The Semi-Markov model of operation and maintenance of gas analytical system

V Bobkov¹, O Kanishchev² and I Men'shova³

¹Department of Higher Mathematics, Branch of the National Research University Moscow Power Engineering Institute in Smolensk, 1 Energeticheskiy Avenue, 214013, Smolensk, Russian Federation
²Department of design, SPA “Analitpribor”, 3 Babushkina Street, 214031, Smolensk, Russian Federation
³Department of Logistics and Economic Informatics, D Mendeleev University of Chemical Technology of Russia Laboratory of Materials Science, 9 Miusskaya Square, 125047, Moscow, Russian Federation

*E-mail: VM@sbmpei.ru

Abstract. Gas analytical systems are one of the key elements for the safe operation of airports and ground airspace infrastructure facilities where there is a risk to life or property caused by the possible combustible gases leaks. The gas analytical systems provides safety operation of airspace infrastructure facilities through the early detection of combustible gases and the issuance of warning signal; therefore the gas analytical systems is imposed high operational reliability requirements ensured by an optimal maintenance strategy. Known methods of maintenance management could not consider operation features and gas analytical systems structure complexity therefore increase cost of maintenance of airspace infrastructure facilities. The developed semi-Markov model of operation and maintenance of gas analytical systems, which takes into account the frequency and duration of control, the intensity of operation, the reliability of components, quantity of the spare parts and the gas analytical systems complexity makes it possible to design optimal maintenance strategy and therefore to reduce the cost of maintenance of ground airspace infrastructure facilities by reducing insupportable wear out of gas analytical systems and to reduce capital expenditures by excluding an excessive amount of the spare parts.

1. Introduction

Airports and airspace ground infrastructure are facilities where a significant amount of fuel for aircrafts and launching rockets, industrial gases and neutralization means are stored and used. Most of this liquids and gases are combustible or toxic therefore there are risks to life and property caused by the possible combustible and toxic gases leaks.

Safety operation of the airspace infrastructure facilities relayed on the gas analytical systems (GAS) that are complex continuous or intermittent duty equipment intended to measure one or more components of the gas mixture in the air or technological streams. GAS comprises sampling (aspirated) equipment and sample lines, detection and alarm equipment, supporting measuring equipment, the operation of which is carried out with special software and information support [1].
The general properties of GAS as an object of reliability research are its multifunctionality and structural complexity, so the process of operation and maintenance of GAS is characterized by a large number of possible states “operation, fault, control and repair”.

The GAS provides safety operation space infrastructure facilities through the early detection of combustible gases and the issuance of warning signal therefore it’s imposed high operational reliability requirements which are achieved by hardware and organization ways [2]. The hardware ways use highly reliable components and apply various types of complicating the structure of the system. The organizational ways include different strategies of the maintenance management [3-5]. The practical implementation of which consists in setting such parameters as the frequency and duration of control, the intensity of operation and the quantity of spare parts. Despite the critical role GAS in ensuring the reliable and safety operation of airspace ground infrastructure facilities, there are no studies devoted directly to the development of mathematical models of its operation and maintenance. Currently, international researchers are focused on the development of mathematical models of operation and maintenance of transport, telecommunications, chemical-technological and other complex technical systems [6-9], each of which is distinguished by its own special set of parameters. Applying this generalized or simplified mathematical model for purposes of setting maintenance parameters for GAS might unreasonably increase the capital expenditures and operating cost due to more frequent maintenance and an excessive increase the spare parts.

To exclude an unreasonable increase in the capital expenditures and operating costs, it’s need to develop the mathematical model of operation and maintenance of GAS, which taking into account its structural complexity, reliability, maintainability and various types of recovery strategies, for example, by consuming of spare parts or through emergency delivery. The developing of this model of operation and maintenance of GAS by the property of reliability is a new complex scientific problem, the solution of which is considered in this research.

2. Modern approaches of design of mathematical models of operation and maintenance complex systems

It’s known that the Markov models are a good tool to assess effective maintenance management for complex systems [2,10]. However Markov models have their own requirements and limitations, for instance, the distribution of time “up state” and “fault state” should be exponential. It is an element that is often omitted in the presented analyses, which causes the use of Markov processes to be abused. It is more difficult to estimate parameters in the case of semi-Markov models, which is why they are less popular, but they have less restrictive requirements concerning the form of distributions of studied variables [10-12].

The process of operation and maintenance of complex system (like GAS) by the property of reliability can be described by semi-Markov process, which make it possible to take into account the interdependence of failure states and recovery of individual elements, the structural and technical complexity, the level of reliability and maintainability.

To develop a mathematical semi-Markov model of operation and maintenance of complex system, the system states by the property of reliability must meet the following requirements [11,12]:

1) Changes of the states by the property of reliability of complex system occur in a random way, i.e. the process of changes of states is stochastic;
2) Changes in the states and the time spent by the process in states after transitions are carried out by two independent sets of flows with arbitrary probability distributions;
3) The statistical properties of processes that change states and determine the residence time of the system in states do not depend on the number of transitions already made (the systems are homogeneous) and on the way the system enters each of the possible states;
4) The transition from one state to another occurs instantly.

In the considered formulation of the problem, the semi-Markov process is two-dimensional. The process that changes states is embedded process in relation to the process that forms the time spent by the process in certain states.
The evolution of a semi-Markov process can be defined by several ways, we will use one of them, when the evolution of a semi-Markov process is determined [11,12]:

1) The matrix of conditional stationary probabilities of the transition from the \( i \)-th to the \( j \)-th state \( (w_{ij}) \);

2) The matrix of conditional functions of distribution of probability the time spent by the process in the \( i \)-th state under the condition of the next transition to the \( j \)-th state \( (F_{ij}(t)) \).

The elements of the matrix \( w_{ij} \) and \( F_{ij}(t) \) are determined from the analysis of the physical essence of the process of operation and maintenance of complex system, represented as a semi-Markov process of states changes. In such cases semi-Markov process conveniently represented by the form of oriented graph.

For complex systems (like GAS), a main complex indicator of reliability is the availability factor \( (F_a) \), which represents the stationary probability of the system being in “up state” (or “up states”).

For a semi-Markov process the stationary probability of the process being in a certain state \( \pi_i \) might be expressed by the formula [11-13]:

\[
\pi_i = \frac{p_i m_i}{\sum_{j=1}^{n} p_j m_j}, \quad i = 1, 2, ..., n
\] (1)

\( p_i \) – stationary probabilities of states of the embedded process; \( m_i \) – mean time spent by the semi-Markov process in the \( i \)-th state.

Taking into account the conditional stationary probabilities of the transition from the \( i \)-th to the \( j \)-th state \( (w_{ij}) \) and conditional functions of distribution of probability the time spent by the process in the \( i \)-th state under the condition of the next transition to the \( j \)-th state \( F_{ij}(t) \) the mean time spent by the semi-Markov process in the \( i \)-th state should be found [11,12]:

\[
m_i = \int_0^\infty (1 - \sum_{j=1}^{n} F_{ij}(t) w_{ij}) dt,
\] (2)

\( n \) – total quantity of the semi-Markov process states.

Stationary probabilities of states of the embedded process \( P_i \) might be found from the system of linear equations [11, 12]:

\[
P_i = \sum_{j=1, j \neq i}^{n} P_j w_{ji}, \quad i = 1, 2, ..., n,
\] (3)

where one of the equations is replaced by the normalization condition [11,12]:

\[
\sum_{j=1}^{n} P_j = 1.
\] (4)

It can be shown that the analytical expression for the availability factor \( (F_a) \) [14] of the complex recovered system might be expressed by [11-13]:

\[
F_a = \frac{\sum_{i=1}^{n} p_i m_i}{\sum_{i=1}^{n} p_i m_i}
\] (5)

where summation in the numerator is carried out over all “up states” and in the denominator for all states (“up state”, “fault state”, “recovery state”).

Since the expression (5) is the availability factor, it could be argued that expression (5) is a mathematical semi-Markov model of operation and maintenance of complex system by the property of reliability.

3. Design and application of the semi-Markov model of operation and maintenance of GAS

3.1 Design of the semi-Markov model of operation and maintenance of GAS

Let’s suppose, that maintenance consists of checking up one generalized parameter, if the parameter matches the specified values the GAS (or it’s element) is “up state”, otherwise the GAS (or it’s element) is “fault state”. Let’s make the assumption that the completeness of control and, consequently, its reliability remains unchanged. This assumption is valid also if maintenance consists of checking up fixed number parameters.

Let’s make an assumptions that the detection of the GAS (or its elements) failures should be detected only during maintenance and the control operations during the maintenance are troubleproof, therefore the GAS reliability would be function of the maintenance frequency \( (T_m) \) [15]:

...
\[ R = 1 - \exp\left(-\int_0^{T_m} \lambda(t)\,dt\right), \]  
\[ R = 1 - \exp\left(-\lambda T_m\right) \]  

\( \lambda(t) \) – failure rate of elements of GAS, if assumed \( \lambda(t) = \text{const} \) the GAS reliability could be simplified as [15]:

A characteristic feature of modern GAS is the use double redundancy or duplication of its elements and the mathematical model of operation and maintenance of GAS should consider this feature.

The process of operation and maintenance of the GAS can be characterized by four time parameters: frequency of control \( T_m \), duration of control \( T_c \), recovery time \( T_r \), and emergency delivery time \( T_{ed} \). In the general case, these parameters are random values, which, nevertheless, are conveniently assumed to be constant values.

To survey the mathematical model operation and maintenance of GAS, let's describe the GAS operation and maintenance as a sequential change of the following situations:

1. Before maintenance both the main and duplicated GAS’s elements are “up state” with a probability \((1 - R)^2\). Then maintenance both elements takes place, after that GAS continues to normally operate;

2. Before maintenance the main GAS’s element is “fault state” with a probability \(R\), duplicated element is “up state” with a probability \((1 - R)\). In this case maintenance elicits failed element which replaced by consume of the spare parts with a probability \((1 - P)\). Here \(P\) is probability of “SPTA package failure”, i.e. – probability of the event consisting in the fact that an order for a spare part is not satisfied due to the fact that at the time of receipt of the order there are no this type of the spare parts. After that GAS will continue to operate normally. Otherwise, if SPTA failure, GAS will continue to operate without duplication element;

3. Before maintenance both the main and duplicated GAS’s elements are “fault state” with a probability \(R^2\). In this case maintenance elicits failed elements which are recovered by consume of the spare parts with a probability \((1 - P)\). After that GAS will continue to operate normally. Otherwise, if SPTA failure, GAS will discontinue to operate until emergency delivery of spare parts \(T_{ed}\).

The semi-Markov process of operation and maintenance of GAS considers all mention above situation and consists of followed states: E1 - main and duplicated elements are “up state”, E2 - main and duplicated elements are “fault state”, E3 – maintenance provided in case of main and duplicated elements are “fault state”, E4 - main element is “fault state”, duplicated is “up state”, E5 - maintenance elicits failed element which replaced by consume of the spare parts, E6 – recovering the GAS by emergency delivery of spare parts, E7 - maintenance elicits failed main and duplicated elements, then GAS is recovered by consume of spare parts or emergency delivery of spare parts.

The graph of semi-Markov process of operation and maintenance of GAS is shown on figure 1.

![Figure 1. The graph of semi-Markov process of operation and maintenance of GAS.](image)

The matrix of conditional stationary probabilities of the transition from the \(i\)-th to the \(j\)-th state \(W = \{w_{ij}\}\) might be express as follows:
expressed with follows:

\[ W = \begin{pmatrix} 0 & R^2 & (1 - R)^2 & 2R(1 - R) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ (1 - P) & 0 & 0 & 0 & 0 & 0 & P \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P & 0 \\ 0 & R & 0 & 0 & 0 & 0 & (1 - R) \end{pmatrix} \] (8)

It can be shown if GAS is recovered by emergency delivery of spare parts and consuming of spare parts, probability of SPTA failure \( P \) might be expressed with follows:

\[ P = (1 + v(a_0^{e1} + \mu(a_0a_1) + \cdots + \mu^n(a_0a_1 \cdots a_{n+1})^{-1})^{-1}, \] (9)

\( m, n \) - quantity of identical elements in the GAS and quantity identical spare parts; where \( a_i \) could be expressed with follows:

\[ a_i = m(K_e + (1 - K_e)\lambda^{off}) + (n - i)\lambda^{off}, \] (10)

\( \lambda, \lambda^{off} \) - GAS’s elements failure rate in the “on state” and the “off state” respectively; \( K_e \) - intensity of operation:

\[ K_e = T_{on}T_m^{-1}, \] (11)

\( T_{on} \) - time of being GAS in the “on state”.

The elements of the matrix of conditional functions of distribution of probability the time spent by the process in the \( i \)-th state under the condition of the next transition to the \( j \)-th state \( F_{ij}(t) \) might be expressed with follows:

\[ F_{12}(t) = \begin{cases} 0, & t < 0 \\ (1 - e^{-\lambda t})^2 R^2 - 2R(1 - R) & 0 < t < T_m \\ 1, & t \geq T_m \end{cases} \]

\[ F_{13}(t) = F_{87}(t) = \begin{cases} 0, & t < T_m \\ 1, & t \geq T_m \end{cases} \]

\[ F_{14}(t) = \begin{cases} e^{-\lambda t} & 0 < t < T_m, \\ 0, & t \leq 0 \end{cases} \]

\[ F_{31}(t) = \begin{cases} 0, & t < T_c, \\ 1, & t \geq T_c \end{cases} \]

\[ F_{27} = \begin{cases} 2(e^{-\lambda t} - 1) & 0 < t < T_m \\ 1, & t \geq T_m \end{cases} \]

\[ F_{45}(t) = \begin{cases} e^{-\lambda t} & 0 < t < T_m, \\ 0, & t \leq 0 \end{cases} \]

\[ F_{51}(t) = F_{78}(t) = \begin{cases} 0, & t < T_c + T_r, \\ 1, & t \geq T_c + T_r \end{cases} \]

\[ F_{65}(t) = \begin{cases} 0, & t < T_c + T_r + T_{td} \\ 1, & t \geq T_c + T_r + T_{td} \end{cases} \]

\[ F_{58}(t) = F_{76}(t) = \begin{cases} 0, & t < T_c, \\ 1, & t \geq T_c \end{cases} \]

\[ F_{82}(t) = \begin{cases} (1 - e^{-\lambda t}) & 0 < t < T_m \\ 1, & t \geq T_m \end{cases} \]

Stationary probabilities of states of the embedded process \( P_i \) might be found by (3), (4):
Let us apply the developed semi-Markov model to determine the optimal value of the intensity of operation, which, according to (11), is determined by the time of being GAS in the “on state” ($T_{on}$). The method for determining the optimal $T_{on}$ consists of two stages:

Stage 1. Using as the initial data the values of frequency ($T_m$) and duration of control ($T_c$), recovery time ($T_{ed}$), failure rate ($\lambda$), availability of spare parts the value of the availability factor ($F_A$) should be calculated according to (11) as a function of the $T_{on}$ as figure 2.

For the process of operation and maintenance of GAS states that might be considered as “up state” are states 1, 4, and 8 (fig. 1) so the availability factor ($F_A$) of GAS might be expressed with follow (5):

$$F_A = (P_3 m_3 + P_4 m_4 + P_6 m_8) (\sum_{i=1}^{8} P_i m_i)^{-1},$$  

$m_i$ might be found by (2) and $P_i$ might be found by (12).

Thus (14) is the mathematical semi-Markov model of operation and maintenance of GAS, which taking into account the structural complexity of the GAS, it’s reliability, maintainability and various types of recovery by consume of spare parts or through emergency delivery and allows to explore the effect of parameters model of operation and maintenance of GAS on systems reliability.

### 3.2 Application of the semi-Markov model of operation and maintenance of GAS

As mentioned above, applying of the mathematical model that takes into account the structural complexity of the GAS, its reliability, maintainability and various types of recovery allows to determine the optimal parameters of operation and maintenance and therefore reduce an unreasonable increase in the capital expenditures and operating costs.

Let us apply the developed semi-Markov model to determine the optimal value of the intensity of operation, which, according to (11), is determined by the time of being GAS in the “on state” ($T_{on}$). The method for determining the optimal $T_{on}$ consists of two stages: Stage 1. Using as the initial data the values of frequency ($T_m$) and duration of control ($T_c$), recovery time ($T_{ed}$), failure rate ($\lambda$), availability of spare parts the value of the availability factor ($F_A$) should be calculated according to (11) as a function of the $T_{on}$ as figure 2.
Stage 2. The value of the time of being GAS in the “on state” ($T_{on}$) should be determined to required value of availability factor ($F_a$) is provided. This can be done graphically or by solving the optimization equation:

$$T_{on}^* = \arg \max F_a(T_{on}) \quad \text{or} \quad \frac{dF_a}{dT_{on}} = 0$$  \hspace{1cm} (15)$$

As calculations have shown, the time of being GAS in the “on state” could vary over a wide range, without significantly affecting the value of the availability factor as the consequence of applying for GAS highly reliable components. In particular, increasing the time of being GAS in the “on state” from 10 to 50 days (240 to 1200 hours) with a frequency of control ($T_m$) of one year (appr. 8000 hours), the availability factor will decrease only from 0.95 to 0.9 as figure 3.

![Figure 3. Dependence of availability factor on the time of being GAS in the “on state”](image)

The considered method for determining the optimal value of $T_{on}$ illustrates the possibilities of applying the mathematical semi-Markov model to determine the optimal parameters of operation and maintenance of GAS. The optimal value of $T_{on}$ not only provides the required value of availability factor ($F_a$), but also allows you to minimize operating costs by saving the system resource. Similar reasoning could be apply to determine the optimal quantity of spare parts by using of the mathematical model. The optimal quantification of spare parts allows to optimize capital and maintenance costs of the system.

4. Conclusion
Applying of the mathematical apparatus of semi-Markov processes made it possible to develop mathematical model of operation and maintenance of GAS. The mathematical model allows determining the GAS availability factor depending on the structural complexity of the GAS, its reliability, maintainability and various types of recovery by consume of spare parts or through emergency delivery.

The mathematical model makes it possible to develop methods for determining the optimal parameters of operation and maintenance of GAS to exclude an unreasonable increase in the capital expenditures and operating costs.

The authors believe that the mathematical model could be used not only for GAS, but also to optimize the capital expenditures and operating costs of a wide range of complex technical systems with parameters of model of operation and maintenance are similar to those of the GAS.

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