Some covering properties of the $\alpha$-topology

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Abstract

Recently, Mršević and Reilly discussed some covering properties of a topological space and its associated $\alpha$-topology in both topological and bitopological ways. The main aim of this paper is to investigate some common and controversial covering properties of $T$ and $T^\alpha$.

1 Introduction

In 1965, Njåstad introduced the notion of an $\alpha$-set in a topological space $(X, T)$, and proved that the collection of all $\alpha$-sets in $(X, T)$ is a topology on $X$, finer than $T$. Having two related topologies on the same underlying set, it is quite natural to ask whether they share some topological properties. The sharing of separation axioms has been considered by Dontchev [12], Janković and Reilly [19], Mršević and Reilly [24], etc. as a part of their investigations. Recently, Mršević and Reilly [25] discussed some covering properties of $(X, T)$ and $(X, T^\alpha)$ in both topological and bitopological ways. As is shown in [25], $T$ and $T^\alpha$ do not share compactness, countable compactness, Lindelöfness and paracompactness. On the other hand, Cao and Reilly [6] proved that $(X, T)$ is almost compact (resp. almost paracompact) if and only if $(X, T^\alpha)$ is almost compact (resp. almost paracompact). In light of these results, we study the $S$-closedness of $(X, T^\alpha)$ and show in Section 3 that $T$ and $T^\alpha$ share most of $S$-closed-like properties. Some controversial properties are discussed in Section 3.

Let $(X, T)$ be a topological space and $A \subseteq X$. The interior (resp. closure) of $A$ is denoted by $\text{Int}A$ (resp. $\text{Cl}A$). Recall that $A$ is semi-open (resp. regular open, an $\alpha$-set) if...
A ⊆ Cl(IntA) (resp. A = Int(ClA), A ⊆ Int(Cl(IntA))). In addition, A is called semi-closed (resp. regular closed, α-closed) if its complement X \ A is semi-open (resp. regular open, an α-set). The semi-closure (resp. α-closure) of A, denoted by sClA (resp. Cl_αA), is defined as the intersection of all semi-closed (resp. α-closed) subsets containing A. The α-semi-closure of A, denoted by sCl_αA is defined in a similar way. The family of all semi-open sets (resp. α-sets, regular closed sets) of (X, T) is denoted by SO(X, T) (resp. T^α, RC(X, T)). No separation axioms are assumed unless it is explicitly stated.

A topological space (X, T) is semi-compact \cite{15} if every cover of X by semi-open sets has a finite subcover. Moreover, (X, T) is S-closed \cite{31} (resp. s-closed \cite{10}) if for every cover of X by semi-open sets there is a finite subfamily whose closures (resp. semi-closures) form a cover of X. It is easy to show that (X, T) is S-closed if and only if every cover by regular closed sets has a finite subcover. In a similar manner, a topological space (X, T) is called rc-Lindelöf \cite{18} if every cover of X by regular closed subsets has a countable subcover.

**Lemma 1.1** \cite{26} SO(X, T^α) = SO(X, T) for any space (X, T). □

Semi-open sets are not the only classes of sets that (X, T) and (X, T^α) have always in common. The following classes of sets are also shared by both topologies in question: locally dense (= preopen) sets \cite{8}, nowhere dense sets, dense and codense sets, clopen sets, β-open (= semi-preopen) sets \cite{29} and of course α-open sets.

Recall that a subset A of a topological space (X, T) is called sg-open \cite{3} (resp. g-open \cite{21}) if every semi-closed (resp. closed) subset of A is included in the semi-interior (resp. interior) of A. Complements of sg-open (resp. g-open) sets are called sg-closed (resp. g-closed). The family of all sg-closed (resp. g-closed) subsets of a topological space (X, T) is denoted by SGC(X, T) (resp. GC(X, T)). Although the definitions of sg-closed and g-closed sets are very similar to each other, sg-closed and g-closed sets behave in a very different way. More precisely, sg-closed sets are more close (and related) to semi-closed and β-closed sets, while g-closed sets behave more like the other types of generalized closed sets, i.e. gs-closed, gp-closed, δ-g-closed (see \cite{4} for more details). We will see next that this type of behavior is valid also in connection with the α-topology.
2 S-closed-like properties

Proposition 2.1 Let \((X, \mathcal{T})\) be a topological space. Then:

1. \(s\text{Cl}_\alpha A = s\text{Cl}A\) for any \(A \subseteq X\).
2. \(SGC(X, \mathcal{T}) = SGC(X, \mathcal{T}^\alpha)\).

Proof. (i) follows easily from Lemma [1].

(ii) Let \(A \in SGC(X, \mathcal{T})\) and let \(A \subseteq U\), where \(U\) is semi-open in \((X, \mathcal{T}')\). By Lemma [1], \(U\) is semi-open also in \((X, \mathcal{T})\). Thus, \(s\text{Cl}A \subseteq U\). By (1), \(s\text{Cl}_\alpha A \subseteq U\). Thus \(A \in SGC(X, \mathcal{T}^\alpha)\). Conversely, assume that \(A \in SGC(X, \mathcal{T}^\alpha)\) and \(A \subseteq U\), where \(U\) is semi-open in \((X, \mathcal{T})\). Again by Lemma [1], \(U\) is semi-open in \((X, \mathcal{T})\). Thus, \(s\text{Cl}_\alpha A \subseteq U\). Using again (1), we have \(s\text{Cl}A \subseteq U\). This shows that \(A \in SGC(X, \mathcal{T})\). \(\square\)

We provide an example showing that in general \(GC(X, \mathcal{T}) \neq GC(X, \mathcal{T}^\alpha)\).

Example 2.2 Let \(X = \{a, b, c\}\) and \(\mathcal{T} = \{\emptyset, \{a\}, X\}\). Set \(A = \{a, b\}\). Observe that \(A\) is g-closed in \((X, \mathcal{T})\). Since \(\mathcal{T}^\alpha = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}\), then it is easily checked that \(A\) is not g-closed in \((X, \mathcal{T}^\alpha)\).

Lemma 2.3 [4] Let \((X, \mathcal{T})\) be a topological space. Then \(\text{Cl}_\alpha A = \text{Cl}A\) for any \(A \in SO(X, \mathcal{T})\). \(\square\)

Proposition 2.4 \(RC(X, \mathcal{T}^\alpha) = RC(X, \mathcal{T})\) for any space \((X, \mathcal{T})\).

Proof. Let \(A \in RC(X, \mathcal{T})\). Then \(A = \text{Cl}G\) for some \(G \in \mathcal{T}\). By Lemma [4], \(A = \text{Cl}_\alpha G\). Hence \(A \in RC(X, \mathcal{T}^\alpha)\). Conversely, let \(A \in RC(X, \mathcal{T}^\alpha)\). Then \(A = \text{Cl}_\alpha G\) for some \(G \in \mathcal{T}^\alpha\). Since each \(\alpha\)-set is semi-open, we have \(A = \text{Cl}G = \text{Cl}(\text{Int}G)\). Furthermore, \(G \subseteq A \subseteq \text{Cl}G\) implies \(\text{Cl}(\text{Int}A) = \text{Cl}(\text{Int}G)\). Therefore, \(A = \text{Cl}(\text{Int}A)\), and \(A \in RC(X, \mathcal{T})\). \(\square\)

Recently, the concept of a sg-compact space was introduced independently by Caldas in [7] and by Devi, Balachandran and Maki in [9]. A topological space \((X, \mathcal{T})\) is called
sg-compact [4] if every cover of $X$ by sg-open sets has a finite subcover. In [3], sg-compact spaces are called SGO-compact. Sg-compact spaces are studied in detail in [13].

As consequences of Lemma 1.1, Proposition 2.1 (ii) and Proposition 2.4, we have the following.

**Theorem 2.5** Let $(X, T)$ be a topological space. Then:

(a) $(X, T^a)$ is semi-compact if and only if $(X, T)$ is semi-compact.
(b) $(X, T^a)$ is $S$-closed if and only if $(X, T)$ is $S$-closed.
(c) $(X, T^a)$ is $s$-closed if and only if $(X, T)$ is $s$-closed.
(d) $(X, T^a)$ is rc-Lindelöf if and only if $(X, T)$ is rc-Lindelöf.
(e) $(X, T^a)$ is sg-compact if and only if $(X, T)$ is sg-compact. □

Recall that a subset $A$ of $(X, T)$ is $S$-closed relative to $X$ (resp. $s$-closed relative to $X$) if for every cover of $A$ by semi-open sets in $(X, T)$, there exists a finite subfamily whose closures (resp. semi-closures) in $(X, T)$ from a cover of $A$. Furthermore, $(X, T)$ is locally $S$-closed [27] (resp. locally $s$-closed [2]) if each point of $X$ has a neighbourhood which is $S$-closed relative to $X$ (resp. $s$-closed relative to $X$).

**Corollary 2.6** Let $(X, T)$ be a topological space. Then:

(a) $(X, T^a)$ is locally $S$-closed if and only if $(X, T)$ is locally $S$-closed.
(b) $(X, T^a)$ is locally $s$-closed if and only if $(X, T)$ is locally $s$-closed. □

In [7], Chen defined a space $(X, T)$ to be para-$S$-closed if every cover of $X$ by semi-open sets has a locally finite refinement by semi-open sets whose union is dense in $X$. In a recent paper [18], Janković and Konstadilaki introduced the notion of para-rc-Lindelöfness. Recall that a topological space $(X, T)$ is para-rc-Lindelöf if every cover of $X$ by regular closed sets has a locally countable refinement by regular closed sets. Next, we prove that $T$ and $T^a$ share these properties.

**Theorem 2.7** Let $(X, T)$ be a topological space. Then the following conditions are equivalent:

...
(a) \((X, \mathcal{T})\) is para-S-closed.

(b) Every cover of regular closed sets of \((X, \mathcal{T})\) has a locally finite refinement consisting of regular closed sets of \((X, \mathcal{T})\).

(c) Every cover of regular closed sets of \((X, \mathcal{T}^\alpha)\) has a locally finite refinement consisting of regular closed sets of \((X, \mathcal{T}^\alpha)\).

(d) \((X, \mathcal{T}^\alpha)\) is para-S-closed.

Proof. \((a) \Rightarrow (b)\). Let \(\mathcal{F} = \{F_\gamma : \gamma \in \Delta\}\) be a cover by regular sets of \((X, \mathcal{T})\). Then it is also a cover by semi-open sets of \((X, \mathcal{T})\). Therefore, it has a locally finite refinement \(\mathcal{V}\) by semi-open sets of \((X, \mathcal{T})\) such that \(X = \text{Cl}(\bigcup \mathcal{V})\). Then, \(\{\text{Cl}V : V \in \mathcal{V}\}\) is a locally finite refinement of \(\mathcal{F}\) consisting of regular closed sets of \((X, \mathcal{T})\).

\((b) \Rightarrow (c)\). It is obvious.

\((c) \Rightarrow (d)\). Let \(\mathcal{U} = \{U_\gamma : \gamma \in \Delta\}\) be a cover by semi-open sets of \((X, \mathcal{T}^\alpha)\). Then \(\{\text{Cl}U_\gamma : \gamma \in \Delta\}\) is cover by regular closed sets of \((X, \mathcal{T}^\alpha)\). Thus, it has a locally finite refinement \(\mathcal{F}\) by regular closed sets of \((X, \mathcal{T}^\alpha)\). Without loss of generality, we may assume that \(\mathcal{F} = \{F_\gamma : \gamma \in \Delta\}\) such that \(F_\gamma \subseteq \text{Cl}U_\gamma\) for each \(\gamma \in \Delta\). Set \(V_\gamma = F_\gamma \cap \text{Int}U_\gamma\) for each \(\gamma \in \Delta\), and \(\mathcal{V} = \{V_\gamma : \gamma \in \Delta\}\). Then \(\mathcal{V} \subseteq \text{SO}(X, \mathcal{T}^\alpha)\). Moreover, \(F_\gamma \subseteq \text{Cl}V_\gamma\) for each \(\gamma \in \Delta\). Hence, \(\mathcal{V}\) is a locally finite refinement of \(\mathcal{U}\) consisting of semi-open sets of \((X, \mathcal{T}^\alpha)\) such that \(X = \text{Cl}(\bigcup \mathcal{V})\).

\((d) \Rightarrow (a)\). Let \(\mathcal{U} = \{U_\gamma : \gamma \in \Delta\}\) be a cover by semi-open sets of \((X, \mathcal{T})\). Then \(\mathcal{U}\) is also a cover by semi-open sets of \((X, \mathcal{T}^\alpha)\). Thus, it has a locally finite refinement \(\mathcal{V}\) of semi-open sets of \((X, \mathcal{T}^\alpha)\) such that \(X = \text{Cl}(\bigcup \mathcal{V})\). Note that \(\mathcal{V} \subseteq \text{SO}(X, \mathcal{T})\) and \(\mathcal{V}\) is locally finite in \((X, \mathcal{T})\). It follows that \((X, \mathcal{T})\) is para-S-closed.

Similar to Theorem 2.7, we can obtain the following.

**Theorem 2.8** Let \((X, \mathcal{T})\) be a topological space. Then \((X, \mathcal{T}^\alpha)\) is para-rc-Lindelöf if and only if \((X, \mathcal{T})\) is para-rc-Lindelöf. □

It is well known that every regular Lindelöf space is paracompact. Analogous to this, we have the following result. Recall that a topological space \((X, \tau)\) is called *extremally
disconnected if the closure of every open subset of \( X \) is also open or equivalently if every regular closed set is regular open.

**Theorem 2.9** Every extremally disconnected, \( rc \)-Lindelöf space is para-S-closed.

**Proof.** Suppose that \((X, \mathcal{T})\) is an extremally disconnected and \( rc \)-Lindelöf space. Let \( \mathcal{U} \) be a cover of \( X \) by regular closed sets. Then \( \mathcal{U} \) has a countable subcover \( \{U_n : n \in \omega\} \). Define \( V_n = U_n \setminus \bigcup_{k=1}^{n-1} U_k \) for each \( n \in \omega \). Then, it is easy to see \( \{V_n : n \in \omega\} \subseteq RC(X, \mathcal{T}) \) and \( V_n \subseteq U_n \) for each \( n \in \omega \). For each \( x \in X \), let \( n(x) = \min\{n \in \omega : x \in U_n\} \). Clearly, we have \( x \in V_{n(x)} \). It follows that \( \{V_n : n \in \omega\} \) is a cover of \( X \) by regular closed sets. Since \((X, \mathcal{T})\) is extremally disconnected, \( U_{n(x)} \) is an open neighbourhood of \( x \) for each \( x \in X \). On the other hand, \( U_{n(x)} \cap V_n = \emptyset \) for all \( n > n(x) \). Therefore, \( \{V_n : n \in \omega\} \) is a locally finite refinement of \( \mathcal{U} \). By Theorem 2.7, \((X, \mathcal{T})\) is para-S-closed. \( \square \)

### 3 \( \alpha \)-subparacompact spaces

In this last section we prove a subspace theorem for \( \alpha \)-subparacompact spaces.

**Definition 1** A topological space \((X, \mathcal{T})\) is called \( \alpha \)-subparacompact if every \( \alpha \)-open cover of \( X \) has a \( \sigma \)-discrete closed refinement.

Clearly, every \( \alpha \)-subparacompact space is subparacompact but not vice versa as the following example shows:

**Example 3.1** Let \( X \) be the real line with topology in which the only nontrivial open set is \( \{0\} \). Note that \( \{\{0, y\} : y \neq 0\} \) is an \( \alpha \)-open cover of \( X \) which has no \( \sigma \)-discrete closed refinement. Thus, even a compact space need not be \( \alpha \)-subparacompact.

Next, we provide an example of a connected, Tychonoff, \( \alpha \)-subparacompact space which is not even metacompact.
Example 3.2 Recall that a measurable set $E \subseteq \mathbb{R}$ has density $d$ at $x \in \mathbb{R}$ if
\[
\lim_{h \to 0} \frac{m(E \cap [x-h, x+h])}{2h}
\]
exists and is equal to $d$. Set $\phi(E) = \{x \in \mathbb{R} : d(x, E) = 1\}$. The open sets of the density topology $\tau_d$ are those measurable sets $E$ that satisfy $E \subseteq \phi(E)$. Note that every nowhere dense subset of the density topology is closed [30]. Hence every $\alpha$-open set is open. Thus the subparacompactness of density topology [30] implies automatically its $\alpha$-subparacompactness. On the other hand, the density topology is not paracompact, in fact it is not even metacompact [30].

Recall that a subset $A$ of a topological space $(X, T)$ is called a generalized $\alpha$-closed set (briefly $g\alpha$-closed) [23] if $\text{Cl}_\alpha(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha$-open. We call a subset $A$ of a topological space $(X, \tau)$ $F_\sigma$-$g\alpha$-closed if $A$ is countable union of $g\alpha$-closed subsets of $X$. The set of all rationals $\mathbb{Q}$ (in the Real line) is an example of an $F_\sigma$-$g\alpha$-closed set which is not $g\alpha$-closed.

Theorem 3.3 Let $X$ be an $\alpha$-subparacompact space, and let $A$ be $F_\sigma$-$g\alpha$-closed. Then $A$ is $\alpha$-subparacompact (as a subspace), in particular, $\alpha$-subparacompactness is a $\alpha$-closed hereditarily.

Proof. Let $A = \bigcup_{n \in \omega} A_n$, where each $A_n$ is $g\alpha$-closed. Let $\mathcal{U} = \{U_i : i \in I\}$ be a cover of $\alpha$-open subsets of $(A, T|A)$. Note that for each $i \in I$, there exists $V_i \in T^\alpha$ (i.e. $V_i$ is $\alpha$-open in $(X, T)$) such that $V_i \cap A = U_i$. Since union of $\alpha$-open sets is $\alpha$-open, then $V = \bigcup_{i \in I} V_i$ is $\alpha$-open in $(X, T)$. Since each $A_n$ is $g\alpha$-closed, then $\text{Cl}_\alpha(A_n) \subseteq V$. Observe that $\{X \setminus \text{Cl}_\alpha(A_n) : n \in \omega\} \cup \{V_i : i \in I\}$ is an $\alpha$-open cover of $(X, T)$. Since $X$ is $\alpha$-subparacompact, then there exists a $\sigma$-discrete closed refinement, say $\mathcal{W} = \bigcup_{m \in \omega} \mathcal{W}_m$. For $m, n \in \omega$, set $\mathcal{W}'_{mn} = \{W \cap A_n : W \in \mathcal{W}_m\}$. Clearly, $\bigcup_{m \in \omega} \bigcup_{n \in \omega} \mathcal{W}'_{mn}$ is a $\sigma$-discrete closed refinement of $\mathcal{U}$ in $(A, T|A)$. This shows that $A$ is $\alpha$-subparacompact subspace of $(X, T)$. □

Corollary 3.4 Every closed subspace of an $\alpha$-subparacompact space is also $\alpha$-subparacompact. □
There is a result due to Burke [4] cited in Theorem 5.2 of [32] (see also the reference to that in Theorems 2.14 – 2.16 (pp. 224) of Junnila’s survey [20]): A space is subparacompact if and only if each open cover has a σ-closure preserving closed refinement. In a similar fashion one can prove the following.

**Lemma 3.5** A topological space \((X, \tau)\) is \(\alpha\)-subparacompact if and only if each \(\alpha\)-open cover has a σ-closure preserving closed refinement. □

Recall that a function \(f: (X, \tau) \to (Y, \sigma)\) is called \(\alpha\)-irresolute [22] if the preimage of every \(\alpha\)-open subset of \((Y, \tau)\) is \(\alpha\)-open in \((X, \tau)\).

**Theorem 3.6** Every closed \(\alpha\)-irresolute image of an \(\alpha\)-subparacompact space is also \(\alpha\)-subparacompact.

**Proof.** Let \(f: (X, \tau) \to (Y, \sigma)\) be a closed \(\alpha\)-irresolute (not necessarily continuous) map from the \(\alpha\)-subparacompact space \(X\) onto the topological space \(Y\) and let \(\mathcal{V} = \{V_i : i \in I\}\) be an \(\alpha\)-open cover of \(Y\). From the \(\alpha\)-irresoluteness of \(f\), we have that \(\mathcal{U} = \{f^{-1}(V_i) : i \in I\}\) is an \(\alpha\)-open cover of \(X\). Since \(X\) is \(\alpha\)-subparacompact, then \(\mathcal{U}\) has a σ-closure preserving closed refinement, that is, there exists \(\mathcal{Z} = \bigcup_{i=1}^{\infty} Z_i\), where each \(Z_i\) is a closure preserving family of closed sets and \(\mathcal{Z}\) refines \(\mathcal{U}\). Since \(f\) is closed, then by Lemma 3.5 \(\mathcal{W} = \bigcup_{i=1}^{\infty} W_i\), where \(W_i = \{f(Z) : Z \in Z_i\}\), is a σ-closure preserving closed family in \(Y\). It is straightforward to check that \(\mathcal{W}\) is a refinement of \(\mathcal{V}\). □

The product of an \(\alpha\)-subparacompact space and a compact space need not to be \(\alpha\)-subparacompact (take an infinite indiscrete space that is clearly \(\alpha\)-subparacompact and the compact non-\(\alpha\)-subparacompact space from Example 3.1).

If \((X, T^\alpha)\) is compact, then \((X, T)\) is usually called \(\alpha\)-compact. Properties of \(\alpha\)-compact spaces were studied in 1986 by Noiri and Di Maio [28].

**Question 1.** Is the product of two \(\alpha\)-subparacompact spaces \(\alpha\)-subparacompact? Is the product of an \(\alpha\)-subparacompact space and an \(\alpha\)-compact space necessarily \(\alpha\)-subparacompact?

**Definition 2** A topological space is called \(\alpha\)-paracompact if every \(\alpha\)-open cover of \(X\) has a locally finite open refinement.
Proposition 3.7 If \((X, \mathcal{T})\) is a Hausdorff \(\alpha\)-paracompact space, then \((X, \mathcal{T}^\alpha)\) is normal, in particular if \((X, \mathcal{T})\) is Hausdorff and \(\alpha\)-paracompact, then \(\mathcal{T}^\alpha = \mathcal{T}\), i.e., \(X\) is a nodec space.

Proof. Let \(A \subseteq X\) be an \(\alpha\)-closed set and \(x \notin A\). For every \(y \in A\) there exists an open set \(U_y\) such that \(y \in U \setminus y\) and \(x \notin \overline{U}_y\). Then \(\{U_y; y \in A\} \cup \{X \setminus A\}\) is an \(\alpha\)-open cover of \(X\). Let \(\mathcal{W}\) be a locally finite open refinement and let \(U = \cup \{W \in \mathcal{W}; W \cap A \neq \emptyset\}\). Then \(U\) is open, contains \(A\) and \(\overline{U} = \cup \{\overline{W}; W \cap A \neq \emptyset\}\). But each such set \(W\) is contained in some \(U_y\), and hence \(\overline{W} \subseteq \overline{U}_y\) and thus \(x \notin \overline{U}\). Now let \(A, B\) disjoint \(\alpha\)-open subsets of \(X\). For each \(x \in A\) there exists an open set \(V_x\) such that \(x \in V_x\) and \(\overline{V}_x \cap B = \emptyset\). Then \(\{V_x; x \in A\} \cup \{X \setminus A\}\) is an \(\alpha\)-open cover of \(X\). Let \(\mathcal{W}\) be a locally finite open refinement and let \(V = \cup \{W \in \mathcal{W}; W \cap A \neq \emptyset\}\). Then \(V\) is open, contains \(A\) and \(\overline{V} = \cup \{\overline{W}; W \cap A \neq \emptyset\}\). But each such set \(W\) is contained in some \(V_x\), and hence \(\overline{W} \subseteq \overline{V}_x\) and thus \(\overline{U} \cap B = \emptyset\). \(\blacksquare\)

Theorem 3.8 Suppose \((X, \mathcal{T})\) is Hausdorff and \(\alpha\)-paracompact. Then \((X, \mathcal{T}^\alpha)\) is Hausdorff and paracompact.

Proof. By a theorem of Engelking [16, Theorem 5.1.5, page 373], \((X, \mathcal{T}^\alpha)\) is normal. By a result of Dontchev [11], \(\mathcal{T} = \mathcal{T}^\alpha\). \(\blacksquare\)

Question 2. Let \((X, \mathcal{T}^\alpha)\) be subparacompact. Is \((X, \mathcal{T})\) \(\alpha\)-subparacompact?

References

[1] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 12 (1991), 5–13.
[2] C. Basu, On locally \(s\)-closed spaces, Internat. J. Math. Math. Sci., 19 (1996), 67–73.
[3] P. Bhattacharyya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29 (3) (1987), 375–382.
[4] D.K. Burke, On subparacompact spaces, Proc. Amer. Math. Soc., 23 (1964), 655–663.
[5] M.C. Caldas, Semi-generalized continuous maps in topological spaces, Portug. Math., 52 (4) (1995), 399–407.
[6] J. Cao and I.L. Reilly, \(\alpha\)-continuous and \(\alpha\)-irresolute multifunctions, Math. Bohemica, 121 (1996), 415–424.
[7] B. Chen, *Para-S-closed spaces*, J. Math. Res. Exposition, 5 (1985), 1–5.

[8] H.H. Corson and E. Michael, *Metrizability of certain countable unions*, Illinois J. Math., 8 (1964), 351–360.

[9] R. Devi, K. Balachandran and H. Maki, *Semi-generalized homeomorphisms and generalized semi-homeomorphisms in topological spaces*, Indian J. Pure Appl. Math., 26 (3) (1995), 271–284.

[10] G. Di Maio and T. Noiri, *On s-closed spaces*, Indian J. Pure Appl. Math., 18 (1987), 226–233.

[11] J. Dontchev, An answer to a question of Mršević and Reilly, *Questions Answers Gen. Topology*, 12 (2) (1994), 205–207.

[12] J. Dontchev, *On some separation axioms associated with the α-topology*, Mem. Fac. Sci. Kochi Univ. (Math.), 18 (1997), 31–35.

[13] J. Dontchev and M. Ganster, *More on sg-compact spaces*, Portugal. Math., 55 (1998), to appear.

[14] J. Dontchev and H. Maki, *On sg-closed sets and semi-λ-closed sets*, Questions Answers Gen. Topology, 15 (2) (1997), 259–266.

[15] C. Dorsett, *Semi compact R₁ and product spaces*, Bull. Malaysian Math. Soc., (2) 3 (1980), 15–19.

[16] R. Engelking, General Topology (PWN, Warszawa, 1977).

[17] D. Janković, *A note on mappings of extremally disconnected spaces*, Acta Math. Hung., 46 (1985), 83–92.

[18] D. Janković and Ch. Konstadilaki, *On covering properties by regular closed sets*, Math. Pannonica, 7 (1996), 97–111.

[19] D. Janković and I.L. Reilly, *On semi-separation properties*, Indian J. Pure Appl. Math., 16 (1985), 957–64.

[20] H.J.K. Junnila, *Three covering properties*, Surveys in General Topology. G.M. Reed Ed., Academic Press, 1980, 195–246.

[21] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, 19 (2) (1970), 89–96.

[22] S.N. Maheshwari and S.S. Thakur, *On α-irresolute mappings*, Tamkang J. Math., 11 (1980), 209–214.

[23] H. Maki, R. Devi and K. Balachandran, *Generalized α-closed sets in topology*, Bull. Fukuoka Univ. Ed. Part III, 42 (1993), 13–21.
[24] M. Mršević and I.L. Reilly, *Separation properties of a topological space and its associated topology of α-subsets*, Kyungpook Math. J., **33** (1993), 75–86.

[25] M. Mršević and I.L. Reilly, *Covering and connectedness properties of a topology space and its associated topology of α-subsets*, Indian J. Pure Appl. Math., **27** (1996), 995–1004.

[26] O. Njåstad, *On some classes of nearly open sets*, Pacific. J. Math., **15** (1965), 961–970.

[27] T. Noiri, *On locally S-closed spaces*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., (8) **74** (1983), 66–71.

[28] T. Noiri and G. Di Maio, *Properties of α-compact spaces*, III Convegno Nazionale di Topologia Trieste, 9-12 giugno 1986, Suppl. Rend. Circ. Mat. Palermo Ser. II, **18** (1988), 359–369.

[29] V. Popa and T. Noiri, *On β-continuous functions*, Real Anal. Exchange, **18** (1992/1993), 544–548.

[30] F.D. Tall, *The density topology*, Pacific J. Math., **62** (1976), 275–284.

[31] T. Thompson, *S-closed spaces*, Proc. Amer. Math. Soc., **60** (1976), 335–338.

[32] Y. Yasui, *Generalized paracompactness*, Topics in General Topology, K. Morita and J. Nagata Eds. North-Holland, 1989, 159–202.

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