Coloured Scalars Mediated Rare Charm Meson Decays to Invisible Fermions

Svjetlana Fajfer and Anja Novoseč
Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia and Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia
(Dated: January 27, 2021)

We consider effects of coloured scalar mediators in decays $c \to u$ invisibles. In particular, in these processes, as invisibles, we consider massive right-handed fermions. The coloured scalar $\tilde{S}_1 \equiv (3, 1, -2/3)$, due to its coupling to weak singlets up-quarks and invisible right-handed fermions ($\chi$), is particularly interesting. Then, we consider $\tilde{R}_2 \equiv (3, 2, 1/6)$, which as a weak doublet is a subject of severe low-energy constraints. The $\chi$ mass is considered in the range $(m_K - m_\pi)/2 \leq m_\chi \leq (m_D - m_\pi)/2$. We determine branching ratios for $D \to \chi\chi$, $D \to \chi\gamma$ and $D \to \pi\chi$ for several $\chi$ masses, using most constraining bounds. For $\tilde{S}_1$, the most constraining is $D^0 - D^0$ mixing, while in the case of $\tilde{R}_2$ the strongest constraint comes from $B \to K\overline{E}$. We find in decays mediated by $\tilde{S}_1$ that branching ratios can be $\mathcal{B}(D \to \chi\chi) < 10^{-8}$ for $m_\chi = 0.8$ GeV, $\mathcal{B}(D \to \chi\gamma) \sim 10^{-8}$ for $m_\chi = 0.18$ GeV, while $\mathcal{B}(D^+ \to \pi^+\chi\gamma)$ can reach $\sim 10^{-6}$ for $m_\chi = 0.18$ GeV. In the case of $\tilde{R}_2$ these decay rates are very suppressed. We find that future tau-charm factories and Belle II experiments offer good opportunities to search for such processes. Both $\tilde{S}_1$ and $\tilde{R}_2$ might have masses within LHC reach.

I. INTRODUCTION

Low-energy constraints of physics beyond Standard Model (BSM) are well established for down-like quarks by numerous searches in processes with hadrons containing one $b$ or/and $s$ quark. However, in the up-quark sector, searches are performed in top decays, suitable for LHC studies, while in charm hadron processes at b-factories or/and $\tau$-charm factories. Recently, an extensive study on $c \to w\overline{v}$ appeared in Ref. [1], pointing out that observables very small in the Standard Model (SM) offer unique (null) tests of BSM physics. Namely, for charm flavour changing neutral current (FCNC) processes, severe Glashow-Iliopoulos-Maiani (GIM) suppression occurs. The decay $D^0 \to \nu\overline{\nu}$ amplitude is helicity suppressed in the SM. The authors of [2] made very detailed study of heavy meson decays to invisibles, assuming that the invisibles can be scalars or fermions with both helicities. They found out that in the SM branching ratio $\mathcal{B}(D^0 \to \nu\overline{\nu}) = 1.1 \times 10^{-31}$. Then the authors of [3] found that the decay width of $D^0 \to \nu\overline{\nu}$ in the SM is actually dominated by the contribution of $D^0 \to \nu\overline{\nu}\nu\overline{\nu}$. These studies’ main message is that SM provides no irreducible background to analysis of invisibles in decays of charm (and beauty) mesons. They also suggested [2], that in searches for a Dark Matter candidate, it might be important to investigate process with $\chi\chi\gamma$ in the final state, since a massless photon eliminates the helicity suppression. We also determine branching ratios for such decay modes. The authors of Ref. [1] computed the expected event rate for the charm hadron decays to a final hadronic state and neutrino - anti-neutrino states. They found out that in experiments like Belle II, which can reach per-mile efficiencies or better, these processes can be seen. In addition future FCC-ee might be capable of measuring branching ratios of $O(10^{-6})$ down to $O(10^{-8})$, in particular $D^0$, $D^0_{(s)}$ and $A^-\overline{\tau}$ decay modes.

On the other hand, the Belle collaboration already reached bound of the branching ratio for $\mathcal{B}(D^0 \to \nu\overline{\nu}) = 9.4 \times 10^{-5}$ and the Belle II experiment is expected to improve it [4]. The other $e^+e^-$ machines as BESSIII [5] and future FCC-ee running colliders at the $Z$ energies [6, 7] with a significant charm production with $\mathcal{B}(Z \to c\overline{c}) \approx 0.22$ [7] provide us with excellent tools for precision study of charm decays.

In this work we focus on the particular scenarios with coloured scalars or leptoquarks as mediators of the invisible fermions interaction with quarks. The coloured scalar might have the electric charge of $2/3$ or $-1/3$ depending on the interactions with up or down quarks. Instead of using general assumption on the lepton flavour structure from [1] and justifying Belle bound from [8], we rely on observables coming from the $D^0 - D^0$ oscillations and in the case of weak doublets, we include constraints from other flavour processes.

Motivated by previous works of Refs. [1, 2, 9, 12], we investigate $c \to u\chi\chi$ with $\chi$ being a massive $SU(2)_L$ singlet. Coloured scalars carry out interactions between invisible fermions and quarks. Namely, leptoquarks usually denote the boson interacting with quarks and leptons. However, the state $\tilde{S}_1$ does not interact with the SM leptons and, therefore, it is more appropriate to call it coloured scalar. Our approach is rather minimalistic due to only two Yukawa couplings and the mass of coloured scalar. The effective Lagrangian and coloured scalar mediators are introduced in Sec. II. In Sec. III we describe effects of $\tilde{S}_1$ mediator in rare charm decays, while in Sec. IV we give details of $\tilde{R}_2$ mediation in the same processes. Sec. V contains conclusions and outlook.
II. COLOURED SCALARS IN $c \to u\chi\bar{\chi}$

In experimental searches, the transition $c \to u$ invisibles might be approached in processes $c \to u\bar{E}$ with $\bar{E}$ being missing energy. Therefore, invisibles can be either SM neutrinos or new right-handed neutral fermions (having quantum numbers of right-handed neutrinos), or scalars/vectors as suggested in Ref. [2]. They found that these assumptions allow upper limits as large as few $10^{-5}$, while in the limit of lepton universality branching ratios can be as large as $10^{-6}$. To consider invisible fermions, having quantum numbers of right-handed neutrinos, and being massive, we extend the effective Lagrangian by additional operators as described in Refs. [3, 13]

$$\mathcal{L}_{\text{eff}} = \sqrt{2} G_F \left[ c_{LL}^{\mu \nu} (\bar{\nu}_L \gamma_{\mu \nu} c_L) (\bar{\nu}_L \gamma_{\mu \nu} c_L) \right]$$

$$+ c_{RR}^{\mu \nu} (\nu_R \gamma_{\mu \nu} \bar{c}_R) (\nu_R \gamma_{\mu \nu} \bar{c}_R) + c_{LR}^{\mu \nu} (\bar{\nu}_L \gamma_{\mu \nu} c_R) (\bar{\nu}_L \gamma_{\mu \nu} c_R) + c_{LR}^{\mu \nu} (\bar{\nu}_L \gamma_{\mu \nu} c_L) (\bar{\nu}_L \gamma_{\mu \nu} c_L)$$

$$+ c_{RR}^{\mu \nu} (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_L) (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_L) + c_{RR}^{\mu \nu} (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_R) (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_R)$$

$$+ h_{\text{c..}} \left[ (\bar{\nu}_L \gamma_{\mu \nu} c_R) (\bar{\nu}_L \gamma_{\mu \nu} c_R) + (\bar{\nu}_L \gamma_{\mu \nu} c_L) (\bar{\nu}_L \gamma_{\mu \nu} c_L) + (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_L) (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_L) + (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_R) (\bar{\nu}_R \gamma_{\mu \nu} \bar{c}_R) \right]$$

In Ref. [1] right-handed massless neutrinos are considered. Also, in Ref. [12] authors considered charm meson decays to invisible fermions, which have negligible masses. In the following, we consider massive right-handed fermions and use further the notation $\nu_R \equiv \chi_R$. Following [13], we write in Table I interactions of the coloured scalar $S_1$ and $R_2$ with the up quarks and $\tilde{R}_2$ and $S_1$ with down quarks.

| Coloured Scalar | Invisible fermion |
|-----------------|------------------|
| $S_1 = (3, 1, 1/3)$ | $\tilde{d}_R, \chi^j \bar{\chi}_j S_1$ |
| $\tilde{S}_1 = (3, 1, -2/3)$ | $\tilde{u}_R, \chi^j \bar{\chi}_j S_1$ |
| $\tilde{R}_2 = (3, 2, 1/6)$ | $\tilde{c}_R, \chi^j \bar{\chi}_j \tilde{R}_2^{1/3}$ |
| $\tilde{R}_2 = (3, 2, 1/6)$ | $\tilde{d}_R, \chi^j \bar{\chi}_j \tilde{R}_2^{1/3}$ |

Table I. The coloured scalars $\tilde{S}_1$, $S_1$ and $\tilde{R}_2$ interactions with invisible fermions and quarks. Here we use only right-handed couplings of $S_1$, Indices $i,j$ refer to quark generations.

We concentrate only on coloured scalar and scalar leptoquark due to difficulties with vector leptoquarks. Namely, the simplest way to consider vector leptoquarks in an ultra-violet complete theory is when they play the role of gauge bosons. For example, $U_1$ is one of the gauge bosons in some of Pati-Salam unification schemes [14, 15]. However, other particles with masses close to $U_1$ with many new parameters in such theories, making it rather difficult to use without additional assumptions.

Coloured scalars contributing to transition $c \to u\chi\bar{\chi}$ have following Lagrangians, as already anticipated in in [12]

$$\mathcal{L}(\tilde{S}_1) \supset \tilde{y}_{1ij}^{RR} \tilde{u}_R \chi^i \chi^j \bar{S}_1 + h.c.$$  

$$\mathcal{L}(\tilde{R}_2) \supset (V_{y_2}^{LR})_{ij} \tilde{u}_L \chi_R \bar{\chi}_j R_2^{2/3} + (V_{y_2}^{LR})_{ij} \tilde{d}_L \bar{\chi}_j R_2^{2/3} + h.c..$$

Here, we give only terms containing interactions of quarks with right-handed $\chi$. The $S_1$ scalar leptoquark, in principle, might mediate $c \to u\chi\bar{\chi}$ on the loop level, with one $W$ boson changing down-like quarks to $u$ and $c$. Obviously, such a loop process is also suppressed by loop factor $1/(16\pi^2)$ and $G_F$ making it negligible. Also, due to the right-handed nature of $\chi$, one can immediately see that in the case of $\tilde{S}_1$, the effective Lagrangian has only one contribution

$$\mathcal{L}_{\text{eff}} = \sqrt{2} G_F c^{LR} (\tilde{u}_L \gamma_{\mu} c_R) (\bar{\chi}_R \gamma^\mu \chi_R),$$

with

$$c^{RR} = \frac{v^2}{2M_\chi^2} \tilde{y}_{1ij}^{RR} \tilde{y}_{1ij}^{RR*}.$$  

In the case of $\tilde{R}_2$

$$\mathcal{L}_{\text{eff}} = \sqrt{2} G_F c^{LR} (\tilde{u}_L \gamma_{\mu} c_R) (\bar{\chi}_R \gamma^\mu \chi_R),$$

with

$$c^{LR} = -\frac{v^2}{2M_{\tilde{R}_2}^2} \left( V_{y_2}^{LR} \right)_{ij} \left( V_{y_2}^{LR} \right)_{ij}^*.$$  

For the mass of $\chi$, kinematically allowed, in the $c \to u\chi\bar{\chi}$ decay, one can relate this amplitude to $b \to s\chi\bar{\chi}$ or in $s \to \bar{d}\chi\bar{\chi}$. However, it was found [16] that the experimental rates for $K \to \pi\nu\bar{\nu}$ are very close to the SM rate [17], leaving very little room for NP contributions. Therefore, we avoid this kinematic region and consider mass of $\chi$ to be $m_\chi \geq (m_K - m_\pi)/2$, while the charm decays allow $m_\chi \leq (m_D - m_\pi)/2$. For our further study it is very important that $\chi$ is a weak singlet and therefore LHC searches of high-$p_T$ lepton tails [18, 19] are not applicable for the constraints of interactions in the cases we consider. However, further study of final states containing mono-jets and missing at LHC and future High luminosity colliders will shed more light on these processes.

III. $\tilde{S}_1$ IN $c \to u\chi\bar{\chi}$

Due to its quantum numbers, the coloured scalar $\tilde{S}_1$ and $\chi$ can interact only with up-like quarks. Most generally, the number of $\chi$’s can be three and the matrix $y_{1ij}^{RR}$...
can have \(9 \times 2\) parameters. Here, we consider one \(\chi\), that can couple to both \(u\) and \(c\) quarks. These two couplings might enter in amplitudes for processes with down-like quarks at loop-level, as discussed in [20]. Obviously, due to the right-handed nature of \(\chi\), one can immediately see that in the case of \(S_1\), the effective Lagrangian has only the contribution
\[
\mathcal{L}_{\text{eff}} = \sqrt{2} G_F \frac{v^2}{2 M^2} \bar{y}_{1c}^{RR} \bar{y}_{1w}^{RR*}(\pi R\gamma_\mu c R)(\bar{\chi} \gamma_\mu \chi R). \tag{8}
\]

First, we discuss constraints from \(D^0 - \bar{D}^0\) mixing and then consider exclusive decays \(D^0 \to \chi \bar{\chi}, \bar{D}^0 \to \chi \bar{\chi}\gamma\), and \(D \to \pi \chi \bar{\chi}\). The authors of Ref. [12] considered scalar leptoquarks allowing each up-quark can couple to different flavour of lepton or right-handed neutrino. In such a way, they avoid constraints from the \(D^0 - \bar{D}^0\) mixing.

1. Constraints from \(D^0 - \bar{D}^0\)

The strongest constraints on \(\chi\) interactions with \(u\) and \(c\) comes from the \(D^0 - \bar{D}^0\) oscillations. The interactions in Eqs. (2) and (3) can generate transition \(D^0 - \bar{D}^0\). Coloured scalar \(S_1\) contributes to the operator entering the effective Lagrangian [13] [21]
\[
\mathcal{L}_{\text{eff}}^{\text{mix}} = - C_6 (\bar{c} \gamma_\mu P_R u) (\bar{c} \gamma_\mu P_R u), \tag{9}
\]
with the Wilson coefficient given by
\[
C_6 = \frac{1}{64 \pi^2 M_{S_1}^2} \left( \frac{\bar{y}_{1c}^{RR}}{\bar{y}_{1w}^{RR*}} \right)^2. \tag{10}
\]

The standard way to write the hadronic matrix element is \(\langle \bar{D}^0 | (\bar{u} \gamma_\mu P_R c)(\bar{u} \gamma_\mu P_R c) | D^0 \rangle = \frac{3}{2} m_D f_D^2 B_D\), with the bag parameter \(B_D(3 \text{ GeV}) = 0.757(27)(4)\) computed in the MS scheme, which was computed by the lattice QCD [22] and the D meson decay constant defined as \(\langle 0 | (\bar{u} \gamma_\mu \gamma_5 c) | D(p) \rangle = i f_D p_\mu\), with \(f_D = 0.2042\) GeV [23].

Due to large nonperturbative contributions, the SM contribution is not well known. Therefore, in the absence of CP violation, the robust bound on the product of the couplings can be obtained by requiring that the mixing frequency should be smaller than the world average \(x = 2 |M_{12}|/\Gamma = (0.43^{+0.10}_{-0.11})\%\) by HFLAV [24]. The bound on this Wilson coefficient can be derived following [20] [25]
\[
|r C_6(M_{S_1})| \frac{2 m_D f_D^2 B_D}{3 \Gamma_D} < x, \tag{11}
\]
with a renormalisation factor \(r = 0.76\) due to running of \(C_6\) from scale \(M_{S_1} \approx 1.5\) TeV down to 3 GeV. One can derive \(|C_6| < 2.3 \times 10^{-13} \text{ GeV}^{-2}\) or
\[
|\bar{y}_{1c}^{RR} \bar{y}_{1w}^{RR*}| < 1.2 \times 10^{-5} M_{S_1}/\text{GeV}. \tag{12}
\]
\[
|c^{RR}| < 0.363 \text{ GeV}. \tag{13}
\]

2. \(D^0 \to \chi \bar{\chi}\)

The amplitude for this process can be written as
\[
\mathcal{M}(D^0 \to \chi \bar{\chi}) = \sqrt{2} G_F f_D c^{RR} m_{\chi} \bar{u}_\chi (p_1) \gamma_5 v_\chi (p_2), \tag{14}
\]
giving the branching ratio
\[
B(D^0 \to \chi \bar{\chi}) = \frac{1}{\Gamma_D} \frac{G_F f_D^2 m_D}{16 \pi} |c^{RR}|^2 m_{\chi}^2 \sqrt{1 - \frac{4 m_{\chi}^2}{m_D^2}}. \tag{15}
\]

Using Belle bound \(B(D^0 \to \chi \bar{\chi}) < 9.4 \times 10^{-5}\) [5], one can find easily the bound on Wilson coefficient \(|c^{RR}| < 0.046\). This value is derived for the mass \(m_{\chi} = 0.8\) GeV. We analyse the dependence on the mass of \(S_1\), allowing the mass of \(\chi\) to be \((m_K - m_\pi)/2 < m_{\chi} < (m_D - m_\pi)/2\), and assume the branching ratio for \(B(D^0 \to \chi \bar{\chi}) < 10^{-10}, 10^{-9}\) and \(10^{-8}\), with \(|\bar{y}_{1c}^{RR} \bar{y}_{1w}^{RR*}| = 1\). We present our result in Fig. 2 and find that mass of \(S_1\), using these reasonable assumptions, can be within LHC reach.

| \(m_{\chi}\) (GeV) | \(B(D^0 \to \chi \bar{\chi}))_{D_D}\) |
|----------------|------------------|
| 0.18           | < 1.1 \times 10^{-9} |
| 0.50           | < 7.4 \times 10^{-9} |
| 0.80           | < 1.1 \times 10^{-8} |

Table II. Branching ratios for \(B(D^0 \to \chi \bar{\chi})\) for three selected values of \(m_{\chi}\). The constraints from the \(D^0 - \bar{D}^0\) mixing is used, with \(c^{RR} < 3.63 \times 10^{-4}\), assuming \(M_{S_1} = 1000\) GeV.

Figure 1. The product of Yukawa couplings \(|\bar{y}_{1c}^{RR} \bar{y}_{1w}^{RR*}|\) as a function of the \(S_1\) mass. The pink line denotes the bound derived from Belle result [5], while the turquoise one is obtained with the bound from \(D^0 - \bar{D}^0\) oscillations.
In the second column the constraints from the \( B(D^0 \to \chi \chi) \) are obtained assuming \( B(D^0 \to \chi \chi) < 10^{-10}, 10^{-9} \) and \( 10^{-8} \), for the product \( g_{\chi \gamma} g_{\chi \chi} = 1 \).

3. \( D^0 \to \chi \chi \gamma \)

The authors of Ref. [2] suggested, that the helicity suppression, present in the \( D^0 \to \chi \chi \) amplitude for \( m_{\chi} = 0 \), is lifted by an additional photon in the final state and therefore \( D^0 \to \chi \chi \gamma \) might bring additional information on detection of invisibles in the final state. They found that the branching decay is

\[
B(D^0 \to \chi \chi \gamma) = \frac{G_F^2 F_{DQ}^2 F_{D}^4 |c^{RR}|^2 m_D^2 \alpha}{1152 \pi^2 \Gamma_D \sqrt{1 - 4x^2}} Y(x). \tag{16}
\]

In the above equations \( x = m_\chi / m_D \), \( F_{DQ} = 2/3(-1/(m_D - m_e) + 1/m_e) \), \( F_D = 0.2042 \text{ GeV} \) [23] and \( Y(x_\chi) \) is given in Appendix. Coefficient \( c^{RR} \) is constrained by Eq. [13]. Comparing this results with

| \( m_\chi (\text{GeV}) \) | \( B(D^0 \to \chi \chi \gamma)_{D-\bar{D}} \) | \( B(D^0 \to \chi \chi \gamma)_{Belle} \) |
|-----------------|-----------------|-----------------|
| 0.18            | \( < 2.1 \times 10^{-11} \) | \( < 1.3 \times 10^{-7} \) |
| 0.50            | \( < 6.9 \times 10^{-12} \) | \( < 6.3 \times 10^{-9} \) |
| 0.80            | \( < 8.4 \times 10^{-14} \) | \( < 2.2 \times 10^{-10} \) |

Table III. Bounds on the branching ratio for \( B(D^0 \to \chi \chi \gamma) \). In the second column the constraints from the \( D^0 - \bar{D}^0 \) mixing is used, assuming \( M_{\tilde{S}_i} = 1000 \text{ GeV} \). In the third column Belle bound \( B(D^0 \to \psi) < 9.4 \times 10^{-5} \) is used.

the SM result presented in Ref. [2] \( B(D^0 \to \nu \bar{\nu})_{\text{SM}} = 3.96 \times 10^{-14} \) we see that the existing Belle bound allows significant branching ratio, while the bounds from the \( D^0 - \bar{D}^0 \) mixing, for larger values of \( m_\chi \), lead to the branching ratio to be close to the SM results. Due to the mass of \( \chi \), the photon energy can be in the range \( 0 \leq E_\gamma \leq (m_D^2 - m_\chi^2)/(2m_D) \), which in principle would distinguish the SM contribution from the contributions with massive invisible fermions.

4. \( D \to \pi \chi \bar{\chi} \)

The rare charm decays due to GIM-mechanism cancellation are usually dominated by long distance contributions. Long distance contributions to exclusive decay channel \( D \to \pi \nu \bar{\nu} \) were considered in Ref. [20]. For example, the branching ratio \( BR(D^+ \to \pi^+ \rho^0 \to \pi^+ \nu \bar{\nu}) < 5 \times 10^{-16} \). The authors of [20] discussed another possibility \( D^+ \to \pi^+ \nu \to \pi^+ \nu \bar{\nu} \) and found that the branching ratio should be smaller than \( 1.8 \times 10^{-16} \). An interesting study of these effects was done in Ref. [27], implying that in order to avoid these effects one should make cuts in the invariant \( \chi \bar{\chi} \) mass square, \( q^2 < (m_\tau - m_\pi^0)(m_{\pi^+} - m_\tau)/m_\tau^2 \).

The amplitude for \( D \to \pi \chi \bar{\chi} \) can be written as

\[
\mathcal{M}(D \to \pi \chi \bar{\chi}) = \sqrt{2} G_F c^{RR} \bar{u}_\chi(p_1) \gamma_\mu P_R v_\chi(p_2) \langle \pi(k) | \bar{u}_\mu P_R | D(p) \rangle, \tag{17}
\]

with the standard form-factors definition

\[
\langle \pi(k) | \bar{u}_\mu P_R | D(p) \rangle = f_+(q^2) \left[ (p + k)^\mu - \frac{m_D^2 - m_\tau^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_D^2 - m_\tau^2}{q^2} q^\mu,
\]

with \( q = p - k \). We follow the update of the form-factors in Ref. [28]. This enables us to write the amplitudes in the form given in Ref. [21].

\[
\mathcal{M}(D(p) \to \pi(k) \chi(p_1) \bar{\chi}(p_2)) = \sqrt{2} G_F \langle V(q^2) \bar{u}_\chi(p_1) \gamma_5 v_\chi(p_2) + A(q^2) \bar{u}_\chi(p_1) \gamma_\gamma v_\chi(p_2) + P(q^2) \bar{u}_\chi(p_1) \gamma_\gamma v_\chi(p_2) \rangle, \tag{19}
\]

with the following definitions

\[
V(q^2) = A(q^2) \equiv c^{RR} f_+(q^2)
\]

\[
P(q^2) \equiv -c^{RR} m_\chi \left[ f_+(q^2) - \frac{m_D^2 - m_\tau^2}{q^2} (f_0(q^2) - f_+(q^2)) \right]. \tag{20}
\]

We can the differential decay rate distribution as

\[
\frac{d\Gamma(D \to \pi \chi \bar{\chi})}{dq^2} = \frac{1}{F_D} N \lambda^{1/2} \beta \left[ 2a(q^2) + \frac{2}{3} c(q^2) \right]. \tag{21}
\]

with notation \( \lambda \equiv \lambda(m_D^2, m_\tau^2, q^2) \), \( (\lambda(x,y,z) = (x + y + z)^2 - 4(xy + yz + zx)) \), \( \beta = \sqrt{1 - 4m_\chi^2/q^2} \) and \( N = \frac{G_F^2}{64 \pi^3 m_D} \). Note that in case of charged charm meson there is a multiplication by 2 in the differential decay rate compared to neutral \( D \).
Figure 3. Branching fraction for $D^+ \to \pi^+ \chi \bar{\chi}$ and $D^0 \to \pi^0 \chi \bar{\chi}$ as a function of $m_\chi$.

The integration bounds should be $4m_\chi^2 \leq q^2 \leq (m_D - m_\pi)^2$ in the case of $m_\chi = 0.5, 0.8$, while instead of $m_\chi = 0.18$ GeV, $q^2_{\text{cut}}$ is used from Ref. [27], giving the lowest mass of the invisibles should be searched in the region $m_\chi \geq \sqrt{q^2_{\text{cut}}} \simeq 0.29$ GeV. This enables us to avoid the region in which the effects of the long distance dynamics dominates. One can use

| $m_\chi$ (GeV) | $|B(D^0 \to \pi^0 \chi \bar{\chi})_{D-\bar{D}}|$ | $|B(D^+ \to \pi^+ \chi \bar{\chi})_{D-\bar{D}}|$ |
|---------------|---------------------------------|---------------------------------|
| 0.18          | $< 5.9 \times 10^{-9}$         | $< 3.0 \times 10^{-8}$         |
| 0.50          | $< 3.2 \times 10^{-9}$         | $< 1.6 \times 10^{-8}$         |
| 0.80          | $< 1.5 \times 10^{-10}$        | $< 7.6 \times 10^{-10}$        |

Table IV. Branching ratios for $B(D \to \pi \chi \bar{\chi})$. In the second and the third columns the constraint from the $D^0 - D^0$ mixing is used, assuming the mass of $M_{S1} = 1000$ GeV. In the case $m_\chi = 0.18$, the cut in integration variable is done by taking $q^2_{\text{cut}}$, as described in the text.

The Belle bound [8] for $B(D \to \bar{E})$ and determine $c_{RR}$ from $D^0 \to \chi \bar{\chi}$ for each $\chi$ mass. We obtain $B(D^0 \to \pi^0 \chi \bar{\chi})_{\text{Belle}} \leq 4.9 \times 10^{-4}, 4.0 \times 10^{-5}, 1.2 \times 10^{-6}$ and $B(D^+ \to \pi^+ \chi \bar{\chi})_{\text{Belle}} \leq 2.5 \times 10^{-5}, 2.1 \times 10^{-4}, 6.1 \times 10^{-6}$ for $m_\chi = 0.18, 0.5, 0.8$ GeV respectively. Obviously, the current Belle bound used in the Wilson coefficient leads to the significant increase of the branching ratios for both decay modes. Although the charm meson mixing is very constraining for the relevant couplings, the calculated branching ratios reaching the order $10^{-8}$ might be possible to observe at the future tau-charm factories and Belle II experiment.

IV. $\bar{R}_2$ IN c $\to$ $\nu \chi \bar{\chi}$

The $\bar{R}_2$ leptoquark is a weak doublet and it interacts with quark doublets [3]. Therefore, the appropriate couplings, $g_{2 \chi}^{LR}$, $g_{2 \chi}^{LR \chi}$ can be constrained from the $b \to s \chi \bar{\chi}$ and $s \to d \chi \bar{\chi}$ decays, as well as from observables coming from the $B_s \to \bar{B}_s$, $B_d \to \bar{B}_d$, $K^0 \to \bar{K}^0$ oscillations as in [20]. We consider the most constraining bounds coming from decays $B \to K \bar{E}$ and from the oscillations of $B_s \to \bar{B}_s$, relevant for the $\chi$ mass region $(m_K - m_{\pi})/2 < m_\chi < (m_D - m_\pi)/2$. The decay $B \to K \bar{E}$ was recently studied by the authors of Ref. [29]. They pointed out that current bound on the rate $B \to K \bar{E}$ when the SM branching ratio for $B \to K \bar{\nu}$ is subtracted from the experimental bound on $B(B^+ \to K^+ \bar{E})$ is the most constraining. They derived $B(B \to K \bar{E}) < 9.7 \times 10^{-9}$ as the strongest bound among $B \to H_\chi \bar{E}$ ($H_\chi$ is a hadron containing the $s$ quark).

1. Constraints from $B \to K \chi \bar{\chi}$ and $B_s - \bar{B}_s$ oscillations

The amplitude for $B \to K \chi \bar{\chi}$ can be written as

$$M(B \to K \chi \bar{\chi}) = \sqrt{2} g_F c^{LR}_B u_{1\chi}(p_1) \gamma_\mu P_R v_{\chi}(p_2)$$

$$\langle K(k) \gamma_\mu \bar{P}_L |B(p)\rangle.$$

(22)

In the case of Wilson coefficient $c_B^{LR}$ it is easy to find [14]

$$c_B^{LR} = -\frac{v^2}{2 M_{R_s}^2} g_{2 \chi}^{LR} g_{2 \chi}^{LR \chi}.$$

(23)

The integration over the phase space depends on the mass of $m_\chi$ we chose. Here we can choose a mass $\chi$, which we used in $D$ decays $(m_K - m_{\pi})/2 < m_\chi < (m_D - m_\pi)/2$. The bounds on the Wilson coefficient in Eq. (23) are following $|c_B^{LR}| < 3.3 \times 10^{-4}$, $< 4.9 \times 10^{-4}$ and $< 9.1 \times 10^{-4}$ for $m_\chi = 0.18, 0.50, 0.80$ GeV.

There are two box diagrams with $\chi$ within the box contributing to the $B_s - \bar{B}_s$ oscillations. The contribution of $R_2$ box diagrams to the effective Lagrangian for the $B_s - \bar{B}_s$ oscillation is

$$L_{B=2}^{\chi} = -\frac{1}{128 \pi^2} \left(\frac{g_{2 \chi}^{LR}}{M_{R_s}^2}\right)^2 \left(\frac{g_{2 \chi}^{LR \chi}}{M_{R_s}^2}\right)^2 \left(\bar{\chi} (\bar{\gamma}_\mu P_R) B_s (\bar{\gamma}_\mu P_R) \right).$$

(24)

We can understand this result in terms of the recent study of new physics in the $B_s - \bar{B}_s$ oscillation in [30]. The authors of [30] introduced the following notation of the New Physics (NP) contribution containing the right-handed operators as

$$L_{B=2}^{\chi NP} = -\frac{4 g_F}{\sqrt{2}} (V_{t b}^2 V_{t \bar{s}}^2) C_{R_s}^\chi (\bar{\chi} (\bar{\gamma}_\mu P_R) B_s (\bar{\gamma}_\mu P_R)).$$

(25)

Following their notation, one can write the modification of the SM contribution by the NP as in Ref. [30]

$$\frac{\Delta M_s^{SM+NP}}{\Delta M_s^{SM}} = 1 + \frac{\eta^{1/23}}{R_s^{SM}} C_{R_s}^\chi.$$  (26)

They found that $R_s^{SM} = (1.31 \pm 0.010) \times 10^{-3}$ and $\eta = \alpha_s(\mu_{NP})/\alpha_s(\mu)$. Relying on the Lattice QCD results of the two collaborations FNAL/MILC [31], HPQCD [32], the FLAG averaging group [33] published following results, which we use in our calculations

$$\Delta M_s^{FLAG2019} = (20.1^{+1.2}_{-1.6}) \, ps^{-1} = (1.13^{+0.07}_{-0.09}) \, \Delta M_s^{exp},$$

(27)
From these results, one can easily determine bound
\[
\left| \left( \frac{\tilde{g}^{LR}_{s\chi}}{M_{R_2}} \right)^2 \left( \frac{\tilde{g}^{LR+}_{b\chi}}{M_{R_2}} \right)^2 \right| < 1.39 \times 10^{-8} \text{ GeV}^{-2},
\]
(28)

The same couplings \( \tilde{g}^{LR}_{s\chi}, \tilde{g}^{LR+}_{b\chi} \) enter in the \( D^0 - \bar{D}^0 \) mixing [4] and condition [12], and one can derive
\[
\left| \left( V_{us} \tilde{g}^{LR}_{s\chi} \right) \left( V_{cb} \tilde{g}^{LR+}_{b\chi} \right)^* + \left( V_{cs} \tilde{g}^{LR}_{s\chi} \right) \left( V_{ub} \tilde{g}^{LR+}_{b\chi} \right)^* \right|_{D-\bar{D}} < 1.2 \times 10^{-5} M_{R_2}/\text{GeV}.
\]
(29)

The bound on coefficients in (28) lead to the one order of magnitude stronger constraint then one in (29), \( \tilde{g}^{LR}_{s\chi} \tilde{g}^{LR+}_{b\chi} < 1.58 \times 10^{-6} M_{R_2}/\text{GeV} \). In our numerical calculations we use this bound and do not specify the mass of \( \tilde{R}_2 \). However, one can combine these constraints and determined the \( \tilde{R}_2 \) mass, which can satisfy both conditions. In Fig. 4 we present dependence of the couplings \( \tilde{g}^{LR}_{s\chi}, \tilde{g}^{LR+}_{b\chi} \) as a function of mass \( M_{R_2} \) for masses using constrain from \( B^0_s - \bar{B}^0_s \) mixing and from the bound \( B(B^+ \to K^+ \bar{\psi}) < 9.7 \times 10^{-6} \) for \( m_\chi = 0.18, 0.5, 0.8 \) GeV. From Fig. 4 we see that the largest mass of \( \tilde{R}_2 \), which satisfies both conditions is \( M_{R_2} \approx 4400, 7100, 10800 \) GeV for the masses \( m_\chi = 0.8, 0.5, 0.18 \) GeV respectively. All \( \tilde{R}_2 \) masses below these limiting values are allowed and interestingly, they are within LHC reach.

2. \( \tilde{R}_2 \) in \( B(D^0 \to \chi \bar{\chi}), B(D^0 \to \chi \bar{\chi} \gamma) \) and \( B(D^+ \to \pi^+ \chi \bar{\chi}) \)

Using the same expressions as in the previous section, we calculate branching ratios for \( D^0 \to \chi \bar{\chi}, D^0 \to \chi \bar{\chi} \gamma \) and present them in Table V. The results for \( D \to \pi \chi \bar{\chi} \) are presented in Table VI. The Wilson coefficient \( c_D^{LR} \) is obtained using the constraint from \( B \to K \) missing energy. For \( m_\chi = 0.18, 0.5, 0.8 \) GeV

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\( m_\chi \) (GeV) & \( B(D^0 \to \chi \bar{\chi}) \) & \( B(D^0 \to \chi \bar{\chi} \gamma) \) \\
\hline
0.18 & \(< 1.6 \times 10^{-13}\) & \(< 1.9 \times 10^{-13}\) \\
0.50 & \(< 2.4 \times 10^{-12}\) & \(< 1.4 \times 10^{-15}\) \\
0.80 & \(< 1.3 \times 10^{-11}\) & \(< 2.7 \times 10^{-16}\) \\
\hline
\end{tabular}
\caption{Branching ratios for \( B(D^0 \to \chi \bar{\chi}) \) and \( B(D^0 \to \chi \bar{\chi} \gamma) \). The bounds on the Wilson coefficient \( c_D^{LR} \) derived from \( B(B \to K \bar{\psi}) < 9.7 \times 10^{-6} \) for selected masses of \( \chi \) from the range \( (m_K - m_\pi)/2 < m_\chi < (m_D - m_\pi)/2 \).}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\( m_\chi \) (GeV) & \( B(D^0 \to \pi^0 \chi \bar{\chi}) \) & \( B(D^+ \to \pi^+ \chi \bar{\chi}) \) \\
\hline
0.18 & \(< 8.7 \times 10^{-13}\) & \(< 4.5 \times 10^{-12}\) \\
0.50 & \(< 1.1 \times 10^{-12}\) & \(< 5.4 \times 10^{-12}\) \\
0.80 & \(< 1.7 \times 10^{-13}\) & \(< 8.7 \times 10^{-13}\) \\
\hline
\end{tabular}
\caption{Branching ratios for \( B(D^0 \to \pi^0 \chi \bar{\chi}) \) and \( B(D^+ \to \pi^+ \chi \bar{\chi}) \). The bounds on the Wilson coefficient \( c_D^{LR} \) derived from \( B(B \to K \bar{\psi}) < 9.7 \times 10^{-6} \). In the case \( m_\chi = 0.18 \), the cut in the integration variable is done by taking \( g_{cut}^2 \), as described in the text.}
\end{table}

they are \( c_D^{LR} = |(V_{us} V_{cb}^* + V_{cs} V_{ub}^*) c_D^{LR}| = 4.4 \times 10^{-6}, 6.6 \times 10^{-6}, 1.2 \times 10^{-5} \). Compared with the coloured scalar \( S_1 \) mediation, the branching ratios for all three decay modes are suppressed for several orders of magnitude, indicating the important role of constraints from \( B \) mesons. Such suppressed branching ratios of the all rare charm decays mediated by \( \tilde{R}_2 \) is almost impossible to observe. On the other hand, decays of hadrons containing \( b \) quarks, mediated by \( \tilde{R}_2 \) a much more suitable for searches of invisible fermions.

V. SUMMARY AND OUTLOOK

We have presented a study on rare charm decays with invisible massive fermions \( \chi \) in the final state. The mass of \( \chi \) is taken to be in the range \((m_K - m_\pi)/2 < m_\chi < (m_D - m_\pi)/2\), since the current experimental results on \( B(K \to \pi \nu \bar{\nu}) \) are very close to the SM result, almost excluding the presence of New Physics. We considered two cases with coloured scalar mediators of the up-quarks interaction with \( \chi \). The simplest model is one with \( S_1 = (3, 1, -2/3), \) which couples only to weak up-quark singlets, and the second mediator is \( \tilde{R}_2 = (\bar{3}, 2, 1/6) \) which couples to weak quark doublets.

In the case of \( S_1 \), the relevant constraint comes from the \( D^0 - \bar{D}^0 \) oscillations. We have calculated branching ratios for \( D^0 \to \chi \bar{\chi}, D^0 \to \chi \bar{\chi} \gamma \) and \( D \to \pi \chi \bar{\chi} \). The charm meson mixing severely constrain the branching ratio \( D^0 \to \chi \bar{\chi} \) in comparison with the experimental result for the branching ratio of \( D^0 \to \bar{E} \). For our choice of \( m_\chi \), the branching ratio for \( D^0 \to \chi \bar{\chi} \gamma \) can be calculated using experimental bound on the rate for \( D^0 \to \bar{E} \). In this case, there is an enhancement factor up to three orders of magnitude smaller, depending on the mass of \( \chi \) in comparison with the constraints from the \( D^0 - \bar{D}^0 \) os-
cillations. The branching ratios for $D \to \pi \chi \bar{\chi}$, based on charm mixing constraint, are of the order $10^{-9} - 10^{-7}$, suitable for searches at future tau-charm factories, BES III and Belle II experiments.

In the case of $\bar{R}_3$, for the mass range of $\chi$ relevant for charm meson rare decays, we rely on constraints coming from $B(B \to K\bar{\ell})$ and from the $B^0_s - \bar{B}^0_s$ mixing. We find that the all three decay modes $D^0 \to \chi \bar{\chi}$, $D^0 \to \chi \bar{\chi}^\gamma$ and $D^+ \to \pi^+ \chi \bar{\chi}$ are now having branching ratios for a factor $3 - 4$ orders of magnitude smaller then in the case of coloured scalar $\bar{S}_1$ mediation, making them very difficult for the observation.

Interestingly, the mass of both mediators $\bar{S}_1$ and $\bar{R}_2$ are in the range of LHC reach, and hopefully, searches for mono-jets and missing energy might put constraints on their masses.

**VI. ACKNOWLEDGMENT**

The work of SF was in part financially supported by the Slovenian Research Agency (research core funding No. P1-0035). The work of AN was partially supported by the Advanced Grant of European Research Council (ERC) 884719 — FAIME.

**VII. APPENDIX**

**A. Phase space factors**

In eq. (16) phase space function $Y(x_\chi)$ is used

$$Y(x_\chi) = 1 - 2x_\chi^2 + 3x_\chi^3(3 - 6x_\chi^2 + 4x_\chi^4)\sqrt{1 - 4x_\chi^2} \times \log \left( \frac{2x_\chi}{1 + \sqrt{1 - 4x_\chi^2}} \right) - 11x_\chi^4 + 12x_\chi^6.$$  (30)

In Eq. (21) $a(q^2)$ and $c(q^2)$ are introduced denoting

$$a(q^2) = \frac{\lambda}{2}[(|V(q^2)|^2 + |A(q^2)|^2) + 8m_\chi^2m_0^2|A(q^2)|^2] + 2q^2|P(q^2)|^2 + 4m_\chi(m_0^2 - m_\chi^2 + q^2)\text{Re}[A(q^2)P(q^2)]^*,$$

$$c(q^2) = -\frac{\lambda q^2}{2}(|V(q^2)|^2 + |A(q^2)|^2).$$  (31)

**B. $D \to \pi$ form factors**

Following (31) one can use $z$-expansion with

$$z = \frac{t_+ - q^2 - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2 + \sqrt{t_+ - t_0}},}$$  (32)

with $t_+ = (m_D + m_\pi)^2$ and $t_0 = (m_D + m_\pi)(\sqrt{m_D} - \sqrt{m_\pi})$. The form factors can be written as

$$f_{+}^{D\to\pi}(q^2) = \frac{f_{+}^{D\to\pi}(0) + c_{+}^{D\to\pi}(z) - 3(z - z_0)(1 + \frac{1}{2}(z + z_0))}{1 - P_V q^2},$$

$$f_{0}^{D\to\pi}(q^2) = \frac{f_{0}^{D\to\pi}(0) + c_{0}^{D\to\pi}(z) - 2(z - z_0)(1 + \frac{1}{2}(z + z_0))}{1 - P_S q^2},$$

where $z_0 = z(0, t_0^0)$. The fit parameters are given in Table VII. For the most recent discussion on form-factors see also [35].

| $f(0)$ | $c_+$ | $P_V$ (GeV)$^{-2}$ | $c_0$ | $P_S$ (GeV)$^{-2}$ |
|--------|-------|--------------------|-------|--------------------|
| 0.6117 (354) | -1.985 (347) | 0.1314 (127) | -1.188 (256) | 0.0342 (122) |

Table VII. Fit parameters for $f_0$, $f_+$ in the $z$-series expansion for $D \to \pi$ [34].

**C. $B \to K$ form factors**

Most recent results are presented in FLAG report [33]

$$f_{+}^{BK}(q^2) = \frac{r_1}{1 - \frac{q^2}{m_R^2}} + \frac{r_2}{1 - \frac{q^2}{m_R^2}},$$  (35)

$$f_{0}^{BK}(q^2) = \frac{r_1}{1 - \frac{q^2}{m_R^2}}.$$  (36)

The parameters are $r_1 = 0.162$, $r_2 = 0.173$, $m_R = 5.41$ GeV and $m_{R'} = 6.12$ GeV, as in [33].

[1] R. Bause, H. Gisbert, M. Golz, G. Hiller, Rare charm $c \to u\nu\bar{\nu}$ dineutrino null tests for $e^+e^-$-machines (10 2020). arXiv:2010.02225

[2] A. Badin, A. A. Petrov, Searching for light Dark Matter in heavy meson decays, Phys. Rev. D 82 (2010) 034005. arXiv:1005.1277 doi:10.1103/PhysRevD.82.034005

[3] B. Bhattacharya, C. M. Grant, A. A. Petrov, Invisible widths of heavy mesons, Phys. Rev. D 99 (9) (2019) 093010. arXiv:1809.04606 doi:10.1103/PhysRevD.99.093010

[4] W. Altmannshofer, et al., The Belle II Physics Book, PTEP 2019 (12) (2019) 123C01, [Erratum: PTEP 2020, 029201 (2020)]. arXiv:1808.10567 doi:10.1093/ptep/
[5] M. Ablikim, et al., Future Physics Programme of BESIII, Chin. Phys. C 44 (4) (2020) 040001. arXiv:1912.05983. doi:10.1088/1674-1137/44/4/040001

[6] A. Abada, et al., FCC Physics Opportunities: Future Circular Collider Conceptual Design Report Volume 1, Eur. Phys. J. C 79 (6) (2019) 474. doi:10.1140/epjc/s10052-019-6904-3

[7] A. Abada, et al., FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2, Eur. Phys. J. ST 228 (2) (2019) 261–623. doi:10.1140/epjst/e2019-900045-4

[8] Y.-T. Lai, et al., Search for $D^0$ decays to invisible final states at Belle, Phys. Rev. D 95 (1) (2017) 011102. arXiv:1611.09455. doi:10.1103/PhysRevD.95.011102

[9] R. Bause, H. Gisbert, M. Golz, G. Hiller, Exploiting $CP$-asymmetries in rare charm decays, Phys. Rev. D 101 (11) (2020) 115006. arXiv:2004.01206. doi:10.1103/PhysRevD.101.115006

[10] T. G. Ruggiero, New Result on $B^0_s$ mixing, Phys. Lett. B 79 (2009) 114303. arXiv:0903.2830. doi:10.1016/j.physletb.2009.09.041

[11] J. Fuentes-Martín, G. Isidori, I. Dorsner, S. Fajfer, A. Greljo, J. Martin Camalich, J. D. Ruiz-Alvarez, Charm physics confronts high-$p_T$ lepton tails, JHEP 11 (2020) 080. arXiv:2003.12421. doi:10.1007/JHEP11(2020)080

[12] S. Fajfer, D. Susić, Coloured Scalar Mediated Nucleon Decays to Invisible Fermion (10 2020). arXiv:2010.08367

[13] A. J. Buras, D. Buttazzo, J. Girrbach-Noe, R. Knegjens, A. Angelescu, D. A. Faroughy, O. Sumensari, Lepton Flavor Violation and Dilepton Tails at the LHC, Eur. Phys. J. C 80 (7) (2020) 641. arXiv:2002.05684. doi:10.1140/epjc/s10052-020-8210-5

[14] J. Fuentes-Martín, A. Greljo, J. Martin Camalich, J. D. Ruiz-Alvarez, Charm physics confronts high-$p_T$ lepton tails, JHEP 11 (2020) 080. arXiv:2003.12421. doi:10.1007/JHEP11(2020)080

[15] S. Fajfer, D. Susic, Coloured Scalar Mediated Nucleon Decays to Invisible Fermion (10 2020). arXiv:2010.08367

[16] A. Abada, et al., FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2, Eur. Phys. J. ST 228 (2) (2019) 261–623. doi:10.1140/epjst/e2019-900045-4

[17] Y.-T. Lai, et al., Search for $D^0$ decays to invisible final states at Belle, Phys. Rev. D 95 (1) (2017) 011102. arXiv:1611.09455. doi:10.1103/PhysRevD.95.011102

[18] R. Bause, H. Gisbert, M. Golz, G. Hiller, Exploiting $CP$-asymmetries in rare charm decays, Phys. Rev. D 101 (11) (2020) 115006. arXiv:2004.01206. doi:10.1103/PhysRevD.101.115006

[19] E. Golowich, J. Hewett, S. Pakvasa, A. A. Petrov, Rare lepton decays as a probe of new physics, Phys. Lett. B 680 (2009) 471–475. arXiv:0908.1174. doi:10.1016/j.physletb.2009.09.041

[20] G. Faisel, J.-Y. Su, J. Tandeau, Exploring charm decays with missing energy in leptoquark models (12 2020). arXiv:2012.15847

[21] J. F. Kamenik, C. Smith, Tree-level contributions to the rare decays $B^+ \to p + n$ anti-$n$, $B^+ \to K^+ + n$ anti-$n$, and $B^+ \to K^+ + n$ anti-$n$ in the Standard Model, Phys. Lett. B 680 (2009) 471–475. arXiv:0908.1174. doi:10.1016/j.physletb.2009.09.041

[22] G. Li, T. Wang, Y. Jiang, J.-B. Zhang, G.-L. Wang, Spin-1/2 invisible particles in heavy meson decays (4 2020). arXiv:2004.10942

[23] L. Di Luzio, M. Kirk, A. Leinzer, T. Rauh, ΔM, theory precision confronts flavour anomalies, JHEP 12 (2019) 009. arXiv:1909.11087. doi:10.1007/JHEP12(2019)009

[24] A. Bazavov, et al., Review of Particle Physics, PTEP 2020 (8) (2020) 1007/JHEP11(2020)080. arXiv:1912.09872

[25] A. Abada, et al., FCC Physics Opportunities: Future Circular Collider Conceptual Design Report Volume 1, Eur. Phys. J. C 80 (7) (2020) 641. arXiv:2002.05684. doi:10.1140/epjc/s10052-020-8210-5

[26] J. Fuentes-Martín, G. Isidori, I. Dorsner, S. Fajfer, A. Greljo, J. Martin Camalich, J. D. Ruiz-Alvarez, Charm physics confronts high-$p_T$ lepton tails, JHEP 11 (2020) 080. arXiv:2003.12421. doi:10.1007/JHEP11(2020)080

[27] S. Fajfer, D. Susić, Coloured Scalar Mediated Nucleon Decays to Invisible Fermion (10 2020). arXiv:2010.08367