The Semi-empirical equation to calculate stopping power of the electrons in human blood and skin tissues

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ABSTRACT

In this work the mass stopping power electron (MSPE) in human blood and skin tissues for electrons at energy range from 0.01MeV to 1000MeV are calculated in MeV cm²/g unit by summation the collisional SPE with adding density effect (δ) and radiative SPE contributions. Through using the Bragg’s additively rule for compound targets (Z/A) and from semi-empirical (SPE) equations in terms of kinetic energy E(MeV) for electrons in the ranges of the low energies from 0.01MeV to 0.7MeV and the high energies from 0.7MeV to 1000MeV then have been fitted by a three-parameter approximation written by MALAB2016 language. From fitting results with experimental SPE data calculated using ESTAR program and the error ration and correlation coefficient are calculated. All the results of these equations were compared with ESTAR program data and written by MALAB2016 language. The purpose of this paper is equipping up to date mass SPE information, with focus on the requirements of electron beam therapy (EBT) where electrons are directed to a tumor position.

Keyword:
Stopping power, Electrons, human blood and skin.

Introduction

In radiation physics, radiation biology and medicine, it is often important to have exact information about the mass SPE of different material and human body parts for charged particles that means the average rate at which the charged particles lose energy along their path [1]. Many experimental and theoretical studies about energy loss, mass SPE, mean free path, straggling of charged particles and ions such as (H, He, Li, C, O) and equivalent dose have been carried in many different human body parts [2]. With the advent of high-energy linear accelerators, electron beam therapy can treat such skin damages as basal-cell cancer because an electron beam penetrates to a limited depth before being absorbed, up to a depth of about 5cm for electron energies in the range 5–20 MeV [3,4]. The total mass SPE of electron is defined as the mean energy loss per unit path due to the ionization and excitation losses as well as the radiation loss [5]. When studying the SPE, one must count two kinds different energy loss techniques: First due to collision electron with the target’s other electrons (collisional SPE) and the second due to interaction electron with field of the target’s nuclei (radiative SPE). The second is significant to light particles because it is of high value at higher energy [6]. The Bethe equation is used to calculate the SPE when the incident light particle energy is higher than 0.01MeV, but there is a reduction of data at lower energies [6,7]. We report calculated total mass SPE of electrons with matter that will be studied, also the effects of electrons interactions with human skin tissues and blood are displayed. The aim of the this study is to calculate the SPE results from the external and internal radiation interactions with material, this will leads the estimation of energy loss and doses which are very important for electron therapy and the possible damage to adjacent tissue cells.
It is well known that the ionization value in tissues is proportional to cells damage. Thus, the main aim of this work is to estimate loss electron energy in target the various entrance layers (human skin and blood). The energy ranges between 0.01MeV and 1000 MeV are used, and calculate the total mass SPE from the total SPE formula (collision and radiation SPE) and semi-empirical as giving in the equation, for these calculations we will use the ESTAR (experimental data) and (MALAB2016) language.

The results for SPE vs energy for the human skin and blood tissues are calculated, this work is very important for knowing energy loss and the dose correlated, and help to formulate the interaction of electrons with material to predict the affectivity of the electron therapy and the possible damage to adjacent body tissue. Electron therapy depends on the type of interaction between the radiation and material (body tissue), the absorb energy and dose which human body might be exposed during electron therapy and knowing the energy loss and dose will help us to protect ourselves during exposed.

Materials and methods

Total mass stopping power SPE of electrons:

The expressions for the energy loss per unit path length for electrons were derived by Bethe formula can be written in the following equation [8, 9].

\[
\frac{dE}{dx}_{col} = \left( \frac{e^2}{4\pi\varepsilon_0} \right) \frac{2\pi N\rho Z}{mc^2\beta^2} \left[ \ln \frac{T(T+mc^2)\beta^2}{2T^2mc^2} + (1-\beta^2) \right] - \frac{1}{8} \left( 1 - \sqrt{1-\beta^2} \right)^2 - \delta(x) \tag{1}
\]

\[
\frac{dE}{dx}_{rad} = \left( \frac{e^2}{4\pi\varepsilon_0} \right) \frac{Z^2N\rho(E+mc^2)}{137m^2c^4A} \left[ 4\ln \frac{2(T+mc^2)}{mc^2} - \frac{4}{3} \right] \tag{2}
\]

where \(\frac{dE}{dx}_{col}\) and \(\frac{dE}{dx}_{rad}\) is the collision and radiation SPE of electron respectively.

E is the kinetic energy of the electron.

\(mc^2\) is the rest energy of the electron=0.511 MeV.

N is the number of atoms per m³ in the absorber medium N=\(\rho N_o/A\).

\(N_o\) is the Avogadro number 6.62×10⁻²³ m³ kg / s.

\(\rho\) is the absorber density.

\(\beta=v/c\) is the ratio of the velocity charged particle to velocity of light 3×10⁸ m/sec.

\(Z/A\) are the ratio effective atomic number to atomic weight and can be find according to the Bragg’s additively rule of the compound target [8,10,11,12,13].

\[
\gamma_A = \frac{\sum j n_j Z_j}{\sum j n_j A_j} \tag{3}
\]

\(T = E/mc^2\) is the ratio of the electron relativistic kinetic energy to its rest energy.

I is the mean excitation potential of the absorber and can be find according to the Bragg’s additively rule of the compound target [8, 10,11,12,13].

\[
I \cong \begin{cases} 
19.0 \text{ eV}, Z = 1(\text{hydrogen}) \\
11.2 + 11.7 Z \text{ eV}, 2 \leq Z \leq 13 \\
52.8 + 8.71 Z \text{ eV}, Z \geq 13
\end{cases} \tag{4}
\]

\[
\ln(I) = \frac{\sum j n_j Z_j n_j}{\sum j n_j Z_j} \tag{5}
\]
δ(x) is the density effect correction, the numerical values of δ were fitted to the formula proposed by Stenheimer in 1952, namely [12],

\[
\delta(X) = \begin{cases} 
4.6052X + a(X_1 - X)M + C & (X_0 < X < X_1), \\
4.6052X + C & (X > X_1), 
\end{cases}
\]

(6)

where

\[
X = \log(\beta\gamma) = \frac{1}{2}\log[(T(T + 2)]
\]

(7)

where \(\gamma = (1 - \beta^2)^{-1/2}\). At \(X_0 < X\), the values of \(\delta(X)\) is zero for the case of an insulator or gas, and the values of \(\delta(X)\) for a metal (conductor) is small, i.e., \(\delta(X) \leq 0.14\). \(X_1\) is the value has essentially attained its asymptotic form.

\(a\) and \(M\) are adjustable parameters which will be determined below, and \(C\) is given by:

\[
C = -2\ln\left(\frac{I}{h\nu_p}\right) - 1
\]

(8)

\(h\nu_p\) is the plasma energy of the electron, and is given by:

\[
h\nu_p = 28.816 \left(\rho < \frac{Z}{A}\right)^{1/2} \text{ eV}
\]

(9)

where \(\rho\) is the density of medium in (g/cm\(^3\)), and \(<Z/A>\) can be find by equation (3) [12].

The total SPE of electrons is just the sum of the collision and radiation SPE [14].

\[
\left(\frac{dE}{dx}\right)_{\text{tot}} = \left(\frac{dE}{dx}\right)_{\text{col}} + \left(\frac{dE}{dx}\right)_{\text{rad}}
\]

(10)

The ratio of radiation to collisional SPE for an electron, varies approximately as [2]:

\[
\frac{\left(\frac{dE}{dx}\right)_{\text{rad}}}{\left(\frac{dE}{dx}\right)_{\text{col}}} \approx \frac{E}{mc^2} \frac{Z}{1600}
\]

(11)

\(mc^2 = 0.511\text{ MeV}\), The energy at which this ratio is unity in MeV for the electron in the particular material.

Results & discussions:

In this work, the results of 0.01MeV-1000MeV incident electron calculations have been reported in human blood and skin tissue. The chemical composition of the materials used here for these tissues was taken from (ICRP) (NIST 1998) (Tables 1, 2, 3) [12, 15]. The results obtained for total mass SPE can be calculated from summing collision SPE with addition density effect correction and radiation SPE as equations (1, 10, 11) with application the Bragg’s additively rule as equations (3, 5) for compound targets, and from semi-empirical SPE equations in terms of kinetic energy E(MeV) for electrons in the ranges of the low energies from 0.01MeV to 0.7 MeV and the high energies from 0.7 MeV to 1000 MeV can be written as the following equation (12):

\[
\left(\frac{dE}{dx}\right)_{\text{tot}} = \begin{cases} 
0.422E^{-0.853} + 1.17 & 0.01\text{MeV} \leq E \leq 0.7\text{MeV} \\
0.0275E^{0.992} + 1.844 & 0.7\text{MeV} \leq E \leq 1000\text{MeV}
\end{cases}
\]

(12)

In actuality, the energy range (0.01MeV-1000MeV) is so wide for electron beam therapy, also of it possibility use in other applications and to show compatibility of the present calculations from eq.12 with those given by eq. 10 and ESTAR (experimental data). Also, these calculations are shown in Figs. 1 and 2, for electrons in human blood and skin tissues, because of the usefulness of these results in electron therapy.
Table 1: Density effect parameters for human blood and skin tissues (ICRP1998)[12,15].

| Tissue | Z/A  | J(eV) | $\rho$(g/cm$^3$) | $h_v$(eV) | -C   | $X_0$ | $X_1$ | a     | M     |
|--------|------|-------|------------------|----------|------|-------|-------|-------|-------|
| blood  | 0.54995 | 75.2  | 1.0600          | 22.001   | 3.4581 | 0.2239 | 2.8017 | 0.08492 | 3.5406 |
| skin   | 0.54932 | 72.7  | 1.1             | 22.400   | 3.3546 | 0.2019 | 2.7526 | 0.09459 | 3.4643 |

Table 2: The chemical composition of blood (ICRP1998).

| Element | Fraction by weight |
|---------|--------------------|
| H       | 0.101866           |
| C       | 0.100020           |
| N       | 0.029640           |
| O       | 0.759414           |
| Na      | 0.001850           |
| Mg      | 0.000040           |
| Si      | 0.000030           |
| P       | 0.000350           |
| S       | 0.001850           |
| Cl      | 0.002780           |
| K       | 0.001630           |
| Ca      | 0.000060           |
| Fe      | 0.000460           |
| Zn      | 0.000010           |

Table 3: The chemical composition of skin tissue (ICRP1998).

| Element | Fraction by weight |
|---------|--------------------|
| H       | 0.100588           |
| C       | 0.228250           |
| N       | 0.046420           |
| O       | 0.619002           |
| Na      | 0.000070           |
| Mg      | 0.000060           |
| P       | 0.000330           |
| S       | 0.001590           |
| Cl      | 0.002670           |
| K       | 0.000850           |
| Ca      | 0.000150           |
| Fe      | 0.000010           |
| Zn      | 0.000010           |
Figs 1 and 2 and Tables 4 and 5 show, for comparison, the semi-empirical equation (12) results well agreement with the ESTAR results within error ratio 10% for blood and 0.58% for skin, and correlation coefficient 0.99 for both. When the incident particle’s energy is in the range 0.01MeV-1000MeV, comparison can be made with the total mass SPE results calculated from equation (10), and gave agreement with error ratio 40% and correlation coefficient 0.84 for total SPE.

In addition, Figs 1 and 2 and Tables 4 and 5 show that the total mass SPE from Eq. (10) has a maximum values at relativistic energies above 0.7 MeV and agree with ESTAR data at lower energies; this is because of the dominance of collisional SPE dominant at lower energies. While the radiative mass SPE has a maximum values at relativistic energies above 0.7MeV and the dominant process at relativistic energies; below 1MeV, the radiation losses are negligible[8]. The influence of density effect correction in the compatibility of the results Eq. (10) with ESTAR data in lower energies when velocity of incident charged particle less than Bohr velocity non-relativistic, while no agreement with ESTAR data in the higher energies. As a consequence, the correction for the density-effect is small at lower energies and the energy loss increasing at relativistic energies because the density effect correction approaches a linear dependence on ln(βγ) as in Eq. (7) [10], this leads to an increase results total mass SPE from Eq.(10) in comparison with ESTAR results.

Table 4: The total mass SPE for electron in blood obtained by present work Eq(10), Semi-empirical Eq.(12) and experimental (ESTAR data).

| E(MeV) | (dE/dx)_tot (MeV cm²/gm) | (dE/dx)_semi (MeV cm²/gm) | (dE/dx)_ESTAR (MeV cm²/gm) |
|--------|-------------------------|-------------------------|-------------------------|
|        |                         |                         |                         |


Table 5: The total mass SPE for electron in skin tissues obtained by present work Eq.(10), Semi-empirical Eq.(12) and experimental (ESTAR data)

| E(MeV) | \(\frac{dE}{dx}_{\text{tot}}\) (MeV cm\(^2\)/gm) | \(\frac{dE}{dx}_{\text{semi}}\) (MeV cm\(^2\)/gm) | \(\frac{dE}{dx}_{\text{ESTAR}}\) (MeV cm\(^2\)/gm) |
|--------|---------------------------------|---------------------------------|---------------------------------|
| 0.01   | 22.60209                        | 22.44177                        | 22.34                           |
| 0.015  | 16.56951                        | 16.23189                        | 16.31                           |
| 0.02   | 13.31529                        | 12.95981                        | 13.05                           |
| 0.025  | 11.26177                        | 10.91984                        | 10.99                           |
| 0.03   | 9.841452                        | 9.518002                        | 9.563                           |
| 0.035  | 8.797846                        | 8.49123                         | 8.513                           |
| 0.045  | 7.363393                        | 7.080966                        | 7.065                           |
| 0.06   | 6.063261                        | 5.796852                        | 5.745                           |
| 0.07   | 5.492709                        | 5.227767                        | 5.161                           |
| 0.08   | 5.060045                        | 4.791703                        | 4.716                           |
| 0.1    | 4.449713                        | 4.165047                        | 4.08                            |
| 0.15   | 3.643558                        | 3.290701                        | 3.211                           |
| 0.2    | 3.266413                        | 2.829995                        | 2.771                           |
| 0.25   | 3.068144                        | 2.542769                        | 2.509                           |
| 0.3    | 2.961929                        | 2.345392                        | 2.338                           |
| 0.35   | 2.89053                         | 2.200823                        | 2.191                           |
| 0.5    | 2.829186                        | 1.930853                        | 2.021                           |
| 0.7    | 2.915343                        | 1.863305                        | 1.905                           |
| 0.8    | 2.990129                        | 1.866039                        | 1.875                           |
| 0.9    | 3.076483                        | 1.868771                        | 1.855                           |
| 1.5    | 3.701521                        | 1.885116                        | 1.819                           |
| 2.5    | 4.845634                        | 1.912248                        | 1.847                           |
| 3.5    | 5.994564                        | 1.932929                        | 1.888                           |
| 4.0    | 6.563431                        | 1.952787                        | 1.909                           |
| 5.0    | 7.688297                        | 1.979741                        | 1.949                           |
| 6.0    | 8.796882                        | 2.006652                        | 1.987                           |
| 7.0    | 9.890874                        | 2.033527                        | 2.024                           |
| 10.0   | 13.09914                        | 2.113981                        | 2.126                           |
| 15.0   | 18.25062                        | 2.247661                        | 2.281                           |
| 20.0   | 23.21696                        | 2.380975                        | 2.427                           |
| 25.0   | 28.04076                        | 2.514022                        | 2.569                           |
| 30.0   | 32.74893                        | 2.646855                        | 2.707                           |
| 35.0   | 37.35993                        | 2.779509                        | 2.843                           |
| 40.0   | 41.88714                        | 2.912012                        | 2.979                           |
| 45.0   | 46.34072                        | 3.044382                        | 3.114                           |
| 60.0   | 59.33193                        | 3.44083                         | 3.515                           |
| 80.0   | 75.96977                        | 3.968213                        | 4.048                           |
| 90.0   | 84.05355                        | 4.231488                        | 4.315                           |
| 100.0  | 92.00408                        | 4.49453                         | 4.581                           |
| 200.0  | 166.2488                        | 7.115746                        | 7.253                           |
| 300.0  | 234.1895                        | 9.72601                         | 9.937                           |
| 400.0  | 298.0266                        | 12.32919                        | 12.63                           |
| 500.0  | 355.8091                        | 14.92711                        | 15.32                           |
| 600.0  | 417.1561                        | 17.53085                        | 18.02                           |
| 700.0  | 473.4768                        | 20.11111                        | 20.72                           |
| 800.0  | 528.0623                        | 22.69841                        | 23.43                           |
| 900.0  | 581.1306                        | 25.28312                        | 26.13                           |
| 1000.0 | 632.8506                        | 27.86552                        | 28.84                           |
| Value | 22.58181 | 22.44177 | 22.47  |
|-------|----------|----------|--------|
| Value | 16.49369 | 16.23189 | 16.39  |
| Value | 13.20824 | 12.95981 | 13.11  |
| Value | 11.13355 | 10.91984 | 11.04  |
| Value | 9.697266 | 9.516002 | 9.605  |
| Value | 8.640748 | 8.49123  | 8.549  |
| Value | 7.185753 | 7.080966 | 7.093  |
| Value | 5.864701 | 5.796852 | 5.767  |
| Value | 5.277609 | 5.227767 | 5.18   |
| Value | 4.832525 | 4.791703 | 4.733  |
| Value | 4.19941  | 4.165047 | 4.094  |
| Value | 3.341819 | 3.290701 | 3.221  |
| Value | 2.915939 | 2.829995 | 2.779  |
| Value | 2.669413 | 2.542769 | 2.516  |
| Value | 2.513611 | 2.345392 | 2.344  |
| Value | 2.392124 | 2.200823 | 2.225  |
| Value | 2.193774 | 1.930853 | 2.024  |
| Value | 2.098168 | 1.863435 | 1.906  |
| Value | 2.081983 | 1.866186 | 1.875  |
| Value | 2.077313 | 1.868934 | 1.854  |
| Value | 2.157515 | 1.885377 | 1.817  |
| Value | 2.409331 | 1.912663 | 1.843  |
| Value | 2.548231 | 1.926268 | 1.863  |
| Value | 2.68975  | 1.939854 | 1.884  |
| Value | 2.832471 | 1.953423 | 1.904  |
| Value | 3.118955 | 1.98052  | 1.944  |
| Value | 3.405046 | 2.00757  | 1.982  |
| Value | 3.68989  | 2.034582 | 2.018  |
| Value | 4.534274 | 2.115435 | 2.118  |
| Value | 5.905073 | 2.249752 | 2.269  |
| Value | 7.235631 | 2.383681 | 2.411  |
| Value | 8.532916 | 2.517323 | 2.548  |
| Value | 9.802219 | 2.650737 | 2.682  |
| Value | 11.04747 | 2.78396  | 2.814  |
| Value | 12.27166 | 2.917022 | 2.946  |
| Value | 13.47713 | 3.049942 | 3.076  |
| Value | 16.99843 | 3.447995 | 3.466  |
| Value | 21.51516 | 3.977437 | 3.983  |
| Value | 23.71157 | 4.241715 | 4.241  |
| Value | 25.87265 | 4.505743 | 4.5    |
| Value | 46.07653 | 7.136213 | 7.09   |
| Value | 64.58073 | 9.755008 | 9.692  |
| Value | 81.97189 | 12.36625 | 12.3   |
| Value | 98.5323 | 14.97189 | 14.92  |
| Value | 114.4293 | 17.57307 | 17.53  |
| Value | 129.7737 | 20.17055 | 20.15  |
| Value | 144.6447 | 22.76487 | 22.77  |
| Value | 159.1014 | 25.35643 | 25.4   |
| Value | 173.1899 | 27.94554 | 28.02  |
Fig. 1: Total mass SPE of blood for incident electrons obtained by present work Eq.(10), Semi-empirical Eq(12) and experimental (ESTAR data).

Fig. 2: Total mass SPE of skin tissues for incident electrons obtained by present work Eq.(10), Semi-empirical Eq.(12) and experimental (ESTAR data).

Conclusions
From the present results obtained using the suggested semi-empirical Eq.(12) for incident electrons in human blood and skin tissues, it is completely apparent that the total mass SPE of electron in this materials can be expressed in terms of energy. We arrive to the conclusion that energy of the incident electron is key parameter for the calculation of total mass SPE in this material. It is also remarkable that suggested Semi-empirical equation is simpler, widely applicable and results are in better agreement with the experimental (ESTAR data) compared to results obtained by Eq.(10). The characteristic of the followed method allows the calculation contributions, both the collisional and radiative SPE analytical way.

The given tables, in the present work, are expected to be helpful in the estimation of dose in electron therapy, the tables consist of basic values for electrons interactions. Which is simply applied
the followed method to the compound target material, or non-compound. This method is only applicable when the projectiles are only electrons and positrons.

1- At low E values, the (-dE/ρdx)total decreases rapidly at the energy increases owing to the 1/β² term.

2- At relativistic energies, when the velocity approaches the speed of light and β=1, the (-dE/ρdx)total curve reaches a minimum.

3- The (-dE/ρdx)total will start to grow again beyond its minimum due to the logarithmic terms. This latter depends on the increase with the incoming momentum.

References:
[1] M. J. Berger and S. M. Seltzer, Stopping powers and ranges of electrons and positrons, Office of health and environmental research, p. 1, 1982.
[2] M. O. El-Ghossain, Calculations of stopping power, and range of electrons interaction with different material and human body parts, International Journal of Scientific and Technology Research, Vol. (1), p.114-115, 2017.
[3] J. Michael Gazda, MS, and Lawrence R. Coia, Principles of radiation therapy, Journal Cancer Network, p. 11, 2007.
[4] A.W. Tigner, Handbook of accelerator physics and engineering, World Scientific, p. 155-188, 1999
[5] R.K. Batra, Approximate stopping power of low energy electrons and positrons in matter, Nuclear Instruments and Methods in Physics Research, p.135, 1987.
[6] M.Ç. Tufan, H. Gumus and T. Namdar, Stopping power and CSDA range calculations for incident electrons and positrons in breast and brain tissues, Springer-Radiation and Environmental Biophysics, p.246, 2013.
[7] S. Tanuma, C.J. Powell and D.R. Penn, Calculations of stopping powers of 100eV-30eV electrons in 31 elemental solids, National Institute for Materials Science, p.1, 2008.
[8] K.S. Krane, Introductor nuclear physics, Oregon state university, p.196, 1987.
[9] H. Shinotsuka, S. Tanuma, C.J. Powell and D.R. Penn, Calculations of electron stopping powers for 41 elemental solids over the 50 eV to 30 keV range with the full Penn algorithm, Nuclear Instruments and Methods in Physics Research B, p. 77, 2012.
[10] P.G. Rancoita and C. Leroy, Principles of radiation interaction in matter and detection, University de montreal, Istituto nazionale di fisica nucleare, World Scientific, p.48-77, 2009.
[11] J. F. Ziegler, The stopping of energetic light ions in elemental matter, J. Appl. Phys / Rev. Appl. Phys., p.6, 1999.
[12] N. Tsoulfanidis, Measurement and detection of radiation, Second Edition, University of Missouri-Rolla, p.262-246, 1995.
[13] G. Tanır, M. H. Böldükdemir, S. Keleş, and I. Gök, On the stopping power for low energy positrons, Chinese journal of physics, p.426, 2011.
[14] M. Y. El-Shaer, Medical effect of radiator interactions, Islamic University of Gaza, p.24, 2015.
[15] R.J. McConn, C.J. Gesh, R.T. Pagh, R.A. Rucker and R.G. Williams, Compendium of material composition data for radiation transport modeling, Homeland Security, p.29-286, 2011.