Excitation of $\Delta$ and $N^*$ resonances in isobaric charge-exchange reactions of heavy nuclei

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Abstract. We present a model for the study of the excitation of $\Delta(1232)$ and $N^*(1440)$ resonances in isobaric charge-exchange ($^{A+1}Z,^{A+1}(Z \pm 1)$) reactions of heavy nuclei. Quasi-elastic and inelastic elementary processes contributing to the double differential cross sections of the reactions are described in terms of the exchange of virtual pions. The inelastic channel includes processes where the resonances are excited both in the target and in the projectile nucleus. We present results for reactions of $^{112}$Sn and $^{124}$Sn on different targets. Our results confirm that the position of the $\Delta$ peak is insensitive to targets with mass number $A \geq 12$, and show that the origin of the $\Delta$ peak shift towards low excitation energies, with respect to its position in reactions with a proton target, can be easily explained in terms of the superposition of the different excitation mechanisms contributing to the reaction.

1 Introduction

Determining the in-medium properties of nucleon resonances is essential for a better understanding of the underlying dynamics governing many nuclear reactions. Isobaric charge-exchange reactions are particularly interesting because, at low energy, they allow the investigation of spin-isospin nuclear excitations (Gamow–Teller, spin-dipole, spin-quadrupole, ...) and, at medium and high energies, the excitation of nucleon resonances such as e.g., the $\Delta(1232)$ isobar or the Roper $N^*(1440)$. In addition, they can provide valuable information on the radial distribution of neutrons and protons in nuclei. All this makes isobaric charge-exchange reactions very promising tools for the study of the spin-isospin dependence of the nuclear force.

The excitation of the $\Delta(1232)$ in $(p,n)$ and $(^3\text{He},t)$ reactions was extensively studied in the 1980’s [1–5]. A experimental program completely devoted to study the excitation of this resonance in reactions with light and medium mass projectiles was performed at the SATURNE accelerator in Saclay during those years [6]. Those experiments showed that the position of the $\Delta$ peak was shifted towards low excitation energies by $\sim 70$ MeV when using a target with mass number $A \geq 10$ as compared to its position in reactions with a proton target. The origin of this shift has been long discussed although there is not a general consensus yet (see e.g., Refs. [7–10] and references therein).

Recent experiments performed at GSI with the FRagment Separator (FRS) using stable and unstable Sn projectiles with different targets [11, 12] has renewed the interest on the excitation of nucleon resonances in these reactions. The use, in these experiments, of relativistic nuclei far off stability allows to explore the isospin degree of freedom enlarging in this way our present knowledge of the properties isospin-rich nuclear systems.

In this paper we present a model to study the excitation of $\Delta(1232)$ and $N^*(1440)$ resonances in isobaric charge-exchange reactions of heavy nuclei such as the ones performed at GSI. The description of the model is made in Sec. 2. Results for reactions of $^{112}$Sn and $^{124}$Sn on different targets are shown and discussed in Sec. 3. Finally, our a summary and the main conclusions are given in Sec. 4.

2 Model for the $(^A Z, ^A(Z + 1))$ reaction

The model includes the contribution to the double differential cross sections of the $(^A Z, ^A(Z \pm 1))$ reaction of quasielastic (Fig. 1a) as well as inelastic (Figs. 1b and 1c) elementary processes which are described only in terms of the exchange of a virtual pion between the interacting nucleons. The contribution of the $\rho$ and other heavy mesons is neglected here, although it will be considered in a near future. Therefore, the basic ingredients of the model are the $NN\pi$, $N\Delta\pi$ and the $NN^*\pi$ vertices [13]. The excitation of the resonances is considered both in the target and in the projectile nucleus.

Without losing any generality, let us assume (see Fig. 1) that $N_1 (N_2)$ is one of the nucleons of the projectile (ejectile) nucleus whereas $N_2 (N_1)$ is a nucleon of the target (recoil) nucleus. The double differential cross section...
of the \((^4Z, ^4(Z \pm 1))\) reaction is calculated in our model as
\[
\frac{d^2\sigma}{dE d\Omega}[^4Z, ^4(Z \pm 1)] = \sum_{N_1=N_p} \sum_{N_2} \frac{d^2\sigma}{dE d\Omega} N_{N_1 N_2},
\]
where \(\left(\frac{d^2\sigma}{dE d\Omega}\right)\) denotes the double differential cross section of the quasi-elastic (qe) or the inelastic (qe) elementary process, and \(N_{N_1 N_2}\) accounts for the effective number of such elementary processes contributing to the \((^4Z, ^4(Z \pm 1))\) reaction. In the following we describe briefly these two pieces of the model.

### 2.1 Elementary cross sections

The double differential differential cross section of the elementary quasi-elastic process \(N_2(N_1, N_3)N_4\) shown in Fig. 1a is given by
\[
\left(\frac{d^2\sigma}{dE d\Omega}\right)_{qe} = \frac{1}{S} \frac{2|p_\Sigma|}{(2\pi)^3} \frac{m^4}{\Lambda^{1/2}(s, m^2, m^2)} \times \frac{1}{E_4} \sum \sum |M|^2 \delta(E_i - E_f),
\]
where \(s\) is the total energy in the center-of-mass frame, \(m\) is the nucleon mass, \(E_i = E_1 + E_2\), \(E_f = E_3 + E_4\), \(\sum \sum\) indicates the average and sum over the initial and final spins, and \(\Lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc\) is the so-called Källen function. Finally, \(S\) is the symmetry factor,
\[
S = \prod_i k_i!
\]
for \(k_i\) identical particles of species \(l\) in the final state. In our case \(S\) can be 1 or 2 depending on the particular reaction channel.

The scattering amplitude \(M\) describing quasi-elastic nucleon particle-hole excitations is calculated in the Born approximation and reads
\[
M = \left(\frac{f_{NN\pi}}{m_\Sigma}\right)^2 F_{NN\pi}(\vec{q})^2 \times \left(\frac{s_1 s_2}{\pi}\right) D_\Sigma(\vec{q}) + g' \left(s_3 s_4 s_1 s_2\right) \left(t_1 t_2\right)^2 |t_1 t_2|, \]
where \(\bar{t} (\vec{t})\) are the Pauli matrices connecting spin (isospin) \(s_N = \pm \frac{1}{2} (t_N = \pm \frac{1}{2})\) states, \(f_{NN\pi} = 1.008\) is the \(NN\pi\) coupling constant, and a monopolar form factor,
\[
F_{NN\pi}(\vec{q}) = \frac{\Lambda_{NN\pi}^2 - m_\Sigma^2}{\Lambda_{NN\pi}^2 + |\vec{q}|^2},
\]
has been introduced in each vertex with a cut-off \(\Lambda_{NN\pi} = 1300\) MeV. The first term in Eq. (4) corresponds to the one-pion-exchange contribution while the second takes into account the effect of short-range correlations by means of the Landau-Migdal parameter \(\Lambda'\) taken here in the range \(0.7 - 0.8\). \(D_\Sigma(\vec{q})\) is the pion propagator taken in the non-relativistic limit
\[
D_\Sigma(\vec{q}) = \frac{1}{|\vec{q}|^2 + m_\Sigma^2}.
\]

The double differential differential cross section for the elementary inelastic reactions \(N_2(N_1, N_3)N_4\) and \(N_2(N_1, N_3, N_4)\) shown in Fig. 1b can be calculated in the Born approximation by
\[
M = \left(\frac{f_{NN\pi}}{m_\Sigma}\right)^2 F_{NN\pi}(\vec{q}) F_{NN\pi}(\vec{q}) F_{NN\pi}(\vec{q}) \times D_\Sigma(\vec{q}) D_\Sigma(\vec{q}) D_\Sigma(\vec{q}) C_R, \]
where \(\sqrt{T_f} = \sqrt{p_f + p_i - p_f}^2\) is the invariant mass of the resonance \((R = \Delta, N')\), \(f_{NN\pi}(\vec{q}) = 2.156(0.477)\), \(F_{NN\pi}(\vec{q})\) is a monopolar form factor as the one of Eq. (5) with \(\Lambda_{NN\pi}\) replaced by \(\Lambda_{NN\pi} = 650(1300)\) MeV, and \(D_\Sigma(\sqrt{T_f})\) is the resonance propagator taken also in the non-relativistic limit
\[
D_\Sigma(\sqrt{T_f}) = \frac{1}{\sqrt{T_f} - m_R + \frac{1}{2}(\sqrt{T_f})}.\]
Table 1. Elementary inelastic $N_1(N_1, N_1)N_1$ and $N_2(N_1, N_1)N_1$ processes contributing to the $(^4Z, ^4(Z + 1))$ reactions. In brackets it is given the corresponding isospin factor of each process.

| $(^4Z, ^4(Z + 1))$ reaction | Projectile excitation |
|------------------------------|------------------------|
| $p(n, p)\Delta^0 = p(n, p)n\pi^0$ | $[2/3]$ |
| $p(n, p)\Delta^+ = p(n, p)n\pi^+$ | $[\sqrt{2}/3]$ |
| $n(n, p)\Delta^+ = n(n, p)n\pi^+$ | $[-2/3]$ |
| $p(n, p)P_{11}^0 = p(n, p)n\pi^0$ | $[-2] |
| $p(n, p)P_{11}^+ = p(n, p)n\pi^+$ | $[2\sqrt{2}]$ |
| $n(n, p)P_{11}^0 = n(n, p)n\pi^0$ | $[2]$ |
| $n(n, p)P_{11}^+ = n(n, p)n\pi^+$ | $[\sqrt{2}]$ |

| $(^4Z, ^4(Z - 1))$ reaction | Projectile excitation |
|------------------------------|------------------------|
| $p(n, n)\Delta^+ = p(n, n)n\pi^+$ | $[\sqrt{2}]$ |
| $n(n, n)\Delta^+ = n(n, n)n\pi^+$ | $[\sqrt{2}/3]$ |
| $n(n, n)\Delta^0 = n(n, n)n\pi^0$ | $[2/3]$ |
| $n(n, n)P_{11}^0 = n(n, n)n\pi^0$ | $[-2\sqrt{2}]$ |
| $n(n, n)P_{11}^+ = n(n, n)n\pi^+$ | $[2]$ |
| $n(n, n)P_{11}^0 = n(n, n)n\pi^0$ | $[\sqrt{2}]$ |

Here $m_R$ is the resonance rest mass, and the energy dependence of its width is given by [14]

$$\Gamma(\sqrt{s_T}) = \Gamma(m_R)\left(\frac{p_{\pi,n,m}(\sqrt{s_T})}{p_{\pi,n,m}(m_R)}\right)^3, \quad (10)$$

with $\Gamma(m_A) = 120$ MeV for the $\Delta$ and $\Gamma(m_{N^*}) = 350$ MeV for the $N^*$, and $p_{\pi,n,m}$ the momentum of the emitted pion measured in the center-of-mass frame of the $\pi N$ system.

Finally, $C_R$ is an spin-isospin factor that reads

$$C_A = \langle s_s s_4|\bar{\sigma}_1 \cdot \bar{q}_2 \cdot \bar{q}_1 s_2|s_4|\bar{S} \cdot \bar{p}_n|s_A\rangle \times \langle t_3 s_{A4}|T_3^+ \cdot \bar{T}_2|t_{12}\rangle<\bar{T}_3 \cdot \bar{\phi}(\Delta) \rangle \quad (11)$$

in the case of the $\Delta$ excitation, with $\bar{S}$ ($\bar{T}$) spin (isospin) $1/2 \to 3/2$ transition operators [13], and

$$C_{N^*} = \langle s_s s_4|\bar{\sigma}_1 \cdot \bar{q}_2 \cdot \bar{q}_1 s_2|s_4|\bar{S} \cdot \bar{p}_n|s_{N^*}\rangle \times \langle t_3 t_{N^*}|T_3^+ \cdot \bar{T}_2|t_{12}\rangle<\bar{T}_3 \cdot \bar{\phi}(N^*) \rangle \quad (12)$$

in the case of the $N^*$.

On the other hand, if the excitation of the resonances happens in the projectile nucleus then the scattering amplitude $M$ needed is that of the process $N_2(N_1, N_1)N_1N_1$ shown in Fig. 1c, whose expression is formally equal to that of Eq. (8) with the change of the invariant mass $\sqrt{s_T}$ by $\sqrt{s_T} = \sqrt{(p_3 + p_\pi)^2}$ and the spin-isospin factors $C_A$ and $C_{N^*}$, respectively, by

$$\tilde{C}_A = \langle s_s s_4|\bar{\sigma}_1 \cdot \bar{q}_2 \cdot \bar{q}_1 s_2|s_4|\bar{S} \cdot \bar{p}_n|s_A\rangle \times \langle t_3 t_{A4}|T_3^+ \cdot \bar{T}_2|t_{12}\rangle<\bar{T}_3 \cdot \bar{\phi}(\Delta) \rangle \quad (13)$$

and

$$\tilde{C}_{N^*} = \langle s_s s_4|\bar{\sigma}_1 \cdot \bar{q}_2 \cdot \bar{q}_1 s_2|s_4|\bar{S} \cdot \bar{p}_n|s_{N^*}\rangle \times \langle t_3 t_{N^*}|\bar{T}_3^+ \cdot \bar{T}_2|t_{12}\rangle<\bar{T}_3 \cdot \bar{\phi}(N^*) \rangle \quad (14).$$

The complete list of all the elementary inelastic processes $N_2(N_1, N_3)N_1\pi$ and $N_2(N_1, N_3)N_1$ contributing to the $(^4Z, ^4(Z \pm 1))$ reactions is shown in Table 1 together with their corresponding isospin factors (last two terms in Eqs. (11)–(14)).

### 2.2 Effective number of elementary processes

The effective number of elementary process contributing to the $(^4Z, ^4(Z \pm 1))$ reaction is given by

$$N_{N_1N_2} = \int d^3b p_{\text{overlap}}^{N_1N_2}(b) \left|1 - T(b)\right| p_{\theta}(b). \quad (15)$$

In this expression,

$$p_{\text{overlap}}^{N_1N_2}(b) = \int dz \int d^3\tilde{r} p_{\theta}^{N_1N_2}(\tilde{r})p_{\theta}(\tilde{b} + \tilde{z} + \tilde{r}) \quad (16)$$
is the density of the overlap region between the projectile and the target nucleus at the impact parameter \( b \), with \( \rho_p(\rho_T) \) the density profile of the nucleons of type \( N_1(N_2) \) in the projectile (target).

The quantity \( 1 - T(b) \) in Eq. (15) is the so-called transmission function and gives the probability that the projectile undergoes a collision with the target at the impact parameter \( b \) where \( T(b) \) is defined as

\[
T(b) = \exp \left[ - \int d\vec{r} \int d\vec{r} \sigma_{NN} \rho_p(\vec{r}) \rho_T(\vec{b} + \vec{z} + \vec{r}) \right]
\] (17)

with \( \sigma_{NN} \) being the nucleon-nucleon cross section, and \( \rho_p \) and \( \rho_T \) the projectile and target total density profiles, respectively. In Fig. 2a, we show \( 1 - T(b) \) as a function of the impact parameter \( b \) for the reaction of \( 112\text{Sn} \) and \( 124\text{Sn} \) projectiles on several targets. The calculation of \( 1 - T(b) \) has been done using density distributions obtained within a relativistic mean field model with the FSU parametrization of Todd-Rutel and Piekarewicz [15]. The nucleon-nucleon cross section has been taken here \( \sigma_{NN} = 40 \text{ mb} \).

Finally, the function \( P_p(b) \) in Eq. (15) is the pion survival probability that accounts for the probability that the pion produced in an inelastic process is not absorbed. It is given by

\[
P_p(b) = \exp \left[ - \int d\vec{r} \int d\vec{r} \sigma_{NN} \rho_p(\vec{r}) \rho_T(\vec{b} + \vec{z} + \vec{r}) \right]
\] (18)

with \( \sigma_{NN} \) the pion-nucleon absorption cross section. We note that \( P_p(b) \) is taken equal to 1 when evaluating the contribution of the quasi-elastic process since no pion is produced in this case. The pion survival probability is shown in Fig. 2b as a function of the impact parameter \( b \) for the same reactions of \( 112\text{Sn} \) and \( 124\text{Sn} \) projectiles on different targets. The same density distributions have been used here. For \( \sigma_{NN} \) we have taken an average value of 200 mb in the region of the \( \Delta \) peak [16].

We note that the product \( [1 - T(b)] P_p(b) \) can in fact be interpreted as the probability of the reaction \( (A,Z) \rightarrow (A,Z+1) \). We show it in Fig. 3 for the same set of reactions with \( 112\text{Sn} \) and \( 124\text{Sn} \) projectiles. As it can be seen, the probability is peak at intermediate impact parameters being very small at low and large values of \( b \). The reason is that at low \( b \) pions are strongly absorbed (see Fig. 2b) because the overlap between the projectile and the target is large, whereas at large \( b \) this overlap is small making \( 1 - T(b) \) negligible (see Fig. 2a). This shows, therefore, that the reaction \( (A,Z) \rightarrow (A,Z+1) \) is clearly peripheral and that only the nucleons sitting on both the projectile and target surfaces effectively contribute to it. We show in Table 2 the effective number of elementary processes contributing to the reaction of \( 112\text{Sn} \) and \( 124\text{Sn} \) projectiles on several targets. The total effective number of elementary processes, defined as \( N_{\text{total}} = \sum_{N_1,N_2} N_{N_1,N_2} \), is shown in the last column.

### Table 2. Effective number of elementary processes \( N_{N_1,N_2} \) contributing to the reaction of \( 112\text{Sn} \) and \( 124\text{Sn} \) on different targets. The total effective number of elementary processes \( N_{\text{total}} \) is shown in the last column.

| Projectile | Target | \( N_{pp} \) | \( N_{pn} \) | \( N_{np} \) | \( N_{nn} \) | \( N_{\text{total}} \) |
|------------|--------|--------------|--------------|--------------|--------------|------------------|
| \( 112\text{Sn} \) | \( p \) | 0.006 | 0 | 0.011 | 0 | 0.017 |
| | \( ^{12}\text{C} \) | 0.003 | 0.003 | 0.007 | 0.006 | 0.019 |
| | \( ^{63}\text{Cu} \) | 0.003 | 0.004 | 0.006 | 0.009 | 0.022 |
| | \( ^{208}\text{Pb} \) | 0.001 | 0.007 | 0.004 | 0.015 | 0.027 |
| \( 124\text{Sn} \) | \( p \) | 0.004 | 0 | 0.015 | 0 | 0.019 |
| | \( ^{12}\text{C} \) | 0.002 | 0.002 | 0.01 | 0.009 | 0.023 |
| | \( ^{63}\text{Cu} \) | 0.001 | 0.002 | 0.009 | 0.01 | 0.022 |
| | \( ^{208}\text{Pb} \) | 0.0006 | 0.003 | 0.005 | 0.02 | 0.029 |

![Figure 2](image-url)  
**Figure 2.** (Color online) Transmission function (panel a) and pion survival probability (panel b) as a function of the impact parameter \( b \) for the reaction of \( 112\text{Sn} \) and \( 124\text{Sn} \) projectiles on several targets.

![Figure 3](image-url)  
**Figure 3.** (Color online) Probability of the reaction \( (A,Z) \rightarrow (A,Z+1) \) as a function of the impact parameter \( b \) for the reaction of \( 112\text{Sn} \) and \( 124\text{Sn} \) projectiles on several targets.
uum. Note that $N_{\text{data}}$ is very similar in all the cases. This seems to indicate that this number is quite insensitive to the global isospin asymmetry of the system projectile-target, and that in fact, it rather reflects the isospin content of the overlap region of the tails of the projectile and target densities. Consequently, isobaric charge-exchange reactions are probably a very good tool to explore the properties of isospin asymmetric nuclear matter and, in particular, the density dependence of the nuclear symmetry energy in the sub-saturation density region.

We want to finish this section by pointing out that, in principle, in-medium $\sigma_{NN}$ and $\sigma_{\pi N}$ cross sections should be used when evaluating the $N_{N,N}$. However, due to the peripheral character of the reaction, the densities explored are small and, therefore, density effects on $\sigma_{NN}$ and $\sigma_{\pi N}$ are expected to be negligible.

3 Results and discussion

Before we start our analysis of the isobar charge-exchange reactions with heavy nuclei, let us focus for a while on a more simple reaction which will allow us to understand later the role of the excitation mechanisms of the resonances in the target (Fig. 1b) and in the projectile (Fig. 1c). This reaction is $pp \rightarrow np\pi^+$. The double differential cross section for this reaction is shown in Fig. 4 as a function of the missing energy $E_n - E_p$ for a kinetic energy of the incident proton of 800 MeV and a scattering angle of zero degrees. Experimental results are taken from Ref. [17]. Three elementary processes contribute to the total double differential cross section of this reaction, the process $p(p, n)\Delta^{++}$ where a $\Delta^{++}$ is excited in the target and then it decays into $pn\pi^+$, and the processes $p(p, \Delta^+)n$ and $p(p, P_{11}^+)n$ in which a $\Delta^+$ or the charge state $P_{11}^+$ of the $N^*$ are excited in the projectile and after they decay in $n\pi^+$. The total double differential cross section of the reaction is clearly dominated by the excitation of the $\Delta^{++}$ in the target due its large isospin factor (see Table 1), although the contribution of the three processes is necessary to reproduce the experimental data. Note that the shape, the position and the width of the $\Delta$ resonance is different if it is excited in the target or in the projectile. This is because its invariant mass is different in both cases (see the definition of $\sqrt{s_{\pi}}$ and $\sqrt{s_T}$ above). When the $\Delta$ is excited on the target, its invariant mass $\sqrt{s_T}$ does not depend on the momentum of the emitted pion and, therefore, the scattering amplitude can be taken out of the integral in Eq. (7). In this case, the $\Delta$ peaks at a value of the missing energy $E_n - E_p \sim m_n - m_{\Delta}$, and its widths is $\Gamma(\sqrt{s_T}) \sim \Gamma(m_{\Delta})$. On the contrary, when the $\Delta$ is excited in the projectile, its invariant mass $\sqrt{s_{\pi}}$ depends explicitly on the momentum of the emitted pion and, therefore, the scattering amplitude should be also integrated. Consequently, the shape of the $\Delta$ becomes more asymmetric, its position is shifted to values of the missing energy smaller than $m_n - m_{\Delta}$ and its width increases. Similar conclusions can be drawn for the $N^*$ from the analysis of reactions where it can be also excited in the projectile and the target.

Let us now analyze the isobar charge-exchange reactions of $^{112}$Sn and $^{124}$Sb projectiles with different targets. The double differential cross sections for the reactions ($^{112}$Sn,$^{112}$In) and ($^{112}$Sn,$^{124}$Sb) are shown in Fig. 5, whereas those for the reactions ($^{124}$Sn,$^{124}$In) and ($^{124}$Sn,$^{124}$Sb) are plot in Fig. 6. The separate contributions of the quasi-elastic and the inelastic channels, with the resonances excited either in the target or in the projectile, are also shown. Results are presented for an incident kinetic energy of 1000 MeV per nucleon and a scattering angle of zero degrees as a function of the missing energy. We have taken the values of $\sigma_{NN} = 40$ mb and $\sigma_{\pi N} = 200$ mb and used the density distributions of the FSU model [15] for the evaluation the effective number of

![Figure 4](image-url) Figure 4. (Color online) Double differential cross section for the reaction $pp \rightarrow np\pi^+$ at 800 MeV and zero degrees as a function of the missing energy $E_n - E_p$. Experimental results are taken from Ref. [17].

![Figure 5](image-url) Figure 5. (Color online) Double differential cross section for the reactions ($^{112}$Sn,$^{112}$In) (left panels) and ($^{112}$Sn,$^{124}$Sb) (right panels) on different targets at 1000 MeV per nucleon and zero degrees as a function of the missing energy. The separate contributions of the quasi-elastic and the inelastic channels, with the resonances excited either in the target or in the projectile, are also shown.
elementary processes contributing to these reactions (see Eqs. (15)-(18). We note first that the size of the cross sections is similar for both projectiles. This is a consequence of the peripheral character of the reaction which makes it to be sensitive only to the overlap of the tails of the target and projectile density distributions. We observe also that the cross sections of the reactions $^{112}\text{Sn},^{124}\text{In}$ and $^{124}\text{Sn},^{124}\text{In}$ are slightly smaller, respectively, than those of the reactions $^{112}\text{Sn},^{112}\text{Sb}$ and $^{124}\text{Sn},^{124}\text{Sb}$. This can be easily understood by looking at Table 2. The effective number of elementary processes contributing to the reactions $^{112}\text{Sn},^{112}\text{In}$ and $^{124}\text{Sn},^{124}\text{In}$, $N_{pp} + N_{pn}$, is smaller than the number, $N_{pp} + N_{pn}$, of processes effectively contributing to the reactions $^{112}\text{Sn},^{112}\text{Sb}$ and $^{124}\text{Sn},^{124}\text{Sb}$. The position of the $\Delta$ peak is shifted towards a low excitation energy by $\sim 65$ MeV, becoming insensitive to targets with mass number $A \geq 12$. As said in the introduction, this shift was already observed on isobaric charge-exchange reactions with lighter nuclei [1-5], and its origin was the subject of intense debate (see e.g., Refs. [7-10] and references therein). The peripheral character of the $(^3\text{He},^4\text{A}(Z \pm 1))$ reaction explains in a natural way the insensitivity of the peak position to the target mass since the only participating nucleons are those in the overlap region of the projectile and target tails (see discussion in Sec. 2.2). As we have said before, the shape, position and width of a resonance depends on the excitation mechanism. Therefore, the shift of the $\Delta$ peak, with respect to its position in reactions with a proton target, can be easily explained as a result of the superposition of the different excitation mechanisms contributing to the reaction. This conclusion was already pointed out by Oset, Shiino and Toki [8] in their analysis of the $(^3\text{He},t)$ reaction on proton, deuteron and $^{12}\text{C}$ targets.

4 Summary and conclusions

In this work we have studied the excitation of $\Delta(1232)$ and $N^*(1440)$ resonances in isobaric charge-exchange reactions of heavy nuclei. The double differential cross sections of the reactions include the contribution of quasi-elastic and inelastic elementary processes which are described in terms of the exchange of virtual pions. The inelastic channel includes processes where the resonances are excited both in the target and in the projectile. Our results confirm that the position of the $\Delta$ peak is insensitive to targets with mass number $A \geq 12$, and show that the origin of its shift towards low excitation energies, with respect to its position in reactions with a proton target, can be easily explained in terms of the superposition of the different excitation mechanisms contributing to the reaction. We want to point out that our model is being used in the analysis and interpretation of the results of several isobaric charge-exchange inclusive reactions recently performed with the FRS at GSI using stable and unstable Sn projectiles on different targets [11, 12]. The results of this analysis are still under evaluation and, therefore, they have not been discussed here.

To finish, we would like to stress that isobaric charge-exchange reactions are sensitive to the isospin content of the overlap region of the projectile and target tails making these reactions a very good tool to explore the properties of isospin asymmetric nuclear systems and, in particular, the density dependence of the nuclear symmetry energy in the subsaturation density region. Future exclusive measurements are very much awaited since they would allow the identification of the different reaction mechanisms giving us access to this valuable information.

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