**B Leptonic Decays and $B - \bar{B}$ Mixing with 2+1 Flavors of Dynamical Quarks**

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Calculations of $B$ leptonic decays and $B - \bar{B}$ mixing using NRQCD heavy and Asqtad light valence quarks on the MILC dynamical configurations are described. Smearing has been implemented to substantially reduce the statistical errors of the matrix elements needed for the determination of $f_B$. The four-fermion matrix elements needed for the determination of $f_B^2$ have been calculated and a preliminary result is given.

1. Introduction

$B - \bar{B}$ mixing is a key process in standard CKM analysis. Calculations of the $B(B_s)$ leptonic decay constant $f_B(f_{B_s})$ and bag parameter $B_B(B_{B_s})$ are needed to constrain $V_{td}(V_{ts})$. In particular, we aim to calculate the ratio $f_{B_s}/f_B$ accurate to the few percent level allowing a determination of $V_{ts}/V_{td}$ in which theoretical errors do not dominate those from experiment.

The Asqtad improved staggered formulation has allowed precise determinations of a variety of quantities [1], and recent progress has allowed the use of improved staggered valence quarks in heavy-light simulations [2]. Here we report on the progress of $B$ leptonic decay and mixing calculations using NRQCD heavy quarks and improved staggered (Asqtad) light quarks. We use the MILC $n_f = 2 + 1$ dynamical configurations [3].

2. $B$ leptonic decays

In previous work [4] the chiral behavior of $f_B, \sqrt{m_B}/f_B, \sqrt{m_B}$ was masked by large statistical errors. Here these errors have been substantially reduced with the introduction of smearing. Two dynamical ensembles were used; one incorporating a light dynamical quark mass $m_f \equiv m_{u,d} = m_s/4$ while the other has $m_f = m_s/2$ where $m_s$ is the physical strange quark mass. The valence light quark mass $m_q$ was varied from $m_s$ to $m_s/8$, i.e. some runs were in the partially quenched approximation. For $b$ quarks, the standard tadpole improved Lattice NRQCD action correct through $O(1/(am_b)^2)$ was used along with zeroth and first order in $1/am_b$ currents. $a^{-1}$ and $m_b$ were fixed by $\Upsilon$ while $m_{u,d}$ and $m_s$ were fixed by $\pi$ and $K$ respectively.

A ground state hydrogenic style wavefunction was used to smear the heavy quark at both source and sink and the optimal radius was found as that which minimized the fit errors while maintaining a reasonable $\chi^2/dof$. For each combination of source and sink smearing, the functional form of the $k^{th}$ order in $1/am_b$ correlator is given by the oscillating function

$$G(t) = \sum_{j=0}^{n_{exp}+1} C^{(kj)}(-1)^j e^{-m_j t}.$$  \hspace{1cm} (1)

Bayesian multi-exponential fits were done to the $k = 0, 1$ correlators, and $\Phi^{(k)}_B$ were determined as

$$\Phi^{(k)}_B = \sqrt{2C^{(k0)}}.$$  \hspace{1cm} (2)
currents have been considered. A preliminary Bayesian fit has been successful for $n_{\text{exp}} = 4$, as can be seen in Figure 8 which compares the data and fit function of the effective amplitude (correlator with ground energy contributions removed). The oscillations in both $t_B$ and $t_B$ can clearly be seen to be nicely represented by the fit function, and the fit has a good $\chi^2$/dof. Work is underway to obtain fits at other $n_{\text{exp}}$ in order to solidify these results.

3. $B - \bar{B}$ Mixing

3-point correlators were generated with the creation of a $B$ at $t = -t_B$, conversion to a $\bar{B}$ with the use of a four-fermion operator $O$ at $t = 0$ and destruction of the $B$ at $t = t_B$. The same simulation parameters as in the $B$ leptonic decay case were used but only so far with $m_f = 0.01$, $m_q = 0.04$ (i.e the $B_s$). No smearing has yet been performed, and so far only zeroth order in $1/am_b$ currents have been considered.

The continuum four fermion operator $\langle O_L \rangle^{MS}$ is given in terms of lattice operators at 1-loop by

$$a^6 \langle O_L \rangle^{MS} = [1 + \rho_{LL} \alpha_s] \langle O_L \rangle_{\text{lat}} + \rho_{LS} \alpha_s \langle O_S \rangle_{\text{lat}}$$

where

$$O_L = [\bar{\psi}_Q \gamma^\mu (1 - \gamma_5) \psi_q] \ [\bar{\psi}_Q \gamma^\mu (1 - \gamma_5) \psi_q]$$
$$O_S = [\bar{\psi}_Q (1 - \gamma_5) \psi_q] \ [\bar{\psi}_Q (1 - \gamma_5) \psi_q]$$

and $\rho_{LL}$ and $\rho_{LS}$ have been calculated perturbatively [5].

The lattice correlators have the functional form

$$C(t_B, t_B) = \sum_{j,k=0}^{n_{\text{exp}}-1} A_{jk} (-1)^{t_B} e^{-m_j t_B} \ * (-1)^{kt_B} e^{-m_k t_B}.$$ (7)

A preliminary Bayesian fit has been successful for $n_{\text{exp}} = 4$, as can be seen in Figure 8 which compares the data and fit function of the effective amplitude (correlator with ground energy contributions removed). The oscillations in both $t_B$ and $t_B$ can clearly be seen to be nicely represented by the fit function, and the fit has a good $\chi^2$/dof. Work is underway to obtain fits at other $n_{\text{exp}}$ in order to solidify these results.
Figure 2. $\xi\Phi$ vs. ratio of light and strange valence quark masses. Closed symbols indicate that smearing has been used while open symbols indicate smearing has not been used. Squares represent fully unquenched results and triangles represent results in the partially quenched approximation.

where the correlator has good statistical noise, are possible using multiple exponentials without the inefficiency and ambiguity of needing a plateau.

$B_B$ is defined through

$$\langle O_L \rangle_{\overline{MS}} = \frac{8}{3} f^2_B M_B^2 B_B$$

and our results preliminarily give

$$f_B, \sqrt{B_B}(m_b) = 0.197(16)(28)\text{GeV}.$$  \hfill (9)

The first error arises from the fact that the fits are still preliminary and the second error is from systematics, the main contributions being from the neglect of $O(\Lambda_{QCD}/m_b)$ and $O(\alpha_s^2)$ contributions.

4. Conclusions

We report here on the status of $B$ leptonic decays and $B - \bar{B}$ mixing simulations with NRQCD heavy and Asqtad light quarks. The use of smearing has significantly reduced the statistical errors for the leptonic decay case. Chiral extrapolations and more fully unquenched simulations are in progress. A successful fit to the $B\bar{B}$ correlator has allowed a preliminary result for $f_B, \sqrt{B_B}$.

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REFERENCES

1. MILC/HPQCD, C. T. H. Davies et al., Phys. Rev. Lett. 92, 022001 (2004), [hep-lat/0304004].
2. M. Wingate, J. Shigemitsu, C. T. H. Davies, G. P. Lepage and H. D. Trottier, Phys. Rev. D67, 054505 (2003), [hep-lat/0211014].
3. C. W. Bernard et al., Phys. Rev. D64, 054506 (2001), [hep-lat/0104002].
4. M. Wingate et al., Nucl. Phys. Proc. Suppl. 129, 325 (2004), [hep-lat/0309092].
5. E. Gulez et al., these proceedings; E. Gulez, J. Shigemitsu, M. Wingate Phys. Rev. D69, 074501 (2004), [hep-lat/0312017].
6. C. Aubin and C. Bernard, these proceedings and private communication.