A Sequential Student’s t-Based Robust Kalman Filter for Multi-GNSS PPP/INS Tightly Coupled Model in the Urban Environment

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Abstract: The proper stochastic model of a global navigation satellite system (GNSS) makes a significant difference on the precise point positioning (PPP)/inertial navigation system (INS) tightly coupled solutions. The empirical Gaussian stochastic model is easily biased by massive gross errors, deteriorating the positioning precisions, especially in the severe GNSS blockage urban environment. In this paper, the distributional characteristics of the gross-error-contaminated observation noise are analyzed by the epoch-differenced (ED) geometry-free (GF) model. The Student’s t distribution is used to express the heavy tails of the gross-error-contaminated observation noise. Consequently, a novel sequential Student’s t-based robust Kalman filter (SSTRKF) is put forward to adjust the GNSS stochastic model in the urban environment. The proposed method pre-weights all the observations with the a priori residual-derived robust factors. The chi-square test is adopted to distinguish the unreasonable variances. The proposed sequential Student’s t-based Kalman filter is conducted for the PPP/INS solutions instead of the standard Kalman filter (KF) when the null hypothesis of the chi-square test is rejected. The results of the road experiments with urban environment simulations reveal that the SSTRKF improves the horizontal and vertical positioning precisions by 57.5% and 62.0% on average compared with the robust Kalman filter (RKF).

Keywords: PPP/INS tightly coupled; urban environment; student’s t distribution; robust Kalman filter

1. Introduction

Nowadays, GNSS is widely applied for vehicle navigation due to its high accuracy, high availability, and global coverage. Multi-GNSS PPP can achieve a high-accuracy kinematic positioning performance with a sole receiver, attributing to the good satellite geometry. However, high buildings, tunnels, and other typical urban environments can severely impact the observation quality, leading to the biased positioning solutions. The short-term well-performed INS is widely applied to assist environment-dependent PPP in the sophisticated urban environment; furthermore, the PPP/INS tightly coupled system can effectively improve the long-term precise navigation. In the urban environment, the line-of-sight (LOS) multipath effect and the non-line-of-sight (NLOS) reception are two common error sources impacting on the GNSS observation quality. The LOS multipath errors happen when the receiver antenna intercepts both the LOS and multipath satellite signal. This case is mainly attributed to a mixture of reflection and scattering echoes [1]. The LOS multipath errors can reach up to tens of meters in code and a quarter cycle in phase [2,3], which can be modelled by the observation noise variance changes [4]. In the NLOS case, an obstacle completely blocks the LOS satellite signal, but the signal still reaches the antenna mainly through reflection. The NLOS reception is a more serious problem in the urban environment, also leading to significant biases in both code and phase observations [1]. Reference [4] modelled the NLOS reception errors as the observation noise mean value jumps. Hence, due to the impacts of the LOS multipath and NLOS reception
errors on the GNSS observations analyzed above, they should be properly handled for the PPP/INS solutions. With the booming GNSS applications in the urban environment, many studies have contributed to mitigating the LOS multipath and NLOS reception errors.

The antenna and receiver modifications can reduce the LOS multipath errors, while these methods are invalid for tackling the NLOS reception [5–7]. Surrounding information can assist in determining the NLOS observations, whereas the auxiliary devices (e.g., 3D LiDAR) and the a priori information (e.g., 3D city model) limit these methods [8–10]. For a static receiver, the sidereal filtering was proposed based on the orbital period of satellites, improving the positioning precisions by 56% [11]. Generating the multipath stacking maps can also deal with the site-specific multipath errors, which can reduce the residual standard deviation by about 20% [12]. However, the static methods fail to aid the kinematic applications. Meanwhile, the variance jumps model of the LOS multipath errors and the mean value jumps model of the NLOS reception errors induce heavy tails in the Gaussian observation noise, leading to the KF losing the optimality [13]. Hence, many robust filtering methods were proposed for kinematic applications in the presence of the LOS multipath and NLOS reception errors. Reference [14] applied marginalized likelihood ratio test to correct the NLOS reception errors biased a priori residuals, improving the horizontal and vertical precisions by 41.2% and 60.0%, respectively. The Institute Of Geodesy And Geophysics (IGG-III) function depending on the observation residuals is applied to adjust the biased Gaussian stochastic model, improving the positioning precisions by 49.8% [15]. Reference [16] improved the GPS/BDS PPP/INS positioning solutions by 17.8%, 50.2%, and 22.4% in the north, east, and down components by the RKF. However, the severe GNSS blockages enlarge the covariance matrices of the prior residuals due to the bad satellite geometry, deteriorating the conventional RKF’s validity to the gross errors. Reference [17] derived the robust Student’s t-based Kalman filter by the variational Bayes (VB) method to adapt the heavy tail observation noise, whereas it highly depends on the pre-setting degree of freedom (DOF) parameter of Student’s t distribution. Then, the Student’s t-based Kalman filter with an adaptive DOF parameter was put forward to GNSS/INS and INS/ ultra-short baseline acoustic navigation system integrations [18,19]. However, almost all research of the existing Student’s t-based methods simultaneously adjusts the stochastic model of all the observations, ignoring the satellite-specific observation noise stochastic characteristics in the urban environment. Therefore, a novel robust Kalman filter is indispensable for the multi-GNSS undifferenced and uncombined (UDUC) PPP/INS solutions in the urban environment.

In this paper, we first analyze the code and phase observation distributional characteristics by the epoch-differenced geometry-free model. The observation noise can be described by the Gaussian distribution in the absence of the gross errors, while the heavy tails of the gross-error-contaminated observation noise prefer Student’s t distribution. Although the Student’s t distribution will degenerate to the Gaussian distribution as the DOF inclines to be infinite, it will be dramatical calculation burdens to adapt the DOF for every observation in PPP/INS tightly coupled estimation. Hence, we propose a SSTRKF to meet the needs of the kinematic application. We first determine the observation weights using the IGG-III scheme. Then, the chi-square test is introduced to validate the rationality of the observation weights derived by the IGG-III scheme. The unreasonable group is processed by the sequential Student’s t Kalman filter, while others are still processed by the RKF. The numerical comparisons with real kinematic data validate the proposed method.

The paper is organized as follows. In Section 2, we formulate the mathematical model of the multi-GNSS PPP/INS tightly coupled system. Then, the distributional characteristic analysis is conducted in Section 3. Moreover, the proposed sequential Student’s t-based robust Kalman filter is detailed in Section 4. Subsequently, the road experiments with the urban environment simulations are conducted to validate the proposed method in Section 5. The concluding remarks are summarized in Section 6.
2. Multi-GNSS PPP/INS Tightly Coupled Model

In the following, the PPP/INS tightly coupled functional and stochastic model are described. We first derive the multi-GNSS UDUC PPP observation model, involving GPS, GLONASS, Galileo, and BDS. Then, the north-oriented INS state model is formulated.

2.1. Multi-GNSS PPP/INS Tightly Coupled Observation Model

After applying the known corrections of the precise satellite products, i.e., orbits, clock offsets, and satellite differential code biases, the linearized UDUC PPP observation equations of code $P_{k,r}$ and phase $L_{k,r}$ for $m$ available satellites read [20,21]

$$
P_{k,r} = (e_2 \otimes A_{k,r})p_{k,r}^G + \left( e_2 \otimes g_{k,r} \right) T_{k,r}^G + e_2 m \delta{\iota}_{k,r,\text{IF}} + \left( e_2 \otimes \Lambda_m \right) \Sigma_k + U_{k} + \epsilon_{k,r} \tag{1}$$

$$
L_{k,r} = (e_2 \otimes A_{k,r})p_{k,r}^R + \left( e_2 \otimes g_{k,r} \right) T_{k,r}^R + e_2 m \delta{\iota}_{k,r,\text{IF}} + \left( e_2 \otimes \Lambda_m \right) \Sigma_k - U_{k} - \bar{\alpha}_r + \epsilon_{L_{k,r}} \tag{2}$$

where the subscripts $k$ and $r$ and the superscript $s$ refer to epoch, receiver, and satellite, respectively. In the following, the superscripts G, R, E, and C represent GPS, GLONASS, Galileo, and BDS, respectively. $P_{k,r}$ is the vector of the dual-frequency multi-GNSS code observations with $P_j = \begin{bmatrix} (P^G_{k,r})^T & (P^R_{k,r})^T \end{bmatrix}^T$ and $L_{k,r}$ is the receiver clock error first parameterized as $\delta{\iota}_{k,r,\text{IF}}$. The code and phase hardware delays absorbed into the clock error is first parameterized as $\delta{\iota}_{k,r,\text{IF}} = \bar{\alpha}_r + \epsilon_{L_{k,r}}$.

The code and phase hardware delays are absorbed in the code and phase hardware residuals. The code and phase noise vectors are $\epsilon_{P_{k,r}}$ and $\epsilon_{L_{k,r}}$. Moreover, $\otimes$ denotes the Kronecker product.
The PPP/INS tightly coupled state parameters is organized as
\[ x_k = \begin{bmatrix} \phi_k^T & (\omega_k^n)^T & (\omega_k^e)^T & \nabla_k^T & \delta \omega_k^{ib,IE} & \Sigma_k^T & \nabla_k^T & \delta \omega_k^{en,IR} & \delta \omega_k^{en,IR} \end{bmatrix}^T, \]
where \( \phi_k \), \( \delta \omega_k^{en} \), \( \delta \omega_k^{ib} \), \( \nabla_k \), and \( \epsilon_k \) are the attitude error vector, velocity and position error vectors under the navigation frame (n-frame) and the accelerometer and gyroscope bias vectors, respectively. The observation functional model can be constructed as follows

\[
y_{\text{GNSS},k} = \begin{bmatrix} \rho_{\text{INS},k} - P_{h,j} \\ \rho_{\text{INS},k} - L_{h,j} \end{bmatrix} = H_{y,k} x_k + \epsilon_{\text{GNSS},k} \tag{3}
\]

where \( H_{y,k} \) is the design matrix to \( x_k \). Here, \( \rho_{\text{INS},k} \) is the INS-derived geometry distance vector. \( G_k = \Lambda_i C_i^n \) denotes the INS position error design matrix in the navigation frame (n-frame), where \( C_i^n \) is the rotation matrix from the n-frame to the e-frame.

The empirical multi-GNSS Gaussian stochastic model is constructed as \( Q_k = Q_m \otimes Q_0 \) with \( Q_m = \text{blkdiag}(Q_{\rho}, Q_L) \) and \( Q_0 = \text{blkdiag}(Q_G, Q_R, Q_E, Q_C) \) defining the elevation-dependent dispersion for the four systems [22]. Moreover, \( Q_p = \text{diag}(\sigma_p^2, \sigma_p^2) \) and \( Q_L = \text{diag}(\sigma_L^2, \sigma_L^2) \) describe the code and phase precisions.

### 2.2. North-Oriented INS State Model

The PPP/INS tightly coupled state functional model is based on the north-oriented INS error function as follows [23]

\[
\begin{bmatrix} \delta \phi \\ \delta v_k^v \\ \delta p_k^b \end{bmatrix} = \begin{bmatrix} f^n \times \phi + v^n \times (2 \delta \omega_k^{en} + \delta \omega_k^{ib}) - (2 \omega_k^{en} + \omega_k^{ib}) \times \delta v^n + \delta f^n \\ M_{pv} \delta v^n + M_{pp} \delta p_k^b \end{bmatrix} \tag{5}
\]

where the points upon \( \delta \phi, \delta v_k^v, \) and \( \delta p_k^b \) denote the derivative; the superscript n represents the n-frame; and \( \delta \) denotes the error. \( \omega_k^{en}, \omega_k^{ib}, \) and \( \omega_k^{en} \) are the n-frame, e-frame, and body frame (b-frame) angular rate relative to the inertial frame (i-frame), projected to the n-frame. \( \omega_k^{en} \) can be expressed in the same way above. \( f^n \) is the specific force in the n-frame. Moreover, \( \nabla_k \) and \( \epsilon_k \) are contained in \( \delta f^n \) and \( \delta \omega_k^{ib} \), respectively. \( M_{pv} \) and \( M_{pp} \) are the coefficient matrices of \( \delta v^n \) and \( \delta p \) in the position error function, which read

\[
M_{pv} = \begin{bmatrix} 0 & 1/R_m & 0 \\ R_m & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad M_{pp} = \begin{bmatrix} 0 & -\frac{\epsilon_N}{(R_m + h)^2} \\ -\frac{\epsilon_E}{R_m} & 0 & -\frac{\epsilon_N}{R_m + h} \\ \frac{\epsilon_E}{R_m + h} & \frac{\epsilon_E}{R_m} & 0 \end{bmatrix} \tag{6}
\]

where \( \phi \) and \( h \) are the latitude and altitude, respectively. \( \epsilon_E \) and \( \epsilon_N \) are the east and north velocity. \( R_m \) and \( R_N \) represent the principal radii of curvature along the meridional and prime-vertical normal sections, respectively.

### 3. PPP/INS Observation Noise Distributional Characteristic Analysis

The PPP/INS observation noise distributional characteristic is analyzed as follows. First, the raw GNSS observation noise is extracted and analyzed by the ED GF model. Then, we analyze the distribution characteristics of the gross-error-contaminated observation noise.
3.1. Multi-GNSS Observation Noise Extraction

Two types of GF models are formulated to extract the phase and code observation noise [24]

\[
GF_{1,k,r} = L_{k,r,1}^{s} - L_{k,r,2}^{s} = a_{s,2}^{r} - a_{s,1}^{r} + (\mu_{2} - 1)\delta_{k,r} + \delta B_{k,r,12} + \delta b_{k,r,12} + \epsilon_{GF_{1,k,r}}
\]

(7)

\[
GF_{2,k,r,j} = P_{k,r,j}^{s} - L_{k,r,1}^{s} = (1 - \mu_{j})\delta_{k,r} + D_{k,r,j} - B_{k,r} - \delta b_{k,r,j} + \epsilon_{GF_{2,k,r,j}}
\]

(8)

where \(\delta B_{k,r,12} = B_{k,r,1} - B_{k,r,2}\) and \(\delta b_{k,r,12} = b_{k,r,1} - b_{k,r,2}\) denote the inter-frequency receiver and satellite phase hardware delay differences, respectively. \(D_{k,r,j}\) is the frequency-specific receiver code hardware delay. \(\epsilon_{GF_{1,k,r}}\) and \(\epsilon_{GF_{2,k,r,j}}\) are the unmodelled error residuals of \(GF_{1,k,r}^{s}\) and \(GF_{2,k,r,j}^{s}\) involving the LOS multipath errors, the NLOS reception errors, and the GF observation noise. Moreover, the phase hardware delays and unmodelled error residuals are evidently smaller than those of code [25]. Thus, Equation (8) can be simplified as

\[
GF_{2,k,r,j} = (1 - \mu_{j})\delta_{k,r} + D_{k,r,j} - \delta b_{k,r,j} + \epsilon_{GF_{2,k,r,j}}
\]

(9)

All cycle slips are detected and repaired by applying the TurboEdit method in the post processing [26,27]. Hence, the hardware delays and the phase ambiguities can be eliminated in the epoch difference GF combinations \(\Delta GF_{1,k,r}^{s}\) and \(\Delta GF_{2,k,r,j}^{s}\), which read

\[
\Delta GF_{1,k,r}^{s} = GF_{1,k+1,r}^{s} - GF_{1,k,r}^{s}
\]

(10)

\[
\Delta GF_{2,k,r,j}^{s} = GF_{2,k+1,r,j}^{s} - GF_{2,k,r,j}^{s}
\]

(11)

where only the ionospheric delay residuals and the unmodelled error residuals are contained in \(\Delta GF_{1,k,r}^{s}\) and \(\Delta GF_{2,k,r,j}^{s}\). Moreover, satellites with large a priori residuals (an empirical threshold 30 m) are rejected to guarantee the observations to be gross-error-free. Afterwards, 1Hz sampling static multi-GNSS data from the IGS MGEX GAMG station (7:00:00–7:59:59, 21 February 2022) is applied to analyze the observation noise. To eliminate the influence of the multipath errors as much as possible, the cut-off elevation is set to 35°. Moreover, the ionospheric delay residuals in \(\Delta GF_{1,k,r}^{s}\) and \(\Delta GF_{2,k,r,j}^{s}\) are ignored in this case, as the stability of ED ionospheric delays within high-rate (1Hz) data [28,29]. Eventually, the phase and code observation noise are extracted by \(\Delta GF_{1,k,j}^{s}\) and \(\Delta GF_{2,k,r,j}^{s}\). Considering that the observation precisions are satellite-type-specific, the distributional characteristics of GPS, GLONASS, Galileo, BDS geostationary earth orbit (GEO), BDS inclined geosynchronous orbit (IGSO), and BDS medium earth orbit (MEO) observation noise are analyzed afterwards in Figures 1 and 2. In the following, \(N(\text{mean}, \text{cov})\) denotes the Gaussian probability density functions (PDF) with the mean vector \(\text{mean}\) and the covariance matrix \(\text{cov}\). Moreover, \(\text{var}(\cdots)\) is the variance derivation operation.

Figure 1 indicates that the phase observation noise can be well fitted by the zero-mean-Gaussian distribution in absence of the gross errors. Moreover, the same conclusion is obtained for the code observation noise by analyzing Figure 2. Note that the Gaussian characteristic can also be extracted from \(\Delta GF_{2,k,r,2}^{s}\), which is not further elaborated here. In this case, the Gaussian-based KF can properly handle the PPP/INS solutions. The distributional characteristics of the gross-error-contaminated observations are discussed in the next section.
3.2. Distributional Characteristic Analysis on Gross-Error-Contaminated Observation Noise

The gross error simulations were introduced into the raw observations. According to the research by reference [1], in a typical city urban environment (Shanghai–Lujiazui area), 95% of the LOS multipath error lifetime is shorter than 13.6 s when the rover velocity is 0–3 km/h. Hence, the 10 s LOS multipath errors were introduced into the G08 raw
observations from epoch 1801 to 1810. The LOS multipath errors are modelled as the observation noise variance jumps [4] as

\[
\begin{align*}
P_{k,r,j}^c &= P_{k,r,j} + N(0, (15\text{m})^2), 1800 < k \leq 1810 \\
I_{k,r,j}^s &= I_{k,r,j}^s + N(0, (0.01\text{m})^2), 1800 < k \leq 1810
\end{align*}
\]

Moreover, reference [1] reveals that most LOS multipath errors are less than 350 m, while the observations with much too large multipath errors are always eliminated by the a priori residual test (e.g., an empirical threshold 30 m) in PPP data pre-processing. Hence, a constant km/h contaminated man management where 4.1.

As for the NLOS reception errors, 95% of their lifetime is shorter than 68.2 s with 0–3 km/h rover velocity [1]. Hence, the NLOS reception errors can be modelled as the 60 s constant mean value jumps in the code and phase. However, \(\Delta GF_{1,k,r}^s\) and \(\Delta GF_{2,k,r}^s\) cannot extract the NLOS reception errors modelled as constant mean value jumps which are eliminated by the epoch-differenced strategy. Hence, only the LOS multipath errors are simulated to accomplish the analysis. The \(\Delta GF_{1,k,r}^s\) PDF of G08, Student’s t distribution PDF (DOF = 3), and \(N(0, \text{var} \left(\Delta GF_{1,k,r}^s\right))\) are shown in Figure 3.

![Figure 3. \(\Delta GF_{1,k,r}^s\) PDF of G08, \(N\left(0, \text{var} \left(\Delta GF_{1,k,r}^s\right)\right)\), and Student’s t PDF (DOF = 3).](image)

Figure 3 demonstrates that the distributional characteristics of the gross-error-contaminated observation noise can be divided into the Gaussian part and the heavy tail. When fitting the heavy tail, the Student’s t distribution with a proper DOF parameter is preferred to the Gaussian distribution. However, the Gaussian part is obviously better fitted by the Gaussian distribution than the Student’s t distribution. Hence, the Gaussian and Student’s t combined processing strategy will outperform any single distribution-based strategy. Eventually, if the NLOS reception errors were introduced, the true distribution would obtain a heavier tail due to the mean value jump model.
4. Sequential Student’s t-Based Robust Kalman Filter

First, the robust Kalman filter and the Student’s t based Kalman filter (STKF) are introduced briefly. Then, the SSTRKF is derived by incorporating the RKF and the sequential Student’s t Kalman filter. Eventually, we provide the SSTRKF processing strategy.

4.1. Robust Kalman Filter Based on IGG-III Function

The RKF tunes the gross-error-contaminated observation noise in the standard Kalman filter by setting the robust factors as follows

\[ \tilde{Q}_k = \text{diag}(r_{k,1}, \ldots, r_{k,i})Q_k \]  

where \( \tilde{Q}_k \) is the new stochastic model with \( \tilde{Q}_{k,i} = r_{k,i}Q_k \). \( r_{k,i} \) represents the \( i \)th element in the main diagonal of the raw observation stochastic model \( Q_k \). \( r_{k,i} \) is the \( i \)th observation robust factor, derived by the IGG-III function [30]

\[ r_{k,i} = \begin{cases} 1 & |\tilde{v}_{k,i}| \leq k_0 \\ \frac{\sqrt{v_{k,i}}}{k_0 - |\tilde{v}_{k,i}|}^2 & k_0 < |\tilde{v}_{k,i}| \leq k_1 \\ \infty & |\tilde{v}_{k,i}| > k_1 \end{cases} \]  

where \( k_0 \) and \( k_1 \) are presetting constants, chosen from 1.0 to 2.5 and from 3.5 to 8.0, respectively. The \( i \)th standardized residual vector \( \tilde{v}_{k,i} \) reads

\[ \tilde{v}_{k,i} = v_{k,i}/\sqrt{Q_{k,i}} \]  

where \( v_{k,i} \) and \( Q_{k,i} \) are the \( i \)th observation a priori residual and the corresponding variance, respectively. Note that \( Q_{k,i} \) is the \( i \)th element in the main diagonal of the a priori residual covariance matrix \( Q_k = H_{g,k}Q_{g,k}H_{g,k}^T + Q_k \). Here, \( Q_k \) is the covariance matrix of the one-step prediction \( \hat{x}_k \). After redertermining the stochastic model of the observation system, the modified Kalman gain matrix can be written as

\[ K_k = \tilde{Q}_kH_{g,k}^T \left( H_{g,k}^T\tilde{Q}_kH_{g,k} + \tilde{Q}_k \right)^{-1} \]  

The RKF state estimators and the corresponding covariance matrices will be obtained after updating the Kalman gain matrix. As shown in Equation (14), the robust factors vary with \( \tilde{v}_{k,i} \), which is determined by \( Q_{k,i} \) in Equation (15). Meanwhile \( Q_{k,i} \) is significantly susceptible to the worse PPP/INS positioning solutions with poor satellite geometry.

4.2. Student’s t-Based Kalman Filter

The Student’s t distribution is applied to fit the gross-error-contaminated observation noise. The STKF can be described as follows [18,19]

\[ p(x_k|y_{1:k-1}) = N(x_k; \bar{x}_k, \bar{Q}_k) \]  

\[ p(y_k|x_k) = St(y_k; H_{g,k}x_k, Q_k, \theta_k) = \int N(y_k; H_{g,k}x_k, Q_k)G(\xi_k; \frac{\theta_k}{2}, \frac{\theta_k}{2})G(\theta_k; p_k, q_k)d\xi_k \]  

where \( St(y_k; H_{g,k}x_k, Q_k, \theta_k) \) denotes the Student’s t PDF with mean vector \( H_{g,k}x_k \), scale matrix \( Q_k \), and DOF \( \theta_k \). \( \xi_k \) is an auxiliary random variable obeying Gamma distribution \( G(\xi_k; \frac{\theta_k}{2}, \frac{\theta_k}{2}) \). The STKF adjusts the observation stochastic model mainly by \( \xi_k \). Moreover, \( \theta_k \) is also modelled as Gamma distribution with shape parameters \( p_k \) and rate parameter \( q_k \). The estimation problem can be solved by the VB method as follows [19]

\[ \{q(x_k), q(\xi_k), q(\theta_k)\} = \text{arg min} \text{KLD}(q(x_k), q(\xi_k), q(\theta_k)) || p(x_k, \xi_k, \theta_k|y_{1:k})) \]
where \( q(\cdots) \) and \( p(\cdots) \) denote the approximate posterior and posterior PDF, respectively. KLD(\cdots) is the Kullback–Leibler divergence. Equations (17) and (18) shows that the STKF simultaneously adjusts the stochastic model of every observation with solely one parameter \( \xi_k \), which can mis-weight the gross-error-free observations. Moreover, the gross-error-contaminated observation noise has diverse stochastic characteristics, requiring separately processing. It is noticed that the computation burden of the STKF is relatively heavy as Equation (19) is solved iteratively. Although the Student’s t distribution degenerates to be Gaussian as the DOF tends to be infinite, conducting STKF with high-dimension matrices in every epoch can lead to disastrous computation burdens. Hence, the SSTRKF is proposed in the next section.

4.3. Sequential Student’s t-Based Robust Kalman Filter

The proposed SSTRKF first pre-weights the observations by Equation (14), which can give the proper observation noise covariance matrix with ideal satellite geometry and gross-error-free observation noise. Hence, after being pre-weighted by the IGG-III function, we conduct the chi-square test satellite-by-satellite as follows

\[
\begin{cases}
(v_k^i)^T (\tilde{Q}_{v,k}^i)^{-1} v_k^i \leq th & H_0 \\
(v_k^i)^T (\tilde{Q}_{v,k}^i)^{-1} v_k^i > th & H_1
\end{cases}
\]  

(20)

where \( v_k^i \) and \( \tilde{Q}_{v,k}^i = H_k^i \tilde{Q}_{\xi,k}^i (H_k^i)^T + \tilde{Q}_{q,k}^i \) are the a priori residual vector and the corresponding IGG-III-modified covariance matrix of the satellite \( s \), respectively. \( th \) is the presetting chi-square threshold. Furthermore, \( H_0 \) and \( H_1 \) represent the null and alternative hypothesis, respectively. If the \( H_0 \) case is accepted, the IGG-III-derived variances of the satellite \( s \) are reasonable. In contrast, the observations are sorted into the unreasonable variance group when accepting \( H_1 \). The reasonable variance group obeys the Gaussian assumption, whereas the unreasonable variance group is modelled by the Student’s t distribution. Hence, we first process the reasonable group by Equation (16).

The unreasonable group is sequentially processed due to the diverse stochastic characteristics of the gross-error-contaminated code and phase observations. Hence, the estimation problem of the unreasonable group can be presented as

\[
p(x_k|y_{1:k-1}, y_{k:N}, y_{k,ST,1:d-1}) = N(x_k,i; \tilde{x}_k,i, \tilde{Q}_k,i)
\]

(21)

\[
p(y_{1:k-1}, y_{k,N}, y_{k,ST,i}; x_k,i) = N(y_{1:k-1}, y_{k,N}, y_{k,ST,i}; H_k x_k,i, Q_k,ST,i, \tilde{Q}_{k,i})
\]

(22)

where \( y_{k,ST,1:i} = [y_{k,ST,1} \cdots y_{k,ST,i}] \) is the observation vector with the first \( i \) observations in the unreasonable group. \( y_{k,N} \) denotes the reasonable variance group observation vector. Note that the subscript \( i \) represents the \( i \)th observation in the unreasonable group. \( Q_{k,ST,i} \) is the \( i \)th unreasonable observation variance, determined by Equation (13). Then, the estimation problem can be solved using the VB method as Equation (19).

The Gamma PDF \( q^{(t+1)}(\xi_{k,i}) = G(\xi_{k,i}; a_{k,i}^{(t+1)}, b_{k,i}^{(t+1)}) \) is chosen as the posterior PDF for the auxiliary random variable \( \xi_{k,i} \) in the \( (t + 1) \)th iteration. Thus, the shape and rate parameter \( a_{k,i}^{(t+1)} \) and \( b_{k,i}^{(t+1)} \) are derived as follows

\[
a_{k,i}^{(t+1)} = \{ dim + E^{(t)}[\theta_{k,i}] \}
\]

(23)

\[
b_{k,i}^{(t+1)} = 0.5 \{ E^{(t)}[\theta_{k,i}] + \text{tr}[E^{(t)} Q_{k,ST,i}^{-1}] \}
\]

(24)

\[
E_{k,i}^{(t)} = \left(y_{k,ST,i} - H_{y,k,i}\xi_{k,i}^{(t)}\right)^T + H_{y,k,i}\tilde{Q}_{k,i}^i H_{y,k,i}^T
\]

(25)
where $x_{k,i}^{(t)}$ and $\hat{Q}_{k,i}^{(t)}$ are the $i$th iteration estimator and the corresponding covariance matrix, respectively. $\text{dim}$ is the observation dimension. $\text{tr} [\cdots]$ is the trace of a matrix. Furthermore, $\zeta_k$ can be calculated through the expectation of the Gamma distribution as

$$E^{(t+1)}[\zeta_{k,i}] = \frac{\alpha_{k,i}^{(t+1)}}{\hat{\beta}_k^{(t+1)}}$$

(26)

Another Gamma PDF $q_k^{(t+1)}(\theta_{k,i}) = G(\theta_{k,i}; p_{k,i}^{(t+1)}, q_{k,i}^{(t+1)})$ is exploited to be the posterior PDF of the DOF $\theta_{k,i}$, resulting in

$$p_{k,i}^{(t+1)} = p_{k,i}^{(0)} + 0.5$$

(27)

$$q_{k,i}^{(t+1)} = q_{k,i}^{(0)} - 0.5 - 0.5E^{(t+1)}[\log(\zeta_{k,i})] + 0.5E^{(t+1)}[\zeta_{k,i}]$$

(28)

$$E^{(t+1)}[\log(\zeta_{k,i})] = \psi(\alpha_{k,i}^{(t+1)}) - \log(\hat{\beta}_k^{(t+1)})$$

(29)

where $\psi(\cdots)$ denotes the digamma function. The expectation $E^{(t+1)}[\theta_{k,i}]$ is obtained as

$$E^{(t+1)}[\theta_{k,i}] = p_{k,i}^{(t+1)}/q_{k,i}^{(t+1)}$$

(30)

Then, the posterior PDF of $x_{k,i}$ is modelled by the Gaussian PDF $N\left(x_{k,i}; x_{k,i}^{(t+1)}, \hat{Q}_{k,i}^{(t+1)}\right)$. Thus, we obtain the measurement update process, given as

$$\tilde{Q}_{k,i}^{(t+1)} = Q_{k,ST,i}/E^{(t+1)}[\zeta_{k,i}]$$

(31)

$$K_{k,i}^{(t+1)} = \tilde{Q}_{k,i}H_{y,k,i}^TH_{y,k,i}\tilde{Q}_{k,i}^{(t+1)}$$

(32)

$$x_{k,i}^{(t+1)} = \hat{x}_{k,i} + K_{k,i}^{(t+1)}(y_{k,ST,i} - H_{y,k,i}\hat{x}_{k,i})$$

(33)

$$\hat{Q}_{k,i}^{(t+1)} = \hat{Q}_{k,i} - K_{k,i}^{(t+1)}H_{y,k,i}\hat{Q}_{k,i}$$

(34)

More derivation details refer to references [18] and [19]. To intuitively demonstrate the process of SSTRKF, the flow chart is shown in Figure 4. Moreover, we elaborate the processing steps in the kinematic applications: (i) After the time update, the IGG-III pre-weighted observation stochastic model is calculated by Equations (12)–(14). (ii) The chi-square test is applied to sort the available satellites into the reasonable and unreasonable group. (iii) The observation stochastic model is calculated by Equations (12)–(14). (ii) The chi-square test is applied to sort the available satellites into the reasonable and unreasonable group. (v) $p_{k,i}^{(0)}$ and $q_{k,i}^{(0)}$ are initialized by two presetting constants. (vi) Conduct the $i$th estimation by Equations (23)–(34). (vii) Assign $x_{k,i}^{(t=\text{itmax})}$ to $x_{k,i+1}$ and $\hat{Q}_{k,i}^{(t=\text{itmax})}$ to $\hat{Q}_{k,i+1}$, where $\text{itmax}$ is the maximum iteration number. (viii) If $i$ is less than the unreasonable group observation number $N$, return to (v) with $i = i + 1$. (ix) If $i = N$, output $x_{k,N}^{(\text{itmax})}$ and $\hat{Q}_{k,N}^{(\text{itmax})}$ as the estimator and the corresponding covariance matrix of the current epoch.
More derivation details refer to references [18] and [19]. To intuitively demonstrate the process of SSTRKF, the flow chart is shown in Figure 4. Moreover, we elaborate the processing steps in the kinematic applications:

(i) After the time update, the IGG-III pre-weighted observation stochastic model is calculated by Equations (12)–(14).

(ii) The chi-square test is applied to sort the available satellites into the reasonable and unreasonable group.

(iii) The measurement update with the Kalman gain obtained by Equation (16) is conducted for the reasonable group.

(iv) The estimator and the corresponding covariance matrix of the reasonable group are assigned to \( x_\text{th}^{(0)}, P_{x_\text{th}}^{(0)} \), respectively.

(v) \( p_{x_\text{th}}^{(0)}, q_{x_\text{th}}^{(0)} (\omega) \) are initialized by two presetting constants.

(vi) Conduct the \( \text{ith} \) estimation by Equations (23)–(34).

(vii) Assign \( x_{\text{th}^{(0)}}, P_{x_{\text{th}}^{(0)}} (\omega) \) to \( x_{\text{th}^{(0)}}, P_{x_{\text{th}}^{(0)}} (\omega) \), where \( \text{iter}_\text{max} \) is the maximum iteration number.

(viii) If \( i \) is less than the unreasonable group observation number \( N \), return to (v) with \( i = i + 1 \).

(ix) If \( i = N \), output \( x_{\text{th}^{(0)}}, P_{x_{\text{th}}^{(0)}} (\omega) \) as the estimator and the corresponding covariance matrix of the current epoch.

**Figure 4.** Flow chart of the SSTRKF.

5. Experiments and Discussions

The road experiments were conducted with SPAN-CPT IMU and a Trimble receiver in March 2019, Xuzhou, China. The dataset is available in the open-access software Ginav [31]. The experiment duration is 1721 s with 100 Hz IMU data and 1 Hz GNSS data. Note that we applied the RTK/INS tightly coupled solutions as the reference to evaluate the navigation performance. The precise clock and orbit products were obtained from the MGEX data-processing center (http://www.igs.org/mgex/products, accessed on 31 October 2021), and the satellite and receiver antenna parameters were gained from igs14.atx. In real-time navigation, the real-time clock and orbit products can be used to support PPP/INS.

Figure 5 is the 2D experiment trajectory, containing two laps on a same road. We select three segments of the road to conduct the six severe urban environment simulations, presented in red in Figure 5. To simulate the blockage urban environment as severe as possible, the cut-off elevation was set to \( 90^\circ \) and \( 60^\circ \) for the satellites with azimuth \( 285^\circ \sim 360^\circ \) and \( 0^\circ \sim 285^\circ \), respectively. This environment can be always found in the urban environment, such as the narrow streets surrounded by high buildings. Moreover, the LOS
multipath and the NLOS reception errors were introduced into an available satellite and the blocked satellite with the highest elevation, respectively. According to the analysis in Section 3.2, the lifetime and simulation model of the LOS multipath and the NLOS reception errors are summarized in Table 1. It is noticed that the elevation of the satellite contaminated by the NLOS reception errors is higher than 50°, which tends to lead the NLOS reception errors lower than 50 m [1]. Therefore, the value of the NLOS reception errors was set to 15 m with the same consideration as the LOS multipath errors in Section 3.2.

Figure 5. Two-dimensional experiment trajectory.

Table 1. The LOS multipath and the NLOS reception error simulation settings.

| Simulation Number | 1     | 2     | 3     | 4     | 5     | 6     |
|-------------------|-------|-------|-------|-------|-------|-------|
| LOS multipath error | Lifetime | 10 s  | 10 s  | 0 s   | 0 s   | 20 s  | 10 s  |
| Model             | \[P_k^r_{k,r,j} = P_k^r_{k,r,j} + N(0,(15m)^2)\] |
|                   | \[L_k^s_{k,r,j} = L_k^s_{k,r,j} + N(0,(0.01m)^2)\] |
| NLOS reception error | Lifetime | 45 s  | 30 s  | 20 s  | 55 s  | 23 s  | 20 s  |
| Model             | \[P_k^r_{k,r,j} = P_k^r_{k,r,j} + 15m\] |
|                   | \[L_k^s_{k,r,j} = L_k^s_{k,r,j} + 0.01m\] |

The available satellite (AS) number of the multi-GNSS (GPS/GLONASS/Galileo/BDS), GPS (G), GLONASS (R), Galileo (E), BDS (C) is shown in Figure 6. During the experiment, the average AS number of multi-GNSS, G, R, E, and C are 18.9, 5.8, 2.5, 3.0, and 7.5, respectively. The first urban environment simulation was from 80 s to 125 s with 6, 1, 0, 1, and 4 average AS number for multi-GNSS, G, R, E, and C. During the 320 s–350 s, we conducted the second urban environment simulation, where the AS number drops to 4.3, 1, 0, 0, and 3.3 for multi-GNSS, G, R, E, and C on average, respectively. As for the third urban environment simulation during 540 s–560 s, the multi-GNSS, G, R, E, and C AS number maintain 4, 1, 0, 0, and 3, respectively. The fourth, fifth and sixth urban environment simulations were from 945 s to 1000 s, from 1227 s to 1260 s, and from 1445 s to 1465 s, respectively, where 4, 1, 0, 0, and 3 satellites for multi-GNSS, G, R, E, and C are available on average. Figure 6 indicates that the AS number reduces significantly during the urban environment simulations. Moreover, the shadow areas in Figures 6–9 represent the urban environment simulations.

To reveal the impacts of the LOS and NLOS multipath errors on the positioning solutions in the urban blockage environment, we first conducted three schemes with the KF, summarized in Table 2. Figure 7 depicts the positioning errors of the three schemes, where the Scheme 1 achieves the best precisions among the three schemes. When suffering the blockage environment, the positioning errors (blue line in Figure 7) increases due to the
barren observation conditions. Moreover, the positioning errors diverge dramatically when the LOS multipath and NLOS reception errors are introduced into the observations, such as the red line in Figure 7. Therefore, the gross errors induced by the LOS multipath effect and NLOS reception are rather serious problems for the PPP/INS tightly coupled system, especially in the urban severe blockage environment.

**Figure 6.** The AS number during the road experiments.

**Figure 7.** Impacts of the LOS multipath and NLOS reception errors on the positioning solutions.

**Figure 8.** Chi-square statistics during the road experiment.
Three simulation schemes to analyze the impacts of the urban environment.

Table 2. Three simulation schemes to analyze the impacts of the urban environment.

| The LOS Multipath and NLOS Reception Errors | Blockage Environment |
|---------------------------------------------|----------------------|
| Scheme 1                                    | No                   | No                   |
| Scheme 2                                    | No                   | Yes                  |
| Scheme 3                                    | Yes                  | Yes                  |

The chi-square test threshold was set to 13.277 with the significance level and the DOF being 1% and 4, respectively. Figure 8 gives the chi-square statistics during the road experiment. The chi-square statistics average 0.23 when neither the LOS multipath errors nor the NLOS reception errors are introduced. Moreover, only 15 in total 31,360 chi-square statistics exceed the chi-square threshold in this case, indicating that the IGG-III scheme can properly adjust the observation noise stochastic model in this case. On the contrary, the average chi-square statistics of the satellites impacted by the LOS multipath and NLOS reception errors are 41.6, 68.1, 89.3, 74.0, 52.8, and 72.0 for the six urban environment simulations, respectively. The chi-square statistics of the gross-error-contaminated satellites evidently exceed the chi-square threshold during the six urban environment simulations. Consequently, the IGG-III-based RKF fails to give proper variances for the gross-error-contaminated observations. Moreover, the SSTRKF avoids an unnecessary computation burden from processing every observation with the sequential Student’s t-based Kalman filter by dividing the observations into the reasonable and unreasonable variance group.

In the proposed SSTRKF, the sequential Student’s t-based Kalman filter is activated to process the observations that cannot be appropriately weighted by the IGG-III model. According to Section 4.3, \( p_{k,i}^{(0)} \) and \( q_{k,i}^{(0)} \) were initialized to 0.3 and 1 before processing every observation, respectively. The maximum iteration number of the SSTRKF was set to 10 [17–19]. To explore the superiority of the proposed SSTRKF, the KF, the RKF, the STKF, and the STKF were applied to the multi-GNSS PPP/INS tightly coupled system in the Scheme 3 setting.

Figure 9 shows the KF, the RKF, the STKF, and the SSTRKF positioning errors during the road experiments. The STKF simultaneously tunes the observation noise covariance matrix with solely one factor, leading to the severely biased PPP stochastic model. Hence, the STKF errors diverge rapidly. It is evident that the positioning precisions are impacted by the urban environment simulation. The root mean square (RMS) positioning errors during the six urban environment simulations are collected in Figure 10. Compared with the KF, the proposed SSTRKF improves the horizontal and vertical precisions by 65.7% and 72.1% on average, respectively. Moreover, the SSTRKF achieves 57.5% and 62.0% average precision improvement relative to the RKF in the horizontal and vertical components, respectively. During simulation 2, although the RKF ameliorates the positioning precisions compared
with the KF, the RKF positioning errors still diverge to 1.78m in maximum. As for the
SSTRKF, the positioning precisions are improved by 29.4% and 32.6% relative to the RKF in
the horizontal and vertical components during simulation 2. Moreover, the RKF impairs
the positioning precisions compared with the KF during simulation 3, indicating that the RKF
fails to derive proper observation variances with the IGG-III function. In contrast, during
simulation 3, the SSTRKF suppresses the error divergence during simulation 3, reducing
the horizontal and vertical positioning errors by 84.9% and 71.0% compared with the RKF
and by 83.9% and 68.6% compared with the KF, respectively. In simulation 4, compared
with RKF and KF, the horizontal and vertical PPP/INS precisions are improved by 80.3%
and 95.3% and by 87.5% and 96.3% after applying the SSTRKF. As for simulation 5, the
horizontal and vertical positioning errors of RKF and KF are reduced by 72.6% and 81.8%
and by 54.2% and 76.7% with the SSTRKF. During simulation 6, the SSTRKF improves by
78.5% and 61.5% the horizontal and vertical positioning precisions compared with the RKF.
As for KF, 89.3% and 76.7% horizontal and vertical precision improvement are achieved
by conducting the SSRTKF. Moreover, the SSTRKF improves the positioning solutions
slightly in the simulation 1. As shown in Figure 6, the observation condition of simulation
1 is better than the other five simulations, contributing to well-independent positioning.
Hence, all three schemes can properly identify and handle the gross-error-contaminated
satellites. According to the analysis above, the proposed SSTRKF evidently outperforms
to the KF and RKF during the urban environment simulations, especially in the severe
blockage environment.

![Figure 10. RMS positioning errors of the three schemes during the six simulations.](image-url)

### 6. Conclusions

The standard Kalman filter is optimal for PPP/INS tightly coupled only when the
observation noise obeys the Gaussian assumption. However, the LOS multipath and NLOS
reception gross errors in the urban environment lead to heavy tails in the observation noise.
In the urban severe blockage environment, the heavy tails cannot be properly fitted by the
IGG-III function. Hence, in this paper, we apply the Student’s t distribution to fit the heavy
tails of the gross-error-contaminated observation noise. Moreover, the SSTRKF is proposed
by sequentially conducting the RKF and the sequential Student’s t-based Kalman filter. The
research findings and conclusions are summarized as follows.

1. The GNSS phase and code observation noise obey the Gaussian assumption in the
   absence of the LOS multipath and NLOS reception errors. Moreover, the Student’s t
   distribution can fit the heavy tails of the gross-error-contaminated observation noise.
2. The proposed SSTRKF can adjust the IGG-III function-derived unreasonable vari-
   ances through the chi-square test and the sequential Student’s t-based Kalman filter,
   respectively.
3. The numerical comparisons have validated our proposed SSTRKF for the gross-error-
   contaminated observations. Compared with the RKF, the proposed SSTRKF improves
   the horizontal and vertical positioning precisions by 57.5% and 62.0% on average.
during the urban environment simulations. Consequently, the proposed SSTRKF is superior to the KF and the RKF in the urban environment.

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