\textit{J/\psi and \( \eta_c \) in strongly magnetized nuclear matter}  
\textit{– a QCD sum rule approach}  

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Abstract  

The mass modifications of the charmonium states \textit{J/\psi} and \textit{\eta_c} in strongly magnetized nuclear matter are calculated using a QCD sum rule approach. The shifts in the masses of these states arise due to the medium modifications of the scalar and twist-2 gluon condensates. The gluon condensates are calculated within a chiral effective model, from the medium modifications of a scalar dilaton field, which is incorporated in the hadronic model to simulate the broken scale invariance of QCD. The effects of the magnetic field, isospin asymmetry and density on the masses of these charmonium states have been investigated. In the presence of the magnetic field, there are contributions from the Landau levels for the proton, the charged baryon. The anomalous magnetic moments of the nucleons have been taken into consideration in the present work, and the contributions of these effects are compared to the case when the anomalous magnetic moments are not taken into account.

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I. INTRODUCTION

The study of the properties of the hadrons in the medium is an important topic of research in strong interaction physics due to its relevance in the ultra relativistic heavy ion collision experiments as well as in nuclear astrophysics, e.g. in the study of bulk properties of neutron stars. The heavy flavour hadrons have also been a topic of research in the recent years, due to the prediction of bound states with the nucleus. Such bound states may be possible due to the attractive interaction of these hadrons in nuclear matter. The effects of magnetic field on the hadron properties are also important to investigate, as strong magnetic fields are known to be created in ultra relativistic heavy ion collision experiments.

The open charm (bottom) mesons, e.g, the $D(\bar{D})$ and $B(\bar{B})$ mesons, as well as, the hidden charm (bottom) mesons, namely the charmonium and bottomonium states have been studied extensively in the literature. In the QCD sum rule approach, the mass modifications of the open charm (bottom) mesons, due to the presence of the light quark (antiquark) in these mesons, arise due to interaction with the light quark condensates. The heavy flavour vector mesons, e.g, the charmonium and bottomonium states are modified due to the gluon condensates in the medium within the QCD sum rule framework. On the other hand, the modifications of the light vector mesons are due to interaction with the light quark condensates. In Ref. [13], the mass modifications of the charmonium states have been studied using the leading order QCD formula and using the linear density approximation for the medium modification of the gluon condensates in the nuclear medium. The QCD sum rule calculations for the charmonium states have been generalized to finite temperatures, where the medium modifications of these states have been studied arising due to the temperature effects on the gluon condensates, extracted from lattice calculations.

Using a chiral effective model, the mass modifications of the open charm mesons have been studied, arising from their interactions with the nucleons, hyperons and scalar mesons. The broken scale invariance of QCD is incorporated in the chiral effective model through a scalar dilaton field. The in-medium masses of the charmonium states have been investigated within the chiral effective model, by computing the scalar gluon condensate in the hadronic medium from the medium modification of a scalar dilaton field. This investigation shows small drop of the $J/\psi$ mass in the medium, whereas the masses
of the excited charmonium states are observed to have appreciable drop at high densities. In the present investigation, we study the in-medium modifications of the vector meson, $J/\psi$ and the pseudoscalar meson, $\eta_c$, using QCD sum rules, with the gluon condensates in the hadronic medium calculated in the chiral effective model. We calculate the contributions of the scalar gluon condensates, $\langle \alpha_s \pi^a G_{\mu\nu}^a G^{a\mu\nu} \rangle$ and twist-2 tensorial gluon operator, $\langle \alpha_s \pi^a \nabla_\sigma G_{\mu\nu}^a G^{a\mu\nu} \rangle$ up to dimension four, to investigate the masses of $J/\psi$ and $\eta_c$ in nuclear matter in the presence of magnetic field. The chiral SU(3) model has been used successfully to study the medium modifications of light vector mesons and strange pseudoscalar mesons (kaons and antikaons), as well as to investigate the bulk matter in the interior of (proto) neutron stars. The model has been generalized to the bottom sector and the in-medium masses of the open (strange) bottom mesons as well as bottomonium states have been studied. The mass modifications of the open charm (bottom) as well as charmonium (bottomonium) states have been used to compute the in-medium partial decay widths of the charmonium (bottomonium) to $D\bar{D}$ ($B\bar{B}$) using a light quark pair model, namely $3P_0$ model as well as field theoretic model for composite hadrons. The effects of magnetic field have also been studied on the masses of the open charm, open bottom, the charmonium as well as the bottomonium states in nuclear matter including the effects of the anomalous magnetic moments of the nucleons. In the present investigation, we study the in-medium masses of the $J/\psi$ and $\eta_c$ mesons in nuclear matter in the presence of strong magnetic fields, using the QCD sum rule approach, with the gluon condensates in the medium calculated from the medium changes of the dilaton field, $\chi$, within the chiral SU(3) model.

The outline of the paper is as follows: In section II, we give a brief introduction of chiral SU(3) model used in the present investigation, to calculate the in-medium gluon condensates needed to study the in-medium masses of charmonium states $J/\psi$ and $\eta_c$, using the QCD sum rule approach. The medium modifications of these charmonium states arise from the medium modification of the scalar and twist-2 gluon condensates. Section III discusses briefly the QCD sum rule approach used to calculate the masses of $J/\psi$ and $\eta_c$. In section IV, we discuss the results of the present investigation. Section V summarizes the conclusions of the present work.
II. THE HADRONIC CHIRAL SU(3) × SU(3) MODEL

In this section, we briefly describe the chiral SU(3) model used to calculate the gluon condensates in the magnetized nuclear matter. The in-medium masses of the lowest charmonium states, J/ψ and ηc, are then computed from these gluon condensates, using a QCD sum rule framework. The chiral SU(3) model is based on the nonlinear realization of chiral symmetry and broken scale invariance. The latter is incorporated with a scale breaking logarithmic potential in terms of a scalar dilaton field, whose medium modification yields the medium dependent gluon condensates. The effective hadronic chiral Lagrangian density is given as

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{BM}} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{\text{SB}} + \mathcal{L}_{\text{mag}}.$$  \hspace{1cm} (1)

In the above Lagrangian density, the first term $\mathcal{L}_{\text{kin}}$ corresponds to the kinetic energy terms of the baryons and the mesons. $\mathcal{L}_{\text{BM}}$ is the baryon-meson interaction term, $\mathcal{L}_{\text{vec}}$ corresponds to the interactions of the vector mesons, $\mathcal{L}_0$ contains the meson-meson interaction terms, $\mathcal{L}_{\text{scalebreak}}$ is a scale invariance breaking logarithmic potential given in terms of a scalar dilaton field, $\mathcal{L}_{\text{SB}}$ is the explicit chiral symmetry breaking term, and $\mathcal{L}_{\text{mag}}$ is the contribution from the magnetic field, given as

$$\mathcal{L}_{\text{mag}} = -\bar{\psi}_i \gamma_\mu A_\mu \psi_i - \frac{1}{4} \kappa_i \mu_\mu \bar{\psi}_i \gamma_\sigma \sigma_{\mu\nu} F_{\mu\nu} \psi_i - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$ \hspace{1cm} (2)

where, $\psi_i$ is the field operator for the $i$-th baryon ($i = p, n$, for nuclear matter, as considered in the present work), and the parameter $\kappa_i$ in the second term in equation (2) is related to the anomalous magnetic moment of the $i$-th baryon. The values of $\kappa_p$ and $\kappa_n$ are given as 3.5856 and -3.8263 respectively, which are the values of the gyromagnetic ratio corresponding to the anomalous magnetic moments of the proton and neutron respectively.

Within the chiral SU(3) model used in the present investigation, the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G_\mu^a G^{a\mu} \rangle$, as well as the twist-2 gluon operator, $\langle \frac{\alpha_s}{\pi} G_\mu^a G^{a\nu} G^{a\nu} \rangle$, are simulated by the scalar dilaton field, $\chi$. These are obtained from the energy momentum tensor

$$T_{\mu\nu} = (\partial_\mu \chi) \left( \frac{\partial \mathcal{L}_\chi}{\partial (\partial_\nu \chi)} \right) - g_{\mu\nu} \mathcal{L}_\chi.$$ \hspace{1cm} (3)
derived from the Lagrangian density for the dilaton field, given as
\[ L_\chi = \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - k_4 \chi^4 \]
\[ - \frac{1}{4} \chi^4 \ln \left( \frac{\chi^4}{\chi_0^4} \right) + \frac{d}{3} \chi^4 \ln \left( \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \delta_0} \left( \frac{\chi}{\chi_0} \right)^3 \right). \]  
(4)

The energy momentum tensor, accounting for the current quark masses, can be written as
\[ T_{\mu \nu} = -ST(G^a_{\mu \sigma}G^a_{\nu}{}^\sigma) + \frac{g_{\mu \nu}}{4} \left( \sum_i m_i \bar{q}_i q_i + \frac{\beta_{QCD}}{2g} G^a_{\sigma \kappa} G^{a \sigma \kappa} \right) \]  
(5)
where the first term is the symmetric traceless part and second term is the trace part of the energy momentum tensor. Writing
\[ \langle \frac{\alpha_s}{\pi} G^a_{\mu \sigma} G^a_{\nu}{}^\sigma \rangle = \left( u_\mu u_\nu - \frac{g_{\mu \nu}}{4} \right) G_2, \]  
(6)
where \( u_\mu \) is the 4-velocity of the nuclear medium, taken as \( u_\mu = (1, 0, 0, 0) \), we obtain the energy momentum tensor in QCD as
\[ T_{\mu \nu} = -\left( \frac{\pi}{\alpha_s} \right) \left( u_\mu u_\nu - \frac{g_{\mu \nu}}{4} \right) G_2 + \frac{g_{\mu \nu}}{4} \left( \sum_i m_i \bar{q}_i q_i + \frac{\beta_{QCD}}{2g} G^a_{\sigma \kappa} G^{a \sigma \kappa} \right) \]  
(7)
Equating the energy-momentum tensors given by equations (3) and (7) and multiplying by \( (u^\mu u^\nu - \frac{g_{\mu \nu}}{4}) \), we obtain the expression for \( G_2 \) as
\[ G_2 = -\frac{\alpha_s}{\pi} \left( \partial_{\alpha} \chi \right) \left( \frac{\partial L_\chi}{\partial \partial_{\alpha} \chi} \right) + \frac{4}{3} \left( \partial_{\alpha} \chi \right) \left( \partial_{\beta} \chi \right). \]  
(8)
Using the Euler-Lagrange’s equation for \( \chi \) and dropping a total divergence term in equation (8), and using the fact that the twist-2 gluon condensate vanishes for vacuum \( \langle \rangle \), i.e., using the condition that \( (G_2)_{vac} = 0 \), the expression for \( G_2 \) is given as
\[ G_2 = \frac{\alpha_s}{\pi} \left[ - (1 - d + 4k_4)(\chi^4 - \chi_0^4) - \chi^4 \ln \left( \frac{\chi^4}{\chi_0^4} \right) \right. \]
\[ + \left. \frac{4}{3} d \chi^4 \ln \left( \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \delta_0} \left( \frac{\chi}{\chi_0} \right)^3 \right) \right]. \]  
(9)
Equating the trace of the energy momentum tensor for the QCD with that of the chiral effective model, gives the relation between the scalar gluon condensate to the scalar dilaton field, \( \chi_i \), as
\[ \langle \frac{\alpha_s}{\pi} G^a_{\mu \nu} G^a_{\mu \nu} \rangle = \frac{8}{9} \left( 1 - d \right) \chi^4 + \left( \frac{\chi}{\chi_0} \right)^2 \left( m_\pi^2 f_\pi \sigma + (\sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi) \zeta \right) \]  
(10)
The scalar gluon condensate also depends on the scalar fields, $\sigma$ and $\zeta$, as we have included the term due to the finite quark masses, in the trace of the energy momentum tensor in QCD. The in-medium masses of charmonium states are modified due to the scalar gluon condensate and the twist-2 gluon operators (through the expression $G_2$).

### III. QCD SUM RULE APPROACH AND IN-MEDIUM MASSES OF $J/\psi$ AND $\eta_c$

In the present section, we shall use the medium modifications of the gluon condensate, calculated from the dilaton field in the chiral effective model, to compute the masses of the charmonium states $J/\psi$ and $\eta_c$ in isospin asymmetric nuclear matter in the presence of magnetic field. These masses have been studied in Ref. [10] for the case of zero magnetic field. Using QCD sum rule approach [9], the in-medium masses of the lowest charmonium states can be written as

$$m^2 \simeq M_{n-1}^J(\xi) - 4m_c^2\xi$$

where $M_{n-1}^J$ is the $n$th moment of the meson and $\xi$ is the normalization scale. Using operator product expansion, the moment $M_{n-1}^J$ can be written as [9]

$$M_{n-1}^J(\xi) = A_{n-1}^J(\xi)[1 + a_n^J(\xi)\alpha_s + b_n^J(\xi)\phi_b + c_n^J(\xi)\phi_c],$$

where $A_n^J(\xi)$, $a_n^J(\xi)$, $b_n^J(\xi)$ and $c_n^J(\xi)$ are the Wilson coefficients. The common factor $A_n^J$ results from the bare loop diagram. The coefficients $a_n^J$ take into account perturbative radiative corrections, while the coefficients $b_n^J$ are associated with the scalar gluon condensate term

$$\phi_b = \frac{4\pi^2}{9} \frac{\langle \alpha_s G_{\mu\nu}G^{\mu\nu} \rangle}{(4m_c^2)^2}$$

As already mentioned, the contribution of the scalar gluon condensate is taken through the dilaton field within the chiral $SU(3)$ model used in the present investigation. Using equation [10], the above equation can be rewritten in terms of the dilaton field $\chi$, as

$$\phi_b = \frac{32\pi^2}{81(4m_c^2)^2} \left[ (1 - d)\chi^4 + \left( \frac{\chi}{\chi_0} \right)^2 \left( m_{\pi}^2 f_{\pi}\sigma + \left( \sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_{\pi}^2 f_{\pi} \right)\zeta \right) \right].$$
The coefficients $A_n^J$, $a_n^J$, and $b_n^J$ are listed in Ref. [53]. The coefficients $c_n^J$ are associated with the value of $\phi_c$, which gives the contribution from twist-2 gluon operator and is given as

$$\phi_c = \frac{4\pi^2}{3(4m_c^2)} G_2,$$

where $G_2$ is given by equation (9). The Wilson coefficients, $c_n^J$, in the vector channel (for $J/\psi$) and the pseudoscalar channel (for $\eta_c$) can be found in Ref. [9]. The parameters $m_c$ and $\alpha_s$ are the running charm quark mass and running coupling constant and are $\xi$ dependent [53]. In the next section, we present and discuss the results of our present work of the investigation of the in-medium masses of $J/\Psi$ and $\eta_c$ in isospin asymmetric nuclear matter in the presence of magnetic field.

IV. RESULTS AND DISCUSSIONS

In this section, we investigate the in-medium masses of the $J/\psi$ and $\eta_c$ mesons in strongly magnetized nuclear matter, using the QCD sum rule approach [8–10, 15] from the scalar and twist-2 gluon condensates calculated within the chiral effective model. The broken scale invariance of QCD is incorporated in the effective hadronic model through a scale breaking logarithmic potential in terms of a scalar dilation field, $\chi$. These masses in hot nuclear matter were calculated within the QCD sum rule framework using the gluon condensates calculated within the chiral effective model, arising due to the medium modification of the scalar dilaton field. In the present work, the effects of magnetic field are studied on the masses of $J/\psi$ and $\eta_c$ in addition to the effects of density as well as isospin asymmetry of the nuclear matter.

In the chiral effective model, the calculations are done in the mean field approximation. In this approximation, the meson fields are treated as classical fields. In the presence of a magnetic field, the proton has contributions from the Landau energy levels. The effects of the anomalous magnetic moments of the nucleons are also taken into consideration in the present work of investigating the in-medium masses of the $J/\psi$ and $\eta_c$ using the QCD sum rule approach. For given values of baryon density, $\rho_B$, isospin asymmetry, $\eta = (\rho_n - \rho_p)/(2\rho_B)$ ($\rho_p$ and $\rho_n$ are the number densities for the proton and neutron respectively), and magnetic field, the coupled equations of motion for the scalar fields, $\sigma$, $\zeta$, $\delta$ and $\chi$ are solved. The
FIG. 1: (Color online) The mass of $J/\psi$ plotted as a function of $n$, at given baryon densities and magnetic fields, for the symmetric nuclear matter ($\eta=0$).
FIG. 2: (Color online) The mass of $\eta_c$ plotted as a function of n, at given baryon densities and magnetic fields, for the symmetric nuclear matter ($\eta=0$).
FIG. 3: (Color online) The mass of $J/\psi$ plotted as a function of $n$, at given baryon densities and magnetic fields, for the isospin asymmetry parameter, $\eta=0.5$. 
FIG. 4: (Color online) The mass of $\eta_c$ plotted as a function of $n$, at given baryon densities and magnetic fields, for the isospin asymmetry parameter, $\eta=0.5$. 
twist-2 and the scalar gluon condensates, are calculated from the equations (9) and (10), which are then used to calculate the values of $\phi_b$ and $\phi_c$ (given by equations (14) and (15) respectively). Using the values of $\phi_b$ and $\phi_c$, the in-medium masses of the $J/\psi$ and $\eta_c$ are calculated using equation (11).

In isospin symmetric nuclear medium, at baryon densities, $\rho_B = 0$ and $\rho_0$, the values of the dilaton field, $\chi$ are 409.76 and 406.38 MeV respectively and hence using equation (10), the values of the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \rangle$ turn out to be (373 MeV)$^4$ and (371.6 MeV)$^4$ for densities $\rho_B = 0$ and $\rho_0$ respectively. The values of $\phi_b$, turn out to be $2.3 \times 10^{-3}$ and $2.27 \times 10^{-3}$ in the vacuum and at nuclear matter saturation density, $\rho_0$, respectively. These may be compared with the values of $\phi_b$ to be equal to $1.8 \times 10^{-3}$ and $1.7 \times 10^{-3}$ respectively for $\rho_B = 0$ and for $\rho_B = \rho_0$, obtained from the values of scalar gluon condensate of (350 MeV)$^4$ and (344.81 MeV)$^4$ respectively in vacuum and at nuclear saturation density, $\rho_0$ in Ref. [9] in the linear density approximation. We might note here that the value of nuclear matter saturation density used in the present calculations is 0.15 fm$^{-3}$ and in Ref. [9], it was taken to be 0.17 fm$^{-3}$.

In the present investigation, we choose $\xi = 1$, leading to the $\xi$ dependent coupling constant and charm quark mass, $\alpha_s = 0.21$ and $m_c = 1.24$ GeV respectively [8, 9, 53]. With these parameters and with $\phi_b$ calculated within the chiral effective model, the vacuum masses of $J/\psi$ and $\eta_c$ are obtained to be 3196.2 and 3066.56 MeV respectively. In figures 1 and 2, the masses of the $J/\psi$ and $\eta_c$ are plotted for isospin symmetric nuclear matter for various values of the magnetic field accounting for the anomalous magnetic effects (AMM) of the nucleons. These results are compared to the case when the AMM effects are not taken into consideration (shown as dotted lines). The values obtained for $J/\psi$ at the nuclear matter saturation density are 3191.9 (3191.77), 3191.9 (3191.757), 3191.91 (3191.74), 3191.88 (3191.7) MeV leading to mass shifts (in MeV) of $-4.3(-4.43)$, $-4.3(-4.43)$, $-4.29(-4.46)$ and $-4.32(-4.5)$ respectively, from the vacuum value of 3196.2 MeV, for values of magnetic field as $|eB|$ equal to $4m^2\pi$, $8m^2\pi$, $10m^2\pi$, $12m^2\pi$ respectively, with (without) the effects of anomalous magnetic moments (AMM) of the nucleons. For $\eta_c$, at $\rho_B = \rho_0$, the values of the mass are obtained as 3062.49 (3062.39), 3062.49 (3062.376), 3062.5 (3062.382), 3062.504 (3062.349) MeV leading to mass shifts (in MeV) of $-4.07(-4.17)$, $-4.07(-4.184)$, $-4.06(-4.178)$ and
−4.056(−4.211) respectively, for values of magnetic field as $|eB|$ equal to $4m_n^2$, $8m_n^2$, $10m_n^2$, $12m_n^2$ respectively, with (without) the effects of anomalous magnetic moments (AMM) of the nucleons. These values of mass shifts for $J/\psi$ and $\eta_c$ mesons may be compared with the mass shifts of $−7$ MeV and $−5$ MeV respectively obtained in the linear density approximation in Ref. [9]. In Ref. [8] the operator product expansion was carried out up to dimension six and mass shift for $J/\psi$ was found to be $−4$ MeV at nuclear saturation density $\rho_0$.

For density of $2\rho_0$, the values for the mass of the $J/\psi$ are obtained as 3186.256 (3185.62), 3186.13 (3185.2) and 3186.315 (3185.171) MeV respectively, leading to mass shifts of $−10.07(−10.945)$, $−10.03(−11)\text{ and }−9.885(−11.029)$ MeV respectively, for values of magnetic field as $|eB|$ equal to $4m_n^2$, $8m_n^2$, $10m_n^2$, $12m_n^2$ respectively, with (without) the effects of anomalous magnetic moments (AMM) of the nucleons. For density of $4\rho_0$, for the same magnetic fields, the values for the mass of the $J/\psi$ are obtained as $3179.27 (3178.331)$, $3180 (3176.96)$ and $3180.97 (3176.29)$ MeV respectively, leading to mass shifts of $−16.93(−17.869)$, $−16.2(−19.24)$, $−15.829(−19.637)$ and $−15.23(−19.91)$ MeV respectively, with (without) the AMM effects.

The values of mass of $\eta_c$ at $\rho_B = 2\rho_0$ are obtained as $3058.556 (3058.153)$, $3058.476 (3057.92)$, $3058.519 (3057.89)$ and $3058.592 (3057.871)$ leading to mass shifts of $−8.004(−8.407)$, $−8.084(−8.64)$, $−8.041(−8.67)$ and $−7.968(−8.689)$ for values of magnetic field as $|eB|$ equal to $4m_n^2$, $8m_n^2$, $10m_n^2$, $12m_n^2$ respectively, with (without) the effects of anomalous magnetic moments (AMM) of the nucleons. For density of $4\rho_0$, these values are modified to $3054.33 (3053.79)$, $3054.76 (3053)$, $3054.98 (3052.78)$ and $3055.33 (3052.62)$ MeV respectively, leading to mass shifts (in MeV) of $−12.23(−12.77)$, $−11.8(−13.56)$, $−11.58(−13.78)$ and $−11.23(−13.94)$ respectively with (without) the AMM effects.

In figures 3 and 4, the in-medium masses of $J/\psi$ and $\eta_c$ are plotted for the asymmetric nuclear matter with $\eta=0.5$, for different values of density and magnetic fields. The effects from isospin asymmetry are observed to be lead to lessen the mass shifts of these mesons. For the isospin asymmetric nuclear matter, the values of $J/\psi$ mass shift for $|eB|$ as $8m_n^2$ and $12m_n^2$ for $\rho_B = \rho_0$ for the cases of AMM (no AMM) effects are obtained as $−3.772(−4.125)$ and $−3.688(−4.125)$ respectively, which may be compared with the values for $\eta = 0$ as
−4.07(−4.184) and −4.056(−4.211) respectively. For $\rho_B = 4\rho_0$, these values for $\eta=0.5$ are modified to −13.827(−15.7) and −12.45(−15.7), as compared to the values of −16.2(−19.24) and −15.23(−19.91) MeV respectively. We might note here that when the AMM effects are not taken into consideration, for the asymmetric nuclear matter with $\eta=0.5$, since there are only neutrons in the system, there is no effect from the magnetic field, since the effect of magnetic field can arise in this situation only due to the anomalous magnetic moment of the neutron.

For the isospin asymmetry parameter, $\eta=0.5$, the masses of $\eta_c$ are shown for different values of the magnetic field and density in figure 4. The values of mass shift of $\eta_c$ for $|eB|$ as $8m_\pi^2$ and $12m_\pi^2$ are obtained as $−3.626(−3.917)$ and $−3.557(−3.917)$ for $\rho_B = \rho_0$, and, $−10.369(−11.462)$ and $−9.538(−11.462)$ for $\rho_B = 4\rho_0$. These may be compared to the values of $−4.07(−4.184)$ and $−4.056(−4.211)$ for $\rho_B = \rho_0$ and, $−11.8(−13.56)$ and $−11.23(−13.94)$ for $\rho_B = 4\rho_0$ in symmetric nuclear matter. The difference in the masses of $J/\psi$ and $\eta_c$, when the AMM effects are taken into account and when these are not considered, is observed to be smaller for the isospin asymmetric nuclear matter as compared to the symmetric nuclear matter case.

V. SUMMARY

In summary, in the present investigation, we have studied the mass modifications of the charmonium states, $J/\psi$ and $\eta_c$ in the nuclear medium using QCD sum rule approach, due to modifications of the scalar gluon condensate and twist-2 tensorial gluon operator, calculated within a chiral effective model. The scalar and twist-2 gluon condensates in the nuclear medium are obtained from the medium modification of a scalar dilaton field, $\chi$, which is introduced in the chiral effective model to simulate the broken scale invariance of QCD. The value of the dilaton field in the nuclear matter in the presence of magnetic field, is obtained by solving the equations of motion of $\sigma$, $\zeta$, $\delta$ and $\chi$ fields. The effects of anomalous magnetic moments of the nucleons are observed to be appreciable at higher densities and higher magnetic fields. The isospin asymmetry effects are observed to lead to smaller mass shifts for both $J/\psi$ and $\eta_c$ in the nuclear medium. The density effects are observed to be the dominant medium effects on the masses of these charmonium states.
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