Calibration of Laser Doppler Anemometer: Fringe Spacing and Velocity

F S Ferreira¹, F O Costa¹, I P Barros¹, S Araújo¹

¹ National Institute of Metrology, Quality and Technology – Inmetro
Directory of Scientific and Technology Metrology - Dimci
Fluid Dynamics Metrology Division – Dinam
Gas Flow Laboratory - Lagas
Av. Nossa Senhora das Graças, 50 – Prédio 6 - Duque de Caxias – RJ
E-mail: fsferreira@inmetro.gov.br

Abstract. This paper presents the most recent mathematical modelling and procedure for Laser-Doppler Anemometers (LDA) calibration used by the Brazilian National Institute of Metrology Institute, the National Institute of Metrology, Quality and Technology (Inmetro). Here we calculate the fringe spacing calibration coefficient (C_f) and the Burst Spectrum Analyser (BSA) calibration coefficient (C_bsa). Using these two calibration coefficients we can calibrate a LDA in a determinate velocity range.

Keyword. Calibration; LDA; Spacing; Fringe; Velocity; Anemometry.

1. Introduction
Most of the National Metrology Institutes use a calibration disk as a primary standard for fluid speed [1]. The working principle is based on the calculation of the linear speed of a rotating disk and the direct comparison with the speed in a Laser Doppler anemometry system. The LDA readings detect some particle moving on the disk, which may be valleys and peaks of the surface roughness [2], a tightly bound tungsten wire [3, 4], or particles embedded in the building material of a glass disk [5, 6].

The LDA calculates the speed by determination of the Doppler frequency, reflected by a particle passing through the control volume under non-slip condition [7]. Knowing the spacing between the fringes, a linear relationship can be easily deduced to calculate fluid speed. The spacing between the fringes can be theoretically determined according to the equation (a.1):

\[ \delta = \frac{\lambda}{2\sin(\theta/2)} \]  

(a.1)

Where \( \lambda \) is the wavelength of the laser beam and \( \theta \) is the angle of intersection.

However, imperfections of lens, slight variations in wavelength and phase shift outside the laser beam waist create distortions in the fringe spacing, making it necessary to characterize the control volume.
As the distance between interfering fringes are independent of the fluid speed, by calibrating the Burst Spectrum Analyzer in frequency, the entire range of velocities can be determined by the equation (a.2):

\[ v = \delta \cdot f_d \] (a.2)

2. Development

2.1. Calibration of the Speed standardization Rotation Disk

The rotation disk was calibrated by Inmetro’s Dimensional Laboratory. The diameter was calibrated in three different heights (figure 1). To each height eight diameters in different directions and the out of roundness were measured.

![Figure 1 – Measurements Heights](image)

For the purpose uncertainty calculation, the reference diameter can be determined as the mean of all calibrated diameters and the out of roundness can be determined as the random variation of the diameter:

\[ d_0 = \frac{1}{n \cdot m} \sum_{i=1}^{n} \sum_{j=1}^{m} d_{i,j} + \delta d \] (a.3)

Where \( n \) is the number of measured diameters at each one of the \( m \) heights and \( \delta d \) is the random variation of the diameter, which can be calculated as the mean out of roundness:

\[ \delta d = \frac{1}{m} \sum_{j=1}^{m} \delta d_{i,j} \] (a.4)

2.2. Reference Speed

The reference speed for the LDA calibration shall be the tangential velocity of the disk at the intersection position between the control volume and the disk surface. Figure 2 shows the used coordinate system.
The reference for calibration will be the plane perpendicular to the plane formed by the laser beams, which can be described by a vector:

\[ \vec{p} = \langle 0, 0, 1 \rangle \]  

This vector will be used as the basis for calculating the alignment between the control volume and the calibration disk. The angular velocity of the calibration disk can also be described by a vector:

\[ \vec{\omega}_d = \langle 0, \omega_c, 0 \rangle \]  

Where \( \omega_c \) is the calibrated angular speed of the calibration disk, and:

\[ V_e = \sqrt{\left( \frac{d_d}{2} \right)^2 - \Delta z^2, 0, \Delta z} \]  

The point where the Control Volume intercepts the surface of the calibration disk, where \( d_d \) is the disk diameter and \( \Delta z \) is the vertical alignment error. The arc length at \( V_e \) is:

\[ \vec{r}_r = \langle 0, 0, 0 \rangle - V_e \]  

The surface velocity will be given by the cross product between \( \vec{\omega}_d \) and \( \vec{r}_r \):

\[ \vec{v}_l = \vec{\omega}_d \times \vec{r}_r = \langle -\Delta z \omega_d, 0, -\frac{d_d^2}{4} - \Delta z^2 \omega_d \rangle \]  

The speed seen by the LDA will be given by the projection of \( \vec{v}_l \) in \( M \cdot \vec{p} \), where \( M \) is the rotation matrix which describes angular misalignment of the system:

Figure 2 – Coordinate System
\[ M = M_z \cdot M_x \cdot M_y \] (6)

\[
M_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\phi_x) & \sin(\phi_x) \\
0 & -\sin(\phi_x) & \cos(\phi_x)
\end{bmatrix}
\] (7)

\[
M_y = \begin{bmatrix}
\cos(\phi_y) & 0 & -\sin(\phi_y) \\
0 & 1 & 0 \\
\sin(\phi_y) & 0 & \cos(\phi_y)
\end{bmatrix}
\] (8)

\[
M_z = \begin{bmatrix}
\cos(\phi_z) & -\sin(\phi_z) & 0 \\
-\sin(\phi_z) & \cos(\phi_z) & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (9)

By the equation (5) we have:

\[
V_{ref} = \frac{\omega_d}{2} \sqrt{d_d^2 - 4\Delta z^2 \cos(\phi_x) \cos(\phi_y) + 2\Delta z \sin(\phi_x) \sin(\phi_z) \cos(\phi_y)} + 2\Delta z \cos(\phi_z) \sin(\phi_y)
\] (10)

\[ \omega_d = 2\pi f_d \] (11)

\[ f_d = \frac{f_c}{n_p + \delta n_p} \] (12)

\[ d_d = (d_0 + \delta d_0) \left(1 + \alpha (T_m + \delta T_m - T_r)\right) \] (13)

\[ T_m = \frac{T_{1i} + T_{2i}}{2}, \] (14)

Where:

- \( \omega_d \): angular velocity of the calibration disk;
- \( \Delta z \): vertical alignment error;
- \( \phi_x, \phi_y, \phi_z \): system angular alignment errors;
- \( d_0 \): calibrated diameter of the disk (mean of the diameters);
- \( f_d \): frequency of the calibration disk;
- \( f_c \): clock frequency of the time and frequency pattern;
- \( n_p \): number of mean clock pulses per revolution;
- \( \delta n_p \): error due to random variation of the number of clock pulses;
- \( \delta d_0 \): error due to the mean circularity of the disk;
- \( \alpha \): coefficient of linear expansion of steel;
- \( T_m \): temperature during measurements;
- \( T_{1i}, T_{2i} \): initial temperature of the disk at spots 1 and 2, respectively.
The calibration coefficient for fringe spacing can be determined as the ratio between the reference velocities and the reconstructed speed of the LDA. The speed of the LDA needs to be reconstructed because, during calibration, only one position of the control volume can be observed. In real flows, particles pass homogeneously throughout its entire length. However, the probability of a particle being detected is not uniform throughout the control volume. Thus, the control volume should not only be characterized as to its distortion, but also as to the relative acquisition rate at a given point [11]. The reconstructed speed is calculated as the average speed weighted by the Data Rate (or by the number detected values) in each position:

\[ C_f = \frac{v_{ref}}{v'_{ida}} + \delta C_f \]  \hspace{1cm} (15)

\[ v'_{ida} = \frac{\sum_{i=1}^{N}(v_i + \delta v_i)n_i}{\sum_{i=1}^{N}n_i} \]  \hspace{1cm} (16)

Where:

- \( C_f \) calibration constant;
- \( v_{ref} \) reference speed (linear velocity of the calibration disk);
- \( v'_{ida} \) speed calculated by scanning the control volume;
- \( \delta C \) error due to repeatability;
- \( N \) number of positions measured;
- \( v_i \) mean speed of \( i \)-th measured position;
- \( \delta v_i \) random variation of the speed of \( i \)-th measured position; and
- \( n_i \) number of points measured in the \( i \)-th measured position.

2.4. Calculation of the BSA constant \( (C_{BSA}) \)

The Burst Spectrum analyzer shall be calibrated for frequency measurement to achieve traceability at speed over the entire desired range. However, many LDA systems do not have an input channel directly connected to the data acquisition board, and this calibration cannot be performed directly by a time and frequency laboratory.

In this way, the calibration disk can be used again to perform this step. Due to the symmetry of the control volume for the acquisition rate and the approximately linear variation of the distortion with the position, the calibration coefficient \( C_f \) will be adjusted for the Doppler frequency obtained at the center (position of the highest data rate) of the control volume. This position can be determined through the two highest data rate points. Let these positions be \( x_1 \) and \( x_2 \) with data rates of \( Dr_1 \) and \( Dr_2 \), respectively. The highest data rate \( (x_{max}) \) position, where the BSA calibration will be performed, will be:

\[ x_{max} = \frac{Dr_1x_1 + Dr_2x_2}{Dr_1 + Dr_2} \]  \hspace{1cm} (17)
By determining a point in the control volume with a high acquisition rate and by varying the frequency of the disk, signals with different Doppler frequencies will be generated, allowing the calibration of the frequency card. The direct relationship between the reference speed and the speed obtained in \( v_{lda} \) determines a BSA coefficient for a given velocity:

\[
C'_{bsa} = \frac{v_{ref}}{v_{lda} + \delta v_{lda}} + \delta C'_{bsa}
\]  

(18)

It is observed in the equation above that the LDA velocity doesn’t need to be reconstructed as in the fringe spacing calibration, since this step is only intended to determine the BSA frequency card. The superscript ‘ ′ ’ in the calibration coefficient of the BSA is relative to the unadjusted constant. This adjustment is necessary so that the fringes distortion is not considered twice:

\[
C_{bsa} = \frac{C'_{bsa}}{C_{bsa,f}}
\]  

(19)

Where:

- \( C'_{bsa} \) unmatched constant;
- \( v_{lda} \) speed read by LDA;
- \( \delta C_{bsa} \) error due to random variation of the unbalanced constant;
- \( C'_{bsa,f} \) constant not compensated for the speed used during the calibration of the spacing between stripes; and
- \( \delta v_{lda} \) error due to the random variation of the speed read by the LDA.

Strictly speaking, an iterative process between the calibration of the spacing between the fringes and the calibration of the BSA should be performed: In performing the calibration of the spacing between the fringes, the differences between the speeds at each position of the control volume should be corrected by the calibration of the BSA. However, the differences between the frequencies are extremely small relative to the stability of the BSA frequency card, so that its coefficient can be considered constant for values around the calibrated frequency.

2.5. Calibrated speed calculation

The speed of the LDA is determined by multiplying the two calibration coefficients at the speed read by the LDA system:

\[
v_c = C_{bsa}C'_{f}v_{lda}
\]  

(20)

2.6. Automation and data acquisition

System automation and data acquisition was developed in LabView environment. Due to the high frequency of the reference clock, the angular frequency measurement of the disk was developed in FPGA logic and compiled in NI cRIO-9073 controller, with data transfer using FIFO structure, as shown in figure 3:
The programming in FPGA is essential for precisely measure the period of rotation of the calibration disk. A 32-bit counter integrates the number of pulses of the time reference, with the beginning and end of the counts determined by the signal coming from the photodetector circuit. In this way, each revolution generates a value of number of pulses and, consequently, period of rotation. The signals are stored in memory, forming a data queue with FIFO structure that is downloaded to the user interface.

A user interface was also created in LabView, operating in Windows environment. After the end of a measurement, the data is exported in a text file.

2.7. Results and Uncertainty calculation
The uncertainty calculation was performed according to “Evaluation of Measurement Data — Guide to the Expression of Uncertainty in Measurement” (ISO-GUM) [9] in Excel worksheet and validated in Mathematica software. [8].

The final uncertainty of the calibration was 0.2%, with major contributions of the repeatability (56%), alignment (20%) and the diameter calibration (16%).

3. Conclusions
The uncertainty of the system showed good agreement to other National Institutes of Metrology. The calibration of the BSA can lead to virtually unlimited speed range, requiring only fringe spacing calibration and a frequency based calibration of the frequency card.

Developments of mathematical models for compensating temperature effects on the probe and lenses can enhance the repeatability of the calibration, while the assessing the effects of the laser intensity and its intensity distribution can be useful for improving the data rate and velocity profile.

The calibration of Laser Doppler Anemometers is the first step to create the traceability chain of fluid speed. Next steps include calibration of Ultrasonic anemometers and Pitot Tubes.
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