Nonlinear anisotropic diffusion filtering for the characterization of stochastic structured surfaces

N Loftfield, M Kästner and E Reithmeier
Institute of Measurement and Automatic Control, Leibniz Universität Hannover, Nienburger Strasse 17, 30167 Hannover
E-mail: nina.loftfield@imr.uni-hannover.de

Abstract. Structured surfaces enhance the functionality of components. Well known is the influence of the surface structure on friction and wear behavior. Beyond this, structured surfaces are widely used for various purposes such as optical, biological or mechanical applications. Therefore, the characterization of structured surfaces and surface features becomes increasingly important. The functionality of a surface can either be tested directly or indirectly. Due to the correlation of geometric surface features and its functionality, an indirect and self-evident way is by measuring the surface topography. To obtain the geometric essentials of these features, they need to be separated from the raw surface data. The standard procedure of decomposing a surface topography is by the use of a Gaussian filter bank, gaining so called scale-limited surfaces. This procedure shows drawbacks when characterizing structured surfaces by introducing distortions to the feature boundaries. To overcome these limitations, this work proposes the use of an automatic nonlinear anisotropic diffusion filter as an initial step to separate the features from the residual surface topography because of its edge preserving properties. It is shown that the nonlinear anisotropic diffusion serves well the separation of the features and their geometrical characterization.

1. Introduction
It is widely known that structured surfaces serve the functionality of components. Well-known functions are of optical or mechanical nature like the Fresnel lens or the influence of surface features on friction and wear behavior. Therefore, structured surfaces become increasingly important, not only technologically but also economically [1]. Testing the functionality of a surface can either be done directly or indirectly. Due to the correlation of surface features and its physical functionality, an indirect yet obvious characterization is possible by geometric measurements of the surface. However, Evans et al. stated already in 1999 that the function of structured surfaces cannot be related to traditional surface finishing parameters [2]. Further, Mathia et al. showed more recently that the traditional concept of determined parameters for roughness and waviness does not satisfy the characterization of structured surfaces [1]. A surface is commonly considered as a superposition of structures with different scales. Scale-limited surface descriptions are most widely gained by spatial linear Gaussian filtering with determined cutoff frequencies. For structured surfaces this holds some drawbacks. For example, when filtering noise: noise is usually represented by high frequencies, however the converse is not necessarily true, not all high frequency portions of the signal are solely due to noise. Therefore, filtering the surface data with a linear Gaussian filter can diminish slopes at crucial
feature boundaries. In contrast, nonlinear anisotropic diffusion has very good edge preserving properties. In the research field of metrology different approaches have been pursued. Zeng et al. use the advantage of the nonlinear anisotropic diffusion to separate the geometrical features of MEMs surfaces [3]. Rather than investigating the roughness or waviness components of the surface data, Zeng et al. state that the focus is on geometrical features like the line width, step height, etc. Their work shows good results but no setting parameters are recommended and the problem of an automatic stopping criterion is not addressed. Also, Wang et al. apply the nonlinear anisotropic diffusion filter to discontinuous surfaces, thereby avoiding smoothing effects around the edges [4]. However, the used stopping criterion is based on the cutoff frequencies of the scale limited surfaces using the correlation of diffusion and wavelengths. As they state, this can only be used as an approximation. Nonlinear filtering cannot be assigned to one specific cutoff frequency. Based on the promising results of the work of Zeng et al., we propose a nonlinear anisotropic diffusion as a preliminary filtering step for structured surfaces. For the stopping criterion of the diffusion process a frequency based approach similar to the one proposed by Ilyevsky et al. is pursued [5].

2. Materials and Methods
Within this work, stochastic structured, porous ceramic layers are characterized. The measurement data of the ceramic layers are filtered to separate the structure from noise and roughness components. This is done by nonlinear anisotropic diffusion. Following, to identify surface features, the data needs to be segmented, e.g. by watershed segmentation. In this work the focus lies on the filtering process and the segmentation is not further discussed.

2.1. Ceramic Layers
The investigated surfaces are comprised of porous aluminum oxide layers with a nominal planar surface. The layers are manufactured by a thermal spraying process and are subsequently polished. We will refer to them as stochastic structured surfaces. The porosity is influenced by the process parameters and the material composition. The measurement data are acquired by a confocal laser scanning microscope VK-X210 from Keyence. All data shown in this work is acquired with a 50x magnification lens, respectively with a lateral pixel resolution of 0.277 µm.

2.2. Anisotropic Diffusion
Anisotropic diffusion was first published by Perona and Malik, suggesting a new definition of scale-space. The scale-space is modeled as a diffusion process [6]. The linear heat diffusion equation describes the dissipation of heat in a specific region. In image processing or signal
processing this can be transferred to a smoothing process at a constant rate dependent on the
diffusion time, similar to Gaussian filtering depending on the variance. With the topography
height data \( I(x, y, t) \), where \( I(x, y, 0) \) is the initial surface topography, the linear diffusion is
described by:

\[
\frac{\partial I(x, y, t)}{\partial t} = \text{div}(c \cdot \nabla I(x, y, t)), \tag{1}
\]

where \( \text{div} \) is the divergence operator, \( \nabla \) is the gradient operator and \( c \) is a constant conductance
coefficient indicating the diffusion speed. Though linear diffusion reduces noise, it also blurs the
signal, which in turn results in smoothed structure boundaries.

The nonlinear anisotropic diffusion is a modified linear diffusion process with an adaptive
conductance coefficient \( c \) which is updated in every iteration step as a function of the gradient:

\[
c(x, y, t) = g(\nabla I(x, y, t)). \tag{2}
\]

Perona and Malik suggest two conductance functions:

\[
g_1(\nabla I(x, y, t)) = e^{(-||\nabla I(x,y,t)||/K)^2} \tag{3}
\]

and

\[
g_2(\nabla I(x, y, t)) = \frac{1}{1 + (||\nabla I(x,y,t)||/K)^2}, \tag{4}
\]

with the diffusion coefficient \( K \) controlling the rate of diffusion. The first conductance
function \( g_1(\nabla I(x, y, t)) \) privileges high-contrast edges over low-contrast ones, the second one
\( g_2(\nabla I(x, y, t)) \) privileges wide regions over small ones [6]. In the following \( g_1(\nabla I(x, y, t)) \) and
\( g_2(\nabla I(x, y, t)) \) will be denoted as \( g_1(\nabla I) \) and \( g_2(\nabla I) \). In any case the conductance function is
chosen to provide fast diffusion within regions and a slow diffusion at sharp edges with a high
gradient to avoid smoothed structure boundaries.

With \( I_t \) the discrete sampled image at the iteration step \( t \) the discretized anisotropic diffusion
is given by Perona and Malik by:

\[
I_{t+1} = I_t + \lambda \cdot [c_N \nabla_N I_t + c_S \nabla_S I_t + c_E \nabla_E I_t + c_W \nabla_W I_t] \tag{5}
\]

with \( 0 \leq \lambda \leq 1/4 \) for the numerical scheme to be stable, \( c_i \) the conductance function and \( \nabla_i I_t \)
the difference of the neighboring pixels. The indices \( i \in \{N, E, S, W\} \) indicate the 4-neighbor
pixel: North, East, South, West, e.g. in the North direction:

\[
\nabla_N I(x, y, t) = I(x, y-1, t) - I(x, y, t). \tag{6}
\]

As Perona and Malik state this is not an exact discretization of the anisotropic diffusion but
a good approximation and is chosen due to its simplicity.

2.3. Choice of Parameters

The choice of the conductance function, the diffusion coefficient \( K \) and the number of diffusion
iterations \( n \) is very decisive for the result of the filter operation. Therefore, these have to be
chosen carefully. Oftentimes, the parameters are determined by trial and error and appointed
manually, however, an automated parameter selection is desirable. A detailed overview of dif-
ferent approaches on the choice of parameters for anisotropic diffusion is given by Tsiotsios and
Petrou, further, they discuss the importance and influence of the different parameters [7].
The conductance function characterizes the diffusion process in terms of which parts of the signal are favored. In any case the conductance function is a monotonically decreasing function of the gradient. Other conductance functions have been proposed, e.g. by Black et al. or Weickert adjusting the diffusion speed or even stopping the diffusion at a given gradient, in respect to avoiding an over-smoothed filter result [8, 9]. However, in this work we focus on the proposed conductance functions of Perona and Malik. The input of the conductance function is the gradient of a smoothed image rather than of the original image to avoid the influence of outliers. The regularization process is done by Gaussian smoothing. The cutoff frequency of the Gaussian filter is picked depending on the resolution of the measurement data according to the DIN Norm 25178 [10].

The diffusion coefficient $K$ controls the rate of diffusion. $K$ is a soft threshold between the image gradients that are attributed to noise or edges. $K$ can either be set to a constant related to the original data or be updated in every iteration step related to the diffused image. Most approaches are based on the gradient distribution, e.g. $K$ is updated as the mean value or the maximum value of the gradients in every iteration [3, 4]. In this work, $K$ is set adaptively in every iteration step according to the Canny noise estimator: the histogram of gradients is determined and the threshold is set to the 90% value of its integral [11]. Further Tsiotsios and Petrou suggest the use of two separate thresholds, evaluated in the horizontal (EW) and in the vertical (NS) direction to individually consider the gradients in the smoothing process oriented along the separate axes [7]. As we determine the 8-neighbors gradient, four different thresholds are used, differentiating also the diagonal directions, $NW\_SE$ and $NE\_SW$. This is done with respect to filtering structured surfaces in general. Isotropic surfaces show a predominant direction, different gradients respectively. The thresholds $K_j$ with $j \in \{NS, EW, NW\_SE, NE\_SW\}$ are, therefore, evaluated based on four different histograms of gradients.

As stated, anisotropic diffusion is an iterative process and the filtering result is highly dependent on the number of iterations. The used stopping criterion for $t$ is based on a frequency approach similar to Ilyevsky et al. by estimating the energy of the signal’s high frequency components and stopping at an ideal stopping time [5]. They define the determination based on a highly denoised smoothed signal and a ratio at which to stop. Commonly, high frequencies are labeled as frequencies in the range of $N/4$ and $N/2$ in the frequency band. The energy of the frequencies is the Euclidean norm of the frequency section, denoted by $L_2^h$. Noise is mainly composed of high frequencies but the opposite is not always true. In our approach we determine the number of iterations $n$ in which the high frequency component energy falls in the first local minimum with a minimum distance of eight or, alternately, when the function runs approximately in a constant course. Instead of choosing the high frequency components in the range of $N/4$ and $N/2$, the cutoff frequency could also be defined according to the DIN 25178 [10]. Just as before, the stopping criterion for the diffusion time would be set to the local minimum of the frequencies’ energy of the specified band width.

3. Results

The results show well preserved features of the stochastic structured surface together with high-level noise suppression or, if desired, roughness separation. In comparison to linear filtering, e.g. Gaussian filtering, the edges are well preserved and the filtered signal follows the features’ structure even in very steep regions. The surfaces were filtered according to the proposed approach with the use of both conductance functions $g_1(\nabla I)$ and $g_2(\nabla I)$. Different results for the decreasing energy and diffusion coefficients were detected for the different conductance functions.

From the filtering results, it can be seen that the energy portion of the high frequencies
Figure 2: Course of energy $L_2^h$ and diffusion coefficients $K_j$ of the filtering process with $g_1(\nabla I)$ and $g_2(\nabla I)$ over 1000 iteration steps: (a) $L_2^h$, (b) $K_j$ with $g_1(\nabla I)$, (c) $K_j$ with $g_2(\nabla I)$.

Figure 3: Profile sections at $y = 175 \, \mu m$ of the aluminum oxide layer (figure 1) and the filter results: (a) with $g_1(\nabla I)$, (b) with $g_2(\nabla I)$.

$L_2^h$ falls with increasing time. Due to the different characteristics of the conductance functions $g_1(\nabla I)$ and $g_2(\nabla I)$ the course of $L_2^h$ also differs. The results of the filtering process with the conductance function $g_1(\nabla I)$, privileging high contrast edges over low-contrast ones, show higher $L_2^h$ preserved in the data, see figure 2a. The sequence of the four different diffusion coefficients $K_{NS}$, $K_{EW}$, $K_{NW,SE}$ and $K_{NE,SW}$ are shown in figures 2b and 2c. The diffusion coefficients show a similar decreasing course. The different directions vary just slightly due to the form of the micro-structure, there is no preferable structure direction, see figure 1.

The filtering results of the stochastic structured aluminum oxide surfaces of the proposed approach are shown in figure 3. With the conductance function $g_1(\nabla I)$ the micro-structure edges are well preserved and the high frequency portion is suppressed. With $g_1(\nabla I)$ the diffusion process is stopped at the iteration step $t_1 = 116$, the first local minimum in the $L_2^h$ course. In contrast, when filtering with $g_2(\nabla I)$, also the edges of big structures are well preserved, however, small regions get smoothed and modified by the filtering process. The diffusion is stopped at the iteration step $t_2 = 273$.

Concluding, filtering results show the advantage of the proposed method with respect to Gaussian filtering. The surface micro-structure shows steep edges, see figure 4a. Filtered by the proposed nonlinear anisotropic diffusion filter, the edges are preserved very well and the roughness and noise components are filtered. Because the advantage to linear filtering is best seen at steep edges the filtered signal is compared to Gaussian filtering. With the same filter result for the roughness and noise with the Gaussian filter the micro-structure edges are smoothed, see section $x = 800 \, \mu m$. 
Figure 4: Filter results of a deterministic micro-structure: (a) deterministic structure, (b) profile section at $y = 140 \, \mu m$ of raw data, filtered by nonlinear anisotropic diffusion and by Gaussian.

4. Discussion and Conclusion
Nonlinear anisotropic diffusion with a frequency based stopping condition was used for the scale-space representation of structured surface data with focus on the features. It is shown that the micro-structures can be well segmented from the other surface components with good edge preserving conditions. The smoothed edges impede the following segmentation of structures, e.g. by watershed segmentation.

However, as stated the filter result of the surfaces is dependent on the choice of the conductance function. Filtered with $g_1(\nabla I)$, the results show sharper edges and preserves small regions, whereas when filtered with $g_2(\nabla I)$ the filter result smooths small regions. For our proposed stochastic structured surfaces we recommend filtering with $g_1(\nabla I)$ rather than $g_2(\nabla I)$. Although, filtering large micro-structures with wide region, good results can also be achieved with $g_2(\nabla I)$, see figure 4b.

While mainly shown based on porous surfaces yet, we believe that for other structured surfaces the anisotropic diffusion is a powerful tool to separate the features from the surfaces.

Acknowledgments
Acknowledgments for the funding of this work go to the "Dr. Jürgen und Irmgard Ulderup Stiftung" and for his contribution to this work to Stefan Siemens.

References
[1] Mathia T G, Pawlus P and Wieczorowski M 2011 Wear 271 494–508 ISSN 00431648
[2] Evans C J and Bryan J B 1999 CIRP Annals - Manufacturing Technology 48 541–556 ISSN 00078506
[3] Zeng, W, Jiang, X, Scott, P J, and Blunt, L Proc. of the 10th International Symposium of Measurement Technology and Intelligent Instruments, 2011
[4] Wang M, Shao Y P, Du S C and Xi L f 2015 International Journal of Precision Engineering and Manufacturing 16 2057–2062 ISSN 2234-7593
[5] Ilyevsky A and Turkel E 2010 Journal of Scientific Computing 45 333–347 ISSN 0885-7474
[6] Perona P and Malik J 1990 IEEE Transactions on Pattern Analysis and Machine Intelligence 12 629–639 ISSN 01628828
[7] Tsotsios C and Petrou M 2013 Pattern Recognition 46 1369–1381 ISSN 00313203
[8] Black M J, Sapiro G, Marimont D H and Heeger D 1998 IEEE transactions on image processing : a publication of the IEEE Signal Processing Society 7 421–432 ISSN 1057-7149
[9] Weickert J 2001 Pattern Recognition 34 1813–1824 ISSN 00313203
[10] DIN Deutsches Institut für Normung e V November 2012 DIN ISO 25178-3 Geometrische Produktspezifikation (GPS) - Oberflächenbeschaffenheit: Flächenhaft URL www.din.de ; www.beuth.de
[11] Canny J 1986 IEEE Transactions on Pattern Analysis and Machine Intelligence PAMI-8 679–698 ISSN 0162-8828