NUMERICAL SIMULATION OF EXCITATION OF SOLAR OSCILLATION MODES FOR DIFFERENT TURBULENT MODELS

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ABSTRACT

The goal of this research is to investigate how well various turbulence models can describe the physical properties of the upper convective boundary layer of the Sun. Accurate modeling of these turbulent motions is necessary for understanding the excitation mechanisms of solar oscillation modes. We have carried out realistic numerical simulations using a hyperviscosity approach and various physical Large-Eddy Simulation (LES) models (Smagorinsky and dynamic models) to investigate how the differences in turbulence modeling affect the damping and excitation of the oscillations and their spectral properties and to compare with observations. We have first calculated the oscillation power spectra of radial and nonradial modes supported by the computational box with the different turbulence models, followed by calculation of the work integral input to the modes to estimate the influence of the turbulence model on the depth and strength of the oscillation sources. We have compared these results with previous studies and with the observed properties of solar oscillations. We find that the dynamic turbulence model provides the best agreement with the helioseismic observations.

Subject headings: convection — methods: numerical — Sun: oscillations

1. INTRODUCTION

The Sun has a resonant cavity between the surface, where the density decreases, and the interior, where the sound speed increases, and standing sound waves are trapped in this cavity. The dominant restoring force of these oscillations (also called p-mode waves) is given by the gradient of pressure. Surface and internal gravity modes (f- and g-modes) are also present in the Sun (see review of Christensen-Dalsgaard. 2002). Solar p-mode oscillations are observed continuously by the Global Oscillation Network Group (GONG; Hill et al. 1994) and from the Solar and Heliospheric Observatory (SOHO; Scherrer et al. 1995). The observed oscillations have extremely small amplitude and hence can be described as linear perturbations. An important task is to understand the interaction between the oscillations and the turbulent convection. The first to propose turbulent motions as the main acoustic sources were Goldreich & Keeley (1977) and Kumar (1997). Further studies (Balmforth 1992; Goldreich et al. 1994; Kumar 1994; Nordlund & Stein 1998) showed that turbulent motions stochastically excite the resonant modes via Reynolds stresses (turbulent pressure) and entropy fluctuations (gas pressure). Many analytical formulations have been proposed (Balmforth 1992; Goldreich et al. 1994; Samadi & Goupil 2001). The problem is that these models introduce free parameters related to the choice of the turbulent medium model. The advantage of three-dimensional realistic numerical simulations comes from the possibility of calculating all the quantities related to turbulent convection.

The modal excitation sources occur close to the surface, mainly in the intergranular lanes and near the boundaries of granules (Stein & Nordlund 2001). Thus, an accurate modeling of the turbulence motions is necessary to understand the excitation mechanisms of solar oscillation modes. Unfortunately, direct numerical simulation (DNS) of these high Reynolds-number turbulent motions is not achievable. It is not possible to resolve all the motion scales even with the most advanced computers. The large-eddy simulation (LES) method allows overcoming the Reynolds-number limits possible for DNS by modeling the effects of the smallest turbulent scales. The most widely used subgrid-scale models (SGS) are the Smagorinsky eddy viscosity model (Smagorinsky 1963) and its dynamic formulation (Germano et al. 1991; Moin et al. 1991).

The objective of this research is to study the influence of turbulence models on the excitation mechanisms by means of realistic numerical simulations. To show the influence on the damping and excitation of the oscillations, we have compared the results obtained using hyperviscosity approach and different subgrid-scale models (Smagorinsky and dynamic models). We note that a correct choice of the turbulence model is also important in many other astrophysical simulations (e.g., Schekochihin et al. 2005; Prentice & Dyt 2003).

2. NUMERICAL MODEL

We use a three-dimensional, compressible, nonlinear radiative-hydrodynamics code developed by A. Wray for simulating the upper solar photosphere and lower atmosphere. This code takes into account several physical phenomena: compressible fluid flow in a highly stratified medium, radiative energy transfer between the fluid elements, and a real-gas equation of state. The equations
we solve are the grid-cell average (henceforth called “average”) conservations of mass (1), momentum (2), and energy (3):

\[
\frac{\partial \rho}{\partial t} + (\rho u_i) _i = 0, \quad (1)
\]

\[
\frac{\partial \rho u_i}{\partial t} + [\rho u_i u_j + (P_i + \rho \tau_{ij})] = -\rho \phi, \quad (2)
\]

\[
\frac{\partial E}{\partial t} + [E u_i + (P_{ij} + \rho \tau_{ij}) u_j - (\kappa + \kappa_T) T_j + F^{rad}_i] = 0, \quad (3)
\]

where \(\rho\) denotes the average mass density, \(u_i\) the Favre-averaged velocity, and \(E\) the average total energy density \(E = \frac{1}{2} \rho u_i u_i + \rho e + \rho \phi\), where \(\phi\) is the gravitational potential and \(e\) is the Favre-averaged internal energy density per unit mass. \(F^{rad}_i\) is the radiative flux, calculated by solving the radiative transfer equation, and \(P_{ij}\) is the average stress tensor \(P_{ij} = (p + 2 \mu \delta_{ij}) / 3 - \mu (u_i u_j + u_j u_i)\), where \(\mu\) is the viscosity. The gas pressure \(p\) is a function of \(e\) and \(\rho\) through a tabulated equation of state (Rogers et al. 1996); \(\tau_{ij}\) is the Reynolds stress, \(\kappa\) is the molecular thermal conductivity, and \(\kappa_T\) is the turbulent thermal conductivity.

We have carried out simulations using three different turbulence models: two subgrid-scale models and a hyperviscosity approach.

The subgrid-scale models used are the original Smagorinsky model (Smagorinsky 1963) and its dynamic formulation (Germano et al. 1991), herein called simply the dynamic model. The Reynolds stresses \(\tau_{ij}\) are modeled by the usual eddy viscosity assumption using the large-scale stress tensor \(S_{ij} = (u_i u_j + u_j u_i) / 2\):

\[
\tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2 \nu_T \left( S_{ij} - \frac{1}{3} S_{kk} \delta_{ij} \right), \quad (4)
\]

The eddy viscosity \(\nu_T\) is defined by the Smagorinsky model (Smagorinsky 1963), \(\nu_T = C_S \Delta^2 \overline{S}\); and the trace of the subgrid Reynolds stress \(\tau_{kk}\) is modeled using the Yoshizawa’s expression (Yoshizawa 1986): \(\tau_{kk} = 2 C_S \Delta^2 \overline{S}^2\), where \(\overline{S} \equiv (2 \Delta y S_{yy})^{1/2}\) and \(\Delta \equiv (\Delta x \Delta y \Delta z)^{1/3}\).
Application of the eddy viscosity, $\nu_T$, and the subgrid kinetic energy, $\tau_{ik}$, results in the compressible Smagorinsky formulation (Moin et al. 1991):

$$\tau_{ij} = -2C_S \Delta^2 |S| (S_{lj} - u_{lk} \delta_{lj}/3) + 2C_C \Delta^2 |S|^2 \delta_{lj}/3,$$

(5)

where $C_S$ is the classical Smagorinsky coefficient as used in incompressible flow; and $C_C$ is a coefficient associated with the trace of the subgrid Reynolds stress (which is absent in incompressible flow). The two parameters must be specified in some way. For the original Smagorinsky model, constant values are used. In the present work, we have chosen $C_S = 0.2$ and $C_C = 0.1$. For the dynamic formulation, these parameters are not constant and determined following the procedure of Moin et al. (1991). A planar average is used to perform the average needed by the dynamic procedure. The turbulent Prandtl number was taken as unity to set $\alpha_T$. The molecular viscosity $\mu$ and thermal conductivity $\kappa$ were neglected as their solar values are exceedingly small.

In the hyperviscosity approach, the Reynolds stresses $\tau_{ij}$ are not modeled directly, but their effects are assumed to be accounted for by an implicit linear smoothing of each dependent variable (density, momentum density, energy density, etc.); the smoothing used leaves polynomials of higher order. In the following parts, the “minimal” hyperviscosity approach describes runs with weak smoothing, just enough to stabilize the simulation, while the “enhanced” hyperviscosity approach describes runs with a slightly stronger smoothing.

We simulate the upper layers of the convection zone using $66 \times 66 \times 40$ grid cells. The region extends $6 \times 6$ Mm horizontally and from 2.5 Mm below the visible surface to 0.5 Mm above the surface. This computational box has been chosen to directly compare with the previous results obtained by Stein & Nordlund (2001), who used a numerical viscosity model not related to a particular turbulence model.

3. KINETIC ENERGY OF RADIAL MODES

First we studied how the kinetic energy is dissipated for the different subgrid-scale models and the hyperviscosity approach. To do this, we calculated the oscillation power spectra of radial modes. Those modes are extracted by horizontal averaging of the vertical velocity and Fourier transforming in time. The results presented here (Figs. 1 and 2) have been obtained from simulations of 60 hr of solar time using instantaneous snapshots saved every 30 s.

Three oscillation modes can be clearly seen in the spectra of the horizontally averaged, depth-integrated kinetic energy obtained with all three turbulence models (Fig. 2, left); they correspond to the peaks in the kinetic energy. The smallest resonant mode frequency is 2.6 mHz. This mode is excited along all the depth. The resonant frequencies supported by the computational box are 2.6, 4.0, 5.6, 7.8, and 9.5 mHz. These frequencies are very close to the values obtained by Stein & Nordlund (2001). We see that for the hyperviscosity approaches and the dynamic model the modes within the computation box are excited with the same magnitude, whereas the Smagorinsky model yields a lower magnitude.

The kinetic energy spectra as a function of frequency and depth (Fig. 1) confirm that the dissipation is weaker with the minimal hyperviscosity approach. In this case, almost all the dissipation is numerical. As a result, the kinetic energy is higher for high frequencies in comparison with the results obtained with the other turbulence models. The spectra obtained with the enhanced hyperviscosity approach and the dynamic model show a slightly higher dissipation compared to the calculation with the minimal hyperviscosity approach. Moreover, the dissipation applied by the hyperviscosity approaches and the dynamic model does not directly affect the three oscillation modes because the dissipation scale is smaller than the wavelength of the acoustic modes. With the Smagorinsky model, the excitation of the modes is weaker, the scale of the dissipation is close to the scale of the third mode, and the patterns do not extend above 6 mHz (compared to 12 mHz with the hyperviscosity approaches and
10 mHz with the dynamic model). The cutoff frequency obtained with the Smagorinsky model is very similar of that obtained by Stein & Nordlund (2001).

These results show that the spectral kinetic energy of the radial modes is rather similar for the hyperviscosity approach and dynamic model, while the Smagorinsky model gives lower values.

4. KINETIC ENERGY OF NONRADIAL MODES

The nonradial modes have been extracted by first performing two-dimensional spatial Fourier transforms of the vertical velocity at each time step to obtain a power spectrum as a function of horizontal wavenumber \( k_x^2 = k_x^2 + k_z^2 \), then taking a Fourier transform in time. We have especially considered modes with horizontal wavelength corresponding to the box size \( L = 6 \, \text{Mm} \). This corresponds to an angular size of \( l \approx 740 \, (k_h = 1 \, \text{Mm}^{-1}) \). Figure 2 (right) shows the power spectra for the different turbulent models. The leftmost peak corresponds to the surface gravity (\( f \)) mode, while the others correspond to acoustic (\( p \)) modes. The \( f \)-mode peak has the same frequency independent of depth in the computational box (2.8 mHz). Note that the Smagorinsky model gives the lowest mode power. The hyperviscosity approach and the dynamic model give very similar results: the modes have approximately the same magnitude. The influence of the turbulence models is thus similar for the radial and nonradial modes.

5. CALCULATION OF THE \( p \)-MODE EXCITATION RATES

Nordlund & Stein (2001) presented a formalism for analyzing the interaction of convection with purely radial oscillations. This formalism seems to be the most accurate because it accounts for phase relations between pressure fluctuations (both turbulent and gas) and the mode compression factor \( \delta \xi / \delta r \) (Stein et al. 2004). In this section the mode excitation rate is calculated using the same method. The rate of energy input to the modes per unit surface area (ergs cm\(^{-2}\) s\(^{-1}\)) is

\[
\frac{\Delta \langle E_\omega \rangle}{\Delta t} = \frac{\omega^2}{8 \Delta \nu E_\omega} \int \left( \delta P^* \right)^2 \, \delta E_\omega / \delta \nu ,
\]

where \( \delta P^* \) is the Fourier transform of the nonadiabatic total pressure; \( \delta \) in front of the pressure means that one computes so-called pseudo-Lagrangian fluctuations relative to a fixed mass radial coordinate system (Nordlund & Stein 2001); \( \Delta \nu \) is the frequency interval for the Fourier transform; and \( \xi_\omega \) is the mode displacement for the radial mode of angular frequency \( \omega \). It is obtained from the eigenmode calculations for a standard solar model of Christensen-Dalsgaard et al. (1996). His spherically symmetric model S gives 35 radial modes that provide much denser frequency spectrum in comparison with the three resonant modes obtained within the simulation box. \( E_\omega \) is the mode energy per unit surface area (ergs cm\(^{-2}\)) defined as

\[
E_\omega = \frac{1}{2} \omega^2 \int dr \rho \xi_\omega^2 \left( \frac{r}{R} \right)^2 .
\]

We have calculated the rate of stochastic energy input to the modes for the entire solar surface in order to compare with the observed results (Fig. 3). The rate of energy input to the solar modes is obtained by multiplying the rate for the modes in the simulation by the ratio of the mode mass of the solar modes to the mode mass in the computational domain (Stein & Nordlund 2001). The numerical results have been compared with the observed rates for \( l = 1 \) modes, obtained from the GOLF (Global Oscillation at Low Frequencies; Gabriel et al. 1997), BISON (Birmingham Solar Oscillation Network; Chaplin et al. 1996), and GONG data (Baundin et al. 2005). It is important to note that there are large uncertainties at high frequencies, related to the complex determination of several parameters: the absolute calibration, the height in the solar photosphere where the modes are observed, the width of the modal lines, etc. (Baundin et al. 2005).

Comparison of the observed and calculated rates shows that the Smagorinsky model gives values too low, more than one decade below the observations, but note that the values of the calculated rates depend on the choice of model parameters \( C_S \) and \( C_C \). At low frequencies, i.e., below 3 mHz, the best agreement is obtained using the dynamic model. Using the minimal hyperviscosity approach, the main discrepancy lies between 2 and 3 mHz. We can see a peak in this range which is much less pronounced in the observed results. This peak is present also in the enhanced hyperviscosity approach; however, it is lower. It is not present in the dynamic model. Above 3 mHz, both the enhanced hyperviscosity and dynamic model provide good results. The observed and calculated rates show a plateau between 3 and 4 mHz. This plateau has been obtained by Roca Cortes et al. (1999) using observed data from GOLF for \( l = 0-3 \). In both the measurements and the calculations, the plateau is followed by a slower decline. The origin of this plateau in the solar power spectrum in terms of mixing length theory was discussed by Gough (1980) and Balmforth & Gough (1990). The distributions of the integrand of the work integral as a function of depth and frequency (Fig. 4) can explain the presence of the peak between 2 and 3 mHz. The distributions are similar to the results obtained by Stein & Nordlund (2001). Most driving is concentrated between the surface and 500 km depth at around 3-4 mHz. We can see that the main differences between

![Fig. 3.—Comparison of observed and calculated rate of stochastic energy input to modes for the entire solar surface (ergs s\(^{-1}\)). Observed distributions: SoHo-GOLF is shown by circles, BiSON by squares; and GONG for \( l = 1 \) by triangles (Baundin et al. 2005).](image-url)
the magnitudes obtained with the different turbulence models are located in the region around 2 Mm. We observe that the excitation obtained using minimal hyperviscosity (Fig. 4, top left) is high between 2 and 3 mHz at each vertical position. Conversely, the excitation decreases with depth using the enhanced hyperviscosity approach and the dynamic model. The excitation magnitude becomes low when the depth is greater than 2 Mm with the enhanced hyperviscosity approach and below 1.5 Mm with the dynamic model. These variations explain the difference in magnitude of the peak between 2 and 3 mHz for the rate of stochastic energy input. The absence of the peak in the observations is consistent with the excitation lying close to the visible surface of the Sun.

6. CONCLUSION

The goals of this research were to investigate how well various turbulence models can describe the convective properties of the upper boundary layer of the Sun and to study the excitation and damping of acoustic oscillations. Results obtained with the hyperviscosity approaches have been compared with those obtained with the Smagorinsky and dynamic turbulence models. We have seen that the dissipation is very high for the Smagorinsky model with the values of the parameters $C_S$ and $C_C$ used, while the hyperviscosity approaches and dynamic model give similar results. In addition, we find that the dynamic turbulence model provides the best overall agreement with observations.

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