Note on tree-level unitarity in the General Two Higgs Doublet Model.

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Abstract

Tree-level unitarity constraints on the masses of the Higgs bosons in the general Two Higgs Doublet Model (THDM) are studied. We first consider the case where the Higgs potential is invariant under a discrete symmetry transformation, and derive strong constraints on the mass of the lightest CP-even Higgs boson ($M_h$) as a function of $\tan \beta$. We then show that the inclusion of the discrete symmetry breaking term weakens the mass bounds considerably. It is suggested that a measurement of $M_h$ and $\tan \beta$ may enable discrimination between the two Higgs potentials.
1 Introduction

The Minimal Standard Model (MSM) of electroweak interactions [1] is in complete agreement with all precision experimental data (LEPII, Tevatron, SLD), with the notable exception of neutrino oscillation experiments [2]. In this minimal version there is one complex $SU(2)_L \otimes U(1)$ doublet which provides mass for the fermions and gauge bosons. After electroweak symmetry breaking three of the four real degrees of freedom initially present in the Higgs doublet become the longitudinal components of the gauge bosons $W^\pm, Z$, leaving one degree of freedom which manifests itself as a physical particle ($\phi^0$) [3]. The Higgs mass ($M_{\phi^0}$) is not fixed by the model, although constraints on $M_{\phi^0}$ can be obtained by making additional theoretical assumptions. These include the unitarity bound ($M_{\phi^0} < 870$ GeV) [4] and the triviality bound [5]. So far no experimental information on the nature of the Higgs particle has been found, and from the negative searches in the Higgsstrahlung channel at LEPII the lower bound $M_{\phi^0} > 107.7$ GeV [6] has been derived.

In recent years there has been growing interest in the study of extended Higgs sectors with more than one doublet [7]. The simplest extension of the MSM is the Two Higgs Doublet Model (THDM), which is formed by adding an extra complex $SU(2)_L \otimes U(1)_Y$ scalar doublet to the MSM Lagrangian. Motivations for such a structure include CP–violation in the Higgs sector, supersymmetry, and a possible solution to the cosmological domain wall problem [8]. In particular, the Minimal Supersymmetric Standard Model (MSSM) [7] takes the form of a constrained THDM.

The most general THDM scalar potential which is renormalizable, gauge invariant and CP invariant depends on ten parameters, but such a potential can still break CP spontaneously [4]. In order to ensure that tree-level flavour changing neutral currents are eliminated, a discrete symmetry ($\Phi_i \rightarrow -\Phi_i$, where $\Phi_i$ is a scalar doublet) may be imposed on the lagrangian [4], which reduces the number of free parameters to 6. The resulting potential was considered in [14], and is referred to as $V_A$ in [12]. We shall be concerned with the potential described in [7] which is equivalent to $V_A$ plus a term which breaks the discrete symmetry (parametrized by $\lambda_5$) and contains 7 free parameters. Such a potential does not break CP spontaneously or explicitly [11], [13] provided that all the parameters are real.

We note here that tree-level unitarity constraints for the THDM scalar potential $V_A$ were studied in [14], [13]. When deriving constraints from unitarity, [14] considered only seven elastic scattering processes $S_1 S_2 \rightarrow S_1 S_2$ (where $S_i$ is a Higgs scalar) while [13] considered a larger scattering ($S$) matrix. In [15], upper bounds on the Higgs masses were derived, in particular $M_h \leq 410$ GeV for $\tan \beta = 1$, with the bound becoming stronger as $\tan \beta$ increases. We improve those studies for $V_A$ by including the full scalar $S$ matrix which includes channels which were absent in [15], and we also show graphically the strong correlation between $M_h$ and $\tan \beta$.

To our knowledge such unitarity constraints have not been considered for the case of $\lambda_5 \neq 0$ and this is the principal aim of this note. We shall see that the presence of a non-zero $\lambda_5$ can significantly weaken the unitarity bounds found in [14], [13].

The paper is organized as follows. In Section 2 we give a short review of the THDM
potential and explain the unitarity approach we will be using. In Section 3 we present our numerical results for the cases of $\lambda_5 = 0$ and $\neq 0$, while Section 4 contains our conclusions.

2. Scalar potential and unitarity constraint

It has been shown [16] that the most general THDM scalar potential which is invariant under $SU(2)_L \otimes U(1)_Y$ and conserves CP is given by:

$$V(\Phi_1, \Phi_2) = \lambda_1(|\Phi_1|^2 - v_1^2)^2 + \lambda_2(|\Phi_2|^2 - v_2^2)^2 + \lambda_3(|\Phi_1|^2 - v_1^2)(|\Phi_2|^2 - v_2^2) + \lambda_4(|\Phi_1|^2|\Phi_2|^2) + \lambda_5(Re(\Phi_1^+\Phi_2) - v_1v_2)^2 + \lambda_6[Im(\Phi_1^+\Phi_2)]^2$$

(2.1)

where $\Phi_1$ and $\Phi_2$ have weak hypercharge $Y = 1$, $v_1$ and $v_2$ are respectively the vacuum expectation values of $\Phi_1$ and $\Phi_2$ and the $\lambda_i$’s are real–valued parameters.

$$\Phi_i = \left( \frac{\varphi_i^+}{v_i + \frac{h_i}{\sqrt{2}}} \right)$$

This potential violates the discrete symmetry $\Phi_i \rightarrow -\Phi_i$ softly by the dimension 2 term $\lambda_5 Re(\Phi_1^+\Phi_2)$ and has the same general structure of the scalar potential of the MSSM. One can prove easily that for $\lambda_5 = 0$ the exact symmetry $\Phi_i \rightarrow -\Phi_i$ is recovered.

After electroweak symmetry breaking, the $W$ and $Z$ gauge bosons acquire masses given by $m_W^2 = \frac{1}{2}g^2v^2$ and $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$, where $g$ and $g'$ are the $SU(2)_{weak}$ and $U(1)_Y$ gauge couplings and $v^2 = v_1^2 + v_2^2$. The combination $v_1^2 + v_2^2$ is thus fixed by the electroweak scale through $v_1^2 + v_2^2 = (2\sqrt{2}G_F)^{-1}$, and we are left with 7 free parameters in eq.(2.1), namely the $(\lambda_i)_{i=1,...,6}$’s and $\tan \beta = v_2/v_1$. Meanwhile, three of the eight degrees of freedom of the two Higgs doublets correspond to the 3 Goldstone bosons ($G^\pm, G^0$) and the remaining five become physical Higgs bosons: $H^0, h^0$ (CP–even), $A^0$ (CP–odd) and $H^\pm$. Their masses are obtained as usual by the shift $\Phi_i \rightarrow \Phi_i + v_i \varphi_i$. After generating the scalar masses in term of scalar parameters $\lambda_i$, using straightforward algebra one can express all the $\lambda_i$ as functions of the physical masses:

$$\lambda_4 = \frac{g^2}{2m_W^2}m_W^2 \pm , \quad \lambda_6 = \frac{g^2}{2m_W^2}m_A^2 \pm , \quad \lambda_3 = \frac{g^2}{8m_W^2}s_{\alpha\beta}(m_H^2 - m_h^2) - \frac{\lambda_5}{4}$$

(2.2)

$$\lambda_1 = \frac{g^2}{8c_Bm_W^2}[c_\alpha^2m_H^2 + s_\alpha^2m_h^2 - \frac{s_{\alpha\beta}}{\tan \beta}(m_H^2 - m_h^2)] - \frac{\lambda_5}{4}(-1 + \tan^2 \beta)$$

(2.3)

$$\lambda_2 = \frac{g^2}{8s_Bm_W^2}[s_\alpha^2m_H^2 + c_\alpha^2m_h^2 - \frac{s_{\alpha\beta}}{\tan \beta}(m_H^2 - m_h^2)] - \frac{\lambda_5}{4}(-1 + \frac{1}{\tan^2 \beta})$$

(2.4)

The angle $\beta$ diagonalizes both the CP–odd and charged scalar mass matrices, leading to the physical states $H^\pm$ and $A^0$. The angle $\alpha$ diagonalizes the CP–even mass matrix leading to the physical states $H^0, h^0$.

We are free to take as 7 independent parameters $(\lambda_i)_{i=1,...,6}$ and $\tan \beta$ or equivalently the four scalar masses, $\tan \beta, \alpha$ and one of the $\lambda_i$. In what follows we will take $\lambda_5$ as a free parameter.
To constrain the scalar potential parameters one can demand that tree-level unitarity is preserved in a variety of scattering processes. This corresponds to the requirement that the $J = 0$ partial waves ($a_0$) for scalar-scalar and gauge boson scalar scattering satisfy $|a_0| < 1/2$ in the high-energy limit. At very high-energy, the equivalence theorem [17] states that the amplitude of a scattering process involving longitudinal gauge bosons $V_{\mu}^{\pm,0}$ may be approximated by the scalar amplitude in which gauge bosons are replaced by their corresponding Goldstone bosons $G^{\pm,0}$. We conclude that unitarity constraints can be implemented by solely considering pure scalar scattering.

In very high energy collisions, it can be shown that the dominant contribution to the amplitude of the two-body scattering $S_1S_2 \rightarrow S_3S_4$ is the one which is mediated by the quartic coupling. Those contributions mediated by trilinear couplings are suppressed on dimensional grounds. Therefore the unitarity constraint $|a_0| \leq 1/2$ reduces to the following constraint on the quartic coupling, $|Q(S_1S_2S_3S_4)| \leq 8\pi$. In what follows our attention will be focussed on the quartic couplings.

In order to derive the unitarity constraints on the scalar masses we will adopt the technique introduced in [13]. It has been shown in previous works [18] that the quartic scalar vertices written in terms of physical fields $H^\pm, G^\pm, h^0, H^0, A^0$ and $G^0$, are very complicated functions of $\lambda_i$, $\alpha$ and $\beta$. However the quartic vertices (computed before electroweak symmetry breaking) written in terms of the non-physical fields $\varphi_i^\pm, h_i$ and $z_i$ (i=1,2) are considerably simpler expressions. The crucial point of [13] is the fact that the $S$ matrix expressed in terms of the physical fields (i.e. the mass eigenstate fields) can be transformed into an $S$ matrix for the non-physical fields $\varphi_i^\pm, h_i$ and $z_i$ by making a unitarity transformation. The latter is relatively easy to compute from eq. 2.1. Therefore the full set of scalar scattering processes can be expressed as an $S$ matrix composed of 4 submatrices which do not couple with each other due to charge conservation and CP-invariance. The entries are the quartic couplings which mediate the scattering processes.

The first submatrix corresponds to scatterings whose initial and final states are one of the following: $(\varphi_1^+, \varphi_2^+, \varphi_1^-, h_1z_2, h_2z_1, z_1z_2, h_1h_2)$. Therefore one obtains a $6 \times 6$ matrix leading to the following 5 distinct eigenvalues:

$$
e_1 = 2\lambda_3 - \lambda_4 - \frac{\lambda_5}{2} + 5\frac{\lambda_6}{2}$$
$$e_2 = 2\lambda_3 + \lambda_4 - \frac{\lambda_5}{2} + \frac{1}{2}\lambda_6$$
$$f_+ = 2\lambda_3 - \lambda_4 + 5\frac{\lambda_5}{2} - \frac{1}{2}\lambda_6$$
$$f_- = 2\lambda_3 + \lambda_4 + \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6$$
$$f_1 = f_2 = 2\lambda_3 + \frac{1}{2}\lambda_5 + \frac{1}{2}\lambda_6 \quad (2.5)$$

The second submatrix corresponds to scatterings with initial and final states one of the following: $(\varphi_1^+, \varphi_2^+, \varphi_3^+, \varphi_4^+, h_1v_2, h_2v_2, h_1h_2, h_1v_2, h_2v_2)$. Again we obtain a $6 \times 6$ matrix with the
following 6 eigenvalues:

\[ a_\pm = 3(\lambda_1 + \lambda_2 + 2\lambda_3) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \frac{1}{2}(\lambda_5 + \lambda_6))^2} \]  
\[ b_\pm = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(-2\lambda_4 + \lambda_5 + \lambda_6)^2} \]  
\[ c_\pm = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(\lambda_5 - \lambda_6)^2} \]

The third submatrix corresponds to the basis: \((h_1 z_1, h_2 z_2)\). The 2 \(\times\) 2 matrix possesses the eigenvalues \( d_\pm \) and \( c_\pm \), with \( d_\pm = c_\pm \). All the above eigenvalues agree with those found in [15], up to a factor of \(1/16\pi\) which we have factorised out. In our analysis we also include the two body scattering between the 8 charged states: \(h_1\varphi_1^+, h_2\varphi_1^+, z_1\varphi_1^+, z_2\varphi_1^+, h_1\varphi_2^+, h_2\varphi_2^+, z_1\varphi_2^+, z_2\varphi_2^+\). Note that these channels were neglected in [15]. The 8\(\times\)8 submatrix obtained from the above scattering processes contains many vanishing elements, and the 8 eigenvalues are straightforward to obtain analytically. They read as follows: \(f_-, e_2, f_1, c_\pm, b_\pm\) and \(p_1\), where

\[ p_1 = 2(\lambda_3 + \lambda_4) - \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6 \]

As one can see, these additional channels lead only to one extra eigenvalue, \(p_1\), although we shall see that this eigenvalue plays an important role in constraining \(M_{H^\pm}\) and \(M_A\).

3. Numerical results and discussion

In this section we present our results for the unitarity constraints on the Higgs masses in the THDM. All the eigenvalues are constrained as follows:

\[ |a_\pm|, |b_\pm|, |c_\pm|, |d_\pm|, |f_\pm|, |e_{1,2}|, |f_{1,2}|, |p_1| \leq 8\pi \]  

In [15] the eigenvalues explicitly contain the factor \(1/16\pi\) and so were constrained to be less than \(1/2\). In section 3.1 we consider the special case of \(\lambda_5 = 0\), while section 3.2 presents results for the general case of \(\lambda_5 \neq 0\).

3.1 Case of \(\lambda_5 = 0\)

For \(\lambda_5 = 0\) our potential is identical to those considered in [14, 15]. We improve those analyses on two accounts:

(i) We have considered extra scattering channels which leads to one more eigenvalue constraint, \(p_1\), as explained in Section 2.

(ii) When finding the allowed parameter space of Higgs masses we simultaneously impose all the eigenvalue constraints. In [15] only the condition \(|a_+| \leq 8\pi\) was applied when deriving mass bounds.
Table 1: A comparison of our bounds (AAN) and those of KKT [15]. All masses are in GeV and $\lambda_5 = 0$.

|       | $M_{H^\pm}$ | $M_A$ | $M_h$ | $M_H$ |
|-------|-------------|-------|-------|-------|
| AAN   | 691         | 695   | 435   | 638   |
| KKT   | 860         | 1220  | 410   | 700   |

In order to obtain the upper bounds on the Higgs masses allowed by the unitarity constraints we vary all the Higgs masses and mixing angles randomly over a very large parameter space. We confirm the result of [15] which states that $a_+$ is comfortably the strongest individual eigenvalue constraint. However, the other eigenvalues impose important constraints on $M_A$ and $M_{H^\pm}$. If only $|a_+| \leq 8\pi$ is imposed we can reproduce the upper bounds on the Higgs masses given in [15], in particular their main result of $M_h \leq 410$ GeV. When all eigenvalue bounds are applied simultaneously we find improved bounds on the Higgs masses, particularly for $M_A$ and $M_{H^\pm}$. We note that the new eigenvalue constraint $p_1 \leq 8\pi$ (eq.2.9) plays a crucial role in determining the upper bound on $M_{H^\pm}$. We write in Table 1 our bounds (AAN) and those of [15], denoted by KKT.

Note that the bounds given in Table 1 are obtained for relatively small $\tan \beta$ (say $\tan \beta \approx 0.5$). For large $\tan \beta$ the bound is stronger, although for the case of $A^0$, $H^0$ and $H^\pm$ the $\tan \beta$ dependence is rather gentle. Of particular interest is the $\tan \beta$ dependence of the bound on $M_h$ which will be covered in the section 3.2.

### 3.2 General case of $\lambda_5 \neq 0$

We now consider $\lambda_5 \neq 0$ which corresponds to the inclusion of the term which softly breaks the discrete symmetry. Such a term was neglected in the analyses of [14, 15], and from perturbative constraints may take values $|\lambda_5| \leq 8\pi$ [19]. In the graphs which follow we do not impose the perturbative requirement $|\lambda_i| \leq 8\pi$ for the remaining $\lambda_i$. Imposing this condition only leads to minor changes in the numerical results which will be commented on when necessary.

We plot in Fig.1 the maximum value of $M_h$ against $\tan \beta$ for increasing values of $\lambda_5$, imposing all the eigenvalue constraints simultaneously as done in Section 3.1. For the case of $\lambda_5 = 0$ one finds a strong correlation, with larger $\tan \beta$ requiring smaller $M_h$. For example, $\tan \beta \geq 7$ corresponds to $M_h \leq 100$ GeV, which is the mass range already being probed by LEPII. However, $h^0$ in the THDM with $M_h \leq 100$ GeV is not guaranteed to be found at LEPII due to the suppression factor of $\sin^2(\beta - \alpha)$ for the main production process $e^+e^- \rightarrow h^0Z$. For the case of $\lambda_5 = 0$ we find that values of $\tan \beta \geq 20$ are strongly disfavoured since they easily violate one of the unitarity constraints. If $\lambda_5 \neq 0$, Fig.1 shows that for a given $\tan \beta$, the action of increasing $\lambda_5$ allows larger maximum values of $M_h$. For $\lambda_5 = 15$ one finds a horizontal line at $M_h \approx 670$ GeV, showing that the upper bound has been increased for all values of $\tan \beta$. In addition, large ($\geq 30$) values of $\tan \beta$ are allowed if $\lambda_5 \neq 0$, in contrast to the case of $\lambda_5 = 0$. For example, $\lambda_5 = 1$ comfortably permits values of $\tan \beta = 60$. However, as pointed out in [20] perturbative constraints on...
the $\lambda_i$ also restrict the allowed values of $\tan \beta$ in the THDM. Using the condition in $[19]$ which requires $|\lambda_i| \leq 8\pi$, we found that $\tan \beta \geq 30$ is strongly disfavoured. In Fig.2 we plot the maximum mass values ($M_S$) of all the Higgs bosons as a function of $\lambda_5$. Again all the Higgs masses and mixing angles have been varied randomly. We see that $\lambda_5 = 0$ corresponds to the values in Table 1, while for $\lambda_5 = 15$ the upper bounds have been increased significantly. Including the perturbative requirement would lower the bounds on $M_A$ and $M_{H^\pm}$ by $10 \rightarrow 20$ GeV. The $\tan \beta$ dependence of $M_S$ for $S = H^\pm, A^0$ and $H^0$ is not very pronounced, and the maximum mass value may be obtained for both small and large $\tan \beta$.

We note that the relaxation of the strong correlation between $M_h$ and $\tan \beta$ with $\lambda_5 \neq 0$ would in principle allow the possibility of distinguishing between the discrete symmetry conserving and violating potentials. If $h^0$ is discovered and the measured values of $M_h$ and $\tan \beta$ lie outside the rather constrained region for $\lambda_5 = 0$, this would signify $\lambda_5 \neq 0$ and thus a soft breaking of the discrete symmetry. Possibilities of measuring $\tan \beta$ at high–energy $e^+e^-$ colliders have been considered in the context of the MSSM in $[21]$. Production and decay of $H^\pm$ and $A^0$ are particularly promising since the rates do not involve the mixing angle $\alpha$. Much of the analysis of $[21]$ is also valid in the THDM.

Figure 1: Maximum $M_h$ in GeV as a function of $\tan \beta$ for various values of $\lambda_5$. 

4. Conclusions

We have derived upper limits on the masses of the Higgs bosons in the general Two Higgs Doublet Model (THDM) by requiring that unitarity is not violated in scalar scattering processes. We first considered the THDM scalar potential which is invariant under a discrete symmetry transformation and improved previous studies by including the complete set of scattering channels. Stronger constraints on the Higgs masses were derived. Of particular interest is the tan $\beta$ dependence of the upper bound on $M_h$, with larger tan $\beta$ requiring a lighter $h^0$ e.g. tan $\beta \geq 7$ implies $M_h \leq 100$ GeV. We then showed that the presence of the discrete symmetry breaking term parametrized by $\lambda_5$ may significantly weaken the upper bounds on the masses. In particular, the aforementioned correlation between tan $\beta$ and maximum $M_h$ is relaxed. It was suggested that a measurement of tan $\beta$ and $M_h$ may allow discrimination between the two potentials.

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