Chiral Four-Dimensional $N=1$ Supersymmetric Type IIA Orientifolds from Intersecting D6-Branes

Mirjam Cvetič\textsuperscript{1,2,3}, Gary Shiu\textsuperscript{1,2} and Angel M. Uranga\textsuperscript{2}

\textsuperscript{1}Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104-6396, USA
\textsuperscript{2}Theory Division, CERN, CH-1211 Geneva 23, Switzerland
\textsuperscript{3}Center for Applied Mathematics and Theoretical Physics, University of Maribor, SI-2000 Maribor, Slovenia
(July 19, 2001)

Abstract

We construct $N = 1$ supersymmetric four-dimensional orientifolds of type IIA on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at angles. The use of D6-branes not fully aligned with the O6-planes in the model allows for a construction of many supersymmetric models with chiral matter, including those with the Standard Model and grand unified gauge groups. We perform a search for realistic gauge sectors, and construct the first example of a supersymmetric type II orientifold with $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and three quark-lepton families. In addition to the supersymmetric Standard Model content, the model contains right-handed neutrinos, a (chiral but anomaly-free) set of exotic multiplets, and diverse vector-like multiplets. The general class of these constructions are related to familiar type II orientifolds by small instanton transitions, which in some cases change the number of generations, as discussed in specific models. These constructions are supersymmetric only for special choices of untwisted moduli. We briefly discuss the supersymmetry breaking effects away from that point. The M-theory lift of this general class of supersymmetric orientifold models should correspond to purely geometrical backgrounds admitting a singular $G_2$ holonomy metric and leading to four-dimensional M-theory vacua with chiral fermions.
Four-dimensional $N = 1$ supersymmetric Type II orientifolds (and references therein) provide a class of consistent open-string solutions which in turn could shed light on the physics of strongly coupled heterotic string models. From a more phenomenological viewpoint, they also provide a natural setup to localized gauge interactions in lower dimensional sub-manifolds of space-time, namely the D-branes, and lead to brane-world models. Unfortunately, the constraints (arising from Ramond-Ramond (RR) tadpole cancellation conditions) on four-dimensional models are rather restrictive in the supersymmetric case, and lead to relatively unrealistic gauge sectors and matter contents in the simplest constructions.

Among the several discrete or continuous deformations of the simplest models that have been explored, often mainly motivated by the search for standard model like theories, we may recall

- Blowing-up of orientifold singularities \cite{12,13}: The resulting models are not described by a free world-sheet conformal field theory (CFT), hence the space-time spectrum can only be computed using field theory techniques. In compact setups, the $\mathbb{Z}_3$ orientifold of \cite{1} is the only example in which this analysis has been performed \cite{12} (it is interesting to point out that results in D-brane models differ in nature from those of perturbative heterotic orbifolds \cite{11}).

- Locating the branes at different point in the internal orbifold/orientifold space (see for instance \cite{8,13,11}). In a T-dual picture these possibilities correspond to the turning of continuous or discrete Wilson lines. Explicit examples of Wilson lines have been extensively considered, see for instance \cite{6,8,9,16} for discrete Wilson lines, and \cite{5,6,15} for continuous ones. For partially successful attempts at supersymmetric model building using Wilson lines see e.g., \cite{11}.

- Introduction of discrete values for the (Neveu-Schwarz-Neveu-Schwarz) NS-NS $B$ field \cite{7,17}. The novel feature of this discrete deformation is that the rank of the gauge group is reduced. This can also be understood in the T-dual picture as tilting the tori, thereby requiring fewer orientifold planes (see Appendix A) and hence fewer D-branes.

- Introduction of gauge fluxes in the D-brane world-volumes (see \cite{18} for an early discussion). In the supersymmetric context this has been explored in six-dimensional examples in \cite{13}. In a T-dual version, models with gauge fluxes correspond to type II
orientifolds with D-branes intersecting at angles. Supersymmetric models in six and four dimensions [20–22] have been constructed in the situation with D6-branes parallel to the O6-planes in the model [1]. Such constructions however lead to non-chiral models. (In six dimensions, chiral supersymmetric models with D-branes not parallel with the O-planes have been obtained in [24], and implicitly in [19]).

Recently it has been realized that the RR tadpoles cancellation conditions turn out to be much less constraining if one gives up the requirement of supersymmetry. This observation in [25,26] was exploited in the context of type IIB orientifold model building in e.g., [27–29]. It has also allowed for a construction of type IIA orientifold and orbifold models with D-branes at supersymmetry breaking angles and realistic theories, with the Standard Model gauge group (or simple extensions thereof) and three quark-lepton families [24,30–35] (for other proposals for realistic model-building using D-branes, without a specific string construction or in a non-compact set-up, see [36,28,37]).

Despite this remarkable success, non-supersymmetric models have more complicated dynamics than supersymmetric ones, hence are less understood. In particular, tree level flat directions are generically lifted by quantum corrections, leading to involved stabilization problems. Also, the models contain uncancelled NS-NS tadpoles which force to redefine the background geometry [38], as is obvious, for example, from the existence of non-zero cosmological constant in the models. For these and other reasons, it is more reassuring to restrict to supersymmetric model building at the stringy level. However, even for such models, the eventual supersymmetry breaking required in any model attempting to describe realistic low-energy physics, will lead to these or analogous omnipresent issues, like the cosmological constant problem.

The purpose of this paper is explore the construction of four-dimensional \( N = 1 \) supersymmetric type IIA orientifolds with D6-branes intersecting at angles, and leading to chiral gauge sectors. The simplest models satisfying those requirements are orientifolds of toroidal type IIA orbifolds \( T^6/Z_N \) or \( T^6/(Z_N \times Z_M) \), with D6-branes not parallel to the O6-planes. In this paper we focus on the \( Z_2 \times Z_2 \) orbifold, for which the general pattern of the chiral spectrum is simple enough. Extension to other orientifolds should be more involved, but

---

1A non-compact version of these theories has been studied in [23].

2Some NS-NS tadpoles may be partially avoided in models where the corresponding moduli are frozen to discrete values, as in [35].
otherwise straightforward.

We succeeded in constructing the first $N = 1$ supersymmetric model with Standard Model gauge group and three quark-lepton families in this setup. (The letter version that summarizes the key results for this model appeared in [39].) Beyond the structure of the minimal supersymmetric Standard Model (MSSM), the model contains some additional gauge factors, right-handed neutrinos, a chiral set of fields with exotic Standard Model gauge quantum numbers, and diverse vector-like multiplets. Despite its lack of fully realistic features, it provides the first construction of phenomenologically appealing supersymmetric compactifications in the setup of intersecting brane worlds. Moreover, a particularly nice feature of such construction is that supersymmetry avoids the hierarchy problem generically present in the (otherwise realistic) models with D6-branes in [30,31,24,33,34]. Namely, our models will have a relatively large string scale (close to the 4d Planck scale) and not very large internal dimensions ($\ell \simeq (\text{TeV})^{-1}$ at the largest).

We also discuss a number of interesting general results involved in the construction. We describe the cancellation or previously unnoticed mixed $U(1)$-gravitational anomalies, present in some orientifold models (even without orbifold projection). They are canceled by a Green-Schwarz mechanism mediated by untwisted RR closed string fields, similar to that in [31] for mixed non-Abelian anomalies.

The models under consideration, in a T-dual version, correspond to chiral supersymmetric versions of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ type IIB orientifold, with D9-branes with magnetic fluxes. We show that such models are related to the familiar non-chiral $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold in [2] by the T-dual of a four-dimensional version of the small instanton transitions [40]. These transitions correspond, in the original picture, to recombinations of D6-branes wrapped on intersecting cycles. This provides a explicit picture of the transitions, which allows to reproduce interesting phenomena. In particular, we explicitly construct transitions where a toy model with Standard Model-like gauge group changes the number of generations, in a manner reminiscent of the chirality changing phase transitions in [41,42].

Finally, the models upon consideration are supersymmetric only for specific choices of the untwisted (complex structure) moduli. Namely, the condition to preserve supersymmetry [43] is that the different D6-branes and O6-planes are related by rotations in $SU(3)$. This implies certain constraints on the angles among objects, in the three complex planes in $T^6$.

\[\text{3} \text{Notice the } N = 1 \text{ supersymmetric D3-brane realistic model in Section 4.3 in [28] in a different context.}\]
For fixed wrapped three-cycles, they imply constraints on the untwisted complex structure moduli, so that supersymmetric solutions exist generically only at isolated points in moduli space. We briefly discuss the interesting question of the dominant supersymmetry breaking effects upon small departures from the supersymmetric points, and of supersymmetry restoration by vacuum restabilization.

The paper is organized as follows. In Section II we describe the construction of $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds with D6-branes at angles, and discuss their spectrum, independently of supersymmetry. In Section III we formulate the conditions to preserve 4d $\mathcal{N} = 1$ supersymmetry. We present several explicit examples, including a four-family Standard Model like theory, and a four-family $SU(5)$ grand unified theory (GUT) model. The construction of three-family models turns out to be very constrained, but we succeed in building a model with SM gauge group (times additional factors) and three quark-lepton families (plus additional exotic and vector-like matter). The requirement of three families demands introducing tilted complex structure in one two-torus, and supersymmetry requires choosing specific ratios for the radii in the remaining two-tori.

In Section IV we discuss how our chiral supersymmetric models are related to the familiar non-chiral $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold [2]. In a T-dual version they are connected through small instanton transitions. In the picture of branes at angles the transition corresponds to a recombination of 3-cycles on which the D6-branes wrap. Such processes can mediate phase transitions changing the number of chiral families, as we illustrate in a toy construction with the Standard Model gauge group. In Section V we briefly discuss the supersymmetry breaking effect in the open string sector when the ratios of radii in the two-tori are chosen slightly away from the supersymmetry-preserving values. These closed string moduli couple as Fayet-Iliopoulos (FI) terms, hence they generate D-term supersymmetry breaking terms whose magnitude is related to a deviation of the untwisted moduli away from their supersymmetric values. Finally, in Section VI we comment on more formal applications of our constructions, which provide examples whose lift to M-theory corresponds to compact 7-dimensional spaces admitting $G_2$ holonomy metrics, and leading to chiral four-dimensional gauge theories. Section VII contains our final comments.
II. MODEL BUILDING RULES FOR $\mathbb{Z}_2 \times \mathbb{Z}_2$ ORIENTIFOLD WITH BRANES AT ANGLES

In this Section we provide the rules to construct consistent orientifolds, and to obtain the spectrum of massless states. We state these rules independently of supersymmetry, so they are valid for non-supersymmetric model building as well.

Our starting point is type IIA theory on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, with generators $\theta$, $\omega$ associated to the twists $v = (\frac{1}{2}, \frac{-1}{2}, 0)$ and $w = (0, \frac{1}{2}, \frac{-1}{2})$, hence acting as

$$
\theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) \\
\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)
$$

(1)

where $z_i$ are complex coordinates in the $T^6$. For simplicity, we consider the case of factorizable $T^6$.

We mod out this theory by the orientifold action $\Omega R$, where $\Omega$ is world-sheet parity, and $R$ acts by

$$
R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3)
$$

(2)

The model contains four kinds of O6-planes, associated to the actions of $\Omega R$, $\Omega R\theta$, $\Omega R\omega$, $\Omega R\theta\omega$, as shown in Figure [1]. We will not be specially interested in the closed string sector, which can anyway be computed using standard techniques. For the case of rectangular two-tori, it is as in [2], by T-duality.

In order to cancel the corresponding RR crosscap tadpoles, we introduce D6-branes wrapped on three-cycles, which we consider factorized, namely obtained as the product of one-cycles in each of the three two-tori. Specifically, we consider $K$ stacks of $N_a$ D6-branes, $a = 1, \ldots, K$, wrapped on the $(n^i_a, m^i_a)$ cycle in the $i^{th}$ two-torus. We also need to include the images of these under the elements in the orientifold group. Assuming for simplicity that our two-tori are rectangular (extension to tilted two-tori is easy, and is discussed below), we include $N_a$ D6-branes with wrapping numbers $(n^i_a, -m^i_a)$. For branes on top of the O6-planes we also count branes and their images independently.

For future convenience, we define the homology class of the corresponding 3-cycles by

$$
[\Pi_a] = \prod_{i=1}^{3} (n^i_a [a_i] + m^i_a [b_i])
$$

(3)

and analogously for $[\Pi'_a]$. We also define the homology classes of the cycles wrapped by the O6-planes, which for rectangular tori read
\[ [\Pi_{\Omega R}] = [a_1] \times [a_2] \times [a_3] \quad [\Pi_{\Omega R\omega}] = -[a_1] \times [b_2] \times [b_3] \]

\[ [\Pi_{\Omega R\theta}] = -[b_1] \times [a_2] \times [b_3] \quad [\Pi_{\Omega R\theta\omega}] = -[b_1] \times [b_2] \times [a_3] \]  

(4)

and we define \([\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\theta}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}]\).

Concerning the orbifold projections, let us focus on the case where the branes pass through fixed points of the orbifold actions, hence \(\theta\) and \(\omega\) map each stack of branes to itself (and with the same 3-cycle orientation). Extension to other cases is simple, and briefly mentioned in Section IIIA. To specify the action of the different actions on the Chan-Paton indices of the branes, for each stack of D6\(_a\)-branes, and their \(\Omega R\) images, denoted D6\(_{a}'\)-branes, we introduce the Chan-Paton actions

\[ \gamma_{\theta,a} = \text{diag} \left( i1_{N_a/2}, -i1_{N_a/2}, -i1_{N_a/2}, i1_{N_a/2} \right) \]

\[ \gamma_{\omega,a} = \text{diag} \left[ \begin{pmatrix} 0 & 1_{N_a/2} \\ -1_{N_a/2} & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1_{N_a/2} \\ -1_{N_a/2} & 0 \end{pmatrix} \right] \]

\[ \gamma_{\Omega R,a} = \begin{pmatrix} 1_{N_a/2} & 0 \\ 0 & 1_{N_a/2} \end{pmatrix} \]

(5)

The actions for the orbifold group form a projective representation, which corresponds to the choice of closed string sector usually known as without discrete torsion \(\text{[4]}\).

The models are constrained by RR tadpole cancellation conditions. Orientifolds by \(\Omega R\) action do not generate twisted crosscaps \(\text{[20, 22]}\), hence the twisted disk tadpoles should vanish. The simplest way to accomplish this is to choose traceless Chan-Paton matrices, as done above\(\text{[5]}\). Cancellation of untwisted RR tadpoles simply requires the cancellation of D6-brane and O6-plane charge, namely

\(4\)Our D6-branes wrap special lagrangian 3-cycles (A-branes), hence they carry projective representations in the model without discrete torsion \(\text{[14]}\). In a T-dual (mirror) version, the system is mapped to a set of D9-branes with holomorphic bundles (B-branes) in a model with discrete torsion, which again carry projective representations \(\text{[15]}\) (for orientifolds with discrete torsion, see \(\text{[16]}\)).

\(5\)As pointed out in \(\text{[24]}\), and implicitly used in \(\text{[19]}\), it is possible to achieve this for non-traceless
\[
\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_a'] + (-4) \times 8 [\Pi_{O6}] = 0
\]  
(6)

It is further discussed in Appendix A, and leads to the constraints (A3).

Such choices define a consistent model, for which the resulting spectrum is discussed in the following. The results for branes at generic angles are shown in Table I.

Let us consider the aa sector (strings stretched within a single stack of D6\(_a\)-branes) which is invariant under \(\theta, \omega\), and which is exchanged with \(a' a'\) by the action of \(\Omega R\). For the gauge group, the \(\theta\) projection breaks \(U(N_a)\) to \(U(N_a/2) \times U(N_a/2)\), and the further \(\omega\) projection identifies both factors, leaving \(U(N_a/2)\). Concerning the matter multiplets, we obtain three adjoint \(N = 1\) chiral multiplets. This sector is however not \(N = 4\) since the superpotential for the adjoints \(\Phi_1, \Phi_2\) and \(\Phi_3\) reads

\[
W = \text{Tr} (\Phi_1 \Phi_2 \Phi_3 + \Phi_1 \Phi_3 \Phi_2)
\]  
(7)

instead of the \(N = 4\) commutator structure. This agrees with the result in [45] in the T-dual (mirror) picture. For branes parallel to some O6-plane the projections are identical to [2], leading to a \(USp(N_a)\) group with three \(N = 1\) chiral multiplets in the two-index antisymmetric representation (in our ‘antisymmetric’ of symplectic factors we also include the singlet).

The \(ab+ba\) sector, strings stretched between D6\(_a\)- and D6\(_b\)-branes, is invariant as a whole under the orbifold projections, and is mapped to the \(b' a' + a' b'\) sector by \(\Omega R\). The matter content before any projection would be given by \(I_{ab}\) chiral fermions in the bifundamental \((N_a, \overline{N}_b)\), where

\[
I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_{i=1}^3 (n^i_a m^i_b - m^i_a n^i_b)
\]  
(8)

is the intersection number of the wrapped cycles, and the sign of \(I_{ab}\) denotes the chirality of the corresponding fermion (our default convention is that negative intersection numbers correspond to left-handed fermions). For supersymmetric intersections, additional massless scalars complete the corresponding supermultiplet. For non-supersymmetric intersections, the masses for light scalars are as in [31].

choices if for any point fixed under the orientifold and some orbifold action the total Chan-Paton trace for the different branes passing through it cancels. We will not consider this case in the present analysis.
In principle, in performing the orbifold quotient one needs to take into account the orbifold action on the intersection point. The final result however turns out to be rather insensitive to this subtlety, as opposed to the six-dimensional case \[24\]. For an intersection point fixed under \(\theta\) and \(\omega\), the orbifold projections reduce the matter content to a bifundamental \((a, b)\) of \(U(N_a/2) \times U(N_b/2)\). For intersection points exchanged by the orbifold actions, for instance two points fixed under \(\omega\) but exchanged under \(\theta\), we should consider just one point (the other being merely its \(\theta\)-image) and not impose the \(\theta\) projection. The resulting fields are two bifundamentals. Due to this compensation, the total number of fields in the \(ab\) sector is simply \(I_{ab}\) chiral fermions in the \((a, b)\) of \(U(N_a/2) \times U(N_b/2)\) (plus scalars, which fill out supermultiplets in the supersymmetric case).

A similar effect takes place in \(ab' + b'a\) sector, for \(a \neq b\), where the final matter content is given by \(I_{ab'}\) chiral fermions in the bifundamental \((a, b)\).

Finally, let us consider the \(aa' + a'a\) sector. In this case, the orbifold action on the intersection point turns out to be crucial. At an intersection point with angle \(\theta_i\) in the \(i^{th}\) two-torus states are labeled, in the bosonized formulation (see \[31\]), by a vector \((r_1 + \theta_1, r_2 + \theta_2, r_3 + \theta_3, r_4)\), where \(r_i = Z, Z + \frac{1}{2}\) in the NS, R sectors, respectively, and \(\sum_i r_i = \text{odd due to the GSO projection}\), and \(r_4 = -1/2, +1/2\) corresponds to respective left-handed and right-handed fermions (in our default convention). For an intersection point invariant under \(\theta\) and \(\omega\), the eigenvalues of such state under \(\theta\), \(\omega\) and \(\Omega R\) are \(\exp(2\pi i r \cdot v), \exp(2\pi i r \cdot w)\), and \(-1\), where recall \(v = (1, -1, 0, 0)/2, w = (0, 1, -1, 0)/2\) are the twist vectors. The projections on the Chan-Paton factors are

\[
\lambda = e^{2\pi i r v} \gamma_{\theta,6} \gamma^{-1}_{\theta,6'} \\
\lambda = e^{2\pi i r w} \gamma_{\theta,6} \gamma^{-1}_{\theta,6'} \\
\lambda = -\gamma_{\Omega R} \lambda^T \gamma^{-1}_{\Omega R} \quad (9)
\]

Before the orientifold projection, one gets a chiral fermion in the bifundamental \((N_a/2, N_a/2')\), regardless of the \(\theta, \omega\) eigenvalues of the state. The orientifold projection, however, distinguishes the different cases and leads to a two-index antisymmetric representation of \(U(N_a/2)\), except for states with \(\theta\) and \(\omega\) eigenvalue \(+1\), where it yields a two-index symmetric representation.

Now consider points not fixed under some orbifold element, say two points fixed under \(\omega\), and exchanged by \(\theta\). Then one simply keeps one point, and does not impose the \(\omega\) projection. Equivalently, one considers all possible eigenvalues for \(\omega\), and applies the above rule to read off whether the symmetric or the antisymmetric representation survives.
It is possible to give a closed formula for the precise chiral matter content in this sector, which basically follows from cancellation of non-Abelian anomalies. The final result for the net number of symmetric and antisymmetric representations in the $aa'$ sector is

$$n_{\Box} = -\frac{1}{2}(I_{aa'} - \frac{4}{2k}I_{a,O6})$$

$$n_{\square} = -\frac{1}{2}(I_{aa'} + \frac{4}{2k}I_{a,O6})$$

(10)

where $k$ is the number of tilted tori and $I_{aa'} = [\Pi_a] \cdot [\Pi'_a]$, $I_{a,O6} = [\Pi_a] \cdot [\Pi_{O6}]$. Notice that the definitions (4) should be modified in the obvious way for tilted tori. The result (10) is then general. Clearly, getting the non-chiral piece requires the full computation of the spectrum.

| Sector   | Representation                      |
|----------|-------------------------------------|
| $aa$     | $U(N_a/2)$ vector multiplet          |
|          |                                     |
|          | 3 Adj. chiral multiplets             |
| $ab + ba$| $I_{ab}$ ($\Box_a, \Box_b$) fermions |
| $ab' + b'a$| $I_{ab'}$ ($\Box_a, \Box_b$) fermions |
| $aa' + a'a$| $-\frac{1}{2}(I_{aa'} - \frac{4}{2k}I_{a,O6})$ fermions |
|          |                                     |
|          | $-\frac{1}{2}(I_{aa'} + \frac{4}{2k}I_{a,O6})$ fermions |

TABLE I. General spectrum on D6-branes at generic angles (namely, not parallel to any O6-plane in all three tori). The spectrum is valid for tilted tori. The models may contain additional non-chiral pieces in the $aa'$ sector and in $ab$, $ab'$ sectors with zero intersection, if the relevant branes overlap. In supersymmetric situations, scalars combine with the fermions given above to form chiral supermultiplets.

---

A sketch of the anomaly argument is as follows. Tadpole cancellation (3) implies that $\sum_b N_b I_{ab} + \sum_b N_b I_{ab'} - \frac{32}{2k}I_{a,O6} = 0$. The first two members, for $b \neq a$ give (minus two times) the $SU(N_a/2)$ anomaly due to $ab$ and $ab'$ fields. The rest, which equals $N_a I_{aa'} - \frac{32}{2k}I_{a,O6}$ must correspond to (minus two times) the anomaly in the $aa'$ sector, which is given by $(-2) \times (n_{\Box} (N_a/2 + 4) + n_{\square} (N_a/2 - 4))$. Equating both for arbitrary $N_a$ yields (10).
III. CONSTRUCTION OF SUPERSYMMETRIC MODELS

In this Section we turn to the construction of $N = 1$ supersymmetric models. The condition that the system of branes preserve the $N = 1$ supersymmetry unbroken in the closed string sector amounts, following [13], to requiring that each stack of D6-branes is related to the O6-planes by a rotation in $SU(3)$. More specifically, denoting by $\theta_i$ the angles the D6-brane forms with the horizontal direction in the $i^{th}$ two-torus, supersymmetry preserving configurations must satisfy

$$\theta_1 + \theta_2 + \theta_3 = 0 \quad (11)$$

For fixed wrapping numbers $(n^i, m^i)$, the condition translates into a constraint on the ratio of the two radii on each torus. For rectangular tori, denoting $\chi_i = (R_2/R_1)_i$, with $R_2, R_1$ the vertical resp. horizontal directions, the constraint is

$$\arctan\left(\chi_1 \frac{m_1}{n_1}\right) + \arctan\left(\chi_2 \frac{m_2}{n_2}\right) + \arctan\left(\chi_3 \frac{m_3}{n_3}\right) = 0 \quad (12)$$

The modification for tilted tori is straightforward and will be discussed later.

It is possible to find sets of D6-branes solving the tadpole conditions and preserving supersymmetry for some choice of $\chi_i$. A simple example with non-trivial angles in all three tori is provided in Section III C. Clearly, as the number of branes at angles increases, the constraints to preserve supersymmetry get more involved and there may not exist solutions to the $\chi_i$ for a given set of RR tadpole-free wrapping numbers.

In order to simplify the supersymmetry conditions, and our search for realistic models, we will consider a particular Ansatz for the kind of D6-branes in our configuration. We will consider that the D6-branes, not parallel to any O6-plane, will have angles (with respect to the O6-plane) of the form $(\theta_1, \theta_2, 0)$, $(\theta_1, 0, \theta_3)$ or $(0, \theta_2, \theta_3)$. Such an angle structure makes the supersymmetry conditions relatively simple. An additional advantage is that having the branes parallel to some O6-plane helps in avoiding chiral matter in $aa'$ sectors (although not completely in all cases). It is interesting to observe that, despite the fact that our brane system is the composition of three different “six-dimensional” configurations, the Ansatz is rich enough to allow for a construction of chiral models, and moreover yields realistic

---

Notice that an overall rescaling of any two-torus leaves the conditions unaffected. This agrees with the expected property that supersymmetric features of A-branes are independent of Kähler moduli, and depend on complex structure moduli alone.
structures, as we show in the remainder of this Section. Further exploration of more general models is left for future research. In the following, we turn to the construction of several explicit, potentially phenomenologically viable models.

A. Four-family model

The general structure of the chiral spectrum in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ models is relatively similar to that of the “un-orbifolded” models. A simple consequence of this observation for the construction of standard model like theories is that, following the argument in [30], the net number of left-handed quarks is even if all tori are rectangular [8].

| Type | $N_a$ | $(n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3)$ |
|------|------|--------------------------------------------------|
| $A_1$ | 6+2 | $(1,1) \times (1,-2) \times (1,0)$ |
| $A_2$ | 2 | $(1,0) \times (1,0) \times (1,0)$ |
| $B_1$ | 4 | $(1,0) \times (1,2) \times (1,-1)$ |
| $B_2$ | 8 | $(1,0) \times (0,1) \times (0,-1)$ |
| $C_1$ | 2 | $(1,2) \times (1,0) \times (1,-2)$ |
| $C_2$ | 8 | $(0,1) \times (1,0) \times (0,-1)$ |

TABLE II. D6-brane configuration for the four-family model

In this Section, we consider the case of rectangular tori, as an illustration and warm-up, and present a model with the Standard Model like group and four generations. The model we present is by no means unique; one can construct rather straightforwardly other four-family models along the lines we discussed here. Let us consider branes wrapping around the 3-cycles shown in Table II. The model preserves supersymmetry if $\chi_1 = 2 \chi_2 = \chi_3$.

The 8 D6-branes labeled $A_1$ are split into two parallel but not overlapping stacks of 6 and 2 branes, giving rise to $U(3) \times U(1)$ gauge group. This can be achieved by placing them at different positions in one of the two-tori.

---

8This assumes that $U(3)$ and $U(2)$ arise from generic D6-branes. Another possibility would be that avoided the $SU(2)$ factor is obtained as $USp(2)$ from D6-branes on top of O6-planes.
| Sector       | \( U(3) \times U(2) \times USp(2) \times USp(8) \times USp(8) \) | \( Q_3 \) | \( Q_1 \) | \( Q_2 \) | \( Q'_1 \) | \( Q_Y \) | Field   |
|--------------|---------------------------------------------------------------|--------|--------|--------|--------|-------|---------|
| \( A_1B_1 \) | \( 4 \times (3, \overline{3}, 1, 1, 1) \)                   | 1      | 0      | -1     | 0      | -1/6  | \( Q_L \) |
|              | \( 4 \times (1, \overline{3}, 1, 1, 1) \)                   | 0      | 1      | -1     | 0      | -1/2  | \( L \)  |
| \( A_1B_2 \) | \( (3, 1, 1, 8, 1) \)                                        | 1      | 0      | 0      | 0      | -\( 1/3 \), -2/3 | \( \mathcal{U}, \mathcal{D} \) |
|              | \( (1, 1, 1, 8, 1) \)                                        | 0      | 1      | 0      | 0, -1  | \( 1/3 \), -2/3 | \( \mathcal{U}, \mathcal{D} \) |
| \( A_1C_1 \) | \( 4 \times (\overline{3}, 1, 1, 1) \)                      | -1     | 0      | 0      | 1      | -1/6  | \( \nu, \mathcal{E} \) |
|              | \( 4 \times (1, 1, 1, 1) \)                                  | 0      | -1     | 0      | 1      | 1/2   | \( \nu, \mathcal{E} \) |
| \( A_1C_2 \) | \( 2 \times (\overline{3}, 1, 1, 1, 1) \)                    | -1     | 0      | 0      | 0      | 1/3   | \( U, D \) |
|              | \( 2 \times (1, 1, 1, 8) \)                                  | 0      | -1     | 0      | 0      | 0, 1  | \( \nu, \mathcal{E} \) |
| \( B_1C_1 \) | \( 4 \times (1, 2, 1, 1, 1) \)                              | 0      | 0      | 1      | -1     | 0     | \( H_U, H_D \) |
| \( B_1C_2 \) | \( 2 \times (1, 2, 1, 1, 8) \)                              | 0      | 0      | 1      | 0      | \( \pm 1/\sqrt{2} \) | \( H_U, H_D \) |
| \( B_2C_1 \) | \( 2 \times (1, 1, 1, 1, 1) \)                              | 0      | 0      | 0      | 1      | \( \pm 1/\sqrt{2} \) | \( H_U, H_D \) |
| \( A_1C'_1 \) | \( 12 \times (\overline{3}, 1, 1, 1, 1) \)                   | -1     | 0      | 0      | -1     | -1/6  | \( \nu, \mathcal{E} \) |
|              | \( 12 \times (1, 1, 1, 1, 1) \)                             | 0      | -1     | 0      | -1     | 1/2   | \( \nu, \mathcal{E} \) |
| \( B_1C'_1 \) | \( 12 \times (1, 2, 1, 1, 1) \)                             | 0      | 0      | 1      | 1      | 0     | \( H_U, H_D \) |
| \( A_1A'_1 \) | \( 2 \times (3, 1, 1, 1) \)                                  | -2     | 0      | 0      | 0      | -1/3  | \( \nu, \mathcal{E} \) |
|              | \( 2 \times (6, 1, 1, 1, 1) \)                              | 2      | 0      | 0      | 0      | 1/3   | \( \nu, \mathcal{E} \) |
|              | \( 2 \times (1, 1, 1, 1, 1) \)                              | 0      | 2      | 0      | 0      | 1     | \( \nu, \mathcal{E} \) |
| \( B_1B'_1 \) | \( 2 \times (1, 1, 1, 1) \)                                  | 0      | 0      | 2      | 0      | 0     | \( \nu, \mathcal{E} \) |
|              | \( 2 \times (1, \overline{3}, 1, 1, 1) \)                    | 0      | 0     | -2     | 0      | 0     | \( \nu, \mathcal{E} \) |

**TABLE III.** Chiral open string spectrum for the four-family model. For convenience we have changed our default convention, so that positive intersections give left handed fermions. The \( U(1) \) are the overall \( U(1) \) motion of the \( A_1 \) branes (split into groups of 6 and 2), \( B_1 \) and \( C_1 \) branes respectively. For clarity, we have not listed the \( aa \) sectors. We distinguish the 2 and the 3 of \( U(2) \) in order to keep track of their \( U(1) \) charges.
Note that one of these two $U(1)$’s, namely $Q_3 - 3Q_1$, is actually a generator within the $SU(4)$ part of the $U(4)$ when the $6 + 2$ branes coincide. This ensures that this $U(1)$ is non-anomalous (see [31,34] for $U(1)$ anomalies in this context), and moreover do not have the linear $B \wedge F$ couplings which make some of the non-anomalous $U(1)$ massive (as pointed out in [34]). We will make use of this fact in seeking for the hyper-charge in the model.

The chiral open string spectrum is tabulated in Table III. For clarity, we have displayed the spectrum when the D6-branes labeled $B_1$, $B_2$, $C_2$ are on top of some O6-planes. Therefore, the corresponding gauge groups are $USp(4)$, $USp(8)$ and $USp(8)$ respectively. In a more generic situation, we can move these $D6$-branes (and their images under $\theta, \omega$ and $\Omega R$) away from the O6-planes, in a way consistent with the orbifold and orientifold projections. The basic unit that can be moved away from the O6-planes is 4 D6-branes and their 4 orientifold images. In the effective field theory, the generic model corresponds to Higgsing of the $USp(4n)$ group to $U(1)^n$, and decomposing the fundamental of $USp(4n)$ as

$$4n = 2 \times (\pm 1, 0, \ldots, 0) + 2 \times (0, \pm 1, \ldots, 0) + \ldots 2 \times (0, \ldots, 0, \pm 1)$$

(13)

We will actually be interested in the generic model when these $D6$-branes are away from the O6-planes, in order to generate additional $U(1)$’s to enter the hyper-charge generator. The gauge group is broken to $SU(3) \times SU(2)$ plus a number of $U(1)$ factors. Let $Q_4$, $Q_{SB}$, $Q'_{SB}$, $Q_{SC}$ and $Q'_{SC}$ be the $U(1)$ generators arising from moving the $B_1$, $B_2$ and $C_2$ branes away from the O6-planes respectively. It is easy to check that these $U(1)$’s are automatically non-anomalous and massless, consistent with the fact that they arise from some non-Abelian gauge groups. Hence, we can form the non-anomalous and massless linear combination

$$Q_Y = \frac{1}{6} Q_3 - \frac{1}{2} Q_1 + \frac{1}{2} (Q_{SB} + Q'_{SB}) + \frac{1}{2} (Q_{SC} + Q'_{SC})$$

(14)

The remaining $U(1)$’s (except those arising from non-Abelian generators) have $B \wedge F$ couplings and become massive. The spectrum for this generic model is easily obtained from that in the Table by splitting the fundamental representations of the symplectic factors as explained above.

The charge (14) gives a good candidate for hyper-charge in this model, as can be seen from the charge assignments for different fields, shown in the Table. In particular, the left-handed quarks and leptons come from $A_1B_1$. Four net families of up and down antiquarks and right-handed leptons and neutrinos, along with some vector-like pairs, arise from the $A_1C_2$ and $A_1B_2$ sectors. There are many candidates for the Higgs fields in the supersymmetric Standard Model, from the $B_1C_2$ and $B_2C_1$ sectors. In addition, there are
exotic chiral (but anomaly-free) sets of fields, plus vector-like multiplets under the Standard Model.

In the Table above we have not computed the non-chiral pieces of the $ab$, $ab'$ and $aa'$ spectrum, because we can get rid of them by a simple mechanism. Non-chiral matter arises whenever two-branes are parallel in some complex plane. Locating them at different positions in that plane makes such a non-chiral matter massive. This can be done consistently with the orbifold projection, as follows. Consider a stack of $2N$ branes mapped to itself under $\theta$ and $\omega$. Moving them off in e.g., the first plane amounts to splitting them in two stacks of $N$ branes, fixed under $\omega$ and exchanged by $\theta$. The choice $\gamma_\omega = i1_N$ for one stack and $\gamma_\omega = -i1_N$ for the other is consistent, and leads to exactly the same chiral spectrum as before the motion. This is not surprising since the deformation is continuous and cannot modify the chiral structure. From the effective field theory viewpoint, this amounts to turning on a nonzero vacuum expectation value (vev) for the singlet part in the $U(N)$ adjoint multiplets in the $aa$ sector; the non-chiral multiplets get massive due to non-trivial superpotentials, which are easy to obtain although we do not discuss them explicitly.

In the construction above, and the subsequent ones we will assume we are performing this operation to avoid phenomenologically undesirable non-chiral matter.

B. Three-family model

Following [24], the even generation number problem can be solved by considering some tori to be tilted. As discussed in [17], the tilting parameter is discrete by the orientifold symmetry, and it can take only one non-trivial value. This mildly modifies the closed string sector, and in particular implies the existence of fewer O6-planes in the model. Concerning the D6-brane sector, a 1-cycle $(n^i_a, m^i_a)$ along a tilted torus is mapped to $(n^i, -m^i - n^i)$, $(n^i, -m^i - n^i)$. It is convenient to define $\tilde{m}^i = m^i - \frac{1}{2}n^i$, and label the cycles as $(n^i, \tilde{m}^i)$. The tadpole cancellation conditions are computed in Appendix A for the case of tilting just the third two-torus, and lead to (A6), which are naturally expressed in terms of the redefined labels. It is also easy to check that intersection numbers are

$$I_{ab} = (n^1_a m^1_b - n^1_b m^1_a) \times (n^2_a m^2_b - n^2_b m^2_a) \times (n^3_a \tilde{m}^3_b - n^3_b \tilde{m}^3_a)$$

Due to the smaller number of O6-planes in tilted configurations, RR tadpole conditions are very constraining for more than one tilted torus. Centering on tilting just the third torus, the search for theories with $U(3)$ and $U(2)$ gauge factors carried by branes at angles and three left-handed quarks, turns out to be very constraining, at least within our Ansatz. We have
found essentially a unique solution, with D6-brane configuration with wrapping numbers $(n^i_a, \tilde{m}^i_a)$ given in Table IV. The configuration is supersymmetric for $\chi_1 : \chi_2 : \chi_3 = 1 : 3 : 2$.

| Type | $N_a$ | $(n^1_a, m^1_a) \times (n^2_a, m^2_a) \times (n^3_a, \tilde{m}^3_a)$ |
|------|-------|------------------------------------------------------------------|
| $A_1$ | 8     | $(0, 1) \times (0, -1) \times (2, \tilde{0})$                   |
| $A_2$ | 2     | $(1, 0) \times (1, 0) \times (2, \tilde{0})$                   |
| $B_1$ | 4     | $(1, 0) \times (1, -1) \times (1, \frac{3}{2})$               |
| $B_2$ | 2     | $(1, 0) \times (0, 1) \times (0, \tilde{1})$                   |
| $C_1$ | 6+2   | $(1, -1) \times (1, 0) \times (1, \frac{1}{2})$               |
| $C_2$ | 4     | $(0, 1) \times (1, 0) \times (0, \tilde{1})$                   |

TABLE IV. D6-brane configuration for the three-family model.

The 8 D6-branes labeled $C_1$ are split in two parallel but not overlapping stacks of 6 and 2 branes, hence lead to a gauge group $U(3) \times U(1)$. For simplicity we may choose them to pass through different $\mathbb{Z}_2$ fixed points in some two-torus (alternatively, we may locate them at generic positions in one two-torus, as described in the previous Section).

It is interesting to observe that one of these two $U(1)$’s is actually a generator within the $SU$ part of the $U(4)$ gauge group that would arise for coincident branes. This feature ensures that this $U(1)$ is automatically non-anomalous, and moreover does not have $B \wedge F$ couplings. This $U(1)$ will turn out to be crucial in the appearance of hyper-charge in this model.

For convenience it is also useful to consider the 8 D6-branes labeled $A_1$ to be away from the O6-planes in all three complex planes. One is left with two D6-branes that can move independently (hence give rise to a group $U(1)^2$), plus their $\theta, \omega$ and $\Omega R$ images. The spectrum is computed applying standard rules, even though they differ slightly from our explicit rules above. In the effective theory, the generic model corresponds to a “Higgsing” of $USp(8)$ down to $U(1)^2$. A possibility to get the chiral spectrum below, with a minimum amount of effort is to merely decompose the chiral spectrum for D6-branes on top of the O6-plane, with respect to the surviving group $U(1)^2$.

The open string spectrum is tabulated in Table IV. The generators $Q_3, Q_1$ and $Q_2$ refer to the $U(1)$ factor within the corresponding $U(n)$, while $Q_8, Q'_8$ are the $U(1)$’s arising from the
| Sector | $U(3) \times U(2) \times USp(2) \times USp(2) \times USp(4)$ | $Q_3$ | $Q_1$ | $Q_2$ | $Q_8$ | $Q_8'$ | $Q_Y$ | $Q_8 - Q_8'$ | Field |
|--------|-------------------------------------------------|------|------|------|-------|-------|------|-------------|-------|
| $A_1B_1$ | $3 \times 2 \times (1, \overline{2}, 1, 1, 1)$ | 0 | 0 | -1 | ±1 | 0 | ±$\frac{1}{2}$ | ±1 | $H_U, H_D$ |
| $A_1C_1$ | $3 \times 2 \times (1, \overline{2}, 1, 1, 1)$ | 0 | 0 | -1 | 0 | ±1 | ±$\frac{1}{2}$ | ±1 | $H_U, H_D$ |
| $A_1C_1$ | $2 \times (\overline{3}, 1, 1, 1, 1)$ | -1 | 0 | 0 | ±1 | 0 | $\frac{1}{3}, -\frac{2}{3}$ | 1, -1 | $U, D$ |
| $A_1C_1$ | $2 \times (\overline{3}, 1, 1, 1, 1)$ | -1 | 0 | 0 | 0 | ±1 | $\frac{1}{3}, -\frac{2}{3}$ | -1, 1 | $U, D$ |
| $A_1C_1$ | $2 \times (1, 1, 1, 1, 1)$ | 0 | -1 | 0 | ±1 | 0 | 1, 0 | 1, -1 | $E, \nu_R$ |
| $A_1C_1$ | $2 \times (1, 1, 1, 1, 1)$ | 0 | -1 | 0 | 0 | ±1 | 1, 0 | -1, 1 | $E, \nu_R$ |
| $B_1C_1$ | $(3, \overline{2}, 1, 1, 1)$ | 1 | 0 | -1 | 0 | 0 | $\frac{1}{6}$ | 0 | $Q_L$ |
| $B_1C_1$ | $(1, \overline{2}, 1, 1, 1)$ | 0 | 1 | -1 | 0 | 0 | $-\frac{1}{2}$ | 0 | $L$ |
| $B_1C_2$ | $(1, 2, 1, 1, 4)$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $B_1C_1'$ | $(3, 1, 2, 1, 1)$ | 1 | 0 | 0 | 0 | 0 | $\frac{1}{6}$ | 0 | 0 |
| $B_1C_1'$ | $(1, 1, 2, 1, 1)$ | 0 | 1 | 0 | 0 | 0 | $-\frac{1}{2}$ | 0 | 0 |
| $B_1C_1'$ | $2 \times (3, 2, 1, 1, 1)$ | 1 | 0 | 1 | 0 | 0 | $\frac{1}{6}$ | 0 | $Q_L$ |
| $B_1C_1'$ | $2 \times (1, 2, 1, 1, 1)$ | 0 | 1 | 1 | 0 | 0 | $-\frac{1}{2}$ | 0 | $L$ |
| $B_1B_1'$ | $2 \times (1, 1, 1, 1, 1)$ | 0 | 0 | -2 | 0 | 0 | 0 | 0 | 0 |
| $B_1B_1'$ | $2 \times (1, 3, 1, 1, 1)$ | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |

**TABLE V.** Chiral Spectrum of the open string sector in the three-family model. Notice that we have not included the $aa$ sector, even though it is generically present in the model. As explained in the text, the non-chiral pieces in the $ab, ab'$ and $aa'$ sectors are generically not present.
The next two columns provide the charges under the only non-anomalous \( U(1)'s \), namely \( Q_8 - Q'_8 \) and

\[
Q_Y = \frac{1}{6} Q_3 - \frac{1}{2} Q_1 + \frac{1}{2} (Q_8 + Q'_8)
\]

It is easy to check that the \( B \wedge F \) couplings for these combinations also vanish, hence the corresponding \( U(1)'s \) remain in the massless spectrum. The combination (16) plays the role of hyper-charge in the present model. The theory contains three Standard Model families, plus one exotic chiral (but anomaly-free) set of fields, plus multiplets with vector-like quantum numbers under the SM group.

In this model, the quarks, leptons and Higgs fields live at different intersections. Hence, the Yukawa couplings \( Y_{ijk} \) among the Higgs fields and two fermions arise from a string world-sheet of area \( A_{ijk} \) (measured in string units) stretching between the three intersections and hence \( Y_{ijk} \sim \exp(-A_{ijk}) \). Note that one family of quarks and leptons do not have renormalizable couplings with the Higgs fields, due to the uncanceled \( Q_2 \) charges, and the only chiral multiplets which carry opposite \( Q_2 \) charges are charged under the weak \( SU(2) \). In addition to the above couplings, there are generically couplings among \( aa, ab \) and \( ba \) fields, and among the \( aa \) fields. This is also the case in all the previous models of intersecting branes in the literature). These couplings are not exponentially suppressed. It would be interesting to examine if these couplings may pose phenomenological challenges for these models.

Let us emphasize that this is the first realization of a semi-realistic spectrum in the context of fully supersymmetric type II orientifold constructions. Clearly, there exist different variants, obtained by changing the additional branes not directly involved in the Standard Model structure. The Standard Model core structure which we have found is however relatively unique, at least within our Ansatz for the angles on branes. It would be interesting to explore such variants to eliminate part of the additional vector-like matter, or the extra exotics.

C. A supersymmetric GUT example

The present setup allows us to consider new possibilities in model building. For instance, we may consider constructing a supersymmetric grand unified model. This possibility is not available in standard type IIB orientifolds, due to the difficulty in getting adjoint representations to break the GUT group to the Standard Model (see e.g., [10]). In the context
of intersecting branes, GUT models have been constructed in e.g., \[30\], but they are non-supersymmetric, hence suffer from a severe hierarchy problem.

In this Section we show it is extremely straightforward to build GUT models in the present setup, as we illustrate with a simple four-family example. Consider the following configuration of branes

| $N_a$ | $((n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3))$ |
|------|---------------------------------------------------------------|
| 10 + 6 | $(1, 1) \times (1, -1) \times (1, 1/2)$ |
| 16 | $(0, 1) \times (1, 0) \times (0, -\frac{1}{2})$ |

which is supersymmetric for $\arctan \chi_1 - \arctan \chi_2 + \arctan(\chi_3/2) = 0$. We consider that the first set of 16 branes is split in two parallel stacks of 10 and 6. The resulting spectrum is

$$U(5) \times U(3) \times USp(16)$$

$$3(24 + 1, 1, 1) + 3(1, 8 + 1, 1) + 3(1, 1, 119 + 1)$$

$$4(\overline{10}, 1, 1) + (5, 1, 16) + 4(\overline{3}, \overline{3}, 1) + (1, 3, 16) + 4(1, 3, 1)$$

(17)

where we have ignored for simplicity that one of the $U(1)$’s is actually anomalous and massive. The model is a four-family $SU(5)$ GUT, with additional gauge groups and matter content. Notice that turning on suitable vev’s for the adjoint multiplets the model corresponds to splitting the $U(5)$ branes. This provides a geometric interpretation of the GUT Higgsing to the Standard Model group upon splitting $U(5) \rightarrow U(3) \times U(2) \times U(1)$. Also, it provides the construction of a new Standard Model with four quark-lepton families with correct quantum numbers. In this framework hyper-charge is given by the linear combination familiar in grand unification. It is important to point out that since all the Standard Model gauge groups would arise from branes wrapped on parallel but otherwise identical cycles, this string construction provides a natural initial condition for the unification of gauge couplings. Hence, such models provide a stringy embedding of the basic philosophy in traditional GUT.

Clearly the above model can be improved by complicating the configuration, but we refrain from doing so. Our purpose is to illustrate it is possible to build such GUT’s with reasonable numbers of families, and adjoint Higgs multiplets. Notice that a generic feature of these GUT constructions is that the adjoints are exact moduli in the model, since they are associated with the brane motion in transverse space. This absence of Higgs self-interactions leads, upon breaking to Standard Model gauge group, to the matter in the adjoint representation of the Standard Model factors. This is very reminiscent of what happened in heterotic string GUT’s \[38\].
IV. SMALL INSTANTON TRANSITIONS

In this Section we briefly discuss how our general class of models with branes at angles is connected to the familiar non-chiral $Z_2 \times Z_2$ type IIB orientifold \cite{2}. In that respect, it is useful to recall the T-duality between configurations of branes at angles and branes carrying gauge magnetic fluxes (see e.g., \cite{30}, and \cite{49} for a recent discussion). In particular, a brane wrapped at angles on cycles $(n^i, m^i)$ correspond to a brane fully wrapped on the two-tori (in fact, multi-wrapped $\prod_i n^i$ times), with total magnetic flux $m^i/n^i$ in the $i^{th}$ two-torus.

It follows that in the $Z_2 \times Z_2$ IIA model, D6-branes along the O6-planes are mapped to D9-, D5$_i$-branes in the $Z_2 \times Z_2$ type IIB orientifold in \cite{2}. On the other hand, our models with D6-branes at non-trivial angles in two directions T-dualize to configurations where some D9-branes carry fluxes in two complex directions. The corresponding non-zero instanton numbers endow the D9-branes with D5$_i$-brane charge (see \cite{19} for discussion).

Specifically, $N$ D6-branes along e.g., $(1,0) \times (n_2, m_2) \times (n_3, m_3)$ T-dualize to a bound state of $N n_2 n_3$ D9-branes and $N m_2 m_3$ D5$_1$-branes. Consequently, such models contain a smaller number of pure D5$_i$-branes. It is clear that these models are connected to the basic $Z_2 \times Z_2$ orientifold by small instanton transitions \cite{40}, in which some of the D5$_i$-branes are dissolved into the D9-branes, and expand to fill the corresponding four-torus in a uniform manner. Obviously, the intermediate steps in the process involve non-constant self-dual field-strength gauge configurations, which are not described by a free world-sheet CFT. However, the final configuration, with constant flux, admits such a description.

The fattening of small instantons can be followed using field theory techniques, in the region of small instanton size. In fact, it is an interesting exercise (left to the reader) to verify that in the six-dimensional context there exist flat directions which connect the $U(16)^2$ type IIB $T^4/Z_2$ model in \cite{22,23} with the models in \cite{19}. Such flat directions represent the effective field theory description of the small instanton transition, analogous to the flat space discussion in \cite{40}. Obviously, the field theory analysis is perturbative in the vevs, and hence valid only close to the small instanton point. Hence it cannot be used to follow the

---

9 A useful hint is that e.g. in the $U(13) \times U(4) \times U(3)$ example in \cite{19}, this gauge group is embedded in the original $U(16)^2$ as follows from the decomposition $U(16)^2 \supset U(13) \times U(4) \times U(3)^5 \supset U(13) \times U(4) \times U(3)_D$, where $U(3)_D$ is the diagonal combination of the five $U(3)$ factors. This breaking to the diagonal accounts for the (Landau level) multiplicities in the final model with fluxes.
flat direction for a finite distance, namely until the instanton has become a uniform flux.

In the picture of branes at angles, the process corresponds to recombining D6-branes wrapping different intersecting cycles. In the intermediate steps the recombined cycle is complicated, hence it is difficult to describe it in detail. After flattening it out (when possible) preserving its homology class, it corresponds to D6-branes in a cycle at non-trivial angles. These recombinations have been considered in the non-supersymmetric case \[31,32,52\], where they are triggered by tachyons. In our present supersymmetric context, they rather correspond to flat directions in moduli space.

Notice the striking feature that using these transitions one can generate chiral models starting from non-chiral ones. The situation is reminiscent of the chirality changing small instanton transitions in \[41,42\]. Consequently, one can also use small instanton transitions to relate different chiral models within our class, differing in their D6-brane wrapping numbers, and with different chiral content (and/or different gauge group). In fact, in the following we discuss a specific Standard Model toy example where the number of families changes by such a process.

| Type     | \(N_a\) | \((n^1_a, m^1_a) \times (n^2_a, m^2_a) \times (n^3_a, m^3_a)\) |
|----------|---------|---------------------------------------------------------------|
| Spectator| 6+2     | \((1, 0) \times (1, 1) \times (1, -1)\)                      |
|          | 4       | \((1, 0) \times (1, 0) \times (1, 0)\)                      |
|          | 8       | \((1, 0) \times (0, 1) \times (0, -1)\)                     |
|          | 16      | \((0, 1) \times (1, 0) \times (0, -1)\)                     |
| Before   | 4       | \((1, 1) \times (1, -1) \times (1, 0)\)                     |
|          | 12      | \((0, 1) \times (0, -1) \times (1, 0)\)                     |
| After    | 4       | \((1, 2) \times (1, -2) \times (1, 0)\)                     |

**TABLE VI.** Wrapping numbers/fluxes for the small instanton transition. The upper piece of the Table lists branes unaffected by the transition. The lower pieces describe the branes existing before and after the transition. The first set of 8 branes is split in 6 + 2 to yield gauge group \(U(3) \times U(1)\).

Let us consider the \(Z_2 \times Z_2\) models, with rectangular two-tori, described in Table \[VI\]. Consider the initial model, where D6-brane wrapping numbers are given by the first six rows.
in the Table. In the T-dual picture the model is understood in terms of instanton bundles as follows. Consider the model without magnetic fluxes in \cite{4}, with group $USp(16)^4$, the four factors arising from D9- and D5$_i$-branes. Consider dissolving 8 D5$_1$- and 4 D5$_3$-branes (and images) as instantons within a $U(1) \times U(1)$ sub-group of $USp(16)_9$. In the decomposition

$$USp(16)_9 \supset U(4) \times U(2) \times USp(4)$$

the two $U(1)$ generators which acquire non-zero flux correspond to the $U(1)$’s within $U(4)$ and $U(2)$, respectively. The surviving group from the D9-branes is the commutant of the gauge background, namely $U(4) \times U(2) \times USp(4)$. The final spectrum can be computed directly in string theory in the flux or angle picture, or in field theory using the index theorem (see \cite{18} for a discussion). The chiral spectrum is

"$U(4)$" $\times$ $U(2) \times USp(12) \times USp(4) \times USp(8) \times USp(16)$

$$2 \times (4, \overline{2}; 1, 1, 1, 1) + (4, 1; 12, 1, 1, 1) + (\overline{4}, 1; 1, 1, 1, 16) +$$

$$(1, \overline{2}; 1, 1, 8, 1) + (1, 2; 1, 1, 1, 16)$$

where the symplectic groups arise from (a) D9-branes without flux, and (b) D5$_i$-branes not dissolved as fat instantons. The quotation marks for $U(4)$ denote that we are actually interested in splitting it as $U(3) \times U(1)$ (by separating the D6-branes, or equivalently by introducing D9-brane Wilson lines). The resulting model corresponds to a two-family Standard-like model. One can even obtain a sensible hyper-charge by splitting the $USp(4n)$ symplectic factors as $U(1)^n$ so as to generate additional $U(1)$’s. In fact the linear combination

$$Q_Y = \frac{1}{6} Q_3 - \frac{1}{2} Q_1 + \frac{1}{2} (Q_{12} + Q_4 + Q_8)$$

(\text{where } Q_3, Q_1 \text{ arise from } "U(4)"’, and Q$_{12}$, Q$_4$, Q$_8$ arise as diagonal combinations of the $U(1)$’s from symplectic factors) is automatically anomaly-free and massless, and plays the role of hyper-charge in the above model. The net chiral content with respect to Standard Model interactions is

$$2(3, 2)_{1/6} + 2(\overline{3}, 1)_{1/3} + 2(\overline{3}, 1)_{-2/3} + 2(1, 2)_{-1/2} + 2(1, 1)_{1} + 2(1, 1)_{0}$$

Namely two standard quark-lepton generation (plus right handed neutrino).

In the angle picture, let us consider the 4 D6-branes along $(1, 1) \times (1, -1) \times (1, 0)$ (and their $\Omega R$ images) combine with the 12 along $(0, 1) \times (0, -1) \times (1, 0)$ (and images) to give
4 D6-branes along $(1, 2) \times (1, -2) \times (1, 0)$ (and images). This process is possible since the total homology class is conserved, and is triggered by vev’s for scalars in strings stretching between the stacks involved. The final model has D6-branes with wrapping numbers given by the first four and the last rows in Table [VI].

Let us describe the process in the T-dual picture. The final model corresponds, in terms of the underlying $USp(16)^4$, to the following. Consider dissolving 8 D5$_1$- and 16 D5$_3$-branes (and images) as instantons in a $U(1) \times U(1)$ sub-group corresponding to the splitting [(18)]. The resulting flux structure corresponds to the final model. Hence the transition connecting both models amounts to dissolving 12 additional D5$_3$-branes (and images) as instantons within the same $U(1)$ sub-group where the previous 8 were already dissolved. In particular this shows that the unbroken D9-brane gauge group has identical generators in both cases. However the chiral fermion content may differ, due to the presence of additional flux modifying the index of the Dirac operator in the internal space (or the intersection number in the angle picture). The final spectrum can be computed with string theory or index theory techniques, and reads

$$"U(4)" \times U(2) \times USp(4) \times USp(8) \times USp(16)$$

$$6 \times (4, \overline{2}; 1, 1, 1, 1) + 2 \times (4, 2; 1, 1, 1, 1) + (\overline{4}, 1; 1, 1, 16) + 2 \times (1, \overline{2}; 1, 1, 8, 1) + 2 \times (1, 2; 1, 1, 1, 16)$$

Upon splitting $U(4) \rightarrow U(3) \times U(1)$ the model corresponds to an eight-family Standard Model. In this case hyper-charge can be obtained by splitting the symplectic factors into $U(1)$'s and considering the anomaly-free and massless combination

$$Q_Y = \frac{1}{6}Q_3 - \frac{1}{2}Q_1 + \frac{1}{2}(Q_4 + Q_8)$$

The chiral spectrum with respect to Standard Model gauge interactions is

$$8(3, 2)_{1/6} + 8(3, 1)_{1/3} + 8(\overline{3}, 1)_{-2/3} + 8(1, 2)_{-1/2} + 8(1, 1)_{1} + 8(1, 1)_{0}$$

Namely, there are eight standard quark-lepton families. Hence the process of dissolving additional D5-branes in the gauge bundle on the D9-branes leads to phase transitions changing the number of chiral families.

A minor difficulty in the above transition is that the hyper-charge generator is not the same in both theories. This can be avoided by removing the $Q_{12}$ term in [(24)], although this yields to exotic hyper-charges in the initial model. Leaving these subtle points aside,
we would like to emphasize that it is remarkable that one can describe quite explicitly these transition in relatively realistic models.

We have succeeded in showing that small instanton transitions can mediate changes in the number of families in a model. Moreover, the T-dual interpretation of these processes as recombination of cycles provides a useful tool in analyzing these transitions in the type II orientifold setup. We believe these techniques can be useful in the study interesting phenomena in a simple geometric setup, and hence are complementary to other realizations of these transitions \[41,42\].

V. D-TERM SUPERSYMMETRY BREAKING

As mentioned above, the general models we have considered are supersymmetric for specific choices of the untwisted moduli \(\chi_i\). In this Section we briefly consider the main supersymmetry breaking effects on the open string sector as one moves away from the special supersymmetric values for \(\chi_i\).

On general grounds, the complex structure moduli \(\chi_i\) are expected to couple open string modes in our D6-branes on 3-cycles (A-branes) as Fayet-Iliopoulos (FI) terms. This is mirror to the statement that Kähler moduli couple to B-branes as FI-terms, a familiar situation for D-branes at singularities \[53\]. (Note that related techniques have been employed in the blowing-up procedure of the type IIB orientifold singularities \[12\]). This has appeared in a related context in \[54\], and in situations where there are D-branes with \((B_{NS}\) or equivalently magnetic field) fluxes in \[55\]. Following the latter, we expect the corresponding FI-terms to be proportional to the deviation from the supersymmetric situation, in particular we expect the terms in the effective \(D = 4\) action

\[
\sum_a \int d^4x \left( \theta_a^1 + \theta_a^2 + \theta_a^3 \right) D_a
\]

(26)

where \(\theta_a^i\) is understood as a function of \(\chi_i\) for fixed wrapping numbers \((n_a^i, m_a^i)\). For instance, for square tori

\[
\theta_a^i = \frac{m_a^i}{n_a^i} \chi_i.
\]

(27)

The FI-term vanishes for the supersymmetric situation \(\theta_1 + \theta_2 + \theta_3 = 0\), hence in general it is proportional to the deviation from this case. It is easy to see that this term reproduces the leading order splitting between scalar and fermion masses, as one would obtain from
the string computation. Namely, in the $ab$ sector, chiral fermions remain massless at tree level, while their scalar partners obtain a mass proportional to $\delta \theta = \sum_i (\theta^i_a - \theta^i_b)$.

In supersymmetric models, the familiar arguments in [56] relate the existence of Green-Schwarz anomaly cancellation mechanism with the existence of FI terms, controlled by the partners of the fields mediating the GS interactions. Their precise determination also requires knowledge about their Kähler potential, which should be easily determined for the untwisted complex structure moduli in our models. However, we skip the derivation of the FI terms, and briefly discuss the physics arising from Eq. (26).

The turning of FI terms when untwisted moduli are shifted from the supersymmetric values actually does not automatically imply breaking of supersymmetry. As is familiar in heterotic constructions, some scalar fields may acquire vev’s so as to make the D-term vanish. Hence, supersymmetry would be restored in the shifted vacuum. An important difference with respect to heterotic models is that the FI-terms are not related to the dilaton, and can be tuned at will by tuning the untwisted moduli $\theta^i$. The physics behind this process is that the original D6-brane configuration is no longer supersymmetric for the new choice of untwisted moduli (since the angles are changed), so some intersecting D6-branes recombine into a smooth 3-cycle which is supersymmetric. This recombination is described by the vev acquired by certain scalar fields at intersections.

It is an interesting question whether supersymmetry can always be restored in this fashion. Despite the lack of a general argument, we strongly suspect that this is the case, at least in compact models. Notice that however non-compact models allow for supersymmetry breaking by this FI-term mechanism (see [54] for discussion).

In order to provide a simple illustrative example, let us consider the string-GUT model in Section III C, where there is only one relevant set of angles, namely those formed by the $U(8)$ branes with the horizontal axes, denoted by $\theta_i$ henceforth. There are two relevant kinds of scalars, those arising at the intersections between the $U(8)$ branes and their images, $\phi_{aa'}$, and between the former and the $USp(16)$ branes, $\phi_{ab}$. The corresponding chiral multiplets carry opposite charges with respect to the single $U(1)$ symmetry in the model, hence the D-term has the schematic structure

$$D = 2|\phi_{aa'}|^2 - |\phi_{ab}|^2 + (\theta_1 + \theta_2 + \theta_3)$$

10 This is analogous to the observation in [27] for FI-terms and twisted moduli in standard type IIB orientifolds.
Hence, for deformations such that \( \sum_i \theta_i > 0 \), the fields \( \phi_{ab} \) become tachyonic and acquire a vev, restoring supersymmetry. This corresponds to recombining the \( USp(16) \) and the \( U(8) \) branes. For \( \sum_i \theta_i < 0 \), it is \( \phi_{aa'} \) which acquire vev’s to restore supersymmetry, triggering the recombination of the \( U(8) \) branes and their images.

It would be interesting to explore these processes in more detail, both in their effective field theory and in their geometric description.

VI. THE RELATION TO COMPACT SINGULAR \( G_2 \) MANIFOLDS

In this Section we briefly outline a different (more formal) aspect of our models. Recently there has been a lot of interest in the study of the dynamics of M-theory on 7-dimensional manifolds \( X_7 \) admitting a \( G_2 \) holonomy metrics \[58-60\]. The interest stems from the fact that such compactifications lead to four-dimensional \( N = 1 \) supersymmetric field theories, with gauge interactions determined by the singularity structure of \( X_7 \). Moreover, such constructions have provided a geometric interpretation \[59\] of the duality between type IIA configurations with D6-branes on the special lagrangian 3-cycle in the deformed conifold, and type IIA on the resolved conifold, without D6-branes but with RR 2-form fluxes \[61\]. These results have been extended in diverse directions (see, e.g., \[62\]), and suggest interesting connections with gauge theory dynamics and string duality.

Topological manifolds admitting a \( G_2 \) metric are not easy to characterize, as opposed to, e.g., spaces admitting a \( SU(n) \) holonomy metric, which can be characterized by a topological condition (the vanishing of the first Chern class). The explicit construction of \( G_2 \) metrics is difficult, and has only been achieved in a few non-compact examples constructed in \[63\] and the more recent generalizations in \[64\]. (For applications to regular configurations of M-theory with \( N = 1 \) supersymmetry, see e.g., \[65\] and references therein.) However, string theory duality provides a simple strategy to obtain topological spaces which admit a \( G_2 \) metric, without constructing it explicitly. Basically, any type IIA configuration preserving \( D = 4 \ N = 1 \) supersymmetry, and including at most D6-branes and O6-planes, will lift to an M-theory compactification on a \( G_2 \) manifold (see \[66,67\] for nice discussions). The topological information can be used to obtain interesting qualitative features of these theories.

\[11\] The comments in this Section lie outside the main line in this paper. We advice readers with more phenomenological interests to safely skip it.
In this respect, the $D = 4$ $N = 1$ supersymmetric type IIA orientifolds with D6-branes and O6-planes at angles, studied in [21,22] and in this paper, correspond to M-theory compactifications on $G_2$ manifolds. In configurations where the RR 7-form charges are locally cancelled (namely, 2 D6-branes and 2 images on top of each O6-plane in the configuration), the M-theory lift is remarkably simple. The M-theory circle is constant over the base space $B_6$, leading to a total variety $(B_6 \times S^1)/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ flips the coordinate parametrizing the M-theory circle, and acts on $B_6$ as an antiholomorphic involution (hence changing the holomorphic 3-form to its conjugate). This is analogous to the discussion in [68].

Unfortunately, such models lead generically to non-chiral spectra, in the sense that even though one obtains chiral multiplets, they arise in real representations of the gauge group. On the other hand, configurations with D6-branes away or not fully aligned with the O6-planes would lead to more involved M-theory lifts. However, they are of great interest since, as shown in our general constructions, they lead to chiral gauge theories.

Certainly, there exist simpler lines of attack to obtain chirality out of $G_2$ singular spaces. In particular, the simplest type IIA supersymmetric configuration leading to chiral fermions is simply two intersecting D6-branes in flat space, related by an $SU(3)$ rotation. Its M-theory lift would correspond to a rigid 7-dimensional $G_2$ singularity, and is the basic building block for engineering chiral theories using $G_2$ geometries; the chiral multiplet arises from M2-brane wrapping a collapsed two-cycle. In this sense, our configurations are more complicated, and what they provide is a consistent embedding of this building block singularity in a compact setup.

We expect that the generic class of models described here may exhibit some interesting phenomena in this context, in particular the existence of non-perturbative equivalences among seemingly different models, which nonetheless share the same M-theory lift, in analogy with [68]. On the other hand the type IIA transitions in which intersecting D6-branes recombine (the T-dual of the small instanton transitions) would have interesting M-theory descriptions, in which the topology of the $G_2$ space changes. It would be interesting to explore possible connections of such process with [58,69]. We hope that our explicit constructions may provide a useful laboratory to probe new ideas in this exciting development.

VII. CONCLUSIONS

In this paper, we constructed four-dimensional $N = 1$ supersymmetric type II orientifolds with branes at angles. We provided the first D-brane construction of a three-
family $N = 1$ supersymmetric vacuum solution with the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as part of the gauge structure. We have also illustrated other possibilities for model building by constructing a supersymmetric GUT model with (four) chiral families and adjoint Higgs multiplets, the first example of its kind in the orientifold setup. Although we have not discussed here, it is quite straightforward to construct other extensions of the Standard Model, such as the left-right symmetric models (e.g., the Pati-Salam type).

Even though our models are explicit string realizations of the brane-world scenario, they generically require a high string scale. In models with intersecting D6-branes, the experimental bounds on masses of Kaluza-Klein replicas of Standard Model gauge bosons imply that the internal dimension cannot be large (since there is no dimension transverse to all Standard Model branes). Hence, a large Planck mass can be generated only from a large string scale, and not from a large volume. Specifically, one obtains

$$g^2_{YM}M_P^{(4d)} = M_s \frac{\sqrt{V_6}}{V_3}$$

where $V_3$ is the volume of the cycle wrapped by the corresponding brane, and $V_6$ is the total internal volume. Moreover, large anisotropies in the internal space would generically reflect in different gauge couplings for different gauge group factors. For nearly isotropic configurations, the string scale is of the order of the Planck scale. There is however more freedom than in the traditional heterotic approach, and it could be used to lower the string scale to, e.g., $10^{16}$ GeV, a certainly desirable choice for GUT models.

Another interesting feature of this class of models with branes at angles is the structure of the Yukawa couplings. Since the quarks, leptons and Higgs fields are located at different intersections of the branes, the Yukawa couplings $Y_{ijk}$ are generically exponentially suppressed by the area $A_{ijk}$ of the string world-sheet stretching between the locations of these fields (measured in string units) \cite{22}, i.e.,

$$Y_{ijk} \sim \exp(-A_{ijk})$$

These exponential factors may provide an interesting geometrical explanation for the observed fermion masses. In order for the Yukawa couplings not to be negligibly small, the area of the string world-sheet (which is typically the compactification scale) cannot be much larger than the string length, so internal dimensions should be of the order of the string scale.

Note that in addition to the above Yukawa couplings, there are generically couplings among $aa$, $ab$ and $ba$ fields, and among $aa$ fields. These couplings are not exponentially suppressed.
Therefore, in studying the resulting phenomenology, one should examine whether they may pose phenomenologically challenges to this general class of models involving branes at angles. Hopefully, they might be useful, e.g., to get rid of the unwanted non-chiral matter in the $aa$ sector, even though we have no concrete proposal in this respect. We also note in passing that unlike the recent models in [33], our models have absolute proton stability due to symmetries (as discussed in [32]).

The basic model building rules that we have constructed, allow for the exploration of a potentially large class of supersymmetric models. In particular, to simplify the conditions from supersymmetry, we have mainly restricted our search to D6-branes at angles of the form $(\theta_1, \theta_2, 0), (\theta_1, 0, \theta_3)$ or $(0, \theta_2, \theta_3)$. It is quite remarkable that within this restricted class of models, there exist three-family Standard-like models. The three-family model is however not fully realistic, as it contains extra vector-like multiplets as well as exotic chiral matter. Nevertheless, it is possible that variants of this (or other) model(s) may eliminate these additional states, and lead to solutions closer to the Standard Model (in particular, it would be interesting to reproduce the very economical spectrum structure in [34]). Clearly, a detailed search of realistic models deserves further investigation.

In this regard, it is interesting to note that supersymmetric D6-branes with three non-trivial angles, say $(n_i, m_i) = (1, 1) \times (1, 1) \times (1, -1)$, contribute to some tadpole conditions (but not all!) with the same sign as that of the O6-planes. This implies that even though the configuration is supersymmetric, the branes can carry negative RR charges under some RR forms in the dimensional reduction on the internal space. This allows for more flexibility in satisfying the tadpole conditions, and may give more room for embedding of the Standard Model.

The general class of models with branes at angles are connected to familiar orientifolds by (the T-dual of) small instanton transitions. For instance, the three-family model we presented is connected by such a transition to the non-chiral $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold in [2]. In the picture of D6-branes at angles, the transition amounts to a recombination of the intersecting cycles, resulting in D6-branes not fully aligned with the O6-planes, and leading to chiral matter. The recombination description provides a simple setup to analyze these transitions, and their field theory interpretation in detail.

We have also discussed the main physical effects when the untwisted moduli are chosen away from the supersymmetric point. The model develops FI terms proportional to the deviation from the supersymmetric situation. The corresponding non-zero D-terms in general force some scalars to acquire vev’s and restore supersymmetry at the restabilized
vacuum. This is the field theory counterpart of a process in which the intersecting D6-branes, non-supersymmetric in the new situation, recombine and wrap a new 3-cycle which is supersymmetric in the new, deformed, complex structure.

As this general class of supersymmetric orientifold models involve only D6-branes and O6-planes, their M-theory lift correspond to compactifications on purely geometrical background, in fact, a compact, singular 7-manifold with $G_2$ holonomy. Given the recent interest in M-theory compactifications on such spaces, we expect our general class of orientifold models may lead to new insights into the construction of spaces with special holonomy leading to four-dimensional gauge theories with chiral fermions, and into new physical phenomena in such compactifications.

There are many promising avenues to explore in supersymmetric orientifolds with D6-branes at angles, both from the phenomenological and the theoretical viewpoints. We hope our results here have provided the first steps in some of these directions.

**ACKNOWLEDGMENTS**

We thank Gerardo Aldazabal, Savas Dimopoulos, Jens Erler, Gary Gibbons, Jaume Gomis, Luis Ibáñez, Paul Langacker, Hong Lü, Chris Pope, Raul Rabadán and Edward Witten for useful discussions. G.S. thanks Ralph Blumenhagen and Boris Körs for email correspondence, and Matt Strassler for sharing his insights on generation-changing transitions. We would like to thank the Theory Division at CERN (M.C. and G.S.), CAMTP, University of Maribor, Slovenia (M.C.) Amsterdam workshop on String theory (M.C.) and Benasque workshop (M.C. and A.M.U.) for hospitality during the course of the work. A.M.U. thanks M. González for kind encouragement and support. This work was supported in part by U.S. Department of Energy Grant No. DOE-EY-76-02-3071 (M.C.), in part by the Class of 1965 Endowed Term Chair (M.C.), UPenn SAS Dean’s funds (G.S.) and the NATO Linkage grant 97061 (M.C.).
APPENDIX A: R-R TADPOLE AND SUPERSYMMETRY CONDITIONS

In this Appendix we derive the tadpole cancellation conditions for the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold model of the type discussed in the main text. The results can be generalized in a rather straightforward way to other types of $\mathbb{Z}_N$ orientifold models.

As obtained in [20–22], $\Omega R$ orientifolds of type IIA toroidal orbifolds do not contain twisted crosscap tadpoles. Correspondingly, D-branes wrapped on factorized three-cycles on the six-torus (i.e. wrapped on one-cycles in each complex plane) do not generate twisted disk tadpoles. Therefore, only cancellation of untwisted RR tadpoles should be imposed, namely, the consistency conditions are the same as for a set of O6-planes and D6-branes (and their images) in the six-torus. This is simply Gauss law, the cancellation of the total charge under the RR 7-form. Such charges are proportional to the homology class of the wrapped three-cycles. Let us consider the case of rectangular two-tori, and denote $[a_i], [b_i]$ the $(1,0)$ and $(0,1)$ homology one-cycles in the $i^{th}$ two-torus. The O6-planes fixed under the orientifold actions $\Omega R, \Omega R\theta, \Omega R\omega, \Omega R\theta\omega$, shown in figure 1, carry an overall charge proportional to

$$-4 \times 8 \times ( [a_1] \times [a_2] \times [a_3] - [b_1] \times [b_2] \times [a_3] - [a_1] \times [b_2] \times [b_3] - [b_1] \times [a_2] \times [b_3] ) \quad (A1)$$

where the $-4$ is the charge of a single O6-plane (in D6-brane charge units, and as counted in the covering space), and $8$ is the number of O6-planes of each kind.

We would like to cancel this charge by introducing sets of $N_a$ D6-branes wrapped on the three-cycle defined by wrapping numbers $(n^i_a, m^i_a)$, and their orientifold images, with wrapping numbers $(n^i_a, -m^i_a)$. The total D6-brane charge is

$$\sum_a N_a \prod_{i=1}^{3} (n^i_a[a_i] + m^i_a[b_i]) + \sum_a N_a \prod_{i=1}^{3} (n^i_a[a_i] - m^i_a[b_i]) \quad (A2)$$
FIG. 2. O6-planes in the orientifold of $\mathbf{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ where the third two-tori is tilted.

The requirement that charges (A1), (A2) add up to zero yields the RR tadpole constraints

$$\sum_a N_a n_a^1 n_a^2 n_a^3 - 16 = 0$$
$$\sum_a N_a n_a^1 m_a^2 m_a^3 + 16 = 0$$
$$\sum_a N_a m_a^1 n_a^2 m_a^3 + 16 = 0$$
$$\sum_a N_a m_a^1 m_a^2 n_a^3 + 16 = 0 \quad (A3)$$

The procedure carries over in the same spirit for the case with some tilted tori. For instance, consider only the third torus is tilted, the total O6-plane RR charge is

$$-4 \times 4 \times \left\{ [a_1] \times [a_2] \times (2[a_3] - [b_3]) - [b_1] \times [b_2] \times (2[a_3] - [b_3]) \right.$$  
$$\left. - [a_1] \times [b_2] \times [b_3] - [b_a] \times [a_2] \times [b_3] \right\} \quad (A4)$$

where we have taken into account that due to the tilt there are only four O6-planes of each kind, see figure 3. The charge of sets of $N_a$ branes with wrapping numbers $(n_a^i, m_a^i)$, and their images, with wrapping numbers $(n_a^i, -m_a^i - n_a^i)$ are

$$\sum_a N_a \prod_{i=1}^{3} (n_a^i [a_i] + m_a^i [b_i]) + \sum_a N_a \prod_{i=1}^{3} (n_a^i [a_i] - (m_a^i + n_a^i) [b_i]) \quad (A5)$$

Cancellation of RR charges leads to

$$\sum_a N_a n_a^1 n_a^2 n_a^3 - 16 = 0$$
$$\sum_a N_a n_a^1 m_a^2 m_a^3 + 8 = 0$$
$$\sum_a N_a m_a^1 n_a^2 m_a^3 + 8 = 0$$
$$\sum_a N_a m_a^1 m_a^2 n_a^3 + 16 = 0 \quad (A6)$$
where we have defined \( \tilde{m}_a^i = m_a^i + \frac{1}{2} n_a^i \). The general rule is to modify tadpole conditions involving the \( m_a^i \) if the \( i^{th} \) two-torus is tilted. The modification is simply to replace \( m_a^i \rightarrow \tilde{m}_a^i \), and cut by half the corresponding crosscap contribution.

**APPENDIX B: CANCELLATION OF MIXED GRAVITATIONAL ANOMALIES**

We first consider the case of \( \Omega R \) orientifolds of type IIA on \( T^6 \) [30], and turn to the case with additional \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold projections towards the end.

Consider the \( \Omega R \) orientifold of type IIA on \( T^6 \). For simplicity we center on a six-torus factorizable into three rectangular two-tori. Other cases are worked out analogously. In order to cancel the tadpoles, we introduce sets of \( N_a \) D6-branes wrapped on 3-cycles in the homology class \( \{\Pi_a\} \), defined by the wrapping numbers \((n_a^i, m_a^i)\). We also introduce their \( \Omega R \) images, namely \( \tilde{N}_a \) branes on cycles \( \{\Pi'_a\} \), defined by wrapping numbers \((n_a^i, -m_a^i)\). For convenience, we define \( \{\Pi_{O6}\} \) the homology class associated to the 3-cycles with wrapping numbers \((1, 0)\) along each two-torus.

The RR tadpole cancellation conditions amount to cancellation of RR charge in homology, namely

\[
\sum_a N_a [\Pi_a] + \sum_{a'} N_a [\Pi_{a'}] - 32 [\Pi_{O6}] = 0 \quad (B1)
\]

The spectrum of chiral fermions is obtained from [30], and for branes on generic cycles reads

\[
\text{Multiplicity} \quad \text{Representation}
\]

\[
\begin{align*}
\{\Pi_a\} \cdot \{\Pi_b\} &\to (\mathbf{0}, \mathbf{0}) \\
\{\Pi_a\} \cdot \{\Pi'_b\} &\to (\mathbf{0}, \mathbf{0}) \\
\frac{1}{2} \{\Pi_a\} \cdot \{\Pi_{a'}\} + 4 \{\Pi_a\} \cdot \{\Pi_{O6}\} &\to \mathbf{E}_8 \\
\frac{1}{2} \{\Pi_a\} \cdot \{\Pi_{a'}\} - 4 \{\Pi_a\} \cdot \{\Pi_{O6}\} &\to \overline{\mathbf{E}_8}
\end{align*}
\]

where representations are with respect to the gauge group \( \prod_a U(N_a) \). It is a straightforward computation to check that the cubic non-Abelian anomaly vanishes, upon use of the RR tadpole condition.

As discussed in [31] in the absence of the orientifold projections, the mixed \( U(1)_a \)-\( SU(N_b)^2 \) anomaly does not vanish in general, rather it is proportional to

\[
A_{ab} = \frac{1}{2} N_a \{\Pi_a\} \cdot (\{\Pi_b\} + \{\Pi_{b'}\}) \quad (B3)
\]

33
In the toroidal models in [31] the mixed $U(1)_a$-gravitational anomalies vanishes automatically. This is not true in general for $\Omega R$ orientifolded models, where the anomaly is proportional to

$$A^{grav}_a = 24 N_a [\Pi_a] \cdot [\Pi_{O6}]$$  \hfill (B4)

Hence, for any model with branes intersecting with the $O6$-plane the anomaly does not vanish, which is the case for the examples in [30] (notice that in the orientifold models in [34] gravitational anomalies vanish due to the specific choice of D6-branes, which never intersect the $O6$-planes).

These anomalies are canceled by a Green-Schwarz mechanism mediated by untwisted RR fields, as discussed in [31] for mixed non-Abelian anomalies. Our treatment of the gravitational anomalies below is novel.

Expanding the Chern-Simons couplings for the D6-branes (and the images) and $O6$-planes (see e.g., [70])

$$\int_{D6} C \epsilon^F \sqrt{\hat{A}(R)} ; \int_{O6} C \sqrt{\hat{L}(R)}$$  \hfill (B5)

we obtain the following relevant interactions

$$\frac{1}{2} \int_{D6_a} C_3 \wedge \text{tr} (F_a \wedge F_a) ; \int_{D6_a} C_5 \wedge \text{tr} F_a$$

$$- \int_{D6_a} C_3 \wedge \text{tr} (R \wedge R) ; (-4) \times \frac{1}{2} \int_{O6} C_3 \wedge \text{tr} (R \wedge R)$$  \hfill (B6)

Operating as in [31], we introduce two dual basis of homology 3-cycles, $\{[\Sigma_i]\}$, $\{[\Lambda_i]\}$, satisfying $[\Lambda_i] : [\Sigma_j] = \delta_{ij}$, and introduce the expansions

$$[\Pi_a] = \sum_i r_{ai} [\Sigma_i] ; \hspace{0.5cm} [\Pi_a] = \sum_i p_{ai} [\Lambda_i]$$

$$[\Pi_{O6}] = \sum_i r_{i} [\Sigma_i] ; \hspace{0.5cm} [\Pi_{O6}] = \sum_i p_{i} [\Lambda_i]$$  \hfill (B7)

We define the untwisted RR fields $\Phi_i = \int_{[\Lambda_i]} C_3$ ; $B^i_2 = \int_{[\Sigma_i]} C_5$, Hodge duals in four dimensions. The four-dimensional couplings read

$$\frac{1}{2} \sum_i p_{ai} \int_{M_4} \Phi_i \text{tr} (F_a \wedge F_a) ; \quad N_a \sum_i r_{ai} \int_{M_4} B^i_2 \wedge \text{tr} F_a$$

$$- N_a \sum_i p_{ai} \int_{M_4} \Phi_i \text{tr} (R \wedge R) ; \quad \frac{1}{2} \times (-32) \sum_i p_i \int_{M_4} \Phi_i \text{tr} (R \wedge R)$$  \hfill (B8)
These couplings can be combined in GS diagrams where $U(1)_a$ couples to the $i^{th}$ untwisted field, which then couples to either two non-Abelian gauge bosons or two gravitons, and hence may cancel both kinds of mixed anomalies. The coefficients of these amplitudes, taking into account the coupling from $\Omega R$ image branes is, for the mixed non-Abelian anomaly

$$\frac{1}{2} \sum_i N_a (r_{ai} p_{bi} + r_{ai} p_{b'i} - r_{a'i} p_{bi} - r_{a'i} p_{b'i}) = N_a [\Pi_a] \cdot [\Pi_b] + N_a [\Pi_a] \cdot [\Pi_{b'}]$$

For the mixed gravitational anomaly we have

$$- \sum_i N_a N_b (r_{ai} p_{bi} + r_{ai} p_{b'i} - r_{a'i} p_{bi} - r_{a'i} p_{b'i}) + \frac{32}{2} \sum_i N_a (r_{ai} p_i - r_{a'i} p_i)
= N_a [\Pi_a] \cdot (-2 \sum_b N_b [\Pi_b] - 2 \sum_b N_b [\Pi_{b'}] + \frac{32}{2} [\Pi_{O6}]) = -2 \times 48 \times N_a [\Pi_a] \cdot [\Pi_{O6}]$$

where we have used the tadpole cancellation conditions. The final expressions, modulo numerical factors not computed carefully, have precisely the form required to cancel the residual anomaly.

The cancellation of mixed non-Abelian and mixed gravitational anomalies in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model works analogously. In fact, considering the chiral spectrum given in Table I, one can reproduce step by step the above computation and reach analogous results. Namely the anomalies are canceled by a Green-Schwarz mechanism mediated by untwisted RR fields, whose couplings to gauge bosons and gravitons follow from the Chern-Simons interactions for D6-branes and O6-planes. It is also clear that an analogous mechanism will be at work in models with other orbifold groups.
REFERENCES

[1] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Ya.S. Stanev, Phys. Lett. B 385 (1996) 96, hep-th/9606169.

[2] M. Berkooz and R.G. Leigh, Nucl. Phys. B 483 (1997) 187, hep-th/9605049.

[3] Z. Kakushadze, Nucl. Phys. B 512 (1998) 221, hep-th/9704059; Z. Kakushadze and G. Shiu, Phys. Rev. D 56 (1997) 3686, hep-th/9705163; Z. Kakushadze and G. Shiu, Nucl. Phys. B 520 (1998) 75, hep-th/9706051.

[4] G. Zwart, Nucl. Phys. B 526 (1998) 378, hep-th/9708040. D. O’Driscoll, hep-th/9801114.

[5] L. Ibáñez, JHEP 9807 (1998) 002, hep-th/9802103.

[6] G. Aldazabal, A. Font, L.E. Ibáñez and G. Violero, Nucl. Phys. B 536 (1999) 29, hep-th/9802026.

[7] G. Shiu and S.-H.H. Tye, Phys. Rev. D 58 (1998) 106007, hep-th/9805157.

[8] Z. Kakushadze, Phys. Lett. B 434 (1998) 269, hep-th/9804110. Phys. Rev. D 58 (1998) 101901, hep-th/9806044. Nucl. Phys. B 535 (1998) 311, hep-th/9806008.

[9] M. Cvetič, M. Plümer and J. Wang, JHEP 0004 (2000) 004, hep-th/9911021.

[10] M. Klein and R. Rabadán, JHEP 0010 (2000) 049, hep-th/0008173.

[11] M. Cvetić, A. M. Uranga and J. Wang, Nucl. Phys. B 595, 63 (2001), hep-th/0010091.

[12] M. Cvetič, L. Everett, P. Langacker and J. Wang, JHEP 9904 (1999) 020, hep-th/9903051.

[13] J. Park, R. Rabadán and A. M. Uranga, Nucl. Phys. B 570 (2000) 38, hep-th/9907086.

[14] M. Cvetič and L. Dixon, unpublished; M. Cvetič, in Proceedings of Superstrings, Cosmology and Composite Structures, College Park, Maryland, March 1987, S.J. Gates and R. Mohapatra, eds. (World Scientific, Singapore, 1987), Phys. Rev. Lett. 59 (1987) 1795, and Phys. Rev. Lett. 59 (1987) 2829.

[15] M. Cvetič and P. Langacker, Nucl. Phys. B 586 (2000) 287, hep-th/0005049.

[16] J. Lykken, E. Poppitz and S. P. Trivedi, Nucl. Phys. B 543 (1999) 105, hep-th/9806080.
[17] M. Bianchi, G. Pradisi and A. Sagnotti, Nucl. Phys. B 376, 365 (1992); M. Bianchi, Nucl. Phys. B 528, 73 (1998) E. Witten, JHEP 9802, 006 (1998); Z. Kakushadze, G. Shiu and S.-H. H. Tye, Phys. Rev. D 58, 086001 (1998).

[18] C. Bachas, hep-th/9503030.

[19] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B 489 (2000) 223, hep-th/0007090.

[20] R. Blumenhagen, L. Görlich, B. Körs, Nucl. Phys. B 569 (2000) 209, hep-th/9908130.

[21] R. Blumenhagen, L. Gorlich, B. Kors, JHEP 0001 (2000) 040, hep-th/9912204.

[22] S. Förste, G. Honecker, R. Schreyer, Nucl. Phys. B 593 (2001) 127, hep-th/0008250.

[23] B Feng, Y.-H. He, A. Karch, A. M. Uranga, JHEP 0106 (2001) 065, hep-th/0103177.

[24] R. Blumenhagen, B. Körs, D. Lüst, JHEP 0102 (2001) 030, hep-th/0012150.

[25] I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B 464 (1999) 38, hep-th/9908023.

[26] G. Aldazabal and A. M. Uranga, JHEP 9910 (1999) 024, hep-th/9908072.

[27] G. Aldazabal, L. E. Ibáñez and F. Quevedo, JHEP 0001 (2000) 031, hep-th/9909172; JHEP 0002 (2000) 015, hep-ph/0001053.

[28] G. Aldazabal, L. E. Ibáñez, F. Quevedo and A. M. Uranga, hep-th/0005067.

[29] D. Bailin, G.V. Kraniotis and A. Love, Phys. Lett. B 502 (2001) 209, hep-th/0011289.

[30] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP 0010 (2000) 006, hep-th/0007024.

[31] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, hep-th/0011073.

[32] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, JHEP 0102 (2001) 047, hep-ph/0011132.

[33] S. Förste, G. Honecker and R. Schreyer, JHEP 0106 (2001)004, hep-th/0105208.

[34] L. E. Ibáñez, F. Marchesano and R. Rabadán, hep-th/0105155.

[35] R. Blumenhagen, B. Körs, D. Lüst and T. Ott, hep-th/0107138.

[36] I. Antoniadis, E. Kiritsis and T. N. Tomaras, Phys. Lett. B 486, 186 (2000).
[37] D. Berenstein, V. Jejjala and R. G. Leigh, hep-ph/0105042.

[38] E. Dudas and J. Mourad, Phys. Lett. B486 (2000) 172, hep-th/0001165; R. Blumen-
hagen, A. Font, Nucl. Phys. B 599 (2001) 241, hep-th/0011269.

[39] M. Cvetič, G. Shiu and A. M. Uranga, hep-th/0107143.

[40] E. Witten, Nucl. Phys. B 460, 541 (1996), hep-th/9511030.

[41] S. Kachru and E. Silverstein, Nucl. Phys. B 504 (1997) 272, hep-th/9704183.

[42] B. A. Ovrut, T. Pantev and J. Park, JHEP 0005 (2000) 045, hep-th/0001133.

[43] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B 480 (1996) 265, hep-th/9606139.

[44] M. R. Gaberdiel, JHEP 0011 (2000) 026, hep-th/0008230.

[45] M. R. Douglas, hep-th/9807235.

[46] M. Klein and R. Rabadán, JHEP 0007 (2000) 040, hep-th/0002103.

[47] C. Angelantonj and R. Blumenhagen, Phys. Lett. B 473 (2000) 86, hep-th/9911190.

[48] G. Aldazabal, A. Font, L. E. Ibáñez and A. M. Uranga, Nucl. Phys. B 452 (1995) 3, hep-th/9410206; Nucl. Phys. B 465 (1996) 34, hep-th/9508033.

[49] R. Rabadán, hep-th/0107036.

[50] M. Bianchi, A, Sagnotti. Phys. Lett. B247 (1990) 517; Nucl. Phys. B361 (1991) 519.

[51] E. G. Gimon, J. Polchinski, Phys. Rev. D 54 (1996) 1667, hep-th/9601038.

[52] R. Blumenhagen, V. Braun and R. Helling, Phys. Lett. B 510 (2001) 311, hep-th/0012157.

[53] M. R. Douglas and G. Moore, hep-th/9603167; M. R. Douglas, B. R. Greene and D.
R. Morrison, Nucl. Phys. B 506 (1997) 84, hep-th/9704151.

[54] S. Kachru and J. McGreevy, Phys. Rev. D 61 (2000) 026001, hep-th/9908135.

[55] E. Witten, hep-th/0012054.

[56] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289 (1987) 589.

[57] L. E. Ibáñez, R. Rabadán and A. M. Uranga, Nucl. Phys. B 542 (1999) 112, hep-th/9808139; Nucl. Phys. B 576 (2000) 285, hep-th/9905098.

[58] B. S. Acharya, hep-th/0011089; hep-th/0101206.

[59] M. Atiyah, J. Maldacena and C. Vafa, hep-th/0011256.

[60] M. Atiyah and E. Witten, M-theory Dynamics on a Manifold of $G_2$ holonomy, to appear.

[61] C. Vafa, hep-th/0008142.

[62] K. Dasgupta, K. Oh and R. Tatar, hep-th/0106040.

[63] R. L. Bryant and S. Salamon, Duke Math. J. 58 (1989) 829; G. W. Gibbons, D. N. Page and C. N. Pope, Commun. Math. Phys. 127 (1990) 529.

[64] M. Cvetič, G. W. Gibbons, H. Lü and C. N. Pope, hep-th/0106026; A. Brandhuber, J. Gomis, S. S. Gubser and S. Gukov, hep-th/0106034.

[65] M. Cvetič, G. W. Gibbons, H. Lü and C. N. Pope, hep-th/0101096; M. Cvetič, H. Lü and C. N. Pope, hep-th/0105096; M. Cvetič, G. W. Gibbons, J. T. Liu, H. Lü and C. N. Pope, hep-th/0106162; for a review, see: M. Cvetič, G. W. Gibbons, H. Lü and C. N. Pope, hep-th/0106177.

[66] J. Gomis, hep-th/0103115.

[67] J. D. Edelstein and C. Nunez, JHEP 0104, 028 (2001).

[68] S. Kachru and J. McGreevy, JHEP 0106 (2001) 027, hep-th/0103223.

[69] H. Partouche and B. Pioline, JHEP 0103 (2001) 005, hep-th/0011130.

[70] J. F. Morales, C. A. Scrucca and M. Serone, Nucl. Phys. B 552 (1999) 291, hep-th/9812071; B. Stefanski, Jr. Nucl. Phys. B 548 (1999) 275, hep-th/9812088.