1/f noise and slow relaxations in glasses

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Abstract

Recently we have shown that slow relaxations in the electron glass system can be understood in terms of the spectrum of a matrix describing the relaxation of the system close to a metastable state. The model focused on the electron glass system, but its generality was demonstrated on various other examples. Here, we study the noise spectrum in the same framework. We obtain a remarkable relation between the spectrum of relaxation rates $\lambda$ described by the distribution function $P(\lambda) \sim 1/\lambda$ and the 1/f noise in the fluctuating occupancies of the localized electronic sites. This noise can be observed using local capacitance measurements. We confirm our analytic results using numerics, and also show how the Onsager symmetry is fulfilled in the system.

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1. INTRODUCTION

$1/f$ noise is ubiquitous and has been observed in a variety of physical systems, such as metals, semiconductors and superconductors [1, 2], as well as biological and economic systems [3, 4]. Systems displaying slow relaxations and aging are also abundant, and a broad range of materials, such as spin glasses [5], structural glasses [6], electron glasses [7] and biological systems [8] have been shown to exhibit these properties.

There has been much interest in studying $1/f$ noise in electron glasses, both experimentally and theoretically [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Yu [22] has motivated the study of noise in electron glasses, particularly the study of the ‘second noise’ (describing the fluctuations in the noise spectrum), as a tool to distinguish between different glassy models.

In a recent work [23], we have proposed a model showing that the slow relaxations observed in the electrons glasses can be understood in terms of an underlying $1/\lambda$ distribution of relaxation rates, which arise naturally as the eigenvalues of a certain class of random matrices. It is the purpose of this work to show that the same model predicts a $1/f$ noise spectrum for the site occupancies, arising directly from the $1/\lambda$ distribution. This establishes a remarkable relation between $1/f$ noise and the universal slow relaxations. We study in detail the electron glass model, but the established connection between the two should be much more general than this specific system of interest, since the same equations can be used to describe a broad range of systems.

The structure of the manuscript is as follows: we first define the model, and discuss briefly the local mean-field approximation used. We then review the previously derived results for the spectrum of the relaxation rates, and set up the equations describing the near-equilibrium fluctuations. We show that the Onsager symmetry is obeyed in the system, resulting in the specific form of the non-hermitian relaxation matrix, and show that using Onsager’s regression hypothesis we can relate the noise spectrum with the $1/\lambda$ relaxation spectrum. Finally, we show numerical support for the calculation, and propose an experimental test for it.
II. DEFINITION OF THE MODEL AND THE LOCAL MEANFIELD APPROXIMATION

The model typically used to study electron glasses addresses a set of states, which are assumed to be Anderson localized due to the strong disorder present. While many studies have been performed, usually for numerical convenience, on a lattice, we choose to work with localized states positioned randomly in space, which we find more realistic. The number of electrons $M$ is smaller than the number of sites $N$. The states' on-site energies contain disorder, of typical magnitude $W$. Due to the localization, the Coulomb interactions between the electrons are not screened. They cannot be neglected in the analysis and are not neglected within the model. The electrons are coupled to a phonon bath, assumed at thermal equilibrium. Details of the model are found in [24], where we use a local mean-field approximation [25] to study this model. We should emphasize that within this approximation the on-site and spatial disorder are not averaged over. We find that the approximation captures the well-known results for the Coulomb gap [26, 27] in the density-of-states (DOS), a soft gap emerging near the Fermi-energy as a result of the Coulomb interactions. It was also used to describe the slow relaxations experimentally observed in an aging experiment [23]. In a later study we have shown that this model also describes the transition in the hopping conductance, from Mott’s variable range hopping at higher temperatures to Efros-Shklovskii variable range hopping below the crossover [28].

An important point is that within the local mean-field approximation, the occupation numbers are continuous variables, and their dynamics is described by a coupled set of $N$ non-linear differential equations. Metastable states are configurations of occupation numbers for which the dynamics vanishes. In [24], we show that one can understand the dynamics near one of the many metastable states by linearizing the equations of motion close to it. This led us to an equation for the vector of deviations $\mathbf{\delta n}$ from the equilibrium configuration in which the $i$’th site has the occupation $n_i^0$:

$$\frac{d\mathbf{\delta n}}{dt} = A \cdot \mathbf{\delta n},$$  \hspace{1cm} (1)

where the explicit form of the non-diagonal elements of $A$ is:
\[ A_{ij} = \frac{\gamma_{ij}}{n_j^0(1-n_j^0)} - \sum_{k \neq j,i} e^{2\gamma_{ik}} \frac{1}{T} \left( \frac{1}{r_{ij}} - \frac{1}{r_{jk}} \right), \]  

with \( \gamma_{ij} \) the equilibrium current from site \( i \) to site \( j \) (detailed balance is equivalent to \( \gamma \) being a symmetric matrix). From particle number conservation it follows that the sum of each column of \( A \) vanishes. Therefore the diagonal element of the matrix is minus the sum of the rest of the elements in the column, i.e., \( A_{ii} = -\sum_{j \neq i} A_{ij} \).

Notice that exactly at the metastable state there would be absolutely no dynamics (or fluctuations) within this equation, even though it describes a system at finite temperature. This is clearly not physical, and in subsection IV C we add a necessary Langevin noise term, and determine its magnitude from thermodynamic considerations.

III. KNOWN RESULTS FOR THE SPECTRUM

It can be shown that the eigenvalues of the matrix \( A \) are real and negative. For a proof that the eigenvalues are real see subsection IV D, and for the negativity property see Ref. [17]. In [24] we have given numerical evidence that the model described in section II yields a spectrum which is approximately:

\[ P(\lambda) \approx \frac{C}{\lambda}, \quad C = \frac{1}{\log \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}}}, \]  

where \( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \) are the upper and lower cutoffs of the distribution. In a later work [29] we discuss a toy-model for this problem: we define a class of random matrices whose sum of columns vanish, and where the \( i,j \)'th element decays exponentially with the distance of points \( i \) and \( j \), distributed randomly. Notice that these are exactly the properties of the matrix \( A \) described in section II. The merit of this simplification of the problem is that we manage to find an analytical formula for the averaged spectrum of the matrices, and verify the numerical result: up to logarithmic corrections in dimensions higher than one, the spectrum follows the \( 1/\lambda \) spectrum in the low density regime. This is the essential result we would use in the following, and which would lead to the \( 1/f \) noise in the fluctuation of the occupation numbers.
IV. DERIVATION OF THE NOISE SPECTRUM

Onsager’s regression hypothesis, (see the Appendix for a derivation) states that the equation of motion of the correlation function \( \phi_{ij}(t) = \langle \delta n_i(t) \delta n_j(t) \rangle \) is obtained by simply replacing \( \delta n_i(t) \) in the equation of motion (1) by the function \( \phi_{ij}(t) \):

\[
\frac{d\phi_{ij}(t)}{dt} = A_{ik} \phi_{kj}(t). \tag{4}
\]

To find the correlation function one also needs the initial conditions:

\[
\phi_{ij}(0) = \langle \delta n_i(0) \delta n_j(0) \rangle \equiv \beta_{ij}^{-1}. \tag{5}
\]

A. Equilibrium correlations

By definition, the inverse of the matrix \( \beta \) describes the equilibrium correlations in the system. We would now like to calculate \( \beta \). Notice that generally \( \beta_{ij} = \beta_{ji} \) and that, for example, for a free Fermion system we have \( \beta_{ij}^{-1} = n_i(1 - n_j)\delta_{ij} \).

In our case, we consider an interacting Fermion system, and it is not a-priori clear that we can use the above form. One might argue that within the local mean-field approximation the problem is essentially a non-interacting problem, with renormalized on-site energies due to the Coulomb interactions. It is therefore reasonable that the free Fermion result, not depending on the on-site energies, would still be valid. However, we shall now show that there is an additional term in \( \beta \) arising from the interactions. To do this, we write the free energy [25]:

\[
F = \sum_i \epsilon_i \tilde{n}_i + \sum_{i \neq j} \frac{e^2 \tilde{n}_i \tilde{n}_j}{r_{ij}} + kT \sum_i \left( \frac{1}{2} + \tilde{n}_i \right) \log(\frac{1}{2} + \tilde{n}_i) + \left( \frac{1}{2} - \tilde{n}_i \right) \log(\frac{1}{2} - \tilde{n}_i), \tag{6}
\]

with \( \tilde{n}_i \equiv n_i - \frac{1}{2} \). The local mean-field equations can be obtained from the minimization condition \( \frac{\partial F}{\partial \tilde{n}_i} = 0 \). Notice that each site contains a positive background charge of \( \frac{1}{2} \), to keep charge neutrality.

Expanding \( F \) near a metastable state (a local minima), we have:

\[
F = F_0 + \frac{kT}{2} \sum_{i,j} \beta_{ij}\delta n_i \delta n_j, \tag{7}
\]
with:

\[
\beta_{ij} = \delta_{ij} \frac{1}{n_i^0(1-n_i^0)} + \frac{e^2}{kT_{ij}}.
\]  

The correlation matrix is proportional to \( \beta^{-1} \), since we have a quadratic free energy:

\[ \langle \delta n_i \delta n_j \rangle = \beta_{ij}^{-1}. \]  

The free fermion result is corrected for \( i \neq j \) by the interaction term. Notice that at low temperatures \( n_i \) tend to 0 or 1 exponentially, and thus the matrix is nearly diagonal, i.e., the interaction term is negligible.

The matrix \( \beta \) has another physical meaning: if we introduce the conjugate variables to \( n_i \), the so-called thermodynamic forces \( \mu_i \), defined as \( \frac{\partial F}{\partial n_i} \), we find that

\[ \mu_i = \beta_{ik} n_k. \]  

B. Onsager symmetry

According to Onsager’s principle (see Appendix) the kinetic coefficients,

\[ \gamma_{ij} = \sum_k A_{ik} \beta_{kj}^{-1} \]  

are symmetric:

\[ \gamma_{ij} = \gamma_{ji}. \]  

Indeed, Eqs. 2 and 8 are related according to:

\[ A = \gamma/\beta, \]  

where \( \gamma \) is the symmetric matrix describing the transition rates at equilibrium, defined earlier. This puts the results of the linearization procedure of [24] in a much more natural context, and explains why the matrix \( A \) describing the relaxation is not hermitian in the general case: the multiplication of the symmetric matrix \( \gamma \) and the symmetric matrix \( \beta \) is not expected to be (and is generally not) a symmetric matrix.
C. Langevin approach

Due to the finite temperature, we expect the system to fluctuate around the equilibrium distribution. This can be modeled using a Langevin equation. In a similar fashion to the usual treatment of Brownian motion, where the equation governing the motion is \( \frac{dv}{dt} = -\frac{v}{\tau} + f(t) \), we add a Langevin noise term to Eq. (1):

\[ \frac{d\delta n}{dt} = A \cdot \delta n + f(t), \]

where \( f_i \) is a white-noise term. We shall now show, however, that the different components of the noise vector must be correlated, i.e., \( \langle f_i f_j \rangle \equiv F \) is a non-diagonal matrix. As in the case of Brownian motion, thermodynamic considerations determine the magnitude of the noise, as well as the correlations between the different vector components. It is known that \( F = A\beta^{-1} = \gamma \) \[30\]. There is a clear physical intuition for this: correlations between sites \( i \) and \( j \) are contributed from the direct microscopic current between the two sites, \( \gamma_{ij} \). Since \( \gamma \) is not diagonal, there exist correlations between the different components of the noise vector, as stated previously. See the Appendix for an explanation of the physical meaning of the matrix elements of \( \gamma \) as kinetic coefficients.

D. Form of the spectrum

It is useful to diagonalize Eq. (1) by multiplying both sides on the left by \( \beta^{1/2} \). We then have, using \( \{13\} \) and a matrix notation:

\[ \frac{d(\beta^{1/2}\phi(t))}{dt} = \beta^{1/2}\gamma\beta^{1/2}(\beta^{1/2}\phi(t)). \]

Defining \( \tilde{\phi} = \beta^{1/2}\phi \) and \( \tilde{\gamma} = \beta^{1/2}\gamma\beta^{1/2} \) we reduce the equation to a symmetric form (notice that \( \tilde{\gamma} = \tilde{\gamma}^T \)):

\[ \frac{d\tilde{\phi}(t)}{dt} = \tilde{\gamma}\tilde{\phi}(t). \]

We can now diagonalize the symmetric operator \( \tilde{\gamma} \) in a standard way:

\[ \tilde{u}^\dagger\tilde{\gamma}\tilde{u} = \tilde{\Lambda} \]

with \( \tilde{\Lambda} \) a diagonal matrix, and the elements of \( \tilde{u}_{ij} = \psi_i^j \) are the orthonormal eigenbasis \( (\tilde{u}^\dagger\tilde{u} = \tilde{u}\tilde{u}^\dagger = I) \) of \( \tilde{\gamma} \). The eigenvalues of \( \tilde{\gamma} \) turn out to be identical to those of \( A \). Notice
that this proves that the eigenvalues of $A$ are real. To see this, it is useful to utilize the definition $u^{-1} \equiv \tilde{u}^{\dagger} \beta^{1/2}, u = \beta^{-1/2} \tilde{u} \Rightarrow \tilde{u} = \beta^{1/2} u$. Thus:

$$
\tilde{\Lambda} = \tilde{u}^{\dagger} \beta^{1/2} \gamma \beta^{1/2} \tilde{u} = u^{-1} \gamma \beta^{1/2} \tilde{u} = u^{-1} \gamma \beta u = u^{-1} Au
$$

(18)

We can immediately write the solution for $\tilde{\phi}$:

$$
\tilde{\phi} = e^{-\tilde{\gamma} t} \tilde{\phi}(t = 0) = \tilde{u} e^{-\tilde{\Lambda} t} \tilde{\phi}(t = 0) \Rightarrow \phi = \beta^{-1/2} \tilde{u} e^{-\tilde{\Lambda} t} \beta^{-1/2}
$$

Explicitly:

$$
\phi_{ii}(t) = \sum_{\alpha} \frac{1}{\beta_{ii}} |\psi^{\alpha}\rangle |\psi^{\alpha}\rangle^* e^{-\lambda_{\alpha} t}
$$

(19)

This equation is an exact formula for the correlation function at site $i$, and will be later used to numerically find the spectrum. At low temperatures, as mentioned in section IV.A, the occupations tend to 0 and 1 exponentially since they follow Fermi-Dirac statistics. For all sites whose distance from the Fermi-energy is larger than the temperature, we can therefore neglect the off-diagonal elements of the matrix. This is nevertheless a non-trivial approximation, since the sites close to the Fermi-energy are the ones contributing the most to the noise. For this reason we test the validity of this approximation numerically, and show that it doesn’t change the form of the spectrum at the end of this section. Making the approximation we obtain:

$$
\phi_{ii}(t) = \sum_{\alpha} \frac{1}{\beta_{ii}} |\psi^{\alpha}\rangle |\psi^\alpha\rangle e^{-\lambda_{\alpha} t}
$$

Or in the frequency domain:

$$
\phi_{ii}(\omega) = \sum_{\alpha} \frac{1}{\beta_{ii}} |\psi^\alpha\rangle |\psi^\alpha\rangle^* \frac{2\lambda_{\alpha}}{\omega^2 + \lambda_{\alpha}^2}
$$

(20)

This is an explicit formula for the noise spectrum. We can make a further approximation, by noticing that for most sites $1/\beta_{ii} = n_i(1 - n_i)$ is very close to zero, since $n_i$ is the Fermi function of the on-site energy, which is typically much greater than the temperature. Therefore, only a small fraction of sites for which the on-site energy (renormalized by the interaction) is of the order of $kT$ away from the Fermi energy, will contribute to the sum. This is physically clear: it is exactly those sites which are close to the Fermi energy whose charge can fluctuate and contribute to the noise. Other sites are nearly permanently empty or full. For sites with energies $E_i$ a distance much smaller than $kT$ from the Fermi energy,
\( n_i(1-n_i) = 1/4 \). We therefore make an approximation and sum only over a partial number of the sites, for which we replace \( 1/\beta_{i,i} \) by \( 1/4 \). This leads us to the equation:

\[
\phi_{ii}(\omega) \sim \sum_{\alpha} \frac{1}{4} |\psi^\alpha_i|^2 \frac{2\lambda_\alpha}{\omega^2 + \lambda_\alpha^2}.
\]

(21)

Taking another average over the different sites, we can replace \( \langle\langle (\psi^\alpha_i)^2 \rangle\rangle \) by a constant \( \frac{1}{N} \), leading to:

\[
\langle\langle \delta n^2 \rangle\rangle_\omega \sim \frac{1}{N} \sum_{\alpha,i} \frac{2\lambda_\alpha}{1 + (\frac{\omega}{\lambda_\alpha})^2},
\]

(22)

where \( \langle\langle ; \rangle\rangle \) denotes averaging over sites as well as time.

At this stage the equation takes the form leading to \( 1/f \) noise in various other theories: a discrete sum of equally contributing Lorentzians. The crucial thing is that the non-trivial weights arising from the matrix \( \beta \) did not affect the structure of the noise statistics. Finally, turning the sum into an integral and using the fact that \( P(\lambda) \sim \frac{1}{\lambda} \) in a large window \( [24] \), we obtain:

\[
\langle\langle \delta n^2 \rangle\rangle_\omega \sim \frac{1}{N} \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} d\lambda \frac{1}{1 + (\frac{\omega}{\lambda_\alpha})^2} = \frac{1}{N\omega} \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} dm \frac{1}{1 + m^2}.
\]

(23)

This shows that for \( \lambda_{\text{min}} \ll \omega \ll \lambda_{\text{max}} \), a \( 1/f \) spectrum indeed follows for the noise in the average occupation number, which is one of our main results. In the future we intend to study the implications of the fluctuations in the occupation numbers on the conductance fluctuations.

We have performed a numerical test of the calculation, taking the complete form of the matrix \( \beta \), including the off-diagonal elements, and using Eq. (19). Figure 1 shows the results for the spectrum, as well as explanations of the numerical procedure. The best linear fit over more than 4 decades gave a slope of \(-1.03 \pm 0.03\), close to the expected result.

V. A PROPOSAL FOR AN EXPERIMENT

A \( 1/f \) noise spectrum has been observed experimentally in electron glasses \([18, 19, 20, 21]\). The measurements performed so far, however, were always for the conductance. This is not the quantity we calculate in this work, although \([23]\) shows a strong connection between the excess conductance and the deviations of the occupations. We hereby propose another
FIG. 1: A numerical test of the noise spectrum. In the first step, the equilibrium occupation numbers, energies and transition rates were found for an arbitrary metastable state, using the procedure described in [24]. Next, the matrix $\beta$ was constructed using Eq. (8), and the spectrum of $A$ defined in Eq. (13) was found. Finally, the spectrum was computed using the Fourier transform of Eq. (19), and averaged over the different sites. The spectrum was averaged over 100 realizations. The best fit over more than 4 decades gave a slope of $-1.03 \pm 0.03$. The system is one-dimensional, with $N = 100$ sites, and $20T = e^2/r_{nn} = W$, where $T$ is the temperature, $r_{nn}$ the average nearest-neighbor distance, and $W$ the magnitude of the disorder. $\xi/r_{nn} = 0.1$, where $\xi$ is the localization length.

possible experiment, in which the fluctuations in the occupation numbers are measured directly. In this experiment a small capacitor is placed in close proximity to the sample. The fluctuations in the occupation numbers of the neighboring sites would induce fluctuations in the voltage of the capacitor, which can be measured. Averaged over the different location in the sample, these should obey the $1/f$ noise discussed in section IV. A similar experimental technique has been implemented by [31, 32, 33]. It would be very interesting to see whether a change in the nature of the noise is seen across the metal-insulator transition in such a measurement.

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**APPENDIX A: ONSAGER SYMMETRY OF KINETIC COEFFICIENTS**

It is the purpose of this Appendix to give a concise derivation of the Onsager regression hypothesis and the Onsager symmetry of the kinetic coefficients, along the lines of [30].

Let us consider fluctuations of variables $n_i$ around some stable state in thermal equilibrium. For convenience we take the equilibrium values to be 0. Close to the stable point we can expand the free energy to second order (the first order vanishes since we are at an extremum), and arrive at Eq. (7):

$$ F = \sum_{i,j} \frac{kT}{2} \beta_{ij} n_i n_j. $$ \hspace{1cm} (A1)

The Boltzmann distribution immediately gives us, after a gaussian integration, that:

$$ \langle n_i(0) n_j(0) \rangle = \beta^{-1}_{ij}, $$ \hspace{1cm} (A2)

where $\langle \rangle$ is the statistical ensemble average. We see that the inverse of $\beta$ gives the equal-time correlation matrix.

We now define the conjugate variable to $n$ (the thermodynamic forces), as in section IV:

$$ \mu_i = \frac{\partial F}{\partial n_i} = \beta_{ik} n_k. $$ \hspace{1cm} (A3)

A similar calculation to the one done for the correlations of $n$ gives that $\langle \mu_i \mu_j \rangle = \beta_{ij}$, and $\langle n_i \mu_j \rangle = \delta_{ij}$.

Let us assume the linearized equation of motion at the stable state is $\dot{n} = A\tilde{n}$.

Defining $\phi_{ij} \equiv \langle n_i(t) n_j(0) \rangle$, we obtain that:

$$ \dot{\phi}_{ij} = \langle \dot{n}_i n_j(0) \rangle = A_{ik} \langle n_k(t) n_j(0) \rangle. $$ \hspace{1cm} (A4)

Thus we have the matrix equation:

$$ \dot{\phi} = A\phi. $$ \hspace{1cm} (A5)

This is the **Onsager regression principle**, stating that the correlation function obeys the same relaxation equation as that of the microscopic variables.
Let us look at \[ C_{ij}(t) \equiv \frac{d\phi_{ij}}{dt} = \frac{d\langle n_i(t)n_j(0) \rangle}{dt}. \] Assuming time-reversal symmetry, \( C \) is a symmetric matrix: Since we are at equilibrium we have time translation invariance, thus
\[ \frac{d\langle n_i(t)n_j(0) \rangle}{dt} = \frac{d\langle n_i(0)n_j(-t) \rangle}{dt}, \] and using the time-reversal symmetry this is equal to \( \frac{d\langle n_i(0)n_j(t) \rangle}{dt} = C_{ji}. \) Taking \( t = 0 \) we obtain:

\[ C_{ij}(0) = \langle A_{i,k}n_k(0)n_j(0) \rangle = A_{i,k}\beta_{kj}^{-1}. \]  

Thus, \( A\beta^{-1} \equiv \gamma \) is a symmetric matrix.

Notice that \( \dot{\vec{n}} = A\beta^{-1}\beta\vec{n} = \gamma\vec{\mu} \), which is why the elements of \( \gamma \) are called kinetic coefficients, relating the fluxes (derivatives of \( n \)) to the forces \( \mu \).

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