Algorithm for restoration of friction power in drum brake device with a layer from the compositional material on temperature data

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Abstract. A quasilinear two-dimensional mathematical model of the thermal process in a drum braking device is considered, taking into account the rotational motion of the brake cylinder. The basic heat conduction equations and the algorithm for solving by method of conjugate gradients of the boundary inverse heat conduction problem to restore the friction power between the brake pad and the cylinder on temperature data are given.

Keywords: brake cylinder, brake pad, frictional heat generation, mathematical model, friction power, inverse problem, conjugate gradient method.

1. Introduction
The restoration of the total heat flux and, accordingly, the specific friction power resulting from frictional heat generation in a friction pair, according to additional temperature data, belongs to the class of boundary inverse heat transfer problems. An effective method for solving inverse heat transfer problems is an iterative regularization method based on gradient methods for minimizing the target residual functional [1]. Some similar problems for solving inverse problems of identification of frictional heat generation from temperature data for various friction pairs are given, for example, in [2–6].

In this paper, the problem of restoring the total heat flux in the friction zone of a drum braking device widely used in vehicles, for example, in the T-150K tractor and in VAZ-2101 and GAZ-3102 cars is considered [7]. Drum brake device consists of a pair of fixed brake pads, located inside the brake cylinder, fixed with the drive shaft of the wheel and rotating with it. The brake pads are pressed against the cylinder with a certain force during braking, ensuring that the rotation of the wheel axis stops. In the process of braking, the kinetic energy of a moving vehicle is converted into heat by the braking device, dissipated by its elements into the environment. Thermal analyzes carried out by means of temperature measurements for a friction pair pursue such important problems as estimating the power of frictional heat generation and determining the effective thermophysical properties of the materials from which the braking device elements are made [7]. In this regard, the estimation or restoration of the specific power of frictional heat generation from temperature measurements is an actual mathematical problem.

2. Problem statement
Consider a simplified design diagram of a drum brake device in Fig.1, consisting of a brake cylinder, rotating together with a drive shaft, and a fixed pair of brake pads. In the design scheme, the actual
geometrical forms of the braking device are approximately idealized to describe the determining heat conduction equations in cylindrical coordinates. Heat sinks from the ends of the cylinder and pads will be considered negligible compared to the heat sinks from the side surfaces, which will allow us to apply the two-dimensional heat conduction equations in cylindrical coordinates. When braking, the initially rotating cylinder immediately does not stop, and its rotational speed will decelerate from the maximum initial value to zero during the time the vehicle stops. Therefore, we will consider the thermal model for a pair of brake pad-cylinder, taking into account the rotation of the cylinder, which has geometrical symmetry about the horizontal axis in the design diagram shown in Fig.1. Thus, we will set two-dimensional temperature problems for the brake pad and cylinder separately, in the most general case, taking into account the temperature dependence of the thermophysical properties of materials.

The temperature field \( T(r, \varphi, t) \) in the brake pad is described by a quasilinear two-dimensional heat conduction equation in cylindrical coordinates with thermophysical coefficients \( C_T = C_T(T) \), \( \lambda_r = \lambda(T) \):

\[
C_T \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \left( \lambda_r \frac{\partial T}{\partial \varphi} \right),
\]

\( R_1 < r < R_2, \quad \varphi_0 < \varphi < \pi - \varphi_0, \quad 0 < t \leq t_m. \)  \hspace{1cm} (1)

The temperature field \( U(r, \varphi, t) \) in the cylinder is also described by a two-dimensional heat conduction equation with thermophysical coefficients \( C_U = C_U(U) \), \( \lambda_r = \lambda_r(U) \) and with a convective term that takes into account its rotational motion with an angular velocity \( \Omega(t) \):

\[
C_U \frac{\partial U}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_u \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \left( \lambda_u \frac{\partial U}{\partial \varphi} \right) + \Omega(t) C_U \frac{\partial U}{\partial \varphi},
\]

\( R_2 < r < R_3, \quad 0 < \varphi < \pi, \quad 0 < t \leq t_m. \)  \hspace{1cm} (2)

Conditions of non-ideal thermal contact are set in the contact zone of cylinder and brake pad:

\[
\lambda_u \frac{\partial U}{\partial r} \bigg|_{r=R_2} - \lambda_r \frac{\partial T}{\partial r} \bigg|_{r=R_2} = q(\varphi, t), \quad \varphi_0 < \varphi < \pi - \varphi_0 \hspace{1cm} (3)
\]

\[
U(R_2, \varphi, t) = T(R_2, \varphi, t), \quad \varphi_0 < \varphi < \pi - \varphi_0. \hspace{1cm} (4)
\]
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\( \lambda_u \frac{\partial U}{\partial r} \bigg|_{r=R_2} = \alpha_u (U|_{r=R_2} - T_m), \quad 0 \leq \varphi \leq \varphi_0, \quad \pi - \varphi_0 \leq \varphi \leq \pi; \) \quad (5)

\( \lambda_u \frac{\partial U}{\partial r} \bigg|_{r=R_1} = -\alpha_u (U|_{r=R_1} - T_m), \quad 0 \leq \varphi \leq \pi; \) \quad (6)

\( \frac{\partial U}{\partial \varphi} \bigg|_{\varphi=0} = \frac{\partial U}{\partial \varphi} \bigg|_{\varphi=\pi} = 0, \quad R_2 \leq r \leq R_3; \) \quad (7)

\( \lambda_T \frac{\partial T}{\partial r} \bigg|_{r=R_1} = \alpha_T (T|_{r=R_1} - T_m), \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0; \) \quad (8)

\( \lambda_T \frac{\partial T}{r \partial \varphi} \bigg|_{\varphi=\varphi_0} = \alpha_T (T|_{\varphi=\varphi_0} - T_m), \quad R_1 \leq r \leq R_2; \) \quad (9)

\( \lambda_T \frac{\partial T}{r \partial \varphi} \bigg|_{\varphi=\pi-\varphi_0} = -\alpha_T (T|_{\varphi=\pi-\varphi_0} - T_m), \quad R_1 \leq r \leq R_2; \) \quad (10)

The initial temperature distribution in the elements of the braking device is assumed to be equal to the environment temperature

\[ U(r, \varphi, 0) = T(r, \varphi, 0) = T_m. \] \quad (11)

The direct problem (1) - (11) is solved by the finite difference method by splitting into spatial variables, consisting in a step-by-step solution by the one-dimensional difference equation sweep method with iteration over nonlinearity at each time step. In this case, the heat equation for a rotating cylinder with respect to the angular variable is solved using a monotonic difference scheme [8].

3. Algorithm for solving the boundary inverse problem

To solve the boundary inverse problem of identifying the total heat flux \( q(\varphi, t) \) using temperature data, we use the iterative regularization method developed in [1] and successfully implemented in [2-6]. We present the basic relations necessary for the implementation of the method. Let the fixed point \((R_f, \varphi_i)\), \(i = 1, ..., n\), of the measurement of temperature over time be set in a fixed brake pad at a fixed distance \( R_f, \quad R_1 < R_f < R_2 \) from the friction zone. The measured values of the temperature data are denoted by \( f(\varphi_i, t) \).

The problem of identifying the heat flux function from the temperature data is to minimize the following residual functional:

\[ J[q(\varphi, t)] = \frac{1}{2} \int_0^t \sum_{i=1}^n \left[ T(R_f, \varphi_i, t) - f(\varphi_i, t) \right]^2 dt, \] \quad (12)

on the solutions of system (1) - (11). Minimization of the functional (12) by the method of conjugate gradients is the most effective method for solving the inverse problem by iterative regularization. To determine the gradient of the functional (12), the adjoint problem is considered with respect to variables \( \Psi(r, \varphi, t) \), \( \Phi(r, \varphi, t) \) for brake pad and cylinder, respectively:

\[ -C_T \frac{\partial^2 \Phi}{\partial t} = \frac{\lambda_T}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Psi}{\partial r} \right) \left( \frac{\partial \Psi}{\partial r} \right) + \frac{\lambda_T}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{1}{r} \left[ T(r, \varphi, t) - f(\varphi, t) \right] \delta(\varphi - \varphi_0), \quad \pi > \varphi > \varphi_0, \quad 0 < t \leq t_m, \] \quad (13)

where \( \delta(\varphi - \varphi_i) = 1 \) at \( \varphi = \varphi_i \), \( \delta(\varphi - \varphi_i) = 0 \) at \( \varphi \neq \varphi_i \).
\begin{equation}
-C_U \frac{\partial \Phi}{\partial t} = \frac{\lambda_u}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) + \frac{\lambda_u}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2} - \Omega(t) \cdot C_U \frac{\partial \Phi}{\partial \varphi},
\end{equation}

\begin{equation}
R_2 < r < R_3, \quad 0 < \varphi < \pi, \quad 0 < t \leq t_m,
\end{equation}

\begin{equation}
\left( \lambda_U \frac{\partial \Phi}{\partial r} \right)_{r=R_2} - \left( \lambda_T \frac{\partial \Psi}{\partial r} \right)_{r=R_2} = 0, \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0,
\end{equation}

\begin{equation}
\Phi(r, \varphi, t) = \Psi(r, \varphi, t), \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0,
\end{equation}

\begin{equation}
\lambda_U \frac{\partial \Phi}{\partial r} \bigg|_{r=R_2} = \alpha_U \Phi \big|_{r=R_2}, \quad 0 \leq \varphi \leq \pi,
\end{equation}

\begin{equation}
\lambda_U \frac{\partial \Phi}{\partial r} \bigg|_{r=R_3} = -\alpha_U \Phi \big|_{r=R_3}, \quad 0 \leq \varphi \leq \pi,
\end{equation}

\begin{equation}
\frac{\partial \Phi}{\partial \varphi} \bigg|_{\varphi=0} = 0, \quad R_2 \leq r \leq R_3,
\end{equation}

\begin{equation}
\lambda_T \frac{\partial \Psi}{\partial r} \bigg|_{r=R_2} = \alpha_T \Psi \big|_{r=R_2}, \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0,
\end{equation}

\begin{equation}
\lambda_T \frac{\partial \Psi}{r \partial \varphi} \bigg|_{\varphi=\varphi_0} = \alpha_T \Psi \big|_{\varphi=\varphi_0}, \quad R_1 \leq r \leq R_2,
\end{equation}

\begin{equation}
\lambda_T \frac{\partial \Psi}{r \partial \varphi} \bigg|_{\varphi=\pi-\varphi_0} = -\alpha_T \Psi \big|_{\varphi=\pi-\varphi_0}, \quad R_1 \leq r \leq R_2,
\end{equation}

\begin{equation}
\Phi(r, \varphi, t_m) = \Psi(r, \varphi, t_m) = 0.
\end{equation}

The formula for calculating the gradient of the functional (14) is determined by solving the adjoint boundary value problem (13) - (23)

\begin{equation}
J'[q(\varphi, t)] = \Psi(R_2, \varphi, t).
\end{equation}

The algorithm for minimizing the functional (12) using the conjugate gradient method is represented by the following sequence of transition from the k-th approximation \( q^k(\varphi, t) \) to (k + 1) -th \( q^{k+1}(\varphi, t) \):

\begin{equation}
q^k(\varphi, t) \rightarrow T_k(r, \varphi, t) \rightarrow \Psi(r, \varphi, t) \rightarrow J'[q^k] \rightarrow \gamma_k \rightarrow S^k(\varphi, t) \rightarrow V_k(r, \varphi, t) \rightarrow \beta_k \rightarrow q^{k+1}(\varphi, t).
\end{equation}

Where

\begin{equation}
q^{k+1}(\varphi, t) = q^k(\varphi, t) - \beta_k S^k(\varphi, t), \quad k = 0, 1, 2, \ldots;
\end{equation}

\begin{equation}
S^k(\varphi, t) = J'[q^k] + \gamma_k S^{k-1}(\varphi, t), \quad \gamma_0 = 0, \quad \gamma_k = \int_{t_0}^{t_m} \left( J'[q^k(\varphi, t)) \right)^2 d\varphi dt.
\end{equation}

The descent step \( \beta_k \) is determined from the expression

\begin{equation}
\beta_k = \frac{\int_{t_0}^{t_m} [T_k(R_f, \varphi, t) - f(\varphi, t)] V_k(R_f, \varphi, t) d\varphi dt}{\int_{t_0}^{t_m} V_k^2(R_f, \varphi, t) d\varphi dt},
\end{equation}

where \( V_k(r, \varphi, t) \) is determined by the solution of the boundary value problem for increments \( V \) and \( W \) of the temperature the brake pads and cylinder, respectively, at \( \Delta q(\varphi, t) = S^k(\varphi, t) \):
\[
\frac{\partial C_T V}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \lambda_T V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \left( \frac{\partial \lambda_T V}{\partial \varphi} \right),
\]

(25)

\[
R_1 < r < R_2, \quad \varphi_0 < \varphi < \pi - \varphi_0, \quad 0 < t \leq t_m,
\]

\[
\frac{\partial C_U W}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \lambda_U W}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \left( \frac{\partial \lambda_U W}{\partial \varphi} \right) + \Omega(t) \frac{\partial C_U W}{\partial \varphi},
\]

(26)

\[
R_2 < r < R_3, \quad 0 < \varphi < \pi, \quad 0 < t < t_m,
\]

\[
\left. \frac{\partial \lambda_U W}{\partial r} \right|_{r=R_1} - \left. \frac{\partial \lambda_T V}{\partial r} \right|_{r=R_2} = \Delta q(\varphi, t), \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0,
\]

(27)

\[
W(R_2, \varphi, t) = V(R_2, \varphi, t), \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0.
\]

(28)

\[
\left. \frac{\partial \lambda_U W}{\partial r} \right|_{r=R_2} = \alpha_t W \bigg|_{r=R_2}, \quad 0 \leq \varphi \leq \varphi_0, \quad \pi - \varphi_0 \leq \varphi \leq \pi,
\]

(29)

\[
\left. \frac{\partial \lambda_U W}{\partial r} \right|_{r=R_3} = -\alpha_t W \bigg|_{r=R_3}, \quad 0 \leq \varphi \leq \pi,
\]

(30)

\[
\left. \frac{\partial W}{\partial \varphi} \right|_{\varphi=\pi} = \left. \frac{\partial W}{\partial \varphi} \right|_{\varphi=0}, \quad R_2 \leq r \leq R_1,
\]

(31)

\[
\left. \frac{\partial \lambda_T V}{\partial r} \right|_{r=R_1} = \alpha_t V \bigg|_{r=R_1}, \quad \varphi_0 \leq \varphi \leq \pi - \varphi_0,
\]

(32)

\[
\left. \frac{\partial \lambda_T V}{r \partial \varphi} \right|_{\varphi=\varphi_0} = \alpha_t V \bigg|_{\varphi=\varphi_0}, \quad R_1 \leq r \leq R_2,
\]

(33)

\[
\left. \frac{\partial \lambda_T V}{r \partial \varphi} \right|_{\varphi=\pi-\varphi_0} = -\alpha_t V \bigg|_{\varphi=\pi-\varphi_0}, \quad R_1 \leq r \leq R_2,
\]

(34)

\[
W(r, \varphi, 0) = V(r, \varphi, 0) = 0.
\]

(35)

The problems (13) - (23) and (25) - (35) are solved numerically by the finite difference method in the same way as the direct problem (1)-(11).

In addition to the geometric dimensions and thermophysical properties of the materials from which the cylinder and brake pad are made, it is necessary to know the function of the angular velocity of rotation of the cylinder for the numerical realization of the problem. This angular velocity can be calculated by the method given in [7], based on the knowledge of the vehicle mass, its speed at the initial moment of deceleration, and the time of a complete stop.

4. Conclusion
1. An algorithm has been developed to solve the boundary inverse problem of heat transfer to restore the total heat flux, which characterizes the specific friction power between the brake pads and the cylinder according to temperature data.
2. The corresponding boundary nonstationary problems for conjugate variables and temperature increments are obtained, which are necessary for applying the conjugate gradient method to solve the boundary inverse problem.
5. References

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