Generalized RSA/DH/ECC via Geometric Cryptosystem

Gordon Chalmers

e-mail: gordon@quartz.shango.com

Abstract

A scheme is presented based on numbers that represent a manifold in \( d \) dimensions for generalizations of textbook cryptosystems. The interlocking or intersection of geometries, requiring the addition of a series of integers \( q_j \), can be used to enhance a cryptosystem via algorithms based on the form of the geometry. Further operations besides addition of these numbers, e.g. sewing of the geometries, can be given such as rotations or contractions.
Introduction

Textbook RSA, Diffie-Hellman, or elliptic curve cryptostems (ECC) in standard representation requires the factoring of large numbers into two smaller ones. The time consuming nature of the factoring process protects the security of the methods.

Recently, a polytopic definition of a manifold was introduced in which a number specifies a region in space [2, 3]. The presence of the number parameterization allows a geometric manipulation of the surfaces so that two or more numbers may form a more complicated geometry. The geometry may serve as an algorithm for an encryption process; one geometry is specified by a number \( p_1 \), and a second geometry by the number \( p_2 \), with the two together specifying another geometry. The volumes may specify a surface-dependent encryption process.

The volume dependence of the combined geometry due to \( (p_1, p_2) \), or a series of combined geometries \( (p_1, \ldots, p_n) \), allows the message sender to encode the content in a variety of ways. Furthermore, the numbers may be sent in and manipulated via the secure RSA or DH scheme before the geometric gluing of the numbers and subsequent encryption/de-encryption using the geometry.

One adaptation of these protocols allow the user numbers \( p_j \) to be chosen in an arbitrary manner. Depending on the number chosen the information such as a password may be encrypted in different fashions, which require the geometries to unlock. This is analogous to having one large integer \( N \) factoring into \( pq \) (or \( \prod p_j \)) for an arbitrary \( p \) and \( q \); the standard methods require the user to possess one number, which is elliptically multiplied/calculated. The presence of the choice of an arbitrary pair of numbers in the factoring of a number \( N \) results in an exponentiation of the possible combinations.

Polytopes or Polyhedra

The polytopic surface, or polyhedron, as defined in [2] is described in the number basis. Take a series of numbers \( a_1a_2\ldots a_n \) corresponding to the digits of an integer \( p \), with the base of the individual number being \( 2^n \). In this way, upon reduction to base 2 the digits of the number span a square with \( n + 1 \) entries. Each number \( a_j \) parameterizes a column with ones and zeros in it. The lift of the numbers could be taken to base 10 with minor modifications, by converting the base of \( p \) to 10 (with possible remainder issues if the number does not ’fit’ well).

The individual numbers \( a_i \) decompose as \( \sum a_i^m2^m \) with the components \( a_i^m \) being 0 or 1. Then map the individual number to a point on the plane,
\[ \vec{r}_i^m = a_i^m \times m\hat{e}_1 + a_i^m \times i\hat{e}_2, \] (1)

with the original number mapping to a set of points on the plane via all of the entries in \( a_1a_2\ldots a_m \). In doing this, a collection of points on the plane is spanned by the original number \( p \), which could be a base 10 number. The breakdown of the number to a set of points in the plane is represented in figure 1.

A set of further integers \( p_j = a_i^{(j)}a_i^{(j)}\ldots a_i^{(j)} \) are used to label a stack of coplanar lattices with the same procedure to fill in the third dimension. The spacial filling of the disconnected polhedron is assembled through the stacking of the base reduced integers.

The polyhedron is constructed by the single numbers spanning the multiple layers in 3-d, or by one number with the former grouped as \( p_1p_2\ldots p_n \). The generalization to multiple dimensions is straightforward.

The addition of the multiple numbers \( a_i^{(j)} \) in each of the geometric numbers \( q_j \) generate the new geometry and its numbers of \( \tilde{a}_i^{(j)} \). The lattice picture is represented in (1).

Other operations can be implemented in the sewing of the manifolds. There are rotations, contractions, expansions, and displacements of the individual geometries, for example. These operations can be implemented before or after the manifolds are molded together.

*Geometric Manipulation*

The are various ways in which the geometry may be used as an encryption method. A simple one is to take all of the coefficients \( a_i^{(j)} \) and construct a polynomial, for example,

\[ P(z) = \sum p_j z^j, \] (2)

with \( p_j = \sum a_i^{(j)}z^i \), containing the entries of the individual rows on the lattice. This polynomial in (2) is dependent on the geometry of the number and could be used as a map to alter information. Other polynomials may be found via alternate constructions.

The coefficients associated with the geometry may be used to define an L-series, and in turn an elliptic curve. The coefficients \( p_j \) for example may be used to count the solutions to a curve.
\[ y^2 = x^3 + ax + b \mod p, \]  
with \( p \) prime numbers. The geometry defines an L-series

\[
\zeta(C, s) = \prod \left(1 - a_p p^{-s} + p^{1-2s}\right)^{-1},
\]

with the numbers \( a_p = p - b_p \), with \( b_p \) the number of rational solutions to the curve with the modding of \( p \).

Elliptic curves are standard in the RSA and DH schemes, and in alternatives. The construction of the geometries, and their gluing together, naturally define the elliptic curves. They could be used in generalizing the textbook RSA/DH or elliptic schemes.

Another method to make a more direct comparison with the previous techniques is to have the sender use a number that incorporates with the elliptic factorization of these standard methods. The breakup of a number into two smaller ones, i.e. \( N = pq \) for a general pair of numbers rather than one preferred pair, allows the \( p \) and \( q \) to be used as the individual geometries. Various numbers \( q_j \) could be generated this way; these numbers could then be used to enhance the information sharing protocol, for both 'password' and message content.

**Concluding Remarks**

The number representation of a multi-dimensional manifold is used to provide an enhancement, or alternative, to the well known RSA or Diffie-Hellman or elliptic crypto schemes for password encryption. Numbers \( p_j \) are used to define a geometry, and the sewing of these geometries or possible intersection is deduced by adding them.

The geometries are specific to the user, the data, and the receiver and depending on the input manifold a case dependent geometric molding is determined. This is useful in a variety of protected password schemes and information sharing.
References

[1] G. Chalmers, *Polytopes and Knots*, [physics/0503212](http://physics/0503212).

[2] G. Chalmers, *Integer and Rational Solutions to Polynomial Equations*, [physics/0503200](http://physics/0503200).

[3] G. Chalmers, *Algebraic and Polytopic Formulation Cohomology*, preprint.