Two Lectures on FCNC and CP Violation in Supersymmetry

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Abstract
These two lectures constitute a reappraisal and an update of the status of the Flavour Changing Neutral Current (FCNC) and CP violation issues in supersymmetry. The first lecture discusses these points in the framework of the Minimal Supersymmetric Standard Model (MSSM), while the second one provides an analysis in a generic low-energy supersymmetric extension of the Standard Model. The goal of these lectures is twofold: on one hand we present a qualitative and quantitative discussion of the threat that FCNC and CP violation represent on supersymmetry model building; on the other hand, we point out how precious FCNC and CP violation may be in obtaining some signals of the presence of supersymmetry at low energy in the years that separate us from the advent of LHC physics. In particular, concerning this latter point, we emphasize and thoroughly analyze the role of experimental searches for rare $B$ decays and CP violation in $B$ physics.

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1 Introduction

In spite of the extraordinary success of the Standard Model (SM) in accounting for all the existing experimental data, we have several well-motivated theoretical reasons to expect new physics beyond it. In this view the SM may be regarded as the low-energy limit of this more fundamental underlying theory. Indeed, it is likely that we have a “tower” of underlying theories which show up at different energy scales.

If we accept the above point of view we may try to find signals of new physics considering the SM as a truncation to renormalizable operators of an effective low-energy theory which respects the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry and whose fields are just those of the SM. The renormalizable (i.e. of canonical dimension less or equal to four) operators giving rise to the SM enjoy three crucial properties which have no reason to be shared by generic operators of dimension larger than four. They are the conservation (at any order in perturbation theory) of Baryon (B) and Lepton (L) numbers and an adequate suppression of Flavour Changing Neutral Current (FCNC) processes through the GIM mechanism.

Now consider the new physics (directly above the SM in the “tower” of new physics theories) to have a typical energy scale $\Lambda$. In the low-energy effective Lagrangian such scale appears with a positive power only in the quadratic scalar term (scalar mass) and in the dimension zero operator which can be considered a cosmological constant. Notice that $\Lambda$ cannot appear in dimension three operators related to fermion masses because chirality forbids direct fermion mass terms in the Lagrangian. Then in all operators of dimension larger than four $\Lambda$ will show up in the denominator with powers increasing with the dimension of the corresponding operator.

The crucial question that all of us, theorists and experimentalists, ask ourselves is: where is $\Lambda$? Namely is it close to the electroweak scale (i.e. not much above 100 GeV) or is $\Lambda$ of the order of the grand unification scale or the Planck scale? B- and L-violating processes and FCNC phenomena represent a potentially interesting clue to answer this fundamental question.

Take $\Lambda$ to be close to the electroweak scale. Then we may expect non-renormalizable operators with B, L and flavour violations not to be largely suppressed by the presence of powers of $\Lambda$ in the denominator. Actually this constitutes in general a formidable challenge for any model builder who wants to envisage new physics close to $M_W$. Theories with dynamical breaking
of the electroweak symmetry (technicolour) and low-energy supersymmetry constitute examples of new physics with a “small” $\Lambda$. In these lectures we will see that the above general considerations on potentially large $B, L$ and flavour violations apply to the SUSY case (it is well-known that FCNC represent a major problem also in technicolour schemes).

Alternatively, given the abovementioned potential danger of having a small $\Lambda$, one may feel it safer to send $\Lambda$ to superlarge values. Apart from kind of “philosophical” objections related to the unprecedented gap of many orders of magnitude without any new physics, the above discussion points out a typical problem of this approach. Since the quadratic scalar terms have a coefficient in front scaling with $\Lambda^2$ we expect all scalar masses to be of the order of the superlarge scale $\Lambda$. This is the gauge hierarchy problem and it constitutes the main (if not only) reason to believe that SUSY should be a low-energy symmetry.

Notice that the fact that SUSY should be a fundamental symmetry of Nature (something of which we have little doubt given the “beauty” of this symmetry) does not imply by any means that SUSY should be a low-energy symmetry, namely that it should hold unbroken down to the electroweak scale. SUSY may well be present in Nature but be broken at some very large scale (Planck scale or string compactification scale). In that case SUSY would be of no use in tackling the gauge hierarchy problem and its phenomenological relevance would be practically zero. On the other hand if we invoke SUSY to tame the growth of the scalar mass terms with the scale $\Lambda$, then we are forced to take the view that SUSY should hold as a good symmetry down to a scale $\Lambda$ close to the electroweak scale. Then $B, L$ and FCNC may be useful for us to shed some light on the properties of the underlying theory from which the low-energy SUSY Lagrangian resulted. Let us add that there is an independent argument in favour of this view that SUSY should be a low-energy symmetry. The presence of SUSY partners at low energy creates the conditions to have a correct unification of the strong and electroweak interactions. If they were at $M_{\text{Planck}}$ and the SM were all the physics up to superlarge scales, the program of achieving such a unification would largely fail, unless one complicates the non-SUSY GUT scheme with a large number of Higgs representations and/or a breaking chain with intermediate mass scales is invoked.

In these lectures we will tackle some of the major aspects of the issue of FCNC and CP violation in SUSY theories. First of all we will discuss whether
we expect SUSY to be actually related to the flavour problem. Then we will proceed to analyze the different status of SUSY in relation to the FCNC problem according to the class of SUSY models one considers. This discussion is not a mere academic exercise, but it has profound phenomenological implications. When you read or hear sentences starting with “SUSY predicts that...” or “this result would rule out SUSY...” it should be kept in mind that we do not have a low-energy SUSY theory like we have the SM, but rather we have classes of models which differ in the content of superfields, in their couplings, in the nature of the SUSY breaking terms, etc. At the end of these lectures we hope that the following message may emerge: while, undoubtedly, FCNC represents a challenge for SUSY, at the same time it can be seen as one of the major hopes that we have now (where now may actually mean any time before LHC!) to have some signal of the presence of low-energy SUSY.

2 FCNC and SUSY

The generation of fermion masses and mixings (“flavour problem”) gives rise to a first and important distinction among theories of new physics beyond the electroweak standard model.

One may conceive a kind of new physics which is completely “flavour blind”, i.e. new interactions which have nothing to do with the flavour structure. To provide an example of such a situation, consider a scheme where flavour arises at a very large scale (for instance the Planck mass) while new physics is represented by a supersymmetric extension of the SM with supersymmetry broken at a much lower scale and with the SUSY breaking transmitted to the observable sector by flavour-blind gauge interactions. In this case one may think that the new physics does not cause any major change to the original flavour structure of the SM, namely that the pattern of fermion masses and mixings is compatible with the numerous and demanding tests of flavour changing neutral currents.

Alternatively, one can conceive a new physics which is entangled with the flavour problem. As an example consider a technicolour scheme where fermion masses and mixings arise through the exchange of new gauge bosons which mix together ordinary and technifermions. Here we expect (correctly enough) new physics to have potential problems in accommodating the usual
fermion spectrum with the adequate suppression of FCNC. As another example of new physics which is not flavour blind, take a more conventional SUSY model which is derived from a spontaneously broken N=1 supergravity and where the SUSY breaking information is conveyed to the ordinary sector of the theory through gravitational interactions. In this case we may expect that the scale at which flavour arises and the scale of SUSY breaking are not so different and possibly the mechanism itself of SUSY breaking and transmission is flavour-dependent. Under these circumstances we may expect a potential flavour problem to arise, namely that SUSY contributions to FCNC processes are too large.

The potentiality of probing SUSY in FCNC phenomena was readily realized when the era of SUSY phenomenology started in the early 80’s [1]. In particular, the major implication that the scalar partners of quarks of the same electric charge but belonging to different generations had to share a remarkably high mass degeneracy was emphasized.

Throughout the large amount of work in this last decade it became clearer and clearer that generically talking of the implications of low-energy SUSY on FCNC may be rather misleading. We have a minimal SUSY extension of the SM, the so-called Minimal Supersymmetric Standard Model (MSSM) [2], where the FCNC contributions can be computed in terms of a very limited set of unknown new SUSY parameters. Remarkably enough, this minimal model succeeds to pass all the set of FCNC tests unscathed. To be sure, it is possible to severely constrain the SUSY parameter space, for instance using $b \rightarrow s\gamma$, in a way which is complementary to what is achieved by direct SUSY searches at colliders.

However, the MSSM is by no means equivalent to low-energy SUSY. A first sharp distinction concerns the mechanism of SUSY breaking and transmission to the observable sector which is chosen. As we mentioned above, in models with gauge-mediated SUSY breaking (GMSB models [3]-[5]) it may be possible to avoid the FCNC threat “ab initio” (notice that this is not an automatic feature of this class of models, but it depends on the specific choice of the sector which transmits the SUSY breaking information, the so-called messenger sector). The other more “canonical” class of SUSY theories that was mentioned above has gravitational messengers and a very large scale at which SUSY breaking occurs. In this talk we will focus only on this class of gravity-mediated SUSY breaking models. Even sticking to this more limited choice we have a variety of options with very different
implications for the flavour problem.

First, there exists an interesting large class of SUSY realizations where the customary R-parity (which is invoked to suppress proton decay) is replaced by other discrete symmetries which allow either baryon or lepton violating terms in the superpotential. But, even sticking to the more orthodox view of imposing R-parity, we are still left with a large variety of extensions of the MSSM at low energy. The point is that low-energy SUSY “feels” the new physics at the superlarge scale at which supergravity (i.e., local supersymmetry) broke down. In this last couple of years we have witnessed an increasing interest in supergravity realizations without the so-called flavour universality of the terms which break SUSY explicitly. Another class of low-energy SUSY realizations which differ from the MSSM in the FCNC sector is obtained from SUSY-GUT’s. The interactions involving superheavy particles in the energy range between the GUT and the Planck scale bear important implications for the amount and kind of FCNC that we expect at low energy.

3 FCNC in SUSY without R-Parity

It is well known that in the SM case the imposition of gauge symmetry and the usual gauge assignment of the 15 elementary fermions of each family lead to the automatic conservation of baryon and lepton numbers (this is true at any order in perturbation theory).

On the contrary, imposing in addition to the usual $SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetry an N=1 global SUSY does not prevent the appearance of terms which explicitly break B or L [6]. Indeed, the superpotential reads:

$$W = h^U Q H_U u^c + h^D Q H_D d^c + h^L L H_D e^c + \mu H_U H_D$$

$$+ \mu' H_U L + \lambda_{ijk} u^c_i d^c_j d^c_k + \lambda'_{ijk} Q_i L_j e^c_k + \lambda''_{ijk} L_i L_j L_k e^c_k ,$$

where the chiral matter superfields $Q$, $u^c$, $d^c$, $L$, $e^c$, $H_U$ and $H_D$ transform under the above gauge symmetry as:

$$Q \equiv (3, 2, 1/6); \quad u^c \equiv (3, 1, -2/3); \quad d^c \equiv (3, 1, 1/3);$$

$$L \equiv (1, 2, -1/2); \quad e^c \equiv (1, 1, 1); \quad H_U \equiv (1, 2, 1/2); \quad H_D \equiv (1, 2, -1/2).$$

The couplings $h^U$, $h^D$, $h^L$ are $3 \times 3$ matrices in the generation space; $i$, $j$ and $k$ are generation indices. Using the product of $\lambda'$ and $\lambda''$ couplings it
is immediate to construct four-fermion operators leading to proton decay through the exchange of a squark. Even if one allows for the existence of $\lambda'$ and $\lambda''$ couplings only involving the heaviest generation, one can show that the bound on the product $\lambda' \times \lambda''$ of these couplings is very severe (of $O(10^{-7})$) [7].

A solution is that there exists a discrete symmetry, B-parity [8], which forbids the B violating terms in eq. (1) which are proportional to $\lambda''$. In that case it is still possible to produce sizeable effects in FC $B$ decays. For instance, using the product of $\lambda'_{3jk} \lambda_{j^c}$ one can obtain $b \rightarrow s (d) + ll^c$ taking $k = 2 (1)$ and through the mediation of the sneutrino of the $j$-th generation. Two general features of these R-parity violating contributions are:

1. complete loss of any correlation to the CKM elements. For instance, in the above example, the couplings $\lambda'$ and $\lambda$ have nothing to do with the usual angles $V_{tb}$ and $V_{ts}$ which appear in $b \rightarrow sl^+l^-$ in the SM;

2. loss of correlation among different FCNC processes which are tightly correlated in the SM. For instance, in our example $b \rightarrow dl^+l^-$ would depend on $\lambda'$ and $\lambda$ parameters which are different from those appearing in $B_d - \bar{B}_d$ mixing.

In this context it is difficult to make predictions given the arbitrariness of the large number of $\lambda$ and $\lambda'$ parameters. There exist bounds on each individual coupling (i.e. assuming all the other L violating couplings are zero) [9]. With some exception, they are not very stringent for the third generation (generally of $O(10^{-1})$), hence allowing for conspicuous effects. Indeed, one may think of using the experimental bounds on rare $B$ decays to put severe bounds on products of L violating couplings.

Obviously, the most practical way of avoiding any threat of B and L violating operators is to forbid all such terms in eq. (1). This is achieved by imposing the usual R matter parity. This quantum number reads +1 over every ordinary particle and −1 over SUSY partners. We now turn to FCNC in the framework of low-energy SUSY with R parity.
4 FCNC in SUSY with R Parity - MSSM Framework

Even when R parity is imposed the FCNC challenge is not over. It is true that in this case, analogously to what happens in the SM, no tree level FCNC contributions arise. However, it is well-known that this is a necessary but not sufficient condition to consider the FCNC problem overcome. The loop contributions to FCNC in the SM exhibit the presence of the GIM mechanism and we have to make sure that in the SUSY case with R parity some analog of the GIM mechanism is active.

To give a qualitative idea of what we mean by an effective super-GIM mechanism, let us consider the following simplified situation where the main features emerge clearly. Consider the SM box diagram responsible for the $K^0 - \bar{K}^0$ mixing and take only two generations, i.e. only the up and charm quarks run in the loop. In this case the GIM mechanism yields a suppression factor of $O((m_c^2 - m_u^2)/M_W^2)$. If we replace the W boson and the up quarks in the loop with their SUSY partners and we take, for simplicity, all SUSY masses of the same order, we obtain a super-GIM factor which looks like the GIM one with the masses of the superparticles instead of those of the corresponding particles. The problem is that the up and charm squarks have masses which are much larger than those of the corresponding quarks. Hence the super-GIM factor tends to be of $O(1)$ instead of being $O(10^{-3})$ as it is in the SM case. To obtain this small number we would need a high degeneracy between the mass of the charm and up squarks. It is difficult to think that such a degeneracy may be accidental. After all, since we invoked SUSY for a naturalness problem (the gauge hierarchy issue), we should avoid invoking a fine-tuning to solve its problems! Then one can turn to some symmetry reason. For instance, just sticking to this simple example that we are considering, one may think that the main bulk of the charm and up squark masses is the same, i.e. the mechanism of SUSY breaking should have some universality in providing the mass to these two squarks with the same electric charge. Another possibility one may envisage is that the masses of the squarks are quite high, say above few TeV’s. Then even if they are not so degenerate in mass, the overall factor in front of the four-fermion operator responsible for the kaon mixing becomes smaller and smaller (it decreases quadratically with the mass of the squarks) and, consequently, one
can respect the observational result. We see from this simple example that
the issue of FCNC may be closely linked to the crucial problem of the way
we break SUSY.

We now turn to some more quantitative considerations. We start by
discussing the different degree of concern that FCNC originate according to
the specific low-energy SUSY realization one has in mind. In this section we
will consider FCNC in the MSSM realizations. In Sect. 5 we will deal with
CP-violating FCNC phenomena in the same context. After discussing these
aspects in the MSSM we will provide bounds from FCNC and CP violation
in a generic SUSY extension of the SM (Sect. 6 and 7).

Obviously the reference frame for any discussion in a specific SUSY
scheme is the MSSM. Although the name seems to indicate a well-defined
particle model, actually MSSM denotes at least two quite different classes
of low-energy SUSY models. In its most restrictive meaning it denotes the
minimal SUSY extension of the SM (i.e. with the smallest needed number
of superfields) with R-parity, radiative breaking of the electroweak symme-

try, universality of the soft breaking terms and simplifying relations at the
GUT scale among SUSY parameters. In this “minimal” version the MSSM
exhibits only four free parameters in addition to those of the SM. Moreover,
some authors impose specific relations between the two parameters $A$ and
$B$ that appear in the trilinear and bilinear scalar terms of the soft breaking
sector further reducing the number of SUSY free parameters to three. Then,
all SUSY masses are just function of these few independent parameters and,
ence, many relations among them exist. Obviously this very minimal ver-
sion of the MSSM can be very predictive. The most powerful constraint on
this minimal model in the FCNC context comes from $b \to s\gamma$.

In SUSY there are five classes of one-loop diagrams which contribute
to FCNC $B$ decays. They are distinguished according to the virtual parti-
cles running in the loop: $W$ and up-quarks, charged Higgs and up-quarks,
charginos and up-squarks, neutralinos and down-squarks, gluinos and down-
squarks. It turns out that, at least in this “minimal” version of the MSSM,
the charged Higgs and chargino exchanges yield the dominant SUSY con-
tributions. We will deal more on this point in the next section.

As for $b \to s\gamma$ the situation can be summarized as follows. The CLEO
measurement yields $\text{BR}(B \to X_s\gamma) = (2.32 \pm 0.67) \times 10^{-4}$ [14]. On the
theoretical side we have just witnessed a major breakthrough with the com-
putation of the next-to-leading logarithmic result for the BR. This has been
achieved thanks to the calculation of the $O(\alpha_s)$ matrix elements [11] and of the next-to-leading order Wilson coefficients at $\mu \simeq m_b$ [12]. The present theoretical result, $\text{BR}(B \to X_s\gamma) = (3.28 \pm 0.33) \times 10^{-4}$ [12], exhibits an impressive improvement on the size of the error. A substantial improvement also on the experimental error is foreseen for the near future. Hence $b \to s\gamma$ is going to constitute the most relevant place in FCNC $B$ physics to constrain SUSY at least before the advent of $B$ factories. So far this process has helped in ruling out regions of the SUSY parameter space which are even larger than those excluded by LEP I and it is certainly going to be complementary to what LEP II is expected to do in probing the SUSY parameter space. After the detailed analysis in 1991 [13] for small values of $\tan \beta$, there have been recent analyses [14] covering the entire range of $\tan \beta$ and including also other technical improvements (for instance radiative corrections in the Higgs potential). It has been shown [15] that the exclusion plots are very sensitive also to the relation one chooses between $A$ and $B$. It should be kept in mind that the “traditional” relation $B = A - 1$ holds true only in some simplified version of the MSSM. A full discussion is beyond the scope of these lectures and so we refer the interested readers to the vast literature which exists on the subject.

The constraint on the SUSY parameter space of the “minimal” version of the MSSM greatly affects also the potential departures of this model from the SM expectation for $b \to s l^+ l^-$. The present limits on the exclusive channels $\text{BR}(B^0 \to K^{\ast 0} e^+ e^-)$ and $\text{BR}(B^0 \to K^{\ast 0} \mu^+ \mu^-)$ are within an order of magnitude of the SM predictions. On the theoretical side, it has been estimated that the evaluation of $\Gamma(B \to X_s l^+ l^-)$ in the SM is going to be affected by an error which cannot be reduced to less than $10 - 20\%$ due to uncertainties in quark masses and interference effects from excited charmonium states [16]. It turns out that, keeping into account the bound on $b \to s\gamma$, in the MSSM with universal soft breaking terms a 20% departure from the SM expected BR is kind of largest possible value one can obtain [17]. Hence the chances to observe a meaningful deviation in this case are quite slim. However, it has been stressed that in view of the fact that three Wilson coefficients play a relevant role in the effective low-energy Hamiltonian involved in $b \to s\gamma$ and $b \to s l^+ l^-$, a third observable in addition to $\text{BR}(b \to s\gamma)$ and $\text{BR}(b \to s l^+ l^-)$ is needed. This has been identified in some asymmetry of the emitted leptons (see refs. [17, 18] for two different choices of such asymmetry). This quantity, even in the “minimal” MSSM, may undergo a conspicuous deviation from its
SM expectation and, hence, hopes of some manifestation of SUSY, even in this minimal realization, in $b \rightarrow s l^+ l^-$ are still present.

Finally, also for the $B_d - \bar{B}_d$ mixing, in the above-mentioned analysis of rare $B$ physics in the MSSM with universal soft breaking terms \[13\] it was emphasized that, at least in the low tan $\beta$ regime, one cannot expect an enhancement larger than $20\% - 30\%$ over the SM prediction (see also ref. \[19\]). Moreover it was shown that $x_s / x_d$ is expected to be the same as in the SM.

It should be kept in mind that the above stringent results strictly depend not only on the minimality of the model in terms of the superfields that are introduced, but also on the "boundary" conditions that are chosen. All the low-energy SUSY masses are computed in terms of the $M_{Pl}$ four SUSY parameters through the RGE evolution. If one relaxes this tight constraint on the relation of the low-energy quantities and treats the masses of the SUSY particles as independent parameters, then much more freedom is gained. This holds true even if flavour universality is enforced. For instance, BR($b \rightarrow s \gamma$) and $\Delta m_{B_d}$ may vary a lot from the SM expectation, in particular in regions of moderate SUSY masses \[20\] (the most interesting case, i.e. small chargino and stop masses, will be briefly dealt with in next section).

Moreover, flavour universality is by no means a prediction of low-energy SUSY. The absence of flavour universality of soft-breaking terms may result from radiative effects at the GUT scale or from effective supergravities derived from string theory. For instance, even starting with an exact universality of the soft breaking terms at the Planck scale, in a SUSY GUT scheme one has to consider the running from this latter scale and the GUT scale. Due to the large value of the top Yukawa coupling and to the fact that quarks and lepton superfields are in common GUT multiplets, we may expect the tau slepton mass to be conspicuously different from that of the first two generation sleptons at the end of this RG running. This lack of universality at the GUT scale may lead to large violations of flavour lepton number yielding, for instance, $\mu \rightarrow e \gamma$ at a rate in the ball park of observability \[21\]. In the non-universal case, BR($b \rightarrow s l^+ l^-$) is strongly affected by this larger freedom in the parameter space. There are points of this parameter space where the non-resonant BR($B \rightarrow X_s e^+ e^-$) and BR($B \rightarrow X_s \mu^+ \mu^-$) are enhanced by up to 90\% and 110\% while still respecting the constraint coming from $b \rightarrow s \gamma$ \[17\].
5 CP Violation in the MSSM

CP violation has major potentialities to exhibit manifestations of new physics beyond the standard model. Indeed, it is quite a general feature that new physics possesses new CP violating phases in addition to the Cabibbo-Kobayashi-Maskawa (CKM) phase (\(\delta_{\text{CKM}}\)) or, even in those cases where this does not occur, \(\delta_{\text{CKM}}\) shows up in interactions of the new particles, hence with potential departures from the SM expectations. Moreover, although the SM is able to account for the observed CP violation in the kaon system, we cannot say that we have tested so far the SM predictions for CP violation. The detection of CP violation in \(B\) physics will constitute a crucial test of the standard CKM picture within the SM. Again, on general grounds, we expect new physics to provide departures from the SM CKM scenario for CP violation in \(B\) physics. A final remark on reasons that make us optimistic in having new physics playing a major role in CP violation concerns the matter-antimatter asymmetry in the universe. Starting from a baryon-antibaryon symmetric universe, the SM is unable to account for the observed baryon asymmetry. The presence of new CP-violating contributions when one goes beyond the SM looks crucial to produce an efficient mechanism for the generation of a satisfactory \(\Delta B\) asymmetry.

The above considerations apply well to the new physics represented by low-energy supersymmetric extensions of the SM. Indeed, as we will see below, supersymmetry introduces CP violating phases in addition to \(\delta_{\text{CKM}}\) and, even if one envisages particular situations where such extra-phases vanish, the phase \(\delta_{\text{CKM}}\) itself leads to new CP-violating contributions in processes where SUSY particles are exchanged. CP violation in \(B\) decays has all potentialities to exhibit departures from the SM CKM picture in low-energy SUSY extensions, although, as we will discuss, the detectability of such deviations strongly depends on the regions of the SUSY parameter space under consideration.

In this section we will deal with CP violation in the context of the MSSM. In Sec. 7 we will discuss the CP issue in a model-independent approach. For recent reviews on CP violation in SUSY see [22].

In the MSSM two new “genuine” SUSY CP-violating phases are present. They originate from the SUSY parameters \(\mu, M, A\) and \(B\). The first of these parameters is the dimensionful coefficient of the \(H_uH_d\) term of the superpotential. The remaining three parameters are present in the sector that softly
breaks the N=1 global SUSY. $M$ denotes the common value of the gaugino masses, $A$ is the trilinear scalar coupling, while $B$ denotes the bilinear scalar coupling. In our notation all these three parameters are dimensionful. The simplest way to see which combinations of the phases of these four parameters are physical [23] is to notice that for vanishing values of $\mu$, $M$, $A$ and $B$ the theory possesses two additional symmetries [24]. Indeed, letting $B$ and $\mu$ vanish, a $U(1)$ Peccei-Quinn symmetry originates, which in particular rotates $H_u$ and $H_d$. If $M$, $A$ and $B$ are set to zero, the Lagrangian acquires a continuous $U(1)$ $R$ symmetry. Then we can consider $\mu$, $M$, $A$ and $B$ as spurions which break the $U(1)_{PQ}$ and $U(1)_{R}$ symmetries. In this way the question concerning the number and nature of the meaningful phases translates into the problem of finding the independent combinations of the four parameters which are invariant under $U(1)_{PQ}$ and $U(1)_{R}$ and determining their independent phases. There are three such independent combinations, but only two of their phases are independent. We use here the commonly adopted choice:

$$\Phi_A = \text{arg} \left( A^* M \right), \quad \Phi_B = \text{arg} \left( B^* M \right).$$  \hspace{1cm} (3)

The main constraints on $\Phi_A$ and $\Phi_B$ come from their contribution to the electric dipole moments of the neutron and of the electron. For instance, the effect of $\Phi_A$ and $\Phi_B$ on the electric and chromoelectric dipole moments of the light quarks ($u$, $d$, $s$) lead to a contribution to $d_N^e$ of order [25]

$$d_N^e \sim 2 \left( \frac{100 \text{GeV}}{\tilde{m}} \right)^2 \sin \Phi_{A,B} \times 10^{-23} \text{e cm},$$  \hspace{1cm} (4)

where $\tilde{m}$ here denotes a common mass for squarks and gluinos. The present experimental bound, $d_N^e < 1.1^{-25} \text{ e cm}$, implies that $\Phi_{A,B}$ should be $< 10^{-2}$, unless one pushes SUSY masses up to $O(1 \text{ TeV})$. A possible caveat to such an argument calling for a fine-tuning of $\Phi_{A,B}$ is that uncertainties in the estimate of the hadronic matrix elements could relax the severe bound in eq. (4) [26].

In view of the previous considerations most authors dealing with the MSSM prefer to simply put $\Phi_A$ and $\Phi_B$ equal to zero. Actually, one may argue in favour of this choice by considering the soft breaking sector of the MSSM as resulting from SUSY breaking mechanisms which force $\Phi_A$ and $\Phi_B$ to vanish. For instance, it is conceivable that both $A$ and $M$ originate from
one same source of $U(1)_R$ breaking. Since $\Phi_A$ "measures" the relative phase of $A$ and $M$, in this case it would "naturally" vanish. In some specific models it has been shown \cite{27} that through an analogous mechanism also $\Phi_B$ may vanish.

If $\Phi_A = \Phi_B = 0$, then the novelty of SUSY in CP violating contributions merely arises from the presence of the CKM phase in loops where SUSY particles run \cite{28}. The crucial point is that the usual GIM suppression, which plays a major role in evaluating $\varepsilon$ and $\varepsilon'$ in the SM, in the MSSM case is replaced by a super-GIM cancellation which has the same "power" of suppression as the original GIM (see previous section). Again also in the MSSM as it is the case in the SM, the smallness of $\varepsilon$ and $\varepsilon'$ is guaranteed not by the smallness of $\delta_{\text{CKM}}$, but rather by the small CKM angles and/or small Yukawa couplings. By the same token, we do not expect any significant departure of the MSSM from the SM predictions also concerning CP violation in $B$ physics. As a matter of fact, given the large lower bounds on squark and gluino masses, one expects relatively tiny contributions of the SUSY loops in $\varepsilon$ or $\varepsilon'$ in comparison with the normal $W$ loops of the SM. Let us be more detailed on this point.

In the MSSM the gluino exchange contribution to FCNC is subleading with respect to chargino ($\chi^\pm$) and charged Higgs ($H^\pm$) exchanges. Hence when dealing with CP violating FCNC processes in the MSSM with $\Phi_A = \Phi_B = 0$ one can confine the analysis to $\chi^\pm$ and $H^\pm$ loops. If one takes all squarks to be degenerate in mass and heavier than $\sim 200$ GeV, then $\chi^\pm - \tilde{q}$ loops are obviously severely penalized with respect to the SM $W - q$ loops (remember that at the vertices the same CKM angles occur in both cases).

The only chance for the MSSM to produce some sizeable departure from the SM situation in CP violation is in the particular region of the parameter space where one has light $\tilde{q}$, $\chi^\pm$ and/or $H^\pm$. The best candidate (indeed the only one unless $\tan \beta \sim m_t/m_b$) for a light squark is the stop. Hence one can ask the following question: can the MSSM present some novelties in CP-violating phenomena when we consider $\chi^+ - \tilde{t}$ loops with light $\tilde{t}$, $\chi^+$ and/or $H^+$?

Several analyses in the literature tackle the above question or, to be more precise, the more general problem of the effect of light $\tilde{t}$ and $\chi^+$ on FCNC processes \cite{29,30}. A first important observation concerns the relative sign of the $W - t$ loop with respect to the $\chi^+ - \tilde{t}$ and $H^+ - t$ contributions. As it is well known, the latter contribution always interferes positively with the SM one.
Interestingly enough, in the region of the MSSM parameter space that we consider here, also the $\chi^+ - \tilde{t}$ contribution constructively interferes with the SM contribution. The second point regards the composition of the lightest chargino, i.e. whether the gaugino or higgsino component prevails. This is crucial since the light stop is predominantly $\tilde{t}_R$ and, hence, if the lightest chargino is mainly a wino then it couples to $\tilde{t}_R$ mostly through the $LR$ mixing in the stop sector. Consequently, a suppression in the contribution to box diagrams going as $\sin^4 \theta_{LR}$ is present ($\theta_{LR}$ denotes the mixing angle between the lighter and heavier stops). On the other hand, if the lightest chargino is predominantly a higgsino (i.e. $M_2 \gg \mu$ in the chargino mass matrix), then the $\chi^+ - \tilde{t}$ contribution grows. In this case contributions $\propto \theta_{LR}$ become negligible and, moreover, it can be shown that they are independent on the sign of $\mu$. A detailed study is provided in reference [30]. For instance, for $M_2/\mu = 10$ they find that the inclusion of the SUSY contribution to the box diagrams doubles the usual SM contribution for values of the lighter $\tilde{t}$ mass up to $100 - 120$ GeV, using $\tan \beta = 1.8$, $M_{H^+} = 100$ TeV, $m_\chi = 90$ GeV and the mass of the heavier $\tilde{t}$ of 250 GeV. However, if $m_\chi$ is pushed up to 300 GeV, the $\chi^+ - \tilde{t}$ loop yields a contribution which is roughly 3 times less than in the case $m_\chi = 90$ GeV, hence leading to negligible departures from the SM expectation. In the cases where the SUSY contributions are sizeable, one obtains relevant restrictions on the $\rho$ and $\eta$ parameters of the CKM matrix by making a fit of the parameters $A$, $\rho$ and $\eta$ of the CKM matrix and of the total loop contribution to the experimental values of $\varepsilon_K$ and $\Delta M_{B_d}$. For instance, in the above-mentioned case in which the SUSY loop contribution equals the SM $W - t$ loop, hence giving a total loop contribution which is twice as large as in the pure SM case, combining the $\varepsilon_K$ and $\Delta M_{B_d}$ constraints leads to a region in the $\rho - \eta$ plane with $0.15 < \rho < 0.40$ and $0.18 < \eta < 0.32$, excluding negative values of $\rho$.

In conclusion, the situation concerning CP violation in the MSSM case with $\Phi_A = \Phi_B = 0$ and exact universality in the soft-breaking sector can be summarized in the following way: the MSSM does not lead to any significant deviation from the SM expectation for CP-violating phenomena as $d^\prime_N$, $\varepsilon$, $\varepsilon'$ and CP violation in $B$ physics; the only exception to this statement concerns a small portion of the MSSM parameter space where a very light $\tilde{t}$ ($m_\tilde{t} < 100$ GeV) and $\chi^+$ ($m_\chi \sim 90$ GeV) are present. In this latter particular situation sizeable SUSY contributions to $\varepsilon_K$ are possible and, consequently, major restrictions in the $\rho - \eta$ plane can be inferred. Obviously, CP violation in $B$
physics becomes a crucial test for this MSSM case with very light $\tilde{t}$ and $\chi^+$. Interestingly enough, such low values of SUSY masses are at the border of the detectability region at LEP II.

6 Model-Independent Analysis of FCNC Processes in SUSY

Given a specific SUSY model it is in principle possible to make a full computation of all the FCNC phenomena in that context. However, given the variety of options for low-energy SUSY which was mentioned in the Introduction (even confining ourselves here to models with R matter parity), it is important to have a way to extract from the whole host of FCNC processes a set of upper limits on quantities which can be readily computed in any chosen SUSY frame.

The best model-independent parameterization of FCNC effects is the so-called mass insertion approximation [31]. It concerns the most peculiar source of FCNC SUSY contributions that do not arise from the mere supersymmetrization of the FCNC in the SM. They originate from the FC couplings of gluinos and neutralinos to fermions and sfermions [32]. One chooses a basis for the fermion and sfermion states where all the couplings of these particles to neutral gauginos are flavour diagonal, while the FC is exhibited by the non-diagonality of the sfermion propagators. Denoting by $\Delta$ the off-diagonal terms in the sfermion mass matrices (i.e. the mass terms relating sfermion of the same electric charge, but different flavour), the sfermion propagators can be expanded as a series in terms of $\delta = \Delta/\tilde{m}^2$ where $\tilde{m}$ is the average sfermion mass. As long as $\Delta$ is significantly smaller than $\tilde{m}^2$, we can just take the first term of this expansion and, then, the experimental information concerning FCNC and CP violating phenomena translates into upper bounds on these $\delta$’s [33]-[35].

Obviously the above mass insertion method presents the major advantage that one does not need the full diagonalization of the sfermion mass matrices to perform a test of the SUSY model under consideration in the FCNC sector. It is enough to compute ratios of the off-diagonal over the diagonal entries of the sfermion mass matrices and compare the results with the general bounds on the $\delta$’s that we provide here from all available experimental information.
There exist four different $\Delta$ mass insertions connecting flavours $i$ and $j$ along a sfermion propagator: $(\Delta_{ij})_{LL}$, $(\Delta_{ij})_{RR}$, $(\Delta_{ij})_{LR}$ and $(\Delta_{ij})_{RL}$. The indices $L$ and $R$ refer to the helicity of the fermion partners. The size of these $\Delta$’s can be quite different. For instance, it is well known that in the MSSM case, only the $LL$ mass insertion can change flavour, while all the other three above mass insertions are flavour conserving, i.e. they have $i = j$. In this case to realize a $LR$ or $RL$ flavour change one needs a double mass insertion with the flavour changed solely in a $LL$ mass insertion and a subsequent flavour-conserving $LR$ mass insertion. Even worse is the case of a FC $RR$ transition: in the MSSM this can be accomplished only through a laborious set of three mass insertions, two flavour-conserving $LR$ transitions and an $LL$ FC insertion. Instead of the dimensional quantities $\Delta$ it is more useful to provide bounds making use of dimensionless quantities, $\delta$, that are obtained dividing the mass insertions by an average sfermion mass.

Let us first consider CP-conserving $\Delta F = 2$ processes. The amplitudes for gluino-mediated contributions to $\Delta F = 2$ transitions in the mass-insertion approximation have been computed in refs. [34, 35]. Imposing that the contribution to $K^-\bar{K}$, $D^-\bar{D}$ and $B_d-\bar{B}_d$ mixing proportional to each single $\delta$ parameter does not exceed the experimental value, we obtain the constraints on the $\delta$’s reported in table 1, barring accidental cancellations [35] (for a QCD-improved computation of the constraints coming from $K^-\bar{K}$ mixing, see ref. [36]).

We then consider the process $b \rightarrow s\gamma$. This decay requires a helicity flip. In the presence of a $(\delta^d_{23})_{LR}$ mass insertion we can realize this flip in the gluino running in the loop. On the contrary, the $(\delta^d_{23})_{LL}$ insertion requires the helicity flip to occur in the external $b$-quark line. Hence we expect a stronger bound on the $(\delta^d_{23})_{LR}$ quantity. Indeed, this is what happens: $(\delta^d_{23})_{LL}$ is essentially not bounded, while $(\delta^d_{23})_{LR}$ is limited to be $< 10^{-3} - 10^{-2}$ according to the average squark and gluino masses (see table 2) [35]. Given the upper bound on $(\delta^d_{23})_{LR}$ from $b \rightarrow s\gamma$, it turns out that the quantity $x_s$ of the $B_s - \bar{B}_s$ mixing receives contributions from this kind of mass insertions which are very tiny. The only chance to obtain large values of $x_s$ is if $(\delta^d_{23})_{LL}$ is large, say of $O(1)$. In that case $x_s$ can easily jump up to values of $O(10^2)$ or even larger.

Then, imposing the bounds in table 1, we can obtain the largest possible
Table 1: Limits on $\text{Re}(\delta_{ij})_{AB}$ $(\delta_{ij})_{CD}$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 500$GeV and for different values of $x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2$. For different values of $m_{\tilde{q}}$, the limits can be obtained multiplying the ones in the table by $m_{\tilde{q}}$(GeV)/500.
value for BR\((b \rightarrow d\gamma)\) through gluino exchange. As expected, the \((\delta^d_{13})_{LL}\) insertion leads to very small values of this BR of \(O(10^{-7})\) or so, whilst the \((\delta^d_{13})_{LR}\) insertion allows for BR\((b \rightarrow d\gamma)\) ranging from few times \(10^{-4}\) up to few times \(10^{-3}\) for decreasing values of \(x = m_\tilde{q}^2/m_\tilde{g}^2\). In the SM we expect BR\((b \rightarrow d\gamma)\) to be typically \(10-20\) times smaller than BR\((b \rightarrow s\gamma)\), i.e. BR\((b \rightarrow d\gamma)\) = \((1.7 \pm 0.85) \times 10^{-5}\). Hence a large enhancement in the SUSY case is conceivable if \((\delta^d_{13})_{LR}\) is in the \(10^{-2}\) range. Notice that in the MSSM we expect \((\delta^d_{13})_{LR} < m_\tilde{b}^2/m_\tilde{g}^2 \times V_{td} < 10^{-6}\), hence with no hope at all of a sizeable contribution to \(b \rightarrow d\gamma\).

An analysis similar to the one of \(b \rightarrow s\gamma\) decays can be performed in the leptonic sector where the masses \(m_\tilde{q}\) and \(m_\tilde{g}\) are replaced by the average slepton mass \(m_\tilde{l}\) and the photino mass \(m_\tilde{\gamma}\) respectively. In table 3 we exhibit the bounds on \((\delta^l_{ij})_{LL}\) and \((\delta^l_{ij})_{LR}\) coming from the limits on \(\mu \rightarrow e\gamma\), \(\tau \rightarrow e\gamma\) and \(\tau \rightarrow \mu\gamma\), for a slepton mass of \(O(100 \text{ GeV})\) and for different values of \(x = m_\tilde{\gamma}^2/m_\tilde{l}^2\) [33].

| \(x\) | \(|(\delta^d_{23})_{LL}|\) | \(|(\delta^d_{23})_{LR}|\) |
|---|---|---|
| 0.3 | 4.4 | \(1.3 \times 10^{-2}\) |
| 1.0 | 8.2 | \(1.6 \times 10^{-2}\) |
| 4.0 | 26 | \(3.0 \times 10^{-2}\) |

Table 2: Limits on the \(|\delta^d_{23}|\) from \(b \rightarrow s\gamma\) decay for an average squark mass \(m_\tilde{q} = 500\text{GeV}\) and for different values of \(x = m_\tilde{g}^2/m_\tilde{q}^2\). For different values of \(m_\tilde{q}\), the limits can be obtained multiplying the ones in the table by \((m_\tilde{q}\text{GeV})/500)^2\).
Table 3: Limits on the $|\delta d_{ij}|$ from $l_j \rightarrow l_i \gamma$ decays for an average slepton mass $m_\tilde{\nu} = 100$GeV and for different values of $x = m_\tilde{\nu}^2/m_\tilde{\nu}^2$. For different values of $m_\tilde{\nu}$, the limits can be obtained multiplying the ones in the table by $(m_\tilde{\nu}$(GeV)/100)$^2$.

| $x$  | $|\delta d_{12}\rangle_{LL}$ | $|\delta d_{12}\rangle_{LR}$ | $|\delta d_{13}\rangle_{LL}$ | $|\delta d_{13}\rangle_{LR}$ | $|\delta d_{23}\rangle_{LL}$ | $|\delta d_{23}\rangle_{LR}$ |
|------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.3  | $4.1 \times 10^{-3}$ | $1.4 \times 10^{-6}$ | 15 | $8.9 \times 10^{-2}$ | 2.8 | $1.7 \times 10^{-2}$ |
| 1.0  | $7.7 \times 10^{-3}$ | $1.7 \times 10^{-6}$ | 29 | $1.1 \times 10^{-1}$ | 5.3 | $2.0 \times 10^{-2}$ |
| 5.0  | $3.2 \times 10^{-2}$ | $3.8 \times 10^{-6}$ | $1.2 \times 10^2$ | $2.4 \times 10^{-1}$ | 22 | $4.4 \times 10^{-2}$ |
We start by considering CP violation in the kaon system, i.e. $\varepsilon$ and $\varepsilon'$. In reference [35] we provide a very detailed description of how to calculate the effective Hamiltonian for $\Delta s = 2$ and $\Delta s = 1$ processes as well as a determination of the hadronic matrix elements. Asking for each $\tilde{g}$ exchange contribution not to exceed the experimental value $\varepsilon = 2.268 \times 10^{-3}$ we obtain the bounds reported in table 4 for $m_{\tilde{q}} = 500$ GeV. In fig. 1 we plot the bound on $\sqrt{\text{Im}((\delta_{d_{12}}^{d_{12}})_{LL}(\delta_{d_{12}}^{d_{12}})_{RR})}$ as a function of $x$ for $m_{\tilde{q}} = 500$ GeV. It should be noticed that the bounds derived from $\varepsilon$ on the imaginary parts of products of $\delta$'s are one order of magnitude more stringent than the corresponding limits on the real parts which are obtained from $\Delta m_K$.

Coming to $\Delta s = 1$ processes, both superpenguin and superboxes contribute to $\varepsilon'$. It was only very recently [34] that it was realized that superboxes are at least as important as superpenguin diagrams in contributions which proceed through a $((\delta_{d_{12}}^{d_{12}})_{LL})$ insertion. In fig. 2 we report the bound on $\text{Im}((\delta_{d_{12}}^{d_{12}})_{LL})$ as a function of $x$ for $m_{\tilde{q}} = 500$ GeV which comes from the conservative demand that $\varepsilon'/\varepsilon < 2.7 \times 10^{-3}$. The contribution of box and penguin diagrams to the LL terms have opposite signs and a sizeable cancellation occurs for $x$ close to one, where the two contribution are of comparable size (this explains the peak around $x = 1$ in the plot of fig. 2). A much more

| $x$  | $\sqrt{\text{Im}((\delta_{d_{12}}^{d_{12}})_{LL})^2}$ | $\sqrt{\text{Im}((\delta_{d_{12}}^{d_{12}})_{LR})^2}$ | $\sqrt{\text{Im}((\delta_{d_{12}}^{d_{12}})_{LL}(\delta_{d_{12}}^{d_{12}})_{RR})}$ |
|------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|
| 0.3  | $1.5 \times 10^{-3}$                            | $6.3 \times 10^{-4}$                            | $2.0 \times 10^{-4}$                            |
| 1.0  | $3.2 \times 10^{-3}$                            | $3.5 \times 10^{-4}$                            | $2.2 \times 10^{-4}$                            |
| 4.0  | $7.5 \times 10^{-3}$                            | $4.2 \times 10^{-4}$                            | $3.2 \times 10^{-4}$                            |

Table 4: Limits on $\text{Im}((\delta_{d_{12}}^{d_{12}})_{AB}(\delta_{d_{12}}^{d_{12}})_{CD})$, with $A, B, C, D = (L, R)$, for an average squark mass $m_{\tilde{q}} = 500$ GeV and for different values of $x = m_{\tilde{q}}^2/m_{\tilde{q}}^2$. For different values of $m_{\tilde{q}}$, the limits can be obtained multiplying the ones in the table by $m_{\tilde{q}}$(GeV)/500.

7 CP Violation in Low Energy SUSY - First Two Generations

We start by considering CP violation in the kaon system, i.e. $\varepsilon$ and $\varepsilon'$. In reference [35] we provide a very detailed description of how to calculate the effective Hamiltonian for $\Delta s = 2$ and $\Delta s = 1$ processes as well as a determination of the hadronic matrix elements. Asking for each $\tilde{g}$ exchange contribution not to exceed the experimental value $\varepsilon = 2.268 \times 10^{-3}$ we obtain the bounds reported in table 4 for $m_{\tilde{q}} = 500$ GeV. In fig. 1 we plot the bound on $\sqrt{\text{Im}((\delta_{d_{12}}^{d_{12}})_{LL}(\delta_{d_{12}}^{d_{12}})_{RR})}$ as a function of $x$ for $m_{\tilde{q}} = 500$ GeV. It should be noticed that the bounds derived from $\varepsilon$ on the imaginary parts of products of $\delta$'s are one order of magnitude more stringent than the corresponding limits on the real parts which are obtained from $\Delta m_K$.

Coming to $\Delta s = 1$ processes, both superpenguin and superboxes contribute to $\varepsilon'$. It was only very recently [34] that it was realized that superboxes are at least as important as superpenguin diagrams in contributions which proceed through a $((\delta_{d_{12}}^{d_{12}})_{LL})$ insertion. In fig. 2 we report the bound on $\text{Im}((\delta_{d_{12}}^{d_{12}})_{LL})$ as a function of $x$ for $m_{\tilde{q}} = 500$ GeV which comes from the conservative demand that $\varepsilon'/\varepsilon < 2.7 \times 10^{-3}$. The contribution of box and penguin diagrams to the LL terms have opposite signs and a sizeable cancellation occurs for $x$ close to one, where the two contribution are of comparable size (this explains the peak around $x = 1$ in the plot of fig. 2). A much more
Figure 1: The $\sqrt{\text{Im} \left( \delta_{12}^d \right)_{LL} \left( \delta_{12}^d \right)_{RR}}$ as a function of $x = m_\tilde{g}^2/m_\tilde{q}^2$, for an average squark mass $m_\tilde{q} = 500\text{GeV}$. 
stringent limit is obtained for \((\delta_{12}^d)_{LR}\) (fig. 3). For the LR contribution only superpenguins play a relevant role. Speaking of superpenguins, it is interesting to notice that, differently from the SM case, the SUSY contributions are negligibly affected by electroweak penguins, i.e. gluino-mediated \(Z^0\)- or \(\gamma\)-penguins are strongly suppressed with respect to gluino-mediated gluon penguins \[35\].

In table 4 we summarize the bounds on \(\text{Im}\left(\delta_{12}^d\right)_{LL}\) and \(\text{Im}\left(\delta_{12}^d\right)_{LR}\) coming from \(\varepsilon'/\varepsilon < 2.7 \times 10^{-3}\) for the same values of SUSY masses chosen in table 4. The comparison of the two tables leads to the following two conclusions:

1. if we consider a SUSY extension of the SM where the LR insertions are
Figure 3: The $\text{Im} \left( \delta_{12}^{d} \right)_{LR}$ as a function of $x = m_{\tilde{q}}^2 / m_{\tilde{g}}^2$, for an average squark mass $m_{\tilde{q}} = 500\text{GeV}$.
Table 5: Limits from $\varepsilon'/\varepsilon < 2.7 \times 10^{-3}$ on $\text{Im} \left( \delta_{12}^d \right)$, for an average squark mass $m_{\tilde{q}} = 500\text{GeV}$ and for different values of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$. For different values of $m_{\tilde{q}}$, the limits can be obtained multiplying the ones in the table by $(m_{\tilde{q}}/(\text{GeV})/500)^2$.

| $x$  | $|\text{Im}(\delta_{12}^d)_{LL}|$ | $|\text{Im}(\delta_{12}^d)_{LR}|$ |
|------|-------------------------------|-------------------------------|
| 0.3  | $1.0 \times 10^{-1}$           | $1.1 \times 10^{-5}$          |
| 1.0  | $4.8 \times 10^{-1}$           | $2.0 \times 10^{-5}$          |
| 4.0  | $2.6 \times 10^{-1}$           | $6.3 \times 10^{-5}$          |

much smaller than the LL ones (this is what occurs in the MSSM, for instance), then fulfilling the bound coming from $\varepsilon$ implies that $\text{Im} \left( \delta_{12}^d \right)_{LL}$ is too small to provide a sizeable contribution to $\varepsilon'$ unless $\left( \delta_{12}^d \right)_{LL}$ is almost purely imaginary (remember that $\varepsilon$ bounds $\text{Im} \left( \delta_{12}^d \right)_{LL}$). Hence, in this case the SUSY contribution would be of superweak nature;

2. if, on the contrary, we have a SUSY model with sizeable LR $\Delta s = 1$ mass insertions, then it is possible to respect the bound from $\varepsilon$, while obtaining a large contribution to $\varepsilon'/\varepsilon$. In this case we would have a SUSY milliweak contribution to CP violation. For this to occur, we need a SUSY model where $\left( \delta_{12}^d \right)_{LR}$ is no longer proportional to $m_s$, but rather to some much larger mass.

The above latter remark would lead us to the natural conclusion that to have sizeable SUSY contributions to $\varepsilon'/\varepsilon$ one needs a SUSY extension of the SM where $\tilde{q}_L - \tilde{q}_R$ transitions are no longer proportional to $m_q$ (we remind the reader that in the MSSM a mass term $\tilde{q}_L \tilde{q}_R^*$ receives two contributions, one proportional to the parameter $A$ and the other to $\mu$, but both of them are proportional to $m_q$). However, if this enhancement occurs also for flavour-conserving $\tilde{q}_L - \tilde{q}_R$ transitions one may envisage some problem with the very stringent bound on the $d_N^u$. Indeed, imposing this latter constraint yields the
following limits on \(\text{Im} \left( \delta_{11}^d \right)_{LR} \) for \( m_{\tilde{q}} = 500 \text{ GeV} \):

\[
\begin{array}{c|c}
 x & \text{Im} \left( \delta_{11}^d \right)_{LR} \\
 0.3 & 2.4 \times 10^{-6} \\
 1.0 & 3.0 \times 10^{-6} \\
 4.0 & 5.6 \times 10^{-6} \\
\end{array}
\]

A quick comparison of the above numbers with the bounds on \(\text{Im} \left( \delta_{12}^d \right)_{LR} \) from \(\varepsilon'/\varepsilon\) reveals that to get a sizeable SUSY contribution to \(\varepsilon'\) we need values of \(\text{Im} \left( \delta_{12}^d \right)_{LR} \) which exceed the bound on \(\text{Im} \left( \delta_{11}^d \right)_{LR} \) arising from \(d_N^c\). Obviously, strictly speaking it is not forbidden for \(\text{Im} \left( \delta_{12}^d \right)_{LR} \) to be \(\geq\) \(\text{Im} \left( \delta_{11}^d \right)_{LR} \), but certainly such a probability does not look straightforward. In conclusion, although technically it is conceivable that some SUSY extension may provide a large \(\varepsilon'/\varepsilon\), it is rather difficult to imagine how to reconcile such a large enhancement of \(\text{Im} \left( \delta_{12}^d \right)_{LR} \) with the very strong constraint on the flavour-conserving \(\text{Im} \left( \delta_{11}^d \right)_{LR} \) from \(d_N^c\).

8 CP violation in B Physics

We now move to the next frontier for testing the unitarity triangle in general and in particular CP violation in the SM and its SUSY extensions: \(B\) physics. We have seen above that the transitions between 1st and 2nd generation in the down sector put severe constraints on \(\text{Re} \delta_{12}^d\) and \(\text{Im} \delta_{12}^d\) quantities. To be sure, the bounds derived from \(\varepsilon\) and \(\varepsilon'\) are stronger than the corresponding bounds from \(\Delta M_K\). If the same pattern repeats itself in the transition between 3rd and 1st or 3rd and 2nd generation in the down sector we may expect that the constraints inferred from \(B_d - \bar{B}_d\) oscillations or \(b \to s\gamma\) do not prevent conspicuous new contributions also in CP violating processes in \(B\) physics. We are going to see below that this is indeed the case ad we will argue that measurements of CP asymmetries in several \(B\)-decay channels may allow to disentangle SM and SUSY contributions to the CP decay phase.

First, we consider the constraints on \(\delta_{13}^d\) and \(\delta_{23}^d\) from \(B_d - \bar{B}_d\) and \(b \to s\gamma\), respectively. From the former process we obtain the bounds on \(\text{Re} \left( \delta_{13}^d \right)_{LL'}^2\).
Re \((\delta_{13}^d)^2\) and Re \((\delta_{13}^d)_{LL} (\delta_{13}^d)_{RR}\) which are reported in table I. The radiative decay \(b \rightarrow s\gamma\) constraints only the \(|(\delta_{23}^d)_{LR}|\) quantity in a significant way. For \(m_{\tilde{q}} = 500\) GeV we obtain bounds on \(|(\delta_{23}^d)_{LR}|\) in the range \((1.3 \div 3) \times 10^{-2}\) for \(x\) varying from 0.3 to 4, respectively (the bound scales as \(m_{\tilde{q}}^2\)). On the other hand, \(b \rightarrow s\gamma\) does not limit \(|(\delta_{23}^d)_{LL}|\). In the following, we will take \(|(\delta_{23}^d)_{LL}| = 1\) (corresponding to \(x_s = (\Delta M/\Gamma)_{B_s} > 70\) for \(m_{\tilde{q}} = 500\) GeV).

New physics can modify the SM predictions on CP asymmetries in \(B\) decays \([22]\) by changing the phase of the \(B_d - \bar{B}_d\) mixing and the phase and absolute value of the decay amplitude. The general SUSY extension of the SM that we discuss here affects both these quantities.

The remaining part of this chapter tackles the following question of crucial relevance in the next few years: where and how can one possibly distinguish SUSY contributions to CP violation in \(B\) decays \([37]\)? As we said before we want our answer to be as general as possible, i.e. without any commitment to particular SUSY models. Obviously, a preliminary condition to properly answer the above question is to estimate the amount of the uncertainties of the SM predictions for CP asymmetries in \(B\) decays.

To discuss the latter above-mentioned point, we choose to work in the theoretical framework of ref. \([38]\). We use the effective Hamiltonian (\(H_{\text{eff}}\)) formalism, including LO QCD corrections; in the numerical analysis, we use the LO SM Wilson coefficients evaluated at \(\mu = 5\) GeV, as given in ref. \([32]\). In most of the cases, by choosing different scales (within a reasonable range) or by using NLO Wilson coefficients, the results vary by about 20 – 30\%. This is true with the exception of some particular channels where uncertainties are larger. The matrix elements of the operators of \(H_{\text{eff}}\) are given in terms of the following Wick contractions between hadronic states: Disconnected Emission (\(DE\)), Connected Emission (\(CE\)), Disconnected Annihilation (\(DA\)), Connected Annihilation (\(CA\)), Disconnected Penguin (\(DP\)) and Connected Penguin (\(CP\)) (either for left-left (\(LL\)) or for left-right (\(LR\)) current-current operators). Following ref. \([40]\), where a detailed discussion can be found, instead of adopting a specific model for estimating the different diagrams, we let them vary within reasonable ranges. In order to illustrate the relative strength and variation of the different contributions, in table 6 we only show, for six different cases, results obtained by taking the extreme values of these ranges. In the first column only \(DE = DE_{LL} = DE_{LR}\) are
assumed to be different from zero. For simplicity, unless stated otherwise, the same numerical values are used for diagrams corresponding to the insertion of $LL$ or $LR$ operators, i.e. $DE = DE_{LL} = DE_{LR}$, $CE = CE_{LL} = CE_{LR}$, etc. We then consider, in addition to $DE$, the $CE$ contribution by taking $CE = DE/3$. Annihilation diagrams are included in the third column, where we use $DA = 0$ and $CA = 1/2DE$ [41]. Inspired by kaon decays, we allow for some enhancement of the matrix elements of left-right (LR) operators and choose $DE_{LR} = 2DE_{LL}$ and $CE_{LR} = 2CE_{LL}$ (fourth column). Penguin contractions, $CP$ and $DP$, can be interpreted as long-distance penguin contributions to the matrix elements and play an important role: if we take $CP_{LL} = CE$ and $DP_{LL} = DE$ (fifth column), in some decays these terms dominate the amplitude. Finally, in the sixth column, we allow for long distance effects which might differentiate penguin contractions with u and charm quarks in the loop, giving rise to incomplete GIM cancellations (we assume $DP = DP(c) - DP(u) = DE/3$ and $CP = CP(c) - CP(u) = CE/3$).

In addition to the ratios of the different SM contributions to the decay amplitudes given in table 3 obtained letting the matrix elements vary in the broad range defined above, we also give, in table 4, the branching ratios for the channels of interest to us. These branching ratios are obtained following the approach of ref. [41]. We use QCD sum rules form factors [42] to compute the factorizable $DE$ contribution, then fit $CE$ using the available data on $b \to c$ two-body decays; $CP$ and $DP$ are extracted from the measured $B \to K\pi$ branching ratios, $CA$ is varied between 0 and 0.5 and $DA$, $DP$ and $CP$ are set to zero. The range of values in table 4 corresponds to the variation of the CKM angles in the presently allowed range and to the inclusion of the contributions proportional to $CP$ and $DP$ (see ref. [41] for further details).

Coming to the SUSY contributions, we make use of the Wilson coefficients for the gluino contribution (see eq. (12) of ref. [35]) and parameterize the matrix elements as we did before for the SM case. We obtain the ratios of the SUSY to the SM amplitudes as reported in table 3 for $\tilde{q}$ and $\tilde{g}$ masses of 250 GeV and 500 GeV (second and third row, respectively). From the table, one concludes that the inclusion of the various terms in the amplitudes, $DE$, $DA$, etc., can modify the ratio $r$ of SUSY to SM contributions up to one order of magnitude.
Table 6: Ratios of amplitudes for exclusive $B$ decays. For each channel, whenever two terms with different CP phases contribute in the SM, we give the ratio $r$ of the two amplitudes. For each channel, the second and third lines, where present, contain the ratios of SUSY to SM contributions for SUSY masses of 250 and 500 GeV respectively.
In terms of the decay amplitude $A$, the CP asymmetry reads

$$A(t) = \frac{(1 - |\lambda|^2) \cos(\Delta M dt) - 2 \text{Im} \lambda \sin(\Delta M dt)}{1 + |\lambda|^2},$$

with $\lambda = e^{-2i\phi^M} \bar{A}/A$. In order to be able to discuss the results model-independently, we have labeled as $\phi^M$ the generic mixing phase. The ideal case occurs when one decay amplitude only appears in (or dominates) a decay process: the CP violating asymmetry is then determined by the total phase $\phi^T = \phi^M + \phi^D$, where $\phi^D$ is the weak phase of the decay. This ideal situation is spoiled by the presence of several interfering amplitudes. If the ratios $r$ in table 6 are small, then the uncertainty on the sine of the CP phase is $< r$, while if $r$ is $O(1)$ $\phi^T$ receives, in general, large corrections.

The results of our analysis are summarized in tables 7 and 8 which collect the branching ratios and CP phases for the relevant $B$ decays of table 6. $\Phi_{SM}$ denotes the decay phase in the SM; for each channel, when two amplitudes with different weak phases are present, we indicate the SM phase of the Penguin (P) and Tree-level (T) decay amplitudes. The range of variation of $r$ in the SM ($r_{SM}$) is deduced from table 6. For $B \to K_S \pi^0$ the penguin

| Channel                  | BR $\times 10^5$ |
|--------------------------|------------------|
| $B \to J/\psi K_S$       | 40               |
| $B \to \phi K_S$         | 0.6 - 2          |
| $B \to \pi^0 K_S$        | 0.02 - 0.4       |
| $B \to D_{CP}^0 \pi^0$   | 16               |
| $B \to D^+ D^-$          | 30 - 50          |
| $B \to J/\psi \pi^0$     | 2                |
| $B \to \phi \pi^0$       | $1 - 4 \times 10^{-4}$ |
| $B \to K^0 K^0$          | 0.007 - 0.3      |
| $B \to \pi^+ \pi^-$      | 0.2 - 2          |
| $B \to \pi^0 \pi^0$      | 0.003 - 0.09     |
| $B \to K^+ K^-$          | < 0.5            |
| $B \to D^0 D^*$          | < 20             |

Table 7: Branching ratios for $B$ decays.
contributions (with a vanishing phase) dominate over the tree-level amplitude because the latter is Cabibbo suppressed. For the channel $b \to s \bar{s}d$ only penguin operators or penguin contractions of current-current operators contribute. The phase $\gamma$ is present in the penguin contractions of the $(\bar{b}u)(\bar{u}d)$ operator, denoted as $u-P \gamma$ in table 8 [43]. $\bar{b}d \to \bar{q}q$ indicates processes occurring via annihilation diagrams which can be measured from the last two channels of table 8. In the case $B \to K^+K^-$ both current-current and penguin operators contribute. In $B \to D^0\bar{D}^0$ the contributions from the $(\bar{b}u)(\bar{u}d)$ and the $(\bar{b}c)(\bar{c}d)$ current-current operators (proportional to the phase $\gamma$) tend to cancel out.

SUSY contributes to the decay amplitudes with phases induced by $\delta_{13}$ and $\delta_{23}$ which we denote as $\phi_{13}$ and $\phi_{23}$. The ratios of $A_{SUSY}/A_{SM}$ for SUSY masses of 250 and 500 GeV as obtained from table 6 are reported in the $r_{250}$ and $r_{500}$ columns of table 8.

We now draw some conclusions from the results of table 8. In the SM, the first six decays measure directly the mixing phase $\beta$, up to corrections which, in most of the cases, are expected to be small. These corrections, due to the presence of two amplitudes contributing with different phases, produce uncertainties of $\sim 10\%$ in $B \to K_S\pi^0$, and of $\sim 30\%$ in $B \to D^+D^-$ and $B \to J/\psi\pi^0$. In spite of the uncertainties, however, there are cases where the SUSY contribution gives rise to significant changes. For example, for SUSY masses of $O(250)$ GeV, SUSY corrections can shift the measured value of the sine of the phase in $B \to \phi K_S$ by an amount of about 70%. For these decays SUSY effects are sizeable even for masses of 500 GeV. In $B \to J/\psi K_S$ and $B \to \phi\pi^0$ decays, SUSY effects are only about 10% but SM uncertainties are negligible. In $B \to K^0\bar{K}^0$ the larger effect, $\sim 20\%$, is partially covered by the indetermination of about 10% already existing in the SM. Moreover the rate for this channel is expected to be rather small. In $B \to D^+D^-$ and $B \to K^+K^-$, SUSY effects are completely obscured by the errors in the estimates of the SM amplitudes. In $B^0 \to D_{CP}^0\pi^0$ the asymmetry is sensitive to the mixing angle $\phi_M$ only because the decay amplitude is unaffected by SUSY. This result can be used in connection with $B^0 \to K_S\pi^0$, since a difference in the measure of the phase is a manifestation of SUSY effects.

Turning to $B \to \pi\pi$ decays, both the uncertainties in the SM and the SUSY contributions are very large. Here we witness the presence of three independent amplitudes with different phases and of comparable size. The
Incl.     | Excl.     | $\phi_{SM}^D$ | $r_{SM}$ | $\phi_{SUSY}^D$ | $r_{250}$ | $r_{500}$ \\
---|---|---|---|---|---|---
$b \to c\bar{c}s$ & $B \to J/\psi K_S$ & 0 & – & $\phi_{23}$ & 0.03 – 0.1 & 0.008 – 0.04 \\
$b \to s\bar{s}s$ & $B \to \phi K_S$ & 0 & – & $\phi_{23}$ & 0.4 – 0.7 & 0.09 – 0.2 \\
$b \to u\bar{u}s$ & & P 0 & & & \\
$b \to d\bar{d}s$ & | & T $\gamma$ & & & \\
$b \to u\bar{u}d$ & | & 0 & & & \\
$b \to c\bar{c}d$ & $B \to D^0_{CP} \pi^0$ & 0.02 & – & – & – & \\
$b \to s\bar{s}d$ & $B \to D^+ D^-$ & T 0 & 0.03 – 0.3 & 0.007 – 0.02 & 0.002 – 0.006 & \\
$b \to c\bar{c}d$ & $B \to J/\psi\pi^0$ & P $\beta$ & 0.04 – 0.3 & 0.007 – 0.03 & 0.002 – 0.008 & \\
$b \to s\bar{s}d$ & $B \to \phi\pi^0$ & P $\beta$ & – & 0.06 – 0.1 & 0.01 – 0.03 & \\
$b \to u\bar{u}d$ & $B \to K^0 \bar{K}^0$ & u-P $\gamma$ & 0 – 0.07 & 0.08 – 0.2 & 0.02 – 0.06 & \\
$b \to d\bar{d}d$ & $B \to \pi^+ \pi^-$ & T $\gamma$ & 0.09 – 0.9 & $\phi_{13}$ & 0.02 – 0.8 & 0.005 – 0.2 & \\
$bd \to q\bar{q}$ & $B \to K^+ K^-$ & T $\gamma$ & 0.2 – 0.4 & $\phi_{13}$ & 0.04 – 0.1 & 0.01 – 0.03 & \\
$b \to d\bar{d}d$ & $B \to D^0 D^0$ & P $\beta$ & only $\beta$ & $\phi_{13}$ & 0.01 – 0.03 & 0.003 – 0.006 & \\

Table 8: CP phases for $B$ decays. $\phi_{SM}^D$ denotes the decay phase in the SM; T and P denote Tree and Penguin, respectively; for each channel, when two amplitudes with different weak phases are present, one is given in the first row, the other in the last one and the ratio of the two in the $r_{SM}$ column. $\phi_{SUSY}^D$ denotes the phase of the SUSY amplitude, and the ratio of the SUSY to SM contributions is given in the $r_{250}$ and $r_{500}$ columns for the corresponding SUSY masses.
observation of SUSY effects in the $\pi^0\pi^0$ case is hopeless. The possibility of separating SM and SUSY contributions by using the isospin analysis remains an open possibility which deserves further investigation. For a thorough discussion of the SM uncertainties in $B \to \pi\pi$ see ref. [40].

In conclusion, our analysis shows that measurements of CP asymmetries in several channels may allow the extraction of the CP mixing phase and to disentangle SM and SUSY contributions to the CP decay phase. The golden-plated decays in this respect are $B \to \phi K_S$ and $B \to K_S\pi^0$ channels. The size of the SUSY effects is clearly controlled by the the non-diagonal SUSY mass insertions $\delta_{ij}$, which for illustration we have assumed to have the maximal value compatible with the present experimental limits on $B^0_d - \bar{B}^0_d$ mixing.

9 Outlook

In the past major emphasis was given to the fact that the MSSM succeeded to pass all the dangerous FCNC and CP tests unscathed. As important as this point may actually be, we think that what really matters is how much these rare processes can yield us a clue on the low-energy SUSY realization and, hence, on the underlying theory which produces it. In this view the MSSM constitutes an interesting prototype for more general SUSY extensions of the SM: FCNC and CP provide crucial “borders” on the allowed departures from such prototype. In particular, the issue of the way one realizes the breaking of SUSY becomes central for the solution of the flavour problem.

On a more phenomenological basis, we hope that these two lectures may help in correcting a rather common misjudgment on the possibility of achieving experimental hints of the existence of low-energy SUSY. We refer to the statement that if at LEP II there is no SUSY manifestation, then we have to wait for LHC, i.e. quite a few years, before having any answer about the existence of SUSY. We strongly believe that experiments dealing with FCNC and CP violating phenomena have a conspicuous potentiality to give us some hints of new physics, in particular if the latter is represented by low-energy SUSY. We have stressed that such a potentiality is better expressed in three classes of FCNC and CP experiments which promise to give us important improvements well before the advent of LHC: i) CP violation in B physics (and, to some extent, also rare FCNC B decays); ii) measurements of the
electric dipole moments of the neutron and the electron; iii) flavour lepton number violations ($\mu \rightarrow e\gamma$, $\mu \rightarrow e$ conversion in nuclei).

Our conviction that this possibility of manifestation of SUSY is not a wishful thought, but rests on a solid ground is closely linked to a point that we hope to have stressed enough in these lectures. The constrained MSSM extension of the SM is a useful SUSY prototype model to perform quantitative analysis, but it is unlikely to emerge in its “minimality” from an underlying effective supergravity. Indeed, if, for some reason, it should turn out that it is just this constrained MSSM that is realized at low energy, then, as we have seen, even our efforts to discover SUSY in the above three classes of indirect tests would be frustrated. In this sense FCNC and CP “measure” the amount of departure not only from the SM physics, but also from the constrained version of the MSSM.

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