TASI Lectures on Non-BPS D-Brane Systems

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Abstract

In this set of lectures various properties of D-branes are discussed. After reviewing the basics, we discuss unstable D-brane/anti-D-brane systems, a subject pioneered by Sen. Following him, we discuss the construction of the non-BPS D0-brane in type I theory. This state is stable since it carries a conserved $\mathbb{Z}_2$ charge. The general classification of D-brane charges using K-theory is discussed. The results for the type I theory, and the T-dual type $\text{I}'$ theory, are emphasized. Compactification of type I on a circle or torus gives a theory with 16 supersymmetries in 9d or 8d. In each case the moduli space has three branches. The spectrum of non-BPS D-branes are different for each of these branches. We conclude by pointing out some problems with the type I D7-brane and D8-brane predicted by K-theory.

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1 Introduction

In the past couple of years Sen has drawn attention to various non-BPS D-brane configurations in type II superstring theories as well as various type II orbifolds and orientifolds \([1, 2, 3, 4, 5]\). Two review articles have been written \([6, 7]\). One goal is to identify and understand various objects that are stable but not supersymmetric. By now the BPS D-branes that preserve half the ambient supersymmetry have been extensively studied and are quite well understood \([8, 9]\). So it is time to move on and develop an understanding of other objects that carry conserved charges and therefore should have a stable ground state. In these lectures we will concentrate on the simplest examples only — especially those that occur in the type I superstring theory in ten dimensions.

Non-BPS D-brane configurations can be used to give a new outlook on BPS D-branes. For example, a non-BPS system consisting of a type II D\(_p\)-brane and a coincident D\(_p\)-brane can give a novel characterization of a BPS D\((p - 2)\)-brane if the world-volume fields have a suitable vortex-like configuration. One can also use other field configurations to construct unstable D\((p - 1)\)-branes. Even though they are unstable, and carry no conserved charges, such unstable D-branes are useful for certain purposes.

The basic observation that has arisen from these studies is that a system of coincident D-branes and anti-D-branes is characterized by a pair of vector bundles (one for the D-branes and one for the anti-D-branes). However, when keeping track of conserved charges, one wants to allow for the possibility of brane-antibrane annihilation. Thus one is led to introduce equivalence classes of pairs of bundles. Such classes turn out to be elements of K-theory groups. Thus, K-theory is the appropriate mathematical framework for classifying conserved D-brane charges \([10, 11]\). Of course, one has to understand the physics to do this right. For one thing, one must figure out which K-theory group is appropriate for classifying D-brane charges for a particular superstring theory vacuum. Also, the mathematical theory may only apply when certain physical conditions are satisfied. Specifically, one must use care in analyzing D-branes of high dimension (low co-dimension), as we will explain.

We will be considering various unstable brane configurations for which annihilation is expected to occur. The signal of the instability will always be the occurrence of one or more “tachyons” in the world-volume theory appropriate to the D-brane system in question. We will always assume that the tachyon rolls to a stable minimum, as in the usual Higgs mechanism. The arguments we will give are physically motivated and plausible. But they are only qualitative, as in most cases we do not have very good mathematical control. This will be sufficient for our purposes in these lectures. It should be pointed out, however, that there are two other approaches that have been used to give more mathematical control in certain
cases. In the first approach (used extensively in Sen’s papers) the tachyon condensation mechanism is translated into a well-controlled change in the coupling of a marginal operator in a conformal field theory. Another useful technique for characterizing D-brane systems uses the so-called “boundary-state” formalism \[12, 13, 14\]. It has also been applied to the study of non-BPS D-brane systems \[15, 16\]. The idea, roughly, is that open-string partition functions can be viewed as cylindrical world sheets whose boundary is on the D-brane. Therefore the D-brane can be characterized by an appropriate coherent state in the closed-string Fock space.

The stable non-BPS D-branes that we will discuss occur in theories with 16 supercharges. In ten dimensions this is just the type I theory. (Heterotic theories do not have D-branes.) In fewer than 10 dimensions there are more possibilities, however. For example, in 8 or 9 dimensions the moduli space of quantum vacua with 16 supercharges has three disconnected components. We will explain this and show that the different components can have different types of non-BPS D-branes, even though the BPS ones are the same for all components. In discussing the type I theory compactified on a circle, it is sometimes illuminating to consider the T-dual description, called type I'. At several places in our discussion we will comment on the type I' interpretation of the results.

2 Review of Basics

In these lectures we will be concerned with D-branes, which are dynamical objects on which fundamental strings can terminate. (D stands for Dirichlet.) Such objects occur in weakly coupled type I and type II superstring theories, but not in heterotic string theories. The superstring theories have world-sheet supersymmetry for both left-movers and right-movers. Thus, in the Ramond–Neveu–Schwarz (RNS) formulation, the world-sheet spinor has both left and right-moving components, and hence the space-time spectrum has a Ramond-Ramond (RR) sector. In this section we review some basic facts about RR gauge fields and D-branes \[17, 18\].

2.1 Type II RR gauge fields

Superstrings in the RNS formalism have world-sheet fields \(X^\mu(\sigma, \tau)\) and \(\psi^\mu(\sigma, \tau)\) that are related by world-sheet supersymmetry. Here \(X^\mu\), which describes the embedding of the world-sheet in the spacetime, transforms as a space-time vector and a collection of world-sheet scalars. Its superpartner, \(\psi^{\mu}\), is also a spacetime vector but a collection of world-sheet spinors. The 2d world-sheet conformal anomaly cancels for \(D= 10\) (\(\mu = 0, 1, \ldots, 9\)). There are a variety of ways of showing this. One is to note that the anomaly contribution of \(X^\mu\) is
D and of $\psi^\mu$ is $\frac{1}{2}D$ for a total of $\frac{3}{2}D$. The diffeomorphism and supersymmetry ghosts, $b$ and $\beta$, give contributions of $-26$ and $+11$, respectively. Thus the sum vanishes for $D = 10$.

For closed strings, parameterized by a periodic coordinate $0 \leq \sigma < 2\pi$, the world-sheet equations of motion are easily solved in a flat spacetime. For example, $X^\mu$ satisfies the 2d wave equation and therefore

$$X^\mu(\sigma, \tau) = X^\mu_L(\sigma + \tau) + X^\mu_R(\sigma - \tau). \tag{1}$$

Similarly, the two components of $\psi^\mu(\sigma, \tau)$, which satisfy a 2d Dirac equation, are just $\psi^\mu_L(\sigma + \tau)$ and $\psi^\mu_R(\sigma - \tau)$. $X^\mu$ is required to be periodic in $\sigma$, and thus $X^\mu_L$ and $X^\mu_R$ each have period $2\pi$. However, $\psi^\mu_L$ and $\psi^\mu_R$ can belong to the periodic Ramond sector (R) or the antiperiodic Neveu–Schwarz sector (NS).

In the R sector $\psi^\mu_L$ (or $\psi^\mu_R$) has integer Fourier modes, including a zero mode $\psi^\mu_0$. The canonical anticommutation relations derived from the world-sheet action imply that these zero modes satisfy a Dirac algebra

$$\{\psi^\mu_0, \psi^\nu_0\} = i\gamma^{\mu\nu}. \tag{2}$$

Thus, aside from a factor of $\sqrt{2}$, $\psi^\mu_0$ can be identified with Dirac matrices $\gamma^\mu$ in ten dimensions. (Such matrices are $32 \times 32$.) Representations of this algebra give space-time spinors (fermions). Spinors in 10d can be taken to both Majorana and Weyl at the same time. This means that they are real (in a Majorana representation of the Dirac matrices) and eigenstates of $\gamma_{11} = \gamma^0 \ldots \gamma^9$. The eigenvalue ($\pm 1$) determines the chirality. Thus the minimal spinor in 10d has 16 real components. For a massless spinor on-shell particle, also satisfying a Dirac equation, only eight components would describe independent propagating modes. The fact that a minimal spinor has 16 real components explains why the number of conserved supercharges in 10d must be a multiple of 16. It is 16 for the type I theory and 32 for the type II theories. In the IIA theory the two 16-component supercharges have opposite chirality, whereas in the IIB theory they have the same chirality.

The closed-string particle spectrum has four sectors, since $\psi^\mu_L$ and $\psi^\mu_R$ can each have R or NS boundary conditions. The sectors that give spacetime bosons are NS$\otimes$NS and R$\otimes$R, while the ones that give spacetime fermions are NS$\otimes$R and R$\otimes$NS. After Gliozzi–Scherk–Olive (GSO) projection, which requires the world-sheet fermion number to be odd (for left-movers and right-movers separately), the spectrum is supersymmetric [19]. In particular, it contains zero-mass and positive-mass particles, but no tachyons.

The massless bosons in the NS$\otimes$NS sector arise from tensoring two vectors, whereas in the R$\otimes$R sector they arise from tensoring two spinors. Thus in the NS$\otimes$NS sector one finds the metric tensor $g_{\mu\nu}$, an antisymmetric tensor (two-form) $B_{\mu\nu}$, and the dilaton $\phi$. In the
R⊗R sector one obtains a bispinor $C_{ab'}$ (for the IIA case where the chiralities are opposite) or $C_{ab}$ (for the IIB case where the chiralities are the same) [20]. However, these are reducible representations, and one can write

$$C_{ab'} \sim \sum_{n \text{ odd}} \gamma_{ab'}^{\mu_1\ldots\mu_n} C_{\mu_1\ldots\mu_n} \tag{3}$$

$$C_{ab} \sim \sum_{n \text{ even}} \gamma_{ab}^{\mu_1\ldots\mu_n} C_{\mu_1\ldots\mu_n}. \tag{4}$$

Here $\gamma^\mu = \sqrt{2} \psi_0^\mu$ are 10d Dirac matrices and $\gamma^{\mu_1\ldots\mu_n}$ is an antisymmetrized product

$$\gamma^{\mu_1\ldots\mu_n} = \gamma^{[\mu_1} \gamma^{\mu_2} \ldots \gamma^{\mu_n]} \tag{5}.$$

The RR gauge fields $C_{\mu_1\ldots\mu_n}$ are totally antisymmetric in their indices, and therefore they can also be represented as differential forms

$$C^{(n)} \equiv C_{\mu_1\ldots\mu_n} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_n}. \tag{6}$$

### 2.2 BPS D-branes

The defining property of D$p$-branes is that they are $(p+1)$-dimensional dynamical hypersurfaces in 10d spacetime — with $p$ spatial dimensions and time — on which fundamental strings can end [21, 22]. As a result, their dynamics (for small string coupling constant $g$) can be studied using string perturbation theory, even though they themselves are nonperturbative excitations. It turns out that each BPS D$p$-brane is a source for an RR gauge field $C^{(p+1)}$ with one unit of the corresponding RR charge [8]. With this convention, a $\overline{Dp}$-brane has RR charge $-1$.

The stable BPS D$p$-branes in 10d Minkowski spacetime are easily deduced by noting which RR gauge fields $C^{(p+1)}$ occur in the spectrum. The results are

- Type I $p = 1, 5, 9$
- Type IIA $p = 0, 2, 4, 6, 8$
- Type IIB $p = -1, 1, 3, 5, 7, 9. \tag{7}$

Of course, for high-dimension branes ($p = 7, 8, 9$) there are restrictions on whether such D-branes can actually be present. The story for $p = 7$ is given by F theory [23], which will not be discussed in these lectures. The restrictions for $p = 8, 9$ will be discussed later. The case $p = -1$ is the D-instanton, which is point-like in the Euclideanized theory.

The RR charges appear as central charges in the supersymmetry algebra. As a result, the BPS condition ensuring that half of the supersymmetries are unbroken, relates the tension of a D$p$-brane ($T_{Dp}$) to the charge. The result turns out to be [8]

$$T_{Dp} = 2\pi \frac{m_{p+1}}{g}. \tag{8}$$
The conventions are that the universal Regge slope $\alpha'$ is the square of the fundamental string length $\ell_s$ ($\alpha' = \ell_s^2$), and the string coupling constant is determined by the dilaton $\phi$, which we are assuming is constant, by $g = e^\phi$. We also define a string mass scale

$$m_s = \frac{1}{2\pi\ell_s}.$$  

(9)

Usually, we set $\ell_s = 1$.

The world-volume theory of $Dp$-branes is most conveniently presented in the Green–Schwarz (GS) formalism. The type II $Dp$-brane theories contain superspace fields $X^\mu(\sigma)$ and $\theta^a(\sigma)$ as well as a world-volume gauge field $A_\alpha(\sigma)$. Here $\sigma$ represents the $p+1$ coordinates of the world volume, the index $a$ is a spacetime spinor label, and the index $\alpha$ is a world-volume vector label. The end of an open string attached to the D-brane gives a point-charge source for the $U(1)$ gauge field $A_\alpha$.

The effective world-volume action for a $Dp$-brane, $S_{Dp}$, can be written as a sum of two terms

$$S_{Dp} = S_{DBI} + S_{WZ},$$  

(10)

where DBI denotes Dirac-Born-Infeld and WZ denotes Wess–Zumino. Complete world-volume actions with local kappa symmetry have been presented elsewhere [24, 25, 26]. Here we will simply sketch some basic features. Dropping fermions and normalization factors,

$$S_{DBI} \sim \int d^{p+1}\sigma \sqrt{\det(g_{\alpha\beta} + F_{\alpha\beta})}$$

$$S_{WZ} \sim \int (Ce^F)_{p+1}.$$  

(11)

The notation requires some explanation. $g_{\alpha\beta}(\sigma)$ is the induced world-volume metric

$$g_{\alpha\beta}(\sigma) = g_{\mu\nu}(X)\partial_\alpha X^\mu \partial_\beta X^\nu,$$  

(12)

$F$ is the $U(1)$ field strength, and $(Ce^F)_{p+1}$ means the $(p+1)$-form part where

$$C = \sum_k C^{(k)}.$$  

(13)

The index $k$ runs over odd integers in the IIA case and even integers in the IIB case.

$N$ parallel static $Dp$-branes also give a BPS system. The RR charge and the tension are both increased by a factor of $N$, so the ratio is unchanged. It follows that there is a precise cancellation of forces so that the system is in neutral equilibrium for arbitrary separations. When the branes coincide there is an enhanced nonabelian gauge symmetry [27], which is broken when they are separated. In the case of type II $Dp$-branes the enhanced gauge symmetry is $U(N)$ for all values of $p$. The type I theory can be obtained from the IIB
theory by an orientifold projection described later. This projection also influences the gauge
symmetry on the brane system. Thus one obtains $O(N)$ for $p = 1, 9$ and $Sp(N)$ for $p = 5$.
(In writing $O(N)$ I am not being careful about distinguishing $O(N)$, $SO(N)$, Spin $(N)$,
etc. Such issues will be important later.) The WZ term $\int [C_\bullet][p+1] = \int C^{(p+1)}$ shows that the
D$p$-brane is a source for $C^{(p+1)}$ with one unit of RR charge.

Like all dynamical objects, D-branes are sources for the gravitational field giving rise to spacetime curvature. In the famous AdS/CFT duality\cite{28} this geometry plays a crucial role. There one considers a large $N$ limit holding the ‘t Hooft parameter \( \lambda = g^2_{YM} N \) fixed. The string coupling $g = g^2_{YM}$ goes to zero in this limit. The effect of $N$ D-branes on the geometry is controlled by

$$N \cdot T_{Dp} \cdot G_N \sim N \cdot \frac{1}{g} \cdot g^2 \sim gN,$$

(14)

where $G_N$ is Newton’s constant in 10d. This combination is held fixed in the AdS/CFT story. However, we will be concerned with a different limit, namely $g \to 0$ with $N$ fixed. In this limit the effect of the D-branes on the geometry can be ignored at leading order, and one can simply regard them as hypersurfaces embedded in Minkowski spacetime, which is what we will do. This reasoning is reliable for branes of sufficiently low dimension (or high co-dimension) so that gravitational fields fall with distance from the branes. Otherwise, as in the case of D8-branes in the type I’ theory, the effect on the geometry cannot be ignored. It is also worth noting that this decoupling of the geometry would not work for NS 5-branes, which have $T \sim 1/g^2$.

\subsection{2.3 T duality for D-branes}

Let us consider how T duality\cite{29} works for D-branes in the simplest setting, namely when the spacetime is $R^9 \times S^1$, with a single circular dimension $x^9 \equiv x$ of radius $R$. The corresponding string world-sheet field $X(\sigma, \tau)$ then takes the form

$$X = mR\sigma + \frac{n}{R}\tau + \text{periodic terms}.$$  

(15)

This describes a closed string with winding number $= m$ and Kaluza–Klein (KK) momentum
$= n$. Decomposing $X$ into left-moving and right-moving parts $X_L(\sigma + \tau)$ and $X_R(\sigma - \tau)$,

$$X_L = \frac{1}{2} \left( mR + \frac{n}{R} \right) (\sigma + \tau) + \ldots$$

$$X_R = \frac{1}{2} \left( mR - \frac{n}{R} \right) (\sigma - \tau) + \ldots.$$  

(16)

Under a T-duality transformation $X_R \to -X_R$, while $X_L$ is unchanged. Thus

$$X = X_L + X_R \to X_L - X_R = \frac{n}{R}\sigma + mR\tau + \ldots.$$  

(17)
Comparing with the original $X$, we see that this describes a closed string on a circle of radius $1/R$ with winding number $= n$ and KK momentum $= m$.

World-sheet supersymmetry requires that the transformation $X_R \rightarrow -X_R$ be accompanied by $\psi_R^9 \rightarrow -\psi_R^9$. This has the consequence of reversing the right-moving chirality projection, in particular $\gamma_{11} \rightarrow -\gamma_{11}$. It therefore follows that IIA $\leftrightarrow$ IIB. Thus, to recapitulate, the IIA theory compactified on a circle of radius $R$ is equivalent to the IIB theory on a circle of radius $1/R$ and the role of winding number and KK excitation number is interchanged under this map.

Now let us examine the implications for D-branes. Suppose, for example, we start with a pair of parallel $D_p$-branes with positions given by

\begin{align}
\#1 : X^i = d_1^i & \quad i = p + 1, \ldots, 9 \\
\#2 : X^i = d_2^i & \quad i = p + 1, \ldots, 9.
\end{align}

(18) \hspace{1cm} (19)

Thus the branes fill the dimensions $X^m$, $m = 0, 1, \ldots, p$ and are localized in the other $9 - p$ dimensions. An open string connecting the two D-branes satisfies Neumann boundary conditions in the first $p + 1$ dimensions

$$
\partial_\sigma X^m|_{\sigma = \pi} = 0, \quad m = 0, \ldots, p
$$

(20)

and Dirichlet boundary conditions in the remaining $9 - p$ dimensions

$$
X^i|_{\sigma = 0} = d_1^i, \quad X^i|_{\sigma = \pi} = d_2^i, \quad i = p + 1, \ldots, 9.
$$

(21)

Solving the 2d wave equation with these boundary conditions gives

$$
X^m = x^m + p^m \tau + i \sum_{n \neq 0} \frac{\alpha^m_n}{n} \cos n\sigma e^{-in\tau}
$$

(22)

$$
X^i = d_1^i + (d_2^i - d_1^i) \frac{\sigma}{\pi} + \sum_{n \neq 0} \frac{\alpha^i_n}{n} \sin n\sigma e^{-in\tau}.
$$

(23)

Suppose now that $X^9$ is a circle as before. What happens to the $D_p$-branes under the T-duality transformation? It is easy to see that $X_R \rightarrow -X_R$ simply interchanges the Dirichlet and Neumann boundary conditions. Therefore the D-branes, which we assumed were localized on the original circle of radius $R$, are wrapped on the T-dual circle of radius $1/R$. Thus in the T-dual theory there are $D(p+1)$-branes, wrapped on the circular dimension. Thus the general rule is that under T duality: unwrapped $D_p$ $\leftrightarrow$ wrapped $D(p + 1)$. This meshes nicely with the fact that BPS $D_p$-branes exist in the IIA theory for even values of $p$ and in the IIB theory for odd values of $p$. 

9
Now a question arises. In the T-dual description, where the D-branes are wrapped on the
dual circle, how are the positions on the original circle encoded? Clearly, this information
must not be lost, since the transformation should be invertible. The key to answering this
question is to recall that T duality interchanges winding number and KK excitation number.
However, the original configuration has charged states with fractional winding associated to
open strings that connect the two D-branes. So in the dual description these charge states
should have fractional KK number. This is achieved by Wilson lines $\exp[i \oint A_9]$ in the world-
volume gauge theories of the wrapped D-branes. In other words, the component of the gauge
fields along the circle encodes the position of the original D$p$-branes on the circle.

### 2.4 Type I superstrings

The modern construction of type I superstring theory (not the way it was originally devel-
oned) is as an “orientifold projection” of the type IIB theory [30, 31, 32]. Recall that in the
type IIB theory, the fermions associated with left-movers and right-movers have the same
chirality. As a result the world-sheet theory has a $\mathbb{Z}_2$ symmetry corresponding to interchange
of all left-movers and right-movers

\[ X_\mu^L \leftrightarrow X_\mu^R, \quad \psi_\mu^L \leftrightarrow \psi_\mu^R. \tag{24} \]

Equivalently, one can speak of world-sheet parity $\Omega$, which reflects the spatial world-sheet
coordinate ($\sigma \to -\sigma$ or $2\pi - \sigma$). One can, therefore, project the spectrum of the theory onto
a left-right symmetric subspace by using the projection operator $\frac{1}{2}(1 + \Omega)$. The result of this
is to give a closed string with a restricted set of excitations. This is the type I closed-string,
which is unoriented.

The interacting theory of type I closed strings is not consistent by itself. This fact can be
understood in a variety of ways. For one thing it is a chiral theory with $N = 1$ supersymmetry
(16 supercharges), and it has gravitational anomalies. To achieve consistency one must add a
“twisted sector,” which consists of open strings. These are strings whose ends are associated
to the fixed points of the orientifold projection ($\sigma = 0$ and $\sigma = \pi$). They are also unoriented.
It is known from the anomaly cancellation requirement that the open strings should carry
$SO(32)$ Chan–Paton charges at their ends, since this is the only Chan–Paton gauge group
for which the anomalies can be cancelled [33]. ($E_8 \times E_8$ cannot be realized in this way.)

The orientifold projection construction can be interpreted as giving rise to a spacetime
filling orientifold plane — an O9-plane. The reason it fills the entire space-time is that every
$x^\mu$ is a fixed point of $\Omega$, which does not act on $x^\mu$. This O9-plane carries $-32$ units of RR
charge. This charge needs to be cancelled. This is achieved by adding 32 D9-branes, which
also fill the entire spacetime. The $SO(32)$ gauge fields arise as zero modes of open strings.
that connect any pair of D-branes. (These are the same open strings that were introduced earlier.) In type II theories coincident D-branes give \(U(N)\) gauge groups, because the open strings are oriented. In the type I theory they are unoriented, and therefore one obtains orthogonal or symplectic gauge groups.

### 2.5 T-dual description of the type I theory

In Section 2.3 we saw that T duality on a circle \(S^1\) corresponds to \(X_R \rightarrow -X_R\) (and \(\psi_R \rightarrow -\psi_R\)), and this implies that

\[
X = X_L + X_R \rightarrow X' = X_L - X_R.
\]

In the case of type II theories, \(X'\) describes a dual circle \(\tilde{S}^1\) of radius \(R' = 1/R\). The type I theory is constructed by gauging the world-sheet parity symmetry \(\Omega\) of the type IIB theory. This corresponds to \(X_L \leftrightarrow X_R\) (and \(\psi_L \leftrightarrow \psi_R\)). Thus, we see that in the T-dual description the action of \(\Omega\) gives \(X' \rightarrow -X'\). This gauging therefore gives an orbifold projection of the dual circle: \(\tilde{S}^1/Z_2\) \[34\]. This orbifold describes an interval

\[
0 \leq X' \leq \pi R'.
\]

An alternative viewpoint is that the entire circle \(\tilde{S}^1\) is present, but that what happens on the semicircle \(\pi R' \leq X' \leq 2\pi R'\) is determined by the other semicircle. More generally, T duality along an \(n\)-torus \(T^n\) results in an orbifold \(\tilde{T}^n/I_n\). Here \(I_n\) denotes the simultaneous inversion of all \(n\) coordinates of the torus \(\tilde{T}^n\). More precisely, this inversion is carried out simultaneously with the orientation reversal \(\Omega\) of the non-compact coordinates \((X^\mu_L \leftrightarrow X^\mu_R)\). The fermions are transformed at the same time.

In the case of a circle \((n = 1)\), the T-dual theory can be regarded as type IIA orientifold of the form

\[
(R^9 \times \tilde{S}^1)/\Omega \cdot I_1.
\]

The combined operation \(\Omega \cdot I_1\) is the relevant \(Z_2\) projection in this case. The resulting theory — the T-dual description of type I theory compactified on a circle — is called the type I\(^\prime\) theory. (Sometimes the alternative name type IA is used.) T duality in this setting is the equivalence of the type I\(^\prime\) theory with the type IIB orientifold

\[
(R^9 \times S^1)/\Omega,
\]

which is the compactified type I theory.

The ends of the interval \(X' = 0, \pi R'\) are the fixed-point set of the IIA orientifold projection, and, therefore, they are the locations of orientifold planes — O8-planes. Each of
these orientifold planes can be regarded as carrying $-8$ units of RR charge. Consistency of the type I' theory then requires adding 16 D8-branes to cancel the RR charges. They are parallel to the O8-planes, which means that they are localized at points in the interval $0 \leq X' \leq \pi R'$, and fill the other nine dimensions. In addition there are 16 “mirror image” D8-branes on the other half of the circle ($\pi R' \leq X' \leq 2\pi R'$). Altogether, these 32 D8-branes are the T-duals of the 32 D9-branes in the type I description.

The position of the D8-branes along the interval in the type I' description are determined in the type I description by Wilson lines. The Wilson line is an element of $SO(32)$ and by an equivalence transformation can be brought to a canonical form in which it is written as 16 blocks of $SO(2)$ matrices, which are characterized by angles

$$\theta_i = X'_i/R', \quad i = 1, 2, \ldots, 16.$$  \hspace{1cm} (29)

The $X'_i$ are then the positions of the 16 dual D8-branes, and their mirror images are located at $2\pi R' - X'_i$.

By introducing a Wilson line into the compactified type I theory, one breaks the $SO(32)$ gauge group to the subgroup that commutes with the Wilson line matrix. Expressed in terms of the type I' description, this gives the following rules:

- When $n$ D8-branes coincide in the interior of the interval, they give an unbroken $U(n)$ gauge group.
- When $n$ D8-branes coincide with one of the O8-planes, they give an unbroken $SO(2n)$ gauge group.

In both cases the gauge bosons arise as zero modes of D8 – D8 open strings. In the second case, the additional symmetry enhancement can be traced to open strings connecting D8-branes to mirror-image D8-branes.

The case of a trivial Wilson line ($W = \pm 1$) corresponds to having all 16 D8-branes coincide with one of the O8-planes, which gives the expected unbroken $SO(32)$ gauge symmetry. It should be noted that there are two additional $U(1)$'s that cannot be pictured in terms of D-branes. Rather they are associated to gauge fields $g_{\mu 9}$ and $B_{\mu 9}$ that are defined in the bulk. One linear combination of these gauge fields belongs to the 9d supergravity multiplet. Another linear combination belongs to a 9d vector supermultiplet. The latter can participate in further symmetry enhancement in special cases \cite{35, 36}. For example, the Wilson line

$$W = \left( \begin{array}{cc} I_{16+2N} & 0 \\ 0 & -I_{16-2N} \end{array} \right)$$  \hspace{1cm} (30)

generically gives the gauge symmetry

$$SO(16 + 2N) \times SO(16 - 2N) \times U(1)^2.$$  \hspace{1cm} (31)
However, from the S-dual perturbative heterotic theory one knows that there is additional
gauge symmetry enhancement.

$$SO(16 - 2N) \times U(1) \rightarrow E_{9-N},$$  

(32)

for a specific value of the compactification radius. In type I units, this radius is given by
$$R^2 = gN/8.$$ I will not explore this issue further in these lectures.

## 3 D-Brane Anti-D-Brane Systems

When one superposes a BPS $D_p$-brane and a BPS $\overline{D_p}$-brane the system is no longer BPS, and
it is unstable. The basic idea is that the open string connecting them has a spectrum based
on a GSO projection that is the reverse of the usual one, and as a result its ground state
is a tachyon, which signals an instability. One method of deriving this result is to consider
starting with a pair of coincident $D_p$-branes and then imagine rotating one of them by an
angle $\pi$ thereby turning it into an $\overline{D_p}$-brane \[37\]. At intermediate angles $\theta$ the intersection
takes place on $p-1$ spatial dimensions. By working through the implications of Dirichlet and
Neumann boundary conditions for arbitrary angles $\theta$ and continuing from $\theta = 0$ to $\theta = \pi$,
one can deduce the reversal of the GSO projection. The occurrence of a tachyon at $\theta = \pi$ is
physically sensible, of course, as it signals the possibility of annihilation, which is no longer
prevented by charge conservation.

We will begin by discussing such systems for type II theories and then move on to the
type I theory. In each case, we will be interested in exploring the possibility of setting up
field configurations on the $D_p + \overline{D_p}$ system such that something stable survives after the
annihilation takes place.

### 3.1 Type II $D_p + \overline{D_p}$ systems

In the case of type II theories the open strings connecting D-branes are oriented, and as a
result the tachyonic mode of the $D_p - \overline{D_p}$ open string is complex. Denoting the tachyon field
in the $(p+1)$-dimensional world volume by $T$, one can combine it with the gauge fields in a
"superconnection" of the form

$$A = \begin{pmatrix} A & T \\ \overline{T} & A' \end{pmatrix}.$$  

(33)

Here $A$ is the $U(1)$ gauge field associated to the $D_p$-brane and $A'$ is the $U(1)$ gauge field
associated to the $\overline{D_p}$-brane. The tachyon is charged with respect to both of them. More
generally, one could consider $N D_p$-branes and $N' \overline{D_p}$-branes. Then $A$ would be $U(N)$ gauge
fields, \( A' \) would be \( U(N') \) gauge fields, and \( T \) would be an \( N \times N' \) matrix transforming as \( (N, \bar{N}') \). To start with, we will restrict consideration to the case \( N = N' = 1 \).

As an explicit example of what can happen, let us (following Sen [3]) consider a \( D2 + \overline{D2} \) system wrapped on a rectangular torus with radii \( R_1, R_2 \). As we discussed earlier, the WZ term of the D2-brane is

\[
\int (C e^F) = \int_{T^2 \times R} (C_3 + C_1 \wedge F).
\]

This formula shows that magnetic flux through the torus — \( \int_{T^2} F \) — is a source of \( C_1 \). In other words, a D2-brane with magnetic flux carries D0-brane charge.

Suppose that the D2 and the \( \overline{D2} \) each have one unit of D0-brane charge. Assuming constant fields, this means that

\[
F_{12} = F'_{12} = \frac{2\pi}{V},
\]

where \( V = (2\pi R_1)(2\pi R_2) \) is the volume of the torus. Let us now examine the system described above from a T-dual viewpoint. In the T-dual description (which is also type IIA, since the torus has even dimension) we get a dual torus \( \tilde{T}^2 \) with radii \( \tilde{R}_i = 1/R_i \). By matching coupling constants in eight dimensions one learns that

\[
\frac{V}{g^2} = \frac{\tilde{V}}{\tilde{g}^2},
\]

and hence that

\[
\tilde{g} = g/(R_1 R_2).
\]

Since the T-duality transformation interchanges D0 ↔ D2, the fluxes that gave D0-brane charges imply that the dual system has two D2-branes wrapping \( \tilde{T}_2 \). Being two coincident type II D-branes, the system has a \( U(2) \) gauge symmetry. The original D2 and \( \overline{D2} \) give rise to a D0 on one D2 and a \( \overline{D0} \) on the other. Thus the dual magnetic flux is

\[
\tilde{F}_{12} = \frac{2\pi}{\tilde{V}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

The original \( D2 + \overline{D2} \) system had a complex tachyon, signaling instability, and the T-duality transformation certainly doesn’t change that fact. However, now the instability is attributable to structure of the flux matrix. The minimum energy configuration corresponds to \( \tilde{F}_{12} = 0 \), which corresponds to annihilation of the D0 + \( \overline{D0} \). The nontrivial fact, which Sen has demonstrated, is that the flux in eq. (38) is continuously connected to \( \tilde{F}_{12} = 0 \). So this deformation corresponds to the tachyon rolling to a minimum in the dual picture. We are thus left with a pair of wrapped D2-branes and no flux.

The total mass at the minimum (ignoring the contribution of radiation emitted in the annihilation process) is

\[
M = 2\tilde{V} \tilde{T}_{D2} = 2 \cdot 4\pi^2 R_1 R_2 \cdot \frac{1}{4\pi^2 \tilde{g}} = \frac{2}{\tilde{g}} = 2T_{D0}.
\]

(39)
This calculation shows that the mass is just that of two D0-branes in the original picture. This means that on the original torus the mass density $M/V$ vanishes in the limit of large $V$. This implies that the tachyon potential at its minimum precisely cancels the D-brane tension. Specifically

$$2T_{D2} + V(T_0) = 0,$$

where $|T| = T_0$ is the (assumed) position of the minimum. This reasoning generalizes to arbitrary dimensions of the branes. So the general conclusion is that a coincident D$p$-brane $+\overline{D}p$-brane system with the tachyon field at a minimum is equivalent to pure vacuum. This result makes good sense. It is also a very insightful viewpoint.

### 3.2 Vortex solutions

Let us now consider the D2 + $\overline{D}2$ system in $\mathbb{R}^{1,9}$. As we already noted, the tachyon $T$ has charge $(1, -1)$ with respect to the $U(1) \times U(1)$ gauge symmetry. The tachyon potential $V(T)$ is gauge invariant. Therefore, it can only depend on the magnitude $|T|$, and its minimum is assumed to occur for $|T| = T_0$. Its kinetic term is $|D_\mu T|^2$, where

$$D_\mu T = (\partial_\mu - iA_\mu + iA'_\mu)T. \quad (41)$$

A single D0-brane can be described as a vortex solution on the infinite D2 + $\overline{D}2$ system $[4]$. Working in polar coordinates $(r, \theta)$ we want to describe a vortex localized in the vicinity of $r = 0$. We want the field configuration to describe pure vacuum for large $r$, but in a topologically nontrivial way. This is achieved by requiring that as $r \to \infty$

$$T \sim T_0 e^{i\theta}$$

$$A_\theta - A'_\theta \sim 1. \quad (42)$$

These ensure that $D_\mu T \to 0$ and $V(T) \to V(T_0)$, so that the energy density cancels for large $r$ leaving a soliton in the small $r$ region. We can see that this configuration has the quantum numbers of a D0-brane, since

$$\oint_{\text{large circle}} (A_\theta - A'_\theta) d\theta = 2\pi$$

implies that

$$\int (F_{12} - F'_{12}) dx^1 dx^2 = 2\pi. \quad (43)$$

As we see it is only the relative $U(1)$ group that matters. We could, for example, set $A' = 0$. In any case, we conclude that there is one unit of magnetic flux, as required for a D0-brane.

This result extends trivially to larger values of $p$ by simply adding additional inert directions. The conclusion is that a D($p - 2$)-brane can be described as a vortex solution in...
a $Dp + \overline{Dp}$ system. The fact that it carries the correct RR charge can be understood as a consequence of the coupling
\[ \int C_{p-1} \wedge dT \wedge d\overline{T}, \] (45)
which is required for consistency of the picture. This term can be elegantly described using the superconnection $A$, following Kennedy and Wilkins [38]. First one introduces the supercurvature
\[ F = dA + A \wedge A = \left( \begin{array}{cc} F - TT & DT \\ DT & F' - T\overline{T} \end{array} \right), \] (46)
which has even-dimensional forms on the diagonal, and odd-dimensional ones on the off-diagonal. Using this one can construct a suggestive generalization of the WZ term to brane-antibrane systems, namely
\[ S_{WZ} = \int C_{\wedge} STr(e^{F}). \] (47)
The symbol $STr$ is the usual super-trace for graded algebras. This contains the usual terms
\[ \int C_{\wedge}(e^{F} - e^{F'}), \] (48)
where the minus sign reflects the opposite RR charges. It also contains the term in eq. (45), as well as some others. It would be interesting to see whether this nice formula reflects some new symmetry.

3.3 Unstable type II D-branes

We have seen that a suitably chosen vortex in a coincident $Dp + \overline{Dp}$ system describes a stable $D(p - 2)$-brane. Now (following Sen [4]) we would like to consider a different field configuration that describes an unstable soliton. Recall that at its minimum ($|T| = T_0$) the energy density associated with the tachyon potential exactly cancels the tension of the branes, $V(T_0) + 2T_{Dp} = 0$, leaving a configuration equivalent to pure vacuum. The proposal is to construct a $D(p - 1)$-brane as a domain-wall on the $Dp + \overline{Dp}$ system. This means that if $x$ denotes a Cartesian coordinate, we want to construct a soliton localized in the vicinity of $x = 0$.

The way to construct a localized domain wall is by choosing a kink configuration for the tachyon field. Specifically consider choosing $\text{Im} T = 0$ and $\text{Re} T = T_0 \tanh(x/a)$. This has the property that $T \rightarrow T_0$ for $x \rightarrow +\infty$ and $T \rightarrow -T_0$ for $x \rightarrow -\infty$. Also, $|T|$ differs from $T_0$ appreciably only in a region of thickness $a$ in the vicinity of $x = 0$. The precise functional form of $T(x)$ is not important. This configuration gives pure vacuum except in the vicinity of $x = 0$, so it can be interpreted as a $D(p - 1)$-brane. This domain wall soliton is unstable because the field configuration we have chosen is topologically trivial. To understand this,
we note that the vacuum manifold given by $|T| = T_0$ is a circle ($S^1$). However, a circle is connected ($\pi^0(S^1)$ is trivial). Therefore, by turning on $\text{Im } T$ and increasing the thickness $a$ this configuration can be continuously deformed to pure vacuum. Thus we conclude that this D-brane is unstable. Since it carries no conserved charge it can decay into neutral radiation (such as gravitons). Despite their instability, Sen has demonstrated that it is useful to consider these objects. They can be defined in type II theories for all “wrong” values of $p$ (odd for IIA and even for IIB).

The instability of the $D(p - 1)$-brane is reflected in the fact that its world-volume theory contains a real tachyon. The reason there is one real tachyon is that there is one direction of instability for unwinding the kink in the vacuum moduli space. It can also be understood as the ground state of the open string connecting the D-brane to itself. In any case, we can use this tachyon to repeat the kink construction. This gives us a domain wall on the unstable $D(p - 1)$-brane, which is interpreted as a $D(p - 2)$-brane. This kink is topologically stable, however. The tachyon on the $D(p - 1)$-brane is real and the vacuum moduli space in this case is two disconnected points $T = \pm T_0$. So this construction gives a stable $D(p - 2)$-brane, which carries a conserved charge.

We already have a candidate for what this $D(p - 2)$-brane is. It is exactly the same stable BPS D-brane that we constructed as a vortex solution in Section 3.2. Sen has shown that this two-step kink construction is equivalent to the vortex solution.

### 3.4 Type I $D_p + \overline{D_p}$ systems

The type I theory has three types of BPS $D_p$-branes, namely $p = 1, 5, 9$. Just as we constructed the type II $D(p - 2)$-brane as a soliton in a $D_p + \overline{D_p}$ system, so we can construct type I $D$-branes as solitons in higher-dimensional brane-antibrane systems. In this section we will sketch the construction of the $D1$-brane as an instanton-like configuration in a $D5 + \overline{D5}$ system \[^{[4]}\] and the construction of the $D5$-brane as an instanton-like configuration in a $D9 + \overline{D9}$ system \[^{[11]}\]. The two constructions are not completely identical (unlike for the type II theories), and therefore need to be considered separately.

Before proceeding, let us recall a few basic facts about the type I D-branes. First of all, the $D1$-brane or D-string is nothing but the $SO(32)$ heterotic string in disguise. Recall that the $SO(32)$ heterotic theory is S-dual to the type I theory \[^{[34]}\], which means that the coupling constants are related by $g_h = 1/g$. In talking about the type I theory it is implicit that $g$ is small, so that we can use perturbative analysis. In type I string units, the type I fundamental string has tension $1/2\pi$, whereas the D-string has tension $1/(2\pi g)$ and thus appears as a nonperturbative object. Even so, the D-string can be identified as the heterotic string continued to strong coupling. This string is BPS, carrying a conserved
charge. This charge is an RR charge from the type I viewpoint, though it is NS from the heterotic viewpoint. The type I D5-brane is the electromagnetic dual of the D-string and is also BPS. We also need to know the gauge groups that are associated to the D-brane world volumes. It turns out that $N$ coincident D-strings give $O(N)$ gauge symmetry, whereas $N$ coincident D5-branes gives $Sp(N) \equiv USp(2N)$ gauge symmetry \[40\]. These facts can be derived by considering coincident type IIB D-branes and applying the appropriate orientifold projection.

Let us now consider a coincident type I D5 + $\overline{D5}$ system. The world-volume theory has $SU(2) \times SU(2)$ gauge symmetry since each brane has an $Sp(1) = SU(2)$ gauge field on its world volume. The ground state of the open string connecting the D5 to the $\overline{D5}$ gives a tachyon $T$ with $(2, 2)$ quantum numbers. It can be parameterized in the form

$$T = \lambda W,$$

where $\lambda$ is a positive real number and $W$ is an $SU(2)$ matrix. The two $SU(2)$'s act on $T$ by left and right multiplication. The tachyon potential $V(T)$ is required to be gauge invariant, which implies that it is a function of $\lambda$ only. As in the corresponding type II constructions, we assume that it has an isolated minimum at some value $\lambda = \lambda_0$ and that if $T = \lambda_0 W$, with $W$ fixed, and all gauge fields vanishing, then the D5 + $\overline{D5}$ system is equivalent to pure vacuum. As before, this requires that

$$V(\lambda_0) + 2T_{D5} = 0,$$

so that the total energy density cancels at the minimum.

We now wish to construct a field configuration on the D5 + $\overline{D5}$ system that describes a string-like soliton, which can be identified as the D-string. Thus we will assume symmetry in one Cartesian direction. The four orthogonal spatial directions can be described by a distance $r$ from the core of the string and a sphere $S^3$. So we want the soliton's energy to be concentrated in the vicinity of $r = 0$ and to approach zero as $r \to \infty$. At infinity, we certainly need that $\lambda \to \lambda_0$, but we also need that the tachyon kinetic energy

$$Tr[(D_\mu T)^\dagger D^\nu T],$$

should vanish as well. Here

$$D_\mu T = \partial_\mu T + i(A_\mu T - TA'_\mu),$$

and $A_\mu$ is the $SU(2)$ gauge field of the D5-brane and $A'_\mu$ is the $SU(2)$ gauge field of the $\overline{D5}$-brane. The story is very similar to the type II vortex solution, except that it is now based on an instanton-like construction.
We require that for large $r$

$$\text{ } T \sim \lambda_0 U, \quad (53)$$

where the $SU(2)$ element $U$ is identified with the spatial $S^3$. Then the tachyon kinetic energy can be made to vanish at the same time by requiring that

$$A_{\mu} \sim \partial_{\mu} U \cdot U^{-1}$$
$$A_{\mu}' \sim 0. \quad (54)$$

The first formula is the standard instanton configuration and gives one unit of instanton number

$$\frac{1}{8\pi} \int_{\mathbb{R}^4} tr(F \wedge F) = 1. \quad (55)$$

Then the structure of the WZ term implies that this configuration carries one unit of D1-brane RR charge and can be identified as the D-string. The various properties of the D-string, such as its various zero modes, can also be derived from this construction, but we will not pursue that here.

The construction described above is based on the work of Sen. However, it should be pointed out that something closely related was done earlier by Douglas [41]. Specifically, Douglas considered an instanton configuration on a single D5-brane and argued that this describes a D-string bound to the D5-brane.

Let us now turn to the construction of the D5-brane as a soliton. For this purpose we need to consider four D9-branes and four $\overline{D9}$-branes in addition to the usual 32 D9-branes that are present in the type I vacuum configuration. The Chan-Paton group of the $D9$'s, say, is

$$SO(4) = SU(2) \times SU(2). \quad (56)$$

The idea is to introduce an instanton configuration in one of these $SU(2)$’s and accompany it by an appropriate tachyon configuration rather like the one in the previous construction. In this way one makes the D5-brane. Moreover, the $SU(2)$, which is not used in the instanton construction, survives as the gauge symmetry of the D5-brane. The $SO(32)$ gauge symmetry carried by the D9-branes that are inert in this construction survives as a global symmetry of the D5-brane world-volume theory. (This is also true for the D-string in the previous construction.)

### 3.5 Non-BPS D0-brane in type I

The perturbative $SO(32)$ heterotic string spectrum has a spin(32)/$\mathbb{Z}_2$ spinor representation at the first excited level. Even though it is not BPS, and belongs to a long representation
of the supersymmetry algebra, it is nonetheless absolutely stable as a consequence of charge conservation. In heterotic string units, its mass is

\[ M_{\text{spinor}} = \sqrt{2} f(g_h), \]  

(57)

where \( f(g_h) = 1 + O(g_h^2) \). Sen has posed the question “what happens to this state as \( g_h \to \infty \)?” For a BPS state, the answer would be trivial, as the charge would control the mass for all values of the coupling. However, this state is not BPS, so the answer is not at all obvious. However, one can hope to answer the question because large \( g_h \) corresponds to small \( g = 1/g_h \) type I coupling. Thus what one needs to do is to identify a plausible candidate for the lightest gauge group spinor in the type I setting and determine its mass to leading order in \( g \).

Ordinarily, the type IIB theory has no stable D0-branes, though in Section 3.3 we discussed how to construct an unstable D0-brane. However, in the presence of an orientifold (or orbifold) plane they can exist as stable particles embedded in the orientifold (or orbifold) plane. One viewpoint is that the orientifold projection eliminates the tachyonic modes from the world-volume of the unstable type II D-brane. Sen has studied these possibilities for both O5-planes and O9-planes. Here we will only consider the latter case, which is realized by the uncompactified type I theory.

We will construct a type I candidate for the desired gauge group spinor as a D0-brane. The procedure will be based on a setting like that of the preceding sections. Namely, following Sen, we will construct a \( D1 + \overline{D1} \) system with a field configuration that describes a stable soliton.

As we have already remarked, the gauge group on the world volume of \( N \) coincident type I D-strings is \( O(N) \). In particular, for the case of a single D-string, it is \( O(1) = \mathbb{Z}_2 \). This \( \mathbb{Z}_2 \) is the subgroup of the \( U(1) \) of a type IIB D-string that survives the orientifold projection. Being discrete, this gauge group has no associated field, but it does have physical consequences. The point is that the gauge group determines the possible Wilson lines when the string is wrapped on a circular compact dimension. Thus the possible Wilson lines for a wrapped D-string are \( W = \pm 1 \). The D-string has 32 left-moving fermion fields \( \lambda^A \) on its world sheet, which arise as the zero modes of \( D1 - D9 \) open strings. (This is also known from the identification of the D-string with the heterotic string.) The possible Wilson lines (or holonomies) \( W = \pm 1 \) correspond to whether the \( \lambda^A \) fields of the wrapped D-string are periodic or antiperiodic

\[ \lambda^A(x + 2\pi R) = W\lambda^A(x). \]  

(58)

When \( W = 1 \), the \( \lambda^A \)'s have zero modes in their Fourier expansions, which obey a Clifford algebra. Therefore, the quantum states of a wrapped D-string with \( W = 1 \) belong to the
spinor conjugacy class of $\text{Spin}(32)/\mathbb{Z}_2$. Similarly, the states of a wrapped D-string with $W = -1$ belong to the adjoint (or singlet) conjugacy class.

Now consider a $D1 + \overline{D1}$ pair wrapped on the circle and coincident in the other dimensions. To get an overall gauge group spinor one of the strings should have $W = 1$ and the other should have $W = -1$. In this case it is clear (by charge conservation) that complete annihilation of the strings is not possible. The wrapped $D1 + \overline{D1}$ system contains a real tachyon field $T$ with a potential $V(T)$. $T$ is odd under the action of either of the $\mathbb{Z}_2$ gauge groups, and therefore gauge invariance requires that $V(T) = V(-T)$. By the same reasoning as before, $V$ should have an isolated minimum at $|T| = T_0$ with $V(T_0) + 2T_{D1} = 0$.

When both Wilson lines are the same, so that their product is $+1$ and the system can be a gauge group singlet, $T(x)$ is periodic and it is possible to choose $T(x) = T_0$. This would then be equivalent to pure vacuum. However, when the product of the Wilson lines is $-1$, $T(x)$ is forced to be antiperiodic

$$T(x + 2\pi R) = -T(x),$$

so that $T(x) = T_0$ is no longer a possibility. Indeed, in this case the tachyon field has a Fourier series expansion of the form

$$T(x, t) = \sum_n T_{n+1/2}(t)e^{i(n+1/2)x/R}.$$  \tag{60}

This tells us that the KK mass of the $n$th mode is

$$M_n^2 = \frac{(n + 1/2)^2}{R^2} - \frac{1}{2}. \tag{61}$$

This shows that there is actually no instability for $R < R_c$, where

$$R_c = 1/\sqrt{2}. \tag{62}$$

In this case the two-particle system is the ground state. There is no lighter bound state with the same quantum numbers. On the other hand, when $R > R_c$, $T_{\pm 1/2}$ are tachyonic signaling an instability. In this case there is a bound state — the D0-brane — that has the same quantum numbers and lower mass.

The mass of the D0-brane can be estimated for small $g$ by considering the transition point $R = R_c$. At this point the mass of two wrapped strings and the mass of the D0-brane should be degenerate. The two wrapped strings have a mass that is approximately given by

$$M = 2 \cdot 2\pi R_c \cdot T_{D1} = \sqrt{2}/g. \tag{63}$$

Sen argues that this result is exact to leading order in small $g$, and that any (possibly $R$-dependent) corrections are of $O(1)$. As an example of such a correction consider a localized
D0-brane on the circle. It experiences gravitational interactions with its images, which are of order \( G_N M^2 \). However, \( M \sim 1/g \) and \( G_N \sim g^2 \), so this is \( O(1) \). Other effects, due to Wilson lines or KK momenta, are also \( O(1) \).

We have now determined the answer to Sen’s question. The mass of the lightest gauge group spinor is a non-BPS D0-brane of the type I theory whose mass (in type I units) is \( \sqrt{2}/g + O(1) \). To compare to the perturbative heterotic result, we must still convert to heterotic string units. Then we conclude that

\[
f(g_h) \sim \sqrt{g_h} \quad \text{for large } g_h. \tag{64}
\]

To complete the story let us describe the D0-brane soliton of the D1 + D1 system in the decompactification limit. In this case we have a localized kink, like the ones we described in Section 3.3. In the present case \( T(x) \) is real and the vacuum manifold \( |T| = T_0 \) consists of two distinct points. Thus the kink configuration is topologically nontrivial. It follows that the resulting D0-brane carries a conserved \( \mathbb{Z}_2 \) charge. This means that whereas they are individually stable, they are TCP self-conjugate and can annihilate in pairs. This meshes nicely with the fact that they are gauge group spinors.

4 K-Theory Classification of D-Branes

4.1 Motivation and background

Since D-branes carry conserved charges that are sources for RR gauge fields, which are differential forms, one might suppose that the charges could be identified with cohomology classes. This is roughly, but not precisely, correct. Moore and Minasian noted that the more precise mathematical identification of D-brane charges is with elements of K-theory groups \([10]\). This was explained in detail in a subsequent paper by Witten \([11]\). Here, we will summarize some of the basic ideas, though we will not go deeply into the mathematics.

Witten’s explanation begins with the observation that the pattern of BPS D-branes is reminiscent of the Bott periodicity rules for homotopy groups. For example,

\[
\pi_i(U(N)) = \pi_i(U(N + 2)), \tag{65}
\]

when \( N \) is sufficiently large (the “stable regime”). This pattern is reminiscent of the fact that BPS D-branes occur for even values of \( p \) in the IIA theory and odd values of \( p \) in the IIB theory, and that the gauge group of \( N \) coincident type II D-branes is \( U(N) \).

The Bott periodicity rule

\[
\pi_i(O(N)) = \pi_{i+4}(Sp(N)), \tag{66}
\]
for sufficiently large \( N \), is reminiscent of the pattern of BPS D-branes in the type I theory. In type I, \( N \) coincident D1-branes or D9-branes have an \( O(N) \) gauge group (roughly), whereas \( N \) coincident D5-branes have an \( Sp(N) \) gauge group.

The challenge is to find the right mathematical framework to explain these “coincidences” and to account for the non-BPS D-branes, as well. K-theory turns out to be the key. It provides a classification of pairs of vector bundles with equivalence relations that mesh nicely with what we have learned from Sen’s studies.

Let us first discuss the relevance of homotopy groups. The world-volume theories are gauge theories, which (as we have seen) can support solitons. These solitons tend to be stable when there is a topologically nontrivial lump in codimension \( n \). The field configuration represents a nontrivial element of \( \pi_{n-1}(G) \) that is identified as the conserved RR charge. Thus, for example, in the case of the non-BPS D0-brane we used the fact that \( \pi_0(\mathbb{Z}_2) = \mathbb{Z}_2 \) to describe a codimension one lump on a D1 + DT system. Similarly, we used \( \pi_3(SU(2)) = \mathbb{Z} \) to construct the D-string in a D5 + D\( \overline{5} \) system. In the type II theories, the vortex solutions utilized the fact that \( \pi_1(U(1)) = \mathbb{Z} \).

Instead of the constructions listed above, we could have constructed all of these objects as suitable bundles on space-time filling nine-branes. From that point of view, we would understand the existence of the type I D-string as a consequence of

\[
\pi_7(O(32)) = \mathbb{Z},
\]

and the non-BPS D0-brane as a consequence of

\[
\pi_8(O(32)) = \mathbb{Z}_2.
\]

Not everything is group theory and topology, however. Dynamics also matters. When the codimension \( n \) is greater than four, as in these two cases, the lump cannot be described as an extremum of the low-energy effective action

\[
S_{\text{eff}} = \int d^n x \, \text{tr}(F_{ij} F^{ij}).
\]

To see this, suppose \( A_i(x) \) is a field configuration of the desired topology. (More precisely one defines a bundle by a collection of \( A_i \)'s on open sets together with suitable transition functions.) Now let’s rescale the gauge field as follows

\[
A_i(x) \rightarrow \lambda A_i(\lambda x),
\]

which sends \( F_{ij}(x) \rightarrow \lambda^2 F_{ij}(\lambda x) \). Thus, one finds that \( S_{\text{eff}} \rightarrow \lambda^{4-n} S_{\text{eff}} \). Therefore, for \( n > 4 \) the action (or energy) is reduced by shrinking the soliton. Presumably it shrinks to string...
Thus, the string scale should characterize the thickness of the D-string or the D0-brane.

Suppose, on the other hand that the soliton has co-dimension \( n < 4 \). In this case the same scaling argument would suggest that the soliton wants to spread out and become more diffuse. This doesn’t always happen, because there can be other effects that stabilize the soliton, but it is an issue to be considered.

### 4.2 Type II D-branes

Consider now a collection of coincident type II D-branes — \( N \) \( Dp \)-branes and \( N' \) \( \overline{Dp} \)-branes. As we discussed in Section 3.1, the important world-volume fields can be combined in a superconnection

\[
\mathcal{A} = \begin{pmatrix} A & T \\ \bar{T} & A' \end{pmatrix},
\]

where \( A \) is a connection on a \( U(N) \) vector bundle \( E \), \( A' \) is a connection on a \( U(N') \) vector bundle \( E' \), and \( T \) is a section of \( E^* \otimes E' \). The \( (p + 1) \)-dimensional world-volume of the branes, \( X \), is the base of \( E \) and \( E' \).

As we have discussed at some length in Section 3, if \( E \) and \( E' \) are topologically equivalent \( (E \sim E') \) complete annihilation should be possible. This requires \( N = N' \) and a minimum of the tachyon potential \( T = T_0 \), where

\[
V(T_0) + 2NT_{Dp} = 0.
\]

As a specific example, consider the case \( p = 9 \) in the type IIB theory. Consistency of the quantum theory (tadpole cancellation) requires that the total RR 9-brane charge should vanish, and thus \( N = N' \). So we have an equal number of D9-branes and \( \overline{D9} \)-branes filling the 10d spacetime \( X \). Associated to this we have a pair of vector bundles \((E, E')\), where \( E \) and \( E' \) are rank \( N \) complex vector bundles. We now want to define equivalence of pairs \((E, E')\) and \((F, F')\) whenever the associated 9-brane systems can be related by brane-antibrane annihilation and creation. In particular, \( E \sim E' \) corresponds to pure vacuum, and therefore

\[
(E, E') \sim 0 \iff E \sim E'.
\]

If we add more D9-branes and \( \overline{D9} \)-branes with identical vector bundles \( H \), this should not give anything new, since they are allowed to annihilate. This means that

\[
(E \oplus H, E' \oplus H) \sim (E, E').
\]

In this way we form equivalence classes of pairs of bundles. These classes form an abelian group. For example, \((E', E)\) belongs to the inverse class of the class containing \((E, E')\). If
$N$ and $N'$ are unrestricted, the group is called $K(X)$. However, the group that we have constructed above is the subgroup of $K(X)$ defined by requiring $N = N'$. This subgroup is called $\tilde{K}(X)$. Thus type IIB D-brane charges should be classified by elements of $\tilde{K}(X)$. Let’s examine whether this works.

The formalism is quite general, but to begin we will only consider the relatively simple case of Dp-branes that are hyperplanes in flat $\mathbb{R}^{10}$. For this purpose it is natural to decompose the space into tangential and normal directions

$$\mathbb{R}^{10} = \mathbb{R}^{p+1} \times \mathbb{R}^{9-p}, \quad (75)$$

and consider bundles that are independent of the tangential $\mathbb{R}^{p+1}$ coordinates. If the fields fall sufficiently at infinity, so that the energy is normalizable, then we can add the point at infinity thereby compactifying the normal space so that it becomes topologically a sphere $S^{9-p}$. Then the relevant base space for the Dp-brane bundles in $X = S^{9-p}$. We can now invoke the mathematical results:

$$\tilde{K}(S^{9-p}) = \begin{cases} \mathbb{Z} & p = \text{odd} \\ 0 & p = \text{even} \end{cases}. \quad (76)$$

This precisely accounts for the RR charge of all the stable (BPS) Dp-branes of the type IIB theory on $\mathbb{R}^{10}$. The relation to homotopy is

$$\tilde{K}(S^n) = \pi_{n-1}(U(N)) \quad (\text{large } N). \quad (77)$$

Note that $N$ does not appear on the left-hand side. K-theory groups are automatically in the stable regime. It should also be noted that the unstable type IIB D-branes, which we discussed in Section 3.3, carry no conserved charges, and they do not show up in this classification.

Suppose now that some dimensions form a compact manifold $Q$ of dimension $q$, so that the total spacetime is $\mathbb{R}^{10-q} \times Q$. Then the construction of a Dp-brane requires compactifying the normal space $\mathbb{R}^{9-p-q} \times Q$ to give $S^{9-p-q} \times Q$. This involves adjoining a copy of $Q$ at infinity. In this case the appropriate mathematical objects to classify D-brane charges are relative K-theory groups $K(S^{9-p-q} \times Q, Q) \quad [12]$. In particular, if $Q = S^1$, we have $K(S^{8-p} \times S^1, S^1)$. Mathematically, it is known that this relative K-theory group can be decomposed into two pieces

$$K(X \times S^1, S^1) = K^{-1}(X) \oplus \tilde{K}(X). \quad (78)$$

The physical interpretation of this formula is very nice. $\tilde{K}(S^{8-p})$ classifies the type IIB D-branes that are wrapped on the circle, whereas

$$K^{-1}(S^{8-p}) \cong \tilde{K}(S^{9-p}), \quad (79)$$

25
classifies unwrapped D-branes. So, altogether, in nine dimensions there are additive D-brane charges for all $p < 8$.

The type IIA case is somewhat more subtle, since the spacetime filling D9-branes are unstable in this case. Also, they are TCP self-conjugate. The right K-theory group was conjectured by Witten, and subsequently explained by Hořava [43]. The answer is $K^{-1}(X)$, which (as we have already indicated) has

$$K^{-1}(S^{9-p}) = \begin{cases} \mathbb{Z} & \text{for } p = \text{even} \\ 0 & \text{for } p = \text{odd}. \end{cases}$$

(80)

This accounts for all the stable type IIA $D_p$-branes embedded in $\mathbb{R}^{10}$. The relation to homotopy in this case is

$$K^{-1}(S^n) = \pi_{n-1}\left(\frac{U(2N)}{U(N) \times U(N)}\right) \text{ (large } N).$$

(81)

Let me refer you to Hořava’s paper for an explanation of these facts.

Compactifying the type IIA theory on a circle gives the relative K-theory group

$$K^{-1}(X \times S^1, S^1) = \tilde{K}(X) \oplus K^{-1}(X).$$

(82)

This time $K^{-1}(X)$ describes wrapped D-branes and $\tilde{K}(X)$ describes unwrapped ones. This result matches the type IIB result in exactly the way required by T duality (wrapped ↔ unwrapped) [42].

### 4.3 Type I D-branes

Type I D-branes charges can also be classified using K-theory [11]. In this case we should consider $N + 32$ D9-branes with an $O(N + 32)$ vector bundle $E$ and $N$ D9-branes with an $O(N)$ vector bundle $E'$. We define equivalence classes as before

$$(E, E') \sim (E \oplus H, E' \oplus H),$$

(83)

where $H$ is an arbitrary $SO(k)$ vector bundle on $X$. These equivalence classes define the elements of a K-theory group. If rank $E - \text{rank } E'$ were unrestricted the group would be $KO(X)$. One can define a subgroup of $KO(X)$, called $\tilde{KO}(X)$, by requiring rank $E = \text{rank } E'$. However, this is not quite what we want. The type I theory requires rank $E = \text{rank } E' + 32$. This is a coset isomorphic to $\tilde{KO}(X)$, so as far as K-theory is concerned the fact that the type I theory has 32 extra D9-branes is irrelevant. However, later we will show that it is quite relevant to some of the physics. So we already see that K-theory is not the whole story.
In any case, by the same reasoning as before, the conserved charges of type I D-branes in $R^{10}$ should be determined by the groups $\overline{KO}(S^{9-p})$. The connection to homotopy in this case is

$$\overline{KO}(S^n) = \pi_{n-1}(O(N)) \quad \text{(large } N).$$

(84)

The results are as follows:

- $\overline{KO}(S^{9-p}) = \mathbb{Z}$ for $p = 1, 5, 9$.
- $\overline{KO}(S^{9-p}) = \mathbb{Z}_2$ for $p = -1, 0, 7, 8$.
- $\overline{KO}(S^{9-p}) = 0$ for $p = 2, 3, 4, 6$.

These are the three kinds of BPS D-branes which carry additive conserved charges.

There are no conserved D-brane charges in these cases.

Let us now consider compactifying the type I theory on a circle, so that the spacetime is $R^9 \times S^1$. In this case we want to understand the classification of D-brane charges in nine dimensions and the T-dual description in terms of type I' theory. As in the type II cases, the K-theory description of D-brane charges of the compactified theory is given by the relative K-theory group $KO(X \times S^1, S^1)$. As in the type II case, the mathematical identity

$$KO(X \times S^1, S^1) = \overline{KO}^{-1}(X) \oplus \overline{KO}(X),$$

(85)

agrees with our physical expectations. Specifically, $\overline{KO}(S^{8-p})$ classifies the wrapped D-branes and $\overline{KO}^{-1}(S^{8-p})$ describes unwrapped D-branes.

There is a slightly subtle point. The K-theory group elements correspond to conserved charges and not to stable D-branes. In Section 3.5 we saw that the type I non-BPS D0-brane decays into two particles, which can be described as a wrapped D1 + D1 system when the compactification radius $R < 1/\sqrt{2}$. The wrapped D-string that has the trivial Wilson line ($W = 1$) is a gauge group spinor, and it carries the charge in question. The K-theory classification of charges does not distinguish the cases $R < 1/\sqrt{2}$ and $R > 1/\sqrt{2}$. While it classifies charges, it doesn’t identify which object is the ground state with that charge.

Let us now examine the situation from the T-dual type I' perspective [12]. In particular, we would like to achieve a qualitative understanding of the transition at $R = 1/\sqrt{2}$ ($R' = \sqrt{2}$). To transcribe the picture to the type I' perspective recall that under T duality wrapped D-branes map to unwrapped D-branes and vice versa. Recall, too, that the position of an
unwrapped D-brane is encoded in a Wilson line of the corresponding wrapped D-brane. Thus, a non-BPS D0-brane of type I localized on the circle (for $R > 1/\sqrt{2}$) should correspond to a non-BPS D1-brane of type I' stretched across the interval from $X' = 0$ to $X' = \pi R'$. A Wilson line of the $U(1)$ gauge group on this string should encode the position of the D0-brane.

If $R < 1/\sqrt{2}$ (and $R' > \sqrt{2}$), on the other hand, then in the type I picture one has a wrapped D1+$D\bar{1}$ pair. One of the strings has $W = +1$ and the other one has $W = -1$. These strings correspond to D0-branes in the dual type I' picture, and the Wilson lines tell us that those D0-branes are stuck to the orientifold planes. Thus, to be specific, there is a $D\bar{0}$-brane stuck to the O8-plane at $X' = 0$ and a D0-brane stuck to the O8-plane at $X' = \pi R'$. An interesting question is how such a configuration morphs into a string stretched across the interval as $R'$ is decreased through the value $\sqrt{2}$. I have a conjecture for the answer to this question, which goes beyond the perturbative framework in which we have been working. It is reminiscent of similar phenomena found in other contexts, and is the only smooth way that I can imagine the transition taking place. The idea is that as $R'$ approaches $\sqrt{2}$ from above, the orientifold planes develop spikes so that the D0 and $D\bar{0}$ approach one another. Then in the limit $R' \to \sqrt{2}$ they touch and annihilate, leaving a connecting tube between the O8-planes, which in the perturbative limit is identified as a string. This is somewhat analogous to the joining/breaking transition of QCD flux tubes with the annihilation/creation of $q\bar{q}$ pairs. It is also reminiscent of a description of fundamental strings ending on D-branes in terms of a soliton field configuration on the D-brane world-volume [44, 45]. However, it differs from these examples in a rather peculiar way. In this case, the inside of the tube, which is identified as a stretched non-BPS D-string, is not even part of the spacetime!

We have just seen that there can be D0-branes stuck to O8-planes in the type I' theory. This is T-dual to the fact that a wrapped type I D-string can have Wilson line $W = \pm 1$, with the two possibilities corresponding to the two orientifold planes. Incidentally, this implies a certain asymmetry between the orientifold planes, since $W = 1$ gives a gauge group spinor and $W = -1$ does not.

The bulk of the type I' spacetime is indistinguishable from type IIA spacetime, at least in the region where half of the D8-branes are to the left and half are to the right. (In other regions one has a “massive” type IIA spacetime of the type discovered by Romans [46].) We know that the type IIA theory has a D0-brane, so this suggests that the type I' theory should also admit this possibility. The way this works is that a pair of stuck D0-branes on an orientifold plane can pair up and move into the bulk, where the composite object is identified as a single bulk D0-brane. It is instructive to understand the T-dual type I description of this process.
Consider a pair of wrapped type I D-strings. The world-volume gauge group is \(O(2)\), which is nonabelian. This system corresponds to D0-branes in the type I’ description, with positions controlled by the choice of \(O(2)\) Wilson line. Inequivalent choices of Wilson line are classified by conjugacy classes of the group. This group has two types of conjugacy classes. A class of the first type describes a rotation by \(\theta\) or \(2\pi - \theta\), which are equivalent, where \(0 \leq \theta \leq \pi\). This class corresponds to a composite bulk D0-brane located at \(X' = \theta R'\) and its mirror image at \(X' = (2\pi - \theta)R'\). The remaining conjugacy class contains all reflection elements of \(O(2)\). A representative of this class is \(
abla \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \). Thus this class corresponds to having one stuck D0-brane on each O8-plane. Note that the two-body system described by this system is a gauge group spinor, whereas a one-body system described by a \(\theta\) class is not a gauge group spinor. I have reviewed some additional related issues elsewhere [47].

5 Moduli Spaces of Theories with 16 Supercharges

Consistent vacua of string theories typically form spaces, called moduli spaces, which are parameterized by the vacuum values of massless scalar fields with flat potentials. For theories with 16 unbroken supersymmetries (16 conserved supercharges) such as the type I and heterotic theories, the vacuum moduli spaces are always of the Narain type

\[
\mathcal{M}_{m,n} = \Gamma_{m,n}(\mathbb{Z}) \backslash SO(m, n)/SO(m) \times SO(n).
\]

Here \(\Gamma_{m,n}(\mathbb{Z})\) is the standard infinite discrete duality group given by integral \(SO(m, n)\) matrices. Equivalently, it can be described as the subgroup of \(SO(m, n)\) that preserves an appropriate lattice of signature \((m, n)\). When \(d \geq 4, n = 10 - d\) \(U(1)\) gauge fields belong to the supergravity multiplet. The remainder of the gauge fields belong to vector supermultiplets and form a gauge group of rank \(m\). At generic points in the moduli space this group is \([U(1)]^m\) but there is nonabelian symmetry enhancement on various subsurfaces of the moduli space. The case \(d = 10\) is special in that the moduli space consists of just two points — corresponding to the \(E_8 \times E_8\) and \(\text{Spin}(32)/\mathbb{Z}_2\) theories.

5.1 Three components in \(d = 9\)

In \(d = 9\) the moduli space of consistent vacua with 16 unbroken supersymmetries turns out to have three disconnected components. They correspond to the Narain moduli spaces \(\mathcal{M}_{17,1}, \mathcal{M}_{9,1}\), and \(\mathcal{M}_{1,1}\). Since \(m - n\) is a multiple of 8 in each case, they correspond to even self-dual lattices, which is the key to establishing modular invariance of the corresponding one-loop amplitudes. These three cases turn out to have amusing geometric descriptions in
terms of 11-dimensional M theory. They correspond to compactifications $\mathbf{R}^9 \times K$, where $K$ is a cylinder for $m = 17$, a Möbius strip for $m = 9$, and a Klein bottle for $m = 1$ [18]. We observe that each boundary component of $K$ contributes 8 units of rank to the gauge groups. The M theory viewpoint will not be pursued further in these lectures, because our focus is on perturbative superstring descriptions, and the M theory picture is nonperturbative.

The case $m = 17$ also corresponds to compactification of the type I theory on a circle. We have discussed this system, including its D-brane spectrum and the dual type I′ description in the preceding section, so we will not say more about it here. The case $m = 1$ has a perturbative superstring description in $d = 9$, which will be discussed in the next subsection. The case $m = 9$ does not have a perturbative superstring description in $d = 9$ (even through it does exist). However, a further compactification to a $\mathcal{M}_{10,2}$ moduli space in $d = 8$ makes it amenable to perturbative superstring analysis. We will say a little about this case in Section 5.3.

One of our main concerns is to identify the spectrum of D-branes and D-brane charges in each case. The BPS D-branes are always easy to identify, and are the same for all the components of the moduli spaces (that have a perturbative description). The point is that the $C_{\mu\nu}$ RR gauge field of the type I theory in $d = 10$ gives rise to the relevant RR gauge fields in the lower dimensions. Knowing these gauge fields one can immediately read off the corresponding spectrum of BPS D-branes. For example, in $d = 9$ one has $C_{\mu\nu}$ and $C_{\mu 9} \equiv C_{\mu}$. As a result there are BPS D-branes in $d = 9$ for $p = 0, 1, 4, 5$ for both the $m = 17$ and the $m = 1$ branches of the moduli space. There are corresponding stable $p$-branes in the $m = 9$ case, as well. However, in that case there is no perturbative limit, so it is not very meaningful to call them D-branes. In addition, each case requires a certain number of BPS D8-branes for quantum consistency.

5.2 The type $\tilde{I}$ theory

There is a perturbative superstring description of the $\mathcal{M}_{1,1}$ component of the $d = 9$ moduli space, which has been called the type $\tilde{I}$ theory [18]. It is formulated as a type IIB orientifold that differs somewhat from the usual type I construction. One starts with the IIB theory on $\mathbf{R}^9 \times S^1$, and then mods out by the $\mathbf{Z}_2$ symmetry

$$\tilde{\Omega} = \Omega \cdot S_{1/2}. \quad (87)$$

$\Omega$ is the usual world-sheet orientation reversal. But now it is accompanied by the operation $S_{1/2}$, which is translation half way around the circle. Because of this half shift, $\tilde{\Omega}$ has no fixed points, and as a result the type $\tilde{I}$ theory has no orientifold planes. Therefore, no spacetime-filling D-branes should be added to cancel the RR charge of O-planes. It follows that there
is no gauge group, analogous to $SO(32)$, associated to such D-branes. Indeed the only gauge symmetry of the 9d theory is $[U(1)]^2$, where these fields arise from the components $g_{\mu\nu}$ and $C_{\mu\nu}$ of the 10d metric and RR gauge fields. This is what one expects for $\mathcal{M}_{1,1}$. The T-dual description of the type I theory is called type $\tilde{I}$. It consists of an interval $0 \leq X' \leq \pi R'$ (for the same reason as type I'). The interval is bounded by orientifold planes. However, unlike the type I' case, now the orientifold planes carry opposite RR charge, and could be denoted $O_{8+}$ and $O_{8-}$. As a result, the total RR charge vanishes and no parallel D8-branes should be added.

We have already argued that the type $\tilde{I}$ theory has the same spectrum of BPS D-branes as the type I theory compactified on a circle. We will now examine the spectrum of non-BPS D-branes and show that it differs from the spectrum of the compactified type I theory. Bergman, Gimon, and Hořava examined this problem \cite{42} and showed that the relevant K-theory group is $\widetilde{KSC}(X)$ and that this predicts non-BPS $D_p$-branes carrying a $\mathbb{Z}_2$ charge for $p = -1, 3, 7$. I refer you to their paper for the K-theory analysis. What I want to describe here is a physical argument they presented in support of this conclusion.

Because we mod out by $\Omega$, a $D_p$-brane localized on the circle must be accompanied by an $\Omega$-reflected $D_p$-brane at the opposite location half way around the circle. Now, the action of $\Omega$ on type IIB D-branes is as follows:

$$
\begin{align*}
\Omega : & \quad D_p \rightarrow D_p & \quad p = 1, 5, 9 \\
\Omega : & \quad D_p \rightarrow \overline{D_p} & \quad p = -1, 3, 7.
\end{align*}
$$

This is why the type I theory only has BPS D-branes for $p = 1, 5, 9$.

The configurations based on $p = 1, 5$ give expected BPS D-branes in 9d. However, the configurations with a $p = -1, 3, 7$ Dp-brane and an accompanying $\overline{D_p}$-brane half-way around the circle are non-BPS. But they also correspond to stable configurations in 9d, at least when the radius of the circle is large enough. (Otherwise the open string connecting them might develop a tachyonic mode.) So the result agrees with the K-theory prediction.

We can see that these non-BPS $D_p$-branes are conserved modulo 2, so that they carry a $\mathbb{Z}_2$ charge. Imagine two of them localized on the circle. Then one of them (and its image) can be slid around the circle until the $D_p$-brane of the first pair encounters the $\overline{D_p}$-brane of the second pair. At this point the pairs can annihilate into neutral radiation. As an exercise, the reader might want to describe the T-dual type $\tilde{I}'$ description of this annihilation process.

### 5.3 Consistent vacua in eight dimensions

The three 9d branches of the moduli space correspond to three branches in 8d, as well. They are $\mathcal{M}_{18,2}$, $\mathcal{M}_{10,2}$, and $\mathcal{M}_{2,2}$. Below 8d there are additional branches, which we will
not discuss here. The three branches in 8d can be constructed from a type I perspective by compactifying on a torus, so that the spacetime is $\mathbb{R}^8 \times T^2$, and allowing appropriate holonomies associated to the two cycles of the torus. The possible choices of holonomies were analyzed by Witten [50]. His key observation was that it is possible to have holonomies that correspond to gauge bundles without vector structure.

The holonomies associated to the two cycles are elements of the gauge group, which we can write as $32 \times 32$ matrices $g_1$ and $g_2$. In order to describe a flat bundle, which is a necessary requirement, there are two possibilities

$$[g_1, g_2] = 0 \quad \text{or} \quad \{g_1, g_2\} = 0. \quad (89)$$

The first case is the one with vector structure, and the second is the one without vector structure. The point is that holonomies, when written in terms of allowed representations of the gauge group, should commute to give a flat bundle. However, the nonperturbative gauge group is $\text{Spin}(32)/\mathbb{Z}_2$, which does not admit the 32-dimensional vector as an allowed representation. Matrices that anticommute in the $32$ can correspond to ones that commute when expressed in terms of any of the allowed representations.

Witten analyzed the possibilities and found that there are three inequivalent choices, one with vector structure and two without vector structure. The one with vector structure is the obvious choice, which gives $M_{18,2}$. In each case it is illuminating to consider the T-dual description on $\tilde{T}^2/\mathbb{Z}_2$. In this description the $M_{18,2}$ branch of the moduli space corresponds to having four O7-planes, each with RR charge $-4$. Thus 16 D7-branes localized on $\tilde{T}^2/\mathbb{Z}_2$ must be added. This is a straightforward generalization of the type I' description in 9d. The $M_{10,2}$ branch corresponds to having three O7-planes with RR charge $-4$ and one with charge $+4$. In this case eight D7-branes are required to cancel the charge. The last case, $M_{2,2}$ is described by two O7-planes with RR charge $-4$ and two with charge $+4$ and no D7-branes. In each case, one sees that the number of D7-branes agrees with what is required to explain the rank of the gauge group. Incidentally, if there were more than two O7-planes of positive charge, one would need D7-branes to cancel the RR charge. Such a configuration would not be supersymmetric.

An exercise that has not yet been carried out is to determine the spectrum of non-BPS D-branes for the $M_{10,2}$ branch of the moduli space. This should be possible now.

6 Non-BPS D-Branes in Type I
6.1 Determination of the nonperturbative gauge groups

The gauge symmetry of the perturbative type I theory is $O(32)$. (The reflections have no consequence, so there is no harm in including them.) However, the non-BPS D-instanton, which showed up in the K-theory classification in Section 4.3, arises because $\pi_9(SO(32)) = \mathbb{Z}_2$. Witten argued that it is responsible for breaking $O(32) \to SO(32)$ [11]. Moreover, the non-BPS D-particle, which we have discussed at some length, is a gauge group spinor. More precisely, it gives states belonging to one of the two spinorial conjugacy classes of Spin(32).

When all of these facts are taken into account, one concludes that the nonperturbative type I theory has a different gauge group than is visible in perturbation theory. Specifically, it is Spin(32)/$\mathbb{Z}_2$. This agrees with the gauge symmetry that is manifest in the perturbative heterotic theory. This agreement can be regarded as a successful test of S duality.

Incidentally, we also can conclude that there are no nonperturbative effects in the heterotic description that modify the gauge group.

6.2 Instability of D7 and D8

The K-theory classification of type I D-brane charges, presented in Section 4.3, suggests the existence of a non-BPS D7-brane and a non-BPS D8-brane, each of which is supposed to carry a conserved $\mathbb{Z}_2$ charge. However, there is a tachyon in the spectrum of D7 – D9 and D8 – D9 open strings [51]. Therefore, the proposed D7-brane and D8-brane should each have 32 tachyon fields in their world-volume theory, and therefore they must be unstable. (Such instability occurs for D$p$ – D$q$ open strings whenever $0 < |p - q| < 4$.) This instability does not, by itself, constitute a contradiction with the K-theory analysis, which only purports to give the types of conserved D-brane charges, and not the specific objects which carry them. But it is a cause for concern. Clearly, the instability occurs in these cases because their are 32 spacetime filling D9-branes in the type I vacuum. As we pointed out earlier, the K-theory analysis is indifferent to their presence. The question is to what extent the $\mathbb{Z}_2$ charge survives when the D7 or the D8 dissolves in the background D9-branes.

6.3 Further analysis of the D8

By Bott periodicity, one might expect the non-BPS D8-brane to have features in common with the non-BPS D0-brane. There is one essential difference, however, The D0 – D9 open strings do not have tachyonic modes whereas the D8 – D9 open strings do. We can confirm this fact by trying to emulate Sen’s construction of the D0-brane as a kink solution on a D1 + D1 system [52].

In the case of the D1 + D1 system we saw that there is a single real tachyon $T$ and a
potential $V(T)$ with minima at $T = \pm T_0$, which can be regarded as a 0-dimensional sphere. The vacuum manifold has two disconnected components and, therefore, the kink solution that connects them is topologically stable. The closest analog we can construct eight dimensions higher is to consider a system consisting of 33 D9-branes and one $D^9$-brane. The world volume in this case has 33 tachyonic modes $\vec{T}$ belonging to the fundamental representation of the $O(33)$ gauge symmetry carried by the D9-branes. The potential $V(\vec{T})$ must have $O(33)$ gauge symmetry, so the minima at $|\vec{T}| = T_0$ describe a $S^{32}$ vacuum manifold. This moduli space is connected, and therefore does not support a topologically stable kink. The situation is very reminiscent of the unstable type II D-branes discussed in Section 3.3. In that case there was an $S^1$, and as a result the kink solution had one unstable direction, so that the resulting D-brane ended up with one tachyonic mode. In the present case there are 32 unstable directions so that the resulting D8-brane has 32 tachyonic modes in its world volume. Of course, these are the same modes that one discovers by quantizing the D8 − D9 open strings. In the case of the unstable type II D-branes, there was no associated conserved charge, and they did not appear in the K-theory analysis. The type I D8-brane, on the other hand, does appear in the K-theory analysis, even though it has an analogous instability.

Witten has argued in support of the D8-brane as follows: The D-instanton implies that there are two distinct type I vacua distinguished by the sign of the D-instanton amplitude. This is a $\mathbb{Z}_2$ analog of the $\theta$ angle in QCD. One should expect that there is a domain wall connecting the two vacua — the D8-brane. The sign change in the instanton amplitude means that the D-instanton is the electromagnetic dual of the D8-brane. This has been investigated by Gukov [53].

Even though the D8-brane is unstable one could imagine forming it at some moment in time and asking how the vacua on the two sides of its are distinguished. Recalling that the D-instanton was responsible for breaking $O(32) \to SO(32)$, it seems clear that these vacua should be distinguished by a gauge-group reflection. This implies that opposite spinor conjugacy classes would enter into the $\text{Spin}(32)/\mathbb{Z}_2$ gauge group on the two sides. Of course, once the D8-brane decays, the vacuum becomes uniform everywhere.

7 Concluding Remarks

There has been considerable progress in analyzing non-supersymmetric D-brane configurations. In particular, some stable non-BPS D-branes have been identified, and they have been studied in perturbative limits with some mathematical control. However, without the BPS property it is not possible (at present) to make quantitative studies away from weak coupling.
Another significant development has been the identification of K-theory groups as the appropriate mathematical objects for classifying D-brane charges. In the case of BPS D-branes, these charges are sources for RR gauge fields, whereas in non-BPS cases they are not. The non-BPS D-branes that we have discussed carry conserved $\mathbb{Z}_2$ charges. However, Sen has analyzed examples in which other charge groups (such as $\mathbb{Z}$) also appear. One lesson we have learned is that K-theory does not take account of spacetime filling D-branes, such as the 32 D9-branes of the type I theory. Their presence can destabilize other D-branes in certain cases.

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