Scalable time reversal of Raman echo quantum memory and quantum waveform conversion of light pulse

E S Moiseev\textsuperscript{1,2} and S A Moiseev\textsuperscript{1,2,3,4}

\textsuperscript{1} Kazan (Volga Region) Federal University, Kazan, Russia
\textsuperscript{2} Institute for Informatics of Tatarstan Academy of Sciences, Kazan, Russia
\textsuperscript{3} Kazan Physical-Technical Institute of the Russian Academy of Sciences, Kazan, Russia
E-mail: samoi@yandex.ru

New Journal of Physics \textbf{15} (2013) 105005 (20pp)
Received 6 January 2013
Published 8 October 2013
Online at http://www.njp.org/
doi:10.1088/1367-2630/15/10/105005

Abstract. We have found a new hidden symmetry of time reversal light–atom interaction in the photon echo quantum memory with Raman atomic transition. The time-reversed quantum memory creates generalized conditions for the ideal compression/decompression of time duration of the input light pulses and its wavelength. Based on a general analytical approach to this scheme, we have studied the optimal conditions for the light field compression/decompression in resonant atomic systems characterized by realistic spectral properties. The demonstrated necessary conditions for the effective quantum conversion of the light waveform and wavelength are also discussed for various possible realizations of the quantum memory scheme. The performed study promises new capabilities for fundamental study of the light–atom interaction and deterministic quantum manipulation of the light field, significant for quantum communication and quantum computing.

\textsuperscript{4} Author to whom any correspondence should be addressed.
1. Introduction

Time reversal dynamics of light and multiatomic systems is at the heart of the long-term investigation related to the background of classical and quantum thermodynamics [1], charge parity time symmetry [2] and symmetry of quantum mechanics and electrodynamics equations [3]. The discovery of nuclear spin echo [4] and its numerous analogies, such as electron spin [5] and photon echoes [6] have stimulated a comprehensive study of irreversibility and new experimental methods providing longer time reversal dynamics in various multi-particle systems. Great insight has been achieved in experiments on so called magic-echo for dipolar coupled nuclear spins [7]. Here, the surprising recovery of the initial state is realized experimentally in strongly interacted multi-particle systems by using the impact effective inversion of spin Hamiltonians including dipole–dipole interaction of nuclear spins. Such a generic Hamiltonian approach has provided a universal method for the realization of time reversal dynamics in the ensemble of nuclear spins. Similar approaches promise large capabilities for coherent control of resonant atoms coupled with each other, various electromagnetic fields and with single photon fields. The time reversal methods become especially important for optical quantum memory (QM) in quantum repeaters of quantum communications [8] and in quantum computing [9].

Optical QM based on the multi-atomic systems has been studied over the last 10 years [10–12]. Considerable opportunities for storage of arbitrary multi-qubit states of light have been experimentally realized for the photon echo based QMs [12–18] (see footnote 5). The original scheme of photon echo quantum memory (PEQM) [19] has been proposed in the framework of the light–atom interaction characterized by the perfect time reversal of the input signal field absorption in the photon echo emission. It has been clearly recognized in a detailed study [20], then generalized to a nonlinear regime of the light–atom interaction [21] and demonstrated [22] by observation of hidden symmetry of the light–atom equations (referred in this paper as controlled reverse inhomogeneous broadening (CRIB)-protocol, see also the

5 Present state of art in the optical QM based on multi-atomic ensembles are presented in a special issue devoted to QM in [12].
The elaboration of the generic Hamiltonian approach in the Schrödinger picture to this model of light–atom interaction has made it possible to find the quantum state of the echo field for the arbitrary input light field. CRIB-protocol reproduces the basic concepts of Loschmidt echo for time reversal dynamics of atoms and light as the same kind of generalized time reversal quantum mirror to its classical counterparts. QM techniques have been also proposed for delicate manipulation of single photon states. In particular the possibility of light pulse compression/decompression has been found in a special CRIB scheme. Here CRIB protocol is generalized onto the irreversible domain of the light–atom interaction and demonstrated in [29]. Such a technique can provide both acceleration of quantum communication by shortening photon wave packets or resonant interaction with the atomic media characterized by narrower resonant lines. At the same time, incomplete time reversal dynamics restricts the capabilities of this technique, deteriorating its quality and c/d efficiency. A more significant increase of unitary light pulse compression (also generally referred to as quantum waveform conversion) has been found for three-wave mixing of the signal light pulse with additional chirped light fields. These results have stimulated the study for new capabilities and improvements of the c/d-PEQM technique where the quantum storage could be supplemented with deterministic manipulation of the quantum light fields. For these purposes we have focused our attention on the off resonant Raman scheme of PEQM which has already achieved the record-high quantum efficiency. Here, we have found more general conditions providing scalable time reversal (STR) dynamics of the PEQM. The usefulness of STR-symmetry for the quantum waveform conversion of the light pulse i.e. for c/d temporal duration and deterministic wavelength conversion is demonstrated for transverse and longitudinal inhomogeneous broadenings (IBs). We also show a strategy for switching control laser fields providing high quantum efficiency of the studied QM scheme.

2. Basic model and equations

A detailed study of spectral properties, mis-phasematching, absorption and dispersion effects in CRIB-protocol has been performed in [20] for atomic ensembles with \( \Lambda \)-scheme of quantum transitions characterized by Doppler IB. Here, the rephasing of excited atomic coherence is realized via inversion of the atomic detunings on the resonant transition (due to the change of the Doppler shift) and irradiation of the echo field in the backward direction to the signal field propagation. The CRIB-protocol has been extended to the gaseous atomic ensembles with \( \Xi \)-atomic transition and to solid state systems with a microwave resonant line where IB inversion can be realized by changing the sign of the dipole–dipole interactions with surrounding controlling spins. The usage of external electric fields for the inversion of IB has been proposed in [22, 39, 40] for solid state media.

Alexander et al [40] and Hétet et al [40] have experimentally demonstrated such kinds of IB inversion on rare-earth ions in inorganic crystals by switching the polarity of the external gradient electric field. This technique has been further extended for the magnetic field gradient in [41, 42]. Such spectral control of the resonant line characterized by spatially distributed IB (longitudinal broadening) has opened up new valuable opportunities for PEQM since an efficient echo field irradiation becomes possible in the forward geometry. The technique known now as a gradient echo memory (GEM) simplifies experimental realization of PEQM and extends CRIB-protocol to incomplete time reversal light–atom dynamics. Finally the GEM scheme can be realized for the Raman atomic transitions as well [42]. However, any loss
of perfect time-reversibility could decrease the fidelity of the input signal retrieval in GEM-protocol even for large quantum efficiency. This is a result of additional phase modulation of the irradiated echo field [43]. Here, it is worth noting other new elaborated schemes of PEQMs providing atomic rephasing without using all tools inherent in CRIB-protocol, such as the atomic frequency comb [44] and the approaches exploiting even natural IB of resonant transitions [45–47]. Nevertheless, it was found [48] that all these new protocols can also suffer from the loss of perfect time-reversibility by decreased quantum efficiency and fidelity for relatively broadband input light fields. Thus the observation [48] indicates an important role of the time reversal symmetry on the possible new generalization of CRIB-protocol. In particular it is related to the realization of deterministic quantum manipulations of quantum light fields.

In this work, we generalized CRIB-protocol without a principal loss of perfect time-reversibility by using off resonant Raman atomic transitions. It has been shown that the proposed PEQM scheme can work with usual (transverse), longitudinal and with more general types of IB.

Energy and temporal diagrams of the interaction scheme are depicted in figure 1.

At time $t = 0$, the signal light pulse with electric field component $\hat{E}_1(t/\delta t_1, z) = A_1(t/\delta t_1, z) \exp(ik_1 z)$ enters into the atomic medium (where $\delta t_1$ is a temporal duration). The light pulse is also characterized by the quantum state $|\psi_f\rangle$, spectral width $\delta \omega_1 \approx \delta t_1^{-1}$, carrier frequency $\omega_1$ and wave vector $k_1$. The atomic ensemble is prepared on the long-lived ground states $|1_a\rangle = \prod_{j=1}^N |1\rangle_j$ that provides perfect phasematching for the backward echo pulse emission. So the initial state of light and atoms is $|\Psi\rangle = |\psi_f\rangle |1_a\rangle$. The atoms are simultaneously exposed to an intense control (writing) laser field (characterized by index

![Figure 1. Energy level diagram with atomic transitions for signal/echo pulses $A_{1,2}$ with carrier frequencies $\omega_{1,2}$ and temporal durations $\delta t_1$, $\delta t_1/\eta$; control (writing/reading) laser fields have carrier frequencies $\omega_{1,2}^c$ and Rabi frequencies $\Omega_{1,2}$. Depicted temporal diagram shows scaled shapes and temporal duration of the light field envelopes.](http://www.njp.org/)
\( \mu = 1 \) propagating along the wave vector \( \tilde{K}_1 \) with carrier frequency \( \omega_0^1 \) and Rabi frequency \( \Omega_{\mu}(t, r) = \Omega_{\mu}(t) \exp(-i(\omega_{\mu}^i t - \tilde{K}_1 \cdot r)) \) on \( 2 \leftrightarrow 3 \) atomic transition. Indices \( \mu = 1 \) and 2 indicate stages of the quantum storage and retrieval of the input light field. The writing laser field is reduced to zero after complete absorption of the input light field. The signal and writing fields are assumed to be in Raman resonance \( \omega_{\mu} - \omega_0^1 \approx \omega_0^2 \) with sufficiently large optical spectral detuning \( \Delta_{0,1} = \omega_{0,1} - \omega_0^2 \) from the \( 1 \leftrightarrow 3 \) transition. Such off resonant interaction avoids long-lived excitation of the optical level 3. Total atom–light Hamiltonian is

\[
\hat{H} = \hat{H}_a + \hat{H}_f + \hat{V}_{a-f} + \hat{V}_c,
\]

where \( \hat{H}_a = \sum_{j=1}^{N} \sum_{n=1}^{3} E_j^j \hat{P}_n^j \) is an atomic ensemble Hamiltonian, \( \hat{P}_n^j = |n\rangle_{jj}^j \langle m| \) is an energy of \( n \)th level for \( j \)th atom, \( \hat{H}_f = \hbar \int_{-\infty}^{\infty} \text{d}k \omega_k \hat{a}_k^\dagger \hat{a}_k \) is a Hamiltonian of weak input signal and retrieved echo light fields. Hamiltonians of the interaction between the atoms and quantum light fields \( \hat{V}_{a-f} \) and classical laser fields \( \hat{V}_c \) (characterized by its Rabi frequencies \( \Omega_{\mu}(t, r) \)) are

\[
\hat{V}_{a-f} = -\hbar \sum_{j=1}^{N} \sum_{\mu=1}^{2} \{ g \hat{A}_\mu(z_j) \exp(i k_\mu z_j) \hat{P}_3^1 + \text{h.c.} \},
\]

\[
\hat{V}_c = -\hbar \sum_{j=1}^{N} \sum_{\mu=1}^{2} \{ \Omega_{\mu}(t, r) \exp(-i(\omega_0^j t - \tilde{K}_1 \cdot r)) \hat{P}_3^j + \text{h.c.} \}.
\]

[49] We assume the optical transition \( 1 \leftrightarrow 3 \) has a natural IB for the atomic frequency detunings \( \Delta_{2,j} = \omega_{1,j} - \omega_{3,1,0} \) (where \( \omega_{m,1,j} \) is a resonant frequency of \( j \)th atom and \( \omega_{m,1,0} \) is a central frequency of the transition \( 1 \leftrightarrow m \), \( m = 2, 3 \)). In accordance with CRIB-protocol, we suggest that the atomic frequency detunings \( \Delta_{1,j} = \omega_{2,1,j} - \omega_{2,1,0} \) on the transition \( 1 \leftrightarrow 2 \) can be controlled on demand. Here, one can assume various experimental methods for controlling of \( \Delta_{1,j} \), in particular based on the polarity switching of the external electric/magnetic field gradients [40, 41].

We write a complete system of Heisenberg equations for the weak light field and atomic operators [49] where the population of excited optical atomic levels 2 and 3 can be ignored \( \hat{P}_1^1 \langle \Psi \rangle = \langle \Psi \rangle, \hat{P}_2^2 \langle \Psi \rangle = \langle \hat{P}_2^2 \rangle \langle \Psi \rangle = 0 \) as it is for other ensemble based QMs. The system of light–atom equations for absorption (\( \mu = 1 \)) and retrieval (\( \mu = 2 \)) stages are simplified by usual transfer to slowly variable atomic coherences \( \hat{R}_{12,\mu}(t) \) and light field amplitudes \( \hat{A}_{\mu,0}(z,t) \). By taking into account the carrier frequencies of the probe (echo) light fields and writing (reading) laser fields, we have \( \hat{R}_{12,\mu}(t) = \hat{P}_{13}^1(t) \exp(-i\varphi_{\mu}(\vec{r}, z_j) + i(\omega_{\mu} - \omega_0^j)(t + (-1)^{\mu} n_{\mu} z_j/c)), \hat{R}_{13,\mu}(t) = \hat{P}_{13}^1(t) \exp(i(\omega_{\mu}(t + (-1)^{\mu} n_{\mu} z_j/c)) \) where \( \varphi_{\mu}(\vec{r}, z_j) = -((-1)^{\mu} n_{\mu} \omega_0^j z_j/c + \vec{K}_1 \cdot \vec{r})\) and for the field operator \( \hat{A}_{\mu,0}(z,t) = \hat{A}_{\mu}(z,t) \exp\left(i(\omega_{\mu}(t + (-1)^{\mu} n_{\mu} z_j/c))\right) \) of the signal light pulse envelope. In the moving system of coordinates: \( \tau_1 = t - z/v_g \) and \( Z = z \) we get

\[
\frac{\partial \hat{R}_{13,1}}{\partial \tau_1} = -i(\Delta_{0,1} + \delta_1) \hat{R}_{13,1} + i g \hat{A}_{1,1} + i \Omega_{1,1} \hat{R}_{12,1},
\]

\[
\frac{\partial \hat{R}_{12,1}}{\partial \tau_1} = -i \Delta_1 \hat{R}_{12,1} + i \Omega_{1,1} \hat{R}_{13,1},
\]

\[\text{New Journal of Physics 15} (2013) 105005 (http://www.njp.org/)\]
where $v_g$ is a group velocity of signal light in the host medium without dopant resonant atoms; the atomic operators are parameterized by its continuous spatial coordinate $Z$ and spectral detunings $\delta_1$, $\Delta_1$ on the atomic transitions $1 \leftrightarrow 3$ and $1 \leftrightarrow 2$. The atomic detunings are characterized by normalized spectral distribution function $G(\delta_1, \Delta_1; Z)$ of the IBs, so the atomic index $j$ will be dropped for convenience; $n_0$ and $S$ are the atomic density and cross section of the signal (echo) light fields. For simplicity we also assume that the atomic detunings $\delta_1$ and $\Delta_1$ are uncorrelated for each atom. This assumption will not make a significant effect for large optical detuning $\Delta_{0,1}$.

We take interest in two special cases. In the first case, we assume a spatially homogeneous distribution of the atomic parameters $G(\delta_1, \Delta_1; Z) = G_1(\delta_1)G_2(\Delta_1)$ (also called the ‘transverse’ IB). In the second case (GEM [42]), we assume that IB on the atomic transition $1 \leftrightarrow 2$ is realized due to spatial (linear or slightly linear) dependence of the atomic detunings $\Delta_1$ along $Z$-axis of the medium: $G(\delta_1, \Delta_1; Z) = G_1(\delta_1)G_2(\Delta_1; Z)$. Here, we use $G_2(\Delta_1; Z) = \delta(\Delta_1 - \chi(Z))$ for the GEM scheme ($\chi$ is determined by an external field gradient due to the Stark (or Zeeman) effect for storage and retrieval stages ($\mu = 1, 2$); $\delta_{\mu, \text{in}}$ and $\Delta_{\mu, \text{in}}$ are the bandwidths of two IBs for the both stages, respectively. Below we use notation $G_2(\Delta_1; Z)$ for the general case of IB which coincides with each of two mentioned IBs.

3. Ideal scalable time-reversibility

At first we study the case of resonant Raman interaction in equations (3) and (4) characterized by very large optical spectral detuning $|\Delta_{0,\mu}| \gg |\Delta_{1,\mu}|$ and $|\Delta_{0,\mu}| \gg \delta_{\mu}$ (where $\delta_{\mu}$ are the spectral widths of signal and echo fields). Here, by taking into account the initial state (given at $\tau_1 = -\infty$) of light and atoms for equation (3), we get for the excited optical coherence

$$\hat{R}_{13,1}(\tau_1, Z)\Psi \equiv \frac{1}{\Delta_{0,1}} \{ g \hat{A}_{1,0}(\tau_1, Z) + \Omega_1(\tau_1) \hat{R}_{12,1}(\tau_1, Z) \} \Psi, \quad (6)$$

where $\hat{R}_{13}(-\infty, Z)\Psi = 0$ was used.

Equation (6) demonstrates the purely adiabatical disappearance of the optical excitation simultaneously with the disappearance of the signal and control light fields. By using equation (6) in equations (4) and (5) we get

$$\frac{\partial \hat{R}_{12,1}}{\partial \tau_1} = -i \left( \Delta_1 - \frac{\Omega_1^2(\tau_1)}{\Delta_{0,1}} \right) \hat{R}_{12,1} + i \frac{\Omega_1(\tau_1)}{\Delta_{0,1}} g \hat{A}_{1,0}, \quad (7)$$

$$\frac{\partial \hat{A}_{1,0}}{\partial Z} = i \frac{\beta}{2\Delta_{0,1}} \hat{A}_{1,0} + i \frac{\beta \Omega_1(\tau_1)}{2g\Delta_{0,1}} \int_{-\infty}^{\infty} \hat{A}_1 G_2(\Delta_1; Z) \hat{R}_{12,1}, \quad (8)$$

where $\beta = 2\pi n_0 g^2 S / v_g$.

Equations (7) and (8) describe off resonant Raman interaction of the weak signal light field with a three-level atomic medium. We focus our attention on the specific property of equations (7) and (8) demonstrating possible tuning of the efficient absorption coefficient $\alpha_1 = \beta \Omega_1^2 / \Delta_{\mu, \text{in}} \Delta_{0,1}$. It can be realized by changing the optical spectral detuning $\Delta_{0,1}$, bandwidth $\Delta_{1, \text{in}}$ of the Raman transition and by changing the Rabi frequency $\Omega_1$. We show that controllable
changing of these parameters can provide an additional STR symmetry of equations (7) and (8). By taking into account new spectral atomic parameters and echo signal irradiation in the backward direction, we write the light–atom equations for both stages—absorption of signal pulse (µ = 1) and subsequent echo pulse irradiation (µ = 2):

\[
\frac{\partial \hat{M}_{12,\mu}}{\partial \tau_{\mu}} = -i\Delta_\mu \hat{M}_{12,0,\mu} + \Omega_{\mu}(\tau_{\mu}) g \hat{E}_{12,0,\mu},
\]

\[
\frac{\partial \hat{E}_{\mu,0}}{\partial Z} = (-1)^{\mu+1} \frac{\beta \Omega_{\mu}(\tau_{\mu})}{2g\Delta_{0,\mu}} \int_{-\infty}^{\infty} d\Delta_\mu G_{2,\mu}(\Delta_\mu; Z) \hat{M}_{12,\mu},
\]

where \(\tau_2 = t + z/v_{\text{gr}}\) and \(Z = z\) occur for a new moving coordinate system of the echo irradiation, \(\hat{E}_{\mu,0} = \hat{A}_{\mu,0} \exp(-i(\beta Z/2\Delta_{0,\mu}))\) and \(\hat{M}_{12,\mu} = \hat{R}_{12,\mu} \exp(-i(\beta Z/2\Delta_{0,\mu}))\). Also we have assumed slowly varied control fields \(\Omega_{\mu}(\tau_{\mu})\) and appropriate Stark shifts of the Raman resonances for both processes \(\Delta_\mu = \Delta_\mu + \frac{\Omega_{\mu}^2}{2g\Delta_{0,\mu}}\), where \(\Delta_2\) is a spectral detuning on the atomic transition 1 → 2 for the echo emission stage (i.e. signal and echo fields evolve in the medium in a presence of constant control laser fields).

3.1. Transverse broadening

In usual cases, the shape of IB \(G_{2,\mu}(\Delta_\mu)\) is characterized by a smooth Gaussian or Lorentzian profile. It is worth noting additional possibilities for realization of the controlled IB besides those discussed in the introduction. The controlled IBs can be realized in solid state media for atoms (molecules) characterized by stochastically oriented frozen permanent dipole moments [22]. Here, the external electric fields can shift the resonant atomic (molecular) frequencies due to the interaction with the permanent dipole moments. The induced Stark shifts will be different for the atoms (molecules) depending on the spatial orientations of its permanent dipole moments. While changing the polarity of external electric field at \(\tau = \tau_2\) will invert the atomic (molecular) spectral shifts providing the controlled inversion of IB. \(\Delta_\mu(\tau) \rightarrow -\Delta_\mu\). Such spectral inversion of IB will recover the excited atomic coherence leading to the echo signal emission, respectively.

Also one can use the atomic ensembles in optical waveguides [18] or on the interface of two media with different refractive indices [36]. Here, the controlled IBs can be a result of spatially inhomogeneous optical Stark shifts depending on the atomic coordinate in the waveguide cross section or on the atomic distance from the interface. While the opposite Stark shifts can be induced by changing the carrier frequencies of controlled intensive laser fields.

The time-reversibility of equations (9) and (10) is revealed by the following symmetry transformation [22] for the standard CRIB-protocol: (i) \(\Delta_2 \rightarrow -\Delta_1\), (ii) \(Z \rightarrow -z\), (iii) \(\tau_2 \rightarrow -\tau_1\) and (iv) \(\hat{E}_{2,0} \rightarrow -\hat{E}_{1,0}\), where the conditions (i)–(iv) are met for the light–atoms equations (9) and (10) together with coupling condition \(\Omega_2/\Delta_{0,2} = \Omega_1/\Delta_{0,1}\). An opposite sign of the echo field \(\hat{E}_{2,0}\) reflects probably the well-known difference of the relative phases between the light field and excited atomic dipole for the absorption or for the emission processes. This becomes clear if we take into account that equations (9) and (10) are held for a similar transformation but where the condition (iv) is replaced by (iv'): \(\hat{M}_{12,2} \rightarrow -\hat{M}_{12,1}\). Inversion of the excited atomic dipole moments on the resonant transition \((|1\rangle \leftrightarrow |2\rangle)\) can be performed by
additional $2\pi$-pulse on the adjacent atomic transition ($|2\rangle \leftrightarrow |3\rangle$). This well-known property of $4\pi$ symmetry of the two-level system has been realized experimentally in the QED cavity scheme [50].

Preserving the coupling condition by appropriate variation of the detuning $\Delta_{0,2}$ and Rabi frequency $\Omega_2$ provides the unitary wavelength conversion [32]. However, we can break the coupling condition transferring to more general symmetry transformation of equations (9) and (10) that makes possible a STR of echo field retrieval. In turn STR will provide an ideal c/d of the input light temporal duration. In order to find the STR conditions we note that the ideal quantum compression of the light pulse duration $\delta t_e = \delta t_1/\eta$ is accompanied by appropriate enhancement of the light field amplitude given by the factor $\sqrt{\eta}$ [28] ($\langle \hat{E}_2 \rangle = \sqrt{\eta} \langle \hat{E}_1 \rangle$). By taking into account this requirement in equations (9) and (10) and assuming $\hat{G}_{2,\mu}(\Delta_\mu) = G_2(\Delta_1)$, we find five basic STR conditions for the echo emission (written in the first possible form):

(i)(c) $\tilde{\Delta}_2 \rightarrow -\eta \tilde{\Delta}_1$,
(ii)(c) $Z \rightarrow -Z$,
(iii)(c) $\tau_2 \rightarrow -\tau_1/\eta$,
(iv)(c) $\Omega_2(-\tau_2/\eta)/\Delta_{0,2} = \sqrt{\eta} \Omega_1(\tau_1)/\Delta_{0,1}$.
(v)(c) $\hat{E}_{2,0} \rightarrow -\sqrt{\eta} \hat{E}_{1,0}$.

Also we can find a second form of STR transformation. Here, by saving the first three conditions (i)(c), (iii)(c), we rewrite two others as follows: (iv)(c) $\Omega_2(-\tau_2/\eta)/\Delta_{0,2} = -\sqrt{\eta} \Omega_1(\tau_1)/\Delta_{0,1}$ and (v)(c) $\hat{E}_{2,0} \rightarrow -\sqrt{\eta} \hat{E}_{1,0}$. It is seen that STR indicates an increasing of the reading laser field amplitudes by the factor $\sqrt{\eta}$.

Finally we write STR symmetry in a third form. By saving the first four conditions (i)(c)–(iv)(c) we add two new conditions: (v)(c) $\hat{E}_{2,0} \rightarrow \sqrt{\eta} \hat{E}_{1,0}$ and (vi)(c) $\hat{M}_{12,2} \rightarrow -\hat{M}_{12,1}$. Concerning the new STR conditions (i)(c)–(v)(c), we note that the fourth condition (iv)(c) arises for preserving the interaction strength of the echo field with the atomic system characterized by new IB bandwidth on this stage. It is worth noting the used initial relation between the bandwidths of IB $\Delta_{1,\text{in}}$ and input light signal $\delta \omega_1$ is held for the atomic and light parameters $\Delta_{2,\text{in}}$ and $\delta \omega_2$ at the echo emission stage. For example, the scaling spectral factor $\eta > 1$ will decrease the interaction strength of light with atoms due to larger IB bandwidth $\Delta_{2,\text{in}} = \eta \Delta_{1,\text{in}}$. However increasing the Rabi frequency in accordance with (iv)(c) is possible to compensate this negative effect for time reversibility. We change the Rabi frequency of the reading laser pulse by the factor $\sqrt{\eta}$. In this case we preserve the same absorption coefficient for both stages. Thus preserving the condition (iv)(c) (or its similar form (iv)(c)′) via appropriately changing the Rabi frequency of the control laser field $\Omega_2$ or optical detuning $\Delta_{0,2}$, we can realize the unitary c/d with wavelength conversion of the signal light pulse. Similar consideration is valid for the second and third forms of STR transformations based on the additional conditions: (iv)(c)′–(v)(c)′ or (v)(c)′–(vi)(c)′.

The discussed new conditions clearly demonstrate considerable opportunities for the realization of STR dynamics in the studied scheme of PEQM. The question arises about the possibility of such STR symmetry for other realizations of the Raman echo QM scheme.
3.2. Longitudinal broadening

Following [28, 29] in this case we can switch the initial IB \( G_{2,1} = \delta(\hat{A}_1 - \chi_1 Z) \) to the new shape with the inverted and scaled spatial gradient \( G_{2,2} = \delta(\hat{A}_1 + \eta \chi_1 Z) \) with compression factor \( \eta \) \((\chi_2 = -\eta \chi_1 \) so the light pulse compression occurs for \( \eta > 1 \) and decompression is for \( \eta < 1 \). By using IBs \( G_{2,1} \) and \( G_{2,2} \) in equations (9) and (10) we find that all the four basic considerations of STR (i)(c)–(v)(c) with two supplement conditions are met for this type of IB if the echo field is irradiated in the backward direction (see also equation (34)). Thus, the four conditions constitute the generalized time reversal of CRIB based techniques and open a perfect method for quantum c/d of the single photon light fields.

It is worth noting a possibility of QM realization using Raman atomic interactions in the media with time controlled refractive index [51]. So one can also realize the STR of signal pulse in this scheme by combining the time modulation of refractive index with appropriate intensity variation of the control laser field determined by the conditions (i)(c)–(v)(c). For efficient realization of the described schemes, we have to evaluate the optimal physical parameters where the ideal system of equations (9) and (10) can be used for real atomic systems.

4. Efficiency of quantum waveform conversion

4.1. The influence of inhomogeneous broadening on 1 ↔ 3 transition

The discussed ideal scheme of generalized STR assumes a negligibly small IB width of the atomic transition 1 ↔ 3 and sufficiently large optical spectral detuning \( \Delta_{0,1} \). Also it is important to optimize the switching rate of control laser fields in order to provide an accessible quantum efficiency. Below we perform such analysis following the original system of equations (3)–(5) and conditions (i)(c)–(v)(c) of the ideal pulse compression. By using the initial condition \( \langle \hat{R}_{12}(\infty) \rangle = \langle R_{13}(\infty) \rangle = 0 \) in equations (3)–(5) with constant control laser field \( \Omega (\tau) = \Omega_{1,0} \) and applying Fourier transformation \( \tilde{B}(v, 0) = \int_{-\infty}^{\infty} \text{d}t \hat{B}(t, 0) \), where \( \hat{B}(t, 0) \) is atomic or field operator and \( \tilde{B}(v, 0) \) we get the solution

\[
A_{1,0}(\tau_1, 0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{d}v \tilde{A}_{1,0}(v, 0) \exp\left(-iv\tau_1 - \frac{1}{2} \int_{0}^{Z} \alpha_1(v, Z) \text{d}Z\right),
\]

(11)

\[
R_{12}(\delta_1, \Delta_1; \tau_1, 0) = \int_{-\infty}^{\infty} \frac{\text{d}v}{2\pi} \frac{g\Omega_{1,0} \tilde{A}_{1,0}(v, 0) \exp(-iv\tau_1 - \frac{1}{2} \int_{0}^{Z} \alpha_1(v, Z) \text{d}Z)}{(\Delta_{0,1} + \delta_1 - v - iy_{31})(\Delta_1 - v - iy_{21}) - \Omega_{1,0}^2},
\]

(12)

\[
R_{13}(\delta_1, \Delta_1; \tau_1, 0) = \int_{-\infty}^{\infty} \frac{\text{d}v}{2\pi} \frac{g(\Delta_1 - v - iy_{21}) \tilde{A}_{1,0}(v, 0) \exp(-iv\tau_1 - \frac{1}{2} \int_{0}^{Z} \alpha_1(v, Z) \text{d}Z)}{(\Delta_{0,1} + \delta_1 - v - iy_{31})(\Delta_1 - v - iy_{21}) - \Omega_{1,0}^2},
\]

(13)

where for completeness we have added the phenomenological decay constants \( y_{21} \) and \( y_{31} \) for the atomic coherences \( R_{12}, R_{13} \) and effective absorption coefficient on the Raman transition,

\[
\alpha_1(v, 0) = -i \beta \int_{-\infty}^{\infty} \frac{G(\delta_1, \Delta_1; Z)(\Delta_1 - v - iy_{21}) \text{d}\delta_1 \text{d}\Delta_1}{(\Delta_{0,1} + \delta_1 - v - iy_{31})(\Delta_1 - v - iy_{21}) - \Omega_{1,0}^2}.
\]

(14)

Excited atomic coherences are given by general solutions (12), (13) for time \( \tau_1 \gg \delta_4 \) after complete disappearance of the signal light pulse \( \langle A_{1,0}(\tau_1 > \tau_0, 0) = 0 \rangle \). General analysis of

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the atomic coherences can be performed on equations (11)–(13) and on similar equations for the echo emission stage. Here, we take into account that the highest quantum efficiency is possible only for the sufficiently weak decoherence of Raman transition $\gamma_{21} \ll \Delta_{1, \text{in}}$. We also assume a sufficiently large spectral detuning on the optical transition $\Delta_{0,1} \gg \delta \omega_1, \Omega_1, \Delta_{1, \text{in}}$, $\Delta_{1, \text{in}}$ in equations (12) and (13). After such calculation we get the following relations for atomic coherences:

$$ R_{12}(\tau_1) \approx i \zeta_{12} \tilde{A}_{1,0}(v_1, 0) e^{-i\nu_1 \tau_1} \exp \left( -\frac{1}{2} \int_0^Z \alpha_1(v_1, Z) dZ \right) $$

and

$$ R_{13}(\tau_1) \approx \zeta_{13} R_{12}(\tau_1), $$

where $\zeta_{12} = \Omega_{1,0}/(\Delta_{0,1} - \delta_1 - \Delta_1 + 2 \Omega_{1,0}^2 / \Delta_{0,1} + \delta_1 - \Delta_1)$ and $\zeta_{13} = \Omega_{1,0}/(\Delta_{0,1} + \delta_1 - \Delta_1)$ in accordance with equation (6), $v_1 \approx \Delta_1 - \Omega_{1,0}^2 / \Delta_{0,1} + \delta_1 - \Delta_1 - i \gamma_{\text{eff}}$, $\gamma_{\text{eff}} = \gamma_{21} + \gamma_{31} (\Omega_{1,0}^2 / \Delta_{0,1})$. More detailed analysis of $\zeta_{12}(\delta_1)$ and $\zeta_{13}(\delta_1)$ shows that influences of these factors on $\delta_1$ will not lead to essential effects for sufficiently large optical detuning $\Delta_{0,1}$. This property of the Raman photon echo quantum storage reveals a nonadiabatic feature of the light–atom interaction in contrast to the EIT based QM. In this case ($\Delta_{\mu,0} \gg \delta_{1, \text{in}}, \Delta_{1, \text{in}}$), with large accuracy we can take $\zeta_{12} \approx \zeta_{13} \approx \Omega_{1,0}/\Delta_{0,1}$ and to simplify the complex absorption coefficient $\alpha_1(v, Z)$ as follows:

$$ \alpha_1(v, Z) \approx 2 \beta \frac{\Omega_{1,0}^2}{\Delta_{0,1}} \int_{-\infty}^{\infty} d\Delta_1 \frac{G_2(\Delta_1, Z)}{(\Delta_1 - v - i\gamma_{\text{eff}})}, $$

where both the real $\alpha_{\text{abs},1}(v, Z)$ and the imaginary $\alpha_{\text{dis},1}(v, Z)$ parts of the complex absorption coefficient can play a significant role in the efficient quantum storage of broadband signal light fields [48].

In the next step we evaluate the quantum storage for the conditions which are close to the case described by equations (9) and (10). Here, we identify the influence of IB on the optical transition to the excited atomic coherences via the frequency $Re(v_1) \approx \Delta_1(1 - \frac{|\Omega_{1,0}|^2}{\Delta_{0,1}}) - \frac{|\Omega_{1,0}|^2}{\Delta_{0,1}} + \delta_{1,R}$, where $\delta_{1,R} = \delta_1 (\Omega_{1,0}/\Delta_{0,1})^2$ (see for comparison equation (7)). One can see that additional spectral detuning $\delta_{1,R}$ in $Re(v_1)$ will lead to additional irreversible dephasing of the macroscopic atomic coherences $R_{12}(\tau)$ and $R_{13}(\tau)$ due to averaging over the exponential factor $\langle \exp[-i\delta_{1,R}\tau] \rangle$. Thus, taking into account $\delta_{1,R}$ in the macroscopic atomic coherence we find that the echo field will decay proportionally to the factors

$$ \Gamma_G = \exp \left( -\frac{1}{4} (\Omega_{1,0}/\Delta_{0,1})^4 (1 + \eta^2)(\delta_{1, \text{in}}(\tau_{\text{echo}}(\eta) - \tau_\text{st}))^2 \right), $$

$$ \Gamma_L = \exp \left( -\frac{1}{2} (\Omega_{1,0}/\Delta_{0,1})^2 (1 + \eta) \delta_{1, \text{in}}(\tau_{\text{echo}}(\eta) - \tau_\text{st}) \right) $$

for the Gaussian (G) and Lorentzian (L) shapes of the IB line on transition $1 \leftrightarrow 3$, where we have assumed $(\Omega_{2,0}/\Delta_{0,2}) = \sqrt{\eta}(\Omega_{1,0}/\Delta_{0,1})$, $\tau_{\text{echo}}(\eta) = \frac{1+1/\eta}{\frac{1}{2}} \tau_{\text{echo}}(1)$ is the time moment of echo pulse irradiation [28] and $\tau_\text{st}$ is the storage time on the long-lived atomic levels 1 and 2 (see figure 1).

Then we deal with the control laser field switching to transfer of the excited atomic coherence on the long-lived levels 1 and 2. This problem has not been discussed in the previous
4.2. Switching the control fields

We assume sufficiently large times of the interaction when the input light field ($\tau_0 \gg \delta t_s$) and excited macroscopic atomic coherence disappear completely in the medium ($(\tilde{E}_1(\tau_0, Z) = 0)$). At the same time the resonant atoms are excited to the state characterized by nonzero atomic coherences $R_{12}$ and $R_{13}$. Obviously, the ideal quantum storage will occur for complete adiabatic transfer of the optical coherence $R_{13}$ on the long-lived levels 1 and 2. Below we evaluate the realistic conditions of the perfect transfer of the optical coherence during the both stages.

Taking into account the vanished input light field in equations (3) and (4), we evaluate the influence of switching rate with the exponential decay of the control laser field $\Omega_1(\tau_i) = \Omega_{1,0} e^{-k(\tau_i - \tau_0)}$ (where $\tau_0$ is a moment of time when the switching off procedure starts, $k$ is a switching rate). Here, we have the following atomic equations:

$$\frac{dR_{12}}{d\tau_i} = -i(\Delta_1 - i\gamma_{21})R_{12} + i\Omega_{1,0} e^{-k(\tau_i - \tau_0)} R_{13}, \quad (18)$$

$$\frac{dR_{13}}{d\tau_i} = -i(\Delta_{0,1} + \delta_1 - i\gamma_{31})R_{13} + i\Omega_{1,0} e^{-k(\tau_i - \tau_0)} R_{12}. \quad (19)$$

The atomic evolution of equations (18) and (19) will not be accompanied by macroscopic atomic coherence and coherent irradiation of the stored light field, respectively, due to the strong dephasing of excited atoms during the switching procedure.

After the substitutions $R_{12} = \tilde{R}_{12} e^{-(i(\Delta_1 + \delta_1))(\tau_i - \tau_0)}$, $R_{13} = \tilde{R}_{13} e^{-(i(\Delta_1 + \gamma_{21})(\tau_i - \tau_0)}$, we transfer to the new atomic variables $\tilde{R}_{12} = \chi^{(1+i\tilde{\alpha})/2} Y(\chi)$, $\tilde{R}_{13} = \chi^{(1+i\tilde{\alpha})/2} \tilde{Y}(\chi)$ with appropriate timescale $\chi = e^{-k(\tau_i - \tau_0)}\Omega_{1,0}/k$ (where $\tilde{\alpha} \equiv (\Delta_{0,1} + \delta_1 - \Delta_1 - i(\gamma_{31} - \gamma_{21}))/k$) that leads equations (18) and (19) to the form of Bessel equations

$$\frac{d^2Y}{d\chi^2} + \frac{1}{\chi} \frac{dY}{d\chi} + \left(1 - \frac{(1+i\tilde{\alpha})^2}{4\chi^2}\right) Y = 0, \quad (20)$$

$$\frac{d^2\tilde{Y}}{d\chi^2} + \frac{1}{\chi} \frac{d\tilde{Y}}{d\chi} + \left(1 - \frac{(-1+i\tilde{\alpha})^2}{4\chi^2}\right) \tilde{Y} = 0. \quad (21)$$

Using the well-known solutions [52] of these equations, we find the atomic coherences

$$R_{12}(\tau_i) = e^{-(i(\Delta_1 + \gamma_{21}))(\tau_i - \tau_0)} \frac{(2k/\Omega_{1,0})^{1+i\tilde{\alpha}}}{\Gamma\left(\frac{1+i\tilde{\alpha}}{2}\right) M} \left( J_{\frac{1-i\tilde{\alpha}}{2}}\left(\frac{\Omega_{1,0}}{k}\right) R_{12}(\tau_0) + i J_{\frac{1+i\tilde{\alpha}}{2}}\left(\frac{\Omega_{1,0}}{k}\right) R_{13}(\tau_0) \right), \quad (22)$$

$$R_{13}(\tau_i) = i e^{-(i(\Delta_{0,1} + \delta_1 + \gamma_{31}))(\tau_i - \tau_0)} \frac{(2k/\Omega_{1,0})^{1+i\tilde{\alpha}}}{\Gamma\left(\frac{1+i\tilde{\alpha}}{2}\right) M} \left( J_{\frac{1-i\tilde{\alpha}}{2}}\left(\frac{\Omega_{1,0}}{k}\right) R_{12}(\tau_0) - i J_{\frac{1+i\tilde{\alpha}}{2}}\left(\frac{\Omega_{1,0}}{k}\right) R_{13}(\tau_0) \right). \quad (23)$$
where the complete switching off the control laser field occurs for \( \tau_1 > \tilde{\tau}_1 \), \( (\tilde{\tau}_1 - \tau_0) \gg 1/k \), 
\[ J_{1+i\frac{\Omega_{\alpha}}{k}}(\frac{\Omega_{\alpha}}{k}) \] and \( \Gamma \left( \frac{1+i\frac{\Omega_{\alpha}}{2}}{2} \right) \) are the Bessel and Gamma functions [52], 
\[ M = J_{1+i\frac{\Omega_{\alpha}}{k}}(\frac{\Omega_{\alpha}}{k}) J_{2+i\frac{\Omega_{\alpha}}{k}}(\frac{\Omega_{\alpha}}{k}) + J_{-1+i\frac{\Omega_{\alpha}}{k}}(\frac{\Omega_{\alpha}}{k}) J_{-2+i\frac{\Omega_{\alpha}}{k}}(\frac{\Omega_{\alpha}}{k}). \]

Figure 2 demonstrates a decrease of the optical coherence \( R_{13}(\tau) \) with enhancement of switching rate \( k \) for various atomic detunings \( \Delta_{0,1} \) (for weak atomic decoherence \( \gamma_{21}(\tilde{\tau}_1 - \tau_0) \ll 1 \), \( \gamma_{31}(\tilde{\tau}_1 - \tau_0) \ll 1 \) and negligibly small IBs \( \delta_{1,in}, \Delta_{1,in} \ll \Delta_{0,1} \) after complete switching of the control laser field \( (\tilde{\tau}_1 - \tau_0) \gg 1/k \). Effective depopulation of \( R_{13}(\tilde{\tau}_1) \) occurs for large enough detuning \( \Delta_{0,1} \) where almost adiabatic transfer of the optical coherence occurs to the long-lived levels. The transfer efficiency \( \epsilon_1 \) is given by 
\[ \epsilon_1 = \left| R_{12}(\tilde{\tau}_1) \right|^2 \left/ \left( |R_{12}(\tau_0)|^2 + |R_{13}(\tau_0)|^2 \right) \right. \] 
with enhancement \( \epsilon_1 \) that is considerably suppressed for large optical spectral detuning \( \Delta_{0,1} \gg \Omega_{1,0} \). However, the transfer efficiency \( \epsilon_1 \approx 0.96 \) can occur for relatively small optical detuning \( \Delta_{0,1}/\Omega_{1,0} \approx 5 \) and fast switching rate. The switching rate \( k \geq 20 \) leads to an apparently negative impact for intermediate optical detuning \( \Delta_{0,1}/\Omega_{1,0} \approx 10 \) where \( \epsilon_1 \approx 0.99 \). As also seen in figure 3, retardation of the switching off procedure increases the transfer efficiency.
Figure 3. Transfer efficiency to the long-lived coherence after switching off the first control laser field \( \left( \epsilon_t = \frac{|R_{12}(\tau)|^2}{(|R_{12}(\tau_0)|^2 + |R_{13}(\tau_0)|^2)} \right) \), \( \tau - \tau_0 \gg 1/k \). The curves are given for switching rate in the units of spectral detuning \( \frac{k}{\Delta_{0,1}/\Omega_{1,0}} = 3 \) (green dashed line), \( \frac{k}{\Delta_{0,1}/\Omega_{1,0}} = 5 \) (black dot-dashed line), \( \frac{k}{\Delta_{0,1}/\Omega_{1,0}} = 10 \) (red dotted line) and \( \frac{k}{\Delta_{0,1}/\Omega_{1,0}} = 20 \) (blue solid line).

However, using the low-speed switching rate \( k \) is limited by the relaxation processes of the optical atomic coherence. Moreover, the optical spectral detuning is limited since we have provided a large enough optical depth of the atomic medium in accordance with \( \alpha_1 \sim \beta \left( \frac{\Omega_{1,0}}{\Delta_{1,0}} \right)^2 \). Thus, we have to use an optimal exchange for choice of the optimal switching rate in order to get the maximum quantum efficiency for really used parameters of the light–atom interaction and IBs.

Let us assume the control reading laser field is switched exponentially \( \Omega_2(\tau_2) = \Omega_{2,0} e^{k(\tau_2 - \bar{\tau}_2)} \) with the switching rate \( k_r \). Evolution of atomic coherences are described by the atomic equations which are quite similar to equations (16)–(19) where \( k \) is replaced by \( -k_r \) and \( \Delta_1 \) by \( -\eta \Delta_{1,1} \). By taking into account the initial coherence on the long-lived level \( R_{12}(\bar{\tau}_1) \), we find the solution for atomic equations after the reading control field is switched on at the moment of time \( \tau_2 = \bar{\tau}_2 \)

\[
R_{12}(\bar{\tau}_2, \delta_1, \Delta_1, Z) = C_{1,2} R_{12}(\bar{\tau}_1, \delta_1, \Delta_1, Z),
\]

\[
R_{13}(\bar{\tau}_2, \delta_1, \Delta_1, Z) = i C_{1,3} R_{12}(\bar{\tau}_1, \delta_1, \Delta_1, Z),
\]

where the functions \( C_{1,2} \) and \( C_{1,3} \)

\[
C_{1,2} = \left( \frac{\sqrt{\eta} \Omega_{1,0}}{2k_r} \right)^{1-\frac{i \Delta_{1,0}}{2/k_r}} \Gamma \left( 1 + \frac{i \Delta_{1,0}}{2/k_r} \right) J_{\frac{i \Delta_{1,0}}{2/k_r}} \left( \frac{\sqrt{\eta} \Omega_{1,0}}{k_r} \right),
\]

\[
C_{1,3} = i \left( \frac{\sqrt{\eta} \Omega_{1,0}}{2k_r} \right)^{1-\frac{i \Delta_{1,0}}{2/k_r}} \Gamma \left( 1 + \frac{i \Delta_{1,0}}{2/k_r} \right) J_{\frac{i \Delta_{1,0}}{2/k_r}} \left( \frac{\sqrt{\eta} \Omega_{1,0}}{k_r} \right),
\]

are given with accuracy of the same phase factor \( e^{i\phi} \) and it was assumed \( \Omega_{2,0} = \sqrt{\eta} \Omega_{1,0} \). Here it is worth noting an important property for the functions: \( |C_{1,2}|^2 + |C_{1,3}|^2 = 1 \). Further evolution
of atoms and light is accompanied by the rephasing of the macroscopic coherences of the echo field emission. The optimal switching on procedure of the reading laser pulse can be clarified from general analysis of the echo emission.

### 4.3. Echo field emission

Here we use the coupled system of light–atom equations with new atomic detuning \( \Delta_2 = -\eta \Delta_1 \) on the long-lived transition \( 1 \leftrightarrow 2 \) and echo field emission in the backward direction

\[
\frac{\partial R_{13}}{\partial \tau_2} = -i \left( \Delta_{0,2} + \delta_1 \right) R_{13} + ig A_{2,0} + i\Omega_{2,0} R_{12},
\]

\[
\frac{\partial R_{12}}{\partial \tau_2} = i\eta \Delta_1 R_{12} + i\Omega_{2,0} R_{13},
\]

\[
\frac{\partial A_{2,0}}{\partial Z} = -i \frac{\pi n g S}{v_g} \int_{-\infty}^{\infty} d\delta_1 d\Delta_1 G(\delta_1, \Delta_1, Z) R_{13},
\]

where the initial condition for atomic coherences and irradiated field are determined by \( R_{12}(\tilde{\tau}_2) \) and \( R_{13}(\tilde{\tau}_2) \) and \( A_{2,0}(\tilde{\tau}_2, Z) = 0 \) at time \( \tau_2 = \tilde{\tau}_2 \).

We solve the system equations in spectral Fourier representation of the atomic coherences \( R_{12(13)}(\nu, \ldots, Z) = \int_{-\infty}^{\infty} d\tau_2 R_{12(13)}(\tau_2, \ldots, Z) e^{i\nu \tau_2} \) and field amplitude \( A_{2,0}(\nu, Z) = \int_{-\infty}^{\infty} d\tau_2 A_{2,0}(\tilde{\tau}_2, Z) e^{i\nu \tilde{\tau}_2} \). After performing a number of standard calculations [20, 28] we find following equation for the Fourier component of echo field amplitude \( \tilde{A}_{2,0}(\nu, Z) \):

\[
\frac{\partial \tilde{A}_{2,0}(\nu, Z)}{\partial Z} = -\frac{\beta}{2g} \int_{-\infty}^{\infty} d\delta_1 d\Delta_1 G(\delta_1, \Delta_1, Z) \times \left( R_{13}(\tilde{\tau}_2, \delta_1, \Delta_1, Z) (\nu + \eta \Delta_1) + \Omega_{2,0} R_{12}(\tilde{\tau}_2, \delta_1, \Delta_1, Z) \right) \left( \Delta_{0,2} + \delta_1 - \nu \right) (\nu - \eta \Delta_1) + \Omega_{2,0}^2 \right) + ig(\nu + \eta \Delta_1) \tilde{A}_{2,0}(\nu, Z) \left( \Delta_{0,2} + \delta_1 - \nu \right) (\nu + \eta \Delta_1) + \Omega_{2,0}^2 \right) \).
\]

In evaluation of this equation we use the early discussed approximations determined by sufficiently large optical detuning \( \Delta_{0,1} \). Then similarly to the calculation of integrals in equations (11)–(13), we find the following equation for the echo field amplitude:

\[
\frac{\partial \tilde{E}_{2,0}(\nu, Z)}{\partial Z} = \frac{\eta'}{2\eta} \left( \frac{2\sqrt{\epsilon}}{\sqrt{\eta'}} \tilde{E}_{1,0}(-\nu/\eta, 0) e^{i\nu t_{\text{echo}} - \frac{1}{2} j Z \alpha_1(-\nu/\eta, Z)} + \alpha_1(\nu/\eta, Z) \tilde{E}_{2,0}(\nu, Z) \right),
\]

where optical Stark shifts have been taken into account in the new field amplitude \( E_{2,0} = A_{2,0} \exp\{-i(\beta Z/2\Delta_{0,2})\} \) and \( \Omega_{2,0}/\Delta_{0,2} = \sqrt{\eta'} \Omega_{1,0}/\Delta_{0,1} \), \( \alpha_{\text{abs}, 1}(\Delta_{1}/\eta, z) = 2\pi \beta \Omega_{1,0}^2 G_2(\Delta_{1}/\eta, z) \) is an absorption coefficient, \( \sqrt{\epsilon} = \sqrt{\epsilon}(C_{1,2} + i \sqrt{\Omega_{1,0}^2 C_{1,3}}) \Gamma_{\text{G,L}} \).

The solution of equation (32) can be written for the arbitrary ratio \( \eta'/\eta \). However, we discuss only the case \( \eta' = \eta \). Taking into account

\[
\alpha_1(-\nu/\eta, z) + \alpha_1(\nu/\eta, z) = 2\alpha_{\text{abs}, 1}(\nu/\eta, z),
\]

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in equation (32), we get the solution
\[
\tilde{E}_{2,0}(\nu, Z) = \sqrt{\tilde{\epsilon}} \tilde{E}_{1,0}(-\nu/\eta, 0) \sqrt{\eta} e^{i\nu \tau_{\text{echo}} + \frac{1}{2} \int_0^Z dz \alpha_1(-\nu/\eta, z)} \\
\times (e^{-\int_0^Z dz' \alpha_{\text{abs},1}(-\nu/\eta, z')} - e^{-\int_0^Z dz' \alpha_{\text{abs},1}(-\nu/\eta, z')}).
\]
(33)

Equation (33) is valid for the arbitrary type of IB including the transverse and longitudinal IBs. In the case of almost constant optical depth \(\int_0^L dz' \alpha_{\text{abs},1}(-\nu/\eta, z') \approx \tilde{\kappa}\) within the input light pulse spectrum, we get for the irradiated echo field \(E_{2,0}(\tau_2, Z = 0) = \frac{1}{\pi} \int_{-\infty}^\infty d\nu \tilde{E}_{2,0}(\nu, 0) e^{-i\nu \tau_2}\):
\[
E_{2,0}(\tau_2, 0) = \sqrt{\tilde{\epsilon}} \eta \tilde{E}_{1,0} \left( -\frac{(\tau_2 - \tau_{\text{echo}})}{\delta \epsilon}, 0 \right) e^{-\gamma_{21} \tau_{\text{echo}} (1 - e^{-\tilde{\kappa}})},
\]
(34)
where \(\delta \epsilon = \delta t_1 / \eta\) is the temporal duration of the echo pulse, the exponential factor \(e^{-\gamma_{21} \tau_{\text{echo}}}\) is determined by the relaxation of long-lived atomic coherence during storage time. The result confirms the general consideration of section 3 based on the formal using of STR symmetry. Using equation (34) we obtain the quantum efficiency:
\[
\epsilon = \epsilon_r \epsilon_{G.L.} \epsilon^{-2\gamma_{12} \tau_{\text{echo}}} |1 - \exp(-\tilde{\kappa})|^2,
\]
(35)
where the latter two factor in \(\epsilon\) (phase relaxation of the atomic transition 1 \(\leftrightarrow \) 2 and finite optical depth \(\tilde{\kappa}\)) are quite typical for CRIB in particular for the Raman scheme [32, 33, 35]. The first three factors (result of this work) describe an influence of switching procedure and IB on the optical transition. Here, the quantum efficiency of switching on is determined by
\[
\epsilon_r = |C_{1,2}|^2 + \frac{\sqrt{\eta} \Omega_{1,0}}{\Delta_{0,1}} |C_{1,3}|^2.
\]
(36)
Quantity \(\epsilon_r\) demonstrates maximum efficiency for fast switching on of the control laser field in contrast to the switching off procedure.

We can conclude from equation (36), that only a small part \(\sim |\frac{\sqrt{\eta} \Omega_{1,0}}{\Delta_{0,1}} C_{1,3}|^2 \ll |C_{1,3}|^2\) of the excited optical coherence evolves adiabatically and rephases with long-lived atomic coherence.
Such behavior is accompanied by frozen dephasing of this optical atomic coherence. While another leaving part of the atomic coherence will not participate in the echo emission that decreases the echo field retrieval. Particular properties of the switching on efficiency are demonstrated in figures 4–6. Here it is worth noting that the quantum efficiency $\epsilon_r$ is independent of scaling factor $\eta$ for a sufficiently fast switching rate. In figures 7 and 8, we evaluate the overall efficiency only for the fast rate of switching on.

Thus in the case of highly perfect STR, the main influence to the quantum efficiency can be determined by the switching off procedure in accordance with the factor $\sim \epsilon_t\epsilon_r$. All other decoherent processes are almost repeated for the echo emission stage. In particular as seen in equations (16) and (17), it is realized for the influence of extra detuning $\delta_{1,R}$ with weak modification due to the scaling factor $\eta$.

Efficiency of the quantum waveform conversion (35) is depicted in figure 7 using quite typical atomic parameters for experiments with condensed and gaseous atomic systems.
Figure 7. Quantum efficiency of the waveform conversion in STR as a function spectral detuning $\Delta_{0,1}/\Omega_{1,0}$ and interaction time in the units $(\tau_{\text{echo}} - \tau_{\text{st}})\Omega_{1,0}$ for switching rate $k/\Omega_{1,0} = 1$, optical depth $\tilde{\kappa} = 200$ and Gaussian IB shape with spectral width $\delta_{1,\text{in}} = 0.1\Omega_{1,0}$.

Figure 8. Quantum efficiency as a function of switching rate $k/\Omega_{1,0}$ and spectral detuning $\Delta_{0,1}/\Omega_{1,0}$ for short interaction time $(\tau_{\text{echo}} - \tau_{\text{st}})\Omega_{1,0} = 10$, optical depth $\tilde{\kappa} = 200$ and Gaussian IB shape with spectral width $\delta_{1,\text{in}} = 0.1\Omega_{1,0}$.

Figure 7 demonstrates possible optimal values of optical detuning $\Delta_{0,1}$ and interaction time $\tau_{\text{echo}} - \tau_{\text{st}}$ in the presence of the writing and reading control fields. As seen in the figure, maximum efficiency takes place for a relatively small interaction time. For a larger interaction time, one can find a local maximum at the frequency detuning $\Delta_{0,1} \approx 6.5\Omega_{1,0}$. This maximum arises due to suppression of the atomic dephasing caused by IB on $1 \leftrightarrow 3$, however further increasing of $\Delta_{0,1}$ reduces considerably the effective optical depth. For highly efficient waveform conversion (realized at small interaction time), one can see the large influence of the switching rate in figure 8 which provides almost perfect c/d and wavelength conversion of the signal light pulse.
Leaving further discussion of STR symmetry we note that the quantum efficiency $\epsilon$ characterizes general features of perfect transformation for weak quantum light pulses and can be used for studying the delicate interaction of single and multi-pulse light fields with resonant media. We note that the relatively slow rate of the switching off procedure is preferable for the storage stage since we need to transfer the excited coherence to the long-lived levels. However the read-out stage is needed in a fast switching rate. Even the control laser field is turned off without the presence of the signal light field, the perfect storage requires an optimal switching of the writing laser field. The optimization problem of the control field shapes practically reminds the optical QM based on the electromagnetically induced transparency [53].

5. Conclusion

The main idea of the proposed deterministic quantum waveform conversion is based on the observed STR symmetry of the light–atom dynamics inherent to the absorption of input light field and subsequent echo emission in the off resonant Raman echo QM. It is worth noting the Maxwell’s equations reveal enhanced symmetries which have been the subject of thorough long-term studies [3]. The observed STR symmetry is a new type of discrete symmetry for the light–atoms dynamics which arises in the medium with specifically controlled spectral properties of resonant atoms. STR symmetry demonstrates various physical possibilities for its realization that opens up a way for theoretically perfect c/d of signal light pulses. STR technique is naturally integrated with long-lived storage and can be realized with additional manipulations such as wavelength conversion of the light field. The proposed STR technique can be also implemented with transformation of the input photon wave packets into two-color single photon fields. Multi-pulse readout of the stored light pulse can be also used with STR readout that is quite interesting for effective generation of time bin photonic qubits in optical quantum communications. Here, one can use the reversible transfer of excited atomic coherence to an additional long lived transition ($1 \leftrightarrow 4$) with subsequent readout in the photon echo signals. It is worth noting the usefulness of these opportunities for quantum engineering with broadband multi-photon fields [17, 18]. Finally note that the discussed STR dynamics could open up new promising possibilities for studying the fundamental interactions of photons with atoms due to possibly using more intelligent experimental approaches.

Acknowledgments

Authors thank Russian Foundation for Basic Research through grant no. 12-02-91700 for partial financial support of this work.

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