Numerical evaluation of low frequency sound propagation in two layered media

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Abstract. This paper presents the simulation results of the infrasound sound energy transfer through various interfaces and discusses on the acoustic transparency of different media as a function of refraction index. Acoustic transparency is defined as the ratio of acoustic power radiated into the structure to the acoustic power emitted by a wave source. The effect of various surfaces on the transmission of infrasound has been studied by numerical simulation. To examine the transmitted homogeneous and inhomogeneous plane waves at the air - textile interface, we used MATLAB 2016 to calculate and plot the contributions of homogeneous and inhomogeneous plane waves as a function of frequency ranging from 3 to 5 Hz. Textiles (composed of natural or synthetic fibres) such as 100% cotton, 50% cotton – 50% flax, 100% wool (felt), nylon1 and nylon2 and flax showing various wave propagation properties were investigated. We found that almost the entire acoustic power passes through the interface of layered structure in the case of wool, cotton and flax, therefore the reflected power can be neglected. In the case of nylons, an amount of acoustic power ranging from 2% to 25% is reflected into air.

1. Introduction

The behavior of acoustic waves at boundaries is determined by the acoustic boundary conditions. It is generally accepted that at boundaries between two or more structures in equilibrium, both the acoustic pressure and the normal component of the acoustic velocity must be continuous [1]. Brekhovskikh [2] and Brekhovskikh and Godin [3] have demonstrated that the ray-theoretical models cannot be applicable when studying the low frequency sound propagation in layered media produced by a source placed at a distance less than a wavelength from the interface. When there is no sound dispersion in materials of layered structure (i.e. a fluid-solid interface) the classical Fresnel equations are valid. However, reflection at and transmission through a layered structure exhibit strong frequency dependence...
associated with resonances in the layer. When one layer of the structure shows dissipative properties, the frequency components of the acoustic pulse are transformed and spectrum of reflected signal changes [4-6].

This paper studies the acoustic transparency of the interface between two media showing a significant mass density contrast (i.e. air and various textile) at low frequencies (3, 4 and 5 Hz). The monochromatic acoustic point source is located within a fraction of the wavelength from the interfaces. We are devoted to characterize the acoustic transparency, defined as the ratio of acoustic power radiated into structure to the acoustic power emitted by a wave source, by means of the acoustic power into textiles and total emitted power flux. We have computed the exact relation of the coefficients of the acoustic power flux into textile and the total emitted power flux. The coefficients describe the contributions of both homogeneous and inhomogeneous plane waves in the air and they have a very complex form different from the Fresnel formula. The transmission of infrasound through the interface is facilitated by inhomogeneous waves. They turn into homogeneous waves after they pass through the interface.

2. Materials and methods

2.1. Infrasound reflection and refraction at a plane structure air-textile interface

We shall devote our attention to the generated sound waves by point source that is located at \((0,0,-z_0)\) in the air. Let be \(z = 0\) the plane that separates two half-spaces: \(z < 0\) (air) and \(z > 0\) (textile). The sound speeds and densities are \(c_1, \rho_1\) (for air) and \(c_2, \rho_2\) for textile, respectively. The values of the refraction index \(n = c_1/c_2\) and mass density ratio \(m = \rho_1/\rho_2\) are responsible for the peculiarities of the sound transmission through the air/textile interface. We denoted the reflected wave number vector as \(k' = (k_1 \sin \theta, k_1 \cos \theta)\) and the transmitted wave number vector as \(k'' = (k_2 \sin \theta', k_2 \cos \theta')\), where \(\theta'\) is the incident angle with respect to the \(z\) axis and \(\theta\) is the refracted angle. Snell’s law imposes \(k_1 \sin \theta = k_2 \sin \theta'\), namely the horizontal components of the wave vector do not change in the course of reflection and refraction. According to Brekhovskikh [3], \(p_1\) is the acoustic pressure radiated by a simple source or monopole in absence of an interface, \(p_2\) acoustic pressure reflected from the interface into the air and \(p_3\) acoustic pressure refracted into textile:

\[
p_j = \frac{1}{2\pi} \int d^2 q e^{i(q \cdot r + v_1 t)} Q_j(q), \quad j = 1, 2, 3
\]

\[
Q_1 = S_i(q) e^{i(q \cdot z_0)}, \ z > z_0; \quad Q_2 = S_i(q) e^{-i(q \cdot z)}, \ z < z_0
\]

\[
Q_3 = S_i(q) V(q) e^{i(q \cdot z)}, \ z > 0; \quad Q_4 = S_i(q) W(q) e^{-i(q \cdot z)}, \ z < 0
\]

where \(q = (q_1, q_2, 0)\) is ‘vertical wave number’, \(|q| = q = \sqrt{k^2 - q^2}\) are the ‘horizontal wave number’; \(k_s = \frac{\omega}{c_s}, s = 1, 2; \omega\) is the sound frequency. Clearly we have \(\text{Im} v_1 \geq 0\), \(V(q)\) and \(W(q)\) denote arbitrary plane-wave reflection coefficient and arbitrary plane-wave refraction coefficient, respectively.

In the homogeneous half-space \(z < 0\) (air), \(V = \frac{m_1 - v_2}{m_1 v_1 + v_2}, W = \frac{2m_1}{m_1 v_1 + v_2}\) are the Fresnel reflection and transmission coefficients for an incident plane wave with the wave vector \((q_1, q_2, v_1)\). The function \(S_i(q)\) represents the plane-wave spectra of the field emitted by a monopole source downward and upward, respectively.
For a monopole source which radiates sound equally well in all directions, the pressure is given by:

\[ p_i = p_0 = \frac{1}{R} e^{ik_0}, \]

where \( R = \sqrt{x^2 + y^2 + (z - z_0)^2} \). \((q_i, q_2, v_i)\) is the wave vector of reflected plane wave and \((q_1, q_2, v_1, v_2)\) is the wave vector of refracted plane wave. When \( 0 \leq q \leq k_1 \), the plane waves are homogeneous in the air (i.e. \( \text{Im} v_i = 0 \)) and produce homogeneous refracted wave in textile if the refraction angles obey to the rule \( 0 \leq \theta_i \leq \delta \), where \( \delta = \arcsin n^{-1} \) is the critical incidence angle or Brewster’s angle. Plane waves with \( k_i \leq q \leq k_1 \) are evanescent (inhomogeneous) in air (i.e. \( \text{Im} v_i > 0 \)) but they produce homogeneous refracted wave in the textiles if the refraction angles obey to the rule \( \delta \leq \theta_i \leq \pi/2 \). Plane waves with \( q > k_1 \) are inhomogeneous in air and textiles. The amplitude of inhomogeneous waves varies exponentially with \( z \) [7].

Acoustic power in air \( J_a \) is given by:

\[ J_a = \frac{J_0}{4\pi k_1} \int_{v < k_1} d^2 q |S_i(q)|^2 \left[ 1 + V(q) \right] e^{2i\omega v_i} \frac{\text{Re} v_i}{v_i} \]

Acoustic power in textiles \( J_f \) is:

\[ J_f = \frac{J_0}{4\pi k_1} \int_{v < k_1} d^2 q |S_i(q)|^2 \left[ 1 - \left( \frac{\text{Re} v_i}{v_i} \right)^2 + 2i\text{Im} V(q) \right] \]

The total radiated power of the generic source is:

\[ J_r = J_a + J_f \]

where \( J_0 = 2\pi/\rho v_c \) is the acoustic power radiated by a monopole source in the absence of the interface. The acoustic power fluxes \( J_a \) and \( J_f \) can be calculated numerically or evaluated analytically using the parameters \( m \) and \( n^{-1} \). Both integrals (4) and (5) are computed over finite domains and are absolutely convergent. Equation (6) is valid for reflecting surfaces with arbitrary plane-wave reflection coefficient \( V(q) \). For a monopole source of sound near the plane air - textile interface, the acoustic power flux radiated in textile and the total emitted power flux are explicitly written using homogeneous and inhomogeneous components as follows:

\[ J_f = mJ_0 \left[ A_1(n, m, k_1, z_0) + A_2(n, m, k_1, z_0) \right] \]

\[ J_r = mJ_0 \left[ T_1(n, m, k_1, z_0) + T_2(n, m, k_1, z_0) \right] \]

Where the dimensionless quantities \( A_1 \) and \( T_1 \) describe the contributions of homogeneous plane waves in the air, while \( A_2 \) and \( T_2 \) describe the contributions of inhomogeneous plane waves in the air that become homogeneous plane waves in textile after refraction. They are given by the following integrals:

\[ A_1(n, m) = \int \frac{\sqrt{n^2 - u}}{m\sqrt{1 - u} + \sqrt{n^2 - u}} \frac{du}{\sqrt{1 - u}} \]

\[ A_2(n, m, b) = \int \frac{\sqrt{n^2 - u} \exp(-2b\sqrt{1 - u})}{n^2 - m^2 - (1 - m^2)u} \frac{du}{n^2 - m^2 - (1 - m^2)u}, \quad b \geq 0 \]

\[ T_1(n, m, b) = 1 + \frac{1}{2} \int \frac{du}{\sqrt{1 - u}} \frac{m\sqrt{1 - u} - \sqrt{n^2 - u}}{m\sqrt{1 - u} + \sqrt{n^2 - u}} \cos(2b\sqrt{1 - u}) \]
After some calculus, Eq. (11) is rewritten as:

\[ T_z(m, n, b) = mA_z(m, n, b) \]  

(12)

Here \( b = k_z z_0 \leq 1 \) (i.e. low frequency range). According to eq. (6) the power output of a point monopole source depends on the reflexive properties of the interface and on distance to the interface. After some calculus, Eq. (11) is rewritten as:

\[ T_i(m, n, k, z_0) = 1 - \frac{\sin 2k_z z_0}{2k_z z_0} + 2m \int_0^{\pi} \sqrt{n^2 - 1 + u^2} \cos(2k_z u) \, du \]  

(13)

When \( b = k_z z_0 \leq 1 \) is arbitrary but close to zero, the eqs. (10) and (11) become:

\[ A_z(m, n, 0) = \int_0^{\pi} \sqrt{n^2 - u} \, du \left( \frac{m}{m^2 - 1} \right)^{\frac{1}{2}}, \quad b \geq 0 \]

or

\[ A_z(m, n, 0) = -2 \frac{\sqrt{n^2 - 1}}{m^2 - 1} + \frac{2m \sqrt{n^2 - 1} \tanh^{-1} \left( \frac{\sqrt{m^2 - 1}}{m} \right)}{(m^2 - 1)^{3/2}}, \quad b \geq 0 \]  

(14)

or

\[ T_i(m, n, 0) = 1 + \frac{1}{2} \int_0^{\pi} \frac{du}{\sqrt{1 - u} - \sqrt{n^2 - u}} \]

or

\[ T_i(m, n, 0) = 1 - \frac{1}{2} \left( 2 \sqrt{m^2 - 1} \sqrt{1 - n^2} \sqrt{n^2 - 1} - 2m^2 \sqrt{1 - (n^2 - 1)^2} \log \left( \sqrt{m^2 - 1} - \sqrt{n^2 - 1} \right) + \right. \]

\[ 2m^2 \sqrt{-(n^2 - 1)^2} \log \left( \sqrt{m^2 - 1} - \sqrt{n^2 - 1} + \sqrt{m^2 - 1} - \sqrt{n^2 - 1} \right) + \]

\[ \frac{m^2}{2} \left( 2 \sqrt{m^2 - 1} \sqrt{1 - n^2} - 2 \sqrt{m^2 - 1} \sqrt{n^2 - 1} + \sqrt{m^2 - 1} - \sqrt{n^2 - 1} \right) - \]

\[ -m^2 \sqrt{-(n^2 - 1)^2} \log \left( \sqrt{m^2 - 1} + \sqrt{n^2 - 1} \right) + \]

\[ \frac{m^2}{2} \sqrt{-(n^2 - 1)^2} \log \left( \sqrt{m^2 - 1} + \sqrt{n^2 - 1} \right) + \]

\[ + m^2 \sqrt{-(n^2 - 1)^2} \log \left( \sqrt{m^2 - 1} + \sqrt{n^2 - 1} + \sqrt{m^2 - 1} - \sqrt{n^2 - 1} \right) \]

(15)

After some calculus, Eq. (12) is rewritten as:

\[ T_z(m, n, 0) = mA_z(m, n, 0) = -2m \sqrt{\frac{n^2 - 1}{m^2 - 1}} + \frac{2m^2 \sqrt{\frac{n^2 - 1}{m^2 - 1}} \tanh^{-1} \left( \frac{\sqrt{m^2 - 1}}{m} \right)}{(m^2 - 1)^{3/2}}, \quad b \geq 0 \]  

(16)

2.2. Materials

Tables 1 and 2 present the experimental conditions.
Table 1. Physical constants and acoustic properties of textiles and air.

| Textile       | Density (kg/m³) | Acoustic impedance Z [kg/(s m²)] | Speed of sound [m/s] |
|---------------|-----------------|----------------------------------|----------------------|
| 100%cotton    | 563             | 0.28 x 10⁶                       | 500                  |
| 50% cotton – 50% flax | 605             | 0.075 x 10⁶                     | 123                  |
| 100% wool (felt) | 1760           | 0.8 x 10⁶                       | 623                  |
| Nylon1        | 115             | 240                             | 2                    |
| Nylon2        |                 |                                  | 3                    |
| flax          | 1430            | 0.9 x 10⁶                       | 650                  |
| air           | 1.293           | 420                             | 334                  |

Table 2. m and n parameters

| Textile       | m = ρ₂/ρ₁ | Density (kg/m³) | n = c₁/c₂ | Speed of sound [m/s] |
|---------------|-----------|-----------------|-----------|----------------------|
| 100%cotton    | 436.4     | 563             | 0.668     | 500                  |
| 50% cotton – 50% flax | 467.9     | 605             | 2.7       | 123                  |
|                | 874       | 1130            |           |                      |
|                | 928       | 1200            |           |                      |
|                | 1005.4    | 1300            |           |                      |
| 100% wool (felt) | 1082.7    | 1400            | 0.53      | 623                  |
|                | 1160      | 1500            |           |                      |
|                | 1237.4    | 1600            |           |                      |
|                | 1314.8    | 1700            |           |                      |
|                | 1361.1    | 1760            |           |                      |
| nylon         | 89        | 115             | 167       | 2                    |
|                |           |                 | 111.3     | 3                    |
| flax          | 1106      | 1430            | 0.51      | 650                  |
| air           | 1.293     | 420             |           | 334                  |

3. Results of simulation and discussion

Textiles (composed of natural or synthetic fibers) such as 100% cotton, 50% cotton – 50% flax, 100% wool (felt), nylon1 and nylon2 and flax are investigated from the point of view of acoustic transparency at their interface with air. According to the classical geometrical acoustics theory of acoustic wave, the high impedance contrast of the air-textile media interface (see table 1) makes the interface almost fully reflective. When infrasonic frequencies are addressed, a strong increase of the radiated acoustic power flux into textiles has been reported. As an example, in the case of refracted waves in wool, under normal conditions, the critical angle is δ ≈ 21° and some infrasound wave may penetrate wool at angles > 21° from the vertical. In our simulation, we considered the acoustic plane wave normally incident on the interfaces.

The integrals in Eqs. (15) and (16) cannot be calculated analytically at an arbitrary b. They were computed at b = 0 that is the low frequency range. The contribution of homogeneous plane waves in air T₁ as a function of the refraction index for the studied textiles is presented in Figures 1 and 2. Specifically, the atypical of the nylon is presented in figure 2. The contribution of the inhomogeneous
plane waves in air that become homogeneous plane waves in textiles after refraction at air-textile interface as a function of refraction index values for various textiles is showed in figures 3 and 4. The case of nylon is presented in fig. 4, separately.

For the nylon case, $T_2$ or $A_2(n, m, b)$ is a monotonically decreasing function of $n$. Nylon is the only case for which $n >> 1$. The power flux $T_1$ is much larger than $T_2$, but $T_1$ oscillates for $n < 1$ as a result of the interference of incident and reflected waves. According to Eqs. (4) and (5), the relative increase in the power flux into the air half-space, when $b = k_1z_0 \rightarrow 0$ is, unlike the magnitude of the power flux, insensitive to the mass density ratio and is determined by the refraction index $n$ (figs 1 and 2). We have found that almost the entire acoustic power passes through the interface in the case of wool, cotton and flax, therefore the reflected power can be neglected. In the case of nyons, an amount of acoustic power ranging from 2% to 25% is reflected into air.

![Figure 1](image1.png)

**Figure 1.** $T_1$ term describing the contribution of homogeneous plane waves in air for various textiles. The density ratio of the two media is given in table 2.

![Figure 2](image2.png)

**Figure 2.** $T_1$ describing the contribution of homogeneous plane wave in the air for nylon.
Figure 3. T2 term describing the contribution of inhomogeneous plane waves in air for various textiles. The density ratio of the two media is given in table 2.

Figure 4. T2 term describing the contribution of inhomogeneous plane waves in the air for nylon.

We have addressed here to a special case of low-frequency acoustic field that is generated by a sound source localized in air at a distance less than a wavelength. Sound transmission through the interface of air and a dense environment such as various textiles (which have lower and larger sound speed compared to sound speed in air) is unusually strong in this case. Moreover, reflection is weak in this case. The source of this atypical behavior is the inhomogeneous or evanescent waves in air in the vicinity of the interface. When frequency is less than structural frequency characteristic to the textile structures, the acoustic wave remains concentrated near the interface. The amplitude of the evanescent wave decays exponentially with distance to the radiating boundary. The evanescent waves selectively transfer the acoustic power energy into the textiles due to their conversion in textiles into propagating waves.

4. Conclusions
Low-frequency sound generated by a source placed at a distance less than a wavelength from the interface can efficiently pass to textile structure through a planar air surface. The air-to-textile interface is abnormally transparent to infrasound. Transfer through interface of the power flux is facilitated by
inhomogeneous waves which exist in the spatial spectrum of the incident wave. These evanescent waves transform into homogeneous plane waves after refraction at the interface. Simulation results show that almost the entire acoustic power passes through the interface in the case of wool, cotton and flax, therefore the reflected power can be neglected. In the case of nylons, an amount of acoustic power ranging from 2% to 25% is reflected into air.

Acknowledgement

The authors acknowledge funding under the Project MESMERISE 700399 from the European Union’s Horizon 2020 research and innovation programme – the Framework Programme for Research and Innovation (2014-2020).

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