Neutrino mass and oscillation as probes of physics beyond the Standard Model *

S. Khalil\textsuperscript{1,2,†} and E. Torrente-Lujan\textsuperscript{3,†}

\textsuperscript{1} Centre for Theoretical Physics, University of Sussex, Brighton BN1 9QJ, U.K.
\textsuperscript{2} Ain Shams University, Faculty of Science, Cairo 11566, Egypt.
\textsuperscript{3} Departamento de Física Teórica, C-XI, Universidad Autónoma de Madrid, 28049 Cantoblanco, Madrid, Spain.

\textsuperscript{†} E-mail: kafz8@pact.cpes.susx.ac.uk, e.torrente@cern.ch.

Abstract: We present a review of the present status of the problem of neutrino masses and mixing including a survey of theoretical motivations and models, experimental searches and implications of recently appeared solar and atmospheric neutrino data, which strongly indicate nonzero neutrino masses a mixing angles.

Keywords: Neutrino mass, Neutrino Oscillation, Left-Right Models, Supersymmetry, Extra Dimensions, Neutrino Experiments.

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1 Introduction

The existence of a so-called neutrino, a light, neutral, feebly interacting fermion, was first proposed by W. Pauli in 1930 to save the principle of energy conservation in nuclear beta
The idea was promptly adopted by the physics community; in 1933 E. Fermi takes the neutrino hypothesis, gives the neutrino its name and builds his theory of beta decay and weak interactions. But, it was only in 1956 that C. Cowan and F. Reines were able to discover the neutrino, more exactly the anti-neutrino, experimentally [2]. Danby et al. [3] confirmed in 1962 that there exist, at least, two types of neutrinos, the $\nu_e$ and $\nu_\mu$. In 1989, the study of the Z boson lifetime allows to show with great certitude that only three light neutrino species do exist. Only in 2000, it has been confirmed by direct means [4] the existence of the third type of neutrino, the $\nu_\tau$ in addition to the $\nu_e$ and $\nu_\mu$. Until here the history, with the present perspective we can say that the neutrino occupies a unique place among all the fundamental particles in many ways and as such it has shed light on many important aspects of our present understanding of nature and is still believed to hold a key role to the physics beyond the Standard Model (SM).

In what respects its mass, Pauli initially expected the mass of the neutrino to be small but not necessary zero: not very much more than the electron mass, F. Perrin in 1934 showed that its mass has to be less than that of the electron. After more than a half a century, the question of whether the neutrino has mass is still one open question, being one of the outstanding issues in particle physics, astrophysics, cosmology and theoretical physics in general. Presently, there are several theoretical, observational and experimental motivations which justify the searching for possible non-zero neutrino masses (see i.e. [5–11] for excellent older reviews on this matter).

Understanding of fermion masses in general are one of the major problems of the SM and observation of the existence or confirmation of non-existence of neutrino masses could introduce useful new perspectives on the subject. If they are confirmed as massless they would be the only fermions with this property. A property which is not dictated by any known fundamental underlying principle, such as gauge invariance in the case of the photon. If it is concluded that they are massive then the question is why are their masses so much smaller than those of their charged partners. Although theory alone can not predict neutrino masses, it is certainly true that they are strongly suggested by present theoretical models of elementary particles and most extensions of the SM definitively require neutrinos to be massive. They therefore constitute a powerful probe of new physics at a scale larger than the electroweak scale.

If massive, the superposition postulates of quantum theory predict that neutrinos, particles with identical quantum numbers, could oscillate in flavor space. If the absolute difference of masses among them is small enough then these oscillations could have important phenomenological consequences. Some hints at accelerator experiments as well as the observed indications of spectral distortion and deficit of solar neutrinos and the anomalies on the ratio of atmospheric $\nu_e/\nu_\mu$ neutrinos and their zenith distribution are naturally accounted by the oscillations of a massive neutrino. Recent claims of the high-statistics high-precision Super-Kamiokande (SK) experiment are unambiguous and left little room for the scepticism as we are going to see along this review.

Moreover, neutrinos are basic ingredients of astrophysics and cosmology. There may be a hot dark matter component (HDM) to the Universe: simulations of structure formation...
fit the observations only when some significant quantity of HDM is included. If so, neutrinos would be, at least by weight, one of the most important ingredients in the Universe.

Regardless of mass and oscillations, astrophysical interest in the neutrino and their properties arises from the fact that it is copiously produced in high temperature and/or high density environment and it often dominates the physics of those astrophysical objects. The interactions of the neutrino with matter is so weak that it passes freely through any ordinary matter existing in the Universe. This makes neutrinos to be a very efficient carrier of energy drain from optically thick objects becoming very good probes for the interior of such objects. For example, the solar neutrino flux is, together with heliosysmology, one of the two known probes of the solar core. A similar statement applies to objects as the type-II supernovas: the most interesting questions around supernovas, the explosion dynamics itself with the shock revival, and, the synthesis of the heaviest elements by the so-called r-processes, could be positively affected by changes in the neutrino flux, e.g. by MSW active or sterile conversions [12]. Finally, ultra high energy neutrinos are called to be useful probes of diverse distant astrophysical objects. Active Galactic Nuclei (AGN) should be copious emitters of $\nu$’s, providing both detectable point sources and an observable diffuse background which is larger in fact than the atmospheric neutrino background in the very high energy range [13].

This review is organized as follows, in section 2 we discuss the neutrino in the SM. Section 3 is devoted to the possible ways for generating neutrino mass terms and different models for these possibilities are presented. Neutrino oscillation in vacuum and in matter are studied in section 4. The cosmological and the astrophysical constraints on diverse neutrino properties are summarized in section 5. In section 6 we give an introduction to the phenomenological description of neutrino oscillations in vacuum and in matter. In section 7 we give an extensive description of the different neutrino experiments, their results and their interpretation. Finally we present some conclusions and final remarks in section 7.

2 The neutrino in the Standard Model.

The current Standard Model of particles and interactions supposes the existence of three neutrinos. The three neutrinos are represented by two-component Weyl spinors each describing a left-handed fermion. They are the neutral, upper components of doublets $L_i$ with respect the $SU(2)$ group, the weak interaction group, we have,

$$L_i \equiv \left( \begin{array}{c} \nu_i \\ l_i \end{array} \right), \quad i = (\mu, \tau).$$

They have the third component of the weak isospin $I_{3W} = 1/2$ and are assigned an unit of the global $i$th lepton number. The three right-handed charged leptons have however no counterparts in the neutrino sector and transform as singlets with respect the weak interaction.
These SM neutrinos are strictly massless, the reason for this can be understood as follows. The only Lorenz scalar made out of them is the Majorana mass, of the form $\nu^\dagger_i \nu_i$; it has the quantum number of a weak isotriplet, with $I_{3W} = 1$ as well as two units of total lepton number. Thus to generate a renormalizable Majorana mass term at the tree level one needs a Higgs isotriplet with two units of lepton number. Since in the stricter version of the SM the Higgs sector is only constituted by a weak isodoublet, there are no tree-level neutrino masses. When quantum corrections are introduced we should consider effective terms where a weak isotriplet is made out of two isodoublets and which are not invariant under lepton number symmetry. The conclusion is that in the SM neutrinos are kept massless by a global chiral lepton number symmetry (and more general properties as renormalizability of the theory, see Ref.[8] for an applied version of this argument). However this is a rather formal conclusion, there is no any other independent, compelling theoretical argument in favor of such symmetry, or, with other words, there is no reason why we would like to keep it intact.

Independent from mass and charge oddities, in any other respect neutrinos are very well behaved particles within the SM framework and some figures and facts are unambiguously known about them. The LEP $Z$ boson line-shape measurements imply that are only three ordinary (weak interacting) light neutrinos [14, 15]. Big Bang Nucleosynthesis (BBN) constrains the parameters of possible sterile neutrinos, non-weak interacting or those which interact and are produced only by mixing [16]. All the existing data on the weak interaction processes in which neutrinos take part are perfectly described by the SM charged-current (CC) and neutral-current (NC) Lagrangians:

\[
L_{i}^{CC} = -\frac{g}{\sqrt{2}} \sum_{i=e,\mu,\tau} \bar{\nu}_{Li}\gamma_{\alpha}l_{Li}W^{\alpha} + h.c. \tag{2.1}
\]

\[
L_{i}^{NC} = -\frac{g}{2\cos\theta_{W}} \sum_{i=e,\mu,\tau} \bar{\nu}_{Li}\gamma_{\alpha}\nu_{Li}Z^{\alpha} + h.c. \tag{2.2}
\]

where $Z^{\alpha}, W^{\alpha}$ are the neutral and charged vector bosons intermediaries of the weak interaction. The CC and NC interaction Lagrangians conserve three total additive quantum numbers, the lepton numbers $L_{e,\mu,\tau}$ while the structure of the CC interactions is what determine the notion of flavor neutrinos $\nu_{e,\mu,\tau}$.

There are no indications in favor of the violation of the conservation of these lepton numbers in weak processes and very strong bounds on branching ratios of rare, lepton number violating, processes are obtained, for examples see Table 1.

| Process | Branching Ratio |
|---------|----------------|
| $\mu \rightarrow e\gamma$ | $< 4.9 \times 10^{-11}$ |
| $\tau \rightarrow e\gamma$ | $< 2.7 \times 10^{-6}$ |
| $\mu \rightarrow 3e$ | $< 1.0 \times 10^{-12}$ |
| $\tau \rightarrow \mu\gamma$ | $< 3.0 \times 10^{-6}$ |
| $\mu \rightarrow e(2\gamma)$ | $< 7.2 \times 10^{-11}$ |
| $\mu \rightarrow 3e$ | $< 2.9 \times 10^{-6}$ |

**Table 1:** Some lepton number violating processes. See Ref.[15], all limits at 90% CL.

From the theoretical point of view, in the minimal extension of the SM where right-handed neutrinos are introduced and the neutrino gets a mass, the branching ratio of the
\( \mu \to e\gamma \) decay is given by (2 generations are assumed [17]),

\[
R(\mu \to e\gamma) = G_F \left( \frac{\sin 2\theta \Delta m^2_{12}}{2M^2_W} \right)^2
\]

where \( m_{1,2} \) are the neutrino masses, \( M_W \) is the mass of the \( W \) boson and \( \theta \) is the mixing angle in the lepton sector. Using the experimental upper limit on the heaviest \( \nu_\tau \) neutrino one obtains \( R \sim 10^{-18} \), a value far from being measurable at present as we can see from table [1]. The \( \mu \to e\gamma \) and similar processes are sensitive to new particles not contained in the SM. The value is highly model dependent and could change by several orders of magnitude if we modify the neutrino sector for example introducing an extra number of heavy neutrinos.

### 3 Neutrino mass terms and models.

#### 3.1 Model independent neutrino mass terms

Phenomenologically, Lagrangian mass terms can be viewed as terms describing transitions between right (R) and left (L)-handed states. For a given minimal, Lorenz invariant, set of four fields: \( \psi_L, \psi_R, (\psi^c)_L, (\psi^c)_R \), would-be components of a generic Dirac Spinor, the most general mass part of the Lagrangian can be written as:

\[
L_{\text{mass}} = m_D (\bar{\psi}_L \psi_R) + \frac{1}{2} m_T (\bar{(\psi^c)_L} \psi_L) + \frac{1}{2} m_S (\bar{(\psi^c)_R} \psi_R) + h.c. \tag{3.1}
\]

In terms of the newly defined Majorana fields \( (\nu^c = \nu, N^c = N): \nu = (1/\sqrt{2})(\psi_L + (\psi^c)_L) \), \( N = (1/\sqrt{2})(\psi_R + (\psi^c)_R) \), the Lagrangian \( L_{\text{mass}} \) can be rewritten as:

\[
L_{\text{mass}} = (\bar{\nu}, \bar{N}) M \begin{pmatrix} \nu \\ N \end{pmatrix} \tag{3.2}
\]

where \( M \) is the neutrino mass matrix defined as:

\[
M \equiv \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \tag{3.3}
\]

We proceed further and diagonalizing the matrix \( M \) one finds that the physical particle content is given by two Majorana mass eigenstates: the inclusion of the Majorana mass splits the four degenerate states of the Dirac field into two non-degenerate Majorana pairs.

If we assume that the states \( \nu, N \) are respectively active (belonging to weak doublets) and sterile (weak singlets), the terms corresponding to the "Majorana masses" \( m_T \) and \( m_S \) transform as weak triplets and singlets respectively. While the term corresponding to \( m_D \) is an standard, weak singlet in most cases, Dirac mass term.

The neutrino mass matrix can easily be generalized to three or more families, in which case the masses become matrices themselves. The complete flavor mixing comes
from two different parts, the diagonalization of the charged lepton Yukawa couplings and that of the neutrino masses. In most simple extensions of the SM, this CKM-like leptonic mixing is totally arbitrary with parameters only to be determined by experiment. Their prediction, as for the quark hierarchies and mixing, needs further theoretical assumptions (i.e. Ref.[8, 18] predicting $\nu_\mu - \nu_\tau$ maximal mixing).

We can analyze different cases. In the case of a purely Dirac mass term, $m_T = m_S = 0$ in Eq.(3.2), the $\nu, N$ states are degenerate with mass $m_D$ and a four component Dirac field can be recovered as $\nu \equiv \nu + N$. It can be seen that, although violating individual lepton numbers, the Dirac mass term allows a conserved lepton number $L = L_\nu + L_N$.

In the general case, pure Majorana mass transition terms, $m_T$ or $m_S$ terms in Lagrangian (3.2), describe in fact a particle-antiparticle transition violating lepton number by two units ($\Delta L = \pm 2$). They can be viewed as the creation or annihilation of two neutrinos leading therefore to the possibility of the existence of neutrinoless double beta decay.

In the general case where all classes of terms are allowed, it is interesting to consider the so-called ”see-saw” limit in Eq.(3.2). In this limit taking $m_T \sim 1/m_S \sim 0, m_D << m_S$, the two Majorana neutrinos acquire respectively masses $m_1 \sim m_D^2/m_S << m_D, m_2 \sim m_S$. There is one heavy neutrino and one neutrino much lighter than the typical Dirac fermion mass. One of neutrino mass has been automatically suppressed, balanced up (“see-saw”) by the heavy one. The ”see-saw” mechanism is a natural way of generating two well separated mass scales.

### 3.2 Neutrino mass models

Any fully satisfactory model that generates neutrino masses must contain a natural mechanism that explains their small value, relative to that of their charged partners. Given the latest experimental indications it would also be desirable that includes any comprehensive justification for light sterile neutrinos and large, near maximal, mixing.

Different models can be distinguished according to the new particle content or according to the scale. According to the particle content, of the different open possibilities, if we want to break lepton number and to generate neutrino masses without introducing new fermions in the SM, we must do it by adding to the SM Higgs sector fields carrying lepton numbers, one can arrange then to break lepton number explicitly or spontaneously through their interactions. But, possibly, the most straightforward approach to generate neutrino masses is to introduce for each neutrino an additional weak neutral singlet.

This happens naturally in the framework of LR symmetric models where the origin of SM parity ($P$) violation is ascribed to the spontaneous breaking of a baryon-lepton (the $B - L$ quantum number) symmetry.

In the $SO(10)$ GUT the Majorana neutral particle $N$ enters in a natural way in order to complete the matter multiplet, the neutral $N$ is a $SU(3) \times SU(2) \times U(1)$ singlet.

According to the scale where the new physics have relevant effects, Unification (i.e. the aforementioned $SO(10)$ GUT) and weak-scale approaches (i.e. radiative models) are
usually distinguished \[19, 20\].

The anomalies observed in the solar neutrino flux, atmospheric flux and low energy accelerator experiments cannot all be explained consistently without introducing a light, then necessarily sterile, neutrino. If all the Majorana masses are small, active neutrinos can oscillate into the sterile right handed fields. Light sterile neutrinos can appear in particular see-saw mechanisms if additional assumptions are considered (“singular see-saw “ models) with some unavoidable fine tuning. The alternative to such fine tuning would be seesaw-like suppression for sterile neutrinos involving new unknown interactions, i.e. family symmetries, resulting in substantial additions to the SM, (i.e. some sophisticated superstring-inspired models, Ref.[21]).

Finally some example of weak scale models, radiative generated mass models where the neutrino masses are zero at tree level constitute a very different class of models: they explain in principle the smallness of \( m_\nu \) for both active and sterile neutrinos. Different mass scales are generated naturally by different number of loops involved in generating each of them. The actual implementation generally requires however the ad-hoc introduction of new Higgs particles with nonstandard electroweak quantum numbers and lepton number-violating couplings [22].

The origin of the different Dirac and Majorana mass terms \( m_D, m_S, M_T \) appearing above is usually understood by a dynamical mechanism where at some scale or another some symmetry is spontaneously broken as follows.

First we will deal with the Dirac mass term. For the case of interest, \( \nu_L \) and \( \nu_R \) are SU(2) doublets and singlets respectively, the mass term describes then a \( \Delta I = 1/2 \) transition and is generated from SU(2) breaking with a Yukawa coupling:

\[
L_{Yuk} = h_i (\bar{\nu}_i \bar{l}_i)_L \left( \begin{array}{c} \phi^0 \\ \phi^- \end{array} \right) N_{Ri} + \text{h.c.} \tag{3.4}
\]

Where \( \phi^0, \phi^- \) are the components of the Higgs doublet. The coefficient \( h_i \) is the Yukawa coupling. One has that, after symmetry breaking, \( m_D = h_i v/2 \) where \( v \) is the vacuum expectation value of the Higgs doublet. A neutrino Dirac mass is qualitatively just like any other fermion masses, but that leads to the question of why it is so small in comparison with the rest of fermion masses: one would require \( h_{\nu_e} < 10^{-10} \) in order to have \( m_{\nu_e} < 10 \) eV. Or in other words: \( h_{\nu_e}/h_e \sim 10^{-5} \) while for the hadronic sector we have \( h_{up}/h_{down} \sim O(1) \).

Now we will deal with the Majorana mass terms. The \( m_S \) term will appear if \( N \) is a gauge singlet. In this case a renormalizable mass term of the type \( L_N = m_S V_i N_i \) is allowed by the SM gauge group \( SU(3) \times SU(2) \times U(1) \) symmetry. However, it would not be consistent in general with unified symmetries, i.e. with a full SO(10) symmetry and some complicated mechanism should be invoked. A \( m_S \) term is usually associated with the breaking of some larger symmetry, the expected scale for it should be in a range covering from \( \sim \) TeV (LR models) to GUT scales \( \sim 10^{15} - 10^{17} \) GeV.

Finally, the \( m_T \) term will appear if \( \nu_L \) is active, belongs to some gauge doublet. In this case we have \( \Delta I = 1 \) and \( m_T \) must be generated by either a) an elementary Higgs triplet or
b) by an effective operator involving two Higgs doublets arranged to transform as a triplet. In case a), for an elementary triplet $m_T \sim h_T v_T$, where $h_T$ is a Yukawa coupling and $v_T$ is the triplet VEV. The simplest implementation (the old Gelmini-Roncadelli model [23]) is excluded by the LEP data on the Z width: the corresponding Majoron couples to the Z boson increasing significantly its width. Variant models involving explicit lepton number violation or in which the Majoron is mainly a weak singlet ([24], invisible Majoron models) could still be possible. In case b), for an effective operator originated mass, one expects $m_T \sim 1/M$ where $M$ is the scale of the new physics which generates the operator.

A few words about the range of expected neutrino masses for different types of models depending on the values of $m_D, M_{S,T}$. For $m_S \sim 1$ TeV (LR models) and with typical $m_D$'s, one expects masses of order $10^{-1}$ eV, 10 keV, and 1 MeV for the $\nu_{e,\mu,\tau}$ respectively. GUT theories motivates a big range of intermediate scales $10^{12} - 10^{16}$ GeV. In the lower end of this range, for $m_S \sim 10^{12}$ GeV (some superstring-inspired models, GUT with multiple breaking stages) one can obtain light neutrino masses of the order $(10^{-7}$ eV, $10^{-3}$ eV, 10 eV). At the upper end, for $m_S \sim 10^{16}$ (grand unified seesaw with large Higgs representations) one typically finds smaller masses around $(10^{-11}, 10^{-7}, 10^{-2})$ eV somehow more difficult to fit into the present known experimental facts.

3.3 The magnetic dipole moment and neutrino masses

The magnetic dipole moment is another probe of possible new interactions. Majorana neutrinos have identically zero magnetic and electric dipole moments. Flavor transition magnetic moments are allowed however in general for both Dirac and Majorana neutrinos. Limits obtained from laboratory experiments are of the order of a few $\times 10^{-10} \mu_B$ and those from stellar physics or cosmology are $O(10^{-11} - 10^{-13}) \mu_B$. In the SM electroweak theory, extended to allow for Dirac neutrino masses, the neutrino magnetic dipole moment is nonzero and given, as ([15] and references therein):

$$\mu_\nu = \frac{3eG_F m_\nu}{8\pi^2\sqrt{2}} = 3 \times 10^{-19}(m_\nu/1\text{ eV})\mu_B$$

(3.5)

where $\mu_B$ is the Bohr magneton. The proportionality of $\mu_\nu$ to the neutrino mass is due to the absence of any interaction of $\nu_R$ other than its Yukawa coupling which generates its mass. In LR symmetric theories $\mu_\nu$ is proportional to the charged lepton mass: a value of $\mu_\nu \sim 10^{-13} - 10^{-14} \mu_B$ can be reached still too small to have practical astrophysical consequences.

Magnetic moment interactions arise in any renormalizable gauge theory only as finite radiative corrections. The diagrams which generate a magnetic moment will also contribute to the neutrino mass once the external photon line is removed. In the absence of additional symmetries a large magnetic moment is incompatible with a small neutrino mass. The way out suggested by Voloshin consists in defining a SU(2)$_\nu$ symmetry acting on the space ($\nu, \nu'$), magnetic moment terms are singlets under this symmetry. In the limit of exact SU(2)$_\nu$ the neutrino mass is forbidden but $\mu_\nu$ is allowed [25]. Diverse con-
crete models have been proposed where such symmetry is embedded into an extension of the SM (left-right symmetries, SUSY with horizontal gauge symmetries [26]).

4 Aspects of some theoretical models for neutrino mass

4.1 Neutrino masses in LR models

A very natural way to generate neutrino mass is to minimally extend the SM including additional 2-spinors as right handed neutrinos and at the same time extend the, non-QCD, SM gauge symmetry group to \( G_{LR} \equiv SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \). The resulting model, initially proposed in 1973-1974, is known as the left-right (LR) symmetric model [27]. This kind of models were first proposed with the goal of seeking a spontaneous origin for \( P \) violation in weak interactions: CP and \( P \) are conserved at large energies; at low energies, however, the group \( G_{LR} \) breaks down spontaneously at a scale \( M_R \). Any new physics correction to the SM would be of order \( (M_L/M_R)^2 \) where \( M_L \sim m_W \); if we choose \( M_R \gg M_L \) we obtain only small corrections, compatible with present known physics. We can satisfactorily explain in this case the small quantity of CP violation observed in present experiments and why the neutrino mass is so small, as we will see below.

The quarks \((Q)\) and leptons \((L)\) in LR models transform as doublets under the group \( SU(2)_{L,R} \) as follows: \( Q_L, L_L \sim (2, 1) \) and \( Q_R, L_R \sim (1, 2) \). The gauge interactions are symmetric between left and right-handed fermions; therefore before symmetry spontaneous breaking, weak interactions, as the others, conserve parity.

The breaking of the gauge symmetry is implemented by multiplets of LR symmetric Higgs fields, the concrete choosing of these multiplets is not unique. It has been shown that in order to understand the smallness of the neutrino mass, it is convenient to choose respectively one doublet and two triplets as follows:

\[
\phi \sim (2, 2, 0) \\
\Delta_L \sim (3, 1, 2) , \quad \Delta_R \sim (1, 3, 2).
\]

The Yukawa couplings of these Higgs fields to the quarks and leptons are given by

\[
L_{yuk} = h_1 \bar{L}_L \phi L_R + h_2 \bar{L}_L \tilde{\phi} L_R + h'_1 \bar{Q}_L \phi Q_R + h'_2 \bar{Q}_L \tilde{\phi} Q_R + f(L_L L_L \Delta_L + L_R L_R \Delta_R) + h.c. \tag{4.1}
\]

The gauge symmetry breaking proceeds in two steps. The \( SU(2)_R \times U(1)_{B-L} \) is broken down to \( U(1)_Y \) by choosing \( \langle \Delta_R^0 \rangle = v_R \neq 0 \) since this carries both \( SU(2)_R \) and \( U(1)_{B-L} \) quantum numbers. It gives mass to charged and neutral right handed gauge bosons, \( i.e., \)

\[
M_{W_R} = g v_R , \quad M_{Z'} = \sqrt{2} g v_R / \sqrt{1 - \tan^2 \theta_W}.
\]

Furthermore, as consequence of \( f \)-term in the Lagrangian above this stage of symmetry breaking also leads to a mass term for the right-handed neutrinos of the order \( \sim f v_R \).
Next, as we break the SM symmetry by turning on the vev’s for $\phi$ fields as $\langle \phi \rangle = \operatorname{diag}(v_\kappa, v'_\kappa)$, with $v_R >> v'_\kappa >> v_\kappa$, we give masses to the $W_L$ and the $Z$ bosons and also to quarks and leptons ($m_\kappa \sim h v_\kappa$). At the end of the process of spontaneous symmetry breaking the two $W$ bosons of the model will mix, the lowest physical mass eigenstate is identified as the observed $W$ boson. Current experimental limits set the limit (see Ref.[15], at 90% CL) $m_{WR} > 550$ GeV.

In the neutrino sector the above Yukawa couplings after $SU(2)_L$ breaking by $\langle \phi \rangle \neq 0$ leads to the Dirac masses for the neutrino. The full process leads to the following mass matrix for the $\nu, N$, (the matrix M in eq.3.3)

$$M = \begin{pmatrix} \sim 0 & hv_\kappa \\ hv_\kappa & f v_R \end{pmatrix}. \quad (4.2)$$

From the structure of this matrix we can see the see-saw mechanism at work. By diagonalizing $M$, we get a light neutrino corresponding to the eigenvalue $m_\nu \simeq (hv_\kappa)^2/f v_R$ and a heavy one with mass $m_N \simeq f v_R$.

Variants of the basic LR model include the possibility of having Dirac neutrinos as the expense of enlarging the particle content. The introduction of two new singlet fermions and a new set of carefully-chosen Higgs bosons, allows us to write the $4 \times 4$ mass matrix [28]:

$$M = \begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D & 0 & 0 & f v_R \\ 0 & 0 & 0 & \mu \\ 0 & f v_R & \mu & 0 \end{pmatrix}. \quad (4.3)$$

Matrix 4.3 leads to two Dirac neutrinos, one heavy with mass $\sim f v_R$ and another light with mass $m_\nu \sim m_D \mu/f v_R$. This light four component spinor has the correct weak interaction properties to be identified as the neutrino. A variant of this model can be constructed by addition of singlet quarks and leptons. One can arrange these new particles in order that the Dirac mass of the neutrino vanishes at the tree level and arises at the one-loop level via $W_L - W_R$ mixing.

Left-right symmetric models can be embedded in grand unification groups. The simplest GUT model that leads by successive stages of symmetry breaking to left-right symmetric models at low energies is the $SO(10)$-based model. A example of LR embedding GUT Supersymmetric theories will be discussed below in the context of Superstring-inspired models.

4.2 SUSY models: Neutrino masses without right-handed neutrinos

Supersymmetry (SUSY) models with explicit broken $R$-parity provide an interesting example of how we can generate neutrino masses without using a right-handed neutrino but incorporating new particles and enlarging the Higgs sector.

In a generic SUSY model, due to the Higgs and lepton doublet superfields have the same $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers, we have in the superpotential terms,
bilinear or trilinear in the superfields, that violate baryon and lepton number explicitly. They lead to a mass for the neutrino but also to proton decay with unacceptable high rates. One radical possibility is to introduce by hand a symmetry that rule out these terms, this is the role of the $R$-symmetry introduced in the MSSM.

A less radical possibility is to allow for the existence in the superpotential of a bilinear term, i.e. $W = \epsilon_3 L_3 H_2$. This is simplest way to illustrate the idea of generating neutrino mass without spoiling current limits on proton decay. The bilinear violation of $R$-parity implied by the $\epsilon_3$ term leads by a minimization condition to a non-zero sneutrino vev, $v_3$. In such a model the $\tau$ neutrino acquire a mass, due to the mixing between neutrinos and neutralinos. The $\nu_e$ and $\nu_\mu$ neutrinos remain massless in this model, it is supposed that they get masses from scalar loop contributions. The model is phenomenologically equivalent to a three Higgs doublet model where one of these doublets (the sneutrino) carry a lepton number which is broken spontaneously. We have the following mass matrix for the neutralino-neutrino sector, in block form the $5 \times 5$ matrix reads:

$$M = \begin{pmatrix}
G & Q \\
Q^t & \\
0 & -\mu & 0 \\
-\mu & 0 & \epsilon_3 \\
0 & \epsilon_3 & 0
\end{pmatrix}
$$

(4.4)

where $G = diag(M_1, M_2)$ corresponding to the two gauginos masses. The $Q$ is a $2 \times 3$ matrix containing $v_{u,d,3}$ the vevs of $H_1$, $H_2$ and the sneutrino. The next two rows are Higgsinos and the last one denotes the tau neutrino. Let us remind that gauginos and Higgsinos are the supersymmetric fermionic counterparts of the gauge and Higgs fields.

In diagonalizing the mass matrix $M$, a “see-saw” mechanism is again at work, in which the role of $M_D, M_R$ scale masses are easily recognized. It turns out that $\nu_\tau$ mass is given by $(v'_3 \equiv \epsilon_3 v_d + \mu v_3)$,

$$m_{\nu_\tau} \propto \frac{(v'_3)^2}{M},$$

where $M$ is the largest gaugino mass. However, in an arbitrary SUSY model this mechanism leads to (although relatively small if $M$ is large) still too large $\nu_\tau$ masses. To obtain a realistically small $\nu_\tau$ mass we have to assume universality among the soft SUSY breaking terms at GUT scale. In this case the $\nu_\tau$ mass is predicted to be small due to a cancellation between the two terms which makes negligible the $v'_3$.

We consider now the properties of neutrinos in superstring models. In a number of these models, the effective theory imply a supersymmetric $E_6$ grand unified model, with matter fields belonging to the 27 dimensional representations of $E_6$ group plus additional $E_6$-singlet fields. The model contains additional neutral leptons in each generation and neutral $E_6$-singlets, gauginos and Higgsinos. As before but with a larger number of them, all of these neutral particles can mix, making the understanding of neutrino masses quite difficult if no simplifying assumptions are employed.

Several of these mechanisms have been proposed to understand neutrino masses [28]. In some of these mechanisms the huge neutral mixing mass matrix is reduced drastically.
down to a $3 \times 3$ neutrino mass matrix result of the mixing of the $\nu, \nu^c$ with an additional neutral field $T$ whose nature depends on the particular mechanism. In the basis $(\nu, \nu^c, T)$ the mass matrix is of the form (with $\mu$ possibly being zero):

$$M = \begin{pmatrix}
0 & m_D & 0 \\
m_D & 0 & \lambda_2 v_R \\
0 & \lambda_2 v_R & \mu
\end{pmatrix}. \quad (4.5)$$

We distinguish two important cases, the R-parity violating case and the mixing with a singlet, where the sneutrinos, superpartners of $\nu^c$, are assumed to acquire a v.e.v. of order $v_R$.

In the first case the $T$ field corresponds to a gaugino with a Majorana mass $\mu$ that can arise at two-loop order. Usually $\mu \approx 100$ GeV, if we assume $\lambda v_R \approx 1$ TeV additional dangerous mixing with the Higgsinos can be neglected and we are lead to a neutrino mass $m_\nu \approx 10^{-1}$ eV. Thus, smallness of neutrino mass is understood without any fine tuning of parameters.

In the second case the field $T$ corresponds to one of the $E_6$-singlets presents in the model [30, 31]. One has to rely on symmetries that may arise in superstring models on specific Calabi-Yau space to conveniently restrict the Yukawa couplings. If we have $\mu \equiv 0$ in matrix 4.5, this leads to a massless neutrino and a massive Dirac neutrino. There would be neutrino mixing even if the light neutrino remains strictly massless. If we include a possible Majorana mass term for the $S$-fermion of order $\mu \approx 100$ GeV we get similar values of the neutrino mass as in the previous case.

It is worthy to mention that mass matrices as that one appearing in expression 4.5 have been proposed without embedding in a supersymmetric or any other deeper theoretical framework. In this case small tree level neutrino masses are obtained without making use of large scales. For example, the model proposed by Ref.[32] (see also Ref.[33]) which incorporates by hand additional iso-singlet neutral fermions. The smallness of neutrino masses is explained directly from the, otherwise left unexplained, smallness of the parameter $\mu$ in such a model.

4.3 Neutrino masses and extra dimensions

Recently, models where space-time is endowed with extra dimensions ($4+n$) have received some interest [34]. It has been realized that the fundamental scale of gravity need not be the 4-dimensional “effective” Planck scale $M_P$ but a new scale $M_f$, as low as $M_f \sim$ TeV. The observed Planck scale $M_P$ is then related to $M_f$ in $4+n$ dimensions, by

$$\eta^2 \equiv \left( \frac{M_f}{M_P} \right)^2 \sim \frac{1}{M_f^2 R^n}$$

where $R$ is the typical length of the extra dimensions. i.e., the coupling is $M_f/M_P \approx 10^{-16}$ for $M_f \sim 1$ TeV. For $n = 2$, the radii $R$ of the extra dimensions are of the order of the millimeter, which could be hidden from many, extremely precise, measurements that exist
at present but it would give hope to probe the concept of hidden space dimensions (and gravity itself) by experiment in the near future.

According to current theoretical frameworks (see for example Ref. [34]), all the SM group-charged particles are localized on a 3-dimensional hyper-surface ‘brane’ embedded in the bulk of the \( n \) extra dimensions. All the particles split in two categories, those that live on the brane and those which exist everywhere, as ‘bulk modes’. In general, any coupling between the brane and the bulk modes are suppressed by the geometrical factor \( \eta \). Graviton and possible other neutral states belongs to the second category. The observed weakness of gravity can be then interpreted as a result of the new space dimensions in which gravity can propagate.

The small coupling above can also be used to explain the smallness of the neutrino mass [35]. The left handed neutrino \( \nu_L \) having weak isospin and hypercharge must reside on the brane. Thus it can get a naturally small Dirac mass through the mixing with some bulk fermion which can be interpreted as right handed neutrinos \( \nu_R \):

\[
L_{\text{mass,Dirac}} \sim h\eta H \bar{\nu}_L \nu_R.
\]

Here \( H, h \) are the Higgs doublet fields and a Yukawa coupling. After EW breaking this interaction will generate the Dirac mass \( m_D = h\nu_\eta \simeq 10^{-5} \text{eV} \). The right handed neutrino \( \nu_R \) has a whole tower of Kaluza-Klein relatives \( \nu_{iR} \). The masses of these states are given by \( m_i = i/R \), the \( \nu_L \) couples with all with the same mixing mass. We can write the mass Lagrangian as \( L = \bar{\nu}_L M \nu_R \) where \( \nu_L = (\nu_L, \tilde{\nu}_L, \ldots) \), \( \nu_R = (\nu_0 R, \tilde{\nu}_1 R, \ldots) \) and the resulting mass matrix \( M \) being:

\[
M = \begin{pmatrix}
m_D & \sqrt{2}m_D & \sqrt{2}m_D & \sqrt{2}m_D \\
0 & 1/R & 0 & 0 \\
0 & 0 & 2/R & 0 \\
\vdots & \vdots & \vdots & k/R \\
\vdots & \vdots & \vdots & \vdots
\end{pmatrix}_{(4.6)}
\]

The eigenvalues of the matrix \( MM^\dagger \) are given by a transcendental equation. In the limit, \( m_D R \rightarrow 0 \), or \( m_D \rightarrow 0 \), the eigenvalues are \( \sim k/R \), \( k \in Z \) with a doubly-degenerated zero eigenvalue.

Other examples can be considered which incorporates a LR symmetry (see for example Ref. [36]), a \( SU(2)_R \) right handed neutrino is assumed to live on the brane together with the standard one. In this class of models, it has been shown that the left handed neutrino is exactly massless whereas assumed bulk sterile neutrinos have masses related to the size of the extra dimensions. They are of order \( 10^{-3} \text{ eV} \), if there is at least one extra dimension with size in the micrometer range.

### 4.4 Family symmetries and neutrino masses

The observed mass and mixing interfamily hierarchy in the quark and, presumably in the lepton sector might be a consequence of the existence of a number of \( U(1)_F \) family
symmetries [37]. The observed intrafamily hierarchy, the fact that for each family \( m_{up} >> m_{down} \), seem to require one of these to be anomalous [38, 39].

A simple model with one family-dependent anomalous \( U(1) \) beyond the SM was first proposed in Ref.[38] to produce the observed Yukawa hierarchies, the anomalies being canceled by the Green-Schwartz mechanism which as a by-product is able to fix the Weinberg angle (see also Ref.[39]).

Recent developments includes the model proposed in Ref.[18], which is inspired by models generated by the \( E_6 \times E_8 \) heterotic string. The gauge structure of the model is that of the SM augmented by three Abelian \( U(1) \) symmetries \( X,Y \), the first one is anomalous and family independent. Two of the them, the non-anomalous ones, have specific dependences on the three chiral families designed to reproduce the Yukawa hierarchies. There are right handed neutrinos which trigger neutrino masses by the see-saw mechanism.

The three symmetries \( X,Y^{1,2} \) are spontaneously broken at a high scale \( M \) by stringy effects. It is assumed that three fields \( \theta_i \) acquire a vacuum value. The \( \theta_i \) fields are singlets under the SM symmetry but not under the \( X \) and \( Y^{1,2} \) symmetries. In this way, the Yukawa couplings appear as the effective operators after \( U(1) \) spontaneous symmetry breaking.

For neutrinos we have [40] the mass Lagrangian

\[
L_{mass} \sim h_{ij} L_i H_u N_j^c \lambda^{q_i + n_j} + M_N \xi_{ij} N_i^c N_j^c \lambda^{n_i + n_j},
\]

where \( h_{ij}, \xi_{ij} \simeq O(1) \). The parameter \( \lambda \) determine the mass and mixing hierarchy, \( \lambda = (\theta)/M \sim \sin \theta_c \) where \( \theta_c \) is the Cabibbo angle. The \( q_i, n_i \) are the \( U(1) \) charges assigned respectively to left handed leptons \( L \) and right handed neutrinos \( N \).

These coupling generate the following mass matrices for neutrinos:

\[
M^D_{\nu} = \text{diag}(\lambda^{q_1}, \lambda^{q_2}, \lambda^{q_3}) \hat{h} \text{diag}(\lambda^{n_1}, \lambda^{n_2}, \lambda^{n_3}) \langle H_u \rangle,
\]

\[
M_{\nu} = \text{diag}(\lambda^{n_1}, \lambda^{n_2}, \lambda^{n_3}) \hat{\xi} \text{diag}(\lambda^{n_1}, \lambda^{n_2}, \lambda^{n_3}) M_N.
\] (4.7)

From these matrices, the see-saw mechanism gives the formula for light neutrinos:

\[
m_{\nu} \simeq \frac{(H_u)^2}{M} \text{diag}(\lambda^{q_1}, \lambda^{q_2}, \lambda^{q_3}) \hat{h} \hat{\xi}^{-1} \hat{h}^T \text{diag}(\lambda^{n_1}, \lambda^{n_2}, \lambda^{n_3}).
\]

The neutrino mass mixing matrix depends only on the charges assigned to the left handed neutrinos, by a cancellation of right handed neutrino charges by virtue of the see-saw mechanism. There is freedom in assigning charges \( q_i \). If the charges of the second and the third generations of leptons are equal \( q_2 = q_3 \), then one is lead to a mass matrix which have the following structure:

\[
m_{\nu} \sim \begin{pmatrix}
\lambda^6 & \lambda^3 & \lambda^3 \\
\lambda^3 & a & b \\
\lambda^3 & b & c
\end{pmatrix},
\] (4.8)

where \( a, b, c \sim O(1) \). This matrix can be diagonalized by a large \( \nu_2 - \nu_3 \) rotation, it is consistent with a large \( \mu - \tau \) mixing. In this theory, explanation of the large neutrino mixing is reduced to a theory of prefactors in front of powers of the parameter \( \lambda \).
5.1 Cosmological mass limits and Dark Matter

There are some indirect constraints on neutrino masses provided by cosmology. The most relevant is the constraint which follows from demanding that the energy density in neutrinos should not be too high. At the end of this section we will deal with some other limits as the lower mass limit obtained from galactic phase space requirements or limits on the abundance of additional weakly interacting light particles.

Stable neutrinos with low masses ($m_\nu \lesssim 1$ MeV) make a contribution to the total energy density of the universe which is given by:

$$\rho_\nu = m_\nu \rho_c$$  \hspace{1cm} (5.1)

where the total mass $m_{\text{tot}} = \sum \nu (g_\nu / 2) m_\nu$, with the number of degrees of freedom $g_\nu = 4(2)$ for Dirac (Majorana) neutrinos. The number density of the neutrino sea is related to that one of photons by entropy conservation in the adiabatic expansion of the universe, $n_\nu = 3/11 n_\gamma$, and this last one is very accurately obtained from the CMBR measurements, $n_\gamma = 410.5 \text{ cm}^{-3}$ (for a Planck spectrum with $T_0 = 2.725 \pm 0.001 \text{ K} \simeq 2.35 \times 10^{-4} \text{ eV}$). Writing $\Omega_\nu = \rho_\nu / \rho_c$, where $\rho_c$ is the critical energy density of the universe ($\rho_c = 3H_0^2/8\pi G_N$), we have ($m_\nu >> T_0$)

$$\Omega_\nu h^2 = 10^{-2} m_{\text{tot}} \text{ (eV)},$$  \hspace{1cm} (5.2)

where $h$ is the reduced Hubble constant, recent analysis [41] give the favored value: $h = 0.71 \pm 0.08$.

Constrained by requirements from BBN Nucleosynthesis, galactic structure formation and large scale observations, increasing evidence (luminosity-density relations, galactic rotation curves, large scale flows) suggests that [42]

$$\Omega_M h^2 = 0.05 - 0.2,$$  \hspace{1cm} (5.3)

where $\Omega_M$ is the total mass density of the universe, as a fraction of the critical density $\rho_c$. This $\Omega_M$ includes contributions from a variety of sources: photons, baryons, non-baryonic Cold Dark Matter (CDM) and Hot Dark Matter (HDM).

The two first components are rather well known. The photon density is very well known to be quite small: $\Omega_\gamma h^2 = 2.471 \times 10^{-5}$. The deuterium abundance BBN constraints [43] on the baryonic matter density ($\Omega_B$) of the universe $0.017 \leq \Omega_B h^2 \leq 0.021$.

The hot component, HDM is constituted by relativistic long-lived particles with masses much less than $\sim 1$ keV, in this category would enter the neutrinos. Detailed simulations of structure formation fit the observations only when one has some 20% of HDM (plus 80% CDM), the best fit being two neutrinos with a total mass of 4.7 eV. There seems to be however some kind of conflict within cosmology itself: observations of distant objects favor a large cosmological constant instead of HDM (see Ref.[44] and references therein). One may conclude that the HDM part of $\Omega_M$ does not exceed 0.2.
Requiring that \( \Omega_\nu < \Omega_M \), we obtain \( \Omega_\nu h^2 \lesssim 0.1 \). From here and from Eq. 5.2, we obtain the cosmological upper bound on the neutrino mass

\[
m_{\text{tot}} \lesssim 8 \text{ eV}.
\]

Mass limits, in this case lower limits, for heavy neutrinos (\( \sim 1 \text{ GeV} \)) can also be obtained along the same lines. The situation gets very different if the neutrinos are unstable, one gets then joint bounds on mass and lifetime, then mass limits above can be avoided.

There is a limit to the density of neutrinos (or weak interacting dark matter in general) which can be accumulated in the halos of astronomical objects (the Tremaine-Gunn limit): if neutrinos form part of the galactic bulges phase-space restrictions from the Fermi-Dirac distribution implies a lower limit on the neutrino mass [45]:

\[
m_\nu \gtrsim 33 \text{ eV}.
\]

The abundance of additional weakly interacting light particles, such as a light sterile \( \nu_s \), is constrained by BBN since it would enter into equilibrium with the active neutrinos via neutrino oscillations. A limit on the mass differences and mixing angle with another active neutrino of the type \( \Delta m^2 \sin^2 2\theta \lesssim 3 \times 10^{-6} \text{ eV}^2 \) should be fulfilled in principle. From here is deduced that the effective number of neutrino species is

\[
N_{\nu}^{\text{eff}} < 3.5 - 4.5.
\]

However systematical uncertainties in the derivation of the BBN bound make it too unreliable to be taken at face value and can eventually be avoided [46].

### 5.2 Neutrino masses and lepton asymmetry

In supersymmetric LR symmetric models, inflation, baryogenesis (or leptogenesis) and neutrino oscillations can become closely linked.

Baryogenesis in GUT theories is in general inconsistent with an inflationary universe. The exponential expansion during inflation will wash out any baryon asymmetry generated previously at GUT scale. One way out of this difficulty is to generate the baryon or lepton asymmetry during the process of reheating at the end of the inflation. In this case the physics of the scalar field that drives the inflation, the inflaton, would have to violate CP (see Ref.[45] and references therein).

The challenge of any baryogenesis model is to predict the observed asymmetry which is usually written as a baryon to photon (number or entropy) ratio. The baryon asymmetry is defined as

\[
n_B/s \equiv (n_b - n_{\bar{\nu}})/s.
\]

At present there is only matter and not known antimatter, \( n_{\bar{\nu}} \sim 0 \). The entropy density \( s \) is completely dominated by the contribution of relativistic particles so is proportional
to the photon number density which is very well known from CMBR measurements, at present \( s = 7.05 \, n_\gamma \). Thus, \( n_B/s \propto n_b/n_\gamma \). From BBN we know that \( n_b/n_\gamma = (5.1 \pm 0.3) \times 10^{-10} \) so we arrive to \( n_B/s = (7.2 \pm 0.4) \times 10^{-11} \) and from here we obtain equally the lepton asymmetry ratio.

It was shown in Ref. [47] that hybrid inflation can be successfully realized in a SUSY LR symmetric model with gauge group \( G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R \). The inflaton sector of this model consists of the two complex scalar fields \( S \) and \( \theta \) which at the end of inflation oscillate about the SUSY minimum and respectively decay into a pair of right-handed sneutrinos \( (\nu_\ell^c) \) and neutrinos. In this model, a primordial lepton asymmetry is generated [48] by the decay of the superfield \( \nu_2^c \) which emerges as the decay product of the inflaton. The superfield \( \nu_2^c \) decays into electroweak Higgs and (anti)lepton superfields. This lepton asymmetry is subsequently partially converted into baryon asymmetry by non-perturbative EW sphalerons.

The resulting lepton asymmetry [49] can be written as a function of a number of parameters among them the neutrino masses and mixing angles and compared with the observational constraints above.

It is highly non-trivial that solutions satisfying the constraints above and other physical requirements can be found with natural values of the model parameters. In particular, it is shown that the values of the neutrino masses and mixing angles which predict sensible values for the baryon or lepton asymmetry turn out to be also consistent with values required to solve the solar neutrino problem.

6 Phenomenology of Neutrino Oscillations

6.1 Neutrino Oscillation in Vacuum

If the neutrinos have nonzero mass, by the basic postulates of the quantum theory there will be in general mixing among them as in the case of quarks. This mixing will be observable at macroscopic distances from the production point and therefore will have practical consequences only if the difference of masses of the different neutrinos is very small, typically \( \Delta m \lesssim 1 \, \text{eV} \).

In presence of masses, weak \((\nu_w)\) and mass \((\nu_m)\) basis of eigenstates are differentiated. To transform between them we need an unitary matrix \( U \). Neutrinos can only be created and detected as a result of weak processes, at origin we have a weak eigenstate:

\[ \nu_w(0) = U \nu_m(0). \]

We can easily construct an heuristic theory of neutrino oscillations if we ignore spin effects as follows. After a certain time the system has evolved into

\[ \nu_m(t) = \exp(-iHt)\nu_m(0) \]

where \( H \) is the Hamiltonian of the system, free evolution in vacuum is characterized by \( H = \text{diag}(\ldots E_i \ldots) \) where \( E_i^2 = p^2 + m_i^2 \). In most cases of interest \( (E \sim \text{MeV}, m \sim \text{eV}) \),
it is appropriated the ultrarelativistic limit: in this limit \( p \simeq E \) and \( E \simeq p + m^2/2p \). The effective neutrino Hamiltonian can then be written \( H^{\text{eff}} = \text{diag}(\ldots m_i^2 \ldots)/2E \) and

\[
\nu_w(t) = U \exp(-iH^{\text{eff}}t)U^\dagger \nu_w(0) = \exp(-iH^{\text{eff}}_w t)\nu_w(0).
\]

In the last expression we have written the effective Hamiltonians in the weak basis \( H^{\text{eff}}_w \equiv M^2/2E \) with \( M \equiv U \text{diag}(\ldots m_i^2 \ldots)U^\dagger \). This derivation can be put in a firm basis and one finds again the same expressions as the first terms of rigorous expansions in \( E \), see for example the treatment using Foldy-Woodythens transformations in Ref.[104].

The results of the neutrino oscillation experiments are usually analyzed under the simplest assumption of oscillations between two neutrino types, in this case the mixing matrix \( U \) is the well known 2-dimension orthogonal rotation matrix depending on a single parameter \( \theta \). If we repeat all the computation above for this particular case, we find for example that the probability that a weak interaction eigenstate neutrino (\( \nu_e \)) has oscillated to other weak interaction eigenstate neutrino (\( \nu_\mu \)) after traversing a distance \( l(=ct) \) is

\[
P(\nu_e \rightarrow \nu_\mu; l) = \sin^22\theta \sin^2\left(\frac{l}{l_{\text{osc}}}\right)
\]

where the oscillation length is defined by \( 1/l_{\text{osc}} \equiv \frac{\delta m^2}{4E} \) and \( \delta m^2 = m_1^2 - m_2^2 \). Numerically, in practical units, it turns out that

\[
\frac{\delta m^2}{4E} \simeq 1.27 \frac{\delta m^2 (eV^2) l(m)}{E(MeV)}.
\]

These probabilities depend on two factors: a mixing angle factor \( \sin^22\theta \) and a kinematical factor which depends on the distance traveled, on the momentum of the neutrinos, as well as on the difference in the squared mass of the two neutrinos. Both, the mixing factor \( \sin^22\theta \) and the kinematical factor should be of \( O(1) \) to have a significant oscillations.

### 6.2 Neutrino Oscillations in Matter

When neutrinos propagate in matter, a subtle but potentially very important effect, the MSW effect, takes place which alters the way in which neutrinos oscillate into one another.

In matter the neutrino experiences scattering and absorption, this last one is always negligible. At very low energies, coherent elastic forward scattering is the most important process. As in optics, the net effect is the appearance of a phase difference, refractive index or equivalently a neutrino effective mass.

This effective mass can considerable change depending on the densities and composition of the medium, it depends also on the nature of the neutrino. In the neutrino case the medium is flavor-dispersive: the matter is usually nonsymmetric with respect \( e \) and \( \mu, \tau \) and the effective mass is different for the different weak eigenstates [50].

This is explained as follows for the simpler and most important case, the solar electron plasma. The electrons in the solar medium have charged current interactions with \( \nu_e \) but
not with \( \nu_\mu \) or \( \nu_\tau \). The resulting interaction energy is given by 
\[
H_{\text{int}} = \sqrt{2} G_F N_e,
\]
where \( G_F \) and \( N_e \) are the Fermi coupling and the electron density. The corresponding neutral current interactions are identical for all neutrino species and hence have no net effect on their propagation. Hypothetical sterile neutrinos would have no interaction at all. The effective global Hamiltonian in flavor space is now the sum of two terms, the vacuum part we have seen previously and the new interaction energy:
\[
H_{\text{eff,mat}} = H_{\text{eff,vac}} + H_{\text{int}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

The practical consequence of this effect is that the oscillation probabilities of the neutrino in matter could largely increase due to resonance phenomena [51]. In matter, for the two dimensional case and in analogy with vacuum oscillation, one defines an effective mixing angle as
\[
\sin 2\theta_M = \frac{\sin 2\theta / l_{\text{osc}}}{\left[ (\cos 2\theta / l_{\text{osc}} - G_F N_e / \sqrt{2})^2 + (\sin 2\theta / l_{\text{osc}})^2 \right]^{1/2}}. \tag{6.2}
\]
The presence of the term proportional to the electron density can give rise to a resonance. There is a critical density \( N_{e,\text{crit}} \), given by
\[
N_{e,\text{crit}} = \frac{\delta m^2 \cos 2\theta}{2\sqrt{2} E G_F},
\]
for which the matter mixing angle \( \theta_M \) becomes maximal (\( \sin 2\theta_M \rightarrow 1 \)), irrespective of the value of mixing angle \( \theta \). The probability that \( \nu_e \) oscillates into a \( \nu_\mu \) after traversing a distance \( l \) in this medium is given by Eq. (6.1), with two differences. First \( \sin 2\theta \rightarrow \sin 2\theta_M \). Second, the kinematical factor differ by the replacement of \( \delta m^2 \rightarrow \delta m^2 \sin 2\theta \). Hence it follows that, at the critical density,
\[
P_{\text{matter}}(\nu_e \rightarrow \nu_\mu; l)_{(N_e = N_{e,\text{crit}})} = \sin^2 \left( \sin 2\theta \frac{l}{l_{\text{osc}}} \right). \tag{6.3}
\]
This formula shows that one can get full conversion of a \( \nu_e \) weak interaction eigenstate into a \( \nu_\mu \) weak interaction eigenstate, provided that the length \( l \) and the energy \( E \) satisfy the relations
\[
\sin 2\theta \frac{l}{l_{\text{osc}}} = \frac{n\pi}{2}; \quad n = 1, 2, \ldots
\]
There is a second interesting limit to consider. This is when the electron density \( N_e \) is so large such that \( \sin 2\theta_M \rightarrow 0 \) or \( \theta_M \rightarrow \pi/2 \). In this limit, there are no oscillations in matter because \( \sin 2\theta_M \) vanishes and we have
\[
P_{\text{matter}}(\nu_e \rightarrow \nu_\mu; l)_{(N_e \gg \frac{\delta m^2}{2\sqrt{2} E G_F})} \rightarrow 0.
\]
7 Experimental evidence and phenomenological analysis

In the second part of this review, we will consider the existing experimental situation. It is fair to say that at present there are at least an equal number of positive as negative (or better “non-positive”) indications in favor of neutrino masses and oscillations.

7.1 Laboratory, reactor and accelerator results.

No indications in favor of a non-zero neutrino masses have been found in direct kinematical searches for a neutrino mass.

From the measurement of the high energy part of the tritium $\beta$ decay spectrum, upper limits on the electron neutrino mass are obtained. The two more sensitive experiments in this field, Troitsk [52] and Mainz [53], obtain results which are plagued by interpretation problems: apparition of negative mass squared and bumps at the end of the spectrum.

In the Troitsk experiment, the shape of the observed spectrum proves to be in accordance with classical shape besides a region $\sim 15$ eV below the end-point, where a small bump is observed; there are indications of a periodic shift of the position of this bump with a period of “exactly” $0.504 \pm 0.003$ year [54]. After accounting for the bump, they derive the limit $m_{\nu e}^2 = -1.0 \pm 3.0 \pm 2.1$ eV$^2$, or $m_{\nu e} < 2.5$ eV (95% C.L.).

The latest published results by the Mainz group leads to $m_{\nu e}^2 = -0.1 \pm 3.8 \pm 1.8$ eV$^2$ (1998 “Mainz data 1”), From which an upper limit of $m_{\nu e} < 2.9$ eV [53] (95% C.L., unified approach) is obtained. Preliminary data (1998 and 1999 measurements) provide a limit $m_{\nu e} < 2.3$ eV [55]. Some indication for the anomaly, reported by the Troitsk group, was found, but its postulated half year period is not supported by their data.

Diverse exotic explanations have been proposed to explain the Troitsk bump and their seasonal dependence. The main feature of the effect might be “phenomenologically” interpreted, not without problems, as $^3$He capture of relic neutrinos present in a high density cloud around the Sun [52, 56].

The Mainz and Troitsk ultimate sensitivity expected to be limited by systematics lies at the $\sim 2$ eV level. In the near future, it is planned a new large tritium $\beta$ experiment with sensitivity $0.6 - 1$ eV [55].

Regarding the heavier neutrinos, other kinematical limits are the following:

a) Limits for the muon neutrino mass have been derived using the decay channel $\pi^+ \rightarrow \mu^+ \nu_\mu$ at intermediate energy accelerators (PSI, LANL). The present limits are $m_{\nu_\mu} \lesssim 160$ keV [57].

b) A tau neutrino mass of less than 30 MeV is well established and confirmed by several experiments: limits of 28, 30 and 31 MeV have also been obtained by the OPAL, CLEO and ARGUS experiments respectively (see Ref.[58] and references therein). The best upper limit for the $\tau$ neutrino mass has been derived using the decay mode $\tau \rightarrow 5\pi^\pm \nu_\tau$ by the ALEPH collaboration [59]: $m_{\nu_\tau} < 18$ MeV (95% CL).
Many experiments on the search for neutrinoless double-beta decay \([(\beta\beta)_0]\),

\[(A, Z) \rightarrow (A, Z + 2) \rightarrow 2 \, e^-\],

have been performed. This process is possible only if neutrinos are massive and Majorana particles. The matrix element of the process is proportional to the effective Majorana mass \(\langle m \rangle = \sum \eta_i U_{ei}^2 m_i\). Uncertainties in the precise value of upper limits are relatively large since they depend on theoretical calculations of nuclear matrix elements. From the non-observation of \((\beta\beta)_0\), the Heidelberg-Moscow experiment gives the most stringent limit on the Majorana neutrino mass. After 24 kg/year of data [60] (see also earlier results in Ref.[61]), they set a lower limit on the half-life of the neutrinoless double beta decay in \(^{76}\text{Ge}\) of \(T_{1/2} > 5.7 \times 10^{25} \text{ yr at 90\% CL}\), thus excluding an effective Majorana neutrino mass \(| \langle m \rangle | > 0.2 \, \text{eV (90\% CL)}\). This result allows to set strong constraints on degenerate neutrino mass models. In the next years it is expected an increase in sensitivity allowing limits down to the \(| \langle m \rangle | \sim 0.02 - 0.006 \, \text{eV levels (GENIUS I and II experiments, [62])}\).

Many short-baseline (SBL) neutrino oscillation experiments with reactor and accelerator neutrinos did not find any evidence of neutrino oscillations. For example experiments looking for \(\bar{\nu}_e \rightarrow \bar{\nu}_e\) or \(\nu_\mu \rightarrow \nu_\mu\) dissapereance (Bugey, CCFR [63, 64]) or oscillations \(\nu_\mu \rightarrow \nu_e\) (CCFR,E776[64, 65]).

The first reactor long-baseline (L \(\sim 998-1115 \, \text{m}\)) neutrino oscillation experiment CHOOZ found no evidence for neutrino oscillations in the \(\bar{\nu}_e\) disappearance mode [66,67]. CHOOZ results are important for the atmospheric deficit problem: as is seen in Fig.(1) they are incompatible with an \(\nu_e \rightarrow \nu_\mu\) oscillation hypothesis for the solution of the atmospheric problem. Their latest results [67] imply an exclusion region in the plane of the two-generation mixing parameters (with normal or sterile neutrinos) given approximately by \(\Delta m^2 > 0.7 \, 10^{-4} \, \text{eV}^2\) for maximum mixing and \(\sin^2 2\theta > 0.10\) for large \(\Delta m^2\) (as shown approximately in Fig.(1) (left) which corresponds to early results). Lower sensitivity results, based only on the comparison of the positron spectra from the two different-distance nuclear reactors, has also been presented, they are shown in Fig.(1) (right). These are independent of the absolute normalization of the antineutrino flux, the cross section and the target and detector characteristics and are able alone to almost completely exclude the SK allowed oscillation region [67].

The Palo Verde Neutrino Detector searches for neutrino oscillations via the disappearance of electron anti-neutrinos produced by a nuclear reactor at a distance \(L \sim 750 – 890 \, \text{m}\). The experiment has been taking neutrino data since October 1998 and will continue taking data until the end of 2000 reaching its ultimate sensitivity. The analysis of the 1998-1999 data (first 147 days of operation) [68] yielded no evidence for the existence of neutrino oscillations. The ratio of observed to expected number of events:

\[\frac{\bar{\nu}_e,\text{obs}}{\bar{\nu}_e,\text{MC}} = 1.04 \pm 0.03 \pm 0.08.\]

The resulting \(\bar{\nu}_e \rightarrow \bar{\nu}_x\) exclusion plot is very similar to the CHOOZ one. Together with results from CHOOZ and SK, concludes that the atmospheric neutrino anomaly is very unlikely to be due to \(\nu_\mu \rightarrow \nu_e\) oscillation.
Los Alamos LSND experiment has reported indications of possible $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations [69]. They search for $\bar{\nu}_e$’s in excess of the number expected from conventional sources at a liquid scintillator detector located 30 m from a proton beam dump at LAMPF. It has been claimed that a $\bar{\nu}_e$ signal has been detected via the reaction $\bar{\nu}_ep \rightarrow e^+n$ with $e^+$ energy between 36 and 60 MeV, followed by a $\gamma$ from $np \rightarrow d\gamma$ (2.2 MeV).

The LSND experiment took its last beam on December, 1998. The analysis of the complete 1993-1998 data set (see Refs.[70–72]) yields a fitted-estimated excess of $\bar{\nu}_e$ of $90.9 \pm 26.1$. If this excess is attributed to neutrino oscillations of the type $\nu_\mu \rightarrow \nu_e$, it corresponds to an oscillation probability of $3.3 \pm 0.05 \times 10^{-3}$. The results of a similar search for $\nu_\mu \rightarrow \nu_e$ oscillations where the (high energy, $60 < E_\nu < 200\text{ MeV}$) $\nu_e$ are detected via the CC reaction $C(\nu_e,e^-)X$ provide a value for the corresponding oscillation probability of $2.6 \pm 1.0 \times 10^{-3}$ (1993-1997 data).

There are other exotic physics explanations of the observed antineutrino excess. One example is the lepton-number violating decay $\mu^+ \rightarrow e^+\bar{\nu}_e\nu_\mu$, which can explain these observations with a branching ratio $Br \sim 0.3\%$, a value which is lower but not very far from the respective existing upper limits ($Br < 0.2 - 1\%$, [15]).

The surprisingly positive LSND result has not been confirmed by the KARMEN experiment (Rutherford- Karlsruhe Laboratories). This experiment, following a similar experimental setup as LSND, searches for $\bar{\nu}_\mu$ produced by $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at a mean distance of 17.6 m. The time structure of the neutrino beam is important for the identification of the neutrino induced reactions and for the suppression of the cosmic ray background. Systematic time anomalies not completely understood has been reported which rest credibility to any further KARMEN claim. They see an excess of events above the typical muon decay curve, which is 4.3 sigmas off (1990-1999 data, see Ref.[73]) and which could represent an unknown instrumental effect.

Exotic explanations as the existence of a weakly interacting particle “X”, for example a mixing of active and sterile neutrinos, of a mass $m_X = m_\pi - m_\mu \simeq 33.9\text{ MeV}$ have been proposed as an alternative solution to these anomalies and their consequences extensively studied [73, 74]. This particle might be produced in reactions the $\pi^+ \rightarrow \mu^+ + X$ and decay as $X \rightarrow e^+e^-\nu$. KARMEN set upper limits on the visible branching ratio $\Gamma_X = \Gamma(\pi^+ \rightarrow \mu^+ + X)/\Gamma(\pi^+ \rightarrow \mu^+ + \nu_\mu)$and lifetime $\tau_X$. From their results [73] one obtains the relation ($1 << \tau_x(\mu\text{ s}) \sim 10^8$)

$$\frac{\Gamma_X}{\tau_X(\mu\text{ s})} \sim 10^{-18}.$$ 

More concretely, the results are as it follows. About antineutrino signal, the 1990-1995 and early 1997-1998 KARMEN data showed inconclusive results: They found no events, with an expected background of $2.88 \pm 0.13$ events, for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations [75]. The results of the search Feb. 1997- Dec. 1999 which include a 40-fold improvement in suppression of cosmic induced background has been presented in a preliminary way [73, 76]. They find this time 9.5 oscillation candidates in agreement with the, claimed, well known background expectation of $10.6 \pm 0.6$ events. An upper limit for the mixing
angle is deduced: $\sin^2 2\theta < 1.7 \times 10^{-3}$ (90% C.I.) for large $\Delta m^2 (= 100 \text{ eV}^2$). The positive LSND result in this channel could not be completely excluded but they are able to exclude the entire LSND favored regions above 2 eV$^2$ and most of the rest of its favored parameter space.

In the present phase, the KARMEN experiment will take data until spring 2001. At the end of this period, the KARMEN sensitivity is expected to be able to exclude the whole parameter region of evidence suggested by LSND if no oscillation signal were found (Fig.4). The first phase of a third pion beam dump experiment designed to set the LSND-KARMEN controversy has been approved to run at Fermilab. Phase I of "BooNe" (MiniBooNe) expects a 10 $\sigma$ signal ($\sim$ 1000 events) and thus will make a decisive statement either proving or ruling it out. Plans are to run early 2001. Additionally, there is a letter of intent of a similar experiment to be carried out at the CERN PS [77, 78].

The K2K experiment started in 1999 the era of very long-baseline neutrino-oscillation experiment using a well-defined neutrino beam.

In the K2K experiment ($L \sim 250 \text{ km}$), the neutrino beam generated by the KEK proton synchrotron accelerator is aimed at the near and far detectors, which are carefully aligned in a straight line. Then, by comparing the neutrino events recorded in these detectors, they are able to examine the neutrino oscillation phenomenon. Super-Kamiokande detector itself acts as the far detector. The K2K near detector complex essentially consists of a one kiloton water Cerenkov detector (a miniature Super-Kamiokande detector).

A total intensity of $\sim 10^{19}$ protons on target, which is about 7% of the goal of the experiment, was accumulated in 39.4 days of data-taking in 1999 [79]. They obtained 3 neutrino events in the fiducial volume of the Super-Kamiokande detector, whereas the expectation based on observations in the front detectors was $12.3^{+1.7}_{-1.9}$ neutrino events. It corresponds to a ratio of data versus theory $0.84 \pm 0.01$. Although the preliminary results are rather consistent with squared mass difference $8 \times 10^{-3} \text{ eV}^2$ and maximal mixing, it is too early to draw any reliable conclusions about neutrino mixing. An complete analysis of oscillation searches from the view points of absolute event numbers, distortion of neutrino energy spectrum, and $\nu_e/\nu_\mu$ ratio is still in progress.

### 7.2 Solar neutrinos

Indications in the favor of neutrino oscillations were found in "all" solar neutrino experiments (along this section and the following ones, we will make reference to results appeared in Refs. [80–85]): The Homestake Cl radiochemical experiment with sensitivity down to the lower energy parts of the $^8 \text{B}$ neutrino spectrum and to the higher $^7 \text{Be}$ line [82]. The two radiochemical $^{71} \text{Ga}$ experiments, SAGE and GALLEX, which are sensitive to the low energy pp neutrinos and above [80, 81] and the water Cerenkov experiments Kamiokande and Super-Kamiokande (SK) which can observe only the highest energy $^8 \text{B}$ neutrinos. Water Cerenkov experiments in addition demonstrate directly that the neutrinos come from the Sun showing that recoil electrons are scattered in the direction along the sun-earth axis [83–85].
Two important points to remark are: a) The prediction of the existence of a global neutrino deficit is hard to modify due to the constraint of the solar luminosity on pp neutrinos detected at SAGE-GALLEX. b) The different experiments are sensitive to neutrinos with different energy ranges and combined yield spectroscopic information on the neutrino flux. Intermediate energy neutrinos arise from intermediate steps of the thermonuclear solar cycle. It may not be impossible to reduce the flux from the last step (\( ^8B \)), for example by reducing temperature of the center of the Sun, but it seems extremely hard to reduce neutrinos from \( ^7Be \) to a large extent, while keeping a reduction of \( ^8B \) neutrinos production to a modest amount. If minimal standard electroweak theory is correct, the shape of the \( ^8B \) neutrino energy spectrum is independent of all solar influences to very high accuracy.

Unless the experiments are seriously in error, there must be some problems with either our understanding of the Sun or neutrinos. Clearly, the SSM cannot account for the data (see Fig.3) and possible highly nonstandard solar models are strongly constrained by heliosysmology studies [see Fig.(4)].

There are at least two reasonable versions of the neutrino oscillation phenomena which could account for the suppression of intermediate energy neutrinos. The first one, neutrino oscillations in vacuum, requires a large mixing angle and a seemingly unnatural fine tuning of neutrino oscillation length with the Sun-Earth distance for intermediate energy neutrinos. The second possibility, level-crossing effect oscillations in presence of solar matter and/or magnetic fields of regular and/or chaotic nature (MSW, RSFP), requires no fine tuning either for mixing parameter or neutrino mass difference to cause a selective large reduction of the neutrino flux. This mechanism explains naturally the suppression of intermediate energy neutrinos, leaving the low energy pp neutrino flux intact and high energy \( ^8B \) neutrinos only loosely suppressed. Concrete range of parameters obtained including the latest SK (Super-Kamiokande) data will be showed in the next section.

7.3 The SK detector and Results.

The high precision and high statistics Super-Kamiokande (SK) experiment initiated operation in April 1996. A few words about the detector itself. SK is a 50-kiloton water Cerenkov detector located near the old Kamiokande detector under a mean overburden of 2700 meter-water-equivalent. The effective fiducial volume is 22.5 kt. It is a well understood, well calibrated detector. The accuracy of the absolute energy scale is estimated to be \( \pm 2.4\% \) based on several independent calibration sources: cosmic ray through-going and stopping muons, muon decay electrons, the invariant mass of \( \pi^0 \)'s produced by neutrino interactions, radioactive source calibration, and, as a novelty in neutrino experiments, a 5-16 MeV electron LINAC. In addition to the ability of recording higher statistics in less time, due to the much larger dimensions of the detector, SK can contain multi-GeV muon events making possible for the first time a measurement of the spectrum of \( \mu \)-like events up to \( \sim 8 \rightarrow 10 \) GeV.

The results from SK, to be summarized below, combined with data from earlier ex-
periments provide important constraints on the MSW and vacuum oscillation solutions for the solar neutrino problem (SNP), [89–91]:

**Total rates.** The most robust results of the solar neutrino experiments so far are the total observed rates. Preliminary results corresponding to the first 825 days of operation of SK (presented in spring’2000, [89]) with a total number of events $N_{ev} = 11235 \pm 180 \pm 310$ in the energy range $E_{vis} = 6.5 - 20 \text{ MeV}$. predict the following flux of solar $^8\text{B}$ neutrinos:

$$\phi_{^8\text{B}} = (2.45 \pm 0.04 \pm 0.07) \times 10^6 \text{ cm}^{-2} \text{ sec}^{-1},$$

a flux which is clearly below the SSM expectations. The most recent data on rates on all existing experiments are summarized in Table (2). Total rates alone indicate that the $\nu_e$ energy spectrum from the Sun is distorted. The SSM flux predictions are inconsistent with the observed rates in solar neutrino experiments at approximately the 20$\sigma$ level. Furthermore, there is no linear combination of neutrino fluxes that can fit the available data at the 3$\sigma$ level [Fig.(3)].

| Experiment  | Target | E. Th. (MeV) | $S_{Data}/S_{SSM}$ ($\pm 1\sigma$) |
|-------------|--------|-------------|----------------------------------|
| SK-825d     | H$_2$O | $\sim 6.5\text{–}20$ | 0.474 $\pm$ 0.020 |
| Homestake   | $^{37}\text{Cl}$ | 0.8 | 0.33 $\pm$ 0.03 |
| Kamiokande  | H$_2$O | $\sim 7.5$ | 0.54 $\pm$ 0.07 |
| SAGE        | $^{71}\text{Ga}$ | 0.2 | 0.52 $\pm$ 0.06 |
| GALLEX      | $^{71}\text{Ga}$ | 0.2 | 0.60 $\pm$ 0.06 |

Table 2: Ratios of neutrino fluxes by solar neutrino experiments to corresponding predictions from the SSM (see Ref.[93] and references therein, we take the INT normalization for the SSM data).

**Zenith angle: day-night effect.** If MSW oscillations are effective, for a certain range of neutrino parameters the observed event rate will depend upon the zenith angle of the Sun (through a Earth matter regeneration effect). Win present statistics, the most robust estimator of zenith angle dependence is the day-night (or up-down) asymmetry, $A$. The experimental estimation is [89]:

$$A \equiv \frac{N - D}{N + D} = 0.032 \pm 0.015 \pm 0.006, \quad (E_{\text{recoil}} > 6.5 \text{ MeV}). \quad (7.1)$$

The difference is small and not statistically significant but it is in the direction that would be expected from regeneration at Earth (the Sun is apparently neutrino brighter at night). Taken alone the small value observed for $A$ excludes a large part of the parameter region that is allowed if only the total rates would be considered [see Fig.(4)].

**Spectrum Shape.** The shape of the neutrino spectrum determines the shape of the recoil electron energy spectrum produced by neutrino-electron scattering in the detector and is independent of the astrophysical source. All the neutrino oscillation solutions (SMA,LMA,LOW and Vacuum) provide acceptable, although not excellent fits to the
recoil energy spectrum. The simplest test is to investigate whether the ratio, $R$, of the observed to the standard energy spectrum is a constant with increasing energy. The null flatness hypothesis is accepted at the 90% CL ($\chi^2 \sim 1.5$, [89]). However, alternative fits of the ratio $R$ to a linear function of energy yields slope values does not discard the presence of distortion at higher energies [see Figs.(6) and next paragraph].

**Spectrum shape: the hep neutrino problem.** A small but significant discrepancy appears when comparing the predictions from the global best fits for the energy spectrum at high energies ($E_\nu \gtrsim 13$ MeV) with the SK results. From this discrepancy it has been speculated that uncertainties on the hep neutrino fluxes may affect the higher energy solar neutrino energy spectrum. Presently low energy nuclear physics calculations of the rate of the hep reaction are uncertain by a factor of, at least, six. Coincidence between expected and measured ratios is improved when the hep flux is allowed to vary as a free parameter [see Fig.(8) and Ref.[89]]. The best fit is obtained by a combination $\phi \sim 0.45^{+8}_{-5}\text{B} + 16\text{hep}$ ($\chi^2 \sim 1.2$. An upper limit on the ratio of experimental to SSM hep flux is obtained:

$$\frac{\phi^{exp}_{\text{hep}}}{\phi_{BP98}^{\text{hep}}} < 15, \ (90\%CL).$$

**Seasonal Variation.** No evidence for a anomalous seasonal variation of the neutrino flux has been found. The results (SK 825d, $E_{vis} = 10 - 20$ MeV) are consistent with what is expected from a geometrical variation due to the Earth orbital eccentricity ($\chi^2 \sim 0.5$ for the null hypothesis, Ref.[89]).

**Analysis of data.** From a two-flavor analysis (Ref.[89], see also Ref.[93, 94]) of the total event rates in the ClAr, SAGE, GALLEX and SK experiments the best $\chi^2$ fit considering active neutrino oscillations is obtained for $\Delta m^2 = 5.4 \times 10^{-6}$ eV$^2$, $\sin^2 2\theta = 5.0 \times 10^{-3}$ (the so called small mixing angle solution, SMA). Other local $\chi^2$ minima exist. The large mixing angle solution (LMA) occurs at $\Delta m^2 = 3.2 \times 10^{-5}$ eV$^2$, $\sin^2 2\theta = 0.76$, the LOW solution (lower probability, low mass), at $\Delta m^2 = 7.9 \times 10^{-8}$ eV$^2$, $\sin^2 2\theta = 0.96$. The vacuum oscillation solution occurs at $\Delta m^2 = 4.3 \times 10^{-10}$ eV$^2$, $\sin^2 2\theta = 0.79$. At this extremely low value for the mass difference the MSW effect is inoperative.

For oscillations involving sterile neutrinos (the matter effective potential is modified in this case) the LMA and LOW solutions are not allowed and only the (only slightly modified) SMA solution together with the vacuum solution are still possible.

In the case where all data, the total rates, the zenith-angle dependence and the recoil energy spectrum, is combined the best-fit solution is almost identical to what is obtained for the rates-only case. For other solutions, only the SMA and vacuum solution survives (at the 99% CL). The LMA and the LOW solutions are, albeit marginally, ruled out [93].

**Solar magnetic Fields and antineutrino flux bounds.** Analysis which consider neutrino propagation in presence of solar magnetic fields have also been presented. In this case a variant, more complicated, version of the MSW effect, the so called RSFP effect could manifest itself. Typically, these analysis yield solutions with $\Delta m^2 \sim 10^{-7} - 10^{-8}$ eV$^2$ for both small and large mixing angles. Spin flavor or resonant spin flavor (RSFP) solutions are much more ambiguous than pure MSW solutions because of necessity of introducing additional free parameters in order to model the largely unknown intensity and profile
of solar magnetic field. The recognition of the random nature of solar convective fields and recent theoretical developments in the treatment of Schrödinger random equations have partially improved this situation, allowing the obtaining of SNP solutions without the necessity of a detailed model description (see recent analysis in [95–99]).

In addition, random RSFP models predict the production of a sizeable quantity of electron antineutrinos in case the neutrino is a Majorana particle.

Presently, antineutrino searches [100] with negative results in Kamiokande and SK are welcome because restrict significantly the, uncomfortably large, parameter space of RSFP models.

A search [100] for inverse beta decay electron antineutrinos has set limits on the absolute flux of solar antineutrinos originated from the solar $^8$B neutrino component:

$$\Phi_{\bar{\nu}}(^8B) < 1.8 \times 10^5 \text{cm}^{-2}\text{s}^{-1}, \ (95\% \ CL),$$

a number which is equivalent to an averaged conversion probability bound of (with respect the SSM-BP98 model)

$$P < 3.5\% \ (95\% \ CL).$$

In the future such antineutrinos could be identified both in SK or in SNO experiments setting the Majorana nature of the neutrino. In Ref.[97] [see Fig.(5) for illustration] it has been shown that, even for moderate levels of noise, it is possible to obtain a probability for $\nu_e \rightarrow \bar{\nu}_e$ conversions about $\sim 1 - 3\%$ in the energy range 2-10 MeV for large regions of the mixing parameter space while still satisfying present SK antineutrino bounds and observed total rates. In the other hand it would be possible to obtain information about the solar magnetic internal field if antineutrino bounds reach the 1% level and a particle physics solution to the SNP is assumed.

7.4 Atmospheric neutrinos

Atmospheric neutrinos are the decay products of hadronic showers produced by cosmic ray interactions in the atmosphere. The composed ratio $R$

$$R \equiv (\mu/e)_{\text{DATA}} / (\mu/e)_{\text{MC}}$$

where $\mu/e$ denotes the ratio of the number of $\mu$-like to $e$-like neutrino interactions observed in the experiment or predicted by the simulation is considered as an estimator of the atmospheric neutrino flavor ratio $(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$. The calculations of individual absolute neutrino fluxes have large uncertainties at the $\sim 20\%$ level [101]. However, the flavor flux ratio is known to an accuracy of better than 5% in the energy range $0.1 - 10$ GeV. The calculated flux ratio has a value of about 2 for energies $< 1$ GeV and increases with increasing neutrino energy reaching a value $\sim 10$ at 100 GeV. The angle distribution of the different fluxes is also an important ingredient in the existing evidence for atmospheric neutrino oscillations. Calculations show that for neutrino energies higher than a few GeV, the fluxes of upward and downward going neutrinos are expected to be
nearly equal; geomagnetic field effects at these energies are expected to be small because of the relative large geomagnetic rigidity of the primary cosmic rays that produce these neutrinos [101].

Prior to the present era dominated by Super-Kamiokande results, anomalous, statistically significant, low values of the ratio $R$ have been repeatedly obtained previously [102, 103] in the water Cerenkov detectors Kamiokande and IMB-3 and in the calorimeter-based Soudan-2 experiment for “sub-GeV” events ($E_{\text{vis}} < 1$ GeV). The NUSEX and Frejus experiments reported however results consistent with no deviation from unity with smaller data samples. Kamiokande experiment observed a value of $R$ smaller than unity in the multi-GeV ($E_{\text{vis}}>1$ GeV) energy region as well as a dependence of this ratio on the zenith angle. IMB-3, with a smaller data sample, reported inconclusive results in a similar energy range, not in contradiction with Kamiokande observations [102, 103].

Super-Kamiokande (SK) results are completely consistent with previous results at a much higher accuracy level. Specially significant improvements in accuracy have been obtained in measuring the zenith angular dependence of the neutrino events: in summary, the single most significant result obtained by SK is that the flux of muon neutrinos going up is smaller than that of down-going neutrinos.

As we commented before, in addition to the ability of recording higher statistics in less time, due to the much larger dimensions of the detector, the SK detector can contain multi-GeV muon events making possible for the first time a measurement of the spectrum of $\mu$-like events up to $\sim 8 - 10$ GeV. From experimental and phenomenological reasons, the SK experiment uses the following event classification nomenclature. According to their origin, events can be classified as e-like (showering, $\nu_e$ or $\overline{\nu}_e$ events) or $\mu$-like (non-showering, $\nu_\mu$ or $\overline{\nu}_\mu$ events). According to the position of the neutrino interaction, they distinguish contained events (vertex in fiducial volume, 98% muon induced), which, depending on their energy, are typed as sub-GeV ($E < \sim 1$ GeV) or multi-GeV samples ($E < \sim 10$ GeV). Non-contained events can be: Upward through-going muons (vertex outside the detector, muon induced, $E_\nu \sim 500$ GeV) or Upward stopping muons (typically $E_\nu < \sim 50$ GeV). In all cases, the neutrino path-length covers the full range, from $\sim 10^1$ km for down events to $10^4$ km for up events. In what follows we summarize the present results about total and zenith-angle dependent rates.

**Total rates.** In the sub-GeV range ($E_{\text{vis}} < 1.33$ GeV), From an exposure of 61 kiloton-years (kty) (990 days of operation) of the SK detector the measured ratio $R$ is:

$$R_{\text{subgev}} = 0.66 \pm 0.02 \pm 0.05.$$ 

It is not possible to determine from data, whether the observed deviation of $R$ is due to an electron excess of a muon deficit. The distribution of $R$ with momentum in the sub-GeV range is consistent with a flat distribution within the statistical error as happens with zenith angle distributions [see right plots in Fig. (11)].

In the multi-GeV range, it has been obtained (for a similar exposure) a ratio $R$ which is slightly higher than at lower energies

$$R_{\text{multigeV}} = 0.66 \pm 0.04 \pm 0.08.$$
For e-like events, the data is apparently consistent with MC. For \( \mu \)-like events there is a clear discrepancy between measurement and simulation.

**Zenith Angle.** A strong distortion in the shape of the \( \mu \)-like event zenith angle distribution was observed [Plots (9-10)]. The angular correlation between the neutrino direction and the produced charged lepton direction is much better at higher energies (\( \sim 15^\circ -20^\circ \)): the zenith angle distribution of leptons reflects rather accurately that of the neutrinos in this case.

At lower energies, the ratio of the number of upward to downward \( \mu \)-like events was found to be

\[
(N_{\text{up}}/N_{\text{down}})^\mu_{\text{Data}} = 0.52 \pm 0.07
\]

while the expected value is practically one:

\[
(N_{\text{up}}/N_{\text{down}})^\mu_{\text{MC}} = 0.98 \pm 0.03.
\]

The validity of the results has been tested by measuring the azimuth angle distribution of the incoming neutrinos, which is insensitive to a possible influence from neutrino oscillations. This shape agreed with MC predictions which were nearly flat.

Another signal for the presence of neutrino oscillations could be present in the ratio of neutrino events for two well separated energy ranges. This is the case for the ratio between upward throughgoing to upward stopping muon events, both classes correspond to very high energy events. The results and expected values are the following ([92, 101])

\[
(N_{\text{stop}}/N_{\text{through}})^\mu_{\text{Data}} = 0.23 \pm 0.02 \tag{7.2}
\]
\[
(N_{\text{stop}}/N_{\text{through}})^\mu_{\text{MC}} = 0.37 \pm 0.05. \tag{7.3}
\]

The ratio of data to MC is \( \sim 0.6 \). With these results, the probability that they do correspond to no-oscillation scenario is rather low, \( P \sim 10^{-4} - 10^{-3} \) [92].

**Analysis.** Oscillation parameters are measured by several samples (FC, PC, up-stop, up-through). The result is that all samples are overall consistent with each other. This hypothesis fits well to the angular distribution, since there is a large difference in the neutrino path-length between upward-going (\( \sim 10^4 \) Km) and downward-going (\( \sim 20 \) Km): a zenith angle dependence of \( R \) can be interpreted as a clear-cut evidence for neutrino oscillations.

Among the different possibilities, the most obvious solution to the observed discrepancy is \( \nu_\mu \to \nu_\tau \) flavor neutrino oscillations. \( \nu_\mu - \nu_e \) oscillations does not fit however so well, they would also conflict laboratory measurements [CHOOZ, see figs.(1-11)].

Oscillation into sterile neutrinos, \( \nu_\mu \to \nu_s \), could also be in principle a good explanation consistent with data. Different tests has been performed for distinguishing \( \nu_\mu \to \nu_\tau \) from \( \nu_\mu \to \nu_s \) oscillations: A possible test of \( \nu_\mu \to \nu_s \) vs \( \nu_\mu \to \nu_\tau \) oscillations is provided by the study of the \( \pi^0/e \) ratio [128]. In the \( \mu - \tau \) case, the \( \pi^0 \) production due to neutral current interactions do not change, causing the \( \pi^0/e \) ratio to be the same as the expectation without neutrino oscillations. In the sterile case such a ratio should be smaller (\( \sim 83\% \)) than expected because the absence of \( \nu_s \) neutral current interactions.
π⁰ events experimental identification can be performed by study of their invariant mass distributions compared with Monte Carlo simulations. Present results conclude that the $\nu_\mu \rightarrow \nu_s$ oscillation hypothesis is disfavored at the 99\%$CL$.

Evidence for oscillations equals evidence for non-zero neutrino mass within the standard neutrino theory. The allowed neutrino oscillation parameter regions obtained by Kamiokande and SK from different analysis are shown in Fig.11. Under the interpretation as $\nu_\mu \rightarrow \nu_\tau$ oscillations, the best fit provide $\Delta m^2 \sim 2 - 5 \times 10^{-3}$ and a very large mixing angle $\sin^2 2\theta > 0.88$. Unless there is no fine tuning, this suggests a neutrino mass of the order of 0.1 eV. Such a mass implies the neutrino energy density in the universe to be 0.001 of the critical density which is too small to have cosmological consequences. This is of course a very rough argument: specific models, however, may allow larger neutrino masses quite naturally.

7.5 Global multi-fold analysis and the necessity for sterile neutrinos.

From the individual analysis of the data available from neutrino experiments, it follows that there exist three different scales of neutrino mass squared differences and two different ranges of small and maximal mixing angles, namely:

\[
\begin{align*}
\Delta m^2_{\text{sun}} & \sim 10^{-5} - 10^{-8} \text{eV}^2, \quad \sin^2 2\theta \sim 7 \times 10^{-3} (\text{MSW, RSFP}), \\
\Delta m^2_{\text{Atm}} & \sim 10^{-10} \text{eV}^2, \quad \sin^2 2\theta \sim 0.8 - 0.9 \ (\text{Vac.}); \\
\Delta m^2_{\text{LSND}} & \sim 5 \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta \sim 1 \\
\end{align*}
\]

(7.4)\hspace{1cm}(7.5)\hspace{1cm}(7.6)

Fortunately for the sake of simplicity the neutrino mass scale relevant for HDM is roughly similar to the LSND one. The introduction of the former would not change any further conclusion. But for the same reason, the definitive refutation of LSND results by KARMEN or future experiments does not help completely in simplifying the task of finding a consistent framework for all the neutrino phenomenology.

Any combination of experimental data which involves only of the two mass scales can be fitted within a three family scenario, but solving simultaneously the solar and atmospheric problems requires generally some unwelcome fine tuning of parameters at the $10^{-2}$ level. The detailed analysis of Ref.[104] obtains for example that solutions with 3 neutrino families which are compatible with the results from SBL inclusive experiments, LSND and solar neutrino experiments are possible. Moreover it has been shown that it is possible to obtain, under simple assumptions but without a detailed fit of all possible parameters, very concrete expressions for the $3 \times 3$ mixing matrix, see for example the early Ref.[105], of which the called bi-maximal model of Ref.[106] is a particular case. The real problem arises when one add the results from CHOOZ, which rule out large atmospheric $\nu_\mu, \nu_e$ transitions and zenith dependence from SK atmospheric data one comes to the necessity of consideration of schemes with four massive neutrinos including a light sterile neutrino. Among the numerous possibilities, complete mass hierarchy of four neutrinos is not favored by existing data [104] nor four-neutrino mass spectra with one neutrino
mass separated from the group of the three close masses by the "LSND gap" (∼ 1 eV). One is left with two possible options where two double-folded groups of close masses are separated by a ∼ 1 eV gap:

\[ \begin{align*}
&\text{(A)} \quad \nu_e \to \nu_A : m_1 < m_2 \ll \nu_\mu \to \nu_\tau : m_3 < m_4 \\
&\text{LSND} \sim 1 \text{eV}
\end{align*} \tag{7.8} \]

\[ \begin{align*}
&\text{(B)} \quad \nu_e \to \nu_\tau : m_1 < m_2 \ll \nu_\mu \to \nu_\sigma : m_3 < m_4 \\
&\text{LSND} \sim 1 \text{eV}
\end{align*} \tag{7.9} \]

The two models would be distinguishable from the detailed analysis of future solar and atmospheric experiments. For example they may be tested combining future precise recoil electron spectrum in \( \nu e \to \nu e \) measured in SK and SNO (see Ref.[107] for experiment details and Refs.[108] for performing expectations) with the SNO spectrum measured in CC absorption. The SNO experiment (a 1000 t heavy water under-mine detector) will measure the rates of the charged (CC) and neutral (NC) current reactions induced by solar neutrinos in deuterium:

\[ \begin{align*}
\nu_e + d &\to p + p + e^- \quad (\text{CC absorption}) \\
\nu_x + d &\to p + n + \nu_x \quad (\text{NC dissociation}). \tag{7.10}
\end{align*} \]

including the determination of the electron recoil energy in the CC reaction. Only the more energetic \(^8\)B solar neutrinos are expected to be detected since the expected SNO threshold for CC events is an electron kinetic energy of about 5 MeV and the physical threshold for NC dissociation is the binding energy of the deuteron, \( E_b = 2.225 \) MeV. If the (B) model it is true one expects \( \phi^{CC}/\phi^{NC} \sim 0.5 \) while in the (A) model the ratio would be ∼ 1. The schemes (A) and (B) give different predictions for the neutrino mass measured in tritium \( \beta \)-decay and for the effective Majorana mass observed in neutrinoless double \( \beta \) decay. Respectively we have \(| \langle m \rangle | < m_4 \) (A) or << \( m_4 \) (B). Thus, if scheme (A) is realized in nature this kind of experiments can see the effect of the LSND neutrino mass.

From the classical LEP requirement \( N^\text{act}_\nu = 2.994 \pm 0.012 \) [15], it is clear that the fourth neutrino should be a \( SU(2) \otimes U(1) \) singlet in order to ensure that does not affect the invisible Z decay width. The presence of additional weakly interacting light particles, such as a light sterile \( \nu_s \), is constrained by BBN since it would enter into equilibrium with the active neutrinos via neutrino oscillations (see Section 5). The limit \( \Delta m^2 \sin^2 2\theta < 3 \times 10^{-6} \) eV\(^2\) should be fulfilled in principle. However systematical uncertainties in the derivation of the BBN bound make any bound too unreliable to be taken at face value and can eventually be avoided [46]. Taking the most restrictive options (giving \( N^\text{eff}_\nu < 3.5 \)) only the (A) scheme is allowed, one where the sterile neutrino is mainly mixed with the electron neutrino. In the lest restrictive case \( N^\text{eff}_\nu < 4.5 \) both type of models would be allowed.
8 Conclusions and future perspectives.

The theoretical challenges that the present phenomenological situation offers are two at least: to understand origin and, very particularly, the lightness of the sterile neutrino (apparently requiring a radiatively generated mass) and to account for the maximal neutrino mixing indicated by the atmospheric data which is at odd from which one could expect from considerations of the mixing in the quark sector. Actually, the existence of light sterile neutrinos could even be beneficial in diverse astrophysical and cosmological scenarios (supernova nucleosynthesis, hot dark matter, lepton and baryon asymmetries for example).

In the last years different indications in favor of nonzero neutrino masses and mixing angles have been found. These evidences include four solar experiments clearly demonstrating an anomaly compared to the predictions of the Standard Solar Model (SSM) and a number of other atmospheric experiments, including a high statistics, well calibrated one, demonstrating a quite different anomaly at the Earth scale.

One could argue that if we are already beyond the stage of having only ”circumstantial evidence for new physics”, we are still however a long way from having ”conclusive proof of new physics”. Evidence for new physics does not mean the same as evidence for neutrino oscillations but there exists a significant case for neutrino oscillations and hence neutrino masses and mixing as ”one”, indeed the most serious candidate, explanation of the data.

Non-oscillatory alternative explanations of the neutrino anomalies are also possible but any of them will not be specially elegant or economical (see Ref.[109] for a recent summary and references therein): they will involve anyway non-zero neutrino masses and mixing. As a result even if neutrinos have masses and do mix, the observed neutrino anomalies, may be a manifestation of a complicated mixture of effects due to oscillations and effects due to other exotic new physics. The dominant effect would not necessarily be the same in each energy or experimental domain. The list of effects due to exotic physics which have been investigated in some degree in the literature, would include [109]: Oscillation of massless neutrinos via FCNC and Non-Universal neutral currents (NUNC) which has been considered as feasible explanation for the solar neutrino observations [110] and atmospheric neutrinos [138]. It has also been studied the possibility of decaying neutrinos as possible solutions in the solar case [111] and in the atmospheric case [112]. LSND results could be accounted for without oscillations provided that muon conventional decay modes are accompanied by rare modes including standard and/or sterile neutrinos. In this case the energy or distance dependence, typical of the oscillation explanation, would be absent [113].

Finally, explanations of the neutrino experimental data which involve alterations of the basic framework of known physics (quantum and relativity theory) have been proposed: A similar signature to neutrino decay would be produced by a huge, non-standard, quantum decoherence rate along the neutrino propagation [115]. Proposed explanations involving relativity effects include gravitationally induced oscillations (see, for example, Ref.[116]) or violation of Lorenz invariance [117]. In the first case it has been suggested that different
flavors have different coupling to the gravitational potential. In the second case it is claimed the existence of different maximum speeds for each neutrino specie. In both cases, rather than the usual dependence on $L/E$, one finds a $L \times E$ as a characteristic signature.

Of course, one possible alternative is that one or more of the experiments will turn out to be wrong. This is possible, probable and even desirable from a phenomenologist point of view because his/her task would be considerably simplified as we have seen above. What it is little probable, with all the evidence accumulated by now, is that all the experiments turn out to be simultaneously wrong.

Many neutrino experiments are taking data, are going to start or are under preparation: solar neutrino experiments (SNO and Borexino are of major interest, also HERON, HELLAZ, ICARUS, GNO and others); LBL reactor (CHOOZ, Palo Verde, KamLand) and accelerator experiments (K2K, MINOS, ICARUS and others); SBL experiments (LSND, KARMEN, BooNe and many others). The important problem for any next generation experiment is to find specific and unambiguous experimental probe that the ”anomalies” which has been found are indeed signals of neutrino oscillations and to distinguish among the different neutrino oscillation possibilities (this is specially important in the Solar case). Among these probes, we could include:

- Perhaps the most direct test of SM deviation: to measure the ratio of the flux of $\nu_e$’s (via a CC interaction) to the flux of neutrinos of all types ($\nu_e + \nu_\mu + \nu_\tau$, determined by NC interactions). This measurement will be done hopefully by the SNO experiment in the near future [see Fig.(12)].

- Statistically significant demonstration of an energy-dependent modification of the shape of the electron neutrino spectrum arriving at Earth. Besides observing distortion in the shape of $^8$B neutrinos, it will be very important to make direct measurements of the $^7$Be (Borexino experiment) and pp (HERON,HELLAZ) neutrinos.

- Improved observation of a zenith angle effect in atmospheric experiments or their equivalent, a day-night effect in solar experiments.

- And least, but by no means the least, independent confirmation by one or more accelerator experiments.

There is a high probability that in the near future we should know much more than now about the fundamental properties of neutrinos and their masses, mixing and their own nature whether Dirac or Majorana.

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Figure 1: (Left) The 90% C.L. exclusion plot for CHOOZ, compared with previous experimental limits and with the KAMIOKANDE allowed region. (From Ref. [66]). (Right) Exclusion plot contours obtained from the ratios of the positron spectra from the two reactors (From Ref. [67]).
Figure 2: (Left) KARMEN 2 90% CL confidence limits and sensitivity according to the unified approach compared to LSND,BUGEY and BNL776. The full line is the 90% C.I. of the latest available data (feb. 1999) the dotted line the corresponding sensitivity and the dashed line the expected sensitivity in year 2001. (From Ref.[76]). (Right) Comparison of preliminary LSND $\nu_\mu \rightarrow \nu_e$ results (filled region) with those of KARMEN (dashed curve), Bugey (dot-dashed) and NOMAD (dot-x-dash). KARMEN’s exclusion curve is 90% Bayesian confidence level, while the LSND region is bounded by 90% Bayesian upper and lower limits with $36 < E_e < 60 MeV$. The pair of smooth curves surrounding the LSND region gives the LSND 90% C.I. for $\nu_\mu \rightarrow \nu_e$ oscillation. (From Ref.[70]).
**Figure 3:** The severity of the problem with astrophysical solutions. The constraints on $^8\text{B}$ and $^7\text{Be}$ fluxes (considered as free parameters) from the combined Cl, Ga, and Čerenkov experiments (90, 95, and 99% C.L.) are shown. The best fit solutions are obtained for unphysical values. Diverse standard and nonstandard solar models are shown. [From Hata and Langacker, Ref.([129] and references therein.)]

**Figure 4:** The excellent agreement between the calculated (solar model BP95 [130]) and the measured (Sun) sound speeds. The fractional error is much smaller than generic fractional changes in the model, 0.03 to 0.08, that might significantly affect the solar neutrino predictions. [Adapted from Christensen-Dalsgaard, Ref.[131], as it appears in [132].]
Figure 5: Solid lines: antineutrino production probability as a function of the neutrino mixing angle \( \log \sin^2 \theta \) and \( \Delta m^2 \). Dashed thick lines: \( S/S_{SSM} \) signal rates at SK (contours = 0.3, 0.5). The threshold neutrino energy is in this case: \( E_{th} = 5.5 \) MeV. From left to right and from top to bottom: \( P = 0.998, 0.98, 0.95, 0.8, 0.7, 0.5 \). The corresponding r.m.s fields are \( \sqrt{\langle \tilde{B}^2 \rangle} = 15, 45, 65, 150, 220 \) and 600 kG respectively (supposing the scale \( L_0 = 1000 \) Km and \( \mu = 10^{-11} \mu_B \)). (From Ref.[97])
Figure 6: The result of the MSW parameter space (shaded regions) allowed by the combined observations at 95\% C.L. assuming the Bahcall-Pinsonneault SSM with He diffusion. The constraints from Homestake, combined Kamiokande and Super-Kamiokande, and combined SAGE and GALLEX are shown by the dot-dashed, solid, and dashed lines, respectively. Also shown are the regions excluded by the Kamiokande spectrum and day-night data (dotted lines). [From Hata and Langacker, Ref.(129) and references therein.]

Figure 7: Deviation from an undistorted energy spectrum. The $1, 2, 3\sigma$ allowed regions are shown in the figure. The ratio of the observed counting rate as a function of electron recoil energy [90] to the expected undistorted energy spectrum was fit to a linear function of energy, with intercept $R_0$ and slope $S_0$. The five oscillation solutions SMA active and sterile, LMA, LOW, and vacuum oscillations, all provide acceptable fits to the data, although the fits are not particularly good. (From Bahcall and Krastev, Ref.[93]).
Figure 8: Nuclear physics calculations of the rate of the hep reaction are uncertain. The figure show the results for the predicted energy spectrum that is measured by SK ([90]). The total flux of hep neutrinos was varied to obtain the best-fit for each scenario. The calculated curves are global fits to all of the data, the chlorine, GALLEX, SAGE, and SK total event rates, the SK energy spectrum and Day-Night asymmetry. (Figure reproduced from Ref.[133]).

Figure 9: The SK multi-GeV data sample. The ratio of the number of FC (fully contained) data events to FC Monte Carlo events versus reconstructed $L/E_\nu$. Points: absence of oscillations. Dashed lines: expected shape for $\nu_\mu \leftrightarrow \nu_\tau$ at $\Delta m^2 = 2.2 \times 10^{-3}\text{eV}^2$ and $\sin^2 2\theta = 1$. The slight $L/E_\nu$ dependence for $e$-like events is due to contamination (2-7%) of $\nu_\mu$ CC interactions. (From Ref.[134]).
Figure 10: Angular distribution for Super-Kamiokande electron-like and muon-like sub-GeV and multi-GeV events. Predictions in the absence of oscillation (thick solid line), $\nu_\mu \rightarrow \nu_s$ (thin solid line), $\nu_\mu \rightarrow \nu_e$ (dashed line) and $\nu_\mu \rightarrow \nu_\tau$ (dotted line). The errors displayed in the experimental points is only statistical. (From Ref.[135])

Figure 11: The allowed neutrino oscillation parameter regions obtained by Kamiokande and SK (90% C.L.). (1) and (2): the regions obtained by contained event analyses from Super-Kamiokande and Kamiokande, respectively. (3) and (4): upward through-going muons from SK and Kamiokande, respectively. (5) stopping/trough-going ratio analysis of upward going muons from SK. (From Ref.[136])
Figure 12: Conclusive probes of lepton number violation in solar neutrino experiments. Iso-sigma contours for the SNO for the combined CC-shape and CC/NC test, for the representative oscillation cases. Iso-sigma contours for the combined CC-shape and CC/NC test, for representative oscillation cases. STD = standard (no oscillation); SMA = small mixing angle (MSW); LMA = large mixing angle (MSW); VAC = vacuum oscillation. (From Ref.[137])