Active Disturbance Rejection Control for a Class of Non-affine Nonlinear Systems via Neural Networks

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Abstract. An active disturbance rejection controller based on radial basis function (RBF) neural network is proposed for a class of non-affine nonlinear systems in this paper. In which, the active disturbance rejection control (ADRC) is utilized to estimate the full system state and RBF neural network is used to approximate the reference signal. It is proved that, the composite controller has faster response speed and higher tracking accuracy, which effectively improves the control performance of the system and alleviates the adverse effects caused by strong coupling in the nonlinear dynamic system. Effectiveness of the proposed method is verified by MATLAB simulation.

Introduction

Highly uncertain nonlinear systems exist in ship heading control systems, blackbody source temperature control systems and brushless DC motor speed control systems. In recent years, some nonlinear system control methods have been proposed in these fields: adaptive control, active disturbance rejection control (ADRC), fuzzy control, neural network control, etc. Among them, ADRC has many advantages in the control of nonlinear system due to its strong anti-interference ability, simple algorithm, low overshoot, fast convergence and high precision. In [1], the extended state observer (ESO) with nonlinear gain is used to solve the problem of measuring noise, and a more powerful nominal output regulator is proposed according to the internal model principle. Although the ADRC has strong robustness, a fixed set of parameters cannot be applied to system with large and fast variations. In [2], an adaptive sliding mode controller with an extended state observer is proposed which ESO is used to estimate the system state and the adaptive strategy is applied to deal with the estimation error. However, the traditional ADRC still has some limitations in parameter adjustment and performance optimization.

Recently, neural network has received extensive attention from scholars because of their ability to map and approximate nonlinear systems. An ADRC based on neural network with stronger parameter adaptability and better dynamic characteristics is proposed in [3]. A certain structure of the artificial neural network (ANN) used to arbitrarily approximate the properties of the nonlinear function is proposed in [4]. In [5-6], the adaptive ADRC based on BP neural network is designed to solve the problem of controlled objects with the large and fast change range, in which, the BP neural network is used to realize the online adjustment of ESO parameters. However, it only tunes the ESO parameters. There are still a lot of parameters to be tuned in TD and NFLLSF modules.

Inspired by the above researches, in this paper, an adaptive ADRC based on Radical Basis Function Neural Network (RBFNN) is proposed to solve the parameters tuning problem. The composite controller RBF-ADRC is constructed. Compared with the traditional ADRC, the controller uses RBFNN to approximate arbitrary nonlinear functions with arbitrary accuracy, achieves better tracking of controlled objects. In particular, the controller reduces the parameter tuning of the ADRC, has faster response speed, higher tracking accuracy and smaller control oscillation amplitude.
Construction and Analysis of Traditional ADRC

The ADRC technology was designed by Han\(^7\) in the 1990s. Its core technology is to estimate and compensate all uncertainties of the system by attributing them to the total disturbance of the system. The ADRC mainly introduces three main control links in the input, forward channel and feedback-Tracking differentiation (TD), nonlinear state error feedback (NLSEF), and Extended State Observer\(^8\) (ESO).

Tracking Differentiation (TD)

TD refers to the dynamic structure that can not only track the input signal of the system, but also ensure the undistorted differential signal can be obtained, which resolves the contradiction between the response time and overshoot of the system. The TD used in this paper is:

\[
\begin{align*}
  v_1(k+1) &= v_1(k) + h \cdot v_2(k) \\
  v_2(k+1) &= v_2(k) + h \cdot f_{han}(v_1(k) - v(k), v_2(k), r, h)
\end{align*}
\]

where \(v_1(k)\) is the tracking value of the instruction, \(v_2(k)\) is the tracking value of the instruction differential, \(r\) is the fast factor, \(h\) is the sampling time, and \(f_{han}\) is the synthesizing function of the fastest control. Through (1), \(v_1(k) \rightarrow v(k)\), \(v_2(k) \rightarrow \dot{v}(k)\) can be realized.

Extended State Observer (ESO)

For the following non-linear systems, ESO estimates the state and disturbance of the system based on the input and output of the controlled object.

\[
\begin{align*}
  x_1 &= x_2 \\
  x_2 &= f(x_1, x_2) + bu, x_3 = f(x_1, x_2) \\
  y &= x_1
\end{align*}
\]

Establish a state observer for the expanded system shown in (3), \(\beta_i (i=1,2,3)\) is the correction coefficient to be adjusted, and the saturation function \(f_{al}\) is used to suppress the chattering of the signal.

\[
\begin{align*}
  e &= z_1 - y, fe = f_{al}(e, \alpha_1, h), fe_1 = f_{al}(e, \alpha_2, h) \\
  z_1 &= z_1 + h(z_2 - \beta_1 e) \\
  z_2 &= z_2 + h(z_3 - \beta_2 fe + bu) \\
  z_3 &= z_3 + h(-\beta_3 fe_1)
\end{align*}
\]

Nonlinear State Error Feedback (NLSEF)

NLSEF uses the nonlinear combination error and the differential of the error to get the actual control amount of the controlled object. The specific design is as follows:

\[
\begin{align*}
  e_1 &= v_1 - z_1, e_2 = v_2 - z_2 \\
  u_0 &= -f_{han}(e_1, ce_2, r, h) \\
  u &= u_0 - z_3/b
\end{align*}
\]

The controlled value \(y\) can be well estimated by choosing appropriate parameters.

Construction and Analysis of The Composite Controller RBF-ADRC

Parameter tuning is an important link in the design of ADRC, however, it is usually complex and time-consuming and will re-set the parameters after replacing the control object or input signal. The RBF-ADRC composite controller has the ability of fast learning\(^9\). It combines the good approximation performance of RBFNN and the good characteristic of ADRC which does not depend...
on the precise model of the system. Without complex parameter tuning, it is easier to establish an adaptive mechanism, which is equivalent to the system identification with automatic adjustment. It can improve the stability of the system while improving the robustness of the system. The block diagram of the RBF-ADRC composite controller is shown in Figure 1.

![Figure 1. Structural Diagram of RBF-ADRC Composite Controller.](image)

Design of Ideal Controller

Equation (2) shows that \( x_1 \) represents a nonlinear function, its estimated value is \( \hat{x}_1 \). Actuator output \( \Gamma(t) = H(u, \dot{u}) \) is non-linear with respect to the control input. The control objective is to design control input \( u \) to force output \( y \) to track a given desired trajectory \( x_d(t) = [x_d, x_d^{(1)}, \ldots, x_d^{(n-1)}]^T \).

Define tracking error \( e = x - x_d \), filtering tracking error is \( r = [\Lambda^T]e \), where \( \Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T \) is a properly chosen coefficient vector such that \( e \to 0 \) by the time \( r(t) \to 0 \), i.e., \( S + \lambda_{n-1}S^{n-2} + \cdots + \lambda_1 \) is Hurwitz. The time derivative of \( r \) , \( \dot{r} = x_3 - x_d^{(2)} + [0 \Lambda]^T \). Select the ideal tracking controller \( \Gamma_{\text{des}} = -k_a r - \hat{x}_1 + x_d^{(2)} - [0 \Lambda]^T \), here, \( k_a \) is the design parameter.

RBF-ADRC Nonlinear Compensation

Define the error between the expected and actual output of the actuator as \( \tilde{r} = \Gamma_{\text{des}} - \Gamma \), Consider the following state-dependent conversion: \( \tilde{r} = \Psi = H(u, \dot{u}) \). Among them, \( \Psi \) is called pseudo control, Here \( \hat{u} = H^{-1}(\bar{u}, \tilde{\Psi}) \), so its inverse is \( \hat{u} = H^{-1}(\bar{u}, \tilde{\Psi}) \). \( \tilde{\Psi}(\bar{u}, \dot{\bar{u}}) \) stands for nonlinear inverse error and will be approximated by a neural network. The pseudo input is designed as \( \tilde{\Psi} = k_b \tilde{r} + \Gamma_{\text{des}} - \frac{1}{c} \tilde{\zeta} + u_{\text{ad}} + u_t \), where \( u_{\text{ad}} \) is the output of RBFNN, \( u_t \) is the output of ADRC, \( \tilde{\zeta} \) is the filter output. Filter dynamics are given by \( \dot{\xi} + \zeta_c \xi = \xi_{\text{des}} \). Therefore, the pseudo control input can be written as \( \tilde{\Psi} = k_b \tilde{r} + \zeta + u_{\text{ad}} + u_t \).

Then the error dynamic changes to \( \dot{\tilde{r}} = -k_b \tilde{r} + \frac{1}{c} \zeta - H(\bar{u}, \dot{\bar{u}}) - u_{\text{ad}} - u_t \).

Neural Network Approximation

Given a multi-input single-output RBFNN, \( n \) and \( m \) are the number of nodes in the input layer and hidden layer respectively. The excitation functions are Gauss function. Based on the approximation of RBFNN, the nonlinear inverse error and part of the filter can be expressed as \( W^T S(x_{nn}) + \epsilon(x_{nn}) \), where \( |\epsilon(x_{nn})| \leq \epsilon_N \) is the undetermined bound and \( W \in \mathbb{R}^{m \times n} \) is the network weight vector. The input of RBFNN is selected as \( x_{nn} = [x^T, \bar{r}, \bar{G}]^T \) and the output is \( u_{\text{ad}} = \hat{W}^T S(x_{nn}) \).

Simulation Analysis

In order to verify the effectiveness of the proposed scheme, we use MATLAB Simulink to simulate the dynamic system represented by the following differential equations\(^{[10]}\).
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1^2 + 0.1(1 + x_2^2) + 0.1 \sin(x_2)
\end{align*}
\] (5)

The tracking target is \(x_d(t) = \sin(3t) + 1.5\cos(t)\). The neural network input vector is \(x_{in} = [x^T, \tilde{r}, \Gamma]^T\), the number of hidden layer nodes is 10, the initial value of the weight is 0, the initial value of the controlled object is \(x(0) = [0, -0.3]^T\). Filter tracking error \(r(t) = [501]e\), ideally controlled as \(\Gamma = -\dot{x}_3 + \dot{x}_d - [0\ 50]e + u_1\).

In the following, the traditional ADRC and the RBF-ADRC composite controller will be simulated in order to verify the control effect of the composite controller, and the nonlinear ADRC is used to compare the performance difference between the two controllers. Firstly, the traditional ADRC tracking state under random parameters will be shown. The specific parameters are as follows.

Table 1. Traditional ADRC parameter design.

| Parameter | \(r\) | \(h\) | \(\alpha_1\) | \(\alpha_2\) | \(\delta\) | \(\beta_1\) | \(\beta_2\) | \(\beta_3\) |
|-----------|-------|-------|-------------|-------------|---------|-----------|-----------|-----------|
| Figure 2  | 15    | 0.01  | 0.5         | 5           | 0.3     | 115       | 130       | 0.1       |
| Figure 3  | 5     | 0.1   | 0.5         | 6           | 0.5     | 110       | 125       | 0.5       |

![Figure 2](image2.png)  ![Figure 3](image3.png)

Figure 2. (a) Output tracking.  Figure 3. (b) Output tracking.

It can be seen that the traditional ADRC has extremely high requirements for parameter tuning. In order to accurately complete the target tracking, a large number of parameter tuning is needed to be performed, which is complicated and time-consuming. Moreover, the tracking error is large and the control performance is poor from the comparison chart of tracking error. Next, the traditional ADRC chooses Figure 3 parameters to compare the tracking state of traditional ADRC and RBF-ADRC composite control. The simulation results are shown in the following figure 4.

![Figure 4](image4.png)  ![Figure 5](image5.png)

Figure 4. Output tracking.  Figure 5. Tracking error comparison.

The RBF-ADRC composite controller has better control performance than the traditional ADRC. The tracking error of traditional ADRC is 3 rad and that of RBF-ADRC is 0.8 rad, and there is basically no overshoot. What’s more, compound controller does not need complex parameter tuning under the system requires control accuracy. The data show that the control effect of RBF-ADRC is better than that of traditional ADRC in tracking accuracy and robustness.

**Conclusion**

In this paper, a composite control method (RBF-ADRC) is proposed by combining RBF neural network with ADRC. Compared with the traditional ADRC method, this method not only inherits the advantages of ADRC technology, but also enhances its performance and effectively improves the
convergence of the controller. The experimental results show that, compared with the traditional ADRC system, the ADRC system based on RBF neural network has less computation, no need for complex parameter tuning, faster response speed and higher tracking accuracy. It is of great significance to improve the stability accuracy of the non-linear system.

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