Analysis of Morton Effect Induced Vibration based on Transfer Function Model: Influence of L/D ratio of Journal Bearing and Rotational speed

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Abstract. The Morton effect is the vibration phenomena that occur due to a thermal imbalance in the oil film in the journal bearing. In previous studies, the qualitative evaluation was performed based on theoretical modeling and numerical simulation. On the other hand, authors proposed that the characteristic of the journal bearing was considered as a proportional differential controller from a control engineering point of view. In the proposed model, the vibration due to the Morton effect can be evaluated based on the transfer function models. The proposed model is based on frequency characteristics identified from the experimental data directly, therefore, the model can be easily applied to the actual rotating machineries. The authors focus this study on applying the proposed method to the actual design, and investigated the relationship of the parameters of the transfer function between the L/D ratio and rotating speed. The relationship indicates the degree of influence of the L/D ratio and rotating speed to the vibration due to the Morton effect. The author measured the vibration by arbitrarily setting the L/D ratio, and analyzed the influence to the vibration due to the Morton effect based on the transfer function model. As a result, the author was able to predict the influence of the bearing L/D and rotating speed on the Morton effect.

1. Introduction

Recently, as the production capacity of various plants has been increased, blowers used in these plants are required to have higher performance. However, a large blower is disadvantageous in cost and delivery time, therefore, it is necessary to reduce the weight of the rotating body and reduce the size of the entire blower while ensuring the reliability and maintainability of the blower. Thus, adopting the flexible shaft is advantageous in reducing the weight of the rotating body. In other words, the design is changed from the conventional blower design, in which the rotation speed is lower than the critical speed, to the new design in which the rotation speed is higher than the critical speed. However, the rotating machinery may become unstable due to the increase of rotational speed. One of these unstable vibrations is the Morton effect[1], which significantly impairs the reliability of the blower. Many studies of this have been reported since the existence of the Morton effect phenomenon was confirmed.

Usually, the blower is horizontal rotating machinery, so it is desirable to reproduce the state of the horizontal axis for modeling and verification. The influence of gravity cannot be ignored on the horizontal axis unlike the vertical axis. Simulation of a theoretical model complicates modeling...
because gravity affects the rotational movement of the shaft in the journal bearing. Furthermore, not only mechanical vibrations but also fluid and heat combine to occur the Morton effect, which makes quantitative analysis difficult. Modeling approaches using mathematical models and numerical simulations have been used, but they have not yet been established. Further, in the design of a blower, it is difficult to change the rotation speed as a measure against the Morton effect because it affects the fluid performance of the blower. In addition, changing the supply temperature of the lubricating oil requires complicated equipment and power supply equipment. In designing a blower, it is necessary to investigate realistic and effective design parameters.

Due to this, the authors proposed that the characteristic of the journal bearing was considered as a proportional differential controller from a control engineering point of view. In the proposed model, the vibration due to the Morton effect can be evaluated based on the transfer function models. The proposed model is based on frequency characteristics identified from the experimental data directly, therefore, the model can be easily applied to the actual rotating machineries. In this study, regression analysis is used to directly evaluate the effect of any design parameters on the Morton effect. In the design of the blower, it is desirable to take measures for the Morton effect without affecting the fluid performance and without need for any additional complicated device. In this regard, investigation of L/D ratio of the journal bearing is considered to be effective. In addition, the authors believe that the optimal L/D ratio may depend on each rotation speed therefore we investigated the effect of vibration due to the Morton effect on L/D ratio and rotational speed.

2. Quasi-experimental data generated by a simulation model

2.1. Transfer function model

This research uses quasi-experimental data generated by a simulator instead of actual experimentation. The simulation model is a model with transfer function proposed by the authors[2]. This method requires an understanding of a rotor system from a regulative perspective, and a transfer function expressing vibration by the Morton effect is determined with Laplace transform. In an advanced research, Takahashi etc. modeled vibration caused by heat on a magnetic bearing by the transfer function of frequency [3]. The authors considered to apply this due to similarity of vibration caused by heat. The authors consider the vibration phenomenon as Figure 1.

![Figure 1. Overview of a transfer function model for Morton effect](image1)

The transfer function is determined with experimental data. The authors use a test machine (Figure 2) to collect experimental data by Morton effect.

![Figure 2. Test machine](image2)
The test measures the average temperature of the shaft surface around the journal (referred to as the average temperature of shaft surface) and the displacement of the rotation shaft. The model is created by steady rotation at 1500rpm. Experimental data of the shaft is recorded before any temperature difference occurs within the shaft right after starting rotation. The trajectory around the average position of the rotating shaft is shown in Figure 3. From the Figure 3, the eccentricity is about 40%. Vibration at this time is caused by static unbalance applied to the rotation shaft to induce the Morton effect. The average temperature of the shaft surface will be higher over time. Experimental data is recorded per one-degree Celsius increase from the starting temperature. Figure 4 shows the amplitude of the rotation shaft when the average temperature of the shaft surface is 18 degrees Celsius right after starting the test, and at 28 degrees Celsius.

Compared with the 18 degrees without the Morton effect occurring, the amplitude was smaller at 28 degrees because the Morton effect works to remove vibration due to static unbalance. Figure 5 shows the frequency response by defining experimental data at 18 degrees without the Morton effect occurring as input and experimental data at average temperature of each journal when the Morton effect occurs as output.

Authors considered that the Morton effect can be expressed by a third-order transfer function base on the frequency response.

\[
P_M(s) = \frac{1}{s + \tau s^2 + 2\zeta \omega s + \omega^2} \quad (2.1)
\]

2.2. Generating quasi-experimental data by a simulator model

Figure 6 shows the simulator structure under this model.
This Figure shows: \( T \) (average surface temperature of each journal), \( \omega \) (speed), and \( U_m \) (static unbalance). The displacement of the shaft \( X_T \) by the Morton effect after inputting the displacement \( X \) of the rotation shaft by the Morton effect to Morton block in which the Morton effect is expressed by transfer function. The bearing reaction of the sliding bearing against the journal displacement is calculated in the JB block.

\[
JB : C(s) s + K(s)
\] (2.2)

\( C \) matrix and \( K \) matrix are calculated using a separate bearing analysis program in advance. The transfer function under Formula (2.1) is included to Morton block which shows the impact of the Morton effect. Coefficients: representatively represent the function of the average temperature of the shaft surface \( T \). Each coefficient of the transfer function is designated by fitting against experimental data.

\[
\tau = 2\pi \{40 + 0.5(T - 18)\}
\]
\[
\alpha = 0.005 \{1 + 0.3(T - 18)\}
\]
\[
\zeta = 0.015
\]

(2.3)

2.3. Simulation conditions

Vibration by the Morton effect under several L/D conditions is calculated in order to generate quasi-experimental data. Conditions except L/D ratio include the average temperature of shaft surface \( T \) and speed \( \omega \). Table 1 summarizes simulation conditions.

| Simulation conditions | T | L/D | \( \omega \) |
|----------------------|---|-----|--------|
| T                    | 18deg~28deg, per 1deg 11 points | 0.5~1.0, per 0.05 11 points | 17,500~21,000 rpm, per 500rpm 8 points |

In the simulation model, the average temperature of shaft surface \( T \) is designed so that the impact of heat bending can be stronger as the temperature increases. When at the lowest temperature of \( T = 18 \) degrees, it is considered that the Morton effect does not occur. In addition, the speed \( \omega \) is calculated on the high speed side at the critical speed of 16,000 rpm. It should be noted that the simulation model on the low speed side has been validated as reasonable for the same L/D ratio as the experimental machine [2] while the reasonability has not been validated when L/D ratio is changed on the high speed side. Our first purpose is to consider a method by a regression analysis and we rarely focus on the reasonability of the simulation model.

3. Research on the relevance between L/D ratio, rotating speed and Morton effect by a regression analysis

3.1. Regression analysis

The regression analysis is one of methods in machine learning or in statistics that calculates a function while predicting one variable by the other variable. An algorithm, Lasso regression for which a regularization term is added to linear least squares is used this time [4]. Formula (3.1) shows the formula of Lasso regression.
This formula shows: \(N\) (the number of data), \(y_i\) (variable that you want to predict 'objective variable'), \(x_i\) (variable used for prediction 'explanatory variable'), \(\beta_i\) (regression coefficient), \(\beta_0\) (segment), \(p\) (the number of parameters of explanatory variable). Item 2 of Formula (3.1) is called regularization term and the impact level of the regularization term is decided by hyper parameter. Lasso regression, called \(L_1\) Norm regularization, functions (such as preventing over-learning and variable selectivity) can be realized by involving addition of absolute values from regression coefficient (\(L_1\) Norm) to the formula of least squares. This research structures a formula that predicts vibration by the Morton effect by the design parameter under this variable selectivity. We make the hyper parameter higher while watching how the regression coefficient and the prediction model will perform and extract explanatory variables that principally impact on Morton amplitude by reducing variables wherever we can.

### 3.2. Research by regression analysis

We now prepare Morton amplitude, called \(A_{\text{morton}}\), as an objective variable. \(A_{\text{morton}}\) took the amplitude of \(X_T\) in Figure 6. The higher this value is, the stronger the heat bending is shown by the Morton effect. Formula (3.3) shows the explanatory variables. Both objective variables and explanatory variables have been processed in advance for standardization as expressed by Formula (3.2) so that the average can be zero and the standard deviation can be one.

\[
\begin{align*}
\tilde{x}_i &= \frac{x_i - x_{i,\text{mean}}}{s_x} \\
\tilde{y} &= \frac{y - y_{\text{mean}}}{s_y}
\end{align*}
\tag{3.2}
\]

This formula shows: \(x_i, y\) (explanatory variable/objective variable), \(x_{\text{mean}}, y_{\text{mean}}\) (arithmetical mean of explanatory variable/objective variable), \(s_x, s_y\) (standard deviation of explanatory variable/objective variable).

\[
\begin{align*}
y &= A_{\text{morton}} \\
x^T = \begin{bmatrix}
\left(\frac{L}{D}\right)^2 T^2 \omega^4 & \left(\frac{L}{D}\right)^2 T^2 \omega^4 & \left(\frac{L}{D}\right)^2 T^2 \omega^4 & \left(\frac{L}{D}\right)^2 T^2 \omega^4 & \left(\frac{L}{D}\right) T \omega^4 & \left(\frac{L}{D}\right) T \omega^4 & \left(\frac{L}{D}\right) T \omega^4 \\
\left(\frac{L}{D}\right)^2 T^3 \omega^3 & \left(\frac{L}{D}\right)^2 T^3 \omega^3 & \left(\frac{L}{D}\right)^2 T^3 \omega^3 & \left(\frac{L}{D}\right)^2 T^3 \omega^3 & \left(\frac{L}{D}\right) T^2 \omega^3 & \left(\frac{L}{D}\right) T^2 \omega^3 & \left(\frac{L}{D}\right) T^2 \omega^3 \\
\left(\frac{L}{D}\right)^2 T^2 \omega^2 & \left(\frac{L}{D}\right)^2 T^2 \omega^2 & \left(\frac{L}{D}\right)^2 T^2 \omega^2 & \left(\frac{L}{D}\right)^2 T^2 \omega^2 & \left(\frac{L}{D}\right) T \omega^2 & \left(\frac{L}{D}\right) T \omega^2 & \left(\frac{L}{D}\right) T \omega^2 \\
\left(\frac{L}{D}\right)^2 T^2 \omega & \left(\frac{L}{D}\right)^2 T^2 \omega & \left(\frac{L}{D}\right)^2 T^2 \omega & \left(\frac{L}{D}\right)^2 T^2 \omega & \left(\frac{L}{D}\right) T^2 \omega & \left(\frac{L}{D}\right) T^2 \omega & \left(\frac{L}{D}\right) T^2 \omega \\
\left(\frac{L}{D}\right)^2 T \omega & \left(\frac{L}{D}\right)^2 T \omega & \left(\frac{L}{D}\right)^2 T \omega & \left(\frac{L}{D}\right)^2 T \omega & \left(\frac{L}{D}\right) T \omega & \left(\frac{L}{D}\right) T \omega & \left(\frac{L}{D}\right) T \omega \\
\left(\frac{L}{D}\right)^2 T^2 & \left(\frac{L}{D}\right)^2 T^2 & \left(\frac{L}{D}\right)^2 T^2 & \left(\frac{L}{D}\right)^2 T^2 & T^2 & T^2 & T^2
\end{bmatrix}
\end{align*}
\tag{3.3}
\]
3.3. Results and consideration

The regression is conducted to the simulation data. Figure 7 shows the regression coefficient to individual hyper parameter $\lambda$. The Lasso regression will actually push coefficients all the way to zero.

Figure 7 shows that some coefficients have become zero and important explanatory variables have been selected. We see that when $\lambda<0.05$ all 46 variables are included in the model and when $\lambda=0.3$ only 4 variables are retained. Accordingly, the formula calculated by regression is as shown below.

$$A_{\text{morton}} = \beta_0 + \frac{L}{D} + (\beta_{T} + \beta_{\omega} + \beta_{T\omega})T + \beta_{\omega}L + \beta_{\omega}T + \beta_{0}$$ (3.4)

Finally, as per Formula (3.4), the formula estimated by assuming the parameter using or linear least squares is Formula (3.5).

$$A_{\text{morton}} = -0.0169\omega^4 + 0.1222\omega^3 + 0.6711\omega^2 + 1.8661\times10^{-15}$$ (3.5)

Furthermore, Figure 8 and Figure 9 show the plot of estimated values by Formula (3.5) to the learning data. The red line shows the plot of estimation values corresponding to Formula (3.5) while the black line shows the plot of the simulation data used for regression. In Figure 8, one cycle shows the result from the temperature conditions ($T=18\text{deg} \sim 28\text{deg}$) of 11 points and one block shows the result from the L/D ratio conditions ($L/D=0.5 \sim 1.0$) of 11 points. Figure 8 shows the result of the rotational speed conditions ($\omega=17,500 \sim 21,000$) constituted by these blocks. This Figure shows that each parameter increases from low side to high side in the direction of "the Number of Data". Based on the regression results, we can consider that a potential was suggested that the L/D ratio might impact on vibration by the Morton effect on the high speed side to the critical speed, and also the higher the L/D ratio is on the high speed side, the more restrictive the vibration by the Morton effect is. On the other hand, the higher the rotational speed, the smaller the effect of restrictive the vibration due to the L/D ratio. However, it cannot be ignored that the influence of the temperature condition is larger than other parameters. This result is consistent with the phenomenon that the coefficient of temperature is the largest in equation (3.5) and the main factor of the Morton effect is due to heat. Furthermore, it is considered that the dependence on the rotational speed and L/D ratio is a reasonable result because the stiffness and damping coefficient of the journal bearing depend on the rotational speed and L/D ratio.
Figure 8. Lasso Regression result with different values of $T$, $L/D$ and $\omega$. ($\lambda=0.3$)

Figure 9. Regression result of sorting the amplitudes in ascending order. ($\lambda=0.3$)

Figure 9 shows the result of sorting the amplitudes in ascending order. The figure shows that we could express vibration phenomenon by the Morton effect under the regression analysis with functions by design parameters of bearing because the model calculated by the regression is relatively consistent with the simulation data. However, the data used this time is just a quasi-experimental data generated by a simulation, therefore we plan to validate under a true prototype test.

4. Conclusion
Blower design requires investigation of realistic and effective design parameters for the Morton effect. The purpose of this research is to directly evaluate the Morton effect with design elements of a rotating machine. As the method, we performed a regression analysis. The data used for the regression analysis is generated by a simulation using a transfer function model. The regression results showed that we can express vibration of the Morton effect by design parameter functions. In addition, the regression results suggested a potential that the higher the $L/D$ ratio is, the more restrictive the vibration by the Morton effect is when it is driven on the high speed side to the critical speed. On the other hand, it was also shown that the higher the rotational speed, the smaller the effect of restrictive the vibration by the $L/D$ ratio.

References
[1] L. Gu, 2017, “A Review of Morton Effect: From Theory to Industrial Practice”, Society of Tribologists and Lubrication Engineers DOI: 10.1080/10402004.2017.1333663
[2] S. Yabui, J. Chiba, T. Suzuki, S. Tomimatsu and T. Inoue, 2019, “Transfer function modeling and experimental variation of rotor system considering Morton Effect caused in journal bearing”, Proceedings of ASME Turbo Expo 2019, June 17 – 21
[3] Takahashi N., Hiroshima M., Miura H. and Fukushima Y. (2003) Dynamic Instability Induced by Iron Loss Unbalance in Rotor-Active Magnetic Bearing System, Transactions of the Japan Society of Mechanical Engineers Series C, Vol 69 (685), pp. 2287-2294.
[4] T. Hersterberg, N. H Choi, L. Meier and C. Fraley, 2008, “Least angle and l1 penalized regression: A review”, Statics Surveys, Vol. 2, pp.61-93