Abstract: Random plane generators may use various types of the random number algorithms to create multidimensional planes. At the same time, the discrete Descartes random planes have to be uniform. The matter is that using the concept of the uncontrolled random generation may lead to a result of weak quality due to initial sequences having either insufficient uniformity or skipping of the random numbers. This article offers a new approach for creating the absolute twisting uniform two-dimensional Descartes planes based on a model of complete twisting sequences of uniform random variables without repetitions or skipping. The simulation analyses confirm that the resulted random planes have an absolute uniformity. Moreover, combining the parameters of the original complete uniform sequences allows a significant increase in the number of created planes without using additional random access memory.

Keywords: Pseudorandom Number Generator, Stochastic Sequences, Congruential Numbers, Twister Generator, Random Plane, Random Field

Introduction

In our previous studies (Deon and Menyaev, 2016a; 2016b; 2017) there were proposed several pseudorandom number generators, particularly nsDeonYuliTwist32D, which offers a technique of using no congruential twisting array. This generator allows the creation of absolutely complete twister uniform sequences having various lengths.

The direction of Random Plane (RP) Generators (RPG) employs a stochastic process at the time of creating the points distributed on N-dimensional plane. Here we consider a two-Dimensional (2D) plane only. Other discrete-dimensional planes have the same initial properties. Each coordinate of RP-generated points may belong to its own Random Field (RF). An analysis of the last sources sums up the following selected types of random fields: Conditional RF (Quattioni et al., 2004; Sutton and McCallum, 2012), Markov RF (Sarawagi and Cohen, 2004; Bekkerman et al., 2006), Gaussian RF (Rimstad and Omre, 2014), uniformed RF (Xiao, 2010) and others (Qi et al., 2004; Dachian and Nahapetian, 2009). In the application areas the RPGs are often applied in graphical images (Kumar and Hebert, 2003), phone systems (Sung and Jurański, 2009), advertising applications, etc. Next, the RPGs are actively used in fundamental studies, starting from 2D theoretical modeling (Gnedenko, 1998; Feller, 2008), Monte Carlo plane simulation (Newman and Barkema, 1996; Spanos and Zeldin, 1998), factorial development (Kim and Zabih, 2002), realizations for training systems (Sha and Pereira, 2003), etc. and biomedical engineering (Menyaev and Zharov, 2005; 2006a; 2006b; Menyaev et al., 2013; 2016; Koonce et al., 2017).

The principles of all these studies are based on the conception of random planes, in which the Descartes plane features have to satisfy the following properties: (1) The generation process has to provide the uniqueness (i.e., no repetitions) of each point on the plane and (2) the generation process has to keep the completeness (i.e., without skipping) for all created points. These properties should be considered as a ‘natural filter’ for choosing the random number generator.

Let’s consider two of them in brief. If the generation uses the twister generator MT19937 (Matsumoto and Nishimura, 1998; Matsumoto et al., 2006; 2007; Saito and Matsumoto, 2008), then the result of this attempt is very discussable since this generator in DieHard Tests (Berger and Zorn, 2006; Novark and Berger, 2010; Alani, 2010) demonstrates a uniqueness level of 0.7, which is equivalent to the level of repeatability 1-0.7 = 0.3. On the other hand, we may use the twister generator nsDeonYuliTwist32D (Deon and Menyaev, 2017), which is guaranteed to create the complete uniform twisting sequences of an arbitrary size having no repetitions and skipping of elements. Now the question here is: Would it be possible to observe the Descartes properties in the current particular task?
Let's consider this issue more in detail. For this, let us use the aforementioned twister generator, which is capable of creating complete uniform twisting sequences of arbitrary size. Below is the program code for modeling a grid of the discrete Descartes plane $U \times V$.

The program algorithm generates the integer random numbers independently along the $U$ and $V$ independent axes. Matrix $A$ is the discrete plane indicator; its cells $a[u, v] \in A$ with indices $u$ and $v$ correspond to the coordinates of discrete points $<u, v> \in U \times V$. The value of cell $a[u, v] \in A$ indicates the quantity of attempts to create independently the corresponding point $<u, v>$ on the generated plane. Without loss of generality and in order to visualize the result, we assign the amount of discrete coordinates by given sets $U = V = \{0, 1, 2, 3, 4, 5, 6, 7\}$, which in the binary representation corresponds to the length $w = 3$ bits for each coordinate. According to the previous studies (Deon and Menyaev, 2016b; 2017), let's choose a twister generator $nsDeonYuliTwist32D$, which operates on the basis of congruential model $x_{n+1} = (ax_n + c) \mod\ maskW$ with constants $a = 5, c = 1$. The initial values of the twisting sequences are taken here as $x_0 = 1 \in U$ and $x_0 = 4 \in V$. Any other choice of parameters for generation is possible; the essence of the obtained results will not be changed. Program names $P040101$ and $cP040101$ are taken by chance. The chosen programming language is C# available in Microsoft Visual Studio. The use of other dialects of the older C versions (i.e., Win32) or C++ available in dialects of the older C versions (i.e., Win32) or C++ (CLR) provides the same results.

Using $nsDeonYuliTwist32D$; // twister uniform generator namespace P040101
{
    class cP040101
    {
        static void Main(string[] args)
        {
            uint w = 3; // number bit length
cDeonYuliTwist32D GU =
                new cDeonYuliTwist32D();

            GU.x0 = 1; // U sequence beginning
            GU.w = w; // number bit length
            GU.Start(); // GU generator starts
cDeonYuliTwist32D GV =
                new cDeonYuliTwist32D();

            GV.x0 = 4; // V sequence beginning
            GV.w = w; // number bit length
            GV.Start(); // GV generator starts

            int N = 1 << (int)w; // U and V sequences length
            Console.WriteLine("U = \{0\} | V = \{1\}", U[m], V[m]);
            Console.WriteLine();

            for (int m = 0; m < N; m++)
            {
                for (int j = 0; j < N; j++)
                {
                    Console.Write("{0,4}", A[i, j]);
                    Console.WriteLine();
                }
            }

            U = new uint[N]; // U sequence
            V = new uint[N]; // V sequence
            A = new int[N, N]; // result matrix
        }
    }
}

After this code execution the listing below appears:

$$w = 3 \quad N = 8$$

$$\begin{array}{cccccccc}
    i & 0 & & & 1 & 6 & 7 & 4 & 5 & 2 & 3 & 0 \\
    U & 1 & 6 & 7 & 4 & 5 & 2 & 3 & 0 & 1 & 6 & 7 \\
    & V & 4 & 5 & 2 & 3 & 0 & 1 & 6 & 7 & 4 & 3 \\
    i & 1 & & & 3 & 5 & 7 & 1 & 2 & 4 & 6 & 0 \\
    U & 3 & 5 & 7 & 1 & 2 & 4 & 6 & 0 & 3 & 5 & 7 \\
    & V & 1 & 2 & 4 & 6 & 0 & 3 & 5 & 7 & 1 & 4 \\
    i & 2 & & & 7 & 3 & 6 & 2 & 5 & 1 & 4 & 0 \\
    U & 7 & 3 & 6 & 2 & 5 & 1 & 4 & 0 & 7 & 3 & 6 \\
    & V & 2 & 5 & 1 & 4 & 0 & 7 & 3 & 6 & 1 & 5 \\
    i & 3 & & & 6 & 7 & 4 & 5 & 2 & 3 & 0 & 1 \\
    U & 6 & 7 & 4 & 5 & 2 & 3 & 0 & 1 & 6 & 7 & 4 \\
    & V & 5 & 2 & 3 & 0 & 1 & 6 & 7 & 4 & 3 & 2 \\
    i & 4 & & & 5 & 7 & 1 & 2 & 4 & 6 & 0 & 3 \\
    U & 5 & 7 & 1 & 2 & 4 & 6 & 0 & 3 & 5 & 7 & 1 \\
    & V & 2 & 4 & 6 & 0 & 3 & 5 & 7 & 1 & 2 & 3 \\
    i & 5 & & & 3 & 6 & 2 & 5 & 1 & 4 & 0 & 7 \\
    U & 3 & 6 & 2 & 5 & 1 & 4 & 0 & 7 & 3 & 6 & 2 \\
    & V & 5 & 1 & 4 & 0 & 7 & 3 & 6 & 2 & 3 & 4 \\
    i & 6 & & & 7 & 4 & 5 & 2 & 3 & 0 & 1 & 6 \\
    U & 7 & 4 & 5 & 2 & 3 & 0 & 1 & 6 & 7 & 4 & 5 \\
    & V & 2 & 3 & 0 & 1 & 6 & 7 & 4 & 5 & 2 & 1 \\
    i & 7 & & & 7 & 1 & 2 & 4 & 6 & 0 & 3 & 5 \\
    U & 7 & 1 & 2 & 4 & 6 & 0 & 3 & 5 & 7 & 1 & 2 \\
    & V & 4 & 6 & 0 & 3 & 5 & 7 & 1 & 2 & 3 & 4 \\
\end{array}$$

Matrix $A$
In matrix $A$ the cells with values that differed from 1, show that the independent generation of coordinates of the points on the plane doesn’t ensure uniform distribution of random points $<u, v>$. Some points are missed (values 0) and others are present several times (values > 1).

So, the aim of this article is to find a solution for the generation of uniform discrete twisting planes, which possess the Descartes property of a single presence of the random points in nodes of the discretization grid.

**Theory**

One of the options to represent the discrete Descartes plane is an enumeration of all points in the grid nodes, formed from the values of discretization on the corresponding axes. If the location of axes is independent, the grid has a rectangular form. Moreover, if discretization for both axes is the same and uniform, the grid has a square view.

Let’s assume that the square grid includes $N$ points of discretization along each axis. Thus, the total number of grid points is $N \times N = N^2$. To set these points in a random way, an algorithm is required that can provide a random move from one point to another.

Now would be a good time to point out and emphasize the following: The plane is random only if moving from one point to another, while creating the plane, utilizes the stochastic process.

In this case, the requirement of Descartes axes, which prescribes a unique representation of each point, has to be kept. The unambiguity is determined by the discretization of the axes. Uniqueness is provided by an appropriate procedure, which does not allow entering each point of the grid twice or more times. The skipping of vertices of the grid isn’t allowed either. In other words, each point is presented once during generation of all the grid points. In this case, the total enumeration of the points is $N^2$. Following this way, such a grid on the Descartes plane is called uniform and a random Descartes plane, which contains uniform grid, is called uniform Descartes RP.

There are many ways to specify the points on the grid. Let’s name a few of them:

- Rectangular left or right filling of the grid, when one of the axes is selected and at each location of discretization of this axis, the points along the discrete points of the other axis are placed on the grid
- Rectangular top or bottom filling of the grid under the same conditions
- Diagonal filling of the grid under the same conditions
- The secondary indexing of the discrete points along the Descartes axes

This is not a whole list of possible techniques. The options to choose aren’t limited and can be organized by the designer in any possible way. Note that items (1) – (3) create ordinary Descartes planes and item (4) allows obtaining the random Descartes planes, if the secondary index is a result of the stochastic process.

In this article, an option of secondary indexing of the discrete marks on the Descartes axes is adopted. Let’s demonstrate this by an example, in which the congruential generation of random numbers $x_{r+1} = (ax_r + c) \& maskW$ is used as secondary indexing base. In order to visualize the results, we take the complete uniform sequences of random numbers $x \in \{0, 1, 2, 3\} = \{0, 1, 2, 3\}$ having length $w = 2$ bits. In this case, each complete sequence contains $N = 2^w = 2^2 = 4$ elements. Without loss of generality, let’s assume that $a = 1 \in [LN-I]$ and $c = 3 \in [LN-I]$. In total, four congruential sequences are possible: $<0, 3, 2, 1>$, $<1, 0, 3, 2>$, $<2, 0, 1, 3>$, $<3, 2, 1, 0>$. These sequences allow creating various random tracks on uniform Descartes RP.

If random value $x_0 = 1$ is chosen as an initial value, then the designated generator $GU$ creates the sequence $U = <1, 0, 3, 2>$. From this it follows that the initial random vertex will be located on the vertical part of the grid with horizontal discrete mark 1 along the $U$ axis (Fig. 1).

If the second independent generator, which is designated as $GV$, uses the initial random value $x_0 = 3$, then sequence $V = <3, 2, 1, 0>$ is created. From this it’s obvious that the second coordinate has the value of 1 for the initial random point $<1, 3>$. The next vertex has coordinates $<0, 2>$. Both obtained vertices are connected by an arc, forming the beginning of the random track. Then, vertex $<3, 1>$ will be placed on this track. Finally, the vertex with coordinates $<2, 0>$ completes the random track. For clarity, the visual representation of this track is shown in Fig. 1.

Regarding the random sequence of secondary indices $<1, 0, 3, 2>$ along the $U$ axis, four sequences along the $V$ axis are possible:

1) $U = <1, 0, 3, 2>$
   $V = <3, 2, 1, 0>$
2) $U = <1, 0, 3, 2>$
   $V = <2, 1, 0, 3>$
3) $U = <1, 0, 3, 2>$
   $V = <1, 0, 3, 2>$
4) $U = <1, 0, 3, 2>$
   $V = <0, 3, 2, 1>$

These sequences $V$ can be interpreted as the left circular shift (Deon and Menyaev, 2016b) of the original sequence $<3, 2, 1, 0>$. Figure 2 shows four tracks of an interaction of the pairs of sequences on the $U$ and $V$ axes. These tracks provide an exact one-time generation of each vertex on a grid of the Descartes RP.
The program code for this task is presented below, in which all the vertices are generated on a grid of the Descartes RP. Random numbers have the length of \( w = 3 \) bits. In each sequences \( U \) and \( V \) there are \( N = 2^w = 2^3 = 8 \) random numbers with \( x \in [0, N-1] = \{0,1,2,3,4,5,6,7\} \). Both sequences \( U \) and \( V \) are given by a congruential model \( x_{i+1} = (ax_i + c)\&\text{mask}_W \) with coefficients \( a = 5 \) and \( c = 1 \). The first sequence begins with value \( x_0 = 1 \) and has the form \( U = \langle 1, 6, 7, 4, 5, 2, 3, 0 \rangle \). The second sequence begins with value \( x_0 = 4 \) and has the form \( V = \langle 4, 5, 2, 3, 0, 1, 6, 7 \rangle \). Each cell of matrix \( A \) corresponds to one vertex on a grid of RP. The value of cell \( a \in A \) shows how many times the corresponding vertex is generated. Program names \textit{P040201} and \textit{cP040201} are selected by chance.

```csharp
namespace P040201
{
    class cP040201
    {
        static void Main(string[] args)
        {
            uint w = 3;                                // number bit length
            int N = 1 << (int)w;     // U and V sequences length
            Console.WriteLine("w = {0}  N = {1}", w, N);
            uint[] U = new uint[8] {1,6,7,4,5,2,3,0};
            uint[] V = new uint[8] {4,5,2,3,0,1,6,7};
            int[,] A = new int[N, N];                  // result matrix
            for (int i = 0; i < N; i++) {             // one of the axis
                Console.Write("i = {0,3} |   ", i);
                for (int j = 0; j < N; j++) A[i, j] = 0;
            }
            for (int i = 0; i < N; i++)                // another axis
                Console.Write("U = ");
            for (int j = 0; j < N; j++) A[U[j], V[j]]++; // <u,v> generation counter
            for (int i = 0; i < N; i++)                // one of the axis
                Console.Write("V = ");
            for (int j = 0; j < N; j++) A[U[j], V[j]]++; // <u,v> generation counter
            Console.WriteLine("Matrix A");
            for (int i = 0; i < N; i++)
                for (int j = 0; j < N; j++) Console.Write("{0,4}", A[i, j]);
            Console.WriteLine();
            Console.ReadKey();                        // result viewing
        }
    }
}
```

After this code execution the listing below appears:

```
263
```

```plaintext
w = 3  N = 8
i = 0 | U = 1 6 7 4 5 2 3 0
  | V = 4 5 2 3 0 1 6 7
i = 1 | U = 1 6 7 4 5 2 3 0
  | V = 5 2 3 0 1 6 7 4
i = 2 | U = 1 6 7 4 5 2 3 0
  | V = 2 3 0 1 6 7 4 5
i = 3 | U = 1 6 7 4 5 2 3 0
  | V = 3 0 1 6 7 4 5 2
i = 4 | U = 1 6 7 4 5 2 3 0
  | V = 0 1 6 7 4 5 2 3
i = 5 | U = 1 6 7 4 5 2 3 0
  | V = 1 6 7 4 5 2 3 0
i = 6 | U = 1 6 7 4 5 2 3 0
  | V = 6 7 4 5 2 3 0 1
i = 7 | U = 1 6 7 4 5 2 3 0
  | V = 7 4 5 2 3 0 1 6
```
Matrix A

\[
\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

The values of 1 in the cells of matrix A show that each cell was updated once. This result reflects the single initialization of the corresponding vertices on a grid of uniform Descartes RP.

Similar tests of creating uniform planes with an arbitrary bit length \( w \) of the random numbers confirm a uniformity of the received RPs. Replacement of the congruential sequences by the different corresponding twisting sequences shows a similar result when generating the twisting RPs. Thus, the complete uniform twisting sequences can ensure a creation of the complete uniform twisting RPs. Now we may proceed to the generator designing.

**Construction and Results**

Program \( P040201 \), described in the previous section, utilizes two arrays \( U \) and \( V \) as initial congruential sequences. Such arrays can be created using the twister generator \( nsDeonYuliTwist28DA \) (Deon and Menyaev, 2016b). However, this solution will not be perfect for the case of large planes because it requires a lot of available Random Access Memory (RAM). For many reasons it may be unavailable on a computer. This limitation can be overcome by using the twister generator \( nsDeonYuliTwist32D \) (Deon and Menyaev, 2017), which doesn’t use arrays of the twisting sequences. However, a more satisfactory solution is the aforementioned program \( P040201 \), which implies the repeated generation of \( U \) and \( V \), whereas \( nsDeonYuliTwist32D \) doesn’t provide this. Thus, we come to the inference that before performing a generation of the twisting planes, it is necessary to have tools for the trivial operations with the twisting sequences.

**Simple Twister Generator**

Class \( cDeonYuliSTwist32D \), which is presented below, includes in its name a letter \( S \) indicating the meaning of the word ‘simple’. This class provides elementary operations with the twisting sequences and it does not use any arrays. The prototype of class \( cDeonYuliSTwist32D \) is class \( cDeonYuliTwist32D \). They are different in the issue that automatic setting of the congruential parameters \( a \) and \( c \) is excluded in class \( cDeonYuliSTwist32D \). An example of using this class is presented in this section later, in the description of the program code \( P040301 \).

```c
namespace nsDeonYuliSTwist32D
class cDeonYuliSTwist32D
{
public:
  uint w = 16U; // number bit length
  uint N1 = 0U; // max-number
  uint x0 = 1U; // sequence beginning
  uint xB = 1U; // twister beginning
  uint xG = 0U; // created random number
  uint xL = 0U, xR = 1U; // pair numbers
  uint a = 5U; // congruential constant a
  uint c = 1U; // congruential constant c
  uint maskW = 0U; // number mask
  uint maskU = 0U; // elder bit mask
  uint maskT = 0U; // twister bits
  uint nW = 0U; // pair twister number in w

public:
  cDeonYuliSTwist32D()
  {
    N1 = 0xFFFFFFFF >> (32 - (int)w); // max-number
  }

public:
  void StartTwist(uint snW)
  {
    xB = x0;
    uint sxBe = sxB & maskW;
    for (int i = 0; i < sxBe; i++)
      xB = (a * xB + c) & maskW; // shifted beginning
    xR = xB;
    return xG;
  }

public:
  void RepeatCong()
  {
    xR = xB; // repeat track
  }

public:
  void ShiftCong()
  {
    xB = (a * xB + c) & maskW; // shifted beginning
    xR = xB; // for twister beginning
  }

public:
  void StartCong(uint sxB)
  {
    xB = x0;
    uint sxBe = sxB & maskW;
    for (int i = 0; i < sxBe; i++)
      xB = (a * xB + c) & maskW; // shifted beginning
    xR = xB;
    return xG;
  }

public:
  void NextTwist()
  {
    u g = (xR & maskT) >> (int)(w - nW); // elder
    xG = ((xL << (int)nW) & maskW) | g; // younger
    xL = xR; // next pair beginning
  }  
```

```c
```
xR = (a * xL + c) & maskW;  // end of xL,xR pair
return xG;                     // twister of pair
}

public void RepeatTwist()
{  xL = xB;                         // for twister beginning
    xR = (a * xL + c) & maskW;        // end of xL,xR pair
}

public void ShiftTwist()
{  xB = (a * xB + c) & maskW;        // shifted beginning
    xL = xB;                         // for twister beginning
    xR = (a * xL + c) & maskW;        // end of xL,xR pair
}

public void Start()
{  N1 = 0xFFFFFFFF >> (32- (int)w);  //max-number
    maskW = 0xFFFFFFFF >> (32- (int)w);  //n. mask
    maskU = 1U << ((int)w - 1);         // elder bit mask
    maskT = maskU;                      // first twister bit
    DeonYuli_PlusA();                   // a-value
    DeonYuli_SetC();                    // c-value
    x0 = N1 / 7U;                       // sequence beginning
    StartCong(0);                      // for conguerential generation
}

public void TimeStart()
{  x0 = (uint)sx;                        // sequence beginning
    x0 = (uint)(N1 * xd);            // sequence beginning
    Start();                          // generator starts
}

public void SetW(uint sw)
{  w = (uint)Math.Abs(sw);         // number bit length
    public void SetA(uint sa)
{  a = (uint)Math.Abs(sa);        // related set of a
    if (a < 1) a = 1;               // min-value
}

void DeonYuli_PlusA()
{  if (a < 1U) {a = 1U; return; }    // bottom edge for a
    if (ad > 1.0) ad = 1.0;            // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_SetA()
{  if (a < 1) {a = 1U; return; }    // bottom edge for a
    if (ad > 1.0) ad = 1.0;            // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_SetC()
{  if (c % 2U == 0U) c += 1;          // random condition
    else break;
    if (a % 4U != 0U) a--;           // random condition
    for (uint i = 0U; i < 3U; i++)
        a++;                              // true value for constant a
    if (a < 1U) {a = 1U; return; }
    else if (a > N1) a -= 4U;    // on right from bottom edge
    if (a >= N1 - 1) a -= 4U;   // on left from top edge
}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // random condition
    else break;
    if (a % 4U != 0U) a--;           // random condition
    for (uint i = 0U; i < 3U; i++)
        a++;                              // true value for constant a
    if (a < 1U) {a = 1U; return; }
    else if (a > N1) a -= 4U;   // on left from top edge
}

void DeonYuli_SetC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_SetC()
{  if (c < 1) a = 1;                                     // min-value
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_PlusC()
{  if (c < 1) a = 1;                                     // min-value
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

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{  if (c % 2U == 0U) c += 1;          // only odd c
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}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

void DeonYuli_PlusC()
{  if (c % 2U == 0U) c += 1;          // only odd c
    if (c > N1) c = N1;                  // max-value
}

To verify the correct utilization of the presented generator nsDeonYuliSTwist32D, let’s use the program code P040301, in which the sequences of all twisters of the random numbers having length w = 3 bits are generated. The total quantity of sequences is w·N = w·2^w = 3·2^3 = 24. Program names P040301 and cP040301 are taken by chance.

using nsDeonYuliSTwist32D; // s-twister uni-generator
namespace P040301
{
    class cP040301
    {
        static void Main(string[] args)
        { cDeonYuliSTwist32D ST =
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Aleksei F. Deon and Yulian A. Menyaev / Journal of Computer Science 2018, 14 (2): 260.272

265
new cDeonYuliTwist32D();
ST.SetW(3);  // number bit length
ST.SetA(5);  // congruential constant a
ST.SetC(1);  // congruential constant c
ST.SetX0(1); // sequence beginning
ST.Start();   // generator starts
int w = (int)ST.w;  // random bit length
int N = (int)ST.N1 + 1; // sequence length
Console.WriteLine("w = {0}  N = {1}", w, N);
int N1 = (int)ST.N1 + 1; // sequence length
ST.a, ST.c, ST.x0);

int k = 0;
for (int n = 0; n <= ST.N1; n++)
  Console.Write("k = {0,3}  |  Cong = ", k++);

for (int m = 0; m <= ST.N1; m++)
  Console.WriteLine("a = {0}  c = {1}  x0 = {2}",
  ST.a, ST.c, ST.x0);

int k = 0;
for (int m = 0; m <= ST.N1; m++)
  Console.WriteLine("a = {0}  c = {1}  x0 = {2}",
  ST.a, ST.c, ST.x0);

int k = 0;
for (int m = 0; m <= ST.N1; m++)
  Console.WriteLine("a = {0}  c = {1}  x0 = {2}",
  ST.a, ST.c, ST.x0);

ST.ShiftCong(); // congruential shift
ST.RepeatTwist();

Console.Write("{0,4}", z);
for (int nW = 1; nW < ST.w; nW++)
  Console.Write("{0,4}", z);

Console.WriteLine();
for (int nW = 1; nW < ST.w; nW++)
  Console.WriteLine("a = {0}  c = {1}  x0 = {2}",
  ST.a, ST.c, ST.x0);

k = 14 | Twist 2 = 5 1 4 0 7 3 6 2
k = 15 | Cong = 2 3 0 1 6 7 4 5
k = 16 | Twist 1 = 4 6 0 3 5 7 1 2
k = 17 | Twist 2 = 1 4 0 7 3 6 2 5
k = 18 | Cong = 3 0 1 6 7 4 5 2
k = 19 | Twist 1 = 6 0 3 5 7 1 2 4
k = 20 | Twist 2 = 4 0 7 3 6 2 5 1
k = 21 | Cong = 0 1 6 7 4 5 2 3
k = 22 | Twist 1 = 0 3 5 7 1 2 4 6
k = 23 | Twist 2 = 0 7 3 6 2 5 1 4

These congruential twisting sequences coincide with the result of tests of our previously developed twister generator nsDeonYuliTwist32D using a technique of the congruential twisting array.

Running the program P040301 with other values of the bit length w of the random numbers confirms a completeness of the generated sequences. This is sufficient to ensure the development of programs for generating the Descartes uniform twisting RPs.

Twister Generator of Uniform Planes

When constructing a complete generator of uniform twisting planes, let’s use two uniform complete generators GU and GV of the aforementioned instrumental class cDeonYuliTwist32D. It allows organizing the next class nsDeonYuliPlaneTwist32D for the generation of all points on a grid of the twisting plane. By default, the initial track takes a diagonal of the grid points from the left-bottom position to the right-top one, although this may be changed by setting the independent beginnings for the internal generators GU and GV. An example use of class cDeonYuliPlaneTwist32D is presented in this section later, located in the description of the program code P040302.

using nsDeonYuliTwist32D;  // s-twister uni-generator namespace nsDeonYuliPlaneTwist32D

{  public uint w = 16;  // uniform number bit length
  public uint N1 = 0;  // max-number in track
  public uint a = 5U;  // congruential constant a
  public uint c = 1U;  // congruential constant c
  public uint x0 = 1U;  // constant of track beginning
  public uint wU = 0;  // twist shift number in GU
  public uint xRU = 0;  // ring shift number in GU
  public uint xWV = 0;  // twist shift number in GV
  public uint xRV = 0;  // ring shift number in GV
  public uint x63 = 0;  // elements in GU and GV tracks
}

public cDeonYuliPlaneTwist32D()
  {  GU = new cDeonYuliTwist32D();  // generator 1
     GV = new cDeonYuliTwist32D();  // generator 2
  }

public void SetW(int sw)
public void Next(ref uint u, ref uint v)
{
    if (nWV == 0) GU.NextCong();
    else GU.NextTwist();
    if (nWV == 0) GV.NextCong();
    else GV.NextTwist();
    u = GU.xG;  // <u,v> point coordinates
    v = GV.xG;
    if (nG < N1) {nG++; return;}  // inside track
    nG = 0;  // elements number in GU and GV tracks
    GU.Start();  // GU generator starts
    GV.Start();  // GV generator starts
}

public void Start()
{
    DeonYuli_SetN1ACX();  // congruential constants
    GU.Start();  // GU generator starts
    GV.Start();  // GV generator starts
    a = GU.a;  // congruential constant a for GU
    c = GU.c;  // congruential constant c for GU
    x0 = GU.x0;  // beginning of U and V sequences
    nRU = 0;  // ring shift number in GU generator
    nRV = 0;  // ring shift number in GV generator
    nG = 0;  // elements number in GU and GV tracks
    GU.StartCong(0);  // for congruential GU generation
    GV.StartCong(0);  // for congruential GV generation
}

public void Next(ref uint u, ref uint v)
{
    if (nWV == 0) GU.NextCong();
    else GU.NextTwist();
    if (nWV == 0) GV.NextCong();
    else GV.NextTwist();
    u = GU.xG;  // <u,v> point coordinates
    v = GV.xG;
    if (nG < N1) {nG++; return;}  // inside track
    nG = 0;  // regular track beginning
    if (DeonYuli_RingCongGV()) return;  // inside GV
    if (DeonYuli_RingTwistGV()) return;  // inside GV
    if (DeonYuli_RingCongGU()) return;  // inside GU
    DeonYuli_RingTwistGU();  // inside GU
}

bool DeonYuli_RingTwistGU()
{
    if (nWV != 0) return false;  // no congruential ring
    if (nWV == 0) GU.RepeatCong();
    else GU.RepeatTwist();
    if (nRV < N1)  // congruent. ring opportunity in GV
    {
        if (nRV == 0)  // first ring in GV
        {
            GV.ShiftTwist();  // twister starts in GV
            nRV = 1;  // first twister in GV
            GV.StartTwist(nWV);  // twister starts in GV
            return true;  // inside track
        }
        else GU.RepeatTwist();
        if (nWU == 0) GU.RepeatCong();
        return false;  // rings R2 and W2 are over in GV
    }
}

bool DeonYuli_RingTwistGV()
{
    if (nWV != 0) return false;  // no twister ring
    if (nWV == 0) GU.RepeatCong();
    else GU.RepeatTwist();
    nRV = 0;  // new ring in GV
    if (nRV < w - 1)  // continue twisters
    {
        nRV++;  // next bit twister in GV
        GV.StartTwist(nWV);  // twister starts in GV
        return true;  // inside twister regime in GV
    }
    else GV.RepeatTwist();
    if (nWV == 0) return false;  // no congruential ring
    GV.StartCong(nWV);  // generator starts
    return false;  // rings R2 and W2 are over in GV
}

bool DeonYuli_RingCongGU()
{
    if (nWU != 0) return false;  // no congruential ring
    if (nRU < N1)  // congruent. ring opportunity in GU
    {
        if (nRU == 0)  // first ring in GU
        {
            GV.ShiftCong();  // congruential shift in GV
            nRV = 1;  // first ring in GV
            GV.StartTwist(nWV);  // twister starts in GV
            return true;  // inside track
        }
        else GU.RepeatTwist();
        if (nWU == 0) GU.RepeatCong();
        return false;  // rings R2 and W2 are over in GV
    }
}
To test an operation of the presented generator $nsDeonYuliPlaneTwist32D$, let's use the program code $P040302$ shown below, in which the points of the initial twisting plane of the random numbers having length $w = 3$ bits are generated. By default, the initial track of generation is a diagonal of the grid points from the left-bottom position to the right-top one. The uniformity of points on a plane would be confirmed by matrix $A$, in which the cell values are the counters of generation of the corresponding points $<u, v>$ on RP. Program names $P040302$ and $cP040302$ are taken by chance.

```csharp
using nsDeonYuliPlaneTwist32D; // twist-plane generator namespace P040302
{
    static void Main(string[] args)
    {
        cDeonYuliPlaneTwist32D TP =
            new cDeonYuliPlaneTwist32D();
        int w = 3; // random number bit length
        int N = 1 << w; // track length
        TP.SetW(w);
        TP.Start(); // generator starts
        Console.WriteLine("w = {0}   N = {1}", w, N);
        Console.WriteLine("a = {0}   c = {1}   x0 = {2}",
            TP.a, TP.c, TP.x0);
        uint[] u = new uint[N]; // point u-coords on track
        uint[] v = new uint[N]; // point v-coords on track
        int[,] A = new int[N, N]; // result matrix
        for (int i = 0; i < N; i++)
            for (int j = 0; j < N; j++)
                A[i, j] = 0;
        uint uu = 0; // point job coordinates on plane
        uint vv = 0;
        for (int i = 0; i < N; i++)
            for (int j = 0; j < N; j++)
            {
                Console.Write("i = {0}, j = {1}, uu = {2}, vv = {3}",
                    i, j, uu, vv);
                TP.Next(ref uu, ref vv); // point on grid
                u[i] = uu;
                v[j] = vv;
                A[uu, vv]++;// generation counter for <u, v>
                Console.WriteLine("A[uu, vv] = {0}", A[uu, vv]);
            }
            // generation counter for <u, v>
        Console.WriteLine("Matrix A");
        for (int i = 0; i < N; i++)
            for (int j = 0; j < N; j++)
            {
                Console.WriteLine("", A[i, j]);
            }
        Console.ReadKey(); // result viewing
    }
}
```

After the execution of $P040302$ code, the listing below appears. To reduce the listing size, we skipped some strings, which are indicated by a dashed line.

$w = 3   N = 8$
$a = 5   c = 1   x0 = 3$

```
i = 0 nWU = 0 nRU = 0 nNV = 0 nRV = 0
U = 3 0 1 6 7 4 5 2
V = 3 0 1 6 7 4 5 2
i = 1 nWU = 0 nRU = 0 nNV = 0 nRV = 0
U = 3 0 1 6 7 4 5 2
V = 0 1 6 7 4 5 2 3
i = 2 nWU = 0 nRU = 0 nNV = 0 nRV = 2
U = 3 0 1 6 7 4 5 2
V = 1 6 7 4 5 2 3
i = 7 nWU = 0 nRU = 0 nNV = 0 nRV = 7
U = 3 0 1 6 7 4 5 2
V = 2 3 0 1 6 7 4 5

Matrix A
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
```

In this listing, indicator $nWU$ shows the number of a
twister on axis $U$ and indicator $nRU$ is pointed to the
circular shift number on this axis $U$. The values $nWU = 0$
and $nRU = 0$ correspond to the congruential initial sequence
on axis $U$. Similar values of indicators $nWV$ and $nRV$ show
the circular shift of the initial sequence on axis $V$.

The listing of results for the singular matrix $A$ confirms
that each random point $<u, v>$ is created once. The same
results are valid for other $w \leq 32$. This corresponds to a
concept of the Descartes uniform RP. Thus, generator
nsDeonYuliPlaneTwist32D is ready for implementation.

**Discussion**

Twister generator nsDeonYuliPlaneTwist32D is capable creating a set of uniform twisting RPs for each
pair of the congruential constants $a, c \in \{1, N - 1\} = \{0, 2^n - 1\}$ in the sequences of random
numbers having $w$ bit length. All the initial values
$x_0 \in [0, 2^n - 1]$ are automatically presented in circles of the
twisters (Deon and Menyaev, 2016b). However, the
question is how many twisting planes could be obtained
for each pair of parameters $a$ and $c$?

In the previous section, it has been determined that
when creating the initial RP $U \times V$, two random sequences
$U$ and $V$ can be taken, which are created by the corresponding
twisting generators. It is known that
twister 0 is the uniform congruential sequence.
Let’s denote it as $U_0$. Another sequence $V_0$ could be
chosen arbitrarily but on the condition that it is also
uniform. If the sizes or quantity of elements in the initial
sequences are the same \( \text{card}(U_0) = \text{card}(V_0) \), then it can
be stated that among all the possible tracks of the
random uniform plane there has to be a track that creates
points on the second diagonal (from the left-bottom position to the right-top one) of the corresponding square
discrete grid. This is somewhat reminiscent of the idea of a
central abstract element and specifically in our case
this second diagonal track is the central track of the
discrete grid. In generator nsDeonYuliPlaneTwist32D
this is exactly what is done, shown at the beginning of the
listing of results for the previous program P0040302.

\[
U_0 = \begin{bmatrix}
3 & 0 & 1 & 6 & 7 & 4 & 5 & 2
\end{bmatrix}
\]

\[
V_0 = \begin{bmatrix}
3 & 0 & 1 & 6 & 7 & 4 & 5 & 2
\end{bmatrix}
\]

This is not the only solution because as the central
track, one could take the main diagonal (from the left-top
position to the right-bottom one) of a grid. But since the
initial sequences $U$ and $V$ are random, the order of the
vertex passage $<3, 3>, <0, 0>, <1, 1>, <6, 6>, <7, 7>, <4, 4>, <5, 5>, <2, 2>$ is also random.

Now let’s fix the sequence $U$ while the sequence $V$
is shifted to the left by the circular technique by one position.

\[
U_0 = \begin{bmatrix}
3 & 0 & 1 & 6 & 7 & 4 & 5 & 2
\end{bmatrix}
\]

\[
V_1 = \begin{bmatrix}
0 & 1 & 6 & 7 & 4 & 5 & 2 & 3
\end{bmatrix}
\]

This combination of pairs gives the points of the first
track $<3, 0>, <0, 0>, <1, 6>, <6, 7>, <7, 4>, <4, 5>, <5, 2>, <2, 3>$ on RP. None of these points can appear on the
reverse main diagonal of a grid, since sequences $U_0$ and $V_1$
are uniform. Their uniformity follows from the
determined properties of the complete twisting sequences
(Deon and Menyaev, 2016b). Shifting of sequence $V_1$
provided that sequence $U_0$ is fixed, could be continued and
so the second track on a grid might be obtained.

\[
U_0 = \begin{bmatrix}
3 & 0 & 1 & 6 & 7 & 4 & 5 & 2
\end{bmatrix}
\]

\[
V_2 = \begin{bmatrix}
1 & 6 & 4 & 7 & 5 & 2 & 3 & 0
\end{bmatrix}
\]

The second track contains the points, which also
cannot occur on track 1 and track 0 of the reverse diagonal
and again that is because of the properties of the complete
uniform twisting sequences. In this example, only 8
options to present sequence $V$ are possible since the $9^\text{th}$
shift repeats the initial variation $V_2 = V_0$.

So, the shift operations for sequences comply with the
corresponding varieties of the twisting sequences. In the
presented example, the shifts of the congruential sequence,
which comply with the congruential generation from the corresponding initial values, are
considered. Thus, the congruential initial generation of sequences $U_0$ and $V_0$ initiates the creation of one RP
using the complete sequence of shifts of one of the original sequences $V_j$ while another sequence $U_0$ is fixed.

By analyzing the result of the previous program
P040301, it’s easy to see that among all sequences
$V_{k \in \{0, 2, 4, 6, 8, \ldots \}}$ there are all 8 congruential shifts
$V_k = 0, V_k = 3, V_k = 6, V_k = 9, V_k = 12, V_k = 15, V_k = 18, V_k = 21$ in amount of $N = 2^n$
$= 2^3 = 8$ random numbers having length of 3 bits in the
complete sequence. As a result, it turns out that tracks
$U_0, V_j > = < U_0, V_j >$ form the initial plane $L_0$.

Similar arguments apply to plane $L_1$. If we refer again
to the result of the previous program P040301, this example shows that plane $L_1$ is created using the congruential twisting tracks $< U_k = 0, V_k \{4, 7, 10, 13, 16, 19, 22\} >$. Plane $L_2$ is created by using shifts of the next twister
$< U_k = 0, V_k \{5, 8, 11, 14, 17, 20, 22\} >$. Next is plane $L_3$, but to create it we need to perform the congruential circular shift of sequence $U_0$ by one random step to the left,
which leads to obtaining the new distribution of the random values $U_k$ along the axis $U$. Then, the next
plane $L_4$, which can be obtained with the help of tracks
$< U_k = 3, V_k \{3, 6, 9, 12, 15, 18, 21\} >$.

A summary of all the points obtained leads us to the
conclusion that number \( \text{card}(L) \) of the complete set of RPs is defined by multiplication of two things. The first
is \( \text{card}(U_C) = w \cdot N = w \cdot 2^n \), which is a quantity of the
different options of the congruential twisting forms of

269
sequence $U$; the second is $\text{card}(V) = w$, which is a quantity of the various twisters in sequence $V$ including the congruential twister 0, i.e., $\text{card}(L) = \text{card}(U/L)$. $\text{card}(V) = wN = w N$.

Below is the program code $P040401$, in which the uniformity of all the random twisting planes $L(w = 3)$ is checked. Matrix $A$ is helpful for this task. Since in the uniform RPs each vertex is created once, after the full enumeration of all the RPs the quantity of generations of each vertex has to be equal to the amount of planes. In other words, the counters of cells of matrix $A$ have to have the same values and moreover, they have to be equal to the quantity of the generated RPs. To clarify, the program below uses the random numbers with a bit length $w = 3$. The names $P040401$ and $cP040401$ are selected by chance.

```csharp
using nsDeonYuliPlaneTwist32D; // twist-plane generator
namespace P040401
{
    class cP040401
    {
        static void Main(string[] args)
        {
            cDeonYuliPlaneTwist32D TP = new cDeonYuliPlaneTwist32D();
            int w = 3; // random number bit length
            int N = 1 << w; // track length
            TP.SetW(w);
            TP.Start(); // generator starts
            Console.WriteLine("w = {0}   N = {1}", w, N);
            Console.WriteLine("a = {0}   c = {1}   x0 = {2}", TP.a, TP.c, TP.x0);
            uint[] u = new uint[N]; // point u-coords on track
            uint[] v = new uint[N]; // point v-coords on track
            uint[,] A = new uint[N, N];
            for (int i = 0; i < N; i++)
            {
                for (int j = 0; j < N; j++)
                {
                    A[i, j] = 0;
                }
            }
            uint uu = 0; // point job coordinates on plane
            uint vv = 0;
            int plane = 0; // plane number
            int k = 0; // track number
            while (true)
            {
                uint nWU = TP.nWU, nRU = TP.nRU;
                uint nWV = TP.nRV, nRV = TP.nRV;
                for (int i = 0; i < N; i++)
                {
                    u[i] = uu;
                    v[i] = vv;
                    A[uu, vv]++;
                }
                if (k % N == 0) plane++; // plane number
                Console.WriteLine("plane = {0}", plane);
                k++;
                if (k < 569) continue;
                Console.WriteLine("k = {0}   u[j] = {1}   v[j] = {2}", k, u[j], v[j]);
                for (int i = 0; i < N; i++)
                {
                    for (int j = 0; j < N; j++)
                    {
                        Console.WriteLine("A[i, j] = {0}", A[i, j]);
                    }
                }
                Console.WriteLine("Matrix A");
                for (int i = 0; i < N; i++)
                {
                    for (int j = 0; j < N; j++)
                    {
                        Console.WriteLine("A[i, j] = {0}"");
                    }
                }
            }
        }
    }
}
```

After executing the program $P040401$, the following listing below appears on the monitor. The skipped strings are indicated by a dashed line.

```csharp
w = 3   N = 8
a = 5   c = 1   x0 = 3
plane = 1
k = 1   nWU = 0   nRU = 0   nWV = 0   nRV = 0
U = 3 0 1 6 7 4 5 2
V = 3 0 1 6 7 4 5 2
plane = 1
k = 2   nWU = 0   nRU = 0   nWV = 1   nRV = 1
U = 3 0 1 6 7 4 5 2
V = 0 1 6 7 4 5 2 3
plane = 1
k = 8   nWU = 0   nRU = 0   nWV = 7   nRV = 7
U = 3 0 1 6 7 4 5 2
V = 2 3 0 1 6 7 4 5
```

```matlab
Matrix A
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
```

plane = 72
The values of the elements of matrix $A$ confirm the analytical calculations for $\text{card}(L) = w^2N$ and for arbitrary $w \leq 32$. Generator $\text{nsDeonYuliPlaneTwist32D}$ creates a complete set of uniform twisting RPs, which is considered as the primary task of this article.

**Conclusion**

Analysis of the sources indicates that algorithms of the generators of uniform planes do not take into account the potential of the sequences having absolutely uniform distribution. Techniques of those generating algorithms do not guarantee the absolute uniformity of the complete random planes. To overcome this limitation, we proposed here the generators of the complete uniform sequences, which include unique twisting techniques described in our previous works. However, their direct application for the described task is hampered by the required properties of Descartes uniform planes. To satisfy this requirement, a new class $\text{nsDeonYuliStwist32D}$ was constructed and now with its help it is possible to create the dynamic objects of the simplest twisters without using congruential arrays. Applying secondary indexing technique allows for getting a generator of Descartes twisting random planes, which ensures the completeness and uniqueness of all the random variables on a grid of the Descartes plane. The performed tests confirm the absolute uniform distribution of the generated random values on a plane. In addition, a variety of the initial twisting sequences allows getting a set of the twisting planes for each pair of the congruential constants. In perspective, the obtained results can be used in a large number of applied tasks, which use the spatial plane distributions.

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