Collision-induced amplitude dynamics of pulses in linear waveguides with the generic nonlinear loss

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Abstract

We study the effects of the generic weak nonlinear loss on fast two-pulse collisions in linear waveguides. We derive the analytic expression for the collision-induced amplitude shift in a fast two-pulse collision in linear waveguides in the presence of the weak $(2m+1)$–order of loss, for any $m \geq 1$. The analytic calculations are based on a generalization of the perturbation technique for calculating the effects of weak perturbations on fast collisions between solitons of the nonlinear Schrödinger equation. The theoretical predictions are confirmed by the numerical simulations with the propagation model of coupled partial differential equations.

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I. INTRODUCTION

Linear waves are widely studied and used in a variety of physical applications [1–6]. In linear optical waveguides, the dynamics of pulses can be affected by nonlinear loss [2, 7]. Nonlinear loss arises in optical waveguides due to multiphoton absorption or gain/loss saturation [2, 7]. More specifically, the \((2m + 1)\)-order of loss can be a result of \((m + 1)\)-photon absorption in silicon waveguides [2]. The \(M\)-photon absorption with \(2 \leq M \leq 5\) has been the subject of intensively theoretical and experimental research in recent years due to a wide variety of potential applications, including lasing, material processing, and optical data storage, etc. [2, 7–13]. Therefore, it is important to study the impact of nonlinear loss on the propagation and dynamics of pulses in linear and nonlinear waveguides. Recently, in [13], the authors found an expression for collision-induced amplitude shift in a fast collision between two pulses in linear waveguides in case of \(m = 1\), that is, in the presence of the weak cubic loss. Moreover, in this work, the authors also demonstrated that pulses in linear waveguides with weak cubic loss exhibit soliton-like behavior in fast two-pulse collisions. This made an interesting connection between the collision of two quasi-linear pulses with the one of two solitons of the nonlinear Schrödinger equation. However, so far, a comprehensive theoretical study of the effects of the \((2m + 1)\)-order of loss, for any \(m \geq 1\), on two-pulse collisions in linear waveguides is still lacking.

In the current paper, we address this important and interesting problem. First, we derive the equation for amplitude dynamics of a single pulse in the presence of the generic weak nonlinear loss, i.e., the \((2m + 1)\)-order of loss, for any \(m \geq 1\). This can be calculated in a straightforward manner by employing the standard adiabatic perturbation theory. Second, we derive the collision-induced amplitude shift in a fast collision between two pulses in the presence of the generic weak nonlinear loss. The calculations are based on deriving and integrating the partial differential equation for the collision-induced change in the envelope of pulse at the leading order of the perturbative calculation. We show that the nonlinear loss also strongly affects the collisions of pulses, by causing an additional decrease of pulses amplitudes. Finally, we validate our theoretical calculations by numerical simulations with the propagation model for \(m = 2\) and \(m = 3\). The calculations of the collision-induced amplitude shift in the current paper are based on an extension of the perturbation technique, developed in Refs. [14–17] for calculating the effects of weak perturbations on fast collisions.
between solitons of the nonlinear Schrödinger equations and between pulses of coupled partial differential equations (PDEs) with the weak cubic loss in [13].

The rest of the paper is organized as follows. In section II, we introduce the propagation model and derive the equations for amplitude dynamics of a single pulse and for collision-induced amplitude dynamics. In section III, we validate the theoretical calculations by simulations. Section IV is reserved for conclusions.

II. PULSE INTERACTION IN LINEAR WAVEGUIDES WITH THE GENERIC WEAK NONLINEAR LOSS

We consider fast collisions between two optical pulses in linear waveguides in the presence of the weak \((2m + 1)\)-order of the nonlinear loss for \(m \geq 1\). The propagation equations can be given by the following coupled PDEs [7, 13, 16, 17]:

\[
\begin{align*}
    i\partial_z \psi_1 - \text{sgn}(\tilde{\beta}_2)\partial_t^2 \psi_1 &= -i\epsilon_{2m+1}|\psi_1|^{2m}\psi_1 - i\epsilon_{2m+1}\sum_{k=1}^{m} b_k|\psi_2|^{2k}|\psi_1|^{2(m-k)}\psi_1, \\
    i\partial_z \psi_2 + id_1\partial_t \psi_2 - \text{sgn}(\tilde{\beta}_2)\partial_t^2 \psi_2 &= -i\epsilon_{2m+1}|\psi_2|^{2m}\psi_2 - i\epsilon_{2m+1}\sum_{k=1}^{m} b_k|\psi_1|^{2k}|\psi_2|^{2(m-k)}\psi_2,
\end{align*}
\]

where \(b_k = \frac{m!(m+1)!}{(k!)^2(m+1-k)!(m-k)!}\), \(\psi_1\) and \(\psi_2\) are proportional to the envelopes of the electric fields of the pulses, \(z\) is the (normalized) propagation distance, and \(t\) is the time [18]. In Eq. (1), \(\tilde{\beta}_2\) is the second-order dispersion coefficient, \(d_1\) is the group velocity coefficient and \(\epsilon_{2m+1}\) is the generic nonlinear loss coefficient in the weak \((2m + 1)\)-order of loss, \(0 < \epsilon_{2m+1} \ll 1\), for \(m \geq 1\). The first and second terms on the right hand side of Eq. (1) describe intra-pulse and inter-pulse effects due to the \((2m + 1)\)-order of loss.

First, we study the amplitude dynamics of a single pulse in the presence of the generic \((2m + 1)\)-order of nonlinear loss described by the following equation:

\[
i\partial_z \psi_j + id_1\partial_t \psi_j - \text{sgn}(\tilde{\beta}_2)\partial_t^2 \psi_j = -i\epsilon_{2m+1}|\psi_j|^{2m}\psi_j. \tag{2}
\]

By deriving the energy balance of Eq. (2), we arrive at

\[
\partial_z \int_{-\infty}^{\infty} |\psi_j(t, z)|^2 dt = -2\epsilon_{2m+1} \int_{-\infty}^{\infty} |\psi_j(t, z)|^{2m+2} dt.
\]

We express the approximate solution of the propagation equation (2) as \(\psi_j(t, z) = A_j(z)\tilde{\psi}_j(t, z)\), where \(A_j(z)\) is the amplitude and \(\tilde{\psi}_j(t, z) = \hat{\Psi}_j(t, z)\exp[i\chi_j(t, z)]\) is the
solution of the propagation equation in the absence of \((2m + 1)\)-order of loss with initial amplitude \(A_j(0) = 1\). Substituting the relation for \(\psi_j(t, z)\) into Eq. (3), we obtain

\[
\frac{d}{dz} [I_{2j}(z)A_j^2(z)] = -2\epsilon_{2m+1}I_{2m+2,j}(z)A_j^{2m+2}(z),
\]

where \(I_{2j}(z) = \int_{-\infty}^{\infty} |\tilde{\psi}_j(t, z)|^2 dt = I_{2j}(0)\) by the conservation of energy for the unperturbed solution \(\tilde{\psi}_j(t, z)\), and \(I_{2m+2,j}(z) = \int_{-\infty}^{\infty} |\tilde{\psi}_j(t, z)|^{2m+2} dt\). Integrating the differential equation (4) by a change of variable of \(S_j(z) = A_j^2(z)\), one can arrive at the equation for amplitude dynamics of a single pulse:

\[
A_j(z) = \frac{A_j(0)}{1 + 2\epsilon_{2m+1}\int_{-\infty}^{\infty} |\tilde{\psi}_j(t, z)|^2 dt A_j^{2m}(0)} \left[ 1 + 2\epsilon_{2m+1}\int_{-\infty}^{\infty} |\tilde{\psi}_j(t, z)|^{2m+2} dt A_j^{2m}(0) \right]^{-1/(2m)},
\]

where \(\tilde{I}_{2m+2,j}(0, z) = \int_0^z I_{2m+2,j}(z') dz'\).

Second, we calculate the collision-induced amplitude dynamics in a fast collision between two pulses with generic shapes and with tails that exhibit exponential decay. We consider a complete collision, i.e., the two pulses are well separated at the initial distance \(z = 0\) and at the final distance \(z = z_f\). We assume that the pulses can be characterized by initial amplitudes \(A_j(0)\), initial widths \(W_{j0}\), initial positions \(y_{j0}\), and initial phases \(\alpha_{j0}\). We define the collision length \(\Delta z_c\), which is the distance along which the envelopes of the colliding pulses overlap, by \(\Delta z_c = W_0/|d_1|\), where for simplicity we assume \(W_{10} = W_{20} = W_0\). The condition for a fast collision is \(W_0|d_1|/2 \gg 1\). In an analogy with the perturbative calculation approach in [13, 15–17], we look for a solution of Eq. (1) in the form

\[
\psi_j(t, z) = \psi_{j0}(t, z) + \phi_j(t, z),
\]

where \(j = 1, 2\), \(\psi_{j0}\) are the solutions of Eq. (1) without the inter-pulse interaction terms, and \(\phi_j\) describe corrections to \(\psi_{j0}\) due to inter-pulse interaction. That is, \(\psi_{10}\) and \(\psi_{20}\) satisfy

\[
i\partial_z \psi_{10} - \text{sgn}(\tilde{\beta}_2)\partial_t^2 \psi_{10} = -i\epsilon_{2m+1}|\psi_{10}|^{2m} \psi_{10},
\]

and

\[
i\partial_z \psi_{20} + id_1 \partial_t \psi_{20} - \text{sgn}(\tilde{\beta}_2)\partial_t^2 \psi_{20} = -i\epsilon_{2m+1}|\psi_{20}|^{2m} \psi_{20}.
\]

We substitute relation (6) into (1) and use Eqs. (7) and (8) to obtain equations for the \(\phi_j\). Taking into account only leading-order effects of the collision, we can neglect terms
containing $\phi_j$ on the right hand side of the resulting equation. We therefore obtain the equation for $\phi_1$:

$$i\partial_z \phi_1 - \text{sgn}(\tilde{\beta}_2) \partial_t^2 \phi_1 = -i\epsilon_{2m+1} \sum_{k=1}^{m} b_k |\psi_{20}|^{2k} |\psi_{10}|^{2(m-k)} \psi_{10}. \tag{9}$$

We substitute $\psi_{j0}(t, z) = \Psi_{j0}(t, z) \exp[i\chi_{j0}(t, z)]$ and $\phi_1(t, z) = \Phi_1(t, z) \exp[i\chi_1(t, z)]$, where $\Psi_{j0}$ and $\chi_{j0}$ are real-valued, into Eq. (9). This substitution yields the following equation for $\Phi_1$:

$$i\partial_z \Phi_1 - (\partial_t \chi_{10}) \Phi_1 - \text{sgn}(\tilde{\beta}_2) \left[ \partial_t^2 \Phi_1 + 2i (\partial_t \chi_{10}) \partial_t \Phi_1 \right] + i \left( \partial_t^2 \chi_{10} \right) \Phi_1 - (\partial_t \chi_{10})^2 \Phi_1 = -i\epsilon_{2m+1} \sum_{k=1}^{m} b_k \Psi_{20}^{2k} \Psi_{10}^{2(m-k)} \Psi_{10}. \tag{10}$$

Since the collision length $\Delta z_c$ is of order $1/|d_1|$, the term $i\partial_z \Phi_1$ in Eq. (10) is of order $|d_1| \times O(\Phi_1)$. Equating the orders of $\partial_z \Phi_1$ and $\epsilon_{2m+1} \sum_{k=1}^{m} b_k \Psi_{20}^{2k} \Psi_{10}^{2(m-k)} \Psi_{10}$, this yields that $\Phi_1$ is of order $\epsilon_{2m+1}/|d_1|$. Therefore, in the leading order of the perturbative calculation, the equation for the collision-induced change in the envelope of pulse 1 is

$$\partial_z \Phi_1(t, z) = -\epsilon_{2m+1} \sum_{k=1}^{m} b_k \Psi_{20}^{2k} \Psi_{10}^{2(m-k)+1}. \tag{11}$$

Let $z_c$ be the collision distance, which is the distance at which the maxima of $|\psi_j(t, z)|$ coincide. Thus, the fast collision takes place in the small interval $[z_c - \Delta z_c, z_c + \Delta z_c]$. Let $\Delta \Phi_1(t, z_c) = \Phi_1(t, z_c + \Delta z_c) - \Phi_1(t, z_c - \Delta z_c)$ be the net collision-induced change in the envelope of pulse 1. We substitute $\Psi_{j0}(t, z) = A_j(z) \tilde{\Psi}_{j0}(t, z)$ into Eq. (11) and integrate with respect to $z$ over the interval $[z_c - \Delta z_c, z_c + \Delta z_c]$, we have

$$\Delta \Phi_1(t, z_c) = -\epsilon_{2m+1} \sum_{k=1}^{m} b_k J_{k,m}, \tag{12}$$

where $J_{k,m} = \int_{z_c-\Delta z_c}^{z_c+\Delta z_c} A_2^{2k}(z') A_1^{2(m-k)+1}(z') \tilde{\Psi}_{20}^{2k}(t, z') \tilde{\Psi}_{10}^{2(m-k)+1}(t, z') dz'$. To calculate $J_{k,m}$, we note that in the integrand of $J_{k,m}$, there is only the term $\tilde{\Psi}_{20}(t, z')$ that contains the fast changing factor of the form $y = t - y_{20} - d_1 z'$. Therefore, from Eq. (12), we obtain the following approximation:

$$\Delta \Phi_1(t, z_c) = -\epsilon_{2m+1} \sum_{k=1}^{m} b_k A_2^{2k}(z_c) A_1^{2(m-k)+1}(z_c) \tilde{\Psi}_{10}^{2(m-k)+1}(t, z_c) L_{k,m}, \tag{13}$$

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where \( L_{k,m} = \int_{z_c-\Delta z_c}^{z_c+\Delta z_c} \tilde{\Psi}_{20}^k(t, z') dz' \) and \( A_j(z_c^-) \) denotes the limit from the left of \( A_j \) at \( z_c \). In calculating the integral \( L_{k,m} \) one can take into account only the fast dependence of \( \tilde{\Psi}_{20} \) on \( z \), i.e., the \( z \) dependence that is contained in factors of the form \( y = t - y_{20} - d_1 z \). Denoting this approximation of \( \tilde{\Psi}_{20}(t, z) \) by \( \bar{\Psi}_{20}(y, z_c) \), we obtain

\[
\Delta \Phi_1(t, z_c) = -\epsilon_{2m+1} \sum_{k=1}^{m} b_k A_2^{2k}(z_c^-) A_1^{2(m-k)+1}(z_c^-) \tilde{\Psi}_{10}^{2(m-k)+1}(t, z_c) M_{k,m},
\]

where \( M_{k,m} = \int_{z_c-\Delta z_c}^{z_c+\Delta z_c} \tilde{\Psi}_{20}^k(t - y_{20} - d_1 z', z_c) dz' \). Since the integrand of \( M_{k,m} \) is sharply peaked at a small interval about \( z_c \), one can extend the limits of the integral to \(-\infty\) and \( \infty \) and change the integration variable from \( y' \) to \( y = t - y_{20} - d_1 z' \) then obtain \( M_{k,m} = \frac{1}{|d_1|} M'_{k,m} \), where \( M'_{k,m} = \int_{-\infty}^{\infty} \tilde{\Psi}_{20}^k(y, z_c) dy \). Based on Eq. (14) and on the following relation between the net collision-induced change in the envelope and the collision-induced amplitude:

\[
\Delta A_1^{(c)} \int_{-\infty}^{\infty} \tilde{\Psi}_{10}^2(t, z_c) dt = \int_{-\infty}^{\infty} \Delta \Phi_1(t, z_c) \tilde{\Psi}_{10}(t, z_c) dt \quad \text{(see Eq. (12) in [13])},
\]

one can derive the expression for the collision-induced amplitude shift of pulse 1:

\[
\Delta A_1^{(c)} = -\frac{\epsilon_{2m+1}}{|d_1|} \sum_{k=1}^{m} b_k A_2^{2k}(z_c^-) A_1^{2(m-k)+1}(z_c^-) N_{k,m} M'_{k,m},
\]

where \( N_{k,m} = \frac{\int_{-\infty}^{\infty} \tilde{\Psi}_{10}^{2(m-k)+2}(t, z_c) dt}{\int_{-\infty}^{\infty} \tilde{\Psi}_{10}^2(t, z_c) dt} \), for \( 1 \leq k \leq m \). We emphasize that in the specific case of \( m = 1 \), Eq. (15) becomes Eq. (13) in [13]. Also, it is worthy to remark that the analytic expression for \( \Delta A_1^{(c)} \) in Eq. (15) is independent on the pulse shapes of the colliding pulses.

### III. NUMERICAL SIMULATIONS

In this section, we shall validate Eq. (15) by numerical simulations with Eq. (1). As a concrete example, we demonstrate the numerical simulations for a collision of two Gaussian pulses in the presence of quintic loss \((m = 2)\) and septic loss \((m = 3)\).

The initial envelopes of the Gaussian pulses are \( \psi_j(t, 0) = A_j(0) \exp[-(t - y_{j0})^2/(2W_{j0}^2) + i\alpha_{j0}] \), for \( j = 1, 2 \). Thus, we have

\[
\tilde{\Psi}_{10}(t, z) = \frac{W_{10}}{(W_{10}^4 + 4z^2)^{1/4}} \exp \left[ -\frac{W_{10}^2(t - y_{10})^2}{2(W_{10}^4 + 4z^2)} \right],
\]

(16)
and

\[ \tilde{\Psi}_{20}(t, z) = \frac{W_{20}}{(W_{20}^4 + 4z^2)^{1/4}} \exp \left[ -\frac{W_{20}^2(t - d_1 z - y_{20})^2}{2(W_{20}^4 + 4z^2)} \right]. \]  

(17)

Equations (16) and (17) completely determine the theoretical prediction for \( \Delta A_1^{(c)} \) in Eq. (15). On the other hand, we calculate the numerical value of \( \Delta A_1^{(c)} \) from the simulations by using

\[ \Delta A_1^{(c)(num)} = A_1(z_c^+) - A_1(z_c^-). \]  

(18)

where \( A_1(z_c^-) \) is measured from \( A_1(0) \) by Eq. (5) and \( A_1(z_c^+) \) is measured from \( A_1(z_f) \) by integrating Eq. (4), and \( A_1(z_f) \) is calculated by numerical simulations with Eq. (1).

We demonstrate the simulations with the following parameters for \( m = 2 \) and \( m = 3 \):
\( \epsilon_{2m+1} = 0.01, W_{j0} = 2, \alpha_{j0} = 0, y_{10} = 0, y_{20} = \pm 15, \text{sgn}(\tilde{\beta}_2) = 1, \) and \( 8 \leq |d_1| \leq 80 \). The value of \( z_c \) is calculated by \( z_c = |(y_{20} - y_{10})/d_1| \). Equation (1) is numerically integrated by implementing the split-step Fourier method with periodic boundary conditions [19–21].

Figure 1 depicts a particular simulation with Eq. (1) for a fast collision between two Gaussian pulses in the presence of quintic loss \( (m = 2) \) with a specific value of \( d_1 = 30 \) and other parameters described as above. The two pulses collide at the collision distance of \( z_c = 0.5 \). The numerical value for \( \Delta A_1^{(e)} \) measured from Eq. (18) is \( \Delta A_1^{(c)(num)} = -0.007 \) while its theoretical prediction calculated from Eq. (15) is \( \Delta A_1^{(c)} = 0.0072 \).
FIG. 2: (Color online) Collision-induced amplitude shift in fast collisions of two Gaussian pulses. The blue solid curve and red circles correspond to $\Delta A_1^{(c)}$ measured from the theoretical predictions of Eq. (15) and from numerical simulations of Eq. (1) with $m=2$ (a) and with $m=3$ (b), respectively.

Figures 2(a) and 2(b) represent the dependence of the collision-induced amplitude shift $\Delta A_1^{(c)}$ on $d_1$, for $8 \leq |d_1| \leq 80$, obtained by simulations with Eq. (1) with $m=2$ and $m=3$, respectively, along with their analytic predictions of Eq. (15). As can be seen in Fig. 2, the agreement between the simulations and the analytic predictions are very good. Indeed, for $m=2$, the relative error in the approximation, which is defined by $|\Delta A_1^{(c)(num)} - \Delta A_1^{(c)}| \times 100/|\Delta A_1^{(c)}|$, is less than 4% for $|d_1| \geq 18$ and less than 2% for $|d_1| \geq 44$. Even at $|d_1| \simeq 8$, the relative error is only 5.7%. For $m=3$, the relative error is less than 5% for $|d_1| \geq 56$ and less than 9.8% for $|d_1| \geq 24$. Even at $|d_1| \simeq 8$, the relative error is only 15.05%. We note that similar results are also obtained for other values of the physical parameters.

IV. CONCLUSIONS

We derived the analytic expression for the collision-induced amplitude shift in a fast collision between two pulses in linear waveguides in the presence of the generic weak nonlinear loss. The results revealed that the weak nonlinear loss strongly affects the collisions of pulses, by causing an additional downshift of pulse amplitudes. More specifically, the collision-induced amplitude shift is of the order of $\epsilon_{2m+1}/|d_1|$ in the perturbative calculations, where $0 < \epsilon_{2m+1} \ll 1$ and $|d_1| \gg 1$. Moreover, we showed that the analytic expression for the
collision-induced amplitude shift is independent on the pulse shapes of the colliding pulses. The theoretical calculations were confirmed by numerical simulations of the propagation equations in terms of coupled PDEs with $m = 2$ and $m = 3$ for fast collisions of two Gaussian pulses. Our results, which generalized the results in [13], provided the insightful understanding of the effects of high order of nonlinear loss on dynamics of pulses in linear waveguides.

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[18] The dimensionless distance $z$ in Eq. (1) is $z = Z/(2L_D)$, where $Z$ is the dimensional distance, $L_D = \tau_0^2/|\tilde{\beta}_2|$ is the dispersion length, and $\tau_0$ is a reference pulse width. The dimensionless time is $t = \tau/\tau_0$, where $\tau$ is time. $\psi_j = E_j/\sqrt{P_0}$, where $E_j$ is the electric field of the $j$th pulse and $P_0$ is peak power. $d_1 = 2(\tilde{\beta}_{12} - \tilde{\beta}_{11})\tau_0/|\tilde{\beta}_2|$, where $\tilde{\beta}_{1j} = n_{gj}/c = 1/v_{gj}$, $c$ is the speed of light, and $n_{gj}$ and $v_{gj}$ are the group refractive index and the group velocity for the $j$th pulse. $\epsilon_1 = 2\tau_0^2\tilde{\rho}_1/|\tilde{\beta}_2|$ and $\epsilon_{2m+1} = 2P_0^m\tau_0^2\tilde{\rho}_{2m+1}/|\tilde{\beta}_2|$, where $\tilde{\rho}_1$ and $\tilde{\rho}_{2m+1}$ are the dimensional linear and $(2m+1)$-order of loss coefficients.

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