Glueball spectroscopy on $S^3$

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Abstract: For SU(2) gauge theory on the three-sphere we implement the influence of the boundary of the fundamental domain, and in particular the $\theta$-dependence, on a subspace of low-energy modes of the gauge field. We construct a basis of functions that respect these boundary conditions and use these in a variational approximation of the spectrum of the lowest order effective hamiltonian.

1 Introduction

It is our aim to study the dynamics of the low-energy modes of pure SU(2) gauge theory defined in a finite volume \cite{1,2,3}. If we take the volume small, the mechanism of asymptotic freedom results in a small coupling constant and we can use standard perturbation theory. Gradually increasing the volume then allows us to monitor the onset of non-perturbative phenomena. Especially when the non-perturbative effects manifest themselves appreciably only in a small number of low-lying energy modes, this can be described adequately using a hamiltonian formulation. We are interested in the influence of the multiple vacuum structure of the theory on the glueball spectrum: in particular we would like to see the dependence of the energies on the $\theta$-angle which corresponds to the transition over instanton barriers.

In our approach, we impose the Coulomb gauge by restricting the gauge fields to a so-called fundamental domain \cite{4,5}. This is a convex subset of the space of all gauge field configurations that is in one-to-one correspondence (modulo constant gauge transformations) with the space of gauge orbits. The latter is precisely the physical configuration space on which we want to study the dynamics.

Our strategy will be to split up the gauge field in orthogonal modes and to reduce the dynamics of this infinite number of degrees of freedom to a quantum mechanical problem with a finite number of modes. We do this by taking standard harmonic oscillator wave functions for the high-energy modes of the field. For the low-energy modes, the onset of non-perturbative effects means that the wave functional starts to spread out over configuration

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space and will, in particular, become sensitive to conditions imposed on the boundary of
the fundamental domain. We therefore replace the wave functionals for these modes by
functions that satisfy the (θ-dependent) boundary conditions.

The dynamics of these modes can be regarded as an effective low-energy theory. Similar
to a Born-Oppenheimer approximation we integrate out the fast (high-energy) modes and
are left with an effective theory of the slow modes. If we are at energies at which the higher
modes cannot be excited, these modes will, through virtual processes, in first order only
result in a renormalisation of the coupling constant.

To make contact with lattice calculations it would be most natural to take the finite
(spatial) volume to be a 3-torus \( T^3 \). Detailed knowledge of the vacuum structure in this
case is however limited. In particular the instantons, which are the gauge field configu-
rations that describe tunnelling between different vacua, are only known numerically \([6]\).
To circumvent this problem we take our space to be the three-sphere \( S^3 \) on which the
instantons are known analytically.

Previous studies of gauge theory on \( S^3 \) \([7]\) did not use conditions at the boundary of
the fundamental domain, but focussed on the influence of the Gribov horizon: rescaling
the wave function with the square root of the Faddeev-Popov determinant results in a
potential with an infinite barrier at the Gribov horizon.

In this paper, we first construct suitable boundary conditions on the space of low-energy
modes and then give a basis of functions that respect these boundary conditions. Using
this basis we perform a Rayleigh-Ritz analysis to approximate the spectrum of the lowest
order effective Hamiltonian. The obtained energy differences give a first hint at the values
of the glueball masses in the different (scalar, tensor) sectors.

## 2 The Effective Theory

We will briefly review the formalism developed in \([2,3]\) for gauge fields on \( S^3 \) and construct
the effective Hamiltonian. We embed \( S^3 \) in \( \mathbb{R}^4 \) by considering the unit sphere parametrized
by a unit vector \( n_\mu \). Using the scale-invariance of the classical Hamiltonian, we can make
the restriction to a sphere of radius \( R = 1 \). \( R \)-dependence can be reinstated on dimensional
grounds. We introduce the unit quaternions \( \sigma_\mu \) and their conjugates \( \bar{\sigma}_\mu = \sigma_\mu^\dagger \) by
\[
\sigma_\mu = (id, i\vec{\tau}), \quad \bar{\sigma}_\mu = (id, -i\vec{\tau}).
\]
They satisfy the multiplication rules
\[
\sigma_\mu \sigma_\nu = \eta_\mu^\alpha \sigma_\alpha, \quad \bar{\sigma}_\mu \sigma_\nu = \bar{\eta}_\mu^\alpha \sigma_\alpha,
\]
where we used the ’t Hooft \( \eta \) symbols \([3]\), generalised slightly to include a component
symmetric in \( \mu \) and \( \nu \) for \( \alpha = 0 \). We can use \( \eta \) and \( \bar{\eta} \) to define orthonormal framings of \( S^3 \),
which were motivated by the particularly simple form of the instanton vector potentials
in these framings. The framing for \( S^3 \) is obtained from the framing of \( \mathbb{R}^4 \) by restricting
in the following equation the four-index \( \alpha \) to a three-index \( a \) (for \( \alpha = 0 \) one obtains the
normal on \( S^3 \)):
\[
e_\mu^\alpha = \bar{\eta}_\mu^\alpha n_\nu, \quad e_\mu^\alpha = \bar{\eta}_\mu^\alpha n_\nu.
\]
A gauge field on $S^3$ ($n_\mu A_\mu = 0$) can be written with respect to either framing, e.g.,

$$A_\mu = A^i e^i_\mu = A^a e^i_\mu \sigma_a / 2$$  \hfill (4)

A gauge transformation $g$ acts as follows:

$$({_g}^A)_i = g^{-1} A_i g + g^{-1} \partial_i g = g^{-1} D_i^{(A)} g.$$  \hfill (5)

In order to isolate the lowest energy levels, we define the quadratic fluctuation operator $\mathcal{M}$ by

$$\mathcal{V}(A) = - \int_{S^3} \frac{1}{2} \text{tr} (F_{ij}^2) = \int_{S^3} \text{tr} (A_i \mathcal{M}_{ij} A_j) + O (A^3).$$  \hfill (6)

The 18 dimensional space $A(c, d)$ given by

$$A_\mu (c, d) = \left( c^a_i e^i_\mu + d^a_j e^j_\mu \right) \sigma_a / 2$$  \hfill (7)

is the eigenspace of $\mathcal{M}$ corresponding to its lowest eigenvalue 4, whereas the next eigenvalue is 9. As was explained in [2, 3], this space contains gauge copies of the vacuum $A = 0$ and the tunnelling paths inbetween these vacua. In particular, the vacuum $A_a = -\sigma_a$ ($c^a_i = -2 \delta^a_i$, $d^a_i = 0$) is a copy of $A = 0$ under the gauge transformation $g = n \cdot \sigma$ with winding number one. The tunnelling path is $c^a_i = -u \delta^a_i$, $d^a_i = 0$ with $u$ running from 0 to 2. For $u = 1$ it passes through the sphaleron, which is a saddle point of the energy functional. The vacuum $c^a_i = 0$, $d^a_i = -2 \delta^a_i$ is a copy of $A = 0$ under $g = n \cdot \sigma$. The corresponding saddle point is called ”anti-sphaleron” and is actually a copy of the sphaleron under this gauge transformation. The energy functional for these 18 modes is given by

$$\mathcal{V}(c, d) \equiv - \int_{S^3} \frac{1}{2} \text{tr} (F_{ij}^2) = \mathcal{V}(c) + \mathcal{V}(d) + \frac{2 \pi^2}{3} \left\{ (c_i^a)^2 (d_j^b)^2 - (c_i^a d_j^b)^2 \right\},$$  \hfill (8)

$$\mathcal{V}(c) = 2 \pi^2 \left\{ 2 (c_i^a)^2 + 6 \det c + \frac{1}{4} (c_i^a c_j^a)^2 \right\}.$$  \hfill (9)

The lowest order hamiltonian for these modes is

$$R \mathcal{H}(c, d) = - \frac{f}{2} \left( \frac{\partial^2}{\partial c_i^a \partial c_i^a} + \frac{\partial^2}{\partial d_i^a \partial d_i^a} \right) + \frac{1}{f} \frac{\mathcal{V}(c, d)}{2 \pi^2}$$  \hfill (10)

with $f = \frac{g(R)^2}{2 \pi^2}$ and $g(R)$ the renormalized coupling constant.

The boundary of the fundamental domain in the full $(c, d)$ space is not known, but at the energies we are interested in, the boundary conditions are relevant only at those points where the potential is low, i.e. at the sphalerons. At other boundary points the potential will be (much) higher then the energy of the wave functional. This means that the wave functional will have decayed exponentially at these points and that the precise boundary conditions should not have a big influence on the spectrum. By the same token, the precise location of the boundary in these regions is not important either. This gives us the freedom to choose tractable boundary conditions.

At the sphalerons however, the boundary conditions are fixed. Since the gauge transformation connecting the two sphalerons has winding number one, we have to set

$$\Psi(A(\text{Sph}, 0)) = e^{i \theta} \Psi(A(0, \text{Sph})), $$  \hfill (11)

thus introducing the $\theta$-angle.
3 The Rayleigh-Ritz basis

We now want to determine the spectrum of the hamiltonian (10), properly restricted to the fundamental domain. The variational basis we need must incorporate the boundary conditions but also respect the symmetries of the hamiltonian to obtain an optimal block diagonalisation.

We first turn to the boundary conditions. We define radial coordinates $r_c$ and $r_d$ by

$$r_c = [c^a_i c^a_i]^{1/2}, \quad r_d = [d^a_i d^a_i]^{1/2}. \tag{12}$$

The sphaleron has radial coordinates $(\sqrt{3}, 0)$ and angular coordinates $\hat{c}^a_i = -\delta^a_i$ (with $c^a_i = c^a_i / r_c$). It will be connected with the anti-sphaleron at $(0, \sqrt{3})$. We will restrict the $(r_c, r_d)$ plane by $r_c < \sqrt{3}, \quad r_d < \sqrt{3}$ and impose boundary conditions at the edges. This means that we will be working towards basis functions of the form $\phi(r_c, r_d) Y(\hat{c}, \hat{d})$.

As was explained earlier, for values of the coupling constant at which our approximation will be valid, only the effect of the boundary conditions at the sphaleron will be felt. By imposing boundary conditions in the $(r_c, r_d)$ plane we pair up two submanifolds, of which only the sphaleron/anti-sphaleron need belong to the boundary of the fundamental domain.

Consider the following decomposition of the full wave function

$$\Psi = \frac{1}{r_c r_d} \sum_n \psi^{(n)}(c, d) \chi^{(n)}_{c,d}(q), \tag{13}$$

where $q$ denotes all the modes orthogonal to the $c$ and $d$ modes. The functions $\chi^{(n)}_{c,d}(q)$ are chosen to be (perturbative) eigenstates of the transverse hamiltonian. In the adiabatic approximation we assume the transverse wave function to be in its ground state and we assume this ground state to decouple dynamically from the excited states. This results in a hamiltonian for $\psi = \psi^{(0)}$ given by

$$H = -\frac{f}{2} \left( \frac{\partial^2}{\partial r_c^2} + \frac{\partial^2}{\partial r_d^2} - 12 \left( \frac{1}{r_c^2} + \frac{1}{r_d^2} \right) + \frac{1}{r_c^2} \Delta_c + \frac{1}{r_d^2} \Delta_d \right) + \frac{1}{f} \frac{\mathcal{V}(c, d)}{2\pi^2} \tag{14}$$

with $\Delta_c$ the laplacian in the angular coordinates. The boundary condition on $\psi$ follows directly from (11):

$$\psi(\text{Sph}, 0) = e^{i\theta} \psi(0, \text{Sph}). \tag{15}$$

To obtain the condition on the derivative of $\psi$ we must take the residual constant gauge symmetry into account. Focussing on the sphaleron path $c^a_i = -u \delta^a_i$, we take coordinates $c = S(\tilde{\alpha}) H(u, h_i)$, with $S \in \text{SO}(3)$ and $H$ symmetric. Under a constant gauge transformation only $S$ changes: $H(u, h_i)$ forms a set of gauge invariant curvilinear coordinates. Matching along the sphaleron path across the boundary of the fundamental domain, we need to compensate for the curvature with the appropriate jacobian factor $u^{3/2}$ (details will appear elsewhere), which results in the following boundary condition on $\psi$:

$$\frac{\partial (r_c^{-5/2} \psi)}{\partial r_c}(\text{Sph}, 0) = -e^{i\theta} \frac{\partial (r_d^{-5/2} \psi)}{\partial r_d}(0, \text{Sph}). \tag{16}$$

We now turn to the symmetries of the hamiltonian. $H(c, d)$ is invariant under the trans-
**Table 1: Behaviour of the functions of the angular variables**

| Operator       | Eigenvalue       | Operator       | Eigenvalue       |
|----------------|------------------|----------------|------------------|
| $(\hat{L}_c^S)^2$ | $l_s(l_s+1)$     | $(\hat{L}_c^S)^2$ | $l_s(l_s+1)$    |
| $(\hat{L}_c^S)_3$ | $m_s$            | $(\hat{L}_d^S)^2$ | $l_s(l_s+1)$    |
| $(\hat{L}_c^R)^2$ | $l_r(l_r+1)$     | $(\hat{L}_d^S)^2$ | $l_r(l_r+1)$    |
| $(\hat{L}_c^R)_3$ | $m_r$            | $(\hat{J}^S)^2$  | $0$             |
| $\Delta_{\hat{c}}$ | $-L(L+7)$         | $\Delta_{\hat{d}}$ | $-L_1(L_1+7)$  |
| $\Delta_{\hat{d}}$ | $-L_2(L_2+7)$    |

formation $c \to ScR_1$, $d \to SdR_2$ with $S, R_1, R_2 \in SO(3)$ and under the interchange $c \leftrightarrow d$. The generators of left- and right multiplication are $\hat{L}_c^R, \hat{L}_c^S, \hat{L}_d^R$ and $\hat{L}_d^S$. The following set of operators commutes:

$$\{\mathcal{H}, \hat{J}^S, \hat{J}^R, (\hat{L}_c^R)^2, (\hat{L}_d^R)^2, (\hat{L}_c^S)^2, (\hat{L}_d^S)^2, \mathcal{P}\}.$$  \hspace{1cm} (17)

$\mathcal{P}$ is defined by $\mathcal{P} f(c, d) \equiv f(d, c)$. On $S^3$ it corresponds to the parity $(n_0, \vec{n}) \leftrightarrow (n_0, -\vec{n})$. The operator $\hat{J}^S \equiv \hat{L}_c^S + \hat{L}_d^S$ implements constant gauge transformations: we have to demand $(\hat{J}^S)^2 = 0$ for physical wave functions. The operator $\hat{J}^R \equiv \hat{L}_c^R + \hat{L}_d^R$ is the rotation operator. The different sectors under this symmetry correspond to scalar glueballs, vector glueballs, etc.

An orthonormal basis of functions of $\hat{c}$ is given by the set $\{\langle \hat{c}|L; l_s, l_r, \tau; m_s, m_r \rangle\}$. Each of these functions is a certain polynomial in $\hat{c}$, homogeneous of degree $L$. Its eigenvalues under the various symmetries are collected in table 1. A possible degeneracy is labeled by $\tau$. Using algebraic manipulation programs, we were able to construct these functions explicitly for $L \leq 10$. Using Clebsch-Gordan coefficients, we define a function $Y^{i_1i_2}(\hat{c}, \hat{d})$ which is an eigenfunction of $\hat{J}^R$ and of $\hat{J}^S$ ($i$ denotes a set $(L, l_s, l_r, \tau)$):

$$Y^{i_1i_2}(\hat{c}, \hat{d}) = \langle \hat{d}|j, m, i_1, i_2 \rangle$$

$$= \sum_{m_s=-l_s}^{l_s} \sum_{m_1=-l_1}^{l_1} \sum_{m_2=-l_2}^{l_2} (-1)^{l_1-l_2+m} \frac{1}{\sqrt{2l_s+1}} (\hat{c}|L_1; l_s, l_1, \tau_1; m_s, m_1) \langle \hat{d}|L_2; l_s, l_2, \tau_2; -m_s, m_2 \rangle \sqrt{2j+1} \left( \begin{array}{ccc} l_1 & l_2 & j \\ m_1 & m_2 & -m \end{array} \right).$$  \hspace{1cm} (18)

Note that in order to have $\hat{J}^S = 0$, the functions of $c$ and $d$ need to have the same $l_s$. This restricts the possible combinations of $i_1$ and $i_2$.

Let us focus on the case $\theta = 0$. In this case we can in principle take the basis functions to be the eigenfunctions of the kinetic part of the hamiltonian. These functions are given
by
\[ \varphi_{\gamma_1}^{(L_1)}(r_c)\varphi_{\gamma_2}^{(L_2)}(r_d)Y^{\ell_1\ell_2}(\hat{c}, \hat{d}), \]  
where \( L_k \) is determined by \( i_k \) and \( \varphi_{\gamma}(\ell_1) = \gamma r j_{\beta L_1}(\gamma r) \) with \( j_{\beta}(z) \) the spherical Bessel function of order \( \beta \). During the variational stage of the calculation, however, the use of Bessel functions of different order will lead to a large number of integrals. Therefore we take the radial functions to be independent of \( L_1 \) and \( L_2 \) and define
\[ \psi_{\gamma_1\gamma_2}^{i_1i_2}(c,d) = \varphi_{\gamma_1}(r_c)\varphi_{\gamma_2}(r_d)Y^{i_1i_2}(\hat{c}, \hat{d}), \]  
with \( \varphi_{\gamma}(r) = \gamma r j_{\beta}(\gamma r) \). The functions \( \psi_{\gamma_1\gamma_2}^{i_1i_2}(c,d) \) now have singularities at certain points, but these are not felt during the variational calculation. Taking even and odd combinations gives
\[ \psi_{\gamma_1\gamma_2}^{\alpha}(c,d) = \psi_{\gamma_1\gamma_2}^{i_1i_2}(c,d) + p \psi_{\gamma_1\gamma_2}^{i_1i_2}(d,c) \]  
We implement the boundary conditions (15) and (16) for \( \theta = 0 \) by imposing the following conditions on \( \gamma_1 \) and \( \gamma_2 \):
\[ p = -1 : \varphi_{\gamma_1}(\sqrt{3}) = \varphi_{\gamma_2}(\sqrt{3}) = 0, \]  
\[ p = 1 : \frac{\partial (r^{-5/2} \varphi_{\gamma_1})}{\partial r}(\sqrt{3}) = \frac{\partial (r^{-5/2} \varphi_{\gamma_2})}{\partial r}(\sqrt{3}) = 0. \]  
These conditions are expected to be accurate as long as the wave function transverse to the sphaleron path (near the sphalerons) is predominantly in its ground state. For general \( \theta \) we multiply \( \psi_{\gamma_1\gamma_2}^{i_1i_2}(c,d) \) with a phase factor \( \exp(i\theta \alpha(r_c, r_d)) \). The function \( \alpha \) is a kind of Cherns-Simons functional that gives the right behaviour to the wave function under large gauge transformations. The resulting functions no longer have well-defined parity, but they do obey the general boundary conditions for suitable \( \alpha \). Also the hermiticity of the hamiltonian for these functions can be checked explicitly. Sufficient conditions on \( \alpha \) are: \( \alpha(r_c, r_d) = -\alpha(r_d, r_c) \) and \( \alpha(\sqrt{3}, 0) = \frac{1}{2} \). We choose
\[ \alpha(r_c, r_d) = \frac{1}{2} \left( \left( \frac{r_c}{\sqrt{3}} \right)^{\frac{\beta}{2}} - \left( \frac{r_d}{\sqrt{3}} \right)^{\frac{\beta}{2}} \right). \]  
For \( \beta \to \infty \) we approach the situation that the phase factor over the entire edge is constant and equal to \( e^{i\theta} \). But already for the choice \( \beta = 2 \), the boundary conditions at the sphalerons are taken into account properly.

4 The Rayleigh-Ritz analysis

To perform the Rayleigh-Ritz analysis, we take a finite subset of the basis constructed above. We then calculate the matrix of \( \mathcal{H} \) with respect to this truncated basis and diagonalise. The eigenvalues found are upper bounds for the true eigenvalues. In order to get lower bounds, we use Temple’s inequality \[9\] which as input requires the expectation values of \( \mathcal{H} \) for the variational eigenvectors. For the calculation of the matrix elements we essentially need the radial integrals
\[ \int_0^{\sqrt{3}} dr \ r^n \varphi_{\gamma'}(r) \varphi_{\gamma}(r) \]  
\[ (25) \]
with $\gamma$ and $\gamma'$ given by the boundary conditions (23). The angular sector is more difficult: especially $V^2$ contains some non-trivial angular operators. By virtue of the construction (18) all the angular integrations over $\hat{c}$ and $\hat{d}$ can be reduced to integrations over just $\hat{c}$. Both the radial and the angular integrals were tabulated and stored for later use in a Fortran program.

Due to the symmetries, the function space splits up in different sectors characterized by the conserved quantum numbers $j$, $m$, $l_1(l_1+1) + l_2(l_2+1)$ and, for $\theta = 0$, $p$. It is an amusing fact that the only influence of the conserved numbers $j$ and $m$ on the spectrum is through the number $(-1)^j$ and through the fact that $l_1$, $l_2$ and $j$ have to satisfy a triangle inequality. If we denote a sector with a triple $(l_1, l_2, j)$, this for instance means that the sectors $(1, 1, 0)$ and $(1, 1, 2)$ are degenerate.

We found that for the window of coupling constants for which we trust our approximation, the lowest-lying scalar ($j = 0$) and tensor ($j = 2$) levels are found in respectively the sectors $(0, 0, 0)$ and $(1, 1, 2)$. See figures 1 and 2. For $\theta = 0$, the vacuum corresponds to the ground state of the $(0, 0, 0)$-even sector. The scalar glueball $0^+$ can be identified with the first excited state in the $(0, 0, 0)$-even sector. Note that for small $f$ the lowest level in the $(0, 0, 0)$-odd sector $0^-$ has actually a lower energy than the scalar glueball. For small $f$, there is virtually no dependence of the masses on $\theta$, whereas for larger $f$ the $\theta$-dependence is shown in figure 4.

5 Conclusions

Using the Rayleigh-Ritz method we can determine the spectrum of the effective hamiltonian. The use of Temple’s inequality gives us confidence that our results are accurate, especially since experience tells us that the actual error is usually much smaller. The results are also consistent with an ordinary perturbative calculation of the levels that was done up to $O(f^3)$. When we make a preliminary comparison with results on the torus, we find that the mass ratio $m_{2+}/m_{0+}$ comes out rather nice: for a volume of 1 fm$^3$, corresponding roughly to $f = 0.4$, we obtain the value 1.5.

Our model is valid for values of the coupling constant at which only the boundary conditions at and near the sphalerons are felt. Checking this a posteriori with the help of plots of the wave function in the $(r_c, r_d)$ plane indicates that $f$ should not be larger than roughly 0.4. Consider the function

$$|\psi(r_c, r_d)|^2 = \int d\hat{c} d\hat{d} |\psi(c, d)|^2$$

which is a measure of the probability distribution in the $(r_c, r_d)$ plane. Dividing this $|\psi(r_c, r_d)|$ by $r^4_c r^4_d$, we obtain a function with the expected behaviour of the true wave function: it is localised at the sphalerons and decays exponentially in the transverse directions. Although the lowest barrier is the sphaleron, the volume factor $r^4_c r^4_d$ causes the configurations that are close to the sphaleron but have a somewhat higher energy to make the dominant contribution to the tunnelling. The relevant parameter here is the characteristic decay length of the wave function, which in turn is determined by the rise of the potential in the transverse directions.

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Although we are primarily interested in the case \( \theta = 0 \), an appreciable dependence on \( \theta \) for a certain value of \( f \) shows that the non-perturbative influence of the boundary has become important. To explain the fact that the spectrum is not exactly periodic in \( \theta \), note that our implementation of the \( \theta \)-dependence \( (24) \) only has this periodicity in the limit \( \beta \to \infty \). The volume effect described above implies that the relevant distribution \( |\psi(r_c, r_d)| \) samples a piece of the boundary over which the phase difference already starts to depart from \( e^{i\theta} \).

The first improvement that has to be made is to take the higher order corrections of the other modes into account. The leading order taken here gives rise to a renormalisation of the coupling constant. Higher order corrections may alter the potential in our effective model and can for instance result in a different degree of localisation around the sphalerons. We hope to report on these matters in the near future.

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Figure 1: Lowest energy levels for $\theta = 0$. Drawn curves correspond to levels in the $(0, 0, 0)$ sector. The dashed curve denotes the ground level in the $(1, 1, 2)$-even sector. The short-dashed curves are the perturbative expansions, and the individual dots are lower bounds on the levels as obtained by Temple's inequality.
Figure 2: Glueball masses for $\theta = 0$ as a function of the coupling constant. The drawn curves are the masses of resp. the first scalar ($0^+$) and tensor ($2^+$) glueball. The dashed lines denote the lower bounds, the dotted lines the perturbative result.
Figure 3: Scalar glueball mass at $f = 0.4$ as a function of $\theta$. 