Life prediction of highly reliable products based on wiener process based on random shock response

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Abstract. Aiming at the life prediction problem of high reliability products with random shock response, this paper analyzes the two-dimensional degradation of highly reliable products, the nonlinear Wiener process model was used to analyze the degradation process of the product performance index, and the multivariate nonlinear Wiener process model was established. Based on the sliding window mechanism and the variable step length method, a dynamic variable step size nonlinear Wiener process degradation model was established by dynamically evaluating the parameters of the degradation model. The approximate expression of the probability density function of the residual life was further given. Finally, the validity and practicability of the proposed model and method were verified by an example.

1. Introduction
Life prediction has been concerned by researchers and engineering practitioners many years ago. Especially in recent years, more and more long-life products are widely used in various industries with the progress of technology. It has aroused the enthusiasm of people in the field of life prediction method. Scientific and reasonable maintenance strategy is of great significance to ensure the safety and reliability of long-life products. The traditional maintenance strategy is based on the principle of regular maintenance, which specifies that the product will be used at a predetermined time, or before this time, the product must be withdrawn from use and overhauled [1]. Due to failure to consider the real health status of the product, the scheduled maintenance strategy often appears the problem that "the repair is not repaired, but the one should not be repaired" [2]. For example, for some poor quality products, the failure time is likely to occur earlier than the pre-set maintenance time, resulting in accidental shutdown of the equipment and significant economic losses; On the contrary, for some products of better quality, the performance of the product is very good when the maintenance occurs, and the maintenance will be an unnecessary waste at this time. In order to overcome the above-mentioned shortcomings of traditional regular maintenance, American scientists first put forward the concept of condition-based maintenance, which has been widely used in recent years [3]. Different from the passive periodic maintenance, the condition-based maintenance is an active preventive maintenance strategy, which is based on the analysis of the failure mechanism and uses sensors and other monitoring devices to grasp the healthy state of the product and predict the time of the product failure. And the corresponding maintenance plan is made based on this. Reliability assessment and residual life prediction based on performance degradation data belong to the main category of condition-based maintenance research. In recent years, scholars at home and abroad have carried out in-depth and extensive research on it [4].

In recent years, with the product function becoming more and more complex, life is longer and longer,
the law of performance degradation also presents the characteristics of complexity. Especially in the practical use of the product, there are many factors affecting the product degradation, and the degradation law of the key performance parameters is more complex than the degradation law under the laboratory condition. Therefore, in order to improve the precision of life prediction, there is a higher requirement for the degradation modeling in engineering [5]. At present, the research and application of reliability assessment and residual life prediction methods based on complex degradation processes are not widely used, especially for the complex degradation cases of complex stress and time-varying stress, multiple performance parameters and so on, which often occur under the condition of present field [6]. At present, there are still many problems in modeling, reliability assessment, model updating, residual life distribution estimation and so on in complex degradation process modeling, reliability assessment, model updating, residual life distribution estimation and so on.

In this paper, according to the life prediction problem of high reliability products with random shock response, by analyzing the two-dimensional degradation of highly reliable products, the nonlinear Wiener process model is used to analyze the degradation process of the product performance index, and the multivariate nonlinear Wiener process model is established. Based on the sliding window mechanism and the variable step length method, a dynamic variable step size nonlinear Wiener process degradation model is established by dynamically evaluating the parameters of the degradation model. The approximate expression of the probability density function of the residual life is further given. Finally, the validity and practicability of the proposed model and method are verified by an example.

2. Wiener process

2.1 Independent incremental process

Wiener process is a typical stochastic process, which belongs to the so-called independent incremental process. It plays an important role in the theory and application of stochastic process. The independent incremental process is introduced in stochastic process [7].

Definition: \( \{X(t), t \geq 0\} \) is a second-order moment process, then we call \( X(t) - X(s), 0 \leq s \leq t \) the increment of stochastic process on the interval \( (s, t] \). For arbitrary \( n(n \in \mathbb{N}) \) and arbitrary increments \( 0 \leq t_0 < t_1 < \cdots < t_n \):

\[
X(t_1) - X(t_0), X(t_2) - X(t_1), \ldots, X(t_n) - X(t_{n-1})
\]

Which \( \{X(t), t \geq 0\} \) is independent of each other, then \( X(0) = 0 \) is defined as independent incremental process.

The family of finite dimensional distribution functions of independent incremental processes \( X(t) - X(s), (0 \leq s < t) \) can be determined by the distribution of increments. If the distribution of sumn+1h ∈ R and \( 0 \leq s + h < t + h, X(t + h) - X(s + h) \) are the same, \( X(t) - X(s) \) is defined as increment stationary. At this point, then the incremental distribution function \( X(t) - X(s) \) is dependent on the time difference \( t - s(0 \leq s < t) \), and is independent of sum. It is worth noting that the independent incremental process is homogeneous and the increment is stationary.

2.2 Definition of Wiener process

Given the second order moment process \( W(t), t \geq 0 \), if the

1) There is independent incremental;

2) There is \( \forall t > s \geq 0 \), then the incremental is shown as:

\[
W(t) - W(s) \sim N(0, \sigma^2(t - s)), \text{ and } \sigma > 0
\]

\( W(0) = 0 \) process is called as Wiener process. It can be concluded from (ii) that the increment distribution of Wiener process depends only on time difference, so Wiener process \( n(n \geq 1) \) is a homogeneous independent incremental process and obeys normal process \( 0 \leq t_0 < t_1 < \cdots < t_n \). In fact, \( W(t_k) \) is shown as:

\[
W(t_k) = \sum_{i=1}^{k}[W(t_i) - W(t_{i-1})], \quad k = 1, 2, \cdots, n
\]

That know from (i)-(iii) that they are the sum of independent normal random variables. From the
properties of dimensional normal variables, we can conclude that they are dimensional normal variables. \( W(t_1), W(t_2), \cdots, W(t_n) \) is normal processes. So its distribution depends on its expectation function and self-covariance function. Depending on (ii), (iii), the expectation and variance function of Wiener process \( W(t) \sim N(0, \sigma^2 t) \) is the parameter called Wiener process in the above formula[8].

\[
E[W(t)] = 0, D_w(t) = \sigma^2 t
\]

That can estimate the size \( \sigma^2 \) of Wiener process by doing experiments. The self-covariance function is shown as:

\[
C_w(s, t) = R_w(s, t) = \sigma^2 \min\{s, t\} \quad s, t > 0
\]

3. Dynamic Wiener process degradation Modeling

3.1 Multivariate nonlinear Wiener process degradation Modeling

According to the assumption of the model, the degradation process of product’s performance indexes obeys by the multi-dimensional nonlinear Wiener process [9]. The \( X(t) \) has the following related properties:

The main results are as follows:

1) the cumulative degradation process of the \( k \)th performance index is a nonlinear drift Wiener process;

2) Increment within the disjoint time interval is mutual independence

\[
X(t_1), X(t_2) - X(t_1), \cdots, X(t_n) - X(t_{n-1})
\]

3) \( X(t_j) - X(t_l) \) obeys multidimensional normal distribution, the mean value is \( (A(t_j) - A(t_l)) \beta \), and the variance moment is \( \sum t_j - t_l \).

Assume that the overall product degradation information contains measurements of product’s performance at different times. The performance of product has been measured a total of \( n_i \) times, at the time of measurement at \( t_i,j \), the performance measurement value is \( X_{i,j} \). For the convenience of analysis, the data information of product and the degradation information of the product as a whole are assumed to be at the current time, and the degradation information of the target product is as follows.

\[
X_{\mathbf{h}}(t) = (X^{(1)}(t_{\mathbf{h}}), X^{(2)}(t_{\mathbf{h}}), \cdots, X^{(p)}(t_{\mathbf{h}}))
\]

The target product can visually be characterized by its future degradation behavior \( t_{\mathbf{h}} \) as a result of the current time when the index does not fail. It is given the future degradation behavior of the product[10].

\[
X(t) = X(t_{\mathbf{h}}) + (A(t) - A(t_{\mathbf{h}})) \beta + \xi_{t, t}
\]

Which \( t_{\mathbf{e}} \). Obviously, the given prediction equation only makes use of the degradation information of the target product at the current time, and does not utilize the historical degradation information of the target product.

\[
X_{\mathbf{1:h}} = (X(t_1), X(t_2), \cdots, X(t_{\mathbf{h}}))
\]

The linear drift-based Wiener process can reduce the uncertainty of the prediction results by making full use of the historical degradation information of the target products.

3.2 the time-varying tracking factor

In order to make full use of the historical degradation information of the target product, the state space model is described as the evolution process of the degradation, and there sets a certain length and step of sliding window. Add the new data to the sliding window in step size. At the same time, the model is modified by introducing time-varying tracking factor and forgetting coefficient to reduce the reliability of the old data. The weighted coefficient is used to add different reliability to the data at different times to enhance the time-varying tracking ability of the algorithm. Set the time-varying tracking factor to:
\begin{equation}
\begin{aligned}
\{ T(L, k) &= u(L)T(L - 1, k) \\
T(L, L) &= \Lambda(L)
\end{aligned}
\end{equation}

It is a time-varying forgetting coefficient \( u(L) \), for all, \( L \) satisfies \( 0 < u(L) < 1 \). \( \Lambda(L) \) is a weighted coefficient. The time-varying tracking factor \( T(L, k) \) is a step-by-step tracking factor \( \Lambda(K) \), which is related to the weighted coefficient of \( k \) time. The smaller the \( k \) value is, the farther the distance between \( k \) and \( L \) time is, when \( k \) and \( L \) denote the time.

The analytic relations among time-varying tracking factor, forgetting coefficient and weighted coefficient are as follows:

\begin{equation}
T(L, k) = [\prod_{j=k+1}^{L} u(j)]\Lambda(k)
\end{equation}

The oblivion coefficient and weighted coefficient can be considered as time-varying tracking factors in special cases:

\begin{equation}
\begin{aligned}
\{ T(L, k) &= \Lambda(k), u(j) = 1 \\
T(L, k) &= [\prod_{j=k+1}^{L} u(j)]\Lambda(k) = 1
\end{aligned}
\end{equation}

By means of historical degradation information of multivariate nonlinear, Wiener process degradation model can be updated by setting sliding windows and time-varying tracking factors.

\[ X_{1:h} = T(L, k)(X(t_1), X(t_2), \ldots, X(t_h)) \]

4. Example

The vertical NC (Numerical Control) milling machine produced by a large machine tool factory is used as the processing equipment, the carbide vertical shank end milling cutter is used as the test tool, and the workpiece processing material is made of LC4 aluminum alloy. The degradation data of machining surface precision of aluminum alloy milling are collected. The data of surface roughness \( R_a \) and machining time \( k_t \) are recorded, and the cutting parameters are selected as the spindle speed \( n_0 = 1400 r/min \). The feed speed \( F_0 = 190 r/min \) and the amount of back cutters \( P_a = 0.8 mm \), the milling width \( E_a = 4 mm \).

On the premise that the machining condition is stable, the grooves of long 20mm and wide 20mm are machined by end milling method, and there are the test time \( k_t \) and surface roughness \( R_a \). Are recorded 3 times each time. It is found that the degradation parameters of surface roughness can be obtained by measuring \( R_a = 0.385 \) at the beginning of the test. The starting time of test is defined as \( t_0 \), the surface roughness at the monitoring time \( k_t \) is \( R_a \). The degradation parameters of the surface roughness can be obtained and gradually change with the increase of the test data. The degradation trend is shown in Figure1.
As can be seen from Figure 2 and Figure 3, the experimental values are basically coincident with the predicted values, which shows that the degradation trend of machining surface precision can be well predicted by using the degradation model presented in this paper. Thus, the validity of the proposed method for predicting the precision life and residual life of machined surfaces is verified. This method can make full use of the existing data of precision degradation to predict the moment of precision failure of machined surfaces.
5. Conclusion
In order to predict the life of highly reliable products with random shock response, the two-dimensional degradation of highly reliable products is analyzed, the nonlinear Wiener process model is used to analyze the degradation process of the product performance index, and the multivariate nonlinear Wiener process model is established. Based on the sliding window mechanism and the variable step length method, a dynamic variable step size nonlinear Wiener process degradation model is established by dynamically evaluating the parameters of the degradation model. The approximate expression of the probability density function of the residual life is further given. Finally, the validity and practicability of the proposed model and method are verified by an example. It is shown that the stochastic shock response based on wiener process provides a method guidance and theoretical basis for the life prediction and evaluation of highly reliable products.

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