REPRESENTATION OF THE AWARENESS REFLEXIVE STRUCTURE WITH REGARD TO THE CONFLICT INVOLVING SIGNATURES*

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Abstract. This paper offers a method for analysis of conflict situations when utility of agents is replaced by rules of decision making. This approach simplifies review of a conflict situation, though it imposes significant restrictions to conflict conditions. A conflict shall be presented as separate transitions and Boolean readiness of agents to perform those transitions. These restrictions can be seen when a model is formulated. This approach appeared in modeling of conflict situations in security systems and it corresponds to analysis of intruder-defender systems the most. Models provided in the present paper enable to calculate an object’s readiness to take active actions depending on the condition of its awareness structure. Awareness structure includes the view on indicators of readiness to take active actions by other participants of a conflict and their view on the indicators of the others. A notion of a system researcher is introduced based on the model. The system researcher creates its own awareness structure upon researching structures of the other participants. Conditions for a researching task for two awareness structure researchers which analyze structures of each other are provided herein and it is ascertained which of the participants and in which case is able to solve the task. The paper provides examples of applying mathematical models in real life situations.

1. Introduction
The paper offers a specific method of analysis in a theoretic game situation, when the utility function is not used explicitly. In other words, the analysis of a game situation with specific features is observed herein. The difference of the signature model from the classic theoretic game situation task is only in representation form. Actions of players depend on the reflection level. That dependence was described by V.A. Lefebvre in reflexive research and introduced as a logical element in reflexive games [1, 2]. Later on D.A. Nivikov and A.G. Chkhartashvili named that strategic reflection [3] and reflexive games in addition to strategic reflection include informational reflection, that is values of different elements of the awareness structure. This particular definition of reflexive games is used in the present paper. We can see a dependency of a player’s activity throughout the whole awareness structure by replacing the utility function with rules of decision making. In some cases it is more

* The research was done with the financial support of the Ministry of Education and Science of the Russian Federation in accordance with the agreement № 14.574.21.0126, unique identifier RFMEFI57414X0126

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convenient than introduction of a utility function in applied tasks as we can easily find the whole set of awareness structures in the conflict favorable for us.

The signature model, which is presented in this article, evolved out of conflict situations. It mostly corresponds to the analysis of intruder-defender systems. Clearly, the classic theoretical game model is widely considered for these type of systems. Let us consider the following articles for example [4-6]. Regardless that they used a different method of the game theory in these works, all of them used the qualitative method based on the utility function. It’s not always possible for the researcher to get the initial value of the utility function. The qualitative characteristic is more convenient in order to answer the question of whether the agent benefits from this transaction or not. This qualitative characteristic allows us to build a Boolean expression that lets us make decisions.

2. Model Formulation

Not every utility function can be represented as a Boolean signature, but only with two possible actions for each player. That means that a conflict situation has to be reduced to separate transitions, when each of them is related to the activity of one of the players (take actions or stay inactive). Here is an example of such model.

Suppose there are two agents in the system defined as \( x \) and \( y \). Agent \( x \) wants to take some actions (they may be interpreted as attack methods), and agent \( y \) needs to take a counteraction (protection methods) against each action of \( x \). Hence, a conditional payoff matrix for the agents that has the form as shown in fig.1

\[
\begin{array}{c|cc}
\text{Transition} & + & 0 & 1 \\
\hline
0 & 0 & +1 & 0 \\
1 & 0 & -1 & 1 \\
\end{array}
\]

\[
\begin{array}{c|cc}
\text{Transition} & + & 0 & 1 \\
\hline
0 & 0 & -2 & 0 \\
1 & 1 & -1 & -1 \\
\end{array}
\]

**Figure 1.** The payoff matrix of agents.

Columns marked with “+” denote actions of the agent trying to make a transition in the system. In that case it is agent \( x \). Lines marked as “-” denote actions of agent \( y \) that resists the transition in the system.

1 stands for agent taking actions, 0 stands for inactivity.

According to the first matrix when the agent does not take actions (attack) then it gets no payoff. When it takes actions and the counterpart does not oppose then its payoff is +1, when the counterpart opposes to the transition then the payoff is -1 (attack failed + cost of taking actions).

According to the second matrix when the agent takes actions then its payoff is always -1 (costs). When it does not take actions and the counterpart does not take actions, then the respective payoff is 0, when the counterpart acts (attacks), then the damage is -2.

Agents are aware of the actual matrix values of each other. And resistance to transition in the system is always preventive, which is quite reasonable in terms of security systems.

The game above does not have a solution in pure strategies, and thus agents have to be governed by a guaranteeing strategy. In this case guaranteeing strategies are based on the reflection level. Decision of each agent on the choice to act or stay inactive is based on the length of the chain of considerations “I think that he thinks of what I think...”.

Let us unify notation of such strategies with composing clearer conclusion principles of agents. Therefore 0 and 1 shall denote agent’s readiness to take a specific action (counteraction) in the example provided. The initial state of the object in a conflict shall be denoted as $O$ and suppose that the object can be transferred in to the $O'$ state as a result of actions taken by the subject(s). Such transition is possible after actions taken by subjects, for which the $O'$ state of the object is more favorable than the $O$ state. Description of qualitative characteristics (intention) shall involve the following symbols:

(+)$ means the subject is ready for $O \rightarrow O'$ transition (positive intention)

(–)$ means the subject resists $O \rightarrow O'$ transition (negative intention).

(±)$ means the subject is indifferent to $O \rightarrow O'$ transition, which is the result of absence of any winning/loss of the subject after transition to $O'$, or confidence that it is impossible to transfer the object to that state (zero intention).

The subject’s intention $x \in N$ shall be denoted as $(\cdot)_x$, the subject $x$’s perception of the subject $y$’s perception shall be denoted as $(\cdot)_{xy}$, etc. Therefore subject $x$ maybe denoted as a tree (fig.2) that may be considered as the awareness dot structure described in [2].

![Figure 2. Model of the subject x.](image)

Sequential notation of the tree’s vertices $(\cdot)_x$, $(\cdot)_{xy}$, $(\cdot)_{xyz}$, shall be referred to as “signature”. As there is only one path to each of the vertices signatures shall be defined by the identifier of the last vertice, i.e. $(+ – \pm)_{xyz}$. In case general properties of signatures irrespective to specific subjects are researched then signature identifiers will be omitted.

The subject’s readiness to take actions shall be denoted by putting the Boolean readiness function in accordance with the model’s signature $f$: $(\cdot)_x \rightarrow \{0,1\}$. Axioms introduced are $f(–)=1$, $f(+) = 1$, $f(\pm)=0$. Readiness for other signatures is expressed by introducing rules (hypotheses) “rational conclusions”, which agents use for making decisions on the possibility to take actions. These rules can be introduced in an agent’s utility function matrix. In order to make the most beneficial action in terms of the utility function agents predict actions of counterparts. However as it is impossible to find solutions in pure strategies we suppose that choice of agents depends on the reflection level. The example below contains transitions beneficial for agents under the following scheme (fig.3).

![Figure 3. Transitions beneficial for agents.](image)
In the matrix on the left when \( y \) does not take actions (chooses 0 being the readiness value), hence action (1) is beneficial for \( x \) and vice versa, i.e. \( x \) benefits from applying the readiness value opposite to the value of \( y \).

In the matrix on the right agent \( y \) benefits from applying the same readiness as \( x \).

In order to unify notation of such transitions \( S \) shall denote a set of all conceivable linear signatures, i.e. any sequences of elements \{+, −, ±\}. As a result the rules that belong to either matrix for any of \( \lambda \) and \( \gamma \) contained by \( S \) may be represented as shown in fig.4.

\[
\begin{array}{c|cc}
  & + & 0 & 1 \\
\hline
0 & 0 & +1 & \\
1 & 0 & -1 & \\
\end{array}
\]

\[
\begin{array}{c|cc}
  & + & 0 & 1 \\
\hline
0 & 0 & +1 & \\
1 & 0 & -1 & \\
\end{array}
\]

\[f(+(-\lambda)) = \neg f(-\lambda)\]  
\[f(-(-\lambda)) = f(+\lambda)\]

**Figure 4.** Accordance of transitions rules and utility matrix.

Thus rules express the dependency of element’s readiness and the element following it in the awareness structure. Then let us generalize the situation and suppose that we can have a set of agents complying to the model above, which is expressed in an intention, readiness and reflexive structure.

The rules above may be regarded as first-order rules, i.e. ones created on the first branch of a tree (signature).

### 3. First-order rules

**Rule 1.1.** The subject resists the transition with the same readiness as the counterpart is ready to perform it.

**Rule 1.2.** The subject does not resists the transition when there are other subjects resisting the transition and ready to act.

1.1: \( f(-(+\lambda)) = f(+\lambda) \) (fig.4)

1.2: \( f(-(-\lambda)) = \neg f(-\lambda) \). (resource saving rule)

**Rule 2.1.** The subject is ready to perform a transition only when the counterpart is not ready to take actions.

**Rule 2.2.** The subject does not perform a transition when there are subjects with a positive intention ready to take actions.

2.1: \( f(+(-\lambda)) = \neg f(-\lambda) \) (fig.4)

2.2: \( f(+(+\lambda)) = \neg f(-\lambda) \). (resource saving rule)

**Rule 3.** The subject with a negative intention, which does not observe the counterpart is not ready to take actions:

3: \( f(-(-\lambda)) = 0 \).

**Rule 4.** The subject with a positive intention which does not observe the counterpart is always ready to take actions:

4: \( f(+(+\lambda)) = 1 \).

It is evident that \( f(\pm(\lambda)) = 0 \).

Based on the rules introduced it is possible to determine subject \( x \)’s readiness to take actions with \((+ - +)_{xx}\):

The rules above may be regarded as first-order rules, i.e. ones created on the first branch of a tree (signature).
Therefore the signature \((+ - +)_{x_1y_1z_1}\) is a blocking one for \(x\). The subject does not perform actions with such signature. A similar calculation states that \((+ - + -)_{x_1y_1z_1x_2y_2z_2}=1\). The theorems below can be demonstrated:

**Theorem 1.**
\[
f\left(\frac{+ - + - \ldots \pm \sigma}{1 \ldots \frac{n}{2}}\right) = \begin{cases} 1, & \text{when } n \text{ is even,} \\ 0, & \text{when } n \text{ is odd,} \end{cases}
\]
\(\sigma\) equals either to (+) or to an empty set.

Many other theorems can be similarly introduced and proved, which associate a subject’s set of signatures with readiness status.

### 4. Second-order rules

Second order rules are defined not in the linear signature but in two branches of the subject’s awareness structure tree (fig.1).

**Rule 5.** A subject with a negative intention takes actions only when the counterpart is ready for active actions and the other subject with a negative intention is not ready for active actions:

\[5: f(- (+ \lambda)(- \gamma)) = f(+ \lambda) \land \lnot f(- \gamma).\]

**Rule 6.** The subject with a positive intention takes actions only when the counterpart is not ready for active actions and the other subject with a positive intention is not ready to take actions:

\[6: f(+ (+ \lambda)(- \gamma)) = \lnot f(+ \lambda) \land \lnot f(- \gamma).\]

**Rule 7.** The subject with a negative intention observing the two counterparts takes actions only when one of the counterparts is ready to take actions:

\[7: f(+ (- \lambda)(- \gamma)) = \lnot f(- \lambda) \land \lnot f(- \gamma).\]

**Rule 8.** The subject with a negative intention observing the two counterparts takes actions only when one of the counterparts is ready to take actions:

\[8: f(- (+ \lambda)(+ \gamma)) = f(+ \lambda) \lor f(+ \gamma)\]

As a result a generalization can be made:

**Theorem 2.**
\[
f\left(+ (+ \ldots (+ \ldots (\ldots (- \ldots \right)^{n})\right) = 1, \text{ when } n \text{ (number of brackets) is divisible by } 3 \text{ and each nested signature complies to the constraints of } (+ (+ \lambda)(- \gamma)) \lor (- (- \lambda)(+ \gamma)).
\]

Thus the agent with a positive intention observing the two other agents having different intentions and reckoning that they are not mistaken in their realizations is ready to take actions when its reflection level is divisible by 3.

Conclusion from theorem 2. When an agent does not have any preferences in choosing the reflection level then the probability that \(x\) will be ready to take actions for the latter example equals to \(1/3\) and for an example with two agents it’s \(1/2\). Therefore an agent with a negative...
intention may introduce nonexistent agents into the real counterpart’s perception and therefore decrease its readiness in order to decrease the risk of making a transition in the system.

**Theorem 3.** \( f(\mathbf{+}(-)(-))^n = 0 \iff (n - 1) \text{ divisible by 3} \).

**Theorem 4.** \( f(-(+)(+))^n = 1 \iff (n - 1) \text{ divisible by 3} \).

**Theorem 5.** \( f(-(+)(-))^n = 0 \iff (n - 1) \text{ divisible by 3} \).

**Theorem 6.** \( f(+(+)(+))^n = 1 \iff n \text{ divisible by 2} \).

**Conclusion**

The method of conflict situation analysis reviewed in the present paper is more convenient in some cases as it allows reviewing decisions concurrently at all possible agent reflection levels. However it significantly simplifies a conflict which is unacceptable in some cases. For instance in a situation with three agents we observe evident isomorphism for \( k \) agents, which is more likely not to be observed in reality and it resulted from a very rough description of agent types and rules of their decision making. However the model is quite appropriate for predicting behavior of a small number of agents without introduction of a utility function which is sometimes difficult to establish.

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