Secondary star formation within massive star clusters: origin of multiple stellar populations in globular clusters

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ABSTRACT
We numerically investigate whether and how gaseous ejecta from AGB stars can be converted into new stars within originally massive star clusters (MSCs) in order to understand the origin of multiple stellar populations in globular clusters (GCs). We adopt a scenario in which (i) MSCs with masses of $M_s$ can be formed from high-mass, high-density giant molecular clouds (GMCs) in their host galactic building blocks embedded in dark matter haloes at high redshifts, and (ii) their evolution therefore can be significantly influenced by $M_s$, their initial locations and physical properties of their hosts. Our 3D hydrodynamical simulations show that gaseous ejecta from AGB stars can be retained within MSCs and consequently converted into new stars very efficiently in the central regions of MSCs, only if $M_s$ exceeds a threshold mass ($M_{th}$) of $\approx 10^6 M_\odot$. The new stars can correspond to the ‘second generation (SG)’ of stars with higher Na and lower O abundances observed in GCs. Star formation efficiencies during the formation of SG stars within MSCs with $M_s \geq M_{th}$ can be rather high (0.3–0.9) so that very compact new clusters within original MSCs can be formed. $M_s$ should be as large as $10^6–10^7 M_\odot$ to explain the observed large fraction of SG stars in the present ordinary Galactic GCs, because new stars can consist of only 1–4 per cent among all stars for the standard initial mass function. Nuclear MSCs are found to retain much more effectively the AGB ejecta and convert more efficiently the gas into new stars, owing to the much deeper gravitational potential of their hosts. Capture and accretion of cold molecular gas (or small GMCs) by forming MSCs themselves can be mechanisms for mixing (i.e., dilution) of AGB ejecta with cold pristine gas. We suggest that both $M_s$ and their locations within their hosts can determine whether abundance spread can be seen only in light elements or even in heavy ones. We discuss how and in what time-scale MSCs preferentially lose old stars owing to tidal stripping by their host galactic building blocks. We also suggest that the origin of the intermediate-age GCs with possible age spread of $\sim 100$ Myr yet apparently no/little abundance spread in light elements in the LMC is closely associated with their incapability to retain the AGB ejecta owing to their low masses.

Key words: stars: formation – globular clusters: general – galaxies: star clusters: general.

1 INTRODUCTION
It has long been discussed both observationally and theoretically why some of the Galactic GCs show star-to-star inhomogeneity among the light elements of stars and what physical mechanisms are responsible for the inhomogeneity (e.g. Cottrell & Da Costa 1981; Sneden et al. 1992; Norris & Da Costa 1995; Cannon et al. 1998; D’Antona & Caloi 2004; Fenmer et al. 2004; Gratton, Sneden & Carretta 2004; Norris 2004; Lee et al. 2005; Smith, Briley & Harbeck 2005; Bekki et al. 2007b; Alves-Brito et al. 2008; Catelan 2008; Kayser et al. 2008; Piotto 2008; Da Costa et al. 2009; Marcolini et al. 2009; Yong et al. 2009; Carretta et al. 2010; D’Ercole et al. 2010; Romano et al. 2010; van Loon 2010). Although such star-to-star inhomogeneity was discovered in 1970s and 1980s (e.g. Cohen 1978; Peterson 1980; Norris et al. 1981; Leep, Wallerstein & Oke 1986), recent statistical studies for a larger number of the Galactic GCs have established that the presence of multiple stellar populations is a universal phenomena seen across most of the Galactic GCs (e.g. Carretta et al. 2010). The latest observational results by Ferraro et al. (2009) have revealed that the Galactic metal-rich GC Tarzan 5 has two different stellar populations with different abundances of heavy elements. Lee et al. (2009) has also suggested a significant fraction of the Galactic GCs have two different populations with different abundances of heavy elements based on colour–magnitude diagrams of the GC stars in $hk$-bands.
Now large star-to-star abundance variations have been confirmed in almost unevolved stars in some of the Galactic GCs (e.g. Gratton et al. 2001; Ramirez & Cohen 2002; Bedin et al. 2004; Carretta et al. 2004; Gratton 2004; Piotto et al. 2005, 2007; D’Orazi et al. 2010a), which strongly suggests that after the first generation (FG) of stars formed within the forming GCs, gas chemically mixed with gaseous ejecta of FG stars was converted into the second generation (SG) of stars (and even third and fourth generations). Extensive investigations have been carried out on how chemical properties of SG stars (e.g. fractions of SG stars) correlate with their global internal properties (e.g. magnitudes and ellipticities) and with locations and 3D motions with respect to the Galactic centre to understand the origin of SG stars and physical relationships between formation of FG and SG stars (e.g. Carretta 2006; Carretta et al. 2010).

Following these observational developments, theoretical studies have considered that SG stars can form from gaseous ejecta either by AGB stars (‘AGB scenario’; D’Antona & Caloi 2004; Karakas et al. 2006) or by fast rotating massive stars (‘FRMS’ scenario; Prantzos & Charbonnel 2006; Desselin et al. 2007) and thereby investigated whether the observed physical properties of the Galactic GCs can be explained by their models. Although it has not been determined which of the two can explain observations in a self-consistent manner, a number of authors suggested that the AGB scenario is more physically viable because AGB ejecta are more likely to be converted into new stars owing to their low ejection velocities (e.g. Renzini 2008).

Recent observations have provided the following two key results which can give strong constraints on any theory for GC formation: (i) the large fraction (typically 0.67) of SG stars in each individual Galactic GCs and (ii) the observed Na–O and Mg–Al anticorrelations between cluster stars (e.g. Carretta et al. 2010) for a comprehensive study for these two. The first key result suggests that the original stellar systems (i.e., FG stars) need to be much more massive than the present GCs unless we adopt very unusual and unrealistic IMFs (e.g. Smith & Norris 1982; D’Antona & Caloi 2004); the original systems are either very massive star clusters or dwarf galaxies hosting GCs (e.g. Bekki & Norris 2006). If the original systems are really massive ones, then the FG stars need to be preferentially lost while SG ones remain the same so that the observed typical fraction (~0.67) of SG stars can be explained.

Although a number of theoretical works based on the AGB scenario tried to explain the second key observational result in a self-consistent manner (e.g. Fenner et al. 2004; D’Antona et al. 2005; Ventura & D’Antona 2006, 2008; Bekki et al. 2007b; D’Antona & Ventura 2007; D’Ercole et al. 2010), their models appear to have not yet explained all of the relevant observations on chemical abundances of the Galactic GCs in a fully self-consistent manner. Chemical evolution models based on the FRMS scenario have not yet been fully explored so that the validity of the FRMS scenario cannot be currently assessed.

In these previous models, it is assumed that AGB ejecta can be converted into new stars (i.e. formation of SG stars) within already existing clusters. However, it would not be so obvious that such secondary star formation can occur within clusters, given the shallow gravitational potential wells of clusters and the small mass fraction of AGB ejecta. Therefore, secondary star formation processes within clusters should be investigated by numerical simulations that can include various physical processes within clusters (e.g. retention of AGB ejecta). D’Ercole et al. (2008) first investigated whether SG stars can be formed from the gaseous ejecta of AGB stars of FG with $M_\star = 10^7 M_\odot$. However, their models are based on 1D hydrodynamical simulations and have limitations in predicting 3D structures and kinematics of final stellar systems. 3D stellar and gas dynamical numerical simulations with a plausible model for star formation are ideal to investigate secondary star formation within clusters and can furthermore provide theoretical predictions that can be compared with the observed differences in 3D structures and kinematics between FG and SG stars (e.g. Norris et al. 1997; Ferraro, Bellazzini & Pancino 2002; Sollima et al. 2005, 2007; Pancino et al. 2007; Bellini et al. 2009; Anderson & van der Marel 2010).

The purpose of this paper is thus to investigate extensively star formation from gaseous ejecta from AGB stars within MSCs based on self-consistent hydrodynamical simulations with a reasonable model for star formation. We adopt a scenario in which (i) MSCs can be formed in their host galactic building blocks embedded in dark matter haloes at high redshifts and (ii) new stars formed from AGB ejecta can finally become the observed SG stars in the present GCs. Based on this scenario, we investigate (i) how AGB ejecta can be retained within MSCs, (ii) whether and how secondary star formation from AGB ejecta proceeds within MSCs and (iii) how MSCs evolve if they are located in nuclei of their hosts. We also investigate how MSCs lose their stars during tidal interaction with their hosts in order to discuss the observed smaller fractions of FG stars in the Galactic GCs.

A number of previous works discussed a scenario in which GCs were originally stellar galactic nuclei or nuclear star clusters in nucleated galaxies and their host galaxies had been already destroyed by strong tidal fields of much larger galaxies to disappear completely (e.g. Zinnecker et al. 1988; Freeman 1993; Bekki & Freeman 2003; Bellazzini et al. 2008; Boeker 2008). Following this scenario, Bekki et al. (2007b) investigated chemical abundances of FG and SG stars in GCs formed within the central regions of their host galaxies. The evolution of MSCs initially nuclear regions of their hosts in the present study therefore can provide some implications on the validity of the above scenario in explaining observational properties of GCs.

The plan of the paper is as follows. In the next section, we describe the details of the proposed scenario for GC formation. In Section 3, we describe the numerical models for evolution of gaseous ejecta of AGB stars within MSCs. In Section 4, we present the numerical results mainly on physical properties of SG stars. In Section 5, we discuss how long MSCs can retain most of their stars when they are influenced by strong tidal fields of their hosts. In Section 6, we discuss a number of key issues related to the origin of multiple stellar populations in GCs. We summarize our conclusions in Section 7.

2 THE SCENARIO

Although we describe the scenario in the context of the Galaxy formation, the formation processes of GCs within galaxies in general would be similar to those described below. The scenario is based on the results from our present and previous theoretical studies on GC formation (e.g. Bekki et al. 2002, 2004; Bekki 2006; Bekki & Chiba 2007; Bekki, Yahagi & Forbes 2007a; Hurley & Bekki 2008). Since the scenario is based partly on results of cosmological simulations (e.g. Bekki et al. 2007a), the scenario has some implications on the observed correlations between abundance properties of GCs and 3D motion and kinematics of GCs (e.g. Carretta 2006; Lee, Gim & Dinescu 2007). In the present paper, the masses of the present GCs are represented by $M_{gc}$ so that $M_\star$ (original masses of the GCs) and $M_{gc}$ can be discriminated with each other.
2.1 Time sequence

2.1.1 Formation of FG stars of MSCs within host dwarfs

FG stars of MSCs form from high-density GMCs with masses \( (M_{\text{GMC}}) \) as large as or larger than \( 10^7 \, M_\odot \) within the Galactic building blocks which are later destroyed owing to the strong tidal field of the Galaxy when they merge with the Galaxy. Most of the hosts form as massive dwarf galaxies with masses \( (M_h) \) larger than \( 10^9 \, M_\odot \) before reionization \((z > 10)\), and thus their star formation and chemical enrichment histories can be different from those of the present dwarfs within the Galaxy (Bekki et al. 2008). The formation places of MSCs are highly likely to their hosts’ central regions where mass fractions of GMCs can be much higher (i.e. not necessarily in the nuclei of the hosts).

The MSCs formed in the very centres of their hosts start their lives as nuclear MSCs so that their evolution can be different significantly from that for MSCs formed outside the very centres owing to much deeper gravitational potentials of hosts’ central regions. The host GMCs of MSCs may well initially have numerous substructures (smaller GMCs) owing to their large masses (e.g. Efremov 1995; Bonatto & Bica 2010) so that MSCs at their birth can be composed of a number of smaller clusters (i.e. not single entities). These smaller subunits can finally merge with one other to form single massive clusters within the merging time-scale depending on \( M_h \) and their sizes \( (R_h) \).

2.1.2 Evolution of FG stars within GMCs

Massive stars and type II supernovae can strongly influence later evolution of molecular gas left behind from the formation of FG stars. If most of the energy from massive stars and type II supernovae are converted into kinetic energy of gas surrounding the stars, then gaseous ejecta from the stars cannot be mixed well with the residual molecular gas (Bekki & Chiba 2007). Therefore, SF formation from gaseous ejecta from the above energetic stars and the mass fraction of SG stars formed from the mixed gas is very small \((<10^{-4})\). However, if massive stars are FRMSs with very slow ejection radial velocities \((<10 \, \text{km s}^{-1})\), then the gaseous ejecta could be well mixed with the residual gas to form SG stars before supernova explosion expels a significant fraction of gas.

Although type II supernovae can heat up and expel a significant fraction of gas from massive GMCs, some fraction of gas can be immune from the effects of supernovae owing to the clumpy nature of the massive GMCs. The smaller residual clouds may well be later accreted on to the forming clusters to dilute AGB ejecta. While supernova explosions influence GMCs significantly, star formation rates within the GMCs can drop significantly. Gaseous ejecta from massive AGB stars start to be accreted on to the central regions of MSCs after all supernova events finish. However, star formation does not resume until an enough amount of gas can be accumulated within the central regions: there should be a threshold gas mass fraction \((f_{\text{g,b}})\) above which star formation can start within MSCs. Therefore, there can be significant time-delay \((\sim 10^8 \, \text{yr})\) between FG and SF formation in this scenario.

2.1.3 Formation of SG stars

SG stars in an MSC start to form efficiently in the central region of the MSC when the mass fraction of the accumulated AGB ejecta to \( M_s(f_{\text{g,b}}) \) exceeds \( f_{\text{g,b}} \). This efficient secondary star formation from AGB ejecta can occur only if \( M_s \) exceeds a threshold cluster mass \((M_{th})\), because AGB ejecta cannot be efficiently retained in MSCs with smaller \( M_s \). During this secondary star formation, the residual molecular gas can be mixed with AGB ejecta to form new stars. Owing to the presence of deeper gravitational potential wells of MSCs, star formation efficiencies \((\epsilon_{sf})\) can be much higher \((>0.3)\) for this secondary star formation so that very compact clusters can be formed. MSCs thus can initially show ‘nested structures’ with more diffuse distributions of FG stars and more compact ones of SG stars.

Although MSCs with \( M_h \) below \( M_{th} \) can hardly form new stars from AGB ejecta, such less massive clusters can still capture residual molecular gas and smaller nearby GMCs (not chemically contamined by ejecta from FG stars). Cold molecular gas with initially very high-densities obtained by MSCs themselves can be converted into new stars within the central regions of MSCs (Bekki & Mackey 2009). Very low mass MSCs can be destroyed by interaction with GMCs so that secondary star formation cannot happen in their central regions. Therefore, there should be a threshold cluster mass below which MSCs show differences neither in ages nor in light elements among their stellar populations. Secondary star formation processes within nuclear MSCs can be significantly different from those described above owing to much deeper potential wells of their hosts (as described in the present study).

2.1.4 Tidal stripping of MSCs due to their host dwarfs

Secondary star formation can continue until MSCs lose most of their FG stars, though massive stars and type II supernovae of SG stars can also suppress or even truncate further star formation. Strong tidal fields of hosts can efficiently strip stars preferentially from FG stars owing to initially diffuse spatial distributions. This preferential stripping by the Galaxy has been already proposed by D’Ercole et al. (2008) and discussed in the context of origin of the Galactic stellar halo (Vesperini et al. 2010). The present scenario suggests that not the tidal field of the Galaxy but those of hosts of MSCs are responsible for the tidal stripping of most FG stars in MSCs. The time-scale of an MSC to lose most of their FG stars due to tidal stripping by its host depends on its position, with respect to its host \((R_h)\), \( M_h \) and \( R_h \). Therefore, more massive and denser MSCs can continue secondary star formation longer so that AGB stars with lower masses can possibly contribute to further star formation.

2.1.5 Evolution into the Galactic halo GCs

Hosts of MSCs are strongly influenced by the tidal field of the forming Galaxy during their merging with the Galaxy so that they can be completely destroyed by the Galaxy. The stripped stars from hosts, which include FG stars from MSCs, can be some parts of the Galactic stellar halo. MSCs are stripped during disintegration of their hosts to finally become halo GCs. Strong ram pressure of the Galactic halo gas (e.g. Frank & Gisler 1976; Bekki 2006) and presence of no cold gas in the halo prevent almost completely star formation of the MSCs since they become the Galactic halo GCs. Thus, multiple stellar populations can be formed within MSCs only when they are within their hosts.

Gas can be efficiently transferred to the nuclear regions of hosts to be converted into new stars while they are being destroyed by the Galaxy (e.g. Bekki & Freeman 2003). These new stars can have chemical abundances of heavy elements different from those of original nuclear MSCs, because they are from the outer regions,
where chemical enrichment histories are quite different from those of nuclei. Nuclear MSCs can show star-to-star abundance variations not only in light elements but also in heavy ones.

### 2.2 Constraints on original masses of MSCs

A number of authors have already suggested that if the standard IMF is applied for FG stars and if $\epsilon_{d}$ (star formation efficiency for SG formation) is 1.0, then original total masses of FG stars ($M_{*}$) are required to be at least 10 times larger than the present masses of SG stars in the Galactic GCs with multiple stellar populations (e.g. Bekki & Norris 2006). Given that $\epsilon_{d}$ is not like 100 per cent as observed in star-forming regions in the Galaxy, the required $M_{*}$ should be even higher than the above. Recent chemical evolution models have shown that the total mass of pristine gas that can mix with AGB ejecta to form SG stars needs to be comparable to that of AGB ejecta in order to explain the observed levels of star-to-star abundance variations and the O–Na anticorrelation in the Galactic GCs (e.g. D’Ercole et al. 2010): AGB ejecta should not be too much diluted by external gas captured/accreted by MSCs. Thus the required $M_{*}$ estimated in previous studies cannot change significantly even if contribution from external pristine gas to SG formation is considered.

In the present scenario, gaseous ejecta from AGB stars with masses ranging from $m_{u,\text{AGB}}$ to $m_{l,\text{AGB}}$ can contribute to the formation of SG stars within a time-scale of $t_{u}$ (which determines $m_{u,\text{AGB}}$). We here estimate the mass fraction of AGB ejecta to the initial mass of an MSC ($f_{\text{ej}}$) for a given $m_{u,\text{AGB}}$ and (ii) for a given $t_{u}$ by assuming $m_{u,\text{AGB}} = 8M_{\odot}$ and power-law initial mass functions with the slopes of $\alpha$. The adopted IMF in number defined as $\psi(m_{i}) = M_{\odot}m_{i}^{\alpha - 2}$, where $m_{i}$ is the initial mass of each individual star and the slope $\alpha = 2.35$ corresponds to the Salpeter IMF. The normalization factor $M_{\text{tot},i}$ is a function of $M_{i}$, $m_{u}$ (lower mass cut-off) and $m_{o}$ (upper mass cut-off):

$$M_{i,\text{tot}} = \frac{M_{\odot}(2 - \alpha)}{m_{o}^{2 - \alpha} - m_{u}^{2 - \alpha}},$$  \hspace{1cm} (1)

where $m_{o}$ and $m_{u}$ are set to be 0.1 and 120 $M_{\odot}$, respectively. The total mass of AGB ejecta within an MSC ($M_{\text{AGB}}$) is accordingly described as

$$M_{\text{AGB}} = \int_{m_{u,\text{AGB}}}^{m_{l,\text{AGB}}} m_{i}\psi(m_{i})\mathrm{d}m_{i},$$  \hspace{1cm} (2)

where $m_{\text{ej}}$ describes the total gas mass ejected from an AGB star with initial mass $m_{l}$ and final mass $m_{o}$. We derive an analytic form of $m_{\text{ej}}$ ($= m_{l} - m_{o}$) from the observational data by Weidemann (2000) by using the least-squares fitting method, and find

$$m_{\text{ej}} = 0.916M_{l} - 0.444.$$  \hspace{1cm} (3)

In order to calculate the main-sequence turn-off mass ($m_{\text{TOD}}$) we use the following formula (Renzini & Buzzoni 1986):

$$\log m_{\text{TOD}}(t_{u}) = 0.0558(\log t_{u})^{2} - 1.338 \log t_{u} + 7.764,$$  \hspace{1cm} (4)

where $m_{\text{TOD}}$ is in solar units and time $t_{u}$ is in years.

Figure 1 shows that $f_{\text{ej}}$ is larger for smaller $m_{l,\text{AGB}}$ both for the standard IMF ($\alpha = 2.35$) and the top-heavy one ($\alpha = 1.5$). Although the top-heavy model shows a larger $f_{\text{ej}}$ than the standard one for $m_{l,\text{AGB}} < 2M_{\odot}$, the derived $f_{\text{ej}}$ is small: only 0.036 for $m_{l,\text{AGB}} = 5M_{\odot}$ and 0.079 $m_{l,\text{AGB}} = 3M_{\odot}$. Ventura & D’Antona (2008) showed that the observed O–Na anticorrelations can be well reproduced only if AGB stars with masses larger than $5M_{\odot}$ can contribute to secondary star formation within GCs. Thus, it is highly likely that progenitor clusters for the present GCs need to be at least $\sim$25 times more massive than the present GCs.

As shown in Fig. 1, $f_{\text{ej}}$ can be larger than 0.1 for $t_{u} > 1$ Gyr (or $m_{l,\text{AGB}} < 3M_{\odot}$) for the two IMF models. It should be stressed here that MSCs with larger $t_{u}$ can end up with a large age difference ($>1$ Gyr) between FG and SG stars. Original MSCs can well lose most of their FG stars due to tidal stripping of their hosts within less than $\sim$1 Gyr, as shown later in the present study. Therefore gaseous ejecta only from massive AGB stars can participate in secondary star formation within MSCs. It would be reasonable to consider that $m_{l,\text{AGB}}$ is $4$–$5M_{\odot}$, which corresponds to $t_{u} \sim 10^{7}$ yr.

### 3 THE NUMERICAL MODEL

We investigate star formation of gaseous ejecta from FG AGB stars in MSCs with $M_{*}$ ranging from $10^{4}$ to $10^{7}M_{\odot}$ by using the latest version of GRAPE (GRavity PipE, GRAPE-7) which is the special-purpose computer for gravitational dynamics (Sugimoto et al. 1990). We have revised our original GRAPE-SPH code (Bekki 2009) so that we can investigate star formation processes within the above-mentioned massive stellar system. MSCs with very large ($M_{*} > 10^{7}M_{\odot}$) are still referred to as ‘clusters’ just for convenience in the present study, though their stellar masses are as massive as those of dwarf galaxies.

We focus mainly on secondary star formation from AGB ejecta of FG stars within original MSCs and describe the results of numerical simulations of the star formation. We have numerically investigated ram-pressure stripping of AGB ejecta of their hosts by ISM, and the
strength of ISM which is dependent on \( M \) and physical parameters of ISM. We however describe the results in our forthcoming papers (Bekki et al., in preparation), because ram pressure stripping of AGB ejecta of MSCs by typical ISM of hosts is only effective for low-mass MSCs with \( M_f < 10^4 \, M_\odot \) (which cannot become ordinary GCs in the present study) and thus not so important.

We have numerically investigated accretion of warm ISM on to MSCs by Bondi accretion and found that Bondi accretion with the accretion rate of \( \sim 10^{-3} \, M_\odot \, \text{yr}^{-1} \) is possible only if \( M_f \) is larger than \( 3 \times 10^6 \, M_\odot \) for typical ISM with \( n = 1 \, \text{atom cm}^{-3} \), relative velocity of \( \sim 20 \, \text{km s}^{-1} \) and gas temperature of 10000 K. Although this accretion rate is too small for MSCs to obtain fresh gas for the formation of SG stars within \( \sim 10^4 \, \text{yr} \), this Bondi accretion can be an important mechanism for obtaining fresh gas in massive GCs with abundance variations in heavy elements like \( \omega \mathrm{Cen} \), as briefly discussed later in this paper. We will describe the details of the numerical results on Bondi accretion of ISM in our forthcoming papers (Bekki et al., in preparation), mainly because this paper becomes too long if the results are included in this paper.

3.1 Initial structures and kinematics of MSCs

The original MSCs are assumed to have a Plummer density profile (e.g. Binney & Tremaine 1987) with luminosities \( (L_\odot) \) and central velocity dispersions \( (\sigma_s) \) consistent with the relation observed for GCs (Djorgovski et al. 1997):

\[
L_s = K_s \sigma_s^{1.7},
\]

where \( K_s \) is a normalization factor for the relation. The scalelength \( (a_s) \) of an MSC is determined by the formula

\[
a_s = G M_s/6 \sigma_s^2,
\]

where \( G \) is the gravitational constant. Since the mass-to-light ratio \( (M_\odot/L_\odot) \) is assumed to be constant for all SCs, \( a_s \) and \( \sigma_s \) are determined by equations (1) and (2) for a given \( M_\odot \). The normalization factor \( K_s \) in equation (5) is determined such that a cluster with \( M_f = 6 \times 10^7 \, M_\odot \) can have the size of \( R_s = 50 \, \text{pc} \) (\( R_s = 5a_s \)), and the central velocity dispersion of 7 km s\(^{-1}\).

Hasegawa et al. (2005) revealed that massive stellar systems with masses larger than \( 2 \times 10^6 \, M_\odot \) appear to have scaling regions different from those of the present GCs. We accordingly consider that original sizes of MSCs with \( M_f \geq 2 \times 10^6 \, M_\odot \) cannot be determined by equations (5) and (6) and therefore we investigate models with different \( R_s \) for a given \( M_f \) for such MSCs. It is also possible that original MSCs with \( M_f < 2 \times 10^6 \, M_\odot \) have smaller or larger than those described by the above scaling relations because MSCs composed only of FG stars have ages of \( \sim 10^8 \) yr in the present study: an order of \( 10^8 \) yr would not be long enough to form GCs on the present scaling relation owing to dynamical relaxation processes. We therefore investigate models with \( R_s \) smaller and larger than equations (5) and (6) predict for a given \( M_f \).

An MSC is assumed to have a small amount of rotation with the ratio of the initial rotational energy \( (T_{\text{rot}}) \) to the total kinetic one \( (T_{\text{kin}}) \) being a free parameter represented by \( s_{\text{rot}} \). The parameter values of \( s_{\text{rot}} \) range from 0 (no rotation) to 0.3 (rapid rotation). The initial rotational velocity of a particle at a distance of \( R \) from the centre of MSC is \( \omega R \), where \( \omega \) (constant angular velocity) is determined such that \( s_{\text{rot}} \) can be the adopted value. Therefore, the system has random kinetic energy \( (T_{\text{rot}}) \) of \((1 - s_{\text{rot}})T_{\text{kin}} \) due to isotropic velocity dispersion of stars. First we estimate \( T_{\text{kin}} \) for \( \sigma_s \) determined by \( M_f \) and \( R_s \) (in equations 5 and 6) and then reduce \( \sigma_s \) so that the final system can be in virial equilibrium \( (T_{\text{kin}} = T_{\text{rot}} + T_{\text{inter}} = 0.5 |W|) \), where \( W \) is the total potential energy of the system.

3.2 Gas dynamics

The FG stars in an MSC are represented by equal-mass stellar particles with the particle number of \( N_f = \left( 10^5 \right) \) and some fraction of the particles are assigned as 'AGB stars' with initial masses of \( m_0 \) and the total number of \( N_{\text{AGB}} \) that can eject SPH gas particles with ejection velocities of \( V_{ej} \) with respect to the centres of the AGB stars. The mass fraction of AGB particles in an MSC is a parameter represented by \( f_{\text{AGB}} \) that is determined by the adopted IMF. As outlined in Section 2, gaseous ejecta only from massive AGB stars with initial masses of \( \sim 5-8 \, M_\odot \) need to be converted into new stars (i.e. SG stars) to explain the chemical abundances of SG (e.g. Ventura & D’Antona 2008). In the present paper, \( V_{ej} = 20 \, \text{km s}^{-1} \) is adopted, which is a reasonable choice for massive AGB stars (e.g. Marshall et al. 2004).

The present simulations cannot resolve gaseous evolution of each individual AGB star owing to the adopted numerical resolution \((\sim 0.5 \, \text{pc})\). We therefore assume that each AGB particle initially has an expanding gaseous sphere which is much larger than the AGB star itself. The mass, size and temperature of the large gaseous sphere represented by SPH particles with the particle number of \( N_{\text{SPH}} \) are set to be \( m_g \), \( r_g \) and \( T_g \), respectively. Each AGB particle accordingly has a gas sphere represented by \( N_{\text{AGB}} \) SPH particles with radial velocities (with respect to the AGB particle) of \( V_{ej} \). The AGB particle therefore has a mass of \( m_f - m_g \) after gas ejection. We here assume that each AGB particle in a model eject SPH particles at \( T = 0 \) when the simulation starts: we do not consider gradual ejection of gas from AGB stars at each time-step, because an appropriate modelling of such gradual ejection requires a huge number of gas particles and thus is numerically costly (i.e. practically not feasible).

Although we have investigated models with \( T_g = 100-1000 \, \text{K} \), we mainly describe the results of the models with \( T_g = 100 \, \text{K} \) which correspond to star-forming warm molecular clouds (e.g. Wilson, Walker & Thornley 1997). This is because we consider that AGB wind cools down during expansion throughout interstellar space due to radiative cooling (to finally become molecular gas for further star formation) so that \( T_g \) becomes much smaller than the original temperature of the wind \((\sim 1000 \, \text{K})\). We adopt an isothermal equation so that AGB ejecta can initially have low temperatures \((100 \, \text{K}) \). It should be stressed here that if \( T = 1000 \, \text{K} \) is adopted (which is not realistic though), star formation is possible only for models with \( M_f \geq 10^6 \, M_\odot \).

Most of the AGB ejecta can be converted into new stars well before a few Myr after commencement of secondary star formation so that feedback effects of massive stars and type II supernovae cannot significantly influence the gas dynamics during the secondary star formation processes. We therefore consider that the above isothermal assumption is reasonable. In order to estimate the value of \( m_f \) corresponding to the total mass ejected from each AGB star after the main-sequence turn-off, we use the formula given in equation (3). About 83 per cent of an AGB particle with \( m_f = 5 \, M_\odot \) can be ejected to be used for further star formation in the present model.

If we adopt the Salpeter IMF, then \( f_{\text{AGB}} \) (the mass fraction of AGB stars with masses larger than \( 5 \, M_\odot \)) is \( \sim 0.04 \) in the present study. Since initial masses within an MSC are the same between stellar particles, \( N_{\text{AGB}} = f_{\text{AGB}} N_f \). Although we adopt this \( f_{\text{AGB}} = 0.04 \) for most models, we investigate models with different \( f_{\text{AGB}} \) so that we
can find a threshold $f_{\text{AGB}}$ for a given MSC above which secondary star formation can occur in the MSC. It is found that if $f_{\text{AGB}}$ is larger than 0.008, secondary star formation is possible in some models, though the star formation efficiency is low. This means that secondary star formation within an MSC cannot be possible until a certain amount of gas is accumulated within the MSC and thus that there should be a time-delay between the formation of FG stars and that of SG ones.

3.3 Star formation

We investigate whether the gas accumulated in the central regions of MSCs can be sufficient to form new stars by adopting a simple prescription for star formation. In the models with ‘star formation’, a gas particle is converted into a collision-less new stellar one if the gas particle meets the following three conditions: (i) the dynamical time-scale of the SPH gas particle is shorter than the sound-crossing time; (ii) the gas is converging (i.e. $\nabla \rho < 0$, where $\nabla$ is the velocity vector of the gas particle); and (iii) the local gas density exceeds a threshold gas density ($\rho_{th}$) which corresponds to the densities of dense cores of molecular clouds ($n > 10^4$ atoms cm$^{-3}$), where individual star formation is taking place. The first two conditions mimic the Jeans gravitational instability for gaseous collapse. The new stellar particles and the old ones present initially within MSCs are referred to as ‘new stars’ (i.e. SG stars) and ‘old stars’ (i.e. FG ones), respectively. Also, original clusters and new ones formed from AGB ejecta are referred to as ‘old clusters’ and ‘new clusters’, respectively, just for convenience.

We investigate models with $\rho_{th} = 0$ and $10^4$ atoms cm$^{-3}$ in order to investigate (i) whether the threshold gas density is important for determining star formation histories of SG stars and (ii) how the present results, in particular, structural and kinematical properties of final clusters composed of SG stars, depend on $\rho_{th}$: we here consider that $\rho_{th}$ has not yet been observationally well determined and therefore could be different for individual star-forming clouds. Although these star formation models are less realistic in some points (not inclusive of magnetic fields and radiative transfer etc), we consider that they enable us to grasp essential ingredients of secondary star formation from AGB ejecta within original massive clusters.

3.4 External gravitational fields of host galaxies

We mostly investigate ‘isolated MSCs’ which are located outside nuclei of their host galaxies and therefore short-term ($\sim 10^7$ yr) evolution of AGB eject cannot be strongly influenced by the tidal fields of their hosts. However, dynamical evolution of AGB ejecta from nuclear MSCs that are initially located in the very centres of their hosts can be significantly influenced by their hosts owing to the much deeper gravitational potential wells of the hosts. We therefore investigate how hosts influence evolution of AGB ejecta from nuclear MSCs by assuming that the host dwarfs are dominated by dark matter haloes with masses of $M_{h}$ and thus their gravitational potentials are determined only by the dark haloes.

We adopt the density distribution of the NFW halo (Navarro, Frenk & White 1996) suggested from CDM simulations:

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2},$$

where $r$, $\rho_0$, and $r_s$ are the spherical radius, the characteristic density of a dark halo and the scalelength of the halo, respectively. Although we investigate models with $10^8 M_\odot \leq M_{h} \leq 2 \times 10^{10} M_\odot$, we mainly describe the results of the models with $M_{h} = 10^9 M_\odot$, $r_s = 0.6$ kpc and the virial radius ($r_{vir}$) of $9.9$ kpc (i.e. the $c$ parameter of 13.9). These models with $M_{h} = 10^9 M_\odot$ are reasonable in the present scenario in which there is a threshold galaxy mass ($10^7 M_\odot$) above which GCs can be formed. It should be stressed here that low-mass dark matter haloes with $M_{h} = 10^8 M_\odot$ can influence the evolution of AGB ejecta in hosts as significantly as those with $M_{h} = 10^9 M_\odot$.

3.5 Ranges of parameters

First we investigate (i) whether AGB ejecta can be retained within MSCs and (ii) whether and how secondary star formation from AGB ejecta can proceed for models that are consistent with the scaling relation of GCs shown in the equation (2). Secondly, we investigate the structure and kinematics of the simulated clusters in the models with $M_{h} = 10^9 M_\odot$ and $R_{h} = 100$ pc in which the final masses of SG stars can be as large as $\sim 10^5 M_\odot$. Thirdly, we investigate whether much deeper gravitational potential wells of host dwarf galaxies can play a role in retaining AGB ejecta efficiently in the nuclear MSCs.

We have investigated secondary star formation processes for numerous models with different model parameters (e.g. $M_s$, $R_{c}$, $\rho_{th}$ and $s_{th}$); we mainly describe here the results for the ‘standard model’ with $M_{h} = 10^9 M_\odot$, $R_{c} = 44.7$ pc, $s_{th} = 0.0$ and $\rho_{th} = 10^4$ atoms cm$^{-3}$. We assume that $n_{\text{AGB}} = 20$ for most models except those for investigating how AGB ejecta can be retained within MSCs. The reason for this adoption is that as long as $n_{\text{AGB}} > 10$, the results do not depend on $n_{\text{AGB}}$. Since we focus on secondary star formation processes with a time-scale of $\sim 10^7$ yr, we do not intend to discuss long-term dynamical evolution of clusters due to two-body relaxation. We accordingly introduce a gravitational softening length ($\epsilon_g$) for each simulation and $\epsilon_g$ is set to be equal to the mean particle separation at the half-number radius of old stars.

The ranges of model parameters are shown in Table 1. We describe in detail only the results of some representative models showing key parameter-dependences in retention processes of AGB ejecta and secondary star formation processes within clusters among the investigated models. Secondary star formation processes can continue only for $\sim 10^7$ yr so that they cannot be strongly influenced by gravitational fields of their hosts unless MSCs are located in the very centres of their hosts. We thus show mainly the results for models without gravitational fields of hosts in order to much more clearly show the importance of the four key parameters $M_s$, $R_{c}$, $s_{th}$ and $\rho_{th}$ in secondary star formation processes.

3.6 Limitations of the model

As described in Section 3.2, we do not consider that different AGB stars with different masses eject gas at different $T$ owing to different lifetimes of the stars. Therefore all of the gaseous ejecta can be converted into new stars within $\sim 10^5$ Myr. This rapid consumption of the AGB ejecta is not so realistic, given that there should be a time delay of at least several tens of millions of years between the epochs when AGB stars with masses of 8 and 5 $M_\odot$ start to eject their gas. If we consider the different epochs of gas ejection from AGB stars with different masses, then the star formation period would become significantly longer. However, energy feedback effects of numerous supernovae from the SG stars are likely to truncate star formation in gaseous ejecta from lower-mass AGB stars. In our future papers, we will discuss this point in detail by using a more sophisticated numerical model.
Table 1. Range of model parameters for numerical simulations of star-forming MSCs.

| $M_s$ | $R_s$ | $s_{tot}$ | $f_{AGB}$ | Star formation | $\rho_{th}$ | External tidal field | $M_h$ |
|-------|-------|-----------|-----------|---------------|-----------|-------------------|-------|
| $10^4$–$10^6$ | 35–200 | 0–0.32 | 0.002–0.08 | YES/NO | 0–$10^4$ | YES/NO | $10^3$–$2 \times 10^6$ |

$a$ The initial mass of an MSC in units of $M_\odot$.

$b$ The initial size of an MSC in units of pc.

$c$ The ratio of rotational energy to total kinetic one in an MSC.

$d$ The ratio of the total mass of AGB stars to that of stars in an MSC.

$e$ ‘YES/NO’ means that the star formation model is/is not included in the simulation.

$f$ The threshold gas density for star formation in units of atoms cm$^{-3}$.

$g$ ‘YES/NO’ means that an MSC can/cannot be influenced by the tidal field of its host galaxy.

$h$ The total mass of the host galaxy for an MSC in units of $M_\odot$.

The present model does not include the effect of type Ia supernovae (SNe Ia) on star formation processes of SG stars. D’Ercole et al. (2008) have already showed that the cumulative effect of numerous SNe Ia is important for the star formation processes within massive clusters, because it can drastically alter gas dynamics within the clusters. Therefore the present model that does not include such a SN Ia effect possibly overestimates the star formation rates in the accumulated AGB ejecta in the central regions of MSCs. Our future more sophisticated models with SN Ia effect will discuss how SNe Ia can influence gas dynamics and star formation in MSCs.

4 RESULTS

4.1 Retention of AGB ejecta

Fig. 2 shows that time evolution of spatial distributions of AGB ejecta is very different between models with $M_s = 2 \times 10^5 M_\odot$ and $M_s = 10^6 M_\odot$ in which model parameters except $M_s$ are exactly the same. Gaseous particles with $n_{AGB} = 10^4$ ejected from an AGB star locating in the very centre of an MSC can expand quickly ($T = 2.7$ Myr) and finally escape from the cluster and never return to the model with $M_s = 2 \times 10^5 M_\odot$ ($T = 13.6$ Myr). On the other hand, gaseous ejecta from an AGB star in the model with $M_s = 10^6 M_\odot$ can return to the initial location within a time-scale of a few Myr after the ejection owing to the deeper gravitational potential for this more massive cluster. This clear difference suggests that $M_s$ is an important parameter for determining whether AGB ejecta can be retained in MSCs during their early formation phases.

Fig. 3 shows how the mass fractions of AGB ejecta retained within MSCs ($= F_{ret}$) depend on $M_s$. Fig. 3 shows that MSCs with $M_s > 2 \times 10^5 M_\odot$ can retain a significant fraction of AGB ejecta if a scaling relation similar to the observed one is applied for all MSCs. This result implies that there can be a threshold mass above which AGB ejecta can be efficiently retained. For MSCs with $M_s \sim 2 \times 10^5 M_\odot$, AGB ejecta initially in the outer part of the MSC can escape from the MSC whereas that initially in the inner part can be retained within the MSC. As a result of this, a ‘core-halo’ structure of AGB ejecta can be formed within 13.6 Myr. A very compact gas sphere can be formed where star formation would be able to proceed rapidly if star formation is included in the model with $M_s = 10^6 M_\odot$. A larger fraction of AGB ejecta can be retained in MSCs with larger $M_s$.

Gaseous spheres composed of AGB ejecta can interact with one another within an MSC when numerous stars enter into AGB phase almost simultaneously. The present 3D numerical simulations enable us to investigate whether and how hydrodynamical interaction
Figure 3. Dependences of the mass ratios \( F_{ret} = M_{AGB}/M_s \) of AGB ejecta that can finally be within the central 50 pc (blue filled circles) and 10 pc (red filled squares) of MSCs on \( M_s \).

Figure 4. The same as Fig. 2 but for the model with two AGB stars in an MSC with \( M_s = 2 \times 10^5 M_\odot \).

Figure 5. Dependences of \( F_{ret} \) normalized by \( N_{AGB} \) on \( N_{AGB} \) in an MSC with \( M_s = 2 \times 10^5 M_\odot \). This \( F_{ret}/N_{AGB} \) describes how efficiently AGB ejecta can be retained within MSCs.

Figure 6. The same as Fig. 3 but for the models in which all AGB stars can eject gas with \( V_{ej} = 20 \text{ km s}^{-1} \).

interaction of AGB ejecta can cause efficient gaseous dissipation and thus play a great role in retaining AGB ejecta within MSCs.

Fig. 6 shows that (i) there is a threshold \( M_s (\approx 2 \times 10^5 M_\odot \) in this parameter study) above which AGB ejecta can be retained effectively and (ii) \( R_{ret} \) is larger for larger \( M_s \). The reason of these dependences is that MSCs with larger \( M_s \) have deeper gravitational potentials so that they can retain AGB ejecta more effectively. Differences in \( F_{ret} \) between \( R < 10 \text{ pc} \) and \( R < 50 \text{ pc} \) are small for models with \( M_s \geq 6 \times 10^5 M_\odot \), which means that most of the accumulated gas can form very compact gaseous regions within the central regions of MSCs. Thus MSCs with larger \( M_s \) can obtain a larger amount of AGB ejecta for further star formation: \( M_s \) is a key for determining whether MSCs can have SG stars.

4.2 Secondary star formation

Fig. 7 shows how gas ejected from AGB stars is accumulated within the central region of the MSC and consequently converted into new stars (i.e. SG ones) there within 13.6 Myr in the standard model with \( M_s = 10^6 M_\odot, R_s = 44.7 \text{ pc}, v_{rot} = 0.0 \text{ and } \rho_{th} = 10^4 \text{ atoms cm}^{-3} \).

The gaseous ejecta from AGB stars initially located in the central region of the MSC can be accumulated there within \( \approx 1 \text{ Myr} \) so that star formation can start efficiently. The star formation from AGB ejecta can proceed like a ‘starburst’ with most gas being consumed rapidly within the first 3-Myr evolution of the MSC. The star formation rate (SFR) reaches its maximum value (0.03 \( M_\odot \text{ yr}^{-1} \)) at
Figure 7. Time evolution of the distributions of gas (magenta) and new stars (cyan) projected on to the \(xz\) plane for the standard model with \(M = 10^6 M_\odot\), \(R_s = 44.7\) pc, \(s_{\text{rot}} = 0.0\) and \(\rho_{\text{th}} = 10^4\) atoms cm\(^{-3}\). The green circle in each frame represents the original half-mass radius of old stars. Time (\(T\) in units of Myr) that has elapsed since the simulation started is shown in the upper left corner for each frame.

\(T = 1.0\) Myr and then rapidly declines to be 0.001 \(M_\odot\) yr\(^{-1}\) at \(T = 6\) Myr.

About 86 per cent of the gas can be converted into new stars within 13.6 Myr in this model. This very high star formation efficiency \(\epsilon_{\text{sf}}\) results from the fact that almost all of the AGB ejecta can be accumulated very quickly (in less than a few \(10^6\) yr) within the central region of the MSC owing to the deep gravitational potential and can consequently be converted into new stars there. The accumulated gas is strongly self-gravitating (i.e. gas mass comparable to stellar mass there) in the central region of the cluster so that the gas can continue to collapse to form new stars. This result clearly suggests that secondary star formation within MSCs is responsible for the origin of the observed high stellar densities of the present GCs: a GMC cannot be converted into a GC just by one star-formation event.

A new compact star cluster embedded in low-density residual gas can be formed in the central regions of the MSC within 13.6 Myr in this model. Owing to the high star formation efficiency (>0.5), the new cluster is highly likely to survive gas removal by energetic winds of massive stars and supernova explosions. As shown in Fig. 8, the new cluster is much more compact than the original MSC and has a half-mass radius (~3 pc) significantly smaller than that of the original MSC (~11 pc). Owing to its compactness, the new cluster is much less susceptible to tidal destruction by its host galaxy. Thus a ‘nested stellar system’ composed of a diffuse original cluster (FG stars) and a compact new one (SG ones) can be formed as a result of secondary star formation from AGB ejecta within the MSC in this model. This nested structure is one of the common features of the simulated MSCs in the present study.

Fig. 9 shows how the fractions \(F_{\text{ns}}\) of the total mass of new stars \(M_{\text{ns}}\) to the initial total mass of the MSC \(M_s\) depend on \(M_s\) for \(R < 10\) pc and \(R < 50\) pc in the models with \(\rho_{\text{th}} = 10^5\) atoms cm\(^{-3}\). Clearly, there is a threshold cluster mass \(M_{\text{th}}\) above which secondary star formation is possible: \(M_{\text{th}} \sim 6 \times 10^5 M_\odot\) and does not depend on \(\rho_{\text{th}}\) in the present study. Furthermore, \(F_{\text{ns}}\) is higher in MSCs with larger \(M_s\), which implies that \(\epsilon_{\text{sf}}\) is also higher in MSCs with larger \(M_s\). Almost all stars can be formed within half-mass radii (~10 pc) of MSCs with \(M_s > M_{\text{th}}\) so that there can be no differences between \(F_{\text{ns}}\) for \(R < 10\) pc and \(R < 50\) pc.

Although \(\epsilon_{\text{sf}}\) is rather high (0.86) in the model shown in Figs 7 and 8, the total mass of new stars can be at most \(4 \times 10^4 M_\odot\) owing to the adopted \(f_{\text{AGB}} = 0.04\) that is reasonable for the standard IMF. This means that MSCs with \(M_s \sim 10^6 M_\odot\) can become low-mass GCs with multiple stellar populations. Furthermore, \(\epsilon_{\text{sf}}\) is 0.3 for the model with \(M_s = 6 \times 10^5 M_\odot\), which suggests that the new cluster can significantly expand after gas expulsion due to energetic winds of massive stars and supernovae. The cluster is likely to become a low-mass, low-density GC with a smaller fraction of SG stars (like Pal 5). The dependence of \(\epsilon_{\text{sf}}\) on \(M_s\) implies that more massive GCs have larger fractions of SG stars.

The details of star formation histories depend weakly on \(s_{\text{rot}}\) and \(\rho_{\text{th}}\) such that gas can be more rapidly converted into new stars in
MSCs with smaller $s_{\text{rot}}$ and $\rho_{\text{th}}$. Fig. 10 shows that the overall SFR is higher in the model with $s_{\text{rot}} = 0$ ($\epsilon_{sf} = 0.86$) than in that with $s_{\text{rot}} = 0.08$ ($\epsilon_{sf} = 0.47$). This is mainly because initial angular momentum of the MSC in the model with $s_{\text{rot}} = 0.08$ can suppress the formation of a very compact gaseous region at the centre of the MSC. The secondary peak of the SFR in the model with $s_{\text{rot}} = 0.08$ at $T = 2.8$ Myr is due to later infall of gas on to the MSC’s centre where a compact gas disc is formed.

4.3 Structures and kinematics

The formation of nested structures with very compact new clusters and initially more diffuse old ones is one of the essential ingredients of the present simulations. Fig. 11, which shows a typical example of the nested structures, describes how the total stellar mass within $R$ depend on $R$ for old and new stars in the standard model. Clearly, the final cluster is dominated by new stars (SG) within $R < 2$ pc, and the fraction ($F_{\text{SG}}$) of the total mass of new stars ($M_{\text{SG}}$) to the total mass of old stars ($M_{\text{FG}}$) and new ones is more than 0.5 for $R < 2$ pc. Fig. 12 shows how $F_{\text{SG}}$ depends on $R$ for different models with different $\rho_{\text{th}}$ and $s_{\text{rot}}$. Irrespective of these model parameters, $F_{\text{SG}}$ is higher in the inner regions of MSCs and lower in the outer ones. $F_{\text{SG}}$ can be higher for higher $\rho_{\text{th}}$ and smaller $s_{\text{rot}}$.

As discussed in previous sections, $M_s$ should be at least $2.5 \times 10^6 M_\odot$ to explain GCs with $M_{\text{SG}} \sim 10^5 M_\odot$ (for $f_{\text{AGB}} = 0.04$). Given that $\epsilon_{sf}$ is not 1 (0.3–0.9 for most models) and later dynamical evolution can reduce the total masses of MSCs, stellar structures...
and kinematics of the simulated MSCs with rather large $M_\ast$ ($> 5 \times 10^5 \, M_\odot$) can correspond to those of the observed ones of GCs with masses larger than $\sim 10^5 \, M_\odot$. Fig. 13 shows the total mass of new stars ($M_{\text{new}}$) is larger than $10^5 \, M_\odot$ and the effective radius ($R_{\text{eff}}$) of the stars is $5.2 \, \text{pc}$, which is about a factor of 5 smaller than that of the old ones ($R_{\text{old}}$). The half-mass ratios ($= R_{\text{eff}}/R_{\text{old}}$) are very similar ($\sim 0.2$) in the present models with different model parameters. As shown in Fig. 14, the new stars dominate the inner region ($R < 2 \, \text{pc}$) of MSC, though $F_{\text{SG}}$ there cannot become so high as that in the standard model. It is confirmed that more massive models with $M_\ast = 10^7 \, M_\odot$ with different $s_\text{rot}$ and $\rho_0$ also show distinct nested structures.

Fig. 15 shows clear differences in stellar kinematics between old and new stars (thus FG and SG ones, respectively) in the final MSC for the model. New stars clearly have rotational kinematics with the maximum rotational velocity ($V_{\text{rot}}$) of $12.0 \, \text{km} \, \text{s}^{-1}$ and the smaller central velocity dispersion ($\sigma_0$) of $9.5 \, \text{km} \, \text{s}^{-1}$, which means $V_{\text{rot}}/\sigma_0 = 1.25$. This rotational kinematics cannot be seen in old stars, which have $V_{\text{rot}}/\sigma_0 = 0.1$ in this model. The models with larger $s_{\text{rot}}$ show larger $V_{\text{rot}}/\sigma_0$ in new stars, which has already been discussed by Bekki (2010). These results suggest that the present GCs originate from MSCs composed of two stellar populations with significantly different kinematics.

4.4 Influences of host galaxies

Fig. 16 shows the time evolution of the projected spatial distribution of AGB ejecta for a nuclear MSC in the model without star formation yet with the external gravitational field of its host galaxy dominated by dark matter halo ($M_\text{dark} = 10^9 \, M_\odot$). In this model, only one gaseous sphere of an AGB star is included and located initially in the centre of a nuclear MSC with $M_\ast = 2 \times 10^5 \, M_\odot$ ($s_{\text{rot}} = 0.0$) so that the results can be compared with the model with $M_\ast = 2 \times 10^5 \, M_\odot$ shown in Fig. 2: it is much better to use these two comparative models with only one gaseous sphere of an AGB star, because the effects of MSC’s host galaxy can be more clearly demonstrated. As shown in Fig. 16, although the gas sphere expands initially to a larger radius ($R > 50 \, \text{pc}$), the gas can finally return to the central region of the MSC within $\sim 10 \, \text{Myr}$ owing to the deep potential well of its host galaxy.
5 SURVIVAL OF ORIGINALLY MASSIVE STELLAR SYSTEMS

In the present scenario, original MSCs need to keep substantial masses at least for $\sim 10^6$ yr after they form within GMCs so that gaseous ejecta from more massive AGB stars can be retained in MSCs and finally converted into new stars (= SG ones). The hosts (= massive dwarfs) can strip stars from MSCs owing to their tidal fields, even if the fields are not so strong as that of the Galaxy. Therefore, here we investigate how long MSCs can keep their substantial fractions of stars when they are under strong gravitational influences of their hosts.

5.1 The model

We consider how an MSC dynamically evolves after most of the residual gas left behind from the formation of FG stars is expelled from the cluster through supernova events. Accordingly we adopt an assumption that (i) the MSC is assumed to consists purely of stellar particles and (ii) it is not necessarily in virial equilibrium owing to loss of the original mass of the GMC. The cluster with a mass $M_\star$ and a size $R_s$ has a Plummer density profile with the scalelength $\alpha_s$ ($R_s = 5\alpha_s$). Owing to the above (ii), the central velocity dispersion $\sigma_s$ of the MSC can be significantly larger than the one ($\sigma_{\text{vir}}$) which the MSC can have when it is in virial equilibrium. Therefore, we introduce a parameter $f_s$ that is the ratio of $\sigma_s$ to $\sigma_{\text{vir}}$ and ranges from $1.0$ (virial equilibrium) to $1.3$. Thus MSCs can expand significantly for models with $f_s > 1$.

The cluster is influenced by the host with the density distribution of the NFW halo (i.e. equation 7). The initial location of the MSC with respect to the centre of the host is set to be $(x_h, y_h, z_h)$. The initial velocities of the MSC in the $x$, $y$- and $z$-directions are set to be $(u_x, v_x, w_x)$. The orbital plane of the MSC is coincident with the $xy$ plane for all models in the present study (i.e. $z_h = 0$ and $w_x = 0$). The initial direction of the velocity vector of the MSC is only a parameter for the orbit of the MSC owing to the spherically symmetric distributions of the host and the MSC. We therefore assume that the initial direction is parallel to the $y$-axis towards the positive $y$ (i.e. $v_y > 0$ and $u_x = 0$). We assume $x_h = 0$, and thus $x_h$ and $v_x$ are free parameters that determine the orbit of the MSC.

We consider that the orbit of an MSC is not necessarily circular within its host and therefore introduce a free parameter $f_s$ that is the ratio of $u_x$ to the circular velocity $v_{\text{cir}}$ at $x_h$ and ranges from $0.5$ to $1.0$ (i.e. circular orbit). We mainly investigate two representative cluster models with $M_\star = 10^8 M_\odot$ and $R_s = 75$ pc and with $M_\star = 10^7 M_\odot$ and $R_s = 100$ pc for different model parameters $M_{\text{bh}}, f_s, x_h$, and $v_x$. Although we run numerous models, we mainly describe the results of the models with $M_{\text{bh}} = 5 \times 10^3 M_\odot$ and $r_s = 1.6$ kpc ($c = 13.9$).

For each model, we count the total number of stars that are located within $100$ pc from the centre of the MSC at each time-step in order to investigate the time evolution of the total stellar mass of the MSC. Stars that are once stripped and by accident located within the central $100$ pc without being bound by the MSC can be counted as those in the MSC. We consider that since the number fraction of these stars is very small, these stars can only slightly overestimate the total stellar mass of an MSC. The initial stellar mass for an MSC is $(1 - f_s) M_\star$ so that we can estimate the time evolution of the stellar mass for the MSC.
5.2 Results

Fig. 19 shows that the host of an MSC can gradually remove stars from the outer part of the MSC owing to the strong tidal field in the model with \( M_\ast = 10^9 \, M_\odot, R_\ast = 75 \, pc, f_\sigma = 1.2, M_b = 5 \times 10^9 \, M_\odot, r_\ast = 1.6 \, kpc, x_\ast = 200 \, pc \) and \( f_v = 0.75 \). Time \( T \) (in units of Gyr) that has elapsed since the simulation started is shown in the upper left corner of each frame.

![Figure 19](image)

Figure 19. Time evolution of the distribution of stars projected on to the x-y plane for models with \( M_\ast = 10^9 \, M_\odot, R_\ast = 75 \, pc, f_\sigma = 1.2, M_b = 5 \times 10^9 \, M_\odot, r_\ast = 1.6 \, kpc, x_\ast = 200 \, pc \) and \( f_v = 0.75 \). Time \( T \) (in units of Gyr) that has elapsed since the simulation started is shown in the upper left corner of each frame.

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Fig. 20 shows how the time evolution of total stellar masses within 100 pc of MSCs depends on \( f_\sigma \) and \( r_\ast \) for models with \( M_\ast = 10^9 \, M_\odot, R_\ast = 75 \, pc, f_\sigma = 1.2, M_b = 5 \times 10^9 \, M_\odot, r_\ast = 1.6 \, kpc, x_\ast = 200 \, pc \) and \( f_v = 0.75 \). Time evolution of total stellar masses within 100 pc of MSCs depends on \( f_\sigma \) and \( r_\ast \) for models with \( M_\ast = 10^9 \, M_\odot, R_\ast = 75 \, pc, f_\sigma = 1.2, M_b = 5 \times 10^9 \, M_\odot, r_\ast = 1.6 \, kpc, x_\ast = 200 \, pc \) and \( f_v = 0.75 \). About 60 per cent of original stars can be stripped within \( \sim 1 \) Gyr and the stripped stars are distributed widely within the original orbital plane and thus regarded as ‘field stars’. Although the mass and size of the MSC can become much smaller than the original ones, the MSC is still strongly bound at \( T = 1.09 \) Gyr. The stripping process is gradual in this model: only 25 per cent of the original mass of the MSC can be stripped within \( 0.1 \) Gyr. Thus, AGB ejecta might well be retained with the MSC to finally be converted into new stars (= 5G ones) in this model.

Fig. 20. Total stellar masses within 100 pc as a function of time \( T \) for models with \( r_\ast = 1.6 \, kpc \) and \( t_\sigma = 1.0 \) (blue solid), \( r_\ast = 1.6 \, kpc \) and \( t_\sigma = 1.2 \) (green dotted), \( r_\ast = 0.8 \, kpc \) and \( t_\sigma = 1.0 \) (magenta short-dashed), \( r_\ast = 0.8 \, kpc \) and \( t_\sigma = 1.2 \) (red long-dashed). For these for models, \( M_\ast = 10^9 \, M_\odot, M_b = 5 \times 10^9 \, M_\odot, r_\ast = 1.6 \, kpc, x_\ast = 200 \, pc \) and model parameters other than \( r_\ast \) and \( t_\sigma \) are fixed.

![Figure 20](image)

Figure 20. Total stellar masses within 100 pc as a function of time \( T \) for models with \( r_\ast = 1.6 \, kpc \) and \( t_\sigma = 1.0 \) (blue solid), \( r_\ast = 1.6 \, kpc \) and \( t_\sigma = 1.2 \) (green dotted), \( r_\ast = 0.8 \, kpc \) and \( t_\sigma = 1.0 \) (magenta short-dashed), \( r_\ast = 0.8 \, kpc \) and \( t_\sigma = 1.2 \) (red long-dashed). For these for models, \( M_\ast = 10^9 \, M_\odot, M_b = 5 \times 10^9 \, M_\odot, r_\ast = 1.6 \, kpc, x_\ast = 200 \, pc \) and model parameters other than \( r_\ast \) and \( t_\sigma \) are fixed.

6 DISCUSSION

6.1 A threshold cluster mass for secondary star formation

We have shown that \( M_\ast \) can determine whether AGB ejecta from FG stars of MSCs can be retained within the MSCs and consequently converted into new stars efficiently. Fig. 22 briefly summarizes...
multiple stellar populations. Based on the present numerical results, we divide MSCs into the following categories according to their $M_\text{nt}$. It should be stressed here that $M_\text{nt}$ for a cluster is not the present mass ($M_{\text{gs}}$) but its original one.

### 6.2.1 $M_\text{nt} < 10^4 M_\odot$

These low-mass clusters can neither retain AGB ejecta nor convert gas into new stars owing to their shallow gravitational potential wells. Therefore these clusters are highly unlikely to show abundance spread even in light elements. Observations showed that the Galactic open cluster (OCs) show unimodal distributions of CN band strength (Martell & Smith 2009) and do not have abundance spread in light elements (e.g. Carretta, Bragaglia & Gratton 2007) and O–Na anticorrelation (De Silva et al. 2009). These low-mass clusters can be easily destroyed by interaction with GMCs (e.g. Gieles et al. 2006) so that they cannot obtain fresh cold gas for further star formation. The observed low-mass OCs in the Galaxy and the Magellanic Clouds belong to this class.

### 6.2.2 $10^4 M_\odot \leq M_\text{nt} < 2 \times 10^5 M_\odot$

These clusters cannot retain AGB ejecta efficiently within their central regions so that secondary star formation cannot occur. However, they can obtain fresh cold gas for further star formation by capture and accretion of cold molecular gas that are either those left behind from previous star formation events or those located close to them (e.g. Bekki & Mackey 2009). Therefore, if secondary star formation occurs within these clusters, they can only show multiple stellar populations with age differences yet no/little abundance spread even in light elements. The intermediate-age clusters in the LMC and the SMC belong to this class.

### 6.2.3 $2 \times 10^5 M_\odot \leq M_\text{nt} < 6 \times 10^5 M_\odot$

Clusters with this mass range can retain most of their AGB ejecta within the central regions but cannot convert the stars so efficiently ($\epsilon_\text{sf} < 0.3$) into new stars within a short time-scale ($<10^4$ yr). Although dynamical fate of the accumulated AGB ejecta for the cluster remains unclear, ‘triggering mechanisms’ such as cluster mergers (or initially very high-density clusters) would be necessary to convert the gas so efficiently into new stars. Strong ram pressure stripping of the gas by ISM of their host galaxies could remove the gas from the clusters.

### 6.2.4 $6 \times 10^5 M_\odot \leq M_\text{nt} < 10^7 M_\odot$

The ordinary Galactic GCs with abundance spread in light elements can originate from clusters with this mass range, because AGB ejecta can be retained in the clusters and converted into new stars very efficiently ($\epsilon_\text{sf} > 0.5$). The mass fraction of SG stars in a present GC depends mainly on $M_\text{nt}$ and the dynamical evolution of its original cluster (i.e. destruction by its host and the long-term evolution driven by a two-body relaxation). If these massive clusters are initially stellar nuclei of massive dwarf galaxies, then AGB ejecta from SG stars can be converted into new stars (i.e. the formation of a third generation of stars). This sequential formation of stars in the nuclear regions of hosts can continue until the hosts are destroyed by much larger galaxies.
6.2.5 $10^2 M_\odot \leq M_c$

Very massive clusters with this mass range are progenitors of the present massive GCs with masses larger than $10^6 M_\odot$. Stellar systems with $M_c > 10^5 M_\odot$ would be dwarf galaxies themselves rather than clusters, and therefore at least some fraction of massive GCs (e.g. $\omega$ Cen and G1) may well be the remnants of nucleated dwarf galaxies. Owing to much deeper gravitational potential wells of nucleated dwarfs, not only AGB ejecta but also gas from more energetic massive stars and supernovae would be able to be retained and finally converted into new stars. Thus such massive GCs can show abundance spread in both light and heavy elements.

6.3 Mixing of AGB ejecta with fresh cold gas

6.3.1 Bondi accretion

Using analytical models, Pfennig et al. (2009) recently have shown that if the mass of a cluster exceeds a threshold mass ($= 10^6 M_\odot$), then the cluster can obtain a significant amount of warm ISM from outside the cluster by Bondi accretion within a reasonable time-scale (i.e. a few Gyr). We have confirmed their results using 3D hydrodynamical simulations, though the threshold mass for gas accretion is larger ($= 3 \times 10^6 M_\odot$) in our estimation (Bekki et al., in preparation). We suggest that although secondary formation from the mixed gas of AGB ejecta and the accumulated ISM by Bondi accretion would be reasonable for massive GCs with possible age differences (like $\omega$ Cen), such star formation has the following potentially serious problems in explaining chemical abundances of ordinary Galactic GC.

First, if original clusters are within galaxies, then they can obtain gas initially located in different regions with possible different chemical abundances (owing to radial and azimuthal abundance gradients with the galaxies) thus are highly likely to have gas and new stars with different [Fe/H]. Therefore, only a minor fraction of the Galactic GCs may well have SG stars formed from mixed gas of AGB ejecta and ISM obtained by Bondi accretion. Secondly, the Bondi accretion rate is too small ($\sim 10^{-3} M_\odot$ yr$^{-1}$ for $M_c = 3 \times 10^6 M_\odot$) for typical ISM so that an enough amount of SG stars cannot form from gas chemically polluted only by massive AGB stars ($\geq 5 M_\odot$) within an order of $10^7$ yr. It is possible that if densities of warm ISM are as large as $10^2$ atoms cm$^{-3}$, then clusters would be able to obtain an enough amount of gas within a time-scale of less than $10^8$ yr (but if this is the case, then such high-density gas may well be cold molecular clouds rather than warm ISM).

Thirdly, even if warm ISM with an initial temperature of $\sim 10^4$ K can be accreted by clusters, further efficient radiative cooling (to 10–100 K) within a short time-scale ($<10^7$ yr) is required for further star formation. Our previous and present study have not yet investigated how warm ISM and AGB ejecta mix with and consequently convert into cold molecular gas for further star formation using a sophisticated model, and accordingly we cannot currently discuss the possibility of secondary star formation from the accumulated warm ISM. Our future more sophisticated models will investigate whether these three are really serious problems.

6.3.2 Capture and accretion of cold molecular gas by clusters

It is possible that the original MSCs would have a plenty of cold molecular gas left behind from the formation of FG stars, because the star formation efficiencies are highly likely to be well less than 100 per cent. Given that GMCs which are progenitors of FG stars should be very massive ($>10^7 M_\odot$) in the present scenario, the host GMCs would have numerous substructures (i.e. smaller molecular clouds) that remain intact even after the formation of FG stars. These molecular clouds can be captured by or accreted on to MSCs, as a previous simulation has demonstrated (Bekki & Mackey 2009). Owing to initially low temperatures of the captured cold gas, star formation from gas mixed between AGB ejecta and the captured gas would proceed efficiently.

A potential problem of this process is that it remains unclear whether cold molecular clouds left behind the formation of FG stars can really keep the same abundances as those of FG stars (i.e. remain ‘pristine’) without being chemically polluted by massive stars and supernovae of FG stars. A significant fraction of the residual cold gas would be heated up to become high-temperature and low-density warm and hot gas so that they cannot be captured later by MSCs owing to their high temperature and low density. Owing to the clumsy structures of the original massive GMCs, a significant amount of energy from massive stars and supernovae can be expelled from low-density interstructure regions of the GMCs towards the haloes of their hosts. Some smaller cold molecular clouds that can survive thermal and kinetic feedback effects of energetic stars would be able to be captured later by MSCs and consequently be mixed with AGB ejecta.

The above processes of the mixing of cold molecular gas and AGB ejecta have not been investigated in detail by any previous 3D hydrodynamical simulations of star-forming GMCs. Bekki & Chiba (2007) investigated (i) how massive stars and type I supernovae influence gas dynamics within star-forming GMCs and (ii) chemical abundances of gas left behind from star formation. They however did not include ejection of gas so that they could not investigate how AGB ejecta mixed with the residual gas to form new stars. It is thus doubtlessly worthwhile for our future more sophisticated models to investigate whether and what fraction of residual molecular cloud left behind from the formation of FG stars can be converted into new stars without being chemically polluted by massive FG stars.

6.4 Origin of GCs with multiple stellar populations in the Magellanic Clouds

A number of recent observational studies have found that a significant fraction of intermediate-age clusters in the LMC and the SMC show double main-sequence turnoffs (hereafter referred to as DMSTOs for convenience) on their colour–magnitude diagrams (Mackey & Broby Nielsen 2007; Glatt et al. 2008; Mackey et al. 2008; Goudfrooij et al. 2009; Milone et al. 2009). The simplest explanation is that the observed DMSTOs in each of these clusters represent two distinct stellar populations with differences in age of $\sim 100–300$ Myr. Recently Bastian & de Mink (2009) have proposed that stellar rotation in stars with masses between 1.2 and $1.7 M_\odot$ can mimic the effects of DMSTOs without resorting to age differences between stellar populations within clusters of the Magellanic Clouds. However, the presence of a dual red clump of giant stars in the colour–magnitude diagram of NGC 419 in the SMC is suggested to be inconsistent with the claim by Bastian & de Mink (2009) but be explained naturally by the presence of multiple star-formation episodes (Rubele, Kerber & Girardi 2010).

Mucciarelli et al. (2008) investigated chemical abundances of light odd-Z, $\alpha$, iron-peak and neutron-capture elements for four intermediate-age clusters in the LMC and found negligible star-to-star scatter for them. However, Mucciarelli et al. (2009) revealed significant abundance inhomogeneities in [Na/Fe], [Al/Fe], [O/Fe] and [Mg/Fe] and O–Na and Mg–Al anticorrelations in old GCs.
(> 10 Gyr). These observations imply that intermediate-age clusters have (at least) two generations of stars with different ages yet similar chemical abundances whereas old ones are like the Galactic GCs with abundance spread in light elements. Therefore, any theory of GC formation need to explain why the LMC appears to have two different types of GCs with multiple stellar populations.

Bekki & Mackey (2009) showed that low-mass clusters ($M_\ast \sim 5 \times 10^4 M_\odot$) can capture cold GMCs and thereby convert the gas into new stars within their central regions in the LMC. The present study has shown that such low-mass clusters cannot efficiently retain AGB ejecta and thus are unable to use the ejecta for further star formation. The results by Bekki & Mackey (2009) and the present study therefore combine to suggest that if the observed DMSTOs in clusters of the LMC are due to age differences of stellar populations, then the two populations are highly unlikely to show abundance spread in light elements owing to their low masses (< $10^3 M_\odot$).

The present study also suggests that the old clusters of the LMC can show abundance spread in light elements, mainly because their original clusters are significantly more massive than the present ones with masses ranging from $10^5$ to $6 \times 10^5 M_\odot$. Recently Conroy & Spergel (2010) have suggested that clusters with masses larger than a threshold mass of $\sim 10^6 M_\odot$ can form SG stars from AGB ejecta mixed with ISM accreted on to clusters. The observed lack of abundance inhomogeneity in light elements in the intermediate-age clusters of the LMC (Mucciarelli et al. 2008) seems to be inconsistent with their model. The present study suggests that the threshold mass for the formation of SG stars from AGB ejecta mixed with pristine gas is much larger than the one suggested by Conroy & Spergel (2010).

Old and metal-poor ([Fe/H] $< -1.65$) clusters in the LMC have masses more than $10^5 M_\odot$ (Mackey & Gilmore 2003) and would have lost a significant fraction of their masses due to long-term internal evolution effects and external tidal field of the LMC. Therefore recent observations on star-to-star abundance variations in light elements in old and intermediate clusters of the LMC by Mucciarelli et al. (2008, 2009) strongly suggest that the threshold cluster mass ($M_{th}$) for secondary star formation from AGB ejecta mixed with pristine gas would be much more than $10^5 M_\odot$. These observations are consistent with the present model with $M_{th} \sim (6-10) \times 10^5 M_\odot$ and the one by D’Ercole et al. (2008) which showed that clusters with $M_\ast \sim 10^5 M_\odot$ can form SG stars from AGB ejecta.

6.5 Were original massive stellar systems really ‘clusters’?

D’Ercole et al. (2008) first claimed that original single stellar systems (‘clusters’) composed purely of FG stars should have masses of $\sim 10^3 M_\odot$ to explain the observed masses of massive GCs ($\sim 10^5 M_\odot$). Although our present simulations confirm their claim, it remains unclear how such massive single clusters form from very massive GMCs. Given the possible substructures within host GMCs for FG stars, the original stellar systems can still be clusters of smaller clusters when FG AGB stars begin to eject their gaseous winds. If this is the case, evolution of AGB ejecta in such clusters of clusters would be significantly different from what D’Ercole et al. (2008) and the present study describe.

It would also be possible that (i) original massive stellar systems of GCs are dwarf galaxies themselves and therefore (ii) compact stellar systems formed from AGB stars of the dwarfs are identical to their stellar galactic nuclei. In this scenario, stellar nuclei are dominated by SG stars that can be formed more quickly from AGB ejecta of FG ones owing to much deeper gravitational potentials of their host with dark matter haloes. Therefore, stellar nuclei of nucleated dwarfs are likely to be dominated by He-rich, Na-rich and O-poor stars. Future observations on chemical abundances of stars in nuclei of the Sagittarius dwarf and NGC 205 will enable us to discuss whether this scenario is physically viable.

6.6 Implications

6.6.1 A bottom-heavy IMF for SG stars?

The present simulations have shown that secondary star formation from AGB ejecta can occur at the central regions of original clusters, where mean stellar number densities exceed $\sim 10^2$ stars pc$^{-3}$ (for $M_\ast = 10^6 M_\odot$). Recently Krumholz et al. (2009) investigated formation of massive stars by gaseous accretion within a gas cloud with a mass of 100 $M_\odot$ and a size of 0.1 pc and found that two (binary) stars with masses of 29.2 and 41.5 $M_\odot$ can be formed within 5.7 $\times 10^4$ yr. Therefore the present simulations imply that gas clouds for massive stars (as those investigated by Krumholz et al. 2009 above) can interact frequently with stars within the central regions of a dense stellar system owing to the small mean separation of stars ($\sim 0.2$ pc) and the short dynamical time-scale ($\sim 5 \times 10^4$ yr). It would be accordingly possible that massive star formation is severely suppressed owing to this star–cloud interaction which can prevent gas accretion on to clouds forming massive stars: IMFs for the formation of SG stars within MSCs would be bottom-heavy. If formation of binary star-forming gas clouds, which are progenitors of binary stars, can also be prevented owing to star–cloud interaction, then the observed small binary fractions among SG stars in GCs (D’Orazi et al. 2010b) can be naturally explained.

If only SG stars have a bottom-heavy IMF with the power-law slope of 3.35 with FG ones with $\alpha = 2.35$, then the number fraction of low-mass stars with masses lower than 0.5 $M_\odot$ for SG stars to that for FG ones is 1.7 for $m_{\ast} = 120 M_\odot$. This number fraction can become as large as 2 by assuming unrealistically low $m_{\ast} (=1 M_\odot)$ and steeper $\alpha$ (<3). Therefore, if IMFs can be bottom-heavy only for SG stars, then the required original masses of clusters that finally become the present GCs can be up to a factor of 2 smaller than those derived in the present work with the standard IMF both for FG and SG stars. Thus, the possible bottom-heavy IMF cannot change the main conclusion that the progenitor clusters of the present GCs are much more massive than the present GCs.

6.6.2 Origin of rotation, shapes and scaling relations of GCs

Previous works discussed the origin of shapes and rotation (e.g. Frenk & Fall 1982; Einsel & Spurzem 1999), sizes and luminosities (e.g. Murray 2009) and scaling relations (e.g. Djorgovski 1993; Bekki et al. 2004; Gieles et al. 2010) using analytical models and collisionless numerical simulations. They accordingly did not discuss the origin of GC properties in the context of initially nested structures and ignored the importance of gas dynamics in the formation of GC properties. If most GCs really originate from nested stellar systems, then conclusions of the above previous works based on models with ‘non-nested’ stellar structures must be dramatically modified. Thus it would be reasonable to say that the above conclusions can apply only for low-mass clusters with single stellar population and no nested structures.

Bekki (2010) has recently shown that the origin of the observed rotation in GCs (e.g. Meylan & Mayor 1986; Meylan & Heggie 1997; Anderson & King 2003) can be closely associated with dissipative gas dynamics of AGB ejecta from FG stars within the
forming GCs. Given the observed large mass fractions of SG stars in the present GCs (e.g. Carretta et al. 2010), gas dynamics of AGB ejecta within original clusters composed only of FG stars may well be a key determinant for structure and kinematics of the present GCs. Although recent numerical simulations have investigated long-term dynamical evolution of GCs with initially nested stellar structures (e.g. Decressin, Baumgardt & Kroupa 2008; D’Ercole et al. 2008), their models do not include initial rotation and flattened shapes of SG stars so that their results cannot be used for discussing the origin of rotation and shapes of GCs. Our future long-term (>10 Gyr) dynamical simulations of GCs based on structures and kinematics of GCs with nested stellar structures, flattened 3D distributions and rotation simulated in the present study will enable us to discuss structure and kinematics in a consistent manner.

6.6.3 Where are forming GCs with nested structures?
Recently Vinkó et al. (2009) have found two stellar populations with younger and older ones with ages of 10–16 Myr and 32–100 Myr, respectively, in the young, massive stellar cluster Sandage-96 in a spiral arm of NGC 2403. Given the observed possible large age difference (up to ~100 Myr), this type of young massive clusters can be the candidates that are now forming or have just formed SG stars from AGB ejecta from FG ones. Vinkó et al. (2009) also have found that the younger population are located closer to the centre of the cluster in comparison with the older one. If the cluster has a mass as large as 10^7 M⊙, the observation would be consistent with the present results on the nested structure of MSCs, though quantitative comparison (i.e. the mass-ratio and the half-mass-radius-ratio of the two populations) with the present results is not currently possible. The present study suggests that if the total mass of the cluster is as large as 10^6 M⊙, the redder colours in the younger population of the cluster can be due to gas and dust that now surrounds the younger population and were previously ejected from the older one.

If future observations find young clusters with large masses (>10^6 M⊙) and two populations with the age differences as large as 10^3 yr in actively star-forming galaxies, they can be forming GCs with nested structures and thus provide strong constraints on the formation models of GCs like the present one. As suggested above, new compact clusters can be shrouded by gas and dust left behind from the formation of SG stars until supernova events expel the gas and dust: the dust-shrouded new clusters would have very red colour for their young ages and possibly they cannot be even seen in optical bands if dust extinction is so heavy. If FG stars cannot be clearly identified as compact clusters owing to their initially more diffuse distributions, then dust-shrouded new clusters within the FG stars might well be identified as giant H II regions with no optical counterparts. However, it is not clear whether the observed giant H II regions with no near-infrared cluster counterparts in starbursting luminous infrared galaxies with numerous super-star-clusters (e.g. Alonso-Herrero et al. 2002) can be dust-obsured very young compact clusters (SG) embedded in older ones (FG).

Massive stellar systems with masses of ~10^5 M⊙ were discovered in ultraluminous infrared-galaxies with dust-shrouded strong starbursts (e.g. Monreal-Ibero et al. 2007). Numerical simulations showed that such dusty starburst galaxies can evolve into ‘E+A’ galaxies with post-starburst populations with ages of 0.1–1 Gyr (Bekki, Shioya & Couch 2001). The present study has shown that MSCs with Mgc = 10^5 M⊙ and ages of less than 1 Gyr (before destruction of the old clusters) can have nested stellar structures. These results combine to imply that young GCs with nested structures are highly likely to exist in E+A galaxies. It is thus worthwhile for future observational studies to search for massive young clusters in E+A galaxies (e.g. NGC 5102) to provide a clue to the origin of multiple stellar populations of GCs.

7 CONCLUSIONS
We have performed 3D numerical simulations of star formation from AGB ejecta in MSCs formed from GMCs within their host galactic building blocks. We have considered that stars initially in MSCs and those newly formed from AGB ejecta correspond to FG and SG stars, respectively, in the present GCs and thus discussed the origin of multiple stellar populations of GCs. The main results are summarized as follows.

1. Gaseous ejecta from AGB stars can be retained within MSCs with Mgc larger than ~2 x 10^5 M⊙, if MSCs are isolated (i.e. not being influenced by ram pressure of ISM). The mass fraction of gas retained in an MSC to Mgc (Fret) depends on Mgc such that Fret is larger in larger Mgc. The multiple hydrodynamical interaction of gaseous ejecta from numerous AGB stars can significantly increase gas mass accumulated within the central regions of MSCs, which was first found by the present study.

2. If the Mgc of MSCs exceed a threshold cluster mass Mth, gaseous ejecta of AGB stars can sink into the central regions and then form high-density gaseous regions so that new stars (i.e. SG stars) can be formed there with high star formation efficiencies (>0.3). Mth is demonstrated to be (6–10) x 10^5 M⊙ in the present study. Deep gravitational potentials of MSCs play a great role both in retaining the accreting AGB ejecta on to the central regions and in forming high-density gaseous regions there. Therefore secondary star formation within clusters is inevitable in MSCs with Mgc larger than Mth. Owing to the existence of Mth, young open clusters and super-star-clusters with Mgc < 6 x 10^5 M⊙ cannot evolve into stellar systems with abundance spread in light elements.

3. Owing to the high star formation efficiencies (θ = 0.3–0.9), very compact stellar systems can be formed from AGB ejecta within a time-scale of ~10^3 yr (after massive AGB stars start to eject their winds). The half-mass radius of a new compact cluster is by a factor of 5 smaller than that of the old ones for most models in the present study. The nested clusters (or ‘cluster within cluster’) with more diffuse old clusters and very compact new ones are characteristic of the present numerical models.

4. At most 1–4 per cent of all stars can be from new stars formed from AGB ejecta for the canonical IMFs in the present study. Therefore, in order to explain the present total mass of SG stars with masses of ~10^6 M⊙ in a GC, Mg is required to be as large as (3–10) x 10^5 M⊙. The required mass for the progenitor MSC for Ω Cen can be as large as 10^6 M⊙ owing to its larger present mass. Given that Ω = 0.3–0.9 for secondary star formation, the original masses of progenitors (Mgc) for the present GCs with masses of Mgc need to be larger than 25Mgc.

5. Structural and kinematical properties of new compact clusters formed within original old ones depend on Mgc such that new clusters can be more flattened and more strongly supported by rotation in MSCs with larger Mgc. The new clusters are much more compact and more strongly supported by rotation than the old ones in all models with initial rotation in old clusters. These structural and kinematical differences between old and new clusters do not depend on model parameters such as Mgc, Rgc and µgc. Given that the present GCs are dominated by SG stars, the result implies that the physical origins of flattened shapes and internal rotation of GCs are closely associated.
with formation processes of SG stars through gas dynamics within MSCs.

(6) Evolution of MSCs initially in the very centres of their hosts can be significantly different from that of MSCs outside the centres, mainly because AGB ejecta can be more effectively retained within nuclear MSCs owing to much deeper gravitational potentials of their hosts. Irrespective of $M_*$, most AGB ejecta can be retained in these nuclear MSCs. Star formation, however, can proceed within nuclear MSCs, only if $M_*$ exceeds a threshold value that is almost the same as $M_{\rm th}$ described above. Strong gravitational fields of hosts can enhance secondary star formation of nuclear MSCs. Multiple episodes of star formation (i.e. third and fourth generations of stars) are possible in nuclear MSCs with $M_s \geq M_{\rm th}$ until their hosts are destroyed during accretion of the hosts on larger galaxies. The origin of multiple generations of stars observed in $\omega$ Cen and NGC 2808 can be closely associated with evolution of nuclear MSCs.

(7) MSCs can lose gradually ($\sim 0.1$–1 Gyr) significant fractions of their stars during dynamical evolution within their hosts owing to tidal stripping by their hosts. The stripping processes and the timescale depend strongly on $M_s$, orbits and initial locations of MSCs within hosts. If stars in an MSC enter into their AGB phases after the MSC has lost most of their stars because of tidal stripping, then the AGB ejecta cannot be efficiently retained in the MSC owing to its much shallower gravitational potential. Therefore gaseous ejecta from AGB stars with lower masses are less likely to contribute to secondary star formation owing to the destruction of the MSC by its host. Thus the destruction of MSCs due to their hosts can provide a mechanism by which star formation can be truncated in MSCs.

(8) Direct capture and accretion of cold molecular clouds by MSCs themselves can be much more efficient ways to dilute the AGB ejecta of MSCs (i.e. mixing of these gaseous components) in secondary star formation within MSCs. The molecular clouds need to have chemical abundances very similar to those of MSCs, and thus they are either (i) those that are located very close to MSCs or (ii) those left behind from the formation of FG stars. The observed hierarchical structures of GMCs are suggested to be responsible for the dilution processes during the formation of SG stars.

(9) Low-mass MSCs with $M_s < 10^7 M_\odot$ in the LMC can obtain cold molecular gas by capturing the gas but cannot retain AGB ejecta owing to much shallower gravitational potentials of MSCs. Therefore the intermediate-age clusters with DMSTOs recently discovered in the Magellanic Clouds can have multiple stellar populations with age differences yet no abundance spread in light elements. If GMCs really originate from MSCs with large $M_*$, then the host GMCs for such MSCs should be even more massive ($10^8$–$3 \times 10^9 M_\odot$). We plan to investigate how very massive GMCs can form and evolve into MSCs due to efficient star formation within massive dwarf galaxies at high $z$.

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