Hadron production in pA collisions at the LHC from the Color Glass Condensate

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We investigate the contribution of inelastic and elastic processes to single inclusive hadron production in proton-proton and proton (deuteron)-nucleus collisions at RHIC and the LHC. Using the hybrid formulation which includes both elastic and inelastic contributions, supplemented with the running-coupling Balitsky-Kovchegov equation, we get a good description of RHIC data. It is shown that inclusion of the inelastic terms makes the transverse momentum dependence of the production cross section steeper in the mid-rapidity region but does not affect the cross section in the very forward region. The inelastic processes also lead to a sharper increase of the nuclear modification factor $R_{pA}$ with increasing $p_T$. We also make predictions for the nuclear modification factor in proton-nucleus collisions at the LHC ($\sqrt{s} = 4.4$ and $8.8$ TeV) at various rapidities using the Color Glass Condensate framework.

I. INTRODUCTION

The Color Glass Condensate (CGC) formalism\textsuperscript{1} is a self-consistent, effective theory approach to QCD interactions at high energy (or equivalently small $x$). Even though it is a weak coupling approach, it is different from the collinear factorization based approach of pQCD in two important aspects; first, it re-sums quantum corrections which are enhanced by large logarithms of $1/x$ as opposed to large logarithms of $Q^2$ in pQCD and second, it includes high gluon density effects which are important at small $x$ and/or for large nuclei where the physics of gluon saturation may be the dominant.

The CGC formalism has successfully been applied to many QCD processes, from fully inclusive ones such as structure functions in DIS to single and double inclusive particle production in proton-proton and proton-nucleus collisions at high energy, see Ref.\textsuperscript{1} and references therein. The CGC formalism has been also quite successful in providing predictions for particle multiplicities at the LHC\textsuperscript{2-4} and may provide a first-principle way of understanding of isotropization and thermalization of QCD matter produced in high energy heavy ion collisions at RHIC and the LHC\textsuperscript{5}.

The observed suppression of single inclusive hadron production and the disappearance of the away side peak in double hadron production in the forward rapidity region of deuteron-nucleus (dA) collisions at RHIC\textsuperscript{8,7} are perhaps the strongest evidence for the importance and possibly dominance of saturation effects at RHIC. This will soon be further tested at the LHC where one will be able to probe CGC dynamics in a much larger kinematic region due to the larger energy of the collisions at the LHC. Single inclusive hadron production in proton-nucleus (pA) collisions at RHIC and the LHC has been investigated by many authors\textsuperscript{8-13} in the CGC formalism using varying degrees of approximations and models (for an alternative description, see for example Refs.\textsuperscript{14,15}). The most important ingredient of the single inclusive hadron production cross section which captures the saturation dynamics is the fundamental (or adjoint) dipole cross section, the imaginary part of the quark anti-quark scattering amplitude on a proton or nucleus target. This dipole cross section satisfies the JIMWLK/BK evolution equations\textsuperscript{16,17} and re-sums the small $x$ as well as high gluon density effects. The evolution equation for the dipole cross section is now known with next-to-leading-order (NLO) accuracy\textsuperscript{15}, see also Ref.\textsuperscript{16}.

There are two distinct but related approaches to hadron production in high energy asymmetric (such as proton-nucleus or very forward proton-proton and nucleus-nucleus) collisions. One is the well-known $k_T$ factorized approach\textsuperscript{20,21} where partons in both the projectile and target are assumed to be at very small $x$ ($x < 0.01$) so that the CGC formalism is applicable to both the projectile and target. This approach is valid as long as one stays away from the projectile fragmentation region. An alternative approach was developed in\textsuperscript{9} where one treats the projectile wave-function perturbatively, i. e. using the standard DGLAP picture while treating the target by CGC methods. This approach is better suited for the projectile fragmentation region. Very recently this approach has been improved by keeping the inelastic pieces of the cross section which may be important at high transverse momentum.\textsuperscript{22} Here we numerically investigate the contribution of theses inelastic contributions to single inclusive hadron production in proton-nucleus and proton-proton collisions at RHIC and the LHC and show that inclusion of these terms improves the high $p_T$ behavior of the cross section. The nuclear modification factor is also shown to increase faster with increasing $p_T$ than the case where these inelastic contributions are ignored.

The paper is organized as follows: In the next section, we introduce our formalism for the inclusive hadron production
II. SINGLE INCLUSIVE HADRON PRODUCTION; MAIN FORMULATION

The cross section for single inclusive hadron production in asymmetric collisions (scattering of a dilute system of parton with a dense one) at high energy is given by \[ \frac{dN_{pA\to hX}}{d^2p_Td\eta} = K \left[ \int_{x_F}^{1} \frac{dz}{z^2} \left[ x_1 f_q(x_1, Q^2) N_A(x_2, \frac{p_T}{z}) D_{h/q}(z, Q) + \Sigma_g x_1 f_g(x_1, Q^2) N_F(x_2, \frac{p_T}{z}) D_{h/q}(z, Q) \right] + \int_{x_F}^{1} \frac{dz}{z^2} \frac{\alpha_s}{\pi} \int_{k_T^2 < Q^2} \frac{d^2 k_T}{k_T^2} N_F(k_T, x_2) \int_{x_1}^{1} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \sum_{i,j=q,g} P_{i/j}(\xi) x_1 f_j(x_2, \frac{x_1}{\xi}, Q) D_{h/i}(z, Q) \right], \]

where \( f_j(x, Q^2) \) is the parton distribution functions (PDF) of the incoming proton which depends on the light-cone momentum fractions \( x \) and the hard scale \( Q \). The function \( D_{h/q}(z, Q) \) is the hadron fragmentation function (FF) of \( i \)th parton to the final hadron \( h \) with a momentum fraction \( z \). The variables \( \eta \) and \( p_T \) are the pseudo-rapidity and transverse momentum of the produced hadron. The longitudinal momentum fractions \( x_1 \) and \( x_2 \) are defined as follows

\[ x_F \approx \frac{p_T}{\sqrt{s}} e^\eta; \quad x_1 = \frac{x_F}{z}; \quad x_2 = x_1 e^{-2\eta}, \]

where \( \sqrt{s} \) is the collision energy per nucleon. Here we neglect hadron masses since we are only interested in light hadron production at high-\( p_T \) (thereby rapidity and pseudo-rapidity are equal).

It is perhaps useful to remind the reader of the derivation of Eq. (1). The first line of Eq. (1) was first derived in Ref. [6]. The result of [6] has been recently improved in [22] by keeping the inelastic pieces which lead to the second line in Eq. (1). Our main goal in this paper is to consider the effect of this new term in the inclusive hadron production at both RHIC and the LHC. Let us first focus on the first line of Eq. (1), the DHJ term [9]. We refer the reader to Ref. [3] for technical details and just outline the derivation of the DHJ term. One starts by calculating two particle production cross section in proton-nucleus scattering. The simplest process is when a quark from the projectile scatters on the target and radiates a gluon either before or after the scattering (see Fig. (12) in [23]). The incoming quark as well as the outgoing quark and the radiated gluon can all multiply scatter on the target. However, if one is interested in single inclusive production, one needs to integrate over one of the final state partons. Some of the Feynman diagrams will have collinear divergences whereas others do not. There is a collinear divergence in the final state which happens when the outgoing quark and the radiated gluon are nearly parallel and only the initial state quark multiply scatters on the target. This divergent term is absorbed into quark-hadron fragmentation function and lead to its evolution with \( Q^2 \) according to the LO DGLAP evolution equation. The finite (non-collinear divergent) pieces are ignored as they are part of the NLO corrections.

There is also a collinear divergence in the initial state which happens when the incoming quark and the radiated gluon are nearly parallel and only the final state quark multiply scatters on the target. This sort of collinear divergence is absorbed into the incoming parton distribution function and leads to its evolution according to the standard DGLAP evolution equation. Again, the finite parts of these terms were ignored in [9] since they correspond to higher order (in \( \alpha_s \)) corrections. It was pointed out in [22] that the finite pieces which are ignored in [9] may be important at high \( p_T \) and therefore can lead to a modification of the production cross section. Keeping the finite diagrams (which do not have a collinear divergence) and making the high \( p_T \) approximation (gradient expansion of the quadrupole cross section) leads to Eq. (1). While the above argument is for an incoming quark scattering on the target, inclusion of other processes such as an incoming gluon scattering on the target is straightforward [24] where a similar analysis of the collinear divergences can be made.

Now that the origin of these inelastic terms is made more clear, we comment on the relative significance of the two contributions. The first piece of eq. (1), dubbed the elastic part, corresponds to an incoming parton in the proton wave function scattering elastically on the target. This incoming parton initially has zero transverse momentum but picks up transverse momentum of order \( Q_s \) after multiply scattering on the target. This term should be most important when the transverse momentum of the produced hadron is of order \( Q_s \) or perhaps even a bit larger. The second term in eq. (1), dubbed the inelastic piece, corresponds to a high transverse momentum parton radiated from the incoming parton in the projectile wave function. This radiated parton is already at high transverse momentum and interacts with the target only once (higher number of scatterings will be power suppressed). This term is therefore important only when the produced hadron is at transverse momenta much higher that the saturation scale of the target \( Q_s \).
In Eq. (1), we have introduced a K-factor to mimic the effect of higher order corrections. The inelastic weight functions \( w_{i/j} \) are given by

\[
\begin{align*}
    w_{g/g}(\xi) &= 2 \frac{N_c^2}{N_c^2 - 1} (1 - \xi + \xi^2), \\
    w_{g/q}(\xi) &= w_{q/g}(\xi) = \frac{N_c^2}{N_c^2 - 1} \left[ 1 + (1 - \xi)^2 - \frac{\xi^2}{N_c^2} \right], \\
    w_{q/q}(\xi) &= w_{q/q}(\xi) = \frac{N_c^2}{N_c^2 - 1} \left[ 1 + \xi^2 - \frac{(1 - \xi)^2}{N_c^2} \right], \\
    w_{q/g}(\xi) &= w_{q/q}(\xi) = \frac{1}{2} \left[ (1 - \xi)^2 + \xi^2 - \frac{2\xi(1 - \xi)}{N_c^2 - 1} \right],
\end{align*}
\]

where \( N_c \) denotes the number of colors. The function \( P_{i/j} \) in Eq. (1) denotes the Altarelli-Parisi splitting function that describes the probability of a given parton \( j \) splitting into two others. The leading-order splitting functions (for \( N_c = 3 \)) are given by [22],

\[
\begin{align*}
    P_{q/g}(\xi) &= \frac{4}{3} \left[ \frac{1 + \xi^2}{(1 - \xi)^+} \right] + 2\delta(1 - \xi), \\
    P_{qg}(\xi) &= \frac{1}{2} \left[ \xi^2 + (1 - \xi)^2 \right], \\
    P_{gg}(\xi) &= \frac{4}{3} \left[ \frac{1 + (1 - \xi)^2}{\xi} \right], \\
    P_{gg}(\xi) &= 6 \left[ \frac{1 - \xi}{\xi} + \xi(1 - \xi) + \frac{\xi}{(1 - \xi)^+} \right] + \left( \frac{11}{2} - \frac{n_f}{3} \right) \delta(1 - \xi),
\end{align*}
\]

where \( n_f \) is the number of active flavor and subscript + refers to the so-called "\( +^* \)" prescription used to regularize the singularities as \( \xi \to 1 \) [22]. In Eq. (1), the amplitude \( N_{AF}(N_{A}) \) is the two-dimensional Fourier transform of the imaginary part of the forward dipole-target scattering amplitude \( \mathcal{N}_{AF}(r) \) in the fundamental (F) or adjoint (A) representation,

\[
N_{AF}(x, k_T) = \int d^2 \vec{r} e^{-i \vec{k} \cdot \vec{r}} \left( 1 - N_{AF}(r, Y = \ln(x0/x)) \right),
\]

where \( r = |\vec{r}| \) is the dipole transverse size. In the large-\( N_c \) limit, one has the following relation between the adjoint and fundamental dipoles,

\[
\mathcal{N}_A(r, Y) = 2\mathcal{N}_F(r, Y) - \mathcal{N}_F^2(r, Y).
\]

The amplitude \( \mathcal{N}_{AF}(r) \) incorporates all multi-scatterings between a projectile color-dipole and the target and encodes the small-\( x \) dynamics. In the CGC framework, it can be obtained from the solution of JIMWLK/BK evolution equations [16, 17], an infinite set of coupled nonlinear equations for the different Wilson line correlators which systematically incorporate small-\( x \) gluon emission to all orders [16]. In the large-\( N_c \) limit, the JIMWLK evolution equations reduce to the Balitsky-Kovchegov (BK) equation [17], a closed-form equation for the evolution of the dipole amplitude. The running coupling BK (rcBK) equation [17, 26, 27] has the following simple form:

\[
\frac{\partial \mathcal{N}_{AF}(r, x)}{\partial \ln(x_0/x)} = \int d^2 \vec{r}_1 K^{\text{run}}(\vec{r}, \vec{r}_1, \vec{r}_2) \left[ \mathcal{N}_{AF}(r_1, x) + \mathcal{N}_{AF}(r_2, x) - \mathcal{N}_{AF}(r, x) - \mathcal{N}_{AF}(r_1, x)\mathcal{N}_{AF}(r_2, x) \right],
\]

where the evolution kernel \( K^{\text{run}} \) using Balitsky’s prescription [27] for the running coupling is defined as,

\[
K^{\text{run}}(\vec{r}, \vec{r}_1, \vec{r}_2) = \frac{N_c}{2\pi^2} \left[ \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r_1^2}{r_1^2 + r_2^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) \right],
\]

with \( \vec{r}_2 = \vec{r} - \vec{r}_1 \). For the running coupling in the above equation we use the scheme proposed in Ref. 34 at one-loop level. Notice that in the master equations [11, 18], the impact-parameter dependence of the collisions was ignored. However, for the min-bias analysis considered here this may not be important. Nevertheless, it has been shown by several studies that impact-dependence of the BK equation is important at very large rapidities and for fixed centralities [2, 3, 28, 29], though it is very challenging to implement numerically.
III. NUMERICAL RESULTS AND PREDICTIONS

We now evaluate the single inclusive hadron production cross section numerically. To do this, we will use the NLO MSTW 2008 PDFs [30] and the NLO KKP FFs [31]. We have checked that AKK FFs [32] are also consistent with our results. We assume the factorization scale in the FFs and the PDFs to be $Q = \sqrt{s}$.

The only input for the rcBK equation is the initial conditions for the evolution of the dipole amplitude which is commonly taken to be a McLerran-Venugopalan (MV) type model [33]:

$$N_A^{\gamma}(k_T, x) = \frac{1}{\lambda y + d} \log \left( \frac{Q_T^2}{Q_s^2(x)} \right) .$$

with $y = \log(1/x)$ and the scale $Q_T$ in the anomalous dimension is related to the inverse transverse size of the dipole $Q_T^2 \approx 1/x^2$. Saturation scale $Q_s(x)$ is defined as $Q_s^2(x) = A_{s}^{1/3}(x_{0}/x)^{\lambda}$ with $A_{s} = 18.5$ for the minimum bias dAu collisions. Here we are interested in investigating whether the new inelastic contribution, second line in Eq. (1), will affect the description of the RHIC data [10]. To this end, we take the same parameters for the DHJ dipole parametrization [10] for $N_A^{\gamma}(k_T, x)$ which has been used to describe the forward rapidity data in dAu collisions at RHIC. We refer the reader to Ref. [10] for the details of the DHJ dipole model. The dipole scattering amplitude in the DHJ model is simply given by

$$N_A^{\gamma}(Q_T, x) = 1 - \exp \left[ - \frac{1}{4} (r Q_s^2(x))^\gamma \right] ,$$

where the anomalous dimension $\gamma^{\gamma}$ in the DHJ model is parameterized as,

$$\gamma^{\gamma}(Q_T, x) = \gamma_s + (1 - \gamma_s) \frac{\log \left( Q_T^2/Q_s^2(x) \right)}{Q_T^2/Q_s^2(x)} .$$

with $y = \log(1/x)$ and the scale $Q_T$ in the anomalous dimension is related to the inverse transverse size of the dipole $Q_T^2 \approx 1/r^2$. Saturation scale $Q_s(x)$ is defined as $Q_s^2(x) = A_{s}^{1/3}(x_{0}/x)^{\lambda}$ with $A_{s} = 18.5$ for the minimum bias dAu collisions. Here we are interested in investigating whether the new inelastic contribution, second line in Eq. (1), will affect the description of the RHIC data [10]. To this end, we take the same parameters for the DHJ dipole parametrization as employed in Ref. [10] which provides a good description of the RHIC data without the presence of the inelastic contribution namely $\alpha_s = 0$. The parameters $\lambda = 0.3$ and $x_{0}/x = 10^{-4}$ were extracted from a fit to HERA data, and parameter $d$ was fitted to the RHIC data and set to $d = 1.2$ [10]. The anomalous dimension in Eq. (17) runs from the LO BFKL value $\gamma_s = 0.628$ at small $x$ to the DGLAP value $\gamma^{\gamma} \rightarrow 1$. This model incorporates the geometric scaling window as expected from the BK equation [35] consistently.

Here our aim is not to fit the data but to use the best theoretical tools available in low-x physics to highlight the uncertainties involved in making robust predictions from the CGC formalism for the upcoming proton-nucleus collisions at the LHC. Therefore, we take $K = 1$ throughout this paper.

In Fig. 1 we show the single inclusive hadron production yields in pp and dAu collisions at RHIC $\sqrt{s} = 0.2$ TeV at different rapidities using the DHJ parametrization of the dipole cross section as well as the rcBK dipole solution. In order to investigate the contribution of the inelastic term to single inclusive hadron production, we also show the results without this term, namely $\alpha_s = 0$ in Eq. (11), denoted as DHJ in Fig. 1 (left). The inelastic contribution term in Eq. (1) is explicitly proportional to $\alpha_s$. Notice that in the derivation of this formula at the leading-twist order, $\alpha_s$ was assumed to be a fixed parameter. It is not a priori obvious whether the running-coupling corrections to Eq. (11) can be simply incorporated by replacing $\alpha_s$ to a running $\alpha_s(Q)$. It has been shown for example that in $k_T$–factorization formulation, the running coupling effect changes the equation [36]. We have checked that $\alpha_s \approx 0.05 \pm 0.15$ in Eq. (11) gives a reasonable description of RHIC data for both pp and dAu collisions. It is clear that inclusion of the inelastic terms improves the $p_T$ dependence of the cross section closer to mid-rapidity while there is no visible contribution at the most forward rapidity considered. This is more clearly seen in Fig. 1 at the upper-left panel, where the inelastic contribution is seen to significantly improve the description of the data for more central collisions at $\eta = 1$ and makes the $p_T$-spectra steeper in agreement with the data. For more forward collisions, the available phase space is limited and inelastic contributions are consequently negligible independent of the value of the strong coupling $\alpha_s$. 

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FIG. 1: Single inclusive hadron production in proton-proton (upper panel) and deuteron-gold (lower panel) collisions at different pseudo-rapidities at RHIC obtained by the solution of the running-coupling BK equation, the so-called rcBK (right) and the DHJ (left) dipole model. Right: dashed and full lines refer to the results coming from the rcBK equation corresponding to two different initial values for the saturation scale at $x_0 = 0.01$. We have taken $\alpha_s = 0.1$ in Eq. (1) for all curves. Left: dashed and full lines refers to the results when $\alpha_s = 0$ (the DHJ term or the elastic contribution) and $\alpha_s = 0.1$ (for the inelastic term), respectively. We have taken $K = 1$ in all panels. The experimental data are from Ref. [6].

As we have already pointed out, the value of $Q_{0s}$ at $x_0 = 0.01$ is the only free-parameter left to be fixed for a given solution of the rcBK equation. In Fig. 1 (right), we also show the effect of various choices for the initial saturation scale $Q_{0s}$. In the case of pp RHIC data, we found that values in $Q_{0s}^2 = 0.168 \div 0.336 \text{ GeV}^2$ range give a consistent description of data. However, a smaller value of $Q_{0s}^2 = 0.168 \text{ GeV}^2$ may be more preferable, specially at very forward rapidities. This is understandable since the available phase space for multiple re-scattering is limited at very forward rapidity. Therefore, a lower initial saturation scale is required to describe the cross-section. We note that HERA
other mechanisms also partially contribute to the hadron production, see for example Refs. [38, 39].

The parameter set $Q_0^2 = 0.168 \text{ GeV}^2$ and $\gamma = 1.119$ that we used in Fig. 1 also gives excellent description of the structure function data in e+ p collisions with $\chi^2/d.o.f. = 1.104$ [34].

In order to compare our $R_{pA}$ predictions with experimental data, one may need to rescale $R_{pA}$ by matching the normalization $N_{\text{coll}}$ to the experimental value.
In Fig. 2 (right), we show the nuclear modification factor $R_{pA}$ for inclusive charged hadrons $h^+ + h^-$ production at $\sqrt{s} = 4.4$ TeV and $\eta = 4$ obtained from different solutions of the rcBK equation corresponding to different values of $Q_{0s}(x_0 = 0.01)$ extracted from RHIC data (see description of Fig. 1). We also show the contribution of the inelastic term by showing the results due to only the DHJ term ($\alpha_s = 0$). The value of the strong-coupling in the inelastic term in Eq. 1 is set to $\alpha_s = 0.1$ (the same value was taken in Fig. 1). It is obvious that taking different values for the saturation scale $Q_{0s}(x_0 = 0.01)$ for proton and nuclear targets significantly changes the nuclear modification factor. Therefore, the measurement of $R_{pA}$ provides vital information about the initial saturation scale of target and small-x evolution dynamics. Inclusion of the inelastic term changes $R_{pA}$ and makes it increase faster at high-$p_T$, see also Fig. 3. Notice that rcBK solutions taken here approximately reproduce the perturbative power-law behavior of the dipole-amplitude $N_{A(F)} \sim 1/p_T^4$ at high-$p_T$, see Fig. 2 (left). We recall that the parameters of rcBK solutions used here were obtained from a fit to HERA data for virtuality $Q^2 \in [0.25, 45]$ GeV$^2$ [34]. Therefore, our results at very high-$p_T$ may be less reliable.

In Fig. 3 we show our predictions for $R_{pA}$ for $h^+ + h^-$ production at $\sqrt{s} = 4.4$ TeV and $\eta = 4$ using the solution of the rcBK dipole evolution equation, Eq. (13), assuming initial nuclear saturation scales of $Q_{0s}^2 = 0.67$ GeV$^2$ (right panel) and $Q_{0s}^2 = 0.5$ GeV$^2$ (left panel). In both panels we have assumed the initial saturation scale of proton to be $Q_{0s}^2 = 0.168$ GeV$^2$. We note that a larger initial saturation scale for the nucleus leads to a faster rise of $R_{pA}$ with transverse momentum. For comparison, in Fig. 3 (right panel) we show the corresponding $R_{pA}$ obtained using the DHJ dipole model, defined in Eq. (16). It is seen that both approaches lead to a suppression of $R_{pA}$ at forward rapidities at the LHC and that the DHJ parameterization leads to a flatter transverse momentum dependence. We recall that both the rcBK solution and the DHJ model provide a reasonable description of RHIC data.

In order to highlight the uncertainties associated with the different choices of the strong-coupling constant in Eq. 11 more clearly, in Fig. 4 (left) we show $R_{pA}$ for three different values of $\alpha_s$ namely, $\alpha_s = 0$ corresponding to the elastic term only, and $\alpha_s = 0.1, 0.15$ for the inelastic contribution. It is seen from Figs. 2, 3, 4 that at rapidities close to mid-rapidity, increasing $\alpha_s$ reduces $R_{pA}$ while at very forward rapidities and high-$p_T$ the opposite happens.

In Fig. 5 (right), we show our predictions for $R_{pA}$ for inclusive charged hadron production at $\sqrt{s} = 8.8$ TeV and at different rapidities obtained from the rcBK equation (13) with different values of the strong coupling in the master equation (1). It is seen that the energy-dependence of $R_{pA}$ from $\sqrt{s} = 4.4$ to 8.8 TeV is rather weak3. From Figs. 5

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3 Note that our results for $R_{pA}$ at the LHC without inclusion of the inelastic term is different from Ref. [12] mainly due to the fact that
we also note that at very forward rapidities the uncertainty associated with the choice of \( \alpha_s \) is reduced. This is in accordance with the fact that the effect of inelastic contribution at very forward rapidities is negligible, see also Fig. 1.

Our prediction for \( \eta = 0 \) at the LHC energy 4.4 TeV is shown in Fig. 5 (left), using the solution to the rcBK evolution equation and assuming two different initial nuclear saturation scales of \( Q^2_{0s} = 0.5, 0.67 \text{ GeV}^2 \) (extracted from RHIC data). In Fig. 5 we assumed the initial saturation scale for proton to be \( Q^2_{0s} = 0.168 \text{ GeV}^2 \) (extracted from RHIC and HERA data). The theoretical error bars in Fig. 5 show the uncertainties mainly associated with the choice of \( \alpha_s \) in Eq. 11. The observed suppression of \( R_{pA} \) at midrapidity and high \( p_T \) for the case of lower initial nuclear saturation scale \( Q^2_{0s} = 0.5 \text{ GeV}^2 \) is larger compared to the results obtained within the gluon saturation approach with quasi-classical (Glauber) approximation 13. We note that there are large uncertainties in \( R_{pA} \) in midrapidity at the LHC due to the choice of the initial saturation scale for the rcBK evolution equation 13 and the value of strong-coupling constant in Eq. 11. More importantly, large sensitivity of \( R_{pA} \) to the value of \( \alpha_s \) in Eq. 11 in midrapidity at the LHC indicates that higher order corrections in Eq. 11 should be important in midrapidity at the LHC energy. Therefore, we believe that our predictions for \( R_{pA} \) at midrapidity may be less reliable compared to our results for the very forward rapidity collisions.

It should be noted that the particle production cross-section given by Eq. 11 is intrinsically asymmetric, namely it treats the projectile proton approximately in the collinear factorization framework while treating the target proton (or nucleus) in the CGC framework. Strictly speaking, this may be justified only for particle production in the collision of a dilute system on a dense system, such as particle production in mid or forward rapidity in pA collisions or in particle production in the very forward rapidity region in symmetric collisions, such as proton-proton or nucleus-nucleus collisions. Therefore, our formalism can not be reliable for particle production in midrapidity in proton-proton collisions. Unfortunately, this is also what one needs in order to calculate the nuclear modification factor \( R_{pA} \) in midrapidity. A better approach to particle production in midrapidity in symmetric collisions where both projectile and target are dilute (for \( p_t \gg Q_s \)) might be to use the \( k_t \) factorization formalism, proven to LO accuracy for single inclusive hadron production 41. This is beyond the scope of the present work and we leave it for a future study.

we have used a different parameter set for the rcBK solution with \( \gamma = 1.119 \) which was recently suggested in Refs. 8, 32. For the sensitivity of \( R_{pA} \) to the various allowed solutions of the rcBK equation, see Fig. 2 and related discussions.
IV. SUMMARY

We have quantitatively studied, for the first time, the contribution of both elastic and inelastic processes to single inclusive hadron production cross section at RHIC and the LHC using the CGC formalism. We observe that inelastic contributions to single inclusive hadron production are significant at high transverse momentum and close to mid-rapidity. On the other hand, their contribution is very small in the forward rapidity region. Furthermore, we note that inclusion of these inelastic terms makes the nuclear modification factor $R_{pA}$ grow faster with increasing transverse momentum. We make detailed predictions for $R_{pA}$ at the LHC using the numerical solution of the running-coupling BK equation. We have studied various theoretical uncertainties associated with the choice of the initial saturation scale $Q^2_{0s}$ for a proton and nucleus. We have shown that the nuclear modification factor $R_{pA}$ measured at the LHC in the forward rapidity region is a sensitive probe of the low-x dynamics and can help constrain $Q^2_{0s}$ further. We have shown that various theoretical uncertainties in our formalism are minimized at very forward rapidities at the LHC. Therefore, measuring the nuclear modification factor $R_{pA}$ in the very forward region in proton-nucleus collisions at the LHC will be a robust test of gluon saturation dynamics and the Color Glass Condensate formalism.

Acknowledgments

We would like to thank David d’Enterria, Adrian Dumitru, Alex Kovner, Anna Stasto, Mark Strikman and Dionisis Triantafyllopoulos for useful discussions. We are grateful to the organizers of “High-energy QCD after the start of the LHC” workshop in the Galileo Galilei Institute for Theoretical Physics (Florence) and “Frontiers in QCD” workshop in the Institute for Nuclear Theory (Seattle) for their hospitality and invitation to these stimulating workshops where this paper was finalized. J.J-M. is supported in part by the DOE Office of Nuclear Physics through Grant No. DE-FG02-09ER41620, from the “Lab Directed Research and Development” grant LDRD 10-043 (Brookhaven National Laboratory), and from The City University of New York through the PSC-CUNY Research Program, grant 64554-00
42. The work of A.H.R is supported in part by Fondecyt grants 1101781.

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