Skewness as a Test of Non–Gaussian Primordial Density Fluctuations

Peter Coles\textsuperscript{1}, Lauro Moscardini\textsuperscript{2,3}, Francesco Lucchin\textsuperscript{2}, Sabino Matarrese\textsuperscript{4} and Antonio Messina\textsuperscript{5,6}

\textsuperscript{1} Astronomy Unit, School of Mathematical Sciences, Queen Mary \& Westfield College, Mile End Road, London E1 4NS, UK

\textsuperscript{2} Dipartimento di Astronomia, Università di Padova, vicolo dell’Osservatorio 5, I–35122 Padova, Italy

\textsuperscript{3} Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QH, UK

\textsuperscript{4} Dipartimento di Fisica G. Galilei, Università di Padova, via Marzolo 8, I–35131 Padova, Italy

\textsuperscript{5} Centre Européen de Calcul Atomique et Moleculaire, 91405 Orsay CEDEX, France

\textsuperscript{6} Dipartimento di Fisica A. Righi, Università di Bologna, via Irnerio 46, I–40126 Bologna, Italy

Summary. We investigate the evolution of the skewness of the distribution of density fluctuations in CDM models with both Gaussian and non–Gaussian initial fluctuations. We show that the method proposed by Coles \& Frenk (1991), which uses the skewness of galaxy counts to test the hypothesis of Gaussian primordial density fluctuations, is a potentially powerful probe of initial conditions. As expected, the mass distribution in models with initially non–Gaussian fluctuations shows systematic departures from the Gaussian behaviour on intermediate to large scales. We investigate the effect of peculiar velocity distortions and normalisation upon the relationship between skewness and variance. These effects are generally small for the models we consider. Comparing our results to the QDOT measurements of the skewness, we find that our initially positive–skew models are clearly excluded by this analysis, but the available data do not rule out the negative–skew models.

Key Words: Galaxies: formation, clustering – large-scale structure of the Universe – early Universe – dark matter.
1 Introduction

The gravitational instability picture of galaxy formation is coming under increasing pressure from new observational data. In particular, models of structure formation involving dark matter – either hot (HDM) or cold (CDM) – have been found wanting by new information on galaxy clustering and the Cosmic Microwave Background Radiation (CMBR). The “standard” versions of both HDM and CDM incorporate the assumption that present–day structures grew by gravitational instability from small, primordial, Gaussian distributed adiabatic perturbations with the scale–invariant Zel’dovich spectrum. Both these “standard” cosmogonies seem however to be unable to account for all the observational constraints. As far as galaxy clustering data are concerned: HDM succeeds in reproducing the amount of clustering on very large scales but fails in accounting for the age of galaxies and galaxies must be much less clustered than the mass on small scales to reproduce known clustering properties; CDM on the other hand better reproduces small–scale structures but suffers from general lack of power on large scales. In particular, the large–scale problem of CDM is indicated by a number of independent statistical tests such as counts of galaxies in cells (Efstathiou et al. 1990; Saunders et al. 1991), the spatial two–point correlation function of rich clusters (Batuski, Melott & Burns 1987), the angular correlations of projected galaxy distributions (Maddox et al. 1990) and the power–spectrum of the distribution of galaxies and radio–galaxies (Peacock 1991).

In recent months, the COBE detection of large–scale temperature anisotropy in the CMBR has given independent information about the normalisation of fluctuations in these models, and the shape of the power–spectrum on very large scales (Smoot et al. 1992). This poses a particular problem for CDM, since the normalisation implied by COBE is rather high for the standard model. Normalised to match the COBE amplitude, CDM does quite well on large scales but has too high an amplitude on small scales, resulting in runaway clustering and very high peculiar velocities (Davis et al. 1985). Attempts to escape from these difficulties by a straightforward change in either the amplitude or the shape of the primordial power spectrum (e.g. Bardeen, Bond & Efstathiou 1987; Vittorio, Matarrese & Lucchin 1988) are strongly constrained, not only by COBE but also by other observational limits on temperature anisotropies on the microwave background sky (Bond et al. 1991; Vittorio et al. 1991). Nevertheless, it is possible to beat the COBE constraints by invoking a power–law inflationary model (Abbott & Wise 1984, Lucchin & Matarrese 1985a,b) that produces almost, but not quite, scale–invariant density fluctuations but also produces tensor perturbations (i.e. gravitational waves) with large amplitude (Davis et al. 1992; Liddle & Lyth 1992; Lidsey & Coles 1992; Lucchin, Matarrese & Mollerach 1992; Salopek 1992b; Souradeep & Sahni 1992). Such a model can allow a lower amplitude of density fluctuations to be compatible with the COBE anisotropy and can thus reconcile small–scale clustering with the CMBR anisotropy.

Another possible escape route from these constraints is to assume that the relationship between galaxies and mass is different on different mass scales: a scale–dependent bias. Given how little we know about galaxy formation this seems a reasonable choice and specific models have been constructed that can alleviate the large–scale structure problem for CDM without damaging its small–scale successes (Babul & White 1991; Bower et al. 1992). The problem with these models is that they need to invoke collective physical processes acting on a very large scale ∼ 30h⁻¹ Mpc and feedback mechanisms acting on such large scales are hard to find. The need to introduce a physical scale into the problem is demonstrated by Coles (1992): any local biasing effect acting upon Gaussian density fluctuations cannot change the slope of the galaxy correlation function with respect to the mass autocorrelation function.
It is clear that many of the problems with large-scale structure theory can be traced back to the assumption of random-phase fluctuations within the gravitational instability model. Can we construct a self-consistent model for galaxy formation based upon non-Gaussian fluctuations? Within the inflationary picture of the origin of perturbations this issue has been discussed by a number of authors (e.g. Matarrese, Ortolan & Lucchin 1989; Kofman et al. 1991; Barrow & Coles 1990). The actual possibility of getting phase correlations on cosmologically relevant scales is restricted to multiple scalar field models (e.g. Allen, Grinstein & Wise 1987; Salopek, Bond & Bardeen 1989; Salopek & Bond 1991; Salopek 1992a; Fan & Bardeen 1992). Alternatively, non-Gaussian fluctuations can be produced by a discrete, random distribution of seed masses, such as topological defects like monopoles, cosmic strings or textures, provided by a phase-transition in the early universe (Turok 1989; Park, Spergel & Turok 1991; Scherrer & Bertschinger 1991; Scherrer 1992). An alternative way to get non-random phases is the cosmic explosion scenario (Ostriker & Cowie 1981; Ikeuchi 1981), where hydrodynamics rather than gravity plays the main role in the structure formation process.

A typical (though not mandatory) signature of non-Gaussian density fluctuations $\delta_M$, is an initially non-vanishing skewness $\langle \delta_M^3 \rangle \neq 0$. Actually, according to the analysis of the QDOT IRAS–selected data by Saunders et al. (1991), a positive skewness of the distribution of IRAS galaxy counts is observed on quite large scales. A number of recent papers debate the issue of whether such a positive skewness has a primordial origin or is due simply to the non-linear, aggregating action of gravity on a primordial (i.e. unskewed) Gaussian field (e.g. Coles & Frenk 1991, hereafter CF91; Silk & Juszkiewicz 1991; Martel & Freudling 1991; Park 1991; Lahav et al. 1992; Juszkiewicz, Bouchet & Colombi 1992; Bouchet et al. 1992; Bouchet & Hernquist 1992). It has been shown by $N$–body simulations (Moscardini et al. 1991, hereafter MMLM; Matarrese et al. 1991, Weinberg & Cole 1992) that the primordial skewness is a strongly discriminating parameter in determining both the dynamics and the present clustering properties of the universe. MMLM have studied the origin of large-scale structures in skewed CDM models, while Weinberg & Cole (1992) have considered non-Gaussian models obtained by a local non-linear transformation on scale-free Gaussian fields. These studies also show that there are many similarities between skew–positive models and e.g. the texture–seeded CDM model (Park, Spergel & Turok 1991; Cen et al. 1991). Actually, scenarios based on accreting HDM or CDM around seed masses always induce an excess of overdense regions. Cosmic explosions (e.g. Weinberg, Ostriker & Dekel 1989), as well as bubbles left over by a period of extended inflation in the early universe (La & Steinhardt 1989a,b; Liddle & Wands 1991), both give rise to an excess of low density regions, resembling the initial conditions of primordial skew–negative models.

So the initial skewness of non-Gaussian models is a strong indicator of their clustering behaviour. But, as we have mentioned, this primordial skewness is masked to some extent by the effect of gravitational evolution which generally tends to couple the skewness to the variance, which increases in time. Can we disentangle these two possible causes of skewness and use the skewness of the present–day distribution as a test of the hypothesis of Gaussian fluctuations? Based on analytical arguments and $N$–body experiments, CF91 argued that the answer to this question is yes and they proposed a simple test for primordial non-Gaussianity which is almost independent of the primordial spectral index (Bouchet et al. 1992): it is the purpose of this paper to determine how powerful is the test suggested by CF91 against various non-Gaussian alternatives. To this we have used the results of $N$–body simulations with both Gaussian and non–Gaussian initial conditions (Messina et al. 1992; Lucchin et al. in preparation) which represent the universe on a cube of $260 h^{-1}$ Mpc side. We have

---

1As usual, $\delta$ is defined to be the dimensionless density contrast: $\delta = (\rho - \bar{\rho})/\bar{\rho}$, where $\bar{\rho}$ is the mean matter density.
already used these simulations in a study of the large–scale topology of the Universe (Coles et al. 1992, hereafter CMPLMM), which is an alternative way of testing the Gaussian hypothesis (for a review, see Melott 1990).

2 N–body simulations with Non–Gaussian Initial Conditions

The non–Gaussian statistics considered here are the same adopted by MMLM, namely the Lognormal (hereafter LN) and the Chi–squared with one degree of freedom (hereafter $\chi^2$), chosen as distributions for the peculiar gravitational potential, $\Phi$, before the modulation by the CDM transfer function. These distributions actually split in two different types of models: the positive ($LN_p$ and $\chi^2_p$) and negative ($LN_n$ and $\chi^2_n$) models, classified according to the sign of the skewness for linear mass fluctuations, $\langle \delta^3_M \rangle > 0$, for the former and $\langle \delta^3_M \rangle < 0$ for the latter.

We have to restrict the parameter space we analyse in some way, so we set up all our model distributions in such a way that $\Phi$ has the “standard” CDM power–spectrum

$$P_\Phi(k) = \frac{9}{4} P_0 k^{-3} T^2(k), \quad (2.1)$$

where $P_0$ is a normalization constant and $T(k)$ is the CDM transfer function (e.g. Davis et al. 1985)

$$T(k) = [1 + 6.8k + 72.0k^{3/2} + 16.0k^2]^{-1}, \quad (2.2)$$

having considered a flat universe with Hubble constant $h = 0.5$ in units of 100 Km sec$^{-1}$ Mpc$^{-1}$. Such a choice for the spectrum allows a direct comparison with the Gaussian CDM (hereafter G) model.

We used a particle–mesh code with $N_p = 128^3$ particles on $N_g = 128^3$ grid–points [more details will be given in a forthcoming paper (Lucchin et al. in preparation)]. Computations were performed at the CINECA Center (Bologna) on a Cray YMP/432 running under UNICOS. The box–size of our simulations is $L = 260 h^{-1}$ Mpc; each particle has mass $m = 4.7 \times 10^{12} M_\odot$. We run two realizations for each of the five models we consider. We evolve our models starting from the same amplitude up to the “present time” $t_0$. We define $t_0$ as the time when the galaxy two–point function is best fitted by the power–law $\xi(r) = (r/r_0)^{-\gamma}$, with $\gamma = 1.8$ in a suitable interval. To obtain the galaxies in a given simulation we proceed as follows. We filter the initial density field with a Gaussian window function of radius $0.5 h^{-1}$ Mpc and pick up as galaxies the particles closest to each peak, defined as the grid–point with a positive density contrast larger than the 26 nearest grid–points: the result is a galaxy catalogue formed by $\sim 60,000$ per each simulation. Due to the exceedingly high mass of our particles, following from the large box size and low resolution, and to the rather simplified galaxy identification criterion, we can only assume that our peak regions roughly trace the actual galaxy distribution.

Different epochs can be parameterized by the bias factor $b$ defined by the $rms$ linear mass–fluctuation on a sharp–edged sphere of radius $R_8 = 8 h^{-1}$ Mpc, i.e.

$$\sigma^2(R_8) = \frac{P_0}{2\pi^2} \int_0^\infty dk k^3 T^2(k) W^2_{TH}(kR_8) = \frac{1}{b^2}, \quad (2.3)$$

where $W_{TH}(x) = (3/x) j_1(x)$ is a top–hat window function and $j_1$ is the Bessel function of order 1. The present time $t_0$ corresponds to $b = 1$ for the Gaussian model, $b = 1.5$ for both the positive models, $b = 0.5$ for the negative $\chi^2$ and $b = 0.4$ for the negative Lognormal. Note that the method to define the galaxies used in this work is different from the ‘excursion regions’ technique used in CMPLMM, where
a larger galaxy number density, $3 \times 10^{-2} \, h^3 \, \text{Mpc}^{-3}$, was necessary in order to generate simulated Lick catalogs with $\sim 530,000$ galaxies in the whole simulation box. A consequence of this change is, for example, that the present epoch, i.e. the slope $\gamma = 1.8$ for the correlation function, is reached slightly later here than in CMPLMM. Note that a fully consistent normalization of mass fluctuations should give CMBR fluctuations in agreement with those detected by COBE (Smoot et al. 1992), which, for a standard CDM model favour low values of $b$, namely $b \approx 0.8$. On the other hand, the statistical analysis of CMBR anisotropies on large angular scales for non–Gaussian models cannot be reduced to calculating the $\text{rms}$ fluctuation.

The primordial gravitational potential is obtained by the convolution of a real function $\tau(x)$ with a random field $\varphi(x)$,

$$
\Phi(x) = \int d^3 y \, \tau(y-x) \varphi(y).
$$

(2.4)

The field $\varphi$ is obtained by a non–linear transformation on a zero–mean Gaussian process $w$, with unit variance and flicker–noise power–spectrum; the function $\tau$ is fixed by its Fourier transform,

$$
\tilde{\tau}(k) \equiv \int d^3 x e^{-i k \cdot x} \tau(x) = T(k) F(k),
$$

(2.5)

where $T(k)$ is the CDM transfer function of Eq. (2.2) and $F(k)$ a positive correction factor which we applied to have the exact CDM initial power–spectrum of Eq. (2.1) in all our models. The precise form of the non–linear transformations from $w$ to $\varphi$ are $\varphi(x) \propto e^{w(x)}$ and $\varphi(x) \propto w^2(x)$ for LN and $\chi^2$ respectively (Coles & Barrow 1987; Coles & Jones 1991; MMLM).

As MMLM have shown, both the clustering dynamics and the present large–scale structure depend strongly upon the sign of the primordial skewness: positive models rapidly cluster to a lumpy structure with small coherence length, while negative models build up a cellular structure by the slow process of merging of shells around primordial underdense regions, with larger coherence length. The general conclusion of these previous studies is that, of the non–Gaussian alternatives considered, the skew–negative models are the more successful at reproducing the observed properties of the large–scale structure. Indeed, CMPLMM showed that very strong constraints on non–Gaussian models of the types considered here can be placed by the topology test: only Gaussian and skew–negative models survive the rigours of such an analysis.

### 3 The Skewness Test

CF91 describe the physical motivation behind the use of the skewness of cell–counts as a diagnostic of large–scale structure, so we just outline the basics here. Consider the density contrast smoothed on a certain scale: $\delta_M(R)$. In terms of the distribution of $\delta_M(R)$, called $f_R(\delta_M)$, we can define moments as follows:

$$
\overline{\langle [\delta_M(R)]^n \rangle} = \int f_R(\delta_M) \delta_M(R)^n \, d\delta_M(R).
$$

(3.1)

Clearly $\langle \delta_M \rangle = 0$; the quantity $\langle [\delta_M(R)]^2 \rangle = \sigma^2_M(R)$ is the variance of the smoothed mass density fluctuations and $\langle [\delta_M(R)]^3 \rangle$ is the skewness, denoted $\gamma_M(R)$. CF91 found, using a variety of methods in both the quasi–linear and strong clustering regimes, that for initially Gaussian density perturbations, $\gamma_M$ grows according to

$$
\gamma_M(R) \simeq S \sigma^4_M(R),
$$

(3.2)
where $S \simeq 3$ is roughly constant, i.e. almost independent of the scale upon which $\delta_M$ is smoothed, the background cosmology and the power–spectrum of the primordial fluctuations.

Since the publication of CF91, many other others have discussed properties of the skewness and related higher–order moments (Silk & Juszkiewicz 1991; Martel & Freudling 1991; Park 1991; Lahav et al. 1992; Juszkiewicz et al. 1992; Bouchet et al. 1992). These other analyses have confirmed the main conclusions of CF91, but have demonstrated that $S$ is actually a weak function of $R$ and the primordial power spectrum. The errors introduced by taking $S$ to be constant are, however, much smaller than the sampling errors for any real distribution so we shall ignore these refinements here. Moreover, all our non–Gaussian models have the same primordial power–spectrum so we can ignore any dependence on the spectral shape in this particularly paper. Bouchet et al. (1992) have also shown that the effects of redshift space distortion – disregarded by CF91 – should be weak in the interesting regime. The test proposed by CF91, that one should plot $\gamma$ against $\sigma^4$ for different smoothing scales and look for departures from linearity, is therefore confirmed as being a potentially powerful test of non–Gaussian primordial density fluctuations; CF91 found the QDOT data of Saunders et al. (1991) to be consistent with Gaussian initial fluctuations and may be sensitive enough to rule out viable non–Gaussian scenarios. Indeed, Silk & Juszkiewicz (1991) show that $\gamma \propto \sigma^3$ for the cosmic textures model which seems to be incompatible with the QDOT data. We shall see whether the non–Gaussian models described in Section 2 are also incompatible with the data.

There are two main problems when it comes to applying this test in practice. First, the discrete nature of number–counts of galaxies itself introduces a skewness term into the cell–count distribution. Provided that one accepts that the galaxy counts correspond to a Poisson ‘shot–noise’ effect, then one can easily correct for the discreteness terms (see below for a discussion). Secondly, most models of galaxy formation involve some degree of bias in the ratio of luminous galaxies to mass. An arbitrary functional bias of the form discussed by Coles (1992) could seriously interfere with the skewness test. CF91 showed using $N$–body simulations that the standard “high–peak” biasing scenario does in fact produce a galaxy distribution with second– and third–order moments scaling in the same way as (3.2). Nevertheless, different biasing models might produce different behaviours since an arbitrary bias is in some senses equivalent to having non–Gaussian fluctuations.

To check the power of the skewness test in the light of these difficulties, we shall compare the effects of gravitational evolution on the skewness of the Gaussian and non–Gaussian models described in Section 2, for different clustering amplitudes and different levels of bias.

4 Results

It is a relatively straightforward matter to extract estimates, $\hat{\Gamma}(R)$ and $\hat{\Sigma}^2(R)$, of the skewness and variance of cell–counts in cells of different size $R$ from the simulations described in Section 2. For large number, $N$, of cells we have

$$\hat{\Sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} (n_i - \bar{n})^2$$

(4.1a)

and

$$\hat{\Gamma} = \frac{1}{N} \sum_{i=1}^{N} (n_i - \bar{n})^3$$

(4.1b)
where $n_i$ is the number of particles in $i$–th cell and $\bar{n}$ is the mean number per cell, i.e. $\bar{n} = \frac{\sum_{i=1}^{N} n_i}{N}$. Cell–counts are inevitably skewed by virtue of their discrete (integer–valued) nature. To correct for the discreteness terms, one usually subtracts off ‘Poisson’ terms from the estimates, to give estimates of $\gamma$ and $\sigma^2$ which refer to a continuously–distributed variable:

$$\hat{\sigma}^2 = \hat{\Sigma}^2 - \frac{1}{\bar{n}};$$

$$\hat{\gamma} = \hat{\Gamma} - 3\hat{\sigma}^2 - \frac{1}{\bar{n}^2}$$

(Peebles 1980; Saunders et al. (1991); CF91). Possible reasons why this might not be an appropriate scheme, particularly if galaxy formation is in some sense co–operative, are discussed by CF91 (see also Bower et al. 1992). To check whether this correction preserves the shape of the $\gamma$–$\sigma^2$ relation (3.2), we look at both $\hat{\gamma}$–$\hat{\sigma}^2$ (i.e. corrected) and $\hat{\Gamma}$–$\hat{\Sigma}^2$ (i.e. uncorrected) relationships. We need to assign confidence limits to our estimates in order to assess the significance of departures from the predicted behaviour. Approximate methods for placing error limits on the empirically–determined estimates are discussed by Saunders et al. (1991), but these involve a complicated iterative procedure involving high–order moments. We can make a rough estimate of the error following Kendall & Stuart (1977). Suppose each simulation consists of a random sample of $N$ taken from a Gaussian parent distribution with variance $\sigma^2$. The variance from sample to sample of estimates of $\gamma$ and $\sigma^2$ in such a case are $6\sigma^6/N$ and $2\sigma^4/N$ respectively. If we take $\sigma^2 \simeq \hat{\sigma}^2$ then we can place approximate standard errors, $s$, on the estimates $\hat{\gamma}$ and $\hat{\sigma}^2$ as

$$s(\hat{\gamma}) \simeq \hat{\sigma}^2 \sqrt{\frac{6}{N}};$$

$$s(\hat{\sigma}^2) \simeq \hat{\sigma}^2 \sqrt{\frac{2}{N}}.$$  

Of course, our simulations are not random samples from a Gaussian parent: the cells are correlated and the distributions are non–Gaussian. We have also taken the sample variance and the parent population variance to be identical. The estimates (4.3) can be expected to give only a rough order–of–magnitude estimate of the likely confidence limits. We can also calculate error limits by using the two different simulations of each model to calculate an estimate of the ensemble variance analytically. This is still not completely satisfactory – ideally we would wish for many more simulations – but gives results in reasonable accord with the analytic estimates. We find that the estimated errors (4.3a,b) exceed the spread of the simulations on large scales by about 50 per cent, whereas the two estimates agree on intermediate scales.

The simulations also allow us to investigate: (i) the effect of the normalisation of the model upon the skewness–variance relationship; (ii) whether the redshift space relationship is significantly different to that in real space; (iii) whether the relationship for galaxies identified in the manner described in Section 2 is different to that of the dark matter particles (iv) whether the observed QDOT points rule out any of these models. Our results are displayed in Figures 1–3.

Figures 1a & 1b

In Figure 1a we show the $\hat{\Gamma}$–$\hat{\Sigma}^2$ (i.e. uncorrected) relationship for dark matter particles (left) and galaxies (right) for all five models at the present time. The error bars are estimated using eq. (4.3a,b).
First, note that for the Gaussian model the form (3.2) is well obeyed for both the dark matter and the galaxies, as found by CF91. The exception is at very small scales, where $\sigma^2$ is very large. On such small scales our particle code does not describe the fluid nature of the matter very well and this results in large discreteness effects (the cell–size here is comparable to the mesh size). Note that the positively–skewed models have a systematically higher $\Gamma$ for the same $\Sigma^2$; this is expected because they have higher initial skewness and gravity acts so as to increase the skewness further. The trend is somewhat less clear for the negative models. All such models have positive skewness even to very large scales, which shows that even weakly non–linear gravitational effects can wipe out the initial negative skewness. These models, however, need to be evolved for a comparatively long time in order to reach the present time. The result seems to be a much smaller systematic departure from the Gaussian expectation than for the positive–skew models. Figure 1b shows the effect of using the corrected values; we show the $\hat{\gamma}$–$\hat{\sigma}^2$ version. The trends are the same and the quantitative agreement is good, particularly on large scales where the shot–noise terms are small anyway. Note, however, that on small scales the shot–noise correction produces a negative skewness (indicated by the downward arrow plotted at the position of the uncorrected skewness). This confirms our argument that the failure of the relationship (3.2) in these simulations is due to a resolution effect.

Figures 2a & 2b

To consider the effect of evolution we plot, in Figures 2a and 2b, corresponding diagrams for models all normalised at $b = 1$ (2a) and $b = 2.5$ (2b). Only the uncorrected skewness and variance are shown; discreteness effects act similarly on all our simulations. Note that the Gaussian model still follows the relationship (3.2) accurately regardless of the value of $b$; when $b = 2.5$ the skewness on the very largest scale comes out negative but is consistent with zero within the errors. The positively skew models look closer to the Gaussian relationship for smaller $b$. The discrepancies for the negatively–skew models occur in a rather less predictable pattern and the systematic shift is less apparent.

Figure 3

To check the effect of redshift distortions we plotted both the real space and redshift space skewness and variance for all the models at the present time. The results are plotted in Figure 3. It is clear that, with the exception of very small scales where resolution effects dominate, the effect of looking at redshift space rather than real space is minimal.

We can now look at the question of whether the QDOT points place any strong constraints on any of these models. These points are plotted on Figures 1,2 and can be seen to lie on the expected trajectory (3.2). Although these points are on large scales from an observational point of view, they are on quite small scales compared to these simulations. Looking only at the corrected results (1b) – because the QDOT points are in corrected form – we see that the two positive–skew models are clearly excluded at $> 2\sigma$; the two negative–skew models are, however, consistent with the observed points (as is the Gaussian).

5 Discussion & Conclusions

We have investigated the behaviour of the skewness and variance of the distribution of both dark matter and galaxies in a number of models involving non–Gaussian initial conditions.
As expected, we find that non-linear gravitational evolution always acts in such a way that the skewness increases with time. This means that models with negative initial skewness display a positive skewness on cosmologically interesting scales even after very weak evolution. It is difficult therefore to see directly the sign of the initial skewness. Nevertheless, the fact that models with different initial skewness evolve in different ways means that their systematic trends in the behaviour of skewness against variance do remain even into the fairly strongly non-linear regime. In particular, initially positive-skew models seem to obey a similar scaling law to the initially Gaussian models (3.2) but with a higher value of $S$. For initially negative-skew models, the situation is somewhat less clear because systematic trends from the Gaussian are smaller. Part of the reason for this must be that to get models in reasonable accord with observations on small scales, the negative-skew models must be highly evolved whereas the positive-skew models are less strongly evolved (MMLM). This difference in normalisation to the present epoch tends to suppress differences compared to the Gaussian; the extra evolution required by the negative-skew models generates enough skewness to bring them roughly onto the Gaussian locus in the $\gamma - \sigma^2$ plane. Some systematic differences do remain, especially on large scales, but these are generally so small as to make discriminating between models difficult. Moreover, the typical errors for the negative-skew models are somewhat larger than those of the positive-skew models. The reason for this is probably that clustering evolves in the negative-skew models by forming a quasi-cellular network of bubbles characterised by a large coherence length. Since there are relatively few of these structures in the simulations, their presence can produce large fluctuations from simulation to simulation. Structures in the positive-skew models generally have a significantly smaller coherence length and each simulation therefore contains more ‘typical’ structures than the negative-skew case and fluctuations are correspondingly less.

We have confirmed the conclusion of CF91, that the relationship (3.2) seems to be reasonably well obeyed for Gaussian models by both the dark matter and the ‘galaxy’ distributions, at least for our particular scheme for identifying galaxies. Generally speaking the behaviour of galaxy and mass fluctuations is similar for all our models; the most noticeable effect is that in the negative-skew models, the galaxy skewness lies closer to the Gaussian locus than the mass fluctuations. The fact remains, however, that such analyses which rely only on galaxy clustering to test primordial fluctuations do rely on a particular relationship between galaxies and dark matter. Complicated non-linear and/or non-local biasing schemes could produce very different behaviour to that described here (Coles 1992; Bower et al. 1992).

By looking at the distribution of matter and galaxies in both redshift space and real space, we have confirmed that the effect of peculiar velocity distortions on the relationship (3.2) is small for all our models. This confirms the argument given by Bouchet et al. (1992).

In comparison with the QDOT results for skewness discussed by Saunders et al. (1991) we find that initially positive-skew models fare rather poorly and are excluded by $> 2\sigma$. Because the negative-skew models have rather larger errors associated with them and the systematic departures from the Gaussian form are rather smaller than the positive-skew cases, these models cannot be ruled out by the available skewness measurements. To constrain these models more strongly, we would need measurements of skewness with much smaller errors (i.e. from catalogues containing more galaxies) and preferably out to larger scales.

Of course we must stress that we have considered only a very small subset of the space of possible non-Gaussian models. All our models have the CDM power spectrum. Models with more (or less) large (or small) scale power may well behave differently. We have also chosen models with a very particular form of statistical distribution, obtained by locally transforming a Gaussian field. Galaxy
and structure formation involves a complicated interaction between primordial conditions and non-linear gravitational evolution and it would be surprising if the effects of the primordial spectrum and statistics upon the final density distribution could be separated out completely. We suspect however – and other work seems to confirm this idea (Lahav et al. 1992; Bouchet et al. 1992) – that the dominant influence on the skewness-variance relationship at late times is indeed the primordial skewness of the density fluctuations. We have not proved this to be true and to do so would require us to investigate all types of plausible initial power spectra, background cosmologies and so on. Although our work is thus, in a sense, limited it does demonstrate that the skewness of observed galaxy fluctuations is a potentially powerful probe of the initial distribution of density fluctuations.

Acknowledgments
PC acknowledges the SERC for support under the QMW rolling theory grant (GR/H09454) and thanks the Dipartimento di Astronomia at the Università di Padova for its hospitality during the visit when this work was initiated. LM acknowledges the Astronomy Centre at the University of Sussex for the hospitality during the visit when part of this work was done. FL, SM, AM and LM thank the Ministero Italiano dell’Università e della Ricerca Scientifica e Tecnologica for financial support. This work has been partially supported by Consiglio Nazionale delle Ricerche (Progetto Finalizzato: Sistemi Informatici e Calcolo Parallelo). The staff and the management of the CINECA Computer Center are warmly acknowledged for their assistance and for allowing the use of computational facilities.
References

Abbott L.F., Wise M.B., 1984, Nucl. Phys., B244, 541
Allen T.J., Grinstein B., Wise M.B., 1987, Phys. Lett., B197, 66
Babul A., White S.D.M., 1991, MNRAS, 253, 31P
Bardeen J.M., Bond J.R., Efstathiou G., 1987, ApJ, 321, 28
Barrow J.D., Coles P., 1990, MNRAS, 244, 188
Batuski D.J., Melott A.L., Burns J.O., 1987, ApJ, 322, 48
Bond J.R., Efstathiou G., Lubin P.M., Meinhold P.R., 1991, Phys. Rev. Lett., 66, 2179
Bouchet F.R., Hernquist L., 1992, ApJ, 400, 25
Bouchet F.R., Juszkiewicz R., Colombi S., Pellat R., 1992, ApJ, 394, L5
Bower R.G., Coles P., Frenk C.S., White S.D.M., 1992, ApJ, 000, 000
Cen R.Y., Ostriker J.P., Spergel D.N., Turok N., 1991, ApJ, 383, 1
Coles P., 1992, MNRAS, submitted
Coles P., Barrow J.D., 1987, MNRAS, 228, 407
Coles P., Frenk C.S., 1991, MNRAS, 253, 727 (CF91)
Coles P., Jones B.J.T., 1991, MNRAS, 248, 1
Coles P., Moscardini L., Plionis M., Lucchin F., Matarrese S., Messina A., 1992, MNRAS, 000, 000 (CMPLMM)
Davis M., Efstathiou G., Frenk C.S., White S.D.M., 1985, ApJ, 292, 371
Davis R.L., Hodges H.M., Smoot G.F., Steinhardt P.J., Turner M.S., 1992, Phys. Rev. Lett., 69, 1856
Efstathiou G., Kaiser N., Saunders W., Lawrence A., Rowan-Robinson M., Ellis R.S., Frenk C.S., 1990, MNRAS, 247, 10P
Fan Z.H., Bardeen J.M., 1992, preprint
Ikeuchi S., 1981, PASJ, 33, 211
Juszkiewicz R., Bouchet F., Colombi S., 1992, ApJ, 000, 000
Kendall M., Stuart A., 1977, The Advanced Theory of Statistics, Volume 1, 4th Edition, Griffin & Co, London, pp. 257–258
Kofman L., Blumenthal G., Hodges H., Primack J., 1991, in Latham D.W. & da Costa L.N. eds, Proceedings of the Workshop on Large-Scale Structure and Peculiar Motions in the Universe, pp. 339–351, ASP Conference Series
La D., Steinhardt P.J., 1989a, Phys. Rev. Lett., 62, 376
La D., Steinhardt P.J., 1989b, Phys. Lett., B220, 375
Lahav O., Itoh M., Inagaki S., Suto Y., 1992, ApJ, 000, 000
Liddle A.R., Lyth D., 1992, Phys. Lett., B291, 391
Liddle A.R., Wands D., 1991, MNRAS, 253, 637
Lidsey J.E., Coles P., 1992, MNRAS, 258, 57P
Lucchin F., Matarrese S., 1985a, Phys. Rev. D, 32, 1316
Lucchin F., Matarrese S., 1985b, Phys. Lett., B164, 282
Lucchin F., Matarrese S., Mollerach S., 1992, ApJ Lett., 000, 000
Maddox S.J., Efstathiou G., Sutherland W.J., Loveday J., 1990, MNRAS, 242, 43P
Martel H., Freudling W., 1991, ApJ, 371, 1
Matarrese S., Lucchin F., Messina A., Moscardini L., 1991, MNRAS, 252, 35
Matarrese S., Ortolan A., Lucchin F., 1989, Phys. Rev. D., 40, 290
Melott A.L., 1990, Phys. Rep., 193, 1
Messina A., Lucchin F., Matarrese S., Moscardini L., 1992, Astroparticle Phys., 000, 000
Moscardini L., Matarrese S., Lucchin F., Messina A., 1991, MNRAS, 248, 424 (MMLM)
Ostriker J.P., Cowie L.L., 1981, ApJ, 243, L127
Park C., 1991, ApJ, 382, L59
Park C., Spergel D.N., Turok N., 1991, ApJ, 372, L53
Peacock J.A., 1991, MNRAS, 253, 1P
Peebles P.J.E., 1980, *The Large Scale Structure of the Universe*, Princeton University Press, Princeton
Salopek D.S., 1992a, Phys. Rev. D., 45, 1139
Salopek D.S., 1992b, Phys. Rev. Lett., 000, 000
Salopek D.S., Bond J.R., 1991, Phys. Rev. D., 43, 1005
Salopek D.S., Bond J.R., Bardeen J.M., 1989, Phys. Rev. D., 40, 1753
Saunders W., *et al.*, 1991, Nat, 349, 32
Scherrer R.J., 1992, ApJ, 390, 330
Scherrer R.J., Bertschinger E., 1991, ApJ, 381, 349
Silk J., Juszkiewicz R., 1991, Nat, 353, 386
Smoot G.F., *et al.*, 1992, ApJ, 396, L1
Souradeep T., Sahni V., 1992, MNRAS, 000, 000
Turok N., 1989, Phys. Rev. Lett., 63, 2625
Vittorio N., Matarrese S., Lucchin F., 1988, ApJ, 328, 69
Vittorio N., Meinhold P.R., Muciaccia P.F., Lubin P.M., Silk J., 1991, ApJ, 372, L1
Weinberg D.H., Cole S., 1992, MNRAS, 000, 000
Weinberg D.H., Ostriker J.P., Dekel A., 1989, ApJ, 336, 9
Figure Captions

1. The relationship between skewness, $\gamma$, and variance, $\sigma^2$, for our five models. Figure 1a shows the results without shot–noise correction; Figure 1b has shot–noise corrections. The solid line is the theoretical relationship (3.2). The downward–pointing arrows in 1b indicate that the corrected skewness is negative; the arrows are plotted at the corrected variance value and originate at the uncorrected skewness value. All models are normalised to the present time (see Section 2). The three crosses are the QDOT measurements with error bars. Error bars on the simulated results are estimated using eq. (4.3a,b).

2. The effect of normalisation upon the skewness–variance relationship. Figure 2a shows all models normalised to $b = 1$ and Figure 2b shows the normalisation $b = 2.5$. The solid line is the theoretical relationship (3.2). All points are uncorrected for shot–noise. The small downward arrows on the theoretical line indicate that the uncorrected skewness is negative at that point. The three crosses are the QDOT measurements with error bars. Error bars on the simulated results are estimated using eq. (4.3a,b).

3. Skewness–variance relationships in real space and velocity (redshift) space. Filled circles show $\gamma$ and $\sigma^2$ in real space, open circles show velocity space. The two circles generally coincide when $\sigma^2$ is small. All models are normalised to the present time and we plot the shot–noise corrected results. Error bars are estimated using eq. (4.3a,b).