Spontaneous CP Violation and Scalar FCNC

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Abstract. A short overview of general aspects of Spontaneous CP Violation and Scalar Flavour Changing Neutral Couplings is presented. Then, a 2HDM where CP Violation has a spontaneous origin, including the generation of a realistic CKM matrix, and which has Scalar Flavour Changing Neutral Couplings of controlled intensity, is discussed. Phenomenological consequences such as the existence of new scalars with masses below 1 TeV, or the possibility of having flavour changing processes such as $t \rightarrow h c$ or $h \rightarrow b s$ within experimental reach, are explored.

1. Spontaneous CP Violation

The first model of spontaneous T and CP violation was proposed by T.D. Lee in 1973 [1]. At that time, CP Violation had already been established [2] in $K \rightarrow \pi\pi$ decays, only two incomplete quark generations were known (although implications of the existence of the charm quark such as the GIM mechanism [3] had been anticipated), and the possibility to incorporate CP Violation in a three generation context was a recent development [4]. The main motivation for Lee’s seminal work was to put the breaking of CP and T on the same footing as the breaking of gauge symmetry. At the phenomenological, 2 Higgs Doublets Models (2HDMs) could provide scalar mediated contributions to flavour changing neutral $K^0$–$\bar{K}^0$ mixing and thus explain the observed CP Violation in the context of two quark generations.

In Lee’s model, the Lagrangian is CP and T invariant, but the vacuum violates these discrete symmetries. This was achieved through the introduction of two Higgs doublets, with vacuum expectation values with a relative phase which violates T and CP invariance. The scalar potential with two $SU(2)_L$ doublets $\Phi_j$ reads

$$V(\Phi_1, \Phi_2) = \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \left( \mu_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + 2 \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left( \lambda_5 (\Phi_2^\dagger \Phi_2)^2 + \text{H.c.} \right) + \left( \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \text{H.c.} \right),$$

(1)

where $\mu_{11}^2, \mu_{22}^2, \lambda_i=1,...,4$ are real and $\mu_{12}^2, \lambda_j=5,6,7$ could be complex. For a CP transformation

$$\Phi_j(x, t) \rightarrow (CP)\Phi_j(-x, t)(CP)^\dagger = e^{i\tau} \Phi_j^*(x, t),$$

(2)
CP invariance requires real $\mu_2^2$, $\lambda_j=5,6,7$. Vacuum expectation values $\langle \Phi_j \rangle$ break electroweak symmetry spontaneously, and thus Spontaneous CP Violation (SCPV) requires physical relative phases among the different $\langle \Phi_j \rangle$ (that is one single relative phase in the 2HDM).

The basic ingredient for SCPV is a rich scalar sector, and there is a vast literature on the subject which ranges from Left-Right models, Grand Unified Theories or Supersymmetric Models to much more economic scenarios such as [5]. An aspect of Lee’s model which can be considered unsatisfactory is the fact that the vacuum phase is not calculable, that is, it depends on the parameters of the potential and there is a continuum of CP vacuum phases which give SCPV. The possibility that additional symmetry requirements on the scalar sector could reduce or eliminate that freedom was first explored in [6], and is known as geometric CP Violation (for recent work, see for example [7,8]).

Considering now the Yukawa couplings in the 2HDM\(^1\)

$$\mathcal{L}_Y = -\bar{Q}_0^L (\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2) d_R^0 - \bar{Q}_0^R (\Delta_1 \tilde{\Phi}_1 + \Delta_2 \tilde{\Phi}_2) u_R^0 + \text{H.c.},$$

CP invariance also requires real $\Gamma_j$, $\Delta_j$.

The general two Higgs Doublet Model (2HDM) (see [9,10] for recent reviews) has Scalar Flavour Changing Neutral Couplings (SFCNC) at tree level; although Lee’s proposal required them in order to explain CP Violation in kaon mixing, they have to respect stringent experimental constraints. Strategies to put those SFCNC under control range have been widely addressed.

A popular possibility, suggested by Glashow and Weinberg [11], which eliminates tree level SFCNC through symmetry requirements, is Natural Flavour Conservation (NFC). Interesting alternatives include the Branco, Grimus and Lavoura (BGL) models, [12–14], which have SFCNC controlled by the CKM matrix itself, or more general symmetry controlled scenarios [15,16]. Concerning the interplay among SCPV and SFCNC, 2HDM with NFC based on an exact $\mathbb{Z}_2$ symmetry do not allow for SCPV (this can be bypassed, however, by breaking the symmetry softly) and scalar mediated CP violation is absent. Beyond 2 doublets, nevertheless, the possibility to have SCPV is recovered and CP violating contributions due to the exchange of charged scalars are present [17,18].

Overall, no simple 2HDM scenario which incorporates the possibility of SCPV together with SFCNC in agreement with experiment had been explored. With respect to Lee’s original proposal, it is to be noticed that it is now necessary to incorporate a CKM matrix which is irreducibly complex [19]: that is, if CP violation has a spontaneous origin, the vacuum phase should be the source of CP violation of the CKM matrix (which, of course, should be in agreement with experimental data). Such a possibility is discussed in the following.

2. A 2HDM with SCPV and Scalar FCNC

After the overview of the previous section, we now present a realistic 2HDM which realises the original idea of having a spontaneous origin for CP violation, while having SFCNC. The contents of this section are based on [20]. First, the scalar sector is addressed in section 2.1; then, in section 2.2, the flavour structure of the model is described in detail. It is shown that it leads to a CP violating CKM matrix and that SFCNC are unavoidably present, but with controlled intensity. With these ingredients we finally present in section 2.3 a numerical analysis of the model, including the most relevant constraints and phenomenological consequences.

2.1. Scalar Sector

We consider a 2HDM with symmetry under the $\mathbb{Z}_2$ transformation $\Phi_1 \mapsto \Phi_1$, $\Phi_2 \mapsto -\Phi_2$, softly broken by $\Phi_1^4 \Phi_2$, $\Phi_1^2 \Phi_1$ terms. In addition, we impose CP invariance of the lagrangian.

\(^1\) Summation over generation indices is understood and $\Phi_j = i \sigma_2 \Phi_j^\dagger$ (as usual, under $SU(2)_L$, $Q_L^0 = (u_R^0 \ d_R^0)$ transform as doublets while $u_R^0$ and $d_R^0$ transform as singlets).
These two requirements are satisfied by the scalar potential of eq. (1) for \( \lambda_6 = \lambda_7 = 0 \) and both \( \mu_{12}^2 \) and \( \lambda_5 \) real, that is:

\[
\mathcal{V}(\Phi_1, \Phi_2) = \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + 2\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2\lambda_4 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2],
\]

with all parameters real.

Electroweak symmetry is spontaneously broken by the vacuum expectation values

\[
\langle \Phi_1 \rangle = \left( e^{i\theta_1}v_1/\sqrt{2} \right), \quad \langle \Phi_2 \rangle = \left( e^{i\theta_2}v_2/\sqrt{2} \right).
\]

As usual, we use \( \theta = \theta_2 - \theta_1 \), \( v^2 = v_2^2 + v_2^2 \), \( c_\beta = \cos \beta \equiv v_1/v \), \( s_\beta = \sin \beta \equiv v_2/v \) and \( t_\beta = \tan \beta \), with \( v_1 \geq 0 \), \( v_2 \geq 0 \). From the minimization conditions one obtains \( \cos \theta = -\frac{\mu_{12}^2}{2v_1 v_2} \). Notice that, in addition to \( \theta \), \( -\theta \) is also a solution. It is obvious that for \( \theta = 0, \pi \) the vacuum is CP invariant; it has also been shown [18] that for \( \theta = \pm \pi/2 \) the vacuum is CP invariant. One can trade \( \mu_{11}^2 \), \( \mu_{22}^2 \) and \( \mu_{12}^2 \) for other parameters using the minimization conditions. One can in addition choose \( v_1^2 + v_2^2 = (246 \text{ GeV})^2 \) for appropriate electroweak symmetry breaking without loss of generality. Fixing \( v^2 \) in that manner, one is left with a candidate minimum characterised by the values of \( \theta \) and \( \tan \beta = v_2/v_1 \). These are not completely free parameters when requirements such as boundedness from below of the scalar potential or perturbative unitarity bounds on high energy scattering of scalars, are placed on the scalar sector.Expanding \( \Phi_j \) around the candidate vacuum in eq. (5)

\[
\Phi_j = e^{i\theta_j} \left( \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \right),
\]

we can now explore the different mass terms for the charged and neutral scalars. Requiring that the mass parameters of all the physical scalars are positive ensures, at least, that the candidate minimum is a local minimum of the potential. In a “Higgs basis” \( \{H_1, H_2\} [21–23] \)

\[
\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{i\theta_1} \Phi_1 \\ e^{-i\theta_2} \Phi_2 \end{pmatrix}, \quad \text{with} \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \quad \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1},
\]

only \( H_1 \) acquires a vacuum expectation value

\[
\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.
\]

The expansion of the fields reads

\[
H_1 = \left( \begin{pmatrix} v + H^0 + iG^0 \end{pmatrix}/\sqrt{2} \right), \quad H_2 = \left( \begin{pmatrix} H^+ \\ (R^0 + iI^0)/\sqrt{2} \end{pmatrix} \right),
\]

with the would-be Goldstone bosons \( G^\pm \) and \( G^0 \) readily identified, \( G^\pm = c_\beta \varphi_1^\pm - s_\beta \varphi_2^\pm \), \( G^0 = c_\beta \eta_1 - s_\beta \eta_2 \). In 2HDMs, the charged scalar \( H^\pm = s_\beta \varphi_1^\pm - c_\beta \varphi_2^\pm \) is also identified with the transformation into the Higgs basis; it has a mass term

\[
\mathcal{V}(\Phi_1, \Phi_2) \supset v^2(\lambda_5 - \lambda_4)H^+H^- \Rightarrow m_{H^\pm}^2 = v^2(\lambda_5 - \lambda_4).
\]
For the neutral scalar sector, the mass terms are
\[
\mathcal{Y}(\Phi_1, \Phi_2) \supset \frac{1}{2} \begin{pmatrix} H^0 & R^0 & I^0 \end{pmatrix} \mathcal{M}_0^2 \begin{pmatrix} H^0 \n R^0 \n I^0 \end{pmatrix}, \quad \mathcal{M}_0^2 = \mathcal{M}_0^{2T}.
\]

(11)

For \( s_{2\theta} \neq 0 \), there is scalar-pseudoscalar mixing, as it is expected from spontaneous breaking of CP in the scalar sector. \( \mathcal{M}_0^2 \) is diagonalised through a real orthogonal transformation \( \mathcal{R} \)
\[
\mathcal{R}^T \mathcal{M}_0^2 \mathcal{R} = \text{diag}(m_{h_1}^2, m_{H_1}^2, m_{A_1}^2), \quad \mathcal{R}^{-1} = \mathcal{R}^T,
\]

(12)
and the physical neutral scalars are
\[
\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} H^0 \\ R^0 \\ I^0 \end{pmatrix}.
\]

(13)

We assume \( h \) to be the lightest one, the Higgs-like neutral scalar with \( m_h = 125 \text{ GeV} \). \( \mathcal{R} \) “mixes”, a priori, all three neutral scalars. It is interesting to notice that
\[
\text{Tr}[\mathcal{M}_0^2] = m_{h_1}^2 + m_{H_1}^2 + m_{A_1}^2 = 2v^2[\lambda_1 c_{3\beta}^2 + \lambda_2 s_{3\beta}^2 + \lambda_5],
\]
\[
\text{det}[\mathcal{M}_0^2] = m_{h_1}^2 m_{H_1}^2 m_{A_1}^2 = 2v^6 \lambda_5(\lambda_1 \lambda_2 - \lambda_3^2 + \lambda_4^2) \sin^2 2\beta \sin^2 \theta.
\]

(14)

Equations (10) and (14) encode in a transparent manner several interesting properties of the model. The most relevant one is the following: since the different \( \lambda_j \) are bounded by perturbativity requirements (in particular by perturbative unitarity), and \( \sin^2 2\beta \leq 1 \) and \( \sin^2 \theta \leq 1 \), the masses of the new scalars \( H, A, H^\pm \), have a limited allowed range\(^2\). Consequently, in this model there is limited room to have a scalar sector where (i) \( h \) is a Higgs boson with quite SM-like properties and (ii) \( m_{H^\pm}, m_H, m_A \gg m_h \), that is, there is no decoupling regime for the new scalars\(^3\). Having discussed the main characteristics of the scalar sector, we now turn to the flavour structure of the model.

2.2. Yukawa couplings and Flavour Structure

The flavour structure of the model is defined by requiring invariance of the Yukawa lagrangian
\[
\mathcal{L}_Y = -\bar{Q}_L^j(I_1 \Phi_1 + I_2 \Phi_2)d_R^0 - \bar{Q}_L^j(\Delta_1 \Phi_1 + \Delta_2 \Phi_2)e_R^0 + \text{H.c.},
\]

(15)
under the \( \mathbb{Z}_2 \) transformations
\[
\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2,
\]
\[
\bar{Q}_L^{j \ell_3} \rightarrow -\bar{Q}_L^{j \ell_3}, \quad \bar{Q}_L^{j \ell_j} \rightarrow \bar{Q}_L^{j \ell_j}, \quad j = 1, 2,
\]
\[
d_R^0 \leftrightarrow d_R^0, \quad u_R^0 \leftrightarrow u_R^0, \quad k = 1, 2, 3.
\]

(16)

\(^2\) For a very crude estimate, consider for example \( \lambda_1 c_{3\beta}^2 + \lambda_2 s_{3\beta}^2 + \lambda_5 \sim 10 \) in eq. (14): with \( v \approx 2m_h \), \( m_{H_1}^2 + m_{A_1}^2 \sim 80m_h^2 \) and it is clear that the smaller among \( m_{H_1} \) and \( m_{A_1} \) cannot be larger than \( \sim 6m_h \), while the larger among \( m_{H_1} \) and \( m_{A_1} \) cannot be larger than \( \sim 9m_h \).

\(^3\) It is then clear that the new scalars could be produced at the LHC. Nevertheless, the most relevant production and decay modes for their discovery will vary significantly between different regions of parameter space, including the Yukawa couplings discussed in section 2.2, and also on the details of the lepton sector, which is not discussed here.
The transformation of the scalar doublets is, of course, the one considered in the previous section. Apart from $\Phi_2$, only one left-handed doublet, $Q^0_{L3}$, has a non-trivial transformation. Invariance under eq. (16) imposes the following general form of the Yukawa coupling matrices:

$$
\Gamma_1 = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix},
$$

$$
\Delta_1 = \begin{pmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix}.
$$

The symmetry assignment in eq. (16) and the Yukawa matrices in eq. (17) correspond to the generalised BGL models introduced in [15]. Furthermore, in order to impose CP invariance at the Lagrangian level, we also require the Yukawa couplings to be real:

$$
\Gamma^*_j = \Gamma_j, \quad \Delta^*_j = \Delta_j.
$$

In the Higgs basis of eq. (7), eq. (15) gives

$$
\mathcal{L}_Y = -\frac{\sqrt{2}}{v} \bar{Q}_L^0 (M^0_d H_1 + N^0_d H_2) d_R^0 - \frac{\sqrt{2}}{v} \bar{Q}_L^0 (M^0_u \tilde{H}_1 + N^0_u \tilde{H}_2) u_R^0 + \text{H.c.},
$$

which allows the identification of the quark mass matrices $M^0_d$, $M^0_u$. The matrices in eq. (19) read

$$
M^0_d = \frac{v e^{i\theta_1}}{\sqrt{2}} (c_\beta \Gamma_1 + e^{i\theta} s_\beta \Gamma_2), \quad N^0_d = \frac{v e^{i\theta_1}}{\sqrt{2}} (-s_\beta \Gamma_1 + e^{i\theta} c_\beta \Gamma_2),
$$

$$
M^0_u = \frac{v e^{-i\theta_1}}{\sqrt{2}} (c_\beta \Delta_1 + e^{-i\theta} s_\beta \Delta_2), \quad N^0_u = \frac{v e^{-i\theta_1}}{\sqrt{2}} (-s_\beta \Delta_1 + e^{-i\theta} c_\beta \Delta_2).
$$

We set, without loss of generality, $\theta_1 = 0$ for simplicity. Notice that the matrices $N^0_d$, $N^0_u$, following the “row structure” in eq. (17), can be written:

$$
N^0_d = t_\beta M^0_d + e^{i\theta} \frac{v}{\sqrt{2}} (t_\beta + t^{-1}_\beta) s_\beta \Gamma_2 = t_\beta M^0_d - (t_\beta + t^{-1}_\beta) P_3 M^0_d,
$$

$$
N^0_u = t_\beta M^0_u + e^{-i\theta} \frac{v}{\sqrt{2}} (t_\beta + t^{-1}_\beta) s_\beta \Delta_2 = t_\beta M^0_u - (t_\beta + t^{-1}_\beta) P_3 M^0_u,
$$

where $P_3$ is the projector

$$
P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
$$

As emphasized previously, a necessary requirement for the model is that the vacuum phase $\theta$ is capable of generating a complex CKM matrix. Of course, at a quantitative level, we want to reproduce the observed CKM matrix, including the hierarchy of the observed moduli and the right amount of CP violation (in particular the physical phase $\gamma$ which is essentially extracted from tree-level processes): this will be addressed in the numerical analysis of section 2.3. For the moment, let us simply show that the mass matrices in eqs. (20)-(21) can lead to a CP violating CKM matrix. Following eqs. (17)-(18) and (20)-(21), we can write:

$$
M^0_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \hat{M}^0_d, \quad M^0_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \hat{M}^0_u.
$$
with $M^0_d$ and $M^0_u$ real. Then, the bi-diagonalisation of $M^0_d$ and $M^0_u$ reads [20]

$$M_d = \text{diag}(m_d) = U^d_L M^0_d O_R^d, \quad M_u = \text{diag}(m_u) = U^u_L M^0_u O_R^u.$$  \hspace{1cm} (26)

with

$$\text{eq. (22) } U^d_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} O_R^d \quad \text{and} \quad U^u_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} O_R^u,$$  \hspace{1cm} (27)

and $O_R^d$, $O_R^u$, $O_L^d$, $O_L^u$, real orthogonal matrices. Notice the important sign difference in $\theta$ between $U^d_L$ and $U^u_L$ in eq. (22), which gives the following CKM matrix $V = U^d_L U^u_L$,

$$V = O_L^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\theta} \end{pmatrix} O_L^d.$$  \hspace{1cm} (28)

For generic $O_R^d$ and $O_R^u$, if $e^{i2\theta} \neq \pm 1$, $V$ in eq. (28) incorporates an irremovable source of CP violation, that is, the model has spontaneous CP violation as desired. Having established the diagonalisation of the mass matrices that the model can provide a spontaneous origin of CP violation, we now analyse what are the properties of the $N^0_d$, $N^0_u$ matrices in the quark mass bases in order to complete the picture. Starting again with the down quark sector, it follows from eq. (22) that

$$N_d = U^d_L N^0_d O_R^d = t_\beta U^d_L M^0_d O_R^d - (t_\beta + t^{-1}_\beta) U^d_L P_3 M^0_d O_R^d = t_\beta M_d - (t_\beta + t^{-1}_\beta) U^d_L P_3 M^0_d M_d,$$  \hspace{1cm} (29)

with $P_3$ the projector in eq. (24) and $U^d_L$ in eq. (22). In the last term of eq. (29),

$$U^d_L P_3 U^d_L = O_L^T P_3 O_L^d,$$  \hspace{1cm} (30)

that is, $N_d$ in eq. (29) is real. Apart from $t_\beta$, which depends on the details of the scalar sector, this last term encodes all the remaining freedom present in the couplings of (down) quarks to scalars. In order to parametrise it, we introduce a real unit vector $\hat{r}_{[d]}$, and correspondingly, a complex unit vector $\hat{n}_{[d]}$, with components

$$\hat{r}_{[d]} = [O^d_L]_3, \quad \hat{n}_{[d]} = [U^d_L]_3 = e^{i\theta} \hat{r}_{[d]}.$$  \hspace{1cm} (31)

Then

$$[U^d_L P_3 U^d_L]_{ij} = [O^d_L P_3 O^d_L]_{ij} = \hat{n}_{[d]}^* \hat{n}_{[d]} = \hat{r}_{[d]} \hat{r}_{[d]} = \hat{r}_{[d]} \hat{r}_{[d]},$$  \hspace{1cm} (32)

and for $N_u$, similarly, we have

$$N_u = U^u_L N^0_u O_R^u = t_\beta M_u - (t_\beta + t^{-1}_\beta) U^u_L P_3 U^u_L M_u,$$  \hspace{1cm} (33)

with $U^u_L P_3 U^u_L = O_L^u T P_3 O_L^u$ and

$$\hat{r}_{[u]} = [O^u_L]_3, \quad \hat{n}_{[u]} = [U^u_L]_3 = e^{-i\theta} \hat{r}_{[u]} \quad \text{and} \quad [U^u_L P_3 U^u_L]_{ij} = \hat{n}_{[u]}^* \hat{n}_{[u]} = \hat{r}_{[u]} \hat{r}_{[u]}.$$  \hspace{1cm} (34)

Like $N_d$, $N_u$ is real. We finally have the following form of the $N_d$ and $N_u$ matrices:

$$[N_d]_{ij} = t_\beta \delta_{ij} m_d - (t_\beta + t^{-1}_\beta) \hat{n}_{[d]}^* \hat{n}_{[d]} m_d,$$  \hspace{1cm} (35)

$$[N_u]_{ij} = t_\beta \delta_{ij} m_u - (t_\beta + t^{-1}_\beta) \hat{n}_{[u]}^* \hat{n}_{[u]} m_u.$$  \hspace{1cm} (36)
Notice that, since \( V = U_L^d U_L^d \), the complex unitary vectors \( \tilde{n}_{[d]} \) and \( \tilde{n}_{[u]} \) are not independent:

\[
\tilde{n}_{[d]} = \tilde{n}_{[d]} V_{ji}, \quad \tilde{n}_{[u]} = V_{ji}^* \tilde{n}_{[d]}.
\]

Equations (35)-(36) encode the sources of flavour changing couplings of the scalars in the model; it is important to stress that, since they involve explicitly mass factors and elements of \( \tilde{n}_{[d]} \) and \( \tilde{n}_{[u]} \) (which are, of course, bounded from above), these flavour changing entries have a controlled nature, they are not arbitrary.

Furthermore, an important connection between the presence of SFCNC and SCPV arises in this model. Although in eqs. (35)-(36) all possible flavour changing couplings are a priori present and proportional to the products of two components of either \( \tilde{r}_{[u]} \) or \( \tilde{r}_{[d]} \), one may think that they can be parametrically “switched off” (in either the up or the down quark sector, but not in both since \( \tilde{r}_{[u]} \) or \( \tilde{r}_{[d]} \) are related through eq. (37)) by choosing \( \tilde{r}_{[u]} \) or \( \tilde{r}_{[d]} \) to have a single non-vanishing component, for example \( \tilde{r}_{[d]} = (0, 0, 1) \). This kind of limit would correspond to one of the different BGL models [12–14], where SFCNC are only present in one quark sector. However, in that case, since \( \tilde{r}_{[u]} \) is simply the third row of \( O^d \), \( O^d \) would be block-diagonal and \( e^{i2\theta} \) in eq. (28) could be commuted to the right of \( O^d \) and rephased away; that is, in order to have SCPV, it is necessary that SFCNC are present in both sectors. To close this section, we finally write \( \mathcal{L}_Y \) in eq. (15) in terms of the physical fields:

\[
\mathcal{L}_Y = \mathcal{L}_{qq} + \mathcal{L}_{G\bar{q}q} + \mathcal{L}_{H\bar{q}q} + \mathcal{L}_{\Lambda\bar{q}q} + \mathcal{L}_{H^\pm\bar{q}q}.
\]

\[
\mathcal{L}_{qq} = -\bar{d}_L M_d d_R - \bar{u}_L M_u u_R + \text{H.c are the mass terms, } \mathcal{L}_{G\bar{q}q} \text{ gives the couplings to the would-be Goldstone bosons while the Yukawa couplings of the neutral and charged scalars are } \mathcal{L}_{Sqq}, \]

\( S = h, H, A, H^\pm \). Introducing the hermitian and antihermitian combinations

\[
H_q \equiv \frac{N_q + N^\dagger_q}{2}, \quad A_q \equiv \frac{N_q - N^\dagger_q}{2},
\]

we have

\[
\mathcal{L}_{Sqq} = -\frac{S}{v} \left\{ \bar{d} \left[ R_{1s} M_d + R_{2s} H_d + i R_{3s} A_d \right] d + \bar{d} \gamma_5 \left[ R_{2s} A_d + i R_{3s} H_d \right] \right\} + \frac{S}{v} \left\{ \bar{u} \left[ R_{1s} M_u + R_{2s} H_u - i R_{3s} A_u \right] u + \bar{u} \gamma_5 \left[ R_{2s} A_u - i R_{3s} H_u \right] \right\},
\]

with \( s = 1, 2, 3 \) for \( S = h, H, A \), respectively, and

\[
\mathcal{L}_{H^\pm\bar{q}q} = -\frac{\sqrt{2} H^\pm}{v} \left[ \bar{u} L N_d d_R - \bar{u}_R N^\dagger_d V d_L \right] - \frac{\sqrt{2} H^-}{v} \left[ \bar{d}_R N^\dagger_d V^\dagger u_l - \bar{d}_L V^\dagger N_u u_R \right].
\]

With eqs. (35)-(36), \( [H_q]_{ij} \) and \( [A_q]_{ij} \) in eq. (39) read

\[
[H_q]_{ij} = t_\beta \delta_{ij} m_q - (t_\beta + t_\beta^{-1}) \tilde{n}_{[d]}^* \tilde{n}_{[d]} m_{d_i} + m_{d_j} \frac{m_{d_i} + m_{d_j}}{2},
\]

\[
[A_q]_{ij} = (t_\beta + t_\beta^{-1}) \tilde{n}_{[d]}^* \tilde{n}_{[d]} m_{d_i} - m_{d_i} \frac{m_{d_i} - m_{d_j}}{2}.
\]

### 2.3. Phenomenology

In the previous sections we have discussed both the scalar and quark flavour sectors of the model, in this section we finally analyse the model considering simultaneously (i) obtention of an adequate CKM matrix (moduli \( |V_{ij}| \) in the first and second rows and phase \( \gamma \) in agreement with data), (ii) a scalar sector verifying boundedness, perturbative unitarity, oblique parameter constraints and \( m_H, m_A, m_{H^\pm} > 150 \text{ GeV} \), and (iii) a number of constraints, listed in the following, which involve both the quark Yukawa couplings and the scalar sector. (For additional details we refer, again, to [20]).
Production × decay signal strengths of the 125 GeV Higgs-like scalar h.
Neutral meson mixings.
\( \text{Br}(B \to X_s \gamma) \).
Rare top decays \( t \to hq \).

The analysis has two main goals:
(i) to establish that the model is viable after a reasonable set of constraints is imposed;
(ii) to explore the prospects for the observation of some definite non-SM signal. We concentrate in particular on flavour changing decays \( t \to hc, huu \) and \( h \to bs, bd \), of interest, respectively, for the LHC and the ILC. We also consider a representative low energy observable, the time dependent CP violating asymmetry in \( B_s \to J/\Psi \Phi \), \( A^{CP}_{J/\Psi \Phi} \), for which the SM prediction is \( A^{CP}_{J/\Psi \Phi} \sim -0.04 \), while current results [24] give \(-0.030 \pm 0.033\), leaving significant room for New Physics contributions.

Further implications for the phenomenology of H, A and \( H^\pm \), in particular for the observation of these new scalars at the LHC, vary significantly between allowed regions in the parameter space of the model and can also depend on details of the lepton sector (which are not considered here), and thus we do not address these additional implications further.

A selection of results is shown in Figure 1.

Figures 1(a), 1(b) and 1(c) show that the allowed regions have \( \tilde{t}_Q \) with one “dominant” component, the two remaining ones being much smaller. This suppresses effectively the flavour changing couplings, without reaching, as discussed in the previous section, the situation in which they are completely absent in one of the quark sectors. In Figures 1(a), 1(b) and 1(c) that would be the case at the central points in correspondence, respectively, with the flavour structures of the BGL models of types \( d, s \) and \( b \). The regions close to the flavour structures of up type BGL models are also indicated in the Figures. Figures 1(d), 1(e) and 1(f) illustrate straightforwardly the absence of a decoupling regime in this model. Figure 1(g) shows that deviations from \( R_{31} = 1 \) (in that case \( h \) would be exactly SM-like) can be achieved for almost all values of \( t_\beta \) within the allowed range, while Figures 1(h) and 1(i) illustrate, as expected, that for large values of \( |\sin 2\theta| \), \( R_{31} \) (which controls the amount of pseudoscalar \( t^0 \) entering \( h \)), reaches the larger allowed values, while \( R_{11} \) is reduced. Notice in particular that, overall, \( |R_{31}| \geq 10^{-2} \) and that values as large as \( |R_{31}| \sim 0.4 \) are allowed. Finally, Figures 1(j), 1(k) and 1(l) illustrate the prospects for some definite deviations with respect to SM expectations in different flavour changing neutral transitions. Figure 1(j) shows \( \text{Br}(h \to bs) \) vs. \( A^{CP}_{J/\Psi \Phi} \); it is interesting to notice that: (i) \( \text{Br}(h \to bs) \) can reach values as large as \( 10^{-2} \), relevant for searches at the ILC, and (ii) significant deviations of the SM expectation \( A^{CP}_{J/\Psi \Phi} \sim -0.036 \) can arise. An interesting correlation among New Physics effects follows: \( A^{CP}_{J/\Psi \Phi} \) values nearly different from SM expectations (the dashed vertical line in Figure 1(j)) would necessarily require values of \( \text{Br}(h \to bs) \) in the range \( 10^{-4}-10^{-2} \). The origin of such a correlation is clear: the tree level couplings that induce \( h \to bs \) at that level also contribute significantly to the dispersive amplitude \( M_{\tilde{B}_s}^{\tilde{B}_s} \) in \( B^0 \to \bar{B}^0 \) mixing, changing its phase while maintaining \( |M_{\tilde{B}_s}^{\tilde{B}_s}| \) (i.e. \( \Delta M_{\tilde{B}_s} \)).

According to the discussion on the connection of SFCNC and CP violation, the total rate of flavour changing decays of \( h \), \( \text{Br}(h \to q_1 q_2) \equiv \text{Br}(h \to bs) + \text{Br}(h \to bd) + \text{Br}(h \to sd) + \text{Br}(h \to cu) \), is bounded from below: this is clearly illustrated in Figure 1(k).

Finally, Figure 1(l) shows the most relevant flavour decays involving \( h, h \to bs \) in the down quark sector and \( t \to hc \) in the up quark sector: it is clear that there is sufficient available parameter space for the model to have both decays within experimental reach, although, obviously, having both rates in that interesting region is not compulsory.

\footnote{The notation is \( \text{Br}(h \to bs) \equiv \text{Br}(h \to bs + b\bar{s}), \text{Br}(h \to bd) \equiv \text{Br}(h \to bd + b\bar{d}) \), etc.}
Region allowed at 99% C.L. by the requirements of the full analysis.

Figure 1. Region allowed at 99% C.L. by the requirements of the full analysis.
Conclusions
A short overview of general aspects of Spontaneous CP Violation and Scalar Flavour Changing Neutral Couplings has been presented. Then, a 2HDM where CP Violation has a spontaneous origin, which also generates a realistic CKM matrix, and where Scalar Flavour Changing Neutral Couplings are present with controlled intensity, has been discussed. Among the relevant phenomenological consequences of the model, the existence of new scalars with masses below 1 TeV, and the possibility of having flavour changing processes such as $t \rightarrow hc$ or $h \rightarrow bs$ within experimental reach, have been explored.

Acknowledgments
The author thanks the organization of the Discrete 2018 Symposium for the excellent development of the conference and acknowledges support from Fundação para a Ciência e a Tecnologia (FCT, Portugal) through postdoctoral grant SFRH/BPD/112999/2015 and through the projects UID/FIS/00777/2019, CERN/FIS-PAR/0004/2017 and PTDC/FIS-PAR/29436/2017 which are partially funded through POCTI (FEDER), COMPETE, QREN and EU.

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