Strong phases, asymmetries, and SU(3) symmetry breaking in $D \to K\pi$ decays

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Abstract

Motivated by some new experimental data, we carry out a phenomenological analysis of $D \to K\pi$ decays including both Cabibbo favored and doubly Cabibbo suppressed modes. Two asymmetries, $R(D^0)$ and $R(D^+)$, which are generated through interference between Cabibbo favored and doubly Cabibbo suppressed $D \to K\pi$ transitions, are predicted. The relative strong phase, $\delta_{K\pi}$, between $D^0 \to K^-\pi^+$ and $D^0 \to K^+\pi^-$ decays, is estimated. The theoretical results agree well with the current measurements.

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1 Introduction

Non-leptonic $D \to K \pi$ decays and their strong phases have been of great interest as they are essentially related to the studies of CP violation, $D^0 - \bar{D}^0$ mixing, and SU(3) symmetry breaking effects in charm physics [1, 2, 3, 4, 5]. These decay modes contain both Cabibbo favored (CF) and doubly Cabibbo suppressed (DCS) transitions, and the effective Hamiltonian relevant for them is given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \{ V_{ud} V_{cs}^* [ C_1 (\bar{s}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A} + C_2 (\bar{s}_i c_i)_{V-A} (\bar{u}_j d_i)_{V-A} ] + V_{us} V_{cd}^* [ C_1 (\bar{d}_i c_i)_{V-A} (\bar{u}_j s_j)_{V-A} + C_2 (\bar{d}_i c_i)_{V-A} (\bar{u}_j s_i)_{V-A} ] \} + \text{H.c.,}$$

(1)

where $V - A$ denotes $\gamma_{\mu}(1 - \gamma_5)$. The first line in eq. (1) governs CF decays and the second line DCS decays.

Theoretically, factorization hypothesis has been widely utilized in the hadronic $D$ decays. Many studies are based on the naive factorization approach, which simply replaces the matrix elements of a four-fermion operator in a heavy-quark decay by the product of the matrix elements of two currents. This approach has long been used in phenomenological applications, although there is an obvious shortcoming that it cannot lead to the scale and scheme independence for the final physical amplitude. Several years ago, the authors of Ref. [6] have formed an interesting QCD factorization formula for the two-body exclusive non-leptonic $B$ decays, in which the scale and scheme dependence of the hadronic matrix elements is recovered, and the naive factorization can be obtained as the lowest order approximation. The radiative corrections in the strong coupling constant $\alpha_s$, which are dominated by hard gluon exchange, can be calculated systematically using the perturbative QCD in the heavy quark limit. This means the strong final-state interaction phases, which arise from the hard-scattering kernel, are calculable from first principles. Analogously, in the heavy charm quark limit, a similar factorization formula for the matrix elements of the operators $Q_i$’s in the effective weak Hamiltonian (1) can be written as [6]

$$\langle P_1 P_2 | Q_i | D \rangle = \sum_j F_{j}^{D \to P_1} (m_2^2) \int_0^1 du T_{i j}^{I} (u) \phi_{P_2} (u) + (P_1 \leftrightarrow P_2)$$

$$+ \int_0^1 d\xi du T_{i j}^{II} (\xi, u, v) \phi_D (\xi) \phi_{P_1} (v) \phi_{P_2} (u),$$

(2)

where $F_{j}^{D \to P_1} (m_2^2)$ denotes a $D \to P_{1,2}$ form factor, $\phi_X (u)$ is the light-cone distribution amplitude of meson $X$. $T_{i j}^{I} (\xi, u, v)$ and $T_{i j}^{II} (\xi, u, v)$ are hard-scattering functions, which are perturbatively calculable. Then theoretical results for $D$ decays can be obtained straightforwardly. Taking the CF decays $D^0 \to K^- \pi^+$, $D^0 \to K^0 \pi^0$, and $D^+ \to K^0 \pi^+$ as examples, for the leading power contribution, we get

$$\mathcal{B}(D^0 \to K^- \pi^+) = 3.97\%, \quad \mathcal{B}(D^0 \to K^0 \pi^0) = 0.08\%, \quad \mathcal{B}(D^+ \to K^0 \pi^+) = 7.66\%$$

(3)

at the scale $\mu = 1.5$ GeV. (Here we have parameterized $\int_0^1 d\xi \phi_D (\xi) / \xi \equiv m_D / \lambda_D$ and set $\lambda_D = 0.3$ GeV in the numerical calculations.) The corresponding experimental data from [7]...
are  
\[ B(D^0 \to K^-\pi^+) = (3.80 \pm 0.07)\%, \quad B(D^0 \to \bar{K}^0\pi^0) = (2.28 \pm 0.24)\%, \]
\[ B(D^+ \to \bar{K}^0\pi^+) = (2.94 \pm 0.12)\%. \]

It is seen that, although the predicted branching ratio for the color-allowed decay \( D^0 \to K^-\pi^+ \) is in qualitative agreement with the data, the prediction for the color-suppressed decay \( D^0 \to \bar{K}^0\pi^0 \) is too small, and for the charged mode, the theoretical \( B(D^+ \to \bar{K}^0\pi^+) \) is too large. Similar conclusion will be reached when applying the formula (2) to singly Cabibbo-suppressed (SCS) and DCS decays. This seems to indicate that the charm quark mass is not heavy enough to apply the QCD factorization approach [6] or pQCD [8] in \( D \) decays very reliably. Therefore one generally appeals to the phenomenological analysis of these processes.

Experimentally, many new results in \( D \) decays are expected soon from the dedicated experiments conducted at CLEO, E791, FOCUS, SELEX, and the \( B \) factories BaBar and Belle. In particular, as pointed out in [9, 10], there are interesting asymmetries due to interference between CF and DCS \( D \to K\pi \) transitions, defined as

\[ R(D) \equiv \frac{B(D \to K_S\pi) - B(D \to K_L\pi)}{B(D \to K_S\pi) + B(D \to K_L\pi)}, \quad (4) \]

which have been observed by CLEO Collaboration [11] very recently,

\[ R(D^0) = 0.122 \pm 0.024 \pm 0.030, \quad R(D^+) = 0.030 \pm 0.023 \pm 0.025. \quad (5) \]

Also a preliminary result on the relative strong phase between \( D^0 \to K^-\pi^+ \) and \( D^0 \to K^+\pi^- \), which is due to SU(3) symmetry breaking and important in the search for \( D^0 - \bar{D}^0 \) mixing [11, 12], has been reported by CLEO Collaboration as

\[ \cos \delta_{K\pi} = 1.09 \pm 0.66 \quad (13), \]

although with very large uncertainty.

Motivated by the new measurements mentioned above, we would like to perform a phenomenological analysis of both CF and DCS \( D \to K\pi \) decays. As will be shown below, the present data cannot allow us to determine all of the phenomenological parameters appearing in decay amplitudes. Implementing the SU(3) symmetry may constrain the amplitudes, thus largely reduce the number of independent parameters. However, it is known that this symmetry is not well respected in nature, even badly broken in some cases. Therefore, as a conservative way to constrain these amplitudes, in this paper we assume that SU(3) symmetry in \( D \to K\pi \) decays is moderately broken, namely, symmetry breaking effects in decay amplitudes are dominated by decay constants \( f_P \) and \( D \to P \) (\( P = \pi, K \)) form factors, and other SU(3) symmetry breaking sources can be neglected. This is not a general feature of SU(3) symmetry breaking in charmed decays.

## 2 \( D \to K\pi \) decay amplitudes

We begin by considering the \( D \to K\pi \) decay amplitudes in terms of the quark-diagram topologies \( T \) (color-allowed), \( C \) (color-suppressed), \( E \) (\( W \)-exchange), and \( A \) (\( W \)-annihilation)
[14], which are given by

\[ A(D^0 \rightarrow K^+\pi^-) = i \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^{\ast} (T + E), \]  
\[ \sqrt{2} A(D^0 \rightarrow K^0\pi^0) = i \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^{\ast} (C - E), \]  
\[ A(D^+ \rightarrow K^0\pi^+) = i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^{\ast} (T' + E'), \]  
\[ \sqrt{2} A(D^+ \rightarrow K^0\pi^0) = i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^{\ast} (C' - E'), \]  
\[ A(D^0 \rightarrow K^0\pi^+) = i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^{\ast} (C' + A'), \]  
\[ \sqrt{2} A(D^+ \rightarrow K^+\pi^0) = i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^{\ast} (T' - A'), \]  

and two isospin relations

\[ A(D^0 \rightarrow K^+\pi^+) + \sqrt{2} A(D^0 \rightarrow K_0^0\pi^0) = A(D^+ \rightarrow K^0\pi^0), \]  
\[ A(D^0 \rightarrow K^+\pi^-) + \sqrt{2} A(D^0 \rightarrow K_0^0\pi^0) = A(D^+ \rightarrow K^0\pi^+) + \sqrt{2} A(D^+ \rightarrow K^+\pi^0) \]  

are satisfied explicitly. For our notations, we have extracted the CKM matrix elements and factor \( G_F/\sqrt{2} \) from the quark-diagram amplitudes, and the prime is added to DCS amplitudes.

The present experimental status of \( D \rightarrow K\pi \) decays is not very satisfying. Branching ratios of three CF modes and \( \mathcal{B}(D^0 \rightarrow K^0\pi^0) \) have been reported by Particle data group [7], however, there are no measurements for \( \mathcal{B}(D^0 \rightarrow K^0\pi^0) \) yet. Only an upper bound on \( \mathcal{B}(D^+ \rightarrow K^+\pi^0) < 4.2 \times 10^{-4} \) (CL=90\%) is shown in [7]; while, very recently, BaBar Collaboration and CLEO Collaboration have given

\[ \mathcal{B}(D^+ \rightarrow K^+\pi^0) = (2.52 \pm 0.47 \pm 0.25 \pm 0.08) \times 10^{-4} \]  
\[ \mathcal{B}(D^+ \rightarrow K^+\pi^0) = (2.25 \pm 0.36 \pm 0.15 \pm 0.07) \times 10^{-4} \]  

respectively. In general the quark-diagram amplitudes in (7) – (13) could have non-trivial strong phases. Therefore, only using the available experimental data, it is impossible to determine these amplitudes without any theoretical assumptions.

On the other hand, with the help of the factorization hypothesis, \( \mathcal{T}, \mathcal{T}', \mathcal{C}, \) and \( \mathcal{C}' \) can be expressed as

\[ \mathcal{T} = f_\pi(m_D^2 - m_\pi^2) F_0^{D \rightarrow K}(m_\pi^2)a_{1\text{eff}}, \]  
\[ \mathcal{C} = f_K(m_D^2 - m_\pi^2) F_0^{D \rightarrow \pi}(m_\pi^2)a_{2\text{eff}}, \]  
\[ \mathcal{T}' = f_K(m_D^2 - m_\pi^2) F_0^{D \rightarrow \pi}(m_\pi^2)a_{1\text{eff}}, \]  
\[ \mathcal{C}' = f_K(m_D^2 - m_\pi^2) F_0^{D \rightarrow \pi}(m_\pi^2)a_{2\text{eff}}, \]  

(17)
where $a_i^\text{eff}$’s are regarded as the effective Wilson coefficients fixed from the data (in the naive factorization, $a_{1,2} = C_{1,2} + C_{2,1}/N_c$). Generally in the factorization approach $a_i^\text{eff}$’s in DCS amplitudes could be different from the ones in CF amplitudes due to SU(3) symmetry breaking effects. Here we do not distinguish them because, as stated above, we assume that the SU(3) symmetry breaking effects in $D \to K\pi$ modes have been mostly captured by the decays constants ($f_\pi$ and $f_K$) and form factors ($F^{0\to\pi,K}_D(q^2)$) in the amplitudes.

A similar analysis of $W$-exchange and $W$-annihilation amplitudes leads to

$$E = f_D(m_K^2 - m_\pi^2)F^{0\to K\pi}_0(m_D^2)a_2^\text{eff},$$
$$E' = f_D(m_\pi^2 - m_K^2)F^{0\to K\pi}_0(m_D^2)a_2^\text{eff},$$
$$A' = f_D(m_\pi^2 - m_K^2)F^{0\to K\pi}_0(m_D^2)a_1^\text{eff}. \quad (18)$$

This will give $E = -E'$. However, the annihilation form factor $F^{0\to K\pi}_0(m_D^2)$ is expected to be strongly suppressed due to the large $q^2 = m_D^2 \ [17]$, therefore contributions from eq. (18) are believed to be negligible. For $B$ mesons, the weak annihilation amplitudes ($W$-exchange or $W$-annihilation) induced by the topologies of gluon emission arising from the quarks of the weak vertex, as shown in Fig. 1, have been analyzed in [18], which is thought to be numerically important in $B$ decays. The similar study on $D$ mesons have been done in [19, 20], and it has been shown that these contributions could also play important roles in $D$ decays. The $O(\alpha_s)$ contribution can be read directly from Refs. [18, 20],

$$E = f_D f_K f_\pi \frac{C_F}{N_C} \pi \alpha_s C_1 \left[ 18 \left( X_A - 4 + \frac{\pi^2}{3} \right) + 2r_\chi \right] X_A^2 \right], \quad \mathcal{E}' = E, \quad (19)$$

and

$$A' = f_D f_K f_\pi \frac{C_F}{N_C} \pi \alpha_s C_2 \left[ 18 \left( X_A - 4 + \frac{\pi^2}{3} \right) + 2r_\chi \right] X_A^2 \right]. \quad (20)$$

where $X_A = \int_0^1 dy/y$ has been used to parameterize the logarithmically divergent integrals due to the end-point singularity, and $C_1$, $C_2$ are the Wilson coefficients in [11]. Note that the asymptotic form of the light-cone distribution amplitudes for light mesons have been used in the derivation of eq. (19). This is consistent with the assumption on SU(3) symmetry breaking used in deriving eq. (17).

Meanwhile, comparing eq. (20) with eq. (19), one can get an additional constraint

$$A' = \frac{C_2}{C_1} \mathcal{E}'. \quad (21)$$
\( \Lambda_{\text{MS}}^{(4)} = 215 \text{ MeV} \quad \Lambda_{\text{MS}}^{(4)} = 325 \text{ MeV} \quad \Lambda_{\text{MS}}^{(4)} = 435 \text{ MeV} \)

| \( \mu \) [GeV] | NDR | HV | NDR | HV | NDR | HV |
|-----------------|-----|----|-----|----|-----|----|
| 1.0             | -0.339 | -0.390 | -0.400 | -0.464 | -0.464 | -0.548 |
| 1.5             | -0.277 | -0.319 | -0.318 | -0.369 | -0.357 | -0.419 |
| 2.0             | -0.240 | -0.278 | -0.271 | -0.316 | -0.301 | -0.353 |

Table 1: The scale and scheme dependence of \( C_2/C_1 \) at the next-to-leading order. The values of \( C_1 \) and \( C_2 \) are taken directly from [21].

We would like to give some remarks here.

- By combining eq. (21) with eqs. (19) and (17), we will reduce independent complex phenomenological parameters appearing in decay amplitudes as \( a_1^{\text{eff}}, a_2^{\text{eff}} \), and \( \mathcal{E} \). Since only five branching ratios of \( D \to K\pi \) decays are measured up to now, this means that including the additional constraint (21) is important to enable us to determine the \( D \to K\pi \) amplitudes from the present data. From [21], \( C_1 \) and \( C_2 \) have opposite signs, therefore weak-exchange \( \mathcal{E} \) and weak-annihilation \( \mathcal{A} \) amplitudes have opposite signs, consistent with the observations in [22]. Also \( |C_2| < |C_1| \) [21], we have \( |\mathcal{E}'| > |\mathcal{A}'| \); while the contrary conclusion will be obtained if we use eq. (18).

- Strictly speaking, we have to admit that eq. (21) is not very physical since \( C_1 \) and \( C_2 \) are both scale and scheme dependent [21]. The scale and scheme dependence of \( C_2/C_1 \) has been shown in Table 1, from which it is found that \( C_2/C_1 \) is about \(-0.5 \sim -0.3 \) for \( \mu \) around \( 1.0 \sim 1.5 \text{ GeV} \) (Note that the scale in this range is relevant for \( D \) decays). Therefore, we will treat in the following numerical calculations \( C_2/C_1 \) as a negative parameter instead of a ratio of two Wilson coefficients.

- The weak-annihilation contribution is power suppressed in the heavy quark limit. The divergent integral \( X_A \) appearing in (19) and (20) signals that factorization breaks down actually. In the present analysis we therefore use relations \( \mathcal{E} = \mathcal{E}' \) and (21) for these three weak-annihilation amplitudes instead of their explicit expressions shown above. Although they are not model independent relations, one will find that phenomenologically they work very well in \( D \to K\pi \) decays.

### 3 Phenomenological analysis

From now on we study some possible phenomenological applications based on the above theoretical assumptions.
First, the use of eqs. (17) and (19) gives \( C = C' \) and \( E = E' \), hence we have

\[
A(D^0 \to K^0\pi^0) = \frac{V_{us}V_{ud}^*}{V_{ud}V_{cs}} A(D^0 \to \bar{K}^0\pi^0) = -\tan^2 \theta_C A(D^0 \to \bar{K}^0\pi^0),
\]

which implies that the relative strong phase between these two amplitudes vanishes. Here \( \theta_C \) is the Cabibbo angle. Consequently, one gets

\[
R(D^0) = \frac{2\tan^2 \theta_C}{1 + \tan^4 \theta_C} \approx 2 \tan^2 \theta_C.
\]

Using \( \tan \theta_C \approx 0.23 \), \( R(D^0) \approx 0.106 \), which is in agreement with the measurement in eq. (5). The same result has been obtained in Refs. [9, 23]. However, one cannot expect the similar result for \( R(D^+) \) since there is no similar relation as (22) between \( A(D^+ \to \bar{K}^0\pi^+) \) and \( A(D^+ \to K^0\pi^+) \), even in the SU(3) symmetry limit. We will discuss this issue later.

Second, using the constraint (21) together with (20) and (17), one can obtain an interesting relation among the amplitudes of \( D^0 \to K^\pm \pi^\mp \) and \( D^+ \to K^0\pi^0 \) decays, which is

\[
\tan^2 \theta_C A(D^0 \to K^-\pi^+) + \kappa A(D^0 \to K^+\pi^-) = \sqrt{2} \zeta A(D^+ \to K^0\pi^0),
\]

where

\[
\kappa \equiv \frac{1 + C_2/C_1 x f_\pi/f_K}{1 + C_2/C_1}, \quad \zeta \equiv \frac{1 - x f_\pi/f_K}{1 + C_2/C_1}
\]

with

\[
x \equiv \frac{(m_D^2 - m_K^2) F_{D^0 \to \bar{K}(m_\pi^2)}}{(m_D^2 - m_\pi^2) F_{D^0 \to \pi}(m_K^2)}.
\]

Thus the relative strong phase \( \delta_{K\pi} \) between \( A(D^0 \to K^+\pi^-) \) and \( A(D^0 \to K^-\pi^+) \) is given by

\[
\cos \delta_{K\pi} = \frac{\tan^4 \theta_C B(D^0 \to K^-\pi^+) + \kappa^2 B(D^0 \to K^+\pi^-) - 2 \zeta^2 \frac{\tau(D^0)}{\tau(D)} B(D^0 \to K^0\pi^0)}{2 \tan^2 \theta_C \kappa \sqrt{B(D^0 \to K^-\pi^+) B(D^0 \to K^+\pi^-)}},
\]

where \( \tau(D) \) is the lifetime of \( D \). Obviously, in the SU(3) symmetry limit, \( \kappa = x = 1 \), \( \zeta = 0 \), eq. (24) will be \( A(D^0 \to K^+\pi^-) = -\tan^2 \theta_C A(D^0 \to K^-\pi^+) \), and \( \delta_{K\pi} \) vanishes. Note that the relation (24) does not depend on the color-suppressed amplitudes \( C \) and \( C' \) because these amplitudes have nothing to do with the above three decay modes.

In order to go further into the analysis, we need to have information about the form factors \( F_{0}^{D^0 \to P}(q^2) \). For their \( q^2 \) dependence, we adopt the Bauer-Stech-Wirbel model [24], in which the form factors are assumed to behave as a monopole,

\[
F_{0}^{D^0 \to P}(q^2) = \frac{F_{0}^{D^0 \to P}(0)}{1 - q^2/m_*^2},
\]

where \( m_* \) is the pole mass with \( m_* = 2.47 \text{ GeV} \) for \( P = \pi \) and \( m_* = 2.60 \text{ GeV} \) for \( P = K \). \( F_{0}^{D^0 \to P}(0) \) can be obtained via \( F_{0}^{D^0 \to P}(0) = F_{0}^{D^0 \to P}(0) \), since the latter can be measured in
The authors of Ref. [1], by assuming the existence of nearby resonances for the δ meson, have obtained sin δ = 1.0 ∼ δ. It seems that SU(3) symmetry breaking effects are dominated by decay constants (f_π and f_K). By applying it to eq. (25), we get

\[ F_+^{D^0 \to K}(0) = 0.695 \pm 0.023, \quad F_+^{D^0 \to \pi}(0) = 0.624 \pm 0.036, \]

\[ F_+^{D^0 \to \pi}(0)/F_+^{D^0 \to K}(0) = 0.898 \pm 0.045, \] (28)

which are consistent with very recent results from lattice calculation [26] and from the QCD sum rules calculation [27]. In practice, only the ratio of these two form factors in (28) is needed for our numerical calculations. By applying it to eq. (25), we get

\[ x = 1.002 \pm 0.050, \] (29)

which is very close to its value in the SU(3) symmetry limit [the error in (29) is due to the uncertainty of F_+^{D^0 \to \pi}(0)/F_+^{D^0 \to K}(0) only]. Although this may be just a numerical coincidence, it seems that SU(3) symmetry breaking effects are dominated by decay constants (f_π and f_K).

The numerical predictions of δ_Kπ for different values of C_2/C_1 are displayed in Table 2. As mentioned above, C_2/C_1 is regarded as a varying parameter. \( B(D^0 \to K^{-}\pi^+) \) and \( B(D^0 \to K^{+}\pi^-) \) are taken from [17]. Since Particle data group has not given the average for \( B(D^0 \to K^{+}\pi^0) \) yet, both of the measurements listed in eq. [16] have been used, and the results are shown in the second and third lines of Table 2, respectively. The error is due to the uncertainty of x in eq. (29) and the uncertainties of experimental branching ratios mentioned above, in which the contribution from \( B(D^0 \to K^{-}\pi^+) \) and \( B(D^0 \to K^{+}\pi^-) \) is actually very small and can be neglected. From Table 1, for the relevant scale of D decays, i.e. \( \mu \) in the range of 1.0 ∼ 1.5 GeV, C_2/C_1 is about −0.5 ∼ −0.3. Therefore, a not large but nonzero δ_Kπ, whose magnitude is 10° or above, i.e. \( \sin \delta \sim \pm 0.2 \), might be expected from the present analysis. The authors of Ref. [1], by assuming the existence of nearby resonances for the D meson, have obtained \( \sin \delta_K\pi = \pm 0.31 \), namely, \( \cos \delta_K\pi = 0.951 \) and \( \delta_K\pi \) is about ±18°. The

| C_2/C_1 | -0.2 | -0.3 | -0.4 | -0.5 | -0.6 |
|---------|------|------|------|------|------|
| cos δ_{K\pi} | 0.979±0.018 | 0.975±0.021 | 0.969±0.025 | 0.960±0.032 | 0.946±0.043 |
| δ_{K\pi} | 11.7°±4.9° | 12.9°±5.3° | 14.3°±5.8° | 16.2°±6.6° | 18.9°±7.7° |
| cos δ_{K\pi} | 0.983±0.015 | 0.980±0.017 | 0.976±0.021 | 0.970±0.026 | 0.960±0.034 |
| δ_{K\pi} | 10.5°±4.6° | 11.4°±4.9° | 12.6°±5.5° | 14.1°±6.1° | 16.2°±6.9° |

Table 2: The relative strong phase between \( D^0 \to K^+\pi^- \) and \( D^0 \to K^-\pi^+ \) predicted by eq. (26) for different values of C_2/C_1. \( B(D^0 \to K^+\pi^0) \) by BaBar [15] is used for the results in the second line; \( B(D^0 \to K^{+}\pi^0) \) by CLEO [16] is used for the results in the third line. The sign of δ_Kπ could also be minus.
other existing hadronic models which incorporate SU(3) symmetry breaking effects seem to prefer a small value of this phase, \( \sin \delta \leq 0.2 \), with most models giving \( \sin \delta \leq 0.1 \) \cite{28, 29} (see Table I in Ref. \cite{29} for details). Unfortunately, the current measurement of \( \delta_{K\pi} \) is very rough \cite{13}, as shown in eq. (6). Employing the asymmetry \( R(D) \) measurements with some theoretical assumptions, another experimental result \( \delta_{K\pi} \approx (3 \pm 6 \pm 7)^\circ \) with relative small uncertainty is induced in Ref. \cite{11}. Both of them are still consistent with zero.

Finally, we estimate \( D \to K\pi \) decay amplitudes from the currently available data. The three independent complex phenomenological parameters are chosen as \( \mathcal{T}, \mathcal{C}, \) and \( \mathcal{E} \), not as \( a_1^{\text{eff}}, a_2^{\text{eff}}, \) and \( E \), because we will only use the ratio of the form factors [in eq.\cite{28}] instead of their absolute values in the analysis. Without loss of generality, \( \mathcal{T} \) is set to be real. \( \delta_C (\delta_E) \) is the relative strong phase of \( \mathcal{C} (\mathcal{E}) \) to \( \mathcal{T} \). Here we take \( \mathcal{B}(D^0 \to K^-\pi^+) \), \( \mathcal{B}(D^0 \to \bar{K}^0\pi^0) \), \( \mathcal{B}(D^+ \to \bar{K}^0\pi^+) \), and \( \mathcal{B}(D^0 \to K^+\pi^-) \) given by Particle data group \cite{7}, together with \( \mathcal{B}(D^+ \to K^+\pi^0) \) by BaBar Collaboration \cite{15} to illustrate our numerical calculation. The results of \( \mathcal{T}, \mathcal{C}, \mathcal{E} \), and \( |a_2^{\text{eff}}/a_1^{\text{eff}}| \) are summarized in Table 3, and other amplitudes \( \mathcal{T}', \mathcal{C}', \mathcal{E}' \), and \( \mathcal{A}' \) can be easily derived using eqs. (17), (19) and (21). Several observations and remarks are given as follows.

- The color-suppressed amplitude has a phase \( \sim 160^\circ \) relative to the color-allowed amplitude \( \mathcal{T} \), and \( |\mathcal{C}| \) is effectively enhanced. This means that there could exist the strongly destructive interference between \( \mathcal{T} \) and \( \mathcal{C} \). We get \( a_2^{\text{eff}}/a_1^{\text{eff}} \approx 0.56e^{\pm 1160^\circ} \), which is insensitive to the value of \( C_2/C_1 \). \( a_2^{\text{eff}}/a_1^{\text{eff}} = 0.62e^{-1152^\circ} \) is obtained in \cite{22}.

- The \( \mathcal{E} \) amplitude has a relative phase \( \sim 130^\circ \) to \( \mathcal{T} \), \( \sim 70^\circ \) to \( \mathcal{C} \). Its magnitude is relatively large, and \( |\mathcal{E}| > |\mathcal{C}| \). This is contrary to the results in Ref. \cite{22, 3}. As pointed out in \cite{18}, in general, the weak annihilation parameter \( X_A \) in eq. (20) should be of order \( \ln(m_D/\Lambda) \) and \( \Lambda \) is a soft scale. By taking \( \alpha_s \approx 0.5 \), \( C_1 \approx 1.2 \), and \( \mathcal{E} = 0.325e^{\pm 131^\circ} \) for \( C_2/C_1 = -0.3 \), we can roughly estimate \( X_A = 4.09e^{\pm 122^\circ} \) or \( 3.66e^{\pm 171^\circ} \), which indicates \( |X_A| \sim 2 \ln(m_D/\Lambda) \) with \( \Lambda \approx 0.3 \text{ GeV} \). \( |X_A| = 3.84 \) has been obtained in \cite{20}.

- Some of our results are not in agreement with the ones in Refs. \cite{22, 3}, since we do not work in the SU(3) symmetry limit, and we mainly concentrate on \( D \to K\pi \) decay modes in this paper.

We return to discuss the asymmetry \( R(D^+) \). As pointed out before, the charged case is not as simple as the neutral case. Because of

\[
\frac{A(D^+ \to K^0\pi^+)}{A(D^+ \to \bar{K}^0\pi^+)} = -\tan^2 \theta_C \frac{\mathcal{C}' + \mathcal{A}'}{\mathcal{C} + \mathcal{T}} = -\tan^2 \theta_C \frac{\mathcal{C} + C_2/C_1\mathcal{E}}{\mathcal{C} + \mathcal{T}},
\]

one cannot simplify it as a similar analytic relation \cite{22} for neutral modes under \( \mathcal{C} = \mathcal{C}' \) and \( \mathcal{E} = \mathcal{E}' \), even if including the additional constraint \cite{21} already. However, using the values
Therefore $R$ found that, $\delta$

\begin{align*}
\frac{R(D^+ \rightarrow K^0\pi^+)}{R(D^+ \rightarrow K^0\pi^+)} &= \left\{ \begin{array}{ll}
-\tan^2 \theta_C \ 1.538e^{\pm i106^\circ}, & C_2/C_1 = -0.3, \\
-\tan^2 \theta_C \ 1.532e^{\pm i105^\circ}, & C_2/C_1 = -0.4, \\
-\tan^2 \theta_C \ 1.521e^{\pm i103^\circ}, & C_2/C_1 = -0.5,
\end{array} \right. \\
\text{which lead to} \quad R(D^+) &= \begin{cases}
0.044, & C_2/C_1 = -0.3, \\
0.040, & C_2/C_1 = -0.4, \\
0.035, & C_2/C_1 = -0.5.
\end{cases}
\end{align*}

The present observed value by CLEO Collaboration [11] is $R(D^+) = 0.030 \pm 0.023 \pm 0.025$. Also, the suppression of $R(D^+)$ comparing with $R(D^0)$ can be understood. From the definition of $R(D)$ in eq. (4), one will find it is proportional to $2\tan^2 \theta_C \cos \delta$, and $\delta$ is the relative strong phase between the corresponding DCS amplitude and the CF amplitude. Now it is found that, $\delta$ vanishes in the $D^0$ case, as shown in eq. (22), while $\delta$ is about $100^\circ$ in the $D^+$ case [see eq. (31)]. Therefore $R(D^+)$ is suppressed by small $\cos \delta$.

Furthermore, we discuss the possible generalization to the analysis of SCS $D \rightarrow \pi\pi, KK$ decays. Consider the ratio

$$R_1 = 2 \left| \frac{V_{cs}}{V_{cd}} \right|^2 \frac{\Gamma(D^+ \rightarrow \pi^0\pi^+)}{\Gamma(D^+ \rightarrow K^0\pi^+)}.$$  

The recent measurement gives $R_1 = 1.54 \pm 0.27$ [15] and $R_1$ should be unity in the SU(3) symmetry limit. Note that these two modes have only $T$ and $C$ amplitudes. From our analysis, one can get

$$R_1 = 1.073 \left| \frac{F_0^{D \rightarrow \pi}(m^2_{\pi})}{F_0^{D \rightarrow \pi}(m^2_K)} \right|^2 \frac{1 + a_2^{\text{eff}}/a_1^{\text{eff}}}{1/x + (f_K/f_\pi)a_2^{\text{eff}}/a_1^{\text{eff}}}^2,$$  

Table 3: Numerical results of quark-diagram amplitudes $T, C, E$, and of $|a_2^{\text{eff}}/a_1^{\text{eff}}|$ estimated by using the present data with different $C_2/C_1$. Only the central values of the magnitude and the phase are quoted.

| $C_2/C_1$ | $T$ [GeV$^3$] | $C$ [GeV$^3$] | $E$ [GeV$^3$] | $|a_2^{\text{eff}}/a_1^{\text{eff}}|$ |
|----------|--------------|--------------|--------------|----------------|
| -0.3     | 0.417        | 0.289 $e^{\pm i160^\circ}$ | 0.325 $e^{\pm i131^\circ}$ | 0.568 |
| -0.4     | 0.445        | 0.306 $e^{\pm i163^\circ}$ | 0.368 $e^{\pm i135^\circ}$ | 0.563 |
| -0.5     | 0.485        | 0.334 $e^{\pm i167^\circ}$ | 0.425 $e^{\pm i139^\circ}$ | 0.564 |
where the factor 1.073 is from the phase-space differences for the $\pi\pi$ and $K\pi$ final states. Taking $a_2^{\text{eff}}/a_1^{\text{eff}} \simeq 0.56 e^{\pm 160^\circ}$ from Table 3, we obtain $R_1 \simeq 1.44$, in accord with the recent measurement. Likewise, the ratio

$$R_2 = \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \simeq 1.50$$

(35)

can also be estimated using Table 3. Unfortunately, this result is far from the experimental value $\Gamma(D^0 \to K^0K^0)/\Gamma(D^0 \to \pi^+\pi^-) = 2.82 \pm 0.10$ [7], implying that SU(3) symmetry breaking is still not fully accounted for. Since now there exist weak-annihilation contributions, in deriving eq. (35), we have assumed $\mathcal{E}_{K^+K^-} = f_K/f_\pi \mathcal{E}$ and $\mathcal{E}_{\pi\pi} = f_\pi/f_K \mathcal{E}$. This is actually not true because, under this assumption, the amplitude for the pure weak-annihilation $D^0 \to K^0\bar{K}^0$ decay will vanish, whereas $\mathcal{B}(D^0 \to K^0\bar{K}^0) = (7.4 \pm 1.4) \times 10^{-4}$ experimentally [7]. Therefore the failure of reproducing the experimental value in eq. (35) may be unavoidable in the present framework. In the case of $R_1$, the weak-annihilation contribution is fortunately absent. This implies that the above relations for $\mathcal{E}_{K^+K^-}$ and $\mathcal{E}_{\pi\pi}$ need some corrections, and the weak-annihilation amplitudes should be carefully investigated when one would like to generalize the present work to the case of the SCS $D \to \pi\pi$, $KK$ decays including the pure weak-annihilation mode $D^0 \to K^0\bar{K}^0$.

4 Summary

We have presented a phenomenological analysis of $D \to K\pi$ decays including both CF and DCS modes. In order to determine all decay amplitudes for these processes using the present data, a moderate SU(3) symmetry breaking formalism has been assumed. Our analysis indicates this assumption works well in $D \to K\pi$ decays. The color-suppressed amplitude is enhanced, and it has a phase $\sim 160^\circ$ relative to the color-allowed amplitude. A large weak annihilation amplitude is obtained. Both of the asymmetries $R(D^0)$ and $R(D^+)$ have been predicted, which are in good agreement with the experimental data. Our analysis also shows that a not large but nonzero $\delta_{K\pi}$, which is about 10$^\circ$ or above, might be expected. This means that there is no good reason to take $\sin\delta_{K\pi} = 0$ in the experimental analysis of $D^0 \to K^{\pm}\pi^{\mp}$ decays. A precise measurement of $\delta_{K\pi}$ will be welcome both theoretically and experimentally.

We would like to point out that, the relation [21] between $\mathcal{E}'$ and $\mathcal{A}'$ amplitudes, which is important to enable us to calculate $\delta_{K\pi}$ and estimate the $D \to K\pi$ amplitudes in this analysis, is a model dependent assumption. Further tests for this relation will be very useful. But such tests cannot be performed at present because of the lack of suitable data. On the other hand, a similar relation between $W$-exchange and $W$-annihilation amplitudes could occur in SCS $D^0 \to K^+K^-$ and $D^+ \to K^+\bar{K}^0$ decays, which might provide an interesting test. As mentioned above, the present analysis however cannot be generalized to SCS decays straightforwardly when these transitions receive contributions from the weak-annihilation amplitudes. Therefore it would be useful to extend the present framework assuming [21] to include also SCS decays $D \to \pi\pi, KK$. Since here our main analysis concerns $D \to K\pi$ decays, a further discussion of these issues will be left for future work.
Very recently, the similar study for $R(D^0)$ has been obtained in Ref. [23]. Since in our framework, we can employ the currently available data to determine all quark-diagram amplitudes $T(T'), C(C'), E(E'),$ and $A(A')$ including their relative strong phases, $R(D^+)$ and $\delta_{K\pi}$ are also estimated.

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References

[1] A.F. Falk, Y. Nir, and A.A. Petrov, J. High Energy Phys. 9912 (1999) 019.
[2] M. Gronau, Y. Grossman, and J.L. Rosner, Phys. Lett. B 508 (2001) 37.
[3] J.R. Rosner, Phys. Rev. D 60 (1999) 114026.
[4] M. Gronau and J.L. Rosner, Phys. Lett. B 500 (2001) 247; C.W. Chiang and J.L. Rosner, Phys. Rev. D 65 (2002) 054007.
[5] C.W. Chiang, Z. Luo, and J.L. Rosner, Phys. Rev. D 67 (2003) 014001.
[6] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; Nucl. Phys. B591 (2000) 313.
[7] W.M. Yao et al., Particle data group, J.Phys. G 33 (2006) 1.
[8] Y.Y. Keum, H.-n. Li, and A.I. Sanda, Phys. Lett. B 504 (2001) 6; Phys. Rev. D 63 (2001) 054008; Y.Y. Keum and H.-n. Li, Phys. Rev. D 63 (2001) 074006.
[9] I.I. Bigi and H. Yamamoto, Phys. Lett. B 349 (1995) 363.
[10] F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, Z. Phys. C 55 (1992) 243; F. Buccella, M. Lusignoli, G. Miele, A. Pugliese, and P. Santorelli, Phys. Rev. D 51 (1995) 3478.
[11] Q. He et al., CLEO Collaboration, hep-ex/0607068.
[12] L.M. Zhang et al., Belle Collaboration, Phys. Rev. Lett. 96 (2006) 151801; A.J. Schwartz, hep-ex/0605032.
[13] W.M. Sun, for the CLEO Collaboration, hep-ex/0603031; D. Asner et al., CLEO Collaboration, hep-ex/0607078.
[14] L.L. Chau and H.Y. Cheng, Phys. Rev. Lett. 56 (1986) 1655; Phys. Rev. D 36 (1987) 137.
[15] B. Aubert et al., BaBar Collaboration, Phys. Rev. D 74 (2006) 011107.

[16] S. Dytman et al., CLEO Collaboration, [hep-ex/0607075, hep-ex/0609008].

[17] G.P. Lepage and S.J. Brodsky, Phys. Lett. B 87 (1979) 359.

[18] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. B606 (2001) 245; M. Beneke and M. Neubert, Nucl. Phys. B675 (2003) 333.

[19] V.L. Chernyak and A.R. Zhitnitsky, Nucl. Phys. B201 (1982) 492.

[20] J.H. Lai and K.C. Yang, Phys. Rev. D 72 (2005) 096001.

[21] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.

[22] H.-Y. Cheng, Eur. Phys. J. C 26 (2003) 551.

[23] J.L. Rosner, Phys. Rev. D 74 (2006) 057502.

[24] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C 29 (1985) 637; M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34 (1987) 103.

[25] K. Abe et al., Belle Collaboration, [hep-ex/0510003]; L. Widhalm et al., Belle Collaboration, Phys. Rev. Lett. 97 (2006) 061804.

[26] C. Aubin et al., Phys. Rev. Lett. 94 (2005) 011601.

[27] P. Ball, Phys. Lett. B 641 (2006) 50.

[28] L.-L. Chau and H.Y. Cheng, Phys. Lett. B 333 (1994) 514.

[29] T.E. Browder and S. Pakvasa, Phys. Lett. B 383 (1996) 475.