Usadel equation in the presence of intrinsic spin-orbit coupling: A unified theory of magneto-electric effects in normal and superconducting systems

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The Usadel equation is the standard theoretical tool for the description of superconducting structures in the diffusive limit. Here I derive the Usadel equation for gyrotropic materials with a generic linear in momentum spin-orbit coupling. It accounts for the spin-charge/singlet-triplet coupling and in the normal state reduces to the system of spin-charge diffusion equations describing various magneto-electric effects, such as the spin Hall effect (SHE), the spin-galvanic effect (SGE) and their inverses. Therefore the derived Usadel equation establishes a direct connection of these effects to their superconducting counterparts. The working power of the present formalism is illustrated on the example of the bulk SGE.

The influence of spin-orbit coupling (SOC) on transport properties of conducting materials and nanostructures attracts a great deal of attention because of its potential importance for spintronics [1, 2]. The coupling of translational and spin degrees of freedom of charge carriers gives rise to a number magneto-electric effects that allow to control spin dynamics by purely electric means or generate a charge response by acting on the spin subsystem. Probably the most known effect of this sort is the spin Hall effect (SHE) [3] which can be observed in practically any material with a sizable SOC. Other examples of magneto-electric effects driven by SOC are the direct and inverse spin-galvanic effects (SGE and ISGE) also known as the inverse and direct Edelstein effects (IEE and EE), see, for example, a recent review Ref.[4] and references therein. The SGE/IEE is the generation of charge current by producing a nonequilibrium spin polarization [5, 6], while ISGE/EE corresponds to a spin polarization induced by the electric current [7, 8]. The SGE is less universal then the SHE as it can exist only in gyroscopic materials/structures (there are 18 gyroscopic crystal classes out of 21 non-centrosymmetric classes) [9]. The gyroscopic symmetry allows for a second rank pseudotensor that is required to convert an axial vector (the spin polarization) to a polar vector (the electric current) and vice versa.

Magneto-electric effects mediated by SOC are also known for non-centrosymmetric superconductors [10, 11]. In superconducting materials with the gyroscopic symmetry an external Zeeman field creates a charge supercurrent, and, reversely, a supercurrent flowing in such a system induces a spin polarization [12, 13]. These effects are the bulk superconducting analogs of the SGE/IEE and ISGE/EE, respectively [14]. In the Josephson junction geometry the superconducting SGE manifests itself as an anomalous $\varphi_0$-junction that supports a Josephson current at zero phase difference between superconducting leads [15]. In superconducting structures SOC leads to an additional channel of the singlet-triplet conversion, which has been recognized as physical mechanism behind the anomalous Josephson effect [16, 17]. In the last years the spin-charge and singlet-triplet conversion effects are intensively studied in the context of an emerging field of superconducting spintronics [18, 19].

The magneto-electric spin-charge conversion effects in normal conductors are most naturally described theoretically in terms of a coupled spin-charge diffusion equations [20, 21]. In superconductors the analog of the diffusion equation is the Usadel equation [22] for a quasiclassical Green function (GF). Together with the Kupriyanov-Lukichev boundary conditions [24] the Usadel equation constitute a theoretical basis for the description of transport and spectral properties of most experimentally relevant superconducting structures. Unfortunately the full nonlinear form of the Usadel equation accounting for the spin-charge coupling due to intrinsic SOC is not known. Only its linearized version has been established and used in different contexts [15, 17, 25, 26]. While the linearized Usadel equation captures some qualitative physics, such as the anomalous Josephson effect [15, 17], it can not fully describe the low temperature regime and does not give an access to the local spectral properties. In the present paper I fill this gap by deriving the full nonlinear Usadel equation accounting for the spin-charge coupling due to intrinsic SOC. In contrast to the linearized theory this nonlinear equation is valid both for superconducting and for normal systems. In the normal state it reduces to the well known spin-charge diffusion equations [20, 21] thus providing us with a universal theory of magneto-electric effects mediated by SOC.

As a specific model I consider a system of conduction electrons with a linear in momentum SOC described by $\hat{H}_\text{so} = \beta^a p_k \sigma^a$ in the presence of a spin independent disorder potential $V_{\text{dis}}$ and a Zeemann/exchange field, $\hat{H}_Z = h^a \sigma^a$. Here $p_k$, $h^a$ and $\sigma^a$ are the components of the electron momentum, the exchange field and the vector of Pauli matrices, respectively [27]. The SOC is parametrized by a second rank pseudotensor $\beta^a_k$ that is
allowed only in gyrotrropic materials. In fact, the gyrotrropic symmetry is the necessary and sufficient condition for the existence of the linear in $p$ SOC in spin-$1/2$ electronic systems \cite{3,28,29}. Therefore the electron gas with a linear in momentum SOC can be considered as a simple but quite generic model of a gyrotrropic material. A very useful feature of this model is that the exchange field and the SOC coefficients can be interpreted as the time and space components of an effective background SU(2) potential $A_\mu = A_\mu^a \sigma^a/2$, $\mu = (0,k)$ \cite{30,33}. Indeed, by defining $A^a_0 = 2\hbar a$ and $A^a_{\perp} = 2m\beta^a_{\perp}$ one can represent the Hamiltonian of the model in the following form

$$\hat{H} = \frac{1}{2m} (\hat{p}_k - A_k)^2 - A_0 + V_{\text{dis}},$$

which implies a form invariance of the equations of motion under a local SU(2) rotations of the fermionic fields supplemented with the proper gauge transformation of the effective gauge potentials.

Using the gauge field representation of SOC one can formulate a physically transparent SU(2) covariant kinetic (Eilenberger) equation for the quasiclassical GF $\hat{g}(n,r,t,t')$ that depends on the direction $n = p/p_F$ of the Fermi momentum and describes dynamics of the Fermi surface quasiparticles in the presence of spin-dependent forces generated by the SU(2) gauge fields \cite{13,17,34,35}. The covariant Eilenberger equation is the starting point for the derivation of the diffusive Usadel equation presented below.

For superconductors the GF $\hat{g}(n,r,t,t')$ is a $8 \times 8$ matrix in the Keldysh-Nambu-spin space and the Eilenberger equation accounting for the spin-charging coupling reads (see e.g. Eq. (8) in \cite{17})

$$v_F n_k \hat{\nabla}_k \hat{g} + [\hat{\Omega}, \hat{g}] - \frac{1}{2m} \{ F_{ij}, n_i, \frac{\partial \hat{g}}{\partial n_j} \} = -\frac{1}{2\tau} [\hat{g}, \hat{g}] \tag{1}$$

where $\hat{\nabla}_k = \partial_k - i [A_k, \cdot]$ is the covariant derivative, $F_{ij} = \partial_j A_i - \partial_i A_j - i [A_i, A_j]$ is the SU(2) magnetic field tensor, $\tau$ is the momentum relaxation time, $\{ \ldots \}$ stands for the $n$-average, and

$$\hat{\Omega} = (\tilde{\omega} - i A_0) \tau_3 - i \tilde{\Delta} \tag{2}$$

with $\Delta$ being the anomalous self energy describing the superconducting ordering, and $\tilde{\omega}_{t,t'} = \delta_{k} \delta(t - t')$. In the equilibrium Matsubara formalism $\omega = \omega_n = \pi(2n + 1)$ and $\hat{g}(n,r,\omega_n)$ becomes a $4 \times 4$ matrix in the Nambu-spin space. The commutator part of the covariant derivative in Eq. (1) describes the inhomogeneous spin precession due to SOC, which can generate a long range proximity effect \cite{35,60} accompanied with a highly nontrivial modifications of the local density of states in Josephson junctions \cite{37,38}. The last term in the left hand side in Eq. (1) corresponds to the spin dependent Lorentz force produced by the SU(2) magnetic field $B_k = \frac{1}{2} z_{kij} F_{ij}$. An accurate treatment of this term is our main concern as it is responsible for the charge-spin/singlet-triplet coupling and eventually for the SHE, SGE, and the anomalous Josephson effect. The main technical difficulty is related to the anticommutator structure of the Lorentz force term, which leads to the violation of the normalization condition for the quasiclassical GF $\hat{g}(n)$. I will show that the normalization condition reappears in the diffusive limit for the isotropic part of the GF.

The diffusive limit of Eq. (1) is formally obtained as an asymptotic expansion in $\tau$ at $\tau \to 0$. I will perform this expansion to the order $\tau^2$ that is necessary to capture the effects of the SU(2) Lorentz force. With the accuracy of $O(\tau^2)$ it is sufficient to represent the GF $\hat{g}(n)$ by its 0th and 1st moments \cite{40}

$$\hat{g}(n) = \hat{g}_0 + n_k \hat{g}_k \tag{3}$$

where $\hat{g}_0 = \langle \hat{g}(n) \rangle$ and $\hat{g}_k = \langle n_k \hat{g}(n) \rangle / d$ (here $d$ is the dimensionality of space). By taking the 0th and the 1st moment of Eq. (1) and using the representation of $\hat{g}(n)$ given by Eq. (3) we obtain the following system of equations for $\hat{g}_0$ and $\hat{g}_k$, which should be then solved perturbatively in $\tau$

$$\frac{1}{d} v_F \hat{\nabla}_k \hat{g}_0 + [\hat{\Omega}, \hat{g}_0] = 0 \tag{4}$$

$$\tau v_F \hat{\nabla}_k \hat{g}_0 + \tau [\hat{\Omega}, \hat{g}_k] - \frac{\tau}{2m} \{ F_{kj}, \hat{g}_j \} = -\frac{1}{2} [\hat{g}_0, \hat{g}_k] \tag{5}$$

Equation (4) relates the 1st moment $\hat{g}_k$ of the GF to the 0th moment $\hat{g}_0$ (the isotropic part of GF). Importantly, this equation is consistent only when solved up to the second order in $\tau$. The contribution $\sim \tau$ comes from the last two terms in the left hand side of Eq. (4). To make the perturbative structure explicit I represent $\hat{g}_k$ as follows

$$\hat{g}_k = \hat{g}_k^{(1)} + \hat{g}_k^{(2)} + \hat{g}_k^{(3)} \tag{6}$$

where $\hat{g}_k^{(1)} \sim \tau$ and $\hat{g}_k^{(2)}, \hat{g}_k^{(3)} \sim \tau^2$ are determined from the following equations

$$-\tau v_F \hat{\nabla}_k \hat{g}_0^{(1)} = \frac{1}{2} [\hat{g}_0, \hat{g}_k^{(1)}] \tag{7}$$

$$\frac{\tau}{2m} \{ F_{kj}, \hat{g}_j^{(1)} \} = \frac{1}{2} [\hat{g}_0, \hat{g}_k^{(2)}] \tag{8}$$

$$-\tau [\hat{\Omega}, \hat{g}_k^{(1)}] = \frac{1}{2} [\hat{g}_0, \hat{g}_k^{(3)}] \tag{9}$$

In principle Eqs. (7)-(9) and (10) fully determine the diffusive dynamics up to the required order in $\tau$. It is however possible to solve Eqs. (7)-(10) explicitly and obtain a closed equation of motion for $\hat{g}_0$ - the Usadel equation.

The first observation is that the commutator structure of the right hand side in Eq. (7) implies the normalization condition for $\hat{g}_0$,

$$\hat{g}_0^2 = 1 \tag{10}$$
Indeed from Eq. (7) we find the following equation for $\tilde{g}_0^2$

$$-\tau v_F \nabla_k \tilde{g}_0^2 = \frac{1}{2} [\tilde{g}_0^2, \tilde{g}_0^1]$$

which has a unique solution given by Eq. (10) provided it is fulfilled at the spatial infinity. Using the normalization condition we find $\tilde{g}_k^{(1)}$ from Eq. (7)

$$\tilde{g}_k^{(1)} = -\tau v_F \tilde{g}_0 \nabla_k \tilde{g}_0 \equiv \tau v_F (\nabla_k \tilde{g}_0)\tilde{g}_0,$$  \hspace{1cm} (11)

which is the standard expression for the 1st moment of $\tilde{g}$ in the leading $\sim \tau$ order of the diffusive approximation. The $\tau^2$ corrections are obtained from Eqs. (8) and (9). By inserting Eq. (11) into Eq. (5) I rewrite it as follows

$$\frac{v_F \tau^2}{2m} (F_{kj}(\nabla_j \tilde{g}_0)\tilde{g}_0 - \tilde{g}_0(\nabla_k \tilde{g}_0)F_{kj}) = \frac{1}{2} (\tilde{g}_0 \tilde{g}_k^{(2)} - \tilde{g}_k^{(2)} \tilde{g}_0)$$

Multiplying this equation with $\tilde{g}_0$ from both sides, such as $\tilde{g}_0(\ldots)\tilde{g}_0$, and using Eq. (11) I obtain an alternative representation of Eq. (8)

$$\frac{v_F \tau^2}{2m} \left( \tilde{g}_0 F_{kj}(\nabla_j \tilde{g}_0) \tilde{g}_0 - (\nabla_k \tilde{g}_0)F_{kj} \tilde{g}_0 \right) = -\frac{1}{2} (\tilde{g}_0 \tilde{g}_k^{(2)} - \tilde{g}_k^{(2)} \tilde{g}_0)$$

The subtraction of the last two equations from each other yields yet another form of Eq. (8)

$$\frac{v_F \tau^2}{2m} (\tilde{g}_0 \{F_{kj}, \nabla_j \tilde{g}_0\} - \{F_{kj}, \nabla_k \tilde{g}_0\} \tilde{g}_0) = (\tilde{g}_0 \tilde{g}_k^{(2)} - \tilde{g}_k^{(2)} \tilde{g}_0)$$

from which the solution is read out immediately as

$$\tilde{g}_k^{(2)} = -\frac{v_F \tau^2}{2m} \{F_{kj}, \nabla_j \tilde{g}_0\}$$  \hspace{1cm} (12)

Finally I find $\tilde{g}_k^{(3)}$ from Eq. (9) that reads explicitly as

$$-\tau^2 v_F \left( \tilde{g}_0 \nabla_0 (\nabla_k \tilde{g}_0) - \nabla_k (\nabla_0 \tilde{g}_0) \right) = \tilde{g}_0 \tilde{g}_k^{(3)}$$

According to Eq. (4) the commutator $[\tilde{\Omega}, \tilde{g}_0] \sim \tau$. Therefore, to the required accuracy of $\tau^2$, in first term in the above equation one can safely interchange the order of $\tilde{\Omega}$ and $\tilde{g}_0$ to find the following result

$$\tilde{g}_k^{(3)} = -\tau^2 v_F [\tilde{\Omega}, \nabla_k \tilde{g}_0] + O(\tau^3) = \tau^2 v_F [\nabla_k \tilde{\Omega}, \tilde{g}_0] + O(\tau^3)$$

From the explicit form of $\tilde{\Omega}$ in Eq. (2) I identify $\tilde{\nabla}_k \tilde{\Omega} = -i\tau_3 \tilde{\nabla}_k \tilde{\sigma}_0 = -i\tau_3 F_{k0}$ as the SU(2) electric field. Thus

$$\tilde{g}_k^{(3)} = -i\tau^2 v_F [\tau_3 F_{k0}, \tilde{g}_0]$$  \hspace{1cm} (13)

Equations (8), (11)-(12), and (9) define the anisotropic part of the quasiclassical GF to the order of $\tau^2$ in the diffusive limit

$$\tilde{g}_k = -\tau v_F \tilde{g}_0 \nabla_k \tilde{g}_0 - \frac{v_F \tau^2}{2m} \{F_{kj}, \nabla_j \tilde{g}_0\} - i\tau^2 v_F [\tau_3 F_{k0}, \tilde{g}_0]$$  \hspace{1cm} (14)

The Usadel equation is obtained by inserting this result into Eq. (11).

It is convenient to introduce the matrix current $\tilde{J}_k = \frac{1}{2} v_F \tilde{g}_k$ and rewrite the Usadel equation as follows

$$\nabla_k \tilde{J}_k + [(\tilde{\omega} - i\tilde{A}_0)\tau_3 - i\Delta, \tilde{g}] = 0$$  \hspace{1cm} (15)

with the matrix current defined as

$$\tilde{J}_k = -D \left( \tilde{g} \nabla_k \tilde{g} + \frac{\tau}{2m} \{F_{kj}, \nabla_j \tilde{g}\} + i\tau [\tau_3 F_{k0}, \tilde{g}] \right)$$  \hspace{1cm} (16)

where $D$ is the diffusion coefficient and index 0 of the GF is suppressed for brevity. This matrix current $\tilde{J}_k$ enters the boundary conditions at the interfaces [24, 41], while its Keldysh component determines the physical charge $j_k = -\pi N_F \text{tr} \{\tau_3 J_k^F(t, t)/4\}$ and spin $J_k^S = -\pi N_F \text{tr} \{\sigma^a J_k^F(t, t)/4\}$ currents. It is important to emphasize that Eqs. (15)-(16) should be supplemented with the normalization condition $\tilde{g}_0^2 = 1$.

A very appealing property of Eq. (16) is that the intrinsic spin Hall contribution (the second term) has exactly the same structure as the corresponding contribution in the case of the extrinsic SHE (see Eq. (4) in Ref. [11]) with the extrinsic spin Hall angle being replaced by the intrinsic one $\theta_{ij} = \frac{\tau}{m} F_{ij}$. It is therefore natural to expect that, like in the normal case, the Hall angles will add up if both intrinsic and extrinsic SOC is present.

One can argue that the last, proportional to $F_{k0}$ term in Eq. (16) gives a small correction to the effect of the Zeeman/exchange field and therefore can be ignored. Indeed, the contribution of this term to the Usadel equation (14) has the same global (in the Nambu-spin subspace) structure as the Zeeman term. Therefore it simply renormalizes the Zeeman field by a negligible amount $\sim (\Delta_{\sigma} \tau)^2 \ll 1$ where $\Delta_{\sigma} \sim \beta v_F$ is the SOC-induced spin splitting (the inverse spin precession rate).

The Usadel equation simplifies further if SO fields $A_k$ are spatially uniform. In this case the covariant divergence of the spin Hall contribution to Eq. (16) simplifies as

$$\nabla_k \{F_{kj}, \nabla_j \tilde{g}\} = \{\nabla_k F_{kj}, \nabla_j \tilde{g}\} = \{\nabla_k F_{kj}, \partial_j \tilde{g}\},$$

and the Usadel equation (15) takes the following form

$$D \nabla_k (\tilde{g} \nabla_k \tilde{g}) - [(\tilde{\omega} - i\tilde{A}_0)\tau_3 - i\Delta, \tilde{g}] + \frac{\tau D}{2m} \{\tilde{g}\nabla_k F_{kj}, \partial_j \tilde{g}\} = 0$$  \hspace{1cm} (17)

The physical observables are calculated from the Keldysh component of Eq. (16) (without the last term) or the GF. In particular, we have the induced spin density $\delta S^a = S^a - \chi_P \cdot A_0^a$ (its deviation from the Pauli response value $\chi_P \cdot A_0$; where $\chi_P$ is the Pauli susceptibility)

$$\delta S^a = -\frac{\pi}{4} N_F \text{tr} \{\sigma^a \tau_3 \tilde{g} \tilde{g}^K\}$$  \hspace{1cm} (18)

the charge current

$$j_k = \frac{\pi D}{4} N_F \text{tr} \{\tau_3 [\tilde{g} \partial_k \tilde{g}]^K - \frac{D\tau}{m} [F_{kj}, \partial_j \delta S^a + (\nabla_j F_{ik})^a \delta S^a]$$  \hspace{1cm} (19)
and the spin current

\[ J^s_k = \frac{\pi D}{4} N_F \text{tr} \{ \sigma^a \hat{g} \hat{N}_k \hat{g}^{K} \} - \frac{D\tau}{m} F^a_{ki} \partial_t \delta n \]  

(20)

where \( \delta n = -\pi N_F \text{tr} \{ \hat{g}^{K} \} / 4 \) is the induced charge density. It is worth noting that there is no Hall contribution to the spin current in equilibrium because of the vanishing induced charge density.

Equation (17) and expressions for the currents Eqs. (19), (20) are the main results of the present paper. In the normal state \( \hat{g}(t, t') \) becomes diagonal in the Nambu space and \( g^{R,A}(t, t') = \pm \delta(t - t') \). In this case one can set \( t' = t \) directly in the Keldysh component of Eq. (17) to obtain a closed system of equations for \( \delta S^a(r, t) \) and \( \delta n(r, t) \). These equations together with the expressions for the currents recover the known system of coupled spin-charge diffusion equations in normal conductors with intrinsic SOC 20, 22. Hence the general Eqs. (17) - (20) provide us with a unified theoretical tool for addressing magneto-electric phenomena both in superconducting and in normal systems. To illustrate the working power of these equations I will consider the SGE in bulk materials.

For a homogeneous bulk system the Usadel equation (17) reduces the following form

\[ \frac{1}{2} \hat{\Gamma} \hat{g}, \hat{g} - [(\omega - i A_0)\tau_3 - i \partial_t \hat{g} = 0, \]  

(21)

where \( \hat{\Gamma} \) is the Dyakonov-Perel (DP) spin relaxation kernel that is defined as \( \hat{\Gamma} \hat{g} = D [A_0, \{ A_0 \}, \{ \hat{g} \}] \). The kernel \( \hat{\Gamma} \) acts only on the spin part of the GF and has the following explicit form \( \Gamma^{ab} = D (A_{kc} A^c_{kb} g^{ab} - A^c_{kb} A_{kc} g^{ab}) \) [35]. In the expression of Eq. (19) for the charge current only the last term survives, so that we have

\[ j_k = -\frac{D\tau}{m} (\nabla_i F_{ik})^a \delta S^a \]  

(22)

This equation presents a very interesting result. The spin-galvanic relation between the charge current and the induced spin is valid universally both for the normal and for the superconducting state.

In superconductors the Zeeman field induces a nonzero \( \delta S^a \) even in equilibrium because of the Knight shift - the Cooper paring reduces the paramagnetic susceptibility so that it becomes smaller than \( \chi_P \). By solving the equilibrium Matsubara version of Eq. (21) one readily finds the spin \( \delta S^a \) induced by the Zeeman field in the presence of the DP relaxation, \( \delta S^a = -\delta \chi^{ab} A^b_0 \), where

\[ \delta \chi^{ab} = \pi N_F T \sum_{\omega_n} \frac{\Delta}{\omega_n^2 + \Delta^2} \left[ \sqrt{\omega_n^2 + \Delta^2} g^{ab} + \Gamma^{ab} \right]^{-1} \]  

(23)

is the deviation of the paramagnetic susceptibility from \( \chi_P \). This equation generalizes the result of Ref. [42] to the case of diffusive superconductors and arbitrary SOC.

By inserting the induced spin into Eq. (22) we obtain the anomalous supercurrent generated by the Zeeman field

\[ j_k = \frac{D\tau}{m} (\nabla_i F_{ik})^a \delta \chi^{ab} A^a_0 \]  

(24)

For the special case of Rashba SOC and \( T \) close to the critical temperature Eqs. (23), (24) reduce to the Edelstein result [13]. The present formalism clearly demonstrates that the equilibrium SGE in gyrotropic superconductors is directly related to the Knight shift.

In the normal state \( \delta S^a \) entering Eq. (22) can be nonzero only away from equilibrium. In semiconductors the SGE can occur due to optically generated spin polarization [35, 36]. Below I consider the spin generation by a time dependent Zeeman field \( A_0(t) \). In this case from the Keldysh component of Eq. (21) we obtain the following equation of motion for the induced spin \( \delta S = \delta S^a \sigma^a / 2, \)

\[ \partial_t \delta S - i [A_0, \delta S] + \chi_P \partial_t A_0 + \hat{\Gamma} \delta S = 0 \]

The linear response solution of this equation reads

\[ \delta S(\omega) = -i \omega \chi_P [i \omega - \hat{\Gamma}]^{-1} A_0(\omega) \]  

(25)

Equations (25) and (22) determine the charge current generated via the SGE in a normal conductor. At \( \omega \) much smaller than the DP relaxation rate the expression for the current takes the form [43]

\[ j_k = \chi_P \frac{D\tau}{m} (\nabla_i F_{ik})^a (\Gamma^{ab})^{-1} \partial_t A^b_0 = \tau \chi_P \frac{A^a_0}{m} \partial_t A^b_0 \]

which is the expected results for IEE in the presence of intrinsic SOC [44].

In conclusion, I derived the full nonlinear Usadel equation and the expressions for the charge and spin currents for gyrotropic materials with a generic linear in momentum intrinsic SOC. This equation takes into account the spin-charge/singlet-triplet coupling and provides a unified description of magneto-electric effects in diffusive systems. In particular it makes a direct connection of the usual SHE (ISHE) and SGE (ISGE) to their superconducting phase-coherent counterparts. As a simple illustrative example I considered the description of SGE in bulk systems. However the most useful applications of this formalism are expected for inhomogeneous systems, hybrid metallic nanostructures, and Josephson junctions. The presented Usadel equation is perfectly suited for studying various magneto-electric transport phenomena and accompanying them modifications of the local density of states in such systems.

This work is supported by the Spanish Ministerio de Economía y Competitividad (MINECO) Project No. FIS2016-79464-P and by the “Grupos Consolidados UPV/EHU del Gobierno Vasco” (Grant No. IT578-13).

[1] I. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004)
[2] J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Zutic, Acta Physica Slovaca 57, 565 (2007), arXiv:0711.1461.
[3] J. Sinova, S. O. Valenzuela, J. Wunderlich, C. H. Back, and T. Jungwirth, Rev. Mod. Phys. 87, 1213 (2015).
[4] S. D. Ganichev, M. Trushin, and J. Schliemann, arXiv:1606.02043 (2016).
[5] E. L. Ivchenko, Y. B. Lyanda-Geller, and G. E. Pikus, Pis'ma Zh. Eksp. Teor. Fiz. 50, 156 (1989), JETP Lett. 50, 175 (1989).
[6] I. E. Ivchenko, Y. B. Lyanda-Geller, and G. E. Pikus, Zh. Eksp. Teor. Fiz. 98, 989 (1989), Sov. Phys. JETP 71, 550 (1990).
[7] A. G. Aronov and Y. B. Lyanda-Geller, Pis'ma Zh. Eksp. Teor. Phys. 50, 398 (1989), JETP Lett. 50, 431 (1989).
[8] V. M. Edelstein, Solid State Commun. 73, 233 (1990).
[9] E. L. Ivchenko and S. D. Ganichev, in Spin Physics in Semiconductors, edited by M. I. Dyakonov (Springer, Berlin, 2008) Chap. 9, pp. 245–278.
[10] S. Yip, Annu. Rev. Condens. Matter Phys. 5, 15 (2014).
[11] M. Smidman, M. B. Salamon, H. Q. Yuan, and D. F. Agterberg, Rep. Prog. Phys. 80, 036501 (2017).
[12] V. M. Edelstein, Phys. Rev. Lett. 75, 2004 (1995).
[13] V. M. Edelstein, Phys. Rev. B 72, 172501 (2005).
[14] S. K. Yip, Phys. Rev. B 65, 144508 (2002).
[15] E. Konschelle, I. V. Tokatly, and F. S. Bergeret, Phys. Rev. B 92, 125443 (2015).
[16] A. Buzdin, Phys. Rev. Lett. 101, 107005 (2008).
[17] F. S. Bergeret and I. V. Tokatly, EPL 110, 57003 (2015).
[18] J. Linder and J. W. A. Robinson, Nature Phys. 11, 307 (2015).
[19] M. Eschrig, Physics Today 64, 43 (2011).
[20] A. A. Burkov, A. S. Núñez, and A. H. MacDonald, Phys. Rev. B 70, 155308 (2004).
[21] K. Shen, R. Raimondi, and G. Vignale, Phys. Rev. B 90, 245302 (2014).
[22] R. Raimondi, P. Schwab, C. Gorini, and G. Vignale, Annalen der Physik 524, 153 (2012).
[23] K. D. Usadel, Phys. Rev. Lett. 25, 507 (1970).
[24] M. Y. Kuprianov and V. Lukichev, Sov. Phys. JETP 67, 1163 (1988).
[25] A. G. Mal’shukov and C. S. Chu, Phys. Rev. B 78, 104503 (2008).
[26] A. G. Mal’shukov, S. Sadjina, and A. Brataas, Phys. Rev. B 81, 060502 (2010).
[27] Throughout this paper we adopt the summation convention over a pair of repeated indexes.
[28] R. Winkler, in Handbook of Magnetism and Advanced Magnetic Materials, Vol. 5: Spintronics and Magnetoelectronics, edited by H. Kronmüller and S. Parkin (Wiley, 2007).
[29] An explicit form of the linear SOC for all 18 gyrotropic crystal classes can be found, for example, in Table 2 of Ref. [11].
[30] V. Mineev and G. E. Volovik, J. Low Temp. Phys. 89, 823 (1992).
[31] J. Fröhlich and U. M. Studer, Rev. Mod. Phys. 65, 733 (1993).
[32] F.-Q. Jin, Y.-Q. Li, and F.-C. Zhang, J. Phys. A: Math. Gen. 39, 7115 (2006).
[33] I. V. Tokatly, Phys. Rev. Lett. 101, 106601 (2008).
[34] C. Gorini, P. Schwab, R. Raimondi, and A. L. Shelankov, Phys. Rev. B 82, 195316 (2010).
[35] F. S. Bergeret and I. V. Tokatly, Phys. Rev. B 89, 134517 (2014).
[36] F. S. Bergeret and I. V. Tokatly, Phys. Rev. Lett. 110, 117003 (2013).
[37] S. H. Jacobsen and J. Linder, Phys. Rev. B 92, 024501 (2015).
[38] S. H. Jacobsen, J. A. Ouassou, and J. Linder, Phys. Rev. B 92, 024510 (2015).
[39] J. Arjoranta and T. T. Heikkilä, Phys. Rev. B 93, 024522 (2016).
[40] One can show that corrections from higher moments are at least of the order of $\tau^3$.
[41] F. S. Bergeret and I. V. Tokatly, Phys. Rev. B 94, 180502 (2016).
[42] L. P. Gor’kov and E. I. Rashba, Phys. Rev. Lett. 87, 037004 (2001).
[43] The identity $D\nabla F_{\alpha} = D[A_{\alpha}, [A_{\alpha}, A_{\alpha}]] = \hat{\Gamma} A_{\alpha}$ is used in the second equality.
[44] K. Shen, G. Vignale, and R. Raimondi, Phys. Rev. Lett. 112, 096601 (2014).