Computational Simulation of the Shear Test of a Multi-layered Long-Fibre Composite with a Polymer Matrix

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Abstract. This paper deals with the creation of a computational model of a multi-layered composite made of long fibres embedded in a polymer matrix and its use to simulate the response of the composite to a shear test. The research involves the determination of material parameters of the matrix (elastomer, in the context of the current paper) as well as of fibres (textile-cords and steel-cords). Careful attention is given to the Mooney-Rivlin parameters obtained from the elastomers tensile test for these simulations since shear tests have not been performed. Then by getting advantage of APDL (Ansys Parametric Design Language) in ANSYS, the computational model was successfully created and lastly simulated. The outputs obtained from the computational modelling will greatly help later to refine the orthotropic parameters of the composite, needed in future works to build up and achieve computational simulations of tires.

1. Introduction

The current research work is concerned with composite materials used in the tire manufacturing processes and their structural and mechanical behaviours in operating conditions. In automobile transport and aeronautics, tires play a major role in ensuring respectively the comfort and safe transport of persons and goods and the durability of road infrastructures. In this regard, tires are essential elements in the interaction between vehicles and roads [1], which secure the transmission of forces with the ability to mitigate the inequality or disparities of roads. A standard automobile radial tire casing consists of elastomer parts and embedded parts represented by textile-cords and steel-cords in a tire crown and carcass as reinforcements. These composite parts are called namely steel-cord belt, textile carcass ply or plies and textile cap ply, sometimes called overlapping belts [2, 3]. It is important to experimentally determine material parameters used for the description of each part of a casing for computational modelling of tires. E.g., material parameters as the modulus of elasticity and Poisson’s ratio describe the textile fibre whereas elastomers exhibit nonlinear response.

Therefore, for the description of elastomer parts (rubber matrix and drift elastomer of cords), several material constitutive models have been considered in existing works to capture the hyper elasticity of elastomers. Among which the Mooney-Rivlin model is the most used for computational modelling of tires. It shows satisfactory results for the range of elongation up to 150%. Since it is known experimentally that tire casings reach the elongation of about 10–15% in operating conditions where most of it is in the top and bottom tread layer, in the sidewalls of a casing and on the contact area with the road. There exists a large family of Mooney-Rivlin material models developed for a pool of hyper elastic...
materials. This variety of material models are differentiated in terms of the number of parameters needed to fully describe the deviatoric part of the strain density energy. In the scope of this paper and for the description of the behaviour of elastomers in tire casing, the most Mooney-Rivlin material model used is the one with two parameters [4], which is sufficient to cover the range of elongation up to 150%. These parameters suffice to describe the strain-stress behaviour of elastomer parts of a tire casing.

2. Mooney-Rivlin model

The tests of elastomer parts are needed for the determination of parameters \( C_{10} \) and \( C_{01} \) of the Mooney-Rivlin model. To this end, it is necessary to carry out the tensile test on elastomer samples. The stress-strain dependence for an elastomer has a specific course of sigmoid shape with a relatively significant inflexion point. Consequently, according to Equation (1), the relationships for the calculation of the material parameters \( C_{10} \) and \( C_{01} \) are derived:

\[
\sigma = 2C_{01}\left(\alpha - \alpha^{-2}\right) + 2C_{10}\left(1 - \alpha^{-3}\right)
\]

(1)

Where \( \sigma \) [MPa] is the Cauchy stress or true stress, \( C_{10} \) and \( C_{01} \) [MPa] = Mooney-Rivlin parameters, \( \alpha [-] = \) relative length. By simple modifying, Equation (1) is linearized into the form (2):

\[
\frac{\sigma_i}{2\left(1 - \frac{1}{\lambda_i^3}\right)} = C_{01}\lambda + C_{10}
\]

(2)

Where \( \lambda_i [-] \) is relative deformation for individual values of \( \lambda \).

From a straight-line equation, one obtains:

\[
Y = A \cdot X + B \iff \frac{\sigma}{2\left(1 - \frac{1}{\lambda^3}\right)} = C_{10} \cdot \lambda + C_{01} , \text{ where } x = \lambda \text{ and } \lambda = \varepsilon + 1
\]

\[
\sigma = \left(C_{10} \cdot \lambda + C_{01}\right) \cdot 2 \cdot \left(1 - \frac{1}{\lambda^3}\right)
\]

(3)

\[
\sigma = \left[C_{10} \cdot (\varepsilon + 1) + C_{01}\right] \cdot 2 \cdot \left(1 - \frac{1}{(\varepsilon + 1)^3}\right)
\]

Subsequently, an incompressibility parameter is calculated by Equation (4):

\[
d = \frac{2\left(1 - 2 \cdot PR\right)}{C_{10} \left(5 \cdot PR - 2\right) + C_{01} \left(11 \cdot PR - 5\right)}
\]

(4)

Where \( d \) [MPa\(^{-1}\)] is the incompressibility parameter, \( PR [-] = \) Poisson’s ratio (for an elastomer it is 0.5 or approximately 0.49 or 0.4995).

The research aim of the authors is to design computational models on which it would be possible to appropriately verify the influence of material parameters of elastomers (Mooney-Rivlin parameters obtained from the elastomer tensile tests) on the results. Only a computational simulation of the specific shear test of hybrid [5] and multi-layered long-fibre composites can provide this information as a verification analysis. Since it is necessary to quickly create computational models with the required cord geometry parameters, the computational models for strain-stress analyses were created using APDL (ANSYS Graphical User Interface) procedures for the automatic creation of models from geometric parameters such as a cord diameter, cord distance and one-layer thickness, width and length of the layer. First, the calculation with steel-cords was realized then subsequently, the material parameters were changed to PA66 cord.

3. Materials and computational model design

3.1. Material of cords
The steel-cord belts of radial passenger tire casing consist of two plies with a density ranging between 650–680 m\(^{-1}\) (it is a number of ends per one meter of width, marked as EPM) and the cord constructions are 2+2x0.30 mm or 2+2x0.28 mm (the cord consists of 4 filaments with diameter 0.3 or 0.28 mm). The thickness of the steel-cord belt (two plies altogether) is between 1.4–2.8 mm. Sometimes, construction 2x0.30 mm HT (high tensile) with EPM 961 m\(^{-1}\) is used, e.g. tire casing Matador 165/65 R13 (Table 1). It is necessary to consider the twisting of the filaments, therefore the modulus of elasticity is 190 GPa [3]. The Poisson’s ratio is 0.3.

**Table 1.** Geometric and material parameters of composite parts of Matador 165/65 R13.

| Composite part | Steel-cord belt | Textile carcass | Textile cap |
|----------------|-----------------|----------------|------------|
| Material of cord | Steel HT | PES | PA66 |
| Number of layers [-] | 2 | 1 | 1 |
| Thickness of one layer [mm] | 0.95 | 0.95 | 0.80 |
| Diameter of cord [mm] | 0.60 | 0.48 | 0.40 |
| EPM [m\(^{-1}\)] | 961 | 1 160 | 420 |
| Spacing between cords [mm] | 1.04 | 0.86 | 2.38 |
| Modulus of elasticity [GPa] | 190 | 4 | 3.4 |
| Poisson’s ratio [-] | 0.3 | 0.4 | 0.4 |

Next, PA66 cord is treated. The geometric parameters of PA66 cord are shown in Table 1. For example, the tire casing Dunlop 215/40 R17 has a diameter of cords of 0.44 mm, the thickness of the cap ply of 1.2 mm and EPM of 1400 m\(^{-1}\). Tire casing Continental 245/40 R18 has a diameter of cords of 0.69 mm, the thickness of the cap ply of 1.1 and 1.75 mm (because the tire casing consists of two plies of textile cap) and EPM of 775 m\(^{-1}\). The modulus of elasticity of PA66 varies from 900 to 3 450 MPa or from 9 to 50 cN dtex\(^{-1}\). The Poisson’s ration of PA66 is approximately, 0.4. The PA6.6 textile yarns with construction 470x2 are used, which means linear density – fineness 940 dtex. This single end cord name is Kordsa T-728 [6]. The material parameters of T-728 with 940 dtex are breaking force: 81.4 N [7], tenacity: 86.2 cN tex\(^{-1}\) [6], breaking strength [6]: 8.3 kg, elongation break (ductility): 18.6 % [6, 7], elongation (strain) at 45 N set force: 9.6 % [7], elongation at 4.5 kg set force: 9.6 % [6], number of filaments: 140 [7]. If we use force, cord diameter 0.4 mm, we get the strength and if we divide the strength by the given value of ductility, we get the orientation value of the modulus of elasticity of approx. 3.4 GPa.

### 3.2. Material properties of the matrix

The values of Mooney-Rivlin (called M-R) parameters of elastomer parts and elastomer matrix (drift) are depicted in Table 2 for Matador 165/65 R13 tire casing.

**Table 2.** Mooney-Rivlin parameters for elastomer parts of Matador 165/65 R13 [3].

| Mooney-Rivlin parameters | \(C_{10}\) [MPa] | \(C_{01}\) [MPa] | \(d\) [MPa\(^{-1}\)] |
|--------------------------|------------------|------------------|----------------|
| Tread                    | 0.417            | 0.519            | 0.103          |
| Inner liner              | 0.109            | 0.259            | 0.206          |
| Bead elastomer           | 0.692            | 0.371            | 0.267          |
| Sidewall with a tread side edge | 0.532 | 0.065 | 0.138 |
| Bead bundle              | -0.111           | 1.945            | 0.088          |
| Elastomer drift for a steel-cord belt | 0.638 | 0.284 | 0.151 |
| Elastomer drift for a textile carcass | 0.328 | 0.119 | 0.101 |
| Elastomer drift for a textile cap | 0.548 | 0.112 | 0.056 |
Two parametric M-R model is used as stated earlier. Material M-R parameters of elastomer drift for a steel-cord belt and elastomer drift for a textile cap were taken into account in the calculations.

3.3. Computational model design
The first computational model is a model with a thin-wire steel-cord with diameter 0.94 mm. Next, the diameter 0.6 of steel-cord of Matador 165/65 R13 is chosen. The third variant is PA66 with 0.4 mm which is used for the tire casing Matador. The FEA software ANSYS is adopted for the simulation. The Solid 186 element type with Mixed U/P (meant for a mixed variational formulation with two field: the displacement U and hydraulic pressure P. Recalling that the second field is introduced in the formulation to enforce the incompressibility condition to the potential energy of the variational problem) setting is used. One layer has a length and width of 20 mm.

The APDL procedure includes parameterization with the following parameters:
* cset,1,3,Distance,'Distance between cord [mm]',1.04 !for steel-cord
* cset,4,6,Diameter,'Cord diameter [mm]',0.60 !for steel-cord
* cset,7,9,Thickness,'Thickness of layer [mm]',0.95 !for steel-cord
* cset,10,12,Width,'Width of layer [mm]',20
* cset,13,15,Length,'Length of layer [mm]',20
* cset,16,18,Angle,'Cord angle [degree]',0
* cset,19,21,Modulus of elasticity of cord [GPa]',190 !for steel-cord
* cset,22,24,Poisson ratio [-]',0.30 !for steel-cord

The APDL procedure includes the computation of rubber modulus based on M-R parameters which can be entered directly or are determined based on data from a tensile test:
\[ D = \frac{(2-2*PR_E)}{\text{CONST1}(1)*\left(5*PR_E-2\right)+\text{CONST1}(2)*\left(11*PR_E-5\right)} \]  
\[ \text{D,HYPE,2,1,2,MOON} \]  
\[ \text{TBDATA, CONST1(1), CONST1(2), D, } \] !parameters are in MPa;
\[ E_E = 6*(\text{CONST1}(1)+\text{CONST1}(2)) \] !modulus of elasticity
\[ G_E = 2*(\text{CONST1}(1)+\text{CONST1}(2)) \] !shear modulus
\[ K_E = 2/D \] !volume modulus

The orientation values of modulus of elasticity are 5.53 MPa for elastomer drift for a steel-cord belt and 3.96 MPa for elastomer drift for a textile cap. Next, this layer is duplicated, because computational models consist of a defined number of layers. The first model consists of 9 plus 9 layers (marked “9+9”), see Figure 1. The second and third models consist of 2 plus 2 layers (marked “2+2”), see Figure 2.

Figure 1. The first computational model with a steel-cord diameter of 0.94 mm.
Figure 2. The computational model with a steel-cord diameter of 0.60 mm and a textile-cord diameter of 0.40 mm (down) with details of meshing.

The models are reverse loaded, the displacement in z-axis is defined and the sum reaction forces at the area of steel edges (using these edges, the specimen will be clamped in the jaws of the testing machine) is searched. For computational conveniences, this choice was adopted since it leads to better results with quick convergence and a stable scheme. Defined loads: the prescribed displacement of the middle edge is 5 mm in z-axis (the first model “9 plus 9” used 4.2 mm based on previous results), and the edges are set to be fixed. The edges are made of steel. Solver considered: the PCG solver with tolerance $10^{-6}$ is selected. Solution control settings: calculated prestress is switched on, nonlinear geometric effects are in a state on, time at the end of load step is 5 as defined displacement, number of substeps is 20 (it means that the increment of every substep is 0.25 mm in z-axis).

4. Results
The results from the computational modelling of strain-stress state of variant “9+9” layers with steel-cord diameter of 0.94 mm are represented as the sum of displacement and stress $\sigma_1$ ($1^{st}$ principal stress) for rubber matrix in Figures 3 and 4. Reaction force-displacement dependence is shown in Figure 5. Where we have obtained the reaction forces in z-axis of 341 N for deformation of 4.2 mm and approx. 201 N for deformation of 2.5 mm. While $\sigma_1$ reaches a maximum value of 2.05 MPa for deformation of 4.2 mm.

The results for steel-cord with diameter 0.6 mm, variant “2+2” are outlined in Figures 6–8 wherein the resulting reaction force in z-axis for the displacement of 2.5 mm is 2865.6 N for a corresponding maximal principal stress $\sigma_1$ of 12.79 MPa.
As for the results for PA66 cord with a diameter 0.4 mm, variant “2+2” are shown in Figures 9–11 wherein the reaction force in z-axis for a displacement of 2.5 mm is 1753.2 N for a stress component $\sigma_1$ that hits a maximum value of 8.26 MPa.

**Figure 3.** Sum displacement – computational model “9+9”.

**Figure 4.** Sigma1 – computational model “9+9”.
Figure 5. Force-displacement dependence – computational model “9+9”.

Figure 6. Sum displacement – computational model with steel-cord “2+2”.

Figure 7. Sigma1 – computational model with steel-cord “2+2”.
Figure 8. Force-displacement dependence – computational model with steel-cord “2+2”.

Figure 9. Sum displacement – computational model with PA66 cord “2+2”.

Figure 10. Sigma1 – computational model with PA66 cord “2+2”.
The difference in forces between the steel cord and textile PA66 cord is about 61%. The thickness models are 9.8 and 9.2 mm, respectively.

The variants “2+2” are less time-consuming and therefore faster to compute and there is also, based on the results above illustrated, a clear manifestation of the nonlinearity of the matrix on the results.

5. Conclusions and recommendations

The computational models with steel and PA66 long-fibre cords with cord angle 0° were created for the simulation of a specific shear test.

The APDL procedure with parameterization of geometrical and material parameters for the creation of computational models was designed and programmed. The creation of computational models with cord angles was considered. The proposed APDL procedure is ready to create a two-layer steel cord belt with a cord angle of 23°, which is used in tire casings. The APDL procedure allows the user the choice of whether volume or beam elements to model cords.

In upcoming research works and based on the findings of the current simulations, test samples for the shear test with dimensions 20 x 20 (width x length) mm for experiments will be created. Variants “2+2” can be manufactured using LabEcon 600, FONTIJNE PRESSES laboratory vulcanizing press with a maximum production thickness of 20 mm. The Autograph AG-X plus 5 kN, Shimadzu universal test machine will be used for shear tests.

Based on the performed experiments, it will be possible to compare and verify the computational results of the forces for displacements 2.5 and 5 mm. In this way, it is possible to verify and possibly validate the accuracy of the determination of the Mooney-Rivlin parameters from the uniaxial tensile tests (or obtained from the hardness value Shore A).

The outputs from the computational modelling will greatly help later to refine the orthotropic parameters of the composite, which will be exploited in the computational simulations of the tires. The procedure is such that, for example, the two layers of the steel cord belt are homogenized as a whole and replaced by an orthotropic material with direction-dependent material parameters. Similarly, a computational model for shear simulation would be created and considering the comparison of the results, the optimal parameters of the modulus of elasticity in the individual directions would be sought, which would give similar reaction forces.

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