Research Article

Belt Conveyor Dynamic Characteristics and Influential Factors

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Received 13 October 2017; Accepted 30 January 2018; Published 30 April 2018

Academic Editor: Francesco Pellicano

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This paper uses the Kelvin-Voigt viscoelastic model to establish the continuous dynamic equations for tail hammer tension belt conveyors. The viscoelastic continuity equations are solved using the generalized coordinate method. We analyze various factors influencing longitudinal vibration of the belt conveyor by simulation and propose a control strategy to limit the vibration. The proposed approach and control strategy were verified by several experimental researches and cases. The proposed approach provides improved accuracy for dynamic design of belt conveyors.

1. Introduction

Belt conveyors are an integrated transmission and carrying mechanism with length sometimes extending several thousand meters. In traditional design and analysis of belt conveyors, vibration and impact are usually ignored and only static design is considered. However, to ensure the safety of conveyor operation for this restricted analysis, designers must increase the safety factor, which increases production costs.

Many research groups have conducted dynamic analysis of large belt conveyors to reduce production cost and optimize conveyor performance [1, 2]. The conveyor belt was first modeled as an elastic body and then a viscoelastic body to incorporate viscoelastic characteristics of belt cover layer.

In the early 1960s, the former Soviet Union began to study conveyor dynamics. However, due to the limited science and technology at that time, the starting characteristics for constant acceleration or AC motors were studied by using impulse principles and stress wave propagation in the conveyor belt on the basis of a simplified mechanic model [3]. A series of later studies at Hannover University of Technology in Germany established the traveling wave theory. Further studies investigated conveyor dynamic characteristics [4, 5].

Harrison, Robert, and James amongst others investigated starting and braking characteristics of steel wire rope core conveyors and lateral bending vibration of the conveyor belt. They also analyzed stress wave propagation speed in the conveyor belt based on the theory of elastic and stress waves and developed various relevant models [6–8].

Computer simulation of belt conveyor dynamic characteristics was presented by the Taiyuan University of Science and Technology using a Kelvin viscoelastic model, and belt conveyor stability was analyzed in terms of the transverse vibration [9]. Transverse vibration was also studied at Xi'an University of Science and Technology and Shandong University of Science and Technology, obtaining the relationship between transverse vibration and speed and belt conveyor tension to provide a theoretical basis belt conveyor development [10, 11].

Starting and braking curves, horizontal turning, and broken band detection of belt conveyors were studied in the Liaoning Technical University using finite element analysis [12, 13]. A model of the whole belt conveyor was established using the discrete dynamic method and dynamic simulation software was constructed at Northeastern University. The conveyor dynamic characteristics under different boundary conditions were studied experimentally, and the conveyor dynamic tension under different conditions was simulated [14, 15].
Modern conveyor design methods were proposed on the basis of analysis of the vibration characteristics by the Shanghai Jiao Tong University and Shanghai Normal University [16–18]. However, previous and current researches have mainly focused on discrete conveyor models. Although viscoelasticity has been considered, it has usually been simplified as an elastic model, and the actual conveyor dynamic model has rarely been considered. The current paper establishes the dynamic equation for tail hammer tension belt conveyor model based on the Kelvin-Voigt viscoelastic model, adopting the generalized coordinate method to solve the viscoelastic continuity equation. Conveyor natural frequency and longitudinal vibration characteristics are analyzed to provide a more accurate method for conveyor dynamic design.

2. Dynamic Equation for Belt Conveyors

The Kelvin-Voigt model comprises a linear spring and damper in parallel. The model is applicable to simulate the stress response of viscoelastic material. However, when \( \frac{de}{dt} = 0 \) (\( e \) is total strain; \( t \) is time), the model can be simplified to the elastomer constitutive relationship, as shown in Figure 1. Strains \( (\epsilon_1, \epsilon_2) \) on the two elements of the model are the same, and the stress \( (\sigma) \) is the sum of the spring \( (\sigma_1) \) and damper stresses \( (\sigma_2) \):

\[ \sigma = \sigma_1 + \sigma_2 = E\epsilon_1 + \theta \frac{d\epsilon_2}{dt}, \]

where

\[ \epsilon = \epsilon_1 = \epsilon_2. \]

\( E \) is elastic modulus of conveyor belt and \( \theta \) is viscosity coefficient.

Solving (1) and (2),

\[ \sigma = E \left(1 + \mu \frac{d}{dt}\right)\epsilon, \]

which is the constitutive differential equation for the Kelvin-Voigt model, reflecting creep behavior of viscoelastic materials.

This paper considers the belt conveyor as a continuous body, and the conveyor dynamic behavior is described by partial differential equations. Figure 2 shows a typical tail hammer tensioning belt conveyor. The following assumptions are made for establishing the conveyor mathematical model [19, 20].

1. Compared to longitudinal vibration of conveyor belt, the effect of transverse vibration is negligible.
2. Conveyor belt shearing and bending stress is negligible.
3. Conveyor belt length change caused by vertical variation is negligible.
4. Transverse deformation caused by conveyor belt longitudinal tension is negligible.

When starting a belt conveyor, besides being subjected to static tension, the conveyor belt is also subjected to dynamic tension, which is influenced by the conveyor speed change. Suppose that, at time \( t \), the displacement of some point from the origin is \( U(x, t) \). The displacement includes static displacement \( u(x, t) \) caused by the conveyor belt weight and hammer and dynamic displacement \( u(x, t) \) arising from dynamic tension. If the conveyor rigid body translational acceleration is \( a(t) \), then the conveyor absolute acceleration is \( \frac{d^2 u}{dt^2} + a(t) \).

From (3), the tension of the conveyor belt at some point \( x \) is

\[ S(x, t) = EB \left( \frac{\partial U}{\partial x} + \mu \frac{\partial^2 U}{\partial t \partial x} \right), \]

where \( B \) is the belt width.

Ignoring conveyor running resistance, the dynamic equation for conveyor longitudinal vibration can be expressed as

\[ qdx \left( \frac{\partial^2 U}{\partial t^2} + a(t) \right) = \frac{\partial S}{\partial x} dx + qg dx, \]

where \( q, q_1 = q_{RO} + q_{B} + q_G \) and \( q_2 = q_{RU} + q_{B} \) are the conveyor belt equivalent mass per unit length in general and for the carrying and return branches, respectively; \( q_{RU} \) is the mass per unit length of the rotary part of the supporting roller of the return branch; \( q_{RO} \) is the mass per unit length of the rotary part of the supporting roller of the carrying branch; \( q_{B} \) is the mass per unit length of conveyor belt; \( q_G \) is the material mass per unit length of conveyor belt; \( g \) is gravitational acceleration.

Substituting (4) into (5) and introducing wave velocity parameter \( c \),

\[ c^2 = \frac{EB}{q}. \]
Considering belt conveyor dynamic characteristics, the differential equation for viscoelastic longitudinal vibration, neglecting static displacement, is [21]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left( 1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial^2 u}{\partial x^2} - a(t),$$  \hspace{1cm} (7)

where $a(t)$ is the rigid body acceleration of conveyor belt.

When $\mu = 0$, (7) can be simplified to the partial differential equation of the elastic rod, that is, the one-dimensional wave equation. The boundary conditions for (7) are

$$u(0, t) = 0$$
$$\frac{\partial u}{\partial x}(l, t) = \frac{Mg}{2EB},$$  \hspace{1cm} (8)

where $M$ is the weight of heavy hammer. And the initial conditions are

$$u(x, 0) = 0$$
$$\frac{\partial u(x, 0)}{\partial t} = 0.$$  \hspace{1cm} (9)

3. Kinetic Equation

3.1. Obtaining Homogeneous Boundary Conditions. Equation (7) is a nonhomogeneous partial differential equation. To obtain a homogeneous equation, the boundary conditions must be homogenized. Suppose

$$u(x, t) = H(x, t) + W(x, t).$$  \hspace{1cm} (10)

Then an appropriate choice for $W(x, t)$ can make the boundary condition of $H(x, t)$ homogeneous:

$$\frac{\partial W(L, t)}{\partial x} = \frac{Mg}{2EB},$$  \hspace{1cm} (11)

$$W(0, t) = 0,$$

and to meet the boundary condition requirements,

$$W(x, t) = \frac{Mg}{2EB}(x - l),$$  \hspace{1cm} (12)

$$u(x, t) = H(x, t) + \frac{Mg}{2EB}(x - l).$$  \hspace{1cm} (13)

Substituting (13) into (9),

$$H(x, 0) = \frac{Mg}{2EB}(x - l),$$  \hspace{1cm} (14)

$$\frac{\partial H(x, t)}{\partial t} = 0,$$  \hspace{1cm} (15)

and solving the partial derivative of $x$ and $t$ from (15),

$$\frac{\partial^2 H}{\partial t^2} = c^2 \left( 1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial^2 H}{\partial x^2} - a(t).$$  \hspace{1cm} (16)

Thus, the homogeneous boundary conditions are

$$\frac{\partial H(0, t)}{\partial x} = 0$$
$$H(l, t) = 0,$$

$$H(x, 0) = \frac{Mg}{2EB}(x - l),$$
$$\frac{\partial H(x, 0)}{\partial t} = 0.$$  \hspace{1cm} (17)

3.2. Solution of Nonhomogeneous Equation. Equation (16) is equivalent to the forced vibration of a single degree of freedom, and its solution is divided into homogeneous and special solutions. Suppose

$$H(x, t) = h(x, t) + w(x, t),$$  \hspace{1cm} (18)

where $h(x, t)$ and $w(x, t)$ are the special and homogeneous solutions, respectively. Then (16) can be separated into a homogeneous solution,

$$\frac{\partial^2 w}{\partial t^2} = c^2 \left( 1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial^2 w}{\partial x^2},$$

$$\frac{\partial w(0, t)}{\partial x} = 0,$$

$$w(l, t) = 0,$$

$$w(x, 0) = -\frac{Mg}{2EB}(x - l),$$

$$\frac{\partial w(x, 0)}{\partial t} = 0.$$  \hspace{1cm} (19)
and special solution,
\[ \frac{\partial^2 h}{\partial t^2} = \omega^2 \left( 1 + \mu \frac{\partial}{\partial t} \right) \frac{\partial^2 h}{\partial x^2} - a(t), \]
\[ \frac{\partial h(0,t)}{\partial x} = 0, \]
\[ h(l,t) = 0, \]
\[ h(x,0) = 0, \]
\[ \frac{\partial h(x,0)}{\partial t} = 0. \]  

Using the method of separation of variables, (18) is used to determine the boundary conditions.

Suppose \( \eta = \lambda L; \) then
\[ \alpha = \eta \tan \eta, \]  
where \( \alpha \) is the ratio of the equivalent mass per unit length of the carrying or return branch to half the mass of the heavy hammer; that is, \( \alpha = 2ql/M. \) Equation (22) is a transcendental equation with infinitely many solutions, whose shape function can be expressed as
\[ X_m = \sin \lambda_m x = \sin \frac{\eta_m}{L} x, \]  
\[ h(x,t) = \sum_{m=1}^{\infty} q_m(t) \sin \frac{\eta_m x}{l}. \]  

where (24) satisfies
\[ \frac{\partial h(0,t)}{\partial x} = 0, \]
\[ h(l,t) = 0. \]  

Let \( h(x,t) = 0 \) and substitute (24) into (21); then differentiate, multiply by \( \sin(\eta_m l) \) on both sides, and integrate:
\[ \sum_{m=1}^{\infty} \left[ q_m'' + \alpha^2 \mu \lambda^2 q_m' + \alpha^2 \lambda^2 q_m \right] \]
\[ + \int_0^l \sin \left( \frac{\eta_m x}{l} \right) \sin \left( \frac{\eta_n x}{l} \right) dx = a(t) \]  
\[ \int_0^l \sin \left( \frac{\eta_m x}{l} \right) dx. \]  

By orthogonality, if \( n \neq m, \)
\[ \int_0^l \sin' \left( \frac{\eta_n x}{l} \right) \sin' \left( \frac{\eta_m x}{l} \right) = 0, \]
and (26) can be simplified to
\[ q_m'' + \alpha^2 \mu \lambda^2 q_m' + \alpha^2 \lambda^2 q_m = 2\lambda M a(t), \]  
which has an infinite number of mutually independent modal equations, and its form is similar to that of damped forced vibration of single degree of freedom. Therefore, it can be solved by the method of single degree of freedom, with general solution:
\[ q_m = \frac{e^{-\xi \omega_m t}}{\omega_d} \int_0^l 2\lambda M e^{\xi \omega_m t} a(t) \sin \omega_d(t - \tau) t \, d\tau, \]  
(29)

where \( \omega_m = \eta_m / l \) is the conveyor natural vibration frequency, \( \xi = \mu \omega_m / 2 \) is the conveyor damping coefficient, and \( \omega_d = \omega_m \sqrt{1 - \xi^2} \) is the conveyor damped natural frequency.

The subscript \( m \) for \( \omega_m \) means that when \( m \) is a natural number, it gets a corresponding natural frequency, all of which are the natural frequencies. From (29), the conveyor natural frequency is not unique but has unlimited discrete values, and there is no natural frequency between any two discrete values. When \( m = 1 \), the natural frequency is the fundamental frequency. Vibration at the fundamental frequency tends to dominate the free and forced vibration of the system. When the conveyor starts, the fundamental frequency is the earliest resonating frequency and the first frequency to avoid or break through when operating. Thus, identifying the fundamental frequency is the top priority.

The dynamic equation for conveyor longitudinal vibration can be expressed as:
\[ u(x,t) = \frac{ql^2}{EB} \sum_{m=1}^{\infty} \sin \lambda_m x \int_0^l e^{-\xi \omega_m (t-\tau)} a(\tau) \sin \omega_d(t - \tau) \, d\tau. \]  

Substituting (30) into (4), the analytical solution for the dynamic tension of the conveyor’s longitudinal vibration is
\[ s(x,t) = \frac{ql^2}{EB} \sum_{m=1}^{\infty} \cos \lambda_m x \int_0^l e^{-\xi \omega_m (t-\tau)} a(\tau) \sin \omega_d(t - \tau) \, d\tau. \]

From (30) and (31), maximum displacement occurs at the pulley-tension position, and maximum dynamic tension occurs at the trend point of the pulley-driving. These factors also affect conveyor vibration characteristics, such as carrying capacity, length, tension, acceleration, and damping coefficient.

### 4. Impact of Various Factors on Vibration

#### 4.1. Viscous Damping Effect

4.1.1. Acceleration Time Response. Assuming that the belt conveyor adopts rectangular acceleration to start, \( a(t) = a_m. \) This is equivalent to a stepped signal input, and time response characteristics can be obtained by substituting into (31),
\[ A(t) = a_m \left[ 1 - \frac{e^{-\xi \omega_d t}}{\sqrt{1 - \xi^2}} \cos (\omega_d t - \phi) \right], \]  
(32)

where \( \phi = \tan^{-1} \xi / \sqrt{1 - \xi^2}. \)
Figure 3 shows the time response characteristics from (32), when $0 < \xi < 1$, and the following conclusions can be drawn from Figure 3 and (32) and (29):

1. The longitudinal vibration dynamical equation under step input is typical of second-order oscillation, where the natural and damping frequencies are functions of the load ratio.

2. Underdamped vibration occurs as the damping coefficient changes from 0 to 1. As damping increases, maximum overshoot and transient time decrease, peak time increases, and stability improves.

3. From the performance index of second-order systems, maximum system overshoot is \[ M_p = \left( \frac{1}{\sqrt{1-\xi^2}} \right) e^{-\left(\frac{(\pi+4\xi)/\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}}\right)} \]. Table 1 shows overshoot for different damping coefficients.

Table 1 shows that maximum overshoot decreases and system stability improves with increasing $\xi$. When $\xi \geq 0.4$, overshoot is <5%. Thus, the system has good stability when $\xi \geq 0.4$.

4.1.2. Dynamic Tension Characteristics. Using (32) and (31), we set $q = 216.7 \text{ kg/m}$, $l = 845 \text{ m}$, $\eta_1 = 1.52$, $\omega_1 = 2.333 \text{ Hz}$, and $\xi = 0.2$, 0.6, and 0.8. Figure 4 shows dynamic tensions at carrying branches and the meeting point of the drive drum and belt conveyor under heavy load.

Maximum dynamic tension in the conveyor belt occurs at the drive drum and gradually reduces away from the drum. Dynamic tension fluctuation amplitude decreases with increasing $\xi$, which is consistent with the time response characteristics and indicates that viscous damping increases the belt dissipation capacity. As $\xi$ increases, energy dissipation increases and stability improves. However, dynamic tension on the conveyor belt increases with increasing $\xi$, and smaller dynamic tension is better for belt conveyor design. Therefore, smaller damping is a desirable design criterion. Combining time response characteristics, $\xi = 0.4–0.6$ produces less dynamic tension and improved stability.

4.2. Tension Effects. Using the heavy hammer tension belt conveyor as an example, the equivalent hammer weight varies with the level of tension. From $\alpha = 2q/M$, the corresponding natural frequencies are 2, 2.13, and 2.19 Hz, while the load ratios between the carrying branches and hammer at full load are 5, 7.5, and 10. Figure 5 shows conveyor dynamic tension under different tensions when $\xi = 0.4$.

Conveyor belt tension decreases with increasing tension. The maximum dynamic tension at the meeting point of the drive drum was 119.7, 113.0, and 108.4 kN, respectively. Consequently, to ensure normal operation, increasing tension can effectively reduce the conveyor belt dynamic tension.

4.3. Transport Capacity Effects. Transport capacity is the major factor that determines the required belt conveyor power. Generally, the designer determines the required power based on full load or the harshest conditions. Under normal
operating conditions, the belt conveyor dynamic characteristics will usually be significantly different for different loadings. Figure 6 shows dynamic tension response for no, half, and full load conditions.

Dynamic tension at the meeting point of the drive drum under no, half, and full load conditions was 47.95, 76.29, and 98.35 kN, respectively. Thus, dynamic tension at the meeting point of the drive drum increases with increasing transport capacity, but the relationship is nonlinear. Because the natural vibration frequencies of the belt conveyor are different for different loads, dynamic tension under a given load cannot be deduced directly from dynamic tension under a different load.

### 4.4. Tension Method Effects.

There are three major belt tension methods: heavy hammer, constant force, and fixed winch. Figure 7 shows the dynamic tension for these methods when starting with constant acceleration. Maximum dynamic tension at the meeting point of the drive drum was 98.35, 100.9, and 107.5 kN, respectively. Considering the peak starting dynamic tension, heavy hammer tension is superior to the methods; fixed winch tension produces the maximum dynamic tension, whereas constant automatic tension is very similar to the heavy hammer method. This is consistent, since the constant force tension method was developed to simulate and replace heavy hammer tension.

### 4.5. Belt Velocity Effects.

Figure 8 shows dynamic tension for different belt velocities, \( v = 2, 3, \) and \( 4 \) m/s, with constant acceleration starting. Maximum dynamic tension increases with increasing \( v \), and maximum dynamic tensions at the meeting point of drive drum were 43.71, 65.57, and 106.2 kN. Therefore, belt conveyor dynamic tension increases with increasing belt velocity.

### 5. Simulations

The dynamic acceleration and tension of the belt conveyor system were simulated for large capacity, high speed, and long distance transport. Table 2 shows the basic parameters for the chosen conveyor system, comprising steel cord ST-5000.

| Parameter                      | Symbol and value |
|--------------------------------|------------------|
| Carrying capacity             | \( Q = 2000 \) t/h |
| Haul distance                 | \( L = 500 \) m |
| Belt width                     | \( B = 1.2 \) m |
| Stable operation speed         | \( V_0 = 3.15 \) m/s |
| Weight of carrying idlers      | \( G_1 = 66.0 \) kg |
| Mounting distance              | \( l_1 = 3 \) m |
| Weight of return idlers        | \( G_2 = 37.0 \) kg |
| Mounting distance              | \( l_2 = 3 \) m |
| Linear mass per unit length    | \( q_B = 62.4 \) kg/m |
| Cross-sectional area           | \( A = 0.024 \times 1.2 = 0.0288 \) \( \text{m}^2 \) |
| Elasticity modulus             | \( E = 65 \times 5000 \times 103/0.024 = 13.54 \times 10^9 \) Pa |
| Rheological constant           | \( \mu = 0.015 \) |
| Initial tension                | \( F = 450 \) kN |
conveyor belt, head drive, hydraulic tail, and constant force automatic tension.

5.1. Simulation for Three Starting Regimes. Let $\beta = T/T_1$, where $T_1 = 2\pi/\omega_1$ is the fundamental vibration period of the conveyor belt. From (32), if $\omega_1 = 0.7 \text{ rad/s}$, then $T_1 = 8.9$ s. Figures 9 and 10 show dynamic acceleration for three starting regimes with $\beta = 3, 5, 10, 15, 20,$ and $25$ and different starting time $T$ for the system ($x = 0.5L$).

The different starting regimes become more similar and acceleration maximum acceleration reduces as time increases. Dynamic tension fluctuation also converges and reduces with increasing time.

The parabolic starting regime and increased starting time to $15–20 \times$ fundamental vibration period effectively reduced fluctuation and maximum dynamic acceleration and tension, effectively limiting longitudinal vibration shock across the whole system.

5.2. Dynamic Simulation of the Parabolic Starting Regime. As shown in Figures 11 and 12, the simulation responses of the belt conveyor to dynamic acceleration and tension versus time and displacement were obtained with parabolic acceleration.

When starting time $T = 80$ s, although conveyor belt acceleration fluctuations are significant at different displacements, maximum acceleration (Figure 11) and dynamic tension (Figure 12) are relatively stable. However, maximum dynamic tension changes significantly with displacement. Acceleration fluctuation at $x = 0$ is very small, whereas it is very large at $x = L$. Maximum dynamic acceleration is relatively stable, while changes and maximum dynamic tension at $x = 0$ and $x = L$ show opposite trend to dynamic acceleration.

Acceleration of the carrying segment for the whole system is increasing, moving from the head drive drum to tail tensioning, but the dynamic tension of the bearing segment decreases during the process of closing the tail. Therefore, in addition to choosing the starting regime and controlling starting time, further strategies were required to limit dynamic tension and acceleration volatility and maximum. The auxiliary equipment can be used in the middle part of the system, such as driving the middle linear friction wheel, arranging rational position and spacing and number of idlers, and choosing appropriate drums, to further enhance control of longitudinal vibration shocks for the overall system.

6. Experiments

The belt conveyor simulation, run at 2 m/s, starts for 10–15 × fundamental period. Using a parabolic starting regime, dynamic experimental data of tension and acceleration were collected, for the physical conveyor system described in Table 3.

6.1. Acceleration Measurement. Figure 13 shows the hall sensor used to measure the belt running speed. The feedback pulse signal was collected by using a data acquisition system to obtain dynamic speed measurements. If $m_1$ feedback pulses are received from the hall sensor in $T_1$ seconds, $p$ is the number of pulses for each roller rotations, and $D$ is the roller diameter; then the speed of rotation is

$$n = \frac{60m_1}{\beta T_1 c},$$  \hspace{1cm} (33)

and the belt dynamic acceleration is

$$a = \frac{2\pi m_1 D}{p T_1 ^2 c}.$$  \hspace{1cm} (34)

6.2. Tension Measurement. Figure 14 shows the tension testing device, composed of a tension sensor and rollers. Dynamic tension can be derived from the detected pressure device geometry. If $\theta$ is the surrounding angle of the belt and roller in the device, $F_N$ is the measured pressure, and $G$ is the gravity force of the rollers; then the belt tension can be expressed as

$$S = \frac{1}{2} (F_N - G) \csc \theta.$$  \hspace{1cm} (35)

6.3. Simulation Site. To collect suitable data during the experiment, the dynamic testing device was installed close to the drive drum of the tail. Yet testing points of dynamic tension are close to the head and drum of the tail; the devices in the dynamic simulation experiment of acceleration and tension are shown in Figures 15 and 16.

6.4. Experimental Results. Figures 17 and 18 show the measured belt acceleration and tension at the tail, starting for 10 × fundamental period, and Figure 19 shows the dynamic tension at the head.

The tension cylinder oil inlet of the hydraulic automatic constant tension device starts to supply oil to the former at 5 s,
Figure 10
which makes belt tension approximately linearly increase to operationally required initial tension (6 kN). In the following 5–31 s, the belt starts to move, and the system starts, reaching the normal running speed (2 m/s). When the system is running smoothly, tension is small, with the automatic tension cylinder providing a relatively stable 4.3 kN. Belt tension at the tail is relatively stable, due to dynamic buffering of the hydraulic automatic constant tension device.
Figures 20 and 21 show the measured belt acceleration and tension at the tail, starting for $15 \times$ fundamental period, and Figure 22 shows the dynamic tension at the head.

When belt conveyor starts for $10 \times$ fundamental period, peak belt acceleration at the tail and tension at the head are 0.018 m/s$^2$ and 6.29 kN, respectively. When the belt conveyor starts for $15 \times$ fundamental period, peak belt acceleration at the tail and tension at the head are 0.075 m/s$^2$ and 6.079 kN, respectively. Thus, tension and acceleration are smaller when starting for $10 \times$ fundamental period than 15, and starting is more stable.

7. Conclusions

We can conclude the following:
A general solution was presented to simplify the viscoelastic rod system into partial differential continuous longitudinal vibration equations for longitudinal vibration. Also, a general theoretical approach was provided to analyze similar systems.

Based on the longitudinal vibration dynamic equations, belt conveyor time response characteristics were analyzed under different starting control strategies. It was shown that the difference between maximum acceleration and maximum acceleration response was less than 5% when starting time was more than $15 \times$ fundamental vibration period, and it can effectively reduce longitudinal vibration.

Simulation results were experimentally verified. The effects on belt conveyor vibration characteristics from damping, tension, carrying capacity, tension mode, and belt speed were investigated and provide a theoretical basis for design and manufacture of belt conveyors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The research was supported by the New Century Excellent Talents (Grant no. NECT-12-1038), NSFC-Shanxi Coal Based Low Carbon Joint Fund Focused on Supporting Project (Grant no. U1510205), and Science and Technology Project of Shanxi Province (Grant no. 2015031006-2). The authors gratefully acknowledge the helpful discussions with the research group and colleagues of Taiyuan University of technology.

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