Tachyonic Intermediate Inflation in DGP Cosmology; consistency with new observations

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(Dated: March 24, 2015)

Abstract

In this article we study an intermediate inflationary model in the context of Dvali-Gabadadze-Porrati (DGP) cosmology caused by a tachyon scalar field. Considering slow-roll inflation we discuss the dynamics of the Universe. Using perturbation theory, we estimate some of the model parameters numerically and compare them with observations, particularly with Planck Temperature data released in 2013 (PT13), nine-years data set of Wilkinson Microwave Anisotropy Probe (WMAP9) and data from second Background Imaging of Cosmic Extragalactic Polarization instrument (BICEP2).

PACS numbers: 98.80.Cq; 11.25.-w

Keywords: intermediate inflation; DGP; tachyon; slow-roll; perturbation; observational data.

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1. INTRODUCTION

Inflation is a very short epoch at very early stages of the history of the Universe in which the Universe experiences a very rapidly accelerated expansion. This phase first proposed in [1], to solve some of the problems of the hot big bang model of cosmology, such as flatness problem, horizon problem and monopole problem. This model of inflation improved gradually to a more accurate scenario called slow-roll inflation which supports a long enough period of inflation [2, 3]. But maybe, the true merit of inflation is that it provides some inhomogeneities in the Universe arisen from vacuum fluctuations and so can explain the large scale structure of the Universe and the anisotropies in the cosmic microwave background (CMB) radiation [4, 5].

There are several inflationary models which differ in their expression of scale factor parameter, \(a(t)\). Among them, the intermediate inflation is of particular interest because it arises from an effective theory at low dimension of a more fundamental string theory [6]. In this class of inflationary models, the scale factor varies with time faster than the scale factor of power law inflation in which \(a(t) = t^p; p > 1\), but still slower than the scale factor of standard de-Sitter inflation in which \(a(t) = \exp(Ht)\), where \(H = \dot{a}/a\), is the Hubble parameter. The behavior of scale factor parameter in terms of time in intermediate inflationary models is as follows

\[
a(t) = \exp(At^f) \quad (1)
\]

in which \(0 < f < 1\) and \(A > 0\), are constant parameters.

In general, inflation drives by the potential of a standard scalar field, the inflaton, where it obeys the Klein-Gordon (KG) equation. But, there is a nonstandard scalar field action motivated from string theory which can be used to drive an inflationary phase, called tachyon field [7]-[9]. Because its equation of state parameter is bounded as, \(-1 < w < 0\), it can play the role of inflaton field, well [10]-[16]. Tachyon field has some other applications in cosmology, too. It can play the role of dark sectors of the Universe [17]-[28]. Also, in [29], it has been shown that the tachyon field can drive inflation and then behave as dark matter or a nonrelativistic fluid. In principle, the tachyon inflation is a \(k\)-inflationary model with its own features. It has a positive potential \(V(\phi)\), where has a maximum at \(\phi = 0\) and approaches zero when \(|\phi| \to \infty\). Meanwhile, during the entire of this process, \(\frac{dV(\phi)}{d\phi} < 0\).

On the other hand, the theory of extra dimensions which has come out of the string
theory, has attracted a great amount of attention in the past two decades. Several five dimensional cosmological models have been proposed to explain the weakness of gravity and hierarchy problem \cite{30}-\cite{32}. In these models our four dimensional Universe is a surface dubbed brane, embedded into a higher dimensional bulk spacetime. It is assumed that the standard model of particle physics is confined to the brane and only gravitons can propagate into the bulk. The important effect of considering extra dimensions is that they modify the Friedmann equations by adding some new terms. These theories, specially the Randall-Sundrum type II model (RSII), with a fine-tuning relation as \( \Lambda = \frac{1}{2}(\Lambda + \frac{1}{6}\kappa_5^4 \lambda^2) \), in which \( \Lambda \), \( \Lambda \), \( \kappa_5^2 \) and \( \lambda \), are cosmological constant on the brane, cosmological constant of the bulk, five dimensional (5D) gravitational constant and the tension of the brane respectively, have been widely utilized in the literature to explain the dynamics of the Universe, more precisely \cite{16}, \cite{24}, \cite{33}-\cite{40}.

One way to generalize the gravitational action of a five dimensional theory is to bring in an induced gravity correction through considering a four dimensional scalar curvature term in the brane action in addition to the matter Lagrangian in it. A well-known example of induced-brane gravity model is the DGP model \cite{41}. In this model, our four dimensional (4D) brane is embedded into an infinite 5D Minkowskian bulk. Also, in DGP model the cosmological constant of the bulk and of the brane and the brane tension set to zero, simultaneously. DGP model consists of two separate branches depending on how the brane embeds into the bulk. These branches where distinguish with a parameter \( \epsilon = \pm 1 \), have distinct characteristics. For instance, the case \( \epsilon = 1 \), called self-accelerating branch, induces a late-time acceleration without any need to a dark energy component and the case \( \epsilon = -1 \), called normal branch against the prior needs a dark energy component to explain the late-time acceleration.

DGP model has been utilized in studying the dynamics of the Universe in its dark dominated stages \cite{42}-\cite{48}. But, the inflationary era has been usually investigated in the context of a generalized version of DGP model, called warped DGP \cite{49}-\cite{53} and the loss of studying inflation in an original DGP model would be felt more than ever.

In this manuscript we will investigate an intermediate tachyonic inflationary model in the context of an original DGP cosmology. Our motivation to considering such a model is that all the pillars of our model, i.e., intermediate inflation, tachyon scalar field and the higher dimensional DGP model, are come from string theory. Perhaps this model may lead to more
conformity with observations. The outline of this work is as follows. In the next section we start with the action of the DGP model. Considering the slow-roll inflationary conditions and in the high energy regime the effective Friedmann equation and KG equation of the tachyon field will be obtained. After introducing the slow-roll parameters, in section III, we derive some important parameters in terms of them related to perturbation theory in our model. Also, in this section a special situation relevant to tachyon field will be discussed. Section IV deals with numeric approaches to test the validity of our model. To this aim we use a few combinations of many observational data such as PT13, WMAP9 and BICEP2 data sets, the Baryon Acoustic Oscillation data (BAO) and also some of the local cosmological observations such as Hubble constant direct measurement data from Hubble Space Telescope ($H_0$), Sunyaev-Zeldovich cluster counts data (SZ), Cosmic Microwave Background Lensing data (CMBL) and the Cosmic Shear data (CS). Then, we check how realistic is our model, again numerically. The last section will demonstrate a summary of our work and its results.

2. THE MODEL

Our starting point is the action of DGP brane-world model which can be written as

\[ S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g^{(5)}} R^{(5)} + \int d^4x \sqrt{-g} L. \]  

(2)

The first term in the above is corresponding to the Einstein-Hilbert action in a 5D Minkowskian bulk and the second term is the contribution of induced gravity localized on the brane. Here, $^{(5)}R$, is the 5D Ricci scalar and $L$ is the effective 4D Lagrangian on the brane which can be expressed as

\[ L = \frac{\mu^2}{2} R + L_m, \]  

(3)

where $\mu$ is a mass parameter which may correspond to the 4D Planck mass. Also, $R$ and $L_m$ are the Ricci scalar and the matter Lagrangian on the brane, respectively. In fact, our action is a special case of a more general induced gravity action in which the cosmological constants in the bulk and on the brane and the tension of the brane have been set to zero.

Considering the most relevant metric, i.e., the spatially flat FRW metric on the brane and introducing $\rho_0 = \frac{6}{\kappa^2\mu^2}$, we obtain the Friedmann equation of our model as

\[ H^2 = \frac{1}{3\mu^2} \left( \sqrt{\rho + \frac{\rho_0}{2}} + \sqrt{\frac{\rho_0}{2}} \right)^2, \]  

(4)
In which $\epsilon$ can take the values +1 and -1, as we mentioned earlier.

It is obvious from the action that in our model the matter is only confined to the brane, so it obeys the standard form of conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0,$$

where the dot means derivative with respect to the cosmic time $t$. In the following we will consider a tachyon scalar field as the matter on the brane which plays the role of inflaton field in the inflationary era. For a tachyon field, the energy density and the pressure are given by

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2},$$

where $V(\phi)$ is the tachyon potential. Replacing equation (6) in (5), one can obtain the equation of motion of the tachyon field as below

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{V'(\phi)}{V} = 0,$$

in which $V' = \partial V(\phi)/\partial \phi$.

Since inflation is a period in the very early universe, we impose the high energy condition $\rho \gg \rho_0$, in our model. So, the effective Friedmann equation becomes

$$H^2 = \frac{1}{3\mu^2}(\sqrt{\rho} + \epsilon \sqrt{\frac{\rho_0}{2}})^2.$$  \hspace{1cm} (8)

Using this equation and with attention to equations (5) and (6), we reach to

$$\dot{\phi}(t)^2 = -\frac{2\mu\dot{H}}{\sqrt{3H(\sqrt{3\mu H} - \epsilon \sqrt{\frac{\rho_0}{2}})}}.$$

For intermediate inflationary scenario with the scale factor as $a(t) = \exp(At^f)$, we have

$$\phi(t) = \int \left[\frac{2\mu(1 - f)}{\sqrt{3t(\beta t^f - 1 - \epsilon \sqrt{\frac{\rho_0}{2}})}}\right]^{1/2} dt,$$

where $\beta = \sqrt{3}\mu A f$. Using (6), (8) and (9), the effective potential of our model can be written as

$$V(t) = \left(\beta t_f - 1 - \epsilon \sqrt{\frac{\rho_0}{2}}\right)^2 \left(1 + \frac{2\mu(f - 1)}{\sqrt{3t(\beta t_f - 1 - \epsilon \sqrt{\frac{\rho_0}{2}})}}\right).$$
Supporting a long enough period of inflation the tachyon field must slowly rolls down its potential. In this scenario which is called slow-roll inflation the energy density of the inflaton field and its potential satisfy $\rho_\phi \sim V$. If the tachyon field is the inflaton field as in our model, the slow-roll conditions will be $\dot{\phi}^2 \ll 1$ and $\ddot{\phi} \ll 3H\dot{\phi}$. Under these conditions, equations (7) and (8) reduce to

$$\frac{V'}{V} \approx -3H\dot{\phi}$$

(12)

and

$$(\sqrt{3}\mu H - \epsilon \sqrt{\frac{\rho_0}{2}})^2 \approx V,$$

(13)

respectively. Also, the tachyon potential (11), becomes

$$V(t) \approx (\beta t^{f-1} - \epsilon \sqrt{\frac{\rho_0}{2}})^2.$$ 

(14)

Slow-roll parameters are a few dimensionless parameters one can introduce in any slow-roll inflationary model. In our model, they will be as

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{(1-f)}{Af} t^{-f}$$

(15)

and

$$\eta = -\frac{\ddot{H}}{HH} = \frac{(2-f)}{Af} t^{-f}.$$ 

(16)

The inflationary phase takes place whenever $\ddot{a} > 0$ which is proportional to $\varepsilon < 1$. Consequently, in intermediate inflationary models, $t > (\frac{1-f}{Af})^{1/f}$, is the necessary and sufficient condition of inflation. Also, with attention to behavior of $\varepsilon$ in terms of $t$, we can assume inflation begins when $\varepsilon = 1$, which yields to $t_b = (\frac{1-f}{Af})^{1/f}$, as the time in which inflation starts.

The number of $e$-folds between two cosmological times is defined as the logarithm of the amount of expansion between them. In our model and during the period of inflation, it can be expressed as

$$N = \int_{t_b}^t H dt = A(t^f_c - t^f_b),$$

(17)

for $t > t_b$. 

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3. PERTURBATION

A homogeneous and isotropic Universe model is a very good assumption in studying cosmology, but nowadays the existence of some deviations from this assumption are unquestioned. Therefore, it seems necessary to investigate the perturbation theory in cosmology. The attractive feature of gravity can cause growing the inhomogeneities with time. Thus, we can say that in the early Universe they were very smaller than today and because of the smallness of them, using the linear perturbation theory is permissible. But, we must use a relativistic version of perturbation theory because of the relativistic features of cosmology due to Einstein’s equations. So, we will investigate a perturbed inflaton field in a perturbed geometry.

The most general linearly perturbed flat FRW metric which includes both scalar and tensor perturbations can be written as

\[
ds^2 = -(1 + 2C)dt^2 + 2a(t)D_i dx^i dt + a(t)^2 [(1 - 2\psi)\delta_{ij} + 2E_{,ij} + 2h_{ij}]dx^i dx^j,
\]

where \(C, D, \psi \) and \(E \) are the scalar metric perturbations and \(h_{ij} \) is the transverse-traceless tensor perturbation. For describing the distinctive nature of perturbations, maybe, the power spectrum of the curvature perturbation, \(P_R \), which appears in deriving the correlation function of the inflaton field in the vacuum state and the power spectrum of the tensor perturbation, \(P_g \), which indicates the contribution of tensor perturbation, are the most useful quantities.

For the tachyon field, \(P_R \), is defined as \(P_R = \left(\frac{H^2}{2\pi^2}\right)^2 \frac{1}{Z_s} \), where \(Z_s = V(1 - \dot{\phi}^2)^{-3/2} \). In slow-roll approximation, it reduces to \(P_R \approx \left(\frac{H^2}{2\pi^2}\right)^2 \frac{1}{V} \), and then using equations (12) and (13), the power spectrum of curvature perturbation will be

\[
P_R \approx -\frac{\sqrt{3}H^5}{8\pi^2\mu H} \left(\frac{\sqrt{3}\mu H - \epsilon \sqrt{\frac{\rho_0}{2}}}{\epsilon} \right).
\]

Considering intermediate inflation, one can reach to

\[
P_R \approx \frac{\sqrt{3}(Af)^4t^{4f-3}}{8\pi^2\mu(1 - f) \left(\beta tf - f \sqrt{\frac{\rho_0}{2}}\right)}
\]

and also in terms of the number of e-folds \(N \), as

\[
P_R \approx \frac{\sqrt{3}(Af)^4(N^2_f + 1 - f)^{(4f-3)/f}}{8\pi^2\mu(1 - f) \left(\beta(N - 1 + f)^{(f-1)/f - \epsilon \sqrt{\frac{\rho_0}{2}})}\right)}.
\]
For given values of $\epsilon, f, \mu, \rho_0, N$ and $\mathcal{P}_R$, we can find a constraint on the parameter $A$, using the latter equation. In the following we will work in the normal branch of DGP model in which $\epsilon = -1$. Also, in [49], applying high energy and low energy conditions into the Friedmann equation, respectively, the authors have shown that the parameter $\mu$, must have an energy scale in the order $10^{18} GeV$, as the Planck scale and the parameter $\rho_0$, should be at least in the order of $(10^{-3} eV)^4$, as the current critical energy density of our Universe. Using $\mathcal{P}_R \simeq 2.4 \times 10^{-9}$ and $N \simeq 60$, we found constraints on $A$, related to different values of $f$. The results have been indicated in table I.

| $f$ | 0.1 | 0.5 | 0.7 | 0.8 |
|-----|-----|-----|-----|-----|
| $A$ | 12960 | $4 \times 10^{12}$ | $5 \times 10^{16}$ | $4.9 \times 10^{18}$ |

After scalar perturbation we set to the contribution of tensor perturbation in our model. Tensor perturbation would produce gravitational waves during inflation and is more involved in our model, since existing an extra dimension allows gravitons to propagate into the bulk, as noted in introduction. The amplitude of tensor perturbations in an induced gravity model has been calculated in [54], as

$$\mathcal{P}_g = \frac{8}{m_p^2} \left( \frac{H}{2\pi} \right)^2 G_\gamma^2(x),$$

in which $m_p$, is the 4D Planck mass. We should note that here, we have used the notation of [55]. This form of amplitude of tensor perturbations differs from its expression in a standard 4D general theory of relativity by the coefficient $G_\gamma^2(x) = \gamma + (1 - \gamma)F(x)^{-2}$, in which $\gamma = \mu^2/m_p^2$ and $F(x)$ is defined as

$$F(x) = \left[ \sqrt{1 + x^2} - x^2 \sinh^{-1}(1/x) \right]^{-1/2},$$

where $x = H/\mu$, and $\mu$ is the energy scale associated with the bulk curvature.

Because we are working with a original DGP model in which the bulk is Minkowskian, then $\mu \rightarrow 0$. Therefore, one can say that the energy scale at which inflation occurs and the bulk curvature scale, satisfy the condition $H \gg \mu$ and so according to [54], under this condition, equation (22), reduces to

$$\mathcal{P}_g \approx \frac{8}{m_p^2} \left( \frac{H}{2\pi} \right)^2 \frac{1}{\gamma},$$

(24)
where in the context of intermediate inflation we obtain

\[ P_g \approx \frac{2}{\gamma} \left( \frac{Af}{\pi m_p} \right)^2 t^{2(f-1)}. \]  

(25)

Also, we should note that another useful quantity in studying perturbation theory is

\[ r = \frac{P_g}{P_R}, \]  

(26)

which is the ratio between power spectrum of tensor perturbation and power spectrum of scalar perturbation. In our model, equation (26), reduces to

\[ r \approx \frac{16(1-f)(\beta t^{f-1} - \epsilon \sqrt{\frac{m_p}{2}})}{\beta Af} t^{1-2f}. \]  

(27)

Identification of two other parameters in the topic of scalar perturbation in cosmology is of particular interest. These are the scalar spectral index \( n_s \), which is related to \( P_R \), through

\[ n_s - 1 = \frac{d \ln P_R}{d \ln k}, \]

and the running in the scalar spectral index parameter \( n_{\text{run}} \), where can be obtained via \( n_{\text{run}} = d n_s / d \ln k \). Here, \( d \ln k = dN = H dt \).

In our model and under slow-roll approximation, they become

\[ n_s \approx 1 + \frac{5H}{H^2} - \frac{\dot{H}}{H^2} - \frac{\sqrt{3\mu}}{H \left( \sqrt{3\mu H - \epsilon \sqrt{\frac{m_p}{2}}} \right)} = 1 - 5\epsilon + \eta + \epsilon \left( \frac{\sqrt{3\mu H}}{(\sqrt{3\mu H - \epsilon \sqrt{\frac{m_p}{2}}})} \right) \]  

(28)

and

\[ n_{\text{run}} \approx \frac{1}{H} \left( -5\ddot{\epsilon} + \dot{\eta} + \dot{\epsilon} \frac{\sqrt{3\mu H}}{(\sqrt{3\mu H - \epsilon \sqrt{\frac{m_p}{2}}})} + \epsilon (\sqrt{3\mu H}) \left( 1 - \frac{\sqrt{3\mu H}}{(\sqrt{3\mu H - \epsilon \sqrt{\frac{m_p}{2}}})} \right) \right) . \]  

(29)

In [56], it has been investigated an inflationary model in which one of the slow-roll parameters, \( \eta = V'' / V \), plays the essential role and \( |\eta| = 1 \), is the condition of beginning of inflation. There is such a situation in tachyon inflationary models as well, if one assume that the inflation starts from the top of the tachyon potential for small \( \phi \). With this assumption we can claim that for small \( \phi \), we have \( \epsilon \ll \eta \). In [24] and [57], it has been shown that in this situation one can impose a new constraint on the parameter \( f \). To this aim, we should obtain the ratio between \( \epsilon \) and \( \eta \), from (15) and (16) as

\[ \left| \frac{\eta}{\epsilon} \right| = \frac{2-f}{1-f} \]  

(30)

and since \( 0 < f < 1 \), this condition does not give us a new constraint on \( f \), in our model. But other parameters which obtained in the context of perturbation theory, reduce to some simpler expressions. For instance, equation (28) reduces to

\[ n_s \approx 1 + \eta. \]  

(31)
Now, assuming $|\eta| = 1$ as the condition of beginning of inflation we can rewrite (31), in terms of the number of e-folds as

$$n_s \approx 1 + \frac{2 - f}{N f + (2 - f)}.$$  \hspace{1cm} (32)

It is obvious from the equation above that in this situation for all values of $f$, we have $n_s > 1$ and thus, against our original model, i.e., equation (28), having a Harrison-Zel’dovich spectrum is impossible.

Running in the scalar spectral index can be achieved as preceding. So we reach to

$$n_{\text{run}} \approx -\frac{f(2 - f)}{[N f + (2 - f)]^2}$$  \hspace{1cm} (33)

and a straight relation between $n_s$ and $n_{\text{run}}$, from the two prior equations can be obtained as

$$n_{\text{run}} \approx -\frac{f(n_s - 1)^2}{2 - f},$$  \hspace{1cm} (34)

which is just the same as the expression obtained in [57]. Also, in contrast to the result in [24], it depends on the parameter $f$.

4. NUMERICAL DISCUSSION

In this section we do some numeric calculations to check the consistency of our model. At first we try to show that is our model realistic or not. In other words, is our model potential a well-motivated tachyonic potential? From string theory we know the general behavior of a tachyonic potential. As we mentioned in introduction, considering $\frac{dV(\phi)}{d\phi} < 0$, it has an unstable maximum at $\phi \to 0$ and tends to zero when $\phi \to \infty$. Because of difficulties in obtaining an exact form for $V(\phi)$, we will use numeric calculations. In figure (1), we have plotted the behavior of $V(\phi)$ for $f = 0.1, 0.5, 0.7$ and 0.8. It is obvious that in all the cases, our model tachyonic potential is compatible with the features mentioned earlier.

Then, we will focus on some other figures in which we plot some trajectories in $r - n_s$, $r - n_{\text{run}}$ and $n_{\text{run}} - n_s$ planes obtained from our model and compare them with some confidence regions extracted from [58]-[60], in which the authors have used different combinations of observational data mentioned earlier in different basic cosmological models we explain in the following. In all of the figures, the dashed black, long dashed green, dotted red and solid yellow curves are related to $f = 0.1, 0.5, 0.7$ and 0.8, respectively.
FIG. 1: The behavior of our model tachyon potential $V(\phi)$, for different values of parameter $f$. Upper-left for $f = 0.1$, upper-right for $f = 0.5$, bottom-left for $f = 0.7$ and bottom-right for $f = 0.8$, have been illustrated. They satisfy the general conditions of a tachyonic potential.

In the base standard cosmology we apply a spatially-flat six-parameter $\Lambda$CDM model. These parameters include baryon density today ($\Omega_b h^2$), cold dark matter density today ($\Omega_c h^2$), angular scale of the sound horizon at last-scattering ($\theta_s$), Thomson scattering optical depth due to reionization ($\tau$), scalar spectrum power-law index ($n_s$) and log power of the primordial curvature perturbations ($\ln(10^{10} A_s)$). There are many extensions for this basic $\Lambda$CDM model including the scale dependence of primordial scalar fluctuations, primordial tensor fluctuations, spatial curvature of the Universe, neutrino physics, the abundance of
light elements, dynamical dark energy and different combinations of them. Among these scenarios, ΛCDM+r model which consider the contribution of relic gravitational waves in temperature and polarization anisotropy of CMB, is of particular interest, specially after detecting the B-mode polarization of the CMB by the BICEP2 experiment.

Afterward, it was appeared a tension about determining the constraint on the tensor-to-scalar ratio parameter, $r$, between the BICEP2 and PT13. To relieve this tension a few methods were proposed. For instance, we can involve another new parameter in ΛCDM+r model, if we consider the running of the scalar spectral index, $n_{\text{run}}$, as a free parameter. Although the new model which can be shown as ΛCDM+$r+n_{\text{run}}$, can significantly reduce this tension, but because of the large amount of $n_{\text{run}}$, of order $10^{-2}$, it causes trouble to inflation.

Involving a sterile neutrino species is another choice in which we include two new parameters, the effective number of relativistic species, $N_{\text{eff}}$ and the effective sterile neutrino mass, $m_{\nu,\text{sterile}}^\text{eff}$. We represent this model by ΛCDM+$r+\nu_s$, as it has been shown in [58]. In addition to reducing the tension about $r$, between PT13 and BICEP2 results and also more consistency of this model with inflation, it can strongly relax the other tensions between Planck and some of the local astrophysical data, such as $H_0$, SZ and CS data, we noted earlier. The cost of reducing these tensions in ΛCDM+$r+\nu_s$ model, is an increasing in the value of $n_s$.

Another useful model in relaxing these tensions is $w$CDM+$r+\nu_s$ model, as it has been argued in [59]. In this model the equation of state parameter of dark energy, $w$, assumed to be a constant parameter. Although the authors in [60], have been shown that the tension between BICEP2 and PT13 cannot be reconciled in a $w$CDM+r model, but after importing parameters related to the sterile neutrino model, i.e., $N_{\text{eff}}$ and $m_{\nu,\text{sterile}}^\text{eff}$, the model is successful in relieving all the tensions. In the following we compare our model with these three extensions of ΛCDM model, separately.

### 4.1. ΛCDM+$r+\nu_s$ model

First of all, we address to ΛCDM+$r+\nu_s$ model in which three new parameters, $r$, $N_{\text{eff}}$ and $m_{\nu,\text{sterile}}^\text{eff}$, have been added to the base standard ΛCDM model. This model can significantly reduce all the tensions between PT13 with other observational data. In figure [2], we have
FIG. 2: The comparison between the curves of $r(n_s)$ in our model and observational data. The dashed black, long dashed green, dotted red and solid yellow curves, have obtained from our model and are related to $f = 0.1, 0.5, 0.7$ and $0.8$, respectively. The blue contours show $68\%$ and $95\%$ confidence regions related to the strongest observational constraints from combining PT13, WMAP9, BAO, $H_0$, CMBL, SZ, CS and BICEP2 data sets. Our model fits the observation very well specially for $f = 0.5$ and $f = 0.7$.

compared the trajectories in $r - n_s$ plane of our original model with the related confidence regions from [58], in which the authors have used the PT13, WMAP9, BAO, $H_0$, CMBL, SZ, CS and BICEP2 data sets to perform their numerical analysis. In this figure the blue and red contours are related to the case of considering all the astrophysical data in the above with and without considering BICEP2 data. Looking at this figure we can see that our model fits the observation very well, specially for $f = 0.5$ and $f = 0.7$.

Also, we see that the curve $r(n_s)$, for $f = 0.5$, enters the $0.95\%$ confidence region of the blue contours at $r \simeq 0.26$. With attention to this value of $r$ and using the condition of beginning of inflation, $\varepsilon = 1$ and the equations (17) and (20), one can obtain the corresponding number of $e$-folds as $N \approx 61$, which indicates the time spent by the tachyon field to enter the $95\%$ confidence region in the $r - n_s$ plane. On the other hand, this trajectory leaves the $95\%$ confidence region at $r \simeq 0.1$, which corresponds to $N \approx 159$. So, we have found a lower and an upper limit for the value of $N$. A similar procedure for $f = 0.7$, can give rise to limits $20 \lesssim N \lesssim 62$. 
FIG. 3: The comparison between the curves of $r(n_s)$ in our special model and observational data. The dashed black, long dashed green, dotted red and solid yellow curves, have obtained from our model and are related to $f = 0.1, 0.5, 0.7$ and $0.8$, respectively. The blue contours show 68% and 95% confidence regions related to the strongest observational constraints from combining PT13, WMAP9, BAO, $H_0$, CMBL, SZ, CS and BICEP2 data sets. Our special model is not in good agreement with the observations.

On the other hand for our special model in which $\varepsilon \ll \eta$, the model is not in good agreement with observations. Our trajectories does not cross the blue contours at all and does not fit with the red ones very well. See figure (3).

4.2. $w_{\Lambda CDM}+r+\nu_s$ model

In [59], the authors extended the $\Lambda CDM+r+\nu_s$ model replacing the cosmological constant, $\Lambda$, with a dynamical dark energy with constant equation of state parameter that we demonstrated it with $w_{\Lambda CDM}+r+\nu_s$. Similar to their previous work [58], they utilized the PT13, WMAP9, BAO, $H_0$, CMBL, SZ, CS and BICEP2 data sets for numerical calculations and plotted many confidence regions considering different combinations of these data sets. In figure (4), the red contours are related to the case of combining PT13, WMAP9 and BAO. The pink contours show the case PT13+WMAP9+BAO+$H_0$+CMBL+SZ+CS and the blue ones are related to the case of considering all the priors plus BICEP2 data. When
FIG. 4: The comparison between the curves of \( r(n_s) \) in our model and observational data. The dashed black, long dashed green, dotted red and solid yellow curves, have obtained from our model and are related to \( f = 0.1, 0.5, 0.7 \) and \( 0.8 \), respectively. The blue contours show 68% and 95% confidence regions related to the strongest observational constraints from combining PT13, WMAP9, BAO, \( H_0 \), CMBL, SZ, CS and BICEP2 data sets. Our model fits the observation very well specially for \( f = 0.1 \) and \( f = 0.5 \).

we compare our model trajectories with the blue confidence regions, we find that our model fits the observations nicely, for \( f < 0.7 \).

In a similar manner of what we did in the previous subsection, the constraints on the number of e-folds obtained as \( 52 \lesssim N \lesssim 144 \), for \( f = 0.5 \).

Also, from figure (5), it is obvious that our special model does not work well again and the trajectories does not go through the blue contours at all.

4.3. ΛCDM+\( r + n_{run} \) model

Finally, we address to the ΛCDM+\( r + n_{run} \) model in which in addition to \( r \), a new parameter \( n_{run} \), has been added to the base ΛCDM model. As we mentioned earlier this model can significantly relieve the tension between PT13 and BICEP2, as it has been argued in [60], but also may lead to challenge with inflation. We are interested in this model, because in addition to \( r - n_s \) plane, we can do two more comparisons in \( r - n_{run} \) and \( n_{run} - n_s \) planes. When we compare our results with the confidence regions obtained in [60], from
FIG. 5: The comparison between the curves of $r(n_s)$ in our special model and observational data. The dashed black, long dashed green, dotted red and solid yellow curves, have obtained from our model and are related to $f = 0.1, 0.5, 0.7$ and $0.8$, respectively. The blue contours show 68% and 95% confidence regions related to the strongest observational constraints from combining PT13, WMAP9, BAO, $H_0$, CMBL, SZ, CS and BICEP2 data sets. Our special model is not in good agreement with the observations.

In the $r - n_s$ plane we find that our model works very well. But comparing with $n_{run} - n_s$ plane, we deduce that our model fails for the smaller values of $f$. Eventually, from $r - n_{run}$ plane in figure (6), we see that our model fits observations only for $f = 0.8$. Albeit, for this value the constraint on the number of e-folds is obtained as $12 \lesssim N \lesssim 44$.

In figure (7), we have compared our special model with ΛCDM+$r+n_{run}$ model. Although with attention to $r - n_s$ and $r - n_{run}$ planes one can claim that this special model is acceptable but from $n_{run} - n_s$ plane, it is clear that this special case does not fit observations, at all.

5. SUMMARY

In this work we have analyzed an intermediate tachyonic inflationary model in the context of the normal branch of DGP cosmology. The main motivation for considering this model is that all the pillars of our work, i.e., an intermediate inflationary model, a tachyon scalar field and the DGP cosmology, all come from string theory and yield a more conformity with observations. By applying slow-roll inflation and high energy conditions we obtained
FIG. 6: The comparison between the curves of $n_s(r)$, $n_{run}(r)$ and $n_{run}(n_s)$ in our model and observational data. The dashed black, long dashed green, dotted red and solid yellow curves, have obtained from our model and are related to $f = 0.1, 0.5, 0.7$ and $0.8$, respectively. The blue contours show 68% and 95% confidence regions related to the strongest observational constraints from combining PT13, WMAP9 and BICEP2 data sets. Our model fits the observation very well specially for $f = 0.9$.

the effective Friedmann and KG equations in our model. Considering a general perturbed FRW metric, we derived the explicit expressions of some parameters in perturbation theory, such as power spectrum of scalar perturbation, $P_R$, power spectrum of tensor perturbation, $P_g$ and the ratio between them, $r$. Also, we obtained the scalar spectral index, $n_s$ and its running $n_{run}$, in terms of the slow-roll parameters.

To check how well motivated is our model tachyonic potential, we plotted the curve $V(\phi)$ for different values of parameter $f$. We found from figure (1), that the behavior of $V(\phi)$
FIG. 7: The comparison between the curves of $n_s(r)$, $n_{run}(r)$ and $n_{run}(n_s)$ in our special model and observational data. The dashed black, long dashed green, dotted red and solid yellow curves, have obtained from our model and are related to $f = 0.1, 0.5, 0.7$ and $0.8$, respectively. The blue contours show 68% and 95% confidence regions related to the strongest observational constraints from combining PT13, WMAP9 and BICEP2 data sets. Our special model is not in good agreement with the observations.

is consistent with the expected global features of a tachyonic potential. Then we did some other numeric calculations to compare our model with observations. We illustrated the trajectories $r - n_s$, $r - n_{run}$ and $n_{run} - n_s$ of our model and used the related confidence regions from [58]-[60]. In these articles, different extensions of the base ΛCDM cosmological model have been investigated and distinct observational data sets have been utilized to perform analysis related to perturbations. The important feature of these extended models is that they can relieve the tensions between some of the observational data. From figures
and (4), it is obvious that our model fits the observations very well, although in the case of considering running of the scalar spectral index, there is less agreement with observations (See figure (6)).

Also, a particular case of interest in tachyonic inflationary models, i.e., rolling from the top of the potential was discussed. This special case did not impose any new constraint in our model parameters and as it can be understood from figures (3), (5) and (7), the numeric results are not in good agreement with observations. So, we concluded that the DGP model is not consistent with this special case.

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