A chiral theory of strange sea distributions in the nucleon

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Abstract

Theoretical predictions are given for the strange sea distributions in the nucleon based on the flavor SU(3) chiral quark soliton model, with emphasis upon the asymmetry of quark and antiquark distributions. We find that the quark-antiquark asymmetry of the strange sea is much larger for the longitudinally polarized distribution functions than for the unpolarized ones. A preliminary comparison with the CCFR data for the unpolarized $s$-quark distribution and with the LSS fits of the longitudinally polarized distribution functions is encouraging.

An incomparable feature of the chiral quark soliton model (CQSM) as compared with many other effective models of QCD like the MIT bag model is that it can give reasonable predictions not only for the quark distributions but also for the antiquark distributions $[1,2]$. This crucially owes to the field theoretical nature of the model that enables us to carry out nonperturbative evaluation of the parton distribution functions with full inclusion of the vacuum polarization effects in the rotating mean field of hedgehog shape $[1,3]$. It was already shown that, without introducing any adjustable parameter except for the initial-energy scale of the $Q^2$-evolution, the CQSM can explain almost all the qualitatively noticeable features of the recent high-energy deep-inelastic scattering observables. It naturally explains the excess of $\bar{d}$-sea over the $\bar{u}$-sea in the proton $[1,3]$. It also reproduces qualitative behavior of the observed longitudinally polarized structure functions for the proton, the neutron and the deuteron $[1,2]$. The most

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puzzling observation, i.e. unexpectedly small quark spin fraction of the nucleon, can also be explained in no need of large gluon polarization at the low renormalization scale \[6, 7\]. Finally, the model predicts quite large isospin-asymmetry also for the spin-dependent sea-quark distributions \[1, 2, 3, 8\], which we expect will be confirmed by near future experiments.

Our theoretical analyses so far are based on the flavor SU(2) CQSM so that no account has been taken of the possibility of strange quark excitations in the nucleon. However, there have been several experimental indications that $s\bar{s}$ pairs in the nucleon are responsible for the numbers of non-trivial effects \[8\]. An interesting question here is how large the magnitude of this admixture is, and/or how large the quark-antiquark asymmetry of the nucleon strange sea distributions is. Also interesting is whether we do expect asymmetry of $s$- and $\bar{s}$-quarks also for the spin-dependent distributions.

To answer these questions, we here use the CQSM generalized to flavor SU(3) \[10, 11\]. To proceed, we first recall some basics of the SU(2) CQSM. It is specified by the effective lagrangian,

$$\mathcal{L}_0 = \bar{\psi} \left( i \gamma_5 \partial^\mu - M e^{i \gamma_5 \tau \cdot \mathbf{\hat{r}} F(r) / f_\pi} \right) \psi,$$

which describes the effective quark fields with a dynamically generated mass $M$, interacting with massless pions. The nucleon (or $\Delta$) in this model appears as a rotational state of a symmetry-breaking hedgehog object, which itself is obtained as a solution of self-consistent Hartree problem with infinitely many Dirac-sea quarks \[12, 6\]. The theory is not a renormalizable one and it is defined with some ultraviolet cutoff. In the Pauli-Villars regularization scheme, which is used throughout the present analysis, what plays the role of a ultraviolet cutoff is the Pauli-Villars mass $M_{PV}$ obeying the relation $(N_c M^2 / 4\pi^2) \ln (M_{PV} / M)^2 = f_\pi^2$ with $f_\pi$ the pion weak decay constant \[3\]. Using the value of $M \simeq 375$ MeV, which is favored from the phenomenology of nucleon low energy observables, this relation fixes the Pauli-Villars mass as $M_{PV} \simeq 562$ MeV. Since we are to use these values of $M$ and $M_{PV}$, there is no free parameter additionally introduced into the calculation of distribution functions \[2\].

Now, the principle dynamical assumption of the SU(3) CQSM is as follows. The first is the embedding of the SU(2) self-consistent mean-field (of hedgehog shape) into the SU(3) matrix as

$$U_0^{\gamma_5}(x) = \begin{pmatrix} e^{i \gamma_5 \tau \cdot \mathbf{\hat{r}} F(r)} & 0 \\ 0 & 1 \end{pmatrix}.\tag{2}$$

The next is the semiclassical quantization of the rotational motion in the SU(3) collective space
represented as

\[ U_0^{\gamma_5}(\mathbf{x}, t) = A(t) U_0^{\gamma_5}(\mathbf{x}) A^\dagger(t), \tag{3} \]

with

\[ A(t) = e^{-i\Omega t}, \quad \Omega = \frac{1}{2} \Omega_a \lambda_a \in \text{SU}(3). \tag{4} \]

The semiclassical quantization of this collective rotation leads to a systematic method of calculation of any nucleon observables including the parton distribution functions, which is given as a perturbative series in the collective angular velocity operator \( \Omega \). (This takes the form of a 1/\( N_c \) expansion, since \( \Omega \) itself is a 1/\( N_c \) quantity.) In the present study, all the terms up to the first order in \( \Omega \) are consistently taken into account, according to the formalism explained in [1]. Since the resultant expressions for the quark (and/or antiquark) distribution functions are pretty lengthy, we decided to show them elsewhere and demonstrate only the main results here. Several comments are in order, however. The SU(3) symmetry breaking effects arising from the effective mass difference between the strange and nonstrange quarks (it should be an additional parameter of the model) can in principle be taken into account by using a perturbation method. To carry it out for parton distribution functions with full account of the vacuum polarization effects is quite involved, however. We therefore leave it to future studies. (This means that we are still continuing parameter-free analyses of the parton distribution functions, although the magnitude of strange quark mixture under this approximation should rather be taken as upper limits.) Secondly, some inconsistency is known to exist between the basic dynamical assumption of the SU(3) CQSM and the time-order-keeping collective quantization procedure of the rotational motion, although the latter is believed to resolve the long-standing \( g_A \) problem in the SU(2) model [13, 14]. Here, we simply follow the symmetry conserving approach advocated in [15], which amounts to dropping some theoretically contradictory terms by hand.

Now we show in Fig.1 the theoretical s- and \( \bar{s} \)-quark distribution functions evaluated at the model energy scale. Here, (a) represents the unpolarized distributions, while (b) does the longitudinally polarized distributions. One confirms that both \( s(x) \) and \( \bar{s}(x) \) satisfy the positivity constraint as they should do, in sharp contrast to the previous result obtained by Tübingen group in the so-called “valence-quark-only” approximation [16]. This proves our assertion that the proper account of the vacuum polarization effects is vital to give any reliable prediction for anti-quark distributions. One also notices that the distributions \( \bar{s}(x) \) has softer (lower-x) component than \( s(x) \) in qualitatively consistent with the argument of Brodsky and Ma based on the light-cone meson-baryon fluctuation model [17]. Note, however, that the
Figure 1: The theoretical predictions of the SU(3) CQSM for the strange sea distributions at the model energy scale. (a) unpolarized case and (b) longitudinally polarized case.

Asymmetry cannot be extremely large due to the restriction of the strange-quantum-number conservation in the nucleon, i.e. \( \int_0^1 s(x) \, dx = \int_0^1 \bar{s}(x) \, dx \) with \( s(x) > 0 \) and \( \bar{s}(x) > 0 \). In contrast, one observes rather large quark-antiquark asymmetry for the longitudinally polarized distributions. One sees that the \( s \)- and \( \bar{s} \)-quarks are both negatively polarized, but \( |\Delta \bar{s}(x)| \) is much smaller than \( |\Delta s(x)| \). This feature is again consistent with Brodsky and Ma’s conjecture, at least qualitatively. In fact, they argue that, if the intrinsic strange fluctuations in the proton are mainly due to the intermediate \( K^+ \Lambda \) configuration, \( s \)-quark is negatively polarized but the polarization of \( \bar{s} \) is zero. Their argument goes as follows. Since the \( K^+ \) meson is a pseudoscalar particle with negative parity and the parity of \( \Lambda \) is positive, the parity conservation dictates that the relative orbital angular momentum of the intermediate \( K^+ \Lambda \) state must be odd, most probably be a p-wave state. This gives the total angular momentum wave function in the following form:

\[
|K^+ \Lambda(J = \frac{1}{2}, J_z = \frac{1}{2})\rangle = \sqrt{\frac{2}{3}} |L = 1, L_z = 1\rangle |\Lambda(S = \frac{1}{2}, S_z = -\frac{1}{2})\rangle \\
- \sqrt{\frac{1}{3}} |L = 1, L_z = 0\rangle |\Lambda(S = \frac{1}{2}, S_z = \frac{1}{2})\rangle.
\]

(5)

The point here is that the probability of \( \Lambda \)-spin being opposite to the proton spin is twice as large as being parallel to it. Combining it with the observation that the spin of \( \Lambda \) almost comes from its constituent \( s \)-quark, one immediately conclude that the virtually mixed \( s \)-quark in
the proton is negatively polarized against the proton spin direction. The situation is quite different for the $\bar{s}$-quark generated through the same intrinsic fluctuation $p \to K^+\Lambda$. Since the $\bar{s}$ is contained in the pseudoscalar meson $K^+$ without spin, the net spin of $\bar{s}$ in the $K^+$ and consequently in the proton would be zero. Although qualitatively consistent with this argument of Brodsky and Ma, the CQSM predicts sizable amount of negative polarization also for the $\bar{s}$-quark. Such nonzero polarization of $\bar{s}$-quark may be obtained by introducing more complicated virtual process like $p \to K^{*+}\Lambda$. However, the precise estimation of the size of polarization in meson cloud models would be quite hard, since there are many competing processes.

Just by considering intermediate $p\pi^0$ and $n\pi^+$ configurations instead of $K^+\Lambda$ fluctuation, the meson-baryon fluctuation model (or the meson cloud convolution model) can naturally explain the excess of $\bar{d}$-sea over the $\bar{u}$-sea in the proton [18]. Note, however, that by the same reason as the net polarization of $\bar{s}$ is zero, one must conclude that the net polarizations of $\bar{d}$- and $\bar{u}$-seas are zero (or at least very small). This clearly contradicts the previously-mentioned predictions of the SU(2) CQSM in which $\Delta\bar{u}(x)$ is large and positive, while $\Delta\bar{d}(x)$ is large and negative [3, 4]. In our opinion, what is responsible for this remarkable difference is the nontrivial correlation between spin and isospin quantum numbers embedded in the CQSM. At least, one should recognize that the physical contents of the pion cloud model and the CQSM are not necessarily the same, as naively expected.

There are two popular ways to extract unpolarized strange sea distributions from the deep-inelastic-scattering data. The first method uses the neutrino-induced charm production, while the second relies upon a global fit (like the other flavor densities). The first direct determination of the strange quark distribution based on the neutrino-induced charm productions was carried out by the CCFR collaboration some years ago [19]. Here, we perform a very preliminary comparison of the theoretical predictions of the SU(3) CQSM with the strange quark distribution obtained by the CCFR next-to-leading-order (NLO) analysis. The comparison should be taken as preliminary, since the hidden strangeness excitation in the nucleon is thought to be very sensitive to the inclusion of the SU(3) breaking effects due to the mass difference between the strange and nonstrange quarks which we have not yet included [16]. To carry out the comparison, we have taken account of the scale dependence of the distribution functions by using the Fortran code of NLO evolution provided by Saga group [20]. The initial energy scale of this evolution is taken to be $Q^2_{ini} = 0.25 GeV^2$ and the gluon distribution at this scale is simply set to be zero, although may not be completely justified. In Fig.2, we
show the theoretical distributions \( s(x) \) and \( \bar{s}(x) \) together with the result of the CCFR NLO analysis at \( Q^2 = 4 \text{GeV}^2 \) with the constraint \( s(x) = \bar{s}(x) \). Considering that yet-to-be-included SU(3) breaking effects is expected to suppress the magnitude of strange quark excitations, it can be said that the theory reproduces the order of magnitude of the observed strange sea distribution.

Figure 2: The theoretical unpolarized distribution functions \( s(x) \) and \( \bar{s}(x) \) at \( Q^2 = 4.0 \text{GeV}^2 \) in comparison with the corresponding CCFR data.

Turning to the spin-dependent distribution functions, the quality of the presently-available semi-inclusive data is rather poor, so that the main source of analyses is limited to the inclusive DIS data alone, which forces to introduce several simplifying assumption in the analyses. For instance, most analyses in the past adopt apparently groundless assumption of flavor-symmetric polarized sea, \( \Delta \bar{u}(x) = \Delta \bar{d}(x) = \Delta \bar{s}(x) \) [21, 22]. The other analyses assumes that \( \Delta q_3(x, Q^2) = c \Delta q_8(x, Q^2) \) with \( c \) being a constant. Probably, the most ambitious analyses free from these \textit{ad hoc} assumptions on the distribution functions are those by Leader, Sidrov and Stamenov (LSS) [23]. (See also [24].) They also investigated the sensitivity of their analysis on the size of SU(3) symmetry breaking effect. (Although they did not take account of the possibility \( \Delta s(x) \neq \Delta \bar{s}(x) \), it is harmless because only the combination \( \Delta s(x) + \Delta \bar{s}(x) \) appears in their analysis of inclusive DIS data.)

To compare the theoretical distributions of the SU(3) CQSM with the LSS fits given at \( Q^2 = 1 \text{GeV}^2 \), a care must be paid to the fact that their analyses is carried out in the so-called JET scheme (or the chirally invariant scheme [25]). To take account of it, we start with the theoretical distribution functions \( \Delta u(x), \Delta \bar{u}(x), \Delta d(x), \Delta \bar{d}(x), \Delta s(x), \Delta \bar{s}(x) \), which are taken as initial distribution functions given at \( Q^2_{\text{ini}} = 0.25 \text{GeV}^2 \). Under the assumption that \( \Delta g(x) = 0 \) at this initial energy scale, we solve the DGLAP equation in the standard \( \overline{\text{MS}} \)
scheme to obtain the distributions at $Q^2 = 1 GeV^2$. The corresponding distribution functions in the JET scheme are then obtained by using the following transformation:

$$\Delta \Sigma(x, Q^2)_{JET} = \Delta \Sigma(x, Q^2)_{\overline{MS}} + \frac{\alpha_s(Q^2)}{\pi} N_f (1 - x) \otimes \Delta g(x, Q^2)_{\overline{MS}}, \quad (6)$$

$$\Delta g(x, Q^2)_{JET} = \Delta g(x, Q^2)_{\overline{MS}}, \quad (7)$$

with $\Delta \Sigma(x, Q^2) = \sum_{i=1}^{N_f} (\Delta q_i(x, Q^2) + \Delta \bar{q}_i(x, Q^2))$.

Figure 3: The theoretical distribution functions (a) $x (\Delta u(x) + \Delta \bar{u}(x))$, (b) $x (\Delta d(x) + \Delta \bar{d}(x))$, (c) $x (\Delta s(x) + \Delta \bar{s}(x))$, and (d) $x \Delta g(x)$ at $Q^2 = 1.0 GeV^2$ in comparison with the corresponding LSS fits in the JET scheme.

The solid curves in Fig.3 stand for the theoretical distributions $x (\Delta u(x) + \Delta \bar{u}(x))$, $x (\Delta d(x) + \Delta \bar{d}(x))$, $x (\Delta s(x) + \Delta \bar{s}(x))$ and $x \Delta g(x)$ at $Q^2 = 1 GeV^2$ in comparison with the corresponding LSS fits. The long-dashed, dotted and dash-dotted curves in (c) and (d) are their fits, respectively obtained by imposing a constraint on the value of the axial charge $a_8$ to be 0.58 (SU(3) limit), 0.86 and 0.40, while only the case of $a_8 = 0.58$ is shown in (a) and (b) since these distributions are insensitive to the variation of $a_8$. One sees that the distributions $\Delta s(x) + \Delta \bar{s}(x)$ as
well as $\Delta g(x)$ are fairly sensitive to the effects of SU(3) symmetry breaking and their magnitude cannot be determined with good precision from inclusive DIS data alone. Bearing in mind this large uncertainties in the magnitudes of $x(\Delta s(x) + \Delta \bar{s}(x))$ and $x\Delta g(x)$, the predictions of the SU(3) CQSM are qualitatively consistent with the results of LSS analyses.

![Figure 4: The theoretical predictions of the SU(2) and SU(3) CQSM for (a) $x(\bar{d}(x) - \bar{u}(x))$ and (b) $x(\Delta \bar{u}(x) - \Delta \bar{d}(x))$ at $Q^2 = 0.88$ GeV$^2$ in comparison with Bhalerao's semi-theoretical predictions.](image)

As already emphasized, a noteworthy feature of the SU(2) CQSM is that it predicts quite large violation of SU(2) symmetry not only for the spin-independent sea-quark distributions but also for longitudinally polarized one. Why is this observation so important? This is because the NMC observation $\bar{d}(x) - \bar{u}(x) > 0$ in the proton can be explained equally well by the CQSM and the naive meson cloud convolution model, whereas the latter essentially predicts $\Delta \bar{u}(x) \simeq \Delta \bar{d}(x) \simeq 0$ in contrast to the prediction of the CQSM such that $\Delta \bar{u}(x) > 0 > \Delta \bar{d}(x), |\Delta \bar{u}(x) - \Delta \bar{d}(x)| \simeq |\bar{u}(x) - \bar{d}(x)|$. Now the question is whether this feature of the SU(2) CQSM is also shared by the SU(3) CQSM. In Fig.4, we compare the predictions of the SU(2) and SU(3) CQSM for $x(\bar{d}(x) - \bar{u}(x))$ and $x(\Delta \bar{u}(x) - \Delta \bar{d}(x))$ with the corresponding predictions given by Bhalerao based on what-he-call the statistical model [26]. Note that his model is a semi-phenomenological one which uses several experimental information as inputs. One sees that both predictions of the SU(2) and SU(3) CQSM for $x(\bar{d}(x) - \bar{u}(x))$ are fairly close to Bhalerao’s prediction which reproduces the NMC data. On the other hand, the magnitude of $x(\Delta \bar{u}(x) - \Delta \bar{d}(x))$ is much smaller in the SU(3) model than in the SU(2) model. This
reduction is due to a delicate cancellation of several terms in this more complicated theory. We however conjecture that the introduction of the SU(3) symmetry breaking effect in the latter model partially pull back its prediction for \( x(\Delta \bar{u}(x) - \Delta \bar{d}(x)) \) toward that of the SU(2) model, thereby leading to the result, which is not extremely far from Bhalerao’s prediction.

In summary, we have given no-free-parameter theoretical predictions for the strange sea distributions in the nucleon on the basis of the flavor SU(3) CQSM. It has been shown that the \( s^- \) and \( \bar{s}^- \)-quarks are both negatively polarized but the magnitude of \( \Delta s(x) \) is much larger than that of \( \Delta \bar{s}(x) \), while the quark-antiquark asymmetry of the unpolarized strange sea is not extremely large because of the strange-quantum-number conservation in the nucleon. A preliminary comparison with the CCFR data for the unpolarized \( s^- \)-quark distribution is encouraging. The theory also reproduces the characteristic features of the recent LSS fits of the longitudinally polarized distribution functions including the negatively polarized strange sea. We also emphasize that the SU(2) symmetry of the polarized nonstrange seas is likely to be significantly violated such that \( \Delta \bar{u}(x) > 0 > \Delta \bar{d}(x) \). At any rate, the spin and flavor dependence of the antiquark distributions in the nucleon seems very sensitive observables to the nonperturbative dynamics of QCD at low energy. To reveal this interesting aspect of QCD, it is vital to carry out various types of high-energy DIS experiments, which enables us to perform flavor and valence plus sea quark decompositions of the parton distribution functions.

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**References**

[1] M. Wakamatsu and T. Kubota, Phys. Rev. D 60, 034020 (1999).

[2] M. Wakamatsu and T. Watabe, Phys. Rev. D 62, 054009 (2000).

[3] D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, M.V. Polyakov, and C. Weiss, Nucl. Phys. B 480, 341 (1996) ; *ibid.*, Phys. Rev. D 56, 4069 (1997).

[4] M. Wakamatsu, Phys. Rev. D 46, 3762 (1992) ; M. Wakamatsu and T. Kubota, Phys. Rev. D 57, 5755 (1998).

[5] P.V. Pobylitsa, M.V. Polyakov, K. Goeke, T. Watabe and C. Weiss, Phys. Rev. D 59, 034024 (1999).
[6] M. Wakamatsu and H. Yoshiki, Nucl. Phys. A 524, 561 (1991).
[7] M. Wakamatsu and T. Watabe, Phys. Rev. D 62, 054009 (2000).
[8] B. Dressler, K. Goeke, M.V. Polyakov, and C. Weiss, Eur. Phys. J. C 14, 147 (2000).
[9] J.R. Ellis, Nucl. Phys. A 684, 53 (2001).
[10] H. Weigel, R. Alkofer, and H. Reinhardt, Nucl. Phys. B 378 638 (1992).
[11] A. Blotz, D. Diakonov, K. Goeke, N.W. Park, V. Yu Petrov, and P.V. Pobylitsa, Nucl. Phys. A 555, 765 (1993).
[12] D.I. Diakonov, V.Yu. Petrov, and P.V. Pobylitsa, Nucl. Phys. B 306, 809 (1988).
[13] M. Wakamatsu and T. Watabe, Phys. Lett. B 312, 184 (1993) ;
   Chr.V. Christov, A. Blotz, K. Goeke, P. Pobylitsa, V.Yu. Petrov, M. Wakamatsu,
   and T. Watabe, Phys. Lett. B 325, 467 (1994).
[14] M. Wakamatsu, Prog. Theor. Phys. 95, 143 (1996).
[15] M. Praszałowicz, T. Watabe, and K. Goeke, Nucl. Phys. A 647, 49 (1999).
[16] O. Schröder, H. Reinhardt, and H. Weigel, Nucl. Phys. A 651, 174 (1999).
[17] S.J. Brodsky and B.-Q. Ma, Phys. Lett. B 381, 317 (1996).
[18] S. Kumano, Phys. Rep. 303, 183 (1998).
[19] CCFR Collaboration, A.O. Bazarko et al., Z. Phys. C 65, 189 (1995).
[20] M. Hirai, S. Kumano, and M. Miyama, Compt. Phys. Commun. 108, 38 (1998) ;
   ibid., 111, 150 (1998).
[21] T. Gehrmann and W.J. Stirling, Phys. Rev. D 53, 6100 (1996).
[22] M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D 53, 4775 (1996).
[23] E. Leader, A.V. Sidorov, D.B. Stamenov, Phys. Lett. B 488, 283 (2000).
[24] M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D 63, 094005 (2001).
[25] H.-Y. Cheng, Phys. Lett. B 427, 371 (1998).
[26] R.S. Bhalerao, Phys. Rev. C 63, 025208 (2001).