The semileptonic decays $\tau^- \to \mu^-\pi^0(\eta, \eta')$ could be sensitive probe for new physics scenarios with lepton flavor violation (LFV). Motivated by the recent Belle measurement, we investigate these decays in type III two-Higgs-doublet model (2HDM III), R-parity violating supersymmetric models (RPV SUSY) and flavor changing $Z'$ models with family non-universal couplings, respectively. In these new physics scenarios, there are LFV couplings at tree level. Our results have shown that the decays are very sensitive to the LFV couplings and could be enhanced to the present experimental sensitivities. We have derived strong bounds on relevant couplings of these models, which may be useful for further relevant studies.

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I. INTRODUCTION

The flavor physics of fermions are among the most mysterious fundamental problems in particle physics. In the standard model (SM), neutrinos are exactly massless due to the absence of the right handed chiral states ($\nu_R$) and the requirement of $SU(2)_L$ gauge invariance and renormalizability, so that the chirality conservation implies lepton number conservation. In the past decade, the most exciting progress in understanding of these issues has been the observation\cite{1} of oscillation of atmospheric neutrinos with very large mixing. The observation shows that neutrinos are massive and the lepton flavor violating (LFV) exists in the neutral lepton sector.

In the SM supplemented with massive neutrinos, the neutrino mixing will induce, at loop level, rare LFV processes in charge lepton sector such as $\tau \to \mu \gamma$, $\tau \to \mu M (M=$ light hadrons), etc. These processes are expected to be proportional to the ratio of masses of neutrinos over the masses of the W bosons, which is negligible small. However, the $\tau \to \mu$ mixing could be large in new physics models\cite{2}, thus the LFV $\tau$ decays provide some sensitive probes for new physics beyond the Standard Model.

It is interesting to note that the B factories BaBar and Belle are also $\tau$ factories. The $\tau$ production cross section at BaBar and Belle is as large as $\sigma_{e^+e^-\to \tau^-\tau^+} = 0.89\text{nb}$. The integrated luminosity up to now at Belle is about $540\text{fb}^{-1}$, which corresponding to about $4 \times 10^8 \tau^-\tau^+$ pairs. While integrated luminosity at BaBar is about $320\text{fb}^{-1}$. As BaBar

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and Belle are steadily accumulating more data, it would be very promising to search for rare \(\tau\) decays at BaBar and Belle to constrain or reveal new physics effects.

Experimental searches have been performed for \(\tau\) rare decays associated LFV such as \(\tau \rightarrow e(\mu)\eta\) [3, 4, 5], \(\tau \rightarrow e(\mu)\mu^+\mu^-\) [6], \(\tau \rightarrow e(\mu)\pi^+\pi^-\) and \(\tau \rightarrow e(\mu)\gamma\) [8]. The primary theoretical studies are focused on \(\mu\) and \(\tau\) radiative decays and their decays to three charged leptons in different models [2, 10]. There are also many studies on decays \(\tau \rightarrow \ell M\). Ilakovac et al., have studied \(\tau \rightarrow e(\mu)M\) decays in models with heavy Dirac or Majorana neutrinos and found \(\tau \rightarrow e\phi(\rho^0,\pi^0)\) with branching ratios of order \(10^{-6}\) [11]. Sher has investigated the decays \(\tau \rightarrow \mu\eta\) in the large \(\tan\beta\) region in seesaw MSSM [15]. In MSSM model, these decays are analyzed by Fukuyama et al [12]. Brignole and Rossi [13] has presented a comprehensive theoretical study on these decays recently in a general unconstrained MSSM. A model independent study has been performed by Black et al [14] to bound new physics scales.

In order to get information on LFV couplings, an important observable, the anomalous magnetic moment of the muon \((g - 2)_\mu\), should be considered additionally. Several works constraining LFV processes from previous estimations on \(a_\mu\) have been carried out in Refs [16, 17].

The most recent experimental search for semileptonic LFV \(\tau\) transitions has performed by Belle [1] using only 153.8\(fb^{-1}\) data

\[
B(\tau \rightarrow \mu\pi^0) < 4.1 \times 10^{-7}, \quad 90\% \, CL
\]

\[
B(\tau \rightarrow \mu\eta) < 1.5 \times 10^{-7}, \quad 90\% \, CL
\]

\[
B(\tau \rightarrow \mu\eta') < 4.7 \times 10^{-7}, \quad 90\% \, CL.
\]  

(1)

These results are already 10 to 64 times more restrictive than previous CLEO limits [3]. Using the theoretical formula derived by Sher [13] in a seesaw MSSM, the Belle new results have improved the constraints of the allowed parameter space for \(m_A - \tan\beta\) in MSSM. It would be very worthy to study these decays in other new physics models to derive bounds on relevant parameters. In this paper, we will study these decays in three new physics scenarios, namely,

- the 2HDM III [18, 19, 20], where FCNC and LFV could arise at the tree level in the Yukawa sector when the up-type quarks and down quarks are allowed simultaneously to couple more than one scalar doublet,

- SUSY theories with R-parity broken [21, 22, 23], in which the R parity odd interactions can violate the lepton and baryon numbers as well as couple the different generations or flavors of leptons and quarks,

- flavor changing \(Z'\) models [25, 26, 27] with family non-universal couplings.

We have shown that the LFV semileptonic \(\tau\) decays could be enhanced to the present B factories sensitivities in the above three scenarios, thus we have derived bounds on the LFV couplings in the models, which are tighter than exiting ones in the literature.
In next section, we present calculations of the decays and bounds on the LFV couplings in the aforementioned three scenarios. Finally in Sec.III, we give our conclusions.

II. MODEL CALCULATIONS

A. Hadronic matrices of local operators

Before detail model calculations of \( \tau \to \mu M(M = \pi^0, \eta, \eta') \), we would specify hadronic matrices elements which are inputs for calculating these decay amplitudes.

At first, we need

\[
\langle \pi^0(p)|q\gamma_\mu\gamma_5 q|0\rangle = -\frac{i}{\sqrt{2}} f_\pi p_\mu
\]

with \( f_\pi = 130 \pm 5 \text{ MeV} \), and the so-called chiral condensation matrix

\[
\langle \pi^0(p)|q\gamma_5 q|0\rangle = -\frac{i}{\sqrt{2}} f_\pi \frac{m_q^2}{2m_q}
\]

where \( q = u \) or \( d \). We will see that the \( 1/2m_q \) factor will cancel the corresponding quark mass coupling of the weak scalar interaction operators, and thus enhances scalar interaction contributions.

As for \( \eta \) and \( \eta' \), the situation is much more complicated than \( \pi^0 \). The relevant matrices are defined by

\[
\langle M(p)|\bar{q}\gamma_\mu\gamma_5 q|0\rangle = -\frac{i}{\sqrt{2}} f_M^q p_\mu, \quad 2m_q \langle M(p)|\bar{q}\gamma_5 q|0\rangle = -\frac{i}{\sqrt{2}} f_M^q,
\]

\[
\langle M(p)|\bar{s}\gamma_\mu\gamma_5 s|0\rangle = -if_M^s p_\mu, \quad 2m_s \langle M(p)|\bar{s}\gamma_5 s|0\rangle = -ih_M^s
\]

where \( M = \eta \) or \( \eta' \). These equations define eight non-perturbative parameters, which however are not all independent. They could be related to fewer independent non-perturbative parameters by \( \eta - \eta' \) mixing scheme. In this paper, we will take the Feldmann-Kroll-Stech (FKS) mixing scheme [30]. In FKS mixing scheme the parton Fock state decomposition can be expressed as

\[
\begin{pmatrix}
|\eta\rangle \\
|\eta'\rangle
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
|\eta_q\rangle \\
|\eta_s\rangle
\end{pmatrix},
\]

where \( \phi \) is the mixing angle, \( |\eta_q\rangle = (|u\bar{u}| + |d\bar{d}|)/\sqrt{2} \) and \( |\eta_s\rangle = |s\bar{s}| \). The four parameters \( f_M^i \) are therefore related by

\[
f_M^q = f_q \cos \phi, \quad f_M^s = -f_s \sin \phi,
\]

\[
f_M^{q'} = f_q \sin \phi, \quad f_M^{s'} = f_s \cos \phi,
\]

and an analogous set of equations for the \( h_M^i \)

\[
h_M^q = h_q \cos \phi, \quad h_M^s = -h_s \sin \phi,
\]
From these parameters and $\sigma$ parameters with 1 data with results \[30\]

It should noted that the three remaining parameters $f_q$, $f_s$ and $\phi$ in FKS scheme have been constrained from the available experimental data with results \[30\]

$$f_q = (1.07 \pm 0.02) f_{\pi}, \quad f_s = (1.34 \pm 0.06) f_{\pi}, \quad \phi = 39.3^\circ \pm 1.0^\circ. \quad (9)$$

From these parameters and $f_{\pi}$, Beneke and Neubert \[31\] have derived

$$f_q^0 = 108 \pm 3 MeV, \quad f_q^s = -111 \pm 6 MeV, \quad h_q^0 = 0.001 \pm 0.003 GeV^3, \quad h_q^s = -0.055 \pm 0.003 GeV^3,$$

$$f_q^3 = 89 \pm 3 MeV, \quad f_q^s = 136 \pm 6 MeV, \quad h_q^0 = 0.001 \pm 0.002 GeV^3, \quad h_q^s = 0.068 \pm 0.005 GeV^3. \quad (10)$$

It should noted that $h_q^0$ and $h_q^s$ are poorly determined. In our numerical calculations, we take these hadronic parameters with $1\sigma$ variant to display their uncertainty effects on our bounding on LFV couplings. Now we are ready to calculate the decays in the aforementioned three new physics scenarios.

### B. $r \to \mu M$ decays in 2HDM III

Two Higgs Doublet Model(2HDM) is the popular and the most simplest extension of the SM with a scalar sector made of two instead of one complex scalar doublets. In order to build a 2HDM without FCNC at tree level, it is achieved by requiring either that $u$-type and $d$-type quarks couple to the same doublet(2HDMI) or that $u$-type quarks couple to one scalar doublet and $d$-type quarks to the other(2HDMIII). The case in which scalar FCNC not forbidden is dubbed 2HDMIII, where the Higgs doublets could couple to both the $u$- and $d$- type quarks at the same time \[19, 20, 28\].

Generally one can write Yukawa Lagrangian of 2HDMIII \[19, 20, 28\]

$$\mathcal{L}_Y = \eta_{ij}^U \bar{Q}_{i,L} H_1 U_{j,R} + \eta_{ij}^D \bar{Q}_{i,L} H_2 D_{j,R} + \xi_{ij}^U \bar{Q}_{i,L} \tilde{H} U_{j,R} + \xi_{ij}^D \bar{Q}_{i,L} \tilde{H} D_{j,R} + h.c., \quad (11)$$

where $H_i(i = 1, 2)$ are the two Higgs doublets. $Q_{i,L}$ is the left-handed fermion doublet, $U_{j,R}$ and $D_{j,R}$ are the right-handed singlets, respectively. These $Q_{i,L}, U_{j,R}$ and $D_{j,R}$ are weak eigenstates, which can be rotated into mass eigenstates, while $\eta^{U,D}$ and $\xi^{U,D}$ are the non-diagonal matrices of the Yukawa couplings.

For convenience one can express $H_1$ and $H_2$ in a suitable basis such that only the $\eta^{U,D}_{ij}$ couplings generate the fermion masses, i.e.,

$$\langle H_1 \rangle = \left( \begin{array}{c} 0 \\ \frac{1}{\sqrt{2}} \end{array} \right), \quad \langle H_2 \rangle = 0. \quad (12)$$
The two doublets in this basis have the form

\[ H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \sqrt{2}G^+ \\ v + \phi_1^0 & iG^0 \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ \phi_2^0 + iA^0 \end{pmatrix}, \tag{13} \]

where \( G^{0,\pm} \) are the Goldstone bosons, \( H^{\pm} \) and \( A^0 \) are the physical charged-Higgs boson and CP-odd neutral Higgs boson respectively. The advantage of choosing the basis is the first doublet \( H_1 \) corresponding to the scalar doublet of the SM while the new Higgs fields arising from the second doublet \( H_2 \). So \( H_2^0 \) does not couple to gauge bosons of the form \( H_2^{0} ZZ \) and \( H_2^{0} W^+ W^- \).

In Eq.\( \text{(13)} \), \( \phi_1^0 \) and \( \phi_2^0 \) are not the neutral mass eigenstates but linear combinations of the CP-even neutral Higgs boson mass eigenstates \( H^0 \) and \( h^0 \)

\[ H^0 = \phi_1^0 \cos \alpha + \phi_2^0 \sin \alpha, \tag{14} \]

\[ h^0 = -\phi_1^0 \sin \alpha + \phi_2^0 \cos \alpha, \tag{15} \]

where \( \alpha \) is the mixing angle. In the case of \( \alpha = 0 \), \( (\phi_1^0, \phi_2^0) \) coincide with the mass eigenstates of \( H^0 \) and \( h^0 \).

After diagonalizing the mass matrix of the fermion fields, the Yukawa Lagrangian becomes

\[
\mathcal{L}_{Y,C} = \bar{h}_i \xi_{iL} Q_{iL} H_1 U_{jL} R_j + \bar{h}_j \xi_{jR} Q_{jR} H_1 D_{jR} + \bar{h}_i \xi_{iL} Q_{iL} H_2 U_{jL} R_j + \bar{h}_j \xi_{jR} Q_{jR} H_2 D_{jR} + \text{h.c.}, \tag{16}
\]

where \( Q_{iL}, U_{jL}, \) and \( D_{jR} \) now denote the fermion mass eigenstates and

\[
\eta^{U,D} = (V_{U,D})^{-1} \cdot \eta^{U,D} \cdot V_{U,D} = \frac{\sqrt{2}}{v} M^{U,D}(M_{ij}^{U,D} = \delta_{ij} m_{ij}^{U,D}), \tag{17}
\]

\[
\xi^{U,D} = (V_{U,D})^{-1} \cdot \xi^{U,D} \cdot V_{U,D}. \tag{18}
\]

In Eq.\( \text{(18)} \), \( V_{U,D} \) are the rotation matrices acting on up and down-type quarks, with left and right chiralities respectively. Thus \( V_{\text{CKM}} = (V_{U}^L)^{\dagger} V_{L}^D \) is the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix. We can see from Eq.\( \text{(16)} \) that the matrices \( \xi^{U,D} \), as defined by Eq.\( \text{(18)} \), allow scalar-mediated FCNC. That is, in the quark mass basis only the matrices \( \eta^{U,D} \) of Eq.\( \text{(17)} \) are diagonal, but the matrices \( \xi^{U,D} \) are in general not diagonal. The FCNC part of the Yukawa Lagrangian is

\[
\mathcal{L}_{Y,\text{FCNC}} = -\frac{H^0 \sin \alpha + h^0 \cos \alpha}{\sqrt{2}} \left\{ \bar{U} \left[ \xi_{ij}^{U,D} \frac{1}{2}(1 + \gamma_5) + \xi_{ij}^{D,1} \frac{1}{2}(1 - \gamma_5) \right] U + \bar{D} \left[ \xi_{ij}^{D,1} \frac{1}{2}(1 + \gamma_5) + \xi_{ij}^{D,1} \frac{1}{2}(1 - \gamma_5) \right] D \right\} + \frac{iA^0}{\sqrt{2}} \left\{ \bar{U} \left[ \xi_{ij}^{U,D} \frac{1}{2}(1 + \gamma_5) - \xi_{ij}^{D,1} \frac{1}{2}(1 - \gamma_5) \right] U - \bar{D} \left[ \xi_{ij}^{D,1} \frac{1}{2}(1 + \gamma_5) - \xi_{ij}^{D,1} \frac{1}{2}(1 - \gamma_5) \right] D \right\}. \tag{19}
\]

The corresponding Feynman rules from Eq.\( \text{(19)} \) can be found in Refs.\( \text{19, 29}. \)

Because the definition of \( \xi_{ij}^{U,D} \) couplings is arbitrary, we can take the rotated couplings as the original ones and shall write \( \xi_{ij}^{U,D} \) in stead of \( \xi^{U,D} \) hereafter.

In this paper, we use the Cheng-Sher ansatz\( \text{29} \)

\[
\xi_{ij}^{U,D} = \lambda_{ij} \sqrt{m_i m_j} \tag{20}
\]
which ensures that the FCNC within the first two generations are naturally suppressed by small fermions masses. This ansatz suggests that LFV couplings involving the electron are naturally suppressed, while LFV transition involving muon and tau are much less suppressed and may generate sizeable effects.

The decay amplitudes are given by

\[
A(\tau^+ \rightarrow \mu^- \pi^0) = \frac{G_F}{4} \sqrt{m_{\mu} m_{\tau}} f_{\pi^0} m_{\pi^0}^2 \left\{ E \lambda_{\tau \mu}(\bar{\mu} \tau)_{S+P} + F \lambda_{\tau \mu}^*(\bar{\mu} \tau)_{S-P} \right\},
\]

(21)

\[
A(\tau^+ \rightarrow \mu^- \eta) = \frac{G_F}{2 \sqrt{2}} \sqrt{m_{\mu} m_{\tau}} \left\{ h^0_{\eta} J_q + h^0_{\eta} J_s \lambda_{\tau \mu}(\bar{\mu} \tau)_{S+P} + \left( h^0_{\eta} K_q + h^0_{\eta} K_s \right) \lambda_{\tau \mu}^*(\bar{\mu} \tau)_{S-P} \right\},
\]

(22)

\[
A(\tau^+ \rightarrow \mu^- \eta') = \frac{G_F}{2 \sqrt{2}} \sqrt{m_{\mu} m_{\tau}} \left\{ h^0_{\eta'} J_q + h^0_{\eta'} J_s \lambda_{\tau \mu}(\bar{\mu} \tau)_{S+P} + \left( h^0_{\eta'} K_q + h^0_{\eta'} K_s \right) \lambda_{\tau \mu}^*(\bar{\mu} \tau)_{S-P} \right\},
\]

(23)

where \( h^i_M \) are defined by Eq.4 and the auxiliary functions are

\[
E = \frac{1}{m_{A^0}^2} \left( \Re \lambda_{uu} + \Re \lambda_{dd} \right) + i \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{H^0}^2} \right) \left( \Im \lambda_{uu} - \Im \lambda_{dd} \right),
\]

\[
F = - \frac{1}{m_{A^0}^2} \left( \Re \lambda_{uu} + \Re \lambda_{dd} \right) + i \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{H^0}^2} \right) \left( \Im \lambda_{uu} - \Im \lambda_{dd} \right),
\]

(24)

\[
J_q = \frac{1}{m_u + m_d} \left[ \frac{1}{m_{A^0}^2} \left( m_u \Re \lambda_{uu} - m_d \Re \lambda_{dd} \right) + i \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{H^0}^2} \right) \left( m_u \Im \lambda_{uu} + m_d \Im \lambda_{dd} \right) \right],
\]

\[
K_q = \frac{1}{m_u + m_d} \left[ \frac{1}{m_{A^0}^2} \left( m_u \Re \lambda_{uu} - m_d \Re \lambda_{dd} \right) + i \left( \frac{\sin^2 \alpha}{m_{H^0}^2} + \frac{\cos^2 \alpha}{m_{H^0}^2} \right) \left( m_u \Im \lambda_{uu} + m_d \Im \lambda_{dd} \right) \right],
\]

From Eq.21-23 we can see that there are two types operators \((\bar{\mu} \tau)_{S \pm P}\) contribute to decays at tree-level. And it should be noted that the small quark mass factor in the neutral Higgs-quark couplings are cancelled by the one in \(\langle P^0 | q \gamma_5 q | 0 \rangle\).

| Decay modes | Bounds on \(\lambda_{\tau \mu}\) | Previous Bounds |
|-------------|------------------------------|------------------|
| \(\tau \rightarrow \mu^0\) | \(\lambda_{\tau \mu} \sim O(1)\) \(\{28\}\), \(\lambda_{\tau \mu} \sim O(10) - O(10^2)\) \(\{34\}\) | |
| \(\tau \rightarrow \mu\eta\) | \(\lambda_{\tau \mu} \sim O(10^2) - O(10^3)\) \(\{35\}\) | |
| \(\tau \rightarrow \mu\eta'\) | \(\lambda_{\mu\mu} = |\lambda_{\tau\tau}| = |\lambda_{\mu\mu}| = |\lambda_{\tau\mu}| = 10\) \(\{36\}\) | |

In this work we choose \(\xi^{U,D}\) to be complex for the sake of simplicity, so that besides Higgs boson masses, only \(\lambda_{uu}, \lambda_{dd}\) and \(\lambda_{ss}\) in the quark sector and \(\lambda_{\tau \mu}\) in the lepton sector are parameters relevant to the semileptonic \(\tau\) decays. Totally there are seven parameters in the amplitudes: \(\lambda_{qq}(q = u, d, s), \lambda_{\tau \mu}\), their phases \(\theta\), the masses of neutral Higgs \(m_{h^0}, m_{H^0}, m_{A^0}\) and the mixing angle \(\alpha\). Taking into account of constraints on parameters from experimental data and theoretical limits, the values of parameters can be taken as \(\{32,33,34\}\).

\(\theta = \pi/4, \ |\lambda_{uu}| = 150, \ |\lambda_{dd}| = 120, \ |\lambda_{ss}| = 100, \ m_{h^0} = 115 GeV, \ m_{A^0} = 120 GeV, \ m_{H^0} = 150 GeV, \ \alpha = \pi/4\). \(\{25\}\)
FIG. 1: The branching ratios as functions of $|\lambda_{\tau\mu}|$ for fixed $m_{H^0} = 150\text{GeV}, m_{A^0} = 120\text{GeV}, m_{h^0} = 115\text{GeV}, \alpha = \pi/4$, the phase of $\lambda_{\tau\mu}, \theta = \pi/4$ in the 2HDM III. (a) for $\tau^- \to \mu^- \pi^0$ decay, (b) for $\tau^- \to \mu^- \eta$ decay, and (c) for $\tau^- \to \mu^- \eta'$ decay. The upper horizontal lines denote Belle upper bounds. The lower horizontal lines denote the possible upper bounds obtained naively by scaling Belle upper bounds with $540 fb^{-1}$ integrated luminosity. The dotted bands correspond to the effects of hadronic inputs uncertainties.

where the higgs masses satisfy the relation $115\text{GeV} \leq m_{h^0} < m_{A^0} < m_{H^0} \leq 200\text{GeV}$ [32, 33, 34].

In the literature, the effects of $\lambda_{ij}$ have been studied under different phenomenological considerations [19, 28, 32, 33, 34, 35, 36, 37]. For comparison we list bounds on the $\lambda_{\tau\mu}$ parameter of 2HDM III in Table.I. In Ref. [28], it is suggested $\lambda_{\tau\mu} \sim O(1)$. However, based on limits from $(g - 2)_\mu$ results [34] and tau decay [35], $\lambda_{\tau\mu}$ could have much larger value $\sim O(10^{-1} - 10^{-3})$. The branching ratios for $\tau^- \to \mu^- \pi^0(\eta, \eta')$ versus $|\lambda_{\tau\mu}|$ in 2HDM III are presented in Fig.1, where Fig.1.(a), Fig.1.(b) and Fig.1.(c) are the results for $\tau^- \to \mu^- \pi^0$, $\tau^- \to \mu^- \eta$ and $\tau^- \to \mu^- \eta'$ decays, respectively. The dotted bands show our theoretical uncertainties due to the variants of hadronic inputs as in Eq.(10).

The hadronic uncertainties in $B(\tau \to \mu \pi^0)$ arise from $f_\pi$ and are very small. Although $B(\tau \to \mu \eta, \mu \eta'^0)$ involve the poorly known parameters $h_q^\eta$ and $h_q^{\eta'}$, the two decays are dominated by $h_s^\eta$ and $h_s^{\eta'}$, respectively. So the hadronic uncertainties are moderate. The upper horizontal lines denote current Belle upper limits. As shown in Fig.1, the branching ratios, as functions of $|\lambda_{\tau\mu}|$, rise rapidly with the increase of $|\lambda_{\tau\mu}|$. We find that $\tau^- \to \mu^- \eta(\eta')$ decays are more sensitive to $|\lambda_{\tau\mu}|$ than $\tau^- \to \mu^- \pi^0$ decay. From the Belle measurement [4], we get bounds on the LFV coupling $|\lambda_{\tau\mu}|$ which are listed in Table.I for hadronic inputs with central values defaulted. $\tau^- \to \mu^- \eta, \eta'$ decays give upper bounds on the strength of $|\lambda_{\tau\mu}|$ at order of $O(1)$, while $\tau^- \to \mu^- \pi^0$ gives a looser bound. As shown in Table.I, comparing to the former constraints from other processes, we have obtained much more stringent bound for $|\lambda_{\tau\mu}|$ from $\tau \to \mu \eta^{(0)}$. Through calculations, we find that the branching ratios of $\tau^- \to \mu^- \pi$ can be as low as $2.0 \times 10^{-9}$ when $\lambda_{\tau\mu} \simeq 4.5$. It should be noted that the current Belle bounds are based on only $153.8 fb^{-1}$ data. Up to now, Belle has accumulated about $540 fb^{-1}$ data already. To show the potential of bounding on LFV couplings with the full data at Belle, we scale the current upper bounds naively by a factor $153.8 fb^{-1}/540 fb^{-1}$. As a benchmark, the
potential are presented by lower horizontal lines in Fig.1, which would give bounds few times more restrictive than these from current Belle upper limits.

To conclude this subsection, we have shown that, at the similar experimental sensitivity to the three decay modes, searching for \( \tau \to \mu \eta' \) would put more tighten constraints on the Higgs couplings than these from \( \tau \to \mu \pi^0 \) decays.

C. \( \tau \to \mu M \) in RPV SUSY model

The Supersymmetry model with explicit R-Parity breaking provides a simple framework for neutrino masses and mixing angles in agreement with the experimental data \[23, 24\]. The R parity quantum number is defined by \[21\]

\[
R = (-1)^{3B+L+2S},
\]

(26)

where B is the baryon number, L the lepton number and S the spin, respectively.

Apparently, the lepton and/or baryon number violation could lead to R-Parity violation. The explicit R-Parity breaking would introduce renormalizable bilinear higgsino-lepton field mixings and trilinear Yukawa couplings between fermions and their super-partners \[23\].

The amplitudes of these decays are calculated to be

\[
\mathcal{A}(\tau^+ \to \mu^- \pi^0) = \frac{1}{8\sqrt{2}} f_{\pi^0} \left\{ -\frac{m_{\pi^0}^2}{2m_d} L_1 (\bar{\mu}\tau)_{S-P} + L_2 p_{\tau}^\mu (\bar{\mu}\tau)_{V+A} + L_3 p_{\tau}^\mu (\bar{\mu}\tau)_{V-A} \right\},
\]

(29)

\[
\mathcal{A}(\tau^- \to \mu^- \eta) = \frac{1}{8} \left\{ \frac{p_{\eta}^\mu f_{\eta}}{2} L_2 (\bar{\mu}\tau)_{V+A} - \left[ p_{\eta}^\mu f_{\eta} L_4 + \frac{p_{\eta}^\mu f_{\eta}}{2} L_3 \right] (\bar{\mu}\tau)_{V-A} + \frac{h_{\eta}^\mu}{2(m_u + m_d)} L_1 + \frac{h_{\eta}^s}{2m_s} L_5 \right\} (\bar{\mu}\tau)_{S-P},
\]

(30)
From Eq. (29-31), we can see that the couplings relevant to lepton-number violation are \( \lambda \) and \( \lambda' \) couplings. Therefore, in the decays \( \tau \to \mu M \) the parameters are the products of them: \( \lambda^i_{21k}\lambda^*_{31k} \), \( \lambda^i_{111}\lambda^*_{123} \), \( \lambda^i_{3j1}\lambda^*_{2j1} \), \( \lambda^i_{3j2}\lambda^*_{2j2} \), and \( \lambda^i_{222}\lambda^*_{123} \) which are denoted by \( L_i (i = 1, 2, 3, 4, 5) \), respectively. Among these coupling products, the first three

\[
A(\tau^- \to \mu^{-}\eta') = \frac{1}{8} \left\{ \frac{\mu^P}{m^2} \frac{f_{\eta '}}{2} L_2 (\bar{\mu} \tau)_{V + A} - \left[ \frac{\mu^P}{m^2} \frac{f_{\eta '}}{2} L_4 + \frac{\mu^P}{m^2} \frac{f_{\eta '}}{2} L_3 \right] (\bar{\mu} \tau)_{V - A} \right\} \left[ \frac{h^q}{2 (m_u + m_d)} L_1 + \frac{h^q}{2 m_s} L_5 \right] (\bar{\mu} \tau)_{S - P},
\]

with

\[
L_1 = \frac{\lambda^i_{111}\lambda^*_{123}}{m^2}, \quad L_2 = \frac{\lambda^i_{21k}\lambda^*_{31k}}{m^2}, \quad L_3 = \frac{\lambda^i_{3j1}\lambda^*_{2j1}}{m^2}, \quad L_4 = \frac{\lambda^i_{3j2}\lambda^*_{2j2}}{m^2}, \quad L_5 = \frac{\lambda^i_{222}\lambda^*_{123}}{m^2}. \tag{31}
\]

From Eq. (29-31), we can see that the couplings relevant to lepton-number violation are \( \lambda \) and \( \lambda' \) couplings. Therefore, in the decays \( \tau \to \mu M \) the parameters are the products of them: \( \lambda^i_{21k}\lambda^*_{31k} \), \( \lambda^i_{111}\lambda^*_{123} \), \( \lambda^i_{3j1}\lambda^*_{2j1} \), \( \lambda^i_{3j2}\lambda^*_{2j2} \), and \( \lambda^i_{222}\lambda^*_{123} \) which are denoted by \( L_i (i = 1, 2, 3, 4, 5) \), respectively. Among these coupling products, the first three

| Couplings          | \( \tau \to \mu \pi^0 \) | \( \tau \to \mu \eta \) | \( \tau \to \mu \eta' \) | Previous Bounds            |
|--------------------|--------------------------|--------------------------|--------------------------|---------------------------|
| \( \lambda^i_{111}\lambda^*_{123} \) | \( \leq 1.46 \times 10^{-3} \) | \( \leq 3.6 \times 10^{-2} \) | \( \leq 1.1 \times 10^{-2} \) | \( [45] \) |
| \( \lambda^i_{21k}\lambda^*_{31k} \) | \( \leq 1.80 \times 10^{-3} \) | \( \leq 1.93 \times 10^{-3} \) | \( \leq 4.69 \times 10^{-3} \) | \( \leq 1.2 \times 10^{-2} \) | \( [46] \) |
| \( \lambda^i_{3j1}\lambda^*_{2j1} \) | \( \leq 1.80 \times 10^{-3} \) | \( \leq 1.93 \times 10^{-3} \) | \( \leq 4.69 \times 10^{-3} \) | \( \leq 9.1 \times 10^{-2} \) | \( [47] \) |
| \( \lambda^i_{3j2}\lambda^*_{2j2} \) | \( \leq 9.38 \times 10^{-4} \) | \( \leq 1.53 \times 10^{-3} \) | \( \leq 1.2 \times 10^{-2} \) | \( [46] \) |
| \( \lambda^i_{222}\lambda^*_{123} \) | \( \leq 8.03 \times 10^{-4} \) | \( \leq 1.3 \times 10^{-3} \) | \( \leq 4.5 \times 10^{-2} \) | \( [45] \) | \( \leq 3.0 \times 10^{-2} \) | \( [48] \) |
FIG. 3: $\mathcal{B}(\tau \to \mu \eta)$ as function of R-parity violation couplings. (a) for $|\lambda'_{11}\lambda_{i23}|$, (b) for $|\lambda'_{21k}\lambda^*_{31k}|$ and $|\lambda'_{3j1}\lambda^*_{2j1}|$, (c) for $|\lambda'_{3j2}\lambda^*_{2j2}|$, and (d) for $|\lambda'_{i22}\lambda_{i23}|$, respectively. Others are the same as in Fig.1.

Contribute to $\tau \to \mu \pi^0$ decay and all of them contribute to $\tau \to \mu \eta(\eta')$ decays. Besides the type of $(\bar{\mu}\tau)_{S-P}$ operators, $(\bar{\mu}\tau)_{V_{\pm A}}$ operators also appear in the amplitudes of these decays due to Fierz re-arrangements. Because of $\langle P^0|\bar{q}\gamma_\mu q|0\rangle = 0$, $\lambda'_{21k}\lambda^*_{31k}$ and $\lambda'_{3j1}\lambda^*_{2j1}$ will contribute to these decays with the same coefficient.

As in literature, we assume that only one sfermion contributes one time with universal mass 100GeV. We present our results in Fig.2-4 for $\tau^- \to \mu^- \pi^0$, $\tau^- \to \mu^- \eta$ and $\tau^- \to \mu^- \eta'$ decays, respectively.

From Fig.2-4, we find that these decays could be enhanced to the present Belle sensitivities with the presences of RPV couplings constrained by other processes [23], thus we can derive tighter bounds on the relevant RPV couplings.

In Table II, we present bounds on RPV couplings derived from the Belle upper limits on the three $\tau$ LFV decay modes [4] at 90% CL with central values defaulted for hadronic inputs. Most of them are stronger than before.

For $\tau \to \mu \pi^0$ decay, only three coupling products $|\lambda'_{11}\lambda_{i23}|$, $|\lambda'_{21k}\lambda^*_{31k}|$ and $|\lambda'_{3j1}\lambda^*_{2j1}|$ contribute to branching ratios. As shown by Fig.2, the upper bounds for $|\lambda'_{11}\lambda_{i23}|$, $|\lambda'_{21k}\lambda^*_{31k}|$ and $|\lambda'_{3j1}\lambda^*_{2j1}|$ are $O(10^{-3})$ which are more stringent.
FIG. 4: $B(\tau \to \mu \eta')$ as function of R-parity violation couplings. (a) for $|\lambda'_{11} \lambda_{123}|$, (b) for $|\lambda'_{21k} \lambda_{31k}^*|$, (c) for $|\lambda'_{3j2} \lambda_{2j}^*|$, (d) for $|\lambda'_{11\lambda_{i23}}|$, respectively. Others are the same as in Fig.1.

than previous bounds ($O(10^{-2})$)\cite{46,47}. It is noted that the contribution of $|\lambda'_{11} \lambda_{123}|$ is enhanced by a factor $1/m_d$. In numerical calculation, we take $m_d = (4.2 \pm 1.0)$MeV \cite{48}, as shown by Fig.2(a), which causes large uncertainties for our theoretical prediction of $|\lambda'_{11} \lambda_{123}|$ contribution to $\tau \to \mu \pi^0$. With defaulted value $m_d = 4.2$MeV, we get $|\lambda'_{11} \lambda_{123}| \leq 1.46 \times 10^{-3}$.

All the five PRV coupling products contributing to $\tau \to \mu \eta, \mu \eta'$ decays. The sensitivities of the decays $\tau \to \mu \eta$ and $\tau \to \mu \eta'$ to these five RPV coupling products are depicted by Fig.3 and Fig.4, respectively. As shown by Fig.3.(a) and Fig.4.(a), the contributions of $|\lambda'_{11} \lambda_{123}|(L_1)$ are subjected to huge theoretical uncertainties which arise from the poorly known $h^\eta_{\chi}(\eta')$ and $m_q$ as shown by Eq. \cite{46,47}. We could not get meaningful upper bounds on $|\lambda'_{11} \lambda_{123}|$ from $\tau \to \mu \eta, \mu \eta'$ decays.

As shown by Fig.3.b-d and Fig.4.b-d, $\tau \to \mu \eta, \mu \eta'$ decays are very sensitive to the contributions of $|\lambda'_{21k} \lambda_{31k}^*|$, $|\lambda'_{3j1} \lambda_{2j1}|$, $|\lambda'_{3j2} \lambda_{2j2}|$ and $|\lambda'_{3j2} \lambda_{2j2}|$. Therefor we get strong bounds on these four products which have improved the existing ones by one order \cite{45,46,47,47}. From Fig.3.b-d and Fig.4.b-d, we can see that theoretical uncertainties are
It is noted that our study of $\tau \to \mu \eta'$ in RPV SUSY is new. The decays $\tau \to \mu \pi^0, \mu \eta$ have been studied by Kim, Ko and Lee \cite{45}, however, we have used up-to-date hadronic inputs for $\eta$ and $\eta'$ \cite{30}.

D. $\tau \to \mu M$ decays in $Z'$ model with family non-universal couplings

Many extensions of the standard model, especially grand unified theories and supersymmetry models, have additional $Z'$ bosons. In models with an extra $U(1)'$ gauge boson, the $Z'$ family non-universal couplings with the SM fermions generally induce flavor-changing neutral currents. In this paper, we refer to the basic formalism of $Z'$ model elaborated in Ref. \cite{49}.

In the gauge basis, the $Z'$ neutral-current Lagrangian can be written as

$$\mathcal{L}^{Z'} = -g' J'_\mu Z'_\mu,$$

with $g'$ the gauge coupling constant of the $U(1)'$ group at the $M_W$ scale. Here the renormalization group running between the $M_W$ and $M_{Z'}$ scales is neglected considering the uncertainties of parameters. We assume that the $Z'$ boson has no mixing with the SM $Z$ boson \cite{50}. The $Z'$ current can be written as

$$J'_\mu = \sum_{i,j} \tilde{\phi}_I^j \gamma_\mu [(\epsilon_{\phi_L})_{ij} P_L + (\epsilon_{\phi_R})_{ij} P_R] \phi^I_j,$$

where $I$ denotes the gauge interaction eigenstates and $\epsilon_{\phi_L(R)}$ refers to the left(right)-handed chiral coupling matrix.

The fermion Yukawa matrices $Y_\phi$ in the weak eigenstate basis can be diagonalized by unitary matrices $V_{\phi_L,R}$

$$Y^{diag}_\phi = V_{\phi_R} Y_\phi V_{\phi_L}^\dagger.$$

$V_{\phi_L,R}$ could transform $\phi^I$ into mass eigenstate fields $\phi_{L,R} = V_{\phi_L,R} P_{L,R} \phi^I$. The CKM matrix is given by the combination

$$V_{CKM} = V_{u,L} V_{d,L}^\dagger.$$

Flavor changing effects (FCNC) will present when $\epsilon_{\phi_L(R)}$ are nondiagonal matrices. The chiral $Z'$ coupling matrices in the physical basis of fermions thus read

$$B^X_u = V_{u,X} \epsilon_{u,X} V_{u,X}^\dagger, \quad B^X_d = V_{d,X} \epsilon_{d,X} V_{d,X}^\dagger, (X = L, R)$$

where $B^X_{u(d)}$ are hermitian.

The low energy effective Hamiltonian of $\tau \to \mu M$ decays medicated by $Z'$ is

$$\mathcal{H}_{\text{eff}}^{Z'} = -\frac{4G_F}{\sqrt{2}} \left( \frac{g' M_Z}{g_{\text{eff}} M_{Z'}} \right)^2 \left[ B^L_{\tau_L}(\bar{\mu} \gamma_\mu P_L \tau) + B^R_{\tau_L}(\bar{\mu} \gamma_\mu P_R \tau) \right] \sum_q \left[ B^L_{qq}(\bar{q} \gamma_\mu P_L q) \right] + h.c.$$
where $g_1 = e/(\sin \theta_W \cos \theta_W)$. The diagonal elements of $B_{\tau \mu}^{L,R}$ are real for hermiticity of the effective Hamiltonian, but the off-diagonal elements may contain weak phases. We introduce new weak phases $\phi^{L,R}$ for $B_{\tau \mu}^{L,R}$ ($B_{\tau \mu}^{L,R} = |B_{\tau \mu}^{L,R}| e^{i\phi^{L,R}}$) under assumption of neglecting $B_{\eta \eta'}^R$. Now we can write out the decay amplitudes

\[
A(\tau^- \rightarrow \mu^- \pi^0) = -i \frac{G_F}{\sqrt{2}} f_{\pi} \left\{ X(\bar{\mu}\tau)_{V-A} + Y(\bar{\mu}\tau)_{V+A} \right\},
\]

\[
A(\tau^- \rightarrow \mu^- \eta) = -i \frac{G_F}{\sqrt{2}} f_{\eta} \left\{ \left[ \frac{1}{2} f_{\eta}^2 \Delta_1 + f_{\eta}^2 \Delta_2 \right] (\bar{\mu}\tau)_{V-A} + \left[ \frac{1}{2} f_{\eta}^2 \Gamma_1 + f_{\eta}^2 \Gamma_2 \right] (\bar{\mu}\tau)_{V+A} \right\},
\]

\[
A(\tau^- \rightarrow \mu^- \eta') = -i \frac{G_F}{\sqrt{2}} f_{\eta'} \left\{ \left[ \frac{1}{2} f_{\eta'}^2 \Delta_1 + f_{\eta'}^2 \Delta_2 \right] (\bar{\mu}\tau)_{V-A} + \left[ \frac{1}{2} f_{\eta'}^2 \Gamma_1 + f_{\eta'}^2 \Gamma_2 \right] (\bar{\mu}\tau)_{V+A} \right\},
\]

with

\[
X = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 \left( B_{\tau \mu}^L B_{uu}^L - B_{\tau \mu}^L B_{dd}^L \right), \quad Y = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 \left( B_{\tau \mu}^R B_{uu}^L - B_{\tau \mu}^R B_{dd}^L \right),
\]

\[
\Delta_1 = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 \left( B_{\tau \mu}^L B_{uu}^L + B_{\tau \mu}^L B_{dd}^L \right), \quad \Delta_2 = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 B_{\tau \mu}^L B_{ss}^L,
\]

\[
\Gamma_1 = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 \left( B_{\tau \mu}^R B_{uu}^L + B_{\tau \mu}^R B_{dd}^L \right), \quad \Gamma_2 = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 B_{\tau \mu}^R B_{ss}^L.
\]

As in literature, we take $50$ as benchmark. At first, we assume that only one parameter contributes one time and then both $\xi^{LL}$ and $\xi^{RR}$ contribute with the same strength. We present the correlation of branching ratio versus parameter $|\xi|$ in Fig.5-7. The branching ratios get the same contributions from $|\xi^{LL}|$ and $|\xi^{RL}|$. The upper bounds on parameters $|\xi|$ from experimental data are $O(10^{-3})$ which are listed in Table III for comparison. Compared to former two new physics scenarios, there is no scalar operator induced by $Z'$ family non-universal couplings. Thus the poorly known hadronic parameters $h_q^{s,s}$ and factor $1/m_q$ are absent. Theoretical uncertainties are due to decay constants $f_{\pi}$ and $f_{\eta,\eta'}^{q,s}$ which have been determined with few percentages accuracy as listed in Eq.(10). So theoretical estimations of these decays could be made quite accurate.

### Table III: Constraints on the values of $\xi$ in the flavor changing $Z'$ model with family nonuniversal couplings

| Parameters | $\tau \rightarrow \mu \pi^0$ | $\tau \rightarrow \mu \eta$ | $\tau \rightarrow \mu \eta'$ | Previous Bounds |
|------------|-----------------|-----------------|-----------------|----------------|
| $|\xi^{LL}|$ | $\leq 1.81 \times 10^{-3}$ | $\leq 1.91 \times 10^{-3}$ | $\leq 6.91 \times 10^{-3}$ | $\leq 0.02([-70^0, -55^0])$, $\leq 0.005([-80^0, -30^0])$ $[50]$, $\leq 0.0055(110^0)$, $\leq 0.0098(-97^0)$ $[51]$ |
| $|\xi^{RL}|$ | $\leq 1.81 \times 10^{-3}$ | $\leq 1.91 \times 10^{-3}$ | $\leq 6.91 \times 10^{-3}$ | $\leq 0.02([5^0, 15^0])$, $\leq 0.005([-80^0, -30^0])$ $[50]$, $\leq 0.0104(-70^0)$, $\leq 0.0186(83^0)$ $[51]$ |

\[
\xi^{LL} = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 B_{\tau \mu}^L B_{uu}^L e^{i\phi_L}, \quad \xi^{RL} = \left( \frac{g' M_{Z'}}{g_1 M_{Z'}} \right)^2 B_{\tau \mu}^R B_{uu}^L e^{i\phi_R},
\]

and $B_{uu}^L \simeq -2B_{dd}^L$ $[51]$. We find that the variation of phase $\phi_{L,R}$ have nearly negligible influence on the branching ratios. So we take $\phi_{L,R} = 10^0$ $[50]$ as benchmark. At first, we assume that only one parameter contributes one time and then both $\xi^{LL}$ and $\xi^{RR}$ contribute with the same strength. We present the correlation of branching ratio versus parameter $|\xi|$ in Fig.5-7. The branching ratios get the same contributions from $|\xi^{LL}|$ and $|\xi^{RL}|$. The upper bounds on parameters $|\xi|$ from experimental data are $O(10^{-3})$ which are listed in Table III for comparison. Compared to former two new physics scenarios, there is no scalar operator induced by $Z'$ family non-universal couplings. Thus the poorly known hadronic parameters $h_q^{s,s}$ and factor $1/m_q$ are absent. Theoretical uncertainties are due to decay constants $f_{\pi}$ and $f_{\eta,\eta'}^{q,s}$ which have been determined with few percentages accuracy as listed in Eq.(10). So theoretical estimations of these decays could be made quite accurate.
As shown by Fig.5 and 6, the sensitivities of $B(\tau \to \mu \pi^0)$ and $B(\tau \to \mu \eta)$ to the $Z'$ LFV FCNC couplings are quite similar. From Belle upper limit on $B(\tau \to \mu \pi^0)$, we get $\xi < 2 \times 10^{-3}$ which are comparable with previous bounds\cite{50,51}. However, from Fig.7, we can see that $\tau \to \mu \eta'$ decay gives weaker bounds than $\tau \to \mu \eta(\pi)$.

Anyhow the Belle searching for $\tau \to \mu M$ decays have already put strong bounds on the parameter spaces. In other words, these decays could be enhanced to the present B factories sensitives by the $Z'$ family nonuniversal FCNC couplings without conflict with bounds from other exist measurements.
III. CONCLUSION

The measurement of LFV processes would be a definite evidence for physics beyond the SM. In this paper, we have studied LFV processes $\tau^- \rightarrow \mu^- M (M = \pi^0, \eta, \eta')$ at the tree level in the 2HDM III, RPV SUSY and flavor changing $Z'$ model with family non-universal couplings. Since these decays are very sensitive to the presence of LFV couplings, we have derived constraints on parameter space of the three New Physics scenarios from the recent Belle limits\cite{4}. Our main findings can be summarized as follows.

1. In 2HDM III, the strongest bound on $\lambda_{\tau \mu}$ comes from $\tau \rightarrow \mu \eta$ decay. The bound is consistent with those in literature, however, improve these by several times.

2. In RPV SUSY, $\tau \rightarrow \mu \eta, \mu \eta'$ decays are very sensitive to the contributions of $|\lambda'_{21k} \lambda'_{31k}|$, $|\lambda'_{3j1} \lambda'_{2j1}|$, $|\lambda'_{3j2} \lambda'_{2j2}|$ and $|\lambda'_{122} \lambda'_{123}|$. Therefore, we get strong bounds on these three products which have improved the existing ones by one order \cite{45, 46, 47, 48}. However, there are large uncertainties in calculating contributions of these RPV coupling products due to hadronic parameters $h^{*}_{\eta, \eta'}$. We could not get bounds on $|\lambda'_{111} \lambda'_{233}|$ from the two decays because of poorly known $h^{*}_{\eta, \eta'}$. However, we can get strong bound on $|\lambda'_{111} \lambda'_{233}|$ with $\tau \rightarrow \mu \pi^0$.

3. In $Z'$ model, theoretical predictions of $Z'$ LFV coupling contributions could be made quite accurate. $\tau \rightarrow \mu \pi, \mu \eta$ and $\mu \eta'$ decays have similar sensitivities to the LFV couplings. Belle current upper limits on $B(\tau \rightarrow \mu \eta, \mu \eta')$ have already constrained $\xi$ as small as $O(10^{-3})$.

In summary, we have shown that the LFV semileptonic decays $\tau \rightarrow \mu \pi^0, \mu \eta$ and $\mu \eta'$ are very sensitive to the presence of LFV couplings in 2HDM III, RPV SUSY and $Z'$ models. Using only 153.8 fb$^{-1}$ data, Belle recent upper
limits for these decays have already given quite tight bounds on the LFV couplings in the aforementioned three new physics scenarios. It should be noted that Belle and BaBar had accumulated about $540 f b^{-1}$ and $320 f b^{-1}$, respectively, till the end of year 2005. With refined measurements with the these data at Belle and BaBar, we could get more crucial information on LFV, at least more stringent bounds on the parameter spaces of models with LFV couplings. The results derived in this paper from the recent measurements at Belle would be useful for phenomenological studies of the scenarios in other interesting processes.

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