EMBEDDING LENS SPACES IN DEFINITE 4-MANIFOLDS

PAOLO ACETO AND JUNGHWAN PARK

Abstract. Every lens space has a locally flat embedding in a connected sum of 8 copies of the complex projective plane and a smooth embedding in n copies of the complex projective plane for some positive integer n. We show that there is no n such that every lens space smoothly embeds in n copies of the complex projective plane.

1. Introduction

Every closed 3-manifold embeds in $S^5$ [Hir61, Roh65, Wal65]. On the other hand, there are strong restrictions on which closed 3-manifolds can embed in $S^4$. It was shown by Hantzsche [Han38] that if a rational homology 3-sphere $Y$ embeds in $S^4$, then $H_1(Y;\mathbb{Z}) \cong G \oplus G$. In particular, no lens space (other than $S^3$ and $S^1 \times S^2$) embeds in $S^4$. Further, a punctured lens space $L(p,q)_0$ admits an embedding in $S^4$ if and only if $p$ is odd [Zee65, Eps65]. Also, we have a complete understanding of which connected sums of lens spaces can be smoothly embedded in $S^4$ by Donald [Don15] (see also [KK83, GL83, FSS7]). There are also various interesting results on embedding other 3-manifolds in $S^4$ [Kaw77, CH81, Hil96, CH98, BB08, Hil09, IM18].

Even though the embedding problem of lens spaces in $S^4$ is completely solved, there are many interesting generalizations. In this paper, we focus on the embedding problem of lens spaces in definite 4-manifolds (the embedding problem of lens spaces in spin 4-manifolds has been studied in [AGL17]). In [EL96], Edmonds and Livingston showed that every lens space smoothly embeds in $\#_n\mathbb{CP}^2$ for some positive integer $n$. Further, they showed that there is a family of lens spaces that do not have locally flat embeddings in $\#_n\mathbb{CP}^2$. Later, Edmonds [Edm05] showed that every lens space has a locally flat embedding in $\#_8\mathbb{CP}^2$ using independent works of Boyer [Boy93] and Stong [Ste93] which extend Freedman’s [Fre82] realization result. In contrast, we show that there is no $n$ such that every lens space smoothly embeds in $\#_n\mathbb{CP}^2$. Our main argument relies on Donaldson’s diagonalization theorem [Don87] and is based on the combinatorics of integral lattices.

Theorem 1.1. Let $L(p,q)$ be a lens space bounded by the canonical positive definite plumbed manifold $P(p,q)$ with the plumbing graph

$$
\begin{array}{cccc}
& a_1 & a_2 & \cdots & a_m \\
\bullet & \bullet & \cdots & \bullet
\end{array}
$$

Suppose that $a_i \geq 6$ for all $i$. If $L(p,q)$ smoothly embeds in a definite 4-manifold $W$ with $b_1(W) = 0$, then $b_2(W) > m$. In particular, if $L(p,q)$ smoothly embeds in $\#_n\mathbb{CP}^2$, then $n > m$.

Furthermore, we show that if a lens space $L(p,q)$ as in Theorem 1.1 bounds a smooth, positive definite 4-manifold, there is a strong restriction on its intersection form. This also reflects the big discrepancy between the smooth and the topological category in dimension 4 since for every 3-manifold $Y$ and a $\mathbb{Z}$-valued symmetric bilinear form $Q$ that presents the linking form of $Y$, $Q$ is realized as the intersection form of a simply

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connected, topological 4-manifold bounded by $Y$. For instance, every lens space bounds a simply connected, positive definite, topological 4-manifold $X$ with $b_2(X) \leq 6$. Recall that an integral lattice is a pair $(G, Q)$, where $G$ is a finitely generated free abelian group and $Q$ is a $\mathbb{Z}$-valued symmetric bilinear form defined on $G$. The integral lattice with the standard positive definite form is denoted by $(\mathbb{Z}^N, \text{Id})$. A morphism of integral lattices is a homomorphism of abelian groups which preserves the form. An embedding is an injective morphism. To a given 4-manifold $X$, we associate the integral lattice $(H_2(X; \mathbb{Z})/\text{Tors}, Q_X)$, where $Q_X$ is the intersection form on $X$.

**Theorem 1.2.** Let $L(p, q)$ be a lens space that satisfies the assumption of Theorem 1.1. If $L(p, q)$ bounds a smooth, positive definite 4-manifold $X$, then $b_2(X) \geq m$ and there is an embedding

$$(H_2(X; \mathbb{Z})/\text{Tors}, Q_X) \hookrightarrow (H_2(P(p, q); \mathbb{Z}), Q_{P(p, q)}) \oplus (\mathbb{Z}^{b_2(X)-m}, \text{Id}).$$

Moreover, $b_2(X) = m$ if and only if $X$ and $P(p, q)$ have isomorphic intersection forms.

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## 2. Proof of Theorem 1.1 and 1.2

We work in the smooth category and all manifolds are oriented. Recall that the lens space $L(p, q)$ is the result of $-p/q$ Dehn surgery on the unknot. Up to orientation preserving diffeomorphism, we may assume that $p > q > 0$. For the rest of this article we only consider lens spaces $L(p, q)$ that bound the canonical positive definite plumbed manifolds $P(p, q)$ with the plumbing graph $\Gamma_{p,q} :=$

\[
\begin{array}{cccc}
\bullet & \cdots & \bullet & \cdots & \cdots & \cdots & \cdots & \cdots & \bullet \\
\end{array}
\]

where $a_i \geq 6$ for all $i$.

Let $-L(p, q)$ be the lens space $L(p, q)$ with the reversed orientation, then there is an orientation preserving diffeomorphism between $-L(p, q)$ and $L(p, p-q)$. Using Riemenschneider’s point rule [Rie74] (see also [Lis07, Lec12]), we see that $L(p, p-q)$ bounds a canonical positive definite plumbed manifold $P(p, p-q)$ with the plumbing graph $\Gamma_{p,p-q} :=$

\[
\begin{array}{cccc}
\bullet & \cdots & \bullet & \cdots & \cdots & \cdots & \cdots & \cdots & \bullet \\
a_1 & a_2 & \cdots & a_{m-2} & a_{m-1} & a_m \\
\end{array}
\]

We denote the integral lattice associated to $P(p, q)$ as $(\mathbb{Z}\Gamma_{p,q}, Q_{p,q})$ and call it the integral lattice associated with $L(p, q)$. Similarly, we also have a dual positive definite integral lattice $(\mathbb{Z}\Gamma_{p,p-q}, Q_{p,p-q})$ associated with $L(p, p-q)$.

**Proposition 2.1.** If there is an embedding from $(\mathbb{Z}\Gamma_{p,p-q}, Q_{p,p-q})$ to $(\mathbb{Z}^N, \text{Id})$, then $N \geq m + \text{rk}(\mathbb{Z}\Gamma_{p,p-q})$.

**Proof.** We label the first $a_1$ vertices of $\Gamma_{p,p-q}$ as follows

\[
\begin{array}{cccc}
\bullet & \cdots & \bullet & \cdots & \cdots & \cdots \\
x_1 & x_{a_1-2} & x_{a_1-1} & x_{a_1} \\
\end{array}
\]

...
Let \( \{e_1, \ldots, e_N\} \) be the standard basis for \((\mathbb{Z}^N, \text{Id})\). By abuse of notation we identify \((\mathbb{Z} \Gamma_{p,p-q}, Q_{p,p-q})\) with its image in the standard lattice. It is straightforward to see that a chain of 2’s with length longer than 3 has a unique embedding. Hence up to reordering and changing sign of the standard basis elements we may write

\[ x_i = e_i + e_{i+1} \quad \text{for} \quad 1 \leq i \leq a_1 - 2. \]

Further, since \( x_{a_1-1} \) intersects with \( x_{a_1-2} \) once and has norm 3,

\[ x_{a_1-1} = e_{a_1-1} + \sum_{a_1-1 < j} e_j e_j. \]

Lastly, \( x_{a_1} \) has a trivial intersection with \( e_{a_1-1} \) since it is disjoint from the first chain of 2’s and it has norm 2. Therefore, \( x_{a_1-1} - e_{a_1-1} \) is disjoint from the first chain of 2’s and intersects \( x_{a_1} \) once. Now, if we only consider \( x_{a_1-1} - e_{a_1-1} \) and all the vertices that reside on the right hand side of \( x_{a_1-1} \), we get to the same situation as we have started with. Hence we can repeat the same argument to get the following identifications

\[
\begin{align*}
2 & \quad 2 & \quad \cdots & \quad 2 \\
\bullet & \quad \bullet & \quad \cdots & \quad \bullet
\end{align*}
\]

for each chain of 2’s where \( n_\ell = \sum_{k=1}^{\ell-1} a_k + (3 - \ell) \) and \( 2 \leq \ell \leq m - 1 \), and

\[
\begin{align*}
2 & \quad 2 & \quad \cdots & \quad 2 \\
\bullet & \quad \bullet & \quad \cdots & \quad \bullet
\end{align*}
\]

for the \( m \)-th chain of 2’s where \( n_m = \sum_{k=1}^{m-1} a_k + (3 - m) \). The \( \ell \)-th vertex with weight 3 shares one coordinate from its left chain and one coordinate from its right chain. Further, it needs an extra coordinate, which we denote it by \( e_{n_{\ell+1}-1} \).

In total, we have used \( \sum_{k=1}^{m} a_k - m + 1 \) coordinates which implies that \( N \geq \sum_{k=1}^{m} a_k - m + 1 \).

The proof is complete by observing that \( m + \text{rk}(\mathbb{Z} \Gamma_{p,p-q}) = \sum_{k=1}^{m} a_k - m + 1 \). \( \square \)

**Remark 2.2.** In fact, the proof of Proposition 2.1 shows that there is a **unique** embedding up to change of basis from \((\mathbb{Z} \Gamma_{p,p-q}, Q_{p,p-q})\) to \((\mathbb{Z}^N, \text{Id})\) when \( N \geq m + \text{rk}(\mathbb{Z} \Gamma_{p,p-q})\).

**Proposition 2.3.** If \( L(p,q) \) bounds a positive definite 4-manifold \( X \), then \( b_2(X) \geq m \).

**Proof.** Let \( W \) be the closed 4-manifold obtained by gluing \( X \) with \( P(p,p-q) \) along \( L(p,q) \). We obtain the following embedding

\[ (H_2(X;\mathbb{Z})/\text{Tors}, Q_X) \oplus (\mathbb{Z} \Gamma_{p,p-q}, Q_{p,p-q}) \hookrightarrow (H_2(W;\mathbb{Z})/\text{Tors}, Q_W). \]

Further, by Donaldson’s diagonalization theorem \([\text{Don87}]\) we have

\[ (H_2(W;\mathbb{Z})/\text{Tors}, Q_W) \cong (\mathbb{Z}^{b_2(W)}, \text{Id}). \]

Combining Proposition 2.1 with \( b_2(W) = b_2(X) + \text{rk}(\mathbb{Z} \Gamma_{p,p-q}) \) completes the proof. \( \square \)

**Proof of Theorem 1.1.** Suppose \( L(p,q) \) smoothly embeds in a definite 4-manifold \( W \). Since \( L(p,q) \) embeds in \( W \) if and only if \( L(p,q) \) embeds is \(-W\), we may assume that \( W \) is positive definite. Then \( L(p,q) \) separates \( W \) into two positive definite components, the closures of which we denote by \( X_1 \) and \( X_2 \). Note that by the MayerVietoris sequence we have \( b_2(X_1) + b_2(X_2) = b_2(W) \) and the result follows from Proposition 2.3 and the fact that \( L(p,q) \) does not bound a rational ball (see \([\text{Lis07}]\)). \( \square \)
The rest of the section is devoted to proving Theorem 1.2. Suppose there is an embedding of an integral lattice \( \phi : \Gamma \hookrightarrow \Gamma' \). Then the orthogonal complement of \( \Gamma \) with respect to \( \phi \) is defined as follows,

\[
\Gamma_{\phi}^\perp := \{ x \in \Gamma' \mid x \cdot \phi(y) = 0 \text{ for all } y \in \Gamma \}.
\]

**Proposition 2.4.** Suppose there is an embedding

\[
\phi : (Z\Gamma_{p,p,q}, Q_{p,p,q}) \hookrightarrow (Z^N, \text{Id}),
\]

then \((Z\Gamma_{p,p,q}, Q_{p,p,q})_{\phi}^\perp \cong (Z\Gamma_{p,q}, Q_{p,q}) \oplus (Z^{N-m-rk(Z\Gamma_{p,p,q})}, \text{Id})\). In particular, if \( N = m + rk(Z\Gamma_{p,p,q}) \), then \((Z\Gamma_{p,p,q}, Q_{p,p,q})_{\phi}^\perp \cong (Z\Gamma_{p,q}, Q_{p,q})\).

**Proof.** From Proposition 2.1 and Remark 2.2, we know that there is a unique embedding up to change of basis from \((Z\Gamma_{p,p,q}, Q_{p,q})\) to \((Z^N, \text{Id})\). Hence we may decompose \( \phi \) as follows

\[
\phi : (Z\Gamma_{p,p,q}, Q_{p,p,q}) \hookrightarrow (Z^{m+rk(Z\Gamma_{p,p,q})}, \text{Id}) \oplus (Z^{N-m-rk(Z\Gamma_{p,p,q})}, \text{Id}),
\]

where the image of \( \phi \) on the second summand is trivial. Let \( \pi \) be the projection map from \((Z^N, \text{Id})\) to \((Z^{m+rk(Z\Gamma_{p,p,q})}, \text{Id})\), then we have the following identification

\[
(Z\Gamma_{p,p,q}, Q_{p,p,q})_{\phi}^\perp \cong (Z\Gamma_{p,p,q}, Q_{p,p,q})_{\phi \circ \pi} \oplus (Z^{N-m-rk(Z\Gamma_{p,p,q})}, \text{Id}).
\]

Let \( W \) be the closed 4-manifold obtained by gluing \( P(p,q) \) with \( P(p,p-q) \) along \( L(p,q) \). Using Donaldson’s diagonalization theorem [Don77], we have an embedding

\[
\psi : (Z\Gamma_{p,q}, Q_{p,q}) \oplus (Z\Gamma_{p,p,q}, Q_{p,p-q}) \hookrightarrow (Z^{m+rk(Z\Gamma_{p,p,q})}, \text{Id}).
\]

Again, since there is a unique embedding up to change of basis from \((Z\Gamma_{p,p,q}, Q_{p,p-q})\) to \((Z^{m+rk(Z\Gamma_{p,p,q})}, \text{Id})\), we may assume that the embedding \( \psi \) restricted to \((Z\Gamma_{p,p,q}, Q_{p,p-q})\), denoted by \( \psi_{p,p,q} \), coincides with \( \pi \circ \phi \). In particular,

\[
(Z\Gamma_{p,p,q}, Q_{p,p-q})_{\psi,\phi}^\perp \cong (Z\Gamma_{p,p,q}, Q_{p,p-q})_{\psi_{p,p,q}}^\perp.
\]

Further, we can use the coordinates from the proof of Proposition 2.1

By restricting \( \psi \) to \( Z\Gamma_{p,q} \), denoted by \( \psi_{p,q} \), we have

\[
\psi_{p,q} : (Z\Gamma_{p,q}, Q_{p,q}) \hookrightarrow (Z\Gamma_{p,q}, Q_{p,q})_{\phi_{p,q}}^\perp.
\]

Now, suppose \( x = \sum_{i=1}^{N} c_i e_i \in (Z\Gamma_{p,p,q}, Q_{p,p-q})_{\psi_{p,p,q}}^\perp \). Since \( x \) needs to have trivial intersections with all the chain of \( 2's \), we have

\[
c_1 = \cdots = c_{a_1-1}, c_{a_\ell} = \cdots = c_{n_\ell+a_\ell-3}, \text{ for } 2 \leq \ell \leq m-1, \text{ and } c_{n_m} = \cdots = c_{n_m+a_m-2}
\]

where \( n_\ell = \sum_{\ell=1}^{\ell-1} a_\ell + (3-\ell) \) for \( 2 \leq \ell \leq m \). Further, \( x \) has trivial intersections with vertices with weight 3. This implies

\[
c_{n_{\ell+1}-2} + c_{n_{\ell+1}-1} + c_{n_{\ell+1}} = 0, \text{ for } 1 \leq \ell \leq m-1.
\]

From the above relations, we see that \( \{ x_\ell \} \) forms a basis for \((Z\Gamma_{p,p,q}, Q_{p,p-q})_{\psi_{p,p,q}}^\perp\), where

\[
x_1 = e_1 - e_2 + \cdots + (-1)^{a_1} e_{n_2-1},
\]

\[
x_\ell = e_{n_\ell-1} - e_{n_\ell} + \cdots + (-1)^{a_\ell} e_{n_{\ell+1}-1}, \text{ for } 2 \leq \ell \leq m-1,
\]

\[
x_m = e_{n_m-1} - e_{n_m} + \cdots + (-1)^{a_m} e_{n_m+a_m-2}.
\]

Finally, it is straightforward to check that the matrix, denoted by \( M \), that represents the intersection form of \((Z\Gamma_{p,p,q}, Q_{p,p-q})_{\psi_{p,p,q}}^\perp\) with respect to the basis \( \{ x_\ell \} \) coincides with the matrix, denoted by \( M_{p,q} \), that represents \( Q_{p,q} \) with respect to the obvious basis.
for $\mathbb{Z}\Gamma_{p,q}$. Note that we have $M = P^T M_{p,q} P$ where $P$ is a matrix that represents $\psi_{p,q}$. This implies that $P$ is unimodular and $\psi_{p,q}$ is an isomorphism. Then the result follows from (2.1) and (2.2). □

Proposition 2.4 is motivated by [ACP18] Proposition 4.1. By restricting to a smaller family of lens spaces, Proposition 2.4 gives the same conclusion as [ACP18] Proposition 4.1 with a weaker assumption. We are now ready to prove Theorem 1.2.

Proof of Theorem 1.2. Let $W$ be the closed 4-manifold resulting from gluing $X$ and $P(p, p - q)$ along $L(p, q)$. Again, by Donaldson’s theorem [Don87] we have an embedding

$$\psi: (H_2(X; \mathbb{Z})/\text{Tors}, Q_X) \oplus (\mathbb{Z}\Gamma_{p,p-q}, Q_{p,p-q}) \hookrightarrow (\mathbb{Z}^{b_2(X)} + \text{rk}(\mathbb{Z}\Gamma_{p,p-q}), \text{Id}).$$

Let $\psi_{p,p-q}$ be the embedding obtained by restricting $\psi$ to $\mathbb{Z}\Gamma_{p,p-q}$. Further, by restricting $\psi$ to $H_2(Z; \mathbb{Z})/\text{Tors}$ we have the following embedding

$$\psi|_{H_2(X;\mathbb{Z})/\text{Tors}}: (H_2(X; \mathbb{Z})/\text{Tors}, Q_X) \hookrightarrow (\mathbb{Z}\Gamma_{p,p-q}, Q_{p,p-q})^\perp_{\psi_{p,p-q}}.$$

Then the first part of the theorem follows from Proposition 2.3 and Proposition 2.4.

Suppose now that $m = b_2(X)$, then by Proposition 2.4 we have

$$(\mathbb{Z}\Gamma_{p,p-q}, Q_{p,p-q})^\perp_{\psi_{p,p-q}} \cong (\mathbb{Z}\Gamma_{p,q}, Q_{p,q}).$$

Let $M_X$ and $M_{p,q}$ be matrices that represent $Q_X$ and $Q_{p,q}$, respectively. Then $M_X = P^T M_{p,q} P$ where $P$ is a matrix that represents $\psi|_{H_2(X;\mathbb{Z})/\text{Tors}}$. Recall that $M_X$ presents a subgroup of $H_1(L(p,q); \mathbb{Z})$ (see, for instance, [OS06] Section 2) and $M_{p,q}$ presents $H_1(L(p,q); \mathbb{Z})$. In particular, $\det(M_X) \leq p$ and $\det(M_{p,q}) = p$, which implies that $P$ is unimodular and concludes the proof. □

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Mathematical Institute University of Oxford, Oxford, United Kingdom
E-mail address: paoloaceto@gmail.com
URL: www.maths.ox.ac.uk/people/paolo.aceto

School of Mathematics, Georgia Institute of Technology, Atlanta, GA, USA
E-mail address: junghwan.park@math.gatech.edu
URL: people.math.gatech.edu/~jpark929/