ASTROPHYSICAL AND TERRESTRIAL
CONSTRANTS
ON SINGLET MAJORON MODELS

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ABSTRACT

The general Lagrangian containing the couplings of the Higgs scalars to Majorana neutrinos is presented in the context of singlet Majoron models with intergenerational mixings. The analytical expressions for the coupling of the Majoron field to fermions are derived within these models. Astrophysical considerations imply severe restrictions on the parameters of the model if the singlet Majoron model with three generations is assumed to be embedded in grand unified theories. Bounds that originate from analyzing possible charged lepton-violating decays in terrestrial experiments are also discussed. In particular, we find that experimental searches for muon decays by Majoron emission cannot generally be precluded by astrophysical requirements.

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Astrophysical considerations play an important role in constraining the strength of the coupling of Nambu-Goldstone bosons to the matter [1]. Among the various types of such extraordinary light particles (e.g. axions, familons, etc.) [2], which are associated with the spontaneous breakdown of some global symmetry, Majoron $J^0$ is a massless pseudoscalar boson arising from the breaking of the baryon–lepton ($B−L$) symmetry [3]-[7].

In such scenarios, apart from the Standard Model (SM) Higgs doublet $\Phi$, an $SU(2)\otimes U(1)$ singlet $\Sigma$ is present which gives rise to $\Delta L = 2$ Majorana mass terms $m_M$ when $\Sigma$ couples to right-handed neutrinos [3]. In general, models with right-handed neutrinos can naturally account for possible lepton-number violating decays of $Z^0$ and $H^0$ particles [8]-[11] induced by Majorana neutrinos at the first electroweak loop order. The non-decoupling physics that the heavy Majorana neutrinos can introduce [8, 9, 11] lead to combined constraints both on neutrino masses and lepton-violating mixings. Similar non-decoupling effects occur when one considers Majoron couplings to fermions. On the other hand, Majoron models can naturally be embedded in grand unified theories (GUT) like the $SO(10)$ model [12]. In such models the $B−L$ scale is determined by the vacuum expectation value (VEV) of the singlet scalar, i.e. $<\Sigma> = w/\sqrt{2}$ and the neutrino Dirac mass matrix $m_D$ may be related to the $u$-quark mass matrix $M_U$ by $m_D = M_U/k$, where $k = \mathcal{O}(1)$ represents the running of the Yukawa couplings between the GUT and the low-energy scale [13]. There are also scenarios where $m_D$ can be proportional to charged lepton mass matrix $M_l$ [14, 15]. Since the $B−L$ scale strongly depends on the mechanism that the $SO(10)$ gauge group breaks down to $U(1)_{em}$ [16], we will treat $w$ as a free parameter of the theory that should be constrained by our forthcoming considerations. The light neutrino masses $m_{\nu_i}$ can generally be estimated by

$$m_{\nu_i} \approx - m_D \frac{1}{m_M} m^T_D.$$  \hspace{1cm} (1)

The mass hierarchical pattern mentioned above should also hold for the heaviest family which implies that the biggest eigenvalue of $m_D$ will be in the range between 10 and
100 GeV for $100 \leq m_t \leq 180$ GeV [13]. If astrophysical constraints are imposed on the $J^0 - e - e$, $J^0 - u - u$ and $J^0 - d - d$ couplings, one derives the bound on

$$\tan \beta = \frac{\langle \Phi \rangle}{\langle \Sigma \rangle} = \frac{v}{w} \leq 10^{-2},$$

(2)

for models without interfamily mixings. The situation becomes more involved if mixings between families are considered. This realization will be illustrated by the present work.

We will finally discuss the bounds from low-energy experiments [15, 16] on the off-diagonal coupling of the Majoron to two different charged leptons.

First, let us briefly describe the low-energy structure of the singlet Majoron model. The scalar potential of this model which should be non-trivial under the $U(1)_Y$ group is given by [3, 8]

$$- \mathcal{L}_V = \mu^2 (\Phi^\dagger \Phi) + \mu_\Sigma^2 (\Sigma^\dagger \Sigma) + \frac{\lambda_1}{2} (\Phi^\dagger \Phi)^2 + \frac{\lambda_2}{2} (\Sigma^\dagger \Sigma)^2 + \delta (\Phi^\dagger \Phi)(\Sigma^\dagger \Sigma).$$

(3)

If all the stability conditions in this model are satisfied (i.e. $\lambda_1, \lambda_2 > 0$ and $\lambda_1 \lambda_2 > \delta$) [3], the above potential can always be minimized by the following Higgs-field configurations:

$$\Phi = \left( \frac{G^+}{\sqrt{2}} + \frac{\phi^0 + i G^0}{\sqrt{2}} \right) \quad \text{and} \quad \Sigma = \frac{w}{\sqrt{2}} + \frac{\sigma^0 + i J^0}{\sqrt{2}}.$$

(4)

After the spontaneous breakdown of the $SU(2)_L \otimes U(1)_Y$ gauge group and diagonalizing the Higgs mass matrix, one obtains two $CP$-even Higgs fields (denoted by $H^0$ and $S^0$) and one massless $CP$-odd scalar, the Majoron $J^0$, while the would-be Goldstone bosons $G^+, G^0$ give simply mass to $W^+, Z^0$ bosons, respectively. The weak eigenstates $\phi^0$ and $\sigma^0$ are related to the corresponding physical mass eigenstates through

$$\begin{pmatrix}
\phi^0 \\
\sigma^0
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
H^0 \\
S^0
\end{pmatrix},$$

(5)
where
\[
\tan 2\theta = \frac{2\delta \tan \beta}{\lambda_2 - \lambda_1 \tan^2 \beta}.
\]
The Yukawa sector containing all the relevant Higgs couplings to neutrinos reads
\[
- \mathcal{L}^{\text{Higgs}}_Y = \bar{\nu}_L^0 m_{D_{ij}} \nu_{R_j}^0 \frac{\phi^0}{v} + \bar{\nu}_{R_i}^0 m_{M_{ij}}^\dagger \nu_{L_i}^0 \frac{\phi^0}{v} + \frac{1}{2} \bar{\nu}_{R_i}^0 m_{M_{ij}}^\dagger \nu_{L_j}^0 \sigma^0 + iJ^0_w + \frac{1}{2} \bar{\nu}_{R_i}^0 m_{M_{ij}}^\dagger \nu_{R_j}^0 \sigma^0 - iJ^0_w.
\]

In Eq. (7) we have assumed the absence of Higgs triplets [17], which seem now to be ruled out by the LEP data on the invisible $Z^0$ width. The interactions of $J^0, H^0$ and $S^0$ with the $2n_G$ Majorana neutrinos $n_i$ — $n_G$ denotes the number of generations — are generally described by the following Lagrangians:

\[
\mathcal{L}_{int}^J = \frac{i g_W t_\beta}{4M_W} J^0 \bar{\nu}_i \left[ \gamma_5 (m_{n_i} + m_{n_j}) \left( \frac{1}{2} \delta_{ij} - \text{Re} C_{ij} \right) + i (m_{n_i} - m_{n_j}) \text{Im} C_{ij} \right] n_j,
\]

\[
\mathcal{L}_{int}^H = - \frac{g_W}{4M_W} (c_\theta - s_\theta t_\beta) H^0 \bar{\nu}_i \left[ (m_{n_i} + m_{n_j}) \left( \text{Re} C_{ij} + \frac{t_\beta s_\theta \delta_{ij}}{2(c_\theta - s_\theta t_\beta)} \right) \right. \\
\left. + i \gamma_5 (m_{n_j} - m_{n_i}) \text{Im} C_{ij} \right] n_j,
\]

\[
\mathcal{L}_{int}^S = \frac{g_W}{4M_W} (s_\theta + c_\theta t_\beta) S^0 \bar{\nu}_i \left[ (m_{n_i} + m_{n_j}) \left( \text{Re} C_{ij} - \frac{t_\beta \delta_{ij}}{2(t_\theta + t_\beta)} \right) \right. \\
\left. + i \gamma_5 (m_{n_j} - m_{n_i}) \text{Im} C_{ij} \right] n_j,
\]

where we have used the abbreviations $s_x = \sin x$, $c_x = \cos x$, $t_x = \tan x$ and defined

\[
C_{ij} = \sum_{k=1}^{n_G} U_{ki}^\nu U_{kj}^{\nu^*}.
\]

The $2n_G \times 2n_G$ unitary matrix $U^\nu$ is responsible for the diagonalization of the $2n_G \times 2n_G$ neutrino mass matrix $M^\nu$ which is of the ”see-saw” form [18]

\[
M^\nu = \begin{pmatrix}
0 & m_D \\
m_D^T & m_M
\end{pmatrix}.
\]
The first $n_G$ eigenvalues of $M^\nu$ are identified with the ordinary light neutrinos, $\nu_e$, $\nu_\mu$, etc., whereas the remaining $n_G$ Majorana neutrino states are new particles provided in this model and should be heavier than the $Z^0$ boson to escape detection at LEP experiments. The $2n_G$ neutral leptons $n_i$ are related to their weak eigenstates $\nu^0_{L,R_i}$ and $\nu^{0C}_{L,R_i}$ through the following unitary transformations (assuming the convention of summation for repeated indices):

\[
\begin{pmatrix}
\nu^0_{L_i} \\
\nu^{0C}_{L_i}
\end{pmatrix}_i = U^{\nu *}_{ij} n_{L_j}, \quad \begin{pmatrix}
\nu^0_{R_i} \\
\nu^{0C}_{R_i}
\end{pmatrix}_i = U^{\nu}_{ij} n_{R_j}.
\]

(13)

It is now easy to see that in the limit of $\tan \beta \to 0$ and $\theta \to 0$, the fields $S^0$ and $J^0$ decouple fully from matter and only one Higgs field, $H^0$, couples to Majorana neutrinos. This scenario has explicitly been described in [19], where for our purposes we will repeat here the interactions of the $W^+$ and $Z^0$ boson with the Majorana neutrinos. They are given by the Lagrangians:

\[
L^{W^+}_{\text{int}} = -\frac{g_W}{2\sqrt{2}} W^{-\mu} \bar{l}_i B_{li,j} \gamma_{\mu}(1 - \gamma_5) n_j + \text{h.c.},
\]

(14)

\[
L^{Z}_{\text{int}} = -\frac{g_W}{4\cos \theta_W} Z^{0\mu} \bar{n}_i \gamma_{\mu} [\text{Im}(C_{ij}) - \gamma_5 \text{Re}(C_{ij})] n_j.
\]

(15)

Moreover, the couplings of the charged would-be Goldstone bosons $G^\mp$ are written down

\[
L^{G^\mp}_{\text{int}} = -\frac{g_W}{2\sqrt{2} M_W} G^- \bar{l}_i [m_i B_{li,j}(1 - \gamma_5) - B_{li,j} (1 + \gamma_5) m_{nj}] n_j + \text{h.c.},
\]

(16)

where

\[
B_{li,j} = \sum_{k=1}^{n_G} V^l_{ik} U^{\nu *}_{kj}.
\]

(17)

In Eq. (17) $V^l$ is a unitary matrix relevant for the bi-diagonalization of the charged lepton mass matrix $M^l$.

At this point it is important to mention that the presence of Majorana neutrino interactions in the Lagrangians (8)–(10) violates the $CP$ symmetry of the model. In particular, the fact that $H^0$, $S^0$ and $J^0$ couple simultaneously to $CP$-even ($\bar{n}_i n_j$) and
CP-odd \(\bar{n}_i\gamma_5n_j\) operators gives rise to CP-violating transitions between states with different CP-quantum numbers \[20\]. For example, one finds a non-zero contribution when computing selfenergy graphs induced by Majorana neutrinos between the CP-even Higgs fields \(H^0\), \(S^0\) and the CP-odd states \(Z^0\), \(J^0\). All these transitions turn out to be proportional to the CP-odd combinations \(\text{Im}C^2_{ij}\) which change sign when a CP conjugation is applied to the vacuum polarization terms. As a consequence, the general Majoron coupling to charged leptons and quarks possesses a scalar and pseudoscalar part. We will include these CP-odd effects in our theoretical considerations, although they seem not to influence our numerical predictions.

Armed with the Lagrangians (8)–(10), (14) and (15) it is now straightforward to calculate the coupling \(J^0 - f_1 - f_2\) given by the Feynman graphs shown in Figs. 1(a)–1(c). The additional CP-violating diagrams of the Majoron coupling to fermions are depicted in Figs. 2(d) and 2(e). In our analytical calculations we have neglected terms proportional to the small quantities \(m_f^2/M_W^2\) for \(f = e, u\) or \(d\). The individual amplitudes contributing to the \(J^0 - f_1 - f_2\) coupling are given by

\[
\mathcal{T}^{l_1l_2}_a = \Delta^S_{ij} \left[ -\frac{1}{2} (\lambda_i + \lambda_j) (\delta_{ij} - C^*_{ij}) I_1(\lambda_i, \lambda_j) + C_{ij} \sqrt{\lambda_i \lambda_j} I_1(\lambda_i, \lambda_j) 
\right.
+ \frac{1}{2} (\lambda_i - \lambda_j) C^*_{ij} I_3(\lambda_i, \lambda_j) 
\left. + \frac{1}{2} \Delta^A_{ij} (\lambda_i - \lambda_j) C^*_{ij} \right] 
\]

\[
\mathcal{T}^{l_1l_2}_b = \frac{1}{2} \Delta^S_{ij} \left[ \left( C_{UV} - \frac{1}{2} \right) (\lambda_i \delta_{ij} - C_{ij} \sqrt{\lambda_i \lambda_j} - \frac{1}{2} (\lambda_i + \lambda_j) C^*_{ij}) - (\lambda_i + \lambda_j) (\delta_{ij} - C^*_{ij} L_2(\lambda_i, \lambda_j) + C_{ij} \sqrt{\lambda_i \lambda_j} \left( -2 L_2(\lambda_i, \lambda_j) + \frac{1}{2} (\lambda_j - \lambda_i) I_3(\lambda_i, \lambda_j) \right) \right] 
\right. 
\left. + \frac{1}{2} \Delta^A_{ij} (\lambda_i - \lambda_j) \left( \frac{1}{2} (C_{UV} - \frac{1}{2}) C^*_{ij} - C_{ij} \sqrt{\lambda_i \lambda_j} I_2(\lambda_i, \lambda_j) 
\right. 
\left. + C_{ij} L_2(\lambda_i, \lambda_j) \right) , \right. 
\]

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\[ T_{e}^{ff} = -2\delta^S(2T_z^f)(\lambda_jC_{ij}(\delta_{ij} - C_{ij}^*) - \sqrt{\lambda_i\lambda_j}\text{Re}C_{ij}^2)\left(-\frac{1}{2}C_{UV} + L_1(\lambda_i, \lambda_j)\right), \] (20)

\[ T_{d}^{ff} = -i\delta^A\text{Im}C_{ij}^2(c_\theta - s_\theta t_\beta)(\lambda_i - \lambda_j)\sqrt{\lambda_i\lambda_j}\lambda_H^{-1}\left(C_{UV} - L_0(\lambda_i, \lambda_j)\right), \] (21)

\[ T_{e}^{ff} = i\delta^A\text{Im}C_{ij}^2(s_\theta + c_\theta t_\beta)(\lambda_i - \lambda_j)\sqrt{\lambda_i\lambda_j}\lambda_S^{-1}\left(C_{UV} - L_0(\lambda_i, \lambda_j)\right), \] (22)

where

\[ \lambda_i = \frac{m_{iW}^2}{M_W^2}, \quad \lambda_H = \frac{M_H^2}{M_W^2}, \quad \lambda_S = \frac{M_S^2}{M_W^2}, \] (23)

\[ C_{UV} = \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi - \ln \frac{M_W^2}{\mu^2}, \] (24)

\[ \Delta_{ij}^A = -\frac{g_{W\alpha W} \tan \beta}{16\pi} B_{i1}^* B_{l2j} \bar{u}_{l2} \left[ \frac{m_{l1}}{M_W} (1 + \gamma_5) + \frac{m_{l2}}{M_W} (1 - \gamma_5) \right] u_{l1}, \] \[ \Delta_{ij}^S = -\frac{g_{W\alpha W} \tan \beta}{16\pi} B_{i1}^* B_{l2j} \bar{u}_{l2} \left[ \frac{m_{l1}}{M_W} (1 + \gamma_5) - \frac{m_{l2}}{M_W} (1 - \gamma_5) \right] u_{l1}, \] (25)

\[ \delta^A = -\frac{g_{W\alpha W}}{16\pi} \tan \beta \bar{u}_f u_f, \] \[ \delta^S = -\frac{g_{W\alpha W}}{16\pi} \tan \beta \bar{u}_f \gamma_5 u_f. \] (26)

The analytical expressions for the one-loop functions \( I_1, I_2, I_3, L_0, L_1 \) and \( L_2 \) are given in Appendix A. In Eq. (20) \( T_z^f \) stands for the third component of the weak isospin and takes the values: \( T_z^u = 1/2, T_z^{ld} = -1/2 \). The UV divergences in the amplitudes (19)–(22) vanish identically due to the following equalities [19, 2]:

\[ \sum_{i=1}^{2n_G} B_{li} C_{ij} = B_{ij}, \] \[ \sum_{k=1}^{2n_G} C_{ik} C_{jk}^* = C_{ij}, \] \[ \sum_{i=1}^{2n_G} m_{ni} B_{li} C_{ij}^* = 0, \] \[ \sum_{k=1}^{2n_G} m_{nk} C_{ik} C_{jk} = 0. \] (27) \[ \sum_{k=1}^{2n_G} m_{nk} C_{ik} C_{jk} = 0. \] (28) \[ \sum_{i=1}^{2n_G} m_{ni} B_{li} C_{ij}^* = 0, \] \[ \sum_{i=1}^{2n_G} m_{ni} B_{li} C_{ij}^* = 0. \] (29) \[ \sum_{k=1}^{2n_G} m_{nk} C_{ik} C_{jk} = 0. \] (30)

Note also that the amplitudes \( T_{d}^{ff}, T_{e}^{ff} \) induce a scalar piece in the \( J^0 - f - f \) coupling.
This scalar part, however, is suppressed for astrophysical reasons by a factor of 10 at least as compared to the pseudoscalar part of the coupling [21].

In order to pin down numerical predictions, we first consider the conservative case of a model with one generation or equivalently a three-generation model without interfamily mixings. Then, the coupling $J^0 - e - e$, $g_{Jee}$, defined by the relation

$$
\mathcal{T}^{ee} = g_{Jee} \bar{e}_i \gamma_5 e,
$$

(31)

takes the simple form

$$
g_{Jee} \simeq \frac{g_W \alpha_W}{16\pi} t_\beta \frac{m_e}{M_W} \left[ (s_L^e)^2 \frac{\lambda_N^2}{1 - \lambda_N} \left( 1 + \ln \frac{\lambda_N}{1 - \lambda_N} \right) + \frac{1}{2} \sum_{e,\mu,\tau} (s_L^l)^2 \lambda_{N_l} \right].
$$

(32)

The mixings $(s_L^l)^2$ are defined by

$$
(s_L^l)^2 = \sum_{i=n_G+1}^{2n_G} |B_{li}|^2.
$$

(33)

In this scenario $(s_L^l)^2 = m_{D_{li}}^2/m_{N_l}^2$ and $\lambda_{N_l} \gg 1$, since the heavy Majorana neutrinos $N_l$ have to satisfy Eq. (1) and $m_{D_{li}} \simeq m_{l_i}$ or $m_{u_i}$. On the other hand, astrophysical constraints arising from helium ignition in red giants or the observational evidence of white dwarf cooling rates are given by the bound [1]

$$
g_{Jee} \leq (9. - 1.4) \times 10^{-13},
$$

(34)

However, the range $3.10^{-13} \leq g_{Jee} \leq 6.10^{-7}$ is excluded from the helium ignition argument mentioned above, if the radius of giant core or dwarf is bigger than the mean free path of the pseudoscalars that these particles require to freely escape from them [1]. It is now obvious that the most stringent constraint on $\tan \beta$ arises from the heaviest family. Thus, for $m_D \simeq m_\tau$ one obtains that

$$
g_{Jee} \simeq \frac{g_W \alpha_W}{32} \tan \beta \frac{m_e}{M_W} \frac{m_\tau}{M_W^2},
$$

(35)
yielding because of Eq. (34)

$$\tan \beta \leq 0.4 .$$  \hspace{1cm} (36)

Of course, if $m_D \simeq m_t/k \simeq 10$ GeV, one finds a much stronger bound, i.e.

$$\tan \beta \leq 10^{-2} .$$  \hspace{1cm} (37)

Note also that such low-energy realizations make unlikely the invisible decay of massive Higgses into Majoron pairs 6, 7.

The afore-mentioned hierarchical scheme, however, is in general not valid if one introduces intergenerational mixings in the singlet Majoron model. This situation seems to be a natural possibility that can be realized by GUT models, since $m_D$ and $M_U$ matrices may get related in such high-energy scenarios (i.e. $m_D(M_X) = M_U(M_X)$ with $M_X$ indicating the grand unification scale). In addition, it has explicitly been demonstrated in 22, 19 that the scale of $m_M$ can be $O(100)$ GeV without contradicting experimental bounds on neutrino masses. For instance, democratic-type mass matrices for the form of $m_D$ 24 can lead to patterns with such a low scale for $m_M$. Then, the mixings $(s^{\nu}_L)^2$ can be treated as purely phenomenological parameters, since Eq. (33) should now read

$$(s^{\nu}_L)^2 \simeq m_D \frac{1}{m_M^2} m_D^\dagger$$  \hspace{1cm} (38)

and cannot therefore be related with the light-neutrino mass matrix of Eq. (1). The mixing angles $(s^{\nu}_L)^2$ can generally be constrained by a global analysis of a great number of low-energy experiments and LEP data 23. In this scenario one makes the remarkable observation that $g_{Jee}$ can severely be suppressed for a certain choice of the mass parameters $\lambda_{N_i}$ and mixings $(s^{\nu}_L)^2$. For example, if all heavy neutrino masses $m_{N_i}$ are approximately equal and $\lambda_{N_i} \gg 1$, then the choice

$$(s^{\nu}_L)^2 \simeq (s^{\nu}_L)^2$$  \hspace{1cm} (39)
leads to $g_{Jee} = 0$. However, even if the Majoron couplings to electrons vanish, the corresponding coupling to nucleons $N$, $g_{JNN}$, is not zero anymore. The reason is that the destructive first term in the bracket of Eq. (32) does not exist anymore and such a fine-tuning is thus not possible. Since $g_{JNN}/g_{Jee} \simeq m_N/m_e \simeq 2 \times 10^3$, one may derive useful constraints from the consideration of cooling rates of neutron stars due to the energy loss mechanism by Majoron emission. In Fig. (3) we present exclusion plots of the parameters $\tan \beta$ versus $m_N$ for three different values of $(s_L^e)^2$ by considering that 

$$g_{JNN} \lesssim 10^{-9}. \quad (40)$$

For a discussion of additional uncertainties on the upper bound of the coupling $g_{JNN}$ that can arise from various reasons like the so-called "Turner's window" etc., we refer the reader to [23]. Ultimately, we must notice that the astrophysical bounds arising from the Majoron coupling to two photons, $C_{J\gamma\gamma}$, should be weaker than that coming from $g_{JNN}$, since $C_{J\gamma\gamma}$ can only be generated at two-loop electroweak order.

In the following we will focus our attention on bounds resulting solely from terrestrial experiments [15, 16] by analyzing lepton-flavor violating decays, i.e. $l_1 \rightarrow J^0 l_2$ with $l_1 \neq l_2$. To the leading order of the heavy neutrino limit one finds from Eqs. (18) and (19) that

$$\text{BR}(l_1^- \rightarrow J^0 l_2^-) \simeq \frac{3\alpha_W}{8\pi} \tan^2 \beta |B_{11}^* N B_{12} N|^2 \lambda^2 N \frac{M_W^2}{m_{l_1}^2}. \quad (41)$$

The experimental information we have for the above lepton-violating decays are the following upper bounds:

$$\text{BR}(\mu \rightarrow J^0 e) \leq 2.6 \times 10^{-6} \ [15],$$

$$\text{BR}(\tau \rightarrow J^0 e) \leq 7.1 \times 10^{-3} \ [16],$$

$$\text{BR}(\tau \rightarrow J^0 \mu) \leq 2.3 \times 10^{-3} \ [16]. \quad (42)$$

In order to quantitatively estimate the magnitude of the lepton-violating effects that could be constrained by the branching ratios stated in (42), we use the upper bound of
The quantity

$$|B_{l_1N}^* B_{l_2N}| \leq (s_{L_1}^R)^2 = \max((s_{L_1}^R)^2, (s_{L_2}^R)^2).$$

The exclusion plots implied by these experiments are presented in Fig. (4) for the three different decay channels. For comparison, we have taken the astrophysical bound coming from Eq. (40) into account in Fig. (4), from which one easily concludes that experimental searches for the decay $\mu \to J^0 e$ may not be excluded by astrophysical constraints and can hence be sensitive to new physics beyond the $SM$.

In conclusion, astrophysical considerations may lead to useful constraints on the parameters of singlet Majoron models with intergenerational mixings. It has been demonstrated that three-generation Majoron models can indeed be constrained if these models are assumed to be embedded in $GUT$ scenarios. Possibilities of how to evade from some of the astrophysical constraints have also been discussed. For example, $g_{Jee}$ vanishes for a specific choice of parameters. Furthermore, terrestrial experiments give independently severe restrictions on the lepton-violating mixings and heavy neutrino masses. Aside from rather involved $R$-parity broken models [28], this minimal extension of the $SM$, the singlet Majoron model, may also naturally account for possible lepton-flavor violating signals in precision experiments. We emphasize again the fact that measurements of the $TRIUMF$ collaboration [15] for exotic decay modes, like $\mu \to J^0 e$, lie in area which may not be excluded by astrophysics and have substantial chances to establish new physics beyond the $SM$. Finally, due to the $CP$-odd interactions that Majorona neutrinos introduce in such models (see e.g. Eqs. (9), (14) and (15)), one may be motivated to discuss their phenomenological impact of possible $CP$-violating effects in the decays of the Higgs particle $H^0$ into top, $W$ or $Z$ pairs [29].

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ported by a grant from the Postdoctoral Graduate College of Mainz.
A The loop integrals

We first define the useful functions $B_1(\lambda_i, \lambda_j)$ and $B_2(\lambda_i, \lambda_j)$ as:

$$B_1(\lambda_i, \lambda_j) = \lambda_i(1 - x) + \lambda_j x ,$$  \hspace{1cm} (A1)  

$$B_2(\lambda_i, \lambda_j) = 1 - y + y[\lambda_i(1 - x) + \lambda_j x] ,$$  \hspace{1cm} (A2)  

where $x$ and $y$ are Feynman parameters. The loop integrals $L_0$, $L_1$, $L_2$, $I_1$, $I_2$ and $I_3$ are then given by

$$L_0(\lambda_i, \lambda_j) = \int dx \ln B_1(\lambda_i, \lambda_j)$$  

$$= -1 + \frac{1}{2} \ln \lambda_i \lambda_j - \frac{\lambda_i + \lambda_j}{2(\lambda_i - \lambda_j)} \ln \frac{\lambda_j}{\lambda_i} ,$$  \hspace{1cm} (A3)  

$$L_1(\lambda_i, \lambda_j) = \int dx x \ln B_1(\lambda_i, \lambda_j)$$  

$$= -\frac{1}{4} + \frac{\lambda_i^2}{2(\lambda_i - \lambda_j)^2} \ln \frac{\lambda_i}{\lambda_j} + \frac{1}{2} \ln \lambda_j - \frac{\lambda_i}{2(\lambda_i - \lambda_j)} ,$$  \hspace{1cm} (A4)  

$$L_2(\lambda_i, \lambda_j) = \int dxdy y \ln B_2(\lambda_i, \lambda_j)$$  

$$= \frac{3}{4(\lambda_i - \lambda_j)} \left[ \frac{\lambda_j}{1 - \lambda_i} - \frac{\lambda_i}{1 - \lambda_j} \right] - \frac{3\lambda_i\lambda_j}{4(1 - \lambda_i)(1 - \lambda_j)}$$  

$$+ \frac{1}{2(\lambda_i - \lambda_j)} \left[ \lambda_i^2 \ln \lambda_j - \lambda_i^2 \ln \lambda_i \right] ,$$  \hspace{1cm} (A5)  

$$I_1(\lambda_i, \lambda_j) = \int dxdy \frac{y}{B_1(\lambda_i, \lambda_j)}$$  

$$= \frac{\lambda_i \lambda_j \ln(\lambda_i/\lambda_j) + \lambda_j \ln \lambda_j - \lambda_i \ln \lambda_i}{(1 - \lambda_i)(1 - \lambda_j)(\lambda_i - \lambda_j)} ,$$  \hspace{1cm} (A6)  

$$I_2(\lambda_i, \lambda_j) = \int dxdy \frac{y^2}{B_2(\lambda_i, \lambda_j)}$$  

$$= -\frac{1}{2(1 - \lambda_i)(1 - \lambda_j)} + \frac{1}{2(\lambda_i - \lambda_j)} \left[ \ln \frac{\lambda_i}{\lambda_j} - \ln \frac{\lambda_i}{(1 - \lambda_i)^2} \right]$$  

$$+ \frac{\ln \lambda_j}{(1 - \lambda_j)^2} ,$$  \hspace{1cm} (A7)  

$$I_3(\lambda_i, \lambda_j) = \int dxdy \frac{y^2(1 - 2x)}{B_2(\lambda_i, \lambda_j)}$$  

$$= -\frac{1}{2(1 - \lambda_i)(1 - \lambda_j)} \left[ \ln \frac{\lambda_i}{1 - \lambda_i} - \ln \frac{\lambda_j}{1 - \lambda_j} \right] - \frac{1}{2(\lambda_i - \lambda_j)} \left[ \frac{\lambda_j}{1 - \lambda_j} \right] .$$
\[
\left[ \frac{\lambda_i}{1 - \lambda_i} \right] - \frac{\ln(\lambda_j/\lambda_i)}{2(\lambda_i - \lambda_j)^2} \left[ \frac{\lambda_j^2}{1 - \lambda_j} + \frac{\lambda_i^2}{1 - \lambda_i} \right].
\]

(A8)

The integration interval of the variables \(x\) and \(y\) is \([0, 1]\).
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Figure Captions

Fig. 1: Feynman graphs responsible for the coupling of Majorons to fermions, $J^0 - f_1 - f_2$.

Fig. 2: $CP$-odd graphs giving rise to a scalar part in the coupling $J^0 - f - f$.

Fig. 3: Exclusion plots from astrophysical requirements. We have considered the values: $(s_{L}^\nu)^2 = 5.10^{-2}$ (solid line), $(s_{L}^\nu)^2 = 10^{-2}$ (dashed line), $(s_{L}^\nu)^2 = 10^{-3}$ (dot-dashed line). The area lying above of the curves is excluded by the restriction $g_{JNN} < 10^{-9}$. In addition, we assume that all heavy neutrino masses are approximately equal with $m_N$.

Fig. 4: Exclusion plots originating from the decays: $\mu \rightarrow J^0 e$ (solid line), $\tau \rightarrow J^0 e$ (dashed line), $\tau \rightarrow J^0 \mu$ (dot-dashed line). For comparison, we have considered the astrophysical bound $g_{JNN} \leq 10^{-9}$ (dotted line). The areas lying above of the curves are excluded by the afore-mentioned conditions.