A Techniques to Downgrade Objective Function in Parallel Robot Kinematics Problem

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ABSTRACT

Quadratic functions and quaternary are preferable forms solving parallel robot kinematics problems. In order to simplify and to use only one method to solve all forms of the objective functions, this article introduces prevalent techniques to downgrade mathematical model of the objective functions from quaternary function to quadratic function in parallel robot kinematics problem. By using equivalent alternative kinematic structure and additional mathematical constraints, we will change from the study of parallel robot kinematics problem with the form of quaternary objective function as the original configuration to equivalent alternative configuration in the form of quadratic objective function. The parallel robot kinematics problem based on alternative configuration with quadratic objective function is not only simpler but also helps to quickly identify the mathematical relationships in the joint space and work space of parallel robot. Then, more accurate control solution of parallel robot kinematics problem can be identified. Moreover, by using techniques in this study, all forms of objective functions of parallel robot of any structure can be easily solved. The result from numerical simulation has been used to prove the presented approaches.

Keyword:
Equivalent structure
Kinematics model
Kinematics problem
Objective functions
Parallel robot

1. INTRODUCTION

Day by day, the applications of the parallel manipulator in various field is become apparent and with a rapid rate utilized in precise manufacturing, medical science and in space exploration equipment [1]. The parallel manipulator can be defined as a closed-chain mechanism which has two platforms (base and moving platform), connected together by at least two independent kinematic chains [2]. The main advantages of parallel manipulator over serial robot include higher stiffness, better accuracy, larger load capacity, higher velocity and acceleration; higher throughput and non-accumulation of joint errors [3, 4]. Despite of its advantages, the parallel manipulator poses some challenges regarding to the limited workspace, singularities within the workspace and, finally, their kinematics model resolution [5]. Since the kinematics of parallel robot problem often contains the solving of the non-linear transcendental equations, it is not always possible to obtain a closed form solution [6]. Therefore, the selection of an efficient mathematical model is very crucial to simplify the kinematics problems in parallel robots [3].

Kinematic models of parallel robots can be built up by using geometric methods or the Denavit Hartenberg (DH) rule. However, to avoid the complicated factors of applying the DH rule, a geometric method is commonly used for modeling by changing the screw joints and spherical joints to the joints of 5-type [7]. By using this method, kinematic models are established, and investigating the kinematic problems of parallel robots can be done by applying optimized algorithm [8]. The obtained mathematical model for
revolute joints and prismatic joints will be quadratic functions and quaternary functions respectively [9]. For the applications of the parallel structures having limb configurations of prismatic joints (P) is active joint example RPR or SPS types, the optimization algorithm is no longer appropriate when it encounters quaternary objective functions. In addition, if the quaternary objective functions are kept, it is necessary to use the technique from [10]. Because of desiring continue to use optimized algorithm to solve kinematic problems of parallel robots. This paper presents an alternative configuration to downgrade the order of complicated objective functions to the simpler form. The new technique of changing variables introduced in this paper is suitable for parallel robots with different structure. Therefore, the objective desire of using a single method to study the kinematics for all kinds of parallel robots will gradually become real.

2. THE FORM OF THE EQUATION WHEN TRANSFORMING PARALLEL ROBOT KINEMATICS PROBLEM INTO THE OPTIMIZATION PROBLEM

Suppose that \( \hat{P}(p_x, p_y, p_z, \alpha, \beta, \gamma) \) is the vector representing position and orientation of end effector of the robot. The aim of kinematic control is to achieve high accuracy of position and orientation of the end effector. Therefore, it is necessary to determine the joint variable \( \theta_i \) so that the error of position and orientation is smallest while satisfying the conditions of structural constraints.

\[
\theta = \left( \theta_1, \theta_2, \ldots, \theta_n \right)
\]

Joint space \( D \) determines the domain of the following joint variables:

\[
\begin{align*}
& a_1 \leq \theta_1 \leq b_1 \\
& a_2 \leq \theta_2 \leq b_2 \\
& \vdots \\
& a_n \leq \theta_n \leq b_n
\end{align*}
\]

(1)

where: \( a_i \) and \( b_i \) are constraints of joint variables \( \theta_i \).

\( T = f(\theta) \): function describing the deviation of position and orientation of the end effector.

The problem determining the value of the joint variables is written as follows:

\[
\begin{align*}
& T = f(\theta_1, \theta_2, \ldots, \theta_n) \rightarrow \text{minimize} \\
& a_i \leq \theta_i \leq b_i \\
& \theta_i \in D; i = 1 \pm n
\end{align*}
\]

(2)

This is the equation of parallel robot kinematics problem when transformed into problem optimizing \( n \) unknown nonlinear function, with linear constraints.

3. ASSOCIATED EQUATION OF PARALLEL ROBOT WHEN DRIVEN BY REVOLUTE JOINTS OR PRISMATIC JOINT

3.1. Associated equation when driven by revolute joints

In parallel robot 3RRR driven by revolute joints as Figure 1.

![Figure 1. Parallel robot with 3RRR structure](image)
Because the robot has symmetrical structure when taking into consideration the closed loop vector equation passing through the points $O_0 = P, D, A, O_1$ of the $1^{st}$ limb:

$$\overrightarrow{O_0 O_1} = \overrightarrow{O_0 D} + \overrightarrow{D A} + \overrightarrow{A O_1}$$

(3)

The detailed equation has form:

$$\begin{align*}
x_{01} &= a_1 \cos(\theta_1) + b_1 \cos(\theta_1 + \alpha_1) + \frac{h\sqrt{3}}{3} \cos(\varphi + 30) \\
y_{01} &= a_1 \sin(\theta_1) + b_1 \sin(\theta_1 + \alpha_1) + \frac{h\sqrt{3}}{3} \sin(\varphi + 30)
\end{align*}$$

(4)

When kinematic problem is structured based on (2), value of joint variables $\theta_1, \alpha_1$ is determined so that position error $x_{01}, y_{01}$ and orientation error $\varphi$ of the end effector reach minimum value. Therefore, objective function $T$ describing deviation of position and orientation of the end effector is determined as follows:

$$T = [x_{01} - (a_1 \cos(\theta_1) + b_1 \cos(\theta_1 + \alpha_1) + \frac{h\sqrt{3}}{3} \cos(\varphi + 30))]^2 +$$

$$[y_{01} - (a_1 \sin(\theta_1) + b_1 \sin(\theta_1 + \alpha_1) + \frac{h\sqrt{3}}{3} \sin(\varphi + 30))]^2 \Rightarrow \text{minimize}$$

(5)

Obviously, (5) is always $\geq 0$, thus minimize value achieved is 0 equivalent to (4) satisfied. Function (5) has its own name called Rosenbrock-Banana function [11], the most appropriate algorithm to solve this functional is Generalized Reduced Gradient (GRG) algorithm [9, 12].

3.2. Associated equation when driven by prismatic joints

Consider detailed diagram of one limb OA$_3$B$_3$P of parallel robot with 3RPS structure (Figure 2).

![Figure 2. Parallel robot with 3RPS structure](image)

Associated vector equation has the following form:

$$\overrightarrow{a_i} = -\overrightarrow{a_i} + \overrightarrow{P} + R_{RPY} \overrightarrow{P_i}$$

(6)
The length of $i^{th}$ limb that is the distance between two ends of vector $\vec{c}_i$ and $\vec{a}_i$ is determined as follows:

$$d_i = \sqrt{(c_{ix} - a_{ix})^2 + (c_{iy} - a_{iy})^2 + (c_{iz} - a_{iz})^2} = \sqrt{(c_i - a_i)(c_i - a_i)}$$  \hspace{1cm} (7)

When the objective function is structured based on (2), similar to section A, objective function is in the form of quaternary function due to the presence of prismatic joint variables $d_i$:

$$r = \sum_{i=1}^{n} \left[ (c_{ix} - a_{ix})^2 + (c_{iy} - a_{iy})^2 + (c_{iz} - a_{iz})^2 - d_i^2 \right]^2 \Rightarrow \text{minimize}$$  \hspace{1cm} (8)

4. EQUIVALENT ALTERNATIVE KINEMATIC MODEL

From section 3, it can be seen that for the parallel robot driven by the prismatic joints, the objective function of the parallel robot kinematics problem always is in quaternary form, thus GRG algorithm is no longer appropriate to solve this form of function. To solve objective function containing fourth order variables, there are some appropriate methods such as Newton Raphson, Hook-Jeeves, Fletcher Powell [10].

In order to simplify and to use only one method - the GRG algorithm to solve the quaternary objective function in parallel robot kinematics problem, this article presents a method to downgrade the objective function based on equivalent alternative kinematic configuration. The downgradation of objective function is not merely a change of variable in mathematical models, but also requires appropriate kinematic structure between the two configurations.

4.1. Methods to determine equivalent alternative configuration

The determination of equivalent alternative configuration should be based on the structure of the robot to switch to an equivalent kinematic structure driven by the revolute joint instead of structure driven by the prismatic joint to avoid quaternary function as mentioned above.

4.1.1. In case of using one revolute joint to replace one prismatic joint

In case of the movement of the prismatic joint axis changes as parallel robot 3RPS, 6S6PS, equivalent alternative configuration proposed for this form of structure show in Figure 3.

![Figure 3. Equivalent structure when prismatic joints are replaced by revolute joints](image)

The alternative configuration is used to create mathematical models having the form of quadratic objective function. It is required to retain the robot’s actuators; only the drive part is replaced to get a structure with similar kinematic functions.

In the alternative configuration requires calculation of factors such as the direction of joint axis, activity limitations of revolute joints in specific relation with movement limitations of prismatic joints in the original configuration so that shape and volume of the workspace of the two structures is the same.

In Figure 3, AB limb originally is a prismatic joint (full association of the limb is SPS) with control variable is the distance between two points A, B, and equal to $d$. AB limb can be replaced by ACB limb with SRS configuration, the degrees of freedom of the structure remains unchanged. In order to keep the volume and workspace of the robot constant it is necessary to convert keep overhead equivalently range of variation of $d$ and $q$ variables.

Figure 4 is examples of equivalent alternative configurations for the parallel robot 3RPS and 6S6PS.
4.1.2. In case of using two revolute joints to replace one prismatic

In case the direction of prismatic joint axis is fixed as in robot 3PRRR (Figure 5a), the equivalent alternative configuration using two revolute joints RR to replace one prismatic joint P has constraints conditions attached as Figure 5b.

Because the direction of prismatic joint is fixed, only the distance between point A and point C in the direction of prismatic joint is changing. Hence, alternative configuration needs further description by other constraints conditions to create equivalency in terms of kinematics between the two configurations.

Consider alternative configuration of Figure 5b, to ensure that the movement of the prismatic joint is in accordance with the direction of Y-axis, the additional mathematical constraints that the alternative configuration needs is:

\[
AB = BC = a \quad \text{and} \quad 2q_1 = q_2 \quad (9)
\]

Verify the mathematical constraints, the trajectory equation of point C is in the following form:

\[
\begin{align*}
    x_c &= AB \cdot \cos(q_1) - BC \cdot \cos\left(\frac{q_2}{2}\right) \\
    y_c &= AB \cdot \sin(q_1) + BC \cdot \sin\left(\frac{q_2}{2}\right)
\end{align*}
\]

Substitute (9) to (10) has:

\[
\begin{align*}
    x_c &= 0 \\
    y_c &= 2a \cdot \sin(q_1) \quad \forall q_1
\end{align*}
\]

Thus with the constraint (9), the locus of point C is the line coincides with the vertical Y-axis or is the movement direction of prismatic joint in the original configuration.
4.2. Form of objective function in equivalent alternative configuration

After equivalent alternative configuration for each type of robot driven by prismatic joint has been identified, the next step needed to be done is to develop the objective function of the alternative function. Displacement graph of the original configuration will completely be inferred from the displacement graph of alternative configuration based on geometric relations between the two configurations.

Suppose that in the original configuration objective function has the form:

\[
T_i(d_1, d_2, ..., d_n)^2 - a_i^2 = 0
\]

The change of variables formula of two configurations and mathematical constraints:

\[
d_i = f(\theta_i)
\]

The variation of the control variables in the two configurations is:

\[
d_{i_{\text{min}}} \leq d_i \leq d_{i_{\text{max}}} \Leftrightarrow \theta_{i_{\text{min}}} \leq \theta_i \leq \theta_{i_{\text{max}}}
\]

Objective function in alternative configuration has the following form:

\[
T'_i(\theta_1, \theta_2, ..., \theta_n) - a'_i = 0
\]

Equation (15) is always in the form of quadratic function and it is the equivalent alternative kinematic model.

5. NUMERICAL SIMULATION EXAMPLE

Consider control example of Stewart Gough robot (Figure 4d) following a trajectory in space (Figure 6), there is an equation:

\[
\begin{align*}
x &= \pm \sqrt{10196. \sin^2(t) - 2187. \sin(t) - 3898.828} \\
y &= \pm 100. \cos(t) \\
z &= 100. \sin(t) \\
0.826 \leq t \leq 1.5708
\end{align*}
\]

Figure 6. Trajectory of movable platen Stewart Gough robot that needs to control

Since prismatic joints axis of the Stewart Gough robot has direction that can be changed so the alternative configuration selected is the type in which a revolute joint replaces a prismatic joint Figure 7.

Figure 7. Geometric relation between original variable \(d_i\) and new variable \(\theta_i\)
Firstly, the change of variables formula must be developed. In the original configuration, control variable is the length of direction axis $d_i$, in the alternative configuration control variable is the angle $\theta_{2i}$.

To simplify, in alternative configuration suppose that the length of 2 links $A_iC_i = C_iB_i = a = \text{const}$, then $\Delta A_iC_iB_i$ is isosceles triangle with apex $C_i$, apply the cosine function in $\Delta$ we have:

$$
\begin{align*}
A_iB_i^2 &= A_iC_i^2 + C_iB_i^2 - 2 \times A_iC_i \times C_iB_i \times \cos \angle A_iC_iB_i \\
\Leftrightarrow \angle A_iC_iB_i &= \arccos \left( \frac{A_iC_i^2 + C_iB_i^2 - A_iB_i^2}{2 \times A_iC_i \times C_iB_i} \right)
\end{align*}
$$

(17)

On the other hand $\hat{\theta}_{2i} = 180^\circ - \angle A_iC_iB_i$ so

$$
\hat{\theta}_{2i} = 180^\circ - \arccos \left( \frac{A_iC_i^2 + C_iB_i^2 - A_iB_i^2}{2 \times A_iC_i \times C_iB_i} \right) \Leftrightarrow \hat{\theta}_{2i} = 180^\circ - \arccos \left( \frac{2a^2 - d^2}{2a^2} \right)
$$

(18)

Equation (18) is the formula to change variables between two configurations.

Since the limbs of Stewart Gough robot have identical structures, detailed development of $i^{th}$ limb of robot is as shown in Figure 8.

![Figure 8. Detailed development diagram of $i^{th}$ limb of Stewart Gough robot in alternative configuration](image)

Associated vector equation has the following form:

$$
\overline{OP} + R_{RPY} \overline{PB_i} - \overline{OA_i} = \overline{A_iC_i} + \overline{C_iB_i} \Leftrightarrow \overline{OP} = \overline{A_iC_i} + \overline{C_iB_i} - R_{RPY} \overline{PB_i} + \overline{OA_i}
$$

(19)

where $R_{RPY}$ is matrix Roll-Pitch-Yaw.

$$
R_{RPY} = R(z, \gamma)R(y, \beta)R(x, \alpha) = \begin{bmatrix}
c \beta s \gamma & s \alpha s \beta c \gamma - c \alpha s \gamma & c \alpha s \beta c \gamma + s \alpha s \gamma \\
c \beta s \gamma & s \alpha s \beta c \gamma + c \alpha s \gamma & c \alpha s \beta c \gamma - s \alpha s \gamma \\
- s \beta & s \alpha c \beta & c \alpha c \beta
\end{bmatrix}
$$

(20)

Detailed development (19):

$$
\begin{bmatrix}
p_x \\
p_y \\
p_z
\end{bmatrix} = \begin{bmatrix}
a s \theta_{3i} + a s \theta_{3i} c (\theta_{3i} + \theta_{2i}) \\
a c \theta_{3i} + a c \theta_{3i} s (\theta_{3i} + \theta_{2i}) \\
a s \theta_{3i} s \theta_{3i} + a s \theta_{3i} c (\theta_{3i} + \theta_{2i})
\end{bmatrix} \cdot \begin{bmatrix}
x_{Ai} \\
y_{Ai} \\
z_{Ai}
\end{bmatrix} + \begin{bmatrix}
c \beta s \gamma & s \alpha s \beta c \gamma - c \alpha s \gamma & c \alpha s \beta c \gamma + s \alpha s \gamma \\
c \beta s \gamma & s \alpha s \beta c \gamma + c \alpha s \gamma & c \alpha s \beta c \gamma - s \alpha s \gamma \\
- s \beta & s \alpha c \beta & c \alpha c \beta
\end{bmatrix} \cdot \begin{bmatrix}
x_{ci} \\
y_{ci} \\
z_{ci}
\end{bmatrix}
$$

(21)
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where “c” and “s” are symbols of cosine and sin; \( \alpha, \beta, \gamma \) are the orientation angle of moving platform. When the objective function is structured based on (2), objective function of robot is determined as follows:

\[
T = \sum_{i=1}^{6} \left[ (c \alpha s \beta c \gamma + s \alpha s \gamma) z_{Ci} - x_{Ai} \right]^2 + [p_x - a_s \theta_{i_1} + c \theta_{i_1} - a_s(\theta_{i_1} + \theta_{i_2}) + c \beta r \gamma, x_{Ci} + (s \alpha s \beta c \gamma - c \alpha s \gamma) y_{Ci} +

- s \beta_r x_{Ci} + s \alpha c \beta c \gamma + c \alpha c \beta c \gamma \right] \right] \Rightarrow \text{minimize}

Using the GRG algorithm to solve equation (22), the results are displacement graph of 6 control joints \( \theta_{i_1} \) passing through 24 key points of the trajectory in alternative configuration and converted displacement graph of prismatic joint \( d_i \) in original configuration as in Figure 9.

Figure 9. The converted displacement graph of 6 equivalent joint variables between the replacement configuration and the original structure
By using alternate configuration to find displacement graph of 6 limbs use revolute joint, use the formula changed to redefine the displacement graph of 6 limbs use prismatic joint in the original configuration. This is the kinematics control information of Stewart Gough robot.

6. CONCLUSION

By using the technique to downgrade objective function based on equivalent alternative configuration, the difficulties encountered regarding the order of the objective function forms solving parallel robot kinematics problems has been resolved. Equivalent alternative configurations proposed in the article use one revolute joint or a combination of two revolute joints to replace a prismatic joint along with different kinematic constraints described in solution selection conditions of optimization problem. All parallel robots having the form of quaternary objective function can be downgraded to quadratic objective function, from which the parallel robot kinematics problem can be simplified and its solutions can be quickly found. This technique has been tested on many different types of parallel robot with different structures and the results show that the applicability of this technique in practice is very high.

Parallel robot kinematics problem often has many mathematical solutions, in addition to finding accurate control solution, identification of extra parameters ($\bar{\theta}_1, \bar{\theta}_2$ as in example of section 5) in limbs of parallel robot to establish a unique solution relation in joint space and workspace is an issue needed discussed. This is a research direction that we will discuss in another paper.

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