The Quantum Eraser Paradox

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The delayed-choice quantum eraser experiment is commonly interpreted as implying that in quantum mechanics a choice made at one time can influence an earlier event. We here suggest an extension of the experiment that results in a paradox when interpreted using a local realist interpretation combined with backward causation (“retrocausality”). We argue that resolving the paradox requires giving up the idea that, in quantum mechanics, a choice can influence the past, and that it instead requires a violation of Statistical Independence without retrocausality. We speculate what the outcome of the experiment would be.

I. INTRODUCTION

In the famous quantum eraser experiment [62], an interference pattern can be re-created by erasing which-way information. Even more remarkable, this information can be successfully erased after the which-way information was already imprinted on the state. This delayed-choice quantum eraser [35, 60, 70] is frequently interpreted as showing the possibility of an influence on the past [72].

In [28], it was argued that from the \( \psi \)-ensemble perspective this retrocausal interpretation makes no sense. Instead, if one wants to explain the outcome in a local realist framework, then the information about what measurement would take place must have been available already at the time of preparation. In a \( \psi \)-ensemble, the wave-function is not itself fundamental, it merely describes a collection of ontic states in an underlying theory.

Here and in the following we will take a local realist theory to mean a hidden variables theory that fulfils Bell’s criterion of local causality. Bell’s theorem [5, 6, 43] requires that any such local realist theory must violate one of the other assumptions of the theorem to reproduce the observed violations of Bell’s inequality. We will here consider the case where Statistical Independence is violated. This is possible through two options. The first involves assuming that the information about the measurement settings at detection is contained in the state of the hidden variables at preparation. This option is known as superdeterminism [80, 31]. The other option is that the measurement setting at detection influences the initial state, which is known as retrocausality.

We want to emphasise that this type of retrocausality does not use the notion of causality which is common in many parts of physics, in which of two causally related events, one is the cause of the other if it is in the past light cone of the other. According to this notion of causality, retrocausality does not exist by definition [7]. Instead, to analyse experiments in the foundations of quantum mechanics, it has become common to use the notion of causality derived from causal diagrams [44, 65, 73] usually referred to as “interventionist causality”. The relation between these two notions of causality has yet to be entirely clarified. For a recent discussion see [61].

The purpose of this paper is to propose and discuss an extension of the delayed-choice quantum eraser suggested by one of us (CB). We will argue that this extension supports the hypothesis that Statistical Independence is violated but that retrocausality is either internally inconsistent or, when internal consistency is enforced, becomes inconsistent with observation.

A variety of retrocausal approaches to quantum mechanics have been put forward in [1, 11, 20, 24, 46, 51, 63, 69, 71]. We will not discuss any one of those approaches specifically here, but will instead generally focus on a common feature that all local realist explanations which exploit retrocausality must share. Our argument differs from previously made ones [2, 8, 38] in that we propose a concrete experimental setup that can be realised in the near future.

This paper is organised as follows. After recalling how the quantum eraser works in Section II and exploring why it is puzzling in Section III, we will in Section IV propose an extension of the experiment that, if one believes the retrocausal explanation, results in a causality paradox. In Section V we will discuss what the outcome of such an experiment would be and in Section VI offer some considerations about the feasibility of the experiment.

The reader who is familiar with the quantum eraser experiment, Bell’s theorem, and the Elitzur-Vaidman bomb experiment [22], can jump right to section IV.

II. THE QUANTUM ERASER

We here briefly summarise the experiment as first realised in [55], which is close to the original proposal from [62]. For illustration, see Figure 1. Photons are emitted from a source (S) and sent through a double slit (black). After the double-slit, they hit a nonlinear optical crystal (grey) which, by spontaneous parametric down conversion, creates an entan-
gled pair of photons from each photon incident on the crystal (solid/pink and dashed/green lines).

One of the photons in each entangled pair is sent directly towards a detector screen (D), depicted as the upper path in Figure 1. The other two photons are either measured at detectors D1 and D2 which reveal where the photon came from (the “which-way information”), or they are combined using mirrors and a semi-transparent plate, and then measured at detectors D3 and D4, without revealing the which-way information. The question is whether the photons arriving at the screen D, coincident with arriving at one of the detectors, will or won’t create an interference pattern.

Theoretically one can understand the setup as follows. The wave-function $|\Psi\rangle$ exiting a double-slit is normally a superposition of two contributions $|\Psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$, where the indices 1 and 2 refer to the two slits. When time-evolved forward to the detection screen, these two contributions are non-orthogonal, giving rise to an interference pattern.

However, creating an entangled pair from each incident photon at a particular location has the result of imprinting information about the position of the photon on the wavefunction. We can write this schematically as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\psi_1\rangle |D_1\rangle + |\psi_2\rangle |D_2\rangle),$$

where $|\psi_1\rangle$ ($|\psi_2\rangle$) is the photon going to the screen coming from the upper (lower) slit, and $|D_1\rangle$ and $|D_2\rangle$ are those going to detector D1 and D2. The states $|D_1\rangle$ and $|D_2\rangle$ are orthogonal because the locations are spatially separate.

This wave-function, when forward-evolved to the screen cannot interfere, because it has the orthogonal which-way information $|D_1\rangle$ and $|D_2\rangle$ imprinted on it. This means, the distribution of photons on the screen will be given by $|\psi_1|^2 + |\psi_2|^2$, that is, it will be a combination of the diffraction patterns of each slit separately, with slightly off-set peaks.

The eraser works by instead using a measurement that projects the state $|\Psi\rangle$ on a symmetric and an asymmetric combinations of the which-way information $|D_1\rangle$ and $|D_2\rangle$, for example

$$|D_3\rangle = \frac{1}{\sqrt{2}} (|D_1\rangle + |D_2\rangle)$$
$$|D_4\rangle = \frac{1}{\sqrt{2}} (|D_1\rangle - |D_2\rangle).$$

The states $|D_3\rangle$ and $|D_4\rangle$ also constitute an orthogonal measurement basis, and it’s this basis which is measured at D3 and D4, respectively.

In this basis, the state $|\Psi\rangle$ takes the form

$$\frac{1}{2} [(|\psi_1\rangle + |\psi_2\rangle)|D_3\rangle + (|\psi_1\rangle - |\psi_2\rangle)|D_4\rangle].$$

One sees from this expression that when one projects the state on either $|D_3\rangle$ or $|D_4\rangle$ by measuring the entangled partner, then the respective contribution which goes to the screen can interfere. This is just because the orthogonal states $|D_1\rangle$ and $|D_2\rangle$ are not orthogonal to $|D_3\rangle$ and $|D_4\rangle$. Loosely speaking, thus, they each have contributions that can interfere.

One further sees from expression (3), that the two interference patterns, created by projecting on $|D_3\rangle$ or $|D_4\rangle$, respectively, are not identical. Because of the minus in the second term, the interference patterns are phase-shifted to each other. When one adds them together, they re-create the original non-interference pattern. This has to be so because we just made a basis transformation.

For the Delayed-Choice Quantum Eraser (hereafter DCQF), one makes the path of the photons to the eraser long enough so that they are detected after the partner particle appeared on the screen. An interference pattern is still observed for photons correlated with those going to either D3 or D4.

It is worth stressing that the measurement outcome on the screen does not depend on the settings on the other path, that is, whether the eraser is on or off does not affect the outcome on the screen. It’s just that one can sample the photons in two different groups in two different ways [23, 34, 52, 53].

When one uses detectors D1 and D2, then the entangled partners of photons reaching each detector separately do not create an interference pattern on D. When one, on the other hand, uses detectors D3 and D4, then the partner particles of photons arriving at only D3 or only D4 create an interference pattern. But together they do not. This is also the case in setup which employs polarising plates to imprint the which-way information, used, for example, in [70], though the symmetric and anti-symmetric combinations differ by a complex phase.

It has to be the case that the result on the screen is independent of whether one measures the which-way information or not, because in quantum mechanics the outcome does not depend on how long after the one photon arrived on the screen its entangled partner is measured. Therefore, these measure-
ments could be causally disconnected (which indeed they are in [37]). Then, if the pattern on $D_4$ did depend on the measurement settings ($D_{1/2}$ or $D_{3/4}$), that would violate the no-signalling theorem [64].

III. INTERPRETATION

First, we want to stress that the measurement results of the DCQE experiments can full well be understood in the Copenhagen interpretation. If one is willing to accept a non-local update of the wave-function upon measurement, then there is nothing puzzling about the experiment. The wave-function simply propagates until it hits the detectors, and then it ‘collapses’ to a localised state with a probability given by Born’s rule.

However, many people find the DCQE results more disturbing (weird, strange, counter-intuitive) than, say, those of Bell-type experiments. We believe this is because a Bell-type experiment can be visualised (mentally or actually) by a local process. One simply creates two entangled particles that one can think of as little balls, those propagate in two different directions, and then they are detected at two separate locations. It is easy to imagine that when the particles hit the detectors, that measurement merely reveals which spin or polarisation they have, and those outcomes are correlated because the particles had a common origin.

The real puzzle of a Bell-type test lies in the fact that the correlations in the outcomes of both detectors are, for certain relative orientations of the measurement settings, more strongly correlated than one would expect from a local, classical theory in which the outcome of the measurement was determined by hidden, internal properties of the particles. But this puzzle is buried in the mathematics underlying Bell’s theorem and cannot be straight-forwardly visualised.

In experiments like the Elitzur-Vaidman bomb detector [22] or the DCQE, on the other hand, it becomes visually apparent that local realist explanations are difficult to come by because these experiments make use of superpositions in position space (particles that “take two paths at once”).

In the bomb experiment (Figure 2), one uses a Mach-Zehnder interferometer with a “bomb” in one path. The bomb plays the role of an additional detector which reveals which-way information. If the detector is on (the bomb is live) and doesn’t click (doesn’t explode), one knows the particle is on the other path.

To explain what happens in the bomb experiment, one first has to account for the destructive interference at detector $D_2$ when the bomb is a dud. In this case, by assumption, no interaction between the photon and the bomb takes place at all, and we just have the usual Mach-Zehnder interferometer. The destructive interference at $D_2$ then one to accept that the photon acts like a wave and takes both possible paths. But if the bomb is in place, live, and yet didn’t explode, then we know the photon must have gone the path on which there is no bomb. This means, if one wants a local explanation, then the propagation of the photon must depend on the detector setting (whether the bomb is live or not) – before the photon even reached the detector (the bomb).

Of course one could argue that we knew this already: a local realist hidden variables explanation of quantum mechanical phenomena requires that the path of a particle depends on the measurement setting at the time of measurement. This, after all, is what Bell’s theorem tells us: If we insist on a local explanation, then Statistical Independence must be violated [31], either by superdeterminism or by retrocausality. However, the bomb experiment makes it much more apparent just what this entails. Most notably, it makes apparent just how easy it is to explain the observations by violating Statistical Independence: If one measures the which-way information (bomb is live), the photon either goes via one path or the other, but not both. Then, it has a 50:50 chance of being transmitted:reflected by the beamsplitter. That’s it.

The quantum eraser seems perplexing for a similar reason. If one measures the which-way information at $D_1$ and $D_2$, then a local realist explanation requires that the original photon either went through the upper slit, triggering the upper entangled pair (pink/solid line) or the lower slit, triggering the lower entangled pair (green/dashed line), but not both. If we, however, ignore the which-way information and use instead detectors $D_3$ or $D_4$, then it seems a particle going to, say, $D_3$, must have gone through both slits, so it can interfere with itself (and similarly for $D_4$). That is, the choice of whether to use the eraser or not seems to decide what happened on the screen earlier (retrocausality) or the photons’ paths depend on the measurement setting all along (superdeterminism).

This argument doesn’t work as well for the quantum eraser as for the bomb experiment, because, in combining the two

2 The measurement settings in Bell’s theorem are those at the time of measurement. It does not matter at which time or how the settings were chosen, it only matters what they are when the actual measurement takes place.

3 In such a case, interaction-free/ “counterfactual” communication/ computation/ imaging, such as that in [20, 27, 32, 55–59], is possible locally specifically because of this statistical independence violation - the message/information is to some degree encoded in the detector settings.
paths they necessarily have to come together in one place. One can therefore explain the quantum eraser just by postulating that the photons always go via only one path, it’s just that at the beam splitter, half of them pass through and the other half of them do not, and the positions their entangled partners go to on the screen are correlated with whether the photons at the eraser go through the beam splitter or not.

We will in the following argue that the DCQE experiment can be extended so that, like the bomb experiment, a local realist explanation is possible only if the path of the photon either (a) allows a retrocausal influence or (b) depends on the measurement setting all along. We will then add another twist to rule out option (a).

IV. THE PARADOX

What goes on in the DCQE experiment becomes clearer when we replace the interference screen D₄ with another eraser quartet of detectors (see Figure 3; ignore the black arrow for now). That is, we create the entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left( |U_1\rangle |D_1\rangle + |U_2\rangle |D_2\rangle \right),$$

where each component of the wave-function is labelled by the detector it evolves towards. On both paths of the entangled photons we can chose whether to measure the which-way information (D₁₂, U₁₂) or whether to erase it (D₃₄, U₃₄). The detectors U₃ and U₄ measure the same superpositions as defined in (2) just with U’s instead of D’s. Note, in this local-realist setting, we assume measurement at the detector collapses the wavefunction.

This modification makes the similarity to Bell-type experiments obvious. We could now either measure the which-way information on both sides (D₁₂, U₁₂), and find that the results are perfectly correlated. Or we could erase the which-way information on both sides (D₃₄, U₃₄) and find the results are also perfectly correlated. Or we could measure the which-way information on one side and not on the other (D₁₂ and U₃₄ or D₃₄ and U₁₂) and find the results are entirely uncorrelated. So far, so unsurprising.

In this case it is still possible to come up with a local realist explanation as previously. One just imagines the photons are particles going via one specific path, which go through the beam splitter half of the time. The entangled photons do the same on each of their beam splitters. No retrocausality or superdeterminism required.

One thing one could do now is to create a mixture of which-way and erasure settings. That would constitute a Bell-type experiment. Then we could use violations of Bell’s inequality to draw conclusions about violations of Statistical Independence. However, this setup it is technically cumbersome and also teaches us nothing new.

Instead, we can make a third type of measurement, which is similar to that in the bomb experiment: Measure on the upper path with U₃ and U₄, but measure on the lower path with D₃, D₄ and D₁. Now remember that to explain the previous measurements with a local realist visualisation, we had to settle on the case where we have a particle going one particular path (pink or green, but not both), and at a beam splitter, it goes through only half of the time, and pairs of entangled particles do the same at their respective splitters. This explanation would tell us that in this third type of measurement, for those photons that do not appear in detector D₁, outcomes for their entangled partners at U₃/₄ and D₃/₄ are still correlated. Alas, in quantum mechanics they should now be uncorrelated.

To fix this problem (in a locally-realist way), we can then either (a) require that turning on detector D₁ determines what happened at U₃/₄, that is, in the past. This is the retrocausal option whose causal relations are depicted in Figure 4. Or we (b) accept that the paths of the photons were dependent on the detector setting already at emission. This is the superdeterministic option whose causal relations are depicted in Figure

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4 Again we want to stress that those are the detector settings at the time of...
To rule out (a), we propose to use a detection at $U_4$ to turn on $D_1$, which creates the causal relations depicted in Figure 6. If using detector $D_1$ indeed retrocausally determined that the entangled partner of a photon which did not go to $D_1$ must have had a 50% chance of going to $U_3$, then in half of the runs we create a causal paradox: The photon went to $U_4$, so we turned on $D_1$, but as a result it went to $U_3$!

This feedback loop seems to realise an instance of the well-known grandfather paradox, in which you go back in time and kill your own grandfather, so you’re never born and can’t go back in time to kill your own grandfather. Did you, or did you not, kill your grandfather? We can ask here the same way: “Did, or didn’t, the photon go to $U_4$?”

V. RESOLVING THE PARADOX

It seems that retrocausality cannot predict what will happen in this experiment. Clearly the contradiction itself will not manifest, for what would that mean? Would the world end in a poof? Would an error message pop up, crashing the simulation that is our reality? Certainly not (we hope). But then what would happen?

In a retrocausal framework, there is only one consistent outcome, which is that the photon on the upper path just never goes to $U_4$. If it doesn’t go to $U_4$, $D_1$ never turns on, and the result is always $U_3$ and $D_3$ together. This is internally consistent – no causal paradox occurs – but it’s not what quantum mechanics predicts. A sequence of photons that go to $U_3$ and $D_3$ is exponentially unlikely. The longer the sequence, the less likely. The retrocausal explanation can hence be ruled out experimentally. If this was achieved, we could concluded that Statistical Independence must have been violated all along. As previously discussed in [28], this also solves the measurement problem and removes other known inconsistencies of quantum mechanics, such as the contradictions resulting in the Extended Wigner’s Friend scenario.

VI. FEASIBILITY

By combining the bomb experiment and the quantum eraser, the experiment proposed above allows us to perform a new test of local realist theories. We will now discuss what it would take to perform this experiment.

The most practical method to create high-intensity polarisation-entangled photon states is to use spontaneous parametric down-conversion (SPDC), as demonstrated by [36], for example. The constraints of energy conservation in SPDC requires that the sum of the energies of the two down-converted photons must equal the energy of the pump photon [33]. This technique typically utilises nonlinear crystals such as barium borate (BBO) [21], or radiative cascade of calcium atoms [3]. We will focus on the former for our purposes.

When a monochromatic light source illuminates the BBO crystal with wavelength $\lambda_0$, it places the crystal in an energy state above ground state. When the atoms within the crystal then de-excite, a pair of down-converted photons are emitted at wavelength $\lambda_{DC}$. Because of the difficulty in producing short-wavelength lasers, and the limits on the operational wavelength range of non-linear crystals, experiments to date [35, 37, 60] have typically used optical or ultraviolet $\lambda_0 \approx 350 − 450 \text{ nm}’$pump’ frequencies to excite the crystals. The resulting down-converted photons then each exhibit a wavelength of precisely $\lambda_{DC} = 2\lambda_0$. Each of the photons in the entangled pair has exactly half the energy of the incident photon, typically in the red or infrared range (depending on the pump $\lambda_0$).

At such low photon energies, conventional detector arrays such as charge-coupled devices (CCDs) simply can not reliably count individual photons due to read noise – the semiconductor energy band gap is the same order of magnitude as the energy of the photons themselves ($\approx 1 \text{ eV}$). Because of this, noise contributions can be mistaken for photon events, and only a large number of photon events can generate a reliable signal. This is obviously not ideal for photon-counting experiments such as we would like to do with the quantum eraser.

We believe the proposed measurements to be possible due to the significant advances in detector technology for optical/near-infrared photons in recent years. If large-format multiplexing is required, the most promising of these technologies is the microwave kinetic inductance detector (MKID) array [4, 25, 39, 42, 66]. An array of MKIDs
can sample the spatial domain to a resolution of better than 100 microns, while providing time resolution of around 100 ns on each pixel with zero read noise. The only false counts result from cosmic rays, but these events display a different profile and can easily be filtered. Thus, it is feasible to monitor the full measurement plane if required (for a quantum-eraser type experiment), as MKIDs are easily multiplexed to arrays of many thousands of pixels through frequency division multiplexing [4][11].

Conveniently though, in the novel set-up proposed here, it becomes unnecessary to measure a large plane, because we do not need the entire interference pattern. We can simply measure correlations in particular detector combinations using coincidence detection. For this, a single pixel (or a small array) on each detector location will suffice. What will be of paramount importance though, is speed of response and the ability to minimise noise. Single-photon avalanche diode (SPAD) arrays have undergone rapid and significant development over the past 18 years since first demonstrated in 2003 [54]. SPADs are solid-state detectors that allow photon counting, with unparalleled time-resolution on the order of a few picoseconds [9, 10, 45, 74, 75]. While multiplexing to large arrays is challenging with this detector technology, this will not be an issue for the experiment outlined in Figure 3 as large-format multiplexing will not be required.

VII. CONCLUSION

We have here proposed a new experiment that combines the Delayed-Choice Quantum Eraser with the Elitzur-Vaidman thought/gedanken bomb experiment. When one tries to explain what happens in this experiment using a local realist interpretation, one is left with two choices: superdeterminism or retrocausality. The retrocausal option can lead to a causality paradox. When one prevents the paradox by requiring a consistent time-evolution, then retrocausality requires measurement outcomes which differ from the predictions of quantum mechanics. The difference is measurable and we have laid out a way to do this measurement in the near future.

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