Time Dependent Convective Non-Orthogonal Hiemenz Flow of Viscoelastic Walter’s B Fluid towards a Non-Uniformly Heated Vertical Surface: Using Spectral Method

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Study of non-orthogonal time dependent mixed convection Hiemenz flow of viscoelastic Walter’s B fluid with thermal radiation is the major focus of this article. The surface intact with the fluid particle is assumed to be oscillating-stretching and heated with sinusoidal surface temperature. This physical process is modeled in mathematical form into system of PDEs, which are simulated numerically by Chebyshev spectral method. The obtained solution is firstly validated for reduced case with already published results, and then further results are achieved for governing physical parameters. The effects of dimensionless emerging constants are described through tables and graphs. In this study, it is concluded that in assisting flow case velocity enhances as compared to opposing flow case. By increasing the Weissenberg number velocity of the fluid decreases in both assisting and opposing flow cases while in opposing flow case these effects are more prominent as compared to assisting flow case. The streamlines in both assisting and opposing flow cases come closer to each other with the passage of time and at $t = \frac{3\pi}{2}$, they overlap each other and stagnation points also get coincide.

Key Words: Nonaligned stagnation point / Oscillating surface / Sinusoidal wall temperature / Mixed convection

1. INTRODUCTION

The study of non-aligned Hiemenz flow getting more attention than orthogonal stagnation point flow due its generalization and one can get extra information for the better understanding of flow behavior. The term Hiemenz is generally used for the study of orthogonal stagnation point flow; the region near the point, where the fluid strike to the surface at right angle and velocity of the fluid become zero. We call it Hiemenz flow because of the reason that Hiemenz was the first who studied such type of flow. The term non-aligned Hiemenz flow (oblique stagnation point flow) is the generalization of the Hiemenz flow where the fluid strikes to surface at a point in any arbitrary angle; velocity becomes zero, the pressure becomes static and heat transfer gain its maximum amount. The leading studies1-8) on oblique stagnation point flow had been carried out by many researchers. In Recent era, Husain et al.9) considered the problem of non-aligned stagnation point flow for a non-Newtonian fluid model and they numerically investigated that how the viscous properties effected the flow near stagnation point region. Mahapatra et al.10) studied the effects of thermal radiation near the oblique stagnation point flow towards a shrinking sheet. They simulated their physical model numerically. They analyzed the dual solution and draw the streamlines showing the reverse behavior near the surface. Lok et al.11) considered non-aligned stagnation point flow over the smooth surface. They considered micropolar fluid for their targeted study. In this fluid small elements process both translation and microrotation. Yajun et al.12) studied the thermal radiation effect on hydro-magnetic flow of micropolar fluid towards moving plate. Javed et al.13) carried out time dependent oblique stagnation point flow over an oscillating plate. In another study, Ghaffari et al.13) found a numerical solution of oblique stagnation point flow of non-Newtonian elastic fluid over a horizontal oscillating-stretching surface. They examined the unsteady flow behavior for different time steps.

Mixed convection is one of the basic heat transfer phenomena in which heat transfers due to external agent and buoyance effect. The study of mixed convection flow is very important due to its fundamental nature and many
engineering applications. In 1942, Martinelli et al.\textsuperscript{15} were among the earlier scientists, who studied the free and forced convection flow in a vertical pipe. Later on\textsuperscript{16-20}, have reported fully developed solutions for convective flow problems. Many investigations have been done for the study of mixed convection flow, which are very difficult to summarize in one paragraph. However, we have tried to mention few of them which are related to present investigation. Earlier investigations on mixed convection flow near stagnation point region was carried out by Ramachandran et al.\textsuperscript{21}. Later on, different studies\textsuperscript{22-27} on convective heat transfer near stagnation point region over different geometries were the major contributions of the researches.

Above mentioned valuable work by different researchers is witnessed that the study of mixed convection flow has major contribution in field of science and technology. Keeping in view we extended the work of Lok et al.\textsuperscript{28, 29}, by considering oscillating stretching surface with sinusoidal wall temperature. The study was performed numerically by a Chebyshev spectral method. The results are authenticated by comparison with old published studies as a limiting case and excellent agreement was found. New results are presented in form of velocity, temperature, and streamlines.

2. CONSTITUTIVE EQUATIONS

Non-Newtonian fluids have much importance due to enrich application in mechanical engineering, industries for the manufacturing of polymers and useful products. Behavior of non-Newtonian fluid model can’t be predicted by a unique constitutive equation. Due to this reason different non-Newtonian fluid models based on their features have been developed experimentally for various categories. Some of models are good for predicting shear thinning, shear thinning some of them are reliable for prediction of time relaxation, time retardation etc. The non-Newtonian Walter’s liquid model-B is one of the viscoelastic fluid model suggested by Walter\textsuperscript{30}. This viscoelastic fluid model can accurately simulate the complex behavior of the various industrial liquids. This can accurately predict the elastic behavior and extensional polymer’s behavior. The constitutive equations of Water’s B fluid model can be represented as

\begin{equation}
\text{div} \vec{V} = 0,
\end{equation}

\begin{equation}
\rho \frac{d \vec{V}}{dt} = \text{div} \vec{T} + \rho g,
\end{equation}

Where

\begin{equation}
\vec{T} = -p \vec{I} + 2 \mu_0 \vec{A} - 2 k_0 \frac{d \vec{A}}{dt}
\end{equation}

\( \mu_0 \) is the dynamic viscosity, \( k_0 \) is elasticity (short memory coefficient). Li\textsuperscript{31} found that “the elasticity in the liquid not only can destabilize the flow, but it can also stabilize the flow for certain values of depth ratio, viscosity ratio and elasticity ratio” and \( \vec{A} \) is the deformation rate tensor. The upper convective derivative \( d \vec{A}/dt \) can be defined as

\begin{equation}
\frac{d \vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + V \nabla \vec{A} - \vec{A} \nabla V - \nabla V . \vec{A}
\end{equation}

3. MATHEMATICAL FORMULATION

The flow of Walter’s B fluid impinging obliquely towards a vertical flat plate (Fig. 1) is considered in the present problem. It is also assumed that the plate is oscillating with velocity \( U_0 \cos \Omega t \) about its mean position and stretching linearly in vertical direction and heated with sinusoidal surface temperature. The temperature surface varies about its mean value \( T_w \), which is higher than the surrounding temperature. The equations which represent the flow and heat transfer can be represented in cartesian coordinate system as\textsuperscript{30, 32, 33}

\begin{equation}
\frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial y} = 0,
\end{equation}

\begin{equation}
\frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{u}}{\partial y} + \frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial \vec{v}}{\partial y} \cdot \frac{\partial \vec{v}}{\partial x} - \frac{\partial \vec{v}}{\partial y} \cdot \frac{\partial \vec{v}}{\partial x} + \frac{\partial \vec{u}}{\partial x} \cdot \frac{\partial \vec{u}}{\partial y} - \frac{\partial \vec{u}}{\partial y} \cdot \frac{\partial \vec{u}}{\partial x} = - \frac{1}{\rho} \frac{\partial q}{\partial x},
\end{equation}

\begin{equation}
\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{\partial \vec{u}}{\partial y} = \frac{k \left( \frac{\partial^2 \vec{u}}{\partial x^2} + \frac{\partial^2 \vec{u}}{\partial y^2} \right)}{\rho c_p} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}.
\end{equation}

\( \vec{u}(x, y) \) in \( x \)-directions and \( \vec{v}(x, y) \) in \( y \)-directions are velocity components, \( T(x, y) \) represent temperature variation, \( \vec{p}(x, y) \) is the pressure, \( q_y = - \left( 16 \sigma \omega T^3/3 a_c \right) \partial T/\partial y \) is the radiative heat flux. The relevant boundary conditions can be defined as

\begin{equation}
\vec{y} = 0 : \quad \vec{u} = U_c = c \vec{c} + U_c \cos \Omega t, \quad \vec{y} = 0, \quad T = T_w + \Delta T(1 + a sin \Omega t),
\end{equation}

\begin{equation}
\vec{y} \to \infty : \quad \vec{u} = U_c = a \vec{c} + b \vec{y}, \quad T \to T_{\infty},
\end{equation}

where \( T_{\infty} \) is the surrounding temperature, \( a \), \( b \) and \( c \) are positive constants and \( \Delta T = T_w - T_{\infty} \) is the temperature difference.
of mean surface temperature to the surroundings temperature. Using \( q_r = -\left(16\sigma_{SB}T^3/3a_R\right)\partial T/\partial y \) in Eq. (8), we get

\[
0 = \frac{\partial^2 T}{\partial y^2} + k \frac{\partial T}{\partial y} + \left(1 + \frac{16\sigma_{SB}T^3}{3a_R}\right) \frac{\partial^2 T}{\partial y^2}.
\]

(10)

For the simplification, \( k \) is considered as constant and small temperature gradient within the flow. Linearization of heat flux \( q_r \) about the surrounding temperature \( T_\infty \), the Eq. (10) acquires the following form

\[
0 = \frac{\partial^2 T}{\partial y^2} + \frac{k}{\rho c_p} \frac{\partial T}{\partial y} + \frac{k}{\rho c_p} \left(1 + \frac{16\sigma_{SB}T^3}{3a_R}\right) \frac{\partial^2 T}{\partial y^2}.
\]

(11)

The important variables which are used to dimensionless the entire system of equations including the boundary constraints are

\[
x = \frac{x}{\sqrt{c}}, \quad y = \frac{y}{\sqrt{c}}, \quad u = \frac{1}{\sqrt{c}} \frac{\partial T}{\partial y}, \quad \psi = \frac{1}{\sqrt{c}} \frac{\partial^2 T}{\partial y^2}, \quad \tau = \frac{1}{\sqrt{c}} \frac{\partial^2 T}{\partial y^2}.
\]

(12)

in Eqs. (5-7), (9) and (11), in dimensionless form

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

(13)

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{k}{\rho c_p} \left(1 + \frac{16\sigma_{SB}T^3}{3a_R}\right) \frac{\partial^2 T}{\partial y^2}.
\]

(14)

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}.
\]

(15)

\[
\frac{\partial T}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{1}{\sqrt{c}} \frac{\partial^2 T}{\partial y^2} + \frac{N_r}{\sqrt{c}} \frac{\partial^2 T}{\partial y^2}.
\]

(16)

By pressure elimination from Eqs. (14) and (15), we get a single equation in form of stream function and one equation for temperature distribution

\[
\frac{\partial \psi}{\partial y} = \frac{1}{\sqrt{c}} \frac{\partial^2 T}{\partial y^2} + \frac{N_r}{\sqrt{c}} \frac{\partial^2 T}{\partial y^2},
\]

(19)

where \( \gamma = b/c \) represents the obliqueness factor in flow, \( \beta = \Omega/c \) is unsteady parameter in dimensionless form, \( We = k_0/c \) be the Wissenberg number which compares the viscous forces to the elastic forces, \( \lambda = \frac{9\beta T_{w-T_\infty}}{c\sqrt{c}} \) is the mixed convection parameter. In above Eq. (19) positive sign represents the assisting flow i.e. buoyancy force assist the flow and negative sign represents the opposing flow i.e. buoyancy force retards the flow. \( \lambda = 0 \) corresponds to forced convection flow when the buoyancy force is absent and \( \lambda \neq 0 \) corresponds to mixed convection flow when the buoyancy force is dominant. Pr = \( \mu c_p/k \) be the Prandtl number and \( N_r = 16\sigma_{SB}T_{w-T_\infty}^3/3a_R \) is the radiation parameter. Let us assume that the solution in form of stream function and temperature profile of Eqs. (19) and (20) subject to boundary constraints (21) is

\[
\psi = xf(y), \quad T = \theta(y, t),
\]

(22)

\( h(y, t) \) represents the oblique component of the flow. Using the Eq. (22) in Eqs. (19-21), and then comparing the coefficient of like powers of \( x^0 \) and \( x^1 \), we get

\[
ff'' + ff'' - f'f'' + We(f' - f'') = 0,
\]

(23)
Where \( \text{Pr}_{\text{eff}} = \frac{\text{Pr}}{1 + \text{Nr}} \) represent the effective Prandtl, which is the combined parameter of Prandtl and radiation as suggested by Magyari et al. \(^{34}\). For the smaller values of \( \text{Pr}_{\text{eff}} \) gets the maximum radiation contribution. Integrating Eqs. (23) and (24) with respect to \( y \) and removing the constant with help of boundary conditions prescribed at infinity, Eqs. (23-26) reduced to

\[
\begin{align*}
\frac{\partial^2}{\partial y^2} + \text{Pr}_{\text{eff}} \left( f \frac{\partial}{\partial y} + \beta \frac{\partial}{\partial t} \right) &= 0, \\
y = 0: & \quad f(y) = 0, \quad f'(y) = 1, \quad h(y,t) = 0, \\
y \to \infty: & \quad f(y) = a/c, \quad \frac{\partial h(y,t)}{\partial y} = y, \quad \theta(y,t) = 0.
\end{align*}
\]

Where \( \frac{\partial}{\partial y} \) is constant, measures the boundary layer displacement. Velocity components in dimensionless form are

\[
\begin{align*}
f'' + f'f'' - \left( f' \right)^2 + \text{We} \left( f'' - 2 f f'' + f' \right) + \left( \frac{a}{c} \right)^2 &= 0, \\
\frac{\partial^2}{\partial y^2} + \text{Pr}_{\text{eff}} \left( f \frac{\partial}{\partial y} + \beta \frac{\partial}{\partial t} \right) &= 0, \\
y = 0: & \quad f(y) = 0, \quad f'(y) = 1, \quad h(y,t) = 0, \\
y \to \infty: & \quad f(y) = a/c, \quad \frac{\partial h(y,t)}{\partial y} = y, \quad \theta(y,t) = 0.
\end{align*}
\]

The skin friction coefficients \( C_f \) and the local Nusselt number \( \text{Nu}_x \), can be expressed as

\[
C_f = \frac{\tau_w}{\rho u_w^2}, \quad \text{Nu}_x = \frac{k_{\text{eff}}(T_u - T_\infty)}{k_{\text{eff}}},
\]

where \( \tau_w \) is shear stress at the wall and \( k_{\text{eff}} \) is the effective conduction-radiation flux at the wall. These are defined in dimensionless form as

\[
\begin{align*}
\tau_w = \mu \left( u' v' - \nu \partial u'/\partial y \right), \\
q_{\text{eff}} &= -k_{\text{eff}} \left( T_u - T_\infty \right) \sqrt{c} \left( \frac{\partial T}{\partial y} \right)_{y=0}.
\end{align*}
\]

By using Eqs. (18) and (22), \( C_f \) and \( \text{Nu}_x \) take the following form

\[
\text{Re}_x C_f = x \left( 1 - 3 \epsilon \text{We} - \epsilon \text{We} \right) f'(0) + (1 - 2 \text{We}) \frac{\partial^2 h(0,t)}{\partial y^2} - \beta \text{We} \frac{\partial^2 h(0,t)}{\partial y^2},
\]

\[
\text{Re}_x^{1/3} \text{Nu}_x = -\partial h(0,t),
\]

where \( \text{Re}_x = c x^2 / v \) and \( u = \partial \psi / \partial y \) intersect the plate at the stagnation point. The location of stagnation point \( x_s \), can be find at zero skin friction or shear stress from Eq. (36) as follows

\[
x_s = \frac{-(1 - 2 \text{We}) \frac{\partial^2 h(0,t)}{\partial y^2} - \beta \text{We} \frac{\partial^2 h(0,t)}{\partial y^2} + \text{We} \frac{\partial^2 g(0,t)}{\partial y^2}}{(1 - 3 \epsilon \text{We} - \epsilon \text{We} f'(0) + (1 - 2 \text{We}) \frac{\partial^2 h(0,t)}{\partial y^2}).
\]

### 4. RESULTS AND DISCUSSION

The numerical simulation for non-linear partial differential equations is carried out by an efficient numerical technique CSNIS\(^{14}\), and the results are presented against all dimensionless parameters. The results with previous studies as limiting case are verified through numerical values, which are given in Table I. It is shown that our results are convergent and highly accurate. In Table II, the values of stagnation point \( (x_s) \) are shown at different time steps. It can be noted from the table that the point where the fluid velocity become zero and dividing stream line strike to the plate oscillates in to and fro motion against time. The interesting results are also plotted in term of velocity and temperature profiles, skin friction coefficient and Nusselt number in Figs. 2-12. In Fig. 13 temporal variation of streamlines for both assisting and opposing flow, cases are plotted. In Figs. 2-8, the values of the parameters are kept fixed \( \epsilon = 1, \epsilon_1 = 1, \gamma = 1, \beta = 0.1, \text{We} = 0.2, a/c = 0.2, \text{Pr}_{\text{eff}} = 3.3333 \). However, in Figs. 9-12, the values of the parameters are fixed \( \epsilon = 1, \epsilon_1 = 1, \gamma = 1, \beta = 0.2, \text{We} = 0.1, a/c = 0.2, \text{Pr}_{\text{eff}} = 3.3333 \) and in Fig. 13 fixed parameters are \( \epsilon = 1, \epsilon_1 = 1, \gamma = 2, \beta = 0.2, \text{We} = 0.1, a/c = 0.3, \text{Pr}_{\text{eff}} = 3.3333 \). The values of the parameters, which are used for solution of the problem other than mentioned above, are given in the figures. In Fig. 2, variation of velocity...
The buoyant force retards the flow having opposite direction. Which is due to the reason that in opposed flow region reduces as compare to the assisted flow region. It is noted from the figure that the velocity of the fluid is also observed that for fluid velocity having the same direction as of the flow field. It is also observed that for \(a/c > 1\), the velocity of the fluid along \(y\) oscillate due to viscoelastic behavior of the fluid.

Table I Comparison for \(x_s\), when \(We = \varepsilon = \varepsilon_i = \beta = 0, \gamma = 1\), and \(Pr_{eff} = Pr = 0.72\) (\(N_r=0\)). The results reported by Lok et al.\(^{29}\) are given in braces.

| \(\lambda\) | \(\frac{a}{c} = 0.1\) | \(\frac{a}{c} = 0.5\) | \(\frac{a}{c} = 1.5\) | \(\frac{a}{c} = 0.1\) | \(\frac{a}{c} = 0.5\) | \(\frac{a}{c} = 1.5\) |
|---|---|---|---|---|---|---|
| 0.1 | 0.3563 | 1.2653 | -1.2612 | 0.1871 | 1.0807 | -1.1662 |
| | (0.3564) | (1.2652) | (-1.2612) | (0.1872) | (1.0806) | (-1.1661) |
| 0.2 | 0.4409 | 1.3575 | -1.3087 | 0.1025 | 0.9884 | -1.1186 |
| | (0.4410) | (1.3575) | (-1.3087) | (0.1026) | (0.9884) | (-1.1186) |
| 0.5 | 0.6947 | 1.6344 | -1.4513 | -0.1513 | 0.7116 | -0.9761 |
| | (0.6948) | (1.6343) | (-1.4513) | (-0.1513) | (0.7115) | (-0.9761) |
| 1.0 | 1.1178 | 2.0958 | -1.6889 | -0.5743 | 0.2501 | -0.7385 |
| | (1.1178) | (2.0958) | (-1.6888) | (-0.5743) | (0.2501) | (-0.7385) |
| 1.5 | 1.5408 | 2.5572 | -1.9265 | -0.9973 | -0.2113 | -0.5009 |
| | (1.5408) | (2.5572) | (-1.9264) | (-0.9973) | (-0.2113) | (-0.5009) |
| 2.0 | 1.9638 | 3.0186 | -2.1641 | -1.4203 | -0.6727 | -0.2633 |
| | (1.9638) | (3.0186) | (-2.1640) | (-1.4203) | (-0.6727) | (-0.2633) |
| 3.0 | 2.8098 | 3.9415 | -2.6393 | -2.2663 | -1.5956 | 0.2119 |
| | (2.8098) | (3.9414) | (-2.6392) | (-2.2663) | (-1.5956) | (0.2119) |
| 5.0 | 4.5018 | 5.7871 | -3.5897 | -3.9583 | -3.4412 | 1.1623 |
| | (4.5019) | (5.7871) | (-3.5896) | (-3.9583) | (-3.4413) | (1.1623) |
| 10.0 | 8.7519 | 10.4013 | -5.9656 | -8.1884 | -8.0553 | 3.5383 |
| | (8.7320) | (10.4013) | (-5.9656) | (-8.1884) | (-8.0554) | (3.5383) |

Table II Values of \(x_s\) at different time steps when \(We = 0.1, \varepsilon = 1, \varepsilon_i = 1, \gamma = 1, \beta = 0.1\) and \(Pr_{eff} = 3.3333\).

| \(a/c\) | \(\frac{\lambda}{t}\) | \(t = 0\) | \(t = \pi/4\) | \(t = \pi/2\) | \(t = \pi\) | \(t = 0\) | \(t = \pi/4\) | \(t = \pi/2\) | \(t = \pi\) |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | -1.0675 | -0.5770 | 0.4553 | 1.7371 | -1.1827 | -0.7671 | 0.2262 | 1.6140 |
| | 0.5 | -0.8372 | -0.1967 | 0.9135 | 1.9833 | -1.4130 | -1.1474 | -0.2321 | 1.3677 |
| | 1 | -0.5494 | 0.2787 | 1.4862 | 2.2911 | -1.7008 | -1.6228 | -0.8048 | 1.0599 |
| | 5 | 1.7536 | 4.0817 | 6.0683 | 4.7535 | -4.0038 | -5.4258 | -5.3869 | -1.4024 |
| 0.5 | 0.1 | -0.7365 | -0.0006 | 1.5902 | 3.6310 | -0.8832 | -0.2451 | 1.2978 | 3.4770 |
| | 0.5 | -0.4433 | 0.4884 | 2.1751 | 3.9390 | -1.1764 | -0.7340 | 0.7130 | 1.6900 |
| | 1 | -0.0768 | 1.0996 | 2.9061 | 4.3241 | -1.5429 | -1.3453 | -0.0180 | 2.7839 |
| | 5 | 2.8556 | 5.9894 | 8.7543 | 7.4043 | -4.4753 | -6.2350 | -5.8663 | -0.2963 |
| 1.5 | 0.1 | 0.4276 | -0.1567 | -1.4724 | -3.2301 | 0.5086 | -0.0202 | -1.3106 | -3.1467 |
| | 0.5 | 0.2655 | -0.4297 | -1.7961 | -3.3969 | 0.6706 | 0.2527 | -0.9869 | -2.9799 |
| | 1 | 0.0630 | -0.7709 | -2.2008 | -3.6054 | 0.8732 | 0.5940 | -0.5823 | -2.7714 |
| | 5 | -1.5574 | -3.5007 | -5.4377 | -5.2735 | 2.4935 | 3.3237 | 2.6547 | -1.1034 |

Figure 3 elaborate the velocity variation against \(y\) for different values of \(Pr_{eff}\) at \(t = \pi/4\), for aiding and opposing flow cases. Here it is necessary to mention that we have combined the radiation and Prandtl number in Prandtl effective as proposed by Magyari et al.\(^{29}\) i.e. \(Pr_{eff} = Pr/(1+N_r)\) where Pr is the Prandtl number and \(N_r\) is the thermal radiation parameter. In their article, they suggested that the effect of radiation is always the same, it shift formally the value of Prandtl number from Pr to the smaller value of \(Pr_{eff}\). In assisted flow region, the velocity is observed as decreasing function of effective Prandtl number where in opposed flow region an opposite profile against \(y\) is shown for various values of \(a/c\). In the figure, the curves are drawn at \(t = \pi/4\) for assisting and opposing cases. It is noted from the figure that the velocity of the fluid increases with increase of \(a/c\). Further, the velocity in opposed flow region reduces as compare to the assisted flow region. Which is due to the reason that in opposed flow region the buoyant force retards the flow having opposite direction, where in assisted flow region it helps to enhance the fluid velocity having the same direction as of the flow field. It is also observed that for \(a/c > 1\), the velocity of the fluid along \(y\) oscillate due to viscoelastic behavior of the fluid.
behavior is observed. In other words, with the increase of radiation parameter $Nr$, the velocity of the fluid enhances in assisted flow region where as in opposed flow region, the velocity of the fluid decreases. It is due to the reason that radiation enhances the temperature, which results in enhancement of buoyant force. Therefore, velocity increases in assisted flow region and decreases in opposed flow region. Figure 4 is plotted to predict the temperature variation for the various values of $a/c$ and temperature is found to be a decreasing function of $a/c$. It is because of, stagnation point encounters the highest heat transfer rate and therefore the temperature reduces with increase of free stream velocity. Figure 5 presents the temperature variation against $y$ at $t = \pi/4$ for the various values of $Pr_{eff}$. With increase of $Pr_{eff}$, the temperature of the fluid decreases. As effective Prandtl number and radiation parameter having inverse relation, so it can also be depict from Fig. 5, that temperature is increasing function of radiation parameter $N_r$. In Fig. 6, the velocity profile is plotted against $y$ at $t = \pi/4$ for the various values of $We$. In this figure, both cases of boundary layer structure ($a/c > 1$) and inverted boundary layer structure ($a/c < 1$) are discussed. It is observed that in case of boundary structure, the velocity of the fluid increases with increase of viscoelastic parameter $We$. On the other hand, in inverted boundary layer case, the velocity of fluid decreases with increase of viscoelastic parameter $We$. Similarly, in Fig. 7, the temperature profile is plotted against $y$ at $t = \pi/4$ for the various values of $We$ for both case of $a/c < 1$ and $a/c > 1$. In case of boundary layer structure, the temperature of the fluid falls due to increase of viscoelastic parameter $We$, while on the other hand, for $a/c < 1$ temperature enhances with the increase in the value of viscoelastic parameter. Figure 8, shows the effect of Weissenberg number on velocity profile for both assisting and opposing flow cases. Further it is noted that in opposing flow case velocity profile rapidly change as compared to assisting flow case. In Figs. 9 and 10, variation of skin friction coefficient and Nusselt number are plotted against $t$ for various values of $Pr_{eff}$ at $\lambda = 1$. With increase of $Pr_{eff}$ the skin friction coefficient decreases and heat transfer rate increases. It is also depicted form Fig. 10 that the increase in radiation parameter results in reduction of heat transfer rate. In Figs. 11 and 12, the variation of skin friction is shown against $t$, for the various values of mixed convection parameter $\lambda$ in assisted and opposed flow region is plotted.
Figure 11 is plotted for non-oscillating flat plate and observed that the skin friction coefficient increases with increase of mixed convection parameter $\lambda$ in assisting flow case, while on the other hand the skin friction decreases in opposing flow case. Figure 12 is plotted for oscillating flat plate. In this figure, same behavior is observed for both assisting and opposing flow cases as in Fig. 11. The temporal variation of streamlines is shown in Fig. 13 (a-h) which are calculated by Eq. (22) at different time steps $t = 0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$ respectively for the both assisting and opposing flow cases. The solid lines are drawn for assisting flow case and
Fig. 13(a-h) Temporal variation of streamlines with $\epsilon_1 = 1, \epsilon = 1, \gamma = 2, \beta = 0.2, We = 0.1, a/c = 0.3$ and $Pr_{eff} = 3.3333$ at different time steps for both assisting and opposing flow cases.
dotted lines are for opposing flow case. The stagnation point is calculated with help of Eq. (37) where the velocity of the fluid become zero. Dot (•) represents the position of stagnation point in assisting flow case and asterisks (*) represents the position of stagnation point in opposing flow case. It is seen from the Fig. 13 (a-h) that the stagnation point oscillates due to the oscillation of plate. The streamlines in both assisting and opposing flow cases come closer to each other with the passage of time and at \( t = 3\pi/2 \), they overlap each other, and stagnation points also get coincide.

5. CONCLUSION

In this article, the influence of thermal radiation and conduction in the region of oblique stagnation point flow is investigated subjected to the sinusoidal surface temperature of the vertical flat plate. The obtained dimensionless PDEs are solved numerically by using CSNIS. To check the validity of our results, the numerical values of \( f''(0), \theta'(0) \) and \( x \), are verified with the old studied as a special case. Furthermore, the result for sundry parameters are given graphically and in tabular form. It is observed that skin friction coefficient, local Nusselt number near stagnation point oscillate because of sinusoidal nature of the plate and surface temperature in both assisted and opposed flow region. It is also concluded that

- The velocity is found as an increasing function of \( a/c \), where an opposite behavior is observed for temperature profile.
- In assisted flow region, velocity is observed as decreasing function of effective Prandtl number where in opposed flow region it increases.
- Temperature is decreasing with increase of effective Prandtl number, which means that radiation enhances the fluid temperature.
- For large straining velocity (\( a/c > 1 \)), the velocity of the fluid increases while the temperature falls due to increase of viscoelastic parameter \( We \).
- For large stretching velocity (\( a/c < 1 \)), the velocity of fluid decreases while the temperature enhances due to increment of viscoelastic parameter \( We \).
- With increase of \( Pr_{eff} \) the skin friction decreases and heat transfer rate increases.

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| Nomenclature                                      | Greek symbols                     |
|--------------------------------------------------|-----------------------------------|
| Constant having dimension [1/time]                |                                   |
| $a$, $b$, $c$                                     |                                   |
| $C_p$ Specific heat constant                      | $\beta$ Unsteady parameter       |
| $f$ Normal component of flow                      | $\beta_T$ thermal expansion coefficient |
| $g$ Gravitational acceleration                    |                                   |
| $h$ Oblique component of flow                     | $\rho$ Fluid density              |
| $k$ Thermal conductivity of the fluid             | $\Omega$ Frequency of oscillation |
| $k_o$ Elasticity of fluid                         |                                   |
| $Nu$ Nusselt number                               |                                   |
| $p$ Pressure                                      | $\epsilon$ Amplitude of the plate oscillation |
| $Pr_{eff}$ Effective Prandtl number                |                                   |
| $q_{eff}$ Effective conduction-radiation flux     | $\lambda$ Mixed convection parameter |
| $q_r$ Radiative heat flux                         | $\epsilon_1$ Amplitude of imposed temperature oscillation |
| $R_e$ Local Reynolds number                       | $\gamma$ Obliqueness parameter    |
| $T$ Temperature of the fluid in the boundary layer| $\mu$ Dynamic viscosity           |
| $t$ Time                                          | $\nu$ Kinematic viscosity         |
| $T_s$ Ambient fluid temperature                   | $\Theta$ Dimensionless temperature |
| $T_w$ Surface temperature                         | $\Psi$ Stream function            |
| $U_e$ Free stream velocity                        |                                   |