Magnetic field induced incommensurate resonance in cuprate superconductors

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The influence of a uniform external magnetic field on the dynamical spin response of cuprate superconductors in the superconducting state is studied based on the kinetic energy driven superconducting mechanism. It is shown that the magnetic scattering around low and intermediate energies is dramatically changed with a modest external magnetic field. With increasing the external magnetic field, although the incommensurate magnetic scattering from both low and high energies is rather robust, the commensurate magnetic resonance scattering peak is broadened. The part of the spin excitation dispersion seems to be an hourglass-like dispersion, which breaks down at the heavily low energy regime. The theory also predicts that the commensurate resonance scattering at zero external magnetic field is induced into the incommensurate resonance scattering by applying an external magnetic field large enough.

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I. INTRODUCTION

The intimate relationship between the short-range antiferromagnetic (AF) correlation and superconductivity is one of the most striking features of cuprate superconductors. This is followed an experimental fact that the parent compounds of cuprate superconductors are Mott insulators with the AF long-range order (AFLRO). However, when holes or electrons are doped into these Mott insulators, the ground state of the systems is fundamentally altered from a Mott insulator with AFLRO to a superconductor with persistent short-range correlations. The evidence for this closed link is provided from the inelastic neutron scattering (INS) experiments that show the unambiguous presence of the short-range AF correlation in cuprate superconductors in the superconducting (SC) state.

At zero external magnetic field, the dynamical spin response of cuprate superconductors exhibits a number of universal features where the magnetic excitations form an hourglass-like dispersion centered at the AF ordering wave vector \( Q = [\pi, \pi] \) (in units of inverse lattice constant). At the saddle point, the dispersing incommensurate (IC) branches merge into a sharp commensurate feature, which is dramatically enhanced upon entering the SC state and commonly referred as the magnetic resonance scattering. In particular, it has been argued that this commensurate magnetic resonance plays a crucial role for the SC mechanism in cuprate superconductors, since the commensurate magnetic resonance with the magnetic resonance energy scales with the SC transition temperature forming a universal plot for all cuprate superconductors. To test the connection between the commensurate magnetic resonance phenomenon and SC mechanism, it is desirable to perform further characterization. Since a uniform external magnetic field can serve as a weak perturbation helping to probe the nature of the short-range AF correlation and superconductivity, therefore the dynamical spin response of cuprate superconductors in the SC state has been studied experimentally by application of a uniform external magnetic field. However, there is no a general consensus. Some experimental results show that applying a uniform external magnetic field enhances the amplitude of the IC magnetic scattering already present in the system. On the other hand, other experiments indicate that the intensity gain of the IC magnetic scattering is suppressed by application of a uniform external magnetic field. In particular, the influence of a uniform external magnetic field has been investigated on the resonance scattering peak by using INS technique. The early INS measurements shows that under a modest external magnetic field (\( \sim 11 \) Tesla), the resonance scattering peak remains almost unaffected, i.e., although a line broadening occurs without change of the resonance scattering peak amplitude, no shifting of the resonance scattering peak energy is observed. However, the later INS experiments show that a modest external magnetic field applied to cuprate superconductors in the SC state yields a very significant reduction in the incommensurate magnetic resonance scattering. To the best of our knowledge, there are no explicit microscopic predictions about the effect of a uniform external magnetic field large enough on the magnetic resonance scattering.

For the case of zero external magnetic field, the dynamical spin response of cuprate superconductors has been discussed based on the framework of the kinetic energy driven SC mechanism, and all main features of the INS experiments are reproduced, including the doping and energy dependence of the IC magnetic scattering at both low and high energies and commensurate magnetic resonance at intermediate energy. In this paper, we study the influence of a uniform external magnetic field on the dynamical spin response of cuprate superconductors in the SC state along with this line. We calculate explicitly the dynamical spin structure factor of cuprate...
superconductors under a uniform external magnetic field, and show that the magnetic scattering around low and intermediate energies is dramatically changed with a modest external magnetic field. With increasing the external magnetic field, although the IC magnetic scattering from both low and high energies is rather robust, the commensurate magnetic resonance scattering peak is broadened. The part of the spin excitation dispersion seems to be an hourglass-like dispersion, which breaks down at the heavily low energy regime.

The rest of this paper is organized as follows. The basic formalism is presented in Sec. II, where we generalize the calculation of the dynamical spin structure factor from the previous zero external magnetic field case to the present case with a uniform external magnetic field. Within this theoretical framework, we discuss the influence of a uniform external magnetic field on the dynamical spin response of cuprate superconductors in the SC state in Sec. III, where we predict that the commensurate magnetic resonance scattering at zero external magnetic field is induced into the IC magnetic resonance scattering by an applied external magnetic field large enough. Finally, we give a summary and discussions in Sec. IV.

II. THEORETICAL FRAMEWORK

In cuprate superconductors, the characteristic feature is the presence of the CuO$_2$ plane. It has been shown from ARPES experiments that the essential physics of the doped CuO$_2$ plane is properly accounted by the t-J model on a square lattice. However, for discussions of the influence of a uniform external magnetic field on the dynamical spin response of cuprate superconductors in the SC state, the t-J model can be expressed by including the Zeeman term as,

\[
H = -t \sum_{\langle i\sigma \rangle} C_{i\sigma}^\dagger C_{i+\sigma} + t' \sum_{\langle i\sigma \rangle} C_{i\sigma}^\dagger C_{i+\sigma} + \mu \sum_{\langle i\sigma \rangle} C_{i\sigma}^\dagger C_{i\sigma} + J \sum_{\langle i\sigma \rangle} S_i \cdot S_{i+\sigma} - \varepsilon_B \sum_{\langle i\sigma \rangle} \sigma C_{i\sigma}^\dagger C_{i\sigma},
\]

where \( \hbar = \pm \hat{x}, \pm \hat{y}, \hat{\tau} = \pm \hat{x} \pm \hat{y} \), \( C_{i\sigma}^\dagger \) \( C_{i\sigma} \) \( (C_{i\sigma}) \) is the electron creation (annihilation) operator, \( S_i = (S_i^x, S_i^y, S_i^z) \) are spin operators, \( \mu \) is the chemical potential, and \( \varepsilon_B = g\mu_B B \) is the Zeeman magnetic energy, with the Lande factor \( g \), Bohr magneton \( \mu_B \), and a uniform external magnetic field \( B \). This t-J model with a uniform external magnetic field is subject to an important local constraint \( \sum_{\sigma} C_{i\sigma}^\dagger C_{i\sigma} \leq 1 \) to avoid the double occupancy. The strong electron correlation in the t-J model manifests itself by this local constraint, which can be treated properly in analytical calculations within the charge-spin separation (CSS) fermion-spin theory, where the constrained electron operators are decoupled as \( C_{i\sigma}^\dagger = h_{i\sigma}^\dagger S_i^\dagger \) and \( C_{i\sigma} = h_{i\sigma} S_i \), with the spinful fermion operator \( h_{i\sigma} = e^{-i\phi_{i\sigma}} h_i \) represents the charge degree of freedom together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), while the spin operator \( S_i \) represents the spin degree of freedom (spin), then the t-J model with a uniform external magnetic field (1) can be expressed in this CSS fermion-spin representation as,

\[
H = -t \sum_{\langle i\sigma \rangle} (h_{i\sigma} S_i^+ h_{i+\sigma}^\dagger S_{i+\sigma}^+ + h_{i\sigma} S_i^- h_{i+\sigma}^\dagger S_{i+\sigma}^-) + t' \sum_{\langle i\sigma \rangle} (h_{i\sigma} S_i^+ h_{i+\sigma}^\dagger S_{i+\sigma}^- + h_{i\sigma} S_i^- h_{i+\sigma}^\dagger S_{i+\sigma}^+) - \mu \sum_{\langle i\sigma \rangle} h_{i\sigma}^\dagger h_{i\sigma} + J_{\text{eff}} \sum_{\langle i\sigma \rangle} S_i \cdot S_{i+\sigma} - 2\varepsilon_B \sum_i S_i^z,
\]

with \( J_{\text{eff}} = (1 - x)^2 J \), and \( x = \langle h_{i\sigma}^\dagger h_{i\sigma} \rangle = \langle h_i h_i \rangle \) is the hole doping concentration. It has been shown that the electron local constraint for the single occupancy is satisfied in analytical calculations in this CSS fermion-spin theory.

Within the framework of the CSS fermion-spin theory, the kinetic energy driven superconductivity has been developed. It has been shown that the interaction from the kinetic energy term in the t-J model (2) is quite strong, and can induce the d-wave charge carrier pairing state by exchanging spin excitations in the higher power of the doping concentration, then the d-wave electron Cooper pairs originating from the d-wave charge carrier pairing state are due to the charge-spin recombination, and their condensation reveals the d-wave SC ground-state. However, the SC-state is controlled by both d-wave SC gap function and quasiparticle coherence, which leads to that the SC transition temperature increases with increasing doping in the underdoped regime, and reaches a maximum in the optimal doping, then decreases in the overdoped regime. Furthermore, for the case of zero external magnetic field, the doping and energy dependent dynamical spin response of cuprate superconductors in the SC-state has been discussed in terms of the collective mode in the charge carrier-particle-channel, and the results are in qualitative agreement with the INS experimental data on cuprate superconductors in the SC state. Following their discussions, the full spin Green’s function in the presence of a uniform external field is obtained as,

\[
D(k, \omega) = \frac{1}{D^{(0)}(k, \omega) - \Sigma^{(s)}(k, \omega)},
\]

with the mean-field (MF) spin Green’s function,

\[
D^{(0)}(k, \omega) = \frac{B_k}{2\omega_k} \left( \frac{1}{\omega - \omega_k^{(1)}} - \frac{1}{\omega - \omega_k^{(2)}} \right) = \sum_{\nu=1,2} (-1)^{\nu-1} B_k \frac{1}{2\omega_k - \omega_k^{(\nu)}},
\]

where \( B_k = 2\lambda_1 (A_1 \gamma_{10} - A_2) - \lambda_2 (2\lambda \chi_{1\nu}^\dagger - \chi_2), \lambda_1 = 2\gamma J_{\text{eff}}, \lambda_2 = 4Z\phi_2 t', \gamma_{10} = (1/Z) \sum_{\vec{k}} \epsilon^{(k)}, Z \) is the number of the nearest neighbor or next nearest neighbor sites of a square lattice, \( A_1 = \epsilon \chi_{1\nu}^\dagger + \chi_{1\nu}, A_2 = \chi_{1\nu}^\dagger + \epsilon \chi_{1\nu}/2, \epsilon = 1 + 2t\phi_1 / J_{\text{eff}}, \) the charge carrier’s particle-hole parameters \( \phi_1 = \langle h_{i\sigma}^\dagger h_{i+\sigma} \rangle \) and \( \phi_2 = \langle h_{i\sigma}^\dagger h_{i+\sigma} \rangle \), and the spin correlation functions \( \chi_1 = \langle S_i^x S_{i+\sigma} \rangle, \chi_2 = \langle S_i^x S_{i+\sigma} \rangle, \chi_1^\dagger = \langle S_i^\dagger S_{i+\sigma} \rangle, \chi_2^\dagger = \langle S_i^\dagger S_{i+\sigma} \rangle \).
\[
\omega_k^2 = \lambda_k^2 [(A_4 - \alpha \epsilon_1 \gamma_k - \frac{1}{2} \alpha \epsilon_1)(1 - \epsilon \gamma_k) + \frac{1}{2} \epsilon_1 (A_3 - \frac{1}{2} \alpha \epsilon_1 - \alpha \epsilon_1 \gamma_k)(\epsilon - \gamma_k)] + \lambda \lambda_2 [\alpha \epsilon_2 \gamma_\nu' = \frac{3}{2 \omega_1}(\epsilon_3 + \frac{1}{2})(A_5 - \frac{1}{2} \alpha \epsilon_2)] + \lambda_1 \lambda_2 [\alpha \epsilon_1 (1 - \epsilon \gamma_k) \gamma_p' - \frac{1}{2} \alpha \epsilon(C_3 - \epsilon \gamma_k)] + \frac{1}{2} \alpha \epsilon_1 (\gamma_k' - C_3)(\epsilon - \gamma_p) + \alpha \epsilon_2 (C_3^\ast - \epsilon \gamma_k)], \tag{5}
\]

where \(A_3 = \alpha C_1 + (1 - \alpha)/(2Z), A_4 = \alpha C_2 + (1 - \alpha)/(2Z),\) and the spin correlations \(C_1 = (1/Z^2) \sum \eta_{i\eta'}^\prime \langle S_{i\eta i\eta'}^z \rangle, C_2 = (1/Z^2) \sum \eta_{i\eta'}^\prime \langle S_{i\eta i\eta'}^z \rangle, C_3 = (1/Z) \sum \zeta_{i\eta i\eta'}^\prime \langle S_{i\eta i\eta'}^z \rangle,\) and \(C_4 = (1/Z) \sum \zeta_{i\eta i\eta'}^\prime \langle S_{i\eta i\eta'}^z \rangle,\) \(\eta_{i\eta'}^\prime = 1/2\) in the case without AFLRO, the important decoupling parameter \(\alpha\) has been introduced in the MF calculation, which can be regarded as the vertex correction. The spin self-energy function \(\Sigma_{i\eta i\eta'}^\prime(k, \omega)\) in the SC-state is obtained from the charge carrier bubble in the charge carrier particle-particle channel.

\[
\Sigma^{(s)}(k, \omega) = \frac{1}{N^2} \sum_{\eta_{i\eta'}, \eta_{i\eta'}, \eta_{i\eta'}} \langle \eta_{i\eta'}(q, p, k) B_{q, k} \frac{E_{q, k} \Delta_{h, z}^{(d)}(p) \Delta_{h, z}^{(d)}(p + q)}{E_p E_{p+q}} \frac{F_1^{(s)}(k, p, q)}{\omega - (E_p - E_{p+q} + \omega_{q+k})} + \frac{F_2^{(s)}(k, p, q)}{\omega - (E_p + E_{p+q} + \omega_{q+k})} - \frac{F_3^{(s)}(k, p, q)}{\omega + (E_p + E_{p+q} - \omega_{q+k})} \rangle, \tag{6}
\]

where \(\Delta_{h, z}(k) = \{\Delta_{h, z}^\prime(k)\},\) the effective charge carrier gap function in the d-wave symmetry with \(z_{q+k}^\prime = (\cos k_x - \cos k_y)/2, F_1^{(s)}(k, p, q) = n_B(\omega_{q+k}) n_F(E_p) - n_F(E_{p+q}) - n_F(E_p) n_F(E_{p+q}), F_2^{(s)}(k, p, q) = n_B(\omega_{q+k}) n_F(E_{p+q}) - n_F(E_p) n_F(E_{p+q}).\)
n_B(ω_p^{(2)})] - U_{hp-q+k_0}^2 n_B(ω_p^{(1)})n_B(-ω_q^{(1)}) - V_{hp-q+k_0}^2 n_B(ω_p^{(2)})n_B(-ω_q^{(2)}), R_2(p, q) = \end{eqnarray}

\begin{eqnarray}
n_B(ω^{(2)})] + V_{hp-q+k_0}^2 n_B(ω_p^{(1)})n_B(-ω_q^{(2)}) - U_{hp-q+k_0}^2 n_B(ω_p^{(2)})n_B(-ω_q^{(2)}) - V_{hp-q+k_0}^2 n_B(ω_p^{(2)})n_B(-ω_q^{(2)}), R_3(p, q) = \end{eqnarray}

\begin{eqnarray}
n_B(ω^{(2)})] + V_{hp-q+k_0}^2 n_B(ω_p^{(1)})n_B(-ω_q^{(2)}) - U_{hp-q+k_0}^2 n_B(ω_p^{(2)})n_B(-ω_q^{(2)}), R_4(p, q) = \end{eqnarray}

\begin{eqnarray}
\frac{2[1 + n_B(ω)]B_0^2 \text{Im} \Sigma^{(s)}(k, ω)}{[(ω - 2ε_B)^2 - ω_k^2 - B_0 \text{Re} \Sigma^{(s)}(k, ω)]^2 + [B_0 \text{Im} \Sigma^{(s)}(k, ω)]^2},
\end{eqnarray}

where \text{Im} \Sigma^{(s)}(k, ω) and \text{Re} \Sigma^{(s)}(k, ω) are the imaginary and real parts of the spin self-energy function (6), respectively.

### III. MAGNETIC FIELD INDUCED INCOMMENSURATE MAGNETIC RESONANCE

We are now ready to discuss the influence of a uniform external magnetic field on the dynamical spin response of cuprate superconductors in the SC state. For cuprate superconductors, the commonly used parameters in this paper are chosen as \( t/J = 2.5 \) and \( t'/t = 0.3 \) with a reasonably estimative value of \( J \sim 120 \text{ meV} \). At zero external magnetic field \( (B = 0) \), we have reproduced the previous results\(^{16} \). Furthermore, we have also performed the calculation for the dynamical spin structure factor \( S(k, ω) \) in Eq. (8) with a uniform external magnetic field, and the results of \( S(k, ω) \) in the \((k_x, k_y)\) plane for doping \( x = 0.15 \) with temperature \( T = 0.002J \) and Zeeman magnetic energy \( ε_B = 0.01J = 1.2 \text{ meV} \) (then the corresponding external magnetic field \( B \approx 20 \text{ Tesla} \)) at energy \( (a) ω = 0.08J = 9.6 \text{ meV} \), \( (b) ω = 0.31J = 37.2 \text{ meV} \), and \( (c) ω = 0.59J = 70.8 \text{ meV} \) are plotted in Fig. 1. In comparison with the previous results without a uniform external magnetic field\(^{16} \), our present most surprising results involve the external magnetic field dependence of the resonance scattering form, i.e., with increasing the external magnetic field \( B \), although the IC magnetic scattering from both low and high energies is rather robust, the incommensurate magnetic resonance scattering peak is broadened, and is shifted from the AF ordering wave vector \( Q \) to the IC magnetic scattering peaks with the incommensurability \( δ \). The main difference is that the resonance response occurs at an IC in the presence of a uniform external magnetic field, rather than commensurate in the case of zero external magnetic field. In this sense, we call such magnetic resonance as the IC magnetic resonance. Experimentally, the growth of the low energy IC magnetic resonance scattering due to the presence of an external magnetic field has been observed from the cuprate superconductor \( \text{La}_{2−x}\text{Sr}_x\text{CuO}_4 \), which is qualitatively consistent with our theoretical predictions. For cuprate superconductors, the upper critical magnetic field at which superconductivity is completely destroyed is \( 50 \text{ Tesla} \) or greater around the optimal doping\(^{24} \). Therefore the present result is remarkable because the magnitude of the applied external magnetic field is much less than the upper critical magnetic field of cuprate superconductors. It has been shown that the magnetic resonance scattering is very sensitive to the SC pairing, and the external magnetic field induced the IC magnetic resonance scattering is always accompanied with a breaking of the SC pairing\(^{24} \), this leads to a reduction of the SC transition temperature in cuprate superconductors.

Having shown the presence of the IC magnetic resonance scattering under a uniform external magnetic field, it is important to determine its dispersion as the outcome will allow a direct comparison of the magnetic excitation spectra with and without a uniform external magnetic field. In Fig. 2, we plot the evolution of the magnetic scattering peaks with energy for \( x = 0.15 \) in \( T = 0.002J \) with \( ε_B = 0.01J = 1.2 \text{ meV} \) (solid line). For comparison, the corresponding result for \( x = 0.15 \) in \( T = 0.002J \) with the same set of parameters except for \( ε_B = 0 \) (\( B = 0 \)) is also shown in Fig. 2 (dashed line). As in the previous work\(^{15} \), the dispersion of the magnetic scattering in the case of zero external magnetic field has
an hourglass shape. However, under a modest external magnetic field \( B \approx 20 \) Tesla, although there is no strong external magnetic field induced change for the IC magnetic scattering at higher energy \( \omega \sim 0.7J \), the magnetic scattering around both intermediate and low energies is dramatically changed, in qualitative agreement with the INS experiments.\(^{13,14,15}\) In particular, although the part above \( 0.16J \approx 19 \) meV seems to be an hourglass-like dispersion, this hourglass-like dispersion breaks down at lower energy \( \omega < 0.16J \approx 19 \) meV. These are much different from the dispersion in the case of zero external magnetic field.

Now we turn to discuss that how strong external magnetic field can induce the IC resonance scattering in cuprate superconductors in the SC state. We have made a series of calculations for the resonance energy at different external magnetic fields, and the result of the incommensurability of the IC resonance scattering \( \epsilon_r \) for \( x = 0.15 \) in \( T = 0.002J \) as a function of a uniform external magnetic field \( B \) is plotted in Fig. 3. Obviously, the incommensurability \( \epsilon_r \) increases with increasing the external magnetic field. For a better understanding of the influence of a uniform external magnetic field on the resonance scattering, we plot the dynamical spin structure factor \( S(k, \omega) \) in the \((k_x, k_y)\) plane for \( x = 0.15 \) and \( T = 0.002J \) with (a) \( \epsilon_B = 0.002J = 0.24 \) meV (then the corresponding external magnetic field \( B \approx 4 \) Tesla) and (b) \( \epsilon_B = 0.005J = 0.6 \) meV (then the corresponding external magnetic field \( B \approx 10 \) Tesla) at \( \omega = 0.31J = 37.2 \) meV in Fig. 4. In comparison with Fig. 1(b), we therefore find that there are two critical values of the Zeeman magnetic energy \( \epsilon^{(c)}_{B_1} \approx 0.002J = 0.24 \) meV (the corresponding critical external magnetic field \( B_{c1} \approx 4 \) Tesla) and \( \epsilon^{(c)}_{B_2} \approx 0.005J = 0.6 \) meV (the corresponding critical external magnetic field \( B_{c2} \approx 10 \) Tesla). When \( B > B_{c2} \), the external magnetic field is strong enough to induce the IC resonance scattering. On the other hand, when \( B_{c1} < B < B_{c2} \), the commensurate resonance scattering peak is broadened, and remains at the same energy position as the zero external magnetic field case with a comparable amplitude, which is furthermore in qualitative agreement with the INS experiments.\(^{14,15}\)
ergy, respectively. In this sense, the essential physics for certain critical wave vectors $k_i$ with $\omega_{i} = \omega_{k_i}$, the inverse of the imaginary part of the spin self-energy $\omega_{s}$ is obtained from Eqs. (6), (8), and (9), then the renormalized spin excitation spectrum at high energy can be reduced approximately as $(\omega - 2\varepsilon_B)^2 = \omega_s^2 + \text{Re}\Sigma(s)(k, \omega) \approx \omega$ in Eqs. (6), (8), and (9). This is why there is only a small influence of a modest external magnetic field on the IC magnetic scattering at high energy. However, around low and intermediate energies, this small Zeeman magnetic energy $\varepsilon_B$ in Eqs. (6), (8), and (9) plays an important role that reduces the range of the IC magnetic scattering at low energy and splits the commensurate resonance peak at zero external magnetic field into the IC resonance peaks, then the IC magnetic resonance scattering appears. Furthermore, at the heavily low energy regime $\omega \ll 0.16J$, the magnitude of the Zeeman magnetic energy $2\varepsilon_B$, the strong Zeeman magnetic energy $2\varepsilon_B = 0.02J$ is comparable with these incoming neutron energies, where both incoming lower neutron energy and Zeeman magnetic energy dominate the IC magnetic scattering, then the hourglass-like dispersion breaks down.

The physical interpretation to the above obtained results can be found from the property of the spin excitation spectrum. In contrast to the case of zero external magnetic field, the MF spin excitation spectrum has two branches, $\omega^{(1)}_k = \omega_k + 2\varepsilon_B$ and $\omega^{(2)}_k = \omega_k - 2\varepsilon_B$, in Eq. (4) under a uniform external magnetic field as mentioned in Sec. II. Since both MF spin excitation spectra $\omega^{(1)}(k)$ and $\omega^{(2)}(k)$ are strong external magnetic field dependent, this leads to that the renormalized spin excitation spectrum $(\Omega_k - 2\varepsilon_B)^2 = \omega_s^2 + \text{Re}\Sigma(s)(k, \Omega_k)$ in Eqs. (3) and (8) also is strong external magnetic field dependent. As in the case of zero external magnetic field, the dynamical spin structure factor $S(k, \omega)$ in Eq. (8) under a uniform external magnetic field has a well-defined resonance character, where $S(k, \omega)$ exhibits peaks when the incoming neutron energy $\omega$ is equal to the renormalized spin excitation, i.e.,

$$W(k_c, \omega) \equiv |(\omega - 2\varepsilon_B)^2 - \omega_{k_c}^2 - B_{k_c} \text{Re}\Sigma(s)(k_c, \omega)|^2 \approx 0,$$

(9)

for certain critical wave vectors $k_c = k^{(L)}_c$ at low energy, $k_c = k^{(I)}_c$ at intermediate energy, and $k_c = k^{(H)}_c$ at high energy, then the weight of these peaks is dominated by the inverse of the imaginary part of the spin self-energy $1/\text{Im}\Sigma(s)(k^{(L)}_c, \omega)$ at low energy, $1/\text{Im}\Sigma(s)(k^{(I)}_c, \omega)$ at intermediate energy, and $1/\text{Im}\Sigma(s)(k^{(H)}_c, \omega)$ at high energy, respectively. In this sense, the essential physics of the external magnetic field dependence of the dynamical spin response is almost the same as in the case of zero magnetic field. However, as seen from Eqs. (6), (8), and (9), a modest external magnetic field mainly affects the behavior of the dynamical spin response around low and intermediate energies, and therefore leads to some changes of the dynamical spin response around low and intermediate energies. This is followed by a fact that the magnitude of the applied uniform external magnetic field is much less than the upper critical magnetic field of cuprate superconductors, i.e., the Zeeman magnetic energy $2\varepsilon_B / J = 0.02 \ll 1$ in Eqs. (6), (8), and (9), then the renormalized spin excitation spectrum at high energy can be reduced approximately as $(\omega - 2\varepsilon_B)^2 = \omega_s^2 + \text{Re}\Sigma(s)(k, \omega) \approx \omega$ in Eqs. (6), (8), and (9). This is why there is only a small influence of a modest external magnetic field on the IC magnetic scattering at high energy. However, around low and intermediate energies, this small Zeeman magnetic energy $\varepsilon_B$ in Eqs. (6), (8), and (9) plays an important role that reduces the range of the IC magnetic scattering at low energy and splits the commensurate resonance peak at zero external magnetic field into the IC resonance peaks, then the IC magnetic resonance scattering appears. Furthermore, at the heavily low energy regime $\omega \ll 0.16J$, the magnitude of the Zeeman magnetic energy $2\varepsilon_B = 0.02J$ is comparable with these incoming neutron energies, where both incoming lower neutron energy and Zeeman magnetic energy dominate the IC magnetic scattering, then the hourglass-like dispersion breaks down.

**IV. SUMMARY AND DISCUSSIONS**

In summary, we have discussed the influence of a uniform external magnetic field on the dynamical spin response of cuprate superconductors in the SC state based on the kinetic energy driven SC mechanism. Our results show that the magnetic scattering around low and intermediate energies is dramatically changed with a modest external magnetic field. With increasing the external magnetic field, although the IC magnetic scattering from both low and high energies is rather robust, the commensurate magnetic resonance scattering peak is broadened. In particular, the part above $0.16J \approx 19 \text{ meV}$ seems to be an hourglass-like dispersion, which breaks down at the heavily low energy regime $\omega < 0.16J \approx 19 \text{ meV}$. The theory also predicts that the commensurate magnetic resonance scattering at zero external magnetic field is induced into the IC magnetic resonance scattering by applying a uniform external magnetic field large enough, which should be verified by further experiments.

From the INS experimental results, it is shown that although some of the IC magnetic scattering properties have been observed in the normal state, the magnetic resonance scattering is the main new feature that appears into the SC state. In particular, applying a uniform external magnetic field large enough to suppress superconductivity would yield a spectrum identical to that measured at normal state. Incorporating
these experimental results, our present result seems to show that the external magnetic field causes the behavior of the dynamical spin response to become more like that of the normal state. Moreover, in our present discussions, the magnitude of an applied external magnetic field is much less than the upper critical magnetic field of cuprate superconductors as mentioned above, and therefore we believe that both commensurate magnetic resonance scattering at zero external magnetic field and IC magnetic resonance scattering at an applied modest external magnetic field are universal features of cuprate superconductors.

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