Subcritical Assembly AURES-01 for training purposes; Practical measurements of the effective multiplication factor and the material buckling

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Abstract
AURES-01 is an Algerian subcritical assembly that uses natural uranium as fuel and light water as moderator and neutron reflector, its effective multiplication factor is less than 1. For didactic purposes, such an assembly has certain advantages compared to a critical assembly: It is economical, easy to use, has a lower risk of accidents and low dose rates. This work presents the approach, the method and the results of two experiments, which we realized on the subcritical assembly AURES-01. The first experiment is mainly to determine the effective multiplication factor $k_{\text{eff}}$ of the subcritical assembly AURES-01. The method consists of gradually charging the medium by fuel elements and counting at each step, the value of the $k_{\text{eff}}$ is deduced by extrapolation [1, 2, 4].

The purpose of the second experiment is to determine the vertical material buckling of subcritical assembly AURES-01. The method is based on measuring the vertical distribution of the neutron flux in different channels. A source of neutrons is used for both experiments.

Keywords: Subcritical, effective multiplication factor, Material buckling, $^3$He Counter.

1. Introduction
For the purposes of training in the field of nuclear engineering, AURES-01 is a subcritical assembly with fewer risks and constraints than a reactor is widely recommended [1, 2, 3].

In this framework and to study the physical characteristics of the subcritical assembly AURES-01, two experiments were conducted. The purpose of the first experiment is to determine the effective multiplication factor of the network. The approach consists of gradually loading the medium into fuel elements and counting at each step, the value of the $k_{\text{eff}}$ is deduced by extrapolation [1, 2, 4].

The purpose of the second experiment is to determine the vertical material buckling of the network. The method is based on measuring the vertical distribution of the neutron flux in different channels [1].
2. Experiment 1: Practical measurement of the effective multiplication factor $k_{\text{eff}}$

2.1. Theoretical aspect
The method presented here consists in studying the decay of the neutron population after a source impulse. If a neutron source is induced in a subcritical multiplier medium we will have a flux distribution due to the multiplication of neutrons within this medium. The multiplication rate $M$ is defined as the ratio of the flux density due to fission to that due to the source [1, 2, 5].

$$M = \frac{I}{I_0}$$  \hspace{1cm} (1)

Where $I$ is the average number of counts after each step; $I_0$ is the number of counts of the neutron source alone. This expression can be written as:

$$M = \frac{S + S k_{\text{eff}} + S k_{\text{eff}}^2 + \ldots + S k_{\text{eff}}^n}{S}$$  \hspace{1cm} (2)

Where $S$ is the neutron source term, the expression (2) become:

$$M = 1 + k_{\text{eff}} + k_{\text{eff}}^2 + \ldots + k_{\text{eff}}^n$$  \hspace{1cm} (3)

The previous expression is a geometric series whose sum is:

$$M = \frac{1}{1 - k_{\text{eff}}}$$  \hspace{1cm} (4)

$$k_{\text{eff}} = 1 - \frac{1}{M}$$  \hspace{1cm} (5)

Using expression (1), we obtain the expression linking between the multiplication factor and the numbers of counts.

$$k_{\text{eff}} = 1 - \frac{I_0}{I}$$  \hspace{1cm} (6)

2.2. Experimental aspect
2.2.1. Equipment used
In this experience we used the subcritical assembly AURES-01 (figure 1), a $Pu-Be (\alpha, n)$ neutron source, fuel elements made of natural uranium and a neutron measurement system (consisting of a previously calibrated $^3He$ proportional counter, a preamplifier, a high voltage, an amplifier, an SCA single channel analyzer and a counting scale).

2.2.2. Manipulation
(1) Neutron source is set to position 1 (figure 1).
(2) The neutron detector is set to position 2 of the horizontal plane, its vertical position was chosen so as to have a maximum number of counts.
(3) The medium is progressively charged with fuel and at each load the counting rate is recorded.
(4) The positions of the detector and the source are changed to have other data during unloading the fuel elements.
(5) The neutron source has been set at position 2, the neutron detector at position 3.
(6) The fuel elements are progressively discharged and the counting rate is recorded at each stage.
2.2.3. Results

The variation of the multiplication factor $k_{\text{eff}}$ as a function of the inverse of the number of fuel elements is given in figure 2.

The linear extrapolation gave the following values of the $k_{\text{eff}}$ corresponding to an infinite number of fuel elements $(1/N)$ tends to 0:
- $k_{eff} = 0.84702$ for the first positions of the source and the detector (for the fuel loading phase),
- $k_{eff} = 0.88018$ for the second positions of the source and the detector (for the fuel unloading phase).

Taking the average value, we obtain the effective multiplication factor of the network AURES-01, $k_{eff} = 0.8636$

3. Experiment 2: Practical measurement of material buckling

3.1. Theoretical aspect

The concept of buckling is used to describe the relationship between requirements on fissile material inside a reactor core and dimensions and shape of that core. Geometrical buckling is a measure of neutron leakage, while material buckling is a measure of neutron production minus absorption. With this terminology the criticality condition may also be stated as the material and geometric buckling being equal.

Considering an infinite parallelepiped multiplier (large enough) of dimensions $a$, $b$ and $c$ containing a point source of neutrons $S$ located in the center that we take as the origin of the coordinates. The parallelepiped material is homogeneous and uniform. If we use the method of diffusion to a group, we obtain the equation of the flux $\nabla^2 \Phi + B^2 \Phi = 0$ [5-7], so:

$$B^2 = - \frac{\nabla^2 \Phi}{\Phi} \tag{7}$$

Where $B^2$ is called material buckling, it describes the characteristics of the fuel material in an infinite medium [4].

$$B^2 = \frac{k_{in}}{D} \Sigma_a \tag{8}$$

Where $k_{in}$ is the infinite neutron multiplication factor, $D$ is the diffusion factor and $\Sigma_a$ is the macroscopic cross section for absorption of the medium.

$$B^2 = B_x^2 + B_y^2 - B_z^2 \tag{9}$$

$B_x^2$, $B_y^2$, $B_z^2$: The material buckling of the medium follows the directions $x$, $y$ and $z$ respectively. The separation of the variables makes it possible to write [6]:

$$\Phi(x,y,z) = X(x)Y(y)Z(z) \tag{10}$$

$$\frac{d^2X}{dx^2} = -B_x^2 X \tag{11}$$

$$\frac{d^2Y}{dy^2} = -B_y^2 Y$$

$$\frac{d^2Z}{dz^2} = \gamma^2 Z$$

In this experience we were interested by the buckling in the vertical direction $\gamma^2$. Boundary conditions require the neutron flux to vanish at $x = \pm \frac{a}{2}$, $y = \pm \frac{b}{2}$ and $z = \infty$ ($z$ is big enough). The solution for the flux along the vertical axis is therefore [6]:
\[ \Phi(z) = C \cdot \exp(-\gamma z) \] (12)

\[ \ln \Phi(z) = \ln \Phi(0) - \gamma z \] (13)

To determine the value of \( \gamma \) we plot the variation of \( \ln \Phi(z) \) as a function of \( z \). \( -\gamma \) is the slope of the line found [6, 7].

3.2. Experimental aspect

3.2.1. Manipulation

The same equipment as the first experience is used. The procedure of this experiment is as follows:

1. \( Pu-Be \) neutron source has been placed in position \( S \) (figure 3), this position is considered as origin of the coordinates \((x; y; z)\), so we have \( S = (0; 0; 0) \).
2. The first 6 rows were loaded into fuel elements (167 fuel elements in total).
3. The neutron detector was set firstly to channel \( A \), at the position \((33.6; 33.6; 0)\) cm, (same level as the neutron source) and counted.
4. Change the position of the neutron detector vertically in the same channel in steps of 5 cm. We counted for the positions of \( z = 5 \) cm to \( z = 35 \) cm (below the level of the neutron source), and for the positions of \( z = 5 \) cm to \( z = 35 \) cm (above the level of the neutron source).
5. The neutron detector was placed in another channel \( B \), at the position \((30.8; 33.6; 0)\) cm. We counted.
6. The operations mentioned in Step 4 have been redone.
7. The neutron detector was placed in another channel \( C \), at the position \((0; 64.4; 0)\) cm. We counted.
8. The operations mentioned in step 4 have been redone.

Considering the sensitivity of our detector which is \( s = 1.8 \), we deduce the neutron flux \( \Phi \) for each count measurement \( I \), so \( \Phi = I / s \) [6, 7].

Figure 3. Neutron flux measurement positions according to the \( XY \) plane (left), with example of case of detector position in channel \( A \) according to the \( XZ \) plane.
3.2.2. Results
The variation of \( \ln \Phi \) as a function of the vertical distance \( z \) between the source and the detector for the 3 channels (below and above the level of the source) is given in the figures 4 to 6. For each curve the line and the slope were determined.

Figure 4. Variation of the neutron flux as a function of the distance from the plane of the neutron source. (a) Case of channel A below level of the source \( (\gamma=0.04133) \); (b) case of channel A above level of the source \( (\gamma=0.04902) \).

Figure 5. Variation of the neutron flux as a function of the distance from the plane of the neutron source. (a) Case of channel B below level of the source \( (\gamma=0.04176) \); (b) case of channel B above level of the source \( (\gamma=0.05045) \).
Figure 6. Variation of the neutron flux as a function of the distance from the plane of the neutron source. (a) Case of channel C below level of the source (\( \gamma = 0.03221 \)); (b) case of channel C above level of the source (\( \gamma = 0.03690 \)).

Taking the average value, we obtain the material buckling for the network of AURES-01: \( \gamma^* = 0.0018 \)

4. Conclusion
The value of the \( k_{ef} \) found in this work is close to that found previously experimentally [8-10], and by theoretical simulation using the WIMS and CITATION and others codes which is about 0.84 [3, 8, 10-13]. The vertical material buckling was determined using three channels so only three horizontal positions (\( x, y \)), it is recommended to use a higher number of points in order to refine the results. Similarly, for a number of fuel elements.

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