Probing Transverse Momentum Broadening in Heavy Ion Collisions

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Abstract

We study the dijet azimuthal de-correlation in relativistic heavy ion collisions as an important probe of the transverse momentum broadening effects of a high energy jet traversing the quark-gluon plasma. We take into account both the soft gluon radiation in vacuum associated with the Sudakov logarithms and the jet $P_T$-broadening effects in the QCD medium. We find that the Sudakov effects are dominant at the LHC, while the medium effects can play an important role at RHIC energies. This explains why the LHC experiments have not yet observed sizable $P_T$-broadening effects in the measurement of dijet azimuthal correlations in heavy ion collisions. Future investigations at RHIC will provide a unique opportunity to study the $P_T$-broadening effects and help to pin down the underlying mechanism for jet energy loss in a hot and dense medium.

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Introduction. One of the most important discoveries in the relativistic heavy ion experiments at RHIC at Brookhaven National Laboratory and the LHC at CERN is the jet quenching phenomena \[1–4\], where high energy partons lose tremendous energy through their interactions with the quark-gluon plasma created in heavy ion collisions. Theoretically, the jet energy loss can be understood as a result of the induced gluon radiation when the parton traverses the hot QCD matter, and has been well formulated in the QCD framework \[5–9\]. Alternatively, the strong coupling feature of the medium can be described by models based on the Ads/CFT correspondence in string theory \[10–15\]. These calculations have been successfully applied to heavy ion phenomenology in order to understand the jet quenching related experimental data from the RHIC and LHC \[16\].

Meanwhile, there has been a strong theoretical argument that the jet energy loss is associated with the \(P_T\)-broadening phenomena \[7\], where the energetic jet accumulates additional transverse momentum perpendicular to the jet direction. Combining the analysis of the jet energy loss and \(P_T\)-broadening is of crucial importance to consolidate the underlying mechanism for the jet energy loss. Dijet production is an ideal process for this physics, where we can use the leading jet as a reference. The jet energy loss can be studied by measuring the energy of the away side jet, and the \(P_T\)-broadening effects can be accessed through the azimuthal angular correlation. The former has been investigated by the ATLAS and CMS collaborations at the LHC through the so-called \(A_J\) distribution measurements, where the theoretical interpretations are consistent with the jet energy loss \[17–19\]. Similar conclusion has been reached also for photon-jet events \[20\], see, e.g., Ref. \[21\]. Both experiments have also studied the azimuthal angular correlation between the two jets, but found no difference as compared to the \(pp\) collisions. The goal of this paper is to perform a systematic study on the dijet azimuthal de-correlation in heavy ion collisions. In particular, we find that the \(P_T\)-broadening effects play a negligible role at the LHC energy, whereas, it will become an important contribution and should be observed at the RHIC energy, since Sudakov effects at the LHC are much stronger than that at RHIC. The experimental investigation of this \(P_T\)-broadening effects in dijet production is a crucial step forward to identify the underlying mechanism for the jet energy loss in heavy ion collisions.

As illustrated in Fig. 1 dijets are produced in partonic scattering, which go through the hot QCD medium before reaching the detector,

\[
A + A \rightarrow \text{Jet}_1 + \text{Jet}_2 + X.
\]  

Most of the dijet events are produced in the back-to-back azimuthal correlation configuration with the azimuthal angle: \(\Delta \phi = \phi_1 - \phi_2 \sim \pi\), where \(\phi_{1,2}\) are the azimuthal angles of these two final state jets with transverse momenta \(k_{1\perp}\) and \(k_{2\perp}\), respectively. There are two important contributions to the azimuthal de-correlation of the two jets in heavy ion collisions: one is the soft and collinear gluon radiation associated with the partonic \(2 \rightarrow 2\) subprocesses, which is referred to as the Sudakov effects; the other is the \(P_T\)-broadening effects due to multiple scattering and medium induced radiation when high energy jets propagate through the medium. Therefore, in order to unequivocally determine the \(P_T\)-broadening effects from the medium, we have to first understand the Sudakov effects in dijet production. Because this comes from the partonic scattering, we can study it in dijet production in \(pp\) collisions, which has been extensively investigated by the experiments at the Tevatron \[22\] and the LHC \[23, 24\]. The theoretical developments \[25, 26\] in the last few years have also advanced, where a successful description of these data was found \[26\]. In the following, we will compare the relative importance of the Sudakov and \(P_T\)-broadening effects at the LHC.
FIG. 1: Dijet production and azimuthal angular de-correlation in heavy ion collisions: both soft
 gluon radiation when the two jets are produced from the partonic scattering processes and the
 multiple scattering between the high energy jet and the medium induced gluon radiation contribute
to the de-correlation.

and RHIC. This will provide a benchmark calculation for the $P_T$-broadening in high energy
 hard scattering processes in $AA$ and $pA$ collisions.

**Sudakov and $P_T$-broadening effects.** We follow the BDMPS framework\[6–8\] to analyze
the $P_T$-broadening effects in the heavy ion collisions, and compare that with the Sudakov
effects from gluon radiation in vacuum. As illustrated in the right panel of Fig. 1 (a),
when a high energy jet traverses the medium, it suffers multiple scatterings and medium
induced gluon radiation. These effects can be represented by a characteristic scale $Q_s^2 = \hat{q}L$,
which depends on the transport coefficient $\hat{q}$\[7\], and the length of the jet path in the
medium $L$. The physics behind the $P_T$-broadening is that each scattering randomly gives
a small transverse momentum kick to the jet, which in turn accumulates a total transverse
momentum of order $Q_s$ along the path in the medium. In the BDMPS framework, this
effects is computed in a Glauber multiple scattering theory, and the result is expressed in the
Fourier transformation conjugate $b_\perp$-space as $e^{-Q_s^2 b_\perp^2 /4}$. When Fourier transforming back to
the transverse momentum space, it leads to a Gaussian-like distribution of $e^{-q_\perp^2 /Q_s^2}$, where $q_\perp$
represents the transverse momentum perpendicular to the jet direction. In addition, recent
studies\[27–31\] reveal that additional medium induced gluon radiation can also contribute to
the jet $P_T$-broadening, and leads to slightly larger values of $Q_s^2$.

The numerical $\hat{q}$ parameter has been a subject of intensive studies in jet quenching
phenomenology, see a recent report from the JET-collaboration [16], which gives roughly
$Q_s^2 = \hat{q}L \simeq 6 \text{GeV}^2$ at RHIC and $\hat{q}L \simeq 10 \text{GeV}^2$ at the LHC for quark jets and medium
length $L = 5 \text{fm}$. For gluon jets, $Q_s^2$ is $2N_c^2 / N_c^2 - 1$ times of that for quark jets due to different
Casimir factors.

The medium related $P_T$-broadening effect is physically different from the Sudakov effects
computed from the collinear and soft gluon radiation in hard scattering processes [32]. To
see this more clearly, we compare the effects from the gluon radiation contributions in the
right panel of Fig. 1. The vacuum radiation diagram of (b) has been excluded in the medium
induced radiation contribution in the BDMPS calculations, which, on the other hand, is part
of the collinear and soft gluon radiation contribution to the imbalance between the two jets in the dijet production process. In particular, this final state gluon radiation will contribute to a term depending the jet size \[26\]: \[ \frac{\alpha_s}{2\pi^2} \frac{1}{q_2^2} C_f \ln \frac{1}{R^2} \], where \( q_\perp \) represents the transverse momentum of the radiated gluon, \( R \) the jet size, and \( C_f \) the color factor for the associated jet (\( C_F \) for quark jet and \( C_A \) for gluon jet). This contribution can be factorized into the soft factor in the dijet production. When we Fourier transform the above expression into \( b_\perp \)-space, we obtain a logarithmic dependence \( \ln(b_\perp^2 \mu^2) \). In the factorization formula, the scale \( \mu \) will be set around the hard momentum scale (such as the leading jet energy) to resum the associated large logarithms.

On the other hand, when we consider the medium induced gluon radiation from the diagram (c) of the right panel of Fig. 1 it is an infrared safe contribution as demonstrated in the BDMPS calculation \[7\]. This is because the famous Landau-Pomeranchuk-Migdal (LPM) effect suppress the small transverse momentum gluon radiation in the medium. There is no such \( 1/q_2^2 \) behavior from this diagram, and it does not contribute to a logarithmic term of \( \ln(b_\perp^2 \mu^2) \), although it will contribute to a high order corrections to \( Q_s \) \[27, 28, 33\]. The bottom line is that the Sudakov effects only take into account the gluon radiation in the vacuum which contains no information of medium effects. In the meantime, it is common practice to subtract the vacuum contribution when we compute any medium effects in the vacuum which contains no information of medium effects. In the meantime, it is common practice to subtract the vacuum contribution when we compute any medium effects in the vacuum which contains no information of medium effects. In the meantime, it is common practice to subtract the vacuum contribution when we compute any medium effects in the vacuum which contains no information of medium effects.

The vacuum radiation diagram is part of all collinear and soft gluon radiation contributions in dijet production, which has been calculated recently in Refs. \[25, 26\]. From these calculations, it was found that there is a simple power counting rule, which allows us to predict that each incoming parton contributes to the leading double logarithm with the associated color factor. They can be factorized into the so-called transverse momentum distributions from the incoming nucleons and the soft factor associated with final state jets. Because of the short distance hard scattering for dijet production, these contributions will not be modified in heavy ion collisions. We extend the resummation formula derived in Ref. \[26\] in our study as follows

\[
\frac{d^4\sigma}{dy_1 dy_2 dk_{1\perp}^2 d^2 k_{2\perp}} = \sum_{ab} \sigma_0 \int \frac{d^2b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot \hat{b}_\perp} W(b_\perp),
\]

where we focus in the small \( q_\perp \ll k_{1\perp} \sim k_{2\perp} \) region. Away from this region, we have to include a fixed order perturbative correction. In the low \( q_\perp \) region, we apply an all order resummation formula for \( W(b_\perp) \),

\[
W(b_\perp) = x_1 f_a(x_1, \mu_b) x_2 f_b(x_2, \mu_b) e^{-S(Q^2, b_\perp)},
\]

where \( \sigma_0 \) represents normalization of the differential cross section, \( y_1 \) and \( y_2 \) are rapidities of the two jets, \( Q^2 = \hat{s} = x_1 x_2 S \) is the partonic center of mass energy squared, \( b_0 = 2e^{-\gamma_e} \), \( f_{a,b}(x, \mu_b = b_0/b_a) \) are parton distributions for the incoming partons \( a \) and \( b \), \( x_{1,2} = k_{1\perp} (e^{\pm y_1} + e^{\pm y_2}) / \sqrt{S} \) are momentum fractions of the incoming hadrons carried by the partons. By introducing the \( b_e \)-prescription \[32\] which sets \( b_e = b_\perp / \sqrt{1 + b_\perp^2 / b_{\max}^2} \) with \( b_{\max} = 0.5 \text{GeV}^{-1} \), we separate the Sudakov form factor \( S(Q, b_\perp) \) into perturbative
and non-perturbative parts in pp collisions: \( S(Q, b_\perp) = S_{\text{pert}}(Q, b_\perp) + S_{\text{NP}}(Q, b_\perp) \) with the perturbative part defined as,

\[
S_{\text{pert}}(Q^2, b_\perp) = \int_{\mu^2_0}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ A \ln \left( \frac{Q^2}{\mu^2} \right) + B + (D_1 + D_2) \ln \frac{1}{R^2} \right],
\]

where \( R \) represents the jet size. We have applied the anti-\( k_t \) algorithm to define the final state jets in our calculations. Here the coefficients \( A, B, D_1, D_2 \) can be expanded perturbatively in terms of powers of \( \alpha_s \). At one-loop order, \( A = C_A \frac{\alpha_s}{\pi} \), \( B = -2C_A\beta_0 \frac{\alpha_s}{\pi} \) for gluon-gluon initial state, \( A = C_F \frac{\alpha_s}{\pi} \), \( B = -\frac{3C_F}{2} \frac{\alpha_s}{\pi} \) for quark-quark initial state, and \( A = \frac{(C_F+C_A)}{2} \frac{\alpha_s}{\pi} \), \( B = -\frac{3C_F}{4} - C_A\beta_0 \frac{\alpha_s}{\pi} \) for gluon-quark initial state. \( D_i \) is \( \frac{\alpha_s}{\pi} C_F \) for quark jet, and \( \frac{\alpha_s}{2\pi} \) for gluon jet. For the non-perturbative part, we follow those in Ref. [26].

**Probing the \( P_T \)-broadening effect.** In the following discussion, we focus on dijet productions in mid-rapidity. In this kinematics, the \( P_T \)-broadening effects also contribute to the longitudinal momentum along the incoming beam direction, and therefore modify the rapidity of the final state jets. However, in the region of interest for our study, this is a sub-leading order effects, which can be neglected in our calculations. We also notice that the \( P_T \)-broadening effect is along the direction perpendicular to the jet, so that it will not affect the transverse momentum along the jet direction. However, for convenience in implementing the \( P_T \)-broadening effects in a single formula together with the Sudakov resummation, we modify Eq. (4) as

\[
S(Q, b)|_{AA} = S_{\text{pert}}(Q, b_\perp) + S_{\text{NP}}(Q, b) + Q_s^2 b^2/4 ,
\]

where \( Q_s^2 \) encodes the medium \( P_T \) broadening effects. In the correlation calculation, we will integrate out the sub-leading jet energy in a certain range, which effectively integrates out the transverse momentum along the jet direction. As a result, Eq. (5) is reduced to the form that the \( P_T \)-broadening effects only apply to the transverse direction perpendicular to the jet direction.

The first two terms in Eq. (5) are the same as that in pp collisions \(^1\). In order to observe the \( P_T \)-broadening effects, we have to find the right kinematics where the last term will be important. This can be achieved by varying the jet energy (which will modify the perturbative Sudakov term) or the medium effects (by changing the centrality or the energy of the collisions).

Let us first examine the typical dijet production at the LHC. In Fig. 2, we plot \( b_\perp \times W(b_\perp) \) as function of \( b_\perp \) for a leading jet energy \( P_\perp = 120 \) and 50GeV, respectively, at mid-rapidity at \( \sqrt{s} = 2.76TeV \), where \( W(b_\perp) \) is defined as in Eq. (4). The Fourier transformation of \( W(b) \) yields the imbalance \( q_\perp \) distribution for the dijet. In the numeric calculations, we have taken into account the perturbative form factor at one-loop order: \( A^{(1)} \), \( B^{(1)} \), and \( D^{(1)} \). We have also checked the complete next-to-leading logarithmic corrections do not change significantly the behavior of these distributions. The three curves in this plot correspond to \( Q_s^2 = 0, 8, 20GeV^2 \), respectively. From these plots, we can see that the dominant contribution of \( W(b_\perp) \) comes from small-\( b \) region, where the \( P_T \)-broadening effects do not affect the results at all. Clearly, at the LHC, the perturbative Sudakov form factor \( S_{\text{pert}}(b) \) dominates the small-\( b \) contribution. More importantly, in the LHC energy region, the dijet productions probe

\(^1\) Here we neglect the \( P_T \)-broadening from the cold nuclei effects, which is much smaller than that in hot QCD matter [7-10].
FIG. 2: Impact of the $P_T$-broadening effects on dijet production at mid-rapidity at the LHC, where we plot the $b_\perp \times W(b_\perp)$ of Eq. (3) as functions of $b_\perp$ with $S(Q, b)$ in Eq. (5) and three different values of $Q^2_s = 0, 8, 20\text{GeV}^2$. The Fourier transformation of $W(b_\perp)$ would give the imbalance transverse momentum $\vec{q}_L = \vec{k}_1 + \vec{k}_2$ distributions, where $k_{1\perp}$ and $k_{2\perp}$ are the leading jet and sub-leading jet transverse momenta. Comparison between the two choices of the leading jet transverse momentum $P_\perp = 120, 50\text{GeV}$ at the LHC, respectively.

FIG. 3: $P_T$-broadening effects in Dijet azimuthal angular distributions in central PbPb collisions at the LHC.

relatively small-$x$ parton distributions, where the $x f_a(x, \mu_b)$ factor in Eq. (3) significantly push the contributions into the small-$b$ region. Therefore, even if we lower the leading jet energy to $50\text{GeV}$, it will still be dominated by small-$b$ contribution as shown in the right panel of Fig. 2, where, again, we find that the medium effects are negligible.

To see the medium effects on the azimuthal angular distribution, we apply Eqs. (2, 3, 4, 5) to calculate the $\Delta \phi$ distribution,

$$\frac{1}{\sigma_{\text{dijet}}} \frac{d\sigma_{\text{dijet}}}{d\Delta \phi},$$

(6)
FIG. 4: $P_T$-broadening effects at RHIC: (left) plot of $b_{\perp}W(b_{\perp})$ as function of $b_{\perp}$; (right) azimuthal de-correlation for dijet production at RHIC for a leading jet $P_{\perp} = 35\text{GeV}$.

where $\sigma_{dijet}$ is the dijet cross-section and the numerator is calculated from Eq. (2) after integrating over other kinematic variables. As shown in Fig. 3, we find that the shape of the angular correlation is consistent with the CMS data for back-to-back dijet configurations. More importantly, our results show that $P_T$-broadening effects are negligible at the LHC, where the three curves (corresponding to three different choices for $Q_s$) almost lay on top of each other. This also explains why the azimuthal angular correlation in dijet productions does not change from $pp$ to $AA$ collisions at the LHC for the kinematical region studied in the ATLAS and CMS measurements.

Nevertheless, the above conclusions can dramatically change when we switch from the LHC to RHIC. As shown in Fig. 4, we plot the same distributions for a typical dijet production at RHIC with $\sqrt{S} = 200\text{GeV}$. Here, clearly, we can see that the medium induced $P_T$-broadening contribution is very important in the $b \sim 0.5\text{GeV}^{-1}$ region. As a result, significant $P_T$-broadening effects can be found in Fig. 4 for RHIC experiments. In particular, the $P_T$ broadening effects changes not only the shape but also the magnitude of the dijet azimuthal correlations in heavy ion collisions at RHIC. We are looking forward to these measurements in the near future [34].

Conclusions. We have performed a systematic study of dijet azimuthal de-correlation in heavy ion collision to probe the $P_T$-broadening effects in the quark-gluon plasma. By taking into account additional Sudakov effects, we found that at the LHC, the medium $P_T$-broadening effects are negligible in the dijet azimuthal angular distribution, which is consistent with the observations from the ATLAS and CMS experiments. By contrast, we demonstrated that the $P_T$-broadening effects can be important at the RHIC energy and we should be able to observe it in experiments. Future study of this physics at RHIC would provide a unique opportunity to probe the $P_T$-broadening effects and help to identify the underlying mechanism for the jet energy loss in relativistic heavy ion collisions.

We also note that there have been significant progress in the development of the Monte Carlo event generator “JEWEL” [35, 36], which incorporates both the parton shower effects and medium effects, such as the LPM effects. By and large, our theoretical work is complementary to these numerical studies.

Further theoretical investigations should follow along the direction of this paper. In a
recent calculation at the next-to-leading order \cite{28}, a double logarithmic term depending on the length $L$ was found in $Q_s$. Since we are dealing with jet propagation in this paper, we need to consider the modification of the finite jet size on the $P_T$-broadening calculations. These contributions will depend on the details of gluon radiation in the medium and could provide an unique way to distinguish different mechanisms. We should also combine the above analysis with the jet energy loss calculations. For that, we need to carry out a next-to-leading order perturbative calculation combined with the jet energy loss with jet size dependence (see, e.g., recent calculations in Ref. \cite{31, 37}). Early attempts have been made in Refs. \cite{17–19} to calculate the $A_J$ asymmetries from the LHC measurements. By combining the theoretical studies of the azimuthal de-correlation and the energy asymmetry $A_J$ for dijet production in heavy ion collisions, together with the sophisticated Monte Carlo simulations, we should be able to unambiguously decode the underlying mechanism for jet quenching phenomena in the strongly interaction quark-gluon plasma.

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