Demand Forecast of Restoring Air Material of Helicopter Based on NHPP and Weibull Model

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Abstract—The demand forecast of spare parts for helicopter air material is one of the key links in the supply and guarantee of helicopter air material. In order to forecast the demand of helicopter restoring air material effectively, a method based on NHPP (non-homogeneous Poisson process) and Weibull model is proposed in this paper. In this method, the reparable air material is fully considered, and the repaired air materials are divided into two categories: basically repairable and completely repairable. Then, reliability assessment is carried out based on NHPP and Weibull model, and the demand of spare parts for air material is predicted on this basis. The effectiveness is proved by an example.

1. INTRODUCTION
Air material supply support of helicopter is an important and complex management. The cost of air material accounts for a considerable proportion in the cost of the whole helicopter maintenance cycle. For airlines, the management mode of air material directly affects the cost control and operation efficiency level. For the helicopter force, the management mode of air material directly affects the rate of task completion and military efficiency. Rational allocation of air material has always been one of the hot issues in the research of accurate guarantee, and the forecast of air material demand is the core and key to formulating the support plan. According to its nature, forecast methods can be divided into qualitative and quantitative forecast. Qualitative forecast is easy to be affected by subjective factors, so the quantitative forecast method is more meaningful [1-4]. At present, the main quantitative prediction methods include monadic linear regression method, econometric method, time series analysis method, neural network method and gray forecasting method. But restoring air material can continue to use after repair, because it is repairable. Therefore the relevant characteristics of air material failure samples will be directly affected by the repaired depth. There are amounts of air materials on the helicopter which are basically repairable. After repair, their reliability status is not exactly the same as that of new air materials, that is to say, the fault samples before and after repair are related. In this case, it is not appropriate to use
the traditional probability distribution model, and the above quantitative prediction methods have certain limitations. Therefore, in this paper, considering the repairability of the repairable air material of helicopter, starting from the reliability evaluation of the repaired air material, a demand forecast of restoring air material of helicopter based on Weibull and NHPP model is proposed.

2. DEMAND FORECAST OF RESTORING AIR MATERIAL OF HELICOPTER BASED ON NHPP AND WEIBULL MODEL

Through in-depth investigation and combined with engineering practice, the reliability evaluation model of restoring air material can be determined, as shown in Figure 1. Completely repaired and the first failure sample of newly installed air materials are processed by the traditional probability distribution model, among which Weibull distribution can be selected; the basic repaired sample is processed by NHPP (non-homogeneous Poisson process) [5-6]. After confirming the reliability model, we can evaluate the reliability of air material, and then forecast the demand of air material.

Figure 1. Reliability evaluation model of restoring air material

2.1. Demand forecast of basically repairable air material

2.1.1. NHPP (inhomogeneous Poisson process) model: The point in time where the restoring air material became unavailable is random. If the maintenance time is ignored, arrange each point in time of going wrong \( t_0 \rightarrow \cdots \rightarrow t_2 > t_1 \geq 0 \), they can be expressed by stochastic point process, as shown in Figure 2.

Figure 2. Failure process of restoring air material

Assume the failure number of air materials in \( (0, t] \) as \( N(t) \), and \( t \geq 0 \), the process is \( \{N(t), t \geq 0\} \). If the failure number meet the following conditions: \( 1 \) \( N(0) = 0 \); \( 2 \) \( \{N(t), t \geq 0\} \) and has independent increment; \( 3 \) for \( t_2 > t_1 \geq 0 \), in any time interval, the failure number \( N(t_2) - N(t_1) \) obeys Poisson distribution, and the mean value is \( \int_{t_1}^{t_2} u(t) \, dt \). The probability of \( j \) failures occurred is

\[
P[N(t_2) - N(t_1) = j] = \left( \int_{t_1}^{t_2} u(t) \, dt \right)^j \exp\{- \int_{t_1}^{t_2} u(t) \, dt\}/j!.
\]

(1)

Then the process \( \{N(t), t \geq 0\} \) is NHPP (non-homogeneous Poisson process).

In NHPP, \( u(t) \) changes with time, so the mean value of the process is also a variable, which shows that it is suitable for dealing with fault samples in different time intervals, which are not independent or distributed, and it is a better model for reliability evaluation of fault samples of restoring air material which is basic repaired. Although the assumption of independent increment may have some influence on the calculation accuracy of NHPP, the model is simpler and more practical on the premise of approximately conforming to the characteristics of restoring air material.

Because the superposition of two or more independent NHPP is still NHPP, so even if there is not a large number of fault samples of a certain air material, the amount of fault samples will still be large due to the large number of air material, which makes NHPP very useful. This is especially true for handling the reliability information of restoring air material.

In NHPP, when the function \( u(t) = u(t) = \beta t^{\beta-1} (t > 0) \), the special NHPP is called Weibull process, where \( u(t) \) is called fault intensity function.
The fault intensity function shows the change rate of the average number of faults with time in each time interval \((t_i, t_i + dt)\), which can reflect the probability of a fault occurring in the time interval \((t_i, t_i + dt)\).

In the failure intensity function, \(\beta\) is the shape parameter, \(\lambda\) is the scale parameter and \(t\) is the working time. The shape parameter \(\beta\) represents the change trend of fault. When \(\beta > 1\), the time interval between faults decreases, that is, faults occur more frequently, and faults have loss characteristics; when \(\beta < 1\), the time interval between faults increases, and faults probability may decrease. It is suitable to describe the fault change rule of complex restoring air material.

The fault rate in the probability distribution model reflects the condition probability of the first fault (also the only fault) of the air material which works well after a period of time. It deals with simple random samples and requires fault samples to be independent and identically distributed, which is not available for most restoring air materials. Therefore, for a large amount of restoring air materials with basically repairable samples, the fault intensity, rather than the traditional fault rate, is more accurate to describe the rule of fault change. NHPP corresponding to the fault intensity is a better model for the reliability evaluation of complex restoring air material.

2.1.2. Parameter estimation method. It is assumed that there are \(k\) restoring air materials in total, in which the No. \(q\) air material has \(N_q\) faults in the statistical time interval \([S_q, T_q]\), and the time which the No. \(i\) fault occurred is \(t_{q,i}(i = 1, 2, \ldots, N_q; \quad q = 1, 2, \ldots, k)\).

The function parameter of NHPP process fault intensity \(\hat{u}(t)\) can be obtained by maximum likelihood estimation method

Scale parameter:
\[
\hat{\lambda} = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k} (T_q^{\hat{\beta}} - S_q^{\hat{\beta}})}. \quad (2)
\]

Shape parameter:
\[
\hat{\beta} = \frac{\sum_{q=1}^{k} N_q / \hat{\lambda} \sum_{q=1}^{k} (T_q^{\hat{\beta}} \ln T_q - S_q^{\hat{\beta}} \ln S_q) - \sum_{q=1}^{k} \sum_{i=1}^{N_q} \ln t_{q,i}}{\sum_{q=1}^{k} \sum_{i=1}^{N_q} \ln t_{q,i}}. \quad (3)
\]

\(\hat{\lambda}, \hat{\beta}\) can be calculated by iterative method.

2.1.3. Distribution test method. There are many methods of distribution test. For example, Cramer von Mises goodness-of-fit test method can be used. Arrange the No. \(q\) air material’s fault time \(t_{q,i}(i = 1, 2, \ldots, k)\) in time interval \([0, T_q]\) in sequence, and then calculate the observation value \(C_m^2\) as follows
\[
C_m^2 = 1/(12M) + \sum_{j=1}^{M} \left\{ \frac{2^j - (2j - 1)/(2M)}{M} \right\}^2. \quad (4)
\]

Find out the critical value of \(C_m^2\) from the table. When it is less than the critical value, accept the NHPP assumption with the fault intensity is \(\hat{u}(t)\), otherwise reject the assumption.

2.1.4. Other relevant calculations. The average number of predicted faults of air materials during \((0, t)\)
\[
E[N(t)] = \hat{\lambda} t^{\hat{\beta}} \quad (t > 0). \quad (5)
\]

Probability of \(N\) faults estimated during length of service \((0, t)\) of air material
\[
P[N(t) = n] = \left\{ (\hat{\lambda}t^{\hat{\beta}})^n e^{-\hat{\lambda}t^{\hat{\beta}}} \right\} / n!, \quad n = 0, 1, 2, \ldots \quad (6)
\]

When the working time of air material is \(t\) and the task time is \(d\), task reliability
\[
R(t) = - \exp\left\{ \hat{\lambda}(t + d)^{\hat{\beta}} - \hat{\lambda}t^{\hat{\beta}} \right\} \quad (7)
\]
According to the result of reliability evaluation, the demand of spare parts can be predicted.
2.2. Demand forecast of completely repairable air material

Weibull distribution is widely used in reliability engineering because it has greater adaptability. In present, when reliability analysis and evaluation of the given life test or data in field observation are produced, two-parameter Weibull distribution is often used which is simple and easy, but its parameter estimation often leads to large errors. Thus, in order to better reflect the actual situation of air material, three-parameter Weibull model [7-8] is adopted for the reliability evaluation and demand forecast of completely repairable air material.

2.2.1. Three-parameter Weibull model. The distribution function of three-parameter Weibull distribution is

\[ F(x) = 1 - \exp\left\{-\left[\frac{(x - \gamma)}{\eta}\right]^{\beta}\right\}, \quad x \geq \gamma. \]  

(8)

In the formula, \( \eta, \beta \) and \( \gamma \) are called scale parameter, shape parameter and position parameter respectively, and \( \eta, \beta > 0, \gamma \geq 0 \).

The density function and failure rate function of Weibull distribution are respectively

\[ f(x) = \left(\frac{\beta \eta}{\gamma}\right)\left(\frac{1}{\gamma}\right)\left[\frac{(x - \gamma)}{\eta}\right]^{\beta-1} \exp\left\{-\left[\frac{(x - \gamma)}{\eta}\right]^{\beta}\right\}. \]

(9)

\[ r(x) = \left(\frac{\beta \eta}{\gamma}\right)\left[\frac{(x - \gamma)}{\eta}\right]^{\beta-1}. \]

(10)

When \( \gamma = 0 \), the model degenerates into a two-parameter model.

In the above formulas, when \( \beta < 1 \), the failure rate given by equation (10) is reduced, which is suitable for modeling early failure; when \( \beta = 1 \), the fault rate is constant, which is suitable for modeling failure in random; when \( \beta > 1 \), the failure rate is increased, which is suitable for modeling wear or ageing failure.

2.2.2. Parameter estimation of three-parameter Weibull distribution. Maximum likelihood method can be adopted where. Maximum likelihood method is a very effective and general parameter estimation method. Its basic idea is to select the undetermined parameter to maximize the probability of the sample appearing in the field of observation value, and take this value as the point estimation value of unknown parameter.

First \( \eta = \theta - \gamma \),

(11)

Rewrite equation (9), and then according to the basic principle of maximum likelihood estimation, form the likelihood function:

\[
\ln L = n \ln \beta - n \ln (\theta - \gamma) + (\beta - 1) \cdot \sum_{i=1}^{n} \ln (x_i - \gamma) - \frac{1}{(\theta - \gamma)^\beta} \cdot \sum_{i=1}^{n} (x_i - \gamma)^\beta 
\]

(12)

In the formula, \( n \) is the sample capacity during the test, and \( x_i (i = 1, 2, \cdots, n) \) is the observed value of the sample, such as the failure time. The likelihood equation of the three-parameter Weibull distribution is as follows:

\[
\frac{\partial \ln L}{\partial \gamma} = \frac{n \beta}{\theta - \gamma} - (\beta - 1) \sum_{i=1}^{n} \frac{1}{x_i - \gamma} - \frac{\beta}{(\theta - \gamma)(\beta - 1)} \sum_{i=1}^{n} (x_i - \gamma)^\beta + \frac{\beta}{(\theta - \gamma)^\beta} \sum_{i=1}^{n} (x_i - \gamma)^{(\beta - 1)} = 0
\]

(13)

\[
\frac{\partial \ln L}{\partial \gamma} = \frac{n}{\beta} - n \ln (\theta - \gamma) + \sum_{i=1}^{n} \ln (x_i - \gamma) + \frac{\ln (\theta - \gamma)}{(\theta - \gamma)^\beta} \sum_{i=1}^{n} (x_i - \gamma)^\beta - \frac{\sum_{i=1}^{n} (x_i - \gamma)^\beta \ln (x_i - \gamma)}{(\theta - \gamma)^\beta} = 0
\]

\[
\frac{\partial \ln L}{\partial \theta} = \frac{n \beta}{\theta - \gamma} + \frac{\beta}{(\theta - \gamma)^{\beta + 1}} \cdot \sum_{i=1}^{n} (x_i - \gamma)^\beta = 0
\]
Obviously, the likelihood equation system is a nonlinear equation system, and its structure is very complex. It is impossible to obtain the analytical solution directly, so it must be solved by computer. In fact, using computer to realize maximum likelihood estimation is to solve the nonlinear equations given in formula (13). Newton-Raphson iterative method is usually used to solve nonlinear equations on computer.

Based on the basic idea of Newton-Raphson iterative method, the primary values of the parameters $\gamma$, $\beta$ and $\theta$ to be estimated are given firstly, which are $\gamma_0$, $\beta_0$, $\theta_0$, and let $\gamma_0 + \Delta \gamma = \gamma$; $\beta_0 + \Delta \beta = \beta$; $\theta_0 + \Delta \theta = \theta$. Then the left-hand items of the likelihood equations are expanded in series at $\gamma_0$, $\beta_0$, $\theta_0$, and make the first-order approximation, then the equations (13) are transformed into linear equations.

After the solutions of $\Delta \gamma$, $\Delta \beta$ and $\Delta \theta$ are obtained by solving the linear equations, the solutions are judged. If $\Delta \gamma$, $\Delta \beta$ and $\Delta \theta$ are all smaller than the given error limits, then the corresponding $\Delta \gamma$, $\Delta \beta$ and $\Delta \theta$ are the estimated values. Otherwise, $\gamma_0 + \Delta \gamma$, $\beta_0 + \Delta \beta$ and $\theta_0 + \Delta \theta$ are used to replace $\Delta \gamma$, $\Delta \beta$ and $\Delta \theta$, and then recalculate. The estimated values of $\gamma$, $\beta$ and $\theta$ are obtained through repeated operations.

According to the result of reliability evaluation, the demand of spare parts can be predicted.

**3. EXAMPLE VERIFICATION**

Taking the demand rate of a restoring air material of a helicopter regiment as an example, the above algorithm is verified. According to the historical data of the air material consumed by three squadrons of the maintenance brigade, the demand of each squadron in January, February, March and April in a certain year is forecasted according to the above algorithm. The results are shown in table 1-table 3. It can be seen from the table that the average forecast error is less than 0.002, and the relative error is also small.

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**TABLE 1. THE PREDICTION RESULTS OF SQUADRON 1**

| Month | 1     | 2     | 3     | 4     |
|-------|-------|-------|-------|-------|
| actual demand rate (quantity/days) | 0.18  | 0.12  | 0.17  | 0.20  |
| prediction results | 0.179 | 0.123 | 0.172 | 0.202 |
| forecast error | 0.001 | 0.003 | 0.002 | 0.002 |
| relative error | 0.6%  | 2.5%  | 1.2%  | 1.0%  |

**TABLE 2. THE PREDICTION RESULTS OF SQUADRON 2**

| Month | 1     | 2     | 3     | 4     |
|-------|-------|-------|-------|-------|
| actual demand rate (quantity/days) | 0.17  | 0.15  | 0.14  | 0.14  |
| prediction results | 0.167 | 0.152 | 0.141 | 0.138 |
| forecast error | 0.003 | 0.002 | 0.001 | 0.002 |
| relative error | 1.8%  | 1.3%  | 0.7%  | 1.4%  |

**TABLE 3. THE PREDICTION RESULTS OF SQUADRON 3**

| Month | 1     | 2     | 3     | 4     |
|-------|-------|-------|-------|-------|
| actual demand rate (quantity/days) | 0.16  | 0.12  | 0.15  | 0.18  |
| prediction results | 0.161 | 0.123 | 0.149 | 0.179 |
| forecast error | 0.001 | 0.003 | 0.001 | 0.001 |
| relative error | 0.6%  | 2.5%  | 0.7%  | 0.6%  |
4. CONCLUSION
The demand forecast of spare parts for helicopter air material is one of the key links in the supply and guarantee of helicopter air material. In order to forecast the demand of helicopter restoring air material effectively, based on the statistical analysis of reliability information, combined with the characteristics of equipment and components, this paper proposes a forecast method of restoring air material demand based on NHPP and Weibull model. First, the NHPP model is used to evaluate the reliability of basically repairable air material, and the Weibull model is used to evaluate the reliability of completely repairable air material. Then, the spare parts demand of air material is forecasted. The validity of the method is proved by an example.

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