0\(^+\) fully-charmed tetraquark states

Jian-Rong Zhang

Department of Physics, College of Liberal Arts and Sciences,
National University of Defense Technology, Changsha 410073, Hunan, People’s Republic of China

Motivated by the LHCb’s new observation of structures in the J/\(\psi\)-pair invariant mass spectrum, for which could be classified as possible \(cc\bar{c}\bar{c}\) tetraquark candidates, we systematically study 0\(^+\) fully-charmed tetraquark states through QCD sum rules. Making the development of calculation techniques to fourfold heavy hadronic systems, four different configuration currents with 0\(^+\) are considered and vacuum condensates up to dimension 6 are included in the operator product expansion (OPE). Finally, mass values acquired for 0\(^+\) \(cc\bar{c}\bar{c}\) tetraquark states agree well with the experimental data of the broad structure, which supports that it could be a 0\(^+\) fully-charmed tetraquark state.

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I. INTRODUCTION

By far, the topic of fully-charmed tetraquark state has attracted much attention. For example, a variety of phenomenological models were employed to predict the existence of some states merely made up of four heavy quarks [1–28]. Particularly, without any light quark contamination, fully-charmed tetraquark states are ideal prototypes to refine one’s understanding on heavy quark dynamics.

Recently, the invariant mass spectrum of double-J/\(\psi\) was researched using proton-proton collision data recorded by the LHCb experiment, which shows a broad structure just above twice the J/\(\psi\) mass ranging from 6.2 to 6.8 GeV and a narrower structure around 6.9 GeV, referred to as X(6900) [29]. Soon after the LHCb’s new results, various investigations were presented to explain them via different approaches [30–57]. To probe a real hadron, one inevitably has to face the sophisticated nonperturbative QCD problem. As one trustable way for evaluating nonperturbative effects, the QCD sum rule [58] is firmly established on the basic theory, and has been widely applied to hadronic systems (for reviews see Refs. [59–62] and references therein). On 0\(^+\) \(cc\bar{c}\bar{c}\) tetraquark states, there have been some related works from different versions of QCD sum rules. For instance, in moment QCD sum rules, Ref. [11] explored doubly hidden-charm/bottom tetraquark states; With the Finite Energy version of the QCD Inverse Laplace sum rules, Ref. [41] investigated doubly-hidden scalar heavy molecules and tetraquarks states. Besides, some other works made analysis of fully-heavy tetraquark states with QCD sum rules involving condensates to dimension 4, specially by choosing the axial vector-axial vector current [12], by considering the scalar’s first radial excited states [31], or by introducing a relative P-wave to construct the diquark operator [50].

Inspired by the so exciting structures observed in the di-J/\(\psi\) mass spectrum, we endeavor to make an extension of the calculation techniques [63–64] to fourfold heavy hadrons and study 0\(^+\) fully-charmed tetraquark states systematically by QCD sum rules. We would take into account four possible currents with 0\(^+\) and include condensates up to dimension 6 which may be helpful to the stability of Borel curves to some extent.

The paper is organized as follows. After the Introduction, the QCD sum rule is derived for 0\(^+\) fully-charmed tetraquark states in Sec. II along with numerical analysis and discussions in Sec. III. The last part contains a brief summary.

II. 0\(^+\) FULLY-CHARMED TETRAQUARK STATE QCD SUM RULES

Considering a tetraquark state, its interpolating current can ordinarily be represented by a diquark-antidiquark configuration. Thus, following forms of currents could be constructed for 0\(^+\) \(cc\bar{c}\bar{c}\) tetraquark
where states, 
\[ j = (Q_a^T C \gamma_5 Q_b)(\bar{Q}_a \gamma_5 C \bar{Q}_b^T) \]
for the scalar-scalar configuration,
\[ j = (Q_a^T C Q_b)(\bar{Q}_a C \bar{Q}_b^T) \]
for the pseudoscalar-pseudoscalar one,
\[ j = (Q_a^T C \gamma_\mu Q_b)(\bar{Q}_a \gamma_\mu C \bar{Q}_b^T) \]
for the axial vector-axial vector (shortened to axial-axial) one, and
\[ j = (Q_a^T C \gamma_\mu Q_b)(\bar{Q}_a \gamma_\mu \gamma_5 C \bar{Q}_b^T) \]
for the vector-vector one. Here the index \( T \) indicates matrix transposition, \( C \) means the charge conjugation matrix, \( Q \) is the heavy charm quark, as well as \( a \) and \( b \) are color indices.

To derive QCD sum rules, one can start with the two-point correlator
\[ \Pi(q^2) = i \int d^4xe^{i q \cdot x} \langle 0 | T [j(x)j^\dagger(0)] | 0 \rangle. \]  
(1)

In phenomenology, it can be expressed as
\[ \Pi(q^2) = \frac{\lambda_H^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^\infty \frac{\text{Im} \left[ \Pi^\text{phen}(s) \right]}{s - q^2} ds, \]  
(2)
in which \( M_H \) is the hadron’s mass, \( s_0 \) denotes the continuum threshold, and \( \lambda_H \) displays the coupling of the current to the hadron \( \langle 0 | j | H \rangle = \lambda_H \). Moreover, it can theoretically be rewritten as
\[ \Pi(q^2) = \int_{(4m_Q)^2}^{s_0} \frac{\rho}{s - q^2} ds, \]  
(3)
where \( m_Q \) is the heavy charm mass, and the spectral density \( \rho = \frac{\lambda_H^2}{\pi} \text{Im} \left[ \Pi(s) \right] \). After equating Eqs. (2) and (3), adopting quark-hadron duality, and applying a Borel transform, it yields
\[ \lambda_H^2 e^{-M_H^2/m_Q^2} = \int_{(4m_Q)^2}^{s_0} \rho e^{-s/M^2} ds, \]  
(4)
with \( M^2 \) the Borel parameter. Taking the derivative of Eq. (4) to \(-\frac{1}{M^2}\) and then dividing the result by Eq. (4) itself, one could achieve the mass sum rule
\[ M_H = \sqrt{\int_{(4m_Q)^2}^{s_0} \rho s e^{-s/M^2} ds / \int_{(4m_Q)^2}^{s_0} \rho e^{-s/M^2} ds}. \]  
(5)

In the OPE calculation, one could work at the momentum-space with the help of heavy-quark propagator \[ \frac{1}{2 \alpha \pi^3} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{\beta_{\max}} \frac{d\beta}{\beta^3} \int_{\gamma_{\min}}^{\gamma_{\max}} \frac{d\gamma}{\gamma^3} \frac{1}{H} \left[ m_Q^2 - \frac{(1 - \alpha - \beta - \gamma)s^2}{H} \right]^2 \]
\[ \times \left\{ -3(1 - \alpha - \beta - \gamma) + 4\left(\gamma(1 - \alpha - \beta - \gamma) + \alpha \beta \right)H - 6\alpha \beta \gamma H^2 \right\} H^2 m_Q^4 \]
\[ + (1 - \alpha - \beta - \gamma)\left[ 18(1 - \alpha - \beta - \gamma) - 10\left(3(1 - \alpha - \beta - \gamma) + \alpha \beta \right)H \right] H m_Q^2 s - 21(1 - \alpha - \beta - \gamma)^3 s^2 \}. \]
\[ \rho(g^2G^2) = -\frac{m_Q^2(g^2G^2)}{3 \cdot 2^{9/2}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{\beta_{max}} \frac{d\beta}{\beta^3} \int_{\gamma_{min}}^{\gamma_{max}} \frac{d\gamma}{\gamma^3} \frac{(1-\alpha-\beta-\gamma)^2}{H^5} \]
\[ \times \left\{ -6(1-\alpha-\beta-\gamma)^2 + (1-\alpha-\beta-\gamma)(6\gamma + 2\gamma(1-\alpha-\beta-\gamma) + 2\alpha\beta)H - 3\alpha\beta\gamma H^2 \right\} \]
\[ \rho^\text{pert} = -\frac{1}{2^{10/2}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{\beta_{max}} \frac{d\beta}{\beta^3} \int_{\gamma_{min}}^{\gamma_{max}} \frac{d\gamma}{\gamma^3} \frac{1}{H^6} \left[ m_Q^2 - \frac{(1-\alpha-\beta-\gamma)s}{H} \right] \]
\[ \times \left\{ -3(1-\alpha-\beta-\gamma) - 4\left( \gamma(1-\alpha-\beta-\gamma) + \alpha\beta \right)H - 6\alpha\beta\gamma H^2 \right\} \]
\[ + (1-\alpha-\beta-\gamma) \left[ 18(1-\alpha-\beta-\gamma) + 10\left( \gamma(1-\alpha-\beta-\gamma) + \alpha\beta \right)H - 6m_Q^2 - 21(1-\alpha-\beta-\gamma)^3s^2 \right] \]

\[ \rho^sG^2 = -\frac{m_Q^2(g^2G^2)}{3 \cdot 2^{9/2}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{\beta_{max}} \frac{d\beta}{\beta^3} \int_{\gamma_{min}}^{\gamma_{max}} \frac{d\gamma}{\gamma^3} \frac{(1-\alpha-\beta-\gamma)^2}{H^5} \]
\[ \times \left\{ -6(1-\alpha-\beta-\gamma)^2 + (1-\alpha-\beta-\gamma)(6\gamma + 2\gamma(1-\alpha-\beta-\gamma) + 2\alpha\beta)H - 3\alpha\beta\gamma H^2 \right\} \]
\[ + 3(1-\alpha-\beta-\gamma)^2 \left[ 4(1-\alpha-\beta-\gamma) + 3\gamma H \right] \]

\[ \rho^sG^3 = -\frac{m_Q^2(g^2G^2)}{3 \cdot 2^{9/2}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{\beta_{max}} \frac{d\beta}{\beta^3} \int_{\gamma_{min}}^{\gamma_{max}} \frac{d\gamma}{\gamma^3} \frac{(1-\alpha-\beta-\gamma)^3}{H^5} \]
\[ \times \left\{ -3(1-\alpha-\beta-\gamma) - 3\gamma H \right\} \]
\[ + 6(1-\alpha-\beta-\gamma)^2 - 6\gamma(1-\alpha-\beta-\gamma)H - 6m_Q^2 + 6(1-\alpha-\beta-\gamma)^2s \]

for the pseudoscalar-pseudoscalar current,
\[
\rho^{(g^3 G^2)} = -\frac{(g^3 G^3)}{3 \cdot 2^{9/2} \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta^3} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{d\gamma}{\gamma^3} \left( \frac{1 - \alpha - \beta - \gamma}{\gamma^3} \right)^3 \left\{ -6(1 - \alpha - \beta - \gamma) \\
-12(1 - \alpha - \beta - \gamma)^2 + 6\gamma(1 - \alpha - \beta - \gamma)H + \alpha\beta H \right\} \left( m_Q^2 + 12(1 - \alpha - \beta - \gamma)^2 s \right),
\]
for the axial-axial current, and
\[
\rho^{\text{pert}} = -\frac{1}{2^{8/3}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta^3} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{d\gamma}{\gamma^3} \left[ m_Q^2 \frac{(1 - \alpha - \beta - \gamma) s}{H} \right]^2 \times \left\{ -3(1 - \alpha - \beta - \gamma) - 2\gamma(1 - \alpha - \beta - \gamma) + \alpha\beta \right\} \left( H - 6\alpha\beta H^2 \right) H^2 m_Q^4 \\
+ (1 - \alpha - \beta - \gamma) \left[ 18(1 - \alpha - \beta - \gamma) + 5\gamma(1 - \alpha - \beta - \gamma) + \alpha\beta \right] H \left( m_Q^2 + 21(1 - \alpha - \beta - \gamma)^3 s \right)^2,
\]
\[
\rho^{(g^2 G^2)} = -\frac{m_Q^2 (g^2 G^2)}{3 \cdot 2^{9/2} \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta^3} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{d\gamma}{\gamma^3} \left( \frac{1 - \alpha - \beta - \gamma}{\gamma^3} \right)^2 \left\{ -12(1 - \alpha - \beta - \gamma)^2 - (1 - \alpha - \beta - \gamma) \left( 6\gamma + 2\gamma(1 - \alpha - \beta - \gamma) + 2\alpha\beta \right) H - 6\alpha\beta H^2 \right\} H m_Q^4 \\
+ 3(1 - \alpha - \beta - \gamma)^2 \left[ 8(1 - \alpha - \beta - \gamma) + 3\gamma \right] H \left( m_Q^2 + 21(1 - \alpha - \beta - \gamma)^3 s \right)^2,
\]
\[
\rho^{(g^3 G^3)} = -\frac{(g^3 G^3)}{3 \cdot 2^{9/2} \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \frac{d\beta}{\beta^3} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \frac{d\gamma}{\gamma^3} \left( \frac{1 - \alpha - \beta - \gamma}{\gamma^3} \right)^3 \left\{ -6(1 - \alpha - \beta - \gamma) \\
-12(1 - \alpha - \beta - \gamma)^2 - 6\gamma(1 - \alpha - \beta - \gamma)H - \alpha\beta H \right\} \left( m_Q^2 + 12(1 - \alpha - \beta - \gamma)^2 s \right),
\]
for the vector-vector current. It is defined as
\[
H = 1 + (1 - \alpha - \beta - \gamma) \left( \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} \right),
\]
\[
\alpha_{\text{min}} = \frac{1}{2} \left[ \left( 1 - \frac{8m_Q^2}{s} \right) - \sqrt{\left( 1 - \frac{8m_Q^2}{s} \right)^2 - 4m_Q^2} \right],
\]
\[
\alpha_{\text{max}} = \frac{1}{2} \left[ \left( 1 - \frac{8m_Q^2}{s} \right) + \sqrt{\left( 1 - \frac{8m_Q^2}{s} \right)^2 - 4m_Q^2} \right],
\]
\[
\beta_{\text{min}} = \frac{1}{2} \left[ 1 + 2\alpha - \frac{3\alpha^2 s}{\alpha s - m_Q^2} \right] - \sqrt{\frac{\alpha(1 - \alpha) s - m_Q^2}{\alpha s - m_Q^2} \left( \alpha s - m_Q^2 \right)^2},
\]
\[
\beta_{\text{max}} = \frac{1}{2} \left[ 1 + 2\alpha - \frac{3\alpha^2 s}{\alpha s - m_Q^2} \right] + \sqrt{\frac{\alpha(1 - \alpha) s - m_Q^2}{\alpha s - m_Q^2} \left( \alpha s - m_Q^2 \right)^2},
\]
\[
\gamma_{\text{min}} = \frac{1}{2} \left[ 1 + \alpha - \beta \right] - \sqrt{\frac{(1 - \alpha - \beta) \left( (1 - \alpha - \beta) \left( \alpha\beta s - (\alpha + \beta) m_Q^2 \right) - 4\alpha\beta m_Q^2 \right)}{\alpha\beta s - (\alpha + \beta) m_Q^2}},
\]
and
\[
\gamma_{\text{max}} = \frac{1}{2} \left[ 1 + \alpha - \beta \right] + \sqrt{\frac{(1 - \alpha - \beta) \left( (1 - \alpha - \beta) \left( \alpha\beta s - (\alpha + \beta) m_Q^2 \right) - 4\alpha\beta m_Q^2 \right)}{\alpha\beta s - (\alpha + \beta) m_Q^2}}.
\]
III. NUMERICAL ANALYSIS AND DISCUSSIONS

In this part, the heavy \( m_Q \) is taken as the running charm mass \( m_c = 1.27 \pm 0.02 \text{ GeV} \), and other input parameters are \( (g^2G^2) = 0.88 \pm 0.25 \text{ GeV}^4 \) and \( (g^3G^3) = 0.58 \pm 0.18 \text{ GeV}^6 \). Following the standard criterion of sum rule analysis, one could find proper work windows for the threshold parameter \( \sqrt{s_0} \) and the Borel parameter \( M^2 \). The lower bound of \( M^2 \) could be gained in view of the OPE convergence, and the upper one is obtained from the pole dominance. Meanwhile, the threshold \( \sqrt{s_0} \) is around \( 0.4 \sim 0.6 \text{ GeV} \) higher than the extracted \( M_H \) empirically, for which describes the beginning of continuum state.

Taking the scalar-scalar case as an example, the inputs are kept at their central values at the start. To find the lower bound of \( M^2 \), its OPE convergence is shown in FIG. 1 by comparing the relative contributions of various parts from sum rule (1) for \( \sqrt{s_0} = 7.0 \text{ GeV} \), and one could note that the relative contributions of two-gluon condensate \( \langle g^2G^2 \rangle \) and three-gluon condensate \( \langle g^3G^3 \rangle \) are very small. Numerically, it is taken as \( M^2 \geq 2.5 \text{ GeV}^2 \) with an eye to the OPE convergence analysis. On the other hand, the upper one of \( M^2 \) is obtained from the pole contribution dominance phenomenologically. FIG. 2 makes the comparison between pole and continuum contribution from sum rule (4) for \( \sqrt{s_0} = 7.0 \text{ GeV} \). The relative pole contribution is about 50% at \( M^2 = 3.0 \text{ GeV}^2 \) and decreasing with \( M^2 \). Thereby, it could satisfy the pole dominance requirement while \( M^2 \leq 3.0 \text{ GeV}^2 \), and the Borel window is fixed as \( M^2 = 2.5 \sim 3.0 \text{ GeV}^2 \) for \( \sqrt{s_0} = 7.0 \text{ GeV} \). Analogously, they are taken as \( M^2 = 2.5 \sim 2.9 \text{ GeV}^2 \) for \( \sqrt{s_0} = 6.9 \text{ GeV} \), and \( M^2 = 2.5 \sim 3.2 \text{ GeV}^2 \) for \( \sqrt{s_0} = 7.1 \text{ GeV} \), respectively. The mass \( M_H \) dependence on \( M^2 \) is shown in FIG. 3 for the scalar-scalar case, and it is computed to be \( 6.44 \pm 0.13 \text{ GeV} \) in the chosen windows. And then varying all the input values, the attained mass is \( 6.44 \pm 0.13 \pm 0.02 \text{ GeV} \) (the first error from variation of \( s_0 \) and \( M^2 \), and the second from the uncertainty of QCD parameters) or \( 6.44^{+0.15}_{-0.16} \text{ GeV} \) in a short form.

In the very similar analyzing processes, proper work windows for other three cases could also be found and their corresponding Borel curves are respectively given in FIG. 4-6. After considering the uncertainty both from work windows and variation of input parameters, the mass values of \( 0^+ \) ccc\( \bar{c} \) tetraquark states are gained as \( 6.45^{+0.14}_{-0.16} \text{ GeV} \) for the pseudoscalar-pseudoscalar configuration, \( 6.46^{+0.13}_{-0.17} \text{ GeV} \) for the axial-axial one, and \( 6.47^{+0.12}_{-0.18} \text{ GeV} \) for the vector-vector one, respectively.

![FIG. 1: The OPE convergence for the 0+ fully-charmed tetraquark state with a scalar-scalar configuration is shown by comparing the relative contributions of perturbative, two-gluon condensate \( \langle g^2G^2 \rangle \), and three-gluon condensate \( \langle g^3G^3 \rangle \) from sum rule (1) for \( \sqrt{s_0} = 7.0 \text{ GeV} \).](image)
FIG. 2: The phenomenological contribution in sum rule for $\sqrt{s_0} = 7.0$ GeV for the $0^+$ fully-charmed tetraquark state with a scalar-scalar configuration. The solid line is the relative pole contribution as a function of $M^2$ and the dashed line is the relative continuum contribution.

FIG. 3: The mass $M_H$ dependence on $M^2$ for the $0^+$ fully-charmed tetraquark state with a scalar-scalar configuration from sum rule is shown. The Borel windows of $M^2$ are $2.5 \sim 2.9$ GeV$^2$ for $\sqrt{s_0} = 6.9$ GeV, $2.5 \sim 3.0$ GeV$^2$ for $\sqrt{s_0} = 7.0$ GeV, and $2.5 \sim 3.2$ GeV$^2$ for $\sqrt{s_0} = 7.1$ GeV, respectively.

FIG. 4: The mass $M_H$ dependence on $M^2$ for the $0^+$ fully-charmed tetraquark state with a pseudoscalar-pseudoscalar configuration from sum rule is shown. The Borel windows of $M^2$ are $2.5 \sim 2.9$ GeV$^2$ for $\sqrt{s_0} = 6.9$ GeV, $2.5 \sim 3.0$ GeV$^2$ for $\sqrt{s_0} = 7.0$ GeV, and $2.5 \sim 3.2$ GeV$^2$ for $\sqrt{s_0} = 7.1$ GeV, respectively.
Focusing on the LHCb’s new observation in the di-$J/\psi$ mass spectrum, we systematically investigate $0^+$ fully-charmed tetraquark states in the framework of QCD sum rules. By developing related calculation techniques to fourfold heavy tetraquark states, four types of currents with different configurations are taken into consideration and condensates up to dimension 6 are involved in the OPE side. At last, the mass values of $0^+$ $cc\bar{c}\bar{c}$ tetraquark states are calculated to be $6.44^{+0.15}_{-0.16}$ GeV for the scalar-scalar case, $6.45^{+0.14}_{-0.16}$ GeV for the pseudoscalar-pseudoscalar case, $6.46^{+0.13}_{-0.17}$ GeV for the axial-axial case, and $6.47^{+0.12}_{-0.18}$ GeV for the vector-vector case, respectively. All these results are numerically consistent with the experimental data $6.2 \sim 6.8$ GeV of the broad structure, which could support its internal structure as a $0^+$ $cc\bar{c}\bar{c}$ tetraquark state. For the future, it is expected that further experimental and theoretical efforts may reveal more on the nature of the exotic states.

**FIG. 5:** The mass $M_H$ dependence on $M^2$ for the $0^+$ fully-charmed tetraquark state with a axial-axial configuration from sum rule (5) is shown. The Borel windows of $M^2$ are $2.5 \sim 2.9$ GeV$^2$ for $\sqrt{s_0} = 6.9$ GeV, $2.5 \sim 3.0$ GeV$^2$ for $\sqrt{s_0} = 7.0$ GeV, and $2.5 \sim 3.2$ GeV$^2$ for $\sqrt{s_0} = 7.1$ GeV, respectively.

**FIG. 6:** The mass $M_H$ dependence on $M^2$ for the $0^+$ fully-charmed tetraquark state with a vector-vector configuration from sum rule (5) is shown. The Borel windows of $M^2$ are $2.5 \sim 2.9$ GeV$^2$ for $\sqrt{s_0} = 6.9$ GeV, $2.5 \sim 3.0$ GeV$^2$ for $\sqrt{s_0} = 7.0$ GeV, and $2.5 \sim 3.2$ GeV$^2$ for $\sqrt{s_0} = 7.1$ GeV, respectively.

**IV. SUMMARY**
Acknowledgments

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