Reevaluation and correction of Maxwell’s Equations: a magnetic field has a source, a moving electric charge [version 1; peer review: awaiting peer review]

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Abstract
Maxwell’s Equations are considered to summarize the world of electromagnetism in four elegant equations. They summarize how electric and magnetic fields propagate, interact, how they are influenced by other objects and what their sources are. While it is widely accepted that the source of a magnetic field is a moving charge, one of the equations instead states that the magnetic field has no source. However, it is widely accepted that a magnetic field cannot be created without a moving electric charge. As such, here, after carefully reevaluating how Maxwell derived his equation, a limitation was identified. After adjustments, a new equation was derived that instead demonstrates that the source of a magnetic field is a moving charge, confirming experimentally verified and widely accepted observations.

Keywords
Maxwell Equation, magnetism, electromagnetism, field
**Introduction**

James Clerk Maxwell [1831–1879] was a pioneer in studying the propagation of electromagnetic waves\(^1\). He unified his observations with Faraday’s, Gauss’s, and Ampere’s Laws into a set of four equations, known as Maxwell’s Equations (Figure 1)\(^2\). Maxwell’s Equations summarize the world of electromagnetics in four elegant equations that state: (a) an electric charge is the source of an electric field; (b) a moving electric charge results in a magnetic field; (c) a changing magnetic field results in an electric field; (d) a magnetic field has no source (Figure 1). As such, Maxwell’s Equations summarize the source of electric and magnetic fields, and also describe how they are interlinked. Simultaneously, Maxwell’s Equations also demonstrated for the first time that electromagnetic waves and visible light have the same speed and are essentially the same\(^3,4\). As such, the equations provide also an equation for light and its origin, making them of fundamental importance to physics and life. To illustrate their importance, the biblical quote “And God said let there be light, and there was light”\(^5\) has been jokingly altered by physicists and is also found displayed on various merchandise: “And God said [Maxwell’s equations], and there was light”\(^6\).

Here, I revisit one of Maxwell’s Equations, which shows that a magnetic field has no source. By carefully reanalyzing how this equation was derived, I show that this equation ought to be adjusted as a substitution has not been made. After the revisions presented here, the equation now demonstrates that the magnetic field has an actual source, instead of showing that a magnetic field has no source, the equation now demonstrates that the magnetic field has an actual source, and that the source is a moving electric charge.

**Electric and magnetic fields can be described by different vectors**

In physics, a field is a region in which each point is affected by a force\(^6,7,9\). For example, objects fall to the ground as they are affected by the force of a gravitational field\(^7\). An electromagnetic field surrounds an electrical charge and it repels or attracts other charges\(^5,7,9\). An electromagnetic field is caused by the motion of an electric charge\(^5,7,9\). A stationary charge will only give rise to an electric field. Only if such charge is moving, a magnetic field is produced\(^1,7,9\).

An electric field can be described by two vectors: the external electric field \(E\) and the electric displacement field \(D\), which is also known as the dielectric displacement field. \(D\) describes the vector field in a non-conducting medium, a dielectric, which is an electrically insulating material\(^7\). \(D\) is proportional to \(E\), but they differ: External materials such as dielectrics change the effective electric field \(E\), while \(D\) remains constant\(^7\). In other words, \(D\) can be regarded as a constant of the system being studied, while \(E\) is the variable vector of the electric field in that system. The relationship between \(E\) and \(D\) is summarized with the equation \(D = E \cdot \varepsilon_0 \cdot \varepsilon_r\), with \(\varepsilon_0\) being the dielectricity constant of the vacuum and \(\varepsilon_r\) the dielectricity of the material\(^7\).

Analogous to the electric fields \(E\) and \(D\), two different vectors are used to describe magnetic fields: one is called magnetic flux density or magnetic induction, and is symbolized by \(B\); the other is called magnetic field strength or magnetic field intensity, abbreviated as \(H\). \(B\) is proportional to \(H\), but they differ; external materials such as diamagnetics or ferromagnetics

\[
\begin{align*}
(a) & \quad \oint E \cdot dA = \frac{Q}{\varepsilon_0 \cdot \varepsilon_r} \\
(b) & \quad \oint B \cdot dx = \mu_0 \cdot \mu_r \cdot I + \mu_0 \cdot \mu_r \cdot \varepsilon_0 \cdot \varepsilon_r \cdot \frac{d}{dt} \int E \cdot dA \\
(c) & \quad \oint E \cdot dx = -\frac{d}{dt} \int B \cdot dA \\
(d) & \quad \oint B \cdot dA = 0
\end{align*}
\]

**Figure 1.** Maxwell Equations: four equations that summarize the field of electromagnetism\(^2\). \(a\), an electric charge is the source of an electric field\(^1\) (\(E\): external electric field, \(A\): area, \(Q\): electric charge, \(\varepsilon_r\): dielectricity constant of the vacuum, \(\varepsilon_r\): dielectricity constant of the material). \(b\), a moving electric charge results in a magnetic field\(^1\) (\(B\): magnetic field intensity, \(x\): distance from current, \(\mu_r\): permeability constant of material, \(\varepsilon_r\): dielectricity constant of the vacuum, \(\varepsilon_r\): dielectricity constant of the material, \(I\): electric current, \(t\): time). \(c\), a changing magnetic field results in an electric field\(^1\) (\(E\): external electric field, \(x\): distance from current, \(t\): time). \(d\), a magnetic field has no source\(^1\) (\(B\): magnetic field intensity, \(A\): area).
change the magnetic field intensity $H$, while $B$ remains constant (Figure 2b)\textsuperscript{5,6}. As such, $B$ can be regarded as a constant of the system being studied, while $H$ is the variable vector of the electric field in that system. The connection between $B$ and $H$ is represented by $B = H \cdot \mu_0 \cdot \mu_r$, with $\mu_0$ being the permeability constant of the vacuum and $\mu_r$ the permeability constant of the material\textsuperscript{5,6}.

Even though $E$ and $B$ are the most commonly used field vectors to describe electric and magnetic fields, respectively, they are not equivalents. This is important to highlight. Instead, $D$ and $B$ are equivalents, as $D$ and $B$ both do not change upon the addition of external materials such as dielectrics or diamagnetics. Similarly, $E$ and $H$ are equivalents, as they are not influenced by such external materials (Figure 3)\textsuperscript{5,6}.

**Flow definition of electric and magnetic fields through a surface are not equivalent**

To describe the flow of an electric field through a given surface $A$, the electric flux $\psi$ is used. Similarly, the magnetic flux $\phi$ is a measure of a magnetic field through a given surface $A$\textsuperscript{5,6}. Both depend on the field’s vectors: $\psi$ has been defined by $E$ through $\psi = \oint E \cdot dA$, while $\phi$ has been defined through $B$, with $\phi = \oint B \cdot dA$\textsuperscript{5,6}. Although these flux definitions might seem equivalent, they are not: $\psi$ has been defined using the electric field vector $E$ that changes with external materials, while $\phi$ has been defined based on the magnetic field vector $B$, which remains constant if for example diamagnetics are introduced into the system. As such, within a given system, the flux of the electric field is defined by a variable vector of the electric field, namely $E$, which can change. However, the flux of the magnetic field has been defined by a constant vector of the magnetic field, $B$, which remains constant even if diamagnetics are introduced into the system. Hence, the flux of electricity and magnetisms are not equivalent, as $\psi$ is based on the variable vector $E$, while $\phi$ is based on a constant vector $B$\textsuperscript{5,6}. However, $B$ depends on the variable vector $H$, with $B = H \cdot \mu_0 \cdot \mu_r$, and $E$ on the constant vector $D$, with $D = E \cdot \varepsilon_0 \cdot \varepsilon_r$\textsuperscript{5,6}. As such, both $\psi$ and $\phi$ can also be described with constant or variable field vectors:

\begin{align*}
\psi &= \oint E \cdot dA = \oint \frac{D}{\varepsilon_0 \cdot \varepsilon_r} \cdot dA \quad \text{with } E \neq \text{constant and } D = \text{constant} \\
\phi &= \oint B \cdot dA = \oint H \cdot \mu_0 \cdot \mu_r \cdot dA \quad \text{with } H \neq \text{constant and } B = \text{constant}
\end{align*}

**Maxwell’s Equations: electric field source**

One of Maxwell’s Equation focuses on $\psi$, and shows $\psi = \oint E \cdot dA = \frac{Q}{\varepsilon_0 \cdot \varepsilon_r}$, that which demonstrates that an electric charge is the source of an electric field (Figure 1a)\textsuperscript{5,6}. This equation is derived from $\psi = \oint E \cdot dA$, and is applied onto one
electric point charge $Q^{1,6}$: the electric field of an electric point charge is defined by radial coordinates. As such, $E$ is proportionate to the point charge $Q$, but decreases with increased radial distance away from that charge. The electric field dependency on an electric charge is summarized by the following equation $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$ (Figure 3)\(^{1,6}\). Substituting $E$ within the $\psi$ equation, gives rise to Maxwell’s equation that shows that an electric charge it the source of an electric field\(^{1,2,5,9}\).

$$
\psi = \oint E \cdot dA = \oint \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \cdot dA = \frac{Q}{\varepsilon_0} \tag{3}
$$

Maxwell’s Equations: magnetic field source

Analogous to the above, another of Maxwell’s equations focuses on $\phi$, and states that $\phi = \oint B \cdot dA = 0$\(^{1,9}\). This equation illustrates that a magnetic field has no source\(^{1,6}\). This equation is derived based on the argument that $B$ is constant, and does not change. As such, $B$ entering an area ($B_1$) and $B$ exiting ($B_2$) the same area is the same. With $B_1 = B_2$, the following Maxwell equation was derived\(^{1,2,6,9}\):

$$
\phi = \oint B \cdot dA = B_1 \cdot dA - B_2 \cdot dA = 0 \quad \text{with } B_1 = B_2 \text{ as } B \text{ is constant} \tag{4}
$$

$$
\oint B \cdot dA = 0 \tag{5}
$$

Effect on Maxwell’s equation when substituting variable vector with constant vector in electric flux calculation.

Both of the above equations by Maxwell are derived from the equation describing the flux of the electric or magnetic field through a surface $A$, as shown in Equation (3) and Equation (5)\(^{1,2,6,9}\). However, unlike $\psi$, which is usually represented by the variable vector $E$, $\phi$ is most commonly represented by the constant vector $B$\(^{1,2,6,9}\). However, both electric and magnetic flux can ultimately be described by variables of the electric or magnetic fields, as shown in Equation (1) and Equation (2). As such, they also ultimately depend on such variables and have to be taken into account. If not, relationships and dependence of variables are eliminated incorrectly.

For example, as outlined above in Equation (3), Maxwell has shown that $\psi$ can be expressed through $\psi = \oint E \cdot dA = \frac{Q}{\varepsilon_0}$. To derive this equation, the variable $E$ was replaced by the electric charge $Q$ (Figure 3)\(^{1,6}\). He applied Coulomb’s law\(^{1,9}\). However, what if he instead of $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$ used $E = \frac{Q}{\varepsilon_0 \varepsilon_r}$ for the substitution, which is derived from the relationship $D = E \cdot \varepsilon_r \cdot \varepsilon_0$. If $E$ is substituted by $E = \frac{D}{\varepsilon_0 \varepsilon_r}$ in the $\psi$ equation, the following relationship is obtained, in which $\psi$ is only represented by constants and a constant vector.

$$
\psi = \oint E \cdot dA = \oint \frac{D}{\varepsilon_0 \varepsilon_r} \cdot dA \tag{6}
$$

Next, following Maxwell’s argument that he used in deriving his Equation (4): $D$, as well as $\varepsilon_0$ and $\varepsilon_r$ are constants that do not change. As such, $D$ entering an area ($D_1$) and $D$ exiting ($D_2$) the same area is the same. Also, the material constants $\varepsilon_0$ and $\varepsilon_r$ remain the same. As such, $\frac{D_1}{\varepsilon_0 \varepsilon_r} = \frac{D_2}{\varepsilon_0 \varepsilon_r}$. Following Maxwell’s reasoning and calculation (4), the following conclusions could be made:

$$
\psi = \oint E \cdot dA = \oint \frac{D}{\varepsilon_0 \varepsilon_r} \cdot dA \cdot \frac{D_1}{\varepsilon_0 \varepsilon_r} \cdot dA = 0 \quad \text{with } D_1 = D_2 \text{ as } D \text{ is constant} \tag{7}
$$

The above reasoning leading to calculation (7), which Maxwell applied to his calculations with $\phi$\(^{1,9}\), one would obtain an equation shown below (8), which illustrates that an electric field has no source.

$$
\oint E \cdot dA = 0 \tag{8}
$$

| Electric Field Vectors | Magnetic Field Vectors |
|------------------------|------------------------|
| $D = E \cdot \varepsilon_0 \cdot \varepsilon_r$ | $B = \mu_0 \cdot \mu_r$ |
| $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$ | $H = \frac{1}{2\pi r} \frac{Q}{t}$ |

Figure 3. Electric and magnetic fields can be both be described by two vectors\(^{1,6}\). The vectors $D$ and $E$ both describe electric fields, while the vectors $B$ and $H$ describe magnetic fields. $D$ and $B$ are equivalent vectors, as both do not change upon the addition of external materials such as dielectrics or diamagnetics, respectively\(^{1,6}\). In contrast, $E$ and $H$ are equivalent vectors, as they are not influenced by such external materials. The relationships between $D$ and $E$, and between $B$ and $H$ are interlinked with permeability ($\mu_0$ and $\mu_r$) and dielectricity ($\varepsilon_0$ and $\varepsilon_r$) constants, respectively. Both $E$ and $H$ depend on the electric charge $Q$, and its distance $r$ away from that charge, and in the case of $H$ also on how the electric charge $Q$ changes over time $t$\(^{1,6}\).
Although Equation (8) might appear mathematically correct, it is not: $\psi$ is experimentally dependent on the variable $E$, which in turn is dependent on the electric charge $Q$, summarized in Coulomb’s law $E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$ (Figure 3)\textsuperscript{1-2}. Hence, because of $E$’s dependence on $Q$, $\psi(Q)$, $\int E \cdot dA \neq 0$, if $Q$ is not zero.

Maxwell’s equation changes when substituting constant vector with variable vector in magnetic flux calculation

Overall, functions are defined by variables that cannot be ignored as otherwise wrong conclusions are drawn. However, this is what occurred when the equation $\oint B \cdot dA = 0$ was derived\textsuperscript{1-2}: Even though $B$ is a constant, it is dependent on $H$, a variable of the magnetic field. This dependence was not considered in the calculation of one of Maxwell’s equations that analyzed the source of the magnetic flux $\phi$\textsuperscript{19}.

As such, here, the $\phi$ calculation is reanalyzed: $\phi$ is defined by $B$ in a given area $A$, through $\phi = \oint B \cdot dA$\textsuperscript{19}. The magnetic field and the vector $B$ are known to depend on an electric current $I$, which is a variable of $B$ and cannot be ignored\textsuperscript{5,6}. Analogous to Coulomb’s law that describes $E$\textsuperscript{5,6}, the Biot-Savart law, $B = \frac{\mu_0 I}{2\pi r}$, describes the dependence of $B$ on $I$\textsuperscript{5,6}: $B$ is proportional to the electric current $I$ along its path $x$, but decreases with increased radial distance $r$ away from $I$ (Figure 4a)\textsuperscript{5,6}. This is true for $B$ along a straight current-carrying wire. For an arbitrary shape current, for example surrounding a moving charge, the magnetic field (B) is dependent on several variables: the electric current ($I$) and the radial distance away from $I$, as well as $\mu_0$ and $\mu_r$, with $\mu_0$ being the permeability constant of the vacuum and $\mu_r$ the permeability constant of the material$^{-5,13}$. $B$, Illustration of the magnetic induction field going through a surface surrounding a point charge$^{-13}$. The magnetic induction field (B) going through a circular surface $A$ perpendicular to $B$ can be described by the electric current ($I$) and the vectors $r$ and $dx$ defining the circular surface $A$, with $d\mathbf{A} = 2\pi r \cdot dx$. All vectors $r$, $x$ and $B$ are perpendicular to each other.

Figure 4. The magnetic induction field surrounding a wire$^{-5,13}$. a, The magnetic induction field (B) is dependent on several variables: the electric current ($I$) and the radial distance away from $I$, as well as $\mu_0$ and $\mu_r$, with $\mu_0$ being the permeability constant of the vacuum and $\mu_r$ the permeability constant of the material$^{-5,13}$. b, Illustration of the magnetic induction field going through a surface surrounding a point charge$^{-13}$. The magnetic induction field (B) going through a circular surface $A$ perpendicular to $B$ can be described by the electric current ($I$) and the vectors $r$ and $dx$ defining the circular surface $A$, with $d\mathbf{A} = 2\pi r \cdot dx$. All vectors $r$, $x$ and $B$ are perpendicular to each other.
an integral calculation can be carried out (Figure 4b)\(^6\). Since the electric current \( I \) is defined by the rate of flow of an electric charge through \( t \), \( \frac{Q}{2\pi r} \), \( B \) can also be defined as:

\[
\phi = \oint B \cdot da = \oint \frac{\mu_i}{2\pi r} \, dA = \oint \frac{\mu_i}{2\pi r} \, r \cdot ds
\]

with Biot - Savart law and \( I = \frac{Q}{2\pi r} \) (9)

In the integral \( \phi \) equation over an area, the area A can be also be replaced by two vectors representing the area, \( r \) and \( x \), with \( x \) being the vector along the path of the electric charge \( Q \), while \( r \) is the distance between \( Q \) and the position of \( B \) (Figure 4b). All vectors are perpendicular to each other. In case of one electric point charge, the magnetic field at that point can be best described by a circular surface area \( dA = 2\pi \cdot r \cdot dx \) (Figure 4b). Furthermore, since \( ds/dr \) describe the velocity \( v \) of the electric charge \( Q \), the following equation can be derived:

\[
\phi = \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv
\]

Therefore, the following relationship for the magnetic field can be derived:

\[
\phi = \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv
\]

The equation \( \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv \) illustrates that a charge in motion is the source of the magnetic field. As such, one could also write the magnetic field as \( \phi(Q, v) \), analogous to \( \psi(Q) \), which summarizes the electric field’s dependence on the electric charge \( Q \). The above equation summarizes what is already well known in physics: a moving electric charge is the source of a magnetic field\(^5\),\(^6\),\(^7\),\(^8\)-\(^14\): only when no electric charge is in motion, no magnetic field exists\(^5\),\(^6\),\(^7\),\(^8\)-\(^14\). This has been also experimentally verified and is widely accepted\(^5\),\(^6\),\(^7\),\(^8\)-\(^14\).

As a control for the calculations above that lead to \( \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv \) one could also consider Ampere’s law: Ampere’s law states \( \oint B \cdot ds = \mu_i \cdot \oint t = \mu_i \cdot \oint \frac{Q}{2\pi r} dt \)\(^5\). As a simple control, when adding an additional dimension to Ampere’s law through the addition of another vector \( dx \), newly derived equation from above is derived, confirming Equation (11) in a different way \( \phi = \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv \).

### Discussion

Maxwell’s Equations summarize the electromagnetic field\(^5\)-\(^9\). One of the equation’s, \( \oint B \cdot da = 0 \), deduces that a magnetic field has no source\(^1\). This is in contrast to what is experimentally known, namely that a moving electric charge gives rise to the magnetic field\(^6\)-\(^7\),\(^12\)-\(^14\). Also, a magnetic field cannot be created without this source, hence, it has a source, namely a moving electric charge. This is essential, as without this source, no magnetic field exists\(^6\),\(^7\),\(^12\)-\(^14\).

Here, after carefully reevaluating how Maxwell derived the equation \( \oint B \cdot da = 0 \), a shortcoming was identified: A substitution has not been made, eliminating an existing experimental finding\(^1\). Although \( \oint B \cdot da = 0 \) still applies when a charge is not in motion, Maxwell’s equation ignored the dependence of the magnetic field on variables\(^1\). Here, when taking into account such variables and making appropriate substitutions, the adjusted Maxwell’s equation instead demonstrates that the source of a magnetic field is a moving charge: \( \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv \)

This adjustment has consequences on science. For example, Schrödinger’s Equation that describes the probability waves of small particles, has been for example applied to describe the hydrogen atoms characteristics\(^5\)-\(^9\). It used Maxwell’s Equation \( \oint B \cdot da = 0 \) to derive such characteristics\(^6\)-\(^10\). However, with \( \oint B \cdot da = \mu_i \cdot \oint Q \cdot dv \), Schrödinger’s Equation ought to be revisited, as it is only true when a moving charge is not in motion.

Overall, it is important to adjust one of Maxwell’s Equations\(^1\), to have a more complete understanding and representation of the world through formulas. After all, if “God said \( \oint B \cdot da = 0 \), there would be no light”\(^1\).

### Data availability

All data underlying the results are available as part of the article and no additional source data are required.

### Acknowledgements

I am grateful to my colleagues in CIBR for their continuous support.

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