Channeling Effects in Direct Dark Matter Detectors

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Abstract. The channeling of the ion recoiling after a collision with a WIMP changes the ionization signal in direct detection experiments, producing a larger signal than otherwise expected. We give estimates of the fraction of channeled recoiling ions in NaI (Tl), Si and Ge crystals using analytic models produced since the 1960’s and 70’s to describe channeling and blocking effects. We find that the channeling fraction of recoiling lattice nuclei is smaller than that of ions that are injected into the crystal and that it is strongly temperature dependent.

1. Introduction

Channeling and blocking effects in crystals refer to the orientation dependence of charged ion penetration in crystals. In the “channeling effect” ions incident upon a crystal along symmetry axes and planes suffer a series of small-angle scatterings that maintain them in the open “channels” between the rows or planes of lattice atoms and thus penetrate much further into the crystal than in other directions. Channeled incoming ions do not get close to lattice sites, where they would be deflected at large angles. The “blocking effect” consists in a reduction of the flux of ions originating in lattice sites along symmetry axes and planes, creating what is called a “blocking dip” in the flux of ions exiting from a thin enough crystal as a function of the angle with respect to a symmetry axis or plane. These effects were first observed in the 1960’s and are used in crystallography, in the study of lattice disorder, ion implantation, surfaces, interfaces and epitaxial layers, in measurements of short nuclear lifetimes etc. In particular, avoiding channeling is essential in the manufacturing of semiconductor devices, since ion implantation at a controlled depth is the primary technique.

Ion channeling in NaI (Tl) was first observed in 1973 by Altman et al. [1]. They observed that channeled ions produce more scintillation light because they lose most of their energy via electronic stopping rather than nuclear stopping. The potential importance of the channeling effect for direct dark matter detection was first pointed out for NaI (Tl) by Drobyshevski [2] and by the DAMA collaboration [3]. When Na or I ions recoiling after a collision with a dark matter WIMP (Weakly Interacting Massive Particle) move along crystal axes and planes, their quenching factor is approximately $Q = 1$ instead of $Q_{Na} = 0.3$ and $Q_{I} = 0.09$, since they give their energy to electrons.

Ion channeling in crystals could give rise to a daily modulation due to the preferred direction of the dark matter flux arriving on the Earth. Earth’s daily rotation naturally changes the direction of the “WIMP wind” with respect to the crystal axes, which produces a daily modulation in the measured recoil energy (equivalent to a modulation of the factor $Q$). Avignone, Creswick, and Nussinov [4] pointed this out for NaI (Tl) crystals. My collaborators, Graciela Gelmini and Paolo Gondolo, and I [5] present here our analytic calculations of the channeling fraction in NaI (Tl), Si and Ge crystals.
2. Modeling of channeling

Our calculation is based on the classical analytic models developed in the 1960's and 70's, in particular by Lindhard [6, 7, 8, 9, 10, 11, 12, 13]. We use the continuum string and plane model, in which the screened Thomas-Fermi potential is averaged over a direction parallel to a row or a plane. The averaged potential, $U$, is considered to be uniformly smeared along the row or plane of atoms, which is a good approximation if the propagating ion interacts with many lattice atoms in the row or plane by a correlated series of many consecutive glancing collisions. Just one row or plane is considered in this model. Lindhard proved that for an ion propagating with kinetic energy $E$, and for small angle $\phi$ between the ion’s trajectory and the atomic row (or plane) in the direction perpendicular to the row (or plane), the so called “transverse energy”, $E_{\perp} = E \sin^2 \phi + U \approx E \phi^2 + U$ is conserved.

The conservation of the transverse energy provides a definition of the minimum distance of approach to the string (or plane) of atoms, $\rho_{\text{min}}$, at which the trajectory of the ion makes a zero angle with the string (or plane), and also of the angle $\psi$ at which the ion exits from the string (or plane), i.e. far away from it where $U \approx 0$. In reality the furthest position from a string or plane of atoms is the middle of the channel, whose width we call $d_{\text{ch}}$. Thus,

$$E_{\perp} = U(\rho_{\text{min}}) = E \psi^2 + U(d_{\text{ch}}/2).$$

Channeling requires that $\rho_{\text{min}} > \rho_c(E)$, where $\rho_c(E)$ is the smallest possible minimum distance of approach of the propagating ion with the row (or plane) for a given energy $E$. Since the potential $U(\rho)$ decreases monotonically with increasing $\rho$, $U(\rho_{\text{min}}) < U(\rho_c(E))$. Using Eq. 1, this can be further translated into an upper bound on $E_{\perp}$ and on $\psi$,

$$\psi < \psi_c(E) = \sqrt{\frac{U(\rho_c(E)) - U(d_{\text{ch}}/2)}{E}}.$$  

$\psi_c(E)$ is the maximum angle the ion can make with the string (or plane) far away from it (i.e. in the middle of the channel) if the ion is channeled. At low enough $E$, $\rho_c(E)$ becomes close to $d_{\text{ch}}/2$, and the critical angle $\psi_c(E)$ goes to zero. This means that there is a minimum energy below which channeling cannot happen, even for ions moving initially in the middle of the channel.

So far we have been considering static strings and planes, but the atoms in a crystal are actually vibrating. We use the Debye model, and take into account thermal effects in the crystal through a modification of the critical distances which was found originally by Morgan and Van Vliet [8] and later by Hobler [13] to provide good agreement with simulations and data. It consists of taking the temperature corrected critical distance $\rho_c(T)$ to be,

$$\rho_c(T) = \sqrt{\rho_c^2(E) + |cu_1(T)|^2},$$

where $u_1$ is the one dimensional rms vibration amplitude of the atoms in a crystal, and the factor $c$ is a number between 1 and 2 in different references [8, 9]. As shown in Fig. 1, with this formalism and using $c = 2$ we fit relatively well the critical angles measured at room temperature for B and P ions in a Si crystal in several channels, for energies between 20 keV and 600 keV that Hobler [13] extracted from thermal wave measurements.

As an example, the static ($c = 0$) axial and planar critical distances are presented in Fig. 2(a) for the 100 channel in NaI crystal, together with the amplitude of thermal vibration $u_1$ at 20 °C, and the Thomas-Fermi screening distances for Na and I ions. Fig. 2(b) shows the temperature corrected axial and planar critical angles at 20 °C (with $c = 1$) for the same channel as functions of energy of the traveling Na and I ions.

3. Channeling of incident particles

The channeling of ions in a crystal depends not only on the angle their initial trajectory makes with strings or planes in the crystal, but also on their initial position. Ions which start their motion close to the center of a channel, far from a string or plane, where they make an angle
iso-Latitude Pixelization (HEALPix) method [14]. The HEALPix method uses an algorithm to

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\psi_{\text{inc}} \text{ over the incident direction. The integrals cannot be solved analytically, so we integrate numerically by performing a Riemann sum once the sphere of directions has been divided using the Hierarchical Equal Area iso-Latitude Pixelization (HEALPix) method [14]. The HEALPix method uses an algorithm to}
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deal with the pixelization of data on a sphere, and it is useful to compute integrals over direction by dividing the surface of a sphere into many pixels, computing the integrand at each pixel (i.e. each direction, see Fig. 3(a)), and finally summing up the values of all pixels over the sphere.

A channeled ion can be pushed out of a channel by an interaction with an impurity such as the atoms of Tl in NaI (Tl). Here we will simply assume that if a channeled ion interacts with a Tl atom it becomes dechanneled and thus it does not contribute to the fully channeled fraction any longer. We thus neglect the possibility that after the interaction the ion may re-enter into a channel, either the same or another. Fig. 3(a) shows the axial and planar channels of the NaI crystal for incoming Na ions with an energy of 50 keV. We include here only the channels with lower crystallographic indices 100, 110 and 111. The fraction of channeled incoming Na and I ions after including dechanneling in the way just described is shown in Fig. 3(b). Our results agree well with those published by the DAMA collaboration [3], also included in the figure.

4. Channeling of recoiling lattice nuclei
The recoiling nuclei start initially from lattice sites (or very close to them), thus blocking effects are important. In fact, as argued originally by Lindhard [6], in a perfect lattice and in the absence of energy-loss processes the probability that a particle starting from a lattice site is channeled would be zero. The argument uses statistical mechanics in which the probability of particle paths related by time-reversal is the same. Thus the probability of an incoming ion to have a particular path within the crystal is the same as the probability of the same ion to move backwards along the same path [15]. This is what Lindhard called the “Rule of Reversibility”. Using this rule, since the probability of an incoming channeled ion to get very close to a lattice site is zero, the probability of the same ion to move in the time-reversed path, starting at a nuclear site and ending inside a channel, is zero too. However, any departure of the actual lattice from a perfect lattice, for example due to vibrations of the atoms in the lattice, would violate the conditions of this argument and allow for some of the recoiling lattice nuclei to be channeled, as already understood in the 70’s [11, 16]. We now estimate the effect using the formalism presented so far.

In our model, a recoiling ion is channeled if the collision ion-WIMP happens at a distance large enough from the string or plane to which the ion belongs. Namely, channeling happens if the initial position of the recoiling motion is $\rho_i > \rho_{i,\text{min}}$. We define $E_{\perp}$ in terms of the initial
Figure 4. Channeling fractions at T=293 K for Na (solid lines) and I (dashed lines) ions for c = 1 (black) and c = 2 (green/gray) cases (a) without and (b) with dechanneling included.

recoil energy $E$ of the propagating ion, the angle of the initial recoil momentum with respect to the particular string or plane of atoms $\phi_i$, and the initial position $\rho_i$. The condition

$$E_{\perp}(E, \phi_i, \rho_{i \min}) = U(\rho_{i \min}) < U(\rho_c),$$

i.e. $E_{\perp}(E, \phi_i, \rho_{i \min}) = U(\rho_c(E))$ defines the distance $\rho_{i \min}$ in terms of $\rho_c$.

We take the initial distance distribution, $\rho$ of the colliding atom to be a Gaussian with a one dimensional dispersion $u_1$, and we obtain the probability of channeling for each individual channel by integrating the Gaussian between the minimum initial distance and infinity (a good approximation to the radius of the channel). The dependence of these probabilities on the critical distances enter in the argument of an exponential (for axial channels) or an erfc function (for planar channels). In order to obtain the total geometric channeling fraction we sum over all the individual channels we consider. The integral over initial directions is computed using HEALPix [14].

Figure 4 shows what we consider to be our main predictions for the range expected as an upper limit to the channeling fraction in NaI (Tl) at 239 K for two different assumptions for the effect of thermal vibrations in the lattice, which depend on the value of the parameter $c$ used in the temperature corrected critical distances of approach. Dechanneling is ignored in Fig. 4(a) and taken into account in Fig. 4(b).

Our results for the total geometric channeling fraction for Si ions propagating in a Si crystal and Ge ions propagating in a Ge crystal at different temperatures are shown in Figs. 5 and 6 for the two cases of $c = 1$ and $c = 2$, respectively. Note that we have not considered the possibility of dechanneling of initially channeled ions, due to imperfections in Si and Ge crystals. Any mechanism of dechanneling will decrease the fractions obtained here for Si and Ge.

5. Conclusions

We have studied the channeling of ions recoiling after collisions with WIMPs within NaI (Tl), Si and Ge crystals. Channeled ions move within the crystal along symmetry axes and planes and suffer a series of small-angle scatterings that maintain them in the open “channels” between the rows or planes of lattice atoms and thus penetrate much further into the crystal than in other directions. Ions which start their motion close to the center of a channel, at an initial angle $\psi$, are channeled if the initial angle is smaller than the critical angle in Eq. 2, and are not channeled otherwise. We have found that the channeling of lattice ions recoiling after a collision with a WIMP is very different from the case of incident ions, and that the channeling fraction is smaller.

As argued originally by Lindhard [6], in a perfect lattice and in the absence of energy-loss processes, the probability that a particle starting from a lattice site is channeled would be zero.
Figure 5. Channeling fractions of (a) Si and (b) Ge recoils in a Si and a Ge crystal respectively, as a function of the ion energy for temperatures $T=900 \degree C$ (orange or medium gray), $T=600 \degree C$ (green or light gray), 293 K (black), and 44 mK (blue or dark gray) in the approximation of $c=1$.

Figure 6. Same as Fig. 5 but for $c=2$.

However, due to vibrations in the crystal, the atom that interacts with a WIMP may be displaced from its position in a perfect lattice, and there is a non-zero probability of channeling.

As seen in Fig. 4(a) for NaI, without including dechanneling, the channeling fraction is never larger than 5% and the maximum happens at 100's of keV. This maximum occurs because the critical distances decrease with the ion energy $E$, making channeling more probable, and the critical angles also decrease with $E$, making channeling less probable. The simple extreme model of dechanneling we used for NaI predicts much smaller fractions, at most in the 0.1% level, with the maximum shifted to small energies, less than 10 keV (see Fig. 4(b)). This reduction may eventually prove to be too extreme and at present we do not have a better formalism to model dechanneling. With the simple model of dechanneling we used for NaI, we could reproduce the channeling fractions computed by the DAMA collaboration (which, however, apply to ions which start their motion close to the middle of a channel and not to the case of WIMP direct detection).

If the values found by Hobler [13], and also by us above (see Fig. 1), to reproduce measured channeling angles in B and P propagating in Si apply also to the propagation of Si ions in Si, then the case of $c=2$ should be chosen and the channeling fractions would never be larger than 0.3%. Moreover, increasing the temperature of a crystal usually increases the fraction of channeled recoiling ions (see Fig. 5), but not always. Sometimes the opposite happens (see
Fig. 6). The $c = 1$ choice leads to channeling fractions close to 1% for Si and Ge.

The analytical approach used here can successfully describe qualitative features of the channeling and blocking effects, but should be complemented by data fitting of parameters and by simulations to obtain a good quantitative description too. Thus our results should in the last instance be checked by using some of the many sophisticated Monte Carlo simulation programs.

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References

[1] Altman R, Dietrich H B, Murray R B and Rock T J 1973 Phys. Rev. B 7 1743
[2] Drobyshevski E M 2008 Mod. Phys. Lett. A 23 3077 (Preprint 0706.3095)
[3] Bernabei R et al 2008 Eur. Phys. J. C 53 205 (Preprint 0710.0288)
[4] Avignone F T, Creswick R J and Nussinov S 2008 Preprint 0807.3758
Creswick R J, Nussinov S and Avignone F T 2010 Preprint 1007.0214
[5] Bozorgnia N, Gelmini G B and Gondolo P 2010 J. Cosmol. Astropart. Phys. JCAP11(2010)019 (Preprint 1006.3110)
Bozorgnia N, Gelmini G B and Gondolo P 2010 J. Cosmol. Astropart. Phys. JCAP11(2010)028 (Preprint 1008.3676)
[6] Lindhard J 1965 Kongel. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 34 No. 14
[7] Andersen J U 1967 Kongel. Dan. Vidensk. Selsk., Mat.-Fys. Medd. 36 No. 7
[8] Morgan D V and Van Vliet D 1963 Can. J. Phys. 46 503
Morgan D V and Van Vliet D 1971 Radiat. Effects and Defects in Solids 8 51
[9] Van Vliet D 1973 Channeling, ed D V Morgan (London: Wiley) p 37
[10] Andersen J U and Feldman L C 1970 Phys. Rev. B 1 2063
[11] Komaki K and Fujimoto F 1970 Phys. Stat. Sol. (a) 2 875
[12] Appleton B R and Foti G 1977 Ion Beam Handbook for Material Analysis, ed J W Mayer and E Rimini (New York: Academic) p 67
[13] Hobler G 1996 Radiat. effects and defects in solids 139 21
Hobler G 1996 Nucl. Instrum. Methods Phys. Research (NIM) B 115 323
[14] Görski K M et al 2005 ApJ 622 759
[15] Gemmell D S 1974 Rev. Mod. Phys. 46 129
[16] Komaki K et al 1971 Phys. Stat. Sol. (a) 4 495