Glueball Spin

D. Singleton *

Dept. of Physics, CSU Fresno, 2345 East San Ramon Ave. M/S 37, Fresno, CA 93740-8031

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Abstract

The spin of a glueball is usually taken as coming from the spin (and possibly the orbital angular momentum) of its constituent gluons. In light of the difficulties in accounting for the spin of the proton from its constituent quarks, the spin of glueballs is reexamined. The starting point is the fundamental QCD field angular momentum operator written in terms of the chromoelectric and chromomagnetic fields. First, we look at the possible restrictions placed on the structure of glueballs from the requirement that the QCD field angular momentum operator should satisfy the standard commutation relationships. This analysis can be compared to the electromagnetic charge/monopole system, where the requirement that the total field angular momentum obey the angular momentum commutation relationships places restrictions (i.e. the Dirac condition) on the system. Second, we look at the expectation value of the field angular momentum operator under some simplifying assumptions.

*E-mail address : dougs@csufresno.edu
I. INTRODUCTION

One of the results of the EMC experiment \[1\] was to show that the spin of the proton did not come solely from the spin of the valence quarks as naive quark models indicated. After the EMC experiment other possible contributing sources to the proton’s spin were considered, such as angular momentum from sea quarks, angular momentum from the orbital motion of valence quarks, and angular momentum from the chromodynamic field.

In light of this “surprise”\[1\] in the make up of the spin of the proton, we want to take a closer look at the spin of pure glue bound states. In the case of glueballs there is currently no firm experimental evidence for their existence. Ref. \[3\] gives the status of various candidates. Thus, presently there is no need to explain anything in regards to the spin of the glueballs, since there is no clear cut discrepancy between experiment and the theoretical models. However, since the simple quark models did not work in accounting for the spin structure of the proton, it is reasonable to ask if a similar problem may exist for the simple theoretical models of the glueball’s spin. While most glueball studies focus on the mass spectrum of glueballs, here we want to examine the glueball’s spin, starting from the fundamental gluonic field angular momentum operator given below in Eq. (1).

The gauge-invariant QCD field angular momentum operator \[4\] which is responsible for the glueball’s spin is

\[
J_{GB} = \int d^3x \left[ x \times (E^a \times B^a) \right]
\]

where \(E^a, B^a\) are the chromoelectric and chromomagnetic fields, and \(a\) is an SU(3) color index. Except for the color indices this form of the pure gauge field angular momentum is similar to the Abelian case \[5\], and like the Abelian form contains both spin and orbital contributions. It is not possible to split Eq. (1) into a separate spin and orbital part \textit{in a gauge-invariant way} \[6\] \[7\]. Eq. (1) is the total angular momentum of the glueball, and so it should satisfy the angular momentum commutation relationship

\[
[J^i_{GB}, J^j_{GB}] = i\epsilon^{ijk} J^k_{GB}
\]

The main point of this paper is that the requirement of Eq. (2) will place some restrictions on the form of the chromodynamic fields, \(E^a, B^a\), and on the structure of glueballs.

Before proceeding with the details of the calculation of \([J^i_{GB}, J^j_{GB}]\) we should ask if it is physically reasonable that Eq. (2) should place restrictions on the color fields. For the electromagnetic interaction there are also systems which carry field angular momentum, such as the charge/monopole configuration. For this example the angular momentum commutators do place a restriction on the system \[8\] : the magnetic charge is restricted to take on Dirac values (\(i.e. e = n/2\) with \(n\) an integer). Based on this electromagnetic example one might similarly expect that Eq. (2) would place restrictions on QCD systems. One difference between QCD and QED is that for QED systems with field angular momentum \(e.g.\) the

\[1\] In Ref. \[2\] there was already an early hint that the spin of the proton might not be as simple as quark models indicated.
charge/monopole system or the charge/magnetic dipole system \[9\]) one generally knows to a very good approximation the form of the fields. Thus the functional form of the field angular momentum can be determined analytically, and the commutation rules can be worked out explicitly. If on the other hand the functional form of the electromagnetic fields were not known to such a good approximation (as is the case with the color fields of QCD) then the angular momentum algebra might lead to broader restrictions (i.e. ruling out a large class of possible electromagnetic field configurations). It would be a greater surprise if any arbitrary form for the color fields satisfied the angular momentum algebra. There is a subtle point in the electromagnetic systems with field angular momentum which needs mentioning: the field angular momentum by itself does not satisfy the commutation relationships. Only the combination of field plus particle angular momentum satisfies Eq. (2). Here we will find that a similar restriction might apply to \( J_{GB} \) — that the pure glueball field angular momentum must combine with some other angular momentum in order to satisfy Eq. (2).

II. STANDARD PICTURE OF GLUEBALL SPIN

There are several methods for investigating glueballs. The most rigorous method is via the numerical calculations of lattice gauge theories. The properties of the glueballs which are most commonly investigated in this way are their masses, string tension, and deconfinement temperature. In either lattice calculations or in phenomenological models, the spin of the glueball is determined based on some simplifying assumptions. For example, glueballs can be associated with gauge invariant QCD field strength operators \[10\] such as \( F^a_{\mu\nu}, \bar{F}^a_{\mu\nu}, \) which are spin 0, or \( F^a_{\mu\lambda} F_{\lambda\nu}^a, \bar{F}^a_{\mu\lambda} \bar{F}_{\lambda\nu}^a, \) which are spin 2. In this last case there is some ambiguity as to whether these operators should be associated with one glueball of spin 2, or two spin 0 glueballs in a d-wave state. Usually one chooses the former case for simplicity. However, we will argue that the angular momentum algebra may restrict glueballs to always be spin 0, in which case the latter choice would be required. There is also some uncertainty in the spin assignment of glueballs in lattice studies as pointed out in a recent lattice glueball review \[11\]. Essentially for a given irreducible representation of the lattice rotational group there are states with different values of total angular momentum which reduce to the same continuum state. Usually one makes the simplest choice and associates a given continuum state with the lattice state with the minimum value of total angular momentum.

One can also investigate glueballs using various phenomenological models \[12\], such as bag models \[13\] or using massive constituent gluons \[14\]. Both of these approaches roughly picture the glueballs as composed of two or three valence gluons. The glueball then has a total spin which is taken as coming from the sum of the orbital and spin angular momentum of these valence gluons (i.e. \( J_{GB} = \sum (L_i + S_i) \)). This kind of picture is similar in spirit to the quark models which had the spin of the proton coming only from the valence quarks. Since the simple quark models had difficulties giving the correct proton spin structure, a similar problem may occur with the glueball spin structure in these simple models.

III. GLUONIC FIELD ANGULAR MOMENTUM

Eq. (1) can be recast in index notation as \( J^i_{GB} = \frac{1}{2} \varepsilon^{ijk} J^j J^k \) where
\[ J^{jk} = \int d^3 x M^{0jk}(\vec{x}) \]  

(3)

\[ M^{0jk} \] are some of the components of the rank 3 tensor \( M^{\alpha \mu \nu} \)

\[ M^{\alpha \mu \nu} = T^{\alpha \nu} x^\mu - T^{\alpha \mu} x^\nu \rightarrow M^{0jk} = T^{0k} x^j - T^{0j} x^k \]  

(4)

\( T^{0k}, T^{0j} \) are the time-space components of the energy-momentum tensor \( T^{\mu \nu} \) of the Yang-Mills gauge fields

\[ T^{\mu \nu} = \frac{1}{4} g^{\mu \nu} F_{\alpha \beta}^a F_{\alpha \beta}^a - F^{\mu a} F^{\nu a} \rightarrow T^{0k} = - F^{0h} a F^{k a} \]  

(5)

the Yang-Mills fields tensor components, \( F^{0h} a, F^{ki} a \), are the color electric and color magnetic fields respectively. In terms of the gauge fields these are

\[ F_{0i}^a = \partial_0 A_i^a - \partial_i A_0^a + g f^{abc} A_0^b A_i^c \equiv E_i^a \]

\[ F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g f^{abc} A_i^b A_j^c \equiv \epsilon_{ijk} B_k^a \]  

(6)

Using this string of definitions we can write down the gluon field angular momentum of Eq. (4) in index form as

\[ J_{GB}^l = \epsilon^{ilm} \int d^3 x \left( F^{0n} a F^{m a} \right) x^l \]  

(7)

**IV. ANGULAR MOMENTUM COMMUTATION RELATIONSHIPS**

In this section we will use the expression in Eq. (3) to calculate the commutator for the pure glueball angular momentum. We will not write out the integrals \( \int d^3 x \int d^3 y \ldots \) since these can always be applied after taking the commutator of the integrand in Eq. (4). From Eq. (4) we have

\[ [J_{GB}^i(x), J_{GB}^j(y)] = \epsilon^{ilm} \epsilon^{ipq} x^l x^p \left[ (F^{0n} a(x) F^{m a}(x)), (F^{0r} b(y) F^{q b}(y)) \right] \]

\[ = \epsilon^{ilm} \epsilon^{ipq} x^l x^p (F^{0n} a F^{0r} b F^{m a} F^{p b}) + F^{0n} a [F^{m a}, F^{0r} b] F^{q b} \]

\[ + F^{0r} b [F^{m a}, F^{n q}] F^{m a} + [F^{0n} a, F^{0r} b] F^{m a} F^{q b} \]  

(8)

In the last line the \( x, y \) dependences are not written out explicitly. The \( \epsilon \)'s and \( x^i \)'s have been pulled outside the commutator brackets since they commute with one another and with \( F^{a}_{\mu \nu} \). Standard commutator algebra has been used to write things in terms of two-term commutators. None of the commutators above have any non-trivial contribution coming from the SU(3) group structure of the fields, since the \( F^{a}_{\mu \nu} \)'s are all simply SU(3) components. Another way to see this is to write \( F^{a}_{\mu \nu} \) in terms of the matrix field strength tensor, \( F_{\mu \nu} \equiv F^{a}_{\mu \nu} T^a \). Using the standard normalization for the group generators \( (Tr[T^a T^b]) = \frac{1}{2} \delta^{ab} \) where \( Tr = \text{Trace} \) \( F^{a}_{\mu \nu} \) can be written as \( F^{a}_{\mu \nu} = 2Tr[F^{a}_{\mu \nu} T^a] \). Since the group factors are traced over in this expression for \( F^{a}_{\mu \nu} \) they only give trivial commutators in Eq. (8). Any possible non-trivial commutation structure in Eq. (8) arises, as in Ref. [15], from representing the gauge potentials in terms of creation/annihilation operators.
In the canonical formalism, where the gauge fields are given by creation/annihilation operators, the SU(3) gauge fields satisfy the following commutation relationships

\[
[A^a_i(x,t), E^b_j(y,t)] = i\delta_{ij}\delta^{ab}\delta^3(x-y)
\]

\[
[E^a_i(x,t), E^b_j(y,t)] = [A^a_i(x,t), A^b_j(y,t)] = 0
\]

Expanding the chromoelectric field in the above commutators as \( E^b_j = F^b_{ij} = \partial_0 A^b_j - \partial_j A^b_0 + gf^{bcd} A^c_k A^d_l \), and using the last commutator in Eq. (8) one finds that only the time derivative term in the chromoelectric field expansion gives a nontrivial commutator since

\[
[A^a_i(x,t), \partial^j A^b_0(y,t)] = \partial^j [A^a_i(x,t), A^b_0(y,t)] = 0.\]

Thus the commutation relationships involving the chromoelectric field can be rewritten as

\[
[A^a_i(x,t), \partial_0 A^b_j(y,t)] = i\delta_{ij}\delta^{ab}\delta^3(x-y)
\]

\[
[\partial_0 A^a_i(x,t), \partial_0 A^b_j(y,t)] = 0
\]

(10)

Except for the group index Kronecker delta these are identical to the field commutators in an Abelian theory.

When the commutator, \([F^{m,a}_n, F^{q,b}_r]\), in Eq. (8) is expanded in terms of the potentials one finds commutators like \([\partial^m A^a_n, g f^{bef} A^e q A^f_r]\) or \([\partial^m A^a_n, \partial^q A^b_r]\) or \([g f^{acd} \partial^m A^a_n, g f^{bef} \partial^q A^b_r]\). By commutator algebra these can all be reduced to sums of commutators like \([A^c_n, A^f_r]\) or \([\partial^m A^a_n, \partial^q A^b_r]\) or \([\partial^m A^a_n, A^e q]\) = \(\partial^m A^a_n, A^e q\) (i.e. commutators of the gauge potential with itself or with its spatial derivatives). All the these vanish by the last commutation relationship in Eq. (8) thus in Eq. (8) \([F^{m,a}_n, F^{q,b}_r] = 0\)

The remaining three terms from Eq. (8) involve commutators between \(F^{oi,a}\) and itself or between \(F^{0i,a}\) and \(F^{jk,b}\). These three commutators can be evaluated by expanding \(F^{oi,a}\) and \(F^{jk,b}\) in terms of the gauge potentials. It is easy to see that the only non-trivial part of these three commutators comes from the commutation of the time derivative term in \(F^{oi,a}\) (i.e. \(\partial^0 A^i a\)) with the gauge potential \(A^k b\) or its spatial derivative \(\partial^0 A^k b\). Since glueballs are bound states one has the extra physical constraint that the gauge potentials are time independent \(\partial^0 A^i a = 0\) (at least in the glueball rest frame). Taking this physical constraint into account the chromoelectric part of the field strength tensor for pure glueballs simplifies to \(F^{oi,a} = -\partial_i A^a_0 + g f^{abc} A^b_0 A^c_i\). (Strictly one can not set the operator, \(\partial^0 A^i a\), equal to zero, but this condition should be applied to the glueball states, \(|G\rangle\), as is done in the following section. This Gupta-Bleuler-like procedure could easily be applied here and would not change the final result in Eq. (11) below). From Eqs. (10) and (11) one can see that each of these terms commutes with other gauge potential components, \(A^i a\), or with spatial derivatives of gauge potential components. Thus the commutators for the glueball angular momentum operator yields

\[
[J^i_{GB}, J^j_{GB}] = 0
\]

The vanishing of this commutator comes directly from the physical requirement that for bound states the gauge field should be time-independent \((\partial^0 A^i a = 0)\).

The result given in Eq. (11) may seem strange, but an analogous result arises in the electromagnetic system of a charge and monopole at rest with respect to one another. For
this system one can calculate the field angular momentum as $J_{EM} = egr/r$, where $e, g, r$ are the electric charge, magnetic charge and displacement between the two charges respectively. The commutator of the components of $J_{EM}$ with itself gives zero since $r/r$ commutes with itself. Alternatively, one could proceed in the electromagnetic case as in the color field case by writing $J_{EM}$ out as in Eq. (1) in terms of the normal electric and magnetic fields. The commutator of the components of $J_{EM}$ among themselves could then be calculated using the electromagnetic version of the gauge potential commutators given by Eqs. (9) (10) with the color indices dropped. The only non-trivial terms would come from commutators between $\partial^0 A^i$ and $A^j$ or $\partial^j A^k$. However, in the frame where the electric and magnetic charges are at rest, the gauge fields are time-independent so that $\partial^0 A^i = 0$, and one again finds that the commutator for the electromagnetic field angular momentum is zero. In the color field case this latter method is the only viable one, since for the color interaction an explicit form for $E^a$ and $B^a$ is not known so that an explicit calculation of $J_{QCD}$ is not possible. For the electromagnetic system $E$ and $B$ are Coulomb fields, and so $J_{EM}$ can be calculated directly.

There are several ways of looking at this apparent conflict between Eqs. (2) and (11).

• First, Eqs. (2) and (11) are consistent if glueballs are restricted to be spin 0 ($J_{GB} = 0$) bound states. This is not an unreasonable restriction since the identification of the spin of a non-zero spin glueball is usually based on some simplifying assumptions. As mentioned previously spin 2 glueballs are usually connected with QCD operators like $F^a_{\mu\lambda} F^\lambda_{\mu\nu}$ or $F^a_{\mu\lambda} \tilde{F}^\lambda_{\mu\nu}$. However, one could instead associate these objects with two spin 0 glueballs in a d-wave orbital angular momentum state. The former choice is a reasonable, simplifying assumption, but may not be correct in light of the restriction coming from $[J_{GB}, J_{GB}] = 0$.

• A second explanation of the zero commutator is that glueballs do not occur as pure glue bound states, but always have some admixture of quarks. If the glueball states are mixed with $\bar{q}q$ states, these would contribute to the overall spin of the bound state. To take into account the contribution coming from the quarks, one would add the total quark angular momentum operator $\vec{J}_Q$ to the pure QCD term of Eq. (1) so that $\vec{J}_{GB} \rightarrow \vec{J}_{GB} + \vec{J}_Q$. (The two terms in Eq. (12) are associated with the spin and orbital angular momentum of the quarks respectively.) This total angular momentum might then satisfy Eq. (2) with the proper non-zero term on the right hand side. Such mixing of quarks with glueballs is thought to be suppressed via the $1/N_c$ expansion approach [10]. However, the restriction coming from the angular momentum commutators as discussed above, might be an indication that pure glue states can not exist – that there will always be some substantial admixture of quarks in order to have the correct angular momentum commutators. This postulated mixing might explain the difficulty in experimentally distinguishing an object as a pure glueball. This explanation for Eq. (11) is also reminiscent of the electromagnetic charge/monopole system, where it is only the sum of the field angular momentum plus the other contributions which satisfies Eq. (2).
Finally, recent work [18][19] has shown that the choice of the Lorentz frame in which the angular momentum of a system is evaluated can have an impact on the interpretation of the angular momentum. Thus, the peculiar result of Eq. (11) may be connected with the choice of Lorentz frame in which the glueball angular momentum is evaluated (up to this point we have implicitly assumed that \( J_{GB} \) is evaluated in the rest frame of the glueball). In Ref. [18] an investigation of the \((\frac{1}{2}, \frac{1}{2})\) representation of the Lorentz group, showed that it is not possible, in a general Lorentz frame, to split the \((\frac{1}{2}, \frac{1}{2})\) space into spin 1 and spin 0 sectors. Two special cases were found in which the spin 1 / spin 0 decomposition was possible: (a) in the rest frame (b) in the helicity frame where the quantization axis for spin projections is aligned with the boost direction. In Ref. [19] a related result was found in a somewhat different context. Ref. [19] examined the break up of the total nucleon spin into terms for the quark spin and orbital angular momentum, and gluon spin and orbital angular momentum. Under a general Lorentz transformation it was found that these individual terms did not transform properly, thus making their interpretation frame dependent. Ref. [19] also found that the two special frames listed above (i.e. the rest frame and the helicity frame) were picked out as being particularly useful in studying the spin structure of the nucleon. Although the central concerns of each of these works differ from each other, they both point out the importance that choice of the Lorentz frame makes in the study of the angular momentum. This may imply that the difficulty in formulating a clear picture of the spin structure of the glueball may rest in our implicit choice of the rest frame.

There are certain phenomenological bag models of glueballs [20] where the assumption that the color fields are time independent is not true. In these models, to lowest order, the chromoelectric and chromomagnetic fields are analogous to the ordinary electric and magnetic fields inside a resonant cavity so that the chromoelectric and chromomagnetic fields, and the gauge potentials have oscillatory time-dependences. However the chromoelectric and chromomagnetic fields in these models are 90° out of phase with one another. Therefore at some particular time the chromoelectric field will be at a maximum while the chromomagnetic field is zero, and at some later time the chromomagnetic field will be at a maximum while the chromoelectric field vanishes. At these times, when either \( E^a \) or \( B^a \) = 0, \( J_{GB} = 0 \) from Eq. (1). Since \( J_{GB} \) is conserved (i.e. is time independent) this implies that \( J_{GB} = 0 \) at any time. This vanishing of \( J_{GB} \) was one of the possible resolutions that we arrived at by applying the angular momentum commutators to \( J_{GB} \) and assuming that the fields were time independent. Thus, even in the phenomenological bag models where the time independence assumption does not hold, we still find the restriction \( J_{GB} = 0 \). One can also present a more general argument to support the time independence of the fields. Since \( J_{GB} \) is conserved \( \rightarrow dJ_{GB}/dt = 0 \). From Eq. (1) this implies that

\[
\int d^3x \left[ x \times \left( \frac{dE^a}{dt} \times B^a + E^a \times \frac{dB^a}{dt} \right) \right] = 0 \quad (13)
\]

\( (x \) is not differentiated since it is the time independent position vector from the origin to some particular part of the momentum density, \( E^a \times B^a \)). There are two simple ways in which Eq. (13) can be satisfied. First, if \( dE^a/dt = dB^a/dt = 0 \). This implies that the fields are time independent which was the assumption in our analysis. Second, if \( E^a = \pm B^a \) then
Eqs. (13) is satisfied. However, in this case, even though the fields do not need to be time independent, one finds that $J_{GB}^i = 0$ from Eq. (1). There is apparently no simple way to have both $J_{GB}^i \neq 0$ and $dE^a_i / dt \neq 0$, $dB^a_i / dt \neq 0$.

V. EXPECTATION OF $J_{GB}$

In this section we calculate the expectation value of $J_{GB}^i$ with respect to a glueball state, $|G\rangle$, under certain simplifying assumptions. From Eq. (7) the expectation value is given by

$$\langle G| J_{GB}^i |G\rangle = e^{ilm} \int d^3 x \langle G| \left( F^{0n}_m a(x) F^{m n}_n a(x) \right) |G\rangle x^l$$  \hspace{1cm} (14)

The glueball state is taken to be normalized so that $\langle G|G\rangle = 1$. Since the field angular momentum operator in Eq. (7) is gauge invariant we are free to choose a gauge without effecting the physically measurable quantities like the spin of the glueball. We choose the temporal gauge where $A^0_i = 0$. In terms of the operator $A^0_i$ we implement this gauge choice using a Gupta-Bleuler-like approach by making it a condition on the glueball state – $\langle G|A^0_i |G\rangle = 0$. $A^0_i$ can be split into positive and negative frequency parts : $A^0_i = A^{0 (+)}_i + A^{0 (-)}_i$. The positive frequency part can be written as a sum of annihilation operators, while the negative frequency part can be written as a sum of creation operators. Thus the gauge condition $\langle G|A^0_i |G\rangle = 0$ becomes $A^{0 (+)}_i |G\rangle = 0$ and $\langle G|A^{0 (-)}_i |G\rangle = 0$. Next, since the glueball is a bound state, we have the physical (as opposed to gauge) restriction that $\partial_0 A^a_i = 0$ which we will also implement as a condition on the glueball state – $\langle G|\partial_0 A^a_i |G\rangle = 0$. This can also be written in terms of the positive and negative frequency parts as $\partial_0 A^{0 (+)}_i |G\rangle = 0$ and $\langle G|\partial_0 A^{0 (-)}_i |G\rangle = 0$. Expanding $F^{0n}_m a$ in term of the gauge potentials we can write out a portion of the right hand side of Eq. (14) as

$$\left( \partial^b A^{na} (y) - \partial^n A^{0a} (y) + g f^{abc} A^{ob} (y) A^{nc} (y) \right) F^{m a}_n (x) |G\rangle$$  \hspace{1cm} (15)

By using the commutation rules for the non-Abelian gauge potentials given by Eq. (10) we will move the various terms coming from $F^{0n}_m a$ through $F^{m a}_n$ and then apply either the gauge condition $A^{0 (+)}_i |G\rangle = 0$ or the physical condition $\partial_0 A^{0 (+)}_i |G\rangle = 0$. In order to apply the commutators of Eq. (10) we have changed $F^{0n}_m a(x)$ to $F^{0n}_m a(y)$. In the end we will let $y \to x$. In Eq. (15) $F^{0n}_m a$, expanded in terms of the $A^{ua}_i$’s, has both positive and negative frequency parts, but the negative frequency parts will vanish directly when $\langle G\rangle$ is applied to the left side of Eq. (13). Thus as we pull $F^{0n}_m a$ through $F^{m a}_n$ we are only dealing with the positive frequency parts even though the superscript (+) will not be written out explicitly. The last term in $F^{0n}_m a \left( g f^{abc} A^{ob} (y) A^{nc} (y) \right)$ can be trivially commuted past $F^{m a}_n (x)$ since both $A^{ob} (y)$ and $A^{nc} (y)$ commute with other gauge fields components or their spatial derivatives by the last commutator in Eq. (7). After this term is commuted through $F^{m a}_n (x)$ one has

$$F^{m a}_n (x) [A^{ob} (y) A^{nc} (y)] |G\rangle \to F^{m a}_n (x) [A^{nc} (y) A^{ob} (y)] |G\rangle = 0$$  \hspace{1cm} (16)

where the gauge condition $A^0_i |G\rangle = 0$ was applied. The second term from Eq. (16), $\partial^n A^{0a} (y)$, can also be commuted past $F^{m a}_n (x)$ using the last commutator in Eq. (7) since only a spatial derivative is involved. This yields
\[ F^n_m a(x)[\partial^n A^a_m(y)]|G\rangle = 0 \] (17)

where we have taken the gauge condition \( A^n_0|G\rangle = 0 \) to imply \( \partial^n(A^n_0|G\rangle) = 0 \). Finally we commute the term, \( \partial^0 A^a_m(y) \) through \( F^n_m a(x) \) by expanding \( F^n_m a(x) \) as

\[ F^n_m a(x) = \partial^n A^n_m(x) - \partial^n A^a_m(x) + g f^{ade} A^m(x) A^n(x) \] (18)

From Eq. (11) \( [\partial^0 A^a_m(y), \partial^n A^a_n(x)] = -i \delta_{mn} \partial^a [\delta^3(y - x)] = 0 \) since \( m \neq n \). Thus \( \partial^0 A^a_m(y) \) commutes with the \( \partial^n A^a_n(x) \) term from \( F^n_m a(x) \). Also \( \partial^0 A^a_m(y) \) will commute past both fields operators in the term \( f^{ade} A^m(x) A^n(x) \). This is due to \( a \neq d \neq e \) from the antisymmetry of \( f^{ade} \), and the Kronecker delta in the group indices in the commutator of Eq. (11) (i.e. \( f^{ade} A^a_m(y), A^n_m(x) = -i \delta_{mn} \delta^{ae} \delta^3(y - x) = 0 \) since \( a \neq e \)). Thus

\[
\begin{align*}
\partial^0 A^a_m(y) \partial^n A^a_n(x)|G\rangle &\rightarrow \partial^n A^a_n(x) \left( \partial^0 A^a_m(y)|G\rangle \right) = 0 \\
\partial^0 A^a_m(y) f^{ade} A^m(x) A^n(x)|G\rangle &\rightarrow f^{ade} A^m(x) A^n(x) \left( \partial^0 A^a_m(y)|G\rangle \right) = 0
\end{align*}
\] (19)

where we have used the physical condition \( \partial_0 A^a_4|G\rangle = 0 \). Finally we examine the last term, \( \partial^0 A^a_m(y) \partial^n A^a_n(x)|G\rangle \). Using the first commutator in Eq. (11) (so \( \partial^0 A^a_m(y) \partial^n A^a_n(x) = \partial^n A^n_m(x) \partial^0 A^a_m(y) - i \delta_{mn} \delta^3(y - x) \)) where the subscript \( (x) \) means that the derivative is taken with respect to \( x \) rather than \( y \). Using this gives

\[
\begin{align*}
\partial^0 A^a_m(y) \partial^n A^a_n(x)|G\rangle &= \left[ \partial^n A^n_m(x) \partial^0 A^a_m(y) - i \delta_{mn} \delta^3(y - x) \right] |G\rangle \\
&= -i \delta_{mn} \delta^3(y - x) |G\rangle
\end{align*}
\] (20)

where the physical condition \( \partial^0 A^a_4|G\rangle = 0 \) was used. Using Eq. (20) one obtains

\[
\langle G| J^i_{GB} |G\rangle = -\epsilon^{ilm} \int d^3x \langle x | \partial^0 A^a_4(x) \delta^3(y - x) |G\rangle = -\epsilon^{ilm} \int d^3x \left( i \delta^a_m \delta^3(y - x) \right) x^l
\] (21)

where \( \langle G|G\rangle = 1 \) was used. Integrating Eq. (21) by parts gives

\[
-\epsilon^{ilm} \left[ x^l \delta^3(y - x) \right]^{+\infty}_{-\infty} + \int \delta^3(y - x) \partial^0 A^a_4(x) d^3x
\] (22)

The first term above is zero because of the \( \delta \) function. The integral in the second term evaluates to the Kronecker delta, \( \delta^{ml} \). Thus the expectation of \( J^i_{GB} \) becomes

\[
\langle G| J^i_{GB} |G\rangle = \epsilon^{ilm} \delta^{ml} = \epsilon^{ill} = 0
\] (23)

using the properties of the \( \epsilon \)'s. Thus by imposing the temporal gauge condition \( A^{a(+)}_0|G\rangle = 0 \; \langle G| A^{a(-)}_0 = 0 \) and the physical condition that for a bound state the gauge fields should be time independent \( (\partial^0 A^{a(+)}|G\rangle = 0 \; \langle G|\partial^0 A^{a(-)} = 0 \) it is found that \( \langle G| J^i_{GB} |G\rangle = 0 \).

VI. CONCLUSIONS

By examining the commutation relationships \( ([J^i_{GB}, J^j_{GB}]) \) and the expectation value \( \langle G| J^j_{GB} |G\rangle \) of the glueball angular momentum operator it was found that both of these quantities were zero. The main assumption in both of these results was that the gauge fields
were time-independent \( (\partial^0 A_{ia} = 0) \), since the glueball is a bound state. The non-Abelian nature of the gauge fields played little role in the commutation relationships of \( J_{GB}^i \), since the group indices are traced over in the expression of \( J_{GB}^i \). Thus one would expect that a similar result \( ([J_{EM}^i, J_{EM}^j] = 0 \) where \( J_{EM}^i \) is the electromagnetic field angular momentum) should hold for Abelian gauge fields \( (A_{\mu}) \) under similar conditions \( (\partial^0 A^i = 0) \). For the two Abelian examples where this has been worked out explicitly (the monopole/charge system and the magnetic dipole/charge system \[^9]\) it is in fact found that the field angular momentum, by itself, does not satisfy the angular momentum commutation relationships. In light of these Abelian examples, and the similarity between \( J_{GB}^i \) and \( J_{EM}^i \) the results for the glueball angular momentum operator are not so surprising.

We have suggested several possible resolutions/explanations for these results for \( J_{GB}^i \).

- First, the fact that both \( [J_{GB}^i, J_{GB}^j] \) and \( \langle G|J_{GB}^i|G \rangle \) are zero may imply that pure glueballs can only be spin 0.

- Second, the above results may indicate that glueballs will always have some significant admixture of quarks, which will contribute to the spin structure of the glueball.

- Finally, one needs to consider the effect that the choice of Lorentz frame makes in the study and interpretation of the angular momentum of the glueball system.

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