A Relativistic Approach to
Deep Inelastic Scattering on the Deuteron.

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Abstract

A covariant field theoretical approach to deep inelastic scattering on the deuteron is presented. The deuteron structure function is calculated in terms of the Bethe-Salpeter amplitude. Numerical calculations for the nucleon contribution are made with a realistic model of the $NN$-interaction, including $\pi$, $\rho$, $\omega$, $\eta$, $\delta$- and $\sigma$-mesons, and results are compared with previous non-relativistic calculations.

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1. Introduction

The relativistic bound state problem appears as one of the most important and interesting in the modern theory of strong interactions. Since the bound state problem in QCD is still unresolved, it is important to study the capability of the effective theories both in the quark and hadron sectors. The most direct way to describe the bound states in a field theory is to consider the Bethe-Salpeter (BS) equation [1]. Approaches based on the BS equation, or its approximations, within models using effective potentials, allow for a successful description of the mass spectrum and the decay widths of mesons as bound systems of two quarks (for recent development see e.g. refs. [2]). The BS equation has also been applied to describe properties of the deuteron [3] and effective NN-forces [4, 5].

This letter presents a relativistic approach to deep inelastic lepton scattering on the deuteron based on the BS equation. This process was previously studied on the basis of various relativistic equations [6]-[10], approximations to the BS equation, and the estimates of the nuclear effects in the deuteron structure function (SF) $F_{D_2}^D$ display quite strong dependence on the approximation are made. We present a consistent covariant description of the nuclear effects in SF of the deuteron based on the BS equation with a realistic NN-interaction, including $\pi$, $\rho$, $\omega$, $\eta$, $\delta$- and $\sigma$-mesons. We calculate $F_{D_2}^D$ in terms of the BS amplitude, using the operator product expansion method within the effective meson-nucleon theory. More details about formalism may be found in ref. [9].

Our investigation is motivated partly by a number of existing and forthcoming experiments on deep inelastic scattering of leptons by deuterons (SLAC, CERN, DESY, CEBAF). The points of interest are (i) to extract the neutron SF from the combined proton-deuteron data and (ii) to estimate the “EMC-effect” on the deuteron, combining the electroproduction and neutrino data [11] (c.f. [3, 12]). A relativistic theory of this process will be beneficial in the analysis of the experimental data.

2. Deep inelastic scattering on the deuteron in the Bethe-Salpeter formalism

The hadronic tensor, $W_{\mu\nu}$, is proportional to the imaginary part of the virtual photon-hadron elastic scattering amplitude:

$$W_{\mu\nu}(P, q) = i \int \frac{d^4x}{x} e^{iqx} \langle P | T(J_\mu(x)J_\nu(0)) | P \rangle,$$  

where $P$ is the momentum of the target and $q$ is the virtual photon momentum ($Q^2 \equiv -q^2$). Thus the calculation of $W_{\mu\nu}$ is split into two relatively independent parts: a description of the bound state, and of the operator to be sandwiched between this state.
An accurate description of both the \(NN\)-interaction at energies up to \(\sim 1\) GeV, and the basic properties of the deuteron, can be provided within the meson-nucleon theory \[3, 5\]. The covariant description is based on the BS equation or its various approximations. We use the ladder approximation for the kernel of the BS equation \[3\]:

\[
\Phi(p, P_D) = i\hat{S}(p_1) \cdot \hat{S}(p_2) \cdot \sum_B \int \frac{d^4 p'}{(2\pi)^4} \cdot \frac{g_B^2 \Gamma_B^{(1)} \otimes \Gamma_B^{(2)} \cdot \Phi(p', P_D),}{(p - p')^2 - \mu_B^2}.
\]

where \(\mu_B\) is the mass of meson \(B\); \(\Gamma_B\) is the meson-nucleon vertex, corresponding to the meson \(B\), \(\hat{S}(p) = (\hat{p} - m)^{-1}\), \(m\) is the nucleon mass. The amplitude \(\Phi\) appears as a \(4 \times 4\)-matrix in the spinor space, so in the most general case the two-spinor BS amplitude consists of 16 independent components. However, due to the parity invariance of the strong interactions, these amplitudes are split into two independent sets with different parities. Only eight components, with a positive parity are relevant to describe the deuteron. Decomposing \(\Phi\) in terms of the complete set of the Dirac matrixes, their bilinear combinations and the \(4 \times 4\)-identity matrix, \(\hat{1}\), and performing the partial wave analysis we obtain for the deuteron a system of eight coupled two-dimensional integral equations with singular kernels. To remove the singularities from the kernels we use the well-known Wick rotation (see, e.g. \[1\] and references therein). More technical details may be found in \[3\].

The meson parameters, such as masses, coupling constants, cut-off parameters are taken similar to those in ref. \[3\], with a minor adjustment of the coupling constant of the scalar \(\sigma\)-meson so as to provide a numerical solution of the BS equation. All parameters are presented in Table 1, where coupling constants are shown in accordance with our definition of the meson-nucleon form-factors, \(F_B(k) = (\Lambda^2 - \mu_B^2)/(\Lambda^2 - k^2)\).

As to the second part of the problem, to describe the interaction operator in \(\mathbb{I}\) we utilize the Wilson Operator Product Expansion (OPE) method within the same meson-nucleon theory \[14, 15, 9\]. The OPE of the spin-averaged Compton amplitude \(\mathbb{I}\) has the form \[13\]:

\[
T_{\mu\nu}^{D}(P_D, q) = \sum_{a;n=2,4,\ldots}^{\infty} C_{a,n}^{(1)} \left( -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) \frac{2^n q_{\mu_1} \cdots q_{\mu_n}}{(-q^2)^n} \langle P_D | O_{a}^{\mu_1 \cdots \mu_n} (0) | P_D \rangle \tag{3}
\]

\[
+ \sum_{a;n=2,4,\ldots}^{\infty} C_{a,n}^{(2)} \left( q_{\mu_{\mu_1}} - \frac{q_{\mu} q_{\mu_1}}{q^2} \right) \left( q_{\nu_{\mu_2}} - \frac{q_{\nu} q_{\mu_2}}{q^2} \right) \frac{2^n q_{\mu_3} \cdots q_{\mu_n}}{(-q^2)^{n-1}} \langle P_D | O_{a}^{\mu_1 \cdots \mu_n} (0) | P_D \rangle,
\]

where \(O_{a}^{\mu_1 \cdots \mu_n} (0)\) is a set of local operators providing the basis for the OPE; \(a\) enumerates operators of different fields of the theory; \(C_{a,n}^{(1,2)}\) are the Wilson’s coefficient functions. The
structure functions, e.g., $F_2$, are obtained by using the dispersion technique \[13\] and is presented in the form:

$$M_{n-1}(F_2^D) = \sum_a C_{a,n}^{(2)} \cdot \mu_{n/D}^a, \quad \text{with} \quad M_n(F) = \int_0^1 F(x)x^{n-1}dx, \quad (4)$$

where $x$ is the Bjorken scaling variable, $0 \leq x \leq 1$ and $\mu_{n/D}^a$ is defined by:

$$\langle P_D | O_{\mu_1...\mu_n}^a | P_D \rangle \equiv \mu_{n/D}^a \cdot P_D^{\mu_1} \cdots P_D^{\mu_n}, \quad (a = N, B). \quad (5)$$

To obtain the moments of the deuteron SF (4) in the meson-nucleon theory we, first, define the set $O_{\mu_1...\mu_n}^a(0)$. For deep inelastic scattering the leading operators, twist two, for nucleon fields have the form (symmetrisation and subtraction of traces are assumed):

$$O_{\psi_1...\psi_n}^\mu = \left(\frac{i}{2}\right)^{n-1} \psi(0)\gamma_\mu \leftrightarrow \partial_\mu ... \leftrightarrow \partial_n \psi(0). \quad (6)$$

The operators for meson fields are similar to (6) (c.f. [14, 15, 9]). Secondly, the coefficients, $C_{a,n}^{(1,2)}$, are target-independent and by consideration of the scattering on the free nucleons and mesons, we find that in the ladder approximation these coefficients are identical to moments of the SF $F_2^a$ of nucleons ($a = N$) or mesons ($a = B$): $C_{N,n}^{(2)} = M_n(F_2^N)$, $C_{B,n}^{(2)} = M_n(F_2^B)$.

From eqs. (5)-(6) we get the following form for moments of the deuteron SF:

$$M_n(F_2^D) = M_n(F_2^N) \cdot \mu_{n+1}^{N/D} + \sum_B M_n(F_2^B) \cdot \mu_{n+1}^{B/D}, \quad (7)$$

where $\mu_{n/D}^{N}$ and $\mu_{n/D}^{B}$ are interpreted as moments of the effective distribution functions of the nucleons and mesons in the deuteron, respectively. In the framework of our approach the meson and nucleon SF are considered as given and moments of effective distributions are the matrix elements of the twist-2 operators on the deuteron state, computed by the Mandelstam method \[16\]. For the nucleon contribution to the moments of the deuteron structure function we get:

$$\mu_{n/D}^{N} = \frac{1}{M_D^n} \int \frac{d^4p}{(2\pi)^4} \Phi_D(p, P_D) \left\{ \hat{S}^{-1}(p_2) \left( \gamma_0^{(1)} + \gamma_3^{(1)} \right) \cdot (p_{10} + p_{13})^{n-1} \right\} \Phi_D(p, P_D) \quad (8)$$

where the kinematical variables in the rest frame are defined by

$$p = (p_0, \mathbf{p}), \quad p' = (p'_0, \mathbf{p'}), \quad P_D = (M_D, \mathbf{0}), \quad p_1 = \frac{P_D}{2} + p, \quad p_2 = \frac{P_D}{2} - p, \quad (9)$$
where $M_D$ is the deuteron mass, $p_{1k}$ is $k^{th}$ component of four-vector $p_1$ and in the deep inelastic kinematics $pq \approx q_0(p_0 + p_3)$.

Using the explicit form of the moments (7)-(8) and the inverse Mellin transform, we obtain the deuteron SF in the convolution form:

$$F_D^2(x) = \int_0^1 f^{N/D}(\xi)F_N^2(x/\xi)d\xi + \text{meson exchange terms},$$

(10)

$$f^{N/D}(\xi) = \frac{1}{M_D}\int \frac{d^4p}{(2\pi)^4} \Phi_D(p, P_D) \hat{S}^{-1}(p_2) \left(\gamma_0^{(1)} + \gamma_3^{(1)}\right) \Phi_D(p, P_D)$$

$$\left\{\theta(p_{10} + p_{13})\delta\left(\xi - \frac{p_{10} + p_{13}}{M_D}\right) + \theta(-p_{10} - p_{13})\delta\left(\xi + \frac{p_{10} + p_{13}}{M_D}\right)\right\}.$$  

(11)

This formula has clear and obvious physical interpretation. The first term on the r.h.s. represents the contribution of the nucleons to the full deuteron SF, $F_D^2$, the relativistic impulse approximation. The second term is the contribution of the meson exchange current, which can be also presented in convolution form \cite{9}.

3. Numerical results and discussion

Solving exactly the dynamical problem for the deuteron, described by eq. (2), and computing the deuteron structure function, eqs. (10)-(11), we obtain realistic estimates for the nucleon contribution to the deuteron SF, $F_D^2$, including off-mass-shell effects (also referred to as “binding effects”) and effects of Fermi motion in the deuteron. In fig. 1 we present our results in form of the ratio, $F_D^2(x)/F_N^2(x)$. The solid line is the result of calculation with the BS approach. We compare our calculations with non-relativistic results and calculation in the light-cone kinematics. The nonrelativistic calculations are presented within two approaches (c.f. \cite{15}), the $x$-rescaling model, referred to in fig. 1 as “nonrelativistic-I” and OPE-OBE model, “nonrelativistic-II”. Both these approaches describe fairly well deep inelastic scattering off heavy nuclei. It is seen that in the relativistic model the effects of nuclear structure of the deuteron is larger in comparison with that in the non-relativistic calculations. Apparently, this difference is due to a more consistent consideration of the off-mass-shell behavior of bound nucleons within the BS formalism. The nuclear structure effects in the deuteron are taken into account in the BS amplitude through the binding energy $\varepsilon_D$ and the dependence of the amplitude on the relative four-momentum of nucleons.

As an illustration of the influence of binding effects in the deuteron we present in fig. 1 the calculation in the light-cone kinematics (dotted line), disregarding the binding effects in the deuteron. In this case the ratio, $F_D^2(x)/F_N^2(x)$, is consistent with 1 at $x < 0.5$ and, due to
Fermi motion, becomes greater than 1 at \( x > 0.5 \). A comparison of the our results with this approach shows that the relativistic off-mass-shell effects lead to an “EMC-effect” on the deuteron, similar to that experimentally observed for heavy nuclei and theoretically obtained in non-relativistic approaches for the deuteron. All approaches may be compared also by computing \( \mu_{N/D}^{N/D} \), which is interpreted as the fraction of the total momentum of the deuteron carried by nucleons \[3, 14, 15, 16\]. Defining \( \delta_N \) as the deviation of the \( \mu_{N/D}^{N/D} \) from unity, we obtain \( \delta_N \approx 1 \cdot 10^{-2} \) which is twice the corresponding nonrelativistic result, \( \delta_N \approx 5 \cdot 10^{-3} \).

At the same time for the light-cone calculation we have \( \delta_N = 0 \).

All curves in fig. 1 are calculated with a rather simple parametrization of the nucleon SF, \( F_{2}^{N} \), used in previous non-relativistic calculations \[15\]. To emphasize the dependence of the ratio, \( F_{2}^{D}(x)/F_{2}^{N}(x) \), on the parametrization for \( F_{2}^{N} \) we show in fig. 2 calculations with a realistic parametrization of the nucleon structure function \( F_{2}^{N}(x) \) \[17\], which allows a consideration of the \( Q^2 \)-dependence of the ratio as well. Results for \( Q^2 = 10 \) \( GeV^2 \) and \( 90 \) \( GeV^2 \) are presented by solid lines, 1 and 2 respectively. The dashed line is the same as the solid line in fig. 1, where the parametrization of \( F_{2}^{N}(x) \) corresponds to low \( Q^2 \) and its behavior differs from the previous parametrization as \( x \to 1 \). Figure 2 demonstrates that the ratio \( F_{2}^{D}(x)/F_{2}^{N}(x) \) is almost \( Q^2 \)-independent and its shape is sensitive to the parametrization of the nucleon structure function as \( x \to 1 \).

The absolute value of the deuteron structure function, \( F_{2}^{D} \), as \( x \to 1 \) is shown in fig. 3 (solid line). The relativistic structure function displays a harder behavior of the “tail” in the vicinity of \( x \sim 1 \) compared to the non-relativistic \( x \)-rescaling calculations (dashed line).

A more detailed analysis of the relativistic effects in the deuteron SF, including the spin-dependent SF, will be presented elsewhere \[18\].

4. Conclusions

We have presented a relativistic approach to the deep inelastic lepton-deuteron scattering based on the Bethe-Salpeter formalism within an effective meson-nucleon theory. In particular,

- The spinor-spinor Bethe-Salpeter equation for the deuteron is solved in the ladder approximation for a realistic meson exchange potential, including \( \pi-, \rho-, \omega-, \eta-, \delta- \) and \( \sigma \)-mesons.
- The structure function \( F_{2}^{D} \) of the deuteron is calculated in terms of the Bethe-Salpeter amplitudes by the Operator Product Expansion method within the meson-nucleon theory.
• Our numerical calculations of the structure function $F_2^D$ emphasize a qualitative agreement with previous non-relativistic results, the magnitude of the effects of nuclear structure have been found to be larger in the relativistic approach.

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Table 1. The parameters of the model.

| meson | coupling constants $g_B^2/(4\pi); [g_t/g_v]$ | mass $\mu_B$, GeV | cut-off $\Lambda$, GeV | isospin |
|-------|----------------------------------------|--------------------|-------------------|--------|
| $\sigma$ | 12.2 | 0.571 | 1.29 | 0 |
| $\delta$ | 1.6 | 0.961 | 1.29 | 1 |
| $\pi$ | 14.5 | 0.139 | 1.29 | 1 |
| $\eta$ | 4.5 | 0.549 | 1.29 | 0 |
| $\omega$ | 27.0; [0] | 0.783 | 1.29 | 0 |
| $\rho$ | 1.0; [6] | 0.764 | 1.29 | 1 |

$m = 0.939$ GeV, $\epsilon_D = -2.225$ MeV
Figure 1. The ratio of the deuteron and nucleon structure functions $F_2^D(x)/F_2^N(x)$ calculated in different theoretical approaches. Curves: Solid is the present result of the BS formalism; Dashed and dot-dashed are previous non-relativistic estimates (see text); Dotted is calculation in the light-cone kinematics. The nucleon SF, $F_2^N$ is taken from ref. [15].

Figure 2. The ratio $F_2^D(x)/F_2^N(x)$ calculated with different parametrization for $F_2^N$. Solid lines present calculations with realistic parametrization from ref. [17] at $Q^2 = 10$ GeV$^2$ (curve 1) and $Q^2 = 90$ GeV$^2$ (curve 2). Dashed line is the same as the solid line in fig. 1.

Figure 3. The deuteron structure function $F_2^D$ as $x \to 1$. Curves: BS approach (solid), non-relativistic $x$-rescaling model (dashed) and the free nucleon structure function (dotted). The nucleon SF, $F_2^N$ is taken from ref. [17] at $Q^2 = 50$ GeV$^2$. 
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