Two/Three-Flavor Oscillation and MSW/Vacuum Oscillation Solution of Neutrinos in the SO(3) Gauge Model

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AS-ITP-99-04

Abstract

Three interesting scenarios for neutrino mixing, i.e., (i) small-large mixing scenario, (ii) nearly bi-maximal mixing scenario and (iii) three-flavor oscillation scenario, are analyzed in connection with three possible assignments of the maximal CP-violating phase after spontaneous symmetry breaking of SO(3) in the model with gauged SO(3) lepton flavor symmetry. As a consequence, it is found that the scenario (ii) is more reliable to be constructed to reconcile both solar and atmospheric neutrino data. Though three Majorana neutrino masses in all scenarios can be nearly degenerate, in the scenarios (ii) and (iii) masses of the neutrinos are allowed to be large enough to play a significant cosmological role and in the scenario (i) the fraction $\Omega_\nu/\Omega_m$ is bounded to be $\Omega_\nu/\Omega_m < (4 - 10)\%$ for $\Omega_m = (1.0 - 0.4)$ and $h = 0.6$.

PACS numbers: 12.15F, 11.30H
1 Introduction

It has been shown that the current neutrino data on both atmospheric[1] and solar[2, 3] neutrino flux anomalies (the solar neutrino flux measured recently by the Super-Kamiokande collaboration is only about half of that expected from the ‘BP’ standard solar model (‘BP’ SSM)[4]) can be explained by a nearly ‘bi-maximal’ mixing pattern (that include the bi-maximal mixing pattern[6] and democratic mixing pattern[7]). In the recent papers[5], we have investigated gauged SO(3) symmetry for three lepton families and derived a realistic scenario for such patterns. In this paper, we consider three possible scenarios for neutrino mixing: (i) small-large mixing scenario and (ii) nearly bi-maximal mixing scenario as well as (iii) three-flavor oscillation scenario due to three possible assignments of the maximal CP-violating phase after spontaneous symmetry breaking of SO(3). The SO(3) symmetry can naturally lead three Majorana neutrino masses in all scenarios to be nearly degenerate[8, 9, 10, 11]. In the scenarios (ii) and (iii), masses of the neutrinos are allowed to be large enough to play a significant cosmological role and in the scenario (i) they are constrained by the neutrinoless double beta decay. In particular, it is seen that the scenario (ii) is more reliable to be constructed to reconcile both solar and atmospheric neutrino data. Our paper is organized as follows: in the section 2, we provide a description of the model based on the SO(3) gauge symmetry of lepton flavor. The small-large mixing scenario is discussed in the section 3. In the section 4, the nearly bi-maximal mixing scenario is analysed. The three-flavor oscillation scenario is discussed in the section 5. Conclusions and remarks are presented in the last section.

2 The Model

We begin with the following SO(3)F×SU(2)L×U(1)Y invariant effective lagrangian for leptons

\[ \mathcal{L} = \frac{1}{2} g'_3 A^k_{\mu} \left( \bar{L}_i \gamma^\mu (t^k)_{ij} L_j + \bar{e}_{Ri} \gamma^\mu (t^k)_{ij} e_{Rj} \right) + D_\mu \varphi^* D^\mu \varphi + D_\mu \varphi'^* D^\mu \varphi' + \left( C_1 \frac{\varphi_i \varphi_j}{M_1 M_2} + C'_1 \frac{\varphi'_i \varphi'_j}{M'_1 M'_2} \chi + C''_1 \frac{\chi'}{M'} \delta_{ij} \right) \bar{L}_i \varphi e_{Rj} + h.c. \]

which is assumed to be resulted from integrating out heavy particles. Where \( \mathcal{L}_{SM} \) denotes the lagrangian of the standard model. \( \bar{L}_i(x) = (\bar{\nu}_i, \bar{e}_i)_L \) (i=1,2,3) are the SU(2)L doublet leptons. \( e_{Ri} \) (i=1,2,3) are the three right-handed charged leptons. \( \varphi_1(x) \) and \( \varphi_2(x) \) are two Higgs doublets. \( \varphi^T = (\varphi_1(x), \varphi_2(x), \varphi_3(x)) \) and \( \varphi'^T = (\varphi'_1(x), \varphi'_2(x), \varphi'_3(x)) \) are two complex SO(3) triplet scalars. \( \chi(x) \) and \( \chi'(x) \) are two singlet scalars. \( M_1, M_2, M', M'_1, M'_2 \) and \( M_N \) are possible mass scales concerning heavy fermions. \( C_a, C'_a \) and \( C''_a \) \((a = 0,1)\) are six coupling constants. The structure of the above effective lagrangian can be obtained by imposing an additional U(1) symmetry, which is analogous to the
construction of the $C_0$ and $C_1$ terms discussed in detail in ref.\[3\]. After the symmetry $SO(3)_F \times SU(2)_L \times U(1)_Y$ is broken down to the $U(1)_{em}$ symmetry, we obtain mass matrices of the neutrinos and charged leptons as follows

\[
(M_e)_{ij} = m_1 \frac{\hat{\sigma}_i \delta_j}{\sigma^2} + m'_1 \frac{\delta_j' \hat{\sigma}_i'}{\sigma^2} + m''_1 \delta_{ij}
\]

\[
(M_\nu)_{ij} = m_0 \delta_{ij} + m'_0 \frac{\hat{\sigma}_i \delta_j + \delta_j \hat{\sigma}_i'}{2\sigma^2} + m''_0 \frac{\delta_j' \hat{\sigma}_i^* + \hat{\sigma}_i \delta_j^*}{2\sigma^2}
\]

(2)

where the mass matrices $M_e$ and $M_\nu$ are defined in the basis $L_M = \bar{e}_L M_e e_R + \bar{\nu}_L M_\nu \nu_R^c + h.c.$.

The constants $\hat{\sigma}_i = \angle \varphi_i(x)$ and $\hat{\sigma}_i' = \angle \varphi'_i(x)$ represent the vacuum expectation values of the two triplet scalars $\varphi(x)$ and $\varphi'(x)$. The six mass parameters are defined as: $m_0 = C_0 v_2^2 / M_N$, $m'_0 = C'_0 (\sigma^2 / M_2^2) (v_2^2 / M_N)$, $m''_0 = C''_0 (\sigma^2 / M_2^2) (v_2^2 / M_N)$, $m_1 = C_1 v_1 \sigma^2 / M_1 M_2$, $m'_1 = C'_1 (\xi / M) (v_1 \sigma^2 / M_1 M'_2)$ and $m''_1 = C''_1 v_1 \xi / M$. Here $\sigma = \sqrt{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 + \hat{\sigma}_3^2}$ and $\sigma' = \sqrt{\hat{\sigma}_1'^2 + \hat{\sigma}_2'^2 + \hat{\sigma}_3'^2}$. $\xi = \angle \chi(x)$ and $\xi' = \angle \chi'(x)$ denote the vacuum expectation values of the two singlet scalars.

Utilizing the gauge symmetry property, it is convenient to reexpress the complex triplet scalar fields $\varphi_i(x)$ and $\varphi'_i(x)$ in terms of the $SO(3)$ rotational fields $O(x) = e^{i \eta_i(x) t^i}$, $O'(x) = e^{i \eta'_i(x) t^i} \in SO(3)$. In general, there exist three different vacuum structures in connection with three possible assignments of the imaginary amplitude field (for completeness and comparison, we will also include the case discussed in ref.\[5\])

\[
A: \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \end{pmatrix} = \frac{e^{i \eta_i(x) t^i}}{\sqrt{2}} \begin{pmatrix} \rho_1(x) \\ \rho_2(x) \\ i \rho_3(x) \end{pmatrix}, \quad \begin{pmatrix} \varphi'_1(x) \\ \varphi'_2(x) \\ \varphi'_3(x) \end{pmatrix} = \frac{e^{i \eta'_i(x) t^i}}{\sqrt{2}} \begin{pmatrix} \rho'_1(x) \\ \rho'_2(x) \\ i \rho'_3(x) \end{pmatrix} \]

\[
B: \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \end{pmatrix} = \frac{e^{i \eta_i(x) t^i}}{\sqrt{2}} \begin{pmatrix} \rho_1(x) \\ i \rho_2(x) \\ \rho_3(x) \end{pmatrix}, \quad \begin{pmatrix} \varphi'_1(x) \\ \varphi'_2(x) \\ \varphi'_3(x) \end{pmatrix} = \frac{e^{i \eta'_i(x) t^i}}{\sqrt{2}} \begin{pmatrix} \rho'_1(x) \\ i \rho'_2(x) \\ \rho'_3(x) \end{pmatrix} \]

\[
C: \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \end{pmatrix} = \frac{e^{i \eta_i(x) t^i}}{\sqrt{2}} \begin{pmatrix} i \rho_1(x) \\ \rho_2(x) \\ \rho_3(x) \end{pmatrix}, \quad \begin{pmatrix} \varphi'_1(x) \\ \varphi'_2(x) \\ \varphi'_3(x) \end{pmatrix} = \frac{e^{i \eta'_i(x) t^i}}{\sqrt{2}} \begin{pmatrix} i \rho'_1(x) \\ \rho'_2(x) \\ \rho'_3(x) \end{pmatrix} \]

(3)

Here the three rotational fields $\eta_i(x)$ ($\eta_i'(x)$) and the three amplitude fields $\rho_i(x)$ ($\rho_i'(x)$) reparameterize the six real fields of the complex triplet scalar field $\varphi(x)$ ($\varphi'(x)$). $SO(3)$ gauge symmetry allows one to remove three degrees of freedom from the six rotational fields. Thus the vacuum structure of the $SO(3)$ symmetry is expected to be determined only by nine degrees of freedom. These nine degrees of freedom can be taken as $\rho_i(x)$, $\rho_i'(x)$ and $(\eta_i(x) - \eta_i'(x))$ without lossing generality. Here we will consider the following vacuum structure for the $SO(3)$ symmetry breaking

\[
< \rho_i(x) > = \sigma_i, \quad < \rho_i'(x) > = \sigma'_i, \quad < (\eta_i(x) - \eta_i'(x)) > = 0
\]

(4)

With this vacuum structure, the mass matrices of the neutrinos and charged leptons can be reexpressed as

\[
A: \quad M_e = m_1 \begin{pmatrix} s_1^2 s_2^2 & c_1 s_1 s_2^2 & i s_1 c_2 s_2 \\ c_1 s_1 s_2^2 & c_1^2 s_2^2 & i c_1 c_2 s_2 \\ i s_1 c_2 s_2 & i c_1 c_2 s_2 & -c_2^2 \end{pmatrix}
\]
Note that the two non-diagonal matrices in the mass matrix $M$ which correspond to the three vacuum structures

\[ 1 = \sigma_B : M_C \quad \text{and} \quad M_B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

and

\[ A : M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_0' \begin{pmatrix} s_1^2 s_2^2 & s_1 c_1 s_2^2 & 0 \\ s_1 c_1 s_2^2 & c_1^2 s_2^2 & 0 \\ 0 & 0 & c_2^2 \end{pmatrix} + m_0'' \begin{pmatrix} s_1^2 s_2^2 & s_1' c_1 s_2^2 & 0 \\ s_1' c_1 s_2^2 & c_1^2 s_2^2 & 0 \\ 0 & 0 & c_2^2 \end{pmatrix} \]

\[ B : M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_0' \begin{pmatrix} s_1^2 s_2^2 & 0 & s_1 c_2 s_2 \\ 0 & c_1^2 s_2^2 & 0 \\ s_1 c_2 s_2 & 0 & c_2^2 \end{pmatrix} + m_0'' \begin{pmatrix} s_1^2 s_2^2 & 0 & s_1' c_2 s_2 \\ 0 & c_1^2 s_2^2 & 0 \\ s_1' c_2 s_2 & 0 & c_2^2 \end{pmatrix} \]

\[ C : M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m_0' \begin{pmatrix} s_1^2 s_2^2 & 0 & 0 \\ 0 & c_1^2 s_2^2 & c_1 c_2 s_2 \\ 0 & c_1 c_2 s_2 & c_2^2 \end{pmatrix} + m_0'' \begin{pmatrix} s_1^2 s_2^2 & 0 & 0 \\ 0 & c_1^2 s_2^2 & c_1' c_2 s_2 \\ 0 & c_1' c_2 s_2 & c_2^2 \end{pmatrix} \]

which correspond to the three vacuum structures $A$, $B$ and $C$ in eq.(3). Where $s_1 = \sin \theta_1 = \sigma_1/\sigma_{12}$ and $s_2 = \sin \theta_2 = \sigma_{12}/\sigma$ with $\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2}$ and $\sigma = \sqrt{\sigma_{12}^2 + \sigma_3^2}$. Similar definitions are for $s_1'$ and $s_2'$.

Note that the two non-diagonal matrices in the mass matrix $M_e$ are rank one matrices. While it is interesting to observe that when the four angles $\theta_1$, $\theta_2$, $\theta_1'$ and $\theta_2'$ satisfy the
Note that the case B is dual to case C via

\[ \frac{s_1}{c_1} = \frac{s'_1}{c'_1}, \quad \frac{c_2}{s_2} = -\frac{s'_2}{c'_2} \]  

which is equivalent to \( \sigma_1'/\sigma_2' = \sigma_1/\sigma_2, \quad \sigma_1'\sigma_2' = -\sigma_3/\sigma_12, \) the two non-diagonal matrices in the mass matrix \( M_e \) can be simultaneously diagonalized by a unitary matrix \( U_e \) via \( M'_e = U_e^\dagger M_e U_e^* \). Here

\[ M'_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m'_1 & 0 \\ 0 & 0 & m_1 \end{pmatrix} + m''_1 U_e^\dagger U_e^* \]  

and

\[ A: \quad U^\dagger_e = \begin{pmatrix} c_1 & -s_1 & 0 \\ ic_2 s_1 & ic_1 c_2 & -s_2 \\ s_1 s_2 & c_1 s_2 & -ic_2 \end{pmatrix} \]  

\[ B: \quad U^\dagger_e = \begin{pmatrix} ic_1 & -s_1 & 0 \\ c_2 s_1 & -ic_1 c_2 & -s_2 \\ s_1 s_2 & -ic_1 s_2 & c_2 \end{pmatrix} \]  

\[ C: \quad U^\dagger_e = \begin{pmatrix} c_1 & -is_1 & 0 \\ -ic_2 s_1 & c_1 c_2 & -s_2 \\ -is_1 s_2 & c_1 s_2 & c_2 \end{pmatrix} \]

where \( U_e^\dagger U_e^* \) has the following explicit form

\[ A: \quad U^\dagger_e U_e^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s_2^2 - c_2^2 & 2ic_2 s_2 \\ 0 & 2ic_2 s_2 & s_2^2 - c_2^2 \end{pmatrix} \]  

\[ B: \quad U^\dagger_e U_e^* = \begin{pmatrix} s_1^2 - c_1^2 & 2ic_1 s_1 c_2 & 2ic_1 s_1 s_2 \\ 2ic_1 s_1 c_2 & c_2 s_1^2 - c_1^2 + s_2^2 & c_2 s_2 (s_1^2 - c_1^2) - c_2 s_2 \\ 2ic_1 s_1 s_2 & c_2 s_2 (s_1^2 - c_1^2) - c_2 s_2 & s_2^2 (s_1^2 - c_1^2) + c_2^2 \end{pmatrix} \]  

\[ C: \quad U^\dagger_e U_e^* = \begin{pmatrix} c_1^2 - s_1^2 & -2ic_1 s_1 c_2 & -2ic_1 s_1 s_2 \\ -2ic_1 s_1 c_2 & c_2 (s_1^2 - s_2^2) + s_2^2 & c_2 s_2 (s_2^2 - c_1^2) - c_2 s_2 \\ -2ic_1 s_1 s_2 & c_2 s_2 (s_2^2 - c_1^2) - c_2 s_2 & s_2^2 (c_1^2 - s_2^2) + c_2^2 \end{pmatrix} \]

Note that the case B is dual to case C via \( c_1 \leftrightarrow -s_1 \). The hierarchical structure of the charged lepton mass implies that \( m''_\ell << m'_1 << m_1 \), it is then not difficult to see that the matrix \( M'_e \) will be further diagonalized by a unitary matrix \( U'_e \) via \( D_e = U_e^\dagger M'_e U_e^* = U_e^\dagger U'_e M_e U_e U_e^* \) with

\[ D_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \]

and

\[ A: \quad U'_e \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & iO (m''_1/m_1) \\ 0 & iO (m''_1/m_1) & 1 \end{pmatrix} \]
obtain the CKM-type lepton mixing matrix $U_{\nu}$.

When going to the physical mass basis of the neutrinos and charged leptons, we then find that the unitary matrix $U_{\nu}$ can be rewritten as

$$\begin{align*}
B : & \quad U_{\nu} \simeq \\
& = \begin{pmatrix}
1 + O(m''_1/m'_1) & iO(m''_1/m'_1) & iO(m''_1/m_1) \\
iO(m''_1/m'_1) & 1 + O(m''_1/m'_1) & O(m''_1/m_1) \\
iO(m''_1/m_1) & O(m''_1/m_1) & 1 + O(m''_1/m_1)
\end{pmatrix} \quad (12)
\end{align*}$$

$$\begin{align*}
C : & \quad U_{\nu} \simeq \\
& = \begin{pmatrix}
1 + O(m''_1/m'_1) & -iO(m''_1/m'_1) & -iO(m''_1/m_1) \\
iO(m''_1/m'_1) & 1 + O(m''_1/m'_1) & O(m''_1/m_1) \\
iO(m''_1/m_1) & O(m''_1/m_1) & 1 + O(m''_1/m_1)
\end{pmatrix}
\end{align*}$$

where $m_e = O(m''_1)$, $m_\mu = m'_1 + O(m''_1)$ and $m_\tau = m_1 + O(m''_1)$ define the three charged lepton masses. This indicates that the unitary matrix $U_{\nu}$ does not significantly differ from the unit matrix. Applying the same conditions given in eq.(7), the three neutrino mass matrices can be rewritten as

$$\begin{align*}
A : & \quad M_\nu = m_0 \begin{pmatrix}
1 + \Delta_- s_1^2 & \Delta_- s_1 c_1 & 0 \\
\Delta_- s_1 c_1 & 1 + \Delta_- c_1^2 & 0 \\
0 & 0 & 1 + \Delta_+
\end{pmatrix} \\
B : & \quad M_\nu = m_0 \begin{pmatrix}
1 + \Delta_- s_1^2 & 0 & 2\delta_- s_2 c_2 s_1 \\
0 & 1 + \Delta_- c_1^2 & 0 \\
2\delta_- s_2 c_2 s_1 & 0 & 1 + \Delta_+
\end{pmatrix} \quad (13)
\end{align*}$$

$$\begin{align*}
C : & \quad M_\nu = m_0 \begin{pmatrix}
1 + \Delta_- s_1^2 & 0 & 0 \\
0 & 1 + \Delta_- c_1^2 & 2\delta_- s_2 c_2 c_1 \\
0 & 2\delta_- s_2 c_2 c_1 & 1 + \Delta_+
\end{pmatrix}
\end{align*}$$

with

$$\Delta_\pm = \delta_+ \pm \delta_- \cos 2\theta_2, \quad \delta_\pm = (m'_0 \pm m''_0)/2m_0 \quad (14)$$

The three type neutrino mass matrices can be easily diagonalized by the orthogonal matrix $O_\nu$ via $O_\nu^T M_\nu O_\nu$. Explicitly, the matrix $O_\nu$ has the following forms for the cases $A$, $B$ and $C$

$$\begin{align*}
A : & \quad O_\nu = \begin{pmatrix}
c_\nu & s_\nu & 0 \\
-s_\nu & c_\nu & 0 \\
0 & 0 & 1
\end{pmatrix} \\
B : & \quad O_\nu = \begin{pmatrix}
c_\nu & 0 & s_\nu \\
0 & 1 & 0 \\
-s_\nu & 0 & c_\nu
\end{pmatrix} \\
C : & \quad O_\nu = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_\nu & s_\nu \\
0 & -s_\nu & c_\nu
\end{pmatrix}
\end{align*}$$

with

$$\begin{align*}
A : & \quad \tan 2\theta_\nu = \tan 2\theta_1 \\
B : & \quad \tan 2\theta_\nu = 2\delta_- s_1 \sin 2\theta_2/(\Delta_+ - \Delta_- s_1^2) \\
C : & \quad \tan 2\theta_\nu = 2\delta_- c_1 \sin 2\theta_2/(\Delta_+ - \Delta_- c_1^2)
\end{align*}$$

When going to the physical mass basis of the neutrinos and charged leptons, we then obtain the CKM-type lepton mixing matrix $U_{\nu\nu}$ appearing in the interactions of the
charged weak gauge bosons and leptons, i.e., $\mathcal{L}_W = \bar{e}_L \gamma^\mu U_{LEP} \nu_L W^-_\mu + h.c.$ Explicitly, we have for the case A

$$\begin{align*}
A: \quad U_{LEP} &= U^e_\nu U^\dagger_e O_\nu = U^e_\nu \begin{pmatrix} 1 & 0 \i & 0 \\ 0 & s_2 & -i c_2 \end{pmatrix} \\
&\simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & i c_2 & -s_2 \\ 0 & s_2 & -i c_2 \end{pmatrix}
\end{align*}$$

(17)

Where we have neglected the relative small terms of order $O(m_1^u/m_1)$. The three neutrino masses are found to be

$$\begin{align*}
m_{\nu_e} &= m_0 \\
m_{\nu_\mu} &= m_0[1 + \Delta_-] \\
m_{\nu_\tau} &= m_0[1 + \Delta_+] 
\end{align*}$$

(18)

For the case B, the lepton mixing matrix and three neutrino masses are given by

$$\begin{align*}
B: \quad U_{LEP} &= U^e_\nu U^\dagger_e O_\nu = U^e_\nu \begin{pmatrix} i c_1 c_\nu & -s_1 & i c_1 s_\nu \\ c_2 s_1 c_\nu + s_2 s_\nu & -i c_2 c_1 c_\nu & c_2 s_1 s_\nu - s_2 c_\nu \\ s_1 s_2 c_\nu - c_2 s_\nu & -i c_1 s_2 & s_1 s_2 s_\nu + c_2 c_\nu \end{pmatrix} \\
&= U^e_\nu \begin{pmatrix} \sqrt{1 + \tan^2 \theta_\nu} \end{pmatrix}
\end{align*}$$

(19)

and

$$\begin{align*}
m_{\nu_e} &= m_0[1 + \frac{1}{2}(\Delta_+ + \Delta_- s_1^2) - \frac{1}{2}(\Delta_+ - \Delta_- s_1^2)\sqrt{1 + \tan^2 \theta_\nu}] \\
m_{\nu_\mu} &= m_0[1 + \Delta_- c_1^2] \\
m_{\nu_\tau} &= m_0[1 + \frac{1}{2}(\Delta_+ + \Delta_- s_1^2) + \frac{1}{2}(\Delta_+ - \Delta_- s_1^2)\sqrt{1 + \tan^2 \theta_\nu}] 
\end{align*}$$

(20)

For the case C, the lepton mixing matrix and three neutrino masses are obtained as follows

$$\begin{align*}
C: \quad U_{LEP} &= U^e_\nu U^\dagger_e O_\nu = U^e_\nu \begin{pmatrix} c_1 & -i s_1 c_\nu & -i s_1 s_\nu \\ -i c_2 s_1 & c_1 c_2 c_\nu + s_2 s_\nu & c_1 c_2 s_\nu - s_2 c_\nu \\ -i s_1 s_2 & c_1 s_2 c_\nu - c_2 s_\nu & c_1 s_2 s_\nu + c_2 c_\nu \end{pmatrix} \\
&= U^e_\nu \begin{pmatrix} \sqrt{1 + \tan^2 \theta_\nu} \end{pmatrix}
\end{align*}$$

(21)

and

$$\begin{align*}
m_{\nu_e} &= m_0[1 + \Delta_- s_1^2] \\
m_{\nu_\mu} &= m_0[1 + \frac{1}{2}(\Delta_+ + \Delta_- c_1^2) - \frac{1}{2}(\Delta_+ - \Delta_- c_1^2)\sqrt{1 + \tan^2 \theta_\nu}] \\
m_{\nu_\tau} &= m_0[1 + \frac{1}{2}(\Delta_+ + \Delta_- c_1^2) + \frac{1}{2}(\Delta_+ - \Delta_- c_1^2)\sqrt{1 + \tan^2 \theta_\nu}] 
\end{align*}$$

(22)

Before going to a detailed analysis, we would like to provide a brief summary of recent experiments. When the $\nu_\mu$ anomaly reported recently by Super-Kamiokande (SK) experiment[1] is interpreted as oscillation of $\nu_\mu \rightarrow \nu_\tau$ with nearly maximal mixing angle
in a two flavor oscillation. The allowed range for mass-squared difference and mixing angle \[1, 12, 13\] is
\[
\Delta m^2_{\text{atm}} = |\Delta m^2_{\nu\tau}| \simeq (0.5 - 6) \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{\text{atm}} > 0.82 \ (90\% \text{C.L.}) \quad (23)
\]
For three-flavor mixing case, the best fit to the sub-GeV, multi-GeV and upward-going muon data from SK is obtained at \[13\]
\[
|\Delta m^2_{\nu\tau}| \simeq 2.5 \times 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{\nu\tau} = 0.63, \quad \sin^2 2\theta_{e\tau} = 0.14 \quad (24)
\]
The deficit in the solar neutrino experiments suggests the following best-fit solution from a global analysis \[12\]
\begin{enumerate}
\item MSW\[14\] small angle solution: \(\Delta m^2_{\text{sun}} \simeq 5 \times 10^{-6} \text{eV}^2, \quad \sin^2 2\theta_{\text{sun}} \simeq 5.5 \times 10^{-3}\);
\item “Just-so” vacuum\[15\] solution: \(\Delta m^2_{\text{sun}} \simeq 6.5 \times 10^{-11} \text{eV}^2, \quad \sin^2 2\theta_{\text{sun}} \simeq 0.75\)
\end{enumerate}
although it allows three possible MSW solutions\[1\] from rates only and a large range of mass-squared difference and mixing angle from spectrum only. For comparison, we list the allowed regions (99% C.L.) from rates only (see Figs.(2) and (5) in ref. \[12\]),
\begin{enumerate}
\item MSW small angle solution: \(\Delta m^2_{\text{sun}} \simeq (3 - 10) \times 10^{-6} \text{eV}^2, \quad \sin^2 2\theta_{\text{sun}} \simeq (1.5 - 10) \times 10^{-3}\);
\item MSW large angle solution: \(\Delta m^2_{\text{sun}} \simeq (6 - 30) \times 10^{-5} \text{eV}^2, \quad \sin^2 2\theta_{\text{sun}} \simeq 0.4 \sim 1\);
\item MSW Low solution: \(\Delta m^2_{\text{sun}} \simeq (3.5 - 20) \times 10^{-8} \text{eV}^2, \quad \sin^2 2\theta_{\text{sun}} \simeq 0.8 \sim 1\);
\item Vacuum Oscillation solution: \(\Delta m^2_{\text{sun}} \simeq (5 - 100) \times 10^{-11} \text{eV}^2, \quad \sin^2 2\theta_{\text{sun}} > 0.55\)
\end{enumerate}

We now turn to discuss possible interesting scenarios in the present model.

### 3 Small-Large Mixing Scenario

It is easily seen that a small-large mixing scenario is most likely to occur in the case A. For simplicity, we may call this scenario as scenario (i). If the solar neutrino data is explained via \(\nu_e \leftrightarrow \nu_\mu\) oscillation and the atmospheric neutrino data via \(\nu_\nu \leftrightarrow \nu_\tau\) oscillation, the mass parameters are fixed to be
\[
\delta_- = \frac{\Delta m^2_{\text{atm}}}{4m^2_0(c_2^2 - s_2^2)}
\]
\[
\delta_+ = \frac{\Delta m^2_{\text{atm}}}{4m^2_0} \left(1 + \frac{\Delta m^2_{\text{sun}}}{2\Delta m^2_{\text{atm}}}ight)
\]

Nevertheless, within the present simple scheme the mixing elements \(U_{ei}\) and \(U_{ie}\) vanish. To obtain a realistic scheme, it is necessary to extend the present simple scheme and introduce new contributions or consider possible higher order corrections.

\[1\]It was shown in ref.\[16\] that if the low energy cross section for \(He + p \rightarrow He + e^+ + \nu_e\) reaction is about 20 times larger than the best (but uncertain) theoretical estimates, it allows three MSW solutions at 95% C.L.
It is seen that as long as the mass scale $m_0 > \sqrt{\Delta m_{\text{atm}}^2 / 2} \simeq (0.02 \sim 0.04) \text{ eV}$, the three neutrino masses are almost degenerate. $m_0$ is mainly constrained from the neutrinoless double beta decay. In this scenario (i), it must satisfy \[ \[17\]

$$m_0 < 0.46\text{ eV} \tag{25}$$

The relation between the total neutrino mass $m(\nu)$ and the fraction $\Omega_\nu$ of critical density that neutrinos contribute is \[18\]

$$\frac{\Omega_\nu}{\Omega_m} = 0.03 \frac{m(\nu)}{1\text{ eV}} \left(\frac{0.6}{h}\right)^2 \frac{1}{\Omega_m} \simeq 0.09 \frac{m_0}{1\text{ eV}} \left(\frac{0.6}{h}\right)^2 \frac{1}{\Omega_m} \tag{26}$$

with $h = 0.5 - 0.8$ the expansion rate of the universe (Hubble constant $H_0$) in units of 100 km/s/Mpc. $\Omega_m$ is the fraction of critical density that matter contributes. In the second equality, we have used the relation $m(\nu) \simeq 3m_0$ for nearly degenerate neutrino mass. Thus the upper bound implies that in the scenario (i) the fraction $\Omega_\nu/\Omega_m < 4\%$ for $\Omega_m = 1$ and $\Omega_\nu/\Omega_m < 10\%$ for $\Omega_m = 0.4$.

4 Nearly Bi-maximal Mixing Scenario

The scenario (i) discussed in the previous section can be regarded as a two-flavor oscillation of a small-large mixing pattern since the mixing matrix element $(U_{LEP})_{13}$ is much smaller than other mixing matrix elements. Another interesting two-flavor oscillation scenario occurs in a nearly bi-maximal mixing pattern. We will show that such a two-flavor oscillation is easily realized in the cases B and C when $\tan^2 2\theta_\nu << 1$.

By taking $\tan^2 2\theta_\nu << 1$ and noting that $\sin^2 2\theta_2 > 0.8$ suggested from the recent atmospheric neutrino data\[11\], we have, to a good approximation, the simple relations: $\Delta_+ \simeq \Delta_- \simeq \delta_+$ and

$$B : \quad \tan 2\theta_\nu \simeq 2s_1\delta_-/\delta_+c_1^2 << 1$$

$$C : \quad \tan 2\theta_\nu \simeq 2c_1\delta_-/\delta_+s_1^2 << 1 \tag{27}$$

Thus masses of the three neutrinos are simply given by

$$B : \quad m_{\nu_e} \simeq m_0[1 + \delta_+s_1^2 - \frac{\delta_+^2s_1^2}{\delta_+c_1^2}]$$

$$m_{\nu_\mu} \simeq m_0[1 + \delta_+c_1^2]$$

$$m_{\nu_\tau} \simeq m_0[1 + \delta_+] \tag{28}$$

$$C : \quad m_{\nu_e} \simeq m_0[1 + \delta_+c_1^2 - \frac{\delta_+^2c_1^2}{\delta_+s_1^2}]$$

$$m_{\nu_\mu} \simeq m_0[1 + \delta_+s_1^2 - \frac{\delta_+^2s_1^2}{\delta_+c_1^2}]$$

$$m_{\nu_\tau} \simeq m_0[1 + \delta_+ + \frac{\delta_+^2c_1^2}{\delta_+s_1^2}]$$
From these masses, one easily reads off the mass-squared differences

\[ B : \quad \Delta m_{\mu e}^2 = m_{\mu}^2 - m_{\nu_e}^2 \simeq m_0^2 \delta_+ [c_1^2 - s_1^2 + \left( \frac{\delta - s_1}{\delta + c_1} \right)^2 ] [2 + \delta_+] \]

\[ \Delta m_{\tau \mu}^2 = m_{\tau}^2 - m_{\nu_\mu}^2 \simeq m_0^2 \delta_+ [s_1^2 + \left( \frac{\delta - s_1}{\delta + c_1} \right)^2 ] [2 + \delta_+ (1 + c_1^2)] \]

\[ C : \quad \Delta m_{\mu e}^2 = m_{\mu}^2 - m_{\nu_e}^2 \simeq m_0^2 \delta_+ [c_1^2 - s_1^2 - \left( \frac{\delta - c_1}{\delta + s_1} \right)^2 ] [2 + \delta_+] \]

\[ \Delta m_{\tau \mu}^2 = m_{\tau}^2 - m_{\nu_\mu}^2 \simeq m_0^2 \delta_+ [s_1^2 + \left( \frac{\sqrt{2} \delta - c_1}{\delta + s_1} \right)^2 ] [2 + \delta_+ (1 + c_1^2)] \] (29)

To explain the atmospheric neutrino anomaly and the observed deficit of the solar neutrino fluxes in comparison with the solar neutrino fluxes computed from the solar standard model, the required neutrino mass-squared difference \( \Delta m_{\tau \mu}^2 \) must satisfy

\[ 5 \times 10^{-4} \text{eV}^2 < |\Delta m_{\tau \mu}^2| < 6 \times 10^{-3} \text{eV}^2 \] (30)

and the mass-squared difference \( \Delta m_{\mu e}^2 \) shall fall into the range

\[ 6 \times 10^{-11} \text{eV}^2 < \Delta m_{\mu e}^2 < 2 \times 10^{-5} \text{eV}^2 \] (31)

Here the larger and smaller values of \( \Delta m_{\mu e}^2 \) provide MSW and vacuum oscillation explanations for the solar neutrino puzzle respectively. It is seen that the ratio between the two mass-squared differences satisfy \( \Delta m_{\mu e}^2 / \Delta m_{\tau \mu}^2 < 0.04 \). With this condition and \(|\delta_-| << |\delta_+|\), we then obtain from eq.(29) the following constraint on the mixing angle \( \theta_1 \)

\[ |c_1^2/s_1^2 - 1| < 0.04 \] (32)

Note that this constraint is independent of the mass scale \( m_0 \). With these constraints, we arrive at the following relations

\[ \frac{m_1''}{m_1'} \sim \sqrt{\frac{m_e}{m_\mu}} = 0.07, \quad \frac{m_1''}{m_1} \sim \sqrt{\frac{m_e m_\mu}{m_\tau}} = 0.004 \] (33)

Because of the smallness of the mixing angles in \( U_e' \) and \( \theta_\nu \), we may conclude that the neutrino mixing between \( \nu_e \) and \( \nu_\mu \) is almost maximal

\[ \sin^2 2\theta_1 > 0.998 \] (34)

which may leave vacuum oscillation as the only viable explanation of the solar neutrino data as it can be seen from the analyses in [13]. This requires that \( \sigma_1 \simeq \sigma_2 \) and \( m'_0 \simeq m''_0 \) which may need a fine-tuning if they are not ensured by symmetries.

With the above analyses, we may come to the conclusion that with two-flavor oscillation and the hierarchical mass-squared differences \( \Delta m_{\mu e}^2 << \Delta m_{\tau \mu}^2 \), both cases B and C lead
to a nearly ‘bi-maximal’ neutrino mixing pattern for the explanations of the solar and atmospheric neutrino flux anomalies.

The smallness of the mass-squared difference $\Delta m^2_{\mu e}$ implies that $\sin \theta_\nu < 0.001$ for $m_0 \sim 2$ eV. To a good approximation, we may neglect the small mixing angle $\theta_\nu$ and the small mixing of order $m''_1/m_1$ in $U'$. Thus the CKM-type lepton mixing matrices for the cases B and C are simply given by

$$B : \quad U_{LEP} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} i & \frac{1}{\sqrt{2}} & -i \frac{m_{\mu} s_2}{m_{\mu}} \\ \frac{1}{\sqrt{2}} c_2 & -\frac{1}{\sqrt{2}} c_2 i & -s_2 \\ \frac{1}{\sqrt{2}} s_2 i & c_2 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$C : \quad U_{LEP} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} i & -\frac{1}{\sqrt{2}} i \frac{m_{\mu} s_2}{m_{\mu}} \\ \frac{1}{\sqrt{2}} c_2 & \frac{1}{\sqrt{2}} c_2 & -s_2 \\ -\frac{1}{\sqrt{2}} s_2 i & \frac{1}{\sqrt{2}} s_2 & c_2 \end{pmatrix}$$

which arrive at the pattern suggested by Vissani[20] once neglecting the small mixing angle at the order of magnitude $\sqrt{m_e/m_\mu s_2}$. When going back to the weak gauge and charged-lepton mass basis, we find that the neutrino mass matrices for the cases B and C have the following simple forms

$$B : \quad M_\nu \simeq m_0 \begin{pmatrix} \frac{m s_2}{m_\mu} & i c_2 & i s_2 \\ i c_2 & \frac{s_2}{2} & -2 s_2 \\ i s_2 & -c_2 s_2 & c_2^2 \end{pmatrix}$$

$$C : \quad M_\nu \simeq m_0 \begin{pmatrix} -\frac{m s_2}{m_\mu} & -i c_2 & -i s_2 \\ -i c_2 & \frac{s_2}{2} & -2 s_2 \\ -i s_2 & -c_2 s_2 & c_2^2 \end{pmatrix}$$

Suggested by the recent atmospheric neutrino data, we are motivated to consider two particular interesting cases: Firstly, setting the vacuum expectation values to be $\sigma^2 = \sigma_1^2 + \sigma_2^2$ and $\sigma_1 = \sigma_2$, namely, $s_1 = s_2 = 1/\sqrt{2}$ ($\sin^2 2\theta_1 = \sin^2 2\theta_2 = 1$), we then obtain a realistic bi-maximal mixing pattern with a maximal CP-violating phase. Explicitly, the neutrino mass and mixing matrices read

$$B : \quad M_\nu \simeq m_0 \begin{pmatrix} -0.002 & \frac{1}{\sqrt{2}} i & \frac{1}{\sqrt{2}} i \\ \frac{1}{\sqrt{2}} i & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} i & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$C : \quad M_\nu \simeq m_0 \begin{pmatrix} -0.002 & -\frac{1}{\sqrt{2}} i & \frac{1}{\sqrt{2}} i \\ -\frac{1}{\sqrt{2}} i & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} i & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

and

$$B : \quad U_{LEP} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} i & -\frac{1}{\sqrt{2}} i & -0.05 i \\ \frac{1}{\sqrt{2}} i & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} i & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$
\[
C : \quad U_{\text{LEP}} \simeq \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0.05i \\
-\frac{i}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\
-\frac{i}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\]

when neglecting the small mixing angle at the order of magnitude \(\sqrt{m_e/m_\mu}\), we then yield
the pattern suggested by Georgi and Glashow\[21\].

Secondly, setting the three vacuum expectation values \(\sigma_i (i=1,2,3)\) to be democratic, i.e.,
\(\sigma_1 = \sigma_2 = \sigma_3\), hence \(s_1 = 1/\sqrt{2}\) and \(s_2 = \sqrt{2}/3\) (sin\(^2\theta_1 = 1\) and sin\(^2\theta_2 = 8/9\), we then
arrive at a realistic democratic mixing pattern with a maximal CP-violating phase. The
explicit neutrino mass and mixing matrices are given by

\[
B : \quad M_\nu \simeq m_0 \begin{pmatrix}
-0.003 & \frac{1}{\sqrt{3}} i & \frac{2}{\sqrt{6}} i \\
\frac{1}{\sqrt{3}} i & \frac{2}{3} & -\frac{\sqrt{2}}{3} \\
\frac{1}{\sqrt{6}} i & -\frac{\sqrt{2}}{3} & \frac{1}{3}
\end{pmatrix}
\]

\[
C : \quad M_\nu \simeq m_0 \begin{pmatrix}
-0.003 & -\frac{1}{\sqrt{3}} i & -\frac{2}{\sqrt{6}} i \\
\frac{1}{\sqrt{3}} i & \frac{2}{3} & -\frac{\sqrt{2}}{3} \\
\frac{2}{\sqrt{6}} i & -\frac{\sqrt{2}}{3} & \frac{1}{3}
\end{pmatrix}
\]

(39)

and

\[
B : \quad U_{\text{LEP}} \simeq \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & -0.057i \\
\frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{6}} & \frac{2}{\sqrt{3}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix}
\]

\[
C : \quad U_{\text{LEP}} \simeq \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0.057i \\
\frac{1}{\sqrt{6}} & -\frac{i}{\sqrt{6}} & -\frac{2}{\sqrt{3}} \\
\frac{i}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix}
\]

(40)

when further neglecting the small mixing angle at the order of magnitude \(\sqrt{m_e/m_\mu}\), we
obtain a similar form provided by Mohapatra\[22\].

We may call the above scenario as ‘scenario (ii)’. From the hierarchical feature in \(\Delta m^2\),
i.e., \(\Delta m^2_{\mu e} \ll \Delta m^2_{\tau \mu} \approx \Delta m^2_{\tau e}\), formulae for the oscillation probabilities in the scenario
(ii) can be greatly simplified to be

\[
P_{\nu_e \rightarrow \nu_e}\bigg|_{\text{solar}} \simeq 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\left(\frac{\Delta m^2_{\mu e} L}{4E}\right)
\]

\[
P_{\nu_\mu \rightarrow \nu_\mu}\bigg|_{\text{atmospheric}} \simeq 1 - 4(1 - |U_{\mu 3}|^2)|U_{\mu 3}|^2 \sin^2\left(\frac{\Delta m^2_{\tau \mu} L}{4E}\right)
\]

(41)

\[
P_{\nu_\beta \rightarrow \nu_\alpha} \simeq 4|U_{\beta 3}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m^2_{\tau \mu} L}{4E}\right)
\]

and

\[
\frac{P_{\nu_\mu \rightarrow \nu_\mu}}{P_{\nu_\mu \rightarrow \nu_\tau}}\bigg|_{\text{atmospheric}} \simeq \frac{|U_{e3}|^2}{|U_{\tau 3}|^2} << 1
\]

(42)
This may present the simplest scheme for reconciling both solar and atmospheric neutrino fluxes via oscillations of three neutrinos.

Furthermore, the resulting ‘bi-maximal’ neutrino mixing pattern allows the three neutrino masses to be nearly degenerate and large enough for hot dark matter without conflict with the current data on neutrinoless double beta decay. This is because the failure of detecting neutrinoless double beta decay provide bounds on an effective electron neutrino mass $< m_{\nu_e} > = \sum_i m_{\nu_i} (U_{LEP})_{ei}^2 < 0.46 \text{ eV}$. In the present scenario, we have $| (M_\nu)_{ee} | =< m_{\nu_e} > \simeq (0.002 - 0.03) m_0$. It shows that the mass scale can be as large as $m_0 \sim 15 \text{ eV}$. From the relation between the total neutrino mass $m(\nu)$ and the fraction $\Omega_\nu$ of critical density that neutrinos contribute, we find that for $m_0 \sim 2 \text{ eV}$ and $h = 0.6$ the fraction $\Omega_\nu \simeq 18\%$ for $\Omega_m = 1$.

5 Three-Flavor Oscillation Scenario

As a general case, it is interesting to consider three-flavor oscillation which may occur in the cases B and C when the mixing angle $\theta_\nu$ is not small. As shown in ref.[13], nonzero values of $U_{e3}^2$ may explain part of the electron excess in the Sub-GeV and Multi-GeV samples and can also contribute to distort the zenith distributions. Direct constraints on $U_{e3}^2$ arise from the CHOOZ experiment. The CHOOZ data can provide a strong constraint on the higher range of $\Delta m_{\text{atm}}^2$, but its role decreases rapidly as $\Delta m_{\text{atm}}^2$ goes down to $10^{-3} \text{ eV}^2$. It was pointed out in ref.[13] that within the framework of three-flavor oscillation, for $\Delta m_{\text{atm}}^2$ close to $10^{-3}$ (or slightly below) $10^{-3} \text{ eV}^2$, the allowed region at 99\% C.L. by the SK and CHOOZ as well as solar neutrino data could be

$$U_{e2}^2 = \frac{8}{25}, \quad U_{e3}^2 = \frac{1}{5}, \quad U_{\mu3}^2 = \frac{4}{5}$$

If using this set of mixing as input, we find that the three mixing angles $\theta_1, \theta_\nu$ and $\theta_2$ for the cases B and C in the present model are fixed to be

$$B : \quad \tan^2 \theta_1 = \frac{8}{17}, \quad \tan 2 \theta_\nu = \frac{-4\sqrt{15}}{7}, \quad \tan 2 \theta_2 = \frac{13}{2\sqrt{30}}$$

$$C : \quad \tan^2 \theta_1 = \frac{13}{12}, \quad \tan 2 \theta_\nu = \frac{-4\sqrt{10}}{3}, \quad \tan 2 \theta_2 = \frac{91}{2\sqrt{5070}}$$

Thus the corresponding mixing matrices for the cases B and C are

$$B : \quad U_{\text{LEP}} \simeq U_e^\dagger \left( \begin{array}{ccc} -0.693i & -0.566 & 0.447i \\ -0.202 & -0.748i & 0.632 \\ -0.692 & -0.348i & -0.632 \end{array} \right)$$

$$C : \quad U_{\text{LEP}} \simeq U_e^\dagger \left( \begin{array}{ccc} 0.693 & 0.566i & -0.447i \\ -0.692i & -0.348 & 0.632 \\ -0.202i & -0.748 & -0.632 \end{array} \right)$$

(45)
where one can neglect the small mixing in $U^\dagger_e$ since the largest mixing in $U^\dagger_e$ is $s_e \approx m_e'^1 \sin 2\theta_1 c_2 / m_\mu \approx 0.049$ as $m_e'^1 \approx 5.56 \text{eV}$ for the case C and $s_e \approx -0.011$ as $m_e'^1 \approx -1.36 \text{eV}$ for the case B. This mixing pattern may be simply called as scenario (iii).

It is interesting to note that once the three angles are given, the ratio of the two mass-squared differences $\Delta m^2_{\text{atm}}$ and $\Delta m^2_{\text{sun}}$ is completely determined in the present model to be

$$B : \quad \frac{|\Delta m^2_{\text{sun}}|}{|\Delta m^2_{\text{atm}}|} \approx 0.036; \quad C : \quad \frac{|\Delta m^2_{\text{sun}}|}{|\Delta m^2_{\text{atm}}|} \approx 0.09$$

(46)

For $\Delta m^2_{\text{atm}} \approx 1.0 \times 10^{-3} \text{eV}^2$, the resulting mass-squared difference $\Delta m^2_{\text{sun}} \approx 0.9 \times 10^{-4} \text{eV}^2$ for the case C and is found to be consistent with the experimental data for the allowed region at 99% C.L., while the numerical result $\Delta m^2_{\text{sun}} \approx 0.36 \times 10^{-4} \text{eV}^2$ for the case B is quite near the low bound allowed from the solar neutrino data[23] at 99% C.L..

As $c_1^2 - s_1^2 = 1/12$ for the case C, the degenerate neutrino mass can be large enough to play a significant cosmological role. For the case B, the mass scale $m_0$ is bounded to be $m_0 < 0.46 V/(c_1^2 - s_1^2) = 1.28 \text{ eV}$ from the neutrinoless double beta decay. Thus the fraction $\Omega_\nu/\Omega_m$ is bounded to be $\Omega_\nu/\Omega_m < (12 - 29)\%$ for $\Omega_m = (1.0 - 0.4)$ and $h = 0.6$.

6 Conclusions and Remarks

Starting from the effective lagrangian in eq.(1) with $\text{SO}(3)_F \times \text{SU}(2)_L \times \text{U}(1)_Y$ symmetry, we have investigated three possible interesting scenarios: small-large mixing scenario (the scenario (i)), nearly bi-maximal mixing scenario (the scenario (ii)) and three-flavor oscillation scenario (the scenario (iii)) to reconcile the solar and atmospheric neutrino anomalies when LSND results[24] are not considered. This is because including the LSND results, it likely needs to introduce a sterile neutrino[23]. The three scenarios are found in connection with the three possible assignments (i.e., cases A, B and C ) of the maximal CP-violating phase to one of three real amplitude fields of the $\text{SO}(3)$ triplet scalar after spontaneous symmetry breaking of the $\text{SO}(3)$ gauge symmetry. The scenario (i) may only arise from the case A, but within the present simple model, one cannot yet obtain the realistic small mixing angle needed for MSW solution of solar neutrino. In contrast, it is of interest to see that both cases B and C can easily lead to the scenario (ii). The scenario (iii) may also be derived from the cases B and C to accomodate the solar and atmospheric neutrino data for the allowed region at 99% C.L., while the parameter space for the case B is limited to be quite small even if for the allowed region at 99% C.L. and this scenario seems rather unlikely in the case B. We therefore conclude that within the present simple model the scenario (ii) should be more reliable to be constructed to reconcile both solar and atmospheric neutrino data. It would be interesting to investigate the vacuum structures of spontaneous $\text{SO}(3)$ symmetry breaking to understand the real case. For that, it may be useful to work in a supersymmetric $\text{SO}(3)_F \times \text{SU}(2)_L \times \text{U}(1)_Y$ gauge theory and regard the complex triplet scalar fields as superfields[23].
Acknowledgments: The author would like to thank J. Bahcall for bringing his attention on the paper [16] and E. Lisi and S. Pakvasa for kind remarks. He would also like to thank C.S. Lam for fruitful communication. This work was supported in part by the NSF of China under the grant No. 19625514.

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