Influence of Parameters of Railway Track Construction on Vertical Dynamic Interaction Vehicle/Track

By analysing the dynamic interaction between the railway vehicle and the track we are able to explain complex interactive phenomena that are particularly important in the case of higher and high speed of a moving rail vehicle on railway track. The paper focuses on the solution of the above mentioned problem with the help of numerical computer simulations. The important step on the way to create a respective simulation model is setting objective entry parameters of railway track as well as taking its imperfections into consideration. These imperfections significantly influence the results of dynamic interaction. Out of the considered track imperfections, we particularly solve deformation resistance inhomogeneity of track grid subgrade and vertical irregularities of rail belt.

1. Introduction

The railway infrastructure presents a strategical component of Slovak transport infrastructure from the point of view of securing continuous national and transit transport within the European Union. The most important transit rail tracks crossing the Slovak Republic are defined according to international conventions AGC (1985) and AGTC (1993). These rail tracks have to fulfill Technical Interoperability Specifications (TSI). From the point of view of rail track construction, for the chosen track corridors it means that there has to be enabled reaching the operating speed 160 km/h at the maximal axle weight 22.5 tons (loading capacity class D4UIC) in the whole longest possible rail localities. The rail track, as it is a significantly inhomogeneous construction, is a subject to influence of operation and climatic factors. Together with the passing time they result in geometric and material degradations by wearing of all the components of track grid and particularly construction layers of sleeper subgrade. By the influence of load increase on modernised rail localities, these geometrical and material degradations are more obvious as a consequence of higher force effects. Recognition and subsequent development prediction of these degradations are conditioned by perfect recognition of acting force effects of rail vehicles. Particularly in the case of high and higher speed, the dynamic interaction force effects are significantly noticeable. They influence the durability and reliability of rail construction in a crucial way and therefore they ought to be expressly considered in the process of construction design and dimensioning.

Lately, the computer simulation of dynamic interaction of complex mechanical system vehicle/track has had significant influence on the knowledge of structure of interactive wheel forces. In the ties of a mechanical system defined in such a way, dynamic interactive forces are produced whose character and size are necessary to be precisely specified. In this paper we focus not only on the determination of a suitable model for simulation of dynamic interaction vehicle/track, but also on setting objective track parameters that will form a basis for wide application in various application simulations. There is also reference to the present deficiencies in the process of design of rail track construction and there are implied solution possibilities in the form of new approaches to rail track construction dimensioning. They are a result of theoretical simulation analyses that we suppose to verify by experimental measurements in near future in the new laboratory of Faculty of Civil Engineering in Veľký Diel campus. In this laboratory there will also be located a dynamic pulsator for cyclical loading tests.

2. Basic Model Characteristics of Track and Rail Vehicle in the SR

The theoretical solution of mechanical problems of rail tracks is carried out on a certain physical model that should sufficiently characterize the basic properties of a mechanical system being solved. These basic properties should also be determinant for the correct interpretation of behaviour of the construction in the given model and they should keep the geometrical similarity with the real construction. The parameters that characterize the construction should at the same time correlate with the parameters that could be determined by standard or other simple diagnostic methods.
2.1 Rail Vehicle

To make a suitable physical model of a train unit it is necessary to use a model of such train unit that is determinant from the point of view of size of effective dynamic forces and the most sensitive from the point of view of feedback e.g. in the form of comfort by travelling. In the conditions of ŽSR, these are mainly passenger train units EPJ 671 that were purchased in 2010 for the operation in corridor tracks with maximum speed up to 160 km h⁻¹. The mentioned train unit consists of electrical double decker rail car EPV 671, double decker trailer car PVV 071 and double decker managing car PRV 971. From the point of view of possible activity of dynamic force effects on the rail track, the car EPV 671 is determinant. Its weight according to Tab. 1 is 87.5 tons, which presents static wheel loading 107.3 kN for the four-axle symmetrical bogie.

To evaluate the dynamic effects of rail vehicles on the construction of rail track, the symmetry of vehicle/track system is used. This task is practically always solved as a plane problem i.e. applied load of one rail belt and the respective plane interactive model of vehicle/track system are considered (analyzed in detail in chapter 4). In the process of transformation of solution of a given system from three-dimensional to plane one, it is desirable to take several factors into account. They influence the results of solution and are often neglected in plane interactive tasks. The basic factors influencing the force symmetry of rail car system are particularly real irregular crosswise mass distribution in the rail cars, influence of centrifugal force in arc with unbalanced crosswise acceleration, change of rail nivelity, transition to superelevation, etc. In Tab. 2 there is a summary of maximum effects of increase of wheel force for the car EPV 671.

We have to be aware of the fact that the maximum loading capacity of a train unit is not always utilised and the occurrence of increase of wheel force from Tab. 2 even in the case of reaching high values is only in specific short track localities, possibly in the case of specific climatic influences. Therefore the synergic coaction of these effects is rare and for the objective analysis of dynamic interaction vehicle/track it is enough to consider the standard wheel

| Parameter                          | EPJ 671 | PVV 071 | PRV 971 |
|------------------------------------|---------|---------|---------|
| Maximum operational speed          | v = 160 km h⁻¹ = 44.45 m s⁻¹ |
| Length of car along the buffer plane | 3 × 26,400 mm |
| Height of car from the rail surface | 4,635 mm |
| Width of car                       | 2,820 mm |
| Arrangement of bogies              | Bo'Bo'  | 2'2'    | 2'2'    |
| Gauge                              | 1,435 mm |
| Wheel base of bogie                | L_w = 2,600 mm | L_w = 2,400 mm | L_w = 2,400 mm |
| Distance of centre of bogies´rotation | L_B = 19,000 mm | L_B = 19,000 mm | L_B = 19,000 mm |
| Distance of external bogies         | 21,600 mm | 21,600 mm | 21,600 mm |
| Number of sitting/standing passengers | 61/67   | 134/134 | 106/134 |
| Weight of empty car                | 74 t    | 45.3 t  | 47.3 t  |
| Maximum car weight at full loading capacity | 87.5 t  | 69.5 t  | 69.5 t  |
| Maximum bogie weight              | 22 t    | 18 t    | 18 t    |

| Effect of increase of wheel force as a result of... | Increase of wheel force ΔP_k at maximum loading EPV 671 | Maximum size of wheel force P_k with increase ΔP_k |
|-----------------------------------------------------|----------------------------------------------------------|---------------------------------------------------|
| transition to the superelevation of maximum inclination 1:1,600 and 1:1,280 according to STN 73 6360 [1] for RP4 | 0.3 kN/0.5 kN | 107.6 kN/107.8 kN |
| maximum side wind according to III. and IV. wind area in accordance with STN 73 0035 at average centre height according to STN P ENV 13803-1 [2] | 12.0 kN/15.5 kN | 119.3 kN/122.8 kN |
| centrifugal force in arc with maximum irregular acceleration 0.65 ms⁻² and 0.85 ms⁻² according to STN 73 6360 [1] | 9.5 kN/12.0 kN | 117.1 kN/119.3 kN |
| change of the rail nivelity (taper to the elevation arc) with vertical acceleration 0.22 ms⁻² and 0.31 ms⁻² according to STN P ENV 13803-1 [2] | 2.5 kN/3.5 kN | 109.8 kN/112.3 kN |
| irregular mass distribution in the rail car at the height of 5 % and 7 % (estimated) | 5.5 kN/7.7 kN | 112.8 kN/115.0 kN |
The rail superstructure on already modernised corridor tracks of ZSR is formed by the system that consists of rails 60 EI on concrete sleepers with inclination of upper areas 1:40 (classic superstructure), non-plate flexible rail fastening system VOSSLOH Sk11A, with sleeper spacing of type UIC (600mm). With the rail of shape 60 EI there is most often used typal ferroconcrete sleeper BP3 or B9/1S/1. A certificate has been issued for using this type in ZSR. Between the rail and sleeper there is placed an elastic rubber pad, most frequently WU7 or WS7. The concrete sleepers are embedded in railway ballast of gravel of fraction 31.5/63 mm, while the width of railway ballast under the lower area of sleepers is 350 mm. The subbase has an unstable width depending on the initial deformation resistance of subgrade. According to STN 73 6312 [3] there is required an equivalent value of static deformation module on the level of subgrade surface 30 MPa and on the level of surface of railway substructure 50 MPa. By calculating in accordance with dimensioning methodology SOUZDORNII it is possible to determine the static deformation module on the level of lower area of sleepers to 95 MPa. Reaching the normative values depends on season, technology of construction manufacturing, and on the state of compaction of construction layers by operation [4]. The summary of basic parameters of a modernised track (state of new building) is given in Tab. 3.

The size of pad stiffness coefficient \( K_p \) and attenuation \( C_p \) (parameters stated according to STN EN 13464-3 and STN 13464-9), depend on the type of pad material (HDPE, rubber, rubbercork), width, environment temperature, its age (i.e. entire amount and size of loading cycles applied to the pad), as well as contact force of rail fastening. Because of this, values of these parameters are given in wide application interval in various studies (\( K_p = 60 \times 10^5 \) to \( 1000 \times 10^5 \) kNm\(^{-1}\) and \( C_p = 40 \) to 120 kNsm\(^{-1}\)). However, for dynamic evaluation of rail track construction in the model there should be used values of the pad stiffness set by dynamic measurement. According to STN EN 13481-2, in case of using concrete sleepers in the main track they should reach values 100 to 200 MNm\(^{-1}\). In Tab. 3 there are used the average parameter values for the pad leading from comparison of several studies.

The biggest deficiencies in determining parameters given in Tab. 3 are in stating the values of the module elasticity, stiffness, and attenuation of construction layers of sleeper subgrade. The equivalent modules of elasticity of a given construction layer of subgrade defined in Tab. 3 were determined from minimum required values of deformation modules, on the basis of analogy of determination of equivalent elasticity modules with the static loading test according to STN 73 6190/5 [5] (the relation leads from the definition of Bousinesque’s elastic half-space). The elastic deformations for the calculation of elasticity modules were considered from the second lightening branch, in the interval of tension according to anticipated tension range in a given construction layer (geostatic tension to the addition of geostatic and incremental tension). This approach to determination of equivalent elasticity modules takes into account the theoretical principles of Hook’s law (the dependence of tension and deformation in a small interval is linear and this interval is also the interval of real operation of tension in a given construction layer). The indicative elasticity intervals are on the level of lower area of sleeper in the interval <0.01 to 0.30 MPa>, on the level of surface of rail substructure for the width of railway ballast 0.35 m in the interval <0.02 to 0.30 MPa> and on the level of subgrade surface for the width of subbase 0.30 m in the interval <0.03 to 0.05 MPa>. The problem of using loading diagrams from the static loading test according to regulation ZSR S4 [6] is the fact that the most frequently used tension interval under loading board <0 to 0.20 MPa> on the level of subgrade surface with the deformation module 30 MPa is significantly overestimated concerning the interval of operation tension in this depth. Because of this, in Tab. 3 there are used entire tension intervals from the average static loading test for the required level of deformation module 30 MPa, 50 MPa and 95 MPa to calculate elasticity modules. From an elasticity module deduced in such a way it is possible to determine the static stiffness of subbase (elastic constant) from the following theoretical relation [7]:

\[
K_p = \frac{E_i}{(1 - \nu_i^2)} \times 2r \text{ (kNm}^{-1}\text{)}
\]

where:

- \( E_i \) – is elasticity module in strain of i-th construction layer material,
- \( r_i \) – is radius of loading board (0.15 m),
- \( \nu_i \) – is Poisson’s constant of i-th construction layer material.

The theoretical relation does not count with the fact that the elasticity module, used in the relation, is determined from the second lightening branch according to STN 73 6190/5, while the coefficient of compressibility (elastic constant) defines the dependence of loading and deformation from steady hysteresis curve (n-th load branch) without permanent deformations (linear elastic constant).The compressibility coefficients should also be determined in the interval of assumed tension state for the given construction layer. The required increase of value of elastic constant from relation (1) as a result of elimination of plastic transformation is approx. 20 % and the elastic constant parameters stated in Tab. 3 are also changed according to this value.

In the construction layers of sleeper subgrade, damping takes place as a result of internal friction of particles (dissipation).The diagnostics of attenuation characteristics of sleeper subgrade construction has not been stated yet, while these parameters are vital for creation of a respective simulation model for determining the dynamic response vehicle/track. For damping of oscillation of mechanical system we introduce a supposition that the resistance
of system movement is proportional to the speed of movement (linear possibly viscous attenuation). According to [8] it is possible to use the following theoretical relation to calculate the coefficient of attenuation:

\[ C_i = \frac{\sqrt{E_i \rho_i}}{1 - \nu_i^2} \text{ (kNsm}^{-1}) \]  

(2)

where:
- \( E_i \) - is elasticity module in strain of i-th construction layer material,
- \( \rho_i \) - is volume weight of i-th construction layer material,
- \( \nu_i \) - is Poisson’s constant of i-th construction layer material.

All the values of elasticity and deformation modules, Poisson’s constants, coefficients of stiffness and damping given in Tab.3 are stated by static loading. The dynamic characteristics of soil of sleeper subgrade are necessary to be found out by experimental measurements e.g. via cyclic loading tests with different frequency composition and deviation (dynamic pulsator). The analysis of dynamic soil characteristics is carried out e.g. in [9]. From this one we can conclude that all the dynamic quantities have higher values as a result of shorter and cyclically active force and in this way smaller deformation (at frequency 5 to 10 Hz dynamic elasticity modules and Poisson's constants are bigger in 10 to 35 % when compared to static ones). By assembling a model for simulation of dynamic
interaction vehicle/track with consideration of dynamic characteristics of entry parameters, there is obvious higher model stiffness of train construction and in this way also possibly higher values of dynamic interactive forces. There is a recent plan to observe the dynamic characteristics of common materials constituting sleeper subgrade in accordance with valid norms and regulations with the help of newly-built dynamic pulsator of the University of Žilina. These results will significantly help specify the entry parameters in order to create a relevant dynamic interactive track model.

3. Mathematical Model of Vertical Dynamic Interaction Vehicle/Track

The method of mathematical modeling of the vehicle and rail track is used for understanding the complicated dynamic interactions between vehicle and track. The solution of dynamic response of construction of rail and vehicle as a plane task gives in the case of correct formulation and setting entry parameters almost the same results as solutions of complex three-dimensional tasks. To observe the vertical dynamic interactions of wheel and rail track it is thus enough to use the plane calculation model of vehicle and track.

3.1 Calculation Model of Rail Vehicle

The calculation model of rail vehicle is usually considered as a 2-level plane model with sprung bogies and car body. In the design of the mathematical model that serves to study dynamic behaviour of vehicles, it is usually supposed that the system components are solid, and the elastic deformations and vibrations are ignored. All the relations between wheel and bogie and bogie and car body (viscous dampers, spring elements) are considered linear. A plane vehicle model, made in this way, enables to model the influence of vertical movement of sprung mass (heavy effect) and the influence of swinging of sprung mass in longitudinal direction (pitch effect). The influence of inclination of sprung mass in crosswise direction (roll effect) on the formation of kinematic quantities of the model is not taken into consideration. However, it is included in quasi-static speculation of wheel force as in part 2.1. The contact of wheel and rail takes into account contact deformation on the basis of Hertz’ theory, considering the contact stiffness \( K_H \). The Hertz’s theory assumes elastic deformations on the contact of wheel and rail depending on vertical interactive force and the way of contact of rail surface and wheel rim [10]. As a consequence, in the model it is necessary to consider the Hertz’ contact stiffness as a function of vertical interactive force that has inconstant value (non-linear Hertz contact spring). To solve the equations of entire system with non-linear Hertz theory, it is suitable to use the iteration method to calculate \( K_H \) in each time step. The example of interactive model of system vehicle/track that is sufficient to analyse the standard characteristics of dynamic interaction of rail car EPV 671 is demonstrated in Fig. 1. The vehicle model has 10 degrees of freedom. The bogie frame is connected to the wheels of unsprung mass through the primary springs of suspension and connected to the car body through secondary springs of suspension. The designed discreet calculation model has to respect geometrical and physical parameters of vehicle and track as well as the most important imperfections influencing dynamic interaction vehicle/track (Tab. 1 and Tab. 3).

Fig. 1 Example of interactive model of system vehicle/track
3.2 Calculation Model of Track Construction

The calculation model of rail track is considered as a plane model of one rail belt that is modelled by simple beam component (rail), solid support component (sleepers), connecting and damping components. The connecting components (ties)- rail/pad, pad/sleeper, and sleeper/subgrade layers are double-nodal spring-damping components. The characteristics of these components could be set as either linear or non-linear characteristics, although in practical calculations they are solved as linear ones. The parameter values of these components should unequivocally be set in dynamic loading mode (see part 2.2). The rail is assumed as a homogenous beam with $N_J$ nodes and $N_C$ components. Each beam mode (rail structure) has two degrees of freedom. In the model we consider the effect of half-sleeper on viscous-elastic basis. The number of sleepers in model $N_J$ depends on the way of analysing the dynamic interaction. In the case of considering stochastic stiffness of beam and imperfection of rail geometry, it is more advantageous to consider a higher number of sleepers in the model ($50$ to $100$ sleepers). To model the subgrade unhomogeneity, it is better to use a multi-layer model consisting of railway ballast, subbase, possibly soil of subgrade surface. The model of sleeper subgrade contains additional gravel mass under each sleeper. The sleepers are connected through the spring and damper in skid ($K_W$ and $C_W$). According to several well-known studies, with this type of model there could be gained better correlations between the calculated and measured responses. The model is usually limited in vertical direction on the level of subgrade surface (solid plane). To this there are connected spring-damping components with parameters of subbase $K_F$, $C_F$ (surface of railway substructure) according to Tab. 3. The main movement equations for discreet system with several degrees of freedom (dynamic balance equations) are composed according to a following relation [11]:

$$[M][w] + [C_I][w] + [K][w] = [P]$$

where:

- $w(x,t)$ - is a vector of generalised shifts in nodal points of final components,
- $[M]$ - is a global matrix of system weight,
- $[C_I]$ - is a global matrix of system damping,
- $[K]$ - is a global matrix of system stiffness,
- $[P]$ - is external node load.

The components of shift vector $w(x,t)$ undoubtedly determine the deformation state of track in any place of construction $x$ and in any time $t$. The shift vectors are primarily composed by unknown quantities, that serve to calculate the internal forces of construction and reactions within the system ties. The global matrices of weight, damping and stiffness of the system are generated from component matrices (connecting components and masses). The numerical time integration is applied to differential equations of dynamic balance. As a result, the movement of rail vehicle on the track is based on the use of so-called topical position of the vehicle in a movable coordinate system. This involves automatic leaving out the sleeper and its ties on one side and addition of sleepers with ties on the second one. After the sleeper shift from the end to the beginning of the examined area, the update takes place – new system parameters are introduced to the matrix of stiffness and attenuation (in the case of stochastic model). This means that the dynamic calculation model works inversely.

4. Software Systems for Solution of Dynamic Interaction Vehicle/Track

The computer simulation of vertical dynamic interaction rail vehicle/track helps clarify complex interaction phenomena and at the same time evaluate the influence of chosen parameters on the quantities of dynamic response of rail track. The task formulated in such way cannot be effectively solved using conventional business numeric programme systems (FEAT, NEXIS, SCIA, ANSYS, ABAQUS, LUSAS, DIANA, ATENA, SOFISTIK, NASTRAN, ADINA, PLAXIS, COSMOS etc.), but has to use special calculation programmes that are individually programmed prevailingly for the purposes of calculation, most frequently in the programming languages C++, FORTRAN, VISUAL BASIC or MATLAB. On this purpose several programmes were created, short overview of these is listed in Tab. 4. Most of them enables to take all the requirements defined in this paper into account.

5. Conclusion

The experimental measurements of dynamic interaction vehicle/track realized in situ present the only way of verification of numeric results and also the only way of revealing objective reality. At the same time it is necessary to emphasize that carrying out of such experimental measurements in the rail track under operation is often limited by the possibility of sensor installation in one measured place only. In the case of placing the measuring devices on the car bogies we cannot reliably describe the rail track parameters that cause the given dynamic reaction [17]. Because of this, the computer simulations of dynamic interaction vehicle/track are really important. They enable us to analyse specific problems of dynamic interaction. From the parametric analysis of several programmes of dynamic interaction vehicle/track as well as from several studies carried out in this area we could conclude that the programmes in use are sensitive to the values of entry parameters of subgrade of rail track. The authors of several studies deal freely with the subgrade parameters as with variable quantity and they subsequently use these parameters to match them with simulation results so they would correlate with the results of experimental measurements or other simulations. The entry characteristics of rail track should be expressly defined on the basis of carried out diagnostic methods realized in real rail track construction possibly from limit standard values that are tolerable in operational conditions. The diagnostic methods of construction layers of sleeper subgrade should also consider dynamic mode of construction loading in the range of frequency and tension that will be active in the given construction.
layer during operation. To determine the dynamic parameters in
the given construction layer, the simulation of dynamic interaction
vehicle/track is notably applicable. By advancing iteration of values
of operational frequencies and tensions determined from computer
simulations, and values of dynamic characteristics of materials in
individual construction layers determined e.g. by experimental mea-
surements with dynamic pulsator, it will be possible to design
a mathematical model with entry parameters in accordance with
real operational conditions. From the simulation results, it will be
subsequently possible to verify the criteria for geometrical posi-
tion and rail arrangement in synergy with criteria for deformation
resistance of subgrade. It can also be assumed that it will be nec-
essary to define the requirements of reaching a certain homogene-
ity level of deformation resistance of subgrade, possibly to set such
technological methods of manufacturing the construction layers
of subgrade (e.g. by intelligent compaction cylinders [18]) that will
secure the required and verifiable level of subgrade homogeneity.
The important part of considering these and defining new criteria
is applying stochastic parameters of track and vehicle in models.
These could be stiffness of rail fastening, sleeper spacing, stiffness
and attenuation of subgrade, regular and irregular imperfection of
rail geometry, wheel imperfection, floating sleepers, unstable speed
of rail vehicle etc. The dynamic analysis with stochastic rail para-
eters (created by Monte Carlo method or by sampling from the
real track) will enable to identify the critical states of dynamic
interaction in a more actual way. Their result is exceeding the level
of safety, comfort or critical degradation of components of rail
track and vehicle.

| Name of programme | Responsible author | Institution where programme was developed |
|-------------------|--------------------|-------------------------------------------|
| DIFF [12]         | Dr. Jens Nielsen   | centre of excellence in Railway Mechanics at Chalmers University of Technology, Sweden |
| NUCARS [12]      | Dr. Nick Wilson    | American Association of Railroads (AAR), Transportation Technology Centre Inc. (TTCI) |
| 3DWTSD [12]      | Dr. Yan Quan Sun   | Central Queensland University, Australia |
| TRACK [12]        | Dr. Stuart Grassie | Stuart Grassie Engineering Solutions, Cambridge University, United Kingdom |
| DTRACK [13]       | Dr. Zhengi Cai     | DynTrack Systems, USA and Queen’s University, Canada |
| DARTS [14]        | Prof. Coenraad Esveld | Esveld Consulting Services, Delft University of Technology, Netherlands |
| SUBTTI [12]       | Dr. Ulf Gersberger | Technical University Berlin, Germany |
| VICT [12]         | Prof. Wanhuming Zhai | Southwest Jiaotong University, China |
| RAIL [14]         | Dr. Valeri Markine | Delft University, Netherlands |
| DATI [15]         | Dr. Jabbar Ali Zakari | University of Science and Technology, Iran and Beijing Jiaotong University, China |
| IIT [12]          | Dr. G. Kumuran     | Indian Institute of Technology, India |
| VIA [13]          | Dr. Buch A. Roda   | University of Valencia, Spain |
| RTRI [12]         | Dr. Makoto Ishida  | Railway Research Institute, Japan |
| TRADYS [15]       | Dr. Mwa Mashiro    | Central Japan Railway Company and Tokyo University of Technology |
| DIASTARS [16]     | Dr. Makoto Tanabe  | Kanagawa Institute of Technology and Railway Technical Research Institute, Japan |
| C32/Interakcia [11] | Prof. Milan Moravcik | University of Zilina, Slovakia |

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