The wheels for final processing of parts

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Abstract. Microrelief characteristics of elastic abrasive wheel and parameters of its interaction with processed surface are studied (depth of penetration and quantity of penetrated grains). The received equitations of mathematical expectation of average depth of penetration of cutting logs and number of grains of elastic wheel being in contact allow to generate mathematical models of material removal and formation of parameters of roughness of the processed surface.

1. Introduction
Elastic circles for final processing of parts are used to assign the required roughness to the processed surface, its preparation for paint coatings, removal of burrs, rounding of sharp edges, preliminary processing before polishing and brightening etc. At this different metals and their alloys, plastic, wood, grass, ceramics and stones can be processed.

The formation of the quality of the surface layer of parts after milling, as well as processing with various types of abrasive tools, including those on a flexible bond, has been the subject of many works, for example [1–6]. It should be noted that processing by the elastic instruments has a number of specific features if compare with processing by "rigid" wheels. Such instruments are not fixed for the defined depth of cutting and necessary conditions for operation of separate grains are created due to their preliminary loading. At this the basis (bottom) of the instrument pressed to the processed surface is deformed.

Fixation of the abrasive in rigid coupling changes the character of its interaction with the processed material radically, namely:

- the stroke of the grain against material is amortized, thereof its resistance is increasing;
- tension of heat flow is decreased right up to the total elimination of burns;
- exclusion of microcracking of surface layer of the frigid materials;
- conditions are created for increasing of quantity of simultaneously operating grains;
- speed of blunting of the operation surface of the instrument is decreased;
- the process of self-cleaning of the instrument is improved;
- time of interaction of the abrasive grain with processed surface is increased.

The quality of the processed surface is improved as a result of the abovementioned factors.

The peculiarity of operation of the grain on the elastic foundation is that the grain has a possibility "to be squeezed" during the operation. When rigid fixation the trajectory of the grain movement is defined only through kinematics. Each abrasive grain has a possibility to shift both when deformation of all weight of the coupling and when deformation of its small volume that are attached directly to the grain.
The scheme of classification of elastic instruments used in different fields of science is given on Figure 1.

**Figure 1.** The scheme of classification of elastic wheels.
In the present report characteristics of microrelief of elastic abrasive wheel and parameters of its interaction with processed surface are studied (depth of penetration and quantity of penetrated grains).

2. Microrelief characteristics of elastic abrasive wheel and parameters of its interaction with processed surface

As a method of the receipt of the raw information, a strip chart recording is taken that differs from other methods by simplicity and large amount of received information. At this microprofile of the elastic abrasive wheel can be taken as a random stationary process with ergodic properties. The strip chart recording in that case will be realization of this random process.

It is known [7] that for full description of the normal random process it is necessary to know only its mathematical expectation $E(h(x))$ and correlation function $K(\tau) = \sigma^2 \cdot \rho(\tau)$, where $\sigma$ - root-mean-square deviation of the random process; $\rho(\tau)$ – normalized correlation function; $\tau$ - distance between points of the profile.

For surface of microrelief the mathematical expectation is equal to zero. Thereof the task of description comes to study of approximation of correlation function. Having used feature of special points [7]:

\[
\begin{align*}
\text{number of zeroes (crossing with mean line): } & E\left(n(0)\right) = \frac{1}{\pi} \cdot \left(-\rho^2(0)\right)^{1/2}, \\
\text{number of maxima: } & E\left(m\right) = \frac{1}{2\pi} \cdot \left(-\rho^4(0) \div \rho^2(0)\right)^{1/2}, \\
\text{number of bends: } & E\left(s\right) = \frac{1}{\pi} \cdot \left(-\rho^6(0) \div \rho^4(0)\right)^{1/2},
\end{align*}
\]

and having set as per [8, 9] approximation of normalized correlation function:

\[
\rho(\tau) = e^{-\alpha \tau^2} = e^{-\left[0.5\pi^2 \cdot E^2(n(0)) \cdot \tau^2\right]},
\]

we received the possibility to detect the required parameters of the micro relief (center-line-average surface finish, maximum height of the profile, relative bearing length, corner radius, pressure angle, quantity of legs, simultaneously staying in the contact and others). Here $\rho^2(0)$, $\rho^4(0)$, $\rho^6(0)$ - correspondingly second, fourth and sixth derivatives of normalized correlation function $\rho(\tau)$ at $\tau = 0$.

Processing of profilograms is simplified at this and results in detection of root-mean-square deviation $\sigma$ and values of special points $m$, $n(0)$ and $s$.

Study of profilograms of instrument from abrasive cloths 14A80H358 GOST13344-79, 91A6HM39 GOST 5009-75, 14A20HK500 GOST 5009-82, 14A1191V1C GOST 13344-79, 14A P1005 СФЖ У1С8 GOST 13344-79, 14А P1023 СФЖ Х1С25Н GOST 13344-79, 14А P1919СФЖ6Н GOST 13344-79, 14А P1919СФЖ6Н GOST 13344-79, 14А A6HC204 GOST 5003-82, 14А P76UT25СФЖ GOST13344-79, 14А P6HM 831 GOST 6456-75 have demonstrated that their values of microrelief are within the following limits: $\sigma = (0.00312 - 0.04129)$ mm; $m = (0.8434 - 5.9036)$ mm$^{-1}$; $n(0) = (1.2 - 6.6265)$ mm$^{-1}$; $\lambda = n(0)/m = (1.122 - 1.958)$.

Study of the process of interaction of abrasive elastic wheel with processed surface is easy to start with solving the task of penetration of absolutely rigid microrelief into the material with ideally smooth surface.

Let the rough profile in the circle section have a mean line in the form of circle with radius $R_x$ and processed surface is ideally smooth. In connection with deformation of the elastic circle the surface
that is in contact with part will take its form (line $m_2m_2$). To simplify the solution, do the centering of the random process relative the mean line that conceive in the form of the straight line $m_1m_1$ (Figure 2).

![Figure 2. Scheme of interaction of elastic wheel with processed part.](image)

3. **Mathematical expectation of the average depth of the penetration of the cutting logs**

If consider convergence of the rough profile with ideally smooth surface at straight lines $m_1m_1$ and $m_2m_2$, then the mathematical expectation of the length of penetration of logs, plastically deformed material, can be written as follows:

$$E(y_e) = \sigma \{E(\gamma_o) - \gamma\},$$  

(1)

where $E(\gamma_o)$ – mathematical expectation of relative heights of maxima,

$$E(\gamma_0) = \frac{\gamma_1}{\gamma} \cdot f_2(\gamma) \cdot d\gamma.$$  

(2)

Here $f_2(\gamma)$ – density of possibility of spreading of relative altitudes of maxima within the convergence:

$$f_2(\gamma) = C \cdot f_1(\gamma),$$  

(3)

where $C$ – normalizing factor defined out of the condition:

$$C \cdot \frac{\gamma_1}{\gamma} \cdot f_1(\gamma) \cdot d\gamma = 1;$$  

(4)

$f_1(\gamma)$ – density of possibility of spreading of the heights for all the limit $-\infty$ to $+\infty$.

In accordance [8, 9]:

$$f_1(\gamma) = \frac{1}{\sqrt{2\pi}} \left[ \nu \cdot \exp\left( -\frac{\gamma^2}{2\nu^2} \right) + \sqrt{2\pi} \cdot \lambda \cdot \gamma \cdot \exp\left( -\frac{\gamma^2}{2} \right) \cdot \Phi\left( \frac{\lambda}{2\nu} \cdot \gamma \right) \right],$$  

(5)

where $\lambda = n(o)/m$, $\nu^2 = 1-0,25\lambda$ – parameters of microrelief;

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{z} e^{-t^2/2} \cdot dt$$ - Laplace’s function.

Putting equations (3) and (4) to formula (2), get the following:

$$E(\gamma_0) = \frac{\gamma_1}{\gamma} \int_{\gamma}^{\gamma} f_1(\gamma) \cdot d\gamma$$  

$$\int_{\gamma}^{\gamma} f_1(\gamma) \cdot d\gamma$$  

(6)

Having put (5) to (6), get the following:
The analysis of the equitation (7) demonstrated that the mathematical expectation of penetration depth if exclude elastic material deformation depends only on parameter \( \lambda \) circle microrelief and relative convergence after contacting \( \gamma \).

It turned out to be impossible to do the equation (7) through elementary functions. That's why the desired values were calculated by numerical method on computer. The results of these calculations are given in the Table 1.

**Table 1. Expectation value of relative pitches of maxima.**

| Parameters | \( \lambda \) | 0,1 | 0,6 | 1,1 | 1,6 | 2,0 |
|------------|-----------------|-----|-----|-----|-----|-----|
| 0          | 0.805           | 0.895 | 0.979 | 1.043 | 1.081 |
| 0,5        | 1.14            | 1.201 | 1.256 | 1.294 | 1.317 |
| \( \gamma \) | 1.515           | 1.556 | 1.59 | 1.611 | 1.623 |
| at \( \gamma_1=3 \) | 1.913           | 1.939 | 1.959 | 1.969 | 1.974 |
| 2          | 2.317           | 2.331 | 2.34 | 2.334 | 2.345 |
| 2,5        | 2.695           | 2.699 | 2.702 | 2.702 | 2.703 |
| 2,8        | 2.891           | 2.892 | 2.892 | 2.892 | 2.892 |

On Figure 3 there are dependencies (received by this method) of relative altitude of maxima excluding elastic material deformation \( E(\gamma_0) \) and mathematical expectation of the average penetration depth of cutting logs referred to the root mean square deviation of the profile \( \sigma \) \( E(y_E)/\sigma \) from convergence \( \gamma \).

**Figure 3.** Dependence of mathematical expectation of relative pitches of maxima \( E(\gamma_0) \) and relative average depth of penetration \( E(y_E)/\sigma \) from convergence: 1 – at \( \lambda = 0,6 \); 2 – at \( \lambda = 1,6 \).
As it is seen from Fig. E(γ_0) and E(γE)/σ depend essentially from the convergence value γ and slightly from microrelief parameter λ = n(0)/m.

To simplify the following calculations, the equitation (7) is approximated by the dependency:

\[ E(γ_0) = (0.72 - 0.095 \cdot λ) \cdot γ \left(0.064 λ + 1.084 \right) + (0.18 \cdot λ + 0.71 + 0.785) \cdot γ. \]

Error of such an approximation does not exceed 2%.

By equitation (1) it is easy to detect the mathematical expectation of average depth of penetration of grains into the material of part.

4. Quantity of grain of the elastic wheel

For the case of convergence of rough relief that obtains the flat average surface with ideally smooth surface author [8] has generated the equitation for average quantity of heights of logs for one unit of square (1 mm²) at normalized correlation function, approximated expression \( ρ(τ) = e^{-α \cdot τ^2} \) in case of homogeneous and isotropic field:

\[ \mathcal{E}(N) = \frac{π \cdot n^2(0)}{2\sqrt{2π}} \cdot γ \cdot e^{-\left(\frac{γ^2}{2}\right)} \]

or

\[ \mathcal{E}(N) = 0.6266 \cdot n^2(0) \cdot γ \cdot e^{-\left(\frac{γ^2}{2}\right)} \]

Dependence of value \( E(N)/n^2(0) \) as it is seen from Figure 4 has an extreme character with maximum at γ =1 that demonstrated that number of spikes for random field differ from spikes of section of this field that are spread along the monotonically decreasing dependency.

![Figure 4](image)

**Figure 4.** Dependence of parameter \( E(N)/n^2(0) \) from convergence.

5. Conclusion

The received equitations of mathematical expectation of average depth of penetration of cutting logs and number of grains of elastic wheel being in contact allow to generate mathematical models of material removal and formation of parameters of roughness of the processed surface.

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