Low-lying $2^+$ states generated by $pn$-quadrupole correlation and $N=28$ shell quenching

Shuichiro Ebata$^1$ and Masaaki Kimura$^2$

$^1$Meme Media Laboratory, Hokkaido University, Sapporo, 060-0813, Japan
$^2$Department of Physics, Hokkaido University, Sapporo, 060-0810, Japan

(Dated: March 18, 2014)

The quadrupole vibrational modes of neutron-rich $N=28$ isotones ($^{48}$Ca, $^{46}$Ar, $^{44}$S and $^{42}$Si) are investigated using the canonical-basis time-dependent Hartree-Fock-Bogoliubov theory including nuclear pairing correlation. It is found that the quenching of $N=28$ shell gap and the proton holes in the sd-shell trigger strong quadrupole correlation and generate the low-lying $2^+$ states in $^{46}$Ar. The pairing correlation further increases the collectivity of the $2^+_1$ state and the observed $B(E2)$ is reproduced well. We also demonstrate that the same mechanism to enhance the low-lying collectivity applies to other $N=28$ isotones $^{44}$S and $^{42}$Si and it generates a couple of low-lying $2^+$ states which may be associated with the observed $2^+$ states. However the evaluated excitation energies of $2^+_1$ states of $^{44}$S and $^{42}$Si and $B(E2)$ of $^{44}$S disagree with the observed values, implying the onset of deformation in $^{44}$S and $^{42}$Si.

As represented by the pygmy dipole mode, there are novel excitation modes peculiar to the unstable nuclei. In particular, the low-lying collective excitation modes are quite sensitive to the underlying shell structure and the pairing correlations, and hence, the quenched magic shell gaps of unstable nuclei should generate unique collective modes.

One of the interesting examples is the $N=28$ shell gap that is known to be quenched in the vicinity of $^{44}$S, and has been paid considerable experimental[11-21] and theoretical attention[11-21]. Since the orbital angular momentum of $f_{7/2}$ and $p_{3/2}$ differ by two, the quench of $N=28$ shell gap will lead to the strong quadrupole correlation in the low-lying state. Furthermore, the protons in Si, S and Ar isotopes occupy the middle of the sd-shell, and hence, the strong quadrupole correlation should also exist in the proton side. Therefore, once the $N=28$ shell gap is quenched, the strong quadrupole correlations amongst protons and neutrons will be ignited and lead to a novel variety of the excitation modes. Indeed, various exotic phenomena such as the shape transition in Si and S isotopes and the coexistence of various deformed states are theoretically suggested[19-21].

In this rapid communication, we report the low-lying quadrupole excitation modes in $^{46}$Ar, $^{44}$S and $^{42}$Si generated by the strong quadrupole correlation between protons and neutrons. To access the low-lying quadrupole modes of these isotopes, we apply the Canonical-basis time-dependent Hartree-Fock-Bogoliubov (Cb-TDHFB) theory[22] which has been successfully applied to the study of the dipole[22,23] and quadrupole[24] resonances of isotopes. The Cb-TDHFB can describe self-consistently the dynamical effects of pairing correlation which has a significant role to generate the low-lying quadrupole strength as reported in Ref. [22-27]. By assuming the diagonal form of pairing functional, the Cb-TDHFB equations are derived from the full TDHF equation represented in the canonical basis $\{\phi_I(t), \phi_I(t)\}$ which diagonalize a density matrix. The Cb-TDHFB equations describe the time-evolution of the canonical pair $\{\phi_i(t), \phi_I(t)\}$, its occupation probability $\rho_I(t)$ and pair probability $\kappa_I(t)$,

$$\frac{\partial}{\partial t} \phi_I(t) = \left(h(t) - \eta_I(t)\right) \phi_I(t),$$

$$\frac{d}{dt} \rho_I(t) = \kappa_I(t) \Delta_I^*(t) - \kappa_I^*(t) \Delta_I(t),$$

$$\frac{d}{dt} \kappa_I(t) = \left(\eta_I(t) + \eta_I(t)\right) \kappa_I(t) + \Delta_I(t) \left(2 \rho_I(t) - 1\right),$$

where $\eta_I(t) \equiv \langle \phi_I(t)|h(t)|\phi_I(t)\rangle$, and the $h(t)$ and $\Delta_I(t)$ are the single-particle Hamiltonian and the gap energy, respectively. We apply the Skyrme interaction with SkM$^*$ parameter set to $ph$-channel and the simple pairing form $\Delta_I(t) \equiv \sum K \delta_{K,K}(t)$, where $G_{K\ell}$ is constant in real-time evolution, to $pp(hh)$-channel as same as Ref. [22]. In accordance with Ref. [29,30], we apply the absorbing boundary condition to eliminate unphysical modes. The canonical basis $\phi_I(\vec{r}, \sigma; t) = \langle \vec{r}, \sigma|\phi_I(t)\rangle$ is represented in three dimensional Cartesian coordinate discretized in a square mesh of 1.0 fm in a sphere with radius of 18 fm including 6 fm for the absorbing potential whose terminator depth is $-3.75$ MeV.

In order to induce quadrupole responses, we add a weak instantaneous external field $V_{ext}(\vec{r}, t) = \vec{F}_K(\vec{r}) \delta(t)$ to initial states of the time evolution. Here the quadrupole external field acting on proton, neutron, isoscalar and isovector channels are given as $\vec{F}_K(\vec{r}) \equiv (\vec{F}_{K,1} \text{ or } \vec{F}_{K,2}) \otimes (i \vec{r} Y_{2K} + i \vec{r} Y_{2-K})/\sqrt{2(1+\delta_{K,0})}$. The amplitude of the external field is so chosen to be a small number $\eta = 1 \sim 3 \times 10^{-3}$ fm$^{-2}$ to guarantee the linearity. The strength function $S(E; \vec{F}_K)$ in each channel is obtained through the Fourier transformation of the time dependent expectation value $F_K(t) \equiv \langle \Psi(t)|\vec{F}_K(t)|\Psi(t)\rangle$:

$$S(E; \vec{F}_K) \equiv \sum_n \frac{\mid\langle \Psi_n|\vec{F}_K|\Psi_0\rangle^2 \delta(E_n - E)}{\pi \eta}$$

$$= -\frac{1}{\pi \eta} \text{Im} \int_0^\infty \left\{ F_K(t) - F_K(0) \right\} e^{i(E+\Gamma/2)t} dt,$$

where $|\Psi_0\rangle$, $|\Psi_n\rangle$ and $|\Psi(t)\rangle$ are the ground and excited states and a time-dependent many body wave function represented in the canonical form, respectively. $\Gamma$ is a
smoothing parameter set to 1 MeV. We also performed unperturbed calculations in which \( h(t) \) in Eq. (1) is replaced with the static single particle hamiltonian \( h(t = 0) \) computed using the ground state density. By comparing the results obtained by the fully-selfconsistent and unperturbed calculations, we investigate the effects of the residual interaction and the collectivity of the excited states.

First, we examine the quadrupole responses of double-closed-shell nucleus \(^{48}\text{Ca}\) which is spherical and has no superfluid phase. The strength functions in the IS and IV channels shown in Fig. 1 (a) have two peaks at 3.40 and 9.12 MeV in addition to the IS giant quadrupole resonance (GQR) at 17 MeV and the IVGQR having broad distribution around 30 MeV. The properties of these two peaks below 10 MeV become clear by comparing the results in the proton and neutron channels (Fig. 1 (b) and (c)). Their strengths in the neutron channel are much larger than those in the proton channel showing the dominance of neutron excitation and they are understood as the neutron single-particle excitations of \( N=28 \) shell gap. Indeed, they correspond to the 3.86 and 9.06 MeV peaks in the unperturbed results (dashed line) that are the neutron single-particle excitations of \( f_{7/2} \rightarrow p_{3/2} \) and \( f_{5/2} \), respectively (see Tab. I). On the other hand, in the proton channel, there is no peak below 10 MeV in the unperturbed result, since the proton excitation with \( \Delta l = 2 \) costs much larger energy due to the \( Z = 20 \) shell closure. Thus we can conclude that these low-lying \( 2^+ \) states have almost no collectivity but have single-particle nature as the result of the double-closed shells \( Z = 20 \) and \( N=28 \).

\(^{48}\text{Ar}\) has different nature of the ground and low-lying \( 2^+ \) states. The ground state of \(^{48}\text{Ar}\) is also spherical but superfluid phases appear in both of proton (\( \Delta p = 1.16 \) MeV) and neutron (\( \Delta n = 1.77 \) MeV). And if the proton pairing is switched off the ground state is oblately deformed (\( \beta = -0.15 \)). These results imply the weaker neutron-magicity in \(^{48}\text{Ar}\) than in \(^{48}\text{Ca}\). Actually, the calculated \( N=28 \) shell gap is slightly reduced in \(^{48}\text{Ar}\) (3.5 MeV) compared to that in \(^{48}\text{Ca}\) (3.86 MeV). The unperturbed strengths in the proton and neutron channels in the Fig. 1 (e) and (f) are quite similar to those of \(^{48}\text{Ca}\) except for minor differences; (1) the reduction of the peak energies below 10 MeV in the neutron channel due to the quenching of the \( N=28 \) shell gap, and (2) the very weak strength distributed below 10 MeV in the proton channel that are generated by the proton hole-states in \( sd\)-shell and fluctuated by the pairing correlation. These differences generate a novel type of low-lying collective mode when the residual interaction is switched on. In the fully self-consistent results, very strong peaks emerge around 1 MeV in all channels as shown in Fig. 1 (d)-(f). Note that \(^{48}\text{Ar}\) has a neutron magic number \( N=28 \) and the shape is spherical; nevertheless, the very strong collectivity appears at very low energy and their strengths are comparable with GQR.

To analyze the peak structure, we fit the strength function \( S(E; \hat{T}_K) \) below 25 MeV by the sum of Lorentzian \( f_k(E; \hat{T}_K) \):

\[
S(E; \hat{T}_K) \simeq \sum_k f_k(E; \hat{T}_K) = \sum_k \frac{a_k (\Gamma/2)^2}{(E - E(2_k^+))^2 + (\Gamma/2)^2} \tag{2}
\]

where \( k \) is a label of \( 2^+ \) state and \( \Gamma = 1 \) MeV corresponding to smoothing parameter in Eq. (1). The reduced transition probabilities in the proton and neutron channels, that are denoted as \( B_k(E2 \uparrow) \) and \( B_k(N2 \uparrow) \) in the following, are evaluated by integrating \( f_k(E; \hat{T}_K) \) for each state,

\[
B_k(E2 \uparrow \text{ or } N2 \uparrow) \equiv \sum_K |\langle 2_k^+ | \hat{Q}_{2K} \frac{1 \pm \tau_z}{2} | 0_1^+ \rangle|^2 \\
\simeq 5 \int_0^\infty f_k(E; \hat{T}_K=0) \, dE. \tag{3}
\]

This evaluation is reasonable for spherical nuclei such as \(^{48}\text{Ca}\) and \(^{48}\text{Ar}\), but gets worse for deformed nuclei \(^{44}\text{Si}\) and \(^{42}\text{Si}\). The results of fitting, evaluated \( B(E2 \uparrow) \) and \( B(N2 \uparrow) \) are shown in Tab. III and Fig. 2. As already discussed, \(^{48}\text{Ca}\) has two peaks at 3.40 and 9.12 MeV corresponding to the neutron single-particle excitations to \( p_{3/2} \) and \( f_{5/2} \), and it is found that their \( B(E2 \uparrow) \) are 60 and 27 \( e^2 f m^4 \), respectively. Experimentally, the \( 2^+ \) state locates at 3.83 MeV and \( B(E2 \uparrow)=56 \pm 32 \) \( e^2 f m^4 \) \(^{31}\) that are in good agreement with the calculation. In the case of \(^{48}\text{Ar}\), the states corresponding to the same single-particle excitations also exist at 3.30 and 9.04 MeV, but have much smaller \( B(E2 \uparrow) \) and \( B(N2 \uparrow) \) values than \(^{48}\text{Ca}\). This missing strength is exhausted by the low-lying peaks which do not exist in \(^{48}\text{Ca}\). They consist of the \( 2^+_1 \) state at 0.78 MeV and the \( 2^+_2 \) state at 1.78 MeV. In particular, the \( 2^+_2 \) state has pronounced collectivity in both of proton and neutron channels, i.e. it has small excitation energy and large values of \( B(E2 \uparrow) = 179 \) \( e^2 f m^4 \) and \( B(N2 \uparrow) = 562 \) \( e^2 f m^4 \). The pairing correlation reinforces the collectivity of the \( 2^+_1 \) state. By comparing the results with and without the pairing in the neutron channel, it turns out that the pairing correlation enhances both of \( B(E2 \uparrow) \) and \( B(N2 \uparrow) \) \(^{25}\) \(^{26}\) by about 20%. These calculated values nicely explain the observed values of \( E(2^+_1) = 1.58 \) MeV and \( B(E2 \uparrow) = 196 \pm 39 \) \( e^2 f m^4 \) \(^{5}\) and indicate collective nature of the \( 2^+_1 \) state of \(^{48}\text{Ar}\). On the other hand, the \( 2^+_2 \) state has much smaller values of \( B(E2 \uparrow) \) and \( B(N2 \uparrow) \) that are comparable with the single-particle unit. Thus, we demonstrated that two very low-lying \( 2^+ \) states appear in \(^{46}\text{Ar}\) and one of them has enhanced collectivity, while the other do not.

To understand the mechanism of the low-lying \( 2^+ \) states of \(^{46}\text{Ar}\), the difference between the results of fully-selfconsistent and unperturbed calculations should be noted. The unperturbed result merely has very weak low-lying strength originates in the proton transitions within the \( sd\)-shell. Nevertheless the fully self-consistent result
FIG. 1. (Color online) Strength functions of quadrupole vibrational modes of $^{48}$Ca and $^{46}$Ar. The strength functions in the isoscalar (solid line) and isovector (dotted line) channels are shown in the panels (a) and (d), while those in the proton and neutron channels are shown in (b), (e) and (c), (f). The solid and dashed lines in the panels (b), (c), (e) and (f) compare the fully self-consistent and unperturbed results. The filled arrows show the experimental one- and two-neutron separation energies $^{32}$, while the open arrows show the calculated two-neutron separation energies.

TABLE I. Properties of the single-particle levels of $^{48}$Ca and $^{46}$Ar around the chemical potential. $J$, $\varepsilon_{sp}$, and $\rho_l$ denote their angular momentums, energies and occupation probabilities, respectively. The single-particle states more than 7.5 MeV above the chemical potential are not shown due to the cutoff in the pairing channel $^{22}$.

|       | $^{48}$Ca          | $^{46}$Ar          |
|-------|--------------------|--------------------|
|       | proton             | neutron            | proton | neutron |
| $f_{5/2}$ | $-1.36$, 0.00     | $-3.90$, 0.05     | $-5.28$, 0.21 |
| $p_{1/2}$ | $-4.43$, 0.00     | $-4.43$, 0.00     | $-8.08$, 0.21 |
| $p_{3/2}$ | $-6.55$, 0.00     | $-6.55$, 0.00     | $-7.87$, 0.00 |
| $f_{7/2}$ | $-9.93$, 0.00     | $-10.41$, 1.00    | $-10.43$, 0.01 |
| $d_{3/2}$ | $-14.69$, 1.00    | $-14.92$, 1.00    | $-12.86$, 0.98 |
| $s_{1/2}$ | $-15.65$, 1.00    | $-15.67$, 0.90    | $-15.61$, 0.99 |
| $d_{5/2}$ | $-21.16$, 1.00    | $-21.39$, 1.00    | $-19.81$, 0.99 |

has strong collectivity below 1 MeV in both of the proton and neutron channels. Obviously, it indicates the importance of residual interaction, especially the quadrupole residual interaction amongst proton and neutron. The mechanism of the enhancement of collectivity may be understood as follows. Different from $^{48}$Ca, $^{46}$Ar has low-lying proton excitations owing to the proton holes in $sd$-shell that are the "seed" of the low-lying collectivity. Even though their strengths are rather weak, they couple to the neutron excitations across $N=28$ shell gap through the residual quadrupole interaction between proton and neutron, and give rise to the low-lying collectivity. The relatively reduced $N=28$ shell gap and pairing correlation further increase low-lying collectivity. This also explains the absence of such collective mode in $^{48}$Ca.

The same mechanism also applies to other $N=28$ isotones $^{44}$S and $^{42}$Si which are experimentally known to have deformed shape and low-lying $2^+$ states $^{[6, 8, 9, 33]}$. In the present calculation, $^{44}$S has obscure prolate shape ($\beta = 0.08$) and the superfluid phase appear in both proton ($\Delta_p = 0.61$ MeV) and neutron ($\Delta_n = 2.00$ MeV), while $^{42}$Si has oblate shape ($\beta = -0.19$) and only neutrons are in the superfluid phase ($\Delta_n = 1.56$ MeV). It is noted that

FIG. 2. (a) $B$(E2) and (b) $B$(N2) calculated for $^{48}$Ca (circles), $^{46}$Ar with neutron-pairing (filled squares) and without neutron-pairing (open square). The lines are drawn to guide the eye. The experimental data are taken from Ref. $^{5, 31}$. 

Further increase low-lying collectivity. This also explains the absence of such collective mode in $^{48}$Ca.

The same mechanism also applies to other $N=28$ isotones $^{44}$S and $^{42}$Si which are experimentally known to have deformed shape and low-lying $2^+$ states $^{[6, 8, 9, 33]}$. In the present calculation, $^{44}$S has obscure prolate shape ($\beta = 0.08$) and the superfluid phase appear in both proton ($\Delta_p = 0.61$ MeV) and neutron ($\Delta_n = 2.00$ MeV), while $^{42}$Si has oblate shape ($\beta = -0.19$) and only neutrons are in the superfluid phase ($\Delta_n = 1.56$ MeV). It is noted that
TABLE II. Peak position \(E(2^+_J; F_{K=0})\) [MeV] and height \(a_k^J\) [\(e^2f_m^4/\text{MeV}\)] of Lorentzian obtained by fitting the strength functions below 25 MeV. The evaluated transition strengths \(B_k(E2^+)\) and \(B_n(N2^+)\) [\(e^2f_m^4\)] are also shown for \(^{48}\text{Ca}, \ ^{46}\text{Ar}\) with and without neutron-pairing.

|       | \(E(2^+_J)\) | \(a_k^J\) | \(a_k^n\) | \(B_k(E2^+)\) | \(B_n(N2^+)\) |
|-------|---------------|-----------|-----------|---------------|---------------|
| \(^{48}\text{Ca}\) | 3.40 | 8.04 | 59.35 | 60 | 443 |
|       | 9.12 | 3.52 | 15.15 | 27 | 305 |
|       | 17.34 | 39.33 | 82.11 | 116 | 637 |
| \(^{46}\text{Ar}\) | 0.76 | 28.10 | 88.32 | 179 | 562 |
|       | 1.78 | 0.89 | 16.91 | 6 | 120 |
|       | 3.30 | 1.76 | 22.41 | 13 | 167 |
|       | 9.04 | 1.32 | 15.23 | 10 | 117 |
|       | 17.48 | 30.03 | 92.20 | 233 | 716 |
| \(^{46}\text{Ar}_{w/o}\) | 0.66 | 21.88 | 68.31 | 135 | 423 |
|       | 1.72 | 3.87 | 29.94 | 27 | 213 |
|       | 3.18 | 1.89 | 26.62 | 14 | 198 |
|       | 9.10 | 1.53 | 13.34 | 11 | 102 |
|       | 17.58 | 29.00 | 94.51 | 225 | 734 |

The pattern of the deformed shape of the \(N=28\) isotope is consistent with other theoretical results [18, 21, 35, 36]. Figure 3 shows its strength functions in the proton and neutron channels. Deformation of these nuclei splits the strength functions in the \(K=0\) and \(2\) modes, and makes their strength distributions more complicated than those in \(^{46}\text{Ar}\). However, we can still identify very low-lying peaks which correspond to a couple of \(2^+\) states with enhanced collectivity. It is noted that, for example, three \(2^+\) states are observed in \(^{44}\text{S}\) [34] and our results may be associated with some of them.

We also estimated \(2^+\) energies and \(B(E2^+)\) of \(^{44}\text{S}\) and \(^{42}\text{Si}\) by using Eq. (2) and (3), although quantitative comparison with observation is not possible because of deformation of these nuclei. The first peak of \(^{44}\text{S}\) at 0.6 MeV has a value of \(B(E2^+) = 193 e^2f_m^4\) and that of \(^{42}\text{Si}\) at 1.68 MeV has 1.68 MeV has 1.68 MeV has \(B(E2^+) = 124 e^2f_m^4\). The energies of these peaks are considerably shifted toward lower energies and the \(B(E2^+)\) are fairly increased compared to the unperturbed results indicating the enhancement of low-lying collectivity similar to \(^{46}\text{Ar}\). In the \(^{44}\text{S}\) experiment [3], the larger \(B(E2^+) = 310 \pm 90 e^2f_m^4\) than our result is measured, which indicates some collectivity is lacked in our evaluation. Then, the evaluated \(2^+\) energies do not agree with the observed values of 1.33 MeV (\(^{44}\text{S}\) and 0.77 MeV (\(^{42}\text{Si}\)). This mismatch may be due to the rotational mode originates in deformation of \(^{44}\text{S}\) and \(^{42}\text{Si}\). In other words, the present results suggest the onset of deformation in \(^{44}\text{S}\) and \(^{42}\text{Si}\) as reported in Refs. [6, 8, 9, 33].

In summary, we have investigated the low-lying quadrupole vibrational modes of \(N=28\) isotones by using Cb-TDHFB theory. While the low-lying states of \(^{48}\text{Ca}\) have single-particle nature, other isotones have strongly enhanced low-lying collectivity. It is also found that the pairing correlation further enhances collectivity and \(B(E2^+)\) of \(^{46}\text{Ar}\) is increased by about 20 %. As a mechanism of this enhancement, we pointed out the importance of quadrupole correlation between protons and neutrons that couples proton hole states with neutron excitations across \(N=28\) shell gap. The evaluated \(E(2^+)\) and \(B(E2^+)\) of \(^{46}\text{Ar}\) reasonably agree with the observed data. However, the discrepancy was found for \(^{44}\text{S}\) and \(^{42}\text{Si}\) that implies the onset of deformation in these nuclei.

Acknowledgement. Computational resources were partially provided by the High Performance Computing system at Research Center for Nuclear Physics, Osaka University.

[1] Y. Suzuki, K. Ikeda and H. Sato, Prog. Theor. Phys. 83, 180 (1990).
[2] A. Leistenschneider et al., Phys. Rev. Lett. 86, 5442 (2001).
[3] P. Adrich et al., Phys. Rev. Lett. 95, 132501 (2005).
[4] J. Gibelin, et al., Phys. Rev. Lett. 101, 212503 (2008).
[5] H. Scheit et al., Phys. Rev. Lett. 77, 3967 (1996).
[6] T. Glasmacher et al., Phys. Lett. B 395, 163 (1997).
[7] F. Sarazin et al., Phys. Rev. Lett. 84, 5062 (2000).
[8] D. Sohler et al., Phys. Rev. C66, 054302 (2002).
[9] B. Bastin et al., Phys. Rev. Lett. 99, 022503 (2007).
[10] L. Gaudefroy et al., Phys. Rev. C78, 034307 (2008).
[11] J. Retamosa, E. Caurier, F. Nowacki, and A. Poves, Phys. Rev.C55, 1266 (1997).
[12] D. J. Dean et al., Phys. Rev. C\textbf{59}, 2474 (1999).
[13] G. A. Lalazissis, D. Vretenar, P. Ring, M. Stoitsov, and L. Robledo, Phys. Rev. C\textbf{60}, 014310 (1999).
[14] S. Péru, M. Girod, and J. F. Ferger, Eur. Phys. J. A\textbf{9}, 35 (2000).
[15] R. Rodríguez-Guzmán, J. L. Egido, and L. M. Robledo, Phys. Rev. C\textbf{65}, 024304 (2002).
[16] E. Caurier et al., Rev. Mod. Phys. \textbf{77}, 427 (2005).
[17] L. Gaudefroy, Phys. Rev. C\textbf{81}, 064329 (2010).
[18] K. Kaneko, Y. Sun, T. Mizusaki, and M. Hasegawa, Phys. Rev. C\textbf{83}, 014320 (2011).
[19] Z. P. Li et al., Phys. Rev. C\textbf{84}, 054304 (2011).
[20] T. R. Rodríguez and J. L. Egido, Phys. Rev. C\textbf{84}, 051307 (2011).
[21] M. Kimura, Y. Taniguchi, Y. Kanada-En’yo, H. Horiuchi, and K. Ikeda, Phys. Rev. C\textbf{87}, 011301 (2013).
[22] S. Ebata, T. Nakatsukasa, T. Inakura, K. Yoshida, Y. Hashimoto and K. Yabana, Phys. Rev. C\textbf{82}, 034306 (2010).
[23] S. Ebata, T. Nakatsukasa and T. Inakura, AIP Conf. Proc. \textbf{1484}, 427 (2012).
[24] G. Scamps and D. Lacroix, Phys.Rev. C\textbf{88}, 044310 (2013).
[25] M. Yamagami and Nguyen Van Giai, Phys. Rev. C\textbf{69}, 034301 (2004).
[26] K. Yoshida, M. Yamagami and K. Matsuyanagi, Nucl. Phys. A\textbf{779} 99 (2006).
[27] Y. Hashimoto, Eur. Phys. J. A\textbf{48}, 55 (2012).
[28] J. Bartel, P. Quentin, M. Brack, C. Guet and H. -B. Håkansson, Nucl. Phys. A\textbf{386} 79 (1982).
[29] T. Nakatsukasa and K. Yabana, J. Chem. Phys. \textbf{114}, 2550 (2001).
[30] T. Nakatsukasa and K. Yabana, Phys. Rev. C\textbf{71}, 024301 (2005).
[31] S. Raman, C. W. Nestor Jr and P. Tikkanen, At. Data. Nucl. Data Tables \textbf{78}, 1 (2001).
[32] M. Wang \textit{et al}., Chin. Phys. C\textbf{36}, 1603 (2012).
[33] S. Takeuchi \textit{et al}., Phys. Rev. Lett. \textbf{109}, 182501 (2012).
[34] D. Santiago-Gonzalez \textit{et al}., Phys. Rev. C\textbf{83}, 061305 (2011).
[35] P.-G. Reinhard, D. J. Dean, W. Nazarewicz, J. Dobaczewski, J. A. Maruhn, and M. R. Strayer, Phys. Rev. C\textbf{60}, 014316 (1999).
[36] Y. Utsuno \textit{et al}., Phys. Rev. C\textbf{86}, 051301 (2012).