High-accuracy resolver-to-digital conversion via phase locked loop based on PID controller

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Abstract. The problem of resolver-to-digital conversion (RDC) is transformed into the problem of angle tracking control, and a phase locked loop (PLL) method based on PID controller is proposed in this paper. This controller comprises a typical PI controller plus an incomplete differential which can avoid the amplification of higher-frequency noise components by filtering the phase detection error with a low-pass filter. Compared with conventional ones, the proposed PLL method makes the converter a system of type III and thus the conversion accuracy can be improved. Experimental results demonstrate the effectiveness of the proposed method.

1. Introduction
Shaft sensors are often used to obtain the information of angular position and velocity for servo control systems [1]. Compared with the other shaft sensors, resolver is widely used owing to its superior properties in ruggedness, reliability and high accuracy [2]. However, resolvers only can output amplitude-modulated orthogonal pair signals containing angular position information. High-accuracy resolver-to-digital converters (RDCs) should be developed to obtain the information of angular position and velocity for servo applications.

The most convenient solution to RDC is the special integrated circuit (IC), such as AD2S80A, AD2S1210. RDC ICs can offer simple interfaces between resolvers and computers, but the bandwidth of RDC ICs can not be adjusted flexibly as it depends on the external resistances and capacitances.

Software-based RDC can avoid the problems of RDC ICs. Software-based RDC methods can be classified into two kinds: arctangent method and phase locked loop (PLL) method [3][4][5][6]. Arctangent method is the most straightforward one, but the angular speed is usually derived by differential operation which will amplify higher-frequency noise. PLL method is essentially a closed-loop system and has been widely used in RDC ICs and software-based RDC. Most of RDC ICs adopt type II tracking loop as PLL structure, and it has been extended to software-based RDC. Besides, angle tracking observer (ATO) proposed in [7] is also a PLL system. Both type II tracking loop and ATO have better performance in disturbance attenuation. However, the conversion accuracy is still restricted by the system structure of conventional PLL methods.

In order to improve RDC accuracy, the problem of RDC is transformed into the problem of angle tracking control, and a PLL method based on PID controller is proposed for software-based RDC. This controller consists of a typical PI controller and an incomplete differential with a low-pass filter. Since the low-pass filter can remove higher-frequency noise components from signals, the incomplete differential often be implemented in engineering. The proposed PLL method makes RDC become a system of type III so that the demodulation accuracy can be improved.
2. Review of typical PLL-based RDCs

The ideal outputs of resolver are two orthogonal amplitude-modulated signals [8]. In software-based RDCs, the envelope detection is always implemented by the peripheral hardware circuits to relieve the pressure on sampling. The envelopes of resolver after detection can be written as:

\[ y_s = \sin \theta, \quad y_c = \cos \theta \] (1)

The information of shaft angle and speed can be obtained from (1) by PLL-based RDCs [9]. Figure 1(a) shows the typical structure of a PLL-based RDC which includes a phase detector (PD), a low-pass filter (LPF), and a voltage-controlled oscillator (VCO). PD is in charge of detecting the phase error \( \hat{\theta} \) between the shaft angle \( \theta \) and its estimate \( \hat{\theta} \). The phase error \( \hat{\theta} \) is given in a nonlinear form as:

\[ \varepsilon = \sin \theta \cos \hat{\theta} - \cos \theta \sin \hat{\theta} = \sin(\theta - \hat{\theta}) = \sin \hat{\theta} \] (2)

For very small \( \hat{\theta} \), \( \sin \hat{\theta} \approx \hat{\theta} \). Therefore, figure 1(a) can be simplified into figure 1(b).

![Figure 1. Block diagram of conventional PLL-based RDC.](image)

Type II tracking loop method and angle tracking observer (ATO) method are two main kinds of PLL-based RDCs. Their performance will be analyzed as follows.

2.1. Type II tracking loop based RDC

According to the datasheet of AD2S1210 [10], AD2S1210 is a type II tracking loop based PLL in fact. Two error transfer functions concerning shaft angle and speed can be derived as:

\[ \Phi_{\theta \theta}(s) = \frac{\hat{\theta}(s)}{\theta(s)} = \frac{\tau_1 s^3 + s^2}{\tau_1 s^3 + s^2 + k_p \tau_1 s + k_a}, \quad \Phi_{\omega \omega}(s) = \frac{\hat{\omega}(s)}{\omega(s)} = \frac{\tau_1 s^3 + s^2}{\tau_1 s^3 + s^2 + k_p \tau_1 s + k_a} \] (3)

where \( k_a > 0 \) is the open-loop gain, and \( \tau_1 > \tau_2 \) are compensation time constants.

When the resolver rotates at a constant speed of \( C \) rad/s, from the final-value theorem, the steady state errors of shaft angle and speed estimation are both equal to 0. And when the resolver rotates at an acceleration of \( A \) rad/s\(^2\), from the final-value theorem, the steady state error of shaft speed estimation is also equal to 0, but the estimate of shaft angle contains steady state error which is equal to \( A / K_a \).

2.2. ATO based RDC

ATO method based on nonlinear state observer for RDC has been proposed in [11]. Two error transfer functions concerning shaft angle and speed also can be obtained as:

\[ \Phi_{\theta \theta}(s) = \frac{\hat{\theta}(s)}{\theta(s)} = \frac{s^2}{s^2 + k_p s + k_a}, \quad \Phi_{\omega \omega}(s) = \frac{\hat{\omega}(s)}{\omega(s)} = \frac{s^2 + k_\omega s}{s^2 + k_\omega s + k_a} \] (4)

where \( k_\omega, k_a \) are positive observer gains.

Similarly, when the resolver rotates at a constant speed of \( C \) rad/s, from the final-value theorem, the steady state error of shaft angle and speed estimation are equal to 0. When the resolver rotates at a constant acceleration of \( A \) rad/s\(^2\), the estimates of shaft angle and speed contain steady state errors which are equal to \( A / k_a \) and \( Ak_\omega / k_a \), respectively.
3. Improved PID controller for RDC

To improve the conversion accuracy of typical PLL based RDC, the problem of RDC is transformed into the problem of angular tracking control and a PID controller based RDC is designed as follows.

3.1. Design of PID controller based RDC

Define an extended state as $\theta_i = \int \theta dt$, then an augmented system for RDC can be constructed as:

$$\dot{\theta}_i = \dot{\theta}, \quad \dot{\theta} = \dot{\omega}, \quad \dot{\omega} = u$$  \hspace{1cm} (5)

The problem of RDC is to design the control $u$ subject to $\dot{\theta} \to \theta$ as $t \to \infty$. Actually, an error state feedback can accomplish the above aim. We can design the controller as:

$$u = K_p \dot{\theta} + K_i \theta_i + K_d \omega$$ \hspace{1cm} (6)

where $K_p$, $K_i$, $K_d$ are positive controller gains, $\theta_i = \theta_0 - \dot{\theta}_i$, $\dot{\theta} = \theta - \dot{\theta}$, $\omega = \omega - \dot{\omega}$. It is noted that system (5) is controllable, the poles of the closed-loop system can be configured by choosing suitable controller gains. Therefore, the demodulation error $\dot{\theta}$ can be asymptotically bounded owing to the uncertainties in acceleration. Substituting (6) into (5) gives the PID form of RDC as:

$$\dot{\theta}_i = \dot{\theta}, \quad \dot{\theta} = \dot{\omega}, \quad \dot{\omega} = K_p \dot{\theta} + K_i \theta_i + K_d \omega$$ \hspace{1cm} (7)

3.2. Realization of PID based RDC

To minimize the influence of the differential computation in (6), a low-pass filter $f(s) = \frac{1}{1+Ts}$ can be used to realize an incomplete differential, as shown in Figure 2(a). Considering the phase detection, the actual form of the new RDC can be implemented as shown in Figure 2(b).

![Figure 2. Block diagram of Improved PID Controller for RDC.](image)

According to Figure 2(a), the error transfer functions concerning shaft angle and speed of the improved PID controller based RDC can be written as:

$$\Phi_{\dot{\theta}/\theta}(s) = \frac{\dot{\theta}(s)}{\theta(s)} = \frac{Ts^4 + s^3}{Ts^4 + s^3 + (k_D + k_D T)s^2 + (k_p + k_D T)s + k_I}$$ \hspace{1cm} (8)

$$\Phi_{\dot{\omega}/\omega}(s) = \frac{\dot{\omega}(s)}{\omega(s)} = \frac{Ts^4 + s^3}{Ts^4 + s^3 + (k_D + k_D T)s^2 + (k_p + k_D T)s + k_I}$$  \hspace{1cm} (9)

According to the error transfer functions (8) and (9), when the resolver rotates at a constant speed of $C$ rad/s, from the final-value theorem, the steady state errors of shaft angle and speed estimation are both equal to 0. When the resolver rotates at a constant acceleration of $A$ rad/s$^2$, the steady state error of shaft angle and speed estimation are also equal to 0.

From the above analysis, as the proposed PLL method makes RDC become a system of type III, it can be seen that the conversion accuracy can be improved by the proposed PLL method compared with the typical PLL method.
4. Experimental results
To evaluate the effectiveness of the proposed RDC, experiments have been carried out on a platform including a resolver simulator and a DSP-based detection board. By using the resolver simulator, the actual values of the shaft angle and speed can be preset which is convenient to evaluate the performance of the proposed RDC by comparing the estimated values with the preset actual ones.

![Shaft angle estimation errors.](image1)

![Shaft speed estimation errors.](image2)

**Figure 3.** Shaft angle and speed estimation errors at a constant speed of $4\pi$ rad/s.

![Shaft angle estimation errors.](image3)

![Shaft speed estimation errors.](image4)

**Figure 4.** Shaft angle and speed estimation errors at a constant acceleration of $4\pi$ rad/s$^2$.

![Shaft angle estimation errors.](image5)

![Shaft speed estimation errors.](image6)

**Figure 5.** Shaft angle and speed estimation errors at a sine speed of $(8\pi + 4\pi \sin 2\pi t)$ rad/s.

To compare the performance of type II tracking loop, ATO and the proposed method, the parameters of ATO and the proposed method are adjusted according to the bandwidth of shaft speed given in [10]. Thus, the parameters of type II tracking loop are $k_p = 92.7 \times 10^3$, $t_1 = 8 \times 10^{-3}$ s, $t_2 = 728 \times 10^{-6}$ s; the parameters of ATO are chosen to be $k_\theta = 1200$, $k_\omega = 1000000$; the parameters of
the proposed method are adjusted as \( k_p = 180000 \), \( k_d = 700 \), \( k_i = 3000000 \), \( T = 7 \times 10^{-4} \) s. By using the above parameters of three methods, experimental results in three cases can be obtained, as shown in Figures (3), (4), and (5). It needs to be explained that type II tracking loop, ATO and the proposed method are denoted as RDC-II, ATO, and PID in the figures respectively.

Case 1: When the resolver rotates at a constant speed of \( 4\pi \) rad/s, it can be seen from figure 3(a) that the estimation error of shaft angle by the proposed method is about 0.18° which is smaller than RDC-II and ATO method whose shaft angle errors are both about 0.21°. And from figure 3(b), the performance of the shaft speed estimation by the proposed method is also much better than that by the other two methods.

Case 2: When the resolver rotates at a constant acceleration of \( 4\pi \) rad/s², as we can see from figure 4(a), the shaft angle estimation errors of these three methods both contain steady state errors, while the shaft angle steady state errors of improved PID method is about 0.08° which is much smaller than that of the other two methods whose shaft angle errors are both about 0.16°. Besides, from figure 4(b), it is clear that the shaft speed error of improved PID method is also much smaller than that of RDC-II and ATO method.

Case 3: When the resolver rotates at a sine speed of \((8\pi + 4\pi \sin 2\pi t)\) rad/s, from figure 5(a), it can be seen that the shaft angle error of RDC-II method is slightly larger than that of ATO method and improved PID method. But from figure 5(b), the shaft speed errors of improved PID method are much smaller than that of RDC-II and ATO method.

To sum up, as the proposed PLL method makes RDC become a system of type III, the conversion accuracy can be improved by the proposed PLL method compared with the typical PLL method.

5. Conclusion
In this paper, a PLL method based on improved PID controller has been designed to estimate the shaft angle and speed of a resolver. This method makes RDC become a system of type III, thus the conversion accuracy can be improved. Theoretical analysis and experimental results show that the proposed method has higher steady-state accuracy compared with the typical ones.

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