A comparative analysis of efficiency of nonlinear dynamics control methods for a buck converter

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Abstract. The paper presents a comparative analysis of efficiency of methods of nonlinear dynamics control with feedback: a method of linearization of Poincaré map, a method of time-delay feedback, a method of target-oriented control. Efficiency of each method is evaluated on basis of the achieved domain of the desired dynamic mode. Dynamics modes maps and diagrams allowing evaluation of a particular method efficiency are given. It is shown that a target-oriented control has the greatest efficiency, and when it is applied in the selected range of system parameters, undesired dynamic modes do not exist. The method of Poincaré map linearization and the method of time-delay feedback control have approximately equal efficiency. A comparative analysis is performed for the first time, and the obtained results are of great importance for practice.

1. Introduction

At present, pulse-width converters find a widespread application in systems that require conversion of DC voltage to another value of DC voltage [1].

Considered devices are closed-loop automatic control systems with voltage feedback that are prone to complex dynamics [2, 3]. In these systems, bifurcations occur with changing of one or several parameters, which may result in origination of undesired dynamic modes accompanied by large amplitude oscillations leading to an increase of electromagnetic interference during the load, current overload of power switches and appearance of acoustic effects during the device operation. To prevent the appearance of these modes, specific control algorithms for nonlinear dynamics are required.

In this case, it is necessary to provide a so-called desired dynamic mode. Under the desired dynamic mode, we mean a mode in which the period of switching processes coincides with the period of pulse width modulation (PWM). This mode is called a 1-cycle [2]. In addition, the undesired \( m \)-cycles can appear, when the period of the system switching processes is \( m \) times greater than the PWM period. Further, index \( m \) will be called the cycle ratio.

There is a large number of works on control of nonlinear dynamics of discrete systems [4-9]. Among control approaches, two main directions can be marked:

– methods with feedback: Poincaré map linearization method (PML) [4], time-delay feedback control methods (TDFC) [5, 6], a method of target-oriented control (TOC) [7, 8].

– methods without feedback, based on resonant perturbation of parameters [9].

Application of these methods in power electronics was widely reported in scientific literature, and the first group of methods has found the widest application. However, one of the main disadvantages
of the existing works is the absence of systematic studies on efficiency of these methods. For example, authors can restrict themselves by demonstration of one timing diagram, or a bifurcation diagram, whereas the drift of two parameters can occur in the system, and at this point, 2-D dynamic modes maps are much more informative. In addition, there is no comparative analysis of efficiency of different control methods.

This paper contains a comparative analysis of efficiency of methods of nonlinear dynamics control with feedback by the example of an automatic control system with a buck converter. The evaluation will be conducted on the relative area of the desired dynamic modes in the space of two system parameters with the use of one or another method. The buck converter relates to switching converters of the first kind, that later on will allow the application of the obtained results in the design of a wide class of systems.

2. Nonlinear dynamics control systems of the buck converter

Three methods of nonlinear dynamics control will be briefly reviewed in this section. One of them (PML) is based on the introduction of perturbation of one parameter, calculated on the basis of Poincaré map linearization [4], and two others (TDFC, TOC) are based on the introduction of corrective actions, calculated on the basis of differential correction functions [5, 7].

The generalized functional diagram of the automatic control system (ACS) with switching converters is shown in Figure 1. The following designations are accepted here: SC – switching converter, $U_L$ – load voltage, $x_1, x_2, ..., x_n$ – state variables, $U_{in}$ – input voltage, $U_{ref}$ – reference signal, $U_p$ – power switches control pulses, $U_{con}$ – control signal, $U_{rmp}$ – ramp voltage with amplitude $U_{rmp,m}$, $U_{ck}$ – corrective action; $\Delta p_k$ – perturbation parameter for PML, NDCS – nonlinear dynamics control system, MC – master oscillator, RG – ramp generator, SH – sample-and-hold device, SB – subtractor, SM – summator, $U_{err}$ – error of ACS, $U_{ld}$ – load voltage, $I_{ld}$ – load current.

![Figure 1. The closed-loop automatic control system with nonlinear dynamics control.](image)

Two subsystems can be allocated here. The main control subsystem provides stabilization of the average value of the output voltage without considering system nonlinear dynamic properties. The
nonlinear dynamics control system (NDCS) provides dynamic stabilization of the desired mode (1-cycle).

TOC and TDFC belong to methods with differential corrective functions. The essence of the methods with differential correction functions is in the following. Let there be given some abstract discrete system described by a stroboscopic map which is presented as

$$x_k = f(x_{k-1}),$$

(1)

where \(x_k\) – the value of the variable on the \(k\)-th iteration of the stroboscopic map.

Stabilization of an unstable fixed point of stroboscopic map (1) \(x^*\) can be accomplished by introduction of a stabilizing action, during which the stroboscopic mapping function of the nonlinear dynamic system takes the following form:

$$x_k = f(x_{k-1}) + f_{cf}(x_{k-1}),$$

where function \(f_{cf}\) is the correction function (CF).

In case of TDFC, the correction function has the following form [5]:

$$f_{cf}(x_{k-1}) = c_p(x_{k-1} - f(x_{k-1})),\quad (2)$$

In case of TOC, the correction function has form [7]

$$f_{cf}(x_{k-1}) = c_p(x^* - f(x_{k-1})),\quad (2)$$

where \(c_p\) is the adjustable coefficient.

Here, the main property of the correction function should be noted. As the representative point approaches the stabilized fixed point of stroboscopic map \(f(x_{k-1})\), the value of the correction function tends to zero, and after reaching the desired mode, the correction is actually absent, which corresponds to \(f_{cf}(x_{k-1})=0\). This approach guarantees that the use of the correction function has no effect on the position of the stabilized fixed point of original stroboscopic map \(f(x_{k-1})\), which is a necessary condition for correct nonlinear dynamics control.

As can be seen from (2), in case of TDFC, the value of the variable in previous iteration \(x_{k-1}\) of the stroboscopic map is used to calculate the corrective action. In case of TOC, fixed point \(x^*\) is used instead of \(x_{k-1}\) and is stabilized. This is a disadvantage of TOC, since \(x^*\) should be calculated in advance, which requires higher computational resources.

Let us consider the mathematical description of ACS of the buck converter in the form of a stroboscopic map. The conventional automatic control system with output voltage average value feedback for DC-to-DC converters, operating in a discontinuous current mode, is described by a stroboscopic map as follows [10]:

$$X_k = \Psi(X_{k-1}) = e^{A_1(1-z_i)}u e^{A_2(z_{i2}-z_{i1})}u e^{A_3(z_{i1})}u X_{k-1} +$$

$$+ e^{A_1(1-z_i)}u e^{A_2(z_{i2}-z_{i1})}u (e^{A_3(z_{i1})}u - E)V_{AB1} +$$

$$+ e^{A_1(1-z_i)}u e^{A_2(z_{i2}-z_{i1})}u (e^{A_3(z_{i1})}u - E)V_{AB2} + (e^{A_1(1-z_i)}u - E)V_{AB3},$$

(3)

where \(X=[x_1, x_2]^T=[i_L, u_c]^T\); \(i_L\) – the inductor current; \(u_c\) – the capacitor voltage; \(z_{k1}, z_{k2}\) – switching moments in the \(k\)-th clock period in relative time: \(z_{k1} = t_{k1} - (k-1)a, \quad z_{k2} = t_{k2} - (k-1)a\), where \(a\) – the duration of the clock period; \(X_{k-1}\) – the vector of system state variables at the beginning of the \(k\)-th clock period.

The circuit of a buck converter and the diagram of processes occurring in clock period [1, 10] are presented in Figure 2, where \(VT\) – power switch, \(VD\) – power diode, \(L\) – inductor, \(R\) – inductor resistance, \(C\) – capacitor, \(R_L\) – load resistance.

Analysis of Figure 2 shows that the clock period for the buck converter is divided into three sections of system structure constancy.
1. Section 1: \((k-1)a < z < z_{k1}\). Here the power switch is at the OFF-state, and inductor current \(i_L\) increases linearly.

2. Section 2: \(z_{k1} < z < z_{k2}\). Here the power switch is in the ON-state, and the inductor current starts to decrease from moment \(z_{k1}\); the energy is transferred from the input to the load.

3. Section 3: \(z_{k2} < z < ka\). At this section, the inductor current is equal to zero, and the output is being discharged in this case, transferring energy to the load.

![Figure 2. Illustration of a buck converter operating principle: a) the buck converter circuit, b) waveforms of processes in a clock period.](image)

Switching moments \(z_{k1}\) and \(z_{k2}\) can be calculated on basis of switching functions [10]:

\[
\xi_{k1}(X_{ik1}, z_{k1}) = aU - \beta c_1^T X_{ik1}(z_{k1}) - U_{rmp, m} z_{k1};
\]

\[
\xi_{k2}(X_{ik2}, z_{k2}) = c_2^T X_{ik2}(z_{k2}),
\]

where \(X_{ik}\) – values of the vector of state variables at the \(i\)-th switching point in \(k\)-th clock period \((i=1, 2); c_1=[0; 1]\) and \(c_2=[1; 0]\) – constant vectors that define the component of the vector of state variables that is used in expressions. At that, it is necessary to solve the so-called switching manifolds equations:

\[
\xi_{k1}(X_{ik1}, z_{k1}) = 0; \quad \xi_{k2}(X_{ik2}, z_{k2}) = 0.
\]

When using NDCS with correction functions, additional control actions \(u_{ik}\) are introduced, which, for the \(k\)-th clock period, are determined by correction functions according to the following expressions:

– in case of TDFC,

\[
u_{ik} = K_i(x_{ik-1} - x_{ik})
\]

– in case of TOC,

\[
u_{ik} = K_i(x_{icref} - x_{ik})
\]

Therefore, the stroboscopic map for the ACS with the correction function takes the following form:

\[
X_k = \Psi(X_{k-1}) = e^{A_1(1-z_{i2})}e^{A_2(1-z_{i1} - \Delta z_1)}e^{A_3(z_{i1} + \Delta z_1)}X_{k-1} + \\
+ e^{A_1(1-z_{i2})}e^{A_2(1-z_{i1} - \Delta z_1)}(e^{A_3(z_{i1} + \Delta z_1)} - E)V_{AB1} + \\
+ e^{A_1(1-z_{i2})}e^{A_2(1-z_{i1} - \Delta z_1)}(e^{A_3(z_{i1} + \Delta z_1)} - E)V_{AB2} + (e^{A_3(1-z_{i2})} - E)V_{AB3},
\]

where \(\Delta z\) – the increment of a duty cycle in the \(k\)-th clock period.

The specified increment can be found on the basis of expression
\[ \Delta z_k = \frac{\alpha U_{ck}}{U_{rmp,m}} = \frac{\alpha \sum_{i=1}^{n} u_{ik}}{U_{rmp,m}}, \]

where \( U_{ck} \) is the corrective action (Figure 1), calculated as the sum of \( u_{ik} \).

Switching function \( \xi_{k1}(X, z_{k1}) \), defining the switching moment of the power switch [11] for the ACS with correction functions, described by a system of differential equations of the \( n \)-th order, takes the following form:

\[ \xi_{k1}(X, z_{k1}) = \alpha \left( U_3 - \beta_{c1}^T X_{k-1} - \sum_{i=1}^{n} \Delta u_{ik} \right) - U_{ref,m} z_{k1}, \]

where \( n \) – the order of the system of differential equations describing the ACS (in the case considered, when \( n=2 \)).

The generalized functional diagram of NDCS (Figure 1) is presented in Figure 3.

**Figure 3.** The generalized nonlinear dynamics control system.

PML blocks are shown with red lines, TOC and TDFC blocks – with blue and green ones, accordingly. Blocks that are common both for TOC and for TDFC are shown with black lines. It is evident that with application of a particular method only the corresponding blocks operate. The following designations are accepted in the Figure 3: \( \beta_i \) – a scale factor of the feedback circuit for the \( i \)-th state variable, \( SH_i \) – a sample-and-hold device for the \( i \)-th state variable, \( DB \) – a delay unit, \( AB \) – an adaptation block, \( K_i \) – proportional coefficients for the \( i \)-th state variable, \( x_{ik} \) – the \( i \)-th state variable in the \( k \)-th stroboscopic moment, \( x_{ik}^{ref} \) – the \( i \)-th state variable of the vector of reference signals for the fixed point, \( u_{ik} \) – a deviation component for the \( i \)-th state variable, \( SM \) – a summator, \( PPC \) – a parameter perturbation calculator, \( CF \) – a correction function block (function (4) or (5)).

The main task of the design of the ACS based on differential correction functions is selection of such coefficients \( K_i \), under which the desired dynamic mode is stable. This problem can be solved
using methods of optimization with constraints. The author has used the method of Nelder-Mead with constraints $K_{\min,i}$ and $K_{\max,i}$ set empirically. In fact, this means that calculated optimal value $K_i$ lies within $K_{\min,i} < K_i < K_{\max,i}$. In this case, the optimality criterion is the target value of the largest multiplier of the desired dynamic mode [5] that should be less than unity.

In case of the use of the method of Poincaré map linearization (PML) for stabilization of the unstable mode, the perturbation of one or another system parameter is performed. Therefore, in Figure 3, the relationship between NDCS and the proportional controller is marked by the dashed line. In this case, the optimality criterion is the target value of the largest multiplier of the desired dynamic mode [5] that should be less than unity.

For calculation of the necessary increment, stroboscopic map (3) is replaced by a similar linearized map at point $(X^*, \rho^*)$:

$$Y_k = M^*Y_{k-1} + C^*u_{k-1},$$  \hspace{1cm} (6)

where $M^* = \frac{\partial \Psi(X^*, \rho^*)}{\partial X_{k-1}}$ – a monodromy matrix of the stabilized 1-cycle; $C^* = \frac{\partial \Psi(X^*, \rho^*)}{\partial \rho}$ – a stroboscopic map derivative with respect to the perturbed parameter; $Y_{k-1} = X_{k-1} - X^*$, $u_{k-1} = p_{k-1} - \rho^*$ – the required parameter perturbation, where $p_{k-1}$ – the value of the perturbed parameter in the $k$-th clock period.

As we have mentioned previously, gain $\alpha$ of the proportional controller is selected as parameter $p$. For linear system (6), stabilizing control $u_{k-1}$ is chosen in the form of linear state feedback

$$u_{k-1} = -K^*Y_{k-1}. \hspace{1cm} (7)$$

Taking into account (7), from (6), we obtain the following expression:

$$Y_k = (M^* - C^*K^*)Y_{k-1}.$$  \hspace{1cm} (8)

Thus, unstable point $X^*$ of stroboscopic map (3) will be stabilized if matrix $K^*$ is defined so that matrix $M^* - C^*K^*$ has all the eigenvalues (multipliers) that are less than one.

To calculate desired matrix $M^* - C^*K^*$, technique [12] is used:

$$M^* - C^*K^* = \frac{\max(\rho_i)}{100}(1 - \delta), \hspace{1cm} (8)$$

where $i=1, 2, \rho_i$ – $i$-th matrix multiplier $M^*$, $\delta=[\%]$ – stability margin.

When using expression (8), the eigenvalues will always be less than one, at that the system properties do not change significantly because the multipliers of the desired matrix remain complex or real numbers, as they do in original matrix $M^*$. Calculation of feedback matrix $K^*$ based on known matrix $M^* - C^*K^*$, as well as calculation of matrices $M^*$ and $C^*$, can be implemented using Ackermann’s formula [13], widely known in automatic control theory. Calculation of required parameter perturbation $u_{k-1}$ is carried out according to expression (7).

3. Study of nonlinear dynamics of the switching converter with PWM

The dynamic modes map of the ACS without nonlinear dynamics control based on a buck converter with PWM-I is shown in Figure 4, a.

The simulation was performed under the following system parameters: $L=0.1$ Hn; $C=1$ $\mu$ F; $R=10$ Ohm; $R_L=100$ Ohm; $\alpha=56$; $\beta=0.01$; $U_{ref}=5$ V; $U_{nmp_{mp_{nmp}}}=10$ V; $\alpha=0.0001$ s.

In Figure 4, symbols $D_{m,j}$ mark domains of existence of different dynamic modes ($i$ stands for the m-cycle that characterizes a particular domain, $j$ is the number of the domain on the dynamic modes map). Domains $D_{m,j}$ correspond to nondeterministic modes of the converter operation, in which large amplitude quasiperiodic oscillations are observed in the system.

As can be seen from the Figure 4, in the whole range of variation of the input voltage, there are undesired modes that limit the possibility of reference voltage regulation in a wide range of values. As
the analysis of Figure 4, a shows, there are multistability domains on the dynamic modes map. In particular, a certain part of domains $D_{4,1}$ and $D_{0,1}$ coexists with the desired 1-cycle. The relative area of the domain of the desired 1-cycle in Figure 4, a is 52.74%.

Next, we will consider the results of the control system operation based on TDFC. The two-parameter diagrams for the system with TDFC-control with adaptation of parameters of the method are presented in Figure 4, b. TDFC parameters have the following values: $K_{1\text{min}}=-3; K_{1\text{max}}=3; K_{2\text{min}}=-3; K_{2\text{max}}=3$, $\beta_1=0.08; \beta_2=0.01$, the target value of the multiplier is $\rho_{tg}=0.3$. As follows from the figure, the domain of undesired modes at high control voltages decreases, which, in its turn, leads to an increase of the domain of desired mode $D_{1,1}$ (96.56% of the map total area).

![Figure 4.](image)

**Figure 4.** The results of nonlinear dynamics analysis of ACS based on a buck converter: a) without nonlinear dynamics control; b) using TDFC; c) using PML; d) using TOC.

With the introduction of adaptation domains, $D_{ch,2}$ and $D_{2,1}$, disappear completely, and the area of undesired modes at high reference voltages is replaced by domains, where, along with the stable 1-cycle, there are chaotic modes. Special attention should be paid to this case, because it shows the disadvantages of the adaptation algorithm based on linearization of the stroboscopic map. In the case considered, the author managed to ensure the stability of the desired mode within a given margin, but at the same time, the system properties changed in such a way that the undesired modes occurred in coexistence with the desired one. For this reason, it is necessary to carry out additional research at the
design stage. Building a more complex objective function that is able to exclude such a situation is difficult for nonlinear dynamic systems.

The map of ACS dynamic modes based on PML, with selected stability margin δ=20% (see (8)) is presented in Figure 4, c. As is seen from the figure, the introduction of PML has led to a significant expansion of the domain of the desired 1-cycle, but the domains of undesired dynamic modes remained. At small values of the reference voltage, there is domain $D_{2,1}$ of a little area, and in case of large values of the reference voltage, there are domains $D_{4,2}$, $D_{4,1}$, $D_{3,1}$, $D_{6,1}$. A characteristic feature of the obtained map is the impossibility of regulation of the reference voltage throughout the whole range of values without appearance of undesired modes, but at this, the area of the domain of the desired mode on the map (96.63% of the map total area) is somewhat larger than on the dynamic modes map with the TDFC control method (Figure 5, b).

In case of TOC with parameters adaptation, the simulation has been performed for the system with algorithm parameters: $K_{min}=-0.7$; $K_{max}=-0.7$; $\beta_1=0.08$; $\beta_2=0.01$. Dynamic modes maps with the use of TOC are shown in Figure 4, d. As is seen from the figure, the use of TOC solves the problem of nonlinear dynamics control completely. The desired 1-cycle occupies 100% of the total area of the map of dynamic modes.

Bar charts for evaluation of efficiency of nonlinear dynamics control methods are shown in Figure 5. As shown in the figure, all control methods have led to the increase of the area of the domain of the desired mode, but the highest efficiency has been demonstrated by TOC.

![Figure 5](image)

**Figure 5.** Bar charts of the relative area of the domain of the desired dynamic mode in percentage terms for a buck converter.

4. **Conclusion**

The conducted research allows drawing the following conclusions.

1. TOC demonstrates the highest efficiency, for, as a result, the area of the domain of the desired mode is maximal, but meanwhile, TOC requires calculation of the fixed stroboscopic map point for the stabilized dynamic mode that demands the use of high-performance MCUs.

2. TDFC has acceptable (but slightly worse than TOC) control efficiency, but at the same time, its application may cause appearance of multistability domains. This method has the minimum requirements for computing resources of MCU.

3. PML shows the worst performance, but with that, as follows from [12], its application is appropriate in multistability domains, where it provides an effective retention of the system in the desired mode and, therefore, ensures a minimal influence of system parameters deviation on the fixed stroboscopic map point. In the unstable area of the desired mode, the control efficiency may be low.
The obtained results can be used at the stage of schematic design of a wide class of switching DC-to-DC converters. On the basis of the results presented, the method of nonlinear dynamics control that is most suitable in each case can be chosen.

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