Quark Diagram Analysis of Bottom Meson Emitting Two Pseudoscalar Mesons

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Abstract
We investigate decay modes of $B$ mesons emitting two pseudoscalar ($P$) mesons in the Quark Diagram Scheme at SU(3) level. Several relations among the decay amplitudes are obtained and various branching ratios are predicted.

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I. Introduction

Understanding nonleptonic weak decays is crucial for testing the Standard Model, and on potential effects of physics beyond it. Nonleptonic weak decays of $B$ mesons are interesting because their studies are vital for extracting information about strong interaction interplay with the weak decays. However, due to the non-perturbative strong interactions involved in these decays, the task is hampered by the computation of matrix elements between the initial and the final hadron states. In order to deal with these complicated matrix elements reliably, several methods based on the factorization scheme [1, 2], including probable final state interactions (FSI) and W-exchange process [3], have been employed to predict branching ratios of nonleptonic decays of $B$ mesons with their semileptonic decays. For some channels of $B$ decay, however, the factorization calculations appear to be in clear disagreement with current measurement [4]. The semi-phenomenological analyses in two-body decays of heavy flavor mesons have indicated the presence of large nonfactorizable contributions [5], especially for the color suppressed decays. On the other hand, the experimental progress [6] for weak semileptonic and nonleptonic decays of bottom mesons during these last years is really astounding. A good amount of experimental data now exist [7] for $B$ decays. Earlier, C.W. Chiang, M. Gronau, J. L. Rosner [8, 9], has studied charmless $B \to PP$ decays using flavor SU(3) symmetry.

In the present work, we have studied $B \to PP$ weak decays investigating contributions arising from various quark level processes. Due to the strong interaction interference, like FSI and nonfactorizable contributions, on these processes, it is not possible to calculate their contributions reliably. Therefore, we employ the model independent Quark-Diagram Scheme (hereafter referred to as QDS) [10-12]; wherein decay amplitudes can be expressed independently in terms of a few quark level diagrams- like: a) the external W-emission diagram, b) the internal W-emission diagram, c) the W-exchange diagram, d) the W-annihilation, and d) the W-loop diagram, and parameterize the various quark level contributions to $B$ meson decays. The QDS has already been shown [11] to be a useful technique for the study of $B \to VV$ and $B_c$ decays.

In section II, we construct the weak Hamiltonian responsible for the $B \to PP$ decays. We obtain several sum rules for decay amplitudes in section III. In section IV, we give straightforward relations among branching ratios of $B$ meson decays in various modes. Several predictions of some of crucial decay modes are also presented. Summary and discussion of the results are given in the last section V.

II. Weak Hamiltonian

To the lowest order in weak interaction, the nonleptonic Hamiltonian has the usual current form

$$H_w = \frac{G_F}{\sqrt{2}} J_\mu J^\mu + \text{h.c.} \quad (2.1)$$

Where the weak current $J_\mu$ is given by
The weak Hamiltonian generating the b-quark decays [12] is given by

\[ J_\mu = (\bar{u} \gamma_\mu \tau_3 u) (1 - \gamma_5) \gamma_\mu (\bar{d} b) \text{s} \]  \hspace{1cm} (2.2)
\[ H_w^{\Delta b=1} = [a(B^m P_m^i P_n^k) + d(B^m P_m^i P_n^k)] H_{i(k)}^n 
+ [a'(B^m P_m^i P_n^k) + d'(B^m P_m^i P_n^k)] H_{i(k)}^n 
+ [c(B^m P_m^i P_n^k)] H_i. \] (2.4)

The brackets [.] and (.), respectively, denote antisymmetrization and symmetrization among the indices \( i, k \). In the flavor SU(4) the \( b \)-quark is singlet, and \( u, d, s, \) and \( c \) quarks form a quartet. The Hamiltonian for \( \Delta b=1 \) process belongs to the representations appearing in

\[ 4' \otimes 1 \otimes 4' \otimes 4 = 4' \otimes 4' \otimes 20' \otimes 36^* \] (2.5)

The weak spurion \( H_i, H_{i(k)}^n, \) and \( H_{i(k)}^n \) belong to the \( 4^*, 20' \), and \( 36^* \) representations, respectively [14]. However, we do not use the complete SU(4)-QDS due to SU(4) being badly broken, and exploit the QDS at the SU(3) level through the following SU(4) \( \rightarrow \) SU(3) decomposition:

\[ 4^* \supset 3^* \oplus 1, \]
\[ 20' \supset 8^* \oplus (3^* \oplus 6) \oplus 3, \] (2.6)
\[ 36' \supset 6^* \oplus (15^* \oplus 3^*) \oplus (8 \oplus 1) \oplus 3. \]

The SU(3)-QDS relates \( \Delta C = 1, \Delta S = 0 \) mode with \( \Delta C = 1, \Delta S = -1, \Delta C = 0, \Delta S = -1 \) mode with \( \Delta C = 1, \Delta S = , \) and \( \Delta C = \Delta S = -1 \) mode with \( \Delta C = -1, \Delta S = 0. \)

The tensor \( B' \) denotes the parent B mesons:

\[ B' = B(\bar{u} b), B' = B(\bar{d} b), B' = B(\bar{s} b), B' = B(\bar{c} b). \] (2.7)

The \( P^i_j \) denotes bottomless pseudoscalar mesons with the quark content \( (\bar{q'} q_j) \). The diagonal states are taken to be:

\[ \frac{\pi_0 + \eta \sin \theta_p + \eta' \cos \theta_p}{\sqrt{2}}, \quad \frac{-\pi_0 + \eta \sin \theta_p + \eta' \cos \theta_p}{\sqrt{2}}, \quad -\eta \cos \theta_p + \eta' \sin \theta_p, \eta, \] (2.8)

where the mixing angle \( \theta_p = \theta_{\text{ideal}} - \phi_p; \phi_p = -15.4^\circ \) follow from the quadratic mass formula, and \( \eta_c(\bar{c}c) \), is the charmonium singlet. The physical mesons \( \eta, \eta' \) are defined as follows [7]:

\[ \left( \begin{array}{c} \eta \\ \eta' \end{array} \right) = \left( \begin{array}{cc} \cos \phi_p & -\sin \phi_p \\ \sin \phi_p & \cos \phi_p \end{array} \right) \left( \begin{array}{c} \eta_h \\ \eta_i \end{array} \right). \] (2.9)

where the flavor wave functions of \( \eta_h \) and \( \eta_i \) are given by

\[ \eta_h = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \] (2.10)
\[ \eta_i = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}). \] (2.11)
There exists a straight correspondence between the terms appearing in (2.4) and various quark level diagrams. The terms with coefficients \((a + a')\) represent external W-emission, \((a - a')\) represent internal W-emission, the terms with coefficients \((d - d')\) represent W-exchange and \((d + d')\) for W-annihilation processes. The last term having coefficient \(c\) represents the W-loop penguin diagram contributions. In addition, the following contractions may also be constructed:

\[
+h(B^\prime P_n^k P_m^m) H_{(k)}^n + [h'(B^\prime P_n^k P_m^m)] H_{(k)}^n \tag{2.12}
\]

\[
+f(B^\prime P_n^m P_m^m) + f(B^\prime P_m^m P_n^m) + f(B^\prime P_n^m P_m^m) ] H_{(i)}^i \tag{2.13}
\]

Since, these terms correspond to OZI violating diagrams are expected to be suppressed [15], and hence are ignored in the present scheme.

**III. Decay Amplitudes Sum rules**

Choosing the relevant components of the Hamiltonian in (2.3), we obtain the decay amplitudes for various modes of \(B \to PP\) in the quark-diagram scheme. We categorize them according to the diagrams involved as follows:

**W-external emission:**

\[
\sqrt{2} A(B^\prime \to \eta, \pi^0) = A( B^\prime \to \eta, \pi^0) \tag{3.1}
\]

\[
A( B^\prime \to K^0 \eta_c) = A( B^\prime \to K^0 \eta_c) \tag{3.2}
\]

\[
A( B^\prime \to K^0 \eta_c) = cot\theta_c A( B^\prime \to K^0 \eta_c) \tag{3.3}
\]

\[
A( B^\prime \to K^0 \eta_c) = cot\theta_c A( B^\prime \to K^0 \eta_c) \tag{3.4}
\]

\[
A( B^\prime \to D^\prime D^\prime) = -V_{ud} V_{us} A( B^\prime \to D^\prime D^\prime) \tag{3.5}
\]

**W-internal emission:**

\[
\sqrt{2} A( B^0 \to \eta, \pi^0) = -\sqrt{2} A( B^0 \to \eta, \pi^0) \tag{3.6}
\]

\[
A( B^0 \to K^0 \eta_c) = A( B^0 \to K^0 \eta_c) \tag{3.7}
\]

\[
A( B^0 \to K^0 \eta_c) = cot\theta_c A( B^0 \to K^0 \eta_c) \tag{3.8}
\]

\[
A( B^0 \to K^0 \eta_c) = cot\theta_c A( B^0 \to K^0 \eta_c) \tag{3.9}
\]

\[
A( B^0 \to K^0 D^0) = -V_{us} V_{cd} V_{cs} V_{as} A( B^0 \to K^0 D^0) \tag{3.10}
\]

\[
\sqrt{2} A( B^0 \to \eta, \eta) = -sin\theta_c A( B^0 \to \eta, \eta) \tag{3.11}
\]

\[
A( B^0 \to \eta, \eta) = -cot\theta_c A( B^0 \to \eta, \eta) \tag{3.12}
\]

\[
A( B^0 \to \eta, \pi^0) = A( B^0 \to \eta, \pi^0) \tag{3.13}
\]

\[
\sqrt{2} A( B^0 \to \eta, \eta) = -sin\theta_c A( B^0 \to \eta, \eta) = -cot\theta_c A( B^0 \to \eta, \eta) \tag{3.14}
\]

\[
\sqrt{2} A( B^0 \to \eta, \eta) = cot\theta_c A( B^0 \to \eta, \eta) \tag{3.15}
\]

\[
\sqrt{2} A( B^0 \to \eta, \pi^0) = tan\theta_c A( B^0 \to \eta, \pi^0) \tag{3.16}
\]

\[
\sqrt{2} A( B^0 \to \eta, \pi^0) = cot\theta_c A( B^0 \to \eta, \pi^0) \tag{3.17}
\]

\[
A( B^0 \to K^0 \pi^0) = (1/sin\theta_c) A( B^0 \to K^0 \eta_c) \tag{3.18}
\]

\[
A( B^0 \to K^0 \pi^0) = (1/sin\theta_c) A( B^0 \to K^0 \eta_c) \tag{3.19}
\]

\[
B( B^0 \to \eta, \pi^0) = V_{us} V_{cd} B( B^0 \to \eta, \pi^0) \tag{3.20}
\]
**W-emission (Both):**

\[ A(B \rightarrow \pi^- D^0) = V_{ud}/V_{us} \cdot A(B \rightarrow K^- D^0) \]  \hspace{1cm} (3.21)

\[ A(B_\tau \rightarrow \bar{D}^0 D^+) = -V_{cd}/V_{cs} \cdot A(B_\tau \rightarrow \bar{D}^0 D_s^+) \]  \hspace{1cm} (3.22)

**W-exchange only:**

\[ A(\bar{B}_s^0 \rightarrow \pi^- D^+) = \sqrt{2} A(\bar{B}_s^0 \rightarrow \pi^0 D^0) \]  \hspace{1cm} (3.23)

\[ \sqrt{2} A(\bar{B}_s^0 \rightarrow \bar{D}^0 \pi^0) = A(\bar{B}_s^0 \rightarrow D^- \pi^-) \]  \hspace{1cm} (3.24)

\[ A(\bar{B}_s^0 \rightarrow \pi^0 \pi^0) = A(\bar{B}_s^0 \rightarrow \pi^+ \pi^-) \]  \hspace{1cm} (3.25)

\[ A(\bar{B}_s^0 \rightarrow \bar{D}^0 D^0) = (V_{cb} V_{cs} + V_{ub} V_{us})/V_{cb} V_{cs} \cdot A(\bar{B}_s^0 \rightarrow D^- D^+) \]  \hspace{1cm} (3.26)

\[ A(\bar{B}_s^0 \rightarrow \pi^0 \eta') = \cot \theta_p A(\bar{B}_s^0 \rightarrow \pi^0 \eta) \]  \hspace{1cm} (3.27)

**W-annihilation only:**

\[ A(B_\tau \rightarrow \eta \pi^-) = \cot \theta_p A(B_\tau \rightarrow \eta \pi^-) \]  \hspace{1cm} (3.34)

\[ A(B_\tau \rightarrow \eta K^-) = (\sin \theta_p - \sqrt{2} \cos \theta_p)/(\cos \theta_p + \sqrt{2} \sin \theta_p).A(B_\tau \rightarrow \eta K^-) \]  \hspace{1cm} (3.35)

\[ A(B_\tau \rightarrow \pi^0 K^-) = \sqrt{2}.A(B_\tau \rightarrow K^0 \pi^-) \]  \hspace{1cm} (3.36)

\[ A(B_\tau \rightarrow \bar{K}^0 D^-) = -V_{cd}/V_{cs} \cdot A(B_\tau \rightarrow K^0 D_s^-) \]  \hspace{1cm} (3.37)

\[ A(B_\tau \rightarrow \eta \pi^-) = \sqrt{2}.\sin \theta_p A(B_\tau \rightarrow K^0 \pi^-) \]  \hspace{1cm} (3.38)

**W-external emission and W-exchange:**

\[ A(\bar{B}_s^0 \rightarrow D_s^+ K^+) = -V_{ub} V_{cs}/V_{cd} V_{ub} \cdot A(\bar{B}_s^0 \rightarrow \pi^+ D^-) \]  \hspace{1cm} (3.45)

\[ A(\bar{B}_s^0 \rightarrow D_s^+ K^+) = V_{us}/V_{us} \cdot A(\bar{B}_s^0 \rightarrow \pi^+ D^-) \]  \hspace{1cm} (3.46)

**W-external emission and W-annihilation:**

\[ A(B^- \rightarrow D^- \eta') = \tan \theta_p /\sqrt{2} A(B^- \rightarrow D^- \eta') \]  \hspace{1cm} (3.49)
**W-internal emission and W-exchange:**

\[ A( \bar{B}^0 \to \eta \eta') = \cot \theta_p A( \bar{B}^0 \to \eta \eta') \quad (3.50) \]

\[ A( B^+ \to \eta \eta') = \cos \theta_p A( B^+ \to \eta \eta') \quad (3.51) \]

\[ A( B^0 \to \eta \eta D^0) = \tan \theta_p A( B^0 \to \eta \eta D^0) \quad (3.52) \]

**W-internal emission and W-annihilation:**

\[ A( B^+ \to \bar{B}^0 K^-) = -\frac{V_{cs}}{V_{cd}} A( B^+ \to \bar{B}^0 \pi^-) \quad (3.53) \]

\[ A( B_c^- \to D^- D^0) = V_{cb} V_{ud}/V_{cb} V_{us} A( B_c^- \to D^0 D_s^-) \quad (3.54) \]

\[ A( B_c^- \to D^- \eta') = \tan \theta_p A( B_c^- \to D^- \eta') \quad (3.55) \]

**W-emission Both and W-annihilation:**

\[ \sqrt{2} A( B^+ \to \pi^- \eta') = \tan \theta_p A( B^+ \to \pi^- \eta') \quad (3.56) \]

**IV. Branching ratios:**

The decay rate formula for \( B \to PP \) has the generic form [16]:

\[ \Gamma( B \to PP) = (\text{non-kinematic factors})^2 \times \left( \frac{k}{8\pi m_B^2} \right)^2 \quad (4.1) \]

\( k \) is the 3-momentum of the final states and is given by

\[ k = \left| p_1 \right| = \left| p_2 \right| = \frac{1}{2m_B} \left\{ (m_B^2 - (m_1 + m_2)^2) \left( m_B^2 - (m_1 - m_2)^2 \right) \right\}^{1/2}. \quad (4.2) \]

Several relations are obtained between branching ratios of various possible decay modes of \( B^+ \), \( \bar{B}^0 \), and \( B_c^- \) mesons and are presented in the following. It may be remarked that decay amplitudes given below would remain unaffected by any change of phase of decay amplitudes, which may arise due to elastic FSI. We have used the available experimental values to check the consistency of the relations and to predict the branching ratios of several decay modes. The experimental numbers on the right hand side, given in the parenthesis below the branching relations, are our predictions. These values are obtained by multiplying the known experimental value with the factor (given on R.H.S.), *e.g.* in relation (4.3), branching ratio of \( B( B^- \to \pi^0 D_s^-) \) is predicted by multiplying the experimental value \( B( \bar{B}^0 \to D_s^- \pi^+) = (1.53 \pm 0.35) \times 10^{-5} \) with the factor 0.529. It may be noted that the relations (3.1), (3.6)-(3.9), (3.23) –(3.27), (3.35)-(3.37) and (3.50)-(3.52) follows from QDS at isospin level. There are many other \( B \to PP \) processes which receive contributions from more than one amplitude. The cases in which these do not appear in the same linear combination have not been discussed in the present manuscript.

**W-external emission:**

\[ B(B^- \to \pi^0 D_s^-) = 0.529 B( \bar{B}^0 \to D_s^- \pi^+) \]
\[ (1.6 \pm 0.5) \times 10^{-5} \quad (1.26 \pm 0.21) \times 10^{-5} \]
\[ B( \overline{B}_s^0 \to \pi^- D_s^+) = 18.65 \times 10^{-3} \quad (3.2 \pm 0.5) \times 10^{-3} \quad (3.73 \pm 1.12) \times 10^{-3} \] (4.4)

\[ B( \overline{B}^0 \to K^- \pi^+) = 0.003 \times 10^{-4} \quad (3.61 \pm 0.11) \times 10^{-4} \quad (3.74 \pm 0.42) \times 10^{-4} \] (4.5)

\[ B( \overline{B}_s^0 \to K^+ \pi^-) = 18.63 \times 10^{-6} \quad (4.9 \pm 1.0) \times 10^{-6} \quad (3.61 \pm 0.11) \times 10^{-4} \] (4.6)

\[ B( \overline{B}_s^0 \to K^+ \pi^-) = 0.052 \times 10^{-4} \quad (4.9 \pm 1.0) \times 10^{-6} \quad (3.74 \pm 0.42) \times 10^{-4} \] (4.7)

\[ B( \overline{B}_s^0 \to K^+ D^-) = 0.051 \times 10^{-7} \quad (1.22 \pm 0.20) \times 10^{-7} \] (4.8)

**W-internal emission:**

\[ B( \overline{B}^0 \to \overline{K}^0 \eta_c) = 0.935 \times 10^{-4} \quad (8.9 \pm 1.6) \times 10^{-4} \quad (8.51 \pm 1.21) \times 10^{-4} \] (4.9)

\[ B( \overline{B}^0 \to \overline{K}^0 \eta) = 1.53 \times 10^{-6} \quad (1.1 \pm 0.4) \times 10^{-6} \quad (0.66 \pm 0.04) \times 10^{-6} \] (4.10)

\[ B(B \to \eta_c \pi^-) = 2.115 \times 10^{-4} \quad (2.115) \times 10^{-4} \] (4.11)

\[ B( \overline{B}_s^0 \to \overline{K}^0 \eta_c) = 1.525 \times 10^{-4} \quad (1.525) \times 10^{-4} \] (4.12)

\[ B( \overline{B}_s^0 \to \eta_c \eta) = 0.394 \times 10^{-4} \quad (0.394) \times 10^{-4} \] (4.13)

\[ B( \overline{B}_s^0 \to \overline{K}^0 D^0) = 18.73 \times 10^{-4} \quad (9.74 \pm 1.31) \times 10^{-4} \] (4.14)

\[ B( \overline{B}_s^0 \to D^0 \overline{K}^0) = 0.008 \times 10^{-7} \quad (4.16 \pm 0.56) \times 10^{-7} \] (4.15)

\[ B( \overline{B}_s^0 \to \overline{K}^0 \pi^0) = 18.64 \times 10^{-4} \quad (1.77 \pm 0.15) \times 10^{-4} \] (4.16)

\[ B( \overline{B}^0 \to \eta_c \eta) = 0.301 \times 10^{-4} \quad (0.301) \times 10^{-4} \] (4.17)

\[ B( \overline{B}_s^0 \to \eta_c \eta) = 0.716 \times 10^{-4} \quad (0.716) \times 10^{-4} \] (4.18)

\[ B(B \to \eta_c \pi^-) = 1.097 \times 10^{-4} \quad (1.097) \times 10^{-4} \] (4.19)

\[ B( \overline{B}^0 \to \eta_c \eta) = 0.624 \times 10^{-4} \quad (0.624) \times 10^{-4} \] (4.20)

\[ B( \overline{B}_s^0 \to \eta_c \eta) = 0.581 \times 10^{-4} \quad (0.581) \times 10^{-4} \] (4.21)

\[ B( \overline{B}_s^0 \to \overline{K}^0 \pi^0) = 1.69 \times 10^{-4} \quad (1.69) \times 10^{-4} \] (4.22)

\[ B( \overline{B}_s^0 \to \overline{K}^0 \pi^0) = 1.69 \times 10^{-4} \quad (1.69) \times 10^{-4} \] (4.23)

\[ B( \overline{B}_s^0 \to \overline{K}^0 \pi^0) = 1.69 \times 10^{-4} \quad (1.69) \times 10^{-4} \] (4.23)
**W-emission (Both):**

\[
B(B' \rightarrow \pi^- D^0_+) = 19.42 \times \frac{B(B' \rightarrow K^- D^0_+)}{10^{-3}}
\]
\[
(4.84 \pm 0.15) \times 10^{-3} \quad (7.14 \pm 0.64) \times 10^{-3}
\]

\[
B(B'_s \rightarrow D^0_+ D^-) = 0.053 \times B(B'_s \rightarrow D^0_+ D^-)
\]

\[
(4.25)
\]

**W-exchange only:**

\[
B(\bar{B}^0 \rightarrow \pi^0 \eta) = 1.53 \times \frac{B(\bar{B}^0 \rightarrow \pi^0 \eta)}{10^{-6}}
\]
\[
< 1.5 \times 10^{-6} \quad (1.84 \pm 0.92) \times 10^{-6}
\]

\[
B(\bar{B}^0 \rightarrow \pi^0 \pi^0) = 0.500 \times B(\bar{B}^0 \rightarrow \pi^+ \pi^-)
\]
\[
B(\bar{B}^0 \rightarrow \pi^+ D^0) = 2.00 \times B(\bar{B}^0 \rightarrow \pi^0 D^0)
\]
\[
B(\bar{B}^0 \rightarrow \pi^0 D^0) = 0.500 \times B(\bar{B}^0 \rightarrow D^- \pi^+)
\]
\[
B(\bar{B}^0 \rightarrow D^0 D^0) = 4 \times B(\bar{B}^0 \rightarrow D^- D^0)
\]
\[
B(\bar{B}^0 \rightarrow \pi^0 \pi^0) = 0.026 \times B(\bar{B}^0 \rightarrow K^- K^+)
\]
\[
B(\bar{B}^0 \rightarrow \pi^0 D^0) = 0.026 \times B(\bar{B}^0 \rightarrow K^- D^0_+)
\]
\[
(7.8 \pm 1.0) \times 10^{-7}
\]

\[
B(\bar{B}^0 \rightarrow \pi^0 \eta) = 0.460 \times \frac{B(\bar{B}^0 \rightarrow \pi^0 \eta)}{10^{-5}}
\]
\[
(1.38 \pm 0.18) \times 10^{-5}
\]

\[
B(\bar{B}^0 \rightarrow \pi^0 \eta) = 3.62 \times 10^{-7} \times \frac{B(\bar{B}^0 \rightarrow \pi^0 \eta)}{10^{-7}}
\]
\[
(1.09 \pm 0.14) \times 10^{-7}
\]

**W-annihilation:**

\[
B(B'_s \rightarrow \eta^- \pi^+) = 0.66 \times B(B'_s \rightarrow \eta^- \pi^+)
\]
\[
B(B'_s \rightarrow \eta^- K^-) = 0.005 \times B(B'_s \rightarrow \eta^- K^-)
\]
\[
B(B'_s \rightarrow \eta^- K^-) = 0.500 \times B(B'_s \rightarrow \eta^- K^-)
\]
\[
B(B' \rightarrow K^0 K^-) = 19.03 \times B(B' \rightarrow K^0 K^-)
\]
\[
(1.36 \pm 0.27) \times 10^{-6} \quad (4.40 \pm 0.19) \times 10^{-6}
\]

\[
B(B' \rightarrow K^0 D^-) = 19.48 \times B(B' \rightarrow K^0 D^-)
\]
\[
B(B'_s \rightarrow \eta^- \pi^+) = 1.203 \times B(B'_s \rightarrow \eta^- \pi^+)
\]
\[
B(B'_s \rightarrow \eta^- \pi^+) = 0.044 \times B(B'_s \rightarrow \eta^- \pi^+)
\]
\[
B(B'_s \rightarrow \eta^- K^-) = 3.21 \times 10^{-4} \times B(B'_s \rightarrow \eta^- K^-)
\]

**W-external emission and W-exchange:**

\[
B(\bar{B}^0 \rightarrow K^- K^+) = 0.050 \times B(\bar{B}^0 \rightarrow \pi^+ \pi^-)
\]
\[
(3.3 \pm 0.9) \times 10^{-5} \quad (2.56 \pm 0.12) \times 10^{-7}
\]
\[ B(\bar{B}^0 \rightarrow D^+_s D^+_s) = 17.92 B(\bar{B}^0 \rightarrow D^- D^+) \]
\[ (1.04 \pm 0.35\%) \quad (0.38 \pm 0.06\%) \] (4.46)

\[ B(\bar{B}^0 \rightarrow D^+_s K^-) = 0.049 B(\bar{B}^0 \rightarrow \pi^- D^+) \]
\[ (1.31 \pm 0.06) \times 10^{-4} \] (4.47)

\[ B(\bar{B}^0 \rightarrow \pi^+ D^-) = 0.055 B(\bar{B}^0 \rightarrow D^+_s K^+) \]
\[ (1.65 \pm 0.38) \times 10^{-5} \] (4.48)

**W-external emission and W-annihilation:**

\[ B(B^+ \rightarrow D^- \eta) = 1.55 B(B^+ \rightarrow D^- \eta^+) \] (4.49)

**W-internal emission and W-exchange:**

\[ B(\bar{B}^0 \rightarrow \eta \eta^+) = 0.430 B(\bar{B}^0 \rightarrow \eta \eta) = 0.330 B(\bar{B}^0 \rightarrow \eta \eta^+) \] (4.50)

\[ B(\bar{B}^0 \rightarrow \eta D^0) = 1.543 B(\bar{B}^0 \rightarrow \eta D^0) \]
\[ (2.02 \pm 0.35) \times 10^{-4} \quad (1.93 \pm 0.35) \times 10^{-4} \] (4.51)

**W-internal emission and W-annihilation:**

\[ B(B^- \rightarrow D^0 K^-) = 18.97 B(B^- \rightarrow D^0 \pi^-) \] (4.52)

\[ B(B^- \rightarrow D^- D^0) = 19.48 B(B^- \rightarrow D^0 D^+) \] (4.53)

\[ B(B^- \rightarrow D^- \eta) = 1.524 B(B^- \rightarrow D^- \eta^+) \] (4.54)

**W-emission Both and W-annihilation:**

\[ B(B^- \rightarrow \pi^- \eta) = 1.530 B(B^- \rightarrow \pi^- \eta^+) \]
\[ (4.0 \pm 0.32) \times 10^{-6} \quad (4.13 \pm 1.38) \times 10^{-6} \] (4.55)

The branching ratios thus obtained are generally found to be consistent with experimental values [7].

**V. Summary and Conclusion**

Reasonably good amount of data is now available for B meson decaying to two pseudoscalar mesons. We have given the amplitude relations among various \( B \rightarrow PP \) decays in SU(2) and SU(3)-QDS. Relations among the branching ratios for the various observed decay modes have been given in Sec. IV, and are found to hold true within the experimental limits. Our conclusions are as follows:

- The relations (4.3), (4.4), (4.5), (4.9), (4.13), (4.24), (4.51), and (4.55) are consistent with the experimental data within the errors. The relations based upon SU(2) are expected to be more reliable as compared to the SU(3) symmetry, however the SU(3) breaking may produce 10–15% deviation in the results [8]. Using the branching ratios of observed decays, we also predict the branching ratio of several decays (see (4.8), (4.13), (4.14), (4.15) and (4.16)). It may further be noted that the relations (4.10) and (4.23) have the same order of branching ratios but different magnitudes. The inconsistency between the branching ratios may be attributed to penguin contributions.
to these decays. The same is true for branching relations (4.12) and (4.22) and shall be treated in a separate communication.

- Generally, W-exchange and W-annihilation diagrams are expected to be suppressed due to the helicity and color considerations. In the SU(3) based analysis of charmless decays of B mesons, these diagrams are expected to be suppressed because of the factor \( \frac{f_B}{m_B} \sim 5\% \). Though in some cases their contribution may not be negligible, particularly when nonfactorizable soft gluon exchanges occur. Experimentally, only a few branching ratios and upper limits are available for the decays occurring through these processes. In case of the W-exchange diagrams, using the available branching ratios for \( B^0 \to \pi^0 \eta \) and \( B^0 \to K^+ D^0_s^+ \) decays, we predict the branching ratio of \( B_s^0 \to \pi^0 D^0 \), \( B^0 \to \pi^0 \eta^* \), \( B_c^0 \to D^0 \eta \), and \( B_s^0 \to D^0 \eta \) to be \((1.31 \pm 0.06) \times 10^{-5}\), \((1.65 \pm 0.38) \times 10^{-5}\), \((7.53 \pm 1.00) \times 10^{-7}\) respectively. The prediction of branching ratio in relation (4.26) seems to be consistent with experimental upper limit within errors.

- We also give the predictions for B decays, which involve both W-emission and W-exchange/annihilation processes. The present data are insufficient to explicitly separate their relative strengths. The possible phase difference of the reduced amplitudes, due to the strong interaction interference, further makes it difficult. Using the measured branching ratio \( B(\overline{B}^0 \to \pi D^+ \eta) \) and \( B(\overline{B}^0_s \to D_s^+ K^+) \), we predict the branchings of \( \overline{B}_s^0 \to D_s^+ K^- \) and \( \overline{B}^0 \to \pi^+ D^- \) decay to be \((1.51 \pm 0.20) \times 10^{-4}\) and \((1.65 \pm 0.38) \times 10^{-5}\) respectively. Here also, branching ratio for the decay emitting two charm mesons (see (4.46)) is found to be of the same order, but less than the observed value. Some of the charm conserving decays such as \( \overline{B}^0 \to K K^+ \) and \( \overline{B}^0 \to \pi^+ \pi^- \) are likely to occur through penguins [17].

- The decays \( \overline{B}^0 \to \eta \eta \), \( B^0 \to \eta \eta \), and \( \overline{B}^0 \to \eta \eta \) are suppressed in the present quark diagram scheme. Experimentally, upper limits on their branching are \( 1.7 \times 10^{-6} \), \( 1.0 \times 10^{-6} \), and \( 1.2 \times 10^{-6} \) respectively. These decays may arise through the OZI violating channels, and therefore their accurate measurements would decide the extent of such contributions. Similarly, \( \overline{B}^0 \to K^0 \eta \), \( \overline{B}^0_s \to K^0 \eta \), \( B^0 \to D^0 \eta \), \( B_s \to K^+ D^- \) \( B_s^0 \to K^+ \pi^- \) decays are also forbidden in our analysis. If these decays are seen, their measurements may be employed to decide the strength of the penguin diagrams. This is consistent with the observations of C.W. Chiang, M. Gronau, J. L. Rosner [9]. These decays, therefore, can be preferable modes for the future experimental investigations.

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References:

1. M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34, 103 (1987); N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D 39, 799 (1989); M. Wirbel, Prog. Part. Nucl. Phys. 21, 33 (1988).

2. A. Deandrea et al., Phys. Lett. B 318, 549 (1993); T. E. Browder and K. Honscheid, Prog. Part. Nucl. Phys. 35, 81 (1995); A. C. Katoch and R. C. Verma, Phys. Rev. D 52, 1717 (1995); A. C. Katoch and R. C. Verma, Int. J. Mod. Phys. A 11, 1291 (1996); A. Ali, Acta Phys. Pol. 27, 3529 (1996); M. Beneke and G. Bichella, Phys. Rev. D 53, 4991 (1996); A. Ali et al., ibid. 58, 094009 (1998); Zhi-zhong Xing, Phys. Lett. B 443, 365 (1998); X. G. He, W. S. Hou, and K. C. Yang, Phys. Rev. Lett. 83, 1100 (1999); Y. H. Chen et al., Phys. Rev. D 60, 094014 (1999); P. Ball et al., “B Decays at the LHC,” CERNTH/2000-101, hep-ph/0003238; M. Suzuki, Phys. Rev. D 62, 091502 (R) (2000); M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000); M. Neubert, “Introduction of B Physics,” hep-ph/0001334.

3. J. F. Donoghue et al., Phys. Rev. Lett. 77 (1996) 2178; P. Zenczykowski, Acta Phys. Pol. B 28 (1997) 1605; M. Neubert, Phys. Letts. B 424 (1998) 152; M. Suzuki, The final state interaction in the two body nonleptonic decay of a heavy particle, hep-ph/9807414 (1998); P. Zenczykowski, Phys. Lett. B 460, 390 (1999).

4. M. Beneke et al., hep-ph/0007256; CLEO Collaboration, D. Cronin-Hennessy et al., hep-ex/0001010.

5. J. M. Soares, Phys. Rev. D 51, 3518 (1995); A. N. Kamal et al., ibid. 53, 2506 (1996); R. C. Verma, Phys. Lett. B 365, 377 (1996); Z. Phys. C 69, 253 (1996); A. C. Katoch et al., J. Phys. G 23, 807 (1997); Z. Phys. C 76, 311 (1997); K. Terasaki, “Nonfactorizable long distance contributions in colorsuppressed decays of B mesons,” Report No. YITP-97-52; K. Terasaki, Phys. Rev. D 59, 114001 (1999); F. M. Al-Shamali and A. N. Kamal, ibid. 59, 054020 (1999); R. C. Verma, Indian J. Pure Appl. Phys. 38, 395 (2000).

6. J.N. Butler, “Spectroscopy and Decays of Charm and Bottom,” FERMILAB-Conf.97/052 (1997); Fermilab E835 Collaboration, M. Ambrogiani et al., Phys. Rev. D 62, 032004 (2000).

7. C. Amsler et al., Particle Data Group, Physics Letters B 667, 1 (2008).

8. C.W. Chiang et al., Physi. Rev. D 70, 034020 (2004); Phys. Rev. D 69, 034001 (2004) and references therein.

9. C.W. Chiang et al., Physi. Rev. D 68, 074012 (2003) and references therein.
10. D. Zeppenfeld, Z. Phys. C 8, 77 (1981); L.L. Chau and H. Y. Cheng, Phys. Rev. Letts. 59, 958 (1987); L.L. Chau and H. Y. Cheng, Phys. Letts. B 197, 244 (1987); M. Savage and M. Wise, Phys. Rev D 39, 3346 (1989); 40, 3127(E) (1989); L.L. Chau et al., Phys. Rev. D 43, 2176 (1991); M. Gronau et al., Phys. Rev. D 50, 4529 (1994); 52, 6356 (1995); 52, 6374 (1995).

11. R.C. Verma and A. Sharma, Phys. Rev. D 64, 114018 (2001); R.C. Verma and A. Sharma, Phys. Rev. D 65 (2002) 114007, and references therein.

12. S. Oh, Phys. Rev. D 60, 034006 (1999).

13. A. Khodjamirian and R. Ruckel, Nucl. Phys. B (Proc. Supp) 38 (1994) 396.

14. S. M. Sheikholeslami and R. C. Verma, Int. J. Mod. Phys. A 7, 3691 (1992).

15. A. A. Petrov, “Final state interactions: from strangeness to beauty”, Invited plenary talk at the Chicago Conference on Kaon Physics (Kao'n99), June 21-26, (1999), hep-ph/9909312.

16. R. Dhir and R.C. Verma, J. Phys. G. 34 (2007) 637; R. Dhir, N. Sharma and R.C. Verma, J. Phys. G. 35 (2008) 085002.

17. Guohuai Zhu, arXiv:1106.4709v1 [hep-ph].