Large-Scale Fluid Antenna Systems With Linear MMSE Channel Estimation

Christodoulos Skouroumounis and Ioannis Krikidis
IRIDA Research Centre for Communication Technologies,
Department of Electrical and Computer Engineering, University of Cyprus, Cyprus
Email: {cskour03, krikidis}@ucy.ac.cy

Abstract—In this paper, we investigate the outage performance of fluid antenna (FA)-based user equipments (UEs) in the context of large-scale downlink cellular networks, where all UEs employ linear minimum mean-squared error (LMMSE) channel estimation method. In contrast to existing studies, which assume the existence of perfect channel state information, we develop a novel mathematical framework that accurately captures the channel estimation errors on the performance of the considered network deployment. Specifically, we focus on the limited coherence interval scenario, where a sequential LMMSE-based channel estimation procedure is performed for all FA ports, followed by data reception from the port with the strongest estimated channel. By using stochastic geometry tools, we derive both analytical and closed-form expressions for the achieved outage probability, highlighting the impact of channel estimation on the performance of FA-based UEs. Our results reveal the trade-off imposed between improving the network’s outage performance and reducing the channel estimation quality, indicating new insights for the design of FA-based wireless systems.

Index Terms—Fluid antenna, outage probability, LMMSE, cellular networks, stochastic geometry.

I. INTRODUCTION

With the advent of the sixth generation (6G) wireless communication systems, the concept of high-performance fluid antennas (FAs) has become an increasingly appealing feature in future communication devices [1]. In particular, FAs consist of liquid radiating elements (e.g., Mercury, EGaIn, Galinstan, etc.), which are contained in a dielectric holder and can thus flow in different locations (ports) within its topological boundaries in a programable and controllable manner [2]. Hence, FAs are capable to reversibly re-configure their physical configuration (i.e., size, shape and feeding) as well as their electrical properties (i.e., resonant frequency, bandwidth), providing a new degree of freedom in the design of wireless communication systems [3].

Recently, researchers have acknowledged the potential benefits of the reconfigurable liquid-based antennas, and studied the concept of FAs from an information/communication theoretic standpoint. Specifically, FAs have been investigated for point-to-point communication systems, where the achieved performance in terms of outage [4] and ergodic capacity [5] has been evaluated under spatially correlated channels. An important observation is that a single FA with half-wavelength or less separation between ports can attain capacity and outage performance similar to the conventional multi-antenna maximum ratio combining (MRC) system, if the number of ports is sufficiently large. These works are further extended into the context of multi-user communications in [6], where a mathematical framework has been proposed that takes into consideration multiple pairs of transmitters and FA-based receivers. In particular, both the outage performance, the achievable rate, and the multiplexing gain of the considered topology have been characterized, demonstrating that the overall network performance improves with the number of ports at each receiver.

The aforementioned studies assume that the FA-based systems have perfect knowledge of the channel state information (CSI) for all the FA-associated links. In practice, such CSI needs to be acquired in each channel coherence interval by the transmitter at the cost of channel training overhead that increases with the number of FA ports [7]. In the context of limited coherence interval scenario, this inevitably results in a reduced data transmission duration and, consequently, in an alleviated overall network performance. Hence, the concept of channel estimation in large-scale FA-based systems, triggers a non-trivial trade-off between the channel training duration and the overall network performance. Several research studies have investigated the impact of channel estimation on the performance of different network architectures, including simple network settings [8], [9] and cellular networks [10], [11]. However, a major limitation of the above-mentioned works is the adopted deterministic network configuration, which does not captures the irregularity associated with the actual deployments of the cellular networks. This was the key motivating factor for introducing a powerful mathematical tool, namely stochastic geometry (SG). Specifically, SG consists of a branch of applied probability tools, which captures the random nature of large-scale networks and permits the analytical characterization of numerous performance metrics [14]. However, a limited number of prior works addresses the impact of channel estimation error on the network performance from a macroscopic point-of-view. The authors in [12], evaluated the impact of channel estimation in the context of point-to-point single-input single-output ad-hoc systems, by using linear minimum mean square error (LMMSE) channel estimation. The coverage probability and the impact of channel estimation on the performance of random networks have been studied in [13], capturing the dependence of the optimal training-pilot length on the ratio between the receiver and transmitter densities.

The goal of this paper is to assess the effect of FA tech-
ology on large-scale cellular networks, and study the trade-off imposed by the channel estimation on the outage performance. Initially, we shed light on the modeling and analysis of FA-based wireless communications from a macroscopic point-of-view. Based on the proposed SG-based framework, the outage performance of large-scale FA-based cellular networks is evaluated under a limited coherence interval scenario. In particular, a sequential LMMSE-based channel estimation scheme for all FA ports is adopted, followed by the data reception from the port with the strongest estimated channel. Both analytical and closed-form expressions for the outage performance of FA-based UEs are derived and the performance of the considered system is assessed under different network parameters. Numerical results unveil that an optimal number of FAs’ ports maximizes the network performance with respect to the LMMSE-based channel estimation procedure.

Notation: Vectors are represented by boldface letters (\( \mathbf{A} \)), and their conjugate transpose vectors are denoted by \( \mathbf{A}^\dagger \). The scenario where \( X \) is distributed as \( Y \) is denoted as \( X \overset{\text{iid}}{\sim} Y \). \( J_0(\cdot) \) is the zero-order Bessel function of the first kind.

II. SYSTEM MODEL

A. Network model

We consider a downlink single-tier cellular network, where the locations of the BSs are modeled as points of a homogeneous Poisson point process (PPP), denoted as \( \Phi = \{ \mathbf{v}_k \in \mathbb{R}^2, k \in \mathbb{N}^+ \} \) with spatial density \( \lambda_b \) BS/km². Moreover, the locations of the UEs follow an arbitrary independent point process \( \Psi \) with spatial density \( \lambda_u \gg \lambda_b \). Each BS is equipped with a single omnidirectional antenna, while all UEs are equipped with a single FA (see Section II-B). We assume that an orthogonal multiple access technique is utilized within a cell, e.g. time division multiple access, so that each BS serves one UE at a time and no intra-cell interference exists. Without loss of generality and by following Slivnyak’s theorem [14], the analysis concerns the typical UE located at the origin but the results hold for all UEs. We consider the nearest-BS association policy i.e., the typical UE at the origin communicates with its closest BS at \( \mathbf{v}_0 \in \mathbb{R}^2 \), referred as tagged BS, and its link with the typical UE is denoted as typical link.

B. Fluid antenna model

Fig. 1 depicts the FA architecture adopted by each UE, where a conductive fluid is installed on a tube-like structure within which the fluid is free to move. In particular, the fluid’s location can be promptly switched to one of the \( N \) preset locations (also known as “ports”) that are evenly distributed along a linear dimension of length, \( \kappa \lambda \), where \( \lambda \) is the communication wavelength and \( \kappa \) is a scaling constant. As depicted in Fig. 1, the first port is considered as the reference location, from which the displacement of the \( i \)-th port can be measured as

\[
d_i = \left( \frac{i-1}{N-1} \right) \kappa \lambda, \quad \forall \ i \in \mathcal{N},
\]

where \( \mathcal{N} = \{1,2,\ldots,N\} \). Hence, the distance of the typical link between the \( i \)-th port and the tagged BS is given by

\[
r_i(\rho) = \sqrt{\rho^2 + \frac{\kappa^2 \lambda^2 b}{4} \left( \frac{N-2i+1}{N-1} \right)^2}, \quad \forall \ i \in \mathcal{N},
\]

where \( \rho \) is the distance of the typical link i.e., \( \rho = ||\mathbf{v}_0|| \), with probability density function (pdf) that is given by [14]

\[
f(\rho) = 2\pi \lambda_b \rho \exp \left( -\pi \lambda_b \rho^2 \right).
\]

C. Channel model

We assume that the large-scale attenuation of the transmitted signals follows an unbounded singular path-loss model based on the distance \( r \) between a transmitter and a receiver i.e., \( \ell(r) = r^\alpha \), where \( \alpha > 2 \) denotes the path-loss exponent. Regarding small-scale fading, we consider a block fading channel model. In other words, the channel remains constant during a limited block of \( L = W_c T_c \) consecutive symbols, also known as channel coherence interval, where \( T_c \) and \( W_c \) correspond to the coherence time and bandwidth, respectively, and evolves independently from block to block. In particular, we assume that the small-scale fading between a transmitter and a receiver follows a circularly symmetric complex Gaussian distribution with zero mean and variance of \( \sigma^2 \). Hence, the channel’s amplitude of the \( i \)-th port of the typical UE with respect to its tagged BS, \( ||g_i|| \), is Rayleigh distributed. Since the FA’s ports can be arbitrarily close to each other, the channels are considered to be correlated. In particular, the channels observed by the \( N \) antenna ports of the typical UE can be evaluated as [4]

\[
g_i = \begin{cases} 
\sigma_x i + j \sigma_x y_i & \text{if } i = 1, \\
\sigma_x \left( 1 - \mu_z x_i + \mu_z x_0 \right) + j \sigma_x \left( 1 - \mu_z y_i + \mu_z y_0 \right) & \text{otherwise,}
\end{cases}
\]

where \( i \in \mathcal{N}, \; x_1,\ldots,x_N, y_1,\ldots,y_N \) are all independent Gaussian random variables with zero mean and variance of \( \frac{\gamma}{\tau} \); \( \mu_z \) is the autocorrelation parameter that can be chosen appropriately to determine the channel correlation between the \( i \)-th and the reference (i.e., first) port. In this work, we assume that \( \mu_z = 0 \) if \( i = 1 \), otherwise \( \mu_z = J_0 \left( \frac{2\pi(i-1)}{N-1} \kappa \right) \) [6].

D. Channel estimation period

In the context of the considered limited coherence interval scenario, channel estimation between the UEs’ ports and their
serving BSs is performed via pilot-training symbols in a sequential manner. The pilot-training symbols, which are initially known at both the BS and the UEs in the same cell, are broadcasted by each BS to its connecting UEs\(^1\). In particular, we consider that each channel block is comprised of \(L_T\) pilot-training symbols, followed by \(L - L_T\) data symbols, as depicted in Fig. 2. The key idea of the adopted channel estimation procedure is to divide the entire channel estimation period into \(N\) symmetric segments of \(L = L_T/N\) consecutive symbols. During each segment, the channel between a single FA port and the serving BS is estimated\(^2\). Therefore, the baseband equivalent received pilot signal at the \(i\)-th port of the typical UE, is given by

\[
y_i = \frac{L_T P}{N \ell (r_i)} g_i X_0 + \sum_{k \in \mathbb{N}^+} \sum_{v_k \in \mathcal{P} \setminus v_0} \frac{P}{\ell (\| v_k \|)} g_{ki} X_k + \eta_0,
\]

where \(X_0\) is a deterministic \(\Lambda \times 1\) training symbol vector\(^9\), \(X_k \sim \mathcal{CN}(\Omega_{\Lambda \times 1}, I_{\Lambda})\) is a pilot symbol vector from the \(k\)-th interfering BS with channel \(g_{ki} \sim \mathcal{CN}(0,1)\), and \(\eta_0 \sim \mathcal{CN}(0_{\Lambda \times 1}, N_0 I_{\Lambda})\) is the additive white Gaussian noise (AWGN) vector. By using the low-complexity LMMSE estimator, which is optimal among the class of linear estimators, the estimate of \(g_i\) conditioned on \(\rho\), is given by

\[
\hat{g}_i |_\rho = \frac{L_T P}{N \ell (r_i)} \sigma^2 + N_0 + P E(I_i | \rho) \tilde{y}_i,
\]

where \(\tilde{y}_i\) is the observation scalar signal at the \(i\)-th port of the typical UE i.e., \(\tilde{y}_i = X_0^T \gamma_i\), and \(E(I_i | \rho)\) denotes the instantaneous power of the interference at the \(i\)-th port of the typical UE, that is given by

\[
E(I_i | \rho) = \sum_{k \in \mathbb{N}^+} \frac{1}{\ell (\| v_k \|)} | g_{ki} |^2
\]

Based on the Campbell’s theorem\(^14\) and conditioning on \(\rho\), \(E(I_i | \rho)\) can be expressed as

\[
E(I_i | \rho) = 2 \pi \lambda_b \rho \int_{r_i}^{\infty} r^{-1} e^{-\frac{2}{a - 2} \rho \sigma^2 r_i^2} dr = \frac{2 \pi \lambda_b \sigma^2 \rho^2 r_i^2}{a - 2 \rho r_i^2}.
\]

The channel estimation error can then be derived as

\[
e_i | \rho = g_i - \hat{g}_i |_\rho,
\]

where \(e_i | \rho \sim \mathcal{CN}(0, \sigma^2_e | \rho)\), and \(g_i \sim \mathcal{CN}(0, \sigma^2_e)\), are uncorrelated\(^9\). An explicit expression for \(\sigma^2_e | \rho\) is

\[
\sigma^2_e | \rho = E \left( | g_i - \hat{g}_i |_\rho \right)^2,
\]

which depicts the variance of the channel estimation conditioned on \(\rho\), is given in Section III.

E. Data transmission period

During the data transmission period, all BSs transmit data to their associated UEs, whose FA’s location is switched to the port that is estimated to provide the strongest channel in order to have the best reception performance. Hence, the FA’s location is instantly switched to the port that satisfies

\[
\hat{i} = \arg \max_{i \in \mathbb{N}} \left\{ | \hat{g}_i |_\rho \right\}.
\]

For simplicity, we consider that both the data and the pilot-training symbols are transmitted with the same power \(P\). Thus, the received signal at the selected \(i\)-th port of the typical UE during the \(n\)-th channel use, is given by

\[
d_i[n] = \frac{1}{r_i^2(\rho)} \hat{g}_i [s_0[n] + e_1[n] | \rho s_0[n] + \sum_{k \in \mathbb{N}^+} \sum_{v_k \in \mathcal{P} \setminus v_0} \frac{1}{\| v_k \|} g_{ki} s_k[n] + \eta_0[n]],
\]

where \(n = L_T + 1, \ldots, L\), \(s_0[n]\) and \(s_k[n]\) represent independent Gaussian distributed data symbols from the tagged and the \(k\)-th interfering BS, respectively, satisfying \(E[| s_0[n] |^2] = E[| s_k[n] |^2] = \rho, \forall k \in \mathbb{N}^+; \eta_0[n] \sim \mathcal{CN}(0, N_0)\) is AWGN. Note that, the first term of (7) is known at the receiver, while the remaining terms are unknown and are treated as additive noise. Hence, an estimate of \(s_0[n]\) can be formulated as

\[
s_0[n] = \sqrt{r_i^2(\rho)} \frac{\hat{g}_i |_{\rho}^T}{\hat{g}_i |_{\rho}} d_i[n],
\]

where \(\nu = \frac{N_0}{\rho}\) is the transmit signal-to-noise ratio.

III. FA SYSTEMS UNDER LMMSE CHANNEL ESTIMATION

In this section, we analytically evaluate the outage performance of a downlink homogeneous cellular network under LMMSE channel estimation, where all UEs are equipped with a single linear FA. Initially, we study the statistical properties of the estimated channels under the LMMSE estimator. Finally, the outage performance of the considered system model is evaluated, by leveraging tools from SG.

A. Preliminary Results

In this section, we state some preliminary results, which assist in the derivation of the main analytical framework. We start with the investigation of the variance of the channel estimation, conditioned on \(\rho\), that is given in the following proposition.

**Proposition 1.** By using the LMMSE estimator, the variance of the channel estimation error conditioned on \(\rho\), is given by

\[
\sigma^2_e | \rho = \frac{L_T}{N} \frac{1}{\rho} + \frac{2 \pi \lambda_b \rho}{a - 2}
\]

**Proof.** The proof is omitted due to space limitations.

---

\(^1\)\)Different mutually orthogonal pilot sequences are used by the BSs in all cells, and thus, no pilot contamination occurs.

\(^2\)\)More sophisticated vector-based channel estimation schemes can be used but this estimation process is sufficient for the purpose of this work.
It is clear from (9), that the channel estimation error variance is a non-negative increasing concave function with respect to the number of the FA ports \( N \). For the extreme case of infinite number of FA ports i.e., \( N \to \infty \), the variance of the channel estimation error (i.e., channel estimation quality) becomes one i.e., \( \sigma_{\hat{g}|1|}^2 \approx 1 \). According to (8), this inadequate channel estimation quality leads to a significant reduction of the SINR observed by FA-based UEs, compromising the network performance. Therefore, although the increased number of FA ports initially enhances the receive diversity gain and thereby the network performance, beyond a critical point \( N^* \in \mathbb{N} \), a further increase of the number of FA ports leads to an attenuated channel estimation quality, jeopardizing the overall network performance. Based on the aforementioned discussion, the number of FA ports triggers a trade-off between improving the network’s outage performance and reducing the channel estimation quality.

Regarding the statistical properties of the estimated channels under the LMMSE estimator, the joint pdf and cumulative distribution function (cdf) of \( |g_1|, \ldots, |g_N| \), conditioned on \( \rho \), are given in the following lemma.

**Lemma 1.** The conditional joint cdf and pdf of \( |g_1|, \ldots, |g_N| \) for \( i \in \mathbb{N} \), are given by

\[
F_{|g_1|,\ldots,|g_N|}(\tau_1, \ldots, \tau_N|\rho) = \frac{2\tau_i}{\sigma_i^2} \exp\left(-\frac{\tau_i^2}{\sigma_i^2}+\frac{2\mu_i\tau_i}{\sigma_i^2}\right) I_0\left(\frac{2\mu_i\tau_i}{\sigma_i^2}\right)
\]

and

\[
f_{|g_1|,\ldots,|g_N|}(\tau_1, \ldots, \tau_N|\rho) = \prod_{j=0}^{N} \frac{2\tau_j}{\sigma_j^2} \exp\left(-\frac{\tau_j^2+\mu_j^2}{\sigma_j^2}\right) I_0\left(\frac{2\mu_j\tau_j}{\sigma_j^2}\right),
\]

respectively, where \( \tau_1, \ldots, \tau_N \geq 0 \), \( I_0(\cdot) \) represents the zero-order modified Bessel function of the first kind, \( K_1(\cdot) \) is the first-order Marcum Q-function, and \( \sigma_i^2 = \sigma^2(1-\mu_i^2) + \sigma_{\hat{g}|1|}^2 \).

**Proof.** See Appendix A. \( \square \)

The performance of a FA-based UE in large-scale multi-cell networks is mainly compromised by the existence of multi-user interference [14]. Although the performance of a communication network can be easily evaluated for the PPP case with independent fading channels, in most relevant (realistic) models, it is either impossible to analytically analyze or cumbersome to evaluate numerically. Thus, in this paper, we assume that the multi-user interference of large-scale wireless networks is approximated by its mean value i.e., \( \mathbb{E}(\mathcal{S}) \), aiming to provide simple and tractable expressions for the networks performance.

### B. Outage performance

The downlink outage performance \( \mathcal{P}_c(\vartheta) \) of a network can be described as the probability that the mutual information of the channel between the typical UE and its serving BS is smaller than a target rate \( R \) (data bits/channel use) i.e.,

\[
\mathbb{P}\left((1-\frac{L}{T}) \log (1+\text{SINR}) < R\right).
\]

Based on the adopted system model and the aforementioned condition, the conditional downlink outage probability for the typical UE, can be expressed as follows

\[
\mathcal{P}_c(\vartheta|\rho) = \mathbb{P}\left(|\hat{g}| < \sqrt{\theta r^2_1(\rho) \left(\mathbb{E}(\mathcal{S}) + \frac{\sigma_{\hat{g}|1|}^2}{r^2_1(\rho)} + \frac{1}{\nu}\right)} | \rho\right)
\]

where \( \mathcal{I}_1 = \mathbb{E}(\mathcal{S}) + \frac{\sigma_{\hat{g}|1|}^2}{r^2_1(\rho)} + \frac{1}{\nu} \) and \( \vartheta = 2^{1-\frac{\mu}{\sigma^2}} - 1 \). In the following theorem, an analytical expression for the conditional outage performance of the considered system model is derived.

**Lemma 2.** The conditional outage probability when utilizing the LMMSE estimator for channel estimation in cellular networks with FA-based UEs, is given by

\[
\mathcal{P}_c(\vartheta|\rho) = \int_0^\infty \exp(-t) \prod_{j=2}^N \left[1 - Q_1\left(\frac{2\mu_j\sigma_j^2 t}{\sigma_j^2} \sqrt{\frac{2}{\sigma_j^2}} \Theta_j\right)\right] dt,
\]

where \( \Theta_j = \vartheta r^2_j(\rho) \left(\mathbb{E}(\mathcal{S}) + \frac{\sigma_{\hat{g}|1|}^2}{r^2_j(\rho)} + \frac{1}{\nu}\right) \), \( \vartheta = 2^{1-\frac{\mu}{\sigma^2}} - 1 \), \( \sigma_j^2 = \sigma^2(1-\mu^2) + \sigma_{\hat{g}|1|}^2 \), and \( \sigma_{\hat{g}|1|}^2 \) depicts the channel estimation conditioned on \( r_j \), that is given by (9).

**Proof.** See Appendix A. \( \square \)

Even though the expression in Lemma 2 can be evaluated by using numerical tools, gaining insights by that expression is tedious. Motivated by this, we evaluate the achieved performance in the asymptotic regime. In particular, the following lemma provides an upper and a lower bound for the conditional outage probability by considering the interference-limited scenario (i.e., \( \nu \to \infty \)), and for the special case where \( \mu_i = \mu \forall i \in \mathbb{N} \).

**Lemma 3.** The conditional outage probability when utilizing the LMMSE estimator for channel estimation in cellular networks with FA-based UEs, is upper bounded by

\[
\mathcal{P}_c^u(\vartheta|\rho) = 1 - \exp(-\Xi_1) - \Upsilon^u(\Xi_1) \sum_{j=2}^N \exp(-\Xi_j),
\]

and lower bounded by

\[
\mathcal{P}_c^l(\vartheta|\rho) = 1 - \exp(-\Xi_1) - \Upsilon^l(\Xi_1) \sum_{j=2}^N \exp(-\Xi_j),
\]

where \( \Xi_1 = \frac{\mu^2}{\sigma^2} \) for \( i \in \mathbb{N} \), \( \Upsilon^u(x) = \frac{1}{1+\mu^2}\exp(-x(1+\mu^2)^2) \) and \( \Upsilon^l(x) = \frac{1}{1+\mu^2}\exp(-x(1+\mu^2)^2) \).

**Proof.** By definition, \( 0 \leq Q_1(\alpha, \beta) \leq 1 \), and therefore \( \prod_i (1-Q_1(\alpha, \beta)) \approx 1 - \sum_i Q_1(\alpha, \beta) \). In addition, for the considered interference-limited scenario with \( \mu_2 = \cdots = \mu_N = \mu \), \( \Theta_j \approx \vartheta r^2_j(\rho) \left(\mathbb{E}(\mathcal{S}) + \sigma_{\hat{g}|1|}^2 \right) \), \( \sigma_j^2 = \sigma^2 \), \( \forall j \in \mathbb{N}\setminus\{1\} \), and \( \sigma_{\hat{g}|1|}^2 \approx 2\pi\lambda_0^N \frac{T r^2_1}{\sigma^2} \). Then, based on the tight upper and lower bounds of \( Q_1(\alpha, \beta) \) that are given in [15, C.23], and after
some algebraic manipulation, the upper and lower bounds of the conditional outage probability can be derived.

In the following Theorem, the outage performance of the considered network topology is evaluated, by un-conditioning the expression in Lemma 2 with the pdf of the distance from the typical UE to its serving BS.

**Theorem 1.** The outage probability when utilizing a LMMSE estimator for channel estimation in cellular networks with FA-based UEs, is given by

$$P_c(\vartheta) = \prod_{i=0}^{N} \mathcal{P}_c(\vartheta | \rho) 2 \pi \lambda_b \rho \exp \left(-\pi \lambda_b \rho^2\right) d\rho,$$

where \(\mathcal{P}_c(\vartheta | \rho)\) represents the achieved outage probability conditioned on \(\rho\), that is given in Lemma 2.

**Proof.** By un-conditioning \(\mathcal{P}_c(\vartheta | \rho)\) with respect to the parameter \(\rho\) and by using (2), the final expression can be derived.

**C. Optimal Number of FA ports**

From the network operator point-of-view, the concept of FA technology unlocks a new degree of freedom in the design of wireless communication systems, without compromising the reliability of the network. Based on the discussion in Section III-A, the FA architecture, and specifically the number of ports \(N\), triggers a non-trivial trade-off between improving the outage performance and reducing the channel estimation quality. This trade-off motivates the investigation of the optimal number of FA ports that provides ultra-reliable connectivity i.e., minimize the outage probability.

Let \(N^*\) represents the number of FA ports that minimizes the expression (12) i.e., \(N^* = \arg \max_N \mathcal{P}_c(\vartheta), \) conditioned on \(L_T, L\), and the density of the BSs \(\lambda_b\). The aforementioned problem formulation is a maximization over multiple integrals, and is therefore computationally cumbersome. As a result, an exact closed-form solution cannot be obtained and the problem can be tackled numerically.

**IV. NUMERICAL RESULTS**

In this section, we provide numerical results to verify our model and illustrate the performance of FA-based UEs in large-scale cellular networks. In particular, we consider the following parameters: \(\lambda_b = 15 \text{ BS/km}^2\), \(\lambda_u = 30 \text{ UE/km}^2\), \(\lambda = 6 \text{ cm}\), \(a = 4\), \(\sigma^2 = -60 \text{ dB}\), \(W_c = 500 \text{ kHz}\), and \(T_c = 2.5 \text{ ms}\). Note that, by using different values will lead to a shifted network performance, but with the same conclusions.

Fig. 3 illustrates the achieved network performance of a FA-based large-scale downlink cellular network with respect to the transmit power \(P\), for different scaling constants \(\kappa = \{0.1, 0.5\}\) and number of FAs’ ports \(N = \{5, 15, 25\}\). We can observe that larger FA architectures i.e., a larger \(\kappa\), lead to a reduced outage probability. This was expected since, as the size of the FAs increases, the distance between their ports is also increases, limiting the negative effect of the spatial correlation between the ports’ channels on the network performance. Another important observation is that, by increasing the number of FA ports, the outage performance drops. This observation is based on the fact that, the increased number of FA ports results in a higher receive diversity gain, and hence, a greater observed SINR at the UEs. Additionally, it is clear from the plot that the outage probability asymptotically converges to a constant value which is tightly approximated by the proposed upper and lower bounds derived in Lemma 3. This behavior of the outage performance is based on the fact that as the transmission power of the nodes increases, the additive noise in the network becomes negligible. Finally, the agreement between the theoretical curves (solid and dashed lines) and the simulation results (markers) validates our analysis.

Fig. 4 plots the outage performance versus the number of FA ports, \(N\), for different ratios \(\frac{L_T}{L} = \{0.15, 0.25\}\). It is interesting to note that for small number of FA ports, the existence of more ports leads to the enhancement of the network performance. However, by increasing the number of FA ports beyond a critical point \(N^*\), the outage performance increases. This was expected since, for a large number of FA ports, the dedicated number of training-pilot symbols for the channel estimation of each port is reduced, and therefore the channel estimation quality is decreased (i.e., \(\sigma^2_{e/p} \rightarrow 1\)), alleviating the achieved network performance. Furthermore, we can easily observe that the critical number of ports \(N^*\),
increases with the decrease of the ratio $L_T/L$. On the other hand, the outage performance increases with the increase of the ratio $L_T/L$, since a FA-based UE allocates more time for pilot-based training, shortening the length of data transmission period. For comparison purposes, we also present the outage performance obtained with a perfect (a-priori) CSI \cite{6}, denoted as “Perfect CSI”. We can easily observe that, in contrast to the scenario considered with channel estimation error, the network performance with a perfect CSI is constantly increasing with the increase of FA ports. This is due to the fact that the negative effect of channel estimation quality on the network’s performance is neglected.

Fig. 5 evaluates the outage performance with respect to the density of BSs for different scaling constants $\kappa = \{0.05, 0.5\}$. As mentioned before, the experienced outage performance of a FA-based UE can be reduced by adopting larger FA architectures i.e., a larger $\kappa$. Another interesting observation is that the outage performance initially decreases with $\lambda_b$ but, after a certain value of $\lambda_b$, it starts to increase. This observation is based on the fact that at low density values, the increased density of BSs for different scaling constants $\kappa$ has a negative effect of channel estimation quality on the network’s performance obtained with a perfect (a-priori) CSI \cite{6}, denoted as “Perfect CSI”. We can easily observe that, in contrast to the scenario considered with channel estimation error, the network performance with a perfect CSI is constantly increasing with the increase of FA ports. This is due to the fact that the negative effect of channel estimation quality on the network’s performance is neglected.

V. CONCLUSION

In this paper, we proposed an analytical framework based on SG to study the outage performance of FA-based UEs in the context of downlink cellular networks. The developed mathematical framework captures the presence of both channel estimation error and channel correlation effects. The outage performance was analytically derived and the impact of nodes density, block length, and number of FAs’ ports has been discussed. Our results highlight the impact of the FAs’ architecture and the network topology on the optimal number of ports, providing guidance for the planning of cellular networks in order to achieve enhanced network performance. A future extension of this work is the consideration of multiple FAs at the UEs and the exploitation of spatial correlation in the channel estimation process.

APPENDIX A

PROOF OF LEMMA 1

Under the LMMSE estimator, the channel experienced by the $i$-th port of the typical UE can be expressed as $\hat{g}_i|\rho = g_i + e_i|\rho$, where $g_i \sim CN(0, \sigma^2)$ and $e_i|\rho \sim CN(0, \sigma^2|\rho)$, and therefore, $\hat{g}_i|\rho \sim CN(0, \tilde{\sigma}_i^2)$, with $\tilde{\sigma}_i^2 = \sigma^2(1 - \mu_i^2) + \sigma^2|\rho|$. Under this model, the amplitude of the estimated channels, $|\hat{g}_i|$, is Rayleigh distributed, with a pdf $f_{|\hat{g}_i|}(\tau) = \frac{2\tau}{\sigma_i^2} \exp \left( -\frac{\tau^2}{\sigma_i^2} \right)$, where $\mathbb{E} [|\hat{g}_i|^2] = \tilde{\sigma}_i$. As it already mentioned, the channels $\{\hat{g}_i\}$ are correlated due to the capability of FA’s ports to be arbitrarily close to each other. In particular, the amplitude of the estimated channel $|\hat{g}_j|$, conditioned on $\rho$, $x_0$, and $y_0$, follows a Rice distribution i.e.

$$f_{|\hat{g}_j||x_0,y_0}(\tau_2|\rho, x_0, y_0) = \frac{2\tau_2}{\sigma_i^2} \exp \left( -\frac{\tau_2^2 + \mu_j^2(x_0^2 + y_0^2)}{\sigma_i^2} \right) I_0 \left( \frac{2\mu_j \tau_2 \sqrt{x_0^2 + y_0^2}}{\sigma_i^2} \right),$$

where $\tau_2 \geq 0$. By substituting $\tau_1 = \sqrt{x_0^2 + y_0^2}$ and since $x_0, y_0, |\hat{g}_j|, \ldots, |\hat{g}_N|$ are all independent between each other, the joint pdf of the estimated channels, conditioned on $|\hat{g}_1|$, can be expressed as

$$f_{|\hat{g}_2|,\ldots,|\hat{g}_N|||\hat{g}_1|(\tau_2, \ldots, \tau_N|\rho, \tau_1) = \prod_{i=2}^{N} \frac{2\tau_i}{\sigma_i^2} \exp \left( -\frac{\tau_i^2 + \mu_i^2 \tau_1^2}{\sigma_i^2} \right) I_0 \left( \frac{2\mu_i \tau_1 \tau_i}{\sigma_i^2} \right).$$

Then, the final expression can be achieved by un-conditioning the above expression with the pdf of $|\hat{g}_1|$ i.e.

$$f_{|\hat{g}_2|,\ldots,|\hat{g}_N|||\hat{g}_1|(\tau_2, \ldots, \tau_N|\rho, \tau_1) f_{|\hat{g}_1|}(\tau_1),$$

which gives the desired expression.

The joint cdf of $|\hat{g}_1|$, is given by

$$F_{|\hat{g}_1|,...,|\hat{g}_N|(\tau_1, \ldots, \tau_N|\rho) = \int_0^\tau \cdots \int_0^\tau \int_0^\tau ... \int_0^\tau f_{|\hat{g}_1|,...,|\hat{g}_N|(\tau_1, \ldots, \tau_N|\rho) d\tau_1 \cdots d\tau_N$$

By using the derived expression for the joint pdf and by using the transformation $t = \frac{\tau}{\sqrt{\sigma_i^2}}$, the final expression can be derived.

REFERENCES

[1] I. F. Akyildiz, A. Kak, and S. Nie, “6G and beyond: The future of wireless communications systems,” IEEE Access, vol. 8, pp. 133995–134030, 2020.
[2] Y. Huang, L. Xing, C. Song, S. Wang, and F. Elhouni, “Liquid antennas: Past, present and future,” IEEE Open J. Antennas Propag., vol. 2, pp. 473–487, 2021.
[3] F. Tariq, M. R. A. Khandaker, K.-K. Wong, M. A. Imran, M. Bennis, and M. Debbah, “A speculative study on 6G,” IEEE Wireless Commun., vol. 27, no. 4, pp. 118–125, Aug. 2020.
[4] K. K. Wong, A. Shojaieifard, K. F. Tong, and Y. Zhang, “Fluid antenna systems,” IEEE Trans. Wireless Commun., vol. 20, no. 3, pp. 1950–1962, Mar. 2021.
[5] K. K. Wong, A. Shojaieifard, K. F. Tong, and Y. Zhang, “Performance limits of fluid antenna systems,” IEEE Commun. Lett., vol. 24, no. 11, pp. 2469–2472, Nov. 2020.
[6] K. K. Wong and K. F. Tong, “Fluid antenna multiple access,” [Online] arXiv: 2006.05908 [cs.IT]
[7] H. Yazdani, A. Vosoughi, and X. Gong, “Achievable rates of opportunistic cognitive radio systems using reconfigurable antennas with imperfect sensing and channel estimation,” IEEE Trans. Cogn. Commun. Networking, vol. 7, no. 3, pp. 802–817, Sept. 2021.
[8] R. H. Y. Louie, M. R. McKay, and I. B. Collings, “Maximum sum-rate of MIMO multiuser scheduling with linear receivers,” IEEE Trans. Commun., vol. 57, no. 11, pp. 3500–3510, Nov. 2009.
[9] M. R. McKay, I. B. Collings, and A. M. Tulino, “Achievable sum rate of MIMO MMSE receivers: A general analytic framework,” IEEE Trans. Inf. Theory, vol. 56, no. 1, pp. 396–410, Jan. 2010.
[10] J. Jose, A. Ashikhmin, P. Whiting, and S. Vishwanath, “Channel estimation and linear precoding in multi-user multiple-antenna TDD systems,” IEEE Trans. Veh. Technol., vol. 60, no. 5, pp. 2102–2116, Jun. 2011.
[11] H. Yin, D. Gesbert, M. Filipponi, and Y. Liu, “A coordinated approach to channel estimation in large-scale multiple-antenna systems,” IEEE J. Sel. Areas Commun., vol. 31, no. 2, pp. 264–273, Feb. 2013.
[12] Y. Wu, R. H. Y. Louie, and M. R. McKay, “Analysis and design of wireless ad hoc networks with channel estimation errors,” IEEE Trans. Sig. Process., vol. 61, no. 6, pp. 1447–1459, Mar. 2013.
[13] Y. Wu, M. R. McKay, and R. W. Heath, “Coverage and area spectral efficiency in downlink random cellular networks with channel estimation error,” in Proc. IEEE Int. Conf. Acoustics, Speech and Sig. Process., Vancouver, BC, Canada, May 2013, pp. 4404-4408.
[14] M. Haenggi, Stochastic geometry for wireless networks, in Cambridge, U.K.: Cambridge Univ. Press, 2012.
[15] M. K. Simon, Probability distributions involving Gaussian random variables: A Handbook for Engineers and Scientists, Boston, MA, USA: Springer, 2002.