Force, acceleration and velocity during trampoline jumps—a challenging assignment

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Abstract
Bouncing on a trampoline lets the jumper experience the interplay between weightlessness and large forces on the body, as the motion changes between free fall and large acceleration in contact with the trampoline bed. In this work, several groups of students were asked to draw graphs of elevation, velocity and acceleration as a function of time, for two full jumps of the 2012 Olympic gold medal trampoline routine by Rosannagh MacLennan. We hoped that earlier kinaesthetic experiences of trampoline bouncing would help students make connections between the mathematical descriptions of elevation, velocity and acceleration, which is known to be challenging. However, very few of the student responses made reference to personal experiences of forces during bouncing. Most of the responses could be grouped into a few categories, which are presented and discussed in the paper. Although the time dependence of elevation was drawn relatively correctly in most cases, many of the graphs of velocity and acceleration display a lack of understanding of the relation between these different aspects of motion.

1. Introduction
The motion on a trampoline can be described by a combination of free fall and harmonic motion: While in the air, the trampolinist is only affected by the force of gravity, and during the contact time the trampolinist also experiences a large upward force from the trampoline bed. The stronger the push from the trampoline, the larger the acceleration and the higher the jump. In earlier work [1, 2], the motion on a trampoline was analysed in detail, showing the relation between the maximum force on the rider and the ratio between ‘flight time’ and contact time. The mathematical description was found to be in good agreement with measurement.

In this paper, we present an assignment based on Rosannagh MacLennan’s gold medal trampoline routine from the London Olympics in 2012. She used 19s (as extracted from the video, [3]) to complete the routine of 10 jumps. The score board shows that 16 of these seconds were ‘flight time’ [3]. Several groups of students were asked to draw approximate graphs of elevation, velocity and acceleration during two full jumps of her routine (with the approximation that all jumps are similar). To help the respondents,
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an empty graph with suitable axes was provided. In preparing this assignment, we hoped that earlier kinaesthetic experiences of trampoline bouncing might help students recognise the strong forces during the acceleration at the bottom of the jump.

The assignment requires students to distinguish between the two different types of motion and the forces involved. They also need to work out a strategy for applying familiar relations for height, velocity and acceleration, as well as using the relations between these different aspects of motion. How do students deal with this assignment? What aspects are most challenging?

This trampoline example was first used in 2014 as part of the Swedish national competition for the European union science olympiad (EUSO) [4], with a few scaffolding questions, presented below. The following year, a multiple-choice version of the question was used, as discussed in section 4. Different versions of the problem have then been used for group discussions during workshops for physics teachers but also as an exam problem for first-year university physics students, as discussed in section 5.

Our goal with this investigation is to outline and map students’ understanding of representations of these kinematics concepts as a support for the development of teaching strategies.

1.1. Understanding of graphs of position, velocity, and acceleration versus time

Graphs of position, velocity and acceleration are fundamental in teaching about motion, and many researchers have studied student difficulties, as reviewed e.g. in [5]. Even if students entering introductory physics classes understand the basic construction of graphs, they often find it difficult to apply those skills to the tasks they encounter in the physics courses [6]. These difficulties often arise due to lack in using multiple representations in problem solving [7], but also due to limitations in reading representation such as graphs [8]. Student limitations in discerning critical aspects, are often found to hinder students from being able to understand a problem on a deeper level [9]. Students simply do not see the same things in those representations as their teachers do, while at the same time teachers have forgotten how difficult it was to read representations being a novice. However, proper scaffolding by experienced teachers is found to improve student representational competency and hence increase the success by the students [10]. Meltzer [11] compared students’ problem solving performance depending on representations. They found that the proportion of correct responses was consistently higher for questions posed verbally than for analogous questions posed in a diagrammatic representation.

McDermott et al. [6] and Beichner [12] noted that students see little difference between distance, velocity, and acceleration, and often believe that graphs of these variables should look identical: if students believe that a ‘graph is like a picture’, it should not matter what is graphed, and they may expect all graphs to look like a replication of the object’s physical motion. McDermott et al. also found that, although there was differences in severity, the types of difficulties were similar for high-school students, and for physics and physical sciences college students.

Planinic et al. [13, 14] compared student responses to analogous problems in mathematics and physics. They also asked physics teachers to rank the difficulties of the tasks. While most physics teachers were found to rank the context-free mathematics problems as more difficult, students were considerably more likely to give correct responses for the context-free versions of the problems, supporting the suggestion of McDermott et al. [6] that the lack of mathematical skills is not the main cause of student difficulties with graphs in physics. Training students to use multiple representations is known to be important, as emphasized e.g. by van Heuvelen [7].

Acceleration is typically introduced without reference to force, as seen e.g. in Basson’s analysis [15] of the ‘hierarchy of physics concepts to deal with the concept acceleration’. In the trampoline example, we can make use of the personal experience of the varying forces acting on your own body as you bounce. This kinaesthetic experience can be enhanced by accelerometer data [1, 2] e.g. from a smartphone [16], as well as a slinky for a visual measure of the force [17], and video analysis of the motion [18] to provide a wider variety of representations.

2. Understanding trampoline jumps

To analyse Rosannagh MacLennan’s gold medal trampoline jumps, a few scaffolding questions can be used, e.g.
(i) What time is required to reach the highest point after the feet have left the trampoline?
(ii) Draw a rough graph of how her height varies during two full jumps.
(iii) Draw a graph of the variation of velocity during two jumps.
(iv) What is her velocity as she lands on the trampoline?
(v) What is her velocity as she leaves the trampoline again?
(vi) In what positions is her speed zero?
(vii) What is her acceleration when she does not touch the trampoline?
(viii) Estimate her acceleration during the time her feet are touching the trampoline.

These questions were included as the problem was first used to select the 24 finalists for the Swedish national competition for the European union science olympiad (EUSO) in 2014. The Swedish competition was open for students in the last year of lower secondary school and in the first year of upper secondary school, (15 and 16-years old, respectively in the autumn of the school year), with approximately 12 finalists selected from each age group. The first tests were graded by the local teachers, who then sent up to three tests from each school for final grading. The response sheets from 18 schools with potential finalists were analysed, giving a total of 42 response sheets. Of the 22 (20) responses from 15(16)-year-olds, only 11(8) had graphs. All elevation graphs were essentially correct, whereas most of the velocity graphs indicated confusion between velocity and speed.

The scaffolding questions above have also been used to initiate small-group discussions during teacher workshops. During these occasions, we have found that teachers typically first conclude that each jump has a flight time of around 1.6s and thus reaches the highest point after \( t = 0.8 \text{s} \). From this information, the maximum elevation is obtained using the well-known relation \( s = gt^2/2 \approx 3.2 \text{m} \)—often after some discussion to clarify that the same time \( t = 0.8 \text{s} \) is used to reach the trampoline after passing the highest point. Other groups may instead start by identifying points where the velocity is zero (i.e. at the highest and lowest points). The initial strategies also include drawing the acceleration \( (a = -|g|) \) for the part of the motion when the trampolinist is in the air. Figure 1 shows an example of how these values related to the free-free fall parts of the jumps can be inserted in the diagram, before attempting an estimate of the acceleration during contact time.

In the next section we show redrawn versions of a number of common responses from students challenged with this task, and describe briefly the features of the different types of responses.

### 3. Examples of student responses

The types of responses shown in this section have been found in most student groups, although the university students were more likely to draw the later examples. The responses have been grouped into a number of categories, and have been redrawn for the presentations in this section. These examples may be used for group discussions as a way to elicit and clarify thoughts about the relation between elevation, velocity and acceleration. Selected velocity graphs have also been used for a multiple-choice question, as discussed in the next section.

#### 3.1. The same graphs for the same motion

In the first set of graphs, shown in figure 2, the respondents have correctly assigned the value zero at the turning points. However, as in many
other types of responses the velocity is never drawn to be negative—the students seem to forget that velocity is a vector and that it continues below zero as the jumper is on the way down after the highest point. From responses handed in on paper, it is often obvious that students have tried to avoid discontinuous derivatives, adjusting their graphs to making the velocity change smoothly at the turning points and changing their original graph to something like a sine function. Many of these respondents drawing that type of velocity graph make the acceleration graph very similar, as in figure 2. This set of graphs also illustrates the tendency, found by McDermott et al (1987) and Beichner (1994) of students to make graphs representing the same motion to look similar. Other responses, indeed, showed the velocity and acceleration graphs essentially identical also to the elevation graph.

We also note that in the graphs in figure 2 the acceleration is drawn as zero in the same points as the velocity, which is known to be common problem e.g. from conceptual questions asking about the acceleration of a ball in the highest point.

Some of the responses similar to figure 2 have omitted the short time, 0.3 s, when the trampolinist is in contact with the trampoline, or have forgotten that the elevation is negative during the short contact time (choosing the level of the unloaded trampoline mat as zero). The omission was found also for the responses that were otherwise similar to those in figure 3.

3.2. Positive and negative acceleration during free fall

The velocity in the graph in figure 3 is always positive and has a slope $-|g|$ while the trampolinist is on the way up and a slope $+|g|$ on the way down, indicating a common confusion between velocity and speed. In this case, the acceleration is still drawn as $-|g|$ for the whole free-fall part of the motion. For the somewhat easier question about acceleration and velocity for a ball thrown into the air, students often draw the acceleration changing sign at the highest point. Some textbooks emphasize the difference between speeding up and slowing down, and refer to the slowing down as ‘negative acceleration’, which may encourage that type of velocity graphs, (as well as the common incorrect belief that the acceleration must be zero at the turning point).
The relation between the integral of velocity and the change in elevation does not hold for the graphs in figure 3. Nor do the acceleration graph during contact time correspond to the derivative of velocity.

It should be noted that it is more common for students to draw a constant acceleration during the contact time, typically at $±|g|$, but sometimes at larger values, with an area under the positive part of the acceleration graph, giving the same magnitude as the integrals of the negative part. Of course, neither version is consistent with the velocity graph in figure 3.

### 3.3. Uniform acceleration

As indicated in figure 1 the behaviour during contact time requires additional information or approximations. What is known is the velocity at the beginning and end of the contact time, and that the integral of the acceleration during that time must lead to this change. A reasonable, albeit nonrealistic, approximation is to use constant acceleration during the contact time, as done in the set of graphs in figure 4, where elevation, velocity and acceleration graphs are consistent. A version of this graph drawn by many students is to simply change the sign of the acceleration for the contact time (giving $+|g|$), which, however, leads to a broken relation between the integrals of positive and negative acceleration.

The velocity graph in figure 4 looks very similar to the graphs when Hooke’s law is used to describe the force during the contact time [1], which results in the graphs in figure 5. The main difference is that the acceleration is described by a continuous function (even if the derivative is discontinuous [2]), and that the maximum acceleration is larger.

### 4. From free-text responses to multiple-choice question

The trampoline assignment was converted to multiple-choice version used in the 2015 competition, as part of the written test used to select 24 students to take part in the finals. Typical velocity graph responses were drawn (figure 6) and students were asked to mark which one would best describe how velocity varied with time during the jumps. The competitors were asked to explain their choices, after writing down what forces act during the time in the air and in contact with the mat.

The graphs in figure 6 were based on the responses in the 2014 competition, as well as on discussions with first-year university students who were asked to draw elevation, velocity and acceleration for a ball thrown up into the air.

The graphs in figure 6 have also been used for initial group discussions during teacher workshops. The dotted ‘zero’ lines were not included in the graphs used for the competition, but were added after a few teachers in a workshop suggested that it would make the assignment easier.

Below, we summarize the responses to this question from the 24 students, who made it to the Swedish final. Two of the finalists made no choice of graph.

Graph A was chosen by seven of the students. All their motivations were along the line ‘The velocity has a minimum both at the highest and lowest point’ or ‘The velocity should be zero twice per bounce’.

Three students (all from lower secondary school) chose alternative C, noting e.g. that ‘the velocity first decreases and then increases equally fast, since the force of gravity is constant’.

Two students (both from upper secondary school) chose alternative D, noting the strong acceleration in contact with the mat. Their comments indicate that they discuss speed rather than velocity.
None of the finalists had chosen alternative E, which could be seen as a first approximation to the correct graph B. One of the four students choosing alternative F excluded the graph in E (and also C and D), due to the sharp changes, and rejected B for lack of symmetry. The motivations for graph F describe the motion, fast on the way up, stopping at the top, moving down faster and faster until you hit the mat—without noting that the graph does not correspond to their description of the velocity.

Five students chose the correct response, B, noting e.g. that ‘The acceleration is constant, except when you are in contact with the trampoline’, one of the students also adding a comment about the acceleration being large during the contact time. One student first chose D, due to the number of zero-velocity instances, and then crossed over that answer, adding comments ‘If it is velocity, rather than speed, B is correct’ and ‘The acceleration is not constant during the contact time, as it is shown in D’.

5. First-year university students

In October 2016, the trampoline problem was used as one of six problems for a five-hour written exam for the first-semester physics students at Lund university. The initial scaffolding questions

(i) What forces act on the trampolinist while she is in the air and when she is at the lowest point?

(ii) What time is required to reach the highest point after the feet have left the trampoline?

were followed by an instruction to draw approximative graphs over elevation, velocity and acceleration during two full jumps, with an empty version of figure 1 provided. In addition, students were asked to estimate the maximum force from the trampoline acting on her, using $m$ for her mass.

The initial scaffolding question about forces acting on the trampolinist seems to have steered most students away from the common incorrect assignment of zero acceleration at the highest point.

To get full score, the free fall parts of the diagrams had to be correct and the integrals over positive acceleration must compensate the longer free-fall periods of negative acceleration. Of the 93 students taking the exam, only 12 scored the maximum 5 points on the problem. No deduction was made for using the approximation of constant acceleration during contact time, but four students
discussed going beyond that approximation. Eight students had 1 point deducted, subtracting \(mg\) rather than adding \(mg\) to get the force (or forgetting \(mg\)), or using incorrect scales on the axes. Of the ten students receiving 3 points, several had drawn a positive acceleration of the same magnitude as the acceleration of gravity, drawn speed rather than velocity, or failed in other ways to use the relations between elevation, velocity and acceleration. Many of the students receiving 3 points or less had not drawn any graph, but only provided numerical answers.

6. Discussion

The results in this work confirm earlier research showing that students do not necessarily make the connection between kinematics concepts and the corresponding mathematical description. Even if students draw a reasonably correct velocity diagram, the large slope during the contact time is often not reflected in a large value for the acceleration, even for first-semester university students. With a too small value for the upward acceleration, the area under the graph is insufficient to compensate for the downward acceleration during the longer free fall part.

Difficulties in interpreting integrals have been found in other contexts. E.g. Nguyen and Rebello [19] found that ‘only a few students could recognize that the concept of area under the curve was applicable in physics problems’ and Planinic et al [14] found that students have much less difficulty with the concept of graph slope than for the concept of area under the graph.

Connecting a conceptual understanding with the mathematical description should be an important aspects of physics teaching. Indeed, first-year undergraduates who were taught about integrals with a focus on conceptual understanding have been found to score ‘significantly higher than the students in procedural-based environment on assessment that measures conceptual understanding as well as procedural skills’ [20]. Similar results were found by Kohl and Finkelstein [21]. That students are often able to solve difficult end-of-chapter problems without managing apparently simpler conceptual questions is well documented by physics education research (see e.g. [22]) and keeps being rediscovered by new generations of physics teachers.

Woolnough [23] suggested that senior secondary students operate in ‘three distinct contexts: the real world, the physics world, and the mathematical world, each with different characteristics and belief systems’ and found that students resisted applying their mathematical knowledge to physics. Many everyday situations are too complicated for simple mathematics or physics descriptions and often need so many approximations that the mathematics and physics may no longer seem relevant. However, the easy access to electronic data taking using students’ own smartphones [16] makes it easy to obtain graphs describing familiar events.

The analysis of trampoline jump offers the possibility to discuss many common problems concerning the understanding of position, velocity and acceleration. Students’ personal experiences of trampolining can be used to bridge mathematical modelling to standard physics textbook cases, as well as to authentic measurement data. We hope that the examples in this paper illustrate the rich opportunities offered by the trampoline assignment and would like to invite others use the problem and to collect and share data on responses for different student groups.

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