Discovery learning model with geogebra assisted for improvement mathematical visual thinking ability

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Abstract. The main goal of this study is to improve the mathematical visual thinking ability of high school student through implementation the Discovery Learning Model with Geogebra Assisted. This objective can be achieved through study used quasi-experimental method, with non-random pretest-posttest control design. The sample subject of this research consist of 62 senior school student grade XI in one of school in Bandung district. The required data will be collected through documentation, observation, written tests, interviews, daily journals, and student worksheets. The results of this study are: 1) Improvement students Mathematical Visual Thinking Ability who obtain learning with applied the Discovery Learning Model with Geogebra assisted is significantly higher than students who obtain conventional learning; 2) There is a difference in the improvement of students' Mathematical Visual Thinking ability between groups based on prior knowledge mathematical abilities (high, medium, and low) who obtained the treatment. 3) The Mathematical Visual Thinking Ability improvement of the high group is significantly higher than in the medium and low groups. 4) The quality of improvement ability of high and low prior knowledge is moderate category, in while the quality of improvement ability in the high category achieved by student with medium prior knowledge.

1. Introduction
Thinking ability is a major factor in every aspect of life. Smart thinking is one of the main reason why humans can survive [1]. The practice of mathematics is more often associated with symbolic or algebraic modes of expression than with those are visual [2]. One of the competencies expected to have high school students in learning mathematics is able to show a logical, critical, analytical, or visual thinking, especially in solving mathematical problems solving process. The ability of visual thinking is important to note, because the results of previous study show visual thinking ability have high contributed to improving mathematical understanding, problem solving, and in making connection process [3].

Visual imagery used in mathematics is frequently of a personal nature, not only related with conceptual knowledge and belief systems but often laden with affect [4-6]. However, it is very personal aspects that may enable or contrain the mathematical solution processes of individual [7,8]. That there have been claims in the literature that students are reluctant to visualize when they do mathematics [9]. Eisenberg wrote:” A vast majority of student do not like thinking in term picture-and their dislike is well documented in the literature [10]. Although algebraic expression can easily routine and processes of its
construction be made open for public inspection, visual image are less well-defined and are difficult to capture or communicate without ambiguity [2]. A while the evolution of technology has opened up new possibilities for visual expression in the process of mathematical reasoning. A visual image can be made open to inspection, an object of reflection, which can serve as a building block in an argument-something concrete rather than transitory and fleeting [2].

According to Van Hiele, one of five learning stages in geometry is the introduction stage as visualization. There is a link between the ability of visualization and understanding of concepts, where the ability of visualization of students give a positive influence on understanding the concept. Therefore is very important to investigate these issues. It is necessary to develop a deep discovery into students’ visual thinking in problem solving process. Discovery learning assisted computer software is seen as a promising method of learning for several reasons, the main being that students are actively involved in learning that will result in a structured knowledge base in learning [11,12], discovery learning is a learning where students build their own knowledge by experimenting, and drawing conclusions on rules/concepts from their experimental results. While Dewey [13] said that discovery learning is a model and learning strategy that focuses on liveliness, giving learning opportunities to students. Discovery is a method by which students learn more effectively to construct their own knowledge [14]. Bicknell-Holmes & Hoffman [13] describes three main features of discovery learning: 1) exploring and solving problems to create, integrate, and generalize knowledge; 2) interest-based activities in which students determine stages and frequencies; 3) activities that encourage the integration of new knowledge into the existing knowledge base of students. Discovery learning can be facilitated through various strategies in the classroom.

Discovery Learning is learning with students building their own knowledge by experimenting, and drawing conclusions of rules/concepts from their experimental results. Furthermore, Joolingen argues that discovery learning is seen as a promising way of learning for several reasons, the main being that active learner involvement will result in a structured knowledge base in learners better than traditional learning, where knowledge is said to be transferred to learners only [15]. In line the description and explanation above, Bicknell-Holmes & Hoffman in Castronova illustrates three main features of the Discovery Learning model: 1) exploring and solving problems to create, integrate, and generalize knowledge; 2) interest-based activities in which students determine stages and frequencies; 3) activities that encourage the integration of new knowledge into the knowledge base of existing students [13]. According [16] in applying the model Discovery Learning completely teacher must have facilitate the students to conduct Stimulation, Data collection, Data processing, Verification, and Generalization.

To answer the research question, a quasi-experiment study was a suitable for investigating this issue. The treatment of the experiment is discovery learning with assisted geogebra software.

2. Methods
This study used quasi-experimental method with non-random pretest-post control design. The study conducted in one of the high schools in Bandung district. Time of research is in January-July 2017. The subject population in this study is all students of class XI from one of the State Senior High School Bandung in 2016-2017 even semester. There are 62 students as sample was selected from class XI in one of the State Senior High School in Bandung, then determined as experimental group and control group, 31 students in each group. One of the objectives of this study is to examine whether there is an increase in students’ visual thinking ability after learning is done. Toward the experiment class applied Discovery learning with assisted geogebra software. The test instrument used in this study consists of five essay problems. Each question has been validated by a geometry expert and empirically tested, and
processed using Rasch model-based Winsteps software. Mathematical topics studied include, functions, graphs, tangents, curve equations, and problem solving.

The steps to compare the differences visual thinking ability of the experimental class and control class students are as follows: 1) Testing the normality distribution of population used the Shapiro-Wilk test; 2) Testing the homogeneity of two variance used the Levene test; 3) To compare the differences of mean of independent sample used the T test; 4) To examine the differences of three means based on the prior knowledge students used the one-way ANOVA test.

3. Results and Discussion

3.1. Mathematical visual thinking abilities

The ability of visual thinking mathematically obtained through pretest and postest present in table 1.

| Table 1. Distribution of Qualification of Visual Thinking Abilities |
|-----------------------|-------------|----------------|
| Class | Qualifications (g) | Number of students |
|-------|-------------------|--------------------|
| Experiment | High (g > 0.7) | 7 |
| | Middle (0.3 < g ≤ 0.7) | 16 |
| | Low (g ≤ 0.3) | 8 |
| Control | High (g > 0.7) | 6 |
| | Middle (0.3 < g ≤ 0.7) | 19 |
| | Low (g ≤ 0.3) | 7 |

Based on the table above we can see that number of student distribution of both classes on each n-Gain qualification is relatively homogeneous. The result of normality test (Saviro-Wilk) and homogeneity test (Levene), proven that both population class, experiment and control present in table 2 and table 3 as follow.

| Table 2. Normality Test Results N-gain Score Visual Thinking Mathematical Ability |
|------------------|----------------|----------------|
| Class | Shapiro-Wilk | Conclusion |
| | Statistic | df | Sig. |
|-------|-------------|-----|-----|
| Experiment | 0.978 | 31 | 0.763 | Ho Accepted |
| Control | 0.975 | 31 | 0.658 | Ho Accepted |

| Table 3. Homogeneity Test Results of Variance N-gain Scorrians the ability of Visual thinking Mathematically |
|------------------|----------------|----------------|
| Levene Statistic | df | Sig. | Conclusion |
|-------------------|-----|-----|-------------|
| 0.127 | 60 | 0.679 | Ho Accepted |

From table 2 shows that the N-gain score of visual thinking ability mathematically of experimental class obtains sig = 0.763 ≥ α, and the control class obtains the sig. = 0.658 ≥ α then Ho is accepted, meaning the both class are normally distributed. From table 3 shows that N-gain score of visual thinking ability mathematically has Sig = 0.679 ≥ α so that Ho is accepted. That is, the N-gain score of the visual
thinking ability of the experimental class students and the control class comes from the homogeneous variance. This result implicates that the test the difference of both classes use parametric test, the T test. The following show descriptive statistics and result difference compare mean of N-gain Score Visual thinking ability.

Table 4. Descriptive Statistics of visual thinking ability mathematically

| Test     | Control class | Experiment Class |
|----------|---------------|-----------------|
|          | $\bar{x}$    | $s$             | $\bar{x}$    | $s$         |
| Pretest  | 3.47          | 3.05            | 3.65          | 2.94        |
| n-Gain   | 0.49          | 0.38            | 0.56          | 0.37        |

Table 5. T-test difference mean of N-gain score of visual thinking ability mathematically

|        | T         | Sig. (2-tailed) | $H_0$   |
|--------|-----------|----------------|---------|
|        | 0.275     | 0.063          | Accepted|
|        | 3.366     | 0.019          | Rejected|

The descriptive statistic in table 4 shows that of visual thinking ability mathematically before learning instruction for both class is almost no different, and the result of differences testing in table 5 show that the mean difference is not significant. The following is result of visual thinking ability enhancement test from n-Gain data. The test used parametric statistics t-test. Based on table 4, it can be seen that the significance value is 0.019. Because it is a one-party test, the value of Sig. (2-tailed) divided by two into 0.019 / 2 = 0.0095. The value is less than 0.05 so $H_0$ is rejected. This means enhancement of visual thinking ability mathematically of students that apply the Discovery Learning assisted Geogebra significantly higher than students who received conventional learning.

According to [17], visual context provides opportunity for the students to relate what they learn with their mind. In addition, it is suggested to deliver students to questions such as: What made you think that? Why does that make sense? Where have we seen a problem like this before? How are these ideas related? Did anyone think about this in different way? How does today's work related to what we have done in earlier units of study? [18] states by looking in our mind (visual thinking), we are more confident to solve a problem. To find solutions to mathematical problems, students generally use drawings, diagrams to explain concepts and problems when information problems (words) do not succeed in explaining in the idea of solving mathematical problems. Mathematics learning at school is done verbally, symbolically and numerically. How to learn needs emphasis on improving visual thinking. Student visual thinking can be represented by maps, charts, graphs and expresses in written language or student papers.

Visual Thinking becomes an integral part of problem solving, for example using diagrams to explain, document, calculate or demonstrate the steps involved in reaching a solution. Visual representation can play a role in communication, for example using diagrams and visual forms to convey information, represent data and show relationships.

3.2. Mathematical visual thinking ability based on pkma

Hypothesis testing was done to find out the difference of abstraction ability of experimental class mathematics between high, medium, and low group prior knowledge mathematical ability (PKMA).
with assumed that N-gain score of both group is normally distributed with (P-value) > 0.354 ≥ α and homogeneity in their variances with (P-value)=0.954 ≥ α, Thus to test the hypothesis using ANOVA parametric test. The decision criteria for the ANOVA test are sig. (P-value) ≥ α, then Ho is accepted, whereas sig. (P-value) < α, then Ho is rejected. One Way ANOVA test calculation results are listed in the following table:

**Table 6. ANOVA Test Results N-gain Score Visual thinking Mathematical Ability based on Prior Knowledge Group in the Experiment Class**

|              | df  | F     | Sig.  | Conclusion     |
|--------------|-----|-------|-------|----------------|
| Between Groups | 2   | 14.572| 0.000 | There is differences |
| Within Groups | 28  |       |       |                |
| Total        | 31  |       |       |                |

Table 6 shows that the sig. Smaller than α, that is 0.000 < α = 0.05, so Ho is rejected means there is a significant difference in enhancement of visual thinking ability between groups prior knowledge (high, medium, and low) of students who obtain the Discovery Learning with Geogebra Assisted.

Because of the results of ANOVA trials show there are differences so to know which prior groups are significantly different then conducted a follow-up test (Post Hoc). Post Hoc test used is Tukeys test. Recapitulation of advanced test result of N-gain data of mathematical visual thinking ability of students based on prior knowledge in experiment class can be seen in the following table:

**Table 7. Post Hoc Test of Improvement Mathematical Visual Thinking Ability Based on PKMA in the Experiment Class**

| Group PKMA (I) | Group PKMA (J) | Mean Difference (I-J) | Std. Error | Sig. Error |
|----------------|----------------|-----------------------|------------|------------|
| 1              | 2              | .14317                | .04683     | .013       |
| 2              | 3              | .28875                | .05349     | .000       |
| 2              | 3              | -.14317               | .04683     | .013       |
| 3              | 1              | .14558                | .04683     | .012       |
| 3              | 2              | .28875                | .05349     | .000       |
| 3              | 2              | -.14558               | .04683     | .012       |

From Table 7, the Post Hoc Test using the Tukey test above the Sig value. For upper and moderate groups is 0.013 < Sig. For the upper and lower groups is 0.000 <, and the value of Sig.. For the medium group and the lower group is 0.012 > 0.05. This means that there is a difference in the improvement of visual thinking ability between upper and lower groups. When viewed from the average N-gain, the highest increase in mathematical visual thinking ability between upper and lower group is upper group students. Likewise with the upper and lower groups, there is a difference between the two and the improvement in the ability of mathematical visual thinking is higher occurs in upper group students. For medium and low groups, there was also a difference in the improvement of mathematical visual thinking ability between middle and lower groups, and improved mathematical visual thinking ability were higher in middle group students. i.e: high, medium, and low. This grouping is based on the results of the students' initial mathematical ability tests provided. The problems that are made as a matter of PKMA are questions related to the prerequisite material.
Based on the results of the research, PKMA students give a significant influence on the students' Visual Thinking abilities. The enhancement of high group Mathematical Visual Thinking abilities was significantly higher than in the middle and lower groups. Likewise, when the middle and lower groups we compare, then the group increased mathematical thinking abilities are significantly higher than the lower group. The above description illustrates that prior knowledge mathematical ability contributes well to the acquisition of new knowledge of students. So it can be concluded that the students' early mathematical abilities determine the learning outcomes. If the students' early ability to learn better will also get better.

3.3. Discovery learning model with Geogebra assisted

The results obtained from this study indicate that Discovery Learning model with Geogebra Assisted plays a role in developing the ability of Visual Thinking Ability. The ability of Visual Thinking is an important activity in learning mathematics. In solving the problem, in addition to demanding students to think can also cause students more creative. Mathematics is basically a problem solving, therefore teachers should teach the art of problem solving. Problems faced by students should make students motivated to do it. The ability of Visual Thinking in mathematics emphasizes the critical thinking in mathematics problems solving.

Geogebra software provides an opportunity for experiment class students to explore their own existing problem solving, and disclose their findings with a variety of alternative answers, since the use of computer software for learning activities is very unlimited [19]. In addition, many students received training at the stage of investigating student activity sheets (LKS). At this stage they collect information, investigate the meaning of the non-routine problem presented and solve the problem itself. Discovery Learning strategy was not only able to improve the mastery of students' concepts and attitudes as disclosed [20], but the proper use of computer is also able to improve the Visual Thinking mathematically, because as revealed by Glass [21] in learning with Discovery Learning assisted geogebra strategy there are practice, tutorial, game, simulation, discovery and problem solving exercises.

The results of the interviews show that the use of geogebra software is very helpful for students in understanding a mathematical concept, especially in capturing movable visualized mathematical ideas, such as the change of tangent gradient values through a point in the parabolic curve. Students admitted to having many difficulties before using the software. The most impressive analysis of student work is when they are asked to sketch some of the curves in a Cartesian field, the users of geogebra software show better competence, they are able to draw finer sketches. This is thought to be because the visualization provided by the teacher is helpful construct their visual thinking skills. Meanwhile, when they were asked to fill in their daily journals, they revealed how much geometry they learned with the help of software, and hoped to continue using it in every subsequent geometry lesson. A possible explanation makes sense there is a dialectical relationship in the computer environment between the expression and the expression itself, which is not visible in the case of paper and pencil. This is in accordance with the theory of Wertsch [22], who argue that the object must be "individuals with mediational means", not just their own people. It is the dialectic of people and software that has a tendency to produce a consistent approach.

4. Conclusion

Based on the results of research and discussion of the results of research as described then obtained conclusions from the results of the study that is: 1) Increasing the Mathematical Visual Thinking Ability (MVBA) students who have applied Discovery Learning model with Geogebra assisted is significantly
higher than students who have received conventional learning; 2) There is a difference in the ability of Mathematical Visual Thinking Ability (MVBA) among students with Prior Knowledge Mathematical Ability (PMKA) group (high, medium, and low) who applied Discovery Learning model with Geogebra assisted; 3) The improvement in Mathematical Visual Thinking Ability (MVBA) of the high groups is significantly higher than in the medium and low groups; 4) The quality of improvement ability of high and low PKMA is moderate category, while the quality of improvement ability in the high category achieved by student with medium group.

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