Sensitivity analysis and investment decisions: 
NPV-consistency of 
Straight-Line rate of return

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Abstract

The economic reliability of a performance metric depends on its consistency with the Net Present Value (NPV). We use the new notion of strong NPV-consistency for comparing the Straight-Line rate of return (belonging to the class of AIRR metrics) and the traditional Internal Rate of Return (IRR). We show that the IRR is unreliable and its degree of NPV-inconsistency is not negligible via Spearman’s (1904) correlation coefficient and Iman and Conover’s (1987) top-down coefficient. In contrast, Straight-Line rate of return is strongly NPV-consistent, always exists, is unique, and has an unambiguous financial nature.

Keywords. Finance, sensitivity analysis, investment decisions, NPV, straight-line.

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On the same topic, please see also the companion paper

Investment decisions and sensitivity analysis: NPV-consistency of rates of return
1 Introduction

In this note, we deal with the AIRR (Average Internal Rate of Return) class of performance metrics, developed by Magni (2010, 2013), based on a capital-weighted mean of holding period rates. The AIRR approach consists in associating the capital amounts invested in each period with the corresponding period returns by means of a weighted arithmetic mean. Magni (2010, 2013) showed that any AIRR is NPV-consistent: decisions made by an investor who adopts NPV are the same as those made by an investor who adopts AIRR. As a special case, this paper introduces the Straight-Line (SL) rate of return, denoted as SL-AIRR, which is the ratio of the total project return to the total invested capital, assuming the capital depreciates uniformly over time.

Sensitivity analysis is a tool for measuring the impact on key parameters on a given objective function (see Pianosi et al. 2016, Borgonovo and Plischke 2016) and, given a technique, different objective functions may or may not lead to different results. We use sensitivity analysis (SA) and the new notion of strong NPV-consistency to measure the coherence of three performance metrics: SL rate of return, NPV, and IRR. To this end, we make use of the notion of strong consistency introduced in Marchioni and Magni (2018). If a metric is not NPV-consistent, we measure the degree of inconsistency by means of two alternative indices: Spearman’s (1904) coefficient or Iman and Conover’s (1987) top-down coefficient.

We find that the SL rate of return is strongly NPV-consistent under many techniques, even in a strict sense (the relevances of the parameters are the same). In particular, the SL-AIRR is more reliable than the IRR in more than one sense. First, we show that SL-AIRR is an affine transformation of NPV, which implies that it is strongly coherent with the NPV under many techniques, even in a strict sense (the relevances of the parameters are the same), whereas IRR is not strongly NPV-consistent and the degree of inconsistency may be remarkable. Furthermore, while IRR may not exist or may not be unique, the SL-AIRR always exists and is unique. Moreover, the IRR may exist and be unique for a given set of values of the key drivers, but it may not exist or be multiple if a different set of values is selected. This has the unpleasant implication that a sensitivity analysis cannot be performed. Finally, the IRR may change its financial nature (investment rate versus financing rate) under changes in the key drivers, whereas the SL-AIRR has an unambiguous financial nature determined by the sign of the first cash flow: An investment rate if the initial amount is negative, a financing rate if it is positive.

As a result, the use of IRR for investment evaluation should be discouraged. The SL-AIRR is a more reliable measure of worth, which can coherently be associated with NPV in investment evaluation and decisions. Indeed, the SL-AIRR even provides information that the traditional NPV analysis cannot provide. In particular, the SL-AIRR (i) supplies information about the return per unit of total capital committed, (ii) enables interpreting

\[1\] Marchioni and Magni (2018) deal with the average ROI, implicitly assuming that the operating assets consist of net fixed assets alone. Under this assumption, if net fixed assets are depreciated evenly, the average ROI is equal to the SL-AIRR. If working capital exists, the average ROI is not strongly consistent with NPV unless working capital is estimated exogenously, that is, independently of revenues and costs (e.g., constant or a given percentage of the net fixed assets).
the project as an investment or a financing, and, therefore, whether value is created because funds are invested at a rate of return which is greater than the cost of capital (COC) or because funds are borrowed at financing rate which is smaller than the COC, (iii) decomposes the economic value created into economic efficiency (the difference between SL-AIRR and COC) and the investment scale (the sum of the committed amounts).

The remaining part of the paper is structured as follows. Section 2 introduces the SL-AIRR and describes its properties. Section 3 describes some well-known SA methods and Section 4 introduces the notion of pairwise coherence according to which any two functions are strongly coherent if the ranking of the model parameters coincides. This section shows that, under many SA techniques, a function \( f \) and an affine transformation of it share the same (ranking and) relevances of parameters, so they are strongly coherent in a strict sense. Section 5 shows that the SL-AIRR is strongly NPV-consistent in a strict sense under many SA techniques, whereas IRR is incompatible with NPV. Some numerical counterexamples in section 6 show that, even in very simple cases, the discrepancy between IRR and NPV may be not negligible, ambiguities may arise, and sometimes the SA cannot even be performed. This suggests that the use of IRR should be discouraged. Some concluding remarks end the paper.

## 2 IRR and SL-AIRR as special cases of AIRR

Let \( P \) be a project and let \( F = (F_0, F_1, \ldots, F_p) \), \( F \neq 0 \), its estimated stream of free cash flows (FCFs), where \( p \) is the lifetime of the project. Let \( c_t \) be the capital invested (or borrowed, if negative) in the interval \([t, t + 1]\). Then, the capital evolves recursively as

\[
c_t = c_{t-1} + I_t - F_t
\]  

where \( I_t \) is the net operating profit (after taxes). The boundary conditions are \( F_0 = -c_0 \) and, after liquidation, \( c_p = 0 \). Therefore, the stream of capital amounts is \( c = (-F_0, c_1, \ldots, c_{p-1}, 0) \). For corporate projects, pro forma financial statements are available which report the base values of revenues, costs, interest, and accruals (working capital, net fixed assets, debt). In particular, the capital can be decomposed into working capital (WC) and net fixed assets (NFA) and the following accounting identity holds in every period:

\[
NFA_t + WC_t = D_t + E_t
\]

where \( D_t \) denotes net financial obligations and \( E_t \) denotes equity. Let \( R_t \) and \( C_t \) be the revenues and the operating costs. Then, \( I_t = (R_t - C_t - Dep_t)(1 - \tau) \) where \( Dep_t \) is the depreciation charge for the fixed assets and \( \tau \) is the company tax rate. Using (1), FCF is then derived as

\[
F_t = (R_t - C_t - Dep_t)(1 - \tau) - \Delta NFA_t - \Delta WC_t \tag{2}
\]

where \( \Delta NFA_t = NFA_t - NFA_{t-1}, \Delta WC_t = WC_t - WC_{t-1} \). Let \( k \) be the (assumed constant) cost of capital (COC), that is, the minimum attractive rate of return (if \( c_t > 0 \))
or the maximum attractive financing rate (if $c_t < 0$). We assume that the COC is exogenously fixed by the decision-maker/analyst. It is well-known that net present value (NPV) measures the economic value created: $\text{NPV} = \sum_{t=0}^{p} F_t (1 + k)^{-t}$. Therefore, the NPV decision criterion may be stated as follows:

**Definition 1.** (NPV criterion) A project creates value (i.e., it is worth undertaking) if and only if the project NPV, computed at the discount rate $k$, is positive: $\text{NPV}(k) > 0$.

An AIRR, denoted as $\bar{i}$, is defined as the ratio of the overall income $I = \sum_{t=1}^{p} I_t (1 + k)^{-(t-1)}$ earned by the investor to the overall capital committed $C = \sum_{t=1}^{p} c_{t-1} (1 + k)^{-(t-1)}$:

$$\bar{i} = \frac{I}{C} \quad (3)$$

or, equivalently, as the weighted mean of period rates associated with the capital stream $c$:

$$\bar{i} = \frac{\sum_{t=1}^{p} i_t c_{t-1} (1 + k)^{-(t-1)}}{\sum_{t=1}^{p} c_{t-1} (1 + k)^{-(t-1)}}$$

where $i_t = I_t/c_{t-1}$ is the growth rate for capital (see Magni 2010, 2013).

Magni (2010, 2013) defined a project as a net investment if $C > 0$ and a net financing if $C < 0$. In such a way, the financial nature of any project (and its associated rate of return) can be identified as an investment project or a financing project (respectively, an investment rate or a financing rate).

Traditionally, it is widely accepted that a metric/criterion $\varphi$ is to be NPV-consistent if and only if a decision maker adopting $\varphi$ makes the same decision suggested by the NPV criterion. We can formalize this standard notion as follows.

**Definition 2.** (NPV-consistency) A metric/criterion $\varphi$ is NPV-consistent if, given a cutoff rate $k$, the following statements are true:

(i) an investment project creates value if and only if $\varphi > k$

(ii) a financing project creates value if and only if $\varphi < k$.

Magni (2010, 2013) showed that, if $\varphi = \bar{i}$, then the metric is NPV-consistent, since, for any $c$,

$$\text{NPV}(1 + k) = C(\bar{i} - k). \quad (4)$$

The above definition and eq. (4) are particularly interesting because they show that the AIRR approach enables a deeper inspection of the economic content of the project than the traditional NPV analysis. Indeed, NPV is rewritten in terms of product of a capital base $C$ and an excess return $\bar{i} - k$. This means that the economic value created is determined by two factors: The project scale ($C$) and the project $\bar{i} - k$. The same NPV can be created either by investing a large capital amount at a small rate or investing a small capital at a high rate. Furthermore, the general definition stated above enables the analyst to understand whether value is created because capital is invested at a rate of return which is higher than the COC or because capital is borrowed at a financing rate which is smaller than the COC.
A shortcut for computing of AIRR is available from (4):

\[
i(C) = k + \frac{\text{NPV}}{C} (1 + k)
\]  

(5)

where dependence on the overall capital \( C \) is highlighted.

The internal rate of return (IRR), here denoted as \( x \), is the discount rate such that \( \text{NPV} \) is zero: \( \text{NPV}(x) = 0 \). Magni (2010, 2013) showed that IRR is a special case of AIRR obtained by assuming that the capital base is equal to the sum of Hotelling values. Hotelling value, here denoted as \( c_t(x) \), is the capital associated under the assumption that the force of interest is constant:

\[
c_t(x) = c_{t-1}(x)(1 + x) - F_t
\]

(see Magni 2010, 2013). This means that the capital is assumed to appreciate exponentially between two cash-flow dates and then decreases (or increases) by the distributed (or contributed) amount \( F_t \). Therefore, from (5),

\[
x = i(C^x) = k + \frac{\text{NPV}}{C^x} (1 + k)
\]

(6)

where \( C^x = \sum_{t=1}^{p} c_{t-1}(x)(1 + k)^{-(t-1)} \).

We now consider the (somewhat opposite) assumption of capital that depreciates linearly with time: \( c_t = c_0 (1 - \gamma t) \) where \( \gamma = 1/p \). In essence, this means that a straight-line (SL) depreciation for capital is assumed: The associated AIRR is

\[
i(C^{sl}) = k + \frac{\text{NPV}}{C^{sl}} (1 + k)
\]

(7)

where \( C^{sl} = \sum_{t=1}^{p} (c_0 (1 - \frac{t-1}{p}))(1 + k)^{-(t-1)} \).

IRR and the straight-line AIRR (SL-AIRR) are two special cases of AIRR, associated with different classes of capital streams. As (4) holds no matter what the capital stream is, both IRR and SL-AIRR (both belonging to the AIRR class) are NPV-consistent (see also Hazen 2003 on the NPV-consistency of IRR).

However, note that IRR suffers from many well-known or lesser-known difficulties (see Magni 2013 for a compendium). For example, it may or may not exist or multiple IRRs may exist. Also, the financial nature of the IRR depends upon the COC, \( k \), as the sign of \( C \) is not necessarily invariant under changes in \( k \). Contrary to IRR, the SL-AIRR has the nice property of existence and uniqueness for any project. Also, its financial nature is unambiguously determined by the sign of \( c_0 \), which coincides with the sign of \( C \) for any given \( k \): \( C^{sl} > 0 \) if and only if \( c_0 > 0 \).

This makes SL-AIRR an interesting candidate as a reliable measure of worth, consistent with NPV and immune from the difficulties that mar the IRR.

**Example 1.** Consider a project \( P \) such that \( F = (-10, 23, -17, 24, -22) \) and a COC equal to \( k = 32\% \). Two IRRs exist: \( x(1) = 11.2\% \) and \( x(2) = 67\% \). The former is associated
with the Hotelling stream $c = (10, -6.3, 6.5, -13.2, 0)$, the latter is associated with the Hotelling stream $c = (10, -11.9, 3.8, -19.8, 0)$. The overall capital associated with $x(1)$ is $C_x(1) = 2.4$, the overall capital associated with $x(2)$ is $C_x(2) = -4.1$. Therefore, IRR does not unambiguously determine the financial nature of the project: According to the first IRR, the project is an investment, according to the second IRR the project is a financing. Conversely, the SL-AIRR exists and is unique in any case, and unambiguously identifies the project as an investment, since the associated capital stream is $c = (10, 7.5, 5.25, 0)$ so that the total capital invested is $C_{sl} = 14.9 > 0$. The SL-AIRR is then $\bar{i}(C_{sl}) = 0.32 + 0.86(1 + 0.32)/14.9 = 37.8\%$. The investment is worth undertaking, given that $\bar{i}(C_{sl}) > k$.

Owing to (2), the NPV is a function of several value drivers: (i) The revenues ($R_t$), (ii) the cost of goods sold, the selling, general and administrative costs (all included in $C_t$), (iii) the accounts receivable and payable, the inventory, the liquid assets (all included in $WC_t$), (iv) the depreciation charge for $NFA$ and the capital expenditures (included in $NFA_t - NFA_{t-1}$), (v) the tax rate ($\tau$).

The IRR is an implicit function of the value drivers as well:

$$\sum_{t=0}^{p} \left( (R_t - C_t - Dep_t)(1 - \tau) - (NFA_t - NFA_{t-1}) - (WC_t - WC_{t-1}) \right)(1 + x)^{-t} = 0.$$ 

It is also evident that the SL-AIRR depends on value drivers as well, being a function of NPV.

Our aim is to check whether the coherence of IRR, SL-AIRR and NPV, which is guaranteed in a traditional sense, remains valid if changes in value drivers are considered to take account of the uncertainty of the estimated value drivers. The analysis of change in a model’s inputs and the impact on the model output is the purpose of Sensitivity Analysis (SA).

From now on, we will only deal with SL-AIRR, so we will use the symbol $i$ to denote it, omitting the dependence on $C_{sl}$ to avoid notational pedantry.

### 3 Sensitivity analysis

In the definition of Saltelli et al. (2004, p. 45), sensitivity analysis (SA) is the “study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input.”

Given a model and a set of inputs (parameters), the SA investigates the relevance of parameters in terms of variability of the model output. In the literature there exist many SA techniques (see Borgonovo and Plischke 2016, Pianosi et al. 2016, for review of SA methods). The choice of an SA technique which best suits the model depends on several factors, among which the purpose of the analysis, the size of the variation of the parameters and the computational cost of the analysis.

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3Dep$_t$ is included in both the income statement (it affects $I_t$) and the accruals (it affects $NFA_t - NFA_{t-1}$)
A model can be described as consisting of an objective function \( f \) defined on the parameter space \( A \), which maps vector of inputs onto an output model \( y \):

\[
    f : A \subset \mathbb{R}^n \to \mathbb{R}, \quad y = f(\alpha), \quad \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n).
\]

The vector \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in A \subset \mathbb{R}^n \) is the vector of inputs or parameters or key drivers and \( y(\alpha) \) is the output of the model. Let \( \alpha^0 = (\alpha^0_1, \alpha^0_2, \ldots, \alpha^0_n) \in A \) be the base-case, a representative value (e.g., mean value, most probable value, etc.). The relevance of a parameter \( \alpha_i \) (also known as importance measure) quantifies the impact of \( \alpha_i \) on the output variation. Let \( R^f_i = (R^f_{i1}, R^f_{i2}, \ldots, R^f_{in}) \) be the vector of the relevances. The latter determines the ranking of the parameters in the following way. Input \( \alpha_i \) is defined to be more relevant than \( \alpha_j \) if and only if \( |R^f_i| > |R^f_j| \). The parameters are equally relevant for \( f \) if \( |R^f_i| = |R^f_j| \). The rank of \( \alpha_i \), denoted as \( r^f_i \), depends on the importance measure: \( \alpha_i \) has a higher rank (it has a greater impact on the output) than \( \alpha_j \) if it has greater relevance. Let \( r^f = (r^f_1, r^f_2, \ldots, r^f_n) \) be the vector of ranks.

The average rank is \( r^f_M = \sum_{i=1}^{n+1} i \cdot \frac{\alpha_i}{n} = \frac{n+1}{2} \).

The high parameters (or high-relevance parameters) are those whose rank is higher than the average rank \( r^f_M \); the low parameters are those parameters whose rank is smaller than \( r^f_M \).

**Example 2.** Consider \( A = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\} \) and the two objective functions \( f \) and \( g \). Given an SA technique, suppose the vector of importance measures for \( f \) is \( R^f = (0.1, 0.3, 0.2, 0.05, 0.35) \). No relevance is equal (i.e., there are no ties) so the rank vector is \( r^f = (4, 2, 3, 5, 1) \). Suppose the vector of importance measures for \( g \) is \( R^g = (0.05, 0.4, 0.2, 0.2, 0.15) \). Two relevances are equal (there are ties). Input \( \alpha_3 \) and \( \alpha_4 \) are equally ranked. In this case, the rank is \( \frac{2+3}{2} = 2.5 \) for both. The rank vector is then \( r^g = (5, 1, 2.5, 2.5, 4) \).

In the literature, SA techniques can be divided into global SA techniques and local SA techniques.

Global SA measures the importance of key parameters within the entire parameter space \( A \). Local SA measures the importance of parameters in a neighborhood of \( \alpha^0 \) and makes use of Taylor approximation. This approach presupposes that the objective function is differentiable in \( \alpha^0 \) and the changes in the inputs are small.

Following we briefly describe some well-known SA techniques.

(i) **Standardized regression coefficient** (global SA)

Let \( V \) denote variance and \( \sigma \) denote standard deviation. Consider the linear regression with dependent variable \( f \) and explanatory variables \( \alpha_i \), \( \forall i = 1, \ldots, n \), estimated with OLS method: \( f = \beta^0_0 + \sum_{i=1}^{n} \beta^0_i \cdot \alpha_i + u \). The standardized regression coefficient \( SRC^f_i \) measures the importance of \( \alpha_i \) (Saltelli and Marivoet 1990, Bring 1994, Saltelli et al. 2008):

\[
    SRC^f_i = \frac{\beta^0_i \cdot \sigma(\alpha_i)}{\sigma(f)}.
\]

(ii) **Sensitivity indices in variance-based decomposition methods** (global SA)
Variance-based methods study how the variance of the output is affected by (and apportioned to) the uncertain input factors. In variance-based methods, the importance of a parameter is generally represented through the First Order Sensitivity Index (FOSI) and the Total Order Sensitivity Index (TOSI) (Saltelli et al. 2008). The FOSI, here denoted as $SI_{i}^{1:j}$, measures the individual effect of the parameter on the output variance:

$$SI_{i}^{1:j} = \frac{V(E(f|\alpha_i))}{V(f)},$$  \hspace{1cm} (10)

where $V(E(f|\alpha_i))$ is the variance of the expectation of $f$ upon a fixed value of $\alpha_i$. The TOSI, here denoted as $SI_{i}^{T:j}$, measures the total contribution of $\alpha_i$ to the output variability, i.e., it is inclusive of the interaction effects with other parameters or groups of parameters. $SI_{i}^{T:j}$ can be calculated as (Saltelli et al. 2008)

$$SI_{i}^{T:j} = \frac{E(V(f|\alpha_{-i})))}{V(f)},$$  \hspace{1cm} (11)

where $f|\alpha_{-i} = f|\alpha_1, \alpha_2, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n$.  

(iii) Finite Change Sensitivity Indices (global SA)

The Finite Change Sensitivity Indices (FCSIs), introduced in Borgonovo (2010a, 2010b), focus on the output change due to a finite input change; they are based on the properties of functional ANOVA decomposition for finite changes (Rabitz and Alis 1999, Borgonovo 2010a). There exist two versions of FCSIs: First Order FCSI and Total Order FCSI.

The First Order FCSI of a parameter measures the individual effect of the parameter’s variation on $f$; the Total Order FCSI considers the total effect of a parameter’s variation on $f$, including both the individual contribution and the interactions between a parameter and the other parameters.

Consider the base value $\alpha^0 = (\alpha_0^0, \ldots, \alpha_n^0)$; the corresponding output is $f(\alpha^0)$. The parameters change from $\alpha^0$ to $\alpha^1 = (\alpha_1^1, \alpha_2^1, \ldots, \alpha_n^1) \in A$ and the corresponding output is $f(\alpha^1)$. The output variation is $\Delta f = f(\alpha^1) - f(\alpha^0)$.

Let $(\alpha_i^1, \alpha_{(-i)}^0) = (\alpha_0^0, \alpha_2^0, \ldots, \alpha_{i-1}^0, \alpha_i^1, \alpha_{i+1}^0, \ldots, \alpha_n^0)$ be obtained by varying the parameter $\alpha_i$ to the new value $\alpha_i^1$, while the remaining $n - 1$ parameters are fixed at $\alpha^0$. Similarly, $(\alpha_i^1, \alpha_j^1, \alpha_{(-i-j)}^0) = (\alpha_1^0, \alpha_2^0, \ldots, \alpha_{i-1}^0, \alpha_i^1, \alpha_{i+1}^0, \ldots, \alpha_{j-1}^0, \alpha_j^1, \alpha_{j+1}^0, \ldots, \alpha_n^0)$ is the vector of inputs assuming $\alpha_i$ and $\alpha_j$ are set to the new values, while the remaining $n - 2$ are unvaried, and so forth for all $j$-tuples of inputs, $j = 1, 2, \ldots, n$.

The individual effect of $\alpha_i$ on $\Delta f$ is $\Delta_i f = f(\alpha_i^1, \alpha_{(-i)}^0) - f(\alpha^0)$ and the First Order

\footnote{It can be shown that $V(E(f|\alpha_i)) = V(f) - E[V(f|\alpha_i)]$ (see Satelli et al. 2008).}

\footnote{Among the various variance-based decomposition methods, the High Dimensional Model Representation (HDMR) theory allows a complete decomposition of the output variance through a finite number of terms, under the assumption of independence (i.e., orthogonality) of the input factors (Sobol’ 1993, Sobol’ 2001, Saltelli et al. 2008):}

$$V(f) = \sum_{i=1}^{n} V_i^f + \sum_{i<j} V_{ij}^f + \sum_{i<j<m} V_{ij,m}^f + \cdots + V_{1,2,\ldots,n}^f,$$

where $V_i^f = V(E(f|\alpha_i))$; $V_{ij}^f$ is the interaction between $\alpha_i$ and $\alpha_j$; $V_{ij,m}^f$ is the interaction among $\alpha_i$, $\alpha_j$ and $\alpha_m$ and, finally, $V_{1,2,\ldots,n}^f$ is the residual portion of variance, explained by the interaction among all the $n$ parameters.

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FCSI of $\alpha_i$, denoted as $\Phi_{\alpha_i}^{1,f}$, is (Borgonovo 2010a):

$$\Phi_{\alpha_i}^{1,f} = \frac{\Delta_i f}{\Delta f}. \quad (12)$$

The interaction between $\alpha_i$ and $\alpha_j$, $\Delta_{ij} f$, is the portion of $f(\alpha_i^1, \alpha_j^1, \alpha^0_{(-i,j)}) - f(\alpha^0)$ that is not explained by the individual effects $\Delta_i f$ and $\Delta_j f$: $\Delta_{ij} f = f(\alpha_i^1, \alpha_j^1, \alpha^0_{(-i,j)}) - f(\alpha^0) - \Delta_i f - \Delta_j f$. Likewise, the interaction between the triplet of inputs $\alpha_i$, $\alpha_j$ and $\alpha_w$, denoted as $\Delta_{ijw} f$, is the portion of $f(\alpha_i^1, \alpha_j^1, \alpha_w^1, \alpha^0_{(-i,j,w)}) - f(\alpha^0)$ that is not explained by the individual effects and the interactions between all the possible pairs of inputs $\alpha_i$, $\alpha_j$ and $\alpha_w$:

$$\Delta_{ijw} f = f(\alpha_i^1, \alpha_j^1, \alpha_w^1, \alpha^0_{(-i,j,w)}) - f(\alpha^0) - \Delta_i f - \Delta_j f - \Delta_iw f - \Delta_{ijw} f$$

(analogously for a group of $s > 3$ parameters).

$\Delta f$ is equal to the sum of individual effects and interactions between parameters and groups of parameters (Borgonovo 2010a):

$$\Delta f = \sum_{i=1}^{n} \Delta_i f + \sum_{s=2}^{n} \sum_{i_1 < i_2 < \ldots < i_s} \Delta_{i_1,i_2,\ldots,i_s} f,$$

where $\sum_{i_1 < i_2 < \ldots < i_s} \Delta_{i_1,i_2,\ldots,i_s} f$ is the sum of the interactions between groups of $s$ parameters.

The total effect of the parameter $\alpha_i$, denoted as $\Delta_T f$, is the sum of the individual effect of $\alpha_i$ and of the interactions that involve $\alpha_i$:

$$\Delta_T f = \Delta_i f + \sum_{s=2}^{n} \sum_{i_1 < i_2 < \ldots < i_s \in \{i_1,i_2,\ldots,i_s\}} \Delta_{i_1,i_2,\ldots,i_s} f.$$ 

Borgonovo (2010a, Proposition 1) showed that $\Delta_T f$ is also obtained as

$$\Delta_T f = f(\alpha^1) - f(\alpha^0, \alpha_{(i)}^1), \quad \forall i = 1, 2, \ldots, n,$$

where $(\alpha^0, \alpha_{(i)}^1)$ is the point with all the parameters equal to the new value $\alpha^1$, except the parameter $\alpha_i$, which is equal to $\alpha^0$. The Total Order FCSI of the parameter $\alpha_i$, denoted as $\Phi_{\alpha_i}^{T,f}$, is (Borgonovo 2010a):

$$\Phi_{\alpha_i}^{T,f} = \frac{\Delta_T f}{\Delta f} = \frac{f(\alpha^1) - f(\alpha^0, \alpha_{(i)}^1)}{\Delta f}. \quad (13)$$

(iv) **Helton’s index** (local SA)

Helton (1993) proposed a variance decomposition of $f$ based on Taylor approximation. He assumed parameters are not correlated, so the variance of $f$ can be approximated by

$$\tilde{V}(f) = \sum_{i=1}^{n} \left[ f'_{\alpha_i}(\alpha^0) \right]^2 \cdot V(\alpha_i). \quad (14)$$
The impact of input $\alpha_i$ on $V(f)$ can be measured by

$$H_i^f(\alpha^0) = \frac{[f'_{\alpha_i}(\alpha^0)]^2 \cdot V(\alpha_i)}{V(f)}. \quad (15)$$

(v) **Normalized Partial Derivatives** (local SA)

Helton (1993) also proposed the adoption of normalized partial derivatives as sensitivity measures. He defined two versions of normalized partial derivatives (NPDs):

$$NPD^1_i(\alpha^0) = f'_{\alpha_i}(\alpha^0) \cdot \frac{\alpha_i^0}{f(\alpha^0)}, \quad (16)$$

$$NPD^2_i(\alpha^0) = f'_{\alpha_i}(\alpha^0) \cdot \frac{\sigma(\alpha_i)}{\hat{\sigma}(f)}, \quad (17)$$

where $\hat{\sigma}(f)$ is the square root of $\hat{V}(f)$ defined in (14). $NPD^1_i(\alpha^0)$ measures the elasticity of $f$ with respect to $\alpha$ in $\alpha^0$ assuming that the relative change in $\alpha_i$ is fixed for $i = 1, 2, \ldots, n$ (Helton 1993, p. 329). $NPD^2_i(\alpha^0)$ is the square root of (15).

(vi) **Differential Importance Measure** (local SA)

The total variation $f(\alpha^0 + d\alpha) - f(\alpha^0)$ of a differentiable function $f$ due to a local change $d\alpha$ can be approximated by the total differential

$$df = \sum_{i=1}^{n} f'_{\alpha_i}(\alpha^0) \cdot d\alpha_i.$$ 

The Differential Importance Measure (DIM) of parameter $\alpha_i$ is the ratio of the partial differential of $f$ with respect to $\alpha_i$ to the total differential of $f$ (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004):

$$DIM^f_i(\alpha^0, d\alpha) = \frac{df_{\alpha_i}}{df} = \frac{f'_{\alpha_i}(\alpha^0) \cdot d\alpha_i}{\sum_{j=1}^{n} f'_{\alpha_j}(\alpha^0) \cdot d\alpha_j}. \quad (18)$$

The DIM of a parameter represents the percentage of the function’s variation due to the variation of that parameter (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004).

There are two versions of DIM, according to the assumption made upon the variation structure of parameters: Uniform variation assumption (H1) or proportional variation assumption (H2). H1 implies $d\alpha_i = d\alpha_j$, $\forall \alpha_i, \alpha_j$. This assumption can be validly accepted only if all the parameters are expressed in the same unit of measure. The resulting DIM is

$$DIM^1_i(\alpha^0) = \frac{f'_{\alpha_i}(\alpha^0) \cdot d\alpha_i}{\sum_{j=1}^{n} f'_{\alpha_j}(\alpha^0) \cdot d\alpha_i} = \frac{f'_{\alpha_i}(\alpha^0)}{\sum_{j=1}^{n} f'_{\alpha_j}(\alpha^0)}. \quad (19)$$

H2 implies $d\alpha_i = \xi \cdot \alpha^0_i$ for some $\xi \neq 0$. This assumption can be adopted even when the parameters are expressed in different units of measure, because the parameters’ variation is defined with respect to the base value of each parameter (Borgonovo and Apostolakis...
2001, Borgonovo and Peccati 2004). The resulting DIM is

\[
\text{DIM}_1^f(\alpha^0) = \frac{\sum_{j=1}^{n} f'_\alpha(x^0_i) \cdot \alpha^0_j}{\sum_{j=1}^{n} f'_\alpha(x^0_i) \cdot \alpha^0_j}
\]

\[
\text{DIM}_2^f(\alpha^0) = \frac{\sum_{j=1}^{n} f'_\alpha(x^0_i) \cdot \xi \cdot \alpha^0_j}{\sum_{j=1}^{n} f'_\alpha(x^0_i) \cdot \alpha^0_j}
\]

\[
(20)
\]

4 Coherence between objective functions

Risk management problems are often characterized by the definition of more than one objective function (Borgonovo and Peccati 2006, Borgonovo, Gatti and Peccati 2010). For a given technique, the analysis can be applied using different objective functions. A relevant aspect is the evaluation of the coherence (or compatibility) between the results of the sensitivity analysis for different functions.

We consider the objective functions \( f, g : A \rightarrow \mathbb{R} \). The vector of importance measures are respectively \( R^f = (R^f_1, R^f_2, \ldots, R^f_n) \) and \( R^g = (R^g_1, R^g_2, \ldots, R^g_n) \); the ranking vectors are \( r^f = (r^f_1, r^f_2, \ldots, r^f_n) \) and \( r^g = (r^g_1, r^g_2, \ldots, r^g_n) \).

**Definition 3.** (Coherence) Given a technique of SA and two objective functions \( f \) and \( g \), they are coherent if the ranking vectors coincide: \( r^f = r^g \). If, in addition, the vectors of the relevances coincide, \( R^f = R^g \), they are strictly coherent.

**Example 3.** Consider the functions \( f \) and \( g \) and assume \( R^f = (0.1, -0.3, 0.2, 0.05, 0.35) \) and \( R^g = (0.07, 0.35, 0.15, 0.03, 0.40) \). Recalling that the rank is determined by the absolute value of the importance measure, the functions \( f \) and \( g \) determine the same ranking, \( r^f = r^g = (4, 2, 3, 5, 1) \), so \( f \) and \( g \) are coherent but not strictly coherent.

If two functions \( f \) and \( g \) are not coherent, the degree of incoherence can be alternatively measured through Spearman’s rank correlation coefficient (Spearman 1904) or top-down correlation coefficient (Iman and Conover 1987).

Spearman’s rank correlation coefficient (henceforth, Spearman’s coefficient) between two stochastic variables is the correlation coefficient between the ranks of the stochastic variables (Spearman 1904). In SA, Spearman’s coefficient between two objective functions \( f \) and \( g \), denoted as \( \rho_{f,g} \), is the correlation coefficient of the ranking vectors \( r^f \) and \( r^g \):

\[
\rho_{f,g} = \frac{\text{Cov}(r^f, r^g)}{\sigma(r^f) \cdot \sigma(r^g)} = \frac{\sum_{i=1}^{n} (r^f_i - \bar{r}^f) \cdot (r^g_i - \bar{r}^g)}{\sqrt{\sum_{i=1}^{n} (r^f_i - \bar{r}^f)^2} \cdot \sqrt{\sum_{i=1}^{n} (r^g_i - \bar{r}^g)^2}},
\]

where, as seen, \( \bar{r}^f_M = \bar{r}^g_M = \frac{n+1}{2} \). The coefficient \( \rho_{f,g} \) attributes the same weight to top and low parameters and lies in the interval \([-1, 1]\). The coefficient \( \rho_{f,g} \) is equal to 1 if and only if \( f \) and \( g \) are coherent according to Definition 3. Therefore, a value of \( \rho_{f,g} \) smaller than 1 signals incoherence between \( f \) and \( g \): The smaller the value of \( \rho_{f,g} \), the higher the degree of incoherence. The difference \( 1 - \rho_{f,g} \) can be taken as representative of the degree of incoherence.

Iman and Conover (1987) introduced the top-down correlation coefficient, a compatibility measure that attributes a higher weight to top parameters than to low parameters. This measure is based on **Savage Score** (Savage 1956). The Savage score of parameter \( \alpha_i \),
denoted as $S^f_i$, is

$$S^f_i = \sum_{h=r^f_i}^{n} \frac{1}{h}.$$  \hspace{1cm} (22)

$S^f = (S^f_1, S^f_2, \ldots, S^f_n)$ is the Savage scores’ vector. The average Savage score is $S^f_M = \frac{\sum_{i=1}^{n} S^f_i}{n} = 1$. Table 1 shows the value of Savage scores for $n = 5$.

| $r^f_i$ | $S^f_i$ |
|---------|---------|
| 1       | $\sum_{h=1}^{5} \frac{1}{h} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 2.283$ |
| 2       | $\sum_{h=2}^{5} \frac{1}{h} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 1.283$ |
| 3       | $\sum_{h=3}^{5} \frac{1}{h} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 0.783$ |
| 4       | $\sum_{h=4}^{5} \frac{1}{h} = \frac{1}{4} + \frac{1}{5} = 0.45$ |
| 5       | $\sum_{h=5}^{5} \frac{1}{h} = \frac{1}{5} = 0.2$ |

The top-down correlation coefficient between the objective functions $f$ and $g$, denoted as $\rho_{S^f, S^g}$, is the correlation coefficient between the Savage scores’ vectors $S^f$ and $S^g$ (Iman and Conover 1987):

$$\rho_{S^f, S^g} = \frac{\text{Cov}(S^f, S^g)}{\sigma(S^f) \cdot \sigma(S^g)} = \frac{\sum_{i=1}^{n} (S^f_i - S^f_M) \cdot (S^g_i - S^g_M)}{\sqrt{\sum_{i=1}^{n} (S^f_i - S^f_M)^2} \cdot \sqrt{\sum_{i=1}^{n} (S^g_i - S^g_M)^2}},$$  \hspace{1cm} (23)

where $S^f_M = S^g_M = 1$. The coefficient $\rho_{S^f, S^g}$ measures the compatibility between the parameters’ ranking of $f$ and $g$: The accordance between top parameters determines a remarkable influence on $\rho_{S^f, S^g}$, while the discordance between low parameters has a weak influence on $\rho_{S^f, S^g}$ (Iman and Conover 1987).

If the aim of the analysis is factor prioritization (i.e., identification of the most relevant parameters), the top-down coefficient should be preferred to Spearman’s coefficient.

The maximum value of $\rho_{S^f, S^g}$ is equal to 1. In case $f$ and $g$ have no ties, the minimum value is $-1$ for $n = 2$, it increases as $n$ increases, and it tends to $-0.645$ as $n$ tends to infinity (Iman and Conover 1987).

$\rho_{S^f, S^g}$ is equal to 1 if and only if $f$ and $g$ are strictly coherent. Therefore, a value of $\rho_{S^f, S^g}$ smaller than 1 signals incompatibility between $f$ and $g$. The smaller the value of $\rho_{S^f, S^g}$, the higher the incoherence level. The degree of incoherence of $f$ and $g$ can then be measured by $1 - \rho_{S^f, S^g}$.

Borgonovo et al. (2014) showed that an objective function $f$ and a monotonic transformation $g$ of it generate the same ranking of the parameters under several techniques.
This means that they are coherent according to Definition 3.

We now show that, if \( g \) is an affine transformation of \( f \), that is, \( g(\alpha) = l \cdot f(\alpha) + q \) for all \( \alpha \in A \), then \( f \) and \( g \) are strictly coherent under several techniques.

**Proposition 1.** A function and an affine transformation of it are strictly coherent under the following techniques:

(i) Standardized regression coefficient

(ii) Sensitivity Indices in variance-based decomposition models

(iii) Finite Change Sensitivity Indices

(iv) Helton’s index

(v) Normalized Partial Derivative (NPD2)

(vi) Differential Importance Measure.

**Proof.** By hypothesis, \( g(\alpha) = l \cdot f(\alpha) + q \). Therefore,

(i) \( g = l \cdot (\beta^l_0 + \sum_{i=1}^n \beta^l_i \cdot \alpha_i + u) + q = (l \cdot \beta^l_0 + q) + \sum_{i=1}^n (l \cdot \beta^l_i) \cdot \alpha_i + l \cdot u \), whence

\[
\beta^g_0 = l \cdot \beta^l_0 + q, \\
\beta^g_i = l \cdot \beta^l_i
\]

so that

\[
SRC^g_i = \frac{\beta^g_i \cdot \sigma(\alpha_i)}{\sigma(g)} = \frac{l \cdot \beta^l_i \cdot \sigma(\alpha_i)}{l \cdot \sigma(f)} = SRC^f_i.
\]

(ii) Denoting as \( f|\alpha_i \) (and \( g|\alpha_i \)) the function \( f \) (and \( g \)) conditional to a specific value of \( \alpha_i \), \( g|\alpha_i = (l \cdot f + q)|\alpha_i = (l \cdot f)|\alpha_i + q = l \cdot f|\alpha_i + q \). Therefore,

\[
SI^q_i = \frac{V(E(g|\alpha_i))}{V(g)} = \frac{V(E(l \cdot f|\alpha_i + q))}{V(l \cdot f + q)} = \frac{l^2 \cdot V(E(f|\alpha_i))}{l^2 \cdot V(f)} = SI^f_i.
\]

Analogously, \( g|\alpha_{-i} = (l \cdot f + q)|\alpha_{-i} = l \cdot f|\alpha_{-i} + q \). Hence,

\[
SI^{T,q}_i = \frac{E(V(g|\alpha_{-i}))}{V(f)} = \frac{E(V(l \cdot f|\alpha_{-i} + q))}{V(l \cdot f + q)} = \frac{l^2 \cdot E(V(f|\alpha_{-i}))}{l^2 \cdot V(f)} = SI^{T,f}_i.
\]

(iii) Since \( \Delta g = g(\alpha^1) - g(\alpha^0) = l \cdot f(\alpha^1) + q - l \cdot f(\alpha^0) - q = l \cdot (f(\alpha^1) - f(\alpha^0)) = l \cdot \Delta f \), and

\[
\Delta_i g = g(\alpha^1_i,\alpha^0_{-i}) - g(\alpha^0_i,\alpha^0_{-i}) = l \cdot f(\alpha^1_i,\alpha^0_{-i}) + q - l \cdot f(\alpha^0_{-i}) - q = l \cdot (f(\alpha^1_i,\alpha^0_{-i}) - f(\alpha^0_i)) = l \cdot \Delta_i f,
\]

then

\[
\Phi^{q,g}_i = \frac{\Delta_i g}{\Delta g} = \frac{l \cdot \Delta_i f}{l \cdot \Delta f} = \frac{\Delta_i f}{\Delta f} = \Phi^{f,g}_i.
\]

As for the Total Indices

\[
\Delta^T_i g = g(\alpha^1_i,\alpha^0_{-i}) - g(\alpha^0_i,\alpha^1_{-i}) = l \cdot f(\alpha^1_i) + q - l \cdot f(\alpha^0_i,\alpha^1_{-i}) - q = l \cdot (f(\alpha^1_i) - f(\alpha^0_i,\alpha^1_{-i})) = l \cdot \Delta^T_i f
\]
so that
\[ \Phi_i^{g,f} = \frac{\Delta_i^g}{\Delta g} = \frac{l \cdot \Delta_i^f}{\Delta f} = \Delta_i^f = \Phi_i^{f,g}. \]

(iv) From (14),
\[ \hat{V}(g) = \sum_{i=1}^{n} \left| g_{\alpha_i}(\alpha^0) \right|^2 \cdot V(\alpha_i) \]
\[ = \sum_{i=1}^{n} \left| l \cdot f_{\alpha_i}(\alpha^0) \right|^2 \cdot V(\alpha_i) = l^2 \cdot \sum_{i=1}^{n} \left| f_{\alpha_i}(\alpha^0) \right|^2 \cdot V(\alpha_i) = l^2 \cdot \hat{V}(f). \]

Hence,
\[ H_i^g(\alpha^0) = \frac{\left| g_{\alpha_i}(\alpha^0) \right|^2 \cdot V(\alpha_i)}{\hat{V}(g)} = \frac{l^2 \cdot \left| f_{\alpha_i}(\alpha^0) \right|^2 \cdot V(\alpha_i)}{l^2 \cdot \hat{V}(f)} = H_i^f(\alpha^0). \]

(v) Straightforward, since \( NPD2_i^f \) is the square root of \( H_i^f(\alpha^0) \).\(^6\)

(vi) From (18),
\[ DIM_i^g(\alpha^0, d\alpha) = \frac{g_{\alpha_i}(\alpha^0) \cdot d\alpha_i}{\sum_{j=1}^{n} g_{\alpha_j}(\alpha^0) \cdot d\alpha_j} = \frac{l \cdot f_{\alpha_i}(\alpha^0) \cdot d\alpha_i}{\sum_{j=1}^{n} l \cdot f_{\alpha_j}(\alpha^0) \cdot d\alpha_j} = \frac{f_{\alpha_i}(\alpha^0) \cdot d\alpha_i}{\sum_{j=1}^{n} f_{\alpha_j}(\alpha^0) \cdot d\alpha_j} = DIM_i^f(\alpha^0, d\alpha). \]
(This result is independent of the structure of \( d\alpha \).)

Remark 1. While we have proved that, for several SA techniques, a function and its affine transformation are coherent (even in a strict sense), it is intuitive to inductively believe that a function and its affine transformation share an absolute coherence, in that they are coherent for every existing SA technique. We leave the proof of this more general statement for future research.

5 Coherence between return rates and NPV

The investment risk can be defined as “the potential variability of financial outcomes” (White et al. 1997). The future outcomes of an investment are stochastic and the investor has limited information. Referring to NPV and IRR, Joy and Bradley (1973, p. 1255) wrote: “It has often been suggested that capital budgeting theory has over-emphasized the development of such techniques with little regard for the typically poor data used in project evaluation and the effect that errors in capital budgeting inputs have on project

\(^6\)It is worth noting that \( f \) and \( g \) are coherent but not strictly coherent under \( NPD2_i^f \) technique:
\[ NPD1_i^g(\alpha^0) = g_{\alpha_i}(\alpha^0) \cdot \frac{\alpha_i}{g(\alpha^0)} = l \cdot f_{\alpha_i}(\alpha^0) \cdot \frac{\alpha_i}{g(\alpha^0)} = l \cdot \frac{f(\alpha^0)}{g(\alpha^0)} \cdot NPD1_i^f(\alpha^0) \]
so that \(|NPD1_i^f| > |NPD1_i^g| \) implies \(|NPD1_i^g| > |NPD1_i^f| \). Therefore, the parameters’ ranking in \( f \) and \( g \) is equal: \( r_f = r_g \).
profitability.” The practise of valuation criteria should be corroborated by a careful investment risk analysis.

Given an investment model based on a set of value drivers, SA allows the evaluator to identify the most relevant parameters in terms of variation of the value. The most relevant parameters are the risk factors that mainly influence the investment. After SA has been performed, the investment risk can be reduced through information insights on the main risk factors identified by the analysis; the collection of extra information on these parameters allows more precise estimates and a remarkable uncertainty reduction (Borgonovo and Peccati 2006). Furthermore, the potential investor is able to appreciate the convenience of possible hedging strategies.

As the NPV is the main decision criterion in capital budgeting theory, the analysis of the parameters’ relevance on NPV variability is fundamental. Any relative measure of worth should be consistent with NPV not only in terms of classical consistency but also in terms of output variability with respect to changes in the inputs. We give the following definition (see also Marchioni and Magni 2018).

**Definition 4.** (strong NPV-consistency) Given an analysis technique $T$, a metric $\varphi$ (and its associated decision criterion) is strongly NPV-consistent (or NPV-compatible) under $T$ if it fulfills Definition 2 and NPV and $\varphi$ are coherent functions. The metric $\varphi$ is strictly NPV-consistent if the coherence is strict.

If a metric/criterion possesses strong NPV-consistency, the investor can equivalently adopt NPV or such criterion for measuring the value creation under uncertainty. In case a metric is not strongly NPV-consistent, the degree of incompatibility can be measured through Spearman’s coefficient or through top-down coefficient, as seen in section 4.

We now show that, contrary to IRR, the SL-AIRR possesses strong NPV-consistency. To this end, we maintain the symbol $\alpha = (\alpha_1, \ldots, \alpha_n)$ as the vector of the project’s value drivers (revenues, costs, interest, taxes, working capital, fixed assets etc.) and $\alpha^0$ is the base value. We assume that the initial invested capital (or borrowed amount) is exogenously given, as well as the COC (and $p$). The economic profitability of $P$ depends on the realization of the value drivers, which affect the FCFs, as seen in section 2: $F_t = F_t(\alpha)$, $t = 1, 2, \ldots, p$. We now let $f(\alpha) = \text{NPV}(\alpha) = -c_0 + \sum_{t=1}^{p} F_t(\alpha)(1 + k)^{-t}$ and $g(\alpha) = \bar{i}(\alpha)$ be the SL-AIRR (as anticipated, we omit the dependence on $C^{sl}$ for simplicity).

**Proposition 2.** For any fixed $k$, $c_0$, and $p$, SL-AIRR and NPV are strongly consistent in a strict sense under the following techniques:

(i) Standardized regression coefficient

(ii) Sensitivity Indices in variance-based decomposition models

(iii) Finite Change Sensitivity Indices

(iv) Helton’s index

(v) Normalized Partial Derivative $(NPD^2_i)$

(vi) Differential Importance Measure.
Proof. The capital $C^sl$ does not depend on $\alpha$ so, using (7), $i(\alpha) = q + l \cdot \text{NPV}(\alpha)$ where $q = k$ and $l = (1 + k)/C^sl$. The thesis follows from Proposition 1.

The above proposition guarantees that the value drivers’ effect on the variability of SL-AIRR and NPV is the same, not only in terms of ranks ($r_{\text{npv}} = r^i$) but also in terms of relevances ($R_{\text{npv}} = R^i$). Therefore, $\rho_{\text{npv}} = \rho_{S^i, \text{npv}} = 1$. This means that an investor can equivalently employ SL-AIRR or NPV to analyze an investment under uncertainty. By contrast, it should be evident that such a nice property is not satisfied by the IRR and, in general, it is not possible to determine an analytical relationship between NPV and IRR in a sensitivity analysis (see also Borgonovo and Peccati 2006, Percoco and Borgonovo 2012). Indeed, let $\alpha^* \in A$ be a given value of parameters and $x^*$ be the associated IRR, such that $\text{NPV}(\alpha^*, x^*) = 0$. If there exists a neighbourhood of $\alpha^*$ where function $\text{NPV}(\alpha, k)$ is a continuously differentiable function and $\text{NPV}'_k(\alpha^*, x^*) \neq 0$, then there exists a neighbourhood $V(\alpha^*) \subset A$ and a neighbourhood $W(x^*) \subset \mathbb{R}$ such that $x(\alpha) : V \rightarrow W$ is the implicitly-defined function from the equation $\text{NPV}(\alpha, k) = 0$ and

$$x(\alpha^*) = x^*, \quad \text{NPV}(\alpha, x(\alpha)) = 0, \forall \alpha \in V,$$

$$x'_a(\alpha) = -\frac{\text{NPV}'_a(\alpha, x(\alpha))}{\text{NPV}'_k(\alpha, x(\alpha))}, \forall \alpha \in V.$$ 

In particular,

$$(24)$$

$$x'_a(\alpha^*) = -\frac{\text{NPV}'_a(\alpha^*, x^*)}{\text{NPV}'_k(\alpha^*, x^*)}.$$ 

6 Worked examples and comparison of SL-AIRR and IRR

The aim of this section is to contrast SL-AIRR with IRR via some worked examples. They show that the IRR is unreliable for the following reasons: (i) it is not strongly NPV-compatible, (ii) it may not exist in some scenario, (iii) multiple IRRs may arise, and (iv) the financial nature of IRR may easily change under changes in the value drivers. By contrast, the same examples illustrate what we have proved in the previous sections: The SL-AIRR exists and is unique, it is financially unambiguous, and it is strongly NPV-consistent. To this end, we will discuss a simple model, consisting of a firm facing the opportunity of investing in a 4-period project whose estimated revenues and costs are denoted as $R_i$ and $C_i$. We assume that clients pay in cash and suppliers are paid in cash, and also assume that the tax rate is zero, $\tau = 0$. This implies $\text{Dep}_t = -\Delta WC_t + \Delta NFA_t$, whence $F_t = R_t - C_t - \text{Dep}_t - (\Delta WC_t + \Delta NFA_t) = R_t - C_t$. The project’s value drivers are then $\alpha_i = R_i$ for $i = 1, 2, 3, 4$ and $\alpha_i = C_{i-4}$ for $i = 5, 6, 7, 8$. Hence, the value driver’s

---

5Evidently, SL-AIRR is strongly NPV-consistent under $NPD^1_i$ as well but not in a strict sense.

6Let $\alpha$ be a generic value belonging to a neighbourhood of $\alpha^*$. $\text{NPV}(\alpha, k)$ is the NPV calculated with discount rate $k$. 

---
vector is \( \alpha = \{ R_1, R_2, R_3, R_4, C_1, C_2, C_3, C_4 \} \). NPV is computed as:

\[
\text{NPV}(\alpha) = -c_0 + \frac{R_1 - C_1}{1 + k} + \frac{R_2 - C_2}{(1 + k)^2} + \frac{R_3 - C_3}{(1 + k)^3} + \frac{R_4 - C_4}{(1 + k)^4}.
\]

**Example 4.** Assume \( c_0 = 750 \) and \( k = 10\% \). Table 2 describes the base value \( \alpha^0 = (R^0_1, R^0_2, R^0_3, R^0_4, C^0_1, C^0_2, C^0_3, C^0_4) \) and reports the corresponding Free Cash Flows and valuation metrics. The NPV is 157.37 = \(-750 + 380/1.1 + 270/(1.1)^2 + 360/(1.1)^3 + 100/(1.1)^4\). Considering that 750/4 = 187.5, the vector of capitals associated with SL-AIRR is \( c^\text{sl} = (750, 562.5, 375, 187.5, 0) \) and \( C^\text{sl} = 1712.15 = 750 + 562.5/1.1 + 375/(1.1)^2 + 187.5/(1.1)^3 \). Therefore, SL-AIRR is equal to \( \bar{i} = 10\% + 157.37/1712.15 \cdot 1.1 = 20.11\% \). The IRR exists and is unique: \( x = 20.86\% \).

| Table 2: Investment evaluated in \( \alpha^0 \) |
|-------|
| 0  | 1  | 2  | 3  | 4  |
| \( R^0_t \) | 580 | 570 | 560 | 400 |
| \( C^0_t \) | 200 | 300 | 200 | 300 |
| \( F_t \) | -750 | 380 | 270 | 360 | 100 |
| Valuation | | | | |
| NPV | 157.37 |
| \( \bar{i} \) | 20.11\% |
| \( x \) | 20.86\% |

We now illustrate some SA applications to NPV, SL-AIRR and IRR focusing on two techniques: FCSI and DIM. They give an illustration of the problems of the IRR and, by contrast, the reliability of SL-AIRR. Among other things, it will turn out that the degree of incoherence between IRR and NPV can be rather high even in very simple cases.

### 6.1 IRR versus SL-AIRR using FCSIs

We show four numerical applications, aimed at illustrating the aforementioned problems of the IRR:

1. in the first application, IRR exists and is unique but is not strongly NPV-consistent
2. in the second application, despite IRR exists and is unique in \( \alpha^0 \), it does not exist in \( \alpha^1 \), making it impossible to perform the SA
3. in the third application, multiple IRRs arise for \( \alpha = \alpha^1 \); hence, the SA with IRR is ambiguous
4. in the fourth application, IRR changes its financial nature from investment rate (in \( \alpha^0 \)) to financing rate (in \( \alpha^1 \)).

No such problems will arise with SL-AIRR.

**Example 5.** (First application: NPV inconsistency) Consider project \( P \) described in Example 4 (see Table 2). Let \( \alpha^1 \) be the vector of new values of revenues and costs (see Table 3), with the corresponding new values of \( F_t, \) NPV, SL-AIRR, and IRR. In \( \alpha^1 \), NPV is 442.92, SL-AIRR is 38.46\%, IRR is 41.12\% (it exists and is unique). The observed
variations are: $\Delta{NPV} = 285.55 = 442.92 - 157.37$; $\Delta{i} = 18.35\% = 38.46\% - 20.11\%$; $\Delta{x} = 20.25\% = 41.12\% - 20.86\%$. Table 4 shows the First Order FCSIs ($\Phi_{i}^{1,f}$), the ranks ($r_{f_i}$), and the Savage Scores of parameters ($S_{i}^{f}$) for NPV, SL-AIRR and IRR.

The First Order FCSIs of NPV and SL-AIRR are equal: $\Phi_{i}^{1,\text{npv}} = \Phi_{i}^{1,\bar{i}}$. SL-AIRR and NPV are strongly coherent in a strict sense. Evidently the parameters’ ranking of NPV and SL-AIRR is equal, and $\rho_{i,\text{npv}} = \rho_{S_{i},\text{npv}} = 1$.

The First Order FCSIs of NPV and IRR are different: $\Phi_{i}^{1,\text{npv}} \neq \Phi_{i}^{1,x}$. NPV ranking and IRR ranking of parameters are also different. Therefore, IRR is not NPV-consistent according to Definition 4. The degree of inconsistency may be measured via (one minus) Spearman’s coefficient or top-down coefficient: $1 - \rho_{x,\text{npv}} = 1 - 0.857 = 0.143$ and $1 - \rho_{S_{x},\text{npv}} = 1 - 0.77 = 0.23$ are the degrees of incompatibility of NPV and IRR according to the two alternative measures.

Table 5 shows Total Order FCSIs ($\Phi_{i}^{T,f}$), ranks ($r_{f_i}$), and Savage scores ($S_{i}^{f}$) for the three metrics. The Total Order FCSIs of NPV and SL-AIRR are equal: $\Phi_{i}^{T,\text{npv}} = \Phi_{i}^{T,\bar{i}}$. SL-AIRR and NPV are strictly coherent.

The Total Order FCSIs of NPV and IRR are different: $\Phi_{i}^{T,\text{npv}} \neq \Phi_{i}^{T,x}$. The parameters’ ranking differs as well. It is clear that, even in such a simple example, IRR’s inconsistency is not negligible: The degree of incoherence using the Spearman’s coefficient is $1 - \rho_{x,\text{npv}} = 1 - 0.667 = 0.333$ and is even greater if top-down coefficient is used, $1 - \rho_{S_{x},\text{npv}} = 1 - 0.409 = 0.591$.

Table 3: Investment evaluated in $\alpha^1$

|       | 0   | 1   | 2   | 3   | 4   |
|-------|-----|-----|-----|-----|-----|
| $R_{i}$ | 800 | 810 | 780 | 630 |
| $C_{i}$ | 350 | 250 | 380 | 600 |
| $F_{i}$ | -750 | 450 | 560 | 400 | 30 |

| Valuation |       |
|-----------|-------|
| NPV       | 442.92 |
| $i$       | 38.46\% |
| $x$       | 41.12\% |

Example 6. (Second application: Nonexistence) Consider a project $P$ such that $c_0 = 750$ and $k = 10\%$. Hence $C^{sl} = 1712.15$. The base value is described in the input vector $\alpha^0 = (630, 740, 850, 600, 180, 390, 490, 550)$; the input vector for the realized scenario is $\alpha^1 = (600, 700, 800, 500, 200, 400, 500, 850)$, a worse situation in terms of both revenues and costs. Table 6 reports cash flows, NPV, SL-AIRR, and IRR. In $\alpha^0$ IRR exists, is unique, and is equal to 28.52\%. In $\alpha^1$ IRR does not exist. This implies that the sensitivity analysis cannot be applied for IRR in $\alpha^1$: $\Delta{x}$ is not defined, hence the First Order FCSIs of IRR are not calculable; the total effects of parameters on IRR are not defined either, therefore the Total Order FCSIs of IRR do not exist.

SL-AIRR does not suffer from this problem because it always exists and is unique. Table 7 shows the First Order and Total Order FCSIs of NPV and SL-AIRR: As expected, SL-AIRR and NPV are strongly coherent in a strict from (in this case, the First Order and Total Order FCSIs of NPV are equal: Interaction effects are nihil. Obviously, the same holds for SL-AIRR).
Table 4: First Order FCSI

| Parameter | NPV | SL-AIRR | IRR |
|-----------|-----|---------|-----|
|           | $\Phi_1^{NPV}$ | $r_1^{NPV}$ | $S_1^{NPV}$ | $\Phi_1^{SL}$ | $r_1^{SL}$ | $S_1^{SL}$ | $\Phi_1^{IRR}$ | $r_1^{IRR}$ | $S_1^{IRR}$ |
| $R_1$     | 70.04% | 2 | 1.718 | 70.04% | 2 | 1.718 | 70.78% | 1 | 2.718 |
| $R_2$     | 69.46% | 3 | 1.218 | 69.46% | 3 | 1.218 | 64.05% | 3 | 1.218 |
| $R_3$     | 57.89% | 4 | 0.885 | 57.89% | 4 | 0.885 | 45.56% | 5 | 0.635 |
| $R_4$     | 55.01% | 5 | 0.635 | 55.01% | 5 | 0.635 | 37.68% | 7 | 0.268 |
| $C_1$     | -47.76% | 6 | 0.435 | -47.76% | 6 | 0.435 | -46.93% | 4 | 0.885 |
| $C_2$     | 14.47% | 8 | 0.125 | 14.47% | 8 | 0.125 | 13.68% | 8 | 0.125 |
| $C_3$     | -47.36% | 7 | 0.268 | -47.36% | 7 | 0.268 | -45.25% | 6 | 0.435 |
| $C_4$     | -71.76% | 1 | 2.718 | -71.76% | 1 | 2.718 | -76.83% | 2 | 1.718 |

Correlations

$\rho_{\Phi, NPV} = 1$
$\rho_{S, NPV} = 1$
$\rho_{x, NPV} = 0.857$
$\rho_{S, S NPV} = 0.770$

Table 5: Total Order FCSI

| Parameter | NPV | SL-AIRR | IRR |
|-----------|-----|---------|-----|
|           | $\Phi_1^{NPV}$ | $r_1^{NPV}$ | $S_1^{NPV}$ | $\Phi_1^{SL}$ | $r_1^{SL}$ | $S_1^{SL}$ | $\Phi_1^{IRR}$ | $r_1^{IRR}$ | $S_1^{IRR}$ |
| $R_1$     | 70.04% | 2 | 1.718 | 70.04% | 2 | 1.718 | 75.70% | 1 | 2.718 |
| $R_2$     | 69.46% | 3 | 1.218 | 69.46% | 3 | 1.218 | 65.33% | 2 | 1.718 |
| $R_3$     | 57.89% | 4 | 0.885 | 57.89% | 4 | 0.885 | 44.78% | 4 | 0.885 |
| $R_4$     | 55.01% | 5 | 0.635 | 55.01% | 5 | 0.635 | 34.09% | 6 | 0.435 |
| $C_1$     | -47.76% | 6 | 0.435 | -47.76% | 6 | 0.435 | -57.78% | 3 | 1.218 |
| $C_2$     | 14.47% | 8 | 0.125 | 14.47% | 8 | 0.125 | 13.18% | 8 | 0.125 |
| $C_3$     | -47.36% | 7 | 0.268 | -47.36% | 7 | 0.268 | -31.29% | 7 | 0.268 |
| $C_4$     | -71.76% | 1 | 2.718 | -71.76% | 1 | 2.718 | -34.93% | 5 | 0.635 |

Correlations

$\rho_{\Phi, NPV} = 1$
$\rho_{S, NPV} = 1$
$\rho_{x, NPV} = 0.667$
$\rho_{S, S NPV} = 0.409$
Table 6: IRR not existing in $\alpha$

|       | $\alpha^0$ | $\alpha^1$ |
|-------|-------------|-------------|
| $F_0$ | -750        | -750        |
| $F_1$ | 450         | 400         |
| $F_2$ | 350         | 300         |
| $F_3$ | 360         | 300         |
| $F_4$ | 50          | -350        |

| Valuation | $\alpha^0$ | $\alpha^1$ |
|-----------|-------------|-------------|
| NPV       | 252.97      | -152.09     |
| $\bar{i}$ | 26.25%      | 0.23%       |
| $x$       | 28.52%      | -           |

Table 7: IRR not existing in $\alpha^1$: First Order and Total Order FCSIs

| Parameter | $\Phi_{i, npv}^{T}$ | $r_{i, npv}^{T}$ | $\Phi_{i, irr}^{T}$ | $r_{i, irr}^{T}$ | $\Phi_{i, x}^{T}$ | $r_{i, x}^{T}$ |
|-----------|----------------------|------------------|---------------------|------------------|------------------|--------------|
| $R_1$     | 6.73%                | 5                | 6.73%               | 5                | -                | -            |
| $R_2$     | 8.16%                | 4                | 8.16%               | 4                | -                | -            |
| $R_3$     | 9.27%                | 3                | 9.27%               | 3                | -                | -            |
| $R_4$     | 16.86%               | 2                | 16.86%              | 2                | -                | -            |
| $C_1$     | 4.49%                | 6                | 4.49%               | 6                | -                | -            |
| $C_2$     | 2.04%                | 7                | 2.04%               | 7                | -                | -            |
| $C_3$     | 1.85%                | 8                | 1.85%               | 8                | -                | -            |
| $C_4$     | 50.59%               | 1                | 50.59%              | 1                | -                | -            |

Example 7. (Third application: Nonuniqueness). Consider a project $P$, with $c_0 = 800$ and $k = 15\%$. Therefore, $C^{sl} = 1755.70$. The base value is described in the input vector $\alpha^0 = (2300, 1100, 1400, 2000, 1300, 1200, 1600, 1300)$; the new input vector is $\alpha^1 = (2960, 500, 400, 2300, 600, 1440, 2750, 550)$. Table 8 shows cash flows, NPV, SL-AIRR, and IRR in $\alpha^0$ and $\alpha^1$. In $\alpha^0$, the IRR function supplies a unique value and is equal to 36.72%. For $\alpha^1$, there exist three different IRRs: $x_1(\alpha^1) = 8.07\%$, $x_2(\alpha^1) = 25.0\%$, $x_3(\alpha^1) = 61.93\%$ so the sensitivity analysis is problematic: It is not clear which one IRR should be the relevant one.

Table 9 shows the First Order and Total Order FCSIs of NPV and SL-AIRR: As obvious, SL-AIRR and NPV are strongly coherent in strict sense.
Table 9: Multiple IRR in $\alpha^1$: First Order and Total Order FCSIs

| Parameter | \(\Phi_{T,\text{npv}}\) | \(\Phi_{T,\bar{i}}\) | \(\Phi_{T,x}\) |
|-----------|-----------------|-----------------|-----------------|
| \(R_1\)   | 215.86%         | 4               | 215.86%         |
| \(R_2\)   | 170.64%         | 5               | 170.64%         |
| \(R_3\)   | 247.31%         | 2               | 247.31%         |
| \(R_4\)   | 64.51%          | 8               | 64.51%          |
| \(C_1\)   | 228.94%         | 3               | 228.94%         |
| \(C_2\)   | 68.26%          | 7               | 68.26%          |
| \(C_3\)   | 284.40%         | 1               | 284.40%         |
| \(C_4\)   | 161.29%         | 6               | 161.29%         |

Table 10: IRR changes its financial nature

| \(\alpha^0\) | \(\alpha^1\) |
|-------------|-------------|
| \(F_0\)     | -500       | -500       |
| \(F_1\)     | -700       | -400       |
| \(F_2\)     | 1345       | 1695       |
| \(F_3\)     | 35         | 385        |
| \(F_4\)     | 340        | -1210      |

Table 10: IRR changes its financial nature

Example 8. (Fourth application: Financial nature). Consider a project \(P\) such that \(c_0 = 500\) and \(k = 5\%\). Therefore \(C^{cl} = 1191.88\). The base case is described in the input vector \(\alpha^0 = (800, 2150, 950, 850, 1500, 805, 915, 510)\). The realized vector is \(\alpha^1 = (600, 2000, 800, 800, 1000, 305, 415, 210)\). The difference between \(\alpha^0\) and \(\alpha^1\) lies in lower revenues for \(\alpha^1\) and in intertemporal cost allocation: The total amount of costs is the same in the two cases, but in \(\alpha^1\) costs are highly concentrated in period 4 (one may assume remedial costs at the end of the project have been paid). Table 10 shows the project’s cash flows and the corresponding NPV, SL-AIRR, and IRR in \(\alpha^0\) and \(\alpha^1\). In the base case IRR exists, is unique, and is equal to 22.17% and the Hotelling capital vector is \((500, 1310.85, 256.45, 278.30, 0)\) whence \(C^x(\alpha^0) = 2221.44\); therefore, IRR is an investment rate in \(\alpha^0\). In \(\alpha^1\), IRR exists and is unique, and it is equal to 10%, associated with the Hotelling vector \((500, 950, -650, -1100, 0)\); this implies \(C^x(\alpha^1) = -135.03 < 0\) which means that IRR is a financing rate in \(\alpha^1\). This shows that a change in the value drivers’ vector may cause IRR to change financial nature (from investment rate to financing rate or viceversa). The decomposition of the change of the output in FCSI is economically dubious, as the model output does not merely change in quantitative terms, but it changes in meaning: No more a rate of return but a financing rate.

SL-AIRR does not suffer from this problem, because its financial nature only depends on the sign of \(c_0\). In this case, SL-AIRR is an investment rate, regardless of changes in the value drivers.

It is worth noting that two or more of the above mentioned problems may occur simultaneously. For example, in the latter example IRR changes financial nature from
scenario $\alpha_0$ to scenario $\alpha_1$ and, at the same time, it suffers from a problem of nonexistence of the importance measure of one of the value drivers, namely, the costs in period 4: $x(\alpha_1^4, \alpha_0^{0-8})$ is not defined because the associated cash flows vector $(-500, -700, 1345, 35, -1160)$ does not admit any real $\text{IRR} > -1$. Hence, $\Phi_8^{1, x}$ does not exist. Consequently, the parameters ranking for $\text{IRR}$ is not possible and correlation coefficients are not computable (see Table 11).

### Table 11: First Order FCSI: IRR changes its financial nature

| Parameter | $\Phi_1^{1, npv}$ | $r_i^{1, npv}$ | $S_i^{1, npv}$ | $\Phi_1^{1, S}$ | $r_i^{1, S}$ | $S_i^{1, S}$ | $\Phi_1^{1, x}$ | $r_i^{1, x}$ | $S_i^{1, x}$ |
|-----------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $R_1$     | 51.53%          | 5              | 0.635          | 51.53%         | 5              | 0.635          | 79.64%         |                |                |
| $R_2$     | 36.80%          | 6              | 0.435          | 36.80%         | 6              | 0.435          | 53.16%         |                |                |
| $R_3$     | 35.05%          | 7              | 0.268          | 35.05%         | 7              | 0.268          | 45.59%         |                |                |
| $R_4$     | 11.13%          | 8              | 0.125          | 11.13%         | 8              | 0.125          | 12.17%         |                |                |
| $C_1$     | $-128.81\%$    | 2              | 1.718          | $-128.81\%$   | 2              | 1.718          | $-268.88\%$   |                |                |
| $C_2$     | $-122.68\%$    | 3              | 1.218          | $-122.68\%$   | 3              | 1.218          | $-175.42\%$   |                |                |
| $C_3$     | $-116.84\%$    | 4              | 0.885          | $-116.84\%$   | 4              | 0.885          | $-127.90\%$   |                |                |
| $C_4$     | 333.82%         | 1              | 2.718          | 333.82%        | 1              | 2.718          |                |                |                |

Correlations

| $\rho_{\text{npv}}$ | 1 |
| $\rho_{S \text{npv}}$ | 1 |
| $\rho_{x \text{npv}}$ | - |
| $\rho_{S \text{npv}}$ | - |

#### 6.2 IRR versus SL-AIRR using DIMs

The DIM technique is a local SA technique, so it measures the value drivers’ impact on the objective function in a neighbourhood of $\alpha^0$. We assume that changes in the inputs are proportional, so the DIM is described in eq. (20). In particular, the first partial derivatives of $NPV(\alpha)$, evaluated in $\alpha^0$, are

$$NPV_\alpha'(\alpha^0) = \begin{cases} (1 + k)^{-i}, & i = 1, 2, 3, 4; \\ -(1 + k)^{-(i-4)}, & i = 5, 6, 7, 8. \end{cases} \tag{25}$$

The first partial derivatives of SL-AIRR, evaluated in $\alpha^0$, are

$$\bar{i}_\alpha'(\alpha^0) = NPV_\alpha'(\alpha^0) \cdot \frac{(1 + k)}{C^*}. $$

The partial derivative of $NPV(\alpha, k)$ with respect to $k$ is

$$NPV_k^\alpha(\alpha, k) = \frac{-R_1 - C_1}{(1 + k)^2} - 2 \cdot \frac{R_2 - C_2}{(1 + k)^3} - 3 \cdot \frac{R_3 - C_3}{(1 + k)^4} - 4 \cdot \frac{R_4 - C_4}{(1 + k)^5}$$

and

$$NPV_k^\alpha(\alpha^0, x^0) = \frac{-R_1^0 - C_1^0}{(1 + x^0)^2} - 2 \cdot \frac{R_2^0 - C_2^0}{(1 + x^0)^3} - 3 \cdot \frac{R_3^0 - C_3^0}{(1 + x^0)^4} - 4 \cdot \frac{R_4^0 - C_4^0}{(1 + x^0)^5}. $$
From (24) and (25)
\[
x'_\alpha(\alpha^0) = \begin{cases} 
-(1 + x^0)^{-i} \cdot (NPV'_k(\alpha^0, x^0))^{-1}, & i = 1, 2, 3, 4; \\
(1 + x^0)^{-(i-4)} \cdot (NPV'_k(\alpha^0, x^0))^{-1}, & i = 5, 6, 7, 8.
\end{cases}
\]

We illustrate a numerical application of DIM technique where IRR and NPV are not coherent according to Definition 4. It is evident that SL-AIRR is, again, strongly coherent in a strict form with NPV.

**Example 9.** We consider an investment \( P \), with \( c_0 = 900 \) and \( k = 8\% \). Therefore \( C^{sl} = 2089.41 \). The base value is \( \alpha^0 = (900, 1000, 1100, 1200, 600, 700, 800, 900) \). The corresponding cash-flow vector is \( F = (-900, 300, 300, 300, 300) \) and \( NPV(\alpha^0) = 93.64 \), \( \bar{i}(\alpha^0) = 12.84\% \), \( x(\alpha^0) = 12.59\% \). Table 12 shows the DIMs, the ranks, and the Savage scores. The DIMs for NPV and IRR are different: \( DIM_i^{npv}(\alpha^0) \neq DIM_i^{x}(\alpha^0) \). Not even the ranking is equal; for example, \( R_1 \) has rank 4 for NPV, while it has rank 1 for IRR; \( R_4 \) has rank 1 for NPV, while it has rank 4 for IRR. IRR and NPV are not coherent according to Definition 4. As \( 1 - \rho_{x, npv} = 0.262 \) and \( 1 - \rho_{Sx, Snpv} = 0.691 \), the NPV-inconsistency of IRR is remarkable, especially when using top-down coefficient.

**Table 12: Coherence under DIM technique**

| Parameter | \( \alpha^0 \) | NPV | SL-AIRR | IRR |
|-----------|---------------|-----|---------|-----|
|           |               | \( DIM_i^{npv}(\alpha^0) \) | \( r_i^{npv} \) | \( S_i^{npv} \) | \( DIM_i^x(\alpha^0) \) | \( r_i^x \) | \( S_i^x \) |
| \( R_1 \) | 900          | 83.87\% | 4 | 0.885 | 83.87\% | 4 | 0.885 | 88.82\% | 1 | 2.718 |
| \( R_2 \) | 1000         | 86.28\% | 3 | 1.218 | 86.28\% | 3 | 1.218 | 87.65\% | 2 | 1.718 |
| \( R_3 \) | 1100         | 87.88\% | 2 | 1.718 | 87.88\% | 2 | 1.718 | 85.64\% | 3 | 1.218 |
| \( R_4 \) | 1200         | 88.77\% | 1 | 2.718 | 88.77\% | 1 | 2.718 | 82.97\% | 4 | 0.885 |
| \( C_1 \) | 600          | -55.91\% | 8 | 0.125 | -55.91\% | 8 | 0.125 | -59.21\% | 8 | 0.125 |
| \( C_2 \) | 700          | -60.40\% | 7 | 0.268 | -60.40\% | 7 | 0.268 | -61.36\% | 7 | 0.268 |
| \( C_3 \) | 800          | -63.91\% | 6 | 0.435 | -63.91\% | 6 | 0.435 | -62.28\% | 5 | 0.635 |
| \( C_4 \) | 900          | -66.58\% | 5 | 0.635 | -66.58\% | 5 | 0.635 | -62.23\% | 6 | 0.435 |

**Correlations**

\[
\begin{align*}
\rho_{x, npv} &= 1 \\
\rho_{Sx, Snpv} &= 1 \\
\rho_{x, xpv} &= 0.738 \\
\rho_{Sx, Snpv} &= 0.309
\end{align*}
\]

### 7 Concluding remarks

Many different investment criteria are available to managers, professionals and practitioners. NPV is considered a theoretically reliable measure of economic profitability. Industrial and financial investments are often evaluated through relative measures of worth as well. The most widely used relative measure is the IRR, which assumes that capital appreciates exponentially between two cash-flow dates and decreases, at the end of the period, by the distributed amount (or increases by the contributed amount). Recently, it has been introduced a new class of return rates named AIRR (Magni 2010, Magni 2013). Among these, this paper introduces the SL-AIRR, which assumes that the capital depreciates linearly (in a straight-line fashion).
Both IRR and SL-AIRR are coherent with NPV in the sense that both IRR criterion and SL-AIRR criterion correctly signal value creation or value destruction, just like the NPV (and, therefore, the decision made using one of the metrics is the same).

However, the IRR suffers from several difficulties: Existence, uniqueness and financial nature of IRR (investment rate or financing rate) depend on the project value drivers. A change in the value drivers may modify the financial nature of IRR or generate multiple IRRs or make the IRR nonexistent. SL-AIRR exists and is unique, and it has an unambiguous financial nature, independent of the value drivers.

This work provides a new definition of NPV-consistency making use of sensitivity analysis (SA). Given an SA technique, a metric is strongly consistent or compatible with NPV if it fulfills the classical definition of NPV-consistency and generates the same ranking of the value drivers as that generated by the NPV. If, in addition, the parameters’ relevances are equal to the ones associated with NPV, then the metric and NPV are strongly consistent in a strict form.

We assume that the COC is exogenously fixed by the decision maker, as well as the initial investment and the lifetime of the project. After proving that an affine transformation of a function preserves the ranking, we show that SL-AIRR, being an affine transformation of NPV, is strongly NPV-consistent under several (possibly, all) different techniques of SA.

On the contrary, it is not possible to determine a general relationship between IRR and NPV, since the IRR is an implicit function of NPV.

We have illustrated some simple numerical examples using FCSI (Borgonovo 2010a) and DIM (Borgonovo and Apostolakis 2001, Borgonovo and Peccati 2004), and have measured the degree of NPV-inconsistency of IRR via Spearman’s (1904) coefficient and Iman and Conover’s (1987) top-down coefficient. While SL-AIRR and NPV show perfect correlation, the incompatibility level between IRR and NPV can be remarkable. Furthermore, even when IRR exists and is unique in the base case, the value drivers’ variation might be such that the sensitivity analysis is impossible (owing to nonexistence or multiplicity of IRR).

The properties of SL-AIRR and IRR are summarized in the following table.

| Property               | SL-AIRR | IRR |
|------------------------|---------|-----|
| Classical NPV consistency | yes     | yes |
| Existence              | yes     | no  |
| Uniqueness             | yes     | no  |
| Unambiguous financial nature | yes     | no  |
| Strong NPV consistency  | yes     | no  |
| Strict NPV consistency  | yes     | no  |

These findings cast further shadows on the reliability of IRR as a relative measure of worth even in very simple cases. Most probably, more complex and realistic models might determine even higher incoherence levels between IRR and NPV: Further researches will be conducted for verifying this hypothesis. Conversely, the findings allow us to claim that the SL-AIRR can be reliably associated with NPV, providing consistent pieces of
information. Also, the SL-AIRR is a good candidate for absolute NPV-consistency, to be intended as a perfect coherence under any possible technique of SA (this should hold, given the affine relation between SL-AIRR and NPV).

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