Majorana-assisted nonlocal electron transport through a floating topological superconductor

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The nonlocal nature of the fermionic mode spanned by a pair of Majorana bound states in a one-dimensional topological superconductor has inspired many proposals aiming at demonstrating this property in transport. In particular, transport through the mode from a lead attached to the left bound state to a lead attached to the right will result in current cross-correlations. For ideal zero modes on a grounded superconductor, the cross-correlations are however completely suppressed in favor of purely local Andreev reflection. In order to obtain a non-vanishing cross-correlation, previous studies have required the presence of an additional global charging energy. Adding nonlocal terms in the form of a global charging energy to the Hamiltonian when testing the intrinsic nonlocality of the Majorana modes seems to be conceptually troublesome. Here, we show that a floating superconductor allows to observe nonlocal current correlations in the absence of charging energy. We show that the non-interacting and the Coulomb-blockade regime have the same peak conductance $e^2/h$ but different shot-noise power; while the shot noise is sub-Poissonian in the Coulomb-blockade regime in the large bias limit, Poissonian shot noise is generically obtained in the non-interacting case.

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I. INTRODUCTION

A pair of Majorana bound states at the ends of one-dimensional $p$-wave superconductors hosts a nonlocal fermionic mode at an energy close to the middle of the energy gap. The nonlocality of the fermionic mode has inspired proposals to observe nonlocal current correlations which arise when electrons are transported through the mode from a lead contacting the left bound state to a lead contacting the right. Unfortunately, nonlocal transport is completely suppressed in favor of local Andreev reflection for ideal Majorana bound states at zero energy and the currents through the left and right lead are uncorrelated with characteristic peak conductances $2e^2/h$. Later, it has been pointed out that a coupling $t$ between the Majorana end states that is larger then the lead-induced level broadening $\Gamma$ is required for a recovery of nonlocal current correlations. By increasing the length of the superconductor $L$ the coupling constant $t$ decays exponentially such that for sufficiently long topological superconductors the transport remains local despite the nonlocal nature of the fermionic zero mode.

It was first shown in Ref. 8 that to observe nonlocal current cross-correlations for ideal Majorana zero modes with $t = 0$, it is important that the superconductor carrying the Majorana zero modes is disconnected from the ground plate, resulting in a finite charging energy. In the Coulomb blockade regime, local Andreev reflection is blocked energetically and nonlocal transport with a system size independent level broadening $\Gamma$ and a peak conductance of $e^2/h$ at zero temperature is obtained. Due to the suppression of local Andreev reflection, the analysis can be carried out in the usual mean-field setup of superconductivity without violating current conservation. Subsequent studies have considered the behavior of the peak conductance as the charging energy is lowered away from the Coulomb blockade regime, finding a smooth increase from the peak conductance $e^2/h$ in the Coulomb blockade regime to $2e^2/h$ in the absence of charging energy.\textsuperscript{10,11}

The system size independence of $\Gamma$ in Ref. 8 must be contrasted with the transmission of spinless electrons through a double barrier, where the level broadening $\Gamma$ scales with system size as $L^{-1}$ such that the conductance quantization at a value $e^2/h$ is lost for any finite

![FIG. 1. (a) Setup consisting of a one-dimensional topological superconductor (TS) that is floating and hosts ideal zero-energy Majorana bound states $\gamma_1$, $\gamma_2$ contacted by two normal-conducting, non-interacting leads (N). The superconductor is subject to a charging energy $E_C$ such that a preferred number of charges is controllable through a gate voltage $V_g$. We study the system in the case $E_C = 0$ and compare it to known results for large $E_C$. (b) In absence of charging energy, $E_C = 0$, the conductance and shot noise of the system can be understood in terms of an equivalent circuit consisting of (noiseless) conductances $G_i$ which are associated with local Andreev reflection off the bound state $\gamma_i$ and current sources generating noise currents $\delta I_i$.](image-url)
temperature in the infinite system limit.\footnote{In contrast, the peak conductance $e^2/h$ survives at finite temperatures in the Majorana case, justifying the terminology of calling it 'nonlocal transport'. However, the presence of the large global charging energy, which is a strong, nonlocal perturbation at the level of the Hamiltonian, makes it difficult to unambiguously attribute the nonlocal current correlations to the presence of the Majorana bound states.\footnote{In this sense, a large charging energy can be considered ill-suited for probing the Majorana-induced nonlocality.}} In this work, we consider a floating superconductor where the charging energy $E_C$ vanishes. We will contrast the results to the well-studied limit of Coulomb-blockade $E_C \to \infty$ where the model maps onto a resonant-level model.\footnote{We show that consideration of a floating topological superconductor in a setup as depicted in Fig. (a) allows to observe nonlocal current correlations for ideal Majorana bound states at zero energy in the absence of charging effects. Interestingly, we find that the peak conductance is given by $e^2/h$ both in the non-interacting and in the Coulomb blockade limit. The contradiction to the results of Refs. 10 and 11 can be resolved by noting that away from the Coulomb blockade regime, transport into the Cooper pair condensate is no longer blocked energetically and current conservation has to be enforced self-consistently.\footnote{This appears to be technically demanding in the interacting setup of finite charging energy. Even though the peak conductances does not distinguish the two regimes, the transport mechanisms differ strongly. In the Coulomb-blockade regime, the Cooper-pair condensate is irrelevant and transport proceeds by sequential tunneling which can be modeled as coherent transport through a double barrier.\footnote{In contrast, in the non-interacting case, transport is incoherent due to the presence of the Cooper-pair condensate which acts as a phase-breaking scatterer.\footnote{We will show that while the difference between the two transport regimes is not captured by the peak conductance, it is reflected in the shot noise properties.}} \footnote{The effective phase-breaking in electron transport due to the Cooper-pair condensate allows to understand the transport in terms of an equivalent, classical circuit depicted in Fig. (b). In this circuit, Andreev reflection events at the left and the right terminal are modeled by noiseless conductances $G_1$, $G_2$ and their corresponding noise currents $\delta I_i$ are generated by current sources in parallel to the conductances. Straightforward application of Kirchhoff rules yields the total conductance $G = \langle I \rangle / V = G_1 G_2 / (G_1 + G_2)$ since the resistances $G_i^{-1}$ simply add up. In presence of ideal Majorana bound states, the each of the two conductances $G_1$ and $G_2$ in series peak at a value equal to the conductance quantum $2e^2/h$. Accordingly, the total conductance $G$ has a peak value of half a conductance quantum $e^2/h$. Similarly, assuming uncorrelated noise currents $\langle \delta I(t) \delta I(t') \rangle = 0$, one obtains the total current noise-power $S = \int dt \langle \delta I(t) \delta I(t) \rangle = (G_2^2 S_1^{(0)} + G_1^2 S_2^{(0)}) / (G_1 + G_2)^2$ with $S_j^{(0)} = \int dt \langle \delta I_j(t) \delta I_j(t) \rangle$; here, the prefactors in front of the factors $S_j^{(0)}$ originate from current division and give the (squared) fraction of the noise currents $\delta I_j$ contributing to the total current $I$. The above results correspond directly to our results Eqs. (7) and (9), yielding an appealingly simple picture of transport through the system. By analyzing the interplay between the bias dependencies of the conductances and the noise currents, we will show that the shot noise in the non-interacting limit $E_C = 0$ generically becomes Poissonian in the large-bias limit. In contrast, the shot noise is sub-Poissonian in the Coulomb-blockade regime. The outline of the paper is as follows. We use Sec. III to fix our notation and give a brief review of results obtained in previous studies for the grounded case. We discuss the conductance and shot noise obtained in the floating case for $E_C = 0$ in Sec. III. We will compare the results to the well-studied case of a superconducting island in the Coulomb-blockade regime in Sec. IV and finish with a discussion of our results.}}

In previous studies for the grounded case. We discuss the transport properties are described by the differential conductances $G_{ij}$ which have the form

$$G_{ij} = \frac{\partial I_i}{\partial V_j} = \frac{e^2}{h} \left[ \delta_{ij} - T_{ij}^{ee}(eV_j) + T_{ij}^{he}(eV_j) \right]$$



**II. REVIEW OF THE GROUNDED CASE**

Our system of interest is a one-dimensional $p$-wave spinless superconductor that hosts a pair of Majorana bound states $\gamma_i$, $i = 1, 2$, each of them in tunnel contact with a single normal-conducting lead $i$ as depicted in Fig. (a). Before discussing the case of a floating superconductor that is disconnected from the ground, we start with a review of the grounded case. For the grounded case, we describe the system using a mean-field Bogoliubov-de-Gennes (BdG) Hamiltonian and calculate the transport properties using a scattering approach. The entries $s_{ij}^{\alpha \beta}$ of the scattering matrix describe the transport of particle of type $\beta$ from contact $j$ to contact $i$ as a particle of type $\alpha$, where $i,j \in \{1,2,S\}$ can be either of the normal-conducting terminals or the superconductor ($S$) and $\alpha, \beta \in \{e, h\}$ denote electrons and holes. With the transfer probabilities $T_{ij}^{\alpha \beta} = |s_{ij}^{\alpha \beta}|^2$, the steady-state current at the normal-conducting terminal $i$ into the system can be written as

$$I_i = e \hbar \int dE \sum_{j \neq S} \left( \delta_{ij} - T_{ij}^{ee} + T_{ij}^{he} \right) (f_{j,E} - f_{S,E})$$

with $f_{j,E} = \Theta(\mu_j - E)$ the Fermi distribution of contact $j$ at an electro-chemical potential $\mu_j$ at zero temperatures with $\Theta(x)$ the unit step function. In the following, we choose a gauge where the energy $E$ is measured with respect to the chemical potential $\mu_S$ of the superconductor such that $\mu_j = eV_j$ with $V_j$ denoting the voltage difference between the lead $j$ and the superconductor. The transport properties are described by the differential conductances $G_{ij} = \partial I_i / \partial V_j$ which have the form

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at zero temperature; here, $T_{i}^{\text{esc}}(eV)$ denotes the transmission probability at energy $eV$. For ideal Majorana bound states and low voltages with energy below the gap of the superconductor, contributions from crossed Andreev reflection ($T_{i}^{\text{be}}$), normal transmission ($T_{i}^{\text{nc}}$), and quasiparticle transport into the superconductor ($T_{ij}^{\alpha\beta}$) vanish. Consequently, the only processes which are possible are normal- ($T_{ij}^{n}$) and Andreev-reflection ($T_{ij}^{a}$) such that the quasiparticle conservation leads to the sum rule $T_{ij}^{n} + T_{ij}^{a} = 1$. As a result, the conductance matrix becomes local, $G_{ij} = \delta_{ij} G_{j}$, with

$$G_{j} = \frac{2e^{2}}{h} T_{jj}^{\text{be}}(eV_{j}).$$

(3)

As shown in App. $\Delta$, explicit calculations of the scattering matrix for ideal Majorana bound states yield $T_{ij}^{\text{be}}(0) = 1$, implying resonant Andreev reflection with a peak conductance of $2e^{2}/h$ at zero bias $V_{j} = 0$.\footnote{Note that the expression of the shot noise (3) can directly be understood in terms of the equivalent circuit Fig. (1b) that was discussed in the introduction. However, in order to determine the concrete bias dependence of the noise}

The nonlocal current correlations can be investigated through the zero-frequency shot noise

$$S_{ij} = \int dt \left[ (\hat{I}_{i}(0)\hat{I}_{j}(t)) - \langle \hat{I}_{i}(0)\hat{I}_{j}(t) \rangle \right]^{2},$$

(4)

defined in terms of the current operator $\hat{I}_{i}(t)$ at terminal $i$. For ideal Majorana bound states in a three-terminal NSN setup where the superconductor is grounded, the shot noise at each terminal takes the same form as in a conventional NS junction $S_{ij}^{(0)} = \delta_{ij} S_{j}^{(0)}$ with\footnote{Note that the expression of the shot noise (3) can directly be understood in terms of the equivalent circuit Fig. (1b) that was discussed in the introduction. However, in order to determine the concrete bias dependence of the noise}

$$S_{j}^{(0)} = \frac{4e^{2}}{h} \int dE \ T_{jj}^{\text{be}}(1 - T_{jj}^{\text{be}}) (f_{j,E} - f_{S,E})^{2},$$

(5)

and no current cross-correlation is present.\footnote{Note that the expression of the shot noise (3) can directly be understood in terms of the equivalent circuit Fig. (1b) that was discussed in the introduction. However, in order to determine the concrete bias dependence of the noise}

III. FLOATING CASE WITHOUT INTERACTIONS

In the case of the floating superconductor without interactions $E_{\text{C}} = 0$, the mean-field BdG formalism which neglects the Cooper-pair condensate dynamics cannot be applied naively since electron pairs can be converted into Cooper pairs, resulting in a net current $I_{S} = -(I_{1} + I_{2})$ out of the superconducting reservoir. For a floating superconductor in the stationary regime, no net current $I_{S}$ can be drawn from the reservoir.\footnote{Note that the expression of the shot noise (3) can directly be understood in terms of the equivalent circuit Fig. (1b) that was discussed in the introduction. However, in order to determine the concrete bias dependence of the noise} The condition $I_{S} = 0$ can always be fulfilled by varying the sum of the voltage $V_{1} + V_{2}$ (which corresponds to changing the chemical potential of the superconductors with respect to the leads) while keeping the voltage difference $V = V_{1} - V_{2}$ between the two terminals fixed. The superconducting reservoir then serves as a voltage probe breaking the phase coherence of the quasiparticles without affecting the current balance.

For a general floating NSN setup, one obtains the conductance\footnote{Note that the expression of the shot noise (3) can directly be understood in terms of the equivalent circuit Fig. (1b) that was discussed in the introduction. However, in order to determine the concrete bias dependence of the noise}

$$G = \frac{\partial I_{1}}{\partial V} |_{I_{S}=0} = \frac{G_{11}G_{22} - G_{12}G_{21}}{G_{11} + G_{12} + G_{21} + G_{22}},$$

(6)

where all the conductances $G_{ij}$ have to be evaluated at the voltages $V_{i}$ corresponding to $I_{S} = 0$. For ideal Majorana bound states, we have $G_{12} = G_{21} = 0$ and the formula simplifies to

$$G = \frac{G_{1}G_{2}}{G_{1} + G_{2}},$$

(7)

which corresponds directly to the conductance that we have derived for the introduction for the equivalent circuit Fig. (1b). As previously explained, the total resistance $R_{\text{total}}$ is simply determined by the sum of the resistances $R_{1}, R_{2}$ associated with Andreev reflection processes at the left and right terminal and the total conductance $G$ peaks when the conductances $G_{1}, G_{2}$ attain their maximum values at $V_{1} = V_{2} = 0$. At this point, current conservation follows trivially from $I_{1} = I_{2} = 0$ and one obtains a peak conductance of $e^{2}/h$ at zero bias $V = 0$. The peak conductance in the floating case is thus precisely one half of the peak conductance expected for a grounded superconductor.

For the calculation of the noise in the floating case, we follow the approach of Ref. 3 which assumes that the current fluctuations can be modeled as Langevin forces $\delta I_{j}$ that drive the fluctuations $-eV$ in the chemical potential of the superconductor which in our gauge corresponds to modifying the voltages according to $\delta V_{1/2} = \delta V$. Although Ref. 3 presents the results in linear response in the voltages $V_{j}$ where they can be rigorously justified through fluctuation-dissipation relations, the treatment formally only requires linear response in the fluctuations $\delta V$ of the voltages $V_{j}$. This can be seen by expanding the zero-frequency current operator $\hat{I}_{i}$ of lead $i$ in the form

$$\hat{I}_{i} = I_{i} + \sum_{j \neq S} G_{ij} \delta V + \delta \hat{I}_{i},$$

(8)

where the first term is just the average current Eq. (1) and $\delta \hat{I}_{i}$ are the Langevin forces defined by $\langle \delta \hat{I}_{i}(t) \rangle = 0$, $\int dt \langle \delta \hat{I}_{i}(t) \delta \hat{I}_{j}(t) \rangle = S_{ij}^{(0)}$ with the shot noise $S_{ij}^{(0)}$ of the grounded case. In equation (5), the expressions $I_{i}, G_{ij}$ and $S_{ij}^{(0)}$ all have to be evaluated at the voltages corresponding to $I_{S} = 0$. For long measurement times which corresponds to low frequencies, current conservation has to hold not only on average but as an operator identity $\sum_{i} I_{i} = 0$.\footnote{Note that the expression of the shot noise (3) can directly be understood in terms of the equivalent circuit Fig. (1b) that was discussed in the introduction. However, in order to determine the concrete bias dependence of the noise}

Using the current conservation, the result of Ref. 3 for the shot noise in a floating two-terminal setup is recovered. The shot noise is maximally cross-correlated with $S_{11} = S_{22} = -S_{21} = -S_{12} = S$ where

$$S = \frac{G_{1}^{2}S_{1}^{(0)} + G_{2}^{2}S_{2}^{(0)}}{(G_{1} + G_{2})^{2}}.$$
we need to solve the self-consistency equation $I_S = 0$ determining the voltages $V_j$ given the bias $V$. We determine this relation using a microscopic model of two ideal Majorana bound states $\gamma_1$, $\gamma_2$ that are each connected to a lead. Because there is no coupling between the ideal Majorana modes, the transport is local and we can use the results of App. A in the expressions Eqs. (1), (3) and (5) for $I_j$, $G_j$, and $S_j^{(0)}$. We obtain

$$G_j = \frac{2e^2}{h} (1 + v_j^2), \quad I_j = \frac{4e}{h} \Gamma_j \arctan(v_j), \quad (10)$$

$$S_j^{(0)} = \frac{4e^2}{h} \Gamma_j (\arctan |v_j| - |v_j|/(1 + v_j^2)), \quad (11)$$

with $v_j = eV_j/2\Gamma_j$ where we have introduced the coupling strength $\Gamma_j$ of the Majorana fermion $\gamma_j$ to the nearby lead.

For the following, a possible asymmetry between the lead couplings $\Gamma_j$ will be important. Thus, we will label the leads by the indices $j \in \{\text{min}, \text{max}\}$ such that $\Gamma_{\text{min}} \leq \Gamma_{\text{max}}$. Note, however, that for the voltage drop $V_{\text{min}}$, the label ‘min’ merely means that the voltage drops at the terminal with the smaller lead coupling $\Gamma_{\text{min}}$ which implies that $V_{\text{min}} \geq V_{\text{max}}$ as we will show below.

Solving the constraint $I_S = 0$ valid for a floating superconducting in terms of $v_{\text{max}}$ yields

$$v_{\text{max}} = -\tan[\Gamma_{\text{min}} \arctan(v_{\text{min}})/\Gamma_{\text{max}}]. \quad (12)$$

The physics behind the transport can be entirely understood through the voltage relation (12), which is plotted in Fig. 2 for different ratios $\Gamma_{\text{min}}/\Gamma_{\text{max}}$. We find that for small applied bias voltages $V \ll V_- = 2\Gamma_{\text{min}}/e$ the voltage drop is symmetric with $|V_{\text{max}}| = |V_{\text{min}}| = 1/2V$. This results corresponds to the linearized regime in Fig. 2. Note that for the special case of symmetric couplings this relation holds exactly for arbitrary voltages. At larger voltage bias, the voltage $V_{\text{max}}$ saturates at a value

$$V_{\text{sat}} = 2\Gamma_1 \tan(\pi \Gamma_{\text{min}}/2\Gamma_{\text{max}}).$$

The origin of the saturation is the effect that a single Andreev level can carry at most a current $2\pi e/h$. The total current through the device is thus limited by $2\pi e \Gamma_{\text{min}}/h$. In order for $I_S = 0$ to hold, the current through the junction with the coupling $\Gamma_{\text{max}}$ must not exceed this value. Using (10), with $I_{\text{max}} = 2\pi e \Gamma_{\text{min}}/h$, we obtain that the voltage $V_{\text{max}}$ has to saturate at the value $V_{\text{sat}}$. If the bias voltage exceeds this saturation value almost all the voltage drop will be across the junction with the smaller coupling, i.e., we have $V = V_{\text{min}}$ for $V \gg V_{\text{sat}}$. We highlight that the existence of this regime is a consequence of the resonant structure of Andreev reflection at the NS interfaces.

From the behavior of the voltages, all the other transport characteristics follow. In the regime of low bias $V \ll V_-$, the relation have $V_{\text{min}} = -V_{\text{max}}$ leads to a non-trivial relation between the conductances $G_{\text{min}}/G_{\text{max}} = 1 - (\Gamma_{\text{min}}^2 + \Gamma_{\text{max}}^2)e^2V^2/16\Gamma_{\text{min}}^2 \Gamma_{\text{max}}^2$, and the behavior of the shot noise in the floating case is determined both by the bias dependence of the conductances and the shot noises $S_{\text{min}}^{(0)}$, $S_{\text{max}}^{(0)}$. For $V \ll V_-$, we obtain a Fano factor $F = S/eI = (1 + \Gamma_{\text{min}}^2/\Gamma_{\text{max}}^2)V^2/24V_-^2 \ll 1$ which indicates sub-Poissonian noise. For $V \rightarrow 0$, we obtain the result $F = 0$ consistent with the fact that the local transport is carried by a resonant level with unit transmission probability implying a vanishing Fano factor due to the Pauli exclusion principle.

Since for $V \gg V_{\text{sat}}$, the voltage drop $V_{\text{min}}$ at the terminal with the smaller coupling $\Gamma_{\text{min}}$ is much larger than the voltage drop $V_{\text{max}}$ at the other terminal, the conductances in that regime obey $G_{\text{min}} \ll G_{\text{max}}$. Consequently, we find $S = S_{\text{min}}^{(0)}$ which reflects the fact that according to the circuit Fig. (b), it is the noise current gen-
erated at the terminal associated with $\Gamma_{\text{min}}$ which flows through the larger conductance and thus dominates the shot noise for $V \gg V_{\text{sat}}$. Using the explicit form of the noise $\Pi$ and the current $I$, one finds that the Fano factor $F = S/eI$ tends to one for $V \gg V_{\text{sat}}$. This behavior is illustrated in Fig. 3 for different ratios $\Gamma_{\text{min}}/\Gamma_{\text{max}}$ close to one.

**IV. COMPARISON TO THE COULOMB-BLOCKADE CASE**

The behavior of the shot noise power $\Pi$ in the non-interacting case $E_C = 0$ must be contrasted with the shot noise power obtained for the Coulomb-blockade regime $E_C \to \infty$ of Ref. [8]. In the case where only two charge states are important, transport proceeds by sequential tunneling and can be mapped onto transport through a double barrier where tunneling through the barriers occurs at a rate $\Gamma_{\text{min}}/h$, $\Gamma_{\text{max}}/h$. Current and shot noise in that case can be obtained using the classical circuit Fig. 1(b) and its corresponding shot noise expression $\rho$ if the conductances $G_{\text{min}}, G_{\text{max}}$ model the transmission through a single barrier $\chi$. In this case, the conductances $G_{\text{min}}, G_{\text{max}}$ are simply proportional to the couplings $\Gamma_{\text{min}}, \Gamma_{\text{max}}$ which are energy-independent for wide leads, showing that the voltage drops $V_{\text{min}}, V_{\text{max}}$ at the left and the right barrier must be distributed according to $V_{\text{min}}/V_{\text{max}} = \Gamma_{\text{max}}/\Gamma_{\text{min}}$. Since neither of the voltage drops saturates, both the Fano factors associated with transmission through the left and right barrier $\rho_{eR}^{(0)}/eI$, $\rho_{eL}^{(0)}/eI$ will tend to one in the large bias regime $eV \gg \Gamma_{\text{min}}, \Gamma_{\text{max}}$ and one obtains the limiting expression $F_{\text{db}} = (\Gamma_{\text{min}}^2 + \Gamma_{\text{max}}^2)/(\Gamma_{\text{min}}^2 + \Gamma_{\text{max}}^2) < 1$ for the Fano factor of the double barrier in the large bias limit $\chi$. This shows that the Fano factor is suppressed as compared to the single barrier case with a maximal suppression of 1/2 obtained for symmetric coupling $\Gamma_{\text{min}} = \Gamma_{\text{max}}$.

In the following, we want to give an intuitive picture for the reduction of the Fano factor in the Coulomb-blockade regime with respect to the case of a noninteracting floating superconductor with $E_C = 0$. In the former case, the Fano factor is given by the generic result $F_{\text{db}}$ for an asymmetric double barrier which is smaller than one. In the latter case, for large bias, the voltage generically drops solely over the junction with coupling $\Gamma_{\text{min}}$ and the shot noise as well as the current can be evaluated in an NS-setup involving only this junction. The transport through the NS junction involves Andreev reflections. It is a well-known fact that the Andreev reflection processes in an NS-junction can also be mapped onto the problem of transmission of electrons through a symmetric double barrier with the concreted identification explained in Fig. 3. It is however important to realize that the elementary processes in the transport involve Cooper pairs with charge $2e$ such that the Fano factor is a twice large than the one for the symmetric double barrier. Given the expression $F_{\text{db}}$ above, we thus obtain the result that the Fano factor of the noninteracting setup tends to one for large bias.

**V. CONCLUSIONS**

In this work, we have studied two-terminal transport between two normal-conducting metallic leads contacting the Majorana bound states of a floating topological superconductor in absence of charging energy. The nonlocal current correlations reflecting Majorana-assisted nonlocal electron transport that were previously shown for the Coulomb-blockade regime are also obtained in the non-interacting case. The presence of the phase-breaking Cooper-pair condensate allows modeling the transport processes as an incoherent combination of Andreev reflection events at the left and the right NS junction. Using this model, we have shown that the peak conductance $2e^2/h$ previously obtained for the Coulomb-blockade regime also follows in the non-interacting case as a simple consequence of current division. We have shown that the difference between the non-interacting case and the Coulomb-blockade regime is reflected in the shot noise power, which generically becomes Poissonian in the high bias limit of the non-interacting case, whereas it stays sub-Poissonian in the Coulomb-blockade regime.

![Fig. 4](image-url)
thus find that the two transport regimes can be distinguished by their shot noise properties. Our findings for the peak conductance make it tempting to speculate that the peak conductance will actually remain at value $e^2/h$ for any finite value of the charging energy which however remains a problem for future studies.

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Appendix A: S-matrix for ideal Majorana bound states

The scattering matrix for an ideal Majorana bound state at a NS-interface can be straightforwardly computed through the Weidenmüller formula for the scattering matrix at energy $\omega$:

$$s(\omega) = 1 - 2\pi i W^\dagger \frac{1}{\omega/2 + i\pi WW^\dagger} W,$$  \hspace{1cm} (A1)

where the vector $W = (w, w^*)$ describes the coupling of the Majorana bound state to the electron and hole degrees in the lead which is assumed to have unit density of states. The scattering matrix can be written as

$$s = \begin{pmatrix} s^{ee} & s^{eh} \\ s^{he} & s^{hh} \end{pmatrix} = \begin{pmatrix} 1 + A & A \\ A & 1 + A \end{pmatrix},$$  \hspace{1cm} (A2)

with $A = (i\omega/2\Gamma - 1)^{-1}$ where we have introduced the Fermi’s golden rule coupling strength $\Gamma = 2|w|^2$ to the lead. From this one obtains the Andreev reflection probability $T^{hc} = (1 + \omega^2/4\Gamma^2)^{-1}$.

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