Constraints on warm dark matter from weak lensing in anomalous quadruple lenses

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Abstract
We investigate the weak lensing effect by line-of-sight structures with a surface mass density of \(\lesssim 10^8 \text{M}_\odot/\text{arcsec}^2\) in QSO-galaxy quadruple lens systems. Using high-resolution \(N\)-body simulations in warm dark matter (WDM) models and observed four quadruple lenses that show anomalies in the flux ratios, we obtain constraints on the mass of thermal WDM, \(m_{\text{WDM}} \geq 1.3\text{keV}(95\%\text{CL})\), which is consistent with those from Lyman-\(\alpha\) forests and the number counts of high-redshift galaxies at \(z \geq 4\). Our results show that WDM with a free-streaming comoving wavenumber \(k_{fs} \lesssim 27h/\text{Mpc}\) is disfavored as the major component of cosmological density at redshifts 0.5 \(\lesssim z \lesssim 4\).

Key words: cosmology: theory - gravitational lensing - dark matter - galaxies: formation

1 Introduction
The clustering property of dark halos at spatial scales of \(\lesssim 1\text{Mpc}\) is far from being understood. In particular, the number of satellite galaxies in our Galaxy is by far smaller than expected from theory, which is so-called the "missing satellite problem." As a solution, we may consider: 1. Baryonic solution - the star formation in the satellite galaxy is suppressed due to some baryonic process. 2. Dark matter solution - a number of satellite galaxies are suppressed due to some baryonic process. 3. Dark matter subhalos in the lensing galaxies should be also smaller than the expected values obtained from \(N\)-body simulations.

Indeed, taking into account of astrometric shifts, recent studies have found that the observed anomalous flux ratios can be explained solely by these line-of-sight structures with surface mass density \(\sim 10^{-8}\text{M}_\odot/\text{arcsec}^2\) (Inoue & Takahashi 2012; Takahashi & Inoue 2014) without dark subhalos in the lensing galaxies taken into account. The observed increase in the amplitude of magnification perturbations strongly implies that the origin is associated with sources rather than lenses. If this is the case, one does not need to care about the suppression of dark satellites in the lensing galaxy due to baryonic processes as a number of mini-halos in the line-of-sight are not belonging to massive galaxies.

Another mechanism that can suppress the number of dwarf galaxies is the free-streaming of dark matter particles. If the thermal velocity at the decoupling from the thermal

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bath is large enough, dark matter particles would erase the primordial fluctuations at scales of dwarf galaxies. Warm dark matter (WDM) particles are candidates for achieving such suppression.

However, if the suppression is too strong, the amount of neutral hydrogen such as Lyman-α clouds is also significantly reduced. In fact, the best constraint on the mass scale of WDM comes from the observations of Lyman-α forests at redshifts \( z \gtrsim 4 \) (Viel et al. 2005; Seljak et al. 2006; Boyarsky et al. 2009; Viel et al. 2013).

In a similar manner, one can constrain the mass or the free-streaming scale of dark matter particles using anomalous quadruple lenses, whose sources are at \( z \lesssim 4 \) (Miranda \\& Maccio 2007). If the free-streaming scale is too large, or equivalently, the particle mass is too small, the amplitude of fluctuations of a surface mass density in the line-of-sight becomes so small that the weak gravitational lensing effect, which acts as a perturbation to the flux ratios, becomes negligible. Therefore, the observed anomalous flux ratios cannot be explained in such dark matter models.

In this paper, we revisit the weak lensing effect by the line-of-sight structures in WDM models taking into account two important non-linear effects that have been overlooked in the literature. One is the quick regeneration of the suppressed power of WDM models and the catching up with the linear and non-linear power of the CDM models (Boehm et al. 2005; Schneider et al. 2012). This effect might make WDM models difficult to exclude using QSO-galaxy lensing systems. Another is the weak lensing effect due to non-linear objects such as walls, voids, and filaments. In CDM models, it turned out that the weak lensing effect from locally underdense region are also important for estimating magnification perturbation by the line-of-sight structures (Takahashi & Inoue 2014). The weak lensing effect due to walls and filaments could be also important. Therefore, we need to incorporate lensing effects due to non-linear objects in WDM models as well. For simplicity, however, we do not consider lensing effects due to subhalos in the lensing galaxies.

To take into account such non-linear effects, we first calculate the non-linear power spectra of matter fluctuations down to mass scales of \( \sim 10^9 h^{-1} M_\odot \) using \( N \)-body simulations. For simplicity, we do not consider baryonic effects in our simulations. Then we estimate the probability distribution of magnification perturbation for each lens using the semi-analytic formulae developed in (Takahashi \\& Inoue 2014). We also take into account the astrometric shifts due to line-of-sight structures, which are often overlooked in the literature.

In section 2, we describe our semi-analytic formulation for calculating the magnification perturbation due to line-of-sight structures. In section 3, we show the results of our \( N \)-body simulations and the obtained non-linear power spectra in WDM models. In Section 4, we describe our samples of QSO-galaxy lensing systems that show anomalies in the flux ratios. In section 5, we present our results on the constraints on the mass of WDM particles and the free-streaming scales of dark matter particles. In section 6, we conclude and discuss some relevant issues.

In what follows, we assume a cosmology with a current matter density \( \Omega_{m,0} = 0.3134 \), a baryon density \( \Omega_{b,0} = 0.0487 \), a cosmological constant \( \Omega_{\Lambda,0} = 0.6866 \), a Hubble constant \( H_0 = 67.3 \text{ km/s/Mpc} \), a spectrum index \( n_s = 0.9603 \), and the root-mean-square (rms) amplitude of matter fluctuations at \( 8 h^{-1} \text{ Mpc} \), \( 
abla \sigma_8 = 0.8421 \), which are obtained from the observed CMB (Planck+WMAP polarization, Planck Collaboration et al. 2013).

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\section*{2 SEMI-ANALYTIC FORMULATION}

In this section, we briefly describe our semi-analytical formulation (for details, see Inoue \\& Takahashi (2012); Takahashi \\& Inoue (2014)).

We use a statistic \( \eta \) to measure the magnification perturbation of lensed images in QSO-galaxy lens systems:

\[ \eta \equiv \left[ \frac{1}{2N_{\text{pair}}} \sum_{i \neq j} \left[ \delta'_{\text{(minimum)}} - \delta'_{\text{(saddle)}} \right]^2 \right]^{1/2} \tag{1} \]

where \( \delta'_{\text{(minimum)}} \) and \( \delta'_{\text{(saddle)}} \) are magnification (denoted by \( \mu \)) contrasts \( \delta' \equiv \delta \mu/\mu \) corresponding to the minimum and saddle images and \( N_{\text{pair}} \) denotes the number of pairs of lensed images. If the correlation of magnification between pairs of images is negligible, then \( \eta \) corresponds to the mean magnification perturbation of one of lensed images. For instance, \( \eta = 0.1 \) means that the magnification is expected to change by 10 percent. Note that we need to fix the primary lens model (i.e., a best-fitted model without line-of-sight structures) in order to calculate \( \eta \). In other words, \( \eta \) is a model dependent statistic.

The second moment of the magnification perturbation \( \eta \) can be calculated as follows. First, we need to estimate a perturbation \( \varepsilon \) to the largest angular separation \( \theta_{\text{max}} \) between a pair of lensed images X and Y due to the line-of-sight structures,

\[ \varepsilon = |\Delta \theta(X) - \Delta \theta(Y)|, \tag{2} \]

where \( \Delta \theta \) represents the astrometric shift perturbation of a lensed image at \( \theta \) in the lens plane. We then assume that the perturbation satisfies \( \varepsilon \leq \varepsilon_0 \) where \( \varepsilon_0 \) is the observational error for the largest angular separation. In order to satisfy such a condition, we assume that small-scale modes with a wavelength larger than the mean comoving separation \( b \) between the lens center and lensed images at the primary lens plane are significantly suppressed. Any modes whose fluctuation scales are larger than \( b \), which is roughly the size of the comoving Einstein radius, contribute to the smooth component of a primary lens, namely, a constant convergence and shear (Fig. 1). Therefore, we

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{diagram.png}
\caption{Schematic diagram of unperturbed light rays. The wave number \( k_{\text{ion}} \) is defined as \( k_{\text{ion}} = \pi/2b \) where \( b \) is the mean separation between lensed images and the centroid of the primary lens galaxy.}
\end{figure}
consider only modes whose wavenumbers satisfy \( k > k_{\text{lens}} \) where \( k_{\text{lens}} \equiv \pi/2b \). Otherwise, double-counting of the constant convergence and shear leads to a systematically large perturbation. Furthermore, we also assume that modes with wavenumbers \( k_{\text{lens}} < k < k_{\text{cut}} \) are suppressed to some extent. The cut off scale \( k_{\text{cut}} \) is determined by the condition that the perturbation \( \varepsilon \) of an angular separation \( \theta \) between an arbitrary pair of lensed images should not exceed the observational error \( \varepsilon_{\text{obs}} \) for the maximum separation angle between lensed images. The suppression of these intermediate modes corresponds to the satellites, companion galaxies, possible higher order \((m = 3, 4)\) components of a primary lens.

In what follows, we consider a filtering so called the “constant shift (CS)” filter,

\[
W_{\text{CS}}(k; k_{\text{cut}}) = \begin{cases} \text{W}_{\text{int}}(k), & k < k_{\text{cut}}; \\ 1, & k \geq k_{\text{cut}}, \end{cases}
\] (3)

in which the corresponding contribution to the angular shifts between a pair of images with the maximum separation angle \( \theta_{\text{max}} \) are constant in logarithmic interval in \( k \) for \( k < k_{\text{cut}} \). In this model, contribution from modes with \( k_{\text{lens}} < k < k_{\text{cut}} \) to an angular shift \( \varepsilon \) does not depend on the wavenumber \( k \). \( W_{\text{int}} \) is explicitly given by

\[
W_{\text{int}}^2(k; k_{\text{cut}}) = \left( \frac{\partial^2 \varepsilon^2}{\partial \ln k} \right)_{k = k_{\text{cut}}}, \] (4)

where

\[
\langle \varepsilon^2 \rangle = 2\langle \delta \theta^2(0) \rangle - 2\langle \delta \theta(0) \delta \theta(\theta_{\text{max}}) \rangle,
\] (5)

and

\[
\langle \delta \theta(0) \delta \theta(\theta) \rangle = \frac{9H_0^2 \Omega_m^2}{4c^4} \int_0^{\pi} dr \left( \frac{r - r_S}{r_S} \right)^2 [1 + z(r)]^2 
\times \int_{k_{\text{lens}}}^{k_{\text{cut}}} \frac{dk}{2\pi} W_{\text{CS}}^2(k; k_{\text{cut}}) P_3(k, r) J_0(g(r)k\theta),
\] (6)

where

\[
g(r) = \begin{cases} r, & r < r_L \\ r_L(r_S - r)/(r_S - r_L), & r \geq r_L \end{cases}
\] (7)

and \( P_3(k, r) \) is the power spectrum of dark matter density fluctuations as a function of the wavenumber \( k \) and the comoving distance \( r \), \( r_S \) is the comoving distance to the source and \( r_L \) to the lens from an observer and \( z(r) \) is the redshift of a point at a comoving distance \( r \). \( \langle \cdot \rangle \) represents an ensemble average. \( J_0 \) is the zero-th order Bessel function. \( g(r) \theta \) denotes the tangential separation between two unperturbed light-rays at a comoving distance \( r \) from the observer.

Once \( k_{\text{lens}} \) and \( k_{\text{cut}} \) are determined, we can compute the constrained perturbed convergence \( \delta \kappa \) and shear \( \delta \gamma_1 \) as functions of a separation angle \( \theta \) between a pair of lensed images. For instance, the constrained two-point correlation of \( \delta \kappa \) as a function of a separation angle \( \theta \) is

\[
\xi_{\kappa\kappa}(\theta) \equiv \langle \delta \kappa(0) \delta \kappa(\theta) \rangle
\]

\[
= \frac{9H_0^2 \Omega_m^2}{4c^4} \int_0^{\pi} dr \left( \frac{r - r_S}{r_S} \right)^2 [1 + z(r)]^2 
\times \int_{k_{\text{lens}}}^{k_{\text{cut}}} \frac{dk}{2\pi} W_{\text{CS}}^2(k; k_{\text{cut}}) P_3(k, r) J_0(g(r)k\theta). \] (8)

To calculate \( P_3 \), we use a fitting function obtained from high resolution cosmological simulations (see also Smith et al. (2003); Takahashi et al. (2012); Inoue & Takahashi (2012); Takahashi & Inoue (2014)). The fitting function for the warm dark matter model can be used up to a wavenumber \( k \approx 300 \) hMpc\(^{-1} \) at \( 0 \leq z \leq 3 \) within \( \sim 20\% \) accuracy (see section 3).

The two-point correlation functions for the other perturbed quantities are obtained by the following substitution in equation (8):

\[
\langle \delta \gamma_i(0) \delta \gamma_j(\theta) \rangle = J_0 \rightarrow \frac{1}{2} [J_0 + J_1 \cos(4\phi_\theta)],
\]

\[
\langle \delta \gamma_2(0) \delta \gamma_2(\theta) \rangle = J_0 \rightarrow \frac{1}{2} [J_0 - J_1 \cos(4\phi_\theta)],
\]

\[
\langle \delta \kappa(0) \delta \gamma_1(\theta) \rangle = J_0 \rightarrow -J_2 \sin(2\phi_\theta),
\]

\[
\langle \delta \kappa(0) \delta \gamma_2(\theta) \rangle = J_0 \rightarrow \frac{1}{2} J_4 \sin(4\phi_\theta),
\] (9)

where \( \theta = (\theta \cos \phi_\theta, \theta \sin \phi_\theta) \) and the Bessel functions \( J_{0,2,4} \) are functions of \( g(r)k\theta \). From (1), (8) and (9), we can obtain the second moment of \( \eta \).

For example, let us consider three images with two minima A and C and one saddle B with \( k_B < 1 \). Choosing coordinates where the separation angle is perpendicular to plus mode (i.e., \( \theta \sin \phi_\theta = 0 \)), we have \( \langle \delta \kappa \delta \gamma_2 \rangle = \langle \delta \gamma_1 \delta \gamma_2 \rangle = 0 \). Then, for \( |\delta R| \ll 1 \), the second moment \( \langle \eta^2 \rangle \) can be written as

\[
\langle \eta^2 \rangle = \frac{1}{4} \left[ (I_A + I_B) - 2I_{AB}(\theta_{AB}) + (I_B + I_C) \right] - 2J_{BC}(\theta_{BC}),
\] (10)

where

\[
I_i \equiv \mu_i^2(4(1 - \kappa_i)^2 + 2\gamma_{1i}^2 + 2\gamma_{2i}^2)\xi_{\kappa}(0),
\] (11)

and

\[
I_{ij}(\theta) \equiv 4\mu_i \mu_j \left[ (1 - \kappa_i)(1 - \kappa_j)\xi_{\kappa}(\theta) \right.
\]

\[
+ \gamma_{1i} \gamma_{1j} \langle \delta \gamma_1(0) \delta \gamma_1(\theta) \rangle + 2\gamma_{1i} \gamma_{2j} \langle \delta \gamma_2(0) \delta \gamma_2(\theta) \rangle
\]

\[
+ (1 - \kappa_i) \gamma_{1j} \langle \delta \kappa_i(0) \delta \gamma_1(\theta) \rangle,
\]

\[
+ (1 - \kappa_j) \gamma_{1i} \langle \delta \kappa_j(0) \delta \gamma_1(\theta) \rangle \right],
\]

(12)

for \( i = A, B, C \). In a similar manner, for a four-image system with two minima A and C and two saddles B and D with...
\[ \kappa_B < 1 \text{ and } \kappa_D < 1, \text{ the second moment is given by} \]
\[ \langle \eta^2 \rangle = \frac{1}{8} \left[ I_A + I_B - 2I_{AB} (\theta_{AB}) + (I_C + I_B) - 2I_{CB} (\theta_{CB}) + (I_A + I_D) - 2I_{AD} (\theta_{AD}) + (I_C + I_D) - 2I_{CD} (\theta_{CD}) \right], \]

where \( I_i \) and \( I_{ij}(\theta) \), \( i = A, B, C, D \) are given by (11) and (12). Note that we are using coordinates in which \( \phi_\nu = 0 \).

### 3 NON-LINEAR POWER SPECTRUM

#### 3.1 Initial condition

We calculate the initial power spectrum in models with WDM by using the modified version of CAMB (Lewis et al. 2000). We assume thermal distribution for WDM and all dark matter component being WDM. Since we fix the abundance of WDM, its mass \( m_{\text{WDM}} \) and the temperature of WDM species \( T_{\text{WDM}} \), we declare as

\[ \Omega_{\text{WDM}} h^2 = \left( \frac{T_{\text{WDM}}}{T_e} \right)^3 \frac{m_{\text{WDM}}}{0.46 \text{eV}}, \]

where \( T_e \) is the temperature of neutrinos. By the effect of the free-streaming of WDM particles, the cosmic structure can be erased and the matter power spectrum damps on small scales, which is commonly characterised by the free-streaming scale \( \lambda_{fs} \), defined by the comoving length that WDM particles free-stream until the radiation-matter equality time. \( \lambda_{fs} \) is explicitly given by (Kolb & Turner 1990)

\[ \lambda_{fs} = 0.114 \text{ Mpc} \left( \frac{1 \text{ keV}}{m_{\text{WDM}}} \right) \left( \frac{10.75}{g_*(T_D)} \right)^{1/3} \times \left[ 2 + \log \left( \frac{t_{\text{eq}}}{t_{\text{NR}}} \right) \right], \]

where \( t_{\text{eq}} \) and \( t_{\text{NR}} \) are the time of radiation-matter equality and that when WDM particles become non-relativistic, respectively. \( g_*(T_D) \) is the effective number of degrees of freedom at the time of decoupling of WDM particles, denoted by the temperature \( T = T_D \). In the following analysis, we fix the energy density of WDM as \( \Omega_{\text{WDM}} = 0.2647 \), hence the temperature \( T_{\text{WDM}} \) (or \( g_*(T_D) \)) and the mass \( m_{\text{WDM}} \) are related through equation (14).

Above arguments are valid for thermally produced WDM species. However, other candidates for WDM such as sterile neutrinos (Dodelson & Widrow 1994) have also been discussed in the literature. For the sterile neutrinos produced via active-sterile neutrino oscillations, its distribution function can be approximated by a generalized Fermi-Dirac distribution, then the effect of sterile neutrino can be regarded as the same as the one for WDM by using the following identification for the mass (Colombi et al. 1996; Viel et al. 2005):

\[ m_\nu = 4.46 \text{ keV} \left( \frac{m_{\text{WDM}}}{1 \text{ keV}} \right)^{4/3} \left( \frac{0.12}{\Omega_{\text{WDM}} h^2} \right)^{1/3}. \]

From this formula, one can derive the constraint for the mass of sterile neutrino once we obtain that for thermally produced WDM.

![Figure 2. Plots of linear matter power spectrum for the cases with LCDM model (red solid line), WDM with \( k_{fs} = 2\pi/\lambda_{fs} = 140 \text{ h/Mpc} \) (grey), 44 h/Mpc (blue), 15 h/Mpc (orange) and 4.8 h/Mpc (green).](image)

In Fig. 2, we show the linear matter power spectra in the LCDM model, and WDM models with \( k_{fs} = 2\pi/\lambda_{fs} = 140 \text{ h/Mpc}, 44 \text{ h/Mpc}, 15 \text{ h/Mpc} \) and 4.8 h/Mpc. The corresponding WDM masses are listed in Table 1.

### 3.2 N-body simulation

We run cosmological N-body simulations to investigate the non-linear matter power spectra of WDM models. Our purpose is to obtain the fitting formula of non-linear power spectra used in our analytical formula (see section 2). In order to cover a wide-range scale of gravitational evolution, we run simulations with two different boxes with a side of 100h^{-1} \text{ Mpc} and 10h^{-1} \text{ Mpc}, hereinafter referred to as L100 and L10, respectively. The number of particles in the boxes is set to 1024^3. The initial positions and velocities of particles are given at redshift \( z_{\text{init}} = 24 \) based on second-order Lagrangian perturbation theory (Crocce et al. 2006; Nishimichi et al. 2009). We adopt a concordant CDM model and four WDM models with free-streaming wavenumbers \( k_{fs} = 2\pi/\lambda_{fs} = 140 \text{ h/Mpc}, 44 \text{ h/Mpc}, 15 \text{ h/Mpc} \) and 4.8 h/Mpc in our simulations. The CDM and WDM models are summarised in Table 1. The input linear power spectra of the CDM and WDM models are evaluated using CAMB (see section 3.1). In our simulations, we ignore the thermal motion of WDM particles, which can be verified as follows. The rms thermal velocity of WDM particles at the initial redshift \( z_{\text{init}} = 24 \) is \( \sigma_v \simeq 1.1 \text{ km/s} / (g_{\text{WDM}} / 1.5)^{1/3} (m_{\text{WDM}} / \text{keV})^{-4/3} \) in our cosmological model, where \( g_{\text{WDM}} \) is the degree of freedom of the WDM particle (Bode et al. 2001). On the other hand, the rms physical peculiar velocity of the particles at the initial time is \( \gtrsim 10 \text{ km/s} \) in our WDM models. Thus, we can ignore the thermal motion of WDM particles (see also similar discussion in Angulo et al. (2013)).

To follow the gravitational evolution of the dark matter particles, we employ publicly available tree-PM codes, Gadget2 (Springel et al. 2001; Springel 2005) for the large-box simulation (L100) and GreeM (Ishiyama et al. 2009, 2012) for the small-box simulation (L10). GreeM is tuned to ac-
cerate the tree gravitational calculation, and it is faster than Gadget2 especially in the strongly non-linear regime. Hence, we employ GreeM for the small-box simulation. The PM meshes are $2048^3(512^3)$ for the L100 (L10). The particle Nyquist wavenumbers are $k_{\text{Nyquist}} = 32.2(322)\,h/\text{Mpc}$ for the L100 (L10). The gravitational softening length is set to 3% of the mean particle separation. The simulation snapshots are dumped at redshifts $z = 0, 0.3, 0.6, 1, 2$ and 3. We prepare 3(5) independent realizations for the L100 (L10) for each CDM or WDM model to reduce the sample variance. Our simulation settings are summarised in Table 2.

We check the accuracy of our simulation results as follows. For Gadget2, we use the same simulation parameters (time step, force accuracy and so on) in Takahashi et al. (2012) (section 2) in which we achieved a few percent accuracy for $k < 300h/\text{Mpc}$. For GreeM, we run simulations with finer simulation parameters and confirmed that the power spectra have $<1\%$ accuracy for $k < 300h/\text{Mpc}$.

To evaluate the matter power spectra from the particle distribution, we assign 1024$^3$ particles into 1536$^3$ grids using the CIC (Cloud-in-Cell) method (Hockney & Eastwood 1988) to obtain the density fluctuations. Then, we perform FFT$^1$ and calculate the power spectrum:

\[
P(k) = \frac{1}{N_k} \sum_{k'} |\delta(k')|^2, \tag{17}
\]

where the summation is done over a range of $k - \Delta k/2 < |k'| < k + \Delta k/2$ with a bin-width $\Delta k$, and $N_k$ is a number of modes in the $k$ bin. We also employ the holding method (e.g. Jenkins et al. 1998; Smith et al. 2003) to probe smaller scales. We calculate the mean power spectra and one-sigma errors from 5(3) realizations in the L100(L10).

Fig.3 shows our simulation results for the matter power spectra in the CDM and WDM models shown in Table 1 at redshifts $z = 0, 0.3, 1$ and 2. The filled circles with error bars are the mean power spectra with the errors obtained from the realisations of simulations. The results are taken from the large-box simulations (L100) for $k < 300h/\text{Mpc}$ and from the small-box simulations (L10) for $k > 60h/\text{Mpc}$. Here, $k = 300h/\text{Mpc}$ is the Nyquist wavenumber of the L100, and $k = 60h/\text{Mpc}$ corresponds to a scale of 1/10 times smaller than the small box-size (L10)$^2$. The vertical dotted line denotes the Nyquist wavenumber of the small-box simulation (L10). The solid curves are obtained from our fitting formula based on the Halofit model for a LCDM model (Smith et al. 2003; Takahashi et al. 2012), but slightly modified in WDM models. Details of the model fitting parameters are given in Appendix A. The simulation box-size should be much larger than the free-streaming scales in WDM models to follow gravitational evolution accurately. Thus, we do not use the simulation results for the WDM4.8 in the small-box simulation (L10), because its free-streaming scale ($\lambda_{\text{fs}} = 2\pi/k_{\text{fs}} = 1.3h^{-1}\text{Mpc}$) is close to the box size ($10h^{-1}\text{Mpc}$). As shown in Fig. 3, the suppression due to free-streaming of WDM particles becomes less prominent at low redshifts even though the initial power spectra of the WDM models are exponentially suppressed at small scales $k > k_{\text{fs}}$. For example, the initial power spectrum $P(k)$ of the WDM15 is ten orders of magnitude smaller than that of the CDM at $k = 300h/\text{Mpc}$, but the ratios become only $\approx 2(4)$ at low redshifts $z = 0(2)$. This result exhibits power transfer from large to small scales via the mode coupling during the non-linear evolution (Bagla & Padmanabhan 1997; White & Croft 2000; Smith & Markovic 2011; Viel et al. 2012). The quick regeneration of the suppressed power of WDM models and catching up with the linear and non-linear power of the CDM play an important role for estimating the lensing effects due to line-of-sight structures.

### Table 1. CDM and WDM models

| Model       | $k_{\text{fs}}(6h^{-1}\text{Mpc})$ | $m_{\text{WDM}}(\text{keV})$ |
|-------------|------------------------------------|-------------------------------|
| CDM         | --                                 | --                            |
| WDM140      | 140                                | 5.0                           |
| WDM44       | 44                                 | 1.9                           |
| WDM15       | 15                                 | 0.77                          |
| WDM4.8      | 4.8                                | 0.29                          |

Note: The CDM and WDM models in our simulations. We show the free-streaming wavenumbers $k_{\text{fs}}$ and WDM particle masses $m_{\text{WDM}}$.

### 4 LENSMODEL

As a fiducial model of lensing galaxies, we adopt a singular isothermal ellipsoid (SIE) (Kormann et al. 1994), which can explain flat rotation curves. We use the fluxes of lensed images, the relative positions of lensed quadruple images and the centroid of lensing galaxies and time delay of lensed images if available. The contribution from groups, clusters, and large-scale structures at angular scales larger than the Einstein radius of the primary lens is taken into account as an external shear (ES). The parameters of the SIE(′s) plus ES model are the angular scale of the critical curve or the mass scale inside the critical curve $b'$, the apparent ellipticity $\epsilon$ of the lens and its position angle $\theta_e$, the strength and the direction of the external shear $(\gamma, \theta_\gamma)$, the lens position $(x_G, y_G)$, the source position $(x_s, y_s)$, and the time delay of lensed images $\tau_{\text{del}}$. The Hubble constant $h$ is also treated as a model parameter. The angles $\theta_e$ and $\theta_\gamma$ are measured in East of North expressed in the observer’s coordinates (see Table 4).

To find a set of best-fit parameters, we use a numerical code called GRAVLENS$^3$ developed by Keeton in order to implement the $\chi^2$ fitting of the fluxes, positions, and time delay of lensed images and the positions of centroid of lensing galaxies. The total $\chi^2_{\text{tot}}$ is equal to the sum of $\chi^2_{\text{flx}}$ for lensed images, $\chi^2_{\text{del}}$ for time delays, and $\chi^2_{\text{gal}}$ for the positions of lensing galaxies.

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1. FFTW home page: http://www.fftw.org/
2. The box-size of the L10 is very small ($L = 10h^{-1}\text{Mpc}$ on a side) and hence it does not include density fluctuations larger than the box size that may affect small-scale clustering via mode coupling. To avoid this, we use only modes much smaller than the box size.
3. See http://redfive.rutgers.edu/~keeton/gravlens/
images A, B, C, and a faint image D. Images A, C are minima, and B and D are saddles. The quasar at $z_S = 3.62$ is the largest in our 4 samples and the primary lensing galaxy is at $z_L = 0.34$ (Kundic et al. 1997; Tonry 1998). We use the radio flux ratios (Koopmans et al. 2003) of 4 images at 5 GHz averaged over a period of 8.5 months, which are consistent with the MIR counterparts (Chiba 2002). We use the astrometry of lensed images and the centroid of the primary lensing galaxy in Courbin et al. (2011) obtained from the use of Magain-Courbin-Sohy (MCS) deconvolution algorithm applied in an iterative way (ISMCS) to near-IR HST images. The maximum total error in the positions of lensed images is 1.05 mas. Therefore, we assume an error of $\sqrt{2} \times 1.05 \sim 1.4$ mas for the angular separations of lensed images. The positions of lensed images and the centroid of the primary lensing galaxy are well fitted by an SIE and an external shear assuming that the error in the angular position of G is 0.01 arcsec. However, the flux ratios are not well fitted. Chiba (2002) and Nierenberg et al. (2014) argue a presence of substructure around A. Alternatively, the possible perturber may be a halo or some other objects in the line-of-sight.

### 5.2 B0128+437

The fold-caustic lens B0128+437 consists of one bright image A, and three fainter images B, C, and D. The images A and C are minima, and B and D are saddles. The quasar redshift is $z_S = 3.124$ (McKean et al. 2004) and that of the primary lensing galaxy is either $z_L = 0.645$ or 1.145 (Lagattuta et al. 2010). Combining with the previous photometric and spectroscopic data, the latter choice is favored than the former (McKean et al. 2004; Lagattuta et al. 2010). Therefore, we assume $z_L = 1.145$ in what follows. We use the radio flux ratios (Koopmans et al. 2003) of 4 images at 5 GHz averaged over a period of 8.5 months, and the astrometry in Lagattuta et al. (2010) obtained from ground-based near-IR imaging coupled with laser guide-star adaptive optics. We also assume that the astrometric errors of each lensed image are 0.005 arcsec Lagattuta et al. (2010). Although the positions of lensed images and the centroid of the primary lensing galaxy G can be fitted by an SIE plus an external shear, the predicted flux ratios show discrepancy with the data. There might be a sub/line-of-sight halo around C.

### 5.3 MG0414+0534

The fold-caustic lens MG0414+0534 consists of two bright images A1, A2 and two faint images B, C. The images A1 and B are minima, and A2 and C are saddles. A source quasar at $z_S = 2.64$ is lensed by an elliptical galaxy at $z_L = 0.96$ (Hewitt et al. 1992; Lawrence et al. 1995; Tonry & Kochanek 1999). A simple lens model, an SIE with an external shear (SIE-ES) cannot fit the image positions as well as the flux ratios. Schechter & Moore (1993) and Ros et al. (2000) suggested that another galaxy called “X” is necessary for fitting the relative image positions. We use the MIR flux ratios A1/A2 and A/B/A1 measured by Minezaki et al. (2009) and MacLeod et al. (2013) since the radio fluxes might be hampered by Galactic refractive scintillation (Koopmans et al. 2003). For the astrometry, we use the data from CASTLES database. Although the positions are well fitted by an SIE and an external shear (ES) plus a singular isothermal sphere (SIS) that accounts for object X, the flux ratios are not well fitted. A possible sub/line-of-sight

### 5 QUADRUPLE LENS SYSTEMS

In the following, we shortly describe 6 quadruple lens systems that shows a large cusp relation $R_{\text{cusp}}$ or fold relation $R_{\text{fold}}$. It is shown that 8 quadruple lens systems show apparent anomalies in the radio flux ratios (Xu et al. 2013). However, we exclude B1555+375 and B1933+503 in our analysis since the redshifts of the lens and source of B1555+375 are not measured and the spiral lens B1933+503 has a very complex structure (Suyu et al. 2012). In our analysis, we use observed the MIR fluxes for MG0414+0534 and the radio fluxes averaged over a certain period for other 5 systems. It should be noted that the MIR fluxes are microlens free especially for high-redshift sources. For the astrometry, we use optical or NIR data in order to avoid bias due to complex structures of jets. We also use time delay for modeling B1608+656. We find that B2045+265 and B1608+656 with large $R_{\text{cusp}} \sim 0.5$ are no longer anomalous if a companion galaxy G2 at the redshift of the primary lensing galaxy G1 is taken into account (McKean et al. 2007). Therefore, we use 4 anomalous quadruple lenses B1422+231, B0128+437, MG0414+0534, and B0712+472 for constraining the mass of WDM particles. In what follows, we describe the property of each lens.

### 5.1 B1422+231

The cusp-caustic lens B1422+231 consists of three bright images A, B, C, and a faint image D. Images A, C are minima and B, D are saddles. The quasar redshift $z_S = 3.62$ is the largest in our 4 samples and the primary lensing galaxy is at $z_L = 0.34$ (Kundic et al. 1997; Tonry 1998). We use the radio flux ratios (Koopmans et al. 2003) of 4 images at 5 GHz averaged over a period of 8.5 months, which are consistent with the MIR counterparts (Chiba 2002). We use the astrometry of lensed images and the centroid of the primary lensing galaxy in Courbin et al. (2011) obtained from the use of Magain-Courbin-Sohy (MCS) deconvolution algorithm applied in an iterative way (ISMCS) to near-IR HST images. The maximum total error in the positions of lensed images is 1.05 mas. Therefore, we assume an error of $\sqrt{2} \times 1.05 \sim 1.4$ mas for the angular separations of lensed images. The positions of lensed images and the centroid G of the primary lensing galaxy are well fitted by an SIE and

### Note:

Parameters in our simulations are: side length of simulation box $L$, number of dark matter particles $N_p^3$, Nyquist wavenumber $k_{\text{Nyq}}$, particle mass $m_p$, redshifts of the simulation outputs $z$, and number of realisations $N_r$.

### Table 2. Our Simulation Setting

| $L (h^{-1}\text{Mpc})$ | $N_p^3$ | $k_{\text{Nyq}} (h\text{Mpc}^{-1})$ | $m_p (h^{-1}\text{M}_\odot)$ | $z$ | $N_r$ |
|-------------------------|---------|-----------------------------------|-----------------------------|-----|------|
| L100                    | 100     | 1024$^3$                          | $32.2$                      | $8.1 \times 10^7$ | 0, 0.3, 0.6, 1, 2, 3 | 3    |
| L10                     | 10      | 1024$^3$                          | $322$                       | $8.1 \times 10^4$ | 0, 0.3, 0.6, 1, 2, 3 | 5    |
The lens galaxy G1 belongs to a low-mass group of eight members (Fassnacht et al. 2006). We use the astrometry of lensed images and the centroid of G1 and G2 in Courbin et al. (2011). The fluxes and time delays between these four images are based on radio-wavelength monitoring with the Very Large Array (VLA) at 8.5 GHz (Fassnacht et al. 2002). Time delay between image A and B is denoted as $t_A - t_B = \Delta t_{BA}$. All the observed data are fitted well by an SIE(for G1)+ES(for environment)+SIE(for G2) model $\chi^2$/dof = 1.6 though the best-fit ellipticities $\epsilon(G1) = 0.621, \epsilon(G2) = 0.759$ are somewhat larger than the observed values $\epsilon(G1) = 0.45\pm0.01, \epsilon(G2) = 0.55\pm0.01$ in Courbin et al. (2011). The best-fit Hubble constant $h = 0.905$ is too large. However, $\sim 20\%$ deviation can be explained by mass-sheet degeneracy due to uncertainty of mass distribution in the line-of-sight (Schneider & Sluse 2013). We conclude that this system is not anomalous in the flux ratios though the cusp relation is violated as $R_{\text{cusp}} \sim 0.492$.

### 5.5 B0712+472

The fold-caustic lens B0712+472 consists of two bright image A and B, and two fainter images C and D. The source and lens redshifts are $z_S = 1.339$ and $z_L = 0.4060$ (Fassnacht & Cohen 1998). We use the radio flux ratios in Koopmans et al. (2003) of 4 images at 5 GHz averaged over a period of 8.5 months. For the astrometry, we use the data from CASTLES database. The positions of lensed images and the centroid G of the primary lensing galaxy are well fitted by an SIS and an external shear assuming that the error in the position of G is 0.05 arcsec. However, the flux ratios are not well fitted.

### 5.6 B2045+265

The cusp-caustic lens B2045+265 consists of three bright images A, B, and C and one faint image D. The source and lens redshifts are $z_S = 1.28$ and $z_L = 0.8673$ (Fassnacht et al. 1999). We use the radio flux ratios Koopmans et al. (2003) of 4 images at 5 GHz averaged over a period of 5.5 months. For the astrometry, we use the infrared components of B2045 obtained by adaptive optics imaging at 2.2{$\mu$m}(McKean et al. 2007). In addition to a primary lensing galaxy G1 at $z_L$, a possible companion galaxy G2 may reside near G1 though the redshift has not been known(McKean et al. 2007). All the observed data are fitted extremely well by an SIE(for G1)+ES(for environment)+SIE(for G2) model giving $\chi^2$/dof = 0.03. However, the best-fit ellipticity of G2 seems too large $\epsilon(G2) = 0.867$. Such a large value can only be expected from either an edge-on disk system or a tidally disrupted dwarf galaxy. We conclude that this system is not anomalous in the flux ratios though the cusp relation is significantly violated as $R_{\text{cusp}} \sim 0.501$.

### 6 RESULTS

As shown in table 5, the observed magnification perturbations $\hat{\eta}$ with respect to the best-fit lens models in section 4 are in the range of $0.063 < \hat{\eta} < 0.13$. Using 3 lensed images ($N_{\text{image}} = 3$), we find that $\eta$ for B1422+231 is non-zero at $\sim 20\sigma$ level, implying that the flux-ratio anomaly is most

![Figure 3. Non-linear matter power spectra for CDM and WDM models for various redshifts $z = 0, 0.3, 1$ and 2. The filled circles with the error bars are the simulation results for CDM(red), WDM140(grey), WDM44(blue), WDM15(orange) and WDM4.6(green) in Table 1. The results are taken from the large-box simulations (L100) for $k < 30h$/Mpc and the small-box simulations (L10) for $k > 60h$/Mpc. Note that the vertical axis is $P(k)k^2$ (not $P(k)$) to show the differences among the models clearly. The vertical dotted line denotes the Nyquist wavenumber of the small-box (L10). The solid curves are obtained from our fitting formula for the WDM models (see main text and Appendix A).](image-url)
prominent. For the other three lensing systems, the significance of non-zero \( \eta \) are 2 ~ 3\( \sigma \).

In order to estimate the second moment of \( \eta \), we have to consider a cut-off scale \( k_{max} \) that corresponds to the smallest fluctuations due to the finite size of the source. From dust reverberation, the radius of the MIR emitting region of MG0414+0534 is estimated as \( \sim 2 \) pc (Minezaki et al. 2009). As the magnification of A1 and A2 images are \( \sim 17 \), the apparent comoving size of the lensed source at the lens plane is \( r_s \sim (1 + z_s) \times 2 \times \sqrt{\Omega_M} \sim 30 \) pc. Assuming that \( k_{max} = 2\pi/(4r_s) \), we have \( k_{max} \sim 8 \times 10^3 h/\text{Mpc} \). For radio sources, we can obtain \( k_{max} \) from the apparent angular sizes (typically \( \sim 2 \) ~ 3 mas in radius) of lensed VLBI images. Then we find that \( 10000 \lesssim k_{max} \lesssim 70000 \) in units of \( h/\text{Mpc} \). Taking into account ambiguity in the source size, we consider two types of choices \( k_{max} = 3 \times 10^4 h/\text{Mpc} \) and \( 10^5 h/\text{Mpc} \). In the CDM model, we find that contribution from modes with wavelength \( k > 3 \times 10^3 h/\text{Mpc} \) is significant, especially for high-redshift sources (Table 5). This suggests that \( \eta \) in CDM models is sensitive on the property of small-scale fluctuations in systems with a high-redshift source. However, for the WDM models with a large free-streaming scale, contribution from modes on small scales is small (Fig. 4). For instance, the difference in the second moment of \( \eta \) between the model with \( k_{max} = 3 \times 10^4 h/\text{Mpc} \) and that with \( k_{max} = 10^5 h/\text{Mpc} \) is less than 15 \% for \( k_{fs} = 30 h/\text{Mpc} \) (Fig. 5). However, it should be emphasised that intermediate-scale modes with wavenumbers \( k_{fs} < k < 3 \times 10^4 h/\text{Mpc} \) are not negligible. In other words, powers regenerated due to the coupling between large and small scale modes play an important role for understanding the nature of the weak lensing effects.

In order to constrain the mass of WDM particles with a free-streaming scale \( k_{fs} \), we use the PDFs of magnifica-

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**Table 3. Quadruple lens systems**

| lens system       | image(type) | position(obs.) (") | flux ratio(obs.) | \( \mu \)(model) | flux ratio(model) | references |
|-------------------|-------------|---------------------|------------------|------------------|------------------|------------|
| B1422+231         | A(I)        | (-0.3860 ± 0.0004, 0.3169 ± 0.0003) | A/B=0.9416 ± 0.0080 | 6.892           | A/B=0.7882       | (1) (2)    |
|                  | B(I)        | (0, 0)              |                  |                  |                  |            |
|                  | C(I)        | (0.3360 ± 0.0003, -0.7516 ± 0.0005) | C/B=0.5188 ± 0.0079 | 4.327           | C/B=0.5070       |            |
|                  | D(I)        | (-0.9470 ± 0.0006, -0.8012 ± 0.0005) |                  |                  |                  |            |
|                  | G           | (-0.7321 ± 0.0037, -0.6390 ± 0.0054) |                  |                  |                  |            |
| B0128+437         | A(I)        | (0.000 ± 0.002, 0.0000 ± 0.0003) |                  |                  |                  | (1) (3) (4) |
|                  | B(I)        | (-0.099 ± 0.003, 0.095 ± 0.003) |                  |                  |                  | (3) (4)    |
|                  | C(I)        | (-0.521 ± 0.004, -0.170 ± 0.002) |                  |                  |                  | (4)        |
|                  | D(I)        | (-0.109 ± 0.003, -0.260 ± 0.002) |                  |                  |                  |            |
|                  | G           | (-0.217 ± 0.01, -0.104 ± 0.01) |                  |                  |                  |            |
| MG0414+0534       | A(I)        | (-6.000 ± 0.003, -1.942 ± 0.003) |                  |                  |                  | (5) (6) (7)|
|                  | A2(I)       | (-0.732 ± 0.003, -1.549 ± 0.003) |                  |                  |                  |            |
|                  | B(I)        | (0, 0)              |                  |                  |                  |            |
|                  | C(I)        | (1.342 ± 0.003, -1.650 ± 0.003) |                  |                  |                  |            |
|                  | D(I)        | (0.472 ± 0.003, -1.277 ± 0.003) |                  |                  |                  |            |
|                  | G           | (0.857 ± 0.011, 0.180 ± 0.009) |                  |                  |                  |            |
| B1608+656         | A(I)        | (0, 0)              |                  |                  |                  | (2) (8)    |
|                  | B(I)        | (-0.7464 ± 0.0026, -1.9578 ± 0.0026) |                  |                  |                  |            |
|                  | C(I)        | (-0.7483 ± 0.0038, -0.4465 ± 0.0033) |                  |                  |                  |            |
|                  | D(I)        | (-0.7483 ± 0.0038, -0.4465 ± 0.0033) |                  |                  |                  |            |
|                  | G           | (0.590 ± 0.0031, -0.9539 ± 0.0023) |                  |                  |                  |            |
|                  | X           | (31.512 ± 0.17, 36.0 ± 1.5, 77.0 ± 7) |                  |                  |                  |            |
| B0712+472         | A(I)        | (0.795 ± 0.003, -0.156 ± 0.003) |                  |                  |                  | (1) (5)    |
|                  | B(I)        | (0.747 ± 0.003, -0.292 ± 0.006) |                  |                  |                  |            |
|                  | C(I)        | (-0.013 ± 0.004, -0.804 ± 0.003) |                  |                  |                  |            |
|                  | G           | (0.391 ± 0.006, -0.082 ± 0.003) |                  |                  |                  |            |
|                  |                      | (0, 0)              |                  |                  |                  |            |
| B2045+265         | A(I)        | (0.0000 ± 0.0005, 0.0000 ± 0.0005) |                  |                  |                  | (1) (9)    |
|                  | B(I)        | (0.1316 ± 0.0006, -0.2448 ± 0.0006) |                  |                  |                  |            |
|                  | C(I)        | (0.2869 ± 0.0005, -0.7885 ± 0.0005) |                  |                  |                  |            |
|                  | G           | (-1.084 ± 0.0011, -0.8065 ± 0.0011) |                  |                  |                  |            |
|                  |                      | (0, 0)              |                  |                  |                  |            |

Note: (*) The lens redshift \( z_L \) is obtained from a best-fit model. References: (1) Koopmans et al. (2003) (2) Courbin et al. (2011) (3) Biggs et al. (2004) (4) Lagattuta et al. (2010) (5) CASTLES data base:http://www.cfa.harvard.edu/castles (6) Minezaki et al. (2009) (7) MacLeod et al. (2013) (8) Fassnacht et al. (2002) (9) McKean et al. (2007) Type I and II correspond to minimum and saddle, respectively. \( \mu \) represents magnification.
Constraints on warm dark matter from weak lensing in anomalous quadruple lenses

Table 4. Best-fit model parameters

| Model          | B1422+231 | B0128+437 | MG0414+0534 | B1608+656 | B0712+472 | B2045+265 |
|----------------|-----------|-----------|-------------|-----------|-----------|-----------|
|                | SIE-ES+(0.01") | SIE-ES    | SIE-ES-SIS  | SIE-ES-SIE| SIE-ES+(0.05")| SIE-ES-SIE|
| \( b_G' (")   | 0.754     | 0.207     | 1.07        | 0.737     | 0.543     | 1.012     |
| \((x_s,y_s) (")| (-0.3854,-0.4144) | (-0.2549,-1.0001) | (0.4037,-1.0268) | (0.1027,-1.0981) | (0.0184,-1.0503) | (-0.6387,-0.6533) |
| \( e(G1)     | 0.300     | 0.577     | 0.300       | 0.621     | 0.735     | 0.358     |
| \( \theta(G1) (deg) | -56.6     | -20.2     | -87.9       | 73.1      | 57.2      | 22.0      |
| \( \gamma     | 0.168     | 0.230     | 0.0870      | 0.135     | 0.199     | 0.220     |
| \( \theta(\gamma) (deg) | -52.4     | 46.3      | 47.4        | -84.3     | -23.7     | -70.1     |
| \( b_G'' (")  | 0.192     | 0.212     | 0.102       | 0.212     | 0.449     | 0.449     |
| \( e(G2)     | 0.759     | 0.867     | 0.759       | 0.867     | 0.867     | 0.867     |
| \( \theta(G2) (deg) | 63.6      | -58.8     | 63.6        | -58.8     | 63.6      | -58.8     |
| \((x_G2,y_G2) (")| (0.856,0.183) | (-0.282,-0.936) | (-0.450,-0.642) | (0.0000,0.0000) | (0.0000,0.0000) | (0.0000,0.0000) |
| \( \Delta t_{BA} (days) | 32.9     | 32.9     | 32.9        | 32.9     | 32.9     | 32.9     |
| \( \Delta t_{BC} (days) | 37.1     | 37.1     | 37.1        | 37.1     | 37.1     | 37.1     |
| \( \Delta t_{BD} (days) | 75.8     | 75.8     | 75.8        | 75.8     | 75.8     | 75.8     |
| \( h (Hubble constant) | 0.905     | 0.905     | 0.905       | 0.905     | 0.905     | 0.905     |
| \( \chi^2_{\text{imag}} | 1.4      | 2.1      | 5.1         | 0.1      | 4.2      | 0.00      |
| \( \chi^2_{\text{flux}} | 340.3    | 3.7      | 22.9        | 2.5      | 4.4      | 0.03      |
| \( \chi^2_{\text{del}} | 2.0      | 2.0      | 2.0         | 2.0      | 2.0      | 2.0       |
| \( \chi^2_{\text{tot}} | 344.1   | 7.7      | 29.2        | 4.8      | 14.1     | 0.03      |

The thermal WDM mass, the constraint can be expressed as

\[ m_{\text{WDM}} \geq 1.3 \text{ keV}. \]

For the mass of sterile neutrinos, the constraint corresponds to \( m_s \geq 6.3 \text{ keV}. \) For smaller \( k_{\text{max}} \), the constraint becomes more stringent.

Figure 4. Suppression of magnification perturbation due to free-streaming. The square-root of second moment \( \eta_i \) is plotted as a function of \( k_{\text{fs}} \) for \( k_{\text{max}} = 10^5 h/\text{Mpc} \) (full curve) and \( k_{\text{max}} = 3 \times 10^3 h/\text{Mpc} \) (dashed curve).
We have investigated the weak lensing effect by line-of-sight structures in a concordant CDM and WDM models based on N-body simulations. We have found that 4 quadruple lenses with source redshifts at $1 \leq z_s \leq 4$ out of 6 show anomalies in the flux ratios of lensed images. The magnitudes of expected magnification perturbation due to the line-of-sight structures in the concordant ACDM model are consistent with the observed ones. Using 4 anomalous samples, a constraint on the free-streaming scale of WDM particles, $k_{fs} \geq 27 h/$Mpc(95%CL) has been obtained. For thermally produced WDMs, we have a constraint $m_{WDM} \geq 1.3$ keV(95%CL).

Our result for fluctuations at low-redshifts $0 < z < 4$ is consistent with constraints from Lyman-$\alpha$ forests at $4 < z < 6$ (Viel et al. 2005, 2013) and those from high-redshift ($4 \leq z \leq 10$) galaxy counts (Schultz et al. 2014). Therefore, WDM models with $m_{WDM} > 1.3$ keV are ruled out at redshifts $0.5 \leq z \leq 10$. Thus, WDM models as solutions for the “missing satellite problem” are disfavored virtually at all the redshifts.

Our calculations are based on a semi-analytic formalism that has been used for estimating magnification perturbations due to line-of-sight structures in the CDM models. In order to verify the formalism in the WDM models more rigorously, we need ray-tracing Monte-Carlo simulations where the lens parameter fitting is done with the presence of line-of-sight structures, which will be our future work.

In our simulations, we did not take into account non-luminous subhalos in lensing galaxies. In CDM models, it has been shown that the surface mass density of subhalos in lensing galaxies is not enough for explaining the observed flux-ratio anomalies (Maccio & Miranda 2006; Amara et al. 2006; Xu et al. 2009, 2010; Chen 2009; Chen et al. 2011). As the number density of subhalos with sizes that are comparable to or less than the free-streaming scale $\sim 1/k_{fs}$ is significantly reduced, the role of dark subhalos in lensing galaxies would be minor. However, we may need to check the lensing effects of subhalos in WDM models as well.

We expect that baryonic feedback effects on the line-of-sight structures are limited to the central region of minihalos. Therefore, the weak lensing effects that are relevant to the property of outskirts of minihalos may not change so much. Although, the power spectra from simulations with baryons may significantly differ, those calculated from fluctuations obtained by masking the central regions of halos would be less affected. In order to verify it, however, it is very important to incorporate baryonic effects in our lensing simulations, which will be also our future work.

If intervening perturbers are massive enough ($\gtrsim 10^{10} M_\odot$), we may directly detect the presence and the redshift of perturbers from the extended-source effects (Inoue & Chiba 2005a,b). In order to do so, observation of anomalous quadruple lenses by ALMA is important. Emission from neutral hydrogen (HI) may be another clue for detecting the line-of-sight structures. Measuring correlation between flux-ratio anomaly and HI emission may be a new test for confirming the presence of line-of-sight structures.
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APPENDIX A: FITTING FORMULA FOR NON-LINEAR MATTER POWER SPECTRA IN WDM MODELS

In this Appendix, we present our fitting formula for the matter power spectra in WDM models. Our formula is based on the halofit model (Smith et al. 2003; Takahashi et al. 2012), but slightly modified for WDM models.

To find the best fitting parameters in the theoretical model, we use the standard chi-square fitting, which is defined as

$$\chi^2 = \sum_i \sum_{z=0}^{3} \sum_{k=k_{\text{min}}}^{k_{\text{max}}} \frac{(P_{i,\text{model}}(k, z) - P_{i,\text{sim}}(k, z))^2}{P_{i,\text{sim}}(k, z)^2},$$  \hspace{1cm} (A1)

where $P_{i,\text{model}}$ is the power in the theoretical model, $P_{i,\text{sim}}$’s are those in simulation results, and $i$ denotes the CDM model ($i = 0$) and the four WDM models ($i = 1-4$) in Table 1. The $\chi^2$ is summed over redshifts $z = 0, 0.3, 0.6, 1, 2$ and 3. We use the wave number $k$ larger than $2h/\text{Mpc}(= k_{\text{min}})$ where the Gaussian error of $P(k)$ is less than 1%. The maximum wavenumber is $k_{\text{max}} = 300(30)h/\text{Mpc}$ for the L10(L100) so that the measured power spectrum is much larger than at smaller scales $k > 100h/\text{Mpc}$. The discrepancy becomes more prominent at smaller scales $k \gtrsim 100h/\text{Mpc}$. This is probably due to the fact that the Nyquist wavenumber $k_{\text{Nyq}} = 60h/\text{Mpc}$ (see Fig.7 in Viel et al. (2012)) in their simulations is smaller than ours $k_{\text{Nyq}} = 322h/\text{Mpc}$.

First, we fit the simulation results of the CDM model. The fitting parameters for the CDM model are the same as in Takahashi et al. (2012), except for the following three parameters:

\begin{align*}
\log_{10} a_0 &= 0.9221 + 2.0595 n_{\text{eff}} + 2.4447 n_{\text{eff}}^2 + 1.2625 n_{\text{eff}}^3 + 0.2874 n_{\text{eff}}^4 - 0.7601 C, \\
\log_{10} c_0 &= 0.4747 + 2.1542 n_{\text{eff}} + 0.8582 n_{\text{eff}}^2 + 0.8329 C, \\
\gamma_0 &= 0.2247 - 0.2287 n_{\text{eff}} + 0.9726 C - 0.0533 \ln \left( \frac{k}{h/\text{Mpc}} \right). \hspace{1cm} (A2)
\end{align*}

The ratios of the power spectra of the WDM to that of the CDM are fitted as,

$$\frac{P_{\text{WDM}}(k, z)}{P_{\text{CDM}}(k, z)} = \frac{1}{(1 + k/k_0)^{0.7447}},$$  \hspace{1cm} (A3)

with

$$k_0(k_{\text{fs}}, z) = 2.066 \text{hMpc}^{-1} \left( \frac{k_{\text{fs}}}{h/\text{Mpc}} \right)^{1.703} D(z)^{1.583}. \hspace{1cm} (A4)$$

where $k_{\text{fs}}$ is the free-streaming wavenumber and $D(z)$ is the linear growth factor at $z$, which is normalised as $D(z = 0) = 1$. Equation (A4) can be rewritten in terms of the WDM particle mass $m_{\text{WDM}}$ as

$$k_0(k_{\text{fs}}, z) = 388.8 \text{hMpc}^{-1} \left( \frac{m_{\text{WDM}}}{\text{keV}} \right)^{2.027} D(z)^{1.583}. \hspace{1cm} (A5)$$

The RHS of equation (A3) corresponds to a damping factor. Note that the parameters $n_{\text{eff}}$ and $C$ in equation (A2) are evaluated in the CDM model even when computing $P_{\text{sim}}(k, z)$. Using our fitting formula, the simulation results can be reproduced within an relative error of 19%. The root-mean-square deviation between the theoretical model and the simulation results is about 4.5%.

Fig.1A shows the ratio of the WDM power spectrum $P_{\text{WDM}}(k, z)$ to the CDM power spectrum $P_{\text{CDM}}(k, z)$ at redshifts $z = 0, 0.3, 1$ and 2. The solid curves represent the decay of power spectra described by our damping factor in Eq.(A3), which reproduces our simulation results very well. For comparison, the predicted power spectra based on the previous fitting formula in Viel et al. (2012) are plotted as dotted curves. As shown in Fig.1A, the predicted powers based on the previous fitting formula are too large at scales $k \sim 100h/\text{Mpc}$. The discrepancy becomes more prominent at smaller scales $k \gtrsim 100h/\text{Mpc}$. This is probably due to the fact that the Nyquist wavenumber $k_{\text{Nyq}} = 60h/\text{Mpc}$ (see Fig.7 in Viel et al. (2012)) in their simulations is smaller than ours $k_{\text{Nyq}} = 322h/\text{Mpc}$.

APPENDIX B: FUNCTIONAL FORM OF PDF

In this Appendix, we provide the PDFs of $\eta$. We assume that the PDFs are approximated by the log-normal function as

$$P(\eta) = N \exp \left[ -\frac{1}{2\sigma^2} \left( \ln \left( \frac{1 + \frac{\eta}{\eta_0}}{\mu} \right) - \ln \mu \right)^2 \right] \frac{1}{\eta + \eta_0},$$  \hspace{1cm} (B1)

where $N$ is a normalization constant, $\eta_0$ describes a dispersion scale of $\eta$, and $\mu$ and $\sigma$ are constants. We assume that $\eta_0$ depends only on the second moment $\langle \eta^2 \rangle$ and that $\sigma$ and $\mu$ do not depend on $\langle \eta^2 \rangle$. Using lay-tracing simulations for a concordant ΛCDM model, we find that the best-fit parameters are (Takahashi & Inoue 2014)

$$\mu = 4.10, \sigma = 0.279, \eta_0 = 0.228\langle \eta^2 \rangle^{1/2}. \hspace{1cm} (B2)$$

As the formula (B1) does not depend on the grid size $r_{\text{grid}}$ of the simulation (see Fig.9 in Takahashi & Inoue (2014)), we assume that the formula (B1) is also applicable to WDM models where small-scale fluctuations are suppressed due to free-streaming. We find that the suppression in the power due to the finite grid size $\sim 4.8kpc/h$ is comparable to the one by free-streaming for $k_{\text{fs}} \sim 20h/\text{Mpc}$.

In real setting, we need to take into account errors in observation as well. If the variance of observational error is $\delta \eta^2$, and the mean value is vanishing, we should change $\eta_0$ as

$$\eta_0 = 0.228\langle \eta^2 \rangle + \delta \eta^2)^{1/2}. \hspace{1cm} (B3)$$

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Figure A1. Same as Fig.3, but the ratio of the WDM power spectra $P_{\text{WDM}}(k)$ to the CDM $P_{\text{CDM}}(k)$. The filled circles are the simulation results for WDM140 (grey), WDM44 (blue), WDM15 (orange) and WDM4.8 (green). The solid curves correspond to our fitting formula in equation (A3), while the dotted curves to the previous one in Viel et al. (2012).
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