Doubly Special Relativity in Position Space
Starting from the Conformal Group.

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Abstract

We propose version of doubly special relativity theory starting from position space. The version is based on deformation of ordinary Lorentz transformations due to the special conformal transformation. There is unique deformation which does not modify rotations. In contrast to the Fock-Lorentz realization (as well as to recent position-space proposals), maximum signal velocity is position (and observer) independent scale in our formulation by construction. The formulation admits one more invariant scale identified with radius of three-dimensional space-like hyper-section of space-time. We present and discuss the Lagrangian action for geodesic motion of a particle on the DSR space. For the present formulation, one needs to distinguish the canonical (conjugated to $x^\mu$) momentum $p^\mu$ from the conserved energy-momentum. Deformed Lorentz transformations for $x^\mu$ induce complicated transformation law in space of canonical momentum. $p^\mu$ is not a conserved quantity and obeys to deformed dispersion relation. The conserved energy-momentum $P^\mu$ turns out to be different from the canonical one, in particular, $P^\mu$-space is equipped with nontrivial commutator. The nonlinear transformations for $x^\mu$ induce the standard Lorentz transformations in space of $P^\mu$. It means, in particular, that composite rule for $P^\mu$ is ordinary sum. There is no problem of total momentum in the theory. $P^\mu$ obeys the standard energy-momentum relation (while has nonstandard dependence on velocity).

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1 Introduction.

Doubly special relativity (DSR) proposals [1-5] might be specified as the theories with underlying symmetry group being the Lorentz group $^1$, but with kinematical predictions different from that of special relativity. It can be achieved by taking of some deformation of the Lorentz group realization in space of conserved energy-momentum. In particular, Magueijo-Smolin (MS) suggestion [2, 3] is to take the momentum space realization of the group in the form $\Lambda_U = U^{-1} \Lambda U$, where $\Lambda$ represents ordinary Lorentz transformation and $U(P^\mu)$ is some operator. Ordinary energy-momentum relation $(P^\mu)^2 = -m^2$ is not invariant under the realization and is replaced by $[U(P^\mu)]^2 = -m^2$. It suggests kinematical predictions different from that of special relativity. There is a number of attractive motivations for such a modification (see discussion in [1-5]), in particular, one can believe on DSR as an intermediate theory where the quantum gravity effects are presented even in the regime of negligible gravitational field [9, 3]. In turn, it implies that the formulation includes dimensional parameter $(U = U(P^\mu, \lambda))$ in such a way that one recovers special relativity in some limit. The parameter (or parameters, see the recent work [6]) turns out to be one more (in addition to speed of light) observer independent scale present in the formulation. The scale was identified with the Planck energy in [2, 3]. The emergence of a new scale was taken as the guiding principle for construction of different DSR models in a number of papers. Modifications with various dimensional scales has been proposed [1-3, 4, 5, 6, 13]. In particular, in the work [6] it was discussed an algebraic construction which implies three scales $c, E_p, R$, with $E_p$ identified with the Planck energy and $R$ being the cosmological constant.

To complete the picture, it is desirable to find underlying space-time interpretation for the DSR kinematics, that is to construct position space realization of the Lorentz group which generates one or another DSR kinematics. Then one could be able to formulate dynamical problems on DSR space in the standard framework, starting from the action functional, which suggests physical interpretation of the results obtained in momentum space formulation. Actually, in this case the spaces of velocities, of canonical (conjugated to the

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$^1$For the early proposals, based on $\kappa$-Poincare algebra, see [7].
position) momentum, the energy-momentum space as well as their properties and map of one to another can be obtained by direct computations (the issue being rather delicate question in the formulation with energy-momentum space as the starting point [8, 9, 10]). One expects also that the central problem of the DSR kinematics (the problem of total momentum for multi-particle system) can be clarified in the position space formulation.

To find a position version for the given DSR kinematics one needs to decide, in fact, what is the relation among the energy-momentum $P^\mu$ and the position variables $x^\mu$ (as well as the canonical momenta $p^\mu$), the approach undertaken in [5, 3, 8, 9, 11, 13]. Let us enumerate some of the results.

Assuming coincidence of $P^\mu$ and $p^\mu$ (equivalently, invariance of $Pdx$) [5, 10], one obtains the energy-momentum dependent Lorentz transformations in position space.

In the algebraic approach [11], position version is encoded in the Poisson brackets of an algebra which unifies the Poincare algebra and the phase space one. It implies noncommutativity\(^2\) of position variables [6].

For the MS kinematics it is possible to take ordinary Lorentz realization on $x^\mu$ and then to deform standard relation among $x$ and $P$ in some particular way [13]. Then the MS invariant and the MS transformations are generated on the momentum space from $\dot{x}^2 = -m^2c^2$, $x^\mu' = \Lambda^\mu_\nu x^\nu$, which gives a consistent picture in one-particle sector. Being quite simple, this point of view seems to be unreasonable, mainly due to the fact that it is difficult to construct an addition rule with acceptable physical properties in the multi-particle sector of the theory (see [13] for detailed discussion).

Besides the nonlinear MS transformations, the MS energy - momentum relation is invariant also under some inhomogeneous linear transformations [13]. The latter are induced starting from linearly realized Lorentz group in five-dimensional position space. For the case, there are different possibilities to relate new scale with fundamental constants. In particular, identification with vacuum energy suggests emergence of minimum quantum of mass [13].

The abovementioned works are devoted to search for space-time interpretation of a given DSR kinematics. Instead of this, one can

\(^2\)Appearance of noncommutative geometry in the DSR framework might be starting point to treat the problem of Lorentz (rotational) invariance in different noncommutative quantum mechanical models [12].
ask on reasonable deformations of the Lorentz group realization in position space without reference on a particular DSR kinematics [14, 5]. We follow this line in the present work. We propose deformation of the Lorentz transformations based on the conformal group. By construction, maximum signal velocity is observer (and space-time) independent scale of the formulation, the latter is described in Section 2. In Section 3 we discuss geodesic motion of a particle, with the Lagrangian action being invariant interval of the DSR space. Kinematics corresponding to the theory is constructed and discussed in some details. In particular, the present formulation turns out to be free of the problem of total momentum in many-particle sector.

2 Deformation of the Lorentz transformations due to the special conformal transformation.

In this Section we motivate that the conformal group in four dimensions seems to be an appropriate framework to formulate the DSR models in position space.

In ordinary special relativity the requirement of invariance of the Minkowski interval: \( ds'^2 = ds^2 \) immediately leads to the observer independent scale \( |v^i| = c \). To construct a theory with one more scale, the invariance condition seems to be too restrictive. Actually, the most general transformations \( x^\mu \rightarrow x'^\mu(x'^\nu) \) which preserve the interval are known to be the Lorentz transformations in the standard realization [15] \( x'^\mu = \Lambda^\mu_{\nu} x^\nu \), the latter does not admit one more invariant scale. So, one needs to relax the invariance condition keeping, as before, the speed of light invariant. It would be the case if \( ds^2 = 0 \) will imply \( ds'^2 = 0 \), which guarantees appearance of the invariant scale \( c \) (in the case of linear relation \( x^0 = ct \)).

Thus, supposing existence of one more observer independent scale \( R \), one assumes deformation of the invariance condition: \( ds'^2 = A(x, R)ds^2 \), where \( A \xrightarrow{R \rightarrow \infty} 1 \). By construction, the maximum velocity remains the invariant scale of the formulation. In the limit \( R \rightarrow \infty \) one obtains the ordinary special relativity theory.

Complete symmetry group for the case is the conformal group

\[ \text{It was observed in [18] that the MS operator } U \text{ and the special conformal transformation (on momentum space) with } b^\mu = (l_\mu, 0, 0, 0) \text{ coincide on the surface } p^2 = 0. \]
sist of the dilatations \( x'\mu = \rho x^\mu \) and the special conformal transformations with the parameter \( b^\mu \)

\[
SC_b : x^\mu \longrightarrow \frac{1}{\Omega} (x^\mu + b^\mu x^2),
\]
\[
\Omega(x, b) \equiv 1 + 2bx + b^2 x^2. \tag{1}
\]

Similarly to the previous DSR proposals [2, 3, 5], let us deform the Lorentz group realization in accordance with the rule \( \Lambda_{def} = U^{-1}\Lambda U \). We take the special conformal transformation with some fixed \( b^\mu \) being the similarity operator \( U \Lambda b \equiv (SC_b)^{-1} \Lambda SC_b \),

\[
\Lambda_b : x^\mu \longrightarrow \frac{1}{G} \left[ (\Lambda x)^\mu + [(1 - \Lambda)b]^\mu x^2 \right],
\]
\[
G(x, b, \Lambda) \equiv 1 - 2b(1 - \Lambda)x + 2b(1 - \Lambda)bx^2. \tag{2}
\]

The above mentioned proportionality factor for the case is \( A = G^{-2} \). The parameters \( b^\mu \) can be further specified by the requirement that space rotations \( \Lambda^\mu_\nu = (\Lambda^0_0 = 1, \Lambda^0_i = \Lambda^i_0 = 0, \Lambda^i_0 \equiv R^i_j, R^T = R^{-1}) \) are not deformed by \( b^\mu \). Then the only choice is \( b^\mu = (\lambda, 0, 0, 0) \), which gives the final form of the deformed Lorentz group realization

\[
\Lambda_\lambda : x^\mu \longrightarrow \frac{1}{G} \left[ (\Lambda x)^\mu + (\delta^\mu_0 - \Lambda^\mu_0)\lambda x^2 \right], \tag{3}
\]
\[
G(x, \lambda, \Lambda) \equiv 1 + 2\lambda(x^0 - \Lambda^0_\mu x^\mu) - 2\lambda^2(1 - \Lambda^0_0)x^2. \tag{4}
\]

Our convention for the Minkowski metric is \( \eta_{\mu\nu} = (-, +, +, +) \). One confirms now emergence of one more observer independent scale: there is exist unique vector \( x^\mu \) with zero component unaltered by the transformations (3). Namely, from the condition \( x^0 = x^0 \) one has the only solution \( x^\mu = (R \equiv -\frac{1}{\lambda}, 0, 0, 0) \) (the latter turns out to be the fixed vector). Thus all observers should agree to identify \( R \) as the invariant scale. Let us point that the transformations (3) are not equivalent to either the Fock-Lorentz realization [15], or to recent DSR proposals (the realizations lead to varying speed of light).

Invariant interval under the transformations (3) can be find by inspection of transformation properties of the following quantities:

\[
dx'\mu = \frac{1}{G^2} \left[ ((\Lambda dx)^\mu + 2\lambda(\delta^\mu_0 - \Lambda^\mu_0)(xdx))G - \right.
\]

\[
\left. \right]
\]

[^4]: Invariance under the complete conformal group leads to the massless particle, which is not of our interest here.
2\lambda(dx^0 - \Lambda^0_\nu dx^\nu - 2\lambda(1 - \Lambda^0_0)(x dx))(\Lambda x)^\mu + \lambda(\delta^\mu_0 - \Lambda^\mu_0 x^2)], \quad (5)

(d\xi^\nu) = \frac{(dx^\mu)^2}{G^2}, \quad \tilde{\Omega} = \frac{\Omega}{G}, \quad (6)

where

\tilde{\Omega} \equiv \Omega(-\lambda) = 1 + 2\lambda x^0 - \lambda^2 x^2. \quad (7)

Then the quantity

\[ ds^2 = \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{(1 + 2\lambda x^0 - \lambda^2 x^2)^2} \equiv g_{\mu\nu}(x)dx^\mu dx^\nu, \quad (8) \]

represents the invariant interval. On the domain where the metric is non degenerated, the corresponding four dimensional scalar curvature is zero, while three-dimensional space-like slice \( x^0 = 0 \) is curved space with constant curvature \( R_{(3)} = -\frac{24}{R^2} \).

To conclude this Section, let us note that deformations of the special relativity in some domain by means of the transformation \( \Lambda_{def} = U^{-1}\Lambda U \) suggests the (singular) change of variables \( X = U^{-1} x \). The variable \( X \) has the standard transformation law under \( \Lambda_{def} \): \( X' = \Lambda X \). It is true for the Fock-Lorentz realization [14] and for the recent DSR proposal [5] (see discussion in [14, 17]). Moreover, different DSR proposals in the momentum space can be considered either as different definitions of the conserved momentum \( p^\mu \) in terms of the de Sitter momentum space variables \( \eta^A \) [11], or as different definitions of \( p^\mu \) in terms of the special relativity velocities \( v^\mu = \frac{dx^\mu}{d\tau} \) [13]). Thus the known DSR proposals state, in fact, that experimentally measurable coordinates can be different from the ones specified as ”measurable” by the special relativity theory. For the case under consideration, the transformation \( [3] \) acts as ordinary Lorentz transformation on the variables

\[ X^\mu \equiv \frac{x^\mu - \lambda x^2}{1 + 2\lambda x^0 - \lambda^2 x^2}. \quad (9) \]

Geodesic motion of a particle in the space [3] looks as a free motion in the coordinates [9]: \( \ddot{X} = 0 \), see the next Section. So, Eq. (9) represents coordinates of a locally inertial frame.
3 Particle dynamics and kinematics on the DSR space.

The invariant interval (8) suggests the following action\(^5\) for a particle motion
\[
S = \frac{1}{2} \int d\tau \left[ \frac{\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{e(1 + 2\lambda x^0 - \lambda^2 x^2)^2} - e m^2 \right].
\] (10)

It is invariant under the global symmetry (3), under the ”translations”: \(x'\) \(\mu = (SC\lambda)^{-1}e^a \delta(1 + 2\lambda x^0 - \lambda^2 x^2)^2\) with the parameters \(a\), as well as under the reparametrizations \(\tau \rightarrow \tau'(\tau), \dot{x}'(\tau') = \dot{x}(\tau), e' = \frac{\partial \tau}{\partial \tau'} e(\tau)\). To discuss kinematics corresponding to the theory, it is convenient to use the Hamiltonian formulation for the system. One finds the canonical momenta for the variables \(x, e\)
\[
p^\mu = \frac{\dot{x}^\mu}{\tilde{\Omega}^2}, \quad p_e = 0,
\] (11)
and the Hamiltonian
\[
H = \frac{1}{2e}(\tilde{\Omega}^2 p^2 + m^2) + \sigma_e p_e.
\] (12)

Here and below the expressions of the type \(p^2\) mean contraction with respect to the Minkowski metric \(\eta_{\mu\nu}\). Transformation law for \(p^\mu\) follows from (3)
\[
p'^\mu = ((\Lambda p)^\mu + 2\lambda(\delta^\mu_0 - \Lambda^\mu_0)(xp))G - 2\lambda(p^0 - \Lambda^0 \nu \nu p^\nu - 2\lambda(1 - \Lambda^0_0)(xp))(\Lambda^\nu + \lambda(\delta^\nu_0 - \Lambda^\nu_0)x^2).
\] (13)

On the next step of the Dirac procedure, from the condition of preservation in time of the primary constraint: \(\dot{p}_e = 0\), one finds the secondary constraint, the latter represents deformed dispersion relation for the canonical momenta
\[
p^2 = -\frac{m^2}{(1 + 2\lambda x^0 - \lambda^2 x^2)^2}.
\] (14)

There are no of tertiary constrains in the problem. Then dynamics for the variables \((x, p)\) is governed by the equations
\[
\dot{x}^\mu = e\tilde{\Omega}^2 p^\mu, \quad \dot{p}^\mu = -\frac{2em^2}{\tilde{\Omega}}(\delta^\mu_0 \lambda + \lambda^2 x^\mu), \quad p^2 = -\frac{m^2}{\tilde{\Omega}^2}.
\] (15)

\(^5\)In terms of the variables (9) the Lagrangian acquires the form \(L = \frac{1}{\tilde{\Omega}((X(x))^2 - e m^2)}\).
The equations acquire the most simple form in the gauge $e = \hat{\Omega}^{-2}$ for the primary constraint $p_e = 0$ (the gauge coincides with the standard one in the limit $\lambda \to 0$)

$$\dot{x}^\mu = p^\mu, \quad \dot{p}^\mu = -\frac{2m^2}{\Omega^3}(\delta^\mu_0\lambda + \lambda^2 x^\mu), \quad p^2 = -\frac{m^2}{\Omega^2}. \quad (16)$$

The canonical momentum has now the standard expression in terms of velocity $p^\mu = \dot{x}^\mu$. As a consequence, it’s transformation law coincides with the one for $\dot{x}^\mu$, see Eq. (5). The system (16) implies the following Lagrangian equations for $x^\mu$

$$\ddot{x}^\mu + \frac{2m^2}{\Omega^3}(\delta^\mu_0\lambda + \lambda^2 x^\mu) = 0. \quad (17)$$

The deformed gauge is convenient to study dynamics in a particular reference frame, but implies complicated law for transformation to other frames. Actually, to preserve the gauge, Eq.(3) must be accompanied by reparametrization of the evolution parameter $\tau'$, where $\tau'$ represents a solution of the equation $\frac{\partial \tau'}{\partial \tau} = G^{-2}(\Lambda)$. In contrast, the gauge $e = 1$ retains the initial transformation law (3), and seem to be reasonable to discuss kinematics of the theory.

Kinematical rules must be formulated for conserved energy and momentum. One notes that the canonical momentum (11) is not a conserved quantity, in accordance with Eq. (15). The discussion in the end of the Section 2 prompts that the conserved momentum may be $P^\mu = e^{-1}\dot{X}^\mu(x)$. It’s expression in terms of the canonical momentum (in any gauge) is given by

$$P^\mu = (p^\mu - 2\delta^\mu_0\lambda(xp))\hat{\Omega} - 2(x^\mu - \delta^\mu_0\lambda x^2)(\lambda p^0 - \lambda^2 (xp)). \quad (18)$$

By direct computation one finds that $P^\mu$ is actually conserved on-shell (15) and obeys the ordinary energy-momentum relation as a consequence of Eq. (14). The deformed transformations (3), (13) induce the standard realization of the Lorentz group on $P^\mu$-space. As a consequence, composition rule for the momenta is the standard one, there is no problem of total momentum in the theory. So, the present version of the DSR theory leads to the standard kinematical rules on the space (18)

$$\partial_\tau P^\mu = 0, \quad \eta_{\mu\nu}P^\mu P^\nu = -m^2 \Lambda_\lambda: P^\mu \to \Lambda^\mu_\nu P^\nu, \quad P^\mu_{\sum} = \sum P^\mu_{(i)}. \quad (19)$$
The energy and momentum have nonstandard relation (18), (11) with measurable quantities (velocities and coordinates). It suggests that kinematical predictions of the theory differ from that of the special relativity theory.

The difference among the canonical momentum and the conserved one implies an interesting situation in canonically quantized version of the theory. While the conjugated variables \((x, p)\) have the standard brackets, commutators of the coordinates \(x^\mu\) with the energy and momentum \(P^\mu\) are deformed, as it can be seen from Eq. (18). Thus the phase space \((x, P)\) is endowed with the noncommutative geometry (with the commutators \([x, P]\) and \([P, P]\) being deformed). In particular, the energy-momentum subspace turns out to be noncommutative. The modified bracket \([x, P]\) suggests that the Planck’s constant has slight dependence on \(x\) (similar bracket structure, with energy dependent Planck’s constant, arises in the MS model [3]).

4 Conclusion

In this work we have proposed version of the doubly special relativity theory in position space based on deformation of ordinary Lorentz transformations due to the special conformal transformation. There is unique deformation (3) which does not modify the space rotations, namely, the deformation with the special conformal parameter being \(b^\mu = (\lambda, 0, 0, 0)\). The invariant interval (8) corresponds to the flat four-dimensional space-time (on a domain where the metric is non degenerated). By construction, maximum signal velocity is observer independent scale of the theory. The formulation admits one more independent scale \(R \equiv -\frac{1}{\lambda}\), the latter is identified with radius of three-dimensional hypersection of (8) at \(x^0 = 0\).

Geodesic motion of a particle on the space (8) has been discussed in some details. The conjugated momentum \(p^\mu\) (11) for the coordinate \(x^\mu\) has complicated transformation law (13), and obeys the deformed energy-momentum relation (14). The conserved energy-momentum \(P^\mu\) (18) turns out to be different from the canonical one. The transformations (3), (5) for \((x, p)\) induce the standard Lorentz transformations on the space of conserved momentum. It means, in particular, that composite rule for the energy-momentum is ordinary sum. There is no problem of total momentum in the theory.
The conserved momentum, in contrast to the canonical one, obeys the standard energy-momentum relation. Kinematical rules of the theory are summarized in Eq. (19). One expects that kinematical predictions of the theory differ from that of the special relativity due to nonstandard dependence of energy and momentum on measurable quantities, see Eqs. (18), (11).

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