Autler-Townes Doublet observation via a Cooper-Pair Beam Splitter

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We present a proof-of-principle of how electronic transport measurements permits the observation of the Autler-Townes doublet, an optical property of nanodevices. The quantum physical system consists of one optically pumped quantum dot, a second auxiliary quantum dot, and a superconductor lead which provides an effective coupling between the dots via crossed Andreev reflection. Electrodes, working as source and drain, acts as nonequilibrium electronic reservoirs. Our calculations of the photocurrent at both, transient and stationary regimes, obtained using a density matrix formalism for open quantum systems, shows signatures of the formation of the Autler-Townes doublet, caused by the interplay between the optical pumping and the crossed Andreev reflection.

Keywords: nonequilibrium quantum systems, quantum transport, quantum dots.

Since the seminal work of Loss and DiVincenzo on quantum computation [1], semiconductor quantum dots have become an outstanding system for future development of integrated photonic and electronic scalable devices [2–4]. Such an integration has fundamental importance on the development of quantum computers and quantum internet [5–7]. In the last case, a quantum network is required, where fixed quantum nodes, defined by the quantum dots, exchange information through flying qubits, those codified on the state of the photons [6–7]. Such an implementation is feasible because of the entanglement between spin states on quantum dot and a single photon [8–9].

Quantum dots under the action of laser fields have been intensively investigated in recent years [10–12]. In this respect, the Autler-Townes (AT) effect [13] receives particular attention, which is a phenomenon that occurs in a three-level system, by the formation of a doublet between the dressed states of discrete levels in the presence of a radiation field. The optical response of such a level configuration is rich, counting not only for the appearance of AT doublet (ATD), but also robust states and tunneling induced transparency (TIT) [14–19]. The ATD was first reported in a molecular system composed of gaseous carbonyl sulfide (OCS) being excited by a photon [20]. Since then, it has been observed in atoms [21] and superconductor qubits [22–24]. More recently, it was predicted that a nonlinearity in the current of a photodiode will appear when one of the AT splitted levels crosses the Fermi level of electronic reservoirs [25].

Extending the functionalities of quantum dots, they have also been used to create hybrid systems composed of quantum dots and superconductors [26–29]. One device known as a Cooper pair beam splitter has been implemented experimentally [30–32]. In these systems, a Cooper pair singlet is split into two electrons in different quantum dots. Alternatively, two electrons in distinct dots tunnels to form a Cooper pair in a superconductor. This process takes place via crossed Andreev reflection (CAR) [33–35].

In this letter, we present a proof-of-principle of probing the formation of the ATD, an optical phenomenon, through quantum transport properties. In the literature, the ATD is reported only through spectroscopic measurements [21, 36, 37]. As far as we are concerned, this is the first feasible proposal to detect ATD relying on measurements of photoinduced current, driven by an optical pumping in quantum dots. The interplay between CAR and the electromagnetic field can be mapped into a three-level system, that sustains an Autler-Townes doublet in a nonequilibrium regime.

A schematic illustration of the physical setup is shown in Fig. 1(a), consisting of two quantum dots: quantum dot A (QDA) and quantum dot B (QDB), which are indirectly coupled to each other via a superconductor lead. The quantum dots are also coupled to normal leads, that operate as source or drain of particles. This geometry was proposed in Ref. [33] in order to investigate adiabatic pumping in a Cooper-pair beam splitter. The valence band of QDA (labeled as 3) is coupled, by tunneling, with strength Γ3, to the left (L) lead. Additionally, a monochromatic optical field, with frequency ω, promotes electrons from the valence to conduction band (labeled as 1). Electromagnetically dressed states are then formed, which can be electronically probed by the QDB. The conduction band on QDB (labeled as 2) is also coupled to a right (R) lead, with strength Γ2. Coupling between the conduction bands of both quantum dots to the superconductor (SC) lead is provided by CAR, with strength Δ.

The system is described by the following hamiltonian,

\[ H = \sum_{i=1}^{3} \varepsilon_i d_i^\dagger d_i + \sum_{\eta, k} \varepsilon_{k\eta} c_{k\eta}^\dagger c_{k\eta} + \Omega e^{-i\omega t} d_1^\dagger d_3 \\
+ \Delta d_1^\dagger d_2^\dagger + \sum_{k_3} V_3 d_3 c_{k_3} + \sum_{k_2} V_2 d_2 c_{k_2} + h.c., \]

where the first term is the free hamiltonian for the quan-
The quantum dynamics and the electronic transport are obtained by solving a differential equation for the reduced density matrix $\rho_S(t)$, which carries information of the quantum system, the dots under the action of optical pumping and the CAR. The first step is to use the unitary transformation $U(t) = \exp[ι\omega t(d_1^d d_1 - d_2^d d_2 - d_3^d d_3 - η d_n^d c_{k_n} c_{k_n})/2]$, which drops the time-dependent exponential from the optical pumping. The transformed hamiltonian would read as $H' = H_0 + V$, where

$$H_0 = \sum_{i=1}^3 \tilde{ε}_i d_i^d d_i + \sum_{κ_κ} η κ κ c_{κ κ}^d c_{κ κ} + Ω d_3^d d_3 + Δ d_1^d d_2^d + h.c.$$

$$V = \sum_{κ_κ} V_κ d_κ^d c_{κ κ} + \sum_{κ_κ} V_κ c_{κ κ} d_κ + h.c.,$$

with $\tilde{ε}_1 = ε_1 - Ω/2$, $\tilde{ε}_2(3) = ε_2(3) + Ω/2$ and $\tilde{ε}_{κ κ} = ε_{κ κ} + ω/2$. The evolution of the density matrix $\rho(t)$ for the full system (dots and reservoirs) is given by the Von Neumann equation, $\dot{ρ}(t) = -i[\hat{H}', ρ(t)]$ ($h = 1$). We first write $\dot{ρ}(t) = e^{iH_0 t} ρ(t)e^{-iH_0 t}$, where the hat symbol over the operators stands for the interaction picture. The exact solution for the dynamics of the system is given by $\dot{ρ}(t) = L(t)ρ_0 + \int_0^t dt L(t)L(t_1)\dot{ρ}(t_1)$, where $L(t)$ is the Liouvilian superoperator, $L(t)\dot{ρ}(t_1) = -i[\hat{V}(t), ρ(t_1)]$ and $\hat{V}(t)$ is the dots-to-reservoirs coupling in the interaction picture.

At this point, we use the Born approximation $\dot{ρ}(t) = \dot{ρ}_S(t) ⊗ \dot{ρ}_L ⊗ \dot{ρ}_R$, where $\dot{ρ}_S(t) = \text{Tr}_{L+R}[\dot{ρ}(t)]$ is the reduced matrix for the quantum system after taking the partial trace over the degrees of freedom of the reservoirs. The quantities $\dot{ρ}_L$ and $\dot{ρ}_R$ are the density matrices for the left and right reservoirs, which we have assumed as time-independent. After these calculations, we arrive to the integro-differential equation which describes the dynamics of the reduced density matrix $\dot{ρ}_S(t)$:

$$\dot{ρ}_S(t) = -i \int_0^t dt_1 \sum_{i,j} \left[ g_{ij}^- (t,t_1) d_j^d (t_1) d_j (t_1) \dot{ρ}_S (t_1) - g_{ij}^+ (t,t_1) d_j (t_1) d_j^d (t_1) \dot{ρ}_S (t_1) + g_{ij}^0 (t,t_1) d_j (t_1) d_j^d (t_1) + h.c. \right].$$

where each term contains the first-order correlation functions for the free-electrons on reservoirs defined as $g_{ij}^- (t,t') = δ_{ij}[V_i]^2 Σ_{κκ} i(\hat{c}_{κ κ}^d (t')\hat{c}_{κ κ}(t))$ and $g_{ij}^0 (t,t') = δ_{ij}[V_i]^2 Σ_{κκ} (i\hat{c}_{κ κ}(t')\hat{c}_{κ κ}(t))$, respectively. In the wideband limit approximation [10], they take the form

$$g_{ij}^- (t,t_1) = iδ_{ij} 2π D_i |V_i|^2 f_i δ(t - t_1) \quad g_{ij}^0 (t,t_1) = -iδ_{ij} 2π D_i |V_i|^2 (1-f_i) δ(t - t_1),$$

where the function $f_i$ is the Fermi function and $D_i$ is a constant density of states for $i$-th reservoir. Fermi functions take the values $f_i = 0$ ($f_i = 1$), if the reservoir is a

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we have defined the probabilities of occupation for bands and integrating over time, we obtain
\[
\dot{\rho}_S(t) = \frac{1}{2} \sum_i \Gamma_i \left\{ -(1 - f_i) \left[ d_i^\dagger(t) d_i(t) \rho_S(t) \right] \\
+ d_i(t) \rho_S(t) d_i^\dagger(t) - f_i \left[ d_i(t) d_i^\dagger(t_1) \rho_S(t_1) \right] \\
+ d_i^\dagger(t) \rho_S(t_1) d_i(t_1) + \text{h.c.} \right\},
\]
with $\Gamma_i = 2 \pi D_i |V_i|^2$ being the strength of the coupling with the reservoirs. Going back to the Schrödinger picture, we arrive to the Lindblad equation \cite{11} written as:
\[
\dot{\rho}_S = -i[H_S, \rho_S] - \frac{1}{2} \sum_i \Gamma_i \left[ f_i L_i^+ + (1 - f_i) L_i^- \right],
\]
where $H_S$ is the system Hamiltonian, without the reservoirs terms in Eq. (2). The dissipative terms describing the action of the reservoirs (source or drain) of electrons are given by $L_i^+ = d_i d_i^\dagger \rho_S + \rho_S d_i d_i^\dagger - 2 d_i^\dagger \rho_S d_i$, and $L_i^- = d_i^\dagger d_i \rho_S + \rho_S d_i^\dagger d_i - 2 d_i \rho_S d_i^\dagger$.

To arrive to a final analytical expression, we perform a procedure of vectorization of the density matrix so $\rho_S = \textbf{vec}(\rho_S)$. This procedure takes the Eq. (6) to the form \cite{42}
\[
\dot{\rho}_S(t) = e^{-iM_t} \rho_S(0),
\]
where $M$ is a matrix with dimension $I^{\otimes 3} \otimes I^{\otimes 3}$, for $I$ being the 2D identity matrix. This matrix is defined as $M = M_0 - i \Gamma/2$, where $M_0 = I^{\otimes 3} \otimes H_S - H_S^T \otimes I^{\otimes 3}$ describes the system (superscript $T$ means matrix transposition) and $\Gamma$ contains the effect of reservoirs \cite{43}.

In order to solve Eq. (7), by numerical procedures, we write operators $d_i$ in terms of the Jordan-Wigner transformation \cite{42} as $d_1 = \sigma_x \otimes \sigma_x \otimes \sigma_x, d_2 = I \otimes \sigma_x \otimes \sigma_x$, and $d_3 = I \otimes I \otimes \sigma_x$. This procedure automatically writes any state or operator associated with the system of quantum dots in the basis given by $\{|nlm\rangle = \{[111], [110], [101], [100], [011], [010], [001], [000]\}$, with $n, l, m$ standing for the occupation 1(0), meaning full (empty) conduction band in QDA, conduction band in QDB and valence band in QDA, respectively. This eight elements permit the description of a nonequilibrium scenario where electrons can flow in and out of the quantum dots bands \cite{45}.

In order to calculate the electronic currents through reservoirs $L$ and $R$, we use rate equations \cite{46}, so that $I_R = I_0 \left[ f_2 P_2^0 - (1 - f_2) P_2^0 \right]$, and $I_L = I_0 f_1 P_3^0$, where $I_0 = e \Gamma/\hbar$, with $\Gamma_2 = \Gamma_3 = \Gamma$. Here, we have defined the probabilities of occupation for bands on quantum dots as $P_i^n = \sum_{n,m=0}^{n i} \langle nlm | \rho_S | nlm \rangle$, where $l = 0$ (1) if conduction band on QDB is empty (full), and $P_3^0 = \sum_{n,i=0}^{n i} \langle n0 | \rho_S | n0 \rangle$, for an empty valence band in QDA.

To check the appearance of the ATD on our system, we calculate the evolution of populations of the three levels $n_i = \text{Tr}[d_i^\dagger d_i \rho_S(t)]$ ($i = 1, 2, 3$). In the numerical simulations, we initialized the system in the pure state $\rho_0 = |001\rangle \langle 001|$ (one electron in valence band of QDA). To search for transitions between the states of the system, we also vary the detuning $\delta = \varepsilon_2 - \varepsilon_1$. The results are shown in Fig. 3(b)-(d) as function of time and $\delta$, for $\Gamma = 0$. The first interesting feature to notice is the appearance of fast Rabi oscillations between conduction and valence band in QDA, with time scale $t \sim 1/\Delta$, as one can see from the behavior of $n_1$ and $n_3$, Figs. 3(b) and (d), respectively. The second feature is that the oscillations, caused by the optical pumping, present some beats at a larger time scale $t \sim 1/\Delta$, which coincides with an increase of population $n_2$ (the valence band of QDB), as shown in Fig. 3(d).

This results let us to conclude that, without the action of reservoirs, the elements $|001\rangle$, $|100\rangle$, and $|111\rangle$ form a typical three-level system, with effective coupling $V_{\text{odd}} = \Omega |001\rangle \langle 100| + \Delta |111\rangle \langle 001| + \text{h.c.}$. That means that the crossed Andreev reflection tends to split a Cooper pair, creating a single electron in each quantum dot and an ATD is formed at the condition $\delta = \pm \Omega$. For our particular initialization, the population $n_2$ oscillates around zero and $n_2 \approx 0.5$ as time evolves, due to the coherent nature of the oscillations caused by the optical field and the CAR effect.

We now proceed to demonstrate that the AT effect has signatures on the quantum transport, specifically on the electronic current. For this purpose, we now take into account the effect of the left ($L$) and the right ($R$) reservoirs \cite{43}. The reservoir $L$ operates as a source with $f_3 = 1$. Regarding the right reservoir, if $f_2 = 0$, the level of QDB is set above of the chemical potential $\mu_R$ of the electrode $R$, which acts as a drain. In contrast, if $f_2 = 1$, the level of QDB is set below $\mu_R$, with the electrode working as a source of electrons. Fig. 2 shows the left, $I_L$, and the right, $I_R$, currents for the cases where the right reservoir acts as (i) a drain with $f_2 = 0$, shown in Figs. 2(a)-(b), and (ii) a source with $f_2 = 1$, Figs. 2(c)-(d).

Concerning the first case, Fig. 2(a) shows that $I_L$ has a positive value, meaning the electrons flow from the reservoir $L$ into the valence band of QDA, as they are being photoexcited. The signature of the action of optical pumping is an oscillation at short times that matches with the Rabi oscillations shown in Fig. 1. As time increases, the incoherent coupling between the system and the reservoirs causes the attenuation of these oscillations, with the current being suppressed. Interestingly, the current in the right electrode shows negative values for the condition $\delta = \pm \Omega$ as seen in Fig. 2(b), although both, $I_R$ and $I_L$ go to zero as time evolves. The explanation for such a behavior is that, the $R$ electrode operates as a drain of electrons so, whenever an electron is created in QDB via CAR, it has a finite probability to be drained into the right lead, generating outgoing probability cur-
rent. As soon as the electron leaves QDB, its pairing electron in the conduction band of QDA stays locked, thus forbidding optical transitions or further CAR process. This fact results on a vanishing current at the stationary regime, so we can assert that this configuration permits the detection of ATD from current measurements only at the transient time scale.

The situation changes when $f_2 = 1$, as shown in Fig. 2(c)-(d). Again, we still observe the signatures of Rabi oscillations on both currents, however, two main differences can be noticed: (i) $I_R$ takes positive values only, due to the fact that electrode $R$ acts as a source, (ii) high values of current, even at long times, are predicted at the condition of ATD, $\delta = \pm \Omega$. The differences came from the fact that, when QDB is populated by electrons from the right lead, electrons are optically pumped in QDA. At this point, CAR process annihilates both electrons in the dots, opening the possibility for the injection of one more electron in QDB. Then the sequence repeats again, resulting in a finite current in the stationary regime. The requirement to sustain this stationary current is that QDA must remain photoexcited. Because this stationary current is significatively high at the ATD condition $\delta = \pm \Omega$, our results are a proof-of-principle that this optical phenomena can be detected by performing current measurements.

Finally, in Fig. 3(a) we show the behavior of the stationary current $I = I_R = I_L$ as function of $\delta$, varying the value of optical parameter $\Omega$. The results reveal the characteristic form of a ATD. It shows two resolved peaks at $\delta \approx \pm \Omega$, for $\Omega/\Delta > 1$, while for $\Omega/\Delta \to 1$, these two peaks merge to a single one. This behavior resembles the characteristic profile of the luminescence spectrum for an excitonic system, when laser intensity is varied [14]. In order to clarify the appearance of the peaks on the current, in Fig. 3(b)-(d), we show the eigenvalues associated to the three-level subspace formed by \{100, 001, 111\} for $\Omega = 4\Delta$, $2\Delta$, and $\Delta$, respectively. The eigenvalues present anticrossings around the Rabi energy, revealing a strong coupling between the levels of the quantum dots for this particular detuning. This coupling opens a transmission channel in the system that allows the flow of a stationary current. The values of the strength of optical pumping close to $\Delta$ yields to low resolution of anticrossings, as shown in Fig. 3(d) for $\Omega = \Delta$, which in transport appears as the single peak (blue open circles) in Fig. 3(a).

In summary, we present a proposal for the detection of an optical phenomenon through measurements of quantum transport in a nonequilibrium system. We first describe the process of formation of an Autler-Townes doublet on quantum dots coupled with a superconductor lead, which results from the combination of the action of an optical coupling and crossed Andreev reflection (CAR). Calculations performed with a nonequilibrium formalism shows that signatures of the formation of this doublet can be found on current measurements, even in a stationary regime, with the appropriate parameter conditions.

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