A Simple Method for P-waves Velocity Estimation Using Pore Attributes Shape Factor and Tortuosity

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Abstract – Several researchers have arranged an approach to estimating the P-wave velocity, but none of them specifically relates to the pore attribute. Pore attributes are one of the main factors that affect pore complexity and rock quality. If P-wave velocity is influenced by the pore complexity, then it should be possible to arrange a simple relationship of P-wave velocity with the pore attribute. This study is intended to construct an empirical relationship of P-wave velocity with a combination of pore attributes, shape factor, and tortuosity ($F_s \tau$) so that the P-wave velocity can be easily estimated. This study used two sandstone datasets from 2 different basins, which are the northern part of the West Java basin and the Kutai basin. This research shows that a simple empirical equation can be arranged to relate the P-wave velocity with $F_s \tau$. This relationship provides a good correlation coefficient. It offers an easy and straightforward approach to estimating P-wave velocity.

Keywords: velocity estimation, pore attribute, shape factor and tortuosity, pore complexity, pore geometry.

Introduction

Many studies have been conducted related to the prediction of P-wave velocity based on the physical properties of rocks. One researcher relates the velocity with porosity and shows a linear relationship (Wyllie et al., 1956). Wyllie's time average equation is widely used for velocity predictions based on porosity. However, the linear relationship between porosity and velocity is only valid for porosity below 47% (Raymer et al., 1980). Theoretically, the maximum porosity of sandstone for simple cubic packing is 0.476. Nur proved that there is a boundary between consolidated sediments and suspensions called critical porosity. Above critical porosity, the relationship of porosity with P-wave velocity is no longer linear and can be represented by the suspension curve of Reuss bound. Several studies have shown that an empirical relationship can be arranged between P-wave velocity, porosity, and clay volume (Han et al., 1986; Marion et al., 1989; Catagna et al., 1985; Klimetos, 2002). The presence of clay in rocks will affect the pore complexity and rock rigidity. Related to pore complexity, Prakoso shows that there is an impact of pore geometry and pore structure on P-wave velocity (Prakoso et al., 2016). Pore geometry and pore structure can be stated simply by a combination of porosity and permeability (Wibowo & Permadi, 2015). The same rock quality is expressed to have similar pore geometry and pore structure. The pore geometry and pore structure are functions of pore attributes such as shape factor ($F_s$), tortuosity ($\tau$), and specific surface area ($S$) (Wibowo & Permadi, 2015). The general combination of pore attributes $F_s \tau$ is known as Kozeny constant c. Mortensen arranges a simple mathematical equation for estimation c as a function of porosity (Mortensen et al., 2007).
Regarding rock quality, Prakoso shows that P-wave velocity can be predicted based on pore geometry and pore structure for each rock type (Prakoso et al., 2016). At the same time, Prakoso arranges equations that can be used for estimating P-wave velocity based on critical porosity (Prakoso et al., 2018a). These studies show that P-wave velocity is strongly influenced by the pore complexity. Prakoso shows that the reservoir quality index (RQI) has a linear relationship with the Kozeny constant c (Prakoso et al., 2018b). The Kozeny constant is a function of shape factors and tortuosity (Fsτ). This study is intended to establish an empirical relationship of P-wave velocity with pore attributes and easy to use for estimating P-wave velocity.

**Materials and Methods**

**Data**

This study used two data sets from two sedimentary basins. The available data included petrophysical properties of rock, such as porosity and permeability. The P-wave velocity data is measured using sonic viewer SX equipment (Figure 1). P-wave velocity is measured at room pressure and temperature. Porosity and permeability data were measured at the laboratory using standard laboratory rock analysis procedures. The data set 1 is the sandstone of the Cibulakan formation from the North West Java Basin. While the data set, two are sandstones of Balikpapan formation from the Kutai basin.

![Figure 1. Sonic viewer SX](image)

Data set 1 was taken from the Talangakar formation, from the early Oligocene to late Oligocene. Talangakar Formation is dominated by fine to coarse-grained sandstones, which are alternately deposited with shale, siltstone, coal, and limestone. The entire rock samples were composed of predominantly by quartz minerals. While data set, 2 is the Kutai Basin sandstone taken from the Balikpapan Formation, from Middle Miocene to Late Miocene. The Balikpapan Formation is deposited in deltaic plain - deltaic front associated with fluvial deposits, distributary channels, and mouth bars.

**Rock Quality**

Wibowo and Permadi arranged of rock quality identification based on power-law models (Wibowo & Permadi, 2015). The power-law model is a modification of Kozeny’s equation. Modifications to these equations can be arranged simply in the form of pore geometry and pore structure relationship as follows:
\[
\left( \frac{k}{\phi} \right)^{0.5} = \phi \left( \frac{1}{(F_s \tau S_b^2)^{0.5}} \right)
\]

where \(k\) is permeability, \(\phi\) is porosity, \(F_s\) is shape factor, \(\tau\) is tortuosity, and \(S_b\) is specific internal surface area. \((k/\phi)^{0.5}\) describes characteristics of the pore geometry, and \((k/\phi)^{b}\) represents the pore structure, which is a function of the internal parameters of pore such as shape factor \(F_s\), tortuosity \(\tau\), and the specific surface area \(S_b\).

Equation 1 can be written in the form of power-law as follows:

\[
\left( \frac{k}{\phi} \right) = a \left( \frac{k}{\phi^3} \right)^b
\]

Equation 2 is a power-law model that can be used to identify rock quality. \(1/F_s\) is widely known as the Kozeny’s constant \((c)\). This constant can be estimated simply from the porosity (Mortensen et al., 2007). The Mortensen equation is written as follows:

\[
c = \left( 4 \cos \left( \frac{1}{3} \arccos \left( \frac{8}{\pi^2} - 1 \right) + \frac{4}{3} \pi \right) + 4 \right)^{-1}
\]

Equation 2 and equation 3 will be used as the basis of analysis for P-wave velocity predictions.

**Results**

Wibowo and Permadi define pore geometry and pore structure in a simple form based on a combination of petrophysical properties of rock, namely porosity and permeability (Wibowo & Permadi, 2015). Pore geometry is expressed as \(\sqrt{(k/\phi)}\) while the pore structure is expressed as \(k/\phi^3\). The same quality of the rock will have a similar relationship between pore geometry and pore structure. Pore geometry and pore structure are influenced by the complexity of pores arrangement. Some pore attributes that affect pore complexities are shape-factor \(F_s\) and tortuosity \(\tau\). Figure 2 shows the relationship of Kozeny constant \((F_s\tau)\) and pore geometry. The pore geometry \(\sqrt{(k/\phi)}\) describes the pore size.

In contrast, Shape Factor describes the shape of the pore. The low shape factor describes a simple pore shape close to a perfect round pore. Tortuosity \(\tau\) on the sample scale is defined as the actual length of the fluid flow path compared to sample length. So that a low value of \(\tau\) describes a simple grain packing, this is because a simple grain arrangement will allow a straight fluid flow like flow in a capillary tube. Figure 2 shows that the simpler pore shape and grain packing will form a large pore size, which is indicated by a large value of \(\sqrt{(k/\phi)}\). Regarding the pore structure, simple pore shapes and low tortuosity show a simple pore arrangement or pore structure indicated by a large value of \(k/\phi^3\) (Figure 3). Thus, the Kozeny constant \(\tau\) represented by a combination of pore attributes \(F_s\tau\) relates to the pore size and complexity of pore arrangement.
Figure 2. Relationship of $F_{s\tau}$ with pore geometry for data set 1 (A) and data set 2 (B)

Figure 3. Relationship of $F_{s\tau}$ with pore structure for data set 1 (A) and data set 2 (B)

Discussion

Some studies show that P-wave velocity is strongly influenced by the complexity of the pore. This complexity is partly due to the presence of pore-filling clay (Han et al., 1986; Marion et al., 1989). Prakoso et al. (2016) show that in 1 rock type, large pore geometry and simple pore structure will tend to increase P-wave velocity (Prakoso et al., 2016). Large pore geometry and simple pore structure are characterized by low shape factors and low tortuosity (Figure 4). Rocks with these characteristics will have good quality, especially the ability to flow the fluid.

Regarding the P-wave velocity, Figure 4 shows large pore geometries. This figure also indicates simple pore structures characterized by low shape factor and low tortuosity tend to decrease the P-wave velocity. Although it looks scatter, there is a clear trend between $F_{s\tau}$ and P-wave velocity. Figure 4 shows a strong relationship between $F_{s\tau}$ and the P-wave velocity indicated by the high correlation coefficient $R^2$ above 0.6.
Figure 4. Relationship of $F_{st}$ with dry P-wave velocity for data set 1 (A), and data set 2 (B)

Increasing the P-wave velocity can be explained that the higher $F_{st}$ value indicates, the more complex of pore geometry and pore structure. Increasing the complexity of pore geometry and pore structure causes rocks to be more rigid so that the P-wave velocity increases (Figure 5). Increasing pore rigidity is shown by increasing bulk modulus $K$.

From the relationship of $F_{st}$ with P-wave velocity (Figure 4), empirical equations are obtained with good correlation coefficients. The simple empirical equation of the relationship $F_{st}$ with P-wave velocity is used to estimate P-wave velocity. By using this method, the accurate P-wave velocity estimation results are obtained. The accuracy of estimation results is indicated by the coefficient correlation value of $R^2$ 0.7245 for data set 1 and 0.6381 for data set 2 (Figure 6). Thus, this can be used as an alternative method for estimating P-wave velocity while considering the complexity of the pore.

Figure 5. Relationship of $F_{st}$ with dry bulk modulus $K$ for data set 1 (A) and data set 2 (B)
Figure 6. Relationship between $V_p$ calculated with $V_p$ measured for data set 1 (A), and data set 2 (B)

Conclusion

The study using two data sets from the North West Java basin and the Kutai basin shows that the P-wave velocity is strongly influenced by both pore attributes shape factor and tortuosity. The increase in pore complexity, as indicated by the increase in the $F_{s\tau}$ value, causes an increase in pore rigidity and subsequently increases the P-wave velocity. P-wave velocity can be easily and predicted based on the $F_{s\tau}$ parameter. The accurate P-wave velocity prediction results are indicated by $R^2$ values above 0.6.

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