Efficiency of Neutrino Annihilation around Spinning Black Holes

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Abstract. A fraction of neutrino emission from GRB accretion disks annihilates above the disk, creating e± plasma that can drive GRB explosions. We calculate the efficiency of this annihilation using the recent detailed model of hyper-accretion disks around Kerr black holes. Our calculation is fully relativistic and based on a geodesic-tracing method. We find that the efficiency is a well-defined function of (1) accretion rate and (2) spin of the black hole. It is practically independent of the details of neutrino transport in the opaque zone of the disk. The results help identify the accretion disks whose neutrino emission can power GRBs.

Keywords: Accretion - accretion disks - gamma ray bursts - Kerr black holes

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INTRODUCTION

The relativistic model of GRB accretion disks was completed recently [1]. It describes the disk matter down to the last stable orbit \( r_{\text{ms}} \) and gives energy fluxes carried away by \( \nu \) and \( \bar{\nu} \) at all radii \( r \). In the present work, we trace the trajectories of emitted neutrinos, and calculate the rate of \( \nu \bar{\nu} \) annihilation around the disk. This process deposits e± plasma and can play a key role in the formation of GRB jets.

Neutrino annihilation was previously calculated in a number of works (e.g. [2], [3], [4]) Our work has two motivations: (1) A relativistic calculation has never been done for a realistic accretion disk around a spinning black hole. Previous works either used a toy model for neutrino source (e.g. an isothermal disk, [4]) or replaced neutrino trajectories by straight lines [2]. (2) The efficiency of \( \nu \bar{\nu} \) annihilation depends strongly on the accretion rate \( \dot{M} \) and the spin parameter \( a \) of the black hole. It is desirable to know this dependence and identify the range of \( \dot{M} \) and \( a \) where \( \nu \bar{\nu} \) annihilation can provide the observed energy of GRB explosions.

NEUTRINO EMISSION FROM THE DISK

Fortunately, our final result depends only on the energy fluxes \( F_\nu \) and \( F_{\bar{\nu}} \) from the disk surface, which are independent of the neutrino-transport details and are already calculated in [1]. The rate of \( \nu \bar{\nu} \) annihilation is insensitive to the exact shapes of \( \nu \) and \( \bar{\nu} \) spectra. This fact can be demonstrated using two extreme models A and B:

Model A: Neutrinos \( \nu \) and \( \bar{\nu} \) are emitted with the same spectrum as found inside the disk (same temperature \( T \) and chemical potential \( \mu_\nu \)). The spectrum is normalized so that the emerging emission carries away the known energy fluxes \( F_\nu \) and \( F_{\bar{\nu}} \).

Model B: Neutrinos are emitted with the effective surface temperature \( T_{\text{eff}} \) defined by \( (7/8)\sigma T_{\text{eff}}^4 = F_\nu + F_{\bar{\nu}} \) \( (\sigma \) is the Stefan-Boltzmann constant). The two models give practically the same \( \nu \bar{\nu} \) annihilation rate (Figure 2). Our results confirm the analytical argument that it is sufficient to know \( T_{\text{eff}} \) to calculate neutrino annihilation rate [5]. When the disk is efficiently cooled (neutrino losses almost balance viscous heating), \( T_{\text{eff}} \) is given by the standard thin-disk model [6]: \( T_{\text{eff}} = T_{\text{eff}}^{\text{standard}} \). This model applies to GRB disks in a broad range of accretion rates \( \dot{M}_{\text{ign}} < \dot{M} < \dot{M}_{\text{trap}} \) [1], where

\[
\dot{M}_{\text{ign}} = K_{\text{ign}} \left( \frac{\alpha}{0.1} \right)^{5/3}, \quad \dot{M}_{\text{trap}} = K_{\text{trap}} \left( \frac{\alpha}{0.1} \right)^{1/3}.
\]
FIGURE 1. Spatial distribution of the energy deposition rate by ν̅ν annihilation. This example assumes the accretion rate $\dot{M} = 1 M_\odot s^{-1}$, the disk viscosity parameter $\alpha = 0.1$, and the black hole mass $M = 3 M_\odot$; Model A is assumed for the neutrino spectrum (see the text). Left panel: $a = 0$. Right panel: $a = 0.95$ ($a = 1$ corresponds to the maximally rotating black hole). Note that the energy deposition rate is much higher in the case of $a = 0.95$. The arrows show the specific momentum of the $e^\pm$ plasma injected by ν̅ν annihilation. The white curve is where the radial component of the injected momentum changes sign. This boundary gives an idea of the region where the injected plasma is lost into the black hole.

Here $\alpha \sim 0.1$ is the standard viscosity parameter of the accretion disk, and the factors $K$ depend on the black hole spin $a$; e.g. for $a = 0.95$ they are $K_{\text{ign}} = 0.021 M_\odot s^{-1}$ and $K_{\text{trap}} = 1.8 M_\odot s^{-1}$. $T_{\text{eff}}$ of neutrino emission can be approximately described as

$$T_{\text{eff}}(\dot{M}, r) \approx T_{\text{eff}}^{\text{standard}}(\dot{M}_{\text{ign}}, r) \times \begin{cases} 0 & \dot{M} < \dot{M}_{\text{ign}} \\ (\dot{M}/\dot{M}_{\text{ign}})^{1/4} & \dot{M}_{\text{ign}} < \dot{M} < \dot{M}_{\text{trap}} \\ (\dot{M}_{\text{trap}}/\dot{M}_{\text{ign}})^{1/4} & \dot{M} > \dot{M}_{\text{trap}} \end{cases}$$

Equation (2) defines our Model C, which reproduces surprisingly well the more detailed numerical results (Figure 2). This model allows us to obtain an explicit approximate formula for the annihilation rate (eq. 3 below).

RESULTS

Figure 1 shows two examples of the spatial distribution of the energy deposition rate by ν̅ν annihilation. Integration of this distribution over volume outside the black hole gives the total energy deposition rate $\dot{E}_{\nu\bar{\nu}}$. We performed this calculation for various $\dot{M}$ (Fig. 2). Our results show that $\dot{E}_{\nu\bar{\nu}}$ is well approximated by a simple formula,

$$\dot{E}_{\nu\bar{\nu}} \approx 9 \times 10^{51} x_{\text{ms}}^{-4.7} \times \begin{cases} 0 & \dot{m} < \dot{m}_{\text{ign}} \\ \dot{m}_{\text{ign}}^{9/4} & \dot{m}_{\text{ign}} < \dot{m} < \dot{m}_{\text{trap}} \\ \dot{m}_{\text{trap}}^{9/4} & \dot{m} > \dot{m}_{\text{trap}} \end{cases} \text{erg s}^{-1}$$

where $\dot{m} = \dot{M}/M_\odot s^{-1}$ and $x_{\text{ms}} = r_{\text{ms}}(a)/(2GM/c^2)^{-1}$. Derivation of the scaling of $\dot{E}_{\nu\bar{\nu}}$ with $\dot{m}$ and $x_{\text{ms}}$ is given in [5]. The dependence of $\dot{E}_{\nu\bar{\nu}}$ on the black hole spin is huge: $x_{\text{ms}}^{-4.7}$ varies by a factor of 170 for $0 < a < 0.95$. Note that $\alpha$ (viscosity parameter of the disk) enters the result only through $\dot{M}_{\text{ign}}$ and $\dot{M}_{\text{trap}}$ (eq. 1).
The efficiency of $\nu \bar{\nu}$ annihilation can be defined as $\varepsilon = \dot{E}_{\nu \bar{\nu}} / L$ where $L$ is the total neutrino luminosity of the disk. For example $a = 0.95$ (which corresponds to $x_{\text{ms}} \approx 1$) gives $L \approx 0.1M c^2$ and

$$\varepsilon \approx 0.05 \left( \frac{M}{M_\odot} \right)^{5/4}, \quad M_{\text{ign}} < \dot{M} < \dot{M}_{\text{trap}}.$$  

The observed GRB luminosity $L_{\text{obs}}$ can be supplied by $\nu \bar{\nu}$ annihilation around a rapidly spinning black hole ($a = 0.95$) if $M > 0.38(M_\odot/s)/L_{\text{obs}}/10^{51}$ erg/s), which is in the range of plausible accretion rates in GRB central engines.

Finally, note that $\dot{E}_{\nu \bar{\nu}}$ is defined as the total energy deposition rate outside the event horizon. A fraction of the created $e^\pm$ plasma falls into the black hole and not contribute to the observed explosion (Figure 1). The corresponding refinement of $\varepsilon$ depends on the plasma dynamics outside the disk, which is affected by magnetic fields and is hard to calculate without additional assumptions.

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