Mathematical modeling of the using of the innovative intermediate products at the stage of production of gross regional product

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Abstract. This article presents a mathematical model describing interindustry industrial interrelation. Special attention is paid to the analysis of the production structure in the context of innovative products. The imitation model of using of the innovative intermediate product at the stage of production of gross regional product is developed on the basis of the research. This model allows to predict the volume of output of innovative intermediate product taking into account the need for production of new goods in the first phase of their life cycle.

1. Introduction
The enormous number of the mathematical balance models characterizing interindustry production relationships is described in modern scientific literature. For example, the issues of improving the structure of production based on the best results of the balancing production using the method Moodie Young and Comsoal are explored in the works of Ikhsan S., Ulina T. Sipangkar A. Prasetio A. [1]. Vasin A.A., Morozov V.V., Morgenschtern O., Mulen A., Petrosyan L.A., Zenkevich N.A., Ouan G., Karaseva A.G., Orlova T.A., Pecherskiy S.L., Belyaeva A.A., Blakuall D., Grischik M.A., Chebotarev V.I., Zolotuhin A.Y., Gubko M.V., Novikov D.A., Dixit A., Skit S. explore the issues of development of the economic-mathematical models and the decision-making methods on the basis of the game theory, linear programming, dynamic programming, queueing theory. Siregar I, Nasution A., Prasetio A., Fadillah K. Analysis research the issue of optimization of the structure of production in the context of the concept “Lean manufacturing” [2]. However the idea of mathematical modeling of the using of the innovative intermediate products in the open literature is unexplored.

The proof of striving to balance production of the intermediate product and the production of the product in the structure of gross regional product with the help of the mathematical methods is one of the main tasks of this study. The study is also analyzed in the context of the production of innovative products.

2. Model description
Let’s present the model of using innovative intermediate product at the stage of production of gross regional product (Figure 1).
If $P$ and $p$ – current production volumes of the intermediate and the gross regional product $N$ – the amount of new products in the first phase of their life cycle. $n$ – the amount of innovative products in the structure of new products in the first phase of their life cycle. $l$ and $l$ – the amount of output of production of goods in the last phase of their life cycle taking into account the intermediate and the gross regional product. $q$ — the volume of production of intermediate on the basis of the production need of the new products in the first phase of their life cycle. $I$ and $i$ — the amount of innovative products in the production structure of the intermediate and the gross regional product. $\mu$ — the ratio of innovative products in the gross regional product.

Then the mathematical model of the dynamics of production volumes of the intermediate and the gross regional product has the form:

\[
\begin{align*}
\frac{dP}{dt} &= N - L - q, \\
\frac{dp}{dt} &= q - l, \\
\frac{dl}{dt} &= n \times N - \mu q, \\
\frac{di}{dt} &= \mu q.
\end{align*}
\]

(1)

The intensity of output from production of goods in the last phase of their life cycle taking into account the gross regional product $L$ is a function of $P$. The intensity of output the goods from production in the last phase of their life cycle taking into account intermediate $l$ is a function of $p$. The intensity of the manufacture intermediate products on the basis of need in the manufacture of new products in the first phase of their life cycle $q$ is a function of $P$ and $p$. Let’s suppose that the function $L(P), l(p)$ and $q(P,p)$ have next features:

\[
L(0) = l(0), \frac{dL}{dP} > 0, \frac{dl}{dp} > 0, \frac{dq}{dP} > 0, \frac{dq}{dp} \leq 0.
\]

(2)

We can talk that all the variables of the system ($P, p, I, i$) are greater than zero so the phase space of the system (1) is $\mathbb{R}^4_+$ (nonnegative orthant of the four-dimensional space). In the current system (1) the first equations (for $P$ and $p$) are closed system with phase space $\mathbb{R}^2_+$ (the first quadrant of the plane):
\[
\frac{dP}{dt} = N - L - q, \quad \frac{dp}{dt} = q - l. \tag{3}
\]

3. The proof of the balance of production of innovative intermediate product and the production of an innovative product in the structure of gross regional product

The states of balance \((P = P', p = p')\) of the system (3) are determined by solving the system of equations:

\[
\begin{align*}
L(P) + q(P, p) &= N, \\
l(p) + q(P, p) &= 0.
\end{align*} \tag{4}
\]

Let’s replace the second equation by the sum of both equations. In the result we can get:

\[
\begin{align*}
L(P) + q(P, p) &= N, \\
L(P) + l(p) &= N. \tag{5}
\end{align*}
\]

The system (5) shows us that there is some implicit function \(p = f_1(P)\), which can be determined with the help of the first equation. Also it can be concluded that there is some implicit function \(p = f_2(P)\), which can be determined with the help of the second equation of this system.

Differentiating the second equation (5) according \((P)\), we get:

\[
\frac{dL}{dP} + \frac{dl}{dp} \frac{dp}{dP} = 0. \tag{6}
\]

Earlier we had determined, that formula. Than from (6) follows, that for the function \(p = f_2(P)\):

\[
\frac{dp}{dP} < 0. \tag{7}
\]

So the function \(p = f_2(P)\) is decreasing. Differentiating the first equation (5) according \((P)\), we get:

\[
\frac{dL}{dP} + \frac{dq}{dP} + \frac{dq}{dp} \frac{dp}{dP} = 0. \tag{8}
\]

So from (2) we see that for the function \(p = f_2(P)\):

\[
\frac{dp}{dP} > 0. \tag{9}
\]

So the function \(p = f_2(P)\) is increasing. On the basis of (6) and (7) we can say that graphs of functions \(p = f_1(P)\) and \(p = f_2(P)\) can intersect no more than in one point. This means that there is the balance of production of the intermediate product and the production of the gross regional product.

We will try to prove that we can speak about balance between the production of the intermediate product and the production of the gross regional product when we have any starting conditions \(R_2^2\) in the system (1). This can be verified by approximation of function values. Let’s change variables:

\(\xi = P - P', \eta = p - p'\). Than \(\dot{\xi} = P\dot{p} = p\) and \(\dot{\eta} = p, \) and the balance variables \((\xi, \eta)\) has coordinate system \((0,0)\). Let’s decompose the right parts of the system of getting variables (2.5) in a Taylor series in the variable \(\xi\) and \(\eta\) near the point \((0,0)\):
\[ \dot{\xi} = \left( -\frac{\partial q}{\partial P} \frac{dL}{dP} \right) \xi - \frac{\partial q}{\partial p} \eta, \]
\[ \dot{\eta} = \frac{\partial q}{\partial P} \xi + \left( \frac{\partial q}{\partial p} - \frac{dl}{dp} \right) \eta \] \tag{10}

where derivatives are calculated at the point \((P = P', p = p')\).

The achieving of the balance of the production of the intermediate product and the production of the gross regional product can be if in the characteristic equation \(2\lambda + A\lambda + B = 0\) the coefficients \(A\) and \(B\) will be more than 0.

So (2), we get:
\[ A = \frac{\partial q}{\partial P} \frac{dL}{dP} - \frac{\partial q}{\partial p} + \frac{dl}{dp} > 0, \]
\[ B = \frac{\partial q}{\partial p} \frac{dL}{dP} + \frac{\partial q}{\partial P} \frac{dl}{dp} + \frac{dL}{dp} > 0. \] \tag{11}

Our first statement is proved. Next let’s discuss the model of the balance of production of innovative intermediate product and production innovation of the gross regional product (the subsystem of equations for \(I\) and \(i\) in the system (1) if \(P = P'\) and \(p = p'\).

The ratio of innovative products in the total volume of the intermediate product \(\mu' = \frac{I'}{P'}\) satisfies the equation:
\[ \mu'q(P', p') - nN = 0. \] \tag{12}

We can find the ratio of innovative products in the total volume of the intermediate product:
\[ \mu' = \frac{nN}{q(P', p')} \] \tag{13}

The meaning \(\mu = \mu'\) is balanced index as the equation for \(\frac{dI}{dt}\) in (1) the value \(\frac{dI}{dt}\) will be affirmative if \(\mu < \mu'\) and negative if \(\mu > \mu'\).

Taking into account that balanced volumes \(P'\) and \(p'\) can be found as the solving of the system (5), so according (13) we can get:
\[ \mu' = \frac{nN}{q(P', p')} = n \left( \frac{q + L'}{q'} = n \left( \frac{L + L'}{q'} \right) \right) \tag{14} \]

4. The realization of mathematical models in the system AnyLogic

A mathematical model of the using of innovative intermediate product at the stage of production of gross regional product was implemented in the system AnyLogic (Figure 2).

There are some oscillograms of changes of the volumes of innovative products in the structure of production of intermediate and gross regional product in the animation window.

The model parameter (the amount of innovative products in the structure of new products in the first phase of their life cycle; the amount of output of production of the intermediate product in the last phase of its life cycle; the amount of output of production of gross regional product in the last phase of its life cycle) can be changed with the help of scrollbars.
Figure 2. A mathematical model of the using of innovative intermediate product at the stage of production of gross regional product

This simulation model allows to predict the volume of production of innovative intermediate product according the need in the manufacture of new products in the first phase of their life cycle.

5. Conclusions

So it was proven that there is the opportunity to achieve a balance between the production of intermediate product and the production of the gross regional product in any initial conditions $R^2$.

When we modeled the balance of production of innovative intermediate product and the production innovation of the gross regional product we determined the method of calculation of a balanced ratio of innovative products in the amount of intermediate product.

The applied significance of our scientific research is to develop software product based on the simulation model of using the innovative intermediate product at the stage of production of gross regional product.

This software product will improve the quality of forecasting the amount of production of innovative intermediate product on the basis of need in the manufacture of new products in the first phase of their life cycle.

6. References

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