1. Introduction

The Madden-Julian oscillation (MJO; Madden & Julian, 1971, 1972) phenomenon is well recognized as the dominant mode of intraseasonal variability in the tropics and one of the primary sources of subseasonal predictability (Zhang, 2005, 2013). The MJO is characterized by planetary-scale (wavenumbers 1–3) patterns of convection, circulation, and humidity that coherently propagate eastward within the subseasonal time scale (30–90 days), particularly during the boreal winter season (Adames & Wallace, 2014, 2015; Adames et al., 2016; Lau & Waliser, 2011; Li et al., 2020). Through the local impacts and Rossby waves excited by diabatic heating from intraseasonal convection anomalies, the MJO exerts a significant influence on the global circulation pattern, extreme weather events and even on other climatic modes of variability (Cassou, 2008; Donald et al., 2006; Henderson et al., 2017; Xavier et al., 2014). Therefore, it is important to develop better MJO predictions, as confirmed by the evidence that improved MJO prediction indeed extends the overall predictability of subseasonal variability (Jones et al., 2004; Lin et al., 2009; Vitart & Robertson, 2018; Zhou et al., 2019).

Over the past few years, dynamic models have become the most powerful tool in MJO predictions, attracting a growing research and operational focus (Rashid et al., 2011; Vitart, 2014; Vitart & Molteni, 2010). Owing to the great improvements in initial conditions (Fu et al., 2011; Liu et al., 2017; Wu et al., 2020), ensemble strategies (Green et al., 2017; Ham et al., 2012; Hudson et al., 2013), air-sea coupling (Fu et al., 2013; Zhu et al., 2018) and physical parameter optimization (Hirons et al., 2013; Liu et al., 2019), the MJO prediction skill has been extended to a lead time of 3–4 weeks (Wang et al., 2014, 2019; Wu et al., 2016; Xiang...
et al., 2015), as measured by the real-time multivariate MJO (RMM) index proposed by Wheeler and Hendon (2004, abbreviated WH04 hereafter). However, dynamic models are known for their errors in MJO dynamics, and many of them still fail to simulate the basic thermodynamic structure of the MJO (Jiang et al., 2015; Wang et al., 2017, 2018), which usually lead to a weaker MJO amplitude and unrealistic propagating phase speeds in practical predictions (Kim et al., 2018; Wang et al., 2019).

Since the MJO has a relatively steady oscillation period and some significant precursors (Li, 2014), a number of statistical MJO prediction studies have also been carried out (Kang & Kim, 2010; Zhu et al., 2015). For example, linear inverse modeling (LIM; Penland & Magorian, 1993), involves the least complex form of a reduced stochastic-dynamic climate model, which has been shown to be a predictive tool for MJO with skills up to approximately 2 weeks (Cavanaugh et al., 2015; von Storch & Xu, 1990). However, the MJO prediction skill of empirical statistical methods is hardly comparable to that of the latest dynamic model, which is partly attributed to the limitations of most statistical methods, which mainly capture only linear dynamics (Ren et al., 2015). Moreover, although the dynamic models are highly nonlinear, most of them still have systematic errors in capturing MJO’s fundamentally linear dynamic features (Kim et al., 2018). Therefore, the question is how to effectively combine the dynamic model and empirical statistical methods, especially taking full advantage of the mass hindcast data to further achieve potential improvements for MJO prediction.

In the present study, by utilizing the linear dynamic operators derived with the LIM approach, we developed a method to partly correct the errors in the model’s MJO linear dynamic operators and thus to further correct the MJO predictions of three operational dynamic S2S models. Instead of using LIM for standalone predications, we used the LIM operators for postprocessing error corrections of dynamic model predictions. We demonstrate that our approach will lead to improvements in MJO prediction skill and a gain in skill through the correction of biases in the model’s MJO linear growth rates and propagation speeds.

2. Data and Method

2.1. Observational and S2S Model Data Set

The daily outgoing longwave radiation (OLR) data from the National Oceanic and Atmospheric Administration (NOAA; Liebmann & Smith, 1996) and 850 and 200 hPa zonal wind (U850 and U200) fields from the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) Reanalysis-1 data set (Kalnay et al., 1996) with a horizontal resolution of 2.5° × 2.5° were used for MJO diagnosis and model verification. Considering the diversity of MJO prediction skills of subseasonal to seasonal (S2S) datasets (Lim et al., 2018; Vitart, 2014; Vitart et al., 2017), three representative models were used in the current study: the Beijing Climate Center climate system model version 1.2 (BCC) model from the China Meteorological Administration (CMA), Climate Forecast System version 2 (CFS) from NCEP and the GloSea 5 model from the UK Met Office (UKMO). The ensemble sizes, forecast lead times, reforecast frequencies and covered years of these three models are summarized in Table S1.

Following Lin et al. (2008) and Gottschalck et al. (2010), the seasonal cycles and interannual variabilities in the observed and predicted U200, U850, and OLR fields are removed by subtracting the first three harmonics of the daily climatology and the previous 121-day running mean. Then, a meridional average over 15°S–15°N was applied to retain the longitudinal variations in the three fields, and each was normalized using its observed standard deviation. Finally, the observed and reforecast real-time multivariate MJO (RMM) indices were calculated by projecting the longitudinal intraseasonal variability of U200, U850, and OLR onto the precomputed combined empirical orthogonal function (EOF) modes following WH04.

2.2. LIM and Error Correction

The fundamental assumption of LIM (Newman et al., 2009; Penland & Magorian, 1993) is to approximate the governing dynamics of the resolved system as follows:

\[
\frac{dx}{dt} = Lx + \xi
\]

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where \( x \) represents the system state vector and \( \xi \) is the Gaussian noise forcing vector, which is white in time. The linear operator matrix \( L \) can be determined as follows:

\[
L = \frac{1}{\tau_0} \ln \left[ \mathbf{C}(\tau_0) \mathbf{C}^{-1}(0) \right].
\]

where \( \tau_0 \) is the chosen lag time and \( \mathbf{C}(\tau_0) = \langle x(t+\tau_0) x^T(t) \rangle \) and \( \mathbf{C}(0) = \langle x(t) x^T(t) \rangle \) are the covariance matrices at fixed lag times \( \tau_0 \) and \( 0 \).

After the linear operator matrix \( L \) is obtained, the forecast of \( x \) can be made for an arbitrary lead time \( \tau \) as

\[
x(t + \tau) = \exp(L\tau)x(t) = \mathbf{G}(\tau)x(t)
\]

where the \textit{Green function} \( \mathbf{G}(\tau) = \exp(L\tau) \). For a linear Markov process, the linear operator \( L \) should be independent of lag time \( \tau_0 \) and share common eigenvectors with matrix \( \mathbf{G} \). However, for a real system where nonlinearities affect dynamics, the Green function must be calculated at a variety of lags to estimate the error inherent in the assumption of linear dynamics (Penland & Magorian, 1993). Then, the observed and model-predicted Green functions \( \mathbf{G}_o(\tau) \) and \( \mathbf{G}_m(\tau) \) are calculated by

\[
\mathbf{G}_o(\tau) = \mathbf{C}_o(\tau)^{-1} = \langle x_o(t + \tau) x_o^T(t) \rangle^{-1} \]

\[
\mathbf{G}_m(\tau) = \mathbf{C}_m(\tau)^{-1} = \langle x_m(t + \tau) x_m^T(t) \rangle^{-1}
\]

where \( \tau \) covers the forecast days, taking 1 to 30 in this study, and \( x_m(t + \tau) \) indicates the model forecast data at \( \tau \) days. The subscripts “o” and “m” indicate observed and model-predicted variables, respectively.

From Equations 4 and 5, we can obtain approximate estimates of the observed and model-predicted linear operators. The model predictions can be improved by replacing the predicted linear operators with the observed operator while retaining the model-predicted nonlinear component. The corrected forecast \( \hat{x}_m(\tau) \) can be written as

\[
\hat{x}_m(\tau) = x_m(\tau) - \mathbf{G}_m(\tau)x_o(0) + \mathbf{G}_o(\tau)x_o(0)
\]

Then the standalone linear predictions by the observed and forecasted linear operators are \( \text{LIM}_\text{obs} = \mathbf{G}_o(\tau)x_o(0) \) and \( \text{LIM}_\text{fcs} = \mathbf{G}_m(\tau)x_o(0) \), respectively.

Prior to the dynamic analysis with LIM approach, EOF truncation was first applied to substantially reduce the degrees of freedom of the system so that LIM is actually performed on the leading principal components (PCs) of EOFs of the equatorially averaged (15°S–15°N) U200, U850, and OLR. In this study, the leading 40 modes are retained since the accumulated explained variances reach above 90% for U200 and U850 and 80% for OLR over most regions, which is significantly higher than those at the truncations at 10 or 20 modes (Figure S1). Beyond 40 modes however, further increases of explained variance are small. After applying the corrections of the leading 40 PCs, we can use their corresponding EOF patterns to reconstruct the physical spaces for U200, U850, and OLR, which are further projected onto the first two WH04 modes to obtain RMM index for skill’s verification.

2.3. Validation Methodology

Following Gottschalck et al. (2010), two scalar metrics, the bivariate anomaly correlation coefficient (COR) and bivariate anomaly root mean square error (RMSE), are used to measure the MJO performance. Steiger’s Z-test (Raghunathan et al., 1996) and Student’s t-test are used to decide whether the skill improvements of COR or RMSE from different prediction methods are significant, respectively. In addition, the predicted errors are further divided into amplitude and phase parts to relate them to the model deficiencies in terms of
the MJO linear growth rate and frequency (Rashid et al., 2011). All of the MJO forecast skills are computed using the forecast ensemble mean in this paper.

3. Results

3.1. The In-Sample Skill

All the data of the cover years of BCC (1994–2014), CFS (1999–2010), and UKMO (1993–2015) model are used in this section. As shown in Figure 1, with the criterion of COR exceeding 0.5, the prediction skills are extended from 16 to 20 days, 18 to 20 days, and 24 to 28 days for the BCC, CFS, and UKMO models, respectively, after our linear dynamics-based corrections. The skill improvements generally pass 95% Steiger’s Z-test significance after five forecast days. The skill from standalone LIM predictions from LIM_obs is approximately 15 days, consistent with the findings of a previous study (Cavanaugh et al., 2015), which is superior to those from LIM_fcs (12 days for BCC, 14 days for CFS, and 13 days for UKMO model). In fact, the skills gained in our correction method come from the differences between standalone observation-based and model-based LIM predictions, which identifying how the deficiencies in model linear dynamics affect the linear dependence of the predictions on the initial condition (seen in the last two term in the first line of Equation 6). Moreover, the RMSE metric gives a consistent conclusion (Figure S2). Therefore, the
corrections to the forecasted MJO linear operators are truly helpful in improving MJO prediction skills, and the improvement is larger for the dynamic model with a poorer linear operator (e.g., BCC model).

To further delineate the reasons behind skill improvements from the linear dynamics-based correction, we analyze the drifting of \( L_m \) with forecast lead days (LD) by eigenvalue analysis in terms of MJO decaying (e-folding) time scale (\( \delta \)) and oscillation periods (\( T \)) (see details in Text S1). Because variations exist in reforecast duration and frequency among the BCC, CFS, and UKMO models, their corresponding observed eigenvectors and eigenvalues are slightly different. Nevertheless, the observed MJO e-folding times and oscillation periods are approximately 18 and 50 days (Figure 2a), respectively, which are roughly consistent with those of previous studies (Zhang, 2005). The tracks in the two-dimensional diagram illustrate the e-folding time and oscillation period of each model and their shifts with lead time. For the BCC model, the MJO decay timescale is too short, especially after a lead of 10 days. Its oscillation period is also shorter than that observed in the first few forecast LD, then becomes much longer thereafter. Since the damping rate inferred from LIM after a lead time of 15 days is very strong, it implies that BCC model is poor in capturing MJO as a distinct oscillatory mode. In contrast, this variable drifting is less obvious in the CFS and UKMO models. In the CFS model, the MJO is slightly overdamped and has a slow oscillation with an e-folding time of 16 days and an oscillation period of approximately 50 days, whereas the UKMO model captures the observed oscillation period well, but it is overdamped with a short e-folding time of approximately 10–12 days. In addition, the basic MJO pattern features are generally well captured as the first pair of principle oscillation patterns (POPs; Hasselmann, 1988; von Storch et al., 1995) by each model at the short lead time (LD0-LD5), but spurious smaller scale features appear in the U850 and OLR fields as the lead time increases (LD10-LD20), especially in the BCC model (Figure S3).
These deficiencies of MJO decaying time scale and oscillation periods are clearly reflected in the MJO predictions. As shown in Figures 2b and 2c, the MJO amplitudes were underpredicted in the BCC and UKMO models as both of them had overdamped growth rates; MJO propagation phase speeds were over- and underpredicted in the BCC and CFS models, corresponding to faster and slower periods than those observed. After correction, the MJO characteristics of intensity and propagation speed are accordingly improved for those models. The relatively better prediction for MJO propagation in the UKMO model appears to contribute to its relatively high skill among the three models. These conclusions can also be confirmed by the improved MJO characteristics in Hovmoller diagrams (Figure S4) and composited RMM 2-dimension phase-space diagrams (Figure S5).

3.2. The Out-Sample Skill

To further verify the potential operational utility of our dynamic correction approach, in this section, we generally select the last third of the data periods of BCC, CFS and UKMO models to conduct the out-sample rolling-independent reforecast experiments, which are the years of 2008–2014, 2006–2010, and 2008–2015, respectively. That is, only the data earlier than the target prediction year are taken as the training period for calculating the EOF bases and the LIM operators. Taking BCC model for example, to predict the year of 2008, only the observational and re-forecasted data of 1994–2007 are used as training period, so on and so forth. In terms of COR skill (Figure S6), although the general skill is slightly lower than that of the in-sample experiments (Figure 1), the prediction skill limits are still extended from 15 to 18 days, 18–20 days, and 22–25 days for the BCC, CFS and UKMO models, respectively. It should be noted that the improvements of the BCC and UKMO models are slightly larger than that of the CFS model. This may be attributed to the fact that the CFS model has a more realistic e-folding day and very low amplitude error than the other models (Figure 2) and a shorter training period as the lengths of the training period are important for obtaining accurate and stable linear operators.

The dependences of the MJO prediction skill and its improvement on the MJO initial phases and calendar months are depicted in Figure 3. Generally, the prediction skills have been systematically improved, but their dependences on the MJO initial phases and seasonal cycle are model dependent. For the BCC and CFS models, the skill improvements are significant when the forecast is initiated from Phases 6–8 and 1–3 but from Phases 4–6 in the UKMO model. In addition, the dynamics-based correction is more effective during early spring (February–April) and late summer (July–September) for the BCC model, but during boreal summer (May–September) for the UKMO model. Therefore, the further study that considers seasonal cycle and MJO initial conditions may bring better results. It is noteworthy that MJO forecasts skills of CFS model display degradation during the first few days when applying the LIM dynamical operator (Figures 3b and 3e). One reason is due to the errors of the LIM dynamical operators derived from shorter duration of data for both observation and CFS model, which make the LIM operators subject to significant random errors that could be larger than the intrinsic difference between observed and model's LIM operators. Another reason may come from the “initial shock” of CFS model, as indicated by the large drift of the eigenvalue of the MJO mode of the derived LIM operator in the first few days (Figure 2a), possibly due to model shocks to initializations.

In addition, the skill improvements are also more significant for the predictions initiated from or targeted on strong MJO cases (Figure S7), which are expected since the correction actually depends on the initial conditions, and larger initial amplitudes will bring stronger corrections. Meanwhile, since the correction is performed on the physical space of U200, U850, and OLR, the temporal correlation coefficient (TCC) skills of both variables have also been systematically improved during the lead time of 5–26 days, with larger improvements mainly over the Indian Ocean-west Pacific (IN-WP) sea surface temperature (SST) warm pool for OLR and the border region for U200 and U850 (Figure S8).

The Year of Tropical Convection (YoTC) MJO E case (Waliser et al., 2012) is selected as a special example for out-sample reforecast of multiple 20-day predictions that initialized every 8 days for the three models over the same events (Figure 4). The forecasted MJO in the BCC and UKMO models obviously decays much faster than the observation, whereas the phase propagation speed is slightly slower than the observed speed in the CFS model, which may be inferred from the differences of reached phases between observation and prediction (e.g., the predicted RMM trajectory that initiated from 1 Nov finally reached Phase 3 after 20-day
forecasting, but in observation the RMM trajectory reached Phase 4 at November 21. All of these findings are consistent with the features noted in Figure 2 based on large numbers of cases. The effect on predictions owing to model deficiencies in linear dynamics has been partly overcome by our correction method, as the trajectories after correction are generally somewhat closer to the observed trajectories. These improvements are further quantified by the reduced RMSE of each 20-day prediction case after correction (Figure 4d). Although only limited improvements are achieved for each prediction, this kind of linear dynamics-based correction indeed systematically narrows the gaps of dynamic memories residing in the observation and models.

4. Summary and Discussion

Most S2S models for MJO forecasting have systematic errors in capturing key dynamic MJO features, such as the propagation speed and growth rate, which inevitably weaken their practical MJO prediction skills. In this study, we propose a method to partly correct the errors of MJO linear dynamics by using LIM to improve the practical MJO prediction of three dynamic S2S models (BCC, CFS and UKMO). The prediction skills are enhanced by approximately 4, 2, and 4 days and reach 20, 20, and 28 days for the BCC, CFS, and UKMO models, respectively, after our dynamics-based corrections. The improvements mainly originated from overcoming the bias in predicting the MJO linear growth rate and propagation speed. With our corrections, the prediction skills for the U200, U850, and OLR fields all have improvements, in particular the convective activity over the IN-WP warm pool region. Details of improvements depend on strengths, phases of MJO cases and the seasonal cycle and are model dependent. A prediction case study for the YoTC MJO E events showed consistent results as well.

Our linear dynamics-based correction approach for comprehensive dynamic model prediction takes advantage of the superiority of LIM linear dynamics and the model's ability to capture nonlinear dynamics. Moreover, since the three models have very different errors in their predictions, the multi-model ensemble based on the corrected forecasts can produce even higher skillful forecasts and further extend the MJO skill.
limit up to 29 days for in-sample test (Figure S9), suggesting that our approach shall be applicable to multi-model ensemble forecasting system as well. Additionally, our approach has potential room for improving the predictions of other types of climate variability in which linear dynamics are important, such as ENSO. More importantly, the nonlinear prediction error, which is viewed as a noise component in the LIM approach, may be objectively isolated, and further corrections through other nonlinear methods (Kondrashov et al., 2013) or deep learning (Ham et al., 2019) may bring further significant improvements.
Data Availability Statement

The atmospheric circulation data came from the NCEP/NCAR Reanalysis-1 data set (https://psl.noaa.gov/data/gridded/data.ncep.reanalysis.pressure.html). The NOAA OLR is available at https://www.esrl.noaa.gov/psd/data/gridded/data.interp_ORL.html. The reforecast data of the S2S project are downloaded from the archiving center of CMA (http://s2s.cma.cn/centers). The empirical orthogonal function (EOF) modes for calculating the RMM index are available from http://cawcr.gov.au/staff/mwheeler/maproom/RMM/eofcode.htm.

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