TASI Lectures on M Theory Phenomenology

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Abstract

These lectures discuss some of the general issues in developing a phenomenology for Superstring Theory/M Theory. The focus is on the question: how might one obtain robust, generic predictions. For example, does the theory predict low energy supersymmetry breaking? In the course of these explorations, basics of supersymmetry and supersymmetry breaking, string moduli, cosmological issues, and other questions are addressed. The notion of approximate moduli and their possible role plays a central role in the discussion.
1 Introduction

How might string/M theory make contact with nature? In one view, we might imagine that some day, one will find some “true ground state,” or set of ground states, and that we will simply calculate all of the quantities of low energy physics. Another viewpoint holds that these problems are impossibly hard, at least with any theoretical tools we have today or which we can see on the horizon, and that we should focus exclusively on theoretical issues: theoretical consistency, problems of black holes and other quantum gravity questions, and the like.

In these lectures, I advocate a middle ground. While I don’t believe it is likely that we will succeed in solving the theory completely any time soon, I believe it might be possible to make a few robust, qualitative statements, and perhaps a small number of quantitative ones. If we could reliably assert, for example (preferably before its discovery), that low energy supersymmetry is a prediction of string theory, with some rough pattern of soft breakings, this would be a triumph. If we could predict one or two mass ratios, or the value of the gauge couplings, this would be spectacular. If string theory resolved some of the problems of cosmology, this would be a major achievement.

In these lectures I will not succeed in accomplishing any of these goals, but I do hope to outline the major issues in bringing string theory into contact with nature. Our strategy will be to focus on the major issues in developing a string phenomenology:

- The cosmological constant
- The problem of vacuum degeneracy
- The Hierarchy problem
- The role of supersymmetry
- The smallness of the gauge couplings and gauge coupling unification
- The size (and shape) of extra dimensions
- The question of CP violation and the strong CP problem
- Issues of flavor
- Questions in Cosmology
In line with our remarks above, the goals of a superstring phenomenology should be to obtain qualitative, generic predictions, such as

- Low energy supersymmetry
- Extra light particles
- The pattern of supersymmetry breaking
- Statements about early universe cosmology, including dark matter, the evolution of moduli, possible inflaton candidates, and perhaps the value of the cosmological constant.
- Axions
- Predictions for rare processes

To appreciate the difficulties, consider the problem of low energy supersymmetry. Low energy supersymmetry has long been touted as a possible solution of the hierarchy problem. Supersymmetry seems to play a fundamental role in string theory, and numerous solutions to the classical string equations with $N = 1$ supersymmetry in four dimensions are known. Still, we have no reliable computation of a stable vacuum with broken supersymmetry. Nor do we know of any principle which suggests that vacua with approximate supersymmetry are somehow more special than those without. These problems are closely tied to our lack of understanding of the cosmological constant problem.

It is easiest, as we will see, to study supersymmetric states; the more supersymmetry, the easier. But this by itself is hardly an argument for supersymmetry. We will need some more persuasive argument if we are to make a statement that string theory does or does not predict low energy supersymmetry. An overriding question should be: can we make such a statement without knowledge of the precise ground state?

We might hope to get some insight into these questions by considering two of the problems posed above: the cosmological constant problem and the hierarchy problem. Both represent failures of dimensional analysis. It is possible to add to the effective action of the standard model the terms:

$$
\int d^4x \sqrt{g} (\Lambda + M^2 |H|^2).
$$

Here $\Lambda$ is a quantity with dimensions of $M^4$, and so one might expect that it is of order some large scale in nature, $\Lambda \sim M_p^4$. On the other hand, there are recent claims that a cosmological
constant has been observed, with a value of order

$$\Lambda \approx 10^{-47} \text{ GeV}^4.$$  \hspace{1cm} (2)

Even if one is skeptical of this result (and evidence is steadily mounting that it is correct), the cosmological constant is at least as small as this. Similarly, dimensional analysis suggests that the Higgs mass should be of order $M_p$, but in the electroweak theory it must be many orders of magnitude smaller.

't Hooft gave a precise statement of a widely held notion of naturalness[2]. He argued that if a quantity is much smaller than expected from dimensional analysis, this should be because the theory becomes more symmetric in the limit that the quantity tends to zero. This is a familiar story for the quark and lepton masses; in the limit that the corresponding Yukawa couplings tend to zero, the standard model acquires additional chiral symmetries. As a result, quantum corrections to the Yukawa couplings vanish as the couplings tend to zero.

String theory, on the other hand, sometimes provides exceptions to this rule, and indeed it does provide an exception in the case of the cosmological constant. There are many (non-supersymmetric) vacua of string theory in which the cosmological constant vanishes at tree level. This is already surprising, and a violation of 't Hooft’s rule. The vanishing of the cosmological constant in these theories is a consequence not of a symmetry in space-time, but of a world-sheet symmetry of string theory, \textit{conformal invariance}. It is an example of what is technically known as a “string miracle”[3].

On the other hand, for most such states, a cosmological constant is generated at one loop, and this is far too large to be compatible with the observed number. At one loop, in field theory, the cosmological constant is given by

$$\Lambda = \sum_{\text{helicities}} (-1)^F \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \sqrt{\vec{p}^2 + m_i^2}$$ \hspace{1cm} (3)

This expression is correct in (weakly coupled) string theory, provided that the sum over states is suitably interpreted[4, 5]. For non-supersymmetric states, it is typically of order coupling constants times the string tension to the appropriate power.

In supersymmetric states, $\Lambda = 0$, typically, at least in perturbation theory. On the other hand, supersymmetry must be broken in nature, and from the formula above, we expect that

$$\Lambda = m_{\text{susy}}^4$$ \hspace{1cm} (4)
This is compatible with ’t Hooft’s naturalness principle. But we expect that $m_{\text{susy}}$ is not smaller than 100 GeV, so we are still at least 55 orders of magnitude off.

This problem is perhaps the most serious obstacle to understanding string phenomenology. In these lectures, we will not offer any solutions. Only a small number of ideas have been proposed, and they are, as yet (at best) incomplete. Here we mention two:

- Witten has noted that in three dimensions, supersymmetry can be unbroken without degeneracy between bosons and fermions\[6\]. He imagines a three dimensional theory in which there is a single modulus. Now suppose that one takes the limit of strong coupling. Typically, one expects that this is a theory with one additional dimension. Perhaps this four dimensional limit is a theory with broken supersymmetry but vanishing cosmological constant. Dabholkar and Harvey have constructed models in various dimensions with small numbers of moduli\[7\].

- Kachru and Silverstein, motivated in part by the AdS/CFT correspondence, have constructed models without supersymmetry in which the cosmological constant vanishes at low orders of perturbation theory\[8\]. Conformal invariance plays a crucial role in these constructions, and the existing examples have Bose-Fermi degeneracy. Still, this is perhaps the most successful proposal to date.

Let us turn now to the hierarchy problem. This is a similar problem of dimensional analysis. We might expect that $m_{H}^{2} \sim M_{p}^{2}$, but we know that if there is a fundamental Higgs scalar, its mass must be less than about 1 TeV. The standard model does not become more symmetric in the limit that the Higgs mass becomes small, so this would seem to be a violation of ’t Hooft’s notion of naturalness.

String theory again offers an interesting perspective on this question. In many weak ground states, at weak coupling, there are massless particles, whose masses are not protected by any symmetry. This is already a violation of naturalness, and can usually be understood in terms of world sheet symmetries. In general, however, one expects radiative corrections to these masses, just as for the cosmological constant. For the Higgs particle in the standard model, for example, loops of gauge bosons give corrections to the Higgs mass:

$$m_{H}^{2} \propto \frac{g^{2}}{16\pi^{2}} \int \frac{d^{4}k}{(2\pi)^{4}k^{2}}$$  \hspace{1cm} (5)$$

In string theory, we expect this quadratically divergent integral will be cut off at the string scale. Unlike the case of the cosmological constant, however, for the problem of scalar masses
supersymmetry can offer a resolution. If the state is approximately supersymmetric, then the integrals are cut off, not at the string scale, but at the scale of supersymmetry breaking, $m_{s \text{usy}}^2$. In terms of Feynman diagrams, one has, due to supersymmetry, more types of particles, and one finds cancellations\[9\].

If this is the correct explanation, it requires that $m_{s \text{usy}}^2$ not be significantly larger than, say, 1 TeV. So we can hope for discovery soon, perhaps even at LEPII or at the Tevatron, and certainly at the LHC.

In string theory, these considerations aside, supersymmetry seems to play an important role, perhaps suggestive of low energy supersymmetry. It is easy to find states which, in some approximation, have low energy supersymmetry ($N = 1, 2, 4, 8$ in four dimensional counting). We will see, also, that there are mechanisms for breaking supersymmetry at low energies. But we will need to ask: to what degree is unbroken supersymmetry a fundamental property of string theory, and to what degree is it simply a crutch which gives us some theoretical control?

In any case, supersymmetry is important to our present understanding of string/M theory. It provides us with a great deal of control over dynamics. It is, for example, the basic of all of our current understanding of the many dualities of the theory, as well as of possible non-perturbative formulations (also dualities) such as matrix theory and the AdS/CFT correspondence. We will use this power throughout these lectures.

An alternative proposal for understanding the hierarchy problem is to suppose that the fundamental scale is not $M_p$ but actually of order 1 TeV. This requires that there be some large internal dimensions$[10, 11]$, or a suitable “warp factor” in the extra dimensions$[12]$. These possibilities have received much attention in the last year. At this point, they do not appear significantly less plausible than low energy supersymmetry as a solution. We will focus in these lectures mostly on supersymmetry, since, as we will see, it is, within our current understanding, easier to develop scenarios for the realization of supersymmetry in string theory where there is some understanding of why couplings are weak and of which quantities might be calculable. But given our lack of a complete picture, this may well reflect simply “the state of the art.” We will make some comments on these ideas, particularly in section 9.

The rest of these lectures will be devoted to developing tools for thinking about string dynamics and string phenomenology, based largely on supersymmetry. The next section presents a brief overview of supersymmetry, its representations, the structure of supersymmetric lagrangians (global and local), and the use of superspace. Section 3 discusses some quantum
aspects of supersymmetric theories. For theories with $N = 2$ and $N = 4$ supersymmetry, we will see that moduli are exact quantum mechanically, and prove certain non-renormalization theorems. For the case of $N = 1$ supersymmetry, we will see that fields which are moduli in some approximation generically have non-trivial potentials in the full theory. We will introduce the notion of \textit{approximate moduli}, discuss supersymmetry breaking, and also consider circumstances under which moduli are exact. The fourth section consists of a brief review of $N = 1$ supersymmetry phenomenology: the Minimal Supersymmetric Standard Model (MSSM), soft breakings, counting parameters, constraints, direct detection and theories of soft breakings.

After this, we turn to string theory. Section 5 focuses on string moduli. It is possible to make many exact statements about the full, microscopic theory by focusing on a low energy effective lagrangian for the light fields. We explain why, in much the same way as for field theory, one can discuss the exactness of string moduli and issues of non-renormalization. Even in theories with $N = 1$ supersymmetry, in some cases the problem of supersymmetry breakdown is a problem of low energy physics. We discuss the issue of modulus stabilization, and moduli in cosmology. We make some tentative statements about string phenomenology.

The sixth section is devoted to string phenomenology in the light of duality. We will focus particularly on the Horava-Witten picture, the role of branes, and on recent proposals that the string scale might be as low as 1 TeV.

In the seventh section we will assess the outlook for achieving the goal we set forth in the beginning, of obtaining a few robust predictions from string theory. Overall, we will cover many topics, and offer some speculations, but we won’t provide any real answers to the big questions.

2 An Overview of Supersymmetry

2.1 The Supersymmetry Algebra and its Representations

In this lecture, we will collect a few facts that will be useful in the subsequent discussion. We won’t attempt a thorough introduction to the subject. This is provided, for example, by Lykken’s lectures\cite{13,14}, Wess and Bagger’s text\cite{15}, and Appendix B of Polchinski’s text\cite{16}.

Supersymmetry, even at the global level, is remarkable, in that the basic algebra involves the translation generators:

$$\{Q^A_\alpha, Q^B_\beta\} = 2\sigma_{\alpha\beta}^{AB}\delta^{AB}P_\mu \quad (6)$$
\[ \{Q^A_\alpha, Q^*_B \beta \} = \epsilon_{\alpha\beta} X^{AB}. \]  

The \( X^{AB} \)'s are Lorentz scalars, antisymmetric in \( A, B \), known as central charges.

If nature is supersymmetric, it is likely that the low energy symmetry is \( N = 1 \), corresponding to only one possible value for the index \( A \) above. Only \( N = 1 \) supersymmetry has chiral representations. In addition, \( N > 1 \) supersymmetry, as we will see, is essentially impossible to break; this is not the case for \( N = 1 \). For \( N = 1 \), the basic representations of the supersymmetry algebra, on massless fields, are

- Chiral superfields fields: \((\phi, \psi_\alpha)\), a complex fermion and a chiral scalar
- Vector superfields: \((\lambda, A_\mu)\), a chiral fermion and a vector meson, both, in general, in the adjoint representation of the gauge group
- The gravity supermultiplet: \((\psi_\mu, \alpha, g_{\mu\nu})\), a spin-3/2 particle, the gravitino, and the graviton.

\( N = 1 \) supersymmetric field theories are conveniently described using superspace. The space consists of bosonic coordinates, \( x^\mu \), and Grassman coordinates, \( \theta_\alpha, \theta^*_\dot{\alpha} \). In the case of global supersymmetry, the description is particularly simple. The supersymmetry generators, classically, can be thought of as operators on functions of \( x^\mu, \theta, \theta^* \):

\[ Q_\alpha = \partial_\alpha - i \sigma^{\mu}_{\dot{\alpha} \alpha} \theta^{\dot{\alpha}} \partial_\mu; \quad \bar{Q}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} + i \theta_\alpha \sigma^\mu_{\alpha \dot{\alpha}} \partial_\mu. \]  

A general superfield, \( \Phi(x, \theta, \bar{\theta}) \) contains many terms, but can be decomposed into two irreducible representations of the algebra, corresponding to the chiral and vector superfields described above. To understand these, we need to introduce one more set of objects, the covariant derivatives, \( D_\alpha \) and \( \bar{D}_{\dot{\alpha}} \). These are objects which anti-commute with the supersymmetry operators, thus are useful for writing down invariant expressions. They are given by

\[ D_\alpha = \partial_\alpha + i \sigma^\mu_{\alpha \dot{\alpha}} \theta^{\dot{\alpha}} \partial_\mu; \quad \bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i \theta_\alpha \sigma^\mu_{\alpha \dot{\alpha}} \partial_\mu. \]  

With this definition, chiral fields are defined by the covariant condition:

\[ \bar{D}_{\dot{\alpha}} \Phi = 0. \]  

Chiral fields are annihilated by the covariant derivative operators. In general, these covariant derivatives anticommute with the supersymmetry operators, \( Q_\alpha \), so the condition is a
covariant condition. This is solved by writing

$$\Phi = \Phi(y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y).$$

where

$$y = x^\mu + i\theta\sigma^\mu \bar{\theta}.$$  \hfill (12)

Vector superfields form another irreducible representation of the algebra; they satisfy the condition

$$V = V^\dagger$$

Again, it is easy to check that this condition is preserved by supersymmetry transformations. $V$ can be expanded in a power series in $\theta$'s:

$$V = i\chi - i\chi^\dagger - \theta\sigma^\mu \theta^\dagger A_\mu + i\theta^2 \bar{\theta}\lambda - i\bar{\theta}^2 \theta\lambda + \frac{1}{2} \theta^2 \bar{\theta}^2 D.$$ \hfill (14)

In the case of a $U(1)$ theory, gauge transformations act by

$$V \to V + i\Lambda - i\Lambda^\dagger$$ \hfill (15)

where $\Lambda$ is a chiral field. So, by a gauge transformation, one can eliminate $\chi$. This gauge choice is known as the Wess-Zumino gauge. This gauge choice breaks supersymmetry, much as choice of Coulomb gauge in electrodynamics breaks Lorentz invariance.

In the case of a $U(1)$ theory, one can define a gauge-invariant field strength,

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V.$$ \hfill (16)

In Wess-Zumino gauge, this takes the form

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \sigma^{\mu\nu\beta}_\alpha F_{\mu\nu} \theta_\beta + \theta^2 \sigma^{\mu\beta}_\alpha \partial_\mu \lambda^\beta.$$ \hfill (17)

This construction has a straightforward non-Abelian generalization in superspace, which is described in the references. When we write the lagrangian in terms of component fields below, the non-abelian generalization will be obvious.
2.2 N=1 Lagrangians

One can construct invariant lagrangians by noting that integrals over superspace are invariant up to total derivatives:

$$\delta \int d^4x \int d^4\theta \ h(\Phi, \Phi^\dagger, V) = \int d^4xd^4\theta \ (\epsilon_\alpha Q^\alpha + \epsilon_{\dot{\alpha}} Q^{\dot{\alpha}})h(\Phi, \Phi^\dagger, V) = 0.$$  \hspace{1cm} (18)

For chiral fields, integrals over half of superspace are invariant:

$$\delta \int d^4xd^2\theta f(\Phi) = (\epsilon_\alpha Q^\alpha + \epsilon_{\dot{\alpha}} Q^{\dot{\alpha}})f(\Phi).$$  \hspace{1cm} (19)

The integrals over the $Q_\alpha$ terms vanish when integrated over $x$ and $d^2\theta$. The $Q^*_\alpha$ terms also give zero. To see this, note that $f(\Phi)$ is itself chiral (check), so

$$Q^*_\alpha f \propto \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu f.$$  \hspace{1cm} (20)

We can then write down the general renormalizable, supersymmetric lagrangian:

$$\mathcal{L} = \frac{1}{g^{(4)}} \int d^2\theta W^{(i)}_\alpha + \int d^4\theta \Phi^\dagger_i e^{q_i V} \Phi_i \int d^2\theta W(\Phi_i) + c.c.$$  \hspace{1cm} (21)

The first term on the right hand side is summed over all of the gauge groups, abelian and non-abelian. The second term is summed over all of the chiral fields; again, we have written this for a $U(1)$ theory, where the gauge group acts on the $\Phi_i$'s by

$$\Phi_i \rightarrow e^{-q_i \Lambda} \Phi_i$$  \hspace{1cm} (22)

but this has a simple non-abelian generalization. $W(\Phi)$ is a holomorphic function of the $\Phi_i$'s (it is a function of $\Phi_i$, not $\Phi_i^\dagger$).

In terms of component fields, his lagrangian takes the form, in the Wess-Zumino gauge:

$$\mathcal{L} = -\frac{1}{4}g_a^{-2} F^a_{\mu\nu} - i\lambda^a \sigma^\mu D_\mu \lambda^a + i \psi_i \sigma^\mu D_\mu \psi^*_i + \frac{1}{2g^2} (D^a)^2 + D^a \sum_i \phi^*_i T^a \phi_i$$  \hspace{1cm} (23)

$$+ F^*_i F_i - F_i \frac{\partial W}{\partial \phi_i} + cc + \sum_{ij} \frac{1}{2} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_i \psi_j + i\sqrt{2} \sum \lambda^a \psi_i T^a \phi^*_i.$$  \hspace{1cm} (23)

The scalar potential is found by solving for the auxiliary $D$ and $F$ fields:

$$V = |F_i|^2 + \frac{1}{2g_a^2} (D^a)^2.$$  \hspace{1cm} (24)
with
\[ F_i = \frac{\partial W}{\partial \phi_i^*}, \quad D^a = \sum_i (g^a_i \phi_i^* T^a \phi_i). \] (25)

In this equation, \( V \geq 0 \). This fact can be traced back to the supersymmetry algebra. Starting with the equation
\[ \{ Q_\alpha, Q_\beta \} = 2 P_\mu \sigma^\mu_{\alpha\beta}, \] (26)

Multiplying by \( \sigma^\alpha \) and take the trace:
\[ Q_\alpha Q_\alpha + Q_\beta Q_\beta = E. \] (27)

If supersymmetry is unbroken, \( Q_\alpha |0\rangle = 0 \), so the ground state energy vanishes if and only if supersymmetry is unbroken. Alternatively, consider the supersymmetry transformation laws for \( \lambda \) and \( \psi \). One has, under a supersymmetry transformation with parameter \( \epsilon \),
\[ \delta \psi = \sqrt{2\epsilon} F + \ldots \quad \delta \lambda = i\epsilon D + \ldots \] (28)

So if either \( F \) or \( D \) has an expectation value, supersymmetry is broken.

We should stress that these statements apply to global supersymmetry. We will discuss the case of local supersymmetry later, but, as we will see, many of the lessons from the global case extend in a simple way to the case in which the symmetry is a gauge symmetry.

We can now very easily construct a supersymmetric version of the standard model. For each of the gauge fields of the usual standard model, we introduce a vector superfield. For each of the fermions (quarks and leptons) we introduce a chiral superfield with the same gauge quantum numbers. Finally, we need at least two Higgs doublet chiral fields; if we introduce only one, as in the simplest version of the standard model, the resulting theory possesses gauge anomalies and is inconsistent. In other words, the theory is specified by giving the gauge group \((SU(3) \times SU(2) \times U(1))\) and enumerating the chiral fields:
\[ Q_f, \bar{u}_f, \bar{d}_f, L_f, \bar{e}_f, H_U, H_D. \] (29)

The gauge invariant kinetic terms, auxiliary \( D \) terms, and gaugino-matter Yukawa couplings are completely specified by the gauge symmetries. The superpotential can be taken to be:
\[ W = H_U (\Gamma_U)_{f,f'} Q_f \bar{U}_{f'} + H_D (\Gamma_D)_{f,f'} Q_f \bar{D}_{f'} + H_D (\Gamma_E)_{f,f'} L_f \bar{e}_{f'}. \] (30)
As we will discuss shortly, this is not the most general lagrangian consistent with the gauge symmetries. It does yield the desired quark and lepton mass matrices, without other disastrous consequences.

**Exercise:** Consider the case of one generation. Show that if

\[
\langle H_U \rangle = \langle H_D \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix},
\]

(all others vanishing), then

\[
\langle D^a \rangle = 0; \langle F^i \rangle = 0.
\]

Study the spectrum of the model. Show that the superpartners of the \(W\) and \(Z\) are degenerate with the corresponding gauge bosons. (Note that for the massive gauge bosons, the multiplet includes an additional scalar). Show that the quarks and leptons gain mass, and are degenerate with their scalar partners.

The fact that the states fall into degenerate multiplets reflects that for this set of ground states (parameterized by \(v\)), supersymmetry is unbroken. That supersymmetry is unbroken follows from the fact that the energy is zero, by our earlier argument. It can also be understood by examining the transformation laws for the fields. For example,

\[
\delta \phi_i = \zeta_\alpha [Q^\alpha, \phi_i] = \sqrt{2}i \zeta \psi
\]

but the right hand side has no expectation value. Similarly,

\[
\delta \psi_i = \sqrt{2} \zeta F_i + \sqrt{2}i \sigma^\mu \zeta \partial_\mu \phi.
\]

The last term vanishes by virtue of the homogeneity of the ground state; the first vanishes because \(F_i = 0\). Similar statements hold for the other possible transformations.

This is our first example of a moduli space. Classically, at least, the energy is zero for any value of \(v\). So we have a one parameter family of ground states. These states are physically inequivalent, since, for example, the mass of the gauge bosons depends on \(v\). We will shortly explain why, in field theory, it is necessary to choose a particular \(v\), and why there are not transitions between states of different \(v\) (in any approximation in which degeneracy holds). As we will see later in these lectures, generically classical moduli spaces are not moduli spaces at the quantum level.

One can also read off from the lagrangian the couplings, not only of ordinary fields, but of their superpartners. For example, there is a Yukawa coupling of the gauginos to fermions
and scalars, whose strength is governed by the corresponding gauge couplings. There are also quartic couplings of the scalars, with gauge strength. These are indicated in fig. 1.

Before turning to the phenomenology of this “Minimal Supersymmetric Standard Model,” (MSSM), it is useful to get some more experience with the properties of supersymmetric theories. With the limited things we know, we can already derive some dramatic results. First, we can write down the most general globally supersymmetric lagrangian, with terms with at most two derivatives, but not restricted by renormalizability (in the rest of this section, the lower case \( \phi \) refers both to the chiral field and its scalar component):

\[
\mathcal{L} = \int d^4\theta K(\phi_i^\dagger, \phi_i) + \int d^2\theta W(\phi_i) + cc + \int d^2\theta f(\phi_i) W_\alpha^2 + cc.
\]

(35)

Here \( K \) is a general function known as the Kahler potential. \( W \) and \( f \) are necessarily holomorphic functions of the chiral fields. One can consider terms involving the covariant derivatives, \( D_\alpha \), but these correspond to terms with more than two derivatives, when written in terms of component fields.

We will often be interested in effective lagrangians of this sort, for example, in studying the low energy limit of string theory. From the holomorphy of \( W \) and \( f \), as well as from the symmetries of the models, one can often derive remarkable results. Consider, for example, the “Wess-Zumino” model, a model with a single chiral field with superpotential

\[
W = \frac{1}{2}m\phi^2 + \frac{1}{3}\lambda\phi^3.
\]

(36)

For general \( m \) and \( \lambda \), this model has no continuous global symmetries. If \( m = 0 \), it has an “\( R \)” symmetry, a symmetry which does not commute with supersymmetry:

\[
\phi \to e^{\frac{2i}{3}\alpha} \phi \quad \theta \to e^{i\alpha} \theta \quad d\theta \to e^{-i\alpha} \theta.
\]

(37)

Under this transformation,

\[
W \to e^{2i\alpha} W
\]

(38)
so \( \int d^2 \theta W \) is invariant. This transformation does not commute with supersymmetry; recalling the form of \( Q_\alpha \) in terms of \( \theta \)'s, one sees that

\[
Q_\alpha \sim \frac{\partial}{\partial \theta_\alpha} + \ldots \to e^{i\alpha} Q_\alpha. \tag{39}
\]

Correspondingly, the fermions and scalars in the multiplet transform differently: the scalar has the same \( R \) charge as the superfield, \( 2/3 \), while \( \psi \) has \( R \) charge one unit less than that of the scalar, i.e.

\[
\phi \to e^{2i\alpha/3} \phi \quad \psi \to e^{-i\alpha/3} \psi. \tag{40}
\]

It is easy to check that this a symmetry of the lagrangian, written in terms of the component fields. Correspondingly, in the quantum theory,

\[
Q_\alpha \approx \int d^3 x (\sigma^\mu_{\alpha \dot{\alpha}} \partial_\mu \phi \psi^{* \dot{\alpha}} + \psi_\alpha F) \to e^{i\alpha} Q_\alpha. \tag{41}
\]

Symmetries of this type will play an important role in much of what follows. In general, in a theory with several chiral fields, one has

\[
\phi_i \to e^{i\alpha_{R_i}} \phi_i \quad W(\phi_i) \to e^{2i\alpha} W(\phi_i). \tag{42}
\]

If there are vector multiplets in the model, the gauge bosons are neutral under the symmetry, while the gauginos have charge +1. We will also be interested in discrete versions of these symmetries (in which, essentially, the parameter \( \alpha \) takes on only some discrete values). In the case of the Wess-Zumino model, for non-zero \( m \), a discrete subgroup survives for which \( \alpha = 3n\pi \), i.e. \( \phi \to \phi \), \( \psi \to -\psi \). More elaborate discrete symmetries will play an important role in our discussions.

Even in the Wess-Zumino model with non-zero \( m \), we can exploit the power of continuous symmetries, by thinking of the couplings as if they were themselves background values for some chiral fields. If we assign \( R \) charge +1 to \( \phi \), we can make the theory \( R \)-invariant if we assign \( R \) charge −1 to \( \lambda \). In practice, we might be interested in theories where \( \lambda \) is the scalar component of a dynamical field (this will often be the case in string theory) or we may simply view this as a trick[17]. In either case, we can immediately see that there are no corrections to the \( \phi^3 \) term in the superpotential in powers of \( \lambda \). The reason is that the superpotential must be a holomorphic function of \( \phi \) and \( \lambda \) (so it cannot involve, say, \( \lambda \lambda^\dagger \)), and it must respect the \( R \) symmetry. Because \( \lambda \) is the small parameter of the theory, we have proven a powerful non-renormalization theorem: the superpotential cannot be corrected to any order in the coupling constant. This
non-renormalization theorem was originally derived by detailed consideration of the properties of Feynman graphs. What is crucial to this argument is that $W$ is a holomorphic function of $\phi$ and the parameters of the lagrangian.

This is not to say that nothing in the effective action of the theory is corrected from its lowest order value; non-holomorphic quantities are renormalized. For example:

$$\int d^4\theta \phi^\dagger \phi f(\lambda^\dagger \lambda)$$

is allowed. In the Wess-Zumino model, this means that all of the renormalizations are determined by wave function renormalization. Finally, we should note that if $m = 0$ at tree level, no masses are generated for fermions or scalars in loops.

### 2.3 N=2 Theories: Exact Moduli Spaces

We have already encountered an extensive vacuum degeneracy in the case of the MSSM. Actually, the degeneracy is much larger; there is a multiparameter family of such flat directions involving the squark, slepton and Higgs fields. For the particular example, we saw that classically the possible ground states of the theory are labeled by a quantity $v$. States with different $v$ are physically distinct; the masses of particles, for example, depend on $v$. In non-supersymmetric theories, one doesn’t usually contemplate such degeneracies, and even if one had such a degeneracy, say, at the classical level, one would expect it to be eliminated by quantum effects. We will see that in supersymmetric theories, these flat directions almost always remain flat in perturbation theory; non-perturbatively, they are sometimes lifted, sometimes not. Moreover, such directions are ubiquitous The space of degenerate ground states of a theory is called the “moduli space.” The fields whose expectation values label these states are called the moduli. In supersymmetric theories, such degeneracies are common, and are often not spoiled by quantum corrections.

In theories with $N = 1$ supersymmetry, detailed analysis is usually required to determine whether the moduli acquire potentials at the quantum level. For theories with more supersymmetries ($N > 1$ in four dimensions; $N \geq 1$ in five or more dimensions), one can usually show rather easily that the moduli space is exact. Here we consider the case of $N = 2$ supersymmetry in four dimensions. These theories can also be described by a superspace, in this case built from two Grassman spinors, $\theta$ and $\bar{\theta}$. There are two basic types of superfields, called vector and hyper multiplets. The vectors are chiral with respect to both $D_\alpha$ and $\bar{D}_\alpha$, and have an
expansion, in the case of a $U(1)$ field:

$$\psi = \phi + \tilde{\theta}^\alpha W_\alpha + \tilde{\theta}^2 \tilde{D}^2 \phi^\dagger,$$

where $\phi$ is an $N = 1$ chiral multiplet and $W_\alpha$ is an $N = 1$ vector multiplet. The fact that $\phi^\dagger$ appears as the coefficient of the $\tilde{\theta}^2$ term is related to an additional constraint satisfied by $\psi$\footnote{13}. This expression can be generalized to non-abelian symmetries; the expression for the highest component of $\psi$ is then somewhat more complicated\footnote{13}; we won’t need this here.

The theory possesses an $SU(2)$ R symmetry under which $\theta$ and $\tilde{\theta}$ form a doublet. Under this symmetry, the scalar component of $\phi$, and the gauge field, are singlets, while $\psi$ and $\lambda$ form a doublet.

I won’t describe the hypermultiplets in detail, except to note that from the perspective of $N = 1$, they consist of two chiral multiplets. The two chiral multiplets transform as a doublet of the $SU(2)$. The superspace description of these multiplets is more complicated\footnote{13, 14}.

In the case of a non-abelian theory, the vector field, $\psi^a$, is in the adjoint representation of the gauge group. For these fields, the lagrangian has a very simple expression in superspace:

$$L = \int d^2 \theta d^2 \tilde{\theta} \psi^a \psi^a, \quad \text{(45)}$$

or, in terms of $N = 1$ components,

$$L = \int d^2 \theta \ W_\alpha^2 + \int d^4 \theta \phi^\dagger e^V \phi. \quad \text{(46)}$$

The theory with vector fields alone has a classical moduli space, given by the values of the fields for which the scalar potential vanishes. Here this just means that the $D$ fields vanish. Written as a matrix,

$$D = [\phi, \phi^\dagger], \quad \text{(47)}$$

which vanishes for diagonal $\phi$, i.e. for

$$\phi = \frac{a}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad \text{(48)}$$

In quantum field theory, one must choose a value of $a$. This is different than in the case of quantum mechanical systems with a finite number of degrees of freedom; this difference will be explained below. As in the case of the MSSM, the spectrum depends on $a$. For a given value of $a$, the massless states consist of a $U(1)$ gauge boson, two fermions, and a complex scalar

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(essentially a), i.e. there is one light vector multiplet. The masses of the states in the massive multiplets depend on a.

For many physically interesting questions, one can focus on the effective theory for the light fields. In the present case, the light field is the vector multiplet, $\psi$. Roughly,

$$\psi \approx \psi^a \psi^a = a^2 + a \delta \psi^3 + \ldots$$

(49)

What kind of effective action can we write for $\psi$? At the level of terms with up to four derivatives, the most general effective lagrangian has the form:

$$L = \int d^2 \theta d^2 \tilde{\theta} f(\psi) + \int d^8 \theta \mathcal{H}(\psi, \psi^\dagger).$$

(50)

Terms with covariant derivatives correspond to terms with more than four derivatives, when written in terms of ordinary component fields.

The first striking result we can read off from this lagrangian, with no knowledge of $\mathcal{H}$ and $f$, is that there is no potential for $\phi$, i.e. the moduli space is exact. This statement is true perturbatively and non-perturbatively!

One can next ask about the function $f$. This function determines the effective coupling in the low energy theory, and is the object studied by Seiberg and Witten\cite{18}. We won’t review this whole story here, but indicate how symmetries and the holomorphy of $f$ provide significant constraints (Michael Peskin’s TASI 96 lectures provide a concise introduction to this topic\cite{19}). It is helpful, first, to introduce a background field, $\tau$, which we will refer to as the “dilaton,” with coupling

$$L = \int d^2 \theta d^2 \tilde{\theta} \tau \psi^a \psi^a$$

(51)

where

$$\tau = \theta + \frac{i}{g^2} + \ldots$$

(52)

$\tau$ is a chiral field. For our purposes, $\tau$ need not be subject to the same constraint as the vector superfield. Classically, the theory has an R-symmetry under which $\psi^a$ rotates by a phase, $\psi^a \rightarrow e^{i\alpha} \psi^a$. But this symmetry is anomalous. Similarly, shifts in the real part of $\tau (\theta)$ are symmetries of perturbation theory. This insures that there is only a one-loop correction to $f$.

\footnote{This, and essentially all of the effective actions we will discuss, should be thought of as Wilsonian effective actions, obtained by integrating out heavy fields and high momentum modes.}
This follows, first, from the fact that any perturbative corrections to $f$ must be $\tau$-independent. A term of the form

$$c \ln(a) \psi^2$$

respects the symmetry, since the shift of the logarithm just generates a contribution proportional to $F \tilde{F}$, which vanishes in perturbation theory. Beyond perturbation theory, however, we expect corrections proportional to $ae^{-\tau}$, since this is invariant under the non-anomalous symmetry. It is these corrections which were worked out by Seiberg and Witten.

### 2.4 A Still Simpler Theory: N=4 Yang Mills

$N = 4$ Yang Mills theory is an interesting theory in its own right: it is finite and conformally invariant. It also plays an important role in Matrix theory, and is central to our understanding of the AdS/CFT correspondence. $N = 4$ Yang Mills has sixteen supercharges, and is even more tightly constrained than the $N = 2$ theories. There does not exist a convenient superspace formulation for this theory, so we will find it necessary to resort to various tricks. First, we should describe the theory. In the language of $N = 2$ supersymmetry, it consists of one vector and one hyper multiplet. In terms of $N = 1$ superfields, it contains three chiral superfields, $\phi_i$, and a vector multiplet. The lagrangian is

$$\mathcal{L} = \int d^2\theta W_a^2 + \int d^4\theta \sum \phi_i^\dagger e^V \phi_i + \int d^2\theta \phi_i^a \phi_j^b \phi_k^c \varepsilon_{ijk} \varepsilon^{abc}. \quad (54)$$

In the above description, there is a manifest $SU(3) \times U(1)$ R symmetry. Under this symmetry, the $\phi_i$’s have $U(1)_R$ charge $2/3$, and form a triplet of the $SU(3)$. But the real symmetry is larger – it is $SU(4)$. Under this symmetry, the four Weyl fermions form a 4, while the six scalars transform in the 6. Thinking of these theories as the low energy limits of toroidal compactifications of the heterotic string will later give us a heuristic understanding of this $SU(4)$: it reflects the $O(6)$ symmetry of the compactified six dimensions. In string theory, this symmetry is broken by the compactification lattice; this reflects itself in higher derivative, symmetry breaking operators.

In the $N = 4$ theory, there is, again, no modification of the moduli space, perturbatively or non-perturbatively. This can be understood in a variety of ways. We can use the $N = 2$ description of the theory, defining the vector multiplet to contain the $N = 1$ vector and one (arbitrarily chosen) chiral multiplet. Then an identical argument to that given above insures that there is no superpotential for the chiral multiplet alone. The $SU(3)$ symmetry then insures
that there is no superpotential for any of the chiral multiplets. Indeed, we can make an argument directly in the language of $N = 1$ supersymmetry. If we try to construct a superpotential for the low energy theory in the flat directions, it must be an $SU(3)$-invariant, holomorphic function of the $\phi_i$’s. But there is no such object.

Similarly, it is easy to see that there no corrections to the gauge couplings. For example, in the $N = 2$ language, we want to ask what sort of function, $f$, is allowed in

$$\mathcal{L} = \int d^2 \theta d^2 \tilde{\theta} f(\psi).$$

But the theory has a $U(1) R$ invariance under which

$$\psi \to e^{2/3 i \alpha} \psi \quad \theta \to e^{i \alpha \theta} \quad \tilde{\theta} \to e^{-i \alpha \tilde{\theta}}$$

Already, then

$$\int d^2 \theta d^2 \tilde{\theta} \psi \psi$$

is the unique structure which respects these symmetries. Now we can introduce a background dilaton field, $\tau$. Classically, the theory is invariant under shifts in the real part of $\tau$, $\tau \to \tau + \beta$. This insures that there are no perturbative corrections to the gauge couplings. More work is required to show that there are no non-perturbative corrections either.

One can also show that the quantity $\mathcal{H}$ in eqn. 50 is unique in this theory, again using the symmetries. The expression 20, 21:

$$\mathcal{H} = c \ln(\psi) \ln(\psi^\dagger),$$

respects all of the symmetries. At first sight, it might appear to violate scale invariance; given that $\psi$ is dimensionful, one would expect a scale, $\Lambda$, sitting in the logarithm. However, it is easy to see that one integrates over the full superspace, any $\Lambda$-dependence disappears, since $\psi$ is chiral. Similarly, if one considers the $U(1) R$-transformation, the shift in the lagrangian vanishes after the integration over superspace. To see that this expression is not renormalized, one merely needs to note that any non-trivial $\tau$-dependence spoils these two properties. As a result, in the case of $SU(2)$, the four derivative terms in the lagrangian are not renormalized. Note that this argument is non-perturbative.

2.5 Aside: Choosing a Vacuum

It is natural to ask: why in field theory, in the presence of moduli, does one have to choose a vacuum? In other words, why aren’t their transition between states with different expectation
values for the moduli?

\[ |+\rangle \langle +| = \cos(\theta/2). \] (59)

If there are \( N \) such sites, the overlap behaves as

\[ \langle \theta | 0 \rangle \sim (\cos(\theta/2))^N \] (60)

i.e. it vanishes exponentially rapidly with the volume.

For a continuum field theory, states with differing values of the order parameter, \( v \), also have no overlap in the infinite volume limit. This is illustrated by the theory of a scalar field with lagrangian:

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2. \] (61)

For this system, the expectation value \( \phi = v \) is not fixed. The lagrangian has a symmetry, \( \phi \rightarrow \phi + \delta \), for which the charge is just

\[ Q = \int d^3x \Pi(\vec{x}) \] (62)
where $\Pi$ is the canonical momentum. So we want to study

$$\langle v|0 \rangle = \langle 0|e^{iQ}|0 \rangle.$$  \hspace{1cm} (63)

We must be careful how we take the infinite volume limit. We will insist that this be done in a smooth fashion, so we will define:

$$Q = \int d^3x \partial_t \phi e^{-x^2/V^{2/3}}$$

$$= -i \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\omega_k}{2}} \left( \frac{V^{1/3}}{\sqrt{\pi}} \right)^3 e^{-\vec{k}^2V^{2/3}/4} [a(\vec{k}) - a^{\dagger}(\vec{k})].$$

Now, one can evaluate the matrix element, using

$$e^{A+B} = e^A e^B e^{\frac{1}{2}[A,B]}$$

(provided that the commutator is a c-number), giving

$$\langle 0|e^{iQ}|0 \rangle = e^{-cv^2V^{-2/3}},$$  \hspace{1cm} (65)

where $c$ is a numerical constant. So the overlap vanishes with the volume. You can convince yourself that the same holds for matrix elements of local operators. This result does not hold in 0+1 and 1+1 dimensions, because of the severe infrared behavior of theories in low dimensions. This is known to particle physicists as Coleman’s theorem, and to condensed matter theorists as the Mermin-Wagner theorem.

### 3 N=1: Supersymmetry Breaking?

In four dimensions, we have seen that in theories with more than two supersymmetries, moduli are exact and supersymmetry remains unbroken exactly. We turn, now, to $N = 1$ theories.

We will prove that if supersymmetry is unbroken at tree level, it remains unbroken to all orders in perturbation theory. Non-perturbatively, however, we will see that supersymmetry is often broken; supersymmetry breaking is typically of order $e^{-c8\pi^2/g^2}$, where $g$ is some gauge coupling, and $c$ is a numerical constant. The potential to generate large hierarchies under such circumstances was first stressed by Witten\[22\].

If supersymmetry is spontaneously broken, there is necessarily a massless fermion, just as breaking of an ordinary global symmetry implies the existence of a Goldstone boson. The proof closely parallels the usual proof of Goldstone’s theorem, and the corresponding massless fermion
is called the Goldstino. For example, for chiral fields, under a supersymmetry transformation with parameter $\epsilon$,

$$\delta \psi = \epsilon F + \ldots$$  \hspace{1cm} (66)

So if $\langle F \rangle \neq 0$, supersymmetry is broken. In terms of the supersymmetry current:

$$j_\alpha^\mu = F^{\alpha\beta} \sigma_\alpha^\mu \psi^\star \dot{\alpha} + \ldots$$  \hspace{1cm} (67)

$$\approx \langle F \rangle \sigma_\alpha^\mu \psi^\star \dot{\alpha}.$$  \hspace{1cm} (68)

Since

$$\partial_\mu j_\alpha^\mu = 0$$  \hspace{1cm} (69)

we have

$$\partial_\mu \sigma_\alpha^\mu \psi^\star \dot{\alpha} = 0.$$  \hspace{1cm} (70)

This has an immediate consequence: if supersymmetry is to be broken in some model, there had better be a light fermion which can play the role of the Goldstino$^{[22]}$.

**Exercise:** Prove the Goldstino theorem in generality.

Let us turn now to a particular example: supersymmetric QCD with gauge group $SU(N)$. Consider first the case with no flavors, $n_f = 0$, i.e. a pure supersymmetric gauge theory. The dynamical fields are the gauge bosons and gauginos, $A^\mu$ and $\lambda$. The lagrangian is simply

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + i \lambda D_\mu \sigma^\mu \lambda^\star.$$  \hspace{1cm} (71)

Classically, this theory has no ground state degeneracy. One expects that the spectrum has a mass gap, like QCD, so there is no candidate Goldstino and one does not expect that supersymmetry is broken in this theory. There is a continuous global $U(1)$ (R) symmetry. This symmetry, however, is anomalous and only a discrete $Z_N$ subgroup survives in the quantum theory. This can be seen, for example, by looking at instanton amplitudes. An instanton (fig. 3) in this theory has $2N$ fermion zero modes, meaning that there are expectation values for operators of the form

$$\langle \lambda(x_1) \ldots \lambda(x_{2N}) \rangle$$  \hspace{1cm} (72)

This breaks (explicitly) the $U(1)$ symmetry to a discrete subgroup under which:

$$\lambda \rightarrow e^{\frac{2\pi i}{N}} \lambda.$$  \hspace{1cm} (73)
Figure 3: Instanton in supersymmetric QCD has four gaugino zero modes and two quark zero modes at lowest order. As indicated in the figure, the scalar vev can be used to tie some of these together, generating the two fermion term in the superpotential.

Just as quarks condense in QCD, it is reasonable to expect that gluinos condense as well, i.e.

$$\langle \lambda \lambda \rangle = \Lambda^3.$$  

(73)

As we will see later, one can in fact prove that gaugino condensation occurs in this theory. Note, that $\lambda \lambda$ is the lowest component of a chiral superfield, $W_\alpha W^\alpha$, so the condensate is not an order parameter for supersymmetry breaking, consistent with our expectation that supersymmetry is unbroken.

Consider now the effect of adding “quarks” to the theory, i.e. fields $Q_f$ and $\bar{Q}_f$ in the $N$ and $\bar{N}$ representation of $SU(N)$. Here $f$ is a flavor index, $f = 1, \ldots, N_f$. If these fields are massive, one does not expect supersymmetry breaking, since the theory is much like the theory with no quarks. The massless case is more interesting.

$$V = \frac{1}{2} \sum (D^a)^2 = \text{tr} \, D^2$$  

(74)

where $D_{ij}$ is the matrix:

$$D_{ij} = \hat{D}_{ij} - \delta_{ij} \text{tr} \hat{D} \quad \hat{D}_{ij} = \sum_f Q_{ijf}^* Q_{jff} - \bar{Q}_{ijf} \bar{Q}_{jff}^*.$$  

(75)

Take, for example, the case of two massless doublets in $SU(2)$, $Q$ and $\bar{Q}$ (one “flavor”). Classically there is a moduli space of vacua. Consider the case $N_f < N_c$. By a gauge transformation,
we can always take (writing \( Q \) as a matrix in flavor and color space)

\[
Q = \begin{pmatrix}
v_1 & 0 & 0 & \ldots & 0 \\
0 & v_2 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & v_n
\end{pmatrix}.
\] (76)

Then the vanishing of \( D_{ij} \) requires that

\[
\bar{Q} = \begin{pmatrix}
v_1 e^{i\phi_1} & 0 & 0 & \ldots & 0 \\
0 & v_2 e^{i\phi_2} & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & v_{nf} e^{i\phi_{nf}}
\end{pmatrix}.
\] (77)

These expectation values correspond to moduli, which can be described by the gauge-invariant fields

\[
M_{f f'} = \bar{Q}_f Q_{f'}.
\] (78)

The cases with \( N_f \geq N_c \) are different. For example, in the case \( N_f = N_c \), one has an additional solution, where \( Q \) is proportional to a unit matrix, and \( \bar{Q} = 0 \).

**Exercise:** Write the general flat direction for \( N_f > N_c \). What gauge invariant fields are needed to describe it?

We would like to understand, quantum mechanically, what happens in these flat directions. Consider, first, the case of \( SU(2) \) with one flavor. Here one has just the fields \( Q \) and \( \bar{Q} \); the flat direction is the one we encountered in the MSSM,

\[
Q = \begin{pmatrix} 0 \\ v \end{pmatrix} = e^{i\alpha} \bar{Q}.
\] (79)

For non-zero \( v \), the \( SU(2) \) gauge symmetry is completely broken. States labeled by different \( v \) are physically inequivalent. For example, the masses of the gauge fields are different. For large \( v \), the effective coupling is \( g^2(v) \), and so the theory is weakly coupled, and we should be able to analyze it completely. The modulus which describes this flat direction we will call \( \Phi \),

\[
\Phi = \bar{Q} Q.
\] (80)

To determine what happens to the flat directions quantum mechanically, we should integrate out the massive fields and study an effective action for \( \Phi \). Since the theory is supersymmetric in the limit of weak coupling, we expect this lagrangian to be supersymmetric. This follows from at least two considerations. First, if one gauges the supersymmetry, then one has a
theory with a massless, spin-3/2 particle at low energies. Such a theory must be supersymmetric. Alternatively, one can make the argument purely in the global theory. If supersymmetry is to be spontaneously broken, there must be a massless fermion in the theory to play the role of the Goldstino. Supersymmetry breaking corresponds, precisely, to the generation of an $F$ or $D$-term for this field in the low energy lagrangian.

Given these statements, the low energy effective action has the form,

$$\mathcal{L}_{\text{eff}} = \int d^4\theta f(\Phi^\dagger \Phi) + \int d^2\theta W(\Phi).$$  \hfill (81)

Remarkably, the form of $W$ is completely determined by the symmetries of the theory. At a microscopic level, the theory has a non-anomalous $U(1)_R$ symmetry, under which

$$Q \rightarrow e^{-i\alpha} Q \quad \bar{Q} \rightarrow e^{-i\alpha} \bar{Q} \quad \theta \rightarrow e^{-i\alpha} \theta.$$  \hfill (82)

Under this symmetry, the $R$-charge of the quarks (i.e. the fermionic components of $Q$ and $\bar{Q}$) is $-2$, and the contribution to the anomaly cancels against that of the gauginos with charge 1.

$\Phi$ transforms as

$$\Phi \rightarrow e^{-2i\alpha} \Phi.$$  \hfill (83)

The only holomorphic function, $W(\Phi)$, with $R$-charge 2 is:

$$W = \frac{\Lambda^5}{\Phi}.$$  \hfill (84)

Here $\Lambda$ is the renormalization group scale of the $SU(N)$ theory,

$$\Lambda = e^{\frac{g^2}{b_N g_s^2}} = e^{\frac{g^2}{b_s g_s^2}}.$$  \hfill (85)

As a check, we can determine the $\Lambda$ dependence from a different argument. We can describe the gauge coupling as the background value of a chiral field, $S$,

$$-\frac{1}{4} \int d^2\theta SW^2_\alpha.$$  \hfill (86)

with

$$S = \frac{1}{g^2} + i a.$$  \hfill (87)

In the presence of the background field, the theory has an additional symmetry:

$$S \rightarrow S + i \frac{\alpha}{16\pi^2}.$$  \hfill (88)
\[ Q \rightarrow Q \quad \bar{Q} \rightarrow \bar{Q}. \]

The rotation of the fermions cancels the shift of the lagrangian from the shift in \( S \). \( e^{-S} \) has \( R \)-charge 2, so

\[ W = \frac{e^{-S}}{\Phi}. \quad (89) \]

This agrees with our expression above.

While the symmetry arguments are powerful, they may seem a bit slick, and it is desirable to check that this term is in fact present. Since we are interested in the action of the (at least approximately) massless states, it is appropriate to study the Euclidean functional integral:

\[ \int [d\phi] e^{-S} \quad (90) \]

where \( \phi \) refers to all of the various fields in the (full microscopic) theory. In the classical vacuum characterized by the expectation value \( v \), the field \( \Phi \) is

\[ \Phi = v^2 + v(\psi_Q + \psi_{\bar{Q}})\theta + \ldots \quad (91) \]

so the expected interaction includes terms such as

\[ \Lambda^5 \int d^2\theta \frac{d^2\theta}{\Phi^2} = \cdots \frac{\Lambda^5}{v^4} \psi_Q \psi_{\bar{Q}} \quad (92) \]

We wish to see if such terms appear in the path integral.

It is not hard to show that the interaction of eqn. \( \text{[92]} \) is generated by instantons. Instantons are Euclidean solutions of the classical equations of motion \( \text{[23]} \). They are expected to dominate the Euclidean functional integral at weak coupling. Actually, a simple scaling argument shows that there are no such solutions for non-zero \( v \); however, as ’t Hooft explained in his original paper on instantons \( \text{[24]} \), approximate solutions can be constructed. Starting with the instanton of the pure gauge theory, which has action \( 8\pi^2/g^2 \) and a scale \( \rho \), these solutions can be constructed perturbatively in \( \rho v \). Similarly, starting with the fermion zero modes of the unperturbed solution (there are four associated with the gauginos and two with the \( Q, \bar{Q} \) fields), one can construct two zero modes \( \text{[25]} \). The structure of the calculation is indicated in fig. 3. In the figure, each of the lines emerging from the blob denotes one of the unperturbed fermionic zero modes; the scalar background is treated as a perturbation. The actual calculation is straightforward, and yields a non-zero coefficient for the expected operator \( \text{[26]} \). Other terms generated by the superpotential can be calculated as well \( \text{[29]} \).
The potential generated by the non-perturbative superpotential in this model tends to zero at infinity. In this regime, the calculation is completely reliable. To understand this, note that at the microscopic level, for large \(v\), the theory consists of particles with mass of order \(v\), and the massless multiplet \(\Phi\); there are no direct couplings between the \(\Phi\) fields themselves. As a result, loop diagrams are dominated by physics at the scale \(M_V = gv\), and thus the effective coupling is \(g^2(v)\). For sufficiently large \(v\), this coupling can be made arbitrarily small. The Kahler potential receives only small corrections; the superpotential, we have argued is exact (and in any case, the semiclassical instanton calculation becomes more and more reliable). At strong coupling, it is conceivable that the Kahler potential has some complicated behavior, leading, perhaps, to a local minimum of the potential. We do not have reliable methods to explore this regime: in this region of \(\Phi\), it is not even clear that \(\Phi\) is the appropriate degree of freedom to study. On the other hand, for large \(\Phi\), we have an approximate moduli space.

Consider, now, the effect of adding a mass term, \(mQ\bar{Q}\). This breaks the \(U(1)_R\) symmetry of the theory, leaving over a \(Z_2\).

**Exercise:** Determine the symmetries of \(SU(N)\) supersymmetric QCD with \(N_f\) flavors massless flavors. Show that if all of the quarks have mass, the theory has a \(Z_N\) \(R\)-symmetry. How do the \(Q_\alpha\)'s transform?

For small \(m\), the superpotential is

\[
W = \frac{\Lambda^5}{\Phi} + m\Phi. \quad (93)
\]

The equation \(\frac{\partial W}{\partial \Phi} = 0\) has two roots

\[
\Phi = v^2 = \pm \left(\frac{\Lambda^5}{m}\right)^{1/2}. \quad (94)
\]

These two roots correspond to the spontaneous breaking of the \(Z_2\) symmetry of the theory.

As \(m \to 0\), this computation is completely reliable, since \(v \to \infty\), so the coupling becomes weak. It would seem, that we could say nothing about stronger coupling. However, treating \(m\) as a background field, we can make a simple argument that the superpotential of eqn. [93] is exact. We can assign charge +4 to \(m\) under the original \(U(1)\) symmetry. Then higher powers of \(m\) must be accompanied by higher powers of \(\Phi\). But perturbative and non-perturbative contributions do not lead to such terms. Alternatively, under the symmetry under which \(\Phi\) is neutral and \(S\) transforms (eqn. [88]), \(m\) carries charge 2. So higher powers of \(m\) must come with inverse powers of \(\Lambda\).
So, for general \( m \), we can compute the expectation value of the superpotential:

\[
\langle W \rangle = \pm m \left( \frac{\Lambda^5}{m} \right)^{1/2}
\]

\[
= \pm (m\Lambda^5)^{1/2}.
\]

But for large \( m \), the theory is just pure gauge \( SU(2) \). The “expectation value of the superpotential” in this theory is naturally identified with

\[
\int d^2\theta W^2 \approx \int d^2\theta \langle W^2 \rangle = \int d^2\theta \langle \lambda\lambda \rangle = \Lambda_{LE}^3.
\]

In this expression, \( \Lambda_{LE} \) is the \( \Lambda \)-parameter of the low energy theory. To determine the connection between this quantity and the \( \Lambda \) parameter of the full theory, note that

\[
\Lambda_{LE} = m e^{-\frac{8\pi^2}{b_{LE}g^2(m)}}
\]

while

\[
\Lambda = M e^{-\frac{8\pi^2}{8\pi^2(M)}}
\]

where \( M \) is some high energy scale. \( g^2(m) \) is determined from

\[
8\pi^2 g^{-2}(m) = 8\pi^2 g^{-2}(M) + b\ln(m/M).
\]

Using \( b_{LE} = 6 \), \( b = 5 \) gives

\[
\Lambda_{LE}^3 = (m\Lambda^5)^{1/2}.
\]

So the superpotential computed by these two different arguments agree. We can view this result in several ways. First, the calculation for small quark mass was completely reliable; we then used holomorphy and symmetries to argue that the result was exact, even for large quark mass. The consistency of these computations is a confirmation of these arguments. Alternatively, we can view the holomorphy arguments as reliable, and then argue that we have \( \text{computed} \langle \lambda\lambda \rangle \) in QCD with a fermion in the adjoint representation!

Models of this kind, with different gauge groups and various numbers of flavors, exhibit a rich array of phenomena. These are reviewed, for example, in \[19\]. Of particular interest are the cases:

- \( SU(N), N_f < N \): In these theories, as in our example above, flat directions are lifted, and their is an approximate moduli space at weak coupling.

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• $SU(N)$, $N_f = N$: These theories have an exact moduli space, but the quantum moduli space is not equal to the classical one.

• $SU(N)$, $N_f > N$: In these theories, the quantum moduli space is equivalent to the classical one. These models exhibit quite non-trivial dualities.

3.1 Supersymmetry Breaking

So far, we have seen that moduli can be lifted, but that potentials in such cases tend to zero at weak coupling. We might hope to obtain “real” supersymmetry breaking, i.e. supersymmetry breaking with a unique, nicely behaved ground state. The simplest example of this phenomena is provided by the “3-2 Model” \[^{19}\]. This model has gauge group $SU(3) \times SU(2)$, with fields $Q(3,2), \bar{U}(\bar{3},1), \bar{D}(\bar{3},1), L(1,2)$ (the numbers in parenthesis denote the $SU(3) \times SU(2)$ representations), and superpotential

$$W = \lambda QL\bar{D}. \tag{101}$$

With $\lambda = 0$, this theory has flat directions. If the $SU(3)$ coupling is large compared to the $SU(2)$ coupling, the theory is like the $N_f = N - 1$ theories, and instantons generate a superpotential. On the other hand, if $\lambda \neq 0$, there are no flat directions.

**Exercise:** Check that for non-zero $\lambda$, there are no flat directions in the model. Determine the two non-anomalous global $U(1)$ symmetries of the theory. For the case $\Lambda_3 \gg \Lambda_2$, write down the superpotential which describes the low energy theory and argue that it has a minimum with broken supersymmetry.

**Exercise: (Advanced)**: Consider the case $\Lambda_2 \gg \Lambda_3$. Show that with $\lambda = 0$ this is a theory with a quantum modified moduli space, and argue that with $\lambda \neq 0$ supersymmetry is broken.

For small $\lambda$, the superpotential is the sum of the non-perturbative superpotential and that of eqn. \[^{101}\]. It is straightforward to show that this potential does not have a supersymmetric minimum, and indeed has simply an isolated, supersymmetry breaking minimum. One can understand why this happens by noting that any expectation values of the fields break one or both of the non-anomalous $U(1)$ symmetries of the theory. As a result, there are Goldstone bosons. If supersymmetry is to be unbroken, these Goldstone bosons must have superpartners. The scalar superpartners would parameterize flat directions, but there are no flat directions in the model. This criterion, that if there are broken symmetries in a theory without flat directions, supersymmetry is broken, has proven useful for finding examples of spontaneous supersymmetry
breaking. In recent years, many more examples of theories with supersymmetry breaking have been exhibited, and other criteria for such theories have been established. This subject is thoroughly reviewed in [27].

4 Supersymmetry Phenomenology and Model Building

Before speculating about the dynamics of supersymmetry breaking in nature, there are some features of the MSSM which we must discuss. Not only are these features important phenomenologically, but they are also possible clues to the supersymmetry breaking mechanism. First, while the model has some formal resemblance to the Standard Model, it has some important differences. One of the virtues of the SM is that there are no renormalizable operators in the model which violate baryon number or the separate lepton numbers. As a result, it is not necessary to impose these symmetries by hand, provided that the scales of new physics are well above the scale of weak interactions. This is not the case in the MSSM. There are dimension four operators, such as

$$\int d^2 \theta [aQQQ + b\bar{u}\bar{d}d + cQL\bar{e}],$$

(102)

which violate these symmetries. To suppress these, one usually supposes that the model has an additional symmetry. The simplest hypothesis is that there is a discrete, $Z_2$, $R$ symmetry, known as $R$ parity, under which all of the “ordinary” particles (quarks, leptons, etc.) are invariant, while the “new” particles are odd. This eliminates all of the dangerous couplings above. Weaker versions of this idea are possible, and lead to a different phenomenology[28]. We will adopt the $R$-parity violating hypothesis here, for simplicity. It is disappointing that from the start we must superpose some additional symmetry to make this structure work, but we can take comfort from the fact that such discrete symmetries are quite common in string theory. Even with this hypothesis, there are other potential difficulties. In the SM, the leading baryon number violating operators are of dimension 6. However, even if we suppress the dimension four operators in the MSSM, there are potential problems from operators of dimension six, for example

$$QQQL, \bar{u}\bar{u}\bar{d}\bar{e}.$$  

(103)

With rather mild assumptions, these operators are highly suppressed, however, and can be compatible with current experimental bounds.

The $R$ parity hypothesis has an important and desirable consequence: the lightest new particle predicted by supersymmetry is stable. This “LSP” is, it turns out, a natural candidate
for the dark matter of the universe. Cross sections for its production and annihilation are such that one automatically produces a density of order the closure density.

Figure 4: Infrared divergent contributions to the effective action.

If we take the MSSM as our basic framework for thinking about supersymmetry and nature, we obviously need to give masses to the squarks, sleptons, gauginos and Higgs. We could try to build some supersymmetry-breaking model including these fields, along the lines of the last section, but this turns out to be a challenging problem, so we first adopt a simpler approach: we just add masses for these fields (along with certain cubic couplings of scalar fields). These “soft breakings” don’t reintroduce quadratic divergences. In particular, they don’t introduce quadratic divergences. This follows, more or less, from dimensional analysis. At very high energies, corrections to masses should be proportional to the soft masses themselves, and the resulting Feynman integrals should be correspondingly less divergent. Consider, for example, the (massless) Wess-Zumino model, with \( W = \frac{1}{3} \phi^3 \). The Yukawa and quartic couplings of this theory are:

\[
\mathcal{L}_{\text{int}} = 2\lambda \phi \psi \bar{\psi} + \text{c.c.} + |\lambda \phi^2|^2. \tag{104}
\]

At one loop there are two Feynman diagrams, indicated in fig. 4, which contribute to the scalar mass:

\[
\delta m^2 = \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{k^2} - \frac{1}{k^2} \right]. \tag{105}
\]

Including a small supersymmetry-breaking mass for the scalars, \( m_s^2 \), changes this expression to:

\[
\delta m^2 = \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{k^2 + m_s^2} - \frac{1}{k^2} \right] = \frac{-m_s^2 \ln(\Lambda^2/m_s^2)}{16\pi^2}. \tag{106}
\]

What is crucial, here, is that this is only logarithmically sensitive to the cutoff \( \Lambda \).

We can understand this result another way. Introduce a field, \( X \), which we will refer to as a spurion. This field is similar in many respects to the background fields we have introduced
in earlier sections to describe coupling constants, and in practice we may want to think of it as dynamical or we may not. What is crucial is that $F_X$ has a non-vanishing expectation value, i.e.

$$\langle X \rangle = \ldots + \theta^2 \langle F_X \rangle.$$  \hfill (107)

Consider, then, an operator of the form

$$\frac{1}{M^2} \int d^4 \theta X^\dagger X \phi^\dagger \phi.$$  \hfill (108)

Integrating over $\theta$, this yields a mass term for the scalar field,

$$\frac{\langle F_X \rangle}{M^2} \phi \phi = m_s^2 \phi \phi.$$  \hfill (109)

Note that this is a Kahler potential term, so it can (and will, in general) be renormalized. Working to quadratic order in $X$, however, these renormalizations are at most logarithmic. ($M$ should be thought of as comparable to the cutoff scale).

In terms of $X$, one can enumerate other possible soft breakings:

- $\int d^2 \theta \frac{X}{M} m \phi \phi = m_s m \phi \phi + c.c.$
- $\int d^2 \theta \frac{X}{M} \phi \phi \phi + c.c.$
- $\int d^2 \theta \frac{X}{M} W_\alpha^2 = m_s \lambda \lambda + c.c.$

To develop a phenomenology, then, we add to the MSSM soft breaking terms corresponding to masses for squarks and sleptons, as well as for gauginos. We also add cubic couplings of Higgs fields to squarks and sleptons. We write the soft breaking lagrangian, as:

$$L_{soft} = \sum_{f,f'} Q_f^\dagger (m_Q)_{f,f'}^2 Q'_{f'} + \sum_{f,f'} \bar{U}_f^\dagger (m_U)_{f,f'}^2 \bar{U}'_{f'}$$

$$+ \sum_{f,f'} \bar{D}_f^\dagger (m_D)_{f,f'}^2 \bar{D}'_{f'} + \sum_{f,f'} \bar{e}_f^\dagger (m_E)_{f,f'}^2 \bar{e}'_f$$

$$+ \sum_{f,f'} \bar{L}_f^\dagger (m_L)_{f,f'}^2 \bar{L}'_f$$

$$+ \sum_{f,f'} A_{U,f}^f Q_f \bar{u}'_f H_U + c.c. + \sum_{f,f'} A_{D,f}^f Q_f \bar{d}'_f H_D + c.c.$$  \hfill (110)

$$+ \sum_{f,f'} A_{L,f}^f L_f \bar{e}'_f H_D + c.c. + \sum_i m_{\lambda_i} \lambda_i \lambda_i$$

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If nature is supersymmetric, determining these parameters and understanding their origin will be a principle goal of particle physics. The first question one might ask is: how many parameters are there? It is easiest to count relative to the Standard Model. There, in defining the usual KM phases, one has already used most of the freedom to make field redefinitions. In order to count the remaining parameters, one must ask what is the remaining freedom. Before considering $L_{soft}$, the theory has several global $U(1)$ symmetries. These are

- An R symmetry which rotates all the fields, under which one can define the $R$ charge of the Higgs fields to be two, and of all other matter fields to be zero.
- Peccei-Quinn symmetry (not an R symmetry):
  \[
  H_U \rightarrow e^{i\alpha} H_U \quad H_D \rightarrow e^{i\alpha} H_D
  \]
  \[
  Q\bar{u} \rightarrow e^{-i\alpha} Q\bar{u} \quad Q\bar{d} \rightarrow e^{-i\alpha} Q\bar{d}
  \]
- Three lepton number symmetries.

So all together there are five global symmetries. One of these (the overall lepton number) is preserved by $L_{soft}$, so we have the freedom to make four redefinitions. Now we can count. Each of the five matrices, $m_{Q}^2, m_{U}^2, \ldots$, etc., is a $3 \times 3$ Hermitian matrix, with nine parameters. $A_U$, $A_D$ and $A_L$ are general complex matrices with 18 parameters. The three gaugino masses (complex) represent six additional parameters. In the Higgs sector, we have six more real parameters. So all together, there are 111 parameters, from which we must subtract four possible redefinitions, and the two Higgs parameters of the usual Standard Model. This leaves 105 new parameters.

The parameter space of the MSSM is enormous. It is possible that at some point we will have a compelling theory which will predict the values of these quantities, but this is not the case today. To explore this space, we need to make some hypotheses. Perhaps the simplest possibility is to assume some simple structure for the soft-breaking masses at some very high energy scale. The following relations are often referred to as the “MSSM” or the “supergravity model:”

\[
m_Q^2 = m_U^2 = m_D^2 = m_{\tilde{\nu}}^2
\]
\[
m_{\lambda_i} = m_{1/2}
\]
\[ A_U = A_D = A_L = A, \]

i.e. all of the various matrices are supposed proportional to the unit matrix. These choices have several virtues. First, one can do phenomenology with a small set of parameters. Second, the various flavor-changing neutral current constraints, which we will discuss below, are automatically satisfied. Experiments, especially at LEP and the Tevatron rule out much of this parameter space. TeVII and the LHC will explore much of what remains.

One of the questions we will address in the remainder of this lecture is: “How plausible is this structure.” Before doing so, however, it is important to mention the other prediction of this framework, apart from dark matter: Coupling Unification. If we assume, consistent with the hierarchy problem, that the susy thresholds are at scales of order 100’s of GeV, and if we run the observed gauge couplings from their values at \( M_Z \), one finds that the couplings unify. This unification occurs at a scale of order \( 2 \times 10^{16} \) GeV. It can be thought of as a prediction of \( \alpha_s \) given \( \alpha_W \) and \( \alpha \), and this prediction is good to about 2%.

**Exercise:** These results are quite easy to derive. At one loop, if the couplings, \( \alpha_i \), are equal at a scale \( M_{GUT} \), one has

\[
2\pi\alpha_i^{-1}(M_Z) = 2\pi\alpha_i^{-1}(M_{GUT}) + b_i \ln(M_Z/M_{GUT}).
\]

If unification is in \( SU(5) \), then the hypercharge of the standard model, \( Y \), is proportional to an \( SU(5) \) generator, \( \tilde{Y} \), which is normalized just like any \( SU(N) \) generator:

\[
\text{tr} (\tilde{Y}^2) = \frac{1}{2}.
\]

Compare this with the conventional values of the hypercharge, to show that at \( M_{GUT} \), the gauge couplings are related by

\[
\sin^2(\theta_W) = \frac{3}{5}.
\]

Using the renormalization group equations, show that the couplings are equal at a scale of about \( 2 \times 10^{16} \) GeV.

The simple hypothesis of eqn. [112] for the soft breaking masses avoids a number of potential problems. Apart from proton decay, it also means that separate lepton numbers are conserved; off-diagonal elements in the matrices \( A^L, m^2_E, m^2_L \) in eqn. [110] (in the basis where the charged lepton masses are diagonal) could lead to \( \mu \to e\gamma \), for example. Other rare processes are also suppressed. \( K - \bar{K} \) mixing is a tiny effect, which the SM predicts at roughly the correct
level (fig. 5). In the SM,

\[ \mathcal{L}_{\text{eff}} \propto \frac{\alpha_W}{4\pi} \frac{m^2_c}{M_W^2} G_f \sin^2(\theta_c) \ln(m^2_c/m^2_u) \mathcal{O}. \]  

(116)

\( \mathcal{O} \) is a certain four-fermi operator which violates strangeness by two units. This formula is meant to show the various sources of suppression. The GIM mechanism leads to a suppression by a factor of \( m^2_c/M_W^2 \); there is further suppression by Cabbibo angles. The matrix elements of \( \mathcal{O} \) are also suppressed by powers of \( m^2_K \).

In supersymmetry, there are new contributions, which are potentially quite large. An example is indicated in fig. 6. In general, this graph has no suppression by \( \frac{m^2_c}{M_W^2} \); it involves \( \alpha_s \) rather than \( \alpha_W \) (a factor of about 10 in the rate); the chiral symmetry suppression is absent (i.e. the factor of \( m^2_K \)). If there are no special cancellations, one requires that the scale of supersymmetry breaking be of order 100’s of TeV to adequately suppress this graph. But cancellations are automatic if the hypothesis of eqn. [112] is satisfied. Some degree of
degeneracy seems almost inevitable if one is to understand these facts.

Figure 7: Diagram involving $\gamma$ or $Z$ exchange giving rise to squark and slepton production in $e^+e^-$ annihilation.

Apart from the constraints on soft masses which come from these rare processes, there are by now significant constraints from direct searches. I won’t review these in detail here, but one has, roughly:

- Gluinos: $m_\lambda > 225$ GeV
- Neutralinos (the lightest neutral fermions from the Higgsino/wino/zino sector) $m_{\chi^0} > 30$ GeV
- Charginos: $m_{\chi^\pm} > 100 – 150$ GeV
- Sleptons: $m_{\tilde{e}} > 90$ GeV

An example of a process leading to selectron production in $e^+e^-$ annihilation is indicated in fig. 7.

5 Approaches to Understanding Soft Breakings

While it is good to have a parameterization of supersymmetry breaking, one might hope to understand these phenomena at some more fundamental level, and to make predictions for these soft breaking parameters. There are a few ideas about how supersymmetry might be broken in nature, and about how this breaking is manifest in the spectrum of the MSSM or some generalization. In this section, I will briefly review these. As we will see, while any of these ideas might be correct, none, as they currently stand are completely compelling. Each has a possible realization in string theory.
5.1 “Supergravity” Breaking

So far, we have treated supersymmetry as a global symmetry. In a theory with gravity (and in particular in string theory), we expect that it should be a local symmetry. In other words, under supersymmetry, the graviton should have a fermionic partner, of spin $3/2$, the gravitino, $\psi_{\mu\alpha}$. In $N = 1$ supergravity, terms with up to two derivatives in the effective lagrangian are specified, as in global supersymmetry, by three functions, the Kahler potential, $K(\phi_i, \phi_i^\dagger)$, and two (sets of) holomorphic functions, the superpotential, $W$, and the gauge coupling functions, $f_a$. The scalar potential is given by

$$V = e^K [ (\frac{\partial W}{\partial \phi_i} + \frac{\partial K}{\partial \phi_i} W) g^{ij} (\frac{\partial W^*}{\partial \phi_j^\dagger} + \frac{\partial K}{\partial \phi_j^\dagger} W^*) - 3|W|^2 ].$$

(117)

In this expression, the Kahler metric, $g_{ij}$ is given by

$$g_{ij} = \partial_i \partial_j K.$$  

(118)

The reasons for putting “supergravity” in quotes in the title of this section are twofold. First, because supergravity theories are not renormalizable theories, they are at best low energy descriptions of some more fundamental theory (string theory). Second, what are usually called supergravity models actually involve a quite specific set of assumptions about the structure of the Kahler potential (and often the functions $f_a$). In particular, these models assume that the various chiral fields fall into two types, called “visible sector” and “hidden sector” fields, and which we will denote by $y_i$ and $z_A$, respectively. The $y_i$’s include the quarks, leptons and Higgs, while the $z_A$’s are called “hidden sector” fields, and are supposed to be associated with supersymmetry breaking. The $z_A$’s are assumed to be responsible for supersymmetry breaking. The superpotential is supposed to break up into two pieces:

$$W = W(y_i) + W(z_A).$$

(119)

This is a plausible assumption, which, given the holormophy of $W$, might be enforced by symmetries, at least at the level of relatively low dimension operators. The Kahler potential is also supposed to break up in such a fashion:

$$K = \sum_i y_i^\dagger y_i + \sum_A z_A^\dagger z_A.$$  

(120)

This latter assumption yields universality. It is often said to follow from the universality of gravity, but this is clearly not the case; no symmetry forbids a more complicated form. Generically, this doesn’t hold in string theory (though it is sometimes true in some approximation).
**Exercise:** An example of this sort of model is provided by the “Polonyi” model. Here one just has a single hidden sector field, \( z \). The superpotential is given by

\[
W_{hid} = m^2(z + \beta).
\]  

(121)

Show that for \( \beta = (2 + \sqrt{3}M) \), where \( M \) is the reduced Planck mass,

\[
G_N = \frac{1}{8\pi M^2},
\]

the potential has a minimum for \( V = 0 \), with

\[
Z = (\sqrt{3} - 1)M \quad m_\varphi^2 = 2\sqrt{3}m_{3/2}^2 \quad A = (3 - \sqrt{3})m_{3/2} \quad m_{3/2} = \frac{m^2}{M}e^{(\sqrt{3} - 1)^2/2}
\]

Here \( m_{3/2} \) is the gravitino mass. Gaugino masses in such models are assumed to arise from a coupling

\[
\int d^2\theta Z W^2_\alpha.
\]  

(122)

### 5.2 Incorporating Dynamical Supersymmetry Breaking

One would like to understand supersymmetry breaking dynamically. Within the framework of gravity mediation, one can suppose that the hidden sector fields, \( z_A \), correspond to some supersymmetry-breaking theory, such as the 3 – 2 model discussed earlier. Assuming that the Kahler potential has, say, the form of eqn. [127], one can work out the scalar spectrum in detail, and it is not qualitatively different than that of the Polonyi model discussed above. There is, however, a problem when one discusses the masses of the gauginos. In models in which the hidden sector is dynamical, operators of the type

\[
\int d^2\theta f(\phi) W^2_\alpha
\]  

(123)

are typically highly suppressed. In the 3 – 2 model, for example, the operator \( f \) of lowest dimension is \( \frac{Q L d}{M^3} \). This leads to a gaugino mass of order

\[
m_{1/2} \sim \frac{\Lambda^4}{M^3} \quad (124)
\]

as opposed to scalar masses of order

\[
m_\varphi^2 \sim \frac{\Lambda^4}{M^2}. \quad (125)
\]

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Generically, these are far too small.

Recently, a solution has been proposed to this problem, referred to as “anomaly mediation” [29]. Suppose that the Kahler potential is not that of eqn [120], but instead has the property that the scalar masses vanish to some degree of approximation. Then there are contributions to both scalar and gaugino masses proportional to gauge couplings. These are associated with certain anomalous transformations in the theory, which require a modification of the argument above that fermion masses are extremely small. One finds (here $\alpha$ is the unified coupling at the high energy scale)

$$m_{1/2} \sim \left( \frac{\alpha}{4\pi} \right) m_o$$

while the scalar masses are given by

$$m^2_q = \frac{1}{32\pi^2} c_o b_o g^4 m_o^2$$

where $b_o$ is the lowest order contribution to the $\beta$ function and $c_o$ is related to the anomalous dimension of the scalar field through

$$\gamma = c_o \frac{g^2}{16\pi^2}.$$  

Examining these formulas, it is clear that there are two issues which must be addressed. First, one needs to understand why the scalar masses are so much smaller than the value one naively expects. Second, the simplest formula predicts that some of the scalar masses are negative, leading to breaking of electric charge. A number of ideas have been proposed for resolving both questions[30].

![Figure 8: One loop diagrams which give rise to “anomaly mediation.” Σ denotes a heavy chiral field.](image)

To understand what goes wrong with the naive arguments for the gaugino masses requires careful consideration of various issues in supergravity theories, but it can be understood in the
following heuristic way. Suppose that in the theory there is a very massive chiral multiplet. The lagrangian for this field includes the term

$$\int d^2 \theta M \Phi^2,$$

as well as a soft-breaking “B-term,”

$$m_{3/2} M \Phi^2 + c.c.$$

(129) (130)

So examine the diagram of 8, where a gaugino couples to the heavy field. The diagram gives a result proportional to $\frac{2}{9} m_{3/2}$. But this already violates our earlier arguments about gaugino masses. To understand what went wrong, note first that this diagram is independent of the mass of the heavy field. Suppose now one introduces a Pauli-Villars regulator with mass $\Lambda$. Because the diagram with the Pauli Villars field, like the original diagram, is mass independent, it survives in the limit $\Lambda \rightarrow \infty$ and yields overall a vanishing gaugino mass. This is in accord with the symmetry argument. On the other hand, for a massless field, the Pauli-Villars term now introduces a contribution, which agrees with the anomaly mediation formula. When this observation was first made in [31], its theoretical underpinnings were not pursued, as it did not seem of great importance; the mass seemed too small. Fermion masses would be loop suppressed relative to scalar masses. As we have noted, the authors of [29, 30], in addition to putting the theory on a clearer footing, have suggested some scenarios where a suitable spectrum might naturally result.

5.3 Gauge Mediation

As an alternative to the gravity mediation hypothesis, which has been discussed in the previous section, suppose that supersymmetry is broken at lower energies, smaller than $\sqrt{M_Z M_P}$. For reasons that will become clear shortly, we will refer to this possibility as “gauge mediation” [32]. The basic model building strategy is indicated in fig. 9. We will not review these models here, but simply mention the basic ideas, and refer the interested reader to some of the excellent reviews which are now available [33]. One supposes that supersymmetry is broken by some set of fields. Some of these fields carry ordinary gauge quantum numbers, so that masses of squarks, sleptons, and gauginos can arise through loops involving ordinary gauge fields (and their superpartners). A typical mass formula (associated with the case of “minimal gauge mediation”) has the form:

$$\tilde{m}^2 = 2 \Lambda^2 [c_3 (\frac{\alpha_3}{4\pi})^2 + c_2 (\frac{\alpha_2}{4\pi})^2 + \frac{5}{3} (\frac{Y}{2})^2 (\frac{\alpha_1}{4\pi})^2]\]

(131)
\[ m_{\lambda_i} = \frac{\alpha_i}{4\pi} \Lambda. \]

Here the parameter \( \Lambda \) is typically the ratio of the Goldstino decay constant \( (F, \text{ with dimensions of mass-squared}), \) to the energy scale of the susy-breaking interactions, \( M_{sb}, \Lambda = F/M_{sb}. \) Such an approach automatically satisfies the constraints imposed by strangeness changing neutral currents, since the masses are, to a good approximation, only functions of gauge quantum numbers.

Figure 9: The general structure of gauge-mediated models. The box indicates dynamics at the supersymmetry breaking scale. The wavy lines indicate various types of gauge exchanges, including gauginos and scalars.

Because their dynamics is controlled by renormalizable interactions at low energies, gauge-mediated models tend to be highly predictive. Such models typically have only a small number of parameters beyond those of the SM. The phenomenology of these theories is also quite distinctive. In particular, the gravitino is far lighter than expected in gravity-mediated models. Indeed, examining eqn. [131], one sees that the parameter \( \Lambda \), and thus the scale of these new interactions and of the Goldstino decay constant, could be as low as 10 TeV. In this case, the gravitino mass is of order

\[ m_{3/2} \sim \frac{(10 \text{ TeV})^2}{M_p} \sim 0.1 \text{ eV}. \]

The couplings of this particle are suppressed only by the 10 TeV scale, rather than the Planck scale. Moreover, this particle is now the LSP. The next to lightest supersymmetric particle now may be charged or neutral, in principle. It can decay with a track length as short as a fraction of a cm to final states containing an (unobserved) gravitino[34].

The detailed phenomenology of these models is quite rich, and model building possibilities have only been partially explored. It should be noted that some of the most elegant models which have been constructed so far have supersymmetry broken at a scale much larger than 10 TeV[35]. Others have features such as composite quarks and/or Higgs fields. No model is
yet totally compelling by itself, but it seems possible that a truly “Standard” supersymmetric model might emerge from this framework.

6 String/M Theory Phenomenology

If we are trying to develop a superstring phenomenology, the first question we might ask is: what is string theory? As you know, we have good reason to believe that there is some overarching theory, various limits of which look like weakly coupled string theory, or eleven dimensional supergravity, or other things. All we really understand, however, is what the theory looks like on certain moduli spaces with a great deal of supersymmetry. It is highly unlikely that the state of this theory which describes nature sits on this moduli space. All thinking about string phenomenology to date assumes that this state is a stationary point of some potential in an approximate moduli space. We will see that we have some understanding of these approximate moduli spaces as well. However, even this need not be correct. It could be that the ground state is some truly isolated point. In that case, the question: what is string theory? assumes greater urgency. It is not clear that this state need be connected, in any way, to the states we understand, nor that there is any small parameter which might allow us to study this state. Saying simply that “string theory is the theory of quantum gravity,” in this situation, will have little content.

We won’t offer any general answers to this question here. Instead, we will assume that nature is approximately supersymmetric, and that indeed we sit at a point in some approximate moduli space. We now understand much about supersymmetry dynamics and phenomenology. In the remaining lectures we will apply this understanding to String/M theory. We will use supersymmetry both to constrain possible dynamics, and to consider phenomenology.

Apart from hierarchy, we will see some reasons to believe that low energy supersymmetry might play some role in string theory, but these will not be compelling. The question of whether string theory predicts low energy supersymmetry, or something dramatically different, is the most important question of string phenomenology, and the reader should keep this in mind throughout. There was, for a long time, a prejudice that string theory and low energy supersymmetry somehow go together. Recently, there have been proposals to solve the hierarchy problem with large extra dimensions[10, 11] or warped extra dimensions[12]. In these proposals, the standard model lives on a brane or wall of some sort, and gravity propagates in the bulk.

\footnote{I thank Leonard Susskind for conversations which sharpened this question.}
The first set of proposals generally invoke bulk supersymmetry as part of the explanation of the hierarchy, but typically there is no relic of supersymmetry at low energies in the conventional sense. The second set of proposals don’t seem to require supersymmetry at all! These lectures may reinforce the prejudice that supersymmetry is important, mainly because it will be easiest to analyze states which are supersymmetric, in some approximation. Indeed, it has been hard, up to now, to relate the extra dimension proposals to detailed constructions in string theory (with the exception of [37]). It is unclear what the parameters of these constructions are, and whether any of these would permit a systematic computation of physical quantities, i.e. would permit the prediction of physical quantities directly from some underlying (string) theory. In the framework of low energy supersymmetry will see that, even though the true vacuum of $M$-theory cannot be described in a systematic weak coupling expansion, there may be a small parameter, which would allow computation of some quantities. Perhaps, at the moment, the best thing we can say is that this picture is consistent with things we know about nature, and might permit some predictions. It is your challenge to do better.

6.1 Weakly Coupled Strings

We would like first to get some feeling for the moduli spaces of string/M theory. We begin by considering the heterotic string theory, compactified on $T^6$. This theory has 16 supersymmetries ($N = 4$ in four dimensional counting). It is not hard to see how the ten dimensional fields fall into $N = 4$ multiplets. Vector indices decompose as four dimensional Minkowski indices and internal indices, associated with the 6 dimensional internal space. We have seen that the two-derivative terms of $N = 4$ theories respect an $SU(4)$ symmetry; this is just inherited from the $O(6)$ symmetry of the compact dimensions, at infinite radius. Clearly it is not exact; it is broken by the finite size of the torus, and these effects will show up at the level of higher derivative operators. This $SU(4)$ is convenient for classifying fields. The vector fields of 10 dimensions, for example, decompose as four dimensional vectors and scalars:

$$\begin{align*}
A_M &\rightarrow A_\mu, \phi_i, i = 1 \ldots 6.
\end{align*}$$

(133)

The gauginos decompose as

$$\begin{align*}
\lambda^A &\rightarrow \lambda^{\alpha,i} \quad i = 1, \ldots 4
\end{align*}$$

(134)

where $\alpha$ is a space-time spinor index and $i$ an $SU(4)$ index. The supersymmetry generators have a similar decomposition. The metric decomposes into the four dimensional metric, 6 vectors, and 21 scalars; the antisymmetric tensor of ten dimensions decomposes as a four dimensional
antisymmetric tensor (dual to a scalar), 6 vectors, and 15 scalars. The gravitino decomposes into four four-dimensional gravitinos, $\psi^{a,i}_\mu$, and 24 spinors, $\psi^{a,i}_j$. There is also a scalar from the ten-dimensional dilaton. To see how these states fit into $N = 4$ multiplets, note first that the ten-dimensional vectors give four dimensional vector multiplets. The four dimensional supergravity multiplet contains a graviton, four gravitinos, 6 vectors, four Weyl spinors, and two scalars (see [16] for example). So the state counting is correct to correspond to a supergravity multiplet and 6 additional vector multiplets.

What does characterize the theory we call string theory or $M$-theory is the existence of limits in which there is a perturbation expansion corresponding to a weakly coupled string theory. In each case, one can identify the modulus which corresponds to the string expansion parameter in a variety of ways. For the heterotic case, one can examine the low energy effective lagrangian. This lagrangian can be presented in a variety of ways. One has the freedom to redefine the metric by a Weyl-rescaling, i.e. by $g_{MN} \rightarrow f(\phi)g_{MN}$, where $\phi$ is the dilaton field. A particularly convenient choice is the “string metric,” in which the ten-dimensional action has the form:

$$\int d^{10}x \sqrt{g} e^{2\phi} \left[ -\frac{1}{4} F_{MN}^2 + i\lambda D_M \Gamma^M \lambda + \mathcal{R} + \ldots \right]$$

Here the dilaton appears out in front of the action (“string frame”), and the unit of length is $\ell_s$, the string scale. This is also the cutoff scale for this effective lagrangian. As a result, $e^{-2\phi}$ plays the role of a dimensionless expansion parameter. $\phi \rightarrow \infty$ corresponds to weak coupling; $\phi \rightarrow +\infty$ to strong coupling. That $\phi$ plays the role of the string coupling can also be seen directly in string theory, by studying the conditions for conformal invariance, for example. A more conventional presentation is provided by the Einstein metric (Einstein Frame), in which the curvature term in the action is independent of the dilaton.

**Exercise:** Determine the transformation between the string metric and the Einstein metric.

If we now compactify this theory on a six torus, with

$$\int d^6y \sqrt{g} = V$$

then the four dimensional coupling is

$$g_4^{-2} = e^{2\phi}V.$$  

These compactifications have many interesting features. For example, they exhibit electromagnetic duality, as well as a duality to type II theory compactified on $K_3 \times T_2$. However, for our
discussion, one feature is particularly striking: this moduli space is exact, both perturbatively and non-perturbatively. This follows, as in the case of field theory, by studying the constraints imposed by supersymmetry on the low energy effective action. This is a generic feature of $N \geq 2$ supersymmetry in four dimensions, and $N \geq 1$ in higher dimension. Thus we have our first observation relevant to phenomenology: There may be some states of string theory which resemble our world, but there are certainly many which do not!

One of the interesting features of the moduli space is the existence of various duality symmetries which relate different points. One of the most familiar is $T$-duality, under which, in the case of compactification on a circle of radius $R$, $R \rightarrow \frac{1}{R}$. In the case of the heterotic string, at the self-dual point, the gauge symmetry is enlarged to $SU(2)$. At the enhanced symmetry point, $R$ is a component of an $SU(2)$ triplet (or more properly, $\delta R$ where $R = \sqrt{2} + \delta R$). Since $\delta R$ changes sign under the symmetry, this can be identified with a gauge transformation, a rotation by $\pi$ about the $y$ axis in isospace. Indeed, all of the $T$-dualities of the weak coupling limit can be identified as gauge transformations.

Electric-magnetic duality, or $S$-duality, exchanges $g^2 \rightarrow 1/g^2$. This symmetry can be understood in a variety of ways. For example, under the duality which connects the heterotic string theory on $T^4 \times T^2$ and Type II theory on $K_3 \times T_2$, $S$ duality is mapped to $T$ duality\[38\]. For a particular choice of $g^2$, one has an unbroken discrete symmetry which exchanges $\vec{E}$ and $\vec{B}$.

It is interesting that one can find points of enhanced symmetry under which all of the moduli transform. The simplest example of such a maximally enhanced symmetry is provided by the IIB theory in ten dimensions. There, one has an $SL(2,\mathbb{Z})$ symmetry, one of whose generators transforms the dilaton as multiplet, $\tau = a + \frac{1}{g}$, as

\[\tau \rightarrow -\frac{1}{\tau}\] \hfill (138)

For a special value of $g$, this symmetry is restored, and since this is the only modulus, all of the moduli transform. Such enhanced symmetry points, in theories with less supersymmetry, are potentially interesting for a variety of reasons. First, they are automatically stationary points of whatever may be the effective action, so they are candidate ground states. They are also of interesting, as we will discuss later, for various issues in cosmology.

What about four dimensional compactifications with $N = 1$ supersymmetry? Based on our field theory experience, we might expect that the classical moduli are not moduli; at best, there are approximate moduli for weak couplings and/or large compactification radii. We consider
this question first in the context of Calabi-Yau compactifications of the heterotic string at large radius (see [39, 40]). For large $R$, one constructs solutions perturbatively in $\alpha' R^2$, either by looking for renormalization group fixed points in two dimensions, or by solving the field equations in the effective field theory. In the two dimensional description, one has equations for conformal invariance, constructed order by order in $\alpha'$. In the ten dimensional description, one first solves the equations including operators with at most two derivatives, and then considers the effect of higher derivative operators. One finds that these schemes are equivalent, and that to lowest order, Ricci-flat backgrounds along with appropriate gauge field backgrounds solve the equations of motion.

The first question one might ask is: under what circumstances can these lowest order solutions be generalized to exact solutions? Are the corresponding moduli found at lowest order truly moduli of the classical theory? And what is their quantum mechanical fate? These questions can both be addressed by considering the low energy, four dimensional field theory. Consider, first, the question of constructing solutions given a lowest order solution. What is the issue? Suppose one has solved the classical equations of one of the superstring theories to lowest order in $\alpha'$, or more precisely $\alpha'/R^2$, and suppose that supersymmetry is preserved to this order. Then the spectrum of fluctuations about this background includes states with mass of order $1/R$, mass of order $\ell_s$ (the string scale), and some finite number of states of zero mass. The question of finding a solution is the question of whether there are tadpoles for these states in higher orders of approximation.

For the massive states, tadpoles do not represent obstructions to finding solutions. This is already clear in simple field theories. If

$$\mathcal{L} = m^2 \phi^2 + \Gamma \phi,$$

the massive field simply shifts so as to cancel the tadpole. In the two dimensional, conformal field theory description, this is just the statement that if one adds a small term proportional to an irrelevant operator, the system flows to the fixed point. However, for massless states, tadpoles are potentially more serious. There is no guarantee that one can find a (static) solution, and in general one cannot. But this statement also makes clear that to investigate the existence of solutions, one should integrate out the massive fields and examine the low energy effective lagrangian. This effective lagrangian must be supersymmetric and respect the various symmetries, and as usual, this provides powerful constraints.

In constructing weak-coupling compactifications of the heterotic string, one must not only choose, at lowest order, a Ricci-flat background for the gravitational field, but one must also
choose a background gauge field. The general problem is discussed in [39, 40]. The simplest choice is to take the gauge field to be in an $SU(3)$ subgroup of the gauge group, and to set this field equal to the spin connection. In the $\alpha'$ expansion for these configurations, there is then a simple argument that there can be no obstruction to the construction of an exact solution. Consider the problem first from the conformal field theory point of view. The string propagation in a given background metric is described, for large radius, by a $\sigma$-model,

$$\int d^2\sigma g_{ij}(x^k, \bar{x}^{\bar{k}}) \partial x^i \partial \bar{x}^\bar{j} + \text{fermionic terms, etc.} \tag{140}$$

In this form, it is clear that the expansion parameter is $R^2$, where $g_{ij} = R^2 g^o_{ij}$, where $g^o$ is a reference metric of order the string scale. To lowest order in $R^2$, the condition for conformal invariance is vanishing of the $\beta$-function,

$$\beta_{ij} = \mathcal{R}_{ij} = 0 \tag{141}$$

where $\mathcal{R}$ is the Ricci tensor.

The conformal field theories which describe Calabi-Yau backgrounds have two left-moving and two right-moving supersymmetries. The question of obstructions, here, is whether the conformal field theory constructed as a solution of the lowest order equations generalizes to a solution to all orders. Tadpoles for massive fields correspond to corrections to the $\beta$-functions of irrelevant operators, and are not an issue; the question is whether there are corrections to the $\beta$-functions for the marginal operators. To see that there cannot be, note that these are suitable backgrounds not just for the heterotic string theory but for the Type II theory; the Type II theories have both left and right moving supersymmetries. In the Type II theory, in space-time, these solutions describe backgrounds with $N = 2$ supersymmetry. But we have seen that for $N = 2$ supersymmetry, the moduli cannot be exact. This implies that the $\sigma$-model is an exact conformal theory, and this remains true whether it is considered as a background for the heterotic theory or Type II theory. These statements are supported by detailed perturbative computations, as well as by analyses of instanton effects in the $\sigma$-model.

In situations with less world-sheet supersymmetry, the situation is more complex. To understand the nature of the problem, we can give another argument, more directly in the heterotic string. Among the various moduli of the theory are deformations of the metric, $\delta g_{ij}$ of eqn. [140] which preserve the conformal-invariance condition. These are in one to one correspondence with $(1,1)$ forms, $b_{i\bar{j}}$,

$$b_{ij} = \delta g_{ij} = -b_{ji}. \tag{142}$$
The simplest example is $\delta g \propto g$, corresponding to an overall dilation of the metric; this is clearly a solution, at lowest order, since the $\mathcal{R} = 0$ condition does not determine the radius of the compact space. In general, the Ricci-flat metrics of interest are Kahler,

$$g_{\bar{g}} = \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^\bar{j}} K.$$

(143)

The vertex operator for $b$ is

$$V_b = \int d^2 \sigma \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} K(\partial x^i \partial x^j - \partial \bar{x}^i \partial x^j)e^{ik \cdot x} + \text{terms which vanish at } k^\mu = 0. \quad (144)$$

Here $k$ is the ordinary momentum four vector, and $x^\mu$ refers to the free fields which describe the non-compact dimensions. At zero momentum, the integrand is a total derivative,

$$V_b = \int d^2 \sigma \partial(\frac{\partial K}{\partial x^j} \partial x^j) - \partial(\frac{\partial K}{\partial x^i} \partial x^i). \quad (145)$$

In $\sigma$-model perturbation theory, then, this mode of the metric decouples at zero momentum. This means that the effective four-dimensional theory has a symmetry under which $b$ shifts by a constant. $b$ is part of a chiral multiplet. In the case of the radial dilaton, the scalar component of this multiplet is $R^2 + ib$. But $W$ is an analytic function of this field; because it is independent of $b$ it is also independent of $R^2$. In other words, there are no corrections to the superpotential in $\sigma$-model perturbation theory!

These statements hold for both the “standard embedding,” with spin connection equal to the gauge field, and more generally. One can also ask what happens beyond perturbation theory. We have already given an argument that for the standard embedding there are no non-perturbative corrections either. This can be verified at the level of world-sheet instantons. For more general compactifications, it appears that generically there should be corrections. The actual situation is more complicated, however [41].

Arguments of this type can also be used to establish the existence of massless states other than moduli, not protected by any space-time symmetry. This could well be relevant to the MSSM, where one wants to understand the absence (at the level of the superpotential) of a mass for the Higgs fields.

6.2 Beyond the Classical Approximation

Using space-time arguments, there is a good deal one can say about these theories non-perturbatively. In the Type II case, $N = 2$ supersymmetry again ensures that there is no
potential for the moduli. In the heterotic case one has only $N = 1$ supersymmetry. In this theory, the dilaton $(1/g^2)$ is in a multiplet with the axion, $B_{\mu\nu} \leftrightarrow a$,

$$\int d^2\theta [\frac{1}{g^2} + ia] W^2_\alpha = \int d^2\theta SW^2_\alpha. \quad (146)$$

Again, there is a shift symmetry, $a \rightarrow a + \delta$. This can be understood by looking at vertex operators, or by noting that $\partial a \propto *H$, where $*H$ denotes the dual of $H$ ($*H_\mu = \frac{1}{4!} \epsilon_{\mu\nu\rho\sigma} H_{\nu\rho\sigma}$), and that the $B$ gauge symmetry forbids non-derivative terms for $H$.

So, in string perturbation theory, since $W$ is holomorphic, there are no dilaton dependent corrections to the superpotential. Moduli remain moduli, massless particles remain massless.

What happens beyond perturbation theory? The axion shift symmetry is anomalous. Instantons of the low energy field theory, for example, can generate effects which behave as $e^{-S} = e^{-8\pi^2/g^2+i\alpha}$. Non-perturbative, stringy effects surely generate similar terms. Given our lack of understanding of the theory at a fundamental level, one might think that there is little one can say. However, it turns out that using symmetries and holomorphy, it is often possible to make striking statements.

First, we should note that we do not expect that in a theory of gravity there should be continuous global symmetries. In weakly coupled string theory, this is a theorem. In all of the recent work on strongly coupled theories, no examples of continuous global symmetries have been found. We will thus adopt as a working hypothesis that the only allowed continuous symmetries are gauge symmetries.

Discrete symmetries, on the other hand, abound in string theory. They can usually (and probably always) be thought of as gauge symmetries. It is easy to construct examples. In toroidal compactification on a square lattice there is a symmetry which interchanges the two lattice generators. This $Z_2$ symmetry traces back to general coordinate transformations in the compact space; it is an $R$ symmetry, since it rotates fermions differently than bosons. Much more intricate versions of such symmetries are discussed in the case of Calabi-Yau compactification. Again, generically these transformations, which originate as coordinate transformations in the internal dimensions, act differently on fermions than bosons, and so are $R$-symmetries. The $Z_2$ which exchanges the two $E_8$’s of the heterotic string theory is another discrete symmetry (it can also be shown to be a gauge symmetry).

Such discrete symmetries place powerful constraints on the form of the low energy theory. As an example, consider the (Wilsonian) effective action at some high energy scale for a
compactification of the heterotic string with some set of discrete $R$ symmetries. Typically, the
dilaton, $S$, is neutral. On the other hand, the superpotential must transform by a phase under
the various $R$ symmetries. So

$$\int d^2\theta f(e^{-S}).$$

(147)

Provided that there are not couplings of the type $Me^{-S}$, for some other modulus, $M$, no
perturbative or non-perturbative effects can lift the flat directions. In many cases, such couplings
are forbidden by symmetries. Examples of this phenomena occur already in textbook models,
such as the quintic in $CP^4$ and the $Z_3$ orbifold[3]. Recall that this is a statement about some
Wilsonian action with a large cutoff. What this says is that SUSY breaking, if it occurs at all,
must be a phenomenon of the low energy field theory!

Let us focus, then, on this low energy field theory. For example, in the case of the standard
embedding, the low energy gauge group is $E_6 \times E_8$ (or a subgroup of $E_6$ obtained from Wilson
lines). For the $E_8$, there are no matter fields. As we have seen, in such a theory, there is
gluino condensation. The dilaton couples to the gauge fields through $\int d^2\theta SW^2_\alpha$, which leads
to a superpotential for $S$ proportional to $\langle \lambda \lambda \rangle$. One can, in fact, determine the form of $W(S)$
completely from symmetries. The low energy theory has an approximate symmetry under which

$$\lambda \to e^{i\alpha} \lambda \quad S \to S - i\frac{b_0}{3} \alpha.$$ 

(148)

To be consistent with this, $W$ must take the form

$$W = ce^{-\frac{3S}{b_0}}.$$ 

(149)

We have argued that the global symmetry cannot be exact, and we can at this point ask
about the form of possible corrections. Suppose, for example, that in the high energy theory
there is a term

$$\zeta \int d^2\theta W^2_\alpha W^2_\beta.$$ 

(150)

Such a term could arise at one loop. Treat $\zeta$ as a spurion. It then has $R = -2$, so one might
expect corrections to the superpotential of the form

$$\zeta e^{-\frac{6S}{b_0}}.$$ 

(151)

This correction is systematically smaller than that of eqn. [149] at weak coupling.

The main features of eqn. [149] and its possible corrections should not come as a big
surprise. In general, we expect $V(S) \to 0$ as $S \to \infty$. Similarly, as $R \to 0$ (say with fixed
The value of the ten-dimensional dilaton), the theory has more supersymmetry, and the potential vanishes. What is interesting is how much control we have over the theory in these cases.

The fact that the potential tends to zero at weak coupling and large radius means that stable minima of string theory exist, if at all, at strong string coupling and compactification radii of order one (except in those cases where the moduli are not lifted at all). This is the big puzzle of string phenomenology. Why are the gauge couplings we observe weak? Why is $M_{\text{gut}}/M_p$ small? And finally, is anything calculable in any kind of controlled expansion?

Note that duality, by itself, is not much help in addressing these questions. Any weak coupling or large radius description would lead to the same difficulty. However, holomorphy, as we will argue shortly, may offer, again, some hope for understanding.

### 6.3 Alternatives to the Weak Coupling Viewpoint

There are at least two troubling features of the weak coupling picture of string theory. The first is theoretical, the second more phenomenological. These are

- The strong coupling problem which we have described above, i.e. the fact that one can’t stabilize the moduli in a systematic, weak-coupling approximation.

- Even assuming the moduli are somehow stabilized at weak coupling, there is a difficulty. In the heterotic string, the four dimensional coupling is related to the dimensionless string coupling through

$$g_4^2 = g_s^2 \left( \frac{l_s^2}{V} \right). \quad (152)$$

The four dimensional gauge couplings are numbers of order 1 (typical unified couplings are about $1/\sqrt{2}$). If the theory is weakly coupled, $g_s \leq 1$. So one requires $V/l_s^6 \sim 1$. But this means that $M_p \approx M_s \approx M_{\text{GUT}}$, since $M_{\text{GUT}} \approx R^{-1}$, where $R$ is the compactification scale. This is hard to reconcile with the fact of perturbative unification. Alternatively, if one takes $2 \times 10^{16}$ GeV as the unification scale, one predicts that the string coupling is gigantic!

One can object that perhaps one should consider weakly coupled string theories other than the heterotic string. But difficulties arise in these cases as well. In particular, in the Type I
theory,

\[ G_N \propto \frac{\ell_8^8}{V} g^2 \quad g^2 \sim 1 \sim \frac{\ell_8^2}{Vg} \]  

(153)

so \( g^2 \sim 10^{-12} \) ! It is hard to imagine how the coupling could be stabilized at such a small value, but this is not a question which has been extensively explored.

\[ L = -\frac{1}{2\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g}(R - \sum_{i=1}^{2} \frac{1}{8\pi(4\pi\kappa^2)^{2/3}} \int_{M^{10}} d^{10}x \sqrt{g}(\text{tr} F_i^2 + \ldots) \]  

(154)

This action is useful only in the limit that all length scales are large compared to the eleven dimensional Planck length, \( \ell_{11} \).

Now suppose that six dimensions are compactified, say on a Calabi-Yau space. The small parameters are \( 1/R_{11} \), where \( R_{11} \) is the inverse separation of the two walls (the size of the eleventh dimension), and \( 1/V \), the volume of the Calabi-Yau space. From eqn. 154 it follows that

\[ M_{11} = R^{-1}(2(4\pi)^{-2/3}\alpha_{\text{GUT}})^{-1/6} \quad R_{11} = \frac{\alpha_{\text{GUT}}^3 V}{512\pi^4 G_N^2}. \]  

(155)

Plugging in numbers, one finds that \( R \sim 2M_{11}^{-1}, M_{11}R_{11} \sim 72 \). So this is clearly a better picture than the weak string coupling picture! Both \( 1/R_{11} \) and \( 1/V \) are reasonably small.
But now there is a puzzle, connected with how the moduli are fixed, similar to those we discussed from a weak coupling string viewpoint. We can, again, attempt to construct a low energy effective lagrangian for this theory. Indeed, we can identify fields similar to those we identified for weak coupling compactifications. As one can see from our formulas for the couplings, \(1/V \sim S\), the usual weak coupling dilaton. \(C_{11\mu} \sim a\), what is usually called the model-independent axion. \(T \sim R_{11} R^2\) (\(R\) here is the Calabi-Yau radius), while \(C_{11\ IJ} \sim b\). The lagrangian is constructed from \(dC\), so it is invariant under shifts by harmonic forms. This is the same shift symmetry as in the weak coupling theory. Finally, just as at weak coupling, if the Calabi-Yau has discrete \(R\) symmetries (as for the quintic in \(CP^4\) discussed in [39]), supersymmetry breaking must be a phenomenon of the low energy theory.

The question of supersymmetry breaking and of moduli stabilization is, in such cases, a question of whether the low energy theory generates a superpotential. One picture for how such breaking might arise, which closely mirrors the weak coupling picture, is to suppose that the interactions of the standard model reside on one wall, while the other wall contains some additional, “hidden sector” gauge interactions which give rise to gluino condensation. Gluino condensation, again, will generate a superpotential for a linear combination of \(S\) and \(T\). The superpotential one obtains agrees exactly, for a given Calabi-Yau space, with the weak coupling result. This is not surprising. The parameter for the superpotential calculation is \(e^{-S}, e^{-T}\). It is possible for \(S\) and \(T\) to be large, while the string coupling \((S^{-1}T^3)\) is large. So while the weak coupling string theory and eleven dimensional supergravity limits of the theory do not have overlapping regions of validity, the superpotential calculations do. It is thus important that they agree, and supports not only the duality between these regimes but the very existence of the theory with reduced supersymmetry.

To actually determine the potential requires knowledge of the Kahler potential. In the weak coupling limit, one could easily read this off at tree level. At strong coupling, the situation is similar. At large \(R_{11}\), the theory goes over to a five dimensional, \(N = 1\) theory, i.e. a theory with eight supersymmetries. These symmetries are highly restrictive. In particular, the Kahler potential is necessarily the same at strong as at weak coupling. As a result, just as at weak coupling, one has a potential,

\[
V \sim \frac{1}{R_{11}^{a}} e^{-aR_{11} - S}.
\]

Here, the term \(aR_{11}\) in the exponent arises because the gauge coupling in the hidden sector depends not only on \(S\) but on \(T\). This is true both at weak coupling[15, 16] and strong coupling[14]. Again, this coupling, which is holomorphic, agrees in the two limits[16].

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We see, as we might have expected, that the moduli can be stabilized, if at all, for $S$, $R_{11} \sim 1$. So, while the Horava-Witten limit certainly yields a better qualitative description of the theory than does the weakly coupled string, but the stabilization issue remains.

7 Moduli and Cosmology

A possible clue to understanding the fate of the moduli of string theory comes from cosmology. Suppose that there is a stable vacuum of string theory, with broken supersymmetry and in which the moduli gain mass of order $m_{\text{susy}}$. One might imagine that the modulus potential looks as in fig. 11. Such a picture has two possible implications for cosmology:

- The potentials for the moduli are likely to be rather flat. Thus moduli are candidate inflatons\[47, 48\].
- The moduli have the potential to carry far too much energy, overclosing the universe\[47\]. This is called the “moduli problem” of string cosmology. Related problems include the question of whether the system can even find the correct vacuum\[49\].

![Figure 11: A plausible potential for the moduli.](image)

Through much of these lectures, we have stressed the possibility that the vacuum which describes our universe lies in an approximate moduli space. If one assumes these moduli have masses of order $M_W$ (or smaller), and that they range over values of order $M_p$, one encounters a serious cosmological difficulty. The fluctuations of the microwave background suggest that the Hubble parameter, $H \sim \frac{1}{t}$, was once of order $10^{16}$ GeV $\gg M_W$. At such times, effects other than the zero temperature, low curvature, effective action were probably important. For example, if there was an earlier period of inflation, associated with an inflaton, one would expect large corrections to the potential for the scalar fields. Calling $\mathcal{I}$ the inflaton, and assuming that
the underlying laws are supersymmetric, one might expect \( \frac{\partial W}{\partial T} \sim M_p H \sim (10^{16} \text{ GeV})^2 \). Then couplings such as

\[
\frac{1}{M_p^2} \int d^4 \theta \bar{T} T M^4 \mathcal{M}
\]

(156)
give big corrections to the moduli potential. Generically, there is no reason that \( \mathcal{M} \) should sit at the minimum of its potential at these early times. Indeed, until \( H \sim M_W \), the zero temperature, zero curvature potential is presumably irrelevant. At \( H \sim M_W \), then, it is natural to suppose that \( \mathcal{M} - \mathcal{M}_i \sim M_p \). The equation of motion for \( \mathcal{M} \) is just \( D^2_t \mathcal{M} + V' \), which in the Robertson-Walker background becomes:

\[
\mathcal{M} + 3H \dot{\mathcal{M}} + V'(\mathcal{M}) = 0.
\]

(157)

Assuming that the universe is matter dominated at this time (e.g. due to the oscillations of the inflaton field),

\[
H = \frac{2}{3t}
\]

(158)
corresponding to a scale factor growing as \( R \sim t^{2/3} \). Eqn. (157) then has the solution

\[
\mathcal{M} \approx M_o \sin(mt)(\frac{t_o}{t}).
\]

(159)

where we have written the quadratic term in potential near the minimum as \( m^2(\mathcal{M} - M_o)^2 \).

The energy is approximately

\[
\rho = \frac{1}{2} [\dot{\mathcal{M}}^2 + m^2 \mathcal{M}^2] \propto (\frac{R_o}{R})^3.
\]

(160)

One can think of this as a coherent state of massive particles, oscillating in phase, and diluted by the expansion of the universe.

**Exercise:** Show that the same is true in the radiation dominated case, i.e. when \( R \propto t^{1/2} \).

The difficulty is that these particles typically come to dominate the energy density of the universe long before nucleosynthesis. One expects that they have Planck scale couplings, so their lifetimes are not likely to be shorter than

\[
\Gamma \leq \frac{1}{4\pi^2} \frac{m^3}{M_p^2} \sim 10^{-30}
\]

(161)

for TeV mass moduli. This corresponds to a lifetime of order \( 10^7 \) sec, i.e. much later than typical times associated with nucleosynthesis. The density at the time of decay can be estimated as
follows. By assumption, $\mathcal{M}$ starts at a Planck distance (in field space) from its minimum. It starts to oscillate when the Hubble constant is comparable to its mass, i.e. when $t_o \sim \frac{1}{m}$. Thus initially the energy density is $\rho \sim m^2 M_p^2$, so when the moduli decay,

$$\rho = m^2 M_p^2 \left( \frac{\Gamma}{m} \right)^2.$$  

(162)

Taking $m = 10^3$ GeV, this is about $10^{-20}$ GeV. Thus when the decay products thermalize, their temperature will be of order 1KeV. This is well below the temperature of nucleosynthesis. The successful predictions of nucleosynthesis are thus spoiled (these are based on the assumption that the universe is radiation dominated at nucleosynthesis; moreover, the decay of the moduli destroys most of the light nuclei).

This problem is called the moduli problem of string cosmology. Various solutions have been proposed

- The moduli masses are much larger than one TeV. If the mass is 100 TeV or so, the reheat temperature is higher than a few MeV, and nucleosynthesis can occur again. The possible difficulty here is that one must also produce the baryons in the decays of these particles. This might occur through baryon number violating couplings, or through the Affleck-Dine mechanism\[50\]

- Late inflation\[51\]. The idea here is that a late stage of inflation drives the moduli to their minima. The difficulty with this proposal is that the minima, as we argued above, will not necessarily coincide with the zero curvature minima, so this possibility seems to be fine tuned.

- Enhanced symmetries. We have seen that sometimes in string theory, all moduli are charged under unbroken symmetries at some points in the moduli space. These points of “Maximally enhanced symmetry” are automatically stationary points of the full quantum effective action. They are also naturally minima at early times, so it is plausible that the high curvature (temperature) and zero curvature (temperature) minima coincide.

- There are no approximate moduli. This possibility, in some ways, is not so different than the previous one. It obviously avoids the moduli problem. But it has the inherent problem that there is not likely to be a small parameter in such a picture. Moreover, as we have discussed above, it is unclear how one would connect such a possibility to string theory.

There are other troubling issues in string cosmology connected with moduli, which we do not have time to review here\[49, 52, 53\]. But for the moment, we would argue...
8 Stabilization of Moduli

We turn, then, to the question of stabilization of the moduli. No complete model exists, but there are some ideas. Given the poor state of our present understanding, we should try, as we review these ideas, to keep in mind certain questions:

- Are there generic predictions in any particular scheme? For example, is there low energy supersymmetry with some pattern of soft breakings?

- Why are the gauge couplings weak? Why are the radii large?

- What quantities, if any, are calculable, even in principle?

We will discuss four proposals which have been put forward from this viewpoint: Kahler stabilization, the racetrack scheme, maximal symmetry, and topological stabilization in large dimensions.

8.1 Kahler stabilization

In weak coupling, the string perturbation expansion is believed to be less convergent than that of ordinary field theory. This motivates the hypothesis that the Kahler potential of the moduli, $K(M)$, is much different from its weak coupling form when $e^{-S}$ ($S$ is the modulus which determines the four dimensional gauge couplings, which we will call the dilaton). Indeed, this is compatible with our discussion of the Horava-Witten scheme, where we saw that there is a regime of large $S$ and $R_{11}$ where the weak coupling picture is certainly not valid. (In that limit, the question is to understand why the corrections to the $T$ Kahler potential are so large in a regime where $T$ itself would seem to be large). Focusing on the dilaton, suppose $W = e^{-cS}$ as in models of gaugino condensation. One can easily invent Kahler potentials such that the potential of eqn. [117] has a minimum at some point $S_o$ such that $e^{-S_o}$ is hierarchically small and with vanishing cosmological constant [54]. These models, of course, are finely tuned, but at least the various terms in the potential are comparable in order of magnitude.

What predictions does this hypothesis make? Beyond the starting assumption that the theory is approximately supersymmetric (i.e. low energy supersymmetry), there are two generic outputs of this scheme:
• Coupling unification: if the couplings are unified for large $S$, as in weakly coupled strings,

$$\int d^2 \theta (S + e^{-S} + \ldots) W_\alpha^2$$

is hardly corrected from its large $S$ form. Note, however, that in this scheme one has no control over threshold corrections. In practice, this means that at most one can trust the leading log contributions to coupling unification.

• For similar reasons, the terms in the superpotential are the same as in the large $S$ limit, for similar reasons as above.

However, in this scheme, the Kahler potential, by assumption, is modified from its large $S$ form. So quantities like soft breaking masses are inherently uncomputable.

### 8.2 Racetrack Models

Suppose one has two gauge groups without matter fields, each of whose couplings is determined by a modulus $S$. Then the superpotential has the form

$$W = b_1 \alpha e^{-S/b_1} - b_2 \beta e^{-S/b_2}.$$  \hfill (164)

The equation $\frac{\partial W}{\partial S} = 0$ yields

$$S = \frac{b_1 b_2}{b_2 - b_1} \ln(\beta/\alpha).$$ \hfill (165)

If $b_1$ and $b_2$ are large, then $S$ is large. But unless $b_1 \approx b_2$, $e^{-S/b_1} \sim e^{-S/b_2} \sim 1$, and the low energy analysis is inappropriate. So one requires $b_1 = b_2 + \delta$. This represents a discrete fine tuning. For example, if the gauge groups are $SU(10)$ and $SU(11)$, $S \approx 100$, which is not too far from a grand unified coupling.

In supergravity, this minimum yields a state with a non-zero cosmological constant. This problem can be solved, however, if the model has an $R$ symmetry, unbroken at the minimum. The one existing model requires many singlets, with constraints on their couplings which presumably require rather elaborate discrete symmetries. One might hope to find a more economical model.

While this scheme might generate a small gauge coupling with unbroken supersymmetry and one less modulus, there may be no sense in which there is a weak coupling expansion. Consider, for example, the corrections to the $S$ Kahler potential, indicated in fig. [2].
the point of view of the heterotic string theory. Because $N^2$ gauge fields run in the loop, diagrams like this and its generalizations behave as $(g^2 N^2)^m$, i.e. the would-be expansion parameter is of order one. Still, as for Kahler stabilization, holomorphy can ameliorate this problem. $e^{-S/b_1} \ll 1$. So holomorphic quantities computed at large $S$ maintain there form even at the would-be minimum. In the Type I theory, the situation is somewhat different (I thank I Antoniadis for comments which lead to consideration of this point.) Since the string coupling, $g$, is equal to $g^2_{YM}$, loop corrections might be small (modulus factors can also enter in the heterotic string argument above). It is not completely clear what one should make of this latter observation, since in any case there is no sense in which one can take the gauge group arbitrarily large. We will assume, for the rest of this section, that in the racetrack picture with a discrete fine tuning, one can compute holomorphic but probably not non-holomorphic quantities.

The result is not so unappealing. One discrete fine tuning is the main thing which is required, to fix the “dilaton.” Gauge couplings may be calculable. The mass of the dynamics which fixes the modulus is larger than the scale of supersymmetry breaking. Supersymmetry breaking itself should be through low energy dynamics. Physical quantities related to holomorphic quantities should be calculable.

### 8.3 Maximally Enhanced Symmetry

We have seen that there exist points in moduli spaces at which all moduli are charged under symmetries. The particular examples we discussed had at least 16 supersymmetries. One can find an example with 8 supersymmetries ($N=2$ in four dimensions) by considering the Type II theory on a Calabi-Yau space at a “Gepner Point.” At the Gepner point, all of the geometrical moduli are charged under symmetries. The theory inherits the $SL(2,Z)$ duality of the higher dimensional theory, so by going to the self-dual point, one has a situation where all moduli transform under symmetries.

What about $N = 1$? Such points presumably exist, and they are interesting from at least
two points of view:

- They are naturally stationary points of the effective action.
- If the ground state of the system is such a point, this provides a natural solution of the cosmological moduli problem.

The main objections to such a possibility are that one expects \( \alpha \sim 1 \), as in the self-dual point of electric-magnetic duality, and that one does not expect hierarchies, nor that any sort of systematic approximation schemes should be available. Still, we can adopt the hypothesis that there exist such states, with gauge couplings unified and \( \alpha \ll 1 \), and with \( e^{-2\pi/\alpha} \) small, allowing for hierarchies. This is at least consistent with the observed facts, has the two virtues listed above, and has some definite consequences:

- SUSY breaking is a low energy phenomenon. This is because the symmetries forbid terms in the superpotential linear in any of the moduli.
- \( \int d^2 \theta M \alpha W_\alpha^2 \) is also forbidden, so gluino condensation, in the conventional sense, is also irrelevant for supersymmetry breaking.
- Related to the previous point, gaugino masses cannot be generated by moduli \( F \)-terms. Thus it is probably necessary that supersymmetry be broken as in gauge mediated models at quite low energy.

An interesting observation concerns the identity of the moduli in this picture. They could well be fields of the MSSM. For example, at the level of renormalizable interactions, there is a flat direction in the MSSM with

\[
Q^{[1]} = \begin{pmatrix} v & v \\ 0 & 0 \end{pmatrix}, \quad Q^{[2]} = \begin{pmatrix} 0 & 0 \\ v & v \end{pmatrix}, \quad L = \begin{pmatrix} 0 \\ v \end{pmatrix}.
\]

(166)

Here the numbers in braces are \( SU(2) \) indices. This flat direction has a gauge invariant description in terms of the chiral field \( QQQL \). This direction can be exactly flat if the fields transform suitably under \( R \) symmetries. So the moduli might well be superpartners of known particles, exactly flat due to discrete \( R \) symmetries.

Two related possibilities should be mentioned. First, it could be that there are no moduli, even in some approximate sense. Operationally, this is not too much different than the maximal symmetry hypothesis. Again, there is the puzzle: why are the couplings small? Why are
there hierarchies? One can also consider the racetrack model in combination with enhanced symmetries. The modulus which controls the gauge coupling might be fixed by supersymmetry-conserving dynamics, and very massive, with the rest of the moduli sitting at enhanced symmetry points.

9 Large Dimensions and TeV Scale Strings

It has usually been assumed that the compactification scale of string theory should be similar to the string scale (or the eleven dimensional Planck scale, etc.). The underlying prejudice is simply that dimensionless ratios shouldn’t be terribly large. In the heterotic string, one can make this idea a bit more precise, by noting, as we did earlier, that weak string coupling, and the observed values of the gauge couplings, imply that the scales cannot be too different. Similarly, in the Horava-Witten description, none of the scales are wildly different.

Over the past year, however, there has been much interest in a different possibility. It has been argued that perhaps the solution to the hierarchy problem lies not in supersymmetry, but rather in the possibility that the fundamental scale of interactions is roughly 1 TeV. Much as in the Horava-Witten picture, the Planck scale is a derived quantity, large only because some internal dimensions are now (very) large. The standard model fields should live on a brane or wall, so that the standard model couplings do not become extremely small as the volume of the internal space tends to infinity. The picture, then, is essentially as in fig. II, with the understanding that the fields on the branes are the standard model fields, and perhaps those associated with additional interactions.

How large is the internal space? This depends on the number of compact dimensions. Suppose, for definiteness, that the underlying theory is ten dimensional. Suppose that there are six compact dimensions. The four dimensional Newton’s constant is given by

\[ G_N = \frac{(\ell_{10})^8}{V_6} = \frac{1}{M_p^2} \quad l_{10} \sim \text{TeV}^{-1}. \]  

(167)

Then if there are six large dimensions of comparable size, \( r, r \approx (\text{MeV})^{-1} \), while if there are two large dimensions, with the others of size \( l_{10}, r \approx \text{mm}! \)

This latter possibility is quite amazing, and even more surprising it is not so easy to rule out. In particular, it means that if one probes distances shorter than a millimeter, one will see...
a modification of Newton’s laws, appropriate to five spatial dimensions rather than three, i.e.

\[ F \sim \frac{1}{r^4} \quad (168) \]

This is a dramatic prediction, just barely compatible with current experimental limits, and accessible to improved experiments. As we will see shortly, however, there are astrophysical constraints which suggest that the scale, in the case of two large dimensions, must be significantly larger, probably placing the modification of Newton’s law out of reach.

For any number of compact dimensions, however, there are also dramatic effects in high energy collisions as one approaches the TeV scale. At these energies, one “sees” the extra dimensions. This is because, while each Kaluza-Klein state has coupling of order \( 1/M_p \), there are many states. The fact that individual states couple with strength \( 1/M_p \) follows from the form of the terms in the lagrangian

\[ \ell^{-8} \int d^6y d^4x \sqrt{g} R = M_p^2 \int d^4x \sqrt{g} R, \quad (169) \]

so the coupling of each Kaluza-Klein mode is suppressed by a factor of the volume. However, there are lots of modes; once \( E \gg R^{-1} \), one has a phase space integral appropriate to the higher dimensional field theory, i.e. amplitudes behave like

\[ V_6 \int \frac{d^4k}{(2\pi)^d} G_N \times \text{kinematic factors} \quad (170) \]

\[ \sim \frac{1}{\text{TeV}^8} \int \frac{d^4k}{(2\pi)^d} \times \ldots \]

In other words, the extra dimensions “open up.”

Other proposals have also emerged over the past year, including a particularly interesting one by Randall and Sundrum\[12\], in which the extra dimensions, in some sense, are not large, but there are exponentially large differences in the metric on the different branes. These subjects are developing rapidly, and it is not possible to review them here. On the theoretical side, there are many of intriguing questions. To solve the hierarchy problem in this framework, understand why the radii are large (or the equivalent statement in the Randall-Sundrum picture). In the large dimension case, if one is to avoid introducing very small numbers by hand, it seems necessary to have supersymmetry at least in the bulk. This supersymmetry seems to have little direct consequence for low energy physics. In the Randall-Sundrum case, it seems possible to achieve hierarchies without even bulk supersymmetry\[36\]. The exploration of these extreme regions of the (approximate) moduli space is still in its infancy, and it is quite possible that these issues will be better understood.
On a more phenomenological level, there are a number of issues which any scheme in which the fundamental scale much below the conventional unification scale. These include

- **Proton decay**: In order to suppress proton decay to acceptable levels, it is almost certainly necessary to have a large discrete group. For example, if

  \[ Q \rightarrow e^{2\pi i}Q \quad L \rightarrow e^{2\pi i}L \]  

  then the leading operator, of the form \( L^3Q^9 \), has dimension 18, and the lifetime is of order

  \[ \Gamma = \frac{m_p^{20}}{M^{20}} \times \text{other small factors} \]  

  where \( m_p \) is the proton mass, and \( M \) is the fundamental scale, now supposed to be of order a few TeV.

- **Other problems of flavor**: In order to resolve other problems of flavor, such as rare decays, it is probably necessary that there be additional approximate flavor symmetries. A number of authors have explored the possibility that there is indeed some large (discrete) non-abelian flavor symmetry, perhaps spontaneously broken on distant branes, both in order to understand the absence of flavor violation, and to understand the quark and lepton mass hierarchies\(^{[56]}\).

- **Coupling Unification**: There has been much work on the question of coupling constant unification in this picture. At first sight, one might think that the usual field theoretic analysis of unification is lost, but this is not quite true. For example, in the case of two large dimensions, the log of the large mass scale is replaced by the log of the radius of compactification. Still, obtaining the supersymmetric unification predictions is not generic, requiring, among other things, a spectrum similar to that of the MSSM. This is perhaps a bit troubling, since it sounds as if one needs supersymmetry again as part of the story.

- **Astrophysical Constraints**: In the case of \( d = 2 \), there are significant constraints from the supernova SN87a. The problem is that emission of Kaluza-Klein modes, in this case, can carry off most of the energy. This yields a constraint \( M > 50 \) TeV\(^{[57]}\). Other tests, such as high precision electroweak measurements, suggest that, more or less independent of \( d \), the scale must be larger than several TeV.

What these proposals have shown is that there are plausible solutions to the hierarchy problem which do not involve conventional low energy supersymmetry. They have dramatic
consequences for experiment. It is important to decide whether any of these ideas (including low energy supersymmetry) is really compatible with string theory – or better, whether any is a robust prediction of the theory.

10 What is String Theory and How Would We Know It?

It is not likely that, sometime soon, someone will simply exhibit a solution of string theory with a spectrum and interactions identical with what we see in nature. What we should probably be striving for is a robust, general prediction such as:

- String theory predicts low energy supersymmetry
- String theory predicts large dimensions without low energy supersymmetry
- String theory predicts a warped geometry, with large or infinite dimensions, without low energy supersymmetry.

At the moment, we seem simultaneously close and far from these goals. We understand a great deal about supersymmetry in string theory. We also understand extreme regions of the moduli space which give a brane picture and very large dimensions. What is now crucial is to formulate some principle which might select among these possibilities. While it will be exciting if experiments discover a new symmetry or new dimensions, it would be wonderful if string theorists could commit themselves beforehand to one or another (or some still unknown) possibility.

Supersymmetry has been the focus of much of these lectures. Here, we have proposed some ideas of how a complete picture might look. Perhaps we are in some approximate moduli space, and, while no sort of weak coupling analysis is applicable, still, certain quantities can be computed starting from a weak coupling approximation. We have argued that there are some experimental hints that this is the case: the existence of hierarchies, the smallness of the gauge couplings, and their unification. I am optimistic that we can go farther, perhaps even before our experimental colleagues discover — or fail to discover — supersymmetry at the Tevatron and LHC. Much effort is being devoted at the present time to fleshing out the large/warped dimension pictures in a similar fashion.

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