Detection of damage in welded structure using experimental modal data

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Abstract. A typical automotive structure could contain thousands of spot weld joints that contribute significantly to the vehicle’s structural stiffness and dynamic characteristics. However, some of these joints may be imperfect or even absent during the manufacturing process and they are also highly susceptible to damage due to operational and environmental conditions during the vehicle lifetime. Therefore, early detection and estimation of damage are important so necessary actions can be taken to avoid further problems. Changes in physical parameters due to existence of damage in a structure often leads to alteration of vibration modes; thus demonstrating the dependency between the vibration characteristics and the physical properties of structures. A sensitivity-based model updating method, performed using a combination of MATLAB and NASTRAN, has been selected for the purpose of this work. The updating procedure is regarded as parameter identification which aims to bring the numerical prediction to be as closely as possible to the measured natural frequencies and mode shapes data of the damaged structure in order to identify the damage parameters (characterised by the reductions in the Young’s modulus of the weld patches to indicate the loss of material/stiffness at the damage region).

1. Introduction
Damage detection of aerospace, civil, and mechanical engineering structures are essential in determining their safety, reliability, and operational life. For example, a typical automotive body-in-white (BIW) could contain thousands of spot weld joints that are not only required for connections between layers of metal sheets but also contributing significantly to the vehicle’s structural stiffness and dynamic characteristics. However, some of these joints may be imperfect or even absent during the manufacturing process and they are also highly susceptible to damage due to operational and environmental conditions during the vehicle lifetime. So, because of their significance to the dynamics of vehicles, early detection and estimation of damage are important so necessary actions (such as structural repair and/or part replacement) can be made to avoid further problems.

Damage identification procedure consists of obtaining information about the existence, location, and extent of damage in structures using non-destructive methods. The damage identification methods using dynamic properties information are closely related to the updating of a mathematical model using test
data, hence can be classified as an inverse minimisation problem [1]. Therefore, FE model updating has been considered as one of the most frequently used methods in damage identifications, as reported by Fritzen et al [2] and Pascual et al [3], just to name a few. Existence of damage or a defect often leads to alteration of vibration modes in a structure, which can be manifested by changes in the vibration data. For instance, loss of stiffness due to damage would result in lower natural frequencies; thus demonstrating the dependency between the vibration characteristics and the physical properties of structures. Therefore, an adequate FE model that is able to detect changes in the structural properties should be used in order for the damage to be identified accurately. Nonetheless, uncertainties in the FE model along with errors in the measured vibration data could limit the success of the method [4]. For instance, less pronounced damages may not be detected owing to the presence of measurement noise, and healthy structural elements could be identified as damaged instead [5].

A brief summary of vibration-based structural damage detection methods are presented in this paper, which is followed by thorough explanation regarding the work conducted on the identification of damage in a laser welded structure. In this research, the identification procedure is carried out using natural frequencies and mode shapes data, which is performed with the FE model updating method using a combination of MATLAB and NASTRAN.

2. Damage identification methods
Various methods for damage identification of structures has been developed in the last few decades, mainly for health monitoring purposes [6]. The methods can be generally classified as local damage detection [7] and global damage detection [8, 9], as explained in the following:

**Local damage detection techniques** refer to non-destructive testing, where existence and location of local damage can be detected. Originally, visual inspection has been the most commonly used method in observing structural damage; however as structures become more complicated, the efficiency of the conventional visual inspection is reduced. Furthermore, it is also impossible to identify damage that is hidden or invisible to human eyes, hence other local methods such as CT scanning, ultrasonic, acoustic and X-ray inspections are employed as better alternatives. One of the main advantages of these methods is that they only require data obtained from the damaged structure, so there is no need for information from the intact/healthy structures. Nevertheless, these methods are effective only on small and regular structures. For large complicated structures (especially in closed and invisible environments), these approaches require some knowledge of damage location in order to avoid a costly and time-consuming procedure.

**Global damage detection techniques** (or vibration-based damage detection) has been employed to detect damage across the whole structure (regardless of the size and shapes) using dynamic characteristics (notably natural frequencies, mode shapes, modal damping and FRFs) of the structure. It is known that structural parameters will change once damage emerges in the structure, which consequently alters the dynamic properties of the structure. Therefore, these methods are more appropriate as the structural state can be assessed globally by monitoring the changes in the dynamic properties of the structure. These dynamic-based identification methods have received significant interest (especially by civil, mechanical and aerospace communities) that consequently results in literature being extensively published, as summarised by Doebling et al [10] and Sohn et al [11].

The vibration-based damage detection can be divided into [12]: (1) traditional, and (2) modern types. The former utilises dynamic characteristics of structures, which requires experimental modal analysis in order to measure the natural frequencies, mode shapes, modal damping, FRFs, etc.. This type of damage identification is applied in this research using the FE model updating technique. The modern-type (e.g., Wavelet analysis, Genetic algorithm and Artificial Neural Network), on the other hand, is based on online measured response signal of structures in service. One main advantage of this type of vibration-based
damage detection is that it can monitor the dynamic response signals of the structure in operation, hence structural shutdown or production halt can be avoided. Furthermore, it is possible to establish universal methodology for detection of damage, regardless of the shape of the structure. Nevertheless, there are still some issues to be resolved in the modern-type method, including possible contamination of measured signals by noise that may hinder the detection of small damages. The modern type damage identification is not employed in this research, hence it is not explained further in the paper.

The eigenvalue problem in the presence of damage can be expressed as follows,

\[ (-\tilde{\lambda}\tilde{M} + \tilde{K})\tilde{\phi} = 0 \]  (1)

where \( \tilde{\phi} \) indicates the properties in the damaged state. Assuming that the mass remains unchanged (i.e., \( \tilde{M} = M \)), therefore

\[ \tilde{K} = K + \Delta K \]
\[ \tilde{\lambda} = \lambda + \Delta\lambda \]
\[ \tilde{\phi} = \phi + \Delta\phi \]  (2)

where \( \Delta\lambda \) and \( \Delta\phi \) are changes in modal data due to changes in stiffness (\( \Delta K \)) that is caused by changes in structural parameters. It is shown by the above two equations that the damaged structural parameters can be identified by knowing the measured eigenvalues and eigenvectors in the damaged state. Additionally, the changes in modal characteristics may be different for each mode because the changes depend on the nature, location and severity of the damage. Therefore, the information of the changes in vibration characteristics is very useful in: (1) detecting the occurrence of damage, (2) localising the damage zones, and (3) quantifying the extent of damage in the structure.

The dependency of vibration characteristics on the physical properties of structures offers the possibility to indicate the state of health or quality of structures that leads to increasing interest in the traditional-type vibration-based damage detection methods. Consequently, many techniques have been researched to include the measured vibration data for detecting damage in structures; for example, using natural frequencies [13], mode shapes [14] and mode shape curvatures [15], and others [16–21]. A thorough review of these methods has been reported extensively by Doebling et al [10].

3. FE model updating for identification of damage

The procedure for identification of damage in a welded structure is illustrated in Fig. 1. It is important to begin the damage identification procedure with an FE model that is well-correlated with a benchmark (or healthy/undamaged) structure [22]. Thus, a welded structure consists of twenty spot weld joints (see Fig. 2), with overall dimension of 564 mm in length, 110 mm in width and 40 mm in height, was tested and its measured data was utilised as point of reference when updating initial FE model of the healthy structure so errors in geometrical and/or physical parameters and/or boundary conditions of the FE model can be minimised. Then, damage was introduced to the structure and modal testing was once more performed. Next, the developed FE model of the healthy structure was updated to match the measured data of the damaged structure to localise and quantify the damage.

3.1. Description of damaged structures

The welds studied in this investigation fasten the components very tightly and are hidden underneath. Damage in the structure was deliberately introduced by drilling out spot weld 13 (see Fig. 3) to completely remove the weld connection between the flat plate and the hat-shaped shell. Please note that there does not seem a better way to remove the hidden weld. Modal parameters (i.e., the natural frequencies and mode shapes) of the damaged structure were identified by conducting 80-point roving hammer test. The first five natural frequencies and mode shapes of the damaged structure are compared with the healthy data, as shown in Tables 1 and 2 respectively. It can be seen from Table 1 that the existence of damage does not really affect the fundamental mode, with a very small difference of 0.69%
between the healthy and damaged structure. However, deviations of the natural frequencies for the fourth and fifth modes are very noticeable (with almost 15% for the fourth mode). In addition, the second and third modes appear to have small deviations from the healthy data, but it should be noted that these modes (plus the fourth modes) are swapped - see Table 2. Furthermore, the mode shapes are no longer symmetrical in the longitudinal direction due to the presence of damage.

![Figure 1. Identification of damaged procedure](image1)

**Figure 1. Identification of damaged procedure**

![Figure 2. The laser welded structure](image2)

**Figure 2. The laser welded structure**

![Figure 3. Damaged structure](image3)

**Figure 3. Damaged structure**

3.2. *Identification of damage in welded structure*

In actual damage problem (especially in complicated assembled structures), it is usually very difficult to know if there have been any changes in the mass distribution or not. Moreover, the mass removed in this work is very small in comparison to the mass of the whole structure. Hence, the mass is assumed to be unchanged throughout the detection process, as described in Eqs. 1 and 2. When identifying damage
in the welded structure, only changes in the Young’s modulus of patches surrounding the twenty spot 
welds are considered since they are proven to be quite sensitive to changes in the modal properties [23]. 
Therefore, damage in the welded structure is characterised by reductions in the Young’s modulus to 
indicate the loss of material/stiffness at the damaged patches.

| Mode | Healthy | Damaged | Difference (%) |
|------|---------|---------|----------------|
| 1    | 513.95  | 510.40  | 0.69           |
| 2    | 550.46  | 545.70  | 0.86           |
| 3    | 578.69  | 572.6   | 1.05           |
| 4    | 624.86  | 532.10  | 14.84          |
| 5    | 639.07  | 609.00  | 4.71           |

The Young’s modulus is expressed as

\[ E_k^d = \alpha_k E_k^h \quad (k = 1, 2, \ldots, 20) \]  

where \( E_k^d \) denotes the Young’s modulus in damaged state, \( E_k^h \) represents the Young’s modulus in healthy state, and \( \alpha_k \) is a coefficient whose value ranges from 0 to 1, with a value of 0 meaning 100% damage and a value of 1 indicating no damage at all. If there is no prior knowledge of the integrity of the structure, then \( \alpha_k = 1 \) is set at the start of the identification work, which is the case in this work.

Because of the symmetrical nature of the welded structure, more information is required to assist the identification process. The experimental mode shapes of the damaged structure are observed and it is acknowledged that the damage occurs only at one quarter of the welded structure, as highlighted in Fig. 4. Based on this information, it is decided that only one quarter of the structure would be considered as damage region, thus reducing the number of possible damage locations. In this work, spot welds 11 to 15 are regarded as possible damaged spot welds in the damage region.

The damage coefficients can then be assigned as,

\[ \alpha_k^h \leq \alpha_k \leq \alpha_k^d \quad \text{and} \quad \alpha_k^d \leq \alpha_k \leq \alpha_k^h \]  

where \( \alpha_k^h \) represents the coefficient for healthy regions, while \( \alpha_k^d \) denotes the coefficient for effected region. In this work, \( 0.8 \leq \alpha_k^h \leq 1 \) is used for the healthy regions whereas \( 0.001 \leq \alpha_k^d \leq 1 \) is assigned for the damage area. Note that a very small value is used for \( \alpha_k^d \) instead of 0 in order to avoid poor geometrical and/or material properties (hence potential numerical difficulties) at the spot weld connections in the FE model. It should also be pointed out that an overlap in terms of the value of coefficients for both damage and healthy areas exists since actual damaged spot weld is not known, and there may be healthy spot welds in the specified damage area.

Inverse algorithm for the identification of damage used in this work is shown in Fig. 5. Note that an initial value of 1 is set for all the coefficients in the updating procedure. However, the coefficients are bounded to:

\[ \alpha_k = \begin{cases} 
1 & \text{if } \alpha_k > 1 \\
\alpha_k^h & \text{if } \alpha_k^h < \alpha_k^h \\
\alpha_k^d & \text{if } \alpha_k^d < \alpha_k^d 
\end{cases} \]  

after each iteration.

An objective function of

\[ J = \sum_{i=1}^{n} \left( \frac{n}{\lambda_{i}^{\text{damage}}} - \lambda_{i}^{\text{damage exp}} \right)^2 + \sum_{i=1}^{n} \sum_{k=1}^{20} \left( \frac{\phi_{ik}^{\text{damage}}}{{\phi_{ik}^{\text{damage exp}}}} - 1 \right)^2 \]  

5
is considered in the identification work, where $w^\lambda_i$ and $w^\phi_{ik}$ are weighting coefficients assigned to the objective function (Eq. (6)). The least squares formulation of the objective function allows the residuals to be weighted separately according to their importance and accuracy. For instance, (1) the mode shapes are typically measured with less accuracy than the natural frequencies, and (2) higher natural frequencies are also measured with less accuracy than the lower frequencies. The weighting matrix is usually given by the reciprocals of the measurement variances [24]; nevertheless, in practice this information is often not available. Hence, the analyst has to rely on his/her experience in choosing the appropriate weightings.

The identification procedure is performed using a combination of MATLAB and SOL200 of NASTRAN to update the initial (i.e., the healthy) FE model until the computed natural frequencies and mode shapes match those from the damage experimental data. The FE model updating procedure is conducted based on the first five natural frequencies and the first mode shape of the damaged structure. Some trial runs have been performed in order to find the appropriate weighting factors for both modal data since the statistical information of the measurements is not available. In the trial runs, the weighting factors for both frequencies and the first mode are varied accordingly to find the best value. If the theoretical frequencies match the experimental frequencies very well but a considerable discrepancy can still be seen in the mode shape, it can be assumed that too much weight is given to the frequency residuals, and vice versa. After several runs, it is decided to give the same level of confidence to all frequencies and mode shape data at those twenty DOFs. Hence, $w^\lambda_i = w^\phi_{ik} = 1$.

Consequently, Eq. (6) reduces to

$$ J = \sum_{i=1}^{5} \left( \frac{\lambda^\text{damage}_i}{\lambda^\text{damageexp}_i} - 1 \right)^2 + \sum_{k=1}^{20} \left( \frac{\phi^\text{damage}_{1k}}{\phi^\text{damageexp}_{1k}} - 1 \right)^2 $$

As mentioned earlier, changes in the Young’s modulus of the patch for the twenty spot welds are observed to indicate the loss of stiffness at those twenty spot weld locations. These changes are represented by the coefficient as defined in Eqs. (3) to (5). Coefficients that belong to the five spot welds in the damage region are treated as potential damage, while the rest of the coefficients are assumed to be healthy.

Natural frequencies from the initial FE model are given in Table 3 (Column II), while the initial MAC values are given in Table 4. It can be seen that the natural frequencies of the first five modes show errors of up to approximately 14.5%, while the MAC values show good correlation only for the first and fifth modes. The MAC values for modes 2, 3 and 4 highlight the mode swapping problem between the healthy and damaged experimental mode shapes (see Table 2). In order to improve the correlation between the modes and to reduce the discrepancies of the initial FE model against the experimental data of the damaged structure, the FE model updating is performed to identify and quantify the damage parameters.

The damage identification procedure converges after fifteen iterations and the identified damage coefficients are tabulated in Table 5. It can be seen that the damage has been successfully located and quantified at spot weld 13. However, the supposedly healthy spot welds are also affected by the procedure with the biggest error of approximately 50% at the adjacent spot welds (i.e., spot welds 12 and 14), which is believed to be due to the spillover effect of FE model updating.

Using these identified parameters, the updated natural frequencies are computed as shown in Table 3 (Column III). Although the error for the fundamental natural frequency is increased, the errors for the other frequencies are reduced significantly. The total error of the natural frequencies is also decreased to approximately 6%, as compared with initial error of about 30%. The updated MAC values are obtained as in Table 6. It can be observed that the correlation has improved for all the modes, which means that the identification procedure has successfully match the FE model to the damage measured data. It should also be noted that the correlation between the measured and analytical modes is checked by calculating the MAC values before and after the updating procedure. However, a better procedure may be employed by conducting cross-orthogonality check in order to assess the degree of correlation between the measured and numerical mode shapes during the updating loop.
| Mode | Healthy | Damaged |
|------|---------|---------|
| 1    | ![Healthy Mode 1](image1) | ![Damaged Mode 1](image2) |
| 2    | ![Healthy Mode 2](image3) | ![Damaged Mode 2](image4) |
| 3    | ![Healthy Mode 3](image5) | ![Damaged Mode 3](image6) |
| 4    | ![Healthy Mode 4](image7) | ![Damaged Mode 4](image8) |
| 5    | ![Healthy Mode 5](image9) | ![Damaged Mode 5](image10) |
Figure 4. Observation on the experimental mode shapes of the damaged structure to identify damage region

Figure 5. FE model updating for damage identification

Table 3. Experimental and FE results (in Hz) of the damaged structure

| Mode | Experiment | II Initial | III Updated |
|------|------------|------------|-------------|
|      |            | FE         | Error (%)   | FE          | Error (%)   |
| 1    | 510.40     | 508.39     | 0.39        | 501.41      | 1.76        |
| 2    | 532.10     | 580.89     | 9.17        | 523.00      | 1.71        |
| 3    | 545.70     | 624.47     | 14.43       | 548.15      | 0.45        |
| 4    | 572.60     | 557.71     | 2.60        | 575.12      | 0.44        |
| 5    | 609.00     | 631.63     | 3.72        | 618.88      | 1.62        |
| Total error | 30.31 | 5.98        |
Table 4. Initial MAC values of the damaged structures

| FE modes | Experimental modes |
|----------|-------------------|
| 1        | 0.82 0.05 0.00 0.00 0.02 |
| 2        | 0.00 0.00 0.53 0.04 0.01 |
| 3        | 0.00 0.01 0.08 0.84 0.01 |
| 4        | 0.09 0.55 0.08 0.00 0.01 |
| 5        | 0.01 0.08 0.12 0.14 0.89 |

Table 5. Damage coefficients for spot weld 11 to 15

| Parameter | Initial value | Identified value |
|-----------|---------------|------------------|
| $\alpha_{11}$ | 1             | 0.9032           |
| $\alpha_{12}$ | 1             | 0.5421           |
| $\alpha_{13}$ | 1             | 0.0017           |
| $\alpha_{14}$ | 1             | 0.4363           |
| $\alpha_{15}$ | 1             | 0.8764           |

Table 6. Updated MAC values of the damaged structures

| FE modes | Experimental modes |
|----------|-------------------|
| 1        | 0.87 0.17 0.00 0.00 0.02 |
| 2        | 0.20 0.93 0.00 0.06 0.04 |
| 3        | 0.00 0.01 0.55 0.09 0.02 |
| 4        | 0.02 0.01 0.05 0.89 0.37 |
| 5        | 0.02 0.01 0.16 0.23 0.92 |

4. Conclusions

This paper has presented a brief overview of vibration- or frequency-based damage identification using inverse methods. The sensitivity-based model updating methods have been selected as a practical approach for the purpose of this work, and the updating procedure is regarded as parameter identification which aims to bring the numerical prediction to be as closely as possible to the damage experimental data.

In this work, the identification of damage in a spot welded structure has been carried out. Damage was introduced to a healthy structure by drilling out spot weld 13. Then, the developed FE model of the healthy structure is updated to match the measured data of the damaged structure to localise and quantify the damage. The identification procedure is conducted based on the first five natural frequencies and the first mode shape of the damaged structure, and damage is characterised by the reductions in the Young’s modulus of patches surrounding the spot welds to indicate the loss of material/stiffness at the damage region.

Due to the symmetrical nature of the welded structure, more information is required to assist the identification process. The experimental mode shapes of the damaged structure are observed and it is acknowledged that the damage occurs only at one quarter of the welded structure. Based on this information, it is decided that only one quarter of the structure would be considered as damage region, thus reducing the number of possible damage locations. In this work, spot welds 11 to 15 are regarded as possible damaged spot welds in the damage region. Based on the updating results, it can be concluded that damage certainly exists at the location of spot weld 13. However, the supposedly healthy spot welds are also affected by the procedure with the biggest error of approximately 50% at the adjacent spot.
welds (i.e., spot welds 12 and 14), which is believed to be due to the spillover effect of FE model updating. The identification procedure also brings the predicted natural frequencies closer to their measured counterparts, with total error of only 6%. Furthermore, the correlation between the numerical and experimental modes are very good, which demonstrates the success of the identification procedure.

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References

[1] Teughels A and De Roeck G 2004 Structural damage identification of the highway bridge Z24 by FE model updating Journal of Sound and Vibration 278 589
[2] Fritzen C P, Jennnewein D and Kiefer T 1998 Damage detection based on model updating methods Mechanical Systems and Signal Processing 12 163
[3] Pascual R, Trendafilova I, Golinalj C and Heylen W 1999 Damage detection using model updating and identification techniques Identification in Engineering Systems: Proc. of the 2nd Int. Conf. (Swansea, UK)
[4] Friswell M I, Penny J E T and Garvey S D 1997 Parameter subset selection in damage location Inverse Problems in Engineering 5 189
[5] Farrar C R and Doebbling S W 1998 Damage detection II: field applications to large structures (Los Alamos National Laboratory, New Mexico)
[6] Sinha J, Friswell M I and Edwards S 2002 Simplified models for the location of cracks in beam structures using measured vibration data Journal of Sound and Vibration 251 13
[7] Yoon M K, Heider D, Gillespie J W, Ratcliffe C P and Crane R M 2005 Local damage detection using the two-dimensional gapped smoothing method Journal of Sound and Vibration 279 119
[8] Carden E P and Fanning P 2004 Vibration based condition monitoring: a review Structural Health Monitoring 3(4) 355
[9] van der Auweraer H and Peeters B 2003 International research projects on structural health monitoring: an overview Structural Health Monitoring 2(4) 341
[10] Doebbling S W, Farrar C R and Prime M B 1998 A summary review of vibration-based damage identification methods The Shock and Vibration Digest 30(2) 91
[11] Sohn H, Farrar C R, Hemez F M, Shunk D D, Stinemates D W, Nadler B R and Czarnecki J J 2004 A review of structural health monitoring literature: 19962004 (Los Alamos National Laboratory, New Mexico)
[12] Yan Y J, Cheng L, Wu Z Y and Yam L H 2007 Development in vibration-based structural damage detection technique Mechanical Systems and Signal Processing 21 2198
[13] Lee Y S and Chung M J 2000 A study on crack detection using eigenfrequency test data Computers and Structures 77 327
[14] Kousk M and Zimmerman D C 1992 Structural damage assessment using a generalized minimum rank perturbation theory AIAA Journal 32 836
[15] Pandey A K, Biswas M and Samman M M 1991 Damage detection from changes in curvature mode shapes Journal of Sound and Vibration 145(2) 321
[16] James III G H 1996 Development of structural health monitoring techniques using dynamics testing (Sandia National Laboratories, Albuquerque)
[17] Banks H T, Imman D J, Leo D J and Wang Y 1996 An experimentally validated damage detection theory in smart structures Journal of Sound and Vibration 191(5) 859
[18] Farrar C R and Jauregui D A 1998 Comparative study of damage identification algorithms applied to a bridge: 1. Experiment Smart Mater. Struct. 7 704
[19] Lee U and Shin J 2002 A frequency response function-based structural damage identification method Computers and Structures 80 117
[20] Wang Z, Lin R M and Lim M K 1997 Structural damage detection using measured FRF data Comput. Methods Appl. Mech. Engrg. 147 187
[21] Sampaio R P C, Maia N M M and Silva J M M 1999 Damage detection using the frequency-response-function curvature method Journal of Sound and Vibration 226(5) 1029
[22] Thyagarajan S K, Schulz M J and Pai P F 1998 Detecting structural damage using frequency response functions Journal of Sound and Vibration 210(1) 162
[23] Abu Husain N, Haddad Khodaparast H, Snaylam A, James S and Ouyang H 2009 Finite element modelling and updating of laser spot weld joints in a top-hat structure for dynamic analysis Proc. IMechE Part C: J. Mechanical Engineering Science 224(4) 851

[24] Friswell M I and Mottershead J E 1995 Finite element model updating in structural dynamics Kluwer Academic Publishers Dordrecht, The Netherlands