Toward flow control: An assessment of the curled wake model in the FLORIS framework

Christopher J Bay¹, Jennifer King¹, Luis A Martinez-Tossas¹, Rafael Mudafort¹, Paul Hulsman², Martin Kühn², and Paul Fleming¹

¹ National Wind Technology Center, National Renewable Energy Laboratory, Golden, CO, 80401, USA
² ForWind – University of Oldenburg, Institute of Physics, Küppersweg 70, 26129 Oldenburg, Germany
E-mail: christopher.bay@nrel.gov

Abstract. In this work, a new controls-oriented wake model is modified and compared to an analytical Gaussian wake model, high-fidelity simulation data, and experimental wind tunnel campaign. This model, called the curled wake model, captures a wake phenomenon that occurs behind yawed turbines, modeled as a collection of vortices shed from the rotor plane. Through turbine simulations, these vortices are shown to have a significant impact on the prediction of the wake steering’s performance. Overall, the results support the concept of secondary steering, or a yawed turbine’s ability to deflect the wake of a downstream turbine, and suggest that future turbine wake studies and yaw optimizations should include the curled wake phenomenon.

1. Introduction
Flow control in wind power plants aims to manipulate a turbine’s influence on the wind to achieve an increase in performance at the plant level. One method of flow control known as wake steering intentionally misaligns turbines from the incoming flow to deflect their wakes away from downstream turbines [1, 2, 3]. A tutorial by [4] provides an overview of the key concepts and methods of wind plant control, while [5] provides a survey of wind farm power and fatigue information. Research has shown that with certain misalignments the overall production of a wind plant can increase, even though the misaligned turbines experience an individual power loss [6].

Numerous techniques have been used to study wake steering, including theoretical, experimental, and field campaigns. In computational fluid dynamics (CFD) simulations, [7] observed an overall increase in power production of an offshore wind turbine array of two turbines using wake steering. Large eddy simulations (LES) by [8] were used to investigate the feasibility of wake steering under different atmospheric conditions, finding that stable atmospheric conditions provide for the largest performance increases with wake steering. In a porous disk experiment, as well as LES, [9] characterized the shape of yawed turbine wakes.

Wake deflection due to yaw misalignment has also been studied in wind tunnel experiments [10, 11, 12]. An optimal yaw angle study was conducted by [13] for six scaled turbines in a wind tunnel using a data-driven Bayesian Ascent method. The results showed that the optimal yaw settings resulted in the progressively smaller yaw angles for turbines that were further downstream. Additionally, a handful of field campaigns have been conducted to
evaluate the effectiveness of wake steering [1, 14]. A 13% increase in energy was found by [15] on a downstream turbine for two closely spaced turbines within a 10° bin, consistent with prior predictions from different models. Site-specific, historical operational data was leveraged by [16] to develop a wake control scheme that was tested in a land-based wind power plant in Alberta, Canada. The controller resulted in power increases of 7%–47% for wind conditions that commonly occur at night as well as a decrease in variability of power production of up to 72%.

Alongside theoretical and experimental studies, engineering models have been crafted to predict wake steering and assess its overall benefit. One controls-oriented tool used in the design and study of turbine wake controls is the FLOw Redirection and Induction in Steady State (FLORIS) model [3]. Originally based on the Jensen [17] and Jiménez [18] models, FLORIS has evolved into a software repository that contains several wake models. More recently, FLORIS uses the wake recovery and redirection models of [19, 20], and [21]. A further description of FLORIS and its models is covered in the next section, but the reader is referred to [22] for a detailed description of the current FLORIS models. The FLORIS model is open source, and the latest version is available for download and collaborative development (https://github.com/NREL/FLORIS).

These models have shown increasing ability to accurately design wake steering controllers for the case of one turbine waking a second; however, their success in modeling larger arrays of turbines implementing wake steering is less established. For one reason, there is limited field data for this case and, although there are some LES studies, it is expensive to run many simulations of a large farm. As such, many questions remain around designing wake steering strategies at the full plant level. Additionally, the physics of wake steering at this scale are not fully understood. In papers such as [10], [9], and [8] it is observed that a pair of counter-rotating vortices are generated from wake steering and that these vortices deform the shape of the wake over time. However, it was not immediately apparent if accounting for these wakes in engineering models was necessary. Some recent research suggests these vortices are integral to accurately design wake steering strategies for larger numbers of turbines [23, 24].

First, [25] note that the scale of the vortices have implications for the wake steering’s effectiveness, which can be determined by the scale of the rotor to atmospheric scales. They state that larger turbines induce larger wake deflections due to the larger rotor’s effects on the length and time scales of the vortex structures. Specifically, larger coherent structures dissipate slower, thus having a longer lasting effect on the wake.

LES simulations were used by [23] to demonstrate that the vortices lead to discrepancies in existing models of wake steering behavior for arrays of turbines larger than two. One effect, named “secondary steering,” shows that the wake of an aligned turbine will be deflected if it overlaps with a steered wake. This was not accounted for in engineering models available at the time the paper was written and most likely could be explained by vortex interactions.

Very recent papers look at wake steering for larger arrays of turbines. In [26], the authors use scaled wind turbines in a wind tunnel to investigate the optimal distribution of yaw angles for an array of 3–5 evenly spaced turbines. The authors consider various strategies, including only yawing the first turbine or yawing all but the final turbine an equal amount. However, when all but the downstream turbine are free to independently select yaw angles, a more or less linearly decreasing yaw angle assignment from the front to the rear turbine is found to be optimal.

To determine if wake steering design and analysis could be improved by directly modeling the vortices in a control-oriented model, [27] proposed a theoretical model of vortex-based wake steering. Referred to here as the curled wake model, this model has now been incorporated into the FLORIS controls-oriented modeling framework as an option for wake modeling.

In this paper, we utilize a modified version of the curled wake model in FLORIS (and the FLORIS implementation of [19, 20], referred to here as the Gaussian wake model). Results for
the NREL 5-MW [28] show that improvements to control design are achieved when using the curled wake model. In the first section, the Gaussian and curled wake models are discussed, including the modifications made to the curled wake model in FLORIS. The LES of three and five turbines are then used to indicate the ability of the models to capture wakes for different arrays of turbines. We show that the curled wake model is able to predict secondary steering effects. The authors note that the curled wake model comparison with the Gaussian wake model is intended to show the value of modeling these additional effects and is not intended as a detraction from the Gaussian model, which represented a critical step forward in models of wake steering.

We believe that this most recent model addition within FLORIS captures important wake effects that need to be considered in wind plant and wake control. The implication of secondary steering on downstream turbines is nontrivial. By including these effects in wind plant controller design, we believe this will enable larger energy gains for larger arrays of turbines.

2. FLORIS: A Controls-Oriented Modeling Platform

FLORIS is a controls-oriented wake simulation and wind plant controls analysis tool used to study and optimize wind plant control. FLORIS has been jointly developed by the National Renewable Energy Laboratory (NREL) and Delft University of Technology and is available for download and collaborative development (https://github.com/NREL/FLORIS). Below is a brief description of some models in FLORIS, followed by an explanation of a recent model, the curled wake model. For more details on the models in FLORIS, see [29]. While these are the models included with FLORIS, any wake model can be substituted into the FLORIS framework.

2.1. Wake models

FLORIS includes three models: the Jensen model, the FLORIS (multizone) model, and the Gaussian model. The Jensen model [17] is a well-known, simplified model for calculating wake deficits behind a turbine. The multizone model is built upon the Jensen model, as detailed in [30]. This model splits the wake into three different zones: the near-wake zone, the far-wake zone, and the mixing-wake zone. These models, paired with a wake deflection model such as the Jiménez model [18], were used in the early development of FLORIS, but all of the recent research has utilized the Gaussian model. The theoretical basis for the Gaussian model was developed across several papers [19, 20, 21, 31]. The Gaussian model is used in this paper in the comparisons to the curled wake model in FLORIS. The velocity deficit in the wake is described using a Gaussian profile. This model also takes into account added turbulence from the turbines as well as atmospheric stability. For more model details, the reader is referred to [22].

2.2. Curled wake with vortex decay model

The curled wake model described here is an advancement of the model first proposed in [27]. The reader is referred to [27] for a detailed derivation of the model. This model is based on a phenomenon in turbine wakes identified by [8], [9], [10], and [20]. This phenomenon is observed as a curling of the wake (i.e., deforming the initially circular wake of the rotor to a kidney shape). In addition to this deformation, the wake is displaced laterally. This is due to counter-rotating vortices that are shed from the rotor when it is not aligned with the wind [8, 20, 32].

The model is a simplified and linearized version of the Reynolds-averaged Navier-Stokes streamwise momentum equation for incompressible flow. The simplifications lead to:

\[
U \frac{\partial u'}{\partial x} + V \frac{\partial u'}{\partial y} + W \frac{\partial (u')}{\partial z} = \nu_{eff} \left( \frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2} + \frac{\partial^2 u'}{\partial z^2} \right)
\]

which describes the evolution of the wake deficit, \(u'\), as it moves downstream. \(U\), \(V\), and \(W\) are the streamwise, spanwise, and vertical velocity components from the base flow, respectively.
while $u'$, $v'$, and $w'$ are perturbation velocities around the base flow. Specifically, the perturbation velocities represent the wake deficits that are convected by the base flow. $v_{\text{eff}}$ is the effective viscosity of the flow. Equation 2 shows the mixing length model used to compute the effective viscosity, where $\kappa$ is the von Kármán constant, $z$ is the distance from the ground, $du/dz$ is the velocity gradient from the boundary layer profile, and $\lambda$ is the value of the mixing length in the free atmosphere [27, 33].

$$
\nu_T = \ell_m^2 \left| \frac{du}{dz} \right|, \quad \ell_m = \frac{\kappa z}{1 + \kappa z / \lambda}
$$

(2)

The vortices shed by a yawed turbine introduce motions in the spanwise directions. These large-scale motions are responsible for convecting the wind turbine wake to the sides and generating the curled wake. These vortices decay as they travel downstream based on the turbulent viscosity, assumed here to be the same in streamwise and spanwise directions.

The curled wake effect is captured by an elliptic distribution of vortices added to the base flow [27, 32]. The vortices are described as Lamb-Oseen vortices [34, 27] with a tangential velocity of:

$$
u_T = \frac{\Gamma}{2\pi r} \left( 1 - \exp \left( \frac{-r^2}{4\nu_{\text{eff}}t + r_0^2} \right) \right),
$$

(3)

where $u_t$ is the tangential component of the velocity, $\Gamma$ is the vortex (or circulation) strength, $r$ is the radial distance from the core of the vortex, $r_0$ is the initial vortex core radius, and $t$ is time. Time here is the time it takes for the flow to travel downstream, approximated as $t = x/U_\infty$.

The curled wake model as presented in [27] has vortices that do not decay as they move downstream. Here, vortex decay is included in the curled wake model as a scaling factor dependent on the distance downstream from the vortex’s formation, based on the work in [35]. The spanwise velocities are computed only once and scaled to obtain downstream values to reduce computation. The ratio of the initial vortex velocity to the velocity of a vortex at some later time, $t$, can be used to decay the vortices and approximated as:

$$
\frac{u_t(t)}{u_t(0)} = \frac{1 - \exp \left( \frac{-r^2}{4\nu_{\text{eff}}t + r_0^2} \right)}{1 - \exp \left( \frac{-r^2}{r_0^2} \right)} \approx \frac{r_0^2}{4\nu_{\text{eff}}U_\infty + r_0^2}.
$$

(4)

This is accomplished by expanding the exponential values using their Taylor series expansions, keeping the first two terms of the series, and substituting $t = x/U_\infty$. This decay factor is then applied to the vortex velocities, $V$ and $W$, starting at each turbine and marching through the downstream wake.

In addition to the vortex decay factor, modifications were made to the turbulent viscosity to account for turbulence added to the flow from turbines. Specifically, the effective viscosity, $\nu_{\text{eff}}$, term in Eq. 1, which is represented as a mixing length model, was modified by the turbine turbulence model. This essentially changes the length scale of the turbulence generated by the turbine. A turbine turbulence model that determines the amount of turbulence added by a turbine was used from [36]. The turbulence model implemented is defined as:

$$
I_+ = 0.73a^{0.5} I_0^{0.1} \frac{x^{-0.275}}{d}.
$$

(5)

The empirical values used in this model vary from those originally proposed by [36]. The previously mentioned values have been determined from previous FLORIS studies, as this same turbine turbulence model is used in the FLORIS Gaussian model. This turbulence factor, $I_+$,
was applied to the effective viscosity term in Eq. 1, along with a tunable dissipation scaling parameter, $\alpha$, as shown in Eq. 6.

$$U \frac{\partial u'}{\partial x} + V \frac{\partial u'}{\partial y} + W \frac{\partial (u')}{\partial z} = \alpha I_{\text{eff}} \left( \frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2} + \frac{\partial^2 u'}{\partial z^2} \right)$$

(6)

The curled wake model solves Eq. 6 using a forward-time, centered-space method [27, 37].

3. Validation

The power output of turbines within the curled wake model is compared against the Gaussian model and LES for three- and five-turbine arrays. The large-eddy simulations are performed using NREL’s Simulator for Wind Farm Applications (SOWFA). SOWFA solves the filtered Navier-Stokes equations with a temperature equation, and the wind turbine is modeled using the actuator disk model [38, 39, 40]. First, a neutrally stratified precursor simulation is performed with periodic boundary conditions on the sides, Coriolis effects, and a temperature capping inversion. The simulation domain size is 5 km by 2 km by 1 km with a 10-m grid resolution in all directions. A wall model is used with a roughness height of $z_0 = 0.15$ [m], and the latitude is set to 41.3°, which is representative of conditions in the northern hemisphere. After running the precursor simulation and saving the planes of data, a simulation with inflow/outflow boundary conditions is performed with the turbine. The simulations are time-averaged for 1,700 seconds after an initial transient of 300 seconds.

The turbine used in the simulations is the NREL 5-MW turbine [28]. Both the Gaussian and curled wake models were tuned using SOWFA simulations of a three-turbine array. The details and values of the tuning parameters are provided in Table 1. It is important to note that there will be differences between the engineering models and the SOWFA data. While near-zero percent error could be achieved, we did not want to overfit the models to these handful of SOWFA simulations as there can be significant variations between SOWFA cases depending on inflow definition, spanwise location of the turbines, etc. Thus, a threshold of 5% error was targeted.

The SOWFA simulations had a mean wind speed of 8.0 m/s but, due to the variations in inflow, the wind speed for the Gaussian and curled wake models was tuned such that the lead turbine produced approximately the same power as the lead SOWFA turbine. These variations in flow are due to the turbulent fluctuations that are seen by the rotor in the SOWFA simulations but are not present in FLORIS. Thus, the tuned wind speed was found to be 8.38 m/s. The other atmospheric inputs were held constant.

The FLORIS model parameters were then adjusted for each of the wake models until there was reasonable agreement in the power differences between the models and the respective SOWFA simulations. This was done for the three-turbine cases, then the parameters were kept the same for the five-turbine cases. Results of this tuning process are described in the next sections.

3.1. Three-turbine wake comparison

For the three-turbine case, the turbines were spaced 7 rotor diameters (D) apart. This is a common spacing observed in built wind farms. Four yaw cases were used for tuning the models to SOWFA data. The results are summarized in Table 2. The total power of each case was tuned to +/- 5% of the SOWFA data, which was determined to be sufficiently accurate for control-oriented applications.

Overall, the Gaussian model compares well to the SOWFA results when examining turbine powers. This is further bolstered by looking at cross sections of the flow immediately upstream of the second turbine. Fig. 1a displays flow cross sections 5D downstream of the first turbine where the lead turbine is yawed 20 degrees. Both the Gaussian and curled wake models emulate
the wake shape shown in the SOWFA simulations quite well behind the first turbine, with some
differences in the magnitudes of the velocity deficit (shown by the difference in color).

However, disparities exist between the Gaussian model and SOWFA in the flow downstream
of the second turbine. [23] showed that the wake behind the second turbine continues to deflect
and deform when the first turbine is positively yawed away from the incoming wind direction,
termed secondary steering. Fig. 1b shows a similar depiction, displaying cross sections of the
flow 5D behind the second turbine. With the first turbine yawed, the Gaussian model fails to
predict the continued deflection behind the second turbine. Because of its inclusion of counter-
rotating vortices, the curled wake model better captures this deflection and the overall shape of
the wake as shown in SOWFA. These vortices work to continually deflect the wake of the yawed
turbine over distances greater than the normal spacing between turbines. As such, the impact
of these vortices is even more important when considering anything more than a two-turbine
array. This impact on power in a five-turbine array is discussed in the following section.

3.2. Five-turbine wake comparison

Simulations were performed for a five-turbine case with 6D spacing. The same tuning parameters
from the three-turbine case, listed in Table 1, were used for the five-turbine simulations. Overall,
the models matched the three SOWFA simulations, with a few key exceptions, shown in Fig. 2.
The Gaussian model suffers in the cases where a yawed turbine is upstream of multiple turbines,
as expected. On the other hand, the curled wake model has some difficulty predicting the
baseline, but is able to closely match the yawed turbine cases. This suggests that further tuning
and/or improvement is needed in the local turbine turbulence model.

Fig. 3 shows the relative powers of the individual turbines from the same three yaw cases.
The Gaussian model captures the trends for the upstream turbines, but again miscalculates the
effect of yaw on the most downstream turbines, T3 and T4. The additional physics in the curl
model captures the relative trends for all of the turbines, while again showing that the TI model

Table 1: Atmospheric and model parameters used in wake model comparisons.

| Atmospheric inputs         | Description                                                                 |
|---------------------------|-----------------------------------------------------------------------------|
| Wind speed (m/s)          | 8.38 The freestream velocity of the flow field.                            |
| Wind direction (deg)      | 270.0 The direction of the freestream velocity.                             |
| TI                        | 0.09 A measure of the turbulence intensity.                                |
| Shear                     | 0.12 The coefficient of shear.                                             |
| Dissipation (α)           | 0.06 A tunable parameter for the flow viscosity.                            |

Table 2: Results of the tuning process compared to the respective SOWFA simulations.

| T1 Yaw | T2 Yaw | T3 Yaw | Difference in Power from SOWFA to Gauss | Difference in Power from SOWFA to Curl |
|--------|--------|--------|----------------------------------------|---------------------------------------|
| 0.0°   | 0.0°   | 0.0°   | 4.05%                                  | 3.13%                                 |
| 20.0°  | 0.0°   | 0.0°   | -1.10%                                 | -2.42%                                |
| 0.0°   | 20.0°  | 0.0°   | 4.32%                                  | 3.29%                                 |
| 20.0°  | 20.0°  | 0.0°   | -2.48%                                 | -2.38%                                |
(a) The flow 5D downstream of the first turbine with yaw settings of $T_0 = 20.0^\circ$ and $T_1, T_2 = 0.0^\circ$.

(b) The second turbine with yaw settings of $T_0 = 20.0^\circ$ and $T_1, T_2 = 0.0^\circ$.

Figure 1: Cross sections of the flow 5D behind different turbines in a yawed setting.

Figure 2: Individual turbine powers in a five-turbine array for three different yaw cases.

Figure 3: Relative power trends shown for each turbine in the five-turbine array with that turbine’s yaw setting on the x-axis. Three different yaw cases are shown: an all-aligned case, a $T_0 = 25^\circ$ case, and a $T_0, T_1 = 25^\circ$ case.

can be improved. Next, the curl model’s abilities to predict wakes is compared against a wind tunnel campaign.

4. Wind Tunnel Comparison
The measurement campaign was conducted in the large wind tunnel at ForWind, University of Oldenburg, using the MoWiTO 0.6 (Model Wind Turbine Oldenburg), with a hub height of 0.77 m and a rotor diameter ($D$) of 0.58 m [41]. Furthermore, a short-range Lidar WindScanner was
used to obtain high spatial and temporal resolution of the wake from 1 $D$ up to 10 $D$ downstream. The nozzle of the wind tunnel is fitted with an active grid in a passive mode at the test section entrance to alter the inflow condition. This resulted in a sheared inflow condition with a wind speed of 7.5 m/s, a turbulence intensity of 1.5%, and a shear exponent of $\alpha = 0.27$ at hub height. Furthermore, the turbine was yawed at $\psi = -30^\circ$, $0^\circ$, and $30^\circ$. The WindScanner performed multiple vertical scans ($3 \times 3D$) at multiple downstream distances behind the wind turbine model, providing a 3D representation of the evolution of the streamwise velocity component behind the wind turbine ranging between $-1.5D < y < 1.5D$ and $0D < z < 3D$. Each vertical scan is performed sequentially with a duration of 10 min, using a sampling frequency of 451.2 Hz. The WindScanner measured the line-of-sight wind speed with a Lissajous pattern, from which the streamwise component is extracted with the assumption that the lateral ($v \approx 0$) and vertical velocity components ($w \approx 0$) are negligible and interpolated onto a grid with a spacing of 7 cm x 7 cm during the assessment of the wake flow.

Figure 4: Cross sections of the flow 5D behind different turbines in a yawed setting.

Results at 5D, 7D, and 10D behind one turbine that is yawed at $\psi = 30^\circ$ are shown in Fig. 4. The curled shape of the wake is very apparent in the simulation, matching the measured
cross streams very closely. The right column shows what the effective rotor velocity would be, calculated at several points in the spanwise direction at the three downstream distances. The curl model compares well, with slightly less deflection overall, and a more shallow wake at 5D. These trends can also be seen in horizontal scans of the wake at hub height, shown at several downstream distances in Fig. 5. Of note is the underprediction of the wake depth in the near wake, which the curl model does not currently model correctly, but a near-wake correction is being developed.

Figure 5: Downstream comparison of the FLORIS curled wake model to test data.

5. Conclusions
In this paper, the curled wake model was improved and compared to the existing Gaussian model within the FLORIS software repository and respective high-fidelity SOWFA data sets. This curled wake model features counter-rotating vortices within the flow structures of yawed turbines. Recent research has shown that these vortices are important to include in turbine wake control design and investigation. This new curled wake model was shown to capture the secondary steering effects present in SOWFA simulations. The addition of vortex decay, as well as a local turbulence model to account for turbulence added by operational turbines, allowed the new curled wake model to accurately predict wake steering performance for three- and five-turbine arrays.

Development of the curled wake model is continuing at NREL, including refinement of the local turbulence model, validation of the curled wake model in predicting deep array effects, and investigation of optimized wind plant control strategies for large wind power plants leveraging the new controls-oriented curled wake model. Furthermore, comparison of the curled wake model to additional SOWFA cases, and refinement of the tuning process, are in progress, with the aim of increasing agreement between predictions of each. Accounting for the curved wake phenomenon has the potential to unlock even more performance gains through wake steering, increasing renewable energy production and decreasing the overall cost of wind energy.
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