VIRTUAL LINKS WHICH ARE EQUIVALENT AS TWISTED LINKS

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Abstract. A virtual link is a generalization of a classical link that is defined as an equivalence class of certain diagrams, called virtual link diagrams. It is further generalized to a twisted link. Twisted links are in one-to-one correspondence with stable equivalence classes of links in oriented thickenings of (possibly non-orientable) closed surfaces. By definition, equivalent virtual links are also equivalent as twisted links. In this paper, we discuss when two virtual links are equivalent as twisted links, and give a necessary and sufficient condition for this to be the case.

1. Introduction

A virtual link is a generalization of a classical link introduced by Kauffman [8], which is defined as an equivalence class of certain diagrams, called virtual link diagrams. Virtual link theory is quite natural when we discuss Gauss chord diagrams, since every Gauss chord diagram is realized as a virtual link diagram up to virtual Reidemeister moves [5, 8]. Moreover, virtual links are in one-to-one correspondence with abstract links on oriented surfaces [6], and in one-to-one correspondence with stable equivalence classes of links in oriented thickenings of oriented closed surfaces [2, 4, 6]. It is known that the set of classical links is a subset of the set of virtual links, i.e., two classical link diagrams are equivalent as virtual links if and only if they are equivalent as classical links [5, 8, 9]. This fact is obtained by considering knot groups with peripheral structures [5], or by assuming a stronger fact due to Kuperburg [9] that a virtual link has a unique irreducible representative as a link in an oriented thickening of an oriented surface. For details and related topics on virtual knot theory, refer to [3, 5, 6, 8, 10].

Bourgoin [1] generalized virtual links to twisted links. Twisted links are in one-to-one correspondence with abstract links on surfaces [1, 7], and in one-to-one correspondence with stable equivalence classes of links in oriented thickenings of closed surfaces [1, 7].

A virtual link diagram is a link diagram in $\mathbb{R}^2$ that may have some virtual crossings, which are crossings without over/under information but which are decorated with a small circle surrounding it. A twisted link diagram is a virtual link diagram possibly with bars on arcs. Referring to Figure 1, the moves $R_1, R_2, R_3$ are called classical Reidemeister moves, the moves $V_1, \ldots, V_4$ are called virtual Reidemeister moves, and the moves $T_1, T_2, T_3$ are called twisted Reidemeister moves. All of these are called extended Reidemeister moves.

A virtual link is an equivalence class of virtual link diagrams under classical and virtual Reidemeister moves. A twisted link is an equivalence class of twisted link diagrams under extended Reidemeister moves.

By definition, virtual link diagrams are twisted link diagrams, and if two virtual link diagrams are equivalent as virtual links then they are equivalent as twisted.
Figure 1. Classical, virtual and twisted Reidemeister moves

Thus the inclusion map
\[ \iota : \{ \text{virtual link diagrams} \} \to \{ \text{twisted link diagrams} \} \]
yields a natural map
\[ f : \{ \text{virtual links} \} \to \{ \text{twisted links} \}. \]

In this paper, we discuss when two elements are mapped to the same element by \( f \), and give a necessary and sufficient condition for this to be the case. This clarifies a remark made in \cite[p.1251]{1}, which claims that virtual link theory injects into the theory of links in oriented thickenings; see Remark \ref{remark:virtualinktheory}.

For a virtual link \( L \), let \( s(L) \) denote the virtual link represented by a diagram \( s(D) \) that is obtained from a diagram \( D \) of \( L \) by a reflection along a line in \( \mathbb{R}^2 \) and by switching over/under information on all classical crossings. See Figure 2, where \( r \) is a reflection along a line in \( \mathbb{R}^2 \) and \( c \) is switching over/under information.

\[ D \xrightarrow{r} r(D) \xrightarrow{c} s(D) = c \circ r(D) \]

Figure 2.

**Theorem 1.1.** Two virtual knots \( L \) and \( L' \) are equivalent as twisted knots if and only if \( L' \) is equivalent to \( L \) or \( s(L) \) as a virtual knot.

This theorem is a special case of our main theorem (Theorem \ref{maintheorem}), stated in Section \ref{mainsection}.

It is known that there is a virtual knot \( L \) such that \( L \) and \( s(L) \) are not equivalent as virtual knots \cite{11}. Thus the map \( f \) is not injective.

A link diagram (without virtual crossings nor bars) is referred to as a classical link diagram, and a classical link means an equivalence class of classical link diagrams under classical Reidemeister moves. Recall that the set of classical links is a subset of the set of virtual links. It is also a subset of the set of twisted links.
**Theorem 1.1.** The map $f$ restricted to the set of classical links is injective, i.e., two classical links are equivalent as twisted links if and only if they are equivalent as classical links.

In this paper all (classical, virtual or twisted) links are oriented. A link is called a knot if it consists of one component. Although virtual links are equivalence classes of virtual link diagrams, we often say that two virtual links $L$ and $L'$ are equivalent as virtual links (or as twisted links, respectively) if their representatives are equivalent as virtual link diagrams (or as twisted link diagrams, respectively).

The paper is organized as follows: In Section 2 we give necessary definitions and state the main results (Proposition 2.1 and Theorem 2.4). Proofs of the latter are given in Section 3. In Section 4 we introduce Gauss chord diagrams for twisted links and apply them to give an alternative proof of Proposition 2.1.

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## 2. Definitions and the main theorem

Let $D$ be a (classical, virtual or twisted) diagram. A **split decomposition** of $D$ is a collection of subdiagrams $D_1, \ldots, D_n$ such that $D = D_1 \sqcup \cdots \sqcup D_n$ and $D_i \cap D_j = \emptyset$ for all $i \neq j$. We denote it by $D = D_1 \sqcup \cdots \sqcup D_n$.

Let $L$ be a (classical, virtual or twisted) link. A **split decomposition** of $L$ is a collection of sublinks $L_1, \ldots, L_n$ such that there is a diagram $D$ of $L$ with a split decomposition $D = D_1 \sqcup \cdots \sqcup D_n$ such that $L_i$ is represented by $D_i$ for $i = 1, \ldots, n$. We denote it by $L = L_1 \sqcup \cdots \sqcup L_n$. A (classical, virtual or twisted) link $L$ is **splittable** if there is a split decomposition $L = L_1 \sqcup \cdots \sqcup L_n$ with $n \geq 2$; otherwise, $L$ is **non-splittable**. A split decomposition $L = L_1 \sqcup \cdots \sqcup L_n$ is called **maximal** if for each $i = 1, \ldots, n$, $L_i$ is non-splittable. Note that a maximal split decomposition is unique up to reordering (Lemma 3.1).

For a (classical, virtual or twisted) link diagram $D$ in $\mathbb{R}^2$, as in Section 1 we let $s(D)$ denote a diagram obtained from $D$ by a reflection along a line in $\mathbb{R}^2$ and switching over/under information on all classical crossings. If $D$ and $D'$ are equivalent as (classical, virtual or twisted) link diagrams, so are $s(D)$ and $s(D')$. Thus, for a (classical, virtual or twisted) link $L$, we have that $s(L)$ is well defined as a (classical, virtual or twisted) link. Note that while a classical link $L$ and its counterpart $s(L)$ are equivalent as classical links, a virtual link $L$ and its counterpart $s(L)$ may not be equivalent as virtual links.

We prove the following proposition in Section 3.

**Proposition 2.1.** For any twisted link $L$, we have that $L$ and $s(L)$ are equivalent as twisted links.

**Corollary 2.2.** For any virtual link $L$, we have that $L$ and $s(L)$ are equivalent as twisted links. Thus, $f(L) = f(s(L))$.

**Definition 2.3.** Two virtual links $L$ and $L'$ are **s-congruent** if there are maximal split decompositions $L = L_1 \sqcup \cdots \sqcup L_n$ and $L' = L'_1 \sqcup \cdots \sqcup L'_n$ such that for each $i = 1, \ldots, n$, $L'_i$ is equivalent to $L_i$ or $s(L_i)$ as a virtual link.

The following is our main theorem.

**Theorem 2.4.** Let $L$ and $L'$ be virtual links. Then $L$ and $L'$ are equivalent as twisted links if and only if they are s-congruent.

Theorem 1.1 is a special case of Theorem 2.4. Theorem 1.2 follows from Theorem 2.4, since classical links $L$ and $L'$ are s-congruent if and only if they are equivalent as classical links.
Remark 2.5. In [1], p.125) it is stated that virtual link theory injects into the theory of links in oriented thickenings. It should be understood that virtual link theory modulo $s$-congruence injects into twisted link theory. There is an alternative proof of Theorem 3.4 using a uniqueness theorem ([11, Theorem 1]) of irreducible representatives of links in oriented thickenings of closed surfaces. Our proof given in Section 3 is a direct argument using diagrams.

3. Proofs

Lemma 3.1. A maximal split decomposition is unique up to reordering. That is, if $L = L_1 \cup \cdots \cup L_n$ and $L' = L'_1 \cup \cdots \cup L'_{n'}$ are maximal split decompositions of equivalent (classical, virtual or twisted) links $L$ and $L'$, then $n = n'$ and there exists a permutation $\sigma$ of $\{1, \ldots, n\}$ such that for each $i = 1, \ldots, n$, $L_i$ is equivalent to $L'_{\sigma(i)}$ as an (classical, virtual or twisted) link.

Proof. Fix an equivalence between $L$ and $L'$. (An equivalence between $L$ and $L'$ is a sequence of diagrams $D = D^0, D^1, D^2, \ldots, D^m = D'$ for some $m$ such that $D$ and $D'$ are diagrams of $L$ and $L'$, respectively, and where $D^{k+1}$, $k = 0, 1, \ldots, m - 1$, is obtained from $D^k$ by a single extended Reidemeister move. Fixing such an equivalence, we have a bijection between the components of $L$ and the components of $L'$, and we may consider, for any sublink of $L$, the corresponding sublink of $L'$.) Since $L_1$ is non-splittable, the corresponding sublink of $L'$ is a sublink of $L'_{\sigma(1)}$ for some $\sigma(1) \in \{1, \ldots, n'\}$. Since $L'_{\sigma(1)}$ is non-splittable, the corresponding sublink of $L$ is a sublink of $L_1$. Thus $L_1$ and $L'_{\sigma(1)}$ are equivalent to each other via the equivalence between $L$ and $L'$. Continuing by the same reasoning, we see that $n = n'$ and there is a permutation $\sigma$ of $\{1, \ldots, n\}$ such that $L_i$ and $L'_{\sigma(i)}$ are equivalent for $i = 2, \ldots, n$. \qed

Here we give a direct proof of Proposition 4.1 using diagrams and extended Reidemeister moves. An alternative proof, using Gauss chord diagrams, is given in Section 3.

Proof of Proposition 4.1. We show that for any twisted link diagram $D$, the diagrams $D$ and $s(D)$ are equivalent as twisted link diagrams. By an ambient isotopy of $\mathbb{R}^2$, we may assume that $D$ lies in the half plane $\{x < 0\}$ of the $xy$-plane, and that it lies in general position with respect to the $y$-component. By slicing along finitely many horizontal lines, $D$ has a decomposition into pieces of the types depicted in Figure 3: (i) there is a maximal point, (ii) there is a minimal point, (iii) and (iv) there is a classical crossing, (v) there is a virtual crossing, (vi) there is a bar. We call these pieces standard pieces and denote them by $M_{a,b}$, $m_{a,b}$, $X_{a,b}^+$, $X_{a,b}^-$, $V_{a,b}$ and $T_{a,b}$, respectively, where $a$ (or $b$, respectively) is the number of vertical arcs appearing on the left (or right, respectively) of the event: a maximal point, a minimal point, a classical crossing, a virtual crossing or a bar.

For $k \in \mathbb{Z}$, we denote by $\ell_k$ the horizontal line determined by the equality $y = k$, and denote by $C_k$ the region of $\mathbb{R}^2$ determined by the inequalities $k - 1 \leq y \leq k$. We call $C_k$ the $k$th chamber.

Let $m$ be the total number of maximal points, minimal points, classical crossings, virtual crossings and bars of $D$. Modifying $D$ by an isotopy of $\mathbb{R}^2$, we may assume that $D$ lies in $\bigcup_{k=1}^m C_k$, and for each $k = 1, \ldots, m$, the restriction of $D$ to $C_k$ is a standard piece. For example, for the diagram in Figure 4 $m = 9$ and $D \cap C_k$, $k = 1, \ldots, 9$, is $m_{0,0}$, $m_{1,1}$, $X_{2,0}^+$, $X_{2,0}^-$, $T_{2,1}$, $T_{3,0}$, $V_{2,0}$, $M_{1,1}$ or $M_{0,0}$, respectively.

Let $s(D)$ be the diagram obtained from $D$ by the reflection along the $y$-axis and switching over/under information on all classical crossings. We show that $D$ is equivalent to $s(D)$ by a sequence of extended Reidemeister moves.
Let δ be a sufficiently small positive number and, for each k = 1, . . . , m − 1, let N(ℓk) be the regular neighborhood of ℓk determined by the inequalities k − δ ≤ y ≤ k + δ. We denote by ℓk (or ℓk, respectively) the horizontal line determined by the equality y = k + δ (or y = k − δ, respectively).

We may assume that the intersection D ∩ N(ℓk) is a collection of dk (≥ 0) vertical arcs, say Ak,1, . . . , Ak,dk. Assume that Ak,1, . . . , Ak,dk appear in this order from left to right. Let Pk,j, j = 1, . . . , dk, be the intersection point of Ak,j and ℓk. See Figure 5 (Left), where dk = 4, and Ak,j and Pk,j are denoted by ˜Ajk and P′j, respectively.

Let P′k,j, j = 1, . . . , dk, denote the image of Pk,j under reflection along the y-axis. By virtual Reidemeister moves, we deform Ak,1, . . . , Ak,dk into arcs ˜Ak,1, . . . , ˜Ak,dk as in Figure 5 such that ˜Ak,j ∩ ℓk = P′k,j and ∂ ˜Ak,j = ∂Ak,j for all j = 1, . . . , dk.

In Figure 5 (Right), ˜Ak,j and P′k,j are denoted by ˜Aj and P′j, respectively.

Let D1 be the virtual link diagram obtained from D by this modification for all k = 1, . . . , m − 1.

For each chamber Ck such that D ∩ Ck is of the form Ma,b, ma,b, Va,b or Ta,b, we can deform D1 ∩ Ck into Mb,a, mb,a, Vb,a or Tb,a respectively, by extended Reidemeister moves in Ck rel ℓk−1 ∪ ℓk. Apply this for all chambers Ck such that D ∩ Ck is of the form Ma,b, ma,b, Va,b or Ta,b.

For each chamber Ck such that D ∩ Ck is of the form X±a,b, we can deform D1 ∩ Ck into the composition Vb,aX±b,aVb,a by extended Reidemeister moves in Ck rel ℓk−1 ∪ ℓk (see Figure 5 for Vb,aX±b,aVb,a). Apply this for all chambers Ck such that D ∩ Ck is of the form X±a,b.
Let $D_2$ be the diagram obtained this way, which is equivalent to $D_1$ and hence equivalent to $D$ as a twisted link diagram. We may assume that for each $k = 1, \ldots, m$, $D_2 \cap N(\ell_k)$ is the union of vertical arcs $A'_{k,1}, \ldots, A'_{k,d_k}$, where $A'_{k,j}$ is the image of $A_{k,j}$ under reflection along the $y$-axis. See Figure 7 (Left).

Let $D_3$ be the diagram obtained from $D_2$ by adding a pair of bars on each $A'_{k,j}$, for $k = 1, \ldots, m$ and $j = 1, \ldots, d_k$, such that one of the bars lies in $N(\ell_k) \cap C_k$ and the other bar lies in $N(\ell_k) \cap C_{k+1}$. See Figure 7. In each chamber $C_k$ such that $D_2 \cap C_k$ is of the form $M_{a,b}$, $m_{a,b}$, $V_{a,b}$ or $T_{a,b}$, these bars are canceled. In each chamber $C_k$ such that $D_2 \cap C_k$ is of the form $V_{b,a}X_{b,a}^{\pm}$, we have that $D_3 \cap C_k$ changes to $X_{b,a}^{\pm}$. We then obtain $s(D)$. \hfill \Box

Given a twisted link diagram $D$, [7] describes a method for constructing a virtual link diagram $\tilde{D}$, called the double covering diagram of $D$, and the following result is obtained.

**Theorem 3.2** ([7]). Let $D$ or $D'$ be twisted link diagrams, and let $\tilde{D}$ and $\tilde{D}'$ be double covering diagrams of $D$ and $D'$, respectively. If $D$ and $D'$ are equivalent as twisted link diagrams, then $\tilde{D}$ and $\tilde{D}'$ are equivalent as virtual link diagrams.

Therefore, for a twisted link $L$ represented by a diagram $D$, we may define the double covering $\tilde{L}$ of $L$ to be the virtual link represented by $\tilde{D}$, and there is a map

$$\{\text{twisted links}\} \to \{\text{virtual links}\}, \quad L \mapsto \tilde{L},$$
called the double covering. When \( D \) is a virtual link diagram, it follows from the construction in [1] that the double covering diagram \( \tilde{D} \) is precisely \( D \sqcup s(D) \). Thus, for a virtual link \( L \), the double covering \( \tilde{L} \) is \( L \sqcup s(L) \).

Theorem 3.2 is used in the following proof of our main theorem.

**Proof of Theorem 3.2** We first prove sufficiency. Let \( L \) and \( L' \) be virtual links that are s-congruent. Then there exist maximal split decompositions \( L = L_1 \sqcup \cdots \sqcup L_n \) and \( L' = L'_1 \sqcup \cdots \sqcup L'_n \) such that for each \( i = 1, \ldots, n \), \( L'_i \) is equivalent to \( L_i \) or \( s(L_i) \) as a virtual link. By Corollary 2.2, \( L'_1 \) is equivalent to \( L_1 \), as a twisted link. Thus \( L' \) is equivalent to \( \tilde{L} \) as a twisted link.

We next prove necessity. Let \( L \) and \( L' \) be virtual links that are equivalent as twisted links. Since \( L \) is a virtual link, the double covering \( \tilde{L} \) is the split union \( L \sqcup s(L) \). Similarly, the double covering \( \tilde{L}' \) of \( L' \) is the split union \( L' \sqcup s(L') \). By Theorem 3.2, \( \tilde{L} = L \sqcup s(L) \) is equivalent to \( \tilde{L}' = L' \sqcup s(L') \) as a virtual link.

Let \( L = L_1 \sqcup \cdots \sqcup L_n \) and \( L' = L'_1 \sqcup \cdots \sqcup L'_n \) be maximal split decompositions. Then \( L_1 \sqcup \cdots \sqcup L_n \sqcup s(L_1) \sqcup \cdots \sqcup s(L_n) \) is a maximal split decomposition of \( \tilde{L} \) and \( L'_1 \sqcup \cdots \sqcup L'_{n'} \sqcup s(L'_1) \sqcup \cdots \sqcup s(L'_{n'}) \) is a maximal split decomposition of \( \tilde{L}' \). By the uniqueness of a maximal split decomposition (Lemma 3.1), we see that \( L \) and \( L' \) are s-congruent.

\[ \square \]

4. Gauss chord diagrams and an alternative proof of Proposition 2.1

We introduce Gauss chord diagrams for twisted links and use them to give an alternative proof of Proposition 2.1 For readers who are familiar with Gauss chord diagrams, the proof in this section might be preferred.

A Gauss chord diagram for a twisted link diagram is a diagram in \( \mathbb{R}^2 \) consisting of oriented circles, called base circles, some arcs attaching to the base circles, called chords, and some small bars intersecting base circles, called bars. Base circles are the source 1-manifold for a twisted link diagram \( D \), i.e., there is an immersion from the base circles to \( \mathbb{R}^2 \) whose image is the underlying immersed loops of \( D \), and chords (or bars, respectively) correspond to classical crossings (or bars, respectively) of \( D \). For each classical crossing of \( D \) (two arcs intersecting at a point), the preimage consists of two disjoint arcs on the base circles, over and under crossings, which are neighborhoods of the endpoints of a chord. A chord is oriented and signed such that the initial point lies in the over crossing and the terminal point lies in the under crossing, and the sign is the sign of the crossing. Bars of a Gauss chord diagram correspond to bars of \( D \).

Two Gauss chord diagrams are isomorphic if they are related by a finite sequence of the following transformations: (1) changing by an isotopy of \( \mathbb{R}^2 \), (2) changing a chord without changing the endpoints, the direction nor the sign, and (3) changing the position of base circles in \( \mathbb{R}^2 \). In what follows we consider Gauss chord diagrams up to isomorphism.

For any twisted link diagram \( D \), a Gauss chord diagram \( G \) is uniquely determined up to isomorphism. Conversely, for any Gauss chord diagram \( G \), there exists a twisted link diagram \( D \) which is uniquely determined up to \( V_1, \ldots, V_4 \) and \( T_1 \) moves. By \( R_1, R_2, R_3, T_2 \) and \( T_3 \) moves on twisted link diagrams, Gauss diagrams change as in Figure 8, where vertical arrows are subsets of base circles. (The \( R_3 \) move of Figure 8 corresponds to a special case of \( R_3 \) moves of link diagrams. Other \( R_3 \) moves are consequences of this move and \( R_2 \) moves.) Conversely, the moves of Figure 8 are always applicable to twisted link diagrams after applying suitable \( V_1, \ldots, V_4 \) and \( T_1 \) moves to the link diagrams.

Therefore, we have bijections

\[ \{ \text{twisted link diagrams} \}/(V_1, \ldots, V_4 \text{ and } T_1) \longleftrightarrow \{ \text{Gauss chord diagrams} \} \]
and
\[ \{\text{twisted links}\} \leftrightarrow \{\text{Gauss chord diagrams}\}/(R_1, R_2, R_3, T_2 \text{ and } T_3). \]

Figure 8. Moves on Gauss chord diagrams

**Proof of Proposition 2.1.** Let \( D \) be a virtual link diagram and let \( G \) be a Gauss chord diagram of \( D \). Let \( s(G) \) be a Gauss chord diagram of \( s(D) \). Note that \( s(G) \) is obtained from \( G \) by reversing the orientation of every chord of \( G \), without changing the signs. We show that \( G \) may be transformed into \( s(G) \) by the moves \( T_2 \) and \( T_3 \) of Figure 8. Let \( G_1 \) be the Gauss chord diagram obtained from \( G \) by applying \( T_3 \) moves (from right to left in the figure) for all chords of \( G \), and let \( T \) be the set of bars introduced by the \( T_3 \) moves. Note that \( T \) is the difference between \( s(G) \) and \( G_1 \). Let \( a_1, \ldots, a_k \) be the arcs obtained by cutting the base circles of \( G \) at the endpoints of all chords and by removing base circles where no chords are attached. For each \( i = 1, \ldots, k \), there are exactly two bars on \( a_i \) belonging to \( T \), which can be removed by a \( T_2 \) move of Figure 8. In this way, we can remove all bars of \( T \) and we have obtained \( s(G) \). \( \square \)

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