Adiabatic expansion, early X-ray data and the central engine in GRBs

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ABSTRACT

The \textit{Swift} satellite early X-ray data show a very steep decay in most of the gamma-ray bursts light curves. This decay is either produced by the rapidly declining continuation of the central engine activity or by some leftover radiation starting right after the central engine shuts off. The latter scenario consists of the emission from an ‘ember’ that cools via adiabatic expansion and, if the jet angle is larger than the inverse of the source Lorentz factor, the large angle emission. In this work, we calculate the temporal and spectral properties of the emission from such a cooling ember, providing a new treatment for the microphysics of the adiabatic expansion. We use the adiabatic invariance of $p_{\perp}^2/B$ ($p_{\perp}$ is the component of the electrons’ momentum normal to the magnetic field, $B$) to calculate the electrons’ Lorentz factor during the adiabatic expansion; the electron momentum becomes more and more aligned with the local magnetic field as the expansion develops. We compare the theoretical expectations of the adiabatic expansion (and the large angle emission) with the current observations of the early X-ray data and find that only $\sim 20$ per cent of our sample of 107 bursts are potentially consistent with this model. This leads us to believe that, for most bursts, the central engine does not turn off completely during the steep decay of the X-ray light curve; therefore, this phase is produced by the continued rapidly declining activity of the central engine.

Key words: radiation mechanisms: non-thermal – methods: analytical – gamma-rays: bursts – gamma-rays: theory.

1 INTRODUCTION

The central engine of the gamma-ray bursts (GRBs; see Piran 2005 for a review) is hidden to direct observations and its workings are largely unknown. The only information that we currently have about the GRB is obtained from its electromagnetic radiation. We have to look for signatures in the radiation mechanism to understand how Nature produces these outbursts.

\textit{Swift} has provided very early X-ray data that show that for most bursts there is a very steep decay lasting for about 10 min (Tagliaferri et al. 2005; Nousek et al. 2006; see Zhang et al. 2006 and references therein for possible physical explanations). These observations suggest that the rapidly declining X-ray light curve (LC) and the burst are produced by the same source, because the X-ray LC, when extrapolated backwards in time, matches the gamma-ray LC (O’Brien et al. 2006). Therefore, a natural question arises: Are the early X-ray data really just a rapidly declining continuation of the central engine activity, originally seen in the gamma-ray band, but now seen at lower energy? Or does the central engine switch off abruptly and the early X-ray data do not reflect the activity of the central engine? If the central engine completely shuts off when the gamma-ray photons’ flux falls below the gamma-ray detector sensitivity, then the emission from a cooling ‘ember’ would be responsible for the early X-ray steep decay. This source, which had just produced the gamma-ray emission, would be cooling by adiabatic expansion (AE). In this paper, we study the flux properties of a ‘hot’ shell that undergoes AE and cools. The goal is to determine if the observed X-ray steep decay data are consistent with the AE scenario. If so, then the central engine did shut off abruptly right after the gamma-ray emission ceased. On the other hand, if it is not, then the data are reflecting the rapidly declining activity of the central engine. The reason for this is that any other process that does not invoke central engine activity to explain the early X-ray data has problems explaining the smooth temporal connection observed in the LC between the prompt emission and the early X-ray data.

AE has been studied before to predict the long-wavelength afterglow from GRBs (Mészáros & Rees 1997) and also the optical flashes from internal and reverse shocks (RSs) (Mészáros & Rees 1999; Sari & Piran 1999a). In this work, we describe the evolution of a collisionless plasma due to AE and show that this is in general different from AE of an ideal gas.
We first present the microphysics of the AE for a collisionless plasma in Section 2. Then, we use it to calculate the flux properties of a source undergoing AE in Section 3. We look at what the current observations tell us in Section 4, and we put them in the context of the central engine in Section 5. We summarize our results and give our conclusions in Section 6.

2 MICROPHYSICS OF THE ADIABATIC EXPANSION

For an ideal gas, the pressure evolves due to AE as \( P_0 \propto \rho v^4 \propto V^{-2} \), where \( \alpha_e = 4/3 \) for a relativistic gas and \( \rho \) and \( V \) are the comoving density and ejecta volume, respectively. The collisions between electrons are extremely rare in GRB relativistic shocks, therefore, one needs to be careful in the use of this formula.\(^1\)

For a collisionless magnetized plasma, assuming that no collective plasma processes randomize the particles’ velocity due to scattering, particles move along a magnetic field line and, by using the concept of adiabatic invariant (Rybicki & Lightman 1979; Jackson 1998), we can calculate the particles’ energy. This invariant describes that, for slowly varying fields, the magnetic flux through the orbit of the particle is a constant, or \( p_\perp^2 / B \) is an adiabatic invariant, where \( p_\perp \) is the component of the particle’s momentum transverse to \( B \), the magnetic field. For highly relativistic particles, \( p_\perp \approx m_e c \gamma_\perp \), so that

\[
\gamma_\perp^2 / B = \text{constant} \quad (1)
\]

can be used, where \( \gamma_\perp \) is the Lorentz factor (LF) of the electron in the transverse direction, \( m_e \) is the electron’s mass and \( c \) is the speed of light (from now on, \( c = 1 \)). This relationship can be used because the magnetic field decays on a much larger length-scale than the electron’s gyroradius (see Appendix A). It is worth noting that the parallel component of the electron’s momentum remains unchanged; this will be briefly discussed in the last section.

In the next sections, we will make use of (1) to predict the evolution of the electrons’ LF in a hot shell that undergoes AE. We will use it to calculate the properties of its synchrotron and synchrotron-self-Compton (SSC) radiation.

3 ANALYTICAL LIGHT CURVES OF AN ADIABATICALLY COOLING EMBER

Let’s assume that the GRB ejecta was heated by some process (shocks or magnetic dissipation) and suddenly the central engine switches off completely. There is no other energy injection mechanism at hand, so it begins to coast (the LF of the ejecta is constant, see Section 3.6) and cools via AE. We will calculate the flux properties of this cooling ember.

3.1 Ejecta width and magnetic field

In the following subsections, we will provide the basic ingredients for the radiation calculation. First, we need to determine the comoving thickness of the ejecta, which could be \( \Delta' = R / \Gamma \) or \( \Delta' = \Delta_0 \).

We will call these cases: thin ejecta shell (an ejecta that undergoes significant spreading) and thick ejecta shell (an ejecta that experiences no significant spreading), respectively. The observed time is \( t \propto R / \Gamma^2 \), where \( R \) is the distance of the source from the centre of the explosion and \( \Gamma \) is the LF of the source with respect to the rest frame of the GRB host galaxy.

The magnetic field in the GRB ejecta could be a combination of frozen-in field from the central explosion and field generated locally. We prescribe the decay of the field by using the flux-freezing condition (Panaitescu & Kumar 2004), which gives

\[
B_\perp \propto (R \Delta')^{-1} \quad \text{and} \quad B_\parallel \propto R^{-2}.
\]

We will use the field that decays slower, although this is highly uncertain. This is because the magnetic field generation mechanism for GRBs is still not well understood, so the relative strength of \( B_\perp \) and \( B_\parallel \) is unknown.

3.2 Electrons’ energy distribution

For an adiabatically expanding source, no more energetic electrons are injected in the system when the shock has run its course. This means that, after some time, there will be few electrons with energies higher than the cooling electron LF, \( \gamma_e \). Therefore, the electron population above \( \gamma_e \) will be truncated due to radiation losses and the emission for \( \gamma_e < \nu \) will rapidly shut off (\( \nu \) is the synchrotron frequency corresponding to electrons with \( \gamma_e \)). At this point, the electron distribution will follow \( \propto \gamma^{-p} \) for \( \gamma_e < \gamma < \gamma_{\gamma} \), where \( \gamma_{\gamma} \) is the typical LF of the electrons, since we would be dealing only with adiabatic electrons. Moreover, since the radiative cooling quickly becomes less important than the adiabatic cooling because the magnetic field decays rapidly with the expansion of the ejecta, both \( \gamma_e \) and \( \gamma_{\gamma} \) will evolve in the same way.

For the case \( \gamma_e < \gamma_{\gamma} \), the radiation losses would dominate and, after some time, they would make the whole electron distribution collapse to a value close to \( \gamma_e \). A narrower range in the electron distribution would be responsible for the radiation. In this paper, we focus on the \( \gamma_e < \gamma_{\gamma} \) case.

3.3 Basics of synchrotron and SSC

The electrons’ four-momentum is given by \( P = m_e (\gamma, \gamma \cos \alpha', \gamma \sin \alpha', 0) \), which can be also written as \( P = m_e (\gamma, \gamma_{\gamma}, \gamma_{\gamma}, 0) \), where \( \gamma_{\gamma} \) is the component of the electrons’ momentum parallel to \( B \). The pitch angle, which is the angle between \( B \) and the velocity of the electrons, is \( \alpha' \). In this notation, the electron’s LF is then \( \gamma^2 = \gamma_{\gamma}^2 + \gamma_{\gamma}^2 \). According to the prescription of the adiabatic invariance, \( \gamma_{\gamma} \) evolves following (1) and \( \gamma_{\gamma} \) remains unchanged. We assume that at the onset of the AE \( \gamma_{\gamma} \sim \gamma_{\gamma} \), then quickly when time doubles, the radius would have also doubled, making the magnetic field decrease by at least that factor and reducing \( \gamma_{\gamma} \) making \( \gamma_{\gamma} > \gamma_{\gamma} \), which gives

\[
\gamma = \gamma_0 \sqrt{1 + \gamma_{\gamma}^2 / \gamma_0^2} \approx \gamma_0 \quad \text{and} \quad \sin \alpha' = \gamma_{\gamma} / \gamma \approx \gamma_{\gamma} / \gamma_0.
\]

\(^1\) We have estimated the mean free path for Coulomb scattering between a hot electron and a cold electron and find it to be much larger compared to the shell thickness. The electric field associated with a relativistic hot electron is calculated using the Liénard–Wiechert potential, and we find the cross-section for a significant interaction, i.e. leading to a fraction of the energy of the hot electron transferred to a cold electron, is close to the Thomson cross-section (\( \sigma \approx \sigma_T / 3 \)). The mean free path is \( \lambda = (n \sigma_T)^{-1} \), where \( n \) is the electron density. From the total energy \( E \), the distance of the shell from the centre of explosion \( R \), the source LF \( \Gamma \) and the comoving shell width \( \Delta \), we find \( n = E / (\Gamma m_e c^2 4\pi R^2 \Delta') \). Using the usual notation \( Q_0 = Q / 10^9 \), we obtain: \( \lambda / \Delta' \approx 700 R_5 \Gamma_2 \Delta'_2 \). For scattering between hot electrons, the conclusion is the same. Therefore, Coulomb scattering between the electrons is not significant.
As the transverse component of the momentum decreases, due to the decay of the magnetic field, the pitch angle decreases, which makes the electron’s momentum more aligned with the local magnetic field.

Knowing the electrons’ energy distribution, the emission at any given frequency and time can be calculated using the synchrotron spectrum:

\[
F_\nu = F_\nu \left\{ \begin{array}{l}
\frac{(v/v_d)(v_\nu/v_\iota)^{1/3}}{\nu < \nu_d} \\
\frac{(v/v_\iota)^{1/3}}{\nu_d < \nu < \nu_\iota} \\
\frac{(v/v_\iota)^{-1(p-1)/2}}{\nu > \nu_\iota},
\end{array} \right.
\]

for the case \( \nu < \nu_d < \nu_\iota \), where \( \nu_\iota \) is the self-absorption frequency and it is obtained using equation (52) of Panaitescu & Kumar (2000) (see e.g. Katz & Piran 1997; Sari & Piran 1999b). The characteristic synchrotron frequencies are obtained from the corresponding electrons LF's:

\[
v_{\iota,c} = \frac{eB\gamma^{3}c}{2\pi m_e(1+z)} \sin \alpha',
\]

where \( y_{\iota,c} \) and \( \sin \alpha' \) are given by (2). The observed peak flux is

\[
F_\nu = \frac{1}{4\pi d_l^2(z)\sin \iota} \frac{B_0^2}{\gamma^{5}} \sin \alpha',
\]

where \( d_l \) is the luminosity distance, \( N_c \) is the number of radiating electrons (which in this scenario is constant), \( z \) is the redshift and \( e \) is the electron’s charge.

For the SSC case, the flux peaks at \( v_\nu y_{\iota,c}^2 \) with magnitude \( \tau_{\nu} F_{\nu} \), where \( \tau_{\nu} = N_c \sigma_T/(4\pi R^2) \) is the optical depth to electron scattering. We will calculate SSC emission for photons above \( \nu_d \). We will use the same synchrotron piece-wise spectrum, which is just a very crude approximation.

### 3.4 Temporal and spectral properties

For synchrotron emission of a cooling ember undergoing AE, we find

\[
F_\nu \propto t^{-3} \left( t^{-3/2} \right), \quad v_{\iota,c} \propto t^{-3} \left( t^{-3/2} \right), \quad v_\iota \propto t^{-12/5} \left( t^{-9/5} \right)
\]

and

\[
F_\nu \propto \left\{ \begin{array}{l}
\frac{t^2 (t^{-1})^3 v^2}{\nu < \nu_d} \\
\frac{t^{-1}(t^{-1})^{-1/3} v_{\iota}^3}{\nu_d < \nu < \nu_\iota} \\
\frac{t^{-3(p+1)/2}(t^{-3(p+1)/4})^{p-1/2} v^{-1(p-1)/2}}{\nu > \nu_\iota},
\end{array} \right.
\]

where the time dependences are reported for the thin ejecta shell and parenthesis are used for the thick ejecta shell.

Using the same notation as above, for SSC emission, we find

\[
F_{\nu y_{\iota,c}^2} \propto t^{-3} \left( t^{-7/2} \right), \quad v_{\iota,c} y_{\iota,c}^2 \propto t^{-3} \left( t^{-3/2} \right)
\]

and

\[
F_\nu \propto \left\{ \begin{array}{l}
\frac{t^2 (t^{-1})^3 y_{\iota,c}^2}{v < v_d} \\
\frac{t^{-1}(t^{-1})^{-1/3} v_{\iota}^3}{v_d < v < v_\iota} \\
\frac{t^{-3(p+1)/2}(t^{-3(p+1)/4})^{p-1/2} v^{-1(p-1)/2}}{v > v_\iota},
\end{array} \right.
\]

One can see that for the synchrotron case, the flux decays rapidly for \( v_{\iota} < v < v_d \); for the SSC case, the flux decays even more rapidly for \( v_d y_{\iota}^2 \approx v \approx v_\iota y_{\iota,c}^2 \). For both cases, the peak frequencies of the spectrum also show a fast decrease with time. Also, for both cases, the thin ejecta case gives a faster time decay, since the shell spreads significantly, allowing the ejecta to cool faster.

To compare our theory with the observations, we provide relations between the temporal decay index \( \alpha \) and the spectral index \( \beta \) in Table 1, using the convention \( F_\nu \propto t^{-\alpha} v^{-\beta} \).

To summarize, the emission from an adiabatically cooling source has the following properties:

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### Table 1. Closure relations between \( \alpha \) (decay index) and \( \beta \) (spectral index) for a cooling ember undergoing AE (\( t_0 = t_c \)).

|                | Synchrotron | SSC |
|----------------|-------------|-----|
| Thick ejecta   |              |     |
| (\( \Delta' = \Delta_0 \)) | \( \alpha = 1.5 \beta + 1.5 \) | \( \alpha = 1.5 \beta + 3.5 \) |
| Thin ejecta    |              |     |
| (\( \Delta' = \Gamma / \Delta \)) | \( \alpha = 3 \beta + 3 \) | \( \alpha = 3 \beta + 5 \) |

(i) Its spectral index must be equal to the one at the end of the prompt emission phase of the gamma-ray burst, \( \beta_{\gamma} \).

(ii) The temporal decay index must obey one of the closure relations in Table 1.

(iii) The peak frequency of the spectrum should decrease with time as predicted in (6) and (8).

(iv) After some time, on the order of \( t_\iota \) (defined in Section 3.5), the spectrum should have an exponential cut-off at frequencies greater than the cooling frequency (Section 3.2).

Points (ii) and (iii) have to correspond to the same radiation mechanism (synchrotron or SSC) and the same ejecta width case (thin or thick).

If one were to consider the electrons’ energy as given by the AE of an ideal relativistic gas, instead of using the methods of adiabatic invariance, then \( \gamma \propto V^{-1/3} \) (Section 2, see also section 3 of Mészáros & Rees 1999). In this case, the velocity distribution of the electrons during the AE phase is isotropic, therefore, \( \sin \alpha' \) is a time-independent constant of order unity. We calculate the temporal decay indices as done above and find the following results. For synchrotron: \( \alpha = 2.3 \beta + 1 \) and \( \alpha = 4 \beta + 2 \); and for SSC: \( \alpha = 3.7 \beta + 3 \) and \( \alpha = 6 \beta + 4 \), for the thick and thin ejecta cases, respectively. The difference in the temporal decay indices for the synchrotron case compared to the ones in Table 1 is \( \lesssim 20 \) per cent for \( \beta \in [0.5 - 2] \). Because SSC has a stronger dependence on \( \gamma \), the difference we find in \( \alpha \) is larger.

So far, we have considered only the flux-freezing condition to prescribe the evolution of the magnetic field, but we can also determine the magnetic field using the equipartition consideration, i.e. the energy density in the magnetic field is a constant fraction of the electrons’ internal energy density (Sari, Piran & Narayan 1998). To obtain this last quantity, one needs to know \( \gamma \), which could be obtained either using the adiabatic invariance methods or by the ideal gas law, as mentioned in the last paragraph. But for the equipartition consideration, we will only consider the ideal gas law, because if there is a mechanism that maintains an equipartition between magnetic energy and electron energy, then that same process is also likely to keep different components of electron momentum coupled and that will lead to an ideal gas expansion law for electrons. Therefore, the magnetic field in this case is given by \( B^2 \propto V^{-4/3} \), where \( V \propto R^3 \Delta' \). The synchrotron and SSC emission decays, for the thick case, are both steeper by \( 0.3 \beta + 0.3 \) than the ones presented on the last paragraph, but both thin cases remain unchanged.

### 3.5 Large angle emission

If the central engine switches off abruptly and the gamma-ray producing ejecta has a opening angle \( \theta_j \), such as \( \theta_j > \Gamma^{-1} \), large angle emission (LAE) will also be present (Fenimore & Sunner 1997; Kumar & Panaitescu 2000). The LAE flux declines as \( \alpha = 2 + \beta \).
and the peak frequency of the spectrum decays as $t^{-1}$. Therefore, the AE flux generally decays faster than LAE’s and the AE’s peak frequencies always decrease faster than LAE’s.

The time-scales for these two phenomena, LAE and AE, are essentially the same, they are set by

$$t_c = \frac{R}{2\Gamma c}.$$  

(10)

LAE and AE start at the same time, $t_0$, and same site, $R_0$: right after the central engine has switched off, and the fluxes decline with time as

$$F_\nu = F_0 \left(1 + \frac{t - t_0}{t_c}\right)^{-\alpha},$$

(11)

where $F_0$ is the flux at $t_0$ and $\alpha$ is the decay index of either LAE or AE. The shape of the LC depends on the values for $t_0$ and $t_c$ (Fig. 1); for Section 3.4, $t_0 = t_c$. The case $t_0 < t_c$ is unphysical, since it implies that when AE starts, the shell’s electrons have not yet cooled substantially, i.e. the shell’s radius has not doubled.

If $\theta_j \gtrsim \Gamma^{-1}$, then there will be no LAE, so AE would be the only emission present after the central engine turns off. On the other hand, if $\theta_j > \Gamma^{-1}$, then LAE will dominate over AE (see footnote 2). LAE will cease with the detection of the last photons coming from $\theta_j$ and, at this time, the flux will smoothly become dominated by the AE emission, i.e. there will be a break in the LC to the power-law decay for AE (Fig. 1: bottom). The photons from $\theta_j$ will arrive at a time $t_j \approx t_0 + \theta_j^2/2 = t_0 + \theta_j^2\Gamma^2 t_c$.

3.6 Electron–positron pair-enriched ejecta

When the ejecta cools by AE, the thermal energy of the protons and electrons is converted back to bulk kinetic energy of the shell, increasing $\Gamma$. Even in an extreme case where all the electrons’ energy goes into the shell expansion, $\Gamma$ increases only by a factor of $\sim 2$, if the protons’ and electrons’ energy is $\lesssim m_p c^2$. Therefore, the effect of this change to the observed flux is less than a factor of 2, a relatively small effect. For this reason, we have used a constant $\Gamma$ for the calculations done so far.

On the other hand, if the ejecta consists of $e^\pm$ pairs, then the increase in $\Gamma$ during the AE would be considerable, and it would scale as $\alpha \Gamma^{-1}$ (the observed energy in the shell is a constant and scales as $\propto \gamma \Gamma$). Since $\Gamma$ increases, then the observed time is $t - t_0 = \int_{R_0}^{R} dR/(2\Gamma^2)$. For the thin ejecta case, we find

$$F_\nu = F_0 \left(1 - \frac{t - t_0}{3t_c}\right)^{\delta},$$

(12)

with characteristic frequencies $\propto [1 - (t - t_0)/(3t_c)]^\delta$, where $\delta = (4\beta^2 + 4\beta + 10)$ for the synchrotron ($\nu_i < \nu < \nu_f$) and SSC ($\nu_i \gamma_f^2 < \nu < \nu_f \gamma_f^2$) cases, respectively. The time decay index, for the $t_0 = t_c$ case, is $\delta(t/t_0)/(4 - t/t_0)$, therefore, the LC steepens continuously. It decays even faster than the AE-Baryonic case, since

2 Except when $\beta < 1$ for the AE case of synchrotron emission from a thick ejecta shell.

3 We have assumed a comoving observer sitting in the middle of an infinite parallel shell that sees the left and right halves of the shell move away from him. Assuming the electrons’ LF in the shell rest frame is $10^9$ (and that the protons are essentially cold since the heating mechanism energized all particles equally), the LF of the shells would be $1 + 10^5 m_e/m_p = 1.5$. An observer far away would mainly detect radiation from the half moving towards him, since the radiation is beamed. This observer would see that the LF of this half has increased by a factor of $\sim 2$.

Figure 1. The normalized flux density, equations (11) and (12), plotted versus observed time, assuming that the observed frequency $\nu$ is always between $\nu_i$ and $\nu_f$ (if $\nu_i < \nu_f$, then there is no AE, only LAE if $\theta_j > \Gamma^{-1}$, see Section 3.2). This emission is produced by the last ejected shell, because contributions from previously ejected shells would be buried in the emission of subsequent shells, since both LAE and AE decay very fast. Top: using $t_0 = 100 s$ and $t_c = 10 s$. The LAE and AE-Baryonic decay indices correspond to $\alpha = (3, 6)$, respectively, and the AE-Pair has $\delta = 8 (\beta = 1$, using AE: Synchrotron - thin ejecta). Bottom: using $t_0 = t_c = 100 s$ and the same $\alpha$’s and $\delta$ as above. In this illustrative example, we have $\theta_j = 2/\Gamma$, therefore, LAE dominates over AE until $t_j = 500 s$, when AE-Baryonic takes over. This break in the LC from the LAE to the AE power-law decay (which should be a smooth transition and it is done in the figure for display purposes) has never been observed.

3.7 Reverse shock emission

In this short subsection, we explore the possibility that the GRB ejecta, which just produced the prompt emission, interacts with the interstellar medium (ISM) and a RS crosses it. The ejecta cools adiabatically after the RS has passed through it and we assume that it follows the Blandford–McKee self-similar solution (Blandford & McKee 1976), during which $\Gamma$ decays in time.

Using the same methods as in the previous subsections, we can calculate the LF of the electrons in the ejecta after the passage of the RS (see Appendix B). We find that the RS flux decays as

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\[ \alpha^{-41\rho/1+5/60} = t^{-2.53} \] (synchrotron emission: \( v_i < v < v_c \), for a thin shell, using \( B_0 \) and \( \rho = 2.5 \)), which gives the closure relation: 
\[ \alpha = 1.45 \beta + 1.45 \] (and \( \alpha = 1.45 \beta + 1.67 \) for SSC). If we determine the electrons’ LF using the ideal gas law, then the RS synchrotron flux would decay as 
\[ \alpha t^{-(20\rho+7)/24} = t^{-2.38} \] (for the same case as above), which is still steeper than the 
\[ \alpha t^{-(73\rho+21)/96} \approx t^{-2.12} \] derived by Sari & Piran (1999a), where they used \( \gamma \propto V^{-1/3} \) and the equipartition consideration.

4 APPLICATION TO THE GRB EARLY ‘AFTERGLOW’

AE, together with LAE, dictates the emission of the source after the central engine has completely turned off. In this section, we will determine if the early X-ray steep decay observed by Swift obeys our theoretical LCs for AE and LAE.4 We will do this for each one of the cases considered in the previous section.

The early X-ray data show a single power-law decay with \( 3 \lesssim \alpha \lesssim 5 \) (Nousek et al. 2006; O’Brien et al. 2006; Willingale et al. 2007). With this information, the \( t_0 > t_1 \) case can be ruled out, since, for this case, the theoretical shape of the LCs for LAE and AE-Baryonic is inconsistent with the early X-ray observations and the AE-Pair LC decays extremely fast (Fig. 1: top). Therefore, we focus on the \( t_0 = t_1 \) case only.

The next possibility we explore is to see if the early X-ray data obey LAE or AE (from a baryonic ejecta). To check the validity of these two scenarios, respectively, we will take a sample of bursts and see how many cases are possibly consistent with either LAE or AE.

Our sample consists of 107 GRBs for which their spectral index during the early X-ray decay (\( \beta_i \)) and their temporal decay index during this phase have been previously determined (the sample of Willingale et al. 2007). We first select the bursts for which \( \beta_i = \beta_s \), which cuts down the sample to 55 bursts. Eight of these bursts show strong spectral evolution, inconsistent with LAE and AE (Zhang, Liang & Zhang 2007: Zhang et al.’s sample essentially contains all our sample), which leaves us with 47 bursts. Moreover, we check how many of these satisfy the relations between \( \alpha \) and \( \beta \) for LAE or AE (Table 1) within about a 90 per cent confidence level, and that narrows down the sample to 20 bursts. In conclusion, only a small percentage of the sample, 19 per cent, is consistent with LAE or AE, which leads us to suggest that, for most early X-ray data result from some other process, and the most natural conclusion is continued central engine activity.

It has also been claimed that the gamma-ray emission extrapolated to X-ray energies, together with the early X-ray data, can be well fitted with a falling exponential followed by a power law (O’Brien et al. 2006). At first, this could be thought to be explained by a pair ejecta with \( \theta_j \lesssim \Gamma^{-4} \) undergoing AE, since its LC also steepens continuously (Fig. 1: bottom). However, for three bursts that show this continuous steepening: GRB 050315 (Lazzati & Begelman 2006; Vaughan et al. 2006), GRB 050724 (Barthelmy et al. 2005) and GRB 060614 (Mangano et al. 2007), the theoretical LC decays too fast and cannot fit the observed early X-ray LC.

Therefore, we rule out the possibility that the observed early X-ray decay is from a pair ejecta undergoing AE.

Finally, we use our sample to test if the observed early X-ray steep decay is consistent with the closure relations derived for the RS (Section 3.7). In this case, the condition \( \beta_v = \beta_s \) is not necessary, therefore we start with the entire sample: 107 bursts. We eliminate 19 of these, which show strong spectral evolution, inconsistent with RS. Out of the remainder, only 26 bursts (24 per cent) are possibly consistent with the RS (16 of these are simultaneously consistent with LAE and AE). However, this scenario could be ruled out since it would be difficult to explain the connection observed in the LC between the prompt emission and the early X-ray data. The only way they could be smoothly connected is if the prompt gamma-ray emission is produced also by the RS, for which we lack evidence.

5 DISCUSSION: THE CENTRAL ENGINE

The results of the last section lead us to believe that the observed early X-ray decay for \( \gtrsim 70 \) per cent of GRBs is produced by the rapidly declining continued activity of the central engine, if we assume that there is a one-to-one correspondence between the temporal behaviour of the central engine activity and the observed emission (Fig. 2).

There are two other arguments that support the idea that, for most bursts, neither LAE nor AE might be consistent with the observed early X-ray decay. First, for some bursts, a break frequency has been seen passing through the X-ray band during the early steep decay, and it evolves faster than the \( \propto t^{-1} \) expected in LAE: \( \propto t^{-2} \) for

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Two possible scenarios for the contribution of LAE (dotted), AE (dashed) and the rapidly declining central engine activity (solid), as seen in the X-ray band. The two ‘humps’ represent the gamma-ray detection (from Swift BAT), extrapolated to the X-ray band, attributed to activity of the central engine. Top: the case for which the central engine activity drops extremely fast and LAE and AE appear. Only \( \sim 20 \) per cent of our sample is possibly consistent with this scenario. Bottom: our preferred scenario, where the central engine activity is the dominant contribution and it decays slower than the theoretical LCs for LAE and AE.
GRB 060904A (Yonetoku et al. 2008). Secondly, if AE is entirely responsible for this phase, then $\theta_j$ has to be very small. If this is the case, then we should have observed the edge of the jet (a jet break with a $\propto t^{-p}$ optical LC) very early on. After inspecting many optical LCs (Butler & Kocerki 2007; Liang, Zhang & Zhang 2007; Liang et al. 2008; Melandri et al. 2008; Panaitescu & Vestrand 2008), we can conclude that most bursts with available early optical data do not show this expected jet break starting at very early times, i.e. $t < \text{a few hours}$.

One way that we might have missed these early jet breaks could be explained by the ‘porcupine’ model. In this model, the central engine ejects many very small angle ($\theta_j \lesssim \Gamma^{-1}$) jets. One of these jets, directed towards the observer, produces the gamma-ray emission. The central engine shuts off, and then the AE emission is produced. After a short time, all these small jets combine and give rise to a single jet with $\theta_j > \Gamma^{-1}$ that interacts with the ISM, giving rise to a forward shock (FS) (optical afterglow). This model leaves no sign of a jet break. Another possible explanation for the lack of a very early jet break is that there is energy injection to the FS, making the $\propto t^{-p}$ optical LC more shallow. This last scenario is unlikely, as a large amount of energy is required – more than a factor of 10 increase – to make a $\propto t^{-p}$ LC as shallow as $\sim t^{-3}$, which is the usual observed optical LC decay.

It has been suggested that the observed early X-ray decay is produced by the FS driven by the jet interacting with the ISM (Panaitescu 2007). This scenario has problems explaining the smooth temporal connection in the LC between the prompt emission and the early X-ray steep decay.

The model presented in this paper has several uncertainties. For example, we are not specifying how the magnetic field is generated during the prompt emission. We are assuming: (i) that the magnetic field coherence length-scale is larger than the electron gyroradius, based on the observations (see Appendix A), and (ii) that there are no collective plasma effects that randomize the electrons’ velocity. These assumptions allow us to use the adiabatic invariant presented in Section 2. If, for some reason, these conditions are violated, then the adiabatic invariance of electron magnetic moment cannot be used. However, the LC from an expanding shell is similar whether we follow electron cooling via adiabatic invariance or ideal gas law and therefore, the main conclusions we have presented here are unchanged. Another assumption made in the application of this model is that Swift’s X-ray telescope band lies between $v_j$ and $v_i$ (in that order), which can be inferred from the spectral information of most bursts during this phase. If, however, $v_i < v_j$, then there will be no emission coming from the part of the shell that lies within an angle of $\Gamma^{-1}$ to the observer line of sight – the only emission would be from LAE (if $\theta_j > \Gamma^{-1}$).

### 6 CONCLUSIONS

We have explored the situation in which the central engine shuts off and the ejecta cools via AE. We have derived and discussed this emission’s temporal and spectral properties using a new treatment for the microphysics of the AE: the adiabatic invariant $v_i^2/B$, describing the electron momentum normal to the magnetic field (see summary in Section 3.4). At the onset of the AE, as $B$ decays rapidly, this component of the momentum decreases while the parallel one remains unchanged, making the electrons more and more aligned with the local magnetic field as the ejecta expands. The adiabatic invariant enables us to calculate the electrons’ energy for a collisionless magnetized plasma, if no other collective plasma effects that randomize the electrons’ velocity are present.

In regards to the central engine activity, we can draw a conclusion: the fastest way that the observed flux can decline after the central engine shuts off is set by the LAE and the AE cooling (depending on the value of $\theta_j$).

The early X-ray steep decay shown in most of Swift bursts has been attributed to LAE. In this paper, we consider both AE and LAE for the very early X-ray data. LAE and AE both start with the assumption that the central engine shuts off abruptly. Only $\sim 20$ per cent of our sample of 107 bursts is possibly consistent with either LAE or AE, thereby suggesting that the observed early X-ray steep decay for a large fraction of bursts might be produced by the rapidly declining continuation of the central engine activity.

The component of the electron’s momentum parallel to the magnetic field is unconstrained by the adiabatic invariance. This component would probably cool via inverse Compton scattering with synchrotron photons. Another possibility is that the electrons are scattered by small-scale fluctuations in the magnetic field, which would effectively couple the parallel and perpendicular components of their momentum, resulting in an adiabatic cooling similar to that in the ideal gas case (Section 2; Granot, personal communication).

Any process that does not rely on the central engine activity to explain the observed early X-ray steep decay (i.e. RS, FS) has problems explaining the smooth temporal connection observed in the LC between the prompt emission and the early X-ray steep decay.

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5 Mangano et al. mention in their work that this could be attributed to the $v_j$ decrease due to adiabatic cooling of shock heated shells after an internal shock.
Adiabatic expansion and the engine in GRBs

APPENDIX A: ELECTRON GYRORADIUS VERSUS MAGNETIC FIELD LENGTH-SCALE

The observed peak energy in the prompt phase of a GRB is given by (4) and it is \( \gamma_\| = (1.15 \times 10^{-8} \text{eV}) B \gamma_\|^2 \Gamma (1 + \gamma_\|^z)^{-1} \), assuming synchrotron emission. The value of \( \Gamma \) can be constrained to be a few hundred. There is a wide range of allowed values for \( \gamma_i \). For \( \gamma_i = 10^5 \), \( v_\perp = 100 \text{keV} \), \( \Gamma = 100 \) and \( z = 1 \), then \( B = 2 \times 10^3 \text{G} \), and in that case, the electrons’ gyroradius is \( r = m_e c \gamma_i / (e B) \approx 10 \text{cm} \).

If the magnetic field responsible for the prompt emission is the frozen-in field from the central explosion, then it decays on a length-scale on the order of the source size, which is much larger than the electrons’ gyroradius and the adiabatic invariant presented in Section 1 can be used.

If the field is produced locally (e.g. by the Weibel instability), then we need to estimate its coherence length and compare it with the electrons’ gyroradius. For instance, magnetic field generated by the Weibel instability will have a coherence length on the order of the plasma length \( \lambda_B = c / \omega_p \), where \( \omega_p = (4 \pi e^2 n / m_e)^{1/2} = 6 \times 10^5 \text{eV} \text{s}^{-1} \) and \( n \) is the comoving electron number density in units of cm\(^{-3} \). For the prompt emission, recent studies have shown that the radius of emission is on the order of \( 10^{15} \text{–} 16 \text{ cm} \) or even larger (Kumar et al. 2007; Kumar & Narayan 2008; Racusin et al. 2008; Zou, Piran & Sari 2009), therefore \( n = 5 \times 10^{5.8} \text{ cm}^{-3} \) (see footnote 1), which gives \( \lambda_B = 20 \text{–} 700 \text{ cm} \), making \( \lambda_B \) larger than \( r \) by at least a factor of 2. Moreover, this magnetic field decays extremely fast in time (in about \( \omega_p/r_p \)), which would be less than \( 10^{-8} \text{s} \) in the source comoving frame or \( 10^{-10} \text{s} \) in the observer frame. This locally generated magnetic field cannot be responsible for the prompt emission, unless it is sustained for at least \( \sim 1 \text{s} \) (the comoving time-scale of a few millisecond prompt pulse), which would require a much larger \( \lambda_B \) [Keshet et al. (2008) mention that the field must persist over \( 10^{10} \lambda_B \) downstream, which in this case would be \( \sim 10^{12} \text{ cm} \)]. Therefore, it is safe to assume that even if the field is generated locally \( r < \lambda_B \), allowing us to use the adiabatic invariant.

The prompt emission phase could also be attributed to the SSC emission, which requires smaller values for \( \gamma_\| \) and \( \beta \). For \( \gamma_\| = 10^2 \), then \( B = 2 \times 10^3 \text{G} \), which gives \( r \approx 100 \text{ cm} \sim \lambda_B \). But as mentioned above, the field has to be coherent on length-scale \( \sim 10^{12} \text{ cm} \) in order that it does not decay away on time \( \lesssim 1 \text{s} \).

APPENDIX B: REVERSE SHOCK EMISSION CALCULATION

After the RS has crossed the ejecta, it follows roughly the Blandford–McKee self-similar solution (Blandford & McKee 1976), in which the bulk LF and pressure of the shocked ISM are given by

\[
\gamma_i(t, r) = \gamma(t) \chi \left( \frac{1}{\gamma} \right), \quad \rho_i(t, r) = 4 m_p c^2 n \left( \gamma(t) \right)^2 \chi \left( \frac{1}{\gamma} \right),
\]

where \( \gamma(t) \propto R^{-3/2} \) is the LF of material just behind the shock, \( n \propto R^{-z} \) is the ISM particle number density, \( \chi \) is the similarity variable and \( m_p \) is the proton mass (see e.g. Sari 1997).

Let us assume that the ejecta is at \( \chi_\| \) and it has a pressure \( P_{\|} \) and a LF \( \Gamma_{\|} \), which – because of pressure and velocity equilibrium at the contact discontinuity – should be the same as the bulk LF and pressure of the shocked ISM at \( \chi_\| \) (B1). The pressure in the ejecta for the thin case (\( \Delta' = R / \Gamma_{\|} \)) is given by

\[
P_{\|} \propto V^{-1} \chi \left( \frac{1}{\gamma} \right), \quad \rho_{\|} \propto R^{-3} \Gamma_{\|}^{-3/2},
\]

and using (B1) and the contact discontinuity equilibrium conditions we obtain

\[
\Gamma_{\|} = \gamma(t) \chi_\| \frac{1}{\gamma} \Gamma_{\|} = 4 m_p c^2 n \left( \gamma(t) \right)^2 \chi_\| \frac{1}{\gamma}.
\]

We can solve for \( \Gamma_{\|} \) in terms of \( R \), which gives \( \Gamma_{\|} \propto R^{-3/2} \chi_\| \). For uniform ISM (\( s = 0 \)), we can determine the observed time by

\[
t_{\text{obs}} = \frac{t}{\Gamma_{\|} - \Delta' / R} = \frac{t}{\Gamma_{\|} (1 + \gamma_\|^2)},
\]

for \( p = 2.5 \) (\( v_i < v < v_i \)).

We can also determine the electrons’ LF by using the ideal gas law.

This paper has been typeset from a T\textsc{latex}/\textsc{tex} file prepared by the author.