Searching for cavities of various densities in the Earth's crust with a low-energy $\bar{\nu}_e$ $\beta$-beam

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We propose searching for deep underground cavities of different densities in the Earth's crust using a long-baseline $\bar{\nu}_e$ disappearance experiment, realised through a low-energy $\beta$-beam with highly enhanced luminosity. We focus on four real-world cases: water-filled cavities, iron-banded formations, heavier mineral deposits, and regions of abnormal charge accumulation that, supposedly, appear prior to the occurrence of an intense earthquake. The sensitivity to identify cavities attains confidence levels higher than 3$\sigma$ and 5$\sigma$ for exposures times of 3 months and 1.5 years, respectively, and cavity densities below 1 g cm$^{-3}$ or above 5 g cm$^{-3}$, with widths greater than 200 km. We reconstruct the cavity density, width, and position, assuming one of them known while keeping the other two free, in each of the aforementioned cases. Finally, we introduce an observable to quantify the presence of a cavity by changing the orientation of the $\bar{\nu}_e$ beam.

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Introduction.— Experiments performed during the last decade have confirmed that neutrinos can change flavour with a probability that depends on neutrino energy, distance travelled, squared-mass differences $\Delta m^2_{21} \equiv m_2^2 - m_1^2$ and $\Delta m^2_{31} \equiv m_3^2 - m_1^2$, a CP-violation phase $\delta$, and on the mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ that describe the rotation between the mass and flavour eigenstates. In the presence of matter, coherent forward scattering modifies the mass-squared differences and angles, and so the value of this probability is altered [1, 2].

In this letter, we have used neutrino oscillations in matter to discover regions of under- and over-density of electric charge accumulation in seismic faults prior to earthquakes [3], employing either a superbeam [3] or the flux of $\beta$-beam with $\bar{\nu}_e$ [4], and of electric charge accumulation in seismic faults prior to earthquakes [5], employing reactor neutrinos. In contrast, we have kept the cavity density as a free parameter, and studied how different values affect our ability to locate the cavity and to determine its density. Our experimental arrangement considers long-baseline (1500 km) $\bar{\nu}_e$ disappearance using a low-energy (5–150 MeV) $\beta$-beam, which provides a high-purity sample of $\bar{\nu}_e$ and allows for intense matter effects.

Neutrino propagation in matter.— The probability amplitudes for the transitions $\bar{\nu}_e \to \bar{\nu}_\beta$ can be arranged in a column vector $\Psi_e = (\psi_{ee} \psi_{e\mu} \psi_{e\tau})^T$ which evolves according to $i d\Psi_e / dx = H \Psi_e$, where $x$ is the distance travelled since creation and the effective Hamiltonian in the flavour basis is given by $H (x) = 1/(2E_e) U^\dagger \text{diag} (0, \Delta m^2_{21}, \Delta m^2_{31}) U + A (x)$, with $E_e$ the neutrino energy and $U$ the lepton mixing matrix [6]. The matter effects are encoded in the matrix $A (x) = \text{diag} \left(-\sqrt{2} G_F N_e (x), 0, 0\right)$, where $N_e (x) = y_e \rho (x) N_{AV}$ is the electron number density, with $\rho$ the matter density, $N_{AV}$ Avogadro’s number, and $y_e = 0.494$ in the crust [7]. We have assumed a normal mass hierarchy ($\Delta m^2_{32} > 0$), fixed the values of the squared-mass differences and angles to the best-fit values of Ref. [7], and the CP phase to zero. Our results are obtained by solving numerically the evolution equation described above.

For a qualitative understanding, we will use for the flavour-transition probability the well-known [3] formula $P_{\bar{\nu}_e \to \bar{\nu}_\mu} = P_{\bar{\nu}_e \to \bar{\nu}_\tau} \cos^2 \theta_{13} + \sin^2 \theta_{13}$ (with $A \to A \cos^2 \theta_{13}$), where $P_{\bar{\nu}_e \to \bar{\nu}_\tau}$ is the two-flavour slab approximation, valid on a piecewise constant density pro-
file [9], in the three-layer case that will be pertinent to this work. It is described by \( P_{\nu_i \rightarrow \nu_j}^{2\nu} = \frac{1}{|\mathbf{K}|^2} \), where \( \mathbf{U} = \mathbf{U}_k \times \mathbf{U}_l \), \( \mathbf{U}_k = \cos \phi_k - i(\mathbf{n}_x \cdot \mathbf{\sigma}) \sin \phi_k \), \( \mathbf{\sigma} \) is the vector of Pauli matrices, and \( \mathbf{n}_k = (\sin 2\theta_{M_k}, 0, -\cos 2\theta_{M_k}) \), with \( \theta_{M_k} \) the matter mixing angle in the \( k \)-th slab. The frequencies \( \nu_k \) are given by \( \Delta m_{31}^2 x_k / (4E_\nu) \), where \( x_k \) is the width of the \( k \)-th slab, and \( \Delta m_{31}^2 \) is the squared-mass difference. The numerical calculation and this approximation are in reasonable agreement when using for the latter the dominant oscillation parameters \( \Delta m_{31}^2 \) and \( \theta_{12} \).

**Location and shape of the cavity inside the Earth.**– We have assumed the existence of a cavity of uniform density \( \rho_{\text{cavity}} \) located within the Earth’s crust, itself of density given by the Preliminary Reference Earth Model (PREM) [10]. Together, the ocean, crust, and LID layers of the PREM have a depth of up to 80 km and an average density of \( \langle \rho_0 \rangle = 3.3 \text{ g cm}^{-3} \). Interesting geological features such as porous rock cavities, mineral deposits and seismic faults lie in the crust and LID layers. Therefore, we have supposed that no part of the cavity is below 80 km. The cavity itself has been modelled as an ellipsoid, and we have supposed that no part of the cavity is below 80 km. The cavity itself has been modelled as an ellipsoid, and we have studied an elliptic cross section of it, with major axis length \( w_0 \) and minor axis length 0.2\( w_0 \), as shown in Fig. 1.

Since the density profile of the Earth is radially symmetric, the position of the neutrino source (S) can be fixed at an arbitrary point on its surface, for a given source-detector baseline, \( L_0 \). In order to specify the location of the cavity, we need to set the distance \( d_0 \) measured along the baseline from the source to the cavity’s surface.

**Low-energy \( \beta \)-beams.**– There is currently a proposal to use a pure, collimated beam of low-energy \( \bar{\nu}_e \) generated by means of the well-understood \( \beta \) decay of boosted exotic ions [11] and detected through \( \bar{\nu}_e + ^{12}C \rightarrow e^+ + ^{12}B \) [12]. Our \( \beta \)-beam setup contemplates an ion storage ring of total length \( L_{\text{tot}} = 1885 \text{ m} \), with two straight sections of length \( L_{\text{straight}} = 678 \text{ m} \) each [13]. Inside the ring, \( ^4\text{He} \) ions boosted up to a Lorentz factor \( \gamma = 25 \) decay through \( ^4\text{He}^{++} \rightarrow ^{6}\text{Li}^{++} e^- e^- e^- \) with a half-life \( t_{1/2} = 0.8067 \text{ s} \). Ion production with an ISOLDE technique [14] is expected to provide a rate of ion injection of \( g = 2 \times 10^{13} \text{ s}^{-1} \) for \( ^4\text{He} \); we have assumed a highly optimistic 5000-fold enhancement of this value, which has not even been considered in the literature.

The neutrino flux from the \( \beta \) decay of a nucleus in its rest frame is given by the formula [15] \( \Phi_{\text{e.m.}}(E_\nu) = bE_\nu^2 E_\nu \sqrt{E_\nu^2 - m_e^2 F(\pm Z, E_\nu)} \Theta(E_\nu - m_e) \), where \( b = \ln 2 / (m_e^2 f t_{1/2}) \), with \( m_e \) the electron mass and \( f t_{1/2} = 806.7 \) the comparative half-life. The energy of the emitted electron is given by \( E_\nu = Q - E_\nu \), with \( Q = 3.5078 \text{ MeV} \) the Q-value of the reaction, and \( F(\pm Z, E_\nu) \) the Fermi function.

Based on the formalism of Ref. [16], we have considered a cylindrical detector made of carbon, of radius \( R = 4\sqrt{5} \text{ m} \) and length \( h = 100 \text{ m} \), co-axial to the straight sections of the storage ring, and located at a distance \( L_0 = 1500 \text{ km} \) from it. The integrated number of \( e^+ \) at the detector, after an exposure time \( t \), is calculated as

\[
N = t g \frac{t_{1/2}}{\ln 2} \int dE_\nu \Phi_{\text{e.m.}}(E_\nu) (L_0, E_\nu) \sigma(E_\nu) \, ,
\]

with \( n \approx 6.03 \times 10^{23} \text{ cm}^{-3} \) the density of carbon nuclei in the detector and \( \sigma \) the detection cross section [12].

**Sensitivity to cavities.**– Assuming that there is no cavity along the baseline \( L_0 \), we evaluate the sensitivity to differentiate this situation from the hypothesis that the neutrino beam traverses a cavity of width \( w \), position \( d \), and density \( \rho \). To do this, we define

\[
\chi^2(w, d, \rho) = \sum_i \frac{N_i^{\text{cav}}(w, d, \rho) - N_i^{\text{no-cav}}}{N_i^{\text{no-cav}}}^2 ,
\]

with \( N^{\text{cav}}_i(w, d, \rho) \) the number of \( e^+ \), in the \( i \)-th energy bin, that reach the detector in the case where the beam traverses the cavity, and \( N^{\text{no-cav}}_i \) the corresponding number in the no-cavity case. Given that \( \gamma = 25 \), the neutrino spectrum extends from 5 to 150 MeV, in bins of 5 MeV. On account of the maximum energy considered, the production of muons, from \( \bar{\nu}_e + ^{12}C \rightarrow \mu^+ + X \), is inhibited, and thus the \( \bar{\nu}_e \rightarrow \nu_\mu \) channel is not included in this work.

Before presenting the results, it is convenient to mention that, within the slab approximation, the first and third slabs correspond to the crust, and the second one, to the cavity, so that \( x_1 = d, x_2 = w, \) and \( x_3 = L_0 - d - w. \)
Table I: Minimum value of $\rho_{\text{cavity}}$ to have a statistical significance of $5\sigma$ or $10\sigma$ C.L. with respect to the no-cavity case, for a given width $w$ and exposures of 3 months or 1.5 years.

| Cavity width ($w$) | $\rho_{\text{cavity}}$ [g cm$^{-3}$] |
|-------------------|----------------------------------|
|                   | 3 months | 1.5 years |
|                   | $5\sigma$ | $10\sigma$ | $5\sigma$ | $10\sigma$ |
| 50 km             | 18        | > 25     | 9        | 15        |
| 100 km            | 10        | 18       | 6        | 9         |
| 250 km            | 6.5       | 9        | 4.5      | 6         |

Fig. 2 shows the sensitivity in the form of isocontours of $\chi^2 = 1\sigma$, $3\sigma$, $5\sigma$, and $10\sigma$, with the cavity located at the center of the baseline, so that $d = (L_0 - w)/2$ (which is tantamount to $\phi_1 = \phi_3$), and detector exposure times of (a) 3 months and (b) 1.5 years. Because of this relation between $d$ and $w$, the analysis can be reduced to two parameters: $\rho$ and $w$ (or $d$). It is observed that the sensitivity improves as $|\rho_{\text{cavity}} - \langle \rho_0 \rangle|$ increases. Also, the statistical significance, for the cavity hypothesis, grows with $w$, given that the cumulative matter density differences are greater.

In the slab approximation, the behaviour of the oscillation probability, that dictates the shape of the isocontours, is dominated by the $\phi_2$ frequency, which is the only one that depends on both $\rho$ and $w$. In fact, for a given energy, and since we have observed that the combinations of sines and cosines of $\phi_1$ in the probability vary slowly with $w$, each isocontour approximately corresponds to a constant value of $w \sqrt{c_0^2/E_0^2 + 2c_0c_1\rho\cos 2\theta_{12}/E_0 + c_1^2\rho^2}$, where $c_0 = \Delta m^2_{31}/4$ and $c_1 = -G_F y_e N_{\Lambda N_{\Lambda}} \cos^2 \theta_{13}/\sqrt{2}$.

Table II shows the minimum value of $\rho_{\text{cavity}}$ needed to reach $5\sigma$ and $10\sigma$ separations, for small ($w = 50$ km), medium (100 km), and large (250 km) cavities, after detector exposure times of 3 months and 1.5 years.

Determination of cavity parameters.—The next step in the analysis is to change the assumption of no-cavity to that of a real cavity. This implies replacing $N_{\text{no-cav}} \rightarrow N_{\text{cav}}^\rho (w_0, d_0, \rho_{\text{cavity}})$ in Eq. 4. Firstly, we consider a known cavity position (i.e., $d = d_0$, with the hypothetical cavity no longer centered) and, heretofore, we set 1.5 years as exposure time, unless otherwise specified, and keep the other details of the study equal to those presented in the sensitivity analysis. We will study the four points, motivated by centered real-world cavities, marked in Fig. 2. A ($\rho_{\text{cavity}} = 1$ g cm$^{-3}$, $y_e = 0.555$, $w_0 = 250$ km), corresponding to a deep water-filled underground cavity, B ($\rho_{\text{cavity}} = 5$ g cm$^{-3}$, $y_e = 0.5$, $w_0 = 250$ km), to an iron banded formation [17], C ($\rho_{\text{cavity}} = 10$ g cm$^{-3}$, $y_e = 0.5$, $w_0 = 100$ km), to a heavier mineral deposit, and D ($\rho_{\text{cavity}} = 25$ g cm$^{-3}$, $y_e = 0.5$, $w_0 = 50$ km), representing a zone of seismic faults with the typical charge accumulation that supposedly exists prior to an earthquake of magnitude 7 in the Richter scale [18]. In Ref. 3, a value of $\rho_{\text{cavity}} \approx \langle \rho_0 \rangle$ and a maximum value of $y_e = 4$ at the fault are used, while, to keep in line with the analysis of cavities A–C, we have equivalently taken for cavity D $\rho_{\text{cavity}} = 25$ g cm$^{-3}$ and $y_e = 0.5$.

Notice that the closest distance from the cavities to the Earth surface, for A and B, is about 19 km, and, for cavities C and D, 34 km and 39 km, respectively. Wider or uncentered cavities would lie closer to the surface; for instance, a centered cavity with $w_0 = 323$ km would lie only 12 km deep, which is roughly the current maximum drilling depth [19].

Secondly, in Fig. 3 we observe that values of $\rho$ close to $\langle \rho_0 \rangle$ (equivalent to the no-cavity case), regardless of $w$, are excluded at the same statistical significance as shown in Fig. 2. The shape of the isocontours is explained by the same argument as in said figure. The uncertainty in $w$ is large in cases A and B, since in that density region the contours are parallel to the asymptote at $\rho = \langle \rho_0 \rangle$. In cases C and D, the uncertainty is smaller given that at higher densities the contour regions tend to be parallel to the horizontal axis. It is interesting to point out that in a real-case scenario, where there could be some prior knowledge about the density, it would be possible to constrain $w$ significantly. For instance, for $\rho = 1$ g cm$^{-3}$ (case A), $w = 420^{+30}_{-20}$ km at $1\sigma$ uncertainties, while for $\rho = 25$ g cm$^{-3}$ (case D), $w = 50 \pm 10$ km.

In Fig. 4 we display the case where the cavity position is unknown, while its density is known (i.e., $\rho = \rho_{\text{cavity}}$) and, thus, the contour regions are described by $w$ and $d$ (with $w + d \leq L_0$). In this figure, the size of the uncertainty in $w$ follows from Fig. 3. Therefore, we are capable of determining $w$ with reasonable precision in cases C and D. On the other hand, the determination of $d$ improves mildly as the density increases. For example, in case A, $d = 600^{+375}_{-475}$ km and, in D, $d = 725^{+225}_{-275}$ km at $1\sigma$ uncertainties. Using the slab approximation, this larger uncertainty can be explained by the weak dependence of

![FIG. 3: (Color online) Confidence levels in the $w$ vs. $\rho$ plane. Plots (a)–(d) correspond to central cavities A–D from Fig. 2. The real values ($\rho_0, w_0$) are marked by crosses and the best-fit values by stars.](image-url)
the probability, for most of the relevant energy range, on $\phi_1$ and $\phi_3$, and, therefore, on $d$.

**Searching for a cavity.** We have explored the possibility of varying the beam orientation, defined by the angle $\alpha$ measured with respect to the tangent to Earth at $S$, as a way of finding a cavity. This is shown in Fig. 4, and could be implemented by using a mobile neutrino detector, either on land or at sea [20]. At an angle $\alpha$, the beam travels a baseline $L_\alpha$; for some values, it will cross a portion of the cavity with position $d_\alpha$ and width $w_\alpha$. To quantify the size of the traversed portion, we have defined

$$A(L_\alpha, w_\alpha, d_\alpha, \rho_{\text{cavity}}) = \frac{N_{\text{cav}}(L_\alpha, w_\alpha, d_\alpha, \rho_{\text{cavity}})}{N_{\text{no-cav}}(L_\alpha)},$$

(3)

with $N_{\text{cav}}$ and $N_{\text{no-cav}}$ the total numbers of $e^+$ between 5 and 150 MeV, and $L_\alpha$, $w_\alpha$, $d_\alpha$ functions of $L_\alpha$, $w_\alpha$, $d_\alpha$, and $\alpha$. For this analysis, we have set the exposure time at $t = 3$ months.

Fig. 5 shows the $A$ vs. $\alpha$ curves for the four cavities A–D. The curve for cavity A lies below $A = 1$ because its density is lower than $\langle \rho_S \rangle$, while the densities of cavities B–D are higher. The $1\sigma_A$ to $5\sigma_A$ uncertainty regions around the curves are included, with $\sigma_A \equiv A \sqrt{(1/N_{\text{cav}})(1 + A)}$ the standard deviation of $A$. Since $N_{\text{cav}} \sim 1/L_\alpha^2$, then $\sigma_A \sim L_\alpha$ and, given that, by geometrical construction, $L_\alpha$ grows with $\alpha$, this means that $\sigma_A$ also grows with $\alpha$, a feature that is observed in Fig. 5. Similarly to previous figures, the separation from $A = 1$ grows as $\rho_{\text{cavity}}$ moves away from $\langle \rho_S \rangle$. The C.L. achieved at the maximum deviation point is not as high as those shown in Fig. 2 due to the fact the analysis of $A$ does not take into account the spectral shape, but only the total $e^+$ count.

Separation from $A = 1$ at the $2\sigma_A$ C.L. is achieved for all cavities, with A and C reaching $3\sigma_A$, and D, $5\sigma_A$. Thus, the parameter $A$ could be used as a quick estimator of the presence of a cavity, while a more detailed analysis, similar to the one performed for Fig. 4, could be used to estimate the cavity shape, i.e., its position and width, and, from the latter, its volume.

**Conclusions.** We studied the use of a low-energy ($5–150$ MeV) $\beta$-beam of $\bar{\nu}_e$, with a baseline of 1500 km and a large luminosity enhancement, to find the presence of deep underground cavities in the Earth’s crust. We have determined the sensitivity as a function of the cavity density $\rho$ and size $w$ (dimension of the cavity aligned with the neutrino beamline), which reaches significances in the order of $5\sigma$ ($3\sigma$) for baseline-centered cavities with densities lower than 1 g cm$^{-3}$ or greater than 5 g cm$^{-3}$, exposure time of 1.5 years (3 months), and $w$ greater than 200 km, rendering our analysis highly competitive. We analysed the C.L. regions of the reconstructed parameters of four real-world cavities in the $\rho$ vs. $w$ plane, for a known cavity position $d$, and also in the $w$ vs. $d$ plane, when $\rho$ is known. Finally, we have considered sweeping the Earth in search of a cavity using an orientable neutrino beam, which proves to be a useful tool to detect its presence.

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