Effects of Electromagnetic Field on Gravitational Collapse

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Abstract

In this paper, the effect of electromagnetic field has been investigated on the spherically symmetric collapse with the perfect fluid in the presence of positive cosmological constant. Junction conditions between the static exterior and non-static interior spherically symmetric spacetimes are discussed. We study the apparent horizons and their physical significance. It is found that electromagnetic field reduces the bound of cosmological constant by reducing the pressure and hence collapsing process is faster as compared to the perfect fluid case. This work gives the generalization of the perfect fluid case to the charged perfect fluid. Results for the perfect fluid case are recovered.

Keywords: Electromagnetic Field, Gravitational Collapse, Cosmological Constant.

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1 Introduction

General Relativity (GR) predicts that gravitational collapse of massive objects (having mass $= 10^6 M_\odot - 10^8 M_\odot$, where $M_\odot$ is mass of the Sun) results

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to the formation of spacetime singularities in our universe \(^1\). The singularity theorems \(^2\) of Hawking and Penrose reveal that if a trapped surface forms during the collapse of compact object, such a collapse will develop a space-time singularity. According to these theorems, the occurrence of spacetime singularity (can be observed or not) is the generic property of the spacetime in GR. An observable singularity is called *naked singularity* while other is called *black hole or covered singularity*.

An open and un-resolved problem in GR is to determine the final fate of the gravitational collapse (i.e., end product of collapse is either covered or naked singularity). To resolve this problem, Penrose \(^3\) suggested a hypothesis so-called Cosmic Censorship Conjecture (CCC). This conjecture states that the singularities that appear in the gravitational collapse are always covered by the event horizon. It has two versions, i.e., weak and strong version \(^4\). The weak version states that the gravitational collapse from the regular initial conditions never creates spacetime singularity visible to distant observer. On the other hand, the strong version says that no singularity is visible to any observer at all, even some one close to it. There is no mathematical or theoretical proof for either of the version of the CCC.

The singularity (at the end stage of the gravitational collapse) can be black hole or naked depending upon the initial data and equation of the state. To prove or disprove this hypothesis, many efforts have been made but no final conclusion is drawn. It would be easier to find the counter example that would enable us to claim that the hypothesis is not correct. For this purpose, Virbhadra et al. \(^5\) introduced a new theoretical tool using the gravitational lensing phenomena. Also, Virbhadra and Ellis \(^6\) studied the Schwarzschild black hole lensing and found that the relativistic images would confirm the Schwarzschild geometry close to the event horizon. The same authors \(^7\) analyzed the gravitational lensing by a naked singularity and classified it into two kinds: weak naked singularity (those contained within at least one photon sphere) and strong naked singularity (those not contained within any photon sphere).

Claudel et al. \(^8\) showed that spherically symmetric black holes, with reasonable energy conditions, are always covered inside at least one photon sphere. Virbhadra and Keeton \(^9\) studied the time delay and magnification centroid due to gravitational lensing by black hole and naked singularity. It was found that weak CCC can be tested observationally without any ambiguity. Virbhadra \(^10\) explored the useful results to investigate the Seifert’s conjecture for naked singularity. He found that naked singularity forming in
the Vaidya null dust collapse supports the Seifert’s conjecture. In a recent paper [11], the same author used the gravitational lensing phenomena to find the improved form of the CCC. This work is a source of inspiration for many leading researchers.

Oppenheimer and Snyder [12] are the pioneers who investigated gravitational collapse long time ago in 1939. They studied the dust collapse by taking the static Schwarzschild spacetime as exterior and Friedmann like solution as interior spacetime. They found black hole as end product of the gravitational collapse. To study the gravitational collapse, exact solutions of the Einstein field equations with dust provide non-trivial examples of naked singularity formation. Since the effects of pressure cannot be neglected in the singularity formation, therefore dust is not assumed to be a good matter.

There has been a growing interest to study the gravitational collapse in the presence of perfect fluid and other general physical form of the fluid. Misner and Sharp [13] extended the pioneer work for the perfect fluid. Vaidya [14] and Santos [15] used the idea of outgoing radiation of the collapsing body and also included the dissipation in the source by allowing the radial heat flow. The cosmological constant \( \Lambda \) affects the properties of spacetime as it appears in the field equations. It is worthwhile to solve the field equations with non-zero cosmological constant for analyzing the gravitational collapse. Markovic and Shapiro [16] generalized the pioneer work with positive cosmological constant. Lake [17] extended it for both positive and negative cosmological constant.

Sharif and Ahmad [18]-[21] extended the spherically symmetric gravitational collapse with positive cosmological constant for perfect fluid. They discussed the junction conditions, apparent horizons and their physical significance. It is concluded that apparent horizon forms earlier than singularity and positive cosmological constant slows down the collapse. The same authors also investigated the plane symmetric gravitational collapse using junction conditions [22]. In a recent paper [23], Sharif and Iqbal extended plane symmetric gravitational collapse to spherically symmetric case.

Although a lot of work has been done for dust and perfect fluid collapse of spherically symmetric models. However, no such attempt has been made by including the electromagnetic field. We would like to study the gravitational collapse of charged perfect fluid in the presence of positive cosmological constant. For this purpose, we discuss the junction conditions between the non-static interior and static exterior spherically symmetric spacetimes. The main objectives of this work are the following:
To study the effects of electromagnetic field on the rate of collapse.

To see whether or not CCC is valid in this framework.

The plan of the paper is as follows: In the next section, the junction conditions are given. We discuss the solution of the Einstein-Maxwell field equations in section 3. The apparent horizons and their physical significance are presented in section 4. We conclude our discussion in the last section.

We use the geometrized units (i.e., the gravitational constant \( G = 1 \) and speed of light in vacuum \( c = 1 \) so that \( M \equiv \frac{M}{c^2} \) and \( \kappa \equiv \frac{8\pi G}{c^4} = 8\pi \)). All the Latin and Greek indices vary from 0 to 3, otherwise it will be mentioned.

## 2 Junction Conditions

We consider a timelike 3D hypersurface \( \Sigma \) which separates two 4D manifolds \( M^- \) and \( M^+ \) respectively. For the interior manifold \( M^- \), we take spherically symmetric spacetime given by

\[
 ds_-^2 = dt^2 - X^2 dr^2 - Y^2 (d\theta^2 + \sin \theta d\phi^2),
\]

where \( X \) and \( Y \) are functions of \( t \) and \( r \). For the exterior manifold \( M^+ \), we take Reissner-Nordström de-Sitter spacetime

\[
 ds_+^2 = N dT^2 - \frac{1}{N} dR^2 - R^2 (d\theta^2 + \sin \theta d\phi^2),
\]

where

\[
 N(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{\Lambda}{3} R^2,
\]

\( M \) and \( \Lambda \) are constants and \( Q \) is the charge. The Israel junction conditions are the following [24]:

1. The continuity of line element over \( \Sigma \) gives

\[
 (ds_-^2)_\Sigma = (ds_+^2)_\Sigma = ds_\Sigma^2.
\]

2. The continuity of extrinsic curvature over \( \Sigma \) yields

\[
 [K_{ij}] = K_{ij}^+ - K_{ij}^- = 0, \quad (i, j = 0, 2, 3)
\]
\[
K_{ij}^{\pm} = -n_{\sigma}^{\pm} \left( \frac{\partial^2 x_\sigma^{\pm}}{\partial \xi^i \partial \xi^j} + \Gamma_{\mu \nu}^\sigma \frac{\partial x_\sigma^{\mu}}{\partial \xi^i} \frac{\partial x_\sigma^{\nu}}{\partial \xi^j} \right), \quad (\sigma, \mu, \nu = 0, 1, 2, 3). \tag{2.6}
\]

Here \( \xi^i \) correspond to the coordinates on \( \Sigma \), \( x_\sigma^{\pm} \) stand for coordinates in \( M^{\pm} \), the Christoffel symbols \( \Gamma_{\mu \nu}^\sigma \) are calculated from the interior or exterior spacetimes and \( n_{\sigma}^{\pm} \) are components of outward unit normals to \( \Sigma \) in the coordinates \( x_\sigma^{\pm} \).

The equation of hypersurface in terms of interior spacetime \( M^- \) coordinates is

\[
f_-(r, t) = r - r_\Sigma = 0, \tag{2.7}
\]

where \( r_\Sigma \) is a constant. Also, the equation of hypersurface in terms of exterior spacetime \( M^+ \) coordinates is given by

\[
f_+(R, T) = R - R_\Sigma(T) = 0. \tag{2.8}
\]

When we make use of Eq. (2.7) in Eq. (2.1), the metric on \( \Sigma \) takes the form

\[
(ds_-^2)_\Sigma = dt^2 - Y^2(r_\Sigma, t)(d\theta^2 + \sin \theta d\phi^2). \tag{2.9}
\]

Also, Eqs. (2.8) and (2.2) yield

\[
(ds_+^2)_\Sigma = \left[ N(R_\Sigma) - \frac{1}{N(R_\Sigma)} \left( \frac{dR_\Sigma}{dT} \right)^2 \right] dT^2 - R_\Sigma^2(d\theta^2 + \sin \theta d\phi^2), \tag{2.10}
\]

where we assume that

\[
N(R_\Sigma) - \frac{1}{N(R_\Sigma)} \left( \frac{dR_\Sigma}{dT} \right)^2 > 0 \tag{2.11}
\]

so that \( T \) is a timelike coordinate. From Eqs. (2.4), (2.9) and (2.11), it follows that

\[
R_\Sigma = Y(r_\Sigma, t), \tag{2.12}
\]

\[
[ N(R_\Sigma) - \frac{1}{N(R_\Sigma)} \left( \frac{dR_\Sigma}{dT} \right)^2 ]^{\frac{1}{2}} dT = dt. \tag{2.13}
\]

Also, from Eqs. (2.7) and (2.8), the outward unit normals in \( M^- \) and \( M^+ \), respectively, are given by

\[
n^-_\mu = (0, X(r_\Sigma, t), 0, 0), \tag{2.14}
\]

\[
n^+_\mu = (-\dot{R}_\Sigma, \dot{T}, 0, 0). \tag{2.15}
\]
The components of extrinsic curvature $K^\pm_{ij}$ become

\begin{align*}
  K^+_{00} &= 0, \\
  K^+_{22} &= \csc^2 \theta K^+_{33} = \left( \frac{Y Y'}{X} \right) \Sigma, \\
  K^+_{00} &= \left( \dddot{R} - \ddot{R}^2 - \frac{N dN}{2 dR} \dddot{R} + \frac{3}{2} \frac{dN}{dR} \dddot{R} \right) \Sigma, \\
  K^+_{22} &= \csc^2 \theta K^+_{33} = (N \dddot{R}) \Sigma,
\end{align*}

where dot and prime mean differentiation with respect to $t$ and $r$ respectively. From Eq. (2.5), the continuity of extrinsic curvature gives

\begin{align*}
  K^+_{00} &= 0, \\
  K^+_{22} &= K^-_{22}.
\end{align*}

Using Eqs. (2.16)-(2.21) along with Eqs. (2.12) and (2.13), the junction conditions become

\begin{align*}
  (XY' - \dot{X}Y') \Sigma &= 0, \\
  M &= \left( \frac{Y}{2} - \frac{\Lambda}{6} Y^3 + \frac{Q^2}{2Y} + \frac{Y}{2} Y^2 - \frac{Y}{2X^2} Y'^2 \right) \Sigma.
\end{align*}

3 Solution of the Einstein Field Equations

The Einstein field equations with cosmological constant are given by

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = \kappa (T_{\mu\nu} + T^{(em)}_{\mu\nu}) \]

The energy-momentum tensor for perfect fluid is

\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - pg_{\mu\nu}, \]

where $\rho$ is the energy density, $p$ is the pressure and $u_\mu = \delta^0_\mu$ is the four-vector velocity in co-moving coordinates. $T^{(em)}_{\mu\nu}$ is the energy-momentum tensor for the electromagnetic field defined [25] as

\[ T^{(em)}_{\mu\nu} = \frac{1}{4\pi} \left( -g^{\delta\omega} F_{\mu\delta} F_{\nu\omega} + \frac{1}{4} g_{\mu\nu} F^{\delta\omega} F_{\delta\omega} \right). \]
With the help of Eqs. (3.2) and (3.3), Eq. (3.1) takes the form

\[ R_{\mu\nu} = 8\pi[(\rho + p)u_{\mu}u_{\nu} + \frac{1}{2}(p - \rho)g_{\mu\nu} + T_{\mu\nu}^{(em)}] - \frac{1}{2}g_{\mu\nu}T^{(em)} - \Lambda g_{\mu\nu}. \]  

(3.4)

To solve this equation, we need to calculate the non-zero components and trace free form of \( T_{\mu\nu}^{(em)} \). For this purpose, we first solve the Maxwell’s field equations

\[ F_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu}, \]  

(3.5)

\[ F^{\mu\nu};\nu = -4\pi J^\mu, \]  

(3.6)

where \( \phi_\mu \) is the four potential and \( J^\mu \) is the four current. As the charged fluid is in co-moving coordinate system, the magnetic field will be zero in this case. Thus we can choose the four potential and four current as follows

\[ \phi_\mu = (\phi(t, r), 0, 0, 0), \]  

(3.7)

\[ J^\mu = \sigma u^\mu, \]  

(3.8)

where \( \sigma \) is charge density.

Now for the solution of the Maxwell’s field Eq. (3.6), \( \mu \) and \( \nu \) are treated as local coordinates. Using Eqs. (3.5) and (3.7), the non-zero components of the field tensor are given as follows:

\[ F_{01} = -F_{10} = -\frac{\partial \phi}{\partial r}. \]  

(3.9)

Also, from Eqs. (3.6) and (3.8), we have

\[ \frac{1}{X}\frac{\partial^2 \phi}{\partial r^2} - \frac{\partial \phi}{\partial r} \frac{X'}{X} = -4\pi \sigma X, \]  

(3.10)

\[ \frac{1}{X}\frac{\partial^2 \phi}{\partial r \partial t} - \frac{X'}{X^2} \frac{\partial \phi}{\partial r} = 0. \]  

(3.11)

Equation (3.11) implies that

\[ \left( \frac{1}{X} \frac{\partial \phi}{\partial r} \right) = K, \]  

(3.12)

where \( K = K(r) \) is an arbitrary function of \( r \). Equations (3.10) and (3.12) yield

\[ K'(r) = -4\pi \sigma X. \]  

(3.13)
The non-zero components of $T^{(em)}_{\mu\nu}$ and its trace free form turn out to be

$$
T^{(em)}_{00} = \frac{1}{8\pi} K^2, \quad T^{(em)}_{11} = -\frac{1}{8\pi} K^2 X^2, \quad T^{(em)}_{22} = \frac{1}{8\pi} K^2 Y^2; \\
T^{(em)}_{33} = \frac{1}{8\pi} K^2 \sin^2 \theta, \quad T^{(em)} = 0.
$$

When we use these values, the field equations (3.4) for the interior space-time takes the form

$$
R_{00} = -\frac{\dddot{X}}{X} - \frac{2}{Y} \dddot{Y} = 4\pi(\rho + 3p) + K^2 - \Lambda, 
$$

(3.14)

$$
R_{11} = -\frac{\dddot{X}}{X} - 2 \frac{\dddot{X}}{XY} + \frac{2}{X^2} \left[ \frac{Y''}{Y} - \frac{Y'X'}{XY} \right] \\
= 4\pi(p - \rho) + K^2 - \Lambda, 
$$

(3.15)

$$
R_{22} = -\frac{\dddot{Y}}{Y} - \left( \frac{\dddot{Y}}{Y} \right)^2 - \frac{\dddot{X}}{X} + \frac{2}{X^2} \left[ \frac{Y''}{Y} + \left( \frac{Y'}{Y} \right)^2 - \frac{X'Y'}{XY} \right] \\
= 4\pi(p - \rho) - K^2 - \Lambda, 
$$

(3.16)

$$
R_{33} = \sin^2 \theta R_{22}, 
$$

(3.17)

$$
R_{01} = -2 \frac{\dddot{Y}}{Y} + 2 \frac{\dddot{X}}{X} \frac{Y'}{Y} = 0. 
$$

(3.18)

Now we solve Eqs. (3.14)-(3.18). Integration of Eq. (3.18) with respect to $t$ yields

$$
X = \frac{Y'}{H}, 
$$

(3.19)

where $H = H(r)$ is an arbitrary function of $r$. The energy conservation equation

$$
T^\nu_{\mu;\nu} = 0, 
$$

(3.20)

for the perfect fluid with the interior metric shows that pressure is a function of $t$ only, i.e.,

$$
p = p(t). 
$$

(3.21)

Substituting the values of $X$ and $p$ from Eqs. (3.19) and (3.21) in Eqs. (3.14)-(3.18), it follows that

$$
2 \frac{\dddot{Y}}{Y} + \left( \frac{\dddot{Y}}{Y} \right)^2 + \frac{1 - H^2}{Y^2} = \Lambda + K^2 - p(t). 
$$

(3.22)
We consider \( p \) as a polynomial in \( t \) as given by \[ p(t) = p_c t^{-s}, \] (3.23)
where \( p_c \) and \( s \) are positive constants. Further, for simplicity, we take \( s = 0 \) so that
\[ p(t) = p_c. \] (3.24)
Replacing this value in Eq.(3.22), we get
\[
\frac{\ddot{Y}}{Y} + \left(\frac{\dot{Y}}{Y}\right)^2 + \frac{1 - H^2}{Y^2} = \Lambda + K^2 - 8\pi p_c.
\] (3.25)
Integrating this equation with respect to \( t \), it follows that
\[
\dot{Y}^2 = H^2 - 1 + \left(\Lambda + K^2 - 8\pi p_c\right)\frac{Y^2}{3} + 2\frac{m}{Y},
\] (3.26)
where \( m = m(r) \) is an arbitrary function of \( r \) and is related to the mass of the collapsing system. Substituting Eqs. (3.19), (3.26) into Eq. (3.14), we get
\[
m' = \frac{2K'K}{3}Y^3 + Y'Y^2[4\pi(p_c + \rho) + 2K^2].
\] (3.27)
For physical reasons, we assume that pressure and density are strictly positive. Integrating Eq.(3.27) with respect to \( r \), we obtain
\[
m(r) = 4\pi \int_0^r (\rho + p_c)Y'Y^2 \, dr + 2 \int_0^r K^2Y^2Y' \, dr + \frac{2}{3} \int_0^r K'KY^3 \, dr.
\] (3.28)
The function \( m(r) \) must be positive because \( m(r) < 0 \) implies negative mass which is not physical. Using Eqs. (3.19) and (3.26) into the junction condition Eq.(2.23), it follows that
\[
M = \frac{Q^2}{2Y} + m + \frac{1}{6}(\Lambda + K^2 - 8\pi p_c)Y^3.
\] (3.29)
The total energy \( \tilde{M}(r, t) \) up to a radius \( r \) at time \( t \) inside the hypersurface \( \Sigma \) can be evaluated by using the definition of mass function [13] given by
\[
\tilde{M}(r, t) = \frac{1}{2}Y(1 + g^{\mu\nu}Y_{,\mu}Y_{,\nu}).
\] (3.30)
For the interior metric, it takes the form

\[ \tilde{M}(r, t) = \frac{1}{2} Y(1 + \dot{Y}^2 - \left(\frac{Y'}{X}\right)^2). \]  

(3.31)

Replacing Eqs. (3.19) and (3.26) in Eq. (3.31), we obtain

\[ \tilde{M}(r, t) = m(r) + (\Lambda + K^2 - 8\pi p_c) \frac{Y^3}{6}. \]  

(3.32)

Now we take \((\Lambda + K^2 - 8\pi p_c) > 0\) and the assumption

\[ H(r) = 1. \]  

(3.33)

In order to obtain the analytic solutions in closed form, we use Eqs. (3.19), (3.26) and (3.33) so that

\[ Y = \left( \frac{6m}{\Lambda + K^2 - 8\pi p_c} \right)^{\frac{1}{3}} \sinh \alpha(r, t), \]  

(3.34)

\[ X = \left( \frac{6m}{\Lambda + K^2 - 8\pi p_c} \right)^{\frac{1}{3}} \left[ \left\{ \frac{m'}{3m} - \frac{2KK'}{3(\Lambda + K^2 - 8\pi p_c)} \right\} \sinh \alpha(r, t) \right. \]

\[ + \left. \left\{ \frac{2(t_s(r) - t)KK'}{\sqrt{3(\Lambda + K^2 - 8\pi p_c)}} + t'_s(r) \sqrt{\frac{\Lambda + K^2 - 8\pi p_c}{3}} \right. \right. \]

\[ \times \cosh \alpha(r, t) \right] \right] \sinh \alpha(r, t), \]  

(3.35)

where

\[ \alpha(r, t) = \sqrt{3(\Lambda + K^2 - 8\pi p_c)} \left[ \frac{t_s(r) - t}{2} \right]. \]  

(3.36)

Here \( t_s(r) \) is an arbitrary function of \( r \) and is related to the time of formation of singularity of a particular shell at coordinate distance \( r \).

In the limit \((8\pi p_c - K^2) \rightarrow \Lambda\), the above solution corresponds to the Tolman-Bondi solution [26]

\[ \lim_{(8\pi p_c - K^2) \rightarrow \Lambda} X(r, t) = \frac{m'(t_s - t) + 2mt'_s}{\left[\frac{6m^2(t_s - t)}{[6m^2(t_s - t)]^{\frac{1}{3}}} \right]} \]  

(3.37)

\[ \lim_{(8\pi p_c - K^2) \rightarrow \Lambda} Y(r, t) = \left[ \frac{9m}{2}(t_s - t)^{2} \right]^{\frac{1}{3}}. \]  

(3.38)
4 Apparent Horizons

Here we discuss the apparent horizons for the interior spacetime. The boundary of two trapped spheres whose outward normals are null is used to find the apparent horizons. This is given as follows:

\[ g^{\mu\nu} Y_\mu Y_\nu = Y^2 - \left( \frac{Y'}{X} \right)^2 = 0. \] (4.1)

Replacing Eqs. (3.19) and (3.26) in this equation, we get

\[ (\Lambda + K^2 - 8\pi p_c) \frac{Y^3}{3} - 3Y + 6m = 0. \] (4.2)

When we take \( \Lambda = 8\pi p_c - K^2 \), it gives \( Y = 2m \). This is called Schwarzschild horizon. For \( m = p_c = K = 0 \), we have \( Y = \sqrt{\frac{3}{\Lambda}} \), which is called de-Sitter horizon. Equation (4.2) can have the following positive roots.

**Case (i):** For \( 3m < \frac{1}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} \), we obtain two horizons

\[ Y_1 = \frac{2}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} \cos \frac{\varphi}{3}, \] (4.3)

\[ Y_2 = \frac{-1}{\sqrt{(\Lambda + 8\pi K^2 - p_c)}} (\cos \frac{\varphi}{3} - \sqrt{3} \sin \frac{\varphi}{3}), \] (4.4)

where

\[ \cos \varphi = -3m \sqrt{(\Lambda + K^2 - 8\pi p_c)}. \] (4.5)

If we take \( m = 0 \), it follows from Eqs. (4.3) and (4.4) that \( Y_1 = \frac{3}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} \) and \( Y_2 = 0 \). \( Y_1 \) and \( Y_2 \) are called cosmological horizon and black hole horizon respectively. For \( m \neq 0 \) and \( \Lambda \neq 8\pi p_c - K^2 \), \( Y_1 \) and \( Y_2 \) can be generalized [27] respectively.

**Case (ii):** For \( 3m = \frac{1}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} \), there is only one positive root which corresponds to a single horizon i.e.,

\[ Y_1 = Y_2 = \frac{1}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} = Y. \] (4.6)
This shows that both horizons coincide. The range for the cosmological and black hole horizon can be written as follows

\[ 0 \leq Y_2 \leq \frac{1}{\sqrt{(\Lambda + K^2 - 8\pi p_c)}} \leq Y_1 \leq \sqrt{\frac{3}{(\Lambda + K^2 - 8\pi p_c)}}. \]  

(4.7)

The black hole horizon has its largest proper area \(4\pi Y^2 = \frac{4\pi}{(\Lambda+K^2-8\pi p_c)}\) and cosmological horizon has its area between \(\frac{4\pi}{(\Lambda+K^2-8\pi p_c)} \) and \(\frac{12\pi}{(\Lambda+K^2-8\pi p_c)}\).  

Case (iii): For \(3m > \frac{1}{\sqrt{(\Lambda+K^2-8\pi p_c)}}\), there are no positive roots and consequently there are no apparent horizons.

We now calculate the time of formation for the apparent horizon using Eqs. (3.33), (3.34) and (4.2)

\[ t_n = t_s - \frac{2}{\sqrt{3(\Lambda + K^2 - 8\pi p_c)}} \sinh^{-1}\left(\frac{Y_n}{2m} - 1\right)^\frac{1}{3}, \quad (n = 1, 2). \]  

(4.8)

When \(8\pi p_c - K^2 \longrightarrow \Lambda\), this corresponds to Tolman-Bondi [26]

\[ t_{ah} = t_s - \frac{4}{3}m. \]  

(4.9)

From Eq. (4.8), we can write

\[ \frac{Y_n}{2m} = \cosh^2 \alpha_n, \]  

(4.10)

where \(\alpha_n(r, t) = \sqrt{3(\Lambda+K^2-8\pi p_c)}[t_s(r) - t_n]\). Equations (4.7) and (4.8) imply that \(Y_1 \geq Y_2\) and \(t_2 \geq t_1\) respectively. The inequality \(t_2 \geq t_1\) indicates that the cosmological horizon forms earlier than the black hole horizon.

The time difference between the formation of cosmological horizon and singularity and the formation of black hole horizon and singularity respectively can be found as follows. Using Eqs. (4.3)-(4.5), it follows that

\[ \frac{d(Y_{2m})}{dm} = \frac{1}{m} \left( -\sin \frac{\varphi}{3} + \frac{3\cos \frac{\varphi}{3}}{\cos \varphi} \right) < 0, \]  

(4.11)

\[ \frac{d(Y_{2m})}{dm} = \frac{1}{m} \left( -\sin \frac{\varphi+4\pi}{3} + \frac{3\cos \frac{\varphi+4\pi}{3}}{\cos \varphi} \right) > 0. \]  

(4.12)
The time difference between the formation of singularity and apparent horizons is

\[ T_n = t_s - t_n. \quad (4.13) \]

It follows from Eq. (4.10) that

\[ \frac{dT_n}{d\left(\frac{\Lambda}{2m}\right)} = \frac{1}{\sinh \alpha_n \cosh \alpha_n \sqrt{3(\Lambda + K^2 - 8\pi p_c)}}. \quad (4.14) \]

Using Eqs. (4.11) and (4.14), we get

\[ \frac{dT_1}{dm} = \frac{dT_1}{d\left(\frac{\Lambda}{2m}\right)} \frac{d\left(\frac{\Lambda}{2m}\right)}{dm} = \frac{1}{m \sqrt{3(\Lambda + K^2 - 8\pi p_c)} \sinh \alpha_1 \cosh \alpha_1} \times \left(-\frac{\sin \frac{\varphi + 4\pi}{3}}{\sin \varphi} + \frac{3 \cos \frac{\varphi + 4\pi}{3}}{\cos \varphi}\right) < 0. \quad (4.15) \]

It shows that \( T_1 \) is a decreasing function of mass \( m \). This means that time interval between the formation of cosmological horizon and singularity is decreased with the increase of mass. Similarly, from Eqs. (4.12) and (4.14), we get

\[ \frac{dT_2}{dm} = \frac{1}{m \sqrt{3(\Lambda + K^2 - 8\pi p_c)} \sinh \alpha_2 \cosh \alpha_2} \times \left(-\frac{\sin \left(\frac{\varphi + 4\pi}{3}\right)}{\sin \varphi} + \frac{3 \cos \left(\frac{\varphi + 4\pi}{3}\right)}{\cos \varphi}\right) > 0. \quad (4.16) \]

This indicates that \( T_2 \) is an increasing function of mass \( m \) indicating that time difference between the formation of black hole horizon and singularity is increased with the increase of mass.

5 Summary and Conclusion

This paper is devoted to study the effects of electromagnetic field on gravitational collapse with the positive cosmological constant. The cosmological constant acts as Newtonian potential. The relation for the Newtonian potential is \( \phi = \frac{1}{2}(1 - g_{00}) \). Using Eqs. (2.12) and (3.29), for the exterior spacetime, the Newtonian potential turns out to be

\[ \phi(R) = \frac{m}{R} + (\Lambda + K^2 - 8\pi p_c) \frac{R^2}{6}. \quad (5.1) \]
The corresponding Newtonian force is

\[ F = -\frac{m}{R^2} + (\Lambda + K^2 - 8\pi p_c)\frac{R}{3}. \tag{5.2} \]

Now we discuss the consequence of the Newtonian force. This force is zero for the fixed values of \( m = \frac{1}{3\sqrt{(\Lambda+K^2-8\pi p_c)}} \), and \( R = \frac{1}{\sqrt{(\Lambda+K^2-8\pi p_c)^3}} \) and will be positive (repulsive) if the values of \( m \) and \( R \) are taken larger than these values. If we take \( m = \frac{1}{\sqrt{(\Lambda+K^2-8\pi p_c)}} \) and \( R = \frac{3}{\sqrt{(\Lambda+K^2-8\pi p_c)^3}} \), then \( F = \frac{2(\Lambda+K^2-8\pi p_c)}{9} \), which gives positive value if \((\Lambda + K^2 - 8\pi p_c) > 0\), i.e., \( \Lambda > (8\pi p_c - K^2) \) such that \( 8\pi p_c > K^2 \). Thus we conclude that the repulsive force can be generated from \( \Lambda \) if \( \Lambda > (8\pi p_c - K^2) \) such that \( 8\pi p_c > K^2 \) over the entire range of the collapsing sphere. For the perfect fluid and dust cases, \( \Lambda \) can play the role of the repulsive force for \( \Lambda > 8\pi p_c \) and \( \Lambda > 0 \) respectively. Notice that \( K = K(r) \) gives the electromagnetic field contribution. From Eq.\( (3.26) \), the rate of collapse turns out be

\[ \ddot{Y} = -\frac{m}{Y^2} + (\Lambda + K^2 - 8\pi p_c)\frac{Y}{3}. \tag{5.3} \]

This shows that we have re-formulated the Newtonian model which represents the acceleration of the collapsing process. The analysis of positive and negative acceleration would give the same results as for the Newtonian force.

It is worthwhile to mention that the electromagnetic field reduces the bound of the positive cosmological constant by reducing the pressure. Thus the positive cosmological constant is bounded below as compared to the perfect fluid case. This would decrease the repulsive force which slows down the collapsing process. Making the analysis of the smaller values of \( m \) and \( R \) than the values used for the repulsive force, we find that the attractive force is larger than the perfect fluid case. Since the attractive force favors the collapse while the repulsive force resists against the collapse, thus the collapsing process is faster as compared to perfect fluid case when we include the electromagnetic field.

Further, we have found two apparent horizons (cosmological and black hole horizons) whose area decreases in the presence of electromagnetic field. It is found that the cosmological horizon forms earlier than the black hole horizon. Also, Eq.\( (4.8) \) shows that apparent horizon forms earlier than singularity. In this sense, we can conclude that the end state of gravitational collapse is a singularity covered by the apparent horizons (i.e., black hole).
It is interesting to mention here that our study supports the CCC and would be considered as one of its counter example. Also, it would be possible that the electromagnetic field reduces the range of apparent horizons to extreme limits and singularity would be locally naked. Thus the weak version of the CCC seems to be valid in this case.

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