We study $I = 3/2$ elastic $K\pi$ scattering to Born order using nonrelativistic quark wavefunctions in a constituent-exchange model. This channel is ideal for the study of nonresonant meson-meson scattering amplitudes since $s$-channel resonances do not contribute significantly. Standard quark model parameters yield good agreement with the measured $S$- and $P$-wave phase shifts and with PCAC calculations of the scattering length. The $P$-wave phase shift is especially interesting because it is nonzero solely due to $SU(3)_f$ symmetry breaking effects, and is found to be in good agreement with experiment given conventional values for the strange and nonstrange constituent quark masses.
I. INTRODUCTION

The derivation of hadronic interactions from QCD has been a goal of nuclear physics for many years. At present this appears to be a very difficult problem; even the more modest goal of deriving hadronic interactions from the nonrelativistic quark model is difficult, due in part to the variety of mechanisms which contribute to scattering. In a typical hadronic scattering process these mechanisms include s-channel resonance production, t-channel resonance exchange, and nonresonant scattering. Despite the apparent complexity, there is considerable evidence that some scattering amplitudes are dominated by relatively simple perturbative QCD processes. One well known example is the short-range part of the NN interaction; many groups have concluded that the NN repulsive core is due to the combined effects of the Pauli principle and the color magnetic spin-spin component of one gluon exchange. Similarly, the intermediate-range attractive interaction may be due to a relatively simple effect at the quark level, specifically a color-dipole interaction induced by the spatial distortion of the three-quark clusters [1]. Of course one pion exchange dominates at sufficiently large separations, and in a more complete description one should adjoin this to the short-range quark interaction.

In this paper we discuss $I = 3/2 K\pi$ elastic scattering in the nonrelativistic quark model. This process resembles $NN$ scattering in that s-channel resonances are not expected to give important contributions, assuming that multiquark resonances are not in evidence. The Born-order QCD scattering amplitude for this process involves one gluon exchange followed by quark exchange. In a previous paper [2] it was shown that this simple description of hadron-hadron scattering leads to good agreement with the nonperturbative variational results of Weinstein and Isgur [3] near threshold, and with the measured $I = 2 \pi\pi$ S-wave phase shift throughout the full range of $M_{\pi\pi}$ for which data exists. A similar method was used in Ref. [4] to extract effective potentials for many low-lying meson-meson channels. These potentials have recently been applied to several problems in low-energy meson physics. In particular, Dooley et al. [5] used the results of Ref. [4] to suggest that the $IJ^{PC} = 00^{++} \theta(1720)$ may be a $(K^*\bar{K}^*)-(\omega\phi)$ vector-vector molecular bound state. Simple estimates of branching ratios in this model find good agreement with Particle Data Group values and predict new decay modes. In another application it has been argued that the $f_1(1420)$ “E” effect may be a threshold enhancement which is due to an attractive $(K^*\bar{K})-(\omega\phi)$ interaction in the $01^{++}$ channel [4].

The $I = 3/2 K\pi$ channel is an ideal one for testing our model of Born scattering amplitudes, since we expect it to be largely unaffected by $s$-channel resonances. The success of the previous application of the model to $I = 2 \pi\pi$ scattering and the assumption of flavor symmetry suggest that we may also find reasonable agreement with the experimental S-wave $K\pi$ phase shift. The P-wave, however, is driven entirely by flavor symmetry breaking, and hence the interplay of these two waves provides an interesting and nontrivial test of the model.

II. METHOD

The calculation is based on a standard quark model Hamiltonian of the form

2
\[ H = \sum_{i=1}^{4} \left( \frac{p_i^2}{2m_i} + m_i \right) + \sum_{i<j} \left[ V_{\text{conf}}(r_{ij}) + V_{\text{cont}}(r_{ij}) \vec{S}_i \cdot \vec{S}_j + V_{\text{orb}} \right] \vec{F}_i \cdot \vec{F}_j \]  

(1)

where

\[ V_{\text{cont}} = \frac{-8\pi\alpha_s}{3m_i m_j} \delta(\vec{r}_{ij}) \]  

(2)

is the contact color-hyperfine interaction, \( V_{\text{conf}} \) is a confinement potential, and \( V_{\text{orb}} \) represents spin-orbit and tensor interactions. We neglect \( V_{\text{orb}} \) in this paper since its effects are generally found to be numerically small in meson spectroscopy [3], and are expected to be unimportant in the scattering of two \( \ell = 0 \) mesons as well. We shall also ignore the contribution of \( V_{\text{conf}} \) to the scattering interaction. This may appear to be a questionable approximation; however, resonating group calculations have found that the exchange (scattering) kernel due to \( V_{\text{conf}} \) is much smaller than the corresponding kernel for the hyperfine term [4]. This result was also found in the variational calculation of Ref. [3] and the perturbative calculation of Ref. [4]. The latter reference noted that the small \( V_{\text{conf}} \) contribution to scattering is due to a color-factor cancellation in the matrix element of \( V_{\text{conf}} \). However, this result only applies to certain channels; one should not neglect the effects of the confinement term in scattering involving vector mesons.

Although we calculate the scattering amplitude only to Born order, there is evidence that this is a useful and even accurate approximation in systems which are not dominated by s-channel resonances or t-channel meson exchange. First, as there is little evidence for flavor mixing in meson spectroscopy outside the \( \eta - \eta' \) system, one anticipates that neglecting higher terms in the Born series (such as \( q\bar{q} \to gg \to q\bar{q} \)) is not a bad approximation. In addition, the Born-approximation \( I = 2 \pi \pi \) effective potentials derived in Refs. [2] and [3] are numerically very similar to the nonperturbative potentials derived by Weinstein and Isgur. Finally, comparison of perturbative phase shifts to those found in a variational resonating group calculation shows good numerical agreement [4].

For simplicity we use single Gaussians for the asymptotic pion and kaon wavefunctions,

\[ \psi_{\pi(K)}(r) = \left( \frac{\beta^2_{\pi(K)}}{\pi} \right)^{3/4} e^{-\beta^2_{\pi(K)} r^2/2}, \]

(3)

where \( r = |\vec{r}_q - \vec{r}_{\bar{q}}| \). The corresponding momentum-space wavefunction \( \phi(k_{rel}) \) is a function of the magnitude of the relative momentum vector \( k_{rel} = (m_q \vec{k}_q - m_{\bar{q}} \vec{k}_{\bar{q}})/(m_q + m_{\bar{q}}) \).

Flavor symmetry breaking is incorporated through unequal strange and nonstrange quark masses (we introduce a mass ratio \( \rho = m_u/m_s \)) and a meson width parameter \( \xi \) (defined by \( \xi = \beta^2_u/\beta^2_K ; \xi < 1 \) corresponds to a smaller kaon than pion). Of course these parameters are related. For instance if we take \( V_{\text{conf}}(r_{ij}) = C + \kappa r_{ij}^2/2 \) then \( \xi = \sqrt{(1 + \rho)/2} \). Standard quark model values for the constituent masses, \( m_u = 0.33 \) GeV and \( m_s = 0.55 \) GeV, give \( \rho = 0.6 \), and from the SHO relation above we might anticipate \( \xi \approx 0.9 \). A fit to light meson spectroscopy in a Coulomb plus linear potential model with a contact hyperfine term finds a similar \( \rho \) value of \( \rho = 0.58 \). With this \( \rho \) a single-Gaussian variational calculation [4] actually finds a value for \( \xi \) slightly above unity, \( \xi = 1.05 \), because the stronger pion hyperfine attraction leads to a smaller pion than kaon despite the heavier strange quark mass. In any case we expect \( \xi \) to be near unity.
There are four Born-order quark exchange graphs for $K\pi$ scattering, which we previously classified as two “transfer” or “capture” processes in our discussion of $\pi\pi$ scattering. The transfer diagrams represent scattering due to a spin-spin hyperfine interaction between a quark pair ($T_1$) or an antiquark pair ($T_2$). In the capture diagrams the interaction is between a quark-antiquark pair in different mesons, $u\bar{u}$ for $C_1$ and $u\bar{d}$ for $C_2$. We apply the methods of Ref. [2] (Appendix C) to obtain the Born-order Hamiltonian matrix element $h_{fi}$ for these diagrams, which is

$$h_{fi} = \frac{1}{(2\pi)^3} \frac{4\pi\alpha_s}{9m_u^2} (T_1 + T_2 + C_1 + C_2) ,$$

where the term contributed by each diagram is

$$T_1 = \exp \left\{ -(1 + \xi(1 + \zeta))^2 \left[ \frac{1 - \mu}{2} \right] \frac{k^2}{4\beta^2_\pi} \right\}$$

$$T_2 = \rho \left( \frac{2\sqrt{\xi}}{1 + \xi} \right)^3 \exp \left\{ -\frac{\xi}{1 + \xi} \left[ 1 + (1 - \zeta)^2 + 2(1 - \zeta)\mu \right] \frac{k^2}{4\beta^2_\pi} \right\}$$

$$C_1 = \rho \left( \frac{4}{2 + \xi} \right)^{3/2} \exp \left\{ -\frac{1}{2 + \xi} \left[ 1 + 3\xi - \zeta(1 - \zeta) + (\xi - 1 - 3\xi\zeta)\mu \right] \frac{k^2}{4\beta^2_\pi} \right\}$$

$$C_2 = \left( \frac{4\zeta}{1 + 2\xi} \right)^{3/2} \exp \left\{ -\frac{\xi}{1 + 2\xi} \left[ 3 - \zeta + \zeta^2 + \xi(1 + \zeta)^2 \right. \right.$$ 

$$\left. + (1 - 3\zeta - \xi(1 + \zeta)^2)\mu \right] \frac{k^2}{4\beta^2_\pi} \right\} .$$

In these matrix elements $\mu = \cos(\theta_{CM})$, where $\theta_{CM}$ is the center of mass scattering angle, the quark mass parameter $\zeta$ is $(m_s - m_u)/(m_s + m_u) = (1 - \rho)/(1 + \rho)$, and $k$ is the magnitude of the asymptotic three-momentum of each meson in the CM frame. The matrix element $h_{fi}$ is related to the $\ell$th partial-wave phase shift by

$$\delta^{(\ell)} = -\frac{2\pi^2 k E_\pi E_K}{(E_\pi + E_K)} \int_{-1}^{1} h_{fi}(\mu) P_\ell(\mu) d\mu ,$$

with the meson energies related to $k$ by relativistic kinematics. This result involves $\delta^{(\ell)}$ rather than $\sin \delta^{(\ell)}$ because we choose to equate our Born amplitude to the leading term in the elastic scattering amplitude ($\exp\{2i\delta^{(\ell)}\} - 1)/2i$ rather than to the full real part. The phase shifts for all partial waves follow from this result through application of the integral $\int_{-1}^{1} e^{i\nu \theta} P_\ell(\mu) d\mu = 2i\ell(a)$ where $i_\ell$ is the modified spherical Bessel function of the first kind.

### III. RESULTS AND DISCUSSION

On evaluating the angular integrals (6) we find $I = 3/2$ $K\pi$ phase shifts for all $\ell$ in Born approximation given SHO wavefunctions; these are functions of the four free parameters $\beta_\pi$, $\alpha_s/m_u^2$, $\rho = m_u/m_s$, and $\xi = \beta_\pi^2/\beta_K^2$, and require the physical meson masses as input. In the following discussion we shall fix the nonstrange quark mass to be $m_u = 0.33$ GeV since the phase shifts actually involve the ratios given above rather than the absolute scale of $m_u$. 

4
We proceed by fitting the predicted phase shifts to the S- and P-wave phase shift data of Estabrooks et al. [8]. Note however that there may be a discrepancy between this data and earlier $I = 3/2 K\pi$ results [9,10] near threshold; two S-wave data sets are shown in Fig. 1.

As an initial “benchmark” prediction we first neglect flavor-symmetry violation (except for the use of physical meson masses in kinematics and phase space) and employ the same parameters we previously used to describe $I = 2 \pi\pi$ scattering in Ref. [2]; $\alpha_s = 0.6$, $m_u = 0.33$ GeV and $\beta_\pi (\text{fitted to } \pi\pi) = 0.337$ GeV, and we set $m_s = m_u$ and $\beta_K = \beta_\pi$ so that $\rho = 1$ and $\xi = 1$. The resulting S-wave phase shift is shown as a dotted line in Fig. 1. Although the shape of the predicted phase shift is qualitatively correct, evidently the predicted magnitude is somewhat larger than the data at invariant masses above 0.9 GeV.

Of course this flavor-symmetric parameter set is unrealistic because it does not assume a heavier strange quark; allowing $m_s$ to vary yields the value $\rho = 0.677$ in a fit to the S-wave data of Estabrooks et al.; this is close to the $\rho \approx 0.33$ GeV/0.55 GeV = 0.6 expected from $q\bar{q}$ quark model spectroscopy. The resulting phase shift is shown as a dashed line in Fig. 1, and the agreement is impressive. The same parameter set gives a P-wave phase shift which is shown as a dashed line in Fig. 2. Evidently the agreement with experiment is again quite good. Note that the predicted P-wave phase shift is zero for $\rho = 1$, so the S-wave data are consistent with approximate flavor symmetry (which implies $\pi\pi$ S-wave $\approx K\pi$ S-wave) and the P-wave data are consistent with the expected amount of flavor symmetry breaking (seen in the nonzero $K\pi$ P-wave).

Although we have found a satisfactory description of the data simply by using $\pi\pi$ parameters and physical meson masses and fitting $m_s$, it is of interest to investigate the sensitivity of our results to changes in the other parameters and to determine their global optimum values. Fixing $\xi = 1$ and $\beta_\pi = 0.337$ GeV and fitting $\alpha_s$ and $\rho$ to the Estabrooks et al. S-wave data gives $\alpha_s = 0.634$ and $\rho = 0.604$, again consistent with standard quark model values. A global fit to the 33 S- and P-wave data points of Estabrooks et al. with all four parameters free gives $\rho = 0.789$, $\alpha_s = 0.577$, $\beta_\pi = 0.293$ GeV and $\xi = 0.568$. The rather large $m_u/m_s$ in this fit is partially compensated by a spatially small kaon wavefunction, but as the phase shifts are rather insensitive to $\xi$, and we expect a value closer to unity, this best fit probably gives less realistic parameter values than the single-parameter fit which found $\rho = 0.677$. The phase shifts predicted by the the global four-parameter set are shown as solid lines in Figs. 1 and 2. Note that the four-parameter S-wave phase shift is essentially indistinguishable from the one-parameter ($m_s$) fit (dashed line); the most important difference in the predictions of the two parameter sets is in the P-wave, which is not yet very well determined experimentally.

Estabrooks et al. also reported measurements of the $I = 3/2 K\pi$ D-wave phase shift. We predict a small negative D-wave phase shift in accord with the data, although the magnitude of our D-wave is somewhat smaller than is observed. A similar discrepancy in the $I = 2 \pi\pi$ D-wave was previously noted [2,4]. It should be stressed that the D-wave is qualitatively different from the P-wave; it is not driven by flavor symmetry breaking and is an intrinsically small effect at these energies, so that other contributions which we have neglected may be important here. Possible contributions to this higher partial wave include the confinement, spin-orbit and tensor interactions. The departure of the actual $q\bar{q}$ wavefunction from the assumed single Gaussian may also be important, although this effect was investigated in Ref. [4] for $I = 2 \pi\pi$ scattering and was found to be small.
Weinberg [11] used PCAC to predict an \( I = \frac{3}{2} K\pi \) scattering length of

\[
a_{S}^{(3)} = -\frac{m_K m_\pi}{m_K + m_\pi} \frac{1}{8\pi f^2}
\]

in his original PCAC paper. Here, \( f \) is the pseudoscalar decay constant which may be identified with \( f_\pi \) in the flavor symmetric limit. The quark Born approximation for the scattering length may be extracted from our expression for the S-wave phase shift, and is

\[
a_{S}^{(3)} = -\frac{m_K m_\pi}{m_K + m_\pi} \frac{2\alpha_s}{9m_u^2} \left[ 1 + \left( \frac{4\xi}{1 + 2\xi} \right)^{3/2} + \rho \left( \frac{4}{2 + \xi} \right)^{3/2} + \rho \left( \frac{2\sqrt{\xi}}{1 + \xi} \right)^{3} \right].
\]

With our various parameter sets we find the following values for the scattering length \( a_{S}^{(3)} \):

- \(-0.092/m_\pi \) (\( \rho = 1, \xi = 1 \));
- \(-0.077/m_\pi \) (\( \rho = 0.677, \alpha_s = 0.6 \));
- \(-0.078/m_\pi \) (\( \rho = 0.604, \alpha_s = 0.634 \));
- \(-0.076/m_\pi \) (global fit).

These are compared to the PCAC prediction, one-loop chiral perturbation theory and various model calculations in Table I. Experimental values for the scattering length range from \(-0.07/m_\pi \) to \(-0.14/m_\pi \), and are also summarized in Table I. Note that we may also interpret our scattering length as a quark Born formula for \( f_\pi \) if we assume the PCAC relation (7). With the flavor-symmetric parameter set we find \( f_\pi = 80 \) MeV, in reasonable agreement with the experimental value of 93 MeV.

Our theoretical values for the scattering length are consistent with most experimental results, but not with the most recent, which is that of Estabrooks et al. Lang and Porod [10] note that the Estabrooks et al. S-wave phase shift agrees with previous data for \( m_{K\pi} \gtrsim 1 \) GeV, but is somewhat larger in magnitude than previous measurements for \( m_{K\pi} \lesssim 1 \) GeV. Presumably this leads to their rather large scattering length. It would clearly be useful to resolve this experimental discrepancy, since only in this mass region is there any indication of a possible disagreement between the S-wave phase shift and our predictions. It would also be very useful to improve the accuracy of the P-wave measurement, which is a sensitive test of flavor symmetry breaking, and to extend the S-wave phase shift measurements to higher invariant masses as a test of the extremum predicted and perhaps observed near 1.4 GeV.

### IV. CONCLUSIONS

We have calculated \( I = \frac{3}{2} K\pi \) elastic scattering phase shifts using a Born-order constituent-exchange description in the framework of the nonrelativistic quark potential model.

Extensive previous work leads us to believe that one gluon exchange combined with quark exchange may accurately describe nonresonant hadron scattering in certain channels including \( I = \frac{3}{2} K\pi \), and that the Born approximation to the scattering amplitude is an acceptable one. This reaction is appropriate for testing this model of scattering because t-channel pion exchange is forbidden by \( G \)-parity and the experimental phase shift shows no evidence for s-channel resonance formation.

Since this model was previously found to describe the related \( I = 2 \pi\pi \) S-wave phase shift accurately [3], approximate flavor symmetry leads us to expect that the predicted \( I = \frac{3}{2} K\pi \) S-wave phase shift should at least be in qualitative agreement with the data.
This is indeed found to be the case. The agreement is considerably improved when flavor symmetry is broken by assigning the strange quark a mass consistent with standard quark model values. The P-wave $K\pi$ phase shift however is generated entirely by flavor symmetry breaking effects (primarily by the strange-nonstrange quark mass difference in our model), and is not present in $I = 2 \pi\pi$ scattering. The very reasonable result we find for the P-wave phase shift using fitted S-wave parameters is therefore a nontrivial and successful test of the model. Finally, the model predicts an S-wave scattering length of about $-0.077/m_\pi$, which is in the range of reported experimental values and is commensurate with the predictions of chiral perturbation theory.

Although we find a small negative D-wave phase shift as has been reported experimentally, the magnitude and energy dependence are not well reproduced. This, however, is a small contribution to the scattering amplitude, and the various other scattering mechanisms which have been neglected in this calculation may be significant in this case, and should be investigated in future.

Weinstein and Isgur have also studied S-wave $I = 3/2 K\pi$ scattering in the nonrelativistic quark model, using a nonperturbative variational technique [21]. (Their method does not allow extraction of higher partial waves at present.) They find good agreement with the S-wave data, although they must first scale the range and strength of their effective $K\pi$ potentials. We perform no such scaling but do employ relativistic phase space; the fact that both methods agree well with experiment suggests that their scaling may actually be compensating for kinematic effects above threshold. This conclusion is supported by a recent reanalysis of the Weinstein-Isgur variational calculations [22].

The obvious extension of this work is to meson-baryon and baryon-baryon scattering, with the caveat that one should specialize to channels such as $K^+\text{-nucleon}$ and baryon-baryon in which $q\bar{q}$ pair creation and annihilation is unimportant. These topics are currently under investigation.

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FIGURES

FIG. 1. S-wave $K\pi$ Phase Shift. The filled squares are data from [8]; the open squares are from [9] (second solution). The dotted line corresponds to $\rho = 1$, the dashed line to $\rho = 0.677$, and the solid line corresponds to the global fit (see text).

FIG. 2. P-wave $K\pi$ Phase Shift. The data are from [8]. The dashed line corresponds to $\rho = 0.677$ and the solid line to the global fit (see text).
### TABLE I. Experimental and Theoretical Values for the S-wave Scattering Length.

| $a_S^{(3)} \cdot m_\pi$  | Ref. | comments             |
|--------------------------|------|----------------------|
| −0.071(10)               | 9    | experimental         |
| −0.076(10)               | 12   | experimental         |
| −0.084(11)               | 13   | experimental         |
| −0.086(24)               | 14   | experimental         |
| −0.091(9)                | 15   | experimental         |
| −0.110(16)               | 16   | experimental         |
| −0.138(7)                | 8    |                      |
| −0.05                    | 17   | 1-loop chiral pert. theory |
| −0.05                    | 18   | pointlike meson model |
| −0.06                    | 19   | crossing-symmetric Regge model |
| −0.07                    | 11   | PCAC                 |
| −0.074                   | 20   | coupled channel model |
| −0.077                   |      | this work (central value) |