One-loop matching factors for staggered bilinear operators with improved gauge actions

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We present results for one-loop perturbative matching factors using bilinear operators composed of improved staggered fermions, using unimproved (Wilson) and improved (Symanzik, Iwasaki, and asqtad) gauge actions. We consider two fermions actions—HYP/Fat7-smeared and “asqtad”. The former is being used in calculations of electroweak matrix elements, while the latter have been used extensively by the MILC collaboration. We observe that using the improved gluon action leads to small reductions in the perturbative corrections, but that these reductions are smaller than those obtained when moving from the tadpole-improved naive staggered action to either HYP-smeared or asqtad action.

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I. INTRODUCTION

Improved staggered fermions are an attractive choice for the numerical study of QCD, and are being used for a variety of calculations relevant to phenomenology. For calculations of electroweak matrix elements, such as our ongoing calculation of $B_K$ [1,2], one needs to match continuum operators in the effective Hamiltonian onto corresponding lattice-regularized operators. Here we calculate such matching factors for fermion bilinears composed of improved staggered fermions with various gluon actions. We work at one-loop level in perturbation theory.

The motivation for this work is three-fold. First, the results are a step on the way to the calculation of matching factors for four-fermion operators, such as that needed for $B_K$, results for which will be presented in an upcoming work [5]. Second, our results allow us to compare the efficacy of improvements to fermion and gauge actions at reducing matching factors. Third, some of our results can be compared to ongoing calculations of matching factors [6] using non-perturbative renormalization (NPR) [7]. We can also check our result for the mass-renormalization for asqtad fermions with that obtained (as a byproduct of a two-loop calculation) in Ref. [8].

Two major problems with unimproved staggered fermions are large taste-symmetry breaking and large perturbative corrections to matching factors. Previous work has shown that both problems are alleviated by smearing the gauge links to which the fermions couple. In particular, it turns out that HYP smearing [9] is most effective at reducing one-loop perturbative corrections [10], and also in reducing the taste symmetry-breaking in the pion spectrum [11]. In light of this we are using such smearing for valence quarks in our ongoing calculations of matrix elements. These calculations make use, however, of the MILC configurations [12], which use a Symanzik-improved gauge action. Thus we have undertaken the extension of the calculation of matching factors to the improved gluon action. We have done so using both the HYP-smeared action but also using the asqtad action. The latter gives further information on the comparison between smearing methods.

The generalization to an improved gluon action is non-trivial. The gluon propagator is diagonal with the Wilson gauge action (in the Feynman gauge), but becomes a full $4 \times 4$ matrix for an improved gauge action. Thus various simplifications that are possible with the Wilson gauge action do not occur with the improved gauge action.

A calculation along similar lines has been done previously in Ref. [13]. The authors consider the asqtad fermion action and Symanzik-improved glue, but use staggered fermion operators containing unsmeared (“thin”) links. This is in contrast to the operators which we use, in which all links are smeared.

The paper is organized as follows. In Sec. II, we explain our notation and conventions for actions and operators. In Sec. III, we describe the renormalization of bilinear operators on the lattice. In Sec. IV, we explain the procedure of matching between the continuum and lattice operators. In Sec. V, we close with a discussion of our numerical results. We relegate technical details to three appendices. Appendix A discusses the gluon propagator links introduced in Ref. [10]. We refer to these simply as HYP links in the following. The two smearings differ at higher-order and non-perturbatively.
for improved actions, App. [B] gives results for the renormalization of HYP fermions, and App. [C] describes the results for asqtad fermions.

A preliminary account of this work has appeared in Ref. [16].

II. ACTIONS, FEYNMAN RULES AND OPERATORS

A general form for the \(O(a^2)\)-improved gluon action is \(17\) (using the labeling convention of Ref. \(18\))

\[
S_g = \frac{6}{g_0^2} \left[ c_0 \sum_{pl} \frac{1}{3} \text{ReTr} (1 - U_{pl}) + c_1 \sum_{rt} \frac{1}{3} \text{ReTr} (1 - U_{rt}) + c_2 \sum_{pg} \frac{1}{3} \text{ReTr} (1 - U_{pg}) + c_3 \sum_{ch} \frac{1}{3} \text{ReTr} (1 - U_{ch}) \right]. \quad (1)
\]

Here \(pl, rt, pg,\) and \(ch\) denote the shape of the Wilson loops—plaquette, rectangle, parallelogram and chair, respectively. The overall normalization of the coefficients is such that

\[
c_0 + 8c_1 + 8c_2 + 16c_3 = 1. \quad (2)
\]

If we consider only on-shell improvement, then one operator is redundant \(18\), and we adopt henceforth the convention of setting \(c_3 = 0\).

The Wilson gauge action corresponds to the choices \(c_0 = 1, c_1 = c_2 = 0\). As shown by L"uscher and Weisz, tree-level on-shell Symanzik-improvement of the pure-gluon theory is obtained if the improvement coefficients take the values \(13, 18\)

\[
c_0 = \frac{5}{3}, \quad c_1 = -\frac{1}{12}, \quad c_2 = 0. \quad (3)
\]

The MILC collaboration use a one-loop improved action determined in Refs. \(19, 20\). In a perturbative calculation, however, the one-loop corrections to the improvement coefficients enter at two-loop order in a calculation of bilinear matching coefficients. Thus, for our one-loop calculation of matching coefficients, the consistent choice is to use the tree-level coefficients \(3\) when determining the gluon propagator.

The propagator for the improved gluon action is well known. We have found a relatively simple form for this propagator, which is given in Appendix \(A\).

The staggered fermion actions that we consider in this paper are the asqtad and HYP actions. The former is

\[
S_{\text{asqtad}} = \sum_n \left[ \bar{\chi}(n) \sum_{n, \mu} \eta_\mu(n) \left( \nabla_\mu^\text{F7L} \chi(n) + \frac{1}{8} [\nabla_\mu^\text{T1} - \nabla_\mu^\text{T3}] \chi(n) \right) + (m/u_0) \bar{\chi}(n) \chi(n) \right], \quad (4)
\]

\[
\nabla_\mu^\text{F7L} \chi(n) = \frac{1}{2} [W_\mu(n) \chi(n + \hat{\mu}) - W_\mu(n - \hat{\mu}) \chi(n - \hat{\mu})], \quad (5)
\]

\[
\nabla_\mu^\text{T1} \chi(n) = \frac{1}{2u_0} [U_\mu(n) \chi(n + \hat{\mu}) - U_\mu(n - \hat{\mu}) \chi(n - \hat{\mu})], \quad (6)
\]

\[
\nabla_\mu^\text{T3} \chi(n) = \frac{1}{6u_0} [U(n, n + 3\hat{\mu}) \chi(n + 3\hat{\mu}) - U(n, n - 3\hat{\mu}) \chi(n - 3\hat{\mu})], \quad (7)
\]

where \(n = (n_1, n_2, n_3, n_4)\) labels lattice sites, \(\eta_\mu(n) = (-1)^{n_1 + \cdots + n_{\mu-1}}\) is the usual staggered phase, \(U_\mu(n)\) is the original, “thin” link, and \(m\) is the quark mass in MILC’s convention. Here and in the following we set the lattice spacing to unity, except where clarity dictates otherwise. \(W_\mu(n)\) is a smeared link constructed using the Fat7 blocking transformation \(21\, 22\) combined with Lepage’s prescription \(23\) and tadpole improvement \(24\). \(U(n, n \pm 3\hat{\mu})\) are products of 3 thin links in the \(\mu\) direction,

\[
U(n, n + 3\hat{\mu}) = U_\mu(n) U_{\mu}(n + \hat{\mu}) U_{\mu}(n + 2\hat{\mu})
\]

\[
U(n, n - 3\hat{\mu}) = U_{\mu}(n - \hat{\mu}) U_{\mu}(n - 2\hat{\mu}) U_{\mu}(n - 3\hat{\mu}),
\]

and appear in the Naik term. Finally, \(u_0\) is the tadpole improvement factor, which we determine here as the fourth-root of the average plaquette. This action is tree-level \(O(a^2)\) improved.

The HYP action is simply the unimproved staggered action using HYP-smeared links:

\[
S_{\text{HYP}} = \sum_n \bar{\chi}(n) \left[ \sum_{n, \mu} \eta_\mu(n) \nabla_\mu^\text{H} \chi(n) + m \right], \quad (8)
\]

\[
\nabla_\mu^\text{H} \chi(n) = \frac{1}{2} [W_\mu(n) \chi(n + \hat{\mu}) - W_\mu(n - \hat{\mu}) \chi(n - \hat{\mu})].
\]
where $V_\mu$ is constructed using the HYP blocking transformation of Ref. [10]. This transformation has the advantage of using only links lying within hypercubes attached to the original thin-link, so that $V_\mu$ is less extended than the fat links $W_\mu$ used in the asqtad action. HYP-blocking also includes SU(3) projection. We set the HYP blocking parameters to the values that remove the tree-level coupling of quarks to gluons having one or more components of momenta equal to $\pi/a$ (with the other components vanishing). In the notation of Ref. [11, 25], these are the HYP(II) parameters. These are the values we have used in our simulations.

The HYP action is only partially improved—taste-breaking $O(a^2)$ interactions are removed, but taste-conserving $O(a^2)$ terms are not. It would thus seem to be a poorer choice than the asqtad action, which is fully $O(a^2)$ improved. It turns out, however, to be a better choice in practice, for two reasons. The most important is that it leads to substantially smaller taste-splittings between pions [13]. Since taste-splitting is the dominant $O(a^2)$ effect with staggered fermions, this means that HYP-smereared quarks have smaller $O(a^2)$ effects than asqtad quarks. The second reason is that the HYP action is more continuum-like, in the sense that loop contributions to matching factors are typically smaller. This is known explicitly at one-loop for bilinears (as found for the Wilson gauge action in Ref. [11], and for the improved gauge action in the present work—see Table II], and is expected also to hold at higher order because the four-fermion operators induced at one-loop have greatly reduced coefficients compared to asqtad quarks [20]. These advantages, as well as the computational simplicity of implementing this action for valence quarks, have led us to pursue calculations using the HYP-smereared action. For completeness, we note that similar reductions in taste-splittings (and presumably similar reductions in one and higher-loop contributions to matching factors) can also be obtained using the more highly improved, and more complicated, HISQ action [20].

The thin links are related to the gauge fields $A_\mu$ in the usual way,

$$U_\mu(x) = \exp[i g_0 A_\mu(x + \hat{\mu}/2)], \quad (9)$$

Writing the HYP links in a similar way in terms of “blocked gauge fields” $B_\mu$,

$$V_\mu(x) = \exp[i g_0 B_\mu(x + \hat{\mu}/2)], \quad (10)$$

the $B_\mu$ can be expressed in terms of the gauge fields as

$$B_\mu = \sum_{n=1}^{\infty} B_\mu^{(n)}(A_\nu). \quad (11)$$

Here $B^{(n)}$ contains all terms with $n$ powers of $A$. It turns out that we need only $B_\mu^{(1)}$ in the one-loop calculation. While $B_\mu^{(2)}$ enters in one-loop “tadpole” diagrams, these contributions vanish because of the SU(3) projection [10, 11, 25]. Thus all we need is the relationship between $B_\mu^{(1)}$ and $A_\nu$:

$$B_\mu^{(1)}(k) = \sum_{\nu} h_{\mu\nu}(k) A_\nu(k), \quad (12)$$

where we have gone over to momentum space. A convenient general form for the kernel $h_{\mu\nu}(k)$ is (following Ref. [11], but using the notation of Ref. [22])

$$h_{\mu\nu}(k) = \delta_{\mu\nu} D_\mu(k) + (1 - \delta_{\mu\nu}) \bar{s}_\mu \bar{s}_\nu \tilde{G}_{\nu,\mu}(k), \quad (13a)$$

$$D_\mu(k) = 1 - d_1 \sum_{\nu \neq \mu} \bar{s}_\nu^2 + d_2 \sum_{\nu < \rho} \bar{s}_\nu^2 \bar{s}_\rho^2 - d_3 \bar{s}_\nu^4, \quad (13b)$$

$$\tilde{G}_{\nu,\mu}(k) = d_1 - d_2 (\bar{s}_\nu^2 + \bar{s}_\nu^\sigma)^2 \alpha + d_3 \bar{s}_\nu^2 \bar{s}_\bar{\nu}^2 + d_4 \bar{s}_\nu^2, \quad (13c)$$

Here $\bar{s}_\mu = \sin(k_\mu/2)$.

The coefficients $d_i$ distinguish different choices of smeared links.

(i) Unimproved (“thin”) links ($U_\mu$):

$$d_1 = 0, \quad d_2 = 0, \quad d_3 = 0, \quad d_4 = 0. \quad (14)$$

(ii) HYP-smereared fat links ($V_\mu$) whose coefficients are chosen to remove $O(a^2)$ taste-symmetry breaking at tree level:

$$d_1 = 1, \quad d_2 = 1, \quad d_3 = 1, \quad d_4 = 0. \quad (15)$$

(iii) Fat7 links with the Lepage term ($W_\mu$), which remove all $O(a^2)$ couplings at tree level, both taste-violating and conserving:

$$d_1 = 0, \quad d_2 = 1, \quad d_3 = 1, \quad d_4 = 1. \quad (16)$$

With the exception of some tadpole diagrams, the perturbative calculation requires the propagator from one smeared-link to another. This takes the form

$$\langle B_\mu^{(1)}(b)(k)B_\nu^{(1)}(c)(-k) \rangle$$

$$= \sum_{\alpha,\beta} h_{\mu\alpha}(k) h_{\nu\beta}(-k) \langle A_\alpha^b(k) A_\beta^c(-k) \rangle$$

$$= \delta^{bc} \sum_{\alpha,\beta} h_{\mu\alpha}(k) h_{\nu\beta}(k) D_{\alpha\beta}^{\text{Imp}}(k)$$

$$= \delta^{bc} T_{\mu\nu}, \quad (17)$$

where $b,c$ are color indices, and $D_{\mu\nu}^{\text{Imp}}$ is the propagator for the improved gluon action given in eq. (1). In

2 The reversal of the indices on $\tilde{G}_{\nu,\mu}$ is intended and follows Ref. [22].
the third line we have used the fact that $h_{\mu \nu}$ is an even function of the momentum. We note that this smeared-smeared propagator includes off-diagonal ($\mu \neq \nu$) terms even if the gluon action is unimproved, because the kernel $h_{\mu \nu}$ has off-diagonal terms. Thus, for those diagrams involving the smeared-smeared propagator, the generalization to an improved gluon action does not introduce any new types of contribution, and we can carry over the form of most of the results from Ref. [11]. Appendix B describes how this works.

For the asqtad action, however, the situation is less simple, since not all the links are smeared. Some diagrams must be calculated anew when using an improved gluon propagator. We discuss this in Appendix C.

We now turn to the bilinear operators. We construct them from the standard hypercube convention [28], in which the spin and tastes of the two continuum fermions are spread over a hypercube:

\[ [S \times F](y) = \frac{1}{16} \sum_{A,B} [\bar{\chi}^a(y + A) (\gamma_S \otimes \xi_F)_{AB} \chi^c(y + B)] V^{bc}(y + A, y + B). \]  

(18)

where $y$ denotes the particular $2^4$ hypercube, and $A, B$ are “hypercube vectors” denoting the positions within the hypercube. The matrices $(\gamma_S \otimes \xi_F)_{AB}$ are in the standard notation of Refs. [29, 30]. The spin ($S$) and taste ($F$) of the bilinear can each be scalar, $S$, vector, $V_\mu$, tensor, $T_{\mu\nu}$, axial vector, $A_\mu$, or pseudoscalar, $P$. The only new feature of these operators compared to those used with unimproved staggered fermions lies in the links used to make them gauge invariant. The factor $V^{bc}(y + A, y + B)$ is constructed by averaging over all of the shortest paths between $y + A$ and $y + B$, and for each path forming the product of smeared gauge links. We use the same smeared links as in the fermion action, i.e. $U_\mu$ for the unimproved action, $V_\mu$ for the HYP action, and $W_\mu$ for the asqtad action. This ensures the conservation of the current $[V \times S]$ for unimproved and HYP-smeared fermions.

For the asqtad action, however, the presence of the three-link (Naik) term in the action means that $[V \times S]$ is not the conserved current. As a check, we have also calculated the matching factor for the asqtad conserved vector current. This is described in Appendix C.

Finally, we have also implemented mean-field improvement for the HYP action [11]. Although the HYP-smeared links fluctuate much less than thin links, residual fluctuations are present and can be partly removed by rescaling the links. The rescaling factor, $u_{SM}^{1/4}$, is chosen to be the fourth-root of the plaquette constructed from smeared links. The details of this procedure are explained in Ref. [27] and we do not repeat them here.

III. RENORMALIZATION OF BILINEAR OPERATORS

The one-loop Feynman diagrams are shown in Fig. 1. The X and Z diagrams are infrared divergent. We regularize this divergence, following Refs. [27, 31] by adding a gluon “mass” term, $\lambda^2$, to the denominator of the gluon propagator. This allows us to set both quark masses and external momenta to zero.

We have undertaken two independent calculations, one based on the approach of Ref. [32], the other following Refs. [11, 27]. These two methods lead to identical results within the accuracy of the numerical integrations. We refer to these references for discussions of the methodology.

One-loop matrix elements of lattice operators take the
general form (with the lattice spacing restored for clarity)

\[
\mathcal{M}_{i}^{\text{Latt.}(1)} = \left\{ \delta_{ij} + \frac{g^2}{(4\pi)^2} \left[ \delta_{ij} \gamma_{i}^{(0)} \log(a\lambda) + C_{ij}^{\text{Latt.(0)}} \right] \right\} \mathcal{M}_{j}^{\text{Latt.(0)}} + O(a) \tag{19}
\]

where the superscript indicates the order in perturbation theory, and the subscript labels the different spins and tastes. Thus \(\mathcal{M}_{j}^{\text{Latt.(0)}}\) is tree-level matrix element of the \(j\)’th bilinear operator. \(\gamma_{i}^{(0)}\) are the one-loop anomalous dimensions (which, for bilinears, are diagonal):

\[
\gamma_{i}^{(0)} = -2C_{F}d_{i}. \tag{20}
\]

Here \(C_{F} = 4/3\), while the \(d_{i}\) depend on the spin, but not on the taste, of the bilinear:

\[
d_{i} = \{3, 0, -1, 0, 3\} \quad \text{for} \quad \{S, V, T, A, P\}. \tag{21}
\]

Finally, \(C_{ij}^{\text{Latt}}\) is the finite part of the correction.

The finite part can be broken down as

\[
C_{ij}^{\text{Latt}} = C_{F}\delta_{ij}d_{i}(F_{0000} - \gamma_{E} + 1) + C_{F}[X_{ij} + \delta_{ij}(Y_{i} + T_{i} + ZT + Z)] . \tag{22}
\]

The first line is the finite coefficient accompanying the \(\log(a\lambda)\), and is thus proportional to the anomalous dimension matrix. The numerical values of the constants are \(F_{0000} = 4.36923(1)\) and \(\gamma_{E} = 0.577216\ldots\). The second line gives the finite contributions from each of the diagrams, and incorporates the result that only the \(X\)-diagrams give rise to mixing between bilinears.

Expressions for the finite contributions are given in Appendices \(B\) (HYP fermions) and \(C\) (asqtad fermions). We present numerical values in Tables \(I\) (diagonal components) and \(II\) (off-diagonal components). Results are shown with both Wilson and improved gauge actions. We quote only two decimal places for brevity; our numerical evaluations are accurate to at least \(1 \times 10^{-3}\). For fermion actions, we compare the tadpole-improved staggered action [i.e. the action of eq. \(5\)] with links \(U_{\mu}/u_{0}\), the HYP action, with and without mean-field improvement, and the asqtad action.\(^3\)

We note that improving the gauge action leads to a moderate decrease in the magnitude of the matching coefficients, except for those which were already small (\(|C_{ii}| \lesssim 1\)).

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\(^3\) The results with the Wilson gauge action agree with those obtained in Refs. \(11, 27\), with the exception of the mixing coefficients for asqtad action (column \(q\)) of Table \(I\), where a (numerically small) error in Ref. \(11\) has been found. We note that the off-diagonal mixing coefficients in Table \(II\) must be multiplied by \(-3/4\) to be compared to those quoted in Refs. \(11, 27\).

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\(^4\) This requires that one does the unitary change of basis to convert from \((\gamma_{S} \otimes \xi_{F})\) to \((\gamma_{S} \otimes \xi_{F})\) matrices, as explained in Ref. \(27\).
TABLE I: Diagonal part of finite coefficients, $C_{1\text{Latt}}^\text{cont}$. Indices $\mu$, $\nu$, $\rho$ and $\sigma$ are all different. Results are given for the following choices of fermion and gauge actions: (a) Tadpole-improved staggered fermions with Wilson gluon action; (b) Tadpole-improved staggered fermions with improved gluon action; (c) HYP fermions with Wilson gluon action; (d) HYP fermions with improved gluon action; (e) Mean-field improved HYP fermions with Wilson gluon action; (f) Mean-field improved HYP fermions with improved gluon action; (g) Asqtad fermions with Wilson gluon action; (h) Asqtad fermions with improved gluon action. For brevity, we quote only two decimal places of the numerical results.

| Operator | (a) | (b) | (c) | (d) | (e) | (f) | (g) | (h) |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|
| $(1 \otimes 1)$ | 42.47 | 34.12 | 3.46 | 2.54 | 2.06 | 1.58 | 6.23 | 4.83 |
| $(1 \otimes \xi_{\mu})$ | 14.86 | 12.28 | 0.05 | -0.24 | 0.05 | -0.24 | 3.77 | 2.84 |
| $(1 \otimes \xi_{\mu\nu})$ | 2.58 | 1.01 | -3.15 | -2.85 | -1.75 | -1.89 | 4.40 | 3.25 |
| $(1 \otimes \xi_{\mu5})$ | -3.65 | -2.39 | -6.36 | -5.44 | -3.55 | -3.51 | 6.19 | 4.62 |
| $(1 \otimes \xi_{5})$ | -8.33 | -7.40 | -9.56 | -8.01 | -5.34 | -5.12 | 8.37 | 6.34 |
| $(\gamma_{\mu} \otimes 1)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -1.89 | -1.91 |
| $(\gamma_{\mu} \otimes \xi_{\mu})$ | 6.55 | 5.32 | 1.57 | 1.21 | 0.17 | 0.24 | -5.70 | -5.17 |
| $(\gamma_{\mu} \otimes \xi_{\mu\nu})$ | -0.23 | -0.40 | -2.46 | -1.90 | -1.06 | -0.93 | 2.01 | 1.32 |
| $(\gamma_{\mu} \otimes \xi_{\mu5})$ | 1.43 | 3.46 | -0.49 | -0.42 | -0.48 | -0.42 | -1.91 | -2.00 |
| $(\gamma_{\mu} \otimes \xi_{5})$ | 4.53 | 3.46 | -0.49 | -0.42 | -0.48 | -0.42 | -1.91 | -2.00 |
| $(\gamma_{\mu} \otimes \xi_{\mu\rho})$ | -3.34 | -3.06 | -4.99 | -3.84 | -2.18 | -1.91 | 5.37 | 4.12 |
| $(\gamma_{\mu} \otimes \xi_{\mu\sigma})$ | -0.25 | -0.51 | -2.80 | -2.22 | -1.40 | -1.25 | 1.37 | 0.74 |
| $(\gamma_{\mu} \otimes \xi_{5\rho})$ | -6.52 | -5.80 | -7.58 | -5.82 | -3.36 | -2.93 | 8.66 | 6.87 |
| $(\gamma_{\mu} \otimes \xi_{5\sigma})$ | -3.69 | -3.44 | -5.29 | -4.13 | -2.48 | -2.20 | 4.77 | 3.58 |
| $(\gamma_{\mu} \otimes \xi_{5\rho\sigma})$ | -1.46 | -1.54 | -2.45 | -1.79 | -1.05 | -0.83 | 0.72 | 0.23 |
| $(\gamma_{\mu} \otimes \xi_{5\rho\mu})$ | -0.42 | -0.63 | -0.50 | -0.34 | -0.50 | -0.34 | -3.79 | -3.59 |
| $(\gamma_{\mu} \otimes \xi_{5\rho\sigma})$ | -3.35 | -3.11 | -4.63 | -3.40 | -1.82 | -1.47 | 4.90 | 3.77 |
| $(\gamma_{\mu} \otimes \xi_{5\rho\rho})$ | -5.43 | -4.28 | 0.94 | 0.76 | -0.46 | -0.20 | -9.68 | -8.50 |
| $(\gamma_{\mu} \otimes \xi_{5\rho\sigma})$ | -1.04 | -1.18 | -2.46 | -1.79 | -1.06 | -0.83 | 0.82 | 0.32 |
| $(\gamma_{\mu} \otimes \xi_{5\rho\rho})$ | -5.92 | -5.26 | -6.92 | -5.10 | -2.70 | -2.21 | 8.76 | 7.05 |

TABLE II: Non-vanishing mixing coefficients, $C_{ij}^{\text{Latt}}$. The notation is as in Table I. Note that mean-field improvement does not change off-diagonal coefficients.

| Operator-i | Operator-j | (a) | (b) | (c)/(e) | (d)/(f) | (g) | (h) |
|------------|------------|-----|-----|---------|---------|-----|-----|
| $(\gamma_{\mu} \otimes \xi_{\nu})$ | $(\gamma_{\nu} \otimes \xi_{\mu})$ | 1.05 | -3.33 | -0.47 | -0.43 | -1.73 | -1.49 |
| $(\gamma_{\mu} \otimes \xi_{\mu5})$ | $(\gamma_{\mu} \otimes \xi_{5\mu})$ | 0.86 | 0.81 | 0.34 | 0.33 | 0.73 | 0.68 |
| $(\gamma_{\mu} \otimes \xi_{\mu\nu})$ | $(\gamma_{\mu} \otimes \xi_{\mu\rho})$ | 1.98 | 1.72 | 0.37 | 0.36 | 1.09 | 0.98 |
| $(\gamma_{\mu} \otimes \xi_{5\nu})$ | $(\gamma_{\mu} \otimes \xi_{5\rho})$ | 0.90 | 0.73 | -0.01 | -0.004 | 0.23 | 0.19 |

Thus Ref. [8] would predict

$$Z_S = 1 + \frac{g^2}{(4\pi)^2} \left[ 8 \log(\mu a) + C_{\text{S}}^{\text{Cont}} - 4.8262 \right], \quad (31)$$

and thus that $C_{1\otimes 1}^{\text{Latt}} = 4.8262$. This agrees with our result, which is given in the first row of column (h) in Table I.

V. DISCUSSION

We can use our results to compare the reduction in the size of one-loop matching factors achieved by different improvement schemes. The starting point is tadpole-improved staggered fermions [column (a) in the Tables]. This comparison is most straightforward for vector (and axial) currents, since for these the anomalous dimensions vanish, so that there is no dependence on renormalization scale or scheme, and $C_{i}^{\text{cont}}$ vanishes. Thus, for these operators, $C_{i}^{\text{Latt}}$ gives a direct measure of the size of the corrections. We note that, for the lattices on which we are presently simulating (with $a \approx 0.045 - 0.12$ fm), the range of values of $g^2/(16\pi^2)$ is $0.017 - 0.026$ (evaluating the coupling at scale $2/a$ in the $\overline{\text{MS}}$ scheme).\footnote{A similar range is obtained using $\alpha$ in the $V$-scheme evaluated at the scale $2/a$, which is the close to the calculated $g^*$ values for $Z_m$.} Thus a finite coefficient of size $|C_{i}^{\text{Latt}}| \approx 5$ corresponds to a 10% one-loop correction.

There are eight different tastes of vector currents in Table I and three in Table II. We see that corrections for all actions are of moderate size, with the largest magnitude being $\approx 9$, so there is not much room for improvement over the simple tadpole-improved action. We do note, however, that, for all fermion types, improving the gauge action does lead to a moderate reduction in the size of the correction, except when the magnitude of the coefficient is already of order unity. For most coefficients the reduction is in the range 10-25%. This reduction is,
TABLE III: Spread of values for the diagonal finite corrections, $\bar{c}_{ii}$, both for a given spin (leading to a scale and scheme dependent result), and between all operators (setting $\mu a = 1$). Notation for columns is as in Table II

| Spin | (a)   | (b)   | (c)   | (d)   | (e)   | (f)   | (g)   | (h)   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| S/P  | 50.8  | 41.6  | 13.0  | 10.6  | 7.4   | 6.7   | 4.6   | 3.5   |
| V/A  | 13.1  | 11.1  | 9.2   | 7.0   | 3.5   | 3.2   | 14.4  | 12.0  |
| T    | 5.5   | 4.6   | 7.9   | 5.9   | 2.2   | 2.0   | 18.4  | 15.6  |
| All  | 50.8  | 41.6  | 14.5  | 12.6  | 8.9   | 8.7   | 19.0  | 16.0  |

however, smaller than that achievable using mean-field improved HYP fermions, where the reduction is close to 50%. Without mean-field improvement, HYP and asqtad vector currents turn out to have slightly larger one-loop corrections than those for tadpole-improved staggered fermions.

Turning now to the operators with anomalous dimensions, we can remove scale and scheme dependence by considering the differences between the $\bar{c}_{ii}$ for fixed spin and varying taste. We list in Table III the spread of values for each of the three choices of spins. We also include the spread of values across all spins and tastes, choosing $\mu = 1/a$. This is a useful measure of the range of corrections, since the variation with $\mu$ is relatively weak.

We see from the table that, for HYP and asqtad fermions, improving the gluon action reduces all the spreads, although by a small amount. Once again, the greatest reduction is achieved by the mean-field improved HYP fermions, where the reduction is close to 50%. Without mean-field improvement, HYP and asqtad fermions lead to smaller coefficients.

TABLE IV: Diagonal part of finite coefficients, $c^{\text{lat}}_{ii}$, for HYP fermions with (c) Wilson, (d) Symanzik, (i) DBW2 and (j) Iwasaki gluon actions. Other notation as in Table II

| Operator | (c)   | (d)   | (i)   | (j)   |
|----------|-------|-------|-------|-------|
| $(1 \otimes 1)$ | 3.46  | 2.54  | -1.89 | 1.03  |
| $(1 \otimes \xi)$ | 0.05  | -0.24 | -3.07 | -0.99 |
| $(1 \otimes \xi_{\omega})$ | -3.15 | -2.85 | -4.18 | -2.90 |
| $(1 \otimes \xi_{\epsilon})$ | -6.36 | -5.44 | -5.27 | -4.78 |
| $(1 \otimes \xi_5)$ | -9.56 | -8.01 | -6.34 | -6.62 |
| $(\gamma_{\mu} \otimes 1)$ | 0.00  | 0.00  | 0.00  | 0.00  |
| $(\gamma_{\mu} \otimes \xi)$ | 1.57  | 1.21  | 0.44  | 0.81  |
| $(\gamma_{\mu} \otimes \xi_{\omega})$ | -2.46 | -1.90 | -0.69 | -1.28 |
| $(\gamma_{\mu} \otimes \xi_{\epsilon})$ | -0.49 | -0.42 | -0.20 | -0.32 |
| $(\gamma_{\mu} \otimes \xi_5)$ | -4.99 | -3.84 | -1.39 | -2.59 |
| $(\gamma_{\mu} \otimes \xi_{\rho})$ | -2.80 | -2.22 | -0.87 | -1.55 |
| $(\gamma_{\mu} \otimes \xi_{\sigma})$ | -7.58 | -5.82 | -2.10 | -3.92 |
| $(\gamma_{\mu} \otimes \xi_5)$ | -5.29 | -4.13 | -1.56 | -2.84 |
| $(\gamma_{\mu} \otimes \xi_{\rho})$ | -2.45 | -1.79 | 0.36  | -0.92 |
| $(\gamma_{\mu} \otimes \xi_{\sigma})$ | -0.50 | -0.34 | 0.83  | 0.01  |
| $(\gamma_{\mu} \otimes \xi_5)$ | -4.63 | -3.40 | -0.15 | -1.94 |
| $(\gamma_{\mu} \otimes \xi_{\rho})$ | 0.94  | 0.76  | 1.22  | 0.74  |
| $(\gamma_{\mu} \otimes \xi_{\sigma})$ | -2.46 | -1.79 | 0.36  | -0.92 |
| $(\gamma_{\mu} \otimes \xi_5)$ | -6.92 | -5.10 | -0.68 | -3.02 |

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Appendix A: Improved gluon propagator

The improved gluon propagator $D^{\text{Imp}}_{\mu \nu}$ was determined in Ref. 17, and presented in a useful general form in Ref. 34. Here we present a simpler form.

The improved gluon action takes the following quadratic form in the gluon fields after covariant gauge fixing:

$$Q_{\mu \nu} = ((D^{\text{Imp}})^{-1})_{\mu \nu}$$

$$= \frac{1}{\alpha} \hat{k}^2 \mathcal{P}_{\mu \nu} + f \hat{k}^2 \delta^T_{\mu \nu} - c \mathcal{M}_{\mu \nu},$$

where $\alpha$ is the gauge-fixing parameter,

$$\hat{k}^a \equiv \sum_{\mu} (\hat{k}_{\mu})^a$$

with $\hat{k}_{\mu} = 2 \bar{s}_{\mu} = 2 \sin(k_{\mu}/2)$, and $\mathcal{P}$ is the longitudinal projector

$$\mathcal{P}_{\mu \nu} = \frac{\hat{k}_{\mu} \hat{k}_{\nu}}{\hat{k}^2} \left[ \mathcal{P}^2 = \mathcal{P} \right],$$

$\delta^T$ is the transverse delta-function

$$\delta^T_{\mu \nu} = \delta_{\mu \nu} - \mathcal{P}_{\mu \nu} \left[ \mathcal{P} \delta^T = 0, (\delta^T)^2 = \delta^T \right],$$

$\mathcal{M}$ is an auxiliary transverse matrix

$$\mathcal{M}_{\mu \nu} = \delta_{\mu \nu} \frac{\hat{k}_{\mu} \hat{k}_{\nu}}{\hat{k}^2} - \frac{\hat{k}_{\mu} \hat{k}_{\nu}}{\hat{k}^2} \frac{\hat{k}_{\nu} \hat{k}_{\mu} \hat{k}^2}{\hat{k}^4}.$$
satisfying $\mathcal{P} \mathcal{M} = 0$ and $\delta^T \mathcal{M} = 0$, and the function $f$ is

$$f = (\omega - c' \hat{k}^2 - c \hat{k}^4 / k^2). \quad (A7)$$

The coefficients $\omega$, $c$ and $c'$ are determined from the parameters of the improved action [see eq. (11)]:

$$\omega = c_0 + 8c_1 + 8c_2 + 16c_3 \quad (A8a)$$
$$c = c_1 - c_2 - c_3 \quad (A8b)$$
$$c' = c_2 + c_3 \quad (A8c)$$

We note that $D$ elements are proportional to $\hat{\omega}$ and off-diagonal parts of the “smeared-smeared propagator” [17], which are defined through

$$T_{\mu\nu} = \sum_{\alpha,\beta} h_{\mu\alpha} h_{\nu\beta} D_{\alpha\beta} \quad (B1)$$
$$= B [\delta_{\mu\nu} P_\mu + (1 - \delta_{\mu\nu}) 4 \bar{s}_\mu s_\nu O_{\mu\nu}] \quad (B2)$$

Here $B$ is the boson propagator

$$B = \left[ 4 \sum_{\mu} \bar{s}_\mu^2 \right]^{-1} = \left[ \hat{k}^2 \right]^{-1}. \quad (B3)$$

Explicit expressions for $P_\mu$ and $O_{\mu\nu}$ can be obtained using the decompositions (A11) and (13a) of the improved gluon propagator and smearing kernel, respectively. They are, however, uninformative and we do not reproduce them here.

As noted in the main text, the expressions for one-loop matching factors for HYP-smeared fermions that are given in Ref. [11] still hold as long as the $P_\mu$ and $O_{\mu\nu}$ defined above are used. This follows because the generalized $P_\mu$ and $O_{\mu\nu}$ still satisfy the property of being symmetric separately in each component of $\hat{k}$. This property is used to simplify the expressions.

We think it useful to repeat the one-loop expressions here, both for the sake of clarity (since Ref. [11] considered other cases not relevant here), and in order to facilitate the subsequent discussion of the results with asqtad fermions.

The diagonal part of the $X$-diagram contribution is

$$X_{ii} = \sum_{\mu,\nu} \int_k \left[ c_2^2 P_\mu(s^N_\nu) B F^2 V_i(k) - \frac{B^2}{4} \right] (-) \bar{s}_\mu \bar{s}_\nu$$
$$+ 2 \sum_{\mu < \nu} \int_k s_\mu \bar{s}_\mu s_\nu \bar{s}_\nu O_{\mu\nu} B F^2 V_i(k)$$
$$\times \left[ 1 - (-) \bar{s}_\mu \bar{s}_\nu \right]. \quad (B4)$$

Appendix B: One-loop results for HYP-smeared fermions

In this appendix we present the one-loop expressions for matching factors for HYP-smeared fermions. The results are presented in a general way such that they include also unimproved staggered fermions, as well as the impact of mean-field improvement.

Key building blocks for these results are the diagonal and off-diagonal parts of the “smeared-smeared propagator” (17), which are defined through

$$T_{\mu\nu} = \sum_{\alpha,\beta} h_{\mu\alpha} h_{\nu\beta} D_{\alpha\beta} \quad (B1)$$
$$= B [\delta_{\mu\nu} P_\mu + (1 - \delta_{\mu\nu}) 4 \bar{s}_\mu s_\nu O_{\mu\nu}] \quad (B2)$$

We note that $D$ is symmetric, and that its off-diagonal elements are proportional to $\hat{k}_\mu \hat{k}_\nu$ multiplied by a function that is even in each of the components of $\hat{k}$. Thus it is convenient to write the propagator as

$$D_{\mu\nu} = \delta_{\mu\nu} T_{\mu\nu} + (1 - \delta_{\mu\nu}) \hat{k}_\mu \hat{k}_\nu D_{\mu\nu}. \quad (A11)$$

$$D_{\mu\nu} = \delta_{\mu\nu} T_{\mu\nu} + (1 - \delta_{\mu\nu}) \bar{k}_\mu \bar{k}_\nu D_{\mu\nu}. \quad (A11)$$

We think it useful to repeat the one-loop expressions here, both for the sake of clarity (since Ref. [11] considered other cases not relevant here), and in order to facilitate the subsequent discussion of the results with asqtad fermions.

The diagonal part of the $X$-diagram contribution is

$$X_{ii} = \sum_{\mu,\nu} \int_k \left[ c_2^2 P_\mu(s^N_\nu) B F^2 V_i(k) - \frac{B^2}{4} \right] (-) \bar{s}_\mu \bar{s}_\nu$$
$$+ 2 \sum_{\mu < \nu} \int_k s_\mu \bar{s}_\mu s_\nu \bar{s}_\nu O_{\mu\nu} B F^2 V_i(k)$$
$$\times \left[ 1 - (-) \bar{s}_\mu \bar{s}_\nu \right]. \quad (B4)$$
Here the integral is

$$\int k \equiv 16\pi^2 \prod_{\mu} \int_{-\pi}^{\pi} \frac{dk_{\mu}}{2\pi}. \quad (B5)$$

The new abbreviations are \(\tilde{c}_i^j = \cos(k_{\mu}/2)\) and \(s_{\mu} = \sin(k_{\mu})\). For HYP fermions \(s_{\mu}^j = s_{\mu}\), although this will not hold for asqtad fermions. The denominator of the fermion propagator is

$$F = \left[ \sum_{\mu} (s_{\mu}^N)^2 \right]^{-1}, \quad (B6)$$

while the vertex factors are

$$V_i(k) = \prod_{\mu} \cos [k_{\mu}(S-F)_{\mu}], \quad (B7)$$

with \(S_{\mu}\) and \(F_{\mu}\) being hypercube four-vectors describing, respectively, the spin and taste of the bilinear (which are collectively labeled \(\tilde{S}^i\)).

The non-zero mixing coefficients are \((i \neq j)^6\)

$$X_{ij} = -\int_k 2B F^2 s_1 s_2 (s_1^N s_2^N P_3 c_3^2 V_{mix}^{P_{ij}} + s_1 s_3^N O_{12} V_{mix}^{P_{ij}}). \quad (B8)$$

The vertex factors for the cases of non-vanishing mixing are collected in Table V.

The contribution of \(Y\)-diagrams depends only on the distance \(\Delta = \sum_{\mu} (S-F)_{\mu}^2\). It vanishes for \(\Delta = 0\), and is otherwise

$$Y_\Delta = \sum_{k=1}^{\Delta} I_k, \quad (\Delta \geq 1), \quad (B9)$$

where

$$I_\Delta = \int_k BF (s_1 s_1^N P_1 + 12 s_1^2 s_2^N O_{21}) V_\gamma(\Delta), \quad (B10)$$

with vertex factors

$$V_\gamma(1) = 1, \quad V_\gamma(2) = \frac{c_2 + c_3 + c_4}{3}, \quad (B11)$$

$$V_\gamma(3) = \frac{c_2 c_3 + c_2 c_4 + c_3 c_4}{3}, \quad V_\gamma(4) = c_2 c_3 c_4.$$

The tadpole contribution also depends only on the distance \(\Delta\). It is conveniently divided into the contribution from gluon propagators beginning and ending on the same smeared link, \(T_\Delta^a\), and the remainder, \(T_\Delta^b\), which requires \(\Delta \geq 2\). The former is naturally combined with the self-energy tadpole to yield

$$T_\Delta^a + ZT = (\Delta - 1) \left[ I_{\text{MF}} - \int_k (B/2) P_1 \right]. \quad (B12)$$

Here \(I_{\text{MF}}\) is present if mean-field improvement is implemented, and is given by

$$I_{\text{MF}} = \int_k B z_\Delta^2 \left[ P_1 - 4z_\Delta^2 O_{12} \right]. \quad (B13)$$

Note that for unimproved staggered fermions, mean-field improvement is commonly called tadpole improvement. The numerical values of \(I_{\text{MF}}\) are \(\pi^2 = 9.869605, 7.229736\) for unimproved staggered fermions with Wilson and improved gauge actions, respectively, and \(1.053786, 0.722795\) for HYP fermions with the same two gauge actions.

The second part of the tadpole contribution is unaffected by mean-field improvement, and is

$$T_\Delta^b = \int_k 4B \tilde{s}_1^2 \tilde{s}_2^2 O_{12} V_\gamma(\Delta), \quad (B14)$$

with vertex factors \(V_\gamma(0) = V_\gamma(1) = 0\) and

$$V_\gamma(2) = 1, \quad V_\gamma(3) = 2 + c_3, \quad V_\gamma(4) = 3 + 2c_3 + 3c_4. \quad (B15)$$

Finally, the non-tadpole self-energy contribution can be obtained from the conservation of the taste-singlet vector current:

$$Z = -X_{ii} - Y_1, \quad i = (\gamma_{\mu} \otimes 1). \quad (B16)$$

These results hold for the HYP action with different choices for smearing kernel (entering through the coefficients \(d_{1-4}\)) and different choices of gauge action (entering through the coefficients \(c_{0-3}\) in the gluon propagator).

### Appendix C: One-loop results for asqtad fermions

Using asqtad rather than HYP-smeared fermions leads to three changes: (i) the links are now \(O(a^2)\) improved, rather than HYP-smeared; (ii) the Naik term is present; and (iii) the hypercube vector current is no longer conserved. The impact of these changes is that, while the \(X\)- and \(Y\)-diagrams can be obtained by simple substitutions from those for HYP-smeared fermions, the tadpole and self-energy contributions must be calculated anew. In detail, the changes from the previous section are as follows:

- The coefficients in the smearing kernel are now \(d_1 = 0, d_2 = d_3 = d_4 = 1\). These enter through \(D_{\mu}\) and \(\bar{G}_{\nu,\mu}\).

---

6 For the asqtad action, the \(O_{12}\) part of \(X_{ij}\) in eq. (B5) corrects an error in eqs. (20-23) of Ref. [11]. The numerical impact of this error is, however, minor: results for the mixing coefficients given in Table II of Ref. [11] are changed by less than \(10^{-3}\). We stress that for the HYP action, the expressions and numerical values given in Ref. [11] are correct.
In all expressions, $s^N_{\mu}$ now differs from $s_\mu$ due to the effect of the Naik term on the propagator:

$$s^N_{\mu} = s_\mu (1 + s^2_\mu/6). \quad (C1)$$

The form of the result for X-diagrams remains unchanged, but $P_\mu$ and $O_{\mu\nu}$ are changed because of the impact of the Naik term on the quark-gluon vertex. They are replaced by $P^{NN}_\mu$ and $O^{NN}_{\mu\nu}$, obtained from

$$T^{NN}_{\mu\nu} = \sum_{\alpha,\beta} h^N_{\mu\alpha} h^N_{\nu\beta} D^{\text{imp}}_{\alpha\beta} \quad (C2)$$

$$= B \left[ \delta_{\mu\nu} P^{NN}_\mu + (1 - \delta_{\mu\nu}) 4 s_\mu s_\nu O^{NN}_{\mu\nu} \right], \quad (C3)$$

where

$$h^N_{\mu\nu} = h_{\mu\nu} + \delta_{\mu\nu} s^2_\mu/6. \quad (C4)$$

The form of the result from Y-diagrams is also unchanged, but now $P_\mu$ and $O_{\mu\nu}$ must be replaced by $P^{N}_\mu$ and $O^{N}_{\mu\nu}$, which are obtained from

$$T^{N}_{\mu\nu} = \sum_{\alpha,\beta} h^N_{\mu\alpha} h^N_{\nu\beta} D^{\text{imp}}_{\alpha\beta} \quad (C5)$$

$$= B \left[ \delta_{\mu\nu} P^{N}_\mu + (1 - \delta_{\mu\nu}) 4 s_\mu s_\nu O^{N}_{\mu\nu} \right]. \quad (C6)$$

These asymmetrical changes reflect the fact that the Naik term enters when the gluon attaches to the external fermion leg but not when it attaches to the operator.

Type-(b) tadpole diagrams are unchanged in form and involve $O_{\mu\nu}$ without any change from the Naik term. In other words, one uses eq. (B14) with $O_{\mu\nu}$ from eq. (B2).

Type-(a) tadpole diagrams are replaced by

$$T^a_Z + Z_T = (\Delta - 1) \left[ \frac{1}{4} \int_k D^{\text{imp}}_{11} (-5 + 3c_1 + 3c_2 - 3c_1c_2) + 12 \int_k D^{\text{imp}}_{11} (\bar{\pi}_1 s^4_1) + \frac{5}{2} T^{Sym} \right]$$

$$+ \frac{1}{4} \left[ T^{Sym} - \int_k D^{\text{imp}}_{11} c_1 (1 + c_1) \right], \quad (C7)$$

where the $T^{Sym}$ terms arise from the tadpole-improvement of the links, with

$$T^{Sym} = \int_k \left( D^{\text{imp}}_{11} (\bar{s}_2) - 4 D^{\text{imp}}_{12} (\bar{s}_1) (\bar{s}_2)^2 \right). \quad (C8)$$

For the Wilson gauge action $T^{Sym} = \pi^2$, but this factor is reduced for the improved gauge action to $7.220736$.

The non-tadpole self-energy is given by

$$Z = \int_k (B^2 + B F_{12}), \quad (C9)$$

where

$$I_Z = c_1^N \left[ 1 - 2 (s^N_1)^2 F \right] \left[ c_1^2 P^{NN}_{1} - 3 c_1^3 P^N_{12} \right]$$

$$- s_1 s_2^N P^{NN}_{11} - 12 s_1^2 s_2^N O^{NN}_{12}$$

$$- s_1 s_2^N (s^2_1/3 - c_1^2) D^{1}_1$$

$$+ s_1^2 s_2^N \left( 3/4 - s_2^N \right) G^1_{12}$$

$$- 12 s_1^3 c_1^N s_2^N O^{NN}_{12}. \quad (C10)$$

Here $c_1^N = c_1 (1 + s^2_\mu/2)$, while the new quantities $D^{1}_1$ and $G_{\nu\mu}$ arise from the propagator from a smeared link to a thin (Naik) link. They are defined by

$$\sum_{\rho} h^N_{\rho\mu} T^{\text{imp}}_{\rho\nu} \equiv B \left[ \delta_{\mu\nu} D^{1}_{\mu} + (1 - \delta_{\mu\nu}) s_\mu \bar{s}_\nu G^{1}_{\nu,\mu} \right]. \quad (C11)$$

As noted in the main text, we have also calculated the matching factor for the asqtad conserved vector current. This is given by adding 1- and 3-link terms to the hypercube current, both containing tadpole-improved thin links:

$$V_{\mu}^{\text{CVC}}(y) = [V_{\mu} \times S](y; W) + \frac{1}{8 s_{10}} [V_{\mu} \times S](y; U)$$

$$- \frac{1}{24} \times \frac{1}{16 s_{10}} \sum_A \eta_{\mu}(y) \times$$

$$\left[ \chi_{y_{1+1}} U_{\mu}(y + \bar{A} - 2 \mu, \bar{y} + \bar{A} + \bar{\mu}) \chi_{y_{1+1}} \bar{A}_{\mu} \right.$$

$$+ \chi_{y_{1+1}} \bar{U}_{\mu}(y + \bar{A} - \mu, \bar{y} + \bar{A} + 2 \mu) \chi_{y_{1+1}}_{\mu} \bar{A} \bar{\mu}$$

$$+ \chi_{y_{1+1}} U_{\mu}(y + \bar{y} + \bar{A} + 3 \mu) \chi_{y_{1+1}}_{y_{1+2}} + h.c. \right]. \quad (C12)$$
where the notation for bilinears is as in eq. (18), except that the second argument of $[V_i \times S]$ indicates the type of links used to create the parallel transporter. In addition, $A$ is a vector running over the 8 positions of the cube perpendicular to $\mu$, while “h.c.” implies interchange of the positions of $\bar{\chi}$ and $\chi$ fields and hermitian conjugation of the gauge fields.

At tree-level, and for physical external momenta, the extra 1- and 3-link terms in the current cancel. At one-loop, however, these terms lead to additional contributions. For X-diagrams, the expression for $T_{\mu\nu}^N$ in (C5) is changed by the substitution

$$h_{\nu\beta} \to h_{\nu\beta} - \delta_{\nu\beta} c_\nu (1 + c_\nu)/2,$$

with $h_{\nu\mu}^N$ unchanged. The T diagrams must be calculated anew, but turn out to exactly cancel the ZT contribution (for any choice of gluon propagator).

The net result, as we have checked analytically, is that the contributions of the X, Y and Z diagrams cancel exactly. This provides an important check on our result for the Z diagram with asqtad fermions.

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