Simulation of take-off angle of a ski jump energy dissipater

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Abstract. Ski jumps has been widely used in water conservancy and hydropower engineering with its simple structure and high energy dissipation. The jet trajectory is a key factor to directly determine the scour location, which are the significant index of hydraulic design. Actually, the take-off angle of jet flow is different from the geometrical take-off angle. Reasonable take-off angle would be quite important for calculation method of horizontal length of jet flow. The object of this work is based on the research of three take-off angels of upper and lower jet trajectories with different outlets (geometrical take-off angle, virtual take-off angle, and computational take-off angle), and provides actual calculation method for horizontal length of ski jumps. On the basis of theoretical analysis and numerical simulation, the main contents and methods of this research report includes: 1) analyzing affective analysis of virtual take-off angles of both the lower and upper jet trajectories, establishing virtual take-off angles of both the lower and upper jet trajectories equations with hydraulic and structure parameters and etc; 2) analyzing by comparing virtual take-off angles of both the lower and upper jet trajectories with different outlets of flip bucket (including circular, triangle, etc.); 3) providing horizontal length of jet flow of ski jumps calculation method with take-off angles of upper and lower jet trajectories.

1. Introduction

Compared with other energy-dissipation methods, ski jump energy dissipation has been adopted by many large and medium-sized hydraulic engineering projects at home and abroad because of its simple structure, high energy dissipation rates, economy and practicality, convenient construction and simple hydraulic calculation. While improving the energy dissipation rate, it can effectively avoid some hazards caused by high speed flow [1]. The jet trajectories of the upper and lower jet of the ski jump energy dissipator is directly related to the position of the impact point downstream of the nappe and
the range of the downstream scour, which is an important hydraulic parameter for the design of the ski jump energy dissipator. For many years, the calculation formulas for the pitch of the theoretical formulas have mostly been based on the theory of the projectile for a rigid body. From these formulas, it can be seen that the jet trajectory is closely related to the take-off angle.

For a circular-shaped bucket, under certain inflow and hydraulic conditions, the actual take-off angles of the upper and lower jet trajectories are smaller than the bucket deflector, and the actual take-off angle of the lower is smaller than the actual take-off angle of the upper. Therefore, in the hydraulic calculation of the ski jump energy dissipation, the research on the influencing factors of the actual take-off angles and its different types is of great significance for the determination of the jet trajectory of the upper and lower jet of the ski jump energy dissipator, and it is worthy of further study. On the basis of the traditional circular-shaped bucket, Steiner et al. proposed a triangular-shaped bucket, which has a simple structure and can be used as a two-dimensional structure based on the research of three-dimensional flip bucket. The authors have made some research on the hydraulic characteristics such as pressure distribution, jet trajectory, jet impact, energy dissipation rate and flow obstruction, which are concerned about engineering application. It is pointed out that the triangular-shaped bucket has good hydraulic characteristics compared with the circular-shaped bucket.

Specific research on the take-off angle of the jet is rare in the already-published literature in China. Based on the results of previous physical model tests, this paper uses numerical simulation methods to study the three types of take-off angle of circular-shaped bucket and triangular-shaped bucket (geometrical take-off angle, virtual take-off angle, and computational take-off angle. We propose a calculation method for calculating the jet trajectory of the ski jump energy dissipater.

2. Numerical model
2.1. Turbulence models
In the standard k~ε model, the viscosity coefficient of the Reynolds stress component is assumed to be an isotropic scalar. In the case of a curved flow, however, the turbulence should be an anisotropic tensor, so distortion may occur when the model is applied to strong swirls, curved wall flows, and curved streamlines. To solve this problem, relevant correction is proposed based on the standard k~ε model -- the current RNG k~ε two-equation model. Fluid flow follows the laws of conservation and turbulent transport equations. The calculation in this paper do not consider heat exchange, so the energy equation is not included and we only calculate the flow field. The numerical simulation chooses the RNG k~ε process and the RNG k~ε turbulence mathematical model of the three-dimensional flow on the flip bucket is established. The important basic equations include continuous equation, momentum equation, turbulent energy k equation and turbulent energy consumption rate ε equation, which are as follows:

\[
\frac{\partial (\rho A)}{\partial t} + \frac{\partial (\rho v A)}{\partial x} + \frac{\partial (\rho w A)}{\partial z} = 0
\]

\[
\frac{\partial u}{\partial t} + \frac{1}{V_F} \left( \frac{\partial (\mu A)}{\partial x} + \frac{\partial (\mu A)}{\partial y} + \frac{\partial (\mu A)}{\partial z} \right)
\]

\[
= -\frac{1}{\rho} \frac{\partial p}{\partial x} + G_x + f_x + \frac{1}{V_F} \left( \frac{\partial (\mu A)}{\partial x} + \frac{\partial (\mu A)}{\partial y} + \frac{\partial (\mu A)}{\partial z} \right)
\]

\[
= -\frac{1}{\rho} \frac{\partial p}{\partial y} + G_y + f_y + \frac{1}{V_F} \left( \frac{\partial (\mu A)}{\partial x} + \frac{\partial (\mu A)}{\partial y} + \frac{\partial (\mu A)}{\partial z} \right)
\]

\[
= -\frac{1}{\rho} \frac{\partial p}{\partial z} + G_z + f_z
\]

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k u)}{\partial x} = \frac{\partial}{\partial x} \left[ \frac{\partial (\alpha_k (\mu + \mu_t) \frac{\partial k}{\partial x})}{\partial x} \right] + G_k - \rho \varepsilon
\]

\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho k u)}{\partial x} = \frac{\partial}{\partial x} \left[ \frac{\partial (\alpha_k \mu \frac{\partial \varepsilon}{\partial x})}{\partial x} \right] + C_1 \frac{\partial \varepsilon}{\partial x} \frac{\partial k}{\partial x} - C_2 \frac{\varepsilon^2}{k}
\]
2.2. Aeration models
In a flow with high air concentrations, the aeration model fully considers the increase of the water volume due to air entrainment and the fact that the air in water leaves the water body due to buoyancy. The aeration principle can be briefly summarized as the combination of mass force, surface tension and turbulent shear force. In view of the strong turbulence inside the flow, the widely-used and more accurate RNG k–ε turbulence model is employed in the simulation calculation. The concept of the air entrainment model is to consider the effect of air at the free surface, that is, a continuous gas dispersed in a continuous phase. The entrainment of air at the liquid surface is caused by the strong development of the turbulence of the flow, before the vortex in the water body rises and the small vortex jumps away from the free surface, dropping a certain amount of air when it falls into the water body from the suspended state. Only under sufficient turbulence intensity can the small vortex overcome the mass force and the surface tension leap, resulting in aeration.

2.3. Test program
As shown in Fig. 1, the numerical simulation considers two forms of flip bucket (triangles and arcs), taking into account eleven different geometrical take-off angles $\alpha$ (10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40). The depth of the incoming flow is $h_0=0.05\text{m}$, and by changing the Froude number, 88 sets of numerical simulation were performed (Table 1). Upper and lower jet trajectories were obtained through numerical simulation (Fig. 2 is a circular-shaped bucket with $\alpha=16^{\circ}$ and $Fr=4$).

![Figure 1. Definition sketch for plane ski jump flow](image)

| Deflector type         | Deflection angle $\alpha$ ($^{\circ}$) | Deflection radius $R$ (m) | Deflection length $t$ (m) | Bucket height $s$ (m) | Froude number $Fr$ | Number |
|------------------------|----------------------------------------|---------------------------|---------------------------|-----------------------|-------------------|--------|
| Circular-shaped bucket | 10                                     | 0.5                       | 0.087                     | 0.0076                | 1~9               | 9      |
|                        | 13                                     | 0.5                       | 0.112                     | 0.0128                | 9                 | 1      |
|                        | 16                                     | 0.5                       | 0.138                     | 0.0194                | 1~9               | 9      |
|                        | 19                                     | 0.5                       | 0.163                     | 0.0272                | 9                 | 1      |
|                        | 22                                     | 0.5                       | 0.187                     | 0.0364                | 9                 | 1      |
|                        | 25                                     | 0.5                       | 0.211                     | 0.0468                | 2~9               | 8      |
|                        | 28                                     | 0.5                       | 0.235                     | 0.0585                | 9                 | 1      |
2.4. Calculation method of $\alpha_{cal}$

There are three types of take-off angle involved in this paper: 1) geometrical take-off angle $\alpha$; 2) the virtual take-off angles $\alpha_U$ ($\alpha_Uc$ for circular arc, $\alpha_Ut$ for triangle) and $\alpha_L$ ($\alpha_Lc$ for circular arc, $\alpha_Lt$ for triangle) of the upper and lower jet trajectories; 3) take-off angle $\alpha_{Ucal}$ ($\alpha_{Ucal}c$ for circular arc, $\alpha_{Ucal}t$ for triangle) and $\alpha_{Lcal}$ ($\alpha_{Lcal}c$ for circular arc, $\alpha_{Lcal}t$ for triangle) of the upper and lower jet trajectories calculated from the theoretically and numerically for the ski jump jet.

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 31 | 0.5 | 0.258 | 0.0714 | 3–10 | 8 |
| 34 | 0.5 | 0.280 | 0.0855 | 9 | 1 |
| 37 | 0.5 | 0.301 | 0.1007 | 5–13 | 9 |

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 10 | \ | 0.30 | 0.0529 | 1–9 | 9 |
| 13 | \ | 0.30 | 0.0693 | 9 | 1 |
| 16 | \ | 0.30 | 0.0860 | 1–9 | 9 |
| 19 | \ | 0.30 | 0.1033 | 9 | 1 |
| 22 | \ | 0.30 | 0.1212 | 9 | 1 |
| 25 | \ | 0.30 | 0.1399 | 2–9 | 8 |
| 28 | \ | 0.30 | 0.1595 | 9 | 1 |
| 31 | \ | 0.30 | 0.1803 | 3–10 | 8 |
| 34 | \ | 0.30 | 0.2024 | 9 | 1 |
| 37 | \ | 0.30 | 0.2261 | 5–13 | 9 |

![Fraction of fluid contours](image)

**Figure 2.** Jet simulation ($\alpha = 16^\circ Fr = 4$)

The first two types of take-off angle can be determined through engineering design and model tests. For the calculation of the take-off angle $\alpha_{cal}$, take a circular-shaped bucket ($\alpha = 16^\circ, Fr = 4$) for example, and draw the coordinates of multiple points on the nappe (Fig. 3) based on the jet trajectories. It can be seen from Fig. 3 that the nappe trajectory follows a parabola, and the correlation coefficient of the fitting curve of the upper and lower of the jets is 0.999.

The specific steps are as follows:
1. Obtain the first derivative of the fitted curve;
2. Substitute $x$ after the outlet to obtain the first derivative value;
3. Use the arctangent function to obtain the take-off angle.
Assuming that the flip bucket is smooth, and that the take-off angle is equal to the geometrical take-off angle $\alpha$, if the effect of air resistance is neglected, then we can theoretically derive the equation of nappe trajectory by calculating the free projection trajectory of a mass point.

$$x = \frac{u_1^2 \sin \theta \cos \theta}{g} \left( 1 + \frac{2gy}{u_1^2 \sin^2 \theta} \right) \left( 1 + \sqrt{1 + \frac{2gy}{u_1^2 \sin^2 \theta}} \right) \quad \text{(5)}$$

Substituting the known $y$, $u_1 = v_0$, $x = X_U$ ($X_L$) into Equation (5), the take-off angles $\alpha_{\text{cal}}$ of the upper and lower of the jets can be calculated.

3. Result and discussion

3.1. Comparison of three types of take-off angle

Figure 4 and 5 show the comparison of the three types of) take-off angles of the circular-shaped bucket, in which the horizontal line is the outlet of the flip bucket, which does not change with the increase of the Froude number. The virtual take-off angles were calculated from the nappe trajectory, and the bottom curve is the take-off angles calculated theoretically. It can be seen that $\alpha > \alpha_U$ ($\alpha_L > \alpha_{\text{cal}}$). The reason is that, due to the influence of nappe aeration, diffusion and air resistance on the theoretical formula, the jet trajectory calculated according to Equation (5) is relatively large, and with the same jet trajectory, the take-off angle will be significantly small. Because of the effect of gravity, aeration and diffusion, the take-off angle after flip bucke will be inconsistent with the geometrical take-off angle, and it is smaller than the geometrical take-off angle.
Figure 5. Comparison of three kinds of angle of circular-shaped bucket for lower jet

Figure 6 and 7 compare the three types of the upper and lower jet of the triangular-shaped bucket and the rules are similar to Figs. 4 and 5, which will not be repeated.

Figure 6. Comparison of three types of angle of triangular-shaped bucket for upper jet

Figure 7. Comparison of three types of angle of triangular-shaped bucket for lower jet

3.2. The influence of Froude number

Figure 8 and 9 show the regularity of the take-off angles of the upper and lower jet trajectories of the circular-shaped bucket with the Froude number, respectively. The representative geometrical take-off angles selected are $\alpha = 10^\circ$, $16^\circ$, $25^\circ$, $31^\circ$ and $37^\circ$ which can show the law more completely.
Figure 8. Virtual take-off angle of upper jets of circular-shaped bucket when $\alpha = 10^\circ, 16^\circ, 25^\circ, 31^\circ, 37^\circ$.

Figure 9. Virtual take-off angle of lower jets of circular-shaped bucket when $\alpha = 10^\circ, 16^\circ, 25^\circ, 31^\circ, 37^\circ$.

Figure 10. Virtual take-off angle of upper jets of triangular-shaped bucket when $\alpha = 10^\circ, 16^\circ, 31^\circ$.

It can be seen from the figures that for the circular-shaped bucket, the virtual take-off angles of the upper and lower jet trajectories increase with the Froude number, and combining with figure 4 and figure 5, we can see that as the Froude number increases, the virtual take-off angles will tend to approach the geometrical take-off angle and be smaller than the geometrical take-off angle.
For the triangular-shaped bucket, three representative angles $\alpha = 10^\circ$, $16^\circ$ and $31^\circ$ are selected. Figure 12 and 13 shows the variation of the virtual take-off angles of the upper and lower jet trajectories with the Froude number. Compared with the circular-shaped bucket (Figs. 10 and 11), the virtual take-off angles of the upper and lower jet trajectories of the triangular-shaped bucket also increase with the Froude number, and eventually approach one value, which is less than the geometrical take-off angle.

3.3. The influence of the geometrical take-off angle

Figure 12 and 13 show the change of upper and lower jet trajectories of circular-shaped bucket and triangular-shaped bucket when the geometrical take-off angle is in the range of $10^\circ$-$40^\circ$. Under the same inflow ($Fr = 9$), for the circular-shaped bucket, when the geometrical take-off angle is small,
the difference between upper and lower jet trajectories is not large, both smaller than the geometrical take-off angle. Under the condition of large geometrical take-off angle, the take-off angle of upper jet trajectory is significantly larger than the lower one; for the triangular-shaped bucket, the take-off angle of upper jet trajectory is larger than the lower one, when the geometrical take-off angle is smaller, and as the geometrical take-off angle increases, the take-off angles of the upper and lower jet trajectories gradually approach one value, which is less than the geometrical take-off angle.

4. Conclusion

In this paper, the take-off angle of the circular-shaped bucket and triangular-shaped bucket is numerically simulated. Regarding the take-off angle, the conclusions are as follows:

(1) The virtual take-off angles of the two kinds of the flip buckets are all smaller than the geometrical take-off angle;
(2) The virtual take-off angles of the two kinds of the flip buckets increase with the Froude number and gradually approach the corresponding geometrical take-off angle;
(3) In terms of the jet trajectories of the circular-shaped bucket and triangular-shaped bucket, take-off angle of the upper jet trajectory is not always greater than the lower one.

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