DIFFRACTION 2000: New Scaling Laws in Shadow Dynamics

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New scaling structure for the shadow corrections in elastic scattering from deuteron at high energies has been presented and discussed. It is shown that this structure corresponds to the experimental data on proton(antiproton)-deuteron total cross sections. The effect of weakening for the inelastic screening at super-high energies has been theoretically predicted.

1. INTRODUCTION

Experimental and theoretical studies of high-energy particle interaction with deuterons have shown that total cross section in scattering from deuteron cannot be treated as equal to the sum of total cross sections in scattering from free proton and neutron even in the range of asymptotically high energies. Glauber was the first who proposed an explanation of this effect. Using the methods of diffraction theory, quasiclassical picture for scattering from composite systems and eikonal approximation for high-energy scattering amplitudes he had found long ago [1] that total cross section in scattering from deuteron can be expressed with the formula

$$\sigma_d = \sigma_p + \sigma_n - \delta\sigma,$$

(1)

where

$$\delta\sigma = \delta\sigma_G = \frac{\sigma_p \cdot \sigma_n}{4\pi} \langle \frac{1}{r^2} \rangle_d.$$

(2)

Here $\sigma_d, \sigma_p, \sigma_n$ are the total cross sections in scattering from deuteron, proton and neutron, $\langle \frac{1}{r^2} \rangle_d$ is the average value for the inverse square of the distance between the nucleons (separation of the nucleons) inside of a deuteron, $\delta\sigma_G$ is the Glauber shadow correction describing the effect of eclipsing or the screening effect in the recent terminology. The Glauber shadow correction has a quite clear physical interpretation. This correction originates from elastic rescattering of an incident particle on the nucleons in a deuteron and corresponds to the configuration when the relative position of the nucleons in a deuteron is such that one casts its "shadow" on the other [1].

It was soon understood that in the range of high energies the shadow effects may arise due to inelastic interactions of incident particle with the nucleons of deuteron [2–6]. Therefore inelastic shadow correction had to be added to the Glauber one.

A simple formula for the total (elastic plus inelastic) shadow correction had been derived by Gribov [4] in the assumption of Pomeron dominance in the dynamics of elastic and inelastic interactions. However it was observed that the calculations performed by Gribov formula did not met the experimental data: the calculated values of the inelastic shadow correction over-estimated the experimental values.

The idea that the Pomeron dominance is not justified at most at the recently available energies has been explored in the papers [5]. The authors of Refs [5] argued that account of the triple-reggeon diagrams for six-point amplitude in addition to the triple-pomeron ones allowed them to obtain a good agreement with the experiment. Alberi and Baldracchini replied [6] and pointed out that discrepancy between theory and experiment cannot be filled by taking into account the triple-reggeon diagrams: in fact, it needed to modify the dynamics of the six-point amplitude with more complicated diagrams than the triple
Regge ones. This means that up to now we have not in the framework of Regge phenomenology a clear understanding for the shadow corrections in elastic scattering from deuteron.

The theoretical understanding of the screening effects in scattering from any composite system has the fundamental importance, because the structure of shadow corrections is deeply related to the structure of composite system as itself. At the same time the shadow corrections structure displays the new sites of the fundamental dynamics.

Here we are concerned with the study of shadow dynamics in scattering from deuteron in some details. The new scaling characteristics of scattering from the constituents in the composite systems, having a clear physical interpretation, are established.

2. SCATTERING FROM DEUTERON, THREE-BODY FORCES AND SINGLE DIFFRACTION DISSOCIATION

In our papers [7–9] the problem of scattering from two-body bound states was treated with the help of dynamic equations obtained on the basis of single-time formalism in QFT [7]. As has been shown in [8,9], the total cross-section in the scattering from deuteron can be expressed by the formula

$$\sigma_{tot}(s) = \sigma_{hp}(s) + \sigma_{hn}(s) - \delta\sigma(s),$$

where $$\sigma_{hp}, \sigma_{hn}$$ are the total cross-sections in scattering from deuteron, proton and neutron,

$$\delta\sigma(s) = \delta\sigma^{el}(s) + \delta\sigma^{inc}(s),$$

$$\delta\sigma^{el}(s) = \frac{\sigma_{hp}(s)\sigma_{hn}(s)}{4\pi(R_d^2 + B_{hp}(s) + B_{hn}(s))}, \hat{s} = \frac{s}{2},$$

$$B_{hN}(s)$$ is the slope of the forward diffraction peak in the elastic scattering from nucleon, 1/$$R_d^2$$ is defined by the deuteron relativistic formfactor [8], $$\delta\sigma^{el}$$ is the shadow correction describing the effect of eclipsing or screening effect during elastic rescatterings of an incident hadron on the nucleons in a deuteron.

The quantity $$\delta\sigma^{inc}$$ in our approach represents the contribution of the three-body forces to the total cross-section in the scattering from deuteron. The definition of three-body forces in relativistic quantum theory see the recent paper [10] and references therein.

For simplicity, we consider the model proposed in [7] where the imaginary part of the three-body forces scattering amplitude has the form

$$Im F_0(s; \vec{p}_1, \vec{p}_2, \vec{p}_3; \vec{q}_1, \vec{q}_2, \vec{q}_3) = f_0(s) \exp \left\{ -\frac{R_d^2(s)}{4} \sum_{i=1}^{3} (\vec{p}_i - \vec{q}_i)^2 \right\}, \quad (6)$$

This model assumption is not so significant for our main conclusions but allows one to make some calculations in a closed form.

From the analysis of the problem of high-energy particle scattering from deuteron we have derived the formula relating one-particle inclusive cross-section with the imaginary part of the three-body forces scattering amplitude (see details in [10] and references therein). In this way we can establish a deep connection of inelastic shadow correction with one-particle inclusive cross-section which allows one to express the inelastic shadow correction via total single diffractive dissociation cross-section. In fact, let us define the total single diffractive dissociation cross-section by the formula

$$\sigma^{\varepsilon\varepsilon}_sd(s) = \pi \int_{M_{min}^2}^{s} \frac{dM_X^2}{s} \int_{(\varepsilon M_X^2)}^{(\varepsilon^+ M_X^2)} dt dM_X^2,$$

Here we have especially labeled the total single diffractive dissociation cross-section by the index $$\varepsilon$$. It’s clear the parameter $$\varepsilon$$ defines the range of integration in the variable $$M_X^2$$. Unfortunately up today there is no common consent in the choice of this parameter. However we would like to point out the exceptional value for the parameter $$\varepsilon$$ which naturally arises from our approach. Namely, let us put $$\varepsilon^{ex} = \sqrt{2\pi/2M_NR_d}$$, then we define the exceptional total single diffractive dissociation cross-section $$\sigma^{\varepsilon^{ex}_sd}_s(s) = \sigma^{\varepsilon^{ex}}_sd(s)|_{\varepsilon=\varepsilon^{ex}}$$. As a result we obtain

$$\delta\sigma^{inc}(s) = 2\sigma^{\varepsilon^{ex}_sd}_s(s) a^{inc}(x_{inel}), \quad (8)$$

where

$$a^{inc}(x_{inel}) = \frac{x_{inel}}{(1 + x_{inel})^{5/2}}.$$
\[ x_{inel} = \frac{R_d^2(s)}{R_d^2} = \frac{2B_{el}(s)}{R_d^2}. \] (10)

See the definition of \( B_{el} \) in [1].

Here is just the place to rewrite the elastic shadow correction \([3]\) in a similar form
\[ \delta \sigma_{el}(s) = 2\sigma_{el}(s)a^{el}(x_{el}), \] (11)
where
\[ a^{el}(x_{el}) = \frac{x_{el}}{1 + x_{el}}. \] (12)
\[ x_{el} = \frac{2B_{hN}(s)}{R_d^2} = \frac{R_h^2(s)}{R_d^2}. \] (13)

The obtained expressions for the shadow corrections have a quite transparent physical meaning, both the elastic \( a^{el} \) and inelastic \( a^{inel} \) scaling functions have a clear physical interpretation. The function \( a^{el} \) measures out a portion of elastic rescattering events among of all events during an interaction of an incident particle with a deuteron as a whole, and this function attached to the total probability of elastic interaction of an incident particle with a separate nucleon in a deuteron. Correspondingly the function \( a^{inel} \) measures out a portion of inelastic events of inclusive type among of all events during an interaction of an incident particle with a deuteron as a whole, and this function attached to the total probability of single diffraction dissociation of an incident particle on a separate nucleon in a deuteron. The scaling variables \( x_{el} \) and \( x_{inel} \) have a quite clear physical meaning too. The dimensionless quantity \( x_{el} \) characterizes the effective distances measured in the units of “fundamental length”, which the deuteron size is, in elastic interactions, but the similar quantity \( x_{inel} \) characterizes the effective distances measured in the units of the same “fundamental length” during inelastic interactions.

The functions \( a^{el} \) and \( a^{inel} \) have a quite different behaviour: \( a^{el} \) is a monotonous function while \( a^{inel} \) has a maximum at the point \( x_{inel}^{\max} = 2 \) where \( a^{inel}(x_{inel}^{\max}) = 2/3\sqrt{3} \). This fact results an interesting physical effect of weakening of the inelastic eclipsing (screening) at superhigh energies. The energy at the maximum of \( a^{inel} \) can easily be calculated from the equation \( R_h^2(s) = 2R_d^2 \).

Account of the real part for the hadron-nucleon elastic scattering amplitude modifies the scaling function \( a^{el} \) in the following way
\[ a^{el}(x_{el}) \rightarrow a^{el}(x_{el}, \rho_{el}) = a^{el}(x_{el}) \frac{1 - \rho_{el}^2}{1 + \rho_{el}^2}. \] (14)

The scaling function \( a^{inel} \) is not modifying.

3. COMPARISON TO THE EXPERIMENTAL DATA

We have tried to make a preliminary comparison of the new structure for the shadow corrections in elastic scattering from deuteron to the existing experimental data on proton-deuteron and antiproton-deuteron total cross sections.

In the first step we analysed the experimental data on antiproton-deuteron total cross sections. We have used our theoretical formula describing the global structure of antiproton-proton total cross sections \([10,12]\) as \( \sigma_{tot}^{\bar{p}p} = \sigma_{tot}^{p\bar{p}} = \sigma_{tot}^{\bar{p}N} \). The new fit to the data on the total single diffraction dissociation cross sections in \( \bar{p}p \) collision with the formula from \([10]\) has been made as well using a wider set of the data (see \([13–18]\)), \( R_d^2 \) was considered as a single free fit parameter. Our fit yielded \( R_d^2 = 66.61 \pm 1.16 GeV^{-2} \). The fit result is shown in Fig. 1. The fitted value for the

![Figure 1](image_url)

Figure 1. The total antiproton-deuteron cross-section versus \( \sqrt{s} \) compared with the theory. Statistical and systematic errors added in quadrature.

\( R_d^2 \) satisfies with a good accuracy to the equality
\[ R_d^2 = 2/3 r_{d,m}^2 (r_{d,m} = 1.963(4) fm) \].
After that it was very intriguing for us to make a comparison to the data on proton-deuteron total cross sections where $R_2^d$ has to be fixed from the previous fit to the data on antiproton-deuteron total cross sections. As in the previous fit we supposed $\sigma_{pp}^\text{tot} = \sigma_{pn}^\text{tot} = \sigma_{pN}^\text{tot}$ and $\sigma_{pp}^\text{tot}$ had been taken from our global description of proton-proton total cross sections [10,12]. We also assumed that $B_{pN}^{el} = B_{\bar{p}N}^{el}$. So, in this case we have not any free parameters. The result of the comparison is shown in Fig. 2. As you can see the correspondence of the theory to the experimental data is quite remarkable apart from the resonance region. The resonance region requires a more careful consideration than that performed here.

4. SUMMARY AND DISCUSSION

In this report we have been concerned with a study of shadow corrections to the total cross section in scattering from deuteron. The dynamic apparatus based on the single-time formalism in QFT has been used as a tool and subsequently applied to describe the properties of high-energy particle interaction in scattering from two-body composite system. In this way we found the new structures for the total shadow correction to the total cross section in scattering from deuteron.

First of all, it was observed that the total shadow correction inherits the general structure of total cross section and contains two inherent parts as well, elastic part and inelastic one. This partitioning has been performed explicitly in the framework of our approach. It turns out that the elastic part can be expressed through the elastic scaling (structure) function and the fundamental dynamical quantity, which is the total elastic cross section in scattering from an isolated constituent (nucleon) in the composite system (deuteron). At the same time the inelastic part is expressed through the inelastic scaling (structure) function and the fundamental dynamical quantity, which is the total single diffractive dissociation cross section in scattering from an isolated constituent in the composite system too. Thus the general formalism in QFT makes it possible to define properly the dynamics of particle scattering from composite system and express this dynamics in terms of the fundamental dynamics of particle scattering from an isolated constituent in the composite system and the structure of the composite system as itself.

What was probably the most important, which we have discovered in the work, elastic and inelastic structure functions have a quite different behaviour. The inelastic structure function has a maximum and tends to zero at infinity, while the elastic structure function is a monotonic function and tends to unity at infinity. This is the most significant difference between the elastic and inelastic structure functions and it has far reaching physical consequences. This difference manifests itself in the effect of weakening of inelastic eclipsing (screening) at superhigh energies. What does it mean physically? To understand it let’s combine the elastic shadow correction and the first terms in Eq. (3) for the hadron-deuteron total cross section

$$\sigma_{hd}^\text{tot} = 2\sigma_{hN}^{inel} + 2\sigma_{hN}^{el}(1 - a^{el}) - \delta\sigma^{inel}. \quad (15)$$

We have in this way that asymptotically

$$\sigma_{hd}^\text{tot} = 2\sigma_{hN}^{inel}, \quad s \rightarrow \infty. \quad (16)$$

Probably the generalization of this result to any many-nucleon systems (nuclei) looks like

$$\sigma_{hA}^\text{tot} = A\sigma_{hN}^{inel}, \quad s \rightarrow \infty. \quad (17)$$
Besides, we would like to emphasize the different range of variation for the elastic and inelastic structure functions

\[ 0 \leq a^{el} \leq 1, \quad 0 \leq a^{inel} \leq 2/3\sqrt{3}. \quad (18) \]

The energy, where the inelastic shadow correction has a maximum, has to be calculated from the equality \( R_0^2(s_m) = 2R_d^2 \). Taking \( R_d^2 = 66.61 \) from the fit and \( R_0^2(s) \) from the paper [12], we obtain \( \sqrt{s_m} = 9.01 \times 10^4 \text{GeV} = 901 \text{PeV} \). Of course such energies are not available at the recently working accelerators. However, we always have a room for a speculative discussion. For example, let us consider the proton as a two-body (quark-diquark) composite system. From the experiment it is known the value for the charge radius of the proton \( r_{p,ch} = 0.88 \text{fm} \). If we put \( R_p^2 = 2/3r_{p,ch}^2 \), then resolving the equation \( R_0^2(s_p) = R_p^2 \) we obtain \( \sqrt{s_p} = 1681 \text{GeV} \). This is just the energy of Tevatron.

Our analysis shows that the magnitude of inelastic shadow correction is about 10 percent from elastic one at recently available energies. That is why the precise measurements of hadron-deuteron total cross-sections at high energies are the most important.

ACKNOWLEDGEMENTS

It is my great pleasure to express thanks to the Organizing Committee for the kind invitation to attend the Workshop “Diffraction 2000”. I would like especially to thank Roberto Fiore, László Jenkovsky, Alessandro Papa, Franca Morrone and Maxim Kotsky for warm and kind hospitality throughout the Workshop.

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