Effect of screening of the Coulomb interaction on the conductivity in the quantum Hall regime

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Abstract

We study variable range hopping in the quantum Hall effect regime in the presence of a metallic gate parallel to the plane of a two-dimensional electron gas. Screening of the Coulomb interaction by the gate causes the partial “filling” of the Coulomb gap in the density of localized states. At low enough temperatures this leads to a substantial enhancement and a new temperature behavior of the hopping conductivity. As a result, the diagonal conductivity peaks become much wider. The power law dependence of the width of the peaks on the temperature changes: the corresponding exponent turns out to be twice as small as that for gateless structures. The width dependences on the current in non-ohmic regime and on the frequency for the absorption of the electromagnetic waves experience a similar modification. The experimental observation of the crossovers predicted may demonstrate the important role of the Coulomb interaction in the integer quantum Hall regime.

PACS numbers: 73.40.Hm
I. INTRODUCTION

The integer quantum Hall effect (IQHE) in a disordered two-dimensional electron gas manifests itself more clearly as the temperature \( T \) is lowered. The steps connecting adjacent plateaus in the dependence of the Hall conductance \( \sigma_{xy} \) on the filling factor \( \nu \) narrow with decreasing \( T \) and so do the peaks in the longitudinal conductance \( \sigma_{xx} \). In a number of experiments [1–6] a remarkable result has been obtained: the width \( \Delta \nu \) of the peaks shrinks as \( T \to 0 \) according to a power law \( \Delta \nu \propto T^\kappa \) in measurements having been performed down to temperatures as low as a few tens of millikelvins. The exponent \( \kappa \approx 0.4 \) was found in Refs. [1,2] to be universal: neither the Landau level index nor the electron mobility is relevant at low temperatures. However, some deviations from \( \kappa \approx 0.4 \) were reported in Refs. [3,4].

A new explanation of the scaling dependence \( \Delta \nu \propto T^\kappa \) has been recently suggested by Polyakov and Shklovskii (PS) [7]. (Similar arguments with respect to insulator-superconductor transition were put forward earlier by Fisher [8].) They noticed the fact that in the low-temperature limit the only possible mechanism of transport away from the peaks where the states are localized is variable-range hopping (VRH). In this regime, due to the existence of the Coulomb gap, the temperature dependence of \( \sigma_{xx} \) should have a form [9–11]

\[
\sigma_{xx} = \sigma_0 e^{-\left(\frac{T_0}{T}\right)^{1/2}},
\]

where

\[
T_0(\nu) = C \frac{1}{k_B \varepsilon \xi(\nu)},
\]

\( \xi(\nu) \) is the localization radius of the states on the Fermi level for a given \( \nu \), \( \varepsilon \) is the dielectric constant, \( k_B \) is the Boltzmann constant, and \( C \approx 6.2 \) in two dimensions [12].

Numerical simulations [13–17] show that the localization length \( \xi(\nu) \) diverges as \( \nu \) approaches a half-integer \( \nu_0 \):

\[
\xi(\nu) = \xi_0 |\nu - \nu_0|^{-\gamma},
\]

where \( \xi_0 \) is of the order of the magnetic length. For the case of spinless electron which corresponds to the spin split Landau level the value of the exponent \( \gamma \) was shown to be given by [13–17]

\[
\gamma = \gamma_0 \approx 2.3.
\]

Recently, the same value of \( \gamma \) has been directly measured by studying how \( \Delta \nu \) scales with the sample size in the low-\( T \) limit [4].

According to Eqs. (1), (3), the value of \( T_0 \) tends to zero as \( \nu \to \nu_0 \). Hence, at a given temperature, there should exist a characteristic value of \( \nu \) at which \( T_0(\nu) \sim T \) and the exponential factor in Eq. (1) becomes of the order of unity. PS assumed that the width of the peak is determined by the difference between this value and \( \nu_0 \). Making use of Eq. (3) immediately yields a power-law dependence of \( \Delta \nu \) on \( T \):

\[
\Delta \nu \propto T^\kappa
\]
\[
\Delta \nu = \left( \frac{T}{T_1} \right)^\kappa
\]
with \( \kappa = 1/\gamma \) and
\[
T_1 = A \frac{1}{k_B} \frac{e^2}{\varepsilon \xi_0},
\]
where \( A \) is a numerical coefficient. For \( \gamma \approx 2.3 \) one obtains \( \kappa \approx 0.4 \) which is in a good agreement with the experimental data.

The approach by PS can be related to the conventional theory of the width of the conductivity peaks which was proposed by Aoki and Ando \[13\] and by Pruisken \[18\]. This theory is based upon the concept of a phase-coherence length \( L_\phi \) which sets a limit for the localization. If \( L_\phi \) is less than the localization length \( \xi \) of the state at the Fermi level, the state is considered to be delocalized and it contributes to the conductivity. Then, the width of the conductivity peak should be determined from condition \( L_\phi \approx \xi \). Using this concept, one arrives at Eqs. (5), (6) if the phase-coherence length is given by
\[
L_\phi \approx \frac{e^2}{\varepsilon k_B T}.
\]
Earlier for the case of the Coulomb interaction the same expression was suggested by Fisher, Girvin and Grinstein \[19\] in the framework of dynamic scaling theory. Additional arguments for Eq. (7) have been given recently by Lee, Wang and Kivelson \[20\].

The discussion above shows an important role of the long range Coulomb correlations even in low mobility samples. These correlations create the Coulomb gap at the density of states suppressing the hopping conductivity between the peaks and causing the substantial modification of the conductivity peaks shapes.

The large spatial scale of the correlations allows one to examine the role of the Coulomb interaction experimentally by making use of the intentionally introduced screening.

In this paper we concentrate on one of the possibilities of such a screening, namely on study of gated structures, i.e. structures with a metallic layer being placed parallel to the plane of the 2D electron gas at a distance \( d \). First measurements of the IQHE on the GaAs gated structures have been already reported \[21–24\].

In Sect. \[ IV \] we start from the discussion of how the screening of the interaction affects the shape of the density of states in the classical limit (i.e. for the localization length of the states smaller than the mean distance between them). The main effect of the screening is the appearance of a non-vanishing value of the density of states at the Fermi level. We use the results of Mogilyanskii and Raikh \[25\] who studied this case quantitatively. We will argue that the results remain valid even in the IQHE regime where the localization length \( \xi \) can be much greater than the distance between the states which is of the order of \( \lambda \).

In Sect. \[ V \] we use these results to derive the temperature dependence of the conductivity in the VRH regime. This dependence appears to have a crossover from the Eq. (1) to Mott’s law
\[
\sigma_{xx} = \sigma_M e^{-\left(\frac{T}{T_M}\right)^{1/3}},
\]
where \( T_M \) is inversely proportional to the density of states at the Fermi level. The density of states, however, is strongly diminished by the Coulomb interaction at distances smaller than
Making use of Eq. (8), we extend PS approach to show that the temperature dependence of the width of the conductivity peaks has a crossover with decreasing temperature from Eqs. (5), (6) to

\[ \Delta \nu = \left( \frac{T}{T_2} \right)^\kappa, \]  

where

\[ \kappa = 1/2\gamma, \quad T_2 = B \frac{1}{k_B \varepsilon \xi_0^d}. \]  

The numerical coefficient \( B \) in the Eq. (10) is approximately equal to 140. Crossover happens at temperature

\[ T_c \approx 0.25 \frac{e^2}{k_B \varepsilon d} \]  

if \( d \gg \xi_0 \). In the opposite case \( d \ll \xi_0 \) the width of the peak is given by Eq. (9) in the whole range \( \Delta \nu < 1/2 \). The function \( \Delta \nu(T) \) at different values of \( d \) is shown schematically in Fig. 1. Observation of such a crossover in scaling dependences would provide evidence for the role of the Coulomb interaction.

PS applied their approach also to get dependences of the peak width upon the frequency \( \omega \) and current \( J \). These dependences were shown to have the power law form \( \Delta \nu \propto J^\mu \) and \( \Delta \nu \propto \omega^\eta \). The same phenomena for the gated structures are addressed in Sections IV and V. We show that the exponent \( \mu \) experiences the crossover between \( \mu = 1/2\gamma \) and \( \mu = 1/3\gamma \) with decreasing current. With decreasing frequency, crossover happens between \( \eta = 1/\gamma \) and \( \eta = 1/2\gamma \).

In Sec. VI we discuss the existing experimental data for GaAs gated structures and Si MOSFETs.

**II. EFFECT OF THE GATE ON THE DENSITY OF STATES IN THE COULOMB GAP**

The variable range hopping is known to depend strongly on the shape of the density states at the energies close to the Fermi level. If the Coulomb interaction at large distances is not screened by the gate, the density of states \( g_C \) is of the form \[ g_C(E) = \frac{2\varepsilon^2}{\pi e^4 |E|}, \]  

with the energy \( E \) being counted from the Fermi level.

It is the long range nature of the Coulomb interaction that makes the density of states vanish at the Fermi level. Therefore, it is natural to expect that the screening of the interaction at large distances leads to the finite value of the density of states at the Fermi level \[ g_F = \frac{2\varepsilon^2}{\pi e^4 |E_f|}. \]  

Indeed, the dependence (12) is determined by the interaction at distances of the order of \( r_E = \frac{e^2}{\varepsilon |E|} \). If the potential is screened at the distances greater than \( r_s \) (i.e. the energy of the interaction decays faster than \( 1/r_s^2 \) at \( r > r_s \)), the interaction between two
electrons of energies $|E| < E_s \equiv \frac{e^2}{\epsilon r_s}$ can be neglected. But this interaction is the only reason for the density of states to be suppressed at the Fermi level. Thus, the energy dependence of the density of states saturates at the energy of the order of $E_s$, so that $g(0) \simeq g_C(E_s)$ with $g_C(E)$ given by Eq. (12).

In the case of gated structures, the Coulomb potential is screened by the gate located at the distance $d$ from the plane of 2D electron gas so that the energy of the electron-electron interaction has a form

$$U(r) = \frac{e^2}{\epsilon} \left( \frac{1}{r} - \frac{1}{\sqrt{r^2 + 4d^2}} \right).$$

(13)

In this case distance $d$ plays the role of the screening radius $r_s$, $E_s \simeq \frac{e^2}{\epsilon d}$ and the density of states at the Fermi level is given by

$$g(0) = \alpha \frac{\epsilon}{e^2 d}.$$  

(14)

where $\alpha$ is a numerical coefficient which is to be determined from more rigorous calculations. Such calculations have been done numerically by Mogilyanskii and Raikh [23]. Using self-consistent approach developed in Ref. [27], they obtained the anomalously small value of the coefficient $\alpha \approx 0.1$. Furthermore, they suggested the following interpolation formula fitting the results of numerical simulations of the density of states for the interaction potential (13):

$$g(E) = \frac{\epsilon^2}{e^2} \frac{2}{\pi} \frac{|E|}{|E| + \alpha \frac{e^2}{\epsilon d}}.$$  

(15)

Eqs. (12), (13) have been derived for the classical limit with localization length of the states $\xi$ being much smaller than the mean distance between them. It is not the case for the filling factor of the Landau level close to a half integer when according to Eq. (3) the localization length at the Fermi level diverges whereas the mean distance between electrons is of the order of the magnetic length $\lambda$. It is easy to realize that the energy of the interaction of two localized states is given by the Coulomb law only if the distance between them exceeds the localization length $\xi$. At smaller distances, the interaction energy saturates at the value of the order of $E_\xi = \frac{e^2}{\epsilon \xi}$. Such a form of the interaction energy would lead to a step-like increase of the density of states at energies larger than $E_\xi$. In other words, a “hard” gap of width of the order of $E_\xi$ would exist at such energies rather than the “soft” gap given by Eq. (12).

However, we believe that, if dielectric constant $\epsilon$ does not grow with spatial scale [4], the “soft” Coulomb gap survives at these energies, too. We would like to mention two papers which support this point of view. Yang and MacDonald [28] performed self consistent Hartree-Fock calculations of the system of interacting electrons in a strong magnetic field in the presence of disorder. They demonstrated the existence of the Coulomb gap even for the Fermi level close to the center of first Landau level. Recently, some theoretical arguments for the Coulomb gap in under these conditions were provided in Ref. [29] where we have argued that at the energies $|E| > E_\xi$ the one-electron wave functions of the Hartree-Fock approximation shrink because of the electron-electron interaction so that their localization length $\zeta(E)$ is determined by $\zeta(E) \geq \frac{e^2}{\epsilon |E|}$ rather than by the one-electron value $\xi$. States
closer to the Fermi level \((|E| < E_\xi)\) still have one-electron localization length \(\xi\). Such a picture self-consistently leads to the existence of the Coulomb gap. Indeed, if Coulomb gap exists, the average distance between states in the band of width \(E\) around the Fermi level is of the order of \(\frac{e^2}{\varepsilon|E|}\) which is not smaller than the localization length of the states \(\zeta(E)\). Thus, one can use the Coulomb interaction energy and arrive back to Eq. (12), which makes the theory self-consistent. This picture allows us to make use of the Eqs. (12), (15) even though the one electron \(\xi\) is large and to treat the Coulomb interaction effects classically.

Below we repeat our arguments \[29\] for the phenomenon of the additional localization of Hartree-Fock functions. We showed that the system in which all the one electron states have the one electron localization length \(\xi\) is unstable with respect to the following transformation. Let us construct wave-packets of the size \(L = \frac{e^2}{\varepsilon|E|}\) from the one-electron states. The necessary work is of the order of \((g_0L^2)^{-1}\) per packet, where \(g_0\) is the density of states for non-interacting electrons. On the other hand, due to the electron-electron interaction, the system will gain from crystal-like Coulomb correlations of the wave packets an energy of the order of \(\frac{e^2}{\varepsilon L}\) per packet. This gain is larger than \((g_0L^2)^{-1}\) if \(|E| < \frac{g_0e^4}{\varepsilon^2}\). It means that the Eq. (12) is valid in the whole range \(|E| < \frac{g_0e^4}{\varepsilon^2}\) and that the localization length for \(|E| > \frac{e^2}{\xi}\) is about \(\frac{e^2}{\varepsilon|E|}\).

It is important to emphasize once more that these arguments essentially rely on assumption \[7\] that in a strong magnetic field dielectric constant \(\varepsilon\) does not diverge with \(\xi\). This situation differs from the case of zero magnetic field in which \(\varepsilon\) does diverge \[30\].

### III. Temperature Dependence of the Width of the Peaks of the Diagonal Conductivity

The modification of the density of states in the gated structure results in a qualitative change of the temperature dependence in VRH. In the limit of low temperatures the activation energy for a typical hop contributing to the conductivity is much less than \(\alpha\frac{e^2}{\varepsilon d}\) and the length of such a hop is much larger than \(d\). It means that the Coulomb interaction is not relevant in this regime for the hopping itself, though it affects the value of the density of states (see Sec. II). Therefore, in this temperature range, the hopping conductivity obeys Mott’s formula (8) with

\[ T_M = \frac{\beta}{k_B g(0)\xi^2}, \]

where \(g(0)\) is given by Eq. (14) and the numerical coefficient \(\beta \simeq 14 \[10\].

In the whole range of temperature VRH is described by

\[ \sigma_{xx} = \sigma_0 \exp \left( -\left\{ \frac{T_0 f(T/T_*)}{T} \right\}^{1/2} \right). \]

Here, \(T_0\) is given by Eq. (2), the characteristic temperature \(T_*\) is defined as

\[ T_* = \frac{e^2\xi}{k_B\varepsilon d^2}, \]

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and $f(x)$ is a dimensionless function. This function can be obtained in the framework of the concept of invariance of the number of connections per site in the percolation problem [10]. The result is shown in Fig. 2a. The asymptotic behavior of $f(x)$ that can be obtained immediately from Eqs. (1), (8), (17) and has a form:

$$f(x) = \begin{cases} 1, & \text{if } x \gg 1, \\ \frac{\beta^{2/3} x^{1/3}}{C \alpha^{2/3}} \approx \sqrt{78}x, & \text{if } x \ll 1, \end{cases}$$ (19)

the coefficients $\alpha$ and $C$ being defined in Eqs. (14) and (2) respectively. Crossover between the asymptotes (19) which are also shown in Fig. 2a happens near $x \approx 0.013$, i.e. around $T = \tilde{T} \approx 0.013T_\ast$. By studying the crossover experimentally one can determine the localization length $\xi$ as a function of $\nu$.

Let us now turn to derivation of the temperature dependence of the width of the diagonal conductivity peaks. In the spirit of PS approach, this dependence can be obtained by equating the term in the exponent in Eq. (8) or in Eq. (17) to the unity. Making use of Eq. (16), we obtain for the lowest temperatures

$$\xi^2(\Delta \nu) = \frac{140 e^2 d}{\varepsilon k_B T},$$ (20)

which with help of Eq. (3) yields Eq. (9). In the whole range of temperatures the temperature dependence of the width of the peak can be written as:

$$\Delta \nu = \left\{ \frac{\xi_0}{d} F(T/T_d) \right\}^{1/\gamma}$$ (21)

with $T_d$ given by

$$T_d = \frac{e^2}{k_B \varepsilon d}.$$ (22)

Dimensionless function $F(x)$ obtained as the solution of the equation

$$Cf(xF(x)) = \frac{x}{F(x)}$$ (23)

is depicted in Fig. 2b. Making use of Eq. (19) allows us to derive the asymptotic behavior of the function $F(x)$

$$F(x) = \begin{cases} x/C \approx x/6.2, & \text{if } x \gg 1; \\ \sqrt{\alpha x/\beta} \approx \sqrt{x/140}, & \text{if } x \ll 1, \end{cases}$$ (24)

which corresponds to Eqs. (3), (6). The crossover between two asymptotics happens at $x \approx 0.25$.

Expression (21) is meaningful only if $\Delta \nu \ll 1$ and scaling formula (3) is valid. Thus, the crossover is observable only if $d \gtrsim 0.05 \xi_0$, what is satisfied in most of realistic situations. Otherwise, the whole dependence is described by Eq. (9).
The crossover can be interpreted in the framework of the dynamical scaling approach of Ref. [13]. According to this approach, the characteristic time $\hbar / T$ scales as $\xi^z$. At low temperatures the relevant spatial scales are large and all Coulomb interaction is screened by the gate. Obtained for this case Eq. (20) corresponds to $z = 2$ in agreement with the result of Ref. [13,31]. With temperature increasing the spatial scale decreases and becomes less than the distance to the gate $d$. At these scales long-range nature of the Coulomb interaction becomes relevant and results in the dynamical exponent $z = 1$.

IV. EFFECT OF GATE ON CURRENT INDUCED BROADENING OF THE CONDUCTIVITY PEAKS

It was found in Refs. [4,32] that the width $\Delta \nu$ of the $\sigma_{xx}$ peaks grows with increasing current $J$, i.e. with the increase of the Hall electric field $E_H$. PS showed [7] that the dependence $\Delta \nu(E_H)$ can be understood in terms of the theory of hopping in a strong electric field [33]. This theory is based on the fact that there exists a quasi-Fermi level inclined by the electric field $E$. Zero-temperature hopping with phonon emission becomes then possible and, even though there are no absorption processes, the local Fermi distribution with an effective temperature $\sim eE\xi$ is formed [33]. On this account, the exponent of the current-voltage characteristics at $T = 0$ may be estimated from that of the ohmic conductivity by replacing $T \rightarrow eE\xi$. If the ohmic transport obeys the law [8], the zero-temperature conductivity should behave with increasing electric field as

$$\sigma_{xx} \propto e^{-(E_M/E_H)^{1/3}}, \quad E_M \approx \frac{kBT_M}{e\xi}.$$  

Similarly to the case of ohmic conductivity the width of the $\sigma_{xx}$ peak is found from the equation $E_M(\xi) \sim E_H$. Solving this equation for $\xi$ we get $\xi \sim \left(\frac{\beta \sigma_{xy}(\nu_0)E_H}{\nu_0}\right)^{1/3}$ ($\beta$ is a numerical coefficient from Eq. (16)), which with help of Eqs. (15) and (3) yields

$$\Delta \nu = \left(\frac{E_H}{E_1}\right)^{\mu} = \left(\frac{J}{J_1}\right)^{\mu},$$

where

$$\mu = \frac{1}{3\gamma}, \quad \xi_1 = B \frac{\beta ed}{\alpha \varepsilon \xi_0}, \quad J_1 = \sigma_{xy}(\nu_0)\xi_1, \quad J = \sigma_{xy}(\nu_0)E_H,$$

$B$ being a numerical coefficient of the order of unity. These results should be compared with the corresponding results for the gateless structures [7]:

$$\mu = \frac{1}{2\gamma}, \quad \xi_1 = B_1 \frac{C\varepsilon}{\xi_0}, \quad J_1 \sim \sigma_{xy}(\nu_0)E_{H1},$$

$B_1$ being a coefficient of the order of unity. We see that the screening of the Coulomb interaction by gate substantially affects the character of the current broadening and changes the exponent $\mu$.

Similar to the case of the temperature broadening, crossover between two exponents $\mu = 1/3\gamma$ and $\mu = 1/2\gamma$ in Eq. (20) takes place with increasing current. Comparing Eqs. (27) and (28), we obtain that this crossover happens at
\[ J_c \simeq 0.01\sigma_{xy}(\nu_0) \frac{e^2}{\varepsilon d^2}. \]  

(29)

V. EFFECT OF GATE ON BROADENING OF THE AC-CONDUCTIVITY PEAKS WITH FREQUENCY

In this section we deal with the ohmic zero-temperature conductivity in the quantum Hall regime at a finite frequency \( \omega \). For the gateless structures broadening of narrow \( \sigma_{xx} \)-peaks has been observed at a few tens millikelvins as the frequency changed in the range \( \sim 0.2 - 15 \) GHz \[34\]. This phenomenon has been also interpreted in terms of hopping conductivity \[7\]. In contrast to the dc case, the hopping conductivity at a frequency smaller than the mean energy level spacing \( \Delta \xi \) in a square of the size \( \xi \), is determined by sparsely distributed pairs of the localized states, the typical separation between two sites of a pair being much shorter than that between pairs. The main contribution to the conductivity is provided by resonant phononless transitions of the electrons from one site of a pair to another if \( \hbar \omega \gg k_B T \). As it is shown below, one can neglect the energy dependence of the density of states if \( d \ll \xi \). In this case the derivation of the diagonal conductivity can be done in a fashion similar to the calculations for the three dimensional case \[11\]. It gives

\[ \sigma_{xx}(\omega) = \frac{\pi^2 e^2}{4 \hbar} \hbar \omega (\hbar \omega + U(r_\omega)) g^2(0) \xi r_\omega^3, \]  

(30)

where \( r_\omega = \xi \ln(\Delta \xi/\hbar \omega) \) is the arm of the typical pair contributing to the absorption and \( U(r) \) is given by Eq. \( (13) \). This formula is a modification of the expression derived by Mott \[35\]. The value of the conductivity for the case of the interacting electrons exceeds that for the noninteracting electrons by a factor of \( (\hbar \omega + U(r_\omega))/\hbar \omega \). It happens because the interaction enhances the probability for a pair to be occupied by only one electron \[11\]. At low enough frequencies \( U(r_\omega) = \frac{2e^2 d^2}{\varepsilon r_\omega^2} \gg \hbar \omega \) and, therefore, the conductivity grows linearly with frequency. At larger frequencies the interaction becomes irrelevant and the conductivity depends quadratically on the frequency. Thus, one can find the width of the conductivity peaks by equating \( \sigma_{xx}(\omega) \) to \( e^2/\hbar \) or

\[ \hbar \omega \simeq \Delta \xi \]  

(31)

where \( \Delta \xi = (g(0)\xi^2)^{-1} \). This condition yields

\[ \Delta \nu = \left( \frac{\omega}{\omega_0} \right)^\eta, \]  

(32)

where

\[ \eta = 1/2\gamma = \kappa, \quad \hbar \omega_0 = \frac{A_1 e^2 d}{\alpha \varepsilon \xi^2} \]  

(33)

\( A_1 \) being a numerical coefficient of the order of unity. Now we can establish the condition for neglecting the energy dependence of the density of states. The width of the peak was determined from the condition that the energy \( \hbar \omega \) is of the order of \( \hbar \omega_0 \simeq \Delta \xi \simeq \frac{e^2 d}{\alpha \xi^2} \). It is
larger than the characteristic scale of the energy dependence of the density of states, \( \alpha \frac{e^2}{\varepsilon_d} \) if inequality \( d \ll \alpha \xi \) is valid. Under this condition the energy of the interaction \( U(\xi) \) is also smaller than \( \Delta \xi \).

If \( d \gg \alpha \xi \), the width of the peak is given [4] by Eq. (32)

\[
\eta = 1/\gamma = \kappa, \quad \hbar \omega_0 = A_2 \frac{e^2}{\varepsilon \xi_0},
\]

(34)

where \( A_2 \) is a numerical coefficient.

We see again the crossover in the exponent of the frequency dependence of peak width at frequency \( \omega_c \approx \frac{1}{\hbar \epsilon_{sd}} \).

Comparing the results of this section with those of Sec. [11] we notice that \( \hbar \omega / k_B \) always plays the role of the “effective” temperature in agreement with dynamic scaling approach.

VI. EXPERIMENTS WITH GATED SAMPLES

In this section we briefly discuss the current situation with the experiments on gated structures. The first group of the experiments we would like to mention is the measurements on GaAs heterostructures. Naturally, it is most convenient to study the modification of the width of the peak with screening in the case of spin split peaks which are usually quite narrow. Unfortunately, the gated structures in this regime have not been studied yet. All the experiments appeared up to now [21–24] deal with low-mobility samples where peaks corresponding to the opposite spin directions are not split. These works are devoted to the investigation of the insulator - quantum Hall conductor - insulator transition in 2D electron gas. According to Khmelnitskii [34] and Laughlin [37] this transition is related to the evolution of energy of the isolated extended states with magnetic field. With increasing magnetic field the energy of the extended state first drifts down and crosses the Fermi level, changes the direction of drift, and moves up crossing the Fermi level for the second time. In the vicinity of both critical points the localization length \( \xi \) diverges presumably according to the scaling law (3) and so two distinct peaks of the conductivity appear. Conductivity away from the peaks was shown to be due to the VRH [21–24]. Therefore, our theory of the peak width can be applied for this situation, too.

In Refs. [21–24] the metallic gate was used to decrease the electron concentration in order to increase the role of disorder. Therefore, we can use the expression (9) with \( \kappa = 1/2 \gamma \) to describe the width of the peak in the low temperature limit. The value of the exponent \( \gamma \) for the spin degenerate case was conjectured [4] to be given by \( \gamma = 2 \gamma_0 \) (\( \gamma_0 \approx 2.3 \)). It leads to \( \kappa = 1/4 \gamma_0 \approx 0.11 \) in experimental situation under discussion. At a particular value of gate voltage two critical points merge, i.e. the level of the extended state only touches the Fermi level at some value of magnetic field \( H_c \), not intersecting it. We conjecture that at this point \( \gamma = 4 \gamma_0 \) [38] which leads to the value of the temperature exponent \( \kappa = 1/8 \gamma_0 \approx 0.05 \). Observation of such small values of the exponent \( \kappa \) presumably requires measurements at temperatures lower than those of Refs. [21–24]. Therefore, we do not attempt to compare their data with our predictions.

The other type of gated structures is the Si MOSFETs where the gate is typically closer to the plane of the 2D electron gas. However, the values of the exponent \( \kappa \) obtained in
Ref. [5,6,39] are not consistent with each other being distributed in a broad range 0.2 – 0.9. Therefore, the comparison of the theory to these experiments seems to be premature.

**VII. CONCLUSION**

In this paper we studied the effects of the screening of the Coulomb interaction on the dependence of the conductivity peak width on temperature, current and frequency. The screening results in substantial broadening of the peaks and, moreover, in change of the corresponding exponents. The temperature and frequency exponents $\kappa$ and $\eta$ (see Eqs. (5), (32)) turn out to be equal to $1/2\gamma$ rather than $1/\gamma$ for the non-screened Coulomb interaction ($\gamma$ is the localization length exponent). We expect that the observation of such a modification of the scaling dependences will prove that the Coulomb interaction plays a crucial role for the conductivity in the minima and for the width of the peaks in the IQHE.

Another important prediction of this paper is the crossover in the temperature dependence of the conductivity in the VRH regime. With decreasing temperature the crossover between the laws $\ln \sigma \propto T^{-1/3}$ and $\ln \sigma \propto T^{-1/2}$ is predicted to take place at $T \simeq 0.013 \frac{e^{2} \xi}{\pi d^{2}}$. The observation of such a crossover could become a new tool to study the behavior of the localization length $\xi(\nu)$.

**ACKNOWLEDGMENTS**

We are grateful to A. J. Dahm, S. Girvin, S. Koch and especially to D. G. Polyakov for useful discussions and reading of the manuscript and to A. J. Dahm, R. J. F. Hughes, M. E. Raikh and R. Reintzsch for providing us with their results prior to the publication. This work was supported by the National Science Foundation under Grant No. DMR-9020587.
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FIGURES

FIG. 1. The schematic dependence of the width $\Delta \nu$ of the conductivity peak on the temperature for different distances $d$ from the plane of 2D electron gas to the gate. At $d \ll \xi_0$ all the dependence is described by Eq. (3) whereas at $d \to \infty$ it is described by Eq. (5). At intermediate values of $d$ crossover between these two dependences takes place with increasing temperature.

FIG. 2. Dimensionless functions $f(x)$ (a) and $F(x)$ (b) from Eqs. (17) and (21) are depicted by solid lines. Their asymptotes given by Eqs. (19) and (24) are shown by dashed lines.