Timelike and spacelike hadron form factors, Fock state components and light-front dynamics

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A unified description of spacelike and timelike hadron form factors within a light-front model was successfully applied to the pion. The model is extended to the nucleon to study the role of $q\bar{q}$ pair production and of nonvalence components in the nucleon form factors. Preliminary results in the spacelike range $0 \leq Q^2 \leq 10 \text{(GeV/c)}^2$ are presented.

The controversial experimental results for the nucleon form factors (FF) in the spacelike (SL) region \cite{1} and the striking differences between data \cite{2} and theoretical expectations from perturbative QCD \cite{3} in the timelike (TL) region motivate a unified investigation of SL and TL nucleon FF for a better knowledge of the nucleon internal structure.

Within the light-front dynamics \cite{4}, we developed an approach for a global description of the pion FF in both the SL and the TL regions, which is able to reproduce successfully the experimental data from $q^2 = -10 \text{ (GeV/c)}^2$ up to $q^2 = 10 \text{ (GeV/c)}^2$ \cite{5}. We extend now the procedure already applied in the pion case to the nucleon, considered as a system of constituent quarks of mass $m = .200 \text{ GeV}$. As a first step, we study the nucleon FF in the SL region. In particular we aim to investigate the relevance of the contribution due to the $q\bar{q}$ pair creation by the incoming virtual photon.

For a unified description of the nucleon FF in both the SL and the TL regions a reference frame with $q^+ \neq 0$ is needed. Therefore, following Ref. \cite{7}, we calculate the nucleon form factor in the SL region in the reference frame where $q_\perp = 0$ and $q^+ = [Q^2]^{1/2}$.

To introduce a proper Dirac structure for the nucleon Bethe-Salpeter amplitude (BSA), we describe the $qqq$-nucleon interaction through an effective Lagrangian, which represents an isospin zero, spin zero coupling for the (1,2) quark pair, as in Ref. \cite{8} but with $\alpha = 1$.

Then, the nucleon Bethe-Salpeter amplitude is approximated as follows

$$
\Phi_{N}(k_1, k_2, k_3, p) = \left[ S(k_1) \, i\tau_y \, \gamma^5 \, S_C(k_2) C \, S(k_3) + S(k_3) \, i\tau_y \, \gamma^5 \, S_C(k_1) C \, S(k_2) + \ldots \right]
$$
Figure 1. Diagram (a) : valence, triangle contribution \((0 < k_1^+ < P_{N'}^-, 0 < k_3^+ + q^+ < P_{N'}^-)\). Diagram (b) : nonvalence contribution \((0 > k_3^+ > -q^+)\). The symbol \(\times\) on a quark line indicates a quark on the mass shell, i.e. \(k_{on} = (m^2 + k_{\perp}^2)/k^+\).

\[
S(k_3) i\tau_y \gamma^5 S_C(k_2)C S(k_1) \right] \Lambda(k_1, k_2, k_3) \chi_{\tau N} U_N(p, \sigma)
\]

where \(\Lambda(k_1, k_2, k_3)\) describes the symmetric momentum dependence of the vertex function upon the quark momentum variables, \(k_i\). In the Bethe-Salpeter amplitude \(U_N(p, \sigma)\) is the nucleon Dirac spinor and \(\chi_{\tau N}\) the nucleon isospin state, while the quantities \(S(k)\) and \(S_C(k)\) are the propagators for the quark and the charge conjugated quark, respectively.

The matrix elements of the macroscopic em nucleon current,

\[
\langle p', \sigma'| j^\mu | p, \sigma \rangle = U_N(p', \sigma') \left[ -F_2^N(Q^2) \frac{p'^\mu + p^\mu}{2M_N} + \left( F_1^N(Q^2) + F_2^N(Q^2) \right) \gamma^\mu \right] U_N(p, \sigma)
\]

where \(F_1^{N(2)}(Q^2)\) is the Dirac (Pauli) nucleon FF, in impulse approximation can be approximated microscopically by the Mandelstam formula \([6]\) as follows

\[
\langle \sigma', p'| j^\mu | p, \sigma \rangle = N_c \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \times

\left\{ \Phi_{N}^\sigma(k_1, k_2, k_3, p') S^{-1}(k_1) S^{-1}(k_2) T_3^\mu N (k_1, k_2, k_3, p) \right\}
\]

where \(N_c\) is the number of colors, the traces are performed over Dirac (\(\Gamma\)) and isospin (\(\tau\)) indexes, the subscripts in the traces indicate the particles involved, and \(T_3^\mu\) is the quark-photon vertex for the third quark. The quark-photon vertex has isoscalar and isovector contributions, namely \(T_3^\mu = (T_a^\mu + T_d^\mu)/2 + \tau_z (T_a^\mu - T_d^\mu)/2 = T_S^\mu + \tau_z T_V^\mu\).

We assume a suitable fall-off of the momentum components of the BSA \(\Lambda(k_1, k_2, k_3)\) and \(\Lambda(k_1, k_2, k_3')\), to make finite the four dimensional integrations in Eq. \(3\). Furthermore, we assume that the singularities of \(\Lambda(k_1, k_2, k_3)\) and \(\Lambda(k_1, k_2, k_3')\) give a negligible contribution to the integrations on \(k_1^-\) and on \(k_3^-\) and perform these integrations taking into account only the poles of the quark propagators. As a result the matrix elements of the current become the sum of: i) a triangle term (Fig. 1 (a)), with the spectator quarks on their mass shell, and both the initial and the final nucleon vertexes in the valence sector, and ii) a nonvalence term (Fig. 1 (b)), where the \(q\bar{q}\) pair production appears and only the final
nucleon vertex is in the valence sector. This latter term can be seen as an higher Fock state contribution to the FF. Then, both the isoscalar and the isovector part of the quark-photon vertex $I^\mu_3$ contain a purely valence contribution and a contribution corresponding to the pair production (Z-diagram), which can be decomposed in a bare, point-like term and a vector meson dominance (VMD) term (according to the decomposition of the photon state in bare, hadronic [and leptonic] contributions):

$$I^\mu_i(k, q) = N^i_i \theta(p^+ - k^+) \theta(k^+) \gamma^\mu + \theta(q^+ + k^+) \theta(-k^+) \left[Z^i_i N^i_i \gamma^\mu + Z^i_V \Gamma_{VMD}^\mu(k, q, i)\right]$$  \hspace{1cm} (4)

with $i = IS, IV$ and $N_{IS} = 1/6, N_{IV} = 1/2$. The first term in (4) is the bare coupling of the triangle contribution, while $Z^i_b, Z^i_V$ are renormalization constants to be determined from the phenomenological analysis of the data.

The term $\Gamma_{VMD}^\mu(k, q, i)$ is obtained through the same microscopical VMD model already used in the pion case with the same VM eigenstates \cite{5}.

In the valence vertexes the 3-momentum dependence is approximated through a nucleon wave function a la Brodsky (PQCD inspired), namely

$$W_N \sim \left(\xi_1 \xi_2 \xi_3\right)^{-0.12} \left[\frac{(A_0^2(1, 2, 3) - 9m^2)}{\beta^2 + M_0^2(1, 2, 3)}\right] \left[1 + A(M_0^2(1, 2, 3) - 9m^2) \exp\left(-\frac{(M_0^2(1, 2, 3) - 9m^2)/2\beta^2}{2}\right)\right]$$  \hspace{1cm} (5)

where $M_0(1, 2, 3)$ is the free mass of the three quark system. The parameter $\beta$ is fixed through the nucleon magnetic moments, for which the values $\mu_p = 2.878$ (Exp. 2.793) and $\mu_n = -1.859$ (Exp. $-1.913$) are obtained.

In the non-valence vertex, needed to evaluate the Z-diagram contribution, the momentum dependence is approximated by

$$G^\mu_N \sim \left[\frac{1}{\beta^2 + M_0^2(1, 2)} \left\{\frac{1}{[\beta^2 + M_0^2(3', 2)]} + \frac{1}{[\beta^2 + M_0^2(3', 1)]}\right\}\right]$$  \hspace{1cm} (6)
with $M_0(i,j)$ the free mass of $i$ and $j$ quarks.

The nucleon Sachs form factors can be obtained from the matrix elements of the current by means of proper traces. The preliminary results of Figs. 2 and 3 show clearly the relevance of the pair production process and then of the nonvalence components for the nucleon FF. In particular the interplay of valence and nonvalence contributions generates the possible zero in the proton electric form factor.

Figure 3. Nucleon magnetic form factors. Left panel: $G_M^p(Q^2)/\mu_p G_D(Q^2)$ vs $Q^2$ with $G_D(Q^2) = [1 + Q^2/(0.71(\text{GeV}/c)^2)]^{-2}$. Right panel: $G_M^n(Q^2)/\mu_n G_D(Q^2)$ vs $Q^2$. Solid lines: full calculation, i.e., sum of triangle plus pair production terms. Dotted lines: triangle contribution only. Data from [9].

REFERENCES

1. J. Arrington, Phys. Rev. C 68 (2003) 034325.
2. Fenice Collaboration, Nucl. Phys. B 517, 3 (1998).
3. J. Ellis, M. Karliner, New J. Phys. 4 (2002) 18.
4. S.J. Brodsky, H.C. Pauli, and S.S. Pinsky, Phys. Rep. 301, 299 (1998).
5. J.P.B.C. de Melo, T. Frederico, E. Pace and G. Salmè, Phys. Lett. B 581, 75 (2004); Phys. Rev. D 73, 074013 (2006).
6. S. Mandelstan, Proc. Royal Soc. (London) A 233, 248 (1956).
7. F.M. Lev, E. Pace and G. Salmè, Nucl. Phys. A 641, 229 (1998); Phys. Rev. Lett. 83, 5250 (1999); Phys. Rev. C 62, 064004 (2000).
8. W.R.B. de Araújo, E.F. Suisso, T. Frederico, M. Beyer and H.J. Weber, Phys. Lett. B 478, 86 (2000); Nucl. Phys. A 694, 351 (2001).
9. www.jlab.org/ cseely/nucleons.html and Refs. therein.