Noise calculation of noncompact bodies with impedance boundary in limited enclosed space

Fang Wang¹, Wensi Zheng²

¹School of Civil Engineering, North Minzu University, Yinchuan, Ningxia, 750021, China
²School of Aeronautics, Northwestern Polytechnical University, Xi’an, Shannxi, 710072, China

*Corresponding author’s e-mail: fangw1211@163.com

Abstract. The present paper mainly investigates noise propagation in limited enclosed space with non-compact bodies. In view of the scattering effect induced by solid boundaries, tailored green’s function methods are used to study the noise propagation. Frequency-domain integral equation is presented to calculate scattered noise under impedance boundary condition in enclosed space. At first, the scattered noise of circular cylinder with solid boundary condition is calculated to verify the validity of the present algorithm. Secondly, the impedance boundary condition is executed on the cylinder wall. Furthermore, the impedance boundary conditions are carried out on the boundaries of enclosed space. Numerical results demonstrate that the total noise drastically reduces in some areas with impedance boundary, but increases in other areas. The phenomenon is related with the cylinder radius, the location of sound sources and the scattering effect. In practical engineering problems, the impedance boundary should be applied locally according to the configuration.

1. Introduction
Due to the huge impact of the noise produced by large-scale bodies, numerical simulations of noise reduction have recently received much more attention in aviation and aerospace fields, civil aviation, engineering, et al. A series of methods is presented to study noise and its production mechanism, such as Lighthill’s analogy theory [1], FW-H equation [2] presented by Ffowcs Williams and Hawkings, vortex sound theory proposed by Howe [3] and Powell [4] et al. These methods explained in detail why the fluid and vortices generate noise in the flow field. In fact, the noise received by observers includes two parts which are called the radiated noise and the scattered noise. The radiated noise propagates from sources to receiver directly, and the scattered noise is produced by the scattering effect of boundaries, especially large for non-compact bodies.

In combination with above theories, some researchers have studied how to reduce noise produced by non-compact bodies in practical engineering problems. The first kind is to calculate noise directly in a large computational domain with high-order methods when the body surface is covered with impedance materials, which is called Computational Aero Acoustic (CAA) Method. However, it needs accurate numerical schemes to reduce dissipation or dispersion errors in this method, which always make it time cost and computational expensive [5,6]. The second kind is based on the Boundary Element Method (BEM) and Green’s function to calculate noise under impedance boundaries, it is named tailored Green’s function by including the scattering effect in the Green’s function solution,
and it also satisfy the corresponding impedance boundary conditions on the body surface. The advantage of tailored Green’s function is that the sound scattering effects are explicitly included in noise calculation which greatly reduces the workload of calculating noise sources in fluid field.

At present, the Green’s function methods have been extensively used to solve the noise propagation problems with non-compact bodies. Takahashi [7] presented a tailored Green’s function method to calculate noise based on vortex sound theory. Hu [8] develop an exact formulation of Green’s function in acoustic analogy. Jones [9] proposed a three-dimensional time-domain boundary element method with exact Green’s function. These methods based on low-order fluid simulation are high efficient and easy to implement. Otherwise, the author also presented tailored Green’s function to study scattered noise produced in half space, which avoided the calculation of scattered sources on long boundary. It can be found that the Green’s function could be developed as an integral function in combination with given boundary condition. In this way, noise reduction with impedance boundary is also studied, similar research work has been carried out. Lui [10,11] used this method to study scattered noise produced by an impedance sphere and a long cylinder in half space, and compared with an analytical solution obtained from Weyl-van der Pol formulation [12]. Li also studied the diffraction sound of an impedance sphere close to the ground surface.

In this paper, the propagation of scattered noise in limited enclosed space with non-compact bodies is mainly studied, and tailored green’s function methods are used to obtain the integral solution with impedance boundary in frequency domain. Two dimensional stationary circular cylinder is chosen as the computational model to investigate noise reduction. In Sec. II, the frequency-domain integral equation is presented to calculate scattered noise under impedance boundary condition in enclosed space. In Sec.III, numerical examples are investigated. At first, the scattered noise of circular cylinder with solid boundary condition is calculated to verify the validity of the present algorithm. Secondly, the impedance boundary condition is executed on the cylinder wall. Furthermore, the impedance boundary conditions are carried out on the boundaries of enclosed space. At last, the conclusion is drawn in Sec.IV.

2. Integral solution for tailored Green’s function with impedance boundary
Under arbitrary boundary condition, the wave equation for tailored Green’s function is given as

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} g_{s}(x,y,t,\tau) - \nabla^2 g_{s}(x,y,t,\tau) = \delta(t-\tau)\delta(x-y)$$

(1)

where x and y are the observer and the source, respectively. \(g_{s}(x,y,t,\tau)\) represents the tailored Green’s function, \(\alpha(y,\tau), \beta(y,\tau)\) are functions related to variable y, n is the normal vector directing from noise field to body surface, \(c\) is the wave speed.

The wave equation with convection effect of free-space green’s function \(g_{o}(z,y,t,\tau)\) is written as

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} g_{o}(z,y,t,\tau) - \nabla^2 g_{o}(z,y,t,\tau) = \delta(t-\tau)\delta(z-y)$$

(2)

Eq.2 describes the linear propagation of sound radiated from sources in free space. In combination with Eq. 1 and Eq. 2, the wave equation related to \(g_{s}(x,y,t,\tau)\) can be formulated as

$$g_{s}(x,z,t,\tau) = g_{o}(x,z,t,\tau) - \int_{-\infty}^{\infty} \int_{\mathbb{R}} g_{s}(x,y,t,\tau) \frac{\partial g_{o}(z,y,t,\tau)}{\partial n} - g_{o}(z,y,t,\tau) \frac{\partial g_{s}(x,y,t,\tau)}{\partial n} ds \, dt$$

(3)

When the Fast Fourier Transformation (FFT) is executed on two sides of Eq.3, the following equation is obtained
where $\omega$ is the circular frequency, $\omega = k/c$, $G_s(x, z, \omega)$ and $G_0(x, z, \omega)$ are the frequency-domain tailored Green’s function and Green’s function in free space, respectively. In frequency domain, Eq. 1 satisfied that

$$\alpha = \beta = i\rho_s c/\omega Z(\omega)$$

$Z(\omega) = R_r + iX_r(\omega) = R_i + i(X_i/\omega + X_r)$,

where $Z(\omega)$ is called impedance function, $R_r$ and $X_r$ are the real and the imaginary part of the resistance and reactance coefficients, respectively. $\rho_s$ represents the density of incoming flow. With above boundary condition, Eq. 4 can be formulated as

$$G_s(x, z, \omega) = G_0(x, z, \omega)$$

$$+ \int_{S} G_s(x, y, \omega) \left[ \frac{\partial G_0(z, y, \omega)}{\partial n} - \frac{\partial G_s(x, y, \omega)}{\partial n} \right] ds_y$$

(5)

where Green’s function $G_s(x, z, \omega)$ in free space is given as

$$G_0(x, z, \omega) = \frac{j}{4} H^{(1)}_0(k |x-z|)$$

3. Numerical examples

In the present paper, the sound propagation of point source in limited enclosed space is mainly studied with different boundaries, and the computational field is shown in Fig.1. In the literature [7], a tailored Green’s under the condition $\frac{\partial G_e}{\partial n} = 0$ is developed, and the analytical solution is given in literature [13] for two-dimensional circular cylinder. In order to verify the validity of the present algorithm, the cylinder is decomposed with 200 element, and the diameter is chosen as $D=0.05$.

3.1 Sound propagation with rigid cylinder

The point source $z$ is located at $(0.1, 0)$, and 180 observers are located uniformly on a circle with a radius $R=60D$. Figure2 shows pressure directivity at different frequency, and the analytical solution $G_e$ agrees well with numerical result $G_s$. On the other hand, the pressure directivity display as a monopole at low wavenumber but petal like pattern at high wavenumber. It demonstrates that the sound field varies dramatically with frequency.
3.2 Sound propagation with impedance cylinder wall

In this paper, two kinds of impedance condition are chosen. The first is given as

\[
R_1 = 0.2 \rho_0 c_0 \\
X_1 = 0.0739 \rho_0 c_0 \\
X_{-1} = -13.48 \rho_0 c_0
\]

and the second is given as

\[
R_2 = 0.2 \rho_0 c_0 \\
X_1 = 0.52469 \rho_0 c_0 \\
X_{-1} = -1.1785 \rho_0 c_0
\]

In the following paper, the numerical simulation is developed with Eq.5 under two kinds of impedance condition. The point source \( z \) is located at \((0.1, 0)\), and 180 observers are located uniformly on a circle with a radius \( R = 60D \). As shown in Fig.3, the pressure with two kinds of impedance boundaries is compared with that of rigid wall. It is obviously that the pressure obtained with impedance boundary reduces at some directions but increases at other directions. The amplitude reduces with wavenumber for three kinds of boundaries on the whole. In the practical engineering problems, the impedance boundary condition could be used locally on the rigid wall to reduce noise.
5

3.3 Sound propagation with impedance wall in enclosed space

As shown in Fig. 1, the cylinder is located in the center of the computational field. In this part, the first impedance condition is used on both cylinder wall and zone boundaries. The point source $z$ is located at $(0.1, 0)$, and 180 observers are located uniformly on a circle with a radius $R=7D$. The zone boundaries are decomposed uniformly with 400 elements.

Figure 3 The pressure directivity at different wavenumber for impedance boundaries

(a) $k=20$  (b) $k=30$

Figure 4 The pressure directivity at different wavenumber in enclosed space

(a) $k=5$  (b) $k=10$

(c) $k=15$  (d) $k=20$
Figure 4 displays the pressure directivity with different wavenumber. At first, the pressure amplitude gradually reduces with wavenumber. Secondly, the sound field varies dramatically compared with rigid wall in free space, and the number of petals increases with wavenumber. Thirdly, the impedance condition reduces the pressure by a large margin, especially for high wavenumber. Finally, it can be deduced that the scattering effect of zone boundaries is very large.

4 Conclusions
This paper mainly studied tailored Green’s function with impedance boundary in limited enclosed space. Under the premise of impedance boundary, the integral solution of Green’s function is obtained, and the sound propagation is predicted in detail. The following conclusions are drawn:

(1) Numerical results obtained by the integral solution with BEM agree well with Gloerfelt’s analytical solution for rigid wall, and the noise field varies dramatically with wave number.

(2) With two kinds of impedance boundaries, the pressure amplitude reduces locally compared with that of rigid wall. It can be concluded that the impedance boundary condition should be used locally on the rigid wall to reduce noise.

(3) Because of the strong scattering effect, the noise field in enclosed space is complicated compared with that in free space. The impedance condition reduces the pressure by a large margin, especially for high wavenumber.

Acknowledgments
This research was supported by the Natural Science Foundation of Ningxia under Program No. 2018AAC03112, General Project of North Minzu University and Personal Scientific Research Projects of North Minzu University.

Reference
[1] Lighthill, M. J. (1952) On sound generated aerodynamically. I, Proc. R. Soc. Lond. A., 211: 564-587.
[2] Ffowcs Williams, J. E., Hawking, D. L. (1969) Sound generation by turbulence and surfaces in arbitrary motion, Phil. Trans. R. Soc. A., 264: 321–342.
[3] Howe, M. S. (2003) Theory of vortex sound. Cambridge university press.
[4] Powell, A. (1964) Theory of vortex sound. J. Acous. Soc. Am., 36(1): 177-195
[5] Ikeda, T., Atobe, T., Takagi., S. (2012) Direct simulation of trailing-edge noise generation from two-dimensional airfoils at low Reynolds numbers. J. Sound Vib., 331: 556-574.
[6] Liu, W., Kim, J. K., Zhang. X. (2013) Landing-gear noise prediction using high-order finite difference schemes. J. Sound Vib., 332: 3517-3534
[7] Takaishi, T., Miyazawa, M., Kato. C. (2007) A computational method of evaluating noncompact sound based on vortex sound theory. J. Acous. Soc. Am., 121(3): 1353-1361.
[8] Hu, F. Q., Guo, Y. P., Jones, A. D. On the computation and application of exact green’s function in acoustic analogy. 11th AIAA Aeroacoustics conference, Monterey, California, May, 2005, 2005-2986.
[9] Jones, A. D., Hu, F. Q., A three-dimensional time-domain boundary element method for the computation of exact green's function in acoustic analogy. 13th AIAA/CEAS Aeroacoustics, Rome, Italy, May, 2007, 2007-3479.
[10] Lui W. K., and Li, K. M., (2010) The Scattering of Sound by a Long Cylinder above an Impedance Boundary. J. Acous. Soc. Am., 127(2): 664-674.
[11] Li, K. M., Lui, W. K., Frommer, G. H.,(2004) The Diffraction of Sound by an Impedance Sphere in the Vicinity of a Ground Surface, J. Acous. Soc. Am., 11(1): 42–56.
[12] Miller, S. A. E., (2014) The Prediction of Jet Noise Ground Effects Using an Acoustic Analogy and a Tailored Green's Function. J. Sound Vib., 333(4): 1193-1207.
[13] Gloerfelt X., Perot F., Bailly C., Juve D., (2005) Flow-induced cylinder noise formulated as a diffraction problem for low Mach numbers. J. Sound Vib., 287: 129-151.