Kinetic energy driven superfluidity and superconductivity and the origin of the Meissner effect

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Superfluidity and superconductivity have many elements in common. However, I argue that their most important commonality has been overlooked: that both are kinetic energy driven. Clear evidence that superfluidity in $^4$He is kinetic energy driven is the shape of the $\lambda$ transition and the negative thermal expansion coefficient below $T_\lambda$. Clear evidence that superconductivity is kinetic energy driven is the Meissner effect: I argue that otherwise the Meissner effect would not take place. Associated with this physics I predict that superconductors expel negative charge from the interior to the surface and that a spin current exists in the ground state of superconductors (spin Meissner effect). I propose that this common physics of superconductors and superfluids originates in rotational zero point motion. This view of superconductivity and superfluidity implies that rotational zero-point motion is a fundamental property of the quantum world that is missed in the current understanding.

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I. INTRODUCTION

That superconductivity and superfluidity have many common elements is certainly well known[1, 2]. An indication of this is that the terms “superfluid electrons” and “superfluid condensate” are commonly used to refer to the charge carriers in the superconducting state of a metal. However I propose that a deep commonality between superconductors and superfluid $^4$He has been overlooked until now: that both phenomena are kinetic energy driven. Figures 1 and 2 show kinetic, potential and total energies versus temperature for the model of hole superconductivity[3] and for superfluid $^4$He computed through Monte Carlo simulations by D. Ceperley[4] (direct experimental data on kinetic and potential energies separately do not exist). The similarity in the two figures is very apparent. The potential energy increases as the system enters the superfluid or superconducting state, while the kinetic energy decreases, hence the “super” state is “kinetic energy driven” in both cases.

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In contrast, within conventional BCS theory the kinetic energy of the carriers always increases upon entering the superconducting state and the interaction energy...
FIG. 3: Same as Fig. 1 for an attractive Hubbard model representative of conventional BCS. The $T_c$ and band filling are the same as in Fig. 1. $U = -0.4$.

decreases by a larger amount overcompensating the kinetic energy increase, as shown in Figure 3, hence superconductivity is “potential energy driven”. The physics displayed in Figure 3 is qualitatively different from the physics shown in Figures 1 and 2. I argue that the Meissner effect results from the physics shown in Fig. 1 and would not occur if the physics was as in Fig. 3, for reasons explained below.

That superfluidity in $^4$He is kinetic energy driven is clear from a variety of experimental data that we will review in the next section. That superconductivity is kinetic energy driven is predicted by the model of hole superconductivity, introduced in 1989[5]. The pairing interaction was denoted by $\Delta t$ to indicate its kinetic origin, and its effect on the kinetic energy was discussed in Ref. [6]. However it was only much later that the fundamental physics of kinetic energy lowering, which is completely analogous to the physics taking place in $^4$He, and its role in the Meissner effect, was understood in this model.

II. SUPERFLUID $^4$He AND WAVEFUNCTION EXPANSION

Figure 4 shows four properties of $^4$He that illustrate the physics of interest here. (a) shows the density versus temperature at constant pressure. Below the superfluid transition, there is a slight decrease in the density, which is clearly not driven by potential energy: the $^4$He atoms are spherical, so there is no directionality to the interatomic forces, and the average distance between atoms in the liquid is 4Å, while the minimum in the potential energy curve between He atoms is at distance 3Å[7]. If the density decreases, the interatomic distance increases and the potential energy increases. Hence the decrease in density seen below the $\lambda$ point has to be associated with lowering of kinetic energy, i.e. is kinetic energy driven. We can think of the $^4$He atoms as being confined in a box of size determined by the interatomic distance. The kinetic energy of quantum confinement will decrease when the density decreases and the interatomic distances increase.

Similarly Figure 4(b) shows the increase in volume as $^4$He goes from the solid to the liquid state. It becomes markedly larger at temperatures below the superfluid transition. At low temperatures the entropy of both states is zero[7], so the expansion is not entropy-driven as in an ordinary solid-liquid transition but energy-driven. Once again, since the potential energy increases upon expansion and the total energy decreases in going from the solid to the superfluid state this is direct evidence that the transition from the solid into the superfluid state is kinetic-energy driven.

Figure 4(c) shows the heat capacity versus temperature, the characteristic shape that gives the $\lambda$ transition its name (it should really be called ‘inverted lambda’ transition. The heat capacity is given by

$$C = \frac{d<K>}{dT} + \frac{d<U>}{dT}$$

with $<K>$ and $<U>$ the average kinetic and potential energies. The second term in this equation is positive above $T_\lambda$, since the system expands as $T$ increases and hence the potential energy increases, and is negative below $T_\lambda$ since the system expands as $T$ decreases. Thus,
the first term in Eq. (1) is even larger below $T_\lambda$ and even smaller above $T_\lambda$ than the full line in Fig. 4(c) shows [8], as indicated by the dashed lines in Fig. 4(c), hence the jump at $T_\lambda$ for the change in kinetic energy with $T$ is even larger. The fact that the rate of decrease of the kinetic energy as the temperature is lowered is so much larger below $T_\lambda$ than above $T_\lambda$ is clear evidence that the transition from the normal liquid into the superfluid state is kinetic energy driven [8].

Finally, Figure 4(d) shows the pressure versus temperature at constant density [9]. Below $T_\lambda$, the pressure increases as the temperature is lowered. This is qualitatively different from what occurs in ordinary Bose condensation: in that case, the condensate exerts no pressure, hence the pressure decreases rapidly as the temperature is lowered and the condensate fraction increases. In $^4$He instead, the pressure increases as the condensate forms, indicating that it exerts more quantum pressure than the normal fluid, causing the liquid to expand.

This physics of $^4$He is qualitatively different from Bose condensation physics. In a Bose gas, increasing the external pressure and hence the density at a fixed temperature will eventually lead to Bose condensation as the interatomic distances become of the order of the de Broglie wavelength. Instead, in $^4$He, increasing the pressure and density at fixed temperature will never lead from the normal liquid into the superfluid state, nor from the solid into the superfluid state. The superfluid transition involves expansion, hence application of pressure or increase in density can only lead out of the superfluid state (either into the solid or into the normal fluid state), never into it. This is clearly seen in the phase diagram of $^4$He.

The properties of $^4$He just summarized indicate that the transition into the superfluid state is associated with wavefunction expansion, kinetic energy lowering and enhanced quantum pressure originating in quantum zero-point motion [10]. We propose that exactly the same is true for superconductors and that this is the physics responsible for the Meissner effect.

III. HOLE SUPERCONDUCTIVITY AND WAVEFUNCTION EXPANSION

The theory of hole superconductivity predicts that superconductivity occurs when electronic energy bands are almost full, hence the carriers in the normal state are holes. When a band is almost full, there are a lot of antibonding electrons, as shown schematically in Fig. 5. They would like to break the solid apart, hence their name, “antibonding”. Their wavefunction is confined over a small spatial dimension, their wavelength $k_F^{-1}$ is short ($k_F$ is the Fermi wavevector), and they exert “quantum pressure” outward. They have highly oscillating wavefunctions and hence high kinetic energy.

Within the theory of hole superconductivity [11], pairing of holes occurs at the critical temperature because it gives rise to kinetic energy lowering [5, 6]. When holes pair, the band becomes locally less full, hence the kinetic energy should decrease according to Figure 5. In addition, the pairing interaction $\Delta \Gamma$ gives rise to kinetic energy lowering for the pair. The transition to superconductivity is associated with expansion of the electronic wavefunction and expulsion of negative charge from the interior of the superconductor to a region within a London penetration depth of the surface, $\lambda_L$ [12, 13]. The expansion of the wavefunction and negative charge expulsion results from an expansion of electronic orbits from microscopic radius $k_F^{-1}$ to mesoscopic radius $2\lambda_L$ [14], which lowers the quantum kinetic energy, and changes the diamagnetic susceptibility from the Landau free electron value to the value appropriate for perfect diamagnetism, $\chi = -1/4\pi$, as shown schematically in Figure 6. The expansion of electronic orbits and associated outward motion of negative charge provides a dynamical ex-
planation of the Meissner effect[15]: in the presence of a magnetic field, the Lorentz force on the radially outgoing electrons deflects them in the azimuthal direction giving rise to the Meissner current that expels the magnetic field from the interior. In other words, the outflowing charge carries with it the magnetic field lines, as in a classical plasma[16]. Instead, if there is no radial motion of charge, as expected within BCS theory, magnetic field lines would not move out, there would be no Meissner effect, and the material would not become a superconductor[17].

The fact that superfluid electrons in the superconducting state reside in orbits of radius $2\lambda_L$ can be seen from the fact that the total angular momentum of electrons in such orbits equals the angular momentum of the Meissner current circulating within a London penetration depth of the surface in a cylindrical geometry, as shown by the following equation:

$$L_{\text{total}} = [m_e v(2\lambda_L)] n_s [\pi R^2 \hbar] = [m_e v R] n_s [2\pi R \lambda_L]$$

(2)

where $R$ and $h$ are the radius and height of the cylinder and $n_s$ is the superfluid density. Electrons in the $2\lambda_L$ orbits traverse these orbits with speed given by[14]

$$v_s^0 = \frac{\hbar}{4 m_e \lambda_L}.$$  

(3)

in opposite direction for opposite spin. The superposition of these motions gives rise to a macroscopic spin current of carrier density $n_s/2$ for each spin direction flowing within a London penetration depth of the surface with speed Eq. (3), a macroscopic zero point motion of the superfluid. This is shown schematically in Figure 7.

As a result of this orbit expansion, the electronic density in the interior of the superconductor is slightly smaller than in the normal state. This is entirely analogous to the density decrease that occurs in $^4\text{He}$ upon the onset of superfluidity. The excess negative charge near the surface has density $\rho_-$, related to the speed of the spin current Eq. (3) through the equation[18]

$$\rho_- = en_s \frac{v_s^0}{c}.$$  

(4)

Thus, we can think equivalently of the entire superfluid charge density $en_s$ flowing with speed Eq. (3) (half in each direction) or just the excess charge density $\rho_-$ flowing at the speed of light. The orbital angular momentum of superfluid electrons in the $2\lambda_L$ orbits is

$$L_{\text{orb}} = m_e v_s^0 (2\lambda_L) = \hbar/2.$$  

(5)

The question arises whether the electronic orbit expansion will give rise to a lower density for the solid as a whole when it becomes superconducting. This is indeed seen in many superconductors[19, 20] but not in all. The situation is more complicated than in $^4\text{He}$ because of the presence of electronic and ionic degrees of freedom.

**IV. ZERO POINT MOTION IN SUPERFLUID $^4\text{He}$ AND IN SUPERCONDUCTORS**

The fact that we have found charge expulsion and macroscopic zero point motion in the superconductor, resulting from expansion of the electronic wavefunction, suggests that similar effects should occur in superfluid $^4\text{He}$. Remarkably, such behavior has been known for a long time: the ‘Onnes effect’[21], the flow of superfluid films (Rollin films)[22] along surfaces without any driving force[23]. A superfluid container will expel mass, just like the superconductor expels charge, as shown schematically in Figure 8. $^4\text{He}$ atoms flow in the Rollin film defying the force of gravity, just as electrons develop the Meissner current defying the Faraday electromotive force[23].

The close connection between superconductors and superfluid $^4\text{He}$ becomes even more apparent when we consider superfluid flow under zero potential difference. This

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**FIG. 7:** The left side shows electronic orbits of radius $2\lambda_L$, with electrons with spin pointing into the paper (out of the paper) circulating in counterclockwise (clockwise) direction. The orbits are highly overlapping. The superposition of these motions (right side) gives rise to a spin current circulating in a layer of thickness $\lambda_L$ near the surface in the ground state of the superconductor, and no net currents in the interior.

**FIG. 8:** The expulsion of charge from the interior of the superconductor (a) has as counterpart the expulsion of mass from the superfluid $^4\text{He}$ container (b), climbing the lateral surfaces and escaping to the exterior (“Onnes effect”).
is achieved in a superconducting wire inserted between normal conductors, and in the \(^4\)He double beaker experiment of Daunt and Mendelssohn\[24\], designed specifically for this purpose, as shown schematically in Figure 9. Mendelssohn\[26, 27\] pointed out the clear analogy between the phenomena shown in Figures 9 (a) and (b) and asked the question, what is the dynamical origin of these motions that occur without potential drop, i.e. without a force? He proposed that they are evidence for zero point motion of the condensed particles in the superfluid and in the superconductor. He points out that “neither case corresponds to a Bose-Einstein condensation since both have an appreciable zero-point energy”.

Furthermore, Daunt and Mendelssohn\[28\] as well as London\[29\] and Bilj et al\[30\] pointed out that the measured speed of \(^4\)He in the films obeys the relation

\[ v = \frac{\hbar}{2m_H e d} \tag{6} \]

where \(d\) is the thickness of the film, typically \(\sim 300\,\text{Å}\), giving a speed \(v \sim 26\,\text{cm/s}\). This relation can be interpreted as arising from Heisenberg’s uncertainty principle for a particle confined to a linear dimension \(d\). Similarly, the critical magnetic field for a superconductor is given by\[31\]

\[ H_{c1} = -\frac{\hbar c}{4e\lambda_L^2} \tag{7} \]

and the critical velocity by

\[ v = \frac{e}{m_e c} \lambda_L H_{c1} = \frac{\hbar}{4m_e \lambda_L} \tag{8} \]

which can be interpreted as the speed of an electron confined to linear dimension \(2\lambda_L\) arising from Heisenberg’s uncertainty principle. Mendelssohn argues\[26\] that these speeds, Eqs. (6) and (8), are the speeds of “zero point diffusion” of particles in the condensate, and that this explains why the transport rate is independent of external forces: the transport occurs because if at one end particles of the condensate are removed, zero point diffusion will give rise to flow in that direction. He furthermore stresses that “the momentum of frictionless transport is not dissipated because it is zero-point energy”.

However, Mendelssohn’s interpretation, even though it reveals very deep intuition, is not internally consistent. Heisenberg’s uncertainty principle predicts that the momentum associated with spatial confinement should be in the same direction of the coordinate that is confined. Instead, both in the superfluid and superconductor the transport with speeds given by Eqs. (6) and (8) is parallel to the surface, i.e. perpendicular to the direction of confinement. It is clear that Heisenberg’s uncertainty principle is not the explanation for superfluid film and superconducting current flow under zero potential difference. So what is it?

Superconductors give us the answer. The London-Mendelssohn transfer speed for superconductors Eq. (8) is nothing other than the speed Eq. (3) of electrons in \(2\lambda_L\) orbits giving rise to the spin current near the surface. The motion described by the speed Eq. (3) is rotational (Fig. 7, left side). Thus we conclude that both superconductors and superfluid \(^4\)He must possess rotational zero point motion in their ground states.\[32\]

If the zero-point motion is rotational, it is easy to understand why spatial confinement in direction perpendicular to the surface gives rise to flow along the surface. Furthermore it is easy to understand the magnitude of the flow velocity, arising from quantization of angular momentum

\[ L = mvd = \frac{\hbar}{2} \tag{9} \]

for both Eq. (6) and Eq. (8). It is also easy to understand the origin of quantum pressure in these systems: the kinetic energy of rotational zero point motion decreases as the radius of the motion increases:

\[ E_{\text{kin}} = \frac{L^2}{2MR^2} \tag{10} \]

for particles of mass \(M\) in orbits of radius \(R\) with angular momentum \(L\). Thus, a rotating particle with fixed
quantized angular momentum exerts quantum pressure to lower its kinetic energy by expanding its orbit, and does so in the transition to the superfluid or superconducting state. The expanded orbits overlap, hence phase coherence is required to avoid collisions of particles in different orbits, which is clearly a lower entropy state than when the phases are incoherent in the normal state, hence the transition will occur at sufficiently low temperatures where the energy decrease dominates over the entropy loss.

V. CONCLUSION

In summary, we conclude that in both superconductors and superfluid $^4$He the transition to the superconducting or superfluid state is driven by quantum pressure originating in rotational zero point motion, i.e. the drive of a rotating system to lower its kinetic energy by expansion. This explains a variety of properties of $^4$He like the decrease in density below the superfluid transition, the shape of the heat capacity curve versus temperature that gives the $\lambda$-transition its name, and the flow of Rollin films, as well as the most fundamental property of superconductors, the Meissner effect.

We should point out that there have been several proposals in the literature that $^4$He possesses macroscopic quantum zero point motion in the ground state[28-30], and that superconductors possess macroscopic zero point motion in the form of charge currents over domains[31]. These workers arrived at these conclusions through arguments different from ours.

Finally, the facts that superconductors and superfluid $^4$He are macroscopic quantum systems and they both display quantum pressure originating in rotational zero point motion at the macroscopic level leads us to conclude that quite generally microscopic quantum systems, which also exhibit quantum pressure, must acquire this quantum pressure through rotational zero point motion. In other words, that the origin of the ubiquitous quantum pressure is not Heisenberg’s uncertainty principle as generally believed but instead rotational zero point motion. Since Schrödinger’s equation does not predict rotational zero point motion, this implies that Schrödinger’s equation needs to be modified. The constant $h$ in Schrödinger’s equation presumably represents the angular momentum of this ubiquitous rotational zero point motion rather than the quantum of action as in the conventional understanding of quantum mechanics.

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