Prediction of charge separation in GaAs/AlAs cylindrical Russian Doll nanostructures

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Recent advances in nanotechnology permit fabrication of complex nanostructures with special electronic and optical properties reflecting dimensional confinement on a nanometer scale.\textsuperscript{1,2} e.g. multiple quantum wells\textsuperscript{3} and core-shell structures.\textsuperscript{4–7} The essential building blocks of such structures are alternating layers of different semiconducting materials, acting as “wells” and “barriers”, and controlling the confinement energies and, thus the localization of charge carriers. Electrons and holes are confined in wells and repelled from barriers much like in “a particle in a box”: as the well narrows, the kinetic energy of the confined particle rises. The materials comprising the wells and barriers are usually flat, two-dimensional semiconductor films\textsuperscript{3} stacked like a deck of cards to produce “multiple quantum wells” or “superlattices”. In this case, wave functions of the conduction band minimum (CBM) and valence band maximum (VBM) at the Brillouin zone center, are localized on the widest wells, having the lowest confinement energy.\textsuperscript{3,8} We have contrasted the quantum confinement of (i) multiple quantum wells of flat GaAs and AlAs layers, \textit{i.e.} \((\text{GaAs})_m/(\text{AlAs})_n/(\text{GaAs})_p/(\text{AlAs})_q\), with (ii) “cylindrical Russian Dolls” – an equivalent sequence of wells and barriers arranged as concentric wires (Fig. 1). Using a pseudopotential plane-wave calculation, we identified theoretically a set of numbers \((m, n, p \text{ and } q)\) such that charge separation can exist in “cylindrical Russian Dolls”: the CBM is localized in the inner GaAs layer, while the VBM is localized in the outer GaAs layer. In contrast, the band edge states of linear multiple quantum wells with
equivalent layer thickness does not exhibit any charge separation, having equal amplitudes in two GaAs layers, if \( m = p \). Thus, a Russian Doll geometry provides a charge separation that is impossible with equivalent linear multiple quantum wells. This study thus identifies a new geometric degree of freedom (curvature) that can be used to manipulate electronic properties of nanostructures.

In order to avoid approximate \( \mathbf{k} \cdot \mathbf{p} \) methods that fail for narrow wells,\(^4\) the electronic structure of the nanostructures is described here using screened atomic pseudopotentials in a plane wave basis.\(^4\) Instead of calculating all eigenstates of the pseudopotential Hamiltonians (a procedure whose computational cost scales as \( N^3 \) for an \( N \)–atom system), we transform the Hamiltonian via the “folded spectrum method”, so that only the physically relevant eigenstates around the band edges are sought and obtained.\(^11\) The linear scaling of the computational cost of the folded spectrum method with system size permits super-cell calculations of rather large, \( 10^3 \sim 10^4 \)-atom nanostructures needed to study the effect discovered here.

Figures 2 shows the calculated confinement energies of the conduction band minimum and the valence band maximum of linear multiple quantum wells as a function of the thickness \( p(III_{Ga}) \) of the outer GaAs segment (see Fig. 1 for definition of the structure). The confinement energies are defined with respect to CBM and VBM of the bulk GaAs whose band gap is 1.5 eV. The innermost GaAs segment is fixed at \( m(I_{Ga}) = 5 \) monolayers (ML). We see that, as expected, both the CBM and VBM are localized on the widest wells. This is the innermost GaAs segment \( (I_{Ga}) \) when \( p(III_{Ga}) < 2m(I_{Ga}) = 10 \) ML, and the \( III_{Ga} \) segment when \( p(III_{Ga}) > 2m(I_{Ga}) \). When the two GaAs wells, \( I_{Ga} \) and \( III_{Ga} \), have the same thickness, \( p = 2m \), the CBM and VBM have equal amplitudes in the two wells and no charge separation is evident. The transition in the localization of the CBM and VBM from \( I_{Ga} \) to \( III_{Ga} \) reflects the dependence of the confinement energy on the size of wells, as schematically illustrated in Fig. 3. The confinement energies in well \( III_{Ga} \) increases as the well thickness, \( p(III_{Ga}) \), decreases, while the confinement energies in well \( I_{Ga} \) remain almost constant. The transition from localization of the CBM and VBM on \( I_{Ga} \) to localization on
III_{Ga} occurs at $p(III_{Ga}) < 2m(I_{Ga})$, when the confinement energy of $I_{Ga}$ dips below that of $III_{Ga}$.

Figure 4 shows the confinement energies of the CBM and VBM in the cylindrical Russian Dolls as a function of $p(III_{Ga})$; the thicknesses of other layers are fixed as before. Similarly to the MQW case of Fig. 2, both the CBM and VBM are localized in $I_{Ga}$ when $p(III_{Ga}) < m(I_{Ga})$ and in $III_{Ga}$ when $p(III_{Ga}) > m(I_{Ga})$. However, differently from the MQW, we observe a charge separation in the wells for $p(III_{Ga}) = m(I_{Ga}) = 10$ ML: the CBM is localized in $I_{Ga}$, while the VBM is localized in $III_{Ga}$. We find the same charge separation when $p(III_{Ga}) = m(I_{Ga}) = 12$ ML, where the confinement energies are 151.2 meV (CBM) and -30.1 meV (VBM).

The wave functions of the VBM and CBM of the multiple quantum well and the CBM of the cylindrical Russian Dolls do not change their symmetries (although their localization can change from $I_{Ga}$ to $III_{Ga}$) as $p$ changes. Indeed, the CBMs of both structures are derived from the zincblende $\Gamma_{1c}$ states at all $p$ values, and the VBM of the MQW is derived from the heavy-hole state at the Brillouin zone center for all $p$ values. Since both the VBM and the CBM of the MQW do not change their identities, their localization transitions occur at the same critical thickness, so no charge separation is evident. In contrast, the VBM of the cylindrical Russian Doll structure exhibits, as $p$ increases, a crossing of two levels with distinct symmetries (circles vs. triangles in Fig. 4). Charge separation occurs when the confinement energy of these two states cross, i.e. $p = m$. We emphasize that the charge separation in the cylindrical Russian Dolls is not due to the band alignment between GaAs and AlAs (which is the same in Russian Dolls and multiple quantum wells) but due to the concentric wire geometry and the valence band structure.

Table I gives the confinement energies (insets in Fig. 2 and 4) of the CBM and VBM for a few structures of cylindrical Russian Dolls and linear multiple quantum wells. We see that given the same layer thicknesses, the confinement energies ($\Delta E$) of cylindrical Russian Dolls are considerably larger than those of linear multiple quantum wells. The reason is that the confinement energies are enhanced by the “two-dimensional” nature of the charge carriers.
in case of the *concentric* layers in the cylindrical Russian Doll geometry, compared to the “one-dimensional” nature on the *flat* layers in the linear multiple quantum well structure. The upper part of Table I shows that the confinement dimension together with the well widths affects the localization of the wave functions, as shown in Fig. 2 and Fig. 4.

In all cases discussed so far, all band edge states are $\Gamma-\text{derived}$. However, the bottom half of Table I show that when $m(I_{Ga}) = 6$ ML, the CBM of cylindrical Russian Dolls is derived from bulk $X_{1c}$ state and is localized on region $IV_{Al}$. Indeed, it has been shown by Franceschetti and Zunger that the VBM of the heterostructures consisting of GaAs/AlAs is always $\Gamma-\text{like}$, while the CBM becomes $X-\text{like}$ as the well width becomes smaller and the confinement increases. In other words, the CBM is $X-\text{like}$, when the GaAs well is smaller than a critical size. This transition is found to occur at different critical layer thickness in cylindrical Russian Dolls and in multiple quantum wells. The CBM of the cylindrical Russian Doll changes from $\Gamma$ to $X$-like when both $m(I_{Ga})$ and $p(III_{Ga})$ become smaller than 10 ML. This critical thickness is consistent with that for the $\Gamma \rightarrow X$ transition in an isolated quantum wire. On the other hand, the critical thickness of the $\Gamma \rightarrow X$ transition in the MQW is $m = 5$ ML. Table I shows therefore that when $m = 6$ ML, the CBM of the MQW is $\Gamma-\text{like}$, while that of the cylindrical Russian Dolls is $X-\text{like}$. This illustrate an extreme difference in electronic properties attainable by different confining geometries of nanostructures having the same quantum sizes.

In summary, we have shown that in analogy with nested (Russian Doll) carbon nanotubes, where new physical properties, absent in the corresponding flat (graphite) sheets are attainable, ordinary semiconductor Russian Doll structures can also exhibit novel properties, absent in the flat multiple quantum well. In particular, Russian Doll GaAs/AlAs structures afford charge separation on different sheets and different ($\Gamma$ vs. $X$) symmetries of states.
REFERENCES

1 *Nanostructures and quantum effects*, edited by H. Sakaki and H. Noge, (Springer-Verlag, Heidelberg, 1993).

2 *Optical properties of semiconductor quantum dots*, edited by U. Woggon (Springer-Verlag, Heidelberg, 1997).

3 *Superlattices and other heterostructures: symmetry and optical phenomena*, edited by E. L. Ivchenko and G. E. Pikus, (Springer-Verlag, Berlin, Heidelberg, 1995).

4 G. W. Bryant, P. S. Julienne, and Y. B. Band, *Surface Science*, **361/362** 801–804 (1996).

5 M. A. Hines and P. Guyot-Sionnest, *J. Phys. Chem.* **100**, 468–471 (1996).

6 D. Schooss, A. Mews, A. Eychmüller, and H. Weller, *Phys. Rev. B* **49**, 17072–17078 (1994).

7 H. S. Zhou, I. Honma, H. Komiyama, and J. W. Haus, *J. Phys. Chem.* **97**, 895–901 (1993).

8 A. Franceschetti and A. Zunger, *Phys. Rev. B* **52** 14664–14670 (1995).

9 D. M. Wood and A. Zunger, *Phys. Rev. B* **53**, 7949–7963 (1996).

10 K. A. Mäder and A. Zunger, *Phys. Rev. B* **50**, 17393–17405 (1994).

11 L.-W. Wang and A. Zunger, *J. Chem. Phys.* **100** 2394–2397 (1994).

12 *Science of fullerenes and carbon nanotubes*, edited by M. Dresselhaus, G. Dresselhaus, and P. C. Eklund (Academic Press, San Diego, 1996); *Carbon nanotubes, preparation and properties*, edited by T. W. Ebbesen (CRC Press, New York, 1996).

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TABLE I. Table I. The confinement energies ($\Delta E$ in meV) of the CBM and VBM for various layer thicknesses, $m, n$ and $P$ (in ML) of the cylindrical Russian Dolls and multiple quantum wells. Band edge states are $\Gamma$–like, unless stated.

| Layer thickness | State | $\Delta E$ | Localization | $\Delta E$ | Localization |
|-----------------|-------|------------|--------------|------------|--------------|
| $m - n - p$     |       |            |              |            |              |
| 10-4-4          | CBM   | 181.9      | $I_{Ga}$     | 84.8       | $I_{Ga}$     |
|                 | VBM   | -49.4      | $I_{Ga}$     | -20.7      | $I_{Ga}$     |
| 10-4-10         | CBM   | 170.2      | $I_{Ga}$     | 83.6       | $I_{Ga}$     |
|                 | VBM   | -37.0      | $III_{Ga}$   | -20.7      | $I_{Ga}$     |
| 6-4-4           | CBM   | 216.1      | $IV_{Al}$ ($X$) | 167.1 | $I_{Ga}$ |
|                 | VBM   | -116.7     | $I_{Ga}$     | -48.7      | $I_{Ga}$     |
| 6-4-6           | CBM   | 215.8      | $IV_{Al}$ ($X$) | 165.0 | $I_{Ga}$ |
|                 | VBM   | -84.5      | $III_{Ga}$   | -48.7      | $I_{Ga}$     |

$^1 q(IV_{Al}) = 10$ ML and $^2 q(IV_{Al}) = 14$ ML.
FIGURES

FIG. 1. Schematics of structure of (a) linear multiple quantum well, whose confinement direction is indicated by an arrow, and (b) a \{100\} cross-section of an equivalent cylindrical Russian Doll. These structures are made of alternating GaAs (red) and AlAs segments with thicknesses \(m, n, p\) and \(q\) monolayers, in the order (starting from the center) \(I_{\text{Ga}} \rightarrow II_{\text{Al}} \rightarrow III_{\text{Ga}} \rightarrow IV_{\text{Al}}\), with thicknesses \(m, n, p\) and \(q\) monolayers, respectively.

FIG. 2. Confinement energies (triangles) and wave-function amplitudes (insets) of the (a) CBM and (b) VBM of linear multiple quantum wells, as a function of the thickness \(p(III_{\text{Ga}})\) of the outer GaAs layer. Other thicknesses are fixed at \(m(I_{\text{Ga}}) = 5\) ML, \(n(II_{\text{Al}}) = 4\) ML and \(q(IV_{\text{Al}}) = 8\) ML. Note that the CBM and VBM are always localized on the \textit{widest} wells: on \(I_{\text{Ga}}\) for small \(p\), and on \(III_{\text{Ga}}\) for large \(p\).

FIG. 3. Band alignment of the GaAs and AlAs layers along the confinement direction for the MQW and along the radial direction for the cylindrical Russian Doll (see Fig. 1). The arrows indicate the movement of confined levels as the size \(p(III_{\text{Ga}})\) decreases, while the thicknesses of other layers are held fixed. In a conventional linear multiple quantum well, both the CBM and VBM levels are localized on the widest well, having the lowest kinetic energy confinement, thus the lowest energy levels in the respective wells (Fig. 2). For the same well thicknesses \(m = p\), the band edge states have similar amplitude on \(I_{\text{Ga}}\) and \(III_{\text{Ga}}\). In contrast, in cylindrical Russian Dolls (Fig. 4), we can have the VBM on region \(I_{\text{Ga}}\), while the CBM is localized in region \(III_{\text{Ga}}\), even though \(m = p\).
FIG. 4. Confinement energies of the (a) CBM and (b) two highest valence bands for cylindrical Russian Dolls vs. the thickness $p(III_{Ga})$. The other parameters are held fixed at $m = 10$ ML, $n = 4$ ML and $q = 8$ ML. Wave-function amplitudes, averaged along the wire direction, are shown as insets for a few structures. Note the change in localization of the wave functions from $I_{Ga}$ to $III_{Ga}$. A charge separation of the electron and hole in the GaAs wells is obtained at $m(I_{Ga}) = p(III_{Ga}) = 10$ ML when the level crossing and attendant change in angular symmetry of the VBM wave functions occur; there is no change in angular symmetry of the CBM.
Recent advances in nanotechnology permit fabrication of complex nanostructures with special electronic and optical properties reflecting dimensional confinement on a nanometer scale.\(^1\,^2\) 
E.g. multiple quantum wells\(^3\) and core-shell structures.\(^4\,^5\) The essential building blocks of such structures are alternating layers of different semiconducting materials, acting as "wells" and "barriers", and controlling the confinement energies and, thus the localization of charge carriers. Electrons and holes are confined in wells and repelled from barriers much like in "a particle in a box": as the well narrows, the kinetic energy of the confined particle rises. The materials comprising the wells and barriers are usually flat, two-dimensional semiconductor films,\(^5\) stacked like a deck of cards to produce "multiple quantum wells" or "superlattices". In this case, wave functions of the conduction band minimum (CBM) and valence band maximum (VBM) at the Brillouin zone center, are localized on the widest wells, having the lowest confinement energy.\(^5\,^6\)

We have contrasted the quantum confinement of (i) multiple quantum wells of flat GaAs and AlAs layers, i.e. \((\text{GaAs})_m/(\text{AlAs})_n/(\text{GaAs})_p/(\text{AlAs})_q\), with (ii) "cylindrical Russian Dolls" – an equivalent sequence of wells and barriers arranged as concentric wires (Fig. 1). Using a pseudopotential plane-wave calculation, we identified theoretically a set of numbers \((m,n,p,q)\) such that charge separation can exist in "cylindrical Russian Dolls": the CBM is localized in the inner GaAs layer, while the VBM is localized in the outer GaAs layer. In contrast, the band edge states of linear multiple quantum wells with equivalent layer thickness does not exhibit any charge separation, having equal amplitudes in two GaAs layers, if \(m = p\). Thus, a Russian Doll geometry provides a charge separation that is impossible with equivalent linear multiple quantum wells. This study thus identifies a new geometric degree of freedom (curvature) that can be used to manipulate electronic properties of nanostructures.

In order to avoid approximate \(k \cdot p\) methods that fail for narrow wells,\(^9\) the electronic structure of the nanostructures is described here using screened atomic pseudopotentials in a plane wave basis.\(^10\) Instead of calculating all eigenstates of the pseudopotential Hamiltonians (a procedure whose computational cost scales as \(N^3\) for an \(N\)-atom system), we transform the Hamiltonian via the "folded spectrum method", so that only the physically relevant eigenstates around the band edges are sought and obtained.\(^11\) The linear scaling of the computational cost of the folded spectrum method with system size permits supercell calculations of rather large, \(10^5 \sim 10^4\)-atom nanostructures needed to study the effect discovered here.

Figures 2 shows the calculated confinement energies of the conduction band minimum and the valence band maximum of linear multiple quantum wells as a function of the thickness \(p(III\text{Ga})\) of the outer GaAs segment (see Fig. 1 for definition of the structure). The confinement energies are defined with respect to CBM and VBM of the bulk GaAs whose band gap is 1.5 eV. The inner-
most GaAs segment is fixed at \(m(I_{Ga}) = 5\) monolayers (ML). We see that, as expected, both the CBM and VBM are localized on the widest wells. This is the innermost GaAs segment \(I_{Ga}\) when \(p(III_{Ga}) < 2m(I_{Ga}) = 10\) ML, and the \(III_{Ga}\) segment when \(p(III_{Ga}) > 2m(I_{Ga})\). When the two GaAs wells, \(I_{Ga}\) and \(III_{Ga}\), have the same thickness, \(p = 2m\), the CBM and VBM have equal amplitudes in the two wells and no charge separation is evident. The transition in the localization of the CBM and VBM from \(I_{Ga}\) to \(III_{Ga}\) reflects the dependence of the confinement energy on the size of wells, as schematically illustrated in Fig. 3. The confinement energies in well \(III_{Ga}\) increase as the well thickness, \(p(III_{Ga})\), decreases, while the confinement energies in well \(I_{Ga}\) remain almost constant. The transition from localization of the CBM and VBM on \(I_{Ga}\) to localization on \(III_{Ga}\) occurs at \(p(III_{Ga}) < 2m(I_{Ga})\), when the confinement energy of \(I_{Ga}\) dips below that of \(III_{Ga}\).

Figure 4 shows the confinement energies of the CBM and VBM in the cylindrical Russian Dolls as a function of \(p(III_{Ga})\); the thicknesses of other layers are fixed as before. Similarly to the MQW case of Fig 2, both the CBM and VBM are localized in \(I_{Ga}\) when \(p(III_{Ga}) < m(I_{Ga})\) and in \(III_{Ga}\) when \(p(III_{Ga}) > m(I_{Ga})\). However, differently from the MQW, we observe a charge separation in the wells for \(p(III_{Ga}) = m(I_{Ga}) = 10\) ML: the CBM is localized in \(I_{Ga}\), while the VBM is localized in \(III_{Ga}\). We find the same charge separation when \(p(III_{Ga}) = m(I_{Ga}) = 12\) ML, where the confinement energies are 151.2 meV (CBM) and -30.1 meV (VBM).

The wave functions of the VBM and CBM of the multiple quantum well and the CBM of the cylindrical Russian Dolls do not change their symmetries (although their localization can change from \(I_{Ga}\) to \(III_{Ga}\)) as \(p\) changes. Indeed, the CBMs of both structures are derived from the zincblende \(\Gamma_c\) states at all \(p\) values, and the VBM of the MQW is derived from the heavy-hole state at the Bril- louin zone center for all \(p\) values. Since both the VBM and the CBM of the MQW do not change their identities, their localization transitions occur at the same critical thickness, so no charge separation is evident. In contrast, the VBM of the cylindrical Russian Doll structure exhibits, as \(p\) increases, a crossing of two levels with distinct symmetries (circles vs. triangles in Fig. 4). Charge separation occurs when the confinement energy of these two states cross, i.e. \(p = m\). We emphasize that the charge separation in the cylindrical Russian Dolls is not due to the band alignment between GaAs and AlAs (which is the same in Russian Dolls and multiple quantum wells) but due to the concentric wire geometry and the valence band structure.

![Figure 2](image1.png)

**FIG. 2.** Confinement energies (triangles) and wave-function amplitudes (insets) of the (a) CBM and (b) VBM of linear multiple quantum wells, as a function of the thickness \(p(III_{Ga})\) of the outer GaAs layer. Other thicknesses are fixed at \(m(I_{Ga}) = 5\) ML, \(m(III_{Ga}) = 4\) ML and \(q(III_{Ga}) = 8\) ML. Note that the CBM and VBM are always localized on the widest well: on \(I_{Ga}\) for small \(p\), and on \(III_{Ga}\) for large \(p\).

![Figure 3](image2.png)

**FIG. 3.** Band alignment of the GaAs and AlAs layers along the confinement direction for the MQW and along the radial direction for the cylindrical Russian Doll (see Fig. 1). The arrows indicate the movement of confined levels as the size \(p(III_{Ga})\) decreases, while the thicknesses of other layers are held fixed. In a conventional linear multiple quantum well, both the CBM and VBM levels are localized on the widest well, having the lowest kinetic energy confinement, thus the lowest energy levels in the respective wells (Fig. 2). For the same well thicknesses \((m = p)\), the band edge states have similar amplitude on \(I_{Ga}\) and \(III_{Ga}\). In contrast, in cylindrical Russian Dolls (Fig. 4), we can have the VBM on region \(III_{Ga}\), while the CBM is localized in region \(I_{Ga}\), even though \(m = p\).
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| Layer thickness | Russian Doll | Quantum Well
|-----------------|--------------|-----------------|
| $m - n - p$     | CBM $I_Ga$  | $\Delta E$ Localization | CBM $I_Ga$
| 10-4-4          | 181.9       | 84.8 $I_Ga$         |
|                 | -49.4       | -20.7 $I_Ga$        |
| 10-4-10         | 170.2       | 83.6 $I_Ga$         |
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dimensional” nature on the flat layers in the linear multiple quantum well structure. The upper part of Table I shows that the confinement dimension together with the well widths affects the localization of the wave functions, as shown in Fig. 2 and Fig. 4.

In all cases discussed so far, all band edge states are $\Gamma$–derived. However, the bottom half of Table I shows that when $m(I_Ga) = 6$ ML, the CBM of cylindrical Russian Dolls is derived from bulk $X_{Lc}$ state and is localized on region $IV_{Al}$. Indeed, it has been shown by Franceschetti and Zunger$^8$ that the CBM of the heterostructures consisting of GaAs/AlAs is always $\Gamma$–like, while the CBM becomes $X$–like as the well width becomes smaller and the confinement increases. In other words, the CBM is $X$–like, when the GaAs well is smaller than a critical size. This transition is found to occur at different critical layer thickness in cylindrical Russian Dolls and in multiple quantum wells. The CBM of the cylindrical Russian Doll changes from $\Gamma$ to $X$–like when both $m(I_Ga)$ and $p(III_{Ga})$ become smaller than 10 ML. This critical thickness is consistent with that for the $\Gamma \rightarrow X$ transition in an isolated quantum wire.$^8$ On the other hand, the critical thickness of the $\Gamma \rightarrow X$ transition in the MQW is $m = 5$ ML. Table I shows therefore that when $m = 6$ ML, the CBM of the MQW is $\Gamma$–like, while that of the cylindrical Russian Dolls is $X$–like. This illustrates an extreme difference in electronic properties attainable by different confining geometries of nanostructures having the same quantum sizes.

In summary, we have shown that in analogy with nested (Russian Doll) carbon nanotubes$^{12}$, where new physical properties, absent in the corresponding flat (graphite) sheets are attainable, ordinary semiconductor Russian Doll structures can also exhibit novel properties, absent in the flat multiple quantum well. In particular, Russian Doll GaAs/AlAs structures afford charge separation on different sheets and different ($\Gamma$ vs. $X$) symmetries of states.
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