Free Vibration Analysis of Delaminated Composite Pretwisted Rotating Shells — A Finite Element Approach*

Amit KARMAKAR** and Kikuo KISHIMOTO***

In this paper a finite element method is presented to study the effects of delamination on free vibration characteristics of graphite-epoxy composite pretwisted rotating shells. Lagrange’s equation of motion is used to derive the dynamic equilibrium equation and moderate rotational speeds are considered wherein the Coriolis effect is negligible. An eight noded isoparametric plate bending element is employed in the formulation incorporating rotary inertia and effects of transverse shear deformation based on Mindlin’s theory. To satisfy the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack front, a multipoint constraint algorithm is incorporated which leads to unsymmetric stiffness matrices. Parametric studies are performed in respect of location of delamination, fibre orientation, rotational speed and twist angle on natural frequencies of cylindrical shallow shells. Numerical results obtained for symmetric and unsymmetric laminates are the first known non-dimensional frequencies for the analyses carried out here.

Key Words: Finite Element, Deformation, Transverse Shear, Composite, Delamination, Free Vibration, Pretwisted Shell

1. Introduction

Pretwisted shells are structural elements of considerable technical significance as turbomachinery blades with low aspect ratio could be idealized as twisted rotating cantilever cylindrical shells or plates (Fig. 1). Extensive practical uses of pretwisted shells can be found in aerospace, mechanical and marine industries. In a weight sensitive application such as turbomachinery engine, composite materials are advantageous because of their light weight, high specific stiffness and high specific strength, but one of the major causes of failure in fibre-reinforced laminated composites is the delamination resulting from interlaminar debonding of constituting laminae. Delamination of a critical size can result in strength degradation and can promote instability of the structure. The delaminated structure also exhibits new vibration characteristics depending on size and location of the delamination. The presence of invisible delamination can be detected with the help of prior knowledge of natural frequencies for a composite containing delamination. Moreover, the initial stress system in a rotating shell due to centrifugal body forces affects natural frequencies appreciably. Failures of turbomachinery blades due to fatigue are predominantly the consequence of resonant vibrations resulting in large operating stresses and can be quite costly both in terms of safety and maintenance of turbine engines. In general, it is conceded that nearly all the vibrations of the blades are closely related to their natural frequencies. Hence, as a precursor to the application of twisted composite shells in the critical parts of aero-engines an extensive knowledge series A, vol. 49, no. 4, 2006 JSME International Journal
of these natural frequencies is essential in order to prevent vibration induced fatigue failures and thereby to ensure long life structurally sound blades.

So far there have been a few investigations related to laminated composite cantilever plates and shells with initial twist and consequently, the research findings in this area are limited and scanty. The pioneering work on pretwisted composite plates involved the determination of the non-dimensional natural frequencies of stationary plates using the laminated shallow shell theory in conjunction with the Ritz method, while two earlier investigations provided the first known results on natural frequencies of rotating composite plates. The analyses of rotating composite blades using the plate model focussed, respectively on the derivation of two approximate analytical models for determining the stress-strain field, the details of fabrication and experimentation for determining natural frequencies and steady state strain along with the effect of rotational speed on it and the vibration characteristics including geometric nonlinearity. These works addressed undamped free vibration only, but an attempt was made to study the effect of damping on vibration behaviour of stationary pretwisted blades of glass fibre reinforced plastics. Later on, the first known three-dimensional continuum vibration analysis including full geometric nonlinearities and centrifugal accelerations in the composite blade kinematics was carried out employing the Ritz method. There exists a good number of references on numerical models and experimental investigations of turbomachinery blades idealized according to the Timoshenko theory of beam. It has been found that the application of beam theory is far from straightforward and extracts limited information only. This led to summarize the quantitative comparison of natural frequencies of metal matrix composite pretwisted blades in stationary condition using beam and plate theories.

Although the finite element method is a powerful numerical technique for analyzing the natural frequencies of complicated structures, the combined effect of rotation and pretwist on composite plates was not demonstrated in the above mentioned works. Idealization of turbomachinery blades from this angle was aimed at recently. In the first investigation a nonlinear finite element technique was introduced for analysing the nonlinear static deflection and vibration behaviour of spinning pretwisted composite plates, while in the second one a nine nodded three-dimensional degenerated composite shell element was developed to study the effects of important kinematical parameters on free vibration characteristics of pretwisted rotating composite plates. Of late, the cylindrical shell model was utilized to carry out the free vibration analyses of rotating composite blades with initial twist. Regarding delamination model two worth mentioning investigations were carried out. It included analytical and experimental determination of natural frequencies of delaminated composite beams and the second one dealt with finite element treatment of the delaminated composite cantilever beam and plate for free vibration analyses. Although delamination is one of the most feared damage modes in laminated composites the impact behaviour of delaminated structures has been addressed only in two investigations wherein simply supported plates with single and multiple delamination were considered for the analyses.

Therefore, considering the open literature it is clear that no attention has been paid to the initially stressed delaminated composite plates or shells. To the best of the authors’ knowledge there is no literature available, which deals with delaminated composite pretwisted rotating shells. The present work is aimed at investigating the free vibration characteristics of graphite-epoxy composite pretwisted rotating shallow shells with delamination. An eight nodded isoparametric plate bending element is employed in the finite element formulation. Effects of transverse shear deformation based on Mindlin’s theory and rotary inertia are included. The undelaminated region is modelled by a single layer of plate elements, while the delaminated region is modelled by two layers of plate elements whose interface contains the delamination. To ensure the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack front a multipoint constraint algorithm is incorporated which leads to unsymmetric element stiffness matrices. The standard eigenvalue problem is solved by QR iteration algorithm. Natural frequencies of pretwisted rotating cylindrical shallow shells with delamination are obtained, and the studies mainly focus on the influence of stacking sequence and location of delamination. This paper presents a finite element based numerical approach for natural frequency determination of composite pretwisted rotating shallow shells having delamination without taking care of the effect of dynamic contact between delaminated layers.

2. Theoretical Formulation

A shallow shell is characterized by its middle surface which is defined by the equation

\[ z = -\frac{1}{2} \left[ \frac{x^2}{R_{x}} + 2 \frac{xy}{R_{xy}} + \frac{y^2}{R_{y}} \right] \]  

where \( R_x, R_y \) and \( R_{xy} \) denote the radii of curvature in the \( x \) and \( y \) directions and the radius of twist, respectively. The radius of twist \( (R_{xy}) \), length \( (L) \) of the shell and twist angle \( (\phi) \) are related by the expression

\[ \tan \phi = -\frac{L}{R_{xy}} \]  

The dynamic equilibrium equation for moderate rotational speeds is derived employing Lagrange’s equation of motion and neglecting Coriolis effect the equation in global
form is expressed as\(^{(20)}\)

\[
[M][\ddot{\delta}] + ([K] + [K_{c}])[\delta] = [F(\Omega^2)]
\]  

(3)

where \([M], [K]\) and \([K_{c}]\) are global mass, elastic stiffness and geometric stiffness matrices, respectively. \([F(\Omega^2)]\) is the nodal equivalent centrifugal forces and \([\delta]\) is the global displacement vector. \([K_{c}]\) depends on the initial stress distribution and is obtained by the iterative procedure\(^{(21)-(23)}\) upon solving

\[
([K] + [K_{c}])[\delta] = [F(\Omega^2)]
\]  

(4)

The matrix of angular velocity components contributing towards acceleration vector is given as\(^{(20),(23)}\)

\[
[A] = \begin{bmatrix}
\Omega_x^2 + \Omega_z^2 & -\Omega_x \Omega_y & -\Omega_x \Omega_z \\
-\Omega_y \Omega_x & \Omega_y^2 + \Omega_z^2 & -\Omega_y \Omega_z \\
-\Omega_z \Omega_x & -\Omega_z \Omega_y & \Omega_z^2 + \Omega_y^2
\end{bmatrix}
\]  

(5)

The element centrifugal force vector is given by\(^{(20),(23)}\)

\[
[F_{ce}] = \rho \int_{vol} [N]^T [A] \begin{bmatrix} h_x + x \\ h_y + y \\ h_z + z \end{bmatrix} d(vol)
\]  

(6)

where \(\rho\) is the mass density, \([N]\) stands for the shape function matrix and \([h_x, h_y, h_z]\) are the fixed translational offsets expressed with reference to the plate coordinate system\(^{(20)}\).

The element geometric stiffness matrix\(^{(24)}\) due to rotation is given by

\[
[K_{sr}] = \int_{vol} [G]^T [M_{r}] [G] d(vol)
\]  

(7)

where the matrix \([G]\) consists of derivatives of shape functions and \([M_{r}]\) is the matrix of initial in-plane stress resultants caused by rotation.

The natural frequencies are determined from the standard eigenvalue problem\(^{(18)}\) which is represented below and is solved by QR iteration algorithm.

\[
[A][\delta] = \lambda[\delta] \text{ where } [A] = ([K] + [K_{c}])^{-1} [M]
\]  

and \(\lambda = 1/\omega_n^2\)

(8)

2.1 Multipoint constraints

Figure 2 shows the plate elements at a delamination crack front. The nodal displacements of elements 2 and 3 at the crack tip are expressed as\(^{(17)}\)

\[
u_j = \overline{u}_j - (z - \overline{z}_j) \theta_{xj} \\
v_j = \overline{v}_j - (z - \overline{z}_j) \theta_{yj} \quad (j = 2, 3)
\]  

(9)

where \(\overline{u}_j, \overline{v}_j\) and \(\overline{w}_j\) are the mid-plane displacements, \(\overline{z}_j\) is the \(z\) coordinate of the mid-plane of element \(j\) and \(\theta_{xj}, \theta_{yj}\) are the rotations about \(x\) and \(y\) axes, respectively. The above equation also holds for element 1 with \(\overline{z}_1\) equal to zero. The transverse displacements and rotations at a common node have values expressed as\(^{(17)}\)

\[
[N] = [N]_1 + [N]_2 + [N]_3
\]  

(16)

\[
[M] = [M]_1 + [M]_2 + [M]_3 + \overline{z}_2 [N]_2 + \overline{z}_3 [N]_3
\]  

(17)

\[
[Q] = [Q]_1 + [Q]_2 + [Q]_3
\]  

(18)

where \([Q]\) denotes the transverse shear resultants.

An eight noded isoparametric quadratic plate bending element with five degrees of freedom at each node (three
translations and two rotations) is employed wherein the shape functions are given as(18)

\[ N_i = (1 + \xi_i)(1 + \eta_i)(\xi_i + \eta_i - 1)/4 \cdots i = 1, 2, 3, 4 \]
\[ N_i = (1 - \xi_i^2)(1 + \eta_i)/2 \cdots i = 5, 7 \]
\[ N_i = (1 - \eta_i^2)(1 + \xi_i)/2 \cdots i = 6, 8 \]

where \( \xi \) and \( \eta \) are the local natural coordinates of the element.

3. Results and Discussion

3.1 Comparison of results

The results obtained from the computer codes developed on the basis of present finite element modelling are validated with those in the literature. Tables 1 and 2 furnish the non-dimensional fundamental frequencies of the graphite-epoxy composite twisted plates(1) with different fibre orientation and the isotropic flat rotating cantilever plate(23), respectively. Figure 3 presents the spanwise variation of the first natural frequency of the composite cantilever beam with relative position of the delamination(14). The predictive capability of the computer programs in respect of twisted, rotating and delamination is confirmed. It is observed from the convergence study that uniform mesh divisions of \( 6 \times 6 \) and \( 8 \times 8 \) considering the complete planform of the shell provide nearly equal results the difference being well within one percent (1%). The lower mesh size \( (6 \times 6) \) consisting of 36 elements and 133 nodes, has been used for the analyses due to computational efficiency. The total number of degrees of freedom involved in the computation is 665 as each node of the isoparametric element is having five degrees of freedom comprising of three translations and two rotations.

### Table 1

| \( \theta \) (deg.) | Present FEM | Qatu and Leissa(27) |
|---------------------|-------------|-------------------|
| 15                  | 0.8818      | 0.8759            |
| 30                  | 0.6790      | 0.6923            |
| 45                  | 0.4732      | 0.4831            |
| 60                  | 0.3234      | 0.3283            |

### Table 2

| Non-dimensional speed (\( \Omega \)) | Present FEM | Sreenivasamurthy and Ramamuri(23) |
|-------------------------------------|-------------|----------------------------------|
| 0.0                                | 3.49196     | 3.43685                          |
| 0.2                                | 3.49992     | 3.51858                          |
| 0.4                                | 3.72887     | 3.75280                          |
| 0.6                                | 4.07813     | 4.12875                          |
| 0.8                                | 4.51561     | 4.56786                          |
| 1.0                                | 5.01415     | 5.09167                          |

* \( \Omega = \omega_0 \sqrt{\rho/\rho} \); \( \omega_0 \) - actual angular speed of rotation, \( \omega_0 \) - fundamental natural frequency of a non-rotating plate.

3.2 Parametric study

Parametric studies are performed in respect of location of delamination, fibre orientation, rotational speed and twist angle on natural frequencies of shallow shells. The non-dimensional frequencies \( \sigma = \omega_0 L^2 \sqrt{\rho/\rho} \) for cylindrical shells \( (R_y/R_z = 0) \) having a square planform \( (L/h = 1) \), curvature ratio \( (b/R_y) \) of 0.5 and a thickness ratio \( (b/h) \) of 100 are obtained corresponding to non-dimensional speeds of rotation, \( \Omega = \omega_0 \sqrt{\rho/\rho} \), 0.0, 0.25, 0.5, 0.75 and 1.0, and relative distance, \( d/L \), 0.33 and 0.66. Shells with three different angles of twist \( \phi \) are considered namely, \( \phi = 15^\circ, 30^\circ \) and \( 45^\circ \), in addition to the untwisted one \( (\phi = 0^\circ) \). The fibre orientation angle \( \theta \) is varied from 0° to 90° with an increment of 15°. The characters \( \Omega, \omega_0, \rho, L, b, h \) and \( d \) represent the actual angular speed of rotation, fundamental natural frequency of a non-rotating shell, density, length, width, thickness and distance of the centreline of delamination from the clamped (fixed) end, respectively. The following material properties of graphite-epoxy composite(25) are adopted for computation for a unidirectional layer (Fig. 4):

- \( E_1 = 138.0 \text{ GPa} \)
- \( E_2 = 8.96 \text{ GPa} \)
- \( G_{12} = 7.1 \text{ GPa} \)
- \( G_{13} = 7.1 \text{ GPa} \)
- \( G_{23} = 2.84 \text{ GPa} \)
- \( \nu_{12} = 0.30 \)

The principal material axes of a lamina are assumed to lie along and transverse to the fibre direction, which are indicated by 1 and 2, respectively. Young’s moduli of a lamina along these two directions are \( E_1 \) and \( E_2 \), respectively and shear moduli in the 1 – 2, 2 – 3 and 1 – 3 planes are denoted by \( G_{12}, G_{23} \) and \( G_{13} \), respectively. For a particular layer when fibres are oriented at an angle \( \theta \) with respect to the x-axis, the off-axis elastic constant matrix of the lamina is obtained from the on-axis elastic constant matrix by appropriate transformation(26). For a laminated anisotropic plate of uniform thickness consisting of unidirectional laminae bonded together to act as an integral part
Fig. 4 An arbitrarily oriented lamina

Fig. 5 A typical laminate with layer details

(Fig. 5), the mid-plane forms the $x-y$ plane of the reference plane and the displacement components of any point at a distance $z$ from the mid-surface are expressed in terms of the mid-plane displacements according to Yang, Norris and Stavsky theory\(^{(27)}\) which is an extension of Mindlin’s theory\(^{(28)}\). The stresses in any lamina are finally expressed in terms of the laminate mid-surface strains, curvatures and shear rotations. The internal force and moment resultsants of the laminate are obtained by integrating the stresses and their products with $z$-coordinates in each lamina through the laminate thickness, as in the classical plate theory.

3.2.1 Stacking sequence

Non-dimensional fundamental frequencies of four layers $[\theta, -\theta, -\theta, \theta]$ graphite-epoxy composite pretwisted rotating cylindrical shells with 33% mid-plane delamination centered at a relative distance, $d/L = 0.33$ from the fixed end, for various non-dimensional speeds ($\Omega = \Omega(\omega_0)$, $L/b = 1$, $b/h = 100$, $b/R_y = 0.5$.

Table 3 Non-dimensional fundamental natural frequencies ($\omega = \omega_0 L^2 \sqrt{\rho/E_1 h^3}$) of four layers $[\theta, -\theta, -\theta, \theta]$ graphite-epoxy composite pretwisted rotating cylindrical shells with 33% mid-plane delamination centered at a relative distance, $d/L = 0.33$ from the fixed end, for various non-dimensional speeds ($\Omega = \Omega(\omega_0)$, $L/b = 1$, $b/h = 100$, $b/R_y = 0.5$.

| $\phi$ (Deg) | $\theta$ (Deg) | $\Omega$ | $\Omega = 0$ | $\Omega = 0.25$ | $\Omega = 0.5$ | $\Omega = 0.75$ | $\Omega = 1.0$ |
|-------------|---------------|---------|-------------|--------------|-------------|--------------|--------------|
| 0           | 1.9579        | 1.7804  | 1.7871      | 1.9657       | 2.1442      | 2.1946       |              |
| 15          | 2.0762        | 1.8579  | 1.9025      | 2.1103       | 2.3532      | 2.6425       |              |
| 30          | 2.0606        | 1.6823  | 1.7672      | 1.8169       | 2.2390      | 2.6236       |              |
| 45          | 1.8622        | 1.3840  | 1.4735      | 1.8954       | 2.9305      | 2.9593       |              |
| 60          | 1.5475        | 1.1625  | 0.6728      | 1.0432       | 1.7378      | 3.0443       |              |
| 75          | 1.2033        | 0.9842  | 1.5650      | 1.4917       | 1.5247      | 2.1466       |              |
| 90          | 1.0240        | 0.8931  | 1.5375      | 1.5382       | 1.8336      | 1.3661       |              |

* Undelaminated Stationary Shells

Fig. 6 Effect of fibre orientation on the ratio of delaminated and undelaminated first natural frequencies

maximum value being attained at $\theta = 75^\circ, 45^\circ$ and $60^\circ$ corresponding to $\phi = 15^\circ, 30^\circ$ and $45^\circ$, respectively. In contrast, the non-rotating twisted shells without delamination always provide the minimum value when the fibres are parallel to the clamped edge (i.e. $\theta = 90^\circ$) as was found earlier for composite twisted plates\(^{(1)}\). The percentage differences between the maximum and minimum frequency values for untwisted ($\phi = 0^\circ$) and twisted ($\phi = 15^\circ, 30^\circ, 45^\circ$) stationary shells with delamination are 51.9, 53.2,
(a) $\phi = 15^\circ$

(b) $\phi = 30^\circ$

(c) $\phi = 45^\circ$

Fig. 7 Effect of fibre orientation on the ratio of rotating and stationary first natural frequencies
Table 4  Non-dimensional second natural frequencies ($\sigma = \omega_0 L^2 \sqrt{\mu / E_1 h^3}$) of four layers $[\theta, -\theta, -\theta, \theta]$ graphite-epoxy composite pretwisted rotating cylindrical shells with 33% mid-plane delamination centered at a relative distance, $d/L = 0.33$ from the fixed end, for various non-dimensional speeds ($\Omega = \Omega / \omega_0$). $L/b = 1$, $b/h = 100$, $b/R_e = 0.5$.

| $\theta$ (Deg) | $\psi$ (Deg) | $\nu$ Damping (Stat.) | $\Omega = 0$ | $\Omega = 0.25$ | $\Omega = 0.5$ | $\Omega = 0.75$ | $\Omega = 1.0$ |
|----------------|---------------|----------------------|-------------|----------------|----------------|----------------|----------------|
| 0              | 0             | 2.3266               | 2.0840      | 2.0315         | 2.2035         | 2.7413         | 2.5848         |
|                | 15            | 2.6186               | 2.2731      | 2.1994         | 2.3419         | 2.5470         | 3.7261         |
|                | 30            | 2.9313               | 2.4492      | 2.3609         | 2.6490         | 3.3239         | 4.8835         |
|                | 45            | 2.6246               | 2.4361      | 2.2464         | 3.7160         | 3.0911         | 4.1230         |
|                | 60            | 2.9210               | 1.9770      | 1.5533         | 2.1457         | 2.7408         | 5.5471         |
|                | 75            | 1.6745               | 1.6631      | 1.3070         | 2.1033         | 3.7699         | 3.9018         |
|                | 90            | 1.5547               | 1.5468      | 1.2046         | 2.1143         | 3.3993         | 5.8827         |

* Undelaminated Stationary Shells

**Fig. 8** Effect of fibre orientation on the ratio of delaminated and undelaminated second natural frequencies corresponding to different fibre angle ($\theta$). Figures 8 and 9 also provide graphical interpretation of some salient features of these results. At stationary condition for cantilever cylindrical shells non-dimensional second frequencies for both undelaminated and delaminated cases attain the maximum value for fibre angle of $30^\circ$ unlike the fundamental ones and gradually decrease to a minimum value for $\theta = 90^\circ$. On the other hand for the non-rotating twisted shells with

**Fig. 9** Effect of fibre orientation on the ratio of rotating and stationary second natural frequencies
and without delamination the maximum second frequency is always obtained when the fibres are perpendicular to the clamped edge (i.e. $\theta = 0^\circ$) but the minimum value is mostly found for fibres parallel ($\theta = 90^\circ$) to the clamped edge excepting for $\phi = 30^\circ$ and $45^\circ$ wherein delaminated shells have the minimum frequency parameter at $\theta = 75^\circ$. In contrast to the isotropic cases\(^{(10)}\) the second frequency at stationary condition for both the delaminated and undelaminated shells does not increase with the angle of twist, as for example the maximum second frequency is obtained for $\phi = 15^\circ$ with fibres at $\theta = 30^\circ$, but the maximum percentage reduction in the second frequency parameter considering different fibre angles increases with the increase of twist angle. Unlike the fundamental frequency, the maximum reduction in second frequency values of delaminated non-rotating shells compared to those of the undelaminated cases is obtained for fibre angle of $30^\circ$ (i.e. $\theta = 30^\circ$) excepting for $\phi = 45^\circ$ in which case this occurs for $\theta = 45^\circ$ (Fig. 8).

For delaminated twisted shells ($\phi = 15^\circ$, $30^\circ$, $45^\circ$) the second frequency parameters like the fundamental frequencies always increase with the increase in rotational speed irrespective of the fibre angle. It is to be noted that corresponding to any particular combination of twist angle and non-dimensional speed of rotation the second frequency parameter has in general a maximum value at $\theta = 0^\circ$ with a decreasing trend attaining the minimum value at $\theta = 75^\circ$ or $90^\circ$, but in case of $\phi = 45^\circ$ there are exceptions for $\Omega = 0.75, 1.0$ wherein the maximum value is observed at $\theta = 30^\circ$. The second frequencies of delaminated twisted shells with higher fibre angles (i.e. $\theta = 60^\circ$, $75^\circ$ and $90^\circ$) show greater values compared to the corresponding undelaminated cases even for lowest non-dimensional speed of rotation (i.e. $\Omega = 0.25$). Figure 9 (a) – (c) presents the second rotating frequencies relative to those at stationary condition for delaminated twisted shells ($\phi = 15^\circ$, $30^\circ$, $45^\circ$) while the fibre angle is varied from $0^\circ$ to $90^\circ$ at an interval of $15^\circ$. Like the fundamental frequencies for any particular value of twist angle the stiffening effect due to rotation is observed to be the minimum when the fibre angle is zero degree (i.e. $\theta = 0^\circ$) corresponding to each non-dimensional speed of rotation. But these variations for $\phi = 15^\circ$ distinctly differ from those of the cases for $\phi = 30^\circ$, $45^\circ$. The lower value of twist angle (i.e. $\phi = 15^\circ$) leads to the maximum stiffening at $\theta = 90^\circ$ for all speeds ($\Omega = 0.25, 0.5, 0.75, 1.0$). Although the frequency parameter gradually increases to the maximum value, but there exists a slight dropping tendency at $\theta = 60^\circ$ excepting for $\Omega = 1.0$. For higher values of twist angle (i.e. $\phi = 30^\circ$, $45^\circ$) the stiffening effect is seen to increase to a maximum value followed by a decreasing trend and the maximum percentage increase of rotating frequency with respect to that at stationary condition is found for fibre angle of $45^\circ$ or $60^\circ$ (i.e. at $\theta = 45^\circ$ or $60^\circ$). Even for the second frequency it is also observed that the centrifugal stiffening increases with the increase in twist angle.

3.2.2 Location of delamination The variation of non-dimensional fundamental and second frequencies of angle ply $[45, -45, -45, 45]$ graphite-epoxy composite pretwisted rotating cylindrical shells for different locations of the same sized delamination along the span are furnished in Tables 5 and 6, respectively. In this case $33\%$ delamination centered at relative distances of $d/L = 0.33, 0.66$ from the fixed end is considered at the interface of (45 and $-45$) layers. In case of twisted shells for any particular value of angle of twist fundamental and second frequency values are observed to increase with rotational speed for both the locations of delamination while at stationary condition fundamental frequency decreases with the increase in twist angle, but for second frequency the maximum value occurs at $\phi = 15^\circ$. It is to be noted that as the location of delamination shifts away from the fixed end higher values of fundamental frequencies and lower values of second frequencies are observed at stationary condition both for cantilever and twisted shells. For twisted shells fundamental rotating frequencies also show the same behaviour (increase) as the delamination moves towards the free end as opposed to the inconsistent nature of the second rotating frequencies and significant variation for both the frequencies can be observed for higher values of twist angles ($\phi = 30^\circ$, $45^\circ$). This corroborates the findings of the composite cantilever beam\(^{(14)}\) as the location of delamination changes along its span. It is also observed that at stationary condition the percentage increase of fundamental frequency is always smaller compared to that at rotating frequencies and significant variation for both the delaminated and undelaminated shells.

| $\phi$ (Deg) | $\Omega$ | $d/L=0.33$ | $d/L=0.66$ |
|-------------|----------|-------------|-------------|
| 0           | 0.0      | 1.4957      | 1.5091      |
|             | 0.25     | 1.5049      | 1.3355      |
|             | 0.5      | 0.9128      | 2.3799      |
|             | 0.75     | 1.4773      | 1.2349      |
|             | 1.0      | 2.8730      | 2.1014      |
| 15          | 0.0      | 0.7072      | 0.7167      |
|             | 0.25     | 0.9870      | 0.9979      |
|             | 0.5      | 1.2945      | 1.3039      |
|             | 0.75     | 1.5539      | 1.5636      |
|             | 1.0      | 1.7815      | 1.7920      |
| 30          | 0.0      | 0.4181      | 0.4673      |
|             | 0.25     | 0.9211      | 0.9316      |
|             | 0.5      | 1.3066      | 1.3565      |
|             | 0.75     | 1.6227      | 1.6869      |
|             | 1.0      | 1.9056      | 1.9671      |
| 45          | 0.0      | 0.3286      | 0.4076      |
|             | 0.25     | 1.0340      | 1.0386      |
|             | 0.5      | 1.5643      | 1.6594      |
|             | 0.75     | 1.9165      | 2.1128      |
|             | 1.0      | 2.3370      | 2.5061      |
Table 6 Non-dimensional second frequencies \( (\sigma = \alpha_0 L^2 \sqrt{\rho/E_1 h^2}) \) of angle ply \([45, -45, 45]\) graphite-epoxy composite pretwisted rotating cylindrical shells with 33\% delamination (interface of 45 & \(-45\)) centered at relative distance, \( d/L = 0.33, 0.66 \) from the fixed end, for various non-dimensional speeds \( (\Omega = \Omega_0/\omega_0) \). \( L/b = 1, b/h = 100, b/R_0 = 0.5 \)

| \( \phi \) (Deg) | \( \Omega \) | \( d/L = 0.33 \) | \( d/L = 0.66 \) |
|-----------------|----------|----------------|----------------|
| 0               |          |                |                |
| 0.0             | 2.4372  | 2.3820         |                |
| 0.25            | 2.2881  | 2.0764         |                |
| 0.5             | 2.2526  | 2.3018         |                |
| 0.75            | 2.5819  | 3.0427         |                |
| 1.0             | 3.5867  | 4.5143         |                |

| 15              |          |                |                |
| 0.0             | 2.9544  | 2.7969         |                |
| 0.25            | 3.2886  | 3.1804         |                |
| 0.5             | 3.6881  | 3.6155         |                |
| 0.75            | 4.0084  | 3.9528         |                |
| 1.0             | 4.2727  | 4.2269         |                |

| 30              |          |                |                |
| 0.0             | 2.4295  | 2.2052         |                |
| 0.25            | 3.1622  | 3.0887         |                |
| 0.5             | 4.0389  | 4.0991         |                |
| 0.75            | 4.8400  | 4.9849         |                |
| 1.0             | 5.5686  | 5.7131         |                |

| 45              |          |                |                |
| 0.0             | 2.1303  | 1.9179         |                |
| 0.25            | 3.0919  | 3.1761         |                |
| 0.5             | 4.1667  | 4.4085         |                |
| 0.75            | 4.8075  | 5.1734         |                |
| 1.0             | 5.5466  | 5.3554         |                |

Table 7 Non-dimensional fundamental frequencies \( (\sigma = \alpha_0 L^2 \sqrt{\rho/E_1 h^2}) \) of cross ply \([0/90]_{2S}\) graphite-epoxy composite pretwisted rotating cylindrical shells with 33\% delamination (mid-plane, interface of 1st & 2nd layers) centered at mid-span, for various non-dimensional speeds \( (\Omega = \Omega_0/\omega_0) \). \( L/b = 1, b/h = 100, b/R_0 = 0.5 \)

| \( \phi \) (Deg) | \( \Omega \) | Mid-plane delamination | Delamination at interface of 1st & 2nd layers |
|-----------------|----------|------------------------|----------------------------------|
| 0               | 0.0      | 1.6055                 | 1.6679                           |
| 0.5             | 2.2334   | 1.3175                 |                                  |
| 1.0             | 2.6529   | 3.1104                 |                                  |

| 15              |          |                        |                                  |
| 0.0             | 1.0542   | 1.1487                 |                                  |
| 0.5             | 1.6066   | 1.7045                 |                                  |
| 1.0             | 2.1399   | 2.2655                 |                                  |

| 30              |          |                        |                                  |
| 0.0             | 0.8168   | 0.9020                 |                                  |
| 0.5             | 1.4485   | 1.5360                 |                                  |
| 1.0             | 1.9818   | 2.0994                 |                                  |

| 45              |          |                        |                                  |
| 0.0             | 0.6850   | 0.7519                 |                                  |
| 0.5             | 1.4326   | 1.4998                 |                                  |
| 1.0             | 2.0319   | 2.1296                 |                                  |

4. Conclusions

Delaminated composite twisted rotating shells provide consistent values of fundamental and second non-dimensional frequencies which exhibit a rising trend with the increase of rotational speed irrespective of the fibre orientation and the location of delamination considered for the analyses.

In general, fundamental frequency parameters at stationary condition decrease with the increase in twist angle for all fibre orientations and the centrifugal stiffening is also observed to increase with the increase in twist angle.

The change in fibre angle leads to a pronounced effect on the fundamental and second frequencies. For any particular combination of twist angle and non-dimensional frequency or decrease of second frequency parameter at \( d/L = 0.66 \) with reference to that at \( d/L = 0.33 \) has an increasing trend for higher values of twist angles e.g. (11.76\% & 24.04\%) in case of fundamental, and (9.31\% & 9.97\%) for the case of second frequency corresponding to \( \phi = 30^\circ \) and \( 45^\circ \), respectively.

The change in non-dimensional fundamental and second frequencies of cross ply \([0/90]_{2S}\) graphite-epoxy composite pretwisted rotating cylindrical shells for different positions of fixed size delamination across the shell thickness are presented in Tables 7 and 8, respectively. The mid-plane and the interface of 1st and 2nd layers are considered as the locations of 33\% delamination which is centered at mid-span in each case. In case of stationary condition both fundamental and second frequency values are always found to be higher when the delamination is located near the free surface as obtained earlier for the composite cantilever beam\(^{14}\). For rotating condition this is also significantly noted only for twisted shells. However, at stationary condition fundamental frequency decrease with the increase in twist angle irrespective of the location of delamination across the thickness, but for the second frequency the maximum value is obtained at \( \phi = 30^\circ \) and the cantilever shell (\( \phi = 0^\circ \)) provides the minimum value. In case of twisted shells at stationary condition, the percentage change in values of the frequency near the surface with respect to that at the mid-plane increases gradually with the twist angle only in case of second frequency, but for fundamental one the maximum change (10.43\%) occurs at \( \phi = 30^\circ \), whereas for any particular value of angle of twist fundamental and second frequency parameters increase with rotational speed for both the locations of delamination.

4. Conclusions

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The change in non-dimensional fundamental and second frequencies of cross ply \([0/90]_{2S}\) graphite-epoxy composite pretwisted rotating cylindrical shells with 33\% delamination (mid-plane, interface of 1st & 2nd layers) centered at mid-span, for various non-dimensional speeds \( (\Omega = \Omega_0/\omega_0) \). \( L/b = 1, b/h = 100, b/R_0 = 0.5 \)
speed of rotation the stiffening effect due to centrifugal forces is always minimum when the fibre angle is zero degree (i.e. $\theta = 0^\circ$). For higher values of twist angle (i.e. $\theta = 30^\circ$, $45^\circ$) this effect is always seen to increase to a maximum value followed by a decreasing trend and the maximum percentage increase of rotating frequency with respect to that at stationary condition is always found for fibre angle of either $45^\circ$ or $60^\circ$ (i.e. at $\theta = 45^\circ$ or $60^\circ$). At stationary condition fundamental and second non-dimensional frequencies of delaminated twisted shells attain the maximum value when the fibres are perpendicular to the clamped edge (i.e. $\theta = 0^\circ$). The maximum reduction in fundamental and second frequency values of delaminated non-rotating shells compared to those of the undelaminated cases is obtained for fibre angle of either $30^\circ$ or $45^\circ$ (i.e. $\theta = 30^\circ$ or $45^\circ$).

Higher values of fundamental frequencies and lower values of second frequencies are obtained at stationary condition both for cantilever and twisted shells when the location of delamination is shifted away from the clamped end whereas the delamination near the free surface leads to higher magnitude of both the frequencies. For twisted shells fundamental rotating frequencies increase as the delamination moves towards the free end as opposed to the inconsistent nature of the second rotating frequencies, but at rotating condition both fundamental and second frequencies of twisted shells provide increased values when the delamination is away from the mid-plane.

The non-dimensional frequencies obtained are the first known results of the type of analyses carried out here and the results could serve as reference solutions for future investigators.

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References

(1) Qatu, M.S. and Leissa, A.W., Vibration Studies for Laminated Composite Twisted Cantilever Plates, International Journal of Mechanical Sciences, Vol.33, No.11 (1991), pp.927–940.
(2) Wang, J.T.S., Shaw, D. and Mahrenholtz, O., Vibration of Rotating Rectangular Plates, Journal of Sound and Vibration, Vol.112, No.3 (1987), pp.455–468.
(3) Shaw, D., Shen, K.Y. and Wang, J.T.S., Flexural Vibration of Rotating Rectangular Plates of Variable Thickness, Journal of Sound and Vibration, Vol.126, No.3 (1988), pp.373–385.
(4) Pagano, N.J. and Soni, S.R., Strength Analysis of Composite Turbine Blades, Journal of Reinforced Plastics and Composites, Vol.7 (1988), pp.558–581.
(5) Seshu, P., Ramamurti, V. and Babu, B.J.C., Theoretical and Experimental Investigations of Composite Blades, Composite Structures, Vol.20 (1992), pp.63–71.
(6) Bhumbla, R., Kosmatka, J.B. and Reddy, J.N., Free Vibration Behavior of Spinning Shear Deformable Plates Composed of Composite Materials, AIAA Journal, Vol.28 (1990), pp.1962–1970.
(7) Nabi, S.M. and Ganesan, N., Vibration and Damping Analysis of Pre-twisted Composite Blades, Computers & Structures, Vol.47, No.2 (1993), pp.275–280.
(8) McGee, O.G. and Chu, H.R., Three-Dimensional Vibration Analysis of Rotating Laminated Composite Blades, Journal of Engineering for Gas Turbines and Power, Trans. ASME, Vol.116 (1994), pp.663–671.
(9) Nabi, S.M. and Ganesan, N., Comparison of Beam and Plate Theories for Free Vibrations of Metal Matrix Composite Pre-Twisted Blades, Journal of Sound and Vibration, Vol.189, No.2 (1996), pp.149–160.
(10) Bhumbla, R. and Kosmatka, J.B., Behavior of Spinning Pretwisted Composite Plates Using a Nonlinear Finite Element Approach, AIAA Journal, Vol.34, No.8 (1996), pp.1686–1695.
(11) Karmakar, A. and Sinha, P.K., Finite Element Free Vibration Analysis of Laminated Composite Pretwisted Cantilever Plates, Journal of Reinforced Plastics and Composites, Vol.16, No.16 (1997), pp.1461–1491.
(12) Kee, Y. and Kim, J., Vibration Characteristics of Initially Twisted Rotating Shell Type Composite Blades, Composite Structures, Vol.64, No.2 (2004), pp.151–159.
(13) Shen, M.-H.H. and Grady, J.E., Free Vibrations of Delaminated Beams, AIAA Journal, Vol.30, No.5 (1992), pp.1361–1370.
(14) Krawczuk, M., Ostachowicz, W. and Zak, A., Dynamics of Cracked Composite Material Structures, Computational Mechanics, Vol.20 (1997), pp.79–83.
(15) Sekine, H., Hu, T., Natsume, T. and Fukunaga, H., Impact Response Analysis of Partially Delaminated Composite Laminates, Trans. Jpn. Soc. Mech. Eng, (in Japanese), Vol.63, No.608, A (1997), pp.787–793.
(16) Hu, N., Sekine, H., Fukunaga, H. and Yao, Z.H., Impact Analysis of Composite Laminates with Multiple Delaminations, International Journal of Impact Engineering, Vol.22 (1999), pp.633–648.
(17) Gim, C.K., Plate Finite Element Modeling of Laminated Plates, Computers & Structures, Vol.52, No.1 (1994), pp.157–168.
(18) Bathe, K.J., Finite Element Procedures in Engineering Analysis, (1990), Prentice Hall of India, New Delhi.
(19) Leissa, A.W., Lee, J.K. and Wang, A.J., Vibrations of Twisted Rotating Blades, Journal of Vibration, Acoustics, Stress, and Reliability in Design, Trans. ASME, Vol.106, No.2 (1984), pp.251–257.
(20) Karmakar, A. and Sinha, P.K., Failure Analysis of Laminated Composite Pretwisted Rotating Plates, Journal of Reinforced Plastics and Composites, Vol.20, No.15 (2001), pp.1326–1357.
(21) Bossak, M.A.J. and Zienkiewicz, O.C., Free Vibration of Initially Stressed Solids with Particular Reference to Centrifugal Force Effects in Rotating Machinery, Journal of Strain Analysis, Vol.8, No.4 (1973), pp.245–252.
(22) Henry, R. and Lalanne, M., Vibration Analysis of Rotating Compressor Blades, Journal of Engineering for Industry, Trans. ASME, Vol.96, No.3 (1974), pp.1028–1035.

(23) Sreenivasamurthy, S. and Ramamurti, V., Coriolis Effect on the Vibration of Flat Rotating Low Aspect Ratio Cantilever Plates, Journal of Strain Analysis, Vol.16, No.2 (1981), pp.97–106.

(24) Cook, R.D., Malkus, D.S. and Plesha, M.E., Concepts and Applications of Finite Element Analysis, (1989), John Wiley and Sons, New York.

(25) Qatu, M.S. and Leissa, A.W., Natural Frequencies for Cantilevered Doubly-Curved Laminated Composite Shallow Shells, Composite Structures, Vol.17 (1991), pp.227–255.

(26) Tsai, S.W. and Hahn, H.T., Introduction to Composite Materials, (1980), Technomic Publishing Co. Inc., New York.

(27) Yang, P.C., Norris, C.M. and Stavsky, Y., Elastic Wave Propagation in Heterogeneous Plates, International Journal of Solids and Structures, Vol.2 (1966), pp.665–684.

(28) Mindlin, R.D., Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic Elastic Plates, Journal of Applied Mechanics, Vol.18 (1951), pp.31–38.