21 cm power spectrum and ionization bias as a probe of long mode modulated non Gaussian sky

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ABSTRACT

The observed hemispherical asymmetry in cosmic microwave background radiation can be explained by long mode modulation. In this work we study the prospect of the detection of this effect in 21 cm brightness temperature fluctuations angular power spectrum. For this task, we study the effect of the neutral Hydrogen distribution on the angular power spectrum. This is done by formulating the bias parameter of ionized fraction to the underlying matter distribution. We also discuss the possibility that the long mode modulation is companied with a primordial non-Gaussianity of local type. In this case we obtain the angular power spectrum with two effects of primordial non-Gaussianity and long mode modulation. Finally, we show that the primordial non-Gaussianity enhances the long mode modulated power of 21 cm via the non-Gaussian scale dependent bias up to four order of magnitude. Accordingly the future observations of 21 cm cosmology such as square kilometer array are capable of detection of large scale anomalies in initial condition of early Universe.

Key words: 21cm Cosmology – Long mode modulation – Bias

1 INTRODUCTION

The standard model of cosmology is in good agreement with the observation of the cosmic microwave background radiation (CMB) Ade et al.[Planck Collaboration] (2015) and the results of the surveys of large scale structure (LSS) Tegmark (2004); Eisenstein (2005); Tegmark (2006). On the other hand, the precise data of the CMB from Planck collaboration shows that the initial conditions of the Universe is very simple: nearly Gaussian, isotropic, adiabatic and nearly scale invariant Ade et al.[Planck Collaboration] (2014), which put serious constraint on the early universe models including inflationary scenario Ade et al.[Planck Collaboration] (2016b). However the unknown physics of the dark energy, dark matter and Early Universe (EU) make the above statement somehow a hazy one. Also there exist further evidences from CMB sky indicating anomalies such as the cold spot, quadrupole-octupole alignment, power deficit in low multipole and hemispherical asymmetry Ade et al.[Planck Collaboration] (2016a). These anomalies could be due to the statistical uncertainties or they can open up a new horizon in the study of the EU beyond the standard picture of ΛCDM cosmology. In the case of these anomalies pointing toward a new physics, we can also use the LSS data in the late time Universe to probe the fingerprint of these deviations Abolhasani et al. (2014); Baghram et al. (2014); Namjoo et al. (2015). However, the LSS data suffers from complex non-linearities, biased observations of baryons, and redshift space distortion effects, which must be controlled. This means that if we find specific fingerprints of the EU physics in LSS observables in this course, we will be able to probe any deviations from the standard picture in sub-CMB scale physics. In this direction, the galaxies are the first targets of LSS observables to probe EU effects Zhai and Blanton (2017). Another promising area of research is the future 21 cm observations which will map the neutral Hydrogen distribution in the universe and also can be used to find the statistics of the distribution of the matter Loeb and Zaldarriaga (2004). Here, we focus on the distribution of neutral Hydrogen in the epoch of reionization (EoR) to probe the probable asymmetry of CMB sky. The idea is that the neutral Hydrogen can be used as a fair sample of matter distribution in large scales Barkana and Loeb (2011). One of the interesting anomalies which is the subject of this work to investigate is the CMB hemispherical asymmetry. This asymmetry was first seen in WMAP data Eriksen et al. (2007); Hansen et al. (2009); Hufton et al. (2009) and it seems that there are indications in Planck analysis as well Akrami et al. (2014); Ade et al.[Planck Collaboration] (2016a). In the theoretical
side, there are different proposals to describe these asymmetry as long mode modulation Erickeck et al. (2008), domain walls inspired models Jazayeri et al. (2014), and etc. In the present study we have concentrated on the long mode modulation. In the case that this asymmetry is due to a new physics, it should affect the late time observables. We assert that the 21 cm cosmology gives the chance to probe the distribution of the matter in high redshifts where the matter distribution and evolution can be studied in linear regime. The only issue that we should study with much care is the bias between distribution of neutral Hydrogen and the underlying dark matter. In this direction there are studies that shows the hemispherical symmetry due to long mode modulations is correlated to the local-type non Gaussianity. In other words, long mode modulation and non-Gaussianity appear simultaneously Namjoo et al. (2013, 2014). Accordingly, we study the effect of the long mode modulation on the angular power spectrum of 21 cm brightness temperature with/without non-Gaussian effect on the bias. Then we assert that large scale surveys of 21 cm can probe the fingerprint of this long mode on the power spectrum of neutral Hydrogen when local type non-Gaussianity is present. It is worth to mention that the angular power spectrum of 21 cm is proposed as a probe of primordial spectral running as well Sekiguchi et al. (2018). Also we should note that Shiraishi et al. (2016) propose to use "off-diagonal components of the angular power spectrum of the 21 cm fluctuations during the dark ages to test this power asymmetry". However in this work we study the angular power spectrum and its correction due to long mode modulation and PNG. The structure of this work is: In Sec.(2.1), we briefly introduce the long mode modulation. In Sec. (2.2) we study the 21 cm power spectrum and scale independent bias. In Sec.(3), we see the effect of the long mode on the angular power spectrum of 21 cm. In Sec.(4), the effect of non Gaussianity is studied and Finally in Sec.(5) we conclude. In App.(A) we study the better approximation for bias parameter and its scale dependent case. In this work we set $\Omega_m = 0.32$, $\Omega_\Lambda = 0.68$, $H_0 = 67 km s^{-1} Mpc^{-1}$ and amplitude of primordial curvature perturbation $A_s = 2.1 \times 10^{-9}$ with curvature perturbation dimensionless power spectrum in the form of $\mathcal{P}_R(k) = A_s \left( \frac{k}{k_0} \right)^{n_s - 1}$ with $n_s = 0.965$ and $k_0 = 0.05 Mpc^{-1}$ Aghanim et al.[Planck Collaboration] (2018).

2 THEORETICAL BACKGROUND

In this section we set the scene and review the theoretical background. The first subsection is devoted to the long mode modulation, in which we mainly follow the proposal of Zibin and Contreras (2015). In the second subsection we go through the 21 cm brightness temperature fluctuations and we introduce the power spectrum via the bias parameter.

2.1 Long mode modulation

As mentioned in the introduction, the recent observations of Planck on the hemispherical asymmetry of the CMB, which was first detected by WMAP Eriksen et al. (2007); Hufnugt et al. (2009); Hansen et al. (2009), may point toward an anomalous primordial universe. Long mode modulations can be considered as one of the probable explanations of this observation. The idea of the long mode modulation comes from the dipole observed in CMB temperature as Gordon (2007)

$$\Delta T_{\text{CMB}}(\hat{n}) = \overline{T_{\text{CMB}}} \left[ 1 + A_d \cos(\theta_{h,p}) \right], \quad (1)$$

in which $\overline{T_{\text{CMB}}} \left[ 1 + A_d \cos(\theta_{h,p}) \right]$ is the statistically isotropic temperature fluctuations, $A_d$ is the amplitude of the dipole asymmetry on temperature template, $\hat{p}$ is the preferred direction, $\hat{n}$ is the direction of the observation and consequently $\theta_{h,p}$ is the angle between $\hat{p}$ and $\hat{n}$. Planck collaboration has found $A_d \approx 0.06$ and $\theta_p(\sim 227, b \sim -27)$ in galactic coordinates Ade et al.[Planck Collaboration] (2014). (For a more detailed analysis see also Ade et al.[Planck Collaboration] (2016a); Akrami et al. (2014); Aslanyan et al. (2014); Notari et al. (2014)). The idea of long mode modulation, which is the best yet known solution for dipole asymmetry Erickeck et al. (2008); Dai et al. (2013), can have observational consequences. In this picture, a long super-horizon mode with the wavelength $1/k_l$ modulates the curvature perturbations at the desired scales (either the CMB or the LSS scales. (e.g. in Abolhasani et al. (2014) the effect of long mode modulation in late time observations is discussed). In this respect, a very important observation is the fact that the modulation dies at large moments say $\ell > 65$, therefore, it seems reasonable to write the curvature perturbation in large and small scales as $\mathcal{R}(\ell) = \mathcal{R}^{lo}(\ell) + \mathcal{R}^{hi}(\ell)$ ("lo" super-spectrum indicating the low wave number (i.e. large scales), and "hi" indicating the high moments (i.e. small scales)). In this picture the large scale curvature in real space is modulated as Zibin and Contreras (2015)

$$\mathcal{R}^{lo}(\hat{n}) = \mathcal{R}^{lo}(\hat{n}) \left[ 1 + A_R \frac{r}{\lambda_{LS}} \hat{p} \right], \quad (2)$$

where $A_R$ is the amplitude of modulation in curvature perturbations (i.e. $A_R = A_g$) and $r_{LS}$ is the distance to the last scattering surface. Note that "^lo" indicates modulated quantities. In this model there is a reasonable assumption that the modulation dies off in low redshift. It is worth to mention that the 21 cm power spectrum is a suitable proposal for study because $r(\sim 10)/\lambda_{LS} \sim 1/3$ and it is much more prominent than the low redshift galaxy distribution. In order to follow this proposal further, we can define the dimensionless power spectrum of low and high moment unmodulated curvature from Fourier modes as

$$\langle \mathcal{R}^{lo}(\ell) \mathcal{R}^{lo}(\ell') \rangle = \frac{2\pi^2}{k^3} p_R^{lo}(k) \delta^3(\hat{k} - \hat{k}') \quad (3)$$

where $p_R^{lo}(k)$ is the dimensionless unmodulated curvature power spectrum of low and high moments. As we mentioned earlier, curvature perturbation is divided to two parts to explain the suppression of the mode in high angular moments; also the total statistically isotropic part $\mathcal{R}(\ell) \equiv \mathcal{R}^{lo}(\ell) + \mathcal{R}^{hi}(\ell)$ should give the standard $\Lambda$CDM power spectrum as $p_R^{\Lambda CDM}(k) = \mathcal{R}^{lo}(k) + p_R^{hi}(k)$. For this reason the unmodulated large scale curvature perturbation could have the dimensionless power spectra as

$$\mathcal{P}_R^{lo}(k) = \frac{1}{2} p_R^{\Lambda CDM}(k) \left[ 1 - \tanh \left( \frac{\ln k - \ln k_c}{\Delta \ln k} \right) \right] \quad (4)$$

where $k_c$ is the cut-off wavenumber. In order to make a modulation on angular scales larger than $l \sim 65$ one should
take $k_c \approx 5 \times 10^{-3} \text{Mpc}^{-1}$ and $\Delta \ln k \to 0$,¹ as mentioned by Zibin and Contreras (2015). The Fourier transform of low momentum curvature is $\tilde{R}(k) = R(\delta) = \frac{\delta Z \bar{n}_k}{\bar{n}_k} \mathcal{R}_{\ell S}^* \tilde{R}(k)$.

Thus the total power to first order of $A_R$ can be deduced as

$$\langle \tilde{R}(\vec{k}) \tilde{R}(\vec{k}') \rangle = \frac{2 \pi^2}{k^3} C_{\text{CDM}}(k) \bar{\delta}(\vec{k}) \bar{\delta}(\vec{k}') + 2 \pi^2 \frac{A_R}{r_{\ell S}} \bar{\delta}(\vec{k}) \bar{\delta}(\vec{k}') D_{\text{CDM}}(\vec{k} - \vec{k}'),$$

(5)

This is essential to relate the 21 cm power spectrum to the primordial curvature power, which will be discussed in next sections.

2.2 21 cm power spectrum and bias

One of the promising observational tools to trace the distribution of the matter is in the form of 21 cm brightness temperature. PNG and long mode modulation can affect the distribution of the early star forming regions. Another advantage of 21 cm maps is that we can explore the matter distribution in larger scales. Accordingly, in this section we will explore the theoretical background of 21 cm statistics.

Here we will focus on the study of the brightness maps at EoR where the spin temperature $T_e$ is much larger than the CMB temperature ($T_e \gg T_{\text{CMB}}$). The brightness temperature signal of intergalactic medium (IGM) at redshift $z$ in EoR is Field (1958); Madau et al. (1997)

$$\Delta T_b = (28 mK) \left( \frac{\Omega_b h^2}{0.02} \right) \frac{0.15 (1 + z)}{\Omega_m h^2} \bar{\delta}_{\text{HI}} (1 + \delta_b),$$

(6)

where $\bar{\delta}_{\text{HI}}$ is neutral fraction of baryons in IGM and $\delta_b$ is density contrast of baryons at IGM. So the 21 cm fluctuations from IGM at line of sight $\hat{n}$ in Fourier space ($\delta T_b(k, \hat{n}, z) = \Delta T_b(k, \hat{n}, z) - \bar{\delta}_b(z)$) is as

$$\delta T_b = \delta T_b \bar{\delta}_{\text{HI}} (1 + \delta_{\rho m} \mu_k^2)$$

(7)

where $\delta_{\rho m}$ is neutral Hydrogen density fluctuations, $\delta_{\rho m}$ is total matter density fluctuations and $\mu_k \equiv \bar{k} \bar{n}_k / \bar{n}_k$ (\hat{n} line of sight (LOS)). The $\bar{\delta}_x$ is the global neutral fraction and $\delta T_b$ is defined as

$$\delta T_b = (28 mK) \left( \frac{\Omega_b h^2}{0.02} \right) \frac{0.15 (1 + z)}{\Omega_m h^2} 10 \bar{\delta}_{\text{HI}} (1 + \delta_{\rho m} \mu_k^2),$$

(8)

where $\Omega_b$ and $\Omega_m$ are baryon and total matter density parameters. Eventually, the power spectrum of 21 cm brightness can be written as

$$\langle P_{\Delta T_b}(k, z) = 2 \bar{\delta}_x^2 \bar{\delta}_h^2 \bar{\delta}_{\text{HI}} P_{\delta_{\rho HI}}(\bar{\delta}_{\text{HI}}, \bar{\delta}_{HI})(k, z) \rangle$$

+ $2 P_{\delta_{\rho HI}} \bar{\delta}_{\text{HI}} P_{\delta_{\rho HI}}(k, z) \mu_k^2,$

(9)

where $P_{\Delta T_b}(k, z)$ is the power spectrum of $x$ and $y$ components ($x$ and $y$ each can be neutral and total Hydrogen). Here we assumed that in large scales and higher redshifts the total Hydrogen density traces the dark matter with a very good approach ($\delta T_b \sim \delta_{\rho m}$). Now, we define the effective Lagrangian bias corresponding to neutral Hydrogen, ionized Hydrogen and ionized ratio as $b_{\rho HI}^L = \bar{\delta}_x(\bar{\delta}_{\text{HI}}) / \delta_b$ where $a = \text{HI}, \text{HII}, \text{HIII}$ and $\rho$ is the total matter density. Therefore, equation (7) and the brightness power spectrum can be written as

$$\delta T_b = \delta T_b \bar{\delta}_{\text{HI}} (b_{\rho HI}^E + \mu_k^2) \delta_{\rho m},$$

(10)

and

$$P_{\Delta T_b}(k, z) = \delta T_b^2 \bar{\delta}_{\text{HI}}^2 \bar{\delta}_{HI} P_{\delta_{\rho HI}}(k, z),$$

(11)

where $b_{\rho HI}^E = 1 + b_{\rho HI}^b$ is the Eulerian bias, (superscript $E$ and $L$ indicate the Eulerian/Lagrangian bias) and $P_{\delta_{\rho HI}}(k, z)$ is the matter power spectrum in linear regime. Note that the neutral Hydrogen density bias is related to the ionized density as Mao et al. (2013)

$$b_{\rho HI}^L = \frac{1 - \bar{\delta}_{\text{HI}}}{\bar{\delta}_{\text{HI}}}.$$  

(12)

in which $\bar{\delta}_{\text{HI}}$ is the ionized fraction. Furlanetto et al. (2004) shows that the Excursion Set Model for reionization epoch (ESMR) works for patchy star forming regions, where $\bar{\delta}_{\text{HI}} = 1 - \bar{\delta}_{\text{HI}}$. By the definition of $b_{\rho HI}^L = \delta_{\text{HI}} / \delta_{\rho m}$, we will have

$$b_{\rho HI}^L = b_{\rho HI}^b + 1.$$  

(13)

Now to find ionized fraction bias we make a basic ansatz that the ionized fraction within spherical volume with radius $R_0$ is proportional to the number of ionizing photons that produced by sources within that volume, so we simply obtain D’Aloisio et al. (2013a,b)

$$x_{\text{HI}}(M_{\text{min}}, r, z, R_0) = f_{\text{coll}}(M_{\text{min}}, r, z, R_0) \bar{\delta}_{\text{HI}}(z),$$

(14)

where $\zeta$ is ionizing efficiency and $f_{\text{coll}}(M_{\text{min}}, r, z, R_0)$ is the collapsed mass fraction of volume $R_0$ into luminous sources. Here $M_{\text{min}}$ is the mass corresponding to a virial temperature of $10^{5} K$, so if we assume that the efficiency depends only on the redshift and is independent of position, $\zeta = \zeta(z),$ then the ionized fraction contrast of volume $R_0$ will be

$$1 + \delta_{\text{HI}} = \frac{F_{\text{coll}}(> M_{\text{min}}, r, z, R_0)}{F_{\text{coll}}(> M_{\text{min}}, z)},$$

(15)

where $\bar{F}_{\text{coll}}(> M_{\text{min}}, z)$ is the mean collapsed fraction of masses larger than $M_{\text{min}}$ at $z$, which is independent of the smoothing scale $R_0$. In the appendix A we will discuss the ionized fraction bias in the context of excursion set theory EST Bond et al. (1990); Nikakhtar and Baghram (2017), where we study the non-Markov effects on the bias parameter Musso et al. (2012).

In the case of scale dependent halo bias in form of equation (A14) and by using equations (12, 13, 15) respectively, one simply finds

$$b_{\rho HI}^L(k) = 2 - \frac{\bar{\delta}_{\text{HI}}}{\bar{\delta}_{\text{HI}}} b_{\rho HI}^L - g(k) \bar{\delta}_{\text{HI}} b_{\rho HI}^L \bar{\delta}_{\text{HI}},$$

(16)

where $g(k)$ is a scale dependent function which can be raised from PNG or non-Markov effects. In this work, for matter of simplicity we put scale independent halo bias $b_{h}^L = 1$ and $\bar{\delta}_{\text{HI}} b_{\rho HI}^b = 1$. Note that this simplification does not change the main results and proposal of this work. In the next section we will discuss long mode modulation effect on angular power spectrum.

¹ In this work we set $\Delta \ln k = 0.1$ to proceed in the calculations. This specific choice only affect the smoothing scale of modulated power spectrum.
3 ANGULAR POWER SPECTRUM OF MODULATED HYDROGEN DISTRIBUTION

In this section we are going to investigate the effect of long mode modulation on the angular power spectrum of matter perturbation. It is known that the Poisson equation in Fourier space relates the matter density to the gravitational potential $k^2\Phi(k) = 4\pi G\rho_0(k)\omega^2$, and the potential is related to the curvature perturbation via transfer function $T(k)$ and growth function $D(z)$ (i.e. $\Phi(k) = \frac{4}{3} T(k) D(z)(1+z)\mathcal{R}(k)$), therefore the matter density perturbation is related to the curvature as

$$\delta(k) = \frac{2}{5} \frac{T(k) D(z)}{\Omega_m H_0^2} \mathcal{R}(k). \quad (17)$$

Now the real space 21cm brightness temperature fluctuations can be expressed in Fourier space and written in terms of curvature perturbation as

$$\delta T_b(\delta) = \delta T_b \mathcal{R}(k) \frac{2 D(z)}{5 \Omega_m H_0^2} \int \frac{k^2 dk d\Omega_k}{(2\pi)^3/2} \rho_{\nu} E k^2 \tilde{\delta} \tilde{\xi} \times k^2 T(k) \left| \tilde{\rho}^{hi} + \tilde{\rho}^{lo} + \frac{i A_\rho}{T_{LS}} \delta \tilde{\xi} \right|, \quad (18)$$

where we omit the $k$ in $\mathcal{R}(k)$ for simplicity in notation. Note that here we neglect redshift space distortion effect. Now we can expand the plane wave in terms of spherical Bessel and spherical harmonics, which gives

$$\delta T_b(\delta) = \delta T_b \mathcal{R}(k) \frac{2 D(z)}{5 \Omega_m H_0^2} \int \frac{k^2 dk d\Omega_k}{(2\pi)^3/2} \rho_{\nu} E k^2 \tilde{\delta} \tilde{\xi} \times k^2 T(k) \left( \tilde{\rho}^{hi} + \tilde{\rho}^{lo} + \frac{i A_\rho}{T_{LS}} \delta \tilde{\xi} \right) j_l(kr) Y_{lm}(\delta), \quad (19)$$

where $R_{lm}(kr) = i^l k^l \int d\Omega_k \mathcal{R}(k) Y_{lm}(\delta)$. \quad (20)

For notation simplicity we omit the subscript $b$ hereafter. Now expanding the 21cm brightness signal fluctuations in terms of spherical harmonics $\delta_l(\delta) = \sum_l \delta_{lm} Y_{lm}(\delta)$, we get

$$\delta T_b(\delta) = \delta T_b \mathcal{R}(k) \frac{2 D(z)}{5 \Omega_m H_0^2} \int \frac{k^2 dk d\Omega_k}{(2\pi)^3/2} \rho_{\nu} E k^2 \tilde{\delta} \tilde{\xi} \times k^2 T(k) \left( \tilde{\rho}^{hi} + \tilde{\rho}^{lo} + \frac{i A_\rho}{T_{LS}} \delta \tilde{\xi} \right) j_l(kr) Y_{lm}(\delta), \quad (19)$$

where

$$R_{lm}(kr) = i^l k^l \int d\Omega_k \mathcal{R}(k) Y_{lm}(\delta). \quad (20)$$

Accordingly the angular power spectrum of 21cm brightness temperature fluctuations becomes

$$\langle \delta T_b \delta T_b'(kr) \rangle = C_{TT}^b \delta T^2 \delta_{mmt'} + \frac{1}{2} \partial_{mmt'}^2 C_{TT}^b \Delta \xi_{lmn}^0, \quad (21)$$

where $C_{TT}^b$ is the isotropic part, and the second term in RHS of equation (23) is the correction term introduced by the modulated power spectrum. As Moss et al. (2011) mentioned one can simply set $\Delta C_{TT}^b = \frac{1}{2} A_\rho \delta C_{TT}^b$, so Zibin and Contreras (2015)

$$C_{TT}^b = (\delta T_b \mathcal{R}(k)) \frac{2 D(z)}{5 \Omega_m H_0^2} \int \frac{dk}{k} k^2 T^2(k) \left( \tilde{\rho}^{hi} + \tilde{\rho}^{lo} + \frac{i A_\rho}{T_{LS}} \delta \tilde{\xi} \right) j_l^2(kr), \quad (24)$$

where $\mathcal{R}$ is the dimensionless power spectrum of curvature perturbation.

$$\Delta C_{TT,LM}^b = \frac{2 D(z)}{5 \Omega_m H_0^2} \int \frac{dk}{k} k^2 T^2(k) \left( \tilde{\rho}^{hi} + \tilde{\rho}^{lo} + \frac{i A_\rho}{T_{LS}} \delta \tilde{\xi} \right) j_l^2(kr). \quad (25)$$

Here we assume that neutral density bias has no $k$-dependence ($\tilde{\rho}^{hi} = 2 - \frac{\Omega_{HI}}{\Omega_m} \tilde{\rho}^{hi}$). As far as the bias parameter is scale independent, the main contribution to the angular power spectrum comes from the matter power spectrum corrections. In Fig.1 we plot the angular power spectrum of 21cm brightness fluctuations versus angular moment $l$ and also we compare the signal with long mode modulation effect (equation (25)) in $z = 10$. In Fig.2 we compare the fractional difference of angular power spectrum of 21cm temperature and matter halo distribution in different redshifts (for details see caption of the figures and footnote $^2$). As one can see in Figs.1 and 2, the long mode modulation dies at large moments similar to CMB observations, but as Zibin and Contreras (2015) has made the cut off by a fixed $k$ ($k_c$) instead of $l$, this suppression cut off will shift to lower moments by decreasing redshift. In other words, the cosmic variance problem is more dominant in lower redshifts. Furthermore, due to the suppression term $r/\ell_{LS}$ in equation (2), the effective amplitude of long mode modulation will reduce in lower redshifts. Although the observing of dipole in 21cm temperature brightness map of EoR is more difficult than CMB map but eventually it’s more promising and reachable than low redshift galaxy surveys. In the next section we will study the effect of local type primordial non-Gaussianity (PNG) on bias parameter and angular power spectrum.

4 NON-GAUSSIANITY: THE BIAS AND ANGULAR POWER SPECTRUM

In this section we investigate the effect of power asymmetry in the presence of primordial local non-Gaussianity. As indicated in the introduction the long mode modulation can be companied by local non-Gaussianity which is related to the long to short mode coupling. In this direction we study the local $f_{NL}$ type non-Gaussianity. The primordial Bardeen potential can be expressed in terms of local non-Gaussian corrections as

$$\Phi = \phi + f_{NL} \phi^2. \quad (26)$$

2 The halo matter distribution angular power spectrum is obviously obtained $C_{TT}^{hi} = \frac{2 D(z)}{5 \Omega_m H_0^2} \int \frac{dk}{k} k^2 T^2(k) \left( \tilde{\rho}^{hi} + \tilde{\rho}^{lo} + \frac{i A_\rho}{T_{LS}} \delta \tilde{\xi} \right) j_l^2(kr)$ and all the corrections due to long mode modulation and non-Gaussianity are calculated with this relation respectively as explained in the main text for the statistics of 21cm brightness temperature.
Figure 1. The angular power spectrum $C_{l}^{TT}$ in units of $[\langle \delta T_{b}^{2} \rangle^{1/2}]$ (red solid line) versus moment $l$ for 21 cm brightness temperature at $z = 10$ is plotted. The difference of the angular power spectrum due to long mode modulation $\Delta C_{l}^{TT,LM}$ at $z = 10$ (Blue dashed) is compared with $\Delta C_{l}^{TT,LM}(z=10) \equiv [\langle \delta T_{b}^{2} \rangle^{1/2}]$ (green solid line) and $f_{NL} = 10$ (green dashed) and for dark matter halo distribution $\Delta C_{l}^{h_{h},LM}$ (green dashed) at $z = 2$ (orange dot dashed) and $z = 0.5$ (blue dotted).

Figure 2. The difference of the angular power spectrum due to long mode modulation for 21 cm brightness temperature with respect to $C_{l}^{TT}$ ($\Delta C_{l}^{TT,LM}$$/C_{l}^{TT}$) is plotted versus moment $l$ at $z = 10$ (red solid line) and $z = 6$ (green dashed) and for dark matter halo distribution $\Delta C_{l}^{h_{h},LM}$ ($C_{l}^{h_{h}}$) at $z = 2$ (orange dot dashed) and $z = 0.5$ (blue dotted).

Figure 3. The angular power spectrum of 21 cm brightness temperature including non-Gaussianity $C_{l}^{TT} + \Delta C_{l}^{TT,NG}$ versus moment $l$ at $z = 10$ for $f_{NL} = 0$ (green solid line), $f_{NL} = +10$ (blue dashed) and $f_{NL} = -10$ (red dotted) is plotted.

$M(k, z) = 3k^{-2}T(k)D(z)/5h^{2}_{0}\Omega_{m}$. It is worth to mention that the bias is a scale dependent quantity due to the non-Gaussian effects, which is discussed in Dalal et al. (2008). The halo density contrast with PNG corrections is defined as

$$\delta_{h} = \frac{n(M, z, X_{1}, X_{2}, \delta_{1}) - \bar{n}(M, z)}{\bar{n}(M, z)},$$

where $\delta_{h}(M, z)$ is the density of structures in a mass range of $M$ and $M + dM$ in redshift $z$. By Taylor expanding the modified number per mass $n(M, z, X_{1}, X_{2}; \delta_{1})$ in equation (29) in the presence of long mode $\delta_{l}$ and non-Gaussian terms, we can find the bias parameter due to the definition of halo bias (see appendix A) as

$$\bar{b}_{h}^{L} \equiv \frac{\beta_{2} f_{NL}}{M(k, z)},$$

where $\beta_{2} = 2 \partial \bar{n}/\partial X_{1}$ and therefore

$$\bar{b}_{h}^{L}(f_{NL}) = \frac{\bar{b}_{h}^{L}(0)}{f_{NL}^{2}} + 2 f_{NL} \mathcal{M}^{-1}(k, z) \frac{\delta_{c}}{\bar{c}}(z),$$

where $\delta_{c} = 1.686$ is the critical density. So according to equation (16), we can extend the definition of halo bias to neutral Hydrogen bias. In this case the Eulerian neutral Hydrogen bias $b_{E}^{h_{HI}}$ can be expressed as

$$b_{E}^{h_{HI}}(k) = 2 \frac{\bar{c}}{\delta_{c}} \frac{\bar{b}_{h}^{L}(0)}{f_{NL}^{2}} + 2 f_{NL} \mathcal{M}^{-1}(k, z) \frac{\delta_{c}}{\bar{c}}(z).$$

Now it is straightforward to incorporate the effect of PNG via the bias parameter in the definition of the angular power spectrum of 21 cm brightness temperature fluctuations. However we should note that one of the important studies in this framework was done by Slosar et al. (2008), which investigate the effect of PNG on EoR observations. However in this work we study the angular power spectrum of temperature brightness fluctuations in presence of both PNG and long
The difference of the angular power spectrum due to non-Gaussianity with respect to $C_l$ for $21$ cm brightness temperature ($\Delta C_{TT}^{LM}/C_{TT}^1$) and for dark matter halo distribution ($\Delta C_{TT}^{NL}/C_{TT}^1$) is plotted versus moment $l$ at different redshifts with $f_{NL} =\pm 10$. Long-dashed (green) and dot-dot-dashed (purple) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 10$, dot-dashed (orange) and dotted (black) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = +10$ at $z = 6$ and finally short-dashed (red) and solid (blue) lines are for dark matter halo distribution with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 0.5$ respectively.

Figure 6. The ratio of the angular power spectrum due to Long mode modulation $\Delta C_{TT,LM}/C_{TT}^1$ with respect to $C_{TT}^1$ (in absence of non-Gaussianity) versus moment $l$ for $21$ cm brightness temperature $\Delta C_{TT,LM}/C_{TT}^1$ at $z = 10$ (blue solid line) is plotted. Also the difference of the angular power spectrum due to LM-NG term (equation (38)) $\Delta C_{TT,LM}^{NL}/C_{TT}^1$ with respect to $C_{TT}^1 + \Delta C_{TT}^{NL}$ with $f_{NL} = +10$ (dotted black) and $f_{NL} = -10$ (dashed red) is compared.

Figure 7. Absolute value of the difference of the angular power spectrum due to each term in equations (34-38) with respect to $C_{TT}^1$ versus moment $l$ for $21$ cm brightness temperature $\Delta C_{TT,LM}/C_{TT}^1$ at $z = 10$ is plotted. Dotted (black) and short-dashed (red) lines are for $\Delta C_{TT,NG}/C_{TT}^1$ with $f_{NL} = +10$ and $f_{NL} = -10$, long-dashed (green) and dot-dashed (orange) lines are for $\Delta C_{TT,LM}^{NL}/C_{TT}^1$ with $f_{NL} = +10$ and $f_{NL} = -10$ and finally blue solid line is for $\Delta C_{TT,LM}/C_{TT}^1$ term respectively.

The difference of the angular power spectrum due to $LM - NG$ term $\Delta C_{TT,LM}^{NL}$ introduced in equation (38) with respect to $C_{TT}^1$ for $21$ cm brightness temperature $\Delta C_{TT,LM}^{NL}/C_{TT}^1$ and for dark matter halo distribution is plotted versus moment $l$ at different redshifts with $f_{NL} = \pm 10$. Long-dashed (green) and dot-dot-dashed (purple) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 10$, dot-dashed (orange) and dotted (black) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 6$ and finally short-dashed (red) and solid (blue) lines are for dark matter halo distribution with $f_{NL} = +10$ at $z = 2$ and $z = 0.5$ respectively.

Figure 4. The difference of the angular power spectrum due to non-Gaussianity with respect to $C_l$ for $21$ cm brightness temperature ($\Delta C_{TT}^{LM}/C_{TT}^1$) and for dark matter halo distribution ($\Delta C_{TT}^{NL}/C_{TT}^1$) is plotted versus moment $l$ at different redshifts with $f_{NL} = \pm 10$. Long-dashed (green) and dot-dot-dashed (purple) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 10$, dot-dashed (orange) and dotted (black) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = +10$ at $z = 6$ and finally short-dashed (red) and solid (blue) lines are for dark matter halo distribution with $f_{NL} = +10$ at $z = 0.5$ respectively.

Figure 5. The difference of the angular power spectrum due to $LM - NG$ term $\Delta C_{TT,LM}^{NL}$ introduced in equation (38) with respect to $C_{TT}^1$ for $21$ cm brightness temperature $\Delta C_{TT,LM}^{NL}/C_{TT}^1$ and for dark matter halo distribution is plotted versus moment $l$ at different redshifts with $f_{NL} = \pm 10$. Long-dashed (green) and dot-dot-dashed (purple) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 10$, dot-dashed (orange) and dotted (black) lines are for $21$ cm brightness temperature with $f_{NL} = +10$ and $f_{NL} = -10$ at $z = 6$ and finally short-dashed (red) and solid (blue) lines are for dark matter halo distribution with $f_{NL} = +10$ at $z = 2$ and $z = 0.5$ respectively.

Mode modulation. In the presence of long mode modulation, the angular power spectrum will be

$$C_{TT}^1 = C_{TT}^1 + \Delta C_{TT,LM} + \Delta C_{TT,NG} + \Delta C_{TT,LM}^{NL},$$

where $C_{TT}^1$ is the standard angular power spectrum defined as

$$C_{TT}^1 = \left(2 - \frac{\delta_{HI}}{\bar{\delta}_{HI}} b_{HI}^L\right)^2 \frac{16\pi}{25} \left(\frac{D(z)}{\Omega_m H_0^2}\right)^2 \times \int \frac{dk}{k} k^2 T^2(k) P_R(k) j^2_l(k r),$$

and $\Delta C_{TT,LM}$ is the term which is related to the pure long mode modulation effect defined as

$$\Delta C_{TT,LM} = \left(2 - \frac{\delta_{HI}}{\bar{\delta}_{HI}} b_{HI}^L\right)^2 \frac{32\pi}{25} \left(\frac{D(z)}{\Omega_m H_0^2}\right)^2 \times A_R \sqrt{\frac{\gamma}{\epsilon}} \int \frac{dk}{k} k^2 T^2(k) P_R(k) j^2_l(k r).$$
and $\Delta C_{l}^{TT,LM-NG}$ is the effect of PNG-scale dependent bias term on standard angular power spectrum defined as

$$
\Delta C_{l}^{TT,LM-NG} = (\delta \tilde{\theta}_{b} \tilde{\delta}_{HI})^{2} \left( \frac{2\pi}{\Omega_{m} H_{0}^{2}} \right)^{2} \times \frac{\ln(\Omega_{b}/\Omega_{m})}{R_{LS}} \int \frac{dk}{k} k^{2} P(k) \bar{f}^{2}(k) \times \left[ -4 f_{NL} \delta_{c} M^{-1}(k,z) \frac{\tilde{\delta}_{HI}^{L} b_{L}^{2}}{\tilde{\delta}_{HI}^{L}} \left( 2 - \frac{\tilde{\delta}_{HI}^{L}}{\tilde{\delta}_{HI}^{L}} b_{L}^{2} \right) + \left( 2 f_{NL} \delta_{c} M^{-1}(k,z) \frac{\tilde{\delta}_{HI}^{L} b_{L}^{2}}{\tilde{\delta}_{HI}^{L}} \right)^{2} \right] j_{2}^{2}(kr).
$$

(38)

In Fig. 3, the angular power spectrum of $21$ cm brightness temperature including the non-Gaussianity $C_{l}^{TT} + \Delta C_{l}^{TT,NG}$ term at $z = 10$ for $f_{NL} = 0, \pm 10$ is plotted. It is interesting to note that due to the term $M^{-1}$ in equation (37) (which is proportional to $k^{-2} T^{-1}(k)$) non-Gaussianity term will have a very large effect on angular power spectra at low moments ($\Delta C_{l}^{TT,NG} > C_{l}^{TT}$ at $l \lesssim 50$ with $f_{NL} = +10$). This is very interesting future, for the PNG studies which can be used as probe for their detection in this direction see Lidz et al. (2013). In Fig. 4, we plot the ratio of the standard angular power spectrum with PNG correction with respect to the standard angular power spectrum. For redshifts $z = 10$ and $z = 6$ we plot the $21$ cm brightness temperature fluctuations angular power, where for $z = 0.5$ the angular power spectrum is for dark matter halo distribution. This plot shows how it is promising to use $21$ cm power spectrum to find out the PNG effects. In Fig. 5 we show the ratio of the PNG-long mode modulated term in angular power spectrum introduced in equation (38) with respect to the standard power. This figure shows that PNG-long mode modulated term with respect to the standard power spectrum, also decreases in lower redshifts. In order to emphasis this result in Fig. 6, which is an important plot for our proposal, we show that PNG not only enhance the long mode modulation effect (up to 4 order of magnitude), but also enhance its contribution with respect to main terms in angular power spectrum (up to 2 order of magnitude at $z = 10$) to an observable signal. In Fig. 7, we plot the contribution of each term introduced in this section. Summing up the result, we show that if the long mode modulation has a physical origin, it can be enhanced by PNG and observed in future surveys.

5 CONCLUSIONS AND FUTURE REMARKS

Deviation from the standard picture of inflationary model with nearly Gaussian, nearly scale invariant, isotropic and adiabatic initial conditions can open up a new horizon to the physics of early universe. In this work we study the effect of long mode modulation as a probable explanation for CMB hemispherical asymmetry with local type of non-Gaussianity, on the distribution of neutral Hydrogen in EoR. This work is a continuation of the idea of deviation for CMB hemispherical asymmetry with local type of non-Gaussianity, on the distribution of neutral Hydrogen in EoR. This work is a continuation of the idea of inflationary model (see Hirata (2009); Baghram et al. (2013); Zibin and Moss (2015); Hassani et al. (2016); Ansari Fard and Baghram (2018); Zhai and Blanton (2017)). We show that a dipole will be introduced due to the long mode modulation. Although the observation of dipole in $21$ cm temperature brightness map of EoR is more difficult than CMB map, but eventually it’s more promising and more reachable than low redshift galaxy surveys. It is shown in Fig. 2 that how two different type of observation predict the large angular scale corrections. In this work, we also study the effect of PNG in the presence of long mode modulation. The local type PNG has a $k^{-2}$ dependency on the wave number in the bias parameter, accordingly the PNG in its local type can enhance the long mode modulation effect on angular power spectrum. As it is discussed, the long mode modulations are companied with local type PNG. Therefore, this seems a useful coincidence which can help to detect this effect. For further study, it is necessary to introduce a more sophisticated bias model for neutral Hydrogen by using the simulations to pin down the physics of ionized patches and the efficiency function of ionization. On the other hand, more realistic models of non-Markov bias terms sets a $k$-dependency which must be taken into account. This correction may have drastic effects on the angular power spectrum of $21$ cm. However, summing up this work we should note that the future observations such as square kilometer array observatory with enough large angle coverage are capable to detect the large scale anomalies of initial condition of early universe.

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APPENDIX A: THE MODULATED BIAS IN EXCURSION SET THEORY

In the first glance, bias is considered as a nuisance parameter, but it can also be studied as a parameter for detecting the anomalous initial condition in the process of collapse. In this appendix we study the ionized fraction bias in the context of EST.

In the context of the Excursion set theory and peak background splitting Sheth and Tormen (1999), bias can be defined by the first up-crossing distribution as

\[ b_n = \frac{(-1)^n \partial^n f(s, \delta_c)}{f(s, \delta_c)} \]  \quad (A1)

where \( f(s, \delta_c) \) is the first up-crossing function, which depends on the variance \( s \) and critical density of spherical collapse \( \delta_c \). Note that \( n = 1 \), corresponds to the linear regime Lagrangian bias and for the Markov first up-crossing \( f(s, \delta_c) = 1/\sqrt{2\pi} \delta_c (s^{3/2}e^{-\delta_c^2/2s}) \), we will have \( b_1 = (\nu^2 - 1) - \delta_c \), where \( \nu = \sqrt{\delta} \) is the height parameter.

Now according to equation (15), in the excursion set theory we can find mean value of the collapsed fraction of the ionization ratio as

\[ \langle 1 + \delta_{HII} \rangle = \frac{F(s(M_{min}) | s_0, \delta_0)}{F(s(M_{min}))} \]  \quad (A2)

in which \( F(s(M_{min})) \) is the variance of density fluctuations related to the minimum mass that can ionize the surroundings with scales larger than the scale \( s_0 \) under consideration. The function \( F \) is the integrated fraction of up-crossing. Now if we assume that the walks are uncorrelated steps and they are generated by Gaussian initial condition the collapse fraction will be Mao et al. (2013):

\[ F(s(M_{min}) | s_0, \delta_0) = \text{erfc}[\frac{\delta_c (z) - \delta_0}{\sqrt{2(s(M_{min}) - s_0)}}] \]  \quad (A3)

In order to find the bias parameter in real space, we can simply set \( s_0 = 0 \) and expand \( \langle 1 + \delta_{HII} \rangle \) around \( \delta_0 = 0 \) to obtain

\[ \frac{F(s(M_{min}) | s_0 = 0, \delta_0)}{F(s(M_{min}))} + 1 = 1 + \sum \frac{\delta_c^n}{n!} b^n_{\text{HII},n} \]  \quad (A4)

and thus the first order bias parameter can be found as

\[ b^{\text{HII},1} = \left( \frac{\sqrt{2 \pi s(M_{min})}}{s(M_{min})} \right) \exp\left( -\frac{\delta_c^2}{2s(M_{min})} \right) \text{erfc}\left( \frac{\delta_c}{\sqrt{2s(M_{min})}} \right)^{-1}, \]  \quad (A5)

where \( s(M_{min}) \) and \( \delta_c \) vary with redshift. It is seen that
similar to the real space bias there is no scale dependency in Fourier space Musso et al. (2012)

\[ b_{\text{HiH}}(k) = b_{\text{HiH},1}(k) = b_{\text{HiH},1}. \]  

(A6)

Note that these coefficients are pure numbers, independent of wavenumber \( k \), and by definition, independent of \( s_0 \).

The bias parameter introduced above corresponds to uncorrelated steps in EST approach which refers to a \( k \)-space sharp smoothing window function for density field. For more realistic physical cases, other window functions such as real space top hat and Gaussian filters are suggested. These filters lead to the correlated steps in EST approach Nikakhtar et al. (2018). Although there is no analytical exact solution for first up-crossing problem with correlated steps, there are some approximate solutions in this case. Here we find the ionized fraction bias in the context of the correlated steps in EST. Note that in the ionized fraction bias we used \( F(s(M_{\text{min}}) | s_0 = 0, \delta_0) \) from EST which is mass fraction of volume \( s_0 \) that collapsed in halos with scales between \( s_0 \) and \( s(M_{\text{min}}) \). Musso et al. (2012) found approximate solutions for correlated steps of halo bias:

\[ \langle 1 + \delta_h \rangle = \frac{f(s | s_0, \delta_0)}{f(s)} = 1 + \sum_{n=0}^{\infty} \frac{\delta_0^n}{n!} b_{h,n}. \]  

(A7)

in which \( f(s | s_0, \delta_0) \) is mass the fraction of volume \( s_0 \) that collapsed in halos with scales between \( s \) and \( s + ds \). With this definition in mind, we simply have

\[ F(s_{\text{min}} | s_0, \delta_0) = \int_{s_0}^{s_{\text{min}}} f(s | s_0, \delta_0) ds = \]

\[ \int_{s_0}^{s_{\text{min}}} \langle 1 + \delta_h \rangle f(s) ds = F(s_{\text{min}}) + \sum_{n=0}^{\infty} \frac{\delta_0^n}{n!} \int_{s_0}^{s_{\text{min}}} f(s) b_{h,n}(s, s_0) ds. \]  

(A8)

Note that we had made no assumption on whether bias parameter has scale dependency or not. By comparing equations (A8) and (A4), one can simply find

\[ b_{\text{HiH},n} = \frac{\int_{s_0}^{s_{\text{min}}} f(s) b_{h,n}(s, s_0) ds}{F(s_{\text{min}})}, \]  

(A9)

where

\[ F(s_{\text{min}}) = \int_{0}^{s_{\text{min}}} f(s) ds. \]  

(A10)

The same relation can be found for Fourier space bias

\[ b_{\text{HiH},n}(k) = \frac{\int_{0}^{s_{\text{min}}} f(s) b_{h,n}(k) ds}{F(s_{\text{min}})}. \]  

(A11)

Now we can use this relations to find ionized fraction bias for top hat and Gaussian filters. As mentioned above, there is no exact solution for these window functions so we use the up-crossing approximation introduced in Musso et al. (2012). Here we only discuss scale independent part of bias (\( b_{10} \)) as below

\[ s_{\text{up}}(s) = \frac{\nu \exp \left( \frac{-s^2}{2\nu} \right)}{2\sqrt{2\pi}} \frac{1 + \text{erf}(\Gamma\sqrt{\nu})}{\sqrt{2\pi\nu}} + \frac{\exp(-\Gamma^2 s^2/2)}{\sqrt{2\pi\nu}} \]  

(A12)

where \( \Gamma^2 = \nu^2/(1 - \nu^2) \) as \( \nu^2 = (\delta \delta')^2/(\langle \delta^2 \rangle \langle \delta' \rangle) \) (A prime indicates the derivative with respect to the smoothing scale). Then the bias will be

\[ b_{\text{up},10}^h = \frac{\nu^2 - 1}{\delta_c} + \frac{1}{s_{\text{up}}(s)} \frac{\exp(-\frac{\nu^2(1+\Gamma^2)}{2})}{4\pi \nu s^{3/2} \nu}. \]

(A13)

In the case of modulated power spectrum, the only change in the bias parameter is introduced via the redefinition of the variance and other moments of power spectrum in terms of mass. Otherwise, the functionality of the bias parameter remains the same. In the case the bias parameter has a \( k \)-dependence due to non-Gaussianity or non-Markovianity, we express it in the form

\[ b_{h}^L(k) = b_{h}^L + g(k) b_{h}^L. \]  

(A14)

where \( b_{h}^L(k) \) and \( b_{h}^L \) are scale dependent and independent halo bias respectively and \( g(k) \) is determined by non-Gaussianity or non-Markovianity extensions. Now by using equation (A11) we obtain the \( k \)-dependent Lagrangian ionized fraction bias parameter

\[ b_{\text{HiH},L}(k) = b_{\text{HiH},L}^L + g(k) b_{\text{HiH},L}^L. \]  

(A15)

The above equation is used in the main text to find the angular correlation of 21 cm brightness temperature in case of a local type non-Gaussianity.

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