Tibia and Fibula Stress Strain Research

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Abstract. This article is devoted to the stress strain research of a biomechanical system formed by the tibia, fibula and muscles under external stress (axial compression). In this study an anatomically accurate geometric model of human bones was developed based on CT analysis data, which was the basis of the calculated finite element model. A numerical evaluation of the stress strain state was carried out using the Finite Element Method (FEM). The developed model of the tibia and fibula was validated based on the results of axial compression field tests. The analysis of mesh independence of finite element models of the tibia and fibula has been carried out.

1. Introduction
Detailed human bones modeling allows us to develop computational models of biomechanical systems to solve a number of problems in many areas related to ensuring human safety and security. This article describes the biomechanical system of the tibia and muscles, which in turn after a number of modifications can represent the lower limb of a person. This model can be used to model external mechanical effects at a wide range of speeds to solve various types of problems (safety of pedestrians and drivers, human safety under stroke, etc.). In this article the main focus is a damage modeling that occurs during axial compressive loading. The aim of this work is to create a detailed finite element model of the tibia and fibula, its physical and mechanical properties research, analyze the stress-strain state of the system, as well as the reaction force that occurs during loading in bones and evaluation of the model mesh independence.

2. Tibia and Fibula Anatomy
Each bone is a living, actively functioning and continually renewing organ. Blood vessels and nerves penetrating into the bone provide its interaction with the whole body, participation in metabolism, fulfillment of functions and necessary restructuring during growth, development and changing living conditions.
The peculiarities of the bone’s internal structure are due to its cortical bone and spongy substance. The cortical bone is in a dense layer located on the periphery of the bone. The spongy substance is under cortical bone and has a porous structure. The tibia is more massive and takes on the entire weight of the body [15]. An illustration the tibia and fibula which is shown in Fig. 1a and Fig. 1b is taken from the Atlas of Anatomy [1].

One of the goals of this article is the creation of a mathematical model with complex geometry and diversity. One of the first steps is to divide the model into 2 layers (cortical bone and spongy substance), giving them different properties, which will be discussed later. The difficulty is in determining the thickness of the cortical bone, since this value depends on the age of person, the geometric features of the bone, diseases and injuries. In the studied reference [3, 5, 9], the values range from 1 mm to 7 mm. This parameter greatly affects bone stiffness. Figure 2 shows the way the layer thickness is approximately calculated.
3. 3D Model creation

One of the goals of the research is the 3D model creation of tibia and fibula using CT data. In the process of preparing a three-dimensional model, we can distinguish two stages: a) Point clouds conversion from the DICOM to the STL b) Creation of STL solid-state geometry [6].

Point clouds conversion from the DICOM to the STL. The CT scanner mode of operation is based on the following simultaneous actions:
- continuous rotation of x-ray tube around human body that emits x-rays
- registration of this radiation with special detectors
- continuous movement of the bed with the person lying on it along the longitudinal axis.

X-rays are captured by detectors, then they are converting into electrical signals and transmitting to a computer. Information received from radiation detectors is processed using a CT scanner. As a result, a two-dimensional image of the human body cross section is created at different levels. (Fig.3a).

Computed tomography imaging pictures are a visualization of DICOM examination data. DICOM (Digital Imaging and Communications in Medicine) is an industry standard for creating, storing, transmitting and visualizing medical images and documents of examined patients.

Using the 3D Slicer an STL model of tibia and fibula was constructed (Fig. 4b) after processing DICOM files. Due to sound effects during tomography, the resulting model has a lot of imperfections and protruding surfaces, so it must be filtered. This was done by MeshLab.
Creation of STL solid-state geometry. The STL format is used to store three-dimensional models of objects to use in rapid prototyping technologies. Information about the object is stored as a list of triangular faces that describe its surface, and their normals. To use the model for engineering calculations, it is necessary to build solid-state CAD (Computer-Aided Design) geometry, as this is required by engineering packages. Using the ANSYS SpaceClaim the described STL model was converted to a CAD model.

4. Physico-mechanical properties of bones
Bone is a solid body, one of the main properties of bone is strength. Bone strength is the ability to withstand external destructive forces. Quantitatively the strength is determined by the tensile strength and depends on the macro- and microscopic design, bony tissue composition. As for the macroscopic design, each bone has a specific shape that can carry the highest loads in a certain part of the skeleton. The strength of the bone is also significantly affected by bone structure. With decalcination the bone is easily bent, compressed and twisted, with an increase of calcium it becomes fragile. The bone strength of a healthy adult is greater than the strength of some construction materials, it is the same as of cast iron [12]. The analysis of scientific publications on the values of density, elastic modulus, Poisson's ratio, ultimate stresses and deformations showed a fairly wide scatter. Young's module of healthy spongy tissue along the fibers is $600 - 1050$ MPa, across the fibers it is $375 - 600$ MPa; tensile strength along fibers is $16 - 22$ MPa, across fibers - $7 - 13$ MPa. However, this data is reliable only if the conditional density of the material is sufficiently high. Experimental data on bone strength is available only for dried compact bones and relatively Young’s module it has a fairly wide range of values ranging from $427$ MPa to $22$ GPa, and tensile strength range is $60 - 80$ MPa. Moreover, in the experiment on a dried compact bone, the difference of the values across $(4.2 - 9.9$ GPa) and along $(8.7 - 16.5$ GPa) of bone trabecula was proved. These studies indicate anisotropy of bone tissue [13].

Two mathematical models can be distinguished which most accurately describe the movement of bones.

4.1. Model of an elastoplastic body with nonlinear hardening
The material is strained elastically until the yield point is reached. If this limit is exceeded under loading (load increase), the connection between stresses and strains will be nonlinear, i.e., the material will be...
strengthened nonlinearly (see Fig. 4). When unloading the material is linearly elastic and has permanent strains.

Figure 4. Model of an elastoplastic body with nonlinear hardening [17].

When loaded again the material will strain linearly-elastic (elastic modulus E) until the stress from which unloading begins is reached. This strain plays the role of a new yield strength ($\sigma_i T$) that exceeds the initial one, and as a result, we consider that the material has received plastic strain hardening [17]. The change in constant strains is described using an elastoplastic dependence for an instantaneous strain response. Equation (1) describes the connection between stress and strain in increments [11]:

$$\dot{\sigma}^0 = D^e \cdot (\dot{\varepsilon} - \dot{\varepsilon}^p)$$

where $D^e$ is the instant elasticity tensor. The associated flow law (2) is used:

$$\dot{\varepsilon}^p = \dot{\lambda} \frac{\partial f}{\partial \sigma}$$

where $\lambda$ – plastic flow coefficient. The increase in stresses relatively to the strain above the elastic limit is taken into account when using the law of hardening (3), written in the form:

$$\sigma_T = f(\dot{\varepsilon}^p)$$

The plasticity criterion (4) is the Mises plasticity condition:

$$f(\sigma) = \sqrt{\frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} = \sigma = \sigma_T$$

4.2. Viscoelastic body model

This model is described by equations (5) - (6), in which the dependence on the strain rate of the material is specified [14]. It is important to consider this parameter in calculations with high impact rates, dynamics problems (shock, explosion). The graphs in Fig. 5 [7] show the way the strain rate affects the stress-strain state.

$$\sigma_{ij} = \sigma_{ij}^0 + \int_0^t G(t - \tau) \frac{\partial \varepsilon_{ij}}{\partial \tau} d\tau; \quad G(t) = \sum_{n=1}^N G_n e^{-\beta_n t},$$

where $\sigma_{ij}^0$ are the components of the elastic stress tensor deviator, $\varepsilon_{ij}$ are the components of the strain tensor.

The relaxation function is shown through the Prony serie

$$\int_0^t G(t - \tau) \frac{\partial \varepsilon_{ij}}{\partial \tau} d\tau = \sum_{n=1}^N G_n \int_0^t e^{-\beta_n (t - \tau)} \frac{\partial \varepsilon_{ij}}{\partial \tau} d\tau,$$

$$\frac{\partial \varepsilon_{ij}}{\partial \tau} = \text{engineering strain rate}.$$
Studies show that by increasing strain rate, Young's modulus and tensile strength increase, and fracture strain decreases [8]. Figure 6 shows the dependence of Young's modulus on the strain rate on a logarithmic scale.

Figure 5. Stress – strain curves under compression for a cortical bone of thigh depending on the strain rate [7].

5. Material characteristics selection
As the main mathematical model of materials used in the finite element model of the components of the biomechanical structure, a model of an elastic-plastic material with a fracture criterion based on the maximum value of effective plastic deformation is considered. The effect of plasticity should be taken into account in the model, since the accumulation of strain is significant, and it can also be an indicator of possible damage. In our article we do not consider the viscosity of a cortical bone, since it is known to be quasistatic.

As mentioned above, the scatter of these bone properties is great. We chose the following values (table 1 and Fig. 7-8.), since they are most often found in current sources [4, 10] when modeling the human lower limb. It should be noted that the bones behave differently when compressed and strained.

Table 1. Physico-mechanical properties of bones.

|                | Density, t / mm³ | Young's module, MPa | Poisson's ratio | Yield Strength, MPa | Fracture strain | Fracture stress, MPa |
|----------------|------------------|---------------------|-----------------|---------------------|-----------------|---------------------|
| Tibia,cortical bone | 2e-9             | 15 000              | 0.3             | 100                 | 1.2%            | 157.2               |
| Tibia,spongy substance | 1.1e-9         | 445                 | 0.3             | -                   | 13.4%           | -                   |
| Fibula,cortical bone | 2e-9             | 14 000              | 0.3             | 70                  | 1.2%            | 127.2               |
| Fibula,spongy substance | 1.1e-9         | 292                 | 0.3             | -                   | 13.4%           | -                   |
When designing vehicles, building constructions and technological processes in scientific research, computer software analysis systems (CAE – Computer – Aided Engineering) based on the finite element method (FEM) are widely used today. The finite element method (FEM) is a numerical method for solving differential equations with partial derivatives, as well as integral equations of applied physics. The method is widely used in mechanics tasks of a deformable solid body, heat transfer, hydrodynamics and electrodynamics. A detailed description of the method and the algorithms used is given in the reference list [16].

Advantages of the method:

**Figure 7.** Hardening curves of compression and strain of Tibia.

**Figure 8.** Hardening curves of compression and strain of Fibula.

### 6. Finite Element Method (FEM)

When designing vehicles, building constructions and technological processes in scientific research, computer software analysis systems (CAE – Computer – Aided Engineering) based on the finite element method (FEM) are widely used today. The finite element method (FEM) is a numerical method for solving differential equations with partial derivatives, as well as integral equations of applied physics. The method is widely used in mechanics tasks of a deformable solid body, heat transfer, hydrodynamics and electrodynamics. A detailed description of the method and the algorithms used is given in the reference list [16].

Advantages of the method:
- the possibility of splitting into finite elements of an area of any shape and complexity;
- the ability to calculate stress and strain fields in real details considering all their design features;
- excludes the variation that occurs during the field test (it is especially important when working with biological objects).

7. Finite element model
Figure 9 shows the finite element model of tibia and fibula, which consists of 30,000 elements of the solid type (spongy substance), 10,000 elements of the shell type (cortical bone). The size of the elements is 4 mm. The thickness of the tibia cortical bone varies from 1 mm to 3.2 mm, counting from the metaphysis, fibula - from 1 mm to 1.9 mm.

![Finite element model of tibia and fibula.](image)

8. Problem Statement
The problem statement (see Fig. 10a) [4] was selected as a task for validating the developed finite element model of bones, which allows to evaluate the movement of bones under axial compression. Tibia and Fibula are filled with epoxy resin above and below (E = 1000 MPa, p = 2.2E-9 t / mm ^ 3, v = 0.45). The upper part is fixed, while the lower part is affected by a steel plate with a constant speed of 2 mm / s. As a result, the validation of the developed finite element model of bones is carried out on the basis of a comparison of the contact force arising from the simulation with the results of field tests [4]. The finite element model of the described problem statement is presented in Fig. 10b. Muscles, fat and other biological elements of the lower limb of a person are modeled using solid-state elements with averaged physical and mechanical parameters (E = 13.2 MPa, p = 1.3E-9E-9 t / mm ^ 3, v = 0.48).
9. Results
The article [4] presents the results of a full-scale test for axial compression of tibia and fibula. Based on the sample, a corridor was created of the minimum allowable and maximum allowable values of the force arising in the bones (the contact force between the plate and the model). Fig. 11 shows a comparison of the obtained results of our model and experimental data.

An analysis of the results showed a good agreement with the results of field tests. The stiffness value of the entire system and the process of bone destruction in the developed model correspond to a biological sample.
When choosing the Finite element method as a numerical method for evaluating the stress-strain state, it is necessary to analyze the mesh independence. Thus, based on the developed finite element model of bones, models with element sizes of 3, 4, 8, 16, 24 mm were created.

In order to evaluate the mesh independence, we studied the dependence of the linear rigidity of the system on the number of elements in the model (NOE-Number of elements). The results of this analysis are shown in Fig. 12.

![Figure 12. Mesh Independence Analysis.](image)

It was found that the optimal element size is 4 mm (NOE = 40,000), since with a further element size decrease, the calculation time increases significantly with a slight increase in the accuracy of the results.

**10. Conclusion**

In this study we described the anatomy of the human tibia and fibula and created CAD and FEM models of bones, mathematical models were studied to describe biological tissues, physicomechanical properties of bones were selected, and a numerical analysis of bone compression was performed. The analysis of mesh independence was carried out. It is shown that the behavior of the developed model under external mechanical action is in good agreement with experimental data.

Thus, the biomechanical system described in this article as part of the tibia and muscle tissue describes the properties of the biological sample well and can be used to solve problems of external mechanical stress, for which the axial load (along the bone) is dominant. In future we plan to develop this model to increase the number of tasks to be solved.

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