Wormholes supported by phantom-like modified Chaplygin gas

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Abstract

We have examined the possible construction of a stationary, spherically symmetric and spatially inhomogeneous wormhole spacetime supported by the phantom energy. The later is supposed to be represented by the modified Chaplygin gas equation of state. The solutions so obtained satisfy the flare out and the asymptotic flatness conditions. It is also shown that the averaged null energy condition has to be violated for the existence of the wormhole.

1 Introduction

One of the most exotic geometries that arise as solutions of Einstein field equations is the wormhole. A typical two mouth wormhole connects two arbitrary points of the same spacetime or two distinct spacetimes. One observes that any typical Schwarzschild spacetime contains a singularity at $r = 0$ making it geodesically incomplete. Ellis [1] first observed that the coupling of the geometry of spacetime to a scalar field can produce a static, spherically symmetric, geodesically complete and horizonless spacetime and thus termed it as a ‘drainhole’ that could serve as tunnel to traverse particles from one side to the other. Later on Morris and Thorne [2, 3] proposed that wormholes could be thought of (imaginary) time machines that could render rapid interstellar travel for human beings. While a black hole possesses single horizon which forbids two way travel (in and out) of the black hole but this problem does not arise in the absence of horizon for a

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wormhole. Unfortunately the existence of wormholes require the violation of the most cherished energy conditions of general relativity (null, weak, strong and dominant) which are in fact satisfied by any normal matter or energy [4]. In particular, matter violating null energy condition is called ‘exotic matter’ [5]. Later it was proposed that wormholes could be constructed with arbitrary small quantities of exotic matter [6, 7]. A commonly known form of matter violating these energy conditions is dubbed as ‘phantom energy’ characterized by the equation of state (EoS) $p = \omega \rho$, where $p$ and $\rho$ are respectively, the pressure and the energy density of the phantom energy, with $\omega < -1$. The existence of this matter remains hypothetical but the astrophysical observations of supernovae of type Ia and cosmic microwave background have suggested the presence of phantom energy in our observable universe [8, 9]. It can exhibit itself as a source that can induce an acceleration in the expansion of the universe. The typical size of a wormhole can be of the order of the Planck length but it can be stretched to a larger size if it is supported by exotic phantom like matter [10, 11]. The accretion of phantom energy can increase the mass and size of the wormhole and hence guarantee the stability of the wormhole [12, 13]. The astrophysical implications of wormholes are not exactly clear but it is suggested that some active galactic nuclei and other galactic objects may be current or former entrances to wormholes [14]. It has been predicted that wormholes can also produce gravitational lensing events [15]. Since wormholes are horizonless, they can avoid undergoing any process of decay like Hawking evaporation and hence can survive over cosmological times. But a wormhole may form a black hole with a certain radial magnetic field (a form of magnetic monopole) if it accretes normal matter and consequently loses its structure.

Earlier, Rahaman et al [16] investigated the evolution of wormhole using an averaged null energy condition (ANEC) violating phantom energy and a variable EoS parameter $\omega(r)$. We here investigate the same problem using a more general EoS for the pressure density namely the modified Chaplygin gas. It is well-known that the wormhole spacetime is inhomogeneous and hence requires inhomogeneous distribution of matter. This can be made by introducing two different pressures namely the radial and the transverse pressure. Our analysis shows that the parameters adopted in the equation of state for phantom energy have to be tuned such that the radial pressure becomes negative in all directions and for all radial distance. This result turns out to be consistent with Sushkov [17].

The paper is organized as follows: In the second section, we have modeled the field equations for the wormhole spacetime and proposed the methodology that is adopted in the later sections. Next, we have investigated the behavior of energy condition ANEC for all the wormhole solutions obtained. Finally, the last section is devoted for the conclusion and discussion of our results.
2 Modeling of system

We start by assuming the static, stationary, spherically symmetric wormhole spacetime specified by (in geometrized units $G = 1 = c$):

\[ ds^2 = -e^{2f(r)} dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \] (1)

Here $f(r)$ is the ‘gravitational potential function’ while $b(r)$ is called ‘shape function’ of the wormhole (see Ref. [18] for the consistent derivation of the above metric). The radial coordinate $r$ ranges over $[r_o, \infty)$ where the minimum value $r_o$ corresponds to the radius of the throat of the wormhole. If $b(r) = 2m(r)$, the later being the mass, then Eq. (1) represents a ‘dark energy star’ which may arise from a density fluctuation in the Chaplygin gas cosmological background [19, 20]. Note that $b(r = r_o) = r_o$ corresponds to the spatial position of the wormhole throat. We shall, in this paper, assume $f(r) = \text{constant}$ for the convenience of our calculations. This choice, as a special case, is also physically motivated and makes the time traveler to feel zero tidal force near the wormhole [16, 23]. A wormhole with small $|f'(r)|$ in the vicinity of the throat is likely to be traversable in the sense of having low tidal forces. It also makes the wormhole to be horizon-free.

We take the inhomogeneous phantom energy which is specified by the stress energy tensor: $T_{00} = \rho$, $T_{11} = p_r$, $T_{22} = T_{33} = p_t$. Here $p_r$ and $p_t$ are, respectively, the radial and transverse component of the pressure while $\rho$ is the energy density of the phantom energy. It represents a perfect fluid (which is homogeneous and isotropic) if $T_{11} = T_{22} = T_{33} = p_r = p_t$ [21]. Note that in the stellar evolution, the difference $p_r - p_t$ creates a surface tension inside star which makes it anisotropic. This feature is generically found in more compact stars like neutron and quark stars [22], contrary to normal stars which are majorally supported by radial pressure only against gravity.

The Einstein field equations ($G_{\alpha\beta} = 8\pi T_{\alpha\beta}$) for the metric (1) are

\[ \frac{b'(r)}{r^2} = 8\pi \rho(r), \] (2)

\[ -\frac{b}{r^3} = 8\pi p_r(r), \] (3)

\[ \left(1 - \frac{b}{r}\right) \left[ \frac{-b'r + b}{2r^2(r-b)} \right] = 8\pi p_t(r). \] (4)

The energy conservation equation is obtained from $T_{\alpha\beta}^{\alpha\beta} = 0$, which gives

\[ p'_r + \frac{2}{r} p_r - \frac{2}{r} p_t = 0. \] (5)
This equation can be considered as the hydrodynamic equilibrium equation for the exotic phantom energy supporting the wormhole.

Eq. (2) can be written in the form

$$\frac{db}{dr} = 8\pi r^2 \rho.$$  \hfill (6)

Let us choose the modified Chaplygin gas (MCG) EoS for the radial pressure \[25\]

$$p_r(r) = A\rho(r) - \frac{B}{\rho(r)^\alpha}.$$  \hfill (7)

Here $A$, $B$ and $\alpha$ are constant parameters. The MCG best fits with the 3–year WMAP and the SDSS data with the choice of parameters $A = -0.085$ and $\alpha = 1.724$ \[26\] which are improved constraints than the previous ones $-0.35 < A < 0.025$ \[27\]. Recently it is shown that the dynamical attractor for the MCG exists at $\omega = -1$, hence MCG crosses this value from either side $\omega > -1$ or $\omega < -1$, independent to the choice of model parameters \[28\]. Generally, $\alpha$ is constrained in the range $[0, 1]$ but here we are assuming it to be a free parameter which can take values outside this narrow range, for instance $\alpha = -1$ as considered below. This later choice $\alpha < 0$ makes Eq. (7) a combination of a barotropic and a polytropic equation of state.

Let us take the transverse pressure $p_t$ to be linearly proportional to the radial pressure $p_r$ as

$$p_t = np_r,$$  \hfill (8)

where $n$ is a non-zero constant. Thus $p_r$ is restricted to satisfy Eq. (7) for a given $\rho$ while $p_t$ is arbitrary in nature due to free parameter $n$. Using Eq. (8) in (5), we obtain

$$p_r' + \frac{2}{r}p_r - \frac{2n}{r}p_r = 0,$$  \hfill (9)

which gives

$$p_r = C r^{2(n-1)}.$$  \hfill (10)

Here $C$ is a constant of integration. Since the wormhole is supported by a negative pressure inducing exotic phantom energy, it yields $p_r < 0$ if $C < 0$ and $1 < n < \infty$ in order to obtain finite negative radial pressure. Consequently $p_t < 0$ if $0 < n < \infty$. Using Eq. (10) in (7), we have

$$A\rho - \frac{B}{\rho^\alpha} = C r^{2(n-1)},$$  \hfill (11)

which can be written as

$$A\rho^{\alpha+1} - C r^{2(n-1)}\rho^\alpha - B = 0.$$  \hfill (12)

Note that Eq. (12) is a polynomial equation of degree $\alpha + 1$ in variable $\rho$, which does not yield solutions for any arbitrary $\alpha$. We shall, henceforth, solve Eq. (12) for specific
choices like $\alpha = -1, 0$ and $1$. We shall further employ the following conditions on our solutions given below [24]:

1. The potential function $f(r)$ must be finite for all values of $r$ for the non-existence of horizon. In our model, this condition is trivially satisfied since $f(r)$ is taken to be a finite constant throughout this paper.

2. The shape function $b(r)$ must satisfy $b'(r = r_o) < 1$ at the wormhole throat with radius $r_o$, the so-called flare-out condition.

3. Further $b(r) < r$ outside the wormhole’s throat $r > r_o$. This condition is a direct consequence of the flare-out condition.

4. The spacetime must be asymptotically flat i.e. $b(r)/r \to 0$ for $|r| \to \infty$.

Now we shall consider the three cases for different choices of parameter $\alpha$:

**Case-a:** If $\alpha = 0$, then (12) gives

$$\rho = \frac{B}{A} + \frac{C}{A}r^{2(n-1)}. \tag{13}$$

Using Eq. (13) in (6), we get

$$b(r) = \frac{8\pi}{A} \left[ \frac{Br^3}{3} + \frac{Cr^{2n+1}}{2n+1} \right] + C_1. \tag{14}$$

Here $C_1$ is a constant of integration. Now $\frac{b(r)}{r} \to 0$ as $|r| \to \infty$ if $n = 1$ and $B = -C$. But here $b(r) = \text{constant}$ and hence gives $\rho = 0$ which is an acceptable solution and represents vacuum (empty space-time) outside the wormhole throat. This corresponds to vanishing pressures i.e. $p_r = p_t = 0$. This vacuum solution requires $C = 0$ which in turn leads to $B = 0$. Note that condition (4) can also be met if only $B = 0$ and $n < 0$. In figures 1 to 4, we have plotted the ratio $b(r)/r$ against the parameter $r$. The Fig. 1 shows that the ratio declines as $r \to \infty$, although $r$ is restricted to a certain range. The parameter $A$ can assume the value in the range $-0.35 \leq A \leq 0.025$ [29], we choose $A = 0.025$ for our work.

Further, flare-out condition (2) implies

$$b'(r_o) = \frac{8\pi C}{A} r_o^{2n} < 1, \tag{15}$$

which gives an upper limit on the size of throat’s radius as

$$r_o < \left( \frac{A}{8\pi C} \right)^{\frac{1}{2n}}. \tag{16}$$
This requires both $A > 0$ and $C > 0$. The throat’s radius can be obtained by solving $b(r_o) = r_o$ which gives

$$r_o = \left[ \frac{A(2n+1)}{8\pi C} \right]^{\frac{1}{2n}}.$$  \hfill (17)

This quantity is positive if $A/C > 0$ and $2n+1 > 0$ or $A/C < 0$ and $2n+1 < 0$. Similarly, condition (3) translates into

$$r < \left[ \frac{A(2n+1)}{8\pi C} \right]^{\frac{1}{2n}}, \quad r > r_o.$$  \hfill (18)

Note that conditions (2) and (3) are satisfied if $C_1 = 0$.

**Case-b:** If $\alpha = 1$, then (12) gives

$$A\rho^2 - Cr^{2(n-1)}\rho - B = 0,$$  \hfill (19)

which is quadratic in $\rho$ and gives two roots of the form

$$\rho_\pm = \frac{C r^{2(n-1)} \pm \sqrt{C^2 r^{4(n-1)} + 4AB}}{2A}. \hfill (20)$$

These roots are real-valued if the quantity inside the square root is positive while the roots will be repeated if it is zero and complex valued otherwise. We next determine the shape function $b(r)$ corresponding to these roots by substituting Eq. (20) in (6) to get

$$b_\pm(r) = \pm \frac{4\pi r (\pm Cr^{2n} + r^2 \sqrt{4AB + C^2 r^{4(n-1)}})}{A(1+2n)} + C_{2\pm}$$

$$\pm [2B\pi r^7 \sqrt{1 + \frac{4ABr^{4(1-n)}}{C^2}} \sqrt{4AB + C^2 r^{4(n-1)}} \Gamma\left(\frac{1+2n}{4(1-n)}\right)]/(n-1)(4ABr^4 + C^2 r^{4n})$$

Here $C_{2\pm}$ are two constants of integration whereas $2F_1$ is the regularized hyper-geometric function. Figures 2 and 3 show the ratios $b_+(r)/r$ and $b_-(r)/r$ versus $r$, respectively. Both ratios decline for large values of $r$ and approach zero, satisfying the asymptotic flatness condition for specific choice of the parameters.

**Case-c:** If $\alpha = -1$, then (12) yields

$$\rho = \frac{C}{A - B} r^{2(n-1)}, \hfill (22)$$

Use of this in (6) enables us to write

$$b(r) = \frac{8\pi C}{2(n+1)(A - B)} r^{2(n+1)}. \hfill (23)$$
In figure 4, the ratio $b(r)/r$ is plotted against $r$, showing its convergence to zero. Further, condition (2) implies

$$b'(r_o) = \frac{8\pi C}{A - B} r_o^{2n+1} < 1,$$

which gives the maximum size of the wormhole’s throat

$$r_o < \left(\frac{A - B}{8\pi C}\right)^{\frac{1}{2n+1}}. \quad (25)$$

In other words, the throat’s radius is given by

$$r_o = \left[\frac{2(n + 1)(A - B)}{8\pi C}\right]^\frac{1}{2n+1}. \quad (26)$$

It requires either $A - B > 0$, $C > 0$ and $n > -1$ or $A - B < 0$ and $C < 0$. This later choice of parameters is consistent with the ones that are required for $p_r < 0$ in Eq. (10).

As we discussed earlier, the relativistic energy conditions are satisfied by ordinary classical matter but there are some physical processes where these conditions are violated. For example, for a black hole evaporation caused by the emission of Hawking radiation [30]. The quantized fields in the surrounding of black hole produce massive particles carrying positive energy density. Due to energy conservation in the whole process, the negative energy density is added to the total energy density of the black hole. Consequently the black hole loses mass and its horizon shrinks. The energy conditions are also violated when an electromagnetic wave is squeezed resulting in the energy density of the wave to become negative, zero and positive at certain wavelengths but the averaged energy density of the wave remains positive [23]. One observes that the notion of violation of energy conditions is quite ubiquitous at the quantum scale. As the quantum effects allow for a localized violation of energy condition, there is a limit to an extent by which these conditions can be violated globally. In this connection, the ‘averaged null energy condition’ (ANEC) is specified which states that [31]

$$\int \gamma T_{\alpha\beta} k^\alpha k^\beta d\lambda \geq 0. \quad (27)$$

Here $T_{\alpha\beta}$ is the stress energy tensor, $k^\alpha$ is the future directed null vector, $\gamma$ is the null geodesic and $\lambda$ is the arc-length parameter. In other words, the integrand must be positive. In an orthonormal frame of reference, we have $k^\alpha = (1, 1, 0, 0)$, so that $T_{\alpha\beta} k^\alpha k^\beta = \rho + p_r$. We here adopt the ANEC integral from Visser et al [7] to analyze its violation in our model:

$$I = \oint (\rho + p_r)dV = 2 \int_{r_o}^{\infty} (\rho + p_r) 4\pi r^2 dr. \quad (28)$$

The above integral is called the ‘volume integral quantifier’ [34]. It is obvious that the above integral becomes negative if $\rho + p_r < 0$, the violation of null energy condition
The violation of ANEC is the requirement for any phantom matter and stability of a wormhole. Now we shall take different $\rho$ and $p_r$ calculated in each of the above cases to evaluate $I$. Our aim will be to find conditions under which $I < 0$. We shall also consider the case of $I \to 0$ which suggests the construction of wormhole with arbitrarily small amounts of a phantom energy.

**Case-a** Using Eqs. (7) and (13) in (28), we obtain

$$I = \frac{8\pi C(1 + A)}{A(1 + 2n)} r_1^{1 + 2n} = \frac{8\pi C}{A} (1 + A) r_1^{1 + 2n} |_{r_o}^{\infty}. \quad (29)$$

Note that the above integral gives a finite value if $n < -1/2$. Hence we obtain

$$I = -(1 + A) \left[ \frac{A(1 + 2n)}{8\pi C} \right]^{\frac{1}{\nu}}. \quad (30)$$

Moreover, the above integral $I < 0$ if $1 + A > 0$ or $A > -1$. Further, $A/C < 0$ which implies either (1) $C < 0$ and $A > 0$ or (2) $C > 0$ and $A < 0$. Again the former case (1) is consistent with $p_r < 0$. Also $I \to 0$, if either $A \to 0$ or $n \to -1/2$.

**Case-b** Using Eqs. (7) and (21) in Eq. (28), we get

$$I_\pm = \frac{4\pi r(1 + 2A)Cr_2^{2n} \pm r^2 \sqrt{4AB + C^2r_4^{4(n-1)}}}{A(1 + 2n)} \pm \left[ 32(n - 1) B_{2\nu} \right]^{\frac{r}{2}} \left[ 1 + \frac{4ABr_4^{4(1-n)}}{C^2} \sqrt{4AB + C^2r_4^{4(n-1)}} \right] \left( \frac{5 - 2n}{4(1 - n)} \right)^{\frac{1}{2}} \left( \frac{9 - 6n}{4(1 - n)} \right)^{\frac{1}{2}} \left( \frac{4ABr_4^{4(1-n)}}{C^2} \right) \right]^{1/[2(n + 1)(2n - 5)(4ABr^4 + C^2r^4n)]} \left| r_o \right|^\infty. \quad (31)$$

The ANEC is violated for particular choice of parameters like $B = -3$, $C = -2$ and $n = 3$. Note that we have assumed $C_{2\nu} = 0$ for the convenience of our calculations. Under this choice of parameters, the two integrals $I_\pm$ will be finite. Also figures (5) and (6) show the behavior of the integrals $I_+ < 0$ and $I_- < 0$, respectively. The plots suggest that the two integrals $I_\pm$ tend to infinity for large $r$, so that an infinite amount of ANEC violating matter is necessary to sustain these geometries, which is a problematic issue. However this problem can be evaded by considering a matching to an exterior vacuum solution which gives a thin-shell wormhole solution $[32, 33, 35]$. Further, the case of $I_\pm \to 0$ arises if $n \to 1$, $A \to -1/2$ and $C^2 + 4AB \to 0$.

**Case-c** Making use of Eqs. (7) and (22) in (28) yields

$$I = \frac{8\pi C(1 + A - B)r_1^{1 + 2n}}{(A - B)(1 + 2n)} |_{r_o}^{\infty}. \quad (32)$$
The above integral is finite if \( n < -1/2 \). Therefore we obtain

\[
I = \frac{1 + A - B}{1 + 2n} \left( \frac{A - B}{8\pi C} \right)^{\frac{1}{2n}}.
\]

Further, the ANEC is violated \( I < 0 \) if either \( C < 0 \) or \( A - B < 0 \). The wormhole is supported by arbitrary small amount of phantom energy if \( A - B \to 0 \).

## 3 Conclusion and discussion

In this paper, we have derived three solutions of wormhole by obtaining different forms of \( b(r) \). This is carried out by employing the modified Chaplygin gas for the pressure and using three specific values of the parameter \( \alpha \). It needs to be mentioned that other values of \( \alpha \) either don’t yield any \( b(r) \) or if it does exist than the stability conditions 1 to 4 are not verified. Hence we have restricted ourselves to these specific cases as shown in the figures as well. The solutions so obtained also satisfy the stability conditions. The pressure and the corresponding energy density obtained in each case, violate the null energy condition \( \rho + p_r < 0 \) and hence the averaged null energy condition is also violated. These conditions need to be violated for the existence of any wormhole solution.

We performed the analysis by taking the wormhole spacetime to be inhomogeneous and anisotropic with non-vanishing transverse pressure. The spacetime needs to be anisotropic as it was found that considering an isotropic pressure \( p_r = p_t = p \), for \( f(r) \) to be finite, one cannot construct asymptotically flat traversable wormhole \([10]\). In our work, we represented the radial pressure by the modified Chaplygin gas and the transverse pressure to be linearly proportional to the radial one. The MCG has phantom nature with negative pressure. Earlier, Lobo \([34]\) studied the Chaplygin traversable wormhole and concluded that the Chaplygin gas needs to be confined around the wormhole throat neighborhood. That work was later extended in \([35]\) using the modified Chaplygin gas and it was deduced that modified Chaplygin wormholes may occur naturally and could be traversable. Our results are in conformity with their results since our solutions meet the criteria of wormhole stability and traversability.

In a recent paper, Gorini et al \([36]\) have presented an interesting theorem which states that in a static spherically symmetric spacetime filled with the phantom Chaplygin gas, the scalar curvature becomes singular at some finite value of the radial coordinate \( r \) and henceforth the spacetime is not asymptotically flat. This result apparently forbids the existence of wormholes which are required to be non-singular. The theorem is based on the assumptions of homogeneity and isotropy of the spacetime. In case of anisotropy \( (p_t \neq 0) \), the above theorem is not applicable and wormhole spacetime appears naturally. For an
isotropic and homogeneous spacetime filled with phantom Chaplygin gas, the asymptotic flatness can be achieved by cutting the spacetime at some spatial position $r = R$ and glued with a vacuum spacetime, in particular, Schwarzschild exterior spacetime can be utilized [34].

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Figure 1: The ratio $b(r)/r$ is plotted against $r$ with $C = 3$ and for different values of $n = -2, -2.5, -3, -3.5, -4, -4.5$ which correspond to curves in right to left order.

Figure 2: The ratio $b_+(r)/r$ is plotted against $r$. The parameters are fixed at $B = 6, C = 7$ and $n = 3$. 
Figure 3: The ratio $b_- (r) / r$ is plotted against $r$. The parameters are fixed at $B = -2, C = 7$ and $n = 3$.

Figure 4: The ratio $b (r) / r$ is plotted against $r$ for different values of $n = -3, -4, -5, -6, -7, -8$. The parameters are taken $B = 1, A = -5$ and $C = 2$ which correspond to curves in right to left order.
Figure 5: The ANEC integral $I_+$ is plotted against $r$. The apparent negative values of $I_+$ show the violation of the ANEC condition. The parameters are chosen as $B = -3$, $C = -2$ and $n = 3$.

Figure 6: The ANEC integral $I_-$ is plotted against $r$. The apparent negative values of $I_-$ show the violation of the ANEC condition. The choice of parameters is the same as in figure 5.