3D model to study transitional regimes of train motion

O Markova\textsuperscript{1,2}, H Kovtun\textsuperscript{1} and V Maliy\textsuperscript{1}

\textsuperscript{1} Institute of Technical Mechanics of the NAS of Ukraine and the SSA of Ukraine, 15 Leshko-Popel street, Dnipro, 49005, Ukraine

\textsuperscript{2} olgamarkova2002@gmail.com

Abstract. When considering issues of increasing train speeds, it is necessary to conduct a preliminary assessment of the entire train dynamic characteristics and forces acting in the elements of cars and a locomotive, as well as in inter-car connections when driving at high speeds along a path of arbitrary shape in plan and profile, and also to consider emergency situations, which may take place during the motion. This in turn requires the development of reliable mathematical models simulating the train motion. The study of dynamic characteristics of railway rolling stock is associated with the consideration of vibrations of complex mechanical systems with a large number of degrees of freedom. The vibrations of cars in the train can be described either by one-dimensional or spatial model. The aim of this work is to show that for some studied cases the modelling of train motion by 3D models more accurately reflects the processes occurring in a moving train. Some results are discussed in the paper.

1. Introduction

When designing a new and modernizing the existing freight rolling stock and choosing the conditions for cargo transportation, it is important to take into account forces acting on the car and locomotive arising during standard and emergency modes of train motion. The process of forces occurrence and distribution in a freight train depends on many reasons, the main of which are the speed and mass of the train, the plan, profile and technical conditions of the rail track. Due to the fact that there are gaps in the inter-car connections of the freight train, shock forces can occur during its motion, affecting the strength of the car structural elements. The presence of loaded and empty cars in a freight train can lead to pulling out or squeezing empty and lightly loaded cars out of the train, which in its turn leads to the car wheelsets derailment. Therefore, when considering issues of increasing freight train speeds, it is necessary to conduct a preliminary assessment of dynamic characteristics of their cars and locomotive and forces acting in inter-car connections at the motion with increased speeds along an arbitrary track shape in plan and profile, as well as to consider emergency situations that may occur in the process of motion. This, in turn, requires the development of reliable mathematical models simulating freight train motion and the corresponding software for carrying out evaluative calculations.

To study vehicles stability of motion, to determine their elements rational parameters, to investigate curvilinear motion mathematical models of separate cars or train parts spatial vibrations are used for decades already [1-6]. But to consider train motion in transitional regimes and its motion along the changes of track gradient one or two dimensional mathematical models are still used [7-9]. Similar models are often used to study trains collision with the obstacle [10-13]. Such models allow to estimate the level of forces in inter-car connections at transitional modes, and cars or locomotives...
strength under collision, but their use do not give possibility to determine the possibility of derailment at the collision of the train with the motionless or moving obstacle.

The aim of this work is to show by some studied cases that with the use of the mathematical models of train spatial vibrations one can have more accurate results describing the processes occurring in a moving train.

2. Mathematical model of train spatial vibrations

As the train vibrations model, in which a locomotive and each car are considered as fairly complete spatial systems will have a very large number of degrees of freedom the authors used the model developed earlier [6], in which to assess the quality of a freight train motion along a path of arbitrary shape in plan modelling option is considered when the calculation model of one part of cars takes into account only their longitudinal movements, and for the other part of the cars full spatial models of their vibrations are used.

Differential equations of the train motion along the track of arbitrary shape are set in the form of Lagrange’s equations of the second kind:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Pi}{\partial \dot{q}_i} + \frac{\partial \Phi}{\partial \dot{q}_i} = Q_i^* + F_i,
\]

where \(q_i, \dot{q}_i\) are the generalized coordinates and their velocities (\(i\) is the number of degrees of freedom); \(T\) is the kinetic energy; \(\Pi\) is the potential energy; \(\Phi\) is the dissipative function; \(Q_i^*\) is the sum of wheel/rail interaction forces \(Q_i\), and forces \(S_i\) acting in inter-car connections during train motion; \(F_i\) are the applied external forces.

When determining the number of degrees of freedom of the considered mechanical system, the restrictions imposed on the movements of bodies due to generally accepted assumptions and design features of the cars are taken into account. The kinetic energy of the system is obtained by adding the kinetic energies of the bodies included in the system and the track. For each of the bodies, the kinetic energy was determined by Koenig’s theorem. The potential energy of the system was determined as the sum of the energy of elastic deformations and changes in energy due to the raising or lowering of the centres of masses of the bodies included in the system. The energy dissipation function in the vehicle takes into account the action of viscous friction forces as well as the effect of dry friction forces.

Generalized forces \(Q_i\) are defined as coefficients of variations of generalized coordinates in expressions of creep forces possible work. In determining the forces acting on the wheel in the horizontal lateral direction, components of the force of gravity are taken into account (in addition to the creep forces). Forces \(S_i\) simulate the inter-car connections work.

The external forces \(F_1, F_i\) acting on the locomotive and the \(i\)-th car are a tractive force \(F_1^*\) developed by the locomotive, braking forces \(B_i\), \(B_i^*\) occurring when brakes are switched on, and resisting forces \(W_1, W_i\). In total the external forces can be written as follows

\[
F_1 = W_1 + B_1 + F_1^* - F_1',
\]

\[
F_i = W_i + B_i + F_i^* - F_i'.
\]

Braking forces are applied to the wheelsets with transport delay depending on speed of braking force distribution and cars dimensions. Both braking and tractive forces are changed in time in accordance with the given characteristics. Resisting forces are divided into main ones which act all the time and additional ones acting only at the motion along separate track segments. In particular, forces occurring at the train motion along curves and grades are the additional resisting forces.

At the simulations of the train motion along the track gradients additional forces \(P_i\) acting on the
locomotive \((i = 1)\) and the \(i\) -th car were taken into account and were described as follows [14]:

\[
P_i = \begin{cases} 
  m_i g i, & \text{if } x_i - L_{ci} \leq L_i; \\
  \text{for } x_i - L_{ci} > L_i; \\
  m_i g (i_1 + y R_i^{-1}) & \text{if } 0 < y \leq l_i (y = x_i - L_{ci} - L_i); \\
  m_i g l_2, & \text{if } 0 < y \leq l_2 (y = x_i - L_{ci} - L_1 - l_1); \\
  m_i g (i_2 + y R_2^{-1}) & \text{if } 0 < y \leq l_2 (y = x_i - L_{ci} - L_1 - l_1 - L_2); \\
  \ldots \\
  m_i g i_k, & \text{if } 0 < y \leq L_k (y = x_i - L_{ci} - \sum_{j=1}^{k-1} [l_j + L_j]); \\
  m_i g (i_k + y R_k^{-1}) & \text{if } 0 < y \leq l_k (y = x_i - L_{ci} - \sum_{j=1}^{k-1} [l_j + L_j] - L_k), 
\end{cases}
\]

where \(x_i\) is the longitudinal displacement of the \(i\) -th car; \(m_i\) is the mass of the \(i\) -th car; \(g\) is the gravitational acceleration; \(L_{ci}\) is the length of the train part before the \(i\) -th car; \(i_k\) is the gradient of the \(k\) -th track segment with the length \(L_k\); \(R_k\) is the curve radius connecting the \(k\) -th and the \((k+1)\) -th track segments; \(l_k = |i_{k+1} - i_k| R_k\) is the length of the curve connecting the \(k\) -th and the \((k+1)\) -th segments.

The track was taken like an inertial elastic-viscous one. It is modelled as a reduced to each wheel mass (eight reduced masses for the car and twelve masses for the locomotive) with only vertical and horizontal lateral displacements and is based in these directions on springs and dampers simulating elastic-viscous features of rails and under-rail basement. The train motion is modelled with taking into account external disturbances because of the rails geometrical imperfections. Perturbations were applied under the wheels of the vehicles with a transport delay, which depends on the distance between the wheelsets and the vehicle speed.

In cars, which motion is modelled by a spatial calculation scheme, bogies of 18-7020 type are used as running gears [15, 16]. Each car was represented by the system of solid bodies connected by rigid, elastic and dissipative elements. The calculated scheme of the separate car presents a mechanical system consisting of eleven solid bodies (a car-body, two bolster, four side frames, four wheelsets) and eight reduced masses of the track. Characteristics of the considered bogies connections are sufficiently non-linear because of the friction elements both in the centre and axle-box suspensions and in simultaneous car-body in plan rotations in relation to the bogies. Taking into account the geometrical constrains imposed to the system each separate car has 42 degrees of freedom. To have possibility to study car motion in transitional regimes, the additions coordinate corresponding to the car absolute displacement in longitudinal direction because of train speed variation was included. So each car has 43+16=59 degrees of freedom.

A six-axle locomotive of the 2TE116 type is considered as a locomotive. The computational scheme is submitted as a branched system of nine rigid bodies (a car-body, two frames, and six wheelsets) and twelve reduced masses of the track joined by linear and non-linear connections with various rheology. Physical and geometrical interaction of wheels and rails is taken into account. With usual constrains this system has 35+24=59 degrees of freedom.

The developed model allows us to evaluate the stability and safety of motion of both separate cars and a locomotive, and the train as a whole. In total it represents a system of \((L + jK + N)\) second-order nonlinear differential equations \((L=59)\) the number of equations describing the motion of the locomotive, \(j=59\) is the number of degrees of freedom for each of the \(K\) cars represented by a spatial branched system, \(N\) is the number of cars, the design scheme of which is represented by one mass: it can vary from zero to the full number of cars in the train.
Based on the developed mathematical model, an algorithm and a computer program have been designed to assess the dynamic characteristics of a moving train. The computer program allows one to simulate a wide range of different cases, covering all kinds of combinations of the number of cars in a train, their loading, technical conditions of the running gears and track, plan and profile of the track along which the train is moving. As a result of calculations, almost any set of necessary output quantities can be obtained.

First the freight train motion along grades was studied. To do it the forces acting in inter-car connections were used as the main dynamic characteristic of the train.

3. Results of studies of a freight train motion along changes of track gradient
A train consisting of a locomotive and 23 cars equipped with Sh-2-V absorbing devices [17] was considered. Here, the motion of each first and last ten cars is modelled by one rigid body, the mass of which is equal to the mass of the car as a whole. The motion of the locomotive and each of the three cars in the middle of the train was modelled using spatial models describing vibrations of these vehicles. Irregularities of the rail track in the vertical and horizontal lateral directions corresponding to a satisfactory condition of the track are used as perturbations. Irregularities are represented as random stationary processes with a Gaussian distribution in amplitude and spectral density with monotonously decreasing character [18].

When solving the problem of assessing forces in the train moving along a variable track profile, in order to determine safe modes of motion, first of all, we consider transient processes caused by the train motion without control actions (braking, traction modes). At the same time, the influence of the rail track and train parameters, the train make-up, and the conditions characterizing the initial state of inter-car connections [6] are evaluated.

In the calculation options the train speed, the grades, the train mass, the initial state of inter-car connections (pre-compressed or pre-stretched train), the load of the train (loaded, empty) and the location of the cars of different loading in its scheme are varied. The train motion along a track section of the following shape is considered: first, there is a straight horizontal section of 400 m long, then the train moves along a downgrade of 700 m, then the motion continues along an upgrade of 700 m length and the train again goes to a horizontal section of a track.

An analysis of the calculated data showed that when the considered train moves along changes of longitudinal track gradients in couplers of those cars which vibrations are modelled by a spatial system of bodies with a large number of degrees of freedom, the largest values of longitudinal forces most often arise. As an example Figure 1 shows plots of the maximum tensile (lines 1, 2) and compressive (lines 3, 4) longitudinal forces distribution along a train moving upgrade at a speed of 60 km/h for a train consisting of all loaded cars (Figure 1, a) and a train in which all cars are empty (Figure 1, b). The spatial model represents the eleventh, twelfth and thirteenth cars. Lines 1 and 3 correspond to a pre-stretched train, and lines 2 and 4 correspond to a pre-compressed train.

![Figure 1](image-url)

**Figure 1.** Maximum longitudinal forces distribution along a moving loaded (a) and empty (b) train.

To assess the influence of the characteristics of the separate car model on the magnitudes of the
forces under study, the distribution of maximum forces in inter-car connections was calculated for the case when all the cars of the train are represented by bodies of the same mass and have only one degree of freedom, so they move only in longitudinal direction. The results obtained were compared with the data of the corresponding calculations, in which each of the three middle cars is represented by a spatial model of nine rigid bodies connected by bonds of various natures with a large number of degrees of freedom. As an example Figure 2 shows the distribution curves of the maximum forces in inter-car connections for the described cases during the motion of a pre-stretched train with empty cars along the downgrade (Figure 2, a) and on the upgrade (Figure 2, b). Here, lines 1 and 3 correspond to the case when the model of middle cars takes into account their spatial vibrations, and lines 2 and 4 for the model when all cars are represented by one mass model with only longitudinal motion. From the above results it follows that the calculated forces in the coupler devices of cars represented by spatial models are significantly differ from the forces obtained for a one-dimensional train model. To assess the nature of the forces change over time, Figure 3 and Figure 4 show, as an example, the time histories of the longitudinal forces changes in the connection of the 11th and 12th cars for the case when the train model is one-dimensional (Figure 3) and spatial (Figure 4). If the time histories are similar in nature for the changes of forces in time, the values of forces for different models are significantly differ from each other. Moreover, in all cases, larger values of both tensile and compressive forces arising in the inter-car connections occur when using the spatial model of train cars.

The calculations showed that when evaluating the forces arising in the inter-car connections of freight train cars during its motion along changes of track gradient, it is better to use spatial calculated schemes of train cars, which allows more accurate determination of the forces acting in automatic coupler devices.

Figure 2. Maximum longitudinal forces distribution in a train moving downgrade (a) and upgrade (b).

Figure 3. Time history of forces between the 11th and 12th car (one-dimensional model).
4. Train collision with an obstacle

It was also of a great interest of the authors to use the developed mathematical model for investigation of an emergency situation. As one of the most often accident for railway trains is locomotive and cars derailment, it was used to study the train collision with the obstacle and derailment possibility. An algorithm to calculate a force occurring between the train locomotive and the obstacle is described in papers [10, 11] and its shape is shown on Figure 5.

Figure 5. The force occurring between the locomotive and the obstacle.

$S_1$ is a force of shock absorber closing; $x_1^*$ is a shock absorber travel; $S_2$ is a force of locomotive automatic coupler cutting; $x_2^*$ is the locomotive car-body structure deformation at the force increase from $S_1$ up to $S_2$; $x_3^*$ is the distance between absorbing devices and an obstacle; $S_3$ is the force at which locomotive absorbing devices begin to deform; $x_4^*$ is the locomotive absorbing devices deformation; $x_5^*$ is the locomotive cabin sacrificial zone deformation; $S_4$ is the force, at which the mutual deformation of absorbing devices and locomotive cabin occurs; $S_5$ is the force, at which locomotive frame plastic deformation begins.
At the initial moment of collision a standard shock absorber of the locomotive automatic coupler begins to work. Energy absorption is by the elastic deformation of the locomotive absorbing device. When it is closed, the force is acting on the locomotive frame. When the force acting on the automatic coupler reaches the given limit value, fitting elements of the automatic coupler to the locomotive frame are broken and the automatic coupler is hiding under the locomotive frame. From this moment and till locomotive passive safety devices are in contact with the obstacle the interaction force is equal to zero. After the full deformation of energy absorbing devices the sacrificial zone of the locomotive driver cabin is working. In dependence of the compression force value elastic or elastic-plastic deformation of the locomotive car-body structure occurs.

To assess dynamic behaviour of the train cars, characteristics describing car dynamic qualities were used. For the freight trains such characteristics are

- car horizontal lateral and vertical accelerations at the central plates zones;
- horizontal lateral forces related to the axle load;
- a coefficient of vertical dynamics determined like dynamic vertical forces in the axle-box suspension divided by the static axle load;
- a derailment coefficient of wheel stability.

Values of the derailment coefficient of wheel stability were determined as follows [19]:

$$k_d = \frac{1g\beta - \mu}{1 + \mu g \beta} \frac{P_n(1,03 - 1,17k^*_{dv} + 0,14k^*_{dv}) + 0,515q + 0,705H_p}{P_n(0,24 + 0,04k^*_{dv} - 0,28k^*_{dv}) + 0,12 q + 0,92H_p}, \tag{5}$$

where $\beta$ is the angle of wheel flange inclination to the horizontal axis; $\mu$ is a friction coefficient; $P_n$ is the static axle journal load; $q$ is the weight of the unsuspended parts; $k^*_{dv}$ and $k^*_{dv}$ are coefficients of vertical dynamics for the leading and trailing wheels.

To insure train safe motion, the minimal value of the derailment coefficient of wheel stability has to be higher than 1.4 [20].

The simulation of the train collision with the obstacle has been done in accordance with the following scheme. At the initial moment the train moved with the constant speed along the straight track segment with vertical and horizontal lateral irregularities. After it has passed 100 m distance, the collision with the obstacle occurred. Vehicles of different masses were taken as the movable obstacles. They were a freight train, a similar locomotive, an empty freight car. At the initial moment the obstacle did not move. Calculations were made for the locomotive speed from 25 km/h up to 80 km/h and different levels of geometrical track irregularities corresponding to excellent, good, satisfied and permissible states of the track.

Firstly, the case was considered when collision takes place on the track segment with the excellent state (track without irregularities). In this situation, forces occurring at moving vehicle collision with an obstacle act only in longitudinal direction coinciding with the track axis. After the collision the speed of the coming locomotive reduces, and the speed of the obstacle vehicle increases. The increase of the obstacle mass leads to the greater reduction of the locomotive speed, and the increase of the locomotive speed leads to the increase of its time to full stop. It is worth to note that in the considered situation wheelset derailment does not occur.

In reality such situation does not exist. As a rule the track state can be estimated as good, satisfied or permissible [21]. Vehicle vibrations occur at the motion along such track segments because of different irregularities (gauge, cross-level, alignment, vertical profile change). It leads to occurrence of changing in time angle of the car-body rotation about its vertical axis in the horizontal plane. So not only longitudinal forces, but also lateral force component exists at the colliding vehicles interaction force. In its turn it leads to the lateral wheel/rail interaction forces occurrence. These forces values depend on the locomotive car-body yaw angle at the moment of collision.

The carried investigations have shown that all the studied dynamic characteristics of the locomotive sufficiently depend on the vehicle positioning at the moment of collision. In dependence on the locomotive car-body yaw angle, its motion speed and the obstacle vehicle mass, locomotive
dynamic characteristics and the coefficient of its stability of motion along the track change dramatically. Mostly these characteristic values are in the impermissible zone, and in some cases the stability is lost and wheelset derailment occurs [22].

As one example of the large number of simulations Figure 6 shows the comparison of plots of changes in the values of derailment coefficients of stability of the locomotive wheelsets when it collides with a single freight car at a speed of 60 km/h. Here, the collision occurs on the track section with excellent conditions (Figure 6, a) and on the track section with irregularities (Figure 6, b). As can be seen from the plots, in the case when the arising collision forces act only in the longitudinal direction, the locomotive does not derail. In the case when the interacting forces have not only a longitudinal component, but also the lateral components the value of the derailment coefficient of stability becomes lower than the permissible one and stability is lost (Figure 6, b). The analysis of the results obtained has shown that the heavier is the obstacle vehicle and the higher is the collision speed, the higher is the probability of locomotive and cars derailment.

![Figure 6](image-url)

**Figure 6.** Dependencies of derailment coefficients of stability for locomotive wheelsets, the track section with excellent conditions (a) and with irregularities (b).

5. **Conclusions**

In this article the mathematical model of freight train spatial vibrations developed on the basis of the earlier presented by the authors model was used to investigate two cases of train motion. The first case was the freight train motion along track gradients. From the results obtained it followed that the calculated forces in the coupler devices of cars represented by spatial models were higher than the forces obtained for a one-dimensional train model. So, for evaluating the forces arising in the inter-car connections of freight train cars during its motion along changes of track gradient, it is better to use spatial calculated schemes of train cars, which allows more accurate determination of the forces acting in automatic coupler devices.

The second studied case was the train collision with the moving obstacle like another train, locomotive and freight car. The carried investigations have shown that the locomotive dynamics sufficiently depends on the vehicle positioning at the moment of collision. It can be changed sufficiently in dependence on the locomotive car-body yaw angle, its motion speed and the obstacle vehicle mass. In some cases the stability was lost and wheelset derailment occurred.

Thus, the obtained results for both studied cases has shown that when considering issues related to the safe motion of railway rolling stock, to have more accurate results it is better to use mathematical model describing spatial vibrations of locomotives and train cars.

As a future step, simulations of much more cases should be done to evaluate dynamics of train elements at its motion along the track of arbitrary shape in plan and profile and also in different regimes of motion.
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