Calculating the anomalous supersymmetry breaking in Super-Yang-Mills theories with local coupling

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Abstract

Supersymmetric Yang-Mills-theories with local gauge coupling have a new type of anomalous breaking, which appears as a breaking of supersymmetry in the Wess-Zumino gauge. The anomalous breaking generates the two-loop order of the gauge $\beta$-function in terms of the one-loop $\beta$-function and the anomaly coefficient. We determine the anomaly coefficient in the Wess-Zumino gauge by solving the relevant supersymmetry identities. For this purpose we use a background gauge and show that the anomaly coefficient is uniquely determined by convergent one-loop integrals. When evaluating the one-loop diagrams in the background gauge, it is seen that the anomaly coefficient is determined by the Feynman-gauge value of the one-loop vertex function to $G^{\mu\nu}G^{\nu\mu}$ at vanishing momenta.

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1 Introduction

It has been noted for a long time that the renormalization of the gauge coupling constant of supersymmetric Yang-Mills theories has special improved properties compared to usual gauge theories. These improved renormalization properties are reflected in the closed all-order expression for the gauge $\beta$ function [1, 2, 3].

The improved renormalization property of the coupling constant which is not apparent in usual perturbation theory has been seen in connection with absence of two-loop and higher-order terms to the coupling constant in the Wilsonian effective action [2, 4]. These statements have been reformulated recently in a much more rigorous way and even independent of the usage of a Wilsonian effective action by extending the coupling of the classical action to an external superfield [5]. For this purpose one introduces a chiral and antichiral field multiplet and identifies their real part with the inverse of the square of the (local) gauge coupling. Due to the construction the complex part of their lowest components couples to a total derivative, explicitly to the divergence of the axial current and to the topological term $\text{Tr} \, G^{\mu\nu} \tilde{G}_{\mu\nu}$. This property can be formulated in form of a Ward identity and yields holomorphicity of symmetric counterterms independent of the usage of a Wilsonian effective action. In particular symmetric counterterms to the coupling are only present in one-loop order independent of the specific subtraction scheme one uses.

As for the Wilsonian approach also in the present construction naive applications of symmetries result in a purely one-loop $\beta$-function. However, when the coupling is extended to an external superfield, supersymmetry has an anomalous breaking in one-loop order [5]. We have shown, that it is the anomalous breaking which generates the 2-loop coefficient of the gauge $\beta$-function. By this analysis the coefficient of the anomaly is implicitly determined by the scheme-independent value of the two-loop $\beta$-function.

It is the purpose of the present paper to determine the anomaly coefficient explicitly in one-loop order. For this purpose we use a background gauge field and background gauge invariance. Solving then the Slavnov–Taylor identity we find an expression for the anomaly coefficient in terms of convergent one-loop integrals. By explicit evaluation of the one-loop diagrams we find that the anomaly coefficient is determined by the Feynman-gauge value of the one loop corrections to $G^{\mu\nu} \tilde{G}_{\mu\nu}$ of background fields. Hence, these contributions effect a supersymmetry breaking of Super-Yang-Mills theories in the Wess-Zumino gauge.

The plan of the paper is as follows: In section 2 we recapitulate renormalization of Supersymmetric Yang-Mills theories with local gauge coupling. In section 3 we determine the symmetry identities which allow the calculation of the anomaly coefficient. We use a background gauge fixing and prove that together with background gauge invariance the anomaly coefficient is uniquely determined from convergent
one-loop integrals. In section 3 we compute the one-loop diagrams to $G^{\mu \nu} \tilde{G}_{\mu \nu}$ and solve the identities in one-loop order. Relations to previous results and calculations are discussed in the conclusions. In the appendices we give the BRS transformations of the fields, the one-loop diagrams contributing to the symmetry identities and the conventions for the one-loop integrals.

2 SYM theories with local gauge coupling

We consider Super-Yang-Mills (SYM) theories with a simple gauge group in the Wess-Zumino gauge. The gauge multiplet

\[(A^\mu, \lambda^\alpha, \bar{\lambda}^\dot{\alpha}, D) = (A^\mu_a, \lambda^\alpha_a, \bar{\lambda}^\dot{\alpha}_a, D_a) \tau_a,\]

\[[\tau_a, \tau_b] = i f_{abc} \tau_c,\]

consists of the physical gauge fields, the gauginos and the auxiliary $D$-fields, which are finally eliminated by their equation of motions. The matrices $\tau_a$ are the hermitian matrices of the fundamental representation. We normalize

\[\text{Tr}(\tau_a \tau_b) = \delta_{ab}.\]

Extending the gauge coupling to an external superfield requires to introduce a chiral field multiplet $\eta$ and its complex conjugate field $\bar{\eta}$:

\[\eta(x, \theta) = \eta + \theta^\alpha \chi^\alpha + \theta^2 f, \quad \bar{\eta}(x, \bar{\theta}) = \bar{\eta} + \bar{\theta}^\dot{\alpha} \bar{\chi}^\dot{\alpha} + \bar{\theta}^2 \bar{f},\]

in the chiral and antichiral representation, respectively. The sum of their lowest components is identified with the inverse of the square of the gauge coupling,

\[\eta + \bar{\eta} = \frac{1}{g^2(x)}.\]

With these definitions the gauge invariant classical action with local gauge coupling takes the following form:

\[\Gamma_{\text{SYM}} = -\frac{1}{4} \int dS \eta L_{\text{SYM}} - \frac{1}{4} \int d\bar{S} \bar{\eta} \bar{L}_{\text{SYM}} \]
\[= \int d^4x \left( \frac{1}{2g^2} (L_{\text{SYM}} + \bar{L}_{\text{SYM}}) + \frac{1}{2}(\eta - \bar{\eta})(L_{\text{SYM}} - \bar{L}_{\text{SYM}}) \right.\]
\[\left. - \frac{1}{2}(\chi^\alpha \Lambda_\alpha + \chi_{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}}) - \frac{1}{2}(f \lambda \lambda + \bar{f} \bar{\lambda} \bar{\lambda}) \right).\]
Here, $\mathcal{L}_{\text{SYM}}$ is the chiral Lagrangian multiplet:

$$\mathcal{L}_{\text{SYM}} = -\frac{1}{2}g^2 \text{Tr} \lambda^\alpha \lambda_\alpha + \Lambda^\alpha \theta_\alpha + \theta^2 L_{\text{SYM}},$$

with the following explicit expressions for the spinor and $F$-components:

$$\Lambda_\alpha = -\frac{i}{2} \text{Tr} \left( g\sigma_\alpha^\mu \gamma_\beta G_{\mu\nu}(gA) + g^2 D\lambda_\alpha \right),$$

$$L_{\text{SYM}} = \text{Tr} \left( -\frac{1}{4} G^{\mu\nu}(gA) G_{\mu\nu}(gA) + ig^4 \sigma_\alpha^\mu D_\mu (g\lambda_\alpha) + \frac{1}{8} g^2 D^2 ight.$$

$$\left. -\frac{i}{8} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}(gA) G_{\rho\sigma}(gA) \right).$$

$\bar{\mathcal{L}}_{\text{SYM}}$ is the respective antichiral multiplet, which is obtained by complex conjugation.

$\Gamma_{\text{SYM}}$ is gauge invariant and invariant under the non-linear supersymmetry transformation of the Wess–Zumino gauge \cite{7, 8}. These transformations are combined in nilpotent BRS transformations \cite{9, 10, 11}:

$$\mathbf{s} \phi = (\delta^{\text{gauge}}_{\epsilon(x)} + \epsilon^\alpha \delta_\alpha + \bar{\delta}_{\dot{\alpha}} \bar{\epsilon}_{\dot{\alpha}} - i \omega^\mu \partial_\mu) \phi,$$

$$\mathbf{s}^2 \phi = 0$$

and

$$\mathbf{s} \Gamma_{\text{SYM}} = 0.$$

The BRS transformations with $D$-fields being eliminated are given in Appendix A.

By means of the BRS transformations it is possible to add a BRS invariant gauge fixing and ghost term to the action:

$$\Gamma_{\text{g.f.}} + \Gamma_{\text{ghost}} = \mathbf{s} \text{ Tr } \int d^4x \left( \frac{\epsilon}{2} \mathbf{\tau} B + \mathbf{\tau} \mathcal{F} \right),$$

where $\mathcal{F}$ denotes a generic linear gauge-fixing function as for example $\mathcal{F} = \partial A$. Evaluating the BRS transformations we find the conventional gauge fixing term with the auxiliary $B = B_a \tau_a$-fields:

$$\Gamma_{\text{g.f.}} = \text{Tr } \int d^4x \left( \frac{\epsilon}{2} BB + B \mathcal{F} \right).$$
Eliminating the $D$-fields of the vector multiplet and adding the external field part

$$
\Gamma_{\text{ext}} = \text{Tr} \int d^4x \left( \rho^\mu s A_\mu + Y_\lambda^\alpha s \lambda_\alpha \bar{\lambda}^\lambda + \sigma s c + \frac{1}{2}(Y_\lambda \epsilon - \bar{\tau} Y_\tau^\lambda)^2 \right),
$$

(15)

the classical action,

$$
\Gamma_{\text{cl}} = \Gamma_{\text{SYM}} + \Gamma_{g.f.} + \Gamma_{\text{ghost}} + \Gamma_{\text{ext.f}},
$$

(16)

satisfies the Slavnov–Taylor identity:

$$
S(\Gamma_{\text{cl}}) = 0.
$$

(17)

The Slavnov–Taylor operator acting on a general functional $F$ is defined as

$$
S(F) = \int d^4x \left( \text{Tr} \left( \frac{\delta F}{\delta \rho_\mu} \frac{\delta F}{\delta A^\mu} + \frac{\delta F}{\delta Y_\lambda^\alpha} \frac{\delta F}{\delta \lambda_\alpha} + \frac{\delta F}{\delta Y_\tau^\lambda} \frac{\delta F}{\delta \bar{\lambda}^\bar{\lambda}} + \frac{\delta F}{\delta \sigma} \frac{\delta F}{\delta c} 
\right.
\right.
\left. + s B \frac{\delta F}{\delta B} + s \bar{c} \frac{\delta F}{\delta \bar{c}} + s G^i \frac{\delta F}{\delta G^i} \right) + s \omega^\nu \frac{\partial F}{\partial \omega^\nu}.
$$

(18)

Here $G^i$ denotes the components of the local coupling

$$
G^i = (g, \eta - \bar{\eta}, \chi, \bar{\chi}, f, \bar{f}).
$$

(19)

In addition the classical action satisfies the identity

$$
\int d^4x \left( \frac{\delta}{\delta \eta} - \frac{\delta}{\delta \bar{\eta}} \right) \Gamma_{\text{cl}} = 0,
$$

(20)

which expresses in functional form that the coupling is the lowest component of a constrained real superfield.

On the basis of the classical action loop calculations are performed by treating the local coupling and its superpartners as external fields. Green functions with the local coupling are determined by differentiation with respect to the local coupling and performing the limit to constant coupling:

$$
\Gamma_{g^n\varphi_{i_1}...\varphi_{i_m}}(x_1, \ldots x_n, y_1, \ldots y_m)
\equiv \lim_{G \to g} \frac{\delta^n + m \Gamma}{\delta g(x_1) \ldots \delta g(x_n) \delta \varphi_{i_1}(y_1) \ldots \delta \varphi_{i_m}(y_m)} \bigg|_{\varphi_i=0}.
$$

(21)

Here the fields $\varphi_i$ summarize propagating and external fields of the theory. For $n = 0$ we obtain the usual Green functions of SYM theories.
The local gauge coupling $g(x)$ is distinguished from ordinary external fields by the property that it is the perturbative expansion parameter. For any 1PI Green function (21) the power of the constant gauge coupling is determined by the loop order $l$, by the number of amputated external legs $N_{\text{amp.legs}}$ and by the number of external field differentiations. This property can be expressed by the topological formula:

$$N_{g(x)} = N_{\text{amp.legs}} + N_Y + 2N_f + 2N_{\chi} + 2N_{\eta-\bar{\eta}} + 2(l - 1) .$$

(22)

Here $N_Y$ denotes the number of BRS insertions, and $N_f$, $N_{\chi}$ and $N_{\eta-\bar{\eta}}$ gives the number of insertions corresponding to the respective external fields and their complex conjugates. In particular it is immediately verified that the topological formula is valid for the classical action

With local gauge coupling the Slavnov–Taylor identity (17) and the identity (20) have an anomalous breaking in one-loop order [5]. We have shown that it is possible to adjust counterterms in such a way that either the Slavnov-Taylor identity or the identity (20) is unbroken. For the Wess-Zumino gauge it is natural to shift the anomalous breaking to the Slavnov-Taylor identity, leaving the identity (20) unmodified to all orders, i.e.,

$$\int d^4x \left( \frac{\delta}{\delta \eta} - \frac{\delta}{\delta \bar{\eta}} \right) \Gamma = 0 ,$$

(23)

$$S(\Gamma) = r_\eta^{(1)} \Delta_{\text{brs}}^{\text{anomaly}} + O(\hbar^2) .$$

(24)

The anomalous field monomial is the variation of a gauge invariant field monomial, which depends on the logarithm of the coupling:

$$\Delta_{\text{brs}}^{\text{anomaly}} = s \int d^4x \ln g(x)(L_{YM} + \bar{L}_{YM})$$

$$= (\epsilon^\alpha \delta_\alpha + \bar{\epsilon}^\dot{\alpha} \delta_{\dot{\alpha}}) \int d^4x \ln g(x)(L_{YM} + \bar{L}_{YM})$$

$$= \int d^4x \left( i \ln g(x) \left( \partial_\mu \Lambda_{\alpha YM}^a \sigma^a_{\alpha \mu} \bar{\epsilon}^\dot{\alpha} - \epsilon^\alpha \sigma^\mu_{\alpha a} \partial_\mu \Lambda_{\dot{\alpha} YM}^a \right) 

- \frac{1}{2} g^2(x)(\epsilon \chi + \bar{\epsilon} \bar{\chi})(L_{YM} + \bar{L}_{YM}) \right) .$$

(25)

For constant coupling and for any differentiation with respect to the local coupling $\Delta_{\text{brs}}^{\text{anomaly}}$ is free of logarithms and can appear as an anomaly of the Slavnov-Taylor identity. Since the perturbative expansion is a power series expansion, it can be proven that the coefficient of the anomaly $r_\eta^{(1)}$ is gauge and scheme independent.

The $\eta - \bar{\eta}$ identity (23) restricts the symmetric counterterms of chiral integrals to holomorphic functions in $\eta$ and $\bar{\eta}$. For this reason the counterterms to the SYM action are exhausted in one-loop order, leading in a naive application of symmetries to
a strictly one-loop $\beta$-function. By an algebraic construction of the Callan–Symanzik equation it was shown that the anomalous breaking \cite{24} generates the 2-loop coefficient of the gauge $\beta$ function:

$$\beta_g = \beta_g^{(1)}(1 + g^2 r^{(1)}_{\eta} + O(h^2)) .$$

Using the explicit expressions for the 2-loop $\beta$-functions one finds

$$r^{(1)}_{\eta} = \frac{C(G)}{8\pi^2}$$

(27)

where $C(G)$ is the quadratic Casimir of the adjoint representation.

It is the purpose of the present paper to determine the coefficient from an explicit one-loop calculation, which in particular clarifies the origin of the anomaly as arising from the the vertex corrections to the topological term $\epsilon^{\mu\nu\rho\sigma} G_{\mu\nu} G_{\rho\sigma}$.

3 The symmetry identities of the anomaly coefficient

For the calculation of the anomaly coefficient $r^{(1)}_{\eta}$ \cite{24} the Slavnov-Taylor identity has to be solved in such a way that $r^{(1)}_{\eta}$ is determined from non-local Green functions, which are not subject of renormalization. Due to the various field redefinitions appearing in the Wess–Zumino gauge \cite{10, 5} this calculation is involved for the theory as it stands. The calculation is simplified when we use a background gauge \cite{12, 13}.

For this purpose we choose the following gauge fixing function $\mathcal{F}$ \cite{13}:

$$\mathcal{F} = \partial^\mu A_\mu + i[\hat{A}^\mu, A_\mu] ,$$

(28)

which is covariant under usual background gauge transformations. The background field $\hat{A}^\mu$ is an external field and is distinguished from the quantum field by its BRS-transformations \cite{14, 15, 16}:

$$s \hat{A}^\mu = C^\mu - i\omega \partial A^\mu .$$

(29)

Redefining at the same time in the SYM action and in the BRS transformations

$$gA_\mu \rightarrow gA_\mu' = gA_\mu + \hat{A}^\mu$$

(30)

the classical action \cite{16} and the vertex functional $\Gamma$ are BRS invariant and in addition invariant under background gauge transformations, which can be expressed by a linear Ward identity:

$$\left( w_a - \partial^\mu \frac{\delta}{\delta A^\mu_a} \right) \Gamma = 0 ,$$

(31)
with
\[ w_a \equiv f_{a b c} \sum_i \varphi_i^a \frac{\delta}{\delta \varphi_i^c} \]
\[ \varphi^i = (A^\mu, \delta A^\mu, \lambda^\alpha, \lambda^\beta, c, B, \bar{c}, \rho^\mu, \sigma, Y^{\alpha}, Y^{\beta}, C^\mu) \]

The number of couplings in a 1PI Green functions with background fields is determined by the topological formula as given in (32).

In the background gauge the anomalous breaking (25) depends on the background field
\[ \Delta^{\text{anomaly}}(gA) \rightarrow \Delta^{\text{anomaly}}(gA + \hat{A}) , \]
and we are able to determine the coefficient \( r^{(1)}_\eta \) on the vertex functions of background fields which are gauge invariant by construction. Explicitly we differentiate the anomalous Slavnov Taylor identity with respect to \( \epsilon^\alpha \) and \( \chi^\alpha \) and two background fields and find
\[ \lim_{G \rightarrow g} \frac{\partial}{\partial \epsilon^\alpha} \frac{\delta}{\delta \chi^\alpha(y)} \frac{\delta}{\delta A^\mu_a(x_1)} \frac{\delta}{\delta A^\mu_b(x_2)} S(\Gamma) = 2 \delta_{ab} r^{(1)}_\eta \eta^{\mu \nu} (\delta_{\mu \nu} - \epsilon^\mu \epsilon^\nu) . \]

After Fourier transformation we get the following identity:
\[ \Gamma_{\epsilon^\alpha \chi^\alpha \lambda^\beta} (q, p_1, p_2) \Gamma_{A^\mu_a A^\lambda_{bc}} (p_2, -p_2) + \Gamma_{\epsilon^\alpha \lambda^\beta A^{\alpha}_{bc} (p_1, -p_1)} \Gamma_{\lambda^\beta A^{\lambda}_{bc} (q, p_2, p_1)} + (\hat{A}^{\mu}_{a}(p_1) \leftrightarrow \hat{A}^{\nu}_{b}(p_2)) - 2 \eta^{\mu \nu} \eta^{\beta} (q, p_1, p_2) = 2 g^2 r^{(1)}_\eta \delta_{\mu \nu} (p_1 p_2 - p_1 p_2) + \mathcal{O}(\hbar^2) . \]

Differentiation with respect to \( \tau^\alpha \) and \( \bar{\chi}_{\dot{\alpha}} \) yields the respective complex conjugate equation.

For proceeding we sum and subtract the identity (35) and its complex conjugate.

For the sum of the identity (35) and its complex conjugate identity we take the momentum of \( \chi \)-fields to zero. Using furthermore
\[ \lim_{G \rightarrow g} \int d^4 x \left( \frac{\delta}{\delta \eta} + \frac{\delta}{\delta \bar{\eta}} \right) \Gamma = -g^3 \partial_\eta \Gamma , \]
we obtain an equation which determines the anomaly coefficient:

\[
\Gamma_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2} (0, p_1, -p_1) + 2 \Gamma_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2} (0, -p_1, p_1) + c.c. + 2g^2 \partial_{r} \Gamma_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2} (p_1, -p_1) = 4 \eta^{(1)} g^2 \delta_{ab} (\eta_{\mu_\nu} p_1 p_2 - p_{1\nu} p_{2\mu}) + \mathcal{O}(h^2). \quad (37)
\]

For the difference of the two identities we obtain

\[
\left( \Gamma_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2} (q, p_1, p_2) + (\hat{A}_{a}^{\mu}(p_1) \leftrightarrow \hat{A}_{b}^{\mu}(p_2)) \right)
\]

\[
-2 \Gamma_{\eta, \pi_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2}} (q, p_1, p_2) = 0. \quad (38)
\]

In the following, we will prove that the two equations (38) and (37) determine together with background gauge invariance the coefficient \( \eta^{(1)} \) from scheme independent, convergent one-loop integrals. The procedure is similar to the one, which has been used to determine the axial anomaly in a scheme-independent framework (see [17] and [18] for a recent review) and is based on the tensor decomposition of the 1PI Green functions appearing in (37) and (38).

Because of background gauge invariance (31) the 1PI Green function \( \Gamma_{\epsilon^{\alpha_\lambda}, r} \) and its complex conjugate are transversal. Thus, using parity conservation we find:

\[
\Gamma_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2} (0, p_1, -p_1) = (\eta_{\mu_\nu} - \frac{p_{1\mu} p_{1\nu}}{p_1^2}) \Sigma_1(p_1^2), \quad (39)
\]

\[
\Gamma_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2} = i \epsilon_{\mu_\nu, \rho_\sigma} p_{1\rho} p_{2\sigma} \Sigma_2(p_1, p_2). \quad (40)
\]

\( \Sigma_1 \) and \( \Sigma_2 \) are scalar functions of the momenta and vanish both in the tree approximation. Hence, the local and superficially divergent contribution \( \eta_{\mu_\nu} z_\rho \) is determined from the non-local tensor part.

The Green function

\[
\Gamma_{\eta, \pi_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2}} (q, p_1, p_2) = i \epsilon^{\mu_\nu, \rho_\sigma} p_{1\rho} p_{2\sigma} \delta_{ab} (-2 + \Sigma_\eta(p_1, p_2)) \quad (41)
\]

is not unambiguously determined by background gauge invariance. At this stage the extension to local gauge coupling becomes important: Differentiating once more with respect to the local gauge coupling, we get the 1PI Green function \( \Gamma_{g_{\eta}, \pi_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2}} \).

Using background gauge invariance

\[
p_1^\mu \Gamma_{g_{\eta}, \pi_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2}} (r, q, p_1, p_2) = p_2^\nu \Gamma_{g_{\eta}, \pi_{\epsilon^{\alpha_\lambda}, r, q, p_1, p_2}} (r, q, p_1, p_2) = 0. \quad (42)
\]
and the identity (23)
\[ \Gamma_{g\eta - \eta\hat{A}^\mu\hat{A}^\nu}(r, 0, p_1, p_2) = 0 . \] (43)

the local counterterms are unambiguously fixed by convergent one-loop integrals. Explicitly we find:
\[
\Gamma_{g\eta - \eta\hat{A}^\mu\hat{A}^\nu}(r, q, p_1, p_2) = i \epsilon^{\mu\lambda\rho\sigma} q_\rho p_1\sigma \left( (\delta_\nu^\lambda p_1^2 - p_2^\lambda p_2^\nu) \Sigma_3 + (\delta_\nu^\lambda p_1^2 - p_2^\lambda p_2^\nu) \Sigma_4 + (\delta_\nu^\lambda p_2^2 - p_2^\lambda q_\nu) \Sigma_5 \right) 
+ (\mu \leftrightarrow \nu, p_1 \leftrightarrow p_2) + i \epsilon^{\mu\nu\rho\sigma} p_1\rho p_2\sigma \Sigma_6 . \] (44)

The \( \Sigma_i = \Sigma_i(p_1, p_2, q) \) are scalar functions of the momenta. In addition one has
\[ \Sigma_6(p_1, p_2, 0) = 0 \] (45)
due to the identity (23) and \( \Sigma_6 \) is therefore convergent.

For loop orders \( l \geq 1 \) the Green function with constant coupling \( \Gamma_{\eta - \eta\hat{A}\hat{A}} \) is determined by the Green function with local coupling at vanishing external momentum \( r \) (see (22)):
\[ \Gamma^{(l)}_{g\eta - \eta\hat{A}^\mu\hat{A}^\nu}(0, q, p_1, p_2) = \partial_g \Gamma^{(l)}_{\eta - \eta\hat{A}^\mu\hat{A}^\nu}(q, p_1, p_2) = \frac{2l}{g} \Gamma^{(l)}_{\eta - \eta\hat{A}^\mu\hat{A}^\nu}(q, p_1, p_2) \] (46)

and we get finally \( (q + p_1 + p_2 = 0, l \geq 1) \):
\[ \Sigma^{(l)}_{\eta - \eta}(p_1, p_2) = \frac{g}{2l} \left( p_2^2 \Sigma^{(l)}_3(p_1, p_2) + p_1^2 \Sigma^{(l)}_3(p_2, p_1) + p_1 p_2 \Sigma^{(l)}_4(p_1, p_2) + \Sigma_6(p_1, p_2) \right) , \] (47)

where
\[ \Sigma_3(p_1, p_2) \equiv \Sigma_3(p_1, p_2) - \Sigma_5(p_1, p_2) \]
\[ \Sigma_4(p_1, p_2) \equiv \Sigma_4(p_2, p_1) \equiv \Sigma_4(p_1, p_2) - \Sigma_5(p_1, p_2) + \Sigma_4(p_2, p_1) - \Sigma_5(p_2, p_1) \] (48)

and
\[ \Sigma_i(p_1, p_2) \equiv \Sigma_i(p_1, p_2, -p_1 - p_2) . \] (49)

Hence, \( \Gamma_{\eta - \eta\hat{A}\hat{A}} \) is determined from its extension to local gauge coupling by the convergent functions \( \Sigma_i, i = 3, 4, 5, 6 \).

It remains to consider Green functions \( \Gamma_{\chi^\alpha\hat{A}^\nu Y^\beta} \) and \( \Gamma_{\chi^\alpha\hat{A}^\nu\lambda^\beta} \). Using background gauge invariance we find
\[ \Gamma_{\chi^\alpha\hat{A}^\nu Y^\beta}(p_1, -p_1) = -\frac{1}{g} \delta_{\alpha\beta} \delta^{\mu\nu} p_1\mu (1 + \Sigma_Y(p_1^2)) \] (50)
\[\sigma^\mu_{\alpha} \Gamma_{\chi_0 A_0 A_0}^{\lambda} = -\frac{g}{2} \delta_{bc} (\text{Tr}(\sigma^\mu \sigma^\nu) p_{2\lambda}(1 + \Sigma_{\chi}(p_2, p_1)) \]

\[-\text{Tr}(\sigma^\mu \sigma^\nu) q_\lambda ((\delta^\nu p_{2}^2 - p_{2\lambda} p_{2}^\nu) \Sigma_7 + (\delta^\nu p_2 p_1 - p_{2\lambda} p_{1}^\nu) \Sigma_8) .\]

\[\Sigma_7 \text{ and } \Sigma_8 \text{ are convergent, but the functions } \Sigma_{\chi \lambda} \text{ and } \Sigma_{\chi \gamma} \text{ arise from linearly divergent diagrams and they are determined by gauge invariance only up to local counterterms. Therefore they depend on the specific regularization and subtraction procedure. However, when we insert (50) and (51) together with (47) and (40) into the identity (38) their sum and thus the sum of their local counterterms is determined by the convergent functions } \Sigma_i \text{ in one-loop order.}\]

To this end we use the identity

\[\text{Tr}(\sigma^\mu \sigma^\nu) = 2(\eta^\mu \eta^\nu - \eta^\mu \eta^\nu + i \epsilon^{\mu \nu \lambda \rho}) (52)\]

and

\[\Gamma_{\hat{A}_a A_b}^{\mu}(p_1, -p) = -\delta_{ab}(\eta^\mu p_{2}^2 - p^\mu p_2)(1 + \Sigma_{\hat{A}_a A_b}) (53)\]

and derive the following expression for the one-loop order of the identity (38):

\[2(\Sigma_{\chi}(p_2^2) + \Sigma_{\chi}(p_2^2) + \Sigma_{\chi}(p_2, p_1) + \Sigma_{\chi}(p_1, p_2)) = -2\Sigma_{\eta-\eta}(p_1, p_2) - \Sigma_{\text{conv}}^{(1)}(p_1, p_2), (54)\]

where \(\Sigma_{\text{conv}}(p_1, p_2)\) summarizes the remaining convergent functions \(\Sigma_i\):

\[\Sigma_{\text{conv}} \equiv \Sigma_{\chi}^{(1)}(p_1, p_2) p_2^2 + 2\Sigma_{\chi}^{(1)}(p_2, p_1) p_1^2 + 2\Sigma_{\gamma}^{(1)}(p_2, p_1) p_1 p_2 + (p_1 \leftrightarrow p_2). (55)\]

The identity (54) can be evaluated even at \(q^2 = 0, p_1 = -p_2\).

Evaluating the anomaly equation (37) with (39), (50) and (51) and using

\[\partial_{\gamma} \Gamma_{\hat{A}_a A_b}^{(1)} = 0 (56)\]

one obtains

\[\eta^{(1)} = \Sigma_{\chi}(p_2^2) + \Sigma_{\chi}(p_1, -p_1) + \frac{1}{2} \Sigma_{\chi}(p_1^2) . (57)\]

Hence, combining (54) and (57) the anomaly coefficient is determined by the convergent one-loop functions \(\Sigma_{\gamma}^{(1)}\) and \(\Sigma_{\eta-\eta}^{(1)}\):

\[4\eta^{(1)} = -2\Sigma_{\eta-\eta}(p_1, -p_1) - \Sigma_{\text{conv}}^{(1)}(p_1, -p_1) + 2\Sigma_{\chi}^{(1)}(p_1^2). (58)\]
We want to note that a similar analysis as the one presented in one-loop applies to higher orders. However, the interpretation of the identities is different: Due to the topological formula (22) the identities include in all loop orders except for the one-loop order the self energy of vector fields:

\[ \partial_g \Gamma_{\hat{A}A} = 2(l - 1) \Gamma_{\hat{A}A} . \]  

(59)

Hence, for \( l \geq 2 \) the above identities determine the normalization of the coupling, and all higher-order breakings are related to a finite redefinition of the gauge coupling (see also [4]).

4 The one-loop calculation

In the present section we want to determine \( r^{(1)}_\eta \) by an explicit one-loop calculation. For this purpose we have to calculate the convergent vertex corrections of eq. (58). In order to simplify the calculation as much as possible we use the Feynman gauge (\( \xi = 1 \)) and a specific parameterization of the tree approximation (cf. (73) below). The anomaly coefficient does not depend on the gauge or the specific parameterization of the tree approximation.

It will be seen that the contributions to the anomaly coefficient are effectively generated by the one-loop correction \( \Sigma_{\eta - \bar{\eta}} \) to the Green functions \( \Gamma_{\eta - \bar{\eta} \hat{A}_a \hat{A}_b} \). Therefore we calculate the corresponding one-loop diagrams in the first step.

Differentiation with respect to \( \eta - \bar{\eta} \) on the 1PI functional \( \Gamma \) yields the functional of 1PI Green functions with the insertion of the divergence of the axial current of gluinos and with the insertion of the topological term \( G^{\mu\nu} \tilde{G}^{\mu\nu}, \tilde{G}^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma} \):

\[
\left( \frac{\delta}{\delta \eta(y)} - \frac{\delta}{\delta \bar{\eta}(y)} \right) \Gamma = i \left\{ \text{Tr} ( \partial (g^2 \lambda \sigma \lambda) - \frac{1}{4} G^{\mu\nu} \tilde{G}_{\mu\nu} (gA + \hat{A})) \right\} \cdot \Gamma .
\]

(60)

As we have shown in the last section, \( \Sigma_{\eta - \bar{\eta}} \) is uniquely determined only, when we consider its extension to local coupling. For this purpose we differentiate eq. (60) once more with respect to \( g(z) \). The differentiation acts first on the insertion and second on \( \Gamma \) producing a double inserted diagram. In the limit to constant coupling, in which we are finally interested, contributions of the double insertions vanish and it remains to evaluate the contribution of the single insertion:

\[
\hat{\Gamma}^{(1)}_{\eta \bar{\eta} \hat{A}_a \hat{A}_b} (r, q, p_1, p_2) \equiv 2i q^\rho g \left( \left[ i (J^\text{axial}_\rho - J^\text{top}_\rho) \right] \cdot \Gamma \right)_{\hat{A}_a \hat{A}_b} (q + r, p_1, p_2) ,
\]

(61)
where

\[ j_{\rho}^{\text{axial}} \equiv \text{Tr}(\lambda \sigma_\rho \bar{\lambda}) , \]
\[ J_{\rho}^{\text{top}} \equiv \epsilon_{\rho\mu'\nu'} (A_{\mu' a} \partial_{\nu'} A^\rho_a - \hat{A}_{\mu' c} A_{\nu' d} A_{\rho e} f_{cde}) . \] (62)

Using (44) we obtain from (61) an unambiguous expression for the 1PI Green function \( \Gamma_{\eta - \eta \hat{A} \hat{A}}^{(1)} \):

\[ \Gamma_{\eta - \eta \hat{A} \hat{A}}^{(1)} (q, p_1, p_2) = \frac{g}{2} \hat{\Gamma}_{\eta - \eta \hat{A} \hat{A}}^{(1)} (0, q, p_1, p_2) \] (63)

For the calculation of (61) we split the Green function with insertion into the fermionic and bosonic loop contributions

\[ \left[ i (j_{\rho}^{\text{axial}} - J_{\rho}^{\text{top}}) \right] \cdot \hat{\Gamma}_{\hat{A}_a \hat{A}_b}^{(1)} (q + r, p_1, p_2) \equiv \delta_{ab} (\Gamma_{\rho\mu\nu}^{\text{ferm}} (p_1, p_2) + \Gamma_{\rho\mu\nu}^{\text{bos}} (p_1, p_2)) . \] (64)

Both parts can be separately adjusted to be transversal by adding local counter-terms.

The fermionic part of (64) is just the usual triangle diagram with an axial current insertion of Majorana fermions. For constant coupling, i.e. \( r = 0 \), it yields the usual value of triangle anomaly as contribution to \( \Gamma_{\eta - \eta \hat{A} \hat{A}}^{(1)} \):

\[ iq^\rho \Gamma_{\rho\mu\nu}^{\text{ferm}} (q, p_1, p_2) = \frac{4i}{16\pi^2} C(G) \epsilon_{\mu\nu\lambda\rho} p_1^\lambda p_2^\rho . \] (65)

To the bosonic part the diagrams of figure 1 contribute. In contrast to the fermionic part, the bosonic part is gauge dependent. For the calculation we use the Feynman gauge \( \xi = 1 \). Using the Feynman rules for the background field and the momentum assignment of figure 1 the first diagram yields:

\[ \Gamma_{\rho\mu\nu}^{\text{bos, a}} (p_1, p_2) = \text{Re} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(k + p_1)^2} \frac{1}{(k - p_2)^2} i \epsilon_{\rho\mu'\nu'} (2k - p_2 + p_1)^\lambda \]
\[ \left( (-2k - p_1)_{\mu} \eta_{\mu'\nu'}' - 2p_{1\mu}^\prime \delta_{\mu}^\nu' + 2p_{1\mu}^\prime \delta_{\nu'}^\mu \right) \]
\[ \left( (2k + p_2)_{\nu} \delta_{\nu'}^\mu' - 2p_{2\nu}^\prime \eta_{\nu\mu'} + 2p_{2\nu}^\prime \delta_{\mu'}^\nu \right) C(G) , \] (66)

and the second and third diagram yield the symmetric contribution

\[ \Gamma_{\rho\mu\nu}^{\text{bos, b}} (p_1, p_2) = \text{Re} \int \frac{d^4k}{(2\pi)^4} 4i C(G) \epsilon_{\rho\mu\nu\lambda} \frac{1}{k^2} \left( p_1^\lambda \frac{1}{(k + p_1)^2} - p_2^\lambda \frac{1}{(k - p_2)^2} \right) . \] (67)
The last diagram of figure 1 vanishes identically.

The single contributions are logarithmically divergent and have to be regularized and subtracted for their evaluation, which is indicated by the sign R in front of the integrals. However, using the methods outlined in section 3 the divergent part is determined uniquely by the tensor part in terms of convergent one-loop integrals and the renormalized Green function \( \Gamma_{\text{bos}}^{\rho\mu\nu} \) can be completely expressed in terms of the finite functions \( C_i(p_1^2, p_2^2, q^2) \) and \( C_{ij}(p_1^2, p_2^2, q^2) \), \( i = 1, 2 \):

\[
\Gamma_{\rho\mu\nu}^{\text{bos}} = \frac{C(G)}{16\pi^2} \left( (\epsilon_{\rho\mu\nu\lambda}p_2^\lambda(-4p_1^2C_1 + 4p_2^2C_2 - 4C_0(p_1^2 + p_1p_2) + 4) 
+ p_1^\mu\epsilon_{\rho\mu\nu\sigma}p_1^\lambda p_2^\sigma(-16C_{11} - 16C_1 - 4C_0) 
+ p_2^\mu\epsilon_{\rho\mu\nu\sigma}p_1^\lambda p_2^\sigma(16C_{12} - 4C_0) \right) + (\mu \leftrightarrow \nu, 1 \leftrightarrow 2) .
\]

Using the relation,

\[
p_1^2C_{11} - p_1p_2C_{12} = -\frac{3}{4}p_1^2C_1 - \frac{1}{4}p_2^2C_2 - \frac{1}{4} \quad \text{for} \quad q^2 = (p_1 + p_2)^2, m_i = 0 ,
\]

it is immediately verified that \( \Gamma_{\rho\mu\nu}^{\text{bos}} \) is transversal as required.

From (68) one obtains the bosonic one-loop contribution to \( \Gamma_{\eta-\eta A\bar{A}}^{\text{bos}} \):

\[
id q^\rho \Gamma_{\rho\mu\nu}^{\text{bos}}(p_1, p_2) = \frac{i}{16\pi^2} C(G)\epsilon_{\mu\nu\rho\lambda}p_1^\rho p_2^\sigma(-8 + 4q^2C_0) .
\]

\[\text{For the definition of } C_-\text{-functions we use the conventions of } [19] \text{ with massless propagators, } m_i = 0 . \text{ They are summarized in Appendix C.}\]
The result agrees with the calculation of [20]. There ([G̃G]-Γ), has been determined in dimensional regularization without arguments on transversality we are able to exploit here due to local coupling. It is important to note that the more general result of [20] gives in addition the gauge parameter dependent contributions to the expression (70).

Adding the fermionic (65) and bosonic (70) contributions we get an unambiguous expression for \( \Gamma_{\eta-\eta} \) in Feynman gauge:

\[
\Gamma^{(1)}_{\eta-\eta} \equiv \Gamma^{(1)}_{\eta-\eta \bar{A} \bar{A}} (q, p_1, p_2, \xi = 1) = iq^\rho g^2 (G^{\text{ferm}}(p_1, p_2) + G^{\text{bos}}(p_1, p_2, \xi = 1)) \delta_{ab} \]
\[
= \frac{i}{16\pi^2} C(G) g^2 \epsilon_{\mu\nu\rho\lambda} p_1^\rho p_2^\sigma (4 - 8 + 4q^2C_0) \delta_{ab}, \quad (71)
\]

i.e.,

\[
\Sigma^{(1)}_{\eta-\eta} (p_1, -p_1) \bigg|_{\xi=1} = -\frac{4}{16\pi^2} C(G) g^2. \quad (72)
\]

We will show in the following that in Feynman gauge \( \Sigma^{(1)}_{\eta-\eta} \) is in fact the only non-vanishing contribution to the anomaly coefficient in the identity (58).

The evaluation of the remaining integrals appearing in (58) requires quite some work in the parameterization given in section 2. The calculation is considerably simplified when we choose the following parameterization in the tree approximation: We redefine the gluinos in the quantum vectors and the fermionic partners of the coupling:

\[
\lambda_\alpha \rightarrow \lambda'_\alpha = \lambda_\alpha + \frac{i}{2} g^2 (\sigma^\mu \chi)_\alpha A_\mu, \quad \bar{\lambda}_\dot{\alpha} \rightarrow \bar{\lambda}'_{\dot{\alpha}} = \bar{\lambda}_{\dot{\alpha}} - \frac{i}{2} g^2 (\chi \sigma^\mu)_{\dot{\alpha}} A_\mu. \quad (73)
\]

Since the redefinition does not depend on the background vectors, the Ward identity of background gauge invariance (31) remains unmodified. In addition we modify the background gauge fixing by adding a linear term with the gluinos to the gauge fixing function:

\[
\mathcal{F} \rightarrow \mathcal{F} = \partial A + i[\hat{A}, A] - \frac{1}{2} (\lambda \chi + \bar{\chi} \bar{\lambda}) \quad (74)
\]

The remaining diagrams together with the corresponding Feynman rules of the parameterization (73) are given in Appendix B. Evaluating them we find that the contributions to the vertex functions \( \Gamma_{\epsilon \chi \rho} \) vanish at all, i.e.,

\[
\Gamma^{(1)}_{\epsilon \chi_\alpha + \epsilon \bar{\chi}_{\dot{\alpha}} \hat{A}_\rho} (0, p_1, p_2) = 0 \quad \Rightarrow \quad \Sigma^{(1)}_1 = 0
\]
\[
\Gamma^{(1)}_{\epsilon \chi_\alpha - \epsilon \bar{\chi}_{\dot{\alpha}} \hat{A}_\rho} (q, p_1, p_2) = 0 \quad \Rightarrow \quad \Sigma^{(1)}_2 = 0. \quad (75)
\]
It remains to evaluate $\Gamma_{\chi^a \lambda^b \hat{A}}$ (71) and in particular the convergent functions $\Sigma_7$ and $\Sigma_8$. There are three diagrams which are shown in figure 3 in Appendix B. The first one is divergent and its nonlocal part cancels just the divergent contributions from $\Gamma_{\epsilon \lambda^b \hat{A}}$ in eq. (71) and (74). The $\sigma^{\mu \rho}$ contributions of the second diagram vanish and the ones of the third diagram are finite yielding an additional non-local term to $\Sigma_{\chi^a}$ which vanishes at $q^2 = 0$ ². Altogether we get $(q + p_1 + p_2 = 0)$

$$\Sigma_{\chi^a}(p_2, p_1) = \Sigma_{\chi^a}(p_2, -p_2) + \frac{4g^2}{16\pi^2} C(G) (qp_1 C_1 - qp_2 C_2 + qp_1 C_0), \quad (76)$$

$$\Sigma_7 = \Sigma_8 = 0 . \quad (77)$$

Inserting the one-loop expressions (77) and (75) of the Feynman gauge into the identity (58) the anomaly equation simplifies to

$$r^{(1)}_{\eta} = -\frac{1}{2} \Sigma^{(1)}_{\eta - \overline{\eta}}(p_1, -p_1) \bigg|_{\xi = 1} , \quad (78)$$

and yields with the one-loop result for $\Sigma^{(1)}_{\eta - \overline{\eta}}$ (72) the anomaly coefficient:

$$r^{(1)}_{\eta} = \frac{C(G)}{8\pi^2} . \quad (79)$$

Recalling the definition of $\Sigma^{(1)}_{\eta - \overline{\eta}}$ (11) and using the explicit expression for the $\eta - \overline{\eta}$-insertion (60),

$$\left[i \ \text{Tr} \left( \partial (g^2 \lambda \sigma \lambda) - \frac{1}{4} G^{\mu \nu} \tilde{C}_{\mu \nu} (gA + \hat{A}) \right) \right] \delta^a_{\hat{A}^b} (q, p_1, p_2) = i \epsilon^{\mu \nu \rho \sigma} p_{1 \rho} p_{2 \sigma} \delta_{ab} (-2 + \Sigma_{\eta - \overline{\eta}}(p_1, p_2)) , \quad (80)$$

it is seen that the anomaly coefficient is determined by the Feynman gauge one-loop correction to $G \tilde{G}(\hat{A})$ evaluated in the limit of vanishing external momenta.

This constitutes our final result expressing that the anomalous supersymmetry breaking (see (24) and (25)) is induced by the vertex corrections to $G \tilde{G}$. In Feynman gauge the anomaly coefficient has the simple form (78). For general gauges $G \tilde{G}$ is gauge parameter dependent and only, when solving the complete identities, its gauge parameter dependent part and its non-local part are canceled by the sum of the individual contributions. We have not been able to extract the simplifications of the Feynman gauge from general principles, but they seem to appear due to accidental cancellations in the explicit calculations.

²Inserting (76) and (72) into the identity (74) one verifies that the non-local contributions cancel for $q \neq 0$. 

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5 Discussion and conclusions

The extension of coupling constants to space-time dependent superfields has been seen to be a crucial step for the formulation and derivation of supersymmetric non-renormalization theorems in a scheme and gauge-independent framework [21, 3, 22, 7]. For gauge theories the extension to local couplings does not only include the non-renormalization theorems of the chiral vertices [23, 24] but also the generalized non-renormalization theorem of the gauge coupling [2]. The latter property constitutes itself in absence of symmetric counterterms to the susy Yang-Mills part of the action in loop orders higher than one. Thus, in naive applications of symmetries one would expect a strictly one-loop gauge $\beta$-function.

Extending the coupling to a space time dependent field supersymmetry has an anomalous breaking in one-loop order [3]. The anomalous breaking is the supervariation of a field monomial depending on the logarithm of the local coupling and a such its coefficient has been shown to be scheme independent and gauge parameter independent. This anomalous breaking has been shown to induce the non-holomorphic dependence in the gauge $\beta$-function of pure Super-Yang-Mills theories, whereas the matter contributions are induced by the Adler-Bardeen anomaly.

In the present paper we have determined the coefficient of the anomaly in a one-loop calculation. The computation has been carried out in a completely scheme and regularization independent framework. For this purpose we have first established the symmetry identities which determine the anomaly coefficient. From these identities all contributing one-loop vertex functions are fixed by symmetries and not by a scheme-dependent subtraction procedure. The analysis has been simplified by the use of a background gauge fixing which allows to exploit transversality of 2-point functions involving the background vector fields. Evaluating the anomaly identities yields the anomaly coefficient in terms of convergent one-loop integrals.

The explicit computation has been carried out in the Feynman gauge. Here the anomaly coefficient is directly determined by the vertex correction to the topological term $\text{Tr} \, \tilde{G} \tilde{G}$ at vanishing momenta. There are two terms which contribute to $\text{Tr} \, G \tilde{G}$ in one-loop order, these are diagrams with the insertion of the axial current of gluinos and with the insertion of the topological Chern-Simons current. The first diagram yields the local contributions known from the triangle anomaly, the second yields a non-local and in general even gauge dependent contribution. In the Feynman gauge and at vanishing momenta the topological current yields a local contribution twice as large as the contribution of the axial current and with opposite sign. Thus, they sum up to the non-vanishing anomaly coefficient of Super-Yang-Mills with local gauge coupling.

The result is valid also for $N = 1$ Super-Yang-Mills theories with matter, when the matter part is extended to local couplings as proposed in [3]. This construction
may be modified when one aims at the construction of $N = 2$ and $N = 4$ theories resulting in a higher symmetry and finally in a vanishing anomaly coefficient.

In the Wilsonian approach the non-holomorphic dependence of the gauge $\beta$-function has been recently related to a rescaling anomaly, which appears due to the redefinition from the holomorphic to the canonical coupling constant [25]. The present results are independent of the notion of a Wilsonian coupling, and thus the relations of these findings to our results are not apparent at the first glance.

In earlier publications on the topic [2], however, a relation between the vertex corrections to $\text{Tr} \, G \tilde{G}$ and the two-loop coefficient of the gauge $\beta$-function has been already suggested, but the classical symmetry, to which $\text{Tr} \, G \tilde{G}$ contributes an anomalous breaking, could not be established. Thus, the anomalous contributions have been identified by an infrared analysis claiming a similarity between the infrared renormalization of the axial current and the topological current [26, 27].

For the present construction infrared effects do not play any role, but what is essential, is the extension to local coupling. With local coupling the gauge invariant counterterms to $\text{Tr} \, G \tilde{G}$ are not a total derivative in higher orders and as such they are excluded from the present construction. Hence, with a non-integrated local coupling transversality can be exploited to fix uniquely the local contributions, i.e. the divergent part of the vertex function, from convergent tensor integrals, finding a unique result also in the limit of constant coupling.

In this respect the construction may be also interesting for non-supersymmetric theories: Renormalization of $\text{Tr} \, G \tilde{G}$ has to be considered for establishing the non-renormalization theorems [23] of the Adler-Bardeen anomaly [17, 24, 30]. As a composite operator its definition is not unique as it stands, but its definition can be traced back to renormalization of a finite operator by the usage of descent equations [31, 32, 33]. These difficulties can be circumvented by extending the gauge coupling to a space-time dependent field. Then the renormalization of $\text{Tr} \, G \tilde{G}$ is unique and can be directly deduced from gauge invariance or more generally BRS invariance and from its property as being a total derivative, in the same way as it was worked out in section 3 of the present paper. Hence, the local and possibly divergent contribution to the anomaly is related to the superficially convergent tensor part stating the Adler-Bardeen non-renormalization theorem. A short outline of the non-renormalization of the Adler-Bardeen anomaly in presence of local couplings have been presented also in refs. [2, 5].

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A The BRS transformations

In this appendix we list the BRS transformations of the fields with background gauge fixing.

• BRS transformations of the vector multiplet

\[ s A_\mu = \frac{1}{g} \partial_\mu (gc) + i [gA_\mu + \hat{A}_\mu, c] + i \epsilon \sigma_\mu \bar{\lambda} - i \lambda \sigma_\mu \bar{\epsilon} \]

\[ + \frac{1}{2} g^2 (\epsilon \chi + \bar{\chi} \epsilon) A_\mu - \frac{1}{g} C_\mu - i \omega^\nu \partial_\nu A_\mu , \]

\[ s \lambda^\alpha = -ig \{ \lambda, c \} + \frac{i}{2g} (\epsilon \sigma^{\rho\sigma})^\alpha G_{\rho\sigma} (gA + \hat{A}) \]

\[ + \frac{1}{2} \epsilon^\alpha g^2 (\chi \lambda - \bar{\chi} \lambda) + \frac{1}{2} g^2 (\epsilon \chi + \bar{\chi} \epsilon) \lambda^\alpha - i \omega^\nu \partial_\nu \lambda^\alpha , \]

\[ s \bar{\lambda}_{\dot{\alpha}} = -ig \{ \bar{\lambda}, c \} - \frac{i}{2g} (\epsilon \sigma^{\rho\sigma})_{\dot{\alpha}} G_{\rho\sigma} (gA + \hat{A}) \]

\[ + \frac{1}{2} \epsilon_{\dot{\alpha}} g^2 (\chi \lambda - \bar{\chi} \lambda) + \frac{1}{2} g^2 (\epsilon \chi + \bar{\chi} \epsilon) \bar{\lambda}_{\dot{\alpha}} - i \omega^\nu \partial_\nu \bar{\lambda}_{\dot{\alpha}} . \]

• The BRS transformations of the background field and its ghost

\[ s \hat{A}^\mu = C^\mu - i \omega^\nu \partial_\nu \hat{A}^\mu , \]

\[ s C^\mu = 2 i \epsilon \sigma^\nu \bar{\epsilon} \partial_\nu \hat{A}^\mu - i \omega^\nu \partial_\nu C^\mu . \]

• The BRS transformations of ghosts

\[ s c = -\frac{1}{2} g \{ c, c \} + \frac{2}{g} i \epsilon \sigma^\nu \bar{\epsilon} (gA_\nu + \hat{A}_\nu) + \frac{1}{2} g^2 (\epsilon \chi + \bar{\chi} \epsilon) c - i \omega^\nu \partial_\nu c , \]

\[ s \epsilon^\alpha = 0 , \]

\[ s \bar{\epsilon}^{\dot{\alpha}} = 0 , \]

\[ s \omega^\nu = 2 \epsilon \sigma^\nu \bar{\epsilon} . \]

• BRS transformations of the B-fields and the anti-ghosts

\[ s B = 2 i \epsilon \sigma^\nu \bar{\epsilon} \partial_\nu \bar{\epsilon} - i \omega^\nu \partial_\nu B , \]

\[ s \bar{\epsilon} = B - i \omega^\nu \partial_\nu \bar{\epsilon} , \]
BRS transformations of the local coupling (5) and its superpartners (4)

\[ s g^2 = -\left(\overline{\epsilon}^\alpha \chi + \overline{\chi}^i \epsilon^i\right)g^4 - i\omega^\nu \partial_\nu g^2 , \]
\[ s (\eta - \overline{\eta}) = (\epsilon^\alpha \chi + \overline{\chi}^i \epsilon^i)(\eta - \overline{\eta}) - i\omega^\nu \partial_\nu (\eta - \overline{\eta}) , \]
\[ s \chi_\alpha = -i(\sigma^\mu \epsilon)_\alpha \left(\frac{1}{g^4} \partial_\mu g^2 - \partial_\mu (\eta - \overline{\eta})\right) + 2\epsilon_\alpha f - i\omega^\mu \partial_\mu \chi_\alpha , \]
\[ s \overline{\chi}_\dot{\alpha} = -i(\epsilon \sigma^\mu)_{\dot{\alpha}} \left(\frac{1}{g^4} \partial_\mu g^2 + \partial_\mu (\eta - \overline{\eta})\right) - 2\overline{\epsilon}_{\dot{\alpha}} \tilde{f} - i\omega^\mu \partial_\mu \overline{\chi}_{\dot{\alpha}} , \]
\[ s f = i\partial_\mu \chi \sigma^\mu \tau - i\omega^\mu \partial_\mu f , \]
\[ s \tilde{f} = -i\epsilon \sigma^\mu \partial_\mu \overline{\chi} - i\omega^\mu \partial_\mu \tilde{f} . \]

B The one-loop diagrams

In figure 2 and 3 we give the one-loop diagrams contributing to the identities (37) and (38). For the calculation we use the parameterization (73) of the classical action and the modified gauge fixing function (74) and eliminate the auxiliary B-fields from the gauge fixing action (14). The vertices of the Super-Yang-Mills action and its BRS-transformations remain unmodified for constant coupling. For the \( \chi \) and \( \epsilon \) tree vertices, which contribute in the diagrams of figure 2 and 3, one has the following expressions:

\[ \Gamma^{(0)}_{\chi_\alpha \lambda_\alpha \chi_\lambda} (q, p_1, p_2) = -g^2 \frac{i}{2} \delta_{ab} (\sigma^\nu \sigma^\mu)_{\beta} q^\nu \]
\[ \Gamma^{(0)}_{\chi_\alpha \lambda_\alpha \chi_\lambda} (q, p_1, p_2, p_3) = g^3 \frac{i}{2} f_{abc} (\sigma^\mu \sigma^\nu)_{\beta} \]
\[ \Gamma^{(0)}_{\epsilon_\alpha \lambda_\alpha \epsilon_\lambda} (p, -p) = \delta_{ab} \sigma^\nu \sigma^\mu p^\nu \]
\[ \Gamma^{(0)}_{\epsilon_\alpha \lambda_\alpha \epsilon_\lambda} (p_1, p_2, p_3) = -if_{abc} \sigma^\mu \sigma^\nu \]

We want to note that the vertex function \( \Gamma_{\chi \lambda \hat{A}} \) indeed vanishes in the tree approximation:

\[ \Gamma^{(0)}_{\chi_\alpha \lambda_\alpha \chi_\lambda} (q, p_1, p_2, p_3) = 0 \]
\[ C_{\{0,\mu,\nu\}}(p_1, p_2, m_0, m_1, m_2) = \frac{1}{i\pi^2} \text{Re} \int d^4k \frac{\{1, k_\mu, k_\mu k_\nu\}}{D_0 D_1 D_2}. \]  

with \( D_0 = q^2 - m_0^2 + i\epsilon, D_i = (q + p_i)^2 - m_i^2 + i\epsilon, i \geq 1. \) The tensor integrals \( C_\mu \) and \( C_{\mu\nu} \) can be decomposed into Lorentz tensors constructed of external momenta \( p_{i\mu} \) and the metric tensor \( \eta_{\mu\nu}. \) The decomposition defines the tensor-coefficient functions \( C_i \) and \( C_{ij} \): 

\[ C_\mu = \sum_{i=1}^{2} C_{i\mu}, \quad C_{\mu\nu} = \sum_{i,j=1}^{2} C_{ij} p_{i\mu} p_{j\nu} + C_{00} \eta_{\mu\nu}. \]
Since the tensor-coefficient functions are scalars, one can write their arguments as:

\[ C_{\cdots}(p_1, p_2, m_0, m_1, m_2) = C_{\cdots}(p_1^2, p_2^2, q^2, m_1, m_0, m_2) \],

(93)

with \( q^2 = (p_1 - p_2)^2 \). The only divergent function is the function \( C_{00} \). Its local part depends on the specific subtraction scheme.

The convergent tensor-coefficient functions of the 3-point integrals \( C_i \) and \( C_{ij}, i, j = 1, 2 \) allow for the following Feynman-parameter representations [19]:

\[
\begin{align*}
C_{\cdots}^{i\ldots j}(p_1, p_2, m_0, m_1, m_2) & = (-1)^{i+j} \int_0^\infty \frac{dx_0 dx_1 dx_2 x_1^i x_2^j \delta(1 - x_0 - x_1 - x_2)}{m_0^2 x_0 + m_1^2 x_1 + m_2^2 x_2 - p_1^2 x_0 x_1 - p_2^2 x_0 x_2 - q^2 x_1 x_2 - i\epsilon}.
\end{align*}
\]

(94)

The Feynman parameter representation of the scalar 3-point function \( C_0 \) is given by (94) with \( i = j = 0 \).

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