Elastic punching of material during flattening of a spherical asperity and restoration of the imprint in unloading

P M Ogar*, D B Gorokhov and E V Ugryumova
Bratsk State University, 40 Makarenko st., Bratsk, 665709, Russia

*ogar@brstu.ru

Abstract. The development of the methodology previously proposed by the authors for determining the displacements of half-space points under the action of an axisymmetric load described by a parabola is presented. It is shown that when the sphere is flattened, the pressure distribution on the contact area is a function of complex shape, which can be represented by a combination of several equations for description in adjacent sections. The knowledge and description of the contact pressure distribution function is of practical importance in solving the issues of elastic punching of the material, the magnitude of the elastic recovery of the contact print during unloading, and determining the curvature of its profile. Equations are proposed for determining displacements inside and outside the contact area when describing the distribution of contact pressure by analytical functions and when using a discrete distribution of contact pressure. The accuracy of the obtained results depends on the rate of the approximating functions to the actual pressure distribution at the contact area. At the same time, the goals of the problems to be solved should be considered. When determining elastic recovery in the center of the contact area, it is sufficient to use a uniform distribution of contact pressure. When determining the profile of the restored contact print and its curvature, increased accuracy of approximating functions is required.

1. Introduction
Most models of engineering surfaces at micro levels are represented by asperities in the form of a set of spherical segments of equal radii with a Gaussian height distribution [1], or with a distribution corresponding to the curve of the supporting surface [2, 3]. To calculate the real contact area and other characteristics, these models suggest that the contact of two rough surfaces can be represented as the contact of an equivalent rough surface with a flat surface. The solution to this problem has applications in tribology [4–6], hermetology [7–9], in electrical [10] and thermal contacts [11].

In most cases, the contact of an individual asperity is also modeled as the contact of a sphere and a plane, which are divided into two main groups [12]: indentation models, when the plane is deformed, and the hemisphere is either rigid or elastic; flattening models when the plane is considered rigid and the sphere is deformed. A review of analytical and numerical studies of elastoplastic contacts is given in [13, 14], which indicated the absence of a closed solution.

When a sphere is intended into a deformable half-space, a distinction is made between an elastic region, a region of limited elastic-plasticity, and a region of developed elastic-plasticity. They differ in the correlation of the role of two independent processes: elastic pushing of materials (sink-in) and plastic displacement of material (pile-up), which is confirmed by the results of [15]. The presence of the indicated areas when flattening the sphere is also obvious. Therefore, in our view of the problems
of studying elastic elasticity, we can relate the issues of elastic punching of materials and elastic restoration of an contact print during flattening of a sphere due to their practical importance. Recall that their solution is relevant when using the kinetic indentation diagram to describe the elastic-plastic contact interaction [16], when determining the joint density of rough surfaces [17], when determining contact characteristics when unloading a pre-loaded joint [18].

To solve these problems, it is necessary to know the pressure distribution at the contact area. It was shown in [16] that if the branch of elastoplastic loading of the sphere of the kinetic indentation diagram has the form

$$P = C_i h^\alpha,$$

where $$C_i = P_m \bar{h}_m^\alpha$$, $$P_m$$ is the maximum load, $$h$$ is the displacement in the contact, $$\bar{h}_m$$ is the maximum displacement, $$\alpha = 1...1.5$$ is a constant, then the pressure distribution at the contact site is described by the equation

$$p(r) = p_0 \left(1 - \frac{r^2}{a^2}\right)^\beta,$$

where $$\beta = \alpha - 1$$, $$p_0 = p_m (\beta + 1)$$, $$p_m = \sqrt{\frac{\rho}{\pi a^2}}$$ is the average pressure in contact, $$a$$ is a contact radius.

There are practically no analytical equations for the distribution of pressure on the contact area during flattening of the sphere in the technical literature. The exception is the work [19,20]. The authors of [19] first proposed an elastoplastic empirical model according to which the distribution of contact pressure for a plastically deformed sphere was assumed to be uniform and equal to the maximum Hertz pressure during critical displacement. A uniform pressure distribution in the central part of the contact region and an elliptical distribution outside this part, starting from the maximum pressure and approaching zero at the contact boundary, were similarly proposed in [20].

In [21, Figure 3] V. Brizmer et al. Presented the distribution of contact pressure obtained as a result of finite element modeling when a sphere is flattened under conditions of stick and under conditions of slip for different values of the relative amount of flattening $$\omega^*$$. As follows from [20, 21], the pressure distribution at the contact area can be complex in shape, i.e. present a combination of several equations for description in different sections or be specified numerically. The question of determining the displacement within and outside the action area of an axisymmetric load of complex shape was considered in [22]. The purpose of this research is to determine the elastic forcing of the material during flattening of a spherical asperity and the restoration of the contact print during unloading.

2. Methodology for solving the problem

The definition of a stress-strain state when a load of the form (2) acts on a half-space was considered in [23]. Earlier, in the technical literature on the theory of elasticity and mechanics of contact interaction [24, 25, etc.], special cases of the stress-strain state were considered: for $$\beta = 0$$ (uniform distribution), for $$\beta = 0.5$$ (hertz distribution), for $$\beta = -0.5$$ (constant normal displacement, or draft). For the data were absent due to the difficulties of obtaining analytical expressions when integrating the products of functions with fractional powers.

Using the method described in [25, Section 4.3] for calculating displacements at an internal point from a load of the form (2), the authors of [23] obtained.

$$\varphi_c(\rho) = \frac{P_m a}{E} \cdot (1 + \beta) \cdot 2^{\beta+1} \cdot B(\beta + 1, \beta + 1) \cdot F_1\left(\frac{-\beta - 1}{2}, \frac{1}{2}; 1; \rho^2\right), \quad 0 \leq \rho \leq 1,$$

where, $$\rho = r/a$$, $$B(a_1, a_2)$$ is the beta function, $$F_1(a, b, c, x)$$ is the Gauss hypergeometric function, $$E$$ is the elastic modulus.

Similarly for a point outside the circular loading area
\[
\bar{u}_z(\rho) = \frac{P_{ma}}{E} \cdot (1 + \beta) \frac{2^{2\beta+1}B(\beta + 1; \beta + 1)}{\pi \cdot \rho^3} \cdot B\left(\frac{1}{2}; \beta + \frac{3}{2}\right) \cdot\left[ F_1\left(\frac{1}{2}; \frac{1}{2}; \beta + 2; \rho^{-2}\right) \right], 1 \leq \rho.
\] (4)

As the analysis of the obtained equations (3) and (4) showed, they are valid for values \(-0.5 \leq \beta \leq 0.5\), however, the method used in [30] is not suitable for the distribution of contact pressures of complex shape.

Figure 1. Schemes of distribution of load pressures of the form (2).

Figure 1 presents graphical diagrams of dependencies of the form (2) for positive and negative values \(\beta\), as well as for \(\beta = 0\). As follows from fig. 1, the load distribution of complex shape (for example, in [21, Fig. 3]) in different sections can be represented by combinations of functions of the form (2) with different degrees.

In [22], the calculation of displacement at an internal point with the coordinate from the load of the form (2) is presented in the form:

\[
\bar{u}_{zn}(\rho_a) = \bar{u}_{m}(\rho_a) + \bar{u}_{z}(\rho_a),
\] (5)

\[
\bar{u}_{m}(\rho_a) = \frac{P_{ma}}{E} \cdot 2(1 + \beta) \int_{\rho_a}^{1} \frac{1}{\rho_a^{\beta}} \left(1 - \rho_a^{2}\right)^\beta \cdot F_1\left(\frac{1}{2}; \frac{1}{2}; \frac{1}{\rho_a^{2}}; \frac{4\rho_a}{\left(1 + \rho_a^{2}\right)^2}\right) \cdot d\rho,
\] (6)

\[
\bar{u}_{z}(\rho_a) = \frac{P_{ma}}{E} \cdot 2(1 + \beta) \int_{\rho_a}^{1} \left(1 - \rho_a^{2}\right)^\beta \cdot F_1\left(\frac{1}{2}; \frac{1}{2}; \frac{1}{\rho_a^{2}}; \frac{4\rho_a}{\left(1 + \rho_a^{2}\right)^2}\right) \cdot d\rho,
\] (7)

where, \(r/a = \rho\), \(r_a/a = \rho_a\).

Displacement at an external point with a coordinate \(\rho_a \geq 1\) is

\[
\bar{u}_z(\rho_a) = \frac{P_{ma}}{E} \cdot 2(1 + \beta) \int_{\rho_a}^{1} \frac{1}{\rho_a^{\beta}} \left(1 - \rho_a^{2}\right)^\beta \cdot F_1\left(\frac{1}{2}; \frac{1}{2}; \frac{1}{\rho_a^{2}}; \frac{4\rho_a}{\left(1 + \rho_a^{2}\right)^2}\right) \cdot d\rho.
\] (8)

To compare the results of calculating the displacements using equations (3), (5) and (4), (8), we will present their left parts in a dimensionless form:

\[
F_z(\rho) = \frac{\bar{u}_z(\rho)E}{ap_m}, \quad F_{zn}(\rho_a) = \frac{\bar{u}_{zn}(\rho_a)E}{ap_m}.
\] (9)
As follows from Figure 2, the dependences $F_z(\rho)$ and $F_{zd}(\rho)$ for $\beta = 0.4$ and $\beta = 0.1$ ideally coincide. This indicates the possibility of using the technique proposed in [22] to determine displacements within and outside the range of action on a half-space of a complex load.

Assume that the load distribution for $10 \leq \rho \leq 1$ is described by a complex function

$$p(\rho) = \begin{cases} p_1(\rho), & 0 \leq \rho \leq \rho_1; \\ p_2(\rho), & \rho_1 \leq \rho < \rho_2; \\ \ldots \ldots \ldots \ldots \\ p_n(\rho), & \rho_{n-1} \leq \rho < 1. \end{cases} \quad (10)$$

In this case, according to [29], the displacement of the internal point for $0 \leq \rho_a < 1$ is equal to

$$\pi_{zd}(\rho_a) = \frac{4a}{\pi E} \left[ \int_{0}^{\rho_a} \frac{p(\rho)}{1+\rho_a/\rho} K(k_1) d\rho + \int_{\rho_a}^{1} p(\rho) K(k_2) d\rho \right], \quad (11)$$

where $k_1 = k_1(\rho, \rho_a) = \frac{2(\rho/\rho_a)^{0.5}}{1+\rho/\rho_a}$, $k_2 = k_2(\rho, \rho_a) = \frac{\rho_a}{\rho}$; $K(k)$ is the full elliptic integral of Legendre of the 1st kind, which for the convenience of calculation in the Mathcad environment can be represented as:

$$K(k) = \frac{\pi}{2} F\left(0.5,0.5;1;k^2\right). \quad (12)$$

Displacements of a point outside the range of load when $\rho_a \geq 1$.

$$\pi_{zd}(\rho_a) = \frac{4a}{\pi E} \frac{p(\rho)}{1+\rho_a/\rho} K(k) d\rho. \quad (13)$$

Let us consider the distribution of contact pressure during flattening of a sphere under sliding and cohesion conditions [21, Fig. 3]. In Figure 2 shows the digitized values of the distribution of dimensionless pressure at the contact area under relative deformation $\omega^* = 100$. The average contact pressure is $\bar{p}_m = p_m/Y_0 = 2.89$, $Y_0$ is the yield strength. The corresponding approximations of the indicated distributions by equations of the form (2) are also given there:

$$\bar{p}(\rho) = \begin{cases} \chi_1(1+\beta_1)(1-\rho^2)^{\psi_1}, & 0 \leq \rho \leq \rho_1; \\ \chi_2(1+\beta_2)(1-\rho^2)^{\psi_2}, & \rho_1 \leq \rho \leq \rho_2; \\ \chi_3(1+\beta_3)(1-\rho^2)^{\psi_3}, & \rho_2 \leq \rho \leq 1. \end{cases} \quad (14)$$
Figure 3. Distribution of dimensionless contact pressure over the contact area under condition of slip (a) and stick (b).

For the slip condition: \( \chi_1 = 3.8, \beta_1 = -0.28, \rho_1 = 0.65, \chi_2 = 3.19, \beta_2 = 0, \rho_2 = 0.876, \chi_3 = 3.8; \) for stick: \( \beta_3 = 0.3, \chi_1 = 13.6, \beta_1 = -0.806, \rho_1 = 0.489, \chi_2 = 3.289, \beta_2 = 0, \rho_2 = 0.836, \chi_3 = 3.8, \beta_3 = 0.45. \)

Figure 4 shows dimensionless displacement calculation results \( F_z \) using equations (9), (11) and (14). When representing the distribution of contact pressure by discrete values \( p_i, \ 0 \leq i \leq k_m \), the following equations are used:

\[
F_z^+(k_a) = F_z^+(k_a) + F_z^-(k_a), \quad 0 \leq k_a \leq k_m; \tag{15}
\]

\[
F_z^+(k_a) = \begin{cases} 0, & k_a = 0, \\ \frac{\Delta \rho}{p_m} \sum_{i=1}^{k_m} \left( p_{i-1} + p_i \right) \frac{i - 0.5}{i - 0.5 + k_a} \binom{1}{2} F_1 \left( \frac{1}{2}, \frac{1}{2}, 1, \frac{4(i - 0.5)k_a}{(1 - 0.5 + k_a)^2} \right), & 1 \leq k_a \leq k_m, \end{cases} \quad 1 \leq k_a \leq k_m, \tag{16}
\]

\[
F_z^-(k_a) = \begin{cases} 0, & k_a = k_m, \\ \frac{\Delta \rho}{p_m} \sum_{i=1}^{k_m-1} \left( p_i + p_{i+1} \right) \frac{1}{2} \binom{1}{2} F_1 \left( 1, 1, 1, \frac{k_a^2}{(i + 0.5)^2} \right), & 0 \leq k_a \leq k_m - 1, \end{cases} \tag{17}
\]

where \( \Delta \rho = 1/k_m \).

To determine the dimensionless displacement inside the contact area with a uniform distribution of contact pressure, equation (3) was used.

Figure 4. Dimensionless elastic displacements \( F_z \) within the contact area during unloading under conditions of slip (a) and stick (b).
Similar results for dimensionless displacements $F_z$ outside the contact area using equations (9), (13) and (14) are presented in Figure 5. With a discrete distribution of contact pressure values $p_i$, $0 \leq i \leq k_m$,

$$F_z(k_i) = \frac{\Delta_{p_{m+1}}}{\Delta p} \sum_{i=1}^{k_m} \left[ \left( p_{i+1} + p_i \right) \cdot \frac{i - 0.5}{1 - 0.5 + k_a} \cdot F \left( \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{1 - 0.5 + k_a} \right) \right], \quad k_m \leq k_i \leq k_f,$$

where $k_f$ is the limit value $k_f$.

![Figure 5. The dimensionless elastic punching of the material $F_z$ outside the contact area under conditions of slip (a) and stick (b).](image)

To determine values $F_z$ outside the contact area with a uniform distribution of contact pressure, equation (4) was used.

2. Conclusion

An analysis of the results suggests the following:

1. It is shown that when the sphere is flattened, the distribution of contact pressure is a function of complex shape. The knowledge and description of the contact pressure distribution function is of practical importance in addressing the issues of elastic punching of the material and elastic recovery of the contact print during unloading.

2. To describe the load of complex shape, i.e. load, which can be represented by a combination of several equations to describe in adjacent sections. For this purpose, the authors propose the use of an axisymmetric load of the form (2).

3. The method previously proposed by the authors of [22] was developed in determining the elastic forcing of a material during flattening of a sphere and restoration of an contact print during unloading. The key points are expressions (11) and (13) for determining displacements inside and outside the contact area when describing the distribution of contact pressure by analytical functions (10) and equations (15) and (18) when using a discrete distribution of contact pressure.

4. The accuracy of the obtained results depends on the rate of the approximating functions to the actual pressure distribution at the contact area. At the same time, the goals of the problems to be solved should be considered. For example, when determining elastic recovery in the center of the contact area, it is sufficient to use a uniform distribution of contact pressure (Figure 4). When determining the elastic punching of the material, which is important when determining the density of gaps at the junction of rough surfaces [17], it is also sufficient to limit ourselves to a uniform distribution of contact pressure (Figure 5). In determining the profile of the reconstructed indentation and its curvature (Figure 4), increased accuracy of the approximating functions is required.

References

[1] Greenwood JA and Williamson JBP 1966 *Proc. R. Soc.** A295** 300

[2] Demkin NB 1970 *Contacting rough surfaces* (Moscow: Nauka) p 227
[3] Kragelskii IV, Dobychin MN and Kombalov VS 1977 *Basics of friction and wear calculations* (Moscow: Mashinostroenie) p 526

[4] Bhushan B 1998 *Tribology letters* 4(1) p 1

[5] Wang Z, Liu X. Model for Elastic–Plastic Contact Between Rough Surfaces // *ASME: Journal of Tribology*. 2018. V. 140. P. 051402.

[6] Wang D., Zhang Z, Jin F and Fan X 2019 *Acta Mech. Solida Sin.* 32 p 148

[7] Polycarpou AA, and Etsion IA 2000 *Tribology transactions* 43 (2) p 237

[8] Ogar P, Belokobytsky S. and Gorokhov D. 2018 *Contact and Fracture Mechanics* (London: InTechOpen Limited) p 3

[9] Zhang Q, Chen X, Huang Y and Chen Y 2018 *AIP Advances*. 8 p 045310.

[10] Ghaednia H, Jackson RL and Gao J 2014 *IEEE 60th Holm Conference on Electrical Contacts* (Holm) p 1

[11] Jackson RL, Ghaednia H, Elkady YA, Bhavnani SH and Knight RW 2012 *IEEE Transactions on Components, Packaging and Manufacturing Technology* 2(7) p 1158

[12] Jackson RL and Kogut LA 2006 *Journal of tribology* 128 (1) p 209

[13] Ghaednia H, Pope SA, Jackson RL and Marghitu DB 2016 *Tribology International* 93 p 78

[14] Ghaednia H, Wang X, Saha S, Xu Y, Sharma A and Jackson RL 2017 *Applied Mechanics Reviews* 69 p 060804

[15] Ogar P, Gorokhov D, Zhuk A and Kushnarev V 2019 *MATEC Web of Conf.* 298 00093

[16] Ogar PM and Tarasov VA 2013 *Adv. Mat. Research* 664 p 625

[17] Ogar P, Gorokhov D, Mamaev L and Fedorov V 2018 *Adv. in Eng. Research* 158 p 313

[18] Ogar P, Gorokhov D and Ugrumova E 2017 *MATEC Web of Conferences* 129 06016

[19] Chang WR, Etsion I and Bogy DB 1987 *ASME J. Tribol.* 109 p 257

[20] Evseev DG, Medvedev BM and Grigoriyan GG 1991 *Wear* 150 p 79

[21] Brizmer V, Kligerman Y and Etsion I 2006 *Int. J. Solids Struct.* 43:18-19 p 5736

[22] Ogar PM, Ugrumova EV and Gerasimov SV 2020 *Systems, Methods Technologies.* 1 p 7

[23] Ogar P, Gorokhov D, Mamaev L and Kushnarev V 2019 *MATEC Web of Conferences.* 298 00094

[24] Timoshenko SP and Goodyer J 1979 *Theory of Elasticity* (Moscow: Nauka) p 560

[25] Johnson K 1989 *Contact mechanics* (Moscow:Mir) p 510