Models of the competitive mechanism at the organization of mechanical engineering production

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Abstract. This article discusses an iterative counter-planning procedure at the organization of mechanical engineering production, according to which each element informs the center of the effect \( \omega_i \), as well as an effect plan \( u_i^0 \), taken by the element if it is not among the winners. As plans for winners, a final iteration estimate is approved. That is, plans are formed by organizing a multistep procedure using information requested from the elements at each step; the normative effect is determined as follows: \( C = \max_{u_i^0} \omega_i \). Studies conducted in the work have shown the similarity of competitive mechanisms with competitive ones at the organization of production, namely, the results of the examples considered in the article are valid for both competitive and competitive models.

1. Introduction
Consider an active system of mechanical engineering production consisting of a center and of \( n \) active elements [1, 2]. The state of each element is determined by a pair of \((u_i, z_i)\), where \( u_i \) – is the effect (for example, output), and \( z_i \) – is the cost. The set of possible states of elements is determined by the inequalities: \( u_i \geq 0 \), \( 0 \leq z_i \leq \varphi_i(u_i) \), where \( \varphi_i(u_i) \) – is the increasing convex function of the effect of \( u_i \), for example, the production cost function [3-5].

We will call the competitive mechanism, which includes at the planning stage of the competitive procedure to identify multiple winners \( Q \). Let the usual system of stimulation of elements looks like:

\[
f_i = M u_i - \varphi_i(u_i), \quad i \notin Q,
\]

where \( M \) – the effect deduction ratio.

For winners of the competition, an increased standard of deductions from the effect is applied. \( \lambda > M \). However, this standard applies only if the real effect \( u_i \) is not less than a certain normative effect \( C \). Thus, the incentive system for competing elements will be:

\[
f_i = \begin{cases} \lambda u_i - \varphi_i(u_i), & u_i \geq C \\ M u_i - \varphi_i(u_i), & u_i < C \end{cases}
\]
The paper discusses an iterative counter-planning procedure [6, 7] at the organization of mechanical engineering production, according to which each element informs the center of the effect \( \omega_i \), as well as an action plan of \( u_i^0 \), taken by the element if it is not among the winners. As plans for winners, a final iteration estimate is approved. That is, plans are formed by organizing a multi-step procedure using information requested from the elements at each step. Then the normative effect is determined as follows: \( C = \max_{i \in Q} \omega_i \).

### 2. Models of competitive and competitive mechanisms at the organization of machine building production

Let the number of winners \( m \) be fixed. Consider the functioning of the system as a game of \( n \) elements whose strategies are the messages of ratings \( \omega_i \), the functions of gain - the objective functions (2), depending on whether the element \( i \) is the winner of the competition. Note that the choice of plan for the effect of \( u_i^0 \), obviously, is made from the maximum condition (1), that is, \( Mu_i^0 - \varphi_i (u_i^0) = \max_{u_i} [Mu_i - \varphi_i (u_i)] = \Delta_i \).

The set \( V_i = \{ u_i : \lambda u_i - \varphi_i (u_i) \geq \Delta_i \} \) will be called the set of competitively profitable plans (CPP). Denote by \( V_i \) the right boundary of this set. Let \( V_i \) ordered in descending order, that is \( V_{i_k} \geq V_{i_{k-1}} \geq ... \geq V_{i_1} \) (there may be several such orderings).

For competitive mechanisms at the organization of mechanical engineering production, the following theorem is valid.

**Theorem.** The Nash equilibrium exists, and for an equilibrium situation \( Q = \{ i_k : k \leq m \} \), and strategies \( \omega_i^* \) are determined by the conditions \( \omega_i^* = V_i \) for \( i \notin Q \) and

\[
\lambda \omega_i^* - \varphi_i (\omega_i^*) = \max_{i \in V_i \cap \{ \lambda \omega_i - \varphi_i (\omega_i) \}, i \in Q} (\lambda \omega_i - \varphi_i (\omega_i)), i \in Q. \tag{3}
\]

In this case, the equilibrium plans have the form

\[
x_i^* = \begin{cases} u_i^0, & i \notin Q \\ \omega_i^*, & i \in Q. \end{cases} \tag{4}
\]

Evidence. Obviously, for anyone \( i \in Q \) holds \( \omega_i^* \geq \max_{i \in Q} \omega_i \) and \( \omega_i^* \leq V_i \). It follows that \( Q = \{ i_k : k \leq m \}, \max_{i \in Q} V_i = V_{i_{m+1}} \), therefore, conditions (3) and (4) are fulfilled.

**Comment.** In contrast to the corresponding theorem for competitive mechanisms, in this case the existence of a Nash equilibrium is determined by the presence of two evaluations: the effect estimate \( \omega_i \) and the evaluation plan \( u_i^0 \), for the effect assigned to the element if it did not enter the number of winners of the competition.

Let the value of the standard of deductions from the effect of \( M \) and the restrictions on the total effect plan be given of \( P \). Let be \( Q \) – the set of winners, determining the decision of the game in the form (4). We define \( m \) and \( \lambda \) so that:

\[
\left[ \sum_{i \in Q} Mu_i^0 + \sum_{i \in Q} \lambda \omega_i^* \right] \rightarrow \min \text{ provided } \sum_{i=1}^n x_i^* = P.
\]

Substantially, this task is the minimization of incentives to achieve a given effect.
Let \( \varphi_i(y_i) = \varphi(y_i) \) for all \( i = 1, n \), then the right boundary of the CPP \( V_i \) sets is the same for all elements \( V_i = V, \ i = 1, n \), and the task is reduced to minimizing
\[
K = m\lambda V + (n - m)\mu u^0
\]
while limiting
\[
P = mV + (n - m)u^0.
\]

From the condition for the right boundary of the CPP sets \( \lambda V - \varphi(V) = \Delta \) and expression (5), after simple calculations we get:
\[
K = \frac{B[\varphi(V) - \varphi(u^0)]}{V - u^0} + nM\mu u^0, \text{ where } B = P - n\mu u^0 > 0.
\]

By virtue of the convexity of fiction \( \varphi \) the expression \( \frac{\varphi(V) - \varphi(u^0)}{V - u^0} \) is an increasing function for \( V > u^0 \); thus, the minimum \( K \) is reached at the minimum \( V \), and therefore the maximum \( m = n - 1 \).

Substantially, the result obtained corresponds to the principle of "not to be the last", when the loser is considered to be the participant having the worst result (for the case of identical elements, the loser will be the element with the last number).

Having \( \varphi(u) = qu^\alpha \) accepted expressing \( V \) through \( u^0 \) with (5), we find the optimal value \( u^0 \):
\[
u_{opt}^0 = \frac{P}{(n - 1)(\alpha n - n + 1)^{\frac{1}{\alpha - 1}} + 1}, \text{ then } K = qP^\alpha \frac{1 + (\alpha - 1)n}{[1 + (n - 1)(\alpha n - n + 1)^{\frac{1}{\alpha - 1}}]^\alpha - 1}.
\]

Results obtained for identical elements in the case of competitive mechanisms are equivalent to results obtained for competitive mechanisms.

Let there be \( \ell \) of different groups of elements. Given that the hypothesis about the behavior of elements remains the same, and the total plan for the effect \( P = \sum_{i=1}^{\ell} P_i \), where \( P_i \) - plan for the effect of the \( i \)-th group, we have for the competition within each group separately
\[
K = \sum_{i=1}^{\ell} q_i P_i^\alpha \frac{1 + (\alpha - 1)n_i}{[1 + (n_i - 1)(\alpha n_i - n_i + 1)^{\frac{1}{\alpha - 1}}]^\alpha - 1}, \quad \text{(6)}
\]
\[
u_{opt}^0 = \frac{P_i}{(n_i - 1)(\alpha n_i - n_i + 1)^{\frac{1}{\alpha - 1}} + 1},
\]
where \( q_i \) - cost factor for the function \( \varphi_i = q_i u^\alpha \), \( n_i \) - is the number of elements in the \( i \)-th group, optimal number of winners in each group \( - (n_i - 1) \).

Let \( \ell = 2 \), that is, there are two groups with different cost factors \( q_1 \) and \( q_2 \) \((q_1 < q_2)\). We find the optimal distribution of the plan according to the effect \( P_1 \) and \( P_2 \) between the groups. To do this, we represent (6) in expanded form, then using simple calculations and substitutions \( P_2 = P - P_1 \), we get:
\[
P_1 = P \frac{(q_1 \beta_1)^{\frac{1}{\alpha-1}}}{(q_1 \beta_1)^{\frac{1}{\alpha-1}} + (q_2 \beta_2)^{\frac{1}{\alpha-1}}}, \quad P_2 = P \frac{(q_1 \beta_1)^{\frac{1}{\alpha-1}}}{(q_1 \beta_1)^{\frac{1}{\alpha-1}} + (q_2 \beta_2)^{\frac{1}{\alpha-1}}},
\]
where \( \beta_i = \frac{1}{1 + (n_1 - 1)(\alpha - 1)n_1 + 1} \).

Then (6) will take the form
\[
K = P^{\alpha} \frac{q_1 q_2 \beta_1 \beta_2}{(q_1 \beta_1)^{\frac{1}{\alpha-1}} + (q_2 \beta_2)^{\frac{1}{\alpha-1}}}. \tag{7}
\]

However, the question is whether the competition mechanism within each group will be more effective than the competition mechanism conducted for all elements together.

If the competition is held for the entire set of elements, then the following two cases are possible.

1. The winners include all elements of the 1st group and some of the elements of the 2nd group, that is:
\[
K_1 = m \lambda V_2 + (n_2 - m) Mu_0^2 + \lambda n_1 \max \{V_2, u_0^2(\lambda)\} \to \min_m
\]
\[
P = m V_2 + (n_2 - m) u_0^2 + n_1 \max \{V_2, u_0^2(\lambda)\}, \tag{8}
\]
where \( u_0^2(\lambda) = \text{Arg} \max \{\lambda u - q_i u^\alpha\} \).

If the ratio of the magnitudes of the coefficients \( q_1 \) and \( q_2 \) is such that \( \max \{V_2, u_0^2(\lambda)\} = V_2 \), then after simple calculations we get:
\[
K_1 = \frac{A[V_2 - u_0^2]}{V_2 - u_0^2} + (n_1 + n_2) Mu_0^2 + n_1 \left[ \phi_2(V_2) - \phi_2(u_0^2) \right],
\]
where \( A = P - n_2 u_0^2 - n_1 V_2 > 0 \).

When \( V_2 > u_0^2 \) due to the convexity of the cost function \( \phi \) the minimum \( K_1 \) is attained at the minimum \( V_2 \), which corresponds to \( m = n_2 - 1 \).

Express \( V_2 \) through \( u_0^2 \) using (8) and find the optimal value \( u_0^2 \).
\[
u_0^2_{opt} = \frac{P}{((n_1 + n_2) - 1)((\alpha n_1 + n_2) - (n_1 + n_2) + 1)^{\frac{1}{\alpha-1}} + 1}, \text{ then}
\]
\[
K_1 = q_2 P^{\alpha} \frac{q_1 q_2 \beta_1 \beta_2}{(q_1 \beta_1)^{\frac{1}{\alpha-1}} + (q_2 \beta_2)^{\frac{1}{\alpha-1}}} \frac{1}{1 + (\alpha - 1)(n_1 + n_2) + 1} \tag{9}
\]

If \( \max \{V_2, u_1^2(\lambda)\} = u_0^2(\lambda) \), then in this situation, a qualitative analysis has shown that competition within groups is most effective here, since otherwise the normative effect \( C \) does not have a stimulating effect on the 1st group. In other words, the value of the cost ratio of the 1st group is such that for it there are no competitors in the competition.

2. The number of winners includes only a part of the elements of the 1st group, then, conducting the reasoning similar to case 1, we finally get: \( K_2 \) minimum with \( m = n_1 - 1 \). In this case, the optimal values \( u_1^0, u_2^0 \) and \( K_2 \) will be:
\[ u_{1_{opt}}^0 = \frac{P}{((\alpha - 1)n_1 + 1)^{\frac{1}{\alpha - 1}} \left( n_2 \left( \frac{q_1}{q_2} \right)^{\frac{1}{\alpha - 1}} + n_1 - 1 \right) + 1}, \]

\[ u_{2_{opt}}^0 = \frac{P \left( \frac{q_1}{q_2} \right)^{\frac{1}{\alpha - 1}} ((\alpha - 1)n_1 + 1)^{\frac{1}{\alpha - 1}}}{((\alpha - 1)n_1 + 1)^{\frac{1}{\alpha - 1}} \left( n_2 \left( \frac{q_1}{q_2} \right)^{\frac{1}{\alpha - 1}} + n_1 - 1 \right) + 1}, \]

\[ q_1(n_1 - 1) \left( \frac{q_1}{q_2} \right)^{\frac{\alpha}{\alpha - 1}} ((\alpha - 1)n_1 + 1)^{\frac{\alpha}{\alpha - 1}} + q_1((\alpha - 1)n_1 + 1)q_2 \alpha n_2 \left( \frac{q_1}{q_2} \right)^{\frac{\alpha}{\alpha - 1}} ((\alpha - 1)n_1 + 1)^{\frac{\alpha}{\alpha - 1}} \]

\[ K_2 = P^\alpha \left[ ((\alpha - 1)n_1 + 1)^{\frac{1}{\alpha - 1}} \left( n_2 \left( \frac{q_1}{q_2} \right)^{\frac{1}{\alpha - 1}} + n_1 - 1 \right) + 1 \right]^{-\frac{1}{\alpha - 1}}. \tag{10} \]

Using formulas (7), (9), (10), it is possible in each case to determine the most optimal type of competition.

Let \( n_i \) be large, then formulas (7), (9), (10) take the following form:

\[ K_1 = q_2 P^\alpha \left( \frac{1}{n_1 + n_2} \right)^{\frac{1}{\alpha - 1}}, \tag{11} \]

\[ K_2 = P^\alpha q_1^{\frac{\alpha}{\alpha - 1}} \left( \frac{q_1 n_1 + \alpha q_2 n_2}{n_2 q_1^{\frac{1}{\alpha - 1}} + n_1 q_2^{\frac{1}{\alpha - 1}}} \right)^{-\frac{1}{\alpha - 1}}, \tag{12} \]

\[ K = P^\alpha \left[ \frac{q_1 q_2}{q_1^{\frac{1}{\alpha - 1}} n_2 + q_2^{\frac{1}{\alpha - 1}} n_1} \right]^{\frac{1}{\alpha - 1}}. \tag{13} \]

Comparing in pairs the formulas (11) - (13), we obtain that for all \( \alpha \geq 2 \) the competition within the groups is more effective than the competition, when the winners are all elements except one from the 2nd group. When compared to \( K \) with \( K_2 \) we get: if \( \frac{q_2 q_1^{\frac{1}{\alpha - 1}} ((\alpha - 1)n_1 + 1)}{n_1} > \frac{n_2}{q_1^{\frac{1}{\alpha - 1}} n_2 + q_2^{\frac{1}{\alpha - 1}} n_1} \), then the competition is more effective, when the winners will be all (except one) elements of the group with a lower cost coefficient. Otherwise, the competition within each group will be more effective. (in particular, if \( \alpha = 2 \) for the greater efficiency of the competition within the groups, the condition \( n_1 \leq n_2 \), is sufficient, that is, the number of elements in the group with a lower cost factor is not more than in the other group).
3. Conclusion
In conclusion, we note that the conducted studies have shown the similarity of competitive mechanisms with competitive ones, namely, the results of the examples given in this article are valid for both competitive and competitive models.

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