Mathematical abstraction ability of prospective math teacher students

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Abstract. This paper discusses mathematical abstraction ability of prospective Math teacher students. There are two processes as the requirement of mathematical abstraction process, they are generalize and synthesize Math teacher ideally can integrate mathematical thinking skill in their every learning process. The importance of abstraction ability development are in line with the importance of mathematical competence mastery for the students' life. The use of developing mathematical representation ability are (1) students can find the close correlation between the branches of Math, (2) students can find the results of one branch that could trigger the conjecture on related branches. This research is a qualitative study that explores mathematical abstraction ability of prospective math teacher students'. The subject in this study was a six-student prospective math teacher with different mathematical ability. In the case of mathematical abstraction ability, it was known that students had understood the concept well but had not been able to give structured and systematic reason. For the mathematical abstraction ability, it was known that the lack of two skills is caused by (1) students had not used to give reason correspond to the structured and systematic concept; they tend to answer it directly by giving direct illustration with numbers. (2) Students forgot the argument stages in explaining a mathematical concept.

1. Introduction

Math learning has an important role for students to raise their competences. [6] States that math learning is aimed to provide students with not only logical, analytical, systematical, critical, innovative, and creative thinking skills but also teamwork skills. Those competences are needed for students to develop their self-quality to have a better life in the competitive and changing situation. Math learning in senior high school is a continuation of Math learning in elementary and junior high school. In their previous education, students learn Math in a learning situation designed by the teachers with the support of props as the concrete picture of Math objects. After that process, students are guided to think formally. It is done as the adjustment to the students’ thinking skills so that they can easily understand the meaning of those abstract objects of Math. However, in university, Math material is getting difficult to be learned.

Students are used to thinking negatively of Math learning. They often face a big deal in some mathematical process such as reasoning, non-routine problem solving and verifying. According to [2], the change from elementary thinking to advanced mathematical thinking involves a significant transition i.e. from describing to defining, from convincing to verifying logically based on a definition.
The transition process is reputed to be a problem for the students. Math in school is viewed as a combination of visual representation including geometry and graph as well as calculation and symbolic manipulation, while Math in university moves to formal framework of the axiomatic system and mathematical evidence. Therefore, that is why students need mathematical abstraction ability.

Here are some results of surveys from some experts who state that mathematical abstraction ability is still low, such as David’s research [3] on the students of Tennessee Technological University who had already passed Calculus course in which the students were given non-routine questions. “Apparently, there is no student can answer it correctly and even most of them can do nothing.” The situation was not much different from the other four questions. Besides, Davis [3] have done a test towards fresh-students who was excellent in their previous education (senior high school) and it turns out that most of them make mistakes in the conception of a mathematical concept.

According to [16] there are several causes why it was happening, such as: generally, Math instruction, from elementary school to university, may be taught in what is called as ritual: “Do this and then do that,” and usually the teachers will give appraisal to the students who do as told. The issue happened to the students of Faculty of Teacher Training and Education of Universitas Riau Kepulauan majoring in Mathematics Education, especially those who took Abstract Algebra course. This course is one of the compulsory courses which needs advanced mathematical thinking ability. Prerequisite to taking the course is Theory of Numbers and Basic Algebra. The course studies the structure of ring including ring, domain integral, field, ideal, polynomial ring, homomorphism ring and its application. Therefore, it is needed conceptual understanding, formal definitions, mathematical model ability, related theorems, and verification.

Based on the interview results from several students, obtained information showed that Abstract Algebra course was a difficult course. The students admitted that they found it hard; to understand the characteristics of algebra in the form of a symbol; to verified the characteristics of algebra; to relate one concept to another, and to solve Algebra problem if there was no example from the lecturer. It showed that students’ mathematical abstraction ability needs to be improved regarding Math learning in school. Math learned in school is provided on a lesson which consists of several field of study, such as arithmetic, Geometry, Algebra, Statistics, etc. The importance of mathematical abstraction ability in the present demands the mathematics prospective teachers to be able to relate mathematical abstraction ability to every lesson given to the students.

2. Research Methodology

This research is a qualitative study that explores mathematical abstraction ability of students’ prospective math teacher. The subject in this study was a six-student prospective math teacher with different mathematical ability. Data were collected from test and interview. The test is a problem about abstract algebra to showing mathematical abstraction ability of the subject. While the interview is focused to explore the constraints faced by students in solving the problems given. The test results are presented to the subject as an interview guide to explore potential factors related to the settling of problems. The test is given to the respondent before the interview. The test is then discussed to clarify the error of the subject answering the matter. Furthermore, the interview record is discussed again together with the respondent to reveal the limitations of understanding, misconception, or misinterpretation. The interview result is transcribed and then justified by the subject as a form of data validity.

3. Result and Discussion

Mathematical abstraction ability belongs to one part of advanced mathematical thinking. The opinion of [1,4] who defines advanced mathematical thinking as a mathematical thinking process such as the process of representation, abstraction, the connection of representation and abstraction, creativity and mathematical evidence. Besides, [14] tentatively defines advanced mathematical thinking as the ability of representation, abstraction, connecting representation and abstraction, thinking mathematically and creatively as well as constructing mathematical evidence. [11,12] defines abstraction of mathematics
as a process for deriving the essence of mathematical concepts, eliminating its dependence on real-world objects that might initially be interconnected, and broadening it to have wider application or to have application which in correspondence to other abstract explanations for equivalent phenomena. [7, 8, 9, 10, 13, 15, 18, 19, 20] explains that abstraction is the transition from a concrete operational model to a structural (abstract) model. According to [6] abstraction has two meanings, first as a process of ‘describing’ a situation, and secondly a concept as a result of a process. According to [5] if we take into account the development of the mathematics of both points ie. history and learning, we can see that abstractions are often a major step in creating new and frequent concepts new object appears.

There are two processes as the requirement of abstraction process; they are generalizing and synthesizing.

(1) Generalize

According to [16] generalizing is defined as showing or inducting the specifics for identifying the similarities.

Example of abstraction as a generalization

Suppose that Zn association is an integer modulo n. \( n \in \mathbb{Z}^+ \)

For \( n=2 \) (Zn,+) group
For \( n=3 \) (Zn,+) group
For \( n=4 \) (Zn,+) non-group
For \( n=5 \) (Zn,+) group
For \( n=6 \) (Zn,+) non-group

What is the conclusion of those statements?

(2) Synthesize

Synthesize means combining or arranging parts where the parts shape a unity, i.e., totality [17]. For example, in a linear algebra course, there are several materials that are taught separately, such as vector orthogonalization, matrix diagonalization, base transformation, system solutions of linear equations, and so on. In learning, all the unrelated materials are expected to be combined into an image where all materials are interrelated. According to [17] this combining process is called syntheses.

The example of mathematical abstraction ability.

(1) Let \( P = \{\text{genap}, \text{ganjil}\} \) and \( P \subset \mathbb{Z} \). Show that the elements of "genap" and "ganjil" numbers are a commutative ring, then check whether the commutative ring is a field.

Example of respondent’s answer

![Figure 1](image-url)

Figure 1. Example respondent's answer for the first question.
Synthesizing process is a process of combining or arranging parts to make a totality, unity or entity. The totality is not just a total amount of its parts, but it is more than that because the process also relates the separated parts into one related entity. Based on the given questions, students were expected to be able to arrange arguments (parts) to show that “odd” and “even” numbers elements were one commutative ring. According to existing information, students were expected to be able to check whether the ring was included as a field or not. Based on the students’ answers, it could be seen that the given arguments were not entirely correct. For example, in writing mathematic symbol and argument systematics students were tend to write the answer based on the spoken language instead of the good written explanation; whether it was in preparing arguments (parts) to show that “even” and “odd” elements are a commutative ring or check if the commutative ring is a field. In fact, there were some students who did not continue their answer to check whether the commutative ring was a field or not. When the students were asked, it was due to two main reasons; they were: (1) students did not use to give the reasons in accordance to the structured and systematic concept; they tend to respond directly by giving a direct illustration with numbers. (2) Students forgot the stages of the argument in explaining a mathematical concept (commutative ring). The reasons of students, who did not continue their answer to check whether the commutative ring was the field, were: (1) students forgot the field concept, they just remembered a little, (2) students knew the field concept, but they did not know how to give the right answer.

The example of mathematical abstraction ability.

(2) Give an example of group G and the original subgroup H so that G is isomorphic to H. Then show that your answers meet the required requirements.

Example of respondent’s answer

![Example respondent’s answer](image)

Figure 2. Example respondent’s answer for the second question.

Based on the given answer, there was only one answer while the other did not answer. It could be seen that fresh student had enough skill on the sensitivity stage or the skill to understand the problem. For the next stage, students were looked confused and doubt. The students who did not answer stated that they did not know how and what to answer. After they were tracked, it is caused by two major reasons: (1) students are not accustomed to providing reason according to the concept in a structured and systematic way; they tend to respond directly by giving a direct illustration with the numbers. (2) Students forget about the stages of argumentation in explaining a mathematical concept.
4. Conclusion
Based on the study result, in case of mathematical abstraction ability, it was known that students had understood the concept well but had not been able to give a structured and systematic reason. For the mathematical abstraction ability, it was known that the lack of two skills is caused by (a) students had not used to give reason correspond to the structured and systematic concept; they tend to answer it directly by giving direct illustration with numbers. (b) Students forgot the argument stages in explaining a mathematical concept. It is needed a learning design which can facilitate students to develop their mathematical abstraction ability.

Acknowledgments
This research was supported by Direktorat Jenderal Sumber Daya Iptek dan Dikti through Direktorat Kualifikasi Sumber Daya Manusia in collaboration with Lembaga Pengelola Dana Pendidikan that presents Beasiswa Unggulan Dosen Indonesia. We thank who provided opportunities and funding that greatly assisted the research, although they may not agree with all of the interpretations/conclusions of this paper.

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