THE ISGUR-WISE FUNCTION TO $O(\alpha_s)$ FROM SUM RULES IN THE HEAVY QUARK EFFECTIVE THEORY

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Abstract:
Radiative corrections to both perturbative and non-perturbative contributions are added to existing calculations of the Isgur-Wise function $\xi_{IW}$. To this end, we develop a method for calculating two-loop integrals in the heavy quark effective theory involving two different scales. The inclusion of $O(\alpha_s)$ terms causes $\xi_{IW}$ to decrease as compared to the lowest order result and shows the importance of quantum effects. The slope parameter $\rho^2$ violates the bound given by de Rafael and Taron.

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1 Introduction

The heavy quark effective theory (HQET) [1, 2] now seems established as a useful tool for investigating the low energy regime of QCD in the limit of infinitely heavy quarks. New flavour symmetries appearing in the Lagrangian allow for a considerable simplification of the treatment of transition amplitudes of heavy particles. Since the heavy quark decouples from the light degrees of freedom, the (heavy) meson to (heavy) meson amplitudes, for instance, are completely determined by a single universal function, the Isgur-Wise function $\xi_{IW}$ [2]. $\xi_{IW}$ describes the dynamics of the light quark cloud and depends on the “momentum-transfer” $y := v \cdot v'$, where $v$ and $v'$ are the four-velocities of the two heavy mesons, respectively. It satisfies the normalization condition $\xi_{IW}(y)|_{y=1} = 1$ and one can show that the slope at the normalization point $y = 1$ is determined to be $\rho^2 := -d\xi_{IW}(y)/dy|_{y=1} \geq 1/4$ [3]. In addition, it has been claimed that $\rho^2 \leq 0.45$ [4]. As far as we know, the latter bound is in contradiction to almost all parametrizations to be found in the literature [5, 6]. In view of this discrepancy it seems worthwhile to address the problem by means of HQET sum rules which were first developed for the (heavy) meson-to-vacuum transition amplitude [7, 8, 9] and yield nice agreement with lattice calculations. Since the former calculations stress the importance of radiative corrections, it is of great interest to include them in the Isgur-Wise function as well, whereas they have been omitted from existing sum rule calculations of $\xi_{IW}$ [6, 9, 10]. In this letter we now present a calculation of the Isgur-Wise function in the framework of HQET sum rules including radiative corrections up to $O(\alpha_s)$.

Our paper is organized as follows: in Sect. 2 we develop a method for calculating two-loop integrals in the HQET involving two different scales and velocities as a series in $r := (y - 1)/(y + 1)$ and give our result to $O(\alpha_s)$ for the three-point function relevant to the determination of $\xi_{IW}$. In Sect. 3 we derive a sum rule for the renormalization group invariant quantity $\hat{\xi}_{IW}$. Finally, Sect. 4 is devoted to the discussion of our results.

2 Two-loop integrals in the HQET

In order to obtain the Isgur-Wise function within the framework of HQET sum rules we first need to evaluate the three-point function

$$\tilde{\Gamma}(\omega = \tilde{q} \cdot v, \omega' = \tilde{q}' \cdot v', y) (v + v')_{\mu} = i^2 \int d^4x d^4y e^{i\tilde{q} \cdot x - i\tilde{q}' \cdot y} \langle 0 | T \tilde{J}_5(x) \tilde{V}_\mu(0) \tilde{J}_5(y) | 0 \rangle ,$$

with the HQET-currents $\tilde{J}_5 = \bar{q}i\gamma_5 h_v$ and $\tilde{V}_\mu = \bar{h}_v \gamma_\mu h_v'$, where $h_v$ denotes a heavy quark field in the HQET with four-velocity $v$ and $q$ denotes a light quark field.

In this section we shall be mainly concerned with the $O(\alpha_s)$ (two-loop) correction, $\tilde{\Gamma}^{(1)}_{pert}$, to the perturbative part of the three-point function. Throughout this section we will drop the subscript $pert$, for no confusion can arise. After some algebra, one can write
Thus, only the cases $p$-coefficients $A$ and introduce the notation of the form integral in eq. (5) with the appropriate number of metric tensors, $A$ This will provide a set of equations for the result of these integrations in a closed form convenient enough for numerical analysis. We have overcome this difficulty by expanding $v'$ and $v$ around $(v' + v)/2$, thus obtaining an expansion of the integrals in powers of $r = (y - 1)/(y + 1)$. For the numerical analysis of Sect. [4], it will be sufficient to truncate the series at order $v^2$. Since this expansion technique can certainly be useful in other $O(\alpha_s)$ calculations, we shall present it here briefly.

We begin by introducing the variables $v_+ = (v + v')/2$ and $v_- = (v - v')/2$. Because of $v^2 = v'^2 = 1$, we have $v_+ \cdot v_- = 0$. We now expand all $v$- and $v'$-dependent factors in eq. (2) in powers of $v_+ \cdot k$ or $v_- \cdot l$. To get the required accuracy, we need to keep only the first five terms of the expansion. We end up with integrals of the form

$$J = \int \frac{d^Dl \, d^Dk (v_- \cdot l)^p (v_- \cdot k)^q}{(l^2)^{a}(k^2)^b((l - k)^2)^c(v_- \cdot l + \omega)^a(v_- \cdot l + \omega')^n(v_+ \cdot k + \omega)^m(v_+ \cdot k + \omega')^{m'}}. \tag{3}$$

This expansion removes the main source of trouble, namely the presence of two different velocities in the denominators. We next scale the loop variables as

$$l^\mu \to l^\mu \sqrt{v_+^2}, \quad k^\mu \to k^\mu \sqrt{v_+^2}, \tag{4}$$

and introduce the notation $\hat{v}^\mu \equiv v_+^\mu / \sqrt{v_+^2}$, so that $\hat{v}^2 = 1$. Eq. (3) now reads

$$J = \left(\frac{1 + y}{2}\right)^{(2a + 2b + 2c - 2D - p - q)/2} v_+^{\mu_1} \cdots v_+^{\mu_p} v_-^{\nu_1} \cdots v_-^{\nu_q}, \tag{5}$$

$$\times \int \frac{d^Dl \, d^Dk \, l_{\mu_1} \cdots l_{\mu_p} k_{\nu_1} \cdots k_{\nu_q}}{(l^2)^{a}(k^2)^b((l - k)^2)^c(\hat{v} \cdot l + \omega)^a(\hat{v} \cdot l + \omega')^n(\hat{v} \cdot k + \omega)^m(\hat{v} \cdot k + \omega')^{m'}},$$

where the indices in parenthesis are to be symmetrized. The integral in the last equation must be of the form $\sum_k A_k (T_k)_{\mu_1 \cdots \nu_q}$ where $(T_k)_{\mu_1 \cdots \nu_q}$ are totally symmetric tensors built up from $g_{\mu\nu}$ and $\hat{v}_\alpha$. Note that those tensors containing $\hat{v}_\mu$, $\hat{v}_\nu$ can be ignored, since $\hat{v} \cdot v_- = 0$. As a consequence, $J_{(p + q = \text{odd})} = 0$, whereas one can show that $J_{(p + q = 2n)} = O(r^n)$. Thus, only the cases $p + q = 0$, $p + q = 2$ and $p + q = 4$ need be considered. The coefficients $A_k$ can be obtained by the usual procedure of contracting all indices of the integral in eq. (5) with the appropriate number of metric tensors, $g^{\mu\nu}$, and velocities, $\hat{v}^\alpha$. This will provide a set of equations for $A_k$ which can be solved in terms of scalar integrals of the form

$$I^{(1)}(a, b, c, p, q) = \int d^Dl d^Dk \left(\frac{-1}{k^2}\right)^a \left(\frac{-1}{l^2}\right)^b \left(\frac{-1}{(k - l)^2}\right)^c \left(\frac{\omega}{v \cdot k + \omega}\right)^p \left(\frac{\omega^{(c)}}{v \cdot l + \omega^{(c)}}\right)^q. \tag{6}$$
Those integrals \( I \) depending on only one scale \( \omega \) have been evaluated in [11]. Only \( I'(a, b, c, p, q) \) deserves some comments. For \( p > 1 \) and/or \( q > 1 \), \( I'(a, b, c, p, q) \) can be obtained from \( I'(a, b, c, 1, 1) \) by simply taking derivatives with respect to \( \omega \) and/or \( \omega' \).

To evaluate \( I'(a, b, c, 1, 1) \) one can make use of the following master equation:

\[
\left[ c - a + \frac{\omega'}{\omega}(2a + c + p - D) \right] I'(a, b, c, p, q) = \left[ c(1 - \frac{\omega'}{\omega})C^+(A^- - B^-) + aA^+(B^- - C^-) + \frac{\omega'}{\omega}pP^+Q^- \right] I'(a, b, c, p, q),
\]

where the action of \( A^\pm \) is given by \( A^\pm I'(a, b, c, p, q) = I'(a \pm 1, b, c, p, q) \), with similar definitions of \( B^\pm, C^\pm, P^\pm \) and \( Q^\pm \). We have deduced eq. (7) by following the “integration by parts” procedure [12] also used in [11]. By means of eq. (7), one can reduce any \( I'(a, b, c, p, q) \) appearing in the calculation of \( \tilde{\Gamma}^{(1)} \) to cases where at least one of the arguments \( a, b, c, p, \) or \( q \) vanishes. The cases \( p = 0, q = 0, c = 0 \) can be evaluated using the formulae given in [11]. For the cases \( a = 0 \) and \( b = 0 \) we make the change \( k, l \to (l + k) \), and use the results of [11] as well as

\[
\int d^Dk \left( \frac{-1}{k^2} \right)^a \left( \frac{\omega}{\omega + v \cdot k} \right)^p \left( \frac{\omega'}{\omega' + v \cdot k} \right)^q = i\pi^{D/2} \frac{\Gamma(2a + p + q - D)\Gamma(D/2 - a)}{\Gamma(a)\Gamma(p + q)} \left( -\frac{2\omega'}{\omega} \right)^{D - 2a} 2F_1(p + q + 2a - D, p, p + q, 1 - \frac{\omega}{\omega'}),
\]

where \( 2F_1(a, b, c, t) \) is the hypergeometric function.

Using a similar technique, it is straightforward to obtain the lowest order contribution, \( \tilde{\Gamma}^{(0)} \). Up to \( O(\alpha_s) \), the full three-point function can be written as

\[
\tilde{\Gamma} = \frac{1}{\epsilon} \tilde{\Gamma}^{(0)}_{\text{div}} + \epsilon \tilde{\Gamma}^{(0)}_{\text{fin}} + \frac{1}{\epsilon^2} \tilde{\Gamma}^{(1)}_{\text{div I}} + \frac{1}{\epsilon^2} \tilde{\Gamma}^{(1)}_{\text{div II}} + \frac{1}{\epsilon} \tilde{\Gamma}^{(1)}_{\text{div I}} + \tilde{\Gamma}^{(1)}_{\text{fin}} + \cdots,
\]

where the dependence in \( \epsilon \) is given explicitly. It is important to remember that the \( O(\epsilon \alpha_s^0) \)-terms in eq. (8), i.e. \( \tilde{\Gamma}^{(0)}_\epsilon \), cannot be neglected since they give a finite contribution of order \( \alpha_s \) to \( \tilde{\Gamma} \) through the external current renormalization. The terms \( \tilde{\Gamma}^{(0)}_{\text{div}}, \tilde{\Gamma}^{(1)}_{\text{div I}}, \tilde{\Gamma}^{(1)}_{\text{div II}} \) are polynomials in \( \omega, \omega' \) and \( r \) whereas the other contributions in eq. (8) contain logarithms and dilogarithms as well as negative powers of \( \omega - \omega' \). By expanding these functions in powers of \( \omega' - \omega \), one can check that the limit \( \omega' \to \omega \) of \( \tilde{\Gamma} \) exists. Actually, \( \tilde{\Gamma} \) is analytic in \( \omega' - \omega \) for \( \omega, \omega' < 0 \). Note that the UV-divergent portion of \( \tilde{\Gamma}^{(1)} \), i.e. \( \tilde{\Gamma}^{(1)}_{\text{div I}} \) has got a non-polynomial structure in \( \omega, \omega' \). As in ordinary QCD, these non-localities disappear upon renormalization of the currents \( \bar{J}_5 \) and \( \bar{V}^\mu \). The corresponding renormalization constants, \( Z_{HL} \) and \( Z_{HH} \) have been computed by [11, 13, 14] to be

\[
Z_{HL} = 1 + \frac{\alpha_s}{4\pi\epsilon}(-2), \quad Z_{HH} = 1 + \frac{\alpha_s}{4\pi\epsilon} \left[ \frac{32}{9} r + \frac{64}{45} r^2 + O(r^3) \right].
\]
We have checked that the UV-divergent part of the three-point function of the renormalized currents is a polynomial in the external variables $\omega, \omega'$, as it should. This is a nontrivial check of our result. Hereafter, $\tilde{\Gamma}$ will stand for the renormalized three-point function.

As usual in the sum rule approach, one would like to express $\tilde{\Gamma}$ as a dispersion integral, i.e.

$$\tilde{\Gamma}(\omega, \omega', y) = -\frac{\pi^2}{\pi^2} \int_0^\infty ds \int_0^\infty ds' \frac{\rho_3(s, s', y)}{(s - \omega)(s' - \omega')} + \text{subtraction terms} \quad (11)$$

with $\rho_3(\omega, \omega', r) = -\operatorname{Im} \left\{ \operatorname{Im} \Gamma(\omega, s' + i0, r) \bigg|_{\omega = s + i0} \right\}$. \quad (12)

Applying eq. (12) to our result for $\tilde{\Gamma}$ we obtain

$$-\frac{\pi^2}{\pi^2} \rho_3(s, s', r) = Q_0(s, s', r) \frac{d^6}{ds^6} \delta(s - s')$$

$$+ Q_1(s, s', r) \frac{d^7}{ds^7} \log |s - s'| + Q_2(s, s', r) \frac{d^7}{ds^7} \left[ \operatorname{sgn}(s - s') \log |s - s'| \right],$$

where $Q_0, Q_1, Q_2$ are polynomials in $s, s'$ and $r$. By substituting the explicit result summarized in eq. (13) in the partial derivative of eq. (11) with respect to $\omega$ and $\omega'$, we have checked that one recovers the full expression for $\partial^2 \tilde{\Gamma}(\omega, \omega', r)$ computed directly by differentiating our original result for $\tilde{\Gamma}(\omega, \omega', r)$. The differentiation is necessary in order to eliminate the subtraction terms in eq. (11).

Using the explicit form of eq. (13) it is quite straightforward to compute a quantity that plays a central role in the next section: the Borel transform of $\tilde{\Gamma}$, $\hat{B}^{M_3} \hat{B}^{M_3} \tilde{\Gamma}$. One just has to evaluate the integral

$$\hat{B}^{M_3} \hat{B}^{M_3} \tilde{\Gamma} = -\frac{1}{\pi^2 M_3^2} \int_0^\infty ds \int_0^\infty ds' \rho_3(s, s', y) e^{-(s + s')/M_3}, \quad (14)$$

where $M_3$ is the Borel parameter for the three-point function. The final result is a very simple expression which can be cast into the following form:

$$\hat{B}^{M_3} \hat{B}^{M_3} \tilde{\Gamma} = \frac{2}{\pi^2 M_3^2} \int_0^\infty ds \, s^2 e^{-2s/M_3} \left\{ \frac{3}{8} + \left( \frac{\alpha_s}{\pi} \right) \left[ \frac{17}{8} - \frac{3 \log(2s/\mu)}{4} + \zeta(2) \right] \right\}$$

$$+ r \left\{ \frac{3}{4} + \left( \frac{\alpha_s}{\pi} \right) \left[ -\frac{355}{72} + \frac{4 \log 2}{3} + \frac{13}{6} \log(2s/\mu) - 2\zeta(2) \right] \right\}$$

$$+ r^2 \left\{ \frac{3}{8} + \left( \frac{\alpha_s}{\pi} \right) \left[ \frac{11873}{3600} - \frac{32}{15} \log 2 - \frac{109}{60} \log(2s/\mu) + \zeta(2) \right] \right\}. \quad (15)$$
3 Evaluation of the Isgur-Wise function

We are now in a position to derive a sum rule for the Isgur-Wise function including the lowest order radiative corrections. Sum rules for the meson-to-vacuum amplitudes in the HQET were introduced by [7, 8] and applied to the Isgur-Wise function by [6, 9, 10]. We will not go into details about the method of HQET sum rules, for which we refer to [8], but simply review the main points.

One can saturate eq. (1) with physical states of the HQET yielding

\[
\tilde{\Gamma}(v + v') = \tilde{\Gamma}_\text{had} + \tilde{\Gamma}_\text{cont} + \langle 0 | \bar{J}_5 | \tilde{P}(v') \rangle \langle \tilde{P}(v') | \bar{V}_\mu | \tilde{P}(v) \rangle \langle \tilde{P}(v) | \bar{J}_5 | 0 \rangle \frac{1}{4(\Delta m - \omega)(\Delta m - \omega')}
\]

+ contributions of higher states, \(\Delta m = m_M - m_Q \approx 0.5 \text{ GeV}\) is the difference between the meson and the quark mass in the heavy quark limit (HQL). The matrix elements in the numerator can be expressed in terms of the leptonic decay constant in the HQL, \(\tilde{f}\),

\[
\langle 0 | \bar{J}_5 | \tilde{P} \rangle = \tilde{f},
\]

and the universal Isgur-Wise function

\[
\langle \tilde{P} | \bar{V}_\mu | \tilde{P} \rangle = \xi_{IW}(y)(v + v')\mu.
\]

On the other hand, \(\tilde{\Gamma}\) can be calculated by means of the operator product expansion (OPE) à la [15], away from the physical region, i.e. for \(\omega, \omega' \ll 0\). Both expressions are matched after application of the Borel transformation defined in (14) which suppresses the contributions of both higher operators in the OPE and higher states in (16).

Due to the scale-dependence of \(\hat{B}_M^3 \hat{B}_M^3 \tilde{\Gamma}\), we shall derive a sum rule for the renormalization group independent (to two-loop accuracy) quantity

\[
\hat{B}_M^3 \hat{B}_M^3 \tilde{\Gamma} = \hat{B}_M^3 \hat{B}_M^3 \tilde{\Gamma}(\mu)\alpha_s(\mu)^{-2(\gamma_0^H + \gamma_1^H)/(2\beta_0)} \left(1 - \frac{\alpha_s(\mu)}{\pi} \left\{2\Delta^H + \Delta^H\right\}\right)
\]

with \(\Delta = \frac{\gamma_0}{\beta_0} \left(\frac{\gamma_1}{\gamma_0} - \frac{\beta_1}{\beta_0}\right)\) and \(\gamma^H(\gamma^H) = \frac{\alpha_s}{4\pi} \left(\gamma^H(\gamma^H) + \alpha_s \gamma^H(\gamma^H) + \ldots\right)\),

where \(\gamma^H\) denotes the anomalous dimension of the effective current with two heavy (one heavy and one light) quarks [11, 13, 14]. \(\beta_0\) and \(\beta_1\) are the lowest order coefficients of the usual \(\beta\)-function of QCD. Following the analysis of [8], we choose \(\mu = M_3\) which is the natural scale for the Borel transformed expression.

As for the OPE, we take into account operators up to dimension 5 or, to be specific, the quark condensate \(\langle \bar{q}q \rangle\) and the mixed condensate \(\langle \bar{q}\sigma G Gq \rangle\). The gluon condensate gives a small contribution and only at \(O(y - 1)\) [9]. Likewise, the contribution of the four-quark condensate is expected to be small. Thus we write

\[
\hat{B}_M^3 \hat{B}_M^3 \tilde{\Gamma} = \tilde{\Gamma}_{\text{pert}} + \tilde{\Gamma}^{(3)}(\tilde{O}_3) + \tilde{\Gamma}^{(5)}(\tilde{O}_5) + \ldots
\]
with the renormalization group invariant quark and mixed condensate $O_3$ and $O_5$, respectively, as defined in [8]. The perturbative contribution follows from (21):

$$\hat{\Gamma}^{\text{pert}} = \alpha_s(M_3)\frac{3}{\pi^2 M_3^2(1+y)^2} \int_0^\infty ds \frac{s^2 e^{-2s/M_3}}{(s + \frac{1}{2} + \frac{\gamma^H(y)}{4}) \ln \frac{2s}{M_3} + \frac{4}{9} \pi^2 - 2\Delta^H - \Delta^H(y)} \left( \frac{1}{y+1} \right)$$

$$
\left\{ 1 + \frac{\alpha_s(M_3)}{\pi} \left[ \frac{\gamma^H(y)}{2} + \frac{\gamma^H_0(y)}{4} \right] \ln \frac{2s}{M_3} + \frac{4}{9} \pi^2 - 2\Delta^H - \Delta^H(y) + \frac{17}{3} + (y-1) \left( \frac{16}{9} \ln 2 - \frac{49}{54} \right) + (y-1)^2 \left( \frac{8}{15} \ln 2 + \frac{197}{600} \right) \right\}. \tag{22}
$$

Here we have factored out an overall factor $1/(y+1)^2$ which appears in the exact expression for the bare loop diagram. Furthermore, we have summed up the coefficients in front of the logarithm to get the anomalous dimension as specified by the scale-dependence of $\hat{\Gamma}$. The numbers under the underbraces show the good convergence of the series in $(y-1)$.

The lowest order expression for $\hat{\Gamma}^{(3)}$ is simply given by $-1/(4M_3^2)$, whereas for the $O(\alpha_s)$ corrections we take the expression given by [3]. Radiative corrections to $\hat{\Gamma}^{(5)}$ have not been calculated so far, so we use the lowest order expression $\hat{\Gamma}^{(5)} = (2y+1)/(192M_3^2)$.

Using the usual argument of quark-hadron duality, the contribution of higher states to (16) is modelled by the perturbative contribution to the OPE (21) above certain threshold $\Delta s$, the so-called continuum threshold. Following the discussion of [16], we choose the threshold in such a way that undesirable contributions of P-waves or even higher states do not mix in. It was already noticed by [3, 8] that the slope of the Isgur-Wise function is quite sensitive to the choice of the continuum model and introduces a rather large uncertainty. This problem gets resolved by choosing the continuum model according to [16] as

$$\hat{\Gamma}^{\text{cont}} = \int_{\Delta s}^\infty ds \times \text{Integrand of (22)}. \tag{23}$$

Taking all together we get the following sum rule for the renormalization group invariant Isgur-Wise function:

$$\hat{\xi}_{IW}(y) = \frac{4M_3^2 (\hat{B}^{M_3} \hat{B}^{M_3} \hat{\Gamma}(y) - \hat{\Gamma}^{\text{cont}})}{\hat{B}^{M_2} \hat{S} \hat{R}} \tag{24}$$

where $\hat{B}^{M_2} \hat{S} \hat{R}$ denotes the corresponding sum rule for the square of the leptonic decay constant, $\hat{f}^2$, and can be found in [8]. Note that the exponential $\exp(-\Delta m/M_3)$, that formally appears on the left hand side of (24) due to the Borel transformation, cancels against the corresponding factor in $\hat{B}^{M_2} \hat{S} \hat{R}$ and, hence $\hat{\xi}_{IW}$ as calculated in (24) becomes
independent of $\Delta m$. The sum rule (24) automatically fulfills the normalization condition $\hat{\xi}_{IW}(y)|_{y=1} = 1$ for any value of continuum threshold and the Borel parameter provided we take the same value for $\Delta s_2 \equiv \Delta s_3 \equiv \Delta s$ in both the numerator and denominator and take $M_3 \equiv 2M_2 \equiv 2M$. This equivalence will be used throughout the next section.

4 Results and Discussion

We now turn to the numerical analysis of the sum rule eq. (24). The numerical values of the condensates we use are $\langle \bar{q}q \rangle = (-0.24 \text{ GeV})^3$ and $\langle \bar{q} \sigma G G q \rangle = (0.8 \text{ GeV}^2)\langle \bar{q}q \rangle$ at the scale of 1 GeV. In Fig. 1 we show $\hat{\xi}_{IW}$ for different values of $\Delta s$ and $M$ as a function of $y$. For phenomenological applications, such as the decay $B \to D^{*}e\nu$, it is sufficient to know the Isgur-Wise function within the range $1 \leq y \leq 2$. In this region, the sensitivity of eq. (24) to the continuum threshold and the Borel parameter is found to be quite small and to amount to at most 6% for $y = 2$. In the evaluation we have taken $1.1 \text{ GeV} \leq \Delta s \leq 1.4 \text{ GeV}$ and $0.5 \text{ GeV} \leq M \leq 1.0 \text{ GeV}$ as suggested by the analysis of the sum rule for $\hat{f}$ [8]. The resulting curves may be parametrized by a second order polynomial in $(y - 1)$ as

$$\hat{\xi}_{IW}(y) = 1 - (0.54 \pm 0.01)(y - 1) + (0.17 \pm 0.01)(y - 1)^2$$

(25)

where the errors reflect the uncertainty due to $M$ and $\Delta s$. The result is mainly determined by the perturbative term which contributes between 50% and 85% at $y = 1$, depending strongly on $M$, and increasing with $y$.

In Fig. 2 we depict the effect of neglecting radiative corrections which is less than 10% for $y \leq 2$. It is, however, clearly seen that the inclusion of radiative corrections lowers the values of $\hat{\xi}_{IW}$. This is not unexpected an effect, since the coefficients of powers of $(y - 1)$ in eq. (22) are quite small and overcome by the $y$-dependent terms in front of the logarithm. On the other hand, the large constant term $\sim \alpha_s/\pi (17/3 + 4\pi^2/9)$, that was attributed to Coulombic corrections in [8], is cancelled to a large extent by the corresponding term in $\hat{f}$. In other words: the radiative corrections are mainly determined by the one-gluon exchange between the heavy-quark lines, i.e. radiative corrections to the weak vertex, and thus pure quantum effects (and not classical ones such as Coulombic corrections).

In order to determine the slope parameter $\rho^2$ of the Isgur-Wise function, $\rho^2 = - (d\xi_{IW}(y))/(dy)|_{y=1}$, we scale $\hat{\xi}_{IW}$ to a physically meaningful scale as

$$\xi_{IW}(y, \bar{m}) = [\alpha_s(\bar{m})]^{\gamma_{HH}/(2\beta_0)} \left(1 + \frac{\alpha_s(\bar{m})}{\pi} \Delta_{HH}^{\Delta HH} \right) \hat{\xi}_{IW}(y) .$$

(26)

To be specific, we choose $\bar{m} = m_B m_D/(m_B + m_D) \approx 1.4 \text{ GeV}$, the harmonic mean of the masses of the B- and D-meson.
From the above formula we get $\rho^2 = (0.84 \pm 0.02)$ where the error stems from the uncertainty in the choice of parameters. This value is nearly twice as big as the upper bound $\rho_{\text{max}}^2 = 0.45$ given by [4]. Disregarding all radiative corrections but the overall leading-log scaling factors, we get $\rho^2 = (0.78 \pm 0.02)$ which still is far away from $\rho_{\text{max}}^2$. We see no possibility to get such a small value for the slope parameter, but rather agree with the existing calculations and fits [5] which yield $\rho^2 \approx 1.0 - 1.5$. The resolution of this disagreement surely requires further investigations.
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Figure Captions

Figure 1: $\hat{\xi}_{IW}$ as a function of $y$, eq. (24), for $0.5 \ \text{GeV} \leq M \leq 1.1 \ \text{GeV}$, $1.1 \ \text{GeV} \leq \Delta s \leq 1.4 \ \text{GeV}$. The spread of the lines reflects the uncertainty due to the choice of parameters.

Figure 2: Solid line: $\hat{\xi}_{IW}$ as a function of $y$, eq. (24), at $M = 0.9 \ \text{GeV}$, $\Delta s = 1.23 \ \text{GeV}$. Dashed line: $\hat{\xi}_{IW}$ without radiative corrections. The influence of $O(\alpha_s)$ corrections causes a stronger falling-off of the form factor mainly due to the corrections to the weak vertex.