Most of the energy density of the universe, is accelerating (Perlmutter et al. 1999; Riess et al. 1998, 2001). The cosmological constant, "standard candles" suggest that the universe has a nonzero density of Type Ia supernovae (SNe Ia) up to a redshift as large as 1.5. This effect of nonzero $\Lambda$ on angular diameter is large, 40% at $z = 2$, so mapping angular diameter distances versus redshift will give $\Lambda$ with (relative) ease. In principle, these measurements could be made in the UV, optical, near-infrared, or even X-ray bands. Distance measurements using the brightness of astronomical objects, such as radio-quiet quasars that can measure the cosmological constant, $\Lambda$, with minimal assumptions. The largest scale use of astronomical distances is the measurement of the geometry of the universe. Distance measurements using the brightness of Type Ia supernovae (SNe Ia) up to a redshift $z \sim 1.5$ as "standard candles" suggest that the universe has a nonzero cosmological constant, $\Lambda$, so that the expansion of the universe is accelerating (Perlmutter et al. 1999; Riess et al. 1998, 2001). Most of the energy density of the universe, $\Omega$, would then lie in this repulsive "dark energy," $\Omega_\Lambda \sim 0.7$, rather than in "normal dark matter," $\Omega_m \sim 0.3$. This measurement may be key to new physics and new cosmology (Carroll 2001). However, like most methods for finding astronomical distances, the SN Ia method has inherent uncertainties due to model-dependent assumptions, the small size of the effect ($\sim 10\%$), and difficulties of measurement (Rowan-Robinson 2002; Wright 2002; Branch et al. 2001; Richardson et al. 2002; Linder 2001; Goobar, Bergström, & Mörtsell 2002). The need for a second, independent, means of determining $\Lambda$ is clear and urgent.

The most direct and model-independent method for finding astronomical distances involves simple geometry. This method, parallax (Bessel & Rath 1839), uses the motion of the Earth around the Sun to measure the angular displacement of a nearby star, against an effectively unmoving distant background of stars. By this means, we use a known length, in this case the Earth’s orbital radius (1 AU), and a known angle to solve the isosceles triangle created by these two quantities and so solve for the distance to the star (Fig. 1a). Here we propose an equivalent geometric method to determine distances to quasars (Fig. 1b). In this case, the triangle is inverted, with the known length being at the distant quasar, instead of at the Earth. The “standard length” we propose to employ is the size of the quasar broad emission line region (BELR; Peterson 1997), which is known from light-travel time measurements. The angle in the triangle ($\theta$, Fig. 1b) is subtended by the same BELR, as measured with an interferometer. By measuring both quantities to a series of quasars at different redshifts, the “angular diameter distance,” $D_c$, can be mapped out against redshift, $z$. The resulting angular diameter–redshift relation is a basic characteristic of the spacetime metric of the universe. From this relation, we can determine $\Lambda$. An equivalent method was used to measure the distance to SN 1987a (Panagia et al. 1991; Binney & Merrifield 1998) to 6%. Because this method uses a standard length rather than a standard candle, it is less dependent on physical models and on changes in the fundamental constants, other than $c$, the speed of light (c is needed to measure the standard length at the quasar).

Quasars are highly luminous objects and are readily studied in detail up to the highest redshifts (currently $z > 6$; Fan et al. 2001). There have been previous proposals to use quasars as distance indicators (Baldwin 1977; Rudge & Raine 1999; Collier et al. 1999; Homan & Wardle 2000), but these methods are, to a greater or lesser degree, model dependent. A defining characteristic of quasars is a series of strong broad emission lines (BELs) from permitted transitions, whose breadths indicate Doppler velocities of $(1\text{--}5\%)c$ (3000–15,000 km s$^{-1}$). These broad emission lines (BELs) arise from gas that is photoionized by a highly luminous source of radiation with a spectrum that is broader than any single blackbody, thus producing a wide range of ionization in the BELR. Even in radio-quiet quasars, where the observed continuum is not dominated by relativistically beamed jet emission, this continuum is highly variable on short timescales ($\sim 1$ day), and so the source of the continuum is small, $\sim 100$ AU (1.5 $\times$ 10$^5$ cm) across (Peterson 1997). The response of the BELs to continuum changes is delayed by the time light takes to travel from the continuum source to the BELR. This light-travel time gives the size of the BELR (Blandford & McKee 1982; Peterson 1993, 2001; Netzer & Peterson 1997). This technique is called “reverberation mapping.” A typical BELR radius in nearby active galaxies (for $C_\IV \lambda 1550$; Netzer & Peterson 1997), is 10 lt-days (2.5 $\times$ 10$^{16}$ cm; Wandel, Peterson, & Malkan 1999; Kaspi et al. 2000).

In the top panel of Figure 2, we show the angular diameter versus redshift, $D_c$ versus $z$, relation for a $\Lambda = 0$, $\Omega = 1$ cosmology and a $\Lambda = 0.7$, $\Omega_m = 0.3$ cosmology, for a nominal size of 10 lt-days. The turnaround in angular diameter at $z \sim 1--2$ is present in both cosmologies (and is a feature of all $\Omega = 1$ spacetimes) but is notably weaker in the positive $\Lambda$ model. The crucial value for determining $\Lambda$ is the ratio of $D_c$ in the two cosmologies. This ratio is shown in the bottom panel of Figure 2. The deviation between the two $D_c(z)$ relations is a relatively large effect, reaching 40% by $z = 2$ and then continuing to grow slowly to 55% at $z = 6$.

To resolve a radius of 10 lt-days at $z > 2$ requires 0.001 mas resolution (Fig. 2). This appears to imply a long baseline, $d$,
of 100 km at a wavelength $\lambda = 5000 \AA$, using the Rayleigh criterion ($\theta = 1.22\lambda/d$). This is currently a demanding requirement, although by no means an unachievable one (Labeyrie 1999). Fortunately, quasar BELRs at $z = 2$ are significantly larger than those nearby. This is because the radius of the BELR, $r_{\text{BELR}}$, increases with quasar luminosities, $L$, as $L^{0.5}$ (Wandel et al. 1999), and $L$ evolves with redshift as $(1+z)^3$ (Boyle et al. 1988, 2000), so $r_{\text{BELR}} \propto (1+z)^{3/2}$. Hence, at $z = 3$ a typical quasar (i.e., one at the break luminosity, $L^*$, in the optical luminosity function) has a BELR 8 times larger than one at $z = 0$, with typical diameters at $z = 2$–3 of $\sim 0.01$ mas. Although this is still a demanding resolution, quasar evolution gives an order of magnitude reduction in the baseline of the interferometers required, from 5 to 1 km at 5000 Å.

At low redshifts ($z \sim 0.2$), the BELR angular diameters are some 10 times larger (0.05–0.15 mas; Fig. 3). Nearby active galaxies would not give a measurement of $\Lambda$ but would determine $H_0$, independent of all other “distance ladder” methods (e.g., Freedman et al. 2001). Diameters of $\sim 0.1$ mas can be measured using an $\sim 200$ m baseline in the UV (at C IV $\lambda 1549 \times 1.2$), or $\sim 500$ m in the optical (at H$\beta$ $\lambda 4861 \times 1.2$).

The choice of strong BELs to measure is reasonably limited (Osterbrock 1989): Ly$\alpha$ (at 1215 Å in the rest frame), Mg II ($\lambda 2800$ Å), C IV ($\lambda 1549$ Å), H$\beta$ ($\lambda 4861$ Å), and He II ($\lambda 4686$ Å). The hydrogen lines and C IV vary quickly, while Mg II varies more slowly, in response to continuum changes. Weaker lines are also seen (e.g., O IV [1031 Å], He II [1640 Å, 10830 Å], H Pa$\alpha$ [12818 Å], H Pa$\beta$ [18751 Å]) and would be candidates under special constraints of redshift and observing band. If the strong narrow X-ray Fe K emission line in active galaxy spectra at 6.4 keV comes from a BELR-sized region or larger, as seems increasingly likely (Chiang et al. 2000; Elvis 2000; Takahashi, Inoue, & Dotani 2002), then X-rays too would become attractive for measuring $\Lambda$.

Despite this limited choice of emission lines, the optimum wavelength band of the interferometer to use is not obvious. The observed wavelength of any line increases as $(1+z)$, so that the baseline of an interferometer that can resolve the BELR increases with $z$ for a fixed physical size. So in general shorter wavelength emission lines are preferred. However, when these lie shortward of 3000 Å, in the UV or X-ray bands, they can be observed only from space. Several long-baseline space-based interferometers are currently in the planning stages. These include interferometry projects under development such as the Stellar Imager (Carpenter et al. 2002), with a 0.1 mas resolution at UV wavelengths, and long-term concepts such as the Terrestrial Planet Imager with 6000 km baselines. In the X-ray domain, baselines $\sim 1000$ times shorter than in the optical can be used: $\sim 10$ m for $\sim 0.01$ mas resolution at 2 keV (i.e., the energy of the Fe K line at $z = 2$–3). A diffraction-limited version of the Chandra X-Ray Observatory could directly resolve $\sim 20$ mas. Building such a telescope may be feasible (L. P. Van Speybroeck 2002, private communication). However, to measure $H_0$ and $\Lambda$ would require X-ray interferometry, which is under development (Cash et al. 2000).

From the ground, interferometry is easier at longer wavelengths because the atmospheric patches that limit angular resolution are larger there. The near-infrared (1–2 $\mu$m; the $JHK$ bands) is especially promising but requires 10 times longer baselines than in the UV. However, several ground-based optical and near-infrared interferometers employing large-aperture
telescopes (e.g., 8 m) on baselines of several hundred meters are currently operating or being built (the Very Large Telescope Interferometer [VLTI], the Center for High Angular Resolution Astronomy interferometer, and the Optical Hawaiian Array for Nanoradian Astronomy [OHANA]). The telescope sizes needed for interferometry may be large, since interferometers require narrow bandwidths and these narrow bands may sample only a fraction of the emission-line profile (e.g., VLTI: Petrov et al. 2000; OHANA: Perrin et al. 2001). Finding an optimal practical choice of BEL, redshift interval, and observing technique is a complex problem.

Using high signal-to-noise ratio visibility measurements and a model brightness distribution, the limiting resolution of a telescope can significantly exceed the Rayleigh criterion. For example, Karovska et al. (1989) measured diameters of several stars and of SN 1987A with the Cerro Tololo Inter-American Observatory 4 m telescope and obtained angular diameters of 5–10 mas, with uncertainty of ±1 mas, i.e., as much as 5 times smaller than the Rayleigh diffraction limit (~30 mas at optical wavelengths). Similarly, the Hubble Space Telescope (HST) fine guidance sensors have been used to measure the sizes of objects to scales of 8 mas, well below the ~50 mas optical diffraction limit of HST (e.g., Hook, Schreier, & Miley 2000).

Accurate measurements of the reverberation times of fairly high redshift quasars are required to carry out this distance determination project. Since the size of the BELR depends systematically on the line measured, with lower ionization lines coming from larger regions, the lag time and angular size measurements must be carried out using the same emission line. Moreover, different velocities within a line likely come from different locations, so the same velocity slice of the emission lines must be observed by both techniques.

Radio-quiet quasars at $z \sim 2–3$ seem to be sufficiently variable. Optical continuum variations of at least 0.5 mag are seen in at least 40% of bright ($m < 18$) radio-quiet quasars (Netzer & Sheffer 1983; Maoz et al. 1994), although Kaspi (2002) finds little variability in another sample. Quasars at $z \sim 2–3$ are also sufficiently bright. Most of the difference between $\Lambda = 0$ and $\Lambda = 0.7$ cosmologies has been reached by $z = 2–3$ and, as these redshifts span the peak of quasar evolution, there are many optically bright ($B = 17–18$ mag) objects to observe at these redshifts (Wisotzki 2000). The timescale of BEL variations increases with redshift as $(1+z)^\alpha$, since not only does the BELR size increase but also the intrinsic variation timescale is dilated by $(1+z)$. As a result, even for the fastest varying lines, e.g., C iv, to observe for 3–5 years the typical reverberation time for quasars at $z = 3$ requires observing programs spanning 3–5 yr. While this lengthens the total program, the time span is not prohibitive, and the frequency of observations is less demanding than in the smaller, more rapidly varying, active galaxies so far measured by reverberation mapping (Peterson 2001).

To measure $\Lambda$ well requires a final accuracy of order 10% (Fig. 2). The error on measured BELR radii is currently quite large, even up to factors of 2 for the moderately large redshift quasars in Kaspi et al. (2000). However, this is not intrinsic to the method, and the smallest error derived so far is 8% (Peterson et al. 1998). The lag time error is a function of observation density, spectral signal-to-noise ratio, and calibration accuracy, and of catching a quasar in a strong outburst. For example, the 1993 HST campaign on NGC 5548 found only weak (75%) continuum variations, but had the campaign lasted twice as long (~80 days) a factor of 2.5 change would have been found, leading to much improved reverberation lag measurements. A factor of 4.5 variation was recorded in an earlier, less intensive, campaign (Clavel et al. 1991) on the same object. Dedicated observing facilities may be required to measure $\Lambda$ by this method.

To achieve 5% accuracy in lag times, and so leave room for an 8% error in measuring $\theta$, requires sampling at 0.02–0.1 the BELR light crossing time, $r_{BEL}/c$ (Peterson et al. 2002; Edelson et al. 2001). Several lag times must be sampled to give an unambiguous result (Horne et al. 2002), giving a nominal minimum of order 200 observations per quasar, i.e., of order once per week at $z = 2–3$. These estimates are probably somewhat optimistic, and a well-designed program would exceed them by a factor of a few. Large telescopes are not needed to measure high-redshift quasar reverberation times. At $z = 2$, the 9.9 times longer lag times help to compensate for the 100 times decreased flux compared with $z = 0.2$. A 1.5 m telescope is enough to achieve a signal-to-noise ratio of 100 in the C iv line at this sampling rate, even allowing for the slight weakening of the high-ionization emission lines with luminosity (Baldwin 1977). (This estimate applies to space-based instruments where sources of noise other than counting statistics can be rendered unimportant.) Velocity-resolved spectra, to match interferometric bands, will require larger telescopes.

Fortunately, observations indicate that a single timescale does dominate for each emission line (Netzer & Peterson 1997). Even so, there is an inherent problem in relating reverberation lag times as currently measured to the angular sizes that an interferometer would measure. Even perfectly sampled data will produce a broadened cross-correlation function (CCF), and it is not obvious which delay time, e.g., the CCF peak or the CCF centroid, corresponds to the angular size from an image. Moreover, reverberation mapping measures a “responsivity-weighted” size of the light delay surface, while interferometry measures an “emission-weighted” size. That these are different is shown by the fact that the amplitude of variation of the emission lines is much less than that of the continuum. Quite often, factor of 2 changes in the UV continuum produce only 20% changes in the emission lines (e.g., Fig. 31 in Peterson 2001). The high-ionization and high-order lines vary most strongly, but they also tend to be weak (Ulrich et al. 1991).

Not only does this weak response require larger telescopes but also the gas being studied is not the same in the two types of measurement.

We suggest that this difficulty can be removed by observing the target quasar with an interferometer in both low- and high-continuum states, with appropriate lags included, so that the difference map will be a measure only of the “responsive fraction” of the line, i.e., that part that responds to continuum changes. The two methods would then be measuring the same thing. This approach would require monitoring a number of quasars for continuum changes and then using them to trigger “target of opportunity” interferometer measurements.

A more ambitious approach would be to monitor quasars with a true imaging interferometer. We could then watch continuum changes propagating outward and creating emission-line responses as a function of wavelength. Since the geometry of quasar BELRs is not known, this “imaging reverberation mapping” method would go a long way to removing the ambiguities in the method. Possibilities for the BELR geometry range from a simple orderly wind (Elvis 2000) to a maximally chaotic distribution of clouds (Baldwin et al. 1995). The BELR has six dimensions of structure: three of position plus three of...
velocity. Imaging of the BELR reverberations in several velocity bands while the continuum varies would provide four observed dimensions (right ascension, declination, velocity, and lag time) directly. If there is an axis of symmetry, then the BELR will be underdetermined only by one dimension. The angular diameter distances derived would then have minimal room for uncertainty. Currently, reverberation mapping measures only one-plus dimensions (lag time and minimal velocity information), so four dimensions would be a great advance.

However, even this method remains one dimension short of being fully determined. Long-term (years) variations of the BELR may provide the missing sixth dimension. BEL profile changes occur in some objects (e.g., Ark 120; Kollatschny et al. 1981). Moreover, the size of the BELR for particular emission lines seems to respond to changes in the central continuum. But changes occur in some objects (e.g., Ark 120; Kollatschny et al. 1981). Moreover, the size of the BELR for particular emission lines seems to respond to changes in the central continuum. But changes occur in some objects (e.g., Ark 120; Kollatschny et al. 1981). Moreover, the size of the BELR can be determined only by one dimension. The angular diameter distances derived would then have minimal room for uncertainty. Currently, reverberation mapping measures only one-plus dimensions (lag time and minimal velocity information), so four dimensions would be a great advance.

Although interferometry is presently not ready to make a measurement of $\Lambda$ using quasar parallax, or perform true imaging of quasar BELRs, ground-based interferometry has reached a stage in which initial determinations of diameters of the BELRs of nearby active galactic nuclei (AGNs) is feasible. If the geometry of a sample of nearby AGNs can be explored with these long-baseline interferometers, then they will lay the foundations for a long-term program to determine cosmological parameters by this method. In the near future, sub-milliarcsecond resolution will be achieved with the long-baseline (200–800 m) interferometers VLTI and OHANA in the near-infrared, where the Paschen lines are available at low $z$. In the following few years, imaging should become possible in the optical as well (e.g., at $\Lambda\alpha$). Imaging of nearby AGN BELRs in the lines used in reverberation measurements will determine their spectral/spatial morphology and allow the technique to be refined.

This will provide a crucial base for parallax measurements of high-redshift quasars to determine $\Lambda$, when the next generation of long-baseline interferometers with <0.01 mas resolution becomes available.

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