Three-Body Affairs in the Outer Solar System

Yoko Funato
General System Studies, University of Tokyo, Tokyo 153, Japan
funato@chianti.c.u-tokyo.ac.jp

Junichiro Makino
Department of Astronomy, University of Tokyo, Tokyo 113, Japan

Piet Hut
Institute for Advanced Study, Princeton, NJ 08540, USA

Eiichiro Kokubo & Daisuke Kinoshita
National Astronomical Observatory, Tokyo 180, Japan

Recent observations\textsuperscript{1, 2, 3} have revealed an unexpectedly high
binary fraction among the Trans-Neptunian Objects (TNOs) that populate the Kuiper Belt. The TNO binaries are strikingly
different from asteroid binaries in four respects\textsuperscript{2}: their frequency
is an order of magnitude larger, the mass ratio of their compo-
nents is closer to unity, and their orbits are wider and highly
eccentric. Two explanations have been proposed for their for-
mation, one assuming large numbers of massive bodies\textsuperscript{4}, and one
assuming large numbers of light bodies\textsuperscript{5}. We argue that both as-
sumptions are unwarranted, and we show how TNO binaries can
be produced from a modest number of intermediate-mass bodies
of the type predicted by the gravitational instability theory for
the formation of planetesimals\textsuperscript{6}. We start with a TNO binary
population similar to the asteroid binary population, but sub-
sequently modified by three-body exchange reactions, a process
that is far more efficient in the Kuiper belt, because of the much
smaller tidal perturbations by the Sun. Our mechanism can nat-
urally account for all four characteristics that distinguish TNO
binaries from main-belt asteroid binaries.

The TNO binary 1998WW31 has\textsuperscript{2} a mass ratio $m_2/m_1 \sim 0.7$, eccentricity $e \sim 0.8,$
semimajor axis $a \sim 2 \times 10^4$ km, and inferred radii $r_1 \sim 1.1r_2 \sim 10^2$ km, hence
$a/r_1 > 10^2$, in stark contrast to main belt asteroid binaries\textsuperscript{7}, where $m_2/m_1 \ll 1,$
e $\sim 0,$ and $a/r_1 \lesssim 10.$

Asteroid binaries are probably formed by collisions\textsuperscript{8}, as in the leading scenario for
the formation of the Moon\textsuperscript{9, 10}. The observed characteristics, $m_2/m_1 \ll 1,$ $e \sim 0,$ and
\( a/r_1 \lesssim 10 \), are all natural consequences of this scenario\(^{11}\). For a different scenario for 1998WW31, we can look at dynamical binary formation in star clusters, where there are three channels: 1) tidal capture\(^{12}\); 2) three-body binary formation\(^{13}\); and 3) exchange reactions\(^{13}\).

Channel 1 is analogous to the standard scenario for asteroid binary formation. It will indeed occur: each TNO has grown through accretion, and much of this accretion has happened through collisions with an object comparable in mass to the growing TNO itself\(^{14,15}\).

Channel 2 would require a near-simultaneous encounter of three massive objects with low enough velocities to allow an appreciable chance to leave two of the objects bound. For this to work, the random velocities of the most massive objects should be significantly lower than their Hill velocities. Under such conditions, this channel could play a role, as pointed out by Goldreich \textit{et al.}\(^{5}\), who assumed that there are \( \sim 10^5 \) 100 km–sized object embedded in a sea of small (\(< 1\)km) objects. This assumption, however, is at odds with Goldreich and Ward’s theory for the formation of planetesimals\(^{6}\) through gravitational instability, and it is hard to see how objects in the Kuiper Belt could form from non-gravitational coagulation, because the time scales are far too long\(^{16}\). In contrast, the gravitational instability theory predicts the size of the initial bodies to be \( 10 – 100\)km. Starting with these larger bodies would make channel 2 ineffective, because the velocity dispersion would be higher than the Hill velocity\(^{14,17}\).

Recently, Weidenschilling\(^4\) proposed a variation on the idea of using interactions between three unbound bodies in order to create a binary. He studied how a third massive body could capture the merger remnant from a collision of two massive bodies if the third body were near enough during the time of the collision. This mechanism seems unlikely to work, however, since it requires a number density of massive objects about two orders of magnitude higher than the value consistent with present observations\(^5\).

Goldreich \textit{et al.}\(^5\) have proposed another mechanism, based on the dynamical friction from a sea of smaller bodies that can turn a hyperbolic encounter between two massive bodies into a bound orbit under favorable conditions. Effectively, this mechanism makes use of a superposition of three-body encounters, since each light body interacts independently with the two heavier ones, and in that sense it is another variant on channel 2. As we mentioned above, the gravitational instability theory for the formation of planetesimals\(^{6}\) would exclude the existence of such a sea of small objects, and since the alternative theory of nongravitational agglomeration does not seem to work, we will explore the consequences of dropping channel 2.

Channel 3 can operate on the binaries formed through channel 1, so we should check whether channel 1 and 3 together produce the right binaries in the right numbers.

Starting with the first task, consider a relatively massive TNO primary in a binary orbit with a much less massive secondary. If the binary encounters a particle with a mass \( m \) that is comparable to the mass of the primary component (\( m_1 \sim m \gg m_2 \)), the most likely result is an exchange reaction, in which the incoming object replaces the original secondary\(^{18}\). Figure 1 shows an example of such a reaction.

The binding energy of the binary will not change much during the exchange, hence \( m_1m_2/a_0 \approx m_1m/a \) where \( a \) is the new semimajor axis after the exchange.
Figure 1: An example of a binary–single-body exchange interaction, in the ‘(massive, light) meets massive’ category discussed in this paper. Bodies 1 and 2 have masses $m_1 = 1$ and $m_2 = 0.1$, respectively, forming a binary with an initially circular orbit. Body 3, with mass $m_3 = 1$, encounters the binary on an initially parabolic orbit. In panel (a), the whole scattering process is shown. Panel (b) shows the complex central interaction in more detail, while panels (c) and (d) show the orbits of the initial and final binary, respectively. Note that the final binary orbit is highly eccentric and much wider than the initial circular binary orbit.
Table 1: Cross sections $\sigma$ for various configuration-changing channels in binary–single-body scattering. The gravitational focusing factor $v^2$ is scaled out in order to obtain finite values in the parabolic limit, where $v$ is the initial relative velocity between binary and single body at infinity. We use units in which $G = m_1 = m_3 = a = 1$, where $G$ is the gravitational constant, $m_1$ and $m_3$ are the masses of the heaviest body in the binary and the single body, respectively, and $a$ is the initial semi-major axis of the binary. The mass of the lighter body in the binary is $m_2 = 0.05$. The radii are $r_1 = r_3 = 0.05$ and $r_2 = r_1 (m_2 / m_1)^{1/3} \approx 0.01842$. The scattering processes are coded as follows: $(x, y)$ indicates a binary in the final state with components $x$ and $y$, while $a + p + q$ indicates the product of a merger between bodies $p$ and $q$. A single body $z$ in the final state is indicated by $(,), z$. The physical meaning of the six channels is as follows: (a) an exchange reaction resulting in a massive–massive binary; (b) an exchange reaction resulting in a massive–light binary; (c) a merger resulting in a massive–massive binary; (d) a merger resulting in a twice-as-massive–light binary; (e) a merger resulting in a massive–massive binary; (f) no binary is left, after three-body merging or two-body merging followed by escape.

This implies $a/a_0 \approx m/m_2 \gg 1$. Under the impulse approximation, the interaction happens in a space small compared to the distance $a_0$ to the primary. Conservation of specific angular momentum of the system gives $m_2 a_0 (1 - e_0) \approx ma (1 - e)$ which gives $1 - e \approx m_2/m \ll 1$.

We have run a series of scattering experiments to obtain the relevant cross sections, for an initial binary with mass ratio of 20:1 and semimajor axis $a_0 = 20 r_1$, where $r_1$ is the radius of the primary. These values are typical for main-belt binary asteroids, with $m_2/m_1 < 0.1$, and separations 5–40 times the radius of the primary. We choose parabolic relative orbits for the single body approaching the binary, with periastron distances uniformly distributed between 0 and $20 a_0$. We only followed the system as long as all three bodies stayed within their Hill radius, $1000 a_0$.

Table 1 gives cross sections for processes in which initial binary membership is altered. Channels (a), (c) and (e) result in binaries with two massive components, and together comprise about 80% of the total cross section. We checked these results through a comparison with the starlab three-body scattering package[19].

In figure 2 the distribution for the semi-major axis is strongly peaked at $a = 20$, in good agreement with the simple argument presented above. Similarly, the eccentricity peaks at 0.95, as expected. We assumed $r_1 = 75$km, the estimated radius of the primary of 1998WW31. In figure 3, the orbital elements of 1998WW31 are consistent with the binary having formed through the processes modeled here.

We now confront our second task: to check whether the exchange channel is efficient enough to produce the observed binaries. Starting with TNOs of intermediate mass, as predicted by Goldreich and Ward’s theory for the formation of planetesimals[6], the heaviest TNOs will accrete mass primarily through collisions with TNOs of comparable mass[14, 15]. Many of these collisions are of the ‘giant impact’ type
Figure 2: Normalized differential cross sections for the formation of a ‘massive-massive’ binary, under the conditions specified in the text (channels a, c and e in table 1), with respect to the semi-major axis $a$ (top panel), and eccentricity $e$ (bottom panel) of the final binary. The initially circular binary has $a = 1$ in the dimensionless units used for $d\sigma/da$, while the physical units are given for reference at the top of the figure. The filled points are the total values for the differential cross sections, while the open circles are the contributions from the merger channels (c and e in table 1). Note the double-peaked structure in the top panel: the sharp peak toward $a \sim 20$ arises from non-resonant exchanges, where the final binary has an energy comparable to that of the initial binary; the broad peak around $a \sim 10$ arises from resonant exchanges, where the memory of the initial binary is wiped out, leading on average to more strongly hyperbolic escape in which a harder binary is formed.
Figure 3: Orbital properties of ‘massive-massive’ binaries formed in our scattering experiments: $a$ and $e$ have the same meaning and units as in fig. 2. Contributions from exchange reactions, channel (a) in table 1, are limited by energy conservation to $a \lesssim 20$, and give rise to the horizontal rim in the middle of the figure. Contributions involving mergers, channels (c) and (e) in table 1, can lead to $a$ values all the way to the Hill radius $a \approx 10^3$, but are limited by angular momentum conservation to increasingly high $e$ for increasing $a$. The star symbol shows the observed orbit for 1998WW31. Boxes around the star indicate the observational 1- and 2-$\sigma$ error bars.

that form a tight circular strongly unequal-mass binary (channel 1). Let us estimate what fraction of encounters between comparable-mass TNOs will give rise to ‘giant impact’ type binaries, and how long such binaries survive on average before they are destroyed again.

We assume that one in three collisions between comparable TNOs gives rise to a binary. When no binary is produced, we have to wait for a typical time $T$ until another major collision occurs. When a binary is formed, gravitational focusing implies a cross section for three body interactions of order $a_0$. Therefore, our newly-formed binary will undergo an exchange reaction on a time scale $(r/a_0)T \ll T$, leading to a significant increase in $a$. Strong three-body interactions will subsequently occur on a much shorter time scale $(r/a)T \ll T$. As a result, the semimajor axis will shrink systematically, while the ‘thermal’ distribution $f(e) = 2e$ favors high eccentricity.

When the orbit becomes small enough, $r/a \sim 0.03$, the chance for collisions in resonant encounters becomes significant. Let us assume that an exchange reaction turns a ‘giant impact’ binary into a binary with a semi-major axis of $a \sim 300r$. Each subsequent strong encounter will on average decrease $a$ by a factor $2^{1/2} \sim 1.2$. After a dozen encounters, $a \sim 30r$ and collision is likely to occur. The time scale for each encounter to occur is $\sim (r/a)T$. The waiting time for the last encounter in this series to occur is $(1/30)T$, while each previous waiting time was less by a factor 1.2. Summing this series, we get a total waiting time of $(T/30)/(1-(1/1.2)) = 0.2T$ before
a collision between two or three massive TNOs. If all three collide, we are back where we started, and the resulting system may be a single body (with an assumed chance of $2/3$) or a strongly unequal-mass binary (chance $1/3$). If two of the bodies collide, the third one may remain in orbit, or it may escape. In the latter case, we again are back where we started. In the former case, we still have an equal-mass and likely highly eccentric wide binary.

Under these assumptions, in $1/3$ of the cases, we wind up with an equal-mass TNO binary with the observed properties for a period $\sim 0.2T$, compared to a $2/3$ chance to wind up with a single TNO for a period $\sim T$. This allows us to derive the rate equation for the formation and destruction of the binaries. If we denote by $N_S$ and $N_B$ the number of single bodies and the number of binaries, respectively, we have

$$\frac{dN_B}{dt} = \frac{1}{3} N_S - \frac{1}{2} \frac{2}{3} N_B$$
$$\frac{dN_S}{dt} = -\frac{2}{3} \frac{dN_B}{dt}$$

if we measure time in unit of $T$. So for the stationary state we have $dN_B/dt = dN_S/dt = 0$, and $N_B = 0.2N_S/2 = 0.1N_S$. Therefore, the binary fraction is $\sim 10\%$. When accretion in the Kuiper belt region diminished, the number of single and binary objects was frozen, with a ratio similar to this steady-state value.

While our arguments are only approximate, it is clear that after cessation of the accretion stage at least several percent or more of the TNOs were accidentally left in such a binary phase. The fact that more than $1\%$ of the known TNOs are found to be in wide roughly equal-mass binaries is thus a natural consequence of any accretion model independent of the assumed parameters for the density and velocity dispersion of the protoplanetary disk or the duration of the accretion phase. As a corollary, we predict that future discoveries of TNO binaries will similarly show roughly equal masses, large separations, and high eccentricities.

We conclude that we have found a robust and in fact unavoidable way to produce the type of TNO binaries that have been found, as long as we start from the plausible assumption that TNOs were formed through gravitational instabilities.$^6$

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