Study of exotic hadrons in s-wave chiral dynamics

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We study the exotic hadrons in s-wave scattering of the Nambu-Goldstone boson with a target hadron based on chiral dynamics. Utilizing the low energy theorem of chiral symmetry, we show that the s-wave interaction is not strong enough to generate bound states in exotic channels in flavor SU(3) symmetric limit, although the interaction is responsible for generating some nonexotic hadron resonances dynamically. We discuss the renormalization condition adopted in this analysis.

§1. Introduction

One of the nontrivial issues in hadron physics is almost complete absence of flavor exotic hadrons. Experimentally, we have been observing more than hundred of hadrons whose flavor quantum numbers can be expressed by minimal valence quark contents of ¯q q or q q q. The only one exception is the exotic baryon Θ⁺ with S = +1 which is composed of at least five valence quarks. In this way, the exotic hadrons are indeed “exotic” as an experimental fact. On the other hand, there is no clear theoretical explanation for the nonobservation of the exotic hadrons. Our current knowledge does not forbid to construct four or five quark states in QCD and in effective models. Moreover, the multiquark components in nonexotic hadrons are evident, as seen in the antiquark distribution (or pion cloud) in nucleon and successful descriptions of some excited hadrons as resonances in two-hadron scatterings. In view of these facts, it is fair to say that the nonobservation of the exotic hadrons is not fully understood theoretically.

§2. Exotic hadrons in s-wave chiral dynamics

In chiral coupled-channel dynamics, some hadron resonances have been successfully described in s-wave scattering of a hadron and the Nambu-Goldstone (NG) boson, along the same line with the old studies with phenomenological vector meson exchange interaction. It was found that the generated resonances turned into bound states in flavor SU(3) symmetric limit. We therefore conjecture that the bound states in the SU(3) limit are the origin of a certain class of physical resonances, and we examine the possible existence of exotic hadrons as hadron-NG boson bound states.

The low energy interaction of the NG boson (Ad) with a target hadron (T) in

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s-wave is given by

\[ V_\alpha = -\frac{\omega^2}{2f^2} C_{\alpha,T}, \tag{2.1} \]

with the decay constant \( f \) and the energy \( \omega \) of the NG boson. The factor \( C_{\alpha,T} \) is determined by specifying the flavor representations of the target \( T \) and the scattering system \( \alpha \in T \otimes \text{Ad} \):

\[ C_{\alpha,T} = -\langle 2F_T \cdot F_{\text{Ad}} \rangle_\alpha = C_2(T) - C_2(\alpha) + 3, \tag{2.2} \]

where \( C_2(R) \) is the quadratic Casimir of SU(3) for the representation \( R \). Eq. (2.1) is the model-independent consequence of chiral symmetry, known as the Weinberg-Tomozawa theorem.[12, 13]

We have written down the general expression of the coupling strengths (2.2) for arbitrary representations of target hadrons in SU(3). In order to specify the exotic channels, we introduced the exoticness quantum number, as the number of valence antiquarks to construct the given flavor multiplet for the states with positive baryon number. Then we find that the Weinberg-Tomozawa interaction in the exotic channels is repulsive in most cases, and that possible strength of the attractive interaction is given by a universal value

\[ C_{\text{exotic}} = 1, \tag{2.3} \]

with \( \alpha = [p-1, 2] \) for \( T = [p, 0] \) and \( p \geq 3B \).[10, 11]

Next we construct the scattering amplitude with unitarity condition using the N/D method.[9] The unitarized amplitude is given by

\[ t_\alpha(\sqrt{s}) = \frac{1}{1 - V_\alpha(\sqrt{s})G(\sqrt{s})V_\alpha(\sqrt{s})}, \]

as a function of the center-of-mass energy \( \sqrt{s} \). The loop function \( G(\sqrt{s}) \) is regularized by the once subtraction as

\[ G(\sqrt{s}) = -\tilde{a}(s_0) - \frac{1}{2\pi} \int_{s^+}^{\infty} ds' \left( \frac{\rho(s')}{s' - s} - \frac{\rho(s')}{s' - s_0} \right), \tag{2.4} \]

where the phase space integrand is \( \rho(s) = 2M_T \sqrt{(s - s^+)(s - s^-)/(8\pi s)} \), \( s^\pm = (m \pm M_T)^2 \), and \( m \) and \( M_T \) are the masses of the target hadron and the NG boson.

In order to determine the subtraction constant \( \tilde{a}(s_0) \) and the subtraction point \( s_0 \), we adopt the renormalization condition given in Refs. [14, 6],

\[ G(\mu) = 0, \quad \mu = M_T, \tag{2.5} \]

which is equivalent to \( t_\alpha(\mu) = V_\alpha(\mu) \) at this scale. We will discuss the implication of this prescription in section 3. With the condition (2.5), we show that the bound state can be obtained if the coupling strength (2.2) is larger than the critical value

\[ C_{\text{crit}} = \frac{2f^2}{m[-G(M_T + m)]}. \]
Varying the parameters $m$, $M_T$, and $f$ in the physically allowed region, we show that the attraction in the exotic channels (2.3) is always smaller than the critical value $C_{\text{crit}}$. Thus, it is not possible to generate bound states in exotic channels in the SU(3) symmetric limit.

We would like to emphasize that this conclusion is model independent in the SU(3) limit, as far as we respect chiral symmetry. In this study, we only consider the exotic hadrons composed of the NG boson and a hadron, so the existence of exotic states generated by quark dynamics or rotational excitations of chiral solitons is not excluded. In practice, one should bear in mind that the SU(3) breaking effect and higher order terms in the chiral Lagrangian would play a substantial role. Nevertheless, the study of exotic hadrons in a simple extension of a successful model of hadron resonances as we have done in the present work can partly explain difficulty of observation of the exotic hadrons.

§3. Interpretation of the renormalization condition

Here we discuss the renormalization condition (2.5) in this analysis. The interaction kernel $V_\alpha(\sqrt{s})$ is constructed from chiral perturbation theory so as to satisfy the low energy theorem. The low energy theorem also constrains the behavior of the full unitarized amplitude $t_\alpha(\sqrt{s})$ at a scale $\sqrt{s} = \mu_m$ where the chiral expansion is valid. Therefore we can match the unitarized amplitude $t_\alpha(\sqrt{s})$ with the tree level one $V_\alpha(\sqrt{s})$ at the scale $\mu_m$:

$$t_\alpha(\mu_m) = V_\alpha(\mu_m) + V_\alpha(\mu_m)G(\mu_m)V_\alpha(\mu_m) + \cdots = V_\alpha(\mu_m),$$

This condition determines the subtraction constant such that the loop function $G(\mu_m)$ vanishes. This is only possible within the region

$$M_T - m \leq \mu_m \leq M_T + m,$$

since the loop function has an imaginary part outside this region and the subtraction constant is a real number. We consider that by employing this renormalization condition, a natural unitarization of the kernel interaction based on chiral symmetry is realized. Interestingly, if we apply this prescription for the case of the octet baryon target, the subtraction constant turns out to be “natural size” which was found in the comparison with three-momentum cutoff and the experimental observables in the $S = -1$ meson-baryon channel are successfully reproduced.

We take $\mu_m = M_T$ in the present study. The dependence on $\mu_m$ within the region (3.2) is found that the binding energy of the bound state increases if we shift the matching scale $\mu_m$ to the lower energy region. This is discussed also in Refs. 17, 18) by varying the subtraction constant. The region $\mu_m \leq M_T$ corresponds to the $u$-channel scattering. Thus $\mu_m = M_T$ is the most favorable to generate a bound state within the $s$-channel regime.

The unitarized amplitudes in the above prescription do not always reproduce experimental data. In such a case, the subtraction constants $\tilde{a}(s_0)$ should be adjusted in order to satisfy experimental data. The subtraction constants determined in this
way supplement the role of the higher order chiral Lagrangians, which is lacking in the kernel interaction. As shown for the $\rho$ meson effect in the meson-meson scattering, the higher order terms may contain the effect of the resonances. Therefore, if the natural condition (3.1) is badly violated, one may speculate that the seeds of resonances in the higher order Lagrangian, which are possibly the genuine quark states, appear in the unitarized amplitude, as in the study of Ref. 19.

In summary, we have argued the following issues.

- In order for the unitarized amplitude $t_\alpha(\sqrt{s})$ to satisfy the low energy theorem, the loop function should vanish in the region (3.2).
- This requirement can be regarded as a natural unitarization, without introducing the effect of resonances in the higher order Lagrangian.
- Lower matching scale $\mu_m$ is more favorable to generate a bound state.

Turning to the problem of exotic hadrons, what we have shown is the nonexistence of the exotic bound states with the most favorable condition to generate a bound state, without introducing the seed of genuine quark state.

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