NON-DEBYE SCREENING IN HIGH TEMPERATURE QCD?

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Abstract

We study the non-abelian, gauge invariant, screening potential between two charges in a quark-gluon plasma. Using Braaten and Pisarski resummed perturbation theory in temporal axial gauge, we find a repulsive power screened potential which behaves as \( \sim 1/r^6 \) at large distance. This questions either the physical meaning of the Debye mass or the known pragmatic treatment of the temporal axial gauge propagator.

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1 Introduction

Charges in a medium are understood to be screened. At large distance, this screening is usually taken to be an exponential with the screening length given by the inverse of the Debye mass $m$. In QED, according to linear response theory, this screening is related to the two-point function of the photon. This is due to the field strength being linear in the vector potential. The small momentum behaviour of this function gives rise to imaginary poles which leads to the exponentially screened potential, from which the Debye mass derives its physical significance of inverse screening length.

In the last few years, the plasmon puzzle or the gauge dependence of the gluon damping constant has made it clear that it is necessary to employ resummed perturbation theory of Braaten and Pisarski whenever momenta much less than the temperature $T$ of the system under study are involved. Since the long distance behaviour of the screening potential and hence the two-point function is required, it is necessary to use this resummed theory.

Unlike QED, in QCD only the leading term of the static time-time component of the gluon self energy, i.e. the Debye mass squared, calculated in the framework of this resummed theory, is gauge independent. Therefore it is not clear that the Debye mass is a physical quantity in the non-abelian theory. By working out the leading correction to the Debye mass in the resummed theory, Rebhan argued that in covariant gauge, it was possible to have a gauge parameter independent and hence gauge independent Debye mass by defining

$$m^2 = \lim_{q^2 \to -m^2} \Pi_{00}(0, q),$$

instead of

$$m^2 = \lim_{q \to 0} \Pi_{00}(0, q).$$

That is one obtains $m$ self-consistently from the pole of the propagator.

The leading correction to this gauge independent Debye mass was found to be

$$\delta m^2 \sim gm^2 \ln(1/g).$$

The $\ln(1/g)$ is due to the introduction of the magnetic mass as a infrared cut off in the transverse propagator. As a result, the correction is rather large. The new definition Eq. (1) is possible because the gauge parameter dependent term in the leading correction in covariant gauge is proportional to $q^2 + m^2$. However, it is not clear why $m$ found this way in a gauge such as covariant gauge can be readily labelled as the Debye mass: the latter is generally understood to be the inverse screening length in a Yukawa type screening potential between charges and in covariant gauge, the potential is related to more than

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2 In the bare theory, in addition to the gauge invariant leading term, there is a linear term in the modulus of the external momentum which is also gauge independent.
just the longitudinal propagator. On the other hand, the temporal axial gauge (TAG) is the only gauge in which we have this simplification. Baier and Kalashnikov [7] have recently shown, by some unique construction, that the gauge-invariant definition of the Debye mass might not extend to TAG: instead of being exponentially screened, the potential is repulsive with power screening $\sim 1/r^6$ in TAG[4]. It then seems appropriate to investigate this problem in TAG in the framework of Braaten and Pisarski resummed theory.

The choice of TAG at finite temperature may be controversial, because it is known to have problems with the double pole $1/p_0^2$ when $p_0$ is zero. Nevertheless, TAG has the advantage that one can relate unambiguously the potential between two charges to the two-point function. Also it has been shown in non-resummed theory [8] that the same result for $\Pi_{00}(0, q \to 0)$ could be obtained either in Coulomb gauge or in TAG. Basing on this, we will adopt here a pragmatic attitude and use the ansatz Eq. (32). It will be further explained in section 4. We use imaginary time formalism throughout.

2 Linear Response Theory

The potential of a charge $Q_1$ at $x_1$ in the presence of another charge $Q_2$ at $x_2$ is given by [3, 4]

$$V(r) = \frac{1}{2} \sum_a \int d^3x \left( \mathcal{E}_1^{a \text{eff}}(x) \cdot \mathcal{E}_2^a(x) + \mathcal{E}_2^{a \text{eff}}(x) \cdot \mathcal{E}_1^a(x) \right)$$

(4)

where the effective field $\mathcal{E}_1^{a \text{eff}}$ created by the non-abelian charge $Q_1^a$ is the sum of the applied field $\mathcal{E}_1^a$ and the induced field $\delta \langle E_1^a \rangle$:

$$\mathcal{E}_1^{a \text{eff}}(x) = \mathcal{E}_1^a(x) + \delta \langle E_1^a \rangle$$

(5)

$\mathcal{E}_1^a$ is solution of the Gauss’ law:

$$\nabla \cdot \mathcal{E}^a - g f^{abc} A^b \cdot \mathcal{A}^c = Q_1^a \delta^3(x - x_1),$$

(6)

where $\mathcal{A}$ is the vector potential associated with $\mathcal{E}$.

Eq. (4) is manifestly gauge invariant since a trace is being taken over tensor fields (or equivalently colour is being summed over) and is therefore a physical potential.

When we put the charge $Q_1^a$ in the plasma, the electric field $E_1^a(x, t)$ in the medium is coupled to the applied field $\mathcal{E}_1^a$ through the coupling Hamiltonian:

$$H^{\text{ext}}(t) = \int d^3x E_1^a(x, t) \cdot \mathcal{E}_1^a(x).$$

(7)

$^3$Another example of non-Debye screening with a $1/r^{10}$ behaviour in the charge-charge correlation has been found in [4]
From linear response theory, the induced field is
\[
\delta \langle E_i^a(x, t) \rangle = i \int_{-\infty}^{t} dt' \langle [H^{ext}(t'), E_i^a(x, t)] \rangle \\
= i \int_{-\infty}^{t} dt' \int d^3x' \mathcal{E}_j^b(x') \langle [E_j^b(x'), E_i^a(x, t)] \rangle.
\]
(8)

In TAG,
\[
E_i^a(x, t) = -\partial_0 A_i^a(x, t),
\]
(9)
the correlator of electric fields \( \langle [E, E] \rangle \) can be easily related to the retarded gluon propagator and the Gauss’ law satisfied by the applied field in the presence of a static charge \( Q_1^a \), is simply
\[
\nabla \cdot \mathcal{E}_i^a = Q_1^a \delta^3(x - x_1),
\]
(10)
where the non-abelian term of Eq. (8) vanishes for a static \( \mathcal{E} \) field. The solution is
\[
\mathcal{E}_i^a(x) = -iQ_1^a \int \frac{d^3P}{(2\pi)^3} e^{iP \cdot (x - x_1)} \frac{P_i}{P^2}.
\]
(11)

In gauges other than TAG, Gauss’ law itself is gauge parameter dependent in a parametrized gauge. The non-abelian part of Eq. (8) does not vanish, and thus the solution \( \mathcal{E}_i^a(x) \) is not of the same simple form as above and is not necessarily gauge parameter independent. In parametrized Coulomb gauge, although Gauss’ law is not gauge parameter dependent due to \( A^0 \) independence of the gauge fixing term, the solution is not of the simple form of Eq. (11) because the non-abelian term does not drop out. Therefore \( V(r) \) is not simply related only to \( \langle [E_L, E_L] \rangle \) as in TAG. So that the gauge parameter dependence of \( \langle [E_L, E_L] \rangle \) shown in [11, 20] does not mean that \( V(r) \) is gauge dependent. On the contrary, from the definition in Eq. (4), \( V(r) \) is gauge independent.

In TAG, as a consequence of Eq. (8), (9), (11), \( V(r) \) is directly related to the longitudinal propagator so we have
\[
V(r) = Q_1 \cdot Q_2 \int \frac{d^3q}{(2\pi)^3} \frac{\exp(iqr)}{q^2 + \Pi_{00}(0, q)}
\]
with \( r = x_1 - x_2 \). We stress that in other gauges, this advantage of TAG is gone and \( V(r) \) is related to more than just the two-point function [1].

This is the form of Gauss’ law in any gauge only at leading order.

\[\text{It has been argued [12] that in the presence of only one static colour charge, the response of the vector potential } \langle A \rangle \text{ must point only in that direction in colour space, so that a non-abelian term like } \langle A \rangle \land \langle A \rangle \text{ vanishes. For this reason, the response field } \langle E \rangle \text{ and Gauss’ law both become abelian. Using this line of argument Rebhan [4, 5] argued that the potential of a static colour charge is given by a form similar to Eq. (12). This is actually flawed because the response field } \langle E \rangle \text{ cannot be expressed directly in terms of the response vector field } \langle A \rangle. \text{ The response field is actually } \langle E \rangle = \langle \partial A \rangle - \langle \partial A \rangle + \langle A \land \rangle, \text{ i.e. the non-abelian term should be the average of the cross product and not the cross product of the average, so it does not vanish. Likewise, the non-abelian term in Gauss’ law cannot be zero.}\]
After going through the usual steps, we get for the colour singlet quark-antiquark potential [3],

\[ V(r) = -\frac{N^2 - 1}{2N} \frac{g^2(T)}{2\pi^2 r} \int_0^\infty \frac{q dq \sin qr}{q^2 + \Pi_{00}(0, q)} . \]  

(13)

From now on, \( \Pi_{00} \) is the temperature dependent part of \( \Pi_{00} \).

If \( \Pi_{00}(0, q) \) is not even in \( q \) as found in [4, 13], we have to evaluate the integral by closing the contour in the first quadrant of the complex \( q \) plane. Contributions from possible poles or branch cuts in the first quadrant away from the real axis are exponentially suppressed as \( r \to \infty \). Eq. (13) can be rewritten as

\[ V(r \gg m^{-1}) \approx -\frac{N^2 - 1}{2N} \frac{g^2(T)}{2\pi^2 r} \text{Im} \int_0^\infty \frac{z dq e^{-rz}}{-z^2 + \Pi_{00}(0, q = iz)} . \]  

(14)

Eq. (14) shows that we only need the small \( q \) limit of \( \Pi_{00}(0, q) \).

3 Braaten and Pisarski Resummation in Temporal Axial Gauge

In TAG, the gluon propagator is

\[
D_{00}(p) = 0, \quad D_{ij}(p) = 0, \quad D_{0i}(p) = 0
\]

\[
D_{ij}(p) = -\frac{1}{p^2 - G(p)} \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) - \frac{1}{p^2 - F(p)} \frac{p_i p_j}{p^2} \]  

(15)

from which the effective or resummed propagator \( *D_{\mu\nu}(p) \) can be constructed by replacing \( G(p) \) and \( F(p) \) with the corresponding parts of the hard thermal loop (HTL) of the gluon self energy, the latter being given by [14]

\[
\delta \Pi_{\mu\nu}^{ab}(p) = -8N\delta^{ab}(2\pi)^{-3}
(f_{\mu\nu} - 4\pi\delta_{\mu\nu}\delta_{0\nu}) \frac{\pi^2 g^2 T^2}{12}
\]  

(16)

where

\[
f_{\mu\nu}(p) = 4\pi \int \frac{d\Omega}{4\pi} \frac{p_0 \hat{Q}_\mu \hat{Q}_\nu}{p \cdot \hat{Q}} ,
\]  

(17)

and \( \hat{Q} = (1, \hat{Q}) \). From this, with the Debye mass \( m^2 = g^2 T^2 N/3 \), we have for the effective \( *G \) and \( *F \) quantities

\[
\delta \Pi_{00} = -\frac{p^2}{p^2} *F(p) = m^2 (1 - I(p)) ,
\]  

(18)
with
\[ I(p) = \int \frac{d\Omega}{4\pi} \frac{p_0}{p \cdot \hat{Q}} , \]  
(19)  
and
\[ ^*G(p) = \frac{1}{2} \left( \delta \Pi_{\mu}^\mu(p) + \frac{p^2}{p^2} \delta \Pi_{00}(p) \right) \]
\[ = \frac{m^2}{2} \left( \frac{p_0^2}{p^2} - \frac{p^2}{p^2} I(p) \right) . \]  
(20)  
Then \(^*D_{ij}\) is
\[ ^*D_{ij}(p) = -\frac{1}{p^2 - ^*G(p)} \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) - \frac{1}{p^2 + m^2(1 - I(p))} \frac{p_i p_j}{p^2} . \]  
(21)  
To calculate the gluon self-energy, with two Feynman diagrams, we need both the 3-gluon and the 4-gluon effective vertices. From \[ [2, 14]\],
\[ ^*\Gamma_{abc}^0 ij(q, -p - q, p) = 2 g i f^{abc} \left( p_0 g_{ij} - m^2 \int \frac{d\Omega}{4\pi} \frac{p_0 \hat{Q}_i \hat{Q}_j}{p \cdot \hat{Q} (p + q) \cdot \hat{Q}} \right) . \]  
(22)  
Here we have set the external energy, \( q_0 \), to zero for the static self-energy. Because of Eq. (15), this is the only required 3-gluon effective vertex component. For the 4-gluon effective vertex, the six permutations of the gluon legs of the 4-gluon hard thermal loop can be reduced to just a sum of two terms using identities given in \[ [14]\]. For static self-energy and for momentum configuration of the tadpole, these two terms give identical contributions so for our purpose, the effective 4-gluon vertex is
\[ \Gamma_{00ij}^{ab}(p, -p, q, -q) = (i g)^2 f^{abc} f^{bcd} \left( -2 g_{ij} + 4m^2 \int \frac{d\Omega}{4\pi} \frac{p_0 \hat{Q}_i \hat{Q}_j}{(p \cdot \hat{Q})^2 (p + q) \cdot \hat{Q}} \right) . \]  
(23)  

4 Leading Momentum Dependence of \( \Pi_{00}(0, q) \)

The leading term in the static self-energy is well known and is the Debye mass squared. To find the leading correction and the momentum dependent part, it is sufficient to consider only the zero mode in the internal loop energy \[ [3]\] of the self-energy. This is easy to see by power counting of soft momenta. Take for example the gluon self-energy graph with two 3-gluon vertices. The usual thermal propagator in Euclidean space
\[ \frac{1}{(2\pi nT)^2 + p^2} \]  
(24)
means that for soft momentum and in the zero energy mode it goes as $1/(gT)^2$ while for non-zero modes it is $1/T^2$. For a soft loop self-energy graph, we then have $1/(gT)^4$ for two zero mode propagators, $(gT)^2$ for momenta from the two vertices, $(gT)^4$ from the loop 4-momentum, $g^2$ from the vertices and $1/g$ from the distribution function. This yields $g(gT)^2$, which is $g$ times the leading term. Similar soft power counting can be done to establish that there is a contribution of the same order from the tadpole graph.

For hard loop, there appears to be $g(gT)^2$ terms left over from the leading hard thermal loop $(gT)^2$ terms but they are proportional to $q \cdot \hat{Q}/|Q|$, where $Q$ is the hard internal loop momentum vector, and so vanish by virtue of the angular integration.

With the effective quantities of the previous section, the leading correction is

$$\Delta \Pi_{00}^{ab}(0, \mathbf{q}) = \left\{ \Delta \Pi_{00}^{3g\, ab}(0, \mathbf{q}) + \Delta \Pi_{00}^{4g\, ab}(0, \mathbf{q}) \right\}_{\text{zero mode}},$$

with

$$\Delta \Pi_{00}^{3g\, ab}(0, \mathbf{q}) = \frac{1}{2} T \sum_{p_0} \int_{\text{soft}} \frac{d^3p}{(2\pi)^3} \, T_{0ij}^{acd}(q, -p-q, p) \cdot D_{jik}(p) \cdot T_{0lj}^{bcd}(-q, -p, p+q) \cdot D_{lji}(p+q),$$

$$\Delta \Pi_{00}^{4g\, ab}(0, \mathbf{q}) = \frac{1}{2} T \sum_{p_0} \int_{\text{soft}} \frac{d^3p}{(2\pi)^3} \, T_{0ij}^{ab}(p, -p, q, -q) \cdot D_{ij}(p).$$

As mentioned in the end of section 2, we need only the small $\mathbf{q}$ limit of the static self-energy when looking for a possible odd term in $|\mathbf{q}|$ [7, 13]. This simplifies our problem considerably. In fact we can drop the bare vertex part of Eq. (23) because its contribution is independent of $\mathbf{q}$.

We note that since, on the one hand, the effective vertex Eq. (22) and the hard thermal loop part of Eq. (23) are proportional to $p_0$ and, on the other hand, the transverse part of the gluon propagator has no singular factor in $p_0$, the zero mode part of the transverse-transverse gluon contribution of Eq. (26) vanishes. The same is true for the transverse gluon contribution of Eq. (27).

For the transverse-longitudinal gluon contribution, it turns out that, in the zero mode, the bare-HTL vertex and the HTL-HTL vertex combinations are also zero because

$$\int \frac{d\Omega}{4\pi} \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) \frac{(p+q)_i \hat{Q}_j}{p \cdot \hat{Q}} = 0,$$

and

$$\int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} \left( \frac{\hat{Q}_1 \cdot \hat{Q}_2}{p \cdot \hat{Q}_1 p \cdot \hat{Q}_2} - \frac{1}{p^2} \right) = 0,$$

where we integrate over the directions of $\hat{Q}_i$ through $d\Omega_i$, $i = 1, 2$. The remaining bare-bare vertex combination does not give any odd power of $|\mathbf{q}|$: the integral in $\mathbf{p}$ reads

$$\int \frac{d^3p}{(2\pi)^3} \left( p \cdot (p + \mathbf{q})^2 - p^2 (p + \mathbf{q})^2 \right) \frac{1}{(p^2)^2 ((p + \mathbf{q})^2 + m^2)}. $$

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Because the denominator of Eq. (30) is protected by the mass from infra-red divergence, we can expand at small $|\mathbf{q}| < m$. It is clear that odd powers of $|\mathbf{q}|$ always come with odd powers of $\cos \theta$, where $\theta$ is the angle between $\mathbf{q}$ and $\mathbf{p}$, therefore no odd power of $|\mathbf{q}|$ appears in the small $|\mathbf{q}|$ limit of the longitudinal-transverse contribution.

Before discussing the longitudinal-longitudinal contribution, it must be mentioned that the Matsubara frequency sum is now ill defined for certain terms because of the presence of two longitudinal propagators: each has a $1/p_0^2$ factor and we have only a factor of $p_0$ from each vertex. Therefore terms which do not have any extra factor of $p_0^i, i \geq 2$ in the numerator are singular.

In these cases, as was done in [6, 8], we will perform the frequency sums by contour integrals using the usual equation [1]:

$$T \sum_n f(p_0) = \frac{1}{2\pi i} \left\{ \int_{i\infty}^{i\infty} dz f(z) + \int_{-i\infty}^{-i\infty+\epsilon} dz \left( f(z) + f(-z) \right) \frac{1}{e^{z/T} - 1} \right\}$$  

(31)

This equation is valid only if $f(p_0)$ has no singularities on the imaginary axis. This is not the case if $f(p_0)$ contains $1/p_0^2$ or $1/p_0$ factors. We therefore employ the usual ansatz, namely, that the sum is still given by the above integral. Thus the $1/p_0$ and $1/p_0^2$ poles, if existing on their own with no other poles, are vanishing.

$$T \sum 1/p_0 = 0, \quad T \sum 1/p_0^2 = 0.$$  

(32)

Let us point out that if one takes the finite T axial gauge propagator directly from zero T (i.e. that is one ignores the problem with periodicity of the longitudinal propagator at finite T [16], see comment at the end of section [5]) then this is the only known way at finite T to treat the $1/p_0^2$ singularities. The alternative suggested by Leibbrandt and Staley [17] amounts to performing the uniform replacement

$$\frac{1}{p_0} \to \lim_{\epsilon \to 0} \frac{p_0}{p_0^2 + i\epsilon}, \quad \epsilon > 0$$

$$\frac{1}{p_0^2} \to \lim_{\epsilon \to 0} \frac{p_0^2}{(p_0^2 + i\epsilon)^2}, \quad \epsilon > 0$$  

(33)

and dropping the zero mode.

The only justification of this prescription is that it leads to the correct answer for the leading order Debye mass. This is however not surprising since this calculation is only sensitive to hard loop momenta. Furthermore, it is known that due to the plasmon effect [4], the leading correction to $m^2$ starts at $O(g^3)$ and not at $O(g^4)$ as is expected in naive perturbation theory. We know by using soft power counting that the $O(g^3)$ correction comes only from the zero mode, therefore we argue that any prescription which removes the zero mode cannot be the correct one.
We calculate the longitudinal-longitudinal contribution by expanding the longitudinal propagator, making use of \( m^2/(p^2 + m^2) < 1 \). We obtain from Eq. (21) the expanded form

\[
*D_{Lij}(p) = -\frac{p_ip_j}{p^2 + m^2} \left( \frac{1}{p_0^2} \right) \sum_{n=0}^{\infty} \left( \frac{m^2I(p)}{p^2 + m^2} \right)^n. \tag{34}
\]

The longitudinal-longitudinal contribution is then

\[
\Delta \Pi_{00\,LL}(0, q) = 2g^2NT\delta_{ab} \int \frac{d^3p}{(2\pi)^3(p^2 + m^2)((p + q)^2 + m^2)} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{p_0}^{\infty} \frac{1}{p_0^2} \left( \frac{m^2I(p)}{p^2 + m^2} \right)^{n_1} \left( \frac{m^2I(p + q)}{(p + q)^2 + m^2} \right)^{n_2} \left\{ p \cdot (p + q) + m^2 - m^2(I(p) + I(p + q)) + m^2p_0^2J(p, p + q) \right\}^2 \tag{35}
\]

where \( I(p) \) is given in Eq. (13) and \( J(p, q) \) is

\[
J(p, q) = \int \frac{d\Omega}{4\pi} \frac{1}{p \cdot q \cdot q}. \tag{36}
\]

(for more details of this function, see [14]). As the function \( I(p) \) is proportional to \( p_0 \), all frequency sums in Eq. (35) with \( n_1 + n_2 > 2 \) have vanishing zero mode contributions. We are left with the following terms in Eq. (35):

- one frequency sum with a singular zero mode proportional to \( 1/p_0^2 \), which vanishes according to the ansatz Eq. (22).

- frequency sums with zero mode proportional to \( 1/p_0 \). These are performed using Eq. (31) and Eq. (32). The Bose-Einstein distribution function is approximated \( N(p) \simeq T/p \) for \( p << T \), in order to extract the leading contributions. These, for the same reason as Eq. (30), yield no odd power of \( q \) in the \( p \)-integrals.

- frequency sums with well-defined zero modes: we perform the angular integrals first and then take the zero mode. The only odd terms in \( |q| \) originate from the product \( I(p)I(p + q) \) proportional to

\[
\int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} \frac{1}{p \cdot q \cdot q}$

\[
= \frac{\ln \left( \frac{p_0 + |p|}{p_0 - |p|} \right) \ln \left( \frac{p_0 + |p + q|}{p_0 - |p + q|} \right)}{4|p||p + q|} \rightarrow -\frac{\pi^2}{4} \frac{1}{|p||p + q|}. \tag{37}
\]

Collecting the latters and adding an analogous contribution coming from the longitudinal part of the tadpole diagram Eq. (27), we get

\[
-\frac{g^2NTm^4\pi^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{|p||p + q|(p^2 + m^2)^2(p^2 + m^2)^2} \times \left( \frac{q^2}{2} + |q|^2 - (q \cdot p)^2 \right). \tag{38}
\]
From this we find no linear term but a cubic term in $|q|$ which is $g^2NT|q|^3/8m^2$. So that, in the small $|q|$ limit, we get the following behaviour for $\Pi_{00}$:

$$\lim_{|q| \to 0} \Pi_{00}(0,q) = m^2 + \alpha g^2T|q| + \beta g^2TNq^2 + \gamma g^2TN|q|^3/m^2 + O(|q|^4)$$

(39)

where $\alpha$, $\beta$ and $\gamma$ are numerical factors with $\alpha = 0$, $\gamma = 1/8$. The factor $\beta$ is not as important in front of the $q^2$ term already present in the potential Eq. (13) since it is smaller by a factor of $g$. Had the factor Eq. (37) not been present, no odd $|q|$ term would have appeared. The positive cubic term in Eq. (39) is responsible for the algebraic decrease of the potential $V(r)$ of Eq. (14):

$$V(r \to \infty) \simeq \frac{N^2 - 1}{4\pi^2} \frac{3g^4(T)T}{(rm)^6}$$

(40)

which behaves as $\sim 1/r^6$ in agreement with [7]. The presence of this cubic term means that the screening potential is no longer dominated by a pole but by some cut starting at $|q| = 0$. On a physical level, the self-energy in TAG is very different from that in covariant gauge: we get a longitudinal-longitudinal contribution whereas in covariant gauge, this contribution is zero. We find that we have no need to include the magnetic mass since transverse gluon does not yield any odd term in the zero mode.

5 Discussion and Conclusion

In this paper, using Braaten and Pisarski resummed perturbation theory we study the leading correction to the Debye mass in TAG. Indeed, it is the only gauge where the potential between two charges is unambiguously related to the two-point function. We find that provided there is no branch cut touching the real axis in the first quadrant of the complex $q$ plane, one only requires the time-time component of the gluon self-energy at small $q$. This gives a cubic term which yields a repulsive power screening potential for the non-abelian theory. This is in direct contrast with the accepted notion of Debye mass. Nevertheless, one has to be careful in trusting such a result. We have merely looked at the small $|q|$ behaviour of $\Pi_{00}(0,q)$ for the dominant behaviour of the potential. We expect the same pole as found by Rebhan [3, 4] to be present in this gauge due to the theorem [18] that the poles of the propagator are gauge invariant. But in this case, the behaviour of the potential is not dominated by this pole and therefore it could not be interpreted as the Debye mass.

Attempts to understand the difference of our result with other gauges may lead one to question the use of the ansatz summarised in Eq. (32) which is known to be true up to subleading order in the non-resummed theory [8] and is untested at higher order. However, notice that the crucial cubic term in $\Pi_{00}$ comes from frequency sum with well-defined zero mode terms.
Also, James and Landshoff [16] have pointed out that the TAG imaginary time longitudinal propagator should not be periodic in time as in the usual Matsubara framework used in this paper: the requirement to include only physical states in the thermal average breaks periodicity. They have initiated a rather complex formalism appropriate to this gauge based on this. Their new longitudinal propagator has the advantage that it is free of \( p_0 = 0 \) pole and therefore one does not have to use any pragmatic ansatz. Calculating the correction to the Debye mass in this formalism is necessary to confirm our result.

After finishing this work, we noticed Braaten and Nieto [19] and Rebhan [20] have extracted the Debye mass from the gauge invariant correlator of two Polyakov loops. They found the expected exponential behaviour of the potential and no power screening. We do not know exactly where is the source of this disagreement. It could be due to one of the problems that we have already discussed above. We leave this as further investigation.

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