1. Introduction

Design of load bearing metallic components at service temperatures where deformation is time dependent, is usually done on the basis of mechanical property data obtained from laboratory tests performed in air. The test temperatures are often high enough to oxidise the metal surface. Therefore, as the material creeps, the specimen would continue to lose its load bearing cross sectional area because of oxidation as well. It has been found that a specimen having a large cross sectional area gives a longer creep life because of the above interaction.1)

A clear understanding of the effects of section size and specimen geometry on creep properties is, therefore, of considerable technological significance to the design of engineering components. The effect of section size and specimen geometry in an oxidising environment has been reported by many researchers in literature.1–4) Usually a specimen having smaller section size exhibits a higher creep rate and consequently a shorter life. The effect is more pronounced as the section size decreases. On the other hand, for steel specimens having more than 50 mm diameter, almost identical creep strain–time plots are obtained both in air and in vacuum.1)

The objective of the present work is to suggest a set of constitutive equations of creep in oxidising environment for predicting the performance of a component in actual service. A large volume of existing data on nickel base superalloys and steels having different section sizes have been used for validation.

2. Damage Model

Following Dyson and Osgerby,5) creep strain (ε) accumulation in steels and superalloys can be represented as:

\[ \dot{\varepsilon} = \dot{\varepsilon}_0 \left( \frac{\sigma}{\sigma_0} \right)^n \exp(C\varepsilon) \]..........................(1)

where \( \dot{\varepsilon}_0 \) is the initial creep rate at stress (\( \sigma = \sigma_0 \) and \( \varepsilon = 0 \)), \( C \) describes the net contribution of several mechanisms responsible for strain softening. For Ni base super alloys it has been shown to be a composite term given by:

\[ C = C_{\text{int}} + n + n/(3\varepsilon_f) \]..........................(2)

where \( C_{\text{int}} \) represents the contribution to tertiary creep due to the intrinsic deformation mechanisms of the superalloy; \( n \) is the contribution due to stress increment associated with strain accumulation in a constant load test; and \( (n/3\varepsilon_f) \) is the maximum contribution from intergranular cavitation causing fracture.5) \( C_{\text{int}} \) is zero for materials exhibiting predominantly steady state creep, such as pure metals and simple alloys.

The oxide layers that forms during creep test on a test piece of most commercial alloys are brittle and cannot accommodate the creep strain that keeps accumulating in the central core.1) Therefore fine cracks are developed in these layers and the effective cross section area of specimen decreases. As a result it is reasonable to assume that, the stress on the specimen continues to increase with time be-
cause of the reduction in its load bearing section size. The simplest way to couple the environmental interaction is to assume that the thickness of the oxide layer is a measure of the damage and consequently the stress on the remaining metallic section rises proportionately. Thus, if a specimen of circular cross section with radius \( r_o \) is subjected to a force \( F \) then the initial stress \( \sigma_o \) to which the specimen is subjected to is given by:

\[
\sigma_o = F/\left(\pi r_o^2\right) \quad \text{(3)}
\]

Whereas the stress \( \sigma \) at any instant, when an oxide layer of thickness \( x \) has built up on the specimen is given by

\[
\sigma = F/\left(\pi \left(r_o^2 - x^2\right)\right) \quad \text{(4)}
\]

The ratio \( \sigma/\sigma_o \) is, therefore, expected to provide an estimate of stress intensification because of the progressive growth of the thickness of the oxide layer. This is given by

\[
\sigma/\sigma_o = 1/(1-2x/r_o)^2 \quad \text{(5)}
\]

The expressions for \( \sigma/\sigma_o \) for different geometries can be obtained in similar manner. The physical basis has been diagrammatically represented in Fig. 1. The final forms of \( \sigma/\sigma_o \) for a variety of specimen geometry are listed in Table 1. The above formulation is based on the assumption that the oxidation rate is the result of environmental interaction has no load bearing capacity. Therefore it will be applicable where the oxide layers are reported to be brittle and prone to cracking as the sample creeps.

Oxidation being a diffusion controlled process, the thickness \( x \) of oxide layer is expected to follow a parabolic growth law. Several expressions have been proposed in the literature to describe oxide scale growth kinetics. However, in the presence of stress, as the oxide layer can not accommodate plastic (creep) strain, fine cracks are developed exposing the parent metal. Therefore, under creep conditions a linear growth law may appear to be more appropriate.

2.1. Linear Growth Law

In the case of linear growth law, the oxide thickness \( x \) may be expressed as:

\[
x = Kt \quad \text{(6)}
\]

where \( K \) is a temperature dependent oxidation rate constant.

Thus, for a test piece having a circular cross section, a combination of Eqs. (1), (5) and (6) is capable of describing its creep behaviour. Solution of Eq. (1), under conditions of constant load/stress as described above, gives an expression of strain as a function of time. For example, in the case of a test piece having a circular cross section, integration of the following equation

\[
\dot{e} = \dot{e}_o \exp\left(C \varepsilon \left(1 - Kt/r_o\right)^{-2n}\right) \quad \text{(7)}
\]

can generate a set of creep curves of any section size. Since \( Kt \ll r_o \), an algebraic simplification of the Eq. (7) gives:

\[
\ln\left(\dot{e}/\dot{e}_o\right) = C \varepsilon + 2nKt/r_o \quad \text{(8)}
\]

The above equation suggests a simple method for estimating the values of \( C \) and \( K \) from an analysis of the creep curve. The details are given elsewhere.

In Fig. 2,
(r_o/2n)[ln(\dot{e}/\dot{e}_o)−Ct] is plotted against t for a set of creep strain–time plots of 0.5CrMoV steel specimens having diameters in the range (2.5–50.0) mm. The fact that the plot is linear with a very high value of the correlation coefficient (>0.9) demonstrates that the approach as suggested could indeed be used for the evaluation of the relevant material constants.

2.2. Parabolic Growth Law

In the case of parabolic growth law, the oxide thickness x should be represented as:

\[ x = K\sqrt{t} \] ..........................(9)

Consequently the equation describing the evolution of creep strain in this case should be as follows:

\[ \dot{e} = \dot{e}_o \exp(Ce)(1−K\sqrt{t/r_o})^{-2n} \] ..............(10)

Since \( K\sqrt{t} \lesssim r_o \), an algebraic simplification of the Eq. (7) gives:

\[ \ln(\dot{e}/\dot{e}_o) = Ce + 2nK\sqrt{t/r_o} \] ..............(11)

The parabolic rate constant \( K \) can therefore be estimated from the creep strain–time plots in very much the same way as before. Figure 3 shows creep strain–time plots of Inconel X-750 with both linear and parabolic growth laws. Most of the cases predictions based on linear growth law are found closure to the experimental plots. This substantiates the findings earlier reported by Dyson and Osgerby.5)

In the case of a specimen having either a tubular or a flat cross section the only change that is required to be introduced is the incorporation of an appropriate expression for \( s/s_o \) in Eq. (5). The exact forms of \( s/s_o \) to be included are given in Table 1. The rest of the procedure is identical to that of a circular section.

3. Model Validation

Experimental data on 0.5CrMoV steel and X-750 nickel
base superalloy has been used for validation. The material parameters for 0.5CrMoV steel and Inconel X-750 are given in Table 3. All these have been estimated from the respective experimental creep strain–time plots using the procedure described above. Integration of Eq. (8) gives a general expression of the creep curve of an alloy in an oxidizing environment. This is as follows:

\[
e = - \ln\left(\left(\frac{C}{2nK}\left(1 - \exp(2nKt/\rho_o)\right) + 1\right)\right)/C\ldots(12)
\]

The accuracy of Eq. (8) can be judged from the results given in Fig. 4(a)–4(d). The experimental data of 0.5CrMoV steel specimens having different section sizes are indeed very close to the predicted plots. Predictions are based on linear growth law. The values of \(C\) and \(K\) given in Table 3 have been used to generate the creep curves of Inconel X-750 specimens of various cross sectional size and geometry at two different stress levels and Fig. 3 represents a set of such plots. Irrespective of the size and geometry, predictions are fairly close to the experimental plots.

4. Discussion

An understanding of the effects of section size and specimen geometry on creep properties is of considerable technological significance. The approach described above provides a basic framework for simulating the effect of these on the creep strain–time plots of metals and alloys. Having established validity of this approach with respect to the available experimental data\(^1\) on section size effect in 0.5CrMoV steel it is worthwhile to examine the influence

| Material          | \(\sigma\) (MPa) | \(n\) | \(\rho_o\) (h\(^2\)) | \(C\)  | \(K\) (mm h\(^{-2}\)) |
|-------------------|-----------------|------|----------------------|-------|-----------------------|
| 0.5CrMoV          | 70              | 4    | 9.0x10\(^{-6}\)     | 12.6  | 1.1x10\(^{3}\)        |
| Inconel X-750     | 400             | 10   | 6.5x10\(^{-8}\)     | 21    | 2.0x10\(^{3}\)        |

Table 3. Material parameters of 0.5CrMoV steel and Inconel X-750.

![Comparison of experimental and predicted creep curves at 70 MPa, 675°C for 0.5CrMoV steel specimen having different diameters](image-url)

(a) 2.5 mm, (b) 5 mm, (c) 10 mm and (d) 50 mm.
of specimen geometry on creep curves. This has been done for circular, flat and tabular specimens of Inconel X-750. Figure 5 provides a set of such plots.

The set of equations used to model the effect of section size and geometry of test specimens are based on the concept of continuum damage mechanics. The work of Ashby and Dyson7) provides the basis for the development of constitutive relations for creep and environmental interaction. The loss in section size during creep due to strain accumulation, oxidation and cavitation has been considered. Using this model it is possible to explain why a circular specimen in comparison to either a flat or a tabular specimen having a section size (thickness) equal to the diameter of the circular specimen4) has a shorter creep life (Fig. 5). This is primarily because the former has a relatively greater exposed surface area.

The ratio of the perimeter \( P \) to the cross sectional area \( A \) is an important parameter to understand the effect of specimen geometry on the rupture lifetime. Its importance was pointed out by Pandey et al.4) The cross sectional area is a measure of the load bearing capacity whereas the perimeter determines the surface area, which is exposed to environmental attack during creep testing. Since the cross sectional area is proportional to the square of the linear dimension (e.g., radius in case of a circular specimen) whereas the perimeter is only directly proportional to the linear dimension, a specimen having a higher cross sectional area has a relatively shorter perimeter. Consequently it is expected to exhibit longer creep life. Extending the same argument it may be concluded that specimens having the same \( P/A \) ratio are expected to exhibit identical creep curves irrespective of their shape and size. Recent work of Roy and Ghosh8) does indicate that specimens having different cross sectional shape but the same \( P/A \) ratio have identical creep curves (Fig. 6). The experimental data of Pandey et al.4) also support these observations.

In order to demonstrate the usefulness of this approach, a set of creep curves has been generated using Eq. (12) for circular and tabular specimens having different section sizes. Figure 7(a) and 7(b) present a set of such plots. For both circular and tabular specimens, the creep life is found to be longer with increasing section size. However beyond a
certain section size the effect is less pronounced. Nevertheless, in the range of dimensions of specimens commonly used for generating creep strain–time plots of engineering materials, it is quite sensitive to section size. Therefore the approach suggested is extremely useful in extrapolating laboratory test data for life prediction of engineering components.

It is expected that the general form of \((\sigma/\sigma_o)\) listed in Table 1 for various specimen geometries can be expressed by a common expression containing only the shape factor \(f_o\) (P/A). When the expressions given in Table 1 are expanded and higher order terms in \(x\) are neglected (since it is expected to be much smaller than the physical dimension), it is indeed found to be so. A set of approximate expressions can be derived and the same is listed in Table 2. The table also includes the P/A ratio for various cross sectional shapes. It clearly reveals that regardless of the shape of the test specimen, the stress to which a specimen is subjected to, can be given as

\[
\sigma/\sigma_o = 1/(1-xf_o) \quad \text{..................................(13)}
\]

where \(f_o\) denotes the initial shape function (P/A ratio). A substitution of Eq. (13) in Eq. (1) and subsequent integration provides the following general form of creep curve assuming that oxidation kinetics is governed by linear growth law,

\[
\varepsilon = -\ln[(Ce/nKf_o)(1-\exp(nKf_o)t) + 1]/C \quad \text{......(14)}
\]

While deriving Eq. (14), the higher order terms in \(x\) were neglected. However for a precise solution numerical techniques may be adopted.

The above analysis clearly establishes the influence of specimen shape factor \(f_o\) in determining the nature of the creep strain–time plot. However, since circular geometry is known to have the minimum \(f_o\) for a given cross sectional area, it is but natural to expect that this specimen geometry should have the maximum creep resistance. The data reported by Pandey et al.\(^4\) compares creep curves of different geometries have the same specimen thickness. Therefore, the expected influence of P/A ratio is not quite apparent even though the trend reported by them is consistent with the present model prediction. However, with the available material parameters and using Eq. (14), it is possible to simulate creep curves for specimens having different cross sectional shape but the same initial cross sectional area. Figure 8 represents a set of such creep strain–time plots. Indeed a specimen having a circular cross section is found to be the most creep resistant.

![Fig. 8. Creep curves of circular (diameter, 1.6 mm), tubular and flat specimens having same cross sectional area.](image)

![Fig. 9. Stress dependence of material parameters of Inconel X-750 at 700°C (a) C and (b) K mm hr\(^{-1}\).](image)
The method described is capable of estimating the two important material parameters namely $C$ and $K$. This method was used to estimate the values of $C$ for specimens of various section sizes, tested in air and in vacuum. The values are fairly close. It is therefore possible to use this approach to examine the influence of stress and temperature on parameters $C$ and $K$. Available experimental data on Inconel X-750 at different stress levels were analysed. Figures 9(a) and 9(b) give the nature of variation of $C$ and $K$ due to applied stress. The effect of stress on $K$ is as expected; since the oxide layers are unable to support matching creep strain developing on the alloy, cracks are formed and fresh metallic surface gets exposed. The influence of stress on $C$ is also not unexpected. Similar nature has been reported by several workers.9,10

5. Conclusions

(1) The synergy between creep and oxidation can be adequately modelled by integration of a set of equations incorporating specimen geometry dependent expressions.

(2) The model has been validated using published experimental data set on 0.5CrMoV steel and Inconel X-750. Satisfactory agreement has been achieved in predicting the influence of section size.

(3) A quantitative explanation as to why the time to rupture of a tubular specimen having the same wall thickness as the diameter of the circular specimen is reported to be considerably longer has been provided.

Acknowledgements

The authors wish to express their sincere thanks to Prof. P. Ramchandra Rao, Director, National Metallurgical Laboratory for his permission to publish these results.

REFERENCES

1) B. J. Cane and J. A. Williams: Int. Mater. Rev., 32 (1987), 241.
2) E. G. Richards: J. Inst. Met., 96 (1968), 365.
3) T. B. Gibbons: Met. Technol., 8 (1981), 472.
4) M. C. Pandey, D. M. R. Taplin and P. Rama Rao: Mater. Sci. Eng., A118 (1989), 32.
5) R. Viswanathan: Damage Mechanisms and Life Assessment of High Temperature Components, ASM International, Materials Park, OH, (1989).
6) B. F. Dyson and S. Osgerby: Materials and Engineering Design: The Next Decade, ed. by B. F. Dyson and D. R. Hayhurst, Institute of Metals, London, (1989) 287.
7) M. F. Ashby and B. F. Dyson: Advances in Fracture Research Proc., ed. by A. R. Valliant et al., Pergamon Press, Oxford, (1984), 3.
8) N. Roy and R. N. Ghosh: Scr. Metall., 36 (1997), 1367.
9) R. N. Ghosh, R. V. Curtis and M. McLean: Acta Metall. Mater., 38 (1990), 1977.
10) L.-M. Pan, B. A. Shollock and M. McLean: Proc. R. Soc., 453A (1997), 1689.