Simulation and verification of master S-N curves of titanium alloy welded structures based on bootstrap method

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Abstract
In the small sample fatigue test space, in order to obtain the master S-N curves of each reliability of the titanium alloy welded structures. Firstly, the Bootstrap method is combined with the equivalent structural stress to determine the minimum number of samples under each level of equivalent structural stress, and the appropriate resampling times are obtained through residual analysis. Secondly, based on the fatigue test data of titanium alloy welded joints with different plate thicknesses, materials, types and stress ratios, the linear regression model of master S-N curves are determined by using the traditional group method and Bootstrap method, and its reliability is 99%, 95%, 84%, and 50% respectively. Finally, the comparative analysis shows that the titanium alloy fatigue test samples obtained by Bootstrap method tends to be more stable, which improves the fitting degree of the titanium alloy fatigue life test data and the accuracy of the fatigue life prediction when the sample is small. A more accurate regression model of the master S-N curves of titanium alloy welded structures was obtained and verified.

Keywords
Bootstrap method, titanium alloy, master S-N curves, welded structure, small sample

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Introduction
With the rapid development of our country's economy, people's fast-paced lifestyle and high-quality travel demands have posed unprecedented challenges to the rail vehicle transportation industry.1 In order to achieve a balance between the need to increase the speed of the rail vehicle and the strength requirements of the vehicle itself, it is necessary to make the rail vehicles aim at lightweight and reduce their own weight. Therefore, the lightweight will become the long-term goal of the rail transit industry.2,3 The lightweight of rail transit vehicles can save energy heavily. Relevant studies have shown that if the no-load weight of rail vehicles is reduced by 10%, the energy consumption can be reduced by about 7%. At the same time, the lightweight of rail vehicles can reduce the damage caused by the impact load of the track during the operation of the train. The application of traditional steel and aluminum alloy in rail transit vehicles has been relatively mature, and it is difficult to continue to realize lightweight.4 Titanium alloy with the characteristics of high specific strength, strong corrosion resistance, and low density provides a possible direction for the lightweight of rail transit vehicles.5 Therefore, it is of great significance to

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study the fatigue and life prediction of titanium alloy materials.

In recent years, many researchers have studied the titanium alloy, and put forward and revised the prediction method of its fatigue life. Li et al. obtained the fatigue life of TC21 titanium alloy under different stress concentration factors and different stress ratios, and obtained the S-N curve suitable for TC21 titanium alloy. Wang et al. found that the crack propagation rate of small crack is faster than that of long crack under the same load. Based on the unified fatigue life prediction method, considering the elastic-plastic behavior at the tip of small crack, an improved small crack propagation rate model is proposed, and the fatigue life of small crack is predicted based on the improved constitutive relationship. Moussaoui et al. conducted four point bending test and analyzed the influence of surface integrity defined by geometric, mechanical and metallurgical parameters on the fatigue life of Ti-6Al-4V titanium alloy. Finally, it was concluded that only mechanical parameters had the main influence on the fatigue life. Fang et al. determined the stress intensity factor amplitude of each crack propagation stage based on the full field method, avoided the deviation of the measured crack length caused by the change of propagation direction during crack propagation, and then obtained the exact stress intensity factor amplitude of Ti-2Al-1.5Mn titanium alloy. Yang et al. applied the rough set, combined with the welding fatigue test data of titanium alloy with different plate thickness, different load ratio, different stress ratio, different welding methods and different materials, fitted its master S-N curve through linear regression based on the equivalent structural stress, and proved its higher accuracy through comparison. Wang et al. established a bimodal lognormal distribution model based on the fatigue life test data of DED-TA15 titanium alloy, and accurately predicted the life prediction model within a certain confidence interval by using the Bootstrap method. Liu et al. based on Non-intrusive Polynomial Chaos method, the Basquin stress-life model was solved and the goodness of fit test was carried out by applying the fatigue life data of small sub-samples under less stress levels. Wan et al. used the fatigue test bench to evaluate the fatigue reliability of a small sample of the loader, and obtained the mathematical relationship that the life reliability of the loader working device is a third degree polynomial. Li et al. revised the distribution coefficient of the fatigue probabilistic S-N (P-S-N) curve and proposed an improved backward statistical inference approach (ISIA), which improved the prediction accuracy of small-number sample life data. Due to the long-term fatigue test cycle and high test cost of titanium alloy welded joints, test data sample usually is small sample space, and the analysis results by the traditional group method are not accurate. Therefore, the research on the master S-N curves of titanium alloy welded structures in small sample fatigue test space is relatively few, and then the research on the master S-N curves is of great significance.

In order to study the master S-N curves of titanium alloy welded structures in small sample fatigue test space, combined with the fatigue test data of different plate thickness, different materials and different types of titanium alloy welded joints, combining the non-parametric Bootstrap method and the equivalent structural stress, the master S-N curves of titanium alloy welded structures with different reliability are obtained. Moreover, compared with the master S-N curves obtained by nonparametric Bootstrap method and the master S-N curves obtained by traditional group method, which proves the feasibility of the combination of nonparametric Bootstrap method and equivalent structural stress.

**Equivalent structural stress principle**

The crack propagation process can be divided into a short crack stage and a long crack stage. Assuming that the crack propagation process conforms to the energy law, the Paris formula shown in equation (1) can be used to define the full range of crack propagation modes uniformly.

\[
\frac{da}{dN} = C(M_{th})^n(\Delta K_n)^m
\]

Where \(a\) is the crack length, \(N\) is the fatigue life, \(C\) is the material constant, \(m\) and \(n\) is the crack propagation index considering long and short cracks, and \(M_{th}\) is the stress intensity amplification factor. Integrating equation (1), the fatigue life prediction expression from small crack to penetration thickness \(t\) can be obtained as follows:

\[
N = \int_{a/t=0}^{a/t=1} \frac{t \, d(a/t)}{C(M_{th})^n(\Delta K)^m} = \frac{1}{C} \, t^{1-n/2}(\Delta \sigma_a)^{-m} I(r)
\]

Where \(r\) is the thickness of the plate, and \(\Delta K\) is the variation range of the stress intensity factor that conforms to the distal stress range, and its expression is as follows:

\[
\Delta K = \sqrt{\pi} \left[ \Delta \sigma a_{p} f_m(a/t) + \Delta \sigma b f_b(a/t) \right]
\]

\(f_m(a/t)\) and \(f_b(a/t)\) are dimensionless functions that determine the range of the stress intensity factor when membrane stress and bending stress act alone, and the calculation equations are as follows:

\[
f_m(a/t) = 1.12 \sqrt{\pi(a/t)}
\]
The fundamental idea of the Bootstrap method is to use the observed samples to estimate the population parametric distribution, that is, to use the Bootstrap subsample to simulate the sampling of the population sample, so as to obtain an estimate of the unknown parameter distribution. The specific process of the Bootstrap method is as follows:

1. Some observations in all the data are known, and the observation sample sequence is composed as: \( \bar{X} = \{x_1, \ldots, x_i, \ldots, x_n\} \);
2. Specify the unknown parameter \( \theta \) that needs to be counted;
3. Give an estimator computational model for the unknown parameter \( \theta; \theta = S(X) \);
4. Resample the known observation samples with equal probability to get the parameter \( \theta^* \) of the Bootstrap subsample \( X^* \);
5. Calculate the model \( \theta = S(X) \) according to the estimated \( \theta \), and calculate the parameter \( \theta^* \) of the Bootstrap subsample \( X^* \);
6. Repeat process (4) and process (5) \( m \) times;
7. According to the estimated parameter sequence \( \{\theta^*_1, \ldots, \theta^*_1, \ldots, \theta^*_m\} \) obtained by the resampling calculation, the parameter distribution of \( \theta \) is obtained and used as the estimation of the population parametric distribution. The specific idea of Bootstrap method is shown in Figure 1.

**Application of Bootstrap method in master S-N curves**

In the fatigue assessment of welded structures, the master S-N curves method with the equivalent structural stress as the core parameter is widely used.\(^{19}\) The relationship between the equivalent structural stress amplitude \( \Delta S_s \) and the fatigue life \( N \) in the master S-N curves is as shown in equation (9):

\[
N = (\Delta S_s/C_d)^{-1/h}
\]  

(9)

Where \( C_d \) and \( h \) are experimental constants. So when the master S-N curves is applied in double logarithmic coordinates, the relationship is as follows:

\[
h \log N = \log C_d - \log \Delta S_s
\]  

(10)

Assuming \( Y = \log N, X = \log \Delta S_s \), when the equation (10) is determined, the equivalent structural stress amplitude \( \Delta S_s \) can be accurately measured through experiments, but under an equivalent structural stress amplitude \( \Delta S_s \), the obtained fatigue life is uncertain. A large number of fatigue test data show that \( Y \) obeys a
normal distribution $Y \sim N(\mu_Y(X), \sigma(X))$. According to equation (10), its mean value is obtained as follows:

$$\mu_Y(X) = a + bX \quad (11)$$

where $a$ and $b$ are constants; $\sigma(X)$ is the standard deviation related to the equivalent structural stress.

According to the fatigue tests under different equivalent structural stresses, the sample life $N_i(N_i = \{n_{i1}, \cdots n_{ij}, \cdots n_{im}\})$ can be obtained, and then the corresponding logarithmic fatigue life $Y_i(Y_i = \{y_{i1}, \cdots y_{ij}, \cdots y_{im}\})$ can be obtained. The Bootstrap method is used to increase the sample size to obtain $l$ groups of samples $Y_{1i} \cdots Y_{li}$. According to the Bootstrap method, the corresponding mean values $\bar{Y}_{hi} = \frac{1}{l} \sum_{i=1}^{l} Y_{hi}$ can be estimated. Through the above mean values, the logarithmic fatigue life under the equivalent structural stress can be obtained:

$$\hat{\mu}_Y = \frac{1}{l} \sum_{h=1}^{l} \bar{Y}_{hi} \quad (12)$$

According to the logarithmic equivalent structural stress amplitude and logarithmic fatigue life mean values $(X_i, \hat{\mu}_Y)$ under different equivalent structural stresses, the constants $a$ and $b$ of the median master S-N curves can be obtained.

When determining the master S-N curves under other reliability, the following parameters need to be determined:

$$T = \frac{\mu_Y(X) - \mu_Y(X)}{\hat{\sigma} \sqrt{\frac{n + 1}{n}}} \quad (13)$$

where $\hat{\sigma}$ is the estimator of $\sigma(X)$, and the determination method is as follows:

$$\hat{\sigma} = \sqrt{\frac{1}{n - 1} \sum_{i=1}^{n} (Y_i - \hat{\mu}_Y)^2} \quad (14)$$

According to the sample data obtained by the Bootstrap method, the corresponding standard deviation $\hat{\sigma}_{Y1i}, \cdots, \hat{\sigma}_{Yli}$ can be calculated as follows:

$$\hat{\sigma}_{Yi} = \frac{1}{l} \sum_{h=1}^{l} \hat{\sigma}_{Yhi} \quad (15)$$

According to equations (13)–(15), the fatigue life under different reliability can be obtained, as shown in equation (16):

$$Y_{pi} = \hat{\mu}_Y - u_p\hat{\sigma}_{Yi} \quad (16)$$

Through the derivation of the above equations, the $(X_i, Y_{pi})$ under different equivalent structural stresses and different reliability can be obtained, so as to obtain the master S-N curves.

**Application analysis of Bootstrap method combined with master S-N curves of titanium alloy welded structures**

**Fatigue life prediction of small samples of titanium alloys**

In this paper, 84 titanium alloy welded joints with different plate thicknesses, materials, types and stress ratios are used to make fatigue test specimens. It includes six joint types, each joint type is loaded with different stress levels, and each welded joint contains three fatigue specimens at the same stress level, and 84 fatigue specimens can be divided into 28 groups. SolidWorks is used to model various joints. The geometric shapes are shown in Figure 2.

Literature21–28 gives the fatigue life corresponding to various stress levels of different titanium alloy welded joints. The same constraints and loading methods as the corresponding welded joints in literature21–28 are used to constrain and load the finite element model. It can be seen from Table 1 that the stress level loaded by each type of welded joint is different, and a stress level corresponding to each type of welded joint in Table 1 is selected to describe the loading method, as shown in Figure 3. ANSYS is used to calculate and analyze the entire welded joint. The stress nephograms of the selected welded joint specimens at the corresponding stress levels are shown in Figure 4 (the unit of stress in the stress nephogram is MPa). Combined with the analysis results of ANSYS, the equivalent structural stresses $\Delta S_i$ corresponding to each stress level can be calculated by FE-weld software. The test results are shown in Table 1.

**The master S-N curves obtained by the Bootstrap method**

According to the Bootstrap method described above, the logarithmic fatigue test life under each equivalent structural stress is sampled, and the resampling times is 2000. Partial resampling results are shown in Table 2.

According to the Bootstrap method for resampling life samples and the process described in Section 2.2 of this paper, the master S-N curves of titanium alloy welded structures under different reliability can be obtained, as shown in Figure 5. The coefficient of determination ($R^2$) to measure the goodness of the fitting curve can be obtained through calculation and analysis and its magnitude is 0.9451.
Comparison of the master S-N curves obtained by the Bootstrap method and the group method

When using the Bootstrap method to estimate the average fatigue life under each stress level, it is necessary to count the distribution of the resampled fatigue test samples under each stress level. The distribution of the resampled fatigue test samples at only 4 stress levels is listed below, as shown in Figure 6.

The traditional group method and Bootstrap method are used to fit the master S-N curves of titanium alloy welded structures with different reliability. Among them, the logarithmic fatigue life mean values of titanium alloy welded structures can be obtained by the group method and Bootstrap method, and it is shown in Figure 7(a). Figure 7(b) shows the comparison of the standard deviations between the logarithmic fatigue life of the titanium alloy welded structures at each stress level obtained by the two methods.

It can be seen from Figure 7 that the logarithmic fatigue life mean values calculated by the traditional group method and the Bootstrap method are generally consistent, but the standard deviations of samples obtained by bootstrap method is much smaller than that obtained by group method. The reason for the large difference in the sample standard deviations obtained by the two methods is that the sample size of the traditional group method is small, so the standard deviations are quite different. After resampling by the Bootstrap method, the sample size is expanded to a certain extent. Then the standard deviation stability increases.

The master S-N curves of titanium alloy welded structures at different reliability levels that can be obtained according to the traditional group method are shown in Figure 8. Through calculation and analysis, its coefficient of determination ($R^2$) is 0.8553.

Figure 2. Structural dimensions of welded joints: (a) flat butt joint, (b) cylindrical butt joint, (c) cross joint with longitudinal vertical plate, (d) cross joint, (e) T-joint, and (f) lap joint.
Figure 3. Loading methods of welded joints.

Table 1. Fatigue test results of titanium alloy welded structures.

| Group number | Types of welded joints                        | First principal stress $\lg \sigma_1 / \text{MPa}$ | Equivalent structural stress $\lg \Delta \sigma / \text{MPa}$ | Logarithmic fatigue life $\lg N / \text{cycle}$ |
|--------------|-----------------------------------------------|------------------------------------------------|------------------------------------------------|-----------------------------------------------|
| 1            | Cross joint                                   | 2.2648                                        | 2.3168                                        | 6.405, 6.8202, 6.7662                          |
| 2            |                                                | 2.2788                                        | 2.307                                         | 6.4206, 6.1412, 6.1562                          |
| 3            |                                                | 2.3160                                        | 2.3679                                        | 5.9492, 5.8316, 5.7289                          |
| 4            |                                                | 2.3464                                        | 2.3983                                        | 5.1135, 5.3650, 5.7816                          |
| 5            |                                                | 2.3617                                        | 2.4137                                        | 5.0233, 5.3374, 5.3045                          |
| 6            | Cross joint with longitudinal vertical plate  | 2.3711                                        | 2.4230                                        | 5.4236, 6.0530, 5.5801                          |
| 7            |                                                | 2.3874                                        | 2.4394                                        | 6.0829, 5.7923, 5.9747                          |
| 8            |                                                | 2.4065                                        | 2.4585                                        | 5.3891, 5.6703, 5.2765                          |
| 9            |                                                | 2.4330                                        | 2.4849                                        | 5.4663, 5.1848, 5.2584                          |
| 10           |                                                | 2.4594                                        | 2.5114                                        | 4.6801, 4.8922, 5.0905                          |
| 11           | Lap joint                                     | 2.5549                                        | 2.6652                                        | 4.6423, 6.6435, 6.6544                          |
| 12           |                                                | 2.5682                                        | 2.6785                                        | 4.1500, 4.1700, 4.3000                          |
| 13           |                                                | 2.5911                                        | 2.7014                                        | 4.0414, 4.1067, 4.1289                          |
| 14           |                                                | 2.6232                                        | 2.7335                                        | 3.6149, 3.9306, 3.8156                          |
| 15           | T-joint                                       | 2.6021                                        | 2.6812                                        | 4.7361, 4.7534, 4.5563                          |
| 16           | Cylindrical butt joint                        | 2.4914                                        | 2.4304                                        | 5.0815, 5.8527, 5.9574                          |
| 17           |                                                | 2.5051                                        | 2.4442                                        | 5.1900, 5.1700, 5.2990                          |
| 18           |                                                | 2.5378                                        | 2.4768                                        | 4.6572, 4.6612, 4.6949                          |
| 19           |                                                | 2.5740                                        | 2.5131                                        | 4.2726, 4.6163, 4.6922                          |
| 20           |                                                | 2.6021                                        | 2.6812                                        | 4.9494, 4.8692, 4.9294                          |
| 21           |                                                | 2.6532                                        | 2.5922                                        | 4.5911, 4.8196, 4.6628                          |
| 22           | Flat butt joint                               | 2.4983                                        | 2.4681                                        | 5.0000, 4.9000, 5.1100                          |
| 23           |                                                | 2.5185                                        | 2.4883                                        | 4.6021, 4.6353, 5.0443                          |
| 24           |                                                | 2.5911                                        | 2.7014                                        | 4.2100, 4.2500, 4.300                           |
| 25           |                                                | 2.6675                                        | 2.6442                                        | 4.1100, 4.1700, 4.2000                          |
| 26           |                                                | 2.6990                                        | 2.6687                                        | 4.0140, 4.1500, 4.1300                          |
| 27           |                                                | 2.7243                                        | 2.7010                                        | 3.5300, 3.6000, 3.7000                          |
| 28           |                                                | 2.7782                                        | 2.7549                                        | 3.3600, 3.4700, 3.5000                          |
By comparing Figures 5 and 8, the master S-N curves of the titanium alloy welded structure fitted by the traditional group method and the Bootstrap method can be compared and analyzed under different reliability. In general, the closer the coefficient of determination ($R^2$) of the fitted curve obtained by linear regression of the observed value is to 1, the better the fitting degree of the statistic. Comparing the coefficient of determination ($R^2$) obtained by the two methods, it shows that the Bootstrap method has a better fitting degree to the test data than the group method. It can be judged from this that the Bootstrap method improves the fitting degree of the fatigue life test data of titanium alloy, and largely solves the problem of low accuracy when the sample is small.

**Figure 4.** Stress nephograms of welded joints: (a) flat butt joint, (b) cylindrical butt joint, (c) cross joint with longitudinal vertical plate, (d) cross joint, (e) T-joint, and (f) lap joint.

**Table 2.** Partial resampling life samples of Bootstrap method.

| Group number | Equivalent structural stress $\Delta S$, MPa | Resampling fatigue life samples $\lg N$, cycle |
|--------------|--------------------------------------------|---------------------------------------------|
| 1            | 2.3168                                     | 6.4058, 6.8202, 6.7662, 6.8202, ...         |
| 2            | 2.3307                                     | 6.4206, 6.1412, 6.1562, 6.5162, ...         |
| 3            | 2.3679                                     | 5.9492, 5.8316, 5.7289, 5.9492, ...         |
| 4            | 2.3983                                     | 5.1135, 5.3650, 5.7016, 5.1135, ...         |
| 5            | 2.4137                                     | 5.0233, 5.3374, 5.3045, 5.3045, ...         |
| 6            | 2.4394                                     | 6.4236, 6.0530, 5.5801, 5.4236, ...         |
| 7            | 2.4394                                     | 6.0829, 5.9235, 5.9747, 5.9747, ...         |
| 8            | 2.4585                                     | 5.3891, 5.6703, 5.2765, 5.6703, ...         |
| 9            | 2.4849                                     | 5.4663, 5.1848, 5.2584, 5.4663, ...         |
Resampling times and sample size of Bootstrap method

When the Bootstrap method is applied, the minimum number of samples under equivalent structural stresses at all levels needs to be determined by the following equation:

$$\delta_{\max} = \frac{\bar{x} \sqrt{\frac{1}{n} + \hat{k}^2 (\gamma - 1) - \delta^p_{\max}} \geq \frac{\bar{x}}{\bar{x}}}{1}$$

Where $\delta_{\max}$ is the limits of error, which is usually taken as 5%; $n$ is the number of fatigue test samples under each stress; $\hat{k}$ is the standard correction factor; $t_{\gamma}$ is the distribution value of $t$, which is determined by the number of test samples $n$ and confidence coefficient; $S_x$ is the standard deviation of logarithmic fatigue life; $\bar{x}$ is the mean value of logarithmic fatigue life; $\mu_p$ is the standard normal bias associated with survival rate.

Figure 5. The master S-N curves of titanium alloy welded structures by Bootstrap method.

Figure 6. Sample distribution of resampling fatigue test for partial welded joints: (a) logarithmic equivalent structural stress: 2.7014, (b) logarithmic equivalent structural stress: 2.6532, (c) logarithmic equivalent structural stress: 2.4624, and (d) logarithmic equivalent structural stress: 2.3617.
When the number of fatigue test samples under the equivalent structural stresses at all levels satisfies the equation (17), the fatigue test accuracy meets the requirements. After calculation and analysis, it is found that the minimum number of samples at all levels of stress in this fatigue test is 3, indicating that the number of samples is 3 and meets the accuracy requirements. However, if the sample size is appropriately increased, the comparison results may be more obvious.

Since the fatigue test object is titanium alloy welded joints, the Bootstrap method has a relatively large number of resampling times. Figure 9 shows the variation trend of lower boundary residual with resampling times. As can be seen from Figure 9, when the resampling times are 2000, the lower boundary residual tends to be stable.

**Verification of test statistical constants and fitting accuracy of master S-N curves**

According to the master S-N curves of double logarithm titanium alloy welded structure processed and fitted by bootstrap method and group method, the test statistical constants of master S-N curves with different reliability under the two methods are obtained, as shown in Table 3.
The correctness of the master S-N curves of titanium alloy welded structures obtained by the two methods is verified below. The fatigue life tests of titanium alloy welded structures have been carried out in literature, which is different from the fatigue test obtained from the data in Table 1. It contains the following key information:

1. Materials: TC4, TA2, Ti80, TC18, etc.;
2. Variation range of plate thickness: 2–60 mm;
3. Joint type: butt joint, cross joint, longitudinal reinforcing rib weld, etc;
4. Applied load: high frequency tension and compression, low frequency tension and compression, etc;
5. Welding forms: Vacuum medium pressure electron beam welding, tungsten inert gas welding, vacuum high pressure electron beam welding, tungsten plate argon arc welding, manual TIG welding, etc.

The equivalent structural stress of each joint welding position is calculated according to the method described in Section 3.1. The equivalent structural stress and its corresponding test fatigue life are shown in Table 4.

Using the master S-N curves obtained by bootstrap method and traditional group method and the master S-N curves parameters shown in Table 3. The fatigue life corresponding to each equivalent structural stress in Table 4 is estimated and compared with the test fatigue life. The results are shown in Figure 10.

It can be seen from Figure 10. The fatigue life under each equivalent structural stress predicted by the master S-N curves of titanium alloy welded structures processed by bootstrap method is basically consistent with the test life, and the maximum error is only 10.24%. The fatigue life under each equivalent structural stress predicted by the master S-N curves of titanium alloy welded structures treated by group method is quite different from the test life, with the maximum of 75.64%.

Through the above comparison, it can be seen that Bootstrap method improves the fitting accuracy of the master S-N curves of titanium alloy welded structures, and verifies the correctness of the master S-N curves of titanium alloy welded structures processed by Bootstrap method.

### Table 3. The master S-N curves parameters of titanium alloy welded structures by two methods.

|                | Statistical basis (%) | \( \lg C_d \) | h |
|----------------|-----------------------|---------------|---|
| (a) Group method | Reliability = 99       | 3.2779        | 0.1863 |
|                | Reliability = 95       | 3.2666        | 0.1741 |
|                | Reliability = 84       | 3.2806        | 0.1693 |
|                | Reliability = 50       | 3.2962        | 0.1651 |
| (b) Bootstrap method | Reliability = 99       | 3.1922        | 0.1550 |
|                | Reliability = 95       | 3.2169        | 0.1563 |
|                | Reliability = 84       | 3.2461        | 0.1591 |
|                | Reliability = 50       | 3.2962        | 0.1651 |

### Table 4. Validation test data.

| Number of test piece | Equivalent structural stress \( \Delta S_s \)/MPa | Test fatigue life N/cycle |
|----------------------|-----------------------------------------------|---------------------------|
| 1                    | 567.45                                        | 632                       |
| 2                    | 502.20                                        | 1578                      |
| 3                    | 343.20                                        | 18,408                    |
| 4                    | 300.00                                        | 38,810                    |
| 5                    | 289.75                                        | 51,937                    |
| 6                    | 285.00                                        | 61,131                    |
| 7                    | 281.60                                        | 61,802                    |
| 8                    | 275.50                                        | 73,238                    |
| 9                    | 243.73                                        | 175,350                   |
| 10                   | 218.26                                        | 346,169                   |
| 11                   | 204.34                                        | 520,647                   |
| 12                   | 200.80                                        | 534,280                   |
| 13                   | 196.02                                        | 675,006                   |
| 14                   | 171.95                                        | 1,616,350                 |
| 15                   | 171.00                                        | 1,447,050                 |

![Figure 10. Comparison between test life and predicted life.](https://via.placeholder.com/150)
Conclusions

(1) 84 fatigue test samples of titanium alloy welded structures with different plate thicknesses, materials, types and stress ratios are expanded by Bootstrap method, and the master S-N linear regression models with reliability of 99%, 95%, 84%, and 50% is determined.

(2) By analyzing the master S-N linear regression models of titanium alloys with different reliability determined by the traditional group method and the Bootstrap method, it can be found that the standard deviation of the sample data obtained by the Bootstrap method is smaller and tends to be stable. It largely solves the problem of low accuracy when there are few samples. And the accuracy of the master S-N curves of titanium alloy welded structures obtained by Bootstrap method is verified.

(3) Comparing the coefficient of determination ($R^2$) obtained by the two methods, the coefficient of determination ($R^2$) determined by the Bootstrap method is closer to 1. It can be proved that Bootstrap method improves the fitting degree of titanium alloy fatigue life test data.

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