Knowledge Refactoring for Program Induction

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Abstract
Humans constantly restructure knowledge to use it more efficiently. Our goal is to give a machine learning system similar abilities so that it can learn more efficiently. We introduce the knowledge refactoring problem, where the goal is to restructure a learner’s knowledge base to reduce its size and to minimise redundancy in it. We focus on inductive logic programming, where the knowledge base is a logic program. We introduce Knorf, a system which solves the refactoring problem using constraint optimisation. We evaluate our approach on two program induction domains: real-world string transformations and building Lego structures. Our experiments show that learning from refactored knowledge can improve predictive accuracies fourfold and reduce learning times by half.

1 Introduction
Humans commonly restructure knowledge to use it more efficiently [Rumelhart and Norman, 1976; Vosniadou and Brewer, 1987; Stern, 2005]. For instance, we refactor computer programs to make them more reusable, readable, and efficient. As a running example, consider the Lego structures shown at the top of Figure 1. Both structures have eight bricks of four distinct shapes. We could simplify both structures by introducing an L-shaped brick. The resulting structures shown in the bottom of Figure 1 have 5 and 3 bricks respectively of two distinct shapes. By introducing an L-shaped brick, the two structures, and potentially future structures, are now easier and faster to build. The key to effective restructuring is to find the appropriate abstractions to use.

The majority of learning AI agents, however, never restructure their acquired knowledge, which can have detrimental consequences on their performance [Srinivasan et al., 1995; 2003]. In machine learning, knowledge can be seen as a form of inductive bias [Mitchell, 1997] which determines the hypothesis space, where increasing the amount of knowledge increases the hypothesis space. The major challenge in machine learning is, therefore, how to choose a learner’s inductive bias (i.e. knowledge) so that the hypothesis space is large enough to contain the target hypothesis, yet small enough to be efficiently searched. A few approaches that do revise knowledge do so by either by modifying knowledge to make it consistent with new observations [Adé et al., 1994] or by forgetting certain knowledge which minimally affects their performance [De Raedt et al., 2008; Cropper, 2020].

In this paper, we claim that the human-like ability to restructure knowledge is essential for AI agents. Instead of removing or updating knowledge, we argue that changing its structure, without changing any knowledge, can provide a better inductive bias to a learner. The goal of our work is to tackle the inductive bias problem by (1) reducing the size of the knowledge base, and (2) restructuring it to make it easier to learn from.

To restructure knowledge, we must explicitly store knowledge. This requirement eliminates learning approaches which dissipate knowledge in the parameters of a model, i.e. non-symbolic approaches. We therefore use symbolic learning approaches which combine machine learning and knowledge representation. Specifically, we focus on inductive logic programming (ILP) [Muggleton and De Raedt, 1994], a form of program induction which represents knowledge as a logic program (a set of logical rules).

Our specific contributions are:

- We introduce the knowledge refactoring problem: revising a learner’s knowledge base (a logic program) to reduce its size and minimise redundancy.
- We introduce Knorf, a system that restructures knowledge bases by searching for new, reusable pieces of knowledge. We do this by casting the problem as a constraint optimisation problem.
- We evaluate our approach on two program induction domains: string transformations and Lego structures. Our
experiments show that learning from refactored knowledge can improve predictive accuracies of an ILP system fourfold and reduce learning times by a half.

2 Related work

The goal of program induction is to learn computer programs from input/output examples and background knowledge (BK). The goal of our work is to improve the performance of a program induction system by (1) reducing the size of the BK, and (2) restructuring the BK to make it easier to learn from. We focus on ILP, a form of program induction which learns programs from examples and BK, where the examples, BK, and programs are all logic programs.

Eliminating redundancy in a clausal theory is useful in many areas of AI, e.g. to improve SAT efficiency [Heule et al., 2015]. Most work focuses on removing redundant literals and clauses in a clausal theory [Plotkin, 1971]. Our work is similar, but we also allow for the introduction of new knowledge, as to compress existing knowledge. Theory minimisation approaches to try find a minimum equivalent formula to a given input propositional formula [Hemaspaandra and Schnoor, 2011] and also introduce new formulas. By contrast, we focus on first on first-order (Horn) logic.

Theory revision [Adé et al., 1994; De Raedt, 1992] approaches revise a program so that it is consistent with a new example. Theory compression [De Raedt et al., 2008] selects a subset of clauses such that the performance is minimally affected with respect to certain examples. In contrast to these works, our approach does not consider any examples: we only consider the knowledge base.

Studies show that irrelevant and redundant knowledge is detrimental to learning performance [Srinivasan et al., 1995; 2003; Cropper and Tourret, 2019]. For instance, in the forgetting approach [Cropper, 2020], the goal is to reduce the size of the BK to improve the learning efficiency of a program induction system. However, whereas the forgetting approach only removes clauses from the BK, we additionally introduce new knowledge to compress existing knowledge.

Alps [Dumančić et al., 2019] revises knowledge by compressing factual knowledge, rather than a program itself. Dreamcoder [Ellis et al., 2018] learns libraries of subroutines by compressing a program, but requires a domain-specific neural architectures and context-free grammars. Knorf, by contrast, works out-of-the-box with any domain.

3 Problem Description

To introduce the knowledge refactoring problem, we first provide essential preliminaries on logic programming (LP) [Sterling and Shapiro, 1986], after which we show how knowledge refactoring can aide the ILP Metagol [Cropper and Muggleton, 2016].

3.1 Logic Programming

An atom (e.g. studies(alan,computers)) is a predicate symbol (studies) applied to constants (alan,computers) or variables (X). A clause is a formula of the form H ← B₁,...,Bₙ, where H and each Bᵢ are literals (atoms or their negations). A clause is read as logical implication: H if B₁ and ... and Bₙ. H is the head of a clause and B₁,...,Bₙ is the body. A clause is a set of literals. The size size(c) of a clause c is the number of literals in c (including the head literal). A logic program is a set of clauses. The size size(T) of a logic program T is the number of literals in T. Figure 2 shows an example Lego program.

A key concept in LP is (un)folding [Tamaki and Sato, 1984]. Intuitively, given a set of clauses S, the fold(T,S) operation replaces every occurrence of the body of clause c ∈ S, up to variable renaming, in program T with its head. For instance, folding the top clause in Figure 2 with the clause instructing how to make an L-shaped brick results in the bottom program in Figure 2. The unfold(T) operation essentially inlines all functions: for every clause c in program T which defines a predicate that is used in the body of another clause, it replaces every occurrence of the head of c in T with its body. For instance, unfolding the bottom program in Figure 2 results in the program on top: the body of the second clause is inlined in the first clause. We assume that every inlined clause is removed from the program after unfolding.

3.2 Knowledge Refactoring Problem

We differentiate between three types of predicates in a logic program (a knowledge base):

- **Primitive** predicates appear only in the bodies of clauses after unfolding the program, e.g. place/1 and right/2 in Figure 2
- **Target** predicates appear only in the heads of clauses after unfolding the program, e.g. shp/1 in Figure 2
- **Support** predicates occur in the original program but disappear once unfolded, e.g. placeL/1 in Figure 2

These types can be intuitively understood by relating them to a programming language: primitive predicates are built-in functions of a programming language, target predicates form an API of a library, while support predicates are auxiliary functions. The key difference, in our case, is that the primitive predicates come from the problem domain, rather than a programming language.

Support predicates help us better structure a program. They can be removed from a program without changing its semantics with respect to the target predicates. When refactoring a program, the goal of a programmer is to identify a good set

\[
\text{shp}(X) \leftarrow \text{place}(X), \text{right}(X,Y), \text{place}(Y), \text{right}(Y,Z), \text{place}(Z), \text{place}(Z), \text{place}(Z).
\]

[Figure 2: Generating a simple Lego structure shp. The program places a single Lego piece on position X, and then places an L-shaped piece on position Y on the right of X. To place an L-shaped piece, the program places one piece and stacks three Lego pieces right to it.]

\[
\text{shp}(X) \leftarrow \text{place}(X), \text{right}(X,Y), \text{place}(Y).
\]

\[
\text{placeL}(A) \leftarrow \text{place}(A), \text{right}(A,B), \text{place}(B), \text{place}(B), \text{place}(B).
\]
of support functions to make the program more compact and reusable. Knowledge refactoring shares the same goal.

The goal of refactoring is to introduce support predicates to reduce redundancy in a program, i.e. more compactly expresses the same knowledge. To define a space of possible support predicates, we introduce the following concepts.

**Definition 1 (Connected literals).** A set of literals is connected if it cannot be partitioned into two sets such that the variables appearing in the literals of one set are disjoint from the variables appearing in the literals of the other set.

**Definition 2. (Clausal power-set)** A clausal power-set of the clause \( c, P(c) \), is the power-set of the literals in the body of \( c \), excluding the empty set.

**Definition 3. (Connected clausal power-set)** A connected clausal power-set of the clause \( c, P(c) \), is the maximal subset of \( P(c) \) such that at every \( s \in P(c) \) is a connected set of literals.

**Example 1.** Consider the clause \( h(X,Y) :- a(X,Y), b(Y,Z), c(Z) \). The clausal power-set of the clause would be (with variables dropped) \( \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \). The connected clausal power-set would remove \( \{a,c\} \) as it is not a connected set of literals (variables in \( c \) are disjoint from variables in \( a \)).

**Definition 4 (Space of possible support clauses).** A clause is in the support clause space \( S_j \) of a program \( T \) when (1) it has at least \( i \) and at most \( j \) literals in the body, (2) the set of literals in the body is in \( \bigcup_{T \in \mathcal{C}(c)} \) (up to variable names), and (3) the head predicate symbol is unique and does not appear in \( T \).

We define the knowledge refactoring problem:

**Definition 5 (Knowledge refactoring problem).** Let \( T \) be a logic program and \( S_j \) be a set of support clauses. Then the refactoring problem is to return \( s \in S_j \) such that:

1. \( \text{fold}(\text{unfold}(T),s) \models \text{unfold}(T) \)
2. there is no \( s' \in S_j \) such that \( \text{fold}(\text{unfold}(T),s') \models \text{unfold}(T) \) and \( \text{size}(\text{fold}(\text{unfold}(T),s')) < \text{size}(\text{fold}(\text{unfold}(T),s)) \)

In other words, condition 1 states that we want to identify support clauses that once folded into a program preserve the semantics of the original program, and condition 2 states that we want the support clauses that, once folded into it, lead to the smallest program.

**3.3 Why Refactor?**

Why would refactoring improve the performance of a program induction system? To explain, we formalise the impact of refactoring in an ILP setting.

Metagol [Muggleton et al., 2015; Cropper and Muggleton, 2016; Cropper et al., 2019] is a state-of-the-art ILP system, which we use in our experiments to evaluate the benefits of refactoring. Metagol takes as input (1) positive and negative examples (ground atoms) of a target concept, (2) BK (a logic program), and (3) a set of metarules, higher-order Horn clauses that define the hypothesis space. These inputs define the size of the Metagol hypothesis space [Cropper et al., 2019]:

**Proposition 1 (Hypothesis space).** The size of the Metagol hypothesis space is at most \((mp^{p+1})^n\), where \( m \) is the number of metarules with at most \( j \) body literals, \( p \) is the number of predicate symbols in the BK, and \( n \) is the minimal number of clauses need to express the target hypothesis given the BK.

According to the Blumer bound [Blumer et al., 1987], given two hypothesis spaces of different sizes, searching the smaller space will result in fewer errors, assuming that the target hypothesis is in both spaces. If we assume (1) a fixed set of metarules, and (2) that we do not exclude the target hypothesis, then Proposition 1 implies that we can improve the performance of Metagol by either reducing the number of predicate symbols \( p \) or the size of the target program \( n \). We can reduce both by refactoring. We can reduce \( p \) by removing redundant predicate symbols and also by limiting the number of predicate symbols allowed in a refactored BK. We can reduce \( n \) by restructuring the BK so that we can express the target hypothesis using fewer clauses.

**4 Knorf: A Knowledge Refactoring System**

**Knorf** solves the refactoring problem (Definition 5) by transforming it to a constraint optimisation problem (COP) [Rossi et al., 2006], where the goal is to find an optimal set of support clauses. Given (1) a set of decision variables, (2) a problem description in terms of constraints, and (3) an objective function, a COP solver (CP-SAT [Perron and Furnon, 2019]) finds an assignment to decision variables that satisfies all specified constraints and maximises or minimises the objective function.

Rather than work at the semantic level, Knorf works at the syntactical level by minimising program size and syntactic redundancies:

**Definition 6. (Syntactic redundancy)** A logic program \( T \) has syntactic redundancy if there are two clauses \( c_1, c_2 \in T \) such that \( c_1 \neq c_2, u_1 \in \mathcal{C}(c_1), u_2 \in \mathcal{C}(c_2), \text{size}(u_1) > 1, \text{size}(u_2) > 1, \) and the sets of literals \( u_1 \) and \( u_2 \) are the same up to the variable renaming.

For example in Fig. 3 (bottom), \( \text{place}(P), \text{right}(P,Q) \) is a connected subset that appears in both and hence the program has redundancy.

We will minimize both program size and redundancy. Though minimizing program size should imply the removal of redundancy, we notice empirically that minimizing both better guides the search to good solutions, e.g. within a certain time limit.

To find the best set of support clauses, we build and order them constructively from simple to more complex.

**4.1 Decision Variables: Support Clauses**

The decision variables in the COP problem are Boolean variables which indicate whether a particular support clause is included in the refactored program.

To build the support clause space \( S_j \) of increasingly more complex clauses, we first unfold the initial program \( T \) so that clause bodies only contain primitives (Figure 3 left). Following our Lego example, this results in all structures being described only with \( \text{place}/1 \) and \( \text{right}/2 \) predicates. We
then enumerate all subprograms consisting of at least \( i \) and at most \( j \) literals in the body, i.e., each element of \( \bigcup_{c \in T} C(c) \) with at least \( i \) and at most \( j \) literals forms one support clause. In the Lego example, taking \( i = 1 \) and \( j = 2 \) would result in the three candidates illustrated in Figure 3.

To obtain more complex support clauses, we fold each clause in the program with the newly extracted candidates of the support clause space \( S^i \), until the bodies of all clauses are expressed entirely in terms of extracted candidates (each clause could have multiple foldings), and repeat the same procedure of enumerating all subprograms. In the Lego example, this would result in \( L \)-shaped candidates and longer vertical and horizontal shapes (Figure 3). This step is repeated until each clause has only one body literal. This process creates a hierarchy of support clauses, each one building on simpler support clauses. We refer to these steps as levels of refactoring; the folding the unfolded program yields level one refactoring, folding again yields level two refactoring, and so on.

**Pruning Support Clauses**

Unrestricted candidate enumeration procedures typically result in many candidates because each clause can be expressed in terms of support predicates in many ways. To improve efficiency, we do two things: eliminate singleton clauses and remove provably useless clauses.

We remove support clauses with singleton variables, i.e., clauses with a variable that only appears once. For instance, the clause:

\[
\text{sup1(X)} :\text{- place(X),right(X,Y),place(Y).}
\]

is a valid candidate because all variables appear at least twice, while the clause:

\[
\text{sup2(X)} :\text{- place(X),right(X,Y).}
\]

is not because \( Y \) only appears once. Because we focus on program induction problems, ignoring singleton clauses is not sacrificing expressivity because singleton variables are essentially variables that are never used.

We also remove support clauses that cannot reduce the size of the program. For instance, let \( c \) be a support clause and \( \text{usage}(c, T) \) be the number of clauses in the program \( T \) in which \( c \) can be used. This means that in the best case, we can replace \( \text{usage}(c, T) \times \text{size}(c) \) literals of the original program by \( \text{usage}(c, T) \times 1 \) uses of the head of the support clause and the addition of a clause \( \text{size}(c) \) to the theory \( T \). Hence, if it is not the case that \( \text{usage}(c, T) \times \text{size}(c) > \text{usage}(c, T) \times \text{size}(c) \) we know that the use of this candidate support clause will never lead to a reduction in the size of the program (our overall goal). We remove candidate support clauses that violate this inequality.

**4.2 Constraints: Valid Refactoring**

Each clause has multiple foldings, grouped in different levels due to the support clause generation process. We enforce that at least one of the foldings of the clause \( i \) should be formed by the chosen support clauses. We group the foldings per level and add an additional level indicator \( L_i \) that will be used later. This results in constraints of the form:

\[
\max \text{ levels } \sum_{i=1}^{n} L_i = 1.
\]

This level variable will be part of the objective, where higher levels are typically better.

To decide whether a folding \( f_n \) can be constructed, the solver needs to know which support clauses are needed for that folding. For instance, to construct the initial program in Figure 3, we need a single Lego piece, an \( L \)-shaped piece, and a vertical two-piece component (assume that the selection of these support clauses is indicated with the variables \( \text{sc}_1, \text{sc}_2 \) and \( \text{sc}_3 \)). To ensure this connection, \( \text{Knorf} \) enforces the constraint stating that the folding \( f_n \) can be constructed only if all the necessary pieces are selected as a part of the solution:

\[
f_n \leftrightarrow (\text{sc}_p \land \text{sc}_r \land \text{sc}_q).
\]

Finally, candidates extracted from level \( L \) depend on the candidates from the level \( L - 1 \) (i.e., the bodies of candidates from level \( L \) are composed from candidates at level \( L - 1 \) ) \( \text{Knorf} \) imposes the constraint directly materialising this dependency – if the support clause \( \text{sc}_k \) is selected as a part of the solution, then all support predicate in the body of the clause (assume \( \text{sc}_l \) and \( \text{sc}_m \) ) also have to be a part of the solution

\[
\text{sc}_k \Rightarrow (\text{sc}_l \land \text{sc}_m).
\]

For instance, to make the \( L \)-shaped brick needed in Figure 1, we need to have a vertical and a horizontal 2-brick element available.
4.3 Objective: Size and Redundancy

Our goal is to find the smallest refactored program with the least redundancy. The program size is the total number of literals in it. The size is equal to the sum of the sizes of the foldings that can be constructed and the sizes of the selected support clauses. This is where the level indicator variables $1_i$ become important.

Consider the situation in Figure 3 with two levels of refactoring. If it is possible to construct a folding from level two, it is also possible to construct at least one folding at level one. But in this case, the folding from level one should not be counted in the program size as it will not be used; this is what the level indicator variables communicate. Thus, the objective function has the form of

$$\sum_{i=1}^{|T|} \sum_{l=1}^{\text{max levels}} \sum_{n_i=1}^{\text{foldings of } i} \text{size} \left( f_n^i \right) + \text{size} \left( s_{c_i} \right) \cdot s_{c_i} .$$

To identify redundancies, we identify all foldings with redundant sub-parts (for instance, the redundancy in the bottom structures of Figure 1). We then introduce a new Boolean variable (e.g., $r_i$) indicating whether more than one folding (e.g., $f_n^i$ and $f_m^i$) with such redundancy can be constructed

$$r_i \Leftrightarrow \left( f_n^i + f_m^i > 1 \right).$$

Knorf introduces such constraint for all found redundancies and adds the sum over $r_i$ variables to the objective function.

5 Experiments

We argue that an ILP system can learn better from refactored BK. Our experiments therefore aim to answer the question:

Q: Can an ILP system learn better with refactored BK?

By better, we ask whether it can solve more tasks, learn with higher predictive accuracies, or learn in less time. To answer this question, we compare the performance of Metagol with and without refactored BK.

We consider scenarios where lots of BK is available. To generate lots of BK, we use Playgol [Cropper, 2019]}, an ILP system can generate BK by learning programs for random play tasks which are similar to the user-provided target ones. In other words, we use Playgol to generate lots of BK. We then evaluate Metagol when learning from (1) the generated BK (No Refactoring), and (2) the BK after refactoring, i.e. after Knorf has refactored it (Refactoring).

Settings For refactoring, we set the minimum and maximum length of support clauses to 2 and 3 respectively and impose a timeout of 90 minutes. If refactoring takes longer, we stop the search and take the best solution found so far. We additionally impose a constraint that the refactored BK cannot have more predicates than the original BK. We give Metagol a learning timeout of 60 seconds per task. We repeat each experiment 10 times, and plot the means and 95% confidence intervals.

Knorf introduces such constraint for all found redundancies and adds the sum over $r_i$ variables to the objective function.

5.1 Experiment 1 - Lego

Our first experiment is on learning to build Lego structures in a controlled environment [Cropper, 2020]. In the next section we look at real-world string transformation learning.

Materials

We consider a Lego world with a base dimension of $6 \times 1$ on which bricks can be stacked. We only consider $1 \times 1$ bricks of a single colour. A training example is an atom $f(s_1, s_2)$, where $f$ is the target predicate and $s_1$ and $s_2$ are initial and final states respectively. A state describes a Lego structure as a list of integers. The value $k$ at index $i$ indicates that there are $k$ bricks stacked at position $i$. The goal is to learn a program to build the Lego structure from a blank Lego board (a list of zeros). We generate training examples by generating random final states. The learner can move along the board using the actions $\text{left}$ and $\text{right}$; can place a Lego brick using the action $\text{place}\_\text{brick}$; and can use the fluents $\text{at}\_\text{left}$ and $\text{at}\_\text{right}$ and their negations to determine whether it is at the leftmost or rightmost board position.

Method

For each $n$ in $\{200, 400, \ldots, 2000\}$, we use Playgol to generate BK from $n$ play tasks, where each play task is a Lego board of size 2 to 4. We randomly generate 1000 target tasks for a Lego board of size 6. We compare Metagol’s learning performance on the target tasks with and without refactoring of the BK. We measure the percentage of tasks solved (tasks where the Metagol learns a program) and learning times (total time need to solve all target tasks).

Results

Figure 4a shows that refactoring slightly degrades the ability to solve tasks when BK is small. This result suggests that when given less BK (and so less chance for redundancy), refactoring is eliminating predicates that Metagol could use to help solve tasks. Figure 4a shows that refactoring improves the ability to solve tasks when the BK is large ($\geq 1000$ play tasks). These results appear to corroborate existing results [Cropper, 2020], which show that simple forgetting can improve learning performance but only when learning from lots of BK. Figure 4b also shows that refactoring reduces learning times almost threefold.
5.2 Experiment 2 - String Transformations

Our second experiment is on string-transformations.

Materials

We use 130 real-word string transformation tasks from [Cropper, 2019]. Each task has 10 examples. An example is an atom \( f(x, y) \) where \( f \) is the task name and \( x \) and \( y \) are input and output strings respectively. An example task is to map the full name of a person (input) to its initials (output). We provide as BK the binary predicates \( \text{mk\_uppercase}, \text{mk\_lowercase}, \text{skip}, \text{copy}, \text{write} \), and the unary predicates \( \text{is\_letter}, \text{is\_uppercase}, \text{is\_space}, \text{is\_number} \).

Method

We follow the procedure described in [Cropper, 2019] to obtain the play tasks and thus BK. For each of the 130 tasks, we sample uniformly without replacement 5 examples as training examples and use the remaining 5 as test examples. We measure learning times and predictive accuracy.

Results

Figure 5a shows that refactoring drastically improves predictive accuracies. Metagol’s performance quickly deteriorates given more (unrefactored) BK because, as Proposition 1 shows, Metagol’s search space increases exponentially in the size of the BK. By contrast, when given refactored BK, Metagol has higher predictive accuracy in all cases, eventually four times higher than without refactored BK. Moreover, Figure 5b shows that refactoring reduces learning times by a third. Contrary to our expectation, the gain here does not come from the reduced number of predicates, as both programs have equal number of predicates, though overall program size decreases. Rather, the performance gains (both in accuracy and speed) come from better structured knowledge.

6 Conclusion

The main claim of this work is that the structure of an agent’s knowledge can significantly influence its learning abilities: more knowledge results in larger hypothesis spaces and makes learning more difficult. Focusing on inductive logic programming, we introduced a problem of knowledge refactoring – rewriting an agent’s knowledge base, expressed as a logic program, by removing the redundancies and minimizing its size. We also introduced \text{Knorf}, a system that performs automatic knowledge refactoring by formulating it as a constraint optimisation procedure. We evaluated the proposed approach on two program induction domains: building Lego structures and real-world string transformations. Our experimental results show that learning from the refactored knowledge base results can increase predictive accuracies in fourfold and reduced learning times by a half.

Limitations and Future Work

There are many possibilities for extending this work.

Solvers \text{Knorf} solves the refactoring problem by transforming it to a COP. While the experiments show good results, there may be other formulations that can reduce the size more directly or that are more efficient with fewer timeouts.

Metrics We have focused on eliminating redundancy to improve the performance of an ILP system. However, there are many other properties that we may want to optimise, such as modularity, reusability, or readability.

Domains We evaluated refactoring on two domains: building Lego structures and real-world string transformations. In future work, we want to consider other domains. One exciting idea is to try to induce entire programming languages libraries from examples, where refactoring would be essential to identify reusable abstractions (i.e. sub-libraries).

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