Principle of Local Conservation of Energy-Momentum

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May 26, 2008

Abstract
Starting with Einstein’s theory of special relativity and the principle that whenever a celestial body or an elementary particle, subjected only to the fundamental forces of nature, undergoes a change in its kinetic energy then the mass-energy equivalent of that kinetic energy must be subtracted from the rest-mass of the body or particle, we derive explicit equations of motion for two falling bodies. In the resulting mathematical theory we find that there are no singularities and consequently no blackholes.

Subject Classification AMS 2000: 83A05, 83D05, 83C57, 81V22.
Keywords: black holes, inertial mass, Minkowski spacetime, rest mass, space-time algebra, special relativity, two body problem.

Introduction
Special Relativity has proven itself to be an exceptionally powerful theory that has revolutionized human understanding of the material universe in the 20th Century [1], [2], [3]. The purpose of the present article is to show how by imposing a local conservation of energy-momentum in the special theory of relativity, the theory takes on a new elegance and universality. In [4], the second author considered how a single object would fall in the gravitational field of a celestial object infinitely more massive. In what follows, we derive the exact equations of motion for two bodies of arbitrary masses falling into each other under the influence gravity. We find that there are no singularities, even in the case of idealized point masses.
Section 1, defines the concept of rest-mass utilized in our theory. Whereas Einstein, by his equivalence principle, considers “inertial mass” and “rest-mass” to be equivalent, we believe that there is a clear asymmetry between an accelerating elevator and a gravitational field. An observer must get accelerated to be able to catch up with an accelerating elevator, whereas he has to get decelerated in order to be able to land on the celestial body. In our theory, the first process yields a mass increase, whereas the second one leads to a mass decrease. It follows that the idea that the rest-mass of an object is a fundamental constant of nature, must be replaced by the concept of the instantaneous rest-mass of an object in a non-homogeneous field, as was first done

Section 2, defines the concept of binding energy of a two body system to account for the work done by any one or all of the four fundamental forces of nature. We find explicit formulas both for the masses and also for the velocities of the two masses in terms of the total binding energy. All our calculations are based upon the simple principle that each body, as it moves under the forces of nature, must subtract the mass-equivalent for any change in its kinetic energy. We express the ideas of special relativity in the framework of the Minkowski spacetime algebra (STA) developed by D. Hestenes. In STA, each relative frame of an observer is defined by a unique, future pointing, Minkowski timelike unit vector tangent to the timelike curve called the history of that observer.

Section 3, considers the binding energy due to Newton’s gravitational force between two bodies and derives a Riccati-like differential equation of motion. We find closed form solutions for the case of a celestial body and for the case when two bodies have the same mass. In the general case when an exact solution is not possible, we use a numerical solution.

Section 4, discusses the results of previous sections and concludes that black holes with a well defined Schwarzschild radius cannot exist.

1 The concept of rest-mass

We begin by defining the rest-mass $m_\infty$ of a body to be the mass of the body when it is isolated from all other bodies and forces in the Universe, as measured by an observer traveling at relative rest with respect to that body. The great advantage of the STA of Hestenes, for the most part still unappreciated by the physics community, is that each such inertial frame is uniquely characterized by a constant Minkowski time-like unit vector $u$. See [6] and [7] for details of the spacetime algebra formulation of special relativity which we use throughout this paper.

Let $p_\infty$ be the Minkowski energy-momentum vector of the rest-mass $m_\infty$. Since we have assumed that $m_\infty$ is at rest in the frame defined by $u$, it follows that $p_\infty = m_\infty c^2 u$. Now let $v = \frac{dx}{d\tau}$ be the Minkowski timelike unit vector of an observer with the timelike history $x = x(\tau)$, where $\tau$ is the natural parameter of proper time (arc length). The unit vector $v = v(\tau)$ uniquely defines the instantaneous frame of the observer at the proper time $\tau$.

As measured from the rest-frame $u$ to the instantaneous relative frame $v$, we
have

\[ p_\infty v = m_\infty c^2 u v = m_\infty c^2 (u \cdot v + u \wedge v) = \gamma v m_\infty c^2 \left(1 + \frac{v}{c}\right), \quad (1) \]

where \( \gamma_v = \frac{u \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}} \) and \( \hat{v} = \frac{u \wedge v}{u \cdot v} \). We say that \( E_v = p \cdot v = \gamma_v m_\infty c^2 \) is the instantaneous relative energy, \( p_v = \gamma_v m_\infty c^2 \hat{v} \) is the instantaneous relative momentum, and \( v \) is the instantaneous relative velocity of \( m_\infty \) in the instantaneous frame \( v \) as measured by \( u \). This convention is opposite by a sign to the convention used by Hestenes in his 1974 paper. We use the same convention here as used by Sobczyk in [8]. There are many different languages and offshoots of languages that have been used to formulate the ideas of special relativity. For a discussion of these and related issues, see [9], [10]. A unified language for mathematics and physics has been proposed in [11].

Equation (1) shows that with respect to the relative frame \( v \), the mass \( m_\infty \) has the increased relative energy \( E_v = \gamma_v m_\infty c^2 \). This means that if we want to boost the mass \( m_\infty \) from the rest-frame \( u \) into the instantaneous frame \( v \), we must expend the energy \( \Delta E_1 = (\gamma_v - 1)m_\infty c^2 \) to get the job done. Expanding the right-hand side of this last equation in a Taylor series in \( |v| \), we find that

\[ \Delta E_1 = \frac{m_\infty}{2} \hat{v}^2 + \frac{3m_\infty}{8c^2} \hat{v}^4 + \frac{5m_\infty}{16c^4} \hat{v}^6 + \cdots. \quad (2) \]

For velocities \( |v| << c \), we see that the energy expended to boost the mass \( m_\infty \) into the instantaneous frame \( v \) moving with velocity \( v \) with respect to the rest-frame \( u \) is \( \Delta E_1 = \frac{m_\infty}{2} v^2 \), which is the classical Newtonian expression for kinetic energy of the mass \( m_\infty \) moving with velocity \( |v| \).

If, instead, we pay for the work done by deducting the required energy-equivalent from the mass \( m_\infty \), to get the residual rest-mass \( m = \frac{m_\infty}{\gamma_v} \), then the terminal energy-momentum vector of the mass \( m_\infty \) when it has reached the velocity \( v \) is

\[ p = mc^2 v = \frac{m_\infty}{\gamma_v} c^2 \hat{v} = \frac{p_\infty}{\gamma_v} uv = e^{-\frac{\hat{v}}{c\hat{u}}} \frac{1}{\gamma_v} \frac{p_\infty}{\gamma_v} e^{\frac{\hat{v}}{c\hat{u}}}. \quad (3) \]

In this equation, \( \hat{v} \) is a unit relative vector in the direction of the velocity \( v \), and \( c \tan h(\hat{v}) = |v| \) is the magnitude of the velocity as measured in the rest-frame \( u \).

Equation (3) has some easy but important consequences. We first note that \( m = \frac{m_\infty}{\gamma_v} = 0 \) when \( |v| \to c \). This means that the energy content of each material body is exactly the energy which would be required to accelerate the body to the speed of light \( c \). Assuming that we have a one hundred percent efficient photon drive, the body would reach the speed of light at precisely the moment when its last bit of mass-equivalent is expelled as a photon. A second interesting observation is that when we expand \( (m_\infty - m)c^2 = m_\infty (1 - \frac{1}{\gamma_v})c^2 \) in a Taylor series in \( |v| \) around \( |v| = 0 \), we obtain

\[ \Delta E_2 = (m_\infty - m)c^2 = \frac{m_\infty}{2} \hat{v}^2 + \frac{m_\infty}{8c^2} \hat{v}^4 + \frac{m_\infty}{16c^4} \hat{v}^6 + \cdots = \frac{\Delta E_1}{\gamma_v}. \quad (4) \]
Whereas the expressions $\Delta E_1 = \Delta E_2$ for $|v| < c$, the expression for $\Delta E_2$ is much closer to the classical kinetic energy over a much larger range of velocities $|v| < c$, and differs only by a factor of 2 when $|v| = c$.

The basic premise upon which our theory is built is that when any particle evolves on its timelike curve $x(\tau)$, subjected only to the elementary forces of nature and satisfying the initial condition that $p(0) = m_\infty c^2 u$, then its energy-momentum vector has the form $p(\tau) = m(\tau) c^2 v(\tau)$ for $m(\tau) = \frac{m_\infty}{\gamma_v}$, and satisfies the conservation law

$$p(\tau) \cdot u = m_\infty c^2 = \text{constant} \quad (5)$$

for all values $\tau \geq 0$. This law is a direct consequence of the local conservation of energy requirement $[3]$. We say that

$$m(\tau) = \frac{\sqrt{p^2}}{c^2} = \frac{p(\tau) \cdot v(\tau)}{c^2} = \frac{m_\infty}{\gamma_v} \quad (6)$$

is the instantaneous rest-mass of $m_\infty$ in the instantaneous frame $v(\tau)$.

At the atomic level, our insistence upon the strict local conservation of the energy-momentum of each particle $[3]$, means that whenever an elementary particle undergoes a change in its kinetic energy, it must pay for it with a corresponding change in its instantaneous rest-mass $[3]$. Thus, we do not accept that the rest-mass $m_\infty$ of an isolated particle is an invariant when that particle undergoes interactions. We consider that the field of an elementary particle carries only information about the location of that elementary particle, but does not magically transfer energy across spacetime to affect other elementary particles. Each elementary particle pays for any change in its kinetic energy as it navigates in space, guided by the information supplied by the four elementary forces of Nature. Consequently, an elementary particle annihilates if and only if it reaches the speed of light.

A beautiful discussion and derivation of the basic relationships of relativistic particle dynamics is given in $[7]$ and $[12]$, so we need not rederive them here. We will need, however, a number of special formulas regarding the evolution of a particle whose the energy-momentum vector is given by $p(\tau) = m(\tau) c^2 v(\tau)$ and satisfies $[3]$, as given above. The Minkowski force on such a particle as it moves along its timelike curve $x(\tau)$, is given by $f(\tau) = \frac{dp(\tau)}{d\tau}$. It is very easy to calculate the relative force $F(\tau) = \frac{1}{c^2} uf(\tau)$ as measured in the rest frame $u$. We find that

$$F(\tau) = \frac{1}{c^2} uf(\tau) = \frac{1}{c^2} \frac{dp(\tau)}{d\tau} = \frac{dt}{d\tau} \frac{d}{dt}(m_\infty + m_\infty v) = \gamma_v m_\infty a, \quad (7)$$

where $a = \frac{dv}{dt}$ is the relative acceleration experienced by the particle as measured in the rest frame $u$. Formula $[7]$ is immediately recognized as the relativistic form of Newton’s Second Law. This form of Newton’s Second Law applies to particles subjected only to elementary forces. Noting that $\frac{1}{\gamma_v} = 1 - \frac{v^2}{c^2}$, so that

$$\frac{d}{dt}(\gamma_v^{-2}) = -2 \frac{v^2}{c^2},$$

it is easy to calculate the useful formulas

$$\frac{d\gamma_v}{dt} = \gamma_v^3 \frac{v \cdot a}{c^2} \quad (8)$$

4
and, with the help of (7),
\[
\frac{dm(\tau)}{d\tau} = \gamma_v \frac{dm(\tau)}{dt} = -\gamma_v^2 m_\infty \frac{\mathbf{v} \cdot \mathbf{a}}{c^2} = -\frac{\gamma \mathbf{F} \cdot \mathbf{v}}{c^2},
\]
(9)
or
\[
\frac{dm(\tau)}{dt} = -\frac{1}{c^2} \mathbf{F} \cdot \mathbf{v}.
\]
(10)

It is well-known that the total energy-momentum vector of an isolated n-particle system is a constant of motion in every inertial system [12, p. 634]. Assuming that the only interactions between the particles are the elementary forces, so that (5) applies, it follows that the energy-momentum vector of each particle has the form
\[
\mathbf{p}_i(t) = m_i(t) c^2 \mathbf{v}_i(t),
\]
and
\[
\mathbf{p}_i(0) = m_\infty c^2 u_i,
\]
where \(u_i\) is the parameter of relative time in the rest-frame \(u\). Assuming further that there are no collisions, this conservation law takes the form
\[
\mathbf{P}(t) = \sum_{i=1}^{n} \mathbf{p}_i(t) = P_0 = \sum_{i=1}^{n} \mathbf{p}_i(0)
\]
(11)
for all \(t \geq 0\). Dotting and wedging each side of this equation on the left by \(u\), gives the equivalent statements that
\[
u \cdot \mathbf{P}(t) = \sum_{i=1}^{n} m_\infty c^2 = u \cdot P_0,
\]
meaning that the total energy of the isolated system is constant, and that the total linear momentum
\[
u \wedge \mathbf{P}(t) = \sum_{i=1}^{n} m_\infty c^2 \mathbf{v}_i(t) = u \wedge P_0 = 0
\]
of the isolated system is 0 for all values of \(t \geq 0\).

2 Change of mass due to binding energy

Let us consider an isolated system of two objects \(m_i(r)\), with the respective energy-momentum vectors \(p_i(r) = m_i(r) c^2 \mathbf{v}_i(r)\), for \(i = 1, 2\), when they are a distance \(r\) from each other as measured in the rest-frame \(u\). This means that the objects can only interact with each other, and that they begin at rest in the rest-frame \(u\) when \(r = \infty\). Thus, \(\lim_{r \to \infty} p_i(r) = m_\infty c^2 u_i\) for \(i = 1, 2\).

The conservation law (5) and the conservation law of total energy-momentum (11) applied to our two particle system gives
\[
\mathbf{P}^\infty = \mathbf{P}_1^\infty + \mathbf{P}_2^\infty = p_1^\infty + p_2^\infty = P(r)
\]
(12)
for all values of \(r \geq 0\). Equivalently,
\[
u \cdot \mathbf{P}^\infty = (m_1^\infty + m_2^\infty) c^2 = u \cdot P(r),
\]
which is the conservation of the total energy of the system for all $r \geq 0$, and
\begin{equation}
0 = \frac{u \wedge P^\infty}{c^2} = \frac{u \wedge P(r)}{c^2} = m_1^\infty v_1(r) + m_2^\infty v_2(r),
\end{equation}
which is the conservation of the total linear momentum of the system for all $r \geq 0$.

The quantities
\begin{equation}
E_i^b(r) = p_i(r) \cdot (u - v_i(r)) = m_i^\infty c^2 (1 - \frac{1}{\gamma_i}),
\end{equation}
which are seen in (11) to be closely related to the classical kinetic energy, are called (by the first author) Tolga’s binding energies of the respective bodies $m_i(r)$ when they are brought quasi-statically (very slowly) to a distance $r$ from each other in the rest-frame $u$. The total binding energy $E^b(r) = E_1^b(r) + E_2^b(r)$, is the work done by the gravitational attraction between the two bodies. With the help of formula (10), we can easily calculate
\begin{equation}
\frac{dE^b}{dt} = -c^2 \left( \frac{dm_1(\tau_1)}{dt} + \frac{dm_2(\tau_2)}{dt} \right) = \mathbf{F}_1 \cdot \mathbf{v}_1 + \mathbf{F}_2 \cdot \mathbf{v}_2 = \frac{dE^b}{dr} \frac{dr}{dt}.
\end{equation}
Whereas we are only interested here in the binding energies of the two bodies due to the force of gravity, all our considerations can be applied more broadly [5].

Let us directly calculate the change of the rest-masses $m_1^\infty$ and $m_2^\infty$ as the two masses move under the force of gravity. Very simply, the instantaneous rest-masses $m_i(E^b)$ are specified by
\begin{equation}
m_i(E^b) = m_i^\infty - \frac{E_i^b}{c^2},
\end{equation}
where $E_i^b$ is the instantaneous binding energy of $m_i^\infty$, as follows directly from the binding condition (13). The total binding energy between the instantaneous rest-masses $m_1(E_1^b)$ and $m_2(E_2^b)$ is given by $E^b = E_1^b + E_2^b$. For our considerations below, we will assume that $m_2^\infty = sm_1^\infty$ for a constant value of $s \geq 1$, so that $m_2^\infty \geq m_1^\infty$.

Because of the total binding energy $E^b$ expended by the forces acting between them, as measured in the rest-frame $u$, the bodies will have gained the respective velocities $\mathbf{v}_1(E^b)$ and $\mathbf{v}_2(E^b)$, fueled by the respective losses to their rest-masses $m_1^\infty$ and $m_2^\infty$. Precisely, we can say that
\begin{equation}
m_1^\infty - f_1 \frac{E_1^b}{c^2} = \frac{m_1^\infty}{\gamma_i}
\end{equation}
where $f_i$ is the fraction of the total binding energy $E^b$ given up by $m_i^\infty$ for $i = 1, 2$, respectively. This means that $f_1 + f_2 = 1$, and, by the conservation of linear momentum (13), we also know that 
\begin{equation}
(m_1^\infty)^2 v_1^2 = (m_2^\infty)^2 v_2^2 \text{ or } v_2^2 = \frac{1}{s^2} v_1^2.
\end{equation}
Using this information, leads to the system of equations
\begin{equation}
m_1^\infty \left(1 - \sqrt{1 - \frac{v_1^2}{c^2}}\right) - f_1 \frac{E_1^b}{c^2} = 0 \text{ and } m_1^\infty \left(s - \sqrt{s^2 - \frac{v_1^2}{c^2}}\right) - (1 - f_1) \frac{E_1^b}{c^2} = 0.
\end{equation}
Solving the system of equations (18) for \( f_1 \) and \( v_1^2 \) in terms of the binding energy \( E_b \), we find that

\[
f_1(E_b) = 1 - \frac{m_1^\infty s c^2}{E_b} + \frac{2m_1^\infty s(s + 1)c^4 - 2E_b m_1^\infty (s + 1)c^2 + (E_b)^2}{2E_b(E_b - c^2 m_1^\infty (s + 1))}
\]

and \( v_1^2(E_b) \)

\[
= -\frac{E_b(E_b - 2c^2 m_1^\infty) (4m_1^\infty s(s + 1)c^4 - 2E_b m_1^\infty (2s + 1)c^2 + (E_b)^2)}{4c^2 m_1^\infty (E_b - c^2 m_1^\infty (s + 1))^2}.
\] (19)

In the interesting special case when \( m_2^\infty = sm_1^\infty \) and \( s \to \infty \), we find that the velocity

\[
v_1^2 \to \frac{E_b(2c^2 m_1^\infty - E_b)}{c^2(m_1^\infty)^2}.
\] (20)

We will use this result later.

Similarly, we can now obtain the instantaneous rest-masses

\[
m_1(E_b) = m_1^\infty (1 - f_1 \frac{E_b}{m_1^\infty c^2})
\]

or

\[
m_1(E_b) = m_1^\infty (1 + s) - \frac{E_b}{c^2} - \frac{2m_1^\infty s(s + 1)c^4 - 2m_1^\infty (s + 1)E_b c^2 + (E_b)^2}{2c^2(E_b - c^2 m_1^\infty (s + 1))}
\] (21)

and \( m_2(E_b) = sm_1^\infty (1 - f_1 \frac{E_b}{m_2^\infty c^2}) \) or

\[
m_2(E_b) = \frac{2m_1^\infty s(s + 1)c^4 - 2m_1^\infty (s + 1)E_b c^2 + (E_b)^2}{2c^2(E_b - c^2 m_1^\infty (s + 1))}.
\] (22)

We now calculate for what critical value \( E_c \) of the binding energy \( E_b \) the smaller mass \( m_1(E_c) = 0 \). We find that

\[
E_c = c^2 m_1^\infty \left( s + 1 - \sqrt{s^2 - 1} \right).
\]

For this value of the binding energy \( E_b \), we find that

\[
m_2(E_c) = m_1^\infty \sqrt{s^2 - 1}, \quad v_1^2(E_c) = c^2, \quad \text{and} \quad v_2^2(E_c) = \frac{v_1^2(E_b)}{s^2}.
\]

We also find that \( f_1(E_c) = \frac{1}{1 + s - \sqrt{s^2 - 1}} \).

It is interesting to graph the instantaneous rest-masses \( m_i(E_b) \) for \( i = 1, 2 \), the velocity \( |v_1(E_b)| \) and the fraction \( f_1(E_b) \) of the binding energy being consumed by the first mass, in terms of the total binding energy \( E_b \) being expended. In Figure 1, the velocity of light \( c = 1 \), the mass \( m_1^\infty = 1, m_2^\infty = \sqrt{2} \), and the binding energy \( E_b \) satisfies the constraints \( 0 \leq E_b \leq \sqrt{2} \). At the critical value \( E_b = \sqrt{2} \) the mass \( m_1^\infty \) has entirely consumed itself. Note that up to now, we have made no assumption regarding the nature of the force or forces which produce this binding energy. In the next section, we will assume that the binding energy is due to an inverse square law attractive force such as that due to Newton’s law of gravitational attraction.
Figure 1: The masses \( m_1(E^b) \) and \( m_2(E^b) \), the velocity \( |v_1(E^b)| \) and \( f_1(E^b) \) are plotted as functions of the binding energy \( E^b \). Initially, \( m_1(0) = 1 \), and \( m_2(0) = \sqrt{2} \).

3 Binding energy due to Newton’s gravitational force

Current knowledge tells us that there are four fundamental forces in Nature acting between the two objects. We will consider here only the force due to gravitational attraction between the two objects with histories \( x_i(r) \) and the respective Minkowski energy-momentum vectors \( p_i(r) = m_i(r)c^2v_i(r) \), where the instantaneous rest masses are given by \( m_i(r) = \frac{m_\infty}{\gamma_i(r)} \), and where \( r \) is the distance between their centers as measured in the rest frame \( u \).

Thus, the two bodies \( m_1(r) \) and \( m_2(r) \), in their respective instantaneous frames \( v_1(r) \) and \( v_2(r) \) at a distance of \( r \), will experience a mutually attractive force

\[
F = \frac{Gm_1(r)m_2(r)}{r^2} = \frac{G}{c^4} \sqrt{\frac{p_1^2p_2^2}{(x_1-x_2)^4}},
\]

(23)

where \( G = 6.67 \times 10^{-11} N \frac{m^2}{kg^2} \) is Newton’s constant. It is worth recalling that

\[
r = |x_1 - x_2| = \sqrt{[(x_1 - x_2) \cdot u]^2} = \sqrt{-(x_1 - x_2)^2},
\]

since \( (x_1 - x_2) \cdot u = ct - ct = 0 \) for the simultaneous events \( x_1(r) \) and \( x_2(r) \) at the time \( t \) as measured in the rest frame \( u \). The fact that we can express (23) entirely in terms of the energy-momentum vectors \( p_i \) and the histories \( x_i \) implies that Newton’s Law is Lorentz invariant. An explanation of how the \( \frac{1}{r^2} \) dependency of Newton’s Law becomes a requirement of special relativity can be found in [5].

In the case that the binding energy between the two bodies is totally due to Newton’s gravitational attraction \( 23 \), we can write down the differential
Figure 2: This figure is the same as figure 1, except that the numerical inverse solution $r(E^b)$ for the distance $r$ between the two bodies, acted upon by the force of gravity, is shown as a function of the binding energy $E^b$. Note that the value of $r \to 0$ at exactly the moment the binding energy $E^b = \sqrt{2}$, and that $\lim_{E^b \to 0} r(E^b) = \infty$.

equation for the total binding energy $E^b(r)$ as a function of the distance $r$ between the two bodies as measured in the rest-frame $u$. We get

$$
\frac{dE^b}{dr} = - \frac{Gm_1(E^b(r))m_2(E^b(r))}{r^2}
$$

(24)

where $m_1(E^b(r))$ and $m_2(E^b(r))$ are given in (21) and (22), respectively. Making these substitutions, we arrive at the rather complicated Riccati-like differential equation

$$
4G(m_1^\infty)^4 s(s+1)^2(2s+1)c^8 - 4G(m_1^\infty)^3(s+1)(5s^2+6s+1)E^b(r)c^6
$$

$$
+ 2G(m_1^\infty)^2(11s^2+18s+7)(E^b(r))^2c^4
$$

$$
- 12G(m_1^\infty)(s+1)(E^b(r))^3c^2 + 3G(E^b(r))^4
$$

$$
+ \left(-4(m_1^\infty)^2r^2(s+1)^2c^8 + 8(m_1^\infty)r^2(s+1)E^b(r)c^6 - 4r^2(E^b(r))^2c^4\right)E^{b'}(r)
$$

= 0.

We shall consider the solutions of various special cases of this differential equation.

We first consider a numerical solution in the case that $m_1^\infty = 1$, $m_2 = s = \sqrt{2}$, and the constants $G = c = 1$. For this case, the graph of the solution is given in Figure 2. Note that we are actually plotting the inverse function $r(E^b)$ of the solution. This is permissible because, as can be seen in the figure, $r(E^b)$ is a strictly decreasing function in the physical range of interest for $0 < E^b \leq \sqrt{2}$. 
1
0.8
0.6
0.4
0.2

Figure 3: The mass $m_1(r)$, the binding energy $E^b(r)$, and the velocity $|v_1|$ are shown for $0 \leq r \leq 60000$. This is the case of binding to a celestial body. To make this figure, we have assumed that $m_2^\infty = 10000m_1^\infty$ where $m_1^\infty = 1$.

Note also that $r(\sqrt{2}) = 0$, although the accuracy of the numerical solution does not clearly show this.

In the case that the body $m_2^\infty$ is so massive that $m_2(r) = m_2^\infty$ for all values of $r \geq 0$, the differential equation (24) becomes

$$\frac{dE^b}{dr} = -m_2^\infty \frac{Gm_1(r)}{r^2},$$

which, together with the boundary condition that $E^b(\infty) = 0$, gives the particularly surprising solution

$$E^b(r) = E^b_1(r) = c^2 (1 - e^{-Gm_2^\infty/c^2r}) m_1^\infty,$$

or solving (16) for $m_1(r)$,

$$m_1(r) = e^{-\frac{Gm_2^\infty}{c^2r}} m_1^\infty.$$

Using (20) and the expression for $E^b(r)$ above, we find the velocity

$$|v_1(r)| = c \left(1 - e^{-\frac{Gm_2^\infty}{c^2r}}\right).$$

See Figure 3. The differential equation (25) and its solution, were first derived in [5], and a discussion of how it is related to the total energy found by Einstein can be found therein.

Another interesting two body case is when the masses $m_1^\infty = m_2^\infty$. In this case the differential equation for the binding energy becomes

$$\frac{dE^b(r)}{dr} = 2 \frac{dE^b_1}{dr} = -\frac{Gm_2^\infty}{r^2} = -\frac{G(m_1^\infty - \frac{E^b_1(r)}{c^2})^2}{r^2},$$

(26)
which has the simple solution
\[ E_b^1(r) = \frac{2c^2 G(m_1^\infty)^2}{G m_1^\infty + 2c^2 r}. \]

We also easily find
\[ m_1(r) = m_1^\infty - \frac{E_b^1(r)}{c^2} = \frac{2c^2 m_1^\infty r}{G m_1^\infty + 2c^2 r}, \]
and using (19), the velocity
\[ |v_1(r)| = c \sqrt{\frac{G m_1^\infty (G m_1^\infty + 4c^2 r)}{G m_1^\infty + 2c^2 r}}. \]

See Figure 4. The terminal velocities of the equal bodies \( m_1(r) \) and \( m_2(r) \), when they self-annihilate, are equal to the speed of light \( c \).

Figures 3 and 4 strongly suggest that black holes do not exist. Whenever a light mass approaches an extremely dense object, depending upon initial conditions, it will necessarily self-annihilate or coalesce. There cannot be any critical mass which would define the Schwarzschild radius of a black hole.

### 4 Discussion

A major problem of general relativity is that it does not easily lend itself to quantization, although Einstein, himself, apparently did not believe in quantum mechanics [13]. We have seen that, theoretically, when a mass falls from infinity into a larger mass it will self-annihilate at \( r = 0 \). However, quantum
mechanics implies that the object’s dimensions effectively become that of space itself at $r = 0$. The restrictions of quantum mechanics imply, therefore, that $r$ can never reach the value $r = 0$ for macroscopic objects. In the case of elementary particles, where additional forces other than gravity are known to be at work, self-annihilation does occur. We have already seen that in our approach singularities, even those arising from the inverse square dependency of Newton’s Law, disappear.

Indeed, taking into account how unit lengths quantum mechanically stretch in a gravitational field, the second author obtained the precession of the perihelion of Mercury as well as the deflection of light passing near a celestial body [5]. Typically, these have been considered to be the best proofs of the validity of Einstein’s general theory of relativity.

A consequence of our theory is that black holes of macroscopic objects solely due to the force of gravity do not exist. Rather, when a sufficient amount of mass coalesces in space, the object becomes either invisible or nearly invisible due to the extreme red-shift near such a body. We thus predict that very dark objects, but no black holes, should be found in the center of many galaxies. On the other hand, if a sufficient amount of mass coalesces causing a total collapse to values of $r$ so small that other elementary forces become predominant, then it becomes plausible that there will be a partial or even a total annihilation of the macroscopic body with a corresponding large burst of energy. This may explain the presence of the recently discovered “biggest expanse of nothing”, a billion light years wide, which is the space that would normally be occupied by thousands of galaxies. “No stars, no galaxies, no anything” [14].

Although our theory produces results that are practically the same as those of the General Theory of Relativity, they are only the same up to a third order Taylor expansion. For a further discussion of the issues involved and how a quantum theory of gravity becomes possible in this setting, see [4, 5] and the references therein. Ultimately, the value of any theory rests not upon the conviction or authority of its authors, but on the fruits of its predictions and its ability to encompass and explain experimental results.

Acknowledgements

The first author thanks Dr. Guillermo Romero, Academic Vice-Rector, and Dr. Reyla Navarro, Chairwomen of the Department of Mathematics, at the Universidad de Las Americas for continuing support for this research. He and is a member of SNI 14587. The second author is grateful to Dr. V. Rozanov, Director of the Laser Plasma Theory Division, Lebedev Institute, Russia, to Dr. C. Marchal, Directeur Scientifique de l’ONERA, France, to, Dr. Alwyn Van der Merwe, Editor, Foundations of Physics, to Dr. O. Sinanoglu from Yale University, to Dr. S. Kocak from Anadolu University, and to Dr. E. Hasanov from Isik University; without their sage understanding and encouragement, the seeds of this work could never have born fruit.
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