Domain walls, stabilities, and the mass hierarchy of the Randall-Sundrum Model

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Abstract

Randall-Sundrum model, which has a scalar field, is used to investigate the domain structure of the extra dimension and to obtain a possible solution of the mass hierarchy problem. It is found that when the domain wall size is comparable to that of domains, domains become unstable. To construct a reliable theory, a region of physical parameter space, where domains are stable, is identified. Analytic forms of field configurations are obtained by perturbative expansions in term of a small parameter that is approximately equal to the relative size of domain wall with respect to domains. By placing a single 3-brane in one of the domain, one can solve the mass hierarchy problem.

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Randall-Sundrum model\cite{1} inspired a novel way to the solution of the old mass hierarchy problem, that is, why are the masses of ordinary particles almost negligible compared to the Planck mass? The main idea is that by introducing two worlds\cite{2}, one hidden and another visible, separated by a small distance along an extra dimension, may produce exponentially large Planck mass when branes of two worlds are allowed to interact through the Einstein gravity. In their sequel paper\cite{3} they have shown that the invisible hidden world may be infinitely separated from the visible world, and then the five dimensional gravity interaction produces a single normalizable bound state mode that propagates along the brane where the visible world is\cite{3,4,5}. The continuum modes of massive Kaluza-Klein states contribute a testable correction to the Newtonian limit\cite{6,7,8,9,17}.

After their pioneering works, investigations along various directions were done during the last few years. The detailed analysis of the gravitational fluctuations of the Randall-Sundrum vacuum configuration is one of the next imperative steps\cite{4,5,10}. Another is the inclusion of more than two branes, either thin or thick ones\cite{11,12}. Still another possibility is the introduction of a scalar field which acts as an initiating field of the domain structure\cite{13,14,15}. Ichinose\cite{18} found interesting results that there exists an exact solution of the field configuration when one assumes a series expansion in terms of hyperbolic tangent functions, and that the spacetime has the Randall-Sundrum type vacuum.

In this paper we ask questions such as, “Is it possible to describe the Randall-Sundrum scenario with a single brane, and if it is the case what is the physical meaning of the location of the brane in the extra dimension? Are these field configurations stable? If it is the case, is there any restriction on the values of physical parameters?” To answer this question we make use of a five dimensional Randall-Sundrum type model with a scalar field which Ichinose used. The scalar field, which is different from the Higgs field of the brane world, is introduced to determine the domain structure along the extra dimension. It can be shown that when the flow lines corresponding to the evolutions of phase points dictated by the equations of motion are drawn in a certain phase space, there is a clear-cut
region of a physical parameter space where the domain structure is unstable. One of the two controlling parameters which is critical to the stability is $\epsilon$ which is a measure of the relative size of the domain wall with respect to domains. If the size of domain wall is comparable to that of domains, domains become unstable and tend to squeeze down to the domain wall. But for a relatively narrow domain wall, domains remain stable. In this case it is possible to solve the equations of motion analytically when the usual perturbative expansion in terms of $\epsilon$ is performed. The analytic solutions clearly show the domain structure. When the origin of the extra dimension is chosen to be at the center of the domain wall, the spacetime metric has the correct $AdS_5$ geometry in the domain, and is almost flat at the domain wall. If the relative size of domain wall with respect to domains goes to zero, it reproduces the exact Randall-Sundrum vacuum geometry. By placing a single 3-brane at $y_b$, a location in one of the domains, the Randall-Sundrum type solution of the mass hierarchy problem may be reproduced. The extra coordinate $y_b$ effectively acts as the “compactification radius.” Furthermore there is no need of fine-tuning of the cosmological constants of the visible and hidden worlds that is needed in the original Randall-Sundrum model.

In this paper, the equations of the motion of the model are presented in section 1. By introducing a phase space, stabilities and a region of the parameter space where domains are stable are analyzed in section 2. The perturbative analytical solutions of fields expanded in terms of $\epsilon$ are given in section 3. Domain structures and the mass hierarchy problem are consulted in the last section.

1 Action and the equations of motion

We consider a $D$-dimensional Randall-Sundrum model which has the spacetime coordinate patch given by $x^A = (x^\mu, y), \mu = 0, ..., D-2$, where $y = x^{D-1}$ is the coordinate of the extra dimension which may take any value in $-\infty < y < \infty$. The action we are considering is
given by

\[ S = \int d^D x \sqrt{-g} \left[ \delta(y - y_b)L_{\text{matter}} + \Lambda - \frac{1}{2} M^{D-2} R_D - \frac{1}{2} g^{AB} \partial_A \Phi \partial_B \Phi - V(\Phi) \right] \tag{1} \]

where \( L_{\text{matter}} \) is a Lagrangian of matter fields which reside in a \((D - 2)\)-brane located at \( y = y_b \), and \( M \) is the \( D \)-dimensional Planck mass which one should introduce to leave the action dimensionless. The detailed form of matter Lagrangian is irrelevant for determining the vacuum configuration of graviton, and we assume \( L_{\text{matter}} = 0 \). \( R_D \) is the \( D \)-dimensional curvature scalar, and the cosmological constant, for the convenience of later calculations, is taken to be \(-\Lambda\). For the scalar potential \( V(\Phi) \) and metric tensor \( g_{AB} \), we assume the usual forms

\[ V(\Phi) = \lambda (\Phi^2 - \nu^2)^2, \tag{2} \]
\[ ds^2_D = e^{-2\sigma(y)} ds^2_{D-1} + dy^2. \tag{3} \]

Here \( ds^2_{D-1} = \eta_{\mu\nu}(x) dx^\mu dx^\nu \) is a \((D - 1)\)-dimensional metric tensor such that \( ds^2_{D-1} > 0 \) for spacelike intervals. If the ansatz (3) is truly correct, it can be used in the calculation of the action. In fact, the usual Einstein’s field equations, derived from (1), can be compared to the equations obtained from the Lagrangian using the ansatz first. But the results are the same. For the convenience of calculations we use the later algorithm to obtain the vacuum geometry.

The \( D \)-dimensional curvature scalar written in terms of the \((D - 1)\)-dimensional one is given by

\[ R_D = e^{2\sigma(y)} R_{D-1} - 2(D - 1) \left( \frac{\partial^2 \sigma}{\partial y^2} - \frac{D}{2} \left( \frac{\partial \sigma}{\partial y} \right)^2 \right). \tag{4} \]

The Lagrangian corresponding to (1) is given by

\[ \mathcal{L} = -\frac{1}{2} e^{-(D-3)\sigma} \left[ M^{D-2} R_{D-1} + \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right] - e^{-(D-1)\sigma} \left[ -\Lambda + \frac{1}{2} \left( \frac{\partial \Phi}{\partial y} \right)^2 + \lambda (\Phi^2 - \nu^2)^2 - \frac{1}{2} (D - 1)(D - 2) M^{D-2} \left( \frac{\partial \sigma}{\partial y} \right)^2 \right]. \tag{5} \]

Since we are interested in the domain structure, we assume that \( \Phi \) depends only on \( y \).
The equations of motion corresponding to the Lagrangian (5) are

\[
\frac{1}{2} M^{D-2} e^{2\sigma} R_{D-1} = \frac{1}{2} \left( \frac{d\Phi}{dy} \right)^2 - \lambda (\Phi^2 - \nu^2)^2 + \Lambda - \frac{(D - 1)(D - 2)}{2} M^{D-2} \left( \frac{d\sigma}{dy} \right)^2 .
\]

\[
\left( \frac{d\Phi}{dy} \right)^2 = (D - 2) M^{D-2} \frac{d^2 \sigma}{dy^2} + \frac{1}{D - 1} M^{D-2} e^{2\sigma} R_{D-1} ,
\]

\[
R_{D-1}^{\mu\nu} = \frac{1}{D - 1} \eta^{\mu\nu} R_{D-1} ,
\]

where \( R_{D-1} \) is constant.

In this paper, we restrict our consideration only to \( D = 5 \) case which has the flat four dimensional metric given by \( \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1) \). Then the equations of motion reduce to

\[
6M^3 \left( \frac{d\sigma}{dy} \right)^2 = \frac{1}{2} \left( \frac{d\Phi}{dy} \right)^2 - \lambda (\Phi^2 - \nu^2)^2 + \Lambda ,
\]

\[
3M^3 \frac{d^2 \sigma}{dy^2} = \left( \frac{d\Phi}{dy} \right)^2 .
\]

To pay our attention to the essence of the equations, we introduce dimensionless variables such as

\[
\varphi = \nu^{-1} \Phi ,
\]

\[
z = \sqrt{\frac{3\lambda M^3}{2\nu^4}} y .
\]

Even though \( \sigma \) is dimensionless, it is useful to define new dimensionless variables \( s \) and \( \zeta \) given by

\[
s = \frac{3M^3}{\nu^2} \sigma ,
\]

\[
\zeta = \frac{ds}{dz} .
\]

The equations of motion, when written in terms of these, are

\[
\zeta' = (\varphi')^2 ,
\]

\[
\zeta^2 = 1 + \frac{3M^3}{4\nu^2} \varphi'^2 - \frac{\lambda \nu^4}{\Lambda} (\varphi^2 - 1)^2 ,
\]

where the symbol ' denotes the differentiation with respect to \( z \). These are the basic equations that we begin with to discuss stabilities in the next section.
2 Stability analysis of the Randall-Sundrum model

The basic equations (15) and (16) we derived in the last section are nonlinear coupled differential equations that do not allow us to obtain analytic solutions in closed forms. But fortunately these are first order differential equations whose general behavior can be visualized in a phase space constructed by \((\varphi, \zeta)\). Flows of phase points are determined by

\[
\frac{d\zeta}{d\varphi} = \frac{\zeta'}{\varphi'} = \varphi' = \pm A \sqrt{(1 - \varphi^2)^2 - \epsilon(1 - \zeta^2)},
\]

where the dimensionless parameters \(A\) and \(\epsilon\) are given by

\[
A = \sqrt{\frac{4\lambda \nu^6}{3\Lambda M^3}},
\]

\[
\epsilon = \frac{\Lambda}{\lambda \nu^4}.
\]

It is sufficient to consider only the branch corresponding to the positive sign in (17). Since \(d\zeta/d\varphi\) is a real function, it is clear that the allowed region of the phase space is restricted by the condition \(C_\epsilon(\varphi, \zeta) \geq 0\), where

\[
C_\epsilon(\varphi, \zeta) = (1 - \varphi^2)^2 - \epsilon(1 - \zeta^2).
\]

The shape of the boundary of the forbidden region, which we name it the island, depends on \(\epsilon\). The islands for various values of \(\epsilon\) are drawn in Fig.1. They are symmetric under the separate reflections of \(\varphi\) and \(\zeta\). When we assume that the domain wall is located at \(z = 0\), it becomes clear that the origin of the phase space, \(\varphi = \zeta = 0\), should be in the allowed region of the phase space. It is possible only when \(0 < \epsilon \leq 1\).

The coefficient \(A\) given in (17) determines the initial flow direction at the origin \(\varphi = \zeta = 0\). The same equation shows that for a given shape of island, which is determined by \(\epsilon\), flows in the phase space either terminate at the island or diverge indefinitely depending on \(A\). There is unique \(A(\epsilon)\) by which the flow line starting at the origin terminates at
Figure 1: The shapes of boundaries of the forbidden regions depend only on $\epsilon$. For $\epsilon < 1$, it consists of two disjoint islands. When $\epsilon = 1$, it touches the origin of the phase space, but the origin is still allowed. If $\epsilon > 1$, the two forbidden islands merge to one big island. In this case, the origin is not allowed

$\varphi = \zeta = 1$. When $A$ is less than this critical value $A(\epsilon)$, the flow line reaches the island, and stays there forever. But if it is slightly larger than $A(\epsilon)$, the flow bypasses the island, and runs indefinitely. In this case it becomes unstable. The numerical result of computation of this behaviour is plotted in Fig.2. To understand stabilities one should find $A(\epsilon)$ corresponding to the critical flow. This is investigated in the next section where analytical solutions of field configurations are also found.

### 3 Perturbative analytical solutions of fields

Even though the flow equation is highly nonlinear, one may use the usual perturbation technique to solve it. We assume that $\varphi$ and $\zeta$ both reach the critical values, $\varphi \to 1$ and $\zeta \to 1$, as $z \to \infty$. We have seen, for the stable solution, that $0 < \epsilon \leq 1$. It allows us to
Figure 2: Flows of phase points terminate on the boundary of the forbidden region, that is the island, when \( A \leq A(\epsilon) \). If \( A = A(\epsilon) \), it terminates at the critical point \( \varphi = \zeta = 1 \). But if \( A > A(\epsilon) \), \( \zeta(\varphi) \) grows indefinitely.

Expand the flow equation (17) in terms of \( \epsilon \),

\[
\frac{d\zeta}{d\varphi} = A[1 - \varphi^2 - \frac{\epsilon}{2}(1 - \zeta^2) + \frac{\epsilon^2}{8}(1 - \zeta^2)^2 + \cdots].
\] (21)

One can solve this equation perturbatively under the conditions

\[
\zeta(\varphi = 0) = 0, \quad \zeta(\varphi = 1) = 1.
\] (22)

Firstly, we find \( A(\epsilon) \) corresponding this critical flow, and then solve (21). The curve \( A(\epsilon) \) in the parameter space \((\epsilon, A)\) divides it up into the stable and unstable regions. The general formula for \( A(\epsilon) \) is hidden in

\[
\int_0^1 \frac{d\zeta}{d\varphi} d\varphi = 1.
\] (23)

Using the \( \epsilon \) independent part of (21), we have

\[
1 = A(\epsilon) \int_0^1 (1 - \varphi^2) d\varphi = \frac{2}{3} A(\epsilon).
\] (24)

That is, to the order of \( \mathcal{O}(\epsilon^0) \), \( A(\epsilon) = \frac{3}{2} \). Then, by (21), one has

\[
\zeta = \frac{3}{2}(\varphi - \frac{1}{3}\varphi^3).
\] (25)
Substituting this in (21) again, one gets the following equation,
\[ \frac{d\zeta}{d\varphi} = A(\epsilon)[1 - \varphi^2 - \frac{\epsilon}{2} \frac{9}{4}(\frac{\varphi}{3} - \frac{\varphi^2}{3})^2]. \] (26)

From this we find that \( A(\epsilon) \) and \( \zeta(\varphi) \), to the order \( O(\epsilon) \), are
\[ A(\epsilon) = \frac{3}{1 - \frac{18}{40} \epsilon}, \] (27)
\[ \zeta(\varphi) = \zeta_0(\varphi) + \epsilon \zeta_1(\varphi). \] (28)

Here \( \zeta_0(\varphi) \) is the same as (23), and
\[ \zeta_1(\varphi) = -\frac{3}{80}(\varphi - 2\varphi^3 + \varphi^5). \] (29)

To solve \( \varphi \) as a function of \( z \), we combine (17) and (21),
\[ \frac{d\varphi}{dz} = \frac{d\zeta}{d\varphi} = A(\epsilon)[1 - \varphi^2 - \frac{\epsilon}{2} \frac{9}{4}(\frac{\varphi}{3} - \frac{\varphi^2}{3})^2]. \] (30)

This can be integrated to give the following
\[ z = \frac{\epsilon}{12} \varphi + \frac{10 - \epsilon}{30} \log \frac{1 + \varphi}{1 - \varphi}, \] (31)

It is valid up to the order \( O(\epsilon) \). Solving \( \varphi \) in terms of \( z \), one gets
\[ \varphi(z) = \tanh \frac{3z}{2} + \frac{3}{2} \epsilon \left( \frac{z}{10} - \frac{1}{12} \tanh \frac{3z}{2} \right) \left( 1 - \tanh^2 \frac{3z}{2} \right) + O(\epsilon^2). \] (32)

Note that there is a \( z \)-linear term which is not easy to guess. When (32) is used in (28), one may have \( \zeta = \zeta(z) \). The corresponding graph is plotted in Fig.4. It is clear that both \( \varphi \) and \( \zeta \) approach the domain values quickly. Comparing the result with the original Randall-Sundrum model, it shows that the domain structure of \( \varphi \) smoothes out the singular behavior of \( \zeta \) at the origin.

## 4 Domain structures and the mass hierarchy problem

The original Randall-Sundrum model is a theory where the size of domain wall is zero. In this paper the domains and domain wall have finite sizes whose analytical structures are
Figure 3: The fate of field configuration depends on $\epsilon$ and $A$. At the dark region of the phase space the field configuration is unstable, and $\zeta$ runs indefinitely as $z$ becomes larger. The boundary of this unstable region is described by the curve $A = A(\epsilon)$. If $A \leq A(\epsilon)$, $\zeta$ has the limiting value $\zeta_{\infty}$ as $z \to \infty$. For a given $\epsilon$, there is unique $A = A(\epsilon, \zeta_{\infty})$ by which flows of phase points determined by (17) terminate at $\zeta_{\infty}$. Various curves correspond to various $\zeta_{\infty}$. Note that $A(\epsilon) = A(\epsilon, 1)$. 
Figure 4: For a given $\epsilon$, $\zeta(z)$ saturates if $A \leq A(\epsilon)$. When $A = A(\epsilon)$, $\zeta(z)$ saturates exactly to 1.

determined by the relation (32). Using this equation we determine the domain wall size $\mu$ and the domain size $\xi$. According to (5), an observer who lives in the 3-brane located at $y = y_b$ effectively measures the extra dimension $y$ with an effective scale factor $e^{-(D-3)\sigma(y)}$, where $D = 5$. This means that

$$\int_{-\infty}^{\infty} e^{-2\sigma(y)} dy = \mu + 2\xi. \quad (33)$$

To compute the domain wall size, we use (15) and (16). Fig.4 shows that $\zeta(z)$ is nearly constant, $\zeta_\infty = \zeta(z \to \infty)$ or $-\zeta_\infty$, in the domains, and almost linear at the transitory domain wall. Here, $\zeta_\infty < 1$ for a stable configuration. Both $\varphi$ and $\zeta$ vanish at the center of the domain wall. That is, the slop of $\zeta(z)$-curve at the origin is given by

$$\zeta'_0 = \varphi'_0 = \frac{4v^2}{3M^3} \left( \frac{1}{\epsilon} - 1 \right). \quad (34)$$

The $z$-coordinate $\delta$ of the boundary of the domain wall is given by

$$\delta = \frac{\zeta_\infty}{\zeta'_0} = \frac{3M^3}{4v^2} \frac{\epsilon\zeta_\infty}{1-\epsilon}. \quad (35)$$

For $\epsilon \approx 0$, it can be simplified to

$$\delta \approx \frac{3M^3}{4v^2} \epsilon\zeta_\infty. \quad (36)$$
Note that $z$ is the physical coordinate $y$ measured in the unit of $\ell$, where

$$\ell = \sqrt{\frac{2\nu^4}{3\Lambda M^3}}. \quad (37)$$

We approximate $\zeta(z) = ds/dz$ in the following way,

$$\frac{ds}{dz} = \begin{cases} 
\zeta_\infty & \text{if } z > \delta, \\
\zeta_\infty \frac{1}{\delta} z & \text{if } -\delta < z < \delta, \\
-\zeta_\infty & \text{if } z < -\delta.
\end{cases} \quad (38)$$

This can be solved to give

$$s(z) = \begin{cases} 
\left( \frac{1}{2\delta} z^2 + \frac{1}{2} \delta \right) \zeta_\infty & \text{if } -\delta < z < \delta, \\
\zeta_\infty |z| & \text{otherwise},
\end{cases} \quad (39)$$

where $\frac{1}{2}\delta$ is the integration constant which is needed to leave $s(z)$ both continuous and smooth at the boundary of the domain wall, $z = \pm \delta$.

The domain wall size is given by

$$\mu = \ell \int_{-\delta}^{\delta} e^{-2\left(\frac{z^2}{2\delta^2} + \frac{1}{2}\delta\right)} \zeta_\infty \frac{1}{\delta M} dz \approx 2\delta \ell, \quad (40)$$

where the last approximation is valid when $\epsilon \approx 0$. On the other hand, the domain size $\xi$ is give by

$$\xi = \ell \int_{\delta}^{\infty} e^{-\frac{z^2}{2\delta^2}} \zeta_\infty dz = 2\delta \ell \left( 1 - \frac{1}{\epsilon} - \frac{\zeta_\infty^2}{\epsilon^2} e^{-\frac{\zeta_\infty^2}{2(1-\epsilon)}} \right), \quad (41)$$

which, for $\epsilon \approx 0$, can be reduced to the following simple form

$$\xi \approx \frac{1}{\epsilon \zeta_\infty^2} \mu. \quad (42)$$

When $\epsilon \to 1$, the above equation states that the sizes of domain and domain wall are comparable implying that our approximation is not valid. For unstable field configurations such as $A > A(\epsilon)$, both $\sigma(y)$ and $\Phi(y)$ grow enormously. These mean that the potential energy density is very large in the domain. But the effective scale factor $e^{-4\sigma(y)}$ vanishes rapidly, thus pushing domains to the domain wall. This sustains the total potential energy still to a finite value.
Now consider a possible solution of the mass hierarchy problem. The Planck mass $M_{Pl}$, inferred from the Lagrangian (3), is given by

$$M_{Pl}^2 = M^3 \int_{-\infty}^{\infty} e^{-2\sigma(y)} dy,$$

which, for a relatively narrow domain wall, is equal to $M_{Pl}^2 \approx \frac{1}{\epsilon \zeta} \mu M^3$. To compare physical mass with the Planck mass, we place a 3-brane at the coordinate $y_b = \ell z_b$, and then turn on the interactions among particles which live in the brane. The usual form of the action of a Higgs field is the following,

$$S_H = -\int d^4x \sqrt{-g} \left[ \bar{g}^{\mu \nu} D_\mu H^\dagger D_\nu H - \lambda_H (|H|^2 - \nu_H^2)^2 \right],$$

where $\bar{g}_{\mu \nu}(x) = e^{-2\sigma(y_b)} \eta_{\mu \nu}(x)$. After renormalization of the wave function $H \rightarrow e^{\sigma(y_b)} H$, the Higgs vacuum expectation value is changed to

$$v_{\text{obs}} = e^{-\sigma(y_b)} v_H \approx e^{-\epsilon \zeta^2 \frac{1}{\delta} z_b} v_H,$$

where the last step holds when $\epsilon \zeta^2 \approx 0$. This shows that any mass parameter $m$ of the original theory appears to have

$$m_{\text{obs}} \approx e^{-\epsilon \zeta^2 \frac{1}{\delta} z_b} m.$$ 

Thus, by changing the relative position $z_b/\delta$ of the 3-brane with respect to the domain wall boundary, one gets a theory that has exponentially varying masses relative to the Planck mass.

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