Getting SM and New Physics CP phases from $B$ Decays

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Abstract

A method to extract Standard Model(SM) CP phases involving B decays is presented. The method involves a pair of decays where one decay receives a significant $\bar{b} \to \bar{d}$ penguin contribution while the second has a significant $\bar{b} \to \bar{s}$ penguin contribution, but is dominated by a single amplitude. SM CP phase information is obtained using the fact that the $\bar{b} \to \bar{s}$ transition amplitude is related by SU(3) to a piece of the amplitude with the $\bar{b} \to \bar{d}$ transition. If there is significant new physics only in $\bar{b} \to \bar{s}$ transitions and not in $\bar{b} \to \bar{d}$ transitions then this method can be used to measure new physics(NP) phases.
1 Introduction

The $B$-factories BaBar and Belle have already reported a large number of measurements involving $B$ decays, and this will continue for a number of years. These measurements will test the SM picture of CP violation. Deviations from the SM predictions will indicate new physics. Hence, it becomes necessary to extract the SM phases cleanly and in many different decays. If deviation from the SM and evidence for new CP violating phases are detected one would like to measure the new CP phases also. In this talk we first present a method to measure SM phases which is clean and experimentally feasible. In the second part of the talk we discuss how our method can be used to measure NP phases if they are present.

The SM CP phase information is conveniently represented by the unitarity triangle, whose interior angles are known as $\alpha$, $\beta$ and $\gamma$. Many methods involving nonleptonic $B$ decays have been proposed for measuring the CP phases. In general, these techniques suffer from some degree of theoretical error due to the hadronic uncertainty in calculating nonleptonic $B$ decays. Some methods are quite clean, i.e. they have little theoretical uncertainty. An example is the extraction of $\beta$ in $B_0^0(t) \to J/\psi K_s$ where the corrections from unknown hadronic quantities are highly CKM suppressed. In general the extraction of CP angles require a theoretical input about hadronic quantities. To see this explicitly, consider processes where there is a $\bar{b} \to \bar{d}$ penguin amplitude. Some examples of such processes are $B_d \to \pi^+\pi^-, B_d \to K_0\bar{K}_0$ etc.

The general amplitude for the process $B_0 \to M_1 M_2$ involving, a $\bar{b} \to \bar{d}$ penguin contribution, can be written

$$A(B_0 \to M_1 M_2) = A_u V_{ub}^* V_{ud} + A_c V_{cb}^* V_{cd} + A_t V_{tb}^* V_{td}$$  \hspace{1cm} (1)

Using the CKM unitarity relation:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0,$$

one can eliminate the $V_{tb}^* V_{td}$ term to rewrite the amplitude as

$$A = (A_u - A_t) V_{ub}^* V_{ud} + (A_c - A_t) V_{cb}^* V_{cd}$$

$$\equiv A_{ut} e^{i\gamma} e^{i\delta_{ut}} + A_{ct} e^{i\delta_{ct}}$$  \hspace{1cm} (2)

The time-dependent measurement of $B_0(t) \to M_1 M_2$ allows one to obtain the three observables

$$B \equiv \frac{1}{2} \left( |A|^2 + |\bar{A}|^2 \right) = A_{ct}^2 + A_{ut}^2$$

\footnote{\textbf{$B_0^0(t) \to J/\psi K_s$ measures only $\sin 2\beta$. The measurement of $\cos 2\beta$ is possible in $B_d \to D^{(*)} D^{(*)} K$ decays \cite{3}.}}
\[ a_{\text{dir}} \equiv \frac{1}{2} \left( |A|^2 - |\bar{A}|^2 \right) = -2A_{ct} A_{ut} \sin \delta \sin \gamma \]

\[ a_t \equiv \text{Im} \left( e^{-2i\beta} A^* \bar{A} \right) \]

\[ \begin{align*}
&= A_{ct}^2 \sin 2\beta - 2A_{ct} A_{ut} \cos \delta \sin(2\beta + \gamma) \\
&- A_{ut}^2 \sin(2\beta + 2\gamma) 
\end{align*} \quad (3) \]

where \( \delta \equiv \delta^{ut} - \delta^{ct} \)

The three independent observables depend on five theoretical parameters: \( A_{ut}, A_{ct}, \delta, \beta, \gamma \). Therefore one cannot obtain CP phase information from these measurements. Hence theoretical input about hadronic quantities is necessary to get the CKM phase information.

One popular class of methods relies on flavor SU(3) symmetry \[5\] to provide the theoretical input to get CP phase information. To see how this works, consider the decays \( B_d \to \pi^+ \pi^- \) which has a \( \bar{b} \to \bar{d} \) penguin and \( B_s \to K^+ K^- \) which has a \( \bar{b} \to \bar{s} \) penguin\[6\].

The amplitudes for \( B_d \to \pi^+ \pi^- \) and \( B_s \to K^+ K^- \) can be written as

\[ A_{\pi\pi} = T^\pi e^{i\gamma} e^{i\delta_{ut}} |V_{ub}^* V_{ud}| + P^\pi |V_{cb}^* V_{cd}| e^{i\delta_{ct}} \]

\[ A_{KK} = T^K |V_{ub}^* V_{us}| e^{i\gamma} e^{i\delta'_{ut}} + P^K |V_{cb}^* V_{cs}| e^{i\delta'_{ct}} \quad (4) \]

In these equations there 8 unknowns which may be reduced to 7 by assuming \( \beta \) from \( B_d \to J/\psi K_s \) but there are only 6 measurements. The theoretical input in the form of the SU(3) relation

\[ \frac{P^K}{T^K} = \frac{P^\pi}{T^\pi} \]

allows us to extract \( \gamma \).

This method has several potential problems. First it requires many measurements which can lead to significant errors in the extraction of the CP angle. Moreover, we want to estimate or reduce SU(3) breaking which is around \( \sim 25 \% \) and can also seriously affect the clean measurement of angles of the unitarity triangle. Finally decays involving \( B_s \) mesons may be experimentally difficult because of large \( B_s \) mixing and more importantly \( B_s \) mesons are not available at present B factories.

## 2 New Method for measuring SM CP phase

We now describe the new method to obtain SM phase information which is described in details in Ref. \[7\] and builds on earlier works presented in Ref. \[8, 9\]. This method uses SU(3) and neglects amplitudes that are highly CKM suppressed. This reduces the number of measurements making it easier to obtain the CKM unitarity angles.
from an experimental point of view. With additional measurements one can reduce
SU(3) breaking errors allowing for more precise measurements of the angles of the
unitarity triangle. If annihilation type diagrams are neglected one can use many
decays that do not involve $B_s$ decays and hence would be accessible at present B
factories.

While, in principle the CKM suppressed amplitudes may become significant if
multiplied by highly enhanced strong amplitudes, this is unlikely from our experi-
ence of hadron dynamics. More importantly the assumption that the highly CKM
suppressed amplitudes may be neglected can be tested in experiments and present
experimental data appear to support this[7]. Furthermore, whether annihilation
type contributions are important can also be tested experimentally. In fact these
contributions are typically power suppressed $\sim \frac{\Lambda_{QCD}}{m_{b}}$ and are expected to be negli-
gible specially for vector-vector final states.

To see how this method works, consider the the time-dependent mea-
surement of $B^0(t) \to M_1 M_2$ involving $\bar{b} \to \bar{d}$ penguin. Experimentally we obtain the three
observables
\begin{align*}
B & \equiv \frac{1}{2} (|A|^2 + |\bar{A}|^2) = A_{ct}^2 + A_{ut}^2 \\
+ 2A_{ct} A_{ut} \cos \delta \cos \gamma \\
a_{dir} & \equiv \frac{1}{2} (|A|^2 - |\bar{A}|^2) = -2A_{ct} A_{ut} \sin \delta \sin \gamma \\
a_t & \equiv \text{Im} (e^{-2i\beta} A^* \bar{A}) \\
- A_{ct}^2 \sin 2\beta - 2A_{ct} A_{ut} \cos \delta \sin (2\beta + \gamma) \\
- A_{ut}^2 \sin (2\beta + 2\gamma) \quad (5)
\end{align*}

However, one can partially solve the equations to obtain
\begin{align*}
A_{ct}^2 & = \frac{a_R \cos (2\beta + 2\gamma) - a_t \sin (2\beta + 2\gamma) - B}{\cos 2\gamma - 1} \quad (6) \\
a_R^2 & = B^2 - a_{dir}^2 - a_t^2 \quad (7)
\end{align*}

Hence if $A_{ct}$ is known then we can find $\gamma$. We can obtain $A_{ct}$ from a partner process
related by SU(3).

Consider now a decay $B' \to M'_1 M'_2$ involving $\bar{b} \to \bar{s}$ penguin. We refer to this
as the “partner process.”

The general amplitude for $B' \to M'_1 M'_2$ can be written
\begin{align*}
A' & = A'_{u} V_{ub} V_{us} + A'_{c} V_{cb} V_{cs} + A'_{t} V_{tb} V_{ts} \\
& = (A'_u - A'_c) V_{ub} V_{us} + (A'_c - A'_t) V_{cb} V_{cs} \\
& \equiv A'_{ut} e^{i\phi_{ut}} + A'_{ct} e^{i\phi_{ct}} \quad (8)
\end{align*}
We now assume $A'_{ub} \ll A'_{ct}$ based on the fact that 
\[ |V_{ub}V_{us}/V_{cb}V_{cs}| \approx 2\%. \]

That is, the partner process is assumed to be dominated by a single amplitude.

\[ A' \approx A'_{ct} e^{i\delta_{ct}} \]  

Hence $A'_{ct}$ is obtained just from the rate of $B' \to M'_1 M'_2$ and no time dependent measurement is necessary thus greatly simplifying the experimental situation.

Now, in the SU(3) limit $A_{ct} = \lambda A'_{ct}$ where $\lambda = 0.22$ is the Cabibbo angle and so we can get $\gamma$ from

\[ A_{ct}^2 = \lambda^2 A'_{ct}^2 = \frac{a_R \cos(2\beta + 2\gamma) - a_I \sin(2\beta + 2\gamma) - B}{\cos 2\gamma - 1} \]  

There are many decays to which this method can be applied: the list of decays where the final state can be pseudoscalar-pseudoscalar (PP), pseudoscalar-vector (VP) and vector-vector (VV) are

1. $B^0_d \to D^+ D^-$ and $B^0_s \to D^+_s D^-_s$, $B^0_d \to D^+_s D^-$, or $B^+_u \to D^+_s \bar{D}^0$;
2. $B^0_d \to \pi^+ \pi^-$ and $B^0_s \to K^0 \bar{K}^0$ or $B^+_u \to K^0 \pi^+$;
3. $B^0_d \to K^0 \bar{K}^0$ and $B^0_s \to K^0 \bar{K}^0$ or $B^+_u \to K^0 \pi^+$;
4. $B^0_d \to \rho^0 \rho^0$ and $B^0_d \to K^* \rho^0$;
5. $B^0_s \to \bar{K}^* \rho^0$ and $B^0_d \to K^* \rho^0$;
6. $B^0_s \to \phi \bar{K}^* \rho^0$ and $B^0_d \to \phi K^* \rho^0$, $B^+_u \to \phi K^*$, or $B^0_s \to \phi \phi$.

The decays
1. $B^0_d \to D^+ D^-$ and $B^0_s \to D^+_s D^-_s$, or $B^+_u \to D^+_s \bar{D}^0$;
2. $B^0_d \to \pi^+ \pi^-$ and $B^+_u \to K^0 \pi^+$;

are particularly interesting as data on these processes are available and can be/is being used already.

Finally we can address the issue of dropping the highly suppressed CKM contributions. This dynamical assumption can be tested in a variety of ways. For instance if $B' \to M'_1 M'_2$ is not dominated by a single amplitude and there is a significant subleading piece then one would observe non zero Direct CP asymmetry or measure non zero T-violation asymmetry in VV modes \[10\].

One can use this method to considerably reduce SU(3) by considering two pairs of processes. For example, consider $B^0_d \to D^{+*} D^{-*}$ and $B^0_d \to D^+_s D^-_s$ \[9\] where
we can consider pairs with different helicity states (i) The theoretical input in this case is

\[ A_{cti} = \lambda A'_{cti} \]

\[ A_{cti} / A_{ctj} = A'_{cti} / A'_{ctj} \]

It can be shown that the leading SU(3) (in \(1/N_c\) approach) cancels in double ratio leaving a small residual SU(3) breaking.

3 Measuring New Physics CP phases

CP violation in the standard model is large and therefore one expects large angles of the unitarity triangle as well as significant direct CP violating effects in many decays [11]. There are many reason to believe that the Standard Model is not a complete theory as it leaves several puzzles unresolved, specially in the flavour sector. Since CP is not a symmetry of the SM there is no reason to believe that any new physics would be CP symmetric. One would therefore expect deviations from SM CP predictions, specially in rare decays where NP can compete with the SM contribution. Rare decays where the SM CP violation are tiny are very useful to look for new physics signals as any non zero CP violation would be a smoking gun signal for new physics. There are several hints of possible deviations from the SM [12, 13] and several interesting methods have been proposed to measure the parameters of the underlying new physics [13]. It appears so far that there may be significant new physics in \(\bar{b} \rightarrow \bar{s}\) transitions while the \(\bar{b} \rightarrow \bar{d}\) transitions are relatively unaffected by new physics.

To demonstrate how our method can be turned around to measure NP phases, we consider a specific model of NP. One model which has received much attention is Z-mediated (or \(Z'\)-mediated) flavour-changing neutral currents (FCNC’s) [14]. Here one introduces an additional vector-singlet charge \(-1/3\) quark \(h\) and allows it to mix with the ordinary down-type quarks \(d, s\) and \(b\). Since the weak isospin of the exotic quark is different from that of the ordinary quarks, FCNC’s involving the \(Z\) are induced. The \(Zb\bar{s}\) FCNC coupling, which leads to the \(\bar{b} \rightarrow \bar{s}\) transitions, is parametrized by the independent parameter \(U_{sb}^Z\):

\[
\mathcal{L}_{FCNC}^{Z} = -\frac{g}{2\cos^2\theta_W} U_{sb}^Z \bar{s}_L \gamma_\mu b_L Z_\mu .
\]

Note that it is only the mixing between the left-handed components of the ordinary and exotic quarks which is responsible for the FCNC. Furthermore, the \(Z\) decays to all \(q\bar{q}\) pairs for \(q = u, d, s, c\). These are effectively new contributions to the electroweak penguin operators of the SM.

The new-physics weak phase arises because \(U_{sb}^Z\) can be complex - \(U_{sb}\) can contain a CP violating phase \(\Phi\). However, because this parameter is universal, the weak phase of all NP operators will be the same. Note that \(U_{db}\) is constrained from \(B_d\) mixing so there is possible significant NP only in \(\bar{b} \rightarrow \bar{s}\) decays.
Now consider a $\bar{b} \to \bar{s}$ process, some examples of which are $B_s \to K^0\bar{K}^0$, $B_d \to K^{0*}\rho^0$ etc. The general amplitude for such a process including the NP contribution is

$$A' = A'_{ct} e^{i\beta_{ct}} + A_{NP} e^{i\Phi} e^{i\delta_{NP}} \quad (12)$$

Now as before there are three measurements: BR, $a_{direct}$ and $a_{indirect}$ and we assume $B_s$ mixing is measured in $B_s \to J/\psi\eta(\phi)$. There are four unknowns, $A'_{ct}$, $A_{NP}$, $\Phi$ and $\delta' = \delta'_{ct} - \delta_{NP}$, in Eq. (12). If we use $A'_{ct}$ as input we can solve for the rest including the NP phase $\Phi$.

To obtain $A'_{ct}$, consider the $\bar{b} \to \bar{d}$ partner process: $B_d \to K^0\bar{K}^0$, $B_d \to \rho^0\rho^0$ etc. The amplitude here is

$$A = A_{ut} e^{i\gamma_e} e^{i\delta_{ut}} + A_{ct} e^{i\beta_{ct}} \quad (13)$$

Now $\beta$ is obtained from $B_{d}^{0}(t) \to J/\psi K_{s}$ and $\gamma$ may be obtained from $B \to \pi\pi$ \cite{15} as it is a $\bar{b} \to \bar{d}$ transition and is not significantly affected by new physics according to our assumption. The three unknowns $A_{ct}$, $A_{ut}$ and $\delta = \delta_{ct} - \delta_{ut}$ can then be obtained from the 3 time independent measurements. One can use SU(3) symmetry to obtain $A'_{ct}$ in Eq. (12) from $A_{ct} = \lambda A'_{ct}$.

Note that this model of NP gives the same $\Phi$ from pairs $B_s \to K^0\bar{K}^0$ and $B_d \to K^0\bar{K}^0$ and $B_s \to K^{0*}\rho^0$ and $B_d \to \rho^0\rho^0$ and so on. Hence if this is not found to be the case this model of NP would be ruled out.

There are models of NP which contain more than one NP operator. The general treatment of this case is described in Ref. \cite{13}. As an example of NP model with more than one NP operator, consider supersymmetric models with broken R-parity. The L-violating couplings in this model are given by \cite{16}

$$\mathcal{L}_N = -\lambda'_{ijk} \left[ \bar{d}_L^i d_R^k d_L^j + \bar{d}_L^j d_R^i d_L^k + (\bar{d}_R^k)^* (\bar{d}_L^i)^c d_L^j \right. + \left. -\bar{e}_L^i \bar{e}_R^j w_L - \bar{u}_L^i \bar{e}_R^j e_L^j - (\bar{d}_R^k)^* (\bar{e}_L^j)^c u_L^j \right] + h.c. \quad (14)$$

From this Lagrangian, we see that there are R-parity-violating contributions to all $\bar{b} \to \bar{s}$ transitions \cite{17}. There is a single contribution to the decay $\bar{b} \to \bar{s}u\bar{u}$:

$$L_{eff} = -\frac{\lambda'_{12} \lambda_{13}}{2m_\tilde{u}} \bar{u}_\alpha \gamma_\mu \gamma_L u_\beta \bar{s}_\beta \gamma_\mu \gamma_R b_\alpha . \quad (15)$$

For $\bar{b} \to \bar{s} d\bar{d}$, there are four terms:

$$L_{eff} = \frac{\lambda_{11} \lambda_{23} \lambda_{13}}{m_\tilde{d}} \bar{d}_\gamma_L d_\beta \bar{s}_\gamma_R b + \frac{\lambda_{32} \lambda_{11} \lambda_{13}}{m_\tilde{u}} \bar{d}_\gamma_R d \bar{s}_\gamma_L b$$

$$- \frac{\lambda_{12} \lambda_{13}}{2m_\tilde{d}} \bar{d}_\alpha \gamma_\mu \gamma_L d_\beta \bar{s}_\beta \gamma_\mu \gamma_R b_\alpha - \frac{\lambda_{13} \lambda_{12} \lambda_{13}}{2m_\tilde{u}} \bar{d}_\alpha \gamma_\mu \gamma_L d_\beta \bar{s}_\beta \gamma_\mu \gamma_R b_\alpha . \quad (16)$$
Finally, the relevant Lagrangian for the $\bar{b} \to \bar{s} s \bar{s}$ transition is
\begin{equation}
L_{\text{eff}} = \frac{\lambda'_{i32} \lambda'_{i22}}{m_{\tilde{\nu}_i}^2} \bar{s} \gamma_R s \bar{s} \gamma_L \bar{b} + \frac{\lambda'_{i22} \lambda'_{i23}}{m_{\tilde{\nu}_i}^2} \bar{s} \gamma_L s \bar{s} \gamma_R \bar{b} .
\end{equation}

From the above expressions we can deduce the following predictions of $R$-parity-violating SUSY models. First, since there is only a single term contributing to $\bar{b} \to \bar{s} u \bar{u}$ transitions, the measured value of the new CP phase, $\Phi_{uu}$, should be independent of the decay pairs considered with the same underlying $\bar{b} \to \bar{s} u \bar{u}$ transition. On the other hand, since there is more than one contribution to both $\bar{b} \to \bar{s} d \bar{d}$ and $\bar{b} \to \bar{s} s \bar{s}$, the value of effective CP phases $\Phi_{dd}$ and $\Phi_{ss}$ will be process-dependent. Should this pattern of NP weak phases not be found experimentally, we can rule out this model of new physics.

4 Conclusions

In conclusion we have presented a new method based on SU(3) and a testable dynamical assumption to extract the angles of the unitarity triangle. This method can be applied to many pairs of decays, many of which do not involve $B_s$ decays and can be used in present B-factories. This method can be used to considerably reduce SU(3) breaking error and finally the method can be turned around to measure new physics parameters and rule out or constrain many models of new physics.

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