Some remarks on vortex matter in high-$T_c$

superconductors

I. L. Landau and H. R. Ott

Laboratorium für Festkörperphysik, ETH Hönggerberg, CH-8093 Zürich, Switzerland

We show that some experimentally observed features of vortex matter in high-$T_c$ superconductors may be interpreted in simpler ways than it is usually done. In particular, we consider magnetic flux creep at low temperatures as well as the irreversibility line in the $H - T$ phase diagram. We also discuss a new approach to the analysis of the equilibrium magnetization in the mixed state of type-II superconductors and we suggest an alternative configuration for the mixed state in magnetic fields close to the upper critical field.

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1. INTRODUCTION

The behavior of vortex matter in high-$T_c$ superconductors (HTSC’s) seems to be extremely complex. Many different theoretical models have been developed in order to explain various experimental observations. The complexity of the description of the mixed state in HTSC’s and its dynamics often arises from the apparent inability of traditional approaches for explaining the experimental results. Recent reconsiderations of available data indicate, however, that this may not always be the case. In this brief review we discuss several features of the mixed state in HTSC’s and show that in some cases more traditional and less complex models may perfectly well explain the experimental observations. This is particularly true for the vortex dynamics. We show that thermally activated flux creep as well as very specific features of the vortex dynamics, which are usually related to the irreversibility line and a vortex-glass transition, may quite well be explained by employing a simple Kim-Anderson approach to the flux-creep process, if a profile of the pinning potential well is taken into account. We also con-
Consider a new approach to the analysis of the reversible magnetization $M$ in an external magnetic field $H$ and show that the temperature dependence of the upper critical field $H_{c2}$ as well as the value of superconducting critical temperature $T_c$ may reliably be obtained by scaling the $M(H)$ curves measured at different temperatures without assuming any specific $M(H)$ dependence. Finally, we consider an alternative model for describing the mixed state of type-II superconductors in magnetic fields close to $H_{c2}$.

Most of the results that are presented and discussed below are included in Refs. 3, 4, 5, 6, 7, 8.

2. VORTEX DYNAMICS

Our analysis of the vortex dynamics is based on the assumption of single vortex hopping. This is not a commonly accepted approach to this problem. Nevertheless, the analysis of experimental results presented in subsection 2.1 provides rather strong evidence that the magnetic relaxation in HTSC’s may indeed quite well be described by assuming the motion of single vortex lines. We employ here the Kim-Anderson approach to the flux motion with the only difference that instead of triangular potential barriers, which were implicitly assumed by Kim and Anderson in their original work, we consider a more realistic, smooth profile of the pinning potential well, as shown by the solid line in Fig. 1(a).

![Fig. 1. Schematic profiles of the pinning well for different values of $j/j_c$. (a) $j/j_c \ll 1$. (b) $(1 - j/j_c) \ll 1$. The inset illustrates the meaning of $U_1$ and $U_2$. This simple functional $x$ dependence is chosen only for illustration and has no real physical meaning.](image)

If the current density $j$ in a superconducting sample in the mixed state is less than its critical value $j_c$, all vortices are pinned and their motion is entirely due to thermally activated hopping of the vortex lines or their
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quantum tunneling. At a given temperature $T$, the profile of the potential well for $j = 0$ may be written as

$$u(x) = U(T)f(x,T),$$  \hspace{1cm} (1)$$

where $U(T)$ is the energy fixing the pinning strength, and the function $f(x,T)$ defines the shape of the potential well, which may be temperature dependent as well.

An electric current creates a Lorentz force $F_L$ acting on the vortices. The Lorentz force tilts the potential profile which facilitates the vortex motion in one direction. In the presence of a current the potential profile may be written as

$$u(x,j) = u(x) - xF_L$$  \hspace{1cm} (2)$$

with $F_L = j\delta\Phi_0/c$, where $\delta$ is the sample thickness, $\Phi_0$ is the magnetic flux quantum, and $c$ is the speed of light. The variation of the potential profile with increasing current density is illustrated in Figs. 1(a) and 1(b). It may be seen that at low currents ($j \ll j_c$) the decrease of the activation energy with increasing current is entirely determined by the behavior of $u(x)$ near its maxima, while for currents close to $j_c$, only $u(x)$ in the vicinity of the inflection point, i.e., where $d^2u/dx^2 = 0$, is important. The critical current density is reached if the potential barriers in the direction of the vortex motion vanish. According to Eqs. (1) and (2), this results in

$$j_c = \frac{cU(T)f'_\text{max}}{\delta\Phi_0},$$  \hspace{1cm} (3)$$

where $f'_\text{max}$ is the value of $df/dx$ at the inflection point. Equation (3) represents a formal definition of the critical current density in the mixed state of type-II superconductors.

As may clearly be seen in Figs. 1(a) and 1(b), the distance between the bottom of the well and the adjacent potential maximum along the direction of the flux motion decreases with increasing current and vanishes at $j = j_c$. This is a direct consequence of Eq. (2) and is true for any smooth profile of the potential well. Consequently the flux-creep activation energy is always a non-linear function of the current. This non-linearity of $U_1(j)$ results in a negative curvature of $\ln E$ versus $j$ and in a positive curvature of the logarithmic time dependence of the irreversible magnetic moment $M_{\text{irr}}$. Although these features of the flux-creep process, as has been pointed out by Beasley et al. \cite{Beasley}, are a direct consequence of the Kim-Anderson approach, deviations of $M_{\text{irr}}(\ln t)$ from linearity are often considered in the literature as being incompatible with the Kim-Anderson model.
2.1. Thermally activated flux creep at low temperatures

The experiments that we describe in this section have been carried out on a ring-shaped YBa$_2$Cu$_3$O$_{7-x}$ (YBCO) film with a thickness of 0.3 $\mu$m. The external diameter of the ring was 10 mm and its width was approximately 2 mm. The current in the ring $I$ and its decay due to flux creep were measured with a Hall probe placed in the central part of the ring cavity (see Refs. 3, 4, 5 for details).

The main advantage of choosing a ring geometry for this type of experiments is the possibility to obtain voltage-current ($V-I$) characteristics in the flux creep regime. Both the analysis and the interpretation of the $V-I$ curves are much less influenced by employing different models than is usually the case for the analysis of experimental curves of magnetic relaxation. The voltage around the ring can straightforwardly be evaluated from the experimentally measured $I(t)$ data via $V = L dI/dt$, where $L$ is the sample inductance. The second advantage is that almost all the magnetic flux is concentrated inside the ring cavity, while the current flows around it. In this case, the non-uniformity of the current distribution in the sample’s cross-section can be neglected and all complications arising from the use of critical-state models are avoided. Examples of experimental voltage-current characteristics plotted as $T \ln V$ versus $I$ are shown in Fig. 2.

![Image of voltage-current characteristics](image_url)

**Fig. 2.** Examples of voltage-current characteristics of a ring-shaped YBCO film at $H = 0$.

At temperatures not very close to $T_c$, the probability of vortex hopping is negligible at low currents and therefore, the flux motion can be observed only at current densities sufficiently close to $j_c$. In this case, the vortex hopping in the direction opposite to the Lorentz force can be neglected and the hopping rate $\nu$ is entirely determined by the height of the potential
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barrier $U_1$ in the direction of the Lorentz force (see inset to Fig. 1(a)), i.e.,

$$\nu = \nu_0 \exp(-U_1/k_BT),$$

(4)

where $\nu_0$ is the attempt frequency, $k_B$ is the Boltzmann constant, and $U_1$ plays the role of the activation energy in the flux-creep process. Taking into account that the electric field $E$ is proportional to the hopping rate, we can rewrite Eq. (4) as

$$U_1(j,T) = -k_BT[\ln E(j) - \ln E_0],$$

(5)

where $E_0$ is a parameter which includes the attempt frequency, the hopping distance and the magnetic induction. Because $U_1(j_c,T) = 0$, $\ln E_0 = \ln E(j_c)$.

Eq. (5) provides the possibility to obtain the current dependence of the flux-creep activation energy from the experimental $E - j$ characteristics. The problem is that the experimental data for each temperature cover only a very narrow range of currents and, therefore, only a very small part of the $U_1(j)$ curve at a given temperature can be obtained in this way. An additional complication in this evaluation of $U_1(j)$ is that neither $E_0$ nor $j_c$ are a priori known. However, with some additional assumptions about the temperature dependence of the pinning potential profile, Eq. (5) is adequate for the analysis of flux-creep data obtained at different temperatures. Not only can $U_1(j)$ curves be determined for a much wider range of currents, but also the quantities $E_0$ and $j_c$ may be evaluated. The problem here is to select reasonable assumptions. As will be shown below, the assumption that the function $f$ in Eq. (1) is temperature independent is quite consistent with experimental data in a very wide range of temperatures. In this case, according to Refs. 3 and 5, the flux-creep activation energy may be written as

$$U_1(j,T) = U(T)Y(j/j_c),$$

(6)

where $U(T)$ is the same as in Eq. (1) and the function $Y$ depends only on the ratio $j/j_c$.

It has been demonstrated in Refs. 3 and 5 that, if the flux-creep activation energy may be represented as a product of a temperature dependent and a current dependent term, the transformation

$$\ln E(j/i,T_0) = (T/iT_0)\ln E(j,T) + A$$

(7)

with $i = j_c(T)/j_c(T_0)$ and

$$A = (1 - T/iT_0)\ln E_0$$

(8)
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Fig. 3. Results of the scaling procedure in the form of $T_0 \ln V(T_0)$ versus $I/i$ with $T_0 = 13$ K. The inset shows, on linear scales, the small part of the curve for $H = 0$ which is indicated by the rectangle in the main figure. For clarity only very few points for each temperature are displayed.

may be used to merge the $\ln E - j$ curves measured at different temperatures into a single master curve. Here, $i$ and $A$ are scaling parameters, and $T_0$ is some arbitrary chosen temperature within the investigated temperature range. The resulting master curve represents the current dependence of $\ln E$ at $T = T_0$, as if $E(j)$ could actually be measured over this extended range of currents at this single temperature. For each temperature the values of $i$ and $A$ can be found from forcing the overlapping $T \ln E$ versus $j$ curves for the adjacent temperatures to match each other. In this procedure the relation between $i$ and $A$ given by Eq. (8) is not employed, both quantities are rather considered as independent fitting parameters. Eq. (8) is only used retrospectively in order to check the validity of our approach.

In the following discussion of experimental results we use measurable quantities, such as the voltage $V$ and the current $I$, but will switch to $E$ and $j$ in relevant equations.

The feasibility of the scaling procedure, implicit in Eq. (7), has been demonstrated in Refs. 3 and 5. In Fig. 3 we show again two examples of this procedure, obtained for two different values of external magnetic fields. The scaling procedure provides the corresponding master curves, exhibiting a practically perfect alignment of the $T \ln V$ versus $I$ curves measured at different temperatures between 10 and 80 K. The inset of Fig. 3 emphasizes the matching quality on extended scales.

We note that the behavior of the $\ln V$ versus $I$ curves changes drastically at temperatures $T \leq 10$ K and the low temperature data cannot be scaled using Eq. (7). This is a clear indication for a crossover from the thermally activated vortex hopping to quantum tunneling of the vortex lines.
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with decreasing temperature. As was shown in Ref. 4, our scaling procedure can easily be modified for the analysis of the quantum creep regime.

The resulting temperature dependence of the scaling parameter \(i\) is shown in Fig. 4(a). This plot represents the temperature dependence of the normalized critical current, exhibiting the expected trend to saturation at low temperatures. Note that, according to Eq. (3) and our assumption that \(f\) is temperature independent, \(j_c(T)/j_c(T_0) = U(T)/U(T_0)\).

\(0 \quad 20 \quad 40 \quad 60 \quad 80\)  
\(0.0 \quad 0.5 \quad 1.0\)

\(T\) (K)

\(0 \quad 10 \quad 20 \quad 30\)  
\(-600 \quad -400 \quad 0\)

\(H = 1\) kOe

(a)

\(\tau = T/iT_0\)

Fig. 4. (a) The scaling parameter \(i = j_c(T)/j_c(T_0)\) as a function of temperature. (b) The parameter \(A\) as a function of \(\tau = T/iT_0\) with \(T_0 = 13\) K. The straight line is the best linear fit to \(A(\tau)\) for \(\tau < 3\). The inset shows the low temperature part of the plot on expanded scales.

In Fig. 4(b) we display the temperature dependence of the scaling parameter \(A\) which, according to Eq. (8), depends on the ratio \(T/i\) rather than the temperature alone. Hence \(A\) is plotted as a function of \(\tau = T/iT_0\). If the temperature dependence of \(\ln V_0\) is negligible and our procedure makes sense, we expect the data to lie along a straight line. Although \(V_0\) is proportional to the temperature-dependent attempt frequency, it enters Eq. (8) only as \(\ln V_0\) and therefore, the resulting curve is expected to deviate only weakly from linearity, as is indeed the case. At low temperatures, where the temperature dependence of \(\ln V_0\) may definitely be neglected, the \(A(\tau)\) points are indeed well approximated by a straight line with a slope \(dA/d\tau = -A(0)\), in complete agreement with Eq. (8) and convincingly documented in the inset of Fig. 4(b). Although the temperature dependence of \(A\) itself does not carry much of physical information, this perfect agreement between \(A(\tau)\) obtained from the scaling procedure and \(A\) given by Eq. (8) serves as an important confirmation of the validity of our approach. As is discussed in more detail in Ref. 5, the linearity of \(A(\tau)\) breaks down at temperatures \(T > 0.9T_c \approx 80\) K. Our main assumption about the temperature independence of the function \(f\) entering Eq. (1) is thus not valid at temperatures close to \(T_c\). This is...
to be expected because both the magnetic field penetration depth $\lambda(T)$ and the coherence length $\xi(T)$ diverge at $T_c$.

From the $A(\tau)$ data, a reliable value of $\ln V_0$ may be obtained. According to Eq. (8), $\ln V_0 = dA/d\tau = -A(0)$, and thus $\ln[ V_0 \text{ (nV) } ] = 21.9$. With $\ln V_0$ known, Eq. (5) can now be used to establish the current dependence of the flux-creep activation energy from the data presented in Fig. 3.

As mentioned above, at current densities close to $j_c$, only a small part of the $u(x)$ function in the vicinity of the inflection point represents the essential part of the potential barrier (see Fig. 1(b)). In this case, $u(x)$ can be replaced by its Taylor series expansion. Taking into account that at the inflection point $d^2u/dx^2 = 0$ and keeping only the first two nonzero terms in the expansion of $u(x)$, one obtains

$$U(j/j_c) \propto (1 - j/j_c)^{3/2}. \quad (9)$$

This is the current dependence of the flux-creep activation energy for $(1 - j/j_c) \ll 1$ which does not depend on the particular shape of $u(x)$. Eq. (9), together with Eq. (5) serves to estimate the value of the critical current and we obtain $I_c(1 \text{ kOe}) = 300 \text{ A}$ at $T = 13 \text{ K}$. Taking into account that $I_c$ is almost constant at these low temperatures, this value of $I_c$ may safely be considered as the critical current for $T = 0$. 

![Graph](image-url)

**Fig. 5.** $U_1(j/j_c)$ calculated for $T = 0$ is shown by the solid line. The dotted line is extrapolation of the $U_1(j/j_c)$ curve to $j/j_c = 1$ using Eq. (9).

We may now insert the values $I_c$ and $\ln V_0$ into Eq. (5) and calculate $U_1(j/j_c)$ from the master curves presented in Fig. 3. The result is shown in Fig. 5. The extrapolation of the $U_1(j/j_c)$ curve to $j/j_c = 1$ using Eq. (9) is shown by the dotted line.

Our approach assumes a direct connection between the profile of the potential well $u(x)$ and $U_1(j/j_c)$, which allows for the reconstruction of the
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![Graph showing potential profile](image)

**Fig. 6.** (a) The pinning potential profile for $j=0$ calculated from $U_1(j/j_c)$. (b) The potential profile for different current densities.

potential profile in real space using the $U_1(j/j_c)$ data as they follow from experiment. The result is shown in Fig. 6(a). Here, for simplicity, we have chosen that the inflection point of the $u(x)$ function coincides with the bottom of the potential well. The evolution of the potential profile with increasing current is shown in Fig. 6(b). As may be seen in Fig. 5, our $U_1(j/j_c)$ data are limited to currents $j \geq 0.08j_c$. This is why only a limited section of the potential profile for $|x| < x_{\text{max}} \approx 67 \, \text{Å}$ can be obtained. No information about the potential profile at distances larger than $x_{\text{max}}$ can be gained on the basis of our experimental results.

Here we have used the current dependence of the flux-creep activation energy to calculate the profile of potential barriers. On the other hand, it is well known that HTSC samples are not uniform and one should expect that different barriers have different shapes. Hence the physical relevance of the potential profiles calculated as demonstrated above, is not obvious. In order to clarify the situation, we consider the flux-creep process in more detail. There are very many different trajectories along which the vortices are allowed to cross the ring sample. It is obvious, however, that only those trajectories containing the lowest potential barriers will actually be traced. Along each trajectory many different potential barriers are met, but only one or a very few of them with the largest amplitudes are essential. The next question to be answered is, how many trajectories are needed to let all the vortices pass across the sample. In the ring geometry, the evaluation of the number $N$ of vortices which are leaving or entering the ring cavity per second is straightforward. Taking into account that the experimentally accessible voltages range between $3 \cdot 10^{-5}$ and 0.3 nV, $N$ is between 20 and $2 \cdot 10^5 \, \text{s}^{-1}$ for the lowest and the highest voltage, respectively. For $B = 1$
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kG, the distance between vortices is of the order of $10^{-5}$ cm. This implies an average vortex velocity $w \approx 2$ cm/s, if we force all the $2 \cdot 10^5$ vortices per second to follow the same trajectory across the sample. This value of $w$ is rather low and there is good reason to assume that a single trajectory is in principle sufficient to transfer all the vortices.

An important consequence of this line of thoughts is that the analysis of flux-creep rates provides information only about one particular pinning center, which represents the highest potential barrier for the vortex motion on the energetically most favorable trajectory across the sample. This is true not only for the ring geometry, but for measurements of the magnetization relaxation for other sample shapes, as well.

Although it is claimed by many that single vortex hopping cannot describe experimental observations reflecting flux-creep in HTSC’s, we have demonstrated that this is not really the case. The described procedure for the analysis of flux-creep rates measured at different temperatures allows for at least a partial reconstruction of the pinning potential well profile in real space. All the flux-creep data in the temperature range between 10 and 80 K, where our scaling procedure is applicable, may perfectly well be described by a reasonable potential profile shown in Fig. 6(a), with $U(T)/U(0)$ as shown in Fig. 4.

2.2. Irreversibility line

The irreversibility line (IRL) in the $H-T$ phase diagram separates two regions with distinctly different behaviors. Above the IRL the magnetization of the sample is perfectly reversible, i.e., the sample cannot carry any persistent current. Only below the IRL, irreversible magnetization $M_{irr}$ is observed.

The situation arising near the IRL may much easier be analyzed if we again consider a ring-shaped sample. In this case, instead of magnetization curves, the temperature variations of the persistent current $I_p$ may be considered. Typical experimental data for our ring-shaped YBCO film are presented in Fig. 7, displaying a heating-cooling cycle of $I_p$. These data as well as results of measurements of the irreversible magnetization $M_{irr}$ reveal that above the irreversibility temperature, $T_{irr}$, persistent currents are essentially zero. For this reason, $T_{irr}$ is usually considered as the temperature at which the critical current density vanishes. It is commonly accepted that the melting of the vortex-glass is responsible for such a behavior. However, as we demonstrate below, this type of $I_p(T)$ or $M_{irr}(T)$ curves necessarily follows from the simplest Kim-Anderson approach for describing the thermally activated vortex motion and the critical current density does not really vanish at $T = T_{irr}$ but
Fig. 7. Persistent current $I_p$ in the ring-shaped YBa$_2$Cu$_3$O$_{7-x}$ film as a function of temperature. The vertical arrow indicates the position of the irreversibility temperature. The sample was cooled in an external magnetic field $H = 1$ kOe to $T = 82$ K. The current was subsequently induced by enhancing the magnetic field by 1 Oe.

remains nonzero also above the IRL.

Encouraged by the success of the Kim-Anderson approach in the analysis of low-temperature flux-creep rates, we now consider the same concept at temperatures close to $T_c$. In this case, the vortex hopping in the direction opposite to the Lorentz force cannot be neglected and the electric field in the sample is

$$E = E_0 \left\{ \exp \left[ - \frac{U_1(T, j)}{k_B T} \right] - \exp \left[ - \frac{U_2(T, j)}{k_B T} \right] \right\}. \quad (10)$$

The second term in Eq. (10) describes the vortex hopping in the direction opposite to that of the Lorentz force (see Fig. 1(a) for the definitions of $U_1$ and $U_2$). We note that, because $U_1$ and $U_2$ depend on current differently, Eq. (10) cannot be reduced to a hyperbolic sinus.

The electric field $E$ is proportional to the current decay rate $dj/dt$ and therefore Eq. (10) may be used for its evaluation. In order to calculate the temperature and current dependencies of $dj/dt$, we have to assume some explicit expression for the profile of the potential well $u(x)$. For the following analysis we have chosen two rather different representations for $f(x, T)$, i.e.,

$$f(x, T) = |x| - (1 - T/T_c)^k x^2 \quad (11)$$

and

$$f(x, T) = \left( \sqrt{|x| + x_0} - \sqrt{x_0} \right) - a \cdot \frac{(|x| + x_0)^{3/2} - x_0^{3/2}}{(1 - T/T_c)^m}, \quad (12)$$
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with

\[ U(T) = U_0 \left(1 - \frac{T}{T_c}\right)^{3/2} \]  

(13)

for both cases. According to Eq. (1) the profile of the potential well \( u(x, T) = U(T)f(x, T) \). Our choice of \( f \) functions, shown in Fig. 8, is quite arbitrary and was mainly dictated by the possibility of performing analytical calculations. As will be shown below, the particular choice of \( f(x, T) \) does not influence the main qualitative features of the flux-creep process.

![Fig. 8. (a) and (b) Examples of profiles of the pinning potential wells as given by Eqs. (11) and (12).](image)

With the chosen potential profiles, the dependencies of \( \frac{dj}{dt} \propto E \) on current and temperature may straightforwardly be calculated using Eqs. (1), (2) and (10). Fig. 9(a) displays the results of calculations of \( \frac{dj}{dt} \) for \( f(x, T) \) given by Eq. (11) with \( k = 1.4 \) at several fixed current densities. We use a log-scale for the \( \frac{dj}{dt} \)-axis and the total change in \( \frac{dj}{dt} \) is 100 orders of magnitude. This figure clearly demonstrates that \( \frac{dj}{dt} \) grows extremely fast with increasing temperature which is exactly the experimentally observed behavior of the current decay near the irreversibility temperature. Note that all the data presented in Fig. 9(a) correspond to \( j < j_c \).

Experimentally the persistent current \( I_p \) is usually determined as the current that does not decay during the time of the experiment. With our approach we can calculate the temperature dependence of the persistent current by fixing \( dI/dt \) according to experimental conditions. The corresponding calculations were made for \( dI/dt = 3 \cdot 10^{-4} \) A/s, which is close to the resolution of the experimental data presented in Fig. 7 (see Ref. 3 for details). In order to understand how the \( I_p(T) \) curves depend on the particular choice of the potential profile, the calculations were made for both approximations of \( u(x) \) and for different values of the exponents \( k \) and \( m \) in Eqs. (11) and (12), respectively. The results of the calculations, together with experimental data from Fig. 7, are shown in Fig. 9(b). It may be see that an "irreversibility" temperature exists for all chosen \( u(x) \) functions. At the same time, both the shape of the \( I_p(T) \) curves and the position of the apparent \( T_{irr} \) are rather sensitive to the choice of \( u(x) \). This means that by
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Fig. 9. (a) $dj/dt$ versus $T/T_c$ calculated for $f(x, T)$ given by Eq. (11) with $k = 1.4$. The values of the current density normalized by $j_c$ at $T = 0.9T_c$ are indicated for each curve. (b) The persistent current as a function of temperature. The solid lines are the results of calculations for different representations of $u(x)$. The points are experimental data from Fig. 7.

a proper choice of $U(T)$ and $f(x, T)$ entering Eq. (1), any experimentally observed temperature dependence of the irreversible magnetic moment or the persistent current may sufficiently well be approximated and no specific transition in the vortex system is needed to explain the existence of the experimentally observable irreversibility line.

2.3. Vortex-glass transition

The vortex-glass transition is undoubtedly the most popular interpretation of the IRL in HTSC’s. Many experimental results, especially the scaling of log $E$ – log $j$ curves measured at different temperatures, seem to confirm the concept of a vortex-glass melting at temperatures close to the IRL.\[14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\] Quite recently, however, it has been shown that the same experimental log $E$ – log $j$ data may be scaled equally well by assuming very different temperatures of the vortex-glass transition.\[30\] This observation is important because it demonstrates very clearly that the concept of vortex-glass melting, although commonly accepted, is not actually confirmed by experiment. In this section we show that the distinct variation of the shape of the log $E$ – log $j$ curves, which is usually interpreted as a manifestation of the vortex-glass melting, follows straightforwardly from Eq. (10).

For the calculations, whose results are presented in this section, we used $f(x, T)$, as given by Eq. (12) with $m = 2$. The results of the calculations of $E(j)$ are shown in Figs. 10(a) and 10(b) as $E(j)$ curves at fixed temperatures. Qualitatively the plot shown in Fig. 10(a) is indistinguishable from numerous
Fig. 10. $E - j$ curves calculated for $f(x, T)$ given by Eq. (12) with $m = 2$.

experimental results (see Refs. [20,21,22,23,24,25,26,27,28,29]) and it is clear that, by corresponding adjustments of $U(T)$ and $f(x, T)$, any experimental $E(j, T)$ curve may be approximated even quantitatively. We note that the calculated log $E$ – log $j$ curves remain qualitatively the same for any chosen potential profile.

Fig. 10(b) shows the $E(j)$ curves down to much lower voltages and, as may clearly be seen, the change of the sign of the curvature, which is usually attributed to the vortex-glass transition, is a universal feature of the $E(j)$ curves at any temperature. With decreasing temperature however, the sign change of the curvature is shifted to a range of voltages which is not accessible experimentally.

A very similar explanation of the voltage-current characteristics near the "vortex-glass" transition has been suggested by Coppersmith at al. [31]. In their short comment they considered a sinusoidal potential barrier. It was shown that even with this simple potential all the qualitative features of the experimental $E(j)$ curves could be reproduced. The authors also pointed out that the insignificant quantitative disagreement with the experimental data is simply due to the arbitrary chosen sinusoidal profile of the potential barriers.

3. TEMPERATURE DEPENDENCE OF $H_{c2}$ FROM ISOTHERMAL MAGNETIZATION DATA.

In order to evaluate the upper critical field, $H_{c2}$, from experimental data in complex materials such as HTSC’s, it is very important to introduce an appropriate definition of this parameter. In an ideal type-II superconductor, $H_{c2}$ is the highest value of the magnetic field compatible with superconductivity, i.e., the $H_{c2}(T)$ curve in the $H - T$ phase diagram represents a line
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of second order phase transitions to the normal state. As is well known for HTSC’s, this transition degenerates to a cross-over region because of fluctuation effects. We note that small inclusions of impurity phases with critical temperatures different from that of the bulk cannot always be excluded in HTSC’s and they may also contribute to the broadening of the transition.

At the same time, in magnetic fields well below $H_{c2}(T)$, the effect of fluctuations and possible inclusions of impurity phases on the sample magnetization is small and the $M(H)$ curves in this magnetic field range must practically be the same as for a perfectly uniform sample without fluctuations. This circumstance provides the possibility to evaluate the temperature dependence of $H_{c2}$, in its traditional sense, from magnetization measurements in magnetic fields well below $H_{c2}$.

Here we discuss a new approach to this problem by scaling the $M(H)$ curves measured at different temperatures. This scaling procedure is based on the application of the Ginzburg-Landau (GL) theory in very general terms, without assuming any specific magnetic field dependence of the magnetization. We consider this as an important point because reliable calculations of $M(H)$ are extremely difficult even for uniform and isotropic superconductors. The reliability of approximate models can in most cases not independently be verified and hence their application can easily lead to misinterpretations of experimental results.

The scaling procedure is based on the assumption that the GL parameter $\kappa$ is temperature independent. Although the microscopic theory of superconductivity predicts a temperature dependence of $\kappa$, this dependence is rather weak and is not expected to change the results significantly. From the GL theory it follows straightforwardly that, if $\kappa$ is temperature independent, the magnetic susceptibility $\chi$ of the sample is a universal function of $H/H_{c2}$, i.e., $\chi(H,T) = \chi(h)$ with $h = H/H_{c2}(T)$, and the magnetization density is

$$M(H,T) = H_{c2}(T)h\chi(h).$$

Eq. (14) leads to the following relation between the values of $M$ at two different temperatures

$$M(H,T_0) = M(h_{c2}H,T)/h_{c2}.$$

with $h_{c2} = H_{c2}(T)/H_{c2}(T_0)$. The collapse of individual $M(H)$ curves measured at different temperatures into a single master curve may be achieved by a suitable choice of $h_{c2}(T)$. In this way one can only establish the temperature dependence of the normalized upper critical field $H_{c2}(T)/H_{c2}(T_0)$, while the absolute values of $H_{c2}(T)$ remain unknown. This is the price to pay for the fact that we do not specify the variation of the magnetization upon changing the applied magnetic field.
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The scaling procedure described by Eq. (15) is actually valid for ideal type-II superconductors only and in order to apply it to HTSC’s, we have to introduce the necessary corrections to Eq. (15) that are dictated by some specific features of HTSC’s. Most of the families of HTSC’s exhibit a weak paramagnetic susceptibility in the normal state. Its influence may be accounted for by replacing Eq. (15) by

\[
M(H, T_0) = \frac{M(h_{c2}H, T)}{h_{c2}} + c_0(T)H.
\]

(16)

The term \(c_0(T)H\) also includes contributions to the sample magnetization arising from fluctuations effects and small inclusions of impurity phases which, of course, can only approximately be accounted for. However, as will be shown below, Eq. (16) can be successfully used for the scaling of experimental \(M(H)\) data up to temperatures quite close to \(T_c\). In the following we use the parameter \(c_0(T)\) in Eq. (16) as an additional adjustable parameter in the scaling procedure.

![Figure 11](image)

Fig. 11. (a) The magnetization data for a sample of YBa\(_2\)Cu\(_3\)O\(_{7-x}\) (Ref. 35) after scaling using Eq. (4) with \(T_0 = 82\) K. The inset displays the original data. (b) \(H_{c2}(T)/H_{c2}(82K)\). The solid line is the best fit with Eq. (17).

The result of this scaling procedure for an Y-based cuprate sample is shown in Fig. 11(a). It may be seen that a rather perfect overlap of the individual \(M(H)\) curves, measured at different temperatures, is obtained in this way. Because the variable \(h_{c2}\) enters the denominator of the first term in Eq. (16), the magnetization data for the highest temperatures are considerably expanded in comparison with the low temperature data. This is the reason for the somewhat enhanced scatter in the high temperature data. The resulting temperature dependence of the normalized upper critical field for this sample is shown in Fig. 11(b).
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Fig. 12. $H_{c2}(T)/H_{c2}(0.9T_c)$ versus $T/T_c$ for different samples; the values of $T_c$ are estimated by extrapolating the $h_{c2}(T)$ curves and the corresponding references are indicated near the symbols. The solid lines are guides to the eye. (a) Y-based cuprates. (b) Bi-based cuprates. (c) Hg-based cuprates. (d) Tl-based cuprates.

We note that the uncertainty of $h_{c2}(T)$ increases considerably for temperatures close to $T_c$ as well as for the lowest temperatures. The loss of accuracy for the highest temperatures is due to the obvious enhancement of the experimental uncertainty of the original $M(H)$ data. Although the experimental accuracy is improving with decreasing temperature, the increase of the irreversibility field limits the available magnetic field range. If the experimental data are only collected in a narrow magnetic field range, our scaling procedure is not reliable.

The temperature dependence of the normalized upper critical field, as shown in Fig. 11(b), may also be used to evaluate the critical temperature $T_c$. For this purpose the ratio $H_{c2}(T)/H_{c2}(T_0)$ was approximated by

$$
\frac{H_{c2}(T)}{H_{c2}(T_0)} = \frac{1 - (T/T_c)\mu}{1 - (T_0/T_c)\mu},
$$

(17)
in which $\mu$ and $T_c$ are used as fit parameters. Eq. (17) provides a rather good approximation to $h_{c2}(T)$ curves for $T \geq 0.8T_c$. The corresponding fit is shown as the solid line in Fig. 11(b). The resulting value of $T_c$ is indicated in Fig. 11(b). If the experimental data are obtained up to temperatures close to the critical temperature, the value of $T_c$, estimated by the extrapolation of the $h_{c2}(T)$ curve to $h_{c2} = 0$ is quite accurate. A reliable value of $T_c$ is essential for the comparison of the results that were obtained for samples with different critical temperatures. Using the values of $T_c$ evaluated in such a way, we have plotted $H_{c2}(T)/H_{c2}(0.9T_c)$ versus $T/T_c$ for various Y-based compounds as shown in Fig. 12(a). Quite surprisingly, the temperature variations of $H_{c2}$ for rather different samples turn out to be identical.

The temperature variations of $h_{c2}$ for other families of HTSC’s are plotted in Figs. 12(b-d). Similar to what has been found for Y-based compounds, the scaling procedure again leads to an almost perfect merging of all the data into one single curve for different samples. Furthermore, as may clearly be seen in Fig. 13, the temperature dependencies of the normalized upper critical field for different families of HTSC are virtually identical at all temperatures for which the experimental data are available. The insignificant differences between the $h_{c2}(T/T_c)$ curves for different samples, visible at the lowest temperatures in Figs. 12(b), 12(c), and 13, are due to small errors in the evaluation of the critical temperature.

Fig. 13. The normalized temperature dependence of $H_{c2}$ for different HTSC compounds. Only some selected data points from Figs. 12(a-d) are shown. The solid and the broken line represent the ratios $H_c(T)/H_c(0.9T_c)$ for pure metallic Lead and Tin, respectively.

The scaling procedure based on Eq. (17) turns out to be rather successful for the analysis of the reversible magnetization of HTSC’s. The most surprising and completely unexpected result of our analysis is that for practically all families of HTSC’s, the $h_{c2}(T/T_c)$ curves are virtually identical.
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It is difficult to imagine that this universality of the $h_{c2}(T/T_c)$ dependence is just a coincidence. We strongly believe that the spectacular agreement between the $h_{c2}(T/T_c)$ data for a great variety of different samples is an unambiguous evidence that our approach captures the essential features of the magnetization process in HTSC’s. It does not necessarily mean, however, that the Ginzburg-Landau parameter $\kappa$ is indeed temperature independent. The universality of $h_{c2}(T/T_c)$ is preserved if the temperature dependence of $\kappa$ is the same for the different HTSC compounds studied here.

Our analysis is applicable only to reversible magnetization data and therefore, all the results and conclusions are limited to temperatures more or less close to $T_c$. The lower limit of validity is quite different for different families of HTSC’s as may be seen in Figs. 12(a-d).

The universality of the normalized temperature dependence of $H_{c2}$ implies that the normalized temperature variations of the thermodynamic critical fields $H_c(T)$ for different HTSC’s are also identical. Since $H_{c2}^\varphi/8\pi$ is the difference in the free energy densities between the normal and superconducting states, $H_c(T)$ also reflects the temperature dependence of the energy gap $\Delta$. In other words, our result that the normalized temperature dependence of $H_{c2}$ follows the same universal curve for different families of HTSC’s implies that the normalized temperature variations of the energy gap $\Delta(T)/\Delta(0)$ for different HTSC’s are also identical.

We note that the temperature dependencies of $H_{c2}$ for HTSC’s obtained as outlined above are qualitatively very similar to those for conventional superconductors. They are linear at temperatures close to $T_c$ with a pronounced negative curvature at lower temperatures. Apparently, the positive curvature of $H_{c2}(T)$ for HTSC’s, which is often reported in the literature, is due to the uncertainty of the definition of $H_{c2}$ used in those studies.

4. ALTERNATIVE MODEL OF THE MIXED STATE OF TYPE-II SUPERCONDUCTORS IN MAGNETIC FIELDS CLOSE TO $H_{c2}$

It is commonly accepted that the mixed state of a type-II superconductor is characterized by the penetration of an external magnetic field into the sample along quantized vortex lines or Abrikosov vortices. In most cases the vortices may be considered as thin normal filaments embedded in a superconducting environment. A completely different situation may, however, be established in superconductors with a GL parameter $\kappa \gg 1$ in magnetic fields close to $H_{c2}$. In this case, the distance between adjacent vortex cores is much smaller than the magnetic field penetration depth and the density of shielding currents is negligibly small. Here we suggest that in magnetic fields
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close to $H_{c2}$, the natural alternative to the conventional vortex structure is the formation of superconducting filaments embedded in the matrix of the normal metal.

It is easy to show that the main interaction between such filaments is a short-range repulsion. This is why the filaments should always be separated by regions where the superconducting order parameter $\psi$ is zero. This circumstance makes it possible to analyze the properties of such superconducting filaments by numerically solving the GL equations for a single filament with the condition that $\psi = 0$ along the boundary between the filaments. Subsequently, various characteristics of the mixed state consisting of superconducting filaments such as the density of the free energy, the diamagnetic response, and the equilibrium density of filaments may be evaluated.

The corresponding period $D_f$ of a triangular structure of superconducting filaments is plotted in Fig. 14(a) as a function of $(1 - H/H_{c2})$ together with the same quantity $D_v$ for the vortex lattice. As may be seen, $D_f \gg D_v$.

![Graph](image)

Fig. 14. (a) Equilibrium periods of the system of superconducting filaments $D_f$ and the vortex lattice $D_v$, respectively, as functions of the applied magnetic field. (b) The maximum amplitude of the order parameter for the system of filaments and for the vortex lattice. The data for vortices are taken from Ref. [60].

Because each filament provides a negative contribution to the free energy, they should form a rather dense triangular configuration similar to that for a traditional vortex lattice. In order to use the 1-dimensional GL equations, we have to assume that the filaments are cylindrical. It is obvious, however, that, because of their mutual interaction, the filaments should adopt a hexagonal rather than a circular cross-section. This simplification is expected to lead to only slightly overestimated free energy values.
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In the high-magnetic-field limit, the GL free energy reduces to

$$F = -\frac{H^2}{16\pi} \int \psi^4(r)d^3r$$

(18)

with integration over the volume of the sample. Considering Eq. (18), it seems almost obvious that the vortex structure corresponds to a lower free energy than that for superconducting filaments. Indeed, in the case of the vortex lattice, the order parameter vanishes only along the vortex axes, while for the configuration of filaments the same happens along 2-dimensional inter-filament boundaries. In this case, the integration in Eq. (18) leads to a large numerical factor in favor of the vortex lattice. The actual situation, however, is more complex. Because each vortex line carries one magnetic flux quantum, the vortex density $n_v$ is strictly determined by the value of the applied magnetic field $H$. In the high-field limit, $n_v = H/\Phi_0$. There is no magnetic flux quantization condition for the system of superconducting filaments and therefore the density $n_f$ of filaments is a free parameter which may adjust itself to lower the free energy of the sample. In Fig.14(b) we show the field dependence of the order parameter amplitude $\psi_{\text{max}}^{(f)}$ for the system of superconducting filaments in comparison with the same quantity $\psi_{\text{max}}^{(v)}$ for the vortex lattice calculated in Ref. 60. In both cases $\psi_{\text{max}}$ vanishes at $H = H_{c2}$, however, as may be seen in Fig.14(b), in the case of filaments, $\psi_{\text{max}}^{(f)}$ is proportional to $\sqrt{1 - H/H_{c2}}$, while for the vortex lattice, $\psi_{\text{max}}^{(v)} \propto (1 - H/H_{c2})$. This means that, in spite of the numerical factor mentioned above, the free energy of the system of filaments is lower than that for the vortex lattice in the limit of $H \rightarrow H_{c2}$.

The properties of a mixed state consisting of superconducting filaments are quite different from that which contains Abrikosov vortices. First, because the filaments are always separated by normal conducting regions, the sample resistance for currents perpendicular to the direction of the magnetic field never vanishes and the true zero-resistance superconducting state may be achieved only after a transition to the vortex structure. Second, in the case of filaments, the magnetic flux faces no barriers to move in or out of the sample and the magnetization of the sample must be reversible, independent of whether the filaments are pinned or not. With decreasing external magnetic field, the configuration of superconducting filaments necessarily has to undergo a transition to the conventional mixed state, involving Abrikosov vortices. The value of the transition field is determined by the free-energy balance between these two configurations which cannot be determined without more precise calculations of the free energy for both cases. The transition from one type of mixed state to the other involves a complete change of topology and must be accompanied by discontinuities in both the resistivity
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and the magnetic moment of the sample. We also expect some hysteresis, as well as a latent heat, dictated by the discontinuity of the magnetization. In other words, this transition is expected to exhibit all the features of a first-order phase transition. Similar transitions are observed in high-$T_c$ superconductors at $H < H_c^2$ and are usually attributed to the melting of the vortex lattice.

As we have seen, in the high-magnetic field range, the distance between filaments is several times larger than the separation of vortices in the vortex lattice (Fig. 14(a)). This difference can be the key element to distinguish between these two realizations of the mixed state experimentally. This is not a simple task, however. In the high-magnetic-field range, where the mixed state consisting of superconducting filaments is expected to exist, the magnetic field is distributed almost uniformly across the sample and experiments that might distinguish between the mentioned options are quite difficult.

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