Calculation of Cementation Exponent and Multiplier Using P-Wave Velocity and Porosity for Sandstone Reservoirs

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Abstract. The formation resistivity factor is important as reservoir property because it is required to calculate volume of hydrocarbon in reservoir rocks. It depends on Winsauer’s multiplier (a) which is a function of fluid tortuosity and cementation exponent (m). Cementation exponent is a major source of uncertainty in the calculation of hydrocarbon saturation for a given reservoir condition. A small error in the cementation exponent and/or multiplier could give a huge error in the formation factor and have serious consequences. The empirical relationships between reservoir parameters such as P-wave velocity and formation factor depend on local conditions such as environment of deposition, rock type, mineralogy, and diageneis. For promoting hydrocarbon reservoir development; numerous petrophysical parameters are required. Some of them are measured but the others are sometimes assumed such as cementation exponent (m) and the multiplier (a) of Archie’s general equation. The present work offers some empirical models and charts to be used for resolving the cementation exponent and the Winsauer’s multiplier. Thirty-nine core samples obtained from the Upper Cretaceous reservoir zones; Gebel Nazzazat, Western central Sinai (Raha, Wata and Matulla formations) were laboratory analyzed for porosity, density, electrical resistivity, and ultrasonic wave velocity. They are composed mainly of clean sandstone, with some limestone and dolomitic limestone. This paper introduces seven innovative charts to estimate formation factor parameters using electrical quality index concept (EQI). They are classified according to the value of multiplier (a = 0.5 – 4.0) which is a function of electric tortuosity. The first chart estimates the multiplier (a) from P-wave velocity. The other six charts estimate the cementation exponent (from m = 1.0 up to 3.0) using fractional porosity and P-wave velocity at the previously determined multiplier (a). The present charts are based on a reliable empirical relationship (R² = 0.82) connecting elastic-electrical parameters as formation resistivity factor and P-wave velocity. The model reliability verification is accomplished using 82 laboratory measurements of porosity and P-wave velocity for sandstone core samples from the Bahariya Formation side by side with 51 empirical estimated multiplier and cementation exponent for different lithofacies (sandstones and carbonates) and obtained from several geographic locations.

1. Introduction
Many factors affect cementation exponent such as secondary porosity, reservoir pressure, temperature fluid concentration, mineralogy, pore throat size distribution, pore geometry, and the wettability condition of the reservoir rock. [1] studied the influence of reservoir porosity, permeability and clay content on the joint elastic electrical properties of reservoir sandstones. P- and S-wave velocities were
found to be linearly correlated with an apparent electrical formation factor on a semi-logarithmic scale for both clean and clay-rich sandstones.

Based on the analysis of the theoretical expressions of cementation factor and SDR model, a useful cementation factor prediction model, which is relevant to porosity and logarithmic mean of NMR $T_2$ [2]. [3] introduces a numerical method for cementation exponent determination only for clean sandstones. [4] studied the cementation factor in the Asmari limestone and stated that it is impossible to introduce a constant $m$ value and separate cementation factor correlations versus porosity for each petrofacies. [5] proposed a new parameter for classification of formation factor data to improve estimation of cementation exponent in carbonate reservoirs.

Imprecise estimation of Archie's parameters such cementation factor, saturation exponent and Winsauer's multiplier as function of tortuosity factor may generate large error in calculation of water saturation. In this paper seven charts are nominated to estimate these parameters graphically. The first chart estimates the multiplier ($a$) from P-wave velocity. The other six charts estimate the cementation exponent using fractional porosity and P-wave velocity at the previously determined multiplier as a function of tortuosity. This classification of existed charts according to their multiplier value depends on the electrical quality index (EQI) concept presented by [5] which improves estimation of cementation exponent in sandstone reservoir rocks.

2. Theoretical background

2.1. Formation factor

Archie [6] showed experimentally that resistivity of a brine-saturated rock ($R_o$) increases linearly with resistivity of brine saturant ($R_w$). He called proportionality constant formation resistivity factor ($F$) as:

$$F = \frac{R_o}{R_w}$$

(1)

Archie then plotted formation resistivity factor against porosity ($\phi$) on a log-log paper and identified another linear trend as follow:

$$F = \phi^{-m}$$

(2)

Winsauer et al. [7] introduced a new parameter to Archie's equation, namely tortuosity factor or multiplier ($a$). It is defined as one of the key parameters to characterize the transport properties of porous media:

$$F = 0.62 \phi^{2.15}$$

(3)

Carothers et al. [8] obtained correlation by analyzing 981 sandstone core samples as:

$$F = 1.45 \phi^{1.54}$$

(4)

Itenberg et al. [9] established a relationship for clean sandstone samples as:

$$F = 1.15 \phi^{1.65}$$

(5)

El Sayed [10] obtained correlation by analyzing 26 sandstone core samples from Yusr Member of Lower Miocene in age (Ummel-Yusr oil field, Gulf of Suez, Egypt), where $R_w = 2.0$ ohm.m, as:

$$F = 0.798 \phi^{1.44}$$

(6)

El Sayed [11] using 53 sandstone core samples obtained from 35 drilled wells and belonging to the Algypo-2 reservoir zone of Pliocene in age (Algypo” oil and gas field, Hungary) established a relationship as:

$$F = 0.925 \phi^{2.0}$$

(7)

Timur et al. [14] studied formation factor of 1800 sandstone samples to present the following correlation [14]:

$$F = 1.13 \phi^{1.73}$$

(8)
2.2. Cementation exponent

El Sayed [11] studied cementation factor (m) of 53 sandstone samples versus saturation exponent (n) at two different brine concentrations (R_w = 0.553 and 0.1 ohm.m) for three deltaic rock genetic types (barrier bar, distributary channel and deltaic fringes) as:

\[ m = 0.85 + 0.66 \, n \]  

(9)

Borai [12] studied cementation factor of 64 carbonate core samples offshore in Abu Dhabi and proposed a correlation versus porosity for predicting the value of cementation factor (m):

\[ m = 2.2 - (0.035/ (\Phi + 0.042)) \]  

(10)

Liang [13] used NMR log data to predict the value of m in low-permeability formations, ending up with the following correlation:

\[ m = \left( \frac{a_1 + n_1 \ln(T2lm)}{\ln \Phi} \right) + a_2 \]  

(11)

Where T2lm is logarithmic mean of NMR T2 spectrum (ms) and a1; n1 and a2 are constants to be determined by NMR and resistivity experimental data.

3. Methodology

The porosity and permeability were measured for 39 core plug samples (1.5 inch in diameter and length ranged from 1.5 up to 2.0 inch) belonging to Upper cretaceous. They were cleaned by organic solvents (Toluene and Methanol) and then tested by standard methods at ambient condition. In routine core analysis, porosity values are obtained by Helium porosimeter (model-7542-005) apparatus using the Boyle’s law, and permeability is measured by gas permeameter apparatus using the Darcy’s law, while the Klinkenberg correction method on permeability values is accomplished [14].

To measure electrical resistivity (Ro), samples were fully saturated by a brine salinity (200,000 ppm, NaCl). Using Core Lab.Inc. A.C. Resistivity Bridge (Model- 100A) at a frequency of 1 kHz, electrical resistance (r) of the samples was measured along the axis of cylindrical plugs in reservoir condition. Then, resistivity (R) was calculated from the measured resistance (r) using the cross-sectional area (A) and the length of the core plug (L). Formation factor was obtained as a ratio of rock resistivity (Ro) to brine resistivity (Rw). It ranged from 30 to 1100.

These samples were conducted to laboratory acoustic wave velocity measurements of both compressional and shear wave types using the Sonic Viewer_170 OYO-Instruments (Ain Shams University- Petrophysical Unit). The acoustic velocities were measured at frequencies of 63 kHz and 33 kHz for compressional and shear wave respectively.

4. Results and Discussions

4.1. The proposed model:

The existing model introduces seven charts to estimate formation factor parameters using electrical quality index concept (EQI). They are classified according to the value of multiplier (a = 0.5 – 4.0) which is a function of electric tortuosity. The first chart estimates the multiplier (a) from P-wave velocity. The other six charts estimate the cementation exponent (from m = 1.0 up to 3.0) using fractional porosity and P-wave velocity at the previously determined multiplier (a). The present charts are based on a reliable empirical relationship (R^2 = 0.82) connecting elastic-electrical parameters as formation resistivity factor and P-wave velocity [15]:

\[ \log F = 0.9 \, V_p - 0.69 = \log a - m \log \Phi \]  

(12)

Using equation (9), seven charts were built. The first one (figure 1) indicates the value of multiplier, from the relationship between (a) and P-wave velocity at different values of cementation exponent and fixed porosity. The other six charts (figure-2-7) were constructed, where each one is
characterized with a fixed value of multiplier (a) to calculate cementation exponent (m) from porosity and P-wave velocity. To handle the chart (figure 1) for calculating (a) from Vp you have to follow these steps; 1. from $V_p = 1.8$ to 2.5 use line ($m = 1$), 2. from $V_p > 2.5$ to 3.2 use line ($m = 1.5$), 3. from $V_p > 3.2$ to 4.0 use line ($m = 2.0$), 4. from $V_p > 4.0$ to 4.8 use line ($m = 2.5$), 5. from $V_p > 4.8$ use line ($m = 3.0$).

**Figure 1.** P-wave Velocity ($V_p$, km/s) Vs. Multiplier (a)

**Figure 2.** P-wave Velocity Vs. Porosity at Different Cementation Exponent (m)
Then one can select the chart of the obtained \( a \) and by use of \( V_p \) and \( \Theta \) values calculated from borehole logging, Seismic or measured in laboratory to calculate the value of cementation exponent \( m \).

**Figure 3.** P-wave Velocity Vs. Porosity at Different Cementation Exponent(m)

**Figure 4.** P-wave Velocity Vs. Porosity at Different Cementation Exponent(m)
Figure 5. P-wave Velocity Vs. Porosity at Different Cementation Exponent (m)

Figure 6. P-wave Velocity Vs. Porosity at Different Cementation Exponent (m)
4.2. Model reliability verification

The model reliability verification is accomplished using 82 laboratory measurements of porosity and P-wave velocity for sandstone core samples from the Bahariya Formation [16], side by side with 51 empirical estimated multiplier and cementation exponent (table 1) for different lithofacies (sandstones and carbonates) and obtained from several geographic locations (figure 3). This figure shows that the chart’s multipliers and cementation exponents are plotted altogether and inside them the laboratory measured parameters are located (red colored points). It means that the calculated parameters using these charts are reasonable and reliable.

Table 1. Laboratory Measured Cementation Exponent (m) and Multiplier (a) by Different Authors for Different Sandstone and Carbonate facies in Egypt, Iran and Hungary

| m     | a   | m     | a   | m     | a   | m     | a   | m     | a   | m     | a   |
|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|-------|-----|
| Mabrouk et al. (2013) Sandstone Egypt | 1.71 | 0.68 | 1.1 | 5.58 | 1.02 | 14.8  | 1.9 | 1.8   | 1.52 | .91  | .466 | 147.64 |
| M.R. Rezaee et al. (2007) Carbonates Iran  | 1.69 | 0.72 | 1.18 | 7.01 | 1.8 | 1.2   | 2.9 | 0.48  | 1.38 | .98  |    |       |
| El Sayed and El Dairy (1976, 1981 &1990) Sandstone Egypt | 1.84 | 0.59 | 1.22 | 9.35 | 1.9 | 1.5   | 2.4 | 1.3   | 1.77 | 3.6  |    |       |
| El Sayed and El Dairy (1991) Sandstone Egypt | 1.88 | 0.89 | 1.34 | 14.19 | 2.2 | 1.4   | 2.7 | 0.5   | 1.83 | 3.9  |    |       |
| El Sayed, 1991 Saudi Arabia sandstone | 1.45 | 1.4 | 1.11 | 5.22 | 2.4 | 1.3   | 2.2 | 1     | 1.74 | 3.3  |    |       |
| Ali, (2019) Egypt Carbonates | 1.92 | 1.2 | 2.4 | 0.5 | 2.6 | 1.2   | 2.7 | 0.42  | 1.411 | 2.092 |    |       |
| Mabrouk et al. (2013) Sandstone Egypt | 1.7 | 1.1 | 0.74 | 22.56 | 3 | 1     | 1.8 | 1     | 1.56 | 1.691 |    |       |
| Mabrouk et al. (2013) Sandstone Egypt | 1.79 | 1.01 | 0.94 | 12.53 | 1.44 | 0.798 | 2 | 0.92  | 1.034 | 5.422 |    |       |
| Mabrouk et al. (2013) Sandstone Egypt | 2.2 | 1.3 | 0.55 | 49.5 | 1.26 | 4.4   | 2 | 0.92  | 0.859 | 16.573 |    |       |
| Mabrouk et al. (2013) Sandstone Egypt | 1.65 | 1.6 | 1.03 | 12.58 | 1.36 | 6.81  | 1.49 | 1.1   | 0.728 | 77.481 |    |       |
The calculated regression line equation controlling the laboratory measured (a) and (m) for different lithofacies in different geographic localities (table-1) is:

\[ a = 8.1395 m^{3.091} \]  

According to the zigzag capillary tube model [17] and using the empirical equation (10), therefore; at \( \Theta=45^\circ \), \( a=2.0 \) then (m) vale will be = 1.5747, so the formation factor – porosity relation will be:

\[ F = 2.0 \phi^{-1.57} \]  

By use of 82 laboratory measurements [16] of porosity and P-wave velocity for sandstone core samples from the Bahariya Formation (Egypt) to enter the existing charts for calculating (a) and (m). They are plotted together with the data of Table-1 (figure-8) showing great harmonization with each other’s.

5. Conclusions
This paper introduces seven innovative charts to estimate formation factor parameters using the electrical quality index concept (EQI). They are classified according to the value of multiplier (\( a = 0.5 \) – 4.0) which is a function of electric tortuosity. The first chart estimates the multiplier (a) from P-wave velocity. The other six charts estimate the cementation exponent (from m = 1.0 up to 3.0) using fractional porosity and P-wave velocity at the previously determined multiplier (a). The model reliability verification is accomplished by using 82 laboratory measurements of porosity and P-wave velocity for sandstone core samples from the Bahariya Formation side by side with 51 empirical estimated multiplier and cementation exponent for different lithofacies (sandstones and carbonates) and obtained from several geographic locations. The calculated (a) and (m) parameters using these charts are reasonable and reliable.

Acknowledgements
Authors acknowledge the facilities given by the Petrophysical research unit of Ain Shams University.
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