Interstellar Extinction and Elemental Abundances

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Abstract

Elements in the interstellar medium (ISM) exist in the form of gas or dust. The interstellar extinction and elemental abundances provide crucial constraints on the composition, size, and quantity of interstellar dust. Most of the extinction modeling efforts have assumed the total (gas and dust) abundances of the dust-forming elements—known as the “interstellar abundances,” “interstellar reference abundances,” or “cosmic abundances”—to be solar and the gas-phase abundances to be environmentally independent. However, it remains unclear whether the solar abundances are an appropriate representation of the interstellar abundances. Meanwhile, the gas-phase abundances are known to exhibit appreciable variations with local environments. Here we explore the viability of the abundances of B stars, the solar and protosolar abundances, and the protosolar abundances augmented by Galactic chemical enrichment (GCE) as an appropriate representation of the interstellar abundances by quantitatively examining the extinction and abundances of 10 interstellar sight lines for which both the extinction curves and the gas-phase abundances of all the major dust-forming elements (i.e., C, O, Mg, Si and Fe) have been observationally determined. Instead of assuming a specific dust model and then fitting the observed extinction curves, for each sight line we apply the model-independent Kramers–Kronig relation, which relates the wavelength-integrated extinction to the total dust volume, to place a lower limit on the dust depletion. This, together with the observationally derived gas-phase abundances, allows us to rule out the B-star, solar, and protosolar abundances as the interstellar reference standard and support the GCE-augmented protosolar abundances as a viable representation of the interstellar abundances.

Unified Astronomy Thesaurus concepts: Gas-to-dust ratio (638); Interstellar dust extinction (837); Interstellar dust (836); Cosmic abundances (315); Interstellar abundances (832); Solar abundances (1474)

1. Introduction

Elements in the interstellar medium (ISM) exist in the form of gas or dust. The interstellar gas-phase abundances of elements can be measured from their optical and ultraviolet (UV) spectroscopic absorption lines. The elements “missing” from the gas phase are bound up in dust grains, known as “interstellar depletion.” The dust-phase abundance of an element is derived by assuming a “reference abundance” and then subtracting off the gas-phase abundance. The reference abundance (also known as “interstellar abundance,” or “cosmic abundance”) of an element is the total abundance of this element (both in gas and in dust).

The interstellar abundances of heavy, dust-forming elements such as C, O, Mg, Si, and Fe provide insight into the maximum amount of raw materials available for making the dust to account for the observed extinction, i.e., any dust models should not use more elements than what is available in the ISM. This involves two unknown (or not well-determined) sets of abundances for the dust-forming elements: the interstellar reference abundances and the gas-phase abundances. Although it is well recognized that a viable dust model should satisfy the cosmic abundance constraints, what might be the most appropriate set of interstellar reference abundances and what are the true gas-phase abundances of the dust-forming elements have been subjects of much discussion in the past decades. Historically, the interstellar abundances are commonly assumed to be solar. The gas-phase abundances of C and O are often assumed to be invariable with respect to the local interstellar conditions (see, e.g., Cardelli et al. 1996; Meyer et al. 1998), while Mg, Si, and Fe appear to be fully depleted from the gas phase (i.e., their gas-phase abundances are negligible).

In the late 1990s, it was argued that, because of their young ages, the interstellar abundances might be better represented by those of B stars and young F and G stars, which are “subsolar” (i.e., $\sim 60\%$–$70\%$ of solar; Snow & Witt 1995, 1996; Sofia & Meyer 2001). However, if the interstellar abundances are indeed “subsolar,” the ISM seems to lack enough raw material to make the dust account for the observed interstellar extinction (Li 2005). Meanwhile, the published solar abundances have undergone major changes over the years (see Table 1). The most recently determined solar abundances (Asplund et al. 2009, hereafter A09) are significantly reduced from their earlier values (e.g., Anders & Grevesse 1989). It is interesting to note that, using the non-LTE (NLTE) techniques, Przybilla et al. (2008) and Nieva & Przybilla (2012) derived the photospheric abundances of heavy elements for unevolved early B-type stars. They found that the photospheric abundances of those B stars are in close agreement with the A09 solar abundances (see Table 1).

Lodders (2003) argued that the currently observed solar photospheric abundances (relative to H) must be lower than those of the proto-Sun because helium and other heavy elements have settled toward the solar interior since the time of its formation $\sim4.55$ Gyr ago. Lodders (2003) further suggested that the protosolar abundances derived from the photospheric abundances by considering settling effects are more representative of the solar system abundances. With the
Notes.  

- Przybilla et al. (2008).
- Anders & Grevesse (1989).
- Asplund et al. (2009).
- Lodders (2003).
- Protosolar abundances augmented by GCE (Asplund et al. 2009; Chiappini et al. 2003).

settling effects taken into account, the A09 solar abundances are roughly consistent with the proto-Sun abundances of Lodders (2003). On the other hand, as the Galaxy evolves, heavy elements are expected to be enriched (Chiappini et al. 2003). Draine (2015) suggested that the protosolar abundances augmented by Galactic chemical enrichment (GCE) over the past 4.55 Gyr might be the best estimate for the interstellar abundances in the solar neighborhood.

Regarding the gas-phase abundances of the dust-forming elements C, O, Mg, Si, and Fe, numerous observational studies carried out in the past decade do not appear to favor a constant gas-phase C/H abundance of ~ 140 ppm (Cardelli et al. 1996) and a constant gas-phase O/H abundance of ~ 320 ppm (Meyer et al. 1998), which were commonly adopted by dust models. Also, contrary to what is commonly assumed by dust models, there seem to be nontrivial amounts of gas-phase Si, Mg, and Fe (10% or more in many sight lines; Jensen et al. 2010).

Cardelli et al. (1996) determined the gas-phase C abundance to be \([C/H] \approx 140 \pm 20\) ppm from the weak interstellar line of C II at 3253 Å obtained with the Goddard High Resolution Spectrograph (GHRS) on board the Hubble Space Telescope (HST) for six sight lines that exhibit a wide range of extinction variation. They found that \([C/H] \approx 319 \pm 14\) ppm and found no statistically significant variations among the sight lines and no evidence of density-dependent O depletion. In contrast, using the Space Telescope Imaging Spectrograph (STIS) on board HST, Carle et al. (2001) performed high-resolution observations of the O I 1356 absorption of 11 translucent clouds and found an appreciably lower \([O/H] \approx 319 \pm 14\) ppm, which suggests a trend toward an enhanced O depletion for denser clouds. Jensen et al. (2005) analyzed the HST/STIS and HST/GHRS spectra of the O I 1356 absorption of 10 sight lines and found a trend of increasing O depletion with \(R_V\) and \(f(H_2)\), the fraction of H in molecular form.

Based on the HST/STIS echelle spectra of the \(\lambda \lambda 1239, 1240\) absorption of Mg II, Cartledge et al. (2006) determined \([Mg/H] \approx 33 \pm 9\) ppm, the gas-phase Mg abundance of 47 sight lines extending up to 6.5 kpc through the Galactic disk that probe a variety of interstellar environments, covering ranges of \(\sim 4\) orders of magnitude in \(f(H_2)\) and over two orders of magnitude in \(\langle n_H\rangle\). They found that the depletion of Mg is density dependent. Miller et al. (2007) determined the gas-phase Si and Fe abundances based on the HST/STIS data of the Si II line at 2335 Å and the Fe I lines at 1142, 2234, 2249, 2260, and 2367 Å for six translucent clouds that sample a variety of extinction characteristics as indicated by their \(R_V\) values, which range from 2.6 to 5.8. They found that \([Si/H] \approx 319 \pm 14\) ppm and \([Fe/H] \approx 37 \pm 9 \) ppm vary from one sight line to another. Similarly, Karis et al. (2016) derived \([Si/H] \approx 319 \pm 14\) ppm for 131 sight lines based on the HST/STIS, HST/GHRS, and International Ultraviolet Explorer (IUE) data and found that \([Si/H] \approx 319 \pm 14\) ppm is correlated with \(f(H_2)\) through the Galactic disk that probe a variety of interstellar environments.

The interstellar extinction and elemental abundances provide crucial constraints on the composition and size of interstellar dust. Practically, most of the extinction modeling efforts have been so far directed to the Galactic average extinction curve, which is obtained by averaging over many clouds of different gas and dust properties (see, e.g., Mathis et al. 1977; Draine & Lee 1984; Désert et al. 1990; Mathis 1996; Li & Greenberg 1997; Weingartner & Draine 2001; Zubko et al. 2004, Jones et al. 2013), despite the fact that the interstellar extinction curves actually exhibit considerable variations from one sight line to another. Also, most of the extinction modeling efforts have assumed a solar abundance and a constant gas-phase abundance for the dust-forming elements, even though, as discussed above, the interstellar reference abundance remains unknown and the gas-phase abundances exhibit appreciable variations with the local interstellar environments. Therefore, by modeling the Galactic average extinction curve, unavoidably, any details concerning the relationship between the dust properties and the physical and chemical conditions of the interstellar environments would have been lost.

In this work we will examine the extinction and elemental abundances of 10 interstellar lines of sight for which both the extinction curve and the gas-phase abundances for all the major dust-forming elements C, O, Mg, Si, and Fe have been observationally determined. Our goal is to investigate which

| Element | B Stars\(^a\) | Solar\(^a\) | Solar\(^b\) | Protosolar\(^d\) | GCE-augmented Protosolar\(^d\) |
|---------|---------------|-------------|-------------|----------------|-------------------------------|
| C       | 209 ± 15      | 363 ± 33    | 269 ± 31    | 288 ± 27        | 339 ± 39                      |
| O       | 575 ± 40      | 851 ± 69    | 490 ± 57    | 575 ± 66        | 589 ± 68                      |
| Mg      | 36.3 ± 4.2    | 38.0 ± 4.4  | 39.8 ± 3.7  | 41.7 ± 1.9      | 47.9 ± 4.4                    |
| Si      | 31.6 ± 1.5    | 35.5 ± 4.1  | 32.4 ± 2.2  | 40.7 ± 1.9      | 42.7 ± 4.0                    |
| Fe      | 27.5 ± 2.5    | 46.8 ± 3.2  | 31.6 ± 2.9  | 34.7 ± 2.4      | 47.9 ± 4.4                    |

\(^a\) AB indicates a Solar-based Solarc Protosolare Lodders are roughly consistent with the proto-Sun abundances of Asplund et al. 2009.  
\(^b\) Anders & Grevesse 329.  
\(^c\) C 209.  
\(^d\) Protosolar abundances augmented by GCE (Asplund et al. 2009; Chiappini et al. 2003).  

\(^4\) \(R_V \equiv A_V/E(B - V)\) is the total-to-selective extinction ratio, where \(A_V\) is the visual extinction, \(E(B - V) \approx A_B - A_V\) is the reddening or color excess between \(A_V\) and \(A_B\), where \(A_B\) is the \(B\)-band extinction.
would be the most appropriate set of interstellar reference abundances, those of B stars (Przybilla et al. 2008), solar (A09), protosolar (Lodders 2003), or protosolar+GCE (Draine 2015). To reduce the number of model parameters, we will not model the extinction curves; instead, we will simply apply the model-independent Kramers–Kronig (KK) relation (Purell 1969) to relate the wavelength-integrated extinction to the total dust volume. Then, for each assumed set of reference abundances, we will explore whether the remaining elements, after subtracting off their gas-phase abundances, are sufficient for accounting for the total dust volume derived from the KK relation. Our approach is essentially model independent since it does not require the knowledge of the detailed optical properties and size distribution of the dust. All we need is a general assumption of the dust composition (e.g., silicate, graphite, oxides, and iron).

This paper is organized as follows. We first compile in Section 2 a “gold” sample consisting of all the (10) sight lines for which the extinction curves from the near-IR to the far-UV and the gas-phase abundances of C, O, Mg, Si, and Fe have been observationally determined. Based on the measured gas-phase abundances of the dust-forming elements and the adopted interstellar reference abundances, we derive in Section 3 for each line of sight the total dust volumes. We assume that interstellar dust is made of (i) graphite and iron-containing amorphous silicate, (ii) graphite and iron-lacking amorphous silicate plus iron oxides, or (iii) graphite and iron-lacking amorphous silicate plus iron. We then apply the KK relation to determine in Section 4 the wavelength-integrated extinction from the total dust volume. In Section 5 we compare the wavelength-integrated extinction derived from the KK relation with that derived from observations and find that only if the interstellar abundances are like the GCE-augmented protosolar abundances would the KK-based wavelength-integrated extinction exceed the observation-based wavelength-integrated extinction, which is implied by the KK relation. These results, as well as the “missing O” problem and the relations between the extinction-to-gas ratios and the interstellar physical and chemical conditions of the lines of sight in this “gold” sample, are also discussed in Section 5. Finally, we summarize our major results in Section 6.

2. The Sample

We first search in the literature for as many interstellar sight lines as possible for which both the extinction curves have been determined from the near-IR to the far-UV and the gas-phase abundances have been measured for all the major dust-forming elements C, O, Mg, Si, and Fe. To this end, we find 10 such lines of sight and tabulate in Table 2 the gas-phase abundances of C, O, Mg, Si, and Fe, as well as the column densities of atomic hydrogen $N_{\text{H}}$, molecular hydrogen $N_{\text{H}_2}$, and the total hydrogen column density $N_{\text{HI}} = N_{\text{H}} + 2N_{\text{H}_2}$. The extinction parameters $c_1$, $c_2$, $c_3$, $c_4$, $x_0$, and $\gamma$, as well as $A_V$, $E(B - V)$, and $R_V$, are taken from Valencic et al. (2004) and tabulated in Table 3. These parameters characterize the UV extinction measured by the International Ultraviolet Explorer (IUE) at 3.3 $\mu$m $< \lambda^{-1} < 8.7$ $\mu$m $^{-1}$ as a sum of three components: a linear background, a Drude profile for the 2175 Å extinction bump, and a far-UV nonlinear rise at $\lambda^{-1} > 5.9$ $\mu$m $^{-1}$:

\[ A_V / A_{\lambda} = c_1 + c_2 x + c_3 D(x, \gamma, x_0) + c_4 F(x), \]

\[ D(x, \gamma, x_0) = \frac{x^2}{(x^2 - x_0^2)^2 + x^2 \gamma^2}, \]

\[ F(x) = \begin{cases} 0, & x < 5.9 \mu m^{-1}, \\ 0.5392(x - 5.9)^2 + 0.05644(x - 5.9)^3, & x \geq 5.9 \mu m^{-1}, \end{cases} \]

where $A_{\lambda}$ is the extinction at wavelength $\lambda$; $x \equiv 1/\lambda$ is the inverse wavelength in $\mu$m$^{-1}$; $c_1'$ and $c_2'$ define the linear background; $c_3'$ defines the strength of the 2175 Å extinction bump, which is approximated by $D(x, \gamma, x_0)$, a Drude function that peaks at $x_0 \approx 4.6 \mu$m$^{-1}$ and has an FWHM of $\gamma$; and $c_4'$ defines the nonlinear far-UV rise.\(^5\) In the following, we will refer to the parameterization described by Equations (1)–(3) as the FM parameterization.

For each line of sight, we aim at integrating the extinction over wavelength from 0 to $\infty$ (i.e., $\int_0^\infty A_{\lambda} d\lambda$). For practical reasons, this is not possible since the extinction curve is observationally determined only over a limited wavelength range, usually from the near-IR to the far-UV. We shall therefore “construct,” for each sight line, the extinction curve from 912 Å to 1 cm and then obtain

\[ A_{\text{int}}^{\text{obs}} \equiv \int_{912 \lambda}^{1 \text{cm}} A_{\lambda} d\lambda. \]

We construct the extinction curves as follows (see Figure 1). For $3.3 \mu$m$^{-1} < \lambda^{-1} < 11$ $\mu$m$^{-1}$, we represent the extinction by Equation (1) with the extinction parameters taken from Table 3.\(^6\) For $1.1 \mu$m$^{-1} < \lambda^{-1} < 3.3$ $\mu$m$^{-1}$, we compute the extinction from the parameterization of Cardelli et al. (1989, hereafter CCM). The CCM parameterization involves only one parameter, that this, $R_V$. As illustrated in Figure 1(a), there is often a discontinuity between the FM parameterization at $\lambda^{-1} > 3.3$ $\mu$m$^{-1}$ and the CCM parameterization at $\lambda^{-1} < 3.3$ $\mu$m$^{-1}$. To comply with the observed extinction-to-gas ratio $A_V/N_{\text{HI}}$, we multiply the FM extinction curve by a factor to smoothly join the CCM curve (see Figure 1(b)). For $0.9 \mu$m $< \lambda < 1$ cm, we approximate the extinction by the model extinction calculated from the standard silicate-graphite-PAH model of Weingartner & Draine (2001; WD01) for $R_V = 3.1$ (see Figure 1(c)). Note that the WD01 model extinction curve exhibits a deep minimum at $\sim 5$–8 $\mu$m, whereas numerous observations made with the Infrared Space Observatory (ISO) and the Spitzer Space Telescope have shown that the mid-IR extinction at $3 \mu$m $< \lambda < 8$ $\mu$m is flat for both diffuse and dense environments (Lutz 1999; Indebetouw et al. 2005; Jiang et al. 2006; Flaherty et al. 2007; Gao et al. 2009; Nishiyama et al. 2009; Wang et al. 2013; Xue et al. 2016; Hensley & Draine 2020a). By introducing a population of very large, micron-sized graphitic

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\(^5\) This parameterization was originally introduced by Fitzpatrick & Massa (1990, hereafter FM90) for the interstellar reddening

\[ E(\lambda - V)/E(B - V) = R_V (A_V/A_V - 1) = c_1 + c_2 x + c_3 D(x, \gamma, x_0) + c_4 F(x). \]

\(^6\) We note that although Equation (1) was originally derived from the IUE data over $3.3 \mu$m$^{-1} < \lambda^{-1} < 8.7$ $\mu$m$^{-1}$, Gordon et al. (2009) found that the general shapes of the extinction curves at $8.4$ $\mu$m$^{-1} < \lambda^{-1} < 11$ $\mu$m$^{-1}$, obtained by the Far Ultraviolet Spectroscopic Explorer (FUSE) are broadly consistent with extrapolations from the IUE extinction curves.
### Table 2
Gas Column Densities and Abundances of the 10 Interstellar Lines of Sight in the “Gold” Sample

| Star      | $N_{\rm H}$ (10$^{21}$ cm$^{-2}$) | $N$(HI) (10$^{21}$ cm$^{-2}$) | $N$(H$_2$) (10$^{21}$ cm$^{-2}$) | [C/H]$_{\text{gas}}$ (ppm) | [O/H]$_{\text{gas}}$ (ppm) | [S/H]$_{\text{gas}}$ (ppm) | [Mg/H]$_{\text{gas}}$ (ppm) | [Si/H]$_{\text{gas}}$ (ppm) | [Fe/H]$_{\text{gas}}$ (ppm) |
|-----------|-----------------------------------|--------------------------------|--------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| HD 24912  | 1.98 ± 0.54 (2)                   | 1.20 ± 0.18 (3)                | 0.34 ± 0.07 (3)                 | 163.1 ± 86.5 (16)           | 326.1 ± 97.5 (3)             | 1.75 ± 0.48 (4)             | 1.99 ± 0.17 (2)             | 1.61 ± 0.44 (2)             | 0.92 ± 0.25 (13)            |
| HD 27778  | 2.51 ± 0.44 (7)                   | 0.89 (5)                       | 0.62 (10)                       | 79.2 ± 27.6 (14)            | 269.2 ± 53.8 (3)             | ...                         | ...                         | ...                         | ...                         |
| HD 37021  | 4.79 ± 1.50 (3)                   | 4.79 ± 1.50 (3)                | ...                             | 90.9 ± 37.9 (14)            | 257.0 ± 82.6 (3)             | ...                         | 1.78 ± 0.57 (7)             | 2.99 ± 1.15 (9)             | 0.58 ± 0.19 (9)             |
| HD 37061  | 5.37 ± 1.20 (3)                   | 5.37 ± 1.20 (3)                | ...                             | 98.1 ± 35.5 (14)            | 316.2 ± 72.2 (3)             | 0.38 ± 17.9 (4)             | 1.12 ± 0.26 (7)             | 0.20 ± 0.17 (4)             | 0.06 ± 0.08 (4)             |
| HD 116852 | 1.02 ± 0.09 (1)                   | 0.91 ± 0.08 (1)                | 0.06 ± 0.001 (1)                | 117.3 ± 103.2 (15)          | 537.5 ± 133.2 (1)            | 3.89 ± 27.4 (4)             | 7.76 ± 0.74 (1)             | 2.57 ± 0.39 (4)             | 0.98 ± 0.35 (4)             |
| HD 122879 | 2.45 ± 0.28 (1)                   | 2.04 ± 0.28 (1)                | 0.21 ± 0.01 (1)                 | 324.3 ± 38.2 (15)           | 448.1 ± 72.8 (1)             | ...                         | 6.44 ± 0.76 (1)             | 4.97 ± 0.61 (2)             | 0.53 ± 0.10 (6)             |
| HD 147888 | 5.37 ± 0.26 (1)                   | 4.79 ± 0.28 (1)                | 0.28 ± 0.03 (1)                 | 105.4 ± 22.6 (14)           | 301.7 ± 50.6 (1)             | 0.49 ± 0.33 (4)             | 1.86 ± 0.33 (1)             | 2.44 ± 0.60 (9)             | 0.15 ± 0.08 (6)             |
| HD 149757 | 1.40 ± 0.03 (2)                   | 0.51 ± 0.02 (3)                | 0.45 ± 0.06 (3)                 | 100.9 ± 48.6 (11)           | 307.1 ± 29.1 (8)             | 2.37 ± 4.14 (4)             | 1.86 ± 0.16 (2)             | 1.50 ± 0.04 (2)             | 0.32 ± 0.09 (4)             |
| HD 185418 | 2.63 ± 0.21 (1)                   | 1.55 ± 0.14 (1)                | 0.53 ± 0.07 (1)                 | 167.7 ± 22.8 (15)           | 380.2 ± 43.8 (1)             | 0.87 ± 0.08 (12)            | 4.07 ± 0.42 (1)             | 0.06 ± 0.01 (12)            | 0.32 ± 0.09 (12)            |
| HD 207198 | 3.16 ± 0.40 (1)                   | 1.91 ± 0.31 (1)                | 0.62 ± 0.07 (1)                 | 102.1 ± 25.1 (14)           | 445.9 ± 59.9 (1)             | 1.20 ± 20.2 (4)             | 3.64 ± 0.43 (1)             | 2.19 ± 1.07 (9)             | 0.43 ± 0.08 (9)             |

Notes. (1) Jenkins (2019); (2) Gnaciński & Krogulec (2006); (3) Cartledge et al. (2004); (4) van Steenberg & Shull (1988); (5) Welty & Crowther (2010); (6) Jensen & Snow (2007a); (7) Jensen & Snow (2007b); (8) Knauth et al. (2006); (9) Miller et al. (2007); (10) Sheffer et al. (2007); (11) Sofia et al. (1994); (12) Sonnenbucker et al. (2003); (13) Jenkins et al. (1986); (14) Sofia et al. (2011); (15) Parvathi et al. (2012); (16) Sofia et al. (2004).
### Table 3

Extinction Parameters of the 10 Interstellar Lines of Sight in the Gold Sample

| Star           | $A_U$ (mag) | $A_V$ (mag) | $A_J$ (mag) | $A_H$ (mag) | $A_K$ (mag) | $E(B-V)^*$ | $R_V$ | $c_1^*$ | $c_2^*$ | $c_3^*$ | $c_4^*$ | $x_0$ (μm$^{-1}$) | $\gamma$ (μm$^{-1}$) |
|----------------|-------------|-------------|-------------|-------------|-------------|-------------|------|--------|--------|--------|--------|------------------|---------------------|
| HD 24912       | ...         | ...         | 1.00 ± 0.21 | ...         | ...         | 0.35 ± 0.04 | 2.86 ± 0.51 | 1.187 ± 0.728 | 0.270 ± 0.054 | 0.943 ± 0.219 | 0.050 ± 0.024 | 4.541 ± 0.106 | 0.846 ± 0.028 |
| HD 27778       | 1.58        | 1.25        | 0.91 ± 0.14 | 0.19        | 0.11        | 0.35 ± 0.04 | 2.59 ± 0.24 | 1.421 ± 0.252 | 0.232 ± 0.038 | 0.878 ± 0.180 | 0.386 ± 0.062 | 4.603 ± 0.012 | 0.974 ± 0.032 |
| HD 37021       | ...         | ...         | 2.80 ± 0.17 | ...         | ...         | 0.48 ± 0.02 | 5.84 ± 0.26 | 1.063 ± 1.047 | 0.020 ± 0.008 | 0.225 ± 0.043 | 0.007 ± 0.007 | 4.584 ± 0.047 | 1.081 ± 0.036 |
| HD 37061       | 3.33        | 2.93        | 2.40 ± 0.21 | 0.72        | 0.45        | 0.56 ± 0.04 | 4.29 ± 0.21 | 1.544 ± 0.145 | 0.000 ± 0.100 | 0.310 ± 0.042 | 0.050 ± 0.012 | 4.574 ± 0.014 | 0.901 ± 0.029 |
| HD 116832      | ...         | ...         | 0.51 ± 0.12 | ...         | ...         | 0.21 ± 0.04 | 2.42 ± 0.37 | 0.518 ± 0.249 | 0.376 ± 0.103 | 0.633 ± 0.173 | 0.010 ± 0.015 | 4.548 ± 0.041 | 0.782 ± 0.069 |
| HD 122879      | 1.69        | 1.45        | 1.13 ± 0.20 | ...         | ...         | 0.36 ± 0.05 | 3.15 ± 0.30 | 1.321 ± 0.299 | 0.233 ± 0.040 | 1.243 ± 0.230 | 0.190 ± 0.039 | 4.581 ± 0.004 | 0.831 ± 0.021 |
| HD 147888      | 2.85        | 2.48        | 1.99 ± 0.18 | 0.60        | 0.35        | 0.22        | 0.51 ± 0.04 | 3.89 ± 0.20  | 1.471 ± 0.267 | 0.037 ± 0.012 | 0.665 ± 0.100 | 0.087 ± 0.022 | 4.587 ± 0.013 | 0.879 ± 0.029 |
| HD 149757      | 1.26        | 1.09        | 0.82 ± 0.13 | 0.27        | 0.14        | 0.1         | 0.32 ± 0.04 | 2.55 ± 0.24  | 1.002 ± 0.100 | 0.286 ± 0.037 | 1.872 ± 0.313 | 0.215 ± 0.052 | 4.552 ± 0.010 | 1.186 ± 0.042 |
| HD 185418      | 2.07        | 1.78        | 1.27 ± 0.14 | 0.21        | 0.11        | 0.07        | 0.50 ± 0.04 | 2.54 ± 0.20  | 1.817 ± 0.265 | 0.100 ± 0.018 | 1.156 ± 0.170 | 0.158 ± 0.029 | 4.604 ± 0.005 | 0.819 ± 0.024 |
| HD 207198      | 2.43        | 1.96        | 1.50 ± 0.20 | ...         | ...         | 0.54 ± 0.08 | 2.77 ± 0.35 | 0.811 ± 0.259 | 0.344 ± 0.050 | 0.976 ± 0.169 | 0.277 ± 0.045 | 4.596 ± 0.006 | 0.883 ± 0.024 |

Notes.

* Data taken from Valencic et al. (2004).
1 The $U, B, J, H, K$ broadband photometric extinction data ($A_U, A_B, A_J, A_H$ and $A_K$) are taken from Fitzpatrick & Massa (2007) for those lines of sight marked by “1” and from Gordon et al. (2009) for those marked by “2.”
grains, Wang et al. (2015a, hereafter WLJ15) closely reproduced the observed flat mid-IR extinction. Therefore, for $0.9 \mu m < \lambda < 1$ cm, we will also approximate the extinction by the WLJ15 model extinction (see Figure 1(c)). As a result, for each sight line we derive two extinction curves (which we refer to as “WD01” and “WLJ15”; see Figures 2–4) and integrate the extinction (per hydrogen column) over 912 Å and 1 cm to obtain $A_{\text{int}}^{\text{obs}}(\text{WD})/N_H$ and $A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_H$. We have also tried our best to compile for each line of sight the broadband photometric extinction data (see Table 3). Whenever available, they are displayed as black squares superimposed on the synthesized extinction curves. As shown in Figures 2–4, the synthesized extinction curves of all lines of sight closely agree with the observationally determined $U$, $B$, and $V$ extinction. While the WD01 curve of HD 27778 and the WLJ15 curves of HD 37061 and HD 147888 agree with their $J$, $H$, and $K$ extinction data, the WD01 curve of HD 185418 is somewhat higher and the WLJ15 curve of HD 149757 is somewhat lower than their $J$, $H$, and $K$ extinction data.

3. Total Dust Volumes as Constrained by the Elemental Abundances

For an assumed dust composition, for each line of sight we can estimate the total dust volume per H nucleon ($V_{\text{dust}}/H$) from the adopted set of interstellar reference abundances and the observationally determined gas-phase abundances. For the
of carbon dust, although its exact composition remains obscure. Figure 3. Same as Figure 2, but for HD 116852, HD 122879, and HD 147888. Black squares show the U, B, V extinction data for HD 122879 and the U, B, V, J, H, K extinction data for HD 147888.

dust composition, we will first consider a mixture of amorphous silicate and graphite. It is well recognized that amorphous silicate is a ubiquitous component of the universe as revealed by the 9.7 μm Si–O stretching feature and the 18 μm O–Si–O bending feature seen either in absorption or in emission (see Henning 2010). There must also be a population of carbon dust, although its exact composition remains unknown (see Henning & Salama 1998). This is because, as discussed in Section 1, C is partially depleted from the gas phase and silicate alone is not sufficient to account for the observed extinction (see Mishra & Li 2015, 2017). In this work we assume that all the C atoms missing from the gas phase are locked up in graphite grains since presolar graphite grains have been identified in primitive meteorites and the interstellar 2175 Å extinction bump, the strongest interstellar absorption feature, is generally attributed to small graphitic grains. Let [C/H]_{ISM} be the interstellar C abundance (relative to H).

We calculate the volume per H nucleon \( V_{\text{gra}/H} \) occupied by graphite dust from

\[
\frac{V_{\text{gra}}}{H} = ([\text{C/H}]_{\text{ISM}} - [\text{C/H}]_{\text{gas}}) \frac{\mu_{\text{C}} m_{\text{H}}}{\rho_{\text{gra}}},
\]

where \( \mu_{\text{C}} = 12 \) is the atomic weight of C, \( \rho_{\text{gra}} \approx 2.2 \, \text{g/cm}^3 \) is the mass density of graphite, and \( m_{\text{H}} = 1.66 \times 10^{-24} \, \text{g} \) is the mass of a hydrogen atom. For the silicate component, we assume an even mixture of pyroxene \((\text{Mg}_2\text{Fe}_{1-x}\text{Si}_3\text{O}_8)\) and olivine \((\text{Mg}_2\text{Fe}_{2-x}\text{SiO}_4)\) compositions, where \( 0 \leq x \leq 1 \). Therefore, we assign 3.5 O atoms for each Si atom. We calculate the volume per H nucleon \( V_{\text{sil}/H} \) occupied by silicate dust from

\[
\frac{V_{\text{sil}}}{H} = (\text{[Mg/H]}_{\text{ISM}} - [\text{Mg/H}]_{\text{gas}}) \mu_{\text{Mg}} + ([\text{Fe/H}]_{\text{ISM}} - [\text{Fe/H}]_{\text{gas}}) \mu_{\text{Fe}} + ([\text{Si/H}]_{\text{ISM}} - [\text{Si/H}]_{\text{gas}}) \mu_{\text{Si}} + 3.5([\text{Si/H}]_{\text{ISM}} - [\text{Si/H}]_{\text{gas}}) \mu_{\text{O}} \frac{m_{\text{H}}}{\rho_{\text{sil}}},
\]

where \([\text{Mg/H}]_{\text{ISM}}, [\text{Fe/H}]_{\text{ISM}}, [\text{Si/H}]_{\text{ISM}},\) and \([\text{O/H}]_{\text{ISM}}\) are, respectively, the interstellar Mg, Fe, Si, and O abundances (relative to H); \( \mu_{\text{Mg}}, \mu_{\text{Fe}}, \mu_{\text{Si}}, \) and \( \mu_{\text{O}} \) are, respectively, the atomic weights of Mg, Fe, Si, and O; and \( \rho_{\text{sil}} \approx 3.5 \, \text{g/cm}^3 \) is the mass density of silicate dust.

We consider four sets of interstellar reference abundances, by adopting the abundances of B stars (Przybilla et al. 2008), solar abundances (A09), protosolar abundances (Lodders 2003), and GCE-augmented protosolar abundances (Draine 2015) as the interstellar abundances. For each adopted set of interstellar abundances, we calculate \( V_{\text{gra}/H} \) and \( V_{\text{sil}/H} \) from Equations (7)–(8) for each sight line and tabulate the results in Tables 4–7.

So far, we have assumed that all the Fe atoms missing from the gas phase are depleted in amorphous silicate grains. In the Galactic ISM, typically 90% or more of the Fe is missing from the gas phase (Jenkins 2009), suggesting that Fe is the largest elemental contributor to the interstellar dust mass after O and C and accounts for ~25% of the dust mass in diffuse interstellar regions. However, as yet we know little about the nature of the Fe-containing material. Silicate grains provide a possible reservoir for the Fe in the form of interstellar pyroxene or olivine analogs. Nevertheless, iron abundances and depletions in the ISM often diverge from the pattern shown by Si and Mg, suggesting that Fe is not tied to the same grains as Si, and therefore most silicate grains are likely Mg based. Also, the shape and strength of the interstellar 9.7 μm silicate absorption feature suggest that the silicate material is Mg-rich rather than Fe-rich (Poteet et al. 2015), and hence a substantial fraction (~70%) of the interstellar Fe might be in other forms such as iron oxides, iron sulfides, or metallic iron (see Draine & Hensley 2013; Dwike 2016;...
Hensley & Draine 2017). Therefore, we will also consider the ISM to consist of graphite, Fe-lacking silicates (i.e., forsterite Mg$_2$SiO$_4$ and enstatite MgSiO$_3$), and three types of iron oxides (i.e., wüstite FeO, hematite Fe$_2$O$_3$, and magnetite Fe$_3$O$_4$). We assume that the Fe atoms missing from the gas phase are evenly tied up in FeO, Fe$_2$O$_3$, and Fe$_3$O$_4$. In this case, we calculate the volumes of silicate dust and iron oxides from

\[
\frac{V_{\text{sil}}}{H} = \frac{1}{\rho_{\text{sil}}} \left\{ \left[ (Mg/H)_{\text{ISM}} - (Mg/H)_{\text{gas}} \right] \mu_{\text{Mg}} + \left[ (Si/H)_{\text{ISM}} - (Si/H)_{\text{gas}} \right] \mu_{\text{Si}} + 3.5 \left[ (Si/H)_{\text{ISM}} - (Si/H)_{\text{gas}} \right] \mu_{\text{O}} \right\},
\]

(9)

\[
\frac{V_{\text{FeO}}}{H} = \frac{1}{3} \left\{ \left[ (Fe/H)_{\text{ISM}} - (Fe/H)_{\text{gas}} \right] \mu_{\text{Fe}} + \left[ (Fe/H)_{\text{ISM}} - (Fe/H)_{\text{gas}} \right] \mu_{\text{O}} \right\},
\]

(10)

\[
\frac{V_{\text{Fe}_2\text{O}_3}}{H} = \frac{1}{3} \left\{ \left[ (Fe/H)_{\text{ISM}} - (Fe/H)_{\text{gas}} \right] \mu_{\text{Fe}} + \frac{4}{3} \left[ (Fe/H)_{\text{ISM}} - (Fe/H)_{\text{gas}} \right] \mu_{\text{O}} \right\},
\]

(11)

where $\rho_{\text{sil}} \approx 3.2$ g cm$^{-3}$ is the mass density of Fe-lacking silicate and $\rho_{\text{FeO}} \approx 5.7$ g cm$^{-3}$, $\rho_{\text{Fe}_2\text{O}_3} \approx 5.3$ g cm$^{-3}$, and $\rho_{\text{Fe}_3\text{O}_4} \approx 5.2$ g cm$^{-3}$ are, respectively, the mass densities of FeO, Fe$_2$O$_3$, and Fe$_3$O$_4$. In this work, we adopt the mean densities of these materials of interest in this work (i.e., graphite, iron, FeO, Fe$_2$O$_3$, Fe$_3$O$_4$, MgFeSiO$_4$, Mg$_2$SiO$_4$, and MgSiO$_3$).

To calculate the $F$ factors, we take the dust to be spheroids with semiaxes $a$, $b$, $b$ (prolate if $a/b > 1$, oblate if $a/b < 1$, and then we related it to the static dielectric constant $\varepsilon_0$ of the grain material (Purcell 1969). We will calculate the $\varepsilon_0$ values for all the dust materials of interest in this work (i.e., graphite, iron, FeO, Fe$_2$O$_3$, Fe$_3$O$_4$, MgFeSiO$_4$, Mg$_2$SiO$_4$, and MgSiO$_3$).

4. Total Wavelength-integrated Extinction as Constrained by the Total Dust Volume

Let $A_{\text{int}} = \int_0^{\infty} A_t \, d\lambda$ be the extinction integrated over all wavelengths. If, in the ISM, there are $N$ different types of dust species and $V_i/H$ is the volume (per H nucleon) of the $i$th dust type, the KK relation relates the wavelength-integrated extinction to the total volume (per H nucleon) occupied by dust through

\[
A_{\text{int}}/N_H = \int_0^{\infty} \frac{A_t}{N_H} \, d\lambda
= 1.086 \times 3\pi^2 \sum_{j=1}^{N} F_j (V_j/H),
\]

(14)

where the dimensionless factor $F_j$ is the orientationally averaged polarizability of the $j$th dust type relative to the polarizability of an equal-volume sphere, depending only on the grain shape and the static (zero-frequency) dielectric constant $\varepsilon_0$ of the grain material. In Table 8 we will tabulate the $\varepsilon_0$ values for all the dust materials of interest in this work (i.e., graphite, iron, FeO, Fe$_2$O$_3$, Fe$_3$O$_4$, MgFeSiO$_4$, Mg$_2$SiO$_4$, and MgSiO$_3$).

Finally, we also consider the case of locking up all the Fe atoms missing from the gas phase in iron grains. In this case, the silicate dust volume is the same as Equation (9), and the volume of iron dust is

\[
\frac{V_{\text{Fe}}}{H} = \frac{1}{\rho_{\text{Fe}}} \left\{ \left[ (Fe/H)_{\text{ISM}} - (Fe/H)_{\text{gas}} \right] \mu_{\text{Fe}} \right\},
\]

(13)

where $\rho_{\text{Fe}}$ is the mass density of iron in the ISM. In this case, we calculate the dust-to-gas ratio of iron, which is related to the static dielectric constant $\varepsilon_0$ of the grain material. In Table 8 we will tabulate the $\varepsilon_0$ values for all the dust materials of interest in this work (i.e., graphite, iron, FeO, Fe$_2$O$_3$, Fe$_3$O$_4$, MgFeSiO$_4$, Mg$_2$SiO$_4$, and MgSiO$_3$).

By making use of the $\varepsilon_0$ values shown in Table 8, we calculate the $F$ factors as a function of grain shape (i.e., elongation $a/b$) for all the dust species considered here (see Figure 5). It is apparent that for both dielectric materials (e.g., MgFeSiO$_4$, Mg$_2$SiO$_4$, MgSiO$_3$, FeO, Fe$_2$O$_3$) and conducting materials (e.g., graphite, iron, Fe$_3$O$_4$), the $F$ factors of moderately elongated or flattened grains do not deviate much from unity. In this work, for each dust type we will adopt the mean $F$ value averaged over that calculated for $a/b = 3$ prolate and
### Table 4

Wavelength-Integrated Extinction Obtained from the Observed Extinction Curves [$A_{\text{int}}^{\text{obs}}(\text{WD})$ and $A_{\text{int}}^{\text{obs}}(\text{WLJ})$] and from the Dust Volumes Based on the KK Relation [$A_{\text{int}}^{\text{KK}}$]

| Star             | $A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}}$ | $A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}}$ | Graphite + Fe-containing Silicate | Graphite + Fe-lacking Silicate + FeO$_3$ | Graphite + Fe-lacking Silicate + FeO$_2$ | Graphite + Fe-lacking Silicate + FeO$_3$ |
|------------------|------------------------------------------------------|--------------------------------------------------------|------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
|                  |                                                      |                                                        | $V_{\text{graph}}/H$                | $V_{\text{sil}}/H$                      | $V_{\text{K}}/N_{\text{H}}$            | $V_{\text{graph}}/H$                      | $V_{\text{sil}}/H$                      | $V_{\text{K}}/N_{\text{H}}$            | $V_{\text{graph}}/H$                | $V_{\text{sil}}/H$                      | $V_{\text{K}}/N_{\text{H}}$            |
| HD 24912         | 1.24E-25                                             | 1.52E-25                                               | 1.51E-28                           | 2.29E-27                                 | 7.41E-26                                 | 1.51E-28                                 | 1.73E-27                           | 1.86E-28                                 | 2.24E-28                           | 2.20E-28                                 | 7.46E-26                                 |
| HD 27778         | 9.07E-26                                             | 1.10E-25                                               | 1.15E-27                           | 2.25E-27                                 | 1.22E-25                                 | 1.15E-27                                 | 1.67E-27                           | 1.91E-28                                 | 2.31E-28                           | 2.26E-28                                 | 1.23E-25                                 |
| HD 30721         | 1.42E-25                                             | 1.82E-25                                               | 1.05E-27                           | 2.25E-27                                 | 1.17E-25                                 | 1.05E-27                                 | 1.68E-27                           | 1.88E-28                                 | 2.27E-28                           | 2.22E-28                                 | 1.18E-25                                 |
| HD 30761         | 1.07E-25                                             | 1.35E-25                                               | 9.86E-28                           | 2.38E-27                                 | 1.17E-25                                 | 9.86E-28                                 | 1.81E-27                           | 1.92E-28                                 | 2.31E-28                           | 2.27E-28                                 | 1.18E-25                                 |
| HD 116852        | 1.24E-25                                             | 1.50E-25                                               | 8.16E-28                           | 2.19E-27                                 | 1.03E-25                                 | 8.16E-28                                 | 1.62E-27                           | 1.85E-28                                 | 2.23E-28                           | 2.19E-28                                 | 1.04E-25                                 |
| HD 122879        | 1.15E-25                                             | 1.42E-25                                               | ...                                | 2.12E-27                                 | 6.16E-26                                 | ...                                    | 1.53E-27                           | 1.89E-28                                 | 2.27E-28                           | 2.23E-28                                 | 6.28E-26                                 |
| HD 147888        | 8.96E-26                                             | 1.13E-25                                               | 9.21E-28                           | 2.28E-27                                 | 1.11E-25                                 | 9.21E-28                                 | 1.70E-27                           | 1.91E-28                                 | 2.30E-28                           | 2.26E-28                                 | 1.12E-25                                 |
| HD 149757        | 1.47E-25                                             | 1.77E-25                                               | 9.62E-28                           | 2.31E-27                                 | 1.14E-25                                 | 9.62E-28                                 | 1.74E-27                           | 1.90E-28                                 | 2.29E-28                           | 2.25E-28                                 | 1.15E-25                                 |
| HD 185418        | 1.16E-25                                             | 1.41E-25                                               | 3.68E-28                           | 2.35E-27                                 | 6.62E-26                                 | 3.68E-28                                 | 1.78E-27                           | 1.90E-28                                 | 2.29E-28                           | 2.24E-28                                 | 8.68E-26                                 |
| HD 230198        | 1.22E-25                                             | 1.48E-25                                               | 9.50E-28                           | 2.26E-27                                 | 1.12E-25                                 | 9.50E-28                                 | 1.69E-27                           | 1.89E-28                                 | 2.28E-28                           | 2.24E-28                                 | 1.13E-25                                 |

**Notes.** The dust volumes are derived from the assumption of the abundances of B stars as the interstellar reference abundances and the dust as mixtures of (i) graphite + Fe-containing amorphous silicates, (ii) graphite + Fe-lacking amorphous silicates + iron oxides, and (iii) graphite + Fe-lacking amorphous silicates + iron.

$a$ $A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}}$ (mag cm$^{-3}$ H$^{-1}$) is the observed extinction (per H column) integrated from 912 Å to 1 cm, with the extinction at 0.9 $< \lambda < 1$ cm approximated by the $R_V = 3.1$ model curve of Weingartner & Draine (2001).

$b$ $A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}}$ (mag cm$^{-3}$ H$^{-1}$) is the observed extinction (per H column) integrated from 912 Å to 1 cm, with the extinction at 0.9 $< \lambda < 1$ cm approximated by the $R_V = 3.1$ model curve of Wang et al. (2015a).

$c$ $V_{\text{graph}}/H$ (cm$^3$ H$^{-1}$) is the volume (per H nucleon) of graphite grains.

$d$ $V_{\text{sil}}/H$ (cm$^3$ H$^{-1}$) is the volume (per H nucleon) of Fe-containing amorphous silicate grains.

$e$ $A_{\text{int}}^{\text{KK}}/N_{\text{H}}$ (mag cm$^{-3}$ H$^{-1}$) is the K-band-based wavelength-integrated extinction derived from the dust volumes.

$f$ $V_{\text{K}}/H$ (cm$^3$ H$^{-1}$) is the volume (per H nucleon) of Fe-containing amorphous silicate grains.

$g$ $V_{\text{FeO}}/H$ (cm$^3$ H$^{-1}$) is the volume (per H nucleon) of FeO$_3$ grains.

$h$ $V_{\text{FeO}}/H$ (cm$^3$ H$^{-1}$) is the volume (per H nucleon) of FeO$_2$ grains.

$i$ $V_{\text{FeO}}/H$ (cm$^3$ H$^{-1}$) is the volume (per H nucleon) of iron grains.
| Star      | $A_{\text{WD}}^\text{int}$/N$_{\text{H}}$ | $A_{\text{WL}}^\text{int}$/N$_{\text{H}}$ | Graphite + Fe-containing Silicate | Graphite + Fe-lacking Silicate + Fe$_2$O$_3$ | Graphite + Fe-lacking Silicate + Fe |
|-----------|-------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|           | $V_{\text{RMS}}$/H                  | $V_{\text{sd}}$/H               | $V_{\text{H}}$/H               | $V_{\text{H}2}$/H               | $V_{\text{H}3}$/H               | $A_{\text{WD}}^\text{int}$/N$_{\text{H}}$ | $V_{\text{RMS}}$/H | $V_{\text{sd}}$/H | $V_{\text{H}}$/H               | $V_{\text{H}2}$/H               | $V_{\text{H}3}$/H               | $A_{\text{WD}}^\text{int}$/N$_{\text{H}}$ | $V_{\text{RMS}}$/H | $V_{\text{sd}}$/H | $V_{\text{H}}$/H               | $V_{\text{H}2}$/H               | $V_{\text{H}3}$/H               | $A_{\text{WD}}^\text{int}$/N$_{\text{H}}$ |
| HD 24912  | 1.24E-25                            | 1.52E-25                        | 9.41E-28                       | 2.47E-27                        | 1.18E-25                        | 9.41E-28                       | 1.81E-27                        | 2.14E-28                        | 2.58E-28                        | 2.53E-28                        | 1.19E-25                        | 6.85E-28                        | 1.81E-27                        | 3.66E-28                        | 1.08E-25                        |
| HD 27778  | 9.07E-26                            | 1.10E-25                        | 1.69E-27                       | 2.43E-27                        | 1.53E-25                        | 1.69E-27                       | 1.74E-27                        | 2.20E-28                        | 2.65E-28                        | 2.60E-28                        | 1.55E-25                        | 1.69E-27                        | 1.74E-27                        | 3.75E-28                        | 1.43E-25                        |
| HD 37021  | 1.42E-25                            | 1.82E-25                        | 1.58E-27                       | 2.43E-27                        | 1.48E-25                        | 1.58E-27                       | 1.76E-27                        | 2.17E-28                        | 2.61E-28                        | 2.56E-28                        | 1.49E-25                        | 1.58E-27                        | 1.76E-27                        | 3.70E-28                        | 1.38E-25                        |
| HD 37061  | 1.07E-25                            | 1.35E-25                        | 1.52E-27                       | 2.56E-27                        | 1.49E-25                        | 1.52E-27                       | 1.88E-27                        | 2.20E-28                        | 2.65E-28                        | 2.61E-28                        | 1.50E-25                        | 1.52E-27                        | 1.88E-27                        | 3.76E-28                        | 1.38E-25                        |
| HD 116852 | 1.24E-25                            | 1.50E-25                        | 1.35E-27                       | 2.37E-27                        | 1.35E-25                        | 1.35E-27                       | 1.70E-27                        | 2.14E-28                        | 2.58E-28                        | 2.53E-28                        | 1.36E-25                        | 1.35E-27                        | 1.70E-27                        | 3.65E-28                        | 1.25E-25                        |
| HD 122879 | 1.15E-25                            | 1.42E-25                        | ...                           | 2.30E-27                        | 6.69E-26                        | ...                           | 1.61E-27                        | 2.17E-28                        | 2.62E-28                        | 2.57E-28                        | 8.66E-26                        | ...                           | 1.61E-27                        | 3.70E-28                        | 5.69E-26                        |
| HD 147888 | 8.96E-26                            | 1.13E-25                        | 1.45E-27                       | 2.46E-27                        | 1.43E-25                        | 1.45E-27                       | 1.78E-27                        | 2.20E-28                        | 2.65E-28                        | 2.60E-28                        | 1.44E-25                        | 1.45E-27                        | 1.78E-27                        | 3.75E-28                        | 1.32E-25                        |
| HD 149757 | 1.47E-25                            | 1.77E-25                        | 1.50E-27                       | 2.49E-27                        | 1.46E-25                        | 1.50E-27                       | 1.82E-27                        | 2.19E-28                        | 2.63E-28                        | 2.58E-28                        | 1.47E-25                        | 1.50E-27                        | 1.82E-27                        | 3.73E-28                        | 1.35E-25                        |
| HD 185418 | 1.16E-25                            | 1.41E-25                        | 9.01E-28                       | 2.53E-27                        | 1.18E-25                        | 9.01E-28                       | 1.85E-27                        | 2.19E-28                        | 2.63E-28                        | 2.58E-28                        | 1.19E-25                        | 9.01E-28                       | 1.85E-27                        | 3.73E-28                        | 1.07E-25                        |
| HD 207198 | 1.22E-25                            | 1.48E-25                        | 1.48E-27                       | 2.44E-27                        | 1.44E-25                        | 1.48E-27                       | 1.77E-27                        | 2.18E-28                        | 2.62E-28                        | 2.58E-28                        | 1.45E-25                        | 1.48E-27                        | 1.77E-27                        | 3.72E-28                        | 1.33E-25                        |
Table 6
Same as Table 4, but with the Protosolar Abundances as the Interstellar Reference Abundances

| Star    | $A_{\text{inst}}^{\text{obs}}$(WD)/$N_{\text{H}}$ | $A_{\text{inst}}^{\text{obs}}$(WL)/$N_{\text{H}}$ | $V_{\text{gas}}$/H | $V_{\text{ad}}$/H | $A_{\text{inst}}^{\text{obs}}$/H | $V_{\text{Fe}\alpha}$/H | $V_{\text{Fe}\beta}$/H | $V_{\text{Fe}\gamma}$/H | $A_{\text{inst}}^{\text{obs}}$/H | $V_{\text{gas}}$/H | $V_{\text{ad}}$/H | $V_{\text{Fe}}$/H | $A_{\text{inst}}^{\text{obs}}$/H |
|---------|---------------------------------|-------------------------------|-----------------|-----------------|---------------------|-----------------|----------------|----------------|-------------------|-----------------|----------------|----------------|------------------|
| HD 24912 | 1.24E-25                        | 1.52E-25                      | 1.11E-27        | 2.91E-27        | 1.39E-25            | 2.02E-27        | 2.36E-28        | 2.79E-28        | 1.40E-25          | 2.02E-27        | 2.36E-28        | 2.79E-28        | 1.40E-25         |
| HD 27778 | 9.07E-26                        | 1.10E-25                      | 1.86E-27        | 2.87E-27        | 1.74E-25            | 2.13E-27        | 2.42E-28        | 2.86E-28        | 1.75E-25          | 2.13E-27        | 2.42E-28        | 2.86E-28        | 1.75E-25         |
| HD 37021 | 1.42E-25                        | 1.82E-25                      | 1.75E-27        | 2.86E-27        | 1.69E-25            | 2.14E-27        | 2.38E-28        | 2.82E-28        | 1.70E-25          | 2.14E-27        | 2.38E-28        | 2.82E-28        | 1.70E-25         |
| HD 37061 | 1.07E-25                        | 1.35E-25                      | 1.69E-27        | 3.00E-27        | 1.70E-25            | 2.27E-27        | 2.42E-28        | 2.86E-28        | 1.70E-25          | 2.27E-27        | 2.42E-28        | 2.86E-28        | 1.70E-25         |
| HD 116852| 1.24E-25                        | 1.50E-25                      | 1.52E-27        | 2.80E-27        | 1.66E-25            | 2.08E-27        | 2.36E-28        | 2.79E-28        | 1.57E-25          | 2.08E-27        | 2.36E-28        | 2.79E-28        | 1.57E-25         |
| HD 122879| 1.15E-25                        | 1.42E-25                      | 2.73E-27        | 7.95E-26        | 2.15E-25            | 2.39E-28        | 2.88E-28        | 2.82E-28        | 8.08E-26          | 2.39E-28        | 2.88E-28        | 2.82E-28        | 8.08E-26         |
| HD 147888| 8.96E-26                        | 1.13E-25                      | 1.62E-27        | 2.90E-27        | 1.64E-25            | 2.16E-27        | 2.42E-28        | 2.85E-28        | 1.65E-25          | 2.16E-27        | 2.42E-28        | 2.85E-28        | 1.65E-25         |
| HD 149757| 1.47E-25                        | 1.77E-25                      | 1.66E-27        | 2.93E-27        | 1.67E-25            | 2.20E-27        | 2.40E-28        | 2.84E-28        | 1.67E-25          | 2.20E-27        | 2.40E-28        | 2.84E-28        | 1.67E-25         |
| HD 185418| 1.16E-25                        | 1.41E-25                      | 1.07E-27        | 2.96E-27        | 1.38E-25            | 2.24E-27        | 2.40E-28        | 2.84E-28        | 1.39E-25          | 2.24E-27        | 2.40E-28        | 2.84E-28        | 1.39E-25         |
| HD 207198| 1.22E-25                        | 1.48E-25                      | 1.65E-27        | 2.88E-27        | 1.65E-25            | 2.15E-27        | 2.40E-28        | 2.83E-28        | 1.65E-25          | 2.15E-27        | 2.40E-28        | 2.83E-28        | 1.65E-25         |
Table 7
Same as Table 4, but with the GCE-augmented Protosolar Abundances as the Interstellar Reference Abundances

| Star          | $A_{\text{H}}^{\text{wd}}/N_{\text{H}}$ | $A_{\text{K}}^{\text{wd}}/N_{\text{H}}$ | $A_{\text{H}}^{\text{wil}}/N_{\text{H}}$ | $A_{\text{K}}^{\text{wil}}/N_{\text{H}}$ | Graphite + Fe-containing Silicate | Graphite + Fe-lacking Silicate + Fe$_2$O$_3$ | Graphite + Fe-lacking Silicate + Fe |
|---------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------|----------------------------------------------|----------------------------------------|
| HD 24912      | 1.24E-25                               | 1.52E-25                               | 1.56E-27                               | 3.41E-27                               | 1.76E-25                         | 1.56E-27                                      | 3.28E-27                               | 3.95E-28                               | 3.88E-28                               | 1.78E-25                               | 1.56E-27                                      | 2.36E-27                               | 5.60E-28                               | 1.61E-25                               |
| HD 27778      | 9.07E-26                               | 1.10E-25                               | 2.31E-27                               | 3.37E-27                               | 2.11E-25                         | 2.31E-27                                      | 3.34E-28                               | 4.02E-28                               | 3.95E-28                               | 2.14E-25                               | 2.31E-27                                      | 2.29E-27                               | 5.70E-28                               | 1.96E-25                               |
| HD 37021      | 1.42E-25                               | 1.82E-25                               | 2.21E-27                               | 3.36E-27                               | 2.06E-25                         | 2.21E-27                                      | 3.31E-28                               | 3.98E-28                               | 3.91E-28                               | 2.09E-25                               | 2.21E-27                                      | 2.30E-27                               | 5.64E-28                               | 1.91E-25                               |
| HD 37061      | 1.07E-25                               | 1.35E-25                               | 2.14E-27                               | 3.50E-27                               | 2.06E-25                         | 2.14E-27                                      | 3.34E-28                               | 4.03E-28                               | 3.95E-28                               | 2.09E-25                               | 2.14E-27                                      | 2.43E-27                               | 5.70E-28                               | 1.91E-25                               |
| HD 116852     | 1.24E-25                               | 1.50E-25                               | 1.97E-27                               | 3.30E-27                               | 1.92E-25                         | 1.97E-27                                      | 3.28E-28                               | 3.95E-28                               | 3.88E-28                               | 1.96E-25                               | 1.97E-27                                      | 2.25E-27                               | 5.59E-28                               | 1.78E-25                               |
| HD 122879     | 1.15E-25                               | 1.42E-25                               | 1.31E-28                               | 3.23E-27                               | 1.09E-25                         | 1.31E-28                                      | 3.31E-28                               | 3.99E-28                               | 3.91E-28                               | 1.04E-25                               | 1.31E-28                                      | 2.16E-27                               | 5.65E-28                               | 8.60E-26                               |
| HD 147888     | 8.96E-26                               | 1.13E-25                               | 2.08E-27                               | 3.40E-27                               | 2.00E-25                         | 2.08E-27                                      | 3.34E-28                               | 4.02E-28                               | 3.94E-28                               | 2.03E-25                               | 2.08E-27                                      | 2.33E-27                               | 5.69E-28                               | 1.85E-25                               |
| HD 149757     | 1.47E-25                               | 1.77E-25                               | 2.12E-27                               | 3.43E-27                               | 2.03E-25                         | 2.12E-27                                      | 3.33E-28                               | 4.00E-28                               | 3.93E-28                               | 2.06E-25                               | 2.12E-27                                      | 2.37E-27                               | 5.67E-28                               | 1.88E-25                               |
| HD 185418     | 1.16E-25                               | 1.41E-25                               | 1.52E-27                               | 3.64E-27                               | 1.75E-25                         | 1.52E-27                                      | 3.33E-28                               | 4.00E-28                               | 3.93E-28                               | 1.78E-25                               | 1.52E-27                                      | 2.40E-27                               | 5.67E-28                               | 1.60E-25                               |
| HD 207198     | 1.22E-25                               | 1.48E-25                               | 2.11E-27                               | 3.38E-27                               | 2.01E-25                         | 2.11E-27                                      | 3.32E-28                               | 4.00E-28                               | 3.92E-28                               | 2.04E-25                               | 2.11E-27                                      | 2.32E-27                               | 5.66E-28                               | 1.86E-25                               |
where the dimensionless factor \( F \) is chosen because Greenberg & Li (1996) found that the 9.7 and 18 \( \mu \)m silicate polarization toward the Becklin-Neugebauer (BN) object is best reproduced by \( a/b = 3 \) core-mantle spheroids, while the \( a/b = 1/2 \) oblate shape is selected because Lee & Draine (1985) found that the 3.1 \( \mu \)m ice polarization of the BN object is best explained by \( a/b = 1/2 \) oblates. Also, Hildebrand & Dragovan (1995) found that \( a/b = 1/2 \) bare silicate oblates could fit the 9.7 \( \mu \)m polarization of the BN object.

For each adopted set of interstellar reference abundances, depending on how the Fe atoms missing from the gas phase are divided among amorphous silicates, oxides, and iron grains, we have accordingly calculated in Section 3 the possible total dust volumes for each line of sight (see Tables 4–7). Combining the dust volumes derived in Section 3 and the \( F \) factors derived in this section, we calculate and tabulate in Tables 4–7 for each sight line \( A_{int}^{KK} \), the extinction integrated over all wavelengths, for each of the four interstellar reference abundances (i.e., B star, solar, protosolar, and GCE-augmented protosolar abundances) and each of three different dust mixtures (i.e., graphite + Fe-containing silicate, graphite + Fe-lacking silicate + iron oxides, and graphite + Fe-lacking silicate + Fe).

It is interesting to note that the \( A_{int}^{KK} \) values calculated for different dust mixtures are rather close. Compared with the graphite + Fe-containing silicate mixture, the graphite + Fe-lacking silicate + iron oxide mixture contains more mass (on a per H nucleon basis). But the total dust volume of the latter exceeds that of the former by only several percent (because of the mass density differences between Fe-containing silicates with iron oxides and Fe-lacking silicates). Although the \( F \) factors of Fe-containing silicates are smaller than that of iron oxides, they are larger than that of Fe-lacking silicates. Adding these effects together, the resulting \( A_{int}^{KK} \) values of the graphite + Fe-lacking silicate + iron oxide mixture differ little from that of the graphite + Fe-containing silicate mixture. On the other hand, because of the higher mass density of iron (compared with silicates), the total dust volume of the graphite + Fe-lacking silicate + Fe mixture is smaller than that of the graphite + Fe-containing silicate mixture. The higher \( F \) factor of iron does not sufficiently compensate the smaller dust volume, and therefore the graphite + Fe-lacking silicate + Fe mixture actually has a lower \( A_{int}^{KK} \) value than the graphite + Fe-containing silicate mixture.

The aforementioned results are derived from the KK-based wavelength-integrated extinction of Purcell (1969), who assumed nonmagnetic grains (see Equation (14)). For magnetic grains, incident electromagnetic waves certainly excite the oscillation of magnetic moments and result in the loss of the incident electromagnetic waves. Therefore, the absorption due to magnetic dipole moments from ferromagnetic (Fe) and ferrimagnetic (Fe\(_2\)O\(_3\) and Fe\(_3\)O\(_4\)) grains could be appreciable, and the wavelength integral of extinction becomes

\[
\int_0^\infty A_\lambda / N_H \, d\lambda = 1.086 \times 3\pi^2 \sum_{j=1}^N \left( F_j(a/b, \varepsilon_0) + F_j(a/b, \mu_0) \right) (V_j/H),
\]

where the dimensionless factor \( F(a/b, \varepsilon_0) \), as defined in Equation (15), measures the electric response, while the dimensionless factor \( F(a/b, \mu_0) \) measures the magnetic response (see Draine & Lazarian 1999):

\[
F(a/b, \mu_0) = \frac{(\mu_0 - 1)}{\mu_0} \left[ \frac{1}{(\mu_0 - 1)L_0 + 1} \right] + \frac{2}{(\mu_0 - 1)L_0 + 1},
\]

where \( \mu_0 \) is the static (zero-frequency) magnetic permeability of the grain material (for nonmagnetic grains \( \mu_0 \approx 1 \) and therefore \( F(a/b, \mu_0) \approx 0 \)). For grain materials having \( \varepsilon_0 \approx 3 \) and \( \mu_\circ \approx 3 \), \( F(a/b, \mu_0) \approx F(a/b, \varepsilon_0) \), and we see from Equations (14) and (19) that the wavelength integral of extinction for magnetic grains (e.g., Fe, Fe\(_2\)O\(_3\), and Fe\(_3\)O\(_4\)) would be increased by a factor of \( 1 + F(a/b, \mu_0) / F(a/b, \varepsilon_0) \approx 2 \) more than it would have been had the grains been nonmagnetic. However, as demonstrated in Draine & Lazarian (1999) and Draine & Hensley (2013), the magnetic dipole absorption mostly occurs at
\( \lambda \gtrsim 3 \) mm, and its contribution to the extinction of interest here is not important since we are mostly concerned with the UV, optical, near-IR, and mid-IR interstellar extinction. This justifies the neglect of the magnetic dipole absorption of magnetic grains (i.e., Fe, Fe\(_2\)O\(_3\), and Fe\(_3\)O\(_4\)) in this work.

### 5. Results and Discussion

For a fixed amount of dust materials, \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} \) gives the maximum possible amount of wavelength-integrated extinction per H nucleon (see Equation (14)), while both \( A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}} \) and \( A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}} \) are obtained by integrating the “observed” extinction over a finite wavelength range (see Equation (6)). For an interstellar reference abundance standard to be a viable representation of the “true” interstellar abundances, the amount of dust-forming elements available for making dust has to be sufficient to account for the observed extinction. This implies that \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} \) should exceed \( A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}} \) and \( A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}} \) since the extinction \( A_{\lambda}/N_{\text{H}} \) is always positive and therefore it is always true that \( \int_0^\infty A_{\lambda}/N_{\text{H}} \, d\lambda > \int_{912 \, \text{Å}}^{1 \, \text{cm}} A_{\lambda}/N_{\text{H}} \, d\lambda \). For an adopted set of interstellar reference abundances, if the corresponding \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} \) is smaller than or equal to \( A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}} \) and \( A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}} \), it simply means that the adopted reference abundances are not viable since there would be insufficient amounts of dust-forming elements to make the dust to cause the observed amounts of extinction.

We first consider the abundances of B stars. In Figure 6 we compare the wavelength-integrated “observed” extinction \( A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}} \) and \( A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}} \) with \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} \), the KK-based extinction integrated over all wavelengths obtained by assuming that the interstellar abundances are those of B stars. With \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} \) exceeding \( A_{\text{int}}^{\text{obs}}(\text{WLJ})/N_{\text{H}} \) for only one sight line (over 10 sight lines), it is apparent that the B-star abundances are not a viable representation of the interstellar abundances. Even if we compare \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} \) with \( A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}} \), we still find \( A_{\text{int}}^{\text{KK}}/N_{\text{H}} < A_{\text{int}}^{\text{obs}}(\text{WD})/N_{\text{H}} \) for the majority (7 over 10) of the sight lines. This is true, irrespective of the exact form that the Fe atoms missing from the gas phase take.
We have also considered the solar abundances as the interstellar reference abundances. As illustrated in Figure 7, we reach $A_{\text{int}}^{\text{KK}} / N_{H} > A_{\text{int}}^{\text{obs}} (\text{WLJ}) / N_{H}$ only for a small fraction (3/10) of the sight lines. Even if we assume the interstellar abundances to be those of protosolar, the number of sight lines with $A_{\text{int}}^{\text{KK}} / N_{H} < A_{\text{int}}^{\text{obs}} (\text{WLJ}) / N_{H}$ persists to be substantial. As shown in Figure 8, half of the 10 sight lines still have $A_{\text{int}}^{\text{KK}} / N_{H} < A_{\text{int}}^{\text{obs}} (\text{WLJ}) / N_{H}$. However, this changes when the GCE-augmented protosolar abundances are adopted as the interstellar reference standard. As shown in Figure 9, we achieve $A_{\text{int}}^{\text{KK}} / N_{H} > A_{\text{int}}^{\text{obs}} (\text{WLJ}) / N_{H}$ for all the sight lines except HD 122879. The line of sight toward HD 122879 has an unusually large gas-phase carbon abundance of...
Such a high CH gas abundance is difficult to reconcile with the observed extinction and other C-related interstellar spectral phenomena. Therefore, we argue that the GCE-augmented protosolar abundances are a viable interstellar reference standard.

So far, we have assumed the carbonaceous dust component to be graphite. However, other forms of carbonaceous solid materials (e.g., amorphous carbon) have also been postulated to be a major interstellar dust component (see Henning & Salama 1998). Depending on their H contents, the mass densities of laboratory amorphous carbon materials range from $\sim 1.4$ to $\sim 2.0$ g cm$^{-3}$ (see Jäger et al. 1998; Li & Greenberg 2002). On the other hand, those (H-rich) materials with a low mass density often have no or low DC conductivities (Jäger et al. 1998), and therefore their static dielectric constants $\varepsilon_0$ are much smaller than that of graphite. This implies a smaller $F$ factor than that of graphite for H-rich amorphous carbon. Although a lower mass density leads to a larger dust volume (see Equation (7)), this will be compensated by a smaller $F$ so that the KK-based wavelength-integrated $A_{\text{int}}^{\text{KK}}$ will not be appreciably affected (see Equation (14)). Even if we adopt the $F$ factor of graphite for amorphous carbon and assume a mass density of 1.8 g cm$^{-3}$, the

Figure 7. Same as Figure 6, but with the solar abundances as the interstellar reference abundances.

[C/H]$_{\text{ISM}}$ $\approx$ 324 $\pm$ 38 ppm (Parvathi et al. 2012). Such a high [C/H]$_{\text{ISM}}$ abundance is difficult to reconcile with the observed extinction and other C-related interstellar spectral phenomena. Therefore, we argue that the GCE-augmented protosolar abundances are a viable interstellar reference standard.

10 If the interstellar C/H abundance is like the GCE-augmented protosolar C/H abundance of [C/H]$_{\text{ISM}}$ $\approx$ 339 $\pm$ 39 ppm, there will be only a small amount of C atoms (i.e., [C/H]$_{\text{ISM}}$ $\approx$ [C/H]$_{\text{ISM}}$ $\approx$ 15 ppm) left for making carbon dust, and therefore it is not surprising that it leads to an unusually small $A_{\text{int}}^{\text{KK}}/A_{\text{int}}^{\text{WLJ}}$ ratio. Such a low [C/H]$_{\text{ISM}}$ abundance is troublesome since the ubiquitous and widespread “unidentified” IR emission (UIE) bands at 3.3, 6.2, 6.2, 7.7, 8.6, and 11.3 $\mu$m alone require their carriers to lock up $\sim$40–60 ppm of C/H (see Li & Draine 2001). In addition, other interstellar spectral phenomena, e.g., the so-called extended red emission (ERE; Witt & Vĳh 2004; Witt 2014), the 2175Å extinction bump (Draine 1989), and the 3.4$\mu$m aliphatic C-H stretching absorption band (Pendleton & Allamandola 2002), also require an appreciable amount of C/H to be tied up in their carriers. Unless the line of sight toward HD 122879 is locally substantially enhanced in C, it is difficult to account for the observed extinction, as well as the UIE bands, the 2175Å extinction bump, and the 3.4$\mu$m absorption band, while in the mean time this sight line has such a high [C/H]$_{\text{ISM}}$ abundance.

So far, we have assumed the carbonaceous dust component to be graphite. However, other forms of carbonaceous solid materials (e.g., amorphous carbon) have also been postulated to be a major interstellar dust component (see Henning & Salama 1998). Depending on their H contents, the mass densities of laboratory amorphous carbon materials range from $\sim$1.4 to $\sim$2.0 g cm$^{-3}$ (see Jäger et al. 1998; Li & Greenberg 2002). On the other hand, those (H-rich) materials with a low mass density often have no or low DC conductivities (Jäger et al. 1998), and therefore their static dielectric constants $\varepsilon_0$ are much smaller than that of graphite. This implies a smaller $F$ factor than that of graphite for H-rich amorphous carbon. Although a lower mass density leads to a larger dust volume (see Equation (7)), this will be compensated by a smaller $F$ so that the KK-based wavelength-integrated $A_{\text{int}}^{\text{KK}}$ will not be appreciably affected (see Equation (14)). Even if we adopt the $F$ factor of graphite for amorphous carbon and assume a mass density of 1.8 g cm$^{-3}$, the

11 Jana et al. (2019) found that the mass density of amorphous carbon could range from $\sim$1.4 to $\sim$3.5 g cm$^{-3}$, with a typical value of $\sim$2.25 g cm$^{-3}$, close to that of graphite.
resulting $A_{\text{int}}^{KK}$ would only increase by $\sim 10\%$, and this would not affect our conclusion.

We have so far also assumed that the ISM consists of separate, distinct individual grain populations (i.e., amorphous silicate, graphite, iron oxides, and iron). However, in the literature other dust structural forms have also been proposed, including silicate core-carbon mantle grains (Jones et al. 1990; Li & Greenberg 1997); composite grains consisting of small silicates, amorphous carbon, and vacuum (Mathis 1996); and composite “astrodust” consisting of amorphous silicates, metal oxides, hydrocarbons, and vacuum (Draine & Hensley 2020). Nevertheless, whether the dust takes a core-mantle structure or a mixture of separate, distinct individual grains would not affect the dust volume as long as the amount of dust material is fixed. On the other hand, compared to compact grains of the same amount of material, a larger volume is expected for composite grains. However, composite grains will have smaller $F$ owing to the reduction of their static dielectric constant $\varepsilon_0$ (see Li 2005) and hence are not expected to result in a higher $A_{\text{int}}^{KK}$. Therefore, what exact structural form interstellar dust may take does not affect our conclusion.

As early as the mid-1990s, it has been recognized that if the interstellar abundances are like those of B stars, the amount of C atoms left for dust after subtracting the gas-phase C abundance is insufficient to form the carbonaceous dust species required by dust models (Snow & Witt 1995, 1996). This, known as the “C crisis,” still holds even if one assumes the interstellar C abundance to be solar. For O, there is also a case of “O crisis” or “missing O.” Unlike C, which is insufficient, for O there is as much as $\sim 160$ ppm of O/H unaccounted for in interstellar atoms, molecules, and dust (Jenkins 2009; Whittet et al. 2010a, 2010b; Potte et al. 2015). Wang et al. (2015b) suggested that micron-sized H$_2$O ice grains could accommodate the excess O/H without exhibiting the 3.1 $\mu$m absorption band of H$_2$O ice, and they could be present in the diffuse ISM through rapid exchange of material between dense clouds where they form and diffuse clouds where they are destroyed by photosputtering. Alternatively, H$_2$O ice could be trapped in silicates. Very recently, Potapov et al. (2020) found evidence for the trapping of H$_2$O ice in silicate grains based on an analysis of the Spitzer/IRS spectra in combination with laboratory data. We examine the
depletion of O in the context of the GCE-augmented protosolar abundances as the interstellar reference standard. If we assume that the ISM consists of graphite and Fe-containing silicates (or Fe-lacking silicates plus iron grains), the total amount of O/H that the ISM could accommodate is

\[
\frac{[O/H]_{\text{tot}}}{[O/H]_{\text{gas}}} \approx [O/H]_{\text{gas}} + 3.5 \times ([\text{Si/H}]_{\text{ISM}} - [\text{Si/H}]_{\text{gas}}).
\]

(21)

Similarly, if the ISM consists of graphite and Fe-lacking silicates plus iron oxides, the ISM could accommodate a total amount of

\[
\frac{[O/H]_{\text{tot}}}{[O/H]_{\text{gas}}} \approx [O/H]_{\text{gas}} + 3.5 \times ([\text{Si/H}]_{\text{ISM}} - [\text{Si/H}]_{\text{gas}}) + \frac{23}{18} \times ([\text{Fe/H}]_{\text{ISM}} - [\text{Fe/H}]_{\text{gas}}),
\]

(22)

where we assume equal amounts of FeO, Fe₂O₃, and Fe₃O₄. Taking [Si/H]_{\text{ISM}} and [Fe/H]_{\text{ISM}} to be that of the GCE-augmented protosolar abundances, for each line of sight we calculate [O/H]_{\text{tot}} for different dust mixtures. In Figure 10 we compare [O/H]_{\text{tot}} with the GCE-augmented protosolar O/H abundance. It appears that the majority of the sight lines have no difficulty in accommodating the “missing O,” particularly if the Fe atoms are tied up in oxides. Very recently, Psaradaki et al. (2020) compared the experimental X-ray spectra of the oxygen K edges of various silicate and oxide dust materials with the X-ray spectrum of Cygnus X-2, a bright low-mass X-ray binary, observed by XMM-Newton. They derived a remarkably high gas-phase abundance of [O/H]_{\text{gas}} \approx 610 \pm 60 \text{ ppm} for the line of sight toward Cygnus X-2, although an accurate derivation of the [O/H]_{\text{gas}} abundance relies on an accurate knowledge of the atomic data of the oxygen edge spectral region. Also, they determined the solid-phase O/H abundance to be [O/H]_{\text{dust}} \approx 45 \pm 7 \text{ ppm}, which is smaller than what could be accommodated by silicate dust alone by a factor of \sim 3. Nevertheless, the total O/H abundance falls on the high side of the GCE-augmented protosolar O/H abundance.
It is widely believed that in the ISM dust and gas are well mixed, as evidenced by the tight empirical correlation between reddening $E(B-V)$ and hydrogen column density $N_{\text{HI}}$. Bohlin et al. (1978) derived the hydrogen-to-reddening ratio to be $N_{\text{HI}}/E(B-V) \approx 5.8 \times 10^{21}$ H cm$^{-2}$ mag$^{-1}$ for a sample of 100 stars with $E(B-V)$ up to $\sim0.5$ mag, based on the UV absorption spectra of HI and H$_2$ observed by the Copernicus satellite. With $R_V \approx 3.1$ for the Galactic diffuse ISM, this corresponds to $A_V/N_{\text{HI}} \approx 5.3 \times 10^{-22}$ mag cm$^{-2}$ H$^{-1}$, a ratio long taken to be representative of the ISM in the solar neighborhood. However, appreciably lower extinction-to-hydrogen ratios have been reported (see Hensley & Draine 2020b and references therein), e.g., Lenz et al. (2017) recently derived $A_V/N_{\text{HI}} \approx 3.5 \times 10^{-22}$ mag cm$^{-2}$ H$^{-1}$ for diffuse, low column density regions with $N(\text{HI}) < 4 \times 10^{20}$ H cm$^{-2}$. We have also examined the $A_V/N_{\text{HI}}$ relation for the 10 sight lines of our “gold” sample compiled in this work and determined $A_V/N_{\text{HI}} \approx 4.6 \times 10^{-22}$ mag cm$^{-2}$ H$^{-1}$ (see Figure 11). This extinction-to-hydrogen ratio is intermediate between that of Bohlin et al. (1978) and that of Lenz et al. (2017) and close to that of Zhu et al. (2017), who derived $A_V/N_{\text{HI}} \approx 4.8 \times 10^{-22}$ mag cm$^{-2}$ H$^{-1}$ from X-ray observations of a large sample of Galactic sight lines toward supernova remnants, planetary nebulae, and X-ray binaries.

We have also explored the relationships between $A_V/N_{\text{HI}}$ and the physical and chemical conditions of the interstellar environments, including $A_V$, $R_V$, $N_{\text{HI}}$, $N_{\text{HI}}$, and $f(H_2) = 2N(H_2)/(N(\text{HI}) + 2N(H_2))$, the fraction of molecular hydrogen in the line of sight (see Figure 12). For the 10 lines of sight considered here, $A_V/N_{\text{HI}}$ does not appear to show any appreciable correlations with these parameters. This seems to contradict the conventional belief that, due to grain aggregation (see, e.g., Jura 1980), $A_V/N_{\text{HI}}$ decreases toward denser regions, which are often characterized by larger values of $R_V$, $N_{\text{HI}}$, and $f(H_2)$. Kim & Martin (1996) explored the variation of $A_V/N_{\text{HI}}$ with $R_V$ for several dozen sight lines spanning $2.7 < R_V < 5.6$. Despite a large scatter, they found an increase of $A_V/N_{\text{HI}}$ with $R_V$ for $R_V < 4.4$, and above this value, $A_V/N_{\text{HI}}$ tends to be smaller. With $R_V \approx 2.1$, the high Galactic latitude cloud toward

![Figure 10](figure.png)

**Figure 10.** Comparison of the GCE-augmented protosolar O/H abundance (589 ± 68 ppm; blue horizontal shaded box) with $[O/H]_{\text{tot}}$, the total amounts of O/H that could be accommodated by gas (green vertical boxes) and dust (orange vertical boxes) in each line of sight. The (vertical) error bars are for $[O/H]_{\text{gas}}$, resulting from the uncertainties in $[O/H]_{\text{gas}}$, and the uncertainties in the GCE-augmented protosolar Si/H and Fe/H abundances.

| Candidate      | Mass Density | Static Dielectric Constant | F(a/b; ε₀) |
|----------------|--------------|----------------------------|------------|
|                | ρ (g cm$^{-3}$) | (ε₀) | Prolate (a/ε) | Oblate (b) |
| Dust Material  |              |              | b = 3 | b = 1/2 |
| Forsterite (Mg$_2$SiO$_4$) | 3.2$^a$ | 5.5$^a$ | 0.668 | 0.633 |
| Enstatite (MgSiO$_3$) | 3.2$^b$ | 6.7$^b$ | 0.749 | 0.698 |
| Olivine (Mg$_2$SiO$_4$) | 3.5$^a$ | 10$^a$ | 0.905 | 0.814 |
| Wüstite (FeO) | 5.7$^c$ | 24$^c$ | 1.18 | 0.989 |
| Hematite (Fe$_2$O$_3$) | 5.26$^d$ | 16$^d$ | 1.07 | 0.920 |
| Magnetite (Fe$_3$O$_4$) | 5.18$^e$ | $\infty$ | 1.52 | 1.15 |
| Iron (Fe) | 7.8$^f$ | $\infty$ | 1.52 | 1.15 |
| Graphite (C) | 2.24$^g$ | $\infty$ | 1.52 | 1.15 |

**Notes.**

$^a$ Jäger et al. (2003).

$^b$ Dorschner et al. (1995).

$^c$ Henning & Mutschke (1997).

$^d$ Schreitle et al. (2012).

$^e$ Steyer (1974).

**Table 8** Properties of Dust Materials

Despite a large scatter, they found an increase of $A_V/N_{\text{HI}}$ with $R_V$ for $R_V < 4.4$, and above this value, $A_V/N_{\text{HI}}$ tends to be smaller. With $R_V \approx 2.1$, the high Galactic latitude cloud toward
HD 210121 has $A_V/N_H \approx 4.2 \times 10^{-22}$ mag cm$^2$ H$^{-1}$ (see Li & Greenberg 1998), about 20% lower than that of the canonical value of $A_V/N_H \approx 5.3 \times 10^{-22}$ mag cm$^2$ H$^{-1}$ (Bohlin et al. 1978).

6. Summary

We have compiled a “gold” sample of 10 lines of sight for which the extinction parameters and the gas-phase abundances of C, O, Si, Mg, and Fe have been observationally determined. We have applied the KK relation to this sample to examine the viability of (i) B-star, (ii) solar, (iii) protosolar, and (iv) GCE-augmented solar abundances as the interstellar reference abundances. Except that we assume that the ISM is made of (i) graphite and Fe-containing amorphous silicates, (ii) graphite and Fe-lacking amorphous silicates plus iron oxides, or (iii) graphite and Fe-lacking amorphous silicates plus iron, our approach is model independent in the sense that we do not need to specify the dust size distribution and do not need to reproduce the observed extinction curves. Our principal results are as follows:

1. For each of the (10) lines of sight, each of the (four) assumed sets of interstellar reference abundances, and each of the (three) assumed different dust mixtures, we have calculated the dust volumes and the KK-based wavelength-integrated extinction. We have also calculated the observation-based wavelength-integrated extinction for each sight line. We have found that only the GCE-augmented protosolar abundances could meet the KK criterion that the former must exceed the latter.

2. Although we have assumed the interstellar carbonaceous dust component to be graphite, the exact composition of this component (e.g., amorphous carbon vs. graphite) is not critical. Also, although we have assumed interstellar dust to be a mixture of separate, distinct individual dust species, the exact dust structural form (e.g., core-mantle or composite) does not affect our conclusion.

3. For this sample we have investigated the “missing O” problem (i.e., in the diffuse ISM a substantial fraction of the O/H remains unaccounted for in interstellar atoms, molecules, and dust) and found that for the majority of the lines of sight the ISM does not seem to have difficulty in accommodating the O atoms.

4. For this sample we have derived the extinction-to-hydrogen gas ratio to be $A_V/N_H \approx 4.6 \times 10^{-22}$ mag cm$^2$ H$^{-1}$, $\sim 13\%$.

\[ A_V = 4.34 \times 22 \times N_H + 0.10 \]

\[ A_V = 4.60 \times 22 \times N_H \]

Figure 11. Visual extinction ($A_V$) against the hydrogen column densities ($N_H$) for the “gold” sample of 10 sight lines for which both the extinction parameters and the gas-phase abundances of C, O, Si, Mg, and Fe have been observationally determined. Broken lines fit a linear relationship to the data. The goodness of fit is measured by $\chi^2$/dof, the chi-square per degree of freedom ($\chi^2$/dof $\approx 0.35$ for black dashed line and $\chi^2$/dof $\approx 0.37$ for red dotted–dashed line).

\[ \chi^2/dof \approx 0.35 \text{ for black dashed line and } \chi^2/dof \approx 0.37 \text{ for red dotted–dashed line.} \]

\[ AV/N_H \approx 4.2 \times 10^{-22} \text{ mag cm}^2 \text{ H}^{-1} \text{ (see Li & Greenberg 1998), about 20\% lower than that of the canonical value of } AV/N_H \approx 5.3 \times 10^{-22} \text{ mag cm}^2 \text{ H}^{-1} \text{ (Bohlin et al. 1978).} \]

\[ \text{HD 210121 has } AV/N_H \approx 4.2 \times 10^{-22} \text{ mag cm}^2 \text{ H}^{-1} \text{ (see Li & Greenberg 1998), about 20\% lower than that of the canonical value of } AV/N_H \approx 5.3 \times 10^{-22} \text{ mag cm}^2 \text{ H}^{-1} \text{ (Bohlin et al. 1978).} \]

\[ \text{1. For each of the (10) lines of sight, each of the (four)} \]

\[ \text{assumed sets of interstellar reference abundances, and each of the (three) assumed different dust mixtures, we have calculated the dust volumes and the KK-based}} \]

\[ \text{wavelength-integrated extinction. We have also calculated the observation-based wavelength-integrated extinction for each sight line. We have found that only the}} \]

\[ \text{GCE-augmented protosolar abundances could meet the}} \]

\[ \text{KK criterion that the former must exceed the latter.}} \]

\[ \text{2. Although we have assumed the interstellar carbonaceous}} \]

\[ \text{dust component to be graphite, the exact composition}} \]

\[ \text{of this component (e.g., amorphous carbon vs. graphite)} \]

\[ \text{is not critical. Also, although we have assumed}} \]

\[ \text{interstellar dust to be a mixture of separate, distinct}} \]

\[ \text{individual dust species, the exact dust structural form}} \]

\[ \text{(e.g., core-mantle or composite) does not affect our}} \]

\[ \text{conclusion.}} \]

\[ \text{3. For this sample we have investigated the “missing O”}} \]

\[ \text{problem (i.e., in the diffuse ISM a substantial fraction of}} \]

\[ \text{the O/H remains unaccounted for in interstellar atoms,}} \]

\[ \text{molecules, and dust) and found that for the majority of}} \]

\[ \text{the lines of sight the ISM does not seem to have difficulty}} \]

\[ \text{in accommodating the O atoms.}} \]

\[ \text{4. For this sample we have derived the extinction-to-hydrogen}} \]

\[ \text{gas ratio to be } AV/N_H \approx 4.6 \times 10^{-22} \text{ mag cm}^2 \text{ H}^{-1}, \sim 13\% \]
lower than the canonical value of $A_V/N_H \approx 5.3 \times 10^{-22} \text{ mag cm}^2 \text{ H}^{-1}$. Also, $A_V/N_H$ does not systematically decrease toward denser regions as indicated by larger values of $R_V$, hydrogen volume density ($n_H$), and molecular fraction of hydrogen $f(H_2)$, contrary to the conventional wisdom of more reduced $A_V/N_H$ in denser regions due to grain coagulation.

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