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Beam Damage Assessment Using Natural Frequency Shift and Machine Learning

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Abstract: Damage detection based on modal parameter changes has become popular in the last few decades. Nowadays, there are robust and reliable mathematical relations available to predict natural frequency changes if damage parameters are known. Using these relations, it is possible to create databases containing a large variety of damage scenarios. Damage can be thus assessed by applying an inverse method. The problem is the complexity of the database, especially for structures with more cracks. In this paper, we propose two machine learning methods, namely the random forest (RF), and the artificial neural network (ANN), as search tools. The databases we developed contain damage scenarios for a prismatic cantilever beam with one crack and ideal and non-ideal boundary conditions. The crack assessment was made in two steps. First, a coarse damage location was found from the networks trained for scenarios comprising the whole beam. Afterwards, the assessment was made involving a particular network trained for the segment of the beam on which the crack was previously found. Using the two machine learning methods, we succeeded in estimating the crack location and severity with high accuracy for both simulation and laboratory experiments. Regarding the location of the crack, which was the main goal of the practitioners, the errors were less than 0.6%. Based on these achievements, we concluded that the damage assessment we propose, in conjunction with the machine learning methods, is robust and reliable.

Keywords: damage detection; linear regression; random forest; artificial neural network; training parameters; natural frequency

1. Introduction

Nondestructive damage detection methods have received increasing attention in recent decades and have become a central research topic for scholars and practitioners belonging to the structural health monitoring community. The principles of vibration-based techniques, which are nowadays very popular, can be found in [1–5]. These methods are based on the deterministic relation between the damage characteristics (mainly location and severity), and changes in the modal parameters (natural frequencies, mode shapes, and curvatures). Among the modal parameters, the natural frequencies are the easiest to determine and require the involvement of relatively cheap and very robust instrumentation. Moreover, estimating the natural frequencies requires a limited number of sensors compared to methods based on mode shapes [6]. For this reason, in this paper, we focus on the analysis of natural frequencies to detect damage.
Temperature changes can affect the natural frequencies of structures. For accurate damage detection, it is important to remove the effects of temperature on the natural frequencies. An algorithm with this aim was presented in [7], where a successful damage assessment was performed for the case of changing temperatures. Mass changes also affect the natural frequencies of structures. In [8], an analytical approach was developed for a mass-spring-damper system that demonstrates how to localize mass change.

Detection and measurement of damage features for different crack types in slender beams using modal analysis was studied in [9]. Assessment of L and T-shaped cracks as well as delamination in bi-metal structures using natural frequencies was successfully performed in [10]. An assessment of corrosion and the analysis of the structural capacity of corroded girders that belong to steel bridges was studied in [11].

In some real cases, multiple damages can occur at once, and the structures can have multiple supports. A complex study devoted to calculating the natural frequencies and mode shapes of multi-span beams is found in [12], and in [13], a method to assess cracks in continuous beams was presented. A study devoted to identifying multiple damages of multi-span bridges based on influence lines was presented in [14].

The complexity of damage (the crack shape and the orientation, the associated loss of mass, the number of cracks), as well as the complexity of the structures (multi-supported beams, skeletal structures, the use of nonisotropic materials), increase the dimension of the dataset to be processed for damage detection. However, for all cases mentioned above, we determined reliable relations to also consider the different configurations of the structure (multi-span beams or frames), the temperature effects, and the shape of the crack (L or T shape, delamination). This makes the damage detection methods able to take into consideration particular cases, which usually occur in real applications, but makes the recognition of patterns more difficult. For this reason, researchers who develop methods for detecting damages by employing vibration-based methods [15] are increasingly using artificial intelligence (AI) to analyze large amounts of data. AI has now reached a level where it can recognize the characteristics of objects as well as alterations of structures by considering different types of data and can automatically learn the needed criteria [16]. A study of the use of SHM methods is presented in [17,18] in which the authors presented an image processing model capable of monitoring microseismic events [17] or pavement monitoring [18] both of which achieved great strides in enabling the widespread use of artificial intelligence solutions, although many of these continue to face challenges related to large volumes of graphics data. Another approach is the use of vibration-based damage detection methods, which are global and do not require access to the damaged area.

The use of Random Forests (RF) and data fusion for structural damage detection are proposed in [19]. More recently, an RF model was used to predict the location of potential damage on asphalt pavement [20]. Here, RF data mining was used to analyze the interrelationships of variables. Another approach is found in [21]. The damage-sensitive features were extracted from raw sensor data using the cross-correlation function and wavelet packet decomposition. RF and other ensemble learning algorithms such as XGBoost are used to train the damage pattern classifier.

The artificial neural network (ANN) is a commonly applied technique for SHM. A method to identify the damage location and its severity in a ten-floor structure, employing an auto-associative neural network combined with transmissibility is proposed in [22]. The development of a method to detect damages in a truss structure using an ANN is the subject of the research presented in [23]. A method to predict crack width for thick, as well as for thin, concrete elements based on the feed-forward backpropagation and radial basis neural networks is proposed in [24]. Regarding the use of neural networks for condition monitoring, we mention a technique involving a new convolutional neural network (NCNN) that was proposed and successfully applied to detect bearing faults [25].

Despite successful assessment of damage, it is worth mentioning that the damage considered in most studies presented in the literature either manifests on an element with significant extent of damage along the beam [22] or has significant severity: 22% to 68%
in [26] and 20% to 80% in [27]. For skeletal structures, the damage manifests on one or more structural elements, see, for example, [19–21,23].

In prior research, we deduced a relation to calculate the natural frequencies of beams with known crack parameters. This relation is applicable also for beams with non-ideal boundary conditions. Being an analytical relation, it permits creating, easily and rapidly, a database with patterns for a multitude of damage scenarios. The database contains the relative frequency shifts (RFS) for eight out-of-plane vibration modes for all given damage locations, severities, and fixing conditions. It was used to train RF and ANN with a huge amount of data. The training was first performed for scenarios covering all locations along the beam and afterwards for locations on a specific segment. The proposed damage detection methodology presumes an initial/coarse assessment to find the crack location, followed by a second/fine assessment targeting the accurate crack location and severity.

To evaluate the effectiveness of the proposed approach, two examples, comprising numerical simulations and laboratory experiments on steel beams were performed. The results obtained by involving RF and ANN were compared; both methods led to the correct location and quantification of the damage, regardless of the changes in the fixing conditions. It was possible to detect cracks with a much smaller depth than those reported in the literature because a large amount of the calculated, and consequently noiseless, data was used for training. To the best of our knowledge, there are no artificial intelligence-supported crack detection methods that apply to beams with non-ideal boundary conditions or that can detect the weakening of the clamping.

The paper is organized as follows. After an introductory section, we present the theoretical background which permits the development of a database consisting of an INPUT and a TARGET part. The methodology of RF and ANN, along with the training process based on the database are introduced in Section 3. In the next two sections, we prove the efficiency of the proposed approach. In Section 4 we describe the numerical simulations performed on a steel cantilever beam, done to find the natural frequency shifts due to several damage cases. Then, we apply the two machine learning methods. In Section 5 we repeat the approach for laboratory-scale beams. Eventually, conclusions are formulated.

2. Creating the Database for Machine Learning (ML)

As a crack propagates in a structure, it produces an alteration of structural stiffness. This alteration produces a change in modal parameters, of which the most obvious and easy to measure are changes in natural frequencies. There is a deterministic relationship between stiffness change and the decrease in the natural frequencies of the structure; therefore, the position and depth of the crack can be identified if the changes in the frequencies produced by the defect are known. We proposed a robust and easy-to-use mathematical relation to predicting the natural frequency $f_{i-U}(x, a)$ of a cracked beam [28]. This relation relies on the features of the healthy beam, namely, the natural frequencies $f_{i-U}$ and the modal curvatures $\theta(x)$ of the out-of-plane vibration modes, and the damage severity $\gamma(a)$, respectively. The mathematical relation, following [28], is

$$f_{i-D}(x, a) = f_{i-U} \left[1 - \gamma(a) \left[\theta(x)\right]^2\right]. \quad (1)$$

In this relation, we denoted the crack position with $x$, the crack depth with $a$, and $i$ stays for the mode number. From Equation (1) we can deduce the relative frequency shift (RFS), which is the normalized frequency drop due to a crack, as

$$\Delta f_{i-D}(x, a) = \frac{f_{i-U} - f_{i-D}(x, a)}{f_{i-U}} = \gamma(a) \left[\theta(x)\right]^2. \quad (2)$$

We find the two terms in the right part of Equation (2) as follows. The crack severity is calculated involving an energy method, which says that a beam with a crack will be able
to store less energy than a similar healthy beam and will suffer a greater deformation if subjected to a force \([29]\). So, we can calculate the severity involving the mathematical relation proposed in \([30]\), that is

\[
\gamma(a) = \sqrt{\delta_D(a) - \delta_U}.
\]  

(3)

In this relation, we denoted \(\delta_U\) the deflection of the healthy beam, and with \(\delta_D(a)\) the deflection of the beam with a crack that has depth \(a\). Note that the severity is calculated for the crack located at the position where the biggest curvature (or bending moment, which is proportional to the curvature) is achieved. In consequence, the position of the damage for which the severity is calculated differs depending on the boundary conditions of the beam; for example, for the cantilever beam that is the subject of the study presented hereinafter this position is the fixed end. On the other hand, the severity depends solely on the crack depth \(a\), so that for a given depth it is the same regardless of the boundary conditions. The relationship between the crack depth and the severity can be found before starting the damage detection procedure. When applying the damage detection method, we determine the damage severity and afterward determine the damage depth from the depth-severity relationship known a priori. So, knowing the deflection of the monitored structure is not necessary.

The effect of the crack position is controlled by the local curvature (or bending moment). This happens because the stress present in the affected slice is proportional to the bending moment. In consequence, a smaller effect on the frequency drop is obtained if the crack is not located at the particular position where the biggest bending moment is present. There are even situations in which the crack does not cause a decrease in frequency, namely when it is located at an inflection point \([31]\). To consider the effect of the crack position \(x\) on the frequency drop, we use the normalized squared curvature. By normalization, we assign value one to the curvature or bending moment for the location where the beam is the most requested and a subunit value for the rest of the positions. For the cantilever beam, the normalized curvature is

\[
\phi_i'(x) = 0.5 \left( \cos(\lambda_i x) + \cosh(\lambda_i x) - \frac{\cos \lambda_i + \cosh \lambda_i}{\sin \lambda_i + \sinh \lambda_i} \left[ \sin(\lambda_i x) + \sinh(\lambda_i x) \right] \right).
\]  

(4)

where we denoted with \(\lambda_i\) the eigenvalue of the \(i\)-th vibration mode. These values are indicated in the literature for different boundary conditions of the beam. Note that, other boundary conditions also lead to other curvature functions.

Taking into account the above, we can summarize that:

- The severity \(\gamma(a)\) depends on the crack depth \(a\) and is independent of the crack position \(x\), boundary conditions, and vibration mode number \(i\). Therefore, the severity for a beam with a given crack once calculated using Equation (3) is valid for that beam irrespective of the boundary conditions. In practice, we calculate the severity using data obtained from static finite element analysis for a cantilever beam because it presents an important deflection at the free end. A comprehensive description of the procedure to determine the correct severity is given in \([32]\);

- The value of the normalized modal curvature \(\phi_i'(x)\) at the position \(x\) where the crack is located reduces the effect of the severity since at that position less stress is stored in the beam. This term depends on the vibration mode number \(i\) and the boundary conditions. Therefore, Equation (2) has a large degree of generality; it can be properly used for any support type if the correct curvature function \(\phi_i'(x)\) is employed. Hereinafter we exemplify the case of a cantilever, thus Equation (4) is used.
In Figure 1 we represent the RFS calculated using Equation (2) for vibration modes 1 and 3, for two crack depths. The RFS values for the fixed (left) end reflect the severity of the cracks when the cross-section is reduced by 12 and 20%, respectively. For the other locations, the effect of the crack is reduced in concordance to the local curvature. Because the normalized curvature is the unit for the crack located at the fixed end, here the relative frequency shift is equal with the crack severity and is not dependent on the vibration mode number. It is obvious that the bigger the crack depth, the bigger the severity and in consequence the RFS.

Figure 1. Plotted RFS functions for out-of-plain bending vibrations of a cantilever beam with perfect fixing: (a) vibration; mode 1; (b) vibration mode 3. The functions are plotted for a cross-section reduction of 12% and 20%, respectively.

The above approach is valid for perfect boundary conditions. If the fixing is non-ideal, the frequency drop is bigger. In prior research, we demonstrated that the superposition principle is valid [33]. For a beam with two cracks, which have position $x_1$ and $x_2$, and depths $a_1$ and $a_2$, respectively, the frequency drop is calculated as the sum of the two RFS, that is

$$
\Delta \gamma_D(x_1, a_1, x_2, a_2) = \gamma_1(a_1) \left[ \phi''(x_1) \right]^2 + \gamma_2(a_2) \left[ \phi''(x_2) \right]^2.
$$

(5)

We simulate the non-ideal boundary condition, namely the weak fixing, as a crack at the clamped end. So, an additional rotation is possible [34], which leads to a frequency decrease. Knowing that the normalized modal curvature at the fixed end of the cantilever is $\phi''(0) = 1$, Equation (5) becomes

$$
\Delta \gamma_{D-I}(0, a_1, x_2, a_2) = \gamma_1(a_1) + \gamma_2(a_2) \left[ \phi''(x_2) \right]^2.
$$

(6)

Hence, we can plot the RFS for the beam with non-ideal boundary conditions as shown in Figure 2. These are calculated using Equation (6), again for the vibration modes 1 and 3, and the same two crack depths (cross-section is reduced with 12% and 20%, respectively). In this figure, we can observe that in addition to the RFS caused by the crack we have the RFS caused by the non-ideal clamping. The two RFS, caused by crack and by weak clamping, are superimposed and the contribution of the weak clamping is marked with a red dashed line.
Figure 2. Plotted RFS functions for out-of-plain bending vibrations for a cantilever beam with non-ideal fixing: (a) vibration mode 1; (b) vibration mode 3. The functions are plotted for a cross-section reduction of 12%, respective 20%.

The inverse method we developed to detect cracks implies, as a first step, calculating the RFS for numerous damage scenarios and creating a database. The database thus contains two distinct sections:

- TARGET elements, which are the local value of the curvature for a given position, the severity of the defect, and the severity corresponding to the weak fixing. These are placed on columns, the number of columns \( m \) corresponding to the desired number of scenarios. The index of the column is denoted \( k \), thus \( k = 1, \ldots, m \);
- INPUT elements, which are the relative frequency shifts \( \text{RFS}_{c}^{ik} \) calculated with the mathematical relations (2) or (6) for a chosen number of vibration modes \( n \). These are also arranged in columns, each INPUT column corresponding to a TARGET column.

The second step is monitoring the structure, which presumes to measure the natural frequencies. If changes are observed, the frequency shifts calculated for the measured frequencies (\( \text{RFS}_{m}^{i} \)) are compared with the elements of the INPUT section of the database, i.e., \( \text{RFS}_{c}^{ik} \). The column number \( k \) for which the best fit is obtained between the calculated and measured RFS indicates the crack position and severity. In this study, we propose a machine learning approach to find the best fit. The chosen methods are the random forest and the neural network.

3. Machine Learning Methods

In the last decade, artificial intelligence (AI) has become a frequently used term for applications that perform complex tasks that once required human intervention, such as structural health monitoring methods. In this section, we present the development and testing of two machine algorithms used for damage detection. The training data are generated using the method described in the previous chapter. The database contains 36,573 damage scenarios (number of columns in the INPUT and TARGET files). The number of rows in the INPUT file is eight, comprising the first eight out-of-plain bending vibration modes. The TARGET file consists of three elements, i.e., three rows, which are: the damage location, the damage severity, and the severity of the damage simulating the weak clamping. For this study, we considered 100 locations of the crack evenly distributed along the beam, the cracks having 17 levels of depth. The beam has both ideal and non-ideal clamping, for the latter situation seven cases of weak fixing are generated. It resulted in 36,573 damage scenarios. This number of data was selected after testing the influence of the
dataset dimension on the accuracy of the predictions. We observed that larger databases led to better results. The two files of the database are presented in [35].

Because of the complexity of the application and the accuracy required for determining the exact position and severity of transverse cracks we propose a two-step approach. This means that after a coarse localization of the crack using a network trained for all damage scenarios, we apply a second check, this time for a model trained for the specific section of the beam on which the crack is found at the first step. As an example, if the crack is found involving one of the developed methods at 175 mm from the fixed end, we make the second check for a segment extended between 100 and 250 mm. For this segment, we train again the model for a limited number of inputs, considering just those which contain the crack position between the limits 100 and 250 mm. This facilitates obtaining a better model of the structure and, in consequence, more precise localization of the crack.

To have a fast estimation, we divide the beam into nine segments, partially overlapped, and train the ML models a priori for these segments. Overlapping is used to avoid uncertainty regarding the choice of the right segment. The segments used, and the name of the network, are presented in Table 1.

| Segment limits | Network name |
|----------------|--------------|
| 0–150          | Sector 1     |
| 100–300        | Sector 2     |
| 250–400        | Sector 3     |
| 350–500        | Sector 4     |
| 450–600        | Sector 5     |
| 550–700        | Sector 6     |
| 650–800        | Sector 7     |
| 750–900        | Sector 8     |
| 850–1000       | Sector 9     |

Table 1. Segments used to train the machine learning models for enhanced damage detection.

The two ML methods and the settings applied for the training are presented in the next two sub-sections. For both methods, we use 70% of the data for training, 15% for testing, and 15% for validation.

3.1. Random Forest

Decision trees have proven to be successful at exploring non-linear relationships between input and target variables [36]. Such trees work by splitting the dataset in instances that have a minimum amount of node impurity or are, in other words, homogenous. Purity here means that each leaf node represents data points that are in the same class and is defined as the sum of square deviations in class predictions. The biggest drawback of decision trees is that they can easily overfit. This can be mitigated by aggregating such trees to reduce variance.

RF is a technique that employs an ensemble of decision trees and can be used for regression and classifications tasks. The prediction made by an RF aggregates the output of individual trees into a single variable [37]. When building trees, the algorithm randomly selects a given number of features. This essentially prevents multiple decision trees that rely on the same feature. The process is repeated until a group of regression trees, each trained on a randomly selected subset of data, is created. This induced randomness is what compensates for the weakness of each tree.

The performance of an RF model can be tweaked by tuning a few key parameters, with some studies reporting that there is a significant benefit to tuning RF parameters away from their default settings [38,39]. Table 2 summarizes the most common hyperparameters of the RF model.

| Parameter         | Meaning                                           | Value | Range        |
|-------------------|---------------------------------------------------|-------|--------------|
| n_estimations     | The number of estimators in the forest             | 400   | 200–2000, step 200 |
| max_features      | max number of features considered for splitting a node | sqrt  | -            |
| max_depth         | max number of levels in each decision tree         | None  | None; 10–110, step 10 |
| min_samples_split | min number of data points placed in a node before it is split | 2     | 2, 5, 10     |

Table 2. RF hyperparameters.
The number of trees in the forest should, in principle, be as high as possible but in practice, performance plateaus appear after a few hundred trees. In general, increasing the number of features considered in splitting a node will improve performance as each node will have now have a higher number of options to consider; a lower value will increase the chance of selecting features with small effects which in turn could lead to improved performance in cases where such feature would be masked. Parameter min_samples_leaf represents the minimum size of terminal nodes. A higher number will lead to small trees while a smaller leaf size makes the tree more prone to noise in data.

The hyperparameters of the RF were tuned using randomized search with five-fold cross-validation (RandomizedSearchCV in Scikit-Learn), that chooses one of the possible values for each one of the hyperparameters and scores the estimator. The best estimator is used in the model. Table 2 also shows the tuned values for each parameter. The training results using RF are presented in Figure 3 for the entire beam and sector 8, respectively.

![Random forest training results](https://via.placeholder.com/150)

**Figure 3.** Random forest training results; (a) obtained for coarse estimation involving all input data; (b) obtained for accurate localization involving the Sector 8 input data.

### 3.2 Artificial Neural Networks

Numerous studies have shown the versatility and power of ANNs when applied to different computational tasks such as prediction or classification in many real-world applications [40]. In particular, they are universal approximators capable of detecting non-linearities in an n-dimensional input. This is achieved by including a large number of non-linear transformations between the input to an output mapping. A typical neural network consists of several connected neurons, organized in layers, as shown in Figure 4. A neuron will generally process information coming from its connections using a nonlinear activation function. A neural network is trained to perform a specific function by adjusting the values of the connections between its neurons.
Feedforward neural networks are ANNs where the topology is organized such that every neuron in a layer projects only onto subsequent layers. This topology excludes thus recurrent connections and essentially means that information flows through the network from one layer to another until it reaches the output.

By using the methodology presented in Section 2, we calculated the required data for training. The calculated data involves the INPUT data as the calculated RFS values for the first 8 transverse vibration modes and the TARGET data consisting of 3 values, i.e., crack position, crack severity, and the severity for the weak clamping.

The ANN is developed using Matlab software, as shown in Figure 5. A feedforward-backpropagation network type is employed, for which we choose: the Bayesian regularization training function, the Levenberg–Marquardt learning function, the mean squared error (MSE) performance function, and the hyperbolic tangent sigmoid transfer function. This combination of Bayesian regularization and Levenberg–Marquardt training leads to regularisation and essentially makes the network difficult to overfit [41].

![Figure 4. Typical neural network structure.](image)

![Figure 5. The setup for the training of network 1.](image)
All networks used in this study consist of one input layer with 8 neurons, three hidden layers with 30 neurons each, and an output layer with 3 neurons. An example is shown in Figure 6.

**Figure 6.** Network 1 configuration.

The obtained training performance for network 1 is presented in Figure 7 in comparison with one of the particular networks, Sector 9 shown in Figure 8.

**Figure 7.** Neural network training results obtained for coarse estimation involving network 1.
3.3. Evaluation of the Models

In this study, two criteria were used for evaluating the accuracy of the models relative to the ones obtained through FEM analysis and experimental tests, \( \text{error}(x) \) for the position and \( \text{error}(\gamma) \) for the severity. The error in the position is

\[
\text{error}(x) = \frac{x(r) - x(e)}{1000} \times 100 \%
\]

Here, \( x(r) \) is the real position of the crack expressed in mm, \( x(e) \) is the estimated position using the ML models and 1000 represents the total length of the beam in mm.

The severity error is calculated as

\[
\text{error}(\gamma) = \frac{\gamma(r) - \gamma(e)}{1} \times 100 \%
\]

In this relation, \( \gamma(r) \) is the real severity of the crack, \( \gamma(e) \) is the estimated severity of the crack found using the ML models, and 1 represents the maximum value that can be achieved by the crack severity.

In addition to the two above-mentioned errors, we evaluate the capacity of the models to detect weak clamping. The possible responses are false or true.
4. Numerical Validation

For testing the reliability of the damage detection methods described in the previous section, we first involve the finite element method (FEM). Modal analysis is performed using the ANSYS software. We generate a prismatic steel cantilever beam, like that shown in Figure 9, with dimensions: length \( L = 1000 \) mm, width \( B = 50 \) mm, and thickness \( H = 5 \) mm. The assigned material is S355 JR steel with a modulus of elasticity \( E = 2 \times 10^5 \) MPa and a density of 7850 kg/m\(^3\).

![Figure 9. Main dimensions of the considered beam geometry with a transversal crack.](image)

The target was to find the first eight natural frequencies for the out-of-plane vibration modes, for the healthy beam and the beam with different damages, respectively. Both ideal clamping and non-ideal clamping are simulated.

To implement the crack and simulate the effect of the weak clamping, we involve separate elements which are not fixed on one surface, as shown in Figure 10. In this way, we assign the same mass for the beams with damage and non-ideal clamping and yet obtain discontinuities. For simulating real-life scenarios where in most cases measurement data can contain noise, we have also considered different meshing sizes for the damage scenarios to have small differences in the frequency results. The maximum edge size is set to 2 mm for scenarios 1–5 and 1 mm for scenarios 6–10.

![Figure 10. The finite element model of the damaged beam with weak clamping.](image)

The frequencies obtained from the FEM analysis for the undamaged and damaged beam cases are presented in Table 3. The complete dataset is presented in [42].

![Table 3. Obtained natural frequencies for the defined damage scenarios.](image)
The variables used for the studied damage scenarios are the crack location, which is measured from the fixed end, and the proportion in which the clamped end is affected. The weak clamping is replicated by adding the separate element of known thickness $a_1$ to the fixed end. Thus, $\bar{a} = a_1/H$ (%) replicates the weak clamping. The RFS values obtained for the studied damage scenarios are presented in Table 4, where crack position $x$, crack depth $a_2$ and weak clamping $\bar{a}$ represent the output values.

| Scen. | Crack pos. $x$ (mm) | Crack Depth $a_2$ (mm) | Weak Clamp. $\bar{a}$ (%) | RFS for the First 8 Modes |
|-------|---------------------|------------------------|---------------------------|--------------------------|
| 1     | 100                 | 1                      | 0                         | 0.002393 0.001227 0.000540 0.000099 0.000015 0.000075 0.000336 0.000652 |
| 2     | 150                 | 1                      | 0                         | 0.001899 0.000254 0.000038 0.000552 0.001173 0.001254 0.000798 0.000219 |
| 3     | 400                 | 1                      | 0                         | 0.000644 0.001020 0.000683 0.000299 0.001480 0.000037 0.001166 0.000771 |
| 4     | 550                 | 1                      | 0                         | 0.000240 0.001535 0.000256 0.001080 0.000632 0.000640 0.001072 0.000240 |
| 5     | 613                 | 1                      | 0                         | 0.000141 0.001343 0.000683 0.000299 0.001480 0.000037 0.001166 0.000771 |
| 6     | 133                 | 1                      | 20                        | 0.002224 0.000466 0.000003 0.000336 0.001015 0.001417 0.00131 0.000771 |
| 7     | 280                 | 1.2                    | 20                        | 0.004237 0.002804 0.004348 0.003654 0.002554 0.003922 0.004598 0.002963 |
| 8     | 410                 | 1                      | 20                        | 0.002522 0.003063 0.002483 0.002346 0.003381 0.001844 0.003377 0.002300 |
| 9     | 570                 | 1                      | 20                        | 0.000218 0.00167 0.000507 0.000837 0.00116 0.000208 0.001627 0.000002 |
| 10    | 962                 | 0.6                    | 10                        | 0.000412 0.000412 0.000414 0.000419 0.000429 0.000446 0.000472 0.000507 |

The outputs obtained using the two ML methods based on the RFS values (Table 4) are presented in Tables 5–8. Table 5 contains the results from the preliminary ML random forest model, and Table 6 the results obtained using the refined random forest model. In Table 7 we present the results obtained involving coarse estimation with network 1, and in Table 8 are the results obtained with the specific ANN networks chosen for the particular cases after the coarse estimation.
Table 6. Results obtained using Random forest enhanced model—second step.

| Scen. | Position (mm) | Severity $\gamma_{\text{a(a)}}$ | Position (mm) | Severity $\gamma_{\text{a(a)}}$ | Position Error (%) | Severity Error (%) | Weak Clamping |
|-------|---------------|----------------------------------|---------------|----------------------------------|--------------------|--------------------|---------------|
| 1     | 100           | 0.0033459                        | 97.7          | 0.0030                           | 0.23               | 0.03               | Detected False |
| 2     | 150           | 0.0033459                        | 148.3         | 0.0029                           | 0.17               | 0.04               | Detected True  |
| 3     | 400           | 0.0033459                        | 400.0         | 0.0025                           | 0                 | 0.08               | Not detected True |
| 4     | 550           | 0.0033459                        | 539.6         | 0.0028                           | 1.04               | 0.05               | Detected False |
| 5     | 613           | 0.0033459                        | 615.3         | 0.0026                           | 0.23               | 0.07               | Not detected True |
| 6     | 133           | 0.0033459                        | 136.3         | 0.0024                           | 0.33               | 0.09               | Not detected False |
| 7     | 280           | 0.0051239                        | 303.4         | 0.0087                           | 2.34               | 0.36               | Detected True  |
| 8     | 410           | 0.0033459                        | 410.1         | 0.0046                           | 0.01               | 0.13               | Detected True  |
| 9     | 570           | 0.0033459                        | 570.1         | 0.0058                           | 0.01               | 0.25               | Detected True  |
| 10    | 962           | 0.0011911                        | 976.3         | 0.010                            | 1.43               | 0.88               | Detected True  |

The severity of the cracks with depth $a = 1$ mm, $a = 1.2$ mm, and $a = 0.6$ mm is: $\gamma_{\text{a(1)}} = 0.0033459$, $\gamma_{\text{a(1.2)}} = 0.0051239$ and $\gamma_{\text{a(0.6)}} = 0.0011911$.

One can observe that, after the first check, the largest error in estimating the crack position is 7.51%, which is unacceptable. The severity is precisely found, the errors being less than 1.17%. The method was not able to indicate correctly if the clamping is non-ideal. In consequence, this machine learning method is not qualified to assess the crack position and severity at this stage.

Table 7. Results obtained using network 1—coarse estimation.

| Scen. | Position (mm) | Severity $\gamma_{\text{a(a)}}$ | Position (mm) | Severity $\gamma_{\text{a(a)}}$ | Position Error (%) | Severity Error (%) | Weak Clamping |
|-------|---------------|----------------------------------|---------------|----------------------------------|--------------------|--------------------|---------------|
| 1     | 100           | 0.0033459                        | 99.6          | 0.0027                           | 0.04               | 0.06               | Not detected False |
| 2     | 150           | 0.0033459                        | 147.7         | 0.0023                           | 0.23               | 0.10               | Not detected False |
| 3     | 400           | 0.0033459                        | 392.9         | 0.0011                           | 0.71               | 0.22               | Not detected False |
| 4     | 550           | 0.0033459                        | 553.9         | 0.0031                           | 0.39               | 0.02               | Not detected False |
| 5     | 613           | 0.0033459                        | 604.4         | 0.0022                           | 0.86               | 0.11               | Not detected False |
| 6     | 133           | 0.0033459                        | 132.9         | 0.0047                           | 0.01               | 0.14               | Detected True  |
| 7     | 280           | 0.0051239                        | 261.5         | 0.0098                           | 1.85               | 0.47               | Detected True  |
| 8     | 410           | 0.0033459                        | 377.5         | 0.0033                           | 3.25               | 0.00               | Detected True  |
| 9     | 570           | 0.0033459                        | 569.1         | 0.0072                           | 0.09               | 0.39               | Detected True  |
| 10    | 962           | 0.0011911                        | 971           | 0.0134                           | 0.9                | 1.22               | Detected True  |

Table 8. Results obtained using the particular networks—fine estimation.

| Scen. | Position (mm) | Severity $\gamma_{\text{a(a)}}$ | Network Used | Position (mm) | Severity $\gamma_{\text{a(a)}}$ | Position Error (%) | Severity Error (%) | Weak Clamping |
|-------|---------------|----------------------------------|--------------|---------------|----------------------------------|--------------------|--------------------|---------------|
| 1     | 100           | 0.0033459                        | Sector 1     | 99.6          | 0.003332                         | 0.04               | 0.00               | Not detected True |
| 2     | 150           | 0.0033459                        | Sector 2     | 148.9         | 0.003174                         | 0.11               | 0.02               | Not detected True |
| 3     | 400           | 0.0033459                        | Sector 4     | 398.3         | 0.003031                         | 0.17               | 0.03               | Not detected True |
| 4     | 550           | 0.0033459                        | Sector 5     | 554.7         | 0.002947                         | 0.47               | 0.04               | Not detected False |
| 5     | 613           | 0.0033459                        | Sector 6     | 612.6         | 0.002936                         | 0.04               | 0.04               | Not detected True |
| 6     | 133           | 0.0033459                        | Sector 1     | 132.3         | 0.003385                         | 0.07               | 0.00               | Detected True  |
| 7     | 280           | 0.0051239                        | Sector 3     | 280.8         | 0.004469                         | 0.08               | 0.07               | Detected True  |
| 8     | 410           | 0.0033459                        | Sector 4     | 409.4         | 0.003086                         | 0.06               | 0.03               | Detected True  |
| 9     | 570           | 0.0033459                        | Sector 5     | 570.4         | 0.003353                         | 0.04               | 0.00               | Detected True  |
| 10    | 962           | 0.0011911                        | Sector 9     | 962.2         | 0.001419                         | 0.02               | 0.02               | Detected True  |

The severity of the cracks with depth $a = 1$ mm, $a = 1.2$ mm, and $a = 0.6$ mm is: $\gamma_{\text{a(1)}} = 0.0033459$, $\gamma_{\text{a(1.2)}} = 0.0051239$ and $\gamma_{\text{a(0.6)}} = 0.0011911$. One can observe that, after the first check, the largest error in estimating the crack position is 7.51%, which is unacceptable. The severity is precisely found, the errors being less than 1.17%. The method was not able to indicate correctly if the clamping is non-ideal. In consequence, this machine learning method is not qualified to assess the crack position and severity at this stage.
Localization after the second check with the RF method is accurate, errors being below 1%. Estimating the severity of the defect is done also with small errors, up to 0.88%. As shown in Table 6, weak clamping is detected incorrectly for some damage scenarios.

Assessing the crack with the ANN method is also made in two steps. First, a coarse estimation is made with network 1, which considers the locations along the entire beam. Here, we coarsely estimate the damage location and choose the segment on which the crack is supposed to exist and perform a second check for the network trained for scenarios available on this segment.

After the first check, the largest error obtained is 3.3% for the position, and small errors were obtained for the crack severity. The model wrongly detects non-ideal clamping for the first five damage scenarios. But, at this stage, we can estimate the segment on which the crack occurred with high confidence.

Knowing the approximate position of the crack we can select the appropriate segment and use the network trained for this particular segment. The results obtained in the second step are given in Table 8. One can observe that now the largest error obtained is 0.5% for the position and 0.07% for the crack severity. In addition, the model can detect non-ideal clamping with high accuracy. These results allow us to conclude that the two-step ANN method is more efficient in crack assessment.

5. Experimental Validation

To validate the developed ANN models, experimental studies were carried out by measuring the first natural frequencies for five steel beams with the dimensions indicated in the previous section. In the first five damage scenarios, the beams were perfectly clamped. Furthermore, the capability of the developed method to detect non-ideal clamping was also tested for the damaged beams by mounting rubber blocks between the jaws of the vise and the beam (Figure 11). The transverse cracks are produced by wire-cut electrical discharge machining, a method chosen to obtain cracks width of 0.08 mm.

The acceleration signals are recorded with a Kistler 8772 accelerometer mounted on the tested beam (5). It transmits the measured signal through the analog-to-digital conversion module NI9234 mounted in a compact chassis NIDAQ-9175 (3) which is connected to laptop (2). The beam (5) is fixed into the vise (7) and to excite it at the desired resonant frequencies we used the method described in [43]. An audio excitation system consisting of a loudspeaker (6), amplifier (5), and dedicated software installed on a second laptop are used (1). The experimental setup is shown in Figure 12.

Figure 11. Clamping system: (a) rigid clamping obtained by direct fixing in the vise; (b) weak clamping obtained by intercalating rubber layers.
The data is processed using the LabVIEW software installed on laptop (2). We employ a VI designed to acquire the acceleration signal, plotted in the upper graph in Figure 13. The coarse natural frequencies are estimated; in the bottom graph in Figure 13 we present the spectrum of the signal acquired with the excitation set to produce resonance for the 4th out-of-plain vibration mode, at around 140 Hz.

The natural frequencies are estimated one by one, with high accuracy, after a procedure described in [44]. This is imperious necessary for detecting the small changes induced by the crack. Following the procedure, the acquired signal is imported in a Python application, named PyFEST, developed by our research team that uses a method which truncates the acquired signal and calculates the DFT values for different frequency resolutions, and the results are graphically shown in Figure 14. The Python code to estimate the frequencies is given in [45].
5.1. Perfect Clamping Experiments

In the first experimental study, each beam is mounted into a vise, thus achieving a rigid clamping. The resulted frequencies for the undamaged beams are shown in Table 9.

Table 9. Frequencies estimated for the undamaged beams.

| Test Beam | Natural Frequencies [Hz] |
|-----------|--------------------------|
|           | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
| Beam 1    | 4035   | 25,284 | 70,970 | 139,090| 230,336| 344,196| 481,809| 641,261|
| Beam 2    | 4060   | 25,439 | 71,426 | 139,902| 231,038| 344,750| 482,503| 641,823|
| Beam 3    | 4034   | 25,341 | 71,064 | 139,915| 230,138| 341,868| 480,773| 636,769|
| Beam 4    | 4030   | 25,367 | 71,213 | 139,342| 230,295| 343,254| 480,795| 639,510|
| Beam 5    | 4044   | 25,482 | 71,287 | 139,42  | 228,528| 344,177| 481,213| 641,114|

Several damage scenarios are generated by saw cutting each of the five beams at different positions and depths thus replicating a transversal crack. The position and depth for every scenario are according to Table 10.

Table 10. Estimated natural frequencies for the test beams containing a crack of known position and depth.

| Test Beam | Crack Position | Crack Depth | Natural Frequencies [Hz] |
|-----------|----------------|-------------|--------------------------|
|           | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
| 1         | 98     | 2.5   | 3.952 | 25.086 | 70.854 | 139.086| 229.859| 342.143| 477.105| 633.647|
| 2         | 310    | 1.25  | 4.053 | 25.422 | 71.259 | 139.818| 230.913| 343.835| 481.913| 641.729|
| 3         | 569    | 2.5   | 4.024 | 24.904 | 70.707 | 137.883| 227.494| 340.994| 472.791| 636.758|
| 4         | 126    | 2.5   | 4.200 | 25.226 | 71.208 | 138.982| 228.245| 338.441| 473.771| 632.801|
| 5         | 759    | 2.5   | 4.043 | 25.343 | 70.051 | 137.012| 227.517| 343.834| 475.326| 630.614|

By using the measured frequencies for the beams in an undamaged and damaged state, we obtain the RFS values with Equation (2); the results are presented in Table 11.
Table 11. Calculated RFS value for each test beam.

| Test Beam | Crack Position | Crack Depth | Mode 1  | Mode 2  | Mode 3  | Mode 4  | Mode 5  | Mode 6  | Mode 7  | Mode 8  |
|-----------|----------------|-------------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1         | 98             | 2.5         | 0.020610 | 0.007828| 0.001629| 0.000031| 0.002070| 0.005964| 0.009762| 0.011873|
| 2         | 310            | 1.25        | 0.001795 | 0.000660| 0.002334| 0.000600| 0.000542| 0.002654| 0.001223| 0.000146|
| 3         | 569            | 2.5         | 0.002382 | 0.017252| 0.005019| 0.009109| 0.011488| 0.002556| 0.016603| 0.000017|
| 4         | 126            | 2.5         | 0.023458 | 0.005550| 0.000064| 0.002581| 0.008901| 0.014021| 0.014610| 0.010491|
| 5         | 759            | 2.5         | 0.000288 | 0.005461| 0.017336| 0.017272| 0.004422| 0.000996| 0.012234| 0.016377|

The RFS data is fed to the coarse ML models to obtain the estimated crack position and severity for the five damage scenarios. The results obtained after the first estimation are shown in Table 12 for the RF model and in Table 13 for the ANN model.

Table 12. Results obtained using the preliminary Random Forest model.

| Damage Scenarios | Preliminary Random Forest |
|------------------|----------------------------|
| Scen. | Position (mm) | Severity $\gamma^2(a^2)$ | Position (mm) | Severity $\gamma^2(a^2)$ | Position Error (%) | Severity Error (%) | Weak Clamping |
| 1    | 98            | 0.026224                  | 96.99        | 0.0264                  | 0.10              | 0.02              | Not detected True |
| 2    | 310           | 0.005124                  | 309.85       | 0.0051                  | 0.01              | 0.00              | Not detected True |
| 3    | 569           | 0.026224                  | 564.87       | 0.0331                  | 0.41              | 0.69              | Not detected True |
| 4    | 126           | 0.026224                  | 125.42       | 0.0329                  | 0.06              | 0.67              | Not detected True |
| 5    | 759           | 0.026224                  | 758.25       | 0.0329                  | 0.08              | 0.67              | Not detected True |

Table 13. Results obtained using the ANN—Network 1.

| Damage Scenarios | Coarse ANN—Network 1 Output |
|------------------|-----------------------------|
| Scen. | Position (mm) | Severity $\gamma^2(a^2)$ | Position (mm) | Severity $\gamma^2(a^2)$ | Position Error (%) | Severity Error (%) | Weak Clamping |
| 1    | 98            | 0.026224                  | 97.8         | 0.0282                  | 0.02              | 0.20              | Not detected False |
| 2    | 310           | 0.005124                  | 313.2        | 0.0042                  | 0.32              | 0.09              | Not detected False |
| 3    | 569           | 0.026224                  | 567.2        | 0.0323                  | 0.18              | 0.61              | Not detected False |
| 4    | 126           | 0.026224                  | 126          | 0.0337                  | 0.00              | 0.75              | Not detected False |
| 5    | 759           | 0.026224                  | 757.8        | 0.0331                  | 0.12              | 0.69              | Not detected False |

We identify the segments in which cracks were found and select the corresponding networks. Afterward, by employing the enhanced ML models we obtain more precise results, as presented in Table 14 for the refined random forest model and in Table 15 for the enhanced ANN model.

Table 14. Obtained results using the refined Random Forest model.

| Damage Scenarios | Refined Random Forest |
|------------------|-----------------------|
| Scen. | Position (mm) | Severity $\gamma^2(a^2)$ | Position (mm) | Severity $\gamma^2(a^2)$ | Position Error (%) | Severity Error (%) | Weak Clamping |
| 1    | 98            | 0.026224                  | 99.40        | 0.0260                  | 0.14              | 0.02              | Not detected True |
| 2    | 310           | 0.005124                  | 309.73       | 0.0049                  | 0.03              | 0.02              | Not detected True |
| 3    | 569           | 0.026224                  | 568.22       | 0.0327                  | 0.08              | 0.65              | Not detected True |
| 4    | 126           | 0.026224                  | 127.84       | 0.0329                  | 0.18              | 0.67              | Not detected True |
| 5    | 759           | 0.026224                  | 758.19       | 0.0328                  | 0.08              | 0.66              | Not detected True |
Table 15. Obtained results using the precision sector ANN.

| Scen. | Position (mm) | Severity \(\gamma_{a2}\) | Position (mm) | Severity \(\gamma_{a2}\) | Position Error (%) | Severity Error (%) | Weak Clamping |
|-------|---------------|--------------------------|---------------|--------------------------|-------------------|-------------------|---------------|
| 1     | 98            | 0.026224                 | 98            | 0.0275                  | 0.00              | 0.13              | Not detected   |
| 2     | 310           | 0.005124                 | 310           | 0.0054                  | 0.00              | 0.03              | Not detected   |
| 3     | 569           | 0.026224                 | 568           | 0.0343                  | 0.10              | 0.81              | Not detected   |
| 4     | 126           | 0.026224                 | 126           | 0.0343                  | 0.00              | 0.81              | Not detected   |
| 5     | 759           | 0.026224                 | 758           | 0.0343                  | 0.10              | 0.81              | Not detected   |

One can observe from Tables 14 and 15 that, for the case of ideal clamping, the damage location and severity are found with high accuracy irrespective of the employed method. Remarkably, ideal clamping is detected in all cases.

5.2. Improper Clamping Experiments

This study concerns the ability of the proposed approach to detect cracks in two different cases:

- The structure has initially ideal boundary conditions, but after a while, a crack occurs and an alteration of the fixing system is present, i.e., the clamping becomes non-ideal;
- The structure has from the beginning non-ideal boundary conditions, so the fixing system remains unchanged, and is affected after a while by a crack.

The experiment is made for test beam 1. Improper clamping was ensured by mounting rubber blocks between the jaws of the vise and the test beam, as shown in Figure 10. The crack is generated at distance \(x_2 = 98\) mm and has depth \(a_2 = 2.5\) mm. The natural frequencies were obtained from the measured data. The results for the undamaged beam with ideal and non-ideal clamping, as well as the frequencies for the test beam 1 with non-ideal fixing and a crack are shown in Table 16.

Table 16. Estimated natural frequencies.

| Test Beam | Crack Position | Crack Depth | Calculated Natural Frequencies [Hz] |
|-----------|----------------|-------------|-------------------------------------|
|           |                |             | Mode 1         | Mode 2         | Mode 3         | Mode 4         | Mode 5         | Mode 6         | Mode 7         | Mode 8         |
| Undamaged with ideal clamping | 4035 | 25,284 | 70,970 | 139,090 | 230,336 | 344,196 | 481,809 | 641,261 |
| Undamaged with non-ideal clamping | 4.0051 | 25.111 | 70.48 | 138.95 | 229.21 | 341.18 | 476.93 | 635.49 |
| 1          | 98             | 2.5         | 3.926         | 24.935         | 70.420         | 138.211         | 228.362         | 339.873         | 474.057         | 630.080         |

The calculated RFS values are presented in Table 17, first for the undamaged beam with ideal fixing (case 1i) and afterward, for the undamaged beam with initially non-ideal fixing (case 1n-i). In both cases, the damaged beam has non-ideal clamping.

Table 17. Calculated RFS values for the improperly clamped damaged beam.

| Test Beam | Crack Position | Crack Depth | Calculated RFS Values |
|-----------|----------------|-------------|-----------------------|
|           |                |             | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 | Mode 6 | Mode 7 | Mode 8 |
| 1i        | 98             | 2.5         | 0.026955 | 0.013803 | 0.007749 | 0.006322 | 0.008572 | 0.012559 | 0.016089 | 0.017436 |
| 1n-i      | 98             | 2.5         | 0.019749 | 0.007008 | 0.000851 | 0.005318 | 0.003699 | 0.003830 | 0.006023 | 0.008513 |

The crack assessment is made employing the two ML methods, in compliance with the two evaluation stages. The results for case 1i are presented in Table 18, while the results for case 1n-i are presented in Table 19.
Table 18. Crack assessment for a damaged beam with non-ideal clamping for the case the undamaged beam had initially an ideal fixing (case 1i).

| Damage Scenario | Results Obtained with the ML Models |
|-----------------|------------------------------------|
| Scen.           | Position (mm) | Severity γ^2(a_2) | Position Error (%) | Severity Error (%) | Weak Clamping |
| First-step RF   | 129.36        | 0.038300           | 3.14               | 1.21              | Detected     | True         |
| Second-step RF  | 92.68         | 0.039000           | 0.53               | 1.28              | Detected     | True         |
| First-step ANN  | 102.07        | 0.031283           | 0.41               | 0.51              | Detected     | True         |
| Second-step ANN | 99.13         | 0.027038           | 0.11               | 0.08              | Detected     | True         |

Table 19. Crack assessment for a damaged beam with non-ideal (but unchanged) clamping (case 1ii).

| Damage Scenario | Results Obtained with the ML Models |
|-----------------|------------------------------------|
| Scen.           | Position (mm) | Severity γ^2(a_2) | Position Error (%) | Severity Error (%) | Weak Clamping |
| First-step RF   | 82.54         | 0.0353            | 1.546              | -0.9076            | Not detected  | True         |
| Second-step RF  | 32.05         | 0.0292            | 6.595              | -0.2976            | Not detected  | True         |
| First-step ANN  | 74.63         | 0.0247            | 2.337              | 0.1524             | Not detected  | False        |
| Second-step ANN | 96.4          | 0.0247            | 0.16               | 0.1524             | Not detected  | True         |

Reviewing Tables 18 and 19, we can conclude that for the beam with changing boundary conditions, the crack position and severity were found accurately after the second step. The best result is obtained using the ANN, which located the crack with an error of 0.11% and found the severity with an error below 0.1%.

The changed boundary conditions are detected with both ML methods. Dissimilarly, the case when the beam initially had a weak fixing was properly solved just by the ANN method. The errors accuracy, in this case, is sustained by errors less than 0.16%, for position and severity as well. Both ML methods did not find an alteration of the fixing in the vise, thus we consider the response to not be detected as true.

6. Conclusions

In this paper, we applied RF and ANN to identify the crack location and severity in a steel cantilever beam with a rectangular cross-section. The beam had different levels of fixing, including ideal and non-ideal clamping, and we also aimed to find out if the fixing condition changed. To train the network we used as inputs the RFS found, applying an original method, as the crack parameters and the fixing condition, respectively. To improve the accuracy of the assessment method, we performed the crack assessment in two steps: (i) in the first step we applied coarse estimation for the network trained for all damage scenarios to identify the region in which the crack occurred; (ii) in the second step we applied a fine estimation, for the network trained for a specific segment of the beam which included the damaged region to determine the crack position and severity, along with the changes in clamping, if any.

Both ML methods were able to learn and classify new data with characteristics comparable with that of the training data. After training, the networks were successfully used to assess other scenarios, which came from FEM simulation and laboratory experiments. We found that, for all cases, the ability to accurately detect the crack location and severity increases when using the two-step assessment. After the second step, the errors in locating the crack were below 0.11% for the ANN method and 0.53% for RF, and the errors in assessing the crack severity were less than 0.08% for the ANN method and 1.28% for RF. Thus, the ANN led to better results. The smallest crack we found had a 0.6 mm depth in a beam with a thickness of 5 mm. We did not search for smaller crack depths because of the training data we produced, and the knowledge that the ANN method is more efficient in interpolation than extrapolation.
As a limitation, we mention that the proposed method is based on mathematical relations to calculate the frequency variation due to cracking. Thus, the method is easy to apply for relatively simple structures, such as beams. For more complex structures it is necessary to determine the RFS using FEM analysis or from experiments, which can be a laborious activity and provide results possibly affected by noise.

Further investigation should be conducted to evaluate the ability of the proposed approach to identify cracks in the early stage and on large-scale structures.

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