Viable secret neutrino interactions with ultralight dark matter

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Several anomalies in neutrino oscillation experiments point to the existence of a $\sim 1$ eV sterile neutrino $\nu_s$, mixing with $\nu_e$, at the level of $U_{e4} \cong 0.1$, but such a neutrino is strongly disfavored by constraints on additional light degrees of freedom ($\delta N_{\text{eff}}$) and total neutrino mass ($\sum m_\nu$) from cosmology. “Secret neutrino interactions” that have been invoked to suppress the cosmological production of $\nu_s$ typically falter, but recently it was pointed out that $\nu_s$ could get a large mass in the early universe by coupling to ultralight dark matter $\phi$, which can robustly suppress its production. The model has essentially two free parameters: $m_{\phi}$, and $m_{s,0}$, the mass of the sterile neutrino at early times, enhanced by its coupling to $\phi$. I determine the parameter regions allowed by limits on $\delta N_{\text{eff}}$ and $\sum m_\nu$ from the cosmic microwave background and big bang nucleosynthesis, using a simplified yet accurate treatment of neutrino oscillations in the early universe. This mechanism could have an important impact on laboratory experiments that suggest oscillations with sterile neutrinos.

Introduction. Short baseline (SBL) neutrino oscillation experiments at nuclear reactors suggest at 3$\sigma$ an eV-scale sterile neutrino $\nu_s$ that mixes with $\nu_e$ [1,5]. A persistent deficit of low-energy solar $\nu_e$ flux in gallium experiments lends support to this interpretation. The NEOS [6] and DANSS [7] experiments that also search for $\nu_e$-$\nu_s$ oscillations observe features that could be consistent with the SBL anomalies, though are not yet conclusive. Recent fits to the data favor a mass $m_s = 1.1$ eV and mixing matrix element $U_{e4} = 0.11$ [8]. Moreover there are hints from other experiments, LSND [9] and MiniBooNE [10], of $\nu_\mu \to \nu_\tau$ oscillations via a sterile neutrino with similar mass and mixing parameters. The sterile neutrino interpretation of $\nu_\mu \to \nu_\tau$ is clouded by constraints on $\nu_\mu$-$\nu_s$ oscillations from MINOS [11] and IceCube [12,13]. In this work I focus on the simpler $\nu_\mu$-$\nu_s$ scenario that could explain the SBL deficits. The KATRIN experiment will provide an independent probe in the near future [14].

A generic challenge to the existence of sterile neutrinos in the indicated mass and mixing range are their oscillations in the early universe that would fully equilibrate the sterile species [14,16]. This is strongly excluded by big bang nucleosynthesis (BBN) and cosmic microwave background (CMB) constraints on additional effective neutrino species, $\delta N_{\text{eff}}$, as well as the sum of neutrino masses $\sum m_\nu$. Some means of suppressing oscillations in the early universe while allowing them at the present time is needed.

The use of sterile neutrino interactions to inhibit oscillations has a long history [18,20]. With respect to the current anomalies, refs. [21,22] suggested that self-interactions of the sterile neutrino could impede the oscillations and thereby satisfy the cosmological constraints. This mechanism is referred to as “secret neutrino interactions,” despite the efforts of PRL to censor the name. Subsequent investigation showed that although the self-interactions in this context could prevent $\nu_4$ production until freezeout of the active neutrinos, in accordance with bounds on $N_{\text{eff}}$, at lower temperatures their self-scattering combines with oscillations to convert active neutrinos to $\nu_4$ and violate the CMB bound on $\sum m_\nu$. [23,27]. (An exception is found for self-interactions mediated by a light gauge boson of mass $\lesssim 10$ MeV [28].)

It was recently pointed out that an effective realization of secret interactions is to couple $\nu_s$ to ultralight bosonic dark matter $\phi$ [29]. In that case the scalar behaves as a coherent condensate, that has not yet started oscillating at early times. It can easily give a large mass to $\nu_s$ during this epoch, inhibiting the oscillations. Once the Hubble rate drops below $m_\phi$, the field oscillates and redshifts with scale factor as $a^{-3/2}$ as the universe expands. Its contribution to $m_s$ quickly disappears, leaving only the bare Lagrangian mass of $\sim 1$ eV. The “secret interaction” moniker is especially appropriate in this case, since the required coupling of $\nu_s$ to $\phi$ was shown to be exceedingly weak, $\lambda \sim 10^{-23}$. Similar interactions of light dark matter to standard model neutrinos were considered with respect to their effects on laboratory neutrino oscillations in refs. [30,33].

This model is quite economical, depending only upon $m_s$ and the $\nu_e$-$\phi$ coupling $\lambda$, assuming $\phi$ constitutes all of the dark matter (DM) so that its initial amplitude is determined by its relic density. Equivalently, one can trade $\lambda$ for the new contribution $m_{s,0}$ to the $\nu_s$ mass at early times, before $\phi$ has started to oscillate. The purpose of this note is to determine the allowed parameter space, more quantitatively than was done in ref. [29].

Theoretical framework. Considering mixing between $\nu_s$ and $\nu_e$ only, the neutrino mass matrix is

$$
\begin{pmatrix}
m_{ee} & m_{es} \\
m_{es} & m_{ss}
\end{pmatrix}
$$

(1)

It is assumed that $m_{ss} \gg m_{ee}$. Then for small mixing one can show that $m_{es}$ is related to the mass eigenvalue $m_4 \sim 1$ eV by

$$
m_{es} \cong U_{e4} m_4
$$

(2)

Fits to the SBL data favor $m_4 = 1.13 \pm 0.04$, $U_{e4} \in [0.04, 0.13]$ [4]; for definiteness I adopt the central value $m_4 = 1.1$ eV and $U_{e4} = 0.11$ of ref. [8], giving $m_{es} = 0.12$ eV and $m_{s,0} \cong m_4$. 

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leading to the effective mass $m_{\text{eff}} = m_s + \lambda \phi$ when DM has a VEV. For ultralight DM, such a VEV is presumed to exist \cite{34, 35}, assuming some initial value in the early universe, that persists to account for the present relic density. If $\phi$ is sufficiently weakly coupled, it never thermalizes and remains coherent, behaving like a classical field. Its time dependence in the expanding cosmological background is

$$\phi(t) \equiv 1.08 \phi_0 J_{1/4}(m_{\phi} t) / (m_{\phi} t)^{1/4} \equiv \phi_0 \tilde{\phi}(t) \quad (4)$$

during radiation domination (when $a(t) \sim t^{1/2}$). The relevant combination of parameters affecting neutrino oscillations is

$$m_{s,0} = \lambda \phi_0 \quad (5)$$

so that $m_{\text{eff}} = m_s + m_{s,0} \tilde{\phi}(t)$.

For $t \gg m_{\phi}$ (but before matter-radiation equality) it can be shown that $\rho_{\phi} \approx 0.37 m_{\phi}^2 \phi_0^2 (m_{\phi} t)^{-3/2}$. Matching to the present DM density, one finds

$$\phi_0 = 1.0 \times 10^{15} \text{ GeV} \left( \frac{10^{-15} \text{ eV}}{m_{\phi}} \right)^{1/4} \quad (6)$$

Such a large VEV could arise if $\phi$ is an axion-like particle, the phase of a complex field $\Phi = |\Phi| e^{i\phi}/f_{\phi}$, with decay constant $f_{\phi} > \phi_0$. At early times $\rho_{\phi} \sim m_{\phi}^2 \phi_0^2$ would be negligible compared to the energy density of radiation, and $\phi_0$ could take random values in the interval $[0, 2\pi f_{\phi}]$.

**Production of $\nu_c$.** Although a rigorous study of $\nu_s - \nu_c$ oscillations in the early universe requires solving the Boltzmann equation for the density matrix \cite{36, 37}, a good approximation can be obtained in a simpler approach, described in refs. \cite{20, 38}, which in some regimes leads to analytic results\footnote{The normalization is such that $\tilde{\phi}(0) = 1$}. The method is based upon solving the Schrödinger equation for the two-state system, including an imaginary term $-i \Gamma / 2$ in the Hamiltonian representing scattering of $\nu_c$ in the plasma, that causes decoherence.

The solution yields the probability for a $\nu_s$ to oscillate into $\nu_c$ between an arbitrary initial time and a later time $t$. From this, a rate of $\nu_s$ production is derived, and the associated Boltzmann equation can be solved for the ratio of $\nu_s$ occupation number relative to that of $\nu_c$, as a function of temperature and neutrino momentum\footnote{The quantitative agreement of the two formalisms was recently demonstrated in ref. \cite{39}}

$$R \equiv \frac{n_{\nu_s}}{n_{\nu_c}} = \frac{1}{2} \left( 1 - \exp \left( -2 \int_T^{T_i} \left( \frac{\Gamma \sin^2 \theta_m}{H T'} \right) dT' \right) \right). \quad (7)$$

Here $\theta_m$ is the mixing angle including matter effects, and the initial temperature $T_i$ can be taken to infinity. The total interaction rate, including elastic scattering, is $\Gamma = (7\pi/24)G_F^2 T^4 p$ for a $\nu_c$ of momentum $p$ \cite{20, 40}. For relativistic neutrinos,

$$\sin^2 2\theta_m \approx \frac{4m_{\nu_s}^2}{4m_{\nu_s}^2 + (m_{\text{eff}}^2 + 2V_e p/m_{\text{eff}})^2} \quad (8)$$

(recall that $m_{\text{eff}} (t)$ is the total $\nu_s$ mass and $V_e = (7\pi/90\alpha) \sin^2 (2\theta \nu) G_F^2 T^4 p$ is the thermal self-energy for $\nu_e$). The effective number of extra neutrino species produced by the oscillations requires integrating over momentum, weighted by the massless Fermi-Dirac distribution function $f(p)$ for $\nu_e$,

$$\delta N_{\text{eff}}(T) = \int d^3 p f(p) R(T, p) \int d^3 p f(p) \quad (9)$$

Before numerically evaluating $\delta N_{\text{eff}}$, an analytic result can be found, in the regime where $m_{\phi} \lesssim 10^{-14} \text{ eV}$, sufficiently small that $\phi$ does not start oscillating until the integral in eq. (7) has converged. In that case $m_{\text{eff}} \approx m_{s,0}$ can be treated as constant, and $m_{s,0}^2$ can be ignored in the denominator. The integral can be evaluated analytically (ignoring the weak $T$-dependence of $g_*$ in the Hubble rate $H = 1.66 \sqrt{g_* T^2 / M_p}$), to obtain

$$\delta N_{\text{eff}} \approx \frac{1}{2} \left[ 1 - \exp \left( - \frac{5\sqrt{7} \alpha^{1/2} G_F M_p m_{s,0}^2}{64 s_W c_W g_4^{1/2} m_{s,0}} \right) \right] \quad (10)$$

where $W$ denotes the Weinberg angle, $M_p$ is the reduced Planck mass, and $g_4 \approx 10.75$ for the parameters of interest. The dependence on $T$ and $p$ is negligible for $T \lesssim 1 \text{ MeV}$, making it unnecessary to integrate over momenta.

For larger values of $m_{\phi}$, the DM starts oscillating before nucleosynthesis, which tends to activate the neutrino oscillations. This can be compensated by also increasing $m_{s,0}$, but an analytic treatment is no longer possible. One should numerically integrate over $T'$ and $p$ in eqs. (7)\footnote{The factors of $2^3$ missing in \cite{20}, account for the back-reaction from $\nu_s \rightarrow \nu_c$ \cite{39}}.

Additionally for BBN, we should distinguish between oscillations that produced a real excess in $N_{\text{eff}}$, occurring before the freezeout temperature $T_f = 3.2 \text{ MeV}$ of $\nu_s$, versus the subsequent oscillations that conserve total neutrino number but convert some $\nu_s$ into $\nu_c$. The reduction in $\nu_c$ density impacts BBN by changing the
Figure 1. Contours of $\delta N_{\text{eff}}$ (solid blue for CMB and dashed red for BBN) and corresponding to $\sum m_\nu$ (solid black) in the $m_{s,0}$-$m_\phi$ plane, illustrative of cosmological upper limits as described in the text.

For the CMB constraints, there is an analogous effect. Even though oscillations occurring after freezeout of $\nu_e$ should not change $\delta N_{\text{eff}}$, they can increase the sum of neutrino masses by converting some $\nu_e$ to $\nu_s$. Therefore the extra contribution to $\sum m_\nu$ can be estimated as $m_s$, times the asymptotic value of $\delta N_{\text{eff}}$ that results at low $T \sim 1$ eV, neglecting the conservation of neutrino number below $T_f$.

The results are shown in fig. 1, which displays three contours for $\delta N_{\text{eff}}$ in a region constrained by CMB measurements \[11\]. The exact upper limit determined by the Planck Collaboration depends upon which data sets are combined. At 95% c.l. $\delta N_{\text{eff}} < 0.5$ is a typical value (using TT+lowE or TT,TE,EE+lowE+lensing+BAO+R18), although a more stringent bound $\delta N_{\text{eff}} < 0.23$ is derived from TT,TE,EE+lowE alone. To illustrate the BBN constraint I show the 2$\sigma$ limit from ref. \[44\], which is somewhat weaker than that obtained in ref. \[43\]. The BBN contour at $\delta N_{\text{eff}} = 0.31$ illustrates the effect of conversions $\nu_e \rightarrow \nu_s$ after $\nu_e$ freezeout; for low $m_\phi$ it coincides with the corresponding CMB $\delta N_{\text{eff}}$ (since no such conversions take place), but at higher $m_\phi$, $\delta N_{\text{eff}}^{BBN}$ is seen to deviate from its CMB counterpart, as expected.

The strongest constraint is the CMB limit on neutrino masses. Their sum goes as

$$\sum m_\nu \cong [0.06 \text{eV} + m_4 \delta N_{\text{eff}}]$$

taking account of the standard contribution, assuming normal mass hierarchy. Ref. \[44\] recently constrained $\sum m_\nu < 0.145$ eV for the normal hierarchy, implying $\delta N_{\text{eff}} < 0.08$. This implies a lower limit on $m_{s,0} > 160$ eV, hence $\lambda > 10^{-22} \times (m_\phi/10^{-15} \text{eV})^{1/4}$.

**Discussion.** For DM with $m_\phi \lesssim 10^{-14}$ eV, we have seen that the cosmological analysis is relatively simple, since $\nu_e$ has frozen out before $\phi$ starts to oscillate. A favored value for $m_\phi$ from considerations of cosmological structure formation is considerably lower, $m_\phi \sim 10^{-22}$ eV. In this regime, the de Broglie wavelength is so large that structure at galactic scales can be suppressed, providing a possible solution to the cusp/core problem of DM halos \[44\].

Such light DM has an oscillation frequency of order $1 \text{y}$, which could have interesting consequences for laboratory oscillation experiments, if $\lambda$ is large enough to significantly impact the effective mass $m_{\text{eff}}$ of $\nu_s$ during the timescale of the experiment. For example, if the extra contribution to $m_{\text{eff}}$ is as large as the bare mass $m_4$, one would need $\lambda \sim 10^{-17}$, which is technically natural since there are no significant loop corrections. In this situation, the usual analysis of oscillation data could lead to ambiguous results, since the $\Delta m^2$ being fitted would be varying in time. This effect has already been considered with respect to active neutrinos coupling to $\phi$ in refs. \[30\,32\]. It could be interesting to reconsider the experiments that suggest active-sterile neutrino oscillations in this light.

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