Towards precision distances and 3D dust maps using broadband Period–Magnitude relations of RR Lyrae stars

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ABSTRACT
We determine the period-magnitude relations of RR Lyrae stars in 13 photometric bandpasses from 0.4 to 12 μm using timeseries observations of 134 stars with prior parallax measurements from Hipparcos and the Hubble Space Telescope (HST). The Bayesian formalism, extended from our previous work to include the effects of line-of-sight dust extinction, allows for the simultaneous inference of the posterior distribution of the mean absolute magnitude, slope of the period-magnitude power-law, and intrinsic scatter about a perfect power-law for each bandpass. In addition, the distance modulus and line-of-sight dust extinction to each RR Lyrae star in the calibration sample is determined, yielding a sample median fractional distance error of 0.66 per cent. The intrinsic scatter in all bands appears to be larger than the photometric errors, except in WISE W1 (3.4 μm) and WISE W2 (4.6 μm) where the photometric error (σ ≈ 0.05 mag) appears to be comparable or larger than the intrinsic scatter. This suggests that additional observations at these wavelengths could improve the inferred distances to these sources further. With ∼100,000 RR Lyrae stars expected throughout the Galaxy, the precision dust extinction measurements towards 134 lines-of-sight offer a proof of concept for using such sources to make 3D tomographic maps of dust throughout the Milky Way. We find a small but significant increase (3 per cent) in the effective extinction towards sources far from the Galactic plane relative to the expectation from recent dust maps and we suggest several explanations. As an application of the methodology, we infer the distance to the RRC-type star RZCep at low Galactic latitude (b = 5.5°) to be μ = 8.0397 ± 0.0123 mag (405.4 ± 2.3 pc) with colour excess E(B−V) = 0.2461 ± 0.0089 mag. This distance, equivalent to a parallax of 2467 ± 14 microarcsec, is consistent with the published HST parallax measurement but with an uncertainty that is 13 times smaller than the HST measurement. If our measurements (and methodology) hold up to scrutiny, the distances to these stars have been determined to an accuracy comparable to those expected with Gaia. As RR Lyrae are one of the primary components of the cosmic distance ladder, the achievement of sub-1 per cent distance errors within a formalism that accounts for dust extinction may be considered a strong buttressing of the path to eventual 1 per cent uncertainties in Hubble’s constant.

Key words: methods: statistical – stars: distances – stars: variables: RR Lyrae.

1 INTRODUCTION
RR Lyrae stars are old (age ≥ 10 × 10⁹ yr) Population II pulsating stars that exist throughout the Milky Way Bulge, Disc, and Halo. At optical wavebands they are variable with peak-to-peak amplitudes up to about 1 mag. This amplitude generally diminishes with increasing wavelength to around 0.3 mag in the mid-infrared (λ ∼ 4μm). The heat generation and gravitational support of RR Lyrae stars comes from the fusion of helium in the core and hydrogen in a shell surrounding the core. RR Lyrae stars have specific values of temperature, luminosity, and radius such that they exist in the instability strip of the Hertzsprung–Russell diagram. In this slice of stellar parameter-space stars are unstable to radial oscillation, and RR Lyrae stars oscillate with peri-
ods ranging from about 0.2 to 0.9 d. This, and oscillating temperature changes, leads to periodic luminosity variability. Preston (1964) and Smith (1995) both provide excellent reviews of RR Lyrae pulsating variable stars.

RR Lyrae (and other pulsational variables, namely Cepheids) have inspired more than a century’s worth of close attention because of the correlation of their fundamental oscillation period and luminosity. This empirical relation, supported by theoretical modelling (e.g., Catelan et al. 2004), has led to their use as primary distance indicators within the Milky Way and to the nearest neighbouring galaxies. With an effective range of ~100 kpc (for a limiting AB mag of ~20), they bridge the gap in the cosmic distance ladder between trigonometric parallax and the Tip of the Red Giant Branch (TRGB) method. RR Lyrae stars can also serve to calibrate Cepheid distances, most significantly by precisely measuring the distance and morphology of the Magellanic Clouds.

Distance determinations are achieved by leveraging empirical RR Lyrae period–magnitude relations to infer an RR Lyrae star’s intrinsic luminosity in a given waveband (absolute magnitude, $M$) from its measured oscillation period. In prior work this relation is commonly called the “period–luminosity” relation, but here we prefer to use the term “period–magnitude” relation to distinguish that it is not a predictor of bolometric luminosity, but instead of absolute magnitude in a given waveband.

With absolute magnitude for a given waveband in hand, the distance modulus, $\mu$, is then calculated after observing the mean-flux apparent magnitude of the star in the same waveband, $m$, and using the colour excess, $E(B-V)$, to correct for extinction due to light scattering by interstellar dust grains. For waveband $j$, the equation for distance modulus, which is common for all bands, is

$$\mu = m_j - M_j - E(B-V) \times (a_j R_V + b_j),$$

where $a_j$ and $b_j$ are the wavelength-specific parameters for the interstellar extinction law defined in Cardelli et al. (1989) and $R_V$ is the extinction law factor ($R_V = A_V/E(B-V)$) with value equal to 3.1 for the diffuse interstellar medium adopted from Schultz & Wiemer (1975).

The period–magnitude relations of RR Lyrae stars have previously been primarily studied in the near-infrared $K$-band (Sollima et al. 2006) and mid-infrared bands of the Wide-field Infrared Survey Explorer (WISE) all-sky satellite mission (Klein et al. 2011; Madore et al. 2013; Dambis et al. 2014; Klein et al. 2014). The slope of the period–magnitude relation at the shorter-wavelength optical bands is shallower, and a stronger correlation has been found between metallicity, [Fe/H], and optical (generally $V$-band) absolute magnitude. However, shorter-wavelength observations exhibit large scatter about a linear relation and there is no clear consensus on the necessity of the inclusion of secondary and/or nonlinear terms. The interested reader is referred to Section 5 of Sandage & Tammann (2006) for a review of optical ($V$-band) implementations of RR Lyrae stars as distance indicators.

In this analysis for the first time empirical RR Lyrae period–magnitude relations are simultaneously derived for 13 wavebands between the ultraviolet and mid-infrared. The calibration dataset is comprised of 134 RR Lyrae stars with photometry data combined from four astronomical observing facilities (two ground-based telescopes and the space-based Hipparcos and WISE satellites). Distances for the calibration RR Lyrae stars are determined with median fractional error of 0.66 per cent, and the multi-wavelength data are also used to solve for the colour excess to each calibration star.

The improved waveband-specific period–magnitude relations presented here, as well as the Bayesian methodology for simultaneously calibrating or applying any subset of the 13 relations, represent a significant advancement in the use of RR Lyrae stars to measure distance. The claimed level of precision compares to (and even rivals) the expected astrometric precision of Gaia, the space-based, parallax/astrometry mission launched by the European Space Agency in December 2013 (Clark & Quartz 2012).

This paper is outlined as follows. We present a description of the ground-based optical, Hipparcos, ground-based near-infrared, and WISE datasets in Section 2. In Section 3 we review our light curve analysis methodology, describing how mean-flux magnitudes are measured for the sample. In Section 4 we present our Bayesian simultaneous linear regression formalism, extended from our prior work, and the resultant period–magnitude relations. In Section 5 we relate additional findings of note resulting from the period–magnitude relation fits. In Section 6 we demonstrate how the multi-band period–magnitude relations are applied to estimate the distance to RZCep, which had been excluded from the period–magnitude relation fits owing to high, and poorly constrained, interstellar extinction. Finally, in Section 7 we discuss the conclusions and future implications of this work.

### 2 DATA DESCRIPTION

The RR Lyrae calibration sample used in this work is based upon the catalogue of 144 relatively local ($\lesssim 2.5$ kpc) RR Lyrae variables developed by Fernley et al. (1998). Six of these stars are excluded from our present analysis because of minimal light curve data and poor harmonic model fits determined via the procedure described in Section 3. Another three stars (ARPer, RZCep, and BNVul) are excluded because of their large and poorly constrained colour excess values (these stars lie too close to the Galactic Plane for the Schlegel et al. 1998 and Schlafly & Finkbeiner 2011 dust maps to provide accurate colour excess measurements). And, one additional star, ATSer, was excluded because only Hipparcos and W3 photometry was available for this star, and the W3 magnitude was a significant outlier in prior period–magnitude relation fits.

In total, the calibration sample is 134 stars, with 637 band-specific light curves composed of 33,630 epochs. Table 1 provides complete observable prior and fitted posterior data for the calibration sample. The calibration sample contains RR Lyrae stars belong-
during both of the two most common subtypes: RRab (115) and RRc (19). RRab stars oscillate at their fundamental period, \( P_f \), and RRc stars oscillate at their first overtone period, \( P_{fo} \). The RRc stars’ periods must be “fundamentalised” before deriving the period–magnitude relations. As in Dall’Ora et al. (2004), an RRc star’s fundamentalised period is given by

\[
\log_{10} (P) = \log_{10} (P_f) + 0.127.
\]

### 2.1 Colour excess and distance modulus priors

Line-of-sight \( E(B - V) \) colour excess values published in Schlegel et al. (1998) and Schlafly & Finkbeiner (2011) were retrieved from the NASA/IPAC Infrared Science Archive. These colour excess values estimate the total cumulative interstellar extinction due to dust. In practice, the calibration stars are embedded in the Galaxy and the dust maps, which were derived from far-infrared imaging, are averaged over large (tens of arcminute) scales. The former means that the true colour excess can be significantly less than the published value for that line of sight (even approaching zero if the star is close enough) and the latter implies that the published values should be considered to have significantly larger uncertainty bounds when applied to precise lines of sight terminating at unresolved point sources.

In order to begin the Markov Chain Monte Carlo (MCMC) regression traces (Section 4.1) and ultimately fit for colour excess posteriors, the Schlafly & Finkbeiner (2011) values were adapted into prior colour excess distributions according to the following procedure. If \( E(B - V)_{SP} > 0.125 \), then the prior distribution was set to be uniform

| Measurements and Priors |
|-------------------------|
| Name | AACMi | ABUMa | AEBoo | AFVel | AFVir |
| Type | RRab | RRab | RRc | RRab | RRab |
| Blazhko Affected? | False | False | False | True | False |
| Period (d) | 0.4763 | 0.5996 | 0.3149 | 0.5274 | 0.4837 |
| \([Fe/H] \) | 0.15 | -0.49 | 1.39 | -1.49 | -1.33 |
| \( \mu_{Post} \) | 10.4730 ± 0.1384 | 10.0551 ± 0.1326 | 9.9779 ± 0.1252 | 10.2164 ± 0.1282 | 11.1113 ± 0.1258 |
| \( E(B - V)_{Post} \) | 0.0997 ± 0.0029 | 0.0226 ± 0.0012 | 0.0230 ± 0.0012 | 0.2783 ± 0.0068 | 0.0208 ± 0.0010 |
| \( m_U \) | ... | 11.6752 ± 0.0026 | ... | ... | ... |
| \( m_B \) | ... | 11.3352 ± 0.0017 | ... | ... | ... |
| \( m_{hipp} \) | 11.6734 ± 0.0059 | 11.0439 ± 0.0038 | 10.7220 ± 0.0034 | 11.5419 ± 0.0059 | 11.8920 ± 0.0104 |
| \( m_V \) | ... | 10.8694 ± 0.0016 | ... | ... | ... |
| \( m_R \) | ... | 10.5775 ± 0.0018 | ... | ... | ... |
| \( m_I \) | ... | 10.3370 ± 0.0075 | ... | ... | ... |
| \( m_J \) | ... | 10.7343 ± 0.0066 | ... | ... | ... |
| \( m_H \) | ... | 10.0556 ± 0.0065 | ... | ... | ... |
| \( m_K \) | ... | 9.7284 ± 0.0059 | ... | ... | ... |
| \( m_{W1} \) | 10.2375 ± 0.0057 | 9.5697 ± 0.0047 | 9.7129 ± 0.0042 | 9.9828 ± 0.0067 | 10.6980 ± 0.0062 |
| \( m_{W2} \) | 10.2493 ± 0.0053 | 9.5920 ± 0.0043 | 9.7212 ± 0.0041 | 9.9809 ± 0.0114 | 10.7121 ± 0.0085 |
| \( m_{W3} \) | 10.2210 ± 0.0539 | 9.5304 ± 0.0259 | 9.6652 ± 0.0235 | 9.9081 ± 0.0361 | ... |

| Posterior Inferences |
|-----------------------|
| \( \mu_{Post} \) | 10.5864 ± 0.0135 | 10.1514 ± 0.0146 | 9.9580 ± 0.0167 | 10.4202 ± 0.0132 | 11.0736 ± 0.0136 |
| \( E(B - V)_{Post} \) | 0.1469 ± 0.0157 | 0.0955 ± 0.0088 | 0.0456 ± 0.0157 | 0.1656 ± 0.0159 | 0.0682 ± 0.0158 |
| \( M_U \) | ... | 1.0458 ± 0.0433 | ... | ... | ... |
| \( M_B \) | ... | 0.7872 ± 0.0377 | ... | ... | ... |
| \( M_{hipp} \) | 0.5973 ± 0.0017 | 0.5649 ± 0.0014 | 0.6120 ± 0.0522 | 0.5697 ± 0.0517 | 0.5908 ± 0.0522 |
| \( M_V \) | ... | 0.4189 ± 0.0297 | ... | ... | ... |
| \( M_R \) | ... | 0.1768 ± 0.0258 | ... | ... | ... |
| \( M_I \) | ... | 0.0077 ± 0.0220 | ... | ... | ... |
| \( M_J \) | ... | 0.4830 ± 0.0197 | ... | ... | ... |
| \( M_H \) | ... | -0.1827 ± 0.0171 | ... | ... | ... |
| \( M_K \) | ... | -0.4775 ± 0.0160 | ... | ... | ... |
| \( M_{W1} \) | -0.3746 ± 0.0144 | -0.5036 ± 0.0153 | -0.2530 ± 0.0170 | -0.4663 ± 0.0145 | -0.3875 ± 0.0146 |
| \( M_{W2} \) | -0.3529 ± 0.0143 | -0.5697 ± 0.0151 | -0.2418 ± 0.0170 | -0.4570 ± 0.0172 | -0.3688 ± 0.0158 |
| \( M_{W3} \) | -0.3681 ± 0.0555 | -0.6232 ± 0.0297 | -0.2938 ± 0.0288 | -0.5158 ± 0.0385 | ... |
\( U(0, 2.5 \times E(B - V)_{\text{SF}}) \). Otherwise, if \( E(B - V)_{\text{SF}} \leq 0.125 \), then the prior distribution was set to be \( U(0, 0.125) \).

Prior distributions for the calibrator distance moduli were derived as in Klein et al. (2011) and Klein et al. (2014). Hipparcos photometry (Perryman & ESA 1997) were transformed into \( V \)-band (Gould & Popowski 1998), corrected for dust extinction (using the line-of-sight extinction from Schlafly & Finkbeiner 2011 and the \( R \) factor from Schultz & Wiemer 1975), and combined with the Chaboyer (1999) \( M_V - [\text{Fe/H}] \) relation to yield prior distance moduli, \( \mu_{\text{Prior}} \). Precise trigonometric parallax angles for four of the stars (RRLyrae, UVOct, XZCyg, and Sudra; all of the RRab sub-class) have been previously measured with the Hubble Space Telescope (HST) and published (Benedict et al. 2011).\(^3\) For these four stars the more precise, parallax-derived distance moduli were used in the period–magnitude relation fits. We note that the distance moduli derived from the metallicity–magnitude relation for these four stars is in statistical agreement (within 2\( \sigma \)) with the parallax-derived distances.

### 2.2 Hipparcos photometry

The European Space Agency Hipparcos astrometry satellite was launched in August 1989 and operated until March 1993, ultimately producing a catalogue of photometry, parallax, and, in the case of variable stars, light curves, published in Perryman & ESA (1997). Hipparcos obtained light curves for 186 RR Lyrae stars, 134 of which serve as the calibration sample for the period–magnitude relations derived in this work.

Since Hipparcos was primarily an astrometry mission, its imaging detector used a broadband visible light passband, defined primarily by the response function of the detector, an unfiltered S20 image dissector scanner. Bessell (2000) characterizes the Hipparcos waveband, commonly referred to as \( H_P \). Throughout this work, to reduce potential confusion with the near-infrared \( H \)-band, the Hipparcos waveband is referred to as \( \text{hipp} \). The effective wavelength of the \( \text{hipp} \) waveband is taken to be 0.517 \( \mu \)m, and the bandpass itself is substantially broader than \( V \)-band (see Fig. 2 of Bessell 2000).

Hipparcos was a temporally dense all-sky survey, and thus it provides the most complete and numerous light curve data for the RR Lyrae calibration sample. All 134 calibrator stars have \( \text{hipp} \) light curves, which are composed of 11,822 epochs.

### 2.3 Optical photometry

Ground-based optical light curves were obtained with the Nickel 1-m telescope and Direct Imaging Camera at Lick Observatory in California. Imaging data was collected in the \( U, B, V, R, I, \) and Sloan Digital Sky Survey (SDSS) \( z \) wavebands during 26 nights between 2010 May 4 and 2013 February 4. Standard image reduction was conducted using common Python scientific computing modules [using PyFITS (Barrett & Bridgman 1999) for image reading and writing] and aperture photometry was measured with SExtractor (Bertin & Arnouts 1996). Photometric calibration was performed using observations of Landolt standards in the \( U, B, V, R, \) and \( I \) wavebands (Landolt 1992, updated by Landolt 2009), and SDSS standards for the \( z \) waveband (Smith et al. 2002).

The Direct Imaging Camera filter wheel could only accommodate four filters at one time, and so preference was given to \( U, B, V, \) and \( R \) for the first 21 nights (before 2012). The \( I \) and \( z \) filters replaced the \( U \) and \( B \) filters in the 5 observing nights after and including 2012 November 6. The targets for these last 5 nights were repeats of stars already observed during the first 21 nights, and the primary purpose was to supplement the calibration waveband coverage of the sample.

In the \( U \)-band 22 light curves were obtained, consisting of 1409 epochs. In the \( B \)-band 24 light curves were obtained, consisting of 1599 epochs. In the \( V \)-band 25 light curves were obtained, consisting of 1991 epochs. In the \( R \)-band 25 light curves were obtained, consisting of 2031 epochs. In the \( I \)-band 9 light curves were obtained, consisting of 410 epochs. And, in the \( z \)-band 9 light curves were obtained, consisting of 400 epochs.

### 2.4 Near-infrared photometry

Observations in the \( J, H, \) and \( K_{\text{short}} \) (herein abbreviated simply as \( K \)) wavebands were conducted between 2009 April 14 and 2011 May 18 with the 1.3-m Peters Automated Infrared Telescope (PAIRTEL; Bloom et al. 2006) at Fred Lawrence Whipple Observatory in Arizona. PAIRTEL was the robotized 2MASS North telescope mated with the repurposed 2MASS South camera. As such, the near-infrared wavebands used in the present work are identical to the 2MASS photometric system, and photometric calibration was conducted using reference stars contained within the same 8.53’ × 8.53’ field of view. The near-infrared images were reduced and coadded with the software pipeline described in the following subsection. Aperture photometry was measured with SExtractor.

In the \( J \)-band 18 light curves were obtained, consisting of 1293 epochs. In the \( H \)-band 17 light curves were obtained, consisting of 1247 epochs. And, in the \( K \)-band 22 light curves were obtained, consisting of 1512 epochs.

#### 2.4.1 PAIRTEL reduction pipeline

Because PAIRTEL reused the 2MASS camera and unaltered readout electronics, each epoch consisted of multiple exposure triplets separated by \( \sim \)dozen seconds during which a small dither offset was enforced. Each single exposure in the triplet had an exposure time of 7.8 s. A single epoch generally consisted of 8 or 9 triplets, making for a total integration time of \( \sim 3 \) to \( \sim 3.5 \) minutes.

In support of this work on RR Lyrae period–magnitude relations, as well as the prime science goal of PAIRTEL to followup gamma-ray burst (GRB) afterglows, a new image reduction and co- addition pipeline was developed for the robotic telescope and deployed for near-real time operation. This software was the third and final reduction pipeline...
developed for **PAIRITEL**. It operated autonomously in concert with the telescope as new data was gathered each night, often providing reduced and coadded images within a few minutes of the end of an observation. This was particularly beneficial for quickly reacting to GRBs and issuing GCN circulars. The reduction pipeline also provided invaluable near-real time diagnostic information for the telescope supervisors when troubleshooting mechanical, technical, or telescope control system-related faults.

The 2MASS camera uses two dichroics and three near-infrared detectors to simultaneously record the $J$, $H$, and $K$ exposures. For the most part, the reduction pipeline operates on each waveband independently. However, because the images are taken simultaneously in each band, the relative and absolute astrometric solutions for the images need only be solved for $J$ and can then be applied to the two longer-wavelength (and less sensitive) $H$ and $K$ exposures.

The constrained image readout mode of **PAIRITEL** dictated much of how the reduction pipeline operated. Each 7.8 s integration of a triplet exposure (called a “long read”) was preceded by a “short read” of 0.051 s. The short read served as a bias read for the long read, and was subtracted from the long read as the first step in the reduction process. The short reads themselves were also processed to produce final coadded images with very short total exposure times. The advantage of processing the short reads is recovery of extremely bright sources that otherwise saturate in the long reads. This was the intended avenue for photometering the nearby bright RR Lyrae stars, such as RRLyr itself, but ultimately the photometric precision recoverable from the reduced and coadded short reads was found to be unacceptable.

In the near-infrared, the brightness of the atmosphere is significant and must be subtracted to improve the signal-to-noise ratio of astrophysical sources. The reduction pipeline creates median sky background images by masking pixels suspected to fall on sources and stacking temporally adjacent images. The sky brightness fluctuations on 5- to 10-minute timescales, so for a given “target” exposure the pipeline uses the images recorded within ±5 minutes to create this median sky flux image. (Of course, if the target exposure is within 5 minutes of the beginning or end of the observation period, then fewer adjacent images contribute to its sky flux image.)

It was found that the detector response varied significantly, and in a correlated manner, with the read-cycle position of the long reads in the triplet exposures. To account for this, a different sky flux image is produced for each of the three long reads in a triplet exposure, wherein only the first long read of each contributing triplet exposure is combined into the sky flux image corresponding to the first long read of the target triplet exposure, and so on for the second and third reads in the cycle.

The accuracy of this sky brightness subtraction procedure relies heavily upon correctly masking pixels containing flux from astrophysical sources from contributing to the median sky flux image. The reduction pipeline runs this sky subtraction procedure twice, first with a preliminary source pixel mask and then with a more refined, and conservative, source pixel mask constructed from the images resulting from summing long reads of each triplet exposure after subtracting the first iteration of the sky flux images. The source pixel masks were generated by employing a median absolute deviation outlier detection algorithm in combination with the objects check image output from SExtractor. The raw source pixel masks were then Gaussian smoothed (blurred) to expand the masked pixel area and account for diffuse emission from extended sources and the telescope’s PSF. The dither steps between each triplet exposure were large enough to “step over” the footprints of unsaturated (and most saturated) point sources, as well as most galaxies with radii $\leq 30''$.

After the sky flux subtraction, each triplet exposure is directly pixel-wise summed to create a “triplestack”. Each pixel is $2'' \times 2''$, and the telescope jitter was far smaller, so this does not result in any significant smearing. The final step in the reduction process is to coadd the images and produce mosaics, but before this can be done the relative dither offsets must be measured from the pixel data and written into the FITS header WCS keywords. Note that an absolute astrometric solution is not necessary at this step, only a WCS solution that incorporates precisely correct relative offsets between the triplestacks. To accomplish this, the reduction pipeline runs SExtractor on each $J$-band triplestack and analyzes the resultant catalogs to identify the deepest triplestack image. This deep triplestack, generally the image with the most well-detected sources, serves as the reference image from which the pixel offsets of the other images in the sequence are measured. The relative sky position offsets and rotations between the $J$, $H$, and $K$ detectors are well known and constant, so it is only necessary to measure the offsets in the $J$-band triplestack sequence.

A normalized cross-correlation image-alignment program (specially developed by E. Rosten) is used to measure the pixel offsets between the reference triplestack and all other images in the sequence. In addition to the image pair, the alignment program also requires an approximate pixel offset (derived from the telescope control system’s imprecise pointing data) and a search box width. The computed pixel offsets are accurate at the sub-pixel level.

With relative pixel offsets in hand, the reduction pipeline writes appropriate WCS information into the FITS headers of the $J$, $H$, and $K$ triplestack sequences and then uses Swarp (Bertin et al. 2002) to median-combine and mosaic the reduced imaging data. In the mosaicing process the pixel resolution is changed from $2''$ to $1''$. The final astrometry is solved using Astrometry.net (Lang et al. 2010), although sometimes the pipeline falls back on Scamp (Bertin 2006) and then, if Scamp also fails, a specifically-developed pattern-matching Python program is employed.

### 2.5 WISE photometry

Mid-infrared light curve photometry data were obtained from the AllWISE Data Release of the **Wide-field Infrared Survey Explorer** (**WISE**) and its extended NEOWISE mission (Wright et al. 2010; Mainzer et al. 2011). **WISE** provides imaging data in four mid-infrared wavebands: $W_1$ centred at 3.4 $\mu$m, $W_2$ centred at 4.6 $\mu$m, $W_3$ centred at 12 $\mu$m, and $W_4$ centred at 22 $\mu$m. Although the original **WISE** mission was designed for static science goals, the orbit and survey strategy of the **WISE** spacecraft (described in Wright et al. 2010) are highly conducive to recovering light curves of pe-
3 LIGHT CURVE ANALYSIS METHODS

The light curve analysis methods employed in this work are an evolution of those described in Klein et al. (2011) and Klein et al. (2014). Each band-specific light curve is parametrically resampled (assuming a normal distribution) 500 times to fit 500 harmonic models using the adopted pulsation period from Fernley et al. (1998). Thus, 500 realizations of the mean-flux magnitude are measured, and the standard deviation of this distribution is taken to be the uncertainty on the mean-flux magnitude. These are the observed mean-flux magnitudes reported in Table 1 and are not corrected for interstellar extinction.

The 500 harmonic models generated by the bootstrapping procedure were averaged to produce a mean harmonic model. Fig. 1 shows the phase-folded light curve data and mean harmonic models for ABUMa (which was specifically selected to show a well-observed RR Lyrae calibration star with complete 13-waveband data).

The mean harmonic model yields a robust light curve amplitude. Furthermore, the standard deviation of the 500 harmonic models at each phase value provides a metric of how well the shape of the true light curve is recovered in the photometry data (if there is a lot of spread in the distribution of harmonic models, then the photometry is not accurate enough to reveal the shape of the true brightness oscillation). To improve the quality of the dataset used in the period–magnitude relation fits, any light curve with a bootstrapped harmonic model maximum standard deviation larger than its robust amplitude measurement was excluded. This procedure serves to ensure that only stars with light curves well-fit by the harmonic model (i.e., those exhibiting clear sinusoidal-like oscillation) are used in the period–magnitude relation fits.

The summary information given above in Section 2 for periodic variables with periods $\lesssim 1.5$ d, which is well-matched to RR Lyrae variables.

The AllWISE Data Release (made public 2013 November 13) combines the 4-Band Cryogenic Survey (main WISE mission covering the full sky 1.2 times from 2010 January 7 to 2010 August 6), the 3-Band Cryogenic survey (first three wavebands, 30 per cent of the sky from 2010 August 6 to 2010 September 29), and the NEOWISE post-cryogenic survey (first two wavebands, covering 70 per cent of the sky from 2010 September 20 to 2011 February 1). The individual photometry epochs were retrieved from the AllWISE Multiepoch Photometry Database.

WISE, like Hipparcos, was an all-sky survey and thus the AllWISE Data Release provides very good coverage of the calibration sample. In the W1-band 126 light curves were obtained, consisting of 4202 epochs. In the W2-band 127 light curves were obtained, consisting of 4204 epochs. And, in the W3-band 79 light curves were obtained, consisting of 1510 epochs. Significantly fewer stars were detected and provided light curves accepted into the calibration sample in W3 because the W3 detector was not as sensitive and was not operating for the NEOWISE period. Additionally, all W4 data are rejected from the present work because only the few brightest calibration RR Lyrae stars were detected in that bandpass.

4 PERIOD–MAGNITUDE RELATIONS

The present derivation of period–magnitude relations is similar to the Bayesian approach first described in Klein et al. (2011) and later formalised in Klein et al. (2012). A significant advancement over previous implementations is the inclusion of colour excess as a model parameter. Our statistical model of the period–magnitude relationship is

$$m_{ij} = \mu_i + M_{0,j} + \alpha_j \log_{10} \left( \frac{P_i}{P_0} \right) + (B-V)_i \left( a_j R_V + b_j \right) + \epsilon_{ij},$$

where $m_{ij}$ is the observed apparent magnitude of the $i$th RR Lyrae star in the $j$th waveband, $\mu_i$ is the distance modulus for the $i$th RR Lyrae star, $M_{0,j}$ is the absolute mag-
nitude zero point for the jth waveband, $\alpha_j$ is the slope in the jth waveband, $P_i$ is the fundamental period of the ith RR Lyrae star in days, $P_0$ is a period normalisation factor (we use the mean fundamental period of the calibration sample, $P_0 = 0.52854$ d), $E(B - V)_i$ is the colour excess of the ith RR Lyrae star, $a_j$ and $b_j$ are the wavelength-specific parameters for the interstellar extinction law defined in Cardelli et al. (1989), $R_V$ is the extinction law factor ($R_V = A_V/E(B - V)$) with value equal to 3.1 for the diffuse interstellar medium adopted from Schultz & Wiemer (1975), and the $\epsilon_{ij}$ error terms are independent zero-mean Gaussian random deviates with variance $(\sigma_{\text{intrinsic},ij}^2 + \sigma_{\text{m}_j}^2)$. We note that the $\epsilon_{ij}$ error terms are defined differently than in previous work to allow for the model to fit wavelength-dependent intrinsic period–magnitude relation scatter ($\sigma_{\text{intrinsic},j}$). This additive error term, which we call the intrinsic scatter, describes the residual about the best-fit period–magnitude relation in each waveband which cannot be accounted for by instrumental photometric error. Such scatter would naturally be expected if there are unmodelled wavelength-sensitive dependencies (such as with metallicity) on the period–magnitude relation.

To perform the Bayesian regression a design matrix $X$ is constructed for the model expressed in Equation 3. $X$ has dimensions $637 \times 294$. Each of the 637 light curves produced one mean-flux magnitude measurement which is represented by a row in $X$. The terms in Equation 3 with $i$-dependence ($\mu_i$ and $E(B - V)_i$, where each RR Lyrae star is fit with one value) each require 134 columns. And, the terms in Equation 3 with $j$-dependence ($M_0,j$ and $\alpha_j$, where each waveband is fit with one value) each require 13 columns.

We define the vector of model parameters, $b$, which contains the 134 values of $\mu_i$, the 13 values of $M_0,j$, the 13 values of $\alpha_j$, and the 134 values of $E(B - V)_i$. The vector of observed mean-flux magnitudes, $m_{\text{obs}}$, is then given by the dot product of the design matrix and the vector of model parameters,

$$m_{\text{obs}} = X \cdot b.$$  \hspace{1cm} (4)

The model parameters are fit by an implementation of MCMC sampling (4.1) that iteratively refines the distributions of the model parameters until a converged steady-state is achieved. The fitting algorithm is run with the PyMC (Patil et al. 2010) Python module, which leverages the distribution of the observed data vector $m_{\text{obs}}$ with variance given by $(\sigma_{\text{intrinsic},j}^2 + \sigma_{\text{m}_j}^2)$, as well as the model parameter vector $b$ and the associated variance on each model parameter.

Initially $b$ is populated with prior distributions and the MCMC sampling traces are run until convergence, after which 50,000 additional samples are drawn to record the fitted model parameter distributions (also called the posteriors). To avoid inappropriate biasing of the posterior distributions for the slope and intercepts, a wide normal distribution is adopted:

$$M_{0,j}^{\text{prior}} = \mathcal{N}(0, 2^2),$$  \hspace{1cm} (5)

$$\alpha_j^{\text{prior}} = \mathcal{N}(0, 5^2).$$  \hspace{1cm} (6)

The prior distributions for distance modulus and colour excess are star-dependent and given in subsection 2.1.

The summary results for the simultaneous 13-waveband period–magnitude relation fits are provided in Table 2. In the ensuing subsections more detail is provided for the execution of the MCMC fitting procedure, the posterior joint distributions for the 13 (zero point, intercept) pairs are illustrated and explained, and the comprehensive log$_{10} (P)_M$ plot and a validation $\mu_{\text{prior}} - \mu_{\text{post}}$ plot are furnished.

### 4.1 MCMC fitting details

Seven MCMC sampling traces of the model fit were produced, each iterating 25,200,000 steps and thinned by a factor of 252 to result in traces with 100,000 iterations. As an illustrative example, Figs. 2, 3, and 4 show trace plots for the $H$-band $\sigma_{\text{intrinsic}}$, $M_0$, and $\alpha$, respectively. Additionally, trace plots for the $\mu$ and $E(B - V)$ of ABUMa are shown respectively in Figs. 5 and 6.

The traces are considered to be converged after 50,000 iterations, and these converged portions of each of the seven traces are combined to form a posterior distribution for each model parameter of 350,000 samples. Convergence is verified by computing the Gelman-Rubin multiple sequence convergence diagnostic, $\hat{R}$, (Gelman & Rubin 1992) and ensuring $\hat{R} \lesssim 1.1$ in the portion of the chains considered to be converged. The Gelman-Rubin diagnostic factor is the square root of the weighted sum of the within chain variance, $W$, and between chain variance, $B$, divided by the within chain variance. Here,

$$\hat{R} = \sqrt{\frac{(1 - \frac{1}{m}) W + (\frac{1}{m}) B}{W}},$$  \hspace{1cm} (7)

where $n$ is the of length each chain. In Figs. 2 through 6 the Gelman-Rubin diagnostic is displayed for the first 10,000 iterations (demonstrating the lack of convergence early in the MCMC sampling chain) and also for the final 50,000 iterations (where the traces are considered to be converged).

### 4.2 Zero point and slope joint distributions

One significant advantage of a Bayesian approach to linear regression over frequentist methods is that the posterior model parameters are sampled from final joint distributions. Thus, any covariance in the distributions is accurately recorded and the traditional assumption of Gaussian behaviour is not necessary, but can instead be tested. Indeed, the posterior $M_0$ and $\alpha$ distributions are generally well-approximated by Gaussians, but some waveband-specific pairs exhibit covariance. Figs. 7 through 19 display the posterior contour density plots and histograms for the zero point and slope of the 13 waveband-specific period–magnitude relations.

The pronounced covariance between $M_0$ and $\alpha$ observed for the $I$ and $z$ wavebands is primarily caused by the lopsided distribution of the periods of the RR Lyrae stars for which $I$- and $z$-band data were obtained. Only three of the nine stars observed in these wavebands have $P < P_0$, and thus the covariance between the linear regression intercept and slope was not well-removed.
Table 2. Period–magnitude relation parameters and 1-σ uncertainties. The band-specific form of the period–magnitude equation is $M = M_0 + \alpha \log_{10}(P/P_0)$, where $P_0 = 0.52854$ d. $\sigma_{\text{instrumental}}$ is the average photometric uncertainty for the mean-flux magnitudes in each band and is dominated by the quality of the light curve data (although individual light curve consistency of each star does contribute). Note that $\sigma_{\text{instrumental}}$ is not a model parameter, but is provided in the table for direct comparison with $\sigma_{\text{intrinsic}}$, which is the fitted intrinsic scatter of the period–magnitude relation in each waveband. Fig. 2 plots both $\sigma_{\text{intrinsic}}$ and $\sigma_{\text{instrumental}}$ as a function of wavelength.

| band | $M_0$ (intercept) | $\alpha$ (slope) | $\sigma_{\text{intrinsic}}$ | $\sigma_{\text{instrumental}}$ |
|------|------------------|-----------------|-----------------------------|-------------------------------|
| $U$  | $0.9304 \pm 0.0584$ | $-0.3823 \pm 0.7130$ | $0.2358 \pm 0.0438$ | $0.0232 \pm 0.0175$ |
| $B$  | $0.7099 \pm 0.0237$ | $0.0129 \pm 0.3104$ | $0.0553 \pm 0.0126$ | $0.0145 \pm 0.0118$ |
| $hipp$ | $0.5726 \pm 0.0174$ | $-0.4625 \pm 0.2246$ | $0.0474 \pm 0.0079$ | $0.0098 \pm 0.0085$ |
| $V$  | $0.4319 \pm 0.0184$ | $-0.4091 \pm 0.2370$ | $0.0320 \pm 0.0079$ | $0.0106 \pm 0.0085$ |
| $R$  | $0.2638 \pm 0.0164$ | $-0.7461 \pm 0.2108$ | $0.0274 \pm 0.0072$ | $0.0091 \pm 0.0067$ |
| $I$  | $0.1065 \pm 0.0380$ | $-1.0456 \pm 0.4285$ | $0.0713 \pm 0.0264$ | $0.0188 \pm 0.0170$ |
| $z$  | $0.5406 \pm 0.0539$ | $-0.8770 \pm 0.6547$ | $0.1153 \pm 0.0432$ | $0.0175 \pm 0.0184$ |
| $J$  | $-0.1490 \pm 0.0153$ | $-1.7138 \pm 0.1834$ | $0.0385 \pm 0.0081$ | $0.0058 \pm 0.0017$ |
| $H$  | $-0.3509 \pm 0.0148$ | $-2.1936 \pm 0.1752$ | $0.0312 \pm 0.0068$ | $0.0060 \pm 0.0015$ |
| $K$  | $-0.3472 \pm 0.0160$ | $-2.4599 \pm 0.1849$ | $0.0498 \pm 0.0089$ | $0.0071 \pm 0.0019$ |
| $W1$ | $-0.4703 \pm 0.0112$ | $-2.1968 \pm 0.1252$ | $0.0032 \pm 0.0020$ | $0.0050 \pm 0.0013$ |
| $W2$ | $-0.4583 \pm 0.0112$ | $-2.2337 \pm 0.1249$ | $0.0055 \pm 0.0018$ | $0.0053 \pm 0.0016$ |
| $W3$ | $-0.4924 \pm 0.0119$ | $-2.3026 \pm 0.1342$ | $0.0227 \pm 0.0036$ | $0.0350 \pm 0.0291$ |

Figure 2. MCMC traces of intrinsic scatter, $\sigma_{\text{intrinsic}}$, for the $H$ waveband. All seven traces of 100,000 samples each are plotted simultaneously, coloured by trace. The left panel shows the normalized histogram of the first 10,000 samples from each trace and the right panel shows the normalised histogram of the last 50,000 samples from each trace. In each histogram panel the Gelman-Rubin convergence diagnostic, $\hat{R}$, (Gelman & Rubin 1992) is given. $\hat{R}$ should converge to 1 and traces are generally considered converged when $\hat{R} \leq 1.1$. The first 50,000 samples are rejected as burn-in and the last 50,000 samples are considered to be drawn from the converged posterior distribution.

Figure 3. MCMC traces of $M_0$ for the $H$ waveband. Panels formatted as in Fig. 2.
Figure 4. MCMC traces of $\alpha$ for the $H$ waveband. Panels formatted as in Fig. 2.

Figure 5. MCMC traces of $\mu$ for ABUMa. Panels formatted as in Fig. 2.

Figure 6. MCMC traces of $E(B-V)$ for ABUMa. Panels formatted as in Fig. 2.
Figure 7. Contour density plot and histograms for the $U$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 8. Contour density plot and histograms for the $B$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 9. Contour density plot and histograms for the hipp band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 10. Contour density plot and histograms for the $V$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.
Multi-band RR Lyrae period–magnitude relations

Figure 11. Contour density plot and histograms for the $R$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 12. Contour density plot and histograms for the $I$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions. The exhibited correlation in these parameters for $I$ band is caused by the lopsided period distribution of the RR Lyrae variables for which $I$ band data was obtained (c.f. Fig. 20, only three of nine stars have $P < P_0$).

Figure 13. Contour density plot and histograms for the $z$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions. As in Fig. 12, a correlation in these parameters is obvious. The explanation, an uneven period distribution about $P_0$ in the subset of RR Lyrae stars for which $z$ band data was obtained, is the same as for $I$ band (Fig. 12).

Figure 14. Contour density plot and histograms for the $J$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.
Figure 15. Contour density plot and histograms for the $H$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 16. Contour density plot and histograms for the $K$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 17. Contour density plot and histograms for the $W1$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.

Figure 18. Contour density plot and histograms for the $W2$ band period–magnitude relation magnitude intercept ($M_0$) and slope ($\alpha$). The red circle with associated error bars shows the means and standard deviations of the posterior $M_0$ and $\alpha$ distributions.
as a slow cyclic evolution of the light curve shape, with a period ranging from weeks to months (Smith 1995, chapter 5.2). The nature of the Blazhko effect, a second-order amplitude modulation, does not result in a significant impact on a star’s mean-flux magnitude. In Fig. 20 Blazhko-affected stars, as identified via http://www.univie.ac.at/tops/blazhko/Blazhkolist.html, are shown with diamonds and stars without confirmed evidence of the Blazhko effect are shown with squares.

Because of the longer-period nature of the effect, observational investigations of the RR Lyrae Blazhko effect require considerable telescope resources. To our knowledge, no such investigations have been carried out in near- or mid-infrared wavebands. That the amplitude distribution of RR Lyrae stars is significantly reduced in the near- and mid-infrared, as compared to optical bands, suggests that the magnitude of the Blazhko effect will also be diminished in the infrared (Gavrilchenko et al. 2013). However, observational studies are required to test this hypothesis. In the present analysis, and as indicated in Fig. 20, there is no significant impact on the period–magnitude relation by the inclusion of Blazhko-affected stars in the fit.

As a commonsense check on the period–magnitude relations of Fig. 20 and the applied simultaneous Bayesian linear regression fitting method, a plot of the prior distance moduli versus the posterior distance moduli for the calibration sample is provided in Fig. 21. Any bias or a strongly non-normal distribution of the residuals would indicate overfitting. Since the distance modulus is treated as a model parameter to be fit, it is very important that the fitting method respects the original prior distance modulus values. Fig. 21 shows that the prior and posterior distances are in very good agreement. Specifically, 112 out of the 134 residuals that lie within one-errorbar length of zero. The errors on the posterior distance moduli may thus be slightly overestimated. The null hypothesis that the posterior-prior distance modulus residuals are drawn from a standard normal distribution is accepted by a Kolmogorov-Smirnov test with \( p = 0.2 \).

In Fig. 21 RRLyr itself is the star with the lowest distance modulus. The fitted posterior distance modulus of RRLyr is \( \mu_{\text{Post}} = 6.9962 \pm 0.0143 \) with a prior distance modulus, derived from the measured HST parallax, of \( \mu_{\text{Prior}} = 7.130 \pm 0.075 \). For RRLyr, the residual significance is \( -1.75\sigma \).

## 5 Further Discussion of the Fits

The complex model used in the period–magnitude relation fits (described above in Section 4), which newly incorporates colour excess and intrinsic scatter, allows for a deeper analysis of the results. In the following subsections we present the fit results as spectral energy distributions (SEDs), compare the fitted period–magnitude relation intrinsic scatter and mean photometric error as a function of wavelength, analyse the colour excess results more closely, and discuss the evolution of period–magnitude relation slope with wavelength.
Figure 20. Multi-band period–magnitude relations. RRab stars are in blue, RRc stars in red. Blazhko-affected stars are denoted with diamonds, stars not known to exhibit the Blazhko effect are denoted with squares. Solid black lines are the best-fitting period–magnitude relations in each waveband and dashed lines indicate the 1-σ prediction uncertainty for application of the best-fitting period–magnitude relation to a new star with known period. The noted scatter, σ associated with each band in the figure, is the minimum prediction uncertainty, which is where the dashed line “bowtie pinch” around $P_0 = 0.52854$ d.
5.1 RR Lyrae spectral energy distributions

The period–magnitude relation fits provide the absolute magnitudes (at time of mean-magnitude) of the typical RR Lyrae star in 13 wavebands as a function of period. Another way to present, and think about, this result is by converting the fits to SEDs for RR Lyrae stars at selected periods. This is demonstrated in Fig. 22, along with two model stellar spectra (light grey lines) selected with temperatures and radii to bracket the ranges of these parameters inferred in the RR Lyrae population.

This plot of SEDs for RR Lyrae stars of various fundamental periods illustrates why the period–magnitude relations at optical wavebands (near the SED peak) have a shallower slope than at infrared wavebands (along the Rayleigh-Jeans tail). The vertical distance between two SEDs tracks with the slope of the period–magnitude relation. This vertical distance between the brightest SED (longest period RR Lyrae star) and the dimmest SED (shortest period) is effectively zero shortward $R$-band, and then this distance increases with increasing wavelength until the SEDs become nearly parallel in the Rayleigh-Jeans tail. This near-parallel property of the SEDs in the infrared graphically explains why the period–magnitude relation slope approaches an asymptote with increasing wavelength.

5.2 Intrinsic scatter and photometric error

As described in Section 4 the model used in the period–magnitude relation fits allows for investigation of the intrinsic scatter, and of particular importance is the comparison between intrinsic scatter and the photometric error on the mean-flux magnitude measurements. Fig. 23 shows both intrinsic scatter and mean photometric error as a function of wavelength. At any given wavelength, the maximum of the intrinsic scatter and mean photometric error provides a floor to how tightly the resultant period–magnitude relation can be constrained (and, in effect, sets the precision limit of distance measurements).

If the photometric error dominates over the intrinsic scatter, then a tighter period–magnitude relation can be derived by collecting better light curve data (i.e., with more sensitive instruments and/or more observation epochs).
However, if the intrinsic scatter exceeds the mean photometric error already achieved, then the path towards a tighter period–magnitude relation is not as direct. In this latter case, the period–magnitude relation scatter can be reduced slowly via the augmentation of the calibration sample, but the intrinsic scatter will always dictate the minimum absolute magnitude uncertainty when applying the relation to new stars. The inclusion of a spectroscopically derived metallicity as an additional model parameter could, of course, serve to reduce the intrinsic scatter (e.g., Sandage & Tammann 2006) but we expressly have used a model based upon photometry alone.

Fig. 23 shows that the intrinsic scatter exceeds the photometric error for all wavebands except $W_1$ and $W_2$. This explains why the minimum prediction uncertainty given in Fig. 20, $\sigma$ along the right hand side, is lowest for these wavebands. Furthermore, this finding indicates that continued development and application of RR Lyrae period–magnitude relations at wavebands between 3 and 5 $\mu$m will produce the tightest absolute magnitude constraints.

5.3 Colour excess results

A major improvement to the model fit in the present multi-band period–magnitude relations derivation is simultaneously fitting for colour excess to each of the calibration stars. This is not feasible with light curve data for only one wavelength regime (such as the work published in Klein et al. 2011 and Klein et al. 2014). However, the present investigation spans the optical, near-infrared, and mid-infrared wavelength regimes, and this enables colour excess to be treated as a model parameter.

An all-sky visualisation in Galactic coordinates of the fitted colour excess values, as well as the distance modulus, is shown in Fig. 24. The colourbar is purposefully asymmetric to better conform to the dynamic range of the colour excess values (most of the values are near 0.08 mag, and very few fall between 0.2 mag and 0.35 mag). An obvious feature of this skymap is the lack of RR Lyrae stars near the Galactic plane. This was enforced by the sample selection criteria discussed in Section 2. A second visual trend is that the stars closer to the plane generally have higher colour excess values than those nearer the poles due to higher concentrations of interstellar dust near the Galactic plane.

To further explore the fitted colour excess values, Fig. 25 shows the residual $E(B-V)$ as a function of the absolute value of Galactic latitude, $b$. It is expected that the prior $E(B-V)_{\text{SF}}$ values at low Galactic latitude are greater than the fitted posterior values, since the prior values represent the full colour excess expected along a line of sight to infinite distance whereas the posterior values follow a line of sight that terminates at the star (which presumably lies in front of much of the dust that contributes to the prior colour excess value). This expectation is indeed seen to hold in Fig. 25 for galactic latitudes less than about 15 deg.
An unexpected feature of Fig. 25 is that the mean residual colour excess does not settle around zero at high galactic latitude. At high latitude a calibrator RR Lyrae star should be behind most of the interstellar dust, and thus the posterior value should approach the prior \( E(B-V)_{SF} \) value. However, the mean residual at \( b > 30^\circ \) is 0.033 ± 0.001 (with scatter about the mean of 0.024), indicating that either the \( E(B-V)_{SF} \) values have a systematic bias, the calibrator RR Lyrae stars are more likely to lie behind more dust than nearby lines of sight as measured in the Schlegel et al. (1998) dust map, the value of \( R_V = 3.1 \) is systematically incorrect, or some combination of all three.

5.4 Period–magnitude relation slope

Fig. 26 depicts the period–magnitude relation slope as a function of wavelength. Gray band is a spline interpolation of the new calibrations presented in this paper.

6 EXAMPLE APPLICATION

Light curve data were obtained for RZCep (RRc star with period 0.308645 d, or fundamentalised period 0.413484 d) in the \( U, B, V, R, I, z, W_1, W_2, W_3 \) wavebands (see Table 3). However, due to low galactic latitude (\( b = 5.5^\circ \)) and high \( E(B-V)_{SF} = 0.9054 \pm 0.0148 \) mag, this star was excluded from the period–magnitude relation fits presented in Section 4.

Estimating the distance to RZCep using the period–magnitude relations is an excellent test of the results because an HST parallax measurement, \( \mu_{HST} = 8.02 \pm 0.17 \), was published as part of Benedict et al. (2011).

To apply the period–magnitude relations and fit for a distance modulus to RZCep, Equation 3 can be rearranged to place the new likelihood information (now including the period–magnitude relation zero point and slope terms) on the left hand side and the formula can be simplified to apply only to RZCep (\( i \) subscripts are dropped). The form of the model used for estimating the distance to a single star is
thus
\[ m_j - M_{0,j} - \alpha_j \log_{10} (P/P_0) = \mu + E(B-V) (a_j R_V + b_j) + \epsilon_j, \]
where now \( \epsilon_j \) is a zero-mean Gaussian random deviate with variance
\[ \sigma^2_{mj} + \sigma^2_{M_{0,j}} + \left[ \sigma_{\alpha_j} \log_{10} (P/P_0) \right]^2 + \sigma^2_{j,intrinsic}. \]

A Bayesian linear regression is fit to solve for the two unknowns, \( \mu \) and \( E(B-V) \), and \( R_V = 3.1 \) is again used as the extinction law factor (c.f. Section 4). The prior distributions should be uninformative and wide [for example, \( \mu \sim U(0,14) \) and \( E(B-V) \sim U(0,2) \)]. The fit can proceed with mean-flux magnitude measurements in only two bands, but obviously additional waveband data will improve the distance prediction accuracy.

For RZCep, applying the period–magnitude relations derived in Section 4 with this Bayesian prediction procedure results in a distance modulus estimate of 
\[ \mu_{PLR} = 8.0397 \pm 0.0123 \] (or 405.4 \pm 2.3 pc). This is a fractional prediction distance error of 0.57 per cent, an improvement of \( \sim 13 \) times the reported HST parallax distance precision (Benedict et al. 2011) and nearly equal to the 14 microarcsec parallax precision (0.57 per cent fractional distance error) Gaia is expected to achieve for bright stellar sources in its end-of-mission analysis (de Bruijne 2012). In solving for the distance prediction, the fit also produces a posterior colour excess value for RZCep, \( E(B-V)_{RZCep} = 0.2461 \pm 0.0089 \). This is significantly less than the line of sight to infinite distance colour excess of \( E(B-V)_{SF} = 0.9054 \), and is very much consistent with RZCep lying only about 400 pc away, even if it is only 5.5\( ^\circ \) off the Galactic plane. This example demonstrates that the multi-band period–magnitude relation can be used to accurately simultaneously fit for an RR Lyrae star’s colour excess and distance modulus using only its period and mean-flux magnitude measurements. Fig. 28 is the contour density plot for the predicted colour excess and distance modulus for RZCep. The anti-correlation is as expected; for a given brightness, a larger colour excess value requires that the star be closer, and vice versa.

In theory, the fitted model of Equation 8 can be modified to also fit for the extinction law factor, \( R_V \). Such a model was constructed and fit, with the prior distribution of \( R_V = \mathcal{N}(3.1,1) \). The posterior distance is essentially unchanged: \( \mu_{PLR} = 8.0394 \pm 0.0128 \) (or 405.4 \pm 2.4 pc). The posterior colour excess is similar, but substantially wider: \( E(B-V)_{RZCep} = 0.2398 \pm 0.0399 \). And, the posterior \( R_V \) is highly covariant with colour excess and very wide: \( R_V,_{RZCep} = 3.2768 \pm 0.5820 \). Fig. 29 shows the contour density plots for these posterior distributions. Unlike in Fig. 28, the colour excess and distance modulus are not apparently anti-correlated, suggesting that it is effectively the overall magnitude of the bandpass-dependent extinction (set by the combination of \( E(B-V) \) and \( R_V \)) which is most directly constrained by the data.

7 DISCUSSION AND CONCLUSIONS

We have applied a simultaneous Bayesian linear regression methodology to 637 mean-flux magnitude measurements of a calibration sample of 134 RR Lyrae stars to derive new, tightly-constrained RR Lyrae period–magnitude relations in 13 wavebands. As part of the regression model, the colour excess, \( E(B-V) \), for each star was also determined. The final result is that the distances to the 134 calibration stars are measured with median fractional error of 0.66 per cent. We showed how the period–magnitude relations can be used singly or in combination through the methodology described...
in Section 6 to derive distances to other observed RR Lyrae stars achieving a similar level of precision.

As part of the multi-band fit, the intrinsic scatter, $\sigma_{\text{intrinsic}}$, for each period–magnitude relation was constrained. Intrinsic scatter is the residual about the best-fit period–magnitude relation in each waveband which cannot be accounted for by instrumental photometric error. It was found that $\sigma_{\text{intrinsic}}$ is minimised for the mid-infrared $\mu$ RZCep from the multi-band period–magnitude relation with $R_V$ fitted as a model parameter (instead of adopting $R_V = 3.1$, as was done for the model that produced Fig. 28). 100,000 samples were generated after the MCMC chain converged. The red circle with associated error bars shows the means and standard deviations of the $E(B - V)$, $R_V$, and $\mu$ distributions.

Figure 29. Contour density plot and histograms for the predicted colour excess, $E(B - V)$, extinction law factor, $R_V$, and distance modulus, $\mu$, of RZCep from the multi-band period–magnitude relation with $R_V$ fitted as a model parameter (instead of adopting $R_V = 3.1$, as was done for the model that produced Fig. 28). 100,000 samples were generated after the MCMC chain converged.}

The application of the derived RR Lyrae period–magnitude relations range from nearby Milky Way structure studies to distance measurements at truly cosmic scales (pushing into the Hubble Flow at $d > 100$ Mpc). Ground-based optical surveys (PanSTARRS, iPTF, Catalina Sky Survey, OGLE IV, LSST, etc.) and the proliferation of near-infrared followup facilities (RATIR, NEWFIRM, UKIRT, etc.) are now enabling studies of Milky Way Field and Halo RR Lyrae stars to produce highly accurate distance measurements. Mid-infrared facilities and surveys (SOFIA, Spitzer Space Telescope, MaxWISE, and in the near future,
JWST) can also be leveraged to significantly improve RR Lyrae distance measurement precision. These studies will use the RR Lyrae period–magnitude relations to map Milky Way stellar density, measure the morphology of remnant tidal streams in the Halo, and probe the depth structure of the Magellanic Clouds.

Additionally, as demonstrated in the present work, combining optical and infrared light curve data for an RR Lyrae star can provide a fit for both distance and colour excess along that line of sight to that distance. Given enough RR Lyrae targets (Eyer et al. 2012 predicts 100,000 RR Lyrae stars in the Milky Way), a 3D dust map can be constructed to better understand the distribution of Milky Way dust grains and to also aide in estimating line-of-sight extinction for studies of other objects within the Milky Way. As a cross check and calibrator, we see precision 3D line-of-sight dust measurements (Bailer-Jones 2011) as complementary to the ongoing all-sky efforts using aggregate stellar populations (Sale 2012; Berry et al. 2012; Hanson & Bailer-Jones 2014; Green et al. 2014), which offer aggregate dust measures over arcminute scales and in wide distance bins. With a significantly larger sample, it will be also possible to test how universal the power-law fits are for different subpopulations of RR Lyrae: there may very well be measurable differences in relations as a function of metallicty, environment and/or population origin (e.g., thick disc vs. bulge).

RR Lyrae stars serve as primary distance indicators in the Cosmic Distance Ladder via their period–magnitude relations. As such, RR Lyrae stars are vital to calibrating the relations used for secondary distance indicators that extend out well beyond the Local Group. Error in distance measurement methods propagates up the distance ladder, and thus minimisation of error at the local end can significantly improve the accuracy of secondary indicators and the derived higher-level measurements, such as $H_0$. This effect, as applied through improving the Cepheid Leavitt Law to better constrain Type Ia supernovae luminosity, has recently been very well utilised by both Riess et al. (2011) and Freedman et al. (2012) in their measurements of $H_0$ with ~ 3 per cent precision.

RR Lyrae stars, in combination with the TRGB method to reach distant supernova host galaxies, offer a systematically separate and competitive means for Type Ia supernovae luminosity calibration. Additional physical distance measurement methods such as this are necessary to help resolve the conflict between the $H_0$ values found by the distance ladder methods of Riess et al. (2011) ($73.8 \pm 2.4$ km s$^{-1}$ Mpc$^{-1}$) and Freedman et al. (2012) ($74.3 \pm 2.1$ km s$^{-1}$ Mpc$^{-1}$), and the statistically significantly lower measurement derived by Planck Collaboration et al. (2013) with Cosmic Microwave Background data from the Planck satellite ($67.3 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$). If our methodology holds up to scrutiny the achievement of sub-1 per cent fractional distance errors (herein, 0.66 per cent for the calibration sample) within a formalism that accounts for dust extinction may be considered a strong buttressing of the path to eventual 1 per cent uncertainties in Hubble’s constant.

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