Nonequilibrium Phase Transition in the Kinetic Ising model: Dynamical symmetry breaking by randomly varying magnetic field

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The nonequilibrium dynamic phase transition, in the two dimensional kinetic Ising model in presence of a randomly varying (in time but uniform in space) magnetic field, has been studied both by Monte Carlo simulation and by solving the mean field dynamic equation of motion for the average magnetisation. In both the cases, the time averaged magnetisation vanishes from a nonzero value depending upon the values of the width of randomly varying field and the temperature. The phase boundary lines are drawn in the plane formed by the width of the random field and the temperature. PACS number(s): 05.50.+q

1. INTRODUCTION

The problem of the random field Ising model (RFIM), having quenched random field, has been investigated both theoretically [1–4] and experimentally [5] in the last few years because it helps to simulate many interesting but complicated problems. The effects of a randomly quenched magnetic field on the critical behaviour near the ferromagnetic phase transition is the special focus of the modern research. The recent developments in this field can be found in a review article [6]. However, the dynamical aspects of the Ising system in presence of a randomly varying field is not yet studied thoroughly. It would be interesting to know if there is any dynamical phase transition in the presence of a randomly varying magnetic field.

For completeness and continuity, it would be convenient to review briefly the previous studies on the dynamical transition in the kinetic Ising model. Tomé and Oliviera [7] first observed and studied the dynamic transition in the kinetic Ising model in presence of a sinusoidally oscillating magnetic field. They solved the mean field (MF) dynamic equation of motion (for the average magnetisation) of the kinetic Ising model in presence of a sinusoidally oscillating magnetic field. By defining the order parameter as the time averaged magnetisation over a full cycle of the oscillating magnetic field they showed that the order parameter varies depending upon the value of the temperature and the amplitude of the oscillating field. Precisely, in the field amplitude and temperature plane they have drawn a phase boundary separating dynamic ordered (nonzero value of the order parameter) and disordered (order parameter vanishes) phases. They [7] have also observed and located a tricritical point (TCP), [separating the nature (discontinuous/continuous) of the transition] on the phase boundary line. However, such a transition, observed [7] from the solution of mean field dynamical equation, is not dynamic in true sense. This is because, for the field amplitude less than the coercive field (at temperature less than the transition temperature without any field), the response magnetisation varies periodically but asymmet-
The statistical distribution of the dynamic order parameter, near the dynamic transition point, has been studied by Sides et al. They observed that the distribution widens up (to a double hump from a single hump type) as one crosses the dynamic transition phase boundary. Since, the fluctuation increases as the width of a distribution increases, this observation is consistent with the independent observation of critical fluctuations of dynamic order parameter. They have also observed that the fluctuation of the hysteresis loop area becomes considerably large near the dynamic transition point.

In this paper, the dynamic phase transition has been studied, in the two dimensional kinetic Ising model in the presence of randomly varying (in time but uniform in space) magnetic field, both by MC simulation and by solving the meanfield dynamical equation. This paper is organised as follows: in section II the model and the MC simulation scheme with the results are given, in section III the meanfield dynamical equation and its solution with the numerical results are given and the paper ends with a summary of the work in section IV.

II. MONTE CARLO STUDY

A. The Model and the simulation scheme

The Hamiltonian, of an Ising model (with ferromagnetic nearest neighbour interaction) in presence of a time varying magnetic field, can be written as

\[ H = - \sum_{<ij>} J_{ij} s_i^z s_j^z - h(t) \sum_i s_i^z. \]  (2.1)

Here, \( s_i^z (\pm 1) \) is Ising spin variable, \( J_{ij} \) is the interaction strength and \( h(t) \) is the randomly varying (in time but uniform in space) magnetic field. The time variation of \( h(t) \) can be expressed as

\[ h(t) = \begin{cases} h_0 r(t) & \text{for } t_0 < t < t_0 + \tau \\ 0 & \text{elsewhere} \end{cases} \]  (2.2)

where \( r(t) \) is a random variable distributed uniformly between -1/2 and +1/2. The field \( h(t) \) varies randomly from \(-h_0/2\) to \(h_0/2\) and

\[ \frac{1}{\tau} \int_{t_0}^{t_0+\tau} h(t) dt = 0. \]  (2.3)

The system is in contact with an isothermal heat bath at temperature \( T \). For simplicity the values of all \( J_{ij} \) are taken equal to unity. The periodic boundary condition is used here.

A square lattice of linear size \( L (=100) \) has been considered. Initially all spins are taken directed upward and \( h(t) = 0 \). At any finite temperature \( T \), the dynamics of this system has been studied here by Monte Carlo simulation using Metropolis single spin-flip dynamics. The transition rate \( (s_i^z \rightarrow -s_i^z) \) is specified as

\[ W(s_i^z \rightarrow -s_i^z) = \min[1, \exp(-\Delta H/K_B T)] \]  (2.4)

where \( \Delta H \) is the change in energy due to spin flip and \( K_B \) is the Boltzmann constant which has been taken unity here for simplicity. Each lattice site is updated sequentially and one such full scan over the lattice is defined as the time unit (Monte Carlo step per spin or MCSS) here. The magnitude of the field \( h(t) \) changes after every MCSS following equation (2.2). The instantaneous magnetisation (per site), \( m(t) = (1/L^2) \sum_i s_i^z \) has been calculated. After bringing the system in steady state \((m(t)) \) gets stabilised with some fluctuations, the switch of the randomly varying magnetic field \( h(t) \) has been turned on (at time \( t_0 \) MCSS) and calculated the instantaneous magnetisation. The \( t_0 \) has been taken equal to \( 2 \times 10^8 \) and even more (3.15 \times 10^8) near the static ferro-para transition temperature (\( \sim 2.269.. \)). By inspecting the data and the time variation, it is observed that \( m(t) \) gets stabilised, for this choice of the value of \( t_0 \). The time averaged (over the active period of the magnetic field) magnetisation \( Q = \frac{1}{\tau} \int_{t_0}^{t_0+\tau} m(t) dt \) has been calculated over sufficiently large time \( \tau \) (1.75 \times 10^8). By changing the values of \( \tau \) the stabilisation of \( Q \) has been checked quite carefully. It is observed that, \( Q \) does not change much (apart from the small fluctuations) and this choice of the value of \( \tau \) is considered to be good enough to believe that the value of \( Q \) becomes stable. Here, \( Q \) plays the role of dynamic order parameter. It may be noted that, one measures the value of order parameter i.e., spontaneous magnetisation, in the same way in the case of normal ferro-para transition. But, there is a remarkable difference. In that case, system reaches a steady equilibrium state, however in this case the state lies in a nonequilibrium (time dependent) state. This dynamic order parameter \( Q \) is observed to be a function of width \( (h_0) \) of the randomly varying magnetic field and the temperature \( (T) \) of the system, i.e., \( Q = Q(h_0, T) \). Each value of \( Q \) has been calculated by averaging over at least 25 different random samples.

B. Results

Taking all spins are up \((s_i^z = +1)\) as the initial condition, the simulation was performed using the above form of the time varying field \( h(t) \). It has been observed numerically that, for a fixed values of \( h_0 \), if \( T \) increases, \( Q \) decreases continuously and ultimately vanishes at a fixed value of \( T \). Similarly, for a fixed temperature, if \( h_0 \) increases, the value of \( Q \) decreases and finally vanishes at a particular value of \( h_0 \). Figure 1 shows the time variation of magnetisation \( m(t) \) at a particular temperature \( T \) and for two different values of field width \( h_0 \). For a small value of \( h_0 \), the system remains in a dynamically symmetry broken phase (Fig. 1a) where the magnetisation
oscillates asymmetrically about zero line. As a result, the dynamical order parameter \( Q \) (the time averaged magnetisation) is nonzero. As the field increases, the system gets sufficient energy to flip dynamically in such a way that the magnetisation oscillates symmetrically (Fig. 1b) about zero line and as a result \( Q \) vanishes.

It is possible to let \( Q \) vanish, either by increasing the temperature \( T \) for a fixed field width \( h_0 \) or vice versa. It has been observed that for any fixed value of \( h_0 \) the transition is continuous. Two such transitions (for two different values of \( h_0 \)) is depicted in Fig. 2. So, in the plane formed by the temperature \( T \) and the field width \( (h_0) \), one can think of a boundary line, below which \( Q \) is nonzero and above which it vanishes. Figure 3 displays such a phase boundary in the \( h_0-T \) plane obtained numerically by Monte Carlo simulation. The transition observed here is continuous irrespective of the values of \( h_0 \) and \( T \), i.e., unlike the earlier cases \[9,7\], no tricritical point has been observed. It may be noted here that the transition temperature, for field width going to zero, reduces to the equilibrium (zero field) ferro-para transition point. In fig. 3, the overestimated (from the Onsager value \( T_c = 2.269 \)) value of the transition temperature in \( h_0 \rightarrow 0 \) limit, may be due to the small size of the system. It should be mentioned here, that all the results are obtained by using sequential updating scheme. Although, there is some study \[13\] regarding the updating scheme to simulate the dynamic processes, in the present study no significant deviation was found for these two different (sequential/random) updating techniques. Few data points of the phase boundary (in Fig. 3 marked by bullets) are obtained by using the random updating scheme and no significant deviation was observed.

III. MEANFIELD STUDY

A. Meanfield dynamical equation and solution

The meanfield dynamical equation of motion for the average magnetisation \( \langle m \rangle \) is

\[
\frac{dm}{dt} = -m + \tanh \left( \frac{m(t) + h(t)}{T} \right) \tag{3.1}
\]

where \( h(t) \) is the randomly varying external magnetic field satisfying the following condition

\[
\frac{1}{\tau} \int_{t_0}^{t_0+\tau} h(t) dt = 0, \tag{3.2}
\]

with the same distribution \( P(h) \) discussed earlier.

The equation \[3.1\] has been solved employing the fourth order Runge-Kutta method subjected to the above condition. The initial magnetisation is set equal to unity, which serves as a boundary condition. First, the magnetisation \( m(t) \) has been calculated for \( h(t) = 0 \) and brought the system into an equilibrium state and after that the switch of the randomly varying magnetic field has been turned on. The dynamic order parameter \( Q = \frac{1}{\tau} \int_{0}^{\tau} m(t') dt' \) has been calculated, from the solution \( m(t) \) of equation \[3.1\]. The \( Q \) is averaged over 25 different random samples.

B. Results

Observations similar to the MC case are made in this case. For quite small values of \( h_0 \) and \( T \) systems remains in a dynamically asymmetric phase \( (Q \neq 0) \) and gets into a dynamically symmetric \( (Q = 0) \) phase for higher values of \( h_0 \) and \( T \). It is observed that one can force \( Q \) to vanish continuously by tuning \( h_0 \) and \( T \). A phase boundary line for the dynamic transition is shown in Fig. 4. Here also, the limiting \( (h_0 \rightarrow 0) \) transition temperature reduces to the MF equilibrium (zero field) transition point \( (T_c^{MF} = 1) \). It may be noted here that the coordination number \( z (= 4 \) in two dimension) has been absorbed in the interaction strength \( J \) in calculating the \( T_c^{MF}(= 1) \) and the temperature (shown here in Fig. 4) is measured in the unit of \( Jz \).

IV. SUMMARY

The nonequilibrium dynamic phase transition, in the two dimensional kinetic Ising model in presence of a randomly varying (in time but uniform over the space) magnetic field, is studied both by Monte Carlo simulation and by solving the meanfield dynamical equation of motion. In both the cases, it is observed that the system remains in a dynamically symmetric phase \( (Q = 0) \) for large values of the width of the randomly varying magnetic field and the temperature. By reducing the value of field width and temperature one can bring the system in a dynamically symmetry broken phase \( (Q \neq 0) \). The time averaged magnetisation, i.e., the dynamic order parameter vanishes continuously depending upon the value of width of the randomly varying field and the temperature. The nature of the transition observed here is always continuous and the phase boundaries are drawn. It may be mentioned here that the dynamic responses, of Glauber kinetic Ising model, are studied \[8\] for a quasiperiodic time variation of the magnetic field.

There is some experimental evidence of dynamic transition. Recently, Jiang et al \[14\] observed the indications of dynamic transition, associated with the dynamical symmetry breaking, in the ultrathin Co/Cu(001) sample put in a sinusoidally oscillating magnetic field by magneto-optic Kerr effect. For small values of the amplitude of the oscillating field the \( m - h \) loop lies asymmetrically (dynamic ordered phase) in the upper half plane and becomes symmetric (dynamic disordered phase) for
higher values of the field amplitude. Very recently the dynamical symmetry breaking has been observed experimentally in highly anisotropic (Ising like) ultrathin ferromagnetic Fe/W(110) films and nicely depicted in Fig. 1 of Ref. [15]. However, the detailed quantitative study to draw the phase boundary is not yet done experimentally.

Very recently, the stationary properties of the Ising ferromagnet in presence of a randomly varying (having bimodal distribution) magnetic field are studied [16] in the meanfield approximation. The transition observed from the distribution of stationary magnetization is discontinuous. However this present study deals with the dynamical properties (dynamical symmetry breaking) of the system and the nonequilibrium transitions in the Ising ferromagnet in presence of randomly varying (uniformly distributed) magnetic field. In the former case the transition was observed from the distribution of stationary magnetisation. However, in the present study, this was observed from the temperature variation of dynamic order parameter associated with a dynamical symmetry breaking and the transition observed is continuous. An extensive numerical effort is required to characterise and to know the details of this dynamical phase transition. It would also be important to see whether the different kind of distribution of the randomly varying field would give different results.

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Figure captions

Fig. 1. Monte Carlo results of the time ($t$) variations of randomly varying field $h(t)$ and the response magnetisation $m(t)$ for $T = 1.7$ and (a) $h_0 = 1.0$ and (b) $h_0 = 3.0$. The symmetry (about the zero line) breaking is clear from the figure.

Fig. 2. Monte Carlo results of the temperature ($T$) variations of the dynamic order parameter $Q$ for two different values of $h_0$. (●) for $h_0 = 2.4$ and (◇) for $h_0 = 0.8$.

Fig. 3. The phase boundary for the dynamic transition obtained from MC simulations. The data points represented by (●) are obtained by using random updating and those represented by (◇) are obtained by sequential updating.

Fig. 4. The phase boundary for the dynamic transition obtained from the solution of the MF dynamic equation (equation 3.1).
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Fig. 1b, PRE, Acharyya
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