The Application of Principal Component Analysis in Stability Evaluation of Foundation Pit

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Abstract. The foundation pit stability is the key and difficult point in foundation pit engineering research. The stability and safety of the foundation pit should be ensured not only by its excavation, but also by the surrounding buildings. Because the stability of foundation pit is decided by many complicated factors, it has a certain randomness. This paper adopts the principal component analysis method combined with engineering examples to calculate and analyze the six influencing factors that affect the stability of foundation pit: land subsidence, groundwater level, pile top settlement, horizontal displacement of pile top, reinforced concrete brace and inclination. The results show that inclination and pile top displacement have the most important influence on the stability of foundation pit. According to this study, the principal component analysis method has higher accuracy in the stability evaluation of foundation pit and has certain reference value, which can provide theoretical basis for the stability evaluation of foundation pit in related projects.

1. Introduction

The instability of foundation pit is one of the major engineering problems frequently encountered in geotechnical engineering at present. Due to the stress redistribution of soil mass after foundation pit excavation, the soil mass becomes less stable, which can easily cause deformation of foundation pit and surrounding building foundation. Once the deformation is not effectively controlled, engineering accidents are easy to occur. At present, the main methods of foundation pit stability analysis include limit equilibrium method, limit analysis method and finite element method based on strength reduction. A lot of research on foundation pit stability has been done at home and abroad: Through field observation and numerical simulation, Clough and O’rourke [1] pointed out that the stability and deformation of the foundation pit were greatly affected by the undrained shear strength of the soil layer where the foundation pit was located. Li zhijia [2] used the strength reduction finite element method of FLAC3D software to study the factors affecting the stability of the foundation pit without support, and obtained the sensitivity ranking of the factors through the grey relational degree analysis. Bai jiyong [3] simulated the excavation stage of the foundation pit through MIDAS GTS/NX finite element software, and analyzed the stability of the foundation pit by comparing it with field monitoring data. Li ze et al. [4] studied the stability of foundation pit by combining limit analysis upper limit theory, finite element discretization thought and mathematical programming method.
Since the factors affecting the stability of foundation pit are random to some extent, the safety coefficient is absolutized by numerical simulation. In order to effectively avoid the use of absolutization of safety coefficient, mathematical statistics in system analysis, such as variance analysis, regression analysis, grey relational analysis, principal component analysis and so on, are commonly used. Since the 1980s [5], principal component analysis has been continuously applied to various thematic comprehensive evaluations, making it one of the most widely used comprehensive evaluation methods at present. But using the principal component analysis (PCA) to the study of evaluation in the foundation pit stability influence factors is relatively small, in view of this, this article relying on the ChengDu metro line 17 was carried out on the monitoring data of deep foundation pit engineering, and using principal component analysis (PCA) to evaluate the influence factors of the stability of foundation pit, its purpose is to determine the degree of the influence of different influence factors on the stability of foundation pit, as to provide theoretical basis for related engineering.

2. Principle of principal component analysis

2.1. Basic principle
Principal component analysis (PCA) is a multidimensional multivariate statistical analysis method that selects a few comprehensive variables from multiple variables. The principal component concept was extended from only for non-random variables to random variables. The basic principle is to use a few unrelated comprehensive variables to reflect the information of multiple original variables as comprehensively as possible, eliminate the correlation of the original variables, improve the credibility, and the statistical results can effectively explain the real problems.

2.2. Basic steps
Determine the indicators and collect the data of each indicator. The collected data set:

\[
X = \begin{bmatrix}
x_{i1} & x_{i2} & \cdots & x_{ij} \\
x_{i1} & x_{i2} & \cdots & x_{ij} \\
\vdots & \vdots & \ddots & \vdots \\
x_{i1} & x_{i2} & \cdots & x_{ij}
\end{bmatrix}
\]  

(1)

Where: \(x_{ij}\) is the \(i\)th monitoring data under the \(j\)th influencing factor; \(i\) represents the number of samples, and \(j\) represents the number of influencing factors.

The original data is standardized, and the function direction of the evaluation results is standardized according to the influencing factors:

\[
s_{y} = \frac{x_{y} - \min(x_{y})}{\max(x_{y}) - \min(x_{y})} \quad \text{(Positive effect)}
\]  

(2)

\[
s_{y} = \frac{\max(x_{y}) - x_{y}}{\max(x_{y}) - \min(x_{y})} \quad \text{(Negative effect)}
\]  

(3)

Where: \(s_{y}\) is \(x_{y}\) the standardized data.
Determine the correlation coefficient matrix \(R\):
Where: $r_{ij}$ is the relationship coefficient between influencing factor $i$ and influencing factor $j$.

Find the eigenvalues and eigenvectors. According to the characteristic equation of the correlation coefficient matrix $R$: $|R - \lambda E| = 0$, find the eigenvalues $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_m$, $i=1,2,3,...,m$, and corresponding normalized feature vectors $(u_{i1}, u_{i2}, ..., u_{im})$, $i=1,2,3,...,m$.

Determine the number of principal components. The first $p$ principal components were identified according to the cumulative contribution rate of each principal component, contribution: $\alpha = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i}$, refers to the proportion of the eigenvalue of a principal component in all the eigenvalues. The higher the contribution rate is, the more information the principal component carries. Demand characteristic root $\lambda_i > 1$, cumulative contribution: $\alpha_i = \sum_{i=1}^{p} \frac{\lambda_i}{m}$ to meet the above $75\%$, the cumulative contribution rate, the greater the $p$ before shows a main component to contain more original information.

The comprehensive score is calculated and compared. $P$ principal components are expressed as:

$$f_{ik} = u_{i1}s_1 + u_{i2}s_2 + ... + u_{im}s_m, (i=1,2,3,...,p)$$  

Then the comprehensive evaluation function is obtained:

$$F = f_1a_1 + f_2a_2 + ... + f_pa_p$$

Type: $F$ is the $i$th a comprehensive score of evaluation objects, for the first $i$ first $k$ principal component score evaluation objects, $a_i$ is the contribution for the $i$th evaluation objects, $S_k$ is the standardization of original data values, $u_{ik}$ is the first $k$ principal component score coefficient.

### 3. Case analysis

Chengdu metro line 17 Fengxi station is located at the west side of the intersection of Fengxi avenue and Nanxun avenue, and is laid along Fengxi avenue. The buildings around the station are dense, and the nine branches of the armed police on the northeast side are about 10 meters away from the main body of the station. To the southeast of the station, Rongxing garden community is about 5.6 meters away from the main body of the station. Xiangyang mingyuan residential area on the south side of the station is about 30 meters away from the main body of the station. The layout of some monitoring points in the foundation pit of Fengxi station is shown in Figure 1. According to the construction requirements and research needs of the project, the construction monitoring of the open excavation foundation pit covers the underground and ground buildings, structures, underground pipelines, surface and roads in the station foundation pit and the main and secondary affected areas on both sides of the outer edge of the structure line. The main monitoring items of foundation pit include: horizontal displacement of pile top, deformation of support structure, ground settlement, support axial force, underground water level, displacement and settlement of surrounding buildings, pipeline deformation, vertical displacement of column pile, horizontal displacement of column pile. In this paper, the monitoring sections of 9# and 11# are selected for analysis, and the ground settlement, pile top settlement, horizontal displacement of pile top, underground water level, axial force of concrete support, and inclination are selected as evaluation indexes. The stability of foundation pit is evaluated by principal component analysis method. Table 1 ~ Table 2 are the monitoring data of foundation pit monitoring section of 9# and 11#. 

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1j} \\ r_{21} & r_{22} & \cdots & r_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ r_{ij} & r_{i2} & \cdots & r_{ij} \end{bmatrix}$$  

(4)
Figure 1. Layout of local monitoring points of foundation pit

Table 1. 9th monitoring section monitoring data (mm)

| Land subsidence | Pile top settlement | Horizontal displacement of pile tip | Underground water level | Concrete supports the axial force | Inclination |
|-----------------|---------------------|-------------------------------------|-------------------------|---------------------------------|-------------|
| 0.53            | 0.57                | 3.4                                 | 19.2                    | 20.59                           | 0.03        |
| 4.54            | 3.37                | 1.7                                 | 26.024                  | 26.79                           | 0.19        |
| 4.01            | 4.18                | 6.2                                 | 29.44                   | 33.78                           | 0.14        |
| 6.48            | 3.44                | 2.9                                 | 33.01                   | 38.73                           | 0.1         |
| 8.98            | 1.2                 | 0.4                                 | 31.69                   | 38.12                           | 0.42        |
| 10.49           | 3.63                | 0.3                                 | 31.96                   | 41.25                           | 0.08        |

Table 2. 11th monitoring section monitoring data (mm)

| Land subsidence | Pile top settlement | Horizontal displacement of pile tip | Underground water level | Concrete supports the axial force | Inclination |
|-----------------|---------------------|-------------------------------------|-------------------------|---------------------------------|-------------|
| 0.01            | 0.01                | 1.2                                 | 17.659                  | 10.58                           | 0.43        |
| 0.28            | 1.08                | 4.4                                 | 21.555                  | 14.2                            | 0.22        |
| 6.24            | 0.22                | 7.2                                 | 28.412                  | 12.36                           | 0.04        |
| 4.19            | 0.57                | 7.4                                 | 31.71                   | 21.31                           | 0.47        |
| 6.95            | 0.32                | 6.7                                 | 31.92                   | 35.1                            | 0.34        |
| 11.3            | 2.6                 | 2.9                                 | 31.87                   | 41.27                           | 0.36        |

3.1. Standardized treatment of indicators of various influencing factors
Based on the monitoring results of foundation pit of Chengdu metro line 17, the data from Table 1 to Table 2 are standardized by formula (1) to eliminate the dimension of indicators, as shown in Table 3~Table 4.

Table 3. 9th monitoring section monitoring data standardization

| Land subsidence | Pile top settlement | Horizontal displacement of pile tip | Underground water level | Concrete supports the axial force | Inclination |
|-----------------|---------------------|-------------------------------------|-------------------------|---------------------------------|-------------|
| 1.0000          | 1.0000              | 0.4746                              | 0.0000                  | 1.0000                          | 1.0000      |
| 0.5974          | 0.2244              | 0.7627                              | 0.4941                  | 0.6999                          | 0.5897      |
| 0.6506          | 0.0000              | 0.0000                              | 0.7415                  | 0.3616                          | 0.7179      |
| 0.4026          | 0.2050              | 0.5593                              | 1.0000                  | 0.1220                          | 0.8205      |
| 0.1516          | 0.8255              | 0.9831                              | 0.9044                  | 0.1515                          | 0.0000      |
| 0.0000          | 0.1524              | 1.0000                              | 0.9240                  | 0.0000                          | 0.8718      |
Table 4. 11” monitoring section monitoring data standardization

| Land subsidence | Pile top settlement | Horizontal displacement of pile tip | Underground water level | Concrete supports the axial force | Inclination |
|-----------------|--------------------|------------------------------------|------------------------|----------------------------------|------------|
| 1.0000          | 1.0000             | 1.0000                             | 0.0000                 | 1.0000                           | 0.0930     |
| 0.9761          | 0.5869             | 0.4839                             | 0.2732                 | 0.8820                           | 0.5814     |
| 0.4482          | 0.9189             | 0.0323                             | 0.7540                 | 0.9420                           | 1.0000     |
| 0.6298          | 0.7838             | 0.0000                             | 0.9853                 | 0.6504                           | 0.0000     |
| 0.3853          | 0.8803             | 0.1129                             | 1.0000                 | 0.2010                           | 0.3023     |
| 0.0000          | 0.0000             | 0.7258                             | 0.9965                 | 0.0000                           | 0.2558     |

3.2. Find the correlation matrix R for each influence index

SPSS was used to obtain the correlation coefficient matrix R of each monitoring section from the processed sample data:

9” monitoring section:

\[
R_1 = \begin{bmatrix}
1.000 & 0.294 & -0.677 & 0.849 & 0.905 & 0.438 \\
0.294 & 1.000 & 0.271 & 0.555 & 0.469 & -0.237 \\
-0.677 & 0.271 & 1.000 & -0.267 & -0.337 & -0.397 \\
0.849 & 0.555 & -0.267 & 1.000 & 0.969 & 0.387 \\
0.905 & 0.469 & -0.337 & 0.969 & 1.000 & 0.314 \\
0.438 & -0.237 & -0.397 & 0.387 & 0.314 & 1.000
\end{bmatrix}
\]

11” monitoring section:

\[
R_2 = \begin{bmatrix}
1.000 & 0.607 & 0.229 & 0.837 & 0.831 & -0.078 \\
0.607 & 1.000 & -0.294 & 0.353 & 0.668 & 0.100 \\
0.229 & -0.294 & 1.000 & 0.654 & 0.043 & -0.330 \\
0.837 & 0.353 & 0.654 & 1.000 & 0.734 & 0.027 \\
0.831 & 0.668 & 0.043 & 0.734 & 1.000 & 0.314 \\
-0.078 & 0.100 & -0.330 & 0.027 & 0.314 & 1.000
\end{bmatrix}
\]

3.3. Principal components are determined according to the eigenvalues of R

SPSS was used to calculate the eigenvalues, eigenvectors and variance contribution rates of the correlation coefficient matrix, as shown in Table 5~ Table 6.

It can be seen from Table 5 that the main components with characteristic roots greater than 1 were selected from the 9” monitoring section, and two principal components were extracted. The cumulative contribution rate of the first two principal components was 84.197%. The variance of the first principal component was 3.440, and its contribution rate was 57.331%. The variance of the second principal component was 1.612, and the contribution rate was 26.866%.

It can be seen from Table 6 that the main components with characteristic roots greater than 1 were selected from the 11” monitoring section, and two principal components were extracted. The cumulative contribution rate of the first two principal components was 79.626%. The variance of the first principal component was 3.108, and its contribution rate was 51.805%. The variance of the second principal component was 1.669, and the contribution rate was 27.821%.
Table 5. 9\textsuperscript{th} total variance interpretation of monitoring sections

| Composition | Initial eigenvalue | Extract the sum of the squares of the loads |
|-------------|-------------------|-------------------------------------------|
|             | Total             | Percentage variance | Cumulative (%) | Total | Percentage variance | Cumulative (%) |
| 1           | 3.440             | 57.331              | 57.331         | 3.440 | 57.331              | 57.331         |
| 2           | 1.612             | 26.866              | 84.197         | 1.612 | 26.866              | 84.197         |
| 3           | 0.657             | 10.952              | 95.150         | 0.657 | 10.952              | 95.150         |
| 4           | 0.257             | 4.286               | 99.435         | 0.257 | 4.286               | 99.435         |
| 5           | 0.034             | 0.565               | 100.000        | 0.034 | 0.565               | 100.000        |
| 6           | -7.054E-16        | 1.176E-14           |               |       |                     |                |

Table 6. 11\textsuperscript{th} total variance interpretation of monitoring sections

| Composition | Initial eigenvalue | Extract the sum of the squares of the loads |
|-------------|-------------------|-------------------------------------------|
|             | Total             | Percentage variance | Cumulative (%) | Total | Percentage variance | Cumulative (%) |
| 1           | 3.108             | 51.805              | 51.805         | 3.108 | 51.805              | 51.805         |
| 2           | 1.669             | 27.821              | 79.626         | 1.669 | 27.821              | 79.626         |
| 3           | 0.902             | 15.025              | 94.651         | 0.902 | 15.025              | 94.651         |
| 4           | 0.231             | 3.858               | 98.509         | 0.231 | 3.858               | 98.509         |
| 5           | 0.089             | 1.491               | 100.000        | 0.089 | 1.491               | 100.000        |
| 6           | -1.651E-16        | -2.752E-15          |               |       |                     |                |

3.4. Determination of principal components

The principal component equation is established by formula (5) and formula (6) through load coefficients of each monitoring section, and then the comprehensive evaluation function is obtained, as shown in Table 7 ~ Table 10.

Table 7. 9\textsuperscript{th} Monitoring principal component coefficient of section

| Component                        | Primary principal component | Second principal component |
|----------------------------------|------------------------------|----------------------------|
| Land subsidence                  | 0.5227                       | -0.0930                    |
| Concrete supports the axial force| 0.5130                       | 0.1544                     |
| Underground water level          | 0.5097                       | 0.1959                     |
| Pile top settlement              | 0.2194                       | 0.6632                     |
| Horizontal displacement of pile tip inclination | -0.2908 | 0.5258 |
|                                  | 0.2668                       | -0.4613                    |

Table 8. 9\textsuperscript{th} Monitor the principal component score function of the section

| Component                        | formula                      |
|----------------------------------|------------------------------|
| The first principal component score function | \( f_1 = 0.5227z_1 + 0.5130z_2 + 0.5097z_3 + 0.2194z_4 - 0.2908z_5 + 0.2668z_6 \) |
| The second principal component score function | \( f_2 = -0.0930z_1 + 0.1544z_2 + 0.1959z_3 + 0.6632z_4 + 0.5258z_5 - 0.4613z_6 \) |
| Comprehensive score function    | \( F=57.331\%*f_1+26.866\%*f_2 \) |
Table 9. Monitoring principal component coefficient of section

| Component                                    | Primary principal component | Second principal component |
|----------------------------------------------|------------------------------|---------------------------|
| Land subsidence                              | 0.5356                       | -0.0455                   |
| Concrete supports the axial force            | 0.5228                       | 0.2040                    |
| Underground water level                      | 0.5110                       | -0.2857                   |
| Pile top settlement                          | 0.3851                       | 0.3899                    |
| Horizontal displacement of pile tip          | 0.1648                       | -0.6945                   |
| Inclination                                  | 0.0571                       | 0.4902                    |

Table 10. Monitor the principal component score function of the section

| Component                                                      | formula                                                                 |
|---------------------------------------------------------------|-------------------------------------------------------------------------|
| The first principal component score function                  | \( f_1 = 0.5356z_1 + 0.5228z_2 + 0.5110z_3 + 0.3851z_4 + 0.1648z_5 + 0.0571z_6 \) |
| The second principal component score function                 | \( f_2 = -0.0455z_1 + 0.2040z_2 - 0.2857z_3 + 0.3899z_4 - 0.6945z_5 + 0.4902z_6 \) |
| Comprehensive score function                                  | \( F = 51.805\% f_1 + 27.821\% f_2 \)                                    |

3.5. Sort by the value of the comprehensive evaluation function

The index values of each influencing factor are calculated by the principal component calculation formula and the comprehensive evaluation function, and then sorted. The results are shown in Table 11 ~Table 12.

The comprehensive scores of the monitoring section are listed in Table 9, and the comprehensive scores of various factors are as follows: inclination > pile top settlement of > pile top horizontal displacement > underground water level > ground subsidence of concrete support axial force. The inclination has the greatest influence on the stability, so pay attention to the deformation of surrounding support structure. Secondly, the deformation of pile top has a great influence on the stability of the foundation pit. Therefore, it is necessary to pay attention to the influence of the settlement of pile top on the stability of the foundation pit, dewatering the foundation pit, eliminating the displacement of pile top caused by earth and rock piling near the foundation pit, and taking preventive measures for pile top displacement.

The comprehensive scores of the monitoring sections are listed in Table 10. The comprehensive scores of various factors are as follows: inclination > pile top settlement > concrete support axial force > underground water level > ground subsidence. Inclination has the greatest influence on stability, followed by horizontal displacement of pile tip.

Table 11. The ranking of principal components and the ranking of comprehensive scores of monitoring sections

| Component                        | Primary principal component | Sorting | Second principal component | Sorting | Comprehensive scores | Sorting |
|----------------------------------|-----------------------------|---------|----------------------------|---------|----------------------|---------|
| Land subsidence                  | -0.15                       | 6       | 0.28                       | 5       | -1.34                | 6       |
| Concrete supports the axial force| 0.83                        | 5       | 0.56                       | 4       | 62.33                | 5       |
| Underground water level          | 0.89                        | 4       | 1.27                       | 1       | 85.26                | 4       |
| Pile top settlement              | 1.37                        | 3       | 0.95                       | 2       | 103.92               | 2       |
| Horizontal displacement of pile tip| 1.64                       | 2       | -0.11                      | 6       | 91.13                | 3       |
| Inclination                      | 1.73                        | 1       | 0.75                       | 3       | 119.03               | 1       |
Table 12. The ranking of principal components and the ranking of comprehensive scores of monitoring sections

|                    | Primary principal component | Sorting | Second principal component | Sorting | Comprehensive scores | Sorting |
|--------------------|-----------------------------|---------|----------------------------|---------|----------------------|---------|
| Land subsidence    | 0.05                        | 6       | 0.44                       | 2       | 15.05                | 6       |
| Concrete supports the axial force | 0.48                        | 5       | -0.05                      | 3       | 23.66                | 4       |
| Underground water level | 0.9                         | 4       | -0.87                      | 6       | 22.54                | 5       |
| Pile top settlement | 1.19                        | 3       | -0.35                      | 4       | 51.98                | 3       |
| Horizontal displacement of pile tip inclination | 1.49                        | 2       | -0.38                      | 5       | 66.67                | 2       |
|                    | 2.04                        | 1       | 0.44                       | 1       | 117.89               | 1       |

4. Conclusion
Through sorting out and analyzing the monitoring data of Chengdu metro line 17, and using principal component analysis method to calculate and analyze the influencing factors of foundation pit stability, the main conclusions are as follows:

1) It can be concluded from the comprehensive scores of the two monitoring sections of 9th and 11th that both the inclination and pile top displacement have high scores in the two monitoring sections, which have a major impact on the stability of the foundation pit.

2) Compared with the pile top settlement and horizontal displacement of pile head and concrete strut axial forces such as monitoring project, deep foundation pit retaining structure horizontal displacement (inclination) can reflect the deformation of retaining structure under the action of water and soil pressure continuity characteristics, in the foundation pit monitoring should pay special attention to the deformation monitoring of support structure, in a timely manner to support stability.

3) When principal component analysis method is used to evaluate the stability of foundation pit, it can be seen that there is consistency in the evaluation of influencing factors of foundation pit in a certain region, and the principal component analysis method has high accuracy in the evaluation of foundation pit stability, which has certain reference value in engineering construction and can provide theoretical basis for relevant projects.

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