Nonlinear Algebraic Reynolds Stress Model for Two-Phase Turbulent Flows Laden with Small Heavy Particles in Circular Tube

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Abstract. The purpose of the study is to present an explicit algebraic Reynolds stress (nonlinear turbulent viscosity) model combined with modified $k-\varepsilon$ turbulence model taking into account particles effect on turbulence for calculating the main turbulent characteristics of two-phase flows. For calculating particles distribution in space we used diffusion-inertia model (DIM). The turbulence attenuating in the presence of particles is clearly observed, investigated and compared with the experimental data. The developed model adequately described turbulence anisotropy and the influence of particles inertia and concentration on the turbulence intensity.

1. Introduction

A great number of models intended for describing the turbulent characteristics of a fluid in the presence of particles can be found in literature. The most widespread model is the linear isotropic model of turbulent viscosity in combination with the equations for the kinetic turbulent energy and the rate of its dissipation. Thanks to its simplicity, this model is widely used in calculating two-phase flows of different types (see, for example, [1, 2, 3, 4, 5, 6, 7, 8]). However, in spite of its practical utility, the linear model has some well-known shortcomings. Thus, the model does not describe the turbulent stress anisotropy, as well as secondary flows in noncircular channels, and, moreover, leads to considerable errors in modelling strongly nonequilibrium flows with high velocity gradients due to sharp flow expansion or compression, strongly curved flows, etc. Rigorously speaking, the linear model of turbulent viscosity can be applicable only in calculating quasi-equilibrium flows characterized by an approximate balance between the generation and dissipation of the turbulence energy.

A more accurate and detailed modelling of the turbulent fluid characteristics can be based on the solution of the complete system of the Reynolds stress transport equation. This approach was realized in [9]; in this case, the empirical constants in the second moments equations are determined as a result of the calibration with the DNS data for a homogeneous shear flow. The transport equations for the Reynolds stress components of the continuous phase were also used in [9, 10, 11] in modelling axisymmetric jet and channel gas-disperse turbulent flows. However, modelling of complicated three-dimensional flows on the basis of the system of differential
equations for all components of the turbulent stresses leads to a considerable increase in the computation time, as compared with the use of two-equation turbulence models of the $k - \varepsilon$ type. For this reason, for calculating single-phase flows the so-called nonlinear explicit algebraic Reynolds stress models (models of nonlinear turbulent viscosity) have received wide acceptance. These models, which possess almost the same accuracy as the differential models for the second moments of velocity fluctuations, make it possible to reduce considerably the computation time, increasing at the same time the numerical scheme stability.

In [12] turbulent flows and heat transfer in tubes were calculated using a very simple nonlinear model of turbulent viscosity with constant coefficients, without particles [13]. The original nonlinear model [14] is based on the theory of invariants and takes direct account of the particle effect. However, this model is not completely explicit and needs applying an iteration procedure to determine the turbulent energy production-to-dissipation ratio. As noted in [15], this iteration procedure considerably reduces the computational effectiveness of the algebraic model, since it can lead to nonuniqueness of solutions and, as a consequence, to the convergence to a non-physical solution. In [15, 16] a completely explicit, self-consistent algebraic Reynolds stress model for single-phase turbulence, which does not need an iteration procedure, was proposed. In spite of the fact that this model is rigorously valid only for two-dimensional homogeneous mean flows, this approach has gained a wide acceptance in modelling inhomogeneous and three-dimensional single-phase flows (see, for example, [17, 18]). In this study, a fully explicit, algebraic model for describing the Reynolds stresses of a fluid in the presence of small particles of the disperse phase is used.

The nature of the particles effect on the turbulent flow structure is not unique, small particles can attenuate fluid turbulence, while large particles can augment it, depending on their inertia and size. Thus, the presence of relatively small particles results in additional dissipation and an attenuation of turbulent fluctuations, owing to their decelerating (damping) action caused by their incomplete entrainment into the fluctuating fluid flow. With increase in the particle inertia the additional dissipation due to the fluctuating slip between the phases reduces and becomes unimportant for large particles. Apparently, it is the formation of an unsteady vortex structure (wake) due to flow separation behind a large particle in the flow that should be regarded as the main mechanism of the turbulence generation under the back action of the dispersed phase. Moreover, the turbulent characteristics of the continuous phase can considerably be influenced by the diffusive turbulent particle transport due to non-uniform dispersed phase distribution in space.

At the present time the vast amount of experimental data was gathered on the characteristics of particle-laden turbulent flows in channels, tubes, boundary layers etc. However there are always lots of factors in the experiment that make the flow more complicated, so that it's rather difficult to pick out the information about particular mechanisms of particles effect on turbulence. The impact of small heavy particles on the turbulent air flow in a vertical tube was studied in [19]. In that experiment the mean velocity profile was not distorted in the presence of particles, and no unsteady vortex structure (turbulent trace) due to flow separation behind the particle was formed. Thus the main way of the particles influence on turbulence is the damping effect related to incomplete entrainment of particles into the turbulent motion of the fluid, which causes additional dissipation and decrease in turbulent intensity. Taking into account the circumstances mentioned above, the experimental study performed in [19] is of a great value for validating theoretical models concerning the effect of small heavy particles on the turbulence intensity of the flow. In [20] fully explicit, algebraic model for describing the Reynolds stress of a flow in the presence of small particles of the dispersed phase is presented and validated for homogeneous shear flow. The main purpose of the present paper is to extend results of [20] to turbulent flow laden with small heavy particles in vertical tube and combine with DIM presented in our previous papers [21].
2. Mathematical formulation

2.1. Equations of the Turbulent Fluid

We consider the motion of a two-phase disperse medium consisting of an incompressible viscous fluid and small heavy particles. The volume concentration of the dispersed phase \( \Phi \) is assumed to be sufficiently small so that inter-particle collisions can be neglected; however, the mass concentration \( \hat{M} \equiv \rho_p \Phi / \rho_f \) can be fairly large. The particle density \( \rho_p \) is assumed to be much greater than the fluid density \( \rho_f \), while their diameter, \( d_p \), is not greater than Kolmogorov length scale \( \eta \). In this case, the equations for the continuous and disperse phases can be represented in the point-force approximation applied to the mass centres of individual particles. Moreover, the particle behaviour in a turbulent fluid and their effect on the flow is chiefly determined by the hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected. In this paper for taking into account particle effect on hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected. In this paper for taking into account particle effect on hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected. In this paper for taking into account particle effect on hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected. In this paper for taking into account particle effect on hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected. In this paper for taking into account particle effect on hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected. In this paper for taking into account particle effect on hydrodynamic drag, while the forces due to associated and displaced masses and the memory effect (Basset force) can be neglected.

In order to construct the fully explicit algebraic Reynolds stress model the following self-consistent method [15, 16] can be used, which applies the three tensor basis for Reynolds stress expression in the weak-equilibrium assumption. As a result of this approach being utilized to the Reynolds stress equation for turbulent gas laden by small heavy particles [25] we obtain:

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho_f} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu_f \frac{\partial U_i}{\partial x_j} - \langle u_i'u_j' \rangle \right) + A_i, \tag{1}
\]

where \( t \) is the time, \( x_i \) is the spatial coordinate, \( U_i \) is an average flow velocity, \( P \) is the pressure, \( \nu_f \) is the kinematic viscosity coefficient, \( \langle u_i'u_j' \rangle \) is the Reynolds stress tensor, \( \tau_p \) is the particle relaxation time, \( U_{pi} \) is the mean fluid velocity determined on a particle trajectory (the so-called fluid velocity seen by the particle), \( V_i \) is the mean velocity of the disperse phase.

Terms \( A_i \) and \( \langle u_i' \rangle_p \) in (1) are describing the particle action on the turbulent flow and the drift velocity between the fluid and the particles due to inhomogeneity of the disperse phase distribution [3]. Following [20] these terms written as follows:

\[
A_i = \frac{\rho_p}{2 \rho_f \tau_p} \int \langle (v_i - u_i)p \rangle d\mathbf{v} = \frac{M}{\tau_p} (V_i - U_{pi}),
\]

\[
\langle u_i' \rangle_p = \frac{1}{\Phi} \int \langle u_i'p \rangle d\mathbf{v} = -\tau_p \frac{\partial \ln \Phi}{\partial x_i},
\]

where \( \Phi \) signifies the dynamic probability density of the particle velocity distribution in the phase space of the particle coordinates \( \mathbf{x} \) and velocities \( \mathbf{v} \).

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\[
\langle u_i'u_j' \rangle = \frac{2k}{3} \delta_{ij} - 2C_\mu^* \frac{k^2}{\varepsilon} \left\{ S_{ij} - \frac{k}{\varepsilon} \left[ B_1 \left( S_{ik}^* S_{jk}^* - \frac{1}{3} S_{kn}^* S_{kn}^* \delta_{ij} \right) + B_2 \left( S_{ik}^* W_{jk}^* + S_{jk}^* W_{ik}^* \right) \right] \right\}, \tag{2}
\]

\[
C_\mu^* = \frac{3A_1 A_2}{3A_1^2 - 2A_3 A_2 S_{ll} - 6A_3^2 W_{ll}}, \quad B_1 = 2A_3 / A_1, \quad B_2 = A_4 / A_1, \quad S_{ij} = (1 + M f_{u1}) S_{ij}, \quad W_{ij} = (1 + M f_{u1}) W_{ij},
\]

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \quad W_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right).
\]
where $S_{ij}$ and $W_{ij}$ is a strain and rotation rate tensors for the fluid and $S_{ij}^*$ and $W_{ij}^*$ for the two-phase flow.

Together with linear by strain rate tensor $S_{ij}$ term the Reynolds stress expression (2) includes also quadratic terms on the velocity deformations and rotation. Unlike the standard linear $k - \varepsilon$ model of turbulence, here $C_\mu^*$ is not a constant but depends on the strain and deformation tensor invariants for two-phase flow $\bar{S}_{II}$ and $\bar{W}_{II}$. For $A_1$ coefficient the following cubic equation is obtained:

$$A_1^3 - (C_0^0 - 2) A_1^2 - \left\{ \left[ 2A_2 (C_1^1 + 2) + \frac{2A_3^2}{3} \right] \bar{S}_{II} + 2A_3^2 \bar{W}_{II} \right\} A_1 +$$

$$+ 2 (C_1^1 - 2) \left( \frac{A_3^2 \bar{S}_{II}^*}{3} + A_3^2 \bar{W}_{II}^* \right) = 0,$$

which has a solution:

$$A_{1(1)} = M + N - \frac{a}{3}, A_{1(2,3)} = -\frac{M + N}{2} - \frac{a}{3} \pm \frac{i \sqrt{3}(M - N)}{2},$$

where

$$M = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}}, \quad N = \sqrt[3]{-\frac{q}{2} - \sqrt{Q}}, \quad Q = \left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2, \quad p = -\frac{a^2}{3} + b,$$

$$a = -(C_0^0 - 2), \quad b = -\left\{ \left[ 2A_2 (C_1^1 + 2) + \frac{2A_3^2}{3} \right] \bar{S}_{II} + 2A_3^2 \bar{W}_{II} \right\},$$

$$c = 2 (C_1^1 - 2) \left( \frac{A_3^2 \bar{S}_{II}^*}{3} + A_3^2 \bar{W}_{II}^* \right), \quad q = 2 \left( \frac{a}{3} \right)^3 - \frac{ab}{3} + c.$$

If all the three roots (3) are real, then the largest root must be chosen [26]. Other coefficients in (2) equal:

$$A_2 = \frac{4}{3} - C_2, \quad A_3 = 2 - C_3, \quad A_4 = 2 - C_4,$$

and according to [27] constants have the following values:

$$C_0^0 = 3.4, \quad C_1^1 = 1.8, \quad C_2 = 0.36, \quad C_3 = 1.25, \quad C_4 = 0.4.$$

Compared to the original model [15, 16] the explicit algebraic Reynolds stress model (2) together with (3) contains the strain and deformation rate tensors and their invariants for two-phase flow instead of single-phase ones. The kinetic turbulent energy and its dissipation rate included in (2) can be calculated based on the $k - \varepsilon$ turbulence model for two-phase flows with small heavy particles [25]:

$$\begin{align*}
(1 + M f_{u_1}) \left( \frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} \right) &= \frac{\partial}{\partial x_i} \left\{ \left[ \nu + (1 + M f_{u_1}) \frac{C_{\mu} k^2}{\sigma_k \varepsilon} \right] \frac{\partial k}{\partial x_i} \right\} - \\
&- (1 + M f_{u_1}) \left( u_i' u_j' \right) \frac{\partial U_i}{\partial x_j} - (\varepsilon + \varepsilon_p + G_p) \\
(1 + M f_{u_1}) \left( \frac{\partial \varepsilon}{\partial t} + U_i \frac{\partial \varepsilon}{\partial x_i} \right) &= \frac{\partial}{\partial x_i} \left\{ \left[ \nu + (1 + M f_{u_1}) \frac{C_{\mu} k^2}{\sigma_{\varepsilon} \varepsilon} \right] \frac{\partial \varepsilon}{\partial x_i} \right\} - \\
&- \frac{\varepsilon}{k} \left[ C_{\varepsilon 1} (1 + M f_{u_1}) \left( u_i' u_j' \right) \frac{\partial U_i}{\partial x_j} + C_{\varepsilon 2} (\varepsilon + \varepsilon_p + G_p) \right]
\end{align*}$$
where $\varepsilon_p$ and $G_p$ present relatively the additional dissipation in the presence of particles and the effect of particles nonuniform distribution in space. Equations (4) account for the particles contribution to the convection, diffusion, generation and dissipation of the main turbulent flow characteristics. The constants values in (4) are the same as for the single-phase flows in standard $k - \varepsilon$ turbulence model: $C_{mu} = 0.09$, $\sigma_k = 1.0$, $\sigma_{\varepsilon} = 1.3$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$. To set the boundary conditions when solving (4) we used the wall functions method [28], which is widespread for single-phase flow simulations at high Reynolds numbers.

2.2. Diffusion-inertia model

Transport and dispersion of the particles in two-phase turbulent flows a diffusion-inertia model (DIM) is used. This model was investigated in our previous papers [21, 29, 30, 31], combined with the simulation of the carrying turbulent flow within the framework of Reynolds-Averaged Navier-Stokes (RANS) equations. The model is based on the kinetic equation for the probability density function (PDF) of the particles velocity distribution [22, 23, 24], and is valid for two-phase flows with particles, which dynamic relaxation time does not exceed the Lagrangian integral timescale of the turbulence. The expression of the particle velocity in the form of the phase flows with particles, which dynamic relaxation time does not exceed the Lagrangian integral timescale of the turbulence. The expression of the particle velocity in the form of the expansion in terms of carrying flow characteristics is a feature of this model and, thereby, causes a reduction of the problem of the particles transport and dispersion to the solution of a single equation for particle concentration:

$$\frac{\partial M}{\partial t} + \frac{\partial U_i^n M}{\partial x_i} + \frac{\partial}{\partial x_i} \left[ \tau_p \left( F_i - \frac{DU_i}{Dt} \right) M \right] = \frac{\partial}{\partial x_i} \left[ \left( D_B \delta_{ij} + D_{T_{pij}} \right) \frac{\partial M}{\partial x_j} \right] + \tau_p \left( \frac{\partial}{\partial x_i} \left( M \frac{\partial q_u D_{T_{pij}}}{x_j} \right) \right)$$

$$D_{T_{ij}} = \langle u'_i u'_j \rangle T_{Lp}, \quad q_u = \tau_p \frac{f_u}{T_{Lp}}, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + U_j \frac{\partial}{\partial x_j},$$

where $F_i$ is the acceleration of a body force (for example, the gravity force) acting on a particle, $D_B$ is the Brownian diffusion coefficient, $D_{T_{pij}}$ is the particles turbulent diffusion tensor, $q_u$ is the turbulent migration coefficient of particles, $T_{Lp}$ is Lagrangian integral time of the liquid velocity fluctuations on the particle position.

The third term on the left-hand side of (5) describes the impact of gravity and other body forces, and the transport of by reason of the deviation of particle trajectories from the fluid streamlines. The first term on the right-hand side of (5) describes particles transport due to Brownian and turbulent diffusion and the second term quantifies turbulent migration (turbophoresis) due to the gradients of velocity fluctuations.

In DIM average velocity of particles can be expressed as an expansion in terms of the local characteristics of the turbulent fluid, with the particle response time as a small parameter:

$$V_i = U_i - (D_B \delta_{ij} + D_{T_{pij}} - \frac{\partial}{\partial x_j} \left( \frac{DU_i}{Dt} - \frac{\partial q_u D_{T_{pij}}}{\partial x_j} \right) \frac{\partial M}{\partial x_j} + \tau_p \left( F_i - \frac{DU_i}{Dt} - \frac{\partial q_u D_{T_{pij}}}{\partial x_j} \right) \right).$$

It should be noticed that using of anisotropic nonlinear Reynolds stress model compared with the standart isotropic model can improve the calculation results of particles transport and dispersion based on DIM in two points. Firstly, as a result of refined modelling of mean velocity field $U_i$, which determines the convective particles transport. And secondly because of the direct specifying of the Reynolds stress tensor components $\langle u'_i u'_j \rangle$, that is significant in modelling of particles transport due to turbulent migration (turbophoresis) and turbulent diffusion.

The eddy–particle interaction timescales are determined in an isotropic assumption $T_{Lp} = \left( T_{Lp}^\ell + 2T_{Lp}^n \right) / 3$, where the superscripts $\ell$ and $n$ stand for, respectively, the longitudinal
(streamwise) and normal (spanwise) directions to the relative velocity vector $\mathbf{V}_r = \mathbf{V} - \mathbf{U}_p$. The integral timescales of particles interaction with turbulent vortices are determined from the model [32], that takes into account the particles inertia and different timescales of particle interaction with turbulent eddies in different directions - a phenomenon that arises due to "crossing-trajectories effect" [33],

$$T_{Lp}^r = \left\{ \frac{3\Omega + m(2 + 3\gamma^2)}{3\Omega (1 + m\Omega)^2} [1 - F(St_E)] + \frac{F(St_E)}{1 + m\gamma} \right\} T_E, \quad T_{Lp}^n = \left\{ \frac{6\Omega + m(4 + 3\gamma^2)}{6\Omega (1 + m\Omega)^2} [1 - F(St_E)] + \frac{2 + m\gamma}{2(1 + m\gamma)^2} F(St_E) \right\} T_E, \quad \Omega = \frac{\tau_p}{T_{Lp}}, \quad \gamma = \frac{|\mathbf{V}_r|}{(2k/3)^{1/2}}, \quad T_E = \frac{3(1 + m\Omega)^2}{3 + 2m} T_L, \quad m = \frac{T_E(2k/3)^{1/2}}{L},$$

where $T_E$ is the Eulerian temporal integral scale of turbulence, $L$ is the spatial integral scale of turbulence. The Lagrangian integral timescale is assumed to be equal to

$$T_L = \frac{k}{\varepsilon}, \quad \alpha = \text{const}$$

The quantities $f_{u1}$ and $g_u$ in equations (1), (2) and (4) are the coefficients of the particle entrainment into the turbulent fluid flow. Assuming these coefficients to be scalar are determined on the basis of the two-scale bi-exponential autocorrelation function and take the form [34]:

$$f_{u1} = \frac{2\Omega + z^2}{2\Omega + 2\Omega^2 + z^2}, \quad f_u = \frac{(2\Omega + z^2)^2 - 2\Omega^2 z^2}{(2\Omega + 2\Omega^2 + z^2)^2}, \quad \Omega = \frac{\tau_p}{T_{Lp}}, \quad z = \frac{\tau_T}{T_{Lp}}$$

where the Taylor differential timescale of fluid turbulence is determined as [21]:

$$\tau_T = \left( \frac{2\Re_k \nu}{15^{1/2} a_0 \varepsilon} \right)^{1/2}, \quad a_0 = \frac{a_{01} + a_{0\infty} \Re_k}{a_{02} + \Re_k}, \quad \Re_k = \left( \frac{20k^2}{3\varepsilon \nu} \right)^{1/2}, \quad a_{01} = 11, \quad a_{02} = 205, \quad a_{0\infty} = 7.$$ 

The dynamic response time of a small particle is determined by the relation:

$$\tau_p = \tau_{p0} \left( 1 + 0.15 \Re_p^{0.687} \right)^{-1}, \quad \tau_{p0} = \frac{\rho_p d_p^2}{18 \rho \nu},$$

where $\tau_{p0}$ is the particle response time, $d_p$ is the particle diameter, and the particle Reynolds number $\Re_p$ is evaluated as

$$\Re_p = \frac{d_p [\mathbf{V}_r]^2 + 2(1 - 2f_u) k]}{\nu}.$$

Average relative velocity between particles velocity and flow velocity seen by particles, according to (6) equals:

$$V_{ri} = V_i - U_{pi} = \tau_p \left( F_i - \frac{D U_i}{D t} \right) - \frac{1}{M} \frac{\partial}{\partial x_j} \left[ (F_B + q_u D T_{pij}) M \right].$$
3. Validation

In order to validate the presented nonlinear Reynolds stress model for two-phase turbulent flows the comparison was made with the experimental data published in [19]. In [19] the well-known method of laser anemometry was used to investigate the stationary upward fully developed flow in a vertical tube. Reynolds number based on the mean velocity and the channel diameter was $Re = 25600$. Particles were made of glass ($SiO_2$) with the diameter of 50 and 100 µm and density $2550 \text{ kg/m}^3$ as well as of alluminium oxide ($Al_2O_3$) with the diameter of 50 µm and density $3950 \text{ kg/m}^3$. Mass concentration of particles $M$ varied from 0.12 to 0.39.

Fig. 1 shows the comparison of the calculated results for mean axial gas velocity $U_x$ far away from the inlet section with the experimental data. These are the results in the universal coordinates: $u_+ = U_x/u_*$, $y_+ = yu_*/\nu$, where $u_*$ is the friction velocity, $y$ is the wall distance. Both the calculated and measured results reveal that the particles in use (of such mass concentration and inertia) don’t affect the mean velocity distribution, which can be well described by the log-law [35]:

$$u_+ = \ln y_+ / \kappa + B, \ \ \ \kappa = 0.41, B = 5.2$$  \hspace{1cm} (7)

Fig. 2 presents the distribution of the root mean square (rms) streamwise $\langle u'^2_x \rangle^{1/2}/U_{x0}$ and spanwise $\langle u'^2_y \rangle^{1/2}/U_{x0}$ components of velocity fluctuations, normalized by the mean velocity on the tube axis $U_{x0}$. The turbulence attenuating in the presence of particles is clearly observed. The attenuating effect increases as the mass concentration of particles in increased. Besides if the particle response time increases (within considerable range) the particle effect on turbulence is reduced due to decrease of additional dissipation $\varepsilon_p$. The comparison with the experimental data indicate the developed model to adequately describe turbulence anisotropy as well as effect of particle mass concentration and response time on the turbulence intensity. At the same time the velocity fluctuations predicted by the model are slightly lower than the experimental ones (particularly in the spanwise component of velocity pulsations and in the streamwise component near the wall). To improve the quantitative agreement with the experiment one should presumably use a low-Reynolds number model of turbulence instead of the high-Reynolds one to resolve the pulsations near the wall more accurate.

It should be mentioned that the particles concentration distribution in the cross-sections is considerably non-uniform, that is presented in Fig.3. However, authors in [19] observed uniform distributions of the particles and didn’t observe particles concentration increasing near the wall of a tube presumably because measurement accuracy of particles concentration is decreasing near the wall. Presently, there is no doubt that the preferential concentration of particles laden in turbulent flow arises due to the so-called turbophoresis effect caused by the gradient of the turbulence intensity [36, 37]. But in the same time according to Fig.2 root mean square (rms) components of velocity fluctuations in the near-the-wall region do not depend on particle concentration. It can be explained by that the inertia of the particles (Stokes number $St_L$) strongly increases in the region near the wall, because the characteristic time $T_L$ of carrying energy turbulent eddies becomes much less than that at the pipe axis. Therefore, the rate of particles involving in the fluctuation motion and the damping of the air turbulence intensity near the pipe wall are much smaller compared with the pipe axis region.

4. Conclusions

Nonlinear Reynolds stress turbulent model for two-phase flows laden with small heavy particles is presented. The model is based on the fully explicit algebraic Reynolds stress model, developed by Girimaji [15, 16] for single-phase turbulent flow. The presented model is valid for the particle-laden gas flows where particle relaxation time doesn’t exceed the turbulent time macroscale. The model of nonlinear turbulent viscosity was implemented into a high-Reynolds number turbulence
model for two-phase turbulent flow, including differential equations for kinetic turbulent energy \( k \) and turbulent energy dissipation rate \( \varepsilon \) taking into account particle influence on turbulence. The described nonlinear turbulent model was coupled together with the diffusion-inertia model for particles transport and dispersion in turbulent flow, developed by the authors earlier.

By means of comparison with experimental data in circular tube the presented model was shown to describe adequately the anisotropy of Reynolds stress tensor and influence of the particles concentration and inertia on turbulence intensity as well. The attenuation of turbulence increases as the particles concentration is increased. At the same time it is reduced as the particles response time \( \tau_p \) is increased.

In the near wall region inertia of the particles (their Stokes number \( St_L \)) strongly increases, because the characteristic time \( T_L = \alpha k/\varepsilon \) of carrying energy turbulent eddies becomes much less than that at the pipe axis. Therefore, the rate of particles involving in the fluctuation motion and the damping of the air turbulence intensity near the pipe wall are much smaller compared with the pipe axis region. It leads to the fact that the components of velocity fluctuations in the near wall region do not depend on particle concentration. In order to model velocity fluctuations in the near-the-wall region the low-Reynolds modification of the presented nonlinear ARSM model is to be suggested. However, proposed model can be applied to high-\( Re \) industrial problems on complex geometries.

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Figure 1. Mean velocity profiles: 1 – function (7); 2 - 5 – experiments by [19]; 6 - 9 – calculations; 2, 6 – $M = 0$; 3, 7 – $Al_2O_3$, $M_0 = 0.26$, $d_p = 50 \, \mu m$; 4, 8 – $SiO_2$, $M_0 = 0.39$, $d_p = 50 \, \mu m$; 5, 9 – $SiO_2$, $M_0 = 0.39$, $d_p = 100 \, \mu m$. 
Figure 2. Streamwise rms velocity fluctuations – a, c, e; and spanwise rms velocity fluctuations – b, d, f; a, b – SiO$_2$, $d_p = 50$ µm, $\tau_p = 19.7$ ms; c, d – Al$_2$O$_3$, $d_p = 50$ µm, $\tau_p = 30.5$ ms; e, f – SiO$_2$, $d_p = 100$ µm, $\tau_p = 78.7$ ms; 1-5 – experiments by [19]; 6 - 10 – calculations; 1, 6 – $M_0 = 0$; 2, 7 – $M_0 = 0.12$; 3, 8 – $M_0 = 0.18$; 4, 9 – $M_0 = 0.26$; 5, 10 – $M_0 = 0.39$;
Figure 3. Mass concentration profiles: a – Al₂O₃, \( d_p = 50 \, \mu m \), \( \tau_p = 30.5 \, ms \); b – SiO₂, \( d_p = 50 \, \mu m \), \( \tau_p = 19.7 \, ms \); c – SiO₂, \( d_p = 100 \, \mu m \), \( \tau_p = 78.7 \, ms \); 1 – \( M_0 = 0.12 \); 2 – \( M_0 = 0.18 \); 3 – \( M_0 = 0.26 \);