Radio Galaxy Shape Measurement with Hamiltonian Monte Carlo in the Visibility Domain

M. Rivi$^{1,2\star}$, M. Lochner$^{1,3,4}$, S.T. Balan$^1$, I. Harrison$^5$, F.B. Abdalla$^{1,6}$

$^1$Department of Physics and Astronomy, University College London, Gower Street, London, WC1E 6BT, UK
$^2$INAF - Istituto di Radioastronomia, via Gobetti 101, 40129 Bologna, Italy
$^3$African Institute for Mathematical Sciences, 6 Melrose Road, Muizenberg, 7945, South Africa
$^4$SKA South Africa, The Park, Park Road, Pinelands, Cape Town 7405, South Africa
$^5$Jodrell Bank Centre for Astrophysics, School of Physics & Astronomy, The University of Manchester, Manchester M13 9PL, UK
$^6$Department of Physics and Electronics, Rhodes University, PO Box 94, Grahamstown, 6140, South Africa

18 May 2018

ABSTRACT

Radio weak lensing, while a highly promising complementary probe to optical weak lensing, will require incredible precision in the measurement of galaxy shape parameters. In this paper, we extend the Bayesian Inference for Radio Observations model fitting approach to measure galaxy shapes directly from visibility data of radio continuum surveys, instead of from image data. We apply a Hamiltonian Monte Carlo (HMC) technique for sampling the posterior, which is more efficient than the standard Monte Carlo Markov Chain method when dealing with a large dimensional parameter space. Adopting the exponential profile for galaxy model fitting allows us to analytically calculate the likelihood gradient required by HMC, allowing a faster and more accurate sampling. The method is tested on SKA1-MID simulated observations at 1.4 GHz of a field containing up to 1000 star-forming galaxies. It is also applied to a simulated observation of the weak lensing precursor survey SuperCLASS. In both cases we obtain reliable measurements of the galaxies’ ellipticity and size for all sources with SNR $\geq 10$, and we also find relationships between the convergence properties of the HMC technique and some source parameters. Direct shape measurement in the visibility domain achieves high accuracy at the expected source number densities of the current and next SKA precursor continuum surveys. The proposed method can be easily extended for the fitting of other galaxy and scientific parameters, as well as simultaneously marginalising over systematic and instrumental effects.

Key words: cosmology: observations – radio continuum: galaxies – techniques: interferometric – methods: statistical.

1 INTRODUCTION

The next generation of radio telescopes, such as the Square Kilometre Array (SKA$^1$) will reach high enough sensitivity to provide a density of detected galaxies sufficient for weak lensing measurements in the radio band, with the advantage of reaching higher redshifts compared with optical surveys (Brown et al. 2015; Harrison et al. 2016). Standard techniques for the measurement of cosmic shear are based on the observation of the shapes of faint star-forming galaxies, as it quantifies their coherent distortion by a large-scale foreground matter distribution. Such methods have been developed for optical surveys so far and require source shapes be measured accurately in order for errors to be dominated by statistics, rather than systematics (Mandelbaum et al. 2015). Therefore they assume stringent and specific requirements on image fidelity.

Radio instruments do not detect images of the observed sky, they provide its Fourier Transform at a finite number of points instead. The image reconstruction from these data (visibilities), using iterative de-convolution methods such as CLEAN (Högbo 1974; Schwarz 1978), is a highly non-linear process that does not necessarily converge in a well-defined manner when dealing with extended sources resolved by high resolution telescopes. Moreover the Fourier Transform of visibilities makes the noise in radio images highly correlated. SKA simulations have shown that, even on high signal-to-noise ratio (SNR) objects, this process produces images with structures in the residuals that dominate the cosmological signal, confirming that this bias may be in-
duced by the procedure adopted for turning the visibility data into images (Patel et al. 2015).

Cross-correlation of real data images shows that there is no evidence of high levels of correlation between the optical and radio intrinsic shape of the matched objects (Patel et al. 2010; Tunbridge et al. 2016), possibly due to systematics of the imaging pipeline. However, a significant percentage of contamination (as they carry a much higher shape noise) AGN sources and the astrophysical scatter between optical and radio position angle due to the different emission mechanisms may also be the reasons for such results.

A more natural approach to adopt with radio data is to work directly in the visibility domain, where the noise is purely Gaussian and the data not yet affected by the systematics introduced by the imaging process, with the advantage of an exact modelling of the sampling function. However, this approach faces difficult statistical and computational challenges because sources are no longer localized in the Fourier domain and a telescope like SKA generates a large number of visibilities. Nevertheless, the advantages of being able to reduce, as well as holistically incorporate, systematic effects, coupled with Gaussian noise properties make modelling in the visibility domain a compelling option.

Bayesian Inference for Radio Observations (BIRO) (Lochner et al. 2015) combines simulation tools, such as MeqTrees (?), capable of predicting visibilities from a given sky model and observational setup, with Bayesian inference samplers, like MCMC (??) or Nested Sampling (?). This allows full forward modelling of scientific properties, such as the flux and positions of sources, as well as instrumental effects, like pointing errors in telescopes, that can have strong effects on the scientific parameters. In addition, by making full use of the visibility information (instead of gridding as imagers do), BIRO can dramatically increase the resolving power of a telescope. BIRO could thus potentially be a useful tool for radio weak lensing, if the galaxies in the survey can be correctly modelled.

Other available tools for source fitting in the visibility domain have been developed for general purposes (Martí-Vidal et al. 2014). They provide a variety of galaxy shape models, but they are not sufficiently accurate or capable on wide fields of large numbers of sources for weak lensing measurements. Currently, two analytical galaxy models have been considered for shear measurement in the visibility domain.

The first one uses shapelets (Refregier 2003; Refregier & Bacon 2003), where galaxy images are decomposed through an orthonormal basis of functions corresponding to perturbations around a circular Gaussian. Shapelets are invariant under Fourier Transform (up to a rescaling) allowing the adoption in the visibility domain of the same approaches used with images. Finding the best fitting shape via minimising the chi-squared is nominally linear in the coefficients and therefore can be performed simultaneously for all sources by simple matrix multiplications. However, shapelets introduce a shear bias as they cannot accurately model steep brightness profiles and highly elliptical galaxy shapes (Melchior et al. 2010).

For this reason, Sérsic models are commonly used in optical weak lensing surveys analysis (Mandelbaum et al. 2015), where the disc component is described by the exponential brightness profile (Sérsic index $n = 1$). A reasonable assumption is to use the exponential model also in the radio band because the radio-emitting plasma should follow the distribution of stars in galaxy discs. In Rivi et al. (2016b), the exponential profile has been Fourier transformed analytically and used to extend the optical semi-Bayesian lensfit method (Miller et al. 2013) to radio data in the visibility domain. This method, called RadioLensfit, fits the visibilities of a single source at a time and applies a Bayesian marginalisation of the likelihood over uninteresting parameters. Visibilities of each galaxy are isolated by a source extraction algorithm described in Rivi & Miller (2018). This approach is very fast computationally but may be limited by the source number density in the field of view, because of nearby galaxies residuals in the extraction procedure (“neighbour bias”).

For this reason we investigate in this paper a different approach for galaxy shape measurement in Fourier space. We adopt the same analytical galaxy model used in RadioLensfit and introduce it into the BIRO formalism. Unlike in Lochner et al. (2015), we now have a large number of parameters to cope with so we use a Hamiltonian Monte Carlo (HMC) sampler (Neal 2011) to perform a joint model fitting of all the sources in the field of view. We first test the method on two SKA1-MID simulated data, investigating also the potentiality of fitting a large number of sources. Then we apply the method on the simulation of a real observation: SuperCLASS (Battye et al, in prep), whose UV coverage is quite complex as it is composed of the baselines of two different SKA precursor radio telescopes (e-MERLIN and JVLA). Finally we discuss the convergence of individual parameters depending on source properties.

Our paper is organised as follows. In Section 2 we outline the method and its implementation. In Section 3 and 4 we describe data simulation and results for SKA1 and SuperCLASS respectively. In Section 5 we discuss HMC convergence. Conclusions are presented in Sections 6.

2 THE METHOD

In this work we propose to measure radio galaxy shapes directly from the raw visibilities (i.e. in the Fourier space) by sampling the joint posterior distribution of the shape parameters of all the detected galaxies in the field of view. Since the number of galaxies in a single pointing may be very large, conventional Markov-Chain Monte Carlo (MCMC) methods using random-walk samplers (e.g. Metropolis-Hastings) are not suitable because they require a prohibitive number of samples to explore high-dimensional parameter spaces. We use the Hamiltonian Monte Carlo approach that reduces random walk behaviour by applying the Hamiltonian dynamics of particles in potential wells (Section 2.1). It requires the computation of the likelihood gradient that we can compute analytically for our purpose, avoiding numeri-
The basic idea is to sample a distribution $P(x)$ of $n$ parameters $x$ according to the Hamiltonian of a 2$n$-dimensional dynamical system, where the parameters correspond to the positions of $n$ particles and $P(x)$ is the canonical distribution of the potential energy function $U(x)$. Momentum variables $p$ are introduced to be coupled with positions $x$ to allow Hamiltonian dynamics to operate according to a kinetic energy function $K(p)$ whose canonical distribution is the zero-mean Gaussian with a diagonal covariance matrix $M = \text{diag}(m_1, \ldots, m_n)$, where $m_i$ represents the mass of particle $i$. This means:

$$U(x) = -\log[P(x)], \quad K(p) = \frac{1}{2} p^T M^{-1} p. \quad (1)$$

Samples from the parameters distribution are obtained by marginalising the canonical distribution of the corresponding Hamiltonian function $H(x, p) = U(x) + K(p)$:

$$\exp[-H(x, p)] = P(x) \exp[-\frac{1}{2} p^T M^{-1} p] \quad (2)$$

over $p$.

In practice, at each iteration of HMC a new sample $x'$ is generated from the previous one $x_0$ as follows. First a proposal $p_0$ for the momentum variables is generated randomly according to the canonical distribution of the kinetic energy, i.e. a $n$-dimensional uncorrelated Gaussian. Then, a deterministic proposal for positions $x$ is computed by allowing the system to evolve for a fixed time $\tau$ from the starting point $(x_0, p_0)$ according to Hamilton’s equations:

$$\frac{dx_i}{d\tau} = \frac{\partial H}{\partial p_i} = [M^{-1}p]_i, \quad (3)$$

$$\frac{dp_i}{d\tau} = -\frac{\partial H}{\partial x_i} = -\frac{\partial P(x)}{\partial x_i}. \quad (4)$$

These equations are usually solved numerically by applying the leapfrog method, where Hamilton’s equations are discretized with a time step of small size $\varepsilon$ as follows:

$$p_i(t+\varepsilon) = p_i(t) + \frac{\varepsilon}{2} \frac{\partial P}{\partial x_i} |_{x(t)} \quad (5)$$

$$x_i(t+\varepsilon) = x_i(t) + \frac{\varepsilon}{m_i} p_i(t+\varepsilon/2) \quad (6)$$

$$p_i(t+\varepsilon) = p_i(t+\varepsilon/2) - \frac{\varepsilon}{2} \frac{\partial P}{\partial x_i} |_{x(t+\varepsilon)}. \quad (7)$$

The new proposal for $x$ is obtained after $k$ steps as $x' = x(\tau)$, with $\tau = k\varepsilon$. It is accepted with Metropolis criterion, i.e. with probability:

$$\min(1, \exp(-\delta H)), \quad (8)$$

where

$$\delta H = H(x', p') - H(x_0, p_0).$$

This way the rate of acceptance is close to unity when numerical solutions of Hamilton’s equations are very close to the exact ones because along such a trajectory energy is conserved. Neal (2011) show that $\varepsilon < 2$ must be required in order to have stable trajectories computed with the leapfrog method. Our C++ implementation of the HMC algorithm will soon be available within a BIRO repository.

### 2.2 Likelihood and gradient computation

In order to sample the posterior of the parameters, we need to define the likelihood function obtained by comparing data visibilities $V_i$ with the model visibilities $\tilde{V}_i$ dependent on parameters $x$:

$$L(x) = \left( \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_i^2} \right) \exp \left[ -\frac{\sum_{i} (\tilde{V}_i - V_i(x))^2}{2\sigma_i^2} \right], \quad (9)$$

where $\sigma$ is the standard deviation of the Gaussian noise on the $i$-th data visibility.

Visibilities are evaluated at the interferometer baseline vectors, whose coordinates $(u, v, w)$ are measured in wavelengths at the centre frequency of the signal band with respect to a coordinate system where $w$ points towards the phase tracking centre (Thompson et al., 1986).

For a sky containing $N$ galaxies with exponential profile, we adopt the analytical visibility model presented in Rivi & Miller (2018):

$$V(u, v, w) = \frac{\lambda^{\beta}}{\lambda^{\beta}} \sum_{s=1}^{N} \frac{S_{s,\lambda^{\beta}} 2\pi i (\lambda u + v + w \sqrt{1+r^2-m_z^2})}{(1+4\pi^2\alpha^2|\mathbf{A}_s-T\kappa|^2)^{3/2}} \quad (10)$$

where $\beta = -0.7$ is the spectral index for the synchrotron radiation emitted by the galaxy disc, $\kappa = (u, v, w)$, and for $s = 1, \ldots, N$ we have the following source $s$ parameters: position $(l_s, m_s)$, flux $S_{s,\lambda^{\beta}}$ at reference wavelength $\lambda_{\text{ref}}$, scale-length $\alpha$, and ellipticity components $e_{1,s}, e_{2,s}$. The ellipticity parameters are contained in the matrix $\mathbf{A}_s$ that transforms the circular exponential profile in elliptical

$$\mathbf{A}_s = \begin{pmatrix} 1-e_{1,s} & -e_{2,s} \\ e_{2,s} & 1+e_{1,s} \end{pmatrix}. \quad (11)$$

We assume the following ellipticity definition:

$$e = e_1 + i e_2 = \frac{a-b}{a+b} e^{2i\theta}, \quad (12)$$

where $a$ and $b$ are the galaxy major and minor axes respectively, and $\theta$ is the galaxy orientation.

A joint analysis with the image domain allows the reduction of the number of parameters to $3N$ because the flux and position of sources can be accurately measured from the image. Therefore we assume them well known. Since the visibility model is defined analytically we can compute the exact likelihood gradient with respect to the parameters of our interest: scalelength and ellipticity components of each source in the primary beam.

The computation of the galaxy exponential model and the corresponding likelihood gradient (see Appendix A) has been added to the open-source software Montblanc\footnote{https://github.com/ska-sa/montblanc/tree/chi_sqrd_gradient}.\footnote{$I(r) = I_0 \exp(-r/\alpha)$}
and variance $\sigma$ are estimated from VLA observations:

$$V_{tpq} = \frac{1}{N} \sum_{s=1}^{N} K_{tpq} B_s K^H_{tpq}$$

where uvw points are identified by the baseline formed by antennas $p$ and $q$ at timestep $t$ and channel with centre wavelength $\lambda$, $K_{tpq}$ and $B_s$ are respectively the phase and brightness matrices for source $s$ ($H$ denotes matrix hermitian transpose). We use a brightness matrix defined by the exponential model of an extended source:

$$B_s = \left( \frac{\lambda_{ref}}{\lambda} \right)^{\alpha} \frac{S_{B,\lambda_{ref}}}{(1 + 4\pi^2\alpha^2|\mathbf{A}^T\mathbf{k}|^2)^{3/2}}$$

Given the general formalism of the RIME, any instrumental effects that can be modelled, such as primary beam shape, pointing errors and instrumental noise, can be incorporated and marginalised over with our formalism.

### 2.3 Priors

As source scalelength and ellipticity priors we use the distributions presented in Rivi & Miller (2015), whose parameters are estimated from VLA observations:

- $p(\alpha)$ is a lognormal function with mean $\mu = \ln(\alpha_{med})$ and variance $\sigma \sim 0.3$, where $\alpha_{med}$ is given by equation (12).
- for the modulus $\epsilon$ of the intrinsic galaxy ellipticity

$$p(\epsilon) = \frac{N_\epsilon}{(1 - \exp \frac{-e_{\max}}{e_0})(1 + e)(e^2 + e_0^2)^{3/2}},$$

with $c = 0.2298$, $e_0 = 0.0732$ and $e_{\max} = 0.804$.

These distributions are used for both the galaxy catalog simulation and the prior in the HMC sampling.

### 2.4 HMC tuning

Selecting suitable values of the step size $\varepsilon$ and the number of steps $k$ for the leapfrog method is crucial as discussed in Neal (2011). After preliminary runs, we choose $\varepsilon = 0.05$ and $k = 10$ as a trade-off between the accuracy of the Hamilton’s equations solutions (i.e. high acceptance rate) and the computational time due to a too long trajectory.

Since most of galaxy ellipticities should be close to zero, according to the ellipticity modulus distribution, we choose as starting points for the ellipticity components random numbers uniformly distributed between $-0.1$ and $0.1$. For scalelength parameters we choose as starting points the median values depending on the source flux density according to the relation estimated in Rivi et al. (2016a) from the VLA 20 cm continuum radio source catalogue in the SWIFT field (Owen & Morrison 2008):

$$\ln [\alpha_{med}/\text{arcsec}] = -0.93 + 0.33 \ln [S/\mu\text{Jy}].$$

For the convergence of HMC chains, it is very important to choose good “step sizes” of the momentum samples. They are defined by the inverse mass in the kinetic energy function and typically are given by the variance of the parameters. In this application parameter variance is dependent on the source signal-to-noise ratio, and therefore integrated source flux. We estimated a relation between the standard deviation for source scalelength and ellipticity components as a function of its flux density by fitting the error bars measured from a simulated observation of 100 sources (as described in Section 2.3). For both ellipticity components and scalelength we obtain a power law relation between source flux (in $\mu$Jy) and measured standard deviation. For example, for SKA1-MID observations we obtain the following relations (see Fig. 1):

$$\sigma_{e_1}, \sigma_{e_2} = -0.0022 + 1.77 S^{-0.92},$$

$$\sigma_\alpha [\text{arcsec}] = -0.0914 + 0.61 S^{-0.31}.$$

### 3 SKA1-MID Simulations

We simulate SKA1-MID 8-h observations pointing close to the zenith ($\delta \sim -30^\circ$). Visibilities are sampled every 60 sec at the frequency 1.4 GHz (the corresponding UV coverage is plotted in Fig 2) and we choose a single large channel with bandwidth 240 MHz, as no smearing effects are added. Visibilities are simulated by using equation (10) and adding an uncorrelated Gaussian noise whose variance is dependent on the antenna system equivalent flux density (SEFD) of SKA1 dishes, the frequency channel bandwidth and the time sampling according to the formula given in Wrobel & Walker (1999).

#### 3.1 Source catalogs

The source catalogs are simulated as in Rivi & Miller (2015), where positions are generated randomly according to a uniform distribution over a circular field of view, the flux distribution is $p(S) \propto S^{-1.34}$, scalelength and ellipticity follow the prior distributions listed in Section 2.3.

The minimum integrated source flux detectable at a SNR$_{vis}^2 = 10$ is $10 \mu$Jy, and the source density expected in the current planned SKA1 radio weak lensing survey ($2.7 \text{gal/arcmin}^2$ (Brown et al. 2015)) is obtained at the maximum flux of $200 \mu$Jy. We choose a field of view of 400 arcmin$^2$ to test the method at such density for 1000 galaxies (“SKA1 1000”). To compare measurement accuracy and the speed of convergence as a function of the number of sources to fit, we also test the case of 100 sources with the same flux range and field of view (“SKA1 100”).

We compute the signal-to-noise ratio in the visibility domain as $\text{SNR}_{vis} = \sqrt{\sum_{i=1}^{\text{vis}} |V_i|^2 / \sigma_i^2}$, where $V_i$ are the simulated visibilities without noise.
Radio Galaxy Shape Measurement with HMC

3.2 Results

In Fig. 3 we show the one-dimensional marginalised posterior for a few example parameters of “SKA1-100” showing they are well-constrained and accurately recover input parameters. We show the difference between the measured and input parameters (which should be close to zero) for both SKA1 cases in Fig. 3. As expected, brighter objects have much lower uncertainty in the parameters. We find that the shape parameters are largely uncorrelated although they could be highly correlated with instrumental effects that we do not model here (Lochner et al. 2015).

Table 1 contains the values of the best-fit lines coefficients for the scalelength and the ellipticity components obtained by fitting the simulated observations of 100 and 1000 sources in the field of view respectively. They are almost the same for both test cases, where only the uncertainty is obviously larger for a smaller number of sources. The goodness of fit for all shape parameters, estimated as a straight line fit of measured versus true parameters, shows that the measurements are very precise.

Comparing the best-fit slope of ellipticities for 1000 sources, i.e. at the source density of the proposed SKA1 weak lensing survey, with the one obtained at the same density with RadioLensfit we see a significant improvement. In fact, in [Rivi & Miller] (2015) the best-fit slopes of the ellipticity components, measured from a similar source population and SKA1 UV coverage, were found to be $a_1 = 0.9365 \pm 0.0017$ and $a_2 = 0.9262 \pm 0.0017$ respectively. This improvement is expected because joint fitting avoids the source extraction bias of the RadioLensfit method, but with a large computational cost (see Table 3).

4 APPLICATION TO SUPERCLASS

We also investigate the ability of the method to infer shear using current-generation data, from the SuperCLASS survey. This allows us to both assess the possibilities in near-term data, and also understand the effect of realistic UV coverages, which are not typically observed as a single long track, rather multiple interleaved pointings. We emphasise that the results we generate here should not be used as a measure of the relative shape measurement ability of SuperCLASS and an SKA survey; shape measurement ability will be a strong function of UV coverage and we use a very realistic coverage for SuperCLASS and a highly simplified (and much smaller in volume) coverage for SKA.

4.1 The survey

The SuperCLASS (Super CCluster Assisted Shear Survey) (Battye et al, in prep.) is a legacy survey on the UK’s e-MERLIN radio telescope, with the express goal of making a convincing detection of a weak lensing signal in radio data. For the field, containing four massive galaxy clusters at $z \sim 0.2$, observations are taken at L-Band (around 1.4 GHz) using both e-MERLIN and the JVLA. This gives coverage of a wide range of spatial scales, with sensitivity to angular scales between 1 and 10 arcseconds principally coming from the JVLA, and smaller scales from the longer baselines available to e-MERLIN. The field has also been observed at multiple lower radio frequencies (to assist in Rotation Measure synthesis and source classification), sub-mm, and optical and near-IR wavelengths (to obtain photometric redshifts and optical shear measurements). The full data release of the survey is expected to contain thousands of galaxies.

http://www.e-merlin.ac.uk/legacy/projects/superclass.html
### Figure 3

Example 1-d marginalised posterior plots for the SKA1 100 source case, for the brightest (top panel) and the faintest (bottom panel) sources in the catalogue. The input value for each parameter is shown by a solid line, illustrating the ability of our approach to accurately recover both the parameter value and its uncertainty.

### Table 1

| Observation    | $a_1$       | $c_1$       | $a_2$       | $c_2$       | scalelength |
|----------------|-------------|-------------|-------------|-------------|-------------|
| SKA1-100       | 0.9559 ± 0.0116 | -0.0018 ± 0.0032 | 0.9780 ± 0.0118 | -0.0014 ± 0.0031 | 1.0015 ± 0.0091 | -0.0019 ± 0.0161 |
| SKA1-1000      | 0.9704 ± 0.0043 | 0.0001 ± 0.0010 | 0.9718 ± 0.0040 | -0.0002 ± 0.0010 | 1.0048 ± 0.0030 | -0.0090 ± 0.0051 |

Table 1. Multiplicative ($a$) and additive ($c$) coefficients of the best-fit lines for galaxy shape measurements with SKA1 at SNR$_{vis} \geq 10$. at a density of $\sim 1\text{ gal/arcmin}^2$ over $1\text{ deg}^2$. Shape measurement for the survey is being performed using both calibrated image-plane and hybrid image/visibility-plane techniques.

### 4.2 Data simulation

We create simulated UV coverages which closely match the true ones in the SuperCLASS survey. This provides us with an idealised but realistic version of the experiment; though we do not include effects from calibration errors and missing data from telescope outages or Radio Frequency Interference (RFI, an appreciable problem for e-MERLIN [Peck & Fenech 2013]), we do recreate the real shape of the UV coverage (corresponding to the shape of the PSF). The UV coverages for both telescopes are simulated as measurement sets using the CASA simulator tool [McMullin et al. 2007].

#### 4.2.1 e-MERLIN

For e-MERLIN, we simulate a single IF of 512 channels of width 125 kHz, covering a bandwidth of 512 MHz upwards from a starting frequency of 1.25 GHz. The UV coverage is then generated as it is for a real observation: during a single eight hour observing run (one epoch), seven different pointing centres are observed, along with amplitude and phase calibration sources. Observations cycle around the seven pointing centres, spending alternately 12.5 minutes observing the source and 2.5 minutes observing calibrators.
4.2.2 JVLA

For the Jansky Very Large Array (JVLA) telescope observations of the SuperCLASS field, we simulate the full sixteen IFs of 256 channels each, with channel widths of 250 kHz, covering a bandwidth of 1024 MHz upwards from a starting frequency of 1.01 GHz. For the JVLA, each individual pointing is observed over six epochs, with a total time of 11 minutes 6 seconds on source. The greater filling of the Fourier plane on the sample scales by the JVLA compared to e-MERLIN means that a similar image-plane depth of \( \sim 14 \mu \text{Jy/beam} \) is reached in this time. The UV coverage resulting from this procedure is shown in the right panel of Fig. 5.

4.3 Shape measurements

We are able to recover with a good accuracy the shape parameters of all sources. Fig. 6 shows the difference between the measured and input galaxy shape parameters, and Table 2 contains the best-fit lines coefficients of the measured parameter values. The goodness of fit is better than for

\footnote{Usually the source signal-to-noise in the image domain is given by the ratio between the source peak intensity and the image noise standard deviation. Since we are dealing with extended sources, this SNR tends to be lower than what measured in the visibility domain.}

Figure 4. The measured minus true value of each parameter as a function of integrated flux for the ellipticity parameters \( (\varepsilon_1 \text{ and } \varepsilon_2) \) and the scalelength parameter \( (\alpha_s) \), for SKA1 100 sources (top panel) and SKA1 1000 sources (bottom panel). The error bar is estimated from the 1-d marginalised posterior for that parameter. It is clear that brighter sources have lower uncertainty on the parameters.
SKA1 simulations mainly because in this case the SNR in higher, but also because the UV coverage is much larger (see Table 3).

5 CONVERGENCE ANALYSIS

5.1 Tests for convergence

Any Bayesian inference sampler, such as HMC, is only guaranteed to converge to the true posterior with an infinite number of steps. Given that we obviously can only run a chain for a finite time, convergence must be tested for. We test for convergence of the HMC chains using two different metrics: the commonly-used Gelman-Rubin (GR) statistic (\(\hat{\eta}\)) and the autocorrelation function (\(\hat{c}\)).

5.1.1 The Gelman-Rubin Statistic

The GR is a measure of how similar a set of HMC chains are: it compares the variance within a chain to the variance between chains and should be close to one for a converged set of chains. Ideally, one would run several independent chains, starting from different starting points. However, as we are computation limited in this work, we settle for running one chain that we split up into three separate chains and compare.

For the SKA1 100 source chain and the SuperCLASS 100 source chain, we find GR \(\approx 1.1\) for all parameters. However, for the SKA1 1000 source chain, the parameters converge more slowly and for a few parameters, GR \(\approx 2\). While we still make use of this chain in our analysis, because we observe no strong bias with respect to the true parameters and find the effective sample size to be sufficient (see below), if used on real data a longer chain should ideally be run and convergence ensured.

5.1.2 Auto-correlation function

An additional test for convergence is the lag-\(k\) autocorrelation function which computes the autocorrelation between posterior samples separated by \(k\) steps. The autocorrelation function for parameter \(\theta\), with mean \(\bar{\theta}\), sampled by a length-
Radio Galaxy Shape Measurement with HMC 9

Figure 7. Autocorrelation curves for two example parameters of the SKA1 100 sources chain. When the autocorrelation is close to zero, steps separated by the corresponding lag are uncorrelated, independent samples. A short autocorrelation length corresponds to a well-mixed chain. Plotted here are the autocorrelation functions for the scalelength of a bright, round source (solid, blue line) and faint, highly elliptical source (dashed, orange line), for which the steps of the chain are still highly correlated indicating a difficulty to converge.

\[ N \text{ chain is given by:} \]
\[ \rho_k = \frac{\sum_{i=1}^{N-k} (\theta_i - \bar{\theta})(\theta_{i+k} - \bar{\theta})}{\sum_{i=1}^{N} (\theta_i - \bar{\theta})^2}. \] (19)

\[ \rho_k = 0 \text{ indicates samples separated by } k \text{ are independent.} \]

Table 2. Multiplicative (a) and additive (c) coefficients of the best-fit lines for galaxy shape measurements with SuperCLASS at SNR_{vis} \geq 18 (the lowest SNR in the SuperCLASS sample).

| Observation       | a          | e₁        | c          | a          | e₂        | c          | scalelength |
|-------------------|------------|-----------|------------|------------|-----------|------------|-------------|
| SuperCLASS-100    | 1.0001 ± 0.0010 | 0.0006 ± 0.0002 | 1.0011 ± 0.0020 | 0.0004 ± 0.0002 | 1.0002 ± 0.0005 | −0.0047 ± 0.0079 |

where \( \hat{\rho}_k \) is the normalised autocorrelation function \( \rho_k / \rho_0 \), \( \sigma_{samp}^2 \) is the variance on the estimate of a given parameter which is due to finite sampling and is also sometimes referred to as the Monte Carlo standard error.

The integrated autocorrelation time is thus a key metric for analysing the convergence of a chain in an intuitive way: \( t_{\text{int}} \) dictates the number of steps required before a chain will reach a given level of precision in the estimate of a parameter, since it defines the number of independent samples that are in a chain. For instance, an effective sample size \( (N/t_{\text{int}}) \) of 100 would contribute an additional \( \sim 10\% \) to the uncertainty on a given parameter. As the effective sample size increases, the contribution to the error budget from sampling becomes negligible when compared with the intrinsic posterior uncertainty.

In an ideal world, an HMC chain would always be run long enough for the sampling uncertainty to be too small to be concerned with. However, in reality computational resources are limited and the sampling uncertainty must be taken into account. The required level of precision depends on the final analysis to be done with the sample. MCMC analyses have been done in the context of shear estimation in the case of the Dark Energy Survey (?) where chains of a few thousand steps were found to be sufficient. Estimates of the required sampling uncertainty needed would critically depend on the design and cosmological requirements of that survey, which is beyond the scope of this paper. So, rather than enforcing a particular precision, in this work we use \( t_{\text{int}} \) and \( \sigma_{samp} \) to investigate which parameters are difficult to constrain and gain some physical insight in the problem.

5.2 Convergence of individual parameters

Given that some parameters converge more quickly than others, it is natural to ask what properties about a source make it easy or difficult to constrain its parameters. We investigate this here using the integrated autocorrelation time (see equation (21)) and the sampling uncertainty (equation (20)).

We first test the hypothesis that brighter sources should be easier to constrain. Fig. 8 shows the relationship between the Gelman-Rubin statistic and the peak flux for the SKA1 100 sources case. There appears to be some relationship that brighter sources tend to have “more converged” parameters but it’s difficult to be certain. We investigate this further with the integrated autocorrelation time. As seen in Fig. 9 the integrated autocorrelation time only correlates strongly with flux in the SuperCLASS case where the sources are at a significantly higher signal to noise. For the SKA1 case, there’s a clear indication that the scalelength is easier to constrain for brighter sources, but interestingly there doesn’t seem to be a strong relationship between \( t_{\text{int}} \) and peak flux for the ellipticity parameters. This is not to say the uncer-
the Gelman-Rubin statistic as a function of peak flux of the source for all three parameters for the SKA1 100 sources. This indicates the chain is sufficiently converged and also that there may be a loose relationship between how easily a chain converges (i.e. a lower GR value) and peak flux. This relationship is further investigated using the integrated autocorrelation time in Fig. 8.

We also consider the sampling uncertainty, which is the uncertainty on the estimate of the mean of a parameter, due to having a finite number of samples in the posterior. Fig. 10 shows the plots for “SKA1 100”. As expected, the uncertainty decreases with increased flux, even though there is only a subtle relationship between the effective number of samples and flux as shown in Fig. 8. We also find that the sampling error on the ellipticity parameters only has a weak dependence on the actual value of the ellipticity. What is interesting, however, is the relationship between the ellipticity and the scalelength as shown in the middle two panels of Fig. 10. The sampling uncertainty of the scalelength increases as the ellipticity does, while the sampling uncertainty for the ellipticity is lower for larger scalelength. For highly elliptical sources, both \( e_1 \) and \( e_2 \) need to be determined precisely to get an accurate measure of \( \alpha_e \), whereas for more circular sources, this is less important. The middle right panel suggests that for larger sources, the ellipticity is simply easier to measure, as expected since these sources are both easier to resolve and, for a similar peak flux, have a higher integrated flux. The bottom panel indicates an increase in the maximum value of the sampling uncertainty for sources far from the phase centre, which is likely related to a decrease in sensitivity in the beam at the edges of the field of view.

Finally, within these interesting trends there is a great deal of scatter, implying that the properties of a source alone do not determine how quickly its parameters can be constrained. Because each source interacts with every other source due to the complicated beam pattern of the telescope, it’s likely that the exact position of a source relative to the others will also influence constraining power.

5.3 Computational cost

Table 3 shows, for all the tests presented in the paper, the number of samples taken before a chain is converged, that naturally increases as more sources are added to the problem. It also shows the average CPU time for computing a chain sample on a single core of Intel Xeon E5-2650 exploiting 2 NVIDIA Tesla K40 GPUs. Obviously the computational time increases both with the number of sources and visitibilities. Note that most of the computational time may be wasted in data transfer between CPU and GPU. Since GPU memory size is much smaller compared to the CPU one, large datasets as the ones produced by radio observations must be split in chunks to be sent through PCIe connection to the available GPUs. Montblanc implements this trying to overlap GPU computation with data transfer, however the current serial version of the code is not scalable with the number of GPUs used because of the many data chunks and transfer bandwidth bottleneck. The new generation of NVIDIA Tesla Pascal GPUs should overcome this issue by exploiting the new NVLink interconnection which maximizes the throughput of multi-GPUs and GPU/CPU system configurations through a larger bandwidth, more links and a better scalability. Moreover, a parallel version of the code is in preparation, implementing the Message Passing Interface paradigm (MPI), that will also allow the distribution of data and computation among more CPUs of hybrid multi-node architectures.

6 CONCLUSIONS

We have presented a new Bayesian method for measuring galaxy shape parameters from visibility data of radio observations. The method extends the BIRO technique by implementing a joint model fitting of the ellipticity and scale-length of all exponential sources in the field of view. It applies a Hamiltonian Monte Carlo technique, and can be easily extended for the measurement of other galaxy parameters such as position, flux and in band spectral index. Since we fit the model visibilities, it can also be extended for a simultaneous inference of scientific and instrumental parameters \(^{10}\) (Lochner et al. 2015).

We tested the method on simulated observations of up to 1000 galaxies adopting the SKA-MID Phase 1 UV coverage at 1.4 GHz. We were able to recover with a good accuracy the original galaxy shape parameters at the source density of the proposed SKA1 radio weak lensing survey \((2.7 \text{ gal/arcmin}^2)\) with \( \text{SNR}_{\text{vis}} \geq 10 \). As expected, the joint fitting approach improves measurements of galaxy ellipticities obtained with the RadioLensfit method at the same source density because it removes the neighbour bias introduced by the source extraction procedure. On the other hand, HMC has a long computation time, as shown in Table 4, which can be reduced with the new GPUs architectures and a higher level of code parallelisation exploiting a hybrid multi-node HPC system.

We also applied this method to the simulation of a

---

8 https://www.nvidia.com/en-us/data-center/nvlink/
9 http://www.mpi-forum.org/
fraction (100 arcmin$^2$) of SuperCLASS, a precursor radio weak lensing survey performed combining observations of e-MERLIN and JVLA radio telescopes (although again we caution not to directly compare results from the simplified SKA simulation and realistic SuperCLASS simulation). Since the assumed minimum 10$\sigma$ detection in the image domain corresponds to a SNR$_{vis} \geq 18$, we obtained a faster convergence of the chains and better fitting than with SKA1 experiments. In fact, the convergence analysis showed that the parameters of bright sources are easier to constrain. It also showed that ellipticity sampling uncertainty is strongly
Figure 10. The sampling uncertainty (that is the contribution to the error on a parameter measurement due to having a chain of finite independent samples) on each of the parameters as a function of different quantities. Top left: All parameters as a function of flux. Top right: The ellipticity parameters as a function of the amplitude of $e$. Middle left: The scalelength as a function of the amplitude of $e$. Middle right: The ellipticity parameters as a function of scalelength. Bottom left: The scalelength as a function of the distance of the source from phase centre. Bottom right: The ellipticity parameters as a function of the distance of the source from phase centre. In each plot, lines are added to guide the eye that indicate the mean (dashed line) and minimum/maximum (shaded envelope) in broad bins, interpolated with a cubic spline. All plots use the SKA1 100 sources chain, although the results are similar for the others. The conclusion from this analysis is that the easiest sources to constrain the parameters of tend to be sources that are bright and large. The scalelength is more precisely estimated for round sources, although the corresponding ellipticity uncertainty may be a bit larger. The maximum sampling uncertainty increases with distance from the phase centre, although there is significant scatter.
correlated with the source size and it is lower for large sources.

These results show that working with visibilities may provide a more accurate source characterization, and more reliable and complete source catalogues, potentially offering a novel measurement approach for future SKA surveys. In this case, more computing resources may be required in order to deal with the large volume of data that will be produced by the future surveys and the complexity of the data analysis algorithms.

**ACKNOWLEDGEMENTS**

We thank Simon Perkins for support with Montblanc, Isabella Prandoni for useful discussions about e-MERLIN and JVLA extended source detection. We also thank the SuperCLASS collaboration for access to and assistance with their simulations.

The financial assistance of the National Research Foundation (NRF) towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the authors and are not necessarily to be attributed to the NRF. MR acknowledges the support of the Science and Technology Facilities Council via an SKA grant. FBA acknowledges the support of the Royal Society via a Royal Society URF award.

**REFERENCES**

Becker R., White R., D.J.Helfand, 1995, ApJ, 450, 559
Betancourt M., 2017, preprint (arXiv:1701.02434)
Brown et al. M., 2015, in Advancing Astrophysics with the Square Kilometre Array, Vol. 2, Proc. Sci., SISSA, Trieste, p. 1365
Chang T., Refregier A., 2002, ApJ, 570, 447
Chang T., Refregier A., Helfand D., 2004, ApJ, 617, 794
Hamaker J. P., Bregman J. D., Sault R. J., 1996, A & A Suppl. Ser., 117, 137
Hanson K., 2001, Proc. SPIE, 4322, 456
Harrison I., Camera S., Zuntz J., Brown M., 2016, MNRAS, 463, 3674
Högbom J., 1974, A&A, 15, 417
Lochner M., Natarajan I., Zwart J., Smirnov O., Bassett B., Oozeer N., Kunz M., 2015, MNRAS, 450, 1308
Mandelbaum R., Rowe B., Armstrong R., Bard D., Bertin E., Bosch J., Boutigny D., Courbin F. et al, 2015, MNRAS, 450, 2963
Martí-Vidal I., Vlemmings W. H. T., Muller S., Casey S., 2014, A&A, 563, A136
McMullin J. P., Waters B., Schiebel D., Young W., Golap K., 2007, in Astronomical Society of the Pacific Conference Series, Vol. 376, Astronomical Data Analysis Software and Systems XVI, Shaw R. A., Hill F., Bell D. J., eds., p. 127
Melchior P., Böhnert A., Lombardi M., Bartelmann M., 2010, A&A, 510, A75
Miller L., Heymans C., Kitching T. D., van Waerbeke L., Erben T., Hildebrandt H., Hoekstra H., Mellier Y. et al, 2013, MNRAS, 429, 2858
Neal R. M., 2011, MCMC using Hamiltonian dynamics, Brooks S., Gelman A., Jones G., Meng X.-L., eds., Chapman & Hall/CRC Press
Owen F., Morrison G., 2008, AJ, 136, 1889
Patel P., Bacon D. J., Beswick R. J., Muxlow T. W. B., Hoyle B., 2010, MNRAS, 401, 2572
Patel P., Harrison I., Makhathini S., Abdalla F., Bacon D., Brown M., Heywood I., Jarvis M., Smirnov O., 2015, in Advancing Astrophysics with the Square Kilometre Array, Proc. Sci., SISSA, Trieste, p. 1427
Peck L. W., Fenech D. M., 2013, Astronomy and Computing, 2, 54
Perkins S., Marais P., Zwart J., Natarajan I., Tasse C., Smirnov O., 2015, A&C, 12, 73
Refregier A., 2003, MNRAS, 338, 35
Refregier A., Bacon D., 2003, MNRAS, 338, 48
Rivi M., Miller L., 2018, MNRAS (arXiv:1709.01827)
Rivi M., Miller L., Makhathini S., Abdalla F., 2016a, in EXTRA-RADSUR2015, Proceedings of Science, preprint (arXiv:1602.05836)
Rivi M., Miller L., Makhathini S., Abdalla F. B., 2016b, MNRAS, 463, 1881
Schwarz U., 1978, A&A, 65, 345
Smirnov O. M., 2011a, A&A, 527, A106
—, 2011b, A&A, 527, A107
Taylor J. F., Ashdown M. A. J., Hobson M. P., 2008, MNRAS, 389, 1284
Thompson A., Moran J., Swenson G., 1986, Interferometry and Synthesis in Radio Astronomy. John Wiley & Sons, New York
Tunbridge B., Harrison H., Brown M., 2016, MNRAS, 463, 3339
Wrobel J., Walker R., 1999, in ASP Conf. Ser., Vol. 180, Synthesis Imaging in Radio Astronomy II, Taylor G., Carilli C., Perley R., eds., Astron. Soc. Pac., San Francisco, p. 171
APPENDIX A: LIKELIHOOD GRADIENT

The Montblanc code returns the chi-squared value computed comparing data visibilities \( \widetilde{V}_i \) with the sky model visibilities \( V_i(x) \):

\[
\chi^2(x) = \sum_i \frac{|\widetilde{V}_i - V_i(x)|^2}{\sigma_i^2}. \quad (A1)
\]

From it the likelihood can be easily obtained as \( \mathcal{L}(x) \propto \exp \left[-\chi^2(x)/2\right] \). Similarly we can compute the likelihood gradient with respect to a set of parameters \( p_1, \ldots, p_M \) by adding to Montblanc the computation of the corresponding chi-squared partial derivatives:

\[
\frac{\partial \chi^2(x)}{\partial p_j} = -2 \sum_i \left( \frac{3\widetilde{V}_i - 3V_i(x)}{\sigma_i^2} \right) \frac{\partial V_i(x)}{\partial p_j} + \frac{\left( 3\widetilde{V}_i - 3V_i(x) \right)}{\sigma_i^2} \frac{\partial 3V_i(x)}{\partial p_j}, \quad (A2)
\]

where \( \Re \) and \( \Im \) denote the real and imaginary part of the complex visibilities. Adopting visibility formulation as in equation [13], galaxy shape parameters \( p_{s,k} \) are contained only in the brightness matrix, therefore

\[
\frac{\partial V_{ipq\lambda}}{\partial p_{s,k}} = K_{tpq\lambda} \frac{\partial B_{s\lambda}}{\partial p_{s,k}} K^{-H}_{tpq\lambda}. \quad (A3)
\]

From equation [14], we have the following partial derivatives of the model visibilities with respect to Sérsic shape parameters \( \alpha_s, e_s = (e_{1,s}, e_{2,s}) \) of each galaxy \( s = 1, \ldots, N \) in the field of view:

\[
\frac{\partial V_{ipq\lambda}}{\partial \alpha_s} = -\frac{12\pi^2 \alpha_s^2 |A_s^{-T}k|^2}{1 + 4\pi^2 \alpha_s^2 |A_s^{-T}k|^2} V_{ipq\lambda}, \quad (A4)
\]

\[
\frac{\partial V_{ipq\lambda}}{\partial e_{1,s}} = -\frac{12\pi^2 \alpha_s^2 f_1(e_s)}{1 + 4\pi^2 \alpha_s^2 |A_s^{-T}k|^2} V_{ipq\lambda}, \quad (A5)
\]

\[
\frac{\partial V_{ipq\lambda}}{\partial e_{2,s}} = -\frac{12\pi^2 \alpha_s^2 f_2(e_s)}{1 + 4\pi^2 \alpha_s^2 |A_s^{-T}k|^2} V_{ipq\lambda}, \quad (A6)
\]

with

\[
f_1(e_s) = \frac{[(1 + e_{1,s})u + e_{2,s}v]u}{(1 - e_{1,s}^2 - e_{2,s}^2)^2} + \frac{2e_{1,s}[(1 + e_{1,s})u + e_{2,s}v]^2}{(1 - e_{1,s}^2 - e_{2,s}^2)^3}
\]

\[
- \frac{[e_{2,s}u + (1 - e_{1,s})v]v}{(1 - e_{1,s}^2 - e_{2,s}^2)^2} + \frac{2e_{2,s}[(e_{2,s}u + (1 - e_{1,s})v)^2}{(1 - e_{1,s}^2 - e_{2,s}^2)^3}
\]

and

\[
f_2(e_s) = \frac{[(1 + e_{1,s})u + e_{2,s}v]u}{(1 - e_{1,s}^2 - e_{2,s}^2)^2} + \frac{2e_{2,s}[(1 + e_{1,s})u + e_{2,s}v]^2}{(1 - e_{1,s}^2 - e_{2,s}^2)^3}
\]

\[
+ \frac{[e_{2,s}u + (1 - e_{1,s})v]u}{(1 - e_{1,s}^2 - e_{2,s}^2)^2} + \frac{2e_{2,s}[(e_{2,s}u + (1 - e_{1,s})v)^2}{(1 - e_{1,s}^2 - e_{2,s}^2)^3}.
\]