Flavoured Leptogenesis

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ABSTRACT

Thermal leptogenesis, in the seesaw model, is a popular mechanism for generating the Baryon Asymmetry of the Universe. It was noticed recently, that including lepton flavour can modify significantly the results. These proceedings aim to discuss why and when flavour matters, in the thermal leptogenesis scenario for hierarchical right-handed neutrinos. No Boltzmann Equations are introduced.

1. The Baryon Asymmetry of the Universe

The Standard Model of particle physics is extraordinarily successful. However, among the few observations it cannot explain, is the Baryon Asymmetry of the Universe: it is observed that locally, in our patch of the Universe, there is an excess of visible matter (baryons) over anti-matter, and this excess must extend to the whole observable Universe, because we do not see gamma rays from proton anti-proton annihilation. This visible matter excess implies a baryon asymmetry. The magnitude of a lepton asymmetry is unclear, because there could be an asymmetry stored in the ubiquitous cosmic background of neutrinos.

1.1. Observations

The amount of baryonic matter in the Universe affects the fluctuations in the Cosmic Microwave Background. WMAP data gives the excess of baryons $B$ over anti-baryons $\bar{B}$, normalised to the density of photons today:

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \bigg|_{\text{today}} = \frac{n_B - n_{\bar{B}}}{s} \frac{s_0}{n_{\gamma 0}} = 6.15 \pm 0.25 \times 10^{-10}$$

(1)

It is convenient to calculate $Y_B$, the baryon asymmetry relative to the entropy density $s$, because $s = g_* \frac{2\pi^2}{45} T^3$ is conserved during Universe expansion. The production of light nuclei by Big Bang Nucleosynthesis also depends on $Y_B$, and the observed light element abundances are consistent with eqn (1).

Before embarking on a study of how to generate a baryon asymmetry, one could first wonder if the Universe was born with this excess. However, this is incompatible

$^a g_*$ is the number of relativistic degrees of freedom in thermal equilibrium. Today, $g_* = 2 + \frac{4}{11} \zeta(3)$ accounts for the entropy in photons and neutrinos, so that the entropy today $s_0$ is $\sim 7n_{\gamma 0}$, where $n_\gamma = \frac{2\pi^2}{3} T^3$ and $\zeta(3) \simeq 1.2$. 

with inflation, during which a primordial asymmetry would have been diluted to irrelevance. And since inflation is currently the only candidate to explain the temperature fluctuations in the CMB, coherent across many horizons, one can conclude that the baryon asymmetry must be generated after inflation.

1.2. Ingredients

The three ingredients required to produce a Baryon Asymmetry were given by Sakharov [5]:

• baryon number violation —required to evolve from a state with \( B = 0 \) to \( B \neq 0 \).
• C and CP violation—particles and anti-particles should behave differently, to obtain an asymmetry in their distributions.
• out of equilibrium dynamics—the Universe is often an almost-thermalised bath, and in chemical equilibrium, there are no asymmetries in unconserved quantum numbers (such as \( B \), by the first condition).

These ingredients, including baryon number violation, are all present in the Standard Model. The SM contains C and CP violation, parametrised by the Jarlskog invariant \( \sim 10^{-23} \). To use such a small amount of CP violation to generate \( Y_B \sim 10^{-10} \) is not obvious; maybe it could arise via some kinematic amplification factor, such as gives \( \epsilon \sim 10^{-3} \) in \( \bar{K} - K \) oscillations. However, this is difficult in a thermal bath [6].

The non-equilibrium can arise due to the expansion of the Universe, for instance, from an interaction whose timescale is of order the age of the Universe. This happens in thermal leptogenesis, where the asymmetry generating interactions of the right-handed neutrino occur on the timescale Hubble\(^{-1}\).

The third ingredient, Baryon number violation, is (unexpectedly) present and fast [9] in SM cosmology. Non-perturbative knots in the SU(2) gauge fields can act as sources for \( B + L \), emitting simultaneously one lepton and three quarks of each generation [8]. The rate for tying and untying these knots, somewhat abusively referred to as sphalerons [8], is fast before the electroweak phase transition. This will be the source of baryon number violation used in leptogenesis: a lepton asymmetry is produced by some mechanism, then the fast SM \( B + L \) eating interactions partially reprocess it [10] to a baryon asymmetry.

Although the ingredients for baryogenesis are all present in the SM, a way to combine them to generate the observed baryon asymmetry has not been found. The

\(^{b}\) kinematically, these cannot mediate proton decay. Also, at zero temperature the knots are instantons, with an exponentially suppressed rate \( \sim e^{-8\pi/g^2} \).
baryon asymmetry is therefore taken as evidence for Beyond the SM (BSM) physics.

1.3. Why leptogenesis?

It is interesting to generate the baryon asymmetry in BSM models that are motivated for other reasons, because BSM models usually have many free parameters, and $Y_B$ is just one number.

One of the challenges in building baryogenesis models is the non-observation of proton decay. So it is convenient to use for baryogenesis B violation that does not cause the proton to decay—such as the SM non-perturbative processes.

Neutrinos are observed to have small masses, which could be Majorana, and therefore lepton number violating. The same lepton number violation that generates neutrino masses could be used to generate a cosmological lepton asymmetry, transformed to a baryon asymmetry by the SM non-perturbative processes.

Leptogenesis in the seesaw is therefore an attractive possibility for baryogenesis, because the proton can be stable, and the seesaw is a popular neutrino mass generation mechanism.

2. The Seesaw and Leptogenesis

2.1. The model

The seesaw model\cite{11} naturally explains the small observed neutrino masses. In its most simple formulation, two or three right-handed neutrinos $N_i$, are added the Standard Model. Being gauge singlets, they can have large Majorana masses, and the Lagrangian can be written in the mass basis of the charged leptons and right-handed neutrinos\cite{1} as:

$$\mathcal{L} = \mathcal{L}_{SM} - [\lambda]^{\ast}_{\alpha k} t^\alpha N_k \cdot \phi - \frac{1}{2} N^c_j M_j N^c_j \quad (2)$$

There are then 21 parameters in the lepton sector of the Lagrangian (2): counting in the $N$ and charged lepton mass eigenstate bases, there are the six masses $m_e, m_\mu, m_\tau$, $M_1, M_2, M_3$ and 18 - 3 phases, angles and eigenvalues in $\lambda$ (three unphysical phases can be removed by judicious choice of $\ell$ phases).

At scales $\ll M_1$, this gives an effective light neutrino mass matrix

$$[m_\nu] = \lambda M^{-1} \lambda^T v^2_u \quad (v_u = \langle \phi^0 \rangle \simeq 175 \text{ GeV}) \quad (3)$$

In the leptonic sector of the SM augmented with the Majorana neutrino mass matrix of eqn (3), there are 12 parameters: $m_e, m_\mu, m_\tau$, the neutrinos masses $m_1, m_2, m_3$ and 

\footnote{The Yukawa indices are ordered left-right, and the * on $\lambda$ reproduces the Lagrangian of the superpotential $W = LH\lambda N^c + N^c M \cdot N^c$}
3 angles and 3 phases in the mixing matrix $U_{PMNS}$ between the eigenbases. Seven of these parameters are measured, there is an upper bound on the mixing angle $\theta_{13}$, and the light neutrino mass scale and three phases of $U_{PMNS}$ are unknown. There are in addition 9 unknown parameters in the high scale theory, which hopefully arrange themselves such that leptogenesis can work.

2.2. Leptogenesis Mechanisms

In the context of the seesaw extension of the SM, there are many ways to produce a baryon asymmetry. They differ in the cosmological scenario, and in the values of undetermined seesaw parameters. An incomplete list of possibilities is:

- “Thermal” leptogenesis with hierarchical $M_j$ \cite{12,14,13}, will be discussed in the next sections. The $N_1$ are produced by scattering in the thermal bath.

- Thermal leptogenesis with quasi-degenerate $M_j$ \cite{15} can work for lower reheat temperatures, because the $CP$ violation can be enhanced in $N_i - N_j$ mixing.

- “soft leptogenesis” \cite{16} can work in a one-generational SUSY seesaw. If the soft SUSY-breaking terms are of suitable size, there is enough $CP$ in $\tilde{N} - \tilde{N}^*$ mixing.

- In the Affleck Dine mechanism \cite{17}, an asymmetry arises in a classical scalar field, which later decays to particles. The field starts with a large expectation value, which gives it access to lepton number violation that is suppressed at small scales.

- The $N$ could be produced non-thermally, for instance in inflaton decay \cite{18}, or in preheating \cite{19}.

3. Thermal Leptogenesis

The remainder of this proceedings focusses on the first scenario (thermal production of hierarchical $N$). It assumes

1. the Lagrangian of eqn (2)

2. hierarchical $N$ masses: $M_1 \sim 10^9$ GeV $\ll M_2, M_3$ (a hierarchical spectrum seems indicated, if $\lambda$ is hierarchical. Also, the kinematics is simpler in an effective theory of propagating $N_1$ and effective dimension 5 operator induced by $N_2$ and $N_3$).

3. thermal production of the $N_1$ (and negligeable production of $N_2$) \footnote{If the reheat temperature of the Universe is $> M_2$, then the asymmetry produced in $N_2$ decay can be relevant \cite{20,21,22}}
The idea is that a distribution of $N_1$ is produced by scattering processes at temperatures $T \sim M_1$, and then the $N_1$ decay away, as the temperature drops below their mass, because the equilibrium number density is suppressed $\propto e^{-M_1/T}$. If these decays are CP violating, asymmetries in all the lepton flavours can be produced. If inverse decays are out-of-equilibrium, the asymmetries may survive. They can then be reprocessed into a baryon asymmetry by the SM B+L violating processes.

The baryon asymmetry can be estimated by considering the Sakharov conditions. The maximum baryon asymmetry would arise if,

1. at $T \gg M_1$ there was a thermal distribution of $N_1$,
2. each $N_1$ contributes, when it decays, 1 lepton to the asymmetry in some flavour $\alpha$,
3. there are no inverse decays to wash out the asymmetry produced,
4. each lepton then is converted into a baryon.

More realistically:

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} \simeq \frac{135\zeta(3)}{4\pi^4 g_*} \times (\text{prefactor}) \times \sum_{\alpha} \epsilon_{\alpha \alpha} \times \eta_\alpha$$

where the first fraction is the equilibrium $N_1$ number density divided by the entropy density at $T \gg M_1$, of order $4 \times 10^{-3}$ when the number of relativistic degrees of freedom $g_*$ is taken $\simeq 106$ as in the SM. The non-equilibrium is parametrised in $\eta_\alpha$, which varies from 1 (fully out-of-equilibrium) to 0 (decay in equilibrium, no asymmetry). The asymmetry in lepton flavour $\alpha$ per $N_1$ decay is $\epsilon_{\alpha \alpha} \ll 1$ (double index because formally it is a diagonal element of a matrix), and the prefactor $\sim 1$ converts the lepton asymmetry produced in $N_1$ decay to a baryon asymmetry. The aim is now to estimate these parameters.

The estimates will be performed for a lepton flavour $\alpha$. At the end, we can sum over flavour.

### 3.1. $L$ and $B+L$ violation

The interactions of $N_1$ violate $L$ because lepton number cannot be consistently assigned to $N_1$ in the presence of $\lambda$ and $M$: if $N_1$ is a lepton, then $M$ violates $L$ by two units, if $N$ is not a lepton, then $\lambda$ violates $L$ by one unit. The $N_1$ decay, which depends on $M_1$ and $\lambda$, does not conserve $L$: being its own anti-particle, $N_1$ can decay to either $\ell \phi$ or $\bar{\ell} \phi^*$. If there is an asymmetry in the rates, a net lepton asymmetry will be produced.

The baryon number violation is provided by $B + L$ changing SM non-perturbative processes, which are fast (The rate $\Gamma_{B-L} \sim \alpha^5 T$ is faster than the Hubble expansion...
$H$) in the thermal plasma below $T \lesssim 10^{12}$ GeV. An asymmetry in lepton flavour $\alpha$, produced in the $N_1$ decay, contributes to the density of $B/3 - L_\alpha$, which is conserved by the SM interactions. In equilibrium, this excess of $B - L$ implies (for the SM) a baryon excess

$$Y_B \simeq \frac{12}{37} \sum \alpha Y_{B/3 - L_\alpha}$$

(5)

$12/37$ is the prefactor in the SM.

3.2. CP in the decay

To produce a net lepton asymmetry, the $N_1$ must have different decay rates to final states with particles or anti-particles. The asymmetry in lepton flavour $\alpha$, produced in the decay of $N_1$ is

$$\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \to H\ell_\alpha) - \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell}_\alpha)}{\Gamma(N_1 \to H\ell) + \Gamma(N_1 \to \bar{H}\bar{\ell})}$$

(6)

The $\epsilon_{\alpha\alpha}$ carries an incongruous double flavour index, to remind us that it is the diagonal element of a matrix, and not a component of a vector (this is relevant for writing Boltzmann Equations). It is normalised to the total decay rate, so that the Boltzmann Equations are linear in flavour space.

The CP asymmetry $\epsilon_{\alpha\alpha}$ arises from the interference of the tree level and one-loop amplitudes. Writing the tree + loop amplitude as a product of coupling constants $c$ and everything else: $c^tA^t + c^{\text{loop}}A^{\text{loop}}$, one finds that the CP asymmetry is proportional to $\Im\{c^t c^{\text{loop}}\} \times \Im\{A^t A^{\text{loop}}\}$ (where the imaginary part of the amplitudes arises form putting the loop particles on-shell). This is straightforward to calculate from the diagrams in figure 1 [28]. For hierarchical right-handed neutrinos ($M_{2,3} \gg 10M_1$), the internal $N$ propagator can be collapsed to the effective operator $[m_\nu]_0^2 / v^2$ (see eqn (3) [4]), which gives

$$\epsilon_{\alpha\alpha} = \frac{3M_1}{16\pi v_0^2 \lambda \lambda_{11}} \Im\{[\lambda]_{\alpha1}^*[m_\nu^*]_{\lambda_1}\}$$

(7)

This approximation would not work if $N_1$ propagated in the loop; it does not because the associated coupling constant combination is real.
The total CP asymmetry is bounded above \[ 26^{27} \]

\[
\sum_{\alpha} \epsilon_{\alpha \alpha} < \frac{3}{16\pi} \frac{(m_3 - m_1) M_1}{\langle \phi \rangle^2} \sim 10^{-6} \frac{M_1}{10^9 \text{GeV}} \quad (8)
\]

This suggests a lower bound on the mass of \( N_1 \), and the reheat temperature, of order \( M_1 \gtrsim 10^9 \) GeV. This applies only to hierarchical \( N_i \); the CP asymmetry can be much larger for quasi-degenerate \( N_i \).

### 3.3. Out of Thermal Equilibrium

The non-equilibrium for thermal leptogenesis is provided by the Universe expansion: interaction rates which are of order the Hubble expansion rate \( H \), are not fast enough to equilibrate particle distributions. This is the most delicate part of estimating the baryon asymmetry.

Suppose, as initial conditions, that after inflation the Universe reheats to a thermal bath composed of particles with gauge interactions. The \( N_1 \) can be produced by inverse decays \( \phi \ell_\alpha \rightarrow N \), and most effectively by scattering \( q_L t_R \rightarrow \phi \rightarrow \ell_\alpha N \). A thermal number density of \( N_1 \) (\( n_N \approx n_\gamma \)) will be produced if \( M_1 < T \), and if the production timescale for \( N_1 \), \( 1/\Gamma_{\text{prod}} \), is shorter than the age of the Universe \( \sim 1/H \):

\[
\Gamma_{\text{prod}} \sim \sum_{\alpha} \frac{h_t^2 |\lambda_{\alpha 1}|^2}{4\pi} T > H \quad (9)
\]

If this is satisfied, then, since \( h_t \approx 1 \), the \( N_1 \) decay is also “in equilibrium”:

\[
\Gamma_D \approx \frac{[\lambda^\dagger \lambda]_{11} M_1}{8\pi} > \frac{10T^2}{m_{\text{pl}}} \bigg|_{T=M_1} \quad (10)
\]

It is possible to show that

\[
\tilde{m} = \frac{[\lambda^\dagger \lambda]_{11} v_u^2}{M_1} \quad (11)
\]

the total decay rate rescaled by factors of \( M_1 \) and Higgs vev, is “usually” \( \gtrsim m_{\text{sol}} \), so one expects a maximal initial distribution of \( N_1 \), and a total decay rate that is fast compared to \( H \).

More importantly, as the \( N_1 \) start to decay, the inverse decays \( \ell_\alpha \phi \rightarrow N_1 \), which can wash out the asymmetry, may be fast compared to \( H \). Suppose this is the case for flavour \( \alpha \). Then the asymmetry in lepton flavour \( \alpha \) will survive once inverse decays from flavour \( \alpha \) are “out of equilibrium”:

\[
\Gamma_{ID}(\phi \ell_\alpha \rightarrow N_1) \approx \Gamma_{\alpha \alpha} e^{-M_1/T} < \frac{10T^2}{m_{\text{pl}}} \quad (12)
\]

where \( \Gamma_{\alpha \alpha} \equiv \tilde{m}_{\alpha \alpha} M_1^2/(8\pi v_u^2) \) is the partial decay rate \( \Gamma(N \rightarrow \ell_\alpha \phi) \).
At temperature $T_\alpha$ where eqn (12) is satisfied, the remaining $N_1$ density is Boltzmann suppressed: $\propto e^{-M/T_\alpha}$. Below $T_\alpha$, the $N_1$ decay “out of equilibrium”, and contribute to the lepton flavour asymmetry. So the washout factor $\eta_\alpha$ for flavour $\alpha$ can be approximated as

$$\eta_\alpha \simeq \frac{n_N(T_\alpha)}{n_N(T \gg M_1)} \simeq e^{-M/T_\alpha} \simeq \frac{m_\ast}{\tilde{m}_{\alpha\alpha}}$$ (13)

where $m_\ast/\tilde{m}_{\alpha\alpha}$ is $H/\Gamma_{\alpha\alpha}$ evaluated at $T = M_1$. This approximation applies for $\tilde{m}_{\alpha\alpha} > m_\ast \simeq 10^{-3}\text{eV}$.

4. With or without flavour?

Consider the case of strong washout for all flavours. Combining equations (4), (5), (13) and (7) gives

$$Y_B \sim 10^{-3} \sum_\alpha \epsilon_{\alpha\alpha} \eta_\alpha \sim 10^{-3} m_\ast \sum_\alpha \frac{\epsilon_{\alpha\alpha}}{\tilde{m}_{\alpha\alpha}} \quad \text{(flavoured, strong washout)}$$ (14)

where the flavours summed over are presumably the charged lepton mass eigenstates. However, one might also think to choose $\alpha$ as the direction in flavour space into which $N_1$ decays, which can be called $\hat{y}$. Then $\epsilon_{yy}$ is the total CP asymmetry $\epsilon$, $\Gamma(N \to \phi\ell_y)$ is the total decay rate $\Gamma_D$, and one finds

$$Y_B \sim 10^{-3} \epsilon m_\ast \frac{m_\ast}{m} \quad \text{(single flavour, strong washout)}$$ (15)

Which is simpler but not the same. The remainder of this section discusses why and when the charged lepton mass eigenstate basis is the relevant one.

4.1. But flavour should be irrelevant...?

Flavour effects were ignored in leptogenesis for a long time. One reason is that the small CP asymmetry in $N_1$ decay depends on many powers of $\lambda$. Including the charged lepton Yukawas should be a perturbatively irrelevant correction. Secondly, the $B + L$ violating processes partially transform the total lepton asymmetry to baryons, and the total lepton asymmetry is the trace of an “asymmetry number operator” in flavour space. Since the trace can be evaluated in any basis, the simplest choice would include the direction $\hat{y}$ into which $N_1$ decays.

\footnote{There is a subtlety: the asymmetry produced in $B/3 - L_\alpha$ is spread among different particle species by the SM interactions (for instance, some of the $\ell_\tau$ asymmetry could be stored in $\tau_R$). But washout proceeds only from the asymmetry in doublets, so is mildly reduced, depending on which SM interactions are fast enough to redistribute the asymmetry. As discussed in \cite{24,37}, this can usually by accounted for using the $A$-matrix of \cite{22}.}

\footnote{This is technically not quite correct, because the final lepton states are not CP eigenstates. See \cite{23}.}
Nonetheless, flavour matters\cite{25,23,24} : equations (14) and (15) are different, because the first sums probabilities and the second sums amplitudes. Thermal leptogenesis always involves the production and decay of an $N$, via its Yukawa vertex where also appears the lepton $\ell_y$. If lepton flavours are indistinguishable between these two interactions, then the “single flavour” results are correct. But if the charged lepton Yukawas give distinguishable thermal masses to different flavours, then the asymmetry should be computed flavour by flavour.

The timescale for leptogenesis is $H^{-1}$ (because the “non-equilibrium” is provided by the Universe expansion). One way to take into account the many interactions that are faster than $H$ is to resum them into thermal masses. Comparing $H$ to the rates for $h_\tau$ or $h_\mu$ mediated interactions such as $q_L \ell_R \rightarrow \ell_\tau \ell_R$, one finds

$$\Gamma_\tau \simeq 10^{-2} h_\tau^2 T > H \quad \text{for} \quad T < 10^{12} \text{ GeV}, \quad \Gamma_\mu > H \quad \text{for} \quad T < 10^9 \text{ GeV}$$

So below $T \sim 10^{12}$ GeV, the $h_\tau$ is “in equilibrium”, and contributes to the “thermal mass matrix”\footnote{For instance, in the strong washout example of the previous section, the $N_1$ number density is depleted by decays and repopulated by inverse decays. A lepton doublet is involved in both of these interactions.} of the lepton doublets. So there can be two distinguishable flavours down to $T \sim 10^9$ GeV, and below $T \sim 10^9$ GeV there can be three. In the scenario discussed here, where the asymmetry is generated in the decay of $N_1$, the temperature of leptogenesis $\gtrsim 10^9$ GeV, to obtain a large enough CP asymmetry (see eqn (8)).

The second reason given above, for why flavour is “obviously” irrelevant, is elegant and convincing. A technical way to see what is missing is to write the equations of motion for the asymmetry number operator, which is a matrix in flavour space (see\cite{22}, or the appendix of\cite{25} for a toy model). The diagonal elements of this matrix are the asymmetries, the off-diagonals encode quantum correlations (as is the case for a quantum mechanical density matrix). These equations transform under changes of the flavour basis, and contain terms describing the interactions mediated by the charged lepton Yukawa matrix $[h_e]$. In the charged lepton mass eigenstate basis, the $[h_e]$ terms affect only the off-diagonals, and drop out of the Boltzmann-like equation for the flavoured lepton asymmetries on the diagonal. But in any other basis, these $[h_e]$ terms should remain.

5. In practise, so what?

As discussed after eqn (3), there are (currently) 14 unknown parameters in the seesaw model, of which only a few are accessible to low energy experiments. It is interesting, from a phenomenological perspective, to constrain the unknowns by requiring that leptogenesis works. Including flavour effects in calculating these constraints does not change them very much, as discussed in the following subsection.

\footnote{If $\Gamma(N \rightarrow \phi \ell) > \Gamma_\tau$, it should also be included in determination of mass eigenstates, as noted in\cite{29}}
From a top-down perspective, models give predictions for all the parameters of the seesaw. It is interesting to verify that a given model reproduces the correct baryon asymmetry, as well as the observed neutrino masses. Including flavour in such leptogenesis calculations can make a significant difference. This is discussed later.

5.1. Phenomenological Bottom-up perspective

1. The upper bound on the light neutrino mass scale:
   In the “single flavour” calculation, successful thermal leptogenesis required a light $\nu$ mass scale $\lesssim 0.2 \text{ eV}$ $^{31}$. This is no longer the case in the flavoured calculation—models can be tuned to work for $m_\nu \lesssim \text{few eV}$ (the cosmological bound). This is because there is more $\mathbb{CP}$ available. The upper limit of eqn (8) on the total CP asymmetry, decreases like $\Delta m^2_{\text{atm}}/m_{\text{max}}$, as the light neutrino mass scale $m_{\text{max}}$ increases. There is therefore an upper bound on $m_{\text{max}}$. However, the limit (8) does not apply to the flavoured CP asymmetries, which can increase with the light neutrino mass scale.

2. Sensitivity of the baryon asymmetry to PMNS phases:
   An important, but sad, observation in “single-flavour” leptogenesis was the lack of a model-independent connection between $\mathbb{CP}$ for leptogenesis and MNS phases. It was shown $^{30}$ that thermal leptogenesis can work with no $\mathbb{CP}$ in $U_{\text{PMNS}}$, and conversely, that leptogenesis can fail in spite of phases in $U_{\text{PMNS}}$. In the “flavoured” leptogenesis case, it is still true that the baryon asymmetry is not sensitive to PMNS phases $^{32}$ (=leptogenesis can work for any value of the PMNS phases). However, interesting observations can be made in classes of models $^{23,33,34}$.

3. The lower bound on $T_{\text{reheat}}$:
   There is an envelope, in the space of parameters leptogenesis depends on, inside which leptogenesis can work. In the “single-flavour” calculation, the most important parameters are $M_1$, $\Gamma$ (equivalently $\tilde{m}$), $\epsilon$ and the light neutrino mass scale $^{14}$. Including flavour gives the envelope more dimensions ($M_1, \epsilon, \Gamma, \cdots$), but it can still be projected onto $M_1$, $\tilde{m}$ space. Leptogenesis works for $M_1$ a factor of $\sim 3$ smaller in the “interesting” region of $m_\ast < \tilde{m} \lesssim m_{\text{atm}}$. But the lower bound on $M_1$, in the $m_\ast \sim \tilde{m}$ region $^{4}$, remains $\sim 10^9 \text{ GeV}$ $^{35,36,37}$.

$^1$A smaller $M_1$ could be possible for very degenerate light neutrinos $^{25}$.
5.2. Estimating the Baryon Asymmetry in Models

In the “single flavour” calculation, the baryon asymmetry can be approximated as

\[ Y_B \simeq 4 \times 10^{-3} \epsilon \eta \]

where \( \eta \) is the rescaled decay rate (see eqn (11), and \( m_* \simeq 10^{-3} \text{eV} \) is the value of \( \tilde{m} \) for which \( \Gamma_D = H \) at \( T = M_1 \). So to estimate the baryon asymmetry produced by leptogenesis in a particular model, one merely must calculate \( \epsilon \) and \( \tilde{m} \).

In the flavoured case, one should work two or three times harder: if the tau Yukawa is “in equilibrium” (and \( h_\mu \) not), there are two relevant CP asymmetries \( \epsilon_{ee} + \epsilon_{\mu\mu}, \epsilon_{\tau\tau} \), and two partial decay rates \( \tilde{m}_{ee} + \tilde{m}_{\mu\mu}, \tilde{m}_{\tau\tau} \). Assuming that \( \tilde{m} > m_* \), the final baryon asymmetry can be approximated as

\[ Y_B \simeq 4 \times 10^{-3} \sum_\alpha \epsilon_{\alpha\alpha} \eta^\alpha \]

The flavoured formula (17) can give a significantly larger result than eqn (16), as can be seen in figure 2. There are two intuitive reasons for this. First, the washout of the asymmetry by inverse decays is less efficient with flavour, because inverse decays from flavour \( \alpha \) only can destroy the asymmetry in flavour \( \alpha \). As opposed to the total inverse decay rate eating the whole lepton asymmetry, as in the single flavour case. Second, including the charged lepton Yukawas puts more CP in the theory. So the flavoured CP asymmetries can be larger than the sum, because this additional CP violation must vanish from \( \epsilon \).

Finally, consider the case where the light neutrino masses are non-degenerate, and all (e.g., two) distinguishable flavours are in strong washout. Then there is a pretty approximation to \( Y_B \), which exhibits the amplification of the asymmetry due to flavour effects. The asymmetry can be approximated

\[ Y_B \simeq \frac{12135 \zeta(3)}{37 \cdot 4 \pi^4 g_*} \sum_\alpha \epsilon_{\alpha\alpha} \frac{m_*}{5 \tilde{m}_{\alpha\alpha}} \]

and the ratios in the flavour sum can be expressed

\[ m_* \frac{\epsilon_{\alpha\alpha}}{\tilde{m}_{\alpha\alpha}} = \frac{3 M_1 m_*}{16 \pi^2 \tilde{m}} \sum_\beta \Im \left\{ p_\alpha [m_\nu]_{\alpha\beta} p_\beta \right\} \left| \frac{\lambda_{\beta 1}}{\lambda_{\alpha 1}} \right| \quad \text{where} \quad p_\alpha \equiv \frac{\lambda_{\alpha 1}}{|\lambda_{\alpha 1}|} \]

kThis means at least one flavour is in strong washout. An approximate formula for \( Y_B \) in the case of \( \tilde{m} > m_* \) can be found in [24].
Recall that this equation is only valid in strong washout for all flavours, and that the $p_\sigma$ are the phases of the Yukawa couplings $l$. The flavour $o$ is the projection of $\hat{y}$ (defined after eqn (14)) on $e, \mu$ space. The bracketed term shows how stronger washout in one flavour can increase the baryon asymmetry. So models in which the Yukawa coupling $[\lambda_\tau 1]$ is significantly different from $[\lambda_\mu 1], [\lambda_\tau 1], [\lambda_{\ell 1}]$, can have an enhanced baryon asymmetry (with cooperation from the phases).

This equation is attractive step towards writing the baryon asymmetry as a real function of real parameters (the unflavoured upper bound on $Y_B$, depending on $M_1$ and $\tilde{m}_1$), times a phase factor\textsuperscript{27}. In this case, the phase factor is a sum of three terms, depending on: the phases of the neutrino Yukawa couplings, light neutrino mass matrix elements normalised by the heaviest mass, and a real ratio of Yukawas.

6. Summary

Thermal leptogenesis in the seesaw model is an attractive baryogenesis mechanism. Some density of right-handed neutrinos $N$ is generated by scattering in the plasma,\textsuperscript{28}In general, these phases are related to the light neutrino mass matrix, so should not be chosen independently.
then the $N$ produce a lepton asymmetry in their decay. Standard Model $B + L$
violating processes partially transform this lepton asymmetry to baryons. The right-
handed neutrino masses were taken hierarchical in this proceedings.

When the interaction rates of the charged lepton Yukawas are faster than the
leptogenesis rates, lepton flavours are distinguishable and the production of the lepton
asymmetry should be studied flavour by flavour. The baryon asymmetry calculated
in this way is different from earlier calculations that considered the production of
total lepton number.

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