A deep-learning based hybrid strategy for short-term load forecasting is presented. The strategy proposes a novel tree-based ensemble method Warm-start Gradient Tree Boosting (WGTB). Current strategies either ensemble submodels of a single type, which fail to take advantage of statistical strengths of different inference models. Or they simply sum the outputs from completely different inference models, which doesn’t maximize the potential of ensemble. WGTB is thus proposed and tailored to the great disparity among different inference models in accuracy, volatility and linearity. The complete strategy integrates four different inference models (i.e., auto-regressive integrated moving average, nu support vector regression, extreme learning machine and long short-term memory neural network), both linear and nonlinear models. WGTB then ensembles their outputs by hybridizing linear estimator ElasticNet and nonlinear estimator ExtraTree via boosting algorithm. It is validated on the real historical data of a grid from State Grid Corporation of China of hourly resolution. The result demonstrates the effectiveness of the proposed strategy that hybridizes statistical strengths of both linear and nonlinear inference models.

**Keywords** Short-term load forecasting · Hybrid model · Gradient tree boosting · ElasticNet · Long short-term memory neural network

1 Introduction

Electric power is a non-storable product, electric power utilities have to ensure a precise balance between the electricity production and consumption at any moment. Therefore, load forecasting plays a vital role in the daily operational management of power utility, such as energy transfer scheduling, unit commitment, load dispatch, and so on [1][2][3]. With the emergence of load management strategies, it is highly desirable to develop accurate load forecasting models for these electric utilities to achieve the purposes of higher reliability and management efficiency [4][5].

The inference models for Short-term Load Forecasting (STLF) can be classified into two categories based on linearity [6]. The first type is linear models (or auto regressive models), which are mainly used to forecast data with high linearity.
One of the most common forecasting techniques amongst the auto regressive models is Autoregressive Integrated Moving Average (ARIMA) model [7,8]. The other type is nonlinear models (or machine learning models), which can handle the nonlinear data forecasting. The Support Vector Regression (SVR) is a highly effective model in machine learning and has the capability of solving nonlinear problems, even with small quantities of training data [9,10,11]. As a fresh field of machine learning, Artificial Neural Network (ANN) has attracted much more attention for STLF [12,13,14], especially the Long Short-term Memory (LSTM) neural network [15,16]. The SVR and ANN models usually require a substantial amount of time to train the forecasting model. The Extreme Learning Machine (ELM) can reduce the training time of neural networks and achieve a good accuracy of forecasting. Thus ELM has become a popular forecasting technique for STLF due to its faster performance [17,18,19].

Nonetheless, all inference methods have their own flaws and weaknesses, which, if they are used singly, will impinge on the forecasting precision. Furthermore, most of the forecasting methods rely heavily on the presumed data patterns so that no single model is suitable for all. Hybrid models are thus created to aggregate advantages of individual models [20,21]. Hybrid models can be classified into two categories based on the number of types of inference submodels. Models in the first category contains a single inference submodel, usually ANN. They sum the outputs from multiple identical ANNs using bagging [22], boosting [23] or a combination of both [24]. And a fuzzy logic based approach was recently proposed by Sideratos that ensembles multiple radial basis function-convolutional neural networks [25]. Models in the other category consist of multiple types of inference submodels, such as ARIMA and SVRs [26,27], ARIMA and ANN [28], ANN, SVR and Gaussian process regression [29], ELM, phase space reconstruction and a least squares support vector machine [30].

Whereas the aforementioned hybrid strategies could generate decently accurate and reliable load prediction, they are still defective in some ways. The first type of hybrid models contains only a single inference model and fails to take advantage of statistical strengths of different inference models. While the second type of hybrid models do make the use of both linear and nonlinear inference models (or auto regressive models and machine learning models), the methods they use are essentially a simple summation either weighted sum or iterative sum. There are much room for improvement in balancing the great disparity among different inference models in accuracy, volatility and linearity, which maximizes the potential of ensemble.

To overcome the shortcomings of existing approaches, we propose a novel hybrid strategy that incorporates the following novelty points,

1. Compared with the first type of hybrid model, the proposed hybrid strategy improves on integrating both linear model (ARIMA) and nonlinear models (NuSVR, ELM, and LSTM).
2. Compared with the second type of hybrid model, a novel ensemble model WGTB is proposed. To the best of authors’ knowledge, tree-based ensemble method has never been used in deep load forecasting model.
3. And the proposed WGTB is an entirely new ensemble model. It is a boosting algorithm with linear ElasticNet as initial regressor and nonlinear ExtraTree as iterative regressor. The linear ElasticNet allows WGTB to lower bias by picking sparsely from all inference models and automatically avoid high-bias models. And the nonlinear ExtraTree enables those high-bias models to lower the variance of prediction together with low-bias models, which can be achieved only in tree-based ensemble model that is independent of bias.
4. An empirical study demonstrates that the hybrid model has a better forecasting accuracy than individual submodels, and the ensemble model WGTB performs better than existing ensemble models.

The rest of the paper is organized as follows. Section 2 presents the framework of hybrid strategy, the underlying designs of four individual models, and the novel ensemble model WGTB. Section 3 analyzes the prediction results of the proposed model. Finally, conclusions are given in Section 4.

## 2 The Proposed Hybrid Strategy for STLF

A hybrid strategy is proposed to forecast the load, which integrates effectively the linear regression models and nonlinear regression models. This section gives a succinct review of four regression submodels, and presents a novel ensemble model WGTB.

The framework of the hybrid strategy is presented in Fig. 1. The load dataset is first pre-processed. Then the linear regressor ARIMA and nonlinear regressors NuSVR, ELM and LSTM give four individual predictions. Thirdly, the proposed ensemble model WGTB, which integrates linear ElasticNet and nonlinear model stochastic gradient tree boosting, forecasts the load based on the juxtaposition of outputs from four submodels. Finally, the accurate load forecasting can be obtained by data post-processing.
2.1 Regression Models

To fully use the non-linearity of datasets, four regression models (i.e., ARIMA, NuSVR, ELM and LSTM) are selected through the analysis of the prediction effect.

2.1.1 Autoregressive integrated moving average

ARIMA is a forecasting technique that projects the future values of a series based entirely on the inertia of the series itself [31, 32]. The model is often written as ARIMA \((p,d,q)\). The parameters \(p\), \(d\), \(q\) respectively denote the order of the autoregressive model, the degree of differencing and the order of the moving-average model [33]. The equation of ARIMA is given as follows,

\[
x_t = c + \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t + \sum_{i=0}^{q} \theta_i \epsilon_{t-i}
\]  

where \(x_t\) is \(d^{th}\) original time series, \(c\) is constant term, \(\phi_i\) is autocorrelation coefficient, \(\theta_i\) is partial autocorrelation coefficient, and \(\epsilon_t\) is corresponding error.

2.1.2 Nu support vector regression

To control the number of support vectors and training errors of traditional SVR, NuSVR adds a parameter \(\nu\) to restrict the regularization coefficient, where \(0 \leq \nu \leq 1\) [34]. The regression formula can be expressed as,

\[
f(x) = \sum_{i=1}^{N} w_i \phi_i(x) + b
\]

where \(w_i\) is the coefficient, \(\phi_i(x)\) is named feature, \(N\) is the number of input data, and \(b\) is the bias term. The coefficients \(w_i, i=1, N\) can be obtained by optimizing the following quadratic programming problem,

\[
\min_{w} \left\|w\right\|_2^2 + \frac{1}{m\nu} \sum_{i=1}^{m} L(x_i, f(x))
\]

where \(L\) is the loss function, and \(m\) is corresponding regularization factor.
2.1.3 Extreme learning machine

ELM is known to be robust, highly accurate, and computationally efficient [35, 36]. Unlike traditional neural networks where weights of both layers are trainable, ELM is a special case of single-hidden-layer fully connected neural network. Only the weights of output layer are trainable and the weights of the hidden layer are randomly initialized and immutable during training. This benefits ELM to have a global optimum [37].

The output function \( f_L(x) \) of ELM with \( L \) hidden neurons is

\[
f_L(x) = \sum_{i=1}^{L} b_i (\sigma(x^T a_i) + e_i)
\]

where \( x = [x_1, \ldots, x_d]^T \) is the \( d \)-dimensional input vector, \( b = [b_1, \ldots, b_L]^T \) is the vector of output layer weights, \( a_i = [a_{i1}, \ldots, a_{id}]^T \) is the weight vector of the \( i^{th} \) neuron in hidden layer, \( e_i \) is the bias of the \( i^{th} \) neuron in hidden layer, and \( \sigma \) is sigmoid function.

2.1.4 LSTM neural network model for STLF

The LSTM neural network includes an extra memory cell, which can overcome the gradient vanishing problem of recurrent neural network [38]. It has gain great popularity and is proven to be one of the most successful for load forecasting [15, 39].

The architecture of LSTM load forecasting model is shown in Fig. 2. The original dataset is denoted by \( D = \{(x_0, y_0), \ldots, (x_{N-1}, y_{N-1})\} \), where \( N \) is the number of samples in the dataset. Then input matrix \( Z \) is concatenated by three vectors,

\[
Z = [V, W, H]
\]

where \( V \) is normalization vector of load dataset, \( W \) is weekday indices vector, which is introduced to differentiate from Monday to Sunday, and \( H \) is holiday indices vector (i.e., holiday is 1, and non-holiday is 0).

After that, input vector \( Z \) is fed into arbitrary number of LSTM layers. And the output of the last LSTM layer is fed again into a fully connected neural network with \( N_{FC} \) layers. The last layer of the fully connected neural network has dimension \( R^{T'} \), in which each entry corresponds to forecast value of each timestamp. The final prediction is calculated by an additional de-normalization layer after the fully connected neural network.

2.2 Warm-start Gradient Tree Boosting

Current ensemble models contain linear models and nonlinear models. Both linear and nonlinear models can perform a decent job in ensemble task, especially ElasticNet and SGTB. However, each of them has its drawbacks. While ElasticNet is good at finding linear relationship among feature inputs and can also fit in case of sparse data, it lacks the ability to handle nonlinear data. While SGTB excels ElasticNet in exploring non-linearity, because of the limited size of each base decision tree, the bias of initial value is large. And it usually has to take a number of iterations to refine the initial value, during which iterative refinements could not lower both bias and variance. The proposed WGTB model is inspired by and is a mixture of these two models. It is designed to solve their drawbacks by integrating them into a single model via iterative method. Therefore, it can be viewed from two aspects: one is a nonlinear model (SGTB) with linear add-on (ElasticNet), while the other is a linear model (ElasticNet) with nonlinear peripherals (SGTB).

This section first has a review on SGTB. Then the latter viewpoint of a warm-start tree model is presented, before presenting the former viewpoint of a nonlinear-error-corrected linear model.

2.2.1 Stochastic gradient tree boosting

SGTB was first introduced by Friedman [40] who added random sampling technique to original gradient tree boosting [41]. The model is a generalized greedy function approximator using boosting method. Each weak regressor in the model is a decision tree, namely Classification And Regression Trees (CART). The mathematical formulation is in additive fashion,

\[
F(x) = \gamma_0 + \sum_{m=1}^{M} \gamma_m h_m(x)
\]

where \( x \) is input vector, \( M \) is the number of decision trees, \( h_m \) is the \( m^{th} \) decision tree, and \( \gamma_m \) is weight coefficient for decision tree \( h_m \).
The loss objective of the model is,

$$\min_{\mathbf{y}, \mathbf{x}} \mathbb{E}_{\mathbf{y}, \mathbf{x}} [L(y, x)] := \sum_{i=0}^{N-1} L(y_i, F(x_i))$$  \hspace{1cm} (7)$$

where $L$ is mean square error loss function, $(x_i, y_i) \in D$ is a sample in the dataset $D$, $x, y$ denote arbitrary input and groundtruth output respectively, and $N$ is the number of samples in dataset.

Gradient tree boosting builds its model in iterative method as shown below,

$$F_0(x) = \gamma_0$$
$$F_m(x) = F_{m-1}(x) + \gamma_m h_m(x) \quad \forall m > 0$$ \hspace{1cm} (8)$$

The objective for $m^{th}$ iteration then becomes

$$\min \sum_{i=0}^{N-1} L(y_i, F_m(x_i)) := \sum_{i=0}^{N-1} L(y_i, h_m(x_i) + F_{m-1}(x_i))$$ \hspace{1cm} (9)$$
To optimize the objective in Eq. \( \text{9} \), the algorithm first optimizes \( h_m \) by setting \( \gamma_m \) to 1. Then it optimizes \( \gamma_m \) by fixing \( h_m \). To find \( h_m \), gradient tree boosting uses steepest descent algorithm,

\[
F_m(x) = F_{m-1}(x) - \gamma_m \nabla_F \sum_{i=0}^{N-1} L(y_i, F_{m-1}(x_i))
\]  

(10)

So in each iteration, a new decision tree is fitted to predict the additive inverse of gradient,

\[
h_m(x) \leftarrow -\nabla_F \sum_{i=0}^{N-1} L(y_i, F_{m-1}(x_i))
\]  

(11)

After \( h_m \) is determined, the optimization of \( \gamma \) is shown as follows,

\[
\gamma_0 = \arg \min_{\gamma} \sum_{i=0}^{N-1} L(y_i, \gamma)
\]  

(12)

\[
\gamma_m = \arg \min_{\gamma} \sum_{i=0}^{N-1} L(y_i, \gamma h_m(x_i) + F_{m-1}(x_i)) \quad \forall m > 0
\]  

(13)

Two techniques are also developed together with gradient tree boosting, first of which is shrinkage, an effective technique in controlling learning rate. The shrinkage parameter \( \nu \leq 0.1 \) can lead to better generalization error. Then the Eq. \( \text{8} \) becomes,

\[
F_m(x) = F_{m-1}(x) + \nu \gamma_m h_m(x) \quad \forall m > 0
\]  

(14)

The other technique is random sampling. During each iteration, instead of training the base decision tree with the entire dataset, a subsample of training data is drawn at random (without replacement) and it is used to fit the base decision tree.

The stochastic gradient tree boosting algorithm can be summarized as,

**Algorithm 1**: Stochastic Gradient Tree Boosting

**Data**: Dataset \( D = \{(x_i, y_i)\}^N \), shrinkage parameter \( \nu \), number of decision trees \( M \), subsample ratio \( \eta \)

**Result**: Weight coefficient \( \gamma_m \) and decision tree \( h_m \)

\( N' \leftarrow Floor(\eta N); \)
\( \gamma_0 \leftarrow \arg \min_{\gamma} \sum_{i=0}^{N-1} L(y_i, \gamma); \)

for \( m \leftarrow 1 \) to \( M \) do

\( \{(x'_j, y'_j)\}^{N'} \leftarrow \text{Random/Subsample}(); \)
\( r_{jm} \leftarrow -\nabla_F L(y'_j, F_{m-1}(x'_j)) \quad \forall j; \)

Fit decision tree \( h_m(x') \) with \( \{(x'_j, r_{jm})\}^{N'} \quad \forall j; \)
\( \gamma_m \leftarrow \arg \min_{\gamma} \sum_{j=0}^{N'-1} L(y'_j, \gamma h_m(x'_j) + F_{m-1}(x'_j)); \)
\( F_m(x) \leftarrow F_{m-1}(x) + \nu \gamma_m h_m(x); \)

end

2.2.2 Interpretation based on nonlinear model

This section views WGTB as nonlinear model with linear add-on and regards WGTB as a SGTB improved with a linear warm start. More specifically, SGTB uses a constant, denoted as \( F_0(x) \), as its base predicted value before all iterations. The proposed WGTB replaces \( F_0(x) \) with ElasticNet, the carefully selected linear model. ElasticNet is one of the most generalized version of linear regressions that includes both L1 and L2 regularization terms of the coefficients, which allows it to learn a sparse model while maintaining regularization properties. Eq. \( \text{16} \) is the training of ElasticNet, and \( F_0(x) \) is its predicted value. Mathematically, the equations become

\[
F_0(x) = \gamma_0 = x^T w - y
\]  

(15)

\[
w = \arg \min_w E_{x,y} \left[ \frac{1}{2N} \|x^T w - y\|^2_2 + \alpha \rho \|w\|_1 + \alpha(1 - \rho) \|w\|_2^2 \right]
\]  

(16)
where $\alpha$ stands for regularization coefficient of ElasticNet, and $\rho$ stands for L1 ratio of ElasticNet.

However, experimental analysis shows that the results of the aforementioned algorithm are not decent. An innovative approach invented from the perspective based on linear model is utilized to improve the model. Specifically, in WGTB, ExtraTree replaces decision tree as base estimator. The reason behind is explained in Section 2.2.3 where linear-model-based perspective is discussed at full length.

Besides, the choice of hyperparameters of ElasticNet has great impact on the number of total iterations. Exhaustive search is a popular method in tuning hyperparameters. It is incorporated into the model to search for the best ElasticNet hyperparameters. Besides, the choice of hyperparameters of ElasticNet has great impact on the number of total iterations. Exhaustive search is a popular method in tuning hyperparameters. It is incorporated into the model to search for the best ElasticNet hyperparameters.

Algorithm 2: Warm-start Gradient Tree Boosting

Data: Dataset $D = \{(x_i, y_i)\}^N$, shrinkage parameter $\nu$, number of decision trees $M$, subsample ratio $\eta$, regularization coefficient of ElasticNet $\alpha = \{\alpha_a\}^A$, L1 ratio of ElasticNet $\rho = \{\rho_h\}^M$

Result: Weight coefficient $\gamma_m$ and decision tree $h_m$

\[
\begin{align*}
N' &\leftarrow \text{Floor}(\eta N); \\
\gamma_0 &\leftarrow x^T w - y; \\
&\text{for } m \leftarrow 1 \text{ to } M \text{ do} \\
&\quad \{ (x'_j, y'_j) \}^N' \leftarrow \text{Random_Subsample}(\{(x_i, y_i)\}^N); \\
&\quad r_{jm} \leftarrow -\nabla F(y'_j; F_{m-1}(x'_j)) \quad \forall j; \\
&\quad \text{FitExtraTree } h_m(x') \text{ with } \{(x'_j, r_{jm})\}^{N'} \quad \forall j; \\
&\quad \gamma_m \leftarrow \text{argmin}_{\gamma} \sum_{j=0}^{N'-1} L(y'_j, \gamma h_m(x'_j) + F_{m-1}(x'_j)); \\
&\quad F_m(x) \leftarrow F_{m-1}(x) + \nu \gamma_m h_m(x); \\
&\end{align*}
\]

2.2.3 Interpretation based on linear model

Alternative perspective of WGTB is a nonlinear-error-corrected linear model. The base linear model is ElasticNet. The prediction model is formulated by an ElasticNet and a nonlinear error term,

\[y = f(x) + \epsilon\]

where $f(x)$ is ElasticNet model, and $\epsilon$ is nonlinear error.

The nonlinear error can be predicted using a separate estimator. The tree boosting model is shown as a good estimator in SGTB. By replacing $\epsilon$ with it, the new model $F(x)$ is

\[F(x) = f(x) + \sum_{m=1}^{M} e_m(x)\]

where $e_m(x) \forall m$ is the base tree estimator ExtraTree.

As mentioned in Section 2.2.2 ExtraTree replaces decision tree as the base tree estimator. The reason behind is that decision tree has high variance. Since the model has a warm start, the error term is much smaller than previous model. A high variance model could easily lead the prediction off-track. The extremely randomized tree (ExtraTree) has high bias and low variance, so it is a suitable model. Upon experiment, this model performs well in estimating error after the warm-start.

3 Results and Analysis

To verify the effectiveness of the proposed hybrid strategy for STLF, we tested the models on our dataset. Our dataset records the actual load of a grid from State Grid Corporation of China. The record spans from 1 o’clock on January 1, 2017 to 24 o’clock on December 31, 2017. It’s based on one-hour basis (i.e., 24 data points each day). The first 300 days of the data set are used for training, and the rest are used for testing. Three statistical indices (i.e., mean absolute percent error (MAPE), mean absolute error (MAE) and root mean square error (RMSE)) are utilized to analyze the
forecasting effect.

$$\text{MAPE} = \frac{1}{N} \sum_{i=0}^{N-1} \frac{1}{T} \sum_{t=0}^{T-1} \left| \frac{y_t - y'_t}{y_t} \right| \times 100\% \quad (19)$$

$$\text{MAE} = \frac{1}{N} \sum_{i=0}^{N-1} \frac{1}{T} \sum_{t=0}^{T-1} |y_t - y'_t| \quad (20)$$

$$\text{RMSE} = \frac{1}{N} \sum_{i=0}^{N-1} \sqrt{\frac{1}{T} \sum_{t=0}^{T-1} (y_t - y'_t)^2} \quad (21)$$

where $T$ is the number of load forecasting, $N$ is the number of test samples, $y_t$ is the observed value for the time period $t$, and $y'_t$ is the forecasting value for the corresponding period.

### 3.1 Submodel Selection

As illustrated is Section [2.1] four submodels are first used to forecast the load before ensemble. We present our selection mechanism for the individual submodels in the following subsections. And the Table 1 lists the final selection for submodels as well as their training parameters.

| Submodel | One-hot Inputs | $p$ | $d$ | $q$ | Kernel | $\nu$ | # Neurons | Learning Rate [Epoch] |
|----------|----------------|-----|-----|-----|--------|-------|-----------|-----------------------|
| ARIMA    | None           | 1   | 1   | 1   | -      | -     | -         | -                     |
| NuSVR    | None           | -   | -   | -   | Rbf    | 0.1   | -         | -                     |
| ELM      | Holiday        | -   | -   | -   | -      | 1800  | -         | -                     |
| LSTM     | Holiday, Weekday | - | - | - | 2 × 128 | 0.001[100 epochs]+ | 0.0001[130 epochs] | - |

Table 1: Final selection of submodels

#### 3.1.1 ARIMA

The key of ARIMA model is to determine the three parameters $p$, $d$, $q$. The autocorrelation function and partial autocorrelation function are utilized to choose proper values. Through the corresponding analysis, the model is selected as ARIMA(1,1,1).

#### 3.1.2 NuSVR

In the experiment of our dataset, the RMSE, MAE and MAPE of SVR are twice to three times as that of NuSVR. Thus, NuSVR is selected to represent SVR family in the proposed model. The Rbf kernel is used, because it outperforms other kernels tested, including polynomial, exponential and linear. And the parameter $\nu$ is tested to be 0.1.

#### 3.1.3 ELM

The one-hot inputs of ELM are determined in a similar method as LSTM fully described in Section [3.1.4]. And for each set of inputs, the number of neurons is determined by the offset where training set error is similar to testing set error. This selection strategy comes from the fact that more neurons lead to overfitting while less neurons lead to underfitting. In our final selection, the number of neurons in the hidden layer is 1800, and its inputs are holiday and preceding 168 hourly load data.

#### 3.1.4 LSTM

The LSTM model consisting of two-layer LSTM with 128 hidden neurons and one-hidden-layer fully connected is the most suitable model in testing. The experiment data for one-hot input selection is shown in Table 2. The units of MAE and RMSE in the Tables are MWh. The best two numeric values are highlighted in bold. It is easy to find that holiday indices improve the accuracy, weekday indices alleviate overfitting and underfitting issues, and hour indices generally reduce the performance of prediction. So our choice of the one-hot inputs of LSTM are holiday and weekday.
### 3.2 Forecasting Results

To fully validate the performance of proposed hybrid strategy, we first compare the conventional hybrid models with single submodels, then compare the proposed hybrid model with conventional hybrid models. Lastly, the ExtraTree and decision tree are compared.

#### 3.2.1 Conventional hybrid models versus single submodels

The individual regression models (ARIMA, NuSVR, ELM and LSTM) with the hyperparameters specified in the Section 3.1 are compared with hybrid strategy based on conventional ensemble models (ElasticNet and SGTB). Fig. 3 visualizes the predictions from conventional hybrid models versus single submodels in two arbitrary samples. The evaluation indices calculated by our dataset are listed in Table 3, out of which sixty-five samples compose the test set. Fig. 4 provides the corresponding histograms. As shown in the Table 3 and Fig. 4, LSTM is manifestly the best single estimator for STLF. Still, both ensemble models, ElasticNet and SGTB, outstrip it for all indices tested. MAPE, MAE, RMSE of ElasticNet are 2.692\%, 2.769\% and 2.800\% lower than those of LSTM in test set. And MAPE, MAE, RMSE of SGTB are 1.584\%, 1.591\% and 1.653\% lower than those of LSTM in test set. Accordingly the results show that hybrid method is effective in improving the accuracy by taking both linear model and nonlinear models into account in the inputs of ensemble model. This part corresponds to the first novelty point of the hybrid model.

#### 3.2.2 The proposed hybrid model versus conventional hybrid models

The proposed hybrid model based on WGTB is compared with conventional hybrid models based on ensemble models (Bagging, ExtraTree, Random forest, Adaboost, ElasticNet and SGTB). Fig. 5 shows the prediction result, and evaluation indices are listed in Table 4. Fig. 6 provides the corresponding histograms. It is easy to find that the best two traditional ensemble methods are ElasticNet and SGTB. Whereas other traditional ensemble methods have high error rates. But the proposed WGTB further beats both ElasticNet and SGTB in all aspects. It not merely has lower error rates in three evaluation indices and for training set and testing set, but it also has the smallest generalization errors between training set and testing set, especially in MAPE. More specifically in terms of the first aspect on error rates, MAPE, MAE, RMSE of WGTB are 0.895\%, 1.076\% and 1.080\% lower than those of ElasticNet, and 2.011\%, 2.261\% and 2.234\% lower than those of SGTB in test set. And as for second aspect on the generalization error, generalization error of WGTB are 0.001\%, 53.569 MWh and 56.279 MWh in the order of MAPE, MAE and RMSE. By contrast, generalization errors of ElasticNet are 0.002\%, 56.621 MWh and 59.052 MWh and generalization errors of SGTB are 0.012\%, 68.47 MWh and 74.392 MWh, both in the same order as the preceding sentence. Both aspects prove the effectiveness of the proposed WGTB in taking both linearity (ElasticNet) and nonlinearity (ExtraTree) into account in the process of ensemble. This part corresponds to the second and third novelty points of the hybrid model.
Figure 3: Forecasting results of conventional hybrid models and individual submodels

Table 4: Evaluation indices of proposed hybrid model and conventional hybrid models

| Algorithm  | Train |          | Test |          |
|------------|-------|----------|------|----------|
|            | MAPE  | MAE      | RMSE | MAPE     | MAE      | RMSE  |
| Bagging    | 0.313 | 195.067  | 319.824 | 1.665    | 1125.390 | 1400.803 |
| ExtraTree  | 1.591 | 984.398  | 1225.495 | 1.594    | 1068.562 | 1329.800 |
| Random Forest | 1.057 | 661.133  | 813.568 | 1.496    | 1016.035 | 1268.435 |
| Adaboost   | 2.450 | 1469.618 | 1770.931 | 2.160    | 1396.044 | 1665.956 |
| ElasticNet | 1.231 | 769.381  | 940.674 | 1.229    | 826.002  | 999.726  |
| SGTB       | 1.231 | 767.546  | 937.130 | 1.243    | 836.016  | 1011.522 |
| WGTB       | 1.219 | 763.543  | 932.645 | 1.218    | 817.112  | 988.924  |
Figure 4: The comparison of conventional hybrid models and individual submodels

Figure 5: Forecasting results of proposed hybrid model and conventional hybrid models

3.2.3 ExtraTree versus decision tree

One great innovation of the proposed model is the use of ExtraTree. Although decision tree performs well for error approximation in traditional gradient tree boosting, its high variance forbids its performance after the warm-start. Fig.
Figure 6: The comparison of proposed hybrid model and conventional hybrid models

plots the training errors in the first 1000 iterations when either decision tree (i.e., CART) or ExtraTree is used in iterative error approximation. For decision tree based model, whereas its training error quickly converges, its testing error diverges in the first 300 iterations before converging and reaching its minimum at around 900 iterations. This phenomenon reflects the high variance nature of decision tree. And the high variance harms the model in three ways. Firstly, it causes a severe overfitting issue as shown in the figure. Secondly, even with overfitting, its testing error rate is still higher than that of the proposed ExtraTree based model. Thirdly, it takes more iterations to converge.

On the contrary, the lower variance nature of ExtraTree leads to a stable convergence. Its testing errors drop together with training errors before it reaches its optimum and bounces back in testing errors. Then the model can be tuned easily with early stopping method.

Figure 7: ExtraTree versus decision tree

4 Conclusion

A novel hybrid strategy based on WGTB is proposed to forecast the short-term load. The proposed model outperforms existing models by taking both linearity and nonlinearity into account in two ways. First, outputs of the proposed model are used to forecast based on the juxtaposition of outputs from both linear and nonlinear submodels. Second, during ensemble process, a novel ensemble model hybrids linear estimator ElastinNet and nonlinear estimator WGTB via iterative method. ExtraTree is further carefully chosen to replace decision tree as the base tree estimator, which has low variance nature and fast coverage. Experiments have been conducted to prove the effectiveness of our proposed model in the improvement of accuracy. The three statistical indices MAPE, MAE and RMSE of the proposed WGTB model are 3.563%, 3.816%, and 3.85% lower than those of the best individual submodel LSTM, 2.011%, 2.261%, and 2.23% lower than the best conventional hybrid model based on SGTB. Nonetheless, there are still room for improvement in terms of speed. In future work, we will consider speeding up the algorithm by making a histogram based warm-start gradient tree boosting model.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests.

Acknowledgement

This work was supported by China Scholarship Council.

References

[1] Amber KP, Ahmad R, Aslam, MW, et al. Intelligent techniques for forecasting electricity consumption of buildings. Energy 2018; 157: 886-893.

[2] Wang S, Wang X, Wang S, et al. Bi-directional long short-term memory method based on attention mechanism and rolling update for short-term load forecasting. International Journal of Electrical Power & Energy Systems 2019; 109: 470-479.

[3] Yang Y, Che J, Deng C, et al. Sequential grid approach based support vector regression for short-term electric load forecasting. Applied Energy 2019; 238, 1010-1021.

[4] Fan G, Peng L, Hong W, et al. Electric load forecasting by the SVR model with differential empirical mode decomposition and auto regression. Neurocomputing 2018; 212: 372-385.

[5] Rahman A, Srikumar V, Smith A. Predicting electricity consumption for commercial and residential buildings using deep recurrent neural networks. Applied Energy 2018; 173: 958-970.

[6] Yildiz B, Bilbao J, Sproul A. A review and analysis of regression and machine learning models on commercial building electricity load forecasting. Renewable and Sustainable Energy Reviews 2017; 73: 1104-1122.

[7] Li W, Zhang Z. Based on time sequence of ARIMA model in the application of short-term electricity load forecasting. International Conference on Research Challenges in Computer Science 2009; 11-14.

[8] Amjady N. Short-term hourly load forecasting using time-series modeling with peak load estimation capability. IEEE Transactions on Power Systems 2001; 16(3): 498-505.

[9] Chen Y, Xu P, Chu Y, et al. Short-term electrical load forecasting using the Support Vector Regression (SVR) model to calculate the demand response baseline for office buildings. Applied Energy 2017; 195: 659-670.

[10] Kavoussi-Fard A, Samet H, Marzbani F. A new hybrid modified firefly algorithm and support vector regression model for accurate short term load forecasting. Expert Systems with Applications 2014; 41(13): 6047-6056.

[11] Zhang X, Wang J, Zhang K. Short-term electric load forecasting based on singular spectrum analysis and support vector machine optimized by Cuckoo search algorithm. Electric Power Systems Research 2017; 146: 270-285.

[12] Park D, El-Sharkawi M, Marks R, et al. Electric load forecasting using an artificial neural network. IEEE transactions on Power Systems 1991; 6(2): 442-449.

[13] Lou C W, Dong M C. A novel random fuzzy neural networks for tackling uncertainties of electric load forecasting. International Journal of Electrical Power & Energy Systems 2015; 73: 34-44.

[14] Liang Y, Niu D, Hong W. Short term load forecasting based on feature extraction and improved general regression neural network model. Energy 2019; 166: 653-663.

[15] Kong W, Dong Z, Jia Y, et al. Short-term residential load forecasting based on LSTM recurrent neural network. IEEE Transactions on Smart Grid 2019; 10(1): 841-851.

[16] Jiao R, Zhang T, Jiang Y, et al. Short-Term Non-Residential Load Forecasting Based on Multiple Sequences LSTM Recurrent Neural Network. IEEE Access 2018; 6: 59438-59448.

[17] Ertugrul O F. Forecasting electricity load by a novel recurrent extreme learning machines approach. International Journal of Electrical Power & Energy Systems 2016; 78: 429-435.

[18] Li S, Wang P, Goel L. Short-term load forecasting by wavelet transform and evolutionary extreme learning machine. Electric Power Systems Research 2015; 122: 96-103.

[19] Zeng N, Zhang H, Liu W, et al. A switching delayed PSO optimized extreme learning machine for short-term load forecasting. Neurocomputing 2017; 240: 175-182.

[20] Zhang J, Wei Y, Li D, et al. Short-term electricity load forecasting using a hybrid model. Energy 2018; 158: 774-781.
[21] Hanmandlu M, Chauhan BK. Load forecasting using hybrid models. IEEE Transactions on Power Systems 2011; 26(1): 20-29.

[22] Khwaja AS, Naem M, Anpalagan A, Venetsanopoulos A, Venkatesh B. Improved short-term load forecasting using bagged neural networks. Electric Power Systems Research 2015; 125: 109-115.

[23] Khwaja AS, Zhang X, Anpalagan A, Venkatesh B. Boosted neural networks for improved short-term electric load forecasting. Electric Power Systems Research 2017; 143: 431-437.

[24] Khwaja AS, Anpalagan A, Naem M, Venkatesh B. Joint bagged-boosted artificial neural networks: Using ensemble machine learning to improve short-term electricity load forecasting. Electric Power Systems Research 2020; 179: 106080.

[25] Sideratos G, Ikonomopoulos A, Hatziargyriou ND. A novel fuzzy-based ensemble model for load forecasting using hybrid deep neural networks. Electric Power Systems Research 2020; 178: 106025.

[26] Nie H, Liu, G, Liu X, et al. Hybrid of ARIMA and SVMs for short-term load forecasting. Energy Procedia 2012; 16: 1455-1460.

[27] Fan G, Peng L, Hong W, et al. Electric load forecasting by the SVR model with differential empirical mode decomposition and auto regression. Neurocomputing 2016; 173: 958-970.

[28] Fard AK, Akbari-Zadeh MR. A hybrid method based on wavelet, ANN and ARIMA model for short-term load forecasting. Journal of Experimental and Theoretical Artificial Intelligence 2014; 26(2): 167-182.

[29] Sharifzadeh M, Sikiinoti-Lock A, Shah N. Machine-learning methods for integrated renewable power generation: A comparative study of artificial neural networks, support vector regression, and Gaussian Process Regression. Renewable and Sustainable Energy Reviews 2019; 108: 513-538.

[30] Wang Y, Yang Y, Li C, et al. A novel hybrid model based on least square support vector machine and weight coefficients optimization: a case study of short-term electric load forecasting. Journal of Renewable and Sustainable Energy 2017; 9(2): 025504.

[31] Lee C, Ko C. Short-term load forecasting using lifting scheme and ARIMA models. Expert Systems with Application 2011; 38: 5902-5911.

[32] Kavasseri R, Seetharaman K. Day-ahead wind speed forecasting using f-ARIMA models. Renewable Energy 2009; 34(5): 1388-1393.

[33] Wang J, Hu J. A robust combination approach for short-term wind speed forecasting and analysis–Combination of the ARIMA (Autoregressive Integrated Moving Average), ELM (Extreme Learning Machine), SVM (Support Vector Machine) and LSSVM (Least Square SVM) forecasts using a GPR (Gaussian Process Regression) model. Energy 2015; 93: 41-56.

[34] Schölkopf B, Smola AJ, Bach F. Learning with kernels: support vector machines, regularization, optimization, and beyond. MIT press 2002.

[35] Wu J, Cui Z, Chen Y, et al. A new hybrid model to predict the electrical load in five states of Australia. Energy 2019; 166: 598-609.

[36] Sun W, Zhang C. A hybrid BA-ELM model based on factor analysis and similar-day approach for short-term load forecasting. Energies 2018; 11(5): 1282.

[37] Huang G, Zhou H, Ding X, et al. Extreme learning machine for regression and multiclass classification. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 2011; 42(2): 513-529.

[38] Hochreiter S, Schmidhuber J. Long short-term memory. Neural Computation 1997; 9(8): 1735-1780.

[39] Bouktif S, Fiaz A, Ouni A, et al. Optimal deep learning LSTM model for electric load forecasting using feature selection and genetic algorithm: comparison with machine learning approaches. Energies 2018; 11(7): 1636.

[40] Friedman JH. Stochastic gradient boosting. Computational Statistics & Data Analysis 2002; 38(4): 367-378.

[41] Friedman, JH. Greedy function approximation: a gradient boosting machine. The Annals of Statistics 2001; 29(5): 1189-1232.