On Shadowing the $\kappa$-$\mu$ Fading Model

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Abstract

In this paper, we extensively investigate the way in which $\kappa$-$\mu$ fading channels can be impacted by shadowing. Following from this, a family of shadowed $\kappa$-$\mu$ fading models are introduced, classified according to whether the underlying $\kappa$-$\mu$ fading undergoes single or double shadowing. In total six types of single shadowed $\kappa$-$\mu$ model (denoted Type I through to Type VI) and six types of double shadowed $\kappa$-$\mu$ model (denoted Type I through to Type VI) are discussed. Most importantly, the single shadowed $\kappa$-$\mu$ Type II, Type III, Type IV, and all of the double shadowed models are novel. For the single shadowed models, the taxonomy is dependent upon whether a) the fading model assumes that the dominant component, the scattered waves, or both experience shadowing and b) whether this shadowing is shaped by a Nakagami-$m$ or an inverse Nakagami-$m$ random variable (RV). Analytical formulations are derived for the probability density function (PDF) of the Type II and Type III models, whilst a closed-form expression for the PDF of the Type IV model is obtained. The categorization of the double shadowed models is dependent upon whether a) the envelope experiences shadowing of the dominant component, which is preceded (or succeeded) by a secondary round of multiplicative shadowing, b) the dominant and scattered contributions are fluctuated by two independent shadowing processes, or c) the scattered waves of the envelope are subject to shadowing, which is also preceded (or succeeded) by a secondary round of multiplicative shadowing. In all of these models the shadowing phenomena are shaped by either a Nakagami-$m$ RV, an inverse Nakagami-$m$ RV or their mixture. A novel closed form expression for the PDF of the Type I fading model is presented whilst analytical formulations are obtained for the PDF of Type II - Type VI models. It is worth highlighting that the double shadowed $\kappa$-$\mu$ models offer remarkable flexibility as they include the $\kappa$-$\mu$, $\eta$-$\mu$, and the various types of single shadowed $\kappa$-$\mu$ distribution as special cases. Lastly, we demonstrate a practical application of the double shadowed $\kappa$-$\mu$ Type I model by applying it to some field measurements obtained for device-to-device communications channels.
I. INTRODUCTION

The $\kappa$-$\mu$ fading model \cite{1} is a generalized fading model which was developed to describe envelope fluctuations that arise due to the clustering of scattered multipath waves in addition to the presence of elective dominant components. It is characterized by two physical fading parameters, namely $\kappa$ and $\mu$. Here, $\kappa$ represents the ratio of the total power of the dominant component to the total power of the scattered waves whilst $\mu$ represents the number of multipath clusters. Due to its inherent versatility and general nature, it contains other well-known fading models such as the Rice ($\kappa = k$, $\mu = 1$), Nakagami-$m$ ($\kappa \to 0$, $\mu = m$), Rayleigh ($\kappa \to 0$, $\mu = 1$) and One-Sided Gaussian ($\kappa \to 0$, $\mu = 0.5$) as special cases.

A $\kappa$-$\mu$ fading envelope can be affected by shadowing in many different ways. For instance, the dominant component, the scattered waves, or both can be impacted by this propagation phenomenon. It is also entirely possible that in addition to the dominant component being shadowed, further multiplicative shadowing\footnote{Here, the total power of the dominant and scattered signal components are shadowed.} may occur which impacts the scattered signal, and also administers secondary shadowing to the already perturbed dominant component. Likewise, in addition to the scattered waves being shadowed, further shadowing may occur which impacts the dominant signal component, and administers secondary shadowing to the fluctuated scattered waves. As well as this, both the dominant component and scattered waves can be influenced by individual shadowing processes. These different ways in which shadowing can impact the envelope fading leads to a family of shadowed $\kappa$-$\mu$ fading models that can be classified depending on whether the underlying $\kappa$-$\mu$ fading undergoes single or double shadowing.

Traditionally, shadowing has been modeled using the lognormal distribution \cite{2}. However, due to challenges which exist in relation to its tractability, the authors in \cite{3}–\cite{5} proposed the use of the gamma distribution. Similarly, \cite{6}, \cite{7} proposed the use of the closely related Nakagami-$m$ distribution.
distribution due to its ability to exhibit semi-heavy tailed characteristics [7]. More recently, [8] and [9] used the inverse Nakagami-m and inverse gamma distributions, respectively. It should be noted that similar to the lognormal, gamma and Nakagami-m distributions, the inverse gamma and inverse Nakagami-m distributions can also exhibit the semi heavy-tailed behavior necessary to accurately characterize shadowing. Moreover, they offer much of the analytical tractability available from using the gamma and Nakagami-m distributions.

In this work, we investigate how different types of shadowing can impact $\kappa-\mu$ fading. We discuss six types of single shadowed $\kappa-\mu$ fading model (denoted I through to VI) which assume that the multipath fading is manifested by the propagation mechanisms associated with $\kappa-\mu$ fading. In addition, they assume that either the dominant component, the scattered waves, or both suffer from a single shadowing process, which is shaped by a Nakagami-m or an inverse Nakagami-m random variable (RV). Note that the single shadowed $\kappa-\mu$ Type II (where the dominant component is shadowed by an inverse Nakagami-m RV), Type III (where the scattered waves are shadowed by a Nakagami-m RV) and Type IV (where the scattered waves are shadowed by an inverse Nakagami-m RV) models are presented here for the first time. We also introduce six types of double shadowed $\kappa-\mu$ fading model, denoted I through to VI. The Type I and Type II models assume that the dominant component of a $\kappa-\mu$ signal undergoes variations influenced by a Nakagami-m or an inverse Nakagami-m RV, respectively. Moreover, the Type I model considers that the root-mean-square (rms) power of the dominant component and scattered waves, may also be subject to a secondary round of shadowing induced by an inverse Nakagami-m RV, whilst the Type II model considers that this fluctuation is caused by a Nakagami-m RV. Therefore, these models provide a convenient way to not only control the shadowing of the LOS component, but also any multiplicative shadowing which may be present in the channel. The double shadowed $\kappa-\mu$ Type III and Type IV models assume that the dominant component and scattered waves of a $\kappa-\mu$ fading envelope are perturbed by two different shadowing processes, induced by a Nakagami-m and an inverse Nakagami-m RV, or vice versa. Lastly, the double shadowed $\kappa-\mu$ Type V and Type VI models assume that the scattered waves of a $\kappa-\mu$ signal are influenced by a Nakagami-m and an inverse Nakagami-m RV, respectively. The Type V model further considers that the rms power of the dominant component and scattered
waves experience a secondary round of shadowing brought about by an inverse Nakagami-$m$ RV, whilst the Type VI model considers that this variation is caused by a Nakagami-$m$ RV.

It is worth highlighting that, due to the generality of the analysis presented here and under particular shadowing conditions, a number of the existing composite fading models found in the literature occur as special cases. For example, multiplicative composite fading models such as the $\kappa$-$\mu$/inverse gamma and $\eta$-$\mu$/inverse gamma fading models [9]. These models assume that a $\kappa$-$\mu$ or an $\eta$-$\mu$ RV is responsible for generating the multipath fading, and an inverse gamma RV for shaping the shadowing. In particular [9] obtained closed form expressions for the PDFs of the $\kappa$-$\mu$/inverse gamma and $\eta$-$\mu$/inverse gamma fading models. The utility of these composite fading models was also demonstrated through a series of channel measurements obtained for wearable, cellular and vehicular communications. In a similar fashion, some line-of-sight (LOS) composite models 2 such as the shadowed $\kappa$-$\mu$ [6], [7] and shadowed Rician [10], [11] fading models are also found through the analysis conducted here. The shadowed $\kappa$-$\mu$ fading model presented in [6] and [7] assumes that the multipath fading is due to fluctuations brought about by a $\kappa$-$\mu$ RV, whilst the dominant signal component is fluctuated by a Nakagami-$m$ RV. Notably, it includes the $\kappa$-$\mu$, $\eta$-$\mu$ and shadowed Rician fading models as special cases. Previously, this model has been shown to provide excellent agreement with field measurements obtained for body-centric fading channels [7], land-mobile satellite channels [11] and underwater acoustic channels [12].

The main contributions of this paper are now summarized as follows.

- Firstly, we perform a broad investigation of the way in which $\kappa$-$\mu$ fading can be affected by shadowing. Subsequently, we introduce a family of shadowed $\kappa$-$\mu$ models that are classified as either single or double shadowed models. Six types of single shadowed $\kappa$-$\mu$ fading model (denoted Type I - VI) and six types of double shadowed $\kappa$-$\mu$ fading model (denoted Type I - VI) are discussed. Of these, the single shadowed $\kappa$-$\mu$ Type II - IV and all of the double shadowed models are introduced here for the first time.

2We note here that many of the models presented in the literature for which the dominant signal component is subject to shadowing are often referred to as LOS composite fading models. However, in the strict sense, they are not true composite models because the shadowing is not applied multiplicatively to all constituent parts of the fading envelope.

3Note that the $\kappa$-$\mu$ shadowed fading model presented in [6] and [7] is a type of single shadowed $\kappa$-$\mu$ model.
• Secondly, we derive analytical formulations for the PDFs of the single shadowed Type II and Type III models, whilst closed-form expressions are obtained for the PDF of the single shadowed $\kappa$-$\mu$ Type IV model. As well as this, we derive a closed-form expression for the PDF of the double shadowed $\kappa$-$\mu$ Type I model and obtain analytical formulations for the PDF of Type II - Type VI models.

• Thirdly, the generality of the double shadowed $\kappa$-$\mu$ fading models are highlighted through reduction to a number of well-known special cases. In particular, these fading models unify the $\kappa$-$\mu$, $\eta$-$\mu$ and the various types of single shadowed $\kappa$-$\mu$ model.

• Finally, we provide an example of a practical application of the double shadowed $\kappa$-$\mu$ Type I model by applying it to some device-to-device (D2D) channel measurements obtained at 868 MHz.

The remainder of this paper is organized as follows. Section II and III describe and formulate the various Types of single and double shadowed $\kappa$-$\mu$ model, respectively. Section IV presents some special cases of the double shadowed $\kappa$-$\mu$ models. Section V provides some numerical results and also demonstrates the utility of the double shadowed $\kappa$-$\mu$ Type I fading model for characterizing the shadowing encountered in D2D communications channels. Lastly, Section VI finishes the paper with some concluding remarks.

II. SINGLE SHADOWED $\kappa$-$\mu$ MODELS

In this section, we investigate a number of different ways in which the $\kappa$-$\mu$ fading envelope can be impacted by a single shadowing process. This leads to six single shadowed fading models, denoted Type I through to Type VI, three of which are presented here for the first time.

A. Single Shadowed $\kappa$-$\mu$ Type I and Type II Models

Similar to the $\kappa$-$\mu$ fading model, the single shadowed $\kappa$-$\mu$ Type I and II fading models assume that the received signals are composed of clusters of multipath waves propagating in non-homogeneous environments. Within each multipath cluster, the scattered waves have similar delay times and the delay spreads of different clusters are relatively large. The power of the scattered waves in each cluster is assumed to be identical whilst the power of the dominant
component is assumed to be arbitrary. Unlike the $\kappa - \mu$ model, the single shadowed $\kappa - \mu$ Type I and II models assume that the dominant component of each cluster can randomly fluctuate because of shadowing. Their signal envelope, $R$, can be expressed in terms of the in-phase and quadrature phase components as

$$ R^2 = \sum_{i=1}^{\mu} (X_i + \xi p_i)^2 + (Y_i + \xi q_i)^2 $$

where $\mu$ is a real-valued extension related to the number of multipath clusters, $X_i$ and $Y_i$ are mutually independent Gaussian random processes with mean $\mathbb{E}[X_i] = \mathbb{E}[Y_i] = 0$ and variance $\mathbb{E}[X_i^2] = \mathbb{E}[Y_i^2] = \sigma^2$, where $\mathbb{E}[\cdot]$ denotes the expectation operator. Here $p_i$ and $q_i$ are the mean values of the in-phase and quadrature phase components of the multipath cluster $i$, $\xi$ represents a Nakagami-$m$ RV in the Type I model, and an inverse Nakagami-$m$ RV in the Type II model with shape parameter $m_d$ and $\mathbb{E}[\xi^2] = 1$ for both.\(^4\)

The single shadowed $\kappa - \mu$ Type I model was introduced as a generalization of the $\kappa - \mu$ fading model in [6] and [7]. Thus, the PDF of $R$ for the Type I model can be obtained from [6] and [7] as follows.

$$ f_R(r) = \frac{2m_d^{m_d}(1 + \kappa)^\mu \mu r^{2\mu - 1} e^{-\frac{r^2(1 + \kappa)}{\hat{r}^2}} \Gamma(m_d + \kappa \mu m_d)}{\Gamma(\mu) \Gamma(m_d + \kappa \mu)} \frac{1}{1} \frac{1}{F_1} \mu \frac{\mu^2 \kappa (1 + \kappa) r^2}{\hat{r}^2 (m_d + \kappa \mu)} $$

where, $\kappa > 0$ is the ratio of the total power of the dominant component ($d^2$) to that of the scattered waves ($2\mu \sigma^2$), $\mu > 0$ is related to the number of clusters, $\hat{r} = \sqrt{\mathbb{E}[R^2]}$ represents the rms power of $R$, the mean signal power is given by $\mathbb{E}[R^2] = 2\mu \sigma^2 + d^2$, $\Gamma(\cdot)$ represents the gamma function and $1F_1(\cdot;\cdot;\cdot)$ denotes the confluent hypergeometric function [13, 9.210.1]. Likewise, the PDF of $R$ for the single shadowed $\kappa - \mu$ Type II fading model can be obtained via Theorem II as follows.

**Theorem 1.** For $\kappa, \mu, \hat{r}^2 \in \mathbb{R}^+$ and $m_d > 1$, the PDF of the single shadowed $\kappa - \mu$ Type II fading

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\(^4\)To assist with the understanding of the models presented here, throughout the manuscript, we denote $m_d$, $m_s$ and $m_t$ as the shadowing parameters which are responsible for fluctuating the dominant, scattered or total (i.e. the combined dominant and scattered) components respectively.

\(^5\)While the pioneering work presented in [6] refers to this model as $\kappa - \mu$ shadowed, to maintain consistency with the terminology adopted here we refer to it as the single shadowed $\kappa - \mu$ Type I.
model is expressed as
\[ f_R(r) = \sum_{i=0}^{\infty} 4 \left[ (m_d - 1) \kappa \right] \frac{m_d^{i+\frac{1}{2}} r^{2i+2\mu-1} \mu^\frac{1}{2} (3i+m_d) + \mu}{i! \Gamma(m_d) \Gamma(i+\mu) (1+\kappa)^{-i-\mu}} e^{-r^2(1+\kappa)_\mu} K_{i+m_d} \left( 2 \sqrt{(m_d-1)\mu \kappa} \right) \] (3)

where \( K_{\nu}(\cdot) \) denotes the modified Bessel function of the second kind [14, 9.6].

**Proof:** See Appendix [A].

### B. Single Shadowed \( \kappa-\mu \) Type III and Type IV Models

The single shadowed \( \kappa-\mu \) Type III and IV fading models assume that the scattered waves in each cluster can randomly fluctuate because of shadowing. Their signal envelope, \( R \), can be formulated in terms of the in-phase and quadrature-phase components as
\[ R^2 = \sum_{i=1}^{\mu} (\xi X_i + p_i)^2 + (\xi Y_i + q_i)^2 \] (4)

where \( X_i, Y_i, p_i, q_i \) and \( \mu \) are as defined previously, \( \xi \) denotes a Nakagami-\( m \) RV in the Type III model, and an inverse Nakagami-\( m \) RV in the Type IV model with shape parameter \( m_s \) and \( \mathbb{E}[\xi^2] = 1 \) for both.

The PDF of \( R \) for the single shadowed \( \kappa-\mu \) Type III model can be obtained via Theorem 2 as follows.

**Theorem 2.** For \( \kappa, \mu, m_s, \hat{r}^2 \in \mathbb{R}^+ \), the PDF of the single shadowed \( \kappa-\mu \) Type III fading model can be expressed as
\[ f_R(r) = \sum_{i=0}^{\infty} \frac{4 (m_s \mu)^{\frac{1}{2}(2i+m_s+\mu)} r^{2i+2\mu-1} K^i (1+\kappa)^{i+\mu}}{i! \Gamma(m_s) \Gamma(i+\mu) (r^2 (1+\kappa) + \hat{r}^2 \kappa)^{\frac{1}{2}(2i-m_s+\mu)}} \] (5)

**Proof:** See Appendix [B].

It can be seen from (5) that the derivation of a closed-form expression for the PDF of the single shadowed \( \kappa-\mu \) Type III model was infeasible. This was due to the inherent mathematical complexity of the resulting integral shown in (30). However, this is not the case with the single shadowed \( \kappa-\mu \) Type IV model, whose PDF is derived next.

The PDF of \( R \) for the single shadowed \( \kappa-\mu \) Type IV model can be obtained via Theorem 3
Theorem 3. For $\kappa, \mu, \hat{r}^2 \in \mathbb{R}^+$ and $m_s > 1$, the PDF of the single shadowed $\kappa$-$\mu$ Type IV fading model is expressed as

$$f_R(r) = \frac{2(m_s - 1)^{m_s}(1 + \kappa)^\mu \mu^{2\mu - 1} \hat{r}^{2m_s}}{B(m_s, \mu)[r^2 + \hat{r}^2(m_s - 1 + \kappa\mu)]^{m_s + \mu}} \times 2F_1\left(\frac{m_s + \mu}{2}, \frac{1 + m_s + \mu}{2}; \frac{4\mu^2\kappa(1 + \kappa)r^2\hat{r}^2}{[r^2 + \hat{r}^2(m_s - 1 + \kappa\mu)]^2}\right)$$

where $B(\cdot, \cdot)$ represents the Beta function [13, 8.384] and $2F_1(\cdot, \cdot; \cdot; \cdot)$ denotes the Gauss hypergeometric function [13, 9.100].

Proof: See Appendix B.

C. Single Shadowed $\kappa$-$\mu$ Type V and Type VI Models

The single shadowed $\kappa$-$\mu$ Type V and VI models assume that the rms power of a $\kappa$-$\mu$ signal can randomly fluctuate because of shadowing. In particular, the Type V model considers that the multipath waves (both the dominant component and scattered waves) are subject to variations induced by a Nakagami-$m$ RV, whilst the Type VI model assumes that this variation is brought about by an inverse Nakagami-$m$ RV. Accordingly, their signal envelope $R$ can be formulated in terms of the in-phase and quadrature-phase components as

$$R^2 = \xi^2 \sum_{i=1}^{\mu} (X_i + p_i)^2 + (Y_i + q_i)^2$$

where $X_i, Y_i, p_i, q_i$ and $\mu$ are as defined previously, $\xi$ represents a Nakagami-$m$ RV in the Type V model, and an inverse Nakagami-$m$ RV in the Type VI model with shape parameter $m_t$ and $\mathbb{E}[\xi^2] = 1$ for both. The PDF of $R$ for the single shadowed $\kappa$-$\mu$ Type V model is given by Theorem 4.

Theorem 4. For $\kappa, \mu, m_t, \hat{r}^2 \in \mathbb{R}^+$, the PDF of the single shadowed $\kappa$-$\mu$ Type V fading model can be expressed as

$$f_R(r) = \sum_{i=0}^{\infty} \frac{1}{\Gamma(m_t)\Gamma(i + \mu)} \frac{\Gamma(m_t + i)}{\Gamma(m_t + \mu + i)} \frac{2r \sqrt{m_t \mu(1 + \kappa)}}{\hat{r}} \left(\frac{m_t \mu(1 + \kappa)}{\hat{r}}\right)^{m_t + \mu + i} K_{m_t + \mu + n}(\frac{2r \sqrt{m_t \mu(1 + \kappa)}}{\hat{r}}).$$

where $\kappa, \mu, \hat{r}$ are as defined previously.
Proof: See Appendix C.

It is also possible to derive the PDF of the single shadowed $\kappa$-$\mu$ Type V model as a special case of the statistics of the product of $\kappa$-$\mu$ and Nakagami-$m$ RVs as shown in [13] and [16].

Note that the single shadowed $\kappa$-$\mu$ Type VI model was introduced in [9] as the $\kappa$-$\mu$/inverse gamma fading model in which the mean power of the multipath waves were subject to fluctuations induced by an inverse gamma RV. Furthermore, the PDF of this fading model can be obtained as a special case of the statistics of the ratio of $\kappa$-$\mu$ and inverse Nakagami-$m$ RVs as shown in [17]. Since the inverse Nakagami-$m$ RV used for this analysis is assumed to have $\mathbb{E}[\xi^2] = 1$, the PDF of $R$ for the single shadowed $\kappa$-$\mu$ Type VI fading model can be obtained by substituting $\hat{r}^2 = \frac{(m_t-1)\hat{r}^2}{m_t}$ in [9], as follows

$$f_R(r) = \frac{2(1+\kappa)^\mu \mu e^{-\kappa \mu} ((m_t-1)\hat{r}^2)^{m_t} \hat{r}^{2\mu-1}}{B(m_t, \mu) (\hat{r}^2(m_t-1) + r^2(1+\kappa) \mu)^{m_t+\mu+1}} \text{F}_1 \left( \frac{\mu^2 \kappa (1+\kappa) \hat{r}^2}{r^2(m_t-1) + r^2(1+\kappa) \mu} \right).$$

where $m_t > 1$.

III. Double Shadowed $\kappa$-$\mu$ Models

In this section, we discuss six different ways in which the $\kappa$-$\mu$ fading envelope can be impacted by more than one shadowing process. To this end, we propose the double shadowed $\kappa$-$\mu$ Type I through to Type VI fading models.

A. Double Shadowed $\kappa$-$\mu$ Type I and II Models

The double shadowed $\kappa$-$\mu$ Type I and II models characterize the propagation scenario in which the envelope experiences shadowing of the dominant component, which is preceded (or succeeded) by a secondary round of multiplicative shadowing. Physically, this situation may arise when the signal power delivered through the optical path between the transmitter and receiver is shadowed by objects moving within its locality, whilst further shadowing of the received power (combined multipath and dominant paths) may also occur due to obstacles moving in the vicinity of the transmitter or receiver. Following from this, their signal envelope, $R$, can be expressed in terms of their in-phase and quadrature phase components as
\[ R^2 = A^2 \sum_{i=1}^{\mu} (X_i + \xi p_i)^2 + (Y_i + \xi q_i)^2 \]  \hspace{1cm} (10)

where \( \mu, X_i, Y_i, p_i \) and \( q_i \) are as defined previously, \( \xi \) represents a Nakagami-\( m \) RV in the Type I model, and an inverse Nakagami-\( m \) RV in the Type II model with shape parameter \( m_d \) and \( \mathbb{E} [\xi^2] = 1 \) for both. Likewise, \( A \) denotes an inverse Nakagami-\( m \) RV in the Type I model, and a Nakagami-\( m \) RV in the Type II model with shape parameter \( m_t \) and \( \mathbb{E} [A^2] = 1 \) for both. The PDFs of the double shadowed \( \kappa-\mu \) Type I and Type II fading models can be obtained via Theorems 5 and 6, respectively.

**Theorem 5.** For \( \kappa, \mu, m_d, \hat{r}^2 \in \mathbb{R}^+ \) and \( m_t > 1 \), the PDF of the double shadowed \( \kappa-\mu \) Type I fading model is given as

\[
f_R(r) = \frac{2(m_t - 1)^{m_t} m_d^{m_d} K^{2m_t - 2} \hat{r}^{2m_t - 2} F_1 (m_d, m_t + \mu; \mu; \frac{K \mu \hat{r}^2}{(m_d + \mu \hat{r})^{m_d + \mu}})}{(m_d + \mu \kappa)^{m_d} \Gamma(m_t, \mu) (K \hat{r}^2 + (m_t - 1) \hat{r}^2)^{m_t + \mu}} \hspace{1cm} (11)
\]

where \( K = \mu (1 + \kappa) \).

**Proof:** See Appendix D.

**Theorem 6.** For \( \kappa, \mu, m_t, m_d, \hat{r}^2 \in \mathbb{R}^+ \), the PDF of the double shadowed \( \kappa-\mu \) Type II fading model can be expressed as

\[
f_R(r) = \frac{8 (m_t K)^{\mu + m_t} \hat{r}^{\mu + m_t - 1}}{\Gamma(m_d) \Gamma(m_t) (\kappa \mu (m_d - 1))^{-\frac{m_d}{2} \hat{r}^{\mu + m_t}}} \sum_{i=0}^{\infty} \frac{1}{i! \Gamma(i + \mu)} \left( \frac{r \mu \sqrt{\kappa (m_d - 1) m_t (1 + \kappa)}}{\hat{r}} \right)^i \times K_{m_d - i} \left( 2 \sqrt{(m_d - 1) \mu \kappa} \right) K_{m_t - i} \left( 2 r \sqrt{K m_t \hat{r}} \right) \hspace{1cm} (12)
\]

**Proof:** See Appendix E

**B. Double Shadowed \( \kappa-\mu \) Type III and Type IV Model**

The double shadowed \( \kappa-\mu \) Type III and IV models consider a \( \kappa-\mu \) faded signal in which the dominant component and scattered waves experience two different shadowing processes. Their signal envelope, \( R \), is given by

\[
R^2 = \sum_{i=1}^{\mu} (A X_i + B p_i)^2 + (A Y_i + B q_i)^2 \hspace{1cm} (13)
\]
where $\mu$, $X_i$, $Y_i$, $p_i$ and $q_i$ are as defined previously. In the Type III model, $A$ denotes an inverse Nakagami-\(m\) RV with shape parameter $m_s$, and $B$ represents a Nakagami-\(m\) RV with shape parameter $m_d$. However, in the Type IV model, $A$ denotes a Nakagami-\(m\) RV with shape parameter $m_s$, and $B$ represents an inverse Nakagami-\(m\) RV with shape parameter $m_d$. In both these models, $\mathbb{E}[A^2]$ and $\mathbb{E}[B^2]$ are set equal to 1. An analytical expression for the PDF of the double shadowed Type III fading model can be obtained via Theorem 7 as follows.

**Theorem 7.** For $\kappa$, $\mu$, $m_d$, $r^2 \in \mathbb{R}^+$, and $m_s > 1$ the PDF of the double shadowed $\kappa$-$\mu$ Type III fading model can be expressed as

\[
f_R(r) = \frac{2(m_s - 1)^{m_s}m_d^{m_s+\mu}r^{2\mu-1}x^\mu}{\kappa^{m_s+\mu} \Gamma(m_s)\Gamma(m_d)\mu^{m_s+2\mu}} \sum_{i=0}^{\infty} \frac{2^{2i}(\frac{\mu+m_s}{2})_i}{i!\Gamma(\mu+i)} \Gamma(i+m_d) \times \left( \frac{m_d(1+\kappa)r^2}{r^2\kappa} \right)^i U(2i+\mu+m_s, 1+i+\mu-m_d+m_s, \theta_2)
\]

where $\theta_1 = \frac{1}{2}(1+m_s+\mu)$, $\theta_2 = \frac{m_d(m_s-1)\kappa^2+r^2(1+\kappa)\mu}{r^2\kappa\mu}$, $(a)_i$ is the Pochhammer’s symbol \[14\] Eq. 6.1.22] and $U(\cdot, \cdot, \cdot)$ is the confluent Tricomi hypergeometric function \[14\] eq. 13.1.3]

**Proof:** See Appendix F.

Likewise, the envelope PDF of the double shadowed Type IV model can be obtained via Theorem 8 as follows.

**Theorem 8.** For $\kappa$, $\mu$, $m_s$, $r^2 \in \mathbb{R}^+$, and $m_d > 1$ the PDF of the double shadowed $\kappa$-$\mu$ Type IV fading model can be expressed as

\[
f_R(r) = \frac{2\pi}{\sin(\pi m_d)\Gamma(m_d)\Gamma(m_s)} \left( \frac{\mathcal{M}}{(i-m_d)!} \left( \frac{\mathcal{P}}{r\sqrt{\kappa}} \right)^i - \frac{\mathcal{N}}{(i+m_d)!} \left( \frac{\mathcal{P}}{r\sqrt{\kappa}} \right)^{i+m_d} \right)
\]

where $\mathcal{K}$ is as defined previously, $\mathcal{P} = \kappa \mu r(m_d - 1)\sqrt{m_s}$,

\[
\mathcal{M} = K_{i+\mu-m_s} \left( \frac{2r\sqrt{\kappa m_s}}{\hat{r}} \right) _3 \tilde{F}_1 \left( \frac{-i}{2}, \frac{1-i}{2}, m_d - i; \mu; \frac{4r^2\mathcal{K}}{\kappa \mu r^2(m_d - 1)} \right),
\]

\[
\mathcal{N} = K_{i+\mu+m_d-m_s} \left( \frac{2r\sqrt{\kappa m_s}}{\hat{r}} \right) _3 \tilde{F}_1 \left( \frac{-i}{2}, \frac{1-i}{2}, 1 - i - m_d; \mu; \frac{4r^2\mathcal{K}}{\kappa \mu r^2(m_d - 1)} \right)
\]

and $_3 \tilde{F}_1 (\cdot, \cdot, \cdot; \cdot; \cdot)$ is the generalized hypergeometric function \[18\].
Proof: See Appendix G

For conciseness, it is worth mentioning here that two further types of double shadowed model can be found from (13) which coincidentally lead to PDFs equivalent in form to those given in (11) and (12). These can be found by letting $B = A\xi$, where $A$ and $\xi$ represent either a Nakagami-$m$ and an inverse Nakagami-$m$ RV or vice versa. It is worth highlighting that as shown in [19], $B^2$ follows a Fisher-Snedecor $F$ distribution [20]. Now, substituting for $B$ in (13) we evidently arrive at (10). Then letting $A$ denote an inverse Nakagami-$m$ RV and $\xi$ represent a Nakagami-$m$ RV and following the same statistical procedure highlighted in Section III.A, we arrive at the PDF given in (11). Similarly, if we let $A$ denote a Nakagami-$m$ RV and $\xi$ represent an inverse Nakagami-$m$ RV, we arrive at (12).

C. Double Shadowed $\kappa$-$\mu$ Type V and Type VI Models

The double shadowed $\kappa$-$\mu$ Type V and Type VI fading models consider a $\kappa$-$\mu$ faded signal in which the scattered waves in each cluster are subject to fluctuations caused by shadowing. As well as this, they assume that the rms power of the dominant component and scattered waves may also be subject to random variations induced by shadowing. Their signal envelope, $R$ can be expressed as

$$R^2 = A^2 \sum_{i=1}^{\mu} (\xi X_i + p_i)^2 + (\xi Y_i + q_i)^2$$  \hspace{1cm} (18)

where $\mu$, $X_i$, $Y_i$, $p_i$, and $q_i$ are defined previously, $A$ denotes a Nakagami-$m$ RV in the Type V model and an inverse Nakagami-$m$ RV in the Type VI model with shape parameter $m_t$ and $E[A^2] = 1$ for both. Furthermore, $\xi$ represents an inverse Nakagami-$m$ RV in the Type V model, and a Nakagami-$m$ RV in the Type VI model with shape parameters $m_s$ and $E[\xi^2] = 1$ for both. The PDFs of the double shadowed Type V and Type VI models can be obtained via Theorems 9 and 10 as follows.

**Theorem 9.** For $\kappa$, $\mu$, $m_t$, $r^2 \in \mathbb{R}^+$, and $m_s > 1$ the PDF of the double shadowed $\kappa$-$\mu$ Type V
fading model can be expressed as

\[
f_R(r) = \frac{2 \left( m_s - 1 \right)^{m_s} (m_t) \mu_r^{2 \mu - 1}}{\Gamma(m_t) B \left( m_s, \mu_r \right) \Gamma(2 \mu, m_s)} \sum_{i=0}^{\infty} \left( \frac{1}{2} \Gamma(m_s + \mu) \right) _i \left( \theta_i \right) \Gamma(i + m_s + m_t) \\
\times \left\{ \left( 4 \mu r \kappa m_t \right) ^i \left( m_s - 1 + \kappa \mu \right) ^{2i} \right\} \left( 2i + \mu + m_s, 1 + i + \mu - m_t; \frac{r^2 \kappa m_t}{\Gamma(2 \mu, m_s)} \right) \tag{19}
\]

in which \( \kappa \) and \( \theta_i \) are defined previously.

**Proof:** See Appendix [II]

**Theorem 10.** For \( \kappa, \mu, m_s, \hat{r}^2 \in \mathbb{R}^+ \), and \( m_t > 1 \) the PDF of the double shadowed \( \kappa-\mu \) Type VI fading model can be expressed as

\[
f_R(r) = \frac{2 \pi (\kappa m_s / (m_t - 1))^{m_s} \hat{r}^{2m_s - 1} \hat{r}^{-2m_s}}{\Gamma(m_t) \Gamma(m_s) \sin \left( \frac{\pi \hat{r}}{m_t + m_s} \right)} \sum_{i=0}^{\infty} \frac{(\kappa \mu m_s)^i}{i!} \left( G \left( \frac{\kappa \mu \hat{r}^2 (m_t - 1)}{r^2 \kappa} \right) \right)^{-i} \\
- H \left( \frac{\kappa \mu \hat{r}^2 (m_t - 1)}{r^2 \kappa} \right)^{m_t + m_s} + J \left( \frac{\kappa \mu \hat{r}^2 (m_t - 1)}{r^2 \kappa} \right)^{i + \mu - m_s} \left( \kappa \mu m_s \right)^{-i} \tag{20}
\]

in which \( \kappa \) is defined previously,

\[
G = \frac{(\hat{r}) \pi \csc \left( \frac{\pi (\mu - m_s)}{1 + i - \mu + m_s} \right)}{\Gamma(1 + i - \mu + m_s)} 2 \tilde{F}_2 \left( \begin{array}{c} 1 - i - m_s, -i + \mu - m_s, \mu, 1 - i - m_t - m_s; \\
\frac{- \kappa \mu \hat{r}^2 (m_t - 1)}{r^2 \kappa} \end{array} \right), \tag{21}
\]

\[
H = \frac{\Gamma(m_t + m_s) \Gamma(i + m_s)}{\Gamma(-m_t)} 2 \tilde{F}_2 \left( \begin{array}{c} 1 + m_t, \mu + m_t, 1 + i + m_t + m_s, i + \mu + m_t + m_s; \\
\frac{- \kappa \mu \hat{r}^2 (m_t - 1)}{r^2 \kappa} \end{array} \right), \tag{22}
\]

\[
J = \frac{\Gamma(-i - \mu + m_s) \sin \left( \frac{\pi (m_t + m_s)}{\pi (\mu + m_t)} \right)}{\sin \left( \frac{\pi (m_t + m_s)}{\pi (\mu + m_t)} \right)} 2 \tilde{F}_2 \left( \begin{array}{c} -i, 1 - i - \mu, \mu, 1 - i - \mu - m_t; \\
\frac{- \kappa \mu \hat{r}^2 (m_t - 1)}{r^2 (1 + \kappa)} \end{array} \right), \tag{23}
\]

and \( \tilde{F}_2(a, b; c, d, z) = \tilde{F}_2(a, b; c, d) / (\Gamma(c) \Gamma(d)) \) is a particular case of the generalized hypergeometric function \([18, eq. 7.2.3.1]\).

**Proof:** See Appendix [II]

**IV. SPECIAL CASES OF THE DOUBLE SHADOWED \( \kappa-\mu \) FADING MODELS**

The PDFs given in \([11], [12], [14], [15], [19] \) and \([20] \) represent an extremely versatile set of fading models as they inherit the generalities of the various types of single shadowed \( \kappa-\mu \) fading model. Recall that in the double shadowed \( \kappa-\mu \) Type I and Type II models the \( m_d \) parameter denotes the intensity of shadowing that the dominant signal component undergoes, whilst the
| Fading models       | double shadowed $\kappa$-$\mu$ Type I parameters | double shadowed $\kappa$-$\mu$ Type II parameters | double shadowed $\kappa$-$\mu$ Type III parameters |
|---------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Single shadowed $\kappa$-$\mu$ Type I [6] | $m_t \to \infty, m_d = m_d, \quad \kappa = \kappa, \mu = \mu$ | -                                             | $m_s \to \infty, m_d = m_d, \quad \kappa = \kappa, \mu = \mu$ |
| Single shadowed $\kappa$-$\mu$ Type II | -                                             | $m_t \to \infty, m_d = m_d, \quad \kappa = \kappa, \mu = \mu$ | -                                             |
| Single shadowed $\kappa$-$\mu$ Type III | -                                             | -                                             | -                                             |
| Single shadowed $\kappa$-$\mu$ Type IV | -                                             | -                                             | $m_s \to m_s, m_d \to \infty, \quad \kappa = \kappa, \mu = \mu$ |
| Single shadowed $\kappa$-$\mu$ Type V [15], [16] | -                                             | $m_t \to m_t, m_d \to \infty, \quad \kappa = \kappa, \mu = \mu$ | -                                             |
| Single shadowed $\kappa$-$\mu$ Type VI | $m_t = m_t, m_d \to \infty, \quad \kappa = \kappa, \mu = \mu$ | -                                             | -                                             |
| $\eta - \mu$/inverse gamma [9] | $m_t \to \infty, m_d \to \infty, \quad \kappa \to \frac{(1-\eta)}{2\eta}, \mu = 2\mu, \quad \hat{\nu}^2 = \frac{m_t \hat{r}^2}{(m_t-1)}$ | -                                             | -                                             |
| $\kappa$-$\mu$ | $m_t \to \infty, m_d \to \infty, \quad \kappa = \kappa, \mu = \mu$ | $m_t \to \infty, m_d \to \infty, \quad \kappa = \kappa, \mu = \mu$ | $m_s \to \infty, m_d \to \infty, \quad \kappa = \kappa, \mu = \mu$ |
| $\eta - \mu$ | $m_t \to \infty, m_d \to \infty, \quad \kappa = \frac{(1-\eta)}{2\eta}, \mu = 2\mu$ | -                                             | $m_s \to \infty, m_d \to \mu, \quad \kappa = \frac{(1-\eta)}{2\eta}, \mu = 2\mu$ |
| Shadowed Rician [11] | $m_t \to \infty, m_d = m_d, \quad \kappa = \frac{\kappa}{\mu} = 1$ | -                                             | $m_s \to \infty, m_d = m_d, \quad \kappa = \frac{\kappa}{\mu} = 1$ |
| Rician | $m_t \to \infty, m_d \to \infty, \quad \kappa = \frac{\kappa}{\mu} = 1$ | $m_t \to \infty, m_d \to \infty, \quad \kappa = \frac{\kappa}{\mu} = 1$ | $m_s \to \infty, m_d \to \infty, \quad \kappa = \frac{\kappa}{\mu} = 1$ |
| Nakagami-$q$ (Hoyt) [21] | $m_t \to \infty, m_d = 0.5, \quad \kappa = \frac{(1-q^2)}{2q^2}, \mu = 1$ | -                                             | $m_s \to \infty, m_d = 0.5, \quad \kappa = \frac{(1-q^2)}{2q^2}, \mu = 1$ |
| Nakagami-$m$ | $m_t \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = m$ | $m_t \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = m$ | $m_s \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = m$ |
| Rayleigh | $m_t \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = 1$ | $m_t \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = 1$ | $m_s \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = 1$ |
| One-sided Gaussian | $m_t \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = 0.5$ | $m_t \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = 0.5$ | $m_s \to \infty, m_d \to \infty, \quad \kappa \to 0, \mu = 0.5$ |
### Table II

**Special Cases of the Double Shadowed $\kappa$-$\mu$ Type IV-VI Fading Models**

| Fading models                      | double shadowed $\kappa$-$\mu$ Type IV parameters | double shadowed $\kappa$-$\mu$ Type V parameters | double shadowed $\kappa$-$\mu$ Type VI parameters |
|------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| Single shadowed $\kappa$-$\mu$ Type I [6] | $m_s \to \infty, m_d = m_d$, $\kappa = \kappa, \mu = \mu$ | -                                               | -                                               |
| Single shadowed $\kappa$-$\mu$ Type II | $m_s \to \infty, m_d = m_d$, $\kappa = \kappa, \mu = \mu$ | -                                               | -                                               |
| Single shadowed $\kappa$-$\mu$ Type III | $m_s \to m_s, m_d \to \infty$, $\kappa = \kappa, \mu = \mu$ | $m_s \to \infty, m_s m_t \to \infty$, $\kappa = \kappa, \mu = \mu$ | -                                               |
| Single shadowed $\kappa$-$\mu$ Type IV | -                                               | $m_s \to \infty, m_s m_t \to \infty$, $\kappa = \kappa, \mu = \mu$ | -                                               |
| Single shadowed $\kappa$-$\mu$ Type V [15], [16] | -                                               | $m_s \to \infty, m_s m_t \to \infty$, $\kappa = \kappa, \mu = \mu$ | -                                               |
| Single shadowed $\kappa$-$\mu$ Type VI | -                                               | -                                               | $m_s \to \infty, m_s m_t \to \infty$, $\kappa = \kappa, \mu = \mu$ |
| $\eta$-$\mu$/inverse gamma [9] | -                                               | -                                               | -                                               |
| $\kappa$-$\mu$ | $m_s \to \infty, m_d \to \infty$, $\kappa = \kappa, \mu = \mu$ | $m_s \to \infty, m_t \to \infty$, $\kappa = \kappa, \mu = \mu$ | $m_s \to \infty, m_t \to \infty$, $\kappa = \kappa, \mu = \mu$ |
| $\eta$-$\mu$ | -                                               | -                                               | -                                               |
| Shadowed Rician [11] | -                                               | -                                               | -                                               |
| Rician | $m_s \to \infty, m_d \to \infty$, $\kappa = k, \mu = 1$ | $m_s \to \infty, m_t \to \infty$, $\kappa = k, \mu = 1$ | $m_s \to \infty, m_t \to \infty$, $\kappa = k, \mu = 1$ |
| Nakagami-$q$ (Hoyt) [21] | $m_s \to \infty, m_d \to \infty$, $\kappa = k, \mu = 1$ | $m_s \to \infty, m_t \to \infty$, $\kappa = k, \mu = 1$ | $m_s \to \infty, m_t \to \infty$, $\kappa = k, \mu = 1$ |
| Nakagami-$m$ | $m_s \to \infty, m_d \to \infty$, $\kappa \to 0, \mu = m$ | $m_s \to \infty, m_t \to \infty$, $\kappa \to 0, \mu = m$ | $m_s \to \infty, m_t \to \infty$, $\kappa \to 0, \mu = m$ |
| Rayleigh | $m_s \to \infty, m_d \to \infty$, $\kappa \to 0, \mu = 1$ | $m_s \to \infty, m_t \to \infty$, $\kappa \to 0, \mu = 1$ | $m_s \to \infty, m_t \to \infty$, $\kappa \to 0, \mu = 1$ |
| One-sided Gaussian | $m_s \to \infty, m_d \to \infty$, $\kappa \to 0, \mu = 0.5$ | $m_s \to \infty, m_t \to \infty$, $\kappa \to 0, \mu = 0.5$ | $m_s \to \infty, m_t \to \infty$, $\kappa \to 0, \mu = 0.5$ |
The $m_t$ parameter represents the degree of fluctuations that both the dominant and scattered signal components undergo as a result of the secondary shadowing process. Now, letting $m_t \to \infty$ in (11), we obtain the PDF of the single shadowed $\kappa-\mu$ Type I model, whilst letting $m_d \to \infty$, we obtain the PDF of the single shadowed $\kappa-\mu$ Type VI fading model. Allowing, $m_d \to \infty$ and $\hat{r}^2 = \frac{m_t \hat{r}^2}{(m_t-1)}$ the $\kappa-\mu$/inverse gamma fading model is obtained. Of course, letting $m_t \to \infty$ and $m_d \to \infty$, we obtain the PDF of the $\kappa-\mu$ fading model. These special case results are illustrated in Fig. I and are in exact agreement with the Monte-Carlo simulations. The PDF of the $\eta-\mu$/inverse gamma fading model can also be obtained from the double shadowed $\kappa-\mu$ Type I fading model by setting $m_d \to \mu$, $\kappa = \frac{(1-\eta)}{2\eta}$, $\mu = 2\mu$ and $\hat{r}^2 = \frac{m_t \hat{r}^2}{(m_t-1)}$. Of course, letting $m_t \to \infty$, $m_d \to \mu$, $\kappa = \frac{(1-\eta)}{2\eta}$ and $\mu = 2\mu$ we obtain the PDF of the $\eta-\mu$ fading model. Likewise, the PDFs of the double shadowed Rician Type I, shadowed Rician, and Rician fading models can be obtained from (11) by first setting $\mu = 1$, $\kappa = k$ (the Rician $k$-factor), followed by appropriate substitutions for $m_d$ and $m_t$. Fig. I shows the shape of the PDF for these special cases which are indicated in red.

In a similar manner, the double shadowed $\kappa-\mu$ Type II fading model contains the single shadowed $\kappa-\mu$ Type II and Type V fading models as special cases. Now letting $m_t \to \infty$ in (12), we obtain the PDF of the single shadowed $\kappa-\mu$ Type II model, whilst letting $m_d \to \infty$ we obtain the PDF of the single shadowed $\kappa-\mu$ Type V fading model. Allowing both $m_t \to \infty$ and $m_d \to \infty$, the PDF of the $\kappa-\mu$ fading model is obtained. The PDF given in (14) (double shadowed Type III) also represents an extremely flexible fading model as it contains the single shadowed $\kappa-\mu$ Type I, Type IV, $\kappa-\mu$ and $\eta-\mu$ fading models as special cases. Different from the double shadowed $\kappa-\mu$ Type I and Type II fading models, here the $m_s$ parameter represents the degree of fluctuation that the scattered signal components undergo. Now, letting $m_s \to \infty$ in (14), we obtain the PDF of the single shadowed $\kappa-\mu$ Type I model, whilst letting $m_d \to \infty$ we obtain the PDF of the single shadowed $\kappa-\mu$ Type IV fading model. Allowing both $m_s \to \infty$ and $m_d \to \infty$ in (14), the double shadowed $\kappa-\mu$ Type III fading model coincides with the $\kappa-\mu$ fading model. For the reader’s convenience, Table I summarizes the special cases of the double shadowed $\kappa-\mu$ Type I - III fading models whilst Table II summarizes the special cases of the double shadowed $\kappa-\mu$ Type IV - VI fading models. For the sake of clarity, the double shadowed
Fig. 1. The PDF of the double shadowed $\kappa$-$\mu$ Type I fading model reduced to some of its special cases: $\kappa$-$\mu$ (blue asterisk markers), single shadowed $\kappa$-$\mu$ Type I (blue triangle markers), $\kappa$-$\mu$/inverse gamma (blue square markers), Rician (red square markers), shadowed Rician (red asterisk markers), double shadowed Rician Type I (red circle markers). Here, $\hat{r} = 0.8$, lines represent analytical results, and the markers represent simulation results.

Fig. 2. The PDF of the single shadowed $\kappa$-$\mu$ Type IV (blue and green lines) and double shadowed $\kappa$-$\mu$ Type I (red and black lines) fading models. Lines represent the analytical results, circle markers represent simulation results.

$\kappa$-$\mu$ parameters have been underlined.

V. NUMERICAL RESULTS AND CHANNEL MEASUREMENTS

A. Numerical Results

Figs. 2 and 3 show some example plots of the PDF of the single shadowed $\kappa$-$\mu$ Type IV, and double shadowed Type I and Type II fading models for different values of $\kappa$, $\mu$, $m_s$, $m_d$, $m_g$, $m_d$, $m_g$, and $\hat{r}$. Some of these models are represented by blue markers for analytical results and red markers for simulation results. The $\kappa$-$\mu$ parameters have been underlined.
Fig. 3. The PDF of the double shadowed \(\kappa-\mu\) Type I and Type II fading models for different values of \(m_d\) and \(m_t\). Here, \(\kappa = 3.9\), \(\mu = 2.4\), and \(\hat{\nu} = 2.5\). Solid lines represent the PDF of the double shadowed \(\kappa-\mu\) Type I model and dashed lines represent the PDF of the double shadowed \(\kappa-\mu\) Type II model.

It should be noted that the values of the parameters are chosen to illustrate the wide range of shapes that the new shadowed fading models can exhibit. Fig. 2 shows the PDF of the single shadowed \(\kappa-\mu\) Type IV and double shadowed Type I fading models for \(\{\kappa, \mu\} = \{0.5, 2.0\}, \{4.2, 2.0\}, \{15.1, 5.0\}, \{m_d, m_t, \hat{\nu}\} = \{2.3, 3.8, 1.8\}, \{25.1, 18.9, 3.0\}\) and \(\{m_s, \hat{\nu}\} = \{3.8, 1.8\}, \{18.9, 3.0\}\). In all cases, the analytical results agree with the Monte-Carlo simulations.

### B. Channel Measurements

While a detailed empirical investigation of all of the models presented here is clearly beyond the scope and space constraints of the paper, in this subsection we take the double shadowed \(\kappa-\mu\) Type I as an example and compare it with some D2D channel measurements which were conducted at 868 MHz. Specific details of the measurement hardware and experiments can be found in [7]. From the study reported in [7], we consider the walking in LOS and non-LOS (NLOS) scenarios. For the analysis conducted here, only the path loss was removed from the D2D channel data. Fig. 4 shows the PDF of the double shadowed \(\kappa-\mu\) Type I fading model.

\(^4\)It is worth highlighting here that in [7] it was necessary to not only abstract the path loss from the measurement data, but also the large-scale fading. Using the double shadowed \(\kappa-\mu\) Type I model proposed here circumvents the need to determine an appropriate smoothing window for the computation of the local mean signal, which can fundamentally affect the parameter estimation process and any inference made from the channel data.
fitted to the empirical PDF for the $UE_1$ pocket to $UE_2$ head channel as person 1 walked towards (LOS) and then away (NLOS) from person 2. Likewise, Fig. 5 shows the PDF of the double shadowed $\kappa$-$\mu$ Type I fading model fitted to the empirical PDF for the $UE_1$ head to $UE_2$ pocket channel as person 1 walked towards (LOS) and then away (NLOS) from person 2. It is clear from Figs. 4 and 5 that the double shadowed $\kappa$-$\mu$ Type I model provides an excellent fit for both the LOS and NLOS channel data. Therefore, this fading model can be used to directly characterize the shadowed fading encountered in D2D channels, without the need to separate the signal into its constituent small-scale and shadowed parts.

VI. CONCLUSION

For the first time in the literature, this paper has discussed the various ways in which a $\kappa$-$\mu$ fading envelope can be affected by shadowing. A family of shadowed $\kappa$-$\mu$ fading models were proposed and classified based on whether the underlying $\kappa$-$\mu$ envelope undergoes single or double shadowing. In total, six types of single shadowed $\kappa$-$\mu$ model (Type I - VI) were introduced and formulated. Amongst these, the single shadowed $\kappa$-$\mu$ Type II, Type III and Type IV models are novel. Analytical formulations were derived for the PDF of the Type II and Type III models, whilst a closed-form expression was obtained for the PDF of the Type IV model. A further six types of double shadowed $\kappa$-$\mu$ model (Type I - VI) were also introduced, all of which are novel.
A closed-form expression was derived for the PDF of the double shadowed $\kappa$-$\mu$ Type I model whereas analytical formulations were obtained for the PDF of Type II - Type VI models.

Finally, as an example application, the utility of the double shadowed $\kappa$-$\mu$ Type I model was illustrated for characterizing the shadowed fading encountered in D2D channels. It was shown that this fading model was able to provide an excellent characterization of the field data without the need to determine a smoothing window size to abstract the local mean signal. Finally, as an example application, the utility of the double shadowed $\kappa$-$\mu$ Type I model was illustrated for characterizing the shadowed fading encountered in D2D channels. It was shown that this fading model was able to provide an excellent characterization of the field data without the need to determine a smoothing window size to abstract the local mean signal.

**APPENDIX A**

**PROOF OF THEOREM 1**

Consider the signal model given in (1) where $\xi$ is assumed to be an inverse Nakagami-$m$ RV with shape parameter $m_d$ and $\mathbb{E} [\xi^2] = 1$, its PDF is given by

$$f_\xi (\xi) = \frac{2(m_d - 1)^{m_d}}{\Gamma (m_d) \xi^{2m_d+1}} e^{-\frac{(m_d-1)}{\xi^2}}. \quad (24)$$
To determine the envelope distribution of the single shadowed \( \kappa-\mu \) Type II fading model we average the conditional PDF, \( f_{R|\xi}(r|\xi) \), with the PDF of \( \xi \) given in (24) i.e.

\[
f_R(r) = \int_0^\infty f_{R|\xi}(r|\xi) f_\xi(\xi) \, d\xi.
\]

The signal model for the single shadowed \( \kappa-\mu \) Type II fading model, insinuates that the conditional probability, \( f_{R|\xi}(r|\xi) \), follows a \( \kappa-\mu \) distribution with PDF \[1\]

\[
f_{R|\xi}(r|\xi) = \frac{r^\mu}{\sigma^2(\xi d)^{\mu-1}} e^{-\frac{r^2 - \xi^2 d^2}{2\sigma^2}} I_{\mu-1} \left( \frac{\xi d r}{\sigma^2} \right)
\]

where \( d^2 \) and \( \sigma^2 \) are as defined in section II.A (also see \[1\]).

An analytical expression for the PDF of the single shadowed \( \kappa-\mu \) Type II fading model can be obtained by substituting (26) and (24) in (25) as follows

\[
f_R(r) = \int_0^\infty \frac{2r^\mu(m_d - 1)^{m_d} e^{-\frac{(m_d - 1)\xi^2 d^2}{2\sigma^2}}}{\sigma^2(\xi d)^{\mu-1} \Gamma(m_d) \xi^{2m_d + 1}} I_{\mu-1} \left( \frac{\xi d r}{\sigma^2} \right) \, d\xi.
\]

Replacing the modified Bessel function of the first kind with \[22] \[03.02.02.0001.01\] i.e, \( I_v(x) = \sum_{k=0}^{\infty} \frac{(\frac{x}{2})^{v+2k}}{k!\Gamma(v+k+1)} \) in (27), followed by solving the integral using \[13\] eq. 3.471.9, and finally substituting \( d = \sqrt{2\mu \sigma^2 \kappa}; \sigma = \sqrt{\frac{\xi^2}{2\mu(1+\kappa)}} \) in the resultant expression, we obtain (3).

**APPENDIX B**

**PROOF OF THEOREM 2 AND THEOREM 3**

Let us assume that \( \xi \) is a Nakagami-\( m \) RV with shape parameter \( m_s \) and \( E[\xi^2] = 1 \). Its PDF is given by

\[
f_\xi(\xi) = \frac{2m_s^m \xi^{2m_s - 1}}{\Gamma(m_s)} e^{-m_s \xi^2}.
\]

The signal model presented in (4) insinuates that the conditional probability, \( f_{R|\xi}(r|\xi) \), follows a \( \kappa-\mu \) distribution with PDF \[1\]

\[
f_{R|\xi}(r|\xi) = \frac{r^\mu}{\sigma^2 \xi^2 d^{\mu-1}} e^{-\frac{r^2 - \xi^2 d^2}{2\sigma^2 \xi^2}} I_{\mu-1} \left( \frac{dr}{\sigma^2 \xi^2} \right)
\]
An analytical expression for the PDF of the single shadowed \( \kappa-\mu \) Type III fading model can be obtained by substituting (29) and (28) in (25) as follows

\[
f_R(r) = \int_0^\infty \frac{2\mu m_s \xi^{2m_s-3}}{\sigma^2 d\mu^{-1}\Gamma(m_s)} e^{-m_s \xi^2 - \frac{r^2 + d^2}{2\sigma^2 \xi^2}} I_{\mu-1}\left(\frac{dr}{\sigma^2 \xi^2}\right) d\xi.
\] (30)

Replacing the modified Bessel function of the first kind with its series representation [22, 03.02.02.0001.01] in (30), followed by solving the integral using [13, eq. 3.471.9], and finally substituting \( d = \sqrt{2\mu \sigma^2 \kappa} \); \( \sigma = \sqrt{\frac{\hat{r}^2}{2\mu(1+\kappa)}} \) in the resultant expression, we obtain (5).

Similarly, a closed form expression for the PDF of the single shadowed \( \kappa-\mu \) Type IV fading model, is obtained by substituting (29) and (24) (after replacing \( m_d \) with \( m_s \)) in (25)

\[
f_R(r) = \int_0^\infty \frac{2\mu (m_s - 1) m_s e^{-\hat{r}^2 (1+\kappa)}}{\sigma^2 d\mu^{-1}\Gamma(m_s)} \xi^{2m_s+3} I_{\mu-1}\left(\frac{2\mu \sqrt{\kappa (1+\kappa) \hat{r}^2}}{\sqrt{\xi^2 \hat{r}^2}}\right) d\xi.
\] (31)

The above integral is identical to [23, eq. 2.15.3.2]. Now, substituting for \( d \) and \( \sigma \) in the resultant expression and performing some algebraic manipulations, we obtain (7).

**Appendix C**

**Proof of Theorem 4**

We determine the envelope distribution of the single shadowed \( \kappa-\mu \) Type V fading model using (25). Here, the conditional probability, \( f_{R|\xi}(r|\xi) \) is given by

\[
f_{R|\xi}(r|\xi) = \frac{2\mu (1+\kappa) \frac{1}{2} r^\mu e^{-\frac{(1+\kappa) \hat{r}^2}{2\xi^2}}}{\kappa^{\mu+1/2} e^{\kappa \mu} (\xi^2 \hat{r}^2)^{\mu+1/2}} I_{\mu-1}\left(\frac{2\mu \sqrt{\kappa (1+\kappa) \hat{r}^2}}{\sqrt{\xi^2 \hat{r}^2}}\right).
\] (32)

Substituting (32) and (28) (after replacing \( m_s \) with \( m_t \)) in (25), followed by replacing the modified Bessel function of the first kind with its series representation [22, 03.02.02.0001.01], and solving the resulting integral using [13, eq. 3.471.9], we obtain (8).

**Appendix D**

**Proof of Theorem 5**

We determine the envelope distribution of the double shadowed \( \kappa-\mu \) Type I fading model using (25) (after replacing \( \xi \) with \( \alpha \)). Its signal model insinuates that the conditional probability,
\( f_{R|A}(r|\alpha) \), follows a single shadowed \( \kappa-\mu \) Type I distribution with PDF [6] [7]

\[
f_{R|A}(r|\alpha) = \frac{2\mu^\mu (1+\kappa)^\mu r^{2\mu-1}m_d^m d^{\frac{m}{2}}}{\Gamma(\mu)(m_d+\mu\kappa)^m_d\alpha^{2\mu^2}e^{-\frac{(1+\kappa)^2}{\alpha^2 r^2}}} F_1 \left( m_d; \mu; \frac{\mu^2 \kappa (1+\kappa) r^2}{\alpha^2 r^2 (m_d+\mu\kappa)} \right)
\]  

(33)

where, \( \kappa, \mu, \hat{r} \) and \( m_d \) are as defined in section II A.

Now replacing \( \xi \) with \( \alpha \), and \( m_d \) with \( m_t \) in (24), followed by substituting the resultant expression and (33) in (25) (where \( \xi \) is replaced with \( \alpha \)), we obtain

\[
f_R(r) = 4m_d^m_d (m_t-1)^m_t \mu^\mu (1+\kappa)^\mu r^{2\mu-1} \frac{e^{-\frac{2m_t-2\mu-1}{\alpha^{2\mu}} (m_d+\mu\kappa)}}{\Gamma(\mu)(m_d+\mu\kappa)^m_d}\int_0^\infty \frac{e^{-\frac{\alpha^2 \kappa^2}{\alpha^2 r^2} (m_d+\mu\kappa)}}{\Gamma(\mu)(m_d+\mu\kappa)^m_d}\right) d\alpha.
\]

(34)

The above integral is identical to [13] eq. 7.621.4. Performing the necessary transformation of variables followed by some simple mathematical manipulations, we obtain (11).

**APPENDIX E**

**PROOF OF THEOREM 6**

The signal envelope, \( R \), of the double shadowed \( \kappa-\mu \) Type II fading model is given by (10). Here, \( A \) follows a Nakagami-\( m \) distribution with shape parameter \( m_t \), and \( \xi \) follows an inverse Nakagami-\( m \) distribution with shape parameter \( m_d \). It is noteworthy that the model in (10) may be viewed as a product of a Nakagami-\( m \) RV and a single shadowed \( \kappa-\mu \) Type II RV. According to standard probability procedure, this PDF can be obtained as

\[
f_R(r) = \int_0^\infty \frac{1}{a} f_T(t) f_A(a) da
\]

(35)

where \( f_T(t) \) is given in (3). Replacing the respective PDFs in (35) and changing the order of integration and summation,

\[
f_R(r) = \sum_{i=0}^{\infty} \frac{8 [(m_d-1)\kappa]^m_d \mu^{1+m_d} r^{2i+2\mu-1}m_t^m m_t^{m_t}}{\hat{r}^{2i+2\mu} \Gamma(m_d) \Gamma(i+\mu) \Gamma(m_t)} \left( \frac{2\sqrt{(m_d-1)^2\mu}\kappa}{\alpha^2 m_t+\frac{2\kappa^2}{\alpha^2 r^2}} \right) \int_0^\infty \frac{da^{2(m_t-i-\mu)-1}}{e^{a^2(m_t-i-\mu)+\frac{2\kappa^2}{\alpha^2 r^2}}} da
\]

(36)

Now, solving the above integral using [24] eq. 2.3.16.1] followed by some algebraic manipulations, we obtain (12).
APPENDIX F

PROOF OF THEOREM \[7\]

We determine the envelope distribution, \( R \), of the double shadowed \( \kappa-\mu \) Type III fading model when \( A \) and \( B \) vary according to the inverse Nakagami-\( m \) and Nakagami-\( m \) distributions, respectively, from the following integral

\[
f_R(r) = \int_0^\infty \int_0^\infty f_{R|\alpha,\beta}(r|\alpha,\beta) f_\alpha(\alpha) f_\beta(\beta) \, d\alpha \, d\beta \tag{37}
\]

where

\[
f_{R|\beta}(r|\beta) = \int_0^\infty f_{R|\alpha,\beta}(r|\alpha,\beta) f_\alpha(\alpha) \, d\alpha \tag{38}
\]

and the double shadowed \( \kappa-\mu \) Type III signal model insinuates that \( f_{R|\alpha,\beta}(r|\alpha,\beta) \) follows a \( \kappa-\mu \) distribution with PDF \[1\]

\[
f_{R|\alpha,\beta}(r|\alpha,\beta) = \frac{r^\mu}{\sigma^2 \alpha^2 (\beta d)^{\mu-1} e^{-\frac{r^2}{2\sigma^2}} I_{\mu-1} \left( \frac{\beta dr}{\sigma^2 \alpha^2} \right)} \tag{39}
\]

whilst \( f_\alpha(\alpha) \) is similar to \( (24) \) where \( \xi \) and \( m_d \) are replaced with \( \alpha \) and \( m_s \), respectively.

Likewise, \( f_\beta(\beta) \) is similar to \( (28) \) where \( \xi \) is replaced with \( \beta \), and \( m_s \) is replaced with \( m_d \).

Making appropriate substitutions in \( (24) \), followed by using the resultant expression and \( (39) \) in \( (38) \), and finally solving the integral using \[23, eq. 2.15.3.2\] we obtain

\[
f_{R|\beta}(r|\beta) = \frac{2^{m_s+1}(m_s-1)^{m_s} r^{2\mu-1} \sigma^{2m_s} \Gamma(m_s+\mu)}{\Gamma(m_s) \Gamma(\mu) (r^2 + d^2 \beta^2 + 2(m_s-1)\sigma^2)^{m_s+\mu}} \times 2F_1 \left( \frac{m_s+\mu}{2}; \frac{\theta_1 + \mu}{\mu}; \frac{4d^2 r^2 \beta^2}{(r^2 + d^2 \beta^2 + 2(m_s-1)\sigma^2)^2} \right). \tag{40}
\]

Substituting \( (40) \) and \( (28) \) (after replacing \( \xi \) and \( m_s \) with \( \beta \) and \( m_d \)) in \( (37) \), simplifying the resultant integral, followed by replacing the Gauss hypergeometric function with its series representation \[22, 07.23.02.0001.01\] we obtain

\[
f_R(r) = \frac{2^{m_s+2}(m_s-1)^{m_s} r^{2\mu-1} \sigma^{2m_s} m_d \Gamma(m_s+\mu)}{\Gamma(m_s) \Gamma(m_d) (r^2 + 2(m_s-1)\sigma^2)^{m_s+\mu}} \times \sum_{i=0}^{\infty} \frac{(m_s+\mu+i) (m_s+\mu+i+1)}{i!} \left( \frac{4d^2 r^2}{r^2 + 2(m_s-1)\sigma^2} \right)^i \int_{0}^{\infty} \frac{\beta^{2i+2md-1}}{e^{\beta \sigma^2}} \left( \frac{d^2 \beta^2}{r^2 + 2(m_s-1)\sigma^2 + 1} \right)^{-2i-\mu-m_s} \, d\beta. \tag{41}
\]
Now solving the integral in (41) using [14, eq. 13.2.5], followed by substituting [22, 07.33.17.0007.01] for the hypergeometric U function (Tricomi confluent hypergeometric function), \( d = \sqrt{2\mu \sigma^2 \kappa} \) and \( \sigma = \sqrt{\frac{\sigma^2}{2\mu(1+\kappa)}} \), and finally simplifying the resultant expression we obtain (14).

APPENDIX G

PROOF OF THEOREM 8

The envelope distribution, \( R \), of the double shadowed \( \kappa \)-\( \mu \) Type IV fading model when \( A \) and \( B \) vary according to Nakagami-\( m \) and inverse Nakagami-\( m \) distributions, respectively, can be obtained through (37). The double shadowed \( \kappa \)-\( \mu \) Type IV signal model presented in (13) insinuates that \( f_R |_{\alpha, \beta}(r | \alpha, \beta) \) follows a \( \kappa \)-\( \mu \) distribution with PDF given in (39). Also, \( f_\alpha(\alpha) \) is similar to (28) with \( \xi \) replaced by \( \alpha \), and \( f_\beta(\beta) \) is similar to (24) with \( \xi \) replaced by \( \beta \). Now integrating with respect to \( \beta \), we obtain an expression similar to (3) which is conditioned on \( \alpha \)

\[
f_R(r) = 8K^\mu \left( \kappa \mu(m_d - 1) \right)^{\frac{m_d}{\mu} m_s^m s^r} 2^{m-1} \sum_{i=0}^{\infty} \frac{1}{i! \Gamma(\mu + i)} \left( \frac{r^2 K \sqrt{\kappa \mu(m_d - 1)}}{r^2} \right)^i \times \int_0^\infty \alpha^{1-3i-2\mu-m_d+2m_s} e^{-\left( \alpha^2 m_s + \frac{r^2 K}{\alpha^2} \right)} K_{i-m_d} \left( \frac{2 \sqrt{\kappa \mu(m_d - 1)}}{\alpha} \right) d\alpha. \tag{42}
\]

The above integral can be solved by replacing the Bessel function with its power series representation [22, 03.04.06.0002.01] followed by changing the order of integration and summation. Now using [24, eq. 2.3.16.1] we obtain

\[
f_R(r) = \frac{4\pi K^\mu m_s^m s^r 2^{\mu-1} \hat{r}^{2\mu}}{\sin(\pi m_d) \Gamma(m_d) \Gamma(m_s)} \left( \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^i m_s^i}{i! k! \Gamma(i + \mu) \Gamma(1 + i + k - m_d)} \left( \frac{r^2 K}{r^2 m_s} \right)^{m_s - k - \mu} \right) \times K_{2i+k+\mu-m_s} \left( \frac{2r \sqrt{K m_s}}{\hat{r}} \right) - \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{(-1)^i m_s^i}{i! k! \Gamma(i + \mu) \Gamma(1 - i + k + m_d)} \left( \frac{r^2 K}{r^2 m_s} \right)^{1-k-\mu-m_d+m_s} \times K_{n+i+k+\mu+m_d-m_s} \left( \frac{2r \sqrt{K m_s}}{\hat{r}} \right) \tag{43}
\]

The first double summation in (43) can be simplified by using the index substitution \( 2i+k = n \)
so that the inner sum (after some algebraic manipulations) simplifies to
\[ \sum_{i=0}^{n/2} \left( -\frac{n}{2} \right)_i \left( \frac{1-n}{2} \right)_i \left( -n + m_d \right)_i \left( \frac{4r^2(1 + \kappa)}{\tilde{r}^2 \kappa (m_d - 1)} \right)^i = 3 \tilde{F}_1 \left( -\frac{n}{2}, \frac{1-n}{2}, m_d - n; \frac{4r^2(1 + \kappa)}{\tilde{r}^2 \kappa (m_d - 1)} \right). \]

Further simplifying and changing the index \( n \) to \( i \), we obtain the first part of \( (15) \). The last double summation in \( (43) \) is simplified by summing over the infinite triangle \( k = n - i \). The inner sum now reduces to
\[ \sum_{i=0}^{n} \left( -n \right)_i \left( -\frac{n-m_d}{2} \right)_i \left( \frac{1-n-m_d}{2} \right)_i \left( \frac{4r^2(1 + \kappa)}{\tilde{r}^2 \kappa (m_d - 1)} \right)^i \]
which after some algebraic manipulations simplifies to the second part of \( (15) \).

**APPENDIX H**

**PROOF OF THEOREM 9**

The double shadowed \( \kappa-\mu \) Type V model can be viewed as a product of a Nakagami-\( m \) RV and a single shadowed \( \kappa-\mu \) Type IV RV. Accordingly, its PDF can be obtained by first replacing \( (6) \) with the hypergeometric function expressed in terms of its power series expression \[14, eq. 15.1.1\], then substituting the resultant expression and \( (28) \) (after replacing \( \xi \) and \( m_s \) with \( \alpha \) and \( m_t \)) in \( (35) \), changing the order of integration and summation, and finally followed by some algebraic manipulations as
\[ f_R(r) = \frac{4 (m_s - 1)^{m_s} m_t^{m_t+2m_s}}{K^{m_s} \Gamma (m_t) B (m_s, \mu) \tilde{r}^{2m_s+1}} \sum_{i=0}^{\infty} \left( \frac{m_s+\mu}{2} \right)_i (\theta_1)_i \left( \frac{4\kappa r^2}{\tilde{r}^2 (1 + \kappa)} \right)^i \times \int_0^{\infty} \alpha^{-1+2i+2m_s+2m_t} \left( 1 + \frac{\alpha^2 r^2 (m_s - 1 + \kappa \mu)}{\tilde{r}^2 (1 + \kappa) \mu} \right)^{-2i-\mu-m_s} e^{-m_t \alpha^2} d\alpha. \]

Now solving the above integral using \[14, eq. 13.2.5\] followed by some algebraic manipulations we obtain \( (19) \).

**APPENDIX I**

**PROOF OF THEOREM 10**

The double shadowed \( \kappa-\mu \) Type VI model can be viewed as a single shadowed \( \kappa-\mu \) Type VI model in which the variation of the scattered waves is influenced by a Nakagami-\( m \) RV. The
PDF of the envelope $R$ can be obtained via

$$f_R(r) = \int_0^\infty f_{R|\alpha}(r|\alpha) f_\alpha(\alpha) \, d\alpha$$  \hspace{1cm} (46)$$

where $f_\alpha(\alpha)$ is similar to (28) such that $\xi$ is replaced by $\alpha$. $f_{R|\alpha}(r|\alpha)$ can be obtained from (9) by first replacing $\kappa = d^2/(2\mu^2)$ and $\hat{r} = 2\mu \sigma^2 + d^2$, then multiplying $\sigma$ by $\alpha$ and finally using $d = \sqrt{2\mu \sigma^2 \kappa}$, $\sigma = \sqrt{\frac{\hat{r}^2}{2\mu(1+\kappa)}}$ as follows

$$f_{R|\alpha}(r|\alpha) = \frac{2(m_t - 1)m_t^2 \alpha^{2m_t}}{(1 + \kappa) \mu^m B(m_t, \mu) \hat{r}^{-2m_t}} \left(1 + \frac{\alpha^2 \hat{r}^2 (m_t - 1)}{r^2 (1 + \kappa) \mu}\right)^{-\mu - m_t} \times \exp\left(-\frac{\kappa \mu}{\alpha^2}\right) \textbf{1}_{F_1}\left(\mu + m_t; \frac{\kappa \mu}{\alpha^2} \left(1 + \frac{\alpha^2 \hat{r}^2 (m_t - 1)}{r^2 (1 + \kappa) \mu}\right)^{-1}\right).$$  \hspace{1cm} (47)

Now substituting (47) in (46) we obtain

$$f_R(r) = \frac{4((1 + \kappa) \mu)^{-m_t} m_t(m_t - 1)m_t^m m_s^{-1 - 2m_t}}{B(m_t, \mu) \Gamma(m_s) \hat{r}^{-2m_t}} \int_0^\infty \frac{\alpha^{-1 + 2m_t + 2m_s}}{\exp\left(\alpha^2 m_s + \frac{\kappa \mu}{\alpha^2}\right)} \times \left(1 + \frac{\alpha^2 \hat{r}^2 (m_t - 1)}{r^2 (1 + \kappa) \mu}\right)^{-\mu - m_t} \textbf{1}_{F_1}\left(\mu + m_t; \frac{\kappa \mu}{\alpha^2} \left(1 + \frac{\alpha^2 \hat{r}^2 m_t}{r^2 (1 + \kappa) \mu}\right)^{-1}\right) \, d\alpha.$$  \hspace{1cm} (48)

Now, it is possible to rewrite the hypergeometric function in terms of its Mellin-Barnes contour integral representation using [18 eq. 7.2.3.12], whilst the exponential function can be written as a product of two contour integrals using [18 eq. 8.4.3.1 and 8.4.3.2] and [18 eq. 8.2.1.1]. Now performing some algebraic manipulations we obtain

$$f_R(r) = \frac{4\kappa^{-m_t} m_t^2 \alpha^{2m_t}}{\Gamma(m_t) \Gamma(m_s) r^{1 + 2m_t}} \left(\frac{1}{2\pi j}\right)^3 \int_0^\infty \int_{\mathcal{L}} \Gamma(t_1) \Gamma(-t_2) \Gamma(-t_3) \Gamma(t_3 + m_t) (\kappa \mu)^{t_2 + t_3} \Gamma(t_3 + \mu) (1)^{t_3 m_t^1 - m_s} \times \alpha^{2\theta_3 - 1} \left(1 + \frac{\alpha^2 \hat{r}^2 (m_t - 1)}{r^2 \kappa}\right)^{-t_3 - \mu - m_t} \, dt_1 dt_2 dt_3 d\alpha$$  \hspace{1cm} (49)$$

where $\theta_3 = m_t + m_s - t_1 - t_2 - t_3$, $\kappa$ is as defined previously, $j = \sqrt{-1}$ is the imaginary particle and $\mathcal{L}$ is a suitable contour in the complex space. Now changing the order of integration, the inner integral can be solved using [24 eq. 2.2.5.24], which results in the triple contour integral
as follows

\[ f_R(r) = \frac{2(m_sK)^{m_s+2m_s-1}}{\Gamma(m_t)\Gamma(m_s)(m_t-1)^{m_s+2m_s}} \left( \frac{1}{2\pi j} \right)^3 \int_{\mathcal{C}} \Theta(t_1, t_2, t_3) \left( \frac{r^2Km_s}{r^2(m_t-1)} \right)^{-t_1} \times \left( \frac{r^2K}{\kappa\mu r^2(m_t-1)} \right)^{-t_2} \left( \frac{-r^2K}{\kappa\mu r^2(m_t-1)} \right)^{-t_3} dt_1 dt_2 dt_3 \]

where

\[ \Theta(t_1, t_2, t_3) = \frac{\Gamma(t_1)\Gamma(-t_2)\Gamma(-t_3)\Gamma(\theta_3)\Gamma(\theta_4)}{\Gamma(t_3+\mu)} \]

and \( \theta_4 = t_1 + t_2 + 2t_3 + \mu - m_s \). It is possible to obtain a multi-fold series representation from the above contour integral through the sum of residues theorem. The residues for the variable \( t_1 \) are taken around the poles of \( \Gamma(t_1) \) and \( \Gamma(\theta_4) \); the residues for \( t_2 \) are taken from \( \Gamma(-t_2) \) and \( \Gamma(\theta_3) \); and the residues for \( t_3 \) are taken from \( \Gamma(-t_3) \). This results in

\[ f_R(r) = \frac{2((Km_s/(m_t-1))^{m_s+2m_s-1}}{\Gamma(m_t)\Gamma(m_s)r^{2m_s}}(S_1 + S_2 + S_3) \]

where

\[ S_1 = \sum_{i,j,k'=0}^{\infty} (-1)^{i+j} \Gamma(-i+j+2k'+\mu-m_s) \Gamma(i-j-k'+m_t+m_s) \frac{r^2Km_s}{r^2(m_t-1)}^i \left( \frac{\kappa\mu r^2(m_t-1)}{r^2K} \right)^{j+k'} \frac{1}{i!j!k'!\Gamma(k'+\mu)} \]

\[ S_2 = \sum_{i,j,k'=0}^{\infty} (-1)^{i+j} \Gamma(j+k'+\mu+m_t) \Gamma(-i+j+k'-m_t-m_s) \frac{\kappa\mu m_s}{(\kappa\mu r^2(1+\kappa))}^{j+m_t+m_s} \frac{1}{i!j!k'!\Gamma(k'+\mu)} \]

\[ S_3 = \sum_{i,j,k'=0}^{\infty} (-1)^{i+j} \Gamma(i+k'+\mu+m_t) \Gamma(-i-j-2k'-\mu+m_s) \frac{r^2Km_s}{r^2(m_t-1)}^{i+2k'+\mu-m_s} \left( \frac{\kappa\mu r^2(m_t-1)}{r^2K} \right)^{k'} \frac{1}{i!j!k'!\Gamma(k'+\mu)} \]

The first triple summation \( S_1 \) can be reduced by summing over the infinite triangle \( j = n - k \) and using [24, eq. 4.2.5.55] as

\[ S_1 = \sum_{i,n=0}^{\infty} \frac{\pi \csc(\pi (\mu-m_s)) \Gamma(i-n+m_t+m_s)}{i!n!\Gamma(n+\mu)\Gamma(1+i-n-\mu+m_s)} \frac{\Gamma(i+m_s)}{\Gamma(i-n+m_s)} \left( \frac{\kappa\mu r^2(m_t-1)}{r^2K} \right)^n \left( \frac{r^2Km_s}{r^2(m_t-1)} \right)^i \]

The triple summation \( S_2 \) can be simplified by summing it over index \( k' \). This results in a Gauss
The hypergeometric function whose argument is one. Using [18, eq. 7.3.5.2], we obtain

\[ S_2 = \sum_{i,j=0}^{\infty} \frac{(-1)^i j \Gamma (-i - j - m_t - m_s) \Gamma (i + m_s) \Gamma (j + \mu + m_t)}{i! j! \Gamma (i+j+\mu+m_t+m_s) \Gamma (-j - m_t)} \left( \frac{k \mu r^2 (m_t - 1)}{r^2 \mathcal{K}} \right)^{j+m_t+m_s} (k \mu m_s)^i. \]  

(57)

To reduce \( S_3 \), it is required to first perform the variable transformation \( j = n - k' \) followed by \( i = j - k' \), then performing some algebraic manipulations the inner sum on the index \( k' \) is solved using [23, eq. 4.2.5.25] as follows

\[ \sum_{k'=0}^{n} \frac{1}{(j-k')! k'! (n-k')! \Gamma (k'+\mu)} = \sum_{k'=0}^{n} \frac{n!}{n! \Gamma (j+\mu)} = \frac{\Gamma (j+n+\mu)}{j! n! \Gamma (j+\mu) \Gamma (n+\mu)}. \]  

(58)

Now performing some algebraic manipulations, we obtain

\[ S_3 = \sum_{j,n=0}^{\infty} \frac{(-1)^j n \Gamma (j+n+\mu) \Gamma (j+\mu+m_s)}{j! n! \Gamma (j+\mu) \Gamma (n+\mu)} \Gamma (m_s - \mu - j - n) (k \mu m_s)^n \left( \frac{r^2 \mathcal{K} m_s}{r^2 (m_t - 1)} \right)^{j+m_t+m_s}. \]  

(59)

Now summing (56) over the index \( n \), (57) over index \( j \) and (59) over the infinite triangle \( j = i - n \), followed by some algebraic manipulations the double shadowed \( \kappa-\mu \) Type VI PDF simplifies to (20), completing the proof.

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