QUARK MASSES FROM QUARK–GLUON CONDENSATES IN A MODIFIED PERTURBATIVE QCD

A. Cabo
Theoretical Physics Division, CERN
CH - 1211 Geneva 23, Switzerland

ABSTRACT

In this note, it is argued that the mass matrix for the six quarks can be generated in first approximation by introducing fermion condensates on the same lines as was done before for gluons, within the modified perturbative expansion for QCD proposed in former works. Thus, the results point in the direction of the conjectured link of the approximate ‘Democratic’ symmetry of the quark mass matrix and ‘gap’ effects similar to the ones occurring in superconductivity. The condensates are introduced here non-dynamically and therefore the question of the possibility for their spontaneous generation remains open. However, possible ways out of the predicted lack of the ‘Democratic’ symmetry of the condensates resulting from the spontaneous breaking of the flavour symmetry are suggested. They come from an analysis based on the Cornwall–Jackiw–Tomboulis (CJT) effective potential for composite operators.

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* Permanent address: Group of Theoretical Physics, Instituto de Cibernética, Matemática y Física, Calle E, No. 309, Vedado, La Habana, Cuba. Email: cabo@cident.icmf.inf.cu.
1 Introduction

The nature of the chiral and flavour symmetries in QCD and the Standard Model is one of the open fundamental questions in high energy physics \[1, 2, 3, 4, 5, 6\]. The amount of experimental information in direct relation with chiral and flavour physics that remains to be properly understood is enormous. This situation, implies that these problems deserve to be considered along many directions in search of a better understanding.

In previous works we studied the consequences of modifying the Feynman rules of the perturbative QCD in a way that incorporates the presence of a condensate of zero-momentum gluons in the initial state for constructing the Wick expansion \[7, 8, 9, 10\]. In \[8\] the modified theory included only a new term in the gluon propagator. This form of the change resulted from the adiabatic connection of the interaction on a modified vacuum of the non-interacting theory. The new state was chosen in a form similar to the way the BCS wavefunction of superconductivity is sometimes represented, i.e. as an exponential of a quadratic polynomial in the creation operators. For the BCS wavefunction two electron creation operators are employed. In our discussion a quadratic polynomial in the gluon and ghosts creation operators were considered. The colourless character and the additional requirement that the new vacuum be a physical state of the free theory in the non-interacting limit, strictly defined the form of the polynomial. Also, the Lorentz invariance requirement led us to consider only creation operators near the zero momentum limit.

The above propagator, which was employed in a simple tree calculation directly led to a value of the gluon condensate parameter (defined as the mean value of the gauge Lagrangian) different from zero. Moreover, when fixing the only parameter \( C \) to reproduce a current estimate for \( \langle g^2 G^2 \rangle \) the one-loop corrections for the quark masses surprisingly gave as the result one third of the nucleon mass value \[9\].

The mentioned elements naturally suggested the following idea: if the BCS-like modification of the gluon state could lead to such interesting conclusions, as the prediction of the constituent masses for quarks, it could be reasonable to expected that a similar state for the quarks, introduced in the massless QCD, could describe the Lagrangian quark masses. This is the basic question addressed in this note.

The answer, at least in this preliminary stage, is positive. After modifying the free-quark propagator by introducing the zero momentum terms representing the analogue to Cooper pair condensate in this problem (that is, all the colourless and uncharged quark–antiquark condensates) the simplest approximation for the Dyson equation for quarks produces a diagonal Lagrangian mass matrix by the simple choice of a structure also diagonal, for the quark condensates. A second possibility is also analysed it corresponds to the case in which the ‘Democratic’ symmetry proposed by H. Fritzsch \[2, 3\], is assumed for the condensate matrix. Then, it turns out that this symmetry is translated to the quark mass matrix and the spectrum produces two massive quarks each having two massless counterparts at the level of approximation considered. That is, the six initially equivalent quarks are decomposed in two triplets resembling the \( u \) and \( d \) type sets \( (u, c, t) \) and \( (d, s, b) \).
Further, the inclusion of the zero momentum piece of the gluon propagator reflecting the gluon condensate is also examined in order to check for consistency with the results obtained in [9] for the constituent masses when the Lagrangian quark masses were taken as parameters. The results for the six quarks were qualitatively in correspondence with the former evaluation. Again, the light quark masses turned out to have near by 1/3 of the proton mass and the massive ones practically coincided with their Lagrangian values. Thus, the picture seems to be consistent, up to these first approximations.

Finally, comments are given about the possibility of justifying the pattern of quark condensates reproducing the quark masses, as generated by a dynamical breaking of the chiral symmetry. Possible solutions are suggested of the problem related with the indications of a necessary equality of the dynamically generated fermions condensates [5, 4]. The basic argument given here consists in first underlining the relevance, for the determination of the form of the chiral symmetry breaking, that have the gap equations considered in the CJT effective action approach for composite operators; then, and more importantly, it is to indicate the possible need of the inclusion of more than one loop fermion corrections for the CJT effective action for the quark and gluon propagators. These terms are directly connected with the existence of attractive channels between different fermions and should be of relevance in determining the strength of the condensate. They are linked with the two-particle Bethe–Salpeter propagator whose associated bound state equation is in turn in close correspondence with the existence and strength of the condensate of Cooper pairs in the BCS description of superconductivity. On the other hand, the argument implying the equality of all the condensates rests on what could be at first sight the reasonable assumption, that for a large number of colours, the one loop correction to the effective action should be the leading contribution [5].

The paper is organized as follows: in Section 2, the corrections to the one loop quark self-energy and the related dispersion equations are discussed. Section 3 is devoted to checking the effect of the additional consideration of the gluon condensate term in the propagator. Finally, in this same final section, possible ways are suggested through which non-equal condensates for all the fermions could emerge dynamically.

2 Lagrangian quark masses in terms of condensates

As mentioned in the introduction, we will consider within the first approximation, the effects of employing a modified quark propagator reflecting the presence of condensates of quarks in the vacuum of the free massless QCD. The specific form of the propagator will be

\[
C^{i_1 i_2; f_1 f_2}_q(p) = \left( -\frac{\gamma^\mu p_\mu}{p^2 + i\epsilon} + i\delta(p)\delta^{f_1 f_2} \right)\delta^{i_1 i_2}, \quad i, j = 1, 2, 3,
\]

where the structure has a diagonal form in the colour indices, and the real and symmetric matrix in the flavour indices \(C^{f_1 f_2}\) reflects the consideration of all the possible sorts of colourless quark-antiquark condensates. As shown in Refs. [8, 10] this zero momentum contribution for fermions
can be obtained after considering as the initial state, before the connection of the interaction, a modified vacuum in the form of an exponential of quadratic products of quark and antiquark creation operators as

$$|0\rangle = \lim_{p^* \to 0} \exp \left( \sum_{f_1, f_2} \tilde{C}_q^f (p^*) \, \bar{q}_{f_1}^+ (p) \, q_{f_2}^+ (p^*) \right)$$

in which the limit of a small null momentum $p^* \to 0$ should be taken in the corresponding infinite volume limit $V \sim 1/|\vec{p}^*|^3 \to \infty$, where $\vec{p}^*$ is the spatial part of $p^*$. The structure of this state is similar to the one employed for the representation of the BCS wavefunction, and the attractive and very strong character of the colour forces, at least naively suggest the relevance of this approach to the description of QCD. Even in the presence of the weak phonon attraction with respect with the strong Coulomb repulsion between electrons, the Cooper pairs condensate becomes essential for the description of the superconducting state. Then, it is hard to think that in the case of the attractive and even confining attraction due to colour interaction the vacuum without condensates could be stable.

In order to consider the first implications of the introduced modification, lets us disregard for the moment the gluon condensate and evaluate the contribution of the change in the quark propagator to the one loop self-energy, associated to the first diagram in Fig. 1. The quark and gluon lines with central dots represent the added terms in the quark and gluon propagators. As the loop integration is annihilated by the delta function at zero momentum, the result is simply given by

$$\Sigma_{q}^{i_1 i_2; f_1 f_2} (p) = \frac{\gamma_{\mu} \gamma^\mu g^2}{(2\pi)^4} \frac{T_{i_1 i}^a}{p^2} \frac{T_{l i_2}^a}{p^2} = \frac{4 g^2}{(2\pi)^4} \frac{T_{i_1 i}^a}{p^2} \frac{T_{l i_2}^a}{p^2} = \frac{4 g^2 C_F}{(2\pi)^4} \frac{C_f^f f_2}{p^2}.$$  

(2)

where use has been done of the relations

$$\left( \gamma_{\mu} p^\mu \right)^2 = p^2, \quad T_{i_1 i}^a \, T_{l i_2}^a = C_F \, \delta_{i_1 i_2}, \quad C_F = \frac{N^2 - 1}{2N}.$$

Note that a better procedure could be to sum up all the mass insertions in the internal quark propagator joining the two vertices. However, here we are mainly interested in considering the simplest approximation. It should be noticed that this term can be viewed, in conjunction with its gluonic counterpart in Fig. 1 (to be introduced in the next section for treating gluon condensation), as the only two terms in a sort of ‘modified tree’ approximation for the self-energy in which no ‘modified loop’ radiative corrections are considered. The search for a systematic formulation for an expansion of this kind will be considered elsewhere. The relevance
Figure 1: The condensate contributions to the one loop selfenergies. Note that the zero momentum Dirac delta functions annihilate the only existing integration. This circumstance suggests the existence of a systematic ‘modified loop’ approximation in the problem.

of such loop expansion in determining an alternative and more rapidly convergent perturbative scheme is also suggested, for example, by the similarly existing procedures for Bose condensation phenomena. There, the Bogoliubov shift in the amplitude of the scalar condensate field to the minimum, leads to a satisfactory perturbation expansion in terms of the new modified radiative field.

After substituting the self-energy approximate expression (2) in the Dyson equation for the propagator, and considering the quark wave modes solving the homogeneous version of the equation, it follows

\[ 0 = \left( -p_\mu \gamma^\mu \delta^{ij} \delta_{f_1 f_2} - \Sigma_{q_{ji}} \delta_{f_1 f_2} (p) \right) \Psi_{f_2}^{i_2} (p) \]

\[ = \delta^{ij_2} \left( -p_\mu \gamma^\mu \delta_{f_1 f_2} - \frac{4g^2 C_F}{(2\pi)^4} \frac{C_{f_1 f_2}}{p^2} \right) \Psi_{f_2}^{i_2} (p) \]

\[ = \frac{1}{p^2} \left( -p^2 p_\mu \gamma^\mu \delta_{f_1 f_2} - \frac{4g^2 C_F}{(2\pi)^4} C_{f_1 f_2} \right) \Psi_{f_2}^{i_1} (p), \]

which after being multiplied by the matrix between the parentheses with the second term having opposite sign, results in

\[ \left( p^2 \right)^3 \delta_{f_1 f_2} - \left( \frac{g^2 C_F}{4\pi^4} \right)^2 \left( C_{f_1 f_2} \right) \Psi_{f_2} (p) = 0. \]  

Therefore, it can be noticed that, within the considered approximation, the quark masses are defined by the eigenvalues of the square of the matrix of the condensates. Below, two phenomenological possibilities for fixing the condensate matrix will be treated.

2.0.1 Diagonal condensate matrix

This case corresponds to directly selecting \( C_{f_1 f_2} \) as a diagonal matrix in the flavour indices in order that its square multiplied by \( \left( \frac{g^2 C_F}{4\pi^4} \right)^2 \), substituted in (4) will produce the mass matrix
for the six quarks. Thus $C_{f_1 f_2}$ takes the form

$$g^2 C_{f_1 f_2} = \frac{4\pi^4}{C_F} \begin{bmatrix}
m_u^3 & 0 & 0 & 0 & 0 & 0 \\
0 & m_d^3 & 0 & 0 & 0 & 0 \\
0 & 0 & m_s^3 & 0 & 0 & 0 \\
0 & 0 & 0 & m_c^3 & 0 & 0 \\
0 & 0 & 0 & 0 & m_b^3 & 0 \\
0 & 0 & 0 & 0 & 0 & m_t^3
\end{bmatrix},$$  \hspace{1cm} (5)

which, after using the link between this matrix and the condensate values

$$\langle 0 \mid \Psi_{f_1}^i(0) \overline{\Psi}_{f_2}^i(0) \mid 0 \rangle = \frac{4N}{(2\pi)^4} C_{f_1 f_2},$$  \hspace{1cm} (6)

leads to the following connection between the chiral condensates and the corresponding mass of the quark of flavour $f$:

$$g^2 \langle 0 \mid \Psi_{f}^i(0) \overline{\Psi}_{f}^i(0) \mid 0 \rangle = \frac{2(N^2 - 1)}{N} m_{f}^3, \hspace{1cm} f = 1, ..., 6.$$  \hspace{1cm} (7)

The compatibility of these relations between quark condensates and masses with the existing extensive experimental and numerical information will be considered elsewhere.

2.0.2 ‘Democratic’ condensate matrix

The other selection to be analysed is the one which will produce as a result the ‘Democratic’ symmetry (which relevance was argued in [2]) for each one of two separate quark triplets that we will call $u$ and $d$ type quarks. As has been argued [2], assuming that this first stage of the symmetry breaking can be justified dynamically, then the masses for the other massless modes can be produced at next steps of approximation. Thus, it has some interest to verify that such a structure of the mass matrix can also be generated by the presence of quark condensates. The matrix $C_{f_1^* f_2^*}$, showing the symmetry in the above mentioned sectors, takes the form

$$g^2 C_{f_1^* f_2^*} = \frac{4\pi^4}{3C_F} \begin{bmatrix}
m_b^3 & m_b^3 & m_b^3 & 0 & 0 & 0 \\
m_b^3 & m_b^3 & m_b^3 & 0 & 0 & 0 \\
m_b^3 & m_b^3 & m_b^3 & 0 & 0 & 0 \\
0 & 0 & 0 & m_t^3 & m_t^3 & m_t^3 \\
0 & 0 & 0 & m_t^3 & m_t^3 & m_t^3 \\
0 & 0 & 0 & m_t^3 & m_t^3 & m_t^3
\end{bmatrix},$$

where the components of the flavour index vector used up to now, $f = u, d, s, c, b, t$, have been permuted to define the new flavour index $f^* = d, s, b, u, c, t$. This change was made in order render the notation closer to the one employed in the literature for discussing these matrices. It can be noticed that the square, and even any kind of powers, of this matrix also retains the same structure modulo constant factors. Thus, the mass matrix of the quarks, which can be
obtained as being proportional to the power 1/3 of the matrix $C_{f_1 f_2}$, also have the same form. Henceforth, the quark mass matrices proposed in [2] can also be generated by the presence of appropriate condensates in the vacuum having the same symmetry.

The matrix $C_{f_1 f_2}$ above can be diagonalized by a unitary transformation $U$ to produce, for the quark mass matrix:

$$M_{f_1 f_2} = \left( \frac{g^2 C_F}{4\pi^4} \right)^{\frac{1}{3}} (U^{-1} C^{\frac{1}{3}} U)_{f_1 f_2} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_t \end{bmatrix}.$$  

Thus, the first steps in the approach related with the approximate ‘Democratic’ symmetry of the experimental quark mass spectrum is also implied by the presence of the chiral condensates with the same symmetry as treated in the modified perturbative expansion.

3 Constituent masses from the gluon condensation

Up to now, for the sake of clarity in this first exploration, we have considered only the new element with respect to the gluon condensate effects studied before. Let us now examine how the Lagrangian quark masses, previously fixed in terms of the quark condensates, are modified by the introduction of the gluon condensate. The ideal situation would be that the results in this first step could reproduce the spectrum of masses obtained in [9] qualitatively. We will see that in effect this is what happens.

The modified propagator for the gluon has the form

$$G_{\mu\nu}^{ab}(p) = \left( \frac{1}{p^2 + i\epsilon} - i\delta(p) C \right) \delta^{ab} g^{\mu\nu}, \quad (8)$$

which, after being inserted in the expression associated to the second diagram in Fig. 1 for the quark self-energy and integrated over the loop momentum, gives

$$\Sigma^{i_{12};f_1 f_2}(p) = -\frac{M^2 \delta^{i_{12}} \delta^{f_1 f_2} p_{\mu} \gamma^\mu}{p^2}, \quad (9)$$

where the value of the constant $C$ determined in [9] from reproducing a current estimate for the gluon condensate parameter $\langle g^2 G^2 \rangle$ has been used. The constants that appear have the values

$$M = \sqrt{\frac{2 C_F g^2 C}{(2\pi)^4}} = 333 \text{ MeV}/c^2, \quad (10)$$

$$g^2 C = 65 \text{ (GeV}/c^2)^2.$$
Substituting the two contributions to the self-energy in the Dyson equation, the quark modes then satisfy
\[
\frac{1}{p^2} \left( -p^2 \gamma^\mu \left( 1 - \frac{M^2}{p^2} \right) \delta^{f_1 f_2} - \frac{4g^2 C_F}{(2\pi)^4} C^{f_1 f_2} \right) \Psi_{i f}^f (p) = 0,
\]
which gives, again by taking the matrix obtained by changing the relative sign of the two terms in between the parentheses, and multiplying the equation by the result:
\[
\frac{1}{p^2} \left( (p^2)^3 \left( 1 - \frac{M^2}{p^2} \right) - \left( \frac{g^2 C_F}{4\pi^4} \right)^2 (C_f^2) \right) \Psi_i^f (p) = 0,
\]
where it has been assumed that the quark modes are already diagonalizing the matrix $C_{f_1 f_2}$ with eigenvalues $C_f$ for each flavour $f$.

It can be noticed that the quark mass matrix is again determined by the square of the matrix of the quark condensates, but the new dispersion relations for the different quark polarizations associated to the eigenvalues $C_f$ become
\[
p^2 (p^2 - M^2)^2 \delta^{f_1 f_2} - \left( \frac{g^2 C_F}{4\pi^4} \right)^2 C_f^2 = 0.
\]

Let us consider the resulting mass spectrum under the following assumptions: a) the quark condensate parameters are fixed by determining the Lagrangian mass matrix, as was done in the previous section, and b) the $M$ constant has been fixed as before by reproducing the value of the gluon condensate. Then the solution of the Dyson equation (13) determines the following results for the pole quark masses:

| Quark $q$ | $m_{q_Low}^{Exp}$ (MeV) | $m_{q_Up}^{Exp}$ (MeV) | $m_{q_Theo}^{Theo}$ (MeV) |
|-----------|-----------------|----------------|-----------------|
| $u$       | 1.5             | 5              | 333–0           |
| $d$       | 3               | 9              | 333–0           |
| $s$       | 60              | 170            | 339–326–13      |
| $c$       | 1100            | 1400           | 1255            |
| $b$       | 4100            | 4400           | 4233            |
| $t$       | 168600          | 17900          | 173500          |

where the second and third columns give the top and bottom experimental bounds of the quark Lagrangian masses. As can be observed from the data, the light quarks $u$, $d$ and $s$ got values close to 1/3 of the nucleon mass, as was also the outcome in [9]. Similarly also, the massive quark masses have remained almost invariant under the addition of the gluon condensate. The next salient feature of the results in the table is the fact that the strange quark (making honour to his name!) had three solutions at this approximation. One of them has the very low mass of 13 MeV, and the other two are in the same region as the value of the common mass (333 MeV) for the $u$ and $d$ quarks, but one lies over and the other below. It could be the case that, after
introducing the above mentioned sort of ‘modified’ one loop approximation, the results can tend to approach even more those obtained in [9], where the Lagrangian quark masses were introduced explicitly.

Finally, it is needed to remark that various almost massless modes have been obtained. This outcome should be expected in the case when the symmetry has been spontaneously broken. Therefore, the appearance of massless modes is, at least, not in contradiction with the spontaneous generation of the quark condensates from the vacuum in a similar way, as the gluon condensate seems to be generated from the vacuum in the first approximations [7], closely resembling the picture in the early Savvidi’s chromomagnetic field approach.

3.0.3 Comment on the spontaneous generation of the quark condensates

Finally, the question of the spontaneous generation of the condensates from the vacuum will be analysed. Let us consider for concreteness the effective potential for composites operators introduced in [11] as applied to the massless QCD. This functional can be written in the following form

\[
V[A, \Psi \Psi, G_g, G_q](A, \Psi \Psi = 0) = \frac{i}{2} Tr \left[ \log \left( G_g^{-1} G_g - G_g^{-1} G_g \right) \right] + \nu \left[ \log \left( G_g^{-1} G_g - G_g^{-1} G_g \right) \right] + \nu(2)[A, \Psi \Psi, G_g, G_q](A, \Psi \Psi = 0),
\]

where \( V \) depends on arbitrary values of the mean quantum fields (here the quark and gluon ones) and \( G_g, G_q \) are also arbitrarily fixed expressions for their propagators. The \( \nu(2) \) functional represents contributions higher than one loop in terms of Feynman diagrams. The mean field values will be assumed to vanish in the ground state as required by Lorentz invariance, and they will be omitted below. The free inverse propagators as considered in the previous section correspond to the massless QCD and are given by

\[
G_q^{(0)-1} \equiv -p_\mu \gamma^\mu, q = (u, d, s, c, b, t) \\
G_g^{(0)-1} \equiv G_g^{(0)-1 \mu \nu}(p) = p^2 \delta^{ab} g^{\mu \nu}.
\]

The traces in the fist term in (14) are in the spinor, fundamental colour and spatial indices and for the second term they are over the Lorentz, adjoint colour and spatial indices of the operators.

In order to analyse the spontaneous generation of the condensates from the vacuum, the propagator candidates showing non-vanishing condensates should be substituted in (14) in order to verify if they have or not lower potential than the ones showing zero values for the condensates. It could be helpful to recall that this should be the case, because the effective potential satisfies the condition of giving the minimum mean values of the energy in the subspace of the states of the system satisfying the constraints of having fixed the mean fields and propagators (which also are mean values).
It is interesting to notice that in the really first approximation for the propagators, in which only the delta function modifications are included, the first two terms in the above expression for the effective potential are completely independent of the values of condensate parameters. This is simply because the modifications introduced are also allowed free propagators for the theory, only differing from the Feynman ones in the rounding of the singularity. However, even at the simplest one-loop approximation, the propagator modes get masses and the mentioned two terms of the effective potential will depend on the selected condensation parameters.

The gluon and quark propagators, in the approximation in which the self-energies are defined by (9), (2), take the form

\[
G_{g\mu\nu}(p) = \frac{1}{(p^2 + m_g^2)} \delta^{ab} \delta_{\mu\nu},
\]

\[
G_q(p) = \frac{\delta f_1 f_2}{\left(-p^\mu \gamma^\mu(1 - \frac{M^2}{p^2}) - \frac{S_f}{p^2}\right)},
\]

where the new parameters, which simplify the notation but also characterize the gluon and quark condensates, are related with the already defined constants \(C\) and \(C_f\), \((f = u, d, s, c, b, t)\) through

\[
m_g^2 = \frac{6g^2C}{(2\pi)^4},
\]

\[
S_f = \frac{g^2C_F}{4\pi^4} C_f.
\]

The evaluation of various traces in the first two terms of (14) gives the following integral expression for this approximation of the CJT effective potential:

\[
V[G_g, G_q] = -i \int \frac{dp}{(2\pi)^4} \sum_f tr[\log[p^\mu \gamma^\mu \frac{1}{p^\mu \gamma^\mu(1 - \frac{C_F m_g^2}{3 p^2}) + \frac{S_f}{p^2}}] -
\]

\[
\frac{p^\mu \gamma^\mu}{p^\mu \gamma^\mu(1 - \frac{C_F m_g^2}{3 p^2}) + \frac{S_f}{p^2}}] +
\]

\[
\frac{i}{2} \int \frac{dp}{(2\pi)^4} 3N \left[\log[\frac{p^2}{p^2 + m_g^2}] - \frac{p^2}{p^2 + m_g^2}\right],
\]

in which the only trace remaining to be evaluated is the spinor one. This relation directly shows that the fermion contribution is the sum of identical functions of the quark parameters \(S_f\). Therefore, if the expression has a minimum outside the value zero of these parameters, it should be in a point in which all these constants take identical values. This is the structure that had been predicted in Ref. [5] from an analysis based on the large number of colours limit. In that work the central consideration was given to the plausibility that only the first loop fermion correction should play a relevant role in the standard effective action of the system. However, in the presence of ground state or vacuum instabilities, the gap equations (which determine the real spectrum and ground state of the system and the possible field condensates)
Figure 2: Types of two fermions loop graphs contributing to \( V^{(2)} \) which could be relevant for the determination of the pattern of spontaneous flavor symmetry breaking in massless QCD

higher than one, and in particular the two loop fermion contributions, should play an important role. This can be the case because they are closely linked with the Bethe–Salpeter bound state equations. Therefore, diagrams contributing to the \( V^{(2)} \) term of the effective action, such as the ones illustrated in Fig. 2, can be expected to be relevant to the problem under consideration.

In another way, it is natural that this can be the situation in massless QCD, because the attractive binding forces are strong (unlike the case of the BCS superconductivity, where the attractive phonon force works against the coulomb repulsion between electrons). In QCD, as the quark-antiquark pairs have attractive and strong colour forces, it would be expected that their condensates be spontaneously generated. But, as can be observed from the illustrated diagrams, the participation of different quark loops can introduce contributions that do not reduce to the sum of identical functions of each one of the parameters \( S_f \). This fact seems to open the possibility for a dynamical generation of condensates not having equivalent values for all the quarks. As within the modified expansion discussed here, the equality of all the condensates (within the considered simple approximation) is equivalent to the equality of the quark masses, the suggested possibilities for the spontaneous generation of different quark condensates, would allow also the spontaneous generation of ‘Democratic’ quark mass matrices, as proposed in [2]. If these remarks are appropriate, the question about the possible magnitudes of the condensates remains open as well. These issues should be considered in future extensions of this work.

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