Unbiased Cosmological Parameter Estimation from Emission-line Surveys with Interlopers

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Received 2018 November 16; revised 2019 March 8; accepted 2019 March 22; published 2019 April 30

Abstract

The galaxy catalogs generated from low-resolution emission-line surveys often contain both foreground and background interlopers due to line misidentification, which can bias the cosmological parameter estimation. In this paper, we present a method for correcting the interloper bias by using the joint analysis of auto- and cross-power spectra of the main and the interloper samples. In particular, we can measure the interloper fractions from the cross-correlation between the interlopers and survey galaxies, because the true cross-correlation must be negligibly small. The estimated interloper fractions, in turn, remove the interloper bias in the cosmological parameter estimation. For example, in the Hobby–Eberly Telescope Dark Energy Experiment low-redshift ($z < 0.5$) [O II] $\lambda$3727Å emitters contaminate high-redshift ($1.9 < z < 3.5$) Ly$\alpha$ line emitters. We demonstrate that the joint-analysis method yields a high signal-to-noise ratio measurement of the interloper fractions while only marginally increasing the uncertainties in the cosmological parameters relative to the case without interlopers. We also show that the same is true for the high-latitude spectroscopic survey of the Wide-field Infrared Survey Telescope mission where contamination occurs between the Balmer-α line emitters at lower redshifts ($1.1 < z < 1.9$) and oxygen ([O III] $\lambda$5007Å) line emitters at higher redshifts ($1.7 < z < 2.8$).

Key words: galaxies: distances and redshifts – large-scale structure of universe

1. Introduction

Current and future spectroscopic surveys such as HETDEX (Hobby–Eberly Telescope Dark Energy Experiment; Hill et al. 2008), eBOSS (Extended Baryon Oscillation Spectroscopic Survey; Zhao et al. 2016), DESI (Dark Energy Spectroscopic Instrument; Levi et al. 2013), PFS (Prime Focus Spectroscopy; Takada et al. 2014), WFIRST (Wide-field Infrared Survey Telescope; Spergel et al. 2015), SPHEREx (Spectro-Photometer for the History of the Universe, Epoch of Reionization, and Ices Explorer; Doré et al. 2014), and Euclid (Amendola et al. 2013) are designed to map the large-scale structure of the universe by measuring the positions of millions of galaxies. The galaxy power spectrum, which is the Fourier transform of the galaxy two-point correlation function, is a leading statistical measure of the large-scale structure, which can constrain a number of cosmological parameters. For example, several groups have used the baryon acoustic oscillation (BAO; Cole et al. 2005; Eisenstein et al. 2005) feature as a standard ruler to measure the Hubble expansion rate $H(z)$ and angular diameter distance $d_A(z)$, while redshift-space distortion (RSD; Kaiser 1987) has been used to constrain the linear growth rate parameter $f(z)$. These measurements provide, respectively, the geometrical and dynamical test of dark energy (for a review, see Weinberg et al. 2013). The scale dependence of the galaxy power spectrum relative to the matter power spectrum on large scales also provides constraints on the non-Gaussianities in the initial density fluctuations (Dalal et al. 2008; Desjacques et al. 2018a) and is a unique approach to check the consistency for the general theory of relativity on cosmological scales (Yoo et al. 2009; Jeong et al. 2012; Jeong & Schmidt 2015).

Many of the spectroscopic surveys have a modest spectral resolution ($R \equiv \lambda/\Delta\lambda < 1000$) and limited bandwidth that often leave an ambiguity in emission-line identifications at specific redshifts. As a result, a fraction of the objects in galaxy catalogs constructed from these surveys are foreground or background interlopers. Recently, Pullen et al. (2016) investigated the effect of both foreground and background interlopers on the galaxy power spectrum. They show that the interlopers would induce systematic biases in the cosmological parameter estimation. Lidz & Taylor (2016) and Cheng et al. (2016) explored the possibility of cleaning the interloper effect in the intensity power spectrum from the spurious anisotropies induced by the interlopers.
In this paper, we demonstrate that we can eliminate such interloper bias by considering the statistics of both interlopers and main survey galaxies. By simultaneously analyzing the auto- and cross-power spectra of the main survey galaxies and the interlopers, we can estimate the interloper fraction and the cosmological parameters. The cosmological parameters measured in this joint-analysis method are unbiased, albeit with slightly increased measurement uncertainties.

Interlopers in the primary and secondary samples will cause a non-negligible angular cross-correlation that would otherwise be vanishingly small due to the two samples being widely separated in redshift. This is the case for HETDEX where the interlopers ([O II] λ3727 Å emitters, hereafter OIIEs) are at $z < 0.5$, while the main survey galaxies (Lyα λ1216 Å emitters, or LAEs) are at $1.9 < z < 3.5$. For a program such as the high-latitude spectroscopic survey WFIRST we develop is applicable for any survey if we use HETDEX as our main case study, but the formalism provided will show that our method is robust to a change in this value.

We focus here on galaxy surveys with a small footprint such as HETDEX and WFIRST, for which we can apply the Fourier analysis assuming the flat-sky approximation. For simplicity, we ignore the redshift evolution of the galaxy number density, interloper fraction, as well as the linear growth rate. In our investigation, we mimic the angular cross-correlation by projecting one population (OIIEs, for example) onto the redshift of the other (LAEs). Throughout the paper, we use HETDEX as our main case study, but the formalism we develop is applicable for any survey afflicted with interlopers. As an example, we apply the same formalism to the WFIRST mission.

We assume a flat ΛCDM model for our fiducial cosmology with parameters in the base_plikHM_TTTEEE_lowTEB_ lensing_post_BAO_H080p6_JLA column from Planck 2015 (Planck Collaboration 2016a, 2016b): $\Omega_m = 0.2919$, $\Omega_{\text{cdm}} h^2 = 0.022307$, $\Omega_{\text{b}} h^2 = 0.11865$, $\Omega_{\text{cdm}} h^2 = 0.000638$, $h = 0.6778$, and $n_s = 0.9672$. We calculate the linear power spectrum $P_{\text{L}}(k)$ with CAMB10 and normalize the linear power spectrum by setting the root-mean-squared value of the smoothed (spherical filter with radius $8 h^{-1} \text{Mpc}$) linear density contrast, $\sigma_8 = 0.8166$

We begin in Section 2 by discussing preliminaries, providing details for the HETDEX survey, giving a precise definition of interloper fraction, and discussing the projection effects of misidentification. In Section 3, we present the effect of interlopers on the density contrast, the configuration-space correlation functions, and the galaxy power spectrum measurement, including galaxy bias and redshift-space distortion. We construct the likelihood function and apply our method to HETDEX in Section 4 and to WFIRST in Section 5. We conclude in Section 6. Appendix A discusses the transformation between misidentification and interloper fractions. Appendix B provides a rigorous derivation of the observed galaxy power spectra including the discrete nature of the galaxy density field, and Appendix C derives the measurement uncertainty on the power spectrum. Finally, in Appendix D, we present a formula estimating the systematic bias in cosmological parameters from a systematic shift of the power spectrum.

2. Preliminaries

2.1. HETDEX

HETDEX is a blind, integral-field spectroscopic survey observing $a 434 \text{deg}^2$ footprint (294 deg$^2$ around 53° decl. and 140 deg$^2$ around 0° decl.) with a filling factor of 1/4.5 on sky, over the wavelength range from 3500 to 5500 Å. The primary target population for HETDEX are high-redshift ($1.9 < z < 3.5$) galaxies emitting the Lyα line at rest-frame 1216 Å. With the fiducial cosmological parameters, the total survey volume for LAEs is $V_{\text{survey}} = 2.95 h^{-3} \text{Gpc}^3$, centered around $z = 2.7$, which corresponds to the fundamental frequency of $k_f = 0.00438 h^{-1} \text{Mpc}^{-1}$.

The same wavelength range also detects star-forming galaxies at low redshift ($0 < z < 0.5$) emitting [O II] λ3727 Å. If $z_{\text{O I I E}}$ is the redshift of an OIIE and $z_{\text{LAE}}$ is the redshift of a corresponding LAE, then the observed wavelength of the line is

$$\lambda_{\text{obs}} = \lambda_{\text{O II}}(1 + z_{\text{O II}}) = \lambda_{\alpha}(1 + z_{\text{LAE}}),$$

where $\lambda_{\text{O II}}$ and $\lambda_{\alpha}$ are the rest-frame wavelengths of [O II] λ3727 Å and the Lyα line at 1216 Å, respectively. For the same HETDEX footprint, OIIEs occupy the volume $V_{\text{O II}} = 0.0688 h^{-3} \text{Gpc}^3$.

Based on the observed luminosity function of high-redshift LAEs (Ciardullo et al. 2012; Sobral et al. 2018), we expect HETDEX to observe ~755,000 LAEs and ~1,500,000 OIIEs (also see Comparat et al. 2015), where we assume a flux limit of $5 \times 10^{-17} \text{erg s}^{-1} \text{cm}^{-2}$. We set the linear galaxy bias for LAEs to $b_{\text{LAE}} = 2$, which is consistent with Guaita et al. (2010). For OIIEs, we use the linear bias $b_{\text{O II}} = 1.5$, and we will show that our method is robust to a change in this value. For the Fourier analysis, we include the Fourier modes below the maximum wavenumber $k_{\text{max}} = 0.4 h^{-1} \text{Mpc}^{-1}$ (Jeong & Komatsu 2006), but we also check that the result stays robust for $k_{\text{max}} = 0.3 h^{-1} \text{Mpc}^{-1}$, which is adopted for the planning and design of HETDEX (Hill et al. 2008). For our fiducial cosmological parameters, we find $n_s P_s(k) > 1$ for $k < 0.1 h^{-1} \text{Mpc}^{-1}$.

Confusion arises when the line identification is ambiguous. For the majority of objects detected by HETDEX, [O III] λ5007 Å and Hβ fall outside the spectral range. In addition, although [O II] λ3727 Å is a doublet, the resolution ($R \sim 700$) of the HETDEX spectrographs is too low to resolve it (Hill et al. 2016). Leung et al. (2017) investigated a Bayesian approach to distinguish between OIIEs and LAEs, making use of a number of factors, including the presence of other lines in the spectrum and the rest-frame equivalent width of the candidate Lyα line, which tends to be greater than 20 Å for LAEs and less than that for OIIEs (Gronwall et al. 2007; Ciardullo et al. 2013). Leung et al. (2017) used this method to reduce the interloper fraction to ~0.5% at the expense of missing ~6% of the LAEs. For this interloper fraction, we predict that the joint-analysis method introduced in this paper can measure the interloper fraction with high significance (see, for example, Figure 9).
2.2. Notation

Throughout the paper, we shall use the following notation. First, we denote the fraction of misidentified LAEs and OIIEs by, respectively, $\alpha_{\text{LAE}}$ and $\beta_{\text{OIIE}}$. That is, if there are $N_{\text{LAE}}$ LAEs and $N_{\text{OIIE}}$ OIIEs in the survey volume, the observed number of LAEs ($N_{\text{LAE}}^\text{obs}$) and OIIEs ($N_{\text{OIIE}}^\text{obs}$) are, respectively,

$$N_{\text{LAE}}^\text{obs} = (1 - \alpha_{\text{LAE}})N_{\text{LAE}} + x_{\text{OII}}N_{\text{OII}}.$$  \hspace{0.7cm} (2)

$$N_{\text{OII}}^\text{obs} = x_{\text{LAE}}N_{\text{LAE}} + (1 - x_{\text{OII}})N_{\text{OII}}.$$  \hspace{0.7cm} (3)

Here, we use the superscript “obs” to denote the observed quantities in contrast to their true value. We further define the overall interloper fractions in the observed sample as

$$f \equiv \frac{x_{\text{OII}}N_{\text{OII}}}{N_{\text{LAE}}^\text{obs}} = \frac{x_{\text{OII}}N_{\text{OII}}}{(1 - \alpha_{\text{LAE}})N_{\text{LAE}} + x_{\text{OII}}N_{\text{OII}}},$$  \hspace{0.7cm} (4)

$$g \equiv \frac{x_{\text{LAE}}N_{\text{LAE}}}{N_{\text{OII}}^\text{obs}} = \frac{x_{\text{LAE}}N_{\text{LAE}}}{x_{\text{LAE}}N_{\text{LAE}} + (1 - x_{\text{OII}})N_{\text{OII}}},$$  \hspace{0.7cm} (5)

which will simplify the expressions for the observed density contrast.

For sources in the galaxy survey catalog, the most direct observables are the angular coordinate and the redshift $z$. Misidentifying OIIEs and LAEs will alter the estimated redshift and place the lower redshift objects (at $z_{\text{OII}}$) farther away, at the corresponding LAE redshift $z_{\text{LAE}}$ shown in Equation (1). As a result (see Figure 1), the misidentification stretches the tangential coordinate and the radial coordinate of the lower redshift galaxies, respectively, by the factors of $\alpha_{\text{LAE}}$ and $\beta_{\text{OIIE}}$.

$\Delta$ The angular separation $\Delta \theta$ and redshift difference $\Delta z$ are related to the comoving distances as

$$\Delta x = d_A(z) \Delta \theta, \quad \Delta z = \frac{\Delta z}{H(z)},$$  \hspace{0.7cm} (6)

Therefore, when the redshifts of the galaxies are misidentified, the tangential and parallel separations change with the scaling factors $\alpha$ and $\beta$.

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**Figure 1.** Illustration of the geometry of misidentification. The OIIEs (solid galaxy symbol) at lower redshifts are projected to higher redshifts (dashed galaxy symbol); they occupy a larger volume at a larger radius. In the figure, the observer is located at the bottom vertex. The true separations along the tangential direction ($s_\perp$) and radial direction ($s_\parallel$) are projected, respectively, to $\alpha s_\perp$ and $\beta s_\parallel$ when the OIIEs are misidentified as LAEs. The scaling factors $\alpha$ and $\beta$ are defined in terms of the geometrical quantities in Equation (7).

**Figure 2.** Scaling factors $\alpha$ (transverse) and $\beta$ (radial) as functions of redshifts ($z_{\text{LAE}}$ on the lower axis and $z_{\text{OII}}$ on the upper axis) for the fiducial $\Lambda$CDM model. The gray horizontal line marks no rescaling ($\alpha = \beta = 1$), while the gray vertical line is the limit below which no interloping OIIEs exist.

We shall refer to these variables as scaling factors, where $d_A(z)$ is the comoving angular diameter distance, and $H(z)$ is the Hubble expansion rate. Figure 2 displays the redshift dependence of the scaling factors $\alpha$ and $\beta$ as a function of $z_{\text{OII}}$ (upper abscissa) and $z_{\text{LAE}}$ (lower abscissa) for the fiducial cosmology. At all redshifts of interest, the change of coordinate is more significant in the tangential direction ($\alpha$) than in the radial direction ($\beta$). For example, for LAEs at redshift $z_{\text{LAE}} \sim 2.7$ and OIIEs at redshift $z_{\text{OII}} \sim 0.2$, $\alpha \sim 7.1$, due to the change in angular diameter distance, whereas $\beta \sim 0.84$, because the change in $H(z)$ is largely compensated by the ratio of the wavelengths in Equation (7). The disparity in $\alpha$ and $\beta$ introduces yet another source of anisotropy in the observed galaxy clustering.

### 3. Observed Correlation Functions with Interlopers

The presence of low-redshift interlopers in the high-redshift galaxy sample biases the clustering measurement. Similarly, the observed density contrast has extra contributions produced from the density contrast of the interlopers. In this section, we determine the effect interloping OIIEs have on the observed density contrast and two-point correlation functions of LAEs. First, we derive the effect on the observed density contrast and the two-point correlation function in configuration space, then Fourier-transform these quantities to derive the expression for the power spectrum.

Strictly speaking, one must fix the observed angular position and rescale only the radial position for each interloper. In this paper, however, we focus on the three-dimensional Fourier analysis by ignoring the opening-angle effect and by applying the scaling factors at the median redshift ($z_{\text{LAE}} = 2.7$ and $z_{\text{OII}} = 0.2$) to all interloping galaxies. This approach projects the interloping OIIEs from the true lower redshift cuboid volume to the high-redshift cuboid volume of LAEs. This approximation provides a good description for galaxy surveys with small sky coverage and a narrow range of redshifts, with the correction only proportional to the square of the opening angle, and the redshift bin size. It can be undoubtedly applied to galaxy surveys such as HETDEX ($\approx 400$ deg$^2$) and WFIRST.
the position vector for the galaxies in the observed LAE sample and \( s' \) the true position vector of the OIIE interlopers; i.e., \( s' \) refers to the same angular position on the sky as \( s \), but with redshift \( z_{\text{OII}} \) instead of \( z_{\text{LAE}} \). Using the scaling factors in Equation (7), \( s \) and \( s' \) are related by

\[
\mathbf{s} = (\alpha s'_x, \beta s'_y),
\]

where \( \perp \) and \( || \) represent the components that are tangential and radial to the line of sight, respectively.

Note that Equation (8) also holds for interlopers in redshift space, where the line-of-sight directional peculiar velocity \( v \perp \) shifts the observed redshift \( z_{\text{obs}} \) away from the true redshift \( z \) by \((1 + z_{\text{obs}}) = (1 + z)(1 + v \perp /c)\). As a result, the radial distance \( r_x \) in redshift space is shifted relative to the real radial distance \( r \) by \( s = r + v \perp /aH \). The same scaling factor \( \beta \), therefore, applies to both \( r \) and \( v \perp /aH \), when the observed redshift is misidentified.

With these position vectors and the variables defined in Equations (2)–(5), we write the observed density contrast \( \delta_{\text{LAE}}^\text{obs} \) as a function of the true density contrast of LAEs \( \delta_{\text{LAE}}(s) \) and that of the OIIEs \( \delta_{\text{OII}}(s') \) as:

\[
\delta_{\text{LAE}}^\text{obs}(s) = \frac{\delta_{\text{LAE}}(s)}{n_{\text{LAE}}} - 1 = (1 - f)\delta_{\text{LAE}}(s) + f\delta_{\text{OII}}(s'),
\]

where mean number densities are defined in the LAE volume as \( n_{\text{LAE}} \equiv N_{\text{LAE}}/V_{\text{survey}} \) and \( n_{\text{OII}} \equiv N_{\text{OII}}/V_{\text{survey}} \), where \( V_{\text{survey}} \) is the LAE volume. Analogously, the observed OII density contrast is given by

\[
\delta_{\text{OII}}^\text{obs}(s') = g\delta_{\text{LAE}}(s) + (1 - g)\delta_{\text{OII}}(s').
\]

The observed density contrast is a superposition of the true LAE density contrast and the true OII density contrast, each contributing proportionally by number of galaxies in the sample.

Our analysis assumes that the true mean number densities \( (\bar{n}_{\text{LAE}}, \bar{n}_{\text{OII}}) \), and the misidentification fractions \( x_{\text{LAE}} \) and \( x_{\text{OII}} \), remain constant over the survey volume \( V_{\text{survey}} \). For realistic galaxy surveys, both the mean densities and the overall misidentification fractions may vary across the survey volume, which results in a nontrivial window function. We do not study the ramifications here, because the window function effect can be, in principle, modeled very accurately up to our knowledge of the survey conditions. For more discussion, see, for example, chapter 7 of Jeong (2010) and Chiang et al. (2013).

One subtle but important point is that we define the overall misidentification fractions \( x_{\text{LAE}} \) and \( x_{\text{OII}} \) as well as \( f \) and \( g \), in terms of the underlying, continuous galaxy number density fields \( N_{\text{LAE}} \) and \( N_{\text{OII}} \). In reality, the observed galaxy density fields are the distribution of discrete points (galaxies) that reflect these underlying continuous fields; therefore, locally measured values of \( x_{\text{LAE}} \) and \( x_{\text{OII}} \) are not necessarily the same across the survey volume. For instance, in an infinitesimal volume element we only expect a single LAE. Then, the misidentification fraction can be either unity (if the galaxy is an OIIE) or zero (if the galaxy is a genuine LAE). For the local quantities, therefore, the misidentification fractions \( x_{\text{LAE}} \) and \( x_{\text{OII}} \) are only statistical measures of the probability of misidentification.

Of course, the density contrasts given in Equations (9)–(10) must lead to the correct result for the galaxy two-point correlation function and power spectrum. The subtle difference appears in the treatment of shot noise. For example, for galaxies that are drawn randomly from a given continuous density field, the shot noise is proportional to the reciprocal of the total number density of galaxies, including the interlopers (shown in Appendix B).

### 3.2. Observed Two-point Correlation Function with Interlopers

The derivation in this section extends the Appendix A of Leung et al. (2017), including the two-point auto-correlation functions of both the main sample and the interloper sample. We calculate the observed two-point correlation function \( \xi(s) = \langle \delta(r) \delta(r+s) \rangle \) in the configuration space from Equations (9)–(10) as

\[
\xi^\text{obs}(s) = \langle \xi^\text{obs}_{\text{LAE}}(r) \delta_{\text{LAE}}^\text{obs}(r+s) \rangle \\
= (1 - f)^2 \xi_{\text{LAE}}(s) + f^2 \xi_{\text{OII}}^\text{obs}(s') + 2(1 - f) f \xi_{\text{LAE},\text{OII}}^\text{obs}(s) \tag{11}
\]

and

\[
\xi_{\text{OII}}^\text{obs}(s') = \langle \xi_{\text{OII}}^\text{obs}(r) \delta_{\text{OII}}^\text{obs}(r+s') \rangle = (1 - g)^2 \xi_{\text{OII}}(s') + g^2 \xi_{\text{LAE}}^\text{proj}(s') + 2(1 - g) g \xi_{\text{LAE},\text{OII}}^\text{proj}(s'). \tag{12}
\]

Here,

\[
\xi_{\text{OII}}^\text{proj}(s') = \xi_{\text{OII}}(s') = \xi_{\text{OII}}(\alpha^{-1}s_x, \beta^{-1}s_y),
\]

are the OII and LAE two-point correlation functions projected to the wrongly assigned redshifts; therefore, they contaminate the two-point correlation function of the respective sample. Note that the projection merely relabels the coordinates (thus shifting the separation vector) while keeping intact the amplitude of the two-point correlation function.

The other terms in Equations (11) and (12), \( \xi_{\text{LAE},\text{OII}}^\text{obs} \), denote the cross-correlation between the LAEs and projected OIIEs (evaluated at \( s \)) and between the OIIEs and projected LAEs (evaluated at \( s' \)). They are much smaller than the respective auto-correlation functions because the wide radial separation between LAEs and OIIEs suppresses the true cross-correlation, and the cross-correlation from lensing is small. We can therefore ignore this contribution.

Our final expressions for the observed galaxy two-point correlation functions of LAEs and OIIEs are

\[
\xi^\text{obs}_{\text{LAE}}(s) = (1 - f)^2 \xi_{\text{LAE}}(s) + f^2 \xi_{\text{OII}}(\alpha^{-1}s_x, \beta^{-1}s_y),
\]

\[
\xi^\text{obs}_{\text{OII}}(s') = (1 - g)^2 \xi_{\text{OII}}(s') + g^2 \xi_{\text{LAE}}(\alpha^{-1}s'_x, \beta^{-1}s'_y). \tag{16}
\]

Equations (15)–(16) show that the different scaling factors \( (\alpha = 1/\beta) \) introduce anisotropies into the two-point correlation functions. This is true even when \( \xi(s) \) only depends on \( s \)—for example, without the RSD.

### 3.3. Observed Power Spectrum with Interlopers

We initially calculate the observed galaxy power spectrum by the Fourier transform of the corresponding two-point

\[
\xi^\text{obs}_{\text{LAE}}(k) = (1 - f)^2 \xi_{\text{LAE}}(k) + f^2 \xi_{\text{OII}}^\text{obs}(k'),
\]

\[
\xi^\text{obs}_{\text{OII}}(k') = (1 - g)^2 \xi_{\text{OII}}(k') + g^2 \xi_{\text{LAE}}^\text{proj}(k').
\]
correlation function. The results of this section are consistent with those of Pullen et al. (2016) and Leung et al. (2017).

The Fourier transform integrates over the respective observed coordinates—s for LAEs and s′ for OIIEs—whose volume forms are related by the scaling parameters (Equation (8))

\[ d^3s = d^2sL \, ds_L = \alpha^2 \beta \, d^3s'. \]  

Using Equation (15), we compute the observed LAE power spectrum as

\[ P_{\text{LAE}}^{\text{obs}}(k) = (1 - f)P_{\text{LAE}}(k) + f^2P_{\text{OII}}^{\text{proj}}(k), \]  

where \( P_{\text{OII}}^{\text{proj}}(k) \) is the power spectrum of OIIEs (at s′) projected onto the LAE coordinates (s), or the Fourier transform of Equation (13):

\[ P_{\text{OII}}^{\text{proj}}(k) \equiv \int d^3s' e^{ik \cdot \xi_{\text{LAE}}(s', s''')} = \alpha^2 \beta \int d^3s' e^{i(k - \beta \xi_{\text{LAE}}(s', s'''))} \xi_{\text{LAE}}(s', s''') = \alpha^2 \beta P_{\text{OII}}(\alpha k_L, \beta k_L). \]  

Similarly, the power spectrum of the observed OIIEs is

\[ P_{\text{OII}}^{\text{obs}}(k) = (1 - g)^2P_{\text{OII}}(k) + \frac{g^2}{\alpha^2 \beta}P_{\text{LAE}}(\frac{k_L}{\alpha}, \frac{k_L}{\beta}). \]  

Equations (19) and (20) demonstrate how misinterpreting the emission-line redshifts leads to two effects. First, just as in the case for the two-point correlation functions, the misinterpretation shifts the scales, projecting small (large) scales onto larger (smaller) scales when OIIEs (LAEs) are misinterpreted as LAEs (OIIEs). This effect is illustrated in Figure 1. Again, \( \alpha \approx \beta \) introduces an additional anisotropy into the observed power spectrum beyond RSD. Second, the amplitude of the power spectrum is changed proportionally to the ratio between the true volume and the projected volume. For example, when OIIEs in a small, low-z volume are projected into the larger, high-z volume, the projected power spectrum amplitude is boosted by a factor of \( \alpha^2 \beta^2 \).

3.4. The Observed Cross-correlation Functions

As discussed earlier, we ignore the true cross-correlation between the LAEs and OIIEs. Misidentification can, however, induce a cross-correlation between the OIIEs and LAEs because both observed samples contain high-z and low-z objects. Strictly speaking, such a cross-correlation must be measured in the angular cross-correlation function or in the angular cross-power spectrum. As we are adopting the flat-sky approximation throughout this paper, we mimic the angular cross-correlation by the three-dimensional cross-correlation between \( \xi_{\text{LAE}}^{\text{obs}}(s) \) and \( \xi_{\text{OII}}^{\text{obs}}(\alpha k_L, \beta k_L) \) artificially placed at the corresponding LAE redshifts through Equation (1). This procedure resembles the angular cross-correlation because for a given LAE redshift \( z_{\text{LAE}} \), the OIIE redshift \( z_{\text{OII}} \) is uniquely determined. Of course, one can also choose to correlate \( \xi_{\text{OII}}^{\text{obs}} \) with \( \xi_{\text{LAE}}^{\text{obs}} \) projected to the OIIE redshifts.

The cross-correlation function defined here is

\[ \xi_{\text{LAE}, \text{OII}}^{\text{obs}}(s) = \langle \xi_{\text{LAE}}^{\text{obs}}(r) \xi_{\text{OII}}^{\text{obs}}(r + s) \rangle = (1 - f)g \xi_{\text{LAE}}^{\text{obs}}(s) + f(1 - g)\xi_{\text{OII}}^{\text{proj}}(s), \]  

with \( \xi_{\text{OII}}^{\text{proj}}(s) \) given in Equation (13), and the corresponding cross-power spectrum

\[ P_{\text{LAE}, \text{OII}}^{\text{obs}}(k) = (1 - f)gP_{\text{LAE}}(k) + f(1 - g)\alpha^2 \beta P_{\text{OII}}(\alpha k_L, \beta k_L). \]  

The nonzero cross-correlation in Equation (22) is the key for measuring the interloper fractions \( f \) and \( g \). This property is an effective indicator because we expect vanishingly small (contributions from the true clustering and the lensing magnification) cross-correlation for perfect \( f = g = 0 \) LAE and OIIE samples.

Similarly, we define the observed cross-correlation coefficient using the OIIE power spectrum projected into the LAE volume as

\[ r \equiv \frac{P_{\text{LAE}, \text{OII}}^{\text{obs}}(k)}{\sqrt{P_{\text{LAE}}^{\text{obs}}(k) P_{\text{OII}}^{\text{obs}}(k)}}. \]  

The value of \( r \) varies continuously between 0 and 1 as a function of the interloper fractions \( f \) and \( g \). The cross-correlation coefficient \( r \) reaches maximum \((r = 1)\) when \( f + g = 1 \), and minimum \((r = 0)\) for the completely uncontaminated case \((f = g = 0)\) and the completely confused case \((f = g = 1)\).

3.5. Modeling the Redshift-space Galaxy Power Spectrum

The expressions for the observed auto- and cross-power spectra of LAEs and OIIEs in terms of their true redshift-space power spectra are given in Equations (18), (20), and (22). Thus, we can now complete the calculation with expressions for the true redshift-space power spectra of LAEs and OIIEs.

As a baseline model for the true redshift-space galaxy power spectrum, we adopt the linear bias model \((\xi_b(x) = b_\delta \sigma^2(x), \) where \( \delta_b(x) \) is the galaxy number density contrast, \( b_\delta \) the linear galaxy bias parameter, and \( \sigma^2(x) \) the matter density contrast) with linear RSD (Kaiser 1987) augmented by the Lorentzian Finger-of-God (FoG) damping (Jackson 1972):

\[ P_b(k) = \frac{1 + \beta \mu^2}{1 + f^2(z_a)k^2/\mu^2} b_\delta^2 D^2(z_a)P_{\text{Lin}}(k). \]  

Here, \( D(z) \) is the linear growth factor, \( f(z) \) the linear growth rate \((f(z) \equiv d \ln D/d \ln a), \) and \( \beta_\mu \equiv f(z_a)/\mu, \) where we use the subscript \( x = L \) for LAEs and \( x = O \) for OIIEs.\(^{12}\) We define \( \mu = \mathbf{k} \cdot \mathbf{n} = k_L/\hat{k} \) as the cosine of the angle between the wave vector and the line-of-sight direction. We model the FoG effect (Jackson 1972) with a Lorentzian damping term via the one-dimensional velocity dispersion

\[ \sigma_{v,x}^2 = \frac{P}{3} D^2(z_a) \int \frac{d^3k}{(2\pi)^3} \frac{P_{\text{Lin}}(k)}{k^2}, \]  

with a fudge parameter \( p. \) We adopt \( p = 0.4, \) which Jeong (2010) measured from the two-dimensional redshift-space power spectrum of a suite of \( N \)-body simulations.

The left panels in Figure 3 show the two-dimensional power spectra of LAEs (top) and OIIEs (bottom) in the \( k_L \) plane, with interloper fractions of \( f = g = 0.1 \). Also displayed are the corresponding contamination from projected OIIEs (\( P_{\text{OII}}^{\text{proj}}(k) \)) and LAEs (\( P_{\text{LAE}}^{\text{proj}}(k) \)) in the right panels. Here, we use the linear bias of \( b_{\text{LAE}} = 2 \) and \( b_{\text{OII}} = 1.5 \) and assume that all LAEs are

\(^{12}\) We do not distinguish between subscripts “LAE” and “O,” and subscripts “OII” and “O.” We prefer the former over the latter purely based on convenience.
at \( z = 2.7 \) while all OIIEs are at \( z = 0.2 \), which yields the scaling factors \( \alpha = 7.1 \) and \( \beta = 0.84 \). These anisotropic scaling factors squeeze or stretch the power spectrum shapes. The effect is much larger along the perpendicular direction, due to \( \alpha \gg \beta \simeq 1 \). To facilitate the comparison between the anisotropies from contamination and the total anisotropies in the observed power spectrum, we present the contribution from interlopers. For both LAEs and OIIEs, the small interloper fractions (\( f \) and \( g \)) suppress the contribution from interlopers. The overall contribution, however, is much larger for the projected OIIEs (contaminating LAEs) because of the volume factor \( \alpha^2 \beta \simeq 40 \); when projecting the high-redshift objects (such as LAEs) onto lower redshift (to \( z_{\text{OII}} \)), the power spectrum amplitude is suppressed by the factor \( \alpha^{-2} \beta^{-1} \).

The angle-averaged, monopole power spectra are displayed in the top left panel of Figure 4. These spectra are shown without (black) and with (dashed orange) \( f = g = 10\% \) interloper fractions. For the contaminated spectra, we divide by a factor of \((1-f)^2\) for LAEs and \((1-g)^2\) for OIIEs to better compare the shape of the power spectra. The dotted line represents the observed cross-correlation, and the dashed (dashed–dotted) gray horizontal lines indicate the shot noise for the LAEs (OIIEs). Note that the observed cross-correlation can exceed the LAE power spectrum.

To better demonstrate the effect of interlopers, the relative change of the monopole power spectrum is presented in the two top-right panels of Figure 4; the very top panel for LAEs, the bottom for OIIEs. The gray areas indicate the fiducial error bars on the power spectrum that we discuss in Section 3.6. The colored dashed lines are the change for several contamination fractions as indicated in the legends. The most notable feature is that the LAE power spectrum is affected more strongly by interlopers than the OIIE power spectrum.

To investigate the angular dependence of the effect of interlopers, we present the quadrupole power spectrum in the bottom left panel of Figure 4: the true quadrupoles (black

---

**Figure 3.** Left: the observed two-dimensional power spectra of LAEs (top; assuming \( b_{\text{LAE}} = 2 \) at \( z_{\text{LAE}} = 2.7 \)) and OIIEs (bottom; assuming \( b_{\text{OII}} = 1.5 \) at \( z_{\text{OII}} = 0.207 \)) including RSD, FoG, and the contributions from interlopers. We assume the interloper fraction of \( f = g = 0.1 \). Right: contaminant power spectra from projected OIIEs (top) and LAEs (bottom) as they contribute to the observed power spectra in the left panels, i.e., the right panels contain an additional anisotropic feature, due to interlopers. The scaling factors are \( \alpha = 7.1 \) and \( \beta = 0.84 \).
lines), with 10% interloper fractions (orange dashed for LAE, orange dashed–dotted for OIIE), and the cross-correlation quadrupole (orange dotted). The graph reveals that, as for the monopole, the LAE power spectrum is more strongly affected by interlopers than the OIIE power spectrum.

The two bottom right panels of Figure 4 display the relative change of the power spectra for the quadrupole in the same manner as for the monopole. Again, the LAE quadrupole power spectrum is more strongly affected by contamination than the OIIE quadrupole.

Finally, the left panel of Figure 5 shows the expected cross-correlation for $f = g = 0.1$. The figure demonstrates that for a survey such as HETDEX, where the LAEs are at $z \sim 2.7$ and the OIIEs at $z \sim 0.2$, the contribution from the projected OIIE power spectrum is expected to dominate the observed cross-correlation.

3.5.1. On Using the Linear Model of Equation (24)

Strictly speaking, the linear theory prescription for the galaxy power spectrum (Equation (24)) breaks down on small scales, and we must include the nonlinear terms in our analysis. Indeed, for the HETDEX survey under consideration here, high-redshift ($1.9 < z < 3.5$) LAEs probe the quasilinear scales $k \lesssim k_{\text{max}} = 0.4 h \text{ Mpc}^{-1}$, and the corresponding lower redshift OIIEs probe the fully nonlinear scales to $k_{\text{max}} \approx 2 h \text{ Mpc}^{-1}$ (the volume is smaller by a factor of $(\alpha'' \beta)'^{-1}$; see Figure 2). For the former, the complete next-to-leading order expression is given in Desjacques et al. (2018b), which includes nonlinearities in matter clustering, galaxy bias, and RSD. For the fully nonlinear regime, however, no such expression is known, and we may have to rely on cosmological simulations (Springel et al. 2018).
We therefore focus on analyzing the clustering of high-redshift LAEs rather than low-redshift OIIEs. We further assume that the interlopers do not dominate the high-redshift samples; this situation may be achieved by applying astrophysically motivated classifications of emission lines (Pullen et al. 2016; Leung et al. 2017).

By focusing on the cosmological analysis from the LAE sample only, the model in Equation (24) suffices for the study of the effect of interlopers on cosmological parameter estimation. As shown in Figures 3 and 4, the misidentified low-\(z\) interlopers (a) induce an anisotropy into the power spectrum and (b) increase large-scale power. The interlopers, therefore, affect the cosmological parameters measured from large scales \(k < k_{\text{eq}}\), where \(k_{\text{eq}}\) is the wavenumber corresponding to the matter–radiation equality (e.g., the local primordial non-Gaussianity parameter \(f_{\text{NL}}\)), and from the anisotropies due to RSD (e.g., the angular diameter distance \(d_A(z)\), the Hubble expansion rate \(H(z)\), and the linear growth rate \(f(z)\)). For the former, the linear theory applies because \(k < k_{\text{eq}}\) corresponds to sufficiently large scales. For the latter, the parameter estimation is relatively insensitive to nonlinearities such as the constraint on geometrical quantities from the BAO feature and the Alcock–Paczynski (AP) test (Alcock & Paczynski 1979; Shoji et al. 2009). We explicitly check the effect of nonlinear RSD in Section 4.7 by marginalizing the 2D power spectrum over higher powers in \(\mu\).

### 3.6. Variance of the Power Spectrum Measurement

Ignoring the connected trispectrum, we use only the Gaussian part of the covariance matrix where the power spectra at different wave vectors are statistically independent. We then calculate the variance (diagonal part of the covariance matrix) of the power spectrum as

\[
(\Delta P(k))^2 = \frac{2}{N_k} \left( P(k) + \frac{1}{\bar{n}_g} \right)^2
\]

(see Appendix C for the derivation). Here, \(N_k\) is the number of Fourier modes

\[
N_k = \frac{V_{\text{survey}}}{(2\pi)^3} \Delta V_k,
\]

with \(\Delta V_k\) being the volume in Fourier space contributing to the estimation of the power spectrum. For example, when computing the monopole power spectrum with the Fourier bin of \(\Delta k = 2\pi/V_{\text{survey}}\),

\[
\Delta V_k = \Delta V(k, \Delta k) = 4\pi k^2 \Delta k.
\]

When estimating the two-dimensional power spectrum \(P(k_1, k_2)\) with the Fourier bin of \((\Delta k_1, \Delta k_2)\),

\[
\Delta V_k = \Delta V(k_1, k_2, \Delta k_1, \Delta k_2) = 4\pi [k_1 \Delta k_1 \Delta k_2 + k_2 \Delta k_2 \Delta k_1].
\]

For the monopole matter power spectrum, Equation (26) provides a good approximation to the measured variance from \(N\)-body simulations (Jeong & Komatsu 2009).

The \(1/\bar{n}_g\) term in Equation (26) denotes the shot noise, assuming that the galaxy distribution follows the Poisson statistics of the underlying galaxy density field. When dealing with a galaxy sample containing the interlopers, the shot-noise contribution to the observed auto-power spectrum is related to the number density \(n_{g,\text{total}}\) of the total sample including the interlopers. Also, there is no shot-noise contribution to the observed cross-power spectrum, even though both samples contain a mixture of the two populations. We present the rigorous derivation in Appendix B.

In Appendix C, we calculate the variance of the observed cross-power spectrum as

\[
(\Delta P_{LO}(k))^2 = \frac{1}{N_k} \left[ \left( \frac{\bar{n}_{\text{LAE}}}{\bar{n}_{\text{O II}}} \right)^2 \left( \frac{\alpha^2}{\bar{n}_{\text{O II}}} \right) + \left( \frac{\bar{n}_{\text{LAE} \times \text{O II}}}{\bar{n}_{\text{O II}}} \right)^2 \right].
\]

Here, \(\bar{n}_{\text{LAE} \times \text{O II}}\) arises because we implement the cross-correlation by projecting the OIIEs onto...
the LAE redshifts (see Section 3.4). The same projection adds the factor of $\alpha^2/\beta$ to the shot noise of OIIEs. The right panel of Figure 5 presents the signal-to-noise ratio (S/N) for each $k_x k_y$ mode of the observed cross-power spectrum for the case $f = g = 0.1$. This figure indicates that a high S/N measurement for the interloper fraction is possible from the cross-correlation. We shall quantify this conclusion using a Fisher information matrix formalism in the next section.

4. Statistical Analysis for HETDEX

In this section, we present the statistical analysis for the cosmological parameter estimation from all three power spectra: the auto-power spectra of the main survey galaxies and the interlopers, and the cross-power spectrum between the main galaxies and interlopers.

In Section 4.1, we first construct the likelihood function for the data set consisting of the main sample and the interlopers. We focus on the two geometrical observables measured from cosmological distortion: the Hubble expansion rate $H(z)$ and the angular diameter distance $d_A(z)$. These are primary targets for current and future galaxy surveys. We describe the cosmological distortion in the presence of the interloper population in Section 4.2. In Section 4.3, we present (Case A) the proof of concept for measuring the interloper fractions $f$ and $g$ from the cross-correlation between the main survey galaxies and interlopers. We measure the interloper fractions by assuming only that the true cross-correlation between the two populations vanishes.

In the following subsections, we present the projected uncertainties on $H(z)$ and $d_A(z)$ for several different treatments of nonlinearity in the redshift-space galaxy power spectrum. In Sections 4.4–4.6, we assume that the redshift-space power spectrum is given by the linear Kaiser formula (Kaiser 1987) with the FoG effect (Jackson 1972) as in Equation (24). In Section 4.4, we assume that we know the full shape of the nonlinear power spectrum $P(k)$ for both LAEs and OIIEs (Case B). Because OIIEs are lower redshift objects, their population probes much smaller scales than LAEs, and their nonlinearities are much stronger. We therefore investigate $H(z)$ and $d_A(z)$ after marginalizing over the nonlinear OIIE power spectrum in Section 4.5 (Case C), and we assume this case as a baseline for HETDEX. In Section 4.6, we study the pessimistic case of marginalizing over both the LAE and OIIE power spectra (Case D). In this case, we can still measure the combination $d_A(z)/H(z)$ given the AP test. In Section 4.7, we test for the effect of nonlinear RSD by including the higher order dependence on the angular cosine $\mu \equiv k_x/k$.

Finally, we analyze the effect of interlopers on measuring other cosmological parameters such as the linear growth rate $f = d \ln D/d \ln a$ (Section 4.4) and the primordial non-Gaussianity parameter $f_{NL}$ (Section 4.8).

Throughout, we denote the true interloper fractions that we have assumed for the analysis by $f_{true}$ and $g_{true}$.

### 4.1. Likelihood Function

We construct the likelihood function by assuming that the galaxy power spectrum completely specifies the statistics of the galaxy density contrast; i.e., we ignore the effects from higher order correlation functions. Because we are focused on the galaxy power spectrum, this assumption suffices for the purpose of this paper.

To facilitate the calculation and to incorporate the auto- and cross-power spectra in the same setting, we project all OIIEs into the LAE volume. We could just as well have chosen to project all LAEs into the OIIE volume, or, similarly, project both LAEs and OIIEs into any appropriate volume of our choice. As seen from Equation (34) below, the choice of projection merely adds a constant to the log-likelihood function.

The likelihood function is constructed from the observed density contrasts $\delta_{xy}^{obs}(k)$, with $x = L$ for LAEs and $x = O$ for OIIEs, both of which are contaminated by interlopers. For each observed wave mode $k$, we define the observed data vector at the LAE redshift (with OIIEs projected to that redshift) as

$$\Delta(k) = \begin{pmatrix} \Delta_L(k) \\ \Delta_O^{proj}(k) \end{pmatrix} = \begin{pmatrix} \delta_L^{obs}(k_1) \\ \vdots \\ \delta_L^{obs}(k_{N_k}) \\ \delta_O^{obs}(k_1) \\ \vdots \\ \delta_O^{obs,proj}(k_{N_k}) \end{pmatrix},$$

where $N_k$ is the number of Fourier modes for a given bin centered on $k$. We provide the explicit expression for $N_k$ in Equations (27) and (29).

As shown in Section 3.6, each element of the covariance matrix consists of the observed power spectra plus Poisson shot noise $P_{xy}^{obs+shot}(k) = P_{xy}^{obs}(k) + \delta_k^{obs}/N_k^{tot}$. Here, $\delta_k$ is the Kronecker delta symbol. In block-matrix form, the covariance matrix is

$$C(k) = \langle \Delta(k) \Delta^\dagger(k) \rangle = \frac{(2\pi)^3}{V_k} \begin{pmatrix} P_{LL}^{obs+shot}(k) I_{N_k} & P_{LO}^{obs}(k) I_{N_k} \\ P_{OL}^{obs,proj}(k) I_{N_k} & P_{OO}^{obs+shot,proj}(k) I_{N_k} \end{pmatrix},$$

where $I_{N_k}$ is the $N_k \times N_k$ unit matrix, and $P_{OO}^{obs+shot,proj}(k)$ is the OIIE auto-power spectrum projected into the LAE volume. The factor $V_k/(2\pi)^3$ appears from averaging $\langle \delta^2(q) \delta(q') \rangle = (2\pi)^3 \delta_D(q - q') P(q)$ over a cell of volume $V_k$ in $k$ space. From the covariance matrix, we compute the log-likelihood function

$$-\ln \mathcal{L} = \frac{1}{2} \text{det} C + \frac{1}{2} \Delta^\dagger C^{-1} \Delta,$$

as

$$-\ln \mathcal{L} = \sum_k \frac{N_k}{2} \left[ \ln \left| P_{LL}^{obs+shot} P_{OO}^{obs+shot,proj} - (P_{LO}^{obs})^2 \right| + \frac{P_{LO}^{obs,proj} P_{LL}^{data+shot} + P_{LO}^{obs+shot} P_{OO}^{data+shot,proj} - (P_{OO}^{obs})^2}{P_{LL}^{obs+shot} P_{OO}^{obs+shot,proj}} \right].$$

(34)
Here, we replace the square of the data vector with the maximum-likelihood estimators for the observed auto-power spectra,

\[ \hat{P}_L^{\text{data+shot}}(k) = \frac{V_k}{(2\pi)^3N_k} \sum_i |\delta_L^{\text{obs}}(k_i)|^2, \]

\[ \hat{P}_O^{\text{data+shot,proj}}(k) = \frac{V_k}{(2\pi)^3N_k} \sum_i |\delta_O^{\text{obs,proj}}(k_i)|^2, \]

and the cross-power spectrum,

\[ \hat{P}_L^{\text{data}}(k) = \frac{V_k}{(2\pi)^3N_k} \sum_i \Re(\delta_L^{\text{obs},w}(k_i)\delta_O^{\text{obs,proj}}(k_i)), \]

where \( \Re(z) \) is the real part of a complex number \( z \).

### 4.2. Cosmological Distortion

Cosmological distortion refers to the systematic change in the statistical observables, such as the galaxy power spectrum, induced by adopting an incorrect reference cosmology to convert the observed galaxy coordinate (R.A., decl., \( z \)) into physical coordinates. Cosmological distortion allows us to measure the Hubble expansion rate \( H(z) \) and the angular diameter distance \( d_A(z) \) from features such as those produced by BAO (Blake & Glazebrook 2003; Hu & Haiman 2003; Seo & Eisenstein 2003) and the AP test (Alcock & Paczynski 1979; Shoji et al. 2009) in the redshift-space galaxy power spectrum. In this paper, we do not include the uncertainties in the cosmological parameters, such as \( \Omega_m h^2 \) and \( \Omega_b h^2 \), that determine the shape of the linear matter power spectrum. In the real analysis, one must include appropriate priors on these parameters from, e.g., the CMB analysis in, e.g., Planck Collaboration (2016b) and Planck Collaboration et al. (2018).

We model the cosmological distortion in the galaxy power spectrum as follows. Given a reference cosmology, we calculate the angular diameter distance \( d_A(z) \) as well as the Hubble expansion rate \( H(z) \) at redshift \( z \). In general, the wavenumbers \( k_\perp \) and \( k_\parallel \) measured from the reference cosmology differ from the true wavenumbers \( k_L \) and \( k_O \) by some factors \( v(z) \) and \( w(z) \). We determine the factors \( v(z) \) and \( w(z) \) and their effect on the power spectrum in a manner similar to the projection effect discussed in Section 3 using \( \alpha \) and \( \beta \). For LAEs (subscript “L”) and OIIEs (subscript “O”), we define

\[ v_L = \frac{d_{\text{L,ref}}(z_L)}{d_A(z_L)}, \quad w_L = \frac{H(z_L)}{H_{\text{ref}}(z_L)}, \]

\[ v_O = \frac{d_{\text{O,ref}}(z_O)}{d_A(z_O)}, \quad w_O = \frac{H(z_O)}{H_{\text{ref}}(z_O)}.

Using these variables, the Fourier-space vector \( k_\perp^\text{ref}, k_\parallel^\text{ref} \) inferred from the reference cosmology is related to the true Fourier vector as \( (k_L, k_O) = (v_L k_\perp^\text{ref}, w_L k_\parallel^\text{ref}) \), and the power spectrum \( P_L^{\text{ref}}(k_\perp^\text{ref}, k_\parallel^\text{ref}) \), measured by using the reference cosmology, is

\[ P_L^{\text{ref}}(k_\perp^\text{ref}, k_\parallel^\text{ref}) = v_L^2 w_L P_L(v_L k_\perp^\text{ref}, w_L k_\parallel^\text{ref}). \]

Similarly, we calculate the contribution from the projected interloper power spectrum by defining \( \alpha_{\text{ref}} \) and \( \beta_{\text{ref}} \) as the scaling factors \( \alpha \) and \( \beta \) (Equation (7)) in the reference cosmology. The projected OIIE power spectrum takes the following form, where we include both the projection due to the misidentification (see Equation (19)) and the projection due to a cosmological distortion (Equation (41)):

\[ P_O^{\text{ref,proj}}(k_\perp^\text{ref}, k_\parallel^\text{ref}) = \alpha_{\text{ref}}^2 \beta_{\text{ref}}^2 P_O^{\text{ref}}(\alpha_{\text{ref}} k_\perp^\text{ref}, \beta_{\text{ref}} k_\parallel^\text{ref}) \]

\[ = \alpha_{\text{ref}}^2 \beta_{\text{ref}}^2 v_{\text{O,ref}} w_{\text{O,ref}} P_O(v_{\text{O,ref}} \alpha_{\text{ref}} k_\perp^\text{ref}, w_{\text{O,ref}} \beta_{\text{ref}} k_\parallel^\text{ref}). \]  

The parameters \( \alpha_{\text{ref}} \) and \( \beta_{\text{ref}} \) are completely degenerate with \( v_{\text{L,ref}} \) and \( w_{\text{L,ref}} \); therefore, we only include the latter cosmological distortion parameters in the analysis.

### 4.3. Case A: No Prior Knowledge on the Shape of the Galaxy Power Spectra

How accurately can we measure the interloper fractions \( f \) and \( g \) by requiring only that the true cross-power spectrum vanish? We address this question in the most conservative manner, assuming no prior knowledge about the shape of the galaxy power spectrum, i.e., the case where we measure the interloper fractions \( f \) and \( g \) along with the amplitude of the two-dimensional power spectrum \( P_L(k_L, k_O) \) and \( P_O(k_L, k_O) \) from fitting the observed auto- and cross-power spectra. The expressions for these power spectra are given in Equations (18) and (20) (for the auto-power spectra) and Equation (22) (for the cross-power spectrum).

For the HETDEX survey outlined in Section 2.1, there are 917 = 8281 Fourier modes within the maximum wavenumber \( k_{\text{max}} = 0.4 \text{ h/Mpc at the LAE volume. Thus, there are a total of } 2 (f + g) + 8281 \text{ (for } P_L) + 8281 \text{ (for } P_O) = 16,564 \text{ parameters. For each value of } f \text{ and } g, \text{ we use the Fisher information matrix analysis to calculate the projected uncertainties.}

A cautionary remark is in order here. The analysis in this section only serves as a proof of concept for the measurement of the interloper fractions \( f \) and \( g \). The uncertainties found here are a worst case benchmark, because they are derived from minimal assumptions about the shape of the galaxy power spectrum.
Figure 7. Constraints on the interloper fractions \( f \) and \( g \) for several interloper fractions spaced 10% apart. Because \( N_{\text{OII}}^{\text{true}}/N_{\text{LAE}}^{\text{true}} \approx 2 \) for HETDEX, only the interloper fractions bounded by the gray lines are physically possible (see Appendix A). However, because the true number densities will not be known, and thus cannot be used in the fit, estimates of \( f \) and \( g \) may well fall outside those boundaries. \( f \) and \( g \) can be measured best near \( (0, 0) \) and \((1, 1)\). On the diagonal \( f + g = 1 \), the individual measurements become degenerate.

spectrogram. However, we do not advocate such an analysis in practice. In fact, we carried out a Markov-chain Monte Carlo (MCMC) analysis for the HETDEX case only to find that the chain does not converge in the 16,564 dimensional parameter space. Even for a simpler analysis of measuring \( f \) and \( g \) by iteration, it is nontrivial to construct an unbiased estimator.

Figure 6 presents the projected uncertainties on \( f \) and \( g \) under the null hypothesis, i.e., when the true interloper fractions are zero. The shaded ellipse at the center indicates the cosmic-variance-limited constraint without shot noise. The solid ellipse shows the constraint with the fiducial number density of HETDEX given in Section 2.1. The forecast demonstrates that the cross-correlation constrains both \( f \) and \( g \) to the subpercent level. The constraint is better for \( f \) than for \( g \) because on large scales where the \( S/N \) is the largest (Figure 5), the amplitude of the projected OII power spectrum (the contaminant) is much higher than that for the LAEs; this behavior is clear in the upper two panels in Figure 3. In the figure, the relative contributions of \( P_{\text{LAE}}^{\text{true}} \) and \( P_{\text{OII}}^{\text{true}} \) to the observed auto-correlation are similar on large scales even though the latter is suppressed by \( f^2 \).

We also investigate the effect of reducing the number density of LAEs and OIIEs. The orange dashed–dotted ellipse, the green triple-dotted–dashed line, and the red dashed line show the expected constraint when reducing the number of LAEs, OIIEs, or both galaxy groups, respectively, by 50% of what is predicted. Reducing the number density of galaxies increases the Poisson shot noise, which affects the uncertainties in the power spectrum measurements on small scales. This change affects the constraint on \( g \) more than that on \( f \), due to the different scale dependence of the \( f \) and \( g \) contributions. Figure 3 reflected this behavior in the top- and bottom right panels.

We next address the situation of nonzero \( f \) and \( g \). As shown in Appendix A, unlike the misidentification fractions \( x_{\text{LAE}} \) and \( x_{\text{OII}} \) (defined in Equations (2)–(3)) that can take any value between 0 and 1, the true interloper fractions are limited to two regions: one with \( 0 \leq f_{\text{true}} \leq f_{\text{lim}} \) and \( 0 \leq g_{\text{true}} \leq g_{\text{lim}} \), and one with \( f_{\text{lim}} \leq f_{\text{true}} \leq 1 \) and \( g_{\text{lim}} \leq g_{\text{true}} \leq 1 \). The limiting values \( f_{\text{lim}} = N_{\text{OII}}^{\text{true}} / (N_{\text{LAE}}^{\text{true}} + N_{\text{OII}}^{\text{true}}) \) and \( g_{\text{lim}} = N_{\text{LAE}}^{\text{true}} / (N_{\text{LAE}}^{\text{true}} + N_{\text{OII}}^{\text{true}}) \) occur when \( x_{\text{LAE}} + x_{\text{OII}} = 1 \). For HETDEX, \( f_{\text{lim}} \approx 2/3 \) and \( g_{\text{lim}} \approx 1/3 \).

Figure 7 presents the projected 68% confidence ellipses of \( f \) and \( g \) for several true interloper fractions spaced throughout the allowed region at intervals of 10%. The tightest constraints are obtained when the interloper fractions are either small or extremely large. There is a symmetry of exchanging \( f \leftrightarrow g \), which is equivalent to swapping the two samples. Indeed, a closer inspection of the log-likelihood given in Equation (34) reveals that the cross-correlation alone cannot distinguish which of the two regions a given survey will fall into. However, because we expect the power spectra of LAEs and OIIEs to have very different shapes on large scales, inspection of the measured true power spectra should suffice to discriminate between the two regions in cases far from the diagonal \( f + g = 1 \). Because all three observed power spectra are the same, \( f \) and \( g \) are completely degenerate on the diagonal \( f + g = 1 \). In that case, although the nonzero cross-correlation indicates the existence of interlopers, Equations (18) and (20) cannot be inverted. The cross-correlation alone, therefore, is insufficient to determine the interloper fraction; this is the reason why the error ellipse diverges at \( (f_{\text{lim}}, g_{\text{lim}}) \).

Although we plot the projected constraints on \( f \) and \( g \) only in the limited regions, an analysis with measured data must explore the whole range of \( f \) and \( g \) between 0 and 1. This is because the limits \( f_{\text{lim}} \) and \( g_{\text{lim}} \) are given by the true ratio \( N_{\text{OII}}^{\text{true}} / N_{\text{LAE}}^{\text{true}} \), and in reality only the observed numbers of galaxies will be known; the true number of LAEs and OIIEs will be variables that need to be estimated from the observed numbers and the estimated interloper fractions. Thus, while the true interloper fractions are restricted to the allowed regions, the measured interloper fractions \( f \) and \( g \) can have any value between 0 and 1.

How do the uncertainties in measuring the power spectrum change due to the interlopers? Figure 8 displays the change in the uncertainty on each mode \( P_L(k_i, k_j) \) after marginalizing
Figure 9. 1σ (68% C.L.) ellipses on interloper fractions $f$ and $g$ for several models. The first four models correspond to Case B, where we assume that the shapes of the power spectra are fully known. For the first model in green, only the interloper fractions are being fit; all other parameters including the amplitudes are assumed to be known a priori. For the second, in blue, we marginalize over the amplitudes. For the third (solid cyan), we marginalize over the OIIE amplitude only, and for the fourth (solid magenta), over the RSD, FoG, and AP parameters of the OIIEs. Finally, the thick dashed orange ellipses correspond to the worst case scenario, Case A, where only the cross-correlation is known, marginalizing over both 2D power spectra. We consider two interloper fractions, $f_{\text{true}} = g_{\text{true}} = 0$ and $f_{\text{true}} = g_{\text{true}} = 0.2$, as labeled in the figure. Most of the measurement uncertainty on $f$ and $g$ is a result of marginalizing over the amplitudes.

over the interloper fractions. Here, we use $f_{\text{true}} = g_{\text{true}} = 0.01$. Figure 8 shows that the fractional increase in the uncertainties in the power spectrum closely follows $n^2 P$: the power spectrum uncertainties increase mainly in the shot-noise-dominated regime ($\bar{n}_L P_L \ll 1$), and the increase only depends on the interloper fractions and the number of galaxies.

We can estimate the increase in the uncertainty by inverting Equations (18) and (20) for fixed $f$ and $g$, which is valid when $f$ and $g$ are tightly constrained (which is the case for small values; see Figure 7). Combining this result with Equation (26), to first order in $f$ and $g$, we obtain the uncertainty $\Delta P_L$ in the LAE power spectrum

$$\Delta P_L(k) \approx \sqrt{2} \sqrt{\frac{2}{N_k}} \left[ P_L(k) + \left( 1 + f + g \frac{\bar{n}_O}{\bar{n}_L} \right) \frac{1}{\bar{n}_L} \right],$$

which is the same as Equation (26), except that the shot-noise term is increased due to the interlopers. The fractional increase of the uncertainty is

$$\frac{\Delta P_L(k)}{\Delta P_{L,0}(k)} \approx \left( \bar{n}_L + g \frac{\bar{n}_O}{\bar{n}_L} \right) \frac{1}{\bar{n}_L} P_L^\text{ref}(k_{\text{ref}}),$$

where $\Delta P_{L,0}$ is the uncertainty without interlopers. Equation (43) is consistent with the results seen in Figure 8. The estimate in Equation (43) reproduces the numerical result to $\sim$3% for $f_{\text{true}} = g_{\text{true}} = 0.01$ and to $\sim$30% for $f_{\text{true}} = g_{\text{true}} = 0.1$. This difference is because, for larger values of $f_{\text{true}}$ and $g_{\text{true}}$, the linear expansion is not accurate and the larger uncertainties in $f$ and $g$ also contribute to $\Delta P_L$.

4.4. Case B: Knowing the Full Shape of Both LAE and OIIE Power Spectra

We now address the case of the opposite limit where one can model the full shape of the galaxy power spectrum for both LAEs and OIIEs using Equation (24). This is perhaps an unrealistically optimistic case, but it does allow us to set another benchmark point for the effect of interlopers on the measurement of cosmological parameters such as the angular diameter distance and the Hubble expansion rate. The list of parameters that we include in the analysis is the interloper fractions $f$ and $g$, and for each type of tracer the power spectrum amplitude $\ln A$, the angular diameter distance $d_A$, the Hubble expansion rate $H$, the redshift-space distortion parameter $\beta = f/\beta_1$, and the velocity dispersion $\sigma_v^2$, for a total of up to 12 parameters.

First, we study the projected uncertainties in measuring $f$ and $g$. Figure 9 shows the Fisher forecasts for several cases from maximal a priori knowledge where we assume we have complete knowledge of the power spectra and only the interloper fractions are being fitted, to minimal a priori knowledge, where only the true cross-correlation is known beforehand, i.e., our Case A. For each case, Figure 9 displays the 68% C.L. (1σ) contours for two values: $f_{\text{true}} = g_{\text{true}} = 0$ (inner ellipses) and 0.2 (outer ellipses).

Of course, assuming complete knowledge of the shape of the galaxy power spectrum enhances the constraint on the contamination fractions. Between the optimistic case (green, central ellipses) and the pessimistic case (thick, outermost ellipses) are three cases in which we marginalize over different combinations of parameters. Intriguingly, marginalizing over the full 2D power spectra and marginalizing over just the amplitudes of the power spectra gives rise to similar constraints on the interloper fractions. Indeed, for $f = g = 0$, there is no change between the two, and only at interloper fractions $f = g = 20\%$ is the difference apparent. Marginalizing over the parameters controlling the shape of the redshift-space power spectrum of OIIEs ($\beta_2$, $\sigma_v$ and $d_A$) also changes the $f$ and $g$ constraints significantly. The effects of marginalizing over other parameters are not as dramatic.

Pullen et al. (2016) demonstrated that ignoring interlopers in the galaxy sample biases the estimation of cosmological parameters, such as the linear growth rate $f$ and the galaxy bias $b_c$. Similarly, the left panel of Figure 10 presents the bias on $f_{\text{obs}}$ and $b_{\text{obs}}$, induced by ignoring the interlopers, and the right panel of Figure 10 shows that this interloper bias also plagues the distance measurement. We simulate the interloper bias by generating a realization of the LAE power spectrum with $f_{\text{true}} = 5\%$ but ignore the contamination by fixing $f = 0$ in the analysis. For the analysis, we use an MCMC algorithm with the adaptive Metropolis sampler (Roberts & Rosenthal 2009) and find the interloper bias by running MCMC on the ensemble-averaged log-likelihood function. We also check that the result is consistent with the first-order analytical calculation presented in Appendix D. In the appendix, we also justify the use of the ensemble-averaged log-likelihood.

For each set of parameters (as indicated at the beginning of this section and the figures), we ran the chain with 1000 $N_p (N_p + 1)$ iterations, where $N_p$ is the number of parameters, updating the covariance matrix of the proposal distribution every $N_p (N_p + 1)$ steps with $\beta = 0.95$ (see Equation (2.1) in Roberts & Rosenthal 2009). The first half of the iterations is discarded to allow the proposal distribution
to settle, and the analysis is performed on the second half. We use a flat prior as long as the parameters are within physical limits. The exceptions are that the auto-power spectra are additionally limited to $\lesssim 10^9 h^{-3}$ Mpc$^3$, and in later sections (Case C and Case D), the cosmological distortion parameters $v_L$, $w_L$, $v_O$, and $w_O$ are additionally limited so that the splines of the power spectra are well defined. These limits are enforced by setting the likelihood $\mathcal{L} = 0$ outside the bounds.

The interloper bias disappears when one treats the interloper fractions $f$ and $g$ as free parameters and simultaneously analyzes the LAE and OIIE auto-power spectra and the cross-power spectrum. The left panel of Figure 11 displays the results for $f_{\text{true}}$ from our joint analysis as a function of $f_{\text{true}} = g_{\text{true}}$, after marginalizing over two different sets of parameters as indicated in the figure legend. In addition to showing that the interloper bias disappears, the figure also shows that larger interloper fractions come at the cost of increasing the measurement uncertainty.

The right panel of Figure 11 shows the result for the distance measurements ($d_A$ and $H$) at $z \sim 2.7$ (the LAE redshift). Here, we fix the redshift-space distortion parameters $\beta_L$, $\sigma_L$, and $\sigma_O$, and marginalize over $f$, $g$, $\ln A_L$, and $\ln A_O$.  We consider several interloper fractions with $f = g = 0$, 5%, 10%, 15%, 20%, and 25%, and calculate the error ellipses from the Fisher information matrix. Running MCMC on the ensemble-averaged log-likelihood function produces the same conclusion: the joint analysis removes the interloper bias, although the measurement uncertainty increases for larger interloper fractions.

The primary geometrical observables from the cosmological distortion of the two-dimensional redshift-space power spectrum are the following combinations of $d_A$ and $H$:

$$R \equiv \left( \frac{d_A}{H} \right)^{1/3}, \quad AP \equiv d_A H.$$  

These are sensitive to, respectively, the isotropic and anisotropic stretch/contraction in the $k_L-k_H$ plane (Padmanabhan & White 2008; Shoji et al. 2009), i.e., we measure $R$ from

$$f_{\text{true}} = 0.05.$$  

Figure 12 presents the uncertainties on $R$ and $AP$ for the general case $f_{\text{true}} = g_{\text{true}}$. Because the upper right corner of the $f$–$g$ plane (in Figure 7) should only occur for a catastrophic failure of line identification, we only present the lower allowed region for the true interloper fractions. The left three panels display the constraints for $R$, while the right three are the equivalent figures for $AP = d_A H$. In the top two panels, we only marginalize over $f$, $g$, $\ln A_L$, and $\ln A_O$; in the middle two panels, we additionally marginalize over the redshift-space parameters $\beta_L$ and $\beta_O$. In the bottom two panels, we include marginalizations over the FoG parameters $\sigma_L$ and $\sigma_O$. All plots in Figure 12 have a similar structure: the measurement uncertainty is lowest near the origin $f = g = 0$, then increases slowly at first, then rapidly as one gets closer to the limits $f = g = 0$.

For all cases, the larger interloper fractions degrade the measurement precision of $R$ and $d_A H$. The largest effect is for the AP parameter $d_A H$ when marginalizing over the RSD parameters because both RSD and the interlopers contribute to the observed anisotropies in the two-dimensional power spectrum. For example, in the right panel of Figure 12, the uncertainty for the $d_A H$ measurement changes from 1% (top panel) to $\sim 2\%$ (middle panel) near $f = g = 0$, once we marginalize over $\beta_L$ and $\beta_O$. Conversely, the distance measure $R$ is much less affected by the redshift-space distortion parameters, and the uncertainties at the origin $f = g = 0$ changes from 0.88% (top) to 1.02% (bottom). This behavior arises because the BAO feature, which dominates the measurement of $R$, is less affected by RSD (Seo & Eisenstein 2007; Shoji et al. 2009; Seo et al. 2010). As we have seen in Figure 9, the interloper fraction measurement is most affected by the amplitude of the power spectrum. In addition, because a larger OIIE power spectrum means a larger contamination amplitude, it also generates a larger interloper bias, as shown in the left panel in Figure 13. In the center and right panels of the figure, we show the effect of a
change in OIIE linear galaxy bias on the measurement of the interloper fractions and the LAE distance measurement. The figure contains the result for three different values of the OIIE galaxy bias: $b_0 = 1.1$, $b_0 = 1.5$ (this is the value adopted throughout the paper), and $b_0 = 5$. Because a larger OIIE bias results in a larger interloper signal in the LAE and cross-power spectra, a large OIIE galaxy bias results in a tight constraint on $\hat{f}$. However, when $f = g = 0.1$ the LAE distance measurement is essentially independent of the OIIE bias.

4.5. Case C: Knowing the Full Shape of the LAE Power Spectrum Only

The Case B presented in Section 4.4 assumes that we can accurately model the nonlinearities in both the LAE and OIIE power spectra. For the LAEs at redshift $z \sim 2.7$, perturbation-theory-based analytical calculations (Bernardeau et al. 2002) provide a reliable model for the nonlinear evolution of the density field up to $k \sim 0.4$ $h$ Mpc$^{-1}$ (Jeong & Komatsu 2006). This is also the redshift and wavelength range where we expect the perturbative bias expansion (Desjacques et al. 2018a) to model the nonlinear galaxy bias. In contrast, the corresponding OIIE power spectrum extends to $k \sim 2.8$ $h$ Mpc$^{-1}$ at a mean redshift $z \sim 0.2$, at which point perturbative approaches fail. At this redshift, perturbation theory can be reliable only for $k \sim 0.1$ $h$ Mpc$^{-1}$ or larger scales.

Therefore, it is more realistic to explore the case where we lack any prior knowledge on the nonlinear 1D power spectrum of OIIEs. That is, we treat the nonlinear power spectrum for OIIEs as a set of free parameters. We still adopt the anisotropies due to RSD as in Equation (24) and only parameterize the one-dimensional power spectrum. We shall study the effect of higher order RSD parameters in Section 4.7.

To measure the 1D OIIE power spectrum $P_O(k)$, we use a cubic spline with knots linearly spaced at $k < 0.4$ $h$ Mpc$^{-1}$ (in the original OIIE volume) and logarithmically spaced above that. Equation (41) is then used to project the OIIE power spectrum onto the LAE volume. Because we perform the analysis in the projected LAE volume, we determine the minimum and maximum wavenumbers by scaling the corresponding wavenumbers in the LAE volume, but we extend the range by 10% at each end, in order to provide a buffer for the cosmological distortion measurement. This procedure effectively sets a prior on $R$ and AP of $\sim 10\%$, because we set a hard prior outside of the wavenumber range: when the MCMC chain moves outside of the range, we force the likelihood to be zero.

We find no noticeable difference in the constraint on the interloper fractions $f$ and $g$ between this case and Case B in Section 4.4. This result arises because the main information for constraining $f$ and $g$ comes from the cross-correlation, and the cross-correlation method works without knowing the explicit shape of the nonlinear power spectrum (as shown in Section 4.3). Because the constraints for Case B and Case C are nearly identical, we do not duplicate the figures from Section 4.4 for Case C.

Measuring the nonlinear OIIE power spectrum without the shape information means that it is impossible to measure $R = (d_s^2/H)^{1/3}$, unless we specifically search for the BAO signature. Nevertheless, because the assumed RSD function in Equation (24) dictates the angular dependence of the two-dimensional power spectrum, we can still measure AP. We find that the constraint does not change significantly for interloper fractions $f \sim g \lesssim 20\%$, because the contamination from the LAE power spectrum to the OIIE power spectrum is multiplied by a volume factor $1/(\alpha^2\beta) \sim 0.023$ and thus remains insignificant.

The linear RSD model adopted here is not reliable on scales relevant to the OIIE galaxy power spectrum ($k < 2.8$ $h$ Mpc$^{-1}$). To correct for this effect, a more robust method would be to leave the full two-dimensional OIIE power spectrum completely unconstrained, similar to the analysis in Section 4.3. However, leaving the 2D OIIE power spectrum free would require fitting $\sim 8500$ parameters. Given that the measurement uncertainties in $f$ and $g$ hardly change among the three cases examined in Sections 4.3–4.5, we expect that the key result would still remain true that (a) joint analysis removes the interloper bias, and (b) marginalizing over the interloper fractions increases the uncertainties in the cosmological parameters.
4.6. Case D: Knowing Only the Anisotropy Due to RSD

Now we examine the case when we relax the assumption that the 1D LAE power spectrum shape is known. Without the shape information, we cannot measure $R = (d_A^2/H)^{1/3}$. However, the AP test still allows the measurement of the parameter $AP = d_A H$ from the anisotropy of the redshift-space power spectrum. Here, in order to highlight this point, we exclude the shape information altogether, including the BAO that must enhance the constraint on $R$.

We model the LAE power spectrum similarly to the way we modeled the OIIE power spectrum: $P_{\ell}(k)$ will be a third-order spline with knots linearly spaced for $k < 0.04 \ h \ Mpc^{-1}$ and logarithmically spaced for $0.04 \ h \ Mpc^{-1} < k < 0.4 \ h \ Mpc^{-1}$. This approach is adopted to ensure that all major features of the power spectrum can be represented by the fit. Without a dedicated search for the BAO (Koehler et al. 2007; Shoji et al. 2009), however, we cannot measure $R$.

The left-hand side of Figure 14 shows the projected constraints on $d_A$ and $H$ when $f = g = 5\%$, after marginalizing over the interloper fraction and the amplitude (Case C, strongly shaded blue) or the full 1D power spectrum (Case D, lightly shaded blue). For the green and orange ellipses, we additionally marginalize over the FoG velocity dispersion $\sigma_v$. With our method, the forecast constraints on $R$ remain $\sim 1.5\%$ up to interloper fractions $\sim 20\%$. The best constraints on $R$ are at the origin; from top to bottom, 0.88\%, 0.9\%, and 1.02\%. The best constraints on $d_A H$ at the origin are 0.79\%, 1.82\%, and 1.92\%.

4.7. Effect of Higher Order RSD

We have used Equation (24) to model the RSDs, including the linear theory prediction (Kaiser effect) and the FoG suppression. In this section, we study the effect from the nonlinear contribution of redshift-space distortion. To fully
account for the nonlinear distortion in a consistent manner, we need to include the full perturbation theory expression in, for example, Desjacques et al. (2018b). For the purpose of testing the interplay between the interloper fraction and the nonlinear RSD effect on the distance measurement ($R$ and $d_{h}H$), however, we develop an ansatz motivated by the full expression. Specifically, we add parameters $C_6$, $C_6^O$ and $C_8$, $C_8^O$ to account for the higher order angular dependence, replacing Equation (24) by

$$P_x(k, \mu) = A_x^2 A_{RSD}(k, \mu) A_{Fog}(k, \mu) P_{lin}^{x}(k),$$

where

$$P_{lin}^{x}(k) = b_m^x D^2(z_x) P_{lin}^{x}(k),$$

$$A_{RSD}(k, \mu) = [1 + 2 \beta_3 \mu^2 + \beta_4^2 \mu^4] + C_6^x b_m^{x - 2} \sigma_{W}^2 k^{4} \mu^6 + C_8^x b_m^{x - 2} \sigma_{W}^2 k^{4} \mu^8,$$

$$A_{Fog}(k, \mu) = (1 + f^2 k^2 \mu^2 \sigma_{W}^2)^{-1}. $$

This parameterization naturally reduces to the linear Kaiser formula Equation (24) in the large-scale limit $k \to 0$. We study the projected constraints with fiducial values of $C_6 = C_8 = 0$ (under the null hypothesis); increasing them to $C_6 = C_8 = 1$ does not change the result significantly.

Figure 15 presents the projected constraints on $R$ and $d_{h}H$ as we marginalize over successively more parameters, assuming that the shape of both LAEs and OIIE power spectra are known. Including the nonlinear RSD does not affect the projected constraint on the $R$ parameter, which controls the isotropic shift of the wavenumbers because the features in the monopole galaxy power spectrum such as the BAO provide information orthogonal to the anisotropies. Conversely, the AP test is weakened by the marginalization over $C_6$. When marginalizing over the $C_8$ parameter, however, there is no noticeable difference.

4.8. Interloper Bias and Primordial Non-Gaussianity

Inspiration of the interloper effect on the monopole and quadrupole power spectra in Figure 4 suggests that at large scales, the lower redshift interlopers generically add significant power to the power spectrum of the main higher redshift sample. This behavior arises because the small-scale interloper power spectrum is boosted and added to the main sample power spectrum. Also, the scale-dependent addition to the power spectrum on large scales is the characteristic feature of the scale-dependent bias from primordial non-Gaussianities (Dalal et al. 2008; Desjacques et al. 2018a). In this section, we study the effect of interlopers on measuring the primordial non-Gaussianity parameter $f_{NL}$ of local type (Salopek & Bond 1990; Komatsu & Spergel 2001). The scale-dependent bias generated from local-type primordial non-Gaussianity adds to the linear bias $b_x$ ($x = L$ and $x = O$ for, respectively, LAEs and OIIEs) as

$$\Delta b_{x}(k) = f_{NL} \frac{2 \delta_{i}(b_x - 1) \frac{3\Omega_{m}H_{0}^{2}}{2k^{2}T(k)D_{m}(z)}}{},$$

where $f_{NL}$ is the nonlinearity parameter, $\delta_{i} = 1.686$ is the critical density contrast in the spherical collapse model, $\Omega_{m}$ is the present-day matter density parameter, $H_{0}$ is the Hubble constant, $T(k)$ is the transfer function, and $D_{m}(z)$ is the linear growth function normalized to the scale factor $a$ during the matter-dominated epoch.

As is clear in Equation (49), for a positive nonlinearity parameter $f_{NL}$, the scale-dependent bias increases the power at large scales; this behavior is similar to the effect of interlopers. Figure 16 demonstrates this point by comparing the LAE power spectrum without interlopers (solid lines) and with 10% interlopers (dashed lines). The black lines are for $f_{NL} = 0$ while the orange lines are for $f_{NL} = 25$. The top and bottom panels differ in the value of $k$ that is fixed, as indicated in the figure. The top panel of Figure 16 reveals that interlopers have an effect similar to that of a positive $f_{NL}$. The bottom panel of Figure 16, however, shows that the scale dependence from non-Gaussianities are distinct from the scale dependence from interlopers for different $k$, i.e., the scale dependence of the interloper contamination has a distinctive angular dependence from the isotropic scale dependence of the local primordial non-Gaussianity.
Figure 14. Results from the joint-analysis method. Left: 1σ (68% C.L.) uncertainties on $d_A$ and $H$ at the LAE redshift when $f_{\text{true}} = g_{\text{true}} = 5\%$. The strongly shaded ellipses correspond to Case C, where the shape of the LAE power spectrum is known and we fit for the 1D OIIE power spectrum. The lightly shaded ellipses are the constraints for Case D when fitting both the 1D LAE and 1D OIIE power spectra. The color coding indicates which parameters are marginalized over, as described in the legend. Right: projected constraints for the same cases as the left panel, but for $R = (d_A^2/H)^{1/3}$ and $A\bar{F} = d_A H$.

Figure 15. Results from the joint-analysis method, Case B: 1σ (68% C.L.) uncertainty ranges on $R$ and $d_A H$, successively marginalizing over more parameters as indicated in the legend. Whenever a parameter has an $x$ suffix, it means that we marginalize over both parameters for LAEs and for OIIEs.

We first test the effect of primordial non-Gaussianities on the distance measurement in Figure 15. The projected uncertainties on $R$ and $d_A H$, after marginalizing over the nonlinearity parameter $f_{\text{NL}}$, slightly increase along the $R$ direction.

The fact that the interloper effect and primordial non-Gaussianity produce similar scale dependencies in the galaxy power spectrum causes larger interloper bias. Figure 17 shows the interloper bias in $f_{\text{NL}}$ when ignoring the interlopers (dashed black line) and compares it with the result from the joint fitting method (solid orange). Thus, non-Gaussianity can be distinguished from interlopers, and, once again, Figure 17 clearly demonstrates that the joint fitting method removes the interloper bias.

5. Statistical Analysis for WFIRST

In this section, we apply the joint fitting method to the planned High Latitude Spectroscopic Survey of NASA’s WFIRST mission (Spergel et al. 2015). WFIRST is an emission-line galaxy survey using slitless grism spectroscopy in the infrared wavelength range between 1.35 and 1.89 μm with spectral resolution of $R \equiv \lambda/\Delta\lambda \approx 620–870$. The total sky area coverage of the survey is 2200 deg$^2$ ($f_{\text{sky}} \approx 0.05$) for which we can safely apply the Fourier-based analysis method in the previous section.

Focusing on the two largest emission-line samples, we consider the main galaxy sample of Hα ($\lambda$6563 Å) emitters (HAEs) contaminated by [O III] ($\lambda$5007 Å) emitters (OIIEs). With the wavelength coverage of the survey, the observed HAEs will be in the redshift range 1.05 < $z$ < 1.88, and the observed OIIEs will be at 1.70 < $z$ < 2.77.

We assume that the line intensities for both Hα and [O III] are strong for all emission-line galaxies at the redshift range of WFIRST. This is motivated by Bowman et al. (2019). Using the 3D-HST grism data (Brammer et al. 2012; Skelton et al. 2014; Momcheva et al. 2016), they find that both lines are strong at $z \sim 2$. However, in the local universe, [O III] is often weak. Thus, our assumption may turn out to be an optimistic one. A proper solution would take into account the sample of [O II] emitters and other lines that may be present in the spectrum. We leave this to a future investigation.

The assumption of both lines being strong implies two important points for the line identification. First, in the overlapping redshift range 1.7 < $z$ < 1.88, both Hα and [O III] lines will be observed and the HAE sample coincides with the OIIE sample; the presence of a second line unambiguously identifies the sample. Second, for the lower redshift (1.05 < $z$ < 1.20) HAEs, the interpretation is unambiguous; if we identified the line as [O III], the corresponding Hα must be detected as well. Similarly, the line identification for the higher redshift (2.54 < $z$ < 2.77) OIIEs is also unambiguous.
The relation between the Hα and [O III] redshift bins is illustrated in Figure 18, where we identify three regions in the spectrum. For the main HAE sample: 1.05 < z < 1.2, 1.2 < z < 1.7, and 1.7 < z < 1.88; for the interloper OIIIE sample: 1.7 < z < 1.88, 1.88 < z < 2.54, and 2.54 < z < 2.77. For both cases, we assume no interloper for the first and third bins. For the middle bins (1.2 < z < 1.7 for HAEs) and (1.88 < z < 2.54 for OIIIEs), we apply our method of measuring the interloper fractions f (OIIIEs contaminating HAEs) and g (HAEs contaminating OIIIEs) from the cross-correlation. The HAEs and OIIIEs coincide in the overlapping region (1.7 < z < 1.88) that we analyze only once.

For the Fourier analysis, we use the central redshifts of each of the bins to calculate the geometrical quantities. For HAEs, the survey volume for each bin is V_{survey}^{Ho} = 0.92 \ h^{-3} \ Gpc^3 (for the low-z bin), V_{survey}^{mid} = 3.87 \ h^{-3} \ Gpc^3 (for the middle-z bin), and V_{survey}^{Hi} = 1.53 \ h^{-3} \ Gpc^3 (for the high-z bin). For OIIIEs, they are V_{survey}^{OIIIE,lo} = 1.67 \ h^{-3} \ Gpc^3, V_{survey}^{OIIIE,mid} = 5.77 \ h^{-3} \ Gpc^3, and V_{survey}^{OIIIE,hi} = 2.09 \ h^{-3} \ Gpc^3. For the number of samples, we use 16.4 million HAEs and 1.4 million OIIIEs and adopt the linear galaxy bias of b_{HAE} = 1.5 and b_3 = 2 (Spergel et al. 2015). We spread the galaxies uniformly over the survey volume to calculate the number densities for HAEs and OIIIEs. Just like for HETDEX, we project the OIIIEs onto the HAE bin for the Fourier analysis. The scaling factors in our reference cosmology are α = 0.78 and β = 1.10. We use all Fourier modes below the maximum wavenumber k_{max} = 0.25 \ h \ Mpc^{-1} at the HAE redshift. Because the HAEs

Figure 17. The dashed black line indicates the interloper bias in f_{NL} when interlopers are ignored, while the orange line is the result of our joint fitting method. In both cases, the fiducial f_{NL} = 0. The shaded area represents the 1σ (68% C.L.) range calculated from the MCMC chain using the galaxy power spectrum. We use our Case C calculation (Section 4.5), marginalizing over the interloper fractions (f and g), angular diameter distance (d_a), Hubble expansion rate (H), amplitude of LAE power spectrum (\sigma_{LAE}), 1D OIIE power spectrum, and RSD and FoG parameters.

Figure 18. Schematic indicating how we divide the Hα and [O III] samples to avoid overlap in our joint fitting method applied to the WFIRST mission. We split the spectrum into three wavelength ranges A (1.35 \ \mu m < \lambda_{obs} < 1.44 \ \mu m), B (1.44 \ \mu m < \lambda_{obs} < 1.77 \ \mu m), and C (1.77 \ \mu m < \lambda_{obs} < 1.89 \ \mu m), corresponding to the redshift ranges for Hα and [O III] as shown in the figure. In region A, an [O III] line is identified by the simultaneous presence of Hα in the spectrum. Thus, in region A, the absence of a second line identifies Hα. Similarly, in region C, we either expect to see both lines, or else it must be [O III]. In region B, only one of the two lines is present at a time, and thus we expect misidentification to be a potential problem where our method fitting for f and g may be needed.
and OIIIEs are at high redshifts, we assume that both HAE and OIIIE galaxy power spectra are modeled well by a theoretical template. This assumption corresponds to the Case B of Section 4.4. As for the baseline model, we use the expression in Equation (24).

Finally, after the analysis for each bin, we combine the result by adding the Fisher information matrices. The combined center redshifts are \(z_{\text{HAE}} = 1.47\) and \(z_{\text{OIIIE}} = 2.32\). We count the galaxies in the overlapping range \((1.70 < z < 1.88)\) as part of the main HAE sample. For our baseline analysis, we marginalized over the amplitudes of the HAE and OIIIE power spectra, the angular diameter distances and the Hubble expansion rates at \(z_{\text{HAE}}\) and \(z_{\text{OIII}}\), the RSD parameters \((\beta_s)\), and the FoG velocity dispersions \((\sigma_{v,s})\).

5.1. Power Spectra

The monopole and quadrupole of the observed galaxy power spectra along with the contribution from contaminants (for \(f = 0.05\) and \(g = 0.1\)) are presented in Figure 19. The interloper contribution is quite different from the case for HETDEX (shown in Figure 4) because the WFIRST HAEs (the main sample) and OIIIEs (the interlopers) are at similar redshifts, which makes the projection parameters \(\alpha \sim 0.78\) and \(\beta \sim 1.1\) close to unity.

The main effect is, therefore, to suppress the observed power spectra of HAEs and OIIIEs by the factors of \((1 - f)^2\) and \((1 - g)^2\) (see Equations (18) and (20), and replace LAE with HAE and O II with O III). Because the contributions from the contaminants are suppressed by the square of the contamination fraction and the volume factor \(\alpha^2 \beta = 0.67\) is of order unity, the observed power spectra are mainly affected by a change in their amplitude. The products of the linear galaxy bias and growth factor for the HAE and OIIIE samples are quite similar, so their uncontaminated monopole power spectra lie nearly on top of each other.

Because the contamination effect is quite minor in the auto-power spectra of HAEs and OIIIEs, we expect that the cross-power spectrum is the main driver for the measurement of the contamination fractions \(f\) and \(g\) (see Figure 21 below).

5.2. Interloper Bias

We estimate the systematic changes in the maximum-likelihood value of the cosmological parameters \(p_i\) due to the interloper contamination by using

\[
\Delta p_i = F_{ij}^{-1} \sum_k N_k \frac{P_{ij}(k; \tilde{p}_i)}{P(k; \tilde{p}_j)} \frac{\tilde{D} \tilde{P}(k)}{P(k; \tilde{p}_i)}. \tag{50}
\]

where \(F_{ij}\) is the Fisher information matrix, and \(\tilde{D} \tilde{P}(k)\) is the change in the observed spectrum due to interlopers. We give the derivation of Equations (50) in Appendix D.

Let us first consider the interloper bias for the main HAE sample. From Equation (18), this is

\[
\Delta \tilde{P}_{\text{HAE}}(k) = -f(2 - f)P_{\text{HAE}}(k) + f^2 p_{\text{OIII}}^\text{proj}(k). \tag{51}
\]

Because the OIIIE power spectrum gets projected into a smaller volume \((\alpha^2 \beta \sim 0.67)\) and the interloper fraction is smaller than \(\lim_{\tilde{N}_{\text{OIII}}/\tilde{N}_{\text{HAE}} + \tilde{N}_{\text{OIII}}} = 0.079\) (see Appendix A), the second term in Equation (51) must be negligible compared to the first term. The main interloper effect on the HAE power spectrum, therefore, is to change the observed amplitude. We have also indicated this effect in Figure 19.

Because the interloper contamination does not distort the shape of the power spectrum, we forecast that there will be no significant interloper bias for the measurement of the angular diameter distance and the Hubble expansion rate at the HAE redshift.

On the other hand, both \(f_{\text{los}}\) and \(b_{\text{los}}\) (two direct observables from the dynamical measurement of RSD) would be systematically biased if the presence of interlopers is ignored. We show this in the top panel of Figure 20 for five different values for the interloper fractions. This figure is similar to Figure 4 in Pullen et al. (2016), but for the direct observables from the two-dimensional galaxy power spectrum. From the figure, it is apparent that the interloper bias in the \(b_{\text{los}}\) plane is quite strongly correlated, and the correlation is due to the bias in the amplitude of the observed galaxy power spectrum.

For the OIIIE samples, the story is quite different. Because the contamination fraction can be as high as \(g_{\text{lim}} = \tilde{N}_{\text{OIII}}/\tilde{N}_{\text{HAE}} + \tilde{N}_{\text{OIII}} = 0.92\), a small leakage of HAEs into the OIIIE sample...
can generate significant interloper bias for the distance measure-
ment. We show this in the bottom panel of Figure 20. Here, we fix the ratio \( \frac{f}{g} = 0.1 \) to reflect the sample size ratio between HAEs and OIIIEs.

5.3. Joint Fitting

In this section, we apply the joint-analysis technique to WFIRST. That is, we use all three power spectra (HAE power spectrum, OIIIE power spectrum, and HAE-OIIIE cross-power spectrum) as observables to estimate the cosmological parameters and show that the parameters estimated from the joint analysis are unbiased.

First, Figure 21 shows the projected constraints for the interloper fractions \( f \) and \( g \) for two sets of interloper fractions. Bottom: although there is no interloper bias for the HAE distance measurements, the distances \( d_{AH} \) and \( R \) measured from the OIIIEs (at \( z \sim 2.21 \)) will be biased without a proper account of the HAE interloper effect.

![Figure 21](image)

Figure 21. Result of the joint-analysis method: 1\( \sigma \) (68\% C.L.) constraints for WFIRST on the interloper fractions \( f \) and \( g \) for \( f_{\text{true}} = g_{\text{true}} = 0 \) (solid blue ellipse) and \( (f_{\text{true}}, g_{\text{true}}) = (4\%, 40\%) \) (dashed orange ellipse). We assume Case B.

![Figure 22](image)

Figure 22. Results from the joint-analysis method: projected 1\( \sigma \) (68\% C.L.) range for \( R \) and \( d_{AH} \) for WFIRST, assuming Case B. The solid magenta ellipse shows the confidence interval from the lowest redshift bin assuming no misidentification, the thick dashed blue ellipse from the center redshift bin marginalizing over interloper fractions, and the dashed–dotted green ellipse shows the confidence interval from the highest redshift bin again assuming no misidentification. The gray shaded ellipse shows the combined constraint from all three bins. Here, \( f_{\text{true}} = g_{\text{true}} = 0 \).

First, Figure 21 shows the projected constraints for the interloper fractions \( f \) and \( g \) for two sets of interloper fractions. Compared to HETDEX (Figure 9), the measurement uncertainty on the interloper fractions \( f \) and \( g \) is larger, and they are more highly correlated. This is explained by the fact that the two samples are closer in redshift, and so the two power spectra have less distinct signals in the cross-correlation. This plot shows that we can measure a percent-level interloper fraction from the cross-power spectrum of WFIRST.

With our joint-analysis method, the interloper bias on \( f_{\text{B}} \) and the distance measures shown in Figure 20 is removed. For the \( \text{H}\alpha \) sample, we forecast \(~1.5\% \) to \( 2\% \) constraints, and for
the [O III] sample, ∼4.5% to 7% for interloper fraction $g_{\text{true}} \lesssim 30\%$, and higher for larger interloper fractions.

In Figure 22 we show the projected constraints on $R$ and $d_{\Delta H}$ for all three bins when the true interloper fractions vanish. The bins are labeled in the legend by their central redshifts. We marginalize over the amplitude, RSD, and FoG parameters for all three bins. For the central bin, we additionally marginalize over $f$, $g$, and the amplitude, RSD, and FoG parameters of the corresponding OIIIIE sample. The constraints largely reflect the size of the survey volume. The weakest constraints come from the bin with the smallest volume, the tightest constraints from the bin with the largest volume. The gray shaded ellipse shows the constraints combining all three bins assuming they are statistically independent. Combined, we get ∼0.28% uncertainty on $R$. This is more optimistic than Spergel et al. (2015) because we model the full shape of the power spectrum, including the broadband shape.

On the left panel of Figure 23, we show how the projected uncertainty on $R$ changes by less than a percent over most of the plot. Center: similar to the left plot, but for the change in the uncertainty on the AP parameter $d_{\Delta H}$ starting with $\Delta (d_{\Delta H})_0 = 0.49\%$. Right: here we show the expected constraints on $f_8$, the best constraints being $\Delta (f_8)_0 = 1.2\%$ at $f = g = 0$. In all plots we assume Case B.

In all plots we assume Case B.
focused on two relatively narrow-field surveys, HETDEX and WFIRST.

For HETDEX, we define the interloper fraction $f$ of the LAE sample and the interloper fraction $g$ of the OIIE sample in Equations (4)–(5). For WFIRST, we define $f$ to be the interloper fraction in the Hα sample, and $g$ the interloper fraction in the [O III] sample. We then derive the effect of interlopers on the power spectrum in Equations (18), (20), and (22). The change in the power spectrum is given in terms of two geometrical factors defined in Equation (7): the direction perpendicular to the line of sight gets scaled by the factor $\alpha$, while the direction parallel is scaled by the factor $\beta$. For the two surveys that we study, $\alpha \approx \beta$, and thus the projection introduces anisotropies in the two-dimensional galaxy power spectrum, in addition to RSDs. The volume factor $\alpha^2\beta$ also multiplies the contamination contribution from the interloper power spectrum. In Appendix B, we also provide the rigorous derivation of the shot noise under the assumption that the galaxies are a Poisson sample of the underlying continuous galaxy density field, and the interloper fraction plays the role of the probability of having contamination.

We then investigate the joint-analysis method including the auto-power spectra of both samples as well as the cross-power spectrum as observables. We show that the joint analysis yields robust measurements of the interloper fractions, and it removes the interloper bias, a systematic shift of the best-fitting cosmological parameters when ignoring the interlopers. Although measuring and marginalizing the interloper fractions increase the measurement uncertainties in cosmological parameters, it does not bias their maximum-likelihood values. We explicitly show this for the geometrical parameters (angular diameter distance and Hubble expansion parameter) as well as the dynamical parameters (linear growth rate and $\sigma_8$), higher order RSD parameters (Section 4.7), and non-Gaussianity (Section 4.8).

For the joint analysis, we investigate several models for the power spectra of the main survey galaxies and the interlopers. We consider four cases:

Case A: This case makes minimal assumptions, only assuming that the true cross-correlation vanishes; see Section 4.3.
Case B: This case makes maximal assumptions, assuming that the RSDs and the shapes of the 1D isotropic power spectra are modeled well by theory; see Section 4.4.
Case C: Similar to Case B, but we only assume to be able to model the isotropic auto-correlation power spectrum of the higher redshift sample; see Section 4.5.
Case D: Here we assume we can model only the RSDs of the two samples; see Section 4.6.

By doing a Fisher analysis, we show that the constraints on the interloper fractions $f$ and $g$ are essentially the same for the two extreme cases A and B (see Figure 9), confirming that the information comes primarily from the cross-correlation.

Naturally, there is a large continuum of intermediate cases, and which one to use needs to be assessed on a survey-by-survey basis. For HETDEX, while the main LAE samples probe the quasilinear scales at high redshift, the interlopers are at low redshift and probe scales deep into the nonlinear regime. Thus, we assume Case C to be the baseline, where we marginalize over the 1D OIIE power spectrum. For WFIRST, we take Case B as the baseline, because both the main sample
and interloper sample probe the quasilinear scales at high redshifts.

This paper shows that the better the line classification, the tighter we can constrain the cosmological parameters. For the astrophysical methods of line classification, our joint-analysis method provides an estimate of the total contamination fractions. The usual approach to complement the emission-line surveys is to have follow-up imaging data in the same survey footprint. For HETDEX, Leung et al. (2017) developed a Bayesian framework taking into account the equivalent width distribution of LAEs and OIIEs, and searching for other lines that may be present in the spectrum. For WFIRST, Pullen et al. (2016) investigated the use of sensitive photometric data. All of these methods work for the classification of individual emission-line galaxies. The estimated interloper fractions from the joint analysis then provide a global figure of merit, based on which we can modify the line-identification criteria to reach smaller interloper fractions. The interplay between the two methods will provide a way to optimize the analysis pipeline for emission-line galaxy surveys.

Although we have not investigated further, one can also incorporate cross-correlations into external data sets. For example, for HETDEX, low-redshift galaxy samples from SDSS (Eisenstein et al. 2011) that correlate with the OIIE sample should provide an extra constraint on the contamination fraction $g$ when cross-correlating the low-redshift galaxies with the high-redshift LAEs.

Although we have not investigated further, one can also incorporate cross-correlations with external data sets. For example, for HETDEX, low-redshift galaxy samples from SDSS (Eisenstein et al. 2011), or radio catalogs from LOFAR (Shimwell et al. 2019), or APERTIF (Adams et al. 2018) that correlate with the OIIE sample should provide an extra constraint on the contamination fraction $g$ when cross-correlating the low-redshift galaxies with the high-redshift LAEs.

The results of this paper required two key simplifying assumptions: a flat-sky approximation and no redshift evolution in the interloper fractions, the galaxy bias, or the linear growth rate. To address these caveats in the future and to apply the joint-analysis method to wide-angle galaxy surveys including Euclid and SPHEREx, we must extend the method to spherical harmonic space while incorporating astrophysically motivated assumptions about the redshift evolution of key parameters. For example, Leung et al. (2017) predicted the interloper fractions, $f$ and $g$, as a function of redshift for LAEs and OIIEs, which could be incorporated in the future.

The authors thank Charles Bennett and Graeme Addison for useful discussion and comments. We thank the anonymous referee for useful comments that helped to clarify the paper. This work was supported at Pennsylvania State University by NSF grant (AST-1517363) and NASA ATP program (80NSSC18K1103).

Appendix A

Transformation between Misidentification and Interloper Fractions

In this appendix, we seek a better understanding of the mapping between the misidentification fractions $x_{\text{LAE}}$ and $x_{\text{O II}}$, and the interloper fractions $f$ and $g$. We introduced $f$ and $g$ because they are relevant for relating the true and observed power spectra. However, $x_{\text{LAE}}$ and $x_{\text{O II}}$ are a more natural choice for the process of misidentification. We will assume that the misidentification of LAEs as OIIEs is independent from the misidentification of OIIEs as LAEs. That is, we assume that there are two degrees of freedom describing interlopers.

We assume that when performing a forecast for a survey such as HETDEX, the fiducial values for the true numbers of galaxies $N_{\text{LAE}}$ and $N_{\text{O II}}$ are given, e.g., from previously measured luminosity functions (e.g., Ciardullo et al. 2012). This implies that the true interloper fractions $f_{\text{true}}$ and $g_{\text{true}}$ cannot take on arbitrary values. For example, for HETDEX, we expect there to be about twice as many OIIEs as LAEs. Therefore, the fraction of OIIEs in the LAE sample cannot exceed two-thirds unless some LAEs are misidentified as well. Hence, it is not possible to have more than two-thirds OIIEs in the LAE sample at the same time as an uncontaminated OIIE sample, given the true numbers of galaxies.

To find which interloper fractions are physical, we consider the situation when either the observed LAE number density or the observed OIIE number density vanishes. Then, Equations (4)–(5) imply that the limiting cases are on the two lines $f_{\text{true}} = f_{\text{lim}}$ and $g_{\text{true}} = g_{\text{lim}}$, where

$$f_{\text{lim}} = \frac{N_{\text{O II}}}{N_{\text{LAE}} + N_{\text{O II}}},$$

$$g_{\text{lim}} = \frac{N_{\text{LAE}}}{N_{\text{LAE}} + N_{\text{O II}}} = 1 - f_{\text{lim}},$$

where the numbers are the true numbers of galaxies. The lines $f_{\text{true}} = f_{\text{lim}}$ and $g_{\text{true}} = g_{\text{lim}}$ mark boundaries between allowed and disallowed (unphysical) regions for the interloper fractions. By requiring positive number densities, we find that interloper fractions $f_{\text{true}} < f_{\text{lim}}$ are unphysical unless $g_{\text{true}} < g_{\text{lim}}$. Similarly, the interloper fractions $f_{\text{true}} > f_{\text{lim}}$ are unphysical unless $g_{\text{true}} > g_{\text{lim}}$. Thus, we have two allowed regions and two unphysical regions, as shown in the right panel of Figure 26, where we use $N_{\text{O II}}/N_{\text{LAE}} = 2$. Thus, the true interloper fractions must be chosen from the allowed regions $A_1$ or $B_1$ shown in the figure.

Fixing the ratio of true numbers $N_{\text{O II}}/N_{\text{LAE}}$, we derive the nonlinear transformation between interloper fractions and misidentification fractions from Equations (2)–(5) to get

$$f_{\text{true}} = \left(1 + \frac{1 - x_{\text{O II}}}{x_{\text{O II}} - x_{\text{O II}}^{\text{true}}} N_{\text{LAE}}/N_{\text{O II}}\right)^{-1},$$

$$g_{\text{true}} = \left(1 + \frac{1 - x_{\text{O II}}}{x_{\text{LAE}} - x_{\text{O II}}^{\text{true}}} N_{\text{O II}}/N_{\text{LAE}}\right)^{-1}.$$

To gain a better understanding of the transformation, we show the possible values for $x_{\text{LAE}}$ and $x_{\text{O II}}$ in the left panel of Figure 26. The points $(0, 0)$ and $(1, 1)$ in $x_{\text{LAE}}x_{\text{O II}}$ space become $(0, 0)$ and $(1, 1)$ in fg space. The point $(0, 1)$ becomes the line $f_{\text{true}} = f_{\text{lim}}$ (because $g$ is undetermined, due to the observed number of OIIEs vanishing in this case) and $(1, 0)$ becomes $g_{\text{true}} = g_{\text{lim}}$ (because the observed number of LAEs vanishes). The points on the diagonal $x_{\text{LAE}} + x_{\text{O II}} = 1$ map onto the single point $(f_{\text{lim}}, g_{\text{lim}})$, because the LAE and OIIE samples have the same fraction of LAEs in each, and thus the observed auto-power spectra are the same. Furthermore, we have added three more lines: the diagonal $x_{\text{LAE}} = x_{\text{O II}}$ becomes the curved line in fg space, and the dashed and
dashed–dotted lines, which are curved in $x_{\text{LAE}}x_{\text{OII}}$ space, become diagonals of the allowed regions in fg space. Finally, for clarity, the subregions $A_i$ and $B_i$ have been labeled in the two panels of Figure 26 correspondingly.

The reason two regions appear in fg space is due to the true ratio $N_{\text{OII}}/N_{\text{LAE}}$ being fixed. However, when measuring the interloper fractions $f$ and $g$, this ratio cannot be assumed to be known. Rather, it must be viewed as a parameter to be determined in the fit. However, the ratio

$$r_{\text{obs}} \equiv \frac{N_{\text{OII}}}{N_{\text{LAE}}}$$

will be known, so that the transformation Equations (54)–(55) now become

$$x_{\text{LAE}} = \left(1 + \frac{1 - f}{g} r_{\text{obs}}^{-1}\right)^{-1},$$

$$x_{\text{OII}} = \left(1 + \frac{1 - g}{f} r_{\text{obs}}^{-1}\right)^{-1},$$

where now $r_{\text{obs}}$ is fixed instead of $r_{\text{true}} = N_{\text{OII}}/N_{\text{LAE}}$. This transformation has the same form as Equations (54)–(55), provided that we switch $x_{\text{LAE}} \leftrightarrow f$, $x_{\text{OII}} \leftrightarrow g$, and $r_{\text{true}} \rightarrow r_{\text{obs}}$. Thus, the picture is reversed: when only $r_{\text{obs}}$ is known, the full plane $0 \leq f \leq 1$ and $0 \leq g \leq 1$ are allowed. Indeed, because $r_{\text{true}}$ is allowed to vary, it is possible to find that the measured values $f$, $g$ will be within the unphysical regions. Finally, we note that now there are restrictions on the physically allowed values for $x_{\text{LAE}}$ and $x_{\text{OII}}$, similar to those for $r_{\text{true}}$ and $g_{\text{true}}$ in the right panel of Figure 26.

## Appendix B

### The Statistics of Galaxy Samples Contaminated with Interlopers

In the main text, we have derived the galaxy two-point correlation functions of the contaminated galaxy samples based on the underlying, continuous density fields and the relations between them (Equations (9)–(10)). In this appendix, we will extend the derivation including the discrete, point-like nature of the observed galaxy distribution. This analysis clarifies the shot-noise contribution to the two-point correlation functions from the contaminated galaxy sample.

For the analysis in this appendix, we will assume that the galaxy distribution is a Poisson-sampling of the underlying galaxy density field. Note that in the main text, when considering the relationship between the underlying, smooth galaxy fields, we set the misidentification fractions $x_{\text{LAE}}$ and $x_{\text{OII}}$ to be constant. When dealing with the statistics of galaxies, however, we need to take into account that the misidentification fractions are no longer constants. When estimating the density contrast, the survey volume is often divided into a grid of small cells. If the grid is small enough, for example, then each cell will host zero or one galaxy, and, in such an extreme case, the misidentification fraction in each cell can be either $x = 0$ (when the identification is correct), or $x = 1$ (when the identification is wrong).

Therefore, when calculating the contaminated power spectrum measured from discrete points such as galaxies, we need to take into account the distribution of misidentification fractions. We accommodate this by assuming that the misidentification is a stochastic process governed by the probability given by the mean misidentification fractions $\bar{x}_{\text{LAE}}$ and $\bar{x}_{\text{OII}}$.

In this section, we consider the statistics of two generic galaxy populations that we refer to as 1 and 2, which can be, for example, LAEs and OIIEs. Let us call the number of galaxy population 1 and 2 in a given cell $n_1$ and $n_2$, the probability of misidentification $p_1$ and $p_2$, and the true number of misidentified galaxies $m_1$ and $m_2$. Then, in the observed sample, we register

$$n_1^{\text{obs}} = n_1 - m_1 + m_2,$$

$$n_2^{\text{obs}} = n_2 - m_2 + m_1$$

galaxies as population 1 and 2.

While we only consider the case with constant mean misidentification fraction $\bar{x}$ (thus, constant probability $p$) and the constant mean density $n$ in the main text, here, we
generalize the situation by considering their spatial variation. That is usually the case for realistic galaxy surveys where the survey conditions vary over different telescope pointings. We show that including the spatial variation contributes to the survey window function.

### B.1. One-point Statistics: Distribution of Misidentification Fractions

First, let us focus on a sufficiently small volume of cells with a given mean number of galaxies $\mu_n$. Under the assumption that the galaxies are Poisson draws, the misidentified galaxies (with misidentification probability $p$) are also Poisson draws with a modified mean number $\mu'_m = p\mu_n$. We show that as follows.

Consider a cell with $n$ galaxies. With the probability $p$ of misidentification, the probability of misidentifying $m$ (that is, $n-m$ galaxies are correctly identified) is given by

$$P(m|n) = \left(\frac{n}{m}\right)p^m(1-p)^{n-m} = \frac{n!}{m!(n-m)!}p^m(1-p)^{n-m}$$

when $n \geq m$, and 0 otherwise. We then calculate the probability of having $m$ misidentified galaxies by marginalizing the union probability $P(m \cap n)$ as follows:

$$P(m) = \sum_{n=0}^{\infty} P(m \cap n) = \sum_{n=m}^{\infty} P(m|n)P(n)$$

$$= \sum_{n=m}^{\infty} \frac{n!}{m!(n-m)!}p^m(1-p)^{n-m}e^{-\mu} \frac{\mu^n}{n!}$$

$$= (\mu p)^m e^{-\mu} \sum_{n=m}^{\infty} \frac{1}{(n-m)!}\left[\mu e(1-p)\right]^{n-m}$$

$$= (\mu p)^m e^{-\mu p},$$

which completes the proof. It follows that the mean and the variance of the misidentified galaxies in the cell are

$$\mu_m \equiv \langle m \rangle = p\mu_n,$$

$$\sigma_m^2 \equiv \langle m^2 \rangle - \langle m \rangle^2 = p\mu_n,$$

where $p$ is the misidentification probability in a given cell, which is the same as the mean misidentification fraction $\bar{x}$.

Using the Poisson probability distribution function, we also calculate the one-point covariance between the total number of galaxies $n$ and the misidentified galaxies $m$:

$$\langle nm \rangle - \langle n \rangle \langle m \rangle = \sum_{n=0}^{\infty} \sum_{m=0}^{n} nm \times P(n)P(m|n) - \mu_n \mu_m$$

$$= \sum_{n=0}^{\infty} n e^{-\mu} \frac{\mu^n}{n!} \sum_{m=0}^{n} m \times \frac{n!}{m!(n-m)!}p^m(1-p)^{n-m}$$

$$= \mu p^2.$$

Note that we use the following identity to calculate the second summation:

$$a \partial_a[(a+b)^n] = an(a+b)^{n-1} - \sum_{m=0}^{n} m \frac{n!}{m!(n-m)!}a^mb^{n-m} = an(a+b)^{n-1}.$$
Using the two-point correlators we have calculated above, the observed two-point correlation function \( \xi_1^{\text{obs}}(x) = \langle \delta_1^{\text{obs}}(r) \delta_1^{\text{obs}}(r') \rangle \) as a function of the separation \( x = r - r' \) becomes

\[
\bar{\eta}_1^{\text{obs}}(r) \bar{\eta}_1^{\text{obs}}(r') \xi_1^{\text{obs}}(x) = \left[ \bar{n}_1(r) - \bar{m}_1(r) \right] \times \left[ \bar{n}_1(r') - \bar{m}_1(r') \right] + \{ \left[ \bar{n}_1(r) - \bar{m}_1(r) \right] \bar{m}_2(r') + \bar{m}_2(r) \left[ \bar{n}_1(r') - \bar{m}_1(r') \right] \}
\]

\[
\times \xi_{12}(x) + \bar{\eta}_1^{\text{obs}}(r) \delta_D(x).
\]

(75)

Now we define the local value of \( f(r) \) consistent with the one we define in Equation (4) as

\[
f(r) = \frac{m_2(r)}{\bar{n}_1^{\text{obs}}(r)} = \frac{m_2(r)}{\bar{n}_1(r) - \bar{m}_1(r) + \bar{m}_2(r)},
\]

(76)

with which we simplify the expression for the observed two-point correlation function as

\[
\xi_1^{\text{obs}}(x) = [1 - f(r)] [1 - f(r')] \xi_1(x) + f(r) f(r') \xi_2(x) + \{ [1 - f(r)] f(r') \}
\]

\[
+ f(r) [1 - f(r')] \xi_{12}(x) + \frac{1}{\bar{n}_1^{\text{obs}}(r)} \delta_D(x).
\]

(77)

The spatially varying misidentification fraction, therefore, acts just like the survey window function effect (Feldman et al. 1994). Of course, by analogy, we find the observed two-point correlation function of the second galaxy population as

\[
\xi_2^{\text{obs}}(x) = [1 - g(r)] [1 - g(r')] \xi_2(x) + g(r) g(r') \xi_1(x) + \{ [1 - g(r)] g(r') \}
\]

\[
+ g(r) [1 - g(r')] \xi_{12}(x) + \frac{1}{\bar{n}_2^{\text{obs}}(r)} \delta_D(x).
\]

(78)

For constant \( f \) and \( g \), the equations reduce to, respectively, Equations (11) and (12), except for the shot-noise contributions that we have not included in the main text. Note, however, that the shot-noise contribution is given by the total observed number density of galaxies, not by taking Equations (11) and (12) replacing the individual two-point correlation function with the respective shot-noise contribution, that is, \( \xi_1(x) \rightarrow \xi_1(x) + 1/\bar{n}_1^{\text{obs}} \delta_D(x) \).

As we have discussed in the main text, when considering two populations, such as LAEs and OIIEs, that are far away, the direct correlation \( \xi_{12}(x) \) must be negligible compared to the their autocorrelations. We then find that, by taking the Fourier transform, the observed power spectrum is given by the convolution as

\[
\langle |b_1^{\text{obs}}(k)|^2 \rangle = \int \frac{d^3q}{(2\pi)^3} |(2\pi)^3 \delta_D(k - q) - f(k - q)|^2 P_1(q) + \int \frac{d^3q}{(2\pi)^3} |(k - q)|^2 P_2(q)
\]

\[
+ \int d\kappa \frac{1}{\bar{n}_1^{\text{obs}}(r)}.
\]

(79)

B.3. Two-point Statistics: Cross-correlation

Similarly, we calculate the cross-correlation function by

\[
\bar{\eta}_1^{\text{obs}}(r) \bar{\eta}_2^{\text{obs}}(r') \langle \delta_1^{\text{obs}} \delta_2^{\text{obs}} \rangle
\]

\[
= [1 - f(r)] g(r') \xi_2(x) + f(r) [1 - g(r')] \xi_1(x) + \{ [1 - f(r)] [1 - g(r')] \}
\]

\[
+ f(r) g(r') \xi_{12}(x),
\]

(80)

which generalizes Equation (21).

B.4. False Detections

It is straightforward to model random, uncorrelated false detections. Here, we briefly show how they modify the observed power spectrum. If “0” signifies false detections, “1” signifies LAEs, and “2” signifies OIIEs, then the number of objects classified as false detections, LAEs, and OIIEs are

\[
n_0^{\text{obs}} = n_0 - m_{01} - m_{02} + m_{10} + m_{20},
\]

(81)

\[
n_1^{\text{obs}} = n_1 - m_{10} - m_{12} + m_{01} + m_{21},
\]

(82)

\[
n_2^{\text{obs}} = n_2 - m_{20} - m_{21} + m_{02} + m_{12},
\]

(83)

where \( n_i \) are the true number of objects and \( m_j \) are the number of objects of type \( i \) misidentified as type \( j \). Introducing the six independent average interloper fractions

\[
f_{ij} = \frac{\bar{m}_{ij}}{\bar{n}_j^{\text{obs}}},
\]

(84)

we can write the observed power spectra as

\[
P_0^{\text{obs}} = f_0^2 P_1 + f_0^2 P_2,
\]

(85)

\[
P_1^{\text{obs}} = (1 - f_0 - f_2) P_1 + f_1^2 P_2 + f_2^2 P_2,
\]

(86)

\[
P_2^{\text{obs}} = (1 - f_0 - f_2) P_1 + f_1^2 P_2 + f_2^2 P_2,
\]

(87)

\[
P_{01}^{\text{obs}} = (1 - f_0 - f_2) P_{01} + f_1 P_{02} + f_2 P_{12},
\]

(88)

\[
P_{02}^{\text{obs}} = (1 - f_0 - f_2) P_{02} + f_1 P_{01} + f_2 P_{12},
\]

(89)

\[
P_{12}^{\text{obs}} = (1 - f_0 - f_2) P_{12} + f_1 P_{01} + f_2 P_{02},
\]

(90)

where we assume that the true cross-correlations all vanish, and that the false detections do not cluster, i.e., \( P_0 = 0 \). To first order, the last three equations will allow us to measure \( f_{00}, f_{20}, f_{12}, \) and \( f_{21} \). That is, it will be possible to measure the contribution of LAEs and OIIEs in the false-detection sample \( f_{00} \), but the contribution of false detections in the two galaxy samples \( f_{02}, f_{20} \) will need to be assessed via other methods. For example, in the context of HETDEX, we can assume that \( f_{02} = 0 \), because false detections will not show up in the continuum photometry, and, thus, will be exclusively misclassified as LAEs.

Also apparent from Equations (85)–(90) are the following two effects. First, false detections reduce the observed power in the autocorrelations by a factor \( (1 - f_0)^2 \). This can be seen by writing in Equation (86) \( f_{21} = (1 - f_0) f \), where \( f \equiv \bar{m}_{21}/\bar{n}_1^{\text{obs}} \). With \( f_{12}^{\text{obs}} \equiv \bar{n}_2^{\text{obs}} - \bar{m}_2 \) the number of galaxies (either LAE or OIIIE) in the LAE sample. Second, the shot noise for the LAE sample will be \( 1/\bar{n}_1^{\text{obs}} = (1 - f_0)/\bar{n}_1^{\text{obs}} \). Thus, false detections...
reduce the power of a survey by
\[ \hat{n}_i P_1 \rightarrow (1 - f_{\text{th}}) \hat{n}_i P_1. \] (91)

**Appendix C**

**Gaussian Covariance of Power Spectrum**

We estimate the covariance of the galaxy auto- and cross-power spectra by assuming that the galaxy density fields follow Gaussian statistics; the connected higher order correlators are determined by the multiplications among the disconnected two-point correlators that we calculate by Wick’s theorem. This assumption works well in estimating the uncertainties (diagonal covariance) of nonlinear matter power spectrum in a suite of N-body simulations (Jeong & Komatsu 2009).

To get the general expression applicable for multiple galaxy populations, let us start from the cross-power spectrum between the two populations, labeled x and y, for which we can estimate the cross-power spectrum as
\[ \hat{P}_{xy}(k) = \frac{1}{N_k V_x} \sum_{i=1}^{N_N} \frac{1}{2} (\delta^{*}_{x,i} \delta^{*}_{y,i} + \delta^{*}_{y,i} \delta^{*}_{x,i}). \] (92)

Here, \( \delta^{*}_{x,i} \equiv \delta_{x(i)} \) is the Fourier-space density contrast, and \( N_k \) is the total number of Fourier modes contributing to the estimation. For example, when estimating the monopole, \( P_{xy}(k) \), \( N_k \) equals the number of discrete Fourier vectors satisfying \( |k| \sim k \), \( N_k = 4 \pi k \Delta k V_x/(2\pi)^3 \) for the Fourier-space radial bin size \( \Delta k \); when estimating the two-dimensional power spectrum, \( P(k, k) \), \( N_k \) equals the number of discrete Fourier vectors within the cylinders of total Fourier volume of \( V_k = 4 \pi k^2 \Delta k V_x \Delta k_0 \), including both positive and negative \( k \). The configuration-space volume \( V_x \) appears for the normalization.

The estimator of Equation (92) is unbiased because the statistical homogeneity demands that the ensemble average of each term in Equation (92) must be the cross-power spectrum which is defined as
\[ \langle \delta^{*}_{x,i} \delta^{*}_{y,i} \rangle = \delta^{*}_{x,i} V_x P_{xy}(k), \] (93)

where \( \delta^{*}_{i} \) is a Kronecker delta. Furthermore, being the complex conjugate of each other, each contribution in Equation (92) is a real number, and so is the cross-power spectrum. We, therefore, take only the real part of the cross-power spectrum \( P_{xy}(k) \) that can be in general a complex quantity (Bonvin et al. 2014). Specifically, this is equivalent to assuming an even-parity cross-power spectrum, \( P_{xy}(k) = P_{xy}(-k) \).

In order to calculate the covariance matrix, we need other types of two-point correlators that we can derive from Equation (93) using the reality of the galaxy density field: \( \delta^{*}_{x} = \delta_{x} \), with negative indices standing for the Fourier modes with negative wave vectors \( (k_{-j} = -k_j) \). They are
\[ \langle \delta_{x,i} \delta^{*}_{x,j} \rangle = \langle \delta_{y,i} \delta^{*}_{y,j} \rangle = \langle \delta_{x,i} \delta^{*}_{y,j} \rangle = \langle \delta_{y,i} \delta^{*}_{x,j} \rangle = \delta_{i-j} V_x P_{xy}(k). \] (94)

Using them, finally we calculate the covariance between two cross-power spectra \( \hat{P}_{xy}(k) \) and \( \hat{P}_{zv}(k) \) with the Equation (92) estimator as follows. Note that we consider the same binning scheme for \( P_{xy} \) and \( P_{zw} \) for which case the only nonzero covariance is
\[ \langle \hat{P}_{xy}(k) \hat{P}_{zw}(k) \rangle = \langle \hat{P}_{xy}(k) \rangle \langle \hat{P}_{zw}(k) \rangle = \frac{1}{N_k} (P_{xy}(k) P_{zw}(k) + P_{xv}(k) P_{yz}(k)). \] (95)

We will apply the general expression in Equation (95) to the cases that we use in the main text of the paper.

**C.1. Auto-power Spectrum**

For the variance of the auto-power spectrum (with \( x = y = z = w \)), Equation (95) reduces to
\[ \sigma^2_{P_k(k)} = \frac{2}{N_k} \left( P_k(k) + \frac{1}{n} \right)^2, \] (96)

where we included shot noise as \( P_x(k) = P_k(k) + 1/n \), where \( P_k(k) \) is the power spectrum of the underlying field.

**C.2. Cross-power Spectrum**

The cross-correlation (with \( x = z \) and \( y = w \)) has the variance
\[ \sigma^2_{P_{xy}(k)} = \frac{1}{N_k} \left( P_{xy}(k) + 1/n \right) \left( P_{xy}(k) + 1/n \right) + P_{xy}^2(k). \] (97)

**C.3. Variance of the Multipole Power Spectra**

The multipole power spectrum is defined as the \( k \)-dependent coefficient of the multipole expansion:
\[ P_{xy}(k) = \sum_{\ell=0}^{\infty} P_{xy}^\ell(k) \ell^2 P_{\ell}(\mu), \] (98)

with the Legendre polynomials \( P_{\ell}(\mu) \). Here, the assumption is that the power spectrum \( P_{xy}(k) \) also depends on the wavenumber \( k \) and the polar angle \( \mu = \hat{n} \cdot \hat{k} \). Using the orthogonality of the Legendre polynomials,
\[ \int_{-1}^{1} d\mu P_{\ell}(\mu) P_{\ell}(\mu) = \frac{2}{2\ell + 1} \delta_{\ell,0}, \] (99)

we find the expression for the multipole power spectrum as
\[ P_{xy}^\ell(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu P_{\ell}(\mu) P_{xy}(k). \] (100)

The estimator for the multipole power spectrum is, using \( \int_{-1}^{1} d\mu/2 = 1/N_k \sum_{i=1}^{N_N} \), which works for a sufficiently large number of \( N_k \),
\[ \hat{P}_{xy}^\ell(k) = \frac{2\ell + 1}{\frac{1}{N_k} \sum_{i=1}^{N_N} P_{\ell}(\mu) \frac{1}{2} (\delta^{*}_{x,i} \delta^{*}_{y,i} + \delta^{*}_{y,i} \delta^{*}_{x,i}). \] (101)

Note that \( \ell = 0 \) corresponds to the monopole power spectrum. We calculate the covariance of this estimator using the same assumption that we have adopted earlier in this appendix and obtain that
\[ \langle \hat{P}_{xy}^\ell(k) \hat{P}_{zw}^\ell(k) \rangle = \langle \hat{P}_{xy}^\ell(k) \rangle \langle \hat{P}_{zw}^\ell(k) \rangle = \frac{(2\ell + 1)^2}{N_k} \int_{-1}^{1} d\mu P_{xy}(k) P_{zw}(k) + P_{xv}(k) P_{yz}(k). \] (102)
This equation is consistent with Equation (8) of Taruya et al. (2011). Therefore, the variance of the multipole of the auto-power spectrum is

$$
\sigma^2_{P^r(k)} = \frac{2(2\ell + 1)^2}{N_k} \int_{-1}^{1} \frac{d\mu}{2} P^2_\ell(\mu) \left[ P_r(k, \mu) + \frac{1}{n_x} \right]^2,
$$

and that of the cross-power spectrum is

$$
\sigma^2_{P^c(k)} = \frac{(2\ell + 1)^2}{N_k} \int_{-1}^{1} \frac{d\mu}{2} P^2_\ell(\mu)
\times \left[ P_r(k, \mu) + \frac{1}{n_x} \right] \left( P_c(k, \mu) + \frac{1}{n_x} \right) + P^2_{xy}(k, \mu). \tag{103}
$$

### Appendix D
#### Systematic Bias

In this appendix, we derive the systematic bias of a maximum-likelihood estimator relative to some reference maximum-likelihood estimator. Specifically, we consider two situations: first, when the measured power spectrum differs from the model power spectrum by some $\Delta P(k)$, e.g., due to interlopers, and, second, when the $\Delta P(k)$ encapsulate differences between realizations, e.g., to justify our use of the ensemble-averaged log-likelihood function.

In either case, our goal is to see how the estimated parameters $\hat{\theta}$ differ from those estimated by the reference $\hat{\theta}_{ref}$. At $\hat{\theta}$, the Jacobian $J = -(\ln \mathcal{L})_\theta$ must vanish. Expanding in $\Delta \theta = \hat{\theta} - \hat{\theta}_{ref}$, we get

$$
0 = J(\hat{\theta}) = J(\hat{\theta}_{ref}) + F(\hat{\theta}_{ref}) \Delta \theta + \mathcal{O}(\Delta \theta^2), \tag{105}
$$

where $F = -(\ln \mathcal{L})_{\theta \theta}$ is the Fisher information matrix. Now we assume that $\Delta \theta$ is small. That is, we assume that the Fisher information matrix can be written as

$$
F = F_{ref} + \mathcal{O}(\Delta \theta^2),
$$

where $F_{ref}$ is the Fisher information matrix of the reference log-likelihood function. Equation (105) can then be solved for $\Delta \theta$. Also, in all cases of interest to us, we have that the Jacobian $J(\hat{\theta}_{ref}) \propto \Delta P$ (see Equation (10) below). Thus, the bias is

$$
\Delta \theta = -F_{ref}^{-1}(\hat{\theta}_{ref}) J(\hat{\theta}_{ref}) + \mathcal{O}(\Delta \theta^2, \Delta \theta^2). \tag{107}
$$

### D.1. Interloper Bias

To predict the systematic bias due to interlopers using Equation (107), we need $J(\hat{\theta}_{ref})$. In this case, the reference likelihood function is the unbiased interloper-free likelihood, and the full likelihood is

$$
\ln \mathcal{L}(\theta) = \sum_k \frac{N_k}{2} \ln P(k; \theta) + \sum_k \frac{N_k}{2} \ln \hat{P}(k) + \frac{1}{2} \Delta P(k) + \mathcal{O}(\Delta \theta^2)
\times \frac{1}{2} \Delta P(k) + \mathcal{O}(\Delta \theta^2).
$$

where $\Delta P(k) = \int (\Delta P(k; \hat{\theta}_{ref}) + \mathcal{O}(\Delta \theta^2))$. The change in the LAE power spectrum produced by interlopers is

$$
\Delta P(k) = -(2 - \beta) P_{LAE}(k) + \frac{1}{2} \Delta P_{\text{proj}}(k) + 2(1 - \beta) P_{LAE, \text{non}}(k).
$$

Thus, the Jacobian evaluated at $\hat{\theta}_{ref}$ is

$$
J(\hat{\theta}_{ref}) = -\sum_k \frac{N_k}{2} P(k; \hat{\theta}_{ref}) \frac{\Delta P(k)}{P(k; \hat{\theta}_{ref})},
$$

which allows us to calculate the systematic bias using Equation (107).

### D.2. Ensemble-averaged Log-likelihood Function

The second application of Equation (107) is to justify using the ensemble-averaged log-likelihood function $\langle -\ln \mathcal{L} \rangle$ to assess the bias of our estimator. In this case, the reference estimator is the maximum of the ensemble-averaged log-likelihood function (which may be biased to begin with), and $\Delta P(k)$ encapsulates the differences between realizations. Thus, the likelihood function (Equation (34)) can be written similarly to Equation (108) as

$$
\ln \mathcal{L}(\theta) = \sum_k \frac{N_k}{2} \ln P(k; \theta) + \sum_k \frac{N_k}{2} \ln \hat{P}(k) + \frac{1}{2} \Delta P(k) + \mathcal{O}(\Delta \theta^2).
$$

The likelihood function (Equation (34)) can be written similarly to Equation (108) and, thus, $J \propto \Delta P$. However, this time, we are interested in the ensemble average $\langle \Delta \hat{\theta} \rangle$, which will tell us whether the ensemble of Monte Carlo realizations will give the same bias as a single MCMC run on the ensemble average. Because $\langle \Delta P \rangle = 0$, Equation (107) becomes

$$
\langle \Delta \hat{\theta} \rangle = 0 + \mathcal{O}(\Delta \theta^2, \Delta \theta^2),
$$

which confirms that the two methods agree to first order. We have also verified this result by generating 100 realizations of the power spectra and comparing with a single MCMC run on the ensemble-averaged log-likelihood. We performed this both with and without misidentification. The biases are negligible second-order terms in Equation (112).

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