Cosmological implications of Primordial Black Holes

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Abstract. The possibility that a relevant fraction of the dark matter might be comprised of Primordial Black Holes (PBHs) has been seriously reconsidered after LIGO’s detection of a $\sim 30 M_\odot$ binary black holes merger. Despite the strong interest in the model, there is a lack of studies on possible cosmological implications and effects on cosmological parameters inference. We investigate correlations with the other standard cosmological parameters using cosmic microwave background observations, finding significant degeneracies, especially with the tilt of the primordial power spectrum and the sound horizon at radiation drag. However, these degeneracies can be greatly reduced with the inclusion of small scale polarization data. We also explore if PBHs as dark matter in simple extensions of the standard $\Lambda$CDM cosmological model induces extra degeneracies, especially between the additional parameters and the PBH’s ones. Finally, we present cosmic microwave background constraints on the fraction of dark matter in PBHs, not only for monochromatic PBH mass distributions but also for popular extended mass distributions. Our results show that extended mass distribution’s constraints are tighter, but also that a considerable amount of constraining power comes from the high-$\ell$ polarization data. Moreover, we constrain the shape of such mass distributions in terms of the correspondent constraints on the PBH mass fraction.
1 Introduction

The concept of Primordial Black Holes (PBHs) was introduced in the sixties [1], and subsequently it was suggested that they might make up the dark matter [2]. However, increasingly stringent constraints (see e.g. [3–6]) gave way to theories proposing elementary particles as dark matter. Of the latter, the most popular theory is the Weakly Interactive Massive Particles (see e.g., [7]). Nonetheless, the fact that WIMPs are still undetected while experiments are reaching the background sensitivity [8] joint with the LIGO+VIRGO collaboration’s [9] first detection of gravitational waves emissions from $\sim 30M_\odot$ binary black hole merger, make it timely to reconsider PBHs abundance constraints (see e.g., [10–12]).

It is important to bear in mind that even without accounting for a relevant fraction of the dark matter, the existence of PBHs might be a possible solution for other astrophysical open questions. For instance, PBHs might be the progenitors of the super massive black holes located at the nuclei of galaxies (e.g., [13, 14] and references therein) or the intermediate massive black holes that could inhabit the center of dwarf galaxies (e.g., [15] and references therein).

The abundance of PBHs (and hence the fraction of the total dark matter that they constitute, $f_{\text{PBH}}$) is constrained by several independent observations in a wide range of masses, with present datasets and new observables suggested for future experiments (see e.g. [16–26]). For stellar masses, existing constraints include microlensing by compact objects with masses $\lesssim 10M_\odot$ [27–31], wide binaries disruption [32] or stellar distribution in ultra-faint dwarfs galaxies [33] at slightly larger masses. There are also constraints on $f_{\text{PBH}}$ on this mass range from X-ray and radio observations of the Milky Way [34], although they depend strongly on astrophysical assumptions. In addition to astrophysical observables, the presence of PBHs has also consequences on cosmological observables such as the Cosmic Microwave Background (CMB).

The basic mechanism behind the effects that a PBH population has on the CMB is the following. PBHs accrete primordial gas in the early Universe and inject energy into the primordial plasma via radiation. Therefore, the Universe’s thermal and ionization histories are affected by the presence of PBHs, leaving potentially detectable signatures in the CMB. Given that the medium has more energy because of the PBH energy injection, recombination is delayed. This shifts the acoustic peaks and affects some physical quantities (e.g., the sound horizon at radiation drag, $r_s$). To summarize, the imprints are similar to those imposed by the energy injection of exotic species such as dark matter decaying into photons [35].

Recently, Ali-Haïmoud & Kamionkowski [36] (hereafter AHK) rederived constraints from CMB power spectra and spectral distortions using the recently released Planck power spectra [37], assuming spherical accretion and considering two limiting cases of ionization mechanisms (collisional ionization and photoionization). Compared to the previous analysis, AHK generalize the radiative efficiency
computation accounting for Compton drag and cooling by CMB photons, as well as ionization cooling once the gas is neutral, and use a more precise estimate of the relative velocity between PBHs and baryons. All this leads to significantly smaller accretion rates and PBH luminosities than previous analyses, therefore weakening constraints with respect to previous studies [38]. These updated results leave a window open for PBHs to be the dark matter ($f_{\text{PBH}} = 1$), precisely for masses of tens of $M_\odot$. In contrast to the standard cosmological model, $\Lambda$-Cold Dark Matter ($\Lambda$CDM), hereafter we refer to a $\Lambda$CDM model where a significant fraction of the dark matter is PBH as $\Lambda$PBH. Recently, the authors of [39] revisited CMB constraints assuming disk accretion. They find tighter constraints, which close the mentioned window and exclude the possibility that PBHs of tens of $M_\odot$ account for $f_{\text{PBH}} \gtrsim 0.1$. This would rule out the $\Lambda$PBH model. However, both spherical accretion and disk accretion for all the PBHs are limiting cases. It is therefore reasonable to expect that a realistic scenario would be in between these (Refs.[36] and [39]) limiting cases.

Most of the constraints mentioned above are derived assuming a Dirac delta function for the mass distribution (i.e., monochromatic distribution). While being a good approximation as a first step, this is an idealized case. Extended mass distributions (EMDs) appear naturally in formation mechanisms such as the collapse of large primordial fluctuations [40] or cosmic strings [41], or cosmological phase transitions, like bubble collisions [42], among others [43]. Besides, critical collapse [44] broadens any distribution, even if it is nearly monochromatic, making an EMD for PBHs unavoidable [45]. This effect has been studied numerically in [46–48], showing that it applies over ten orders of magnitude in density contrast (see e.g. figure 1 of [47]).

Given the wide variety of EMDs for PBHs, it is impossible to explore them all in detail in a cosmological context. The authors of [22, 23, 49] have proposed approaches to translate monochromatic constraints to EMDs; for a discussion of the advantages and limitations of these approaches see the above references and [50]. Constraints for simple EMDs have been derived exactly for microlensing observations [51].

Recent works demonstrate that it is possible to interpret very accurately the effects of a population of PBHs with an EMD in each different cosmological probe as a monochromatic population with a corresponding effective equivalent mass. The approach is physically motivated as it accounts for the underlying physics of the effects of PBHs [50]. This is the approach we follow here. We refer the reader to Ref. [50] for more details and advantages compared to other approaches.

Although recently there has been a renewed effort to revisit PBHs constraints, there is still a lack of studies on PBHs cosmological implications and their possible correlations with other cosmological parameters. Given the persistent tensions that exist among some observations at high and low redshift within $\Lambda$CDM, it is worthwhile to explore the possibility that the inclusion of PBHs in the model might reconcile these tensions. Using the formalism described in AHK, we study the impact of a large PBH mass fraction on CMB-derived cosmological parameters, for the standard six parameters of $\Lambda$CDM and for common extensions to this model. We also compute constraints on $f_{\text{PBH}}$ for PBHs with EMDs and test the prescriptions of [50] for the effective equivalent mass for the CMB.

This paper is structured as follows. In Section 2 we introduce the observational data we use and our methodology. Results are presented in Section 3: the results for monochromatic distributions are shown in Section 3.1 and those for EMDs, in Section 3.2. Finally, discussion and conclusions can be found in Section 4.

## 2 Methodology and Data

We use the public Boltzmann code CLASS [52, 53] using the modified version of HyRec [54, 55] introduced in AHK. This modification allows us to compute CMB power spectra accounting for effects due to a monochromatic mass distribution of PBH with mass $M$ and fraction $f_{\text{PBH}}$. We further modify this code to allow also for a variety of PBH EMDs following the prescriptions suggested in AHK.

We use the Markov Chain Monte Carlo (MCMC) public code Monte Python [56] to infer cosmological parameters using the observational data described below. We use uniform priors for $f_{\text{PBH}}$ in the range $0 \leq f_{\text{PBH}} \leq 1$, which is motivated by the linear dependence of the differences in the CMB
power spectra on $f_{\text{PBH}}$, shown in AHK. We also consider logarithmic priors finding that results do no change significantly.

We consider the full Planck 2015 temperature (TT), polarization (EE) and the cross correlation of temperature and polarization (TE) angular power spectra [37], corresponding to the following likelihoods: Planck high $\ell$ TTTEE (for $\ell \geq 30$), Planck low $\ell$ (for $2 \leq \ell \leq 29$) and the lensing power spectrum (CMB lensing). The Planck team identifies the low $\ell$ + high $\ell$ TT as the recommended baseline dataset for models beyond $\Lambda$CDM and the high $\ell$ polarization data as preliminary, because of evidence of low level systematics ($\sim \mu K^2$ in $(\ell + 1)C_\ell$) [57]. While the level of systematic contamination does not appear to affect parameter estimation, we present results both excluding and including the high $\ell$ polarization data. Hereinafter we refer to the data set of Planck high $\ell$ TTTEEE, Planck low $\ell$ and CMB lensing as “Full Planck” (or full for short) and we refer to the Planck recommended baseline as “PlanckTT+lowP+lensing” (or baseline).

The approach we follow builds on AHK work and is similar in spirit. In that work, the authors use the Plik lite best fit $C_\ell$, a provided covariance matrix for the high $\ell$ CMB-only TTTEEE power spectra, and a prior on the optical depth of reionization, $\tau_{\text{reio}}$, from [58]. Note that such covariance matrix is computed for a $\Lambda$CDM model without the presence of PBHs, so no correlation among $f_{\text{PBH}}$ and the standard cosmological parameters is considered. One difference comes from the fact that, while AHK use a Fisher matrix approach to estimate parameters constraints, we use MCMC for parameter inference. The price of being more rigorous comes at a considerable increase in the computation time of the analysis. Computing the constraints on $f_{\text{PBH}}$ for all masses would take an unpractical amount of time. However, given that the dependence of the upper limits on $f_{\text{PBH}}$ on $M_{\text{PBH}}$ is smooth, we choose $M_{\text{PBH}}$ values equally spaced in logarithmic scale and interpolate the constraints in between. With this procedure, for any $M_{\text{PBH}}$ in the interval covered by the sampling, we have an estimated value for the 95% and 68% confidence limit on $f_{\text{PBH}}$. This correspondence will be useful in Sec. 3.2 and Figs. 7 and 8.

2.1 Accounting for EMDs

To consider the effect of an extended mass distribution (EMD), rather than exploring the whole parameter space of all possibilities, we follow the Bellomo et al. [50] prescriptions to interpret EMDs as monochromatic populations with an effective equivalent mass, $M_{\text{eq}}$, which depends on both the shape of the EMD and the observable considered. Here we just briefly introduce the formalism for the CMB and refer the interested reader to Ref. [50] for a full explanation.

For an EMD, we define the PBH mass fraction as:

$$\frac{df_{\text{PBH}}}{dM} = f_{\text{PBH}} \frac{d\Phi_{\text{PBH}}}{dM},$$

(2.1)

in such a way that $\frac{d\Phi_{\text{PBH}}}{dM}$ is normalized to unity. We consider two popular mass distributions: a power law (PL) and a lognormal (LN). The former can be expressed as:

$$\frac{d\Phi_{\text{PBH}}}{dM} = N_{\text{PL}}(\gamma, M_{\text{min}}, M_{\text{max}}) \frac{\Theta(M - M_{\text{min}})\Theta(M_{\text{max}} - M)}{M^{1-\gamma}},$$

(2.2)

characterized by the exponent $\gamma$, a mass range $(M_{\text{min}}, M_{\text{max}})$ and a normalization factor

$$N_{\text{PL}}(\gamma, M_{\text{min}}, M_{\text{max}}) = \begin{cases} \gamma M_{\text{max}}^{\gamma} - M_{\text{min}}^{\gamma}, & \gamma \neq 0, \\ \log^{-1} \left( \frac{M_{\text{max}}}{M_{\text{min}}} \right), & \gamma = 0, \end{cases}$$

(2.3)

where $\gamma = -\frac{2w}{1+w}$, $w$ being the equation of state when PBHs form. If an expanding Universe is assumed ($w > -1/3$), $\gamma$ spans the range ($-1, 1$). We consider two cases of power law distribution: one with $\gamma = 0$ (corresponding to PBHs formed in matter domination epoch) and another one with $\gamma = -0.5$ (corresponding to PBHs formed in radiation dominated epoch).
Lognormal mass distributions can be expressed as:

\[
\frac{d\Phi_{PBH}}{dM} = \frac{1}{\sqrt{2\pi\sigma^2 M^2}} e^{-\log^2(M/\mu)/2\sigma^2},
\]

where \(\mu\) and \(\sigma\) are the mean and the standard deviation of the logarithm of the mass.

In order to account for an EMD, the energy density injection rate has to be integrated over the whole mass range spanned by PBHs:

\[
\dot{\rho}_{\text{inj}} = \rho_{dm} \int dM \frac{d\Phi_{PBH}}{dM} \frac{\langle L(M) \rangle}{M},
\]

where \(\langle L(M) \rangle\) is the velocity-averaged\(^1\) luminosity of a PBH. Starting from the results of AHK, it is possible to estimate the mass dependence of the integrand in Eq. (2.5) as:

\[
\langle L \rangle \propto \frac{\dot{M}^2}{L_{\text{Edd}}} \propto \frac{M^4 \lambda^2(M)/M}{M^2} = M^2 \lambda^2(M),
\]

where \(\lambda\) is the dimensionless accretion rate. In principle the averaged luminosity will depend not only on the mass but also on redshift, gas temperature, free electron fraction and ionization regime. Ref. [50] assumes that these dependencies can be factored out by parametrizing the dimensionless accretion rate as \(\lambda(M) = M^{\alpha/2}\), where \(\alpha\) is a parameter tuned numerically a posteriori to minimize differences in the relevant observable quantity between the EMD case and the equivalent monochromatic case.

Therefore, the effects of an EMD on the CMB power spectra are mimicked by a monochromatic distribution of mass \(M_{\text{eq}}\) when:

\[
f_{\text{MMD}}^{\text{EMD}} M_{\text{eq}}^{2+\alpha} = f_{\text{PBH}}^{\text{EMD}} \begin{cases}
N_{\text{PL}} \frac{M_{\text{max}}^{\gamma+2+\alpha} - M_{\text{min}}^{\gamma+2+\alpha}}{\gamma + 2 + \alpha}, & \text{PL}, \\
\mu^{2+\alpha} e^{(2+\alpha)^2 \sigma^2/2}, & \text{LN},
\end{cases}
\]

where \(f_{\text{PBH}}^{\text{EMD}}\) is the PBH mass fraction associated with a monochromatic distribution. In order to obtain the effective equivalent mass, we fix \(f_{\text{PBH}}^{\text{EMD}} = f_{\text{PBH}}^{\text{EMD}}\). This way, the upper limit on \(f_{\text{PBH}}^{\text{EMD}}\) is the corresponding upper limit for a monochromatic distribution with the effective equivalent mass.

As the authors of [50] indicate, there is no single choice of the parameter \(\alpha\) in Eq. 2.7 able to correctly match the time dependent energy injection of the EMDs with the monochromatic case. Since we require to select one effective value for \(\alpha\), the relation between the “best” value of \(\alpha\) and the redshift range where the match is optimised depends also on the EMD. Given that Planck power spectra have the smallest error bars for \(10^2 \lesssim \ell \lesssim 10^3\), we choose to select \(\alpha\) by minimizing the differences in the CMB power spectra in that multipole range. Although we expect that different values of \(\alpha\) do not have great impact in the performance of the method, we use \(\alpha = 0.3\) for Power Law distributions, and \(\alpha = 0.2\) for Lognormal distributions [50].

In principle, our effective treatment of the EMD (Eq. 2.7) and related results strictly applies only for \(M < 10^4 M_\odot\). This is because AHK modelling includes a steady-state approximation to avoid time dependent fluid, heat and ionization computations. This approximation holds when the characteristic accretion timescale is much shorter than the Hubble timescale, which is fulfilled for PBHs with \(M \lesssim 3 \times 10^4 M_\odot\) [50]. Since AHK formalism breaks down for \(M > 10^4 M_\odot\), Eq. (2.7) is valid only for EMDs which do not have significant contributions beyond that limit.

2.2 Important considerations

Several considerations are in order before we present our results and attempt their interpretation. First of all, the treatment of the effects of PBHs on the CMB (adopted from AHK) only considers

\(^1\)Recall that the accretion rate depends, among others, on the relative velocity PBH-gas.
changes in the ionization and Hydrogen thermal history via energy injection due to PBHs accretion. It does not consider any changes in the background history of the Universe. These changes would be mainly introduced because of the energy transfer from the matter component to the radiation one through black hole mergers, for instance. However there are several indications that these processes are not important in this context. In fact Ref. [60] estimated that no more than 1% of the dark matter can be converted into gravitational waves after recombination; hence this mechanism cannot provide a significant “dark radiation” component affecting the early time expansion history and cosmological parameters estimation. It cannot be invoked therefore to reduce the tension between the inferred value of $H_0$ obtained using CMB observations and assuming $\Lambda$CDM [37] and its direct measurement coming from the distance ladder [61]. However, black hole mergers could affect the expansion history before recombination in a similar way than $N_{\text{eff}}$. This effect would further affect recombination quantities, such as $r_s$, which also affects the inferred value of $H_0$ [62, 63]. This avenue is left to study in future work.

Energy transfer from matter to radiation sectors is not the only effect of PBH mergers. When two PBHs merge, the final state is a single black hole with larger mass. Thus the merger history of the PBH population changes the initial EMD predicted by inflation theories. The extent of this effect depends on the merger rate which in turn depends on the initial clustering and mass and velocity distributions. Given current theoretical uncertainties and observational precision, neglecting changes in the EMD through time is not expected to bias current constraints. However, this may need to be accounted for in the future.

As PBHs behave like a cold dark matter fluid at sufficiently large scales, they only affect cosmological observables via energy injection because of the accreted matter. Therefore, the only other cosmological probe affected by PBHs is reionization. The rest of cosmological probes (i.e., cosmic shear, baryon acoustic oscillations, etc) will not be affected by a large $f_{\text{PBH}}$. For this reason we have not considered other cosmological probes here. In practical applications they could certainly be used to further reduce parameter degeneracies.

While we assume spherical accretion, in [39] disk accretion is assumed. As the radiative efficiency when a disk is formed is much larger than if the accretion is spherical, their constraints are much stronger than ours and those appearing in AHK. However, both scenarios are possible in the early Universe: whether the accretion is spherical or a disk is formed depends on the value of the angular momentum of the accreted gas at the Bondi radius compared with the angular momentum at the innermost stable circular orbit. Given that it is difficult to estimate the angular momentum of the gas accreting onto a PBH, the best approach is to consider these as two limiting cases. We refer the reader to Figure 3 and related discussion in Poulin et al. (2017) [39] in order to see the extent of the differences in the shape of the power spectra between both scenarios. Nonetheless, we expect that our results can be extrapolated to the disk accretion framework. Actually, the degeneracies between $f_{\text{PBH}}$ and other parameters are expected to be approximately the same in both cases.

Therefore, it should be clear from the above discussion that uncertainties in the modelling of processes such as accretion mechanisms of PBHs or velocity distributions imply that any constraint should be interpreted as an order of magnitude estimate, rather than a precise quantity. However, the behaviour of parameters degeneracies as a function of the different data sets considered or ionization regime should be qualitatively captured by our analysis. For this reason, we will also report the ratio between the constraints using two different data sets or ionization models.

As the equivalence relation between the effects of PBH with an EMD and a monochromatic distribution derived in [50] is obtained assuming the formalism of AHK, it is subject to the same caveats as AHK modelling. In particular given that AHK modelling breaks down for $M > 10^4 M_\odot$, Eq. (2.7) should be used strictly only for EMDs which do not have significant contributions beyond that limit. A criterion to decide if a EMD fulfill this condition can be found in Ref. [50]. Finally, while in principle the choice of the parameter $\alpha$ in Eq. 2.7 could be numerically optimized, the equivalent mass is relatively insensitive to small changes of its value for the EMDs considered here; therefore a slightly sub-optimal value of $\alpha$ does not bias our results.
3 Results

3.1 Constraints on $f_{\text{PBH}}$ for monochromatic distributions

We start by directly comparing our approach with that of AHK; we use the same data (Planck lite high TT TTEEE + prior in $\tau$) but a MCMC instead of a Fisher approach to estimate parameter constraints. The constraints we obtain are similar to those of AHK, with variations always below the 10% level, which is smaller than the theoretical uncertainty of the model. Although there is no significant effect introduced by using a Fisher approach, as we are interested in the posterior distribution in the whole parameter space, we use MCMC.

In Figure 1 we show the 68% confidence level upper limits on $f_{\text{PBH}}$ for monochromatic populations of PBHs with different masses for both the full Planck dataset (red) and removing the high multipoles of polarization power spectrum (blue). Note that this color coding is used in all the figures throughout the paper. We also show the results of AHK and the 95% confidence level upper limits obtained assuming disk accretion from [39]. The results using full Planck are very similar to those obtained in AHK. For the recommended baseline of Planck (which is the most conservative case) the constraints are weaker, widening the window where $f_{\text{PBH}} \sim 1$ is allowed. The maximum masses for which $f_{\text{PBH}} \sim 1$ is allowed (i.e., there is no upper limit at 68% confidence level) in each of the cases considered are reported in Tab. 1.

![Figure 1](image-url)

**Figure 1.** 68% confidence level marginalized constraints on $f_{\text{PBH}}$ in the collisional ionization regime (dotted lines) and in the photoionization regime (dashed lines). The results using the full data set of Planck are shown in red and without including the high multipoles of polarization, in blue. We also show, as a reference, the 68% confidence level marginalized constraints of AHK (green) and the 95% confidence level marginalized constraints of [39] (black), where disk accretion is assumed. These external results are obtained including the high multipoles of Planck polarization power spectrum.
Table 1. Maximum mass for which \( f_{\text{PBH}} \sim 1 \) is allowed at 68% confidence level for monochromatic distributions. We report the values for all possible combinations of data sets (either full Planck or baseline) and ionization regimes (either collisional ionization or photoionization). Note that (as it should be clear from the text and from Fig. 1) these values are approximated and limited by the discrete sampling in mass for which we compute the constraints.

| max. \( M \) with \( f_{\text{PBH}} \sim 1 \) | coll. Planck full | coll. baseline | photo. Planck full | photo. baseline |
|---|---|---|---|---|
| \( 30M_\odot \) | \( 300M_\odot \) | \( 10M_\odot \) | \( 30M_\odot \) |

Table 2. Ratio of 68% marginalized upper limits on \( f_{\text{PBH}} \). We compare the upper limits obtained using full Planck against using the baseline PlanckTT+lowP+lensing for the two ionization regimes and the constraints obtained assuming photoionization against assuming collisional ionization for both data sets. In all cases we report the mass range for which the correspondent ratio is valid.

| \( f_{\text{PBH}} \) ratio | Planck full/ baseline (coll) | Planck full/baseline (photo) | photo/coll (Full Planck) | photo/coll (baseline) |
|---|---|---|---|---|
| \( 0.11 \) | \( 0.11 \) | \( 5.2 \times 10^{-3} \) | \( 4.8 \times 10^{-3} \) |

3.1.1 Constraints and degeneracies with cosmological parameters in \( \Lambda \)CDM

In addition to the constraints on \( f_{\text{PBH}} \), we also study the degeneracies of this parameter with the six standard cosmological parameters of the \( \Lambda \)CDM model.

The most notable degeneracy, as already noticed in [38], is between \( f_{\text{PBH}} \) and the scalar spectral index \( n_s \). This positive correlation is induced by the energy injection of the PBH emission, which alters the tails of the CMB power spectra. As decoupling is delayed, the diffusion damping increases and the high multipoles are suppressed through Silk damping. In Figures 2 and 3, we show the results for the collisional ionization and photoionization limits, respectively. We report 68% and 95% confidence level marginalized constraints on the \( f_{\text{PBH}} \)−\( n_s \) (left) and \( f_{\text{PBH}} \)−\( r_s \) (right) planes, for different masses, as these are the most affected parameters; in general, degeneracies become stronger for larger masses. For some cases, a scale independent or even blue tilted primordial power spectrum is allowed. Combined analyses with galaxy surveys can be used to constrain \( n_s \), reducing the degeneracy; we will discuss this in more details elsewhere.

The \( f_{\text{PBH}} \)−\( r_s \) degeneracy is the result of two competing effects. On one hand, the presence of PBHs delays recombination and shifts the peaks of CMB power spectra (see Figure 13 of AHK). This yields a larger sound horizon at radiation drag, leading to a lower value of \( H_0 \) to fit the cosmic distance ladder of supernovae type Ia and Baryon Acoustic Oscillations (BAO), as shown in [62]. On the other hand, the effect of a delayed recombination on the value of \( r_s \) is compensated by small degeneracies in the density parameters of baryons and dark matter. Thus, a sizeable PBH component (with masses \( \gtrsim 300M_\odot \) for collisional ionization or \( \gtrsim 30M_\odot \) for photoionization) can change \( r_s \) by \( \sim 1 \)
Figure 2. 68% and 95% confidence level marginalized constraints on the $n_s$-$f_{\text{PBH}}$ plane (left) and $r_s$-$f_{\text{PBH}}$ plane (right) for $\Lambda$CDM with PBHs as part of the dark matter (i.e., free $f_{\text{PBH}}$) in the collisional ionization regime. We consider three monochromatic distributions: 170$M_\odot$ (top), 300$M_\odot$ (middle) and 520$M_\odot$ (bottom). Results using the full data set of Planck are shown in red and using the recommended baseline, in blue. 68% upper limits on $f_{\text{PBH}}$ correspond to the those shown in Figure 1. Note the change of scale in y-axis.
Figure 3. Same as Fig.2, but in photoionization regime. In this case we consider \(30M_\odot\) (top), \(100M_\odot\) (middle) and \(170M_\odot\) (bottom).

Mpc. This shift is of a magnitude comparable to other effects and, most importantly, non-negligible with current and future errors in distance measurements using the BAO scale.

The effect on the rest of ΛCDM parameters is smaller, mostly resulting in mild relaxations of
the confidence regions. Beyond that, only the amplitude of the scalar modes in the primordial power spectrum, $A_s$, presents a small positive correlation with $f_{\text{PBH}}$.\footnote{While in principle one may expect also a degeneracy with the optical depth to recombination $\tau_{\text{reio}}$, this is not noticeable due to the tight constraints on this parameter imposed by the low $\ell$ polarization data and because $f_{\text{PBH}}$ is positive definite.}

Perhaps unsurprisingly, the inclusion of the high $\ell$ polarization data effectively remove the degeneracies; the effect on the TT damping tail can be mimicked by changes of the cosmological parameters, but this is not anymore the case once also the high $\ell$ polarization is considered. It is well known that the high $\ell$ damping tail is affected by the additional physics introduced (e.g., if dark radiation is allowed and for decaying dark matter models).

### 3.1.2 Implications for extended cosmologies

Here we explore the consequences of varying $f_{\text{PBH}}$ in some common extensions of the $\Lambda$CDM model. In addition to an expected weakening of the constraints, we study the degeneracies with the additional parameters. We consider the following models: free equation of state of dark energy ($w$CDM), free sum of neutrino masses ($\Lambda$CDM+$m_\nu$), free effective number of relativistic species ($\Lambda$CDM+$N_{\text{eff}}$) and free running of the spectral index ($\Lambda$CDM+$\alpha_s$). Finally, we also allow the running of the running of the spectral index, $\beta_s$, to be a free parameter ($\Lambda$CDM+$\alpha_s+\beta_s$). We limit the study to the recommended baseline of Planck. Moreover, following the results from the previous section and in order to appreciate changes in $f_{\text{PBH}}$, we choose masses for which $f_{\text{PBH}} \sim 0.1$: 520$M_\odot$ for collisional ionization and 100$M_\odot$ for photoionization.

Figure 4 shows 68% and 95% confidence level marginalized constraints on the plane of $f_{\text{PBH}}$ and the additional parameters of the extended cosmologies. As it can be seen, there is a correlation between $m_\nu$ and $f_{\text{PBH}}$ and between $N_{\text{eff}}$ and $f_{\text{PBH}}$, and an anti-correlation between $\alpha_s$ and $f_{\text{PBH}}$, but no appreciable correlation with $w$. The degeneracies with $m_\nu$, $N_{\text{eff}}$ and $\alpha_s$ open the possibility that when limiting the theoretical framework to $\Lambda$CDM, one might hide the presence of PBHs. On the other hand, $f_{\text{PBH}} = 0$ is always within the 68% confidence region.

The positive correlation between $f_{\text{PBH}}$ and $N_{\text{eff}}$ can be explained as follows. On one hand, as PBH energy injection delays recombination, $r_s$ is higher (and CMB peaks are displaced towards larger scales). On the other hand, a value of $N_{\text{eff}}$ different than the fiducial value changes the expansion history, especially in the early Universe, hence $r_s$ is modified. Concretely, higher values of $N_{\text{eff}}$ derive on lower values of $r_s$ (and displacements of the CMB peaks towards smaller scales). This is why, if PBHs exist, larger values of $f_{\text{PBH}}$ need larger values of $N_{\text{eff}}$ to fit observations. Varying $m_\nu$ also changes the expansion history, the angular diameter distance to recombination and the redshift of equality, so the relation between $f_{\text{PBH}}$ and $m_\nu$ is similar to that explained above.

We also consider the case in which the running of the spectral index, $\alpha_s$, is scale dependent, hence there is an additional free parameter in this case: the running of the running, $\beta_s$, which we consider scale independent. As it can be seen in Figure 5, varying $\beta_s$ weakens the constraints on $\alpha_s$ and the parameters are highly correlated. What is more surprising is that for small values of $f_{\text{PBH}}$, $\beta_s$ is slightly different than 0. However, this trend is not strong enough to be statistically significant.

Although there is a strong correlation between $f_{\text{PBH}}$ and $n_s$, the correlations with $\alpha_s$ and $\beta_s$ are much smaller. This is due to the suppression of the CMB power spectra on the high $\ell$ tail produced by PBHs can be mimicked by a different constant value of $n_s$, without evidence of the need of a strong scale dependent variation.

Fig. 6 shows how the degeneracies in extended cosmologies weaken the constraints on $f_{\text{PBH}}$. Adding an extra cosmological parameter to the 6-$\Lambda$CDM ones, weakens the constraints on $f_{\text{PBH}}$ approximately by a factor 2, except for $w$CDM, due to the lack of correlation. The shift of the parameters $w$ and $\beta$ from their $\Lambda$CDM-fiducial values is responsible for making the $f_{\text{PBH}}$ constraints in the extended model slightly tighter.
Figure 4. 68% and 95% confidence level marginalized constraints on the plane $f_{PBH}$-$x$ ($x$ being the additional parameter to APBH) for $520M_\odot$ in the collisional ionization regime (left) and $100M_\odot$ in the photoionization (right) for the following extended cosmologies (form top to bottom): $\Lambda$PBH+$m_\nu$, $\Lambda$PBH+$N_{eff}$, $\Lambda$PBH+$\alpha_s$ and $w$PBH.
3.2 Constraints on PBHs with extended mass distributions

So far, we have considered monochromatic populations of PBHs. Although this is an interesting first step, a more realistic case involves a distribution of masses. We follow [50] as described in Section 2.1 and assess numerically the accuracy of the approximation in six specific cases: two Lognormal distributions and four Power Law distributions (two with $\gamma = 0$ and other two with $\gamma = -0.5$). For each EMD case, we consider a narrow and a wide distribution. Our aim is to test the approach of [50] by comparing the results obtained following their prescription with an exact calculation using MCMCs.

The resulting marginalized 68% and 95% confidence level upper limits on $f_{\text{PBH}}$ using full Planck for the six EMDs are shown in Table 3. In the second column we report results for a monochromatic distribution of $30 M_\odot$ and in the following columns we report constraints for extended mass distributions chosen to have an effective equivalent mass (according to Sec. 2.1) of $30 M_\odot$. The differences in the upper limits are well below the theoretical uncertainties of the model. These results demonstrates that, within the current knowledge of how an abundant population of PBHs affects the CMB, there is no difference between computing the constraints using the actual EMD and using an effective equivalent mass following the approach of [50]. In the same spirit, the monochromatic upper

![Figure 5](image-url)
Figure 6. Marginalized probability distribution of \( f_{\text{PBH}} \) obtained assuming the extended cosmologies and those obtained assuming \( \Lambda \text{CDM} \) using PlanckTT+lowP+lensing for collisional ionization and 520\( M_\odot \) (left) and photoionization 100\( M_\odot \) (right).

Table 3. Evaluation of the performance of the effective equivalent mass to match the results of accounting properly for EMDs. The result using the monochromatic approximation is shown in the second column (\( M=30M_\odot \)), results obtained using several extended distributions are shown in the following columns. We compare the 68\% and 95\% marginalized upper limits on \( f_{\text{PBH}} \) using full Planck and in the photoionization regime for six different EMDs with an equivalent mass of 30\( M_\odot \) and the correspondent monochromatic case. We consider lognormal distributions (LN) with \( \alpha = 0.2 \) and \( \{\mu, \sigma\} = \{22.8M_\odot, 0.5\} \) (1) and \( \{\mu, \sigma\} = \{10M_\odot, 1.0\} \) (2). We also consider power law distributions with \( \alpha = 0.3 \) and \( \gamma = 0 \) with \( \{\text{min} M, \text{max} M\} = \{1M_\odot, 82M_\odot\} \) (1) and \( \{\text{min} M, \text{max} M\} = \{0.01M_\odot, 114M_\odot\} \) (2), and \( \gamma = -0.5 \) with \( \{\text{min} M, \text{max} M\} = \{1M_\odot, 150M_\odot\} \) (1) and \( \{\text{min} M, \text{max} M\} = \{0.01M_\odot, 564M_\odot\} \) (2). Note how similar the constraints are and how accurate is the monochromatic approximation.

limits on \( f_{\text{PBH}} \) can be used to constrain the parameters of the EMDs considered here; i.e., \( \mu \) and \( \sigma \) for the Lognormal and \( \gamma \) for the Power Law (for which we consider different mass bounds). We use the marginalized 95\% confidence level upper limits of \( f_{\text{PBH}} \) and translate these into limits for the EMDs parameters as follows. For every choice of EMD and its parameters, we compute the effective equivalent monochromatic mass and look up the \( f_{\text{PBH}} \) value corresponding to the 95\% confidence upper limit (see Sec.2 and Fig. 1). The results for photoionization and collisional ionization limits using both full Planck and PlanckTT+lowP+lensing for a \( \Lambda \text{CDM} \) model are shown in Figure 7 for a Lognormal EMD and Figure 8 for Power Law EMDs. The black shaded region indicates parameters values for which the EMD extends significantly beyond the limit of 10\(^4M_\odot \) where the AHK formalism becomes invalid.

The above results can be understood as follows. If PBHs have an EMD, the largest contribution to the injected energy comes from the high mass tail [50]. Therefore for a large width of a Lognormal mass distribution, \( \mu \) is constrained to be small. In the case of the Power Law mass distribution, the flatter the distribution (i.e., the larger \( \gamma \)), the stronger the constraint on \( f_{\text{PBH}} \). In essence, this implies that the constraints on \( f_{\text{PBH}} \) for EMDs are stronger than for monochromatic distributions. Then, if PBHs are meant to be all the dark matter and have an EMD, this has to be peaked at \( M \lesssim 1 - 50M_\odot \), hence the allowed window shrinks.
Figure 7. 95% confidence level marginalized constraints on $f_{\text{PBH}}$ (in color code) for a Lognormal mass distribution as function of the two parameters of the distribution, $\mu$ and $\sigma$, in the collisional regime (upper panels) and in the photoionization regime (bottom panels). The black shaded region indicates the values of $\mu$ and $\sigma$ for which EMDs extends beyond $10^4 M_\odot$ (the ratio between the values of the distribution in $M = 10^4 M_\odot$ and in the peak is $10^{-5}$), masses for which AHK formalism breaks down and Eq. (2.7) is not valid. Left panels: Results obtained using full Planck. Right panels: results obtained without including the high $\ell$ of polarization.

From Eq. (2.7) and the results shown in Tab. 1 it is possible to define a region in the parameter space of the EMDs for which $f_{\text{PBH}} \sim 1$ is allowed (dark red in Figures 7 and 8). Let us define $M_{\text{lim}}$ to be the maximum mass of a monochromatic distribution for which there are no upper limits on $f_{\text{PBH}}$ (values reported in Tab. 1). We can then constrain the parameter space of EMDs by imposing $M_{\text{eq}} \lesssim M_{\text{lim}}$. For Lognormal distributions, there is an analytic solution:

$$\begin{cases} 
\mu \lesssim M_{\text{lim}}, \\
\sigma \lesssim \sqrt{\frac{2}{2+\alpha}} \log \left( \frac{M_{\text{lim}}}{\mu} \right).
\end{cases}$$

(3.1)

4 Discussion and conclusions

The presence of a population of PBHs constituting a large fraction of the dark matter (i.e., in what we call APBH model) would have injected radiation in the primordial plasma affecting the Universe’s thermal and ionization histories and leaving an imprint on e.g. the CMB. A robustly demonstrated absence of these signatures would put strong limits on the abundance of PBHs of masses $\gtrsim 10 M_\odot$.
Figure 8. 95% confidence level marginalized constraints on $f_{\text{PBH}}$ (in color code) for a Power Law mass distribution as function of two parameters of the distribution, $M_{\text{max}}$ and $\gamma$, for a fixed $M_{\text{min}} = 10^{-2}M_\odot$ in the collisional regime (upper panels) and in the photoionization regime (bottom panels). Left panels: Results obtained using full Planck. Right panels: results obtained without including the high $\ell$ of polarization.

and thus help to exclude a possible dark matter candidate. We find no evidence for the presence of such PBHs with an abundance large enough to be appreciable: $f_{\text{PBH}} = 0$ is always inside the 68% confidence region in all the cases considered. However, the sensitivity of present experiments is still not good enough to rule out PBHs in this mass range as a sizeable fraction of the dark matter. Moreover, on the theoretical side, uncertainties in the modelling of processes such as accretion mechanisms of PBH or velocity distribution imply that any constraint should be interpreted as an order of magnitude estimate, rather than a precise quantity.

In this work we use the formalism introduced in AHK [36], which assumes spherical accretion and two limiting scenarios in which either collisional or photoionization processes completely dominate. We build on AHK by performing a robust statistical analysis, considering extended mass distributions of PBHs and exploring cosmological parameter degeneracies and the cosmological consequences that a large $f_{\text{PBH}}$ would have on other parameters. Beside confirming AHK findings, we note that the current allowed window of $f_{\text{PBH}} \sim 1$ for monochromatic populations of PBHs with masses $\sim 10 - 100M_\odot$ greatly depends on the dataset considered (see Figure 1 and Tab. 1). A significant component of the constraining power of CMB observations comes from the high multipoles of polarization power spectrum: the marginalized constraints on $f_{\text{PBH}}$ are about ten times stronger when included.

For Planck’s recommended baseline data (i.e., not including the high multipoles of polarization power spectrum), CMB observations allow $f_{\text{PBH}} \sim 1$ for $M \lesssim 30M_\odot$ for the most stringent case (photoionization) and $M \lesssim 300M_\odot$ for the most conservative one (collisional ionization). The inclusion
of high ℓ polarization data shrinks significantly this window to $M \lesssim 10 M_\odot$ for photoionization and $M \lesssim 30 M_\odot$ for collisional ionization. The forthcoming release of Planck data with confirmation that the high ℓ polarization power spectrum is suitable to be used in extended cosmologies will greatly constrain the models for PBHs and their role as a dark matter component.

We have also investigated degeneracies between $f_{\text{PBH}}$ for PBH masses in the allowed ΛPBH model window of $\sim O(10) M_\odot$ to $\sim O(500) M_\odot$ and some cosmological parameters in a ΛPBH cosmology (Figures 2 and 3). As expected, we find a large correlation with $n_s$, which gets stronger for larger $M_{\text{PBH}}$, and allows even $n_s \approx 1$. Since future galaxy surveys will measure $n_s$ with astonishing precision, it will become possible to limit this degeneracy and set an indirect constraint on $f_{\text{PBH}}$. Interestingly, we find that, compared to the standard $f_{\text{PBH}} = 0$ model, a sizeable $f_{\text{PBH}}$ would yield a higher sound horizon at radiation drag, $r_s$. A joint analysis with BAO would yield a lower value for $H_0$. Therefore a ΛPBH Universe do not ease some of the existing tensions of ΛCDM (and in particular the $H_0$ one, see e.g., [62]), and possibly even worsen them.

However, a sizeable $f_{\text{PBH}}$, if ignored, would bias the determination of $r_s$ by $\sim 1$ Mpc. It is important to note that this shift is non-negligible compared to expected errors in the determination of the BAO distance scale, hence this possibility should be kept in mind when interpreting forthcoming BAO data.

We have shown that the ratios between marginalized upper limits of $f_{\text{PBH}}$ for two different data sets do not depend on the assumptions about the ionization regime. Moreover, the ratios between the constraints on $f_{\text{PBH}}$ for different ionization regimes do not depend on the inclusion of the high multipoles of polarization power spectrum (Tab. 2). This suggests that using the ratio between marginalized upper limits on $f_{\text{PBH}}$ it is possible to isolate effects of specific choices of modelling or data sets, hence our findings can be reinterpreted for different modelling approaches, such as the disk accretion modelling of [39].

If PBHs as dark matter are studied in the framework of popular extensions to ΛCDM, the allowed parameter space is extended because of degeneracies between $f_{\text{PBH}}$ and i.e., $N_{\text{eff}}$ or the neutrino mass (Figure 4). These degeneracies make it possible that PBH as dark matter might be hidden when assuming a ΛCDM background model, by forcing extended parameters to its fiducial values. The degeneracies with the parameters related with the neutrino sector can be explained by the fact that exotic neutrino physics would change the expansion history of the early Universe, therefore changing the epoch of recombination. A larger $f_{\text{PBH}}$ requires a larger $N_{\text{eff}}$, which yields a higher $H_0$ CMB-inferred value when ΛCDM+$N_{\text{eff}}$ is assumed, as can be seen in Figure 9. This could make the CMB inferred value of $H_0$ fully compatible with direct measurements [62, 64–66].

![Figure 9](image)

*Figure 9.* 68% and 95% constraints on the plane $N_{\text{eff}}$-$H_0$ with a color code to express the value of $f_{\text{PBH}}$ for collisional ionization and $520 M_\odot$ (left) and for photoionization and $100 M_\odot$ (right) using the baseline Planck data set. The marginalized constraints on the plane $N_{\text{eff}}$-$f_{\text{PBH}}$ can be seen in figure 4.

Finally, we successfully test the approach proposed in [50] to convert constraints computed for monochromatic distributions into constraints for EMDs in the case of CMB observations. Given the
simplicity of the approach and the performance, we advocate using it to quickly get precise estimate for any EMD. We also present constraints on the properties of popular EMDs, as to being consistent with current CMB data.

Following this approach, we compute 95% confidence level upper limits on $f_{PBH}$ as function of the parameters of the EMDs (Figures 7 and 8). We find that CMB sets strong constraints on $f_{PBH}$ for EMDs that extends towards large masses. If one takes into account microlensing constraints (which constrain PBHs in the $M \lesssim 1 - 10 M_\odot$ range), $f_{PBH} \sim 1$ and thus a ΛPBH cosmology is allowed only for narrow EMDs and only if most conservative assumptions and data are considered.

We eagerly await the release of the final Planck high $\ell$ polarization data: in combination with other constraints it will be key to boost or rule out the hypothesis of PBHs as dark matter.

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References

[1] Y. B. Zel’dovich and I. D. Novikov, “The Hypothesis of Cores Retarded during Expansion and the Hot Cosmological Model,” Soviet Astronomy 10 (Feb., 1967) 602.

[2] G. F. Chapline, “Cosmological effects of primordial black holes,” Nature 253 (Jan., 1975) 251.

[3] C. Alcock, R. A. Allsman, and e. a. Alves, “EROS and MACHO Combined Limits on Planetary-Mass Dark Matter in the Galactic Halo,” Astrophys. J. Letter 499 (May, 1998) L9–L12, astro-ph/9803082.

[4] C. Flynn, A. Gould, and J. N. Bahcall, “Hubble Deep Field Constraint on Baryonic Dark Matter,” Astrophys. J. Letter 466 (Aug., 1996) L55, astro-ph/9603035.

[5] B. J. Carr and M. Sakellariadou, “Dynamical Constraints on Dark Matter in Compact Objects,” Astrophys. J. 516 (May, 1999) 195–220.

[6] P. N. Wilkinson, D. R. Henstock, I. W. Browne, A. G. Polatidis, P. Augusto, A. C. Readhead, T. J. Pearson, W. Xu, G. B. Taylor, and R. C. Vermeulen, “Limits on the Cosmological Abundance of Supermassive Compact Objects from a Search for Multiple Imaging in Compact Radio Sources,” Physical Review Letters 86 (Jan., 2001) 584–587, astro-ph/0101328.

[7] G. Jungman, M. Kamionkowski, and K. Griest, “Supersymmetric dark matter,” Phys. Rep. 267 (Mar., 1996) 195–373, hep-ph/9506380.

[8] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambreni, M. Pierre, S. Profumo, and F. S. Queiroz, “The Waning of the WIMP? A Review of Models, Searches, and Constraints,” ArXiv e-prints (Mar., 2017), arXiv:1703.07364 [hep-ph].

[9] LIGO Scientific Collaboration and Virgo Collaboration Collaboration, B. P. Abbott et al., “Observation of Gravitational Waves from a Binary Black Hole Merger,” Phys. Rev. Lett. 116 (Feb., 2016) 061102, 1602.03837.
[10] S. Bird, I. Cholis, J. B. Muñoz, Y. Ali-Haïmoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess, “Did LIGO Detect Dark Matter?,” Physical Review Letters 116 no. 20, (May, 2016) 201301, arXiv:1603.00464.

[11] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, “Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914,” Physical Review Letters 117 no. 6, (Aug., 2016) 061101, arXiv:1603.08338.

[12] S. Clesse and J. García-Bellido, “The clustering of massive Primordial Black Holes as Dark Matter: Measuring their mass distribution with advanced LIGO,” Physics of the Dark Universe 15 (Mar., 2017) 142–147, arXiv:1603.05234.

[13] K. Kohri, T. Nakama, and T. Suyama, “Testing scenarios of primordial black holes being the seeds of supermassive black holes by ultracompact minihalos and CMB µ distortions,” Phys. Rev. D 90 no. 8, (Oct., 2014) 083514, arXiv:1405.5999.

[14] J. L. Bernal, A. Raccanelli, J. Silk, E. Kovetz, and L. Verde, “Primordial Black Holes as Seeds of Super Massive Black Holes,” in prep (2017).

[15] J. Silk, “Feedback by Massive Black Holes in Gas-rich Dwarf Galaxies,” Astrophys. J. Letters 839 (Apr., 2017) L13, arXiv:1703.08553.

[16] A. Raccanelli, E. D. Kovetz, S. Bird, I. Cholis, and J. B. Muñoz, “Determining the progenitors of merging black-hole binaries,” Phys. Rev. D94 no. 2, (2016) 023516, arXiv:1605.01405 [astro-ph.CO].

[17] A. Raccanelli, “Gravitational wave astronomy with radio galaxy surveys,” Mon. Not. Roy. Astron. Soc. 469 no. 1, (2017) 656–670, arXiv:1609.09377 [astro-ph.CO].

[18] A. Raccanelli, F. Vidotto, and L. Verde, “Effects of primordial black holes quantum gravity decay on galaxy clustering,” 1708.02588.

[19] I. Cholis, E. D. Kovetz, Y. Ali-Haïmoud, S. Bird, M. Kamionkowski, J. B. Muñoz, and A. Raccanelli, “Orbital eccentricities in primordial black hole binaries,” Phys. Rev. D94 no. 8, (2016) 084013, arXiv:1606.07437 [astro-ph.HE].

[20] E. D. Kovetz, I. Cholis, P. C. Breysse, and M. Kamionkowski, “Black hole mass function from gravitational wave measurements,” Phys. Rev. D95 no. 10, (2017) 103010, arXiv:1611.01157 [astro-ph.CO].

[21] J. B. Muñoz, E. D. Kovetz, L. Dai, and M. Kamionkowski, “Lensing of Fast Radio Bursts as a Probe of Compact Dark Matter,” Phys. Rev. Lett. 117 no. 9, (2016) 091301, arXiv:1605.00008 [astro-ph.CO].

[22] B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen, and H. Veermäe, “Primordial black hole constraints for extended mass functions,” Phys. Rev. D 96 (Jul, 2017) 023514. 1705.05567.

[23] F. Kühnel and K. Freese, “Constraints on primordial black holes with extended mass functions,” Phys. Rev. D 95 no. 8, (Apr., 2017) 083508, arXiv:1701.07223.

[24] K. Schutz and A. Liu, “Pulsar timing can constrain primordial black holes in the LIGO mass window,” Phys. Rev. D 95 no. 2, (Jan., 2017) 023002, arXiv:1610.04234.

[25] Y. Ali-Haïmoud, E. D. Kovetz, and M. Kamionkowski, “The merger rate of primordial-black-hole binaries,” ArXiv e-prints (Sept., 2017) , arXiv:1709.06576.

[26] E. D. Kovetz, “Probing Primordial-Black-Hole Dark Matter with Gravitational Waves,” Phys. Rev. Lett. 119 (Sep, 2017) 131301, arXiv:1705.09182.

[27] K. Griest, A. M. Cieplak, and M. J. Lehner, “Experimental Limits on Primordial Black Hole Dark Matter from the First 2 yr of Kepler Data,” The Astrophysical Journal 786 no. 2, (2014) 158, 1307.5798.

[28] H. Niikura, M. Takada, N. Yasuda, R. H. Lupton, T. Sumi, S. More, A. More, M. Oguri, and M. Chiba, “Microlensing constraints on 10−15 M⊙-scale primordial black holes from high-cadence observation of M31 with Hyper Suprime-Cam,” 1701.02151.

[29] EROS-2 Collaboration, P. Tisserand et al., “Limits on the Macho content of the Galactic Halo from the EROS-2 Survey of the Magellanic Clouds,” A&A 469 no. 2, (2007) 387–404, astro-ph/0607207.

[30] S. Calchi Novati, S. Mirzoyan, P. Jetzer, and G. Scarpetta, “Microlensing towards the SMC: a new
analysis of OGLE and EROS results,” Monthly Notices of the Royal Astronomical Society 435 no. 2, (2013) 1582–1597, 1308.4281.

[31] E. Mediavilla, J. A. Munoz, E. Falco, V. Motta, E. Guerra, H. Canovas, C. Jean, A. Oscoz, and A. M. Mosquera, “Microlensing-based Estimate of the Mass Fraction in Compact Objects in Lens Galaxies,” The Astrophysical Journal 706 no. 2, (2009) 1451, 0910.3645.

[32] D. P. Quinn, M. J. Wilkinson, M. J. Irwin, J. Marshall, A. Koch, and V. Belokurov, “On the reported death of the MACHO era,” Monthly Notices of the Royal Astronomical Society: Letters 396 no. 1, (2009) L11–L15, 0903.1644.

[33] T. D. Brandt, “Constraints on MACHO Dark Matter from Compact Stellar Systems in Ultra-faint Dwarf Galaxies,” The Astrophysical Journal Letters 824 no. 2, (2016) L31, 1605.03665.

[34] D. P. Quinn, M. I. Wilkinson, M. J. Irwin, J. Marshall, A. Koch, and V. Belokurov, “On the reported death of the MACHO era,” Monthly Notices of the Royal Astronomical Society: Letters 396 no. 1, (2009) L11–L15, 0903.1644.

[35] G. Giesen, J. Lesgourgues, B. Audren, and Y. Ali-Haïmoud, “CMB photons shedding light on dark matter,” Journal of Cosmology and Astroparticle Physics 2012 no. 12, (2012) 008, https://arxiv.org/abs/1209.0247.

[36] Y. Ali-Haïmoud and M. Kamionkowski, “Cosmic microwave background limits on accreting primordial black holes,” Phys. Rev. D 95 no. 4, (Feb., 2017) 043534, arXiv:1612.05644.

[37] Planck Collaboration, P. A. R. Ade et al., “Planck 2015 results. XIII. Cosmological parameters,” A&A 594 (2016) A13, arXiv:1502.01589.

[38] M. Ricotti, J. P. Ostriker, and K. J. Mack, “Effect of Primordial Black Holes on the Cosmic Microwave Background and Cosmological Parameter Estimates,” The Astrophysical Journal 680 no. 2, (2008) 829, 0709.0524.

[39] V. Poulin, P. D. Serpico, F. Calore, S. Clesse, and K. Kohri, “Squeezing spherical cows: CMB bounds on disk-accreting massive Primordial Black Holes,” 1707.04206.

[40] B. J. Carr, “The primordial black hole mass spectrum,” Astrophysical Journal 201 (1975) 1. http://adsabs.harvard.edu/abs/1975ApJ...201....1C.

[41] S. W. Hawking, “Black holes from cosmic strings,” Physics Letters B 231 (1989) 237.

[42] S. W. Hawking, I. G. Moss, and J. M. Stewart, “Bubble collisions in the very early universe,” Phys. Rev. D 26 (Nov, 1982) 2681–2693.

[43] S. Clesse and J. García-Bellido, “Massive primordial black holes from hybrid inflation as dark matter,” Phys. Rev. D 92 no. 2, (July, 2015) 023524, arXiv:1501.07565.

[44] M. W. Choptuik, “Universality and scaling in gravitational collapse of a massless scalar field,” Physical Review Letters 70 (Jan., 1993) 9–12.

[45] J. C. Niemeyer and K. Jedamzik, “Near-Critical Gravitational Collapse and the Initial Mass Function of Primordial Black Holes,” Physical Review Letters 80 (June, 1998) 5481–5484, astro-ph/9709072.

[46] I. Musco, J. C. Miller, and L. Rezzolla, “Computations of primordial black-hole formation,” Classical and Quantum Gravity 22 (Apr., 2005) 1405–1424, gr-qc/0412063.

[47] I. Musco, J. C. Miller, and A. G. Polnarev, “Primordial black hole formation in the radiative era: investigation of the critical nature of the collapse,” Classical and Quantum Gravity 26 no. 23, (Dec., 2009) 235001, arXiv:0811.1452 [gr-qc].

[48] I. Musco and J. C. Miller, “Primordial black hole formation in the early universe: critical behaviour and self-similarity,” Classical and Quantum Gravity 30 no. 14, (July, 2013) 145009, arXiv:1201.2379 [gr-qc].

[49] B. Carr, F. Kühl, and M. Sandstad, “Primordial black holes as dark matter,” Phys. Rev. D 94 (Oct, 2016) 083504, 1607.06077.

[50] N. Bellomo, J. L. Bernal, A. Raccanelli, and L. Verde, “Primordial Black Holes as Dark Matter: Converting Constraints from Monochromatic to Extended Mass Distributions,” ArXiv e-prints (Sept., 2017), arXiv:1709.07467.
[51] A. M. Green, “Microlensing and dynamical constraints on primordial black hole dark matter with an extended mass function,” Phys. Rev. D 94 (Sep, 2016) 063530, https://arxiv.org/abs/1609.01143.

[52] J. Lesgourgues, “The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview,” arXiv:1104.2932 [astro-ph.IM].

[53] D. Blas, J. Lesgourgues, and T. Tram, “The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes,” JCAP 1107 (2011) 034, arXiv:1104.2933 [astro-ph.CO].

[54] Y. Ali-Haïmoud and C. M. Hirata, “Ultrafast effective multilevel atom method for primordial hydrogen recombination,” Phys. Rev. D 82 (Sep, 2010) 063521, https://arxiv.org/abs/1006.1355.

[55] Y. Ali-Haïmoud and C. M. Hirata, “HyRec: A fast and highly accurate primordial hydrogen and helium recombination code,” Phys. Rev. D 83 (Feb, 2011) 043513, https://arxiv.org/abs/1011.3758.

[56] B. Audren, J. Lesgourgues, K. Benabed, and S. Prunet, “Conservative constraints on early cosmology with MONTE PYTHON,” JCAP 2 (Feb., 2013) 1, arXiv:1210.7183.

[57] Planck Collaboration, N. Aghanim et al., “Planck 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters,” A&A 594 (2016) A11, arXiv:1507.02704.

[58] Planck Collaboration, N. Aghanim et al., “Planck intermediate results - XLVI. Reduction of large-scale systematic effects in HFI polarization maps and estimation of the reionization optical depth,” A&A 596 (2016) A107, arXiv:1605.02985.

[59] M. Ricotti, “Bondi Accretion in the Early Universe,” Astrophys. J. 662 (June, 2007) 53–61, arXiv:0706.0864.

[60] M. Raidal, V. Vaskonen, and H. Veermäe, “Gravitational Waves from Primordial Black Hole Mergers,” Journal of Cosmology and Astroparticle Physics 2017 no. 09, (2017) 037, arXiv:1707.01480.

[61] A. G. Riess, L. M. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, D. O. Jones, J. M. Silverman, R. Chornock, P. Challis, W. Yuan, P. J. Brown, and R. J. Foley, “A 2.4% Determination of the Local Value of the Hubble Constant,” ApJ 826 (July, 2016) 56, arXiv:1604.01424.

[62] J. L. Bernal, L. Verde, and A. G. Riess, “The trouble with H₀,” JCAP 10 (Oct., 2016) 019, arXiv:1607.05617.

[63] L. Verde, J. L. Bernal, A. F. Heavens, and R. Jimenez, “The length of the low-redshift standard ruler,” MNRAS 467 (May, 2017) 731–736, arXiv:1607.05297.

[64] C. L. Bennett, D. Larson, J. L. Weiland, and G. Hinshaw, “The 1% Concordance Hubble Constant,” Astrophys. J. 794 (Oct., 2014) 135, arXiv:1406.1718.

[65] G. Efstathiou, “H₀ revisited,” Mon. Not. Roy. Astron. Soc. 440 (May, 2014) 1138–1152, arXiv:1311.3461.

[66] W. L. Freedman, “Cosmology at a crossroads,” Nature Astronomy 1 (May, 2017) 0121.