$\rho$ parameter constraints on radion phenomenology and a lower bound on Higgs mass

Prasanta Das
Department of Physics, Indian Institute of Technology, Kanpur 208 016, India.

Uma Mahanta
Mehta Research Institute, Chhatnag Road, Jhusi Allahabad-211019, India.

Abstract

In this paper we determine the contribution of a light stabilized radion to the weak isospin violating $\rho$ parameter by using an ultraviolet momentum cut off as the regulator. The LEP1 bound on $\rho_{\text{new}}$ is then used to derive constraints on the radion mass $m_\phi$ and its vev $\langle \phi \rangle$. Finally by using the beta function of the higgs self coupling we have determined a lower bound on the higgs mass from the rho parameter constraints on $m_\phi$ and $\langle \phi \rangle$. Our results show that for $m_\phi < 600$ GeV the rho parameter bound on $m_h$ is stronger than the present direct bound from LEPII.

---

1 E-mail: pdas@iitk.ac.in
2 E-mail:mahanta@mri.ernet.in
Introduction

Recently several attractive proposals (1, 2) based on theories in extra dimensions have been put forward to explain the hierarchy problem. Among them the Randall-Sundrum (RS) model is particularly interesting because it considers a five dimensional world based on the following non-factorizable metric

\[ ds^2 = e^{-2kr_c|\theta|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\theta^2 \]  

(1)

Here \( r_c \) measures the size of the extra dimensions which is an \( S^1/Z_2 \) orbifold. \( x^\mu \) are the coordinates of the four dimensional space-time. \(-\pi \leq \theta \leq \pi\) is the coordinate of the extra dimension with \( \theta \) and \(-\theta\) identified. \( k \) is a mass parameter of the order of the fundamental five dimensional Planck mass \( M \). Two 3 branes are placed at the orbifold fixed points \( \theta = 0 \) (hidden brane) and \( \theta = \pi \) (visible brane). Randall and Sundrum showed that any field on the visible brane with a fundamental mass parameter \( m_0 \) gets an effective mass

\[ m = m_0 e^{-kr_c \pi} \]  

(2)

due to the exponential warp factor. Therefore for \( kr_c \approx 14 \) the electroweak (EW) scale is generated from the Planck scale by the warp factor.

In the Randall-Sundrum model \( r_c \) is the vacuum expectation value (vev) of a massless scalar field \( T(x) \). The modulus was therefore not stabilized by some dynamics. Goldberger and Wise (3) later showed how to generate a potential for the modulus and stabilize it at the right value (\( kr_c \)) that is needed for solving the hierarchy problem without any excessive fine tuning of the parameters.

In their original model Randall and Sundrum assumed that SM fields are localized on the visible brane located at \( \theta = \pi \). However, small fluctuations of the modulus field from its vev gives rise to non-trivial couplings of the modulus field with the SM fields. Using such couplings the effect of a light radion on low energy phenomenology has been studied (4).

In this report we shall determine the radion contribution to the weak isospin breaking \( \rho \) parameter. The LEP1 data imposes stringent constraints on new physics contribution to the \( \rho \) parameter. We have therefore used the LEP1 bound on \( \rho_{\text{new}} \) to put bounds on the two
unknown parameters $m_\phi$ and $\langle \phi \rangle$. Throughout our analysis the cut off $\Lambda$ will be assumed to be related to the expansion parameter $1/\langle \phi \rangle$ of the non-renormalizable radion interaction to SM particles by the naive dimensional analysis (NDA) estimate $\Lambda = 4\pi \langle \phi \rangle$ [3]. In the RS model the cut-off $\Lambda$ corresponds to the mass of the lightest KK graviton mode. The beta function of the Higgs self-coupling is also modified in the presence of a light stabilized radion. In this paper, we have used this beta function to derive a lower bound on $m_h$ from the LEP1 bounds on $m_\phi$ and $\langle \phi \rangle$.

**Radion couplings to electroweak gauge bosons in unitary gauge**

In order to determine the radion contribution to the T parameter we need to determine the radion couplings to the EW gauge bosons localized on the visible brane. The relevant radion couplings to the EW gauge bosons can be determined from the following action

$$S = \int d^4x \sqrt{-g_v} \left[ (D_\mu H)^\dagger (D_\nu H) g_v^{\mu\nu} - \frac{1}{4} W_\mu^a W_\nu^a g_v^{\mu\nu} - \frac{1}{4} B_\mu^a B_\nu^a g_v^{\mu\nu} + \mathcal{L}_{gf} \right]$$

where $\sqrt{-g_v} = \phi_f^4$, $g_v^{\mu\nu} = (\phi(x)/f)^{-2} \eta^{\mu\nu}$ and

$$\mathcal{L}_{gf} = -\frac{1}{2\xi} \left[ \frac{1}{2} \partial_\mu W_\nu^a g_v^{\mu\nu} + ig_2 \xi \left( H^\dagger \frac{\tau_a}{2} < H > - < H > \frac{\tau_a}{2} H \right) \right]^2$$

$$-\frac{1}{2\xi} \left[ \frac{1}{2} \partial_\mu B_\nu g_v^{\mu\nu} + ig_1 \frac{\xi}{2} \left( H^\dagger < H > - < H > H^\dagger \right) \right]^2$$

The SM higgs field in unitary gauge is given by

$$H = H^\dagger + < H >= \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

Using the above expression of the higgs field it can be shown that the gauge fixing Lagrangian $L_{gf}$ vanishes in the unitary gauge ($\xi \to \infty$). Consider first the radion coupling to the K.E. of the gauge bosons. We have

$$\sqrt{-g_v} V_{\mu\nu} V_{\rho\sigma} g_v^{\mu\rho} g_v^{\nu\sigma} = V_{\mu\nu} V_{\rho\sigma} \eta^{\mu\rho} \eta^{\nu\sigma}$$
where $V_\mu = (W^a_\mu, B_\mu)$. At the classical level the radion therefore does not couple to the gauge boson KE in four dimensions. Note that the Christoffel symbol $\Gamma^\lambda_{\mu\nu}$ in the expression for the general covariant derivative $D_\mu V_\nu$ does not contribute to the field strength tensor of $W^a_\nu$ or $B_\nu$ because $\Gamma^\lambda_{\mu\nu}$ is symmetric in $(\mu, \nu)$.

Consider next the radion coupling to the K.E. of the higgs boson. We have

$$\sqrt{-g} (D_\mu H)^+(D_\nu H) g^\mu\nu = \left(\frac{\phi}{\langle \phi \rangle}\right)^2 (D_\mu H)^+(D^\mu H)$$

where $\bar{H} = H \left(\frac{i\phi}{\langle \phi \rangle}\right)$ and $< \bar{H} > = < H > \left(\frac{i\phi}{\langle \phi \rangle}\right)$. In the following we shall assume that the higgs field and its vev has been properly rescaled as above and drop the tilde sign. We then get

$$\sqrt{-g} (D_\mu H)^+(D_\nu H) g^\mu\nu = ig_2 W^a_\mu \left[< H+ > \frac{\tau_a}{2} \partial^\mu H' - \partial^\mu H'^+ \frac{\tau_a}{2} < H > \right] \left(\frac{\phi}{\langle \phi \rangle}\right)^2$$

$$+ i g_1 \frac{1}{2} B_\mu \left[< H+ > \partial^\mu H' - \partial^\mu H'^+ < H > \right] \left(\frac{\phi}{\langle \phi \rangle}\right)^2$$

$$+ \left[ m^2 w W^a_\mu W^-\mu + \frac{1}{2} m^2 Z^-\mu Z^\mu \right] \left[ 1 + \frac{\phi}{\langle \phi \rangle} + \frac{\dot{\phi}^2}{\langle \phi \rangle^2} + .. \right]$$

(6)

In the unitary gauge the first two terms on the rhs of the above expression vanishes, leaving only the gauge boson mass terms to couple to radion fluctuations. The couplings of one and two radions to the EW gauge bosons that are relevant for computing the radion contribution to $\Pi_{vv}^\mu(q)$ in unitary gauge can therefore be expressed by the Feynman rules shown in Fig1(a) and Fig1(b).

The contribution of new physics to the vectorial isospin violating parameter $\rho$ is given by

$$\rho^{new} = \alpha T^{new} = \frac{\Pi_{ww}(0)}{m^2_w} - \frac{\Pi_{zz}(0)}{m^2_z}$$

(7)

The function $\Pi_{vw}(q)$ is defined through the gauge boson self energy tensor

$$i\Pi_{vw}^{\mu\nu}(q) = i\eta^{\mu\nu}\Pi_{vw}(q) - iq^\mu q^\nu \Pi_{vw}(q)$$

(8)
The Feynman diagrams that give rise to radion contribution to $\rho$ parameter in unitary gauge are shown in Fig 2.

Let $\Pi^{(1)}_{vv}(q)$ and $\Pi^{(2)}_{vv}(q)$ denote the contributions arising from single and two radion vertices to $\Pi_{vv}(q)$. We then have

$$\Pi^{(1)}_{vv}(0) = -\frac{m_v^2}{16\pi^2\langle\phi\rangle^2} \left( \Lambda^2 - m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2} \right) - \frac{m_v^4}{16\pi^2} \left[ 3 \ln \frac{\Lambda^2}{m_\phi^2} - 3 \frac{m_v^2}{m_\phi^2 - m_v^2} \ln \frac{m_v^2}{m_\phi^2} \right]$$

and

$$\Pi^{(2)}_{vv}(0) = \frac{m_v^2}{16\pi^2\langle\phi\rangle^2} \left( \Lambda^2 - m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2} \right)$$

It is clear from the above that $\Pi^{(2)}_{vv}(q)$ will not contribute to the $\rho$ parameter since $\frac{\Pi^{(2)}_{vv}(0)}{m_v^2}$ is independent of $m_v$. Radion contribution to $\rho^{new}$ therefore arises only from $\Pi^{(1)}_{vv}(q)$. We would like to note at this point that since the $\hat{\phi}VV$ coupling is proportional to $m_v^2$, the radion tadpole diagrams do not contribute to the $\rho$ parameter. We also find that although $\Pi^{(1)}_{vv}(0)$ and $\Pi^{(2)}_{vv}(0)$ are individually quadratically divergent the sum $\Pi_{vv}(0)$ is only log divergent. This cancellation of quadratic divergence is a consequence of gauge symmetry which protects gauge boson masses from receiving large power corrections.

**Radion contribution to the $\rho$ parameter**

The radion contribution to the $\rho$ parameter is therefore given by

$$\rho^{new} = \frac{m_w^2}{16\pi^2\langle\phi\rangle^2} \left[ -3\ln \frac{\Lambda^2}{m_\phi^2} + \frac{3m_w^2}{m_v^2 - m_w^2} \ln \frac{m_v^2}{m_w^2} \right] - \frac{m_z^2}{16\pi^2\langle\phi\rangle^2} \left[ -3\ln \frac{\Lambda^2}{m_\phi^2} + \frac{3m_z^2}{m_\phi^2 - m_z^2} \ln \frac{m_\phi^2}{m_z^2} \right]$$

Note that the above expression for $\rho^{new}$ diverges logarithmically with the cut off. The cut off dependence of the radion contribution to the rho parameter is easily understood. It arises from using the non-renormalizable dimension five operator $(D_\mu H)^+(D^\mu H)\hat{\phi}$ in the
calculation of $\Pi_{1r}^1(0)$. Secondly the radion contribution to $\rho^{\text{new}}$ depends on three unknown parameters: the cut off $\Lambda$, $m_\phi$ and $\langle \phi \rangle$. Naive dimensional analysis can however be used to relate $\Lambda$ to the expansion parameter $\frac{1}{\langle \phi \rangle}$ through $\Lambda = 4\pi \langle \phi \rangle$. Physically it means that the radion effective field theory would become non-perturbative and radion induced radiative corrections would become very large above the ultraviolet scale $4\pi \langle \phi \rangle$ thereby implying a breakdown of the low energy effective theory. The NDA estimate of the cut off is known to work quite well for estimating chiral loops that arises in dealing with Chiral Lagrangian. Further Luty et al has shown that it gives reliable estimates for extra dimensional gravity also. We shall assume it to hold good for the radion effective field theory also. This reduces the dependence of $\rho^{\text{new}}$ to two unknowns only: $m_\phi$ and $\langle \phi \rangle$. The LEPI bounds on $\rho^{\text{new}}$ can therefore be used to impose stringent constraints on $m_\phi$ and $\langle \phi \rangle$.[7] Finally the radion contribution to the T parameter must be a gauge invariant quantity, since the radion is a gauge singlet. Therefore although the calculations presented in this paper were done in unitary gauge, the final answer given by eqn (11) must be independent of this gauge choice.

**$\rho$ parameter bounds on $m_\phi$ and $\langle \phi \rangle$**

The present value of the T parameter is given by [8] $T = -0.10 \pm 0.14 (0.09)$. In fig 3 we have shown the contour plots in $m_\phi$ vs $\langle \phi \rangle$ plane for $T=0.04$ and $T =0.18$. The first value corresponds to $+1\sigma$ deviation and the second value to $2\sigma$ deviation from the central value. We have chosen positive values of T only since the radion contribution to T is positive for $\Lambda \gg m_\phi$. The region allowed by the $\rho$ parameter bound lies above both curves. We find that for $T = 0.18$ and $m_\phi = 10$ GeV, $\langle \phi \rangle$ must be greater than about 440 GeV. On the other hand for $T = 0.04$, $\langle \phi \rangle$ must be greater than about 1000 GeV for the same $m_\phi$. The bound on $\langle \phi \rangle$ however decreases monotonically with increasing $m_\phi$ and becomes about 320 GeV (for $T=0.18$) and 810 GeV (for $T=0.04$) when $m_\phi$ increases to 500 GeV. The region allowed by the $\rho$ parameter constraint lies above the relevant curve.
Lower bound on Higgs mass

The beta function for the Higgs self coupling in the presence of a light radion and the $\rho$ parameter constraints on $m_\phi$ and $\langle \phi \rangle$ can be used together to derive a lower bound on $m_h$. In the following we briefly describe how this lower bound on $m_h$ can be determined.

The beta function for the Higgs self coupling $\lambda$ in the presence of a light radion is given by

$$\beta(\lambda) = \mu \frac{d \lambda}{d \mu} = \frac{1}{8\pi^2} \left[ 9\lambda^2 + \frac{402\lambda^2 v^2}{\langle \phi \rangle^2} + \frac{144\lambda^2 v^4}{\langle \phi \rangle^4} + \frac{5\lambda m_\phi^2}{\langle \phi \rangle^2} + \lambda \left( 6 g_Y^2 - 9 g_2^2 - \frac{3}{2} g_1^2 \right) - 6 g_Y^2 \right] \left( \frac{3}{16} \right) \left( g_2^4 + \frac{1}{2} (g_2^2 + g_1^2)^2 \right) \right]$$

(12)

i) For a given value of $m_\phi$ we use the above differential equation to determine the value of the renormalized coupling $\lambda(\mu)$ at $\mu = 100$ GeV. In this paper we shall assume that the Kaluza-Klein modes of the graviton, which are much heavier than the radion, decouples at or above the cut-off scale $\Lambda = 4\pi \langle \phi \rangle$. The value of $\lambda$ at the cut-off $\Lambda$ can be chosen to be either strong and non-perturbative ($\lambda(\Lambda) > \sqrt{4\pi}$) or weak and perturbative ($\lambda(\Lambda) < \sqrt{4\pi}$).

In fig. 4 we have plotted $\lambda(\mu)$ at $\mu = 100$ GeV against $\langle \phi \rangle$ for seven different values of $m_\phi$ starting from 5 GeV and going up to 600 GeV under the UVBC $\lambda(\Lambda) = \infty$.

ii) For a given $m_\phi$ we find the $\rho$ parameter bound on $\langle \phi \rangle$ from fig.3. The value of the renormalized coupling $\lambda(\mu)$ at that value of $\langle \phi \rangle$ is then determined from the curve corresponding to the chosen $m_\phi$ shown in fig.4. For each chosen $m_\phi$ we therefore obtain a value for the renormalized coupling $\lambda(\mu)$ at $\mu = 100$ GeV.

iii) The renormalized Higgs mass at $\mu = 100$ GeV can be determined from the $\lambda(\mu)$ obtained in step (2) via the relation $m_h(\mu) = \sqrt{2\lambda(\mu)} v$. Fig 5 shows the lower bound on the higgs mass as a function of $m_\phi$ for $T=0.04$ and $T=0.18$. We find that for $T=0.18$ the bound on $m_h$ is greater than the lower bound on $m_h$ from direct search at LEPII [10] provided $m_\phi < 600$ GeV. This gives rise to two distinct regions in the $m_\phi$ vs $m_h$ plane: RG1 (which is allowed by LEPII but forbidden by $T=0.18$) and RG2 (which is allowed by $T=0.18$ but forbidden by LEPII). We also find that for $T=0.04$ the lower bound on $m_h$ is greater than the LEPII bound for the entire range of values of $m_\phi$ relevant for a light radion.
This gives rise to two distinct regions: RG3 (which is allowed by LEPII but forbidden by T=0.04) and RG4 (which is allowed both by T=0.04 and LEPII). The bound on $m_h$ decreases monotonically with increasing $m_\phi$ due to the following reasons: i) the bound on $\langle \phi \rangle$ decreases with increasing $m_\phi$ (fig 3) and ii) the value of $\lambda(\mu)$ decreases with decreasing $\langle \phi \rangle$ (fig 4).

The results shown in fig 5 were obtained by using the UVBC $\lambda(\Lambda) = \infty$. It is worthwhile however to investigate the sensitivity of the lower bound on $m_h$ to the UVBC on $\lambda$. In fig 6 we have plotted the bound on $m_h$ for two different UBVC $\lambda(\Lambda) = \infty$ and $\lambda(\Lambda) = e$. We find that for T=0.18 the bound on $m_h$ is insensitive to the UVBC over the entire range of $m_\phi$. However for T=0.04 the bound on $m_h$ is somewhat sensitive to the UVBC. To understand this feature we would like to refer the reader to [9] where it was shown that the value of $\lambda(100)$ does not depend on the UVBC provided $\langle \phi \rangle$ is less than 350 GeV. Although the last condition is more or less satisfied for T=0.18 it does not hold at all for T=0.04 over the entire range of values of $m_\phi$.

**Acknowledgement:** We would like to thank Prof. M. Einhorn, Prof. S. Raychaudhuri and Prof. T. Takeuchi for several useful discussions on this paper. Uma Mahanta would also like to thank the Physics Department of IIT Kanpur for its generous support and hospitality while this work was being done.

**References**

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys.Lett* **B429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys.Lett* **B463**, 257 (1998). Some precursors of the model are V. Rubakov and M. Shaposhnikov, *Phys.Lett* **B125**, 136 (1984); A. Barnaveli and O. Kancheli, *Sov. J. Nucl. Phys.* **51**, 573 (1990); I. Antoniadis, *Phys.Lett* **B246**, 377 (1990); I. Antoniadis, C. Muñoz and M. Quiros, *Nucl.Phys.* **B397**, 515 (1993); I. Antoniadis, K. Benakli and M. Quiros, *Phys.Lett*, **B331** 313 (1994).

[2] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).

[3] W. D. Goldberger and M. B. Wise, *Phys. Rev. Lett.* **83**, 4922 (1999).
[4] C. Csaki, M. Graesser, L. Randall and J. Terning, *Phys. Rev.* **D62**, 045015 (2000); W. D. Goldberger and M. B. Wise, *Phys. Lett.* **B475**, 275 (2000).

For detailed study of radion phenomenology in the context of RS model see: U. Mahanta and S. Rakshit, Phys. Lett. B480, 176 (2000); G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B595, 250 (2001); U. Mahanta and A. Datta, Phys. Lett. B483, 196 (2000).

[5] H. Georgi and A. Manohar, Nucl. Phys. B 234, 189 (1984); Z. Chacko, M. Luty and E. Ponton, JHEP 07, 036 (2000).

[6] M. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990); *Phys. Rev.* **D46**, 381 (1992).

[7] For earlier works on \( \rho \) parameter constraints on models of extra dimension scenerio see: P. Das and S. Raychaudhuri, hep-ph/9908205; T. Han, D. Marfatia and R. Zhang, *Phys. Rev.* **D62**, 125018 (2000), hep-ph/0001320; C. Csaki, M. Graesser and G. D. Kribs, *Phys. Rev.* **D63**, 065002 (2001), hep-th/0008151.

[8] D. E. Groom, Review of Particle Physics, Eur. Phys. J. C 15, 1 (2000).

[9] P. Das and U. Mahanta, Phys. Lett. B 520, 307 (2001).

[10] T. Junk, The LEP Higgs Working Group, at LEP Fest October 10th 2000.
Figure. 1 [a, b] : Feynman rules for one and two radion couplings to EW gauge bosons.
Figure 2: Feynman diagrams giving the radion contribution to $T$ parameter in the unitary gauge.
Figure 3: $\rho$ parameter constraints on radion vev $\langle \phi \rangle$ and radion mass $m_\phi$. The allowed region lies above the relevant curves.
Figure 4: Plots of $\lambda(\mu)$ at $\mu = 100$ GeV against $<\phi>$ for different radion masses under non-perturbative UVBC on $\lambda$. 
Figure. 5: Lower bound on higgs mass from $\rho$ parameter constraint plotted against the radion mass.

- **RG1**: Region forbidden by $T = 0.18$ but allowed by direct search of LEPII
- **RG2**: Region allowed by $T = 0.18$ but forbidden by direct search of LEPII
- **RG3**: Region forbidden by $T = 0.04$ but allowed by direct search of LEPII
- **RG4**: Region allowed both by $T = 0.04$ and direct search of LEPII
Figure 6: Plots showing the sensitivity of the lower bound on $m_h$ to the UVBC on $\lambda$ for different values of $m_\phi$.

Lower bound on $m_h$ (=114 GeV) from the direct search of LEPII.