QCD CONDENSATES AND HADRON PARAMETERS IN NUCLEAR MATTER: SELF-CONSISTENT TREATMENT, SUM RULES AND ALL THAT

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Abstract

We review various approaches to the calculation of QCD condensates and of the nucleon characteristics in nuclear matter. We show the importance of their self-consistent treatment. The first steps in such treatment appeared to be very instructive. It is shown that the alleged pion condensation anyway can not take place earlier than the restoration of the chiral symmetry. We demonstrate how the finite density QCD sum rules for nucleons work and advocate their possible role in providing an additional bridge between the condensate and hadron physics.

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1 Introduction

The nuclear matter, i.e. the infinite system of interacting nucleons was introduced in order to simplify the problem of investigation of finite nuclei. By introducing the nuclear matter the problems of \( NN \) interaction in medium with non-zero baryon density and those of individual features of specific nuclei were separated. However, the problem of the nuclear matter is far from being solved. As we understand now, it cannot be solved in consistent way, being based on conception of \( NN \) interactions only. This is because the short distances, where we cannot help considering nucleons as composite particles, are very important.

There is limited data on the in-medium values of nucleon parameters. These are the quenching of the nucleon mass \( m \) and of the axial coupling constant \( g_A \) at the saturation value \( \rho_0 \) with respect to their vacuum values. The very fact of existing of the saturation point \( \rho_0 \) is also the "experimental data", which is the characteristics of the matter as a whole. The nowadays models succeeded in reproducing the phenomena although the quantitative results differ very often.

On the other hand, the knowledge about the evolution of hadron parameters is important for understanding the evolution of the medium as a whole while the density \( \rho \) of distribution of the baryon charge number increases. (When \( \rho \) is small enough, it is just the density of the distribution of nucleons). There can be numerous phase transitions. At certain value of density \( \rho = \rho_a \) the Fermi momenta of the nucleons will be so large, that it will be energetically favourable to increase \( \rho \) by adding heavier baryons instead of new nucleons. The nuclear or, more generally, hadronic matter may accumulate excitations with the pion quantum numbers, known as pion (or even kaon) condensations. Also the matter can transform to the mixture of hadrons and quark-gluon phase or totally to the quark-gluon plasma, converting thus to baryon matter. The last but not the least is the chiral phase transition. The chiral invariance is assumed to be one of the fundamental symmetries of the strong interactions.

The chiral invariance means that the Lagrangian as well as the characteristics of the system are not altered by the transformation \( \psi \rightarrow \psi e^{i\alpha\gamma_5} \) of the fermion fields \( \psi \). The model, suggested by Nambu and Jona-Lasinio (NJL) [1] provides a well-known example. The model describes the massless fermions with the four-particle interactions. In the simplest version of NJL model the Lagrangian is

\[
L_{NJL} = \bar{\psi}i\partial_\mu \gamma^\mu \psi + \frac{G}{2} \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \right].
\]

(1)

If the coupling constant \( G \) is large enough, the mathematical (empty) vacuum is not the ground state of the system. Due to the strong four-fermion interaction in the Dirac sea the minimum of the energy of the system is reached at a nonzero value of the fermion density. This is the physical vacuum corresponding to the expectation value \( \langle 0 | \bar{\psi}\psi | 0 \rangle \neq 0 \).

This phenomenon is called "spontaneous chiral symmetry breaking". In the physical vacuum the fermion obtains the mass

\[
m = -2G\langle 0 | \bar{\psi}\psi | 0 \rangle
\]

(2)

cased by the interaction with the condensate. On the other hand, the expectation value \( \langle 0 | \bar{\psi}\psi | 0 \rangle \) is expressed through the integral over the Dirac sea of the fermions. Of course, we have to introduce
a cutoff $\Lambda$ to prevent the ultraviolet divergence caused by the four-fermion interaction

$$
\langle 0 | \bar{\psi} \psi | 0 \rangle = -\frac{m}{\pi^2} \int_0^\Lambda dp \frac{p^2}{(p^2 + m^2)^{1/2}}.
$$

Thus Eqs.(2) and (3) compose self-consistent set of equations which determine the values of the condensate and of fermion mass $m$ in the physical vacuum.

Originally the NJL model was suggested for the description of the nucleons. Nowadays it is used for the quarks. The quark in the mathematical vacuum, having either vanishing or very small mass is called the "current" quark. The quark which obtained the mass, following Eq.(2) is called the "constituent" quark. In the nonrelativistic quark model the nucleon consists of three constituent quarks only.

Return to the nuclear matter. To understand, which of the hadron parameters are important, note that we believe nowadays most of the strong interaction phenomena at low and intermediate energies to be described by using effective low-energy pion-nucleon or pion-constituent quark Lagrangians. The $\pi N$ coupling constant is:

$$
\frac{g}{2m} = \frac{g_A}{2f_\pi}
$$

with $f_\pi$ being the pion decay constant. This is the well-known Goldberger- Traiman (GT) relation \[2\]. It means, that the neutron beta decay can be viewed as successive strong decay of neutron to $\pi^- p$ system and the decay of the pion. Thus, except the nuclear mass $m^*(\rho)$, the most important parameters will be the in-medium values $g^*_A(\rho)$, $f^*_\pi(\rho)$ and $m^*_\pi(\rho)$.

On the other hand, the baryonic matter as a whole is characterised by the values of the condensates, i.e. by the expectation values of quark and gluon operators. Even at $\rho = 0$ some of the condensates do not vanish, due to the complicated structure of QCD vacuum. The nonzero value of the scalar quark condensate $\langle 0 | \bar{\psi} \psi | 0 \rangle$ reflects the violation of the chiral symmetry. In the exact chiral limit, when $\langle 0 | \bar{\psi} \psi | 0 \rangle = 0$ (and the current quark masses vanish also), the nucleon mass vanishes too. Thus, it is reasonable to think about the effective nucleon mass $m^*(\rho)$ and about the other parameters as the functions of the condensates. Of course, the values of the condensates change in medium. Also, some condensates which vanish in vacuum may have the nonzero value at finite density.

At the same time, while calculating the expectation value of the quark operator $\bar{\psi} \psi$ in medium, one finds that the contribution of the pion cloud depends on the in-medium values of hadron parameters. Hence, the parameters depend on condensates and vice versa. Thus we came to the idea of self-consistent calculation of hadron parameters and of the values of condensates in medium. The idea of self-consistency is, of course, not a new one. We have seen just now, how NJL model provides an example. We shall try to apply the self-consistent approach to the analysis of more complicated systems.

The paper is organized as follows. In Sec.2 we review the present knowledge on the in-medium condensates. In Sec.3 we present the ideas and results of various approaches to calculation of the hadron parameters in medium. We review briefly the possible saturation mechanisms provided by these models. In Sec.4 we consider the first steps to self-consistent calculation of scalar condensate and hadron parameters. The experience appeared to be very instructive. Say, the analysis led to the conclusion, that in any case the chiral phase transition takes place at the smaller values of density than the pion condensation. Hence, the Goldstone pions never condense. However, analysis of the
behaviour of the solutions of the corresponding dispersion equation at larger densities appears to be useful.

Suggesting QCD sum rules at finite density as a tool for a future complete self-consistent investigation, we show first how the method works. This is done in Sec.5. In Sec.6 we present more detailed self-consistent scenario.

We present the results for symmetric matter, with equal densities of protons and neutrons.

Everywhere through the paper we denote quark field of the flavour $i$ and colour $a$ as $\psi^a_i$. We shall omit the colour indices in most of the cases, having in mind averaging over the colours for colourless objects. As usually, $\sigma_i, \tau_j$ and $\gamma_\mu$ are spin and isospin Pauli matrices and 4 Dirac matrices correspondingly. For any four-vector $A_\mu$ we denote $A_\mu \gamma^\mu = A^\mu \gamma_\mu = \hat{A}$. The system of units with $\hbar = c = 1$ is used.

## 2 Condensates in nuclear matter

### 2.1 Lowest order condensates in vacuum

The quark scalar operator $\bar{\psi}\psi$ is the only operator, containing minimal number of the field operators $\psi$, for which the expectation value, in vacuum has a nonzero value. One can find in the textbooks a remarkable relation, based on partial conservation of axial current (PCAC) and on the soft-pion theorems

$$m_{\pi b}^2 f_\pi^2 = -\frac{1}{3} \langle 0 | \left[ F_5^b(0)[ F_5^b(0), H(0) ] \right] | 0 \rangle$$

with $m_{\pi b}, f_\pi$ standing for the mass and the decay constant of pion, $H$ being the density of the Hamiltonian of the system, while $F_5^b$ are the charge operators, corresponding to the axial currents, $b$ is the isospin index.

Presenting (effective) Hamiltonian

$$H = H_0 + H_b$$

with $H_0(H_b)$ conserving (explicitly breaking) the chiral symmetry, one finds that only $H_b$ piece contributes to Eq.(4). In pure QCD

$$H_b = H_{bQCD} = m_u \bar{u}u + m_d \bar{d}d$$

with $m_u, d$ standing for the current quark masses. This leads to well known Gell-Mann–Oakes–Renner relation (GMOR) [5]

$$\langle 0 | \bar{u}u + \bar{d}d | 0 \rangle = -\frac{2 f_\pi^2 m_\pi^2}{m_u + m_d} .$$

Of course, assuming SU(2) symmetry, which is true with the high accuracy, one finds $\langle 0 | \bar{u}u | 0 \rangle = \langle 0 | \bar{d}d | 0 \rangle$. Numerical value $\langle 0 | \bar{u}u | 0 \rangle = (-240 \, \text{MeV})^3$ can be obtained from Eq.(8).

The quark masses can be obtained from the hadron spectroscopy relations and from QCD sum rules — see the review of Gasser and Leutwyler [4]. Thus the value of the quark condensate was
calculated by using Eq.(8). The data on the lowest order gluon condensate \( \left< 0 | \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \right> \approx (0.33 \text{ GeV})^4 \) was extracted by Vainshtein et al. \cite{5} from the analysis of leptonic decays of \( \rho \) and \( \varphi \) mesons and from QCD sum-rules analysis of charmonium spectrum \cite{6}.

### 2.2 Gas approximation

In this approximation the nuclear matter is treated as ideal Fermi gas of the nucleons. For the spin-dependent operators \( A_s \) the expectation value in the matter \( \left< M|A_s|M \right> = 0 \), although for the separate polarized nucleons \( \left< N|A_s|N \right> \) may have a nonzero value. For the operators \( A \) which do not depend on spin the deviation of the expectation values \( \left< M|A|M \right> \) from \( \left< 0|A|0 \right> \) is determined by incoherent sum of the contributions of the nucleons. Thus for any SU(2) symmetric spin-independent operator \( A \)

\[
\left< M|A|M \right> = \left< 0|A|0 \right> + \rho \left< N|A|N \right>
\]

with \( \rho \) standing for the density of nuclear matter and

\[
\left< N|A|N \right> = \int d^3x \left( \left< N|A(x)|N \right> - \left< 0|A(x)|0 \right> \right).
\]

Since \( \left< 0|A(x)|0 \right> \) does not depend on \( x \), Eq.(11) can be presented as

\[
\left< N|A|N \right> = \int d^3x \left< N|A(x)|N \right> - \left< 0|A|0 \right> \cdot V_N
\]

with \( V_N \) being the volume of the nucleon.

The quark condensates of the same dimension \( d = 3 \) can be built by averaging of the expression \( \bar{\psi} B \psi \) with \( B \) being an arbitrary \( 4 \times 4 \) matrix over the ground state of the matter. However, any of such matrices can be presented as the linear combination of 5 basic matrices \( \Gamma_A \):

\[
\Gamma_1 = I, \quad \Gamma_2 = \gamma_\mu, \quad \Gamma_3 = \gamma_5, \quad \Gamma_4 = \gamma_\mu \gamma_5, \quad \Gamma_5 = \sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)
\]

with \( I \) being the unit matrix. One can see, that expectation value \( \bar{\psi} \Gamma_5 \psi \) vanishes in any uniform system, while those of \( \bar{\psi} \Gamma_{3,4} \psi \) vanish due to conservation of parity.

The expectation value \( \sum_i \left< M|\bar{\psi}_i \gamma_\mu \psi_i|M \right> = v_\mu(\rho) \) takes the form \( v_\mu(\rho) = v(\rho) \delta_{\mu 0} \) in the rest frame of the matter. It can be presented as

\[
v(\rho) = \sum_i \frac{n_i^p + n_i^n}{2} \cdot \rho = \sum_i v_i \]

6
with \( n_i^{p(n)} \) standing for the number of the valence quarks of the flavour "\( i \)" in the proton (neutron). Due to conservation of the vector current Eq.(14) presents exact dependence of this condensate on \( \rho \). For the same reason the linear dependence on \( \rho \) is true in more general case of the baryon matter

\[
v_i(\rho) = \frac{3}{2} \cdot \rho, \quad v(\rho) = 3 \cdot \rho. \tag{16}
\]

As to the expectation value \( \langle M|\bar{\psi}\psi|M \rangle \), it is quite obvious that Eq.(10) is true for the operator \( A = \bar{\psi}_i \psi_i \) if the nucleon density is small enough. The same refers to the condensates of higher dimension. The question is: when will the terms nonlinear in \( \rho \) become important?

Before discussing the problem we consider the lowest dimension condensates in the gas approximation.

### 2.3 Physical meaning of the scalar condensate in a hadron

It has been suggested by Weinberg [7] that the matrix element of the operator \( \bar{\psi}_i \psi_i \) in a hadron is proportional to the total number of the quarks and antiquarks of flavour "\( i \)" in that hadron. The quantitative interpretation is, however, not straightforward. It was noticed by Donoghue and Nappi [8] that such identification cannot be exact, since the operator \( \bar{\psi}_i \psi_i \) is not diagonal and can add quark–antiquark pair to the hadron. It was shown by Anselmino and Forte [9, 10] that reasonable assumptions on the quark distribution inside the hadron eliminate the non-diagonal matrix elements. However there are still problems of interpretation of the diagonal matrix elements.

Present the quark field of any flavour

\[
\psi(x) = \sum_s \frac{d^3p}{(2\pi)^3(2E)^{1/2}} \left[ b_s(p)u_s(p)e^{-i(px)} + d_s^+(p)v_s(p)e^{+i(px)} \right] \tag{17}
\]

with \( b_s(p) \) and \( d_s^+(p) \) eliminating quarks and creating antiquarks with spin projection \( s \), correspondingly. This leads to

\[
\langle h|\bar{\psi}\psi|h \rangle = \sum_s \int d^3p \left[ \frac{\bar{u}_s(p)u_s(p)}{2E_i(p)} N^+_s(p) + \frac{\bar{v}_s(p)v_s(p)}{2E_i(p)} N^-_s(p) \right]. \tag{18}
\]

Here \( N^+_s \) and \( N^-_s \) stand for the number of quarks and antiquarks. In the works [9, 10] this formula was analysed for the nucleon in framework of quasi-free parton model for the quark dynamics. In this case the normalization conditions are \( \bar{u}_s(p)u_s(p) = \bar{v}_s(p)v_s(p) = 2m_i \) with \( m_i \) standing for the current mass. The further analysis required further assumptions.

In the nowadays picture of the nucleon its mass \( m \) is mostly composed of the masses of three valence quarks which are caused by the interactions inside the nucleon. In the orthodox nonrelativistic quark model, in which possible quark–antiquark pairs are ignored, we put \( E_i = m_i \) and find \( \langle N|\sum_i \bar{\psi}_i \psi_i|N \rangle = 3 \). In more realistic, relativistic models, there is also the contribution of the quark–antiquark pairs. Note also that in some approaches, say, in the bag models [11] or in the soliton model [12] the motion of the valence quarks is relativistic. This reduces their contribution to the expectation value \( \langle N|\sum_i \bar{\psi}_i \psi_i|N \rangle \) by about 30%, since \( m_i/E_i < 1 \).
The conventional nowadays picture of the nucleon is that it is the system of three valence quarks with the constituent masses \( M_i \approx m/3 \) and the number of quark–antiquark pairs

\[
\langle N| \sum_i \bar{\psi}_i \psi_i |N \rangle = 3 + \sum_s \int d^3p \frac{a_s(p)}{2E(p)} N_s(p)
\]  

(19)

with \( a_s(p) = \bar{u}_s(p)u_s(p) = \bar{v}_s(p)v_s(p) \), while \( N_s(p) \) stands for the number of quark–antiquark pairs with momentum \( p \). Thus, the right-hand side (rhs) of Eq.(19) can be treated as the total number of quarks and antiquarks only under certain assumptions about the dynamics of the constituents of \( \bar{q}q \) pairs. They should remain light and their motion should be nonrelativistic, with \( a_s \approx 2m \approx 2E \).

In other models the deviation of the left-hand side (lhs) from the number 3 is a characteristic of the role of quark–antiquark pairs in the nucleon.

The value of \( \langle N| \sum_i \bar{\psi}_i \psi_i |N \rangle \) is related to the observables. The pion–nucleon \( \sigma \)-term, defined by analogy with Eq.(5) \[13\]

\[
\sigma = \frac{1}{3} \sum_b \langle N| \left[ F^5_b(0)[F^5_b(0), H(0)] \right] |N \rangle
\]  

(20)

provides by using Eq.(7)

\[
\langle N| \bar{q}q |N \rangle = \frac{2\sigma}{m_u + m_d}
\]  

(21)

with

\[
\bar{q}q = \bar{u}u + \bar{d}d .
\]  

(22)

On the other hand, \[14, 15\] the \( \sigma \)-term is connected to the pion–nucleon elastic scattering amplitude \( T \). Denote \( p, k(p', k') \) as momenta of the nucleon and pion before (after) scattering. Introducing the Mandelstam variables \( s = (p + k)^2, t = (k' - k)^2 \) we find the amplitude \( T(s, t, k'^2, k^2) \) in the unphysical point to be

\[
T(m^2, 0, 0; 0) = -\frac{\sigma}{f^2_\pi} .
\]  

(23)

The experiments provide the data on the physical amplitude

\[
T \left( (m + m_\pi)^2, 2m_\pi^2, m_\pi^2, m_\pi^2 \right) = -\frac{\Sigma}{f^2_\pi}
\]  

(24)

leading to \[16, 17\]

\[
\Sigma = (60 \pm 7) \text{ MeV} .
\]  

(25)

The method of extrapolation of observable on-mass shell-amplitude to the unphysical point was developed by Gasser et al. \[18, 19\]. They found

\[
\sigma = (45 \pm 7) \text{ MeV} .
\]  

(26)

Note that from the point of chiral expansion, the difference \( \Sigma - \sigma \) is of higher order, i.e. \( (\Sigma - \sigma)/\sigma \sim m_\pi \).

The value \( \sigma = 45 \text{ MeV} \) corresponds to \( \langle N| \bar{q}q |N \rangle \approx 8 \). This is the strong support of the presence of \( \bar{q}q \) pairs inside the nucleon. However direct identification of the value \( \langle N| \bar{q}q |N \rangle \) with the total number of quarks and antiquarks is possible only under the assumptions, described above.
2.4 Quark scalar condensate in gas approximation

The formula for the scalar condensate in the gas approximation

\[ \langle M | \bar{q} q | M \rangle = \langle 0 | \bar{q} q | 0 \rangle + \frac{2\sigma}{m_u + m_d} \rho \]  

(27)

or

\[ \langle M | \bar{q} q | M \rangle = \langle 0 | \bar{q} q | 0 \rangle \left( 1 - \frac{\sigma}{f_\pi^2 m_\pi^2} \rho \right) \]  

(28)

was obtained by Drukarev and Levin [20, 21]. Of course, one can just substitute the semi-experimental value of \( \sigma \), given by Eq.(26). However for the further discussion it is instructive to give a brief review of the calculations of the sigma-term.

Most of the early calculations of \( \sigma \)-term were carried out in the framework of NJL model — see Eq.(1). The results were reviewed by Vogl and Weise [22]. In this approach the quarks with initially very small "current" masses \( m_u \approx 4 \text{ MeV}, m_d \approx 7 \text{ MeV} \) obtain relatively large "constituent" masses \( M_i \sim 300 - 400 \text{ MeV} \) by four-fermion interaction, — Eq.(1). If the nucleon is treated as the weakly bound system of three constituent quarks, the \( \sigma \)-term can be calculated as the sum of those of three constituent quarks. The early calculations provided the value of \( \sigma \approx 34 \text{ MeV} \), being somewhat smaller, than the one, determined by Eq.(26). The latter can be reproduced by assuming rather large content of strange quarks in the nucleon [8] or by inclusion of possible coupling of the quarks to diquarks [22, 23].

In effective Lagrangian approach the Hamiltonian of the system is presented by Eq.(6) with \( H_b \) determined by Eq.(7) while \( H_0 \) is written in terms of nucleon (or constituent quarks) and meson degrees of freedom. It was found by Gasser [24] that

\[ \sigma = \tilde{m} \frac{dm}{\tilde{m}} \]  

(29)

with

\[ \tilde{m} = \frac{m_u + m_d}{2} . \]  

(30)

The derivation of Eq.(29) is based in Feynman–Hellmann theorem [25]. The nontrivial point of Eq.(29) is that the derivatives of the state vectors in the equation

\[ m = \langle N | H | N \rangle \]  

(31)

cancel.

Recently Becher and Leutwyler [26] reviewed investigations, based on pion–nucleon nonlinear Lagrangian. In this approach the contribution of \( \bar{q} q \) pairs is

\[ \sigma_{\bar{q}q} = \tilde{m} \frac{\partial m}{\partial m^2} \frac{\partial m^2}{\partial \tilde{m}} \]  

(32)

with the last factor in rhs

\[ \frac{\partial m^2}{\partial \tilde{m}} = \frac{m^2}{\tilde{m}} \]  

(33)
as follows from Eq.(8). The calculations in this approach reproduce the value \( \sigma \approx 45 \text{ MeV} \).

Similar calculations \[27\] were carried out in framework of perturbative chiral quark model of Gutsche and Robson \[28\] which is based on the effective chiral Lagrangian describing quarks as relativistic fermions moving in effective self-consistent field. The \( \bar{q}q \) pairs are contained in pions. The value of the \( \sigma \)-term obtained in this model is also \( \sigma \approx 45 \text{ MeV} \).

The Skyrme-type models provide somewhat larger values \( \sigma = 50 \text{ MeV} \) \[29\] and \( \sigma = 59.6 \text{ MeV} \) \[30\]. The chiral soliton model calculation gave \( \sigma = 54.3 \text{ MeV} \) \[31\].

The results obtained in other approaches are more controversial. Two latest lattice QCD calculations gave \( \sigma = (18 \pm 5) \text{ MeV} \) \[32\] and \( \sigma = (50 \pm 5) \text{ MeV} \) \[33\]. The attempts to extract the value of \( \sigma \)-term directly from QCD sum rules underestimate it, providing \( \sigma = (25 \pm 15) \text{ MeV} \) \[34\] and \( \sigma = (36 \pm 5) \text{ MeV} \) \[35\].

We shall analyse the scalar condensate beyond the gas approximation expressed by Eq.(27), in Subsection 2.7.

### 2.5 Gluon condensate

Following Subsec.2.2 we write in the gas approximation

\[
\langle M | \frac{\alpha_s}{\pi} G^2 | M \rangle = \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle + \rho \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle
\]

with notation \( G^2 = G^a_{\mu \nu} G^a_{\mu \nu} \). Fortunately, the expectation value \( \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle \) can be calculated. This was done \[36\] by averaging of the trace of QCD energy-momentum tensor, including the anomaly, over the nucleon state. The trace is

\[
\theta^\mu = \sum_i m_i \bar{\psi}_i \psi_i - \frac{b \alpha_s}{8 \pi} G^2
\]

with \( b = 11 - \frac{2}{3} n \), where \( n \) stands for the total number of flavours. However, \( \langle N | \theta^\mu | N \rangle \) does not depend on \( n \) due to remarkable cancellation obtained by Shifman et al. \[36\]

\[
\langle N | \sum_h m_h \bar{\psi}_h \psi_h | N \rangle - \frac{2}{3} n_h \langle N | \frac{\alpha_s}{\pi} G^a_{\mu \nu} G^a_{\mu \nu} | N \rangle = 0 \quad (36)
\]

Here ”\( h \)” denotes ”heavy” quarks, i.e. the quarks, whose masses \( m_h \) are much larger than the inverse confinement radius \( \mu \). The accuracy of Eq.(35) is \( (\mu/m_h)^2 \). Thus we only have to consider the light flavors \( u, d, s \) to give a reasonable approximation since \( m_c \approx 1.5 \text{ GeV} \approx 0.3 \text{ Fm}^{-1} \). This leads to

\[
\langle N | \theta^\mu | N \rangle = -\frac{9}{8} \langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle + \sum_i m_i \langle N | \bar{\psi}_i \psi_i | N \rangle
\]

with \( i \) standing for \( u, d \) and \( s \). Since on the other hand \( \langle N | \theta^\mu | N \rangle = m \) one comes to

\[
\langle N | \frac{\alpha_s}{\pi} G^2 | N \rangle = -\frac{8}{9} \left( m - \sum_i m_i \langle N | \bar{\psi}_i \psi_i | N \rangle \right).
\]
For the condensate
\[ g(\rho) = \langle M| \frac{\alpha_s}{\pi} G^2|M \rangle \] (39)
Drukarev and Levin \[20, 21\] obtained in the gas approximation
\[ g(\rho) = g(0) - \frac{8}{9} \rho \left( m - \sum m_i \langle N|\bar{\psi}_i\psi_i|N \rangle \right). \] (40)
In the chiral limit \( m_u = m_d = 0 \) and
\[ g(\rho) = g(0) - \frac{8}{9} \rho \left( m - m_s \langle N|\bar{s}s|N \rangle \right). \] (41)

The expectation value \( \langle N|\bar{s}s|N \rangle \) is not known definitely. Donoghue and Nappi \[8\] obtained \( \langle N|\bar{s}s|N \rangle \approx 1 \) assuming, that the hyperon mass splitting in SU(3) octet, is described by the lowest order perturbation theory in \( m_s \). Approximately the same result \( \langle N|\bar{s}s|N \rangle \approx 0.8 \) was obtained in various versions of chiral perturbation theory with nonlinear Lagrangians \[19\]. The lattice calculations provide larger values, e.g. \( \langle N|\bar{s}s|N \rangle \approx 1.6 \) \[37\]. On the contrary, the Skyrme model \[12\] and perturbative chiral quark model \[27\] lead to smaller values, \( \langle N|\bar{s}s|N \rangle \approx 0.3 \). Since \( m \gg m_s \) it is reasonable to treat the second term in the brackets of rhs of Eq.(41) as a small correction. Thus we can put
\[ g(\rho) = g(0) - \frac{8}{9} \rho \left( m - m_s \langle N|\bar{s}s|N \rangle \right) \] (42)
which is exact in chiral SU(3) limit in gas approximation.

One can estimate the magnitude of nonlinear contributions to the condensate \( g(\rho) \). Averaging \( \theta_\mu^\mu \) over the ground state of the matter one finds
\[ g(\rho) = g(0) - \frac{8}{9} \rho \left( m - m_s \langle N|\bar{s}s|N \rangle \right) - \frac{8}{9} \varepsilon(\rho) \rho + \frac{8}{9} m_s S_q(\rho) \] (43)
with \( \varepsilon(\rho) \) standing for the binding energy of the nucleon in medium, while \( S_q(\rho) \) denotes nonlinear part of the condensate \( \langle M|\bar{s}s|M \rangle \). One can expect the last factor to be small (otherwise we should accept that strange meson exchange plays large role in \( N - N \) interaction). Hence we can assume
\[ g(\rho) = g(0) - \frac{8}{9} \rho \left( m + \varepsilon(\rho) \right) + \frac{8}{9} m_s \langle N|\bar{s}s|N \rangle \rho \] (44)
with nonlinear terms caused by the binding energy \( \varepsilon(\rho) \).

Thus, at least at the densities close to saturation value, corrections to the gas approximation are small. At \( \rho \approx \rho_0 \) the value of the condensate \( g(\rho) \) differs from the vacuum value by about 6%.

### 2.6 Analysis of more complicated condensates

The condensates of higher dimension come from averaging of the products of larger number of operators of quark and (or) gluon fields. Such condensates appear also from the expansion of bilocal
operators of lower dimension. Say, the simplest bilocal condensate \( C(x) = \langle 0 | \bar{\psi}(0) \psi(x) | 0 \rangle \) is gauge-dependent (recall that the quarks interact with the vacuum gluon fields). To obtain the gauge-invariant expression one can substitute

\[
\psi(x) = \psi(0) + x_\mu D_\mu \psi(0) + \frac{1}{2} x_\mu x_\nu D_\mu D_\nu \psi(0) + \cdots \tag{45}
\]

with \( D_\mu \) being the covariant derivatives, which replaced the usual partial derivatives \( \partial_\mu \). Due to the Lorentz invariance the expectation value \( C(x) \) depends on \( x^2 \) only. Hence, only the terms with even powers of \( x \) survive, providing in the chiral limit \( m_q = 0 \)

\[
C(x) = C(0) + x^2 \cdot \frac{1}{16} \langle 0 | \bar{\psi} \frac{\alpha_s}{\pi} \frac{\lambda^a}{2} G^{\mu\nu} \sigma_{\mu\nu} \psi | 0 \rangle + \cdots , \tag{46}
\]

where \( \lambda^a \) are Gell-Mann SU(3) basic matrices. The second term in right-hand side (rhs) of Eq.(46) can be obtained by noticing that \( \langle 0 | \bar{\psi} D_\mu D_\nu \psi | 0 \rangle = \frac{1}{4} g_{\mu\nu} \langle 0 | \bar{\psi} D^2 \psi | 0 \rangle \) and by applying the QCD equation of motion in the form

\[
(D^2 - \frac{1}{2} \frac{\alpha_s}{\pi} G^{\mu\nu} \sigma_{\mu\nu} \cdot \frac{\lambda^a}{2} - m^2_0) \psi = 0 . \tag{47}
\]

The condensate \( \langle 0 | \bar{\psi} \frac{\alpha_s}{\pi} G^{\mu\nu} \sigma_{\mu\nu} \frac{\lambda^a}{2} \psi | 0 \rangle \) is usually presented "in units" of \( \langle 0 | \bar{\psi} \psi | 0 \rangle \), i.e.

\[
\langle 0 | \bar{\psi} \frac{\alpha_s}{\pi} G^{\mu\nu} \sigma_{\mu\nu} \frac{\lambda^a}{2} \psi | 0 \rangle = m^2_0 \langle 0 | \bar{\psi} \psi | 0 \rangle \tag{48}
\]

with \( m_0 \) having the dimension of the mass. The QCD sum rules analysis of Belyaev and Ioffe [39] gives \( m^2_0 \approx 0.8 \) GeV\(^2 \) for \( u \) and \( d \) quarks. However instanton liquid model estimation made by Shuryak [40] provides about three times larger value.

The situation with expectation values averaged over the nucleon is more complicated. There is infinite number of condensates of each dimension. This happens because the nonlocal condensates depend on two variables \( x^2 \) and \( (P\mathbf{x}) \) with \( P \) being the four-dimensional momentum of the nucleon. Thus, even the lowest order term of expansion in powers of \( x^2 \) \( (x^2 = 0) \) contains infinite number of condensates. Say,

\[
\langle N | \bar{\psi}(0) \gamma_\mu \psi(x) | N \rangle = \frac{P_\mu}{m} \tilde{\varphi}_a((P\mathbf{x}), x^2) + i x_\mu m \tilde{\varphi}_b((P\mathbf{x}), x^2) \tag{49}
\]

with \( \tilde{\varphi}(x) \) defined by expansion, presented by Eq.(45). The function \( \tilde{\varphi}_a(0, 0) \) is the number of the valence quarks of the fixed flavour in the nucleon. Presenting

\[
\tilde{\varphi}_{a,b}((P\mathbf{x}), 0) = \varphi_{a,b}((P\mathbf{x})); \quad \varphi_{a,b}((P\mathbf{x})) = \int_0^1 d\alpha e^{-i\alpha(P\mathbf{x})} \tilde{\varphi}_{a,b}(\alpha) \tag{50}
\]

we find the function \( \phi_a(\alpha) \) to be the asymptotics of the nucleon structure function [41] and the expansion of \( \varphi_a \) in powers of \( (P\mathbf{x}) \) is expressed through expansion in the moments of the structure function. The next to leading order of the expansion of \( \tilde{\varphi}_a \) in powers of \( x^2 \) leads to the condensate

\[
\langle N | \bar{\psi}(0) G_{\mu\nu} \gamma_\mu \gamma_5 \psi(0) | N \rangle = 2P_\mu \cdot \xi_a \tag{51}
\]
with $\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\alpha\beta} G^a_{\alpha\beta} \cdot \frac{1}{2} \lambda^a$ and

$$\xi_{a(b)} = \int_0^1 d\alpha \theta_{a(b)}(\alpha, 0); \quad \theta_{a(b)}(\alpha, x^2) = \frac{\partial \delta_{a(b)}(\alpha, x^2)}{\partial x^2}. \quad (52)$$

The QCD sum rules analysis of Braun and Kolesnichenko [42] gave the value $\xi_a = -0.33 \text{ GeV}^2$.

Using QCD equations of motion we obtain relations between the moments of the functions $\phi_a$ and $\phi_b$. Denoting $\langle F \rangle = \int_0^1 d\alpha F(\alpha)$ for any function $F$ we find, following Drukarev and Ryskin [43]

$$\langle \phi_b \rangle = \frac{1}{4} \langle \phi_a \rangle; \quad \langle \phi_b \alpha \rangle = \frac{1}{5} \left( \langle \phi_a \alpha^2 \rangle - \frac{1}{4} \langle \phi_a \rangle \right); \quad \langle \theta_b \rangle = \frac{1}{6} \langle \theta_a \rangle. \quad (53)$$

Situation with the nonlocal scalar condensate is somewhat simpler, since all the matrix elements of the odd order derivatives are proportional to the current masses of the quarks. This can be shown by presenting $D_{\mu} = \frac{1}{2}(\gamma_{\mu} \hat{D} + \hat{D} \gamma_{\mu})$ followed by using the QCD equations of motion. Hence, in the chiral limit such condensates vanish for $u$ and $d$ quarks. The condensate containing one derivative can be expressed through the vector condensate and thus can be obtained beyond the gas approximation

$$\langle M|\bar{\psi}_1 D_{\mu} \psi_1|M \rangle = m_i v_{\mu}(\rho). \quad (54)$$

In the chiral limit $m_u = m_d = 0$ this condensate vanishes for $u$ and $d$ quarks. The even order derivatives contain the matrix elements corresponding to expansion in powers of $x^2$ which do not contain masses. In the lowest order there is the expectation value $\langle N|\bar{\psi} A_s^a \lambda_a^{ij} G_{\alpha}^a \sigma_{\mu\nu} \psi|N \rangle$ — compare Eq.(13). It was estimated by Jin et al. [44] in framework of the bag model

$$\langle N|\bar{\psi} A_s^a \lambda_a^{ij} G_{\alpha}^a \sigma_{\mu\nu} \psi|N \rangle \approx 0.6 \text{ GeV}^2 \quad (55)$$

together with another condensate of the mass dimension 5

$$\langle N|\bar{\psi} A_s^a \lambda_a^{ij} G_{\alpha}^a \gamma_0 \sigma_{\mu\nu} \psi|N \rangle \approx 0.66 \text{ GeV}^2. \quad (56)$$

Considering the four-quark condensates, we limit ourselves to those with colourless diquarks with fixed flavours. The general formula for such expectation values is

$$Q_{ij}^{AB} = \langle M|\bar{\psi}_i \Gamma_A \psi_i \bar{\psi}_j \Gamma_B \psi_j|M \rangle \quad (57)$$

with $A, B = 1 \ldots 5$, matrices $\Gamma_{A,B}$ are introduced in Eq.(13). For two lightest flavours there are thus $5 \cdot 5 \cdot 4 = 100$ condensates. Due to SU(2) symmetry $Q_{uu}^{AB} = Q_{dd}^{AB} = Q^{AB}$. Due to parity conservation only the diagonal condensates $Q_{ij}^{AA}$ and also $Q_{ij}^{12} = Q_{ij}^{21}$ and $Q_{ij}^{34} = Q_{ij}^{43}$ have nonzero value in uniform matter. Since the matter is the eigenstate of the operator $\bar{\psi}_i \Gamma_2 \psi_i$, we immediately find

$$Q_{ij}^{12} = \rho_i \langle M|\bar{\psi}_i \psi_j|M \rangle \quad (58)$$

with $\rho_i$ standing for the density of the quarks of i-th flavour. In the case, when the matter is composed of nucleons distributed with the density $\rho$, we put $\rho_i = n_i \rho$ with $n_i$ being the number of quarks per nucleon.
For the four-quark scalar condensate $Q^{11}$ we can try the gas approximation as the first step — see Eq.(10). Using Eq.(12) we find for each flavour

$$
\langle N|\bar{\psi}\psi\bar{\psi}\psi|N\rangle = \int d^3x \langle N|\bar{\psi}(x)\psi(x) - \langle 0|\bar{\psi}\psi|0\rangle|^2 |N\rangle + 2\langle 0|\bar{\psi}\psi|0\rangle \langle N|\bar{\psi}\psi|N\rangle + V_N \left(\langle 0|\bar{\psi}\psi|0\rangle \right)^2 - \langle 0|\bar{\psi}\psi\bar{\psi}\psi|0\rangle \right) .
$$

(59)

One can immediately estimate the second term to be about $-0.09$ GeV$^3$. This makes the problem of exact vacuum expectation value to be very important. Indeed, one of the usual assumptions is that

$$
\langle 0|\bar{\psi}\psi\bar{\psi}\psi|0\rangle \approx \left(\langle 0|\bar{\psi}\psi|0\rangle \right)^2 .
$$

(60)

This means that we assume the vacuum state $|0\rangle/\langle 0|$ to give the leading contribution to the sum

$$
\langle 0|\bar{\psi}\psi\bar{\psi}\psi|0\rangle = \sum_n \langle 0|\bar{\psi}\psi|n\rangle \langle n|\bar{\psi}\psi|0\rangle
$$

(61)

over the complete set of the states $|n\rangle$ with the quantum numbers of vacuum. Novikov et al. [45] showed, that Eq.(61) becomes exact in the limit of large number of colours $N_c \rightarrow \infty$. However, the contribution of excited states, e.g. of the $\sigma$-meson $|\sigma\rangle\langle \sigma|$ can increase the rhs of Eq.(59). Assuming the nucleon radius to be of the order of 1 Fm we find the second and the third terms of the rhs of Eq.(59) to be of the comparable magnitude. This becomes increasingly important in view of the only calculation of the 4-quarks condensate in the nucleon, carried out by Celenza et al. [46]. In this paper the calculations in the framework of NJL model show that about 75% of the contribution of the second term of rhs of Eq.(59) is cancelled by the other ones.

### 2.7 Quark scalar condensate beyond the gas approximation

Now we denote

$$
\langle M|\bar{q}q|M \rangle = \kappa(\rho)
$$

(62)

and try to find the last term in the rhs of the equation

$$
\kappa(\rho) = \kappa(0) + \frac{2\sigma}{m_u + m_d} \cdot \rho + S(\rho) .
$$

(63)

The first attempt was made by Drukarev and Levin [20, 21] in the framework of the meson-exchange model of nucleon–nucleon (NN) interactions. In the chiral limit $m_\pi^2 \rightarrow 0$ (neglecting also the finite size of the nucleons) one obtains the function $S(\rho)$ as the power series in Fermi momenta $p_F$. The lowest order term comes from Fock one-pion exchange diagram (Fig.1). The result beyond the chiral limit was presented in [13].

In spite of the fact that the contribution of such mechanism to the interaction energy is a minor one, this contribution to the scalar condensate is quite important, since it is enhanced by the large factor (about 12) in the expectation value

$$
\langle \pi|\bar{q}q|\pi \rangle = \frac{2m_\pi^2}{m_u + m_d} \approx 2m_\pi \cdot 12
$$

(64)
obtained by averaging the QCD Hamiltonian over the pion state. Using the lowest order $\pi N$ coupling
terms of the $\pi N$ Lagrangian, we obtain in the chiral limit

$$S(\rho) = -3.2 \frac{p_F}{p_{F0}} \rho$$

with $p_F$ being Fermi momentum of the nucleons, related to the density as

$$\rho = \frac{2}{3\pi^2} p_F^3;$$

$p_{F0} \approx 268$ MeV is Fermi momentum at saturation point. Of course, the chiral limit makes sense only
for $p_F^2 \gg m_\pi^2$. This puts the lower limit for the densities, when Eq.(65) is true. The value, provided
by one-pion exchange depends on the values of $\pi N$ coupling $g = g_A/2f_\pi$ and of the nucleon mass in
medium. If we assume that these parameters are presented as power series in $\rho$ (but not in $p_F$) at
low densities, the contribution of the order $\rho^{5/3}$ comes from two-pion exchange with two nucleons in
the two-baryon intermediate state — Fig.2.

In our paper [47] we found for $p_F^2 \gg m_\pi^2$

$$S(\rho) = -3.2 \frac{p_F}{p_{F0}} \rho - 3.1 \left(\frac{p_F}{p_{F0}}\right)^2 \rho + O(\rho^2).$$

Although at saturation point $m_\pi^2/p_{F0}^2 \approx 1/4$, the discrepancy between the results of calculation of
one-pion exchange term in the chiral limit and that with account of finite value of $m_\pi^2$ is rather large
[43]. However, working in the chiral limit one should use rather the value of $\Sigma$, defined by Eq.(25)
for the sigma–term, since the difference between $\Sigma$ and $\sigma$ terms contains additional powers of $m_\pi$
[18]. This diminishes the difference of the two results strongly. Additional arguments in support of
the use of the chiral limit at $\rho$ close to $\rho_0$ were given recently by Bulgac et al. [48].

The higher order terms of the expansion, coming from the $NN$, $N\Delta$ and $\Delta\Delta$ intermediate states,
compensate the terms, presented in rhs of Eq.(67) to large extent. However these contributions
are much more model-dependent. The finite size of the nucleons should be taken into account to
regularize the logarithmic divergence. Some of the convergent terms are saturated by the pion
momenta of the order $k \sim (m_\Delta - m)1/2 \sim 530$ MeV, corresponding to the distances of the order
Figure 2: The interaction of the operator $\bar{q}q$ with the pion field created by the two-pion exchange between the two nucleons, denoted by the solid lines. The solid lines in the intermediate states stand for the nucleons or for delta isobars. The other notations are the same as in Fig.1.

Figure 3: The behaviour of the quark scalar condensate $\kappa(\rho)/|\kappa(0)|$ as function of the ratio $\rho/\rho_0$ obtained in framework of various models. The solid line shows the gas approximation [20], [21], the long-dashed and dashed curves present the pure [59] and modified [50] NJL model results. The dotted and dash-dotted lines present the result of calculation in hadronic model approach [47]. The dotted line corresponds to the physical value of the pion mass. The dash-dotted line shows the result in the chiral limit $m^2_\pi=0$. 
0.4 Fm, where the finite size of the nucleons should be included as well. Also, the result are sensitive to the density dependence of the effective nucleon mass $m^*$. This prompts, that a more rigorous analysis with the proper treatment of multi-nucleon configurations and of short distance correlations is needed. We shall return to the problem in Sec.4.

Anyway, the results for the calculation of the scalar condensate with the account of the pion cloud, produced by one- and two-pion exchanges looks as following. At very small values of density $\rho \lesssim \rho_0/8$, e.g. $p_F^2 \lesssim m_\pi^2$, only the two-pion exchanges contribute and

$$S(\rho) = 0.8\rho \cdot \frac{\rho}{\rho_0}.$$  \hspace{1cm} (68)

Hence, $S$ is positive for very small densities. However, for $\rho \gtrsim \rho_0$ we found $S < 0$. The numerical results are presented in Fig.3. One can see the effects of interaction to slow down the tendency of restoration of the chiral symmetry, in any case requiring $\kappa(\rho) = 0$. There is also the negative contribution to $\kappa(\rho)$ of the vector meson field. The sign of this term can be understood in the following way. It was noticed by Cohen et al. \cite{50} that the Gasser theorem \cite{24} expressed by Eq.(29), can be generalized for the case of the finite densities with

$$S(\rho) = \frac{d\varepsilon(\rho)}{dm},$$  \hspace{1cm} (69)

while $\varepsilon(\rho)$ is the binding energy. The contribution of vector mesons to rhs of Eq.(67) is $\frac{dV}{dm}\frac{dm}{dm}$ with $m_V$ standing for the vector meson mass. Since the energy caused by the vector meson exchange $V > 0$ drops with growing $m_V$, the contribution is negative indeed.

Another approach to calculation of the scalar condensate based on the soft pion technique was developed by Lyon group. Chanfray and Ericson \cite{51} expressed the contribution of the pion cloud to $\kappa(\rho)$ through the pion number excess in nuclei \cite{52}. The calculation of Chanfray et al. \cite{53} was based on the assumption, that GMOR relation holds in medium

$$f_{\pi}^2 m_{\pi}^2 = -\hat{m} \langle M|\bar{q}q|M\rangle.$$  \hspace{1cm} (70)

This is true indeed, as long as the pion remains to be much lighter than the other bosonic states of unnatural parity. Under several assumptions on the properties of the amplitude of $\pi N$ scattering in medium, the authors found

$$\frac{\kappa(\rho)}{\kappa(0)} = \frac{1}{1 + \rho\sigma/f_{\pi}^2 m_{\pi}^2}$$  \hspace{1cm} (71)

and $\kappa(\rho)$ turns to zero at asymptotically large $\rho$ only. This formula was obtained also by Ericson \cite{54} by attributing the deviations from the linear law to the distortion factor, emerging because of the coherent rescattering of pions by the nucleons.

However, Birse and McGovern \cite{55} and Birse \cite{56} argued, that Eq.(71) is not an exact relation and results from the simplified model which accounts only for nucleon-nucleon interaction, mediated by one pion. In framework of linear sigma model, which accounts for the $\pi\pi$ interaction and the $\sigma$-meson exchange, the higher order terms of $\rho$ expansion differ from those, provided by Eq.(71). The further development of calculation of the scalar condensate in the linear sigma model was made by Dmitrašinović \cite{57}.
In several works the function $\kappa(\rho)$ was obtained in framework of NJL model. In the papers of Bernard et al. [58, 59] the function $\kappa(\rho)$ was calculated for purely quark matter. The approach was improved by Jaminon et al. [60] who combined the Dirac sea of quark–antiquark pairs with Fermi sea of nucleons. In all these papers there is a region of small values of $\rho$, where the interaction inside the matter is negligibly small and thus $\kappa(\rho)$ changes linearly. However, the slope is smaller than the one, predicted by Eq.(27). In the modified treatment Cohen et al. [50] fixed the parameters of NJL model to reproduce the linear term. All the NJL approaches provide $S > 0$.

Recently Lutz et. al [61] suggested another hadronic model, based on chiral effective Lagrangian. The authors calculated the nonlinear contribution to the scalar condensate, provided by one-pion exchange. The value of $S(\rho_0)$ appeared to be close to that, obtained in one-pion approximation of [43]. Hence, all the considered hadronic models provide $S < 0$ except for very small values of $\rho$.

There is a common feature of all the described results. Near the saturation point the nonlinear term $S(\rho)$ is much smaller than the linear contribution. Thus, Eq.(27) can be used for obtaining the numerical values of $\kappa(\rho)$ at $\rho$ close to $\rho_0$. Hence, the condensate $|\kappa(\rho_0)|$ drops by about 30% with respect to $|\kappa(0)|$.

3 Hadron parameters in nuclear matter

3.1 Nuclear many-body theory

Until mid 70-th the analysis of nuclear matter was based on nonrelativistic approach. The Schrödinger phenomenology for the nucleon in nuclear matter employed the Hamiltonian

$$H_{NR} = -\frac{\Delta^2}{2m_{NR}^*} + U(\rho)$$

(72)

and the problem was to find the realistic potential energy $U(\rho)$. The deviation of nonrelativistic effective mass $m_{NR}^*$ from the vacuum value $m$ can be viewed as the dependence of potential energy on the value of three-dimensional momenta or “velocity dependent forces” [62]. The results of nonrelativistic approach were reviewed by Bethe [63] and by Day [64].

Since the pioneering paper of Walecka [65] the nucleon in nuclear matter is treated as a relativistic particle, moving in superposition of vector and scalar fields $V_\mu(\rho)$ and $\Phi(\rho)$. In the rest frame of the matter $V_\mu = \delta_{\mu0} V_0$ and Hamiltonian of the nucleon with the three-dimensional momentum $\vec{p}$ is

$$H = (\vec{\alpha}\vec{p}) + \beta(m + \Phi(\rho)) + V_0(\rho) \cdot I$$

(73)

with $\vec{\alpha} = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ being standard Dirac matrices.

Since the scalar “$\sigma$-meson” is rather an effective way to describe the system of two correlated pions, its mass, as well as the coupling constants of interaction between these mesons and the nucleons are the free parameters. They can be adjusted to fit either nuclear data or to reproduce the data on nucleon–nucleon scattering in vacuum. The numerous references can be found, e.g. in [69]. In both cases the values of $V_0$ and $\Phi$ appeared to be of the order of 300–400 MeV at the density saturation point.
The large values of the fields $V_0$ and $\Phi$ require the relativistic kinematics to be applied for the description of the motion of the nucleons.

In the nonrelativistic limit the Hamiltonian (73) takes the form of Eq. (72) with $m^*_{NR}$ being replaced by Dirac effective mass $m^*$, defined as

$$m^* = m + \Phi$$  \hspace{1cm} (74)

and

$$U = V_0 + \Phi.$$  \hspace{1cm} (75)

At the saturation point the fields $V_0$ and $\Phi$ compensate each other to large extent, providing $U \approx -60$ MeV. This explains the relative success of Schrödinger phenomenology. However, as shown by Brockmann and Weise [67], the quantitative description of the large magnitude of spin-orbit forces in finite nuclei requires rather large values of both $\Phi$ and $V_0$.

In the meson exchange picture the scalar and vector fields originate from the meson exchange between the nucleons of the matter. The model is known as quantum hadrodynamics — QHD. In the simplest version (QHD-1) only scalar $\sigma$-mesons and vector $\omega$ mesons are involved. In somewhat more complicated version, known as QHD-2 [68] some other mesons, e.g. the pions, are included. The matching of QHD-2 Lagrangian with low energy effective Lagrangian was done by Furnstahl and Serot [69].

The vector and scalar fields, generated by nucleons, depend on density in different ways. For the vector field

$$V(\rho) = 4 \int \frac{d^3p}{(2\pi)^3} N_V(p) g_V \theta(p_F - p)$$  \hspace{1cm} (76)

with $g_V$ the coupling constant, while $N_V = (\bar{u}_N \gamma_0 u_N)/2E$ with $u_N(p)$ standing for nucleon bispinors while

$$\varepsilon(p, \rho) = V_0(\rho) + \left(p^2 + m^2(\rho)\right)^{1/2}. \hspace{1cm} (77)$$

One finds immediately that $N_V = 1$ and thus $V(\rho)$ is exactly proportional to the density $\rho$. On the other hand, in the expression for the scalar field

$$\Phi(\rho) = 4 \int \frac{d^3p}{(2\pi)^3} N_s(p) \cdot g_s \theta(p_F - p)$$  \hspace{1cm} (78)

the factor $N_s = m^*/E$. Thus, the scalar field is a complicated function of density $\rho$.

The saturation value of density $\rho_0$ can be found by minimization of the energy functional

$$\mathcal{E}(\rho) = \frac{1}{\rho} \int_0^\rho \varepsilon_F(\rho)d\rho$$  \hspace{1cm} (79)

with $\varepsilon_F(\rho) = \varepsilon(p_F, \rho)$ being the single-particle energy at the Fermi surface. Thus, in QHD the saturation is caused by nonlinear dependence of the scalar field $\Phi$ on density.

The understanding of behaviour of axial coupling constant in nuclear matter $g_A(\rho)$ requires explicit introduction of pionic degrees of freedom. The quenching of $g_A$ at finite densities was predicted
by Ericson [70] from the analysis of the dispersion relations for $\pi N$ scattering. The result was confirmed by the analysis of experimental data on Gamow–Teller $\beta$-decay of a number of nuclei carried out by Wilkinson [71] and by investigation of beta decay of heavier nuclei — see, e.g., [72].

$$g_A(0) = 1.25; \quad g_A(\rho_0) = 1.0.$$  (80)

The quenching of $g_A$ as the result of polarization of medium by the pions was considered by Ericson et al. [73]. The crucial role of isobar-hole excitations in this phenomena was described by Rho [74].

Turning to the characteristics of the pions, one can introduce effective pion mass $m^*_\pi$ by considering the dispersion equation for the pion in nuclear matter (see, e.g., the book of Ericson and Weise [75]):

$$\omega^2 - k^2 - \Pi_p(\omega, k) - m^2_\pi = 0.$$  (81)

Here $\omega$ and $k$ are the pion energy and three-dimensional momenta, $\Pi_p$ is the $p$-wave part of the pion polarization operator. Hence, $\Pi_p$ contains the factor $k^2$. The pion effective mass is

$$m^*_\pi = m^2_\pi + \Pi_s(\omega, k)$$  (82)

with $\Pi_s$ being the $s$-wave part of polarization operator.

Polarization operator $\Pi_s$ (as well as $\Pi_p$) is influenced strongly by the nucleon interactions at the distances, which are much smaller than the average inter-nucleon distances $\approx m^{-1}_\pi$. Strictly speaking, here one should consider the nucleon as a composite particle. However, there is a possibility to consider such correlations in framework of hadron picture of strong interactions by using Finite Fermi System Theory (FFST), introduced by Migdal [76]. In framework of FFST the amplitudes of short-range baryon (nucleons and isobars) interactions are replaced by certain constant parameters. Hence, behaviour of $m^*_\pi$ can be described in terms of QHD and FFST approaches.

As well as any model based on conception of NN interaction, QHD faces difficulties at small distances. The weak points of the approach were reviewed by Negele [77] and by Sliv et al. [78]. Account of the composite structure of nucleon leads to the change of some qualitative results. Say, basing on the straightforward treatment of the Dirac Hamiltonian Brown et al. [79] found a significant term in the equation of state, arising from virtual $N\bar{N}$ pairs, generated by vector fields. The term would have been important for saturation. However, Jaroszewicz and Brodsky [80] and also Cohen [81] found that the composite nature of nucleon suppresses such contributions.

Anyway, to obtain the complete description, we need a complementary approach, accounting for the composite structure of hadrons. For pions it is reasonable to try NJL model.

### 3.2 Calculations in Nambu–Jona–Lasinio model

In NJL model the pion is the Goldstone meson, corresponding to the breaking of the chiral symmetry. The pion can be viewed as the solution of Bethe–Salpeter equation in the pseudoscalar quark–antiquark channel. The pion properties at finite density were investigated in frameworks of SU(2) and SU(3) flavour NJL model [82, 59]. It was found that the pion mass $m^*_\pi(\rho)$ is practically constant at $\rho \lesssim \rho_0$, increasing rapidly at larger densities, while $f^*_\pi(\rho)$ drops rapidly. These results were obtained rather for the quark matter. Anyway, as we mentioned in Subsec.2.7, at small $\rho$ the condensate $\kappa(\rho)$, obtained in this approach, does not satisfy the limiting law, presented by Eq.(27).
However, the qualitatively similar results were obtained in another NJL analysis, carried out by Lutz et al. [82]. The slope of the function $\kappa(\rho)$ satisfied Eq. (27). The pion mass $m_\pi^*(\rho)$ increased with $\rho$ slowly, while $f_\pi^*(\rho)$ dropped rapidly. The in-medium GMOR relation, expressed by Eq. (70) was satisfied as well.

Jaminon and Ripka [83] considered the modified version of NJL model, which includes the dilaton fields. This is the way to include effectively the gluon degrees of freedom. The results appeared to depend qualitatively on the way, the dilation fields are included into the Lagrangian. The pion mass can either increase or drop with growing density. Also the value of the slope of $\kappa(\rho)$ differs strongly in different versions of the approach. In the version, which is consistent with Eq. (27) the behaviour of $f_\pi^*(\rho)$ and $m_\pi^*(\rho)$ is similar to the one, obtained in the other papers, mentioned in this subsection.

Note, however, that the results which predict the fast drop of $f_\pi^*(\rho)$ have, at best, a limited region of validity. This is because the pion charge radius $r_\pi$ is connected to the pion decay constant by the relation obtained by Carlitz and Creamer [84]

$$\langle r_\pi^2 \rangle^{1/2} = \frac{\sqrt{3}}{2\pi f_\pi}$$

providing $\langle r_\pi^2 \rangle^{1/2} \approx 0.6$ Fm. Identifying the size of the pion with its charge radius, we find that at $\langle r_\pi^2(\rho) \rangle^{1/2}$ becoming of the order of the confinement radius $r_c \sim 1$ Fm, the confinement forces should be included and straightforward using of NJL is not possible any more. Thus, NJL is definitely not true for the densities, when the ratio $f_\pi^*(\rho)/f_\pi$ becomes too small. Anyway, one needs

$$\frac{f_\pi^*(\rho)}{f_\pi} \gtrsim 0.6.$$  

For the results, obtained in [82] this means that they can be true for $\rho \leq 1.3\rho_0$ only.

### 3.3 Quark–meson models

This class of models, reviewed by Thomas [11] is the result of development of MIT bag model, considering the nucleon as the system of three quarks in a potential well. One of the weak points of the bag-model approach is the absence of long-ranged forces in NN interactions. In the chiral bag model (CBM) the long-ranged tail is caused by the pions which are introduced into the model by requirement of chiral invariance. In the framework of CBM the pions are as fundamental degrees of freedom as quarks. In the cloudy bag model these pions are considered as the bound states of $\bar{q}q$ pairs. The model succeeded in describing the static properties of free nucleons.

Another model, suggested by Guichon [85] is a more straightforward hybrid of QHD and QCD. The nucleon is considered as a three-quark system in a bag. The quarks are coupled to $\sigma$- and $\omega$-mesons directly. Although this quark-meson coupling model (QMC) was proposed by its author as ”a caricature of nuclear matter”, it was widely used afterwards. The parameters of $\sigma$- and $\omega$-mesons and the bag radius, which are the free parameters of the model were adjusted to describe the saturation parameters of the matter. The fields $\Phi$ and $V$ appear to be somewhat smaller than in QHD. Thus, the values of $m^*/m$ and $g_A^*/g$ are quenched less than in QHD [86]. On the other hand, the unwanted $N\bar{N}$ pairs are suppressed. The nonlinearity of the scalar field is the source of saturation.
The common weak point of these models are well known. Say, there is no consistent procedure to
describe the overlapping of the bags. It is also unclear, how to make their Lorentz transformations.

3.4 Skyrmion models

This is the class of models with much better theoretical foundation. They originate from the old
model, suggested by Skyrme [87]. The model included the pions only, and the nucleon was the
soliton. Later Wess and Zumino [88] added the specific term to the Lagrangian, which provided the
current with the non-vanishing integral of the three-dimensional divergence. That was the way, how
the baryon charge manifested itself.

Thus, in framework of the approach most of the nucleon characteristics are determined by Dirac
sea of quarks and by the quark–antiquark pairs, which are coupled into the pions. The model can
be viewed as the limiting case $R \to 0$ of the chiral bag model, where the description in terms of the
mesons at $r > R$ is replaced by description in terms of the quarks at $r < R$ [89].

In the framework of the Skyrme model Adkins et al. [90, 91] calculated the static characteristics
of isolated nucleons. A little later Jackson et al. [92] investigated NN interaction in this model.
The model did not reproduce the attraction in NN potential. It was included into modified Skyrme
Lagrangian by Rakhimov et al. [93] in order to calculate the renormalization of $g_A$, $m$ and $f_\pi$ in
nuclear matter. The magnitude of renormalization appeared to be somewhat smaller than in QHD.

The approach was improved by Diakonov and Petrov — see a review paper [94] and references
therein. The authors build the chiral quark–soliton model of the nucleon. It is based on quark-
pion Lagrangian with the Wess–Zumino term and with spontaneous chiral symmetry breaking. The
nucleon appeared to be a system of three quarks, moving in a classical self-consistent pion field. The
approach succeeded in describing the static characteristics of nucleon. It provided the proper results
for the parton distributions as well. However, the application of the approach to description of the
values of nucleon parameters in medium is still ahead.

3.5 Brown-Rho scaling

Brown and Rho [95] assumed that all the hadron characteristics, which have the dimension of the
mass change in medium in the same manner. The universal scale was assumed to be

$$\chi(\rho) = (-\kappa(\rho))^{1/3}.$$  

(85)

Thus, the scaling which we refer to as BR1 is

$$\frac{m^*(\rho)}{m} = \frac{f^*_\pi(\rho)}{f_\pi} = \frac{\chi(\rho)}{\chi(0)}.$$  

(86)

The pion mass was assumed to be an exception, scaling as

$$\frac{m^*_\pi(\rho)}{m_\pi} = \left(\frac{\chi(\rho)}{\chi(0)}\right)^{1/2}.$$  

(87)
Thus, BR1 is consistent with in-medium GMOR relation. Also, in contrast to NJL, the pion mass drops with density.

Another point of BR1 scaling is the behaviour

$$g_A'(\rho) = g_A(0) = \text{const}. \quad (88)$$

Consistency of Eqs. (80) and (88) can be explained in such a way. Renormalization expressed by Eq. (80) is due to $\Delta$-hole polarization of medium. It takes place at moderate distances of the order $m_\pi^{-1}$, reflecting rather the properties of the medium, but not the intrinsic properties of the nucleon, which are discussed here.

Another version of Brown–Rho scaling [96], which we call BR2 is based on the in-medium GMOR relation, expressed by Eq. (70). It is still assumed that

$$m^*_\pi m = f^*_\pi f_\pi, \quad (89)$$

but the pion mass is assumed to be constant

$$m^*_\pi \approx m_\pi, \quad (90)$$

and thus

$$\frac{f^*_\pi}{f_\pi} = \left(\frac{\kappa(\rho)}{\kappa(0)}\right)^{1/2} \quad (91)$$

instead of 1/3 law in BR1 version — Eq. (86). Note, however, that assuming $m^*_\pi(\rho_0) = 1.05m_\pi$ [96] we find, using the results of subsection 2.6

$$\frac{f^*_\pi(\rho_0)}{f_\pi} = 0.76. \quad (92)$$

This is not far the limit determined by Eq. (84). At larger densities the size of the pion becomes of the order of the confinement radius. Here the pion does not exist as a Goldstone boson any more. In any case, some new physics should be included at larger densities. If Eq. (91) is assumed to be true, this happens at $\rho \approx 1.6\rho_0$.

### 3.6 QCD sum rules

In this approach we hope to establish some general relations between the in-medium values of QCD condensates and the characteristics of nucleons.

The QCD sum rules were invented by Shifman et al. [6] and applied for the description of the mesonic properties in vacuum. Later Ioffe [97] expanded the method for the description of the characteristics of nucleons in vacuum. The main idea is to build the function $G(q^2)$ which describes the propagation of the system ("current") with the quantum numbers of the proton. (The usual notation is $\Pi(q^2)$. We used another one to avoid confusion with pion polarization operator, expressed by Eq. (81)). The dispersion relation

$$G(q^2) = \frac{1}{\pi} \int \frac{\text{Im} \, G(k^2)}{k^2 - q^2} dk^2 \quad (93)$$
is considered at \( q^2 \to -\infty \). Imaginary part in the rhs is expressed through parameters of observable hadrons. Due to asymptotic freedom of QCD lhs of Eq.\((93)\) can be presented as perturbative series in \(-q^2\) with QCD vacuum condensates as coefficients of the expansion. Convergence of the series means that the condensates of lower dimension are the most important ones.

The method was used for the calculation of characteristics of the lowest lying hadron states. This is why the "pole+continuum" model was employed for the description of \( \text{Im} G(k^2) \) in the rhs of Eq.\((93)\). This means that the contribution of the lowest lying hadron was treated explicitly, while all the other excitations were approximated by continuum. In order to emphasise the contribution of the pole inverse Laplace (Borel) transform was applied to both sides of Eq.\((93)\) in the papers mentioned above. The Borel transform also removes the polynomial divergent terms.

Using QCD sum rules Ioffe \[97\] found that the nucleon mass vanishes if the scalar condensate turns to zero. Numerically, \[39, 97, 98\]

\[
m = \left(-2(2\pi)^2 \langle 0|\bar{q}q|0 \rangle\right)^{1/3}.
\]

Later the method was applied by Drukarev and Levin \[20, 21, 99\] for investigation of properties of nucleons in the nuclear matter. The idea was to express the change of nucleon characteristics through the in-medium change of the values of QCD condensates. The generalization for the case of finite densities was not straightforward. Since the Lorentz invariance is lost, the function \( G^m(q) \) describing the propagation of the system in medium depends on two variables, e.g. \( G^m = G(q^2, q_0) \). Thus, each term of expansion of \( G^m \) in powers of \( q^{-2} \) may contain infinite number of local condensates. In the rhs of dispersion relation it is necessary to separate the singularities, connected with the nucleon from those, connected with excitation of the matter itself.

We shall return to these points in Sec.5. Here we present the main results. The method provided the result for the shift of the position of the nucleon pole. The new value is expressed as a linear combination of several condensates with vector condensate \( v(\rho) \) and scalar condensate \( \kappa(\rho) \) being most important \[20, 21, 99\]

\[
m_m = m + C_1 \kappa(\rho) + C_2 v(\rho).
\]

On the other hand,

\[
m_m - m = U \left(1 + 0 \left(\frac{U}{m}\right)\right)
\]

with \( U \) being single-particle potential energy of the nucleon. Hence, the scalar forces are to large extent determined by the \( \sigma \)-term.

The Dirac effective mass was found to be proportional to the scalar condensate

\[
m^*(\rho) = \kappa(\rho) F(\rho)
\]

with \( F(\rho) \) containing the dependence on the other condensates, e.g. on vector condensate \( v(\rho) \).

Using Eqs.(15) and (16) we see, that \( v(\rho) \) is linear in \( \rho \). Thus, the main nonlinear contributions to the energy \( \mathcal{E}(\rho) \) presented by Eq.(79) come from nonlinearities in the function \( \kappa(\rho) \). For the saturation properties of the matter the sign of the contribution \( S(\rho) \) becomes important. The nonlinearities of the condensate \( \kappa(\rho) \) can be responsible for the saturation if \( S < 0 \). Calculations of Drukarev and Ryskin \[43\] show that the saturation can be obtained at reasonable values of density with reasonable
value of the binding energy. Of course, this result should not be taken too seriously, since it is very sensitive to the exact values of $\sigma$-term. It can be altered also by the account of higher order terms. (However, as noted by Birse \[56\], the QHD saturation picture is also very sensitive to the values of the parameters). Similar saturation mechanism was obtained recently in the approach developed by Lutz et.al \[61\]. Anyway, it can be a good starting point to analyse the problem.

4 First step to self-consistent treatment

As we have seen in Sec.2 in the gas approximation the scalar condensate $\kappa(\rho)$ is expressed through the observables. However, beyond the gas approximation it depends on a set of other parameters. Here we show how such dependence manifests itself in a more rigorous treatment of the hadronic presentation of nuclear matter.

4.1 Account of multi-nucleon effects in the quark scalar condensate

Now we present the main equations, which describe the contribution of the pion cloud to the condensate $\kappa(\rho)$. Recall, that the pions are expected to give the leading contribution to the nonlinear part $S(\rho)$ due to the large expectation value $\langle \pi|\bar{q}q|\pi\rangle$ — Eq.(64).

In order to calculate the contribution we employ the quasiparticle theory, developed by Migdal for the propagation of pions in matter \[100\]. Using Eq.(69), we present $S(\rho)$ through the derivative of the nucleon self-energy with respect to $m_\pi^2$:

$$S = \sum_B S_B;$$

$$S_B = -C_B \Upsilon \int \frac{d^3p}{(2\pi)^3} \frac{d^3kd\omega}{(2\pi)^4} \left( \Gamma_B^2 D^2(\omega, k) g_B(p - k) - \Gamma_B^0 D_0^2(\omega, k) g_B^0(p - k) \right).$$

(98)

Here $B$ labels the excited baryon states with propagators $g_B$ and $\pi NB$ vertices $\Gamma_B$. The pion propagator $D$ includes the multi-nucleon effects ($D^{-1}$ is the lhs of Eq.(61)). The second term of the rhs of Eq.(98), with the index ”0” corresponding to the vacuum values, subtracts the terms, which are included into the expectation value already. The coefficient $C_B$ comes from summation over the spin and the isospin variables. Integration over nucleon momenta $p$ is limited by the condition $p \leq p_F$. The factor $\Upsilon$ stands for the expectation value of the operator $\bar{q}q$ in pion, i.e. $\Upsilon = \langle \pi|\bar{q}q|\pi\rangle = m_\pi^2/\hat{m}$. Of course, Eq.(98), illustrated by Fig.4 corresponds to the Lagrangian which includes the lowest order $\pi N$ interactions only.

The pion propagator in medium can be viewed as the solution of the Dyson equation \[72, 76\] — Fig.5:

$$D = D_0 + D_0 \Pi D$$

(99)

with

$$\Pi(\omega, k) = 4\pi \int_{p_F} \frac{d^3p'}{(2\pi)^3} A(p'; \omega, k),$$

(100)
Figure 4: a) The interaction of the operator $\bar{q}q$ (the dark blob) with the pion field. The solid line denotes the nucleon; the wavy line stands for the pion; b,c) The diagrammatic presentation of Eq.(98) with the nucleon in the intermediate state. The bold wavy line denotes the pion propagator renormalized due to baryon-hole excitations in the framework of FFST; d,e) The diagrammatic presentation of Eq.(98) with the $\Delta$-isobar (double solid line) in the intermediate state.

\[
\begin{align*}
\text{Figure 5: The Dyson equation (99) for the pion propagator in medium in the quasiparticle-hole formalism. Wavy line denotes the vacuum pion propagator, bold wavy line stands for the propagator in matter. The dark angle denotes the correlations.}
\end{align*}
\]
while \( A(p'; \omega, k) \) stands for the amplitude of the forward \( \pi N \) scattering (all the summation over the spin and isospin variables is assumed to be carried out) on the nucleon of the matter with the three-dimensional momentum \( p' \). Of course, the pion is not on the mass-shell.

Neglecting the interactions inside the bubbles of Fig.5 (this is denoted by the upper index "(0)"") we can present

\[
A^{(0)} = \sum A_B^{(0)}; \quad \Pi^{(0)} = \sum \Pi_B^{(0)}
\]

\[
A_B^{(0)} = c_B \bar{\Gamma}_B^2(k) \Lambda_B(p'; \omega, k)
\]  

with \( c_B \) being a numerical coefficient,

\[
\Lambda_B(p'; \omega, k) = g_B(\varepsilon' + \omega, \vec{p}' + \vec{k}) + g_B(\varepsilon' - \omega, \vec{p}' - \vec{k})
\]  

(102)

The factors \( \bar{\Gamma}_B^2(k) \) come from the vertex functions. Considering \( p \)-wave part of polarization operator only (the \( s \)-wave part is expressed through the pion effective mass \( m^*_\pi \) — Eq.(82)), we present

\[
\bar{\Gamma}_B^2(k) = \bar{g}_{\pi NB}^2 k^2 d_{NB}^2(k)
\]  

(103)

with \( d_{NB}^2 \) accounting for the finite size of the baryons, \( \bar{g}_{\pi NB} \) is the coupling constant.

Starting the analysis with the contribution of the nucleon intermediate state \( (B = N) \) to Eqs.(101) and (102), we see that in the nonrelativistic limit we can present the first term in rhs of Eq.(102) as

\[
g_N(\varepsilon' + \omega, \vec{p}' + \vec{k}) = \frac{\theta(|\vec{p}' + \vec{k}| - p_F)}{\omega + \varepsilon_{p' + k}}
\]  

(104)

(similar presentation can be written for the second term) with \( \varepsilon_q = q^2 / 2m^* + U \), while \( U \) stands for the potential energy. Hence, the terms, containing \( U \) cancel and all the dependence on the properties of the matter enters through the effective mass \( m^* \). This enables to obtain the contribution to the polarization operator

\[
\Pi_N^{(0)} = -4\bar{g}_{\pi NN}^2 k^2 d_{NN}^2(k) \frac{m^* p_F}{2\pi^2} \phi_N^{(0)}(\omega, k)
\]  

(105)

with explicit analytical expression for \( \phi_N^{(0)}(\omega, k) \) presented in [75, 104], the static long-wave limit is \( \phi_N^{(0)}(0, 0) = 1 \).

Such approach does not include the particle-hole interactions in the bubble diagram of Fig.5. The short-range correlations can be described with the help of effective FFST constants, as it was mentioned above. Using the Dyson equation for the short-range amplitude of nucleon-hole scattering one finds

\[
\Pi_N = -4\bar{g}_{\pi NN}^2 k^2 d_{NN}^2(k) \frac{m^* p_F}{2\pi^2} \phi_N(\omega, k)
\]  

(106)

with

\[
\phi_N(\omega, k) = \frac{\phi_N^{(0)}(\omega, k)}{1 + g_{NN}^{(0)}(\omega, k)}
\]  

(107)

if only the nucleon intermediate states are included.

The long-ranged correlations inside the bubbles were analysed by Dickhoff et al. [101]. It was shown, that exchange by the renormalized pions inside the bubbles ("bubbles in bubbles") can be
accounted for by the altering of the values of FFST constants. The change in the numerical values does not appear to be large.

The usual approach includes also the $\Delta$-isobar states in the sums in Eq.(98). Until the particle-hole correlations are included, the total $p$-wave operator $\Pi^{(0)}$ is just the additive sum of the nucleon and isobar terms, i.e. $\Pi^{(0)} = \Pi^{(0)}_N + \Pi^{(0)}_\Delta$. Also, one can obtain analytical expression, similar to Eq.(105) for the contribution $\Pi^{(0)}_\Delta$ under a reasonable assumption on the propagation of $\Delta$-isobar in medium (see below). However, account of the short-range correlations makes the expression for the total $p$-wave polarization operator more complicated. We use the explicit form presented by Dickhoff et al. [102]

\[ \Pi = \Pi_N + \Pi_\Delta \]

with

\[
\Pi_N = \Pi^{(0)}_N \left( 1 - (\gamma_\Delta - \gamma_{\Delta\Delta}) \frac{\Pi^{(0)}_\Delta}{k^2} \right) / E \tag{108}
\]

\[
\Pi_\Delta = \Pi^{(0)}_\Delta \left( 1 + (\gamma_\Delta - \gamma_{NN}) \frac{\Pi^{(0)}_N}{k^2} \right) / E \tag{109}
\]

Denominator $E$ has the form

\[ E = 1 - \gamma_{NN} \frac{\Pi^{(0)}_N}{k^2} - \gamma_\Delta \frac{\Pi^{(0)}_\Delta}{k^2} + \left( \gamma_{NN} \gamma_{\Delta\Delta} - \gamma_\Delta^2 \right) \frac{\Pi^{(0)}_N \Pi^{(0)}_\Delta}{k^4}. \tag{110} \]

The effective constants $\Gamma$ are related to FFST parameters $g'$ as follows:

\[
\gamma_{NN} = C_0 \frac{g'_{NN}}{g_{\pi NN}} ; \quad \gamma_\Delta = C_0 \frac{g'_{\Delta\Delta}}{g_{\pi\Delta\pi\Delta}} ; \quad \gamma_{\Delta\Delta} = C_0 \frac{g'_{\Delta\Delta}}{g_{\pi\Delta\pi\Delta}}, \tag{111}
\]

where $C_0$ is the normalization factor for the effective particle–hole interaction in nuclear matter. We use $C_0 = \pi^2 / p_fm^*$, following [76]. (Note, that there is some discrepancy in the notations used by different authors. Our parameters $\gamma$ coincide with those, used in [102]. We use the original FFST parameters $g'$ of [76], which are related to the constants $G'_0$ of [102] as $g' = G'/2$). The short-range interactions require also renormalization of the vertices $\Gamma_{\pi NB}^2 \rightarrow \Gamma_{\pi NB}^2 x_{\pi NB}^2$ with

\[
x_{\pi NN} = \left( 1 + (\gamma_\Delta - \gamma_{\Delta\Delta}) \frac{\Pi^{(0)}_\Delta}{k^2} \right) / E ; \quad x_{\pi N\Delta} = \left( 1 + (\gamma_\Delta - \gamma_{NN}) \frac{\Pi^{(0)}_N}{k^2} \right) / E. \tag{112}
\]

In our paper [103] we calculated the contribution $S(\rho)$, presented by Eq.(98), using nucleons and $\Delta$-isobars as intermediate states. The integration over $\omega$ requires investigation of the solutions of the pion dispersion equation-Eq.(81).

4.2 Interpretation of the pion condensate

The pion dispersion equation [75, 104] is

\[ \omega^2 = m^2_\pi + k^2 \left( 1 + \chi(\omega, k) \right) \tag{113} \]
with the function $\chi$ introduced as $\Pi_p(\omega, k) = -k^2 \chi(\omega, k)$. It is known to have three branches of solutions $\omega_i(k)$ (classified by the behaviour of the functions $\omega_i(k)$ at $k \to 0$). If the function $\chi(\omega, k)$ includes nucleons only as intermediate states and does not include correlations, we find $k^2 \chi \to 0$ at $k \to 0$. This is the pion branch for which $\omega_\pi(0) = m_\pi^*$. If the correlations are included, the denominator in the rhs of Eq.(107) may turn to zero at $k \to 0$, providing the sound branch with $\omega_s(0) = 0$. Inclusion of $\Delta$-isobars causes the contribution to $\chi(\omega, k)$, proportional to $[m_\Delta - m - \omega]^{-1}$. Thus, there is a solution with $\omega_\Delta(0) = m_\Delta - m$, called the isobar branch.

The trajectories of the solutions of Eq.(81) on the physical sheet of Riemann surface were studied by Migdal [104]. Their behaviour on the unphysical sheets was investigated recently by Sadovnikova [105] and by Sadovnikova and Ryskin [106]. In these papers it was shown, that besides the branches, mentioned above, there is one more branch starting from the value $\omega_c(0) = m_\pi^*$ and moving on the unphysical sheet for larger $k > 0$. The branch comes to the physical sheet at certain value of $k$ if the density exceeds certain critical value $\rho_C$. Here $\omega_c$ is either zero or purely imaginary and thus $\omega_c^2 \leq 0$. (However, this is true if the isobar width $\Gamma_\Delta = 0$, for the finite values of $\Gamma_\Delta$ we find $\omega_c$ to be complex and $Re\omega_c^2 \leq 0$.) This corresponds to the instability of the system first found by Migdal [100] and called the "pion condensation". On the physical sheet $\omega_c(k)$ coincides with the solutions, obtained in [100], [104]. However, contrary to [100], [104], the $\omega_c(k)$ is not the part of zero-sound branch.

To follow the solution $\omega_c(k)$, let us present the function $\Phi_N^{(0)}$, which enters Eq.(103) as

$$\Phi_N^{(0)}(\omega, k) = \varphi_N^{(0)}(\omega, k) + \varphi_N^{(0)}(-\omega, k)$$

with the explicit expression for $0 < k < 2p_F$

$$\varphi_N^{(0)}(\omega, k) = \frac{1}{p_F k} \left( \frac{-\omega m^* + k p_F}{2} + \frac{(k p_F)^2 - (\omega m^* - k^2/2)^2}{2k^2} \right) \times \ln \left( \frac{\omega m^* - k p_F - k^2/2}{\omega m^* - k p_F + k^2/2} \right) - \omega m^* \ln \left( \frac{\omega m^*}{\omega m^* - k p_F + k^2/2} \right).$$ (115)

At $k > 2p_F$ the expression for $\varphi_N^{(0)}(\omega, k)$ takes another form (see [104]) but we shall not need it here.

It was shown in [105], [106] that, if the density $\rho$ is large enough ($\rho \geq \rho_C$), there is a branch of solutions $\omega_c^2(k) \leq 0$, which is on the physical sheet for certain interval $k_1 < k < k_2$ of the values of $k$. At smaller values $k < k_1$ the branch goes to the unphysical sheet through the cut

$$0 \leq \omega \leq \frac{k}{m^*} \left( p_F - \frac{k}{2} \right),$$ (116)

generated by the third term in the rhs of Eq.(113). At larger values of $k > k_2$ the solution $\omega_c$ goes away to the unphysical sheet through the same cut. The zero-sound wave goes to the unphysical sheet through another cut:

$$\frac{k}{m^*} \left( p_F - \frac{k}{2} \right) \leq \omega \leq \frac{k}{m^*} \left( p_F + \frac{k}{2} \right),$$ (117)
cauised by the second term in the rhs of Eq.(113).

The value of the density $\rho_C$, for which the solution $\omega_c$ penetrates to the physical sheet, depends strongly on the model assumptions. Say, if the contribution of isobar intermediate states is ignored,
The value of $\rho_C$ is shifted to unrealistically large values $\rho_C > 25\rho_0$. Inclusion of both nucleon and isobar states and employing of realistic values of FFST constants leads to $\rho_C \approx 1.4\rho_0$ under additional assumption $m^*_\pi(\rho) = m^*_\pi(0)$.

The zero values of $\omega_c(k)$ at certain nonzero values of $k$ signals on the instability of the ground state. New components, like baryon-hole excitations with the pion quantum numbers emerge in the ground state of nuclear matter. Thus, the appearance of the singularity corresponding to $\omega_c^2 = 0$ shows, that the phase transition takes place.

Note, however, that the imaginary part of the solution $\omega_c(k)$ is negative. Thus, there is no "accumulation of pions" in the symmetric nuclear matter, contrary to the naive interpretation of the pion condensation.

The situation is much more complicated in the case of asymmetric nuclear matter. In the neutron matter the instability of the system emerges at finite values of $\omega$, because of the conversion $n \rightarrow p + \pi^-$ \cite{13}. This process leads to the real accumulation of pions in the ground state. In the charged matter with the non-zero value of the difference between the neutron and proton densities there is an interplay of the reactions $n \leftrightarrow p + \pi^-$ and beta decays of nucleons.

### 4.3 Quark scalar condensate in the presence of the pion condensate

Now we turn back to the calculation of the condensate $\kappa(\rho)$. Note first, that if the isobar width is neglected, we find $\kappa(\rho) \rightarrow +\infty$ at $\rho \rightarrow \rho_C$. The reason is trivial. When $\rho \rightarrow \rho_C$, the contribution $S(\rho)$, described by Eq.(98) becomes

$$S \sim \int \frac{d\omega d^3k}{[\omega^2 - \omega_c^2(\rho, k)]^2}.$$ \hspace{1cm} (118)

The curve $\omega_c(\rho_c, k)$ turns to zero at certain $k = k_c$, being $\omega_c = a(k - k_c)^2$ at $|k - k_c| \ll k_c$. Thus,

$$S \sim \int \frac{d\omega k_c^2 d|k - k_c|}{[\omega^2 - a^2(k - k_c)^2]^2} \rightarrow \infty .$$ \hspace{1cm} (119)

Hence, $S(\rho_C) = +\infty$ and $\kappa(\rho_C) = +\infty$. Once $\kappa(0) < 0$, we find that at certain $\rho_{ch} < \rho_C$ the scalar condensate $\kappa(\rho)$ turns to zero. This means that the chiral phase transition takes place before the pion condensation. (We shall not discuss more complicated models, for which the condition $\kappa(\rho_{ch}) = 0$ is not sufficient for the chiral symmetry restoration). At larger densities the pion does not exist any more as a collective Goldstone degree of freedom. Also the baryon mass vanishes (if very small current quark masses is neglected), and we have to stop our calculations, based on the selected set of Feynman diagrams (Fig.4) with the exact pion propagator.

The in-medium width of delta isobar $\Gamma_\Delta$ (the probability of the decay) depends strongly on the kinematics of the process. We can put $\Gamma_\Delta(\omega, k) = 0$ due to the limitation on the phase space of the possible decay process \cite{103}. Thus, following the paper of Sadovnikova and Ryskin \cite{106}, we find that the chiral symmetry restoration takes place at the densities, which are smaller, than those, corresponding to the pion condensation:

$$\rho_{ch} < \rho_C .$$ \hspace{1cm} (120)
This means, that the pion condensation point cannot be reached in framework of the models, which do not describe the physics after the restoration of the chiral symmetry.

4.4 Calculation of the scalar condensate

4.4.1 Parameters of the model

Now we must specify the functional dependence and the values of the parameters which are involved into the calculations. The $\pi NN$ coupling constant is

$$\tilde{g}_{\pi NN} = \frac{g_{\pi NN}}{2m} = \frac{g_A}{2f_\pi},$$  

(121)

— see Eq.(4). The $\pi N\Delta$ coupling constant is

$$\tilde{g}_{\pi N\Delta} = c_\Delta \tilde{g}_{\pi NN}$$  

(122)

with the experiments providing $c_\Delta \approx 2$ [75]. This is supported by the value $c_\Delta \approx 1.7$, calculated in the framework of Additive Quark Model (AQM).

The form factor $d_{NB}(k)$ which enters Eq.(103) is taken in a simple pole form [75]

$$d_{NB} = \frac{1 - m^2_p/\Lambda_B^2}{1 + k^2/\Lambda_B^2}$$  

(123)

with $\Lambda_N = 0.67$ GeV, $\Lambda_\Delta = 1.0$ GeV.

We use mostly the values of FFSI parameters, presented in [107]: $g'_{NN} = 1.0$, $g'_{N\Delta} = 0.2$, $g'_{\Delta\Delta} = 0.8$ — referring to these values as to set "a". We shall also check the sensitivity of the results to the variation of these parameters.

It is know from QHD approach that the nucleon effective mass may drop with density very rapidly. Thus, we must adjust our equations for description of the case, when the relativistic kinematics should be employed. We still include only the positive energy of the nucleon propagator, presented by Eq.(104). However we use the relativistic expression for

$$\varepsilon_p - \varepsilon_{p+k} = \sqrt{p^2 + m^*} - \sqrt{(p+k)^2 + m^*}.$$  

(124)

The propagator of $\Delta$-isobar is modified in the same way. The explicit equations for the functions $\Phi_{N,\Delta}^{(0)}$, accounting for the relativistic kinematics are presented in [103].

4.4.2 Fixing the dependence $m^*(\rho)$

As we have seen above, the contribution of nucleon-hole excitations to $S(\rho)$ depends explicitly on the nucleon effective mass $m^*(\rho)$. Here we shall try the models, used in nuclear physics, which determine the direct dependence $m^*(\rho)$. One of them is the Fermi liquid model with the effective mass described by Landau formula [108], [76], [109]

$$\frac{m^*(\rho)}{m} = 1 + \left(1 + \frac{2m_p}{\pi^2 f_1}\right).$$  

(125)
In QHD approach the effective mass $m^*$ is the solution of the equation

$$m^* = m - cm^* \left[ p_F(p_F^2 + m^*+2)^{1/2} - m^* \ln \frac{p_F + (p_F^2 + m^*+2)^{1/2}}{m^*} \right], \tag{126}$$

corresponding to the behaviour

$$m^* = m(1 - f_2 \rho), \tag{127}$$

in the lowest order of expansion in powers of Fermi momentum $p_F$. The coefficients $f_{1,2}$ in Eqs. (125), (127) can be determined by fixing the value $m^*(\rho_0)$.

Assuming, that all the other parameters are not altered in medium: $f^*_\pi = f_\pi, m^*_\Delta - m^* = m_\Delta - m, m^*_\pi = m_\pi, c^*_\Delta = c_\Delta, g^*_A = g_A(\rho_0) \approx 1.0$, we find the point of chiral symmetry restoration to depend strongly on the value $m^*(\rho_0)$, being stable enough under the variation of FFST parameters and of the parameter $c_\Delta$ — Fig.6. Fixing $m^*(\rho_0) = 0.8m$, we find $\rho_{ch} < \rho_0$, in contradiction to experimental data. Even in a simplified model with the width of $\Delta$-isobar being accepted to coincide with its vacuum value $\Gamma_\Delta = 115$ MeV, we find $\rho_{ch} \approx 1.15\rho_0$. The value $|\kappa(\rho_0)| \ll |\kappa(0)|$ looks unrealistic, since there are practically no strong unambiguous signals on partial restoration of the chiral symmetry at the saturation value of density $\rho_0 \approx 1.6$. Hence, here we also come to contradiction with the experimental data.

The situation is less critical for the smaller values of $m^*(\rho_0)/m$. Say, for $\Gamma_\Delta = 115$ MeV we find $\rho_{ch} \approx 1.7\rho_0$, assuming $m^*(\rho_0)/m = 0.7$. However, under realistic assumption $\Gamma_\Delta = 0$ we come to $\rho_{ch} < \rho_0$.

Note, that there is another reason for the point of the pion condensation to be unaccessible by our approach. The perturbative treatment of $\pi N$ interaction becomes invalid for large pion fields. In the chiral $\pi N$ Lagrangians the $\pi N$ interaction is described by the terms of the type

$$L_{\pi N} = \bar{\psi}U^+(i\gamma_\mu \partial^\mu)U\psi \tag{128}$$

with

$$U = \exp \frac{i}{2f_\pi} \gamma_5(\tau\varphi).$$

The conventional version of pseudovector $\pi NN$ Lagrangian employed above may be treated either as the lowest term of expansion of the matrix $U$ in powers of the ratio $\varphi/f_\pi$ (identifying the pion with $\varphi$-field) or as the interaction with the field $\tilde{\varphi} = f_\pi \sin((\tau\varphi)/f_\pi)$. In any case, the whole approach is valid only, when the pion field is not too strong ($\varphi \leq f_\pi$ or $\tilde{\varphi} \leq f_\pi$ correspondingly). However, the strict quantitative criteria for the region of validity of Eq.(128) is still obscure.

The strong dependence of the results on the value of $m^*(\rho_0)/m$ forces us to turn to self-consistent treatment of the hadron parameters and the quark condensates.

### 4.4.3 Self-consistent treatment of nucleon mass and the condensate

Now we shall carry out the calculations in framework of the model, where the nucleon parameters depend on the values of condensates. In other words, instead of the attempt to calculate the condensate $\kappa(\rho, y_i(\rho))$ with $y_i$ standing for the hadron parameters ($y_i = m^*_N, m^*_\Delta, f^*_\pi, \ldots$), we shall try
Figure 6: The function $\kappa(\rho)/|\kappa(0)|$. The solid curve presents the result obtained with $g'_{NN} = 1.0$, $g'_{N\Delta} = 0.2$, $g'_{\Delta\Delta} = 0.8$, $c_\Delta = 2.0$, $\Gamma_\Delta = 0.115$ GeV and nucleon effective mass given by Eq.(125). The other curves are obtained with the values of some of the parameters or the shape of the density dependence of the effective mass being modified: a) Dependence of $\kappa(\rho)/|\kappa(0)|$ on the variation of nuclear parameters. The dashed curve corresponds to the calculation with $c_\Delta = 1.7$, the dotted curve – to $g'_{\Delta\Delta} = 1.2$, dot-dashed curve – to $g'_{NN} = 0.7$. b) Dependence of $\kappa(\rho)/|\kappa(0)|$ on the isobar width. Dotted curve corresponds to the calculation with $\Gamma_\Delta = 0.07$ GeV, dot-dashed curve – to $\Gamma_\Delta = 0.05$ GeV, dashed curve – to $\Gamma_\Delta = 0.01$ GeV. c) Dependence of $\kappa(\rho)/|\kappa(0)|$ on the shape of $m^*(\rho)$. Dashed curve corresponds to Walecka formula (127) with $m^*$ ($m^*(\rho = \rho_0) = 0.8m$). Dot-dashed curve is obtained in framework of Walecka model with $m^*(\rho = \rho_0) = 0.7m$. 

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to solve the equation
\[ \kappa(\rho) = \mathcal{K}\left(\rho, y_i(\kappa(\rho), c_j(\rho))\right) \] (129)

with \( c_j(\rho) \) standing for the other QCD condensates. Here \( \mathcal{K} \) is the rhs of Eq.(62). Strictly speaking, we should try to obtain similar equations for the condensates \( c_j(\rho) \).

We shall assume the physics of nuclear matter to be determined by the condensates of lowest dimension. In other words, we expect that only the condensates, containing the minimal powers of quark and gluon fields are important. The condensates of the lowest dimension are the vector and scalar condensates, determined by Eqs. (15) and (62) and also the gluon condensate — Eq.(34). As we saw in Subsect.2.7, the relative change of the gluon condensate in matter is much smaller than that of the quark scalar condensate. Thus we assume it to play a minor role. Hence, the in-medium values of \( \kappa(\rho) \) and \( v(\rho) \) will be most important for us, and we must solve the set of equations

\[ \begin{align*}
\kappa(\rho) &= \mathcal{K}(\rho, y_i) \\
y_i &= y_i(\kappa(\rho), \kappa(\rho)) .
\end{align*} \] (130)

Fortunately, the vector condensate \( v(\rho) \) is expressed by simple formulas (15) and (16) due to the baryon current conservation.

The idea of self-consistent treatment is not a new one. Indeed, Eqs. (2) and (3) provide an example of Eq.(130) for NJL model in vacuum, with the only parameter \( y_i = m \).

As to parameters \( y_i \), which are \( m^*, m_\Delta, f_\pi^* \), etc., there are several relations which are, to large extent, model-independent. Besides the in-medium GMOR relation — Eq.(70), we can present in-medium GT relation

\[ \bar{g}_{\pi NN} = \frac{g_\Delta}{2f_\pi} . \] (131)

Recalling that GT relation means, that the neutron beta decay can be viewed as the strong decay of neutron to \( \pi^- p^+ \) system followed by the decay of the pion, we see that Eq.(131) is true under the same assumption as Eq.(70). Namely, the pion should be much lighter than any other state with unnatural parity and zero baryon charge. Also the expectation value of the quark scalar operator averaged over pion is

\[ \Upsilon^* = \langle \pi^*|\bar{q}q|\pi^* \rangle = \frac{m_{\pi^*}^2}{m} . \] (132)

The other relations depend on the additional model assumptions. Starting with the ratio \( m^*(\rho)/m \) we find in the straightforward generalization of NJL model

\[ \frac{m^*(\rho)}{m} = \frac{\kappa(\rho)}{\kappa(0)} . \] (133)

This relation is referred to in the paper [36] as Nambu scaling. The QCD sum rules prompt a more complicated dependence, presented by Eq.(97) with the function

\[ F(\rho) = \frac{1}{1 + av(\rho)/\rho_0} \] (134)
where \( a \approx -0.2 \) \cite{24, 35, 43, 99} — see also Sec.5. Another assumption, expressed by Brown–Rho scaling equation (83) predicts a slower decrease of \( m^*(\rho) \). Note, that Eq.(85) is based on existence of a single length scale, while there are two at least: \( p_F^{-1} \) and \( \Lambda_{QCD}^{-1} \).

The experimental situation with \( \Delta \)-isobar mass in nuclear matter is not quite clear at the moment. The result on the total photon-nucleus cross section indicates that the mass \( m_\Delta^* \) does not decrease in the medium \cite{110}, while the nucleon mass \( m^*(\rho) \) diminishes with \( \rho \). On the other hand, the experimental data for total pion–nucleus cross sections are consistent with the mass \( m_\Delta^* \) decreasing in the matter \cite{111}. As to calculations, the description within the Skyrmion model \cite{93} predicts that \( m_\Delta^* \) decreases in nuclear matter and \( m_\Delta^* - m^* < m_\Delta - m \). Assuming the Additive Quark Model prediction for the scalar field-baryon couplings \( g_{sNN} = g_{s\Delta\Delta} \) we come to the equation \( m_\Delta^* - m^* = [m^*(m_\Delta - m)]/m \).

Now we present the results of the self-consistent calculations of the condensate under various assumptions on the dependence \( y_i(\kappa(\rho)) \) — Eq.(130).

In Fig.7 we show the results with BR1 scaling of the nucleon mass — Eq.(86) for different sets of FFST parameters. Following \cite{107} we try the values which were obtained at saturation density \( \rho = \rho_0 \) — set “a” defined in 4.4.1. We assume that they do not change with density. We use also another set of parameters \( \gamma_N = \gamma_{N\Delta} = \gamma_{\Delta\Delta} = 0.7 \) (set “b”), presented in \cite{75}. The dependence on the behaviour \( m_\Delta^*(\rho) \) appears to be more pronounced for set ”a” of the FFST parameters. The calculations were carried out under the assumption \( f^*_\pi = f_\pi \). Thus, the pion mass drops somewhat faster, than in BR1 scaling with decreasing \( f^*_\pi \), in order to save in-medium GMOR relation — Eq.(70). The nucleon effective mass at saturation density appears to be quenched somewhat less, than in QHD models, being closer to the value, preferred by FFST approaches \cite{70, 107}.

Assuming that the pion mass does not change in medium we find strong dependence on the values of FFST parameters. For the choice “a” the self-consistent solution disappears before the density reaches the saturation value— see dotted curve ”2” in Fig. 7. We explained in our paper \cite{103}, how it happens technically.

The results, obtained with Nambu scaling for the nucleon mass being assumed — Eq.(133), are shown in Fig.8. We put \( m_\pi^* = m_\pi \) and thus \( f^*_\pi \sim |\kappa(\rho)|^{1/2} \), following GMOR. The three curves illustrate the dependence of the results on the assumption of the in-medium behaviour of the isobar mass.

One of the results of this subsection is that the improved (self-consistent) approach excludes the possibility of the pion condensation at relatively small densities. On the other hand Dickhoff et al. \cite{104} carried out self-consistent description of the particle-hole interactions by inclusion of the induced interactions to all orders. This shifts the point of the pion condensation to the higher densities. The rigorous analysis should include both aspects of self-consistency.

### 4.4.4 Accumulation of isobars as a possible first phase transition

While the density increases, the Fermi momentum and the energy of the nucleon at the Fermi surface increase too. At some value of \( \rho \) it becomes energetically favourable to start the formation of the Fermi sea of the baryons of another sort instead of adding new nucleons. This phase transition takes
Figure 7: The quark scalar condensate $\kappa(\rho)$ and nucleon effective mass $m^*(\rho)$, calculated in framework of BR1 scaling of the nucleon mass. The dash-double-dotted line shows the gas approximation. The dotted lines 1 and 2 present the results under assumption $m^*_\pi(\rho) = m_\pi$ for the sets "b" and "a" of FFST parameters. The other curves present the results obtained under assumption $f^*_\pi(\rho) = f_\pi$ and illustrate dependence on the choice of the values of FFST parameters and on the assumed behaviour $m^*_\Delta(\rho)$. The solid and dashed curves are obtained for the set "a" of FFST parameters with the BR assumption $m^*_\Delta - m_\Delta = (m^* - m)\frac{m_\Delta}{m}$ and for $m^*_\Delta = m_\Delta$ correspondingly. The two other curves are obtained for the set "b" under BR scaling assumption for the isobar mass (dot-dashed curve) and under assumption that the isobar mass does not change in medium (long-dashed curve).
Figure 8: The dependence $\kappa(\rho)$ and $m^*(\rho)$ under the assumption of Nambu scaling of the nucleon mass and $m^*_\pi(\rho) = m_\pi$. The three curves illustrate the dependence on the behaviour of $m^*_\Delta(\rho)$. Solid line corresponds to BR scaling $m^*_\Delta - m_\Delta = (m^* - m)m^*_\Delta/m$, dashed curve to $m^*_\Delta - m_\Delta = m^* - m$ while the dashed-dotted curve is obtained for $m^*_\Delta = m_\Delta$. The calculations were carried out with the choice "b" of FFST parameters. Long-dashed line in Fig.8a shows the gas approximation.
place at the value $\rho_a$, determined by the condition

$$m_B^*(\rho_a) = \left(p_F^2 + m^2(\rho_a)\right)^{1/2} \quad (135)$$

with $p_F a$ being the value of Fermi momentum, corresponding to $\rho_a$, while $m_B^*$ is the mass of the second lightest baryon at $\rho = \rho_a$.

The vacuum values of $\Lambda$ and $\Sigma^+$ hyperon masses are respectively 115 MeV and 43 MeV smaller than that of $\Delta$-isobar. However, both experimental and theoretical data confirm that the hyperons interact with the scalar fields much weaker than the nucleons. Thus, at least in framework of certain assumptions on the behaviour of $m_\Delta^*(\rho)$, the hierarchy of the baryon masses changes in medium. (The investigations of the problem are devoted mostly to the case of neutron or strongly asymmetric matter because of the astrophysical applications. See, however, the paper of Pandharipande [112]). The delta isobar can become the second lightest baryon state. In this case accumulation of $\Delta$-isobars in the ground state is the first phase transition in nuclear matter. Such possibility was considered in several papers [113]–[116].

Under the assumption of BR1 scaling the accumulation of $\Delta$-isobar takes place at $\rho_a \approx 3\rho_0$, being the first phase transition. The value of $\rho_a$ is consistent with the result of Boguta [114].

## 5 QCD sum rules

### 5.1 QCD sum rules in vacuum

Here we review briefly the main ideas of the method. There are several detailed reviews on the subject — see, e.g., [117]. Here we focus on the points, which will be needed for the application of the approach to the case of nuclear matter.

The main idea is to establish a correspondence between descriptions of the function $G$, introduced in Subsec.3.6 in terms of hadronic and quark-gluon degrees of freedom. (Recall that $G$ describes the propagation of the system with the quantum numbers of the nucleon). The method is based on the fundamental feature of QCD, known as the asymptotic freedom. This means, that at $q^2 \to -\infty$ the function $G(q^2)$ can be presented as the power series of $q^{-2}$ and QCD coupling $\alpha_s$. The coefficients of the expansion are the expectation values of local operators constructed of quark and gluon fields, which are called "condensates". Thus such presentation, known as operator product expansion (OPE) [118], provides the perturbative expansion of short-distance effects, while all the nonperturbative physics is contained in the condensates.

The correspondence between the hadron and quark-gluon descriptions is based on Eq.(133). The empirical data are used for the spectral function $\text{Im} \ G(k^2)$ in the rhs of Eq.(133). Namely, we know, that the lowest lying state is the bound state of three quarks, which manifests itself as a pole in the (unknown) point $k^2 = m^2$. Assuming, that the next singularity is the branching point $k^2 = W^2_{ph} = (m + m_\pi)^2$, one can write exact presentation

$$\text{Im} \ G(k^2) = \tilde{\lambda}^2 \delta(k^2 - m^2) + f(k^2) \theta(k^2 - W^2_{ph}) \quad (136)$$

with $\tilde{\lambda}^2$ being the residue at the pole. Substituting rhs of Eq.(136) into Eq.(133) and employing $q^{-2}$
One finds certain connections between quark-gluon and hadron presentations

\[ G_{\text{OPE}}(q^2) = \frac{\tilde{\lambda}^2}{m^2 - q^2} + \frac{1}{\pi} \int_{W_{ph}^2}^{\infty} \frac{f(k^2)}{k^2 - q^2} \, dk^2. \]  

Of course, the detailed structure of the spectral density \( f(k^2) \) cannot be resolved in such approach. The further approximations can be prompted by asymptotic behaviour

\[ f(k^2) = \frac{1}{2i} \Delta G_{\text{OPE}}(k^2) \]

at \( k^2 \gg |q^2| \) with \( \Delta \) denoting the discontinuity. The discontinuity is caused by the logarithmic contributions of the perturbative OPE terms. The usual ansatz consist in extrapolation of Eq.(139) to the lower values of \( k^2 \), replacing also the physical threshold \( W_{ph}^2 \) by the unknown effective threshold \( W^2 \), i.e.

\[ \frac{1}{\pi} \int_{W_{ph}^2}^{\infty} \frac{f(k^2)}{k^2 - q^2} \, dk^2 = \frac{1}{2\pi i} \int_{W^2}^{\infty} \frac{\Delta G_{\text{OPE}}(k^2)}{k^2 - q^2} \, dk^2 \]  

and thus

\[ G_{\text{OPE}}(q^2) = \frac{\tilde{\lambda}^2}{m^2 - q^2} + \frac{1}{2\pi i} \int_{W^2}^{\infty} \frac{\Delta G_{\text{OPE}}(k^2)}{k^2 - q^2} \, dk^2. \]

The lhs of Eq.(141) contains QCD condensates. The rhs of Eq.(141) contains three unknown parameters: \( m, \tilde{\lambda}^2 \) and \( W^2 \). Of course, Eq.(141) makes sense only if the first term of the rhs, treated exactly is larger than the second term, treated approximately.

The approximation \( G(q^2) \approx G_{\text{OPE}}(q^2) \) becomes increasingly true while the value \( |q^2| \) increases. On the contrary, the ”pole+continuum” model in the rhs of Eq.(141) becomes more accurate while \( |q^2| \) decreases. The analytical dependence of the lhs and rhs of Eq.(141) on \( q^2 \) is quite different. The important assumption is that they are close in certain intermediate region of the values of \( q^2 \), being close also to the true function \( G(q^2) \).

To improve the overlap of the QCD and phenomenological descriptions, one usually applies the Borel transform, defined as

\[ Bf(Q^2) = \lim_{Q^2,n \to \infty} \frac{(Q^2)^{n+1}}{n!} \left( -\frac{d}{dQ^2} \right)^n f(Q^2) \equiv \tilde{f}(M^2) \]

with \( M \) called the Borel mass. There are several useful features of the Borel transform.

1. It removes the divergent terms in the lhs of Eqs. (138) and (141) which are caused by the free quark loops. This happens, since the Borel transform eliminates all the polynomials in \( q^2 \).
2. It emphasize the contribution of the lowest lying states in rhs of Eq. (141) due to the relation

\[ B \left[ \frac{1}{Q^2 + m^2} \right] = e^{-m^2/M^2}. \]  

(143)

3. It improves the OPE series, since

\[ B \left[ (Q^2)^{-n} \right] = \frac{1}{(n - 1)!} \left( M^2 \right)^{1-n}. \]  

(144)

Applying Borel transform to both sides of Eq. (141) one finds

\[ \tilde{G}_{OPE}(M^2) = \lambda^2 e^{-m^2/M^2} + \frac{1}{2\pi i} \int_{W_2} dk^2 e^{-k^2/M^2} \cdot \Delta G_{OPE}(k^2). \]  

(145)

Such relations are known as QCD sum rules. If both rhs and lhs of Eq. (141) were calculated exactly, the relation would be independent on \( M^2 \). However, certain approximations are made in both sides. The basic assumption is that there exists a range of \( M^2 \) for which the two sides have a good overlap, approximating also the true function \( \tilde{G}(M^2) \).

The lhs of Eq. (145) can be obtained by presenting the function \( G(q^2) \), which is often called "correlation function" or "correlator" as (strictly speaking, \( G \) depends on the components of vector \( q \) also through the trivial term \( \hat{q} \))

\[ G(q^2) = i \int d^4x e^{i(q \cdot x)} \langle 0|T\{\eta(x)\bar{\eta}(0)\}|0 \rangle \]  

(146)

with \( \eta \) being the local operator with the proton quantum numbers. It was shown in [119] that there are three independent operators \( \eta \)

\[ \eta_1 = \left( u^T_a C \gamma_\mu u_b \right) \gamma_5 \gamma_\mu d_c \cdot \varepsilon^{abc}, \quad \eta_2 = \left( u^T_a C \sigma_{\mu\nu} u_b \right) \sigma^{\mu\nu} \gamma_5 d_c \varepsilon^{abc}, \]

\[ \eta_{3\mu} = \left[ (u^T_a C \gamma_\mu u_b) \gamma_5 d_c - (u^T_a C \gamma_\mu d_b) \gamma_5 u_c \right] \varepsilon^{abc}, \]  

(147)

where \( T \) denotes the transpose in Dirac space and \( C \) is the charge conjugation matrix. However, the operator \( \eta_2 \) provides strong admixture of the states with negative parity [119]. As to the operator \( \eta_3 \), it provides large contribution of the states with spin 3/2 [119]. Thus, the calculations with \( \eta = \eta_1 \) are most convincing. We shall assume \( \eta = \eta_1 \) in the further analysis.

The correlation function has the form

\[ G(q) = G_q(q^2) \cdot \hat{q} + G_s(q^2) \cdot I \]  

(148)

with \( I \) standing for the unit \( 4 \times 4 \) matrix. The leading OPE contribution to \( G_q \) comes from the loop with three free quarks. If the quark masses \( m_u,d \) are neglected, the leading OPE term in \( G_s \) comes from the exchange by the quarks between the system described by operator \( \eta \) and vacuum. Technically this means, that the contribution comes from the second term of the quark propagator in vacuum

\[ \langle 0|Tq_\alpha(x)\bar{q}_\beta(0)|0 \rangle = \frac{i}{2\pi^2} \frac{\hat{x}_{\alpha\beta}}{x^4} - \frac{1}{4} \sum_A \Gamma^A_{\alpha\beta}(0|q\Gamma^Aq|0) + 0(x^2), \]  

(149)
Figure 9: The Feynman diagrams, describing the lowest order OPE contribution to the nucleon correlator in vacuum. The helix line stands for the system with the quantum numbers of proton. The solid lines denote the quarks. The light circles denote the vacuum expectation value.

where only the contribution with $A = 1$ (see Eq.(13)) survives. This is illustrated by Fig.9. The higher order terms come from exchange by soft gluons between vacuum and free quarks carrying hard momenta. Next comes the four-quark condensate which can be viewed as the expansion of the two-quark propagator, similar to Eq.(149).

Direct calculation provides for massless quarks [98]

$$G^{OPE}_q = -\frac{1}{64\pi^4} (q^2)^2 \ln(-q^2) - \frac{1}{32\pi^2} \ln(-q^2) g(0) - \frac{2}{3q^2} h_0,$$

$$G^{OPE}_s = \left( \frac{1}{8\pi^2} q^2 \ln(-q^2) - \frac{1}{48q^2} g(0) \right) \kappa(0)$$

with the condensates $g(0) = \langle 0 | \bar{q} q | 0 \rangle$, $\kappa(0) = \langle 0 | \bar{q} q | 0 \rangle$ and $h_0 = \langle 0 | \bar{u} u \bar{u} u | 0 \rangle$. The terms, containing polynomials of $q^2$ are omitted, since they will be eliminated by the Borel transform. This leads to the sum rules [98]

$$M^6 E_2 \left( \frac{W^2}{M^2} \right) + \frac{1}{4} b M^2 E_0 \left( \frac{W^2}{M^2} \right) + \frac{4}{3} C_0 = \lambda^2 e^{-m^2/M^2},$$

$$2a \left( M^4 E_1 \left( \frac{W^2}{M^2} \right) - \frac{b}{24} \right) = m\lambda^2 e^{-m^2/M^2}$$

with traditional notations $a = -2\pi^2\kappa(0)$, $b = (2\pi)^2 g(0)$, $\lambda^2 = 32\pi^4 \tilde{\lambda}^2$,

$$E_0(x) = 1 - e^{-x}, \quad E_1(x) = 1 - (1 + x)e^{-x}, \quad E_2(x) = 1 - \left( \frac{x^2}{2} + x + 1 \right) e^{-x}.$$  

Also $C_0 = (2\pi)^4 h_0$. Here we omitted the anomalous dimensions, which account for the most important corrections of the order $\alpha_s$, enhanced by the "large logarithms". The radiative corrections were shown to provide smaller contributions, as well as the higher order power corrections [98]. The matching of the lhs and rhs of Eqs.(152),(153) was found in [98] for the domain

$$0.8 \text{ GeV}^2 < M^2 < 1.4 \text{ GeV}^2.$$  

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As one can see from Eq.(153), the nucleon mass turns to zero if \( \langle 0 | \bar{q} q | 0 \rangle = 0 \). Hence, the mass is determined by the exchange by quarks between the our system and vacuum.

The method was applied successfully to calculation of the static characteristics of the nucleons reproducing the values of its mass [39, 97, 98] as well as of magnetic moment [98] and of the axial coupling constant [121]. The proton structure functions also were analysed in framework of the approach [122].

## 5.2 Proton dynamics in nuclear matter

Now we extent the sum-rule approach to the investigation of the characteristics of the proton in nuclear matter. The extension is not straightforward. This is mostly because the spectrum of correlation function in medium

\[
G^m(q) = i \int d^4x e^{i(qx)} \langle M | T \{ \eta(x) \bar{\eta}(0) \} | M \rangle
\]  

is much more complicated, than that of the vacuum correlator \( G(q^2) \). The singularities of the correlator can be connected with the proton placed into the matter, as well as with the matter itself. One of the problems is to find the proper variables, which would enable us to focus on the properties of our probe proton.

### 5.2.1 Choice of the variables

Searching for the analogy in the earlier investigations one can find two different approaches. Basing on the analogy with the QCD sum rules in vacuum, one should build the dispersion relation in the variable \( q^2 \). The physical meaning of the shift of the position of the proton pole is expressed by Eq.(96). Another analogy is the Lehmann representation [120], which is dispersion relation for the nucleon propagator in medium \( g_N(q_0, |q|) \) in the time component \( q_0 \). Such dispersion relation would contain all possible excited states of the matter in rhs. Thus, we expect the dispersion relations in \( q^2 \) to be a more reasonable choice in our case.

It is instructive to adduce the propagation of the photon with the energy \( \omega \) and three dimensional momentum \( k \) in medium. The vacuum propagator is \( D_\gamma \sim [\omega^2 - k^2]^{-1} \). Being considered as the function of \( q^2 = \omega^2 - k^2 \) it has a pole at \( q^2 = 0 \). The propagator in medium is \( D^m_\gamma \sim [\omega^2 \varepsilon(\omega) - k^2]^{-1} \).

The dielectric function \( \varepsilon(\omega) \) depends on the structure of the matter, making \( D^m_\gamma(\omega) \) a complicated function. However, the function \( D^m_\gamma(q^2) \) still has a simple pole, shifted to the value \( q^2_m = \omega^2(1 - \varepsilon(\omega)) \). A straightforward calculation of the new value \( q^2_m \) is a complicated problem. The same refers to the proton in-medium. The sum rules are expected to provide the value in some indirect way.

Thus, we try to build the dispersion relations in \( q^2 \). Since the Lorentz invariance is lost, the correlator \( G^m(q) \) depends on two variables. Considering the matter as the system of \( A \) nucleons with momenta \( p_i \), introduce vector

\[
p = \frac{\Sigma p_i}{A},
\]

which is thus \( p \approx (m, \mathbf{0}) \) in the rest frame of the matter. The correlator can be presented as \( G^m(q) = G^m(q^2, \varphi(p, q)) \) with the arbitrary choice of the function \( \varphi(p, q) \), which is kept constant in
the dispersion relations. This is rather formal statement, and there should be physical reasons for
the choice.
To make the proper choice of the function \( \varphi(p,q) \), let us consider the matrix element, which enters
Eq. (155)
\[
\langle M | T \{ \eta(x) \bar{\eta}(0) \} | M \rangle = \langle M_A | \eta(x) | M_{A+1} \rangle \langle M_{A+1} | \bar{\eta}(0) | M_A \rangle \theta(x_0) - \\
- \langle M_A | \bar{\eta}(0) | M_{A-1} \rangle \langle M_{A-1} | \eta(x) | M_A \rangle \theta(-x_0)
\]  
(157)
with \( |M_A\rangle \) standing for the ground state of the matter, while \( |M_{A\pm1}\rangle \) are the systems with baryon
numbers \( A \pm 1 \). The summation over these states is implied. The matrix element \( \langle M_{A+1} | \eta | M_A \rangle \)
contains the term \( \langle N | \eta | 0 \rangle \) which adds the nucleon to the Fermi surface of the state \( |M_A\rangle \). If the
interactions between this nucleon and the other ones are neglected, it is just the pole at \( q^2 = m^2 \).
Now we include the interactions. The amplitudes of the nucleon interactions with the nucleons of
the matter are known to have singularities in variables \( s_i = (p_i + q)^2 \). These singularities correspond
to excitation of two nucleons in the state \( |M_{A+1}\rangle \). Thus, they are connected with the properties of
the matter itself. To avoid these singularities we fix
\[
\varphi(p,q) = s = \max s_i = 4E_{0F}^2
\]  
(158)
with \( E_{0F} \) being the relativistic value of nucleon energy at the Fermi surface. Neglecting the terms of
the order \( p_F^2/m^2 \) we can assume \( p_i = (m,0) \) and thus
\[
s = 4m^2.
\]  
(159)
Our choice of the value of \( s \) corresponds to \( |\bar{q}| = p_F \) (in the simplified case, expressed by Eq.(159)
\(|\bar{q}| = 0 \)). However, varying the value of \( s \) we can find the position of the nucleon poles, corresponding
to other values of \( |\bar{q}| \).
Let us look at what happens to the nucleon pole \( q^2 = m^2 \) after we included the interactions with
the matter. The self-energy insertions \( \Sigma \) modify the free nucleon propagator \( g_N^0 \) to \( g_N \) with
\[
(g_N)^{-1} = (g_N^0)^{-1} - \Sigma
\]  
(160)
— see Fig.10.

In the mean field approximation (Fig.10a) the function \( \Sigma \) does not contain additional intermediate
states. It does not cause additional singularities in the correlator \( G^m(q^2,s) \). The position of the pole
is just shifted by the value, which does not depend on \( s \). (Note that this does not mean that in the
mean field approximation the condition \( s = \text{const} \) can be dropped. Some other contributions to
the matrix element \( \langle M_{A+1} | \bar{\eta} | M_A \rangle \) are singular in \( s \). Say, there is the term \( \langle B | \bar{\eta} | 0 \rangle \) with \( B \) standing for
the system, containing the nucleon and mesons. If the mesons are absorbed by the state \( |M_A\rangle \), we
come to the box diagram (Fig.11) with the branching point, starting at \( s = 4m^2 \).

Leaving the framework of the mean-field approximation we find Hartree self-energy diagrams
(Fig.10b) depending on \( s \). The latter is kept constant in our approach. Hence, no additional singularities emerge in this case as well.

The situation becomes more complicated if we take into account the Fock (exchange) diagrams
(Fig.10c). The self-energy insertions depend on the variable \( u = (p - q)^2 \). The contribution of these
Figure 10: Self-energy insertions to the nucleon pole contribution to the correlator. The solid lines denote the nucleons, the helix line stands for the correlator. Fig.10a corresponds to mean-field approximation. Fig.10b shows the self-energy in the direct channel. Fig.10c presents the exchange contribution.

Figure 11: One of the contributions, providing the branching point at $s = 4m^2$. The bold solid line denotes the nucleon of the matter. The dashed lines stands for the meson systems.
terms shifts the nucleon pole and gives birth to additional singularities corresponding to real states with baryon number equal to zero. They are the poles at the points \( u = m_x^2 \), with \( m_x \) denoting the masses of the mesons (\( \pi \), \( \omega \), etc.), and the cuts running to the right from the point \( q^2 = m^2 + 2m_\pi^2 \). The latter value is the position of the branching point corresponding to the real two-pion state in the \( u \)-channel.

Thus the single-nucleon states \( |B_{\pm 1}\rangle \) cause the pole \( q^2 = m_m^2 \), a set of poles corresponding to the states with baryon number \( B = 0 \) and a set of branching points. The lowest-lying one is \( q^2 = m^2 + 2m_\pi^2 \). Note that the antinucleon corresponding to \( q_0 = -m \) generates the pole \( q^2 = 5m^2 \) shifted far to the right from the lowest-lying one.

The lowest-lying branching point \( q^2 = m^2 + 2m_\pi^2 \) is separated from the position of the pole \( q^2 = m^2 \) by a much smaller distance than in the case of the vacuum \( (q^2 = m^2 + 2mm_\pi \) in the latter case). Note, however, that at the very threshold the discounting is quenched since the vertices contain moments of the intermediate pions. Thus the branching points can be considered as a separated from the pole \( q^2 = m^2 \). Note also that for the same reason the residue at the pole \( q^2 = m^2 + 2m_\pi^2 \) in the \( u \)-channel vanishes.

As it was shown in the work \([43]\), all the other singularities of the correlator \( G^m(q^2, s) \) in \( q^2 \) are lying to the right from the nucleon pole until we include the three-nucleon terms. Thus, they are accounted for by the continuum and suppressed by Borel transform. To prove the dispersion relation we must be sure of the possibility of contour integration in the complex \( q^2 \) plane. This cannot be done on an axiomatic level. However a strong argument in support of the possibility is the analytical continuation from the region of large real \( q^2 \). At these values of \( q^2 \) the asymptotic freedom of QCD enables one to find an explicit expression of the integrand. The integral over large circle gives a non-vanishing contribution. However, the latter contains only the finite polynomials in \( q^2 \) which are eliminated by the Borel transform.

Thus we expect ”pole+continuum” model to be valid for the spectrum of the correlator \( G^m(q^2, s) \).

The situation becomes more complicated if we include the three-nucleon interactions \([99]\). The probe proton, created by the operator \( \eta \) can interact with \( n \) nucleons of the matter. The corresponding amplitudes depend on the variable \( s_n = (np+q)^2 \). For \( n \geq 2 \) this causes the cuts, running to the left from the point \( q^2 = m^2 \). This requires somewhat more complicated model of the spectrum. From the point of view of expansion in powers of \( \rho \) this means, that ”pole + continuum” model is legitimate until the terms of the order \( \rho^2 \) are included.

5.2.2 Operator expansion

Following our general strategy, we shall try to obtain the leading terms of expansion of the correlator

\[
G^m(q) = G^m_q(q)\hat{q} + G^m_p(q)\hat{p} + G^m_s(q)I ,
\]

in powers of \( q^{-2} \). Note, that the condition \( s = \text{const} \), which we needed for separation of the singularities, connected with our probe proton, provides

\[
\frac{(pq)}{q^2} \rightarrow \text{const}
\]
at \( q^2 \rightarrow -\infty \). This is just the condition which insures the operator expansion in deep-inelastic scattering (see, e.g., the book of Ioffe et al. [123]). It is not necessary in our case. However, the physical meaning of some of the condensates, say, that of \( \langle \varphi_\alpha(\alpha)\rangle \) — Eq.(51) becomes most transparent in this very kinematics.

The problem is more complicated than in vacuum, since each of the terms of the expansion in powers of \( q^2 \) provides, generally speaking, infinite number of the condensates. Present each of the components \( G^m_i \ (i=q,p,s) \) of the correlator \( G^m \)

\[
G^m_i = \int d^4xe^{i(qx)}T_i(x) .
\]

(163)

The function \( T_i(x) \) contains in-medium expectation values of the products of QCD operators in space-time points "\( 0 \)" and "\( \hat{x} \)" with an operator at the point \( x \) defined by Eq.(164). Each in-medium expectation value, containing covariant derivative \( D_\mu \), is proportional to the vector \( p_\mu \). This can be easily generalized for the case of the larger number of derivatives. Thus the correlators take the form \( G^m_i = \sum_n C_n i(p\nabla q)^n f_i(q^2) \). For the contributions \( f_i(q^2) \sim (q^2)^{-k} \) the terms \( (p\nabla q)^n f_i(q^2) \) are of the same order. This is the "price" for the choice of kinematics \( s = \text{const} \). Fortunately, the leading terms of the operator expansion contain the logarithmic loops and thus can be expressed through the finite number of the condensates [99].

The leading terms of the operator expansion can be obtained by replacing the free quark propagators by those in medium

\[
\langle M|T\psi_\alpha(x)\bar{\psi}_\beta(0)|M\rangle = \frac{i}{2\pi^2} \frac{\hat{x}}{x^4} - \sum_A \frac{1}{4} \Gamma^A_{\alpha\beta} \langle M|\bar{\psi}(0)\Gamma^A\psi(x)|M\rangle
\]

(164)

with the matrices \( \Gamma^A \) being defined by Eq.(13). Operator \( \psi(x) \) is defined by Eq.(13). While looking for the lowest order term of the operator expansion we can put \( x^2 = 0 \) in the second term of the rhs of Eq.(164). In the sum over \( A \) the contributions with \( A = 3, 4 \) vanish due to the parity conservation by strong interactions, the one with \( A = 5 \) turns to zero in any uniform system. Thus, only the terms with \( A = 1, 2 \) survive. Looking for the lowest order density effects, we assume that propagation of one of the quarks of the correlator \( G^m \) is influenced by the medium. Hence, the term with \( A = 1 \) contributes to the scalar structure \( G^m_s \), while that with \( A = 2 \) — to the vector structures \( G^m_q \) and \( G^m_p \).

\[
G^m_s = \frac{1}{2\pi^2} q^2 \ln(-q^2) \kappa(\rho)
\]

(165)

\[
G^m_q = -\frac{1}{64\pi^4} (q^2)^2 \ln(-q^2) + \frac{1}{6\pi^2} (s - m^2 - q^2) \ln(-q^2) v(\rho)
\]

(166)

\[
G^m_p = \frac{2}{3\pi^2} q^2 \ln(-q^2) \nu(\rho)
\]

(167)

Thus the correlator \( G^m_s \) can be just obtained from the vacuum correlator \( G_s \) by replacing of \( \kappa(0) \) by \( \kappa(\rho) \). The correlator \( G^m_q \) obtains additional contribution proportional to the vector condensate \( v(\rho) \). Also, the correlator \( G^m_p \), which vanishes in vacuum is proportional to \( v(\rho) \). These terms are illustrated by Fig.12 a,b.

Turn now to the next OPE terms. Start with the structure \( G^m_s \). In the case of vacuum there is a contribution which behaves as \( \ln(-q^2) \), which is proportional to the condensate \( \langle 0|\bar{\psi} \frac{\alpha_s}{\pi} G^a_{\mu\nu} \sigma^{\mu\nu} \lambda^a_s |0\rangle \).
However, similar term comes from expansion of expectation value $\langle 0 | \bar{q}(0) q(x) | 0 \rangle$ in powers of $x^2$. The two terms cancel \[^{35}\]. Similar cancellation takes place in medium \[^{99}\]. However there is a contribution, caused by the second term of rhs of Eq.(45). It does not vanish identically, but it can be neglected due to Eq.(54). Hence, the next OPE term in rhs of Eq.(165) can be obtained by simple replacement of the condensates $\kappa(0)$ and $g(0)$ in the second term of Eq.(151) by $\kappa(\rho)$ and $g(\rho)$.

The next-to-leading order corrections to the correlators $G_{q,p}^m$ come from expansion of the expectation value $\langle M | \bar{\psi}(0) \gamma_0 \psi(x) | M \rangle$. In the lowest order of $x^2$ expansion, the matrix element can be presented through the moments of the deep-inelastic scattering (DIS) nucleon structure functions – Eqs. \[^{49}\] and \[^{50}\]. Since the medium effects in DIS are known to be small, we limit ourselves to the gas approximation at this point. The main contributions to $q^{-2}$ expansion compose the series of the terms $[(s-m^2)/q^2]^n \langle \alpha^n \rangle$ with $\langle \alpha^n \rangle$ denoting $n$-th moment of the structure function. Being expressed in a closed form, they change Eqs. \[^{166}\] and \[^{167}\] to

$$
G_q^m = - \frac{1}{64\pi^4} (q^2)^2 \ln(-q^2) + \frac{1}{12\pi^2} \frac{(s-m^2-q^2)^2}{m} \int_0^1 d\alpha F(\alpha) \ln(q-p\alpha)^2 \cdot \rho \\
+ \frac{m}{3\pi^2} \int_0^1 d\alpha \phi_b(\alpha) \ln(q-p\alpha)^2 \cdot \rho
$$

Figure 12: The Feynman diagrams, contributing to the leading terms of OPE of the nucleon correlator in medium. The dark blob denotes in-medium expectation values. The dotted lines stand for gluons. The others notations coincide with those of Fig.9.
\[ G_p^m = \frac{2}{3\pi^2} q^2 \int_0^1 d\alpha F(\alpha) \ln(q - p\alpha)^2 \cdot \rho \] (169)

with \( F(\alpha) \) being structure function, normalized as \( \int_0^1 d\alpha F(\alpha) = 3 \). The function \( F(\alpha) \) can be presented also as \( F(\alpha) = \phi_a(\alpha) \) with \( \phi_a \) defined by Eq.(50). Another leading OPE term is caused by modification of the value of gluon condensate. This is expressed by changing of the value \( g(0) \) in the second term in rhs of Eq.(150) to \( g(\rho) \) – Fig.12b.

The higher order OPE terms lead to the contributions which decrease as \( q^{-2} \). One of them is caused by the lowest order power correction to the first moment of the structure function. This term is expressed through the factor \( \xi \) which is determined by Eq.(51), being calculated in [42]. The other corrections of this order come from the four-quark condensates \( Q_{AB} \), defined by Eq.(57). The correlator \( G_m^s \) contains the condensate \( Q_{12}^s \). The vector part includes the condensates \( Q_{11}^p \) and \( Q_{22}^p \).

Another contribution of this order comes from the replacement of the condensates \( g(0) \) and \( \kappa(0) \) in the last term in rhs of Eq.(151) by their in-medium values – Fig.12c,d.

### 5.2.3 Building up the sum rules

To construct the rhs of the sum rules, consider the nucleon propagator \( g_N = (H - E)^{-1} \) with \( E \) standing for the nucleon energy, while the Hamiltonian \( H \) in the mean field approximation is presented by Eq.(153). Beyond the mean field approximation the potentials \( V_\mu \) and \( \Phi \) should be replaced by vector and scalar self-energies

\[ V_\mu = \Sigma^V_\mu \; ; \; \Sigma^V_\mu = p_\mu \Sigma^p + q_\mu \Sigma^q \; ; \; \Phi = \Sigma^s . \] (170)

Thus, under condition \( s = 4m^2 \) — Eq.(159)

\[ g_N = Z \frac{\hat{q}(1 - \Sigma^q) - \hat{p}\Sigma^p + m + \Sigma^s}{q^2 - m_m^2} \] (171)

with

\[ m_m = m + U \; ; \; U = m(\Sigma^q + \Sigma^p) + \Sigma^s , \] (172)

while

\[ Z = \frac{1}{(1 - \Sigma^q)(1 - \Sigma^q + \Sigma^p)} . \] (173)

Of course, \( \Sigma^q = 0 \) in mean field approximation.

The Borel-transformed sum rules for in-medium correlators in the assumed "pole+continuum" model for the spectrum are:

\[ \mathcal{L}_q^m(M^2) = \lambda_m^2 e^{-m_m^2/M^2} (1 - \Sigma^q) \] (174)
\[ \mathcal{L}_p^m(M^2) = -\lambda_m^2 e^{-m_m^2/M^2} \Sigma^p \] (175)
\[ \mathcal{L}_s^m(M^2) = \lambda_m^2 e^{-m_m^2/M^2} (m + \Sigma^s) \] (176)
with \( \lambda_m^2 = 32\pi^4 \bar{\lambda}_m^2 Z \) with \( \bar{\lambda}_m^2 \) standing for the residue in the nucleon pole (see similar definition in vacuum — Eqs.(152) and (153)). The lhs of Eqs. (174)–(176) are:

\[
\mathcal{L}_q^m(M^2) = M^6 E_2 \left( \frac{W_m^2}{M^2} \right) L^{-4/9} - \frac{8\pi^2}{3} \left[ (s - m^2)M^2 E_0 \left( \frac{W_m^2}{M^2} \right) - M^4 E_1 \left( \frac{W_m^2}{M^2} \right) \right] \times \langle F \mu(\alpha) \rangle - 2m^2 M^2 E_0 \left( \frac{W_m^2}{M^2} \right) \langle F \mu(\alpha) \alpha \rangle + 4m^2 M^2 E_0 \left( \frac{W_m^2}{M^2} \right) \langle \phi_b \mu(\alpha) \rangle \right] \rho L^{-4/9} + \pi^2 M^2 E_0 \left( \frac{W_m^2}{M^2} \right) g(\rho) + \frac{3}{4} m^2 (s - m^2) \langle \theta_a \mu \rangle + 2m^4 \langle \theta_b \mu \rangle + \frac{4}{3} (2\pi)^4 Q_{uu}^1(\rho) L^\frac{1}{2} \tag{177}
\]

\[
\mathcal{L}_p^m(M^2) = -\frac{8\pi^2}{3} M^4 E_1 \left( \frac{W_m^2}{M^2} \right) \langle F \mu(\alpha) \rangle \rho L^{-4/9} - (2\pi)^4 Q_{uu}^{22} \tag{178}
\]

\[
\mathcal{L}_s^m(M^2) = (2\pi)^2 \left[ M^4 E_1 \left( \frac{W_m^2}{M^2} \right) - \frac{(2\pi)^2}{12} g(\rho) \right] \kappa(\rho) + \frac{8(2\pi)^3}{3} Q_{ud}^{12}(\rho) \tag{179}
\]

In Eqs. (177)–(179) \( \langle \psi \rangle = \int_0^1 d\alpha \psi(\alpha) \) for any function \( \psi \), \( F \) is the structure function, the function \( \mu(\alpha) = \exp \left( \frac{-(s-m^2)\alpha + m^2 \alpha^2}{M^2(1+\alpha)} \right) \) (180) takes into account the terms \[(s-m^2)/M^2]^n\). The factor

\[
L = \frac{\ln M^2/\Lambda^2}{\ln \nu^2/\Lambda^2} \tag{181}
\]

accounts for the anomalous dimensions. Here \( \Lambda = 0.15 \) GeV is the QCD parameter while \( \nu = 0.5 \) GeV is the normalization point of the characteristics involved.

Recall, that "pole+continuum" model is true until we do not touch the terms of the order \( \rho^2 \). Thus, in the sum rules for the difference between in-medium and vacuum correlators we must limit ourselves to linear shifts of the parameters

\[
\Delta \mathcal{L}_q^m(M^2) = \lambda^2 e^{-m^2/M^2} \left( \frac{\Delta \lambda^2}{\lambda^2} - \Sigma^q - \frac{2m \Delta m}{M^2} \right) - \frac{W^4}{2L^{4/9}} \exp \left( -\frac{W^2}{M^2} \right) \Delta W^2 \tag{182}
\]

\[
\mathcal{L}_p^m(M^2) = -\lambda^2 e^{-m^2/M^2} \Sigma^p \tag{183}
\]

\[
\Delta \mathcal{L}_s^m(M^2) = \lambda^2 e^{-m^2/M^2} \left( m \frac{\Delta \lambda^2}{\lambda^2} + \Sigma^s - \frac{2m^2 \Delta m}{M^2} \right) - 2aW^2 \exp \left( -\frac{W^2}{M^2} \right) \Delta W^2 . \tag{184}
\]

Here \( \Delta \) denotes the difference between in-medium and vacuum values. However the self-energy \( \Sigma^q \) and \( \Sigma^s \) cannot be determined separately, since only the sum \( \Sigma^q + \Sigma^s \) can be extracted.

Anyway, the shift of the position of the nucleon pole \( m_m - m \) can be obtained: using Eq.(172) we find

\[
\Delta \mathcal{L}_s^m - m \mathcal{L}_p^m - m \mathcal{L}_q^m = \Delta m \lambda^2 e^{-m^2/M^2} + W^2 e^{-m^2/M^2} \left( \frac{W^2}{2L^{4/9}} m - 2a \right) \Delta W^2 . \tag{185}
\]
Since the value \((W^2/2L^{4/9})m - 2a\) is numerically small, one can write approximate sum rule, neglecting the second term in rhs of Eq.(185)

\[
U = \frac{e^{m^2/M^2}}{\lambda^2} \left( \Delta \mathcal{L}_s^m - mL_p^m - m\Delta L_q^m \right), \tag{186}
\]

or, assuming the sum rules in vacuum to be perfect

\[
U = \frac{e^{m^2/M^2}}{\lambda^2} \left( \mathcal{L}_s - mL_p - m\mathcal{L}_q \right) \tag{187}
\]

with the vacuum part cancelling exactly.

The two lowest order OPE terms (without perturbative expansion in parameter \(\frac{m^2}{M^2}\)) are presented by the first two terms of rhs of Eqs. (177), (179) and by the first term of rhs of Eq.(178). They are expressed through the condensates \(v(\rho), \kappa(\rho)\) and \(g(\rho)\) and through the moments of the nucleon functions \(\phi_{a,b}\) introduced in Subsec. 2.6. The values of the lowest moments of the structure function \(F(\alpha) = \phi_a(\alpha)\) are well known from experimental data. By using the value of \(\xi_a\) and employing relations, presented by Eq.(53) one can find the lowest moments of the function \(\phi_b\). Only the first moment of the function \(\phi_b\) and thus the first and second moments of the function \(\phi_a\) appeared to be numerically important. Thus, at least in the gas approximation all the contributions to the lhs of the sum rules can be either calculated in the model-independent way or related to the observables [43]. The scalar condensate is the most important parameter beyond the gas approximation [20]. The model calculations have been carried out in this case.

The next order of OPE includes explicitly the moments of the functions \(\theta_{a,b}\) defined in Subsec.2.6. It includes also the four-quark condensates \(Q_{11}, Q_{12}\) and \(Q_{22}\). Using Eqs.(53) one can find that only the first moment of the function \(\theta_a\) is numerically important while the moments of the function \(\theta_b\) can be neglected. The condensate \(Q_{12}\) can be obtained easily by using Eq.(58). The uncertainties of the values of the other four-quark condensates \(Q_{11}\) is the main obstacle for decisive quantitative predictions, based on Eqs. (182)–(187). The scalar four-quark condensate \(Q_{11}\) may appear to be a challenge for the convergence of OPE due to the large value of the second term in rhs of Eq.(59). This may be a signal that large numbers are involved. Fortunately, the only calculation of \(Q_{11}\) carried out in [46] demonstrated that there is a large cancellation between the model-dependent first term in rhs of Eq.(59) and the second one, which is to large extent model-independent. However, assuming the result presented in [46], we still find this contribution to be numerically important.

We can try (at least for illustrative reasons) to get rid of this term in two ways. One of them is to ignore its contribution. The reason is that it corresponds to exchange by a quark system with the quantum numbers of a scalar channel between our probe proton and the matter. On the other hand, it contributes to the vector structure of the correlator \(\hat{G}_q^m\), and thus to the vector structure \(\hat{q}\) of the propagator of the nucleon with the momentum \(q\). Such terms are not forbidden by any physical law. However most of QHD calculations are successful without such contributions. Thus the appearance of the terms with such structure, having a noticeable magnitude is unlikely. (Of course, this is not a physical argument, but rather an excuse for trying this version). The other possibility is to eliminate the contribution by calculation of the derivative with respect to \(M^2\). The two ways provide relatively close results.
5.2.4 The structure of the potential energy

Under the conditions, described above, we find that the rhs of Eq. (186) is a slowly varying function of \( M^2 \) in the interval, defined by Eq. (154). Among the moments of the structure function the two first ones appeared to be numerically important. Thus we find

\[
U(\rho) = \left[ 66 \, v(\rho) + 70 \, v^2(\rho) - \frac{10 \, \Delta g(\rho)}{m} - 32 \, \Delta \kappa(\rho) \right] \text{GeV}^{-2}.
\]

(188)

Here \( v(\rho) = 3 \rho \) is the vector condensate — Eq. (16), \( \Delta \kappa(\rho) = \kappa(\rho) - \kappa(0) \) is the in-medium change of the scalar condensate — Eq. (62). The condensate \( v^2(\rho) \), determined as

\[
\langle M | \bar{\psi} \gamma_\mu D_\nu \psi | M \rangle = \left( g_{\mu\nu} - \frac{4 p_\mu p_\nu}{p^2} \right) v^2(\rho)
\]

is connected to the second moment of the nucleon structure function. Numerically \( v^2(\rho) \approx 0.3 \rho \).

Finally, \( \Delta g(\rho) \) is the shift of the gluon condensate, expressed by Eq. (39).

Thus the problem of presenting the nucleon potential energy through in-medium condensates is solved. At the saturation value \( \rho = \rho_0 \) we find \( U = -36 \text{ MeV} \) in the gas approximation. This should be considered as a satisfactory result for such a rough model. This is increasingly true, since there is a compensation of large positive and negative values in rhs of Eq. (186).

Note that the simplest account of nonlinear terms signals on the possible saturation mechanism. Following the discussion of Subsection 2.7 and assuming the chiral limit, present \( \Delta \kappa(\rho) = \Sigma / \hat{m} - 3.2 (p_F / p_{F0}) \rho \). Thus we obtain the potential

\[
U(\rho) = \left[ \left( 198 - 42 \, \frac{\Sigma}{\hat{m}} \right) \frac{\rho}{\rho_0} + 133 \left( \frac{\rho}{\rho_0} \right)^{4/3} \right] \text{MeV}.
\]

(189)

After adding the kinetic energy it provides the minimum of the functional \( \mathcal{E}(\rho) \) defined by Eq. (73) at \( \Sigma = 62.8 \text{ MeV} \), which is consistent with experimental data — Eq. (25). The binding energy is \( \mathcal{E} = -9 \text{ MeV} \). The incompressibility coefficient \( K = 9 \rho_0 (d^2 \epsilon / dp^2) \) also has a reasonable value \( K = 182 \text{ MeV} \).

Of course, the results for the saturation should not be taken too seriously. As we have seen in Sect. 3, the structure of the nonlinear terms of the condensate is much more complicated. Also, the result is very sensitive to the exact value of \( \Sigma \)-term. Say, assuming it to be larger by the magnitude of 2 MeV, we find the Fermi momentum at the saturation point about \( 1/3 \) larger than \( p_{F0} \). Thus, the value of the saturation density becomes about 2.5 times larger than \( \rho_0 \). Such sharp dependence is caused by the form of the nonlinear term in the potential energy equation — Eq. (189). The form of the term is due to oversimplified treatment of nonlinear effects. However the result can be the sign, that further development of the approach may appear to be fruitful.

5.2.5 Relation to conventional models and new points

We obtained a simple mechanism of formation of the potential energy. Recall that Ioffe analysis of QCD vacuum sum rules \([97]\) provided the mechanism of formation of nucleon mass as due to the
Figure 13: Contribution to the scalar structure of the correlator from the exchange of $\bar{q}q$ pairs with quantum numbers of vector mesons between the correlator and the matter. The dark blob stands for the vector condensate. The light circles denote the vacuum expectation value. The generated term is unusual for QHD.

exchange by quarks between the probe nucleon and the quark–antiquark pairs of vacuum. In the nuclear matter the new mass is formed by the exchange with the modified distribution of the quark–antiquark pairs and with the valence quarks. The modified distribution of $\bar{q}q$ pairs is described by the condensate $\kappa(\rho)$. At $\rho$ close to $\rho_0$ the modification is mostly due to the difference of the densities of $\bar{q}q$ pairs inside the free nucleons and in the free space. Similar exchange with the valence quarks is determined by the vector condensate $v(\rho)$ and is described by the first term of rhs of Eq.(188). The second term describes additional interaction which takes place during such exchange. These exchanges cause the shift of the position of the pole $m_m - m$. While the interactions of the nucleons depend on the condensates $\Delta\kappa(\rho)$ and $v(\rho)$, these condensates emerge due to the presence of the nucleons. Also, the nonlinear part of $\Delta\kappa$ is determined by $NN$ interactions. Thus, there is certain analogy between QCD sum rules picture and NJL mechanism.

As we have seen, the QCD sum rules can be viewed as connection between exchange by uncorrelated $\bar{q}q$ pairs and exchange by strongly correlated pairs with the same quantum numbers (mesons). This results in connection between the Lorentz structures of correlators and in-medium nucleon propagators. In the leading terms of OPE the vector (scalar) structure is determined by vector (scalar) condensate. The large values (of about 250–300 MeV) of the first and the fourth terms in rhs of Eq.(188) provide thus the direct analogy with QHD picture.

Note, however, that the sum rules, presented by Eqs. (177)–(179) contain also the terms, which are unusual for QHD approach. Indeed, the term $Q^{11}$ in Eq.(177) enters the vector structure of the correlator (and thus, of the propagator of the nucleon) corresponding, however, to exchange by the vacuum quantum numbers with the matter. On the other hand, the last term of rhs of Eq.(179) treated in the gas approximation, corresponds to exchange by the quantum numbers of vector mesons. However, it appears in the scalar structure. This term originated from the four-quark condensate $Q^{12}$. While the exact value of the condensate $Q^{11}$ is still obscure, the condensate $Q^{12}$ is easily calculated. This OPE term is shown in Fig. 13. It provides a noticeable contribution.

Such terms do not emerge in the mean-filed approximation of QHD. They can be originated by more complicated structure of the nucleon-meson vertices. (Note that if the nucleons interacted through the four-fermion interaction, such terms would have emerged from the exchange interaction
due to Fierz transform).

Another approach, developed by Maryland group, was reviewed by Cohen et al. \cite{124}. In most of the papers (except \cite{125}) the Lehmann representation was a departure point. In framework of this approach the authors investigated the Lorentz structure of QCD sum rules \cite{126}. They analysed detaily the dependence of the self-energies on the in-medium value of the scalar four-quark condensate \cite{127}. The approach was used for investigation of hyperons in nuclear matter \cite{128}.

The approach is based on the dispersion relations in the time component $q_0$ at fixed three-dimensional momentum $|\vec{q}|$. It is not clear, if in this case the singularities, connected with the probe proton are separated from those of the matter itself. The fixed value of three-dimensional momenta is a proper characteristics for in-medium nucleon. That is why this was the choice of variables in Lehmann dispersion relation with the Fermi energy as a typical scale. The sum rules are dispersion relations rather for the correlation function with the possible states $N$, $N$+ pions, $N^*$, etc. The scale of the energy is a different one and it is not clear, if this choice of variables is reasonable for QCD sum rules.

### 5.3 Charge-symmetry breaking phenomena

#### 5.3.1 Nolen–Schiffer anomaly

The nuclei consisting of equal numbers of protons and neutrons with one more proton or neutron added are known as the mirror nuclei. If the charge symmetry (known also as isospin symmetry) of strong interactions is assumed, the binding energy difference of mirror nuclei is determined by electromagnetic interactions only, the main contribution being caused by the interaction of the odd nucleon. Nolen and Schiffer \cite{129} found the discrepancy between the experimental data and theoretical results on the electromagnetic contribution to the energy difference. This discrepancy appeared to be a growing function of atomic number $A$. It reaches the value of about 0.5 MeV at $A = 40$. Later the effect became known as Nolen–Schiffer anomaly (NSA).

The NSA stimulated more detailed analysis of electromagnetic interactions in such systems. Auerbach et al. \cite{130} studied the influence of Coulomb forces on core polarization. However, this did not explain the NSA. Bulgac and Shaginyan \cite{131} attributed the whole NSA phenomena to the influence of the nuclear surface on the electromagnetic interactions. Thus they predict NSA to vanish in infinite medium.

However most of the publications on the subject contain the attempts to explain NSA by the charge symmetry breaking (CSB) by the strong interactions at the hadronic level. The CSB potentials of $NN$ interactions were reviewed by Miller et al. \cite{132}. Some of phenomenological potentials described the NSA, but contradicted the experimental data on CSB effects in $NN$ scattering. The meson-exchange potentials contain CSB effects by inclusion of $\rho - \omega$ mixing. This explains the large part of NSA, but not the whole effect.

On the quark level the CSB effects in the strong interactions are due to the nonzero value of the difference of the quark masses

$$m_d \approx 7\text{ MeV} ; \quad m_u \approx 4\text{ MeV} ; \quad \mu = m_d - m_u \approx 3\text{ MeV} .$$

(190)
Several quark models have been used for investigation of NSA by calculation of neutron–proton mass difference in nuclear matter (recall that in vacuum \(m_n - m_p = 1.8\) MeV, while the Coulomb energy difference is \(-0.5\) MeV. Hence, the shift caused by strong interactions is \(\delta m = 2.3\) MeV). Henley and Krein \[133\] used the NJL model for the quarks with the finite values of the current masses. The calculated neutron–proton mass difference appeared to be strongly density dependent. The result overestimated the value of NSA. The application of the bag models were considered by Hatsuda et al. \[134\]. The chiral bag model provided the proper sign of the effect, but underestimated its magnitude.

5.3.2 QCD sum rules view

The QCD sum rules look to be a reasonable tool for the calculation of neutron–proton binding energy difference for the nucleons, placed into the isotope–symmetric nuclear matter. Denoting

\[
\delta x = x_n - x_p
\]

for the strong interaction contribution to the neutron–proton difference of any parameter \(x\), we present

\[
\delta \varepsilon = \delta U + \delta T
\]

with \(U(T)\) — the potential (kinetic) energy of the nucleon. To the lowest order in the powers of density one finds

\[
\delta T = -\frac{p_F^2}{m^2} \delta m ,
\]

while the value \(\delta U = \delta m_m\) can be obtained from the sum rules.

The attempts to apply QCD sum rules for solving the NSA problem were made in several papers \[134\]–\[137\]. We shall follow the papers of Drukarev and Ryskin \[137\], which present the direct extension of the approach, discussed above. It is based on Eqs. (182)–(184) with the terms \(L^m_i\) being calculated with the account of the finite values of the current quark masses. Besides the quark mass difference the CSB effects manifest themselves through isospin breaking condensates

\[
\gamma_0 = \frac{\langle 0|\bar{d}d - \bar{u}u|0 \rangle}{\langle 0|\bar{u}u|0 \rangle} ; \quad \gamma_m = \frac{\langle M|\bar{d}d - \bar{u}u|M \rangle}{\langle M|\bar{u}u|M \rangle} .
\]

The characteristics \(\gamma_0\) and \(\gamma_m\) are not independent degrees of freedom. They turn to zero, at \(\mu = 0\), being certain (unknown) functions of \(\mu\). The dependence \(\gamma_0(\mu), \gamma_m(\mu)\) can be obtained in framework of the specific models. Anyway, due to the small values of \(m_{u,d}\) we expect \(|\gamma_0|, |\gamma_m| \ll 1\).

Following the strategy and keeping only the leading terms, which are linear in \(\mu\), we shall obtain the energy shift in the form

\[
\delta \varepsilon = a_1 \mu + a_2 \gamma_0 + a_3 \gamma_m
\]

with \(a_i\) being the functions of the density \(\rho\) (the contribution \(\delta T\) is included into \(a_1\)).

To calculate the mass-dependent terms in lhs of Eqs. (182)–(184) one should include the quark masses to the in-medium quark propagator — Eq.(149). The first term in rhs of Eq.(149), which is just the free quark propagator should be modified into \((i\hat{\gamma}_{\alpha\beta})/(2\pi^2 x^4) - m_q/(2\pi^2 x^2)\). This provides the contribution to the scalar structure of the correlator in the lowest order of OPE. Account of the
finite quark masses in the second term of Eq.\(149\) manifest themselves in the next to leading orders of OPE. Say, during evaluation of the second term in Subsection 5.2, we used the relations expressed by Eq.\(53\), which were obtained for the massless quarks. Now the first of them takes the form

\[
\langle \phi^i_b \rangle = \frac{1}{4} \langle \phi^i_a \alpha \rangle - \frac{m_i}{4m} \langle N|\bar{\psi}_i\psi_i|N \rangle
\]

(196)

for the flavour \(i = u, d\). This leads to the contribution to the vector structure of the correlator proportional to the scalar condensate. Also, the second moment of the scalar distribution is proportional to the vector condensate, contributing to the scalar structure of the correlator.

The leading contribution caused by the scalar structure of the correlator

\[
(\delta U)_1 = 0.18 \frac{\mu}{m} \frac{v(\rho)}{\rho_0} \text{ GeV ,}
\]

(197)

while the CSB term, originated by the vector structure is

\[
(\delta U)_2 = -0.031 \frac{\mu}{m} \frac{\kappa(\rho) - \kappa(\rho_0)}{\rho_0} \text{ GeV}
\]

(198)

with \(\rho_0 = 0.17 \text{ Fm}^{-3}\) being the value of saturation density. The term \(\Delta \mathcal{L}_s^m\) of Eq.\(186\) provides the contribution containing the CSB condensate \(\gamma_m\) — Eq.\(194\)

\[
(\delta U)_3 = 32 (\gamma_m(\kappa(\rho) - \kappa(0)) + (\gamma_m - \gamma_0)\kappa(0)) / \text{GeV}^2.
\]

(199)

For the complete calculation one needs the isospin breaking shifts of vacuum parameters \(\delta \lambda^2\) and \(\delta W^2\) while the empirical value of \(\delta m\) can be used. Thus, the analysis of CSB effects in vacuum should be carried out in framework of the method as well. This was done by Adami et al. \[135\]. The values of \(\delta \lambda^2\), \(\delta W^2\) and of the vacuum isospin breaking value \(\gamma_0\) were obtained through the quark mass difference and the empirical values of the shifts of the baryon masses. This prompts another form of Eq.\(193\)

\[
\delta \varepsilon = b_1 \mu + b_2 \gamma_m
\]

(200)

with \(b_1 = a_1 + a_2(\gamma_0/\mu)\), \(b_2 = a_3\). The expressions for the contributions to \(\delta \varepsilon\), caused by the shifts of the vacuum values \(\delta m\), \(\delta \lambda^2\) and \(\delta W^2\) are rather complicated. At \(\rho = \rho_0\) the corresponding contribution is

\[
(\delta U)_4 = -0.4 \text{ MeV .}
\]

(201)

For the sum \(\sum_i (\delta U)_i\) we find, after adding the contribution \(\delta T\)

\[
b_1(\rho_0) = -0.73 , \quad b_2(\rho_0) = -1.0 \text{ GeV .}
\]

(202)

The numerical results can be obtained if the value of \(\gamma_m\) is calculated. This can be done in framework of certain models. However, even now we can make some conclusions. If we expect the increasing restoration of the isospin symmetry with growing density, it is reasonable to assume that \(|\gamma_m| < |\gamma_0|\). Also, all the model calculations provide \(\gamma_0 < 0\). Thus we expect \(\gamma_0 < \gamma_m < 0\). If \(\gamma_m = 0\) we find \(\delta \varepsilon = -2.4 \text{ MeV}\), eliminating the vacuum value \(\delta m = 2.3 \text{ MeV}\). Hence, the isospin invariance appears to be restored for both the condensates and nucleon masses.
The present analysis enables also to clarify the role of the CSB effects in the scalar channel. Indeed, neglecting these effects, i.e. putting $(\delta U)_2 = (\delta U)_3 = 0$ we obtain $\delta \varepsilon > 0$. This contradicts both experimental values and general theoretical expectations. Thus we came to the importance of CSB effects in the scalar channel.

Adami and Brown [135] used NJL model, combined with BR1 scaling for calculation of parameter $\gamma_m$. They found $\gamma_m/\gamma_0 = (\kappa(\rho)/\kappa(0))^{1/3}$. Substituting this value into Eq.(200) we find

$$\delta \varepsilon = (-0.9 \pm 0.6)\text{MeV}$$

(203)

with the errors caused mostly by uncertainties of the value of $\gamma_0$. A more rapid decrease of the ratio $\gamma_m/\gamma_0$ would lead to larger values $|\delta \varepsilon|$ with $\delta \varepsilon < 0$. Putting $\gamma_m = \gamma_0$ provides $\delta \varepsilon = -0.3\text{MeV}$.

Of course, Eq.(203) is obtained for infinite nuclear matter and it is not clear, if it can be extrapolated for the case $A = 40$. We can state that at least qualitative explanation of the NSA is achieved.

### 5.3.3 New knowledge

As we have stated earlier, the QCD sum rules can be viewed as a connection between exchange of uncorrelated $\bar{q}q$ pairs between our probe nucleon and the matter and the exchange by strongly correlated pairs with the same quantum numbers (the mesons). In the conventional QHD picture this means that in the Dirac equation for the nucleon in the nuclear matter

$$(\hat{q} - \hat{V})\psi = (m + \Phi)\psi$$

(204)

the vector interaction $V$ corresponds to exchange by the vector mesons with the matter while the scalar interaction $\Phi$ is caused by the scalar mesons exchange. In the mean field approximation the vector interaction $V$ is proportional to density $\rho$, while the scalar interaction is proportional to the "scalar density"

$$\rho_s = \int \frac{d^3 p}{(2\pi)^3} \frac{m^*}{\varepsilon(p)}$$

(205)

which is a more complicated function of density $\rho$ — see Eqs. (70),(78). Thus $V = V(\rho)$, while $\Phi = \Phi(\rho_s)$. We have seen that QCD sum rules provide similar picture in the lowest orders of OPE: vector and scalar parts of the correlator $G^m$ depend on vector and scalar condensates correspondingly: $G^{m}_{q,p} = G^{m}_{q,p}(v(\rho)); G^{m}_{s} = G^{m}_{s}(\kappa(\rho))$. However as we have seen in Subsection 5.2.6, we find a somewhat more complicated dependence in the higher order OPE terms, say, $G^{m}_{s} = G^{m}_{s}(\kappa(\rho), v(\rho))$, depending on both scalar and vector condensates. This means that the corresponding scalar interaction $\Phi = \Phi(\rho_s, \rho)$, requiring analysis beyond the mean field approximation.

As one can see from Eqs. (197) and (198) in the case of CSB interactions such complications emerge in the sum rules approach in the leading orders of OPE.

Thus the QCD sum rules motivated CSB nuclear forces $V$ and $\Phi$ in Eq.(204) are expected to contain dependence on both "vector" and "scalar" densities, i.e. $V = V(\rho, \rho_s)$ and $\Phi = \Phi(\rho, \rho_s)$. As we said above, such potentials can emerge due to complicated structure of nucleon–meson vertices. This can provide the guide-lines for building up the CSB nucleon–nucleon potentials.
Another new point is the importance of the CSB in the scalar channel. Neglecting the scalar channel CSB interactions we obtain the wrong sign of the effects, i.e. $\delta \varepsilon > 0$. This contradicts the earlier belief that the vector channel $\omega - \rho$ mixing is the main mechanism of the effect [132]. Our result is supported by the analysis of Hatsuda et al. [139] who found that the $\omega - \rho$ mixing changes sign for the off-shell mesons. This can also help in constructing the CSB nuclear forces.

5.4 EMC effect

The experiments carried out by EMC collaboration [140] demonstrated that deep inelastic scattering function $F^A_2(x_B)$ of nucleus with atomic number $A$ ($x_B$ stands for Bjorken variable) differs from the sum of those of free nucleons. Most of the data were obtained for iron (Fe). The structure function was compared to that of deuteron, which imitates the system of free nucleons. The deviation of the ratio

$$R^A(x_B) = \frac{F^A_2(x_B)}{A} / \frac{F^D_2(x_B)}{2}$$

from unity is caused by deviation of a nucleus from the system of free nucleons. The ratio $R(x_B)$ appeared to be the function of $x_B$ indeed. Exceeding unity at $x_B < 0.2$ it drops at larger $x_B$ reaching the minimum value $R_{Fe}(x_B) \approx 0.85$ at $x_B \approx 0.7$. This behaviour of the ratio was called the EMC effect.

There are several mechanisms which may cause the deviation of the ratio $R(x_B)$ from unity. These are the contribution of quark–antiquark pairs, hidden in pions, originated by the nucleon–nucleon interactions, possible formation of multiquark clusters inside the nucleus, etc. Here we shall try to find how the difference of the quark distributions inside the in-medium and free nucleons changes the ratio $R(x_B)$.

The QCD sum rules method was applied to investigation of the proton deep inelastic structure functions in vacuum in several papers. The second moments of the structure functions were obtained by Kolesnichenko [141] and by Belyaev and Block [142]. The structure function $F_2(x_B)$ at moderate values of $x_B$ was calculated by Belyaev and Ioffe [122]. Here we shall rely on the approach, developed by Braun et al. [143] which can be generalized for the case of finite densities in a natural way. On the other hand, such generalization is the extension of the approach discussed in this section.

To obtain the structure function of the proton, the authors of [143] considered the correlation function $G$, describing the system with the quantum numbers of proton, interacting twice with a strongly virtual hard photon

$$G(q, k) = i^2 \int d^4xd^4ye^{i(qx)+i(ky)} \langle 0|T[\bar{\eta}(x)\eta(0)]H(y, \Delta)|0 \rangle .$$

Here $q$ and $q+k$ are the momenta carried by the correlator in initial and final states, $k = k_1 - k_2$ is the momentum transferred by the photon scattering. The incoming (outgoing) photon carries momentum $k_1(k_2)$, interacting with the correlator in the point $y - \Delta/2$ ($y + \Delta/2$). The quark–photon interaction is presented by the function $H(y, \Delta)$. In the next step the double dispersion relation in variables $q^2_1 = q^2$ and $q^2_2 = (q + k)^2$ is considered. The crucial point is the operator expansion in terms of the nonlocal operators depending on the light-like ($\Delta^2 = 0$) vector $\Delta$ [143]. After the Borel transform in both $q^2_1$ and $q^2_2$ is carried out and the equal Borel masses $M_1^2 = M_2^2$ are considered, the Fourier transform in $\Delta$ provides the momentum distribution of the quarks.
Figure 14: Second order interaction of the hard photon (dashed line) with the correlator. The other notations are the same as in the previous pictures.

Figure 15: The in-medium changes of the $d$ quark distribution (dashed curve) and of the $u$ quark distribution (dash-dotted curve) of the fraction $x$ of the momentum of the target nucleon. The solid curve presents the function $R - 1$ with the ratio $R$, defined by Eq.(206).
This approach was applied by Drukarev and Ryskin [144] for calculation of the quark distributions in the proton, placed into the nuclear matter. The two types of contributions to the correlator should be considered — Fig.14a,b. In the diagram of Fig.14a the photon interacts with the quark of the free loop. In the diagram of Fig.14b it interacts with the quark exchanging with the matter. The modification of the distributions of the quarks was expressed through the vector condensate, which vanishes in vacuum and through the in-medium shifts of the other condensates and of the nucleon parameters $m, \lambda^2$ and $W^2$. The result appeared to be less sensitive to the value of the four-quark condensate than the characteristics of the nucleon considered in Subsection 5.2.

Note that the results are true for the moderate values of $x$ only and cannot be extended to the region $x \ll 1$. This is because the OPE diverges at small $x$ [122].

Omitting the details of calculation, provided in [144] we present the results in Fig.15. One can see that the distributions of $u$ and $d$ quarks in fraction of the target momentum $x$ are modified in a different way and there is no common scale. The fraction of the momentum carried by the $u$ quarks $\langle x^u \rangle$ decreases by about 4%. The ratio $R$, determined by Eq.(206) has a typical EMC shape.

The technique used in [144] can be expanded for the calculation of the quark distributions at $1 < x < 2$. Thus the approach enables to describe the cumulative aspects of the problem as well.

5.5 The difficulties

In spite of the relative success, described above, the approach faces a number of difficulties. Some of them take place in vacuum as well. The other ones emerge in the case of the finite density.

The first problem is the convergence of OPE in the lhs of the sum rules. Fortunately, the condensates which contribute to the lowest order OPE terms can be either calculated in a model-independent way, or expressed through the observables. This is true for both vacuum and nuclear matter — at least, for the values of density which are close to the saturation values $\rho_0$. However the situation is not so simple for the higher order OPE contributions. The four-quark condensate is the well known headache of all the QCD sum rules practitioners. The problem becomes more complicated at finite density, since the conventional form of presentation of this condensate contains the strongly cancelling contributions.

In order to include the higher OPE terms one needs the additional model assumptions. The same is true for the attempts to go beyond the gas approximation at finite densities. Recently Kisslinger [145] suggested a hybrid of QCD sum rules and of the cloudy bag model.

Note, however that QCD sum rules is not a universal tool and there are the cases when OPE does not converge. We mentioned earlier that this takes place for the nucleon structure functions at small $x$ — [122]. Some time ago Eletsky and Ioffe [147] adduced the case when the short distance physics plays important role, making the OPE convergence assumption less convincing. Recently Dutt-Mazumder et al. [146] faced the situation when the ratio of two successive terms of $q^{-2}$ expansion is not quenched.

There are also problems with the rhs of the sum rules. The ”pole+continuum” model is a very simple ansatz, and it may appear to be oversimplified even in vacuum. The spectrum of the nucleon correlator in-medium is much more complicated than in vacuum. The problem is to separate the
singularities of the correlator, connected with the nucleon from those of the medium itself. As we have seen, the "pole+continuum" model can be justified to the same extent as in vacuum until we do not include the three-nucleon interactions. We do not have a simple and convincing model of the spectrum which would include such interactions.

Anyway, the success of vacuum sum rules [117] and reasonable results for the nucleons at finite densities described in this Section prompt that the further development of the approach is worth while.

6 A possible scenario

The shape of the density dependence of the quark scalar condensate in the baryon matter $\kappa(\rho)$ appears to be very important for hadronic physics. It is the characteristics of the matter as a whole, describing the degree of restoration of the chiral symmetry with growing density. On the other hand, the dependence $\kappa(\rho)$ is believed to determine the change of the nucleon effective mass $m^*(\rho)$. The shape of the dependence $m^*(\kappa(\rho))$ differs in the different models.

The lowest order density dependence term in the expansion of the function $\kappa(\rho)$ is model-independent. However for the rigorous calculation of the higher order terms one needs to know the density dependence of the hadron parameters $m^*(\rho), m^*_\Delta(\rho), f^*_\pi(\rho)$, etc. In Sec.4 we presented the results of the calculations of the condensate $\kappa(\rho)$ under certain model assumptions. A more detailed analysis requires the investigation of the dependence of these parameters on QCD condensates.

Such dependence can be obtained by using the approach, based on the in-medium QCD sum rules. In Sec.5 we show how in-medium QCD sum rules for the nucleons work. Even in a somewhat skeptical review of Leinweber [148], where the present state of art of applications of the QCD sum rules is criticised, the method is referred to as "the best fundamentally based approach for investigations of hadrons in nuclear matter". Of course, to proceed further one must try to overcome the difficulties, discussed in Subsec.5.5.

The lowest order condensates can be either calculated, or connected directly to the observables. This is true for both vacuum and nuclear matter. However, neither in vacuum nor in medium the higher order condensates can be obtained without applications of certain models. Thus in further steps we shall need a composition of QCD sum rules with model assumptions.

We have seen that the density dependence of the delta isobar effective mass is important for the calculation of the nonlinear contribution to the scalar condensate $\kappa(\rho)$. The shape of this dependence is still obscure. Thus the extension of the QCD sum rules method for the description of $\Delta$-isobars in-medium dynamics is needed. Such work is going on — see, e.g., the paper of Johnson and Kisslinger [149].

The fundamental in-medium Goldberger-Treiman and Gell-Mann–Oakes–Renner relations are expected to be the other milestones of the approach. The agreement with the results with those of conventional nuclear physics at $\rho \sim \rho_0$ would be the test of the approach.

Further development of the approach would require inclusion of the vector mesons. The vector meson physics at finite densities is widely studied nowadays. Say, various aspects of QCD sum rules application where considered in recent papers [146, 150, 151, 152] while the earlier works are cited
in reviews [124], [148].

We expect the investigation in framework of this scenario to clarify the features of baryon parameters and of the condensates in nuclear matter.

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