Optimal Tuning of a Class of Reset Controllers using Higher-Order Describing Function Analysis: Application in Precision Motion Systems

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Abstract

Currently, the demand for a better alternative to linear PID controllers increases due to the rising expectations of the high-tech industry. This causes many researchers to explore non-linear controllers like reset controllers. In literature, many reset architectures have been proposed to overcome the inherent linear controller limitations. However, an appropriate tuning method for these reset controllers has not been proposed so far. In this paper, an optimal tuning method for a class of reset controllers including a novel low-pass filter, is proposed using the recently developed frequency domain method which is applicable to analyze closed-loop performances of these controllers. In order to show the effectiveness of this approach, the performance of the optimally tuned reset controller is compared with another reset controller which is poorly tuned using the DF. Both controllers are also compared with a PID to showcase the effectiveness of the proposed method. The results re-assure that the DF method is not reliable for tuning reset controllers, and it is possible that a reset controller showing better performance according to a pure DF analysis has worse performances than a linear controller in practice. Between the two reset controllers compared, the proposed approach not only ensures optimal performance of the system, but also results in reduced overall control output. Indeed, this tuning method has the potential for enabling wide-scale application.

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1. Introduction

Proportional Integral Derivative (PID) controllers are the industry standard for several decades now. With linear PID limited by waterbed effect, the fast rising high-tech industry calls for a better alternative [1,3]. One of the alternatives is reset control which has gained a lot of attention due to its simple configuration. A reset controller is implemented by resetting all or subset of its states when its input crosses zero.

In 1958, the first reset element was introduced by Clegg [4]. Clegg Integrator (CI) is an integrator which resets its state to zero when its input crosses the zero. Then, Generalized First Order Reset Element (GFORE) and Generalized Second Order Reset Element (GSORE) have been developed to provide more design freedom and applicability [5]. Other reset conditions such as reset band [6,7] and fixed reset instants [8] have also been studied. In order to soften non-linearities of reset elements, several techniques like Partial Reset and PI+CI approaches have been proposed [9]. The advantages of reset control have been utilized to enhance the performance of several mechatronic systems [10-15]. Based on Describing Function (DF) analysis [16], it can be seen that reset controllers provide less lag phase in comparison with their base linear structures.

This phase advantage is utilized to introduce new compensators [5,12,17,19]. One of these novel reset controllers is 'Constant in gain Lead in phase' (CgLp) whose gain is constant while providing lead phase [5,12,18]. In these works, CgLp has been used as an alternative for the derivative to compensate part of the required phase lead. This is advantageous because the open-loop gets a better shape which results in better closed-loop performances. In all these cases, CgLp is tuned to get a specific amount of phase lead at the cross-over frequency. However, as a result of the design flexibility of reset controllers, various combinations of tuning parameters could be used to provide the same open-loop phase compensation at the cross-over frequency based on the DF
analysis.

However, not all sets of tuning parameters result in performance improvement, and an appropriate tuning method for these reset controllers has not been proposed so far. Although the DF method is straight-forward for tuning non-linear controllers, it is insufficient to use this method for frequency analyses of reset elements, especially when precision applications are considered \cite{20, 21}. Recently, the DF method has been extended to higher-order sinusoidal input describing functions (HOSIDF) for the frequency analyses of non-linearities such as backlash, friction, etc. \cite{22}. Then, HOSIDF is extended for analyzing reset controllers in the open-loop \cite{20}. Furthermore, a frequency approach is proposed to study reset controllers in the closed-loop and provide closed-loop HOSIDF of reset elements \cite{21}. There is no methodology that uses higher-order harmonics to design the reset controller. This paper, for the first time, uses the higher-order harmonic along with the pseudo-sensitivity function to tune the reset compensators. These reset compensators include a novel low-pass filter with lead phase and \(C_{gLp}\). The experimental validation verifies that the optimal tuning methodology proposed in this paper overcomes the limitation of linear controllers and outperform the PID controller.

In the remainder of this paper, preliminaries on reset controllers are given in Section 2. In Section 3, the optimal tuning method is explained based on the frequency domain analyses of reset controllers in the closed-loop. In Section 4, the optimally tuned controller is applied to a precision positioning stage, and its performance is compared with a PID. Conclusions and remarks for further study are provided in Section 5.

2. Preliminaries

2.1. DF and HOSIDF of reset controllers

The state-space representation of reset controllers is:

\[
\Sigma_r := \begin{cases} 
\dot{x}(t) = A_r x(t) + B_r e(t) & e(t) \neq 0 \\
x(t^+) = A_r x(t) & e(t) = 0 \\
u(t) = C_r x(t) + D_r e(t)
\end{cases}
\] (1)
in which $A_r$, $B_r$, $C_r$ and $D_r$ are the state matrices of the base linear system, $e(t)$ and $u(t)$ are the error input and control input, respectively. The reset action is triggered when the error crosses zero. The resetting matrix $A_\rho$ determines states’ value after reset action by which the non-linearity of reset systems can be tuned.

Since reset systems are non-linear, the DF analysis is popularly used in literature to study their frequency behaviour. The sinusoidal input DF of reset systems [1] is given in [15] as:

$$G_{DF}(j\omega) = C_r (j\omega I - A_r)^{-1} B_r (I + j\Theta_D(\omega)) + D_r$$

where $\Theta_D$ is:

$$\Theta_D(\omega) = \frac{-2\omega^2}{\pi} (I + e^{\frac{2\pi}{\omega}}) (I - e^{\frac{2\pi}{\omega}}) A_r (I + e^{\frac{2\pi}{\omega}}) - I) (\omega^2 I + A_r^2)^{-1}$$

In the HOSIDF method, a non-linear system is considered as a virtual harmonic generator, and HOSIDF is defined in the following way [22]:

$$G_n(j\omega) = \frac{a_n(\omega)e^{j\phi_n(\omega)}}{a_0}$$

in which $a_n$ is the $n^{th}$ component of the Fourier series expansion of the steady-state output of the system for a sinusoidal input. To include higher-order harmonics and obtain a more accurate frequency description of reset systems, HOSIDF is provided in [20] as:

$$G(j\omega,n) = \begin{cases} C_r (j\omega n I - A_r)^{-1} j\Theta_D(\omega) B_r & \text{for odd } n \geq 2 \\ 0 & \text{for even } n \geq 2 \end{cases}$$

where $n$ is the order of harmonics.

### 2.2. Pseudo-sensitivity for reset systems

In linear systems, the sensitivity function from reference signal $r(t)$ to error $e(t)$ can be calculated by

$$S(s) = \frac{e}{r} = \frac{1}{1 + G(s)C(s)}$$

where $G(s)$ and $C(s)$ are the transfer function of the plant and controller, respectively. For non-linear controllers, $C(s)$ can be replaced by DF of the controller to obtain sensitivity. However, it is not reliable to predict the precision of tracking performance
since high order harmonics are neglected. In order to analyze reset systems more accurately, a pseudo-sensitivity function \( S_\infty(\omega) \) for a sinusoidal reference \( r = r_0 \sin(\omega t) \) is defined in [21]. It is asserted that the tracking error of the reset system is a periodic function with the period of the first harmonic of the system \( \frac{2\pi}{\omega} \) if \( H_\beta \) condition [9, 23] is satisfied and reset instants have well-posedness property [9, 23]. Thus, the pseudo-sensitivity frequency response of reset system is defined as the ratio of the maximum tracking error of the system to the magnitude of the reference at each frequency as:

**Definition 1.** Pseudo-sensitivity \( S_\infty \)

\[
\forall \omega \in \mathbb{R}^+ \exists! t_{\text{max}} \in (t_{ss_0}, t_{ss_m}) \mid \forall t \in (t_{ss_0}, t_{ss_m}) : e(t_{\text{max}}) = e_{\text{max}} \geq e(t)
\]

\[
\Rightarrow S_\infty(j\omega) = \left( \frac{\max_{t_{ss_0} \leq t \leq t_{ss_m}} (r(t) - y(t))}{|r|} \right) e^{j\phi_{\text{max}}} = \left( \frac{e_{\text{max}}}{r_0} \right) e^{j\phi_{\text{max}}}
\]

where \( \phi_{\text{max}} = \frac{\pi}{2} - \omega t_{\text{max}} \), \( y(t) \) is the response of the closed-loop reset systems, \( t_{ss_0} \) and \( t_{ss_m} = t_{ss_0} + \frac{2\pi}{\omega} \) are the steady-state reset instants of the closed-loop system \( (e(t_{ss_0}) = e(t_{ss_m}) = 0) \). In a similar way, pseudo-control sensitivity \( CS_\infty(\omega) \), pseudo-complementary sensitivity \( T_\infty(\omega) \), and pseudo-process sensitivity \( PS_\infty(\omega) \) are defined in [21].

2.3. Stability

There are several theories to determine stability of reset control systems [9, 23, 26]. Among those, \( H_\beta \) condition [9, 23] gets a lot of attention, and it states that:

**Theorem 1.** A reset system with any bounded input is uniformly bounded-input bounded-state (UBIBS) stable if there exists a \( \beta \in \mathbb{R}^{n_r \times 1} \) and \( P \in \mathbb{R}^{n_r \times n_r}, P \succ 0 \) such that the restricted Lyapunov equation

\[
P > 0, \quad A_{cl}^T P + PA_{cl} < 0
\]

\[
B_0 P = C_0
\]

In the case of FORE: \( |A_p| \leq 1 \)
has a solution for $P$, in which

$$C_0 = \begin{pmatrix} \beta C_G & 0_{n_r \times n_L} & P_P \end{pmatrix}, \quad B_0 = \begin{pmatrix} 0_{n_G \times n_r} \\ 0_{n_L \times n_r} \\ I_{n_r \times n_r} \end{pmatrix},$$

$n_r$ is the number of reset state of the controller, $n_L$ is the number of non-reset state of the controller, $n_G$ is the number of plant state, $C_G$ is the output matrix of the plant, and $A_{cl}$ is the dynamic state-space matrix of the closed-loop of the whole system.

### 3. Optimal Tuning Method

In this section, an optimal tuning guideline is proposed based on the defined pseudo-sensitivity ($S_{\infty}$) using optimization methods. First, the reset controller structure which is introduced.

#### 3.1. Reset Lead Compensators (RLC)

One of these RLC is $C_{gL_P}$ (10) which is made using GFORE with the combination of the first order lead filter in series to construct a constant gain with lead phase behaviour [5, 12, 27]. The structure of $C_{gL_P}$ is

$$C_{CgL_P}(s) = \left( \frac{1}{\omega_r} + 1 \right)^{\gamma} \left( \frac{1}{\omega_d} + 1 \right)$$

(10)

where $\omega_r$ is the corner frequency of the reset element, $A_P = \gamma$ is the reset matrix, and $\omega_d$ and $\omega_t$ are the corner frequency of the lead filter. In order to have a constant gain (figure [1a], $\omega_d$ and $\omega_t$ have to be almost equal (there is a small correction factor which is provided in [5, 12, 27]). However, from the loop-shaping viewpoint [1, 28], the less gain of the lead element of the controller at the cross-over frequency, the more precision is achieved by the controller. Thus, it is better to have a RLC with $|DF_{RLC}(j\omega_c)| < 1$. As it is shown in figure [1b], considering DF of the GFORE, appropriate $\omega_r$ and $\omega_d$ produces a lag filter whose gain has a negative slope while it provides lead phase. This novel RLC can be also categorized as a CompLex Order Controller (CLOC) (for more details see [19, 27]).
3.2. Problem Formulation

The structure of the controller is RLC+PID as

\[
C_{RLC+PID}(s) = K_p \left( \frac{1}{\omega_i s} + 1 \right) \left( \frac{\omega_c}{\omega_i s} + 1 \right) \left( 1 + \frac{\omega_l}{s} \right) \left( \frac{\omega_c}{\omega_i s} + 1 \right) .
\]

In this tuning guideline, the controller is tuned given the following specifications crossover frequency \( \omega_c \), phase margin \( \phi_m \), and modulus margin \( M_m \). Furthermore, \( \omega_i \) is tuned as \( \frac{\omega_i}{10} \) as per guideline provided in [3, 28]. The controller structure (11) is re-written based on \( \omega_c \) as

\[
C_{RLC+PID}(s) = K_p \left( \frac{1}{\omega_c s} + 1 \right) \left( \frac{\omega_c}{\omega_c s} + 1 \right) \left( 1 + \frac{\omega_l}{10s} \right) \left( \frac{\omega_c}{\omega_c s} + 1 \right) .
\]

It is shown that the sequence of controller parts has effects on the performance of non-linear controllers [29]. In this research, the reset element is the first element of the controller, and other linear parts come after it. Similar to optimal tuning guidelines
[30–33], multiple constraints are considered to tune parameters \((a, c, d, e, K_p, b, \gamma)\) in this method. In order to use \(S_{\infty}, H_{\beta}\) condition (Theorem 1) must be satisfied. Thus, the designed reset controllers have stable base linear structures and their stability is guaranteed as described in Section 2.3.

To have a stable open-loop configuration, \(\gamma\) is considered between \(-1\) to 1 [15]. As explained in Section 3.1 the gain of the reset compensator of the controller has to be less or equal to 1 at the cross-over frequency. To have iso-damping behaviour [34–37], the phase behaviour of the system must be:

\[
\frac{d\left(\text{CG}(j\omega)\right)}{d\omega} \bigg|_{\omega = \omega_c} = 0 \Rightarrow \left(\frac{\mathcal{K}'\mathcal{C} + \mathcal{K}'\mathcal{G}}{\mathcal{K}}\right) \bigg|_{\omega = \omega_c} = 0. \tag{13}
\]

For linear plants with a small time delay when the cross-over frequency is very far from the resonance frequency, \(\mathcal{K}' \approx 0\). Consequently,

\[
\mathcal{K}' \bigg|_{\omega = \omega_c} = 0. \tag{14}
\]

Therefore, to make the controller robust against gain variations (iso-damping) for linear plants with small time delay, these controllers are designed so that

\[
\frac{d\left(\text{CG}(j\omega)\right)}{d\omega} \bigg|_{\omega = \omega_c} = 0. \tag{15}
\]
Table 1: Constraints for tuning the controller

| No. | Constraint | Reason |
|-----|------------|--------|
| 1   | $1 \leq c, d, e \leq 10$ | Acceptable Noise and Disturbance Rejection |
| 2   | $a, b \geq 1$ | Providing phase at the Cross-over Frequency |
| 3   | $-1 < \gamma < 1$ | Open-Loop Stability |
| 4   | $|DF_{RC}(j\omega)| \leq 1$ | CgLp or Lag filter with lead phase |
| 5   | $|DF_{RC}(j\omega)\cdot PID(j\omega)\cdot G(j\omega)| = 1$ | Cross-Over Frequency Definition |
| 6   | $|DF_{RC}(\omega) + PID(\omega) + G(\omega)\cdot \varphi_m| \geq 5^\circ$ | Phase Margin Definition |
| 7   | $\frac{d}{d\omega}DF_{RC}(j\omega)\cdot PID(j\omega)|_{\omega=\omega_c} = 0$ | Base Linear Stable |
| 8   | $\max |\frac{1}{1+DF_{RC}(\omega)\cdot PID(\omega)\cdot G(\omega)}| < M_m$ | Iso-damping Behaviour |
| 9   | Checking (Theorem 1) | Stable |

All in all, constraints for tuning this controller are summarized in Table 1. Suppose that there are $N$ sets of parameters $\chi = (a, c, d, e, K_p, b, \gamma)$ which satisfies the mentioned constraints. Now, a cost function is needed to tune controller optimally. For this purpose, since reference tracking signals consist of low frequency, some low frequencies ($\omega_1, \omega_2, \ldots, \omega_q$) are selected for optimization (for some application in which a special tracking path is always applied, it is better to consider the Fourier component frequencies of the path for optimization). The pseudo-sensitivity (Definition 1) at these frequencies is depended on tuning parameters (i.e. $|S_{\omega_k}(j\omega_k)| = f_k(\chi)$). Using $|S_{\omega_k}(j\omega_k)|$, from precision perspective, a multi-objectives optimization problem can be defined as

$$\min_{\chi} \left\{ f_k = |S_{\omega_k}(\omega_k)|, \ k \in \mathbb{N}, k \leq q \right\}$$

subject to Constraints 1-10

This multi-objectives optimization problem can be solved using e-constrain method, genetic algorithm, weighted sum, and so forth [38-41]. In this paper, in order to simplify the problem, this multi-objectives optimization is changed to a simple optimizing...
tion problem utilizing weighted sum technique. In this way, the cost function is defined as

\[ J = \sum_{k=1}^{q} \frac{f_k}{\delta_k} \]  

(17)
in which \( \delta_k \) is weighting coefficient which is equal to maximum value of \( f_k \). In other words, \( \delta_k \) is calculated through the following relation.

\[ \delta_k = \maximize_{\chi} \ f_k(\chi) \]  

subject to Constraints 1-10  

(18)

Using these weighting coefficients, the controller which provides the most differences between the best and worst performance at all selected frequencies is considered as the optimal controller. This single objective optimization method can be solved using fminsearch, genetic algorithm, Lsqmin, etc. [42–47]. Thus, before solving final optimization (19), first, it is needed to solve \( k \) single objective maximization problem (18). Finally, the controller parameters \( \chi = (a, c, d, e, K_p, b, \gamma) \) are tuned solving following optimization problem.

\[ \min_{\chi} J(\chi) \]  

subject to Constraints 1-11  

(19)

4. Practical Example

To show the effectiveness of the proposed tuning method, a precision positioning stage (figure 3) is used as a benchmark in this paper [48]. In this stage, which is termed "Spyder", three actuators are angularly spaced to actuate 3 masses (indicated by B1, B2, and B3) which are constrained by parallel flexures and connected to the central mass D through leaf flexures. Only one of the actuators (A1) is considered and used for controlling the position of mass B1 attached to the same actuator which results in a SISO system. To identify the plant and implement the controller, an FPGA module (CompactRio from National Instruments) has been used. A linear power amplifier is utilized to drive the Lorentz actuator, and Mercury M2000 linear encoder is used to obtain position feedback with a resolution of 0.1 µm. The identified frequency response data of the system is shown in figure 2.
Although the plant is a collocated double mass-spring system, the identified frequency response data is well approximated by a mass-spring-damper system (equation 20) as shown in figure 2 (in order to use relations provided in [21], the time delay
\( e^{-0.00014s} \) is approximated by the first order Pade method \([49]\) as \( -s + \frac{14400}{s + 14400} \).

\[
G(s) \approx \frac{Ke^{-cs}}{s^2 + \frac{2s}{\omega_c} + 1} = \frac{1.14e^{-0.00014s}}{s^2 + \frac{0.08s}{87.3} + 1}
\]  

(20)

The design requirements for this system are:

- the cross-over frequency: \( \omega_c = 100 \text{ Hz} \)
- the phase margin: \( \varphi_m = 30^\circ \)
- the modulus margin: \( M_m \leq 6.5 \text{ dB} \)

Now, to satisfy these requirements (for non-linear structure, the DF is used), the controller structure \((12)\) is tuned based on the method described in Section \([3]\). The low frequencies (1, 3, 5, 7, 10, 15) Hz are selected for optimization. Besides, the weighting coefficients for these frequencies are calculated through \((18)\) as (161.5, 24.1, 14.3, 12.6, 18.2, 51.1), respectively. The optimization problem \((19)\) is solved using fminsearch method, and the optimal controller is obtained as

\[
C_{\text{OR}} = 25.5 \left( \frac{1}{s} + 1 \right)^{0.3} \left( \frac{\frac{s}{105.2\pi} + 1}{\frac{s}{1640\pi} + 1} \right) \left( 1 + \frac{20\pi}{s} \right) \left( \frac{\frac{s}{105.2\pi} + 1}{\frac{s}{260\pi} + 1} \right)
\]  

(21)

In order to compare the performance of optimally tuned controller with a linear controller, a PID structure is also considered to satisfy the mentioned requirements and constraints. To have the maximum phase at the bandwidth, the PID structure is selected similar to the structure introduced in \([3, 28, 50]\) as:

\[
C_{\text{PID}} = K_p \left( \frac{1}{s} + 1 \right) \left( \frac{\frac{s}{\omega_c} + 1}{\frac{s}{g\omega_c} + 1} \right) \left( 1 + \frac{\omega_c}{I_s} \right)
\]  

(22)

To have equal conditions, the PI and low pass filter of the PID is tuned the same as \(C_{\text{OR}}\), and \(K_p\) and \(g\) are set so that the system has the specified bandwidth and phase margin. Finally, the \(C_{\text{PID}}\) becomes

\[
C_{\text{PID}} = 18.46 \left( \frac{1}{s} + 1 \right) \left( \frac{\frac{s}{77\pi} + 1}{\frac{s}{520\pi} + 1} \right) \left( 1 + \frac{20\pi}{s} \right)
\]  

(23)
Furthermore, the controller structure [12] is tuned based on the DF so that it has highest gain at low frequencies while it has the same gain at high frequencies and satisfy the mentioned requirements. Also, its base linear system is stable and it achieves the maximum phase at the cross-over frequency. This controller, which is termed $C_{DR}$, has the better DF than $C_{OR}$ form precision perspective, but it has the higher amplitude of high order harmonics. It is to show that the DF cannot always be trusted to design reset controllers particularly for precision motion systems. $C_{DR}$ is

$$C_{DR} = 40.7 \left( \frac{1}{\frac{s}{10\pi} + 1} \right)^{0.5} \left( \frac{\frac{s}{13.2\pi} + 1}{\frac{s}{1880\pi} + 1} \right) \left( 1 + \frac{20\pi}{s} \right) \left( \frac{s}{\frac{153.8\pi}{s} + 1} \right).$$

(24)

which has to outperform the $C_{OR}$ considering only DF. In the following, the frequency and time domain responses of these designed controllers are compared with each other.

4.1. Pseudo-Sensitivities

In this section, open-loop and closed-loop frequency responses of three designed controllers are presented. The frequency responses of the $C_{PID}$ are obtained through relations for linear controllers [3]. The frequency responses of two other reset controllers are calculated through the Toolbox provided in [21, 51]. Figure 4 shows the open-loop and controller frequency responses. All controllers provide the same phase at the cross-over frequency. Moreover, since the time delay of the plant is very low, the constraint (8) makes the system robust against the gain variation (iso-damping behaviour $L(j\omega_c)' = 0$) as shown in figure 4b. Based on DF analyses (the first harmonic), the $C_{DR}$ has to have better tracking than two other controllers. However, its $3^{rd}$ harmonic is higher than the $C_{OR}$.

The closed-loop frequency responses of the controllers including defined pseudo-sensitivities and DF are illustrated in figure 5 and 6. The pseudo-sensitivity and pseudo-complementary sensitivity frequency response of reset controllers with their DF, and sensitivity and complementary sensitivity of PID are illustrated in figure 5.
Figure 4: Open-loop and controller frequency responses including linear response of $C_{PID}$, DF and 3rd harmonic of $C_{OR}$, and DF and 3rd harmonic of $C_{DR}$

Figure 5: Complementary and sensitivity frequency responses including linear response of $C_{PID}$, DF and pseudo frequency responses ($\omega$) of $C_{OR}$, and DF and pseudo frequency responses ($\omega$) of $C_{DR}$

As was illustrated (figure 5b), whereas the DF analyses, pseudo-sensitivities show that not only the $C_{DR}$ does not have the best tracking performance, but also it has
the worst tracking performance among the controllers. As was expected, the \( C_{OR} \) has the best performance among the controllers and outperforms linear PID. Also, since HOSIDF of \( C_{OR} \) is low at low frequencies, its DF and pseudo-sensitivity is merged at that frequency range. In addition, based on the complementary sensitivity (figure 5a), the noise rejection performance of \( C_{OR} \) is almost the same as \( C_{DR} \) better than PID controller. Figure 6 shows the control and process sensitivity responses of these controllers.

Similarly, unlike DF analysis, pseudo-process sensitivities show that the \( C_{OR} \) attenuates disturbances better than other controllers (figure 6a). Also, \( C_{DR} \) has the worst disturbance rejection among these three controllers. Furthermore, based on pseudo-control sensitivity (figure 6b), there is a significant difference between the real control output and what is predicted by DF. In addition, the control output of reset controllers are larger than linear PID. This is explained by the fact that reset elements produce jumps in their output and differentiation of jumps produces a large control output. Since the HOSIDF of \( C_{OR} \) are smaller than \( C_{DR} \) at low frequencies (figure 4), \( C_{OR} \) has smaller control output than \( C_{DR} \) in that frequency range.

![Diagram](image_url)

(a) Process sensitivity frequency responses  
(b) Control sensitivity frequency responses

Figure 6: Process and control sensitivity frequency responses including linear response of \( CPID \), DF and pseudo frequency responses (•) of \( C_{OR} \), and DF and pseudo frequency responses (---) of \( C_{DR} \).
4.2. Time domain results

In this part, the time domain results of three designed controllers are compared with each other. In order to implement controllers (figure 7), each controller is discretized with sample time $T_s = 100 \mu s$ using Tustin method [3, 28, 52]. Furthermore, in order to provide well-posedness property [9, 23], there is no reset instants in tandem.

![Figure 7: The block diagram of the whole system for implementing three controllers (reset matrices are discretized)](image)

The step responses (step of 10 µm) of the system with these controllers are illustrated in figure 8. To assess iso-damping behaviour of the controllers, the gains of the controllers are varied between 80% to 120% of their nominal values. The three step responses have the same rise time (there are small differences because the closed-loop bandwidths ($|T(j\omega_b)| = 3 \text{ dB}$) of controllers (figure 5a) have small differences. The overshoot of reset controllers $C_{OR}$ and $C_{DR}$ are less than PID because the modulus margin of reset controllers are less than the PID controller. The controller $C_{OR}$ has the least settling time and the best steady-state performance among these controllers. Furthermore, all three controllers show iso-damping behaviour. However, $C_{OR}$ and $C_{DR}$ are more robust against gain variation than PID controller. Based on DF analyses, reset controllers would have had similar overshoot as PID, but they have less overshoot than PID. This may have a relation with $T_{\infty}$. Since the peaks of $T_{\infty}$ of two designed reset controllers are less than the peak of $T$ of the PID controller (figure 5), they have smaller overshoot than the PID controller.

In order to examine the tracking performance of controllers, one triangular reference with the amplitude of 400 µm (figure 9a) and one sinusoidal reference $r(t)$ =
$111 \sin(10\pi t) \, \mu m$ (figure 9b) is applied to the system for tracking.

As was predicted by pseudo-sensitivity (figure 5b), $C_{OR}$ has the best performance at 5Hz among other controllers (figure 9d). As shown in figure 9c, $C_{OR}$ also has the best tracking performance among other controllers for the triangular reference (Table 3) which is a combination of several frequencies. For both tracking trajectories, unlike DF prediction, $C_{DR}$ has the worst tracking performance which is consistent with pseudo-sensitivity frequency response (figure 5b). Although the super-position law does not hold for reset controllers, it can be seen that when the pseudo-sensitivity of one system is better than the other one, that controller has better tracking performance even for references which is a combination of several frequencies (figure 9c).
Moreover, the control output of these controllers for sinusoidal reference are depicted in figure 10. As was expected from pseudo-control sensitivity (figure 6b), $C_{DR}$ has the largest control output among other controllers. It means that this tuning method not only enhances the performance of the system, but also it increases the energy efficiency of the system by reducing the controller output.
Figure 10: Control output of three designed controllers for the sinusoidal reference

Figure 11 compares the noise and disturbance rejection of these controllers. In order to study the noise rejection performance of controllers, a white noise with amplitude of the 5µm is applied to the system. Again, as was expected from pseudo-complementary sensitivity (figure 5a), the noise rejection of $C_{OR}$ is almost the same as $C_{DR}$ and better than PID controller (Table 3).

![Figure 11](image1.png)

(a) Disturbance rejection  (b) Noise rejection

Figure 11: Disturbance and noise rejection of three designed controllers

In order to evaluate the ability of controllers for attenuating disturbance, a sinusoidal disturbance $w(t) = 190 \sin(14\pi t)$ µA is applied to the system. Similar to pseudo-process sensitivity (figure 6a), $C_{OR}$ has the optimal and $C_{DR}$ has the worst disturbance rejection performance among controllers. In addition, the tracking error of the reset controllers at 5 Hz and the real error due to disturbance are obtained accurately through
Table 2: Comparison between theoretical and experimental results for tracking and disturbance rejection

| Performance            | C1TR | C1TR | C1TD |
|------------------------|------|------|------|
|                        | Theory | Experiment | Theory | Experiment | Theory | Experiment |
| Tracking $e_r$ (dB)    | -37.57 | -35.8 | -22.6 | -26.2 | -34.8 | -32.2 |
| Disturbance rejection $e_w$ (dB) | -33.1 | -34 | -18.8 | -24 | -30.4 | -30.5 |

pseudo-sensitivity (figure 5b) and pseudo-process sensitivity (figure 6a), respectively (Table 2). The difference between experimental and theoretical sensitivity results is due to quantization, digitalization of controllers, numerical problems of the toolbox, and existed noise of the whole system.

Table 3: Tracking and noise performance of three controllers

| Performance            | C1TR | C1TR | C1TD |
|------------------------|------|------|------|
|                        | RMS $e(t)$ | Max $e(t)$ | RMS $e(t)$ | Max $e(t)$ | RMS $e(t)$ | Max $e(t)$ |
| Noise                  | 5.02 | 20 | 3.59 | 18 | 7.94 | 35 |
| Triangular Tracking    | 2.64 | 4.17 | 6.68 | 15.20 | 3.98 | 6.13 |

To wrap up, in this paper, the defined pseudo-sensitivity is combined with an optimization method to tune reset controllers, and we show the effectiveness of this approach in a practical example. Based on time and frequency domain results, it is shown that if a reset controller tuned considering only DF, there is a possibility that not only it does not outperform linear controllers, but also it has the worse performance than linear controllers. Therefore, it is insufficient to tune reset controllers only based on DF, and we need to consider pseudo-sensitivity.

5. Conclusion

This paper has proposed an optimal tuning method for reset controllers based on the developed method for the frequency domain study of reset controllers. In this method, a PID structure with a reset element is considered and its parameters is tuned so that the pseudo-sensitivity is minimized under several constraints. The proposed reset element exhibits a lead behavior in phase while its gain decreases. Also, the tuned controllers with this method, make systems with small time delay robust against gain
variations. To show the effectiveness of the proposed approach, the performance of this optimally tuned controller is compared with a linear PID and a reset+PID controller which is tuned based on DF on a precision mechatronic system. The results show that the DF is not reliable for tuning reset elements. Comparing this method with DF, not only this method enhances the performance of the system, but also it reduces the control output significantly. Also, pseudo-sensitivity frequency responses predict the real performance of the system accurately. In addition, the optimally tuned controller outperforms linear PID controller. Indeed, this tuning method opens a gate for tuning reset controllers in the frequency domain.

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