Scaling of critical connectivity of mobile ad hoc communication networks

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In this paper, critical global connectivity of mobile ad hoc communication networks (MAHCN) is investigated. We model the two-dimensional plane on which nodes move randomly with a triangular lattice. Demanding the best communication of the network, we account the global connectivity \( \eta \) as a function of occupancy \( \sigma \) of sites in the lattice by mobile nodes. Critical phenomena of the connectivity for different transmission ranges \( r \) are revealed by numerical simulations, and these results fit well to the analysis based on the assumption of homogeneous mixing. Scaling behavior of the connectivity is found as \( \eta \sim f(R^3\sigma) \), where \( R = (r - r_0)/r_0 \), \( r_0 \) is the length unit of the triangular lattice and \( \beta \) is the scaling index in the universal function \( f(x) \). The model serves as a sort of site percolation on dynamic complex networks relative to geometric distance. Moreover, near each critical \( \sigma_c(r) \) corresponding to certain transmission range \( r \), there exists a cut-off degree \( k_c \) below which the clustering coefficient of such self-organized networks keeps a constant while the averaged nearest neighbor degree exhibits a unique linear variation with the degree \( k \), which may be useful to the designation of real MAHCN.

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Mobile ad hoc communication network (MAHCN) is a new sort of communication circumstance. It consists of many mobile nodes carrying out collective duty while nodes communicate with each other via wireless links. Neither central control authority nor intermediate services such as base stations for cellular mobile-phones exist in the network. Each of its nodes needs to relay packets within its limited transmission range for other participants in multi-hop edges. MAHCN changes its topology with time without prior notice since its nodes are free to move randomly. Therefore, to realize effective communication and to do moving jobs, it should self-organize into a dynamically stationary network by certain local protocols. The study of mobile ad hoc networks has attracted much attention recently due to their potential application in battlefield, disaster relief providing, outdoor assemblies and other settings with temporal, inexpensive usage. Tens of protocols have been proposed by designers. However, investigation on property of the connectivity as viewed from statistical physics is still inadequate, which motivates the work in the present paper.

The theory of complex networks can provide powerful tools to investigate the mobile ad hoc networks. Xie et al. analyzed the formation of complex networks involving both geometric distance and topological degree of nodes. Sarshar et al. noticed that while new nodes are added to the existing network, other nodes might leave the network rapidly and randomly. They presented results about the possible emergence of scale-free structure in ad hoc networks. However, they considered only static cases instead of analyzing the influence from the motion of nodes. Németh and Vattay studied the giant cluster of such networks, and pointed out that the giant component size in the percolation could be described by a single parameter—the average number of neighbors of nodes. On the other hand, models of percolation on networks were often employed to analyze spreading processes, especially epidemics with occupancy threshold \( p_c \) showing drastic transitions.

A communication network may deliver meaningful services only if the network is well connected, or at least has a vast subset that is connected. Therefore, one goal of studying the ad hoc network is to find out how the network can maintain its connectivity. Different from previous study, we demand global connection of all nodes in the MAHCN for the best communication, which means a stricter case than site percolation. For simplicity, we model the two-dimensional plane on which nodes move randomly as a triangular lattice with \( N \) vertices, so that we mimic round transmission ranges with discrete hexagons. We define the probability of global connection \( \eta \) as the ensemble average of \( n/n_0 \), where \( n \) is the number of moving nodes globally connected to the integrated network, and \( n_0 \) is the total number of them. Critical behavior of order parameter \( \eta \) is found to rely on both transmission range \( r \) and the occupancy \( \sigma \)(defined as \( n_0/N \)) of sites (i.e. vertices in the triangular lattice). Scaling behavior of the order parameter is verified in the form of \( \eta \sim f(R^3\sigma) \), where \( R \) is the reduced transmission range, and \( \beta \) is the scal-

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between any two sites means that a path exists from one by a common bond (edge) or two occupied blocks have neighbors when two occupied vertices are directly linked means a block[22]; the other means a vertex[23]. Obviously our knowledge, the term “site” can be used dually: one site occupancy will pass threshold of site percolation. To traditional percolation problems, continuously increasing consumption of whole the network. In the framework of the lattice by mobile nodes, while this changes the energy multi-hop linking is to increase the occupancy of sites on other averaged quantities.

To describe the case of self-organized communication of individual nodes, it is assumed that they move on a two-dimensional plane in a discrete way, and a simple dynamic ad hoc network model is proposed as follows. On the two dimensional triangular lattice(see Fig.1) of size \( L \), individual nodes are assumed to distribute randomly on the sites of it. In our work, the total number of the sites are chosen as \( N = L^2 \) with \( L = 200 \). And, under periodic boundary condition, we restrain the motion of nodes along edges between sites. At the initial time step, assign \( n_0 \) nodes to the sites in the triangular lattice randomly. Every site of such a lattice can be occupied by only one node or nothing. Considering the dynamic topology of the ad hoc network, we suppose that, at every time step, each node can move randomly in one of the six directions to its neighbor site if it is not occupied. Two nodes in the ad hoc network can communicate with each other if the distance between them is less than the minimum of their two transmission ranges[18, 19]. To simplify our study, we assume that all the mobile nodes have the same fixed transmission power[20, 21]. Therefore they all have equal transmission range \( r \) for valid communication of the whole network. The transmission range \( r \) is an important parameter in the designation of ad hoc networks, since it is vital to keep the network globally connected. Proper adoption of range \( r \) could minimize energy consumption since transmission power is proportional to the square of it. Note that neither self-loop nor multiple edge is allowed in the network: (1) A node should not communicate with itself; and (2) technically there is no sense to open another communication channel between any two nodes if they are already neighbors. The motion of nodes at a certain value of occupancy forms different configurations of site-occupation on the lattice, which serves as the ensemble for the calculation on the global connectivity(i.e. the probability of the global connection) and other averaged quantities.

An intuitive way to ensure the best communication by multi-hop linking is to increase the occupancy of sites on the lattice by mobile nodes, while this changes the energy consumption of whole the network. In the framework of traditional percolation problems, continuously increasing site occupancy will pass threshold of site percolation. To our knowledge, the term ”site” can be used dually: one means a block[22]; the other means a vertex[23]. Obviously our usage belongs to the later one. Two sites are neighbors when two occupied vertices are directly linked by a common bond (edge) or two occupied blocks have a common border. Therefore, an indirect connection between any two sites means that a path exists from one to the other neighbor by neighbor. In the present model, however, every node is located at the center of its own transmission range \( r = zr_0 \) (\( z \) is a positive integer, and \( r_0 \) is the length of an edge of any minimal triangle). All the other nodes at sites inside this circle connect with it directly. Taking the circle \( r = 2r_0 \) (blue in Fig.1) as an example, nodes 2, 3 and 4 inside the inscribed hexagon (orange in Fig.1) of it are direct neighbors of node 1. Therefore, neighbors here are determined by the transmission range and in the sense of topological connection, just as what occurred in complex network models[10, 11], which distinguishes them from those in traditional two-dimensional site percolation. It is a special case of topological correlation valid within certain geographic distances[24, 25, 20, 27, 28]. The triangular lattice in our model is a beneficial setting to describe moving nodes and for discrete calculation of network parameters. Moreover, in percolation problem one always focuses on the probability for a site to be included in the giant component which just extends from a border to its opposite one in a lattice with finite size. However, for an ad hoc network bearing search, rescue, tracing or precise attack, the task may concentrate on a few, even a single moving target. It may demand global connection of all nodes, which is quite different from the percolation problem which leaves many nodes scattering outside the giant component.

The order parameter \( \eta \), i.e. the global connectivity of the MAHCN, is calculated with burning algorithm[29]. The evolution of it should rely on the occupancy \( \sigma \) of the sites, and \( \eta \) enhances when \( \sigma \) increases. The ratio of
the enhancement depends on the number \( n \) of the nodes which have been connected into the largest dynamic network at that time step, and it also depends on the number of the disconnected nodes, based on the assumption of homogeneous mixing\([30]\) of randomly moving nodes. Therefore, we have

\[
\frac{dn}{d\sigma} \propto n(n_0 - n)
\]

where \( n_0 \) is the total number of mobile nodes. Using the definition of \( \eta \) and getting the effect of the transmission range included, we arrive at

\[
\frac{d\eta}{d\sigma} = g(r)\eta(1 - \eta)
\]

where \( g(r) \) is the function of transmission range \( r \). This equation can be solved with the uniform initial condition \( \eta(\sigma \rightarrow 0) = \eta_0 \):

\[
\eta(\sigma) = \frac{\eta_0}{\eta_0 + (1 - \eta_0)e^{-g(r)\sigma}}
\]

In fig. 2, simulation results for the global connectivity as a function of the occupancy of the sites on the triangular lattice are illustrated. They are in good agreement with the analytical result of eq.(3) under the condition of dimensionless function \( g(r) \sim r/r_0 \). Actually, it is naturally expected by dimension analysis on the exponent in the denominator: \( g(r) \) should have no dimension since occupancy \( \sigma \) is dimensionless. The difference between simulation and analytical results at bottom parts can be attributed to the deviation from homogeneous assumption by the distribution of nodes at discrete sites on the triangular lattice, and size effect.

Numerical results display the critical behavior of global connectivity, i.e., they show its drastic transitions occurring at critical values \( \sigma_c \) for different transmission ranges. When occupancy \( \sigma \) passes \( \sigma_c \), the dynamically moving nodes self-organize from a disconnected state to a surely globally connected one. For our triangular lattice model, \( \sigma_c = 0.37, 0.21, 0.13, 0.09 \) and 0.065 for \( r = 2r_0, 3r_0, 4r_0, 5r_0 \) and \( 6r_0 \), respectively. Obviously, various critical values of node occupancy \( \sigma_c \) are required to ensure global connection for different transmission ranges in MAHCN, which means that we can also inversely choose proper transmission range to minimize energy consumption of the network for different density of nodes on the lattice.

It is natural to rescale \( \eta(\sigma, r) \) into a universal scaling function from direct observation of Fig. 2. we have

\[
\eta \sim f(R^3\sigma)
\]

with reduced transmission range \( R = (r-r_0)/r_0 \). the scaling index \( \beta \) of the universal function \( f(x) \), respectively. We draw transition curves in Fig.3 to show rescaling process. We can see calculated curves for all \( r \) collapse into the one with \( r = 2r_0 \), and the index \( \beta = -0.49 \) gives perfect convergence of all the curves. This provides the evidence that the transitions at \( \sigma_c(r) \) are really critical phenomena. The inset of Fig.3 shows \( \eta \) versus \( \sigma \) for different sizes of MAHCN with \( r = 2r_0 \). The critical value \( \sigma_c \) is independent of the sizes of lattices, which is also valid for different transmission ranges. Near critical points \( \sigma_c(r) \), nodes with autonomic communication self-organize into time-varying complex networks which are reminiscent of directed dynamic small-world network (DDSWN) model\([31]\) but with different scaling variables. Indeed, the ratio of the number of nodes receiving message to the total number of nodes should vary in the same way as that model provided the nodes move at the same speed, have uniform transmission range and relay message without delay, which will be discussed under another title. But in the present work we check global connectivity with burning algorithm, assuming that "combustion" (similar to message spreading out) is much faster than variation of topological structure. Moreover, the feature of transmission range-dependence distinguishes itself from DDSWN model. It is also noticeable that Hu and Chen\([32]\) investigated scaling functions for bond random percolation on honeycomb lattices with different aspect ratios. By comparison, the present model is pertaining to maximally connected network checked with combustion algorithm on a triangular lattice although we always pay attention to the hexagonal cell within the circular communication range.

When the occupancy of nodes just exceeds the value
of \( \sigma \) for certain transmission range \( r \), they are found to self-organize into a dynamically stationary network with our simulations. We can characterize the connection of an ad hoc network with parameters of complex networks. The simplest and the most intensively studied parameter is degree distribution \( p(k) \) because it governs fundamental properties of the system. Degree \( k \) of a node, as well known, is the total number of its topological edges connecting with others. The dispersion of node degree is characterized by the distribution function \( p(k) \) which gives the probability that a randomly selected node has exactly \( k \) edges. In the present paper, we study parameters of the network when it consists of almost all the nodes of MAHCN together. The degree distribution \( p(k) \) follows Poisson distribution which is different from that in ref.6,7,8. Fig.4 shows the ensemble averaged degree distributions for all simulated transmission ranges \( r \) when the global connectivity of nodes is above 0.9995. A cut-off degree \( k_c = 17 \) appears in it, which means that the probability for any node to have degree \( k > k_c \) is very low. The averaged degree of the whole network can be obtained from direct observation of Fig.1, that is,

\[
<k> = 3r(r + 1)\sigma
\]

where \( r = 2r_0 \) as mentioned above. We recognize MAHCN from this kind of Poisson-like distribution as random networks or small-world networks. By the way, the global connectivity \( \eta \) is also a single-variable function of average degree \( <k> \) for certain \( r \), which is qualitatively like the behavior of component size \( S \) in ref.4 since \( \sigma \) is proportional to \( <k> \) in such cases.

Clustering coefficient \( C_i \) and \( k_{nn} \), the averaged nearest neighbor degree of nodes depicts complex networks as viewed from correlations. For a node \( i \), clustering coefficient \( C_i \) can be defined as the fraction of pairs of node \( i \)'s neighbors that are also neighbors of each other in the topological sense. \( C(k) \) of the network is the clustering coefficient averaged over nodes with the same value of degree \( k \). In our case, \( C(k) = 0.52, 0.55, 0.56, 0.57, \) and 0.57 for transmission range \( r = 2r_0, 3r_0, 4r_0, 5r_0 \) and \( 6r_0 \), respectively, and keep invariant for \( k \leq k_c \). The \( k \)-independent behavior of \( C(k) \) can be understood from the symmetry of hexagonal cells and homogeneous distribution. Let us scrutinize the hexagonal cell (orange in Fig.1) for \( r = 2r_0 \) as an example. Site 1 in it has 18 neighbors (assuming full occupation). Therefore, the largest possible number of closed topological triangles in the sense of complex network should be \( C_{18} \) which serves the denominator of the clustering coefficient of it. Linking occupied sites 1 and 2, we search for the third one with the distance less than \( 2r_0 \) to both of them in the hexagonal cell, which makes 7 triangles. Linking occupied sites 3 and 4 to the center 1, makes 8 and 12 triangles, respectively. Therefore, all equivalent sites \(((7 + 8 + 12) \times 6/2)\) in the cell make 81 closed triplets under the constraint of maximum distance \( 2r_0 \), and yields the clustering coefficient of the site as \( 81/C_{18}^2 = 0.53 \). Under the assumption of homogeneous mixing, this is also valid for any occupancy or averaged \( <k> \) since we only need to multiply both the numerator and the denominator of it by local \( \sigma \) simultaneously, so that we have the same \( C(k) \) for small \( k \leq k_c \). The horizontal line for \( k \leq 17 \) (see Fig. 5) of \( C(k) \) indicates particular constant clustering coefficient of MAHCN for degrees occurring in high possibilities. Here value \( k_c \) reflects the limit case of connection, and it may be pertinent to the structure of the hexagonal cell imbedded in the triangular lattice. The high value(above 0.5) of \( C(k) \) implies that we have dynamic small world networks at critical points, and there is large redundancy in communication if only strategy of increasing occupancy of sites is adopted.
FIG. 5: (Colour online) The clustering coefficient $C(k)$ for different transmission ranges $r$ on the triangular lattice.

The averaged nearest neighbor degree of node $i$ is simply: $k_{nn,i} = \sum_j k_j / k_i$, where $j$ is a neighbor of node $i$. And $k_{nn}(k)$ can be accounted as the function of degree $k$ in the following form: $k_{nn}(k) = \sum_{k_i \in V} k_{nn,i} / \sum_{k_i \in V} 1$, where $V$ is the subset of nodes with the same degree $k$. From figure 6 we can see that the curve of $k_{nn}$ versus $k$ suggests an empirical formula in the linear form for our MAHCN, that is

$$k_{nn}(k) = b(r) + C(k)k$$  \hspace{1cm} (6)

where $b(r)$ is a $k$-independent constant. The positive assortativity (i.e. increase behavior of $k_{nn}(k)$) gives the particular feature distinguishing it from most other technical networks\cite{33, 34, 35}. The appearance of tails in large degrees ($k > 17$) (see Fig.5 and Fig.6) is due to very low probabilistic occurrence in the simulation on 300 realizations.

Interestingly, the model can be thought as a special example for the process of coevolutionary competitive exclusion\cite{36}: taking sites as the state variable of moving nodes, each node tends to compete for links to others, which is demanded by the global connection. Only nodes within other ones’ transmission ranges and with the degrees of both ones less than $k_c$ can successfully gain links, which constitutes threshold conditions of the co-evolution network.

In summary, we present a model for mobile ad hoc communication networks by considering uniform transmission range of nodes and assigning moving nodes randomly on the plane of the triangular lattice. Demanded by the best communication, critical global connectivity is found from simulations for various transmission ranges by adjusting the node occupancy of sites on the lattice. The order parameter scales with transmission ranges, but behaves differently from other models. This forms a kind of percolation in dynamic complex networks pertaining to geographic distance. Moreover, cut-off degree $k_c$, invariant clustering coefficient and linear assortativity as functions of degree are found for self-organized complex networks near critical global connectivity. Our model suggests that transmission range of nodes and average occupancy on the plane should adapt to each other to balance minimization of energy consumption with the global connectivity. In fact, nodes relaying message consume energy continuously. Transmission range of each node usually reduces with time relating to its job load. Therefore, the present model is applicable only for the routing strategy of broadcasting\cite{37}. A more practical model should include random distributed nodes with changing transmission ranges. Meanwhile, nodes are not necessary to move along edges of the triangular lattice. Therefore, a random graph model without any lattice is necessary for better investigation on mobile ad hoc communication networks, which leaves our further work in the future.

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\footnote{1} IETF Mobile Ad-hoc Networks Working Group, \url{http://www.ietf.org/html.charters/manet-charter.html} \footnote{2} Wireless Ad Hoc Networks Bibliography,
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