Analysis of whole-body vibration on rheological models for tissues

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Abstract. Whole body vibrations have become a very popular method in recent years, both in physical therapy and in sports. This popularity is due to the fact that, as a result of analyzing the groups of subjects, the effects of small amplitude vibration and low frequency vibration, it was found an increase in the force developed by the feet, a hardening of bone strength or an increase in bone density. In this paper we propose to give a possible explanation of the stress relieving in muscle and/or bone after whole body vibration treatment. To do this we consider some rheological models which after whole body vibrations and after the analysis of their response lead to various experiments.

1. Introduction
The results of the research on the influence of vibrations on the stress relaxation are controversial because there are no conclusive measurement methods whose effects can be demonstrated. Applying high frequency harmonic strain has been shown to stimulate bone resistance in different animals. However, the effects of whole body vibration on the human skeleton have rarely been studied. The effect of vibration on the whole body is difficult to quantify because the mechanical modelling of vibration behaviour of the whole body is not unitary but also offers many possibilities [1].

In another paper [2], it was hypothesized that through a non-contact vibration device, qualitative changes could be detected in a periodontal ligament and alveolar bone, identifying the mechanical parameters of the system: tooth, periodontal ligament and alveolar bone. In this case, the mechanical modeling of the analyzed system was the starting point of the study.

Whole-body vibration is a neuromuscular training method that recently has been developed and which increase strength compared with resistance training [3].

As a biological material, muscle tissue and bone tissue exhibit a complex microstructure and micro components, which result in complicated mechanical properties. Because of the viscoelastic nature of collagen fibers in the bone matrix, bone itself has remarkable viscoelasticity [4-6].

The vast majority of research has an experimental character and has not led to a clear view of the possibility of using vibrations to stress relaxation. The possibilities of reducing this stress are described in the paper [7] from the stress and deformation dependence, a characteristic of the material. Stress relaxation is based on the reinforcement of the material, meaning that by the influence of vibrations, the flow limit can be exceeded.

In this paper we propose a rheological model describing the elasto-plastic behavior with linear reinforcement. By generalization some elasto-plastic models of solid bodies will be analyzed, which show rigidity characteristics, following the effect of stress relief under the influence of vibrations.
2. Description of elasto-plastic behaviour of solid bodies

After reaching the flow limit, in many solid bodies there is a consolidation (stiffening) followed by increase the resistance. This observation is also valid for bone tissue. This means that the stress-strain diagram $\sigma - \epsilon$ can be considered to consist of linear sections in both elastic behaviour and plastic behaviour. The slope of these segments is equal to the modulus of elasticity $E$ for the first domain, equal to that of the plasticity $E_p$ in the second domain (Figure 1). In the case of the monotone stress increase, for the elastic domain, this can be written analytically by the relation:

$$\sigma = \varepsilon E_1 \text{ for } \sigma \leq \sigma_c.$$  \hspace{1cm} (1)

In the equation 1, $\sigma_c$ is the value of the stress to which the rheological model has a purely elastic behaviour. For the domain in which the rheological model has plastic behaviour it can be written:

$$\sigma = E_p \varepsilon + \frac{E - E_p}{E} \sigma_c \text{, for } \sigma > \sigma_c.$$ \hspace{1cm} (2)

If we denote with $\sigma_1$ the maximum stress reached, then the return will be elastic and can be represented by the following relation:

$$\sigma = \sigma_1 - E(\varepsilon_1 - \varepsilon_2).$$ \hspace{1cm} (3)

![Figure 1. Elastic feature with stiffening](image1)

![Figure 2. Elasto-plastic model with linear stiffening](image2)

The rheological model of elasto-plastic type with linear reinforcement is represented in Figure 2 and consists of two Hooke elements with $E_1$ and $E_2$ constants, as well as the Saint-Venant plastic element.

For the elastic domain in which $\sigma \leq \sigma_c$ the element $E_2$ does not work, the behavior of the pattern is purely elastic and corresponds to the $E_1$ element requirement. If $\sigma > \sigma_c$ then the element $E_2$ is also in operation being connected in series with $E_1$. Since the elastic modulus of the elasto-plastic solid model is $E_1$, and in the plastic domain is $E_p$, resulting by serial binding of the elements $E_1$ and $E_2$, (Figure 3), i.e.
\[
\frac{1}{E_1} + \frac{1}{E_2} = \frac{1}{E_p}.
\] (4)

It can be concluded that the modulus of elasticity of the first element is:

\[E_1 = E,\] (5)

and the second element is

\[\frac{1}{E_2} = \frac{1}{E_p} - \frac{1}{E}.\] (6)

For verification, the constitutive equations can be written, having the following form:

\[\varepsilon = \frac{\sigma}{E_1}, \text{ for } \sigma \leq \sigma_c,\] (7)

\[\varepsilon = \frac{\sigma - \sigma_c}{E_2} + \frac{\sigma}{E_1}, \text{ for } \sigma > \sigma_c.\] (8)

The conditions given by equations (5) and (6) must be justified in order to be in accordance with equations (1) and (2). Otherwise, meaning that the stress \(\sigma\) is decreasing, the tension of the Saint-Venant element would fall below the value of \(\sigma_c\), which would lead to the deformation of the element \(E_2\).

3. Elasto-plastic rheological models with viscous properties

The rheological model describing the elasto-plastic behaviour of viscous bodies is that of Goldsmith [8]. It is composed of two Hooke elements, a Newton element and a Saint-Venant element fused as in Figure 3. Until a stress is applied \(\sigma \leq \sigma_c\), the material behaves elastically, while at a higher stress it will behave as plastic.

![Figure 3. Goldsmith model of tissue](image1)

![Figure 4. Bingham model of tissue](image2)
Another model describing a body with elasto-viscoplastic characteristics is that of Bingham as in Figure 4. To write the constitutive relationships we use a modified Bingham model, which takes into account the consolidation phenomenon.

In the domain of plastic deformations, the behaviour of the model can be written according to the relation:

\[ \sigma_p = \sigma_c + E_p \varepsilon_p, \]  

where \( \sigma_p \) is the value of the stress corresponding to the plastic deformation \( \varepsilon_p \). The constituent equations are:

\[ \varepsilon = \frac{\sigma}{E}, \text{ for } \sigma \leq \sigma_c; \]  \hspace{1cm} (10)

\[ \dot{\varepsilon} = \frac{\sigma - \sigma_c}{\eta}, \text{ for } \sigma > \sigma_c, \]  \hspace{1cm} (11)

where \( \eta \) is the viscosity of damper element.

Plastic deformation can be obtained from total deformation if elastic deformation decreases

\[ \varepsilon_p = \varepsilon - \frac{\sigma}{E}. \]  \hspace{1cm} (12)

If this relationship is taken into account in the equation (10), it becomes the following:

\[ \dot{\varepsilon} + \frac{E_p}{\eta} \varepsilon = \frac{\sigma}{E} + \frac{1}{\eta} \left[ \left( 1 + \frac{E_p}{E} \right) \sigma - \sigma_c \right]. \]  \hspace{1cm} (13)

4. Elasto-plastic rheological models with viscous properties

It is considered that the deformation following the application of the vibrations at a given point can be transcribed by the harmonic law:

\[ \varepsilon = \varepsilon_0 + a \sin \omega t, \]  \hspace{1cm} (14)

where \( \varepsilon_0 \) represents the strain according to the initial state of the stress \( \sigma_0 \), and \( a \) is the small amplitude of the applied vibration and \( \omega \) the circularly frequency (\( \omega = 2\pi f \) is related to its frequency \( f \)). In the strain field of plastic deformation the rheological behaviour of the tissue can be described by equation (13) and the return will occur elastically to correspond to the experimental results. So the equation (13) and a harmonic variation of strain given by equation (14), can also be written as follows:

\[ \dot{\varepsilon} + \frac{E_p}{\eta} \varepsilon = \frac{\sigma}{E} + \frac{1}{\eta} \left[ \left( 1 + \frac{E_p}{E} \right) \sigma - \sigma_c \right] \left[ e^{a t} \right]. \]  \hspace{1cm} (15)

The general solution of this equation is:

\[ \sigma(t) = e^{-\alpha t} \left\{ C + \int_0^t \left[ \sqrt{E_p^2 + \eta^2 \omega^2} \cdot \cos(\omega t - \varphi) + \beta \right] e^{a t} dt \right\}, \]  \hspace{1cm} (16)

where the following notations were made

\[ \alpha = \frac{E + E_p}{\eta}; \]  \hspace{1cm} (17)
\[ \beta = \frac{E_p\sigma_0 + E\sigma_c}{\eta}; \]  \hspace{1cm} (18)

\[ \tan \varphi = \frac{E_p}{\eta\omega}. \]  \hspace{1cm} (19)

Depending on these relationships the equation (15) may also be written as follows:

\[ \sigma(t) = (C - \frac{\beta}{\alpha}) - \frac{E}{\sqrt{E_p^2 + \eta^2\omega^2}}e^{-\alpha t} + \frac{E_a\sigma_c}{\eta\omega} \left[ \sin(\omega t - \varphi) + \frac{\alpha}{\omega} \cos(\omega t - \varphi) \right] + \frac{\beta}{\alpha}. \]  \hspace{1cm} (20)

The integration constant C is calculated for each cycle in the conditions of stress and strain continuity in the transition from the elastic to the plastic state. It is found that in the presence of vibrations the tension component decreases exponentially. It can be concluded that the model shown in Figure 4 shows a relaxation phenomenon.

After a large number of cycles, according to the relaxation time for this model, the maximum value of the tension will correspond to the maximum deformation value, as follows:

\[ \sigma_{\text{max}} = \frac{E_p\sigma_0 + E\sigma_c}{E + E_p} + \frac{E_a\left[ \eta^2\omega^2 + (E + E_p)^2 \sigma_c \right]}{\eta^2\omega^2 + (E_p + E)^2}. \]  \hspace{1cm} (21)

On return, the process will be readily elastic along the straight line with the slope value equal to the elastic modulus E, therefore:

\[ \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2a} = E, \]  \hspace{1cm} (22)

or

\[ \sigma_{\text{min}} = \sigma_{\text{max}} - 2aE. \]  \hspace{1cm} (23)

After stopping the vibration phenomenon, after several cycles, the voltage in the rheological model considered will stabilize in the average position of the extreme states of tension created by the subjection to harmonic vibrations, marked with \( \sigma_0 \)

\[ \sigma_0 = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}, \]  \hspace{1cm} (24)

or

\[ \sigma_0 = \frac{E_p\sigma_0 + E\sigma_c}{\eta} - \frac{E^2a(E_p + E)}{\eta^2\omega^2 + (E_p + E)^2}. \]  \hspace{1cm} (25)

If it is assumed that the pretension of the viscoelastoplastic body tissue model is precisely at the limit of its elastic behaviour, meaning it corresponds to the limit of the plastic deformation input area \( \sigma_0 = \sigma_c \) the end of the vibration process of tensing, the stress having the value given by the expression:

\[ \sigma_0 = \sigma_0 - \frac{E^2a(E_p + E)}{\eta^2\omega^2 + (E_p + E)^2} = \sigma_0 - \frac{E^2a(E_p + E)}{4\pi^2f^2\eta^2 + (E_p + E)^2} < \sigma_0. \]  \hspace{1cm} (26)
The correct evaluation of the residual stress drop by applying to the studied model harmonic vibrations can be done only after identifying the parameters of the rheological model, a model that can be identified according to the procedure in the paper [9].

5. Whole body vibration system and method

Whole Body Vibration (WBV) is a technique used to increase neuromuscular performance, including muscle strength, power, movement velocity and flexibility, among various patient groups, including athletes [10-12]. The focus of other investigations was to determine the acute impact of a bout of whole-body vibration on athletic performance [13-15].

This technique has begun to be applied, with the long-term stay of cosmonauts in space, after which a decrease in muscle strength has been observed, but also a decrease in bone density [16].

The systems used for body vibration are based on the principles of acceleration training to stimulate the body's natural response to vibration. The WBV system creates vibrations of controlled frequencies and amplitudes, and in the case of law-shaped sinusoidal harmonic vibrations (14) they will produce amplitude of acceleration at the platform level given by the relationship:

\[ A_{\text{max}} = \omega^2 a = 4\pi^2 f^2 a \approx 4gf^2 a, \]  

(27)

where by \( A_{\text{max}} \) has been noted by the maximum acceleration, \( a \) is the amplitude of the vibration, \( f \) is the frequency, and \( g \) is the numerical value of the gravitational acceleration. The mechanical characteristics of the plate: the vibration amplitude and frequency can be changed, thus obtaining different levels of acceleration. Acceleration levels usually compare to the gravitational acceleration \( g=9.81\text{m/s}^2 \). These accelerations will produce instability of the whole body, and while these vibrations transmit energy in waves, a variety of muscles contract and relax subconsciously throughout the body. This rapid, contraction and muscle relaxation cycle is what makes exercises with whole body vibration systems so effective.

![Figure 5. Exercise from the sitting position, keeping the heels on the platform](image)

![Figure 6. Exercise in orthostatic position and maintain balance through platform handles](image)

A fraction of the kinetic energy introduced by the vibratory platform into the human body is found in other forms of body energy (kinetics or potential), and the other fraction will dissipate by heat. Dissipated energy is actually energy absorbed by the body as heat:
\[
E_d = \int_0^{2\pi} \sigma \, d\varepsilon = \int_0^{2\pi} \sigma \, \frac{d\varepsilon}{dt} \, dt = \int_0^{2\pi} \sigma \varepsilon \, dt = \pi \mathcal{E} \varepsilon^2 \sin \varphi.
\]

The value of the integral in equation (28) represents the energy dissipated over a single cycle, and the value of \( E_d \) expression represents the energy dissipated in one second.

From the analysis of the equation (27) it can be observed that the amplitudes of the acceleration at the level of the platform can be modified by the independent variables, the amplitude of the vibrations and the frequency. Whole body vibrations have been applied with amplitude between 2.5 mm and 5.5 mm, with a 0.5 mm step, and frequency between 5Hz and 30Hz with a 5Hz step. By modifying the two independent parameters, vibration amplitude and frequency could be worked in a range of accelerations ranging from 0.25g to 6.6g. Due to transmissibility, these accelerations are different at some body parts, and are harder to be highlighted by measurements. Given the objective of the investigations, it can be considered that the vibration level at the vibratory platform is the same as the vibration level at the toe.

A group of few children, insufficient to form a lot of statistical analysis, with problems walking on the toes of the legs was then used by the effect of physical therapy through light intensity exercises. For this purpose, the application of the vibration treatment was conducted using a Power Plate body vibration platform (Power Plate North America Inc., Northbrook, IL). For this group of children was conducted several types of exercises.

The first mode of vibration was from the seated position, maintaining the plantar vault on the surface of the vibrating platform for 30 seconds at a frequency of 5 Hz as can be seen in Figure 5. The exercise was repeated six times, with ten seconds pause between exercises, maintaining the amplitude of the platform vibration at 2.5 mm, but increasing the frequency with the 5 Hz step. In the following days, the exercise was repeated, maintaining the same amplitude, but increased by the 0.5 mm increments daily.

In the next exercise, the patient is in the seated position, flexes the leg and attempts to stretch the Achilles tendon so that the heel remains on the vibratory platform.

In another exercise, the patient is placed orthostatically on the vibratory platform and is maintained in this position with the support of the platform handles (Figure 6). Independent variables, exposure time, vibration amplitude and frequency remain the same as in the first exercise.

In other exercises the patient was in a sitting position to standing on the vibration platform, maintaining the calcaneus on the platform or in ventral cubitus and dorsal cubitus.

For all patients, there was an improvement in walking and other functional activities after only six weeks. In the case of a girl, who was at the beginning of her legs in a major adduction that made it impossible to walk, she found amazing progress, making her first steps without help.

6. Conclusions
Based on the analysis of the formula given by the equation (23), after the stopping of the vibration action, the rheological model remains tense at a lower stress value than the initial one. Therefore, the beneficial effect of vibrations on the whole body is not only a consequence of well-being or a psychological consequence. It should also be noted that the magnitude of the amplitude induces a decrease in the residual stress value while the vibration frequency must be small.

As can be seen from both the rheological models, based on the analytical calculations, and the results of the exercises on the vibratory platform, it is possible to obtain muscle stress relief and flexibility of the tendons.

Future research will focus on the statistical tracking of effects of independent variables, vibration amplitude and frequency, by walking analysis.

On the one hand, vibrations have been studied for a long time, for its dangerous effects on humans. On the other hand, recent works have suggested that mechanical stimulation of the low frequency and low amplitude of the human body is a safe and effective way to strengthen and relax the musculoskeletal structures.
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