GRavitational Pulse Astronomy

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Abstract

Thompson has argued that the Kozai mechanism is primarily responsible for driving white dwarf binary mergers and so generating type Ia supernovae (SNe). If so, the gravitational-wave signal from these systems will be characterized by isolated repeating pulses that are well approximated by parabolic encounters. I show that it is impossible to detect these with searches based on standard assumptions of circular binaries, nor could they be detected by analogs of the repeating-pulse searches that have been carried out at higher frequencies, even if these were modified to barycentric time as a function of putative sky position. Rather, new search algorithms are required that take account of the intrinsic three-body motion of the source as well as the motion of the Earth. If these eccentric binaries account for even a modest fraction of the observed SN rate, then there should be of order 1 pulse every 20 s coming from within 1 kpc, and there should be of order 200 detectable sources in this same volume. I outline methods of identifying these sources both to remove this very pernicious background to other signals and to find candidate SN Ia progenitors, and I sketch practical methods to find optical counterparts to these sources and so measure their masses and distances.

Key words: gravitational waves - supernovae: general - white dwarfs

1. Introduction

Except near final in-spiral, binary-star gravitational-wave (GW) sources are strictly periodic, and therefore a Fourier transform of the amplitude can be used to detect these sources. The only nuance is that one must also search over the sky by converting to a barycentric frame as a function of sky position. And, once a source is located, one can then “tune up” the signal by fitting for the phase and spatial orientation of the binary orbit.

Until very recently, it was believed that the vast majority of individually identifiable binary sources would be on circular orbits. In this case the signal is sinusoidal, so a Fourier transform is not merely convenient, it also contains a matched filter to the signal, and so is “optimal.” However, Thompson (2011) has shown that the great majority of white dwarf (WD) binary sources probably have highly eccentric orbits. Within a “Fourier framework,” the resulting signal is represented by a discrete power spectrum, which covers the range \([2\pi/P, \Omega]\), where \(P\) is the orbital period, \((\Omega P/2\pi) = (1 - e)^{-3/2}\), and \(e\) is the eccentricity.

However, for eccentric orbits, Fourier transform no longer provides anything like a matched filter. For isolated binaries, one could in principle simply extend the search by including eccentricity and time of periastron as additional parameters. Then, since such systems are still strictly periodic (or almost so), one could fold the, say, five-year GW data stream by the period (modulated by the annual motion of Earth as a function of putative sky position) to optimize signal detection.

Unfortunately, the eccentric WD binaries predicted by Thompson (2011) are, by their very nature, not isolated. The high eccentricities are induced by the presence of a third body with a separation that is \(\sim 10–100\) times larger than the semi-major axis of the eccentric binary. The lower limit of this range is required for hierarchical stability and the upper limit to make the Kozai mechanism effective. Thus, several extensions of existing search techniques will be required to detect these objects. Note that the low-mass density of WDs implies low-frequency waves, which are only detectable from space.

Some intuition into this problem is gained by noting that the signal is completely dominated by pulses near periastron and that during these passages the eccentric orbit is hardly distinguishable from a parabolic one. Here, I develop this approximation and apply it to the problem to extracting science from gravitational pulse signals.

2. The Parabolic Pulse

Parabolic orbits may be parameterized by

\[
\frac{r}{b} = 1 + \frac{\psi^2}{2}, \quad \frac{x}{b} = 1 - \frac{\psi^2}{2}, \quad \frac{y}{b} = \sqrt{2}\psi, \quad \Omega t = \psi + \frac{\psi^3}{6}. \tag{1}
\]

Here, \(r \equiv (x, y)\) is the separation between the two masses, \(b\) is the distance at periastron, \(\psi\) is a parameter, \(\Omega = \sqrt{GM/b^3}\), \(M\) is the total mass, and \(t\) is time. In the quadrupole approximation, the gravitational strain tensor \(h_{ij}\) is given by (Misner et al. 1973, hereafter MTW, Equations (36.3) and (36.47))

\[
h_{ij} = \frac{2}{D} \frac{d^2 I_{ij}^{\text{traceless}}}{dt^2} = I_{ij}^{\text{traceless}} - \frac{1}{3} \delta_{ij} \sum_k I_{kk}, \quad (G = c = 1), \tag{2}
\]

where \(I_{ij}^{\text{traceless}}\) is the traceless part of the moment of inertia tensor \(I_{ij}\) and \(D\) is the source distance.

Differentiation yields

\[
\frac{h_{xx}}{A} = \frac{3y^2r - 6x^2b - 2r^2b}{3r^3}, \quad \frac{h_{yy}}{A} = \frac{12xb^2 - 2r^2b}{3r^3}, \quad \frac{h_{zz}}{A} = -\frac{2b}{3r}, \quad \frac{h_{xy}}{A} = \frac{y^3}{r^5} \tag{3}
\]

where

\[
A \equiv \frac{2 M_1 M_2}{bD} = 2.1 \times 10^{-21} \frac{M_1 M_2 M_0}{0.1 R_\odot} \left( \frac{b}{0.1 R_\odot} \right)^{-1} \left( \frac{D}{\text{kpc}} \right)^{-1} \tag{4}
\]
and (b)). The wave-amplitude waveform seen at Earth, one would have to rotate according to the Euler angles and then project out only the transverse components. The wave-amplitude components sum to zero at all times, since the matrix is traceless. To find the normalization of the GW amplitude, is the normalization of the GW amplitude, $M_1$ and $M_2$ are the two masses, and $M = M_1 + M_2$. These waveforms are shown in Figure 1. As expected, they are strongly concentrated in a small interval of time $\pm \Omega^{-1}$ near periastron, where

$$\Omega^{-1} = \sqrt{\frac{b^3}{M}} = 50 \text{s} \left(\frac{b}{0.1 R_\odot}\right)^{3/2} \left(\frac{M}{M_\odot}\right)^{-1/2}. \quad (5)$$

The total energy of the pulse is most easily calculated by taking the limit $e \to 1$ in the standard formula for the mean luminosity of an eccentric binary (MTW; Equations 36.16(a) and (b))

$$E_{\text{pulse}} = \frac{85\pi}{16} M_1^2 M_2^2 \sqrt{\frac{M}{2b^3}}. \quad (6)$$

This energy may be directly compared to the potential energy at periastron,

$$Q \equiv \frac{M_1 M_2}{b} E_{\text{pulse}}$$

$$= 4 \times 10^{10} \left(\frac{M_1}{M_\odot}\right)^{1} \left(\frac{M_2}{M_\odot}\right)^{1} \left(\frac{b}{0.1 R_\odot}\right)^{5/2}. \quad (7)$$

Hence, if the Thompson (2011) Kozai mechanism is responsible for even one Milky Way supernova (SN) Ia per 500 years, then there must be one pulse per (500 years)/$Q \sim 0.2$ s. And even supposing that only 1% of these are within 1 kpc of the Sun, there would still be one such “loud” pulse every 20 s. Hence, disentangling these pulses is important both from the standpoint of understanding the sources and removing them as a background.

In addition, if this mechanism is indeed important for generating SNe Ia, then it will almost certainly create a large population of in-spiraling WDs that lack sufficient combined mass to explode but still contribute to the gravitational pulse cacophony.

3. PULSE SEARCH STRATEGY I: ISOLATED ECCENTRIC BINARIES

I begin by analyzing the simplified case of isolated eccentric binaries. As just noted, the real sources are not isolated, but this permits me to explore most of the relevant physics before introducing this additional complication.

Then because the pulses are periodic, they would in principle turn up in a Fourier-transform type search. However, the signal-to-noise ratio ($S/N$) of such a search would be degraded relative to a matched-filter search by roughly $(P\Omega/2\pi)^{1/2} \sim (1 - e)^{-3/4}$. For example, for $e = 0.99$, the degradation would be a factor $\sim 30$, meaning that roughly 30 times smaller signals would be accessible to a matched-filter search than a Fourier-transform search.

Of course, there is some cost associated with enhanced sensitivity. For each period $P$, one would need to consider $P\Omega$ different independent phases, where I adopt $\Omega^{-1} = 40$ s as the typical value. And one would have to step through the periods at $\Delta P = P/(\Omega T_{\text{mission}})$, leading to a total (for each sky position) of $N_{\text{try}} = T_{\text{mission}} P_{\text{max}}\Omega^2 = 3 \times 10^{10}$ independent trials, where I adopt $T_{\text{mission}} = 5$ years as the lifetime of the mission and $P_{\text{max}} = 3$ days as the maximum period that will be searched. And for each such trial, one would have to try several, perhaps 10, independent pulse widths $\Omega^{-1}$. Efficient algorithms have already been developed for carrying out such searches in studies of both periodic GW sources (Abadie et al. 2010; and references therein) and transiting planets (Kovács et al. 2002). However, there is also a cost associated with the added risk of false positives. If one assumes Gaussian noise, then the minimum $S/N$ required to avoid false positives is $\sim \sqrt{2 \ln N_{\text{try}}}$. If, for example, the number of independent trials goes from $10^4$ to $10^5$, then the minimum $S/N$ goes roughly from 4.3 to 8.3, i.e., a factor of two. This is very modest compared to the factor $\sim 30$ gain in sensitivity due to matched filters.

4. A ROUGH ESTIMATE OF LISA SOURCE COUNTS

To proceed further, I consider the specific example of the Laser Interferometer Space Antenna (LISA; Prince et al. 2009), a proposed space mission composed of three antennae separated by $L = 0.033$ AU in an Earth-like heliocentric orbit. LISA sensitivity peaks for sinusoidal periods at $P \sim 150$ s at a threshold of $h \sim 10^{-23}$, with a full width half-maximum (FWHM) of about one decade. Hence, it is extremely well matched to pairs of $0.7 M_\odot$ WDs with periastra $b \sim 0.1 R_\odot$, which have pulse widths corresponding to “sinusoidal periods” of about $2\pi/\Omega \sim 260$ s. That is, for WD binaries with $M = 1.4 M_\odot$, the FWHM of LISA sensitivity corresponds to

$$b = 0.073 \pm 0.08 \quad (\text{FWHM}). \quad (8)$$

However, when comparing forecasted LISA sensitivity to Equation (4), one must adjust by a factor $\sim (1 - e)^{-3/4}$ to account for the fact that the signal only builds up once per period $\frac{b}{0.1 R_\odot}$.
energy loss is governed by simple estimate by assuming that all pulses are the same. Then beyond the scope of this Letter. However, one can make a interplay of pre-WD and post-WD Kozai, binary evolution, LISA will be sensitive to such binaries to several kpc.

Figure 1 that the signal peaks at a few times \( \Omega \). The Astrophysical Journal Letters

et al. 2009) normalized to unity at maximum sensitivity at \( \sim \). The signal peaks at a few times \( \Omega \). The Astrophysical Journal Letters

integrating the square of the time derivative of the wave forms in Figure 1 yields the timing precision of each pulse, \( \sigma_t \approx 0.8 \Omega^{-1} S_{\text{N pulse}}^{-1} \). Hence, if all the \( N_{\text{pulse}} = T_{\text{mission}}/P \) pulses from a single source can be successfully aligned, the angular direction precision from fitting to the correct barycentric pulse-delay pattern is (in radians)

\[
\sigma_\theta = \sqrt{\frac{2}{N_{\text{pulse}}} \frac{\sigma_t}{\Delta U}} = \frac{0.09}{S/N} (40 \text{ s } \Omega^{-1})^{-1}.
\]  

That is, sources within \( D \lesssim 1 \text{ kpc} \) could be located within a few arcminutes. Position measurements from LISA orbital motion alone would suffer an exact degeneracy in ecliptic latitude.

The directional information relative to the spatial orientation of LISA is about \( AL = 30 \text{ times} \) worse than the orbital information, but it does not suffer from this ecliptic degeneracy. It would therefore be sufficient to break the orbit-based-direction degeneracy for nearer sources, but not more distant ones.

6. PULSE SEARCH STRATEGY II: TRIPLES

As mentioned in the Introduction, the Thompson–Kozai WD binaries are all embedded in triples with semimajor axis ratios \( a_2/a_1 \sim 10–100 \). Hence \( 0.3 \lesssim a_2/\Delta U \lesssim 10 \). To illustrate the problems posed by this, I consider the case \( M_1 = M_2 = M_3 = 0.7 M_\odot \), where \( M_3 \) is the third body. Then, the binary will orbit the center of mass with an amplitude \( a_2/3 \), which means that, if not corrected, the pulse signal will drift by \( \sim 170 \text{ s} (a_2/\Delta U) \sin i \), where \( i \) is the inclination. This is many times larger than the pulse width, especially considering that for \( a_2 \lesssim 6 \text{ AU} \), the system will complete at least half an orbit in 5 years. Hence, if the orbit around the third body is not included, the pulses will not align, even approximately, and the signal cannot be recovered.

Hence, for each sky position (and its corresponding barycentric correction) and each combination of \( (\Omega, P, \ell_{\text{peri}}) \), where \( \ell_{\text{peri}} \) is the time of pericenter of the inner binary, one must conduct at least a three-parameter search, corresponding to the period, phase, and amplitude of a sinusoidal orbit about \( M_3 \). Then the number of such trials would be \( \sim [(a_{2,\max}/3)\Omega]^{3} \sim 10^8 \), where \( a_{2,\max} \sim 6 \text{ AU} \). For \( a_2 > a_{2,\max} \) a simpler one parameter uniform-acceleration model would be adequate.

Thus, the full search would be over \( 4\pi (\Delta U)^2 \) sky positions, \( P_{\text{max}} T_{\text{mission}} \Omega^{-2} \sim 3 \times 10^{10} \) inner binary periods and phases, \( [(a_{2,\max}/3)\Omega]^{3} \sim 10^8 \) outer-binary trials, and perhaps 10 different pulse widths. This is \( 10^{19} \) independent trials, which requires a detection threshold of \( S/N > 9 \). The shear volume of computations is forbidding. As already mentioned, there are efficient algorithms for doing the \( 3 \times 10^{10} \) period search calculations. The problem is that these must each be done for \( 2 \times 10^{10} \) different configurations. Here I will simply assume that Moore’s law will handle this problem, although there is probably room for algorithmic improvements as well. In particular, the shorter-period binaries that dominate the “noise” will be detectable from subintervals \( T \ll T_{\text{mission}} \), which will permit elimination of the outer-binary trials, and even allow standard Fourier techniques in many cases. Removal of this dominant “noise” will be essential in finding the more numerous high-eccentricity binaries.

7. EXTRACTING SCIENCE

What parameters can be extracted from such observations, and how can the remaining degeneracies be resolved? I will

\( P \), not once per \( 2\pi/\Omega \). Even so, for \( e = 0.99 \), LISA is sensitive to strains just 30 times higher than its nominal sensitivity, i.e., \( h \sim 3 \times 10^{-22} \). Comparing to Equation (4) and noting from Figure 1 that the signal peaks at a few times \( A \), it is clear that LISA will be sensitive to such binaries to several kpc. The period distribution of sources is governed by a complex interplay of pre-WD and post-WD Kozai, binary evolution, GW emission, and tidal effects (Thompson 2011), which are beyond the scope of this Letter. However, one can make a simple estimate by assuming that all pulses are the same. Then energy loss is governed by \( dE/dt \propto 1/P \), which implies that the cumulative number of systems is \( N(P) \propto P^{1/3} \). Hence, the population is weakly dominated by high-period systems, while pulse generation is dominated by tight systems: \( (dN/d\ln P)/P \propto P^{-2/3} \). This is important. It means that most of the “noise” is concentrated in the highest \( S/N \) objects, making them the easiest to detect. But it also means that the most interesting sources are in the long-period tail. If we assume a typical “injection eccentricity” \( e_{\max} \approx 0.99 \), then \( 2/3 \) of the systems will have \( e \gtrsim 0.9 \). And if the ensemble of sources is responsible for 1 SNe Ia per 50,000 years within 1 kpc, then there are of order 200 such binaries within 1 kpc. Obviously, this is a very crude estimate, but the point is that there are potentially many more sources than there would be to supply the same SN Ia rate from circular orbits.

5. SIGNAL-TO-NOISE RATIO ESTIMATES

To make a more precise estimate of the expected \( S/N \), I integrate the square of the profiles shown in Figure 1, averaging over all Euler orientations, and restricting the integral to \( \pm 3 \Omega^{-1} \). I assume that the LISA sweet spot permits detection of sources with rms strain \( h = A_0 = 1.0 \times 10^{-23} \) in \( T_0 = 1 \) year of observation at 5\( \sigma \) (Prince et al. 2009, pp 2, 21). I then find a \( S/N \) for a single pulse of

\[
S_{\text{N pulse}} = 20 \frac{A}{A_0} (\Omega T_0)^{-1/2}
\]

\[\approx 5.4 \frac{M_1 M_2}{M_1^{1/4} M_2^{7/12}} \left( \frac{b}{0.1 R_\odot} \right)^{-1/4} \left( \frac{D}{\text{kpc}} \right)^{-1} F(\Omega),
\]

where \( F(\Omega) \) is the functional form of LISA sensitivity (Prince et al. 2009) normalized to unity at maximum sensitivity at \( \Omega^{-1} = 24 \) s. Note that this formula basically scales \( \propto M_1 M_2/D \). The integrated \( S/N \) from summing all the pulses observed during the mission is larger by

\[
S = S_{\text{N pulse}} \sqrt{T_{\text{mission}}/P}.
\]

For example, for \( \Omega^{-1} = 40 \text{ s} \) and \( e = 0.99 \), we have \( P = 2\pi \Omega^{-1}(1 - e)^{-3/2} \approx 3 \) days, so \( S = 25 S_{\text{N pulse}} \). Thus, even requiring a 9\( \sigma \) detection (see below) would permit detections throughout the Galaxy, provided that multiple pulses from the same source could be so identified and thus “co-added.” Moreover, individual pulses from within \( D \lesssim 1 \text{ kpc} \) would be detectable. They and their weaker cousins from farther away would constitute an incredible data mine if they could be interpreted, but a vast cacophony of noise if they could not. Since the great majority of this “noise” is due to the small number of binaries that have already been driven to shorter periods, it is important to note that for \( e < 0.1, S/N > 140 S_{\text{N pulse}}, \) implying that these sources are much easier to extract and “remove.”

Integrating the square of the time derivative of the wave forms in Figure 1 yields the timing precision of each pulse, \( \sigma_t \approx 0.8 \Omega^{-1} S_{\text{N pulse}}^{-1} \). Hence, if all the \( N_{\text{pulse}} = T_{\text{mission}}/P \) pulses from a single source can be successfully aligned, the angular direction precision from fitting to the correct barycentric pulse-delay pattern is (in radians)
argue that full resolution requires identification of optical counterparts. The counterparts that are easiest to identify are three-WD systems, and these are also the most likely to yield key spectroscopic data leading to complete resolution. I therefore focus first on these systems. There will of course be a huge number of systems without counterparts, which could be subjected to statistical analysis, but the analysis of that problem is beyond the scope of this Letter.

First, I review the observables. The best-fit barycentric correction gives the direction on the sky, and the waveform gives the three Euler angles of the binary (up to discrete degeneracies due to the quadrupole nature of GWs). These will be of use further below but I ignore them for the moment. There are also (at least) three orbital parameters for third body. The remaining four parameters that can be measured are \( A, \Omega, P, \) and \( t_{\text{peri}} \).

The period is a parameter of fundamental interest since it helps classify the systems according to their evolutionary state and rate of progression toward WD collisions. Together with \( \Omega \), the period also gives the eccentricity, which is again of independent interest and also enables a more precise waveform calculation (although I expect that this will be a very minor consideration for high eccentricity orbits). The time of periastron will also be important further below, but will be ignored for the moment. This leaves two observables

\[
A = 2 \frac{M_1 M_2}{b D}, \quad \Omega^2 = \frac{M_1 + M_2}{b^3}
\]

(12)

that combine four physical parameters, \( M_1, M_2, b, \) and \( D \). It is important to keep in mind that \( \Omega, P, \) and \( t_{\text{peri}} \) are all robustly determined from distinct features in the data, while \( A \) is somewhat degenerate with the Euler angles, particularly the inclination and longitude of nodes. However, I ignore this degeneracy for the moment and assume that \( A \) is also well determined. Then, since there are four parameters and two measurements, there remain two degeneracies to be broken for which two additional pieces of information are required. Real additional “information” can only be derived from counterparts, but at the outset, one can gain a rough idea of \( b \) and \( D \) simply by assuming, e.g., \( M_1 = M_2 = 0.6 M_\odot \). Of course, such an assumption would make it impossible to derive the masses and thus determine whether the system was a viable SN Ia progenitor, which is arguably the most interesting science potential of the sample. But it would give a rough estimate of the distance.

To make any further progress would require a catalog of WD candidates over the estimated distance range of the sample. Optical identification of WDs would be extremely difficult if the third star in the system were a main-sequence star. However, a substantial fraction of tertiaries are likely to be descendants of intermediate mass stars, and so themselves WDs. The most efficient way to construct a catalog of WDs (or multiple WDs) is a reduced proper motion survey (e.g., Salim & Gould 2003).

This will be a natural by-product of the Large Synoptic Survey Telescope (LSST) for about 3/4 of the sky. Since WDs are typically \( M_V \lesssim 15.5 \), LSST should reach several hundred pc or more, depending on its exact performance. For example, Abell et al. (2009) expect that at \( r = 24 \) (so \( D \sim 400 \) pc, allowing for extinction \( A_r \sim 0.5 \)) the “main” LSST survey (1/2 sky) will achieve a proper motion precision of 1 mas yr\(^{-1}\), corresponding to a transverse velocity error of 2 km s\(^{-1}\). This is about five times better than is required to reliably identify a WD on a reduced proper motion diagram. Hence, even in the less well-covered northern 1/4 of the sky, LSST reduced proper motion diagrams should be adequate to about 400 pc.

Recall from Section 5 at these distances \( S/N \gtrsim 60 \) and hence the position is known to \( \lesssim 5' \). Since the surface density of WDs to this distance is only \( \sim 25 \text{deg}^{-2} \), there would be only a few candidate objects consistent with the position determined from GWs, even allowing for a factor \( \sim 2 \) uncertainty in the reduced-proper-motion distance estimates (and the smaller error in \( A \) for these high S/N objects).

A spectrum taken near periastron would, by itself, positively identify the system as the LISA counterpart because the velocity difference between the two components would be of order 1000 km s\(^{-1}\). Even, if the system turned out to be single-lined, the observed (brighter) component would be the less massive, and therefore would be moving at extremely high velocity. If the system were double lined, the radial velocity curve would give the mass ratio \( M_1/M_2 \), while the spectra themselves would yield individual masses. The ratio of these could be checked against the radial velocity value, while the sum would yield \( \sin i \), which could be checked against the value determined from the GW pulse profile. A similar exercise could be applied to the third WD, yielding a comprehensive picture of the entire system. Even if the more massive (so fainter) WD were beyond the detection limit, the degeneracies could be partly resolved by obtaining a trigonometric parallax.

These determinations can probably be made for a large fraction of WD triples out to 400 pc, which plausibly number in the dozens. Moreover, since their individual pulses can be detected, the algorithms to identify these sources are much simpler than those outlined above.

Finally, I note that WD binaries with main-sequence companions can also be positively identified from the correspondence between the pulse timing residuals and the radial velocity curve of the companion. While the WDs would not be directly observable, precise astrometric measurements combined with a spectroscopic mass estimate of the companion would yield both \( D \) and \( M_1 + M_2 \), which (with Equation (12)) permit a complete solution.

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