AN INVENTORY CONTROL PROBLEM FOR DETERIORATING ITEMS WITH BACK-ORDERING AND FINANCIAL CONSIDERATIONS UNDER TWO LEVELS OF TRADE CREDIT LINKED TO ORDER QUANTITY

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Abstract. The paper deals with an inventory control problem for perishable items where two level credit periods depend on the order quantity over the finite time horizon. We assume that the supplier offers delay in payment on outstanding cost of purchasing goods to the retailer when purchasing amount is more than a fixed large amount. Moreover, the retailer offers a delay period to the customers for payment of their purchasing goods. In the inventory system, shortage is permitted and it is completely backordered. The net present value of the retailer's cost function, including costs of ordering, inventory holding, shortage, purchasing and other opportunities, is optimized. Then, an algorithm is proposed to determine the optimal values of order quantity, shortage quantity, number of cycles and the total cost of the system. Finally, a numerical example with sensitivity analysis of the key parameters is illustrated to show the applicability of the proposed model.

1. Introduction. Sometimes, trade credit financing is applied in the supply chain management systems. In traditional inventory modeling, a part of supply chain, the buyer has to pay a fraction of purchasing cost at the time of receiving the order quantities. In practice, the supplier offers a delay period to the buyer for payment of the purchasing cost of an outstanding amount (predetermined value), if the retailer (buyer) places orders more than the predetermined amount. This strategy attracts the new buyers and price discounting occurs indirectly as well. Many researchers as well as practitioners have enlightened the inventory literature addressing this issue. Dates back to 1985, Goyal [11] was the first who introduced trade credit financing EOQ (economic order quantity) model in inventory literature. Thereafter, many research works have been done in this line of works. Among those,

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some noteworthy works in this field are mentioned as follows. Aggarwal and Jaggi [1] extended Goyal’s [11] model for deteriorating items. Huang [12] studied Goyal’s [11] model under two levels of trade credit policies when the down-stream trade credit period was less than the up-stream trade credit period. Soni et al. [23] presented a comprehensive study on the inventory control models with trade credit policies. Ouyang et al. [17] extended the Huang’s [12] work by relaxing two assumptions. Firstly, the interest rate charged by the supplier was not greater than the interest rate earned by the wholesaler. Secondly, the permissible delay period (M) offered by the supplier was independent of the permissible delay period (N) offered by the wholesaler. Taleizadeh et al. [25] investigated an EOQ model regarding partial delay in payment where shortage was permitted in the case of partial backordering. Sana [20] established an EOQ model with stochastic demand of perfect and imperfect quality products, incorporating trade credit financing. Pal et al. [18] developed a three-layer supply chain consisting of supplier, manufacturer and retailer to determine the optimal replenishment size of the supplier and production rate of the manufacturer under three levels of trade credit policies.

One of the usual assumptions of inventory control system is that all the items can be kept for infinite time in order to satisfy demands although this assumption occurs rarely in practice. As for illustrations, blood banks, drugs, foods, and chemical materials are perishable items those are deteriorated over time in warehouses. On the other hand, some materials such as alcohol or petrol evaporate with time during holding period in storage. Therefore, holding and management of perishable items are the vital issues for controlling of inventory. Valliathal and Uthayakumar [27] studied deterministic inventory model for perishable items which decayed exponentially over time. Sarkar et al. [22] introduced an inventory model for perishable items over finite time horizon, considering partial backlogging and time varying deterioration. The inventory modeling on deteriorating items has a vast literature. The notable review works on the inventory control model developed for deteriorating products were performed by Bakker et al. [2], Li et al. [15] and Khanlarzade et al. [14].

In globalization and competitive environment, inflation and the time value of money should be considered significantly in any business analysis, because it sharply declines the purchasing power of money. Ray and Chadhuri [19] presented an EOQ model over finite planning horizon, considering the time varying demand, shortage, inflation and time value of money. Chen [6] investigated an inventory model for time dependent demand by incorporating backlogging, inflation and time value of money. Jamal et al. [13] introduced the EOQ model for deteriorating items under delay in payment policy where the payment time for the retailer was optimized. Liao and Chen [16] studied an EOQ model over finite planning horizon for perishable items while demand rate was dependent on order size. In this model, delay in payments and constant inflation were assumed. Chang et al. [3] proposed an EOQ model under time varying demand and deterioration rates with delay payments linked to order quantity. Chang [4] suggested an EOQ model over finite planning horizon with credit financing linked to order quantity of perishable items. Chang et al. [5] assessed the economic order of perishable items and optimal conditions of payment policies and prices paid by the retailers. In this model, shortage was not permitted and rates of deterioration and inflation were considered to be fixed, and planning horizon was assumed to be finite. Widyadana et al. [28] studied an EOQ model with and without deterioration and compared their optimal solutions with
the classical methods for solving the model. Chung [8] optimized an EOQ model for perishable products under delayed payment policy linked to order quantity. Sarkar [21] developed an EOQ model with finite replenishment rate in which both the demand and deterioration rate were time-dependent. Taleizadeh and Nematollahi [24] proposed an inventory model for deteriorating items considering inflation, where one level of credit period and shortage were permitted. Chiu et al. [25] investigated an integrated imperfect production inventory model in a single vendor-single buyer environment under permissible delay in payments. Ghoreishi et al. [10] studied an EOQ model for non-instantaneous deteriorating items under the effect of inflation and customer returns when demand rate is dependent on selling price and inflation. Chung and Ting [9] developed an inventory model under supplier’s partial trade credit policy in a supply chain system. Ting [26] discussed an EPQ model for perishable items under two levels of trade credit financing.

All of the above mentioned factors such as inflation, time value of money, delay in payment linked to the order quantity and deterioration are regarded separately in the inventory modeling in the existing literature, but none of them considered all of the above factors comprehensively in one model. In our proposed model, we present an inventory model in a supply chain considering two levels of credit periods, in which the supplier lets the retailer to pay off the purchasing cost in a fixed period of time \( M \), if the ordering quantity is larger than a predetermined (outstanding) order quantity \( Q_d \). Moreover, in order to be more practical, the retailer lets his customers to pay off the money with delay period \( V \). With in the time span \([0, M]\), the retailer may earn interest from selling items and he/she may start his business borrowing loan from bank, if the retailer does not have capital at the initial stage. On the other hand, if retailer fails to pay the outstanding amount of the purchasing cost within the time \( M \), the interest of unpaid amount will be charged until total amount is paid to the supplier. Incorporating inflation and time value of money, net present value of the cost function of the retailer over finite time horizon planning is formulated and minimized analytically as well as numerically.

The rest of the article is organized as follows: Section 2 provides the nomenclature of the parameters and assumptions of the model. Formulation of the model is discussed in section 3. Numerical example with its sensitivity analysis is given in section 4. Section 5 concludes the contribution of the paper.

2. Fundamental notation and assumption. The following notations and assumptions are adopted to depict the proposed model.

2.1. Notation.

2.1.1. Parameters:

\( A \): The fixed ordering cost ($) per replenishment.
\( C \): The unit purchasing cost ($) at the time of placing an order (i.e., \( t = 0 \)).
\( i \): Rate of inflation per currency ($) per unit time (year).
\( r \): Decay rate (interest), reflecting the time value of money.
\( R \): Represents the net rate of constant \( R = (r - i) \) decline in inflation.
\( C(t) \): The unit purchasing cost ($) at time \( t \) \( C(t) = e^{-Rt} \).
\( D \): The fixed rate of demand per unit time.
\( H \): The finite length of planning horizon.
\( I(t) \): The onhand inventory level at time \( t \) (year).
\( I_e \): The interest rate received per ($) per year by the retailer.
\( I_c \): The interest rate paid per ($) per year by the retailer.
2.1.2. Decision variables:

- $F$: The fraction of replenishment cycle, in which, inventory level is positive.
- $N$: The number of replenishment during the planning horizon.

2.1.3. Dependent variables:

- $Q$: The order quantity in each replenishment.
- $I_b$: The maximum shortage quantity.
- $I_m$: The maximum inventory level.
- $IE_{\alpha}$: Earned interest related to case $\alpha, \alpha = 1, 2, \ldots, 6$.
- $IP_{\alpha}$: Payable interest related to case $\alpha, \alpha = 1, 2, \ldots, 6$.
- $CC_{\alpha}$: Net present value of capital cost for case $\alpha, \alpha = 1, 2, \ldots, 6$.
- $TC_{\alpha}$: Net present value of total cost for case $\alpha, \alpha = 1, 2, \ldots, 6$.
- $TC_A$: Net present value of the total fixed ordering cost during $[0, H]$.
- $TC_b$: Net present value of the total holding cost during $[0, H]$.
- $TC_P$: Net present value of the total purchasing cost during $[0, H]$.
- $TC_S$: Net present value of the total shortage cost during $[0, H]$.

2.2. Assumptions. The following assumptions are made in order to describe the problem:

- Planning horizon is finite.
- Inventory control considers single item.
- Demand and decay rates are assumed to be constant.
- As soon as stuffs are stored in warehouses, deterioration occurs.
- Shortage is allowed and it is completely backordered.
- The inflation rate is constant with consideration of time value of money.
- The lead time is zero.
- The inventory level is zero at the end of the planning horizon.
- The number of replenishments is integer.
- The expenditures include the ordering cost, purchasing cost, holding cost, shortage cost, capital cost, interest payable, and interest received from sales of stuffs only.
- Last order is only considered to supply the demand of the last period.
- No interest paid or received for the quantity of stuffs which have been corrupted/deteriorated in the period of $\frac{T_h}{N}$ (In fact, at the moment of $M$, the amount of determined objects are paid).
- During the time span $(M - V)(M > V)$, the retailer sells the products and puts its revenue in the bank to earn interest. At the time of $\frac{M}{2}$, he borrows...
from bank and pays its interest since he/she hasn’t received all the money from the buyers.

3. Model formulation. The planning horizon $H$ is divided into $N$ equal cycle length $T$ (see Fig.1) such that $H = NT$, where $N$, the number of replenishments, is a decision variable and $T$ is the time between two replenishments. The on-hand inventory $I(t)$ depletes due to the demand and the decay of products and it reaches to zero at time $t_1$. Then, shortage continues up to time $T$ and total backlogged during $[t_1, T]$ is satisfied when the next order arrives to the system. Therefore, the governing differential equations of the inventory levels at time ‘$t$’ are as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 \leq t \leq t_1 \text{ with } I(0) = I_m \quad (1)$$

and

$$\frac{dI(t)}{dt} = -D, \quad t_1 \leq t \leq T \text{ with } I(t_1) = 0 \text{ and } I(T) = I_b. \quad (2)$$

Using the boundary conditions, from Eqs.(1) and (2), we have

$$I(t) = \frac{D}{\theta} [e^{\theta(t_1-t)} - 1], \quad 0 \leq t \leq t_1 \quad (3)$$

and

$$I(t) = -D(t - t_1), \quad t_1 \leq t \leq T \quad (4)$$

respectively. According to Eqs. (3) and (4), maximum inventory quantity at the beginning of each period and maximum shortage at the end of each period are

$$I_m = I(0) = \frac{D}{\theta} \left( e^{\theta t_1} - 1 \right) = \frac{D}{\theta} \left( e^{\theta FH/N} - 1 \right) \quad (5)$$

and

$$I_b = D(T - t_1) = D \left( \frac{H}{N} - \frac{FH}{N} \right) \quad (6)$$

respectively.

Since we consider two levels delayed payment policies linked to order quantity, the total cost of inventory for retailer depends on his/her order quantity that results in larger or smaller size than the predetermined value of $Q_d$. Indeed, the supplier offers to the retailer a delay period ($M$) if the order size is more than $Q_d$, otherwise the retailer has to pay the total purchasing cost at the time of receiving the order. But, in both the conditions, the retailer offers delay period ($V$) to the customers.

3.1. Fixed ordering cost. The net present value of the ordering cost for $N$ cycles, taking into account of inflation and time value of money, is

$$TC_A = \sum_{j=0}^{N} A_j T = \sum_{j=0}^{N} A e^{-jRT} = A \left( e^{-\frac{(N+1)RH}{N}} - 1 \right). \quad (7)$$

3.2. Holding cost, except the cost of capital. Firstly, we obtain the average inventory in each period. Therefore, from Eq.(3), we have

$$\bar{I} = \int_0^{t_1} I(t) dt = \int_0^{t_1} \frac{D}{\theta} \left( e^{\theta(t_1-t)} - 1 \right) dt = \frac{D}{\theta^2} \left( e^{\theta t_1} - \theta t_1 - 1 \right). \quad (8)$$

Now, the net present value of the total cost for holding the inventory during $[0, H]$ is

$$TC_h = \sum_{j=0}^{N-1} I_h C_j \bar{I} = \sum_{j=0}^{N-1} I_h C e^{-jRT} \bar{I}$$
3.3. Shortage cost. Similar to the holding cost, to determine the shortage cost, the average shortage would be specified as shown in Equation (10).

$$\bar{B} = \int_{t_1}^{T} I(t)dt = \left( \frac{H}{N} - \frac{FH}{N} \right) * D \left( \frac{H}{N} - \frac{FH}{N} \right) = \frac{D}{2} \left( \frac{H}{N} - \frac{FH}{N} \right)^2.$$ (10)

Therefore, the net present value of the shortage cost is

$$TC_S = \sum_{j=0}^{N-1} \pi_j \bar{B} = \sum_{j=0}^{N-1} \pi e^{-jRT} \bar{B}$$

$$= \pi \left( \frac{e^{-RH}}{e^{-RH/N} - 1} \right) \bar{B}$$

$$= \pi \frac{D}{2} \left( \frac{H}{N} - \frac{FH}{N} \right)^2 \left( \frac{e^{-RH}}{e^{-RH/N} - 1} \right).$$ (11)

3.4. Purchasing cost. The purchasing cost of the retailer in $j^{th}$ period is

$$C_{p(j)} = C_{(j)} I_m + C_{(j+1)} T I_b$$

$$= \frac{C_{(j)} D}{\theta} \left( e^{\theta FH/N} - 1 \right) + C_{(j+1)} T D \left( \frac{H}{N} - \frac{FH}{N} \right); j = 0, 1, 2, \ldots, N.$$ (12)

So, the net present value of the purchasing cost over the planning horizon is

$$TC_P = \sum_{j=0}^{N-1} C_{p(j)} = \frac{CD}{\theta} \left( e^{\theta FH/N} - 1 \right) \left( \frac{e^{-RH}}{e^{-RH/N} - 1} \right)$$

$$+ CD e^{-RH/N} \left( \frac{H}{N} - \frac{FH}{N} \right) \left( \frac{e^{-RH}}{e^{-RH/N} - 1} \right).$$ (13)
3.5. Variables cost.

3.5.1. Case-1. When $Q < Q_d$. In this situation, the retailer’s order quantity does not exceed the predetermined quantity $Q_d$ that results in zero delay period and the supplier does not offer delay in payment to the retailer. However, the retailer is permissible to receive money from buyers (in any situation) during $V$. As a result, the retailer borrows from bank at time zero and at time $V$ is starting to pay back it (Fig. 2). So, payable interest by the retailer and capital cost during a cycle are

$$IP_1 = IcC \left( \frac{DHV}{N} + \frac{DH^2F^2}{2N^2} \right)$$

and

$$CC_1 = \frac{LeCD}{\theta^2} \left( e^{\frac{\theta FH}{N}} - \frac{\theta FH}{N} - 1 \right)$$

respectively. Therefore, the retailer’s total cost in a cycle is

$$TC_1 = A + \frac{\pi D}{2} \left( \frac{H}{N} - \frac{F H}{N} \right)^2 + CD e^{-RH} \left( \frac{H}{N} - \frac{F H}{N} \right) + \frac{CD}{\theta} \left( e^{\frac{\theta FH}{N}} - 1 \right) + \frac{(I_h + I_e)CD}{\theta^2} \left( e^{\frac{\theta FH}{N}} - \frac{\theta FH}{N} - 1 \right) + IC \left( \frac{DHV}{N} + \frac{DH^2F^2}{2N^2} \right).$$

(16)

And, the net present value of the total cost during the planning horizon is

$$TC_1 = A \left( \frac{e^{(N+1)RH/N} - 1}{e^{RH/N} - 1} \right) + \left[ \frac{\pi D}{2} \left( \frac{H}{N} - \frac{F H}{N} \right)^2 + CD e^{-RH} \left( \frac{H}{N} - \frac{F H}{N} \right) \right] + \frac{CD}{\theta} \left( e^{\frac{\theta FH}{N}} - \frac{\theta FH}{N} - 1 \right) + \frac{(I_h + I_e)CD}{\theta^2} \left( e^{\frac{\theta FH}{N}} - \frac{\theta FH}{N} - 1 \right) + IC \left( \frac{DHV}{N} + \frac{DH^2F^2}{2N^2} \right).$$

(17)

3.5.2. When $Q > Q_d$. In this case, the order quantity of the retailer exceeds the predetermined value $Q_d$, and the supplier offers delay period $M (M \neq 0)$. According to the values of $M, t_1$ and $V$, the possible permutation among these are as follows:
by the supplier and the capital cost are as follows:

\[
\begin{pmatrix}
\text{Case-2:} & Q > Q_d & M < V & M < t_1 \\
\text{Case-3:} & Q > Q_d & M < V & t_1 < M \\
\text{Case-4:} & Q > Q_d & M > V & t_1 > M \\
\text{Case-5:} & Q > Q_d & M > V & t_1 < M, M < t_1 + V \\
\text{Case-6:} & Q > Q_d & M > V & t_1 < M, M > t_1 + V
\end{pmatrix}
\]

3.5.3. **Case-2.** When \( Q > Q_d, M < V \) and \( M < t_1 \). This case is similar to the first case with the difference that the diagram of interest payable moved to the right by \( M \) units. This instigates to reduce capital cost of the retailer (Fig.3). In this case, as \( M < V \), the retailer doesn’t have any income. Consequently, the amount of payable interest and capital costs are

\[
IP_2 = I_cC \left( \frac{DH(V-M)}{N} + \frac{DF^2H^2}{2N^2} \right)
\]

and

\[
CC_2 = I_c \int_M^{\ell_1} I_t dt = I_c C(0) \left( \int_M^{\ell_1} \frac{D}{\theta} \left( e^{\theta(t_1-t)} - 1 \right) dt \right)
\]

Therefore, the net present value of the retailer’s total cost during \([0, H]\) is

\[
TC_2 = A \left( e^{-\left(\frac{N+1}{e^{-RH/N-1}}\right)} \right) + \left[ e^{\frac{D}{\theta}} \left( \frac{H}{N} - \frac{FH}{N} \right)^2 + CD e^{-RH} \left( \frac{H}{N} - \frac{FH}{N} \right) \right] \\
\times \left( e^{-\left(\frac{RH}{e^{-RH/N-1}}\right)} \right) + \left[ \frac{D}{\theta} e^{\thetaFH/N} - \frac{DF}{\theta} - 1 \right] + \frac{CD}{\theta} \left( e^{\thetaFH/N} - 1 \right)
\]

\[
+ \frac{CD}{\theta} \left( \frac{H}{N} - \frac{FH}{N} + \frac{1}{\theta} e^{(FH/N-M)} + M \right) + I_c C \left( \frac{DH(V-M)}{N} + \frac{DF^2H^2}{2N^2} \right)
\]

\[
\times \left( e^{-\left(\frac{RH}{e^{-RH/N-1}}\right)} \right)
\]

3.5.4. **Case-3.** When \( Q > Q_d, M < V \) and \( M > t_1 \). Since \( M > t_1 \), the capital cost of this scenario is zero and, due to \( M < V \) (Fig.4), the retailer does not have any income. In this case, there is only payable interest which is

\[
IP_3 = I_cC \left( \frac{DH(V-M)}{N} + \frac{DF^2H^2}{2N^2} \right).
\]

Therefore, the net present value of the total cost is

\[
TC_3 = A \left( e^{-\left(\frac{N+1}{e^{-RH/N-1}}\right)} \right) + \left[ e^{\frac{D}{\theta}} \left( \frac{H}{N} - \frac{FH}{N} \right)^2 + CD e^{-RH} \left( \frac{H}{N} - \frac{FH}{N} \right) \right] \\
+ \frac{CD}{\theta} \left( e^{\thetaFH/N} - 1 \right) + \frac{I_c CD}{\theta^2} \left( e^{\thetaFH/N} - \frac{\theta FH}{N} - 1 \right)
\]

\[
+ I_c C \left( \frac{H(V-M)}{N} + \frac{DF^2H^2}{2N^2} \right) \left( e^{-\left(\frac{RH}{e^{-RH/N-1}}\right)} \right)
\]

3.5.5. **Case-4.** When \( Q > Q_d, V < M \) and \( M < t_1 \). In this situation, retailer’s order quantity exceeds the predetermined quantity \( Q_d \). The retailer can earn interest from selling the items up to time \( M \), and he has to pay interest on purchased items if he couldn’t pay the outstanding amount of purchasing cost within \( M \). According to above explanation and Fig.5, the earned interest by the retailer, interest charged by the supplier and the capital cost are as follows:

\[
IE_4 = I_c S \left[ 2D \left( \frac{H}{N} - \frac{FH}{N} \right) + M(M-V) \right] (M-V),
\]

\[
\]
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Figure 3. Calculation of the interest payable when \( Q > Q_d, M < V < t_1 \)

Figure 4. Calculation of the interest payable when \( Q > Q_d, V > M > t_1 \)

\[
IP_4 = \frac{DI_c}{2} \left( \frac{FH}{N} + V - M \right)^2 \tag{24}
\]

and

\[
CC_4 = I_c C \int_M^{t_1} I_t dt = I_c C(0) \int_M^{t_1} \frac{D}{\theta} \left( e^{\theta(t_1-t)} - 1 \right) dt \\
= \frac{I_c CD}{\theta} \left( \frac{1}{\theta} - \frac{FH}{N} + \frac{1}{\theta} e^{\theta(FH/N-M)} + M \right). \tag{25}
\]

Now, the net present value of the total cost of the retailer during \([0, H]\) is

\[
TC_4 = A \left( e^{-(N+1)RH/N} - 1 \right) + \left[ \frac{zD}{2} \left( \frac{H}{N} - \frac{FH}{N} \right)^2 + CD \left( \frac{H}{N} - \frac{FH}{N} \right) e^{-RH} \right. \\
\left. + \frac{CD}{\theta} \left( e^{\theta(FH/N-M)} - 1 \right) + \frac{I_c CD}{\theta^2} \left( e^{\theta(FH/N) - \theta FH/N} - 1 \right) \right] \\
+ \frac{I_c CD}{\theta} \left( \frac{1}{\theta} - \frac{FH}{N} + \frac{1}{\theta} e^{\theta(FH/N-M)} + M \right) + \frac{I_c CD}{2} \left( \frac{FH}{N} + V - M \right)^2 \\
- \frac{L_s}{2} \left( 2D \left( \frac{H}{N} - \frac{FH}{N} \right) + D(M-V) \right) (M-V) \right] \left( e^{-(N+1)RH/N} - 1 \right). \tag{26}
\]
3.5.6. Case-5. When $Q > Q_d, V < M < t_1$ and $M < t_1 + V$. Since $t_1 < M$, the capital cost is zero. Due to $M < t_1 + V$ (Fig.6), the retailer has not received total revenue from the selling goods. The interest charged by the supplier due to late payment is

$$IP_5 = \frac{I_r C D}{2} \left( \frac{FH}{N} + V - M \right)^2. \quad (27)$$

And, the interest earned by the retailer during $[0, M]$ is

$$IE_5 = \frac{I_r S}{2} \left[ 2D \left( \frac{H}{N} - \frac{FH}{N} \right) + D(M - V) \right] (M - V). \quad (28)$$

Therefore, the net present value of the total cost of the retailer in this case is equal to

$$TC_5 = A \left( \frac{e^{-(N+1)RH/N} - 1}{e^{-RH/N} - 1} \right) + \left[ \frac{\pi D}{2} \left( \frac{H}{N} - \frac{FH}{N} \right) \right]^2.$$
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\[ + CD \left( \frac{H}{N} - \frac{FH}{N} \right) e^{-RH} + \frac{CD}{\theta} \left( e^{\theta FH/N} - 1 \right) \]

\[ + \frac{I_c CD}{\theta^2} \left( e^{\theta FH/N} - 1 - \frac{\theta FH}{N} \right) + \frac{I_c CD}{2} \left( \frac{FH}{N} + V - M \right)^2 \]

\[ - \frac{I_c S}{2} \left( 2D \left( \frac{H}{N} - \frac{FH}{N} \right) + D(M - V) \right) (M - V) \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right). \]

3.5.7. Case 6. When \( Q > Q_d, V < M, t_1 < M \) and \( M > t_1 + V \). Since \( M > t_1 + V \), the retailer has earned total revenues from selling the items to the customers before the time \( M \). In this situation, the retailer can earn interest (Fig. 7) which is equal to

\[ IE_6 = I_c S \left[ \frac{FH}{2N} \left( D \left( \frac{H}{N} - \frac{FH}{N} \right) + \frac{DH}{N} \right) + \frac{DH}{N} \left( M - \frac{FH}{N} - V \right) \right]. \]

Therefore, the net present value of the total cost is

\[ TC_6 = A \left( \frac{e^{-(N+1)RH/N} - 1}{e^{-RH/N} - 1} \right) + \left[ \frac{I_c S}{2} \left( \frac{H}{N} - \frac{FH}{N} \right)^2 \right] \]

\[ + CD \left( \frac{H}{N} - \frac{FH}{N} \right) e^{-RH} + \frac{CD}{\theta} \left( e^{\theta FH/N} - 1 \right) \]

\[ + \frac{I_c CD}{\theta^2} \left( e^{\theta FH/N} - 1 - \frac{\theta FH}{N} \right) - I_c S \left( \frac{FH}{2N} \left( D \left( \frac{H}{N} - \frac{FH}{N} \right) + \frac{DH}{N} \right) \right) \]

\[ + \frac{DH}{N} \left( M - \left( \frac{FH}{N} + V \right) \right) \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right). \]

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Now, our objective is to minimize the following objective function

\[ TC(F, N) = \begin{cases} 
TC_1(F, N) & Q < Q_d \\
TC_2(F, N) & Q > Q_d, \ M < V, \ M < t_1 \\
TC_3(F, N) & Q > Q_d, \ M < V, \ t_1 < M \\
TC_4(F, N) & Q > Q_d, \ V < M, \ M < t_1 \\
TC_5(F, N) & Q > Q_d, \ V < M, \ t_1 < M, \ M < t_1 + V \\
TC_6(F, N) & Q > Q_d, \ V < M, \ t_1 < M, \ M > t_1 + V \\
such that \ F \epsilon [0, 1], \ N \epsilon Z^+ 
\end{cases} \]
3.6. Solution method. In real life problems of inventory control model, deterioration rate is too small in many cases. Then, we may approximate $e^{\theta k}$ up to third term by using Taylor series expansion as follows:

$$e^{\theta k} = 1 + \theta k + \frac{1}{2}(\theta k)^2. \quad (33)$$

Using the above approximate value of $e^{\theta k}$ in Eq.(17), we obtain as follows:

$$TC_1 = Ae^{-RH/N} + \left[ A + \frac{\pi DH^2}{2N^2} (1 - F)^2 + \frac{CDH}{N} (1 - F)e^{-RH} + \frac{CDFH}{N} \left( 1 + \frac{\theta FH}{N} \right) \right] + \frac{CDF^2H^2}{2N^2}(I_h + I_v) + I_C \left( \frac{DHV}{N} + \frac{DF^2H^2}{2N^2} \right) \left( \frac{e^{-RH+1}}{e^{-RH/N-1}} \right) \quad (34)$$

Differentiation of Equation (34) with respect to ‘$F$’ yields

$$\frac{dT C_1}{dF} = \left[ \frac{\pi DH^2}{N^2}(F - 1) - \frac{CDH}{N} e^{-RH} + \frac{CDH}{N} \left( 1 + \frac{\theta FH}{N} \right) \right] + \frac{CDF^2H^2}{2N^2}(I_h + I_v) + I_C \left( \frac{DHV}{N} + \frac{DF^2H^2}{2N^2} \right) \left( \frac{e^{-RH+1}}{e^{-RH/N-1}} \right) \quad (35)$$

And, the second derivative of the cost function with respect to ‘$F$’ is

$$\frac{d^2T C_1}{dF^2} = \left[ \frac{\pi DH^2}{N^2} + \frac{\theta CDFH^2}{N^2} + \frac{CDH^2}{N^2}(I_h + I_v) + \frac{LCDH^2}{N^2} \right] \left( \frac{e^{-RH+1}}{e^{-RH/N-1}} \right) \geq 0. \quad (36)$$

The above derivative is positive always as $F$ is positive integer. Thus $TC_1$ is a convex function over $N$. Equating the expression of Eq. (35) to zero, we have the value of $F$ (say $F_1$) as follows:

$$F_1 = \frac{\pi + CN(e^{-RH} - 1)/H}{\pi + C\theta + C(I_h + 2I_v)} \quad (37)$$

By using the approximation in Eq.(33), we may rewrite Eq.(20) as follows:

$$TC_2 = Ae^{-RH} + \left[ A + \frac{\pi DH^2}{2N^2} (1 - F)^2 + \frac{CDH}{N} (1 - F)e^{-RH} \right.\left. + \frac{CDFH}{N} \left( 1 + \frac{\theta FH}{N} \right) \right] + \frac{CDF^2H^2}{2N^2}(I_h + I_v) + I_C \left( \frac{H(V-M)}{N} \right) \quad (38)$$

The first and the second derivatives of the above cost function with respect to $F$ are

$$\frac{dT C_2}{dF} = \left[ \frac{\pi DH^2}{N^2}(F - 1) - \frac{CDH}{N} e^{-RH} + \frac{CDH}{N} \left( 1 + \frac{\theta FH}{N} \right) \right] + \frac{CDF^2H^2}{2N^2}(I_h + I_v) + I_C \left( \frac{H(V-M)}{N} \right) \left( \frac{e^{-RH+1}}{e^{-RH/N-1}} \right) \quad (39)$$

and

$$\frac{d^2T C_2}{dF^2} = \left[ \frac{\pi DH^2}{N^2} + \frac{\theta CDFH^2}{N^2} + \frac{CDH^2}{N^2}(I_h + I_v) + \frac{LCDH^2}{N^2} \right] \left( \frac{e^{-RH+1}}{e^{-RH/N-1}} \right) \geq 0. \quad (40)$$

Here, $TC_2$ is a convex function. Setting Eq.(39) equal to zero, one can obtain the optimum value of $F$ (say $F_2$) as follows:

$$F_2 = \frac{\pi + CN(e^{-RH} - 1)/H + I_CCMN/H}{\pi + C\theta + C(I_h + 2I_v)}. \quad (41)$$
Similarly, we can rewrite Eq.(22) as follows:

\[
TC_3 = Ae^{-RH} + \left[ A + \frac{\pi DH^2}{2N^2} (1 - F)^2 + \frac{CDH}{N} (1 - F)e^{-RH} + \frac{CDFH}{N^2} \left( 1 + \frac{\theta F}{N} \right) \right. \\
+ \left. \frac{CDF^2H^2}{2N^2} (I_h + I_c) + I_c CD \left( \frac{H(V - M)}{N} \right) \right] \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right).
\] (42)

The first and the second derivatives of \( TC_3 \) are as follows:

\[
\frac{dTC_3}{dF} = \left[ \frac{\pi DH^2}{N^2} (F - 1) - \frac{CDH}{N} e^{-RH} + \frac{CDH}{N} \left( 1 + \frac{\theta F}{N} \right) \right. \\
+ \left. \frac{CDF}{N} \left( 1 + \frac{\theta F}{N} \right) + \frac{I_c CD^2 H^2}{2N^2} + \frac{I_c CD}{N^2} (FH + NV - NM)^2 \right] \\
\times \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right)
\] (43)

and

\[
\frac{d^2TC_3}{dF^2} = \left[ \frac{\pi DH^2}{N^2} + \frac{\theta CDH^2}{N^2} + \frac{CDH^2}{N^2} (I_h + I_c) \right] \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right) \geq 0.
\] (44)

Here, \( TC_3 \) is convex function as \( d^2TC_3/dF^2 \geq 0 \). Setting Eq.(43) equal to zero, the optimum value of \( F \) (say \( F_3 \)) is

\[
F_3 = \frac{\pi + CN(e^{-RH} - 1)/H}{\pi + C\theta + C(I_h + I_c)}.
\] (45)

Using the approximation of Eq.(33) in Eq.(26), we have

\[
TC_4 = Ae^{-RH} + \left[ A + \frac{\pi DH^2}{2N^2} (1 - F)^2 + \frac{CDH}{N} (1 - F)e^{-RH} \right. \\
+ \left. \frac{CDFH}{N} \left( 1 + \frac{\theta F}{N} \right) + \frac{I_c CD^2 H^2}{2N^2} + \frac{I_c CD}{2N^2} (FH + NV - NM)^2 \right] \\
\times \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right).
\] (46)

The first and the second derivatives of the above function are

\[
\frac{dTC_4}{dF} = \left[ \frac{\pi DH^2}{N^2} (F - 1) - \frac{CDH}{N} e^{-RH} + \frac{CDH}{N} \left( 1 + \frac{\theta F}{N} \right) \right. \\
+ \left. \frac{CDF}{N} \left( 1 + \frac{\theta F}{N} \right) + \frac{I_c CD^2 H^2}{N^2} (2FH + VN - 2MN) + \frac{I_c SDH}{N} (M - V) \right] \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right)
\] (47)

and

\[
\frac{d^2TC_4}{dF^2} = \left[ \frac{\pi DH^2}{N^2} + \frac{\theta CDH^2}{N^2} + \frac{CDH^2}{N^2} (I_h + 2I_c) \right] \left( \frac{e^{-RH} - 1}{e^{-RH/N} - 1} \right) \geq 0
\] (48)

respectively. Here, \( TC_4 \) is convex function because the second order derivative of objective function with respect to \( F \) is positive. By setting Eq.(47) equal to zero, the value of \( F \) (say \( F_4 \)) is obtained as below:

\[
F_4 = \frac{\pi + CN(e^{-RH} - 1)/H + I_c CN(2M - V)/H + I_c SN(V - M)/H}{\pi + C\theta + C(I_h + 2I_c)}.
\] (49)
Similarly as before, we may rewrite Eq.(29) as follows:

\[ TC_5 = e^{-RH} + \left[ A + \frac{\pi DH^2}{2N^2}(1-F)^2 + \frac{CDH}{N}(1-F) e^{-RH} \right. \]

\[ + \frac{CDFH}{N} \left( 1 + \frac{\theta F H}{2N} + \frac{I_hCDF^2H^2}{2N^2} + \frac{LCD}{2N^2}(FH + NV - NM)^2 \right. \]

\[ - \left. \frac{I_e S}{2} \left( \frac{2DH}{N}(1-F) + D(M-V) \right)(M-V) \right] \left( \frac{1}{e^{\frac{RH}{N}-\frac{1}{1}}} \right). \]  

The first and the second order derivatives of the objective function with respect to \( F \) are as follows:

\[ \frac{dTC_5}{dF} = \left[ \frac{\pi DH^2}{N^2}(F - 1) - \frac{CDH}{N} e^{-RH} + \frac{CDH}{N} \left( 1 + \frac{\theta F H}{N} \right) + \frac{CDFH^2}{N^2} I_h \right. \]

\[ + \frac{LCDH}{N^2}(FH + VN - MN) + \frac{LSDH}{N}(M-V) \right] \left( \frac{1}{e^{\frac{RH}{N}-\frac{1}{1}}} \right) \]  

and

\[ \frac{d^2TC_5}{dF^2} = \left[ \frac{\pi DH^2}{N^2} + \frac{\theta CDH^2}{N^2} + \frac{CDH^2}{N^2} (I_h + I_e) \right] \left( \frac{1}{e^{\frac{RH}{N}-\frac{1}{1}}} \right) \geq 0 \]  

respectively. Since the second derivative shown in Eq.(52) is ever positive. Hence, the objective function \( TC_5 \) is a convex function. So setting Eq.(51) equal to zero, the optimum value of \( F \) (say \( F_5 \)) becomes as follows:

\[ F_5 = \frac{\pi + CN(e^{-RH} - 1)/H + I_e CN(M - V)/H + I_e SN(V - M)/H}{\pi + C\theta + C(I_h + I_e)}. \]  

Using the approximation of Eq.(33) in Eq.(31), we have as follows:

\[ TC_6 = e^{-RH} + \left[ A + \frac{\pi DH^2}{2N^2}(1-F)^2 + \frac{CDH}{N}(1-F) e^{-RH} \right. \]

\[ + \frac{CDFH}{N} \left( 1 + \frac{\theta F H}{2N} + \frac{I_hCDF^2H^2}{2N^2} \right. \]

\[ - \left. I_e S \left( \frac{2FH}{N^2}(2-F) + \frac{DH}{N^2}(MN - V N - FH) \right) \right] \left( \frac{1}{e^{\frac{RH}{N}-\frac{1}{1}}} \right). \]  

The first and the second order derivatives of the above function with respect to \( F \) are

\[ \frac{dTC_6}{dF} = \left[ \frac{\pi DH^2}{N^2}(F - 1) - \frac{CDH}{N} e^{-RH} + \frac{CDH}{N} \left( 1 + \frac{\theta F H}{N} \right) + \frac{CDFH^2}{N^2} I_h \right. \]

\[ + \left. \frac{LSDH^2}{N^2}(F - 1) \right] \left( \frac{1}{e^{\frac{RH}{N}-\frac{1}{1}}} \right) \]  

and

\[ \frac{d^2TC_6}{dF^2} = \left[ \frac{\pi DH^2}{N^2} + \frac{\theta CDH^2}{N^2} + \frac{DH^2}{N^2} (I_h + I_e) \right] \left( \frac{1}{e^{\frac{RH}{N}-\frac{1}{1}}} \right) \geq 0 \]  

respectively. Thus \( TC_6 \) is a convex function. By setting Equation (55) equal to zero, the optimum value of \( F \) (say \( F_6 \)) is obtain as below:

\[ F_6 = \frac{\pi + CN(e^{-RH} - 1)/H + I_e S}{\pi + C\theta + I_h C + I_e S}. \]  

Finally, the following algorithm is suggested in order to find out the optimal values of \( F \) and \( N \) for all possible cases mentioned earlier.
3.6.1. Algorithm:

Step 1. Using initial parameters, calculate the value of $T_d = Q_d / D$.

Step 2. Calculate the values of $(F_\alpha, n_\alpha^*); \alpha = 1, \ldots, 6$ as follows.

2-1. First, put $n_\alpha = 1$ and calculate $F_1(n_1), F_2(n_2), F_3(n_3), F_4(n_4), F_5(n_5), F_6(n_6)$ from Equations (37), (41), (45), (49), (53), (57) respectively.

2-2. Calculate the values of objective functions $TC_\alpha(n_\alpha, F_\alpha(n_\alpha)); \forall \alpha = 1, \ldots, 6$ using Equations (34), (38), (42), (46), (50), (54), respectively.

2-3. Put $n_\alpha = n_\alpha + 1; \forall \alpha = 1, \ldots, 6$ and calculate the values of $F_\alpha(n_\alpha)$ using Equations (37), (41), (45), (49), (53), (57), respectively.

2-4. Calculate the values of objective functions $TC_\alpha(n_\alpha, F_\alpha(n_\alpha)); \forall \alpha = 1, \ldots, 6$ using Equations (34), (38), (42), (46), (50), (54), respectively.

2-5. If $TC_\alpha(n_\alpha, F_\alpha) > TC_\alpha(n_\alpha - 1, F_\alpha - 1); \forall \alpha = 1, \ldots, 6$, then the optimal values are $N_\alpha^* = n_\alpha - 1, F_\alpha^* = F_\alpha(n_\alpha - 1); \forall \alpha = 1, \ldots, 6$ and go to step 3. Otherwise, return to step 2-3.

Step 3. Compare among $M, N, \frac{F(N_1)}{N_1^*}, T_d$.

3-1. If $M < V$ and $\frac{F(N_1)}{N_1^*} < T_d$, then go to step 4 otherwise go to next sub-step.

3-2. If $M < V$ and $\frac{F(N_1)}{N_1^*} > T_d$, then go to step 5 otherwise go to next sub-step.

3-3. If $M > V$ and $\frac{F(N_1)}{N_1^*} < T_d$, then go to step 6 otherwise go to next sub-step.

3-4. If $M > V$ and $\frac{F(N_1)}{N_1^*} > T_d$, go to step 7.

Step 4. According to the results of the above phases, the following four possible cases may occur.

4-1. When $\frac{H}{N_2} > T_d, \frac{H}{N_3} > T_d, M \leq \frac{F(n_2)H}{n_2^*}$ and $M \geq \frac{F(n_3)H}{n_3^*}$. In this situation, go to the following steps.

4-1-1. Compare the values of the objective functions $TC_1(n_1, F_1(n_1)), TC_2(n_2, F_2(n_2)), TC_3(n_3, F_3(n_3))$.

4-1-2. If $TC_1(n_1, F_1(n_1))$ is the minimum value, then $N_1^* = n_1 - 1, F_1^* = F_1(n_1 - 1)$.

4-1-3. If $TC_2(n_2, F_2(n_2))$ is the minimum value, then $N_2^* = n_2 - 1, F_2^* = F_2(n_2 - 1)$.

4-1-4. If $TC_3(n_3, F_3(n_3))$ is the minimum value, then $N_3^* = n_3 - 1, F_3^* = F_3(n_3 - 1)$.

4-1-5. Go to step 6.

4-2. It should be noted that, if $(n_2, F_2(n_2))$ do not satisfy at least one of the conditions presented in Table 2 (see Appendix A), we need to perform the following sub-steps.

4-2-1. First, set $n_2^* = 1$ and from Equation $F_2(n_2^*) = \frac{n_2^*M}{H}$, obtain the value of $F_2'(n_2^*)$.

4-2-2. Using Equation (38), calculate the value of $TC_2(n_2^*, F_2'(n_2^*))$.

4-2-3. Set $n_2^* = n_2^* + 1$. If $n_2^* \leq \frac{H}{T_d}$ then put $F_2'(n_2^*) = \frac{n_2^*M}{H}$ and go to step 4-2-4, otherwise $N_2^* = \frac{H}{T_d}, F_2^* = \frac{H}{T_d}(M/H)$ and using Eq. (38), calculate the value of $TC_3([H/F_d], [H/T_d](M/H))$. Then, go to Step 4-2-6.

4-2-4. Using Eq. (38), obtain the value of $TC_2(n_2^*, F_2'(n_2^*))$.

4-2-5. If $TC_2(n_2^*, F_2'(n_2^*)) > TC_2(n_2^* - 1, F_2'(n_2^* - 1))$ then $N_2^* = n_2^* - 1, F_2^* = F_2'(n_2^* - 1)$ and go to step 4-2-6, otherwise, return to step 4-2-3.

4-2-6. Compare among the values of the objective functions $TC_1(n_1, F_1(n_1))$, $TC_2(n_2^*, F_2'(n_2^*))$ and $TC_3(n_3, F_3(n_3))$.

4-2-7. If $TC_1(n_1, F_1(n_1))$ is the minimum value, then $N_1^* = n_1 - 1, F_1^* = F_1(n_1 - 1)$.

4-2-8. If $TC_2(n_2^*, F_2'(n_2^*))$ is the minimum value, then $N_2^* = n_2^* - 1, F_2^* = F_2'(n_2^* - 1)$.

4-2-9. If $TC_3(n_3, F_3(n_3))$ is the minimum value, then $N_3^* = n_3 - 1, F_3^* = F_3(n_3 - 1)$.

...
4-2. Go to step 8.

4-3. It should be noted that, since \((n_3, F_3(n_3))\) do not satisfy at least one of the conditions presented in Table 3 (see Appendix B), we need to perform the following sub-steps. So, go to sub-step (4-3-1).

4-3-1. First set \(n'_3 = 1\), and using equation \(F'_3(n'_3) = n'_3 M/H\), obtain the value of \(F'_3(n'_3)\).

4-3-2. Using Equation (42), obtain \(TC_3(n'_3, F'_3(n'_3))\).

4-3-3. Now set \(n'_3 = n'_3 + 1\), if \(n'_3 \leq [H/T_d]\) then put \(F'_3(n'_3) = n'_3 M/H\), and go to step 4-3-4, otherwise \(N'_3 = [H/T_d]\), \(F'_3 = [H/T_d] M/H\) and using Equation (42), calculate the value of \(TC_3 = ([H/T_d], [H/T_d]) M/H\), and go to step 4-3-6.

4-3-4. Using Equation (42), calculate the value of \(TC_3(n'_3, F'_3(n'_3))\).

4-3-5. If \(TC_3(n'_3, F'_3(n'_3)) > TC_3(n'_3 - 1, F'_3(n'_3 - 1))\) then set \(N'_3 = n'_3 - 1, F'_3 = F'_3(n'_3 - 1)\), and go to step 4-3-6, otherwise, return to step 4-3-3.

4-3-6. Compare the values of objective functions \(TC_1(n_1, F_1(n_1))\), \(TC_2(n_2, F_2(n_2))\), \(TC_3(n'_3, F'_3(n'_3))\).

4-3-7. If \(TC_1(n_1, F_1(n_1))\) is the minimum value, then \(N^* = n_1 - 1, F^* = F_1(n_1 - 1)\).

4-3-8. If \(TC_2(n_2, F_2(n_2))\) is the minimum value, then \(N^* = n_2 - 1, F^* = F_2(n_2 - 1)\).

4-3-9. If \(TC_3(n'_3, F'_3(n'_3))\) is the minimum value, then \(N^* = n'_3 - 1, F^* = F'_3(n'_3 - 1)\).

4-3-10. Go to step 8.

4-4. If \((n_2, F_2(n_2))\) and \((n_3, F_3(n_3))\) do not satisfy at least one of the conditions presented in Table 4 (see Appendix C), then go to the following sub-steps.

4-4-1. Go to step 4-2-1 to 4-2-5, and calculate the values of \(n'_2, F'_2(n'_2)\).

4-4-2. Go to step 4-3-1 to 4-3-5, and calculate the values of \(n'_3, F'_3(n'_3)\).

4-4-3. Compare among the values of th following objective functions \(TC_1(n_1, F_1(n_1))\), \(TC_2(n_2, F_2(n_2))\), \(TC_3(n'_3, F'_3(n'_3))\).

4-4-4. If \(TC_1(n_1, F_1(n_1))\) is the minimum value, then \(N^* = n_1 - 1\) and \(F^* = F_1(n_1 - 1)\).

4-4-5. If \(TC_2(n_2, F_2(n_2))\) is the minimum value, then \(N^* = n_2 - 1\) and \(F^* = F_2(n_2 - 1)\).

4-4-6. If \(TC_3(n'_3, F'_3(n'_3))\) is the minimum value, then \(N^* = n'_3 - 1\) and \(F^* = F'_3(n'_3 - 1)\).

4-4-7. Go to step 8.

Step 5. According to the results of the above phases, the following four possible cases may occur.

5-1. If \((n_2, F_2(n_2))\) and \((n_1, F_1(n_1))\) do not satisfy at least one of the conditions presented in Table 2 (see Appendix A), we need to perform the following sub-steps.

5-1-1. First set \(n'_1 = 1\) and using Equation \(F'_1(n'_1) = n'_1 M/H\), calculate the the value of \(F'_1(n'_1)\).

5-1-2. Using Equation (34), calculate the value of \(TC_1(n'_1, F'_1(n'_1))\).

5-1-3. Set \(n'_1 = n'_1 + 1\), and find the value of \(F'_1(n'_1)\).

5-1-4. Using Equation (34), find the value of \(TC_1(n'_1, F'_1(n'_1))\).

5-1-5. If \(TC_1(n'_1, F'_1(n'_1)) > TC_1(n'_1 - 1, F'_1(n'_1 - 1))\) and \(n'_1 \geq [H/T_d] + 1\), then \(N'_1 = n'_1 - 1\) and \(F'_1 = F'_1(n'_1 - 1)\) and go to next sub step, otherwise return to step 5-1-3.

5-1-6. Go to step 4-2-1 to 4-2-5, and calculate the values of \(n'_2, F'_2(n'_2)\).

5-1-7. Compare the values of the following objective functions \(TC_1(n'_1, F'_1(n'_1))\), \(TC_2(n'_2, F'_2(n'_2))\) and \(TC_3(n'_3, F'_3(n'_3))\).
5-1.8. If $TC_1(n'_1, F'_1(n'_1))$ is the minimum value, then $N^* = n'_1 - 1$ and $F^* = F'_1(n'_1 - 1)$.
5-1.9. If $TC_2(n'_2, F'_2(n'_2))$ is the minimum value, then $N^* = n'_2 - 1$ and $F^* = F'_2(n'_2 - 1)$.
5-1.10. If $TC_3(n_3, F_3(n_3))$ is the minimum value, then $N^* = n_3 - 1$ and $F^* = F_3(n_3 - 1)$.
5-1.11. Go to step 8.
5-2. It should be noted that $(n_3, F_3(n_3))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 3 (see Appendix B) and $TC_1(n_1, F_1(n_1))$
we need to perform the following sub-steps.
5-2.1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N'_1$ and $F'_1$.
5-2.2. Go to steps 4-3-1 to 4-3-5 and calculate the values of $N'_3$ and $F'_3$.
5-2.3. Compare the values of the following objective functions $TC_1(n'_1, F'_1(n'_1))$, $TC_2(n_2, F_2(n_2))$ and $TC_3(n_3, F_3(n_3))$.
5-2.4. If $TC_1(n'_1, F'_1(n'_1))$ is the minimum value, then $N^* = n'_1 - 1$ and $F^* = F'_1(n'_1 - 1)$.
5-2.5. If $TC_2(n_2, F_2(n_2))$ is the minimum value, then $N^* = n_2 - 1$ and $F^* = F_2(n_2 - 1)$.
5-2.6. If $TC_3(n_3, F_3(n_3))$ is the minimum value, then $N^* = n'_3 - 1$ and $F^* = F'_3(n'_3 - 1)$.
5-2.7. Go to step 8.
5-3. If $(n_2, F_2(n_2))$, $(n_3, F_3(n_3))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 4 (see Appendix C), we need to execute the following sub-steps.
5-3.1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N'_1$ and $F'_1$.
5-3.2. Go to steps 4-2-1 to 4-2-5 and calculate the values of $N'_2$ and $F'_2$.
5-3.3. Go to steps 4-3-1 to 4-3-5 and calculate the values of $N'_3$ and $F'_3$.
5-3.4. Compare the values of the following objective functions $TC_1(n'_1, F'_1(n'_1))$, $TC_2(n'_2, F'_2(n'_2))$ and $TC_3(n'_3, F'_3(n'_3))$.
5-3.5. If $TC_1(n'_1, F'_1(n'_1))$ is the minimum value, then $N^* = n'_1 - 1$ and $F^* = F'_1(n'_1 - 1)$.
5-3.6. If $TC_2(n'_2, F'_2(n'_2))$ is the minimum value, then $N^* = n'_2 - 1$ and $F^* = F'_2(n'_2 - 1)$.
5-3.7. If $TC_3(n'_3, F'_3(n'_3))$ is the minimum value, then $N^* = n'_3 - 1$ and $F^* = F'_3(n'_3 - 1)$.
5-3.8. Go to step 8.
Step 6. According to the results of the above phases, the following four possible cases may occur.
6-1. If $H/N'_4 > T_d, H/N'_5 > T_d, H/N'_6 > T_d, M \leq F(n_4)H/n_4, F(n_5)H/n_5 < M < F(n_6)H/n_6 + V, M > F(n_6)H/n_6 + V$, then do the following steps.
6-1.1. Compare the values of the following objective functions $TC_4(n_4, F_4(n_4))$, $TC_4(n_5, F_5(n_5))$ and $TC_6(n_6, F_6(n_6))$.
6-1.2. If $TC_4(n_4, F_4(n_4))$ is the minimum value, then $N^* = n_4 - 1$ and $F^* = F_4(n_4 - 1)$.
6-1.3. If $TC_5(n_5, F_5(n_5))$ is the minimum value, then $N^* = n_5 - 1$ and $F^* = F_5(n_5 - 1)$.
6-1.4. If $TC_6(n_6, F_6(n_6))$ is the minimum value, then $N^* = n_6 - 1$ and $F^* = F_6(n_6 - 1)$.
6-1.5. Go to step 8.
6-2. If \((n_4, F_4(n_4))\) do not satisfy at least one of the conditions presented in Table 5 (see Appendix D), then perform the following sub-steps.

6-2-1. First set \( n'_4 = 1 \) and using equation \( F'_4(n'_4) = n'_4 M / H \), calculate the value of \( F'_4(n'_4) \).

6-2-2. Using Equation (46), calculate the value of \( TC_4(n'_4, F'_4(n'_4)) \).

6-2-3. Set \( n'_4 = n'_4 + 1 \), and calculate the value of \( F'_4(n'_4) \).

6-2-4. Using Equation (46), calculate the value of \( TC_4(n'_4, F'_4(n'_4)) \).

6-2-5. If \( TC_4(n'_4, F'_4(n'_4)) > TC_4(n'_4 - 1, F'_4(n'_4 - 1)) \) and \( n'_4 \geq [H/T_d] + 1 \), then \( N'_4 = n'_4 - 1, F'_4 = F'_4(n'_4 - 1) \) and go to step 6-2-6, otherwise return to step 6-2-3.

6-2-6. Compare the values of the following objective functions \( TC_4(n'_4, F'_4(n'_4)) \), \( TC_5(n_5, F_5(n_5)) \) and \( TC_6(n_6, F_6(n_6)) \).

6-2-7. If \( TC_4(n'_4, F'_4(n'_4)) \) is the minimum value, then \( N^* = n'_4 - 1 \) and \( F^* = F'_4(n'_4 - 1) \).

6-2-8. If \( TC_5(n_5, F_5(n_5)) \) is the minimum value, then \( N^* = n_5 - 1 \) and \( F^* = F_5(n_5 - 1) \).

6-2-9. If \( TC_6(n_6, F_6(n_6)) \) is the minimum value, then \( N^* = n_6 - 1 \) and \( F^* = F_6(n_6 - 1) \).

6-2-10. Go to step 8.

6-3. When \((n_5, F_5(n_5))\) do not satisfy at least one of the conditions presented in Table 6 (see Appendix E), we need to perform the following sub-steps.

6-3-1. First, set \( n'_5 = 1 \) and, using \( F'_5(n'_5) = n'_5 M / H \), calculate the value of \( F'_5(n'_5) \).

6-3-2. Using \( F'_5(n'_5) = n'_5 M / H \), calculate the value \( TC_5(n'_5, F'_5(n'_5)) \).

6-3-3. Now set \( n'_5 = n'_5 + 1 \). If \( n'_5 \leq [H/T_d] \), then put \( F'_5(n'_5) = n'_5 M / H \), and go to step 6-3-4, otherwise \( N'_5 = [H/T_d], F'_5 = [H/T_d](M / H) \) and using Eq.(50), calculate the value of \( TC_5([H/T_d], [H/T_d]M / H) \), and go to step 6-3-6.

6-3-4. Using Equation (50), calculate the value of \( TC_5(n'_5, F'_5(n'_5)) \).

6-3-5. If \( TC_5(n'_5, F'_5(n'_5)) > TC_5(n'_5 - 1, F'_5(n'_5 - 1)) \), then \( N'_5 = n'_5 - 1, F'_5 = F'_5(n'_5 - 1) \) and go to step 6-3-6, otherwise return to step 6-3-3.

6-3-6. Compare the values of the following objective functions \( TC_1(n_1, F_1(n_1)) \), \( TC_4(n_4, F_4(n_4)) \), \( TC_5(n'_5, F'_5(n'_5)) \) and \( TC_6(n_6, F_6(n_6)) \).

6-3-7. If \( TC_1(n_1, F_1(n_1)) \) is the minimum value, then \( N^* = n_1 - 1 \) and \( F^* = F_1(n_1 - 1) \).

6-3-8. If \( TC_4(n_4, F_4(n_4)) \) is the minimum value, then \( N^* = n_4 - 1 \) and \( F^* = F_4(n_4 - 1) \).

6-3-9. If \( TC_5(n'_5, F'_5(n'_5)) \) is the minimum value, then \( N^* = n'_5 - 1 \) and \( F^* = F'_5(n'_5 - 1) \).

6-3-10. If \( TC_6(n_6, F_6(n_6)) \) is the minimum value, then \( N^* = n_6 - 1 \) and \( F^* = F_6(n_6 - 1) \).

6-3-11. Go to step 8.

6-4. It should be noted that, since \((n_6, F_6(n_6))\) do not satisfy at least one of the conditions presented in Table 7 (see Appendix F), we need to perform the following sub-steps.

6-4-1. First, set \( n'_6 = 1 \) and using \( F'_6(n'_6) = n'_6 M / H \), calculate the value of \( F'_6(n'_6) \).

6-4-2. Using Equation (54), obtain \( TC_6(n'_6, F'_6(n'_6)) \).

6-4-3. Set \( n'_6 = n'_6 + 1 \). If \( n'_6 \leq [H/T_d] \), then put \( F'_6(n'_6) = n'_6(M - V) / H \), and go to step 6-4-4, otherwise \( N'_6 = [H/T_d], F'_6 = [H/T_d](M - V) / H \) and using Equation (50), obtain \( TC_5([H/T_d], [H/T_d]M / H) \), then go to step 6-4-6.

6-4-4. Using Equation (54), calculate the value of \( TC_5(n'_6, F'_6(n'_6)) \).
6-4-5. If $TC_6(n'_6, F'_6(n'_6)) > TC_6(n'_6 - 1, F'_6(n'_6 - 1))$, then $N^*_6 = n'_6 - 1$, $F^*_6 = F'_6(n'_6 - 1)$, and go to step 6-4-6, otherwise return to step 6-4-3.

6-4-6. Compare the values of the following objectives functions $TC_1(n_1, F_1(n_1))$, $TC_4(n_4, F_4(n_4))$, $TC_5(n_5, F_5(n_5))$, and $TC_6(n'_6, F'_6(n'_6))$.

6-4-7. If $TC_1(n_1, F_1(n_1))$ is the minimum value, then $N^*_1 = n_1 - 1$ and $F^*_1 = F_1(n_1 - 1)$.

6-4-8. If $TC_4(n_4, F_4(n_4))$ is the minimum value, then $N^*_4 = n_4 - 1$ and $F^*_4 = F_4(n_4 - 1)$.

6-4-9. If $TC_5(n_5, F_5(n_5))$ is the minimum value, then $N^*_5 = n_5 - 1$ and $F^*_5 = F_5(n_5 - 1)$.

6-4-10. If $TC_6(n'_6, F'_6(n'_6))$ is the minimum value, then $N^*_6 = n'_6 - 1$ and $F^*_6 = F'_6(n'_6 - 1)$.

6-4-11. Go to step 8.

6-5. It should be noted that $(n_4, F_4(n_4))$ and $(n_5, F_5(n_5))$ do not satisfy at least one of the conditions presented in Table 8 (see Appendix G). So, we need to perform the following sub-steps.

6-5-1. Go to steps 6-2-1 to 6-2-5 and calculate the values of $N^*_4$ and $F^*_4$.

6-5-2. Go to steps 6-3-1 to 6-3-5 and calculate the values of $N^*_5$ and $F^*_5$.

6-5-3. Compare the values of the following objectives functions $TC_1(n_1, F_1(n_1))$, $TC_4(n_4, F_4(n_4))$, $TC_5(n_5, F_5(n_5))$, and $TC_6(n'_6, F'_6(n'_6))$.

6-5-4. If $TC_1(n_1, F_1(n_1))$ is the minimum value, then $N^*_1 = n_1 - 1$ and $F^*_1 = F_1(n_1 - 1)$.

6-5-5. If $TC_4(n'_4, F'_4(n'_4))$ is the minimum value, then $N^*_4 = n'_4 - 1$ and $F^*_4 = F'_4(n'_4 - 1)$.

6-5-6. If $TC_5(n'_5, F'_5(n'_5))$ is the minimum value, then $N^*_5 = n'_5 - 1$ and $F^*_5 = F'_5(n'_5 - 1)$.

6-5-7. If $TC_6(n'_6, F'_6(n'_6))$ is the minimum value, then $N^*_6 = n'_6 - 1$ and $F^*_6 = F'_6(n'_6 - 1)$.

6-5-8. Go to step 8.

6-6. Since $(n_4, F_4(n_4))$ and $(n_6, F_6(n_6))$ do not satisfy at least one of the conditions presented in Table 9 (see Appendix H), we need to perform the following sub-steps.

6-6-1. Go to steps 6-2-1 to 6-2-5 and calculate the values of $N^*_4$ and $F^*_4$.

6-6-2. Go to steps 6-4-1 to 6-4-5 and calculate the values of $N^*_6$ and $F^*_6$.

6-6-3. Compare the values of the following objective functions $TC_1(n_1, F_1(n_1))$, $TC_4(n_4, F_4(n_4))$, $TC_5(n_5, F_5(n_5))$, and $TC_6(n'_6, F'_6(n'_6))$.

6-6-4. If $TC_1(n_1, F_1(n_1))$ is the minimum value, then $N^*_1 = n_1 - 1$ and $F^*_1 = F_1(n_1 - 1)$.

6-6-5. If $TC_4(n'_4, F'_4(n'_4))$ is the minimum value, then $N^*_4 = n'_4 - 1$ and $F^*_4 = F'_4(n'_4 - 1)$.

6-6-6. If $TC_5(n_5, F_5(n_5))$ is the minimum value, then $N^*_5 = n_5 - 1$ and $F^*_5 = F_5(n_5 - 1)$.

6-6-7. If $TC_6(n'_6, F'_6(n'_6))$ is the minimum value, then $N^*_6 = n'_6 - 1$ and $F^*_6 = F'_6(n'_6 - 1)$.

6-6-8. Go to step 8.

6-7. Since $(n_5, F_5(n_5))$ and $(n_6, F_6(n_6))$ do not satisfy at least one of the conditions presented in Table 10 (see Appendix I), we have to perform the following sub-steps.

6-7-1. Go to steps 6-3-1 to 6-3-5 and calculate the values of $N^*_6$ and $F^*_6$. 

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6-7-2. Go to steps 6-4-1 to 6-4-5 and calculate the values of \( N_6^* \) and \( F_6^* \).

6-7-3. Compare the values of the following objective functions \( TC_1(n_1, F_1(n_1)) \), \( TC_4(n_4, F_4(n_4)) \), \( TC_5(n_5^*, F_5^*(n_5^*)) \) and \( TC_6(n_6, F_6(n_6)) \).

6-7-4. If \( TC_1(n_1, F_1(n_1)) \) is the minimum value, then \( N^* = n_1 - 1 \) and \( F^* = F_1(n_1 - 1) \).

6-7-5. If \( TC_4(n_4, F_4(n_4)) \) is the minimum value, then \( N^* = n_4 - 1 \) and \( F^* = F_4(n_4 - 1) \).

6-7-6. If \( TC_5(n_5^*, F_5^*(n_5^*)) \) is the minimum value, then \( N^* = n_5^* - 1 \) and \( F^* = F_5^*(n_5^* - 1) \).

6-7-7. If \( TC_6(n_6^*, F_6(n_6')) \) is the minimum value, then \( N^* = n_6 - 1 \) and \( F^* = F_6(n_6 - 1) \).

6-7-8. Go to step 8.

6-8. Since \((n_4, F_4(n_4))\), \((n_5, F_5(n_5))\) and \((n_6, F_6(n_6))\) do not satisfy at least one of the conditions presented in Table 11 (see Appendix J), we have to execute the following sub-steps.

6-8-1. Go to steps 6-2-1 to 6-2-5 and calculate the values of \( N_4^* \) and \( F_4^* \).

6-8-2. Go to steps 6-3-1 to 6-3-5 and calculate the values of \( N_5^* \) and \( F_5^* \).

6-8-3. Go to steps 6-4-1 to 6-4-5 and calculate the values of \( N_6^* \) and \( F_6^* \).

6-8-4. Compare the values of the following objective functions \( TC_1(n_1, F_1(n_1)) \), \( TC_4(n_4, F_4(n_4)) \), \( TC_5(n_5, F_5(n_5)) \) and \( TC_6(n_6, F_6(n_6)) \).

6-8-5. If \( TC_1(n_1, F_1(n_1)) \) is the minimum value, then \( N^* = n_1 - 1 \) and \( F^* = F_1(n_1 - 1) \).

6-8-6. If \( TC_4(n_4^*, F_4^*(n_4^*)) \) is the minimum value, then \( N^* = n_4^* - 1 \) and \( F^* = F_4^*(n_4^* - 1) \).

6-8-7. If \( TC_5(n_5^*, F_5^*(n_5^*)) \) is the minimum value, then \( N^* = n_5^* - 1 \) and \( F^* = F_5^*(n_5^* - 1) \).

6-8-8. If \( TC_6(n_6^*, F_6(n_6')) \) is the minimum value, then \( N^* = n_6^* - 1 \) and \( F^* = F_6(n_6' - 1) \).

6-8-9. Go to step 8.

Step 7. According to the results of the above phases, the following eight possible cases may occur.

7-1. When \( H/N_4^* > T_d, H/N_5^* > T_d, H/N_6^* > T_d, M < F(n_4)H/n_4, F(n_5)H/n_5 < M < F(n_6)H/n_6 + V, M > F(n_6)H/n_6 + V \). In this situation, do the following steps. Since \( N_1 \) and \( F_1(N_1) \) do not satisfy at least one of the conditions of \( H/N_4^* > T_d, H/N_5^* > T_d, H/N_6^* > T_d, M < F(n_4)H/n_4, F(n_5)H/n_5 < M < F(n_6)H/n_6 + V, M > F(n_6)H/n_6 + V \), we amend it and obtain a new value of \((N_1, F_1(N_1))\).

7-1-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of \( N_1^* \) and \( F_1^* \).

7-1-2. Compare the values of the following objective functions \( TC_1(n_1^*, F_1'(n_1')) \), \( TC_4(n_4, F_4(n_4)) \), \( TC_5(n_5, F_5(n_5)) \) and \( TC_6(n_6, F_6(n_6)) \).

7-1-3. If \( TC_1(n_1^*, F_1'(n_1')) \) is the minimum value, then \( N^* = n_1 - 1 \) and \( F^* = F_1'(n_1' - 1) \).

7-1-4. If \( TC_4(n_4, F_4(n_4)) \) is the minimum value, then \( N^* = n_4 - 1 \) and \( F^* = F_4(n_4 - 1) \).

7-1-5. If \( TC_5(n_5, F_5(n_5)) \) is the minimum value, then \( N^* = n_5 - 1 \) and \( F^* = F_5(n_5 - 1) \).

7-1-6. If \( TC_6(n_6, F_6(n_6)) \) is the minimum value, then \( N^* = n_6 - 1 \) and \( F^* = F_6(n_6 - 1) \).

7-1-7. Go to step 8.

7-2. As \((n_4, F_4(n_4))\) and \((n_1, F_1(n_1))\) do not satisfy at least one of the conditions presented in Table 5 (see Appendix D), we need to execute the following sub-steps.

7-2-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of \( N_1^* \) and \( F_1^* \).
7-2-2. Go to steps 6-2-1 to 6-2-5 and calculate the values of $N_7^*$ and $F_7^*$.
7-2-3. Compare the values of the following objective functions $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4', F_4'(n_4'))$, $TC_5(n_5, F_5(n_5))$ and $TC_6(n_6, F_6(n_6))$.
7-2-4. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-2-5. If $TC_4(n_4', F_4'(n_4'))$ is the minimum value, then $N^* = n_4' - 1$, $F^* = F_4'(n_4' - 1)$.
7-2-6. If $TC_5(n_5, F_5(n_5))$ is the minimum value, then $N^* = n_5 - 1$, $F^* = F_5(n_5 - 1)$.
7-2-7. If $TC_6(n_6, F_6(n_6))$ is the minimum value, then $N^* = n_6 - 1$, $F^* = F_6(n_6 - 1)$.
7-2-8. Go to step 8.
7-3. Here, $(n_5, F_5(n_5))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 6 (see Appendix E). So, we need to perform the following sub-steps.
7-3-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N_1^*$ and $F_1^*$.
7-3-2. Go to steps 6-3-1 to 6-3-5 and calculate the values of $N_5^*$ and $F_5^*$.
7-3-3. Compare the values of the following objective functions $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4, F_4(n_4)), TC_5(n_5', F_5'(n_5'))$ and $TC_6(n_6, F_6(n_6))$.
7-3-4. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-3-5. If $TC_4(n_4, F_4(n_4))$ is the minimum value, then $N^* = n_4 - 1$, $F^* = F_4(n_4 - 1)$.
7-3-6. If $TC_5(n_5', F_5'(n_5'))$ is the minimum value, then $N^* = n_5' - 1$, $F^* = F_5'(n_5' - 1)$.
7-3-7. If $TC_6(n_6, F_6(n_6))$ is the minimum value, then $N^* = n_6 - 1$, $F^* = F_6(n_6 - 1)$.
7-3-8. Go to step 8.
7-4. Since $(n_6, F_6(n_6))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 7 (see Appendix F) and $TC_1(n_1, F_1(n_1))$, we have to execute the following sub-steps.
7-4-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N_1^*$ and $F_1^*$.
7-4-2. Go to steps 6-4-1 to 6-4-5 and calculate the values of $N_6^*$ and $F_6^*$.
7-4-3. Compare the values of the following objective functions $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4, F_4(n_4)), TC_5(n_5', F_5'(n_5'))$ and $TC_6(n_6', F_6'(n_6'))$.
7-4-4. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-4-5. If $TC_4(n_4, F_4(n_4))$ is the minimum value, then $N^* = n_4 - 1$, $F^* = F_4(n_4 - 1)$.
7-4-6. If $TC_5(n_5', F_5'(n_5'))$ is the minimum value, then $N^* = n_5' - 1$, $F^* = F_5'(n_5' - 1)$.
7-4-7. If $TC_6(n_6', F_6'(n_6'))$ is the minimum value, then $N^* = n_6' - 1$, $F^* = F_6'(n_6' - 1)$.
7-4-8. Go to step 8.
7-5. It is noted that $(n_4, F_4(n_4)), (n_5, F_5(n_5))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 8 (see Appendix G). We need to perform the following sub-steps.
7-5-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N_1^*$ and $F_1^*$.
7-5-2. Go to steps 6-2-1 to 6-2-5 and calculate the values of $N_4^*$ and $F_4^*$.
7-5-3. Go to steps 6-3-1 to 6-3-5 and calculate the values of $N_5^*$ and $F_5^*$.
7-5-4. Compare the values of the following objective functions $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4', F_4'(n_4'))$, $TC_5(n_5', F_5'(n_5'))$ and $TC_6(n_6, F_6(n_6))$.
7-5-5. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-5-6. If $TC_4(n_4', F_4'(n_4'))$ is the minimum value, then $N^* = n_4' - 1$, $F^* = F_4'(n_4' - 1)$.
7-5-7. If $TC_5(n_5', F_5'(n_5'))$ is the minimum value, then $N^* = n_5' - 1$, $F^* = F_5'(n_5' - 1)$.
7-5-8. If $TC_6(n_6, F_6(n_6))$ is the minimum value, then $N^* = n_6 - 1$, $F^* = F_6(n_6 - 1)$.
7-5-9. Go to step 8.
7-6. It is noted that $(n_4, F_4(n_4)), (n_6, F_6(n_6))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 9 (see Appendix H). We need to execute the following sub-steps.
7-6-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N_1^*$ and $F_1^*$. 
7-6-2. Go to steps 6-2-1 to 6-2-5 and calculate the values of $N_1^*$ and $F_1^*$.
7-6-3. Go to steps 6-4-1 to 6-4-5 and calculate the values of $N_5^*$ and $F_5^*$.
7-6-4. Compare the values of the following objective functions $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4', F_4'(n_4'))$, $TC_5(n_5, F_5(n_5))$ and $TC_6(n_6', F_6'(n_6'))$.
7-6-5. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-6-6. If $TC_4(n_4', F_4'(n_4'))$ is the minimum value, then $N^* = n_4' - 1$, $F^* = F_4'(n_4' - 1)$.
7-6-7. If $TC_5(n_5, F_5(n_5))$ is the minimum value, then $N^* = n_5 - 1$, $F^* = F_5(n_5 - 1)$.
7-6-8. If $TC_6(n_6', F_6'(n_6'))$ is the minimum value, then $N^* = n_6' - 1$, $F^* = F_6'(n_6' - 1)$.
7-6-9. Go to step 8.
7-7. It is observed that $(n_6, F_6(n_6))$, $(n_5, F_5(n_5))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 10 (see Appendix I). We have to perform the following sub-steps.
7-7-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N_1^*$ and $F_1^*$.
7-7-2. Go to steps 6-3-1 to 6-3-5 and calculate the values of $N_5^*$ and $F_5^*$.
7-7-3. Go to steps 7-4-1 to 7-4-5 and calculate the values of $N_6^*$ and $F_6^*$.
7-7-4. Compare the values of the following objective functions $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4', F_4'(n_4'))$, $TC_5(n_5', F_5'(n_5'))$ and $TC_6(n_6', F_6'(n_6'))$.
7-7-5. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-7-6. If $TC_4(n_4', F_4'(n_4'))$ is the minimum value, then $N^* = n_4' - 1$, $F^* = F_4'(n_4' - 1)$.
7-7-7. If $TC_5(n_5', F_5'(n_5'))$ is the minimum value, then $N^* = n_5' - 1$, $F^* = F_5'(n_5' - 1)$.
7-7-8. If $TC_6(n_6', F_6'(n_6'))$ is the minimum value, then $N^* = n_6' - 1$, $F^* = F_6'(n_6' - 1)$.
7-7-9. Go to step 8.
7-8. Since $(n_4, F_4(n_4))$, $(n_5, F_5(n_5))$, $(n_6, F_6(n_6))$ and $(n_1, F_1(n_1))$ do not satisfy at least one of the conditions presented in Table 11 (see Appendix J), we need to perform the following sub-steps.
7-8-1. Go to steps 5-1-1 to 5-1-5 and calculate the values of $N_1^*$ and $F_1^*$.
7-8-2. Go to steps 6-2-1 to 6-2-5 and calculate the values of $N_4^*$ and $F_4^*$.
7-8-3. Go to steps 6-3-1 to 6-3-5 and calculate the values of $N_5^*$ and $F_5^*$.
7-8-4. Go to steps 6-4-1 to 6-4-5 and calculate the values of $N_6^*$ and $F_6^*$.
7-8-5. Compare the values of $TC_1(n_1', F_1'(n_1'))$, $TC_4(n_4', F_4'(n_4'))$, $TC_5(n_5', F_5'(n_5'))$ and $TC_6(n_6', F_6'(n_6'))$.
7-8-6. If $TC_1(n_1', F_1'(n_1'))$ is the minimum value, then $N^* = n_1' - 1$, $F^* = F_1'(n_1' - 1)$.
7-8-7. If $TC_4(n_4', F_4'(n_4'))$ is the minimum value, then $N^* = n_4' - 1$, $F^* = F_4'(n_4' - 1)$.
7-8-8. If $TC_5(n_5', F_5'(n_5'))$ is the minimum value, then $N^* = n_5' - 1$, $F^* = F_5'(n_5' - 1)$.
7-8-9. If $TC_6(n_6', F_6'(n_6'))$ is the minimum value, then $N^* = n_6' - 1$, $F^* = F_6'(n_6' - 1)$.
7-8-10. Go to step 8.

Step 8. According to obtained results of the last steps, calculate the values of the dependent variables.
8-1. Calculate the maximum inventory, using the formula $I_m = (D/\theta)(e^{\theta F}/N - 1)$.
8-2. Calculate the maximum shortage using the formula $I_b = (D/N)H(1 - F)$.
8-3. Calculate the amount of ordering size, using the formula $Q = I_b + I_m$.
8-4. Calculate the total cost of the related optimal values of $F$ and $N$, obtained from the last steps.

The above algorithm is summarized in Fig.8 as follows:

4. **Numerical example.** Let the values of the parameters in proper units as follows: $A = 500$, $C = 5$, $D = 1000$, $H = 5$, $I_c = 0.12$, $I_b = 0.08$, $I_b = 0.5$, $M = 0.2$, $\pi = 4$, $R = 0.08$, $S = 7$, $V = 0.05$, $\theta = 0.1$ and $Q_d = 500$. According to the proposed method, $N^* = 8$, $F^* = 0.32$ and $TC(N^*, F^*) = 22813$ and according to step 6, $T^* = 0.625$, $I_m = 202.0123$, $I_b = 425$ and $Q^* = 627,013$. 


4.1. **Sensitivity analysis.** The changes in the values of the key parameters lead to uncertainty in the decision variables. To study these changes and their impacts on decision variables, sensitivity analysis is a very useful tool. In this section, sensitivity analysis is performed well, and the results are shown in Table 1, based on the changes in the key parameters $M, V, \theta, I_c, I_e$ and $Q_d$ of the model.

From the sensitivity analysis presented in Table 1, the following features are observed: increasing of deterioration rate increases inventory costs and deterioration cost consistently. Indeed, when deterioration rate increases, products will spoil more that results in more wastage of the products. Then, the lower sales reduce the revenues of the retailer. Consequently, net present value of the total cost reduces with increases in deterioration that is obvious in nature. There are adequate methods to reduce inventory costs and deterioration cost applying production maintenance and better preservation systems. When the delay in payment time ($V$) offered by the retailer to the customer increases, the total cost increases. It happens due to increase in the amount of interest payment at time $V$ or the decreasing value of profit for shorten duration of time at which money is kept in the bank. On the other hand, by increasing in term $M$, total cost reduces for two reasons: 1) increasing on
duration of time at which sales revenues are kept in the bank to earn interest and, 2) reduction on duration where the retailer pays interest for late payment to the supplier or the bank. The total cost decreases with increases in interest rate that is quite rational in practice. Also, the total cost increases with increases in interest rate charged by the supplier.

Table 1- Optimal solutions for different values of $M$, $V$, $I_c$, $I_e$, $\theta$ and $Q_d$.

| $M$ | $V$ | $I_c$ | $I_e$ | $\theta$ | $Q_d$ | $N^*$ | $F^*$ | $TC(N^*,F^*)$ |
|-----|-----|-------|-------|----------|-------|-------|-------|----------------|
| 0.2 | 0.05| 0.008 | 0.12  | 0.1      | 500   | 8     | 0.32  | 22813         |
| 0.1 |       |       |       |          |       |       |       | 22666         |
| 0.15|       |       |       |          |       |       | 9     | 0.1264 22927 |
| 0.2 |       |       |       |          |       |       | 9     | 0.1260 22978 |
| 0.25| 0.05 |       |       |          |      10 | 0.1671| 22742 |
| 0.1 |       |       |       |          |      10 | 0.1671| 22800 |
| 0.15|       |       |       |          |      10 | 0.1671| 22878 |
| 0.2 |       |       |       |          |      9  | 0.1264| 22903 |
| 0.30| 0.05 |       |       |          |      6  | 0.36  | 22620 |
| 0.1 |       |       |       |          |      6  | 0.36  | 22702 |
| 0.15|       |       |       |          |      6  | 0.36  | 22771 |
| 0.2 |       |       |       |          |      6  | 0.36  | 22842 |
| 0.20| 0.05 | 0.04  |       |          |      8  | 0.32  | 22943 |
| 0.10|       |       |       |          |      8  | 0.32  | 22901 |
| 0.12 |      |       |       |          |      7  | 0.28  | 22741 |
| 0.08| 0.08 |       |       |          |      8  | 0.32  | 22811 |
| 0.10|       |       |       |          |      8  | 0.32  | 22812 |
| 0.14 |      |       |       |          |      8  | 0.32  | 22814 |
| 0.16 |      |       |       |          |      8  | 0.32  | 22815 |
| 0.20| 0.05 | 0.08  | 0.12  |          | 300   | 8     | 0.32  | 22813         |
| 0.20| 0.05 | 0.08  | 0.12  |          | 800   | 10    | 0.0858| 23041         |
| 0.20| 0.05 | 0.08  | 0.12  | 0.05    | 500   | 8     | 0.32  | 22779         |
| 0.20| 0.20 | 0.28  | 0.28  |          |       |       |       | 22873         |
| 0.30| 0.20 | 0.28  |       |          |       |       |       | 22933         |
| 0.40| 0.20 | 0.28  |       |          |       |       |       | 22992         |

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5. Conclusion. In this study, we have developed an inventory control model in a supply chain, considering upstream and downstream trade credits. The supplier offers a quantity-related order, and the retailer enjoys downstream trade credit. In addition, inflation and time value of money have been considered. In this model, shortage is permitted and the rates of demand and inflation are constant over the finite planning horizon. At first, the inventory system of the retailer has been modeled and then, a solution algorithm is proposed to minimize total costs of the retailer, evaluating the optimal order quantity, shortage amount and number of replenishments in a finite planning horizon. This model suggests to a manager of an enterprise how to determine the optimal order quantity and number of replenishment in various types of delay periods under the inflation and time value of money. The proposed model can be extended in future incorporating price and time varying
demand. The model can be studied further for stochastic demand and time varying deterioration rate which are the main limitations of the proposed model.

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Appendix A: Table 2- Indicates the number of status that restrictions $TC_2(n_2, F_2(n_2))$ have been violated. (Mark * indicates a constraint violation)

| Sl. No. | $n_2 > T_d$ | $n_3 > T_d$ | $M \leq \frac{F(n_2)}{n_2}$ | $M \geq \frac{F(n_3)}{n_3}$ |
|---------|-------------|-------------|-----------------|-----------------|
| 1       | *           | *           |                 |                 |
| 2       | *           | *           |                 |                 |
| 3       | *           | *           |                 |                 |

Appendix B: Table 3- Indicates the number of status that restrictions in $TC_3(n_3, F_3(n_3))$ have been violated.

| Sl. No. | $n_2 > T_d$ | $n_3 > T_d$ | $M \leq \frac{F(n_2)}{n_2}$ | $M \geq \frac{F(n_3)}{n_3}$ |
|---------|-------------|-------------|-----------------|-----------------|
| 1       | *           | *           |                 |                 |
| 2       | *           | *           |                 |                 |
| 3       | *           | *           |                 |                 |

Appendix C: Table 4- Indicates the number of status that restrictions of $TC_2(n_2, F_2(n_2))$ and $TC_3(n_3, F_3(n_3))$ have been violated.

| Sl. No. | $n_2 > T_d$ | $n_3 > T_d$ | $M \leq \frac{F(n_2)}{n_2}$ | $M \geq \frac{F(n_3)}{n_3}$ |
|---------|-------------|-------------|-----------------|-----------------|
| 1       | *           | *           |                 |                 |
| 2       | *           | *           |                 |                 |
| 3       | *           | *           |                 |                 |
| 4       | *           | *           |                 |                 |
| 5       | *           | *           |                 |                 |
| 6       | *           | *           |                 |                 |
| 7       | *           | *           |                 |                 |
| 8       | *           | *           |                 |                 |
| 9       | *           | *           |                 |                 |

Appendix D: Table 5 - Indicates the number of status that restrictions of $TC_4(n_4, F_4(n_4))$ have been violated.

| Sl. No. | $n_1 > T_d$ | $n_4 > T_d$ | $M \leq \frac{F(n_1)}{n_1}$ | $\frac{F(n_4)}{n_4} < M < \frac{F(n_4)}{n_4} + V$ | $M > \frac{F(n_4)}{n_4} + V$ |
|---------|-------------|-------------|-----------------|-----------------|-----------------|
| 1       | *           | *           |                 |                 |                 |
| 2       | *           | *           |                 |                 |                 |
| 3       | *           | *           |                 |                 |                 |

Appendix E: Table 6 - Indicates the number of status that restrictions of $TC_5(n_5, F_5(n_5))$ have been violated.

| Sl. No. | $n_4 > T_d$ | $n_4 > T_d$ | $M \leq \frac{F(n_4)}{n_4}$ | $\frac{F(n_5)}{n_5} < M < \frac{F(n_5)}{n_5} + V$ | $M > \frac{F(n_5)}{n_5} + V$ |
|---------|-------------|-------------|-----------------|-----------------|-----------------|
| 1       | *           | *           |                 |                 |                 |
| 2       | *           | *           |                 |                 |                 |
| 3       | *           | *           |                 |                 |                 |
| 4       | *           | *           |                 |                 |                 |
| 5       | *           | *           |                 |                 |                 |
Appendix F: Table 7 - Indicates the number of status that restrictions of \(TC_6(n_6, F_6(n_6))\) have been violated.

| Sl. No. | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(M \leq \frac{f_{(n_4)H}}{n_4} - \frac{f_{(n_5)H}}{n_5} < M < \frac{f_{(n_6)H}}{n_6} + V\) | \(M > \frac{f_{(n_6)H}}{n_6} + V\) |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1       |                 |                 |                 |                 |                 |
| 2       | *               |                 |                 |                 |                 |
| 3       |                 |                 |                 |                 |                 |

Appendix G: Table 8 - Indicates the number of status that restrictions of \(TC_4(n_4, F_4(n_4))\) and \(TC_5(n_5, F_5(n_5))\) have been violated.

| Sl. No. | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(M \leq \frac{f_{(n_4)H}}{n_4} - \frac{f_{(n_5)H}}{n_5} < M < \frac{f_{(n_6)H}}{n_6} + V\) | \(M > \frac{f_{(n_6)H}}{n_6} + V\) |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1       |                 |                 |                 |                 |                 |
| 2       | *               |                 |                 |                 |                 |
| 3       |                 |                 |                 |                 |                 |
| 4       | *               | *               |                 |                 |                 |
| 5       | *               | *               |                 |                 |                 |
| 6       |                 | *               | *               |                 |                 |
| 7       | *               |                 | *               |                 |                 |
| 8       | *               | *               | *               |                 |                 |
| 9       | *               | *               | *               |                 |                 |
| 10      | *               |                 | *               | *               |                 |
| 11      | *               | *               | *               | *               |                 |
| 12      |                 | *               | *               | *               |                 |
| 13      | *               |                 | *               | *               |                 |
| 14      | *               | *               | *               | *               |                 |
| 15      |                 | *               | *               | *               |                 |

Appendix H: Table 9 - Indicates the number of status that restrictions of \(TC_4(n_4, F_4(n_4))\) and \(TC_6(n_6, F_6(n_6))\) have been violated.

| Sl. No. | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(M \leq \frac{f_{(n_4)H}}{n_4} - \frac{f_{(n_5)H}}{n_5} < M < \frac{f_{(n_6)H}}{n_6} + V\) | \(M > \frac{f_{(n_6)H}}{n_6} + V\) |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1       |                 |                 |                 |                 |                 |
| 2       | *               |                 |                 |                 |                 |
| 3       |                 |                 |                 |                 |                 |
| 4       | *               | *               |                 |                 |                 |
| 5       | *               | *               |                 |                 |                 |
| 6       | *               |                 | *               |                 |                 |
| 7       | *               | *               | *               |                 |                 |
| 8       | *               | *               | *               |                 |                 |
| 9       | *               | *               | *               | *               |                 |
| 10      | *               | *               | *               | *               |                 |
| 11      | *               | *               | *               | *               |                 |
| 12      | *               | *               | *               | *               |                 |
| 13      | *               | *               | *               | *               |                 |
| 14      | *               | *               | *               | *               |                 |
| 15      | *               | *               | *               | *               |                 |

Appendix I: Table 10 - Indicates the number of status that restrictions of \(TC_5(n_5, F_5(n_5))\) and \(TC_6(n_6, F_6(n_6))\) have been violated.

| Sl. No. | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(\frac{H}{N} \times T_d\) | \(M \leq \frac{f_{(n_4)H}}{n_4} - \frac{f_{(n_5)H}}{n_5} < M < \frac{f_{(n_6)H}}{n_6} + V\) | \(M > \frac{f_{(n_6)H}}{n_6} + V\) |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1       |                 |                 |                 |                 |                 |
| 2       | *               |                 |                 |                 |                 |
| 3       |                 |                 |                 |                 |                 |
| 4       | *               | *               |                 |                 |                 |
| 5       | *               | *               |                 |                 |                 |
| 6       | *               |                 | *               |                 |                 |
| 7       | *               | *               | *               |                 |                 |
| 8       | *               | *               | *               | *               |                 |
| 9       | *               | *               | *               | *               |                 |
| 10      | *               | *               | *               | *               |                 |
| 11      | *               | *               | *               | *               |                 |
| 12      | *               | *               | *               | *               |                 |
| 13      | *               | *               | *               | *               |                 |
| 14      | *               | *               | *               | *               |                 |
| 15      | *               | *               | *               | *               |                 |
Appendix J: Table 11 - Indicates the number of status that restrictions of $TC_4(n_4, F_4(n_4))$, $TC_5(n_5, F_5(n_5))$ and $TC_6(n_6, F_6(n_6))$ have been violated.

| Sl. No | $\frac{n_4}{n_4} > T_d$ | $\frac{n_5}{n_5} > T_d$ | $\frac{M}{M} > T_d$ | $\frac{M}{M} < \frac{F_4(n_4)}{H}$ | $\frac{F_4(n_4)}{H} < M < \frac{F_5(n_5)}{H} + V$ | $M > \frac{F_6(n_6)}{H} + V$ |
|--------|-------------------------|-------------------------|-----------------|---------------------------|---------------------------------|------------------------|
| 1      | *                       | *                       | *               |                           |                                 |                        |
| 2      | *                       | *                       | *               |                           |                                 |                        |
| 3      | *                       | *                       | *               |                           |                                 |                        |
| 4      | *                       | *                       | *               |                           |                                 |                        |
| 5      | *                       | *                       | *               |                           |                                 |                        |
| 6      | *                       | *                       | *               |                           |                                 |                        |
| 7      | *                       | *                       | *               |                           |                                 |                        |
| 8      | *                       | *                       | *               |                           |                                 |                        |
| 9      | *                       | *                       | *               |                           |                                 |                        |
| 10     | *                       | *                       | *               |                           |                                 |                        |
| 11     | *                       | *                       | *               |                           |                                 |                        |
| 12     | *                       | *                       | *               |                           |                                 |                        |
| 13     | *                       | *                       | *               |                           |                                 |                        |
| 14     | *                       | *                       | *               |                           |                                 |                        |
| 15     | *                       | *                       | *               |                           |                                 |                        |
| 16     | *                       | *                       | *               |                           |                                 |                        |
| 17     | *                       | *                       | *               |                           |                                 |                        |
| 18     | *                       | *                       | *               |                           |                                 |                        |
| 19     | *                       | *                       | *               |                           |                                 |                        |
| 20     | *                       | *                       | *               |                           |                                 |                        |
| 21     | *                       | *                       | *               |                           |                                 |                        |
| 22     | *                       | *                       | *               |                           |                                 |                        |
| 23     | *                       | *                       | *               |                           |                                 |                        |
| 24     | *                       | *                       | *               |                           |                                 |                        |
| 25     | *                       | *                       | *               |                           |                                 |                        |
| 26     | *                       | *                       | *               |                           |                                 |                        |
| 27     | *                       | *                       | *               |                           |                                 |                        |
| 28     | *                       | *                       | *               |                           |                                 |                        |
| 29     | *                       | *                       | *               |                           |                                 |                        |
| 30     | *                       | *                       | *               |                           |                                 |                        |
| 31     | *                       | *                       | *               |                           |                                 |                        |
| 32     | *                       | *                       | *               |                           |                                 |                        |
| 33     | *                       | *                       | *               |                           |                                 |                        |
| 34     | *                       | *                       | *               |                           |                                 |                        |
| 35     | *                       | *                       | *               |                           |                                 |                        |
| 36     | *                       | *                       | *               |                           |                                 |                        |
| 37     | *                       | *                       | *               |                           |                                 |                        |
| 38     | *                       | *                       | *               |                           |                                 |                        |
| 39     | *                       | *                       | *               |                           |                                 |                        |
| 40     | *                       | *                       | *               |                           |                                 |                        |
| 41     | *                       | *                       | *               |                           |                                 |                        |
| 42     | *                       | *                       | *               |                           |                                 |                        |
| 43     | *                       | *                       | *               |                           |                                 |                        |
| 44     | *                       | *                       | *               |                           |                                 |                        |
| 45     | *                       | *                       | *               |                           |                                 |                        |

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