Buffers Improve the Performance of Relay Selection

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Abstract—We show that the performance of relay selection can be improved by employing relays with buffers. Under the idealized assumption that no buffer is full or empty, the best source-relay and the best relay-destination channels can be simultaneously exploited by selecting the corresponding relays for reception and transmission, respectively. The resulting relay selection scheme is referred to as max-max relay selection (MMRS). Since for finite buffer sizes, empty and full buffers are practically unavoidable if MMRS is employed, we propose a hybrid relay selection (HRS) scheme, which is a combination of conventional best relay selection (BRS) and MMRS. We analyze the outage probabilities of MMRS and HRS and show that both schemes achieve the same diversity gain as conventional BRS and a superior coding gain. Furthermore, our results show that for moderate buffer sizes (e.g., 30 packets) HRS closely approaches the performance of idealized MMRS and the performance gain compared to BRS approaches 3 dB as the number of relays increases.

I. INTRODUCTION

In cooperative networks with multiple relays, where a number of relays assist a source in transmitting information to a destination, relay selection techniques have gained a lot of interest. Relay selection is attractive because of its high performance, efficient use of power and bandwidth resources, and simplicity. For example, simple relay selection schemes can achieve the same diversity order as more complex cooperative schemes employing space time block coding or orthogonal channels. Many different schemes for single relay selection have been proposed in the literature, see e.g., [2], [4]–[6] and references therein. All these schemes have in common that the selected relay receives a packet from the source and retransmits it to the destination. The two most common schemes are the bottleneck and maximum harmonic mean based best relay selection (BRS) schemes [2] and their performances have been extensively studied in the literature [2], [7]–[9]. Here, we adopt bottleneck based BRS as a benchmark for the proposed schemes.

In order to overcome the limitations imposed by using the same relay for reception and transmission, we propose to employ relays with buffers for applications that are not delay sensitive. If the relays have buffers, the relays with the best source-relay channel and the best relay-destination channel can be selected for reception and transmission, respectively. The corresponding selection scheme is referred to as max-max relay selection (MMRS). For MMRS, we make the idealistic assumption that the buffer of the relay selected for reception (transmission) is not full (empty), which is only possible for buffers of infinite size. For the practical case of finite buffer sizes the buffer of a relay may become empty (full) if the channel conditions are such that the relay is selected repeatedly for transmission (reception) but not for reception (transmission).

To overcome this limitation, we propose a hybrid relay selection (HRS) scheme, which is a combination of conventional BRS and MMRS. In particular, for HRS, if the buffer of the relay selected for reception (transmission) is full (empty), BRS is employed; otherwise, MMRS is used. Although both MMRS and HRS can be combined with both amplify-and-forward and decode-and-forward (DF) relaying, in this paper, we only consider DF relays and derive the corresponding outage probabilities. Analytical and simulation results establish the superiority of MMRS and HRS compared to BRS.

We note that relays with buffers have been considered before in [10] and [11] to improve the throughput of simple three-node networks consisting of a source, a destination, and a single relay. However, to the best of our knowledge, relays with buffers have not been considered in the context of relay selection before.

The remainder of this paper is organized as follows. In Section II the system model is presented, and MMRS and HRS are introduced. An outage analysis of MMRS and HRS is provided in Section III. In Section IV numerical results are presented, and conclusions are drawn in Section V.

II. SYSTEM MODEL AND RELAY SELECTION

In this section, we present the system model, briefly review BRS, and introduce the proposed MMRS and HRS schemes.

A. System Model

We consider a relay network with one source node, S, one destination node, D, and N half-duplex DF relays, \( R_1, \ldots, R_N \). Each relay is equipped with a buffer and transmission is organized in two time slots. In the first time slot, the relay selected for reception receives a packet from the source node and stores it in its buffer. In the second time slot, the relay selected for transmission forwards a packet from its buffer to the destination node.

We assume that a direct link between the source and the destination does not exist or, if it does exist, it is not exploited for simplicity of implementation. Let \( g_i \) and \( h_i \), \( i = 1, \ldots, N \), denote the S-\( R_i \) and \( R_i \)-D channel gains, respectively. We assume that the channel coefficients \( g_i \) and \( h_i \) are mutually independent zero-mean complex Gaussian random variables (Rayleigh fading) with variances \( \sigma^2_{g_i} \) and \( \sigma^2_{h_i} \), respectively. Moreover, we assume that the transmission is organized in packets and the channels are constant for the duration of one packet and vary independently from one packet to the next (block fading model). This behavior can
be achieved through frequency hopping between packets. Let 
\( \gamma_g = |g|^2 E_s / N_0 \) denote the instantaneous signal-to-
noise ratio (SNR) between the source and relay \( R_i \) and 
\( \gamma_h \equiv |h|^2 E_s / N_0 \) the instantaneous SNR between 
\( R_i \) and the destination. Here, \( E_s \) is the energy available at 
the transmitting nodes and \( N_0 \) is the variance of the zero mean 
additive white Gaussian noise (AWGN) at the receiving nodes. \( \gamma_g \) 
and \( \gamma_h \) are exponentially distributed with parameters \( 1/\gamma_g \), 
and \( 1/\gamma_h \), respectively, where 
\[ \gamma_g \equiv E[|g|^2] = \sigma^2 g E_s / N_0, \quad \gamma_h \equiv E[|h|^2] = \sigma^2_h E_s / N_0, \] 
and \( E[\cdot] \) denotes expectation.

We assume that the destination node has perfect channel state 
information (CSI) and selects the relays for transmission 
and reception. The destination node feeds back the information 
about the selected relays to all relays via an error-free feedback 
channel.

B. Best Relay Selection

The BRS scheme achieves full diversity by selecting one 
relay out of the \( N \) available relays. This relay is then used for 
reception and transmission. The selected best relay, \( R_b \), has 
the best bottleneck link \[2\], i.e., 
\[ b \triangleq \arg\max_{i=1,...,N} \min\{\gamma_g, \gamma_h\}. \] (1)

C. Max-Max Relay Selection

The relay selected according to the criterion in \[1\] may not 
simultaneously enjoy the best source-relay and the best relay-
destination channels. If the relays are equipped with buffers, 
they can store the packets received from the source and do not 
have to re-transmit them immediately in the next time slot. As 
a result, it is possible to use the relay with the best source-relay 
channel for reception and the relay with the best relay-
destination channel for transmission. Thus, in the resulting 
MMRS scheme, the best relay for reception, \( R_{br} \), is selected 
based on 
\[ br \triangleq \arg\max_{i=1,...,N} \gamma_g, \] (2)
and the best relay for transmission, \( R_{bt} \), is selected according to 
\[ bt \triangleq \arg\max_{i=1,...,N} \gamma_h. \] (3)

For MMRS to work properly, the buffer of no relay can be 
empty or full at any time such that all relays have always 
the option of receiving and transmitting. Clearly, for buffers 
of finite size this may not be possible since a buffer may 
become empty (full) if a relay enjoys repeatedly the best relay-
destination (source-relay) link but never the best source-relay 
(relay-destination) link. To overcome this problem, in the next 
subsection, we combine MMRS with BRS.

D. Hybrid Relay Selection

If buffer over- and underflows are to be avoided, the relay 
selection criterion cannot only depend on the channel status 
as in MMRS but also has to take into account the status of the 
buffer. The basic idea is to use BRS if either the buffer of 
the relay selected for reception is full or the buffer of the 
relay selected for transmission is empty. In all other cases, 
MMRS is used. We assume that all buffers have \( L_b \) elements 
and each element can store one packet. We denote the number 
of elements of relay \( R_i \)'s buffer that are full by \( N_{e,i} \). For HRS, 
the best relay for reception, \( R_{br} \), is selected according to 
\[ br = \begin{cases} b, & \text{if } N_{e,br} = L_b - 1 \text{ or } N_{e,br} = 0, \\ br, & \text{otherwise}, \end{cases} \] (4)
and the best relay for transmission, \( R_{bt} \), is selected according to 
\[ bt = \begin{cases} b, & \text{if } N_{e,br} = L_b - 1 \text{ or } N_{e,br} = 0, \\ bt, & \text{otherwise}, \end{cases} \] (5)

where \( b, br, \) and \( bt \) are defined in \[1\], \[2\], and \[3\], respectively. 
In \[4\] and \[5\], we always leave one element of each buffer 
empty so that each relay is always able to receive in case it 
is selected for reception in the BRS mode in the next 
transmission interval.

We note that for both MMRS and HRS, since different 
packets may be stored at different relays for different amounts 
of time, the packets transmitted by the source may arrive 
at the destination node in an order different from the order 
at the source node. The original order can be restored at 
the destination node if the order information is contained in 
in the preamble of the packet. Furthermore, MMRS and HRS 
introduce a delay in the network. This issue will be 
investigated in Section \[IV\].

III. Outage Probability Analysis

In this section, we study the outage probability of MMRS 
and HRS with DF relays. The outage probability is defined 
as the probability that the output SNR, \( \gamma_b \), falls below a 
certain SNR threshold, \( \gamma \triangleq 2^2R - 1 \), above which error-free 
transmission with rate \( R \) is possible \[12\], i.e., 
\[ P_{out} \triangleq P(\gamma_b \leq \gamma), \] (6)
where \( P(A) \) denotes the probability of event \( A \). Before we 
consider MMRS and HRS, we first briefly review the outage 
probability of BRS, which will be useful for computation of 
the outage probability of HRS.

A. Best Relay Selection

For BRS and DF relays, \( \gamma_b \triangleq \max_i \{\min(\gamma_g, \gamma_h)\} \). Thus, 
based on \[7\], we obtain 
\[ P_{out}^{BRS} = \prod_{i=1}^{N} \left( 1 - \exp \left( -\frac{\gamma}{\bar{y}_i} \right) \right), \] (7)
where \( \bar{y}_i \triangleq \frac{1}{\gamma_g} + \frac{1}{\gamma_h} \).

At high SNR and assuming independent and identically 
distributed (i.i.d.) fading for both links, i.e., \( \gamma_g \sim \gamma_h \) 
and \( \bar{y}_i \approx 2 \), the outage probability can be simplified to 
\[ P_{out}^{BRS} \approx \left( \frac{2\gamma}{\gamma} \right)^N. \] (8)

Expressing the outage probability now in terms of the diversity 
gain \( G_d \) and coding gain \( G_c \), i.e., \( P_{out} \approx \left( \frac{G_d}{\gamma} \right)^{-G_d} \), we observe that the diversity gain of BRS is \( G_d^{BRS} = N \) and 
its coding gain is \( G_c^{BRS} = 1/2 \).
B. Max-Max Relay Selection

For MMRS and DF relays, we have \( \gamma_b \triangleq \min\{\gamma_{g_b}, \gamma_{h_b}\} \), where \( \gamma_{g_b} \triangleq \max_{i=1, \ldots, N} \gamma_{g_i} \) and \( \gamma_{h_b} \triangleq \max_{i=1, \ldots, N} \gamma_{h_i} \). Hence, the outage probability is given by

\[
P_{\text{out}}^{\text{MMRS}} = P\{\min\{\gamma_{g_b}, \gamma_{h_b}\} \leq \gamma\}
= 1 - P(\gamma_{g_b} > \gamma)P(\gamma_{h_b} > \gamma)
= 1 - \left[1 - P(\gamma_{g_b} \leq \gamma)\right]\left[1 - P(\gamma_{h_b} \leq \gamma)\right]
= 1 - \left[1 - \prod_{i=1}^{N} \left(1 - e^{-\frac{\gamma}{\gamma_{g_i}}}\right)\right]\left[1 - \prod_{i=1}^{N} \left(1 - e^{-\frac{\gamma}{\gamma_{h_i}}}\right)\right].
\]

(9)

In case of i.i.d. fading for both links, i.e., \( \gamma = \gamma_{g_i} = \gamma_{h_i} \), \( i = 1, \ldots, N \), (9) simplifies to

\[
P_{\text{out}}^{\text{MMRS}} = 1 - \left[1 - \left(1 - e^{-\frac{\gamma}{\gamma}}\right)^N\right]^2.
\]

(10)

If we assume furthermore that the SNR is high and use the approximation \( 1 - e^{-x} \approx x \), \( x \rightarrow 0 \), we obtain

\[
P_{\text{out}}^{\text{MMRS}} \approx \left(2\gamma + \frac{\gamma^2}{2}\right)^{-N}.
\]

(11)

From (11), we observe that MMRS achieves a diversity gain of \( G_d^{\text{MMRS}} = N \) and a coding gain of \( G_c^{\text{MMRS}} = 2 - \frac{1}{N} \). Thus, in contrast to BRS, the coding gain of MMRS increases with the number of relays.

Interestingly, BRS and MMRS have the same diversity gain. However, MMRS achieves a higher coding gain for \( N \geq 2 \) relays. For a large number of relays, MMRS yields an SNR gain of \( \lim_{N \rightarrow \infty} 10 \log_{10}(G_c^{\text{MMRS}}/G_c^{\text{BRS}}) = 3 \) dB compared to BRS.

C. Hybrid Relay Selection

For simplicity, for the analysis of HRS, we only consider the i.i.d. fading case. Considering (4) and (5), the outage probability of HRS can be written as

\[
P_{\text{out}}^{\text{HRS}} = P_{\text{out}}^{\text{MMRS}} + P_{\text{out}}^{\text{MMRS}} + P_{\text{BRS}}^{\text{out}},
\]

(12)

where \( P_{\text{BRS}} \) and \( P_{\text{MMRS}} \) are the probabilities that BRS (i.e., \( b = b_t = b \)) and MMRS (i.e., \( b = b_r \) and \( b_t = b_t \)) are used in HRS, respectively. Since \( P_{\text{out}}^{\text{MMRS}} \) and \( P_{\text{out}}^{\text{MMRS}} \) are already known from the previous two subsections, we only have to compute \( P_{\text{BRS}} \) (or \( P_{\text{MMRS}} \)) for evaluation of \( P_{\text{out}}^{\text{HRS}} \).

Clearly, if all the buffers are either full or empty, BRS is used all the time and \( P_{\text{BRS}} = 1 \) (and hence \( P_{\text{MMRS}} = 0 \)). In order to compute \( P_{\text{BRS}} \) for the more interesting case where at least one buffer is neither full nor empty, we model the possible states of the buffers and the transitions between the states as a Markov chain. Let \( S_i \triangleq X_1X_2 \ldots X_N \) denote the \( i \)th state in the Markov chain, where \( X_j, j = 1, \ldots, N \), represents the number of full elements in the \( j \)th buffer. Let \( P_{\text{BRS},i} \) denote the probability of using BRS in state \( S_i \). Then, \( P_{\text{BRS}} \) can be written as

\[
P_{\text{BRS}} = \sum_{i=1}^{N_s} P_{\text{BRS},i} P_{S_i},
\]

(13)

where \( P_{S_i} \) and \( N_s \) denote the probability of being in state \( S_i \) and the total number of states, respectively. Since the buffer size is finite and the total number of buffer elements across all relays that are full is constant, each state has to meet the following two constraints

\[
\sum_{i=1}^{N} X_i = N_e,
\]

(14)

\[
0 \leq X_i \leq L_b - 1, \quad i \in \{1, \ldots, N\},
\]

(15)

where \( N_e \triangleq \sum_{i=1}^{N} N_{e,i} \) is the total number of full elements of all buffers.

The probability of transition from one state to another state is \( 1/N^2 \). This can be seen from the fact that for a two-hop relay network with \( N \) relays there are \( N^2 \) possible selections of the relays for reception and transmission. Since we assume that all channels are i.i.d., the probability of each selection is \( 1/N^2 \). Given that the status of the buffers changes only if the relays selected for reception and transmission are different, each transition from one state to another state corresponds to only one selection, resulting in a transition probability of \( 1/N^2 \). On the other hand, if any one of the \( N \) available relays is selected for reception and transmission, the states of the buffers remain unchanged. Thus, there is more than one selection that allows the buffers to remain in the same state and the probability that the buffer remains in the same state is generally larger than \( 1/N^2 \).

**Proposition 1:** The state transition matrix \( \mathbf{P} \) of the Markov chain that models the buffer states is a doubly stochastic matrix.

**Proof:** For any Markov chain \( \sum_{j=1}^{N} p_{ij} = 1 \) holds, where \( p_{ij} \triangleq [\mathbf{P}]_{ij} \) is the transition probability from state \( S_i \) to state \( S_j \). Furthermore, for the considered case, all transition probabilities from one state to another state are equal to \( 1/N^2 \). If there is a transition from state \( S_i \) to state \( S_j \), there is also a transition from state \( S_j \) to state \( S_i \) and the probability of both transitions is \( 1/N^2 \). Thus, the transition matrix \( \mathbf{P} \) is symmetric and \( \sum_{i=1}^{N} p_{ij} = 1 \) holds. Hence, the transition matrix is doubly stochastic, and the proof is complete.

**Lemma 1** ([Page 65]): For a doubly stochastic transition matrix, the stationary distribution is uniform, i.e., all the states are equally likely. For an \( N_s \)-state Markov chain, the probability of being in state \( S_i, i = 1, \ldots, N_s \), is \( P_{S_i} = \frac{1}{N_s} \), regardless the initial state.

From Lemma 1 (13) reduces to

\[
P_{\text{BRS}} = \frac{1}{N_s} \sum_{i=1}^{N_s} P_{\text{BRS},i}. \]

(16)

Since the computation of \( P_{\text{BRS},i} \) is difficult in the general case, we first consider an example to illustrate the main idea.

**Example 1:** Let us consider a relay network with \( N = 2 \) relays and the buffer at each relay is of size \( L_b = 4 \) and half of the buffer elements are full, i.e., \( N_e = 4 \). Fig. 1 depicts the
block diagram of the network. The states of the corresponding Markov chain have to satisfy the constraints in (14) and (15), i.e.,

\[ X_1 + X_2 = 4, \quad X_1 \leq 3, \quad \text{and} \quad X_2 \leq 3. \] (17)

Therefore, we have \( N_s = 3 \) possible states for the Markov chain, \( X_1X_2 \in \{13, 22, 31\} \), and the probability of transition from one state to another is \( \frac{1}{N_s} = \frac{1}{3} \). Let \( S_1 = 13, S_2 = 22, \) and \( S_3 = 31 \). The state diagram of this Markov chain is shown in Fig. 2. The corresponding state transition matrix is given by

\[ P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/4 & 3/4 \end{bmatrix}. \] (18)

As expected, the transition matrix \( P \) is doubly stochastic. Therefore, from Lemma 1 the probability of each state is \( \frac{1}{N_s} = \frac{1}{3} \).

Let us now define matrix \( D \triangleq \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \), where the first and second column represents the relays selected for reception and transmission, respectively. If \( a_1 = 1 \), relay \( R_i \) is selected for reception and if \( b_1 = 1 \), relay \( R_i \) is selected for transmission, otherwise both \( a_i \) and \( b_i \) are zero. Note that since only one relay is selected for reception and transmission, respectively, in each column of \( D \) only one element is equal to one and all other elements are zero. Hence, \( D \) can assume \( N^2 = 4 \) different values: \( D_1 \triangleq \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, D_2 \triangleq \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \) \( D_3 \triangleq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \) and \( D_4 \triangleq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \) and the probability of occurrence of each value is \( \frac{1}{N_s} = \frac{1}{3} \). Now, we are ready to compute \( P_{BRS,i} \) for each state.

1. State \( S_1 = 13 \): For \( D_1 \) and \( D_3 \), based on (4) and (5) MMRS is used, and hence \( P_{BRS,1} = P_{BRS,3} = 0 \). For \( D_2 \) and \( D_4 \), again based on (4) and (5), BRS is used, and hence \( P_{BRS,2} = P_{BRS,4} = \frac{1}{N_s} = \frac{1}{3} \). Summing up the probabilities of using BRS in state \( S_1 \), we obtain \( P_{BRS,1} = \sum_{i=1}^{4} P_{BRS,i} = \frac{1}{3} \).

2. State \( S_2 = 22 \): In this case, \( P_{BRS,2} = 0 \), since the buffers are neither full nor empty.

3. State \( S_3 = 31 \): This state is symmetric to state \( S_1 \). Thus, \( P_{BRS,3} = \frac{1}{3} \).

Finally, the total probability of using BRS is obtained as

\[ P_{BRS} = \frac{1}{N_s} \sum_{i=1}^{N_s} P_{BRS,i} = \frac{1}{3} \left( \frac{1}{3} + \frac{1}{3} \right) = \frac{1}{3}. \] (19)

Let us now return to the general case.

Proposition 2: Let \( N_{F,i} \) and \( N_{E,i} \) denote the number of full and empty buffers for state \( S_i \), respectively. Note that the buffer of relay \( R_i \) is considered full if \( N_{e,i} = L_b - 1 \). Then, the probability of using BRS in state \( S_i \) is given by

\[ P_{BRS,i} = \frac{1}{N_s^2} ((N_{F,i} + N_{E,i})N - N_{F,i}N_{E,i}). \] (20)

Proof: As mentioned before, for \( N \) relays, there are \( N^2 \) possible selections of the relays for reception and transmission and the probability of each selection is \( 1/N^2 \). Therefore, according to (4) and (5), if we have \( N_{F,i} \) full buffers in state \( S_i \), there are \( N_{F,i}N \) selections for which BRS is used. Moreover, if state \( S_i \) has \( N_{E,i} \) empty buffers, from the remaining \( N^2 - N_{F,i}N \) possible selections, there are \( (N - N_{F,i})N_{E,i} \) selections in which BRS is used. Therefore, since each selection has a probability of occurrence of \( 1/N^2 \), the probability of using BRS in state \( S_i \) is given by

\[ P_{BRS,i} = \frac{1}{N_s^2} (N_{F,i}N + (N - N_{F,i})N_{E,i}) = \frac{1}{N_s^2} ((N_{F,i} + N_{E,i})N - N_{F,i}N_{E,i}). \] This concludes the proof.

Computation of \( N_s, N_{F,i}, \) and \( N_{E,i} \): For computation of \( P_{BRS} \), the number of possible buffer states, \( N_s \), has to be determined. It does not seem possible to obtain a general formula for \( N_s \) valid for any \( N, L_b \), and \( N_e \). However, for \( N = 2 \) and \( N = 3 \), the number of states can be calculated in closed form. In particular, for \( N = 2 \), we obtain

\[ N_s = \begin{cases} N_e + 1, & \text{if } N_e \leq L_b - 1 \\ 2L_b - N_e - 1, & \text{otherwise} \end{cases} \] (21)

and, for \( N = 3 \), we have

\[ N_s = \sum_{i=(N_e - L_b)^+ + 1}^{N_e + 1} (i - 2(i - L_b) + 1)^+, \] (22)

where \( (x)^+ \triangleq \max\{x, 0\} \).

For the general case, for a given total number of full buffer elements, \( N_e \), and a given size of the buffers, \( L_b \), the number of states can be obtained algorithmically as the number of all possible combination of \( X_1X_2...X_N \) that satisfy (14) and (15).
Therefore, the diversity gain of HRS is

\[ G_d^{\text{HRS}} = N \]

and the coding gain is \( G_c^{\text{HRS}} = (2P_{\text{MMRS}} + 2^N P_{\text{BRS}})^{\gamma} \), i.e., the coding gain of HRS increases with increasing \( N \) and increasing \( P_{\text{MMRS}} \).

\[ P_{\text{out}}^H \approx \left( 2P_{\text{MMRS}} + 2^N P_{\text{BRS}} \right)^{\frac{\gamma}{\gamma - 1}} \]

Therefore, the diversity gain of HRS is \( G_d^{\text{HRS}} = N \) and the coding gain is \( G_c^{\text{HRS}} = (2P_{\text{MMRS}} + 2^N P_{\text{BRS}})^{\gamma} \), i.e., the coding gain of HRS increases with increasing \( N \) and increasing \( P_{\text{MMRS}} \).

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\[ \gamma \to 1 \]

Therefore, the diversity gain of HRS is \( G_d^{\text{HRS}} = N \) and the coding gain is \( G_c^{\text{HRS}} = (2P_{\text{MMRS}} + 2^N P_{\text{BRS}})^{\gamma} \), i.e., the coding gain of HRS increases with increasing \( N \) and increasing \( P_{\text{MMRS}} \).
of one means that the packet is stored at the relay in one transmission interval and retransmitted in the next. Fig. 6 shows that, as expected, the average delay increases with increasing buffer size. In fact, for sufficiently large buffer sizes the average delay can be approximated as $N L_b/2$. Thus, considering the results in Figs. 3 and 5, delays of less than 50 (100) transmission intervals are sufficient for HRS to closely approach the performance of MMRS for $N = 3$ ($N = 5$) relays.

V. Conclusion

In this paper, we proposed two new relay selection schemes for relays with buffers. The first scheme, MMRS, always selects the relays with the best source-relay and the best relay-destination channels for reception and transmission, respectively, and operates under the assumption that the buffers at the relays are neither full nor empty. Since this assumption is not practical for finite buffers, we proposed a second scheme, HRS, which employs MMRS if the buffer of the relay selected for reception is not full and the buffer of the relay selected for transmission is not empty, and conventional BRS otherwise. We have analyzed the outage probability of MMRS and HRS and established that while they have the same diversity gain as BRS, they achieve a coding gain advantage of up to 3 dB. More importantly, we showed that, for $N = 3$ relays, HRS can achieve a coding gain advantage of 2 dB compared to BRS if an average delay of 50 transmission intervals can be afforded.

Finally, we note that relays with buffers add additional flexibility to cooperative diversity systems. While, in this paper, we used this flexibility to improve the performance of relay selection, exploring other scenarios (e.g. interference avoidance) where relays with buffers may be advantageous in cooperative networks is an interesting topic for future work.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” IEEE Trans. Inf. Theory, vol. 50, pp. 3062–3080, Dec. 2004.
[2] A. Bletsas, A. Khisti, D. Reed, and A. Lippman, “A simple cooperative diversity method based on network path selection,” IEEE J. Select. Areas Commun., vol. 24, pp. 659–672, Mar. 2006.
[3] J. N. Laneman and G. W. Wornell, “Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks,” IEEE Trans. Inf. Theory, vol. 49, pp. 2415–2425, Nov. 2003.
[4] A. Bletsas, H. Shin, and M. Z. Win, “Outage optimality of opportunistic amplify-and-forward relaying,” IEEE Communications Letters, vol. 11, pp. 261–263, Mar. 2007.
[5] Y. Zhao, R. Adve, and T. J. Lim, “Improving amplify-and-forward relay networks: optimal power allocation versus selection,” IEEE Trans. Wireless Commun., vol. 6, pp. 3114–3123, Aug. 2007.
[6] D. S. Michalopoulos, G. K. Karagiannidis, T. A. Tsiftsis, and R. K. Mallik, “Distributed transmit antenna selection (DTAS) under performance or energy consumption constraints,” IEEE Trans. Wireless Commun., vol. 7, pp. 3168–1173, Apr. 2008.
[7] D. S. Michalopoulos and G. K. Karagiannidis, “Performance analysis of single-relay selection in Rayleigh fading,” IEEE Trans. Wireless Commun., vol. 7, pp. 3718–3724, Oct. 2008.
[8] S. Ikki and M. H. Ahmed, “Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection,” IEEE Trans. Commun., vol. 58, pp. 68–72, Jan. 2010.
[9] ——, “Performance of multiple-relay cooperative diversity systems with best relay selection over Rayleigh fading channels,” EURASIP Journal on Advances in Signal Processing, vol. 2008, pp. Article ID 580 368, 7 pages, 2008. doi:10.1155/2008/580368.
[10] B. Xia, Y. Fan, J. Thompson, and H. V. Poor, “Buffering in a threeneode relay network,” IEEE Trans. Wireless Commun., vol. 7, no. 11, pp. 4492–4496, Nov. 2008.
[11] N. Zlatanov, R. Schober, and P. Popovski, “Throughput and diversity gain of buffer-aided relaying,” Submitted to IEEE Globecom, Mar. 2011.
[12] M. K. Simon and M.-S. Alouini, Digital Communication over Fading Channels: A Unified Approach to Performance Analysis. NY: John Wiley & Sons, 2000.
[13] V. Tarokh, N. Seshadri, and A. R. Calderbank, “Space-time codes for high data rate wireless communication: performance criterion and code construction,” IEEE Trans. Inform. Theory, vol. 44, pp. 744–765, Mar. 1998.
[14] O. C. Ibe, Markov Processes for Stochastic Modeling. Elsevier, 2008.