NEUTRON CAPTURE AND NEUTRON HALOS

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Abstract: The connection between the neutron halo observed in light neutron rich
nuclei and the neutron radiative capture process is outlined. We show how nuclear
structure information such as spectroscopic factors and external components of the
radial wave function of loosely bound states can be derived from the neutron capture
cross section. The link between the direct radiative capture and the Coulomb
dissociation process is elucidated.

1. Introduction

The neutron capture process in a light neutron-rich nucleus may be viewed as a cre-
ation of a halo state whenever the final bound orbit into which the neutron is captured
has a strong single-particle s-wave component and a small binding energy. It is then
possible to investigate the properties of the halo state by analyzing the \((n, \gamma)\) process
which leads to its formation. There are certain conditions which must be satisfied
to apply this technique. For example, the neutron capture must proceed through a
direct transition and should not be mediated by the formation of a compound state.
Such a process is referred to as the direct radiative capture process (DRC). In this sit-
uation, the capture mechanism can be described by a theoretical model in such a way
that the structure information derived does not depend on the capture mechanism.

As an example, we will show in detail in the next paragraphs that the neutron
capture cross section as a function of the incident neutron kinetic energy can be
considered as a Fourier-Bessel transform of the radial wave function. By inverting
the \((n, \gamma)\) cross section it is therefore possible to derive the radial component of the
halo state wave function.

In addition to nuclear structure investigations, the modeling of the neutron cap-
ture process finds major applications in nuclear astrophysics. In fact, several crucial
\((n, \gamma)\) reaction rates are required, for example, in the network calculation of the nucle-
osynthetic processes in the inhomogeneous big-bang scenario\(^1\). Some of these reaction
rates may be derived from laboratory experiments but some others need to be sup-
plied by theoretical calculation. In any case, the theoretical models are necessary to
complement the experimentally derivable quantities to obtain the reaction rates in
the temperature range of interest for astrophysical applications.

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2. Direct radiative capture (DRC)

A minimal description of the direct radiative capture model (DRC) is given here. Details and references to alternative approaches can be found elsewhere \(\textsuperscript{2}-\textsuperscript{4}\). The capture cross section for emission of electric dipole radiation (E1) in the transition from a state in the continuum \((c)\) to one of the bound states of the residual nucleus \((b)\) is given by

\[
\sigma_{n,\gamma} = \frac{16\pi}{9\hbar v} \kappa_\gamma^3 \tilde{e}^2 |Q_{b\rightarrow c}^{(E1)}|^2
\]

(1)

where \(v\) is the neutron-nucleus relative velocity, \(\kappa_\gamma = \epsilon_\gamma / \hbar c\) the emitted \(\gamma\)-ray wave number and \(\tilde{e}\) the neutron effective charge. The calculation of the matrix elements

\[
Q_{b\rightarrow c}^{(E1)} = <\Psi_c | \hat{T}^{E1} | \Psi_b >
\]

(2)

where \(\Psi_b\) is the bound-state wave function and \(\Psi_c\) the wave function for the neutron in the continuum, is not an easy task, in general. They can, however, be readily evaluated under the particular condition in which the bound state has a strong single particle component. In this case, they can be decomposed into the products of three factors

\[
Q_{b\rightarrow c}^{(E1)} \equiv \sqrt{S_b} \ A_{b,c} \ I_{b,c}
\]

(3)

where \(S_b\) is the spectroscopic factor of the bound state, \(A_{b,c}\) is a factor containing only angular momentum and spin coupling coefficients and the radial overlap

\[
I_{l_b l_c} = \int_0^\infty u_{l_b}(r) w_{l_c}(r) dr
\]

(4)

can be evaluated using some potential model for the calculation of the radial wave functions \(u_{l_b}(r)\) and \(w_{l_c}(r)\).

We have recently applied the DRC model to the calculation neutron capture cross sections of several light nuclei \((^{12}\text{C}, \^{13}\text{C}, \^{14}\text{C}, \^{16}\text{O}, \^{10}\text{Be})\). The results have been reported elsewhere \(\textsuperscript{5}-\textsuperscript{7}\) and the general conclusion is that the DRC model can well describe the neutron capture process in the energy region from thermal up to several hundred of keV. Other calculations made using an essentially equivalent treatment have been recently performed \(\textsuperscript{4}\) with results quite similar to those obtained by our group.

3. Neutron capture and Coulomb dissociation

The matrix elements \(Q_{b\rightarrow c}^{(E1)}\) have been written with a double arrow because they are the same matrix elements which can be obtained in the inverse of the \((n, \gamma)\) reaction: the Coulomb dissociation. In a Coulomb dissociation experiment, a break-up process of the nuclei in the incident beam is induced by the Coulomb field generated by a high-Z target. Under defined kinematic conditions, the \(B(E1)\) strength distribution
for the dissociation of the incident $^{A+1}X$ nucleus into $^{A}X+n$ is measured. The $B(E1)$ strength distribution is related to the neutron capture cross section by

$$\frac{dB(E1)}{dE_x} = \frac{9}{16\pi^2} \frac{k_n^2}{k_\gamma^3} \frac{2J_c + 1}{2J_b + 1} \sigma_{n,\gamma}$$

(5)

where $E_x$ is the excitation energy (defined as the sum of the neutron-residual nucleus relative energy plus the neutron binding energy), $k_n$ is the neutron wave number in the continuum, $J_b$ is the total angular momentum of the bound state and $J_c$ the spin of the residual nucleus in the continuum. The $B(E1)$ strength distribution is therefore related to the matrix elements by

$$\frac{dB(E1)}{dE_x} = \frac{k_n^2}{\pi^2\hbar^2} \frac{e^2}{2J_c + 1} \frac{2J_c + 1}{2J_b + 1} |Q_{b\rightarrow c}^{(E1)}|^2$$

(6)

It must be noted here that, while in a neutron capture measurement all the available bound states can be in principle populated by the incident neutron, in a Coulomb dissociation experiment, only transitions originating from the ground state of the nuclei in the incident beam are possible. In turn, the great advantage of the Coulomb dissociation measurement is that the E1 matrix elements of radioactive nuclei can be investigated. In fact, with the recent developments made with radioactive beam facilities, E1 matrix elements of unstable neutron rich nuclei up to the drip-line have been measured. A first noticeable example of the application of this method has been the measurement of the Coulomb dissociation of $^{11}\text{Be}$. This is a very well known example of halo nucleus. In fact, its ground state is bound by only 505 keV and is dominated by the $|^{10}\text{Be}(0^+) \otimes (2s_{1/2})_\nu >$ configuration. The $dB(E1)/dE_x$ strength distribution can be well reproduced by calculation made using Eq. 6 and the matrix elements can be used to derive information on the $^{11}\text{Be}$ ground-state wave function (see below).

### 4. Nuclear structure information

A noticeable property of the overlap integral in Eq. 4 is that whenever the continuum state wave function $\Psi_c$ can be well approximated by a two-body wave function, detailed information on the bound state can be derived from given overlaps. Moreover, it can be seen that when the bound state wave function is a $l = 0$ orbit (and the initial state must be in this case a $l = 1$ state because of the E1 selection rules), the overlap takes place essentially in the region outside the nuclear radius. This is simply due to the fact that $p$-wave neutrons do not penetrate into the internal nuclear region because of the centrifugal barrier. In turn, the wave function of a bound $s$-orbit extends significantly outside the nuclear surface and can be well described by a Yukawa-tail of type $\psi_b(r) \sim \exp(-\chi r)/r$ where $\chi = \sqrt{2\mu E_b}/\hbar$ (here, $\mu$ is the reduced mass of the two-body system and $E_b$ the energy of the bound state). This is the typical representation of a halo-state wave function in the asymptotic region.
Figure 1: Radial part of the E1 matrix elements (integrand of Eq. 4). The parameters used to calculate the wave functions are given in the text.

From the same considerations it also follows that the overlap is insensitive to the neutron-nucleus interaction in the continuum and the wave function $w_l(r)$ can be well described by the $l = 1$ component of a plane-wave. This can be verified numerically. In figure 1 we show the integrand of $I_{l,b,c}$ for a case in which the bound state is a $s$-orbit bound by 500 keV in a Woods-Saxon potential of radius $R = 2.83$ fm, depth $V_0 = 57.8$ MeV and diffuseness $d = 0.62$ fm. Three different potentials have been considered for the neutron-nucleus interaction in the continuum. They represent a kind of limiting cases and are respectively a Woods-Saxon potential with the same parameters as those of the bound state, a hard-sphere potential of radius $R = 2.83$ fm and a case with no-potential (plane-wave). For neutron energies in the continuum up to 5 MeV there is essentially no difference (always less than 5% in the specific case) in the overlap. This demonstrates and extend the validity of our conclusion on the independence of the $p \rightarrow s$ E1 matrix elements on the neutron-nucleus interaction in the continuum, for neutron energies up to 5 MeV.

4.1. Spectroscopic factor

An important consequence of the statement proven above is that, assuming that a reliable model is available to calculate the bound state wave function $u_l(r)$, the spectroscopic factor $S_b$ can be derived from a measurement of the capture cross section (see Eqs. 3 above). This procedure, already established and applied in proton
capture measurements can be used in the \((n, \gamma)\) case as well. In principle, a single-energy experimental value would be sufficient to derive \(S_b\) from a measurement of \(\sigma_{n,\gamma}\). However, cross section values in a wide energy range may improve the reliability of the obtainable values. Of course a Coulomb dissociation measurement would be altogether equivalent. As an example of a possible application of this method we show here the calculation of the E1 strength distribution for the Coulomb dissociation of \(^{19}\text{C}\) into \(^{18}\text{C}+n\). It can be clearly seen from the figure that in this case the structure of the ground-state of \(^{19}\text{C}\) could be deduced from a Coulomb dissociation experiment. In fact, three possible configurations are available for the ground-state: \(|^{18}\text{C}(0^+)(2s_{1/2})_\nu\rangle\) or \(|^{18}\text{C}(0^+)(1d_{5/2})_\nu\rangle\) and \(|^{10}\text{C}(0^+)(1d_{3/2})_\nu\rangle\). The strength distribution for the latter two configurations differs from that of the most likely former one by more than a factor of 40. The energy dependence is also quite different for the different configurations. As has been recently proposed\(^1\), the structure of the \(^{19}\text{C}\) could be that of a halo state and its spectroscopic structure could be deduced from the measurement of the \(B(E1)\) strength distribution.

4.2. Radial wave function

We will now proceed to show how the bound state wave function can be extracted from a measurement of the E1 matrix elements. The principal ingredient in the calculation of these is the overlap integral \(I_{lblc}\). As just shown, a plane-wave approximation for
the continuum wave function is a good approximation. Then, the radial overlap becomes

$$I_{01}(k) = 2i\sqrt{3\pi} \int_0^\infty u_0(r)r^2j_1(kr)dr$$  \hspace{1cm} (7)

where $j_1(kr)$ is a spherical Bessel function. $I_{01}(k)$ is nothing but the Fourier-Bessel transform (or Hankel transform) of the bound state wave function $u_0(r)$. It can be promptly inverted to obtain

$$u_0(r) = \frac{1}{i\pi^{3/2}\sqrt{3}} \int_0^\infty I_{01}(k)k^2j_1(kr)dr.$$  \hspace{1cm} (8)

This equation provides the radial component of the bound state wave function in terms of the radial matrix elements $I_{01}(k)$ as a function of the neutron wave number $k$. These matrix elements can be derived (see Eqs. 2,3) from neutron capture measurements as well as from a Coulomb dissociation experiment. An example of the reconstruction of the radial part of a halo-state wave function is shown here. The $B(E1)$ strength distribution as measured in a Coulomb dissociation experiment is shown in figure 3. For comparison, we show in the figure a calculation made using a DRC model as described in the previous paragraphs. The solid-line of figure 3 has been then utilized to obtain the radial part of the E1 matrix elements as a function of the $n + ^{10}\text{Be}$ relative energy. Then, Eq. 8 has been employed to calculate the radial part of the ground-state wave function. The result is shown in figure 4. The
Figure 4: Reconstruction of the radial wave function of the ground-state of $^{11}$Be. The single-particle wave function calculated from a Woods-Saxon potential is shown for comparison. See text for more explanations.

comparison is made with the wave function obtained from a Woods-Saxon potential with the depth adjusted to reproduce the correct binding energy. A technique called Pantis-integration has been adopted to calculate the integral of Eq. 8. We can conclude that the radial part of the halo-state wave function could be reconstructed down to approximately 7 fm.

In concluding we would like to stress here that this technique is essentially model-independent as far as the plane-wave approximation holds its validity (and it does, for $l = 1$ partial waves) in the calculation of the continuum two-body wave function. The radial wave function of halo states can be therefore derived directly from neutron capture measurements and/or from Coulomb-dissociation experiments.

5. Conclusion

We have shown here the relations which can be established between “exotic” nuclear structure properties (such as neutron halo configurations) and the reaction mechanisms which lead to their formation. In particular we have shown that spectroscopic information of halo states can be derived from the knowledge of E1 matrix elements for transitions between bound states and the continuum. These matrix elements can be derived from neutron capture measurements in the case of stable nuclei. For nuclei far from the stability line, we have shown that their structure can be investi-
gated by Coulomb dissociation experiments. With the advent of machines capable of producing radioactive nuclear beams, this technique already firmly established in experiments with light neutron-rich nuclei will represent a decisive tool for the exploring the structure of intermediate mass and heavy nuclei far-off the \( \beta \)-stability line.

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