JACOB’S LADDERS, CROSSBREEDING IN THE SET OF ζ-FAC TORIZATION FORMULAS AND SELECTION OF FAMILIES OF ζ-KINDRED REAL CONTINUOUS FUNCTIONS

JAN MOSER

ABSTRACT. In this paper we introduce the notion of ζ-crossbreeding in a set of ζ-factorization formulas and also the notion of complete hybrid formula as the final result of that crossbreeding. The last formula is used as a criterion for selection of families of ζ-kindred elements in class of real continuous functions.

Dedicated to recalling of Gregory Mendel’s pea-crossbreeding.

1. Introduction

1.1. Let us remind the following notions we have introduced (see [1] – [7]) within the theory of the Riemann-zeta function:

(A) Jacob’s ladders, [1], comp. [2, 3],

(B) ζ-oscillating system, [4], (1.1),

(C) ζ-factorization formula, [5], (4.3) – (4.18), comp. [7], (2.1) – (2.7),

(D) metamorphoses of the ζ-oscillating systems:

(a) first, the notion of metamorphosis of an ζ-oscillating multiform, [4],

(b) after, the notion of metamorphosis of a quotient of two ζ-oscillating multiforms, [5, 6],

(E) $Z_{ζ, Q^2}$-transformation (or device), [7].

Next, we have introduced (see [8]) the following notions:

(F) functional depending ζ-oscillating systems, [8], beginning of the section 2.4,

(G) interaction between corresponding ζ-oscillating systems, [8], Definition 4.

1.2. In this paper we begin with the following set of functions

$$f_m(t) \in \tilde{C}_0[T, T + U], \quad U = o\left(\frac{T}{\ln T}\right), \quad T \to \infty,$$

$$m = 1, \ldots, M, \quad M \in \mathbb{N},$$

(1.1)

$M$ being arbitrary and then fixed, where

$$f_m \in \tilde{C}_0[T, T + U] \iff f_m \in C[T, T + U] \land f_m \neq 0.$$

Next, by application of the operator $\hat{H}$ (introduced in [8], (3.6)) on elements (1.1) we obtain the following vector-valued functions

$$\hat{H}f_m = (\alpha_0^{m, k_m}, \alpha_1^{m, k_m}, \ldots, \alpha_{k_0}^{m, k_m}, \beta_1^{k_m}, \ldots, \beta_{k_m}^{k_m}),$$

$$m = 1, \ldots, M, \quad 1 \leq k_m \leq k_0, \quad k_0 \in \mathbb{N},$$

(1.2)

Key words and phrases. Riemann zeta-function.
where \(k_0\) is arbitrary and fixed. Simultaneously, we obtain by our algorithm (short survey on this can be found in [8], (3.1) – (3.11)) also the following set of \(\zeta\)-factorization formulas

\[
\prod_{r=1}^{k_m} \left| \frac{\zeta\left( \frac{1}{2} + i\alpha_{m,k}^{r} \right)}{\zeta\left( \frac{1}{2} + i\beta_{m,k}^{r} \right)} \right|^2 \sim E_m(U,T)F_m[\alpha_0^m,k_m],
\]

\(L \to \infty, m = 1, \ldots, M.\)

Now, we will suppose that after the finite number of stages of crossbreeding (every member of (1.3) participates in this) in the set (1.3) - that is: after the finite number of elimination of the external functions

\[
E_m(U,T), m = 1, \ldots, M
\]

from the set (1.3) - we obtain the following complete hybrid formula (i.e. the result of complete elimination of elements of (1.4)):

\[
F \left\{ \prod_{r=1}^{k_1} (\ldots), \prod_{r=1}^{k_2} (\ldots), \ldots, \prod_{r=1}^{k_M} (\ldots), F_1[f_1(\alpha_{1,k_1}^0)], F_2[f_2(\alpha_{2,k_2}^0)], \ldots, F_M[f_M(\alpha_{M,k_M}^0)] \right\}
\]

\[
= 1 + o \left( \frac{\ln \ln T}{\ln T} \right) \sim 1, \ T \to \infty.
\]

Remark 1. Here, we may put, of course,

\[
[T, T + U] \to [L, L + U], [\pi L, \pi L + U], \ldots, L \in \mathbb{N}.
\]

Remark 2. Let

\[
F_1 \sim 1, \ T \to \infty
\]

be the complete hybrid formula for the second set (1.1) on the segment \([T, T + U]\), where

\[
(1.1) \neq (1.1).
\]

Then (see (1.3))

\[
FF_1 \sim 1, \ T \to \infty.
\]

But, of course, (1.7) is not the complete hybrid formula for the set

\[
(1.1) \cup (1.1)
\]

since lack of non-empty set of crossbreeding between sets (1.3) and (1.3).

Now, we see that complete hybrid formula (1.5) (it is simultaneously the interaction formula, comp. (G)) expresses the functional dependence (comp. (F)) of the set of vector-valued functions (1.2). Otherwise, the formula (1.5) is a constraint on the set (1.2), (comp. [10], Remark 9).

Consequently, the back-projection of this functional dependence of the set (1.2) into the generating set (1.1) leaves us to the following

Definition. We will call the subset

\[
\{ f_1(t), f_2(t), \ldots, f_M(t) \}, \ t \in [T, T + U]
\]

of the real continuous functions (comp. 14) for which there is the complete hybrid formula (1.5) as the family of \(\zeta\)-kindred functions.
1.3. In this paper, we prove that the following subset of the set of real continuous functions:

\[ \{ \cos^2 t, \sin^2 t \}, \ t \in [\pi L, \pi L + U], \ U \in (0, \pi/2), \ L \to \infty, \]

next,

\[ \{(t - L)^{\Delta}, (t - L)^{\Delta_1}, \ldots, (t - L)^{\Delta_n} \}, \ t \in [L, L + U], \ U \in (0, a), \]

where

\[ \Delta = \Delta_1 + \cdots + \Delta_n, \ \Delta > \Delta_1 > \cdots > \Delta_n > 0, \]

and more

\[ \sum_{l=1}^{n} (t - L)^{\Delta_l}, (t - L)^{\Delta_1}, \ldots, (t - L)^{\Delta_n}, \]

\[ t \in [L, L + U], \ U \in (0, a), \ a < 1, \]

\[ \Delta_l > 0, \ \Delta_l \neq \Delta_k, \ l \neq k, \ 1 \leq k, l \leq n \]

are the first families of \( \zeta \)-kindred functions.

Remark 3. The selection of families (1.8) – (1.10) represents the completely new type of results in the theory of Riemann’s zeta-function and, simultaneously, also in the theory of real continuous functions.

Finally, we notice explicitly, that also the results of this paper constitute the generic complement to the Riemann’s functional equation on the critical line (comp. [6]).

2. The first family of \( \zeta \)-kindred trigonometric functions

2.1. If we put in [8], (4.1) – (4.10)

\[ \mu = 0, \ \tilde{C} \to \tilde{C}_0 \]

(comp. [9], Definition) then we obtain the following lemmas.

Theorem. For the function

\[ f_1(t) = \sin^2 t \in \tilde{C}_0[\pi L, \pi L + U], \ U \in (0, \pi/2) \]

there are vector-valued functions

\[ (\alpha_0^{1,k_1}, \alpha_1^{1,k_1}, \ldots, \alpha_{k_1}^{1,k_1}, \beta_1^{k_1}, \ldots, \beta_{k_1}^{k_1}) \]

such that the following factorization formula

\[ \prod_{r=1}^{k_1} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{1,k_1} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{k_1} \right)} \right|^2 \sim \left( \frac{1}{2} - \frac{\sin U}{2} \frac{\cos U}{\sin^2(\alpha_0^{1,k_1})} \right) \frac{1}{U}, \ L \to \infty \]

holds true, where

\[ \alpha_r^{1,k_1} = \alpha_r(U, L, k_1; f_1), \ r = 0, 1, \ldots, k_1, \]

\[ \beta_r^{k_1} = \beta_r(U, L, k_1), \ r = 1, \ldots, k_1, \]

\[ 0 < \alpha_0^{1,k_1} - \pi L < U. \]
Lemma 2. For the function

\[(2.5) \quad f_2(t) = \cos^2 t \in \tilde{C}_0[\pi L, \pi L + U], \ U \in (0, \pi/2)\]

there are vector-valued functions

\[(2.6) \quad (\alpha_0^{2,k_2}, \alpha_1^{2,k_2}, \ldots, \alpha_k^{2,k_2}, \beta_1^{k_2}, \ldots, \beta_{k_2}^{k_2})\]

such that the following \(\zeta\)-factorization formula

\[(2.7) \quad \prod_{r=1}^{k_2} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{2,k_2} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{k_2} \right)} \right|^2 \sim \left( \frac{1}{2} - \frac{1}{2} \sin U \cos U \right) \frac{1}{\cos^2(\alpha_0^{2,k_2})}, \ L \to \infty \]

holds true, where

\[(2.8) \quad \alpha_r^{2,k_2} = \alpha_r(U, L, k_2; f_2), \ r = 0, 1, \ldots, k_2, \]

\[\beta_r^{k_2} = \beta_r(U, L, k_2), \ r = 1, \ldots, k_2, \]

\[0 < \alpha_0^{2,k_2} - \pi L < U.\]

2.2. Crossbreeding between the \(\zeta\)-factorization formulae (2.3) and (2.7) – one stage is sufficient in this case – gives the following

Complete Hybrid Formula 1.

\[(2.9) \quad \cos^2(\alpha_0^{2,k_2}) \prod_{r=1}^{k_2} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{2,k_2} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{k_2} \right)} \right|^2 \]

\[+ \sin^2(\alpha_0^{1,k_1}) \prod_{r=1}^{k_1} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{1,k_1} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{k_1} \right)} \right|^2 \sim 1, \ L \to \infty.\]

Now, we obtain from (2.9) by Definition the following

**Theorem 1.** The subset

\[(2.10) \quad \{\cos^2 t, \sin^2 t\}, \ t \in [\pi L, \pi L + U], \ U \in (0, \pi/2), \ L \to \infty\]

is the family of \(\zeta\)-kindred elements in the class of trigonometric functions.

**Remark 4.** The formula (2.9) has already been obtained in our paper [8], (2.6). However, in the present paper the same one is playing the role of complete hybrid formula. Of course, the set

\[\{\cos^2 t, \sin^2 t\}; \ \cos^2 t + \sin^2 t = 1\]

is the known family of *school-kindred* functions.

3. THE FIRST FAMILY OF \(\zeta\)-KINDRED REAL POWER FUNCTIONS

3.1. The following lemma holds true (see [3], (2.4), (2.5)).
Lemma 3. For the function
\[ f_\Delta(t, L) = f_\Delta(t) = (t - L)\Delta \in \mathcal{C}_0[L, L + U], \quad U \in (0, a], \quad a < 1, \quad \Delta > 0 \]
there are vector-valued functions
\[ (\alpha_0^{\Delta, k_\Delta}, \alpha_1^{\Delta, k_\Delta}, \ldots, \alpha_k^{\Delta, k_\Delta}, \beta_1^{\Delta, k_\Delta}, \ldots, \beta_k^{\Delta, k_\Delta}) \]
such that the following \( \zeta \)-factorization formula
\[ \prod_{r=1}^{k_\Delta} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{\Delta, k_\Delta} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{\Delta, k_\Delta} \right)} \right|^2 \sim \frac{1}{1 + \Delta \left( \frac{1}{\alpha_0^{\Delta, k_\Delta} - L} \right)\Delta}, \quad L \to \infty \]
holds true, where
\[ \alpha_r^{\Delta, k_\Delta} = \alpha_r(U, L, k_\Delta; \bar{f}_\Delta), \quad r = 0, 1, \ldots, k_\Delta, \]
\[ \beta_r^{\Delta, k_\Delta} = \beta_r(U, L, k_\Delta), \quad r = 1, \ldots, k_\Delta, \]
\[ 0 < \alpha_0^{\Delta, k_\Delta} - \pi L < U. \]

3.2. Let
\[ \Delta = \sum_{l=1}^{n} \Delta_l, \quad \Delta > \Delta_1 > \cdots > \Delta_n > 0, \quad n \in \mathbb{N} \]
(for every fixed \( n \)). Then we have by Lemma 3 that
\[ (1 + \Delta_l)(\alpha_0^{\Delta_l, k_l} - L)^{\Delta_l} \prod_{r=1}^{k_l} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{\Delta_l, k_l} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{\Delta_l, k_l} \right)} \right|^2 \sim U^{\Delta_l}, \]
\[ L \to \infty, \quad \bar{k}(\Delta_l) \to k_l, \quad \alpha_r^{\Delta, k(\Delta_l)} \to \alpha_r^{\Delta_l, k_l}, \ldots \]
i.e. we have the \( n \)-analogues of (3.1).

Now, after the \( n \)-stages of crossbreeding between the \( \zeta \)-factorization formulae (3.3) and (3.4) where, of course,
\[ U^{\Delta_1 + \cdots + \Delta_n} = U^{\Delta_1} \cdots U^{\Delta_n} \]
we obtain the following result.

Complete Hybrid Formula 2.
\[ \prod_{r=1}^{k_\Delta} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{\Delta, k_\Delta} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{\Delta, k_\Delta} \right)} \right|^2 \sim \]
\[ \frac{1}{1 + \Delta} \prod_{l=1}^{n} (1 + \Delta_l) \left( \frac{1}{\alpha_0^{\Delta, k_\Delta} - L} \right)^{\Delta_l} \prod_{l=1}^{n} (\alpha_0^{\Delta_l, k_l} - L)^{\Delta_l} \times \]
\[ \times \prod_{l=1}^{n} \prod_{r=1}^{k_l} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{\Delta_l, k_l} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{\Delta_l, k_l} \right)} \right|^2, \quad L \to \infty, \]
\[ 1 \leq k_\Delta, k_1, \ldots, k_n \leq k_0; \quad k_1 = \bar{k}(\Delta_1), \ldots, \]
here, of course,
\[ (\ldots) \sim \lfloor \ldots \rfloor \iff 1 \sim \left( \begin{array}{c} \ldots \\ \ldots \end{array} \right); \quad (\ldots), \lfloor \ldots \rfloor \neq 0. \]
Remark 5. The symbol (3.8) contains the set of $(k_0)^{n+1}$ formulas. For example, in the case $k_0 = 100$ and $n = 99$ this number is equal to $10^{200}$.

Consequently, from (3.8) we obtain by Definition the following

**Theorem 2.** The subset

$$
\{(t - L)^{\Delta}, (t - L)^{\Delta_1}, \ldots, (t - L)^{\Delta_n}\},
\ n \geq 1, \ \Delta = \sum_{l=1}^{n} \Delta_l, \ \Delta > \Delta_1 > \cdots > \Delta_n > 0 \tag{3.9}
$$

is the family of $\zeta$-kindred elements in the class of real power functions.

Remark 6. The formula (3.8) has already been obtained in our paper [9], (3.3). However, in the present paper this one is playing the role of the complete hybrid formula.

4. The second family of $\zeta$-kindred real power functions

4.1. The following lemma holds true (see [9], (4.2)).

**Lemma 4.** For the function

$$\tilde{f}(t) = \tilde{f}(t; \Delta_1, \ldots, \Delta_n, L) = \sum_{l=1}^{n} (t - L)^{\Delta_l} \in \tilde{C}_0[L, L + U],
\ U \in (0, a], \ a < 1, \ \Delta_l > 0,
\ \Delta_l \neq \Delta_k, \ l \neq k; \ l, k = 1, \ldots, n \tag{4.1}
$$

there are vector-valued functions

$$\tilde{\alpha}_r, \tilde{\beta}_r, 1 \leq k \leq k_0, \ r = 0, 1, \ldots, k_0 \tag{4.2}
$$

such that the following $\zeta$-factorization formula

$$\prod_{r=1}^{k} \left| \frac{\zeta \left( \frac{1}{2} + i\tilde{\alpha}_r \right)}{\zeta \left( \frac{1}{2} + i\tilde{\beta}_r \right)} \right|^2 \sim \frac{\sum_{l=1}^{n} U^{\Delta_l}}{\sum_{l=1}^{n} (\tilde{\alpha}_0 - L)^{\Delta_l}}, \ L \to \infty \tag{4.3}
$$

holds true, where

$$\tilde{\alpha}_r = \alpha_r(U, L, \Delta_1, \ldots, \Delta_n, k), \ r = 0, 1, \ldots, k,
\ \tilde{\beta}_r = \beta_r(U, L, k), \ r = 1, \ldots, k. \tag{4.4}
$$

4.2. Next, we have (comp. [9], (4.6)) the following formulas

$$\left( \alpha_0^{\Delta_l, k_l} - L \right)^{\Delta_l} \prod_{r=1}^{k} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r^{\Delta_l, k_l} \right)}{\zeta \left( \frac{1}{2} + i\beta_r^{k_l} \right)} \right|^2 \sim \frac{1}{1 + \Delta_l} U^{\Delta_l}, \ \Delta_l > 0, \ 1 \leq k_l \leq k_0, \ l = 1, \ldots, n. \tag{4.5}
$$

Now we obtain after $n$-stages of crossbreeding between the $\zeta$-factorization formulas (4.3) and (4.5) the following result (see [9], (4.4)).
Complete Hybrid Formula 3.
\[
\prod_{r=1}^{k} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha_r \right)}{\zeta \left( \frac{1}{2} + i\beta_r \right)} \right|^2 \sim \frac{1}{\sum_{l=1}^{n} (\alpha_0 - L)^{\Delta_l}} \sum_{l=1}^{n} (\alpha_0^{\Delta_l} - L)^{\Delta_l} \prod_{r=1}^{k} \left| \frac{\zeta \left( \frac{1}{2} + i\alpha^{\Delta_l,k_r} \right)}{\zeta \left( \frac{1}{2} + i\beta^{k_r} \right)} \right|^2.
\]

Consequently, we obtain from (4.6) by Definition the following

**Theorem 3.** The subset
\[
\left\{ \sum_{l=1}^{n} (t - L)^{\Delta_l}, (t - L)^{\Delta_1}, \ldots, (t - L)^{\Delta_l} \right\},
\]
\(t \in [L, L + U], U \in (0, a], a < 1, L \to \infty, \Delta_l \geq 0, \Delta_l \neq \Delta_k, l \neq k, 1 \leq k, l \leq n\)

is the family of \(\zeta\)-kindred elements in the class of real power functions.

5. CONCLUDING REMARKS

5.1. The first remarks are connected with the operator \(\hat{H}\) that we have defined in the paper [8], (see Definition 2 and Definition 5; comp. also [9], Definition). In this direction we have used the following notation

\[
(5.1) \quad \forall f(t) \in C_0[T, T + U] \to \hat{H}f(t) = (\alpha_0, \alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_k), \quad k = 1, \ldots, k_0
\]

for every fixed \(k\), (see [10], section 5).

**Remark 7.** Here we have to use more exact notation

\[
(5.2) \quad (\alpha_0, \alpha_1, \ldots, \alpha_k, \beta_1, \ldots, \beta_k) \to (\alpha_0^k, \alpha_1^k, \ldots, \alpha_k^k, \beta_1^k, \ldots, \beta_k^k)
\]

Namely, we have defined in [10], section 5, the \(\hat{H}\) operator as matrix-valued operator with the following inexact notation based on (5.1):

\[
(5.3) \quad f(t) \xrightarrow{\hat{H}} \begin{pmatrix}
\alpha_0 & \alpha_1 & \beta_1 & 0 & \ldots \\
\alpha_0 & \alpha_1 & \alpha_2 & \beta_1 & \beta_2 & 0 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\alpha_0 & \alpha_1 & \ldots & \alpha_{k_0} & \beta_1 & \beta_2 & \ldots & \beta_{k_0}
\end{pmatrix}_{k_0 \times (2k_0 + 1)}
\]

However, the exact notation (instead of (5.3)) is the following one (comp. [10.4])

\[
(5.4) \quad f(t) \xrightarrow{\hat{H}} \begin{pmatrix}
\alpha_0^1 & \alpha_1^1 & \beta_1^1 & 0 & \ldots \\
\alpha_0^2 & \alpha_1^2 & \alpha_2^1 & \beta_1^1 & \beta_2^1 & 0 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\alpha_0^k & \alpha_1^k & \ldots & \alpha_{k_0}^k & \beta_1^k & \beta_2^k & \ldots & \beta_{k_0}^k
\end{pmatrix}_{k_0 \times (2k_0 + 1)}
\]
Remark 8. Now, it is clear that the inexact notation (5.3) may suggest erroneous impression that the elements of the column
\[
\begin{pmatrix}
\alpha_0 \\
\alpha_0 \\
\vdots \\
\alpha_0
\end{pmatrix}
\]
for example, are necessarily mutually equal.

5.2. The second remark is connected with the asymptotic form:
(a) of the \( \zeta \)-factorization formulas (1.3), (2.3), (2.7), (3.3), (4.1),
(b) of the complete hybrid formulas (1.5), (2.9), (3.8), (4.6).

Remark 9. The following is true: if we use from beginning the exact factorization
formula
\[
\prod_{r=1}^{k} \frac{2^2(\alpha_r)}{2^2(\beta_r)} = \frac{H(T, U; f)}{f(\alpha_0)}, \quad T \to \infty
\]
instead of the asymptotic \( \zeta \)-factorization formula
\[
\prod_{r=1}^{k} \left( \frac{\zeta \left( \frac{1}{2} + i\alpha_r \right)}{\zeta \left( \frac{1}{2} + i\beta_r \right)} \right)^2 = \left\{ 1 + O \left( \frac{\ln \ln T}{\ln T} \right) \right\} \frac{H(T, U; f)}{f(\alpha_0)} \sim \frac{H(T, U; f)}{f(\alpha_0)}, \quad T \to \infty
\]
(see short survey of our algorithm for generating \( \zeta \)-factorization formulas in [8],
(3.7), (3.8)), then we obtain the exact \( \zeta \)-factorization formulas and also the exact
complete hybrid formula instead of these in (a) and (b).

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DEPARTMENT OF MATHEMATICAL ANALYSIS AND NUMERICAL MATHEMATICS, COMENIUS UNIVERSITY, MLYNSKA DOLINA M105, 842 48 BRATISLAVA, SLOVAKIA
E-mail address: jan.moser@fmph.uniba.sk

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