Analytic Solution of Non-resonant Multiphoton Jaynes-Cummings Model with Dissipation at Finite Temperature

Le-Man Kuang

Theoretical Physics Division, Nankai Institute of Mathematics,
Tianjin 300071, People’s Republic of China and
Department of Physics and Institute of Physics, Hunan Normal University,
Hunan 410006, People’s Republic of China

Xin Chen

Institute of Acoustics and Department of Electrical Science and Engineering,
Nanjing University, Nanjing 210093, People’s Republic of China

Guang-Hong Chen and Mo-Lin Ge

Theoretical Physics Division, Nankai Institute of Mathematics,
Tianjin 300071, People’s Republic of China

Abstract

We derive a new master equation with and without Markovian approximation for the reduced density operator of a quantum system. We study a multiphoton Jaynes-Cummings model (JCM) with the dissipation at a finite temperature by making use of the new master equation under Markovian approximation. We present an exact solution of a non-resonant multiphoton JCM with the dissipation by the use of the superoperator technique, and obtain analytic expressions for the atomic inversion and the intensity of the cavity field in the JCM. It is shown that the new master equation under the Markovian approximation can better describe the suppression of the revivals of the inversion and oscillatory behaviors of the photon-number distribution in the JCM. When the damping vanishes, the usual expressions for the inversion and the intensity are recovered.

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1 Introduction

It is well known that the Jaynes-Cummings model (JCM) [1] is a model of fundamental theoretical importance, as the simplest nontrivial model of two coupled dissimilar quantum systems (the atom and the field). Without dissipation, it is exactly solvable under the rotating-wave approximation. The solution in the presence of dissipation is not only of theoretical interest, but also important from a practical point of view since the dissipation would be always present in any experimental realization of the model. It was found that the collapses and revivals of the inversion oscillations predicted by the JCM are in agreement with the experiments done with Rydberg atoms in a microwave cavity [2-4]. In these experiments, the damping of the cavity mode is not negligibly small. Thus for a detailed comparison with experiments, the effects of cavity losses must be taken into account.

In the past few years, a number of authors have treated the JCM with the dissipation by the use of analytic approximations [5,6] and numerical calculations [7-11]. With the dissipation one has to solve a master equation for the reduced density operator of the system. It is well known that this master equation is more difficult to solve. To our knowledge, no analytic solution for this model has been given for general initial conditions and temperature of the environment. Agarwal and Puri [12] present an analytic solution for the initial state of the light field being a vacuum state. Daeubler et al. [13] found an analytic expression for the atomic inversion and the intensity of the field in the JCM with the dissipation when the reservoir is at zero temperature.

In the present paper we derive a new master equation of the reduced density operator for the system with and without Markovian approximation. This master equation differs from the previous one [5,6,14,15] due to different coupling between the system and the reservoir and the presence of the renormalization term in the total Hamiltonian. It enable us to discuss the influence of the reservoir temperature on nonclassical effects in the JCM. We show that the JCM is exactly sovable for the new master equation under the Markovian approximation, and analytic expressions of the atomic inversion and the intensity of the field can be obtained for an arbitrary value of the damping within the approximations.
used in deriving the master equation. When the damping vanishes, the usual results are recovered.

This paper is organized as follows: In Sec. II we derive the new master equation for the reduced density operator of the system with and without Markovian approximation. In Sec. III we present an exact solution of the new master equation under the Markovian approximation for the non-resonant multiphoton JCM where the field is initially in a coherent state and the atom in the excited state by the use of the superoperator technique. In Sec. IV we calculate the atomic inversion and the intensity of the field and discuss the limit of a vanishing damping. We close with a summary.

2 Master Equation

We consider a system described by a Hamiltonian $H$ and interacting with a heat-bath environment (a reservoir) which consists of an infinite set of harmonic oscillators. We assume that the system interacting with the environment can be described by the total Hamiltonian

$$H_T = H + \sum_i \left( \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 x_i^2 \right) + \hbar H \sum_i C_i x_i + \hbar^2 H^2 \sum_i \frac{|C_i|^2}{2m_i \omega_i^2}$$

(1)

where the second term is the Hamiltonian of the reservoir, the third one represents the interaction between the system and the reservoir with the coupling constant $\hbar C_i$, and the last one is the renormalization term [16]. Here we have adopted a simple coupling between the system and the reservoir such that it satisfies the condition $[V, H] = 0$ which is of importance in the back-action evading and quantum-nondemolition schemes [17] and in the study of decoherence of quantum system [18-20].

For later convenience, we rewrite the total Hamiltonian (1) in the form

$$H_T = H + H_R + V$$

(2)

where

$$H_R = \sum_i \hbar \omega_i b_i^+ b_i, \quad V = H \sum_{i=1}^3 F_i$$

(3)

with

$$F_1 = \hbar \sum_i C_i b_i, \quad F_2 = \hbar \sum_i C_i b_i^+, \quad F_3 = \hbar^2 H \sum_i \frac{|C_i|^2}{\omega_i}$$

(4)
where \( b_i, b_i^+ \) are the boson annihilation and creation operators for the reservoir.

The total density operator for the system and the reservoir satisfies the Liouville-von Neumann equation in the interaction picture [15,21]

\[
\frac{d\chi}{dt} = -\frac{i}{\hbar}[V(t), \chi]
\]

(5)

It is easy to see that the above equation has the formal solution

\[
\chi(t) = \chi(0) + \frac{1}{i\hbar} \int_{t_0}^{t} [V(t'), \chi(0)] dt' + \left( \frac{1}{i\hbar} \right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' [V(t'), [V(t''), \chi(t'')]]
\]

(6)

Tracing both sides of the above equation over the reservoir, we obtain the change in the reduced density operator

\[
s(t) - s(t_0) = \frac{1}{i\hbar} \int_{t_0}^{t} dt' Tr_R[H \sum_i F_i(t'), s(t_0) f(t_0)]
\]

\[ + \left( \frac{1}{i\hbar} \right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' Tr_R[V(t'), [V(t''), \chi(t'')]]
\]

(7)

where we have assumed that the system and the reservoir are uncoupled and \( \chi(0) \) can be factorized as \( \chi(0) = s(t_0) f(0) \) at the initial time.

We assume that the reservoir is initially in thermal equilibrium at a temperature \( T \).

Then it is straightforward to see that

\[
\langle F_1 \rangle_R = \langle F_2 \rangle_R = 0, \quad \langle F_3 \rangle_R = \Delta\omega' H
\]

(8)

where

\[
\Delta\omega' = i\hbar \int_{0}^{\infty} d\omega \frac{J(\omega)|C(\omega)|^2}{\omega}
\]

(9)

where \( J(\omega) \) is the spectral density of the reservoir.

The total density operator [22,23] can be expressed as

\[
\chi(t) = s(t) f(t) + \chi_c(t)
\]

(10)

where \( \chi_c(t) \) represents the correlation between the system and the reservoir described by the density operator \( f(t) \) at time \( t \). The reservoir assumption allows us to take \( f(t) = f(0) \).

Then, Eq.(7) becomes

\[
s(t) - s(t_0) = -(t - t_0) \Delta\omega' (H[H, s(t_0)] + [H, s(t_0)] H)
\]

\[ + \left( \frac{1}{i\hbar} \right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \{ Tr_R[V(t'), [V(t''), s(t'') f(0)] + Tr_R[V(t'), [V(t''), \chi(t'')] \chi_c(t'')] \}
\]
where the second term is a higher-order term than the first one and can be neglected for the weak damping case [21]. After dropping the second term in Eq.(6), differentiating both sides of Eq.(11) with respect to time, we have the equation

$$\frac{\partial s(t)}{\partial t} = -\Delta \omega'(H[H, s(t_0)] + [H, s(t_0)]H)$$

$$+ \left(\frac{1}{i\hbar}\right)^2 \int_{t_0}^{t} dt' Tr_R[V(t'), [V(t'), s(t') f(0)]]$$

(12)

where the $t'$ integration is over the correlation functions of the reservoir, which are characterized by a time which is short but finite.

Substituting the interaction term $V = \sum_i HF_i$ with a change of variable $t' = t - \tau$ into Eq.(12), under Markovian approximation $s(t') = s(t)$, we can rewrite Eq.(12) as

$$s(t) = -\Delta \omega'(H[H, s(t_0)] + [H, s(t_0)]H)$$

$$- [H, H, s(t_0)] \sum_{ij} \int_{0}^{\infty} d\tau W_{ij}(\tau) - [H, s(t_0)]H \sum_{ij} \int_{0}^{\infty} d\tau [W_{ij}(\tau) - W_{ji}'(\tau)]$$

(13)

where the correlation functions of the reservoir are defined by

$$W_{ij}(\tau) = Tr_R F_i(t) F_j(t - \tau) f(0) = \langle F_i(t) F_j(t - \tau) \rangle_R$$

(14)

$$W_{ij}'(\tau) = Tr_R F_i(t - \tau) F_j(t) f(0) = \langle F_i(t - \tau) F_j(t) \rangle_R$$

(15)

where all operators are in the interaction picture. These correlation functions have been calculated in ref.[15].

It is not difficult to see that under the higher temperature approximation in which the temperature is assumed high enough to so that the Markovian approximation is valid, Eq.(13) reduces to

$$s(t) = -\gamma [H, H, s(t_0)] - \Delta \omega [H, s(t_0)]H$$

(16)

where $\gamma$ and $\Delta \omega$ are constants which depends on the temperature and the spectral density of the reservoir, and given by

$$\gamma = \Delta \omega' + \frac{kT}{\hbar} \lim_{\omega \to 0} \omega J(\omega)|C(\omega)|^2,$$

(17)

$$\Delta \omega = \Delta \omega' + 2i \mathcal{P} \int_{0}^{\infty} d\omega \frac{J(\omega)|C(\omega)|^2}{\omega}$$

(18)

where $\mathcal{P}$ is the Cauchy principal part of the integration [15].
Eq.(16) is the master equation under the Markovian approximation in the interaction picture for the system. It is easy to convert it to that in the Schrodinger picture with this form:

$$\frac{d\rho_s(t)}{dt} = \frac{1}{i\hbar} [H, \rho(t)] - \gamma[H, [H, \rho(t)]] - \Delta\omega[H, \rho(t)]$$ (19)

which is the desired master equation for the reduced density operator of the system in the Schrodinger picture under the Markovian approximation.

Notice that in the derivation of the master equation (14) we have used the Markovian approximation \(s(t') = s(t)\). Without use of the Markovian approximation, through the transformation of variable \(t' = t - \tau\) in Eq.(12), then taking the approximation \(s(t - \tau) \approx s(t) - \tau \partial s(t)/\partial t\) where \(\tau\) is short but finite. To take the result under the Markovian approximation (16) as \(\partial s(t)/\partial t\) here, we then obtain the non-Markovian master equation for the reduced density operator of the system in the Schrodinger picture with the form

$$\frac{d\rho_s}{dt} = \frac{1}{i\hbar} [H, \rho] - \gamma[H, [H, \rho]] - \Delta\omega[H, \rho]H$$

$$- \gamma\alpha[H, [H, [H, \rho]]] - \Delta\omega\alpha[H, [H, [H, \rho]]]H]$$

$$- \gamma\beta[H, [H, [H, \rho]]]H - \Delta\omega\beta[H, [H, \rho]]H - \Delta\omega\beta[H, [H, \rho]]H$$ (20)

where \(\gamma\) and \(\Delta\omega\) are given by Eq.(17) and \(\alpha\) and \(\beta\) are defined by

$$\alpha = \frac{2kT}{\hbar} \mathcal{P} \int_0^\infty d\omega \frac{J(\omega)|C(\omega)|^2}{\omega^3}, \quad \beta = 2\pi i \frac{d}{d\omega} [J(\omega)|C(\omega)|^2] |_{\omega=0}$$ (21)

Comparison Eq.(20) with Eq.(19) it is easy to see that the second and third terms on the rhs of Eq.(20) are the Markovian part, and the last four terms are the non-Markovian contribution.

### 3 Exact Solution for the JCM with Dissipation

In this section we derive an exact solution of the master equation under the Markovian approximation for the non-resonant multiphoton JCM. If we neglect the Lamb shift term in Eq.(14), the master equation under the approximation becomes

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar} [H, \rho(t)] - \gamma[H, [H, \rho(t)]]$$ (22)
It is noticed that Eq.(22) has the same form as the Milburn’s equation [24] under diffusion approximation, but they come from completely different physical mechanism. The former originates from the dissipation while the latter is from the uncontinuous unitary evolution.

The non-resonant multiphoton Jaynes-Cummings Hamiltonian [25] describing an interaction of a two-level atom with a single-mode cavity field via an \( m \)-photon process in the rotating-wave approximation is given by

\[
\hat{H} = \hbar \omega (\hat{a}^+ \hat{a} + \frac{m}{2} \hat{\sigma}_3) + \frac{\hbar}{2} (\omega_o - m\omega) \hat{\sigma}_3 + \hbar \lambda (\hat{\sigma}_- \hat{a}^+ + \hat{\sigma}_+ \hat{a}^m),
\]

(23)

where \( \omega \) is the frequency of the cavity field; \( \omega_o \) is the atomic transition frequency; \( \lambda \) is the atom-field coupling constant; \( \hat{a} \) and \( \hat{a}^+ \) are the field annihilation and creation operators, respectively; \( \hat{\sigma}_3 \) is the atomic-inversion operator and \( \hat{\sigma}_\pm \) are the atomic “spin flip” operators which satisfy the relations \([\hat{\sigma}_+, \hat{\sigma}_-] = 2\hat{\sigma}_3 \) and \([\hat{\sigma}_3, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm \). For simplicity, we take \( \hbar = 1 \) throughout the paper.

We now start to find the exact solution for the density operator \( \hat{\rho}(t) \) of the master equation (22) applied to the Hamiltonian (23). Following the approach in refs.[26,27] we introduce three auxiliary superoperators \( \hat{R}, \hat{S} \) and \( \hat{T} \) which are defined through their actions on the density operator, respectively,

\[
\begin{align*}
\exp(\hat{R}\tau)\hat{\rho}(t) &= \sum_{k=0}^{\infty} \frac{(2\gamma\tau)^k}{k!} \hat{H}^k \hat{\rho}(t) \hat{H}^k \\
\exp(\hat{S}\tau)\hat{\rho}(t) &= \exp(-i\hat{H}\tau)\hat{\rho}(t) \exp(i\hat{H}\tau) \\
\exp(\hat{T}\tau)\hat{\rho}(t) &= \exp(-\gamma \tau \hat{H}^2)\hat{\rho}(t) \exp(-\gamma \tau \hat{H}^2)
\end{align*}
\]

(24)

(25)

(26)

where the Hamiltonian \( \hat{H} \) is given by Eq.(23).

It is straightforward to obtain the formal solution of of the master equation (22) as follows:

\[
\hat{\rho}(t) = \exp(\hat{R}t) \exp(\hat{S}t) \exp(\hat{T}t) \hat{\rho}(0)
\]

(27)

where \( \hat{\rho}(0) \) is the density operator of the initial atom-field system.

We assume that initially the field is prepared in the coherent state \( |z\rangle \) defined by

\[
|z\rangle = \sum_{n=0}^{\infty} \exp(-\frac{1}{2} |z|^2) \frac{z^n}{\sqrt{n!}} |n\rangle \equiv \sum_{n=0}^{\infty} Q_n |n\rangle
\]

(28)
where
\[ Q_n = \exp\left(-\frac{1}{2}|z|^2\right)\frac{z^n}{\sqrt{n!}} \] (29)
and the atom was prepared in its excited state \(|e\rangle\), so that the initial density operator of the atom-field system takes this form:
\[ \hat{\rho}(0) = \begin{pmatrix} |z\rangle\langle z| & 0 \\ 0 & 0 \end{pmatrix} \] (30)

We divide the Hamiltonian (23) into a sum of two terms which commute with each other, i.e.,
\[ \hat{H} = \hat{H}_o + \hat{H}_I, \quad [\hat{H}_o, \hat{H}_I] = 0 \] (31)
In the two-dimensional atomic basis \(\hat{H}_o\) and \(\hat{H}_I\) take the following form, respectively,
\[ \hat{H}_o = \omega \begin{pmatrix} \hat{n} + \frac{m}{\lambda} & 0 \\ 0 & \hat{n} - \frac{m}{\lambda} \end{pmatrix} \]
\[ \hat{H}_I = \begin{pmatrix} \delta & \lambda \hat{a}^+m \\ \lambda \hat{a}^m & -\delta \end{pmatrix} \] (32)
where \(\hat{n} = \hat{a}^\dagger\hat{a}\) and \(\delta = \frac{1}{2}(\omega_0 - m\omega)\).

Similarly, the square of the Hamiltonian (23) can also be expressed as a sum of two terms which commute with each other,
\[ \hat{H}^2 = \hat{A} + \hat{B}, \quad [\hat{A}, \hat{B}] = 0 \] (33)
where the representations of the operators \(\hat{A}\) and \(\hat{B}\) in the two-dimensional atomic basis take the following forms:
\[ \hat{A} = \begin{pmatrix} \omega^2(\hat{n} + \frac{m}{\lambda})^2 + \lambda^2\hat{a}^+m\hat{a}^+m + \delta^2 & 0 \\ 0 & \omega^2(\hat{n} - \frac{m}{\lambda})^2 + \lambda^2\hat{a}^m\hat{a}^m + \delta^2 \end{pmatrix} \] (34)
\[ \hat{B} = 2\omega \begin{pmatrix} \delta(\hat{n} + \frac{m}{\lambda}) & \lambda \hat{a}^m(\hat{n} - \frac{m}{\lambda}) \\ \lambda(\hat{n} - \frac{m}{\lambda})\hat{a}^+m & -\delta(\hat{n} - \frac{m}{\lambda}) \end{pmatrix} \] (35)

It can be checked that operators \(\hat{A}\) and \(\hat{B}\) also commute with \(\hat{H}_o\) and \(\hat{H}_I\) by the use of the following formulae:
\[ \hat{a}^m\hat{a}^+m = \frac{(\hat{n} + m)!}{\hat{n}!}, \quad \hat{a}^+m\hat{a}^m = \frac{\hat{n}!}{(\hat{n} - m)!} \] (36)
\[ f(\hat{n})\hat{a}^m = \hat{a}^m f(\hat{n} - m), \quad \hat{a}^m f(\hat{n}) = f(\hat{n} + m)\hat{a}^m \] (37)
\[ f(\hat{n})\hat{a}^+m = \hat{a}^+m f(\hat{n} + m), \quad \hat{a}^+m f(\hat{n}) = f(\hat{n} - m)\hat{a}^+m \] (38)
where \( f(\hat{n}) \) is an operator function on the number operator.

For convenience, we introduce the following auxiliary “density” operator:

\[
\hat{\rho}_2(t) = \exp(\hat{S}t) \exp(\hat{T}t) \hat{\rho}(0) \tag{39}
\]

From the definition of the superoperators and the initial condition (30), we find that

\[
\hat{\rho}_2(t) = \exp(-i\hat{H}_1t) \exp(-\gamma t\hat{B}) \hat{\rho}_1(t) \exp(-\gamma t\hat{B}) \exp(i\hat{H}_1t) \tag{40}
\]

where we have used

\[
\exp(-\gamma t\hat{A}) = \begin{pmatrix}
\exp\left[-\gamma t\left(\hat{n}^2 + \frac{m^2}{2}\right)\right] & 0 \\
0 & \exp\left[-\gamma t\left(\hat{n}^2 - \frac{m^2}{2}\right)\right]
\end{pmatrix} \tag{41}
\]

\[
\exp(-i\hat{H}_0t) = \begin{pmatrix}
\exp\left[-i\omega t\left(\hat{n} + \frac{m}{2}\right)\right] & 0 \\
0 & \exp\left[-i\omega t\left(\hat{n} - \frac{m}{2}\right)\right]
\end{pmatrix} \tag{42}
\]

The operator \( \hat{\rho}_1(t) \) in Eq.(40) is defined by

\[
\hat{\rho}_1(t) = |\Psi(t)\rangle\langle\Psi(t)| \otimes |\hat{e}\rangle\langle\hat{e}| \tag{43}
\]

where

\[
|\Psi(t)\rangle = \exp\{-\gamma t[\omega^2(\hat{n} + \frac{m}{2})^2 + \lambda^2 \hat{a}^m \hat{a}^{+m} + \delta^2]\} |ze^{-i\omega t}\rangle \tag{44}
\]

If we notice that the powers of the operator \( \hat{B} \) can be written as

\[
\hat{B}^{2k} = \begin{pmatrix}
A_+^{k}(\hat{n}) & 0 \\
0 & A_-^{k}(\hat{n})
\end{pmatrix} \tag{45}
\]

\[
\hat{B}^{2k+1} = \begin{pmatrix}
\delta(\hat{n} + \frac{m}{2})A_+^{k}(\hat{n}) & \lambda(\hat{n} + \frac{m}{2})A_-^{k}(\hat{n})\hat{a}^m \\
\lambda(\hat{n} - \frac{m}{2})A_-^{k}(\hat{n})\hat{a}^{+m} & -\delta(\hat{n} - \frac{m}{2})A_+^{k}(\hat{n})
\end{pmatrix} \tag{46}
\]

where \( k \) takes zero or an arbitrary positive integer and

\[
A_+(\hat{n}) = (2\omega\delta)^2(\hat{n} + \frac{m}{2})^2 + (2\lambda\omega)^2(\hat{n} + \frac{m}{2})^2 \hat{a}^m \hat{a}^{+m} \tag{47}
\]

\[
A_-(\hat{n}) = (2\omega\delta)^2(\hat{n} - \frac{m}{2})^2 + (2\lambda\omega)^2(\hat{n} - \frac{m}{2})^2 \hat{a}^{+m} \hat{a}^m \tag{48}
\]

Then we can write the operator \( \exp(-2\gamma t\hat{B}) \) in the form

\[
\exp(-2\gamma t\hat{B}) = \begin{pmatrix}
E_+(\hat{n}, t) & \hat{a}^m E_+(\hat{n}, t) \\
E_-(\hat{n}, t)\hat{a}^{+m} & E_-(\hat{n}, t)
\end{pmatrix} \tag{49}
\]
where
\[ E_+ (\hat{n}, t) = \cos(\gamma t \sqrt{A_+ (\hat{n})}) - 2\omega \delta (\hat{n} + \frac{m}{2}) \frac{\sinh(\gamma t \sqrt{A_+ (\hat{n})})}{\sqrt{A_+ (\hat{n})}} \] (50)
\[ E_- (\hat{n}, t) = \cos(\gamma t \sqrt{A_- (\hat{n})}) - 2\omega \delta (\hat{n} - \frac{m}{2}) \frac{\sinh(\gamma t \sqrt{A_- (\hat{n})})}{\sqrt{A_- (\hat{n})}} \] (51)
\[ E_{3+} (\hat{n}, t) = -2\omega \lambda (\hat{n} - \frac{m}{2}) \frac{\sinh(\gamma t \sqrt{A_+ (\hat{n} - m)})}{\sqrt{A_+ (\hat{n} - m)}} \] (52)
\[ E_{3-} (\hat{n}, t) = -2\omega \lambda (\hat{n} - \frac{m}{2}) \frac{\sinh(\gamma t \sqrt{A_- (\hat{n})})}{\sqrt{A_- (\hat{n})}} \] (53)

Similarly, we can write the operator \( \exp(-i\hat{H}_t t) \) in the two-dimensional atomic basis as
\[ \exp(-i\hat{H}_t t) = \begin{pmatrix} D_+ (\hat{n}, t) & D_{3+} (\hat{n}, t) \hat{a}^m \\ D_{3-} (\hat{n}, t) \hat{a}^+ & D_- (\hat{n}, t) \end{pmatrix} \] (54)

where
\[ D_+ (\hat{n}, t) = \cos[t(\delta^2 + \lambda^2 \hat{a}^+ \hat{a}^m)] - \delta \frac{\sin[t(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+)]}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+}} \] (55)
\[ D_- (\hat{n}, t) = \cos[t(\delta^2 + \lambda^2 \hat{a}^+ \hat{a}^m)] - \delta \frac{\sin[t(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+)]}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+}} \] (56)

and
\[ D_{3+} (\hat{n}, t) = -i\lambda \frac{\sin[t(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+)]}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+}} \] (57)
\[ D_{3-} (\hat{n}, t) = -i\lambda \frac{\sin[t(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+)]}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+}} \] (58)

For latter use, we list the following formulae
\[ \hat{a}^+ \begin{pmatrix} D_+ (\hat{n}, t) \\ E_+ (\hat{n}, t) \end{pmatrix} = \begin{pmatrix} D_- (\hat{n}, t) \\ E_- (\hat{n}, t) \end{pmatrix} \hat{a}^+, \quad \hat{a}^m \begin{pmatrix} D_+ (\hat{n}, t) \\ E_+ (\hat{n}, t) \end{pmatrix} = \begin{pmatrix} D_- (\hat{n}, t) \\ E_- (\hat{n}, t) \end{pmatrix} \hat{a}^m \] (59)

and
\[ \hat{a}^+ \begin{pmatrix} D_{3-} (\hat{n}, t) \\ E_{3-} (\hat{n}, t) \end{pmatrix} = \begin{pmatrix} D_{3+} (\hat{n}, t) \\ E_{3+} (\hat{n}, t) \end{pmatrix} \hat{a}^+, \quad \hat{a}^m \begin{pmatrix} D_{3-} (\hat{n}, t) \\ E_{3+} (\hat{n}, t) \end{pmatrix} = \begin{pmatrix} D_{3+} (\hat{n}, t) \\ E_{3+} (\hat{n}, t) \end{pmatrix} \hat{a}^m \] (60)

In the derivation of Eq.(54) we have used
\[ \hat{H}_t^{2k} = \begin{pmatrix} (\delta^2 + \lambda^2 \hat{a}^m \hat{a}^+)^k & 0 \\ 0 & (\delta^2 + \lambda^2 \hat{a}^+ \hat{a}^m)^k \end{pmatrix} \] (61)
\[ \hat{H}_I^{2k+1} = \begin{pmatrix} \delta(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^{+m})^k & \lambda(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^{+m})^k \hat{a}^m \\ \lambda(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^{+m})^k \hat{a}^{+m} & -\delta(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^{+m})^k \hat{a}^m \end{pmatrix} \]  

(62)

where \( k \) takes zero or an arbitrary positive integer.

Then, from Eqs.(49) and (54) it follows that

\[ \exp(-i\hat{H}_I t) \exp(-\gamma t \hat{B}) = \begin{pmatrix} F_+(\hat{n}, t) & F_3(\hat{n}, t) \hat{a}^m \\ F_{3-}(\hat{n}, t) \hat{a}^{+m} & F_-(\hat{n}, t) \end{pmatrix} \]  

(63)

where

\[ F_+(\hat{n}, t) = E_+(\hat{n}, t) D_+(\hat{n}, t) + E_{3+}(\hat{n}, t) D_{3+}(\hat{n}, t) \hat{a}^m \hat{a}^{+m} \]  

(64)

\[ F_-(\hat{n}, t) = E_-(\hat{n}, t) D_-(\hat{n}, t) + E_{3-}(\hat{n}, t) D_{3-}(\hat{n}, t) \hat{a}^{+m} \hat{a}^m \]  

(65)

\[ F_{3+}(\hat{n}, t) = E_{3+}(\hat{n}, t) D_+(\hat{n}, t) + E_+(\hat{n}, t) D_{3+}(\hat{n}, t) \]  

(66)

\[ F_{3-}(\hat{n}, t) = E_{3-}(\hat{n}, t) D_-(\hat{n}, t) + E_-(\hat{n}, t) D_{3-}(\hat{n}, t) \]  

(67)

Substituting Eqs.(43) and (63) into Eq.(40), we can obtain an explicit expression for the operator \( \hat{\rho}_2(t) \) as follows:

\[ \hat{\rho}_2(t) = \left( \begin{array}{cc} \hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t) \\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t) \end{array} \right) \]  

(68)

where we have used the following symbol:

\[ \hat{\Psi}_{ij}(t) = | \Psi_i(t) \rangle \langle \Psi_j(t) |, \quad (i, j = 1, 2) \]  

(69)

with

\[ | \Psi_1(t) \rangle = F_+(\hat{n}, t) | \Psi(t) \rangle, \quad | \Psi_2(t) \rangle = F_{3-}(\hat{n}, t) \hat{a}^{+m} | \Psi(t) \rangle \]  

(70)

where \( | \Psi(t) \rangle \) is given by equation (44).

Taking into account the definition of the superoperator \( \hat{R} \), we can obtain the action of the operator \( \exp(\hat{R}t) \) on the “density” operator \( \hat{\rho}_2(t) \) as follows:

\[ \hat{\rho}(t) = \sum_{k=0}^{\infty} (2\gamma t)^k \frac{1}{k!} \hat{H}^k \hat{\rho}_2(t) \hat{H}^k \]  

(71)

where the operator \( \hat{H} \) and \( \hat{\rho}_2(t) \) are given by Eqs.(23) and (68), respectively.

In fact, up to now we have found the exact solution of the master equation (22) for the non-resonant multiphoton Jaynes-Cummings Hamiltonian (23) in the operator form (71). However, in most cases of practical interest, one needs to know the matrix elements of the
density operator $\hat{\rho}(t)$ in the two-dimensional atomic basis to calculate expectation values of observables. Although the form of the solution (71) is pleasant, it is unconvenient in use. Therefore in what follows we evaluate matrix elements of the density operator in the two-dimensional atomic basis.

Since $\hat{H}_o$ commutes with $\hat{H}_I$, from equation (31) we obtain

$$\hat{H}^k = \sum_{l=0}^k \binom{k}{l} \hat{H}_o^{k-l} \hat{H}_I^l$$ (72)

From Eq.(33) it follows that when $l$ takes an even number, we have

$$\hat{H}_o^{k-l} \hat{H}_I^l = \frac{1}{2} \omega^{k-l} \begin{pmatrix} (\hat{n} + \frac{m}{2})(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m)^{l/2} & 0 \\ 0 & (\hat{n} - \frac{m}{2})(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m)^{l/2} \end{pmatrix}$$ (73)

and when $l$ is an odd number, we have

$$\hat{H}_o^{k-l} \hat{H}_I^l = \frac{1}{2} \omega^{k-l} \begin{pmatrix} \delta(\hat{n} + \frac{m}{2})(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m)^{(l-1)/2} & \lambda(\hat{n} + \frac{m}{2})(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m)^{(l-1)/2} \hat{a}^m \\ \lambda(\hat{n} - \frac{m}{2})(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m)^{(l-1)/2} \hat{a}^m & -\delta(\hat{n} - \frac{m}{2})(\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m)^{(l-1)/2} \end{pmatrix}$$ (74)

Then,

$$\sum_{l_{even}} \binom{k}{l} \hat{H}_o^{k-l} \hat{H}_I^l = \frac{1}{2} \begin{pmatrix} \alpha_+^k(\hat{n}) + \alpha_-^k(\hat{n}) & 0 \\ 0 & \beta_+^k(\hat{n}) + \beta_-^k(\hat{n}) \end{pmatrix}$$ (75)

$$\sum_{l_{odd}} \binom{k}{l} \hat{H}_o^{k-l} \hat{H}_I^l = \frac{1}{2} \begin{pmatrix} \delta[\alpha_+^k(\hat{n}) - \alpha_-^k(\hat{n})] & \lambda \frac{[\alpha_+^k(\hat{n}) - \alpha_-^k(\hat{n})]}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m}} \hat{a}^m \\ \lambda \frac{[\beta_+^k(\hat{n}) - \beta_-^k(\hat{n})]}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m}} \hat{a}^m & -\delta[\beta_+^k(\hat{n}) - \beta_-^k(\hat{n})] \end{pmatrix}$$ (76)

where these operators $\alpha_{\pm}(\hat{n})$ and $\beta_{\pm}(\hat{n})$ are defined by, respectively,

$$\alpha_{\pm}(\hat{n}) = \omega(\hat{n} + \frac{m}{2}) \pm \sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m}, \beta_{\pm}(\hat{n}) = \omega(\hat{n} - \frac{m}{2}) \pm \sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m}$$ (77)

Substitution of Eqs.(75) and (77) into Eq.(72) yields that

$$\hat{H}^k = \begin{pmatrix} \varphi_+^{(k)}(\hat{n}) + \delta \varphi_-^{(k)}(\hat{n}) & \lambda \frac{\varphi_+^{(k)}(\hat{n})}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m}} \hat{a}^m \\ \lambda \frac{\varphi_-^{(k)}(\hat{n})}{\sqrt{\delta^2 + \lambda^2 \hat{a}^m \hat{a}^m}} \hat{a}^m & \varphi_-^{(k)}(\hat{n}) - \delta \varphi_+^{(k)}(\hat{n}) \end{pmatrix}$$ (78)

where

$$\varphi_{\pm}^{(k)}(\hat{n}) = \frac{1}{2} [\alpha_{\pm}^{(k)}(\hat{n}) \pm \alpha_{\mp}^{(k)}(\hat{n})], \varphi_{\pm}^{(k)}(\hat{n}) = \frac{1}{2} [\beta_{\pm}^{(k)}(\hat{n}) \pm \beta_{\mp}^{(k)}(\hat{n})]$$ (79)

For convenience, we introduce

$$f_+^{(k)}(\hat{n}) = \varphi_+^{(k)}(\hat{n}) + \delta \varphi_-^{(k)}(\hat{n}), f_-^{(k)}(\hat{n}) = \varphi_-^{(k)}(\hat{n}) - \delta \varphi_+^{(k)}(\hat{n})$$ (80)
Then, Eq. (78) becomes
\[ \hat{H}^k = \begin{pmatrix} f_+^{(k)}(\hat{n}) & \lambda \frac{\varphi_{-}^{(k)}(\hat{n})}{\sqrt{\beta^2 + \lambda^2 \hat{a}_m^* \hat{a}_m^m}} f_-^{(k)}(\hat{n}) \\ \lambda \frac{\varphi_{+}^{(k)}(\hat{n})}{\sqrt{\beta^2 + \lambda^2 \hat{a}_m^* \hat{a}_m^m}} \hat{a}_m^+ & f_-^{(k)}(\hat{n}) \end{pmatrix} \] (81)

Making use of equations (68) and (81), we can obtain the matrix \( \hat{\mathcal{M}}^{(k)}(t) = \hat{H}^k \hat{\rho}_2(t) \hat{H}^k \) where the matrix elements are explicitly given by

\[
\begin{align*}
\hat{\mathcal{M}}_{11}^{(k)}(t) &= f_+^{(k)}(\hat{n}) \hat{\Psi}_{11}(t) f_+^{(k)}(\hat{n}) + \hat{a}_m^m \varphi_{-}^{(k)\prime}(\hat{n}) \hat{\Psi}_{21}(t) f_+^{(k)}(\hat{n}) \\
&\quad + f_+^{(k)}(\hat{n}) \hat{\Psi}_{12}(t) \varphi_{-}^{(k)\prime}(\hat{n}) \hat{a}_m^+ + \hat{a}_m^m \varphi_{-}^{(k)\prime}(\hat{n}) \hat{\Psi}_{22}(t) f_-^{(k)}(\hat{n}) \\
\hat{\mathcal{M}}_{22}^{(k)}(t) &= \varphi_{+}^{(k)\prime}(\hat{n}) \hat{a}_m^+ \hat{\Psi}_{11}(t) f_-^{(k)}(\hat{n}) + f_-^{(k)}(\hat{n}) \hat{\Psi}_{21}(t) \varphi_{-}^{(k)\prime}(\hat{n}) \\
&\quad + \varphi_{+}^{(k)\prime}(\hat{n}) \hat{a}_m^m \hat{\Psi}_{12}(m, t) f_-^{(k)}(\hat{n}) + f_-^{(k)}(\hat{n}) \hat{\Psi}_{22}(t) f_-^{(k)}(\hat{n}) \\
\hat{\mathcal{M}}_{21}^{(k)}(t) &= (\hat{\mathcal{M}}_{12}^{(k)}(t))^+ \\
&= \varphi_{-}^{(k)\prime}(\hat{n}) \hat{a}_m^+ \hat{\Psi}_{11}(t) f_+^{(k)}(\hat{n}) + f_-^{(k)}(\hat{n}) \hat{\Psi}_{21}(m, t) f_+^{(k)}(\hat{n}) \\
&\quad + \varphi_{-}^{(k)\prime}(\hat{n}) \hat{a}_m^m \hat{\Psi}_{12}(m, t) \varphi_{-}^{(k)\prime}(\hat{n}) \hat{a}_m^+ + f_-^{(k)}(\hat{n}) \hat{\Psi}_{22}(m, t) \varphi_{-}^{(k)\prime}(\hat{n}) \hat{a}_m^m ~ (82)\end{align*}
\]

with
\[ \varphi_{-}^{(k)\prime}(\hat{n}) = \frac{\lambda}{\sqrt{\beta^2 + \lambda^2 \hat{a}_m^* \hat{a}_m^m}} \varphi_{-}^{(k)}(\hat{n}) \] (85)

From Eqs. (71) and (82) to (85) we finally can arrive at the explicit expression of the exact solution of the master equation (22) for the non-resonant multiphoton JCM Hamiltonian (23). With these results, one can further evaluate mean values of operators of interest.

In the next section, we will use it to obtain analytic expressions of the atomic inversion, distribution of the photon number and the intensity of the field.

### 4 Atomic Inversion and Intensity of the Field

In this section, we derive analytic expressions of the atomic inversion and the intensity of the field, and study the influence of the dissipation on nonclassical effects in the JCM.

The atomic inversion is defined as the probability of the atom being the excited state minus the probability of being the ground state, that is
\[ \langle \hat{\sigma}_3(t) \rangle = Tr[\hat{\rho}_A(t) \hat{\sigma}_3] \] (86)

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where \( \hat{\rho}_A(t) \) is the reduced density operator of the atom, and can be obtained through tracing over the field part in (86).

Making use of the solution (86), it is easy to rewrite the inversion (87) as

\[
\langle \hat{\sigma}_3(t) \rangle = \sum_{k,n=0}^{\infty} \frac{(2\gamma t)^k}{k!} \langle n|\hat{A}_{11}^{(k)}|n \rangle - \langle n|\hat{A}_{22}^{(k)}|n \rangle
\]  

(87)

It is not an easy task to calculate the two expectation values on the rhs of Eq.(88). From Eqs.(82) and (83) we get that

\[
\langle n | \hat{A}_{11}^{(k)}(t) | n \rangle = (f_+^{(k)}(n))^2 |\psi_1(n, t)|^2 + (g^{(k)}(n + m))^2 |\psi_2(n + m, t)|^2 + 2Re \{f_+^{(k)}(n)g^{(k)}(n + m)\psi_1^*(n, t)\psi_2(n + m, t)\}
\]

(88)

\[
\langle n | \hat{A}_{22}^{(k)}(t) | n \rangle = (g^{(k)}(n))^2 |\psi_1(n - m, t)|^2 + (f_-^{(k)}(n))^2 |\psi_2(n, t)|^2 + 2Re \{f_-^{(k)}(n)g^{(k)}(n)\psi_2(n, t)\psi_1^*(n - m, t)\}
\]

(89)

where we have introduced the symbols:

\[
g^{(k)}(n) = \sqrt{\frac{n!}{(n-m)!}}\phi_-^{(k)\prime}(n)
\]

(90)

\[
\psi_1(n, t) = \langle n | \Psi_i(t) \rangle \quad (i = 1, 2)
\]

(91)

Here \( |\Psi_i(t)\rangle \) are given by Eq.(70). These functions \( f_\pm^{(k)}(n) \) and \( \phi_-^{(k)\prime}(n) \) can be obtained through replacing the number operator \( \hat{n} \) in the operators \( f_\pm^{(k)}(n) \) and \( \phi_-^{(k)\prime}(n) \) by the number \( n \).

Through a lengthy calculation Eqs.(89) and (90) can be written explicitly as

\[
\langle n|\hat{A}_{11}^{(k)}(t)|n \rangle = \frac{1}{4} \{a_+^{2k}(n)[(1 + \delta)|F_+(n)|^2 + a_2(n)\frac{(n + m)!}{n!}(|F_3-(n + m)|^2
\]

\[
+ 2Re(F_+^*(n)F_3-(n + m)a(n)\sqrt{(n + m)!}{n!})
\]

\[
+ 2\alpha_+^{k}(n)a_+^{k}(n)[(1 - \delta^2)|F_+(n)|^2 - a_2(n)\frac{(n + m)!}{n!}(|F_3-(n + m)|^2
\]

\[
+ 2Re(F_+^*(n)F_3-(n + m))\delta a(n)\sqrt{(n + m)!}{n!})
\]

\[
+ \alpha_-^{2k}(n)[(1 - \delta^2)|F_+(n)|^2 + a_2(n)\frac{(n + m)!}{n!}(|F_3-(n + m)|^2
\]

\[
- 2Re(F_+^*(n - m)F_3-(n))\delta a(n)\sqrt{(n + m)!}{n!})\} |\Psi(n, t)|^2
\]

(92)
\[ \langle n | \hat{M}^{(k)}_{22} (t) | n \rangle = \frac{1}{4} \{ \beta^2_{+} (n) [a_2(n - m)] F_+(n - m)]^2 + (1 - \delta)^2 \frac{n!}{(n - m)!} |F_3-(n)|^2 \]

\[ + 2 \text{Re}(F^*_+(n - m) F_3-(n)) a(n - m)(1 - \delta) \sqrt{\frac{n!}{(n - m)!}} \]

\[ + 2 \beta^k_{+} (n) [a^2(n - m)] F_+(n - m)]^2 - (1 - \delta^2) |F_3-(n)|^2 \]

\[ + 2 \text{Re}(F^*_+(n - m) F_3-(n)) \delta a(n) \sqrt{\frac{n!}{(n - m)!}} \]

\[ + \beta^2_{+} (n) [a^2(n)] F_+(n - m)]^2 + (1 + \delta^2) |F_3-(n)|^2 \]

\[ - 2 \text{Re}(F^*_+(n - m) F_3-(n)) (1 + \delta) a(n) \sqrt{\frac{n!}{(n - m)!}} \} |\Psi(n - m, t)|^2 \] (93)

where the functions \( \alpha_{\pm}, \beta_{\pm}, F(n) \) and \( F_3-(n) \) are given by replacing the number operator \( \hat{n} \) in the their corresponding operators by the number \( n \), and we have introduced

\[ |\Psi(n, t)|^2 = |\langle n | \Psi(t) \rangle|^2 \]

\[ = |Q_n|^2 \exp\{ -\gamma t[\omega^2(n + \frac{m}{2})^2 + \delta^2 + \lambda^2 (n + m)!] \} \] (94)

\[ a(n) = \sqrt{\frac{n! \lambda^2}{(n - m)! \delta^2 + n! \lambda^2}} \] (95)

Substituting Eqs.(93) and (94) into Eq.(88) and taking into account the relation \( \alpha^k_{\pm} (n) = \beta^k_{\pm} (n + m) \) we find that

\[ \langle \hat{\sigma}_3(t) \rangle = \frac{1}{4} \sum_{n=0}^{\infty} |\Psi(n, t)|^2 \{ |F_+(n)|^2 [(1 + \delta^2 - a^2(n)) \exp(2\gamma t \alpha^2_+(n))] \]

\[ + (2 - 2\delta^2 + a^2(n)) \exp(2\gamma t \alpha_+(n) \alpha_-(n)) + ((1 - \delta) - \delta^2) (n + m)! \exp(2\gamma t \alpha^2_-(n)) \]

\[ + |F_3-(n + m)|^2 [(a^2(n) - (1 - \delta)^2) \frac{(n + m)!}{n!} \exp(2\gamma t \alpha^2_+(n))] \]

\[ - 2(1 - \delta^2 + (n + m)! a^2(n)) \exp(2\gamma t \alpha_+(n) \alpha_-(n)) \]

\[ + a(n)(1 + \delta + \sqrt{(n + m)! a^2(n)} \exp(2\gamma t \alpha^2_-(n))] \]

\[ + 2 \text{Re}(F^*_+(n) F_3-(n + m)) a(n) \delta \sqrt{(n + m)! \exp(2\gamma t \alpha^2_+(n)) + \exp(2\gamma t \alpha^2_-(n))} \} \] (96)

As is well known, oscillations of the photon-number distribution in the JCM is a kind of nonclassical effects of the cavity field. To see the influence of the dissipation on this kind of nonclassical behavior, in what follows we discuss photon statistics in the radiation field
in the JCM. The reduced density operator of the cavity field can be obtained by taking the trace of the total density operator \( \hat{\rho}(t) \) over the atomic states, that is, \( \hat{\rho}_F = T_{\mathcal{R}} \hat{\rho}(t) \). Then, the probability of finding \( n \) photons in the radiation field is found to be

\[
p(n, t) = \sum_{n=0}^{\infty} \frac{(2\gamma t)^k}{k!} [\langle n|\hat{\mathcal{M}}(k)_{11}(t)|n\rangle + \langle n|\hat{\mathcal{M}}(k)_{22}(t)|n\rangle]
\]

where the two expectation values on the rhs of Eq.(98) have been given explicitly in Eqs.(93) and (94).

Making use of Eq.(98), one can calculate easily the intensity of the cavity field. The result is

\[
\langle \hat{n} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} n|\Psi(n, t)|^2 \left\{ |F_+(n)|^2 \left[ (1 + \delta^2 + a^2(n)) \exp(2\gamma t\alpha^2_+ (n)) + (2 - 2\delta^2 - a^2(n)) \exp(2\gamma t\alpha_+(n)\alpha_-(n)) + ((1 - \delta)^2 + a^2(n)) \exp(2\gamma t\alpha^2_-(n)) \right] + |F_{-}(n+m)|^2 \left[ (a^2(n) + (1 - \delta)^2) \frac{(n+m)!}{n!} \exp(2\gamma t\alpha^2_+(n)) - 2(1 - \delta^2 + \frac{(n+m)!}{n!} a^2(n)) \exp(2\gamma t\alpha_+(n)\alpha_-(n)) - a(n)(1 + \delta - \sqrt{(n+m)! a^2(n)) \exp(2\gamma t\alpha^2_-(n))} \right. \\
\left. + 2\text{Re}(F^*_+(n)F_{-}(n+m)) a(n) \sqrt{\frac{(n+m)!}{n!}} [(2 - \delta) \exp(2\gamma t\alpha^2_+(n)) - 4\delta \exp(2\gamma t\alpha_+(n)\alpha_-(n)) - 2 \exp(2\gamma t\alpha^2_-(n)))] \right\}
\]

(98)

In order to better understand the influence of the dissipation on nonclassical effects in the JCM, we consider the resonant case, i.e., \( \delta = 0 \). In this case, the expressions of the inversion, photon-number distribution and the intensity can simplified significantly. The inversion reduces to

\[
\langle \hat{\sigma}_3(t) \rangle = \sum_{n=0}^{\infty} |Q_n|^2 \exp[-4\gamma^2 \lambda^2 t (n+m)! \frac{(n+m)!}{n!}] \cos[2\lambda t \sqrt{\frac{(n+m)!}{n!}}]
\]

(99)

The photon-number distribution becomes

\[
p(n, t) = \frac{1}{2} |Q_n|^2 \left\{ 1 + \exp[-4\gamma^2 \lambda^2 t (n+m)! \frac{(n+m)!}{n!}] \cos[2\lambda t \sqrt{\frac{(n+m)!}{n!}}] \right\}
\]

(100)

And the intensity of the cavity field is given by

\[
\langle \hat{n}(t) \rangle = \bar{n} + \frac{m}{2} - \frac{m}{2} e^{-n} \sum_{n=0}^{\infty} \frac{n^n}{n!} \exp[-4\gamma^2 \lambda^2 t (n+m)! \frac{(n+m)!}{n!}] \cos[2\lambda t \sqrt{\frac{(n+m)!}{n!}}]
\]

(101)
where $\bar{n} = |z|^2$ is the initial mean photon number in the field.

From the above expressions we see that the damping term in Eq.(22) leads to the appearance of the decay factors $\exp\{-4\gamma\lambda^2 t \frac{(n+m)!}{n!}\}$ and $\exp\{-4\gamma\lambda^2 t \frac{n!}{(n-m)!}\}$ in Eqs.(100), (101) and (102), which are responsible for the destruction of revivals of the atomic inversion and the weakening of oscillatory behaviors of the photon-number distribution. In Fig.1 we plot the time evolution of the inversion. With increasing the damping $\gamma$, one can observe rapid deterioration of the revivals of the inversion. It is worth mentioning that the inversion, oscillations of photon-number distribution and the intensity are weakened with increasing temperature of the environment due to the relation (17).
FIG.1: The atomic inversion $\langle \hat{\sigma}_3 \rangle$ as a function of $t$ for $m = 1, |z|^2 = 20$ and 
(a) $\gamma = 0.0001$, (b) $\gamma = 0.0005$, (c) $\gamma = 0.001$. Here we have let $\lambda = 1$

It is obvious that when the damping vanishes, Eqs.(100), (101) and (102) take the form

$$\langle \hat{\sigma}_3(t) \rangle = \sum_{n=0}^{\infty} |Q_n|^2 \cos[2\lambda t \sqrt{(n+m)!} n!]$$ (102)

$$p(n, t) = |Q_n|^2 \cos^2[\lambda t \sqrt{(n+m)!} n!] + |Q_{n-m}|^2 \sin^2[\lambda t \sqrt{(n+m)!} n!]$$ (103)
\[ \langle \hat{n}(t) \rangle = \bar{n} + \frac{m}{2} - \frac{m}{2} e^{\frac{n}{2}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos[2\lambda t \sqrt{\frac{n!}{(n-m)!}}] \] (104)

which are just the usual expressions without dissipation.

5 Summary

we have derived a new master equation with and without the Markovian approximation for the reduced density operator of a quantum system. Making use of the new master equation under the Markovian approximation, we we have studied the JCM with the dissipation at a finite temperature. We have found an analytic solution of the non-resonant multiphoton JCM with the dissipation and obtained analytic expressions of the atomic inversion, the photon-number distribution and the intensity of the cavity field, which can reduce to usual expressions when the damping vanishes. We have shown that the damping suppresses the revivals of the atomic inversion and the oscillatory behaviors of the photon-number distribution. It must be mentioned that we do not discuss applications of the non-Markovian equation in the present paper, but it may be useful for the study of the non-Markovian relaxation processes in transient optical phenomena [28].

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References

[1] E. T. Jaynes and F. W. Cummings, *Proc. IEEE* 51, 89 (1963); J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, *Phys. Rev. Lett.* 44, 1323 (1980); For a recent review, see B. W. Shore and P. L. Knight, *J. Mod. Opt.* 40, 1195 (1993)

[2] G. Rempe, H. Walther, and N. Klein, *Phys. Rev. Lett.* 58, 353 (1987)

[3] M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, *Phys. Rev. Lett.* 59, 1899 (1987); M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. A* 35, 154 (1987)
[4] F. Diedrich, J. Kruse, G. Rempe, M. O. Scully and H. Walther, *IEEE J. Quantum Electron.* **24**, 1495 (1988)

[5] S. M. Barnett and P. L. Knight, *Phys. Rev.* **A33**, 2444 (1986)

[6] R. R. Puri and G. S. Agarwal, *Phys. Rev.* **A35**, 3433 (1987)

[7] T. Quang, P. L. Knight, and V. Bužek, *Phys. Rev.* **A44**, 6069 (1991)

[8] J. Eiselt and H. Risken, *Opt. Commun.* **72**, 351 (1989); *Phys. Rev.* **A43**, 346 (1991)

[9] M. J. Werner and H. Risken, *Phys. Rev.* **A44**, 4623 (1991)

[10] J. Gea-Banacloche, *Phys. Rev.* **A47**, 2221 (1993)

[11] B. G. Englert, M. Naraschewski, and A. Schenzle, *Phys. Rev.* **A50** 2667 (1994)

[12] G. S. Agarwal and R. R. Puri, *Phys. Rev.* **A33**, 1757 (1986)

[13] B. Daeubler, H. Risken, and L. Schoendorff, *Phys. Rev.* **A46**, 1654 (1992)

[14] H. J. Carmichael, *Phys. Rev. Lett.* **55**, 2790 (1985)

[15] W. H. Louisell, Quantum Statistical Properties of Radiation (Wiley, New York, 1973)

[16] A. O. Caldeira and A. J. Leggett, *Ann. Phys.* **149**, 374 (1983); and A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg W. Zweger *Rev. Mod. Phys.* **59** (1987) 1;

[17] C. Caves, K. Thorne, R. Drever, V. Sandberg, and M. Zimmermann, *Rev. Mod Phys.* **52**, 341 (1980)

[18] W. Zurek, *Phys. Rev.* **D24**, 1516 (1981)

[19] A. Tameshtit, and J. E. Sipe, *Phys. Rev.* **A45**, 8280 (1992); ibid, **A47**, 1697 (1993)

[20] J. Shao, M. L. Ge, and H. Cheng, *Decoherence of Quantum-non-demolition System*, preprint (1994)

[21] G. Gangopadhyay, and D. S. Ray, *Phys. Rev.* **A46**, 1057 (1992)
[22] M. Lax, *Phys. Rev.* **157**, 213 (1967)

[23] M. Sergant III, M. O. Scully and W. E. Lamb in *Laser Physics* (Addison-Wesley, Reading MA, 1974)

[24] G. J. Milburn, *Phys. Rev.* **A44**, 5401 (1991); ibid, **A47**, 2415 (1993)

[25] S. Singh, *Phys. Rev.* **A25**, 3206 (1982)

[26] H. Moya-Cessa, V. Bužek, M. S. Kim, and P. L. Knight, *Phys. Rev.* **A48**, 3900 (1993); L. M. Kuang, X. Chen, and M. L. Ge, *Influence of Decoherence on Non-classical effects in the Jaynes-Cummings Model*, preprint (1994)

[27] L. M. Kuang and X. Chen, *J. Phys.* **A27**, L633 (1994); X. Chen and L. M. Kuang, *Phys. Lett.* **A191**, 18 (1994)

[28] P. Tchénio, A. Debarre, J. C. Keller, and J. L. Le Gouët, *Phys. Rev. Lett.* **62**, 415 (1989); E. T. J. Nibbering, D. A. Wiersma, and K. Duppen, *Phys. Rev. Lett.* **66**, 2464 (1991); J. R. Brinati, S. S. Mizrahi and G. A. Prataviera, *Phys. Rev.* **A50**, 3304 (1994).