THE EXACT DETERMINATION OF THE COSMOLOGICAL CONSTANT

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Abstract

With cooling of the cosmological plasma during phase transitions in the early Universe vacuum condensates of quantum fields were produced with negative energy density. Probably these condensates have compensated the initial vacuum energy. The present vacuum is the vacuum condensate of the last relativistic phase transition ($T_{cr} \sim 100$ MeV) (its information is carried by pseudo-Goldstone bosons ($\pi$-mesons)). This allows us to use Zeldovich’s formula that can calculate exactly the value of the cosmological constant in the present epoch. If the most likely values $\Omega_\Lambda$ are 0.7-0.8 then $H_0$ falls in the range from 72.5 to 67.8 (km/s)/Mpc.

The cosmological constant problem is one of the intriguing problems of modern physics and cosmology. The suggestions for its solution have attracted a lot of attention (for a review see article 2). In this notice I provide some arguments and calculate the present-day value of cosmological constant as the vacuum energy density of the last relativistic phase transition.

The terms of vacuum energy and cosmological constant ($\Lambda$-term) are used practically synonymously in modern cosmology.

The physical vacuum is a complex heterogeneous system of classical and quantum fields. It has the interior structure and changes its state with the change of existence conditions (a phase transition for example). The reality of vacuum energy forces to insert it in equations of the gravitational theory in the form of $\Lambda$-term. $\Lambda$-term was introduced first by A.Einstein in his equations in order to obtain static cosmological solutions. It was also named the cosmological constant

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}$$

(1)

Here $R_{\mu\nu}$, $R$, $g_{\mu\nu}$ and $T_{\mu\nu}$ are Riemann, Ricchi, metric and energy momentum tensors. $G$ is the Newtonian gravitational constant. Up till now these equations were the central ones in cosmology. The equation (1) leads to the Friedmann equations from which the density parameter of the Universe $\Omega_o$ can be defined.

Currently some cosmological data point out that we live in the Universe with a nonzero vacuum energy density (the lensing optical depth of galaxies at the moderate redshifts is substantially greater than in Einstein-de Sitter Universe). The ”age crisis”, the problem of large-scale structure formation, the spatial curvature are solved better if $\Omega_\Lambda = \rho_\Lambda/\rho_{cr} \equiv$
\[ \frac{\Lambda c^2}{3H_0^2} \neq 0 \] (here \( \Omega_\Lambda \) is the vacuum energy density in units of the critical density \( \rho_{cr} = \frac{3H_0^2}{8\pi G} \), \( H_0 \) - is the Hubble constant). \( \Omega_\Lambda = \Omega_o - \Omega_m \) is one way to resolve the discrepancy between \( \Omega_o \approx 1 \) (\( \Omega_o \) should be the order of unity since the spatial curvature is practically zero in the present epoch) and the density of matter \( \Omega_m = \Omega_b + \Omega_{CDM} + \Omega_{HDM} = 0.2 - 0.3 \). Here \( \Omega_b, \Omega_{CDM}, \Omega_{HDM} \) are the densities of baryons, cold and hot dark matter particles respectively (also all are in units of the critical density).

To be exact it is pertinent to say here few words about the birth of the Universe and its early evolution. It is reasonable to suppose that our Universe nucleates spontaneously out of "nothing" either in a clean vacuum state or in an anisotropic state with some number of particles and some nonequilibrium state of vacuum. Probably the processes of relaxations took place near Planck parameters. The creation of the three-dimensional Universe (as one of these processes) may be the process of spontaneous breaking of the local supersymmetry of a high-dimensional space-time. Naturally subsequent evolution of the Universe was accompanied by the decrease of it symmetry via relativistic phase transitions (RPT). Possible series of RPT were:

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P \implies D_4 \times [SU(5)]_{SUSY} \implies D_4 \times [U(1) \times SU(2) \times SU(3)]_{SUSY}\]

\[10^{19} \text{Gev}\] \[\implies D_4 \times U(1) \times SU(2) \times SU(3) \implies D_4 \times U(1) \times SU(3)\]

\[10^5 - 10^{10} \text{Gev}\] \[\implies D_4 \times U(1)\]

\[10^2 \text{Mev}\]

or more complex initial stage \( P \implies E_6 \implies 0(10) \implies SU(5) \implies .... \) \( P \) here is the group of local supersymmetry joining all physical fields and interactions; \( D_4 \) is the group of diffeomorphisms corresponding to the gravitational interaction; \( [SU(5)]_{SUSY} \) is the group of Grand Unification with the global supersymmetry; \( U(1) \times SU(2) \times SU(3) \) is the group of Standard model symmetry of elementary particles physics.

The spontaneous compactification can be considered as the first RPT as a result of which the gravitational vacuum condensate was produced \( (E \sim 10^{19} \text{ GeV}) \). But this transition is practically impossible to describe well now. Probably after the first RPT Lambda-term must be in the form: \( \Lambda = \Lambda_G + \Lambda_{QF} \), where \( \Lambda_G \) is the gravitational vacuum condensate; \( \Lambda_{QF} \) is the vacuum of quantum fields (zero vibrations of quantum fields, nonperturbative condensates, Higgs condensates). The calculation of \( \Lambda_{QF} \) belongs to group of tasks solved in the frame of renormalizable models of quantum field theory. The rest of RPT are described by modern theories of elementary particles (the reality of two last RPT is evident to everybody).

With the cooling of the cosmological plasma during RPT vacuum condensates with a negative energy density are produced. These condensates have the asymptotic equation of state \( \rho_{vac} = -\epsilon_{vac} = \text{const.} \) Thus RPT series were accompanied by the generation of negative contributions in initial Lambda-term. Therefore Lambda-term was changed during the Universe evolution and can be calculated exactly since the present vacuum is the vacuum condensate of the last RPT \( (T_{cr} \sim 100 \text{ MeV}) \). The condensates in the modern quantum field theory
are macroscopic mediums with quasiclassical properties. The periodic collective motions in this medium are perceived as pseudo-Goldstone bosons. But π-mesons that were produced as a result of this RPT are pseudo-Goldstone bosons that carry the information about this vacuum.

Ya. Zeldovich\textsuperscript{21} attempts to account for a nonzero vacuum energy density of the Universe in terms of quantum fluctuations (the gravitational force between particles in the vacuum fluctuations as a higher-order effect). Thus using Zeldovich’s formula\textsuperscript{21} we can get the value of the cosmological constant (Λ-term) and the vacuum energy density:

$$\Lambda = 8\pi G^2 m^6 \pi^{-4} \approx 1.289 \times 10^{-56} \text{cm}^{-2}$$

$$\rho_\Lambda = Gm^6 \pi^{-4} \approx 6.908 \times 10^{-30} \text{gcm}^{-3}$$

in which $m_\pi$ is the mean mass of π-mesons ($m_\pi = \frac{2m_e + m_\mu + m_\tau}{3} = 138.0387 \text{MeV}$); $h$ is the Planck constant (to use Zeldovich’s formula the author\textsuperscript{17} have suggested the first). Then $\Omega_\Lambda = \rho_\Lambda/\rho_{cr}$ can be calculated for different values of the Hubble constant, which is well unknown now.:

| $H_0$ (kms\(^{−1}\)/Mps) | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 |
|-------------------------|----|----|----|----|----|----|----|----|
| $\Omega_\Lambda$        | 1.47 | 1.21 | 1.02 | 0.87 | 0.75 | 0.65 | 0.57 | 0.51 |

If the most likely values $\Omega_\Lambda$ are 0.7-0.8 then $H_0$ falls in the range from 72.5 to 67.8 (km/s)/Mpc. Substituting the Planck mass into formula (2), the difference in 120 orders between the current value and its value at the Planck time can be found. Here it is also worth noting two important facts. Practically the total compensation of initial Λ-term allows to talk about the selforganization of vacuum\textsuperscript{22}. The processes of the creation particles from vacuum energy have occurred during relativistic phase transitions. The series of RPT must be continued by the phase transition in dark matter medium. Certainly our consideration of Λ-term problem is far from fullness. Zeldovich\textsuperscript{21} has obtained the formula (2) for Λ-term using Eddington\textsuperscript{23} and Dirac\textsuperscript{24} formulas of large numbers. But this was an important result at the qualitative level. Nevertheless some conclusions can be made. There had been phase transitions in the Universe. In the present epoch the vacuum energy density is a nonzero and positive (vacuum is in a excited state). The problem of Λ-term was not solved since which has been the initial Λ-term and how it has transformed during evolution of the Universe we do not know exactly. Probably Λ-term is the key to physics of XXI century.

Finally we note, that all the values of fundamental constants, the limits on $H_0$ and masses of π-mesons were taken from the review of particle properties\textsuperscript{25}. By tradition the term ”Λ-term” is used more often than the term ”the cosmological constant”. The surprising thing is that in the article\textsuperscript{26} the square of the most likely values $H_0$, $\Omega_\Lambda$ and $\Omega_m$ is practically coincided with our values.

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