**Sub-dominant Type II Seesaw as an Origin of Non-zero $\theta_{13}$ in SO(10) model with TeV scale $Z'$ Gauge Boson**

Debasish Borah,1,* Sudhanwa Patra,2,† and Prativa Pritimita2,‡

1Department of Physics, Tezpur University, Tezpur - 784028, India
2Center of Excellence in Theoretical and Mathematical Sciences, Siksha ‘O’ Anusandhan University, Bhubaneswar-751030, India

Abstract

We discuss a class of left-right symmetric models where the light neutrino masses originate dominantly from type I seesaw mechanism along with a sub-dominant type II seesaw contribution. The dominant type I seesaw gives rise to tri-bimaximal type neutrino mixing whereas sub-dominant type II seesaw acts as a small perturbation giving rise to non-zero $\theta_{13}$ in our model which also has TeV scale right-handed neutrinos and $Z'$ gauge boson thereby making the model verifiable at current accelerator experiments. Sub-dominant type II and dominant type I seesaw can be naturally accommodated by allowing spontaneous breaking of D-parity and $SU(2)_R$ gauge symmetry at high scale and allowing TeV scale breaking of $U(1)_R \times U(1)_{B-L}$ into $U(1)_Y$. We also embed the left-right model in a non-supersymmetric $SO(10)$ grand unified theory (GUT) with verifiable TeV scale $Z'$ gauge boson. Drawing it to an end, we scrutinize in detail the evaluation of one-loop renormalization group evolution for relevant gauge couplings and estimation of the proton life time which can be accessible to the foreseeable experiments. And in the aftermost part we make an estimation of branching ratio for lepton flavor violating process $\mu \rightarrow e + \gamma$ as a function of type II seesaw strength due to doubly charged component of the right handed Higgs triplet with mass at the TeV scale, which can be accessible at ongoing experiments.

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*Electronic address: dborah@tezu.ernet.in
†Electronic address: sudha.astro@gmail.com
‡Electronic address: pratibha.pritimita@gmail.com
I. INTRODUCTION

The fact that the most successful phenomenological theory, the standard model (SM) of particle physics, suffers from the inability to address several observed phenomena as well as theoretical questions, has always been a source of excitement for particle physicists. The tiny but non-zero neutrino masses that have been confirmed by the phenomenon of neutrino oscillations detected in solar, atmospheric and reactor experiments [1] is certainly one such phenomena which the SM fails to address. These observations, among others have intensified the urge to ponder beyond the SM which has led to several well motivated beyond SM frameworks. The canonical seesaw mechanism (commonly referred to as type-I seesaw [2]), being the most elegant mechanism for generating small neutrino masses relies on the existence of right-handed (RH) neutrinos. Fundamentally speaking, the RH neutrinos are singlets under the SM gauge symmetry and hence can have arbitrary (can be very large) Majorana masses leading to light neutrino masses as $m_\nu \simeq y_\nu v^2/M_N$, where $y_\nu$ is the Dirac Yukawa coupling, $v = 246$ GeV is the vacuum expectation value (VEV) of the SM Higgs and $M_N$ is the RH Majorana neutrino mass. In order to be compatible with the neutrino oscillation data, i.e, $|m_\nu| \simeq \sqrt{\Delta m_{atm}^2} = 0.0495$ eV, one requires $M_N \simeq 10^{14-15}$ GeV taking the Yukawa couplings in their natural values i.e, $\mathcal{O}(1)$.

In addition to the canonical seesaw, other seesaw mechanisms have been worked upon as well to explain the tiny masses of the active neutrinos. Picking the type-II seesaw mechanism [3] from them which requires the existence of $SU(2)_L$ triplet Higgs fields in addition to the minimal SM particle content, the neutrino mass gets an extra contribution given by $m_\nu^{II} \simeq f v_L$, where $v_L$ is the VEV of the neutral component of the triplet and $f$ is the corresponding Yukawa coupling. Minimizing the scalar potential of such a model, the VEV of the Higgs triplet is found to be $v_L = \mu v^2/M_\Delta^2$, where $M_\Delta$ is the mass of the Higgs triplet and $\mu$ defines the mixing between SM Higgs and triplet. An obvious setting would be $f \simeq \mathcal{O}(1)$ and $\mu \sim M_\Delta \simeq 10^{14-15}$ GeV in order to explain sub-eV scale of light neutrino masses. Although these seesaw mechanisms look promising while explaining neutrino oscillation data, they lack the direct experimental testability in the ongoing experiments like Large Hadron Collider (LHC) or any future experiment like International Linear Collider.

Retreating not here, the particle phenomenology community has explored beyond standard model physics operative at few TeV scale, the results being repetitive attempts to corroborate neutrino masses and mixing. One such highly motivated and one of the most widely discussed
beyond standard model framework is the left-right symmetric model (LRSM) [4] which not only gives a clear description of the origin of parity violation at electroweak scale but also leads a way to the generation of neutrino masses naturally. A thorough study of these models strengthens us with the knowledge that in conventional left-right symmetric model (LRSM), the light neutrino masses arise from two sources: the type-I [2] plus type-II [3] seesaw mechanisms where the parity and $SU(2)_R$ gauge symmetry are spontaneously broken at the same scale.

Earlier explorations of the field imply that a deep relation between high energy collider physics and low energy phenomena like neutrino-less double beta decay as well as other lepton flavor violating processes is enrooted by minimal left-right symmetric model (LRSM) valid at TeV scale [5, 6]. If it happens that the parity and $SU(2)_R$ break at the same scale, then according to the seesaw relation $v_Lv_R = \gamma v^2$ (with $\gamma$ being a dimensionless parameter) the microscopic value of $v_L$ as required for type-II seesaw depends on large value of $v_R$, which further implies $v_R \approx (10^{13} \sim 10^{14})$ GeV making itself incapable of direct detection in near future. In a contrast way, if the right-handed scale is assigned with more moderate values, say in the range of few TeV, one can expect to have observable consequences at experiments in the near future. More willingly, if we assume both parity and $SU(2)_R$ to be broken at TeV scale i.e, $v_R \approx$ TeV that is the scale of RH heavy neutrino mass, then we strictly need to calibrate the Higgs couplings up to the order of $\gamma \leq O(10^{-10})$ in order to fit neutrino data from the seesaw relation. To refine this, studies have been done on left-right symmetric models to come upon with spontaneous D-parity breaking [7–11] where parity gets broken much earlier than $SU(2)_R$ gauge symmetry. In this work, we shall be discussing such a class of left-right symmetric models in which the spontaneous breaking of D-parity occurs at reasonably high scale along with $SU(2)_R \times U(1)_{B-L}$ gauge symmetry breaking down to $U(1)_R \times U(1)_{B-L}$. We then check numerically whether $U(1)_R \times U(1)_{B-L}$ breaking occurs at TeV scale (provided parity breaks at much higher scale) and tiny neutrino masses can be obtained without too much fine tuning. In this class of models, the TeV scale breaking of $U(1)_R \times U(1)_{B-L}$ results in the TeV scale masses of the right-handed neutrinos as well as $Z'$ boson while D-parity breaks at a high energy scale ($\sim 10^{9–11}$ GeV). As will be discussed later, this allows the possibility of dominant type I seesaw contribution to neutrino mass whereas type II seesaw contribution can naturally remain sub-dominant. We use such a sub-dominant type II seesaw contribution as the origin of non-zero $\theta_{13}$, the reactor mixing angle. It should be noted that, most of the earlier attempts to explain the non-zero $\theta_{13}$ incorporate different corrections to the $\mu - \tau$ symmetric tri-bimaximal (TBM) neutrino mass matrix which can naturally originate
in generic flavor symmetry models like $A_4$. Motivated by this, we consider the dominant type I seesaw contribution giving rise to TBM type neutrino mass matrix whereas the sub-dominant type II term giving rise to non-zero $\theta_{13}$. We also constrain the D-parity breaking scale from the demand of generating the experimentally allowed range of $\theta_{13}$. Apart from this, we also investigate whether such a choice of intermediate symmetry breaking scales allows the possibility to unify all the gauge couplings while being embedded in a non-supersymmetric $SO(10)$ grand unified theory.

With all these motivations, we present a $SO(10)$ model with a novel chain of symmetry breaking having left-right symmetry as an intermediate step giving neutrino masses through type-I plus type-II seesaw mechanisms, unification of three fundamental forces, prediction of proton life time accessible to the ongoing search experiments and most importantly, a low mass $Z'$ gauge boson which can be probed at LHC. While preparing this manuscript, an interesting work appeared online [12] with similar symmetry breaking chains and scales as the one we are discussing here. However, the neutrino mass phenomenology in that work is completely different from the one we pursue here. The plan of the paper can be sketched as follows. In section II we briefly discuss the left-right symmetric models, elucidating the spontaneous breaking of D-parity. In section III we discuss neutrino masses and mixing via dominant type-I seesaw giving rise to TBM type neutrino mixing and sub-dominate type-II seesaw giving rise to deviations from TBM mixing and hence non-zero $\theta_{13}$. In Sections IV and V, we give a possible path for embedding the present left-right symmetric models in the non-SUSY $SO(10)$ GUT with its symmetry breaking pattern and one-loop gauge coupling unification. In Section VI, the proton lifetime is estimated using the value gauge coupling at GUT scale. In section VII, we estimate the branching ratio for lepton flavor violating decay $\mu \rightarrow e + \gamma$ as a function of type II seesaw strength and finally conclude in section VIII.

II. LEFT-RIGHT SYMMETRIC MODEL WITH SPONTANEOUS D-PARITY BREAKING

In left-right symmetric models with spontaneous D-parity breaking, the discrete symmetry called D-parity gets broken earlier compared to the $SU(2)_R$ gauge symmetry. Here the gauge group can be written effectively as $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D(G_{2213D})$, where $D$ is the discrete left-right symmetry or D-parity. In matter sector, the left and right handed fermions are doublets under $SU(2)_L$ and $SU(2)_R$ gauge groups, respectively. The transformation of quarks
and leptons under the left-right symmetric group can be summarized as

\[
Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv [2, 1, \frac{1}{3}, 3], \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv [1, 2, \frac{1}{3}, 3],
\]

\[
\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \equiv [2, 1, -1, 1], \quad \ell_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix} \equiv [1, 2, -1, 1].
\]

Notably the difference between Lorentz parity and D-parity is that Lorentz parity acts on the Lorentz group and interchanges left-handed fermions with the right-handed ones but the bosonic fields remain the same whereas D-parity acts on the gauge groups \(SU(2)_L \times SU(2)_R\) interchanging the \(SU(2)_L\) Higgs fields with the \(SU(2)_R\) Higgs fields in addition to the interchange of fermions. The spontaneous breaking of D-parity creates an asymmetry between left and right handed Higgs fields making the coupling constants of \(SU(2)_R\) and \(SU(2)_L\) evolve separately under the renormalization group.

The Higgs sector of the left-right model with spontaneous D-parity breaking mechanism consists of a \(SU(2)\) singlet scalar field \(\sigma\) which is odd under discrete D-parity, two \(SU(2)_L\) triplets \(\Delta_L, \Delta_R\) and a bidoublet \(\Phi\) which contains two copies of SM Higgs transforming under the LR gauge group \(G_{2213} = SU(2)_L \times SU(2)_R \times U(1)_{B−L} \times SU(3)_C\) as

\[
\Delta_L = (3, 1, -2, 1), \quad \Delta_R = (1, 3, -2, 1), \\
\Phi = (2, 2, 0, 1), \quad \sigma = (1, 1, 0, 1).
\]

By assigning a non-zero VEV to D-parity odd singlet \(\langle \sigma \rangle \simeq M_P\), the left-right symmetry is spontaneously broken but the gauge symmetry \(G_{2213}\) remains unbroken resulting in

\[
M_{\Delta_R}^2 = M_\Delta^2 - \lambda_\Delta \langle \sigma \rangle M, \\
M_{\Delta_L}^2 = M_\Delta^2 + \lambda_\Delta \langle \sigma \rangle M, \tag{1}
\]

where \(M_\Delta\) is the mass term for triplets i.e., \(M_\Delta^2 \text{Tr} \left( \Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R \right)\), and \(\lambda_\Delta\) is the trilinear coupling in the term \(M \sigma \text{Tr} \left( \Delta_L^\dagger \Delta_L - \Delta_R^\dagger \Delta_R \right)\). In this scenario \(M_\Delta, M, \langle \sigma \rangle\) all are of order of \(M_P\) which is the scale of D-parity breaking thereby resulting TeV scale masses for right-handed Higgs triplets and D-parity breaking scale for their left-handed counterparts by suitable adjustment of trilinear coupling \(\lambda_\Delta\). In order to have \(W_R\) and \(Z_R\) mass predictions at nearly the same scale along with the generation of Majorana neutrino masses, it is customary to break \(SU(2)_R \times U(1)_{B−L} \rightarrow U(1)_Y\) in a single step by the VEV of the right handed triplet \(\langle \Delta_R^0 \rangle \simeq v_R\).
Instead of pursuing the aforementioned left-right symmetric model with D-parity breaking mechanism, we consider a more appealing phenomenological scenario:

\[ G_{2213D} \xrightarrow{M_F} G_{2113} \xrightarrow{M_0} \mathcal{G}_{213}(\text{SM}) \xrightarrow{\Phi} \mathcal{G}_{13} \]  

(2)

with \( M_{W_R} \gg M_{Z_R} \) via two step breaking of the left-right symmetric gauge theory to the SM. The Higgs sector of the present model with spontaneous D-parity breaking mechanism consists of two \( SU(2)_L \) triplets \( \Delta_L \) and \( \Omega_L \), two \( SU(2)_R \) triplets \( \Delta_R, \Omega_R \) and a bidoublet \( \Phi \) which contains two copies of \( SM \) Higgs transforming under the LR gauge groups is shown in Table I.

| Higgs Fields | Under \( \mathcal{G}_{2213} \) \( (2_L, 2_R, 1_B-L, 3_C) \) | Under \( \mathcal{G}_{2113} \) \( (2_L, 1_R, 1_B-L, 3_C) \) |
|--------------|-------------------------------------------------|-------------------------------------------------|
| \( \Omega_R \) | [1, 3, 0, 1] | [1, 1, 0, 1] |
| \( \Omega_L \) | [3, 1, 0, 1] | [3, 0, 0, 1] |
| \( \Delta_R \) | [1, 3, -2, 1] | [1, 1, -2, 1] |
| \( \Delta_L \) | [3, 1, -2, 1] | [3, 0, -2, 1] |
| \( \Phi \) | [2, 2, 0, 1] | [2, \pm 1/2, 0, 1] |

TABLE I: The Higgs fields transform under relevant gauge group as \( \mathcal{G}_{2213} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \) and \( \mathcal{G}_{2113} = SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \). We have chosen those fields in the third column under \( \mathcal{G}_{2113} \) which acquire a non-zero vacuum expectation value and in particular, the \( U(1)_R \) values corresponds to the z-components of Isospin i.e, \( T_{3R} \) of \( SU(2)_R \) satisfying \( Q = T_{3L} + T_{3R} + (B - L)/2 \) valid both for \( \mathcal{G}_{2113} \) as well as \( \mathcal{G}_{2213} \) gauge groups.
The first step of symmetry breaking i.e., $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D \to SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$ occurs at $W_R$ boson mass scale which is implemented through the VEV of the heavier triplet carrying $B-L = 0$ i.e., $\langle \Omega^0_R(1, 3, 0, 1) \rangle$ around D-parity breaking scale $M_P$. The second step of breaking $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \to G_{SM}$ occurs at $Z_R$ mass scale and is carried out by $\langle \Delta^0_R(1, 1, -2, 1) \rangle \sim v_R$ around $M^0_R \simeq (3 - 5)$ TeV. This unique scenario gives us the knowledge that $W_R$ scale completely decouples from $Z_R$ scale and hence, the LHC signatures of these gauge bosons and corresponding bounds on their mass scales should be revived again. The right handed neutral gauge boson $Z_R$ gets mass around few TeV staying very close to the experimental lower bound $M_{Z'} \geq 1.162$ TeV allowing its visibility at high energy accelerators in near future.

Apart from the right handed triplets whose VEV give masses to the right handed gauge bosons, the left handed triplets can also acquire non-zero VEV due to several scalar mixing terms in the Lagrangian. The analytic expression for VEV of the neutral component of $\Delta_L$ can be expressed as

$$v_L \approx \frac{\beta v^2 v_R}{2M M_P},$$

where we have used $v = 246$ GeV and $\beta$ is a coupling constant of $O(1)$. Noticeably in the above eq.(3), the smallness of the VEV of $\Delta_L$ is decided by the parity breaking scale and not by the $SU(2)_R$ breaking scale thereby putting no constraints on $v_R$ from the type-II seesaw point of view. Therefore, the type-II seesaw relation is modified for left-right models accompanied by spontaneous D-parity breaking scenario instead of its usual expression valid for conventional left-right symmetric model. As a result, the type-I [2] seesaw term decouples completely from D-parity breaking scale and become sensitive to the $U(1)_R \times U(1)_{B-L}$ breaking scale $M^0_R$ while the type-II [3] seesaw contribution becomes sensitive to the D-parity breaking scale. In the following section we shall briefly discuss how a particular value of D-parity breaking scale $M_P = 10^9 - 10^{10}$ GeV leads to sub-dominant type-II seesaw giving rise to correct deviations from TBM neutrino mixing in order to generate non-zero $\theta_{13}$. As we show later, the D-parity breaking scale $M_P \sim M$ is constrained to be greater than around $3 \times 10^9$ GeV. Hence, for $v_R \sim 1$ TeV and order one dimensionless couplings, the type II contribution comes out to be $0.001$ eV or less. The leading order TBM type neutrino mass matrix can originate from usual type I seesaw term due to the TeV scale right handed neutrinos originating from the TeV scale breaking of $U(1)_{B-L}$.
III. NEUTRINO MASS

The renormalizable invariant Yukawa Lagrangian that gives rise to the $G_{2113}$ invariant interactions, near the TeV scale for the model considered in our present analysis, is

$$\mathcal{L}_{\text{Yuk}} = Y_{\ell} \ell_L N_R \Phi + f_R N_R^c N_R \Delta_R + f_L \nu_L^c \nu_L \Delta_L + \text{h.c.},$$

resulting in $6 \times 6$ neutral fermion mass matrix after electroweak symmetry breaking

$$M_\nu = \begin{pmatrix} M_{LL} & M_{LR} \\ M_{LR}^T & M_{RR} \end{pmatrix}, \quad (4)$$

One should note here that all the mass scales used in above mass matrix $M_\nu$ have their dynamical interpretations in this model like $M_{RR} = f_R v_R$, $M_{LL} = f_L v_L$, and $M_{LR} = y_\nu v$ in contrast to the SM where two of them $M_{LL}$, $M_{RR}$ have no dynamical origins. The resulting light neutrino mass can be written as a seesaw formula given by

$$m_{LL} = m_{LL}^{II} + m_{LL}^{I} \quad (5)$$

where the usual type I seesaw formula is given by the expression,

$$m_{LL}^{I} = -M_{LR}^{-1} M_{RR}^T M_{LR}. \quad (6)$$

Here $M_{LR}$ is the Dirac neutrino mass matrix. Thus, for type I seesaw dominance with TeV scale $U(1)_{B-L}$ breaking $v_R \sim 1$ TeV, the Dirac Yukawa couplings should be fine tuned to $y_\nu \sim 10^{-5}$ for $f_R \sim 1$. The type II seesaw term ($m_{LL}^{II} = f_L v_L$) however, is directly proportional to the Majorana Yukawa couplings $f_L$ which have to be large in order to have sizeable contribution to neutrino masses.

The induced VEV for the left handed triplet $v_L$ can be shown for generic LRSM to be

$$v_L = \gamma \frac{M_W^2}{v_R}.$$

This expression for type II seesaw term is valid for those class of minimal models where D-parity and $SU(2)_R \times U(1)_{B-L}$ gauge symmetry get broken spontaneously at the same energy scale. However, as discussed in the previous section, it is possible to break D-parity and $SU(2)_R \times U(1)_{B-L}$ gauge symmetry at two different stages. In the left right symmetric models discussed in the previous sections, D-parity and $SU(2)_R$ gauge symmetry get broken down to $U(1)_R$ at a very
$$\Delta m^2_{21} [10^{-5}\text{eV}^2]$$ & 7.50 & 7.00-8.09 \\
$$|\Delta m^2_{31} (\text{NH})| [10^{-3}\text{eV}^2]$$ & 2.473 & 2.27-2.69 \\
$$|\Delta m^2_{23} (\text{IH})| [10^{-3}\text{eV}^2]$$ & 2.42 & 2.24-2.65 \\
$$\sin^2 \theta_{12}$$ & 0.306 & 0.27-0.34 \\
$$\sin^2 \theta_{23}$$ & 0.42 & 0.34-0.67 \\
$$\sin^2 \theta_{13}$$ & 0.021 & 0.016-0.030 \\

**TABLE II:** The global fit values for the mass squared differences and mixing angles taken from [13]

high scale whereas $U(1)_R \times U(1)_{B-L}$ gets broken down to $U(1)_Y$ of standard model at TeV scale. The VEV of the left handed triplet is given by equation (3) in such a case.

Before doing a numerical analysis of neutrino mass and mixing in our model, we note that prior to the discovery of non-zero $\theta_{13}$, the neutrino oscillation data were compatible with the well motivated TBM form of the neutrino mixing matrix discussed extensively in the literature [14]. However, since the latest data (last five references in [1]) have ruled out $\sin^2 \theta_{13} = 0$, one needs to go beyond the TBM framework to incorporate non-zero $\theta_{13}$. Since the experimental value of $\theta_{13}$ is much smaller than atmospheric and solar neutrino mixing angles, TBM type mixing can still be a valid approximation and the non-zero $\theta_{13}$ can be accounted for by incorporating small perturbations to TBM mixing coming from different mechanisms like charged lepton mass diagonalization, for example. There have already been a great deal of activities in this context [15, 16] which can successfully explain the latest data within the framework of several interesting models.

Since non-zero $\theta_{13}$ can be very naturally explained by incorporating corrections to TBM mixing and our model naturally provides such small correction in the form of type II seesaw term, we find it interesting to explore the possibility of TBM type mixing coming from type I seesaw term and the origin of non-zero $\theta_{13}$ through the type II seesaw term. Similar attempts to study the deviations from TBM mixing by using the interplay of two different seesaw mechanisms were done in [17, 18]. Our analysis here differs from these in the sense that we implement our model within a grand unified theory where the strength of seesaw terms can be naturally explained from gauge coupling unification point of view. We also extend our earlier discussion [18] to include two
different cases: one where the light neutrinos are almost degenerate, and the other in which there exists a moderate hierarchy between them, both obeying the cosmological upper limit on the sum of absolute neutrino masses.

Type I seesaw giving rise to $\mu - \tau$ symmetric TBM mixing pattern for neutrinos have been discussed extensively in the literature. The neutrino mass matrix in these scenarios can be written in a parametric form as

$$ m_{LL} = \begin{pmatrix} x & y & y \\ y & x + z & y - z \\ y & y - z & x + z \end{pmatrix} \quad (7) $$

which is clearly $\mu - \tau$ symmetric with eigenvalues $m_1 = x - y$, $m_2 = x + 2y$, $m_3 = x - y + 2z$. It predicts the mixing angles as $\theta_{12} \simeq 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0$. Although the prediction for first two mixing angles are still allowed from oscillation data, $\theta_{13} = 0$ has been ruled out experimentally at more than $9\sigma$ confidence level. This has led to a significant number of interesting works trying to explain the origin of non-zero $\theta_{13}$. Here we study the possibility of explaining the deviations from TBM mixing and hence from $\theta_{13} = 0$ by allowing the type II seesaw term as a perturbation.

It should be noted that the structure of the type I seesaw mass matrix (7) does not constrain the Dirac neutrino mass matrix $M_{LR}$ or the right handed neutrino mass matrix $M_{RR}$ to have some specific form. However, choosing one to have some particular form restricts the other so as to get the desired type I seesaw structure (7). For example, if we choose the Dirac neutrino mass matrix to have a diagonal structure

$$ M_{LR} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \quad (8) $$

then the $M_{RR}$ is restricted to have the following form

$$ M_{RR} = \begin{pmatrix} \frac{a^2(x+y)}{x^2+xy-2y^2} & -\frac{aby}{x^2+xy-2y^2} & -\frac{acy}{x^2+xy-2y^2} \\ -\frac{ab}{x^2+xy-2y^2} & \frac{b^2(x^2-y^2+xz)}{(x^2+xy-2y^2)(x-y+2z)} & \frac{bc(y^2-xy+xz)}{(x^2+xy-2y^2)(x-y+2z)} \\ -\frac{acy}{x^2+xy-2y^2} & -\frac{bc(y^2-xy+xz)}{(x^2+xy-2y^2)(x-y+2z)} & \frac{c^2(x^2-y^2+xz)}{(x^2+xy-2y^2)(x-y+2z)} \end{pmatrix} \quad (9) $$

Before choosing the minimal structure of the type II seesaw term, we note that the parametrization of the TBM plus corrected neutrino mass matrix can be done as [16].

$$ m_{LL} = \begin{pmatrix} x & y - w & y + w \\ y - w & x + z + w & y - z \\ y + w & y - z & x + z - w \end{pmatrix} \quad (10) $$
TABLE III: Parametrization of the neutrino mass matrix for TBM mixing

where $w$ denotes the deviation of $m_{LL}$ from that within TBM frameworks and setting it to zero, the above matrix boils down to the familiar $\mu - \tau$ symmetric matrix (7). Thus, the minimal structure of the perturbation term to the leading order $\mu - \tau$ symmetric TBM neutrino mass matrix can be taken as

$$m_{II}^{LL} = \begin{pmatrix}
0 & -w & w \\
-w & w & 0 \\
w & 0 & -w
\end{pmatrix}$$

Such a minimal form of the type II seesaw term can be explained by incorporating additional flavor symmetries as outlined in [18].

We first numerically fit the leading order $\mu - \tau$ symmetric neutrino mass matrix (7) by taking the central values of the global fit neutrino oscillation data [13] as presented in table II. We also incorporate the cosmological upper bound on the sum of absolute neutrino masses $\sum_i m_i < 0.23$ eV [19] reported by the Planck collaboration recently. For normal hierarchy, the diagonal mass matrix of the light neutrinos can be written as $m_{\text{diag}} = \text{diag}(m_1, \sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{31}^2})$ whereas for inverted hierarchy it can be written as $m_{\text{diag}} = \text{diag}(\sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{31}^2}, m_3)$. We choose two possible values of the lightest mass eigenstate $m_1, m_3$ for normal and inverted hierarchies respectively. First we choose $m_{\text{lightest}}$ as large as possible such that the sum of the absolute neutrino masses fall just below the cosmological upper bound. For normal and inverted hierarchies, this turns out to be $0.07$ eV and $0.065$ eV respectively. Then we allow moderate hierarchy to exist between the mass eigenvalues and choose the lightest mass eigenvalue to be $0.001$ eV to study the possible changes in our
analysis and results. The parametrization for all these possible cases are shown in table III.

![Graphs of Neutrino Parameters vs sin^2 θ_{13} for Normal Hierarchy]

FIG. 1: Variation of neutrino parameters as a function of sin^2 θ_{13} for Normal Hierarchy

We then incorporate the type II seesaw contribution which breaks µ − τ symmetry and hence gives rise to non-zero θ_{13}. We show the variation of other neutrino parameters with respect to sin^2 θ_{13} in figure 1, 2 for normal and inverted hierarchies respectively. It can be seen that the differences in the lightest active neutrino mass show up only in the variation of Δm_{21}^2. In case of normal hierarchy, all the parameters lie in the 3σ range for m_{lightest} = 0.07 eV whereas for inverted hierarchy we see a preference for lighter m_{lightest} namely, 0.001 eV. We then show the variation of sum of absolute neutrino masses in figure 3 and for all the cases considered, the sum is found to be within the cosmological limit. We also show the variation of sin^2 θ_{13} as a function of type II seesaw strength w in figure 4. It is seen that for higher values of m_{lightest}, we require a lower strength of the type II seesaw term to give rise to the desired θ_{13}. For m_{lightest} = 0.065, 0.07 eV, one can see from figure 4 that w \sim 0.002eV \Rightarrow fβ_{MM}^{NP} = 0.002 eV. Taking the dimensionless couplings to be of order unity and v = 10^2 GeV, v_R = 10^4 GeV, one gets a constraint MM_P \sim 5 \times 10^{19} GeV^2.
Similarly, for $m_{\text{lightest}} = 0.001$ eV, one can estimate this bound to be around $2 \times 10^{19}$ GeV$^2$. Thus, from the constraint of neutrino mass, we get a bound on the $SU(2)_R \times D$ breaking scale to be of the order of $10^9 - 10^{10}$ GeV which is consistent with the gauge coupling unification as will be discussed below.

The variation of the neutrino parameters with the perturbation strength can be understood simply by calculating the diagonalizing matrix of the neutrino mass matrix considered in the study.

\[ m_{\text{LL}} = \begin{pmatrix} x & y - w & y + w \\ y - w & x + z + w & y - z \\ y + w & y - z & x + z - w \end{pmatrix} \]  

which has eigenvalues $m_1 = x - y + z - \sqrt{3w^2 + z^2}$, $m_2 = x + 2y$ and $m_3 = x - y + z + \sqrt{3w^2 + z^2}$. Assuming $m_1 < m_2 < m_3$ we calculate the neutrino parameters by first identifying the diagonalizing matrix. Assuming $w$ to be small such that higher order terms beyond $w^2$ can be neglected, we arrive at the following approximate variations of neutrino parameters.
FIG. 3: Variation of the sum of absolute neutrino masses as a function of $\sin^2 \theta_{13}$

\[
\sin^2 \theta_{13} = \frac{w^2}{2z^2} + \text{h.o.} \quad (13)
\]

\[
\sin^2 \theta_{12} = \frac{1}{3(1 - \frac{w^2}{2z})} + \text{h.o.} \quad (14)
\]

\[
\sin^2 \theta_{23} = \frac{(3y - 2z - (1 - \frac{3w}{2z})w - \frac{3w^2}{2z})^2}{2(3y - 2z)^2} + \text{h.o.} \quad (15)
\]

\[
\Delta m^2_{21} = (x + 2y)^2 - (x - y + z - \sqrt{3w^2 + z^2})^2 \quad (16)
\]

\[
\Delta m^2_{31} = 4(x - y + z)\sqrt{3w^2 + z^2} \quad (17)
\]

where h.o. refers to higher order terms in $w$. It can be easily seen that for $w = 0$, the mixing angles correspond to the values predicted by TBM mixing.
IV. EMBEDDING THE MODEL IN NON-SUSY SO(10) GUT

With the rich phenomenology of the TeV scale asymmetric left-right model discussed in previous sections, we now intend to embed the model in a non-supersymmetric SO(10) grand unified theory. We examine whether the model unifies the three gauge couplings successfully with the proton life time lying close to the experimental lower bound and at the same time allows the possibility of TeV scale $Z'$, RH Majorana neutrinos and RH Higgs triplets which can be directly probed at ongoing experiments like LHC. The desired symmetry breaking pattern of SO(10) gauge group with left-right symmetry as an intermediate step is given by

$$SO(10) \xrightarrow{M_U} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D \quad [G_{2213D}, \quad (g_{2L} = g_{2R})]$$

$$\xrightarrow{M_P} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \quad [G_{2213} (g_{2L} \neq g_{2R})]$$

$$\xrightarrow{M_R^0} SU(2)_L \times U(1)_Y \times SU(3)_C \quad [G_{SM} \equiv G_{213}]$$

$$\xrightarrow{M_Z} U(1)_{em} \times SU(3)_C \quad [G_{13}] .$$

(18)
With the above choice of symmetry breaking, the SO(10) gauge group gets broken down to the Standard Model gauge group via the intermediate symmetry breaking chain as $G_{2213D}$, and $G_{2113}$. The breaking of SO(10) group to LR gauge group is achieved by $\{210_H\}$ representation of SO(10) Higgs. The decomposition of $\{210_H\}$ under Pati-Salam gauge group $SU(2)_L \times SU(2)_R \times SU(4)$ is $\Upsilon\{210_H\} \equiv (1, 1, 1) \oplus (1, 1, 15) \oplus (3, 1, 15) \oplus (1, 3, 15) \oplus (2, 2, 10) \oplus (2, 2, \overline{10}) \oplus (2, 2, 6)$. The SO(10) symmetry can be broken by assigning a VEV to $(1, 1, 15)$ of $\{210_H\}$ being even under D-parity ensuring discrete left-right symmetry (D-parity) intact at this stage. Such a Higgs choice, however, does not affect our mechanism of neutrino mass generation. The second stage of symmetry breaking from $G_{2213D}$ ($g_{2L} = g_{2R}$) to $G_{2113}$ ($g_{2L} \neq g_{2R}$) is done via combination of Higgs representation $\{45\}_H$, and $\{54\}_H$. This is the minimal choice of Higgs representation that is necessary to obtain the required symmetry breaking chain consistent with extended survival hypothesis. The principle of extended survival hypothesis says that at every stage of symmetry breaking chain we allow only those scalars to be present that acquire VEVs at the current or the subsequent levels of spontaneous symmetry breaking. This is equivalent to minimal number of fine-tunings to be imposed on the Higgs scalar potential so that all necessary symmetry breaking steps are executed at the desired scales. Under $G_{224}$ and $G_{2213}$, the Higgs representations $\{45\}_H$, and $\{54\}_H$ can be decomposed as

$$S\{54\}_H \equiv (1, 1, 1) \oplus (3, 3, 1) \oplus (1, 1, 20) \oplus (2, 2, 6) \quad \text{under} \quad G_{224},$$

$$\subset (1, 1, 0, 1) \oplus (3, 3, 0, 1) \oplus (1, 1, 0, 8) \oplus (1, 1, -2/3, 6) \oplus (1, 1, -2/3, \overline{6})$$

$$\oplus (2, 2, 1/3, 3) \oplus (2, 2, -1/3, \overline{3}) \quad \text{under} \quad G_{2213},$$

$$A\{45\}_H \equiv (3, 1, 1) \oplus (1, 3, 1) \oplus (2, 2, 6) \oplus (1, 1, 15) \quad \text{under} \quad G_{224},$$

$$\subset (1, 1, 0, 1) \oplus \Omega_L(3, 1, 0, 1) \oplus \Omega_R(1, 3, 0, 1) \oplus (2, 2, 1/3, 3) \oplus (2, 2, -1/3, \overline{3})$$

$$\oplus (1, 1, 2/3, 3) \oplus (1, 1, -2/3, \overline{3}) \oplus (1, 1, 0, 8) \quad \text{under} \quad G_{2213}. \quad (19)$$

The remaining symmetry breaking $SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C$ to the SM gauge group $SU(2)_L \times U(1)_Y \times SU(3)_C$ is implemented by $\{126\}_H$ Higgs representation. The decomposition of $\{126\}_H$ under Pati-Salam gauge group is $\{126\}_H \equiv (2, 2, 15) \oplus (3, 1, 10) \oplus (1, 3, \overline{10}) \oplus (1, 1, 6)$. Assigning a VEV to $\langle \Delta_R(1, 1, -2, 1) \rangle \subset \Delta_R(1, 3, -2, 1) \subset (1, 3, \overline{10})$, we break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$. The last stage of symmetry breaking of the SM gauge group $SU(2)_L \times U(1)_Y \times SU(3)_C$ to $U(1)_{em} \times SU(3)_C$ is achieved by $\{10_H\}$ where the Higgs field $\Phi(2, 1/2, 1) \subset (2, 2, 0, 1) \subset \{10_H\}$ acquires a VEV breaking $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. In the following sections, we present
the gauge coupling evolution with the evaluation of one-loop beta coefficients and estimate the proton life time \( \tau_p \) using the value of gauge coupling at GUT scale.

V. GAUGE COUPLING EVOLUTION WITH ONE-LOOP ANALYSIS

In this section we study the one loop renormalization group evolution (RGE) equations for gauge couplings relevant for our model. The one loop RGE equations for the gauge couplings can be written as

\[
\frac{d \alpha_i^{-1}}{dt} = -\frac{a_i}{2\pi}
\]

(20)

where \( t = \ln(\mu) \), \( \alpha_i = g_i^2/(4\pi) \) are the fine structure constants and \( a_i \) are the one-loop beta coefficients derived for the corresponding \( i \)th gauge group for which coupling evolution has to be determined. The analytic formula for \( a_i \) is

\[
a_i = -\frac{11}{3} C_2(G_i) + \frac{4}{3} \kappa N_G + \frac{1}{3} \eta T(R_{S_i}) d(S_i),
\]

(21)

with no summation over \( i \). We denote \( C_2 \) and \( T_2 \) as quadratic Casimir of a given representation, \( d_{S_i} \) as the multiplicity factor for a particular gauge group \( G_i \) due to other \( SU(N) \) group present in the model, \( N_G \) as the number of fermion generation (which is 3 in our model). We take \( \kappa = 1, \frac{1}{2} \) for Dirac and Weyl fermions, \( \eta = 1, \frac{1}{2} \) for complex and real scalar fields, respectively.

A. Matching condition and estimations for \( M_U, M_P \) and \( \alpha_U \)

One can write the RGE equations for the standard model gauge couplings in terms of present non-SUSY \( SO(10) \) GUT coupling. Since the model has two intermediate symmetry breaking steps above standard model scale, it is important to know the appropriate matching condition at these two symmetry breaking steps. Denoting \( \alpha_i^{-1} = \frac{4\pi}{g_i} \), the appropriate matching conditions for gauge couplings valid at the gauge group \( G_{2113} = SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \) are

At \( \mu = M_R^0 \) :

\[
\begin{bmatrix}
\alpha_Y^{-1}(M_R^0) \\
\alpha_{2L}^{-1}(M_R^0)
\end{bmatrix}_{G_{SM}} = \begin{bmatrix}
\frac{3}{5} \alpha_{1R}^{-1}(M_R^0) + \frac{2}{5} \alpha_{B-L}^{-1}(M_R^0) \\
\alpha_{2L}^{-1}(M_R^0)
\end{bmatrix}_{G_{2113}},
\]

(22)
Similarly, the appropriate gauge coupling matching conditions at the scale \( M_P \) valid for the gauge group \( G_{2213D} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D \) are

At \( \mu = M_P \):

\[
\begin{align*}
\alpha_{2L}^{-1}(M_P) \bigg|_{G_{2213D}} &= \alpha_{2L}^{-1}(M_P) \bigg|_{G_{2213D}} \\
\alpha_{1R}^{-1}(M_P) \bigg|_{G_{2213D}} &= \alpha_{2R}^{-1}(M_P) \bigg|_{G_{2213D}}, \\
\alpha_{2L}^{-1}(M_P) \bigg|_{G_{2213D}} &= \alpha_{B-L}^{-1}(M_P) \bigg|_{G_{2213D}}, \\
\alpha_{3C}^{-1}(M_P) \bigg|_{G_{2213D}} &= \alpha_{2L}^{-1}(M_P) \bigg|_{G_{2213D}}.
\end{align*}
\]  

(23)

Also, one can write down the gauge coupling matching conditions at the unification scale \( M_U \) as

At \( \mu = M_U \):

\[
\begin{align*}
\alpha_{2L}^{-1}(M_U) \bigg|_{G_{2213D}} &= \alpha_{2L}^{-1}(M_U) \bigg|_{G_{2213D}} = \alpha_{10}^{-1}(M_U) \bigg|_{SO_{10}} \\
\alpha_{B-L}^{-1}(M_U) \bigg|_{G_{2213D}} &= \alpha_{10}^{-1}(M_U) \bigg|_{SO_{10}}, \\
\alpha_{3C}^{-1}(M_U) \bigg|_{G_{2213D}} &= \alpha_{10}^{-1}(M_U) \bigg|_{SO_{10}}.
\end{align*}
\]  

(24)

With the above gauge coupling matching conditions, one can express the RGE equations for \( \alpha_i^{-1} \), \( i=2L, Y, 3C \) for SM valid at one-loop level

\[
\begin{align*}
\alpha_{2L}^{-1}(M_Z) &= \alpha_{10}^{-1}(M_U) + \frac{a_{2L}'}{2\pi} \ln \left( \frac{M_P^0}{M_Z} \right), \\
\alpha_{2L}^{-1}(M_Z) &= \alpha_{10}^{-1}(M_U) + \frac{a_{2L}''}{2\pi} \ln \left( \frac{M_P^0}{M_R^0} \right), \\
\alpha_{3C}^{-1}(M_Z) &= \alpha_{10}^{-1}(M_U) + \frac{a_{3C}'}{2\pi} \ln \left( \frac{M_P^0}{M_R^0} \right), \\
\alpha_{3C}^{-1}(M_Z) &= \alpha_{10}^{-1}(M_U) + \frac{a_{3C}''}{2\pi} \ln \left( \frac{M_P^0}{M_R^0} \right).
\end{align*}
\]  

(25)

(26)

(27)

where the one-loop beta coefficients for our model determined by the particle spectrum in the mass ranges \( M_Z - M^0_R, M^0_R - M_P \) and \( M_P - M_U \) are \( \{a_{2L}, a_Y, a_{3C}\}, \{a_{2L}', a_{1R}', a_{B-L}', a_{3C}'\}, \) and \( \{a_{2L}'', a_{2R}', a_{B-L}'', a_{3C}''\} \), for gauge groups \( G_{2213}, G_{2113}, \) and \( G_{2213D}, \) respectively. Fixing \( M_R^0 \) around few TeV, and using particle data group values [20] \( \sin^2 \theta_W = 0.23166 \pm 0.00005, \alpha_S = 0.1184 \pm 0.003, \) and \( \alpha_{em} = 1/127.94, \) a simple one-loop analytical survey of the gauge coupling running equations yields two important relations for \( M_P \) and \( M_U \) as [11]

\[
\begin{align*}
\ln \left( \frac{M_U}{M_Z} \right) &= \frac{D_1 A_P - D_0 B_P}{B_U A_P - A_U B_P}, \\
\ln \left( \frac{M_P}{M_Z} \right) &= \frac{D_0 B_U - D_1 A_U}{B_U A_P - A_U B_P}.
\end{align*}
\]  

(28)

(29)
with
\[
\mathcal{A}_0 = (8a_{3C} - 3a_{2L} - 5a_Y) - (8a'_{3C} - 3a'_{2L} - 3a'_{1R} - 2a'_{B-L}),
\]
\[
\mathcal{A}_P = (8a'_{3C} - 3a_{2L} - 3a'_{1R} - 2a'_{B-L}) - (8a''_{3C} - 6a''_{2L} - 2a''_{B-L}),
\]
\[
\mathcal{A}_U = (8a''_{3C} - 6a''_{2L} - 2a''_{B-L}),
\]
\[
\mathcal{B}_0 = (5a_{2L} - 5a_Y) - (5a'_{2L} - 3a'_{1R} - 2a'_{B-L}),
\]
\[
\mathcal{B}_P = (5a'_{2L} - 3a'_{1R} - 2a'_{B-L}) - (2a''_{2L} - 2a''_{B-L}),
\]
\[
\mathcal{B}_U = (2a''_{2L} - 2a''_{B-L}),
\]
\[
\mathcal{D}_0 = 16\pi \left( \alpha_s^{-1} - 3 \alpha_{em}^{-1} \right) - \mathcal{A}_0 \ln \left( \frac{M_R^0}{M_Z} \right),
\]
\[
\mathcal{D}_1 = 16\pi \alpha_{em} \left( \sin^2 \theta_W - \frac{3}{8} \right) - \mathcal{B}_0 \ln \left( \frac{M_R^0}{M_Z} \right).
\]

In the following subsection, the value of the D-parity breaking scale $M_P$ and the unification scale $M_U$ are estimated using the above model parameters by fixing the $U(1)_R \times U(1)_{B-L}$ breaking scale $M_R^0$ around 1 TeV to 6 TeV. The estimation of $M_P$, $M_U$ and $\alpha_U$ following from eqn.(25) to eqn.(29) is carried out for different scenarios defined by the spectrum of Higgs fields utilized for the purpose of symmetry breaking.

**Breaking of $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ via only Higgs triplet**

We note that the contributions to one-loop beta coefficients coming from the fermion and gauge sector are well known and simple for a given gauge group while the Higgs contributions to beta-coefficients are complicated due to various Higgs fields present in our model. The economical choice of Higgs spectrum for different mass ranges is presented in table IV. We find the D-parity breaking scale $M_P$ and the unification scale $M_U$ for the set of one-loop beta coefficients given in table IV to be $M_P = 1.6 \times 10^{11}$ GeV and $M_U = 1.2 \times 10^{15}$ GeV. The above calculated value of $M_U$ results in predicting proton life time $1.2 \times 10^{33}$ yrs while the current experimental bound on proton life time is $> 8.2 \times 10^{33}$ yrs. Therefore, it is important to discuss the GUT threshold corrections to this unification mass scale in order to know how far we are from the experimental lower bound on proton life time. However, we do not perform such an exercise of calculating GUT threshold corrections in this work.

Alternatively, one can try to check the gauge coupling unification with higher unification scale by incorporating the presence of additional Higgs fields at different stages of symmetry breaking.
The allowed range of mass scales are $M_Z = 91.187$ GeV, $M_R^0 = 3 - 6$ TeV, $M_P = 1.6 \times 10^{11}$ GeV, and $M_U = 1.2 \times 10^{15}$.

allowed in the model. With this motivation, we include extra Higgs fields $\zeta(1, 0, 8)$ and $\xi(2, 1/2, 8)$ (with SM quantum numbers shown within brackets) to the minimal particle content of table IV and numerical values of $M_P$, $M_U$, and $\alpha_U^{-1}$ are estimated in the following paragraph.

| Group $G_I$ | Range of Masses (GeV) | Higgs content | $a$ |
|------------|------------------------|---------------|-----|
| $G_{2L1Y3C}$ | $M_Z - M_R^0$ | $\Phi(2, \frac{1}{2}, 1)_{10}$ | $a_i = \begin{pmatrix} -19/6 \\ 41/10 \\ -7 \end{pmatrix}$ |
| $G_{2L1R1B-L3C}$ | $M_R^0 - M_P$ | $\Phi_1(2, \frac{1}{2}, 0, 1)_{10} \oplus \Phi_2(2, -\frac{1}{2}, 0, 1)_{10'}$ $\oplus \Delta_R(1, 1, -2, 1)_{126}$ | $a'_i = \begin{pmatrix} -3 \\ 14/3 \\ 9/2 \\ -7 \end{pmatrix}$ |
| $G_{2L2R1B-L3C D}$ | $M_P - M_U$ | $\Phi_1(2, 2, 0, 1)_{10} \oplus \Phi_2(2, 2, 0, 1)_{10'}$ $\oplus \Delta_R(1, 3, -2, 1)_{126} \oplus \Delta_L(3, 1, -2, 1)_{126}$ $\oplus \Sigma_R(1, 3, 0, 1)_{210} \oplus \Sigma_L(3, 1, 0, 1)_{210}$ | $a''_i = \begin{pmatrix} -4/3 \\ -4/3 \\ 7 \\ -7 \end{pmatrix}$ |

TABLE IV: One-loop beta coefficients for different gauge coupling evolutions. The allowed range of mass scales are $M_Z = 91.187$ GeV, $M_R^0 = 3 - 6$ TeV, $M_P = 1.6 \times 10^{11}$ GeV, and $M_U = 1.2 \times 10^{15}$.

TABLE V: Calculated values of one-loop beta coefficients presented by adding an extra Higgs fields to the minimal Higgs content given in table IV. The one-loop beta coefficients are presented as $a_i$, $a'_i$, and $a''_i$ in 1st, 2nd and 3rd row of each column, respectively. The allowed range of mass scales are $M_Z = 91.187$ GeV, $M_R^0 = 3 - 6$ TeV, $M_P = 10^9 - 10^{11}$ GeV, and $M_U = 10^{14.5} - 10^{16.5}$.

For evaluation of $a_i$, $a'_i$, and $a''_i$ presented under column C1 of table V, the Higgs field $\zeta(1, 1, 0, 8)$ (with $G_{2213}$ quantum numbers shown within brackets) is added at or above the symmetry breaking scale $M_R^0$. The gauge coupling unification for such a case is shown in figure 5.
Similarly, for the evaluation of $a_i$, $a_i'$, and $a_i''$ presented under column C2 of table V, the Higgs field $\xi(2, 2, 0, 8)$ is introduced at or above the scale $M_p$.

| Higgs content for | $M_R^0$ (in GeV) | $M_P$ (in GeV) | $M_U$ (in GeV) | $\alpha_U^{-1}$ |
|-------------------|------------------|----------------|----------------|-----------------|
| For Table-IV      | $(3-6)$ TeV      | $1.65 \times 10^{11}$ | $1.2 \times 10^{15}$ | 40.9827         |
| For Table-V: C1   | $(3-6)$ TeV      | $2.6 \times 10^{9}$  | $4.9 \times 10^{16}$ | 40.7687         |
| For Table-V: C2   | $(3-6)$ TeV      | $1.38 \times 10^{11}$ | $1.1 \times 10^{16}$ | 37.9465         |

TABLE VI: Allowed solutions for different mass scales, and inverse fine structure constant ($\alpha_U^{-1}$) at unification scale consistent with gauge coupling unification.

![Graph](image)

FIG. 5: One-loop gauge coupling evolution for left-right model with beta functions given in column C1 of table V.

**Breaking of $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ via Higgs triplet ($\Delta$) plus Higgs doublet ($\chi$)**

It should be noted that, shifting the parity breaking scale $M_P$ towards the GUT scale provides us with more possibilities to achieve unification with more minimal set of additional fields than discussed above. However, to keep a sizable contribution of type II seesaw so that it can give rise to the observed $\theta_{13}$, we intend to keep $M_P$ as low as $10^9 - 10^{10}$ GeV. Here lies the need to
include additional field content discussed in previous subsection. Apart from the scenario where \( U(1)_R \times U(1)_{B-L} \) gauge symmetry is broken by Higgs triplets, there can be one more possibility to achieve the same using both triplets and doublets. For the sake of completeness we discuss this case as well and check the gauge coupling unification.

In such a scenario, we allow the breakdown of the intermediate symmetry \( SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C \rightarrow SU(2)_L \times U(1)_Y \times SU(3)_C \) driven by both triplet \( \Delta \) and doublet \( \chi \) coming from \( 126_H \) and \( 16_H \) representation of \( SO(10) \) respectively. The relevant Higgs spectrum and the corresponding one-loop beta coefficients for \( SO(10) \rightarrow G_{2213D} \rightarrow G_{2113} \rightarrow SM \) are listed in table VII. The predicted values of the mass scales for this ranges of input parameters are \( M_P \) (in GeV) = \( 1.51 \times 10^{11} \), \( M_U \) (in GeV) = \( 1.02 \times 10^{15} \), and \( \alpha_U^{-1} = 42.02. \)

| Group \( G_I \) | Higgs content | \( a_i \) |
|-----------------|----------------|--------|
| \( G_{2L1Y3C} \) | \( \Phi(2, \frac{1}{2}, 1)_{10} \) | \(-19/6 \) |
| \( a_i = \) | | \( 41/10 \) |
| | | \(-7 \) |
| \( G_{2L1R1B-L3C} \) | \( \Phi_1(2, \frac{1}{2}, 0, 1)_{10} \oplus \Phi_2(2, -\frac{1}{2}, 0, 1)_{10'} \) \( \oplus \Delta_R(1, 1, -2, 1)_{126} \oplus \chi_R(1, \frac{1}{2}, -1, 1)_{16} \) | \(-3 \) |
| | | \( 19/4 \) |
| | | \( 37/8 \) |
| | | \(-7 \) |
| \( a_i' = \) | | \(-7/6 \) |
| \( G_{2L2R1B-L3C} \) | \( \Phi_1(2, 0, 1)_{10} \oplus \Phi_2(2, 0, 1)_{10'} + \Delta_R(1, 3, -2, 1)_{126} \oplus \chi_R(1, 2, -1, 1)_{16} \oplus \chi_L(2, 1, -1, 1)_{16} \oplus \Sigma_R(1, 3, 0, 1)_{210} \oplus \Sigma_L(3, 1, 0, 1)_{210} \) | \(-7/6 \) |
| | | \(-7/6 \) |
| | | \( 15/2 \) |
| | | \(-7 \) |
| \( a_i'' = \) | | \(-7/6 \) |
| | | \(-7/6 \) |
| | | \( 15/2 \) |
| | | \(-6 \) |

TABLE VII: The estimated one-loop beta coefficients for different gauge coupling evolutions with Higgs the fields relevant for different stages of symmetry breaking. The allowed range of mass scales are \( M_Z = 91.187 \) GeV, \( M_R^0 = 3 - 6 \) TeV, \( M_P = 1.5 \times 10^{11} \) GeV, and \( M_U = 1.02 \times 10^{16.5} \).

With addition of extra color octet scalar \( \zeta(1, 1, 0, 8) \) from \( M_R^0 \) onwards relevant for symmetry breaking, the derived values of one-loop beta-coefficients are \( a_i = (-19/6, 41/10, -7) \), \( a_i' = (-3, 19/4, 37/8, 6) \), and \( a_i'' = (-7/6, -7/6, 15/2, -6) \). As a result, the numerically estimated...
values of mass scales are $M_P (\text{in GeV}) = 1.9 \times 10^9$, $M_U (\text{in GeV}) = 2.49 \times 10^{16}$, and $\alpha_{U}^{-1} = 41.4236$. The coupling evolution for this case is shown in figure 6.

It is worth mentioning here that the effect of two-loop RG analysis on gauge coupling unification might change the mass scale like $M_R^0 \simeq v_R$, $M_P$ and $M_U$. It is found that the two-loop RG evolution in this particular non-SUSY $SO(10)$ set up having two intermediate symmetry breaking steps changes marginally the values of $M_P$ and $M_U$ as compared to the numerical values derived by one-loop RG analysis. We can take the example of two loop analysis having Higgs spectrum as presented in Table VII along with color octet Higgs scalar $(1,1,0,8)$ where the predicted mass scales are

$$M_R^0 = v_R \simeq (3-10) \text{ TeV}, \ M_P \simeq 10^{8.9} \text{ GeV}, \ M_U \simeq 10^{16.57} \text{ GeV}$$

but the findings for one-loop analysis are

$$M_P (\text{in GeV}) = 1.9 \times 10^9, \ M_U (\text{in GeV}) = 2.49 \times 10^{16}.$$ 

Hence, there will be little modification to the type II seesaw contribution which is $m_{\nu}^{II} = f v_L = f \frac{g^2 v_R}{M_{M_P}}$ if one incudes two-loop RG effect. One can fix the neutrino mass arising from type-II seesaw by suitably adjusting the other free parameters like Higgs coupling $\beta$ and $M$ even if we include the effect of two-loop RG corrections on $v_R$ and $M_P$.

It should be noted that the LRSM where the breaking of $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ occurs through Higgs triplet $(\Delta)$ and $SU(2)_R \times D$ gets broken by Higgs triplet $\Sigma$ can also be constrained from the cosmologial constraints on the successful disappearance of domain walls. Domain walls generically arise in such models (due to the spontaneous breaking of discrete symmetry called D-parity) which, if stable, can overclose the Universe conflicting with standard cosmology. As discussed in [21], for $M_R^0 = 10$ TeV, domain wall disappearance requires $M_P < 10^9$ GeV, which are very close to the symmetry breaking scales in our present model. Similar constraint on the second model (the one with both Higgs triplet and doublet) have not been studied yet and left for future investigations.

VI. ESTIMATION OF PROTON LIFE TIME $\tau_p$

With the knowledge of unification mass scale $M_U$, and corresponding value of $\alpha_{U}^{-1}$ (one exemplary case shown in the plot, $M_U = 1.9 \times 10^{16}$ GeV and $\alpha_{U}^{-1} = 41.4238$), we intend to estimate the proton life time $\tau_p$ and compare with the recent and proposed future experiments and also, if
FIG. 6: One-loop gauge coupling evolution for left-right model with particle content given in table VII plus an additional color octet Higgs from the scale \( M_R^0 \) onwards possible, derive uncertainties relevant for this result. The master formula for the gauge-induced \( d = 6 \) proton decay in the chain \( p \rightarrow e^+\pi^0 \) with the known heavy spectrum in this non-SUSY SO(10) model is

\[
\Gamma (p \rightarrow \pi^0 e^+) = \frac{\pi}{4} A_L^2 \left| \alpha_H \right|^2 m_p \alpha_u^2 \frac{M_4^4}{M_U^4} (1 + \mathcal{F} + \mathcal{D})^2 \mathcal{R}
\]

(30)

where \( A_L = 1.25 \) is the renormalization factor from the electroweak scale to the proton mass, \( \mathcal{D} = 0.81, \mathcal{F} = 0.44, \alpha_H = -0.011 \) GeV\(^3\), and \( f_\pi = 139 \) MeV are extracted as phenomenological parameters by chiral perturbation theory and lattice gauge theory. Also, \( m_p = 938.3 \) MeV is the proton mass, and \( \alpha_U \equiv \alpha_G \) is the gauge fine structure constant derived at the GUT scale. The renormalization factor \( \mathcal{R} = \left[ (A_{SR}^2 + A_{SL}^2) (1 + |V_{ud}|^2) \right] \) for SO(10), the (1, 1) element of \( V_{CKM} \) is \( V_{ud} = 0.974 \) = with \( A_{SL} = A_{SR} \) being the short-distance renormalization factor in the left (right) sectors.

Redefining \( \alpha_H = \overline{\alpha}_H (1 + \mathcal{F} + \mathcal{D}) = 0.012 \) GeV\(^3\), and \( A_R \simeq A_L A_{SL} \simeq A_L A_{SL} \), the proton life time can be expressed as

\[
\tau_p = \Gamma^{-1} (p \rightarrow \pi^0 e^+) = \frac{4}{\pi} \frac{f_\pi^2 M_4^4}{m_p \alpha_u^2 \alpha_H^2 A_R^2 \mathcal{F}_q},
\]

(31)

where \( \mathcal{F}_q \simeq 7.6 \)

24
In order to estimate the proton life time, we should have knowledge about the short distance enhancement renormalization factors which are fully model dependent, a few of which are known while a few others have been already determined in the present model. For the particular choice of symmetry breaking considered in present non-SUSY SO(10) model and assuming no threshold corrections at or below the GUT scale, the short distance renormalization factors evaluated at one loop level are given as

$$A_S = A_{2213}^{2113} \cdot A_{2113}^{2113} \cdot A_{213}^{2113},$$

(32)

where,

$$A_{2213}^{2113} = \left( \frac{\alpha_i(M_P)}{\alpha_i(M_U)} \right)^{-\gamma''_{22i}} = \left( \frac{\alpha_i^{-1}(M_P)}{\alpha_i^{-1}(M_U)} \right)^{-\gamma''_{22i}}, \quad i=2L, 2R, B-L, 3C;$$

$$A_{2113}^{2113} = \left( \frac{\alpha_i(M_R)}{\alpha_i(M_P)} \right)^{-\gamma'_{21i}}, \quad i=2L, 1R, B-L, 3C;$$

$$A_{213}^{2113} = \left( \frac{\alpha_i^{-1}(M_Z)}{\alpha_i^{-1}(M_R)} \right)^{-\gamma'_{21i}}, \quad i=2L, Y, 3C.$$  

(33)

We have used the anomalous dimensions taken from [22, 23] and one-loop beta coefficients derived in our model. The estimated value of $A_R = A_L \cdot A_S$ is $A_R \simeq 2.24$. We have estimated the the proton life time to be $\tau_p = 5.75 \times 10^{35}$ yrs for the model under consideration with $M_U = 1.9 \times 10^{16}$ GeV and $\alpha^{-1} = 41.4238$. The predicted proton life time is out of reach for the current experiment Super-Kamiokande (2011) experiment giving bound on the proton life time for $p \to e^+\pi^0$ channel is $\tau(p \to e^+\pi^0)|_{SK,2011} > 8.2 \times 10^{33}$ yrs [24] while it can be accessible to future planned experiment such as $\tau(p \to e^+\pi^0)|_{HK,2025} > 9.0 \times 10^{34}$ yrs and $\tau(p \to e^+\pi^0)|_{HK,2040} > 2.0 \times 10^{35}$ yrs [25].

VII. LEPTON FLAVOR VIOLATING DECAYS

In the left-right model under consideration, there are different Feynman diagrams contributing to the underlying lepton-flavor violating interactions; (i) from $W_L$ exchanges with the mediation of light-heavy RH Majorana neutrinos shown in figure 7(a), (ii) from $W_R$ exchanges with the mediation on heavy right-handed Majorana neutrinos shown in figure 7(b), and (iii) from the doubly charged RH Higgs triplet ($\Delta_R^{++}$) exchanges as shown in figure 7(c). The analytic expression for
FIG. 7: One loop Feynman diagrams for lepton number violating decays $\ell_i \to \ell_j + \gamma (i \neq j)$. Contribution from the $W_L$ exchanges involving mixing between left- and right-handed neutrinos is presented in (a) whereas contribution from the $W_R$ exchanges with heavy RH Majorana neutrinos is presented in (b). The dominant contribution to lepton flavor violation (LFV) decays via doubly charged RH Higgs triplet exchanges is presented in (c).

These contributions are given below

$$\text{Br} (\mu \to e + \gamma)^{(a)}_{W_L} \simeq \frac{\alpha_W^3 \sin^2 \theta_W}{256 \pi^2} \frac{m_\mu^4}{M_{W_L}^4} \frac{m_\mu^2}{\Gamma_\mu} |G_\gamma^{\mu e}|^2,$$

$$\text{Br} (\mu \to e + \gamma)^{(b)}_{W_R} \simeq \frac{3 \alpha_W}{32 \pi} \left( \frac{M_{W_L}}{M_{W_R}} \right)^8 \left( \sin \theta_R \cos \theta_R \frac{M_2^2 - M_1^2}{M_{W_L}^2} \right)^2,$$

$$\text{Br} (\mu \to e + \gamma)^{(c)}_{\Delta^{++}_R} \simeq \frac{2 \alpha_W M_{W_L}^4}{3 \pi g_{2R}^4} \left[ \frac{(f f')_{12}}{M_{\Delta^{++}_R}^2} \right]^2,$$

where $\theta_W$ is the weak mixing angle, $\theta_R$ is the mixing angle between left and right handed neutrino sector, $\Gamma_\mu = 2.996 \times 10^{-19}$ GeV, $G_\gamma^{\mu e}$ contains left-right neutrino mixing plus the loop factor and $\alpha_W = g_{2L}^2/(4\pi)$ is the fine structure constant for $SU(2)_L$ valid at $M_Z$ scale and is found to be 0.18389. There have been several attempts to calculate the enhanced LFV signal in $\mu \to e\gamma$ process for example, in [26] and recently, it has been pointed out in refs. [27] that the LFV branching ratios can be significant if the heavy-light neutrino mixing is large.

Assuming the left-right mixing to be small, one can neglect the contribution $\text{Br} (\mu \to e + \gamma)^{(a)}_{W_L}$ in comparison to other contributions. Also, in our model the $W_R$ gauge boson mass is found to be $\geq 10^8$ GeV making the $\text{Br} (\mu \to e + \gamma)^{(b)}_{W_R}$ contribution suppressed. The remaining dominant contribution due to TeV scale right-handed Higgs triplet contribution is

$$\text{Br} (\mu \to e + \gamma)^{(c)}_{\Delta^{++}_R} \simeq \frac{2 \alpha_W M_{W_L}^4}{3 \pi g_{2R}^4} \left[ \frac{(m_{\mu} \mu_{M_{\mu}} I_{12})}{M_{\Delta^{++}_R}^2} \right]^2.$$

We have numerically estimated this contribution represented by a plot as shown in figure.
FIG. 8: Variation of the branching ratio \( \text{Br}(\mu \rightarrow e + \gamma)^{(c)}_{\Delta_{R}^{++}} \) arising from the LFV decays via doubly charged RH Higgs triplet exchanges with the type II seesaw strength \((f_{V_L})\).

where we have plotted \( \text{Br}(\mu \rightarrow e + \gamma)^{(c)}_{\Delta_{R}^{++}} \) with the type II seesaw strength and using other allowed range of model parameters. From the plot, it can be seen that the numerical prediction for \( \text{Br}(\mu \rightarrow e + \gamma)^{(c)}_{\Delta_{R}^{++}} \) in our model is same as the current MEG upper limit:

\[
\text{Br}(\mu \rightarrow e + \gamma)^{(c)}_{\Delta_{R}^{++}} \bigg|_{\text{expt.}} \leq 5.7 \times 10^{-13} \quad [28, 29]
\]

for type II seesaw strength \( f_{V_L} = 0.013 \) eV. This is consistent with our model where the required type II seesaw strength is of the order of 0.001 eV.

VIII. CONCLUSIONS

We have studied a left-right symmetric gauge theory \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times D(g_{2L} = g_{2R})(G_{2213D}) \) which breaks down to the standard model gauge symmetry through two intermediate stages: first, the \( SU(2)_R \times D \) breaks down to \( U(1)_R \) at scale \( M_P \) and \( U(1)_R \times U(1)_{B-L} \) breaks down to \( U(1)_Y \) at a latter stage \( M_R^0 \). The motivation behind this set up is two-fold: \((i)\) to allow TeV scale intermediate \( U(1)_R \times U(1)_{B-L} \) symmetry which can be accessed at experiments through \( Z' \), right handed neutrino and heavy Higgs searches, \((ii)\) to naturally allow type I seesaw dominance (which can give rise to TBM type \( \mu - \tau \) symmetric neutrino mass matrix) while keeping type II seesaw term as sub-dominant but sizeable enough to give rise to the required deviation from TBM mixing in order to explain non-zero \( \theta_{13} \). First we have performed a numerical analysis taking type I seesaw term as TBM type and type II seesaw term as a perturbation which breaks \( \mu - \tau \) symmetry. We have done this exercise for both normal and inverted hierarchical
neutrino mass spectra as well as two possible values of lightest neutrino mass (one being close to the maximum allowed by cosmological upper bound and one slightly lower). We have constrained the type II seesaw strength by demanding the required deviation from TBM to produce non-zero $\theta_{13}$. For dimensionless couplings to be of order one and $U(1)_R \times U(1)_{B-L}$ breaking scale of around $10$ TeV, the parity breaking scale has been restricted to be $10^9 - 10^{10}$ GeV.

We have also made an attempt to embed this model within $SO(10)$ GUT and check whether the above mentioned symmetry breaking steps can be naturally realized along with successful gauge coupling unification at a scale which lies close to the bound coming from proton lifetime constraint. We have shown that in the framework of non-SUSY $SO(10)$ GUT invoking spontaneous D-parity breaking, one-loop RGE analysis of gauge couplings allow mass ranges $M_R^0 = 3 - 6$ TeV, $M_P = 10^9 - 10^{11}$ GeV and $M_U = 10^{14.5} - 10^{16.5}$ GeV for several possible additional Higgs structures. We have also calculated the proton lifetime from the unification scale and find it to be within future experimental reach. At the end, we have made an estimate of branching ratio for the LFV decays of $\mu \rightarrow e + \gamma$ due to the presence of TeV scale doubly charged component of right handed triplet Higgs and found it to be lying close to the experimental limit.

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