Speeding up adiabatic population transfer in a Josephson qutrit via counter-diabatic driving

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Keywords: shortcut to adiabaticity, population transfer, Josephson qutrit

Abstract

We propose a theoretical scheme to speed up adiabatic population transfer in a Josephson artificial qutrit by transitionless quantum driving. At a magic working point, an effective three-level subsystem can be chosen to constitute our qutrit. With Stokes and pump driving, adiabatic population transfer can be achieved in the qutrit by means of stimulated Raman adiabatic passage. Assisted by a counter-diabatic driving, the adiabatic population transfer can be sped up drastically with accessible parameters. Moreover, the accelerated operation is flexibly reversible and highly robust against decoherence effects. Thanks to these distinctive advantages, the present protocol could offer a promising avenue for optimal coherent operations in Josephson quantum circuits.

1. Introduction

Superconducting quantum circuits, which behave like artificial atoms, possess many appealing advantages such as an adjustable level structure, flexible controllability and convenient measurement [1–4]. Coherent control over artificial atoms through varying external bias voltages and magnetic fluxes has attracted considerable attention over recent years [5–9]. The fruitful realization of dynamical behavior is closely related to many (Ξ-, Λ-, V- and Δ-type) configurations of interactions between the artificial atoms and the driving fields [3]. Especially, allowed by the level-transition rule, the novel Δ-type interaction only occurs in artificial atoms [10, 11], which has no counterpart in natural atoms. Based on these kinds of interactions, superconducting artificial atoms provide us with an excellent platform from which to explore quantum information science, quantum state engineering and fundamental quantum laws [12–15].

Quantum population transfer (QPT), acting as a coherent operation, is a critical issue in the context of quantum information processing [16–19]. The performance of optimal QPT has drawn increasing attention [20–23]. Based on the powerful method of stimulated Raman adiabatic passage (STIRAP) [16, 24], a great deal of effort has been devoted to studying QPTs with Josephson three-level systems both theoretically and experimentally [25–29]. Although adiabatic operations are not sensitive to timing errors and are robust against fluctuations of control parameters, STIRAP processes generally take relatively long times, which makes them undesirable in some practical cases. Therefore, the realization of faster population transfer is highly sought after [22, 30–32] and from which the desired quantum operation can be performed within a shorter time and decoherence effects can be greatly reduced. Another key point related to optimal QPT is the reversible transferring quantum state [33–35], which is a requisite for the storage and retrieval of quantum information.

A set of attractive techniques called ‘shortcut to adiabaticity’, consisting of invariant-based inverse engineering [36, 37], transitionless quantum driving (TQD) [38–40] and the fast-forward approach [41], has been put forward for speeding up adiabatic evolution processes, which can carry out the same target operation as that in the adiabatic process but within a shorter time. Thus the shortcut approach has attracted considerable attention [42–45]. In two- and three-level systems, the method of inverse engineering has been utilized.
extensively to speed up adiabatic population transfers [46, 47]. By designing counter-diabatic driving, TQD can control and steer the fast evolution of a system mimicking adiabaticity exactly. In recent years, the TQD method has been adopted widely for studying dynamical behaviors and state engineering with different systems [48–53]. Particularly, the application of TQD to the rapid generation of entangled states is a robust method of information processing [54, 55]. Quite recently, by using the TQD technique, the quantum coherent manipulation of superconducting artificial atoms has been investigated [56, 57], which indicated that the shortcut approach is feasible for performing accelerated quantum operations and is more robust against decoherence effects.

In this paper, we develop an effective scheme to speed up adiabatic population transfer in a Josephson qutrit by the TQD method. At a magic bias point, an isolated three-level subsystem of a Josephson quantum circuit constitutes our qutrit. Allowed by the level-transition rule, a Λ-configuration interaction between the qutrit and microwave drivings can be used for adiabatic population transfer by means of STIRAP. We then adopt the TQD technique to speed up the adiabatic QPT. Assisted by counter-diabatic driving, accelerated population transfer is performed in the qutrit with a Δ-configuration interaction. With accessible parameters, we further analyze the flexible reversibility and high robustness of the coherent operation. Through the combination of these advantages, the present scheme could provide an attractive approach to implementing optimal quantum operations on Josephson artificial atoms.

This paper is organized as follows. In section 2, we present an artificial qutrit of a Josephson quantum circuit. Section 3 demonstrates the adiabatic population transfer in the qutrit via STIRAP. In section 4, we focus on the accelerated population transfer assisted by a counter-diabatic driving, and then analyze the reversibility and robustness of the population transfer. Finally, a discussion and a conclusion are presented in section 5.

2. An artificial atom of a Josephson quantum circuit

As schematically depicted in figure 1, the Cooper-pair box (CPB) circuit under consideration contains a superconducting box with \( n \) extra Cooper pairs [1], in which the charging energy scale of the system is \( E_c \).

Through two symmetric Josephson junctions (with identical coupling energies \( E_J \) and capacitances \( C_J \)), the CPB is connected to a segment of a superconducting ring. In the charge-phase regime [58], the characteristic system parameter \( E_J \) has the same order of magnitude as \( E_c \), which satisfy \( \Delta \gg E_J \sim E_c \gg k_B T \), here the large energy gap \( \Delta \) prohibits quasiparticle tunneling, and \( k_B T \) stands for a low energy of thermal excitation. The CPB is biased by a static voltage bias \( V_{d} \) through a gate capacitance \( C_d \). Meanwhile, a static magnetic flux \( \Phi_{d} \) in the ring adjusts the effective Josephson coupling \( E_{Jd} \). AC gate voltages \( V_{s} \) and a time-dependent flux \( \Phi_{p} \) threads the ring. The microwave drivings are used to induce the desired level transitions [5, 59], as mentioned below.

In the absence of the microwave drivings, \( V_{s} \) and \( \Phi_{p} \), the static Hamiltonian of the CPB system is given by \( H_0 = E_c (n - n_d)^2 - E_{Jd} \cos \theta \), in which \( E_c = 2e^2/C \), with \( C_t = 2C_J + C_d + C_a + C_\sigma \) being the total capacitance, and \( \theta \) denotes the average phase difference of the two junctions, which is canonically conjugate to \( n \), i.e., \( \{ \theta, n \} = i \). The polarized gate charge induced by \( V_{d} \) is \( n_d = C_d V_{d}/2e \), and \( E_d = 2E_J \cos \left( \frac{\Phi_{d}}{2} \right) \) is the effective Josephson coupling, in which \( \Phi_0 = h/2e \) indicates the flux quantum. Within the basis of Cooper-pair
are chosen at \( V_t \), in which each state can be expressed as a superposition of Cooper-pair number states, namely, \( |g\rangle, |e\rangle, |a\rangle \) and \( |f\rangle \) are chosen at \( n_d = 0.5 \). Here, \( E_c \) has been taken as the zero-energy reference.

Figure 2. The first four eigenlevels \( E_k \) (in units of \( E_c \)) of the CPB system versus the gate charge \( n_d \), with \( k = g, e, a \) and \( f \). The system parameters are \( E_c / h = 3.0 \text{ GHz} \) and \( E_d = 1.5 E_c \). Level states \( |g\rangle, |e\rangle, |a\rangle \) and \( |f\rangle \) are almost vanishing. Since the amplitude \( E_d \) is much smaller than \( E_c \), the first four eigenlevels are plotted in figure 2. At a magic point of \( n_d = 0.5 \), we deal with four eigenstates \( |k\rangle \), in which each state can be expressed as a superposition of Cooper-pair number states, namely, \( |k\rangle = \sum n e^{i\phi} |n\rangle \), with \( e^{i\phi} \) being the superposition coefficients. The quantum states at the magic point are insensitive to the corresponding Stokes and pump drivings, are applied to induce the desired level couplings \( |e\rangle \leftrightarrow |a\rangle \) and \( |g\rangle \leftrightarrow |e\rangle \), respectively, where the microwave frequency \( \omega_{se} \) and \( \omega_{sg} \) are far away from each other. Sufficient anharmonicity can eliminate the leakage errors induced by the coherent drivings, which is beneficial for performing robust population transfer with the qutrit [27, 59].

3. Adiabatic population transfer with the qutrit via stimulated Raman adiabatic passage

As demonstrated in figure 3, two classical microwave fields \( \tilde{V}_s = V_s \cos(\omega_s t) \) and \( \tilde{V}_d = V_d \cos(\omega_d t) \), behaving as the corresponding Stokes and pump drivings, are applied to induce the desired level couplings \( |e\rangle \leftrightarrow |a\rangle \) and \( |g\rangle \leftrightarrow |e\rangle \), respectively, where the microwave frequency \( \omega_s \) and \( \omega_d \) are resonant with the transition frequency \( \omega_{se} \) and \( \omega_{sg} \). Note that the amplitudes \( V_s \) and \( V_d \) are controllable here. Different from previous works that induced level transitions only via voltage drivings [27, 59], the present scheme adopts both ac voltage and time-dependent bias flux. Due to the sufficient level anharmonicity, there exists a large detuning \( \delta_t = \omega_s - \omega_e \), leading to the \( \tilde{V}_s \)-induced transition between \( |g\rangle \) and \( |e\rangle \) almost vanishing. Since the amplitude \( V_s \) (\( \Phi_p \)) is much smaller than \( V_d \) (\( \Phi_b \)) in the present protocol, the effects of \( V_s \) and \( \Phi_p \) on the eigenlevels are negligible.

In what follows, we treat a \( \Lambda \)-type interaction between the qutrit and the microwave drivings. The interaction Hamiltonian between the microwave pulse \( \tilde{V}_r \) and the CPB system takes a diagonal form

\[
H_i = -2E_e \tilde{n}_e \sum_n (n - n_d) |n\rangle \langle n|,
\]

where \( \tilde{n}_e = n_e \cos(\omega_e t) \), with \( n_e = C_e V_s / 2\epsilon \). Here the fast oscillating term \( \tilde{n}_e^2 \) with higher frequency \( 2\omega_e \) has been omitted well under the rotation wave approximation (RWA) [21, 27]. The \( \tilde{V}_r \)-induced transition matrix element between \( |e\rangle \) and \( |a\rangle \) is \( t_{ea} = \langle e | H_i | a \rangle \). In terms of \( |e\rangle = \sum n e^{i\phi} |n\rangle \) and \( |a\rangle = \sum n e^{i\phi} |n\rangle \), we have \( t_{ea} = \Omega_e \cos(\omega_e t) \), where
\[ \Omega_p = -2E_p O_{oa} n_s \]  

indicates the Rabi coupling, with \( O_{oa} = \sum_a (n - n_a)c_{e a}^\dagger c_{ea} \) being the overlap between \(|e\rangle\) and \(|a\rangle\) at the bias point \( n_d \).

The magnetic coupling between the CPB system and the bias flux \( \Phi_p \) reads

\[ H_p = -\frac{E_p}{2} \sum_n \langle n | (n + 1) | + H.c. \rangle, \]

which has a non-diagonal coupling form different from \( H_s \). The amplitude of \( \Phi_p \) is much smaller than \( \Phi_0 \), which yields \( \cos \left( \frac{\Phi_p}{\Phi_0} \right) \approx 1 \) and \( \sin \left( \frac{\Phi_p}{\Phi_0} \right) \approx \pi \frac{\Phi_p}{\Phi_0} \). In this situation, the Josephson coupling induced by the time-dependent bias flux becomes \( E_p = 2E_p \frac{\Phi_p}{\Phi_0} \sin \left( \frac{\Phi_p}{\Phi_0} \right) \). Allowed by the parity-symmetry determined selection rule [60], the magnetic interaction Hamiltonian \( H_p \) gives rise to the wanted coupling between \(|g\rangle\) and \(|a\rangle\) at \( n_d = 0.5 \), the transition element of which is given by \( \langle g | H_p | a \rangle = \Omega_p \cos (\omega_p t) \), in which

\[ \Omega_p = -E_p \sin \left( \frac{\Phi_p}{\Phi_0} \right) O_{oa} \]  

denotes the relevant Rabi coupling, with \( O_{oa} = \sum_a c_{e a}^\dagger c_{ea} \langle n | (n + 1) | + H.c. \rangle |n\rangle \) being the overlap between \(|g\rangle\) and \(|a\rangle\).

Within the basis of \( \{|g\rangle, |a\rangle, |e\rangle\} \), the \( \Lambda \)-configuration interaction under the reference frame rotating at frequencies \( \omega_p \) and \( \omega_e \) can be expressed as

\[ H_I = \begin{pmatrix} 0 & \Omega_p/2 & 0 \\ \Omega_p/2 & 0 & \Omega_e/2 \\ 0 & \Omega_e/2 & 0 \end{pmatrix}, \]

where the RWA has been adopted [61] and the two-photon resonance is satisfied, i.e. \( \omega_{de} - \omega_p = \omega_{de} - \omega_e = 0 \). Hereafter we take \( \hbar = 1 \) for simple notation. Obviously, the Hamiltonian of equation (4) has a dark eigenstate \( |D\rangle \) (with zero eigenenergy), which is a superposition of \(|g\rangle \) and \(|e\rangle \). Since \( |D\rangle \) is not composed of \(|a\rangle\), the ideal population transfer within the subspace \( \{|g\rangle, |e\rangle\} \) can be performed if the state evolution is only along the \( |D\rangle \) under the adiabatic condition.

With the currently available parameters, we design two Rabi frequencies \( \Omega_e \) and \( \Omega_p \) for performing the desired population transfer. The ac voltage amplitude is set as \( n_s = 0.05 e^{-(-\gamma t)^2/\tau^2} \), where 0.05 is the maximum amplitude of the voltage pulse, and \( \tau_s = 57 \) ns and \( \tau = 27 \) ns are the pulse-related parameters. At the static bias point \( n_d = 0.5 \), we numerically get the overlap \( O_{oa} = 0.54 \). And then the Rabi frequency as a Gaussian-shaped function of time is given by \( \Omega_e/2\pi \simeq -0.16 e^{-(-\gamma t)^2/\tau^2} \) GHz, as shown in figure 4(a). Similarly, we consider the Rabi coupling \( \Omega_p \) induced by the magnetic interaction. The static flux bias is chosen as \( \Phi_0 = \Phi_0/3 \), and the time-dependent flux takes \( \Phi_p = \frac{\Phi_0}{3} e^{-(-\gamma t)^2/\tau^2} \), with \( \tau_p = 93 \) ns. Based on the wavefunction overlap \( O_{oa} \simeq 0.68 \), we have \( \Omega_p/2\pi \simeq -0.16 e^{-(-\gamma t)^2/\tau^2} \) GHz, the time dependence of which is given in figure 4(a).

With these two Gaussian pulses, we address the coherent population transfer by numerically solving the time-dependent Schrödinger equation. An arbitrary state vector can be described by \( \psi(t) = \sum_k b_k(t) |k\rangle \), where \( b_k(t) \) are the normalization coefficients and satisfy the following equation.
As plotted in figure 4(b), the population transfer from the initial state $|g\rangle$ to the target state $|e\rangle$ is accomplished with an inversion probability $P_{in} = 99.99\%$ after an evolution time $t = 150$ ns. During the whole process, the intermediate state $|a\rangle$ is almost not populated. With the effective Rabi frequencies $\Omega_d$, we now take the technique of TQD in the present work. Assume that the system is in $|g\rangle$ initially. As plotted in figure 4(b), the population transfer from the initial state $|g\rangle$ to the target state $|e\rangle$ is accomplished with an inversion probability $P_{in} = 99.99\%$ after an evolution time $t = 150$ ns. During the whole process, the intermediate state $|a\rangle$ is almost not populated. With the effective Rabi frequencies $\Omega_d$ and $\Omega_{dp}$, we have $\int_0^t |\Omega_d| \, dt = 48.93$ and $\int_0^t |\Omega_{dp}| \, dt = 50.13$ ($t_f = 150$ ns), which meet the adiabatic conditions well. Here two partially overlapping pulses are applied in a counterintuitive order, first $\tilde{V}_s$ and then $\tilde{V}_{dp}$, the relevant Rabi frequencies $\Omega_d$ and $\Omega_{dp}$ traverse along a closed loop in the time domain. Therefore, the coherent quantum operation is referred to as the regular STIRAP [16].

4. Speeding up adiabatic population transfer by a counter-diabatic driving

Generally, based on the STIRAP method, the adiabatic operation for transferring population needs a long time. To speed up the adiabatic population transfer from $|g\rangle$ to $|e\rangle$, we now take the technique of TQD in the present qutrit. Apart from the drivings $V_s$ and $\tilde{V}_{dp}$, an auxiliary microwave field $V_a = e^{i\beta}V_a \cos(\omega_d t)$, serving as a counter-diabatic driving, is also applied to the CPB through a gate capacitance $C_{ag}$ as schematically shown in figure 1, where $e^{i\beta}$ represents an initial phase factor [63] and $\omega_d$ is its microwave frequency. Since the frequency $\omega_d$ is resonantly matched with $\omega_{qe}$, the driving $V_a$ causes only coupling between $|g\rangle$ and $|e\rangle$. As a result, there exists a $\Delta$-type interaction between the qutrit and these three drivings, as depicted in figure 5, which is allowed by the level-transition rule.

By applying the driving $V_a$, a supplementary interaction Hamiltonian $H_{ad}$ in the space $\{|g\rangle, |a\rangle, |e\rangle\}$ is given by [30, 62]

$$H_{ad} = \begin{pmatrix} 0 & 0 & i\Omega_a/2 \\ 0 & 0 & 0 \\ -i\Omega_a/2 & 0 & 0 \end{pmatrix}$$

where the Rabi coupling strength satisfies

$$\Omega_a = \frac{2(\Omega_{dp}\Omega_r - \Omega_s\Omega_p)}{\Omega_p^2 + \Omega_s^2}.$$  \hspace{1cm} (7)

Now our concern is the $\tilde{V}_s$-induced Rabi coupling $i\Omega_d$ between $|g\rangle$ and $|e\rangle$. For the interaction Hamiltonian between the microwave pulse $\tilde{V}_s$ and the CPB, it is of the form

$$H_e = -2E_e \tilde{n}_a \sum_i (n - n_d) |n\rangle \langle n|,$$

where $\tilde{n}_a = e^{i\beta}n_a \cos(\omega_d t)$, with $n_a = C_aV_a/2\epsilon$. And the maximum amplitude of the dimensionless variable $n_a$ is taken as 0.07 here. The coupling matrix element reads $t_{ge} = \langle g|H_e|e\rangle$. And the wavefunction overlap induced
by the electrical interaction is $\Omega_a \approx 0.60$ at $n_d = 0.5$. Thus we get $t_{pe} = e^{i\Omega_a \cos(\omega_p t)}$, where $\Omega_a = -2E_F \Omega_g n_a$ indicates the Rabi coupling strength as well. By adjusting the initial phase $\beta = 3\pi/2$ and the time-dependent $n_{wa}$, the coupling matrix element $t_{pe}$ can be set to $i\Omega_a/2$ under the RWA.

Assisted by a counter-diabatic driving, the total Hamiltonian of the system thus becomes $H_t = H_I + H_{cd}$, which governs the dynamical evolution following the Schrödinger equation

$$\frac{d}{dt} \begin{bmatrix} b_2(t) \\ b_4(t) \\ b_6(t) \end{bmatrix} = \begin{bmatrix} 0 & \Omega_p/2 & i\Omega_a/2 \\ \Omega_p/2 & 0 & \Omega_a/2 \\ -i\Omega_a/2 & \Omega_a/2 & 0 \end{bmatrix} \begin{bmatrix} b_2(t) \\ b_4(t) \\ b_6(t) \end{bmatrix}.$$ (8)

In this case, we reproduce the Gaussian-shaped Rabi frequencies as $\Omega_a/2\pi = -0.16e^{-t_1^2/\tau_1^2}$ GHz and $\Omega_p/2\pi = -0.16e^{-t_2^2/\tau_2^2}$ GHz, but with $\tau_1 = 8$ ns, $\tau_2 = 17$ ns and $\tau = 8.3$ ns. According to equation (7), the dependency of $\Omega_a$ on time is explicitly sketched in figure 6(a). With the time-shortened pulses, the population transfer from the initial state $|g\rangle$ to the target state $|e\rangle$ can be almost completed within a short time $t = 25$ ns, see figure 6(b). Compared with the adiabatic transfer in figure 4(b), the operation process has been sped up sharply. Numerically, it is found that the accelerated transfer from $|g\rangle$ to $|e\rangle$ takes a time $t = 25$ ns to reach an inversion probability $P_{99.81} = 99.81\%$. However, to achieve the same inversion probability, the adiabatic operation needs a time $t = 104.58$ ns, much longer than that used in the counter-diabatic protocol. Additionally, there is no
population of $|a\rangle$ during the accelerated process, which is beneficial to remove the decoherence effects caused by $|a\rangle$.

As a practical issue related to coherent control, we are concerned with the reversibility of the above population transfer by adjusting the Rabi frequencies $\Omega_p$ and $\Omega_2$. After exchanging $\tau_2$ and $\tau_2$ with each other, $\Omega_p$ precedes $\Omega_2$ in time domain to reach its maximum amplitude. In this way, the Rabi frequencies naturally change $\Omega_p/2\pi = -0.16 e^{-t/\tau_2^2}$ GHz and $\Omega_2/2\pi = -0.16 e^{-t/\tau_3^2}$ GHz, respectively, in which the maximum amplitudes keep fixed as before. The corresponding coupling strength of $\Omega_p$ is plotted in figure 6(c). Reversibly, we execute the population inversion from the initial state $|e\rangle$ to the target state $|g\rangle$ with the redesigned Rabi frequencies $\Omega_{g,p}$ and $\Omega_{g,a}$, see figure 6(d). As a consequence, the state transfer from $|g\rangle$ to $|e\rangle$ can be carried out reversibly. Thereby, the bidirectional state transfer $|g\rangle \leftrightarrow |e\rangle$ is very flexible by just adjusting the time sequence of $\Omega_p$ and $\Omega_2$ in our scenario, which could significantly reduce the complexity of experimental manipulation.

For evolutions with no dissipation effects, we could obtain an ideal population transfer with inversion probability $P_{in} = 100%$. However, owing to the decoherence effects consisting of energy relaxation and dephasing, the system evolution becomes dissipative. Next, by adopting the standard dissipation theory, we address the decoherence effects on the population transfer. The reduced density matrix (regarding $|g\rangle$, $|e\rangle$ and $|a\rangle$) is $\rho_t$, the dynamical evolution of which is described by the Lindblad-type master equation,

$$\frac{d}{dt} \rho = -i[H_{\text{tr}}, \rho] + \sum_{k=1}^{k=1} \gamma^{|k\rangle|\langle k|} D[|k\rangle\langle k|] \rho + \frac{1}{2} \gamma^{|k\rangle|\langle k|} D[|k\rangle\langle k|] \rho,$$

where $H_{\text{tr}} = H_{\text{tr}}(H_1)$ is the effective Hamiltonian that governs the system evolution subject to $\Delta$- ($\Lambda$-) type interaction. $\gamma^{|k\rangle|\langle k|}$ and $\gamma^{|k\rangle|\langle k|}$ are the relaxation rate and dephasing rate associated with states $|k\rangle$ and $|l\rangle$, respectively, and $D[|k\rangle\langle k|] = (2L\rho L^\dagger - L^\dagger L\rho - \rho L^\dagger L)/2$, with $L = \sigma^{(k)}_z$ and $\sigma^{(k)}_z$. The inversion operator is defined as $\sigma^{(k)}_z = |k\rangle \langle k|$, and the Pauli operator is $\sigma^{(k)}_z = |k\rangle \langle k|$. The energy levels satisfy $E_k > E_l$. Here suppose that $\gamma^{|k\rangle|\langle k|} = \gamma_{\text{rel}} = \gamma_{\text{deph}}$ and $\gamma_{\text{rel}} = \gamma_{\text{deph}} = \gamma_{\text{rel}} = \gamma_{\text{deph}} = \gamma_{\text{deph}}$ for simplicity. By numerically calculating equation (9) with the accessible parameters $\gamma/2\pi = 0.05$ MHz and $\gamma/2\pi = 0.3$ MHz [64], we get a time-dependent $\rho$ of the system with an initial condition of $\rho_{0k} = 1$. For the case of the accelerated process, the matrix elements $\rho_{kk}$ ($k = g, e, a$) of interest, representing the occupation probabilities of states $|k\rangle$, are displayed in figure 7(a). Apparently, even in the presence of decoherence effects, a robust operation with an inversion probability $P_{in} = 96.18%$ can be performed within a duration time $t = 25$ ns by the shortcut approach.

Comparatively, we also consider the decoherence effects on the STIRAP-like population transfer, as shown in figure 7(b). Obviously, the decoherence effects on the adiabatic process become much larger as a result of a longer operation time. The occupation probability of $|e\rangle$ is 82.57% at the final time $t = 150$ ns, and the maximum probability of $|e\rangle$ during the transfer process is 87.23% at most. Therefore, based on the shortcut method of TQD, the target population transfer can be sped up dramatically, meanwhile, the decoherence effects have been reduced significantly due to a shorter evolution time.

Figure 7. With decoherence effects, the time evolutions of the density matrix elements $\rho_{kk}$ in (a) for the accelerated transfer, and in (b) for the STIRAP-based operation, with $k = g, e$ and $a$. 
5. Discussion and conclusion

To verify the experimental feasibility, we now examine the utilized microwave drivings based on the available parameters. It is found that the microwave amplitudes are acceptable within current technology. For example, if the maximum amplitudes of the Gaussian-shaped microwave pulses take max(V(t)) = 3.2 μV and max(V(t)) = 4.48 μV, the corresponding numbers of gate charge are the considered max(CnV(t)/2e) = 0.05 and max(CnV(t)/2e) = 0.07 for the accessible capacitances Cn = 5 × 10^4 aF [65], respectively. Provided that the gate capacitance is chosen as Cg = 1.0 × 10^4 aF [27, 65] and the junction capacitance takes Cj = 2.88 × 10^4 aF, the total capacitance yields C = 2.576 × 10^4 aF, which thus enables the used Ee/h = 3.0 GHz. For the flux driving, the maximum amplitude of the Gaussian-shaped flux meets max(φp(t)) = 0.025μx, in our scenario, which is very close to that mentioned in [66].

The proposed scheme may have the following characteristics and advantages. (i) The process of STIRAP-like population transfer is typically robust against the timing errors and the fluctuations of the control parameters; however, the operation generally takes a long time. On the other hand, the direct Rabi oscillation between |g⟩ and |e⟩ via a resonant driving can be completed quickly, whereas, it is highly sensitive to the operation time and the parameter fluctuations. By combining the advantages of these two methods, the present population transfer is sharply accelerated and still insensitive to timing errors and parameter variations, which thus could have a variety of applications to quantum coherent control and information processing. (ii) Thanks to the prohibition of the parity-symmetry determined transition rule [60], the driving φp cannot cause the magnetic coupling between |g⟩ and |f⟩ due to a vanished overlap Ogf = 0. Similarly, the Vr-induced electrical dipole interaction between |e⟩ and |f⟩ is also forbidden as a result of Oef = 0. Although Ee and Ef are relatively close to each other, the effective qudit {⟨g|, |e|, |a|} is well isolated from the effect caused by |f⟩. In this case, adopting RWA to deal with the interactions Hf and Hex becomes more viable within the qutrit. (iii) Compared to the transmon-regime quantum circuit [67], the present charge-phase CPB has the sufficient level anharmonicity, and thus the leakage effects induced by the utilized microwave drivings can be neglected safely. The suitable level structure is very helpful to implement the robust quantum manipulation. (iv) Different from the previous works [21, 35, 60], the present three-level system is selected at the magic point of nq = 0.5, which contributes to remove the dephasing effect and then to prolong the system decoherence time greatly. (v) During the accelerated and reversible transfer process, the intermediate state |a⟩ is almost not populated within the operation time, which makes the robust inversion insensitive to spontaneous emission caused by |a⟩.

In summary, we propose a promising scheme to speed up the adiabatic population transfer in a Josephson qudit by using the TQD technique. At the magic working point, the first three levels constitute our qudit. Allowed by the level-transition rule, a Λ-type interaction is induced by the microwave drivings, from which we implement a STIRAP-like adiabatic population transfer by designing the Rabi couplings. Based on the shortcut approach of TQD, we further apply a counter-diabatic driving to the qutrit for speeding up the adiabatic transfer. In the qutrit with a Λ-type interaction, the target population transfer can be accelerated significantly with the accessible parameters. Moreover, we analyze in detail the reversibility and robustness of the present strategy. Due to the reversible and robust operation in an accelerated manner, the protocol could have many potential applications for experimentally investigating optimal population transfers with the Josephson artificial atoms.

Acknowledgments

This work is supported by the ‘316’ Project Plan of Xuchang University, the Aid Project for the Leading Young Teachers in Henan Provincial Institutions of Higher Education of China (Grant No. 2015GGJS-145), the Aid Project for the Leading Young Talents of Xuchang University, the Natural Science Foundation of Zhejiang Province (Grant No. LY14A040001), and the Education department of Zhejiang Province (Grant No. Y201534406).

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