Mathematical modeling of the loading process dump truck

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Abstract. The article solves the problem of the impact of the load when loading a dump truck. The car is modeled by a rigid beam on two elastic supports of different stiffness with shock absorbers. The impact of the load is absolutely inelastic in the center of mass of the beam. The mathematical model of the car is based on the Lagrange equations of the 2nd kind in the form of a system of two inhomogeneous linear differential equations of small oscillations of the Beam-Load system. An analytical solution of this system is obtained in the form of the sum of two damped main oscillations, and expressions are obtained for the conditional amplitudes and phase shifts of the main oscillations, taking into account the initial conditions determined by the impact of the load. Based on the results obtained, an algorithm and a program for solving the problem were developed. A numerical experiment was performed on loading a BelAZ-7555B dump truck.

1. Introduction
When calculating any design, an important place is occupied by the analysis of the operating loads. To determine the main loading modes, it is necessary to develop a method for calculating loads. Such methods and standards are available in construction and in a number of branches of mechanical engineering. In the automotive industry, there are no uniform standards yet. The problem of their creation is connected both with the complexity of the design of cars, and with the variety of load modes.

One of the main modes of loading cars is driving on an uneven road [1-4]. For dump trucks, the calculated case may be the unloading of the body. The calculation of loads when unloading a dump truck is considered in the article [5], where the load is modeled as a monolithic block. In the process of operating the trucks will also experience a significant shock loads that occur during loading of the cargo. For example, when loading quarry dump trucks, massive chunks of rock can fall into the body from a height of several meters. This article is devoted to the study of loads in this case of loading.

2. Formulation of the Problem
To study the dynamic processes that occur during loading, imagine a car as a rigid beam weighing $M$ and length $L$. The Beam is supported by two springs with shock absorbers. The stiffness of the springs $C_1$ and $C_2$, is equivalent to the stiffness of the suspensions and tires of the front and rear axles of the car, respectively. On the beam from a height $h$ in the center of mass $C$ falls point weight mass $m$. The calculation scheme of the car before the impact is shown in figure 1.
3. Differential equations of small oscillations of the car.

At the moment of impact on the beam the speed of the load is equal to \( v = \sqrt{2gh} \), where \( g \) – is the acceleration of free fall. The blow will be considered absolutely inelastic.

The speed of the center of mass \( C \), as well as the speed of points \( A \) and \( B \) immediately after the impact are equal:

\[
u_C = u_A = u_B = \frac{mv}{M + m}\]

As a result of the impact, small fluctuations occur (Figure 2).

![Figure 1. Calculation scheme of the car before the impact.](image1)

![Figure 2. Calculation scheme of oscillations of the Beam-Load system.](image2)

The system has two degrees of freedom. Let us take as generalized coordinates the vertical movements of points \( A \) and \( B \) of the beam:

\[ q_1 = y_A, \quad q_2 = y_B. \]

The reference coordinates are taken from the equilibrium position of the empty beam (Ox axis). The Oy axis is directed downwards.
To derive the differential equations of oscillations of the Beam-Load system, we use Lagrange equations of the 2nd kind.

The kinetic energy of the system is represented in a quadratic form \([6,7]\):
\[
T = \frac{1}{2} \left( a_{11} \ddot{y}_A^2 + 2a_{12} \ddot{y}_A \ddot{y}_B + a_{22} \ddot{y}_B^2 \right),
\]
where the inertia coefficients of the Beam-Load system have the form:
\[
a_{11} = \frac{1}{J} \left( M_1 \cdot b^2 + J_1 \right); \quad a_{22} = \frac{1}{J} \left( M_1 \cdot \alpha^2 + J_1 \right); \quad a_{12} = \frac{1}{J} (M_1 \cdot \alpha b - J_1).
\]

Here \( J_1 = J_{cz} = M \rho C^2 \) – moments of inertia of the system \( J_1 \) and beams \( J_{cz} \). (They are equal, because the cargo is a material point); \( \rho C \) – radius of inertia of the beam; \( M_1 = M + m \) – the weight of the system.

The potential energy of the Beam-Load system has the form:
\[
\Pi = -(M + m) g \cdot y_c + c_1 \left[ (y_A + \Delta_A)^2 - \Delta_A^2 \right] + c_2 \left[ (y_B + \Delta_B)^2 - \Delta_B^2 \right],
\]
where \( y_c = \frac{y_A \cdot b - y_B \cdot a}{L} \) – coordinate of the center of mass of the system;
\[
\Delta_A = \frac{M g \cdot b}{L \cdot c_1} \quad \text{and} \quad \Delta_B = \frac{M g \cdot a}{L \cdot c_2} \quad \text{– static deformation of the springs}.
\]

After substitution we get the potential energy in the form \([6,7]\):
\[
\Pi = \frac{1}{2} \left( c_{11} y_A^2 + 2c_{12} y_A y_B + c_{22} y_B^2 \right) - mg \frac{b}{L} y_A - mg \frac{a}{L} y_B,
\]
where \( c_{11} = c_1, \quad c_{22} = c_2, \quad c_{12} = 0 \) – coefficients of the stiffness of the system.

Vibrations of the car are damped due to their damping shock absorbers. In the car suspension, the friction can be liquid (hydraulic shock absorbers) or dry (in springs and suspension hinges). Consider the damping of vibrations due to liquid friction, i.e. due to linear-viscous resistance.

Let \( \mu_1, \mu_2 \) be resistance coefficients of hydraulic shock absorbers of the front and rear suspension. Then the dissipative Rayleigh function of the system has the form \([8]\):
\[
\Phi = \sum_{i=1}^{2} \frac{\mu_i y_i^2}{2} = \frac{\mu_1 y_A^2}{2} + \frac{\mu_2 y_B^2}{2},
\]

Substituting the kinetic energy \((1)\), the potential energy \((3)\) and the dissipative Rayleigh function \((4)\) in the Lagrange equations of the 2nd kind, we obtain a non-uniform system of linear differential equations of small oscillations of the Beam-Load system:
\[
\begin{align*}
a_{11} \ddot{y}_A + \mu_1 \ddot{y}_A + c_{11} y_A + a_{12} \ddot{y}_B &= mg \frac{b}{L}, \\
a_{21} \ddot{y}_A + a_{22} \ddot{y}_B + \mu_2 \ddot{y}_B + c_{22} y_B &= mg \frac{a}{L},
\end{align*}
\]
where \( \mu_1 = \mu_1, \mu_2 = \mu_2, \mu_{12} = 0 \).

The solution of the obtained system of differential equations \((5)\) is sought as the sum of the General solution of a homogeneous system and the partial solution of a non-homogeneous system. We will choose a private solution in the form of:
\[
\ddot{y}_A = \frac{mg b}{c_1 L}, \quad \ddot{y}_B = \frac{mg a}{c_2 L}.
\]
In turn we look for the General solution of a homogeneous system in the form:
\[ y_AO = C_1 e^{\lambda t}, \quad y_BO = C_2 e^{\lambda t}. \]

Substituting this solution into the homogeneous part of equations (5) and reducing by \( e^{\lambda t} \), we obtain a homogeneous linear system of two algebraic equations:
\[
\begin{aligned}
& C_1 (a_{11}\lambda^2 + \mu_{11}\lambda + c_{11}) + a_{12}C_2\lambda^2 = 0, \\
& C_1 a_{21}\lambda^2 + C_2 (a_{22}\lambda^2 + \mu_{22}\lambda + c_{22}) = 0,
\end{aligned}
\]

(6)

For the existence of a non-zero -solution \( C_1, \ C_2 \), the determinant of the system (6) must be zero:
\[
\begin{vmatrix}
 a_{11}\lambda^2 + \mu_{11}\lambda + c_{11} & a_{12}\lambda^2 \\
 a_{21}\lambda^2 & a_{22}\lambda^2 + \mu_{22}\lambda + c_{22}
\end{vmatrix} = 0.
\]

Hence we get the characteristic equation:
\[
(a_{11}a_{22} - a_{12}^2)\lambda^4 + (a_{11}\mu_{22} + a_{22}\mu_{11})\lambda^3 + (a_{11}c_{22} + \mu_{11}\mu_{22} + a_{22}c_{11})\lambda^2 + \\
+ (\mu_{11}c_{22} + \mu_{22}c_{11})\lambda + c_{11}c_{22} = 0
\]

(7)

Since the resistance is small, this is more common in practice, the four roots of equation (6) form complex conjugate pairs of numbers:
\[
\lambda_{1,2} = -n_1 \mp p_1 i, \ \lambda_{3,4} = -n_2 \mp p_2 i \quad (i = \sqrt{-1}).
\]

Each of the four roots corresponds to certain values of the constants \( C_1 \) and \( C_2 \). However, from equations (6) for each \( \lambda_i \), only the ratio of values \( C_1 \) and \( C_2 \):
\[
\frac{C_2^{(i)}}{C_1^{(i)}} = \frac{a_{11}\lambda_i^2 + \mu_{11}\lambda_i + c_{11}}{a_{12}\lambda_i^2} = z_i \quad (i = 1, 2, 3, 4),
\]

(8)

\( z_1, z_2 \) and \( z_3, z_4 \) are complex conjugate numbers.

As a result, the General solution of the homogeneous part of the system (6) has the form:
\[
y_AO = \sum_{i=1}^{4} C_1^{(i)} e^{\lambda_i t}, \quad y_BO = \sum_{i=1}^{4} z_i C_1^{(i)} e^{\lambda_i t},
\]

(9)

(10)

Let's rewrite the solution (9) in the equivalent trigonometric form:
\[
y_AO(t) = e^{-n_1 t} \left( C_{11}^{(1)} \cos p_1 t + C_{12}^{(1)} \sin p_1 t \right) + e^{-n_2 t} \left( C_{11}^{(2)} \cos p_2 t + C_{12}^{(2)} \sin p_2 t \right).
\]

(11)

Here
\[
C_{11}^{(1)} = C_1^{(1)} + C_1^{(2)}, \quad C_{12}^{(1)} = i(-C_1^{(1)} + C_1^{(2)}), \quad C_{11}^{(2)} = C_1^{(3)} + C_1^{(4)}, \quad C_{12}^{(2)} = i(-C_1^{(3)} + C_1^{(4)}).
\]

Let us now represent complex conjugate numbers \( z_i \) in expressions (10) as:
\[
z_1 = a_1 - ib_1, \quad z_2 = a_1 + ib_1, \quad z_3 = a_2 - ib_2, \quad z_4 = a_2 + ib_2.
\]
Then we get the solution (10) in the equivalent trigonometric form:

\[ y_{BO} = e^{-n_1 t} \left[ \left( C_{11}^{(1)} a_1 + C_{12}^{(1)} b_1 \right) \cos p_1 t + \left( C_{11}^{(1)} a_1 - C_{11}^{(1)} b_1 \right) \sin p_1 t \right] + \\
+ e^{-n_2 t} \left[ \left( C_{11}^{(2)} a_2 + C_{12}^{(2)} b_2 \right) \cos p_2 t + \left( C_{11}^{(2)} a_2 - C_{11}^{(2)} b_2 \right) \sin p_2 t \right] \]  

(12)

Let's enter designations:

\[ D_t^{(j)} = \frac{\sqrt{C_{1j}^{(1)}}}{1 + \sqrt{C_{1j}^{(1)}}} , \quad \gamma_1^{(j)} = \arctg \frac{C_{1j}^{(1)}}{C_{1j}^{(2)}} , \quad j = 1, 2 ; \]  

(13)

\[ \gamma_2^{(j)} = \arctg \frac{C_{1j}^{(1)} a_j + C_{1j}^{(2)} b_j}{-C_{1j}^{(1)} b_j + C_{1j}^{(2)} a_j} , \quad j = 1, 2 , \]  

(14)

Finally write out the laws of damped oscillations of points A and B:

\[ y_A(t) = e^{-n_1 t} D_1^{(1)} \sin (p_1 t + \gamma_1^{(1)}) + e^{-n_2 t} D_1^{(2)} \sin (p_2 t + \gamma_1^{(2)}) + \frac{mb}{c_1 L}, \]  

(15)

\[ y_B(t) = e^{-n_1 t} D_1^{(1)} \sqrt{a_1^2 + b_1^2} \sin (p_1 t + \gamma_1^{(1)}) + e^{-n_2 t} D_1^{(2)} \sqrt{a_2^2 + b_2^2} \sin (p_2 t + \gamma_1^{(2)}) + \frac{mg}{c_2 L}, \]  

(16)

Constant integrations \( C_{11}^{(1)} , \ C_{12}^{(1)} , \ C_{11}^{(2)} , \ C_{12}^{(2)} \) are determined by the initial conditions, which at the moment of time \( t = 0 \) after the fall of the load on the beam have the form:

\[ y_A(0) = y_B(0) = 0 , \quad \dot{y}_A(0) = \dot{y}_B(0) = mv / M_1 . \]

Expressions for coefficients \( C_{11}^{(1)} , \ C_{12}^{(1)} , \ C_{11}^{(2)} , \ C_{12}^{(2)} \) are not given because of their bulkiness.

4. Numerical experiment of loading the car BelAZ 7555B.

Based on the developed algorithm we will conduct a numerical simulation of the process of loading the car on the example of a quarry dump truck BelAZ 7555V:

- \( M = 40200 \) kg, \( L = 4.0 \) m, \( a = 2.04 \) m, \( b = 1.96 \) m, \( \rho_c = 1.99 \) kg \( \cdot \) m\(^2\),
- \( c_1 = 830000 \) N/m, \( c_2 = 8300000 \) N/m, \( m = 700 \) kg, \( h = 2 \) m, \( \mu_1 = \mu_2 = 4500 \) kN/s

Static spring deformations at points A and B are 0.233 m and 0.242 m, respectively. The load falls on the car body at a speed \( v = \sqrt{2gh} = 6.26 \) m/s, so that the body acquires an initial forward movement down at a speed of \( \nu = 0.152 \) m/s. This movement causes small damped oscillations of the car.

In the process of oscillations of the system in the elastic elements of the forces arise:

\[ F_A = c_1 (y_A(t)+\Delta_A) , \quad F_B = c_2 (y_B(t)+\Delta_B) . \]

We write out the forces in the elastic suspension elements obtained on the basis of the dependencies (15) and (16):

\[ F_A(t) = e^{-0.11 t} \times 23786.4 \sin(6.46t-0.24) + e^{-0.11 t} \times 3838 \sin(6.31t-0.25) + 198044.3 \]

\[ F_B(t) = e^{-0.11 t} \times 6650.0 \sin(6.46t-0.24) + e^{-0.11 t} \times 13729.1 \sin(6.31t-0.25) + 206127.7 \]

Figures 3 and 4 show graphs of forces in the elastic elements of the suspension:
From the presented graphs, it follows that dynamic additives make up about 20% of static loads. Maximum loads are active for the first 5 seconds and fade by the 15th second. Increasing the damping in the suspension has little effect on these parameters and cannot significantly change the peak load values. However, in order to reduce shock loads, it is advisable to make the initial loading of the body with small pieces of rock, which provides additional damping.

5. Conclusion

1. A mathematical model of the impact of the load when loading a dump truck on the basis of a beam on elastic supports is developed.
2. The analytical solution of the Beam-Load system oscillations is obtained taking into account the resistance of the suspension shock absorbers.
3. Based on the results obtained, an algorithm and a program for calculating the forces in the elastic elements of the suspension were developed.
4. A numerical experiment of loading a BelAZ-7555V quarry dump truck was carried out.

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