Gear Ratio Distribution of Electric Vehicle Reducer

Zhihao Tang, Xinmin Li*

No. 99 Shangda Road, Baoshan District, Shanghai University, Shanghai 200444
601963200@qq.com, 17721233575

Abstract. Based on the contact fatigue strength criterion of tooth surface, the mathematical model of gear ratio distribution of the electric vehicle is established. In the process, there are two objective functions. One is the minimum diameter of electric vehicle reducer, the other is the minimum mass of electric vehicle reducer. Through examples, relevant program is compiled with matlab software. It is obtained that there is a distribution method for gear ratio of the electric vehicle reducer.

1. Introduction
The two-stage gear reducer is the most common type in the electric vehicles reducers, and the reasonable distribution of gear ratio determines the volume and carrying capacity of the reducer. Liu Ziran [1] took a two-stage closed differential reducer as an example. Based on equal contact fatigue strength criterion, the transmission ratio is optimized with the planetary gear volume and radial dimensions as the optimization target; Wang Rong [3] took the lightest total mass of the reducer gear as the optimization principle, and analyzed the transmission ratio of the three-stage bevel cylindrical gear with Matlab; In the optimization design of the two-stage cylindrical gear reducer, liang hua qi [7] proposed a weighted combination method and an objective programming approach to unify the objective function and obtain an optimal solution; Zhou Qi ca [6] compared the conventional design and optimization design. In the optimization design, design variables, objective functions, and constraints are constitutes a design problem and greatly shortens the design cycle; Fan Li mei [8] established the objective function with the minimum volume and high transmission stability, and proposed the multi-objective optimization design method; Xiong Yong [11] used the theory of fuzzy optimization to design the two-stage planetary gear reducer system, and improved the weight coefficient according to the genetic algorithm; Bao He yun [9] took a two-stage star gear transmission as an example to optimize the mixed problem with discrete and mixed design variables; Zhao You hong [10] established a sub objective function based on volume, accuracy, and coordinated each sub objective function with the unified objective function method.

The previous research does not provide a specific distribution formula for the gear ratio distribution of electric vehicle reducer. In this paper, the distribution method is mainly applicable to the electric vehicle reducer. The volume or mass of the electric vehicle reducer is used as the objective function. The purpose of this paper is to provide the gear ratio distribution method for different gear transmission structure schemes in the electric vehicle reducer.
2. Mathematical model of gear ratio distribution

2.1. Design variable
In this paper, based on equal contact surface fatigue strength criterion, the gear ratio of electric vehicles is allocated. The factors such as contact strength factor, gear ratio, width-to-diameter ratio, uneven load factor, and number of planet wheels are determine the function value of the two-stage transmission system. Under the constrint that the total transmission ratio of electric vehicle reducer is fixed, the first-stage gear ratio U1 as the design variable under the requirements of contact surface fatigue strength and tooth root bending fatigue strength. The gear ratio distribution formula takes the total outer diameter or the total mass of the transmission system as the objective function.

2.2. Gear ratio distribution of reducer
The gear ratio distribution formula is proposed for the gear structure scheme of the electric vehicle reducer, which is mainly applicable to gear transmission structure schemes such as two-stage parallel shaft and two-stage planet.

2.2.1. Two-stage parallel shaft
The two-stage parallel shaft is the most common and simplest structure in reducer. The two-stage parallel shaft gear reducer is mainly composed of three parallel shafts and two pairs of gear pairs. One pair is a low-speed gear pair, the other is a high-speed gear pair. When allocating the gear ratio of the two-stage parallel shaft gear reducer, the reference circle diameter or mass of the low-speed and high-speed pinion are calculated, and on this basis, the reference circle diameter or mass of the large gear are converted. Therefore, In the two-stage parallel shaft gear reducer transmission system, the sum of the total outer diameter and the total mass of the large and small gears are the objective function.

The diameter $d_1$ and mass $V_1$ of the reference circle of the low-speed parallel shaft pinion:

$$d_1 = \left(\frac{2 \times T_{in} \times (U_1 + 1) \times 10^3}{K_f \times U_1^2 \times \varphi_1}\right)^{\frac{1}{3}}$$

$$V_1 = \frac{2 \times T_{in} \times (U_1 + 1) \times 10^3}{K_f \times U_1^2 \times \varphi_1}$$

The diameter $d_2$ and mass $V_2$ of the reference circle of the high-speed parallel shaft pinion:

$$d_2 = \left(\frac{2 \times T_{in} \times (U_2 + 1) \times 10^3}{K_f \times U_2^2 \times U_1 \times \varphi_2}\right)^{\frac{1}{3}}$$

$$V_2 = \frac{2 \times T_{in} \times (U_2 + 1) \times 10^3}{K_f \times U_2^2 \times U_1}$$

The relationship between the each stage gear ratio of the two-stage parallel shaft gear reducer and the total design transmission ratio $U$:

$$U = U_1 \times U_2$$

The objective function is established by the sum of the outer diameters of the large and small gears:

$$f_1 = d_1 \times (1 + U_1) + d_2 \times (1 + U_2)$$

The objective function is established by the sum of the total mass of the large and small gears:

$$f_2 = V_1 \times (1 + U_1^2) + V_2 \times (1 + U_2^2)$$

In the formula:
- $T_{in}$: Input torque of two-stage parallel shaft transmission system, Nm; $U$: Total transmission ratio; $U_1$: gear ratio of low-speed parallel shaft; $U_2$: high-speed gear ratio of parallel shaft; $\varphi_1$: The width-to-diameter ratio of the low-speed parallel shaft small wheel; $\varphi_2$: The width-to-diameter ratio of the high-speed parallel shaft small wheel.
### Table 1. Contact intensity factor $K_f$

| Application                               | Number of small rounds | Accuracy class | $K_f$(N/mm²) |
|-------------------------------------------|------------------------|----------------|--------------|
| Turbine driven generator                  | $10^{10}$              | high           | 2.76         |
| Generator-driven compressor               | $10^9$                 | high           | 2.07         |
| Transmission system for general industrial use | $10^6$              | Medium to high | 5.52         |
| Large industrial drive system             | $10^6$                 | medium         | 3.45         |
| Helicopter drive system (helical gear)    | $10^9$                 | high           | 5.86         |
| Helicopter transmission (planet)          | $10^9$                 | high           | 4.14         |
| Auto transmission (helical gear)          | $4 \times 10^7$        | medium         | 6.20         |
| Automotive main reducer (straight tooth)  | $4 \times 10^6$        | medium         | 8.96         |

### Table 2. Recommended values of aspect ratio

| Layout form         | The two supports are arranged symmetrically with respect to the pinion | The two supports are arranged asymmetrically with respect to the pinion | Pinion cantilever arrangement |
|---------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------|------------------------------|
| Aspect ratio        | 0.9-1.4 (1.2-1.9)                                                       | 0.7-1.15 (1.1-1.65)                                                     | 0.4-0.6                      |

When both the large and small gears are hard toothed surfaces, the width-to-diameter ratio is take the lower limit value. If both are soft toothed surfaces or the gears are soft toothed surfaces, the upper limit values in the table may be taken;

The value in the register is used for Double-helical gear. At this time, the tooth width $b$ is the total width of the Double-helical gears;

For metal cutting machine, if the transmitted power is not large, the aspect ratio can be 0.2;

Non-metal gears take 0.5-1.2.

#### 2.2.2. Two stage planets

Planetary gear reducer is a widely used deceleration device, which has the characteristics of large transmission ratio, high efficiency, low noise and compact structure. The main components of its structure are sun gear, planet gear, ring gear and so on. When designing a two-stage planetary gear reducer, the volume of the two-stage planetary gear is mainly related to the diameter of the ring gear. Therefore, The objective function is the sum of the diameter or the mass of the reference circle. In the calculation process, the influence of the contact intensity factor, uneven load factor, number of planet wheels and other factors were considered.

Diameter $d_1$ and mass $V_1$ of the reference circle of the first stage planetary ring gear:

$$d_1 = \frac{2 \times T_m \times u_1^3 \times K_f \times Y_1 \times 10^3}{K_f \times (U_1 - 1) \times N_p_1 \times \phi_1}$$  \hspace{1cm} (8)

$$V_1 = \frac{2 \times T_m \times u_1^2 \times K_f \times Y_1 \times 10^3}{K_f \times (U_1 - 1) \times N_p_1}$$  \hspace{1cm} (9)

The diameter $d_2$ and mass $V_2$ of the reference circle of the second stage planetary ring gear:
The relationship between the two-stage gear ratio and the total design transmission ratio $U$:

$$U = (U_1 + 1) \times (U_2 + 1)$$  \hfill (12)$$

The objective function is established by the total outer diameter of the two-stage planetary gear transmission system:

$$f_3 = d_1 + d_2$$  \hfill (13)$$

The objective function is established by the total mass of the two-stage planetary gear transmission system:

$$f_4 = V_1 + V_2$$  \hfill (14)$$

In the formula:

$T_{in}$: Input torque of two-stage planetary gear transmission system, Nm; $U$: Total design transmission ratio; $U_1$: Gear ratio of the first stage planetary gear; $U_2$: Gear ratio of the second stage planetary gear; $\phi_1$: The width-diameter ratio of the first-stage planetary ring gear; $\phi_2$: The width-diameter ratio of the second-stage planetary ring gear; $N_{p1}$: Number of first stage planetary gears; $N_{p2}$: Number of second-stage planetary gears; $K_{\gamma1}$: Uneven load factor of the first stage planetary gear; $K_{\gamma2}$: Uneven load factor of the second stage planetary gear.

Due to the influence of manufacturing errors and installation errors, the load of each planetary wheel is not uniform. The uneven load factor corresponding to the number of different planetary wheels is shown in the following table.

**Table 3. Uneven load factor of recommended values**

| Application conditions | Uneven load coefficient of different number of planetary gears | AGMA accuracy level |
|------------------------|-------------------------------------------------------------|---------------------|
|                        | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 1                      | 1.16 | 1.23 | 1.32 | 1.35 | 1.38 | 1.47 | 1.60 | -   | A7 or less |
| 2                      | 1  | 1  | 1.25 | 1.35 | 1.44 | 1.47 | 1.60 | 1.61 | A5-A6     |
| 3                      | 1  | 1  | 1.15 | 1.35 | 1.44 | 1.47 | 1.60 | 1.61 | Above A4  |
| 4                      | 1  | 1  | 1.08 | 1.12 | 1.16 | 1.20 | 1.23 | 1.26 | A4 or less |

condition 1 is the low-speed and heavy-duty gear system; condition 2 is the commercial marine gear transmission system; condition 3 and 4 are the higher speed, such as steam turbine, generator, wind power equipment, etc.

2.3. constraint condition

The optimal design method can be adopted for the gear ratio distribution of electric vehicle reducer. Design variables, objective functions, and constraints constitute an optimization model to obtain the optimal solution. According to the mechanical design manual and relevant experience, in the process of gear ratio distribution, the following constraints should be met.

$$1 \leq \frac{d_1}{d_2} \leq 1.2$$  \hfill (15)$$
3. Applications
Take the two-stage parallel shaft gear transmission system as an example, the total transmission ratio is 10. According to the above distribution formula, the gear ratio distribution is carried out, and the result is obtained by MATLAB software. Explore the relationship between the outer diameter or mass of the electric vehicle reducer and the first stage gear ratio, and the results are shown in the figure below.

![Figure 1. Take the sum of outer diameter as optimization target](image)
![Figure 2. Take the sum of mass as optimization target](image)

From the above figure, it can be concluded that the outer diameter or mass of the transmission system will decrease first and then increase with the increase of the first stage gear ratio when the other parameters are kept unchanged. Therefore, the reasonable gear distribution has an impact on the outer diameter and mass of the two-stage parallel shaft system.

4. Conclusion
The distribution formula of gear ratio of electric vehicle reducer is verified with matlab analysis. When the objective function is the smallest outer diameter or the lightest mass of the electric vehicle, the relationship between the first stage gear ratio and the objective function is obtained. From the example, the objective function has a minimum value with the change of the first stage gear ratio. This paper is to provide a distribution method for gear ratio distribution of two-stage typical gear transmission structure in the electric vehicle reducer.

Reference
[1] Liu Ziran, Yu Yuxi. The optimal distributing of star-differential reducer gear ratio[J]. *Modern Manufacturing Engineering*, 2012(10):136-138.
[2] Kihan Kwon, Minsik Seo, Seungjae Min. Efficient multi-objective optimization of gear ratios and motor torque distribution for electric vehicles with two-motor and two-speed powertrain system[J]. Applied Energy, 2020, 259.

[3] Wang Rong, Guo Juncai, Liang Jingjing. An Optimal Allocation of Three Reducer Drive Based on Matlab[J]. The World of Inverters, 2019(1):126-128.

[4] LiLi Zhu, GuangXin Wang. Distribution on Transmission Ratio of Cylindrical Gear Reducer[J]. Advanced Materials Research, 2013, 2109.

[5] Maruti Patil, P Ramkumar, K Shankar. Multi-objective optimization of the two-stage helical gearbox with tribological constraints[J]. Mechanism and Machine Theory, 2019, 138.

[6] Zhou Qicai, Li jing, Yu jing. Comparison between conventional design and optimal design for planetary gear reducers[J]. Chinese Journal of Construction Machinery, 2007, 5(3).

[7] Liang Huaqi, Huang Taisong. The optimal design of transmission ratio distribution on double cylinder gear deceleration case[J]. Machinery Design & Manufacture, 2003(05):82-83.

[8] Fan Limei. Multi-objective reliability optimal design of the helical gear transmission[J]. Machinery Design & Manufacture, 2008(08):15-16.

[9] Bao Heyun, Zhu Rupeng. Optimization design of 2-stage star gear train based on Matlab[J]. Journal of Engineering Design, 2005(04):232-235+239.

[10] Zhao Youhong, Fu Wei, Luo Xianguang. Multi-goal Optimum Design for Two-grade Helical Cylindrical Gear Retarder[J]. Natural Science Journal of Xiangtan University, 2003(02):81-84.

[11] Xiong Yong. Optimal Design of Double-stage Wheel Hub Reducer System based on Fuzzy Theory[J]. Journal of Mechanical Transmission, 2017(07):186-193

[12] Tang Z Q, Ma D P, Gong X W. Transmission Ratio Optimization of Two-Speed Pure Electric Vehicle[J]. Applied Mechanics & Materials, 2014, 722:271-275.

[13] Wang Y M, Du C Q, Li X N, et al. Transmission Ratio Optimization of Electric Vehicle Powertrain[J]. Applied Mechanics & Materials, 2013, 397-400:987-992.

[14] Dengfeng L. New Method of Transmission Ratio Distribution on Cylinder Gear Reducer[J]. Journal of Mechanical Transmission, 2013.

[15] Gao Yanjun, Shi Chunjuan. Optimal Design of Two-level Cone-cylinder Gear Reducer Based on MATLAB [J]. HENAN SCIENCE, 2015, 33(07):1154-1158.

[16] Bingzhao Gao, Qiong Liang, Yu Xiang, Lulu Guo, Hong Chen. Gear ratio optimization and shift control of 2-speed I-AMT in electric vehicle[J]. Mechanical Systems and Signal Processing, 2015, 50-51.

[17] Zhao X, Wang J, Zhang Y, et al. Gear ratio optimization of two-speed transmission in electric off-road vehicle[C]. 2019 Chinese Control And Decision Conference (CCDC). 2019.

[18] Ahssan M R, Ektesabi M M, Gorji S A. Electric Vehicle with Multi-Speed Transmission: A Review on Performances and Complexities[J]. Sae International Journal of Alternative Powertrains, 2018, 7(2).

[19] Darle W Dudley. HANDBOOK OF PRACTICAL GEAR DESIGN[M]. California: CRC PRESS, 1983.

[20] Miler Daniel, et al. Multi-objective spur gear pair optimization focused on volume and efficiency[J]. Mechanism & Machine Theory, 2018.