Abstract—This paper presents a novel Nonlinear Model Predictive Control (NMPC) scheme for underwater robotic vehicles operating in a constrained workspace including static obstacles. The purpose of the controller is to guide the vehicle towards specific way points with guaranteed input and state constraints (i.e. obstacle avoidance, workspace boundaries). The proposed scheme incorporates the full dynamics of the vehicle in which the ocean currents are also involved. Hence, the control inputs calculated by the proposed scheme are formulated in a way that the vehicle will exploit the ocean currents, when these are in favor of the way-point tracking mission which results in reduced energy consumption by the thrusters. The closed-loop system has analytically guaranteed stability and convergence properties. The performance of the proposed control strategy is experimentally verified using a 4 Degrees of Freedom (DoF) underwater robotic vehicle inside a constrained test tank with obstacles.

I. INTRODUCTION

During the last decades, underwater robotic vehicles have been widely used in a variety of marine activities. Applications such as monitoring and inspection, surveillance of underwater facilities, oceanography, search and rescue, are indicative examples of applications that require the underwater robot to operate under various constraints and increased level of autonomy, in terms of energy consumption and endurance [1], [2]. Thus, newest research directions point towards the development of motion control strategies that are able to handle complex missions with reduced energy requirements [3].

The energy consumption of an underwater vehicle can be distinguished in two parts: i) the hotel load which is defined as the energy consumption due to the on-board computers, processing effort, instrumentation and communication devices ii) the energy used by the propulsion system (e.g. thrusters) [3]. The hotel load reduction can be achieved by employing low power devices and lean algorithms that do not require significant processing effort. On the other hand, the optimization of the thrust energy consumption, is mainly a path planning problem where the vehicle must reach the desired goal in an energy optimal manner.

Energy minimization via mission planning has been studied in the past [4]–[6] and it is still an active area of research for underwater robotics. The importance of utilizing ocean currents in underwater vehicle operation was emphasized in [5], where a genetic algorithm planner was proposed for the design of a path with minimum energy requirements. An energy efficient framework was also proposed in [6], where the authors consider quasi-static ocean current information and constant thrust power. In [7], an A* search is used in order to design a continuous path where the ocean currents were incorporated as quadratic drag force terms.

Moreover, the motion control of underwater vehicles is a highly nonlinear problem, where multiple input and state constraints are imposed to the system. Several linear and nonlinear motion control techniques for underwater vehicles can be found in literature. However, input (generalized body forces/torques or thrust) and state (3D obstacles, velocities) constraints are not always considered. Nonlinear Model Predictive Control (NMPC) [8], is an ideal approach for complex underwater missions, as it is able to combine motion planning, obstacle avoidance and workspace restrictions, while handling efficiently input and state constraints.

In [4], the authors propose an MPC scheme in order to design an energy efficient path for a glider, by minimizing a cost function based on the consumed energy. However, only the kinematic model of the vehicle is considered, without any disturbances or noise of the ocean current profile. In order to optimize sawtooth paths for an Autonomous Underwater Vehicle (AUV), an MPC scheme with a least squares cost function is presented in [9]. Interesting results including estimated ocean wave profiles into an NMPC scheme, with an emphasis on real-time execution are presented in [10]. However, the effect of noise and disturbance are not theoretically considered, but instead are presented through simulation testing. An MPC scheme with reduced dynamic model is presented in [11], where in order to avoid computational complexity, simplified linear models were considered for the vertical and horizontal control of the vehicle. In the aforementioned works the validation
of the proposed strategies was conducted via simulation tests. An experimental validation of a visually aided NMPC scheme for an underwater robotic system was presented in [12], where simple kinematic equations of the system was considered.

In this work, a novel NMPC scheme for underwater robotic vehicles is presented. The purpose of the controller is to guide the vehicle towards specific way points (See Fig 1). Various constraints such as: sparse obstacles, workspace boundaries, control input saturation as well as predefined upper bound of the vehicle velocity (requirements for several underwater tasks such as seabed inspection scenario, mosaicking) are considered during the control design. The proposed scheme incorporates the full dynamics of the vehicle in which the ocean currents are also involved. The controller is designed in order to find optimal thrusts required for minimizing the way point tracking error. Moreover, the control inputs calculated by the proposed approach are formulated in a way that the vehicle will exploit the ocean currents, when these are in favor of the way-point tracking mission, which results in reduced energy consumption by the thrusters. The closed-loop system has analytically guaranteed stability and convergence properties. The performance of the proposed control strategy is experimentally verified using a 4 DoF underwater robotic vehicle inside a constrained test tank with obstacles. To the best of the authors knowledge, this is the first time where a NMPC scheme which incorporates the full dynamics of the vehicle is experimentally verified in a constrained workspace including sparse obstacles.

II. PRELIMINARIES

A. Notation

In this work, the vectors are denoted with lower bold letters whereas the matrices by capital bold letters. Given two sets \( A \) and \( B \subset \mathbb{R}^n \), the Minkowski addition set \( C \) of two sets \( A \) and \( B \) is defined as: \( C = A + B = \{a + b : a \in A, b \in B\} \). The Pontryagin difference set \( P \) of two sets \( A \) and \( B \) is defined as \( P = A \sim B = \{x \in \mathbb{R}^n : x \notin B\} \). We define \( B(c,r) \) as the closed sphere with radius \( r \) and center \( c \). For a given set \( A \subset \mathbb{R}^n \) we define as \( \text{cl}(A) \), \( \text{int}(A) \) and \( \partial S = \text{cl}(A) \setminus \text{int}(A) \) its closure, interior and boundary, respectively. Thus, we have \( A = \text{int}(A) \cup \partial A \).

B. Mathematical Modeling

The prior step before analyze the proposed methodology is the presentation of the preliminary aspects of the modeling of underwater vehicles. Firstly, let us define a common body-fixed frame \( V = \{e_x, e_y, e_z\} \) attached to the vehicle center of gravity, as well as the inertial frame \( I = \{e_F, e_R, e_D\} \) as shown in Fig 2. The pose vector of the vehicle with respect to (w.r.t) the inertial frame \( I \) is denoted by \( \eta = [\eta_1^T \eta_2^T]^T \in \mathbb{R}^6 \) including the position (i.e., \( \eta_1 = [x \ y \ z]^T \)) and orientation (i.e., \( \eta_2 = [\phi \ \theta \ \psi]^T \)) vectors. The \( v = [v_F^T \ v_R^T \ v_D^T]^T \in \mathbb{R}^6 \) is the velocity vector of the vehicle expressed in fixed-body frame \( V \) and includes the linear (i.e., \( v_1 = [u \ v \ w]^T \)) and angular (i.e., \( v_2 = [p \ q \ r]^T \)) velocity vectors. In this work, we consider that the vehicle operates under the influence of bounded irrotational ocean currents w.r.t the inertial frame \( I \). An estimation of the ocean currents can be achieved by employing the data obtained from Naval Coastal Ocean Model (NCOM) [13] and Regional Ocean Model Systems (ROMS)[14]. However, an estimation of the ocean currents can be achieved locally using an appropriate estimator [15]. Thus, in the following analysis, we consider the effect of ocean currents during the control design, but we assume that these data are inaccurate and the uncertainties on the ocean current’s profile should be handled during the robustness analysis. In this work, the bounded irrotational ocean current velocities w.r.t the inertial frame \( I \) is denoted by \( \omega_c = [(v_c^F)\ T, (0_{1 \times 3})]^T \in \mathbb{R}^6 \) with \( \omega_c^T = [v_c^F, v_c^R, v_c^D]^T \) to be the vector of linear velocity terms. Therefore, we can define the vehicle velocity vector relative to the water expressed in body frame \( V \) as

\[
v_r = v - v_c \tag{1}
\]

Notice that the vector \( v_c = [u_c, v_c, w_c, 0_{1 \times 3}]^T \) indicates the expression of the ocean currents with respect to the body frame \( V \). Without loss of generality, according to the standard underwater vehicles’ modeling properties [16], assuming that the current velocity is slowly varying with respect to the inertial frame (e.g., \( \frac{\partial v_c}{\partial t} \approx 0 \)), and the vehicle is operating at relative low speeds, the dynamic equations of the vehicle can be given as [16, eq:3.110-3.116]:

\[
\dot{\eta} = J(\eta) \dot{v}_r + v_c^T \tag{2a}
\]

\[
M_\ell \ddot{v}_r + C(v_r) \dot{v}_r + D(v_r)\dot{v}_r + g(\eta) = \tau_v \tag{2b}
\]

where:

- \( \tau_v = [X, Y, Z, K, M, N]^T \in \mathbb{R}^6 \) is the total propulsion force/torque vector (i.e., the body forces and torques generated by the thrusters) applied on the vehicle and expressed in \( V \);
- \( M = M_{RB} + M_A \), where \( M_{RB} \in \mathbb{R}^{6 \times 6} \) and \( M_A \in \mathbb{R}^{6 \times 6} \) are the inertia matrix for the rigid body and added mass respectively;

Fig. 2: Seabotix LBV150 ROV. In red color the inertial frame (I) is indicated, while in green color the body-frame (V) one. The under-actuated DoFs (roll -p- and pitch -q-) are also depicted in blue color.
\[ C(\mathbf{v}_r) = C_{RB}(\mathbf{v}_r) + C_A(\mathbf{v}_r), \] where \( C_{RB}(\mathbf{v}_r) \in \mathbb{R}^{6 \times 6} \) and \( C_A(\mathbf{v}_r) \in \mathbb{R}^{6 \times 6} \) are the coriolis and centripetal matrix for the rigid body and added mass respectively;

\[ D(\mathbf{v}_r) = D_{quad}(\mathbf{v}_r) + D_{lin}(\mathbf{v}_r), \] where \( D_{quad}(\mathbf{v}_r) \in \mathbb{R}^{6 \times 6} \) and \( D_{lin}(\mathbf{v}) \in \mathbb{R}^{6 \times 6} \) are the quadratic and linear drag matrix respectively;

\[ g(\eta) \in \mathbb{R}^6 \] is the hydrostatic restoring force vector;

\[ J(\eta) = \begin{bmatrix} J_1(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J_2(\eta_2) \end{bmatrix} \] is the Jacobian matrix transforming the velocities from the body-fixed (V) to the inertial (I) frame, in which \( J_1(\eta_2) \in SO(3) \) is the well known rotation matrix and \( J_2(\eta_2) \in \mathbb{R}^{3 \times 3} \) denotes the lumped transformation matrix;

Notice that the transformation from ocean current mean velocity defined in the inertial frame I (i.e., \( \mathbf{v}_c^I \)) into body-fixed one (i.e., \( \mathbf{v}_c \)) is achieved using the transposed rotation matrix i.e. \( \mathbf{v}_c = J^T(\eta)\mathbf{v}_c^I \) (See[16]). In [2], the total propulsion force/torque vector (\( \tau_V \)) is computed using the thruster allocator matrix which is formulated by the actuation geometry and properties of the underwater vehicle’s thrusters. The vehicle used in this work is a 4 DoF Seabotix LBV150. It is equipped with 4 thrusters (i.e., Port (p), Starboard (s), Vertical (v), Lateral (l)), which are effective in Surge (X), Sway (Y), Heave (Z) and Yaw (N) motion. Thus, we can define a new thrust vector (\( \tau = [\tau_p, \tau_s, \tau_v, \tau_l]^T \in \mathbb{R}^4 \)) and the appropriate thruster allocator matrix (\( T_A \in \mathbb{R}^{4 \times 4} \)) such as:

\[ \tau_V^{LV} = T_A \tau, \] (3)

where \( \tau_V^{LV} [X, Y, Z, N]^T \in \mathbb{R}^4 \).

**Remark 1**: In the vehicle used in this work, the angles \( \phi, \theta \) and angular velocities \( \omega \) and \( \psi \) are negligible and we can consider them to be equal to zero. Thus, from now on, the \( \eta = [x, y, z, \psi] \) and \( \mathbf{v} = [u, v, w, r] \). The vehicle is symmetric about \( x - z \) plane and close to symmetric about \( y - z \) plane. Therefore, we can safely assume that motions in heave, roll and pitch are decoupled [16].

**III. PROBLEM FORMULATION**

In this Section we formally address the problem under consideration:

**Problem 1**: Given an underwater vehicle with dynamics as described in [3], design a robust feedback control law for the autonomous guidance towards a set of way-points \( \eta^i_k \), \( i = \{1, \ldots, n\} \), while guaranteeing the following specifications:

- Avoid the workspace boundaries and a limited set of obstacles within.
- Respect operational limitations in the form of state (e.g. velocity bounds) and input (thrust saturation) constraints.
- The energy consumed from thrusters to be retained in a reduced level.

**IV. METHODOLOGY**

In this Section we present in detail the methodologies proposed in order to formulate the solution of the Problem defined in [3].

**A. Geometry of Workspace**

Consider an underwater vehicle which operates inside the workspace \( \mathcal{W} \subset \mathbb{R}^3 \) with boundaries \( \partial \mathcal{W} = \{ \mathbf{p} \in \mathbb{R}^3 : \mathbf{p} \in \text{cl}(\mathcal{W}) \setminus \text{int}(\mathcal{W}) \} \) and scattered obstacles located within. Without any loss of the generality, the robot and the obstacles are all modeled by spheres (i.e., we adopt the spherical world representation[17]). In this spirit, let \( \mathcal{B}(\eta_1, \bar{r}) \) to be a closed ball that covers all the vehicle volume (main body and additional equipments). Moreover, the \( \mathcal{M} \) statics obstacles within the workspace are defined as closed spheres described by \( \pi_m = \mathcal{B}(\mathbf{p}_{\pi_m}, r_{\pi_m}) \), \( m \in \{1, \ldots, M\} \), where \( \mathbf{p}_{\pi_m} \in \mathbb{R}^3 \) is the center and the \( r_{\pi_m} > 0 \) the radius of the obstacle \( \pi_m \). Additionally, based on the property of spherical world[17], for each pair of obstacles \( m, m' \in \{1, \ldots, M\} \) the following inequality holds:

\[ ||\pi_m - \pi_{m'}|| > 2\bar{r} + r_{\pi_m} + r_{\pi_m}' \] (4)

which intuitively means that the obstacles \( m \) and \( m' \) are disjoint in such a way that the entire volume of the vehicle can pass through the free space between them. Therefore, there exists a feasible trajectory \( \eta(t) \) for the vehicle that connects the initial configuration \( \eta(t_0) \) with \( \eta^d \) such as:

\[ \mathcal{B}(\eta(t_0), \bar{r}) \cap \{ \mathcal{B}(\mathbf{p}_{\pi_m}, r_{\pi_m}) \cup \partial \mathcal{W} \} = \emptyset, m \in \{1, \ldots, M\} \]

A graphical representation of the feasible trajectory is depicted in Fig. 3.

![Fig. 3: Graphical representation of a feasible transition of AUV from the initial position \( \eta(t_0) \) to the desired position \( \eta^d \).](image-url)
where:

\[
\mathcal{A}(x_k) = \begin{bmatrix}
    u_{r,g} c v_{\phi} - v_{r,g} s v_{\phi} + v_c \\
    u_{r,g} s v_{\phi} + v_{r,g} c v_{\phi} + v_c \\
    w_{r,g} + u_c \\
    \frac{1}{m_21}(m_{22}v_{r,k} + X_{u_r}u_{r,k} + X_{u_{\phi}}|u_{r,k}|u_{r,k}) \\
    \frac{1}{m_22}(-m_{11}v_{r,k} + Y_{v_r}v_{r,k} + Y_{v_{\phi}}|v_{r,k}|v_{r,k}) \\
    \frac{1}{m_{42}}(Z_{w_r}w_{r,k} + Z_{w_{\phi}}|w_{r,k}|w_{r,k}) \\
\end{bmatrix},
\]

with \(c(\cdot) = \cos(\cdot), s(\cdot) = \sin(\cdot)\) and \(x_k = [\eta_k, v_{r,k}]^T = [x_k, y_k, z_k, \psi_k, u_{r,k}, v_{r,k}, w_{r,k}, \tau_k]^T \in \mathbb{R}^8\) denotes the state vector at the time-step \(k\) which includes the position and orientation of the vehicle with respect to the inertial frame \(I\) and the relative linear and angular velocity of the vehicle with respect to the water. In addition, \(m_{ij}, i = 1, \ldots, 4\) are the mass terms including added mass, \(X_{u_r}, X_{u_{\phi}}, Z_{w_r}, N_r < 0\) are the linear drag terms, \(X_{u_{\phi}}, Y_{v_r}, Z_{w_{\phi}}, N_{v_r} < 0\) are the quadratic drag terms, while \(dt\) denotes the sampling period. The control input of the system is \(\tau_k = [\tau_{p_k}, \tau_{s_k}, \tau_{v_{\phi}k}, \tau_{\phi}k]^T \in \mathbb{R}^4\) consisting of the thrusters’ forces. As mentioned previously, the ocean current profile induced to the dynamic model is inaccurate (i.e., in general, it is an estimation achieved by employing data from NCOM or ROMS) and consequently a difference between the real value of the current and the estimated one must be considered. The effect of these uncertainties can be considered as disturbances on the system [5]. Thus, by \(\delta_k = [0_{1 \times 4}; \delta_{u_k}, \delta_{v_k}, \delta_{w_k}, \delta_{\phi_k}]^T \in D \subset \mathbb{R}^7\) we present the effect of ocean current profile uncertainties on the system at the time step \(k\), with \(D\) to be a compact set. Moreover, we have that \(D\) is bounded by \(||\delta_k|| \leq \delta\). Furthermore, it is assumed that vehicle’s dynamic parameters have been identified via a proper identification scheme. However some degree of uncertainties on dynamic parameters denoted by \(\Delta f(x_k, \tau_k)\) should be considered. Taking into consideration the aforementioned disturbances that affect the vehicle, we are now ready to model the perturbed system as follows:

\[
x_{k+1} = f(x_k, \tau_k) + \delta_k = f(x_k, \tau_k) + \Delta f(x_k, \tau_k) + \delta_k = f(x_k, \tau_k) + \gamma_k + \delta_k
\]

with:

\[
\gamma_k = \Delta f(x_k, \tau_k) \in \Gamma, \quad ||\gamma_k|| \leq \bar{\gamma} \quad \forall x_k \in X, \tau_k \in T
\]

where \(\Gamma\) is the compact set of uncertainties and \(\bar{\gamma} \geq 0\) is a positive upper bound for this set. The equation (6) can be rewritten as:

\[
x_{k+1} = f(x_k, \tau_k) + w_k
\]

with \(w_k = \gamma_k + \delta_k \in W \subset \mathbb{R}^8\) as the result of adding a uncertainties and external disturbances of the system. \(W\) is a compact set such that \(W = D \oplus \Gamma\). Since the sets \(D\) and \(\Gamma\) are compact, we have that \(W\) is also a compact set, bounded by \(||w_k|| \leq \bar{w}\) with \(\bar{w} \triangleq \bar{\gamma} + \delta\).

**Remark 2:** Notice that the dynamic (7) is the actual equation of the system since it contains the vector of disturbance affecting on the system. In this way, we consider the dynamic (5) as the nominal model of the system, in which no disturbances are considered.

**Property 1:** The nominal model \(f(x, \tau)\) is locally Lipschitz in \(X\) i.e., there is a positive Lipschitz constant \(L_f < \infty\) such that for every \(\tau \in T, \|f(x_1, \tau) - f(x_2, \tau)\| \leq L_f\|x_1 - x_2\|\).

**C. Constraints**

**State Constraints:**

In this work, we consider that the robot must avoid the obstacles and the workspace boundaries (test tank). Moreover, for the needs of several common underwater tasks (e.g., seabed inspection, mosaicking), the vehicle is required to move with relatively low speeds with upper bound denoted by the velocity vector \(v_p = [u_p, v_p, w_p, r_p]^T\). These requirements are captured by the state constraint set \(X\) of the system, given by:

\[
x_k \in X \subset \mathbb{R}^8
\]

which is formed by the following constraints:

\[
\begin{align*}
    u_p + v_p - |u_r + v_r| & \geq 0 \quad (9a) \\
    w_p - |u_r| & \geq 0 \quad (9b) \\
    r_p - |r_r| & \geq 0 \quad (9c) \\
    B(\eta_k(t), \bar{r}) \cap B(p_{\pi_{\infty}}, r_{\pi_{\infty}}) \cup \partial W & = \emptyset, \quad (9d) \\
    m & \in \{1, \ldots, M\}
\end{align*}
\]

**Input Constraints:**

It is well known that the forces generated by the thrusters. Thus, we define the control constraint set \(T\) as follows:

\[
\tau_k = [\tau_{p_k}, \tau_{s_k}, \tau_{v_{\phi}k}, \tau_{\phi}k]^T \in T \subset \mathbb{R}^4
\]

These constraints are of the form \(||\tau_{p_k}|| \leq \bar{\tau}_p, ||\tau_{s_k}|| \leq \bar{\tau}_s, ||\tau_{v_{\phi}k}|| \leq \bar{\tau}_v, ||\tau_{\phi}k|| \leq \bar{\tau}_\phi\), therefore we get \(||\tau_k|| \leq \bar{T}\) where \(\bar{T} = (\bar{\tau}_p^2 + \bar{\tau}_s^2 + \bar{\tau}_v^2 + \bar{\tau}_\phi^2)^{\frac{1}{2}}\) and \(\bar{\tau}_p, \bar{\tau}_s, \bar{\tau}_v, \bar{\tau}_\phi \in \mathbb{R}_{\geq 0}\).

**D. Control Design**

The control objective is to guide the actual system (7) to a desired compact set around the waypoints \(i = \{1, \ldots, n\}\) that includes the desired state \(x_d \triangleq [\eta_d, v_{r,d}]^T = [x_{d,1}, y_{d,1}, z_{d,1}, \psi_{d,1}, u_{d,1}, v_{d,1}, w_{d,1}, r_{d,1}]^T \in X\), while respecting the state constraints (9a)-(9d) as well as the input constraints (10). A predictive controller is employed in order to achieve this task. In particular, at a given time instant \(k\), the NMPC is assigned to solve an Optimal Control Problem (OCP) with respect to a control sequence \(\tau_f(k) \triangleq [\tau(k|k), \tau(k + 1|k), \ldots, \tau(k + N - 1|k)]\), for a prediction horizon \(N\). The
OCP of the NMPC is given as follows:

\[
\begin{align}
\min_{\tau(k)} J_N(x_k, \tau_f(k)) &= \min \sum_{j=0}^{N-1} F(\hat{x}(k+j|k), \tau(k+j|k)) + E(\hat{x}(k+N|k)) \\
\text{subject to:} & \\
\hat{x}(k+j|k) &\in X_j, \quad \forall j = 1, \ldots, N-1, \quad (11b) \\
\tau(k+j|k) &\in T, \quad \forall j = 0, \ldots, N-1, \quad (11c) \\
\hat{x}(k+N|k) &\in \delta_f \quad (11d)
\end{align}
\]

where \(\delta_f\) is the terminal set and \(F\) and \(E\) are the running and terminal cost functions, respectively. At time instant \(k\), the solution of the OCP (11a)-(11d) is providing an optimal control sequence, denoted as:

\[
\tau_f^*(k) = [\tau(k|k), \tau(k+1|k), \ldots, \tau(k+N-1|k)]
\]

where the first control vector (i.e., \(\tau(k|k)\)) is applied to the system. The disturbance term \(w_k\) can cause discrepancies between the predicted state given from the nominal model (5) subject to a specific sequence of inputs and the actual state, given from (7) for the same sequence of inputs. In order to account for this mismatch we use the double subscript notation for the predicted state of system (5) inside the OCP of the NMPC:

\[
\hat{x}(k+j|k) = f(\hat{x}(k+j-1|k), \tau(k+j-1|k))
\]

where the vector \(\hat{x}(k+j|k)\) denotes the predicted state of the nominal system (5) at sampling time \(k+j\) with \(j \in \mathbb{Z}_{\geq 0}\). The predicted state is based on the measurement of the state \(x_k\) of the actual system at sampling time \(k\) (i.e., provided by on-board navigation system), while applying a sequence of control inputs \([\tau(k|k), \tau(k+1|k), \ldots, \tau(k+N-1|k)]\).

It holds that \(\hat{x}(k|k) \equiv x_k\). The cost function \(F(\cdot)\), as well as the terminal cost \(E(\cdot)\), are both of quadratic form, i.e., \(F(\hat{x}, \tau) = \hat{x}^T Q \hat{x} + \tau^T R \tau\) and \(E(\hat{x}) = \hat{x}^T P \hat{x}\), respectively, with \(P, Q\) and \(R\) being positive definite matrices. Particularly we define \(Q = \text{diag}\{q_1, \ldots, q_8\}\), \(R = \text{diag}\{r_1, \ldots, r_4\}\) and \(P = \text{diag}\{p_1, \ldots, p_8\}\). Obviously for the running cost function \(F\), we have \(F(0,0) = 0\).

Notice, here that the OCP is solved for the nominal system, i.e., the model of the system that is not affected by disturbances. This is the case, due to the fact that the OCP is solved for a prediction horizon, thus, it is not possible to address the disturbances beforehand. However, we distinguish the nominal system that will be denoted as \(\hat{x}(\cdot)\) with the actual system, i.e., the system that is affected by disturbances that will be denoted as \(x(\cdot)\). Thus, it is shown that the difference between the real evolution of the state, given by (7) and the predicted evolution of the state, given by (5), is in fact, bounded:

**Lemma 1:** (18), Lemma 1: The difference between the actual state \(x_{k+j}\) at the time-step \(k+j\) and the predicted state \(\hat{x}(k+j|k)\) at the same time-step, under the same control sequence, is upper bounded by:

\[\|x_{k+j} - \hat{x}(k+j|k)\| \leq \sum_{i=0}^{j-1} L_f \bar{w}\]  

(14)

Notice that in (11b), the state constraint set \(X\) from (8), is being replaced by a restricted constraint set \(X_j \subseteq \dot{X}\). This state constraints' tightening for the nominal system guarantees that the evolution of the real system will be admissible for all time. More specifically, using this technique, it can be guaranteed that the evolution of the state of the perturbed system (7), when the control sequence developed from the NMPC Problem of (11a)-(11d) is applied to it, will necessarily satisfy the state constraint set \(X\). More specifically, given Lemma 1 where a bound on the state prediction error is evaluated, we set \(X_j = X \sim B_j\) where \(B_j = \{x \in \mathbb{R}^8 : ||x|| \leq \sum_{i=0}^{j-1} L_f \bar{w}\}\). More details for this constraint tightening technique are presented in [18] and [19].

**Remark 3** (Thrust’s energy consumption): The OCP of the proposed NMPC scheme is designed in order to find the optimal thrust \(\tau_f^*(\cdot)\) required for minimizing the state error. Thus, owing to the existence of ocean currents profile in the dynamic model (2), the control inputs \(\tau_f^*(\cdot)\) calculated by the OCP (11a)-(11d) may exploit the ocean currents when are in favor of the waypoint tracking mission. Hence, the proposed scheme calculates the optimal commands, in order to retain the energy consumed by the thrusters in a reduced level, as dictated by the requirements of the considered Problem 1.

**Remark 4:** The obstacles within the workspace may be detected while the robot moves inside the workspace using its onboard sensors (e.g., multi beam imaging or side scan sonar). In such a case, it should be assured that the predicted state of the NMPC is always within the sensing region of the robot. This intuitively means that the prediction state is always feasible even in the worst case (i.e., maximum velocity of the robot under maximum sea current). Thus, assuming that the \(R\) denotes the sensing range of the system, the prediction horizon \(N\) should be set as follows:

\[N \leq \frac{R}{(||\bar{v}_1|| + ||\bar{v}_e||)dt}\]

where the \(||\bar{v}_1||\) is the norm of maximum linear velocity of the vehicle and \(||\bar{v}_e||\) is the norm of the upper bound of the sea current velocity.

Before proceeding the necessary robust analysis of the proposed NMPC strategy, we employ some standard stability conditions that are used in MPC frameworks, as the following:

**Assumption 1:** For the nominal system (5), there is an admissible positively invariant set \(\delta' \subseteq X\) such that the terminal region \(\delta_f \subseteq \delta'\), where \(\delta = \{x \in X : ||x|| \leq \bar{e}_0\}\) and \(\bar{e}_0\) being a positive parameter.

**Assumption 2:** We assume that in the terminal set \(\delta_f\), there exists a local stabilizing controller \(\tau_k = h(x_k) \in T\) for all \(x \in \delta\), and that \(E\) satisfies \(E(f(x_k, h(x_k))) - E(x_k) + F(x_k, h(x_k)) \leq 0\) for all \(x \in \delta\).

**Assumption 3:** The terminal cost function \(E\) is Lipschitz in \(\delta\), with Lipschitz constant \(L_E = 2\bar{e}_0\bar{e}_{max}(P)\) for all
$x \in \mathcal{E}$, where $\sigma_{\text{max}}(P)$ denotes the largest singular value of the $P$.

**Assumption 4:** Inside the set $\mathcal{E}$ we have $E(x) = x^T P x \leq \alpha_x$, where $\alpha_x = \max(p_1, \ldots, p_n)\sigma_0^2 > 0$. Assuming that $\mathcal{E} = \{ x \in X_{(N-1)} : h(x) \in T \}$ and taking a positive parameter $\alpha_{\mathcal{E}}$ such that $\alpha_{\mathcal{E}} \in (0, \alpha_x)$, we assume that the terminal set designed as $\mathcal{E}_f = \{ x \in \mathbb{R}^8 : E(x) \leq \alpha_{\mathcal{E}} \}$ is such that $\forall x \in \mathcal{E}$, $f(x, h(x)) \in \mathcal{E}_f$.

**Lemma 2:** The cost function $F(x, \tau)$ is such that $F(0, 0) = 0$ and $\mathcal{F}(||x||) \leq F(x, \tau)$, $\forall x \in X$ and $\forall \tau \in T$ where $\mathcal{F}$ is a $K_{\infty}$ function. Furthermore, $F(x, \tau)$ is Lipschitz continuous with respect to $x$ in $X$, with a Lipschitz constants $L_F \in \mathbb{R}_{>0}$.

Notice that finding a $K_{\infty}$ function as well as the Lipschitz constants $L_F$ and $L_{F_{\tau}}$ of the last Lemma 2 owing to the quadratic form of the cost function $F(x, \tau)$, can be achieved analytically after some mathematic manipulations. However, the analytical derivation of them are omitted due to space limitations.

**V. Stability Analysis of the Proposed NMPC**

Here, the stability analysis of the closed loop system under the NMPC law provided by (11a)-(11d) is presented. The approach for establishing stability consists of two parts: in the first part it is shown that the initial feasibility implies feasibility afterwards and based on this, it is then shown that the state converges to a bounded set, due to the presence of the persistent disturbances. We begin by denoting $\tau^*_1(k-1)$ as the optimal solution that results from (11a)-(11d) at a time-step $k-1$. We, also, denote a feasible control sequence $\bar{\tau}_i(k+j|k)$ of the optimization problem at time-step $k$, such as:

$$\bar{\tau}(k+j|k) = \begin{cases} \tau^*(k+j|k-1) & \text{for } j = 0, \ldots, N-2 \\ h(\hat{x}(k+N-1|k)) & \text{for } j = N-1 \end{cases}$$

where $h(x)$ is the local stabilizing controller defined in Assumption 2. Moreover, from (11d) and Assumption 2 is clear that $\tau(k+j|k) \in T$.

**A. Feasibility analysis**

First we are going to provide a necessary definition: Let $X_{\text{MPC}}$ be the set containing all the state vectors for which a feasible control sequence exists that satisfies the constraints of the optimal control problem. In particular, while having a slight violation of the notation we can define the feasible set $X_{\text{MPC}}$ as follows:

**Definition 1:** $X_{\text{MPC}} = \{ x_0 \in \mathbb{R}^n \exists \text{ a control sequence } \tau_j \in T, \hat{x}(j) \in X_j, \forall j \in \{1, \ldots, N\} \text{ and } \hat{x}(N) \in \mathcal{E}_f \}$.

At this point we want to find a bound $\bar{w}$ for the uncertainties, then the closed-loop system in $X_{\text{MPC}}$ is stable. That means that if $x_k \in X_{\text{MPC}}$ then $x_{k+1} = f(x_k, \tau^*_k) + w_{k+1} \in X_{\text{MPC}}$, for all $w_{k+1} \in W$. In order to derive feasibility it must be guaranteed that $\hat{x}(k+N|k) \in \mathcal{E}_f$. This results to a permitted upper bound of disturbances $\bar{w}$ under which the system is proven to be feasible. In order to prove this, first it will be shown that:

$$||\hat{x}(k) - \hat{x}(k-1)|| = w_k \leq \bar{w}$$

$$||\hat{x}(k+1) - \hat{x}(k+1|k-1)|| =$$

$$= ||f(\hat{x}(k)) - f(\hat{x}(k-1))||$$

$$\leq L_f(||\hat{x}(k) - \hat{x}(k-1)||) \leq L_f \cdot \bar{w}$$

(15)

From Assumption 3 we have:

$$E(\hat{x}(k+N-1|k)) - E(\hat{x}(k+N-1|k-1)) \leq L_E ||\hat{x}(k+N-1|k) - \hat{x}(k+N-1|k-1)|| \leq L_E L_f \cdot \bar{w}$$

For the nominal system and based on the optimal solution $\tau^*(k+j|k-1)$ we have: $E(\hat{x}(k+N-1|k-1)) \in \mathcal{E}_f$. Therefore, taking into account Assumption 4:

$$E(\hat{x}(k+N-1|k)) \leq \alpha_{\mathcal{E}} + L_E \cdot L_f \cdot \bar{w}$$

we want $E(\hat{x}(k+N-1|k))$ to belong to the set $\mathcal{E}$, thus from Assumption 4 it must satisfy $E(\hat{x}(k+N-1|k)) \leq \alpha_x$, which leads to:

$$E(\hat{x}(k+N-1|k)) \leq \alpha_x \leq \alpha_{\mathcal{E}} + L_E \cdot L_f \cdot \bar{w} \leq \alpha_x$$

Therefore, if the uncertainties of the system are bounded by $\bar{w} \leq \frac{\alpha_x - \alpha_{\mathcal{E}}}{L_E \cdot L_f}$ then $E(\hat{x}(k+N-1|k))$ belongs to the set $\mathcal{E}$, and from Assumption 4 we get $E(\hat{x}(k+N|k)) \in \mathcal{E}_f$, which consequently means that $\hat{x}(k+N|k) \in \mathcal{E}_f$.

**B. Convergence analysis**

In order to treat the convergence property, it should be guaranteed that the state of the perturbed system reaches to a desired terminal set. A proper value function must be shown to be decreasing in order to prove stability of the closed loop system and consequently the state convergence of the system. Consider the optimal cost $J_N^*(k-1) = J^*(x_{k-1}, \tau^*_N(k-1))$ from (11a), at the time-step $(k-1)$ as a Lyapunov function candidate. Consider, also, the cost of the feasible sequence at time-step $k$ as $\tilde{J}_N(k) = \tilde{J}(\tilde{x}_k, \tilde{\tau}_f(k))$ evaluated from the control sequence $\tilde{\tau}_f(k+i|k)$. This “feasible” cost will help us to obtain the difference $\tilde{J}_N^*(k) - \tilde{J}_N^*(k-1)$. In particular, the difference between the optimal cost and the feasible cost is:

$$\Delta J = \tilde{J}_N(k) - J_N^*(k-1) =$$

$$= \sum_{i=0}^{N-1} F(\tilde{x}(k+i|k), \tilde{\tau}(k+i|k)) + E(\tilde{x}(k+N|k))$$

$$- \sum_{i=0}^{N-1} F(\tilde{x}(k+i-1|k-1), \tau^*(k+i-1|k-1))$$

$$- E(\tilde{x}(k+N-1|k-1)) = \sum_{i=0}^{N-2} F(\tilde{x}(k+i|k), \tilde{\tau}(k+i|k))$$

$$- F(\tilde{x}(k+i|k-1), \tau^*(k+i|k-1))$$

$$+ F(\tilde{x}(k+N-1|k), \tilde{\tau}(k+N-1|k))$$

$$- F(\tilde{x}(k-1|k-1), \tau^*(k-1|k-1))$$

$$+ E(\tilde{x}(k+N|k)) - E(\tilde{x}(k+N-1|k-1))$$
where $\hat{x}(k + N - 1|k) = h(\hat{x}(k + N - 1|k))$ taken from [15] and $\hat{x}(k + i|k) = \tau^*(k + i|k - 1)$ for $i = 0, \ldots, N - 2$. Also from Lemma 2 combined with Lemma 1 we get:

$$\sum_{i=0}^{i=N-2} F(\hat{x}(k + i|k), \hat{x}(k + i|k)) - F(\hat{x}(k + i|k - 1), \tau^*(k + i|k - 1)) \leq L_F \sum_{i=0}^{i=N-2} L_j^i \cdot \bar{w}$$

From Assumption 3 it yields:

$$E(\hat{x}(k + N|k) - E(\hat{x}(k + N - 1|k - 1)) = E(\hat{x}(k + N|k) - E(\hat{x}(k + N - 1|k)) + E(\hat{x}(k + N - 1|k)) - E(\hat{x}(k + N - 1|k - 1))$$

$$\leq E(\hat{x}(k + N|k) - E(\hat{x}(k + N - 1|k)) + L_E L_j^{N-1} \bar{w}$$

We used the instantaneous difference of the predictive state $\hat{x}(k + N - 1|k)$ and the feasible state $\hat{x}(k + N - 1|k - 1)$ at the time-step $k + N - 1$ such that:

$$||\hat{x}(k + N - 1|k - 1) - \hat{x}(k + N - 1|k)|| \leq L_j^{N-1} \bar{w}$$

Therefore, we obtain:

$$\Delta J \leq \left( L_F \sum_{i=0}^{i=N-2} L_j^i + L_E L_j^{N-1} \right) \bar{w} +$$

$$+ F(\hat{x}(k + N - 1|k), h(\hat{x}(k + N - 1|k)))$$

$$+ E(\hat{x}(k + N|k)) - E(\hat{x}(k + N - 1|k)) \right]$$

$$- F(\hat{x}(k - 1|k - 1), \tau^*(k - 1|k - 1))$$

Finally, taking into account the Assumption 2 and Lemma2:

$$\Delta J \leq \left( L_F \sum_{i=0}^{i=N-2} L_j^i + L_E L_j^{N-1} \right) \bar{w} -$$

$$- F(\hat{x}(k - 1|k - 1), \tau^*(k - 1|k - 1)) \right)$$

From the optimality of the solution, we derive the following:

$$\Delta J^* = J_N^*(k) - J_N^*(k - 1) \leq \Delta J$$

Therefore, the optimal cost $J_N^*(k)$ is an ISS-Lyapunov function of the closed loop system, and hence, the closed-loop system is input-to-state stable.

VI. EXPERIMENTAL RESULTS

This section demonstrates the efficacy of the proposed motion control scheme via a set of real-time experiments employing a small underwater robotic vehicle. In particular, Subsection VI-A introduces the experimental setup and Subsection VI-B presents the detailed results of two experimental studies with the proposed controller.
B. Results

In order to prove the efficacy of the proposed controller two experiments are presented (Scenario A and B). In both experiments, the objective is to follow a set of predefined waypoints while simultaneously avoid two static obstacles and respect the workspace (test tank) boundaries. The location and geometry of the obstacles are considered known. More specific, the position of the obstacles w.r.t the Inertial Frame \( \mathcal{I} \) in \( x-y \) plane is given by: \( \mathbf{x}_{\text{obs}_1} = \begin{bmatrix} -0.625 \\ -0.625 \end{bmatrix}, \mathbf{x}_{\text{obs}_2} = \begin{bmatrix} 0.9375 \\ 0 \end{bmatrix} \).

The state constraints of the (9a)-(9d) which must be satisfied during all the experiments are analytically formulated as follows: i) The obstacles are cylinders (See Fig. 11) with radius \( r_{\pi_i} = 0.16m, \ i = \{1,2\} \) and are modeled together with the workspace boundaries according to the spherical world representations as consecutive spheres. ii) the radius of the sphere \( B(\eta_i, \bar{r}) \) which covers all the vehicle volume (i.e., main body and additional equipment) is defined as \( \bar{r} = 0.3m \). However, for the clarity of presentation, we depict it as a safe zone around the obstacles where the vehicle center \( \eta_1 \) (denoted by blue line See Fig. 5, Fig. 10) should not violated it. iii) the vertical position must be between \( 0 < z < 1.2m \), iv) the vehicle’s body velocity norm of \( |u_r + v_r| \) (planar motion) must not exceed \( 0.5m/s \), v) heave velocity must be retained between \( -0.25 < w_r < 0.25 \) m/s (i.e., \( 0.25 - |w_r| \geq 0 \)), vi) yaw velocity must be retained between \( -1 < r_y < 1 \) rad/s (i.e., \( 1 - |r_y| \geq 0 \)).

Moreover, each of the four thrusters must obey the following input constraint: \( -12 < \tau_i < 12N \), \( i = \{p, s, v, l\} \). The state and input constraints in the following figures are depicted in red dashed lines were applicable. At this point we should mention that each mission is considered as successful only if the vehicle performs the waypoint tracking three consecutive times, hence repeatability is proved. In all times the vehicle is under the influence of the water currents depicted in Fig. 4.

1) Scenario A: Two Way Points Tracking: In this scenario the vehicle must travel via two waypoints which are placed at \( \eta_1^{\text{I}} = \begin{bmatrix} -1.60 m \\ -0.35 m \end{bmatrix}, \eta_2^{\text{I}} = \begin{bmatrix} 1.75m \\ 0m \end{bmatrix} \) respectively. The three consecutive trajectories of the vehicle along the horizontal plane are depicted in Fig. 5. It can be seen that the vehicle performs successfully the waypoint tracking while safely avoids the obstacles and the test tank boundaries. We observe that in one case the vehicle traveled from the second waypoint back to first one following a different trajectory. This can be explained by the fact that the MPC found a different optimal solution at the specific time frame, due to the unmodeled dynamics of the tether which significantly affect the vehicle motion. The vertical and angular motion of the vehicle are depicted in Fig. 5 where it can be seen that the state constraints are always satisfied. The vehicle is considered to reach each waypoint if it has entered a terminal region \( \mathcal{E}_i \) (i.e., spherical region of 0.3m and a offset of \( \pm 0.15 \) rad) around the waypoint. These regions are depicted in circles in Fig. 5 and 6. In Fig. 7 the body velocity norm in planar motion is depicted and the respective constraint is satisfied. The same stands for the heave and yaw velocities as shown in Fig. 8. In Fig. 9 the vehicle’s thruster input are shown. As it can be seen the input constraints are also satisfied.

2) Scenario B: Three Way Points Tracking: This scenario is similar to the previous one except that the vehicle must travel along 3 waypoints which makes the mission more challenging considering the narrow workspace. The locations of the three waypoints are given by: \( \eta_1^{\text{I}} = \begin{bmatrix} -1.50 m, 0.30 m, 0.40 m, \pi rad \end{bmatrix}, \eta_2^{\text{I}} = \begin{bmatrix} 0.45 m, -1m, 0.25 m, 0 rad \end{bmatrix}, \eta_3^{\text{I}} = \begin{bmatrix} 1.20 m, 1m, 0.30 m, -\pi rad \end{bmatrix} \). The three consecutive trajectories of the vehicle along the horizontal plane are depicted in Fig. 10. Although this scenario is more complicated the vehicle again carries it out successfully. As it can be observed, the vehicle also in this mission follows different trajectories, for the same reasons explained in Scenario A. The vertical and angular motion of the vehicle are depicted in Fig. 11 while in Fig. 12 the body velocity norm in planar motion is shown. The heave and yaw velocities are presented in Fig. 13 while in Fig. 14 the vehicle’s thruster inputs are shown. As it can be observed, the vehicle achieved all desired waypoints while simultaneously satisfied all respective state and input constraints.

VII. CONCLUSION

In this paper, we presented a novel Model predictive Control strategy for underwater robotic vehicles operating
Fig. 7: 2 WP tracking scenario: Vehicle body velocity norm $|u_v + v_r|$

Fig. 8: 2 WP tracking scenario: Vehicle heave and yaw velocities

Fig. 9: 2 WP tracking scenario: Thruster Commands

Fig. 10: 3 WP tracking scenario: Vehicle trajectory in horizontal plane

Fig. 11: 3 WP tracking scenario: Vehicle vertical and angular motion

Fig. 12: 3 WP tracking scenario: Vehicle body velocity norm $|u_v + v_r|$

Fig. 13: 3 WP tracking scenario: Vehicle heave and yaw velocities

Fig. 14: 3 WP tracking scenario: Thruster Commands
in a constrained workspace including obstacles. The purpose of this control scheme is to guide the vehicle towards specific way points. Various constraints such as: obstacles, workspace boundary, thruster saturation and predefined upper bound of the vehicle velocity (requirements for various underwater tasks such as seabed inspection scenario, mosaicking) are considered during the control design. Moreover, the proposed control scheme incorporates the dynamics of the vehicle and is designed in order to find optimal thrusts required for minimizing the way point tracking error. Owing to the existence of ocean currents profile in the motion dynamics, the control inputs calculated by the proposed controller may exploit the ocean currents when are in favor of the waypoint tracking mission, which results in retaining the energy consumed by the thrusters in a reduced level. Future research efforts will be devoted towards extending the proposed methodology for multiple AUVs operating in a dynamic environment including static and moving obstacles.

REFERENCES

[1] G. Griffiths, Technology and Applications of Autonomous Underwater Vehicles. Ocean science and technology, CRC Press, 2002.
[2] T. Fossen, Handbook of Marine Craft Hydrodynamics and Motion Control, 2011.
[3] Z. Zeng, L. Lian, K. Sammut, F. He, Y. Tang, and A. Lammas, “A survey on path planning for persistent autonomy of autonomous underwater vehicles,” Ocean Engineering, vol. 110, pp. 303–313, 2015.
[4] V. Huynh, M. Dunbabin, and R. Smith, “Predictive motion planning for auvs subject to strong time-varying currents and forecasting uncertainties,” vol. 2015-June, pp. 1144–1151, 2015.
[5] A. Alvarez, A. Caiti, and R. Onken, “Evolutionary path planning for autonomous underwater vehicles in a variable ocean,” IEEE Journal of Oceanic Engineering, vol. 29, no. 2, pp. 418–429, 2004.
[6] B. Garau, M. Bonet, A. Alvarez, S. Ruiz, and A. Pasqual, “Path planning for autonomous underwater vehicles in realistic oceanic current fields: Application to gliders in the western mediterranean sea,” Journal of Maritime Research, vol. 6, no. 2, pp. 5–21, 2009.
[7] C. Phtrs, Y. Paillhas, P. Patrn, Y. Petillot, J. Evans, and D. Lane, “Path planning for autonomous underwater vehicles,” IEEE Transactions on Robotics, vol. 23, no. 2, pp. 331–341, 2007.
[8] F. Allgwer, R. Findeisen, and Z. Nagy, “Nonlinear model predictive control: From theory to application,” the Chinese Institute of Chemical Engineers, vol. 35, no. 3, pp. 299–315, 2004.
[9] L. Medagoda and S. Williams, “Model predictive control of an autonomous underwater vehicle in an in situ estimated water current profile,” Program Book - OCEANS 2012 MTS/IEEE Yeosu: The Living Ocean and Coast - Diversity of Resources and Sustainable Activities, 2012.
[10] D. Fernandez and G. Hollinger, “Model predictive control for underwater robots in ocean waves,” IEEE Robotics and Automation Letters, vol. 2, no. 1, pp. 88–95, 2017.
[11] P. Jagtap, P. Raut, P. Kumar, A. Gupta, N. Singh, and F. Kazi, “Control of autonomous underwater vehicle using reduced order model predictive control in three dimensional space,” IFAC-PapersOnLine, vol. 49, no. 1, pp. 772–777, 2016.
[12] S. Heshmati-Alamdari, A. Eqtami, G. Karras, D. Dimarogonas, and K. Kyriakopoulos, “A self-triggered visual servoing model predictive control scheme for under-actuated underwater robotic vehicles,” Proceedings - IEEE International Conference on Robotics and Automation, pp. 3826–3831, 2014.
[13] National HF RADAR network - surface currents.
[14] R. Smith, Y. Chao, P. Li, D. Caron, B. Jones, and G. Sukhatme, “Planning and implementing trajectories for autonomous underwater vehicles to track evolving ocean processes based on predictions from a regional ocean model,” International Journal of Robotics Research, vol. 29, no. 12, pp. 1475–1497, 2010.
[15] A. Aguilar and A. Pascoal, “Dynamic positioning and way-point tracking of underactuated auvs in the presence of ocean currents,” International Journal of Control, vol. 80, no. 7, pp. 1092–1108, 2007.

[16] T. Fossen, “Guidance and control of ocean vehicles,” Wiley, New York, 1994.
[17] D. Kotschik and E. Rimon, “Robot navigation functions on manifolds with boundary,” Advances in Applied Mathematics, vol. 11, no. 4, pp. 412–442, 1990.
[18] D. L. Marruedo, T. Alamo, and E. Camacho, “Input-to-state stable mpc for constrained discrete-time nonlinear systems with bounded additive uncertainties,” 41st IEEE Conf. Decision and Control, pp. 4619 – 4624, 2002.
[19] G. Pin, D. Raimondo, L. Magni, and T. Parisini, “Robust model predictive control of nonlinear systems with bounded and state-dependent uncertainties,” IEEE Transactions on Automatic Control, vol. 53, no. 7, pp. 1681–1687, 2009.
[20] S. Garrido-Jurado, R. M. noz Salinas, F. Madrid-Cuevas, and M. Marín-Jiménez, “Automatic generation and detection of highly reliable fiducial markers under occlusion,” Pattern Recognition, vol. 47, no. 6, pp. 2280 – 2292, 2014.
[21] P. Marantos, Y. Koveos, and K. Kyriakopoulos, “Uav state estimation using adaptive complementary filters,” IEEE Transactions on Control Systems Technology, vol. 24, no. 4, pp. 1214–1226, 2016.
[22] M. Quigley, B. Gerkey, K. Conley, J. Faust, T. Foote, J. Leibs, E. Berger, R. Wheeler, and A. Ng, “Ros: an open-source robot operating system,” in Proc. of the IEEE Intl. Conf. on Robotics and Automation (ICRA) Workshop on Open Source Robotics, (Kobe, Japan), May 2009.
[23] V. Asouti, X. Trompoukis, I. Kampolis, and K. Giannakoglou, “Unsteady cfd computations using vertex-centered finite volumes for unstructured grids on graphics processing units,” International Journal for Numerical Methods in Fluids, vol. 67, no. 2, pp. 232–246, 2011.