Reheating constraints on K-inflation

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Abstract

In this work we revisit constraints on K-inflation with DBI kinetic term and power-law kinetic term from reheating. For DBI kinetic term we choose monomial potentials, $V \propto \phi^n$ with $n = 2/3, 1, 2$ and $4$, and natural inflaton potential, and for power-law kinetic term we choose quadratic, quartic and exponential potentials. The phase of reheating can be parameterized in terms of reheating temperature $T_{re}$, number of e-folds during reheating $N_{re}$ and effective equation of state during reheating $w_{re}$. These parameters can be related to the spectral index $n_s$ and other inflationary parameters depending on the choice of inflaton kinetic term and potential. By demanding that $w_{re}$ should have a finite range and $T_{re}$ should be above electroweak scale, one can obtain the bounds on $n_s$ that can provide bounds on tensor-to-scalar ratio $r$. We find, for K-inflation with DBI kinetic term and quadratic and quartic potentials, that the upper bound on $r$ for physically plausible value of $0 \leq w_{re} \leq 0.25$ is slightly larger than the Planck-2018 and BICEP2/Keck array bound, and for $n = 2/3$ and $1$, the reheating equation of state should be less than $0$ to satisfy Planck-2018 joint constraints on $n_s$ and $r$. However, natural inflation with DBI kinetic term is compatible with Planck-2018 bounds on $r$ and joint constraints on $n_s$ and $r$ for physically plausible range $0 \leq w_{re} \leq 0.25$. The quadratic and quartic potential with power-law kinetic term are also compatible with Planck-2018 joint constraints on $n_s$ and $r$ for $0 \leq w_{re} \leq 1$. However, for exponential potential with power-law kinetic term, the equation of state during reheating $w_{re}$ should be greater than $1$ for $r - n_s$ predictions to lie within $68\%$C.L. of joint constraints on $n_s$ and $r$ from Planck-2018 observations.

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1. INTRODUCTION

The idea of inflation [1] is now well accepted solution to the horizon and flatness problem of big-bang cosmology. It also provides seeds for anisotropy of cosmic microwave background and structures in the universe [2–4]. The predictions of inflation, i.e., nearly scale-invariant, Gaussian and adiabatic density perturbations are confirmed by the various CMB observations such as COBE [5], WMAP [6], Planck [7] etc. In standard scenario potential energy of a scalar field, named as inflaton, dominates the energy density of the universe during inflation and provides quasi exponential expansion. Inflaton rolls slowly through its potential during inflation, and the quantum fluctuations in this field, which are coupled to the metric fluctuations, generate the primordial density perturbations (scalar perturbations). The vacuum fluctuations in the tensorial part of the metric generated during inflation are responsible for the primordial gravitational waves (tensor perturbations). The power spectra for scalar and tensor perturbations generated during inflation depend on the inflaton potential, which can be obtained from particle physics models and string theory. Many models of inflation have been explored in recent years (see [8] for details). Although the predictions of inflation are in excellent agreement with the CMB observations, we still lack a unique model. The most popular quadratic and quartic potentials are ruled out by recent Planck observations [7] as they give large tensor-to-scalar ratio.

There is an alternative to the standard scenario of inflation, named as $K$-inflation [9,10], where inflation is achieved by the non-standard kinetic term of the inflaton. The nonstandard kinetic term in the action of inflaton can have monomial and polynomial form [9,11] or Dirac-Born-Infeld form [12], which arises in string theory [13–16] (see [17–20] for various choices of noncannonical kinetic terms and potentials derived from string theory). In [21–23] it was shown that the tensor-to-scalar ratio can be lowered for quadratic and quartic potentials with noncanonical kinetic term. $K$-inflation with pseudo-Nambu-Goldstone-Boson has also been studied in [21,24,25] and it is shown that natural inflation with noncanonical kinetic term is compatible with the Planck CMB observations. Power-law kinetic term with exponential potential has also been studied in [23] and it is found that this model is also compatible with the CMB observations. In [26] power-law kinetic term has also been studied with deformed steepness exponential potentials.

Several generalizations of $K$-inflation have been studied in the literature such as inflaton with non-minimal coupling with Ricci scalar [27,28], inflaton coupled with Gauss-Bonnet invariant [29] and $K$-inflation with $f(R)$ gravity [30]. $K$-inflation with constant-roll conditions has also been studied in [31]. It has been shown in [32] that the action of $R^2$-inflation in the framework of Palatini gravity resembles $K$-inflation models in the Einstein frame.

All these models of noncanonical inflation are in agreement with the current bounds on spectral index and tensor-to-scalar ratio from Planck-2018 observations, and there is no unique choice for noncanonical kinetic term and inflaton potential.

At the end of inflation, the universe reaches to a cold and highly non-thermal state without any matter content. However, for baryogenesis and big-bang nucleosynthesis the universe needs to be in a thermalized state at a very high temperature. This is achieved by reheating, a transition phase between the end of inflation and start of radiation dominated era. During this phase the inflaton energy is transferred to radiation, baryons and leptons, leaving the universe at a reheating temperature $T_{re}$ at the onset of radiation epoch. In the
simplest models of reheating\cite{33,35} inflaton oscillates around the minimum of its potential
and decays perturbatively into the standard model particles through various interactions of
inflaton with other scalars and fermions. However, perturbative reheating is model depen-
dent and cannot give correct description of the process at various states, and it also does
not take into account the coherent nature of the inflaton field\cite{36,37}. In other scenar-
ios the reheating is preceded by preheating, during which the classical inflaton field decays
into massive particles via non-perturbative processes such as parametric resonance\cite{38,39},
tachyonic instability\cite{40,41}, and instant preheating\cite{42}. After preheating these massive
particles decay perturbatively into the standard model particles, which are then thermal-
ized and the universe enters into radiation dominated era with a black body spectrum at a
temperature $T_{re}$, named as reheating temperature.

Although the physical processes involved during reheating are complex, this phase can
be parameterized in terms of three parameters, reheating temperature $T_{re}$, the effective
equation of state of matter during reheating $w_{re}$ and duration of reheating that is given in
terms of number of e-foldings $N_{re}$. The reheating temperature cannot be constrained from
CMB and LSS observations, but, it is assumed that $T_{re}$ should be above the electroweak scale
so that the weak scale dark matter can be produced. In a more conservative approach $T_{re}$
should be above 10 Mev for successful big-bang nucleosynthesis. The reheating temperature
can be as low as 2.5 to 4 MeV, for considering late-time entropy production by massive
particle decay\cite{43,44}. By considering instant reheating we can also put an upper bound
on the reheating temperature $T_{re}$ to be of the order of scale of inflation, which is $10^{16}$ GeV
for current upper bounds on tensor-to-scalar ratio from Planck. The second parameter of
reheating is effective equation of state $w_{re}$ representing evolution of energy density of the
cosmic fluid during reheating. This parameter is, in general, time dependent and its value
changes from $-\frac{1}{3}$ to $\frac{1}{3}$ from the end of inflation to the onset of radiation dominated era.
For the reheating occurring due to perturbative decay of massive inflaton, $w_{re}$ is 0 and for
instant reheating it is $\frac{1}{3}$. The evolution of equation of state during preheating and the early
thermalization state was studied in\cite{45} by using lattice numerical simulation for quadratic
potential interacting with light fields, and it was found that the equation of state starts from
$w_{re} = 0$ after inflation and saturates around $w_{re} \sim 0.2 - 0.3$ long before the thermalization
of the universe. This analysis was generalized in\cite{46,47} for inflaton potentials behaving
as $|\phi|^{2n}$ near $|\phi| = 0$ and flatter beyond some scale $|\phi| = M$ by taking into account the
fragmentation of the inflaton field and ignoring coupling to massless fields, and it was found
that the equation of state $w_{re}$ reaches $1/3$ for $n > 1$ after sufficient long time, while, it
remains 0 for $n = 1$. The third parameter to describe reheating is its duration, which
can be defined in terms of number of e-foldings from the end of inflation to the beginning
of radiation dominated epoch. This duration is incorporated in the number of e-foldings
$N_k$ during inflation from the time, when the Fourier mode $k$ corresponding to the horizon
size of present observable universe leaves the Hubble radius during inflation, to the end of
inflation. The e-foldings $N_k$ depends on the potential of inflaton and it should be between
46 to 70 to solve horizon problem. The upper bound on $N_k$ arises from assuming that the
universe reheats instantaneously, and the lower bound comes from considering the reheating
temperature at the electroweak scale. In\cite{48,49} a detailed analysis of upper bound on $N_k$
we performed for various scenarios and it was shown that, for some cases, $N_k$ can be as large
as 107.
In [50–52] it was shown that the above mentioned reheating parameterization can be used to constrain various models of inflation. The reheating temperature $T_{re}$ and the e-folds during reheating $N_{re}$ can be expressed in terms of spectral index $n_s$ by assuming $w_{re}$ to be constant during reheating [50–52]. By imposing that the effective equation of state during reheating lies between 0 and 0.25 and the temperature at the end of reheating $T > 100$ GeV, one can obtain bounds on spectral index $n_s$ and $N_k$, which translates to bounds on tensor-to-scalar ratio. As various models of inflation predict similar values of $n_s$ and $r$, it has been shown in [53] that by imposing constraints on these reheating parameters this degeneracy can be removed. The bounds on reheating parameters were also used to constrain tachyon inflation [54], where inflaton have a DBI kinetic term with inverse cosh and exponential potential. It was shown that one requires effective equation of state during reheating $w_{re} > 1$ to satisfy Planck-2018 observations.

In this work we use these reheating parameters to constrain K-inflation with DBI kinetic term with monomial potentials and PNGB potential, and K-inflation with power-law kinetic term with monomial and exponential potentials. Reheating constraints on noncanonical inflation with inflaton having DBI kinetic term and PNGB potential are already considered in [24] with Planck-2015 data. Here we revisit tachyon natural inflation with Planck-2018 data along with other potentials with DBI kinetic term.

The work is organized as as follows: in section 2 we discuss the dynamics of K-inflation and present expressions for power spectra. In section 3 we discuss parameterization of reheating phase. We obtain expressions for $T_{re}$ and $N_{re}$ in terms of spectral index by assuming constant effective equation of state during reheating. In section 4 we discuss noncanonical inflation with DBI kinetic term and obtain expressions for $T_{re}$ and $N_{re}$ for monomial and PNGB potential for various choices for $w_{re}$. We use these three parameters to constrain K-inflation with DBI kinetic term. In section 5 we discuss dynamics of noncanonical inflation with power law kinetic term, and obtain $T_{re}$ and $N_{re}$ for monomial and exponential potential with various choices of $w_{re}$. We again use these three parameters to constrain K-inflation with power-law kinetic term. In section 6 we conclude our work.

2. K-INFLATION: GENERAL FRAMEWORK

In K-inflation the inflaton field has a noncanonical kinetic term. The action for inflaton is given as

$$S = \int \sqrt{-g} \left\{ -\frac{1}{16\pi G} R + \mathcal{L}(X, \phi) \right\}, \quad (1)$$

where $\mathcal{L}(X, \phi)$ is the Lagrangian of scalar field, which is a function of kinetic term $X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi$ and the field $\phi$. We can obtain energy-momentum tensor by varying this action with respect to the metric as

$$T_{\mu \nu} = \frac{\partial \mathcal{L}(X, \phi)}{\partial X} \partial_{\mu} \phi \partial_{\nu} \phi - \mathcal{L}(X, \phi) g_{\mu \nu}. \quad (2)$$

This energy-momentum tensor is equivalent to that of a perfect fluid with pressure

$$p = \mathcal{L}(X, \phi), \quad (3)$$
energy density
\[ \rho = 2X \frac{\partial \mathcal{L}}{\partial X} - \mathcal{L} \] (4)
and four-velocity
\[ u_\mu = \sigma \frac{\partial \mu \phi}{\sqrt{2X}}. \] (5)
where \( \sigma \) refers to the sign of \( \dot{\phi} \). The evolution of the universe is described using Friedmann equations
\[ H^2 = \frac{1}{3M_P^2} \rho, \] (6)
\[ \dot{H} = -\frac{1}{2M_P^2} (\rho + p). \] (7)
Here \( M_P = \frac{1}{\sqrt{8\pi G}} \) is the reduced Planck mass. For inflation the second derivative of the scale factor should satisfy the condition \( \ddot{a} a = \dot{H} + H^2 > 0 \), which can be expressed in terms of the slow-roll parameter
\[ \epsilon = -\frac{\dot{H}}{H^2} < 1. \] (8)
For our analysis we define slow-roll parameters in terms of the Hubble flow parameters as
\[ \epsilon_0 \equiv \frac{H_k}{H}, \] (9)
and
\[ \epsilon_i \equiv \frac{d\ln|\epsilon_i|}{dN}, \quad i \geq 0. \] (10)
where \( H_k \) is the Hubble constant during inflation at the time when a particular mode \( k \) leaves the horizon and \( N \) is the number of e-foldings
\[ N = \ln \left( \frac{a}{a_i} \right), \] (11)
where \( a_i \) is the scale factor at the beginning of inflation. The first derivative of Hubble flow parameter with respect to time can be expressed as
\[ \dot{\epsilon}_i = \epsilon_i \epsilon_{i+1}. \] (12)
The first two Hubble flow parameters \( \epsilon_1 \) and \( \epsilon_2 \) can be obtained in terms of energy density and pressure as
\[ \epsilon_1 = \epsilon = \frac{3}{2} \frac{\rho + p}{\rho}, \] (13)
and
\[ \epsilon_2 = \frac{3}{2H} \frac{d}{dt} \left( \frac{\rho + p}{\rho} \right). \] (14)
The power spectra for scalar and tensor perturbations, scalar spectral index $n_s$ and tensor-to-scalar ratio $r$ for K-inflation are computed in [10], and can be expressed in terms of the Hubble flow parameters as

$$P_\zeta = \frac{H^2}{8\pi^2 M_P^2 c_s \epsilon |_{c_S k = aH}}$$

(15)

$$P_h = \frac{2 H^2}{\pi^2 M_P^2 |_{c_S k = aH}}$$

(16)

$$n_s = 1 - 2\epsilon_1 - \epsilon_2,$$

(17)

$$r = 16 c_S \epsilon_1.$$ 

(18)

where

$$c_S^2 = \frac{\partial p}{\partial X} \frac{\partial \rho}{\partial X}$$

(19)

is the sound speed for perturbations. These power spectra are evaluated at the Hubble crossing during inflation for the Fourier mode $k$ of curvature perturbation and tensor perturbation. In K-inflation the condition for Hubble exit is modified as $c_S k = aH$ for scalar perturbations. For CMB analysis the power spectrum for curvature perturbation is expressed as $P_\zeta = A_S \left(\frac{k}{k_0}\right)^{n_s - 1}$, where the amplitude of scalar perturbations $A_S$ is given by Eq. (15). All the three quantities $A_S$, $n_s$ and $r$ are evaluated at pivot scale $k_0$, which is 0.05 Mpc$^{-1}$ for Planck observations, and they depend on the choice of noncanonical kinetic term and potential of inflaton. Bounds on these quantities are provided by CMB and LSS observations, which can be used to put constraints on parameters of the potential and the noncanonical kinetic term of inflaton. Again all these inflationary parameters also appear in reheating temperature and number of e-folds during reheating, which can, along with CMB constraints, be used to analyze models of inflation. In this work we analyze K-inflation having noncanonical kinetic term of DBI form in section and of power-law form. In the next section we obtain relation between reheating parameters, $T_{re}$ and $N_{re}$, and inflationary parameters.

3. PARAMETERIZING REHEATING

As mentioned earlier the reheating phase can be parameterized in terms of thermalization temperature $T_{re}$ at the onset of radiation dominated epoch after reheating, effective equation of state of cosmic fluid $w_{re}$ during reheating and number of e-folds $N_{re}$ for which reheating lasts. In our analysis we consider $w_{re}$ to be constant during reheating. Its value should lie between $-\frac{1}{3}$ to 1. The lower bound on $w_{re}$ comes from the fact that it should be $-\frac{1}{3}$ when inflation ends, and the upper bound arises from the fact that it should be smaller than 1 to satisfy dominant energy condition of general relativity, $\rho \geq |p|$ for the causality condition to be preserved [50, 56, 57].

In this section we express the reheating parameters ($N_{re}$, $T_{re}$ and $w_{re}$) in terms of the quantities that are derivable from inflation models [50, 58, 60]. Assuming a constant equation
of state during reheating and using $\rho \propto a^{-3(1+w)}$, the reheating epoch can be expressed as

$$\frac{\rho_{\text{end}}}{\rho_{\text{re}}} = \left(\frac{a_{\text{end}}}{a_{\text{re}}}\right)^{-3(1+w_{\text{re}})}.$$  

(20)

here the subscript "end" refers to the quantity evaluated at the end of inflation, and the subscript "re" denotes the quantity evaluated at the end of reheating. The number of e-foldings during reheating is obtained using (20) as

$$N_{\text{re}} = \ln \left(\frac{a_{\text{re}}}{a_{\text{end}}}\right) = \frac{1}{3(1+w_{\text{re}})} \ln \left(\frac{\rho_{\text{end}}}{\rho_{\text{re}}}\right).$$  

(21)

where we have used $\rho_{\text{end}} = \frac{3}{2}V_{\text{end}}$ in the last expression as $w = -\frac{1}{3}$ at the end of inflation. At the end of reheating the universe enters into radiation era, hence the energy density at the end of reheating can be expressed in terms of reheating temperature as

$$\rho_{\text{re}} = \frac{\pi^2}{30}g_{\text{re}}T_{\text{re}}^4,$$  

(22)

where $g_{\text{re}}$ is the number of relativistic species at the end of reheating. We will use $g_{\text{re}} = 100$ (the value for standard model of particle physics) for our analysis. Using Eqs. (21) and (22) $N_{\text{re}}$ can be expressed in terms of reheating temperature as

$$N_{\text{re}} = \frac{1}{3(1+w_{\text{re}})} \ln \left(\frac{30\frac{3}{2}V_{\text{end}}}{\pi^2 g_{\text{re}}T_{\text{re}}^4}\right).$$  

(23)

Since the entropy remains conserved between the end of reheating and today, the reheating temperature can be related to the CMB temperature today as

$$T_{\text{re}} = T_0 \left(\frac{a_0}{a_{\text{eq}}}\right) \left(\frac{43}{11g_{\text{re}}}\right)^{1/3} = T_0 \left(\frac{a_0}{a_{\text{eq}}}\right) e^{N_{\text{RD}}} \left(\frac{43}{11g_{\text{re}}}\right)^{1/3},$$  

(24)

where “0” in the subscript denotes the values of the quantities evaluated at present epoch, and “eq” refers to the values evaluated at matter-radiation equality. $N_{\text{RD}}$ in Eq. (24) refers to the number of e-foldings during radiation era, $e^{-N_{\text{RD}}} \equiv \frac{a_0}{a_{\text{eq}}}$. The ratio $\frac{a_0}{a_{\text{eq}}}$ is expressed as

$$\frac{a_0}{a_{\text{eq}}} = \frac{a_0}{a_k} \frac{a_k}{a_{\text{end}}} \frac{a_{\text{end}}}{a_{\text{re}}} a_{\text{re}} a_{\text{eq}} = \frac{a_0H_k}{c_S k} e^{-N_k} e^{N_{\text{re}}} e^{-N_{\text{RD}}}. $$  

(25)

Here the subscript ”k” denotes that the quantity is evaluated at the time when Fourier mode $k$ crosses the Hubble radius during inflation. $N_k$ represents the number of e-folds from this time to the end of inflation, and the condition for horizon crossing $c_S k = a_k H_k$ is also used. Substituting Eq. (25) into Eq. (24), we obtain

$$T_{\text{re}} = \left(\frac{43}{11g_{\text{re}}}\right)^{1/3} \left(\frac{a_0T_0}{c_S k}\right) H_k e^{-N_k} e^{-N_{\text{re}}}.$$

(26)
Again substituting Eq. (26) into Eq. (23), one can find

\[ N_{re} = \frac{4}{3(1 + w_{re})} \left[ \frac{1}{4} \ln \left( \frac{3^2.5}{\pi^2 g_{re}} \right) + \ln \left( \frac{V_{end}^{1/4}}{H_k} \right) + \frac{1}{3} \ln \left( \frac{11g_{re}}{43} \right) + \ln \left( \frac{c_{Sk}}{a_0 T_0} \right) + N_k + N_{re} \right]. \]  

(27)

This, on solving for \( N_{re} \), with assumption \( w_{re} \neq \frac{1}{3} \), gives

\[ N_{re} = \frac{4}{(1 - 3w_{re})} \left[ \frac{-1}{4} \ln \left( \frac{3^2.5}{\pi^2 g_{re}} \right) - \frac{1}{3} \ln \left( \frac{11g_{re}}{43} \right) - \ln \left( \frac{c_{Sk}}{a_0 T_0} \right) - \ln \left( \frac{V_{end}^{1/4}}{H_k} \right) - N_k \right]. \]  

(28)

The reheating process is instantaneous for \( w_{re} = \frac{1}{3} \) and the reheating temperature is at grand unification scale for this case. Hence parameters of reheating cannot be used for constraining models of inflation. Now we use Eq. (26) to obtain the final expression for \( T_{re} \)

\[ T_{re} = \left[ \frac{43}{11g_{re}} \right]^{\frac{1}{4}} a_0 T_0 H_k \exp^{-N_k} \left[ \frac{3^2.5V_{end}}{\pi^2 g_{re}} \right]^{\frac{-1}{3(1+w_{re})}}. \]  

(29)

The expressions for number of e-folds during reheating \( N_{re} \), (28), and reheating temperature \( T_{re} \), (29), are the main results of this section. It is evident that these two quantities depend on inflationary parameters \( H_k \), \( N_k \) and \( V_{end} \), which can be expressed in terms of amplitude of scalar perturbations \( A_s \) and spectral index \( n_s \). Hence bounds on reheating temperature and demanding \( w_{re} \) to lie between \(-\frac{1}{3}\) and 1 provide bounds on \( n_s \). In subsequent sections we use these reheating parameters \( N_{re} \) and \( T_{re} \) to constrain noncanonical inflation with DBI kinetic term and power-law kinetic term.

4. K-INFLATION WITH DBI KINETIC TERM

In this section we consider K-inflation with DBI kinetic term, and monomial potentials and natural inflation potential. The Lagrangian for the scalar field in this case is given as

\[ \mathcal{L} = -V(\phi) \sqrt{1 - \eta^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}. \]  

(30)

Here \( \eta \) has the dimension of \([\text{length}]^2\) and the field \( \phi \) has the dimension of mass. Using this Lagrangian we can obtain the energy density, (4), and pressure, (3), for the background part of the scalar field in a homogeneous and isotropic universe as

\[ \rho = \frac{V(\phi)}{\sqrt{1 - \eta^2 \phi^2}}, \]  

(31)

\[ P = -V(\phi)(1 - \eta^2 \phi^2)^{\frac{1}{2}}. \]  

(32)
Using Eq. (32) we can write the Friedmann equations for Hubble parameter and its first derivative as

\[
H^2 = \frac{1}{3M_P^2} \frac{V(\phi)}{(1 - \eta^2 \dot{\phi}^2)^{\frac{3}{2}}},
\]

\[
\dot{H} = -\frac{V(\phi)\eta^2 \dot{\phi}^2}{2M_P(1 - \eta^2 \dot{\phi}^2)^{\frac{3}{2}}}.
\]

The equation of motion for the background part of the scalar field can be obtained from energy-momentum tensor (2) as

\[
\frac{\ddot{\phi}}{(1 - \eta^2 \dot{\phi}^2)} + 3H\dot{\phi} + \frac{V'(\phi)}{\eta^2 V(\phi)} = 0.
\]

Here "\(r\)" refers to the derivative with respect to \(\phi\). The Hubble flow parameters \(\epsilon_1\) and \(\epsilon_2\), for K-inflation with DBI kinetic term, can be obtained by substituting the expressions for energy density and pressure (32) in Eq. (13) and Eq. (14) as

\[
\epsilon_1 = \frac{3}{2} \eta^2 \dot{\phi}^2,
\]

\[
\epsilon_2 = \frac{2\ddot{\phi}}{H \dot{\phi}}.
\]

Under slow-roll approximation \(\ddot{\phi}\) in Eq. (35) should be smaller than the friction term \(3H\dot{\phi}\), and \(\eta^2 \dot{\phi}^2\) can be neglected in Eq. (33). Hence we obtain

\[
\dot{\phi} = -\frac{V'(\phi)}{3\eta^2 HV(\phi)}, \quad H^2 \sim \frac{V}{3M_P^2},
\]

during inflation. Using these approximations slow-roll parameters \(\epsilon_1\) and \(\epsilon_2\) can be written in terms of the inflaton potential as

\[
\epsilon_1 = \frac{M_P^2}{2} \left(\frac{V'^2}{\eta^2 V^3}\right),
\]

\[
\epsilon_2 = \frac{M_P^2}{\eta^2} \left(-\frac{V''}{V^2} + 3\frac{V'^2}{V^3}\right).
\]

The amplitude of scalar perturbations \(A_S\), spectral index \(n_s\) and tensor-to-scalar ratio can now be obtained in terms of the parameters of inflaton potential using these equations. Another parameter depending on inflaton potential is the number of e-foldings from the time when the Fourier mode \(k\) leaves the Hubble radius during inflation to the end of inflation, which can be obtained using Eq. (38) as

\[
N_k = \int Hdt = -\frac{\eta^2}{M_P} \int_{\phi_k}^{\phi_{end}} \frac{V^2}{V'} d\phi
\]

We now impose reheating constraints on k-inflation having DBI kinetic term with monomial potential and PNGB potential.
4.1. Monomial potential

We consider the following potential

\[ V(\phi) = \frac{1}{2} m^{4-n} \phi^n. \]  

(42)

We choose \( n = \frac{2}{3}, 2 \) and 4 for our analysis. This potential, for canonical single field inflation, in the context of reheating is studied in [8, 50, 52, 56]. Using Eqs. (39) and (40) for potential (42), the slow roll parameters can be obtained as

\[ \epsilon_1 = \frac{M_p^2 n^2}{\eta^2 m^{4-n} \phi^{n+2}}, \]  

(43)

\[ \epsilon_2 = \frac{2 M_p^2 n(n + 2)}{\eta^2 m^{4-n} \phi^{n+2}}. \]  

(44)

At the end of inflation \( \epsilon_1 = 1 \) and hence the value of the scalar field at this time can be obtained using Eq. (43) as

\[ \phi_{\text{end}} = \left( \frac{M_p^2 n^2}{\eta^2 m^{4-n}} \right)^{\frac{1}{n+2}}. \]  

(45)

The number of e-foldings \( N_k \) for monomial potential can be obtained using Eq. (41) as

\[ N_k = -\frac{\eta^2 m^{4-n}}{2 M_p^2 n(n + 2)} (\phi_{\text{end}}^{n+2} - \phi_k^{n+2}). \]  

(46)

Here \( \phi_k \) is the value of inflaton field at the time when mode \( k \) leaves the horizon during inflation. The spectral index \( n_s \) can be obtained by substituting values of \( \epsilon_1 \) and \( \epsilon_2 \) from Eq. (43), Eq. (44) in Eq. (17) at \( \phi = \phi_k \) as

\[ n_s = 1 - \frac{4 M_p^2 n(n + 1)}{\eta^2 m^{4-n} \phi_k^{n+2}}. \]  

(47)

Using this equation we get

\[ \phi_k = \left( \frac{4 M_p^2 n(n + 1)}{(1 - n_s) \eta^2 m^{4-n}} \right)^{\frac{1}{n+2}}, \]  

(48)

and the slow-roll parameter \( \epsilon_1, (43) \) at \( \phi = \phi_k \) is given as

\[ \epsilon_1 = \frac{n^2 (1 - n_s)}{4 n(n + 1)}. \]  

(49)

Putting the values of \( \phi_{\text{end}} \) and \( \phi_k \) from Eq. (45) and Eq. (48) in Eq. (46), the number of e-foldings \( N_k \) can be expressed in terms of spectral index \( n_s \) as

\[ N_k = \frac{n^2 (3 + n_s) + 4 n}{2 n(n + 2)(1 - n_s)}. \]  

(50)
The inflation potential at the end of inflation will be

\[ V_{\text{end}} = \frac{1}{2} m^{4-n} \phi_{\text{end}}^n, \]  

(51)

which can be expressed in terms of \( H_k \) using Eq. (38) as

\[ V_{\text{end}} = 3M_P^2 H_k^2 \phi_{\text{end}}^n. \]  

(52)

Putting the values of \( \phi_{\text{end}} \) and \( \phi_k \) from (45) and (48) we obtain

\[ V_{\text{end}} = 3M_P^2 H_k^2 \left\{ \frac{n^2(1-n_s)}{4n(n+1)} \right\}^{\frac{n}{n+2}}. \]  

(53)

The speed of sound \( c_S \) for monomial potential with DBI kinetic term can be found using Eq. (19) as

\[ c_S = \sqrt{1 - \frac{n^2(1-n_s)}{6n(n+1)}}. \]  

(54)

The Hubble constant \( H_k \) at the time when the mode \( k \) leaves the horizon during inflation can be expressed in terms of scalar amplitude \( A_S \) using Eq. (15) as

\[ H_k = \pi M_P \sqrt{8A_S \epsilon_1 c_S}, \]  

(55)

which can be written in terms of spectral index \( n_s \) and \( A_S \) using Eq. (49) and Eq. (54) as

\[ H_k = \pi M_P \sqrt{8A_S \left\{ \frac{1 - \frac{n^2(1-n_s)}{6n(n+1)}}{4n(n+1)} \right\} \frac{n^2(1-n_s)}{4n(n+1)}}. \]  

(56)

Using the expressions for \( N_k \) (50), \( V_{\text{end}} \) (53) and \( H_k \) (56), we can evaluate reheating temperature \( T_{\text{re}} \) (29) and e-folds during reheating \( N_{\text{re}} \) in terms of spectral index for various equation of state. Fig. 1 depicts the variation of reheating temperature \( T_{\text{re}} \) and \( N_{\text{re}} \) with respect to \( n_s \) for \( n = 2/3, 1, 2, 4 \). We choose four values of effective equation of states during reheating \( w_{\text{re}} \) = \(-1/3, 0, 0.25 \) and 1. The Planck-2018 bounds on \( n_s = 0.9853 \pm 0.0041 \) are also shown in the figure. We have used Planck-2018 value \( A_S = 2.20 \times 10^{-9} \) for scalar amplitude for our analysis. The point, where the curves of all \( w_{\text{re}} \) meets, corresponds to instant reheating, \( N_{\text{re}} \rightarrow 0 \). The curve for \( w_{\text{re}} \) would pass through this point and be vertical.

By demanding that the reheating temperature should be above 100 GeV for weak scale dark matter production, we obtain bounds on spectral index by solving Eqs. (29) and (50) and assuming \(-\frac{1}{3} \leq w_{\text{re}} \leq 1 \) for various choices of \( n \). These bounds on \( n_s \) provides bounds on number of e-folds \( N_k \) from Eq. (50). The tensor-to-scalar ratio \( r \) can be expressed in terms of \( n_s \) using Eqs. (18) and (49) as

\[ r = \frac{4n^2(1-n_s)}{n(n+1)} \left[ 1 - \frac{n^2(1-n_s)}{6n(n+1)} \right]^{\frac{1}{2}}. \]  

(57)
Using this expression the bounds on $n_s$, obtained using reheating temperature and effective equation of state during reheating, can be transferred to the bounds on tensor-to-scalar ratio $r$.

The bounds on $n_s$, $N_k$ and $r$, thus obtained, are listed in Table I. It can be seen from Table I and Fig. 1 that, for $n = 2/3$ and 1, the bounds on $n_s$ lies outside the Planck-2018 bounds, if we demand that the effective equation of state lie between the physically plausible range $0 \leq w_{re} \leq 0.25$. With this range of $w_{re}$ the tensor-to-scalar ratio $r$ for quadratic and quartic potential is slightly greater than joint BICEP2/Keck Array and Planck bounds $r < 0.06$.

The plots between $N_k$ and $n_s$ are shown in the left panel of Fig. 2 for various values of $n$ and $w_{re}$. The tensor-to-scalar ratio $r$ as a function $n_s$ for the four choices of monomial potentials, is shown in right panel of Fig. 2 along with joint 68% and 95% C.L constraints.

FIG. 1: $N_{re}$ and $T_{re}$ as function of $n_s$ for four different values of $n$ of monomial potential. The vertical pink region shows Planck-2018 bounds on $n_s$ and dark pink region represents a precision of $10^{-3}$ from future observations \[61\]. The horizontal purple region corresponds to $T_{re}$ of 10 MeV from BBN and light purple region corresponds to 100GeV of electroweak scale. Red dotted line corresponds to $w_{re} = -\frac{1}{3}$, blue dashed lines corresponds to $w_{re} = 0$, green solid line corresponds to $w_{re} = 0.25$ and black dot-dashed line is for $w_{re} = 1$. 

Using this expression the bounds on $n_s$, obtained using reheating temperature and effective equation of state during reheating, can be transferred to the bounds on tensor-to-scalar ratio $r$.

The bounds on $n_s$, $N_k$ and $r$, thus obtained, are listed in Table I. It can be seen from Table I and Fig. 1 that, for $n = 2/3$ and 1, the bounds on $n_s$ lies outside the Planck-2018 bounds, if we demand that the effective equation of state lie between the physically plausible range $0 \leq w_{re} \leq 0.25$. With this range of $w_{re}$ the tensor-to-scalar ratio $r$ for quadratic and quartic potential is slightly greater than joint BICEP2/Keck Array and Planck bounds $r < 0.06$.

The plots between $N_k$ and $n_s$ are shown in the left panel of Fig. 2 for various values of $n$ and $w_{re}$. The tensor-to-scalar ratio $r$ as a function $n_s$ for the four choices of monomial potentials, is shown in right panel of Fig. 2 along with joint 68% and 95% C.L constraints.
TABLE I: The allowed values of spectral index $n_s$ and number of e-folds $N_k$ for various values of $n$ for monomial potential by demanding $T_{re} \geq 100\text{GeV}$.

| $n$   | Equation of state | $n_s$       | $N_k$       | $r$       |
|-------|-------------------|-------------|-------------|-----------|
| $n = 2/3$ | $-1/3 \leq w_{re} \leq 0$ | $0.9497 \leq n_s \leq 0.9728$ | $24.72 \leq N_k \leq 45.79$ | $0.0804 \geq r \geq 0.0435$ |
|       | $0 \leq w_{re} \leq 0.25$ | $0.9728 \leq n_s \leq 0.9769$ | $45.79 \leq N_k \leq 54.16$ | $0.0435 \geq r \geq 0.0368$ |
|       | $0.25 \leq w_{re} \leq 1$ | $0.9769 \leq n_s \leq 0.9813$ | $54.16 \leq N_k \leq 66.68$ | $0.0368 \geq r \geq 0.0300$ |
| $n = 1$  | $-1/3 \leq w_{re} \leq 0$ | $0.9468 \leq n_s \leq 0.9711$ | $24.89 \leq N_k \leq 45.97$ | $0.1062 \geq r \geq 0.0577$ |
|       | $0 \leq w_{re} \leq 0.25$ | $0.9711 \leq n_s \leq 0.9755$ | $45.97 \leq N_k \leq 54.34$ | $0.0577 \geq r \geq 0.0489$ |
|       | $0.25 \leq w_{re} \leq 1$ | $0.9755 \leq n_s \leq 0.9801$ | $54.34 \leq N_k \leq 66.84$ | $0.0489 \geq r \geq 0.0398$ |
| $n = 2$  | $-1/3 \leq w_{re} \leq 0$ | $0.9411 \leq n_s \leq 0.9678$ | $25.21 \leq N_k \leq 46.28$ | $0.1566 \geq r \geq 0.0858$ |
|       | $0 \leq w_{re} \leq 0.25$ | $0.9678 \leq n_s \leq 0.9727$ | $46.28 \leq N_k \leq 54.63$ | $0.0858 \geq r \geq 0.0728$ |
|       | $0.25 \leq w_{re} \leq 1$ | $0.9727 \leq n_s \leq 0.9777$ | $54.63 \leq N_k \leq 67.11$ | $0.0728 \geq r \geq 0.0593$ |
| $n = 4$  | $-1/3 \leq w_{re} \leq 0$ | $0.9355 \leq n_s \leq 0.9645$ | $25.50 \leq N_k \leq 46.56$ | $0.2055 \geq r \geq 0.1135$ |
|       | $0 \leq w_{re} \leq 0.25$ | $0.9645 \leq n_s \leq 0.9698$ | $46.56 \leq N_k \leq 54.89$ | $0.1135 \geq r \geq 0.0963$ |
|       | $0.25 \leq w_{re} \leq 1$ | $0.9698 \leq n_s \leq 0.9754$ | $54.89 \leq N_k \leq 67.33$ | $0.0963 \geq r \geq 0.0786$ |

FIG. 2: $N_k$ vs $n_s$, and $r$ vs $n_s$ predictions along with joint 68% and 95% C.L. Planck-2018 constraints for monomial potentials with DBI kinetic term. Here in both panels the orange region corresponds to $w_{re} \leq 0$, green region corresponds to $0 \leq w_{re} \leq 0.25$, yellow region shows $0.25 \leq w_{re} \leq 1$ and purple region corresponds to $w_{re} > 1$.

from Planck-2018. It can be seen from Fig. 2 that $r$ vs $n_s$ predictions for the quadratic and quartic potential with DBI kinetic term lie within 95% C.L. but lie outside 68% C.L. of Planck-2018 data for physically plausible range of $0 \leq w_{re} \leq 0.25$. However, potential with $n = \frac{2}{3}$ and $n = 1$ lie well within 68% of Planck-2018 observations, but, for this the equation of state during reheating should be less than 0.
4.2. Natural inflation potential

The potential for Pseudo-Nambu-Goldstone-Boson, natural inflation is given as [63]

\[ V(\phi) = \Lambda^4 \left[ 1 + \cos \left( \frac{\phi}{f} \right) \right], \]  

where \( f \) is the spontaneous symmetry breaking scale and \( \Lambda \) is explicit symmetry breaking scale for pseudo-Nambu-Goldstone boson. Reheating constraints on this potential with noncanonical kinetic term having DBI form are discussed in [24]. Here we revisit these constraints with Planck-2018 data. Defining \( \beta \equiv \eta^2 f^2 \Lambda^4 M_P^{-2} \) the slow-roll parameters for potential given in Eq. (58) can be obtained using Eqs. (39) and (40) as

\[ \epsilon_1 = \frac{1}{2\beta} \frac{1 - \cos \left( \frac{\phi}{f} \right)}{\left( 1 + \cos \left( \frac{\phi}{f} \right) \right)^2}, \]  

\[ \epsilon_2 = \frac{1}{\beta} \frac{3 - \cos \left( \frac{\phi}{f} \right)}{\left( 1 + \cos \left( \frac{\phi}{f} \right) \right)^2}. \]  

The value of inflaton field at the end of inflation can be obtained by setting \( \epsilon_1 = 1 \) as

\[ \cos \left( \frac{\phi_{\text{end}}}{f} \right) = \frac{-(4\beta + 1) + \sqrt{(1 + 16\beta)}}{4\beta}. \]  

The spectral index \( n_s \) can be obtained by substituting values of \( \epsilon_1 \) (59) and \( \epsilon_2 \) (60) in Eq. (17) as

\[ n_s = 1 - \frac{1}{\beta} \frac{1 - \cos \left( \frac{\phi}{f} \right)}{\left( 1 + \cos \left( \frac{\phi}{f} \right) \right)^2} - \frac{1}{\beta} \frac{3 - \cos \left( \frac{\phi}{f} \right)}{\left( 1 + \cos \left( \frac{\phi}{f} \right) \right)^2} \]  

\[ = 1 - \frac{2}{\beta} \frac{2 - \cos \left( \frac{\phi}{f} \right)}{\left( 1 + \cos \left( \frac{\phi}{f} \right) \right)^2}. \]
Number of e-foldings for potential (58) can be expressed using Eq. (41) as: For natural inflation potential Eq. (58), \( N_k \) can be written as:

\[
N_k = \beta \int_{\phi_k}^{\phi_{\text{end}}} \frac{1 + \cos \left( \frac{\phi}{f} \right)}{\sin \left( \frac{\phi}{f} \right)} d\phi
\]

\[
= \beta \left[ \cos \left( \frac{\phi_{\text{end}}}{f} \right) - \cos \left( \frac{\phi_k}{f} \right) \right] + 2\beta \ln \left[ \frac{\cos \left( \frac{\phi_{\text{end}}}{f} \right) - 1}{\cos \left( \frac{\phi_k}{f} \right) - 1} \right],
\]

where again \( \phi_{\text{end}} \) and \( \phi_k \) are the values of inflaton field at the end of inflation and at the time the mode \( k \) leaves inflationary horizon during inflation respectively. Defining \( \cos \left( \frac{\phi_{\text{end}}}{f} \right) = x \) and \( \cos \left( \frac{\phi_k}{f} \right) = y \), Eq. (61) for number of e-folds \( N_k \) can be written as

\[
N_k = \beta x - \beta y + 2\beta \ln (x - 1) - 2\beta \ln (y - 1).
\]

The spectral index \( n_s \), (63), at \( \phi = \phi_k \) will have the form in terms of \( y \) as

\[
n_s = 1 - \frac{2}{\beta} \left( \frac{2 - y}{1 + y} \right)^2.
\]

To express \( N_k \) in terms of \( n_s \), Eq. (66) can be solved for \( y \) as

\[
y = 1 + \frac{1 + 2\beta - 2n_s\beta - \sqrt{1 + 6\beta - 6n_s\beta}}{n_s\beta - \beta},
\]

and \( x \) is given by Eq. (61). Inflaton potential at the end of inflation can be given as

\[
V_{\text{end}} = \Lambda^4 \left[ 1 + \cos \left( \frac{\phi_{\text{end}}}{f} \right) \right],
\]

which can be written using Eq. (38) as

\[
V_{\text{end}} = 3M_p^2H_k^2 \left[ 1 + \cos \left( \frac{\phi_{\text{end}}}{f} \right) \right] \left[ 1 + \cos \left( \frac{\phi_k}{f} \right) \right]^{-1}
\]

\[
= 3M_p^2H_k^2 \frac{(1 + x)}{(1 + y)}.
\]

From Eq. (19), the speed of sound \( c_S \) at \( \phi = \phi_k \) can be written as

\[
c_S = \sqrt{1 - \frac{1}{3\beta} \left( 1 - \cos \left( \frac{\phi_k}{f} \right) \right)^2},
\]

\[
= \sqrt{1 - \frac{1}{3\beta} \left( 1 - y \right)^2}.
\]
The value of the Hubble constant at the time when Fourier mode $k$ leaves the inflationary horizon during inflation can again be expressed in terms of amplitude of scalar perturbations $A_S$ by putting the values of $\epsilon_1$ (59), and $c_s$, (70), in Eq. (15) as

$$H_k = \pi M_P \sqrt{8 A_S \frac{1 - y}{2 \beta (1+y)^2} \sqrt{1 - \frac{1}{3 \beta (1+y)^2}}}.$$  \hspace{1cm} (71)

We can express $N_k$, $V_{end}$ and $H_k$ in terms of spectral index by substituting the value of $y$ from Eq. (67) and $x$ from Eq. (61) in Eq. (55), Eq. (69) and Eq. (71), and then using these expressions the reheating temperature $T_{re}$ and number of e-folds during reheating $N_{re}$ can be obtained in terms of spectral index from Eqs. (28) and (29). We have chosen $\beta = 35, 50, 100$ and 125 for our analysis. Increasing $\beta$ beyond 125 does not affect the results. The variation of $N_{re}$ and $T_{re}$ with respect to $n_s$, along with Planck-2018 bounds on $n_s = 0.9853 \pm 0.0041$, is represented in Fig. 3 for various values of effective equation state during reheating. Again the curves for various values of $w_{re}$ meet at the point corresponding to instant reheating, $N_{re} \rightarrow 0$. The curve for $w_{re} = 1/3$ would pass through this point and be vertical.

By imposing the bounds on $T_{re}$, i.e., $T_{re} > 100 \text{ GeV}$ for weak scale dark matter production, we obtain bounds on $n_s$ for various equation of states $w_{re}$ solving Eq. (29). Again the tensor-to-scalar ration for natural inflation with DBI kinetic term can be obtained from Eqs. (16,59,70) as

$$r = \frac{8}{2 \beta (1+y)^2} \sqrt{1 - \frac{1}{3 \beta (1+y)^2}}.$$  \hspace{1cm} (72)

The bounds on $n_s$ obtained from $T_{re}$ and $w_{re}$ can give bounds on $N_k$ and $r$. These bounds for various choices of $\beta$ are given in Table II.

| $\alpha$ | Equation of state | $n_s$ | $N_k$ | $r$ |
|----------|------------------|-------|-------|-----|
| $\beta = 35$ | $-1/3 \leq w_{re} \leq 0$ | $0.9369 \leq n_s \leq 0.9631$ | $25.08 \leq N_k \leq 46.06$ | $0.1072 \geq r \geq 0.0480$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9631 \leq n_s \leq 0.9678$ | $46.06 \leq N_k \leq 54.37$ | $0.0480 \geq r \geq 0.0377$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9678 \leq n_s \leq 0.9725$ | $54.37 \leq N_k \leq 66.78$ | $0.0377 \geq r \geq 0.0274$ |
| $\beta = 50$ | $-1/3 \leq w_{re} \leq 0$ | $0.9384 \leq n_s \leq 0.9648$ | $25.10 \leq N_k \leq 46.10$ | $0.1159 \geq r \geq 0.0547$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9648 \leq n_s \leq 0.9696$ | $46.10 \leq N_k \leq 54.42$ | $0.0547 \geq r \geq 0.0438$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9696 \leq n_s \leq 0.9745$ | $54.42 \leq N_k \leq 66.86$ | $0.0438 \geq r \geq 0.0329$ |
| $\beta = 100$ | $-1/3 \leq w_{re} \leq 0$ | $0.9398 \leq n_s \leq 0.9665$ | $25.13 \leq N_k \leq 46.16$ | $0.1287 \geq r \geq 0.0645$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9665 \leq n_s \leq 0.9713$ | $46.16 \leq N_k \leq 54.49$ | $0.0645 \geq r \geq 0.0529$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9713 \leq n_s \leq 0.9764$ | $54.49 \leq N_k \leq 66.95$ | $0.0529 \geq r \geq 0.0412$ |
| $\beta = 125$ | $-1/3 \leq w_{re} \leq 0$ | $0.9401 \leq n_s \leq 0.9668$ | $25.14 \leq N_k \leq 46.18$ | $0.1318 \geq r \geq 0.0669$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9668 \leq n_s \leq 0.9717$ | $46.18 \leq N_k \leq 54.51$ | $0.0669 \geq r \geq 0.0552$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9717 \leq n_s \leq 0.9767$ | $54.51 \leq N_k \leq 66.97$ | $0.0552 \geq r \geq 0.0432$ |

TABLE II: The allowed values of spectral index $n_s$ and number of e-folds $N_k$ for various values of $\beta$ for natural inflation potential, obtained by imposing $T_{re} \geq 100 \text{GeV}$.
FIG. 3: $N_{re}$ and $T_{re}$ as function of $n_s$ for natural inflation potential. The vertical pink region shows Planck-2018 bounds on $n_s$ and dark pink region represents a precision of $10^{-3}$ from future observations [61]. The horizontal purple region corresponds to $T_{re}$ of 10 MeV from BBN and light purple region corresponds to 100GeV of electroweak scale. Red dotted line corresponds to $w_{re} = -\frac{1}{3}$, blue dashed lines corresponds to $w_{re} = 0$, green solid line corresponds to $w_{re} = 0.25$ and black dot-dashed line is for $w_{re} = 1$.

It can be seen from Table II that with physically plausible range $0 \leq w_{re} \leq 0.25$ the bounds on $n_s$ and $r$ are compatible with Planck-2018 observations for $\beta < 125$. We also show $N_k$ vs $n_s$ and $r$ vs $n_s$ plots for PNGB potential with DBI kinetic term in Fig. 4. It is evident from the figure that the values of $n_s$ and $r$ predicted in this model lie within 1σ contour of Planck-2018 joint constraints for physically plausible range $0 \leq w_{re} \leq 0.25$ shown by green region in the figure. Our results for natural inflation with DBI kinetic term agree with [24].
5. K-INFLATION WITH POWER-LAW KINETIC TERM

In this section we will analyze K-inflation with power-law kinetic term. The Lagrangian density for this case is given as \[ L(X, \phi) = X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi), \] (73)

where $M$ has dimension of mass and $\alpha$ is dimensionless. For $\alpha = 1$ the Lagrangian reduces to usual canonical scalar field. Using Eqs. (4) and (3) the energy density and pressure can be obtained as

\[ \rho_\phi = (2\alpha - 1) X \left( \frac{X}{M} \right)^{\alpha - 1} + V(\phi). \] (74)

\[ p_\phi = X \left( \frac{X}{M} \right)^{\alpha - 1} - V(\phi), \quad X \equiv \frac{1}{2} \dot{\phi}^2. \] (75)

Thus, the Friedman equations for Hubble constant and its first derivative become

\[ H^2 = \frac{8\pi G}{3} \left[ (2\alpha - 1) X \left( \frac{X}{M^4} \right)^{\alpha - 1} + V(\phi) \right], \] (76)

\[ \dot{H} = -4\pi G(\rho_\phi + p_\phi) = -\frac{1}{3M_P^2} X \left( \frac{X}{M^4} \right)^{\alpha - 1}. \] (77)

FIG. 4: $N_k$ vs $n_s$ and $r$ vs $n_s$ plots for natural inflation potential along with joint 68%CL and 95%CL Planck-2018 constraints. In both the panels the orange region corresponds to $w_{re} < 0$, the green region corresponds to $0 < w_{re} < 0.25$, the yellow region corresponds to $0.25 < w_{re} < 1$ and the purple region corresponds to $w_{re} > 1$. In the right panel of the figure blue dashed line corresponds to $N_k = 46$, black dashed line corresponds to $N_k = 55$ and red dashed line corresponds to $N_k = 67$. These values of $N_k$ corresponds to bounds on $n_s$ obtained by demanding $T_{re} > 100$ GeV for different values of $w_{re}$. The solid black line in both the panels of the figure corresponds to $\beta = 125$ and the filled region corresponds to $\beta < 125$. 

5. K-INFLATION WITH POWER-LAW KINETIC TERM

In this section we will analyze K-inflation with power-law kinetic term. The Lagrangian density for this case is given as \[ L(X, \phi) = X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi), \] (73)

where $M$ has dimension of mass and $\alpha$ is dimensionless. For $\alpha = 1$ the Lagrangian reduces to usual canonical scalar field. Using Eqs. (4) and (3) the energy density and pressure can be obtained as

\[ \rho_\phi = (2\alpha - 1) X \left( \frac{X}{M} \right)^{\alpha - 1} + V(\phi). \] (74)

\[ p_\phi = X \left( \frac{X}{M} \right)^{\alpha - 1} - V(\phi), \quad X \equiv \frac{1}{2} \dot{\phi}^2. \] (75)

Thus, the Friedman equations for Hubble constant and its first derivative become

\[ H^2 = \frac{8\pi G}{3} \left[ (2\alpha - 1) X \left( \frac{X}{M^4} \right)^{\alpha - 1} + V(\phi) \right], \] (76)

\[ \dot{H} = -4\pi G(\rho_\phi + p_\phi) = -\frac{1}{3M_P^2} X \left( \frac{X}{M^4} \right)^{\alpha - 1}. \] (77)
The evolution equation for inflaton $\phi$ can be obtained by energy-momentum tensor (2) as

$$\ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha - 1} + \left(\frac{V'(\phi)}{\alpha(2\alpha - 1)}\right)\left(\frac{2M_4^{4\alpha - 1}}{\dot{\phi}^2}\right) = 0.$$  

(78)

Using the definition of slow-roll parameter $\epsilon = -\dot{H}/H^2$, along with Eq. (77), the Hubble constant (76) can be written as

$$H^2 \left[1 - \left(\frac{2\alpha - 1}{3\alpha}\right)\epsilon\right] = \frac{1}{3M_P^2}V(\phi),$$

(79)

under slow-roll approximation $\epsilon \ll 1$ this reduces to

$$H^2 = \frac{V(\phi)}{3M_P^2}.$$  

(80)

For slow-roll $\ddot{\phi}$ is much smaller than the friction term in Eq. (78), hence using Eq. (80), we obtain

$$\dot{\phi} = \left[\left(\frac{M_P}{\alpha\sqrt{3}}\right)\left(-\frac{V'(\phi)}{\sqrt{V}}\right)\left(2M_4^{4\alpha - 1}\right)^{\frac{1}{2\alpha - 1}}\right].$$

(81)

The two Hubble flow parameters $\epsilon_1$, (13), and $\epsilon_2$, (14), for this case can be obtained using Eqns (74), (75), (80) and Eq. (81), as

$$\epsilon_1 = \frac{1}{\alpha}\left(\frac{3M_4^4}{V}\right)^{\alpha - 1}\left(-\frac{M_PV'}{\sqrt{2V}}\right)^{2\alpha \frac{1}{2\alpha - 1}},$$

(82)

$$\epsilon_2 = \frac{-2\epsilon_1}{2\alpha - 1}\left[2\alpha\left(\frac{V''V}{V'^2}\right) - (3\alpha - 1)\right].$$

(83)

Now the number of e-foldings $N_k$ from the time when mode $k$ leaves the horizon to the end of inflation, in case of power-law kinetic term, can be obtained by using

$$N_k = -\int_{\phi_{\text{end}}}^{\phi_k} \frac{H}{\dot{\phi}} \, d\phi,$$

(84)

and substituting the values of $H$ and $\dot{\phi}$ from Eqs. (80) and (81) respectively in this expression for various choices of potentials. The speed of sound $c_S$, defined in Eq. (19), can be obtained using Eq. (74) and Eq. (75) as

$$c_S^2 = \frac{1}{2\alpha - 1}.$$  

(85)

The speed of sound here is only function of $\alpha$ and independent of choice of potential.
5.1. Monomial potentials

We consider following monomial potential with power-law kinetic term

\[ V(\phi) = \frac{1}{2} m^{4-n} \phi^n, \text{ where } n > 0. \]  

(86)

The two Hubble-flow parameters for this potential can be obtained using Eqs. (82) and (83)

\[ \epsilon_1 = \left[ \frac{1}{\alpha} \left( \frac{6M^4}{m^{4-n}} \right)^{(\alpha-1)} \left( -\frac{nM_p}{\sqrt{2}} \right)^{2\alpha} \frac{1}{\phi^{2a+n\alpha-n}} \right]^{\frac{1}{2\alpha-1}}, \]  

(87)

\[ \epsilon_2 = \frac{2\epsilon_1 \gamma}{n}. \]  

(88)

Here

\[ \gamma = \frac{2\alpha + n(\alpha - 1)}{2\alpha - 1}. \]  

(89)

The value of the inflaton field at the end of inflation, \( \phi_{\text{end}} \), can be obtained by setting \( \epsilon_1 = 1 \) as

\[ \phi_{\text{end}} = \left[ \frac{1}{\alpha} \left( \frac{6M^4}{m^{4-n}} \right)^{(\alpha-1)} \left( -\frac{nM_p}{\sqrt{2}} \right)^{2\alpha} \right]^{\frac{1}{(2\alpha-1)\gamma}}. \]  

(90)

We can obtain the values of \( H \) and \( \dot{\phi} \) from Eqs. (80) and (81) respectively for monomial potential (86) and substitute these values in Eq. (84) to obtain the number of e-foldings \( N_k \) as

\[ N_k = \frac{\phi_k^\gamma - \phi_{\text{end}}^\gamma}{\gamma} \left[ \left( \frac{m^{4-n}}{12M^4} \right)^{\alpha-1} \frac{\alpha}{nM_p^{2\alpha}} (-1)^{2(\alpha-1)} \right]^{\frac{1}{2\alpha-1}}. \]  

(91)

With \( \phi_{\text{end}} \) from Eq. (90), we can obtain the expression for inflaton field \( \phi_k \) when mode \( k \) leaves the horizon as

\[ \phi_k = C_k^{1/\gamma} \left( N_k \gamma + \frac{n}{2} \right)^{\gamma}. \]  

(92)

where,

\[ C_k = \left\{ \left( \frac{n(-M_P)^{2\alpha}}{\alpha} \right) \left( \frac{12M^4}{m^{4-n}} \right)^{\alpha-1} \right\}^{\frac{1}{2\alpha-1}}. \]  

(93)

The first slow roll parameter \( \epsilon_1 \) can be expressed as a function of \( N_k \) by substituting Eq. (92) in Eq. (87) as

\[ \epsilon_1 = \frac{n}{2N_k \gamma + n}. \]  

(94)

Putting value of \( \epsilon_1 \) and \( \epsilon_2 \) from Eq. (94) and Eq. (88) in the definition of scalar spectral index \( n_s \) (17), we obtain

\[ n_s = 1 - 2 \frac{(n + \gamma)}{2N_k \gamma + n}. \]  

(95)
which, on solving for e-folds \( N_k \) becomes
\[
N_k = \frac{1}{2\gamma} \left( \frac{2(\gamma + n)}{1 - n_s} - n \right) .
\] (96)

Using Eq. (80) and Eq. (86), the value of potential at the end of inflation can be obtained as
\[
V_{\text{end}} = 3M_P^2 H_k^2 \left( \frac{\phi_{\text{end}}}{\phi_k} \right)^n .
\] (97)

Substituting Eq. (90) and Eq. (92) in Eq. (97) we get
\[
V_{\text{end}} = 3M_P^2 H_k^2 \left( \frac{n}{2N_k \gamma + n} \right)^{2\gamma} .
\] (98)

By substituting the value of \( c_S \) from Eq. (85) and \( \epsilon_1 \) from Eq. (94) in Eq. (15), we can express Hubble constant \( H_K \) at the time when the Fourier mode \( k \) leaves the inflationary horizon as
\[
H_k = \pi M_P \sqrt{8A_S \left( \frac{n}{2N_k \gamma + n} \right) \left( \frac{1}{\sqrt{2\alpha - 1}} \right)} .
\] (99)

Using Eq. (96) we can express Eq. (98) and Eq. (99) for \( V_{\text{end}} \) and \( H_K \) respectively in terms of \( n_s \). Further, these expressions can be used to obtain the reheating temperature \( T_{\text{re}} \), given by (29), and number of e-folds during reheating \( N_{\text{re}} \), given by (28), as a function of spectral index \( n_s \).

The variation of \( N_{\text{re}} \) and \( T_{\text{re}} \) as a function of \( n_s \), for various values of effective equation of states, is depicted in Fig. 5 along with Planck-2018 bounds \( n_s = 0.9853 \pm 0.0041 \). Here we choose quadratic \( n = 2 \) and quartic \( n = 4 \) potentials for our analysis. It is evident from Fig. 5 that, for both of these potentials, the variation of \( T_{\text{re}} \) and \( N_{\text{re}} \) with respect to \( n_s \) is independent of the power of kinetic term \( \alpha \). Again we imposing the bounds on \( T_{\text{re}} \), i.e., \( T_{\text{re}} > 100 \text{ GeV} \) to obtain bounds on \( n_s \) for various equation of states \( w_{\text{re}} \) by solving Eq. (29). Now the tensor-to-scalar ratio \( r \), (18), for monomial potential with power-law kinetic term can be obtained using the expressions for \( c_S \), (85) and \( \epsilon_1 \) (94), as
\[
r = \left( \frac{1}{\sqrt{2\alpha - 1}} \right) \left( \frac{16n}{2N_k \gamma + n} \right) .
\] (100)

Using the bounds on \( n_s \), obtained from reheating consideration, we get the bounds on \( N_k \) and tensor-to-scalar ratio \( r \) for various \( w_{\text{re}} \) from Eqs. (96) and (100). These bounds on \( n_s \), \( N_k \) and \( r \), thus obtained, for quadratic and quartic potential with power-law kinetic term are provided in Table IV and Table VI. The bounds on \( n_s \) obtained from reheating are independent of \( \alpha \) for quadratic potentials. However, the bounds obtained on tensor-to-scalar ratio \( r \) depend on \( \alpha \) for both the potentials. It can be seen from Table VI that, with \( \alpha = 4 \), the bounds on tensor-to-scalar ratio 0.086 \( \geq r \geq 0.0740 \) lie slightly above than the joint BICEP2/Keck array and Planck-2018 bound \( r < 0.06 \) [62] for physically plausible range 0 \( \leq w_{\text{re}} \leq 0.25 \) for effective equation of state during reheating. But, for larger values of \( \alpha \) the bounds on \( r \) are in agreement with BICEP2/Keck array bound.
FIG. 5: $N_{re}$ and $T_{re}$ as function of $n_s$ for four different values of $\alpha$ of quadratic potential with power-law kinetic term. The vertical pink region shows Planck-2018 bounds on $n_s$ and dark pink region represents a precision of $10^{-3}$ from future observations [61]. The horizontal purple region corresponds to $T_{re}$ of 10 MeV from BBN and light purple region corresponds to 100GeV of electroweak scale. Red dotted line corresponds to $w_{re} = -\frac{1}{3}$, blue dashed lines corresponds to $w_{re} = 0$, green solid line corresponds to $w_{re} = 0.25$ and black dot-dashed line is for $w_{re} = 1$.

Plots for $N_k$ vs $n_s$ for quadratic and quartic potentials are shown in Fig. 7. Here we have chosen only one value $\alpha = 4$ for quadratic potential, as the variation of $N_k$ with respect to $n_s$ is independent of $\alpha$. In case of quartic potential also we have chosen only the smallest and largest values of $\alpha$, because the variation of functional dependence of $N_k$ on $n_s$ with respect to $\alpha$ is very small. Fig. 8 depicts the $r$ vs $n_s$ predictions for quadratic and quartic potential for different values of $\alpha$ and $w_{re}$, along with joint 68% and 95% C.L. constraints from Planck-2018. It can be seen from Fig. 8 that $r$ vs $n_s$ predictions for the quadratic potential with power-law kinetic term lie within 68% C.L. of Planck-2018 constraints for physically plausible range of $0 \leq w_{re} \leq 0.25$. However, for quartic potential the equation of state during reheating should be greater than 0.25 for $r$-$n_s$ predictions to lie within 68% C.L. of Planck-2018 constraints.
FIG. 6: $N_{re}$ and $T_{re}$ as function of $n_s$ for four different values of $\alpha$ of quartic potential with power-law kinetic term. The vertical pink region shows Planck-2018 bounds on $n_s$ and dark pink region represents a precision of $10^{-3}$ from future observations \[61\]. The horizontal purple region corresponds to $T_{re}$ of 10 MeV from BBN and light purple region corresponds to 100GeV of electroweak scale. Red dotted line corresponds to $w_{re} = -\frac{1}{3}$, blue dashed lines corresponds to $w_{re} = 0$, green solid line corresponds to $w_{re} = 0.25$ and black dot-dashed line is for $w_{re} = 1$.

5.2. Exponential potential

We now consider the exponential potential with power-law kinetic term. This potential have the following form

$$V(\phi) = V_0 \exp \left( -\sqrt{\frac{2}{q} \phi M_P} \right). \quad (101)$$

In case of inflation with canonical scalar field this potential provides power-law expansion, $a(t) \propto t^{q/2}$, for flat universe \[64, 66\]. The power-law solutions can also be obtained with this potential in DBI framework \[67\].

We can obtain the slow-roll parameters $\epsilon_1$ and $\epsilon_2$ for this potential using Eq. \[82\] and
\[ \alpha = 4 \]
\[ -1/3 \leq w_{re} \leq 0 \quad 0.9273 \leq n_s \leq 0.9586 \]
\[ 0 \leq w_{re} \leq 0.25 \quad 0.9586 \leq n_s \leq 0.9640 \]
\[ 0.25 \leq w_{re} \leq 1 \quad 0.9640 \leq n_s \leq 0.9709 \]
\[ \alpha = 10 \]
\[ -1/3 \leq w_{re} \leq 0 \quad 0.9287 \leq n_s \leq 0.9589 \]
\[ 0 \leq w_{re} \leq 0.25 \quad 0.9589 \leq n_s \leq 0.9649 \]
\[ 0.25 \leq w_{re} \leq 1 \quad 0.9649 \leq n_s \leq 0.9710 \]
\[ \alpha = 50 \]
\[ -1/3 \leq w_{re} \leq 0 \quad 0.9307 \leq n_s \leq 0.9596 \]
\[ 0 \leq w_{re} \leq 0.25 \quad 0.9596 \leq n_s \leq 0.9653 \]
\[ 0.25 \leq w_{re} \leq 1 \quad 0.9653 \leq n_s \leq 0.9713 \]
\[ \alpha = 100 \]
\[ -1/3 \leq w_{re} \leq 0 \quad 0.9315 \leq n_s \leq 0.9598 \]
\[ 0 \leq w_{re} \leq 0.25 \quad 0.9598 \leq n_s \leq 0.9654 \]
\[ 0.25 \leq w_{re} \leq 1 \quad 0.9654 \leq n_s \leq 0.9714 \]

| Equation of state | \( n_s \) | \( N_k \) | \( r \) |
|-------------------|-----------|-----------|-------|
| \( -1/3 \leq w_{re} \leq 0 \) | \( 0.9152 \leq n_s \leq 0.9510 \) | \( 27.62 \leq N_k \leq 48.36 \) | \( 0.1495 \geq r \geq 0.0863 \) |
| \( 0 \leq w_{re} \leq 0.25 \) | \( 0.9510 \leq n_s \leq 0.9581 \) | \( 48.36 \leq N_k \leq 56.53 \) | \( 0.0863 \geq r \geq 0.0740 \) |
| \( 0.25 \leq w_{re} \leq 1 \) | \( 0.9581 \leq n_s \leq 0.9654 \) | \( 56.53 \leq N_k \leq 68.69 \) | \( 0.0740 \geq r \geq 0.0610 \) |
| \( -1/3 \leq w_{re} \leq 0 \) | \( 0.9180 \leq n_s \leq 0.9522 \) | \( 28.06 \leq N_k \leq 48.73 \) | \( 0.0867 \geq r \geq 0.0505 \) |
| \( 0 \leq w_{re} \leq 0.25 \) | \( 0.9522 \leq n_s \leq 0.9590 \) | \( 48.73 \leq N_k \leq 56.87 \) | \( 0.0505 \geq r \geq 0.0433 \) |
| \( 0.25 \leq w_{re} \leq 1 \) | \( 0.9590 \leq n_s \leq 0.9662 \) | \( 56.87 \leq N_k \leq 68.99 \) | \( 0.0433 \geq r \geq 0.0357 \) |
| \( -1/3 \leq w_{re} \leq 0 \) | \( 0.9209 \leq n_s \leq 0.9533 \) | \( 28.88 \leq N_k \leq 49.10 \) | \( 0.0364 \geq r \geq 0.0215 \) |
| \( 0 \leq w_{re} \leq 0.25 \) | \( 0.9533 \leq n_s \leq 0.9598 \) | \( 49.10 \leq N_k \leq 57.48 \) | \( 0.0215 \geq r \geq 0.0185 \) |
| \( 0.25 \leq w_{re} \leq 1 \) | \( 0.9598 \leq n_s \leq 0.9667 \) | \( 57.48 \leq N_k \leq 69.52 \) | \( 0.0185 \geq r \geq 0.0153 \) |
| \( -1/3 \leq w_{re} \leq 0 \) | \( 0.9219 \leq n_s \leq 0.9536 \) | \( 29.23 \leq N_k \leq 49.69 \) | \( 0.0253 \geq r \geq 0.0150 \) |
| \( 0 \leq w_{re} \leq 0.25 \) | \( 0.9536 \leq n_s \leq 0.9600 \) | \( 49.69 \leq N_k \leq 57.75 \) | \( 0.0150 \geq r \geq 0.0129 \) |
| \( 0.25 \leq w_{re} \leq 1 \) | \( 0.9600 \leq n_s \leq 0.9668 \) | \( 57.75 \leq N_k \leq 69.76 \) | \( 0.0129 \geq r \geq 0.0108 \) |

**Table III:** The allowed values of spectral index \( n_s \) and number of e-folds \( N_k \) for various values of \( \alpha \) for quadratic potential with power-law kinetic term considering \( T_{re} \geq 100GeV \).

| Equation of state | \( n_s \) | \( N_k \) | \( r \) |
|-------------------|-----------|-----------|-------|
| \( -1/3 \leq w_{re} \leq 0 \) | \( 0.9273 \leq n_s \leq 0.9586 \) | \( 27.02 \leq N_k \leq 47.85 \) | \( 0.1098 \geq r \geq 0.0625 \) |
| \( 0 \leq w_{re} \leq 0.25 \) | \( 0.9586 \leq n_s \leq 0.9640 \) | \( 47.85 \leq N_k \leq 56.07 \) | \( 0.0625 \geq r \geq 0.0534 \) |
| \( 0.25 \leq w_{re} \leq 1 \) | \( 0.9640 \leq n_s \leq 0.9709 \) | \( 56.07 \leq N_k \leq 68.33 \) | \( 0.0534 \geq r \geq 0.0439 \) |

**Table IV:** The allowed values of spectral index \( n_s \) and number of e-folds \( N_k \) for various values of \( \alpha \) for quartic potential with power-law kinetic term considering \( T_{re} \geq 100GeV \).

\[
\epsilon_1 = \left[ \frac{1}{\alpha} \left( \frac{3M^4}{V_0} \right)^{\alpha-1} \left( \frac{1}{\sqrt{q}} \right)^{2\alpha} \exp\left(-\frac{1}{\sqrt{q}} \frac{\phi(-1)}{M^p} \right) \right]^{\frac{1}{\alpha-1}},
\]
\[
\epsilon_2 = 2\epsilon_1 \left( \frac{\alpha - 1}{2\alpha - 1} \right).
\]
(a) $N_k$ vs $n_s$ for quadratic potential  
(b) $N_k$ vs $n_s$ for quartic potential

FIG. 7: $N_k$ as function of $n_s$ for quadratic potential and quartic potential with power law kinetic term.

FIG. 8: $r$ vs $n_s$ predictions for quadratic and quartic potentials with four different choice of $\alpha$ along with joint 68% C.L. and 95% C.L. Planck-2018 constraints. Here the orange region corresponds to $w_{re} \leq 0$, green region corresponds to $0 \leq w_{re} \leq 0.25$, yellow region shows $0.25 \leq w_{re} \leq 1$ and purple region corresponds to $w_{re} > 1$.

Now we evaluate $\phi_{end}$, the value of inflaton field at the end of inflation, by setting $\epsilon_1 = 1$ as

$$\phi_{end} = -\frac{M_P}{\alpha - 1} \frac{\sqrt{q}}{2} \ln \left[ \frac{1}{\alpha} \left( \frac{3M^4}{V_0} \right)^{\alpha-1} \left( \sqrt{\frac{1}{q}} \right)^{2\alpha} \right]. \quad (104)$$

To obtain the number of e-foldings $N_k$ from the time when the Fourier mode $k$ leaves the Hubble radius to the end of inflation, for $|\alpha| > 1$, we put values of $H$ and $\dot{\phi}$ from Eqs. (80) and (81) into Eq. (84), and on integrating it we get

$$N_k = \frac{\phi_k^{\frac{\alpha - 1}{2\alpha - 1}} - \phi_{end}^{\frac{\alpha - 1}{2\alpha - 1}}}{\left( \frac{V_0}{3M^4} \right)^{\frac{\alpha - 1}{\alpha}} \left( \sqrt{\frac{q}{2}} \right)^{\frac{2\alpha}{2\alpha - 1}} \alpha^{-\frac{1}{2\alpha - 1}}}. \quad (105)$$
Substituting \( \phi_{\text{end}} \) from Eq. (104) and solving for \( \phi_k \), the value of inflaton field at horizon crossing, we obtain

\[
\phi_k = -\sqrt{\frac{q}{2}} \frac{M_P}{(\alpha - 1)} \ln \left[ \frac{1}{\sqrt[\alpha]{\frac{3M^4}{V_0}}} \left( \sqrt{\frac{2}{q}} \right)^{2\alpha} \left( \frac{1}{2^{\frac{1}{\alpha - 1}}} + N_k \left( \frac{\alpha - 1}{2\alpha - 1} \right) 2^{\frac{2\alpha - 1}{\alpha - 1}} \right)^{2\alpha - 1} \right].
\]  

(106)

Substituting Eq. (106) in Eq. (102), we can evaluate the first slow-roll parameter \( \epsilon_1 \) at \( \phi = \phi_k \) as

\[
\epsilon_1 = \frac{2\alpha - 1}{(2\alpha - 1) + 2N_k(\alpha - 1)}.
\]  

(107)

Putting Eq. (107) and Eq. (103) in Eq. (17), we get the expression for spectral index

\[
n_s = 1 - 2 \frac{(3\alpha - 2)}{(2\alpha - 1) + 2N_k(\alpha - 1)}.
\]  

(108)

Using this equation the number of e-folds \( N_k \) can be expressed in terms of spectral index \( n_s \) as

\[
N_k = \frac{(3\alpha - 2)}{(\alpha - 1)(1 - n_s)} - \frac{(2\alpha - 1)}{2(\alpha - 1)}.
\]  

(109)

The value of the potential at the end of inflation can be expressed in terms of \( H_k \) using Eqs. (80) and (101) as

\[
V_{\text{end}} = 3M^2_PH^2_k \left[ \exp \left( -\sqrt{\frac{2}{q}} \frac{\phi_{\text{end}}}{M_P} \right) \right].
\]  

(110)

Solving above equation with Eq. (104) and Eq. (106)

\[
V_{\text{end}} = 3M^2_PH^2_k \left[ \frac{2\alpha - 1}{2(\alpha - 1) + 2N_k(\alpha - 1)} \right]^{\frac{2\alpha - 1}{\alpha - 1}}.
\]  

(111)

The Hubble constant at \( \phi = \phi_k \), can be obtained by substituting the expression for speed of sound \( c_S \), (85), and slow-roll parameter \( \epsilon_1 \), (107) in Eq. (15) as

\[
H_k = \pi M_P \sqrt{8A_S \frac{2\alpha - 1}{(2\alpha - 1) + 2N_k(\alpha - 1)} \frac{1}{\sqrt{2\alpha - 1}}}. \]

(112)

Using equation Eq. (112), Eq. (110), and Eq. (109), we can obtain \( H_k \) and \( V_{\text{end}} \) as a function of \( n_s \). Again, by using these expressions for \( H_k \) and \( V_{\text{end}} \), the reheating temperature \( T_{\text{re}} \) and the number of e-folds during reheating \( N_{\text{re}} \) can be obtained in terms of \( n_s \) from Eqs. (29) and (28) respectively. The variation of \( N_{\text{re}} \) and \( T_{\text{re}} \) with respect to \( n_s \) for various choices of \( \alpha \) and effective equation of state during reheating is shown in Fig. 9.

By demanding \( T_{\text{re}} > 100 \) GeV we obtain bounds on \( n_s \) using Eq. (29) for various values of \( w_{\text{re}} \). Again from these bounds on \( n_s \), the bounds on \( N_k \) can be obtained using Eq. (109). The tensor-to-scalar ratio \( r \) for exponential potential with power-law kinetic term can be obtained by substituting Eq. (85), Eq. (107) in Eq. (18) as

\[
r = \frac{16\sqrt{2\alpha - 1}}{2\alpha - 1 + 2N_k(\alpha - 1)}, \quad \alpha > 1.
\]  

(113)
FIG. 9: $N_{re}$ and $T_{re}$ as function of $n_s$ for four different values of $\alpha$ of exponential potential with power-law kinetic term. The vertical pink region shows Planck-2018 bounds on $n_s$ and dark pink region represents a precision of $10^{-3}$ from future observations [61]. The horizontal purple region corresponds to $T_{re}$ of 10 MeV from BBN and light purple region corresponds to 100GeV of electroweak scale. Red dotted line corresponds to $w_{re} = -\frac{1}{3}$, blue dashed lines corresponds to $w_{re} = 0$, green solid line corresponds to $w_{re} = 0.25$ and black dot-dashed line is for $w_{re} = 1$.

Using this expression we can get bounds on $r$ from the bounds on $N_k$, obtained by reheating consideration. These bounds on $n_s$, $N_k$ and $r$ for exponential potential are listed in Table [V]. It can be seen from the Table that, with $\alpha = 4$, the bounds $r$, i.e., $0.139 \geq r \geq 0.12$ are higher than the joint BICEP2/Keck array and Planck-2018 bounds $r < 0.06$ [62] for physically plausible range $0 \leq w_{re} \leq 0.25$. However, for this range of $w_{re}$, the bounds on $r$ are compatible with joint BICEP2/Keck array and Planck-2018 bounds for larger values of $\alpha$.

The plots between $N_k$ and $n_s$ are shown in left panel of Fig. [10] for various values of $\alpha$ and $w_{re}$. The $r - n_s$ predictions, along with joint 68% and 95% C.L. Planck-2018 constraints constraints, for this case are shown in the right panel of Fig. [10]. It can bee seen from the figure that, for all values of $\alpha$, the effective equation of state during reheating $w_{re}$ should be greater than 1 to satisfy Planck-2018 joint constraints on $r$ and $n_s$, which violates causality.
TABLE V: The allowed values of spectral index $n_s$ and number of e-folds $N_k$ for various values of $\alpha$ for exponential potential with power-law kinetic term, considering $T_{re} \geq 100\,\text{GeV}$.

| $\alpha$ | Equation of state | $n_s$ | $N_k$ | $r$ |
|----------|------------------|------|-------|-----|
| $\alpha = 4$ | $-1/3 \leq w_{re} \leq 0$ | $0.8888 \leq n_s \leq 0.9341$ | $31.81 \leq N_k \leq 54.45$ | $0.2353 \geq r \geq 0.1395$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9341 \leq n_s \leq 0.9431$ | $54.45 \leq N_k \leq 63.30$ | $0.1395 \geq r \geq 0.1204$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9431 \leq n_s \leq 0.9528$ | $63.30 \leq N_k \leq 76.44$ | $0.1204 \geq r \geq 0.0999$ |
| $\alpha = 10$ | $-1/3 \leq w_{re} \leq 0$ | $0.8967 \leq n_s \leq 0.9385$ | $30.1371 \leq N_k \leq 51.39$ | $0.1286 \geq r \geq 0.0765$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9385 \leq n_s \leq 0.9469$ | $51.39 \leq N_k \leq 59.72$ | $0.0765 \geq r \geq 0.0660$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9469 \leq n_s \leq 0.9559$ | $59.72 \leq N_k \leq 72.09$ | $0.0660 \geq r \geq 0.0549$ |
| $\alpha = 50$ | $-1/3 \leq w_{re} \leq 0$ | $0.9020 \leq n_s \leq 0.9410$ | $30.01 \leq N_k \leq 50.53$ | $0.0527 \geq r \geq 0.0317$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9410 \leq n_s \leq 0.9489$ | $50.53 \leq N_k \leq 58.58$ | $0.0317 \geq r \geq 0.0274$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9489 \leq n_s \leq 0.9575$ | $58.58 \leq N_k \leq 70.54$ | $0.0274 \geq r \geq 0.0229$ |
| $\alpha = 100$ | $-1/3 \leq w_{re} \leq 0$ | $0.9034 \leq n_s \leq 0.9415$ | $30.25 \leq N_k \leq 50.64$ | $0.0366 \geq r \geq 0.0221$ |
| & $0 \leq w_{re} \leq 0.25$ | $0.9415 \leq n_s \leq 0.9493$ | $50.64 \leq N_k \leq 58.63$ | $0.0221 \geq r \geq 0.0192$ |
| & $0.25 \leq w_{re} \leq 1$ | $0.9493 \leq n_s \leq 0.9578$ | $58.63 \leq N_k \leq 70.52$ | $0.0192 \geq r \geq 0.0159$ |

FIG. 10: In the left panel $N_k$ as function of $n_s$ is shown for $\alpha = 4, 50$ and, $100$ of exponential potential with power law kinetic term. In the right panel predictions of $r$ vs $n_s$ for exponential potential with power law kinetic term along with joint 68%CL and 95%CL Planck-2018 constraints is shown for four choices of $\alpha$ ($\alpha = 4, 10, 50$ and, $100$). Here in both panels the orange region corresponds to $w_{re} \leq 0$, green region corresponds to $0 \leq w_{re} \leq 0.25$, yellow region shows $0.25 \leq w_{re} \leq 1$ and purple region corresponds to $w_{re} \geq 1$.

6. CONCLUSION

K-inflation is an alternative to the standard single field slow-roll inflation. In this case the noncanonical kinetic term of the scalar field drives inflation. This scenario has an advantage over the canonical single field inflation as it increases the viability of various
inflaton potentials, ruled out from Planck CMB observations, by reducing the tensor-to-scalar ratio. In this work we analyze models of K-inflation in the light of reheating. The phase of reheating can be parameterized in terms of three parameters, namely reheating temperature $T_{re}$, effective equation of state of cosmic fluid during reheating $w_{re}$ and number of e-folds during reheating $N_{re}$. These three parameters can be related to the amplitude of scalar perturbations, spectral index and other inflationary parameters depending on inflaton kinetic term and potential and can be used to constrain models of inflation (see [50–52] for constraints on canonical single field inflation). We derive expressions for $T_{re}$ and $N_{re}$ in terms of $w_{re}$, $n_s$ and other inflationary parameters, and then use these expressions to constrain models of K-inflation having kinetic term of DBI form and power-law form. With DBI kinetic term we choose monomial and exponential potential. In [45] it was shown that the equation of state during reheating $w_{re}$ should lie between 0 to 0.25 for various reheating scenario. By imposing $0 \leq w_{re} \leq 0.25$ and demanding that the reheating temperature $T_{re} > 100$ GeV for weak scale dark matter production, we find bounds on $n_s$ and number of e-foldings $N_k$ from the time when the mode $k$ corresponding to the pivot scale, $k_0 = 0.05$ Mpc$^{-1}$ leaves the Hubble radius during inflation to the end of inflation. These bounds on $n_s$ and $N_k$ can be transferred the bounds on tensor-to-scalar ratio $r$, and hence the allowed region in $n_s - r$ plane for models of inflation is restricted.

The bounds obtained for $N_k$ and $r$ for K-inflation with DBI kinetic term and monomial potentials $V \sim \phi^n$ are shown in Table: I and the $r - n_s$ predictions for various equation of state during reheating are shown in Fig. 2. We find that the tensor-to-scalar ratio $r > 0.0786$ for $w_{re} \leq 1$ in case of quartic potential, which is greater than the joint BICEP2/Keck array and Planck-2018 bound $r < 0.06$ [62]. The $r - n_s$ predictions for $n = 2/3$ and $n = 1$ lie within the Planck-2018 1σ constraints for $w_{re} < 0$. The bounds on $N_k$ and $r$ for natural inflation potential are shown in Table. III and the predictions for $r - n_s$ are represented in Fig. 4. We find that the natural inflation with DBI kinetic term is compatible with Planck-2018 observations for physically plausible range $0 \leq w_{re} \leq 0.25$.

In case of K-inflation with power-law kinetic term (73) the bounds on $N_k$ and $r$ for quadratic and quartic potential are shown in Table. IV and Table V respectively. We find that, with $\alpha = 4$ for quadratic potential, the tensor-to-scalar ratio $r > 0.0740$ for $w_{re} \leq 0.25$ and $r > 0.0610$ for $w_{re} \leq 1$, which is slightly greater than the joint BICEP2/Keck array and Planck-2018 bound $r < 0.06$ [62]. However, this potential is compatible with Planck-2018 bounds on $r$ for physically plausible range $0 \leq w_{re} \leq 0.25$ with larger values of $\alpha$. The $r - n_s$ predictions for these potentials are shown in Fig. 8. It can be seen from the figure that, for these predictions to lie within Planck-2018 1σ constants, the reheating equation of state $w_{re} \geq 0.25$ for quartic potential. The bounds on $N_k$ and $r$ for exponential potential with power-law kinetic term are shown in Table. VI. We find that, for $\alpha = 4$, the tensor-to-scalar ratio $0.1395 \geq r \geq 0.1204$ for physically plausible range $0 \leq w_{re} \leq 0.25$ and $r \geq 0.0999$ for $w_{re} \leq 1$, which are quite larger than the joint BICEP2/Keck array and Planck-2018 bounds $r < 0.06$ [62]. Again bounds on $r$ are compatible with Planck-2018 bounds for larger values of $\alpha$. The $r - n_s$ predictions for exponential potential are shown in Fig. VII. It is evident from the figure that, for these predictions to lie within joint 68% constraints from Planck-2018 observations, the effective equation of state during reheating should be greater than 1.

These models of K-inflation are well motivated from string theory, and they have similar
predictions. By imposing constraints from reheating we can remove this degeneracy. In [53, 68] it is shown that the spectrum of gravitational waves generated during inflation is sensitive to the equation of state during reheating. We find different allowed values of $w_{re}$ for different models to satisfy joint 68% and 95% C.L. constraints on $r - n_s$ from Planck-2018 observations. Hence, our analysis with future detection of gravitational waves can help us to find suitable model of inflation with noncanonical kinetic term.

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