NUCLEON–NUCLEON AND PION–NUCLEON POTENTIALS
FROM PHASE SHIFTS USING QUANTUM INVERSION

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Abstract

NN and $\pi N$ partial wave radial potentials have been generated using the latest SM94–VPI(NN) and FA93–VPI($\pi N$) phase shifts. The potentials are used to determine the deuteron properties and to compute the $^3$H and $^3$He binding energies. The $\varepsilon_1$ mixing angles of SM94–VPI and NY93–Nijmegen differ significantly and inversion potentials yield a $P_d$ of 6.37% and 5.78%, respectively. Under–binding of $^3$H and $^3$He is enhanced by SM94, which signals more nonlocality and/or three–body potential effects than predicted from Nijmegen phase shifts and boson exchange models. The local pion–nucleon $S_{31}$, $P_{31}$ and $P_{33}$ channel potentials have been generated for guidance and to obtain a quantitative impression in r–space of what the $\pi N$ FA93–VPI phase shifts imply.

1. Introduction and Motivation

The differential form of the Blankenbecler–Sugar equation and the radial Schrödinger equation are used as the equations of motion to determine the underlying potentials of elastic nucleon–nucleon and pion–nucleon scattering from boundary conditions. The most recent phase shifts SM94–VPI and FA93–VPI [1] are used as boundary conditions. They determine the partial wave S–matrix and Jost–functions. Our single and coupled channels inversion algorithms use various forms of Gelfand–Levitan and Marchenko integral equations [2].

The motivation of this analysis comes from differing phase shift analyses results of the VPI and Nijmegen groups irrespective of good fits of almost the same two nucleon data [3, 4]. Within 0—300 MeV their phases differ by as much as one or two degrees. The magnitude of the mixing angle $\varepsilon_1$ is not consistent within the two analyses.

We would like to point out that the VPI analysis has important quantitative implications for three–body or nonlocality effects of boson exchange potential models [5]. To show this we present preliminary results of $^3$H and $^3$He binding energy calculations which
show that the SM94 inversion potentials imply a $P_d$ of 6.37% and an under-binding of three body systems by almost 1 MeV.

2. NN Potential Results

In recent years we followed all changes made to the VPI phase shifts [1] and generated local energy independent $r$–space potentials for partial waves with $L \leq 3$ and this separately for $np$ and $pp$ scattering. In the past we also used inversion potentials whose phase shifts were taken from the Nijmegen phase shift analysis and Bonn or Paris potential models. Fig. 1 contains SM94–VPI inversion potentials which reproduce the phases in the subthreshold domain 0–300 MeV well ($\leq 0.02$ degrees) and thereafter follow closely the data ($-1.6$ GeV) with the implication of a repulsive core potential in all channels. There is no noticeable qualitative difference existing to previous results or NY93 inversion potentials.

![Figure 1: NN inversion potentials computed for SM94–VPI phase shifts. $np$ (full lines) $pp$ (dashed lines). All input phase shifts were retrieved from [1].](image)

In Table 1 are summarized static deuteron properties which either are used as in-
Table 1: Deuteron properties from inversion and genuine AV18 potentials.

|                  | SM94   | SP94   | NY93   | AV18   | Bonn–B | Experiment |
|------------------|--------|--------|--------|--------|---------|------------|
| $E_B$ [MeV]      | 2.22459| 2.224638| 2.22460| 2.224575| 2.224653| 2.22458900(22) |
| $A_s$ [fm$^{1/2}$] | 0.8802 | 0.8860 | 0.8802 | 0.8850 | 0.8860 | 0.8802(20) |
| $\eta$           | 0.0256 | 0.0263 | 0.0256 | 0.0250 | 0.0264 | 0.0256(4)   |
| $P_D$            | 6.3695 | 6.6252 | 5.7881 | 5.76   | 5.8152 |            |
| $Q$ [fm$^2$]     | 0.276415| 0.287457| 0.272714| 0.270  | 0.282740| 0.2859     |
| $\mu$ [$\mu_0$] | 0.84342| 0.84197| 0.84673| 0.847  | 0.84658| 0.857406(1) |
| RMS [fm]         | 1.962706| 1.974389| 1.959989| 1.967  | 1.970987| 1.9650(45) |

Table 2: Three body binding energies from inversion potentials.

| Potential    | Channels | $^3$H | $^3$He | Diff. | $^3$H | $^3$He | Diff. |
|--------------|----------|-------|--------|-------|-------|--------|-------|
|              |          |       |        |       |       |        |       |
|              |          | With CIB $V_{np} \neq V_{pp}$           |
| Nijm–II      | 6        | 7.834 | 7.169  | 0.665 | 7.582 | 6.931  | 0.651 |
|              | 28       | 7.852 | 7.191  | 0.661 | 7.600 | 6.952  | 0.648 |
|              | 34       | 7.849 | 7.188  | 0.661 | 7.597 | 6.949  | 0.648 |
| FA91–VPI     | 6        | 7.413 | 6.770  | 0.643 | 7.242 | 6.604  | 0.638 |
|              | 28       | 7.468 | 6.828  | 0.640 | 7.293 | 6.657  | 0.636 |
|              | 34       | 7.465 | 6.824  | 0.641 | 7.290 | 6.654  | 0.636 |
| SM94–VPI     | 6        | 7.550 | 6.894  | 0.656 | 7.225 | 6.589  | 0.636 |
|              | 20       | 7.476 | 6.827  | 0.649 | 7.151 | 6.521  | 0.630 |
|              | 26       | 7.471 | 6.822  | 0.649 | 7.146 | 6.616  | 0.630 |

version input ($E_B$, $A_s$, $\eta$) or are derived from inversion potentials to the quoted phase shifts. Argonne V18 results are reproduced from a recent preprint [6].

Preliminary results of three body calculations for $^3$H and $^3$He by Y. Wu and S. Ishikawa, using inversion potentials, are given in Table 2. This calculation follows their recent article [7]. Not included in the SM94–VPI calculations is the $^3PF_2$ coupled channel potential and thus the channels are reduced to 20 and 24 for $J \leq 2$ and $J \leq 3$ respectively. This lack of potential causes the binding energies to decrease with increasing channel numbers (a complete analysis is in progress). Three body force effects have not been included. As an important result of this calculation a large difference in binding energy prediction between Nijmegen (not shown Bonn-B) and VPI appears. This is primarily caused by the very different $\varepsilon_1$ values. To eliminate this discrepancy we suggest a critical review of the phase shift analyses.
3. $\pi N$ Potential Results

Inversion potentials for pion–nucleon systems already have a long standing tradition [8]. Due to its importance for medium energy physics, this topic has a very rich literature with several excellent solutions [9].

We have generated local r-space inversion potentials to a selected sample of $\pi N$ phase shifts, FA93–VPI, with the purpose to obtain quantitative results to the latest data in the elastic domain. The used input phases for $S_{31}$, $P_{31}$ and $P_{33}$ are from [1]. Potential results are shown in Fig. 2. Radial dimensions, in which potentials significantly differ from zero, are smaller than accustomed from NN potentials. Of particular interest is the $P_{33}$ resonance as shape–resonance which is localized within the radial region $0–0.5$ fm. A deep attractive well has a radius not larger than $0.15$ fm whereas the depth depends simply from the asymptotic continuation. For future $\pi N$ work we see in inversion techniques generally a possibility to tune theoretical models such that they agree perfectly with the experiment. Also, as a didactic mean, the power of inversion procedures cannot be denied.

![Potential Results Graph](image)

Figure 2: Local r–space potentials in selected channels. The numbers 1, 2 and 3 distinguish solutions with different fall–offs towards high energy, $E>1$ GeV.

References

[1] R.A. Arndt, SAID, Telnet: vtinte.phys.vt.edu/physics/quantum
[2] H. Kohlhoff and H. V. von Geramb, in Quantum inversion theory and applications, Lect. Notes in Physics 427, 285 and 314 (1993)
[3] R.A. Arndt, I.I. Strakovsky and R.L. Workman, VPI–preprint (Aug 5 1994) R.A. Arndt et al., Phys. Rev. D45, 3995 (1992)
[4] V.G.J. Stoks et al., Phys. Rev. C48, 792 (1993)
[5] B.F. Gibson, H. Kohlhoff, H.V. von Geramb and G.L. Payne, submitted PRC (1994)

[6] R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, Argonne-preprint (Aug 15 1994)

[7] Y. Wu, S. Ishikawa and T. Sasakawa, Few Body Systems 15, 145 (1993).

[8] F. Tabakin, Phys. Rev. 177, 1443 (1969)

[9] T. Ericson and W. Weise, in Pions and Nuclei, Clarendon, Oxford (1988)