**Interference Modeling and Analysis of LoRa Network**

Zhiyu Nie\(^1\) and Lingyun Jiang\(^2\)

**ABSTRACT**

With the rapid development of Internet of Things, low power wide area networks have attracted much attention from both industry and academia. In this paper, we investigate the interference issue with the coexistence of LoRa nodes. For this purpose, we adopt Poisson cluster process to model the location of the LoRa nodes and use Poisson point process to model the distribution of non-LoRa nodes. On the other hand, we divide the distance range within the same LoRa network, based on the fact that different spreading factors can extend the transmission distance. By doing this, we can get more comprehensive analysis of the node interference. For the whole network, we analyze coverage probability of the desired node in the network to measure its performance and study how the number of nodes affect area spectrum efficiency. The final simulation results can provide a guidance for the practical deployment.

**INTRODUCTION**

Large scale Internet of Things (IoT) installations are becoming a reality and networks are being deployed to realize smart city, service system or other intelligent applications. Many of these IoT devices rely on Low-Power Wide-Area Network (LPWAN) technologies. New LPWAN technologies such as LoRa\(^1\), NB-IoT\(^2\) and SigFox\(^3\) are emerging which enable power efficient wireless communication over very long distances.

LoRa is a new technology for long-range, low-power, low bit rate, and single-hop wireless communications. It is intended for IoT applications with low throughput

\(^1\) Interference modeling and analysis of LoRa network, Jiangsu Key Lab. of Wireless Commun., Nanjing Univ. of Posts & Telecommun., Nanjing, China
\(^2\) Jiangsu Key Lab. of Wireless Commun., Nanjing Univ. of Posts & Telecommun., Nanjing, China
requirements. Chirp spread spectrum technology is used by LoRa where the digital signal broadens the original signal band to the entire linear spectral range by modulating the chirp signal. Thus gateways can receive and process the data of multiple nodes in parallel, and expanding the system capacity.

As the number of LoRa terminals continues to increase, nodes share the communication medium leading to more interference. The risk of packet collisions remains a problem in uplink. Thus it is necessary to establish model for LoRa network and analyze the interference problem.

**Related Works**

Modeling and analysis of LoRa communication has taken two main directions. The first one concentrates on discussing the change of spreading factor(SF). The differences of the LoRa spread factor is essentially chirp spread spectrum, so the problems of chirp spread spectrum has been discussed. Among them, [4] compared the coexistence of nodes using chirp spread spectrum and other modulation method. Its results can highlight the advantages of chirp spread spectrum. In [5], derived from the mathematical point of view of the chirp spread spectrum. Under these theory, [6]-[8] tried to clarify the impact of changes in SF from the perspective of random collisions.

In general, increasing the SF in LoRa can extend transmission range [9]. Based on the above facts, a single-gateway LoRa network can be divided in some ranges [10]. And for the same node of SF, there is no interference problem. Multi-gateway LoRa research can refer to [11]. In [12], by means of experiments, the interference problem between LoRa network and other wireless technologies is analyzed from the aspect of carrier frequency. However, for a mixed network composed of multiple LoRa gateways and non-LoRa nodes, there is no relevant discussion.

The second direction, which is also more relevant to our work, focuses on characterizing metrics. Based on stochastic geometric, [13] has done some extension research to verify the process. Gateways and nodes are uniformly distributed in an area, Poisson point processes (PPP) model networks. However, for LoRa networks with closely associated nodes and gateways, PPP cannot provide an accurate model for the interference [14]. Different from PPP, Poisson cluster process(PCP) is used to model the transmission process in many scenarios. Multiple parents in all centers generally satisfy the PPP distribution, while sub-nodes generate clusters around the corresponding parent nodes.

In the current studies and networks, PCP is widely used [15]-[17]. [15] is the first time that a PCP process has been applied to the LoRa network. And it also
demonstrated that there exists an optimal number of active LoRa nodes for each cluster, which can maximize the area spectrum efficiency. But the combination of LoRa and PCP networks is relatively simple, without a complete LoRa interference model. It needs a more comprehensive model to describe LoRa network. In the study of D2D terminal coexistence issues, PCP will also be used to establish a research model. For heterogeneous cellular networks, PCP has been adopted to model the phenomena of nodes clustering at hotspots [16]. [17] is a typical application of PCP in D2D scenarios. It studied the direct communication between neighboring nodes and analyzes the change of the capacity performance of the cellular network by establishing a PCP model so as to improve the overall spectrum coverage capability. In [18], the PCP model has been applied to wirelessly powered backscatter communication where gateways form the parent PPP and backscatter nodes are children points in the cluster.

**Motivation and Contributes**

In this paper, we specialize in studying multiple disturbances on the LoRa network and classify different disturbances based on the specific properties of LoRa. First we adopt a PCP cluster process to model the whole network, where PPP cluster center is generated by LoRa gateways and the LoRa nodes in each cluster generate the children process. We also assume that non-LoRa nodes in our network are modeled as PPP. According to the characteristic that larger coverage needs larger SF, then we divide the area affected by the interference of LoRa nodes. Through the above model establishment and division, we can get a more comprehensive and accurate interference model.

The rest of the paper is organized as follows. In section 2, we model the whole network and divide transmission range. In section 3, new mathematical expressions are derived for the coverage probability and area spectrum efficiency. Based on the expressions in section 3, simulation and analysis of results are presented in section 4. In section 5, we draw the conclusion.

**SYSTEM MODEL**

In this section, we introduce propagation models and system settings for the coexistence issues of LoRa networks and other LPWAN technologies in the uplink transmission. Then, we divide the transmission range of single gateway LoRa network.
**Spatial Setup And Key Assumption**

As shown in Figure. 1, we consider a LoRa network $\Phi$. For LoRa terminals located in circular area, the nodes are modeled as a PCP, where the parent points are modeled by PPP $\Phi_{\text{out}}$, with density $\rho_{\text{out}}$. And the offspring point process (one per parent) are conditionally independent.

![Figure 1. System model of LoRa networks and other coexisting radios.](image)

In practice propagation environment, there exist non-LoRa propagation nodes which are modeled as a PPP $\Phi_{\text{other}}$, with density $\rho_{\text{other}}$. LoRa nodes with the same SF will interfere each other when propagating. At same time, since LoRa is working in an unlicensed spectrum, non-LoRa nodes (e.g., NB-IoT nodes) also produce corresponding interference. As Figure. 1 shows, non-LoRa nodes modeled as a PPP.

**Range Division**

LoRa supports adaptive rate modulation, therefore it is possible to make trade-offs between multiple measures to make the most appropriate parameter selection for transmission. In a channel, spreading spectrum can reduce the signal to noise ratio of the channel and SF determines the length of the chirp symbol $T_s=2^{SF}/\text{BW}$, where BW (125kHz) is transmission bandwidth. The higher SF ($SF\in\{7,8,\ldots,12\}$) results in higher receiver sensitivity, which can extends the gateway's communication range. According the above theories, a circular single-gateway LoRa network is divided into six annular areas based on six different SF. Figure 2 shows the divided model. We can find network has been divided into six annular areas $(l_j, l_{j+1}), j=0,1,2,3,4,5.$
In each cluster, every cluster members are assumed to be independent and randomly distributed in various regions based on different SF. For the receiver centered at a cluster $S_y \in \Phi_{out}$, the density function of LoRa nodes in $(l_j, l_{j+1})$ is

$$f(r) = \begin{cases} \frac{1}{\pi(l_{j+1}^2 - l_j^2)} & l_j < r \leq l_{j+1}, j = 0, 1, 2, 3, 4, 5 \\ 0 & \text{else} \end{cases}$$

We assume that nodes are distributed in their own area, and the number of nodes is $m$. On the other hand, the number of simultaneously nodes will be different across clusters, thereby providing sufficient generality to the model.

LoRa protocol states that LoRa nodes with different SF exist orthogonality when transmitting. It means that if gateway receives two or more signals with different SF, the corresponding output waveform can be discerned. Therefore, we will only consider about co-spreading factor interference as described below.

According to the above analysis, three different sources of interference can affect the transmission of the desired node. Combining Figure 1 and Figure 2, a more comprehensive propagation model that interfered with LoRa network can be established.

**Transmission Model**

Assuming transmit power of each nodes to be $P_t$. Each nodes that exist in network can be expressed by
\[ P = P_h h_0 L(d), \]  

(2)

where \( h_0 \sim \text{exp}(1) \) exponential random variable which models Rayleigh fading and \( L(d) \) is path loss attenuation function. In Eq. (2), \( L(d) = \lambda/(4\pi d^\eta) \), which follows from the Friis transmission equation, where \( d \) is distance between desired node and its gateway, \( \lambda \) is the carrier wavelength, and \( \eta \geq 2 \) is the path loss exponent.

In this network \( \Phi \), the total interference caused at the gateways of interest can be described as the sum of three independent terms: (i) In-LoRa cluster interference \( I_{\text{in-LoRa}} \) caused by the interfering nodes inside the LoRa cluster (ii) Out-LoRa cluster interference \( I_{\text{out-LoRa}} \) caused by simultaneously nodes outside the LoRa cluster, (iii) Other-nodes interference \( I_{\text{other}} \) on the same frequency. At a typical LoRa receiver \( S_0 \), interference from LoRa nodes within the same cluster can be expressed as:

\[
I_{\text{in-LoRa}} = \sum_{x \in S_0} P_{x} h_{x,y_0} L(x),
\]  

(3)

where \( P_x \) is the transmit power of the interference LoRa nodes within the same cluster, and \( x \) is distance between nodes and gateway. Similarity, interference from outside the LoRa cluster can expressed as:

\[
I_{\text{out-LoRa}} = \sum_{y \in S_y} \sum_{x \in S} P_{x} h_{x,y} L(x + y).
\]  

(4)

We also consider interference from non-LoRa nodes transmitting

\[
I_{\text{other}} = \sum_{z \in \Phi_{\text{other}}} P_{z} h_{z,y_0} L(z).
\]  

(5)

At same time , the received signal-to-interference-plus-noise ratio (SINR) for the desired LoRa node is

\[
\text{SINR} = \frac{P_{y_0} h_{y_0,y_0} L(x_0)}{I_{\text{in-LoRa}} + I_{\text{out-LoRa}} + I_{\text{other}} + N}.
\]  

(6)
For LoRa nodes, the thermal noise in dBm level is calculated as \( N = -174 + 10 \log_{10}(\text{BW}) \), where \( \text{BW}(125\text{kHz}) \) is transmission bandwidth.

**THE PERFORMANCE OF COVERAGE PROBABILITY AND AREA SPECTRAL EFFICIENCY**

In order to measure the degree of network interference, we will derive coverage probability(\( P_{\text{cov}} \)) and area spectral efficiency(ASE) to analyze the network interference received in this section. \( P_{\text{cov}} \) is defined as the probability that a typical user can successfully decode its signals at the gateway with suitable SINR and threshold \( \beta \). The transmission coverage probability is given by

\[
P_{\text{cov}}(\beta) = E[\Pr\{\text{SINR} \geq \beta\}].
\]  \hspace{1cm} (7)

By applying Eq. (1) into Eq. (7), we can obtain

\[
P_{\text{cov}}(\beta) = E[\Pr\{ \frac{P_s h_{\text{ant}}, g_{I}}{I_{\text{in-LoRa}} + I_{\text{out-LoRa}} + I_{\text{other}} + N} \geq \beta \}]
\]  \hspace{1cm} (8)

\[
= E \left( e^{-\mu N} L_{\text{in-LoRa}}(\mu) L_{\text{out-LoRa}}(\mu) L_{\text{other}}(\mu) \right),
\]

where \( \mu = \frac{(\text{4\pi d})^2 \beta}{L_{\text{d}}^2} \), \( L_{\text{in-LoRa}}(\mu) = E\{e^{-\mu I_{\text{in-LoRa}}}\} \), \( L_{\text{out-LoRa}}(\mu) = E\{e^{-\mu I_{\text{out-LoRa}}}\} \), \( L_{\text{other}}(\mu) = E\{e^{-\mu I_{\text{other}}}\} \) are the Laplace transforms of the \( I_{\text{in-LoRa}}, I_{\text{out-LoRa}} \) and \( I_{\text{other}} \).

**Three Independent Interference**

In this subsection, we commit to getting the Laplace transform of three independent interference.

1) Laplace transform of In-LoRa cluster interference:

\[
L_{\text{in-LoRa}}(s) = \exp(-\frac{2(m-1)}{l_{j+1}^2 - l_j^2}) \int_{l_{j}}^{l_{j+1}} \frac{sp L(x)}{1 + sp L(x)} dx,
\]  \hspace{1cm} (9)
2) Similarity, Laplace transform of Out-LoRa cluster interference is:

\[ L_{\text{Out-LoRa}}(s) = E\{e^{-s\text{Out-LoRa}}} = \exp(-2\pi\rho_{\text{out}} \int_0^{2\pi} 1 - \exp(-mS(x, y)) ydy). \] (10)

In Eq. (10),

\[ S(x, y) = \frac{1}{\pi(\rho_1^2 - \rho_2^2)} \int_0^{\frac{2\pi}{\rho_2^2}} 1 - \frac{1}{1 + s\rho_1 L(Q(x, y, \theta))} d\theta dx, \] (11)

where \[ Q(x, y, \theta) = \sqrt{x^2 + y^2 - 2xy\cos\theta}. \] \( \theta \) is the angle between the distance from the two LoRa gateways \( y \) and the distance from inter-node to its according receiver \( x \). For the situation that \( x << y \), \( Q(x, y, \theta) \) can be approximated as \( y \). Based on Taylor expansion, \( 1 - \exp(-mS(x, y)) \) is approximated as \( mS(x, y) \).

3) we can obtain the Laplace transformers of Other-nodes interference by similar method:

\[ L_{\text{Other}}(s) = E\{e^{-s\text{Other}}} = \exp(-2\pi\rho_{\text{Other}} \int_0^{2\pi} 1 - \frac{1}{1 + s\rho_1 L(z)} dz). \] (12)

According the Laplace transformers of 1), 2) and 3), we can get the expression of coverage probability.

**Coverage Probability**

The transmission coverage probability is given by

\[ P_{\text{cov}}(\beta) = \frac{2}{\pi(\rho_1^2 - \rho_2^2)} \int_{\frac{\beta}{\rho_2^2}}^{\frac{1}{\rho_2^2}} e^{-\mu N} L_{\text{Other}}(\mu) L_{\text{Other}}(\mu) r dr \]

\[ = \frac{2}{\pi(\rho_1^2 - \rho_2^2)} \int_{\frac{1}{\rho_2^2}}^{\frac{1}{\rho_2^2}} e^{-\frac{1}{\rho_2^2}(\frac{1}{\rho_2^2})^2} L_{\text{Other}}(\mu) L_{\text{Other}}(\mu) r dr \] (13)

Coverage probability is a manifestation of network coverage. From the expression of coverage probability, we can find the coverage probability is a monotonically decreasing function of \( d, \beta, m \). This indicates three characteristics:

1) The longer the distance between the node and the gateway, the lower the coverage will be.
2) When the transmission distance is in a far range, it can be improved by lowering the threshold. So for the transmission range of different SFs in \((l_j, l_{j+1})\), we adopt different \(\beta\) to suit different ranges.

3) The more nodes in the area, the more the sources of interference are, and finally the lower the coverage probability.

Area Spectrum Efficiency

Except coverage probability, we will analyze the area spectrum efficiency in this subsection. The ASE denotes the average numbers in bits that all nodes can transmit per unit area. We define the express of ASE is

\[
ASE = m \rho_{\text{out}} \log_{10} (1 + \beta) P_{\text{cov}} (\beta). 
\]

(14)

The area spectrum efficiency is an increasing function of coverage probability and \(m\). However, coverage probability is a decreasing function of \(m\). Therefore, there must exist a suitable \(m\), making ASE reach the maximum.

NUMERICAL RESULTS

Analysis Of Coverage Probability

We first simulate the relationship between the coverage probability and the \(d\). The LoRa nodes within a population are uniformly distributed according to the range of different SFs. We set the carrier frequency of LoRa nodes is 470Mhz. The power of all nodes are 14dBm and the path loss exponent taken to be equal to 3. For the density of the LoRa gateways and non-LoRa nodes are \(\rho_{\text{out}} = \rho_{\text{other}} = 10/(2000^2 \pi)\).

The correspondence between threshold \(\beta\) and \(d\) that the distance between gateway and desired node is shown in Table I.
Table I. The Relationship of Distance and Threshold.

| d         | $\beta_1$ | $\beta_2$ |
|-----------|-----------|-----------|
| $0<d<=200$| -15dB     | -12dB     |
| $200<d<=40$| -18dB     | -15dB     |
| $400<d<=60$| -21dB     | -18dB     |
| $600<d<=80$| -24dB     | -21dB     |
| $800<d<=1000$| -26dB | -24dB |
| $1000<d<=200$ | -28dB | -26dB |

We can find there are two sets of $\beta_1$, $\beta_2$ and $\beta_1$ is lower than $\beta_2$ at the same distance range. And the longer the range in which $d$ is located, the lower the $\beta$ required.

Since the range of distances in which the nodes are located is random, the maximum value generated by the coverage probability is picked. According to the above parameter settings, We compare the trend of the coverage probability with SF division and without SF division. The relationship of them is shown in Figure 3 when $m=6$.

![Figure 3. Coverage probability as a function of d(m) when m=6, assuming $\beta=\beta_1$ and the threshold of curve without SF division is 20dB.](image-url)
Figure 3 forms a downward trend of the ladder. For the curve with SF division, as $d$ becomes longer, the coverage probability decreases gradually and the coverage probability increases a bit after switching to another distance ranges. The reason of these is that longer range require lower $\beta$. At the moment when the distance range of the desired node to increase, the $\beta$ immediately decreases, resulting in an increased coverage probability. In contrast, the curve without SF division presents a normal exponential decay trend. Decreasing threshold will increase the coverage probability to some extent. Even after $d$ reaches a certain value, the performance increase will exceed the problem of degraded performance due to distance. As we can see in Figure 3, the Coverage probability of the curve with SF division is higher than the curve without SF division.

![Graph](image.png)

Figure 4. Coverage probability as a function of $d(m)$ when $m=6$ and $m=12$, assuming $\beta=\beta1$ and the threshold of curve without SF division is 20dB.

The trend in Figure 4 is similar to Figure 3. On the one hand, as $d$ becomes longer, the coverage probability decreases while it rises a bit after switching to another distance ranges. On the other hand, the curve without SF division presents decay trend while curve with SF appears a zigzag shape. Meanwhile, when $d$ is same, the coverage probability of $m=6$ is larger than $m=12$. This is because that more nodes will bring more interference sources so that the ability of coverage will decline.
Figure 5. Coverage probability of different $\beta$ ($\beta$ is $\beta_1$, $\beta_2$) plotted as functions of the distance from the gateway(d), assuming $m=6$.

In Figure 5, the group with $\beta_1$ is the one whose coverage probability is greater than $\beta_2$ as a whole. As discussed before, a higher threshold leads to a lower coverage probability. Figure 5 also verifies the previous analysis.

**Relationship Between ASE And The Number Of Nodes**

As we can see in Figure 6, as the number of LoRa nodes increases, ASE also increases. However, when $m$ exceeds 20, ASE no longer rises and shows a downward trend. This trend validates the previous analysis that monotonic function changes do not occur between $m$ and ASE. Instead, there will be an optimal number of nodes $m$ such that ASE achieves the best. Thus, from perspective of nodes deployment, we optimize the ASE value by deploying the optimal number of nodes in a region.
CONCLUSION

In order to analyze the interference and coexistence issues between LoRa nodes and other nodes, a more comprehensive LoRa network uplink transmission model is developed in this paper. For the whole uplink network, PCP is used to establish the model. Different from other IoT technologies, LoRa has an adaptive rate modulation scheme, so that farther nodes can use higher SF to transmit. Therefore, we use the method of dividing the range within the same cluster to refine the LoRa node interference. Finally, analyzing the change of coverage probability and ASE after the desired node is disturbed. The results can be changed by the relevant indicators to describe the performance of the application scenarios. In addition, an optimal value for the active LoRa node is exist in each cluster. Based on the above conclusions, we can provide a reference for the deployment of the network in the actual scenery.

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