On the stability of black hole event horizons

Bjørn Jensen *

Institute of Physics, University of Oslo, P.O. Box 1048, N-0316 Blindern, Oslo 3, Norway

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Abstract

In this work we study a gedanken experiment constructed in order to test the cosmic censorship hypothesis and the second law of black hole thermodynamics. Matter with a negative gravitating energy is imagined added to a near extremal $U(1)$-charged static black hole in Einstein-Maxwell theory. The dynamics of a similar process is studied and the thermo-dynamical properties of the resulting black hole structure is discussed. A new mechanism which stabilizes black hole event horizons is shown to operate in such processes.

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*Electronic address: BJensen@boson.uio.no
I. INTRODUCTION

In classical general relativity theory strong curvature singularities, i.e. regions of space-time through which no geodesic can be extended, are predicted to appear in space-time. Many believe that such singularities are always hidden behind event-horizons. A cosmic censor will always hide the singular region in such a way as to make the space-time maximally predictable, it is believed. This is the essence of a collection of proposals collectively known as the cosmic censor-hypothesis (for a review see eg. [1]). However, even though general relativity predicts the existence of singular regions accompanied by event-horizons when certain restrictions on the properties of the matter content of the world is made a proof of the stability of black hole event horizons does not exist (see eg. [2]). Even though a singular region is hidden behind an event-horizon initially it is unclear whether the horizon can be removed or not in the future by a physical process so that the singular region is exposed.

Classically no process will shrink the area of a single black hole in an open universe provided that the strong energy condition [3] holds. This is connected with the one way nature of the black hole horizon; nothing can escape from a black hole classically. When quantum effects are taken into account it is well known that black hole horizons are destabilized, i.e. the surface area of the horizon will decrease due to the emission of quantum particles. The decrease of the surface area can also be looked upon as due to the absorption by the black hole of negative energy from the quantum vacuum. However, all computations of the Hawking process relies on the validity of the semi-classical approximation to quantum gravity and at the last stages of the evaporation process this approximation generally breaks down. Hence, what sorts of structures that will remain after the Hawking evaporation process has come to a halt, i.e. whether only radiation remains or a naked singularity or a massive remnant is left behind, is completely unclear.

One of the curious properties of general relativity theory is that it assigns an absolute thermo-dynamic entropy to black holes in terms of the area of the black hole horizon.
Hence, the cosmic censor-ship hypothesis is thus seen to be linked to well known classical thermo-dynamics. The one-way nature of the event horizon on the classical level is in this way connected to the statement that $d(\text{Entropy}) \geq 0$. The Hawking evaporation process violates this relation since it makes the horizon area shrink and hence the associated entropy decrease. However a generalized second law can be formulated in such a way that the entropy loss can be found in the resulting entropy increase in the radiation field outside the black hole.

In this work we study a gedanken experiment devised with the violation of the cosmic censor-ship and the second law of black hole thermo-dynamics in mind. We will focus the attention to a near extremal Reisner-Nordström black hole (i.e. black hole charge $\sim$ black hole metric mass) where thermal quantum effects are highly suppressed. We will be concerned with the situation when macroscopic entities carrying negative gravitating energy (in violation of the strong energy condition) are thrown into the extremal hole. Naively one should expect that the mass of the resulting structure could become less than its charge (in gravitational units $c = \hbar = k_B = G = 1$). When this happens the structure will display a naked singularity.

The structure of this paper is as follows. In the next section we present the gedanken experiment. In section III we discuss some properties of negative mass carrying objects which generate a spherical symmetric geometry and the actual innfall of such an object into a black hole. In section IV we study the consequences of this innfall in relation to the cosmic censor-ship hypothesis and the second law. We conclude with a section where we summarize our findings.

II. A GEDANKEN EXPERIMENT

Consider a near extremal Reisner-Nordström black hole, i.e. an electrically $Q$ charged and metric mass $M$ carrying static black hole in Einstein-Maxwell theory where the equality in
\[ Q^2 \leq M^2 \quad (1) \]

is nearly attained. In a recent work \[4\] an attempt was made to break this relation by adding another \( U(1) \) kind of charge \( q \) to the black hole in such a way that \( q^2 + Q^2 > M^2 \). Different kinds of processes where studied with both classical and quantum mechanical \( q \)-charge carrying entities. It was concluded that \( q^2 + Q^2 \leq M^2 \) probably always holds in such processes due both to classical effects and due to the charge screening effect of the quantum-vacuum. In this work another way to try to break the relation eq.(1) is investigated. We will not try to add more charge of some kind but to try to reduce to mass term in eq.(1). In the Hawking process this is exactly what is done but when the black hole parameters reach a region of the parameter space where equality in eq.(1) is about to be attained the Hawking process shuts down, i.e. the temperature drops to zero. Hence, in this case the Hawking process obeys the cosmic censorship hypothesis. This is not necessarily a general feature of the Hawking radiation process since it is known that electrically charged black holes in the low energy limit of the heterotic string theory may display a naked singularity when it radiates down to extremality \[5\].

One important principle often assumed in the classical singularity theorems in the general theory of relativity is that the strong energy condition is to be satisfied \[3\], i.e. the gravitating mass density \( \rho_G \) satisfies

\[ \rho_G \equiv -T^\hat{\imath}_\hat{\imath} + T^{\hat{i}\hat{i}} \geq 0 \quad (2) \]

relative to an arbitrary tetrad (or null-frame). Here \( T \) denotes the matter energy-momentum tensor \( \hat{\imath} \) the timelike indice and \( \hat{i} \neq \hat{\imath} \). Two initially parallel geodesics will always converge when passing trough matter that obey this “timelike convergence condition” and as a consequence curvature singularities must appear in the theory. However the strong energy condition is probably a to restrictive condition to put on matter in order to classify a specific chunk of matter as physically acceptable. Probably the larger part of the total energy density of the universe is contributed from the quantum vacuum where the pressures in the
three space-like dimensions equals minus a small positive vacuum energy density $\rho$. It follows that the gravitating mass density contribution of the vacuum is $\rho_G = -2\rho$ in violation of eq.(2).

In our gedanken experiment we will specifically imagine that an object of finite spatial extend and with an energy-momentum tensor with the above characteristics of the vacuum is lowered into an extremal Reisner-Nordström black hole. Such objects was probably produced copiously in phase-transitions in the early universe in the form of topological defects [6]. We will briefly review the most important properties of some of these objects in the next section. We expect that adding such ultra-relativistic matter to a specifc system will imply that the resulting gravitating mass of the system will be lower than the mass of the system before the negative energy where added. Is it possible to add sufficient negative mass such that the relation in eq.(1) is violated?  

III. OBJECTS WITH NEGATIVE MASS

In pure Einstein theory it has been proved that the total energy (the ADM mass) carried by an isolated system, i.e. one that generates an asymptotic Minkowski geometry, is non-negative [9]. Objects with a negative mass is therefore expected to generate a curved asymptotic structure. Simple spherically symmetric entities with this property is topological defects in field theories where global symmetries are broken. Examples of such defects are the global monopoles arising in a $\phi^4$-theory with an internal $O(3)$ global symmetry [10] and skyrmions which arise in a non-linear $\sigma$-model with a $SO(4)$ symmetry (see eg. [11]). In the asymptotic region $r \to \infty$ far from the core-region of these defects we find that the

\footnote{In [13] energy which breaks the weak energy condition was imagined injected into a near extremal Reisner-Nordström black hole. It was concluded that quantum field theory prevents an unambiguous violation of cosmic censorship in such a process when the averaged weak energy condition is assumed to hold.}
geometric structures become

\[ ds^2 = -dt^2 + dr^2 + \alpha^2 r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \] (3)

where \( \theta \) and \( \phi \) is the usual polar angles. It is clear that this metric represents a curved space since it can not be transformed to the Minkowski metric in polar coordinates. The Ricci curvature scalar \( R \) is readily computed

\[ R = \frac{(1 - \alpha)(1 + \alpha)}{\alpha^2 r^2}. \] (4)

From this expression it seems that the intrinsic curvature vanishes in the limit \( r \to \infty \) [11]. However this is not correct since the expression for \( R \) refers to the behavior of the scalar curvature relative to a specified point. Any other point in the asymptotic region will also locally be endowed with the metric eq.(3) and an \( R \) on the form eq.(4) will be measured. In fact a \( \alpha \neq 1 \) will induce an effective curvature to the universe [12].

As was mentioned above since the asymptotic regions of the above sources are curved it follows that the total gravitating mass of these objects can be expected to be negative. Indeed, in [13] a model of the interior of a global monopole where presented which was made up of a part of the de-Sitter manifold. This interior region could be matched smoothly to a spherical symmetric geometry on the Schwartshild form but only with a deficit cone in the geometry. This of-course gives rise to a negative mass object since the mass density \( \rho > 0 \) combined with local Lorentz invariance of the source in the interior region implies a negative gravitating mass as explained above [12]. It is worth noting that the “vacuum” outside the global monopole can be modeled by an energy-momentum tensor with vanishing angular components but where \( T^t_t = T^r_r \neq 0 \) [10]. This Lorentz invariance in the radial direction is directly linked to the existence of the deficit cone. The gravitational mass density \( \rho_G \) of the field outside the core-region vanishes due to this invariance property.

The study of the innfall of a particle with a deficit cone into a black hole is very complicated. The process can not be treated in a consistent manner in the test-particle approximation since we never can neglect the effects produced by the deficit cone. We will therefore
Consider a related scenario where a spherically symmetric domain wall $\Sigma$ with a black hole in the center and with a deficit cone in the exterior region collapses into the black hole. The very definition of a domain wall implies that such structures will carry a negative gravitational energy. Even though these processes may seem unrelated the net effects produced in the two scenarios on the hole is expected to be the same since the resulting black hole will not carry any information on the details of the infall process.

We will first study the properties of a static spherically symmetric domain wall with and without a black hole in the center of the spherical geometry. To this end we will utilize the Israel formalism. The energy-stress tensor $S^i{}_{j}$ in $\Sigma$ is determined by Israel’s equations \[ \gamma^i{}_{j} - \delta^i{}_{j} \text{Tr} \gamma = -8\pi G S^i{}_{j} \] where $\gamma^i{}_{j} = \Delta K^i{}_{j}$ denotes the jump of the extrinsic curvatures $K^i{}_{j}$ across $\Sigma$. We will assume that the geometric structure in the region between the black hole and $\Sigma$ (region I) is described by the metric

$$ds_1^2 = -(1 - \frac{2M_1}{r})dt^2 + (1 - \frac{2M_1}{r})^{-1}dr^2 + r^2d\Omega_2^2$$

(6)

where $d\Omega_2^2$ denotes an infinitesimal interval on the unit sphere. The geometric structure between $\Sigma$ and space-like infinity (region II) is taken to be

$$ds_2^2 = -(1 - \frac{2M_2}{R})dT^2 + (1 - \frac{2M_2}{R})^{-1}dR^2 + \alpha^2 R^2d\Omega_2^2.$$  

(7)

In the expression for $ds_2^2$ the angular part of the metric is enhanced by a multiplication by a constant $\alpha$ which is always assumed less that unity. The extrinsic curvatures will be computed relative to a unit space-like vector normal to $\Sigma$ and one which points in the direction of increasing radial coordinate.

Let $\Sigma$ be positioned at a fixed radial distance from the black hole event horizon. Seen from region I we have $\vec{N}^{(1)} = (g_{rr})^{-1}\partial_r$ and from region II $\vec{N}^{(2)} = (g_{RR})^{-1}\partial_R$. The fixed position of the wall outside the event horizon is at $r = r_0$ and $R = R_0$ relative to the inner and outer geometries respectively. We also demand that the time-like and the angular
coordinates coincide on the domain-wall. From the continuity of the gravitational potentials across $\Sigma$ (except the radial one) we have

$$R_0 = \frac{1}{\alpha} r_0\quad (8)$$

$$2r_0(\alpha M_2 - M_1) = 0.\quad (9)$$

It then follows that

$$\gamma^t_t = -\frac{1}{r_0^2} (1 - \frac{2M_1}{r_0})^{1/2} (\alpha^2 M_2 - M_1)$$

$$\gamma^\theta_\theta = \frac{(1 - \alpha)}{r_0} (1 - \frac{2M_1}{r_0})^{1/2} = \gamma^\phi_\phi.\quad (10)$$

From these expressions we see that the angular components of the extrinsic curvatures will vanish when $\alpha = 1$. This means that the pressures in the angular directions in $\Sigma$ also will vanish identically in this limit. In this limit only the energy density will be non-vanishing when $M_1 \neq M_2$. $\Sigma$ can in that case be interpreted as a wall composed of pressure less dust. Relative to a physical observer we will assume that the observed energy density in $\Sigma$ is positive, $S^t_t > 0$ ($\gamma^\theta_\theta > 0$), i.e.

$$1 - \alpha^2 > \frac{2}{r_0^2} (M_1 - \alpha M_2).\quad (12)$$

Combined with the relations which stem from the continuity of the metric above this is easily seen to be generally satisfied provided $\alpha^2 < 1$. Since $\Sigma$ is assumed to be a domain-wall it follows that we must have $S^t_i = S^\theta_\theta = S^\phi_\phi = \sigma = \text{const.} < 0$. This implies in particular that $\gamma^t_t = \gamma^\theta_\theta = \gamma^\phi_\phi$.

We define in general the gravitational mass $M_G$ measure along the following lines. Let $\xi^a$ be a time translation Killing vector field which is time-like near infinity such that $\xi^a \xi_a = -1$ and which has vanishing norm on the event-horizon. Let $\vec{N}$ be a second Killing vector field orthogonal to the event-horizon with normalization such that $N^a N_a = 1$ near infinity. Let $S$ denote the region outside the event-horizon of the black hole. The boundary $\partial S$ of $S$ is taken to be the event-horizon $\partial B$ and a two-surface $\partial S_\infty$ at infinity. It then follows that a
natural measure on the gravitational mass $M_G$ inside $\partial S_\infty$ as measured by a static observer at infinity can be deduced from [13,[12]

$$\int_{\partial B + \partial S_\infty} \xi^{ab} d\Sigma_{ab} = - \int_S R^a_{\ b} \xi^b d\Sigma_a. \quad (13)$$

$R^a_{\ b}$ denotes the Ricci tensor. The integral over $\partial B$ is the surface gravity $\kappa = N_b \xi^a \nabla_a \xi^b$ multiplied with the surface area $A$ of $\partial B$. By the use of this measure it follows that the total gravitating mass $M_G$ of the combined black hole domain wall system relative to a time-like Killing observer $\vec{\xi} = \partial T$ at infinity is given by $M_G = M_\Sigma + M_H = \alpha^2 M_2 = \alpha M_1$ where $M_\Sigma$ is the total gravitating mass of $\Sigma$ and $M_H$ is the mass of the black hole [12]. We have specifically that

$$M_\Sigma = -8\pi \gamma \theta \theta \quad (14)$$

$$M_H = M_1. \quad (15)$$

Hence, $M_G < M_H$. The mass of the domain wall is negative due to the large negative pressures in the angular directions. From these expressions we can obtain an expression for $\alpha$ in terms of the mass of the black hole and the mass of $\Sigma$

$$\alpha = \frac{M_1 - |M_\Sigma|}{M_1}. \quad (16)$$

We note that $\alpha$ vanishes when $M_H = |M_\Sigma|$. This implies that the exterior region outside of $\Sigma$ becomes singular and we will correspondingly confine our attention to the case when the mass of the hole is less than the absolute value of the mass of the domain wall.

We now derive the equation of motion for a shell collapsing into a black hole positioned at the center of the wall. Earlier in this investigation the condition imposed on $\Sigma$ in order for this structure to describe a domain wall was performed relative to a system at rest relative to the wall. In the same vein we therefore introduce Gaussian normal coordinates near every point on $\Sigma$. The timelike coordinate in this system will be the proper time $\tau$ of an observer on the shell while the angular coordinates are inherited from the surrounding space. Hence on $\Sigma$ we have coordinates $(\tau, \theta, \phi)$. In addition we need a radial coordinate
\( \chi \) which measures the proper length from the observer on the wall and to a point in the vicinity of \( \Sigma \). Let \( \vec{N} \) denote the unit space-like vector orthogonal to the collapsing shell \( \Sigma \) and \( \vec{U} \) the four-velocity of a mass element of this surface. The orthogonality condition translates into \( \vec{N} \cdot \vec{U} = 0 \). The four-velocity in region I relative to the coordinatization eq.(6) is taken as \( \vec{U} = i \partial_t + \dot{r} \partial_r \) where \( \cdot \) denotes differentiation with respect to the affine parameter \( \tau \). From the orthogonality condition it then follows that \( \vec{N} = (|g_{tt}|)^{-1} \dot{t} \partial_t + |g_{tt}| \dot{r} \partial_r \) such that \( \vec{N} \cdot \vec{N} = +1 \) when we assume that \( \dot{t} > 0 \). The normalization of the four-velocity, \( \vec{U} \cdot \vec{U} = -1 \), implies

\[
g_{tt} \dot{t} = \pm \sqrt{|g_{tt}| + \dot{r}^2}. \tag{17}
\]

The extrinsic curvatures relative to the Gaussian normal coordinates are simply \( K_{\tau \tau} = N_{\tau \tau} = U^\mu U^\nu N_{\mu \nu} \) and \( K_{ij} = N_{ij} \) \((i, j = \phi, \theta)\). The extrinsic curvature tensor is diagonal such that the \( \tau \tau \) component does not couple to the angular components. However, they are related since energy-momentum is conserved. In deriving the equation of motion it is enough to consider the angular components of Israels equations and we only need to compute the angular components of the extrinsic curvature tensor. The \( \tau \tau \) equation will express the nature of the forces acting on the wall. These are not of interest in our investigation. The gravitational potentials are assumed continuous across the moving wall. Relative to region I we then have

\[
K_{\theta \theta} = \frac{1}{2} |g_{tt}| g_{\theta \theta, \tau} \dot{t} \tag{18}
\]
\[
K_{\phi \phi} = \sin^2 \theta K_{\theta \theta} \tag{19}
\]

Similar expressions are derived in region II expressed in terms of \( T \) and \( R \). The jump \( \gamma_{i,j} \) in the extrinsic curvatures defined by \( \gamma_{i,j} = K_{i,j}^{(II)} - K_{i,j}^{(I)} \) is

\[
\gamma^\theta_\theta = \gamma^\phi_\phi = \frac{\alpha - 1}{r} |g_{tt}| \dot{t} \tag{20}
\]
\[
\text{Tr} \gamma_{ij} = 3 \gamma^\theta_\theta \tag{21}
\]

where the last equality follows since \( \Sigma \) models a domain wall. The \( \theta \theta \)-equation combined with eq.(17) then leads to
\[ \pm \sqrt{|g_{tt}| + r^2} = \kappa r \quad (22) \]

where we have defined

\[ \kappa \equiv \frac{4\pi G\sigma}{(\alpha - 1)} > 0. \quad (23) \]

Equation (22) will be our principal equation of motion. It is useful to define a new radial coordinate by \( z \equiv \kappa r \). We also define \( E \equiv -\kappa^2 \) and

\[ V(z) = -\kappa^2(z^2 + \frac{2M_1\kappa}{z}). \quad (24) \]

The squared of our principal equation of motion then takes the form

\[ \left( \frac{dz}{d\tau} \right)^2 + V(z) = E. \quad (25) \]

Let us first consider the case when there is no black hole present. The potential then reduces to \( V = -\kappa^2 z^2 \). Since \( E \) is always negative and non-zero it follows that \( \Sigma \) always will collapse to a certain minimum radius and then expand. We can understand this behaviour as arising due to repulsive self-forces generated by the large negative pressures. The behaviour of the collapsing shell is somewhat more complicated when a black hole is present. The position of the extremum of the potential function is at

\[ z_{\text{extr}} = \left( \frac{\kappa M_1}{\alpha - 1} \right)^{1/3} \quad (26) \]

where the potential attains its maximal value

\[ V_{\text{extr}} = V(z_{\text{extr}}) = -\frac{3M_1\kappa^3}{(M_1\kappa)^{1/3}}. \quad (27) \]

From these equations it is clear that as long as \( z_{\text{extr}} \) is positioned behind the event horizon of the original black hole the domain wall will collapse into the singularity of the hole. It is also clear that there always exist a class of domain walls for which this holds irrespective of the black hole parameters. An explicit computation reveals that this also holds true in the case of a Reisner-Nordström black hole.
IV. THE COSMIC CENSOR AND THE SECOND LAW

We have seen that provided the black hole mass is large enough compared with the mass of the infalling shell it will be absorbed by the hole and become part of its singularity structure. From our usual experience with black holes which absorbs matter with positive gravitating mass we know that both the pull on test-particles in the vicinity of the event-horizon increases as well as the surface-area of the event-horizon. When a black hole absorbs objects with negative mass we would naively expect that just the opposite processes would take place. Indeed, we would expect that the surface area of the event-horizon where reduced since this quantity is proportional to \((M_G)^2\) in the usual Schwartzshild solution. The reduced mass is also similarly expected to reduce the invariant acceleration felt by a static observer just outside the horizon. Also, consider the Hawking radiation process. In this process we assume that the hole radiates positive energy to infinity which imply that after a sufficient amount of time the amount of the original matter “content” of the singularity which contributed positively to \(M_G\) approaches equality to the negative part of the singularity which supports the deficit cone. Hence in a certain limit we would expect that the gravitating masses of the original singularity pluss the mass of the structure which supports the deficit cone would add up to zero. The singularity would then display an effective vanishing gravitating mass which implies that the temperature of the black hole will diverge to plus infinity. In this limit enough energy will be available to excite the monopole or skyrmion field such that an anti-particle will be created and hence annihilate with the monopole (skyrmion) core of the black hole.

These considerations can be tested in a rather straightforward way. Let us first consider the change of the entropy during a capture process. Before the hole captures the negative mass carrying object the entropy is \(S = 4\pi M_1^2\) and after the capture \(S \equiv S_\alpha = 4\pi \alpha^{-2}(M_G)^2\) [12]. From the matching conditions of the metric coefficients it follows that \(M_G = \alpha^2 M_2 = \ldots\)
\[ \alpha M_1 \] which imply that \( dS \equiv S_\alpha - S = 0 \). The temperature of the hole is \( T \sim M_2^{-1} = \alpha M_4^{-1} < M_1^{-1} \). Hence, when the black hole swallows the object its temperature apparently decreases, i.e. \( dT < 0 \). The consequences of the Hawking process can also be found quite easily along similar lines. We have that \( T \sim M_2^{-1} = \alpha^2 (M_G)^{-1} = \alpha^2 (M_H - |M_\Sigma|)^{-1} \) \( (M_H > |M_\Sigma|) \). In the Hawking process we assume that \( dM_H < 0 \) while \( M_\Sigma \) and \( \alpha \) remains approximately constant for a sufficiently large hole. It then follows trivially that \( dT > 0 \) and \( dS < 0 \) which conforms with the usual result. It is interesting to observe that if the possibility \( M_H < |M_\Sigma| \) where realizable we will have a negative \( M_G \) which would give rise to a negative temperature. If we allow \( \alpha \) also to be time dependent in the radiation process then \( T \) should be written as \( T \sim (M_H - |M_\Sigma|) \) which means that the temperature would drop to zero when the two mass terms coincide. However it is probably not correct to reason along these lines since the relation eq.(16) is derived on the assumption that the wall is static.

V. CONCLUSION AND FINAL REMARKS

In this work we have studied a gedanken experiment constructed specifically in order to violate the cosmic censorship hypothesis and the second law of black hole thermo-dynamics. A near extremal Reisner-Nordström black hole is at the brink of exposing a naked singularity. Either by adding more charge of some kind or some object with negative mass one would naively expect the cosmic censorship to be broken. In [4] it was shown that adding more charge probably will not make the hole expose a singularity. In this work we have shown that the area of the event horizon of a Schwartzshild black hole remains the same when the hole captures a negative mass object with a conic deficit angle. This result can be carried directly over to the Reisner-Nordström black hole. Hence, the cosmic censorship hypothesis is not violated when a near extremal black hole captures negative mass objects.

\[ ^2 \]This result is in sharp contrast to the result announced in [16] where it was argued that the entropy of the black hole would increase due to the capture process.
of the kind considered in this paper. It also follows that the second law of thermo-dynamics is not violated either. The effect of the capture process was apparently to cool down the black hole while the entropy of the black hole remained constant, i.e. \(dT < 0, dS = 0\). These considerations does not prove that the censor-ship hypothesis and the second law holds in any process where a negative mass object is captured by a black hole but we have managed to reveal a new mechanism which must be taken into account when considering such processes. Our treatment has so far been entirely classical. The possibility is open for the occurrence of new processes at the quantum level which may modify our results somewhat. In particular the decrease of the temperature of the black hole will in a quantum treatment probably find its explanation in the fact that the assymptotic region is curved. Assume that the negative mass object is initially sufficiently far from the black hole such that the line-element eq.(3) describes the geometry also at a large distance from the black hole with \(\alpha = 1\). In such a situation one would second quantize in the asymptotic region with respect to the Killing vector \(\vec{\xi} = \partial_t\). After the infall of the object this vector is transformed into \(\vec{\eta} = \alpha \partial_t\). Hence, for a positive frequency mode \(u_k(x)\), i.e. such that \(\mathcal{L}_{\vec{\xi}} u_k = i\omega u_k (\omega > 0)\), we have \(\mathcal{L}_{\vec{\eta}} u_k = \alpha \mathcal{L}_{\vec{\xi}} u_k\) which implies an effective red-shift of the modes in the “out”-vacuum compared to the modes in the “in”-vacuum. This means that the vacuum energy is lowered relative to the initial state of the vacuum something which implies a production of particles out of the quantum vacuum \(^3\). Note that the frequencies are related by a hole multiplum of \(\alpha\) the very same relation that exist between the temperature of the hole before and after the innfall of the negative mass object. We plan to come back to this issue somewhere else.

VI. ACKNOWLEDGMENTS

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\(^3\)In [17][18] the production of scalar particles from the associated vacuum due to the formation of a single global monopole was computed.
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