Quasilocal Quark Models as Effective Theory of Non-perturbative QCD

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We consider the Quasilocal Quark Model of NJL type (QNJLM) as an effective theory of non-perturbative QCD including scalar (S), pseudoscalar (P), vector (V) and axial-vector (A) four-fermion interaction with derivatives. In the presence of a strong attraction in the scalar channel the chiral symmetry is spontaneously broken and as a consequence the composite meson states are generated in all channels. With the help of Operator Product Expansion the appropriate set of Chiral Symmetry Restoration (CSR) Sum Rules in these channels are imposed as matching conditions to QCD at intermediate energies. The mass spectrum and some decay constants for ground and excited meson states are calculated.

Keywords: QCD; Effective Quasilocal Quark Models; OPE; Chiral Symmetry Restoration Sum Rules

1. Introduction

The QCD-inspired quark models with four-fermion interaction are often applied for the effective description of low-energy QCD in the hadronization regime. The local four-fermion interaction is involved to induce the dynamical chiral symmetry breaking (DCSB) due to strong attraction in the scalar channel. As a consequence, the dynamical quark mass \( m_{\text{dyn}} \) is created, as well as an isospin multiplet of pions, massless in the chiral limit, and a massive scalar meson with mass \( m_{\sigma} = 2m_{\text{dyn}} \) arises. However it is known from the experiment [1] that there are series of meson states with equal quantum numbers and heavier masses, in particular, \( 0^{-+}(\pi, \pi', \pi'', ...); 0^{++}(\sigma, \sigma', \sigma'', ...); 1^{-+}(\rho, \rho', \rho'', ...). \) Due to confinement, one expects an infinite number of such excited states with increasing masses. Therefore in order to describe the physics of those resonances at intermediate energies one can extend the quark model with local interaction of the Nambu-Jona-Lasinio (NJL) type [2] taking into account higher-dimensional quark operators with derivatives, i.e. quasilocal quark interactions [3-7,8-10]. For sufficiently strong couplings the new operators pro-
mote formation of additional new meson states. Such a quasilocal approach (see also [11][12][13]) represents a systematic extension of the NJL-model towards the complete effective action of QCD where many-fermion vertices with derivatives possess the manifest chiral symmetry of interaction, motivated by the soft momentum expansion of the high-energy perturbative QCD effective action.

Another idea is to impose CSR Sum Rules at high energies [8]. In particular, at intermediate energies the correlators of QNJLM can be matched to the Operator Product Expansion (OPE) of QCD correlators [9]. This matching realizes the correspondence to QCD and improves the predicability of QNJLM. It is based on the large-$N_c$ approach which is equivalent to planar QCD. In this approximation the correlators of color-singlet quark currents are saturated by infinite number of narrow meson resonances.

On the other hand the high-energy asymptotic is provided [9] by the perturbation theory of QCD and the OPE due to asymptotic freedom of QCD. Therefrom the correlators under discussion increase at large $p^2$: $\Pi(p^2)|_{p^2 \to \infty} \sim p^2 \ln \frac{p^2}{\mu^2}$. When comparing the two approaches one concludes that the infinite series of resonances with the same quantum numbers should exist in order to reproduce the perturbative asymptotic.

Meantime the differences of correlators of opposite-parity currents rapidly decrease at large momenta [8], [12]: $(\Pi_{\text{P}}(p^2) - \Pi_{\text{S}}(p^2))|_{p^2 \to \infty} \equiv \Delta_{\text{SP}} \equiv \Delta_{\text{SP}}(p^2) + O(1/p^6)$,

$\Delta_{\text{SP}} \simeq 24\pi \alpha_s \langle \bar{q}q \rangle^2$ and

$(\Pi_{\text{V}}(p^2) - \Pi_{\text{A}}(p^2))|_{p^2 \to \infty} \equiv \Delta_{\text{VA}} \equiv \Delta_{\text{VA}}(p^2) + O(1/p^8)$,

$\Delta_{\text{VA}} \simeq -16\pi \alpha_s \langle \bar{q}q \rangle^2$, where $\langle \bar{q}q \rangle$ is a quark condensate, we have defined for $V,A$ fields $\Pi_{\mu A}(p^2) \equiv (-\delta_{\mu\nu}p^2 + p_\nu p_\rho)\Pi_{V,A}(p^2)$, and the vacuum dominance hypothesis [9] in the large-$N_c$ limit is adopted.

Therefore the chiral symmetry is restored at high energies and the two above differences represent genuine order parameters of CSB in QCD. As they decrease rapidly at large momenta one can perform the matching of QCD asymptotic by means of few lowest lying resonances that gives a number of constraints from the CSR. They may be used both for obtaining some additional bounds on the model parameters and for calculating of some decay constants (see in [13][14] and references therein). In the present talk the QNJLM is presented with two channels where two pairs of SPVA-mesons are generated. Respectively it is expected to reproduce the lower part of QCD meson spectrum.

2. Quasilocal Quark Model of NJL-type

The minimal $n$-channel lagrangian of the QNJLM has [14][16] the following form,

$$L = \bar{q}i\partial_q + \frac{1}{4N_c A^2} \sum_{k,l=1}^2 \{a_{kl}[\bar{q}f_kq \cdot \bar{q}f_lq + \bar{q}f_ki\gamma_5q \cdot \bar{q}f_lj\gamma_5q]

+ b_{kl}[\bar{q}f_ki\gamma_\mu q \cdot \bar{q}f_lj\gamma_\mu q + \bar{q}f_ki\gamma_\mu \gamma_\nu q \cdot \bar{q}f_lj\gamma_\nu \gamma_\mu q]\}, \quad (1)$$

where $a_{kl}, b_{kl}$ represent symmetric matrices of real coupling constants and $f_k$ are formfactors. We will restrict ourselves by the case $n = 2$ and describe the ground
meson states and their first excitations only.

The observables should not depend on the cutoff \( \Lambda \). The scale invariance is achieved with the help of an appropriate prescription of cutoff dependence for effective coupling constants \( a_{kl}, b_{kl} \). Namely, we require the cancelation of quadratic divergences and parameterize the matrices of coupling constants in the vicinity of poly-critical point as follows: \( 8\pi^2 a_{kl}^{-1} = \delta_{kl} - \frac{\Lambda^2}{4\pi^2}; 16\pi^2 b_{kl}^{-1} = \delta_{kl} - \frac{\Lambda^2}{4\pi^2}; \Delta_{kl}, \Delta_{kl} \ll \Lambda^2 \). The last inequalities provide the masses to be essentially less than the cutoff.

The parameters \( \Delta_{kl} \) just describe the deviation from a critical point and determine the physical masses of scalar mesons. The CSB is generated by the dynamic quark mass function corresponding to nontrivial v.e.v.’s of scalar fields \( \sigma_1, \sigma_2 \):

\[
M(\tau) = \sigma_1 f_1(\tau) + \sigma_2 f_1(\tau); \quad M_0 \equiv M(0) = 2\sigma_1; \quad \tau \equiv -\frac{p_0^2}{m_0^2}.
\]

The physical mass spectrum can be found from solutions of the corresponding secular equation, \( det(Ap^2 + B) = 0; \) \( m^2_{phys} = -p_0^2 \), where \( A \) and \( B \) represent the kinetic term and the momentum independent part correspondingly. Let us display the mass-spectrum for ground meson states and their first excitations. We introduce the notations, \( \sigma^2 \equiv \sigma_1^2 + \frac{\Lambda^2}{3\delta}; \sigma_2 \equiv 3\sigma_2^2 > 0; d \equiv 3\Delta_{11} + 2\sqrt{3}\Delta_{12} + \Delta_{22}, \) and take into account the consistency inequalities \( \Delta_{22} < 0, \Delta_{22} < 0 \). The spectra for scalar and pseudoscalar mesons are:

\[
m_\sigma = 4\sigma_1 = 2M_0; \quad m_\pi = 0; \quad m_\sigma^2 \approx -\frac{4}{3}\Delta_{22} + \sigma^2; \quad m_{\sigma'}^2 - m_\sigma^2 \approx 2\sigma^2 > 0. \]

The spectra for vector and axial-vector mesons are:

\[
m_\rho \approx -\frac{\Lambda^2}{2\Delta_{22} \ln \frac{\Lambda^2}{\rho_0^2}}; \quad m_{a_1} \approx m_\rho^2 + 6M_0^2; \quad m_{\rho'}^2 \approx -\frac{4}{3}\Delta_{22} - \frac{d}{6\ln \frac{\Lambda^2}{\rho_0^2}} - m_{\rho'}^2; \quad m_{a_1}^2 - m_{\rho'}^2 \approx \frac{4}{3}(m_{a_1}^2 - m_{\rho'}^2) \approx 3\sigma^2 > 0. \]

We identify \( \sigma \) with \( f_0(400 \div 1200) \), \( \sigma' \) with \( f_0(1370) \), \( \pi' \) with \( \pi(1300) \), \( \rho \) with \( \rho(770) \), \( \rho' \) with \( \rho(1450) \) and \( a_1 \) with \( a_1(1230) \). The experimental data give us:

\[
m_\sigma = 400 \div 1200 \text{ MeV}; \quad m_\sigma' = 1200 \div 1500 \text{ MeV}; \quad m_\pi' = 1300 \pm 100 \text{ MeV}; \quad m_\rho = 770 \pm 0.8 \text{ MeV}; \quad m_{\rho'} = 1465 \pm 25 \text{ MeV}; \quad m_{a_1} = 1230 \pm 40 \text{ MeV}. \]

The prediction for the mass of \( \sigma \)-meson is then \( m_\sigma \approx 800 \text{ MeV} \), which is close to the averaged experimental value. Furthermore we have the following prediction for the mass of \( a_1'- \)particle, \( m_{a_1'} \approx 1465 \div 1850 \text{ MeV} \). The large range for a possible mass of \( a_1' \)-meson is accounted for by a big experimental uncertainty for the mass of \( \sigma' \) and \( \pi' \) mesons. If we accept the averaged values for them and use the CSR rules, then \( m_{a_1'} - m_{\rho'} \approx 30 \text{ MeV} \). One can confront this value with a phenomenological estimate, \( m_{a_1'} = 1640 \pm 40 \text{ MeV} \).

### 3. Chiral Symmetry Restoration Sum Rules

Let us exploit the constraints based on chiral symmetry restoration in QCD at high energies. Expanding the meson correlators in powers of \( p^2 \) one arrives to the CSR Sum Rules. In the scalar-pseudoscalar case they read:

\[
\sum_n Z_S^S m_{2,n}^2 \equiv \sum_n Z_S^P m_{2,n}^2 = \Delta_S; \quad \sum_n Z_V^S m_{2,n}^2 \equiv \sum_n Z_V^P m_{2,n}^2 = \Delta_V; \quad \sum_n Z_A^S m_{2,n}^2 \equiv \sum_n Z_A^P m_{2,n}^2 = \Delta_A; \quad \sum_n Z_S^0 m_{2,n}^2 \equiv \sum_n Z_V^0 m_{2,n}^2 = \Delta_V; \quad \sum_n Z_A^0 m_{2,n}^2 \equiv \sum_n Z_A^0 m_{2,n}^2 = \Delta_A.
\]

One obtains:

\[
\sum_n Z_S^S m_{2,n}^2 \equiv \sum_n Z_S^P m_{2,n}^2 = \Delta_S; \quad \sum_n Z_V^S m_{2,n}^2 \equiv \sum_n Z_V^P m_{2,n}^2 = \Delta_V; \quad \sum_n Z_A^S m_{2,n}^2 \equiv \sum_n Z_A^P m_{2,n}^2 = \Delta_A; \quad \sum_n Z_S^0 m_{2,n}^2 \equiv \sum_n Z_V^0 m_{2,n}^2 = \Delta_V; \quad \sum_n Z_A^0 m_{2,n}^2 \equiv \sum_n Z_A^0 m_{2,n}^2 = \Delta_A.
\]
Sum Rules, with $f_n$ being the pion decay constant. The residues in resonance pole contributions in the vector and axial-vector correlators have the structure, $Z_{n}^{(V,A)} = 4f_{(V,A),n}^{2}m_{(V,A),n}^{2}$, with $f_{(V,A),n}$ being defined as corresponding decay constants.

In the scalar-pseudoscalar case it has been obtained \cite{12,15} that the residues in poles are of different order of magnitude; the second CSR Sum Rule results in the estimation for splitting between the $\sigma'$- and $\pi'$-meson masses: $m_{\sigma'}^{2} - m_{\pi'}^{2} \approx \frac{1}{6}m_{\sigma}^{2}$; and the value $L_{8} = (0.9 \pm 0.4) \cdot 10^{-3}$ from \cite{16} accepts $m_{\sigma} \approx 800$ MeV.

In the vector-axial-vector case all residues are found to be of the same order of magnitude in contrast to the scalar-pseudoscalar channel \cite{18}. The first and the second Sum Rule is fulfilled identically in the large-log approach. The third one takes the form: $Z_{1}(m_{a_{1}'}^{2} - m_{\rho'}^{2}) \approx 16\pi\alpha_{s} < \bar{q}q >^{2}$. The structure of $Z_{\rho'}$ and $Z_{a_{1}'}$ shows that if $m_{a_{1}'} \approx m_{\rho'}$ then $Z_{a_{1}'} \approx Z_{\rho'}$ and therefore $f_{a_{1}'} \approx f_{\rho'}$. As a consequence these residues approximately cancel each other in Sum Rules and after evaluating we get $f_{\rho} \approx 0.15$ and $f_{a} \approx 0.06$ to be compared with the experimental values \cite{16} $f_{\rho} = 0.20 \pm 0.01$, $f_{a} = 0.10 \pm 0.02$. We have also a reasonable prediction for the chiral constant $L_{10}$ for the $\rho, \alpha_{1}$-mesons and their first excitations (n=2) one gets $L_{10} = \approx -6.0 \cdot 10^{-3}$, which is consistent with that one \cite{16} from hadronic $\tau$ decays: $L_{10} = -(6.36 \pm 0.09) |_{\text{exp}} \pm 0.16 |_{\text{theor}} \cdot 10^{-3}$. It is worth to mention also that within the four-resonance ansatz (n=2) and using two first Weinberg sum rules one obtains the estimation of electromagnetic pion-mass difference $\Delta m_{\pi}^{4} |_{\text{em}} \approx (3.85 \pm 0.16)$ MeV, (see \cite{17}) which improves the agreement between theoretical predictions and the experimental value of $\Delta m_{\pi} |_{\text{exp}} \approx (4.42 \pm 0.03)$ MeV.

4. Summary

Let us summarize the results presented in this talk.

(i) The mass of the second axial-vector particle with $I = 1$ is predicted. It is compatible with the mass of the vector counterpart: $m_{a_{1}'} = 1465 \div 1850$ MeV
and the most plausible value of the mass difference is $m_{a_{1}'} - m_{\rho'} \approx 30$ MeV;
(ii) The estimation on the mass of the $\sigma$-meson does not contradict to existing experimental data \cite{14}, $m_{\sigma} \approx 800$ MeV;
(iii) The couplings $f_{\rho}, f_{a}$ and the chiral constant $L_{10}$ as well as the electromagnetic pion-mass difference $\Delta m_{\pi}^{4} |_{\text{em}}$ \cite{17} are evaluated from CSR Sum Rules as matching rules for QNJLM to QCD at intermediate energies.

Finally we would like to mention that the QNJL Models can be used to describe Higgs particles in extensions of the Standard Model, see \cite{15,19}.

Acknowledgements

We express our gratitude to the organizers of the International Symposium "MENU 2004" in Beijing for hospitality and financial support. This work is supported by
Grant RFBR 04-02-26939, by Grant INFN/IS-PI13 and the Program "Universities of Russia".

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