Discovery of the doubly charmed $T_{cc}^+$ state implies a triply charmed $H_{ccc}$ hexaquark state

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The doubly charmed exotic state $T_{cc}^+$ recently discovered by the LHCb Collaboration could well be a $DD^*$ molecular state long predicted in various theoretical models, in particular, the $DD^*$ isoscalar axial vector molecular state predicted in the one-boson-exchange model. In this work, we study the $DD^*$ system in the Gaussian Expansion Method with the $DD^*$ interaction derived from the one-boson-exchange model and constrained by the precise binding energy of $273 \pm 63$ keV of $T_{cc}^+$ with respect to the $D^+D^0$ threshold. We show the existence of a $DDD^*$ state with a binding energy of a few hundred keV and spin-parity $1^-$. Its main decay modes are $DD\pi$ and $DD\gamma$. The existence of such a state could in principle be confirmed with the upcoming LHC data and will unambiguously determine the nature of the $T_{cc}^+$ state and of the many exotic state of similar kind, thus deepening our understanding of the non-perturbative strong interaction.

Introduction.— Starting from the discovery of $D_{s0}^*(2317)$ and $X(3872)$ in 2003, a large number of the so-called exotic states that do not fit into the conventional quark model have been observed, which have led to intensive studies both theoretically and experimentally [3–8]. The latest addition to this long list is the $T_{cc}^+$ state reported by the LHCb Collaboration at the European Physical Society conference on high energy physics 2021 [9, 10]. This state has a minimum quark content of $cc\bar{u}\bar{d}$ with a binding energy of $B = 273 \pm 61 \pm 5_{11}^{+14}$ keV with respect to the $D^+D^0$ threshold and a decay width of $\Gamma = 410 \pm 165 \pm 43^{+18}_{-38}$ keV. Although such a doubly charmed state has long been anticipated theoretically [11–20], it has remained elusive experimentally until now. Being the first doubly charmed tetraquark state, its discovery will undoubtedly usher in a new era in hadron spectroscopy studies and advance our understanding of the non-perturbative strong interaction.

The measured binding energy and preferred quantum numbers of the $T_{cc}^+$ state are in very good agreement with our predictions based on the one-boson-exchange(OBE) model [16, 20], thus qualifies as a $DD^*$ molecule with $I(J^P) = 0(1^-)$. An urgent question of high relevance is to understand the nature of this state, how to distinguish the various interpretations, and study the consequences.

Being close to some certain two-hadron thresholds is only a necessary but not sufficient condition for a particular hadron to be of molecular nature. Taking $X(3872)$ as one example, after almost 20 years of extensive studies, there is still ongoing heated debate about its true nature being either a conventional $c\bar{c}$ charmonium, a $DD^*$ molecule, a compact tetraquark state, or a combination of them. In a series of recent studies [21–30], we argued that one way to check the molecular nature of certain exotic hadrons is to search for existence of multi-hadron molecules built from the same constituents, in the way that atomic nuclei are bound states of multi-nucleons[1]. More specifically, it was shown that if $D_{s0}^*(2317)$ is dominantly a DK bound state, then $DDKK$ and $DDKK$ states should exist [23, 24, 28]. Similarly, if the latest $T_{cc}^+$ state is indeed a $DD^*$ molecule, then it is very likely that a $DDD^*$ bound state exists (see Fig. 1). Given the capacity of the LHCb experiment, such a state could very well be discovered in the near future and thus not only provide a highly nontrivial check on the molecular nature of the $T_{cc}^+$ state but also deepen our understanding of the strong interaction.

In this work, with the latest experimental measurements [9, 10], we fix the $DD^*$ interaction provided by the time-honored OBE model, and study the $DD^*$ system using the Gaussian Expansion Method.

FIG. 1. From $T_{cc}^+$ (as a $DD^*$ molecule) to $H_{ccc}$ (as a $DDD^*$ molecule).

1 For a concrete demonstration that one can confidently deduce the existence of triton from that of deuterion using either the OBE model or a phenomenological model to describe the nucleon-nucleon interaction, see Ref. [31].
2 The predicted $DDK$ state has a minimum quark content of $cc\bar{s}u/d$, isospin $1/2$, and spin-parity $0^-$. Such a state has recently been searched for by the Belle Collaboration [23].
Theoretical formalism.— The Gaussian Expansion Method has been widely used to solve three-, four- and even five-body problems, because of its high precision and rapid convergence [33]. In this framework, the three-body $DD^*$ system is described by the following Schrödinger equation

$$\hat{H}\Psi = E\Psi,$$

where the Hamiltonian $\hat{H}$ includes the kinetic term and three two-body interaction terms

$$\hat{H} = T + V_{DD} + V_{DD'} + V_{DD''}.$$ (2)

In order to solve the Schrödinger equation, we have to first specify the two-body interactions. In our present work, both the $DD$ interaction and the $DD^*$ interaction are derived from the OBE model. In Ref. [24], the $DD$ OBE potential has been derived with the exchange of $\sigma$, $\rho$, and $\omega$ mesons. For the $DD^*$ interaction, one can also exchange a $\pi$ meson in addition to the $\sigma$, $\rho$, and $\omega$ exchanges [20]. It should be noted that the $DD^*$ interaction of Ref. [20] generates a molecular $DD^*$ state with a cutoff of 1.01 GeV, which was fixed by reproducing the binding energy 4.0 MeV of $X(3872)$ with respect to the $DD^*$ threshold. A detailed description of the OBE potential used can be found in Refs. [20][24]. With the relevant couplings between $DD^{(*)}$ and the exchanged mesons fixed (as shown in Table I), the only free parameter is the cutoff related to the regulator function needed to take into account the finite size of exchanged mesons. More specifically, we use a regulator function of the following form

$$F(q, m, \Lambda) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}\right)$$ (3)

where $m$ is the mass of the exchanged meson (see Table II) and $\Lambda$ the cutoff.

First, we slight fine-tune the cutoff (from the value of 1.01 GeV fixed by reproducing a binding energy of 4 MeV for the $DD^*$ bound state assigned to be $X(3872)$ [20]) taking advantage of the latest experimental data [9][10]. As the $T_{cc}$ state is found about 0.3 MeV below the $D^{(*)}D^0$ threshold and the difference between the thresholds of $D^+D^0$ and $D^{(*)}D^0$ is 1.41 MeV, we study three different binding energy scenarios for the $DD^*$ binding energy, i.e., 0.3 MeV, 1.0 MeV, and 1.7 MeV. The so-determined cutoffs for these three scenarios considering only $S$-wave interactions and $S - D$ mixings are given in Table III. In the same table, we also provide the corresponding root-mean-square (RMS) radius of the $T_{cc}$ state. Two things are noteworthy. First, the RMS radius ranges from 3 to 6 fm, consistent with the expectation for a molecular state whose size should be larger than the sum of its constituents. Second, the impact of $S - D$ mixing is small at the two-body level, consistent with the analysis of Ref. [35]. Based on this observation, we only consider $S$-wave interactions among the $D^{(*)}$ mesons in the following study of the $DDD^*$ system.

### Table I. Couplings of the light mesons of the OBE model ($\pi$, $\sigma$, $\rho$, $\omega$) to the heavy $D/D^*$ mesons. For the magnetic-type coupling of the $\rho$ and $\omega$ vector mesons we have used the decomposition $f_{\rho(\omega)} = \epsilon_{\rho(\omega)} f_{\rho(\omega)}$. $M$ (in units of MeV) refers to the mass scale involved in the magnetic-type couplings [20].

| Coupling | Value for $D/D^*$ |
|---------|-------------------|
| $g$     | 0.60              |
| $g_\sigma$ | 3.4        |
| $g_\rho$  | 2.6              |
| $g_\omega$ | 2.6        |
| $g_\rho$  | 4.5              |
| $\kappa$  | 4.5              |
| $M$      | 1867             |

### Table II. Masses and quantum numbers of the light mesons of the OBE model ($\pi$, $\sigma$, $\rho$, $\omega$) and the heavy mesons $D$ and $D^*$ [34].

| Light Meson | $J^P$ | $M$ (MeV) |
|------------|-------|-----------|
| $\pi$      | 1 $(0^+)$ | 138       |
| $\sigma$   | 0 $(0^+)$ | 600       |
| $\rho$     | 1 $(1^-)$ | 770       |
| $\omega$   | 0 $(1^-)$ | 780       |

| Heavy Meson | $J^P$ | $M$ (MeV) |
|-------------|-------|-----------|
| $D$         | 3 $(0')$ | 1867.24   |
| $D^*$       | 3 $(1')$ | 2008.56   |

### Table III. Cutoffs for three different binding energy scenarios and with/without considering $S - D$ mixing. The binding energies ($B$) are in units of MeV and RMS radii ($r$) in units of fm.

| $\Lambda$(Only $S$) | $B_{\text{DD}}$ | $N(S - D)$ | $B_{\text{DD}}$ | $r_{\text{DD}}$ |
|---------------------|-----------------|------------|-----------------|-----------------|
| 976                 | 0.3 5.94        | 945        | 0.3 6.09        |
| 998                 | 1.0 3.47        | 970        | 1.0 3.55        |
| 1013                | 1.7 2.72        | 986        | 1.7 2.81        |

**FIG. 2.** Three permutations of the Jacobi coordinates for the $DDD^*$ system.

As all the two-body interactions have been specified, we employ the GEM to solve the Schrödinger equation. The three-body wave functions can be constructed in Jacobi coordinates as

$$\Psi = \sum_{c=1}^{3} \Psi_c(r_c, R_c),$$ (4)

where $c = 1 - 3$ is the label of the Jacobi channels shown in Fig.2. In each Jacobi channel the wave function $\Psi(r_c, R_c)$ reads

$$\Psi(r_c, R_c) = C_{cT_1} H_{\Lambda}(r_c, R_c).$$ (5)
where \( C_{\alpha \alpha} \) is the expansion coefficient and the \( \alpha = (nN, IT, ILI) \) labels the basis number with the configuration sets of the Jacobi channels. \( H_{\text{FC}} \) is the three-body isospin wave function where \( t \) is the isospin of the subsystem in Jacobi channel \( c \) and \( T \) is the total isospin.

The three-body spatial wave function \( \Phi(r_c, R_c) \) is constructed by two two-body wave functions as

\[
\Phi_{tL}(r_c, R_c) = \left[ \psi_{\alpha \beta}^G(r_c, \psi_{\alpha \beta}^G(R_c) \right]_{\lambda t},
\]

\[
\psi_{\alpha \beta}^G(r_c) = N_{\alpha \beta} \phi_{\alpha \beta}^G Y_{\ell \nu}(r_c),
\]

\[
\psi_{\alpha \beta}^G(R_c) = N_{\alpha \beta} R_{\lambda \ell}^c e^{-i\frac{\delta R}{\sqrt{\nu}} Y_{\ell \nu}(\hat{R}_c).}
\]

Here \( N_{\alpha \beta} \) is the normalization constant of the Gaussian basis, \( n(N) \) is the number of Gaussian basis used, \( \ell(L) \) is the orbital angular momentum corresponding to the Jacobi coordinates \( r(R) \), and \( \lambda \) is the total orbital angular momentum.

With the constructed wave functions, the Schrödinger equation can be transformed into a generalized matrix eigenvalue problem

\[
[T_{\alpha \beta}^{ab} + V_{\alpha \beta}^{ab} - EN_{\alpha \beta}^{ab}] C_{b \alpha} = 0,
\]

where \( T_{\alpha \beta}^{ab} \) is the matrix element of kinetic energy, \( V_{\alpha \beta}^{ab} \) is the matrix element of potential energy, and \( N_{\alpha \beta}^{ab} \) is the normalization matrix element.

**Results and Discussions.**—Considering only \( S \)-wave interactions, the corresponding configurations of the three Jacobi channels are given in Table IV. With these configurations and the OBE potentials specified above, we solve the Schrödinger equation in the GEM method and obtain the results shown in Table V. It is interesting to note that for all the three scenarios studied, the \( DDD^* \) system is bound. Compared to the \( DD^* \) system, the addition of a second meson only increases the binding energy by about 23\%, 29\%, and 34\%, reflecting the fact that the \( DD \) interaction is less attractive than the \( DDD^* \) interaction. This is corroborated by the observation that for all the three scenarios \( r_{DD} \) is larger than \( r_{DD^*} \) and \( |\langle V_{DD^*} \rangle| \) is much larger than \( |\langle V_{DD} \rangle| \).

**TABLE IV.** Quantum numbers of different Jacobi coordinate channels (\( c = 1 \rightarrow 3 \)) of the \( DDD^* \) \( I(J^P) = \frac{1}{2} (1^-) \) state, considering only \( S \)-wave interactions.

| \( c \) | \( \ell \) | \( L \) | \( T \) | \( J \) | \( P \) | \( n_{\text{max}} \) | \( N_{\text{max}} \) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 1/2 | 1 | 10 |
| 1 | 0 | 0 | 0 | 1 | 1/2 | 10 | 10 |
| 2 | 0 | 0 | 0 | 0 | 1/2 | 10 | 10 |
| 2 | 0 | 0 | 0 | 1 | 1/2 | 10 | 10 |
| 3 | 0 | 0 | 0 | 1 | 1/2 | 10 | 10 |

Since the isospin of the studied \( DDD^* \) system is 1/2, this system consists of two charged states, i.e.,

\[
I(1/2, 1/2) : \sqrt{\frac{2}{3}} D^*D^+D^0 - \sqrt{\frac{1}{6}} (D^+D^0 + D^0D^+D^+),
\]

\[
I(1/2, -1/2) : -\sqrt{\frac{2}{3}} D^0D^*D^+ + \sqrt{\frac{1}{6}} (D^+D^0 + D^0D^+D^+).
\]

**TABLE V.** Binding energies, RMS radii and Hamiltonian expectation values of the \( DDD^* \) system with \( I(J^P) = \frac{1}{2} (1^-) \) and \( S \)-wave OBE interactions.

| \( \Lambda \) (MeV) | \( B \) (MeV) | \( r_{DD} \) | \( r_{DD^*} \) | \( T \) | \( \langle V_{DD} \rangle \) | \( \langle V_{DD^*} \rangle \) |
|---|---|---|---|---|---|---|
| 976 | 0.37 | 8.20 | 10.32 | -10.53 | -0.17 |
| 998 | 1.29 | 5.11 | 6.72 | 20.48 | -21.17 | -0.60 |
| 1013 | 2.27 | 3.87 | 5.06 | 27.81 | -29.11 | -0.98 |

The Coulomb interaction may play a role for the doubly charged state which corresponds to the \( I_3 = 1/2 \) component. We include the Coulomb interaction for this state and find that the binding energies are 0.15, 0.80, and 1.61 MeV corresponding to the cutoff 0.967, 0.998, and 1.013 GeV, respectively. The results are shown in Table VI. The coulomb interaction makes the binding energy of the doubly charged state slightly smaller compared to the singly charged one, but is not strong enough to break it up. The main reason is that the \( DD \) pair is widely separated at a distance of about 10 fm.

**TABLE VI.** Binding energies, RMS radii and Hamiltonian expectation values of the doubly charged \( I(J^P) = \frac{1}{2} (1^-) \) \( DDD^* \) state with \( S \)-wave OBE and Coulomb interactions.

| \( \Lambda \) (MeV) | \( B \) (MeV) | \( r_{DD} \) | \( r_{DD^*} \) | \( T \) | \( \langle V_{DD} \rangle \) | \( \langle V_{DD^*} \rangle \) |
|---|---|---|---|---|---|---|
| 976 | 0.15 | 8.78 | 10.41 | 6.83 | -6.99 | 0.02 |
| 998 | 0.80 | 7.88 | 10.60 | 15.10 | -15.87 | -0.04 |
| 1013 | 1.61 | 5.65 | 7.64 | 23.40 | -24.72 | -0.28 |

**FIG. 3.** Decay mechanism of \( H_{cc} \)

In principle the predicted triply charged \( H_{cc} \) state can decay into \( DDD^\pi \) because the \( DDD^* \) system could be viewed as a weakly bound \( DT_{cc} \) state, in which the \( T_{cc} \) state decays into \( DD\pi \) as observed by the LHCb Collaboration [9, 10]. Such a process is schematically shown in Fig. 3. Theoretically, as \( D^* \) can also decay into \( D \gamma \), the \( H_{cc} \) state can also be observed in the \( DDD^\gamma \) mode. According to the LHCb measurements, the estimated yield of \( T_{cc} \rightarrow DD\pi \) with respect to that of \( X(3872) \rightarrow DD\pi \) is about 1/20 [10]. Naively the yield of \( H_{cc} \rightarrow DDD^\pi \) with respect to that of \( T_{cc} \rightarrow DD\pi \) might only be one order of magnitude smaller, thus accessible to future LHCb experiments.

**Summary and outlook.** — The recently discovered doubly charmed \( T_{cc} \) state is consistent with a \( D^* \) molecule predicted in the OBE model. The precisely measured binding energy

with respect to the $D^{**}D^0$ threshold allows one to fix the $D^*$ interaction. We utilized this valuable information and studied the three-body $D^{**}$ system with the Gaussian Expansion Method. Our studies showed that the $D^{**}$ system is bound even taking into account the Coulomb interaction. We discussed the possible decay modes where the $D^{**}$ states can be discovered. We strongly encourage that this state be directly searched for at present and future experiments.

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