Research Article

Multilayer Orthogonal Beamforming for Priority-Guaranteed Wireless Communications

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To utilize the benefits of cellular systems, wireless machine-to-machine (M2M) communications over cellular systems are being widely considered. In order to support efficient spectrum sharing between M2M devices and normal mobile users, in the paper, we propose a multilayer orthogonal beamforming (MOBF) scheme for M2M communications over orthogonal frequency division multiple access (OFDMA-) based cellular systems. Using MOBF, each subcarrier in OFDMA systems could be efficiently reused by both normal mobile users and machine-type devices which are organized into multiple virtual layers. The users located in higher layers (e.g., mobile users) are not to be interfered by those in lower layers (e.g., machine devices). To improve the performance, the orthogonal deficiency (OD-) based user selection is carried out, where the intralayer fairness and quasimaximal performance can be guaranteed, simultaneously. Moreover, the signal-to-interference plus noise ratio (SINR) is investigated to measure the performance lower bound of different layers. It is demonstrated by both theoretical and numerical results that the proposed approach provides a stable SINR performance for each layer, that is, the interference free ability from lower level layers.

1. Introduction

Wireless machine-to-machine (M2M) communications over cellular systems are being widely considered as an increasingly important interaction mechanism for M2M applications [1, 2]. In order to utilize the benefits of cellular systems, for example, ubiquitous coverage, the beyond 3rd generation (B3G) and the 4th generation (4G) cellular systems are expected to play crucial roles in the future wireless M2M networks [3, 4]. However, due to different services with quality of service (QoS) are required for different users in cellular systems, that is, human users with mobile terminals and a large number of machine devices, it may not be straightforward to consider M2M communications over cellular systems with limited spectrum resources. More specifically, a higher QoS is needed to support instant communications for mobile users, while the delay tolerance of M2M communications could be measured in a wide range (i.e., from a few milliseconds to several hours) [5]. Thus, it is desired to develop a communication strategy that provides different QoS requirements of humans and machines under limited spectrum resources.

Recent studies have shown that the space division multiple access (SDMA) could be an efficient means to exploit the spectrum efficiency over cellular systems. The SDMA approaches are capable of achieving a much higher capacity and wireless resource usage efficiency [6], which have been proposed for the 3rd generation partnership project long-term evolution (3GPP-LTE) standard [7]. As the optimal SDMA strategy, the dirty paper coding (DPC) is proposed in [8, 9], yet it is difficult to be implemented in practical systems due to the prohibitively high complexity. Thus, various suboptimal beamforming (BF) approaches are proposed to reduce the complexity. It is shown in [10] that the zero-forcing beamforming (ZFBF) achieves a large fraction of DPC capacity as the number of users approaches infinity. However, good performance of ZFBF cannot be guaranteed with limited number of users and low signal-to-noise ratio (SNR) regime. Thus, orthogonal beamforming (OBF) is proposed in [11] and further developed in [12] to improve the performance, while a hybrid scheme is studied in [13] to achieve promising performance in various scenarios. In [14], the probability density function (PDF) expressions for the scheduled users and closed-form expression of the first and second scheduled
users are derived to theoretically analyze the performance of OBF. Note that conventional methods of BF have been developed to maximize the total sum rate. If different performance requirements are considered for different users, for example, mobile users require a higher QoS to support instant communications which cannot be interfered by machine users, existing BF approaches may not provide a proper solution.

In this paper, to provide flexible performance for different users with diverse priorities, we propose a multilayer orthogonal beamforming (MOBF) for wireless M2M communications over cellular systems under the orthogonal frequency division multiple access (OFDMA) framework. (As the key technology of the 3GPP-LTE mobile broadband standard [15], OFDMA is considered as a background in this paper.) Although different metrics can be carried out to define the physical layer QoS, we consider the signal-to-interference plus noise ratio (SINR) outage probability as the one used in [16]. Therefore, candidate users with various QoS demands could be organized into multiple layers based on required SINR outage probability. By using the proposed MOBF, users in higher layers (i.e., mobile users) enjoy a sufficiently high SINR and will not be interfered from those in lower layers (i.e., machine users). Although users in lower layers may suffer the interference from higher layers, a user selection strategy is carried out to maximize the performance under the orthogonal constraint. It is shown that our proposed approach supports a stable SINR performance for each layer, which is then illustrated by theoretical and numerical results.

The rest of the paper is organized as follows. Section 2 provides the system model. Our proposed MOBF is introduced in Section 3, while its performance is then analyzed in Section 4. After that, numerical results are presented in Section 5. Finally, we conclude this paper in Section 6.

Notation. The superscripts $T$ and $H$ stand for the transpose and Hermitian transpose, respectively. $I_M$ represents the $M \times M$ identity matrix. Denote by $\| \cdot \|$ and $| \cdot |$ the 2-norm and amplitude of the enclosed complex valued quantity, respectively. $\langle \cdot \rangle$ is the inner product. The statistical expectation is represented by $E[\cdot]$. The statistical distribution of a circularly symmetric complex Gaussian (CSCG) random variable with mean $a$ and covariance $b$ is denoted by $CN(a, b)$.

2. System Model

Consider the downlink channel of cellular systems (as shown in Figure 1) with $K$ users requiring $L$ priorities separately (i.e., $L$ virtual layers needed), where each user is equipped with single receive antenna, and let $A = \{1, 2, \ldots, K\}$ be the index set of all $K$ users. For a given subcarrier (MOBF for one certain subcarrier is emphasized throughout this paper, and the extension method for the whole OFDMA system is straightforward,) the base station (BS) equipped with $M$ transmit antennas simultaneously transmits data to $N$ users selected from $A$, where $N \leq M$ and denote by $\delta$ the index set of $N$ selected users. Since in a typical system, the number of users is larger than the number of transmit antennas, we assume that $M \leq K$ throughout the paper. Thus, the
transmitted signal through one subcarrier from BS can be expressed as
\[ s = \sum_{n=1}^{N} w_n x_n = W_N x, \]  
(1) 
where \( w_n \in \mathbb{C}^{M \times 1} \) is the \( n \)th beamforming vector (which is also regarded as the \( n \)th beam) for the selected user. Let \( W_N = [w_1, w_2, \ldots, w_N] \) be the unitary beamforming matrix with \( W_N^H W_N = I_N \) and denote by \( x = [x_1, x_2, \ldots, x_N]^T \) the transmitted symbol vector in one time slot, where (Equal power allocation over scheduled users is considered, which is equivalent to the optimal water-filling method as the users have high SNR.) \( \mathbb{E}[|x_n|^2] = P_s \). Assume that the channel gain from the BS to the \( n \)th user, denoted by \( h_n \in \mathbb{C}^{M \times 1} \), is independent and identically distributed (i.i.d.) flat Rayleigh fading. Then, the signal received by the \( k_n \) selected user \( k_n \) is given by
\[ y_{k_n} = h_{k_n}^H s + z_{k_n}, \quad k_n \in \mathcal{S}, \]  
(2) 
where \( z_{k_n} \) denotes the additive white Gaussian noise with zero mean and \( \sigma^2 \) variance. According to (2), the SINR of the \( n \)th beam observed by the \( k_n \)th user becomes
\[ \text{SINR}_{k_n} = \frac{\left| h_{k_n}^H w_n \right|^2 P_s}{\sum_{n \neq n} \left| h_{k_n}^H w_n \right|^2 P_s + \sigma^2}, \quad k_n \in \mathcal{S}. \]  
(3) 
Throughout this paper, the perfect channel state information (CSI) at the base station is assumed. For the sake of simplification, we assume \( L = N \) in the paper, which leads to that a single user is considered in each virtual layer at a time. The cases of multiple users selected in one layer are beyond the scope of the paper and can be treated as an extension for future works.

### 3. Multilayer Orthogonal Beamforming with ODS User Selection

In order to avoid the interference from lower layers to higher layers, in this selection, the MOBF is carried out to provide interference-free ability for users with high priorities. Based on that, an orthogonal deficiency (OD) based user selection strategy is considered to improve the performance.

#### 3.1. Multilayer Orthogonal Beamforming

Letting \( \mathcal{S} = \{k_1, \ldots, k_n, \ldots, k_N\} \) and \( h_{k_n} \) be the channel gain of the selected user \( k_n \) for the \( n \)th layer, then the beamforming vector is given by
\[ w_n = \frac{\tilde{w}_n}{||\tilde{w}_n||}, \]  
(4) 
where
\[ \tilde{w}_n = \begin{cases} I_M - \sum_{i=1}^{n-1} \frac{\tilde{w}_i^H \tilde{w}_i}{||\tilde{w}_i||^2} & \text{for } n = 1, 2, \ldots, N. \end{cases} \]  
(5) 
for \( n = 1, 2, \ldots, N \). From (5), we can show that \( W_N \) is a matrix whose column vectors are orthogonal to each other.

Thus, it can be easily derived that \( w_n \) would be orthogonal to \( h_{k_i} \) when \( i < n \), that is, \( h_{k_i}^H w_n = 0 \). In this case, according to the SINR expression in (3), the \( n \)th selected user does not receive any interference from the \( n \)th one, that is, the user \( k_n \) located in the lower layers.

However, \( w_i \) may not be orthogonal to \( h_{k_i} \), as the \( w_i \) is generated without taking \( h_{k_i} \) into consideration when \( i < n \). The well-known orthogonal deficiency (OD) [17, 18] can be used as a metric to measure the orthogonality as
\[ \xi(k_n, i) = \frac{||h_{k_n}^H \tilde{w}_i||^2}{||h_{k_n}||^2 ||\tilde{w}_i||^2} \quad (i < n). \]  
(7) 
Note that two vectors are orthogonal when \( \xi(k_n, i) = 0 \).

**Theorem 1.** Denote by \( \xi(k_n, i) \) the OD of \( h_{k_n} \) and \( w_i \), for the \( n \)th selected user with index \( k_n \), the SINR of the user is given by
\[ \text{SINR}_{k_n} = \frac{1 - \sum_{i=1}^{n-1} \xi(k_n, i)}{\sum_{i=1}^{n-1} \xi(k_n, i) + (1/\gamma_{k_n})}, \quad n \leq N, \]  
(8) 
where \( y_{k_n} = (||h_{k_n}||^2 P_s/\sigma^2) \) denotes the corresponding SNR.

**Proof.** Assuming \( n < j < N \), it can be easily obtained that \( (h_{k_i}, w_j) = 0 \). Therefore, we reshape (3) as
\[ \text{SINR}_{k_n} = \frac{h_{k_n}^H w_n^2 P_s}{\sum_{i=1}^{n-1} \left| h_{k_n}^H w_i \right|^2 P_s + \sigma^2} \]  
(9) 
\[ = \frac{\left( \left| h_{k_n}^H \tilde{w}_i \right|^2 / ||\tilde{w}_i||^2 \right)}{\sum_{i=1}^{n-1} \left( \left| h_{k_n}^H \tilde{w}_i \right|^2 / ||\tilde{w}_i||^2 \right) + (\sigma^2/\gamma_{k_n})}, \quad n \leq N. \]  
According to (5), we can show
\[ \left| h_{k_n}^H \tilde{w}_n \right|^2 = \left| \left| h_{k_n} \right|^2 - \sum_{i=1}^{n-1} \left| h_{k_n}^H \tilde{w}_i \right|^2 \right| \]  
(10) 
\[ \left| \tilde{w}_n \right|^2 = h_{k_n}^H \left( I_M - \sum_{i=1}^{n-1} \frac{\tilde{w}_i^H \tilde{w}_i}{||\tilde{w}_i||^2} \right) h_{k_n} \]  
\[ = \left| h_{k_n} \right|^2 - \sum_{i=1}^{n-1} \left| h_{k_n}^H \tilde{w}_i \right|^2. \]  
Since \( ||\tilde{w}_n||^2 \approx 0 \), we have
\[ \text{SINR}_{k_n} = \frac{\left| h_{k_n} \right|^2 - \sum_{i=1}^{n-1} \left( \left| h_{k_n}^H \tilde{w}_i \right|^2 / ||\tilde{w}_i||^2 \right)}{\sum_{i=1}^{n-1} \left( \left| h_{k_n}^H \tilde{w}_i \right|^2 / ||\tilde{w}_i||^2 \right) + (\sigma^2/\gamma_{k_n})} \]  
(11) 
\[ = \frac{1 - \sum_{i=1}^{n-1} \xi(k_n, i)}{\sum_{i=1}^{n-1} \xi(k_n, i) + (1/\gamma_{k_n})}, \quad n \leq M. \]  
This completes the proof.
It is noteworthy that as γ_{k_n} \gg 1, the SINR of user \( k_n \) at the \( n \)-th orthogonal beam can be approximated by its signal-to-interference ratio (SIR), that is,

\[
\text{SINR}_{k_n} \approx \text{SIR}_{k_n} = \frac{1 - \sum_{i=1}^{n-1} \xi(k_n, i)}{\sum_{i=1}^{n} \xi(k_n, i)},
\]

and upper bounded by

\[
\text{SINR}_{k_n} \leq \min\{\text{SIR}_{k_n}, \text{SNR}_{k_n}\} = \min\left\{\frac{1 - \sum_{i=1}^{n-1} \xi(k_n, i)}{\sum_{i=1}^{n} \xi(k_n, i)}, \gamma_{k_n}\right\}.
\]

In general, \( \xi(k_n, i) \neq 0 \), as \( h_{k_n} \) and the elements of \( W_{n-1} \) are independent. As shown in (12), \( \text{SINR}_{k_n} \) is more likely to decrease with \( n \) increases. Thus, the user with a small \( n \) (at high layer) enjoys a higher SINR thanks to the MOBF, where a high QoS requirement could be guaranteed.

However, the user with a large \( n \) (at low layer) suffers the interference from those with small \( n \) because of OD existing. Note that different channel conditions of users result in different ODs, which may lead to quite different performances of the MOBF. Thus, OD-based user selection plays a crucial role to minimize the interference and meanwhile maximize the system performance.

### 3.2. OD-Based User Selection Criterion.

Let \( A_n \) be the subset of candidate users with the same SINR requirement for layer \( n \), where the user number of subset is \( K_n \), \( n \in \{1, 2, \ldots, N\} \). Note that \( A_1 = A_2 \cup A_3 \cup \cdots \cup A_N \) and \( K = K_1 + K_2 + \cdots + K_N \), where “∪” denotes the set union. In order to maximize the system performance, the user who has the maximum achievable rate can be chosen as

\[
k_n = \arg\max_{k \in A_n} \log_2(1 + \text{SINR}_{k_n})
= \arg\max_{k \in A_n} \text{SINR}_{k_n}.
\]

According to (12) and (14), when \( \gamma_k \) is sufficiently large, the SINR-based user selection criterion is approximately identical to choose the user who has the smallest sum of ODs. Hence, the resulting selection scheme is regarded as the orthogonal deficiency sequential (ODS) user selection, which is summarized as follows.

1. Let \( n = 1 \). According to Theorem 1, the user to be selected in the first layer would not be interfered by other users. Thus, the first user can be selected by

\[
k_1 = \arg\max_{k \in A_1} \gamma_k.
\]

Once the index of the first user is determined, we update \( \delta = \{k_1\} \). In addition, we let \( w_1 = h_{k_1}/\|h_{k_1}\| \).

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2. Let \( n = n + 1 \). The \( n \)-th user is chosen as

\[
k_n = \arg\max_{k \in A_n} \frac{1 - \sum_{i=1}^{n-1} \xi(k_n, i)}{\sum_{i=1}^{n} \xi(k_n, i) + (1/\gamma_k)}
\approx \arg\min_{k \in A_n} \sum_{i=1}^{n-1} \xi(k, i).
\]

Once the \( n \)-th user is determined, we update the user subset \( \delta = \delta \cup \{k_n\} \). After that, we generate \( w_n \) using the proposed MOBF in (4).

3. If \( n = N \), \( \delta = \{k_1, \ldots, k_N\} \) and stop. Otherwise, go back to step (2).

As \( N \) users are selected, the achievable rate for the \( n \)-th layer is given by

\[
R_n = \log_2(1 + \text{SINR}_{k_n}), \quad k_n \in \delta.
\]

And the sum rate for the given subcarrier is carried out as

\[
R_{\text{sum}} = \sum_{n=1}^{N} \log_2(1 + \text{SINR}_{k_n}), \quad k_n \in \delta.
\]

In comparison with SINR-based selection criteria, whose performances are highly dependent on SNR (which may have fairness problem due to the SNR is close related to the distance between BS and users [6]), our proposed ODS user selection could provide an equal chance for each candidate user with the same priority even in the near-far environment. Note that, the ODS user selection neglects the effect of channel gain for the fairness requirement, which means only quasimaximal performance can be guaranteed. However, the performance loss is negligible when the system is interference limited and the SNR is sufficient high.

### 4. Performance Analysis

In this section, the theoretical analysis of our proposed MOBF is carried out. The outage probability of the threshold SINR for each layer is derived to measure the lower bound of the performance. It is assumed that the elements of channel vectors \( h_k, k \in A_n \) are i.i.d. CSCG random variables distributed as \( \mathcal{CN}(0, 1/M) \).

1. The Distribution of OD. Suppose that \( a \) and \( b \) are two independent \( m \times 1 \) complex-valued Gaussian random vectors. Then, we have

\[
\cos^2 \theta = \frac{|a^H b|^2}{\|a\|^2 \|b\|^2} \sim \text{Beta}(1, m - 1),
\]

where \( \theta \) is the angle between \( a \) and \( b \), and Beta \((\alpha, \beta)\) denotes the Beta distribution [19].

**Lemma 2.** \( \xi(k_m, 1), \ldots, \xi(k_m, n - 1) \) are i.i.d. random variables and follow the Beta distribution.
Proof. According to (5), all the column vectors of \( \mathbf{W}_{n-1} \), that is, \( \mathbf{w}_i \) (\( i = 1, 2, \ldots, n - 1 \)), are unitary Gaussian random vectors, as the orthogonalization operation is a linear transformation and \( \mathbf{h}_n \) is a Gaussian random vector.

Besides, it can be derived from (5) that all \( \mathbf{w}_i \) of \( \mathbf{W}_{n-1} \) are uncorrelated with each other. Therefore, they are independent due to equivalence between the uncorrelated and the independent for Gaussian random variables.

Using the above results, different \( \xi(k_n, i) \) in (7) are independent of each other, since (7) is the function of \( \mathbf{h}_n \) and \( \mathbf{w}_n \) when \( i < n \). Thus, according to (19), \( \xi(k_n, 1), \ldots, \xi(k_n, n - 1) \) are i.i.d random variables and follow the Beta distribution, where

\[
\xi(k_n, i) \sim \text{Beta}(1, M - 1).
\] (20)

This completes the proof. \( \square \)

Using Lemma 2, the PDF and the cumulative distribution function (CDF) of \( \xi(k_n, i) \) are given by

\[
\xi(k_n, i) \sim f_\xi(x) = (M - 1)(1 - x)^{M-2}, \quad 0 \leq x \leq 1,
\] (21)

\[
F_\xi(x) = 1 - (1 - x)^{M-1}, \quad 0 \leq x \leq 1,
\] (22)

respectively.

4.2. SINR Outage Probability. In this section, the SINR performance of the MOBF with ODS is analyzed by using the distribution of OD. Since it is not easy to find the PDF and CDF of SINR in (8), we consider SIR as the approximation of SINR in (12) and the upper bound in (13).

Letting \( \Gamma \) be the threshold SINR, the outage probability of SINR in (12) for a random user \( k \in \mathcal{A}_n \) is approximately obtained as

\[
\Pr(\text{SINR} \leq \Gamma) \approx \Pr\left(\frac{1 - \sum_{i=1}^{n-1} \xi(k, i)}{\sum_{i=1}^{n} \xi(k, i)} \leq \Gamma\right) = 1 - \Pr\left(\sum_{i=1}^{n-1} \xi(k, i) \leq \frac{1}{1 + \Gamma}\right),
\] (23)

where \( 0 < \Gamma < +\infty \).

In order to deal with the sum of ODs, we can prove the following lemma.

Lemma 3. Assume \( \xi_1, \xi_2, \ldots, \xi_n \) are i.i.d random variables. Let \( f_{\xi_i}(x_i) \) and \( F_{\xi_i}(x_i) \) be the PDF and CDF of \( \xi_i \), respectively, \( i = 1, \ldots, n \). The CDF of \( \eta_n = \sum_{i=1}^{n} \xi_i \) is given by

\[
F_{\eta_n}(x) = F_{\xi_1} \ast F_{\xi_2} \ast \cdots \ast F_{\xi_n} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{\xi_1}(x_1) f_{\xi_2}(x_2) \cdots f_{\xi_n}(x_n) \cdot F_{\xi_n}(x - x_1 - x_2 - \cdots - x_{n-1}) \cdot dx_{n-1} \cdots dx_2 dx_1,
\] (24)

where “\( \ast \)” represents the convolution operation.

Proof. See Appendix. \( \square \)

Using Lemma 3 together with (21) and (22), the CDF of SIR for a random user in the \( n \)th layer can be rewritten as

\[
F(\Gamma) = 1 - \int_0^x \cdots \int_0^x f_{\xi_1}(x_1) f_{\xi_2}(x_2) \cdots f_{\xi_{n-1}}(x_{n-1}) \cdot F_{\xi_n}(x - t_{n-2}) dt_{n-2} \cdots dx_2 dx_1,
\] (25)

where \( x = 1/(1 + \Gamma) \) and \( t_n \) denotes the sum of \( x_i \), that is, \( t_n = \sum_{i=1}^{n} x_i \).

Since (25) is complicated, it is desired to have a concise expression for analysis. Supposing that \( \delta = 1/(n(n-1)(1+\Gamma)) \), and \( n > 1 \), we have

\[
\Pr(\xi_1 \leq \delta, \xi_2 \leq \delta, \ldots, \xi_{n-1} \leq \delta) \leq \Pr\left(\sum_{i=1}^{n-1} \xi_i \leq \frac{1}{1 + \Gamma}\right),
\] (26)

where \( \xi_i = \xi(k, i) \), \( k \in \mathcal{A}_n \) for notational simplicity. Since \( \xi_1, \xi_2, \ldots, \xi_{n-1} \) are i.i.d. random variables, we have

\[
\Pr(\xi_1 \leq \delta) \cdot \Pr(\xi_2 \leq \delta) \cdots \Pr(\xi_{n-1} \leq \delta) = \Pr^{(n-1)}(\xi_i \leq \delta).
\] (27)

Then, the CDF of SIR in (25) for a random user in the \( n \)th layer is approximated by

\[
F(\Gamma) \approx \tilde{F}(\Gamma)
\]

\[
= 1 - \Pr^{(n-1)}\left(\frac{1}{(n-1)(1+\Gamma)}\right)
\]

\[
= 1 - \left(1 - \left(1 - \frac{1}{(n-1)(1+\Gamma)}\right)^{M-1}\right)^{n-1},
\] (28)

where \( 0 < \Gamma < \infty \) and \( n > 1 \).

Using order statistics [20], the selected user has the maximum SIR among user subset \( \mathcal{A}_n \), which can be derived by

\[
F_{\max}(\Gamma) = \Pr\left(\max_{k \in \mathcal{A}_n} \text{SIR}_{k} \leq \Gamma\right)
\]

\[
= \tilde{F}_{\max}(\Gamma) \approx \tilde{F}_{\max}(\Gamma).
\] (29)

By taking into account of the SNR, from (13), the SINR of the selected user \( k_n \) based on ODS is upper bounded by

\[
\text{SINR}_{k_n} \leq \min\left\{\text{max}\left\{\text{SIR}_{k}, \text{SNR}_{k}\right\}\right\}.
\] (30)

Thus, the lower bound of SINR outage probability for the selected user in the \( n \)th layer is given by

\[
\Pr(\text{SINR}_{k_n} \leq \Gamma) \geq \Pr_{\text{LB}}(\Gamma)
\]

\[
= 1 - (1 - F_{\max}(\Gamma)) \left(1 - F_{\xi(2M)}\left(\frac{2M\sigma^2\Gamma}{P_s}\right)\right),
\] (31)
and the approximation is
\[ P_{LB}(\Gamma) \approx \hat{P}_{LB}(\Gamma) \]
\[ = 1 - \left( 1 - \hat{P}_{K_n}(\Gamma) \right) \left( 1 - F_{\chi(2M)} \left( \frac{2M\sigma^2\Gamma}{P_s} \right) \right), \]
(32)
where \( F_{\chi(n)} \) denotes the chi-square CDF with \( n \) degrees of freedom.

4.3. Achievable Rate Analysis. With the lower bound of SINR outage probability, the achievable rate can be expressed with a given \( \Gamma \). Then, the lower bound of average achievable rate using the outage probability in (31) is shown as
\[ R_n \geq \text{Pr}(\text{SINR}_k \leq \Gamma) \times 0 \]
\[ + \text{Pr}(\text{SINR}_k > \Gamma) \log_2(1 + \Gamma) \]
(33)
\[ = (1 - P_{LB}(\Gamma)) \log_2(1 + \Gamma). \]
A tight lower bound on \( R_n \) can be achieved using integration method with respect to \( \Gamma \) as
\[ R_n = \int_0^{\hat{\Gamma}} \log_2(1 + \Gamma) d(P_{LB}(\Gamma)) \]
(34)
\[ \approx \sum_{\gamma \in \Gamma} \left( \hat{P}_{LB}(\gamma + \delta) - \hat{P}_{LB}(\gamma) \right) \log_2(1 + \gamma), \]
(35)
where \( \hat{\Gamma} \) denotes the practical cutoff limit and \( \delta \) is the quantification step. Note that a smaller \( \delta \) provides a more accurate approximation.

5. Numerical Results
In this section, numerical results are presented where we evaluate the priority-guaranteed ability of the proposed MOBF scheme. We consider homogeneous users in a cell with an \( M \)-antenna base station, that is, user channels are independent unit-variance complex Gaussian vectors. Then, the squared channel magnitude \( \| h_k \|^2 \) follows a chi-square distribution with \( 2M \) degrees of freedom. Additionally, in the simulation of Figures 2–5, we assume that all users experience the same pass-loss condition, while shadow fading is not considered. Moreover, without loss of generality, the number of candidate users \( K_n \) is used ranging from 1 to 1000.

Figure 2: The CDF of SIR for a random user in the \( n \)th layer in a noise-free scenario with \( M = 8 \) antennas and \( N = 4 \) layers.

Figure 3: Comparison of the SINR outage probability simulation and lower bound with subset user number \( K_n = 10, M = 8 \) antennas, \( N = 4 \) layers, and total transmit power \( P_s = 20 \) dB.

Figure 4: Sum rate performance comparison of MOBF with \( M = 4, 8, 12 \) antennas, \( N = M \) layers, and total transmit power \( P_s = 5 \) dB. Equal number of candidate users \( K_n \) is used ranging from 1 to 1000.
In Figure 2, the OD-based analytical CDF of SIR in (25) and the simulation results obtained in a noise-free scenario are compared. Since the first layer is an interference-free layer according to the MOBF, the SIR outage probability comparison is obtained with the parameters $n = 2, 3,$ and 4. It is shown from Figure 2 that the analytical and simulated results exhibit the same behavior.

Figure 3 shows the outage probabilities versus threshold SINR among different layers of MOBF (i.e., $n = 1, 2, 3,$ and 4), where numerical results and the lower bound derived in (31) are compared. Through simulations, $N = 4, K_n = 10,$ and $M = 8$ are considered. It shows that the user with a smaller $n$ (in a higher layer) has lower outage probability since the impact of interference becomes smaller. In particularly, the user in the first layer won’t be interfered by those in lower layers, therefore the first layer SINR is reduced to SNR which could be described as the chi-square distribution. Furthermore, from Figure 3, we can also show that the derived lower bound is able to provide an accurate approximation. Note that the total number of candidate users is $K_n \times N$.

Simulation results of sum rate versus number $K_n$ of each user subset are presented in Figure 4, where $N = M$ for the cases of $M = 4, 8,$ and 12 antennas are considered, respectively. It shows that the sum rate of the system increases with more candidate users (i.e., $K_n$) and transmitting antennas (i.e., $M$). Besides, we can confirm that the appropriate theoretical lower bound in (35) with $\delta_y = 0.1$ dB and $\tilde{\Gamma} = [-10$ dB, $20$ dB] is reasonable accurate, even though a large $K_n$ is considered. In particular, as $K_n$ increases, the sum rates of all schemes increase because of more multi-user diversity.

Additionally, ZFBF with semiorthogonal user selection (SUS) [10] and OBF with greedy user selection algorithm-B (GUS-B) [11] are also considered for performance comparison in Figure 4. It can be observed that both MOBF and OBF outperform ZFBF at the regime with fewer $K_n$ and relatively low SNR, which is more practical for the power-limited M2M networks. Besides, though marginal sum rate gain can be obtained by GUS-B over OBF, a complexity order of $O(K^2)$ is required, which is higher compared to the complexity order of $O(K)$ required by the proposed ODS.

In Figure 5, we compare the performance of our proposed MOBF and the ZFBF-SUS in [10] in terms of the achievable rate versus $N$, where $M = 8$ and total power allocation $P_t = 5$ dB are considered. It shows that the ZFBF provides descending performances as $N$ increases, since the effective channel gain decreases dramatically because of the interlayer interference [10]. On the contrary, our proposed MOBF provides stable performances of all layers even when $N = M$, because the MOBF layer rates are only affected by higher layers. Moreover, it is confirmed that the user with a smaller $n$ (in a higher layer) enjoys a higher performance by using our proposed approach.

6. Conclusion

In this paper, the MOBF strategy for wireless M2M communications over cellular systems is proposed together with
an ODS user selection criterion. It has been shown that users with high priorities enjoy high SINR due to the use of MOBF, since users in higher layers won’t be interfered by those in lower layers. On the other hand, the SINR of users with low priorities can also be maximized using ODS user selection. The performance of different layers has been then analyzed using theoretical tools. Through analysis and numerical results, it has been shown that our proposed scheme provides more stable performance for different users compared to the existing approach. Thus, for wireless M2M communications over cellular systems, the MOBF can be regarded as a stable mechanism for both mobile and machine users.

Appendix

Proof. The proof of lemma 3 is shown as follows. Let \( f_\zeta(x) \) and \( F_\zeta(x) \) be the PDF and CDF of the random variable \( \zeta \), respectively.

First, if \( n = 2 \), \( \zeta_1 \) and \( \zeta_2 \) are i.i.d. random variables, the CDF of \( \eta_2 = \zeta_1 + \zeta_2 \) is given by

\[
F_\eta_2(x) = F_\zeta_1(x_1) * F_\zeta_2(x_2) = \int_{-\infty}^{+\infty} f_\zeta_1(x_1) F_\zeta_2(x-x_1) dx_1,
\]

(A.1)

which is the precise expression of (24) with \( n = 2 \). Provided that (24) is established for \( n = k \) (\( k \geq 2 \)), we have

\[
F_n(x) = F_\zeta * F_\zeta * \cdots * F_\zeta = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_\zeta(x_1) f_\zeta(x_2) \cdots f_\zeta(x_k) dx_1 \cdots dx_k
\]

(A.2)

When \( n = k + 1 \), we can show that

\[
F_{n+1}(x) = F_\zeta * F_\zeta * \cdots * F_\zeta * F_\zeta = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_\zeta(x_1) F_\eta_{n-k+1}(x-x_1) dx_1
\]

\[
= \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_\zeta(x_1) f_\zeta(x_2) \cdots f_\zeta(x_k) \cdot dx_k \cdots dx_2 dx_1.
\]

(A.3)

This completes the proof of lemma 3.

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