Constraints on the new particle in $\Sigma^+ \rightarrow p\mu^+\mu^-$

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(Dated: March 26, 2022)

Abstract

The HyperCP collaboration has presented the branching ratio of $\Sigma^+ \rightarrow p\mu^+\mu^-$ to be $(8.6^{+6.6}_{-5.4} \pm 5.5) \times 10^{-8}$ and suggested a new boson $P^0$ with a mass of $214.3 \pm 0.5$ MeV to induce the flavor changing transition of $s \rightarrow d\mu^+\mu^-$. We demonstrate that to explain the data, the new particle cannot be a scalar but pseudoscalar based on the direct constraints from $K^+ \rightarrow \pi^+\mu^+\mu^-$ and $K_L \rightarrow \mu^+\mu^-$, respectively. Moreover, we determine that the decay width of the pseudoscalar should be in the range of $10^{-7}$ MeV with the lifetime of $10^{-14}$ sec.

PACS numbers: 11.30.Er, 13.25.Hw
According to the report of the HyperCP collaboration [1], the observation of three events for the decay $\Sigma^+ \rightarrow p\mu^+\mu^-$ reveals the possibilities of new physics, as the branching ratio $\text{Br}(\Sigma^+ \rightarrow p\mu^+\mu^-) = (8.6^{+6.6}_{-5.5} \pm 5.5) \times 10^{-8}$ is claimed to be larger than the prediction within the Standard Model [1, 2, 3]. The analysis in Ref. [1] has found an unexpectedly narrow dimuon distribution, which can not be explained by the form factors used to deform the phase space due to their mildly momentum dependences [2, 3]. The plausible explanation can be the threshold effect which is induced as $m_{\mu^+\mu^-} = (p_{\mu^+} + p_{\mu^-})$ is approaching to the pole of some unknown intermediate boson, suggesting a two-body $\Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^-$ decay shown in Fig. 1a, with the $P^0$ mass being $m_{P^0} = 214.3 \pm 0.5$ MeV [1] and the branching ratio [1]

$$\text{Br}(\Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^-) = (3.1^{+2.4}_{-1.9} \pm 1.5) \times 10^{-8}. \quad (1)$$

If the effect is true, the flavor-changing neutral current (FCNC) of the $s \rightarrow d$ transition is discovered at tree level. Clearly, the most important task is to check the reality of the experiment. Note that the observed events are only three and the physical properties of this unknown particle remain ambiguous. Nevertheless, the investigations can proceed via the decays of $K^+ \rightarrow \pi^+\mu^+\mu^-$ and $K_L \rightarrow \mu^+\mu^-$ since they share the same effective four-fermion interaction at quark level as shown in Fig. 1. In this paper, we will explore the constraints on the new particle suggested by the HyperCP collaboration by relating the three decay modes.

We start with the general effective four-fermion interaction for $s \rightarrow d\mu^+\mu^-$ in Fig. 1 by including all possible scalar-type currents, given by

$$\mathcal{L}_{NP} = \frac{\lambda_{ij}}{q^2 - m_{P^0}^2 + i m_{P^0} \Gamma_{P^0}} \bar{d} \Gamma_i s \bar{u} \Gamma_j v + H.C., \quad (2)$$
where \( q = p_{\mu^+} + p_{\mu^-}, \Gamma_{P0} \) is the decay width, \( u (v) \) denotes the \( \mu^- (\mu^+) \) spinor and \( \lambda_{ij} \) are the combined coupling constants with \( i, j = S \) and \( P \) for \( \Gamma_{i,j} = 1 \) and \( \gamma_5 \), representing scalar and pseudoscalar currents, respectively. We stress the necessity of the decay width \( \Gamma_{P0} \) in Eq. (2) for the threshold enhancement around the pole near \( m_{\mu^+\mu^-} = 214.3 \) MeV. In Eq. (2), there are four kinds of couplings through \( S \otimes S, P \otimes P, S \otimes P \) and \( P \otimes S \) currents. Note that the latter two are parity-odd terms with the physical states being the mixtures of scalar and pseudoscalar \([4]\). Moreover, the last one could also violate CP symmetry through the longitudinal muon polarization in \( K_L \rightarrow \mu^+\mu^- \) \([4]\). However, we shall not consider CP violation in the present paper.

We now apply \( \mathcal{L}_{NP} \) in Eq. (2) to the decay of \( \Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^- \). The amplitude is found to be

\[
\mathcal{A}_{\Sigma^+} \equiv \mathcal{A}(\Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^-) = \frac{\lambda_{ij}}{q^2 - m_{P0}^2 + i m_{P0} \Gamma_{P0}} (p|d\Gamma_s|\Sigma^+) \bar{u}\Gamma_j v.
\]

To evaluate the amplitude, we parametrize

\[
\langle p|d\bar{s}|\Sigma^+ \rangle = f_S \bar{u}_\mu u_\Sigma, \quad \langle p|d\gamma_5 s|\Sigma^+ \rangle = g_P \bar{u}_\mu \gamma_5 u_\Sigma,
\]

where the form factors are given by \([5]\)

\[
f_S = f_1(q^2) \frac{m_\Sigma - m_p}{m_s - m_d}, \quad g_P = g_1(q^2) \frac{m_\Sigma + m_p}{m_s + m_d},
\]

with \([5, 6]\)

\[
f_1(q^2) = \frac{f_1(0)}{(1 - \frac{q^2}{m_V^2})^2}, \quad g_1(q^2) = \frac{g_1(0)}{(1 - \frac{q^2}{m_A^2})^2},
\]

\[
f_1(0) = -1.0, \quad g_1(0) = 0.35, \quad m_V = 0.97 \text{ GeV}, \quad m_A = 1.25 \text{ GeV}.
\]

It is noted that the momentum dependences are expressed as the double-pole expansions.

To test possibilities of new physics from \( \Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^- \), we study the decays of \( K^+ \rightarrow \pi^+\mu^+\mu^- \) and \( K_L \rightarrow \mu^+\mu^- \) due to \( \mathcal{L}_{NP} \) in Eq. (2). The amplitudes of \( K^+ \rightarrow \pi^+P^0, P^0 \rightarrow \mu^+\mu^- \) and \( K_L \rightarrow P^0, P^0 \rightarrow \mu^+\mu^- \) are given by

\[
\mathcal{A}_{K^+} = \frac{\lambda_{ij}}{q^2 - m_{P0}^2 + i m_{P0} \Gamma_{P0}} (\pi^+|d\Gamma_s|K^+) \bar{u}\Gamma_j v,
\]

\[
\mathcal{A}_{K_L} = \frac{\lambda_{ij}}{q^2 - m_{P0}^2 + i m_{P0} \Gamma_{P0}} \left[ \langle 0|d\Gamma_s|K_L \rangle + \langle 0|s\bar{\Gamma}_s|K_L \rangle \right] \bar{u}\Gamma_j v,
\]
respectively. Here, we have defined $A_{K^+} \equiv A(K^+ \to \pi^+ P^0, P^0 \to \mu^+ \mu^-)$ and $A_{K_L} \equiv A(K_L \to P^0, P^0 \to \mu^+ \mu^-)$. It is noted that there are no contributions from $\langle \pi^+ | \bar{d} \gamma_5 s | K^+ \rangle$ and $\langle 0 | \bar{s} d | K_L \rangle$ due to the parity conservation in strong interaction. Therefore, $K^+ \to \pi^+ \mu^+ \mu^-$ can only be used to constrain the couplings of $S \otimes S(P)$, while $K_L \to \mu^+ \mu^-$ to those of $P \otimes S(P)$. The matrix elements in Eq. (7) by means of equation of motion are found to be

$$
\langle \pi^+ | \bar{d} s | K^+ \rangle = \frac{m_K^2 - m_\pi^2}{m_s - m_d} f_+, \\
\langle 0 | \bar{d} \gamma_5 s | K_L \rangle + \langle 0 | \bar{s} d | K_L \rangle = i \sqrt{2} f_K \frac{m_K^2}{m_s + m_d},
$$

(8)

where $f_+ \simeq 1$ and $f_K = 160$ MeV.

To proceed, we first concentrate on $S \otimes S(P)$ couplings. The experimental measurement for the decay of $K^+ \to \pi^+ \mu^+ \mu^-$ is

$$
Br(K^+ \to \pi^+ \mu^+ \mu^-) = (8.1 \pm 1.4) \times 10^{-8}.
$$

(9)

It has been demonstrated that the dominate contribution for the decay is from the one-photon exchange in the Standard Model, which can be referred, such as in Ref. [7]. Since the partial branching ratio of the three-body decay is proportional to $|A|^2/m^3 \cdot \tau$, where $|A|^2$ is the squared amplitude and $m (\tau)$ is the mass (lifetime) of the mother particle. For the $S \otimes S (P)$ currents, from Eqs. (3)-(7) we have

$$
|A_{K^+}|^2/|A_{\Sigma^+}|^2 \simeq 0.25,
$$

(10)

$(1/m_K^3)/(1/m_{\Sigma^+}^3) \simeq 14$ and $\tau_{K^+}/\tau_{\Sigma^+} = 1.5 \times 10^2$. After integrating the phase space, we find that

$$
\frac{Br(K^+ \to \pi^+ P^0, P^0 \to \mu^+ \mu^-)}{Br(\Sigma^+ \to p P^0, P^0 \to \mu^+ \mu^-)} \sim O(10^2) \ (O(10^3))
$$

(11)

for the $S \otimes S (P)$ currents. Note that the estimation of the ratio in Eq. (11) is independent of the property of the new particle. When $Br(\Sigma^+ \to p P^0, P^0 \to \mu^+ \mu^-)$ is in the range of $O(10^{-8})$, in any case, $Br(K^+ \to \pi^+ P^0, P^0 \to \mu^+ \mu^-)$ should be of $O(10^{-5} - 10^{-5})$ if the interaction is $S \otimes S (P)$, which is clearly out of the limitation in Eq. (9). As a result, the tree level flavor-changing $s \to d \mu^+ \mu^-$ transition resulting from the new physics of $S \otimes S (P)$ currents is unambiguously ruled out based on the data of $K^+ \to \pi^+ \mu^+ \mu^-$. 

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We now turn to the new physics from $P \otimes P$ and $P \otimes S$ currents. Currently, the experimental measurement on $K_L \to \mu^+\mu^-$ is

$$Br(K_L \to \mu^+\mu^-) = (6.87 \pm 0.12) \times 10^{-9},$$

which is almost saturated by the absorptive (imaginary) part, dominated by the measured mode of $K_L \to \gamma\gamma$ with $Br(K_L \to \gamma\gamma) = (5.56 \pm 0.06) \times 10^{-4}$, i.e., $Br_{\text{abs}}(K_L \to \mu^+\mu^-) = (6.66 \pm 0.07) \times 10^{-9}$. However, there is still a possibility of the cancellation among the short-distance amplitude and the real part of the long-distance part. Nevertheless, it is believed that the new physics contribution to the decay branching ratio cannot exceed of $O(10^{-9})$.

Beginning with a rough estimate, if we assume that $\Gamma_{P^0} \simeq 1$ MeV, we find that the ratio of $Br_{K_L} \equiv Br(K_L \to P^0, P^0 \to \mu^+\mu^-) \leq 10^{-9}$ as our working assumption to constrain the new physics.

While constraints on the coupling constants are mainly from different open windows of $Br(K_L \to P^0, P^0 \to \mu^+\mu^-)$, the sets of $\Gamma_{P^0}$’s are as small as possible to enhance $Br(\Sigma^+ \to pP^0, P^0 \to \mu^+\mu^-)$ to match the experimental value in Eq. (1), where the error has been taken as the larger one between $\sigma_+$ and $\sigma_-$. It is obvious that the pole effect plays the most important role to coincide with both data of $K_L \to \mu^+\mu^-$ and $\Sigma^+ \to pP^0, P^0 \to \mu^+\mu^-$. As a consequence, the coupling constant $\lambda_{PP}$ is in the order of $10^{-13}$ while the decay width $\Gamma_{P^0}$ is in the range of $10^{-7}$ MeV, translated as the lifetime of $\tau_{P^0} \simeq 10^{-14}$ sec. Inasmuch as it is also possible from the $P \otimes S$ current though it would induce CP violation, the window as well is opened as the same as that of the $P \otimes P$ current, and we obtain that $\lambda_{PS}$ around $10^{-13}$ with $\Gamma_{P^0}$ in $10^{-8}$ MeV, which is one order of magnitude smaller than that of the $P \otimes P$ current.
TABLE I: The coupling constant and decay width of $P^0$ for $P \otimes P$ and $P \otimes S$ currents.

| $Br(K_L \rightarrow P^0, P^0 \rightarrow \mu^+\mu^-)(10^{-10})$ | $P \otimes P$ | $P \otimes S$ |
|---------------------------------------------------------------|---------------|---------------|
|                                                              | $|\lambda_{PP}|$ (10$^{-13}$) | $|\lambda_{PS}|$ (10$^{-13}$) | $\Gamma_{po}$ (10$^{-7}$ MeV) | $\Gamma_{po}$ (10$^{-9}$ MeV) |
| 0.3-0.6                                                      | 0.81$^{+0.13}_{-0.15}$ | 0.92$^{+11.75}_{-0.60}$ | 0.90$^{+0.14}_{-0.16}$ | 3.10$^{+39.27}_{-2.01}$ |
| 0.6-1.0                                                      | 1.08$^{+0.13}_{-0.15}$ | 1.64$^{+19.47}_{-0.99}$ | 1.20$^{+0.14}_{-0.16}$ | 5.51$^{+65.69}_{-3.34}$ |
| 1.0-1.5                                                      | 1.35$^{+0.13}_{-0.14}$ | 2.56$^{+29.14}_{-1.49}$ | 1.50$^{+0.14}_{-0.16}$ | 8.61$^{+98.09}_{-4.99}$ |
| 1.5-2.0                                                      | 1.60$^{+0.11}_{-0.12}$ | 3.58$^{+38.62}_{-1.97}$ | 1.77$^{+0.12}_{-0.13}$ | 12.05$^{+130.18}_{-6.62}$ |
| 2.0-10                                                       | 2.97$^{+0.86}_{-1.27}$ | 12.3$^{+198.8}_{-10.2}$ | 3.28$^{+0.95}_{-1.39}$ | 21.7$^{+689.7}_{-14.5}$ |

Among theoretical models, the proposed pseudoscalar particle $P^0$ is not likely the axion for its mass being much heavier than the allowed values [11]. However, it cannot be a leptoquark either since it could not lead to the pole effect. However, the sgoldstino of the supersymmetric model is still allowed as the range of the mass is consistent with the experimental one [12]. Clearly, more efforts of both theory and experiment are needed.

In sum, we have found that as an intermediate boson to the decay of $\Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^-$, $P^0$ can be induced from the $P \otimes P$ or $P \otimes S$-type current, which is testified with $K_L \rightarrow \mu^+\mu^-$, whereas $S \otimes P$ and $S \otimes S$-type currents have been proven to be impossible via the decay of $K^+ \rightarrow \pi^+\mu^+\mu^-$. Moreover, the analysis suggests that in the window of $Br(K_L \rightarrow \mu^+\mu^-) = (0.3-0.6) \times 10^{-10}$, while $Br(\Sigma^+ \rightarrow pP^0, P^0 \rightarrow \mu^+\mu^-) = (3.1 \pm 2.8) \times 10^{-8}$ with $m_{po} = 214.3$ MeV, the decay width and lifetime are $(0.92^{+11.75}_{-0.60}) \times 10^{-7}$ MeV and $(7.2^{+13.2}_{-2.2}) \times 10^{-15}$ sec, respectively, with the coupling constants $\lambda_{PP} = (0.81^{+0.13}_{-0.15}) \times 10^{-13}$. Finally, we remark that our analysis can be generalized to vector and axial-vector currents.

Note added: Before we presented the paper, there were two similar papers by He et al. [13], Deshpande et al. [14] and Gorbunov et al. [15] in arXiv. Here, we make some comparisons as follows:

1. While Refs. [13, 14] begin with the Lagrangian coupled to the new particle, we consider the effective four-quark interaction at quark level, which leads to $\Sigma^+ \rightarrow p\mu^+\mu^-$. The propagator in the interaction includes the decay width to avoid divergence due to the pole. As a result, we can give stronger constraints on the decay width. We point out that in the same effective four-quark interaction for $S \otimes S(P)$ currents, the deviation between $\Sigma^+ \rightarrow p\mu^+\mu^-$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$ is only in $|A|^2/m_{\Sigma(K)}^3 \cdot \tau_{\Sigma(K)}$ with the phase space, given a ratio of
$Br_{K^+}/Br_{\Sigma^+} \simeq 10^3$, such that we completely rule out any possibilities from a scalar coupling of the $s \to d$ transition, which is consistent with Ref. [13] while Ref. [14] leaves ambiguity.

2. We constrain the couplings of $s \to d\mu^+\mu^-$ to avoid the uncertainty while Refs. [13, 14] separately estimate the upper bounds of the coupling constants for $P^0_sd$ and $P^0\mu^+\mu^-$. Borrowing the values from Refs. [13, 14], in which $\lambda_{sd}^P$ and $\lambda_{\mu\mu}^P$ are individually from the $K^0-\bar{K}^0$ mixing and muon magnetic dipole moment, we obtain $\lambda_{PP} \equiv \lambda_{sd}^P\lambda_{\mu\mu}^P < 5 \times 10^{-13}$, which is consistent with the upper values of our windows.

3. We also explicitly consider $S \otimes P$ and $P \otimes S$ couplings which are absent in Refs. [13, 14].

4. We note that our results of $\Gamma_{P^0} \sim 10^{-7}$ MeV and $\tau_{P^0} \sim 10^{-14}$ sec are close to the upper and lower bounds of $\Gamma_{P^0} < 1.6 \times 10^{-6}$ MeV and $\tau_{P^0} \geq 1.7 \times 10^{-15}$ sec, given by Refs. [14] and [15], respectively.

Acknowledgements

We would like to thank Prof. X. G. He for useful discussions. This work is financially supported by the National Science Council of Republic of China under the contract number NSC-94-2112-M-007-004.

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