Numerical Simulation of Plasma Detachment from a Magnetic Nozzle by using Fully Particle-In-Cell Code

R Kawabuchi\textsuperscript{1}, N Matsuda\textsuperscript{1}, Y Kajimura\textsuperscript{1}, H Nakashima\textsuperscript{1} and Y P Zakharov\textsuperscript{2}

\textsuperscript{1) Department of Advanced Energy Engineering Science, Kyushu University, 6-1 Kasugakouen, Kasuga, Fukuoka 816-8580, Japan.
\textsuperscript{2) Institute of Laser Physics, Russian Academy of Sciences, Russia. Novosibirsk 630090, Russia.
E-mail: kawabuchi@aees.kyushu-u.ac.jp

Abstract. A numerical simulation is performed by using a fully 2D3V electromagnetic Particle-In-Cell (TRISTAN) code in order to clarify the plasma behavior and the possibility of the plasma detachment from the magnetic nozzle. As a result of calculation, the plasma detachment was found.

1. Introduction

A magnetic nozzle which can control a plasma behavior and can generate a propulsive force is adopted in some space propulsion systems. Laser Fusion Rocket (LFR) and Variable Specific Impulse Magnetoplasma Rocket (VASIMR) are representative examples for these propulsion systems. The LFR has a magnetic nozzle which can control the plasma flow resulting from a laser fusion. The VASIMR also has a magnetic nozzle which can control the plasma produced by a MPD arc jet. The mechanism of thrust production in a magnetic nozzle of the LFR is given in figure 1. So far, several estimations of thrust efficiency of an LFR have been conducted, for example, Hyde estimated the thrust efficiency using two-dimensional (2D) magnetohydrodynamics (MHD) code [1]. Nagamine and Nakashima calculated plasma behaviors and thrust efficiency by using a three-dimensional (3D) hybrid code [2]. The 3D hybrid numerical simulation code treats ions as individual particles and electrons as a fluid. We have conducted an optimization study for thrust efficiency in a magnetic nozzle [3].

In the plasma controlled by a magnetic nozzle, there is an issue to be overcome, i.e., plasma detachment. The schematic of this phenomenon is shown in figure 2. Electrons have the Larmor radiuses much smaller than ions. So the electrons move along the magnetic field line and come back to the spacecraft. That is, an electric field is generated by a charge separation of ions and electrons. Consequently, ions may be pulled back to the spacecraft. To obtain thrust, it is necessary that electrons should be detached from the magnetic field and exhausted. We can not simulate this phenomenon by the 3D hybrid numerical simulation code which we have been using.

In the present study, we conduct a numerical simulation by using a 2D3V electromagnetic fully Particle-In-Cell (TRISTAN) code in order to clarify the plasma behavior and the possibility of the plasma detachment from a magnetic nozzle.
2. Simulation Code and Model
The calculation model and parameters are based on the experiment performed by Zakharov et al [4]. In the simulation by TRISTAN code, it is necessary that a grid size is nearly equal to the Debye length. However, the plasma in the experiment has only kinetic energy, and it is estimated that the plasma is at a very low temperature. Thus, in this calculation, the plasma temperature is supposed to be zero, and the Debye length is not defined. The Maxwell’s equations and the particle motion equation are the basic equations of this code.

\[
\nabla \times E = -\frac{\partial B}{\partial t} \tag{1}
\]

\[
\nabla \times B = \mu_0 J + \frac{1}{c^2} \frac{\partial E}{\partial t} \tag{2}
\]

\[
\frac{dP_p}{dt} = q_p (E + v_p \times B) \tag{3}
\]

where \( E \) is the electric field, \( B \) the magnetic field, \( \mu_0 \) the vacuum magnetic permeability, \( J \) the current density, \( c \) the velocity of light, \( P_p \) the momentums of particles, \( q_p \) the charges, and \( v_p \) the particle velocities. \( p = i \) (ion) and \( p = e \) (electron). \( J \) and \( P_p \) are respectively given by

\[
J = \sum_p q_p n_p v_p \tag{4}
\]

\[
P_p = m_p \gamma_p v_p \tag{5}
\]

where \( n_p \) is the density of particles, and \( \gamma_p \) is given by

\[
\gamma_p = \left(1 - \frac{v_p^2}{c^2}\right)^{-1/2} \tag{6}
\]

The current density in each grid is calculated by distributing the density and velocity of each particle to each calculation grid. This current density has an important role in connecting the motion of...
charged particles and the electromagnetic field. A second-order central finite difference method is used in the spatial calculation and a second-order Runge-Kutta method is used as the time integration scheme.

The calculation model considered here is illustrated in figure 3. The system size used in the simulation is $L_x = 100\Delta$ and $L_y = 100\Delta$. $\Delta = 1.0$ is a grid width. $\omega_{pe}\Delta t = 0.05$, where $\omega_{pe}$ is the electron plasma frequency, $\Delta t$ the integration time step, and the electron collision-less skin depth is $d_e = c/\omega_{pe} = 10\Delta$. The initial plasma is assumed to have a radius $R_p$ of $2\Delta$, and it is neutral. Particles do not have thermal velocities, and only have isotropic expansion velocities. To obtain the parameters in this calculation, we focus on the parameter $\epsilon_b$. $\epsilon_b$ is an important parameter of a plasma [5]. $\epsilon_b$ is defined as $\epsilon_b = R_{Li}/R_b$, where $R_{Li} = m_i V_0/(Ze B_0)$ and $R_b \approx (3\mu_0 E_0/(2\pi B_0))^{1/3}$. $m_i$ is the ion mass, $V_0$ the initial ion velocity, $Z$ the charge rate, $e$ the elementary electric charge, $B_0$ the magnetic field of the coil in the point of the plasma and $E_0$ the kinetic energy. $R_{Li}$ is the ion Larmor radius. Retardation radius $R_b$ is calculated under the assumption that the kinetic energy density of all the plasma particles is equal to the density of the magnetic field energy. $\epsilon_b$ in this calculation is set equal to that in the experiment. The general calculation parameters used in the simulation are shown in table 1. $R_C$ is the radius of a coil, $L_0$ the distance of the coil to the plasma, and $R_{Le}$ the electron Larmor radius. $m_e$ is the electron mass. Here, we assume a single kind of ion with a charge state $Z$ of +2.5 and a mass of 6.5 amu.

![Figure 3. Schematic calculation model.](image)

| Table 1. Simulation parameters. |
|----------------------------------|
| Parameter | VISTA project[6] | KI-1. experiment[4] | Present calculation |
|----------|-------------------|---------------------|---------------------|
| $L_0/R_C$| 1                 | 3                   | 3                   |
| $R_{Li}/R_C$ | $4.8 \times 10^{-3}$ | 0.76 | 0.65 |
| $R_{Le}/R_C$ | $1.3 \times 10^{-6}$ | $1.6 \times 10^{-4}$ | 0.28 |
| $\epsilon_b$ | $\sim 0.001$ | 1 | 1 |
| $m_i/m_e$ | 3672 | 11934 | 13 |
| $V_0/c$ | 0.001 | 0.00047 | 0.00047 |

3. Simulation Results

The time history of momentums of plasma for each direction component is shown in figure 4. The x-component of it increases with time. Thus, the magnetic field worked as a magnetic nozzle.

Ion and electron particles positions at 120000 time steps and particles trajectories are shown in figure 5. The plasma shape is spherical in the initial stages and the plasma expands isotropically. First, the difference of the velocities between ions and electrons generated the outward electric field, and ions are accelerated. Then the ions moving in the direction of the coil are reflected back by the magnetic field, and the shape of the ion distribution changes to follow the magnetic field line, as is shown in figure 5(a). On the other hand, some electrons stayed at the initial positions because they strongly wind around the magnetic field lines and trapped by the electric field, and other electrons follow ions. Especially, the electrons moving in the ±Y direction are pulled in the direction of exhausted ions by the electric field, that is, the plasma detachment was found.
Figure 4. Momentums of plasma as a function of time.

(a) Position of Ions.
(b) Position of electrons.
(c) Trajectories of ions.
(d) Trajectories of electrons.

Figure 5. Position of ions and electrons at 120000 time steps and their trajectories.

4. Conclusion
Here, we conduct a numerical simulation by using a 2D3V electromagnetic fully Particle-In-Cell (TRISTAN) code in order to clarify the plasma behavior and the possibility of the plasma detachment from a magnetic nozzle. As a result of calculation, the plasma detachment was found. We will improve this code, and calculation condition for a further study.

5. References
[1] Hyde R A 1983 UCRL-88857.
[2] Nagamine Y, Nakashima H 1999 Fusion Technology. 35 62-70.
[3] Sakaguchi N, Kajimura Y and Nakashima H 2005 Trans. Japan Soc. Aero. Space Sci., 48 180
[4] Zakharov Yu P, Orishich A M, Posukh V G, Saikhislamov I F 2001 Abstracts, 4th symposium on Current Trends in International Fusion Research, Washington, DC, 31-34
[5] Zakharov Yu P, Orishich A M, Ponomarenko A G and Posukh V G 1986 Sov. J. Plasma Phys., 12 674