Toroidal solutions in Hořava Gravity

Ahmad Ghodsi
Department of Physics, Ferdowsi University of Mashhad,
P.O. Box 1436, Mashhad, Iran

Abstract

Recently a new four-dimensional non relativistic renormalizable theory of gravity was proposed by Horava. This gravity reduces to Einstein gravity at large distances. In this paper by using the new action for gravity we present different toroidal solutions to the equations of motion. Our solutions contain static and rotational ones.
1 Introduction

Recently a new four-dimensional non relativistic renormalizable theory of gravity was proposed by Hořava [1]. It is believed that this theory is a UV completion for the Einstein theory of gravitation. Recently a lot of efforts have been done to understand this theory, [2-33]. In [2] the solutions with spherical symmetry has been found. It also presents equations of motion for Horava gravity. The topological black hole solution have been found in [16]. In this paper, in section two, by using the same method as [2] we review the static toroidal solution. This is a special solution found in [16]. In section three we try to add rotation to our solutions. We use equations of motion presented in [2] and show that there are different possible solutions to the equations of motion.

We start from the four-dimensional metric written in the ADM formalism, [34]

$$ds^2_4 = -N^2dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt). \quad (1.1)$$

The Einstein-Hilbert action in this formalism is given by

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{g}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda), \quad (1.2)$$

where $G$ is the four dimensional Newton’s constant and $K_{ij}$ is the second fundamental form and is defined by

$$K_{ij} = \frac{1}{2N}(\partial_i g_{ij} - \nabla_i N_j - \nabla_j N_i). \quad (1.3)$$

The action proposed by Hořava is a non-relativistic renormalizable gravitational theory and is given by [1]

$$S = \int dtdx^3 \sqrt{g}N \left\{ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_W R - 3\Lambda_W^2)}{8(1 - 3\lambda)} + \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{32(1 - 3\lambda)} R^2 
- \frac{\kappa^2 \lambda^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R_{il} \nabla_j R^l_k - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} \right\}, \quad (1.4)$$

where $\lambda, \kappa, \mu, w$ and $\Lambda_W$ are constant parameters, and $C_{ij}$ is the Cotton tensor, defined by

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left( R^\ell_j - \frac{1}{4} R^{\ell} \right) = \epsilon^{ik\ell} \nabla_k R^\ell_j - \frac{1}{4} \epsilon^{ikj} \partial_k R. \quad (1.5)$$

Using the relation

$$\epsilon^{ijk} R_{i\ell} \nabla_j R^\ell_k = R_{i\ell} \left[ C^{\ell} - \frac{1}{4} \epsilon^{ij} \partial_j R \right] = R_{i\ell} C^{\ell}, \quad (1.6)$$
one can rewrite the action (1.4) as

\[ S = \int dt d^3x (\mathcal{L}_0 + \mathcal{L}_1), \]

\[ \mathcal{L}_0 = \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_W R - 3 \Lambda_W^2)}{8(1 - 3 \lambda)} \right\}, \]

\[ \mathcal{L}_1 = \sqrt{g} N \left\{ \frac{\kappa^2 \mu^2 (1 - 4 \lambda)}{32(1 - 3 \lambda)} R^2 - \frac{\kappa^2}{2 \mu^2} \left( C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left( C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \right\}. \quad (1.7) \]

By comparing \( \mathcal{L}_0 \) with the general theory of relativity in the ADM formalism, one can read the speed of light, Newton’s constant and the cosmological constant as

\[ c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3 \lambda}}, \quad G = \frac{\kappa^2}{32 \pi c}, \quad \Lambda = \frac{3}{2} \Lambda_W. \quad (1.8) \]

Additionally, demanding that \( \mathcal{L}_0 \) gives the usual four dimensional Einstein-Hilbert Lagrangian (general covariance), one find that \( \lambda = 1 \).

2 Toroidal solution

The topological black hole solution have been found in [16]. We are interested to the special case of toroidal symmetric solutions of this paper. So in this section we review this special solution. We start from the following ansatz

\[ ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + d\phi^2). \quad (2.1) \]

**Special case:** \( \lambda = c = 1 \)

To find the usual toroidal solutions to Einstein gravity we start from the special case \( \lambda = c = 1 \) so that \( \Lambda_W = -\frac{32}{\kappa^2 \mu^2} \) and we just consider \( \mathcal{L}_0 \). The easiest way for finding the solution is to substitute the ansatz into the Lagrangian. Doing this up to some overall constant factor we find

\[ \mathcal{L}_0 = -\frac{2}{\kappa^2} \frac{N(r)}{f^2(r)} \left( 2r \frac{df(r)}{dr} + 2f(r) - 6 \Lambda_W r^2 \right). \quad (2.2) \]

The solution to the equations of motion are given by

\[ N^2(r) = f(r) = \frac{-2Mr - \Lambda_W r^4}{r^2}, \quad (2.3) \]
which has only one real root when $M > 0$ and $\Lambda_W < 0$ at $r_0 = (-\frac{2M}{\Lambda_W})^{\frac{1}{3}}$. Although the value of curvature scalar is constant for this solution, but curvature square term $R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}$ is infinite at $r = 0$. For $\Lambda_W > 0$ there is no real root so we have a naked singularity. The solution (2.3) is a known solution in four dimensions, [35]. Our result agrees to them when the charges and angular momentum is set to zero in [35].

We now consider the total Lagrangian $L_0 + L_1$. Due to the special form of the ansatz, the Cotton tensor is zero and we find the following Lagrangian, which is independent of $\omega$

$$L_0 + L_1 = -\frac{2}{\Lambda_W \kappa^2} \frac{N(r)}{f_\pi(r)} \left( 2r \frac{df(r)}{dr} - f(r) + 3\Lambda_W r^2 \right) \left( \Lambda_W r^2 + f(r) \right). \quad (2.4)$$

For this Lagrangian there are two solutions. The first one is

$$f(r) = -\Lambda_W r^2, \quad (2.5)$$

where $N(r)$ is an arbitrary function. The second one is the following solution

$$N^2(r) = f(r) = -Mr^\frac{4}{3} - \Lambda_W r^2, \quad (2.6)$$

where this solution has two real roots when $M > 0$ and $\Lambda_W < 0$ at $r_- = 0$ and $r_+ = (-\frac{M}{\Lambda_W})^{\frac{4}{3}}$. By computing the scalar curvature, we see that the first root causes singularity but the second one is an event horizon. When $M > 0$ and $\Lambda_W > 0$, there is just one root at $r = 0$ which is a naked singularity.

**The general solution:**

Considering the full Lagrangian with general value for $\lambda$, we find

$$L_0 + L_1 = \frac{3\mu^2 \kappa^2 N}{8(-1 + 3\lambda)r^2 f_\pi^2} \left( \frac{(1 - \lambda)r^2}{6} f^2 + \frac{2}{3} r(\Lambda_W r^2 + \lambda f)f' + \frac{1}{3}(1 - 2\lambda)f^2 + \frac{2}{3}\Lambda_W r^2 f + \Lambda_W^2 r^4 \right), \quad (2.7)$$

where prim denotes the derivative with respect to $r$. The solution to the equations of motion is [16]

$$f(r) = -Mr^n - \Lambda_W r^2, \quad N^2(r) = f(r)(Cr)^{2(1-2n)}, \quad n = \frac{2\lambda - \sqrt{-2 + 6\lambda}}{-1 + \lambda}, \quad (2.8)$$

where, $M$ and $C$ are constants of integrations. The above relation reduces to previous results when $\lambda = 1$. 

3
The above solution has two real roots for $M > 0$ and $\Lambda_W < 0$ at $r_− = 0$ and $r_+ = (-\frac{M}{\Lambda_W})^{\frac{1}{n}}$. The scalar curvature is given by $\mathcal{R} = 2(3\Lambda_W + M(n + 1)r^{n-2})$ and because $\lambda \to +\infty$ then $n \to 2$ so we always have a curvature singularity at $r = 0$. When $\Lambda_W > 0$ then $r = 0$ is a naked singularity.

Our computations also show another value for $n$, which is $n = \frac{2\lambda+\sqrt{-2+6\lambda}}{-1+\lambda}$. The important point about this solution is the fact that when $\lambda \to 1$ then $n \to \infty$.

3 Rotating solutions

After a brief review of toroidal solution, in this section we will find other solutions to the Horava gravity by including the rotation. Because of the rotation we have not enough symmetry to use the previous method (i.e. insert the ansatz into the Lagrangian), instead we must solve equations of motion directly. The equations of motion are very difficult to solve since they are up to six derivatives and the metric in rotating solutions depend to rotation coordinate as well as radial coordinate. To overcome this difficulty we try to find the near horizon geometry of rotating black holes in this paper. This will simplify the equations of motion since as we will see in blow the functionality of the solutions with respect to radial coordinate will be fixed, so it remains to find their dependence on the rotation coordinate.

3.1 Extremality

To find the radial behavior of extremal solutions we start to find the extremality conditon for the general solution found in (2.8). We first find the temperature of the solution (2.8). The temperature of this black hole can be computed by finding the surface gravity at horizon, the result will be

$$T = \frac{1}{2\pi} \left( 2\Lambda_W(n - 1)r_0^{-2n+2} + \left(\frac{3}{2}n - 1\right)Mr_0^{-n}\right)$$

$$= \frac{\Lambda_W(\frac{n}{2} - 1)}{2\pi} \left(-\frac{M}{\Lambda_W}\right)^{\frac{2(n-1)}{n-2}},$$

where the last equality coming from the fact that the location of the horizon is at $r_0 = r_+$. The extremality condition happens when the temperature is zero, so we find the critical value of $M$ for an extremal solution to be zero. In this way the geometry of the extremal
solution will be

\[ ds^2 = r^{4(1-n)}dt^2 - \frac{dr^2}{\Lambda W r^2} + r^2(d\theta^2 + d\phi^2) . \] (3.2)

### 3.2 Two derivative solutions

Before we start to solve the equations of motion, we consider the special case where the equations of motion just contain up to two derivative terms. In this case we expect that we find the known solutions for the Einstein gravity. The solution to the equations of motion for Einstein gravity is given by

\[ ds^2 = -N^2 dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \frac{\Sigma^2}{\rho^2} (d\phi - \varpi dt)^2 , \] (3.3)

with

\[
\begin{align*}
\rho^2 &= r^2 + a^2 \theta^2 , \\
\Delta_\theta &= 1 + \frac{a^2}{l^2} \theta^4 , \\
\Delta_r &= a^2 - 2Mr + \frac{r^4}{l^2} , \\
\Sigma^2 &= r^4 \Delta_\theta - a^2 \theta^4 \Delta_r , \\
\varpi &= \frac{\Delta_r \theta^2 + r^2 \Delta_\theta a}{\Sigma^2} , \\
N^2 &= \frac{\rho^2 \Delta_\theta \Delta_r}{\Sigma^2} ,
\end{align*}
\] (3.4)

where \(a\) is the rotation parameter and in our notation \(l^2 = -\frac{2}{\Lambda W} \). We are interested to find the extremal solution and its near horizon geometry. The extremal condition happens when

\[ M = \frac{a^2 l^2 + r_0^4}{2} , \quad r_0^2 = \frac{1}{\sqrt{3}a} , \] (3.5)

where \(r_0\) is the location of the horizon. For finding the near horizon geometry we need to change our variables to some new dimensionless coordinates as follows

\[ r = r_0 + \frac{\epsilon}{\sqrt{y}a} , \quad t = \frac{c_0}{\epsilon} \tau , \quad \phi = \frac{\sqrt{3}c_0}{l \epsilon} \tau , \quad c_0^2 = \frac{r_0^2}{12} . \] (3.6)

Sending \(\epsilon \to 0\) one find the following metric

\[ ds^2 = (1 + \frac{a^2 \theta^2}{r_0^2}) \left( -\frac{1}{2\sqrt{3} ly^2} d\tau^2 + \frac{l^2}{6y^2} dy^2 + \frac{r_0^2}{1 + \frac{a^2}{l^2} \theta^4} d\theta^2 + \frac{r_0^2}{(1 + \frac{a^2}{l^2} \theta^2)^2} (d\phi + \frac{a}{ly} d\tau)^2 \right) . \] (3.7)

This is the near horizon geometry of the rotating black holes with toroidal symmetry. We expect that this satisfies the equations of motion up to two derivative terms. As a double check, we have inserted this solution to the equations of motion and they satisfy these equations.
3.3 Higher derivative solutions

We are interested to find the effect of higher curvature terms. To find the solutions we use the following steps

1. In rotating solutions the Cotton tensor is not necessarily zero and this make the problem difficult to solve. To find the rotating solution, we consider slow rotation condition, i.e. \( a \ll l \) as a parameter of perturbation and solve the equations of motion up to \( \mathcal{O}(a) \).

2. For finding the extremal rotating solution we use the tree-level solution (3.7) as our guide. We start from the following ansatz

\[
d s^2 = -\frac{A_1^2(\theta)}{y^2} d\tau^2 + \frac{A_2(\theta)}{y^2} dy^2 + A_3(\theta) d\theta^2 + A_4(\theta) (d\hat{\phi} + \frac{a}{ly} d\tau)^2,
\]

where \( y \) is the radial near horizon coordinates and the other functions in the metric are some general functions. This metric satisfies the equation of motion coming from variation of the Lagrangian with respect to \( N \), the laps function. So we just need to insert this general ansatz into the other equations of motion coming from variation with respect to the shift functions \( N^i \) and the metric \( g^{ij} \).

3. One may notice that we have a freedom for time scaling in the metric. We have fixed this by choosing the above proper off-diagonal term.

4. There is another freedom when one chooses the function \( A_3(\theta) \). Because this is just a field redefinition, all different functions of \( \theta \) will be equivalent by a change of coordinate on \( \theta \). For fixing this freedom we suppose the following functionality

\[
A_3(\theta) = \frac{r^2_0}{1 + \frac{a^2}{r^2_0} \theta^2},
\]

where we have chosen it in such a way that we can compare the new metric with the previous two derivative case.

5. For solving the equations of motion perturbativly in terms of rotating parameter \( a \), we choose polynomial functions with unknown constant coefficients as

\[
A_i(\theta) = s_i (1 + b_i a \theta^2), \quad i = 1, 2, 4,
\]

where in writing these functions we have used the fact that we have a symmetry under \( (\theta \leftrightarrow -\theta) \).

6. Similar to (3.7), the regularity condition at \( \theta = 0 \) gives a simple constraint as \( s_4 = z \).

7. Similar to the Einstein gravity solution, we suppose that \( r^2_0 = za \) with \( z \) is a function of the constants of the Horava gravity.
Considering all these facts, we find the four Algebraic equations (see appendix A). As one sees there are four equations and six unknown constants. Because already we have used all symmetries and boundary conditions there are no more constraints left.

### 3.3.1 ω independent solution

One amazing observation of equations shows that when \( b_2 = b_4 \) then the constants are independent of \( ω \). Here we find the following values for a general value of \( λ \)

\[ b_1 = b_2 = b_4 = -\frac{4}{3z}, \quad s_1 = \frac{3}{2} \frac{(3λ - 1)z^3}{l^2(3z^2 + 4l^2λ)}, \quad s_2 = -\frac{3}{4} \frac{l^2(3z^2 + 4l^2λ)z^2}{8l^4(λ - 1) - (24l^2 + 27z^2)z^2}. \]

(3.11)

where \( z \) satisfies in the following equation

\[ (λ - 1)^2l^6 - 6(λ - \frac{3}{4})l^4z^2 - \frac{27}{4}(λ - 1)l^2z^4 + \frac{81}{32}z^6 = 0. \]  

(3.12)

This equation shows that the location of the horizon depends on \( λ \) and \( l \).

One special interesting value in the Horava gravity is when we consider \( λ = 1 \). In this case there are two sets of solutions. When \( l^2 > 0 (Λ_W < 0) \) then

\[ r_0^2 = \pm \frac{2}{3^\frac{λ}{4}}l^2a, \quad b_1 = b_2 = b_4 = \pm \frac{2}{3^\frac{λ}{4}}l, \]

\[ s_1 = \pm \frac{2}{3^\frac{λ}{4}(1 + \sqrt{3})l}, \quad s_2 = \frac{l^2 3 + \sqrt{3}}{12 2 + \sqrt{3}}. \]

(3.13)

For \( l^2 < 0 (Λ_W > 0) \)

\[ r_0^2 = \pm \frac{2}{3^\frac{λ}{4}}\sqrt{-l^2}a, \quad b_1 = b_2 = b_4 = \pm \frac{2}{3^\frac{λ}{4}\sqrt{-l^2}}, \]

\[ s_1 = \pm \frac{2}{3^\frac{λ}{4}(1 - \sqrt{3})\sqrt{-l^2}}, \quad s_2 = \frac{l^2 3 - \sqrt{3}}{12 2 - \sqrt{3}}. \]

(3.14)

### 3.3.2 ω dependent solution

In general, when one chooses \( b_2 \neq b_4 \), the constant values will be \( ω \)-dependent. In this case one may solve the equations of motion and find the first three equations for \( s_1, s_2 \) and \( b_1 \) in terms of \( b_2, b_4 \) and \( z \). Putting them into the fourth equation gives a relation between the remaining free parameters. This an equation of degree 8 for \( z \), 6 for \( b_2 \) and 5 for \( b_4 \), so impossible to solve.
To find a solution we restrict ourselves to a special limit of parameters. One possible solution could be found as a series of $\frac{1}{\omega^4}$. Also we consider the location of the horizon $r_0$, to be independent of $\omega$ and its value is the same as $\omega$-independent solution. With these simplifications we find the following solution to first order, $O(\frac{1}{\omega^4})$, in the case of $\lambda = 1$

\[
\begin{align*}
  b_1 &= -\frac{2}{3^4} l (1 + \frac{x_1}{\omega^4}), \\
  b_2 &= -\frac{2}{3^4} l (1 + \frac{x_2}{\omega^4}), \\
  b_4 &= -\frac{2}{3^4} l (1 + \frac{x_4}{\omega^4}) \\
  s_1 &= \frac{2}{3^4 l (3 + \sqrt{3})} (1 + \frac{y_1}{\omega^4}), \\
  s_2 &= \frac{4 l (3 + \sqrt{3})}{12 (2 + \sqrt{3})} (1 + \frac{y_2}{\omega^4}),
\end{align*}
\]

(3.15)

with

\[
\begin{align*}
  x_1 &= -\frac{1}{13} (105 \sqrt{3} + 217) y_2, \\
  x_2 &= -\frac{1}{13} (45 \sqrt{3} + 67) y_2, \\
  x_4 &= -(5 \sqrt{3} + 7) y_2, \\
  y_1 &= \frac{1}{13} (62 \sqrt{3} - 27) y_2,
\end{align*}
\]

(3.16)

where the constant $y_2$ although is arbitrary but can be absorbed into $\omega$ by a rescaling, so we can set it to one. As we see this will produce the $\omega$-independent solution when one send $\omega$ to infinity.

4 Conclusion

In this paper we have studied the toroidal solutions for non relativistic and renormalizable theory of gravity proposed by Horava [1]. We solved equations of motion by using an ansatz with toroidal symmetry. We show our results for general parameters in the theory and in “detailed balance”.

Our computations in static case show the existence of black hole solutions where their location of horizon depends on the parameters of the theory, when $\Lambda_W < 0$. We also show that for $\Lambda_W > 0$ we have naked singularities.

We find our results with both using the method presented in [2] and using the equations of motion presented in [2] and [7] directly.

Using the equations of motion one may try to find the near horizon geometry for extremal rotating solutions. We find our solutions as a perturbed solution when the rotation parameter is small, $a \ll l$. Our computations show two possible sets of solutions, (in)dependent of the $\omega$ parameter. Comparing these results with those in two derivative case one sees that the location of horizon is shifted due to higher derivative corrections this in agreement with results for spherical solutions found in [2].
Acknowledgment

This work was supported by a grant from Ferdowsi University of Mashhad.

Appendix A

\[
\left\{ (b_2^2 - b_2 b_4 + b_4^2) \lambda - \frac{1}{2} b_1^2 - \frac{1}{2} b_2^2 \right\} s_1 s_2 + \frac{1}{4} (-1 + 3 \lambda) z^3 = 0 ,
\]

\[
l^2 \kappa^4 s_1 s_2 \left\{ - \frac{13}{3} + (b_1 - 5 b_2 - \frac{16}{3} b_4) z + \left( - \frac{14}{3} b_2^2 + (b_1 - \frac{11}{3} b_4) b_2 + (b_1 - 5 b_4) b_4 \right) z^2 \right\} + \frac{4}{3} \left\{ \frac{1}{8} (3 \lambda - 1) z^4 + \left[ - 3 z^3 + l^2 (b_2 + b_4) z^2 + \left( - \frac{9}{2} b_2^2 + (b_1 + \frac{1}{2} b_4) b_2 - \frac{1}{2} (b_1 - 5 b_4) b_4 \right) \lambda \right.ight.
\]

\[
+ \frac{9}{4} b_2^2 - \frac{1}{4} b_4^2 - \frac{1}{2} (b_1 + b_4) b_2) l^4 z - 4 l^4 \left( (b_2 - \frac{1}{2} b_4) \lambda - \frac{1}{2} b_2 \right) \right\} s_1 s_2 \right\} \frac{z^2 \omega^4}{(3 \lambda - 1)(b_2 - b_4)} = 0 ,
\]

\[
l^2 \kappa^4 s_1 (-4 z + s_2 (b_2 - b_4)) (1 + (b_2 + b_4) z) + 2 \left\{ \left[ (b_2 - (2 b_2 - b_4) \lambda) z - \left( \frac{1}{2} (b_2^2 + b_4^2) \lambda \right.ight.ight.
\]

\[
- (b_2^2 - b_2 b_4 + b_4^2) s_2 \left\} l^4 - 2 l^2 z^2 + 6 s_2 z^2 \right\} s_1 + \frac{3}{4} z^3 \left( \frac{1}{3} + \lambda \right) \right\} \frac{z^2 \omega^4}{(3 \lambda - 1)(b_2 - b_4)} = 0 ,
\]

\[
6 l^2 s_1 \kappa^4 \left\{ \frac{4}{3} (b_2 + b_4) z^3 + \left[ \frac{4}{3} + \left( - 5 b_2^2 + (b_1 - \frac{11}{3} b_4) b_2 + (b_1 - \frac{14}{3} b_4) b_4 \right) s_2 \right] z^2
\]

\[
+ s_2 (b_1 - \frac{16}{3} b_2 - 5 b_4) z - \frac{19}{3} s_2 \right\} + 4 \left\{ \left[ - 2 l^2 + 6 s_2 \right] z^3 + \left[ \lambda (b_2 + b_4) l^2 - 2 s_2 (b_1 + b_2) \right] l^2 z^2
\]

\[
+ s_2 \left( (9 b_4^2 + (2 b_1 - b_2) b_4 + b_2 (5 b_2 + b_1)) \lambda - \frac{9}{2} b_2^2 + (b_1 + b_2) b_4 + \frac{1}{2} b_2^2 \right) l^4 z
\]

\[
- 4 \left( (b_2 - 2 b_4) \lambda + b_4) s_2 l^4 \right\} s_1 + \frac{3}{4} \left(3 \lambda - 1 \right) z^4 \right\} \frac{z^2 \omega^4}{(3 \lambda - 1)(b_2 - b_4)} = 0 ,
\]

References

[1] P. Horava, Phys. Rev. D 79, 084008 (2009) [arXiv:0901.3775 [hep-th]].

[2] H. Lu, J. Mei and C. N. Pope, [arXiv:0904.1595 [hep-th]].

[3] P. Horava, JHEP 0903, 020 (2009) [arXiv:0812.4287 [hep-th]].
[4] P. Horava, arXiv:0902.3657 [hep-th].
[5] T. Takahashi and J. Soda, arXiv:0904.0554 [hep-th].
[6] G. Calcagni, arXiv:0904.0829 [hep-th].
[7] E. Kiritsis and G. Kofinas, arXiv:0904.1334 [hep-th].
[8] J. Kluson, arXiv:0904.1343 [hep-th].
[9] S. Mukohyama, arXiv:0904.2190 [hep-th].
[10] H. Nikolic, arXiv:0904.2287 [hep-th].
[11] R. Brandenberger, arXiv:0904.2835 [hep-th].
[12] H. Nikolic, arXiv:0904.3412 [hep-th].
[13] K. I. Izawa, arXiv:0904.3593 [hep-th].
[14] H. Nastase, arXiv:0904.3604 [hep-th].
[15] S. S. Pal, arXiv:0904.3620 [hep-th].
[16] R. G. Cai, L. M. Cao and N. Ohta, arXiv:0904.3670 [hep-th].
[17] R. G. Cai, Y. Liu and Y. W. Sun, arXiv:0904.4104 [hep-th].
[18] G. E. Volovik, arXiv:0904.4113 [gr-qc].
[19] Y. S. Piao, arXiv:0904.4117 [hep-th].
[20] X. Gao, arXiv:0904.4187 [hep-th].
[21] E. O. Colgain and H. Yavartanoo, arXiv:0904.4357 [hep-th].
[22] T. Sotiriou, M. Visser and S. Weinfurtner, arXiv:0904.4464 [hep-th].
[23] B. Chen and Q. G. Huang, arXiv:0904.4563 [hep-th].
[24] S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, arXiv:0905.0055 [hep-th].
[25] Y. S. Myung and Y. W. Kim, arXiv:0905.0179 [hep-th].
[26] R. G. Cai, B. Hu and H. B. Zhang, arXiv:0905.0255 [hep-th].

[27] D. Orlando and S. Reffert, arXiv:0905.0301 [hep-th].

[28] C. Gao, arXiv:0905.0310 [astro-ph.CO].

[29] T. Ha, Y. Huang, Q. Ma, K. D. Pechan, T. J. Renner, Z. Wu and A. Wang, arXiv:0905.0396 [physics.pop-ph].

[30] T. Nishioka, arXiv:0905.0473 [hep-th].

[31] A. Kehagias and K. Sfetsos, arXiv:0905.0477 [hep-th].

[32] R. A. Konoplya, arXiv:0905.1523 [hep-th].

[33] A. Ghodsi and E. Hatefi, arXiv:0906.1237 [hep-th].

[34] R. L. Arnowitt, S. Deser and C. W. Misner, arXiv:gr-qc/0405109.

[35] M. M. Caldarelli and D. Klemm, Nucl. Phys. B 545, 434 (1999) arXiv:hep-th/9808097.