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Real-Time Fair Resource Allocation in Distributed Software Defined Networks

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Abstract—The performance of computer networks relies on how bandwidth is shared among different flows. Fair resource allocation is a challenging problem particularly when the flows evolve over time. To address this issue, bandwidth sharing techniques that quickly react to the traffic fluctuations are of interest, especially in large scale settings with hundreds of nodes and thousands of flows. In this context, we propose a distributed algorithm based on the Alternating Direction Method of Multipliers (ADMM) that tackles the fair resource allocation problem in a distributed SDN control architecture. Our ADMM-based algorithm continuously generates a sequence of resource allocation solutions converging to the fair allocation while always remaining feasible, a property that standard primal-dual decomposition methods often lack. Thanks to the distribution of all computer intensive operations, we demonstrate that we can handle large instances in real-time.

Index Terms—Software-Defined Networking; Fair Resource Allocation; Alternating Direction Method of Multipliers; α-Fairness; Distributed Algorithms; Distributed SDN Control Plane.

I. INTRODUCTION

Software Defined Networking (SDN) technologies are radically transforming network architectures by offloading the control plane (e.g., routing, resource allocation) to powerful remote platforms that gather and keep a local or global view of the network status in real-time and push consistent configuration updates to the network equipment. The computation power of SDN controllers fosters the development of a new generation of control plane architecture that uses computer-intensive operations. Initial design of SDN architectures [24] had envisioned the use of one central controller. However, for obvious scalability and resiliency reasons, the industry has quickly realized that the SDN control plane needs to be partially distributed in large network scenarios [11]. Hence, although logically centralized, in practice the control plane may consist of multiple controllers each in charge of a SDN domain of the network and operating together, in a flat [22] or hierarchical [7] architecture.

In this paper, we study the problem of computing a globally fair (in the sense of α-fairness defined by Mo and Walrand in [16], see Section III) resource allocation in a distributed SDN scenario, where the control plane is distributed over several domain controllers. In this context, flows transiting in the network typically correspond to traffic aggregates of a customer or a class of customers. We consider the traffic engineering use case where the size of flows evolves over time and the bandwidth reserved to each of them has to be quickly adjusted towards the novel fair solution.

In distributed SDN architectures, each controller has full information about its own domain. Moreover, it can communicate with adjacent peer controllers and/or with a central, upper-layer controller entity. However, exchanges between controllers are expensive in terms of communication delay and overhead [20]. This technological limitation translates directly into an algorithmic constraint: distributed algorithms for SDN have a limited budget in terms number of iterations to reach convergence.

A second crucial property for any distributed algorithm for SDN is responsiveness. In fact, the network state may be affected by abrupt changes, e.g., flow size variation, flow arrival/departure, link/node congestion. In this case, convergence for the previous network state may not even be attained when a change occurs in the system. For this reason, it is often preferable to have a quick access to a good quality solution rather than a provably asymptotically optimal solution with poor convergence rate. Hence, it is crucial that the resource allocation computed by a distributed algorithm is feasible, thus implementable, at any iteration.

To recap, we identify two main requirements for a distributed algorithm for fair resource allocation, namely i) converging to a “good” fair solution in a small number of iterations and ii) producing feasible solutions at all iterations.

We claim that none of the current methods that allocate resources in an SDN scenario is able to achieve the two aforementioned goals at the same time. Local mechanisms such as Auto-Bandwidth [18] have been proposed to greedily and distributedly adjust the allocated bandwidth to support time-varying IP traffic in Multi Protocol Label Switching (MPLS) networks. Auto-Bandwidth successfully tackles goal ii) but not i), as it neither ensures fairness nor optimizes resources globally. Also, classic primal-dual algorithms have been proposed to solve the α-fair resource allocation problem in distributed SDN scenarios, as in [15]. However, primal-dual algorithms are known to fail at providing feasible solutions at any iteration step, thus they fail at achieving goal ii).

Recently in the optimization research community, the Alternating Direction Method of Multipliers (ADMM) [3] has captured the attention for its separability and fast convergence properties. We claim that ADMM offers new and yet unexploited possibilities to tackle concurrently the goals i), ii). Indeed, in this paper we show how ADMM serves our purposes, by allowing all controllers to handle their own domains simultaneously, while still converging to a global optimum in the fashion of a general distributed consensus
problem.

Main contributions: We develop an ADMM-based method (FD-ADMM, Algorithm 2) for the \( \alpha \)-fair resource allocation problem over a distributed SDN control plane. It iteratively produces resource allocations that converge to the \( \alpha \)-fair optimal allocation. Heavy computations, requiring projections on polytopes, can be massively parallelized on a link-by-link basis (Algorithm 2, line 7) in each domain. This yields a convergence rate that does not depend on the partitioning of the network into domains.

We show that our FD-ADMM algorithm can function in real-time, as i) close-to-optimal solutions are available since the very first iterations and ii) feasible allocations are available at all iterations (Proposition 1), a property that standard primal-dual decomposition methods generally lack. This permits to adjust within up to a few milliseconds the bandwidth of flows that evolve quickly and need immediate response. Moreover, we show how to achieve a near-optimal convergence rate by providing an explicit and adaptive tuning for the FD-ADMM penalty parameter (Scheme 1).

Finally, we benchmark FD-ADMM with the State-of-the-Art (SoA) approach for computing \( \alpha \)-fair resource allocations in distributed scenarios in [15], which is based on the Lagrangian dual splitting method. We show that our algorithm outperforms the SoA in terms of convergence rate, feasibility preservation and hence, overall, real-time responsiveness.

The remainder of this paper is organized as follows. Section II surveys the related work around the fair resource allocation problem. Section III formulates the \( \alpha \)-fair resource allocation problem and presents the basic centralized application of ADMM. Section IV introduces FD-ADMM, our distributed ADMM-based algorithm that benefits from the distribution of SDN controllers over multiple domains. FD-ADMM is based on a reformulation of the problem in the fashion of a general distributed consensus problem. Section V discusses on a near-optimal tuning of the penalty parameter in FD-ADMM. Section VI provides simulations that validate our approach and finally, Section VII concludes the paper.

For the sake of conciseness, we deferred some of our proofs to the technical report in [1].

II. RELATED WORK

The concept of fair resource allocation has been a central topic in networking. Particularly, max-min fairness has been the classic resource sharing principle [2] and has been studied extensively. The concept of proportional fairness and its weighted variants were introduced in [10]. Later, a spectrum of fairness metrics including the two former ones was introduced by Mo and Walrand in [16] as the family of \( \alpha \)-fair utility functions.

Some early notable works on max-min fairness include [4], where the authors propose an asynchronous distributed algorithm that communicates explicitly with the sources and pays some overhead in exchange for more robustness and faster convergence. Later in [21], a distributed algorithm is defined for the weighted variant of max-min fair resource allocation problem in MPLS networks, based on the well-known property that an allocation is max-min fair if and only if each Label-Switched Path (LSP) either admits a bottleneck link amongst its used links or meets its maximal bandwidth requirement (see Definition 4 there of a bottleneck link). The problem of Network Utility Maximization (NUM) was also addressed with standard decomposition methods that could give efficient and very simple algorithms based on gradient ascent schemes performing their update rules in parallel. In this context, Voice [25], then McCormick et al. [15], tackle the \( \alpha \)-fair resource allocation problem with a gradient descent applied to the dual of the problem.

In these works, no mention is made on the potential (in fact, systematic) feasibility violation of the sequences generated by those algorithms, which is a crucial matter in distributed SDN settings. Regarding this topic, the authors of [12] employ damping techniques to avoid transient infeasibility while reaching the max-min fair point, but cannot guarantee feasibility at all times, especially in dynamic settings. Also motivated by this, more recently the authors of [23] provide a feasibility preserving version of Kelly’s methodology in [10]. Their algorithm introduces a slave that gives at each (master) iteration an optimal solution of a weighted proportionally fair resource allocation problem that is explicitly addressed in only the two cases of polymatroidal and flow aggregating networks. In fact, our paper contributes to this problem by proposing an efficient real-time version of the slave process, for any topology, preserving feasibility at each (slave) iteration. Amongst approximative approaches, one can quote the very recent work [14] where a multiplicative approximation for \( \alpha \neq 1 \) and additive approximation for \( \alpha = 1 \) is provably obtained in poly-logarithmic time in the problem parameters. Moreover, starting from any point, the algorithm reaches feasibility within poly-logarithmic time and remains feasible forever after. The algorithm described in our paper solves the problem optimally and reaches feasibility as from the first iteration from any starting point.

The work around ADMM is currently flourishing. The \( O(\frac{1}{\epsilon^2}) \) best known convergence rate of ADMM [8] failed to explain its empirical fast convergence until very recently, for instance in [6], where global linear convergence rates are established in four scenarios of the strongly convex case. ADMM is also well-known for its performance that highly depends on the parameter tuning, namely, the penalty parameter \( \rho \) (or reciprocal penalty parameter \( \lambda = 1/\rho \)) in the augmented Lagrangian formulation (see Section III-B below). An effective use of this class of algorithms cannot be decoupled from an accurate parameter tuning, as convergence can be extremely slow otherwise. Thus, in the same paper [6], the authors provide a linear convergence proof that yields a convergence rate in a closed form that can be optimized with respect to the problem parameters. Therefore, thanks to these works, we derive in our paper a near-optimal tuning of ADMM for the \( \alpha \)-fair resource allocation problem. Several papers use the distributivity of ADMM to design efficient distributed algorithms solving consensus formulations for e.g. model predictive control [17] and resource allocation in wireless
The success of $\alpha$-fairness is due to its generality: in fact, for $\alpha = 0, 1, 2, \infty$ it is equivalent to max-throughput, proportional fairness, min-delay, and max-min fairness, respectively. We observe that the $\alpha$-fair utility functions are non-decreasing, strictly concave, non-identically equal to $-\infty$, and upper semi-continuous. It is well-known that under these conditions, the function $f^\alpha$ admits a unique maximizer over any convex closed non-empty set.

From now on, we adopt the convex optimization terminology. Define for each $r \in R$ the convex cost function $g_r : x_r \mapsto g_r(x_r) := -f_r^\alpha(x_r)$. Then, $g := \bigoplus_r g_r = -f^\alpha$ is a convex closed proper function over $R_+^R$. Let us introduce $\iota$ as the indicator function of the convex closed set $\{Ax \leq C, x \geq 0\}$:

$$\iota(x) = \begin{cases} 0 & \text{if } Ax \leq C \\ \infty, & \text{otherwise.} \end{cases}$$

Then our $\alpha$-fair problem can equivalently be formulated as the following convex program:

$$\min_{x,z} \sum_{r \in R} g_r(x_r) + \iota(z), \tag{3}$$

s.t. $x - z = 0. \tag{4}$

B. ADMM as an augmented Lagrangian splitting

Let us begin by recalling to the reader the basic principles of the Alternating Direction Method of Multipliers (ADMM), applied to our $\alpha$-fair problem. To this aim, the augmented Lagrangian with penalty $\lambda^{-1} > 0$ for problem (3-4) writes

$$L_{\lambda^{-1}}(x, z, u) = g(x) + \iota(z) + u^T(x - z) + \frac{1}{2\lambda}|x - z|^2 \tag{5}$$

where $u$ is the vector of Lagrange multipliers. The method of multipliers consists in the following update rules, where the superscript $k$ denotes an iteration count:

$$\begin{align*}
(x^{k+1}, z^{k+1}) &= \arg \min_{x,z} L_{\lambda^{-1}}(x, z, u^k) \tag{M1} \\
u^{k+1} &= u^k + \lambda(z^{k+1} - x^{k+1}). \tag{M2}
\end{align*}$$

The main idea in alternating directions is in fact to decouple the variables $(x, z)$ in the optimization stage M1: instead of a global optimization over $(x, z)$, we only optimize $L_{\lambda^{-1}}$ with respect to the variable $x$, then, given the new update of $x$, we optimize $L_{\lambda^{-1}}$, with respect to $z$. Before stating the corresponding update rules of ADMM, let us first remind the following Fact.

Fact 1 ([3]). Let $h : R^n \to \bar{R} = \mathbb{R} \cup \{\infty\}$ be a closed proper convex function. The set $\text{dom}(h)$ denotes the domain of $h$, that is the set upon which $h$ takes real values. Assume $\text{dom}(h) \neq \emptyset$. Then, the following facts hold:

(i) For $u \in R^n$, $\lambda \in R_+^n$, the minimization problem

$$u^*_\lambda = \arg \min_x \left\{ h(x) + \frac{1}{2\lambda}||u - x||^2 \right\}$$

2 closed stands for lower semi-continuous and proper means non-identically equal to $\infty$

3 $a^Tb$ is the Euclidean product of $a$ and $b$ and $|| \cdot ||$ the Euclidean norm.
Main issues with C-ADMM:

such that each local variable $x_i$.

Hence, for general polyhedra, one has to operate alternate closed convex non-empty polyhedron is in general non-trivial. Indeed, computing the projection of a generic point onto a closed convex non-empty polyhedron is in general non-trivial.

Algorithm 1 Centralized ADMM (C-ADMM)

Input: Initial values $z, v$

1: while a suitable termination condition is not met do
2: \[ x \leftarrow \text{prox}_{2\lambda}(z - v) \]
3: \[ z \leftarrow P(x + v) \]
4: \[ v \leftarrow v + x - z \]
5: end while

In Algorithm 1, $P = \text{prox}_{\lambda}$ is the projection on $\{Ax \leq C, x \geq 0\}$, and $v = \lambda u$ the $\lambda$-scaled dual variable. Now, the first step of Algorithm 1 (line 2) can be separated thanks to the separability property of the objective function, see Fact 1. In fact, $g$ is fully separable, as $g(x) = \sum g_i(x_r)$. Thus, the proximal update of line 2 takes the trivially parallelized form:

\[ \forall x \in \mathbb{R}^n \quad \text{prox}_{\lambda g}(x + \lambda u) = \text{prox}_{2\lambda g}(z - v) \]

such that each local variable $x_i$ can be computed separately.

Moreover, although the convergence of C-ADMM may only require some tens of iterations (see Section VI for further details), it may be slow in terms of computation time due the successive application of a projection algorithm that would not scale with respect to the problem size. This also gives rise to a double loop algorithm where each iteration requires the convergence of an inner process that can be time-consuming. Indeed, computing the projection of a generic point onto a closed convex non-empty polyhedron is in general non-trivial.

Hence, for general polyhedra, one has to operate alternate projections, summon quadratic programming solvers or use iterative algorithms such as the one in [9].

We address issues (a,b) in the next section, where we propose FD-ADMM, a distributed version of C-ADMM.

IV. THE GENERAL CONSENSUS FORM OF ADMM: AN EFFICIENT DISTRIBUTED ALGORITHM DESIGN

In this section, we show how to alleviate the cost of the global projection sub-routine in C-ADMM (line 3) by decomposing the formulation with respect to the network links of each SDN domain in the fashion of a consensus problem, and present FD-ADMM. As stated at end of Section III, the global knowledge of the topology and the computational effort required by the projection step (line 3) of C-ADMM are not affordable in the distributed SDN control plane. Thus, the decomposition permits to respect the locality of the different domain controllers that now handle the projections link by link efficiently and in parallel. The decomposition into domains can be orchestrated by the SDN architect without any constraint. Unavoidably though, domains will need to exchange information as routes may traverse different domains.

A. Preliminaries

We organize the network into several domains $J_p, p = 1 \ldots P$ such that $(J_p)_p$ forms a partition of the set of links $J$. Let $R_p$ be the set of routes traversing the domain $J_p$ via some link $j \in J_p$. More formally, $R_p = \{ r \in R : \exists j \in J_p \text{ s.t. } j \in r \}$. Hence, $(R_p)_p$ forms a covering of $R$. Let $t_j$ denote the indicator function for link $j \in J_p$, i.e.,

\[ t_j(x) = \begin{cases} 0 & \text{if } \sum_{r \in j} x_r \leq C_j \\ \infty & \text{otherwise} \end{cases} \]

Also, let us define $S_j := \text{dom}(t_j)$. Thus, for each $j \in J$, $S_j$ is the (convex, closed) capacity set of the link $j$. Finally, for $j \in J$ and $z \in \mathbb{R}^R$, $\text{PROJECTION}(j, z)$ denotes the Euclidean projection of $z$ onto $S_j$.

B. Consensus form

We can now reformulate our objective to a fully separable form. For ease of notation, the variable $x$ will be written $z_0$ and we define $R_0 = R$. We also define an additional variable, $\bar{z}_r$, that will represent the consensus value of $z_{0r}$ found for each route $r$ over all the domains handling the route $r$. We write $I_r = \{ \{ q \in [0, P] \mid r \in R_q \} \}$ to design the set of domain indices (including index 0) which $r$ belongs to. In the same fashion as in Section III-B, we plug the feasibility constraints into the objective. Each constraint being now handled separately, we can formulate Problem (3), (4) as follows:

\[ \min_{r \in R} \sum_{r \in j} g_r(z_{0r}) + \sum_{j \in I} t_j(z_0). \]

In order to obtain a separable objective and fully benefit from the separability property in Fact 1, we artificially create a copy of the variable $z_0$ for each link $j$. This variable will be handled by the unique domain $J_p$ containing $j$. For each $j$, let $z_j \in \mathbb{R}^{|I_j|}$ be the copy of $z_0$ for link $j$.

Creating a complete copy of all the variables for each domain is, nevertheless, of no use. Each domain indeed only
needs information and manipulation over the only variables associated with the routes that they handle completely (the route is included in the domain’s links) or partially (the route meets other domains). Now, $t_j$ actually depends only on the sub-variable $(z_j)_{R_j}$ \(= (z_{jr})_{r \in R_j} \). We erase all the information that is irrelevant to region $J_p$: $z_j \in \mathbb{R}^{R_j}$. We can thus write the objective as follows:

$$
\min G(z) = \sum_{r \in R} g_r(z_{0r}) + \sum_{j \in J} t_j((z_j)_{R_j}).
$$

To sum up, we have artificially separated the objective function by creating a minimal number of copies of the primal variable $z_0$ in order to fully distribute the problem. Now, instead of a global resource allocation variable, several copies of the variable account for how its value is perceived by each link of each domain. To enforce an intra- (local) and inter-(global) domain consistent value of the appropriate allocation, consensus constraints are added to the problem. This new formulation can be interpreted as a multi-agent consensus problem formulation where route $r$ has cost $g_r$, and link $j$ has cost $t_j$. As we separated the global objective on purpose, the separability property of the proximal operator thus gives the following:

$$
\text{prox}_{\lambda G}(u) = \left( (\text{prox}_{\lambda g_r}(u_{0r}))_r, (\text{PROJECTION}(j, u_j))_j \right).
$$

These considerations permit next to write our final distributed consensus model where each agent only has access to local information.

### C. Fast Distributed ADMM

We can finally distribute ADMM by putting into practice the previous section. Then, the general consensus form of the problem can be expressed as follows.

$$
\begin{align*}
\min & \quad \sum_{r \in R} g_r(z_{0r}) + \sum_{j \in J} t_j((z_j)_{R_j}) \\
\text{subject to} & \quad z_{jr} = z_{lr} \quad \forall r \in R_j \cap R_l \quad \forall j, l \in \{0\} \cup J
\end{align*}
$$

where $z_j = (z_{jr})_{r \in R_j} \in \mathbb{R}^{R_j}$. By applying ADMM to this formulation and using again Fact 1 we obtain, after some simplification, Algorithm 2 (Fast Distributed (FD)-ADMM). To update the consensus variables $\bar{z}_r$, we exploit the fact that the Euclidean projection of a point $y \in \mathbb{R}^n$ onto the diagonal is simply its average $\frac{1}{n} \sum y_i B$. Hence, if $I$ denotes the indicator function of the feasible set (11), we have:

$$
\forall r \in R \quad \text{prox}_{\lambda f}(u)_r = \frac{1}{|I_r| + 1} \left( \sum_{l \in I_r} u_{lr} + u_{0r} \right).
$$

This yields the simple update rules at lines 4 and 10.4 Notably, even in the distributed case, each domain $p$ can compute at each iteration a globally feasible allocation $z_{pr}$ for each of the routes $r \in R_p$ (see Proposition 1).

#### Communication among domain controllers:
In FD-ADMM, only domains that do share a route together have to communicate. The communication procedures among the domain controllers are described at lines 2 and 10. In these steps, the domains gather from and broadcast to adjacent domains the sole information related to routes that they share in common. In particular, domains are blind to routes that do not traverse them, and can keep their internal routes secret from others. In details, after each iteration of the algorithm, each domain $J_p$ receives the minimal information from other domains such that $J_p$ is still able to compute a local value $z_{pr}$ and a locally feasible value $z_{spr}$. Next, $J_p$ send them back to neighboring domains $I_r$ that $r$ traverses.

**Communication overhead:** In terms of overhead, we can easily evaluate the number of floats transmitted between each domain at each iteration. At each communication, domain $J_p$ must transmit $z_{pr}$ and $z_{spr}$ for each $r \in R_p$ to each other domain that $r$ traverses. The variable $z_0$ does not need to be centralized or transmitted between controllers. Each domain controller may actually have a copy $z_0$ and perform the (low-cost) computation of their update rule (see line 9 in Algorithm 2) locally. Hence, domain $p$ transmits in total $2 \sum_{q \neq p} |R_p \cap R_q|$ floats to the set of its peers. As a comparison, in a distributed implementation of the algorithm given in [15] and stated in Section VI, each domain $p$ transmits in total $\sum_{q \neq p} |\{j \in J_p : \exists r \in R_q \text{ s.t. } j \in r\}|$ floats to the set of its peers, which is bounded by $(P - 1) |J_p|$ as $|R|$ grows.

**Feasibility preservation:** A potential drawback of the distributed approach is the potential feasibility violation by the iterate $z^k$. However, we have the following positive result.

**Proposition 1.** FD-ADMM provides a sequence of feasible points that converges to the optimum.

**Proof.** Consider the iteration number $k$ and drop the super-script $k$ for lightness. For any link $j$, we have by line 7 of Algorithm 2 that $z_{jr}$ is feasible in link $j$. That is, $\sum_{r \in J} z_{jr} \leq C_j$. Define $z_{sr} = \sum_{j \in J_r} z_{jr}$. Then, for each link $j$:

$$
\sum_{r \in J} z_{sr} \leq \sum_{r \in J} z_{jr} \leq C_j.
$$

Thus, no capacity is violated by the allocation $z_{sr}$. At the optimum, the consensus is reached. Thus $(z_{sr}^*)_{k}$ is a feasible sequence that converges to the optimum.
The number $z_{pr}$ introduced in Proposition 1 above in fact corresponds to the introduced variable of the same name FD-ADMM. Thus, in a certain way, for sufficiently loaded and communicating domains (i.e. the $|R_j \cap R_i|$ are large enough) we sacrifice some overhead (counted on a per iteration basis) compared to standard dual methods, but in exchange for anytime feasibility, a major feature that dual methods do not generically provide.

V. IMPLEMENTATION AND ALGORITHM TUNING

In this section, we discuss two major points in the design of FD-ADMM. First, we precise and justify the choice of the procedure PROJECTION, in line 7 of FD-ADMM Algorithm 2. Next, we derive an explicit adaptive update of the reciprocal penalty parameter $\lambda$ that permits to accelerate the convergence of FD-ADMM on any instance.

A. Projection procedure: A discussion

In Section IV, we advocated a link-wise separation of the formulation because it is non-trivial to project an arbitrary point onto an arbitrary closed convex polyhedron. However, the projection onto the sets $S_j$ (see Section IV-A) can be done with an exact method with a complexity dominated by the one of sorting a list of the size of its dimension. In average, sorting a list of length $q$ is done in $O(q \log q)$. Hence, by operating instead a link-by-link projection, the controllers save a huge amount of time by providing an (generically infeasible) approximate projection point $z_{pr}$ and deriving a locally feasible allocation $z_{pr}$ (see Algorithm 2 line 10). Although the quality of the global iterate $z$ may be altered by further distribution of the projection, the point is quickly generated. Paradoxically enough, FD-ADMM therefore fully adapts to any network distribution into domains because it functions by link, regardless of the network partition into domains. The algorithm we use for PROJECTION in FD-ADMM is presented for instance in [5] in which the authors also give a correctness proof and performance demonstration. It permits to provide an efficient update for each domain $J_p$.

B. Estimating the optimal parameter $\lambda$

It is well-known that the reciprocal penalty parameter $\lambda$ highly conditions the convergence speed of ADMM. An inaccurate tuning can indeed lead to a very slow convergence. For appropriate problems, it is possible to use a result proven in [6] to compute an optimal reciprocal penalty parameter, that we here report. It will help us tune FD-ADMM to optimize its convergence performance.

**Theorem 1 ([6]).** Assume that the following problem:

$$\begin{align*}
\min_{x} & \quad F(x) + G(z) \\
\text{s.t.} & \quad Mx - Pz = 0
\end{align*}$$

(M)

has a saddle point, and both objective functions are convex. Assume that $M$ has full row rank, and that $F$ is $\sigma$-strongly convex and has a $L$-Lipschitz gradient. Then, the sequence of iterates (primal and dual concatenated) of ADMM converges linearly with rate $(1 + \delta)^{-1}$, where

$$\delta = 2 \left( \frac{|M|^2}{\lambda \sigma} + \frac{L \lambda}{\lambda_{\min}(MTM)} \right)^{-1}$$

and $\frac{1}{\lambda}$ is the penalty parameter in the augmented Lagrangian form (see Section III-B).

The following result directly follows.

**Corollary 1.** The optimal reciprocal penalty parameter is

$$\lambda_* = \sqrt{\frac{|M|^2 \lambda_{\min}(MTM)}{\sigma L}}.$$

In order to be able to apply Corollary 1, we still need to express the coefficients of interest $\sigma, L$. Due to page limitation, we defer their calculus to [1] and we here report the result.

**Fact 2.** The function $g = \bigoplus_r g_r$ is $\sigma$-strongly convex and has $L_d$-Lipschitz gradient with:

$$\sigma = \min_r \frac{w_r}{B_r^{\alpha+1}}, \quad L_d = \max_r \frac{w_r}{d_r^{\alpha+1}}$$

on any compact subset of $\text{dom}(g)$ of the form $K_d = \{ x \geq d, Ax \leq C \}$, for $d \geq 0$, and $B_r = \min_{j \in r} C_j$. Unfortunately, Corollary 1 cannot be directly applied to our general consensus formulation. Indeed, its matricial formulation does not provide a full-row rank matrix $M$. The problem which the Theorem 1 applies to is actually the original, centralized one in (3-4). Therefore, we will derive a reciprocal penalty parameter selection for the centralized problem, and use it as a tool to estimate a satisfactory parameter for FD-ADMM.

However, the last difficulty we encounter in choosing the optimal reciprocal penalty parameter is to correctly evaluate the Lipschitz modulus. Unfortunately, $\nabla g$ is not Lipschitz on the feasibility set, because of the singularity of each $g_r$, at 0. In order to circumvent this problem, we introduce the classic concept of disagreement point $d$, according to bargaining theory terminology. A disagreement point $d$ represents the minimal values for an allocation of each route. This allows to reduce the feasibility set to a compact subset of the form $K_d$, $d \gg 0$, on which $\nabla g$ is now Lipschitz. The disagreement point can be naturally defined as the feasible point $z_*$ at the first iteration. Generically, there is no a priori guarantee that the set $K_z$ contains the optimum, but, we remark that at least in the first iterations, the use of $z_*$ provides a good approximation of the best reciprocal penalty parameter. The analytical evaluation of this phenomenon goes beyond the scope of this paper and we keep it for future work.

Thus, finally, we update $\lambda$ in an adaptive fashion in the beginning of the algorithm with the help of those points. We found empirically that operating such update only at the initial steps of FD-ADMM and then fixing $\lambda$ for the rest of the execution provides a good performance in terms of convergence speed. In the next section, we describe this
typical phenomenon in Figure 2. In all our simulations, we use
the simple following update scheme to estimate the optimal
penalty parameter at each execution of the algorithm.

Scheme 1 (Reciprocal Penalty Adaptation). Set threshold $\tau$.
At all iterations below $\tau$, denote by $p$ the last output of
a feasible point. Then, choose the new reciprocal penalty
parameter as:

$$
\lambda^*_\alpha = \frac{1}{\alpha} \left( \min_{r \in R} \frac{w_r}{B^\alpha_{r+1}} \max_{r \in R} \frac{w_r}{p^\alpha_{r+1}} \right)^{\frac{1}{2}}.
$$

After $\tau$ iterations, do not update $\lambda$.

In our numerical evaluations we will set $\tau = 30$. Thus, FD-
ADMM is now fully tuned and we are ready to demonstrate
its performance in the next section, in terms of convergence
speed in real-time scenarios.

VI. PERFORMANCE ANALYSIS

We now evaluate numerically FD-ADMM in terms of its
convergence properties. More specifically, in Section VI-A
we compare the performance of FD-ADMM and C-ADMM
in offline scenarios where the optimum is desired. In Section
VI-B we evaluate FD-ADMM in real-time scenarios, where
good and feasible solutions are needed on-the-fly as weights $w_r$ vary over time. In order to benchmark the transient
properties of FD-ADMM we use the standard Lagrangian dual
decomposition approach (LAGR) for single-path routing in
[25, 15, 19], that we recall in Algorithm 3. We here assume
that domain controllers operate in synchronous mode. In this
case, the decomposition into domains has no impact on FD-
ADMM performance, as projection is on a link-basis. All
simulations are made for the proportional fairness objective
functions ($\alpha = 1$). We used the proximal operation formu-
las found in [3]. The algorithms under investigation were
evaluated using BT’s 21 CN network topology\(^7\), containing
106 nodes and 474 links. The requests were generated by
computing the shortest path between randomly chosen sources
and destinations.

\(^7\)We would like to thank the authors of [15] for their willingness to share
the BT 21 CN topology dataset.

A. Algorithm design

Evaluating the alleviation of the compute-intensive parts
of C-ADMM was a key concern to motivate and validate
the distribution to FD-ADMM. To this aim, we show in
Figure 1 the computation time and iteration count for those two
algorithms on small instances for a number of requests ranging
from 1 to 200. The centralized projection in C-ADMM is
executed using the variation of Hildreth’s projection algorithm
on general polyhedra in [9]. When convergence is desired, a
precise stopping criterion for FD-ADMM is available, as the
optimality gap can be upper-bounded by the primal and dual
residuals, see [3]. In our case, evaluating those residuals results
in computing the absolute variation of two consecutive values

Algorithm 3 Lagrangian-based gradient descent (LAGR)

**Input:** Initial positive values $u_j$

**while** a suitable termination condition is not met **do**

$\begin{align*}
x_r &\leftarrow \arg \max_{x \geq 0} \{ f_r(x) - \sum_{j \in r} x_j u_j \} \quad \forall r \\
u_j &\leftarrow u_j - \frac{\delta}{C_j} (C_j - \sum_{r \in r} x_r) \quad \forall j
\end{align*}$

**end while**
of \( \tilde{z} \), and the consensus accuracy

\[ \max_{r\in R} \sum_{j\in R} |z_{jr} - \tilde{z}_r| \]

This is a first advantage for FD-ADMM implementation as no robust stopping criterion is available for standard gradient descent. When an optimality gap is computed, we thus consider a \( 10^{-6} \)-approximation by FD-ADMM as the reference for all tested algorithms. In Figure 2, we illustrated, on a small instance with 200 requests, the number of iterations of FD-ADMM to reach convergence for a various number of the parameter values, in order to evaluate our adaptive scheme’s accuracy with respect to the empirically best found parameter. It shows that our approximation of \( \lambda^\star \) is fairly satisfactory.

In Figure 1, FD-ADMM shows that distributing the consensus over the links exchanges several more iterations for a reduction of the compute time by two orders of magnitude for small instances. Hence, the distribution does not seem to cost too much convergence rate. Not surprisingly, the use of a central projection sub-routine makes C-ADMM impossible to scale.

The convergence criterion used in Figures 1 and 3 is modest \( (10^{-1}) \). Finally, we plotted a notable behavior of FD-ADMM in Figure 3. One can imagine a link between the convergence rate and the mean link load, i.e., \( \frac{1}{T} \sum |R_j| \). This conjecture requires further investigation that we keep for future work.

B. Comparison against Lagrangian method

We now compare the proposed FD-ADMM algorithm against the classic LAGR Algorithm 3, see [25, 15, 19]. To this aim we perform two experiments, in real-time and static scenarios, respectively.

We start by evaluating the real-time responsiveness of FD-ADMM by considering a small scenario where 200 routes are established and the weights \( (w_t^r)_{r\in R, t\in 0...T} \) vary over discrete time \( t \), following the formula:

\[ w_{t+1}^r \in [(1 - a)w_t^r, (1 + a)w_t^r] \quad a \in [0, 1] \]

where at each event \( t \), \( w_t^r \) is chosen uniformly within the above interval in which \( a \) determines the amplitude of the weight variation. In Figure 4 we illustrated the average optimality gap of the two algorithms achieved over 20 events with 10 iterations between each event. We observe that FD-ADMM outperforms LAGR in terms of optimality gap, although the performance of both algorithms is fairly acceptable. However, remarkably, FD-ADMM remains always feasible whereas LAGR constantly violates the constraints as weights \( w_t^r \) change in real-time. Figure 5 shows the percentage of constraints of the problem that are violated for each value of the amplitude \( a \). In fact, LAGR iteratively approaches the fair resource allocation from the outside of the feasible set. This drawback is commonly amended by projecting the solution onto the feasible set. However, this is not doable in our distributed setting, as projection requires costly on-the-fly operations that require full topological information. For such reasons, we claim that the standard LAGR algorithm is not well suited for computing real-time fair allocations in a distributed SDN setting.

In our last experiment we test the two algorithms under a static scenario, where the weights \( w_r \) do not vary over time and LAGR has enough time to find at least one feasible solution. In Figure 6 we compare the optimality gap of the best feasible solutions found after 5 seconds runtime by FD-ADMM and LAGR, for different instance sizes over BT topology. We observe that FD-ADMM obtains a close-to-optimal feasible solution for all the instance sizes (from 100 to 6000 requests),

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8One can choose any other norm in \( \mathbb{R}^{|R_j|} \).

---
Theorem\textsuperscript{4} In this paper we addressed the real-time fair resource allocation problem in the context of a distributed SDN control plane architecture. Our main contribution is the design of a distributed algorithm that continuously generates a sequence of feasible solutions and adapts to any partitioning of the network into domains. We reformulated the α-fair resource allocation problem in the fashion of a general consensus problem to derive the FD-ADMM algorithm. This algorithm can be massively parallelized on several processors that manage different regions of the network, hence fully benefiting from the computing resources of SDN controllers in distributed architectures. We also provided a strategy for a near-optimal estimation of the penalty parameter of FD-ADMM that boosts its convergence. Finally, we compared FD-ADMM to a standard dual Lagrangian decomposition method (LAGR) and we demonstrated how the former is more adapted to a real-time situation where bandwidth has to be adjusted on-the-fly. In fact, FD-ADMM ensures a smaller optimality gap since the very first iterations and, most importantly, it produces a feasible fair allocation at all iterations.

As a next step, we envision to adapt our formulation to the case where multiple candidate paths are available for each request. Moreover, we plan to run FD-ADMM asynchronously while still guaranteeing near-optimal convergence rate and anytime feasibility.

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