The Energy Density of the Quaternionic Field as Dark Energy in the Universe

V. Majerník
Institute of Mathematics, Slovak Academy of Sciences,
Bratislava, Štefánikova 47, Slovak Republic
and
Department of Theoretical Physics
Palacký University
Tř. 17. listopadu 50
772 07 Olomouc, Czech Republic

Abstract
In this article we describe a model of the universe consisting of a mixture of the ordinary matter and a so-called cosmic quaternionic field. The basic idea here consists in an attempt to interpret $\Lambda$ as the energy density of the quaternionic field whose source is any form of energy including the proper energy density of this field. We set the energy density of this field to $\Lambda$ and show that the ratio of ordinary dark matter energy density assigned to $\Lambda$ is constant during the cosmic evolution. We investigate the interaction of the quaternionic field with the ordinary dark matter and show that this field exerts a force on the moving dark matter which might possible create the dark matter in the early universe. Such determined $\Lambda$ fulfils the requirements asked from the dark energy. In this model of the universe, the cosmical constant, the fine-tuning and the age problems might be solved. Finally, we sketch the evolution of the universe with the cosmic quaternionic field and show that the energy density of the cosmic quaternionic field might be a possible candidate for the dark energy.

1 Introduction
According to the astronomical observation, the evidence continues to mount that the expansion of the universe is accelerating rather than slowing down. New observation suggests a universe that is leigh-weight, is accelerating, and is flat [14] [12] [13]. To induce cosmic acceleration it is necessary to consider some components, whose equations of state are different from baryons, neutrinos, dark matter, or radiation considered in the standard cosmology.
As is well-known, in cosmology a new kind of energy is considered called *quintessence* ("dark energy"). Quintessence represents a dynamical form of energy with negative pressure \[ p_{Q} c^{-2} = w_{Q} \rho_{Q}, \quad -1 < w_{Q} < 0. \] (1)

For the vacuum energy (static cosmological constant), it holds \( w_{Q} = -1 \) and \( \dot{w}_{Q} = 0 \).

An adequate cosmological theory, conform with recent observation, should give answers to the following problems [39] [40]:

(i) The cosmological constant problem. The 'Λ-problem' can be expressed as discrepancies between the negligible value of Λ for the present universe and the value \( 10^{50} \) times larger expected by Glashow-Salam-Weinberg model [1] or by GUT [2] where it should be \( 10^{107} \) times larger.

(ii) The fine-tuning problem. It is a puzzle why the densities of dark matter and dark energy are nearly equal today when they scale so differently during the expansion of the universe. Assuming that the vacuum energy density is constant over time and the matter density decreases as the universe expands it appears that their ratio must be set to immense small value (\( \approx 10^{-120} \)) in the early universe in order for the two densities to nearly coincide today, some billions years later.

(iii) The age problem. This problem expresses the discrepancy connected, on the one side, with the hight estimates of the Hubble parameter and with the age of globular clusters on the other side. The fact that the age of the universe is smaller than the age of globular clusters is unacceptable.

(iv) The flatness problem. Inflation predicts a spatially flat universe. According to Einstein’s theory, the mean energy density determines the spatial curvature of the universe. For a flat universe, it must be equal to the critical energy. The observed energy density is about one-third of critical density. The discrepancy between the value of the observed energy density and the critical energy is called the flatness problem.

(v) Problem of the particle creation. In variable λ models the creation of particles generally takes place. The question what is the mechanism for this process represents the problem of the particle creation.

It is well-known that the Einstein field equations with a non-zero \( \lambda \) can be rearranged so that their right-hand sides consist of two terms: the stress-energy tensor of the ordinary matter and an additional tensor

\[
T_{ij}^{(\nu)} = \left( \frac{c^{4} \lambda}{8\pi G} \right) g_{ij} = \Lambda g_{ij}.
\] (2)

\( \Lambda \) is identified with vacuum energy because this quantity satisfies the requirements asked from Λ, i.e. (i) it should have the dimension of energy density, and (ii) it should be
invariant under Lorentz transformation. The second property is not satisfied for arbitrary systems, e.g. material systems and radiation. Gliner [4] has shown that the energy density of vacuum represents a scalar function of the four-dimensional space-time coordinates so that it satisfies both above requirements. This is why Λ is identified with the vacuum energy.

From what has been said so far it follows that the following properties are required from the vacuum energy density: (i) It should be intrinsically relativistic quantity having the dimension of the energy density. (ii) It should be smoothly distributed throughout the universe. (iii) It should cause the speedup of the universe. (iv) It should balances the total mean energy density to Ω = 1.

In the next Sections we will describe a model of the universe consisting of a mixture of the ordinary matter and a so-called cosmic quaternionic field. The basic idea in this article consists in an attempt to interpret Λ as the energy density of the quaternionic field whose source is any form of energy including the proper energy density of this field. The article is organized as follows. In Section 2 we describe the proposed quaternionic field. In Section 3 we set the energy density of this field to Λ and show that the ratio of ordinary dark matter energy density assigned to Λ is constant during the cosmic evolution. In Section 4 we show that this field exerts force on moving dark matter. In Sections 5 we describe the possible mechanism of the particle creation. We sketch the evolution of the universe with the cosmic quaternionic field. Finally, we show that such defined Λ fulfills the requirements asked from the dark energy and that in this model of the universe the above problems might be solved.

2 The cosmic quaternionic field

In a very recent article [11], Λ has been interpreted as the field energy of a quaternionic field (called Φ-field, for short) [5] [10] [9] (see also the Appendix). The field equation of the Φ-field can be written in the following form

$$\partial_i F_{ij} = J_j,$$

where $J_i = k \rho v_i$ ($v_i$ being the 4-velocity, $\rho$ matter density) is the current of the ordinary matter and $J_0 = k(\epsilon_{self} + \rho)v_0$. $\epsilon_{self}$ is the energy density of the Φ-field and $F_{ij}$ is the field tensor defined as

$$F_{ij} = \begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & \Phi & 0 & 0 \\ 0 & 0 & \Phi & 0 \\ 0 & 0 & 0 & -\Phi \end{pmatrix}.$$

$F_{ij}$ has only diagonal components $F_{ii} = \Phi$ $i = 1, 2, 3$, $F_{i0} = -\Phi$ $i = 0$, and $F_{ij} = 0$ $i \neq j$. These components are transformed under Lorentz transformation as follows
\[ F'_{11} = \frac{1}{1 - \beta^2} (F_{11} + \beta^2 F_{44}) = F_{11} = \Phi, \quad F'_{22} = F_{22} = \Phi \]

\[ F'_{33} = F_{33} = \Phi, \quad F'_{44} = \frac{1}{1 - \beta^2} (F_{44} + \beta^2 F_{11}) = -\Phi = F_{44}. \]

The field variable \( \Phi \) has the dimension of field strength and its square has the dimension of energy density. Two scalars can be formed from \( F_{ij} \): its trace \( F_i^i \) and \( F_{ij} F^{ij} \) the latter has the dimension of the energy density. In the differential form the field equations (3) are

\[ \nabla \Phi = k \vec{J} = k \rho v_i, \quad i = 1, 2, 3 \quad (4) \]

and

\[ -\frac{1}{c} \frac{d\Phi}{dt} = k_0 (\epsilon_{self} + \rho) v_i, \quad i = 0 \quad (5) \]

where \( \epsilon_{self} \) is the energy density of the \( \Phi \)-field given as [5]

\[ \epsilon_{self} = \Phi^2. \]

From the Newton gravitation law it follows that the gravitational "charge" of a point mass \( m_0 \) is \( \sqrt{G}m_0 \) [37] [23]. Accordingly, we set for the coupling constants \( k \) and \( k_0 \)

\[ k = \frac{\sqrt{G}}{c} \quad \text{and} \quad k_0 = \sqrt{G}. \]

These equations are first-order differential equations whose solution can be found given the source terms. Assuming the spacial homogeneity of the \( \Phi \)-field and the absence of any ordinary matter, i.e. \( J_1 = J_2 = J_3 = J_0 = 0 \), the field variable \( \Phi \) becomes independent of spatial coordinates. Here, the only source of the \( \Phi \)-field is its own energy density, i.e. \( \epsilon_{self} = \Phi^2 \). Therefore, Eqs.(4) and (5) become

\[ \nabla \Phi = 0 \quad (6) \]

\[ -\frac{1}{c} \frac{d\Phi}{dt} = \frac{\sqrt{G} \Phi^2}{c^2} = \frac{\sqrt{G} \Phi^2}{c^2}. \quad (7) \]

The solution differential equation (7) has a simple form

\[ \Phi(t) = \frac{c}{\sqrt{G}(t + t_0)}, \quad (8) \]

where \( t_0 \) is the integration constant given by the boundary condition. The energy density \( \epsilon_{self} \) is approximately equal to the observed value of the contemporary cosmological constant.
3 The dark energy modeled by a time-dependent cosmological constant

The theory of the time-dependent cosmological constant in the Friedmann model is well established (see, e.g., [16]). The time-dependent cosmological models in the framework of scalar field theory were first discussed by P. J. E. Peeble and B. Ratra [20], B. Ratra and P. J. E. Peebles ([17]) and M. Özer and M. O. Taha [18]. A number of authors set phenomenologically $\Lambda \propto 1/t^2$ [22-29] (for a review see [21]). Generally, $\Lambda$ contains in its definition the gravitation constant $G$ and velocity of light $c$. From the purely phenomenological point of view the simplest expression for $\Lambda \propto 1/t^2$, having the right dimension, and containing $G$ and $c$ is

$$\Lambda = \frac{\kappa^2 c^2}{8\pi G t^2},$$

where $\kappa$ is a dimensionless constant. Because $\Lambda$ is set equal to the field energy density of the cosmic quaternionic field we have ($t_0 = c = 1$)

$$\Lambda = \frac{1}{8\pi} \left[ \frac{1}{\sqrt{Gt}} \frac{1}{\sqrt{Gt}} \right] = \frac{\Phi^2}{8\pi} \quad \Phi = \frac{1}{\sqrt{Gt}},$$

(9)

Accordingly, we have

$$\Lambda = \frac{\Phi^2}{8\pi} = \frac{1}{8\pi G t^2} \quad \text{and} \quad \lambda = \frac{1}{t^2}.$$  

(10)

The gravitational field equations with a cosmological constant $\lambda$ and the energy conservation law are ($k=0$)

$$H^2 = \frac{8\pi G}{3} (\rho + \Lambda) \quad H = \frac{\dot{R}}{R} \quad \Lambda = \frac{\lambda}{8\pi G}$$

(11)

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} (\rho + 3p + 2\Lambda)$$

(12)

and

$$\dot{\rho} + 3\frac{\dot{R}}{R} (p + \rho) = -\dot{\Lambda}.$$  

(13)

Suppose we have a perfect-gas equation of state

$$p = \alpha \rho$$

(14)

and suppose that the deceleration parameter is constant. If the evolution of the scale factor is given in form $R \propto t^n$ then $q = -(n - 1)/n$, therefore, we set

$$q = -\frac{\dddot{R}}{R^2} = \frac{1}{n} - 1.$$  

(15)
If we suppose the time dependence \( \rho \) and \( \Lambda \) in form
\[
\rho = \frac{A}{t^2} \quad \text{and} \quad \Lambda = \frac{B}{t^2} \quad B = \text{const.} \quad \text{and} \quad A = \text{const.,} \quad (16)
\]
respectively, then, inserting (16) into (12),(13) and (14), gives the following relation between \( A \) and \( B \) [7]
\[
2B = A[(-2 + 3n)(1 + \alpha)]. \quad (17)
\]
Given \( A \) or \( B \) and \( n \) we can uniquely determine \( B \) or \( A \), respectively. For \( \lambda \propto 1/t^2 \), there is a relation between \( \Omega_M \) and the time-dependence of scaling factor \( R(t) \). Assuming that \( \Omega_M \) does not change during the matter-dominated era \( (\alpha = 0) \) [8]
\[
R(t) = \left( \frac{3}{2} \right)^{2/3\Omega_M} (\Omega_M C_1 t)^{2\Omega_M}. \quad (18)
\]
The quantities \( q, R(t) \) and \( \Omega_M \) are mutually related. Given one of them the remained quantities can be determined by means of Eqs. (18), (16) and (15). It seems that \( \Omega_M \) is best determined by the observation, therefore, we take it for the calculation of \( q \) and \( R(t) \). Inserting \( \Omega_M = 1/3 \) into Eq.(18) we obtain \( R \propto t^2 \) which yields \( q = -1/2 = 1/n - 1 \), \( n = 2 \). We see from Eq.(9) that \( B = 1 \) which inserting into Eq.(17) gives \( A = (1/2) \).
The mean energy density \( \rho \) and the cosmological constant is given as \( (\alpha = 0) \)
\[
\rho = \frac{1}{16\pi t^2} \quad \text{and} \quad \lambda = \frac{1}{8\pi G t^2}, \quad (19)
\]
respectively. Their ratio
\[
\frac{\rho}{\lambda} = 1/2. \quad (20)
\]
Supposing the flat space, we have
\[
\Omega_M = \frac{1}{3} \quad \text{and} \quad \Omega_\Lambda = \frac{2}{3}. \quad (21)
\]
There is no “fine tuning” problem in our model since the ratio of the \( \lambda \)-part energy density to the mass-energy density of the ordinary matter remains during the cosmic evolution constant.

4 The force exerting on the moving bodies in the cosmic quaternionic field

When the ordinary matter is present in the \( \Phi \)-field then we have to apply the general field equations (3) and (5) The ordinary matter changes the value of the field variable \( \Phi \) in comparison to the pure \( \Phi \)-field. Simultaneously, \( \Phi \)-field exerts a force \( f_1 \) on the moving matter. This Lorentz-like force is given, similarly as in electrodynamics, by the expression
\[
f_j = \frac{J_i}{c} F_{ij}. \quad (22)
\]
When calculating this Lorentz-like force we take, for the sake of simplicity, only the pure $\Phi$-field, i.e. that $\Phi$-field in which no ordinary matter is present. $J_i^{(m)} = \frac{\sqrt{G}}{c} m_0 v_i$. Inserting $J_i^{(m)}$ in Eq. (22) we find for the Lorentz-like force acting on the moving mass body in the presence of the $\Phi$-field the following formula [15].

$$F_i = c^{-1} \sqrt{G} m_0 \Phi v_i = c^{-1} \sqrt{G} \Phi p_i. \quad (23)$$

This is the fundamental formula which we apply in our further consideration.

It is to be expected that the cosmic $\Phi$-field manifests itself in the present-day only in the following astrophysical situations: (i) at the large mass concentrations, (ii) at the large velocities of massive objects (iii) during the large time and space scales. For the sake of simplicity, we confine ourselves to the non-relativistic case, i.e. we suppose that $m = \text{const.}$ and $v \ll c$. Then Eq.(6) turns out to be

$$m \dot{v} = c^{-1} \sqrt{G} \Phi m v. \quad (24)$$

Below, we present three possible effects of the quaternionic field in the cosmic conditions:

(i) The increase of the velocity of the moving bodies in the $\Phi$-field. Since $c^{-1} \sqrt{G} \Phi = 1/t$ we get a simple differential equation $\dot{v} = \beta v$ where $\beta = 1/t$ the solution of which is $v = Ct$. A free moving object in the quaternionic field is accelerated by a constant acceleration $C$. This acceleration is due to the immense smallness of $\beta \approx 1/10^{18}$ in the present-day extremely small. As is well-known for a given time instant the Hubble constant $H$ is equal in the whole Universe. Supposing $\beta|_0 = H$, the solution of equation $\dot{v} = Hv$ becomes $v = Hr + C$, where $C$ is an integration constant. Setting $C = 0$ we get a Hubble-like law $v = Hr$.

(ii) The increase of the kinetic energy of the moving bodies in the $\Phi$-field. The gain of kinetic energy of a moving body per time unit in the quaternionic field is $(f_i \parallel v_i)$

$$\frac{dE}{dt} = F_i v_i = c^{-1} \sqrt{G} \beta m v^2 = 2 \sqrt{G} c^{-1} \Phi E_{\text{kin}} = 2 \beta E_{\text{kin}}. \quad (25)$$

Again, the increase of the kinetic energy of a moving object is extremely small. However, for a rapid rotating dense body it may represent a considerable value. For example, a pulsar rotating around its axis with the angular velocity $\omega$ having the moment of insertion $I$. Its kinetic energy is $E_{\text{kin}} \approx I \omega^2$ and its change in the quaternionic field is $dE_{\text{kin}}/dt \approx \beta I \omega^2 \approx 10^{32}$ which is a value only of some orders of magnitude smaller than the energy output of a pulsar [36].

(iii) The change of the kinetic parameters of the gravitationally bounded moving bodies. This can be best demonstrated by describing the motion of the Earth around the Sun taking into account Eq.(13). It holds

$$\vec{F}_1 + \vec{F}_2 = -\frac{GM_\odot m_{\oplus}}{r^2} \vec{r}, \quad (26)$$
where \( \vec{F}_1 = m_{\odot} \vec{r} \) and \( \vec{F}_2 = m_{\odot} \beta \dot{r} \). Inserting \( \vec{F}_1 \) and \( \vec{F}_2 \) in Eq.(26) we have

\[
\frac{d}{dt}(\beta \dot{r}) = -\beta \frac{GM_{\odot}}{r^2} \vec{r}
\]

from which it follows

\[
\beta r^2 \dot{\phi} = \text{const.} = h \tag{27}
\]

The reciprocal radius \( u \) satisfies the equation

\[
\frac{d^2 u}{d\phi} + u = \beta^2 \frac{GM_{\odot}}{h^2},
\]

the general solution of which is

\[ u(\phi) = \frac{C}{r^2} + c_2 \cos(\phi) - c_1 \sin(\phi) \]

Setting \( c_1 = c_2 = 0 \), i.e. supposing that the orbit is circle we get

\[ r \sim K t^2 \]

Hence, the distance of the Earth and the Sun varies with time like

\[ r \sim \frac{1}{GM_{\odot} \beta^2} \sim \frac{1}{\beta^2} \sim t^2. \tag{28} \]

According to Eq.(28) the distance between the Earth and the Sun is increasing direct proportional to the square of time. There have been several atomic time measurements of the period of the Moon orbiting around the Earth. A description of the work of some independent research groups can be found in Van Flander’s article [34]. We simply point that after subtracting the gravitational perturbative (tidal) effects, Van Flanders gives

\[ \frac{\dot{P}}{P} = \frac{\dot{n}}{n} = (3.2 \pm 1) \times 10^{-10}/\text{yr}, \]

where \( n = 2\pi/P \) is the angular velocity. Using Eq.(11) we get comparable value

\[ |\dot{n}/n| \approx 4 \times 10^{10}/\text{yr}. \]

However, given the complexity of the data analysis, we must certainly await further confirmation by different, independent test before concluding whether the \( \Phi \)-field really affects the motion of cosmic bodies. Nevertheless, it can be asserted that at present, there exists no evidence against the influence of the \( \Phi \)-field on the moving bodies at the level of the supposed present-day intensity of the \( \Phi \)-field. Due to large value of the cosmic time the present-day effects of the \( \Phi \)-field lie on the limit of the observability. However, they had, probably, strong influence in the early universe. For example, the strong \( \Phi \)-field can destabilize the large rotating mass concentration (e.g. quasars) forming from
them the present-day galaxies. We note that the enlargement of distance of the Earth and the Sun is also suggested by the large numbers hypothesis presented by Dirac in 1937. In this hypothesis Dirac supposed that $G \propto 1/t$. The astrophysical and geological consequences of this hypothesis are discussed in details in [35][33].

It is noteworthy that the force exerting on cosmical body in cosmic quaternionic field is always parallel to the direction of the velocity. This means that velocity of moving bodies in cosmic $\Phi$-field is in all direction increasing. For example, the moving bodies stemming from a exploding cosmic body is equally speedup as those falling in the collapsing center.

5 The creation of particle in the $\Phi$-field

Another interesting effect of the cosmic $\Phi$-field is the possibility of the creation of real particles from the virtual ones. Particle creation in nonstationary strong fields is well-known phenomenon studied intensively in seventies (see, e.g. [32]). There are several proposed ways for the creation of real particles from the virtual ones in the very strong and nonstationary gravitation field. We propose here a new mechanism of the creation of real particles from the virtual ones in the presence of the $\Phi$-field.

We note that our further consideration on the creation of matter from the vacuum quantum excitation are done by a semiclassical way, although we realize that they should be performed in terms of an adequate theory of quantum gravity. However, as is well-known, when constructing a quantum theory of gravity one meets conceptual and technical problems. The usual concepts of field quantization cannot be simple applied to gravity because standard field quantization (e.g., elm field) is normally done in flat spacetime. It is impossible to separate the field equations and the background curved space because the field equations determine the curvature of spacetime. Moreover, the classical quantized field equations are linear and that of gravitation are non-linear and weak-field linearized gravitation field is not renormalizable. There is even no exact criterion on which time and space scales one has necessary to apply quantum laws for gravitation field. Therefore, we take the inequality $|A| \leq \hbar$, where $A$ is the classical action, as a criterion for a possible application of quantum physics in gravity.

According to quantum theory, the vacuum contains many virtual particle-anti-particle pairs whose lifetime $\Delta t$ is bounded by the uncertainty relation $\Delta E \Delta t > \hbar$ The proposed mechanism for the particle creation in the $\Phi$-field is based on the force relation (23). During the lifetime of the virtual particles the Lorentz-like force (23) acts on them and so they gain energy. To estimate this energy we use simple heuristic arguments. As is well-known, any virtual particle can only exist within limited lifetime and its kinetics is bounded to the uncertainty relation $\Delta p \Delta x > \hbar$. Therefore, the momentum of a virtual
particle $p$ is approximately given as $p \approx h \Delta x^{-1}$. If we insert this momentum into Eq.(23) and multiply it by $\Delta x$, then the energy of virtual particle $\Delta E$, gained from the ambient $\Phi$-field during its lifetime, is

$$F \Delta x = \Delta E = \sqrt{G \Phi(t)} \frac{h}{c}.$$  

When the $\Phi$-field is sufficiently strong then it can supply enough energy to the virtual particles during their lifetime and so spontaneously create real particles from the virtual pairs. The energy necessary for a particle to be created is equal to $m_v c^2$ ($m_v$ is the rest mass of the real particle). At least, this energy must be supplied from the ambient $\Phi$-field to a virtual particle during its lifetime. Inserting $\Phi$ into Eq. (29), we have

$$\Delta E \approx \frac{h}{(t + t_0)}.$$  

Two cases may occur: (i) If $m_v c^2 < \Delta E$, then the energy supplied from the $\Phi$-field is sufficient for creating real particles of mass $m_v$ and, eventually, gives them an additional kinetic energy. (ii) If $m_v c^2 > c^2 \Delta E$, then the supplied energy is not sufficient for creating the real particles of mass $m_v$ but only the energy excitations in vacuum. The additional kinetic energy of the created particles, when $\Delta E > m_v c^2$, is

$$E_{\text{kin}} = \Delta E - m_v c^2 = \frac{h}{(t + t_0)} - m_v c^2.$$  

During the time interval ($\approx 0, 10^{-20}$) after the Big Bang, the masses of the created particles lie in the range from $10^{-5}$ to $10^{-27}$ g. Their kinetic energy was $E_{\text{kin}} = \frac{h}{(t + t_0)} - m_0 c^2$. reached values up to $10^{-5}$ erg, which corresponds to the temperature of $10^{21}$ K. Today, energies of the virtual pairs, gained during their lifetime, are negligible small.

## 6 The energy density of the cosmic $\Phi$-field as a possible candidate for the dark energy

As has been shown in the previous Chapters, when taking the field energy density of cosmic quaternionic field as the vacuum energy density, the problems presented in the Introduction may be resolved. The problem of the cosmological constant, because the value of $\Lambda$ is consistent with data, the tuning and age problems because the ratio of mass to vacuum energy density does not vary during the cosmical evolution and the age of the universe is large enough to evolve the globular cluster. The flatness problem is also solved because the sum of the dark energy to the ordinary matter yields just the critical energy density. Moreover, in the cosmic quaternionic field, there exists a plausible mechanism of the particle creation. We conclude that the energy density of the $\Phi$-field represent a relativistic quantity satisfying Gliner’s requirements which is smoothly distributed in
space. It causes the speedup of the universe, balances the total energy density to the critical one and gives a plausible mechanism for the particle creation.

It is generally believed that dark energy was less important in the past and will become more important in the future. In our model the value of dark energy is proportional to $1/t^2$, therefore, its value becomes very large in past and will be adequate large in the future. We remember that the force exerted by the cosmic $\Phi$-field on moving bodies acts always in the direction of the velocity. That means that the high value of the dark energy in the early universe does not interfere with the structure forming, contrarily, it accelerates it.

The evolution of the universe with the cosmic quaternionic field can be briefly sketched as follows: The cosmic evolution started purely field-dominated era with the inflation, after which a massive creation of particles began together with enormous release of entropy. The masses of the created particles reaches values up to $-5$ g and the kinetic energy of the created nucleons values up to $10^{-5}$ erg, which corresponds to the temperature of $10^{21}$ K. The large vacuum energy density of the cosmic quaternionic field at the early stage of the universe accelerates its structure formulation. From what has been said above we conclude that the energy density of the cosmic quaternionic field might be a possible candidate for the dark energy because (i) it has the value consistent with data (ii) it does not suffer from the cosmological constant, fine-tuning, age and flatness problems (iii) it yields a plausible mechanism particle production and (iv) it accelerated the structure formulation in the early universe.

Motivated by the desire to find a possible candidate of the dark energy among the family of quaternionic fields we found the surprisingly simple quaternionic field whose energy density might be considered as the dark energy. (This points out that also the classical field may be interested by study of the quintessence [41]. The energy density of this field has the desired properties of dark energy and changes generally our view of the vacuum energy density modelled by the cosmological constant. It could not be seen as the carrier of repulsive gravity but as an amplifier of velocity of the moving bodies independently of the direction of their motion. The fact that the vacuum density accelerates the expanding galaxies is cause do to fact that they move from the center of the universe. When the galaxies would collapse the vacuum energy would accelerate, likewise, their collapsing. This may eventually lead to change the basic equation of the Friedmann cosmology. As well-know there are many versions of Mach’s principle in the literature and a unique, satisfactory formulation of this principle does not seem to exist as yet. It seem that the influence of the moving bodies changes the value the field variable $\Phi$ which for its part affects other cosmic bodies might be seen as a form of Mach’s principle.
APPENDIX

The quaternionic field equations can be described by a quaternionic equation consisting of the quaternionic differential operator, the field and source quaternions [5].

(i) The quaternionic differential operator is the quaternion (i, j, k are the quaternionic units obeying the following relations $ij = -ji$, $ik = -ki$, $jk = -kj$ and $i^2 = j^2 = k^2 = -1$) [5] [9]

$$\Box = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} + \frac{s}{c} \frac{\partial}{\partial t}, \quad s = \sqrt{-1}. \quad (A1)$$

(ii) The field quaternion the components of which are the field variables is

$$\Phi = i \Psi_1 + j \Psi_2 + k \Psi_3 + \Psi_4, \quad (A2)$$

where

$$\Psi_1 = (\Phi_1 + s\Phi_1') \quad \Psi_2 = (\Phi_2 + s\Phi_2') \quad \Psi_3 = (\Phi_3 + s\Phi_3') \quad \Psi_4 = (\Phi_4 + s\Phi_4').$$

(iii) The source quaternion

$$J = \frac{4\pi}{c} [iJ_1 + jJ_2 + kJ_3 + J_4], \quad (A3)$$

where

$$J_1 = (J'_x - sJ_x) \quad J_2 = (J'_y - sJ_y) \quad J_3 = (J'_z - sJ_z) \quad J_4 = j_0 + sj_0'.$$

The general quaternionic field is described by the field equation of the type (see, e.g. [10])

$$\Box \Phi = J \quad (A4)$$

The field quaternion $\Phi$ consists of the vector part $\vec{\Phi}_1 = \vec{\Phi}_1 + s\vec{\Phi}_2$, where $\vec{\Phi}_1 = (\Phi_1, \Phi_2, \Phi_3)$, $\vec{\Phi}_2 = (\Phi_1, \Phi_2, \Phi_3)$ and the scalar part $\Psi_4 = (\Phi_4 + s\Phi_4)$.

The source quaternion consists likewise of the vector part $\vec{J} = \vec{J}_1 - s\vec{J}_2$, where $\vec{J}_1 = (J'_x, J'_z, J'_z)$ and $\vec{J}_2 = (J_x, J_y, J_z)$ and the scalar part $J_4 = 4\pi(j_0 + sj_0')$.

The energy density of a quaternionic field is [5]

$$E = \sum_{k=1}^{4} \Phi_k \Phi_k^*. \quad (A5)$$

By using the vector notation we can rewrite the quaternionic field equations (A4) in the form

$$\frac{s\partial \Psi_1}{c\partial t} + \nabla \cdot \vec{\Psi} = 4\pi J_4$$

$$\frac{s\partial \vec{\Psi}}{c\partial t} - \nabla \Psi_4 + \nabla \times \Psi = \frac{4\pi}{c} \vec{J}. \quad (A6)$$
According to the specification of the field variables and source components in the field and source quaternion, respectively, we get the following fields:

(i) If we choose $\Phi = -\vec{E}$, $\Phi' = -\vec{B}$, $\vec{J} = -\vec{J}$, $j_0 = -\varrho$, $\Phi_4 = \Phi_4' = J = j'_0 = 0$ and we associate $J, j_0$ with the components of electromagnetic 4-current, we get just the standard Maxwell equations [30] [31].

(ii) If we associate, in addition, $J'$ and $j'$ with the components of the monopole 4-current then we get the Maxwell equations with monopoles [29].

(v) If we associate the components of the field quaternion with the field variables as in (i) and take $J, j_0$ as electric and $J', j'_0$ as monopole currents, $\Phi_4$ and $\Phi_4'$ as the scalar and the pseudoscalar variables, respectively, then we obtain the Ohmura field equations [18].

(vi) If we put $\vec{\Phi} = \vec{\Phi}', \Phi_4 = \Phi_4' = \vec{J} = j'_0 = 0$, and take $J^*, j^*$ as not specified 4-current, we get the scalar quaternionic field equations.

We note that all of these field equations can be written also in form of tensor equation by means of the corresponding field tensors [5]. The scalar quaternionic field represents the $\Phi$-field, whose field equations in vector form we consider in Chapter 2.

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