Non-slow-roll dynamics in $\alpha$-attractors

K. Sravan Kumar, J. Marto, P. Vargas Moniz and Suratna Das

$^a$Departamento de Física, Universidade da Beira Interior (UBI), 6200, Covilhã, Portugal
$^b$Centro de Matemática e Aplicações da Universidade da Beira Interior (CMA-UBI), 6200, Covilhã, Portugal
$^c$Indian Institute of Technology, Kanpur 208016, India

E-mail: sravan@ubi.pt, jmarto@ubi.pt, pmoniz@ubi.pt, suratna@iitk.ac.in

Received July 20, 2015
Revised December 17, 2015
Accepted March 21, 2016
Published April 4, 2016

Abstract. In this paper we consider the $\alpha$—attractor model and study inflation under a non-slow-roll dynamics. More precisely, we follow the approach recently proposed by Gong and Sasaki [1] by means of assuming $N = N(\phi)$. Within this framework we obtain a family of functions describing the local shape of the potential during inflation. We study a specific model and find an inflationary scenario predicting an attractor at $n_s \approx 0.967$ and $r \approx 5.5 \times 10^{-4}$. We further show that considering a non-slow-roll dynamics, the $\alpha$—attractor model can be broaden to a wider class of models that remain compatible with value of $r < 0.1$. We further explore the model parameter space with respect to large and small field inflation and conclude that the inflaton dynamics is connected to the $\alpha$— parameter, which is also related to the Kähler manifold curvature in the supergravity (SUGRA) embedding of this model. We also comment on the stabilization of the inflaton’s trajectory.

Keywords: inflation, primordial gravitational waves (theory), supersymmetry and cosmology

ArXiv ePrint: 1506.05366
## Contents

1 Introduction \hfill 1  
2 $\alpha$-attractor model \hfill 2  
3 Non-slow-roll dynamics \hfill 4  
4 Power spectrum \hfill 6  
   4.1 Scalar power spectrum \hfill 6  
   4.2 Tensor power spectrum \hfill 7  
   4.3 Inflationary predictions for $n = 1$ \hfill 8  
5 Non-slow-roll $\alpha$-attractor \hfill 9  
   5.1 Conditions for small field and large field inflation \hfill 9  
6 Embedding in $\mathcal{N} = 1$ SUGRA \hfill 11  
7 Conclusions and outlook \hfill 12  

## 1 Introduction

Inflationary cosmology has become an extremely convincing paradigm for the early universe after the recent release of Planck data [2–4]. The unprecedented accuracy of the data is consistent with the LCDM, nearly scale invariant and Gaussian nature of curvature perturbations. We now have stringent bounds on the scalar spectral index, namely, $n_s = 0.968 \pm 0.006$ at 95% CL and also for the tensor to scalar ratio, which is severely bounded, $r < 0.1$ ($r < 0.07$ considering recent results of BICEP2/Keck Array [5]). Given these circumstances, single field inflation constitutes a scenario which is in full agreement with the data. However, the nature of the inflaton is still elusive [6].

Among the broad variety of inflationary scenarios, the Starobinsky model, with a $R + R^2$ term, and the Higgs inflation [7–9] stand in a privileged region in the middle of the $(n_s, r)$ plane [2]. Moreover, the Starobinsky model prediction is identified as a target spot for many inflationary models, i.e,

$$n_s = 1 - \frac{2}{N} \quad r = \frac{12}{N^2},$$  \hspace{1cm} (1.1)

where $N$ is the 50–60 number of e-foldings before the end of inflation. The requirement of $r \lesssim \mathcal{O} (10^{-2})$ and the nearly vanishing non-Gaussianities is compatible with inflationary models embedded in a low energy effective field theories derived from a UV-completion physics, such as supergravity (SUGRA) and string theory [10–12].

Since the first release of Planck 2013 data, these two scenarios (Starobinsky model and Higgs inflation) started to attract a lot of attention, they have been extensively studied and realized in the context of conformal symmetries [13, 14], later generalized as $\alpha$– and non-minimal (or) $\xi$–attractors. In addition, these models have been embedded in SUGRA through the use of superconformal symmetries [15–19]. Recently, $\alpha$– attractor models were also realized by means of the inclusion of an auxiliary vector field for the Starobinsky...
model [20]. These two classes of models have also, a posteriori, been unified as cosmological attractor models (CAM) [21–23]. By varying the parameters \((\alpha, \xi)\) in CAM, on the one hand leads to the Starobinsky attractor (1.1) and on the other hand it also reproduces the chaotic inflation predictions with the \(m^2 \phi^2\) potential. In particular, for \(\alpha = \frac{1}{9}\), we retrieve the first model of chaotic inflation in SUGRA proposed in 1983, which is known as the Goncharov-Linde (GL) model, and it is well consistent with the present data [24–26]. CAM were embedded in \(N = 1\) SUGRA using superconformal symmetries by introducing a 3 chiral super multiplets: a conformon \(X^0\), an inflaton \(X^1 = \Phi\) and a sGoldstino \(X^3 = S\) [15, 16, 18]. In this set up single field inflation is achieved at the minimum of the superpotential by the requirement that the fields \(S\) and \(\text{Im}\Phi\) remain heavy during inflation.\(^1\) In recent studies, \(\alpha\)– attractors realized in SUGRA\(^2\) by only requiring a single chiral superfield [35, 36]. A generalization of Kähler potentials for viable single field models with respect to Planck data, plus their connection to open and closed string sector has been investigated in [37].

In this paper we study non-slow-roll inflaton dynamics in the \(\alpha\)–attractor model using the recently proposed approach of Gong and Sasaki (GS) [1], which constitutes, to our knowledge, a new strategy. More concretely, we focus on the non-canonical aspect of the \(\alpha\)–attractor model. We start with the assumption of GS [1], where the number of e-foldings \(\mathcal{N}\) which is counted backward in time is assumed to be a function of the inflaton field \(\phi\) during inflation. We retrieve the local shape of the potential during inflation which can be steep and allowing for 60 e-foldings to occur. More precisely, we restrict our study to the region of the potential where inflation is occurring. We emphasize that both the pre- and post-inflationary dynamics are beyond the scope of this paper. Afterwards, we explore the GS parametrization within our chosen inflaton dynamics showing that inflation occurs for a wider class of potentials. We further show that we can maintain the predictions of the \(\alpha\)–attractor model displayed in [16], but now herein retrieved alternatively within a non-slow-roll. Finally, we study the possibility of realizing this model within \(\mathcal{N} = 1\) SUGRA. We explore the relation between the inflaton dynamics and the corresponding Kähler geometry curvature. We also comment on the stability of inflaton trajectory during inflation.

The paper is organized as follows: in section 2, we revise the \(\alpha\)–attractor model and present arguments supporting a non-slow-roll approach for these models. In section 3, we describe GS parametrization and implement the non-slow-roll dynamics in the context of \(\alpha\)–attractors. In section 4, we present predictions for a specific case of the GS parametrization. In section 5, we complement the previous predictions for a wider class of non-slow-roll dynamics and discuss on large and small field inflation. We show that these scenarios exhibit an attractor in the \((n_s, r)\) plane and discuss the (dis)similarities with standard slow-roll inflaton dynamics. In section 6, we review the SUGRA realization of this scenario and verify the stabilization of the inflaton trajectory during inflation.

## 2 \(\alpha\)-attractor model

In this section, we revise the essentials of \(\alpha\)–attractor models which have been studied under slow-roll frameworks so far as in [16, 21, 32] and provide a baseline for our interest on these

---

\(^1\)This mechanism has also envisaged the multifield inflation with a curvaton, i.e, where we can have generation of isocurvature perturbations when \(S\) or \(\text{Im}\Phi\) are light and play the role of curvaton during or after the end of inflation [27–29].

\(^2\)Obtaining inflation from SUGRA also brings other benefits such as, exploring supersymmetry (SUSY) breaking sector and the presence of dark energy [30–34].
models which we will be exploring in the rest of the manuscript from a new perspective and methodology.

The Lagrangian for $\alpha$–attractor models, in the Einstein frame, is given by\footnote{We assume the units $M_{\text{Pl}} = 1$.} \cite{32}

$$\mathcal{L}_E = \sqrt{-g} \left[ \frac{R}{2} - \frac{1}{(1 - \phi^2/6\alpha)^2} \left( \frac{\partial \phi}{2} - f^2 \left( \phi/\sqrt{6\alpha} \right) \right) \right], \quad (2.1)$$

where $\alpha = 1$ leads to the same prediction of the Starobinsky model (in the Einstein frame), $\alpha = 1/9$ corresponds to GL model \cite{24}, and for large $\alpha$ this model is equivalent to chaotic inflation with quadratic potential \cite{38}. In order to prevent negative gravity in the Jordan frame it is required to have $|\phi| < \sqrt{6\alpha}$ \cite{16, 19}. Furthermore, in the SUGRA embedding of this model, the parameter $\alpha$ is shown to be related to the curvature of Kähler manifold as

$$\mathcal{R}_K = -\frac{2}{3\alpha}. \quad (2.2)$$

The Lagrangian eq. (2.1) is a subclass of $k$-inflationary model where the kinetic term is linear\footnote{$K(\phi) = 1$ gives the canonical kinetic term.} in $X$, i.e.,

$$P(X, \phi) = K(\phi) X - f^2 \left( \phi/\sqrt{6\alpha} \right), \quad (2.3)$$

where $K(\phi) = \frac{1}{(1 - \phi^2/6\alpha)^2}$ and $X = -\frac{(\partial \phi)^2}{2}$. The speed of sound for these class of models is $c_s^2 = 1$ \cite{39}, therefore these models are not expected to show large non-Gaussianities \cite{40}.

In this theory, the Friedmann equation is

$$H^2 = \frac{1}{3} \left[ XK(\phi) + f^2 \left( \phi/\sqrt{6\alpha} \right) \right]. \quad (2.4)$$

The Raychaudhuri equation is

$$\dot{H} = -XP_X \quad \text{with} \quad P_X = \frac{\partial P}{\partial X}, \quad (2.5)$$

and the equation of motion for the scalar field is given by

$$\frac{d}{dt} \left( K(\phi) \dot{\phi} \right) + 3HK(\phi) \dot{\phi} - P_{,\phi} = 0. \quad (2.6)$$

In the literature it is found that inflation in the $\alpha$–attractor model has been realized in terms of a canonically normalized field ($\varphi$) as

$$\frac{d\varphi}{d\phi} = \frac{1}{(1 - \phi^2/6\alpha)} \Rightarrow \frac{\varphi}{\sqrt{6\alpha}} = \tanh \frac{\varphi}{\sqrt{6\alpha}}. \quad (2.7)$$

In this case, flat potentials are natural and subsequent slow-roll dynamics of $\varphi$ lead to viable inflationary scenario with respect to the observational data. The predictions of $(n_s, r)$ for these models are shown to be solely determined by the order and residue of the Laurent series expansion leading pole in the kinetic term \cite{21}. The slow-roll inflationary predictions of $\alpha$–attractor models are

$$n_s = 1 - \frac{2}{N} \quad r = \frac{12\alpha}{N^2}. \quad (2.8)$$
In terms of this canonically normalized field ($\phi$) the equation of motion (2.6) becomes

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0. \quad (2.9)$$

Therefore, under slow-roll assumption this reduces to

$$3H\dot{\phi} \simeq V_{,\phi}. \quad (2.10)$$

Our purpose is to obtain viable inflationary predictions, by means of extending $\alpha-$ attractors towards non-slow-roll dynamics. Therefore, in the present work, we restrict ourselves to the range $\phi^2 < 6\alpha$. We will emphasize similarities and of course the differences with the (canonically normalized field) slow-roll inflation case. In the following section we unveil the context of non-slow-roll towards $\alpha-$attractors.

3 Non-slow-roll dynamics

The recent work by Gong & Sasaki (GS) [1] points out a cautionary remark on applying slow-roll approximation in the context of k-inflation. The argument, presented there, lies in the fact that the second derivative term in the equation of motion (2.6) may not be negligible in general. In this regard, the authors introduce a new parameter

$$p = \frac{\dot{P}_X}{H_P X}, \quad (3.1)$$

which could bring significant differences in the local non-Gaussianity. They have illustrated the role of this new parameter and observationally viable inflationary scenarios in the context of some non-trivial examples.

Let us implement the aforementioned procedure here in the context of $\alpha-$attractors as $\phi$ is a non-canonical scalar field given by eq. (2.3). This new approach enable us to study the $\alpha-$attractors in the context of non-slow-roll by assuming that the inflaton field during inflation behaves as

$$\phi = n \exp (\beta N), \quad (3.2)$$

where $N = \ln \alpha (t)$ is the number of efoldings counted backward in time from the end of inflation and $n$ is treated as a free parameter that specifies the value of the field at $N \to 0$. We assign eq. (3.2) as GS parametrization for subsequent reference. This parametrization is particularly useful in the cases of non-canonical scalar field models, whereas in refs. [41, 42] a different parametrization was applied to the case of canonical scalar field inflation. We declare here that our study of inflation in $\alpha-$attractor model is based on the dynamics for the inflaton assumed in eq. (3.2) parametrized by ($n$, $\beta$). Therefore, we label our approach for the $\alpha-$attractor framework as non-slow-roll, following the same terminology used in ref. [1].

Being more precise, in this paper we do not impose any slow-roll approximation in particular. We note at this point that non-slow-roll does not mean a non-smallness of conventional parameters $\epsilon$, $\eta$ (see ref. [1] for more details). Moreover, and we stress that this is a most important point in our study, we completely relax the choice of the inflaton potential and rather concentrate on the inflaton dynamics that can give rise to viable observational predictions.

Substituting $\phi$ from eq. (3.2) in the Raychaudhuri equation we obtain

$$H' = \frac{\alpha^2 H (N)}{2} \phi^2 K (\phi), \quad (3.3)$$

---

Footnote 5: We start with a similar parametrization as the one used in section 3.2 of [1].
where the prime $'$ denotes differentiation with respect to $N$. Integrating eq. (3.3), we get

$$H(N) = \lambda e^{-\frac{9\alpha^2}{\phi^2 - 6\alpha}},$$

(3.4)

where $\lambda$ is the integration constant. At this point, we should mention that our calculations are similar to the Hamilton-Jacobi like formalism found in [42–44].

Inserting the aforementioned solution in the Friedmann eq. (2.4), we can express the local shape of the potential during inflation as

$$f^2\left(\frac{\phi}{\sqrt{6\alpha}}\right) = \lambda \exp\left(-\frac{18\beta^2}{\phi^2 - 6\alpha}\right) \left[3 - \frac{\beta^2\phi^2}{2\left(1 - \frac{\phi^2}{6\alpha}\right)^2}\right].$$

(3.5)

It should therefore be noted that the suitable choice of potentials considered in the case of slow-roll $\alpha$–attractors are quite different, namely, power law type $V \sim \phi^{2n}$ in terms of original scalar field (or) T-models, i.e., $V \sim \tanh^{2n}\frac{\phi}{\sqrt{6\alpha}}$ in terms of canonically normalized field [16, 21, 32]. In ref. [19] the power law potentials were generalized to the following form of power series

$$f^2\left(\frac{\phi}{\sqrt{6\alpha}}\right) = \sum_n c_n\phi^n,$$

(3.6)

where $c_n$ are non-zero constants and it was argued to be $c_0 \ll 1$. In this class of potentials the inflaton slow-rolls towards the potential minimum$^6$ which is located at $\phi = 0$.

In the subsequent sections, with the assumed GS parametrization, we will show that non-slow-roll inflation occurs to be near the pole of the kinetic term i.e., $|\phi| \to \sqrt{6\alpha}$. Therefore, we can observe from eq. (3.5) that the local shape of the potential in the non-slow-roll approach is different from the power-law (or) T-models and also the power series form given in eq. (3.6). In this regard, our study about the non-slow-roll approach widens the scope for different shapes of inflationary potentials in $\alpha$–attractors.

Subsequently, for the conventional parameters general definitions$^7$

$$\epsilon = \frac{H'}{H}, \quad \eta = -\frac{\epsilon'}{\epsilon},$$

(3.7)

substituting the Hubble parameter from eq. (3.4) and demanding the end of inflation $\epsilon = 1$ at $N = 0$ we get

$$\alpha = \frac{n^2}{3\sqrt{2\beta n + 6}}.$$
Kähler Geometry $\iff$ Inflaton Dynamics

\[ \text{Local shape of the potential} \]

From the above schematic diagram we can decipher that the class of potentials which are obtained by allowing different values for \((n, \beta)\) is related to the family of Kähler geometries, which determine the dynamics of inflaton during inflation. In the next section, we derive the scalar and tensor power spectrum for this model.

4 Power spectrum

In this section, we derive the scalar and tensor spectral indices, the tensor to scalar ratio up to the third order in the parameters \(\epsilon, \eta\). We present our calculations, which are carried by assuming \(\eta \approx \text{constant}\). We closely follow the derivations presented in refs. [46, 47]. Similar results can also be found in refs. [48, 49].

4.1 Scalar power spectrum

The second order action for scalar perturbations in the \(k\)-inflationary model with speed of sound \(c_s = 1\) is given by [46, 47]

\[
S_s^{(2)} = \int dt d^3x \, \epsilon a^3 \left[ \dot{\zeta}^2 - \frac{1}{a^2} (\nabla \zeta)^2 \right],
\]

(4.1)

where \(\zeta\) is the curvature perturbation, \(t\) is the cosmic time.

To quantify the amplitude and tilt of the spectrum we use the variables\(^8\) \(dy_s = \frac{1}{a} dt\), \(z_s \equiv \sqrt{2\epsilon} \, a\) and \(u \equiv z_s \zeta\). The action (4.1) can be canonically normalized as

\[
S_s^{(2)} = \frac{1}{2} \int dy_s d^3x \left[ (u')^2 - (\nabla u)^2 + \frac{z''}{z_s} u^2 \right],
\]

(4.2)

where \(\cdot\) denotes differentiation with respect to \(y_s\). Integrating by parts \(dy_s = (1/a) dt\), by assuming that \(\eta\) is sufficiently small and constant, we get (cf. [47])

\[
y_s = \frac{1}{(\epsilon - 1) \, aH} \left( 1 + \frac{c\eta}{(\epsilon - 1)^2} \right).
\]

(4.3)

The equation of motion for the mode function \(u_k\) is given by,

\[
y_s^2 u_k'' + y_s^2 k^2 u_k - \left( \nu_s^2 - \frac{1}{4} \right) u_k = 0,
\]

(4.4)

where

\[
\nu_s^2 - \frac{1}{4} \equiv y_s^2 \frac{z''}{z_s}.
\]

(4.5)

Imposing the flat spacetime vacuum solution in the subhorizon limit \(k/aH \to \infty\) for the perturbation mode \(u_k\), we find the solution for the eq. (4.4) as

\[
u_k = \frac{\sqrt{\pi}}{2} \sqrt{-y_s H \nu_s} (-ky_s).
\]

(4.6)

\(^8\)We are following a similar notation as the one used in ref. [46].
Using now $\zeta = u/z_s$, we obtain the power spectrum of primordial curvature perturbation as

$$P_\zeta = \frac{\gamma_s}{4\pi^2} \left. \frac{H^2}{k=ah} \right|, \quad \gamma_s \equiv 2^{2\nu_s-3} \frac{\Gamma(\nu_s)^2}{\Gamma(3/2)^2} (1-\epsilon)^2. \quad (4.7)$$

From Planck data [2] the power spectrum amplitude is known to be $P_\zeta^* = 2.2 \times 10^{-9}$. Using this bound, with eqs. (3.4) and (4.7), we constrain $\lambda \sim O\left(10^{-6}\right)$.

Finally the scalar spectral index is given by

$$n_s - 1 = 3 - 2\nu_s. \quad (4.8)$$

Calculating $\nu_s$ up to the third order in the parameters $\epsilon, \eta$, by using the definition of $z_s$ and eq. (4.3), we obtain

$$\nu_s = \left(\frac{3}{2} + \epsilon + \epsilon^2 + \epsilon^3\right) + \left(\frac{1}{2} + 2\epsilon + \frac{29\epsilon^2}{6} + \frac{82\epsilon^3}{9}\right) \eta + \left(-\frac{1}{6} + \frac{23\epsilon}{18} + \frac{1069\epsilon^2}{108} + \frac{5807\epsilon^3}{162}\right) \eta^2 + \left(\frac{1}{18} + \frac{23\epsilon}{54} + \frac{707\epsilon^2}{108} + \frac{19633\epsilon^3}{486}\right) \eta^3 + O\left(\epsilon^4, \eta^4\right). \quad (4.9)$$

### 4.2 Tensor power spectrum

The tensor power spectrum derivation follows closely the scalar power spectrum one. The second order action for tensor perturbations can be written as [46, 47]

$$S^{(2)}_t = \frac{1}{8} \int dt d^3 x a^3 \left[h_{ij}^2 - \frac{1}{a^2} (\nabla h_{ij})^2\right]. \quad (4.10)$$

We use the variables $dy_t \equiv \frac{1}{a} dt$, $z_t \equiv \frac{a}{2}$ and $u_{ij} \equiv z_t h_{ij}$ so that the action (4.10) can be canonically normalized as

$$S^{(2)}_t = \frac{1}{2} \int dy_t d^3 x \left[(u_{ij}')^2 - (\nabla u_{ij})^2 + \frac{z''_t}{z_t} u_{ij}^2\right]. \quad (4.11)$$

Integrating by parts $dy_t = (1/a) dt$, again by assuming that $\eta$ is sufficiently small and constant, we obtain

$$y_t = \frac{1}{(\epsilon - 1)} aH \left(1 + \frac{\epsilon\eta}{(\epsilon - 1)^2}\right). \quad (4.12)$$

Imposing the flat spacetime vacuum solution as in eq. (4.6) we find

$$u_{ij} = \sqrt{\pi} \sqrt{-y_t} H^{(1)}_r (-ky_t) e_{ij}, \quad \nu_t^2 - \frac{1}{4} = y_t^2 \frac{z''_t}{z_t}. \quad (4.13)$$

where $e_{ij}$ is the polarization tensor. Recalling that $h_{ij} = u_{ij}/z_t$ and taking into account the two polarization states, we obtain the power spectrum of primordial tensor perturbation

$$P_T = 8\gamma_t \left. \frac{H^2}{4\pi^2} \right|_{k=ah}, \quad \gamma_t \equiv 2^{2\nu_t-3} \frac{\Gamma(\nu_t)^2}{\Gamma(3/2)^2} (1-\epsilon)^2. \quad (4.14)$$

The tensor tilt ($n_t$) is given by

$$n_t = 3 - 2\nu_t. \quad (4.15)$$
Calculating $\nu_t$ up to the third order in the parameters $\epsilon$, $\eta$, by using the definition of $z_t$ and eq. (4.3), we obtain

$$
\nu_t = \left( \frac{3}{2} + \epsilon + \epsilon^2 + \epsilon^3 \right) + \left( \frac{4\epsilon}{3} + \frac{37\epsilon^2}{9} + \frac{226\epsilon^3}{27} \right) \eta + \left( \epsilon + \frac{227\epsilon^2}{27} + \frac{875\epsilon^3}{27} \right) \eta^2 + \left( \frac{28\epsilon^2}{9} + \frac{6491\epsilon^3}{243} \right) \eta^3 + O(\epsilon^4, \eta^4). \tag{4.16}
$$

From eqs. (4.7) and (4.14) we define the tensor to scalar ratio as

$$
r = \frac{P_T}{P_\zeta} = \frac{16\gamma_t}{\gamma_s} \epsilon. \tag{4.17}
$$

### 4.3 Inflationary predictions for $n = 1$

In this section, we study the inflationary predictions of the model taking $n = 1$. We constrain the parameter $\beta$ to obtain the predictions of ($n_s$, $r$) within current observational range.

Imposing the spectral index $n_s = 0.968 \pm 0.006$, we obtain the constraint $|\beta| \sim O(10^{-3})$ (or equivalently, from eq. (3.8), $\alpha \sim O(10^{-1})$). However, we verify that the inflaton dynamics for the case $\beta > 0$ violates the requirement that $\phi^2 < 6\alpha$. Therefore, we only consider the case with $\beta < 0$ as a viable inflationary paradigm complying with $\phi^2 < 6\alpha$ during inflation. In this case, we find that inflation occurs while approaching asymptotically the kinetic term pole at $|\phi| \to \sqrt{6\alpha}$. The predictions of ($n_s$, $r$) are depicted in the figure 1.

The left panel of figure 2 depict the shape of the potential during which inflation is happening in the non-slow-roll context. In the right panel of figure 2, we plot the parameter $\epsilon$ verses $N$ for a particular value of $\alpha$ corresponds to $n = 1$.

In addition, we compute the energy scale of inflation and mass of the inflaton ($m_\phi^2$) by computing the $V_*^{1/4}$ and the $\partial_\phi^2 V_*$ where $V_*$ is the the potential evaluated at horizon exit. In this context, the shape of the potential during inflation is given by eq. (3.5), consequently we obtain,

$$
V_*^{1/4} \sim 1.2 \times 10^{17} \text{ GeV}, \quad m_\phi^2 < 0. \tag{4.18}
$$
Therefore, since the energy scale of inflation appears to be greater than GUT scale but still below Planck scale, this naturally justify the embedding of this model in SUGRA. Since the mass squared of the inflaton is negative, inflation is driven by a tachyonic field.

5 Non-slow-roll $\alpha$-attractor

In section 4.3, we have studied non-slow-roll inflation with GS parametrization and $n = 1$, in this case we obtained $r \sim O \left(10^{-4}\right)$. The objective, at this point, is to assess inflationary scenarios with any value of $r < 0.1$, by allowing $n \neq 1$ in eq. (3.8).

5.1 Conditions for small field and large field inflation

In this section, we study the parameter space of the model allowing the inflaton to do large and small field excursions during inflation. We address the possibility of large and small field inflation in the context of non-slow-roll dynamics in $\alpha-$attractors.

Using the parametrization from eq. (3.2) the field excursion during the period of inflation is given by

$$\Delta \phi = n \left(1 - \exp \left(60\beta \right) \right).$$

The above relation allows us to identify the parameter space of $(n, \beta)$ to explicit the region of large field ($\Delta \phi > 1$) and small field ($\Delta \phi < 1$) inflation (see figure 3). We further constrain the parameter space using eq. (4.8), by imposing $0.962 < n_s < 0.974$ which is the 95% CL region given by Planck 2015. This constraint on spectral index confine $-0.001 < \beta < -0.01$, and precisely $\beta \sim -0.002$ corresponds to the central value of $n_s \sim 0.967$.

The relation between tensor to scalar ratio and field excursion during the period of inflation is defined by Lyth bound [50] which is

$$\Delta \phi > \sqrt{\frac{r}{8}} N_e,$$

where $N_e = 60$ which is the number of e-foldings before the end of inflation. We can see from the above relation that $r > 0.002$ implies $\Delta \phi > \text{M}_{\text{Pl}}$, i.e, large field inflation. However,
Figure 3. In both plots the overlap of the blue and orange shaded regions satisfies the constraint $0.962 < n_s < 0.974$. The blue shaded region in the left panel is for large field $\Delta \phi > 1$ whereas in the right panel is for small field $\Delta \phi < 1$. We have considered $N = 60$.

this bound gets modified for the $k$-inflationary models [51]. In this case, the generalization of eq. (5.2) is given by

$$\Delta \phi > \int_0^{N_c} \sqrt{\frac{r}{8 c_s^2 P_X}} dN,$$  \hspace{1cm} (5.3)

where the sound speed $c_s = 1$ in the case of $\alpha-$ attractor model. In eq. (5.3) the term $P_X = \left(1 - \frac{\phi^2}{6\alpha}\right)^2$ affects Lyth bound depending on the value of the parameter $\alpha$. From eq. (3.8) we know that the $\alpha$ parameter is directly related to the inflaton dynamics. In figure 3, we depict the parameter space for large and small field inflation overlapped on the region where $0.962 < n_s < 0.974$. Here, we explicitly characterize the possibility of super Planckian excursion of the field $\phi$ attributing to the field value at the end of inflation $n \gtrsim 2$ and the parameter $\beta \sim -0.01$ (see left panel of figure 3). The field $\phi$ is sub Planckian for $0 < n < O(10)$ and the parameter $\beta \sim -0.002$ (see right panel of figure 3). We present the corresponding predictions in figure 5, where we found that the large field inflation in the non-slow-roll context can give rise to the tensor to scalar ratio $0.003 \lesssim r < 0.1$ and the spectral index $0.955 \lesssim n_s \lesssim 0.964$. Whereas in the case of small field we obtain $0 \lesssim r < 0.1$ and the spectral index $0.96 \lesssim n_s \lesssim 0.967$.

The parametrization used in eq. (3.2) leads to an attractor starting at $r \sim 5.5 \times 10^{-4}$ which is the prediction for $n = 1$. We find that $r \to 0$ as $n \to 0$ (or equivalently $\alpha \to 0$). We depict this behavior in figure 4. This attractor behaviour resembles with the recently studied E-models [52]. The most interesting feature of our study is that, even with non-slow-roll dynamics of the inflaton, $\alpha-$attractors still appear to be the most promising models in the $(n_s, r)$ plane. Including the higher order corrections in eqs. (4.9) and (4.16) we have undetectably small deviation from the standard consistency relation $r = -8n_t$ as presented in the right panel of figure 5. However, the validity of the standard consistency relation remains an open question and not even expected to be tested in any future CMB observations [53].
Figure 4. Plot of tensor scalar ratio \( r \) versus \( \alpha \). Here we have taken \( \beta \sim -0.002 \) and \( 0 < n < 10 \). This plot is for \( N = 60 \).

Figure 5. Parametric plots of spectral index \( (n_s) \) verses tensor scalar ratio \( (r) \) (left panel), \( \alpha \) verses the ratio of tensor scalar ratio and tensor tilt (right panel). In these plots the blue line denote predictions for small field inflation for which we take \( \beta \sim -0.002 \) and \( 0 < n < 10 \). In this case \( r \to 0 \) as \( n \to 0 \) (equivalently \( \alpha \to 0 \)). The black line denote predictions for large field inflation for which \( \beta \sim -0.01 \) and \( 2 < n < 10 \). In this case \( r \gtrsim \mathcal{O} \left( 10^{-3} \right) \). We have considered \( N = 60 \).

6 Embedding in \( \mathcal{N} = 1 \) SUGRA

In this section, we revise the embedding of \( \alpha \)-attractor within \( \mathcal{N} = 1 \) SUGRA [16] and verify the stability of inflaton trajectory [27, 28] in the context of non-slow-roll dynamics.

The \( \alpha \)-attractor model can be embedded in SUGRA using 3 chiral multiplets: a conformon \( X^0 \), an inflaton \( X^1 = \Phi = \frac{\phi + i\sigma}{\sqrt{2}} \) and a sGoldstino \( X^2 = S \). In order to extract a Poincaré SUGRA conformon is gauge fixed as \( X^0 = \overline{X^0} = \sqrt{3} \). We write the Kähler and superpotential in the similar way as studied in refs. [16, 32],

\[
\mathcal{K} = -3\alpha \log \left( 1 - ZZ - \frac{SS}{3\alpha} + \frac{g}{3\alpha^2} \left( \frac{SS}{1 - ZZ} \right)^2 - \frac{\gamma}{3\alpha^2} \left( \frac{SS}{1 - ZZ} \right)^2 \right),
\]

\[
W = S f (Z) \left( 1 - Z^2 \right)^{(3\alpha - 1)/2},
\]

where \( Z = \frac{X^1}{X^0} = \frac{\Phi}{\sqrt{6\alpha}} \) and \( f (Z) \) is an arbitrary function and the square of which serves as
the inflaton potential along $S = \text{Im}\Phi = 0$. In the Kähler potential in eq. (6.1) we added an extra term $\frac{sS(2-z)^2}{(1-zZ)^2}$ in order to stabilize the inflaton trajectory in the direction of $\text{Im}\Phi$ for any value of $\alpha$. Although in some cases it is not required to add this extra term [32, 52]. In our case, we only focus our attention to the form of Kähler potential given by eq. (6.1).

The mass squares of $S$ and $\text{Im}\Phi$ for a given Kähler potential are given by [28],

$$
\begin{align*}
m^2_\sigma &= 2 \left( 1 - \kappa_{\Phi S S \bar{S}} \right) f^2 + (\partial_\phi f)^2 - f \partial_\phi^2 f, \\
m^2_s &= -\kappa_{SS\overline{S}S} f^2 + (\partial_\phi f)^2,
\end{align*}
$$

(6.3)

where all the terms in eq. (6.3) are to be evaluated along the inflaton trajectory $S = \text{Im}\Phi = 0$. And here $\kappa_{\alpha\beta\gamma\delta} = \partial_\alpha \partial_\beta \partial_\gamma \partial_\delta \kappa$. For the stability of the inflaton trajectory it is required to have $m^2_\sigma, m^2_s \gg H^2$ during inflation, in order to ensure the absence of isocurvature perturbations and therefore to have inflation solely driven by a single field [28].

For the Kähler potential given by eq. (6.1) we obtain,

$$
\begin{align*}
\kappa_{\Phi S S \bar{S}} &= -\frac{36\alpha^2 (6(\alpha - 2\gamma) + \phi^2)}{(\phi^2 - 6\alpha)^3}, \\
\kappa_{SS\overline{S}S} &= \frac{24\alpha(1 - 6g)}{(\phi^2 - 6\alpha)^2}.
\end{align*}
$$

(6.4)

Evaluating the masses $m^2_\sigma$ and $m^2_s$ for the local shape of inflaton potential given by eq. (3.5) for $n = 1$, we obtain $m^2_\sigma, m^2_s \gg H^2$ for $g, \gamma \geq 0.2$ and for $\alpha \sim 0.17$. For example, in figure 6, we depict the ratio of inflaton mass square to Hubble parameter square during inflation for a chosen values of $(g, \gamma)$.

We can similarly verify the stability of the inflaton trajectory for $n \neq 1$ by appropriate choice of free parameters $(g, \gamma)$.

7 Conclusions and outlook

In this work we have considered the $\alpha$–attractor models from a new perspective, more precisely, employing the framework of non-slow-roll approach in the way it was recently proposed by Gong and Sasaki [1]. We found that the $\alpha$–attractor models are quite compatible
in the \((n_s, r)\) plane of Planck 2015 within non-slow-roll inflaton dynamics. We showed that such a particular inflationary scenario predicts an attractor at \(n_s \approx 0.967\) and \(r \approx 5.5 \times 10^{-4}\). We further found that the model can in principle predict any \(r < 0.1\). In addition, we have extracted relation (3.8) between the \(\alpha-\) parameter, to the curvature of Kähler geometry, and to the inflaton dynamics. In other words, in our model, the curvature of the Kähler geometry defines the local shape of the inflaton potential during inflation. This constitutes an interesting phenomenon which might be useful to understand the pre-inflationary physics. Furthermore, we also studied the possibility of large and small field inflation in the non-slow-roll context and contrasted them in terms of the predictions of the tensor to scalar ratio.

In this work we have not considered any particular form for the inflaton potential, since the assumption of \(N = N(\phi)\) during inflation provides all the necessary ingredients for studying inflation. It would be interesting to look at the reheating phase in this model, given an adequate assumption about the shape of the potential in the post inflationary epoch. In the view of the recent literature about the reheating process regarding Starobinsky inflation [54], it would also be interesting to consider the role of the superfields and their various possible decay modes, as well as of the inflaton. There are other possible directions to be explored, in the context of non-slow-roll dynamics in \(\alpha-\) attractors with respect to SUGRA, such as studying the various mechanisms for SUSY breaking and the origin of dark energy. In the sequence of this work, we are considering the extension of the non-slow-roll dynamics to the \(\xi-\) attractors case [55].

Acknowledgments

We would like to thank the anonymous referee for useful comments. SK is grateful to the IIT Kanpur for the hospitality and to the organizers of the conference on “Primordial Universe after Planck” at the Institut d’Astrophysique de Paris where part of this work was developed. SK acknowledges for the support of grant SFRH/BD/51980/2012 from Portuguese Agency Fundação para a Ciência e Tecnologia. This research work is supported by the grants PTDC/FIS/111032/2009 and UID/MAT/00212/2013. Work of SD is supported by Department of Science and Technology, Government of India under the Grant Agreement number IFA13-PH-77 (INSPIRE Faculty Award).

References

[1] J.-O. Gong and M. Sasaki, *A new parameter in attractor single-field inflation*, Phys. Lett. B 747 (2015) 390 [arXiv:1502.04167] [INSPIRE].

[2] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XX. Constraints on inflation*, arXiv:1502.02114 [INSPIRE].

[3] BICEP2 and PLANCK collaborations, P.A.R. Ade et al., *Joint analysis of BICEP2/Keck Array and Planck data*, Phys. Rev. Lett. 114 (2015) 101301 [arXiv:1502.00612] [INSPIRE].

[4] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XVII. Constraints on primordial non-Gaussianity*, arXiv:1502.01592 [INSPIRE].

[5] BICEP2 and Keck Array collaborations, P.A.R. Ade et al., *Improved constraints on cosmology and foregrounds from BICEP2 and Keck Array cosmic microwave background data with inclusion of 95 GHz band*, Phys. Rev. Lett. 116 (2016) 031302 [arXiv:1510.09217] [INSPIRE].
[6] J. Martin, C. Ringeval and V. Vennin, *Encyclopædia inflationaris*, *Phys. Dark Univ.* **5-6** (2014) 75 [arXiv:1303.3787] [SPIRE].

[7] A.A. Starobinsky, *A new type of isotropic cosmological models without singularity*, *Phys. Lett. B* **91** (1980) 99 [SPIRE].

[8] A.A. Starobinsky, *The perturbation spectrum evolving from a nonsingular initially de-Sitter cosmology and the microwave background anisotropy*, *Sov. Astron. Lett.* **9** (1983) 302 [SPIRE].

[9] F.L. Bezrukov and M. Shaposhnikov, *The standard model Higgs boson as the inflaton*, *Phys. Lett. B* **659** (2008) 703 [arXiv:0710.3755] [SPIRE].

[10] A. Linde, *Inflationary cosmology after Planck 2013*, arXiv:1402.0526 [SPIRE].

[11] C.P. Burgess, M. Cicoli and F. Quevedo, *String inflation after Planck 2013*, *JCAP* **11** (2013) 003 [arXiv:1306.3512] [SPIRE].

[12] J. Martin, *The observational status of cosmic inflation after Planck*, arXiv:1502.05733 [SPIRE].

[13] R. Kallosh and A. Linde, *Universality class in conformal inflation*, *JCAP* **07** (2013) 002 [arXiv:1306.5220] [SPIRE].

[14] R. Kallosh and A. Linde, *Multi-field conformal cosmological attractors*, *JCAP* **12** (2013) 006 [arXiv:1309.2015] [SPIRE].

[15] R. Kallosh and A. Linde, *Superconformal generalizations of the Starobinsky model*, *JCAP* **06** (2013) 028 [arXiv:1306.3214] [SPIRE].

[16] R. Kallosh, A. Linde and D. Roest, *Superconformal inflationary α-attractors*, *JHEP* **11** (2013) 198 [arXiv:1311.0472] [SPIRE].

[17] R. Kallosh and A. Linde, *Non-minimal inflationary attractors*, *JCAP* **10** (2013) 033 [arXiv:1307.7938] [SPIRE].

[18] R. Kallosh and A. Linde, *Superconformal generalization of the chaotic inflation model $\frac{1}{4} \phi^4 - \frac{i}{2} \phi^2 R$*, *JCAP* **06** (2013) 027 [arXiv:1306.3211] [SPIRE].

[19] R. Kallosh, A. Linde and D. Roest, *Large field inflation and double α-attractors*, *JHEP* **08** (2014) 052 [arXiv:1405.3646] [SPIRE].

[20] M. Ozkan, Y. Pang and S. Tsujikawa, *Planck constraints on inflation in auxiliary vector modified f(R) theories*, *Phys. Rev. D* **92** (2015) 023530 [arXiv:1502.06341] [SPIRE].

[21] M. Galante, R. Kallosh, A. Linde and D. Roest, *Unity of cosmological inflation attractors*, *Phys. Rev. Lett.* **114** (2015) 141302 [arXiv:1412.3797] [SPIRE].

[22] S. Cecotti and R. Kallosh, *Cosmological attractor models and higher curvature supergravity*, *JHEP* **05** (2014) 114 [arXiv:1403.2932] [SPIRE].

[23] D. Roest, *Universality classes of inflation*, *JCAP* **01** (2014) 007 [arXiv:1309.1285] [SPIRE].

[24] A. Linde, *Does the first chaotic inflation model in supergravity provide the best fit to the Planck data?*, *JCAP* **02** (2015) 030 [arXiv:1412.7111] [SPIRE].

[25] A.B. Goncharov and A.D. Linde, *Chaotic inflation in supergravity*, *Phys. Lett. B* **139** (1984) 27 [SPIRE].

[26] A.S. Goncharov and A.D. Linde, *Chaotic inflation of the universe in supergravity*, *Sov. Phys. JETP* **59** (1984) 930 [Zh. Eksp. Teor. Fiz. **86** (1984) 1594] [SPIRE].

[27] R. Kallosh and A. Linde, *New models of chaotic inflation in supergravity*, *JCAP* **11** (2010) 011 [arXiv:1008.3375] [SPIRE].

[28] R. Kallosh, A. Linde and T. Rube, *General inflaton potentials in supergravity*, *Phys. Rev. D* **83** (2011) 043507 [arXiv:1011.5945] [SPIRE].
[29] V. Demozzi, A. Linde and V. Mukhanov, *Supercurvaton*, JCAP 04 (2011) 013 [arXiv:1012.0549] [insPIRE].

[30] H. Abe, S. Aoki, F. Hasegawa and Y. Yamada, *Illustrating SUSY breaking effects on various inflation mechanisms*, JHEP 01 (2015) 026 [arXiv:1408.4875] [insPIRE].

[31] A. Linde, D. Roest and M. Scalisi, *Inflation and dark energy with a single superfield*, JCAP 03 (2015) 017 [arXiv:1412.2790] [insPIRE].

[32] R. Kallosh and A. Linde, *Planck, LHC and α-attractors*, Phys. Rev. D 91 (2015) 083528 [arXiv:1502.07733] [insPIRE].

[33] J.J.M. Carrasco, R. Kallosh and A. Linde, *α-attractors: Planck, LHC and dark energy*, JHEP 10 (2015) 147 [arXiv:1506.01708] [insPIRE].

[34] M. Scalisi, *Cosmological α-attractors and de Sitter landscape*, JHEP 12 (2015) 134 [arXiv:1506.01368] [insPIRE].

[35] D. Roest and M. Scalisi, *Cosmological attractors from α-scale supergravity*, Phys. Rev. D 92 (2015) 043525 [arXiv:1503.07909] [insPIRE].

[36] A. Linde, *Single-field α-attractors*, JCAP 05 (2015) 003 [arXiv:1504.00663] [insPIRE].

[37] D. Roest, M. Scalisi and I. Zavala, *Kähler potentials for Planck inflation*, JCAP 11 (2013) 007 [arXiv:1307.4343] [insPIRE].

[38] A.D. Linde, *Chaotic inflation*, Phys. Lett. B 129 (1983) 177 [insPIRE].

[39] C. Armendariz-Picon, T. Damour and V.F. Mukhanov, *k-inflation*, Phys. Lett. B 458 (1999) 209 [hep-th/9904075] [insPIRE].

[40] X. Chen, M.-X. Huang, S. Kachru and G. Shiu, *Observational signatures and non-Gaussianities of general single field inflation*, JCAP 01 (2007) 002 [hep-th/0605045] [insPIRE].

[41] J. Martin, H. Motohashi and T. Suyama, *Ultra slow-roll inflation and the non-Gaussianity consistency relation*, Phys. Rev. D 87 (2013) 023514 [arXiv:1211.0083] [insPIRE].

[42] H. Motohashi, A.A. Starobinsky and J. Yokoyama, *Inflation with a constant rate of roll*, JCAP 09 (2015) 018 [arXiv:1411.5021] [insPIRE].

[43] A.G. Muslimov, *On the scalar field dynamics in a spatially flat Friedman universe*, Class. Quant. Grav. 7 (1990) 231 [insPIRE].

[44] D.S. Salopek and J.R. Bond, *Nonlinear evolution of long wavelength metric fluctuations in inflationary models*, Phys. Rev. D 42 (1990) 3936 [insPIRE].

[45] S. Céspedes and A.-C. Davis, *Non-canonical inflation coupled to matter*, JCAP 11 (2015) 014 [arXiv:1506.01244] [insPIRE].

[46] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Generalized G-inflation: inflation with the most general second-order field equations*, Prog. Theor. Phys. 126 (2011) 511 [arXiv:1105.5723] [insPIRE].

[47] J. Khoury and F. Piazza, *Rapidly-varying speed of sound, scale invariance and non-Gaussian signatures*, JCAP 07 (2009) 026 [arXiv:0811.3633] [insPIRE].

[48] R.H. Ribeiro, *Inflationary signatures of single-field models beyond slow-roll*, JCAP 05 (2012) 037 [arXiv:1202.4453] [insPIRE].

[49] T. Zhu, A. Wang, G. Cleaver, K. Kirsten and Q. Sheng, *Power spectra and spectral indices of k-inflation: high-order corrections*, Phys. Rev. D 90 (2014) 103517 [arXiv:1407.8011] [insPIRE].

[50] D.H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, Phys. Rev. Lett. 78 (1997) 1861 [hep-ph/9606387] [insPIRE].
[51] D. Baumann and L. McAllister, *A microscopic limit on gravitational waves from D-brane inflation*, *Phys. Rev. D* **75** (2007) 123508 [hep-th/0610285] [INSPIRE].

[52] J.J.M. Carrasco, R. Kallosh and A. Linde, *Cosmological attractors and initial conditions for inflation*, *Phys. Rev. D* **92** (2015) 063519 [arXiv:1506.00936] [INSPIRE].

[53] J. Errard, S.M. Feeney, H.V. Peiris and A.H. Jaffe, *Robust forecasts on fundamental physics from the foreground-obscured, gravitationally-lensed CMB polarization*, arXiv:1509.06770 [INSPIRE].

[54] T. Terada, Y. Watanabe, Y. Yamada and J. Yokoyama, *Reheating processes after Starobinsky inflation in old-minimal supergravity*, *JHEP* **02** (2015) 105 [arXiv:1411.6746] [INSPIRE].

[55] K.S. Kumar, J. Marto, P.V. Moniz and S. Das, *Non-slow-roll dynamics in non-minimal inflation*, in preparation.