Offer search deviations from Newton’s law of gravity using low-energy neutrons

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Abstract. Information about the planned experiment at the reactor IBR-2M (installation YuMO) dedicated to finding differs from the Newtonian law of gravitation at small (atomic) distances reported. The experiment is to study the energy dependence of the scattering length on the matter with a very small length of pure nuclear scattering (e.g. by a mixture of isotopes of tungsten), allowing thereby increasing the relative contribution of gravitational scattering. At the concentration of tungsten-186 of about 90% in the mixture, nuclear scattering length may turn to zero at low energies. Relying on the accuracy of existing experiments it can argued that not Newtonian gravitational scattering can be detected if the scattering length is more than 10-16 or 10-17cm. Estimates show that in the theoretical model associated with the length of the electroweak interaction the amplitude of the gravitational scattering of neutrons in the Born approximation may be in the region from 10-11 to 10-21 cm. The magnitude of usual Newtonian gravitational scattering is 10-32 cm, i.e. significantly less.

The possible existence of multidimensional gravity is an idea that creates an increasing interest among physicists. The topicality of this idea is confirmed, in particular, e.g. by the Russian gravitation conferences (Kazan, September 2007 and Moscow, June 2008) and whose program included a number of questions related to multidimensional gravity. Some papers have appeared [1–3] on possible experimental tests of the idea of extra dimensions with the aid of neutron scattering.

The conventional Newtonian law of gravity reads

\[ F = - \frac{G m_1 m_2}{r^2} \]  

where \( F \) is the force of gravity, \( G \) is the Newtonian gravitational constant, \( m_1 \) and \( m_2 \) are the interacting masses and \( r \) is the distance between them. The minus sign means that gravity is an attracting force.

The law (1) has been well confirmed by now up to distances of the order of a few millimeters. Meanwhile, at smaller distances one can expect deviations from it, and, even from a purely experimental viewpoint, testing the Newton law at small distances is highly desirable.

Following [1], one can recall some theoretical reasoning connected with the weak interaction length that can lead to a modification of Newton’s law at small distances. Suppose that the masses as well as all forces (except the gravitational ones) obey the laws established in our usual three-
dimensional space while the gravitational forces should be described in terms of a multidimensional space. Then, according to [1], we can assume that the gravitational force between two particles placed at a distance equal to the electroweak length $R_{e} = \left[\frac{G_{F}(\hbar c)^{2}}{1} \right]^{1/2}$, where $G_{F} = (1.166 \times 10^{-5} \text{ GeV}^{-2}(c\hbar)^{3}$ is the Fermi constant, from each other, and the usual Newtonian force between the same particles at a distance equal to the Planck length $R_{p} = \left[\frac{\hbar G(c^{3})}{1} \right]^{1/2}$, are equal. Then the gravitational force may be written as follows:

$$F_{n} = - \frac{Gm_{1}m_{2}}{R_{c}^{n+2}} r^{-(n+2)}$$  \hspace{1cm} (2)

where $R_{c}^{n}$ is some characteristic distance, and $n = 0, 1, 2, \ldots$ characterizes the multidimensional nature of gravity, so that for $n = 0$ equation (2) passes over to the conventional Newtonian expression (1). Equation (2) should be valid for $r \leq R_{c}^{n}$. At $r > R_{c}^{n}$ the Newton law of gravity should hold. Following [1], one can also obtain that

$$R_{c}^{n} = 2^{-n/2} G_{F}^{-1} \left( \frac{\hbar^{2} + n/2}{1} \right)^{1}$$  \hspace{1cm} (3)

Calculations lead to the following results:

| n  | 1     | 2     | 3     | 4     |
|----|-------|-------|-------|-------|
| $R_{c}, \text{cm}$ | 1.2x10^{-17} | 2.8 | 8x10^{-6} | 1.4x10^{-8} |

It should be noted that the above reasoning and equations (2) and (3) do not form a rigorous theory but can only serve for estimation purposes.

The value $n = 1$ in the above table should be immediately rejected since $R_{c}$ in this case exceeds the size of the Solar system and would lead to instability of the planetary orbits. The values of $R_{c}$ smaller than 1cm are not easily checked by macroscopic experiments but tests with the aid of neutrons are basically possible. At any rate, it is an attractive idea to seek deviations from the inverse square law using neutrons even if the above theoretical reasoning proves to be wrong.

For distances $r \leq R_{c}$, the gravitational potential may be written in the form:

$$V(r) = -\frac{\alpha_{1}m_{2}}{G_{F}((n+1)r^{n+1})}$$  \hspace{1cm} (4)

The gravitational scattering length may be found in Born’s approximation. For $n = 2$ it can be written [4]:

$$a_{\text{coh,grav}} = -2/3 m_{2}^{2} \alpha_{1} G_{F}^{2} c^{7} h \int_{0}^{R_{c}} \frac{\sin(qR)}{q^{2}} \, dq$$  \hspace{1cm} (5)

where $q = 2\sin(\theta/2)$, $k$ is the wave number (unrelativistic $k = 2.197 \times 10^{9} \sqrt{\text{eV}}$ cm if $E$ is in eV), $m_{2}$ and $m_{1}$ are the masses of neutron and target, respectively, $R$ is the nuclear radius, $G_{F} = 1.166 \times 10^{-5} \text{ GeV}^{-2}$

The quantity (5) grows as the neutron energy decreases, and for small $qR$, expanding the first four terms in (5) in a series, we obtain

$$a_{\text{coh,grav}} = -2/3 m_{2}^{2} \alpha_{1} G_{F}^{2} c^{7} h \left[ 0.756 - \ln(qR) \right]$$  \hspace{1cm} (6)

Thus the best conditions for measurements (a comparatively large length $a_{\text{coh,grav}}$) take place at small energies of the scattered neutrons. One can obtain
\[ a_{coh,grav} = -5.44 \times 10^{-21} \text{ cm at } \lambda = 250\text{nm} \quad (7) \]

\[ a_{coh,grav} = -4.56 \times 10^{-21} \text{ cm at } \lambda = 11\text{nm} \quad (8) \]

For \( n = 0 \) holds Newtonian gravitational potential and one can obtain \( a_{coh,grav} = 10^{-32} \text{ cm} \) i.e. quite small. It is almost impossible to measure.

For \( n=1 \) \( V(r) \sim 1/r^2 \) and for example in the electroweak theory form \([1]\) one can be obtained that

\[ a_{coh,grav} = m T_c^{-6} \quad (9) \]

and

\[ a_{coh,grav} = -6.77 \times 10^{-10} \text{ cm at } \lambda = 250\text{nm} \quad (10) \]

\[ a_{coh,grav} = -3.10 \times 10^{-11} \text{ cm at } \lambda = 11\text{nm} \quad (11) \]

For comparison, it should be recalled that the lengths of neutron scattering is \( 5 \times 10^{-13} \text{ cm} \), i.e. significantly less. The truth according to the work \([1]\) \( n = 1 \) option is unlikely to be realized, the solar system probably would not be stable. We should not forget that the electroweak theory of option \([1]\) should be regarded only as estimates - the value of \( R_c \) can be changed for example in 137 times (the \( 1/137 \) is the fine structure constant) even in the work \([1]\).

At \( n>2 \), the quantity \( a_{coh,grav} \) decreases strongly, for \( n=3 \) \( a_{coh,grav} \approx 10^{-28} \text{ cm} \).

In this paper we look for deviations from the formula (1) using neutron scattering at low energies for nuclei with small nuclear scattering length, for example, a mixture of isotopes of tungsten-186 (its content in the mixture should be \( \approx 0.9 \)) The scattering length of one of these mixtures was measured at \( \lambda=150\text{nm} \) and \( a_{coh} = -0.0466(6) \times 10^{-12} \text{ cm} \) \([5]\).

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Figures a and b. Experimental set-up for Christiansen filter measurements.

(a)- beam collimation: a,b-slits; c-stopper, 1-filter, 2-neutron counter.
(b) cross cut of the Christiansen filter: 1-powder, 2-liquid, 3-glass, 4-distance holder, 5-liquid split
One of the accepted methods of measuring the scattering length is the method of Christiansen filters (in optics was first used as early as 1884). The method is as follows (see e.g. [5,6]). In a well-collimated neutron beam (divergence in the horizontal plane did not exceed 2-3 minutes of arc) set so-called Christiansen filter (see figure a, b), which is a vertical cavity of the test substance.

The thickness of the cavity in the direction of the beam is a few millimeters and the width of it fully covers the beam. The space between the grains of powder is filled alternately by several liquids with known refractive indices of neutrons.

At a distance of several meters behind the filter on the beam axis a neutron detector establish so wide that it can detect neutrons scattered by the filter at angles of less than 0.5° in the horizontal plane.

Measurement is to monitor the neutron scattering intensity at small angles by the filter for several values of the refractive index of the liquid filling it. As the liquid can be used such as various mixtures of heavy (Na_{coh} > 0) and light (Na_{coh} <0) water, where N is the number of nuclei per unit volume.

In principle, this scattering should be completely absent if the refractive index of the liquid is equal to the refractive index of the substance of the powder. We can show that the scattering intensity [5,6]:

\[ I \sim (n_{coh})_L \sim (n_{coh})_P \] (12)

where L-refers to the liquid, and P - to the powder.

The absence of small-angle scattering when \( (n_{coh})_L = (n_{coh})_P \), to determine \( a_{coh,P} \). The error of the experiment may not exceed 0.1%.

Coherent scattering length of neutrons by tungsten in the case of low energy neutrons can be written as (see e.g. [5]):

\[ a_{coh} = R - \frac{\alpha f_n}{2k_0E_o}(1 + \frac{E}{E_o}) + Zf_{a_{ne}} + a_{pol} + a_{coh,grav} \] (13)

where R - radius of the nucleus, \( \alpha \) - the concentration of tungsten-186 in the mixture, \( k_0 = 2.197x10^9/\sqrt{E_o} \) - wave number (energy E in eV), \( f_n \) -neutron width of the first resonance of the tungsten-186 (Eo = 18.83 eV), f - atomic form factor (at small scattering angles can be shown that f = 1), \( a_{ne} \) -length neutron scattering on an electron (the value of \( a_{ne} = -1.60 \times 10^{-16} \) cm, it is this line [7] the meson theory of Yukawa). At small scattering angles for tungsten value \( Zf_{a_{ne}} = - 0.118 \times 10^{-13} \) cm. However, the value \( a_{ne} \) insignificant for consideration following the experiment - the only important thing to \( Zf_{a_{ne}} \) does not depend on the neutron energy. \( a_{pol} \) - neutron scattering length induced by the electric polarizability of the neutron. At low energies (\( \lambda \approx 150 \)nm) the

\[ a_{pol} = \frac{ma}{2R} (\frac{Ze}{\hbar})^2(1 - \frac{\pi}{4} qR + ...) \approx 10^{-14}(1 - 2 \times 10^{-6} + ...) \] (14)

and for small scattering angles is almost independent of neutron energy [8]. If you choose a value of \( \alpha \) such that in formula (10) members compensate one another, then the total scattering length is close to zero. At the wavelength of the neutron \( \lambda \approx 150 \)nm value \( \alpha \) should be with about 0.9. In the wavelength range from 10 to 150nm size of the scattering length will vary with energy, but not very strongly, since E <<Eo (at \( \lambda = 150 \)nm E = 0.00036 eV, and E \approx 1 / \lambda^2).

A few words about the possibilities of the proposed method of searching for gravitational scattering length. First of all, it is much more straightforward for experimentalists, since it does not need many of systematic corrections, while not well measured at present, but necessary for a number of other options for proposals (see e.g. [3]).

Building on the work [9], we can assume that used a mixture of isotopes of tungsten deviation of the scattering length of the linear energy dependence due to nuclear interaction is unlikely.
Further, taking the smallest error in the famous experiments by the method of Christiansen filters of $10^{-16}$ cm (see e.g. book [10]), we can assume that the allowable measured length of the gravitational scattering could now make a value of approximately the same order. However, measurement error can be decreased and the method proposed above measurements probably all still close to the most relevant.

The proposed experiment can be performed on YuMO installation. Currently there is a discussion of its details.

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