Study of nuclear giant resonances using a Fermi-liquid method

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The nuclear giant resonances are studied by using a Fermi-liquid method, and the nuclear collective excitation energies of different values of $l$ are obtained, which are fitted with the centroid energies of the giant resonances of spherical nuclei, respectively. In addition, the relation between the isovector giant resonance and the corresponding isoscalar giant resonance is discussed.

Keywords: Collective excitation; Giant resonance; Fermi-liquid method.

In nuclear physics, the Landau parameters are derived microscopically from the ground state energy in the relativistic mean-field approximation, and Landau Fermi liquid theory is used to describe the thermodynamics properties of the nuclear systems, such as the compressibility, the symmetric energy and the hydrodynamics sound velocities.$^1$ This theory is also studied with an effective chiral Lagrangian to obtain the properties of the nuclear ground state and the link between an effective QCD theory and the nuclear many body theory.$^2,3$ Moreover, the low momentum nucleon-nucleon potential is applied to calculate the effective interaction between quasi-particles near the Fermi surface, and then the static properties of the nuclear matter are extracted.$^4$

Landau Fermi liquid theory is an important method to describe the low energy collective excitation properties of many-body systems, and it can be used to study the problem on giant resonances of nuclei, which is still an hot topic in nuclear physics.$^5$ With several typical methods, such as the random phase approximation with Skyrme interactions,$^6$ the relativistic random phase approximation,$^7,9$ the centroid energies and strength distributions of the giant resonances of nuclei are calculated and compared with the experimental data.
In this work, I will try to calculate the collective excitation energy of the nuclear system within a Fermi-liquid model, which is generalized to the three-dimension situation with the spin of nucleons included. In the calculation, a Lagrangian of Walecka model is used as a microscopic input. The calculation results will be compared with the experimental data of the nuclear giant resonance of some spherical nuclei. Furthermore, I hope to check whether the contribution of Dirac sea of nucleons is essential in the study of collective excitations of nuclear many-body systems.

The liquid equation of motion of the quasi-nucleon in the momentum space can be written as

\[ i \frac{\partial}{\partial t} u_\alpha (l, \vec{q}, t) = H u_\alpha (l, \vec{q}, t), \]

where the Hamiltonian

\[ H_{l, \nu} = q (a_l \delta_{l+1, \nu} + a_{l-1} \delta_{l-1, \nu}) \]
\[ \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l, l) \right)^{1/2} \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right)^{1/2} \]

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\[ a_l = \sqrt{\frac{(l + 1)^2}{(2l + 1)(2l + 3)}} \]

and \( v_F \) the Fermi velocity of nucleons in the relativistic mean-field approximation. The eigenvalues of \( H \) would give us the frequencies of the collective excitation modes of the nuclear matter. Moreover, it can be seen from Eq. (2) that the exchange interaction between nucleons \( f_F(l, l) \) causes the collective excitation of the nuclear matter directly.

The parameters in Ref. are used in the calculation, i.e., \( g_\sigma = 10.47 \), \( g_\omega = 13.80 \), \( m_\sigma = 520 MeV \), \( m_\omega = 783 MeV \) and \( M_N = 939 MeV \). Since the nucleon near the Fermi surface would be more possible to be excited, we set the value of nucleon momentum \( |\vec{q}| = k_F = 1.36 fm^{-1} \) in the calculation. When \( f_F(l, l) = 0 \), the Hamiltonian \( H \) has a continuous spectrum and it generates the particle-hole continuum of the nuclear matter in the relativistic mean-field approximation. However, if the value of \( f_F(l, l) \) is large enough, in addition to the continuum eigenvalues, the spectrum of \( H \) has isolated positive and negative eigenvalues. Actually, the mode with a positive eigenvalue corresponds to the creation of a nuclear collective excitation mode, while the the mode with a negative eigenvalue corresponds to the annihilation of a nuclear collective excitation mode.
The nuclear isoscalar giant resonances actually correspond to the nuclear collective excitations with different values of $l$. However, the nuclear isovector giant resonances correspond to the nuclear collective excitation states that the collective excitation of protons with the energy $E_S(l)$ is creating, while the collective excitation of neutrons with the energy $E_S(l)$ is annihilating, and vice versa. Hence, the energy of the nuclear isovector giant resonance is about twice of the corresponding isoscalar giant resonance in the nuclear matter, i.e.,

$$E_V(l) = E_S(l) - (-E_S(l)) = 2E_S(l).$$

The experimental data on the nuclear giant resonances in Ref. 14–19 demonstrate that the relation between the energy of nuclear isovector giant resonance and that of nuclear isoscalar giant resonance in Eq. (3) is correct approximately except for the case $l = 1$, which will be discussed in detail specially. In follows, the giant resonance energies with different values of $l$ in the nucleus will be calculated within the Fermi-liquid model.

The nuclear giant monopole mode, $l = 0$, is also called as breathing mode of the nucleus. Supposed the proton and neutron densities can be calculated approximately:

$$\rho_p = \rho_0 \frac{Z}{A}, \quad \rho_n = \rho_0 \frac{N}{A},$$

the calculated energies for isoscalar and isovector giant monopole resonances of nuclei $^{208}\text{Pb}$, $^{144}\text{Sm}$, $^{116}\text{Sn}$, $^{90}\text{Zr}$, $^{40}\text{Ca}$ and their corresponding experimental values are listed in Table 1. Since the Fermi momentum of protons $k_F(p)$ is different from that of neutrons, the collective excitation energies of protons and neutrons, $E_0(p)$ and $E_0(n)$, are different from each other. When the effective nucleon mass takes a large value, i.e., $M_N^* = 0.742M_N$, the calculation results of the proton excitation energy for heavy nuclei, such as $^{208}\text{Pb}$, $^{144}\text{Sm}$ and $^{116}\text{Sn}$, are fitted with the corresponding experimental centroid energies of the nuclear isoscalar monopole resonance $E_{S\exp}^S$, respectively. However, for those light nuclei, $^{90}\text{Zr}$ and $^{40}\text{Ca}$, we must reduce the effective nucleon mass to $M_N^* = 0.717M_N$, and then the reasonable calculation results fitted with the experimental data are obtained. Moreover, the sum of the excitation energies of protons and neutrons $E_0(p) + E_0(n)$ should correspond to the nuclear isovector giant monopole energy. In Table 1, We can find that calculated values of isovector giant monopole energies for $^{208}\text{Pb}$, $^{90}\text{Zr}$ and $^{40}\text{Ca}$ are in the range of the experimental errors.

The dipole deformation of the nucleus is really a shift of the center of mass. Thus the isovector giant dipole resonance of the nucleus actually
Table 1. The Fermi momenta and the $l = 0$ collective excitation energies of protons and neutrons for different nuclei with different effective nucleon masses. The corresponding experimental values for the nuclear isoscalar giant monopole resonances $E_{0}^{SS}$ are taken from Ref.\textsuperscript{15} and the experimental values for the nuclear isovector giant monopole resonances $E_{0}^{SV}$ from Refs.\textsuperscript{17–19}

| $l = 0$ | $^{208}Pb$ | $^{144}Sm$ | $^{116}Sn$ | $^{90}Zr$ | $^{40}Ca$ |
|---------|---------|--------|--------|--------|--------|
| $M_{N}^{*}/M_{N}$ | 0.742 | 0.742 | 0.742 | 0.717 | 0.717 |
| $k_{F}(p)$ ($fm^{-1}$) | 1.26 | 1.29 | 1.29 | 1.31 | 1.36 |
| $k_{F}(n)$ ($fm^{-1}$) | 1.45 | 1.42 | 1.42 | 1.41 | 1.36 |
| $E_{0}(p)$ (MeV) | 16.28 | 15.26 | 15.26 | 17.57 | 15.58 |
| $E_{0}(n)$ (MeV) | 7.05 | 9.00 | 9.00 | 13.13 | 15.58 |
| $E_{0}(p) + E_{0}(n)$ (MeV) | 23.33 | 24.26 | 24.26 | 30.7 | 31.16 |
| $E_{SS}^{gg}$ (MeV) | 14.17 ± 0.28 | 15.39 ± 0.28 | 16.07 ± 0.12 | 17.89 ± 0.20 | − |
| $E_{SS}^{gg}$ (MeV) | 26.0 ± 3.0 | − | − | 28.5 ± 2.6 | 31.1 ± 2.2 |

The calculation results and the corresponding experimental centroid energies are listed in Table 2. Similarly, we must adjust the effective nucleon mass to obtain the collective excitation energies fitted with the experimental data. It is apparent that in order to obtain a more correct excitation energy, the effective nucleon mass must take a larger value for heavy nuclei, but a smaller value for light nuclei. For the nucleus $^{208}Pb$, the summation of the collective excitation energies of protons and neutrons is equal to the experimental energy of the isoscalar giant dipole resonance, approximately.

The calculation results and the corresponding experimental centroid energies of the giant quadrupole resonances of different nuclei are listed in Table 3. The experimental value of the isovector giant quadrupole resonance energy is just twice of the isoscalar giant quadrupole resonance energy for $^{208}Pb$, and it manifests the relation between the nuclear isovector giant resonance and the corresponding isoscalar giant resonance in Eq. (3) is correct. Actually, the experimental values for the nuclear giant quadrupole resonance of other nuclei, and even for their monopole giant resonance satisfy the relation in Eq. (3), approximately.

In Table 3, Since the average neutron density is larger than the saturation density of the nuclear matter, the collective excitation energies of the
Table 2. The Fermi momenta and the $l = 1$ collective excitation energies of protons and neutrons for different nuclei with different effective nucleon masses. The corresponding experimental values for the nuclear isoscalar giant dipole resonances $E_{\exp}^S$ are taken from Ref.\textsuperscript{20}, and the experimental value for the nuclear isovector giant dipole resonance $E_{\exp}^V$ from Ref.\textsuperscript{16}.

| $l = 1$ | $^{208}$Pb | $^{90}$Zr | $^{40}$Ca |
|---------|-----------|---------|---------|
| $M_N^*/M_N$ | 0.755 | 0.742 | 0.70 |
| $k_F(p)$ (fm$^{-1}$) | 1.26 | 1.31 | 1.36 |
| $k_F(n)$ (fm$^{-1}$) | 1.45 | 1.41 | 1.36 |
| $E_1(p)$ (MeV) | 15.53 | 15.56 | 19.58 |
| $E_1(n)$ (MeV) | 6.57 | 11.37 | 19.58 |
| $E_1(p) + E_1(n)$ (MeV) | 22.1 | 26.93 | 39.16 |
| $E_{\exp}^S$ (MeV) | 22.5 | − | − |
| $E_{\exp}^V$ (MeV) | 13.5 ± 0.2 | 16.5 ± 0.2 | 19.8 ± 0.5 |

nuclei $^{208}$Pb and $^{90}$Zr for $l = 2$ are less than 10MeV. These values might correspond to the low-lying excitation states in heavy nuclei, which are in the range of $2 − 6$MeV for $^{208}$Pb.\textsuperscript{22}

Table 3. The Fermi momenta and the $l = 2$ collective excitation energies of protons and neutrons for different nuclei. The corresponding experimental values for the nuclear isoscalar giant quadrupole resonances $E_{\exp}^S$ are taken from Ref.\textsuperscript{14}, and the experimental values for the nuclear isovector giant quadrupole resonances $E_{\exp}^V$ from Refs.\textsuperscript{7,21}.

| $l = 2$ | $^{208}$Pb | $^{90}$Zr | $^{40}$Ca | $^{16}$O |
|---------|-----------|---------|---------|---------|
| $M_N^*/M_N$ | 0.742 | 0.742 | 0.69 | 0.69 |
| $k_F(p)$ (fm$^{-1}$) | 1.26 | 1.31 | 1.36 | 1.36 |
| $k_F(n)$ (fm$^{-1}$) | 1.45 | 1.41 | 1.36 | 1.36 |
| $E_0(p)$ (MeV) | 15.02 | 13.16 | 18.54 | 18.54 |
| $E_0(n)$ (MeV) | 5.84 | 8.27 | 18.54 | 18.54 |
| $E_0(p) + E_0(n)$ (MeV) | 20.86 | 21.43 | 37.08 | 37.08 |
| $E_{\exp}^S$ (MeV) | 10.9 ± 0.1 | 14.41 ± 0.1 | 17.8 ± 0.3 | 20.7 |
| $E_{\exp}^V$ (MeV) | 22 | − | 32.5 ± 1.5 | − |

The Fermi-liquid model in Ref.\textsuperscript{10} is extended to the 3-dimensional Fermion system with the spin taken into account, and then by using the effective Lagrangian of the linear $\sigma − \omega$ model, the Fermi-liquid function is obtained and the isoscalar and isovector giant resonances of some spherical nuclei are studied. We find the centroid energies of the isoscalar giant resonances just correspond to the positive isolated energy levels of the nuclear collective excitation with different values of $l$, respectively, while the
isovector giant resonances except $l = 1$ correspond to the modes that protons(neutrons) are in the creation state of the collective excitation and neutrons(protons) are in the annihilation state of the same $l$. The low energy excitation of the nuclear giant quadrupole resonance, might be described by the collective excitation of neutrons in the nuclei.

It should be pointed out that the exchange interaction between nucleons plays an important role in the calculation of collective excitations of nuclear systems by using a Fermi-liquid model. Dirac sea of nucleons does not have any influence on the properties of giant resonances of nuclei.

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