Partial order approach to compute shortest paths in multimodal networks

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Abstract

Many networked systems involve multiple modes of transport. Such systems are called multimodal, and examples include logistic networks, biomedical phenomena, manufacturing process and telecommunication networks. Existing techniques for determining optimal paths in multimodal networks have either required heuristics or else application-specific constraints to obtain tractable problems, removing the multimodal traits of the network during analysis. In this paper weighted coloured–edge graphs are introduced to model multimodal networks, where colours represent the modes of transportation. Optimal paths are selected using a partial order that compares the weights in each colour, resulting in a Pareto optimal set of shortest paths. This approach is shown to be tractable through experimental analyses for random and real multimodal networks without the need to apply heuristics or constraints.

Keywords: graph theory, multimodal network, Pareto optimal set, shortest path, weighted coloured–edge graph

1. Introduction

Extensive scientific literature has been devoted in the last three decades to the study of multimodal networks (MMN). During this time, research
has mainly focused on practical applications for freight or urban transporta-
tion (see Jarzemskiene (2007) and Macharis and Bontekoning (2004) for ex-
tensive reviews). As a system in which several means of transport are
available, a multimodal system is able to emulate a wide spectrum of real
life phenomena beyond the field of logistics. Areas such as computer net-
works, biology and manufacturing have begun to utilize multimodal net-
works for studying and modelling situations. Examples can be found in pa-
pers by Abrach et al. (2003), Chen et al. (2005), Heath and Sioson (2007),
Kiesmüller et al. (2005), Nigay and Coutaz (1993), Medeiros et al. (2000)
and Sioson (2005).

From a modelling perspective, a range of techniques have been used to
model MMN. They can be loosely classified into three predominant domains:
mathematical programming, weighted graphs and multi–weighted graphs.

1.1. The mathematical programming approach

These techniques are characterized by making use of linear or non–linear
formulations for representing a MMN by a set of equations.

Linear programming techniques are suitable when each decision variable
is a linear combination of the problem parameters, Hillier and Lieberman
(2009). Integer programming and mixed integer programming stand out
as the most common linear programming techniques used for multimodal
modelling. Sample papers using linear programming as a modelling tool are
given by Min (1991) and Kim et al. (1999).

Non-linear programming is another renowned modelling technique for
MMN. It is mainly used to build intricate cost functions, and principally
deals with second order equations satisfying convex or concave properties.
Examples are provided by Kim and Kim (2006), Horii (2003) and Chang
(2007) which have opted to use non-linear programming as their main mod-
elling approach.

In the mathematical programming approach, mode options are visualized
as decision variable indices, which considerably increases the complexity of
the problem. Relaxation or cutting plane techniques are commonly used to
make the problem tractable. Interesting papers tackling general views of
mathematical programming for intermodal transportation (the transporta-
tion of goods) and urban transportation were by Jarzemskiene (2007) and
Nagurney (1984) respectively.
1.2. The weighted graph approach

In this approach, a node typically represents a location, such as a warehouse, transportation hub or network router, and an edge represents a transportation link, such as rail line, a bus or a wireless connection. A variety of graphs have been used to study these transport systems, such as digraphs, multigraphs, hypergraphs and grid graphs. Ayed et al. (2008) provides a general classification for MMN models based on weighted graph approaches. In particular, the article emphasizes the use of multigraphs, in which there might be multiple edges between two nodes, and the use of grids in which a grid is overlayed on a planimetric map. Both can result in dense graphs, which require edge reduction techniques to make their analysis tractable. In practice such reductions rely on enforcing constraints on feasible edges in order to build a specific path. Studies making use of such graphs are provided by Foo et al. (1999), Qiang Li (2000), Kitamura (1999) and Fragouli and Delis (2002). Hypergraphs are another type of graph used in some articles. In graph theory, a hypergraph is a generalization of a graph, where edges can connect any number of vertices, Lawler (2001). In the multimodal context, such graphs have found interesting applications in biology and urban transportation. Sample papers using hypergraphs to represent MMN are yielded by Heath and Sioson (2007), Sioson (2005) and Lozano and Storch (2002).

In effect, the weighted graph approach only utilizes mode information during the application of constraints, removing the multimodal traits from the network during modelling. The analysis in this approach is very application-dependent as it relies on applying application-specific constraints.

1.3. The multi–weighted graph approach

This approach has been extensively utilized for the Multicriteria Shortest Path Problem (MSPP) which has become a fruitful branch of research since the 1980s, see Tarapata (2007) and Soroush (2008) for a complete review. Basically, the approach assigns multiple weights to each edge. In particular, the bicriteria shortest path problem assigns two weights to each edge, such as cost and time.

Optimality in the multi–weighted graph approach is commonly established by the use of a partial order relation which results in a Pareto optimal set of paths that are candidates for the sought shortest path. There is little literature that directly applies multi–weighted graphs for modelling MMN, but the goal of MSPP is essentially the same as for the shortest path problem in MMN. Although articles developing MSPP formulations
for MMN can be identified, they preferentially use partial orders to optimize route choice decisions mainly based on cost and time, leaving the mode options as an outcome of the optimal route. The tractability of the MSPP is inextricably connected with the cardinality of the Pareto set according to Müller-Hannemann and Weihe (2006). Hansen (1980) showed in his paper that this cardinality is exponential in the worst case, although Loui (1983) points out that Pareto sets for some graphs with multidimensional weights have polynomial average case cardinalities. Constraints are applied during analysis to make the problem tractable, resulting in a Pareto set with manageable cardinality. MMN models whose mainstay is a multi–weighted graph can be found in papers by Androutsopoulos and Zografos (2009), Aifadopoulou et al. (2007) and Modesti and Sciomachen (1998).

The paper is organized as follows. In Section 2 a fourth approach for modeling MMN is introduced and compared with the three previous approaches. An algorithm is described in Section 3 that is used in the paper to compute shortest paths in weighted coloured–edge graphs. In Section 4 the model is experimentally studied to demonstrate the tractability of the approach. Moreover, the algorithm is applied to a real multimodal network in order to assess its practical applicability. Finally, Section 5 provides conclusions about this research.

2. Weighted coloured–edge graph approach

All the approaches described in Section I are heavily application specific and do not actually utilize the multimodal nature of a network. In this paper an approach to model and analyze multimodal networks is introduced. In essence, such approach uses a weighted graph in which edges are endowed with two attributes: a positive weight and a discrete variable called colour.

A weighted coloured–edge graph \( G = (V, E, \omega, \lambda) \) consists of a directed graph \( (V, E) \) with vertex set \( V \) and edge set \( E \), a weight function \( \omega: E \rightarrow \mathbb{R}^+ \) on edges, and a colour function \( \lambda: E \rightarrow M \) on edges, where \( M \) is a set of colours. Typically \( M \) is taken as a finite set with \( k = |M| \). Associated to each edge \( e_{uv} \in E \) from vertex \( u \) to vertex \( v \) there is a positive weight \( \omega(e_{uv}) \) and a colour \( \lambda(e_{uv}) \). For any colour \( i \in M \) and for any path \( p_{uv} = \{e_{x_0x_1}, e_{x_1x_2}, e_{x_2x_3}, \ldots, e_{x_{l-1}x_l}\} \) between two vertices \( u = x_0 \) and \( v = x_1 \), where each \( x_i \in V \), the path weight \( \omega_i(p_{uv}) \) in colour \( i \) is defined as \( \omega_i(p_{uv}) = \sum_{e_{x_ix_{i+1}} \in p} \lambda(e_{x_ix_{i+1}}) = \omega(e_{x_ix_{i+1}}) \). The total path weight is represented as a k–tuple \( (\omega_1(p_{uv}), \ldots, \omega_i(p_{uv}), \ldots, \omega_k(p_{uv})) \), giving the total weight of the path.
in each colour.

Note that there is no restriction placed on the number of edges \( e_{uv} \) from a vertex \( u \) to a vertex \( v \). However, in practice for the shortest path problem attention can be restricted to weighted coloured–edge graphs for which there is at most one edge \( e_{uv} \) in each colour from \( u \) to \( v \).

Let \( u \) and \( v \) be two given vertices of \( G \) and let \( \mathcal{P}_{uv} \) be the set of all paths from \( u \) (source) to \( v \) (destination) in \( G \). A binary relation between two paths \( p_{uv} \) and \( p'_{uv} \), is defined by \( p_{uv} \leq p'_{uv} \) if and only if \( \omega_i(p_{uv}) \leq \omega_i(p'_{uv}) \) for all \( i \). The relation \( \leq \) is clearly reflexive and transitive and gives a partial order on the \( k \)-tuple path weights, but only a preorder on the paths themselves as multiple paths might have the same total path weight.

Let \( \mathcal{M}_{uv} = \{ p_{uv} \in \mathcal{P}_{uv} \mid \forall p'_{uv} \in \mathcal{P}_{uv} \text{ with } \omega(p'_{uv}) \neq \omega(p_{uv}), \exists i \leq k \text{ such that } \omega_i(p_{uv}) < \omega_i(p'_{uv}) \} \) be the set of Pareto optimal paths joining vertices \( u \) and \( v \). This set has an important characteristic: for any \( p_{uv} \in \mathcal{M}_{uv} \), it is impossible to determine a path \( p'_{uv} \) from \( u \) to \( v \) which has smaller weight than \( p_{uv} \) in some of its \( k \) colours without at least one of the other weights being larger, analogously to Martins (1984).

From the above definitions it is apparent that the concept of a weighted coloured–edge graph with \( k \) colours can equivalently be formulated as a multiweighted multigraph where each edge is assigned a \( k \)-tuple of non-negative weights \((w_{c1}, \ldots, w_{ci}, \ldots, w_{ck})\) and exactly one \( w_{ci} > 0 \). However, multiweighted graphs are mostly used in multicriteria optimization applications where the weight components correspond to quantities to be optimized, such as cost and time, so edges typically contribute toward more than just one quantity. For this reason multiweighted graphs whose edge weights are zero in all but one component have not received attention in the literature.

Shortest path analysis in the weighted coloured-edge graph approach is seen in this paper to typically be tractable without the need to apply any application-specific heuristics or constraints, so can be considered a general tool for the study of multimodal networks. Application-specific considerations can still be applied to the resulting set \( \mathcal{M}_{uv} \), or a post-optimal analysis undertaken on it. One facet of this model is that it can be directly applied to multigraph applications, such as transportation networks where there are multiple transportation means between two locations, communication networks where there are multiple links or choice of communication protocols between nodes, or epidemic models which have multiple paths of infection.

However, focusing attention on only the Pareto optimal paths limits the approach to shortest path problems where just the summed contribution of
each colour is important, and where any measure of optimality is presumed to be an increasing (linear or non-linear) function of the summed contribution in each colour. For instance, the approach presumes in a transportation network that the optimal path (such as least cost, time, or distance) is some application-specific increasing function of the total weight in each transportation means, or that the user can apply some application-specific criteria to select a preferred path from the Pareto set once it has been determined.

The approach can be adapted for path constraints such as restricting the number of hops or the number of mode changes by slightly enhancing the algorithm used to determine the Pareto set. For instance, besides using colours to represent the different transportation means, an additional colour can be used to count the number of edges in a path as the path is being built during the analysis and/or to count the number of transfers from one means of transportation to another.

Optimization problems that utilize models similar to weighted coloured-edge graphs have received little attention in the literature. Climaco et al. (2010) experimentally studied the number of spanning trees in a weighted graph whose edges are labelled with a colour. In that work, weight and colour are two criteria to be both minimised and the proposed algorithm generates a set of non-dominated spanning trees. The computation of coloured paths in a weighted coloured-edge graph is investigated in Xu et al. (2009). The main feature of their approach is a graph reduction technique based on a priority rule. This rule basically transforms a weighted coloured-edge multidigraph into a coloured-vertex digraph by applying algebraic operations to the adjacency matrix. Additionally, the authors provide an algorithm to identify coloured source-destination paths. Nevertheless, the algorithm is not intended for general instances because its input is a unit weighted colored multidigraph and only paths not having consecutive edges equally coloured are considered.

This paper investigates the feasibility of the approach as a general tool for multimodal networks by determining the cardinality of the Pareto optimal set $\mathcal{M}_{uv}$ for many diverse networks. It shows that the cardinality is typically a very low order polynomial function of the size of the network, and demonstrates that even dense multimodal graphs with hundreds of thousands of edges can be feasibly analyzed using this approach, without the need for any reduction techniques. In fact, it is seen that the number of modes $k$ is more of a limiting factor of the approach than is the number of vertices or edges in the graph.
3. Algorithm for determining Pareto Optimal Sets

To experimentally study the feasibility of using weighted coloured-edge graphs for multimodal networks an algorithm that determines $M_{uv}$ is required. A simple generalization of Dijkstra’s algorithm from unimodal networks has been developed for the purposes of this paper, although more efficient algorithms might be investigated in the future. The classic Dijkstra’s algorithm for solving the single-source shortest path problem in unimodal networks uses a priority queue $Q$ to store shortest path estimates from a fixed source vertex $s$ to each vertex $v$ in the network until the shortest path to $v$ is determined. Since the weights of any paths $p_{sv}$ from $s$ to $v$ are linearly ordered there is only at most one shortest path estimate in the queue at a time for each vertex $v$. At the start of each iteration of the algorithm the shortest path estimate at the front of the queue is the actual shortest path to one of the vertices in the network.

In a weighted coloured-edge graph Dijkstra’s algorithm must be slightly generalised to handle weights of paths being partially ordered rather than linearly ordered. A priority queue $Q$ can again be used to store shortest path estimates with the requirement that if a path $p_{sv}$ from $s$ to $v$ has smaller weight than another path $p'_{sv}$ then it must appear earlier in the queue. Although the results presented in this paper use such a simple queue instead of a more sophisticated non-linear data structure the performance of the algorithm is seen to be surprisingly good. As in the classical Dijkstra’s algorithm the weighted coloured-edge version of the algorithm takes as input a network $G$ and a source vertex $s$. It commences at $s$ with the empty path $p_{ss}$ and relaxes each edge that is incident from the source vertex $s$, adding the single edge paths to the queue. At the front of the queue will be a shortest path estimate $p_{sv}$ to some vertex $v$ adjacent to $s$. Since all weights are positive in the network $p_{sv}$ must have minimal weight amongst paths from $s$ to $v$ (although it might not be the only minimal path from $s$ to $v$ in the queue), so $p_{sv}$ is added to the set $M_{sv}$ and removed from the queue. The algorithm then relaxes all the edges incident to $v$, extending the path $p_{sv}$ by each edge to a path $p'_{sv} = p_{sv} \cup \{e_{vu}\}$, adding those extended paths $p'_{sv}$ to the queue that have minimal weight amongst paths from $s$ to $u$, and removing from the queue any path $p''_{su}$ from $s$ to $u$ that has greater weight than $p'_{su}$. The algorithm repeats itself until the queue is empty, producing as output the Pareto optimal set $M_{sv}$ for each vertex $v$ in the network. Figure 1 describes the pseudocode of the algorithm using the notation developed by Cormen et al.
Multimodal-Dijkstra($G, s$)

1. $\triangleright$ Initially no Pareto optimal paths known
2. for each vertex $v$
   3. $\triangleright$ Create a queue $Q$ to hold shortest path estimates during processing
   4. $Q \leftarrow \emptyset$
   5. add the empty path $p_{ss}$ from $s$ to $s$ into $Q$
   6. while $Q \neq \emptyset$
      7. $\triangleright$ Relax the edges incident from $v$
      8. for each edge $e_{vu}$ incident from $v$ to a vertex $u$ not in $p_{sv}$
         9. $\triangleright$ Extend the path $p_{sv}$ by the edge $e_{vu}$
         10. $p_{su}' \leftarrow p_{sv} \cup \{e_{vu}\}$
         11. if $p_{su}'$ has minimal weight in $Q$ from $s$ to $u$
            12. then add $p_{su}'$ to $Q$
            13. $\triangleright$ Remove any paths no longer minimal in $Q$
            14. for each $p_{su}'' \in Q$ with $\omega(p_{su}'') > \omega(p_{su}')$
               15. do remove the path $p_{su}''$ from $Q$
   16. return $M_{sv}$

Figure 1: Pseudocode of the algorithm

The number of relaxation steps is an important indicator of the algorithm’s order, so besides finding $M_{sv}$ the experiment discussed in Section 4 also track the number of paths $p_{su}'$ processed by the algorithm.

As an example of an application of the weighted coloured–edge graph approach, the algorithm is run with a multimodal network from Lozano and Storchi (2001) starting at source vertex 0. Figure 2 shows the network which has 21 vertices, 51 edges and 4 different transport choices (bus, metro, private and transfer). The algorithm commences with just the empty path $p_{00}$ on the queue and relaxes two edges: $e_{01}$ with weight (bus, metro, private, transfer) = (15, 0, 0, 0), and $e_{03}$ with weight (0, 0, 5, 0), which are both added to the queue. Since the two weights are incomparable, either could be at the front of the queue, so the next iteration of the algorithm either adds the path $p_{01} = \{e_{01}\}$ to $M_{01}$ and relaxes the four edges incident to vertex 1 by ex-
Figure 2: Multimodal network from Lozano and Storchi (2001)
tending the path \( p_{01} \) by each, or else adds the path \( p_{03} = \{e_{03}\} \) to \( M_{03} \) and relaxes the three edges incident to vertex 3 by extending the path \( p_{03} \) by each. Continuing in this way the Pareto optimal set \( M_{0v} \) is obtained for each vertex \( v \) in the network, resulting in 52 Pareto optimal paths from vertex 0 to vertex 20 whose weights are listed in Table 1. Depending on the application, constraints or heuristics can then be applied to the 52 paths to select a path preferred by the user. Using just a simple priority queue data structure the generalised Dijkstra’s algorithm can determine \( M_{0v} \) for all 21 vertices \( v \) within approximately 10ms. The article of [Lozano and Storchi, 2001] instead uses a weighted graph approach with application-specific constraints and a simple cost function which adds the weights in each mode together to get a single valued total weight, resulting in the paths numbered 2, 25, 33, 47 in the table.

Note that a Pareto set permits a post–optimal analysis to be carried out provided that the total cost is presumed to be an increasing function of the weight in each mode. For instance, suppose in the example network that the edge weights represent distance and for simplicity presume the cost is one dollar per unit distance for each means of transport. A natural optimization question could be how much the unit cost associated to a particular mode could be increased or decreased with the current optimal path remaining optimal. As an illustration, path 25 which has the edges \( \{e_{03}, e_{31}, e_{19}, e_{1014}, e_{1415}, e_{1517}, e_{1716}, e_{1618}, e_{1819}, e_{1920}\} \) and shown in Figure 3 has least total cost $47, but from the Pareto set it is easily seen that an increase of over 20% in the relative metro costing would make path 41 with edges \( \{e_{03}, e_{31}, e_{19}, e_{1313}, e_{1315}, e_{1517}, e_{1716}, e_{1618}, e_{1819}, e_{1920}\} \) a better choice, or a 25% increase in bus prices would make path 16 with edges \( \{e_{03}, e_{32}, e_{210}, e_{1014}, e_{1415}, e_{1517}, e_{1716}, e_{1618}, e_{1819}, e_{1920}\} \) better.

This example demonstrates that the Multimodal Dijkstra’s algorithm can quickly calculate the Pareto set without the need to assign relative costs for the different modes. Then alternative cost functions can be evaluated on just the paths in the Pareto set or a post–optimal analysis conducted without ever having to rerun the algorithm on the network.

4. Experimental Study

In this section the weighted coloured–edge graph approach is applied to multimodal networks in two different scenarios. Firstly, the cardinality of \( M_{uv} \) is analyzed for complete graphs. Secondly, the approach is applied to
Table 1: Pareto set for network with 21 vertices and 51 edges.

| Path Number | Transport Choice Cost | Cost as per Lozano and Storchi (2001) |
|-------------|------------------------|---------------------------------------|
|             | Bus | Metro | Private | Transfer | 55 |
| 1           | 25  | 4     | 21      | 5         | 55 |
| 2           | 0   | 40    | 21      | 5         | 55 |
| 3           | 13  | 11    | 5       | 21        | 55 |
| 4           | 52  | 9     | 5       | 21        | 55 |
| 5           | 24  | 4     | 21      | 5         | 55 |
| 6           | 11  | 26    | 5       | 21        | 55 |
| 7           | 21  | 26    | 5       | 21        | 55 |
| 8           | 21  | 26    | 5       | 21        | 55 |
| 9           | 10  | 30    | 5       | 21        | 55 |
| 10          | 3   | 4     | 5       | 21        | 55 |
| 11          | 3   | 4     | 5       | 21        | 55 |
| 12          | 3   | 4     | 5       | 21        | 55 |
| 13          | 3   | 4     | 5       | 21        | 55 |
| 14          | 3   | 4     | 5       | 21        | 55 |
| 15          | 3   | 4     | 5       | 21        | 55 |
| 16          | 3   | 4     | 5       | 21        | 55 |
| 17          | 3   | 4     | 5       | 21        | 55 |
| 18          | 3   | 4     | 5       | 21        | 55 |
| 19          | 3   | 4     | 5       | 21        | 55 |
| 20          | 3   | 4     | 5       | 21        | 55 |
| 21          | 3   | 4     | 5       | 21        | 55 |
| 22          | 3   | 4     | 5       | 21        | 55 |
| 23          | 3   | 4     | 5       | 21        | 55 |
| 24          | 3   | 4     | 5       | 21        | 55 |
| 25          | 3   | 4     | 5       | 21        | 55 |
| 26          | 3   | 4     | 5       | 21        | 55 |
| 27          | 3   | 4     | 5       | 21        | 55 |
| 28          | 3   | 4     | 5       | 21        | 55 |
| 29          | 3   | 4     | 5       | 21        | 55 |
| 30          | 3   | 4     | 5       | 21        | 55 |
| 31          | 3   | 4     | 5       | 21        | 55 |
| 32          | 3   | 4     | 5       | 21        | 55 |
| 33          | 3   | 4     | 5       | 21        | 55 |
| 34          | 3   | 4     | 5       | 21        | 55 |
| 35          | 3   | 4     | 5       | 21        | 55 |
| 36          | 3   | 4     | 5       | 21        | 55 |
| 37          | 3   | 4     | 5       | 21        | 55 |
| 38          | 3   | 4     | 5       | 21        | 55 |
| 39          | 3   | 4     | 5       | 21        | 55 |
| 40          | 3   | 4     | 5       | 21        | 55 |
| 41          | 3   | 4     | 5       | 21        | 55 |
| 42          | 3   | 4     | 5       | 21        | 55 |
| 43          | 3   | 4     | 5       | 21        | 55 |
| 44          | 3   | 4     | 5       | 21        | 55 |
| 45          | 3   | 4     | 5       | 21        | 55 |
| 46          | 3   | 4     | 5       | 21        | 55 |
| 47          | 3   | 4     | 5       | 21        | 55 |
| 48          | 3   | 4     | 5       | 21        | 55 |
| 49          | 3   | 4     | 5       | 21        | 55 |
| 50          | 3   | 4     | 5       | 21        | 55 |
| 51          | 3   | 4     | 5       | 21        | 55 |
| 52          | 3   | 4     | 5       | 21        | 55 |
a real multimodal network. This network corresponds to the transportation system of France and considers four transport choices.

4.1. Number of processing paths and cardinality of $M_{uv}$

The objective here is to identify general patterns for the number of processing paths and $M_{uv}$ cardinality. In this test a weighted complete multigraph is taken as input so that each analytical scenario is generated by fixing values for $n = |V|$ and $k = |M|$. Such a graph is characterized by having $kn(n - 1)$ edges and the maximum number of possible paths $|P_{uv}| = \sum_{j=0}^{n-2} \binom{n-2}{j} k^{j+1}j!$ for $v \neq u$, which has factorial order $O(k^{n-1}(n-2)!)$.

Specifically, the algorithm is run for complete multigraphs with $k = 2, 3, 4, 5$ colours and $n$ between 20 and 200 vertices. Random edge weights are generated by means of a continuous uniform distribution of positive weights.

Figure 4 depicts the patterns followed by $M_{uv}$ cardinality. The figure uses a logarithmic scale for vertical as well as horizontal axes to demonstrate the average case polynomial behavior as $n$ increases. Table 2 provides the numerical orders determined for different $k$ values. These results demonstrate not only that the Pareto optimal set is calculated in polynomial time but that the resulting set requiring further analysis grows very slowly as a
function of $n$. The results resemble ideas presented in Bentley et al. (1978) and Müller-Hannemann and Weihe (2001), suggesting the applicability of the model in real multimodal network scenarios, even when the networks are dense and without having to apply network reduction techniques or heuristics.

4.2. Performance in a real multimodal network

The approach is now tested on a large multimodal network. In this test, largeness is in the sense of number of vertices and edges. The selected net-

Table 2: Order of processing paths and $M_{uv}$ cardinality for several $k$ values

| $k$ | Processing paths ($p'_{sn}$) | $M_{uv}$ cardinality |
|-----|-----------------------------|----------------------|
| 2   | $O(n^{1.28})$                | $O(n^{0.19})$        |
| 3   | $O(n^{1.37})$                | $O(n^{0.32})$        |
| 4   | $O(n^{1.52})$                | $O(n^{0.46})$        |
| 5   | $O(n^{1.64})$                | $O(n^{0.61})$        |
work scenario corresponds to the multimodal transportation system of France being one of the largest networks in Europe. The multimodal network was obtained from vector data information retrieved from a public GIS library, Geofabrik (2010).

The network dataset for each transport choice was firstly processed in ArcGIS to make it suitable for computation. ArcGIS is a Windows platform application for the analysis and processing of vector geographic information system data. This application has a network analysis extension that permits the identification of junctions and polylines (see Burke (2002) for a definition) in each transport system. In addition, ArcGIS also has a macro for the computation of the adjacency matrix for each system of junctions.

Table 3 summarizes the number of junctions and edges for each transport mode as well as some statistics of the networks. All edge lengths are given in decimal geographic degrees. Four transport modes comprise the France transport system: road, rail, waterways and motorways. The road system mainly consists of primary roads. The rail system is comprised of common train lines disregarding subway and tram. Waterways are the channels and rivers used as transportation links. Finally, the motorway system of France includes toll roads and is considered a different mode of transport in its own right. As an illustration, Figure 5 depicts the France road system.

The construction of the multimodal network requires assembling the data for the four network modes together. This task is accomplished by an ad-hoc algorithm coded in the Java language. The code basically takes two inputs. These are the adjacency matrix of each transport mode network and a list of minimum interjunction distances in each mode. The latter is built in ArcGIS by taking a mode junction dataset and applying the "join and relates" tool with respect to each other mode junction dataset. This

| Modes   | Number of Junctions | Number of polylines | Maximum | Mean    | Stnd. Dev. |
|---------|---------------------|---------------------|---------|---------|------------|
| Roadways| 53562               | 47660               | 0.868028| 0.010674| 0.032452   |
| Railways| 18671               | 20083               | 1.280264| 0.014966| 0.046192   |
| Motorway| 7720                | 7432                | 1.221951| 0.033488| 0.078485   |
| Waterways| 17113              | 11635               | 3.238686| 0.032070| 0.095573   |
information facilitates the performance of a subsequent clustering procedure used inside of the ad–hoc algorithm.

Two parameters need to be specified once the algorithm code is executed. A minimum clustering distance (this generates the network vertices) and a source vertex (a junction number). After entering this information, the Java code invokes the multimodal Dijkstra’s algorithm, reporting at the end a list with the total number of optimal paths to each vertex together with two additional variables: the maximum number of paths found in a particular vertex and the average number of paths.

This dataset was tested by assigning a cluster distances between 0.150 and 0.115 decimal degrees (14 to 11 km). The resulting networks together with running times (minutes) and average number of optimal paths are shown in Table 4. The computations were performed on a standard desktop computer with dual core 2.93 GHz CPU and 8 GB of RAM that was set with the queue version of the multimodal Dijkstra algorithm.

Although the networks shown in Table 4 requiring longer runs of the multimodal Dijkstra’s algorithm when the clustering distance is reduced, it cannot be disregarded that no constraints or reductions were required for obtaining the results in Table 4 and the cardinality of the resulting Pareto
Table 4: Results of France Multimodal Network

| Cluster | Number of Vertices | Number of Edges | Running Time | Avg. Paths | Max Paths |
|---------|--------------------|----------------|--------------|------------|-----------|
| 0.150   | 1501               | 4216           | 0.0182       | 33.0930    | 702       |
| 0.140   | 1869               | 5218           | 0.6953       | 208.270    | 3320      |
| 0.130   | 2343               | 6280           | 4.4010       | 404.380    | 14972     |
| 0.120   | 2948               | 7696           | 245.40       | 2013.82    | 37128     |
| 0.115   | 3108               | 8018           | 823.10       | 3575.03    | 46984     |

sets are quite manageable for any further analysis.

5. Conclusions

In modelling multimodal networks, current approaches for determining shortest paths rely on applying application-specific constraints or heuristics to obtain tractable problems. This paper introduces a modelling approach that keeps the multimodal traits of a network by assigning discrete colour attributes to the edges, uses a partial order to obtain a Pareto set of paths of potential interest, and avoids the need for reduction techniques. Although a straightforward approach to modelling networks in which there are multiple transportation modes, it does appear to give a new perspective and truly general approach for multimodal networks. Another feature of this approach is that it results in a Pareto set, which can be further investigated without rerunning the algorithm. This opens the door to post–optimal analysis in MMN.

The experimental study analyzing the Pareto set indicates that its cardinality is typically a low order polynomial for random uniformly distributed weighted coloured–edge graphs. Furthermore, the approach can deal with networks as large as the multimodal transportation system of France.

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