Coulomb explosions in plasma

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Abstract. Dense and uniformly distributed in volume ions due to uncompensated electrical charge can “explode”, thus creating a directed beam of high-energy particles. The obtained energy during the Coulomb “explosion” can reach high values. It was determined that the energy values and time of the explosion depend on a number of particle system parameters such as charge, density, and dimensions.

In this paper, the calculations of the “explosion” process using a dynamical approach for different configurations are presented: particle–particle, particles in cylindrical and spherical volumes. The “linearization” method is used for calculating the explosion time, particle velocity and energy. The correctness of the applied approach is shown in comparison with the traditional one. The calculations for the configurations are made using the necessary approximations. Numerical calculations are carried out for the case of the double size change.

1. Introduction
Dense and uniformly distributed in volume ions due to uncompensated electrical charge can “explode”, thus creating a directed beam of high-energy particles. In a series of papers [1–3], it has been shown that the effect of target’s Coulomb “explosion”, which is used as an accelerator prototype, plays an important role in particle acceleration. Sometimes the energy of the accelerated using this approach particles reaches values in the MeV range. For a more effective use of such event in the setups, one should provide the corresponding conditions. It is possible to obtain energies that are sufficient to trigger a nuclear fusion process, which is a good aspect for the application of the effects in setups and techniques.

The definition of the Coulomb “explosion” process parameters (dimension, time, velocity, and energy of the particles) is an important problem because the knowledge of the exact parameters is required for constructing setups with fusion process implementation and for other applied problems. Explosion time management is very important for the effective use of the phenomenon effects. Thus, by knowing the characteristic times of “explosions”, it is possible to select the optimal time synchronization of the associated processes, which would increase the efficiency of devices used (for example, in [2], the characteristic times allow us to estimate the effect of laser on a bound cluster that consists of a multitude of atoms). The knowledge of velocities at the output during the “explosion” will allow to calculate energy and use beams for various technical problems. In addition, the knowledge of dimensions will allow to design an optimal configuration scheme of the setup.

Proceeding from these justifications, the purpose of this paper is an analytical calculation of the above mentioned “explosion” processes.

The calculations, which are presented in this paper, were carried out for the systems of similar ions with identical charges. Nevertheless, the results are also applicable to systems of dissimilar particles
with an uncompensated total charge value, partially ionized particles, and can be generalized to different charges.

There are many terms of creation and applications of Coulomb “explosion” phenomenon. The parameters of interaction of two free point charges are calculated in Part 2. The calculation of parameters of some fundamental cases is presented in Parts 3 and 4. The times for double size change in selected direction, average velocity and achievable velocity for this interval are calculated for each configuration. These values are completely applicable as characteristic [2].

2. System with two free point-like identical charged particles

It is known that there is a Coulomb force between charged particles, and it is defined as
\[ F_c = \frac{q_1 q_2}{r^2}, \]
where \( q_1 \) is the charge that creates the electrical field \( E \) at the point where charge \( q_2 \) is situated, and \( r \) is the distance between the charges.

Let us consider a system of two identical free point-like charges \( q_1 = q_2 = q \) with a mass \( m_0 \), which are initially at rest at a distance \( L_0 \) between them (Fig. 1). We "linearize" the distance between particles at any time \( t \), i.e., we represent a linear dependence of the distance as a function of time, in which the constant coefficient is equal to the average velocity that corresponds directly to this interval. \( (L \sim \nu_{av} t) \). The equation of motion for each charge that moves separately with respect to the symmetry axis (dashed line) is
\[ F_c = k \frac{q^2}{2L_0 (L_0/2 + \nu_{av} t)^2} = m_0 \frac{d\nu}{dt}. \]

By solving and integrating with respect to the time between \( t_0 = 0 \) and \( t \), we obtain at an arbitrary moment
\[ \nu = \nu_0 + k \frac{q^2 t}{m_0 L_0 (L_0/2 + 2\nu_{av} t)}. \]

Figure 1. System of two charges with the Coulomb interaction a) in the initial state and b) after the time interval \( t \).

It is well known, that to obtain the average velocity in the time interval \( t \), one should divide the distance traveled by the time taken to travel that distance:
\[ \nu_{av} = \int_{t_1}^{t_2} \frac{\nu \, dt}{t}. \]

Then, using the expression for velocity, we obtain \( \nu_{av} \) in the implicit form:
\[ \nu_{av} = \int_{t_1}^{t_2} \frac{\nu \, dt}{t} = \nu_0 + k \frac{q^2}{m_0 L_0} \left[ \frac{t}{2\nu_{av}} + \frac{L_0}{2\nu_{av}^2} \ln \left( \frac{L_0}{L_0 + 2\nu_{av} t} \right) \right]. \]

In the case of doubling of distance between the particles to \( 2L_0 \), we have \( 2L_0 - L_0 = L_0 = \nu_{av} t \), and for each one, we obtain the average velocity, expansion time and achieved velocity as
\[ \nu_{av,2x} = \sqrt{k \frac{q^2}{2m_0 L_0} (1 - \ln 2)}, \quad t_{2x} = \frac{m_0 L_0^3}{2kq^2 (1 - \ln 2)} \quad \text{and} \quad v_{2x} = \sqrt{k \frac{q^2}{8m_0 L_0 (1 - \ln 2)}} = 0.638 \sqrt{k \frac{q^2}{m_0 L_0}}, \]
respectively.
One can also obtain the velocity of particles in expansion from the energy conservation equation

\[ \frac{k}{L_0} \frac{q^2}{2} = k \frac{q^2}{L} + \frac{2}{m} u^2. \]

Thus, for the double expansion, we have

\[ v_{2x} = \sqrt{\frac{k}{2m_0L_0} \frac{q^2}{2}} \approx 0.707 \sqrt{\frac{k}{m_0L_0} \frac{q^2}{2}}. \]

The results for velocity values in both approaches are approximately equal, which indicates the correctness of the presented method. However, by considering the interaction at each time moment, the first result corresponds to the more accurate value.

3. Cylindrical volume of the charged particles expanding along the axis

Consider a system of particles with the same charges \( q_i = q_{i+1} = q \) and masses \( m_0 \) that are densely and uniformly arranged within a cylindrical volume. We denote the particle concentration as \( n_q \). The average distance between the particles is \( n_q^{-1/3} \). We find the velocity of the test particle at the surface end of the cylinder extending in a longitudinal direction of the axis (Fig. 2a). We assume that the radial expansion is absent, \( R_0 = \text{const} \). Also, due to the conservation of the number of particles, the total charge in the volume is \( q_{\text{total}} = q n_q V = q n_q R_0^2 L_0 = \text{const} \), where \( V \) is the volume, \( R_0 \) is the initial radius of the cylinder, and \( L_0 \) is the initial length of the cylinder. While expanding along the axis to \( L \), for any moment of time according to the Gauss theorem, assuming that \( R >> L > L_0 \), we obtain the electric field of the entire volume that acts on a test particle on the surface of the cylinder (edge effects are ignored)

\[ E = \frac{q n_q L_0}{2 \varepsilon_0}. \]

**Figure 2.** Expansion of the cylindrical volume with similar charged particles along the axis a) in both directions, b) in one direction, c) in radial direction, and d) the expansion in the radial direction of spherical volume with similar charged particles.
The test charge will be acted upon by the force $F = qE = m_0 \frac{d\nu}{dt}$. From it, taking $\nu_0 = 0$, the expression for the velocity is $\nu = \frac{q^2 n_q L_0}{2m_0 e_0} t$. Let us perform the "linearization" of elongation, assuming that on the interval of cylinder’s length change from $L_0$ to $L$ in time $t$, the average velocity of the particle is equal to $\nu_{av}$, which strictly corresponds to exactly this interval. Due to the symmetry of both ends of the cylinder, $L - L_0 = 2\nu_{av} t$. Using the expression for velocity, (1) and “linearization”, we obtain $\nu_{av} = \frac{1}{4} \frac{q^2 n_q L_0 t}{m_0 e_0}$.

For the double expansion to $L = 2L_0$, the average velocity is $\nu_{av2x} = \frac{L_0}{2\sqrt{2}} \sqrt{\frac{q^2 n_q}{2m_0 e_0}}$, the time is $t_{2x} = 2 \sqrt{\frac{m_0 e_0}{q^2 n_q}}$, and the velocity is $\nu_{2x} = L_0 \sqrt{\frac{q^2 n_q}{2m_0 e_0}}$.

Let us consider the same cylindrical volume of the charged particles, but expanding only in one direction along the axis (the second end is fixed). (Fig. 2b). The cylinder expansion times in this and previous problems should be the same because the time is invariant to the reference system (in our non-relativistic case). This can be seen by the example of doubling the length ($L = 2L_0$). We obtain $t_{2x} = 2 \sqrt{\frac{m_0 e_0}{q^2 n_q}}$, which confirms our findings. The velocity is equal to $\nu_{2x} = 2L_0 \sqrt{\frac{q^2 n_q}{m_0 e_0}}$. The derivation and assumptions of expressions are similar to the previous problem.

Now, consider the cylindrical volume of the charged particles that are expanding in the radial direction (Fig. 2c). In this case, it is necessary to take $L_0 >> R > R_0$ while expanding in radial direction to $R$. Neglecting edge effects, for any moment, according to the Gauss theorem for the test particle on the surface of the cylinder, we get $E = \frac{q n_q R_0^2}{2 e_0 R}$.

Taking $\nu_0 = 0$ and “linearizing” as $R - R_0 = \nu_{av} t$, the equation for velocity becomes $\nu = \frac{q^2 n_q R_0^2}{2m_0 e_0 \nu_{av}} \ln \left( \frac{\nu_{av} t}{R_0} + 1 \right)$. Using this expression and (1), we implicitly obtain the expression for $\nu_{av}$:

$$\nu_{av} = \int_0^t \frac{q^2 n_q R_0^2}{2m_0 e_0 \nu_{av}} \ln \left( \frac{\nu_{av} t}{R_0} + 1 \right) dt$$

For the double size increase to $R = 2R_0$, we get $\nu_{av2x} = R_0 \sqrt{\frac{\ln 2 - \frac{1}{2}}{4 \ln 2 - 2}} \sqrt{\frac{q^2 n_q}{m_0 e_0}}$, and then, the time is $t_{2x} = \sqrt{\frac{2}{2 \ln 2 - 1}} \sqrt{\frac{m_0 e_0}{q^2 n_q}}$, and the velocity is $\nu_{2x} = R_0 \sqrt{\frac{1}{4 \ln 2 - 2}} \frac{q^2 n_q}{m_0 e_0}$.

4. Spherical volume of charged particles that are expanding in radial direction

Now, consider a system of identical charged particles that are densely located within a spherical volume. The expansion occurs uniformly in radial direction (Fig. 2d). We will find the velocity of the test particle located on the surface of the sphere in the absence of particle loss.
\[ q_{\text{total}} = q n q V = q n q \frac{4}{3} \pi R_0^3 L_0 = \text{const} . \]  
According to the Gauss theorem, \[ E = \frac{q n q R_0^3}{3\varepsilon_0 R^2} . \]  
Taking \[ v_0 = 0 \] and “linearizing” as \[ R - R_0 = v_{av} t \] for the obtained velocity of the test charge \( q \), we obtain the expression \[ v = \frac{q^2 n_q R_0^3}{3m_0 \varepsilon_0 v_{av}} \left[ \frac{1}{R_0} - \frac{1}{R_0 + v_{av} t} \right] . \]  
Using the expression for velocity and also (1), we implicitly obtain the expression for \( v_{av} \):
\[
v_{av} = \int_0^t \frac{q^2 n_q R_0^2}{3m_0 \varepsilon_0 v_{av}} - \frac{q^2 n_q R_0^3}{3m_0 \varepsilon_0 v_{av}^2} t \ln \left( \frac{v_{av} t}{R_0} + 1 \right) .
\]

In the case of doubling the size to \( R = 2R_0 \), we have \( v_{av} = R_0 \sqrt{\frac{1 - \ln 2}{3} \frac{q^2 n_q}{m_q \varepsilon_0}} \). Then, the time is \( t_{2x} = \left( \frac{3}{1 - \ln 2} \right)^{1/2} \frac{m_q \varepsilon_0}{q^2 n_q} \), and the velocity is \( v_{2x} = R_0 \sqrt{\frac{1}{12(1 - \ln 2)} \frac{q^2 n_q}{m_q \varepsilon_0}} \).

5. Generalization of the results

In calculations, if one assumes that the uniform distribution of particles in the corresponding volumes is preserved at any instance of time, the particle velocities inside the volume will be proportional to the distance from the propagation epicenter. In addition, it is necessary to correctly take into account the constraints when using the Gauss theorem in calculations. The calculations for spherical volume assume no restrictions.

The calculation of kinetic energy acquired for the considered test particles in the case of doubling the dimensions is not difficult, in accordance with the expression for kinetic energy \( E_{\text{kin}} = \frac{1}{2} m v^2 \).

However, for the numerical results, it is necessary to set the initial configuration sizes \( (L_0, R_0) \). For example, in [2], the authors find the cluster size, which allows to achieve fusion energies when doubling the dimension.

In the case of a system of particles with charges of different sign, but with an uncompensated total charge, the configuration is also subject to an "explosion". In this case, the values of the total uncompensated charge should be used instead of \( q_{\text{total}} \) in calculations and results obtained in this paper.

6. Conclusions

In this paper, for the first time, the method of “linearization” of the dependence of the dimension parameters on time was applied for calculating the Coulomb “explosion” characteristics. Based on the particle interactions (part 2), the accuracy of the method has been shown. Considering it, the calculations were made for various configurations (parts 3, 4). Analytical formulas for calculating the parameters are obtained. Some results are presented in an implicit form. An explicit form can be obtained by specifying the finite dimension values of the system. Thus, here, taking into account all of the calculations, the results are shown for the double increase in dimensions. These formulas may be used as a basis for the calculation of “explosion” characteristic parameters.

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