About the Measure of the Bare Cosmological Constant

Massimo Cerdonio

Received: 26 January 2019 / Accepted: 23 July 2019 / Published online: 30 July 2019
© Springer Science+Business Media, LLC, part of Springer Nature 2019

Abstract
I try to revive, and possibly reconcile, a debate started a few years ago, about the relative roles of a bare cosmological constant and of a vacuum energy, by taking the attitude to try to get the most from the physics now available as established. I notice that the bare cosmological constant of the Einstein equations, which is there ever since GR emerged, is actually constrained (if not measured) indirectly combining the effective cosmological constant observed now, as given by ΛCDM Precision Cosmology, with the cumulative vacuum contribution of the particles of the Standard Model, SM. This comes out when the vacuum energy is regularized, as given by many Authors, still within well established Quantum Field Theory, QFT, but without violating Lorentz invariance. The fine tuning, implied by the compensation to a small positive value of the two large contributions, could be seen as offered by Nature, which provides one more fundamental constant, the bare Lambda. The possibility is then discussed of constraining (measuring) directly such a bare cosmological constant by the features of primordial gravitational wave signals coming from epoch’s precedent to the creation of particles. I comment on possibilities that would be lethal, that is if the vacuum does not gravitate. This last issue is often raised, and I discuss the current situation about. Finally a hint is briefly discussed for a possible “bare Lambda inflation” process.

Keywords Cosmological constant · Relativistic aspects of cosmology · Vacuum energy · Primordial gravitational waves · Inflation

1 Introduction

Sometime ago an interesting debate took place about what to invoke in order to explain the speeding up of the expansion of the Universe. On one side Bianchi and Rovelli [1] invoked the role of an Einstein cosmological constant Λ, which, being the other constant allowed in Einstein GR besides the gravitational constant G, by no reason is
to be set to zero. Nature may well be offering the value needed to explain the observed cosmic acceleration. Dadhich [2] remarked that very general guiding principles require having a $\Lambda$ in the Einstein equations, as a true constant of the space–time structure. Both points of view would see its value given by the accelerating expansion of the Universe as observed in $\Lambda$CDM Precision Cosmology. On the opposing side Kolb [3] emphasized how mysterious is the smallness of such a cosmological constant needed by the $\Lambda$CDM model, in respect to that coming from the vacuum energy of particles fields.

I would like to revive the debate with an, until now unnoticed, argument, by taking the attitude to try to get the most from the physics now available. Possibly I reconcile the views summarized above: a bare $\Lambda$ is there in fact, but it is large, not the small one observed now, and the large value predicted by SM + QFT can be accommodated in the picture, without asking for revisions of the theory.

## 2 Constraining Indirectly the Bare Lambda

Recalling the so called semi-classical gravity approach to quantum gravity, Koksma and Prokopec [4] spelled out the distinction between the cosmological constant needed in principle on the left side of Einstein equations, which I call here the bare Lambda $\Lambda_{\text{bare}}$, and the cosmological constant $\Lambda_v$, sourced on the right by the energy density of the vacuum of fields, and consider that the cosmological constant needed by $\Lambda$CDM is actually a combination of the two as a $\Lambda_{\text{eff}}$

$$\Lambda_{\text{eff}} = \Lambda_{\text{bare}} + \Lambda_v$$

(1)

Therefore if we look at the observational situation now [5], we find a $\Lambda_{\text{eff}}$ which is small, while $\Lambda_{\text{bare}}$ and $\Lambda_v$ need not be, as they may appear to be free of being compensated each other.

I make the point here that not only $\Lambda_{\text{eff}}$ but also $\Lambda_v$ can be seen to come from observations, when its value can be calculated with QFT for the SM observed particles, and thus the possibility of a freedom in compensating $\Lambda_{\text{bare}}$ and $\Lambda_v$ is not in fact there anymore.

I remark that, as QFT and SM are well established physics [6] just as GR, and if the SM + QFT provide a full calculation of $\Lambda_v$, then $\Lambda_v$ should be considered as measured, thus also $\Lambda_{\text{bare}}$ would come out to be measured—at least indirectly—from Eq. (1). Then all cosmological constants, $\Lambda_{\text{eff}}, \Lambda_{\text{bare}}$ and $\Lambda_v$, result to be fixed by direct or indirect observations.

In the spirit to squeeze out the most from established physics, I avoid recourse to modifications/extensions of theories as GR and QFT, of modifications/extensions of models as the SM, and of violation of principles as Lorentz invariance [7] and the Equivalence Principle. A full calculation of $\Lambda_v$ is available only at $O(1)$, and so only constrains can be considered at the moment.

To evaluate $\Lambda_v$ within the SM sector, I use the coincident results of the various Lorentz invariant methods of regularization of the energy density of the vacuum of SM particles fields introduced respectively in refs [4, 8]. The motivation for these
regularization procedures was amply discussed and expanded in ref [9]. The issue is that, using the more common method with an ultraviolet cutoff at the Planck scale, one violates Lorentz invariance and gets the wrong equation of state for the vacuum. By contrast [9] “…the zero-point energy…can be made perfectly finite”, when one uses the regularization proposed and discussed in [4, 8, 9].

To get the total SM contribution to $\Lambda_v$, I recall, for convenience of the reader, the calculations of ref [4, 9], and so I use for the present vacuum energy density $\rho_v$ contributed by particles the relation

$$\rho_v = \frac{c}{\hbar}^3 \sum_j n_j (m_j^2/64\pi^2)\ln(m_j/\mu)^2 \text{ with } \Lambda_v = 8\pi G/c^2 \rho_v$$

(2)

where $n_j$ are the degrees of freedom, $m_j$ is the mass of the j particle, $G$ is the gravitational constant, $c$ is the velocity of light, $\hbar$ is the Planck constant (SI units). Notice that the Eq. (2) is demonstrated in [9] to be valid just the same also in curved space–time. The value of the renormalization scale $\mu$ is taken $\mu \sim 3 \times 10^{-25}$ GeV. As the leading term giving the ultraviolet cutoff at the Planck scale for renormalization has been discarded as unphysical, the renormalization scale for $\mu$ is now to be sought at energies below the Planck scale. The value chosen may appear somewhat arbitrary, but in fact the result is quite insensitive, over >30 orders of magnitude, to the value of $\mu$ for $\mu$ below $\sim 10^5$ GeV—see Fig. 5 in [9]. The particles taken in account are bosons (with positive sign)—Higgs, Z and W$^\pm$—and fermions (with negative sign)—quarks and leptons. The result is an overall negative $\rho_{\text{SM}} \sim -2 \times 10^8$ GeV$^{-4}$, which in SI corresponds to a negative $\Lambda_{\text{SM}} \sim -4 \times 10^3$ m$^{-2}$. Photons and neutrinos, as having zero and very small mass respectively, do not contribute.

One must add to $\rho_{\text{SM}}$ the contributions from the EW and QCD phase transitions. Such contributions are model dependent, but of order $O(1)$. Taking the values preferred in [9], all in all the total vacuum contribution from SM piles up to give a total value $\Lambda_v \sim -6 \times 10^3$ m$^{-2}$.

Thus the bare Einstein cosmological constant $\Lambda_{\text{bare}}$ can be evaluated from Eq. (1) with $\Lambda_v$ above and using the observed $\Lambda_{\text{eff}}$. As $\Lambda_{\text{eff}} = +10^{-52}$ m$^{-2}$—just slightly positive—is much smaller in absolute value than $\Lambda_v$, it is seen that $\Lambda_{\text{bare}}$ comes out to be practically equal to $\Lambda_{\text{bare}} = -\Lambda_v = +6 \times 10^3$ m$^{-2}$. This value should be correct at $O(1)$, and should be seen as a constrain from SM and QFT to the value of the bare cosmological constant.

It is commonly believed that GR emerged well before the EW transitions, while, by contrast, until then the energy density of the vacuum of fields was still absent, because, according to Eq. (2) the fields in a state of radiation do not contribute. Therefore, as the large positive $\Lambda_{\text{bare}}$ was acting standalone, it would have impressed to the early Universe a huge accelerated expansion, which may have been actually an inflation without inflatons—see Sect. 5. below.
3 Prospects for Direct Measurements of the Bare Lambda

The constrain discussed above looks however indirect. One may wonder if it would ever be possible to have a direct constrain/measurement. It has been recently considered, see [10] and refs therein, how a non-zero cosmological constant, no matter how small, can affect gravitational waves, GWs. At the moment only post deSitter/Newtonian calculations are available, but efforts for a full GR treatment are announced. Then, should we have available in the future on one side such calculations for a large lambda and on the other side observations of primordial GWs, generated before the vacuum contributions would be in place to balance the bare Lambda, it would be possible to get the bare Lambda in a direct manner, exclusively from the GR sector. There are proposed sources that can lead to cosmological backgrounds of gravitational waves coming from epochs back to inflation and before. A few of them could be within the reach of near-future gravitational wave detectors as LISA and the LIGO/VIRGO/KAGRA ground based observatory, see review [11]. However the only explicit calculation available of the effect on GWs of a positive lambda in a deSitter background concerns periodic GWs [12]. For the observed $\Lambda_{\text{eff}}$, the calculated alterations in periodic GWs, in respect to a $\Lambda_{\text{eff}}$ identically zero, would fail the LIGO/VIRGO/KAGRA and LISA detection levels by more than 20 orders of magnitude. As the bare Lambda considered above would be some 55 order of magnitudes larger than the observed $\Lambda_{\text{eff}}$, one would expect that quite large alterations should show up in primordial GWs, but of course, on one side in these conditions the approximations in [12] break down, and on the other side no extension to a stochastic background is available. So, while as for now a complete framework is not available, still the prospects for the future are encouraging, because on one side theory and calculation may develop definite predictions and on the other side GW detectors may reach adequate sensitivities.

4 Discussion

The logic of Sect. 2 is crucially based on accepting the results of the regularization methods of refs [4, 8, 9]. Usually renormalization procedures, to take care consistently of infinities, connect to physical measures within the sector of relevance. In the case here the connection to physics, to proceed with the regularization, is somewhat less direct. As summarized above it concerns avoiding violation of Lorentz invariance, a violation which however is strongly excluded by a wealth of current experiments/observations.

I searched the literature to find comments/criticisms/rebuttals about this issue, and found increasing consensus. Dadhic remarked [2] that we would have to wait for quantum gravity but meanwhile the important point is that the Lambda coming from that would have no relation with the Planck length. In Ref. [13], where Lorentz invariance is considered for different purposes, but still the issue of the connection with the cosmological constant is discussed at length, it is remarked that “…imposing Lorentz invariance has given us a rather definite finite cut-off estimate for the cosmological
constant”. More recently [14] this result has been used (in a different context) as well known.

It is commonly accepted that the vacuum energy gravitates with minimal coupling in the Einstein equations and that the Casimir effects offer experimental evidence of that—see for instance [9] and refs therein. Such a notion has been questioned recently. On one hand Dadhich [2] proposes that such a vacuum energy should not gravitate through a stress tensor, but rather through enlargement of the framework. On the other hand for Casimir effects, Nikolic contends [15] that the Casimir force cannot originate from the vacuum energy of electromagnetic (EM) field. Cerdonio and Rovelli [16] demonstrated, with a simple gedanken experiment, that the action of the em vacuum in a Casimir cavity is inextricably connected to the (massive) presence of matter in the plates, and that it gives just a (regular, negative) binding energy—nothing to do with the “free” vacuum called in for cosmology. Similar remarks, after different arguments, can be found in [17]. Also, the idea itself of the role of zero-point energies has been contrasted, in favor of relativistic quantum forces within charges in the matter of the plates [18]. In lack of a final clarification of the issue, I warn here how such a semi-classical gravity hypothesis is a crucial assumption for my considerations.

One may feel that the fine tuning which appears, as \( \Lambda_{\text{eff}} = + 10^{-52} \text{ m}^{-2} \) while \( \Lambda_{\text{bare}} \) and \( - \Lambda_v \) are much larger, would be embarrassing. In fact the point made here, that the vacuum energy estimated from the Standard Model—many orders of magnitude larger than the observed cosmological constant—may be compensated by a bare cosmological constant, has been made often in the cosmological literature, but it has been always taken as an unwanted fine-tuning to be dismissed, as, for instance, in refs [4, 9].

At variance with the above attitude, I consider alternatively to be on the table a deceivingly simple notion. As \( \Lambda_{\text{eff}} \) comes from observations, and \( \Lambda_v \) comes also from measured quantities through well-established physical theories, then the logic conclusion is rather that \( \Lambda_{\text{bare}} \) is actually at least heavily constrained, using the observations of current precision cosmology and the measurements coming from realms different from cosmology.

5 A Bare Lambda Inflation?

Finally the above considerations invite to an obvious speculation, that actually the positive and large \( \Lambda_{\text{bare}} \) may have started the inflation process—an inflation without inflatons. The SM physics is well understood and tested up to temperature of the EW transition [6]. Above this temperature the SM particles are massless, and thus do not contribute to the renormalized vacuum energy, according to Eq. (2). It is common view that above the GUT scale the Universe would be filled with radiation at that temperature, somewhat below the Planck scale \( T_P \sim 10^{19} \text{ GeV} \). Constrains on the initial thermal radiation have been considered in [19]. Therefore, if a quasi-DeSitter expansion would be initiated by \( \Lambda_{\text{bare}} \), the Universe would cool down until reaching the EW transition temperature. The particles vacuum contributions would then start to cumulate to give a \( \Lambda_v \). Such a \( \Lambda_v \) would ultimately compensate \( \Lambda_{\text{bare}} \), in sort of a graceful exit from a “bare Lambda inflation”. If I take the temperature of Universe \( T_i \)
at the start of the process somewhat below the Planck temperature, say $T_i \sim 10^{17}$ GeV, and as final temperature $T_f$ that of the completion of particle creation, say indicatively $T_f \sim 1$ MeV when neutrino decoupled, and I use a ratio of expansion rates $a_f/a_i \sim T_i/T_f$ as for radiation, then the number of e-folds would be $N = \ln \left(\frac{a_f}{a_i}\right) \sim 46$. Such an $N$ is close to the values $N \sim 50–60$ preferred by Planck [20] for a generic inflation process. Of course this may be only a numerical coincidence, but the matter may warrant further attention, as the scenario would be pretty rigid, and thus could be more credible in a Bayesian sense than any inflaton model. An elaboration of this hint is however beyond the scope of this paper.

6 Concluding Remarks

In view of the above discussion, it looks to me that the fine tuning is rather offered by Nature. Therefore it is just an observational fact: a set of measurements made now gives actually the value which is built eternal and unchanging in the Einstein equations of GR. Such a Nature given fine tuning appears at the same level of the fine tuning of the fundamental constants, to account for which anthropic reasoning’s have been put forward. Then the result of my considerations should be seen as an observational evidence about a primordial bare Lambda, and thus to be taken in account in modelling the early Universe. My considerations may give a new slant to the Cosmological Constant Problem(s).

Acknowledgements

I am grateful to my wife Annamaria for bearing with me during the preparation of this paper and to Naresh Dadhich for correspondence on the matter. I thank Alessandro Bettini and Gianni Carugno for lively discussions. I thank Philippe Jetzer for a discussion and for helpful suggestions. I am much indebted to Stefano Liberati for a critical reading of the manuscript, with comments I took in due consideration for the present version.

References

1. Bianchi, E., Rovelli, C.: Why all these prejudices against a constant?. arXiv:1002.3966v3[astro-ph. CO] 11 April 2010
2. Dadhich, N.: On the enigmatic $\Lambda$—a true constan of spacetime. arXiv:1006.1552v2 21 Feb 2011; see also of the same Author arXiv:1105.3396 and arXiv:1609.02138
3. Bianchi, E., Rovelli, C., Kolb, R.: Cosmology forum: is dark energy really a mystery? Nature 466, 321–322 (2010)
4. Koksma, J.F., Prokopec, T.: The cosmological constant and lorentz invariance of the vacuum state. arXiv:1105.6296v1 [gr-qc] 31 May 2011
5. by now I mean the cosmic time when the creation of all the SM particles had been completed and the contribution of a $\Lambda_{\text{eff}}$ to the expansion of the Universe is found constant within the precision of current observations
6. “well understood and experimentally tested laws…” from Table I in TASI Lectures on inflation by Baumann D.: arXiv:0907.5424v2 (2012); I am using the term well established physics to summarize what has been remarked therein about the history of the Universe “…from $10^{-10}$ seconds [corresponding to the electroweak unification at an energy of 1 TeV] to today the history of the universe is based on well understood and experimentally tested laws of particle physics, nuclear and atomic physics and gravity”
however Lorentz violating theories as Horava gravity are amenable to renormalization and still compatible with observations, see for instance Wang, A.: Horava gravity at a Lifshitz point: a progress report. Int. J. Mod. Phys. D 26, 1730014 (2017); I thank S. Liberati for pointing this to me

Akhmedov, E.K.: Vacuum energy and relativistic invariance. arXiv:hep-th/0204048v2 (2002)

Martin, J.: Everything you always wanted to know about the cosmological constant problem (but were afraid to ask) in understanding the dark universe Comptes Rendues Physique 13(6-7), 566–665 (2012)

Ashtekar, A.: Implications of a positive cosmological constant for general relativity. Rep. Prog. Phys. 80, 102901 (2017)

Caprini, C., Figueroa, D.G.: Cosmological backgrounds of gravitational waves. arXiv:1801.04268 [astro-ph.CO] 5 Feb 2018

Näf, J., Jetzer, P., Sereno, M.: On gravitational waves in spacetimes with a nonvanishing cosmological constant. Phys. Rev. D 79, 024014 (2009)

Visser, M.: Lorentz invariance and the zero-point stress-energy tensor. Particles 1, 138 (2018)

Lombriser, L.: On the cosmological constant problem. arXiv:1901.08588v1 [gr-qc] 23 Jan 2019

Nikolić, H.: Proof that Casimir force does not originate from vacuum energy. Phys. Lett. B 761, 197 (2016)

Cerdonio M., Rovelli C.: Casimir effects are not an experimental demonstration that free vacuum gravitates: connections to the cosmological constant problem, J Mod Phys D 24, 1544020 (2015) [special issue publishing a selection of Essays awarded with Honorable Mention by the Gravity Research Foundation 2015 Awards]; Cerdonio M. and Rovelli C.: Casimir cavities do not fly. arXiv:1406.1105v3 2 March 2019

Mostepanenko, V.M., Klimchitskaya, G.L.: Whether an enormously large energy density of the quantum vacuum is catastrophic. arXiv:1903.04261v1 [physics.gen-ph] 2 March 2019

Jaffe, R.L.: Casimir effect and the quantum vacuum. Phys. Rev. D 72, 021301 (2005)

Herrera, R., Pavon, D., Saavedra, J.: Constraints on the radiation temperature before inflation. arXiv: 1801.06155v1 [gr-qc] 18 Jan 2018

Planck 2015 results: A&A 594, A13 (2016)

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.