Hints of Dynamical Symmetry Breaking?

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There is current interest in a possible new massive gauge boson $X$ which mixes slightly with the $Z$ boson and accounts for certain anomalies in the LEP data. We show why constraints on models in which the $X$ boson does not couple to the first two families suggest dynamical electroweak symmetry breaking. The associated TeV mass fermions make up a fourth family. Constraints on the effects of the fourth left-handed neutrino also suggest a dynamical origin for its Majorana mass. We finally comment on related implications for the origin of quark masses.

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1 \hspace{1em} Z–X Mixing

We have learned by experience that discrepancies between experiment and the standard model tend to go away over time, and so it is natural to take a cautious attitude toward the present set of anomalies in the data. But we may still ask, from a more theoretical point of view, which of the present anomalies are most likely to survive? To this question we are motivated to consider seriously the following two anomalies in $R_b$ and $\alpha_s$.\[1\]

$$R_b = 0.2202 \pm 0.0016 \text{ when } R_c = R^\text{SM}_c$$  
$$R^\text{SM}_b = 0.2156 \pm 0.003$$  

(1)

$$\alpha_s(M_Z) = 0.126 \pm 0.005 \pm 0.002 \text{ using LEP } R_\ell \text{ only}$$  
$$\alpha_s(M_Z) = 0.113 \pm 0.005 \text{ from deep inelastic scattering}$$  

(2)

The main reason why these particular anomalies are intriguing is that the same piece of new physics would account for both anomalies; namely new physics which slightly enhances the $Zb\bar{b}$ vertex. That is, the shift in this vertex needed to increase $R_b$ from the standard model value by 2\% would shift the total hadronic width of the $Z$ in just such a way so as to reduce the value of $\alpha_s$ from $R_\ell$ by 0.013. This correlation between the two anomalies assumes that $R_c$ is given by the standard model value, since any shift in $R_c$ would affect both $R_\ell$ and $R_b$. Note that $\Gamma_Z$ and $\sigma^0_b$ in combination with $R_\ell$ gives a similar result, $\alpha_s(M_Z) = 0.124 \pm 0.004 \pm 0.002$, but $\Gamma_Z$ and $\sigma^0_b$ may be more sensitive to other new physics in addition to the $Zb\bar{b}$ vertex. If the main effect of new physics is in the $Zb\bar{b}$ vertex, then a clear discrepancy should emerge between $\alpha_s$ from electroweak precision tests and all other measurements of $\alpha_s$.

Another point is that if we believe that the new physics has something to do with the generation of fermion masses, then we should not be surprised to find this new physics coupling most strongly to the heaviest family. In fact the correlation between the $R_b$ and $\alpha_s$ anomalies became evident \[2\] through the study of new physics of this sort. In particular a model \[3\] describing a dynamical origin of the $t$-quark mass contained a new gauge boson $X$ coupling strongly to the third family. This boson
mixed slightly with the $Z$ and thus shifted the $Zb\bar{b}$ vertex. Instead of starting here by describing a model, we will start from the point of view of trying to extend the standard model in a simple way, and see where we are led. We will concentrate on the idea of $Z$–$X$ mixing, although there are of course other possibilities.[4]

Many authors have considered the case of an $X$ (often called $Z'$) coupling to all quarks,[5] with the goal of accounting for a possible anomaly in $R_c$ as well as $R_b$. But we have seen that the correlation between the $R_b$ and $\alpha_s$ anomalies suggests that $R_c$ should stay at its standard model value. In any case the most recent data has the possible anomaly in $R_c$ falling below the 2$\sigma$ level.[1]

We take $X$ to couple predominantly to the third family, with couplings to the light two families generated only because of small mass mixing effects. This case has also been considered in ref. [6], but in a more conventional context of elementary scalar fields. There additional Higgs doublets carrying $X$ charge are postulated to induce the $Z$–$X$ mixing at tree level. It is found that one additional Higgs coupling to both $t$ and $b$ is not sufficient to induce mixing with the correct sign. With two additional Higgs—$H_t$ and $H_b$ coupling to $t$ and $b$ respectively—the desired mixing is possible as long as $H_t$ contributes most of the mixing. Since the $t$ mass violates the $U(1)_X$ gauge symmetry, it must be generated by the coupling to $H_t$ rather than the standard model Higgs.

By considering the diagram with the $Z$ coupling to quarks through an intermediate $X$, via a $Z$–$X$ mass-mixing $\delta M^2$, the magnitude of the shift in the $Z$ coupling to quarks can be written as

$$\delta g_Z = -\delta M^2 \frac{1}{M_X^2} g_X \equiv -\theta g_X.$$  \hspace{1cm} (3)

To be specific we define the shift in the $Z$ couplings to be $-\delta g_Z (\bar{t}\gamma_\mu \gamma_5 t + \bar{b}\gamma_\mu \gamma_5 b)$ and the $X$ coupling to be $-X^\mu (\bar{t}\gamma_\mu \gamma_5 t + \bar{b}\gamma_\mu \gamma_5 b)$. (The reason for axial $X$ couplings to quarks will become clear below.) Since $\delta M^2$ is the off-diagonal element of the $2 \times 2$ mass-squared matrix, $\theta$ is the $Z$–$X$ mixing angle which we may assume to be small.

The $Z$–$X$ mixing also induces a shift in the $Z$ mass, which translates into a contribution to $\delta \rho$,

$$\delta \rho \approx \theta^2 \frac{M_X^2}{M_Z^2}. \hspace{1cm} (4)$$
A possible $Z$–$X$ mixing in the kinetic terms would contribute a term of the opposite sign,\cite{7, 8} but we will assume that this may be neglected. By inserting $\theta$ from (3) into (4) and requiring a large enough $\delta g_Z$ to account for $\delta R_b$, we have the upper bound
\[ \frac{M_X}{g_X} \lesssim 1 \text{ TeV}. \] (5)
This is related to the observation made in ref. \cite{6} that $g_X^2/4\pi \lesssim 1$ implies that $M_X \lesssim 3–4$ TeV.

We further note that $\langle H_t \rangle$ determines $\delta M^2$ and thus
\[ \delta g_Z = -\frac{e}{s_c} \langle H_t \rangle^2 \frac{g_X^2}{M_X^2}. \] (6)
Because of the bound (3) we have
\[ \langle H_t \rangle \lesssim 50 – 100 \text{ GeV}. \] (7)
But since the $t$ mass is generated from $\langle H_t \rangle$, (3) implies that a large Yukawa coupling is required. By imposing an upper bound on the size of this Yukawa coupling we now see that both $\langle H_t \rangle$ and $M_X/g_X$ must come fairly close to saturating the bounds in (3) and (4). We have learned two things; the physics responsible for the $X$ boson mass is characterized by a TeV, and the $t$-quark Yukawa coupling is even stronger than in the standard model.

Now let us recall that new fermions must be introduced to cancel the gauge anomalies involving the $X$. Although new fermions with nonstandard electroweak charges may be added,\cite{6} a question is whether a conventional fourth family would suffice. The answer is yes. If the $X$ has isospin-singlet couplings to the quark doublets $(t, b)$ and $(t', b')$, then all anomalies are canceled if these two doublets have equal and opposite vector $X$ couplings, or equal and opposite axial $X$ couplings. $X$ couplings to leptons are not necessary. In fact the two cases correspond to making different choices for the mass eigenstates. Let us denote by $Q \equiv (U, D)$ and $\bar{Q} \equiv (\bar{U}, \bar{D})$ the two quark doublets with equal and opposite vector $X$ charge. The mass eigenstates have equal and opposite vector $X$ couplings if the mass eigenstates correspond to $Q_LQ_R$ and $\bar{Q}_L\bar{Q}_R$. On the other hand the mass eigenstates have equal and opposite axial couplings if they correspond to $\bar{Q}_LQ_R$ and $\bar{Q}_L\bar{Q}_R$. 

3
We have noted that $M_X/g_X \approx 1$ TeV, which implies that there is some physics at a TeV which breaks the $U(1)_X$ gauge symmetry. We now note that if $t'$ and $b'$ have axial $X$ couplings then their masses do not respect $U(1)_X$. In this case the existence of these masses is naturally linked to the breakdown of the $U(1)_X$ at a TeV, implying that the $t'$ and $b'$ masses are of order a TeV. Given that these new fourth family quarks have conventional weak charges their masses, if fairly degenerate, would imply appropriate masses for the $W$ and $Z$. We are being led to consider dynamical electroweak symmetry breaking.

Given this prompting, let us remove all elementary scalar fields. There is then no tree level contribution to the $Z$–$X$ mass-mixing. There is also little contribution from $t'$ and $b'$ loops due to the required degeneracy of the $t'$ and $b'$ masses. That is, the $X$ couplings to $t'$ and $b'$ are the same whereas the axial $Z$ couplings to $t'$ and $b'$ are equal and opposite, implying that the two contributions in the mass-mixing loop will cancel. The mixing must then come from the $t$-loop. It is interesting that the $t$-loop contribution would vanish if the $X$ had purely vector couplings to the $t$, and so this allows us to reject the vector coupling possibility.

The shift in the $Z$ coupling from the $t$-loop is the same as in (3), but with $\langle H_t \rangle$ replaced by a quantity $f_t$ determined by the $t$-loop. $f_t$ is normalized such that $(f_t/v)^2$, with $v \approx 240$ GeV, gives the fractional contribution of the $t$-loop to $M_Z^2$. The point is that the $t$-loop involves a momentum dependent $t$ mass function which we may assume is fairly constant up to the scale of new physics at a TeV, at which point it falls. We thus calculate the loop with a 1 TeV cutoff and find

$$f_t^2 \approx \frac{3}{8\pi^2} m_t^2 \ln \left( \frac{(1 \text{ TeV})^2}{m_t^2} \right) \approx (60 \text{ GeV})^2.$$  

(8)

This value is consistent with the constraints we found before on $\langle H_t \rangle$. The difference is that $f_t$ is calculated here, whereas $\langle H_t \rangle$ was a free parameter. We conclude that the $t$-loop produces $Z$–$X$ mixing of the correct magnitude and sign to produce the desired shift in the $Zb\bar{b}$ vertex. This provides support for our consideration of dynamical electroweak symmetry breaking.

Leptons of a fourth family are also appearing in the picture, and their masses must also be large. We have mentioned that the $X$ boson does not need to couple
to leptons to cancel anomalies, but it is nevertheless easy to motivate such couplings in the context of quark-lepton unification. It is simplest to expect that the $U(1)_X$ gauge symmetry commutes with the quark-lepton gauge symmetry present at some higher scale, in which case the $X$ boson should couple similarly to quarks and leptons (at least in some basis). This in turn will shift the $Z$ couplings to the third (and fourth) family leptons, and the question is whether such shifts are still allowed by the data. In this connection we note that the $Z$ couplings to charged leptons is mostly axial. Thus if the shifts occurred mostly in the vector couplings then the strongly constrained leptonic partial decay widths of the $Z$ would be little affected.

Anomalies will cancel within the lepton sector if the two families of leptons $(\nu_L, \tau_L, \tau_R; \nu'_L, \nu'_R, \tau'_R)$ have $X$ charges $(+, +, +; -, -)$ or $(+, +, -; -, +)$. The difference again is related to choice of mass eigenstates. As for the quarks we may define the fields $(E_L, E_R)$ and $(\bar{E}_L, \bar{E}_R)$ to have equal and opposite vector $X$ charge. Unlike for quarks these fields must correspond to the mass eigenstates, so that we have vector $X$ couplings to the $\tau$ ($\bar{E}_L$) and $\tau'$ ($\bar{E}_R$), and thus shifts mainly in the vector rather than the axial $Z$ coupling to $\tau$. We emphasize that the underlying strong dynamics at a TeV is responsible for the choice of mass eigenstates, since it determines the fields corresponding to the fourth family quarks and leptons.

We have thus motivated the following $X$ boson coupling to the third family,

$$J^X_\mu = \bar{t}(L_\mu - R_\mu)t + \bar{b}(L_\mu - R_\mu)b + \bar{\nu}_{\tau}(L_\mu + R_\mu)\nu_{\tau}$$

(9)

with $L_\mu, R_\mu \equiv \gamma_\mu(1 \mp \gamma_5)/2$. The most striking prediction is that if the $R_b$ anomaly is to be explained as we have described, then the $\tau$ asymmetry parameter $A_\tau$ would become $\approx 20\%$ higher than in the standard model.[3] If we assume that $A_e$ is given by the standard model (an assumption in good agreement with LEP data), then there are two independent measurements of $A_\tau$. The forward-backward asymmetries and the mean tau polarization give

$$A_{FB}^{0,\tau} \Rightarrow A_\tau/A^{SM}_\tau = 1.28 \pm 0.14, \quad (10)$$

$$A \Rightarrow A_\tau/A^{SM}_\tau = 0.96 \pm 0.05. \quad (11)$$

When combined these results are in perfect agreement with the standard model, but
the discrepancy between the results may suggest that it is too early to completely rule out new physics in the $Z$ coupling to $\tau$.

We note that predicted shifts in other quantities due to $Z$–$X$ mixing, $+0.2\%$, $-1.5\%$, and $-0.5\%$ in $\Gamma_\tau$, $\Gamma_{\nu_\tau}$, and $A_b$ respectively, are all compatible with current measurements. Additional small corrections to $\Gamma_\tau$ from vertex corrections and to $\delta\rho$ from two-loop graphs are discussed in [10] and [11] respectively. In the event that the $X$ couples only to quarks, then only the shifts in $\Gamma_b$ and $A_b$ remain. We also note [2, 4] that flavor changing neutral currents induced by nonuniversal $Z$ couplings are acceptable as long as most mass mixing occurs in the up-quark sector.

2 Neutrino Mass and Quark Mass

We have seen that the $Z$–$X$ mixing can be induced by a Higgs as long as that Higgs has a large Yukawa coupling to the $t$ quark. But then we saw that the $t$-loop would suffice by itself, so that we could remove the Higgs. We now note that a similar situation occurs when we consider the massive fourth-family neutrino. We treat the case where all right-handed neutrinos are absent from the effective theory at a TeV, since we note that the large Majorana masses for right-handed neutrinos are allowed by the $U(1)_X$ gauge symmetry. We must then consider the electroweak corrections induced by the fourth-family left-handed neutrino. Other analyses [12] either have Dirac neutrino masses or right-handed neutrinos involved in a see-saw mechanism. But in the dynamical symmetry breaking context it appears that right-handed neutrinos much more massive than a TeV will more naturally completely decouple.

The fourth-family left-handed neutrino $\nu_L$ must have a Majorana mass greater than $M_Z/2$. If this mass were to come from a vacuum expectation value of a $SU(2)_L$-triplet scalar field $\langle H_M \rangle$, then we have a tree level contribution to $\alpha T \approx -(\langle H_M \rangle / 125\text{ GeV})^2$. This puts a severe upper bound on $\langle H_M \rangle$, which in turn implies a very large $\nu_L$ Yukawa coupling. We therefore are in the same situation as before, leading us again to remove scalar fields and consider the dynamical generation of mass. Of interest now is the neutrino-loop contribution to $T$. 


We have considered the contributions to $S$, $T$, and $U$ from the fourth family leptons $(\nu_L, \tau') = (N, E)$ (we omit the underlines) for a range of masses $m_N$ and $m_E$. The result for $T$ depends on the effective cutoff in the neutrino loop (similar to (8)); this cutoff is supplied by the momentum dependence of the dynamical mass, and we have used $\Lambda = 1.5 m_N$ and $\Lambda = 2 m_N$. From Figs. (1) and (2) we see ranges of masses for which the new contributions to $T$ are negative with reasonable size, while the new contributions to $S$ and $U$ are simultaneously small. We therefore have what appears to be a natural source of negative $T$ within the dynamical symmetry breaking context. This may be useful, given the fact that a dynamically generated $t$-quark mass is typically accompanied by positive contributions to $T$.

Let us consider quark masses further. $t'$ and $b'$ have received TeV masses via dynamics associated with the breakdown of $U(1)_X$. In the absence of scalars there must be four-fermion operators which will feed mass down to the other quarks, and the largest such operator will be the one which provides the $t$ mass. We recall our previous notation for the two quark doublets having equal and opposite vector $X$ charge: $Q \equiv (U, D)$ and $Q \equiv (U', D')$. The fourth family quarks have the form $\overline{Q}_L Q_R$, which breaks $U(1)_X$, and we need an operator to feed this down to $\overline{Q}_L Q_R$. A suitable operator which will feed mass from the $b'$ to the $t$ is $(\overline{Q}_L D_R)\epsilon (\overline{Q}_L U_R)$. (The antisymmetric $\epsilon$ has $SU(2)_L$ indices.) Note that this operator is composed of two Lorentz scalars which are also singlets under $U(1)_X$. This operator is thus of a form which could be expected to be enhanced by strong $U(1)_X$ interactions. If these interactions are of the walking-coupling type, then there can be significant anomalous scaling enhancement of this operator relative to other operators.

This operator has a partner, $(\overline{Q}_L U_R)\epsilon (\overline{Q}_L D_R)$. This operator will feed mass from the $t'$ to the $b$, and it must thus be suppressed relative to the previous operator. The attractive feature is that the isospin breaking implied by the different sizes of these operators does not feed into the TeV quark masses or $T$ in a direct way. In fact four insertions of these operators are needed to produce a contribution to $T$. Other isospin breaking operators which could feed more directly into $T$ are not so enhanced by the anomalous scaling effects. We thus have an example of electroweak breaking physics being protected to some extent from the isospin breaking physics, which is feeding down from higher scales in four-fermion operators.
We may bring in the two light families by embedding $U(1)_X$ into a larger gauge symmetry at a higher scale $\Lambda$ (say 100–1000 TeV), which connects the light families to heavy families. We may then write down various four-fermion operators which can arise at the scale $\Lambda$. It may be shown that such operators are sufficient to produce nontrivial mass mixing and a potentially realistic mass matrix \[11\]. We find that the existence of these operators may also be associated with the breakdown of the $U(1)_X$ gauge symmetry.

Hierarchies develop in the quark mass matrices for three reasons. 1) The various operators have different numbers of fields to which the $U(1)_X$ couples, and thus are enhanced by varying amounts due to anomalous scaling induced by the $U(1)_X$. 2) Some entries in the mass matrices receive mass fed down from a $t$ or $\tau'$ rather than the heavier $t'$ or $b'$. 3) Some entries only receive contributions from loops involving more than one 4-fermion operator. We note from \[11\] that contributions feeding down from the $\tau'$ are important for obtaining realistic quark mass matrices, and that these contributions only arise for the choice of mass eigenstates for $\tau$ and $\tau'$ which we have already motivated.

Acknowledgments

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Figure 1: Lines of constant $T$ as a function of the $N$ and $E$ masses in TeV. Thick and thin lines are for $\Lambda = 1.5 m_N$ and $\Lambda = 2 m_N$ respectively. In each case, from top to bottom, $T = -2, -1, 0$.

Figure 2: Thick and thin lines are lines of constant $S$ and $U$ respectively as a function of the $N$ and $E$ masses in TeV. From top to bottom in each case $S = 1/6\pi, 0, -1/6\pi$ and $U = -1/12\pi, 0, 1/6\pi$. 