Neural network solution of the direct kinematics problem for a hexapod with ball-screw drives of legs

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Abstract. This research is a part of the work implemented by BSTU “VOENMEH” under the financial support of the Ministry of Education and Science of the Russian Federation for design and development of a precision mechanism with the parallel kinematics called “Hexapod”. The object of research is the mechanism, in which the control of leg lengths is implemented by linear drives with ball-screw gears. The article proposes the solution of the extended inverse kinematics of the hexapod. The neural networks solution of the forward kinematics of hexapod is presented, i.e. the calculation of the position and orientation of the moving platform for given rotation angles of the ball-screw drives. The results of the synthesis of neural networks to achieve a given accuracy of solving the forward kinematics are presented. An estimate of the implementation of kinematic problems in the hexapod control system based on an ARM microprocessor is obtained.

1. Introduction

The researchers of BSTU “VOENMEH” and JSC “ISS” named after M.F. Reshetnev jointly created a multi-step mechanism with parallel kinematics [1, 2] (MPK) to ensure accurate positioning and orientation of on-board instruments and devices for space purposes [3].

The object of our research is the MPK (“Hexapod”) based on six linear drives with screw gears, the design of which is shown in figure 1. The hexapod, designed according to the scheme of Stuart’s platform [4], consists of a fixed base and a movable platform, controlled by six identical ball-and-screw drives – legs (bars, poles). Each leg is connected by universal joints with the base and platform. The leg of the hexapod consists of two half-bars, which are interconnected by a ball-screw drive. The kinematic diagram of the hexapod leg is shown in figure 2. Thus, the position and orientation of the platform is determined by the angles of rotation of the ball-screw gears \( \alpha_i \) of each leg of the hexapod.

The main task of the hexapod control system is to test the position of the platform set in Cartesian coordinates relative to the base with an accuracy of ± 10 \( \mu \)m and the orientation of the platform with an accuracy of ± 30 arc sec. Extreme working environment in conditions of open space defines strict requirements for the hardware of the hexapod. The control system is developed on the basis of a domestic radiation-resistant microcontroller [5]. Therefore, the tasks of assessing the feasibility and improving the quality indicators of the designed control algorithms are relevant and important.

When controlling a hexapod with screw gears, two kinematic problems, forward and inverse, are solved. In the majority of existing works on the subject of research, the solution of the inverse problem of kinematics [5, 6] is given as the calculation of the leg lengths of a hexapod for a given position and...
orientation of the platform. Thus, the inverse problem is used for planning trajectories with kinematic control of a hexapod and generating control signals with linear drives [7], for deriving the dynamic equations [6]. In [5] an algorithm for calculating control signals with stepper linear drives based on ball-screw drives is presented; an implementation of the inverse problem based on a domestic ARM-architecture controller is evaluated. The forward kinematics problem is defined [8] as the calculation of the position and orientation of the platform for given leg lengths. In [8, 9] the solution of the forward kinematics problem was obtained using iterative numerical methods. However, of particular interest is the approach based on artificial neural networks (ANN). In [10–14] approaches to the neural network solution of the forward kinematics problem of a hexapod and [13] methods of application in control problems of a hexapod are presented.

The kinematic tasks of the studied MPK with screw pairs differ from those discussed earlier: when controlling the MPK, the upper half bar (“upper leg”, as shown in figure 2) rotates relative to the lower (“lower leg”) at an angle $\gamma$, which affects the length of the leg. Therefore, for a hexapod with screw pairs, we introduce the following definitions of kinematic problems:

1) the extended inverse kinematics problem (EIKP) is to determine the rotation angles of the screw pairs $\alpha_i$, the relative rotation angles of the half-bar $\gamma$, and the leg lengths $L$ from a given position and orientation of the moving platform;

2) forward kinematics problem (FKP) is to determine the position and orientation of the platform for given rotation angles of the screw pairs $\alpha_i$.

Of particular interest is to acquire the FKP of hexapod with screw pairs. There are no analytical solutions to the problem. In this paper, we propose to use the approximation capabilities of ANN to solve the FKP. Thus, the main purpose of this work is to obtain a neural network solution to the forward kinematics problem of a hexapod with screw transmissions and evaluate its feasibility in a control system based on the ARM microcontroller.

Figure 1. Hexapod design sample.  
Figure 2. Kinematic diagram of hexapod leg with ball-screw transmission.

2. Hexapod kinematics

To solve the problems of kinematics, we introduce a coordinate system, as shown in figure 3. The fixed coordinate system OXYZ is connected to the base, the moving coordinate system O’X’Y’Z’ is connected to the platform. We define the initial “zero” position of a symmetrical hexapod, in which the legs have the same elongation. Thus, in this position, the O’X’Y’Z’ coordinate system with respect to the OXYZ coordinate system is shifted along the OZ axis by the $h_0$ parameter.
The length of the legs in the “zero” position will be defined as \( l_0 \) “zero length of the legs”. We introduce the numbers of the legs, as shown in figure 3.

The position of the center \( O' \) of the platform relative to a fixed coordinate system is set using Cartesian coordinates \( X, Y, Z \). Euler angles are used to determine the orientation of the platform \([15]\) \( \varphi, \theta, \psi \). Thus, the linear position and the angular orientation of the platform are given by the vector \( \mathbf{q} = [X, Y, Z, \varphi, \theta, \psi]^T \).

The specified working range of the hexapod is: by coordinate \( X \) – ±100 mm, by coordinate \( Y \) – ±100 mm, by coordinate \( Z \) – ±25 mm, by angular coordinates ±7 deg.

![Figure 3. Kinematic diagram of hexapod.](image1)

![Figure 4. Leg coordinate systems with screw-nut transmission.](image2)

For the connection between the fixed and moving coordinate systems, we use the matrix transformation in homogeneous coordinates \([15]\):

\[
\mathbf{R} = \mathbf{T} \times \mathbf{r}'
\]

where \( \mathbf{r} = [X, Y, Z, 1]^T \) is the vector of homogeneous coordinates of a point in the coordinate system OXYZ, \( \mathbf{T} \) – index denoting vector transposition, \( \mathbf{r}' \) is the vector of homogeneous coordinates of the same point relative to the moving coordinate system.

In the transformation (1), the expanded matrix \( \mathbf{T} \) is defined by \( \mathbf{p} = [X', Y', Z']^T \) – coordinates of the beginning of the moving system are relatively stationary - and the platform rotation matrix \( \mathbf{R}_p \)

\[
\mathbf{R} = \begin{bmatrix}
\mathbf{R}_p & \mathbf{p} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

The rotation matrix for choosing Euler angles is defined by the expression

\[
\mathbf{R}_p = \begin{bmatrix}
\cos \varphi \cos \psi + \sin \varphi \sin \theta \sin \psi & \sin \varphi \sin \theta \cos \psi - \cos \varphi \sin \psi & \sin \varphi \cos \theta \\
\cos \theta \sin \psi & \cos \theta \cos \psi & -\sin \theta \\
\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi & \sin \varphi \sin \psi + \cos \varphi \sin \psi \cos \theta & \cos \varphi \cos \psi
\end{bmatrix}
\]

Calculating from (1–3) the transformation matrix \( \mathbf{T} \) for a given vector \( \mathbf{q} \), according to the coordinates of the points of attachment of the hinges of the legs to the base (\( \mathbf{r}_A \) is the vector of homogeneous coordinates of a point \( A \) in a system OXYZ) and to the platform (\( \mathbf{r}'_B \) is the vector of homogeneous coordinates of the point \( B \) relative to \( O'X'Y'Z' \)), we find the length of the \( i \)-th leg of the hexapod.
For MPK with screw pairs, when implementing control tasks, it is necessary to calculate for each drive the angle of rotation of the screw-nut transmission \( \alpha_{dr} \), the value of which depends on the angle of rotation \( \gamma \), as shown in figure 4.

Given the positions of the centers \( \mathbf{p}_A = [x_A, y_A, z_A]^T \) and \( \mathbf{p}_B = [x_B, y_B, z_B]^T \) of biaxial hinges \( A_i \) and \( B_i \), and the orientation of the biaxial hinges determined by the rotation matrices \( R_{A_i} \) and \( R_{B_i} \), we formulate the matrix kinematics equation relating the transformations of turns of the coordinate systems of the \( i \)-th leg of the:

\[
R_A \cdot R_x (\alpha_{A_i}) \cdot R_y (\beta_{A_i}) \cdot R_z (\gamma_{A_i}) \cdot R_x (\beta_{B_i}) \cdot R_z (\alpha_{B_i}) = R_{B_i},
\]
where

\[
R_x (\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix}, \quad R_y (\beta) = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix}, \quad R_z (\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

We introduce the vector \( S_i = (\mathbf{p}_B - \mathbf{p}_A) / L_i \), defining the ort of the \( z \)-axis of the coordinate systems of the lower and upper half-legs.

Knowing the orientation of the platform \( R_p \), base orientation \( R_o \) and initial orientations of hinges \( R_{A_i}^0, R_{B_i}^0 \) determined by design features, we calculate the orientations of the hinges of the base \( R_{0A_i} \) and the platform \( R_{0B_i} \):

\[
R_{0A_i} = R_x R_{A_i}, \quad R_{0B_i} = R_y R_{B_i}.
\]

Kinematics equations for the lower half-legs

\[
R_A = R_x (\alpha_{A_i}) R_y (\beta_{A_i}) \cdot S_A = R_{0A_i} \cdot S_i = R_{A_i}^3.
\]

where \( R_{A_i}^3 \) is the third column of the matrix \( R_{A_i} \).

Similarly, for the upper half-leg –

\[
R_B = \left( R_y (\beta_{B_i}) R_x (\alpha_{B_i}) \right)^T \cdot S_B = R_{0B_i} \cdot S_i = R_{B_i}^3.
\]

where \( R_{B_i}^3 \) is the third column of the matrix \( R_{B_i} \).

Using the definitions of the rotation matrices, it is easy to calculate the angles

\[
\alpha_{A_i} = \arcsin\left( S_A^T \right),
\]
where $S^1_A$ is the first element of the column $S_A$,

$$
\beta_A = \text{atan}2\left( \begin{array}{c}
-S^2_A \\
\cos\left(\alpha_A\right)
\end{array} ; \begin{array}{c}
-S^3_A \\
\cos\left(\alpha_A\right)
\end{array} \right),
$$

(10)

where $S^2_A, S^3_A$ are second and third elements of the column $S_A$, $\text{atan}2$ is the function of two-argument arctangent, similarly we obtain the angles

$$
\beta_B = \text{as}in\left(S^1_B\right),
$$

(11)

where $S^1_B$ is the first element of the column $S_B$,

$$
\alpha_B = \text{atan}2\left( \begin{array}{c}
-S^2_B \\
\cos\left(\beta_B\right)
\end{array} ; \begin{array}{c}
-S^3_B \\
\cos\left(\beta_B\right)
\end{array} \right),
$$

(12)

where $S^2_B, S^3_B$ are second and third elements of the column $S_B$.

Finally, from equation (5) we get

$$
R_z(\gamma_i) = \left( R_x(a_A) R_y(\beta_A) \right)^T R_x(\beta_B) R_y(A_r) \left( R_x(a_B) \right)^T
$$

(13)

and define the angle

$$
\gamma_i = \text{atan}2\left( R^{2,1}_z , R^{1,1}_z \right),
$$

(14)

where $R^{2,1}_z, R^{1,1}_z$ – the second and first elements of the first column of the matrix $R_z(\gamma_i)$.

Calculated from relations (4), (14) the length $L_i$ and angle $\gamma_i$ are applicable for calculating the total angle of rotation of the screw pair

$$
\alpha_{\phi_i} = 2\pi \frac{L_i - L_0}{h}, \quad \gamma_i = i...6
$$

(15)

where $L_i$ is the length of i-leg, corresponding to the desired position and orientation of the moving platform of the hexapod, $L_0$ is the leg length in the “zero” position, $h$ is the pitch of the screw pair, $\gamma_i$ is the angle of the axial rotation.

Thus, equations (1) – (15) give a solution to the extended inverse kinematics problem for each leg of a hexapod, which can be represented as a system of nonlinear algebraic equations

$$
a = H(q).
$$

(16)
The geometrical parameters needed to solve the extended inverse kinematics problem, the position of the hinges \( p_A, p_B \), and the initial orientation of the hinges \( R_A^0, R_B^0 \) are found from the solid-state model of the hexapod shown in figure 5.

The solution of the forward kinematics problem can be represented by finding the inverse function of (16)

\[
q = H^{-1}(\alpha).
\]

Since the analytical solution of problem (17) is difficult, it is proposed to use the ANN apparatus for approximating (17) the solution with a given accuracy.

3. Neural network solution of the forward kinematics problem

In this paper, for approximation (17), a separate neural network approximation of the components of the vector \( q \) is used, the scheme of which is shown in figure 6.

The use of this approach allows us to speed up and simplify the process of learning and synthesis of ANN for solving the FKP with a given accuracy, during which we determine: the algorithm for the formation of the training sample, the network architecture (network type, number of hidden layers, number of neurons in each layer), learning algorithm, matrix network implementation.

It is convenient to create and train a neural network suitable for the implementation of problems of approximation of multidimensional nonlinear functions using modern mathematical modeling environments, which include the Matlab Neural Network Toolbox [16] extension package. The toolkit of this package is used in the design of neural networks to solve the FKP.

The synthesis procedure was performed for six ANNs approximating the components of the vector \( q \); this paper demonstrates the results for the ANN1 network.

Three implementations of the structures of cascade neural networks of direct distribution for the FKP approximation were studied:

- a cascade network with one hidden layer CFNN1 (Cascade Feedforward Neural Network), which architecture is shown in figure 7,
- a cascade network with two hidden layers CFNN2, the architecture of which is shown in figure 7,
- a cascade network with three hidden layers CFNN3, the architecture of which is shown in figure 8.
Based on the solution of the extended inverse kinematics problem (1) – (15), a training sample was formed as a linearly distributed array for the values of the hexapod working range. The array dimension for the training sample was $6 \times P_6$ ($P_6=7$). For these structures, ANN training was performed by the Levenberg-Marquardt method. The estimated accuracy of the approximation on the test sample is obtained

$$
\Delta_{\text{max}}
$$

is the maximum absolute error of the approximation. The results of the accuracy estimation are summarized in table 1.

| Network Type | Number of Neurons in the Hidden Layer | $\Delta_{\text{max}}, \text{m}$ |
|--------------|--------------------------------------|-------------------------------|
| CFNN1        | 6                                    | $6.86 \times 10^{-4}$         |
| CFNN1        | 12                                   | $1.31 \times 10^{-4}$         |
| CFNN1        | 18                                   | $1.04 \times 10^{-4}$         |
| CFNN1        | 36                                   | $1.81 \times 10^{-5}$         |
| CFNN2        | 6                                    | $7.76 \times 10^{-5}$         |
| CFNN2        | 12                                   | $1.08 \times 10^{-5}$         |
| CFNN2        | 24                                   | $1.40 \times 10^{-6}$         |
| CFNN3        | 6                                    | $9.15 \times 10^{-5}$         |
| CFNN3        | 12                                   | $3.45 \times 10^{-6}$         |
| CFNN3        | 18                                   | $1.25 \times 10^{-6}$         |

Analysis of the obtained results allows us to conclude that in order to achieve the required accuracy of the FKP solution for one coordinate in a given operating range, it is sufficient to use the CFNN2 network with 24 neurons in hidden layers or CFNN3 with 12 neurons in hidden layers.

For the implementation of the ANN, the CFNN2 structure was selected, for which the matrix expression system was obtained.
\[
\begin{align*}
\mathbf{y}_1 &= K_{inp}(\mathbf{x}, \mathbf{x}_{0}) \mathbf{x}_{\text{min}} \\
\mathbf{y}_2 &= F_{th}(W_{11}\mathbf{y}_1, b_1) \\
\mathbf{y}_3 &= F_{th}(W_{22}\mathbf{y}_1, W_{33}\mathbf{y}_2, b_3) \\
\mathbf{y}_{\text{out}} &= W_{31}\mathbf{y}_1 + W_{32}\mathbf{y}_2 + W_{33}\mathbf{y}_3 + b_3 \\
\mathbf{y} &= K_{out}(\mathbf{y}_{\text{out}}, \mathbf{y}_{\text{min}}) \mathbf{y}_{\text{a}} +
\end{align*}
\]  

(18)

Where matrices $W_{11}, W_{22}, W_{33}$ set the synaptic weights in the neural network layers, $b_1, b_2, b_3$ are displacements in each layer, which values are obtained during the training of the ANN; $K_{inp}, \mathbf{x}_0, \mathbf{x}_{\text{min}}$ are matrices of normalization coefficients of input $\mathbf{x}$ values (values of foot length vector $L$); $K_{out}, \mathbf{y}_0, \mathbf{y}_{\text{min}}$ matrices of recovery coefficients for the output of the network $\mathbf{y}$ are determined by normalizing the training samples; $F_{th}$ the activation function of the hyperbolic tangent is found from the expression

\[
F_{th}(z) = \frac{2}{1 + e^{-2z}} - 1.
\]

(19)

4. Evaluation of the implementation of the FKP neural network solution in hexapod control systems

Currently, a digital hexapod control system based on the use of the 32-bit ARM RISC microcontroller K1986BE1T manufactured by JSC “FKK Milandr” [17] is implemented on the basis of the research laboratory of robotic and mechatronic systems of BSTU “VOENMECH”. The microcontroller operates at a clock frequency of up to 144 MHz. It contains 128 Kbytes of one-time programmable ROM of programs with ECC and 48 Kbytes of ECC RAM.

The software for solving the hexapod FKP was made in the Keil µVision 5 environment. The ARM C / C ++ Compiler (armcc) compiler was used to assemble the microcontroller programs. Test results are presented for compilation using optimization (O2 level).

![Hexapod drive control unit.](image)

Figure 9. Hexapod drive control unit.

The implementation of kinematic problems in two versions of the representation of floating-point numbers: float (single precision) and double (double precision) is compared. Based on the results of the synthesis of the cascade neural network (18) and (19), the algorithms for solving the FKP in the high-level C ++ language are obtained, performance and accuracy estimates (maximum positional $\Delta_{\text{max}}$ and angular approximation $E_{\text{max}}$ errors) are presented in table 2.
Table 2. Evaluation of the implementation of the FKP neural network solution.

| Implementation Type | $T$, ms | $\Delta_{\text{max}}$, m | $E_{\text{max}}$, rad |
|---------------------|---------|------------------------|------------------|
| float               | 18,8    | $3,7 \times 10^{-6}$  | $1,1 \times 10^{-5}$ |
| double              | 28,3    | $4,1 \times 10^{-6}$  | $1,1 \times 10^{-5}$ |

Note that the implementation of the synthesized networks in the format of numbers with double precision double time of the algorithm execution increases, but the accuracy does not.

5. Conclusion
This paper presents the solution of the extended inverse problem of kinematics of a hexapod with screw pairs. Based on this, a neural network solution of the forward kinematics problem is obtained. We note the simplicity of the structure of the synthesized networks. For neural network approximation with a given accuracy, it is recommended to use cascade networks of direct distribution with two or three hidden layers based on the hyperbolic tangent as activation functions with a total number of neurons of no more than 48. In the space control hexapod control system under study, based on the domestic ARM microcontroller architecture, the execution time of the algorithms meets the requirements of positional control. For a given accuracy of positioning and orientation of the hexapod, the use of numerical algorithms with the implementation in the single-precision float format is recommended. Thus, the conducted studies have shown the effectiveness and expediency of using an artificial neural network apparatus to solve the forward kinematics problem of a hexapod with screw gears in leg drives.

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