Vibration Reduction of a Stepping Motor
Using a Pre-compensator to Remove Control Delay

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To reduce vibration, overshoot, and tracking error of the rotor in an open-loop micro-step drive of a stepping motor, we propose a novel pre-compensator using a low-pass type filter without control delay. The pre-compensator consists of a low-pass filter that removes the natural frequency component causing the vibration and a zero-phase filter that removes the control delay. Finally, a position control experiment demonstrates the validity of the pre-compensator.

Keywords: stepping motor, vibration reduction, pre-compensator, Butterworth filter, zero-phase filter

1. Introduction

In a stepping motor, the rotor vibration appears even in the micro-step drive enabling it to rotate almost continuously. To reduce the vibration, low-pass filter type pre-compensators have been proposed (1)(2), which remove the natural frequency component of the motor causing the vibration from a reference angle.

The drawback of the low-pass filter type pre-compensators is control delay that is caused by phase delay. To overcome this drawback, methodologies such as a prediction control (1) and a type-2 control system using a proportional-integral controller (2) have been used to remove the delay. However, the prediction control (1) inherently has a delay. Further, in the other reference (2), the response at the final stop position has a large overshoot due to the integral term of the type-2 system.

In this paper, we propose a novel pre-compensator using a Butterworth low-pass filter and a filter cancelling the control delay caused by the Butterworth filter and the motor, which is based on a zero-phase filter (3). The pre-compensator can reduce the rotor vibration at the final stop position and theoretically track a constant-speed reference with no control delay. Furthermore, the result of a position control experiment for a ramp reference using the pre-compensator demonstrates the effectiveness.

2. Control System

The control system of a stepping motor uses a two-phase hybrid type stepping motor, and is controlled in open-loop.

2.1 Control Methodology

Figure 1 shows an open-loop microstepping position control system using the proposed pre-compensator. The pre-compensator calculates the modified reference angle \( \theta'_m \) for a given reference angle \( \theta_t \). The motor then drives according to \( \theta'_m \). \( \theta'_m \) is calculated in advance before driving as the system is controlled in open-loop.

The elements of this system are as follows. The motor \( G_m(s) \) is modelled as the second order system (1):

\[
G_m(s) = \frac{c(J_0 + J_L)}{s^2 + Ds/J_0 + J_L + c/J_0} \quad (1)
\]

where \( J_0 \) and \( J_L \) are moments of inertia with no load and an inertial load, respectively, \( D \) is the viscous friction coefficient, and \( c \) is the constant of the static maximum torque. From Eq. (1), the damping ratio \( \zeta \) and the natural frequency \( \omega_n \) are denoted as

\[
\zeta = D/(2 \sqrt{c(J + J_L)}), \quad \omega_n = \sqrt{c/(J + J_L)} \quad (2)
\]

A low-pass filter used in \( P_1(s) \) and \( P_2(s) \) is a second-order Butterworth low-pass filter \( P(s) \):

\[
P(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} \quad (3)
\]

where \( \omega_n \) is cut-off frequency. The difference between \( P_1(s) \) and \( P_2(s) \) is the cut-off frequency that is set in advance.

The pre-compensator, which is based on a zero-phase filter (3) using Butterworth low-pass filters, is implemented as follows:

(i) the time series of the given reference angle \( \theta_t(t) \) for \( 0 \leq t \leq \lambda \) in time \( t \), as observed in Fig. 1, is filtered using the low-pass filter \( P_1(s) \) set as the cut-off frequency \( \omega_c = \omega_n \). Thus, \( \theta_m(t) \) is obtained;

(ii) the order of \( \theta_m(t) \) is reversed to obtain \( \theta_m(\lambda - t) \);

(iii) the reverse time series \( \theta_m(\lambda - t) \) is filtered using the low-pass filter \( P_2(s) \) set as \( \omega_c = \omega_n \). \( \theta'_m(\lambda - t) \) is thus obtained;

(iv) the order of \( \theta'_m(\lambda - t) \) is reversed to obtain the forward...
time series. Thus, \( \theta'(t) \) is obtained.

Here, the motor drives at a constant-speed \( k \) and then comes to rest at an angle \( \theta_n \). The cut-off frequency \( \omega_c \) of \( P_2(s) \) is set to cancel the control delay caused by \( P_1(s) \) and the motor \( G_m(s) \). The motor then drives according to the modified reference angle \( \theta_0 \), which is the output of the pre-compensator for the reference angle \( \theta_0 \).

### 2.2 Cancelling Tracking Error

If the pre-compensator has only \( P_1(s) \), the drive using it has a tracking error for a ramp reference such as a constant-speed tracking; for \( \Theta_i(s) = 1/s^2 \), the tracking error \( E_{P_i} \) of \( P_1(s)G_m(s) \) is

\[
E_{P_i} = \lim_{s \to 0} s\Theta_i(s)[1 - P_1(s)G_m(s)] = 2\zeta/\omega_n + \sqrt{2}/\omega_c \tag{4}
\]

To eliminate this error, we use a filter to cancel the control delay based on a zero-phase filter \( \Theta_i \). The tracking error of only \( P_2(s) \) for a ramp input is

\[
E_{P_2} = \lim_{s \to 0} s\Theta_i(s)[1 - P_2(s)] = \sqrt{2}/\omega_c \tag{5}
\]

Because the time delay caused by filtering a reverse time series through \( P_2(s) \) is regarded as the time lead in terms of the forward time series, the lead \( E_{P_2} \) can cancel the delay \( E_{P_1} \). Hence, the cut-off frequency \( \omega_c \) of \( P_2(s) \) is set as

\[
\omega_c = \frac{\omega_{c_1}\omega_n}{\omega_n + \sqrt{2}/\omega_{c_1}} \tag{6}
\]

from \( E_{P_2} = E_{P_1} \). Thus, the control delay can be cancelled when \( \omega_c \) is set as shown in Eq. (6).

### 2.3 Pre-compensator Design

When no load is attached to the motor shaft, i.e., \( J_L = 0 \) Nms\(^2\)/rad, the parameters \( J_L, D \), and \( c \) are determined by identifying the step response of Eq. (1) with the actual step response \( \Theta_i \). When an inertial load \( J_L \) is attached to the motor shaft, \( \zeta \) and \( \omega_n \) are obtained from Eq. (2). The cut-off frequencies \( \omega_{c_1} \) and \( \omega_c \) are then set such that the maximum gain of the overall frequency transfer function \( |P_1(j\omega)P_2(j\omega)G_m(j\omega)| \), using the above parameters for all frequency \( \omega \), is no more than 0.82 dB. Here, the gain is chosen such that the overshoot at the final stop position is around 20% of the step angle of the motor. Although \( P_2(s) \) must filter a reverse time series, it is ignored for design simplicity.

### 3. Experiment

An Oriental Motor PK244-02B, which has a step angle of 1.8°, is used as the motor. An inertial load, which has an inertial moment of \( J_L = 10.0 \times 10^{-6} \) Nms\(^2\)/rad, is connected to the motor shaft. The model parameters of Eq. (1) are calculated to be \( c = 8.28 \) Nm/rad, \( D = 0.00141 \) Nms/rad and \( J_0 = 6.86 \times 10^{-6} \) Nms\(^2\)/rad. From Eq. (2) and the attached \( J_L, \zeta = 0.0598 \), and \( \omega_n = 701 \) rad/s are obtained. In the pre-compensator, when the cut-off frequency \( \omega_{c_1} \) of \( P_1(s) \) is set to \( \omega_{c_1} = 444 \) rad/s, \( \omega_c = 421 \) rad/s of \( P_2(s) \) is obtained. Here, the maximum gain of \( |P_1(j\omega)P_2(j\omega)G_m(j\omega)| \) is no more than 0.82 dB.

The motor drives such that the rotor comes to rest at a final stop position \( \theta_0 \) after driving a constant-speed \( k \) using the proposed pre-compensator, as shown in Fig. 1. For comparison, we carry out the drives that use a Butterworth filter \( \Theta_i \), type-2 pre-compensator \( \Theta_i \), and no pre-compensator. Here, the cut-off frequency of the Butterworth filter is set to \( \omega_c = 252 \) rad/s such that the maximum gain of the transfer function of the filter and motor is not more than 0.82 dB. The proportional-integral parameters of the type-2 pre-compensator are set to \( K_p = 390 \) rad/s and \( T_i = 0.0195 \) s/rad. Here, the maximum gain of the overall frequency transfer function of the type-2 pre-compensator and the motor is 12.5 dB, which is a large gain. However, if the gain becomes smaller by setting a smaller cut-off frequency, the response at \( \theta_0 \) has a larger overshoot, i.e., the control performance is worse. This is because of the above parameters.

Figure 2 demonstrated the result of the experiment with \( k = 1386 \) /s and \( \theta_0 = 360 \)° parameters. The response in the constant-speed \( k \) using the proposed pre-compensator shows quicker tracking of the reference angle at the start and smaller tracking error than those using the type-2 and no pre-compensator, whereas the response using the Butterworth filter has a larger error. When the rotor comes to rest at \( \theta_0 \), the response using the proposed pre-compensator exhibits the most suppressed overshoot and vibration. Thus, the results confirm that the drive using the proposed pre-compensator is the best in terms of tracking error in a constant-speed drive and the suppression of overshoot and vibration at a final stop position.

### 4. Conclusion

We proposed a pre-compensator using a low-pass type filter without control delay and a driving experiment has been conducted to demonstrate the effectiveness of the pre-compensator.

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