Characteristics of Students’ Proportional Reasoning In Solving Missing Value Problem

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Abstract. The purpose of this study is to describe the process of students’ proportional reasoning in solving missing value problem. Therefore, this study classified as descriptive qualitative research. The subjects of this study were junior high school students of eighth grade in Malang Regency. The data collection begun with students asked to solve the missing value problem. After obtaining the data of student’s work result in solving the missing value problem, the three subjects with different proportional reasoning level selected. These three subjects were interviewed to strengthen or refine their work result. Furthermore, the work and interview results were used to describe the process of students’ proportional reasoning in solving missing value problem. The results showed that the students’ proportional reasoning is at level 2 (Replicative Proportional Reasoning), level 3 (Pre-Multiplicative Proportional Reasoning) and level 4 (Multiplicative Proportional Reasoning). The Replicative Proportional Reasoning has characterized by the use of repeated addition in solving missing value problem. The Pre-Multiplicative Proportional reasoning has characterized by the use of unit value in solving missing value problem. The Multiplicative Proportional Reasoning has characterized by the use of cross multiplication procedure in solving missing value problem.

1. Introduction
Some researchers have been conducted the study on the importance of reasoning in mathematical learning. From the study obtained the level of students’ proportional reasoning based on their mathematical skill [1]; characteristics of students' proportional reasoning [2]. The reasoning is a thinking process in drawing a conclusion in the form of knowledge [3]. The same thing is expressed by [4] which states that reasoning is a logical thinking process to examine and understand an event which ending in drawing a conclusion. Based on the definitions, it can be concluded that reasoning is a logical thinking process to draw a conclusion. The reasoning is part of thinking but thinking and reasoning are often used synonymously [5]. Thinking is a mental activity performed by students in solving problems that can be seen from their behavior which appears in the form of task completion results [6], [7]. According to [5], thinking can be divided into four categories namely recall thinking, basic thinking, critical thinking, and creative thinking. The reasoning is a thinking process that includes creative thinking, critical thinking, and basic thinking. So, if someone is reasoning, then it can be said that someone is thinking, but if someone is thinking, it can’t be said that someone is the reasoning.
From the explanations, can be seen that the level of students' mathematical reasoning is still low. According to [8], mathematical reasoning is reasoning related to mathematical objects. Mathematical reasoning can also be an interpretation as the foundation for constructing mathematical knowledge. From the definitions can be concluded that mathematical reasoning is a logical thinking process to draw conclusions related to mathematical objects. The low level of mathematical reasoning is one of some factors that cause low mathematical literacy of Indonesian students. Therefore, to improve the mathematical literacy, one thing that can do is to train students' mathematical reasoning. Depdiknas states that mathematical material and mathematical reasoning are two whole things, the mathematical material has understood through mathematical reasoning, and mathematical reasoning understood and trained through learning the mathematical material [9].

There are various types of mathematical reasoning in mathematics learning, one of them is the proportional reasoning. Proportional reasoning is a mental activity that able to understand the relation of a quantity change to another quantity through multiplicative relationship [10]. Proportional reasoning can also be interpreted as logical thinking in proportional situations [2]. From these definitions can be concluded that proportional reasoning is reasoning used by students in solving the proportional problem where students understand the multiplicative relationship in it. The comparable situation relates to the concept of ratio and proportion. A ratio is a number that is connecting two quantities or measure in a given situation to a multiplicative relationship, while a proportion is a statement that two ratios are equal [11]. In the formal form, a proportion can write as \( \frac{a}{b} = \frac{c}{d} \). There are many mathematical materials involving proportional reasoning such as statistics, algebra, and social arithmetic. In addition, fractions, percentages, ratios, decimals, scales, algebra, and probability also require proportional reasoning [12]. Because there is much mathematical material involving proportional reasoning, so proportional reasoning is very important to be trained and developed. In reality, the development of proportional reasoning varies in each student [1]. Therefore, this study examines the process of students' proportional reasoning.

The study result has shown that seventh grader with high mathematical skills using multiplicative reasoning, students with average mathematical skills using pre-multiplicative reasoning and students with low mathematical skills using qualitative reasoning [1]. The proportional reasoning levels in the study refer to the five levels of proportional reasoning developed [13]. The study result of [2] has shown that proportional reasoning of the seventh grader is at level 2 and transition level from level 2 to level 3. At the level 2, students can reason quantitatively; students can manipulate the proportional situation by using a number. While at transition level from level 2 to level 3, students can use cross multiplication procedure to solve proportional problems, but students do not understand the differences between within ratio and between ratio. The proportional reasoning levels in the study refer to the four levels of proportional reasoning developed [13].

To determine the students' proportional reasoning level in solving missing value problem, the researcher proposed the draft of proportional reasoning levels whose idea is from the five levels of students' proportional reasoning developed [4], and four levels of proportional reasoning developed [13]. The categorization of proportional reasoning levels draft based on the characteristics of students proportional reasoning in solving missing value problem. Proportional problems are mathematical problems related to proportional situations. According to the study result of [14], there are two types of proportional problems, namely: (1) problems of finding a missing value, and (2) numerical comparison problem. This study uses the first type of the problems that is to find a value that is missing (missing value problem). In missing value problems, students are required to find a quantity, if given three quantities of proportion (i.e., \( a \), \( b \) and \( c \)), such that \( \frac{a}{b} = \frac{c}{d} \). Proportional reasoning occurs when students are faced with a problem related to the proportional situation. The process of students’ proportional reasoning is different because their proportional reasoning ability is different as well [15]. Therefore, the purpose of this study is to describe the process of students’ proportional reasoning in solving missing value problem.
2. Methodology

This research used descriptive qualitative research design. The qualitative data obtained in this study was described to produce a clear and detailed description of the students' proportional reasoning process in solving missing value problem. The subject in this study were junior high school students of eighth grade in Malang Regency. The instrument of this research is missing value problem. The instruments illustrated below.

Salad Sauce Recipe

- You will make a sauce for salad
- Here is a recipe for 100 milliliter (mL) of sauce.

| Ingredient  | Quantity |
|------------|----------|
| Salad Oil  | 60 mL    |
| Vinegar    | 30 mL    |
| Soy Sauce  | 10 mL    |

How many milliliter (mL) salad oil do you need to make 150 mL of this sauce?

Modified from PISA Released Items 2012

Figure 1. Research Instrument

The procedure began with giving the research instrument to the students to be done within 15 minutes. After obtaining the data of student’s work result in solving the missing value problem, the three subjects with different proportional reasoning level selected. These three subjects were interviewed to strengthen or refine their work result. Furthermore, the work and interview results were used to describe the process of students’ proportional reasoning in solving missing value problem. Qualitative data obtained in the study were analyzed using analysis stage developed [16].

3. Result

3.1. The Proportional Reasoning Process of Subject 1 (S1) at Level 2 (Replicative Proportional Reasoning)

The proportional reasoning process of S1 began with reading missing value problem of salad sauce recipe to understand it. After reading the problem over and over again, finally, S1 understood the problem given. In this case, the understanding of S1 indicated by his ability to explain the problem given. S1 explained that to make 100 mL of salad sauce is needed 60 mL of salad oil, 30 mL of vinegar and 10 mL of soy sauce.

After wrote down the information of the problem, S1 said that the given problem asks about the amount of salad oil needed to make 150 mL of salad sauce. But, S1 only write down 150 mL salad sauce?. In this case, S1 was able to understand the problem by digging the information on the given problem. After understanding the problem, then S1 thought of a way that can be used to solve the problem. After found appropriate way to solve the problem, S1 explained that to be able to find the amount of salad oil needed to make 150 mL of salad sauce, the first thing to do is to look for the amount of salad oil needed to make 50 mL of salad sauce, then after getting the results just add up to the amount of salad oil needed to make 100 mL of salad sauce. Based on the plan he has made, S1 started looking for the amount of salad oil needed to make 50 mL of salad sauce. S1 revealed if 100 mL of salad sauce requires 60 mL of salad oil, then to make 50 mL of salad sauce requires a half of 60 mL of salad oil ie 30 mL of salad oil. S1 wrote down his work result of finding the amount of salad oil needed to make 50 mL of salad sauce.

After knowing that salad oil needed to make 50 mL salad sauce is 30 mL, the next process was S1 looked for the amount of salad oil needed to make 150 mL of salad sauce. S1 added up the amount of salad oil needed to make 100 mL of salad sauce with the amount of salad oil needed to make 50 mL of salad sauce. So, the amount of salad oil needed to make 150 mL of salad sauce is 60 mL +
30 mL = 90 mL. Here is S1’s work in finding the amount of salad oil needed to make 150 mL of salad sauce.

Figure 2. Work Result of S1 in Finding the Amount of Salad Oil for 150 mL Salad Sauce

The Translate of Figure 2.
150 mL sauce salad = 90 mL salad oil

After getting the results 90 mL, S1 re-examined the results of his work with re-read the problem is given. Then S1 looked back at the results of his work carefully, after being sure that his job is correct S1 said that the amount of salad oil needed to make 150 mL of salad sauce is 90 mL. Based on the result of his work, that S1 understood the multiplicative relationship (implicitly) in the problem given, so that S1 has shown proportional reasoning ability. Based on the proportional reasoning levels proposed by a researcher, the proportional reasoning of S1 is at level 2 (Replicative Proportional Reasoning), the students at this level show proportional reasoning ability by using repeated addition.

3.2. The Proportional Reasoning Process of Subject 2 (S2) at Level 3 (Pre-multiplicative Proportional Reasoning)
The proportional reasoning process of S2 began with understanding the problem of missing value about salad sauce recipe. To understand the problem given, S2 read the problem over and over again. S2 showed her understanding by explaining the given problems. S2 explained that to make 100 mL of salad sauce needed 60 mL of salad oil, 30 mL of vinegar and 10 mL of soy sauce. After wrote down the information, S2 said that the thing asked in the given problem is the amount of salad oil needed to make 150 mL of salad sauce.

In this case, the S2 was able to understand the problem by digging the information that exists on the problem given. After understanding the problem given, the next process, S2 thought about the way that can be used to solve the problem. After finding a suitable way to solve the problem, S2 began to explain that to be able to find the amount of salad oil needed to make 150 mL of salad sauce, the first thing to do is to find the amount of salad oil needed to make 1 mL of salad sauce. S2 said if 100 mL of salad sauce requires 60 mL of salad oil then to make 1 mL salad sauce needed \( \frac{60}{100} \) mL of salad oil. Next, S2 said to determine the amount of salad oil needed to make 150 mL of salad sauce, multiply 150 with \( \frac{60}{100} \) mL. S2 used unit value.

Based on the plan that has made, S2 starts multiplying 150 with the unit value that is \( \frac{60}{100} \) mL. S2 did multiplication operations by writing \( \frac{150 \times 60}{100} \). In this case, S2 has experienced proportional reasoning in accordance with the theory [10], which states proportional reasoning is a mental activity that able to understand the relation of a quantity change to another quantity through multiplicative relationship. Here is the result of S2’s work when multiplying 150 with unit values.

Figure 3. Work Result of S2 in Performing Multiplication Operation

The Translate of Figure 3.
Answer: \( \frac{150 \times 60}{100} \)
After writing down \( \frac{150 \times 60}{100} \), S2 performed multiplication operation between 150 and 60 then the result was divided by 100. After getting the result 90 mL, S2 re-examined the results of her work with re-read the problem is given. Then S2 looked back at her work carefully, after being sure that her work was right, then S2 explained that the amount of salad oil needed to make 150 mL of salad sauce is 90 mL. Based on the result of her work, it can that S2 understood the multiplicative relationship in the problem given, so it can that S2 had shown proportional reasoning ability. Based on the proportional reasoning levels proposed by the researcher, the proportional reasoning of S2 is at level 3 (Pre-Multiplicative Proportional Reasoning), the students at this level show proportional reasoning ability by using unit value.

### 3.3. The Proportional Reasoning Process of Subject 3 (S3) at Level 4 (Multiplicative Proportional Reasoning)

The proportional reasoning process of S3 began with reading missing value problem about salad sauce recipe. After reading the problem over and over again finally, S3 understood the problem given. In this case, the understanding of S3 that with her ability to explain verbally the problem given. S3 explained that to make 100 mL of salad sauce is needed 60 mL of salad oil, 30 mL of vinegar and 10 mL of soy sauce. S3 also explained that the problem given ask about the amount of salad oil needed to make 150 mL of salad sauce. In this case, S3 was able to understand the problem by digging the information on the problem is given.

After understanding the problem given, the next process was S3 started to think of a way that can be used to solve the problem. After finding a suitable way to solve the problem, S3 started to explain that to find the amount of salad oil needed to make 150 mL of salad sauce, can be used multiplication. The ratio on the left side of the equation is the ratio between the amount of salad oil needed for 150 mL of salad sauce and the amount of salad oil needed for 100 mL of salad sauce ie 60 mL. While the ratio on the right side of the equation is the ratio between the amount of salad sauce that is 150 mL and 100 mL.

Based on the written plan, S3 began to write a proportion \( \frac{m_{\text{salad}}}{60} = \frac{150}{100} \). According to what was on her plan before the ratio on the left side of the equation is the ratio between the amount of salad oil needed for 150 mL of salad sauce and the amount of salad oil needed for 100 mL of salad sauce ie 60 mL. While the ratio on the right side of the equation is the ratio between the amount of salad sauce that is 150 mL and 100 mL. Here is S3’s work in writing proportion.

![Figure 4. Work in Writing Proportion](image)

After writing down the proportion, S3 then applied the cross multiplication. S3 multiplied salad oil with 100 then wrote it on the left side of the equation. Then, she multiplied 150 with 60 written on the right side of the equation. In this case, S3 has experienced proportional reasoning according to theory [10], which states proportional reasoning is a mental activity that able to understand the relation of a quantity change to another quantity through multiplicative relationship.

After applying the cross multiplication, the next process S3 applied the operation to the number on the right side of the equation, ie 150 \( \times \) 60 which the results are 9000. Here is the work result of S3 when applying the multiplication operation to the number on the right side of the equation. After that S3 evaluated \( \text{salad oil} \cdot 100 = 9000 \). S3 used the concept of division to determine the value of salad
oil. S3 divided each side of the equation with 100, so obtained the equation \( \frac{\text{salad oil}}{100} = \frac{9000}{100} \). Here is the work result of S3 in applying the concept of division to each side of the equation.

After getting the results 90, S3 re-examined her work with re-read the problem given. Then S3 looked back at her work carefully, after being sure that her work is right then S3 said that the amount of salad oil needed to make 150 mL of salad sauce is 90 mL. Based on the result of her work, that S3 understood the multiplicative relationship in the problem given, so that S3 has shown proportional reasoning ability. Based on the proportional reasoning levels proposed by the researcher, the proportional reasoning of S3 is at the highest level, i.e., level 4 (Multiplicative Proportional Reasoning), the student at this level show proportional reasoning ability by using cross multiplication.

4. Conclusion
Based on the result and discussion, we can conclude that proportional reasoning of students in eighth grade is at level 2 (Replicative Proportional Reasoning), level 3 (Pre-Multiplicative Proportional Reasoning) and level 4 (Multiplicative Proportional Reasoning). The replicative proportional reasoning characterized by the use of repeated addition in solving missing value problem. The pre-multiplicative proportional reasoning characterized by the use of unit value in solving missing value problem. The multiplicative proportional reasoning characterized by the use of cross multiplication in solving missing value problem.

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