Hadronization in Nuclear Environment and
Electroproduction of Leading Hadrons

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ABSTRACT

Radiative energy loss of a highly virtual quark originating from a deep-inelastic electron scattering plays a crucial role in production of leading hadrons off nuclei. The density of energy loss for gluon radiation turns out to be time- and energy–dependent in inclusive hadron production. Important phenomena involved are Sudakov’s suppression of no radiation of that part of gluon spectrum, which is forbidden by energy conservation, and color transparency, which suppresses the final state interaction of the produced colorless wave packet.

We model the soft part of hadronization, which usually is supposed to be due to the color strings, using also gluon radiation and nicely reproduce the string parameters.

Our parameter-free calculations provide a good agreement with available data on $z_h$-dependence of the quark fragmentation function in vacuum, as well as $\nu$, $z_h$, and $Q^2$-dependence of nuclear effects. We come to the conclusion that the energy range of ELFE - HERMES is most sensitive to the underlying dynamics of hadronization provided that nuclear targets are used.
1 Why nuclear target?

A quark originated from a hard reaction, eventually converts into colorless hadrons due to confinement. The Lorenz time dilation stretches considerably the duration of this process, and the hadrons carry poor information about the early stage of hadronization. A nuclear target provides a unique opportunity to look inside the process at very short times after it starts. The quark-gluon system produced in a hard collision, interacts while passing through the nucleus. Observation of manifestations of that can bring forth precious information about the structure and the space-time pattern of hadronization.

The modification of the quark fragmentation function by nuclear matter was considered for high-$p_T$ hadron production in \cite{1,2}, for deep-inelastic lepton scattering in \cite{3} - \cite{7}, and for hadroproduction of leading particles on nuclei in \cite{9,8}.

2 Radiative energy loss in vacuum

A highly virtual quark gradually loses energy for radiation of gluons until final hadrons are produced. It was a prominent result of ref. \cite{10} that the density of radiative energy loss $dE/dt$ is energy and time independent, in analogy to the string model. This is a consequence of time ordering of radiation. The emission of a gluon which carries a portion $\alpha$ of the light-cone momentum of the quark and transverse momentum $k_T$ takes a time (called radiation time)

$$t_r \approx \frac{2\nu}{k_T^2} \alpha (1 - \alpha),$$

where $\nu$ is the energy of the quark. Eq. (1) follows from the form of the energy denominator corresponding to such a fluctuation in the infinite momentum frame.

In the case of hadron production with energy $z_h\nu$, energy conservation forbids radiation of gluons with energy higher than $(1 - z_h)\nu$. With this restriction the energy loss during a time interval $t$ reads,

$$\Delta E_{rad}(t) = \int_{\lambda^2}^{Q^2} dk_T^2 \int_0^1 d\alpha \frac{d\nu}{d\alpha dk_T^2} \Theta(1 - z_h - \alpha) \Theta(t - t_r)$$

(2)

Here the $k_T$ and $\alpha$ distribution of the number of emitted gluons is, $dn_g/d\alpha dk_T^2 = \epsilon/\alpha k_T^2$, where $\epsilon = 4\alpha_s(k_T^2)/3\pi$ is calculated perturbatively.

The integration in eq. (2) covers both soft and hard radiation. Usually soft hadronization is described in terms of string model \cite{11}. We, however, model it by radiation as well. To get...
sensible results, we fix the running QCD coupling \( \alpha_s(k_T^2) = \alpha_s(k_0^2) \) at \( k_T^2 \leq k_0^2 \). The parameter \( k_0 \approx 0.7 \, \text{GeV} \) is chosen to reproduce the energy loss density corresponding to the string tension, \( dE/dt \approx 1 \, \text{GeV/fm} \). This value of \( k_0 \) corresponds quite well to the shortness of the gluon-gluon correlation radius suggested by lattice calculation.

Performing the integrations in eq. (2) in the small-\( \alpha \) approximation we find

\[
\Delta E_{\text{rad}}(t) = \frac{\epsilon t}{2}(Q^2 - \lambda^2) \Theta(t_1 - t) + \epsilon \nu(1 - z_h) \Theta(t - t_1) + \\
\epsilon \nu(1 - z_h) \ln \left( \frac{t}{t_1} \right) \Theta(t - t_1) \Theta(t_2 - t) + \epsilon \nu(1 - z_h) \ln \left( \frac{Q^2}{\lambda^2} \right) \Theta(t - t_2),
\]

where \( t_1 = (1 - z_h)/x_{Bj} m_N \), \( t_2 = \frac{Q^2}{\lambda^2} t_1 \), and \( x_{Bj} = Q^2/2m_N \nu \) is the Bjorken variable.

Up to the time \( t = t_1 \), the density of energy loss is constant, \( dE/dt = -\epsilon Q^2/2 \), like when there is no restriction on the radiated energy \[10, 6, 7\]. Then it slows down to \( dE/dt = -\epsilon \nu(1 - z_h)/t \).

Eventually, no radiation is permitted at \( t > t_2 \). However, a color charge cannot propagate long time without radiation. This will be insured by the Sudakov’s type formfactor, \( F(t) = \exp \left[ -\tilde{n}_g(t) \right] \), where \( \tilde{n}_g(t) \) is the number of non-radiated gluons,

\[
\tilde{n}_g(t) = \epsilon \left[ \frac{t}{t_1} - 1 - \ln \left( \frac{t}{t_1} \right) \right] \Theta(t - t_1)
\]

We have arrived at the central question, at what time is the leading hadron (or, better, a colorless ejectile) which does not radiate energy anymore, produced? To answer this question one needs a model for hadronization. In the large \( N_c \) limit, each radiated gluon can be replaced by a \( q\bar{q} \) pair, and the whole system can be seen as made of color dipoles. The fastest \( q\bar{q} \) dipole produced is to be projected over the hadron wave function, \( \Psi_h(\beta, l_T) \), where \( \beta \) and \( 1 - \beta \) are the relative shares of the light-cone momentum carried by the quarks, and \( l_T \) is the relative transverse momentum of the quarks. The corresponding fragmentation function of the quark into the hadron can be represented in the form, \( D(z_h) = \int_0^\infty dt W(t, \nu, z_h) \), where \( W(t, \nu, z_h) \) is a distribution function of the hadron production time intervals, which reads,

\[
W(t, \nu, z_h) \propto \int_0^1 \frac{d\alpha}{\alpha} \delta \left[ \alpha - 2 \left( 1 - \frac{z_h \nu}{E_q(t)} \right) \right] \int \frac{dk_T^2}{k_T^2} \delta \left[ k_T^2 - \frac{2\nu}{t} \alpha(1 - \alpha) \right] \times \\
\int dt_1^2 \delta \left[ l_T^2 - \frac{9}{16} k_T^2 \right] \int d\beta \delta \left[ \beta - \frac{\alpha}{2 - \alpha} \right] |\Psi_h(\beta, l_T)|^2
\]

Here the quark energy \( E_q(t) = \nu - \Delta E_{\text{rad}}(t) \).
The δ-functions in eq. (3) come from the conservation of longitudinal and transverse momenta and from eq. (4).

The hadronic wave function in the light-cone representation, is chosen to satisfy the Regge end-point behaviour $|\Psi_h(l^2, \beta)|^2 \propto \sqrt{\beta}(\sqrt{1-\beta})(1 + l^2 t_h^2/6)^{-1}$, where $r_h$ is the mean square root electromagnetic radius of the hadron.

Some examples of the distribution over time of production, $W(t, \nu, z_h)$ are shown in fig. 1. The mean production time $t_{pr} = \int tW(t, \nu, z_h)dt$ approximately scales in $(1 - z_h)\nu$ and weakly depends on $Q^2$. It vanishes at $z_h \approx 1$.

Once we know $W(t, \nu, z_h)$ we can calculate $D(z_h)$, which nicely agrees with the data from the EMC experiment [13].

3 Nuclear medium

As soon as the colorless wave packet with the desired (detected) momentum is produced inside the nucleus, no subsequent inelastic interaction is permitted, otherwise the leading quark degrades its energy due to further hadronization. Such a restriction means a nuclear suppression of the production rate.

Figure 1: Distribution of the production time at $\nu = 30 \text{ GeV}$ and $Q^2 = 6 \text{ GeV}^2$ vs $z_h = 0.7 - 0.95$

Figure 2: Comparison of our calculations at $Q^2 = 6 \text{ GeV}^2$ with the data [14,15] for the $\nu$-dependence of the ratio of cross sections integrated over $z_h$

On the other hand, soft interactions during hadronization at $t < t_{pr}$ in nuclear matter cannot stop or absorb the leading quark [12]. Since the quark looses much more energy for the
hard gluon radiation following the deep-inelastic scattering than for the induced soft radiation, the latter is a small correction, provided only that $Q^2$ is large.

The produced colorless wave packet may have a small transverse size $\langle \rho^2 \rangle \approx 4/\langle l_T^2 \rangle$. In this case color transparency increases its chances to escape the nucleus without interaction. We also take into account time evolution of the produced wave packet.

The results of our parameter-free calculations are compared in fig. 2 with available data [13, 14] on the energy dependence of nuclear transparency integrated for $z_h > 0.2$.

The $z_h$-dependence of nuclear transparency is supposed to be most sensitive to the underlying dynamics. We compare our predictions with the EMC data in fig. 3.

We predict $Q^2$-dependent effects, which agree with the observed weak $Q^2$-dependence of nuclear transparency observed by the EMC experiment [13]. Much stronger effects are expected at ELFE energies, as shown in fig. 4.

In conclusion, we have developed an approach to electroproduction of leading hadrons on nuclei, based on perturbative QCD. The results of parameter-free calculations agree well with the available data. We expect that nuclear effects are most sensitive to the underlying dynamics of hadronization in the energy range of ELFE or HERMES.

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