Optimal Price-Based Power Allocation Algorithm with Quality of Service Constraints in Non-Orthogonal Multiple Access Networks

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SUMMARY In this letter, we investigate the price-based power allocation for non-orthogonal multiple access (NOMA) networks, where the base station (BS) can admit the users to transmit by pricing their power. Stackelberg game is utilized to model the pricing and power purchasing strategies between the BS and the users. Based on backward induction, the pricing problem of the BS is recast into the non-convex power allocation problem, which is equivalent to the rate allocation problem by variable replacement. Based on the equivalence problem, an optimal price-based power allocation algorithm is proposed to maximize the revenue of the BS. Simulation results show that the proposed algorithm is superior to the existing pricing algorithm in terms of the revenue of BS and the number of admitted users.

key words: non-orthogonal multiple access (NOMA), power allocation, Stackelberg game, price

1. Introduction

Non-orthogonal multiple access (NOMA) has been recognized as an essential candidate technology of the fifth generation (5G) wireless networks. Because superposition coding and successive interference cancellation are utilized at the transmitter and the receiver for NOMA, it can accommodate more users and improve spectrum efficiency compared with traditional orthogonal multiple access (OMA) [1].

As the signals of users in NOMA systems are multiplexed in the power domain, power allocation is an important approach for NOMA to improve the system performance. Moreover, the quality-of-service (QoS) or rate fairness in NOMA systems is an important issue that should be guaranteed. For sum rate maximization problem in NOMA system, the transmit power will be allocated to the user with the best channel condition without the QoS constraints. This allocation scheme will lead to unfair allocation among users, and NOMA will turn into OMA [2].

It was proved that WSRM problem for NOMA is a convex problem if the weights of users satisfied some conditions. In [6], the power allocation for downlink NOMA system was studied under three different performance criteria under QoS or weights constraints. In [7], the sum rate maximization problem was investigated in a NOMA system subject to the minimum user rate requirements and the maximum transmit power of base station (BS). In [8], Fu et al. proposed a distributed power control algorithm to minimize the sum transmit power of a multi-cell NOMA system under user rate constraints. Price-based power allocation in NOMA has been studied by Stackelberg game in [9], [10]. In [9], a sub-optimal price-based power allocation with alternating optimization was proposed to maximize the revenue of the BS. In [10], we have given the optimal price-based power allocation for two user’s case. Then, a novel iterative price-based power allocation algorithm based on block coordinate descent is proposed for more than two user’s case, which is a sub-optimal for the revenue maximization problem of the BS. However, the optimal price-based power allocation for more than two user’s case for the system model in [9], [10] is still an open problem. Moreover, the pricing algorithms in [9], [10] do not consider the minimum rate requirement constraints to ensure the QoS of the users.

This letter investigates the price-based power allocation in NOMA under the minimum rate of each user. The non-convex revenue maximization problem of the BS is firstly expressed as an equivalent convex rate allocation problem by variable replacement. Then, the feasible condition and optimal price-based power allocation algorithm are presented. Thus, we also solve the open problem in [10] for more than two user’s case because it can be viewed as a special case of our model when each user has no QoS constraint. Simulation results show our algorithm outperforms the existing algorithm in [10] concerning the revenue of the BS and the number of admitted users.

2. System Model

We consider a cellular NOMA system in the downlink, where the BS and users have a single antenna. The channel gain between the BS and $m$-th user is given by $h_m, m \in M$, where $M = \{1, 2, \ldots, M\}$ is set of the users. Without loss of generality, we assumed the users are sorted such that $|h_M| \geq |h_2| \geq \cdots \geq |h_1|$. The BS transmits signals for all users on the same spectrum by using superposition coding. Then, the received signals $y_m$ at the $m$-th user from the BS
is given by
\[ y_m = h_m \sum_{i \in M} \sqrt{p_i} s_i + n_m, \quad m \in M \] (1)
where \( p_i \) is the transmit power from the BS to the \( i \)-th user, \( s_i \) is the transmitted signal for \( i \)-th user, and \( n_m \sim \text{CN}(0, \sigma^2) \) represents the additive white Gaussian noise (AWGN) at the \( m \)-th user. The user can cancel the interference power from the other users’ with poorer channel condition by successive interference cancellation (SIC) [3]. The signal to interference and noise ratio (SINR) for user \( i \) can be expressed as
\[ \gamma_i = \frac{|h_i|^2 p_i}{\sigma^2 + |h_i|^2 \sum_{j \neq i} |p_j|}, \quad i = 1, 2, \ldots, M. \] (2)

Therefore, the rate of the \( m \)-th user is expressed as
\[ r_m = \log_2 \left( 1 + \frac{|h_m|^2 p_m}{\sigma^2 + |h_m|^2 \sum_{i=1}^M p_i} \right), \quad m \in M. \] (3)

In the Stackelberg game, the BS is the leader and the users are the follower. The BS will charge user \( m \) by price \( \lambda_m \) per unit transmit power to maximize its revenue. Therefore, the optimization problem for the BS’s revenue maximization can be formulated as
\[ \text{maximize } U_{BS}(\lambda_1, \ldots, \lambda_M) = \sum_{m=1}^M \lambda_m p_m \] (4)
subject to \( \sum_{m=1}^M p_m \leq P_{tol} \), \( r_m \geq r_m^{\min}, \quad m = 1, \ldots, M \), (5)
(6)
where \( p_m \) denotes the buying power for user \( m \) when the price \( \lambda_m \) is set for all users, \( P_{tol} \) is the total transmit power constraint, \( r_m^{\min} \) is the minimum rate requirement for user \( m \). Constraint (5) is the total power constraint at the BS. Constraint (6) is the minimum rate requirement of users to ensure the QoS of users.

The utility function of the user \( m \) includes two parts. The first part is the income for user \( m \) by its rate when the transmit power is \( p_m \), and the second part is the payment \( \lambda_m p_m \) for the BS. Therefore, the revenue function of user \( m \) is expressed as follows.
\[ U_m(p_m) = \log_2 \left( 1 + \frac{|h_m|^2 p_m}{\sigma^2 + |h_m|^2 \sum_{j=1}^M p_j} \right) - \lambda_m p_m \] (7)
Consequently, the optimization problem of the user \( m \) is
\[ \text{maximize } U_m(p_m), \quad \text{subject to } p_m \geq 0. \] (8)

3. Optimal Price-Based Power Allocation Algorithm

In this section, the revenue maximization problem of the BS is first converted to the non-convex power allocation problem by the relationship between the price and transmit power. Then, the non-convex power allocation problem is transformed into an equivalent convex rate allocation problem by variable replacement. Finally, an optimal pricing algorithm is proposed based on the equivalent rate allocation problem.

Using the optimal condition for (8) as [9], the transmit power of users and the price of \( \lambda_i \) \( (i = 1, \ldots, M) \) satisfy the following lemma.

**Lemma 1:** Assuming that \((p_1, \ldots, p_M)\) are the optimal power allocation for all the users when the prices for users are given by \( \lambda_m \) \( (m = 1, \ldots, M) \) such that \( \lambda_m \leq \frac{(\ln 2)^{-1} |h_m|^2}{\sigma^2 + |h_m|^2 \sum_{j=1}^M p_j} \), the relationship about the given price \( \lambda_m \) and the transmit power satisfies the following equation
\[ \lambda_m = \frac{(\ln 2)^{-1} |h_m|^2}{\sigma^2 + |h_m|^2 \sum_{j=1}^M p_j} \] (9)

From lemma 1, substitute (9) into (4), the revenue problem of the BS can be rewritten as follows.
\[ \text{maximize } U_{BS}(p_1, \ldots, p_M) = \sum_{m=1}^M \frac{|h_m|^2 p_m}{\sigma^2 + |h_m|^2 \sum_{i=1}^M p_i} \] (10)
subject to \( \sum_{m=1}^M p_m \leq P_{tol}, \quad p_m \geq 0, \quad m = 1, \ldots, M \), (11)
(12)
where we omit the constant factor \((\ln 2)^{-1}\) in objective function for simplicity.

Because the objective function in (10) is non-concave, (10)–(12) is a non-convex optimization problem. We can’t give the optimal solution to (10)–(12) directly. However, we will show that (10)–(12) is equivalent to a concave rate maximization optimization problem by variable replacement. Therefore, the optimal solution to (10)–(12) can be obtained by the equivalent convex optimization problem.

Substitute \( \gamma_i = \frac{|h_i|^2 p_i}{\sigma^2 + |h_i|^2 \sum_{j=1}^M p_j} \) and let \( r_i = \log_2(1 + \gamma_i) \) be the rate of user \( i \), the revenue of the BS can be rewritten as a function of rate.
\[ U_{BS} = \sum_{i=1}^M \frac{\gamma_i}{\gamma_i + 1} = \sum_{i=1}^M \frac{2^n - 1}{2^n - 1 + 1} = M - \sum_{j=1}^M \frac{1}{2^n} \] (13)

Next, we show that the total power constraint (11) can be rewritten by the rate constraint.

**Lemma 2:** Let \( \sigma_i^2 = \sigma^2 |h_i|^2 \) \( (i = 1, 2, \ldots, M) \) and \( \sigma_{M+1}^2 = 0 \), the transmit power and rate satisfies
\[ \sum_{j=1}^M p_j = \sum_{j=1}^M (\sigma_j^2 - \sigma_{j+1}^2) \times e^{\ln 2 r_i} - \sigma_i^2 \] (14)

**Proof:** Because \( \gamma_i = \frac{|h_i|^2 p_i}{\sigma^2 + |h_i|^2 \sum_{j=1}^M p_j} \) and \( r_i = \log_2(1 + \gamma_i) \), we can write the relationship between rate and power as
\[ e^{\ln 2 r_i} = \frac{|h_i|^2 p_i + \sigma^2 + |h_i|^2 \sum_{j=1}^M p_j}{\sigma^2 + |h_i|^2 \sum_{j=1}^M p_j} \] (15)
if \( i = M \), we have \( e^\ln 2 r_M = \frac{p_M + \sigma_M^2}{\sigma_M^2} \), \( p_M \) can be rewritten as
\[
p_M = \frac{\sigma_M^2 e^{\ln 2 r_M} - \sigma_M^2}{\sigma_M^2}.
\]
(16)

if \( i = M - 1 \), we have
\[
e^{\ln 2 r_{M-1}} = \frac{p_{M-1} + p_M + \sigma_{M-1}^2}{\sigma_{M-1}^2} + p_M.
\]
(17)

Substitute (16) into the denominator of (17), we have
\[
\sum_{i=M-1}^{M} p_i = \sigma_M^2 e^{\ln 2 (r_{M-1} + r_M)} + (\sigma_{M-1}^2 - \sigma_M^2) e^{\ln 2 (r_{M-1} - r_M)} - \sigma_{M-1}^2.
\]
(18)

Equation (16) and (18) satisfy lemma 2. Then, we can prove the following lemma 2 by recursive method for \( i = M - 2, \ldots, 1 \).

By lemma 2, the optimal price-based power allocation algorithm with QoS constraints (OPPAAQC) with the system model in [9], [10] can be used to give the optimal solution to (20)–(22) by interior-point method, then the optimal price \( \lambda_i^* \) for user \( i \) is
\[
\lambda_i^* = \frac{(\ln 2)^2 |h_i|^2}{\sigma^2 + |h_i|^2 (p_i^* + \sum_{j=i+1}^{M} p_j^*)},
\]
where \( p_i^* \) is given as follows:
\[
p_i^* = \sum_{j=i}^{M} (\sigma_j^2 - \sigma_{j+1}^2) e^{\ln 2 \sum_{k=1}^{k_j} r_k^*} - \sigma_j^2 - \sum_{j=i+1}^{M} (\sigma_j^2 - \sigma_{j+1}^2) e^{\ln 2 \sum_{k=1}^{k_j} r_k^*} + \sigma_{j+1}^2.
\]
(25)

Algorithm 1 Optimal Price-based Power Allocation Algorithm with QoS Constraints (OPPAAQC)

Initialization: set \( K = M \) and \( \sigma_i^2 = \frac{\sigma_i^2}{\sigma_i^2} \), \( i = 1, \ldots, M \).

while \( \sum_{j=1}^{K} (\sigma_j^2 - \sigma_{j+1}^2) e^{\ln 2 \sum_{k=1}^{k_j} r_k^*} > \sigma_1^2 + \sum_{j=1}^{K} \lambda_j^* \) and \( K \geq 1 \) do

\( \lambda_k^* = \infty \), \( K = K - 1 \).
end while

for all \( i \in \{1, 2, \ldots, K\} \) do

\( p_i^* = \sum_{j=i}^{K} (\sigma_j^2 - \sigma_{j+1}^2) e^{\ln 2 \sum_{k=1}^{k_j} r_k^*} - \sum_{j=1}^{K} (\sigma_j^2 - \sigma_{j+1}^2) e^{\ln 2 \sum_{k=1}^{k_j} r_k^*} + \sigma_{j+1}^2 - \sigma_j^2 \), where \( (r_1^*, \ldots, r_K^*) \) is the optimal solution to (20)–(22) by the interior-point algorithm.
end for

Output: For \( i = 1, \ldots, K \), price \( \lambda_i^* \) is computed by \( \lambda_i^* = \frac{(\ln 2)^2 |h_i|^2}{\sigma_1^2 + \sum_{k=1}^{k_j} r_k^*} \).

By theorem 1, the optimal price-based power allocation algorithm for NOMA with QoS constraints is given by algorithm 1. In the proposed algorithm, admission control is presented first and set the price for the user \( K \) to be \( \lambda = \infty \) to remove the user \( K \) if the minimum rate is not satisfied by lemma 4. The system model in [9], [10] can be viewed as a special case when \( \gamma_k^* = 0 \), \( k = 1, 2, \ldots, M \). Therefore, the proposed algorithm can be used to give the optimal solution for the system model in [9], [10].

4. Simulation Results

We compare the proposed optimal price-based power allocation algorithm with QoS constraints (OPPAAQC) with the
price-based power allocation algorithm (PPAA) proposed in [10] through simulation. The admission users for the PPAA algorithm is computed by the number of users that satisfying the minimum SINR requirement after applying the PPAA algorithm. The number of users is $M = 4$. The coverage radius of the BS is 500 m, in which the users are randomly distributed. Each small-scale fading channel coefficients follow an $CN(0, 1)$ and the path loss is given by $128.1 + 37.6 \log_{10} d$ dB [5], where $d$ is the distance in kilometers. The maximum transmit power of the BS is 40 dBm. The noise power is $10^{-11}$ W. Each user has the same minimum rate requirement $r_{min}$. Simulation results are averaged by $10^4$ channel realizations.

Figure 1 shows that the revenue of the BS decreases as $r_{min}$ increases for two algorithms. The reason is that the power allocation strategies for the BS decrease as $r_{min}$ increases. The OPPAAQC algorithm can improve the revenue of the BS the PPAA algorithm. Moreover, there is a performance gap between the two algorithms when equals zero, and the PPAA algorithm can obtain 95.5% profit compared with the OPPAAQC algorithm. Therefore, the PPAA is a near-optimal algorithm for more than two users’ case without QoS constraints. Figure 2 shows that the sum rate obtained by the PPAA is larger than OPPAAQC algorithm when there is no minimum rate requirement for users. As the minimum rate requirements increases, the sum rate obtained by the OPPAAQC will outperform the PPAA algorithm. This is because the OPPAAQC algorithm can admit more users and allocate optimal power with admission control as $r_{min}$ increases. Figure 3 shows that the number of admitted users decreases as the increasing of $r_{min}$. The reason is that more users will be not be allowed to transit when the power allocation can’t meet all users’ QoS requirement as $r_{min}$ increases. Moreover, the OPPAAQC algorithm can admit more users than the OPPAAQC algorithm when the $r_{min}$ is larger than zero.

5. Conclusion

Power allocation problem in NOMA with QoS constraints is investigated by Stackelberg game. The revenue of the BS is recast as an equivalent rate allocation problem by variable replacement. Then, the feasible condition of the problem and optimal price-based power allocation algorithm is proposed by solving the convex rate allocation problem. Simulation results show that our algorithm improves the revenue of the BS and the number of admitted users compared with the PPAA algorithm. One of the future work is consideration of the new admission control scheme with some fair scheduling scheme such as PF in [3] to support the user with poor channel condition in the long term.

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