PREDICTING THE ORBIT OF TRAPPIST-1

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The TRAPPIST-1 system provides an exquisite laboratory for advancing our understanding exoplanetary atmospheres, compositions, dynamics and architectures. A remarkable aspect of the TRAPPIST-1 is that it represents the longest known resonance chain (Luger et al. 2017), where all seven planets share near mean motion resonances (MMR) with their neighbors such that

\[(j + k)P_x = jP_{x+1},\]  

where \(P_x\) is the period of the \(x^{th}\) planet such that \(P_x < P_{x+1}\), \(k\) and \(j\) are integers describing the resonance order and spacing respectively. For example, planets b and c are close to the 8:5 resonance, which is a 3\(^{rd}\) order \((k = 3)\) resonance closely packed with \(j = 5\). Because the MMRs are not exact, the location of conjunction circulates with a period commonly called the "super-period" (Cochran et al. 2011; Lithwick et al. 2012) with

\[P_{\text{super}} = \frac{1}{|\frac{j+k}{P_{x+1}} - \frac{1}{P_x}|}.\]  

(2)

More than this, neighboring triples reside in Laplace-like resonances (Luger et al. 2017), characterized by

\[\frac{p}{P_x} - \frac{p+q}{P_{x+1}} + \frac{q}{P_{x+2}} \simeq 0,\]  

(3)

where \(p\) and \(q\) are integers describing the resonances. As a result of the Laplace-like resonances, the outer four pairs share the same super-period, with the inner two having a integer ratio of this global super-period.

Although the period of 1h is now precisely known, the original discovery of this outer planet was based upon just a single transit observation (Gillon et al. 2017). Without repeated events, the period was only very crudely constrained at the time, using the transit duration, to \(P_h = 20^{+15}_{-6}\) days. After the discovery of new transits of 1h using K2, the period was locked down to 18.767 days by Luger et al. (2017). However, the authors noted that this period could have been predicted to several decimal places even before the K2 observations using a resonance chain argument.

Assuming that planet 1h is also in a Laplacian resonance with planets 1f and 1g, and that \(p\) and \(q\) are in the range 1 to 3 (as the others are), then six possible periods result: i) \((p, q) = (1, 3) \rightarrow P_h = 13.941\) d; ii) \((p, q) = (1, 2) \rightarrow P_h = 14.899\) d; iii) \((p, q) = (2, 3) \rightarrow P_h = 15.998\) d; iv) \((p, q) = (1, 1) \rightarrow P_h = 18.766\) d; v) \((p, q) = (3, 2) \rightarrow P_h = 25.345\) d; and vi) \((p, q) = (2, 1) \rightarrow P_h = 39.026\) d. Using the transit duration, one could potentially narrow this choice of six down, although in practise almost all of these are compatible with the observed duration.

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In this work we highlight that an additional, but unproven, way of choosing between these periods is the generalized Titius-Bode (TB) law. The periods of the inner six TRAPPIST-1 planets fall closely onto a power-law relation given by

$$P_x = P_1 \alpha^{x-1}. \quad (4)$$

Although originally posed for the Solar System planets, Bovaird & Lineweaver (2013) showed that for Kepler 4+ planet systems, the TB law also holds. This led the authors to predict the presence of 141 new exoplanets using the TB relation. Taking a subset of 97 of these predictions, Huang & Bakos (2014) found five new planets, but generally concluded the predictive power of the law appeared questionable.

Hayes & Tremaine (1998) showed that the TB law is a natural outcome for multiplanet systems satisfying the condition that they are Hill stable. TRAPPIST-1 is an example of a System with Tightly-spaced Inner Planets (STIP) such that the planets are dynamically packed within a compact region, and thus the argument of Hayes & Tremaine (1998) would indicate that the TB law may be a natural by-product of such an arrangement. Applying the TB law to the inner six planets indeed reveals an excellent fit (as shown in Figure 1), with an extrapolated seventh planet appearing at $18.59 \pm 0.97$ d. Comparing to the six possible resonant periods for $1h$ discussed earlier, the TB + Laplacian arguments together would predict a single unique period of 18.766 d, which is indeed bang on the observed period of $18.767 \pm 0.004$ d (Luger et al. 2017).

A postdiction is never as impactful as a prediction. And so, we take the next step and use this logic to predict the orbital period of TRAPPIST-1i. To be clear, there is presently no literature suggesting an eighth planet but if it should exist, we can predict its period. We first note that the TB law predicts a period for $1i$ of 27.53 $\pm$ 0.83 d. Exploring the possible combinations of $p$ and $q$ below 3, we find five possible periods of i) $(p,q) = (1,3) \rightarrow P_1 = 22.693$ d; ii) $(p,q) = (1,2) \rightarrow P_1 = 25.345$ d; iii) $(p,q) = (2,3) \rightarrow P_1 = 28.699$ d; iv) $(p,q) = (1,1) \rightarrow P_1 = 39.037$ d; and v) $(p,q) = (3,2) \rightarrow P_1 = 84.810$ d. Of these, only options ii) and iii) appear compatible with the TB prediction with the former leading to a 4:3 resonance between $h$ and $i$, and the latter a 3:2 resonance (both yield the same super-period of 489.91 d). Although technically both are compatible with the TB law and the Laplacian resonance argument, the 3:2 configuration - yielding $P_1 = 28.699$ d, would lead to a more generously spaced system.

This work does not address the probability of the existence of an additional planet (see the work of Kipping & Lam 2017 for an example of this). However, if an eighth planet is found with one of the two predicted periods, it would provide some confidence that STIPs with Laplacian resonant chains are prime systems for precise predictions of planetary periods.

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Figure 1. Upper: Postdiction for the orbital period of TRAPPIST-1h using the generalized TB law argument of Bovaird & Lineweaver (2013) combined with the resonant chain argument of Luger et al. (2017). A unique and highly precise period is forecasted, which is ultimately almost exactly the observed value. Lower: Applying the same logic to predict the period of a hypothetical TRAPPIST-1i, where we find two candidate periods satisfying both the TB and Laplacian arguments.