Misconceptions of basic algebra on linear equation in one variable material

H Yansa¹, H Retnawati² and M Janna³

¹ Program Studi Penelitian dan Evaluasi Pendidikan, Pascasarjana Universitas Negeri Yogyakarta
² Fakultas Matematika dan Ilmu Pengetahuan Alam, Universitas Negeri Yogyakarta, Jl. Colombo No.1 Karang Malang, Caturtunggal, DIY, Indonesia
³ Program Studi Fisika, Magister Universitas Gadjah Mada, Jl. Bulaksumur, Caturtunggal, DIY 55281, Indonesia

E-mail: hajrayansa.2019@student.uny.ac.id

Abstract. This research was conducted to identify students' misconceptions of algebraic concepts. The researcher implemented a qualitative method with phenomenological design. The data were collected by conducting a test and an interview. Thirty students at the seventh grade were given a test, and six students were chosen to be interviewed. The subjects were selected by applying a purposive sampling technique. The research results indicated that the students have several misconceptions of basic algebra, such as on the degrees of the equation, variables, coefficients, and constants. Besides, students have difficulties identifying some variables in a function which resulted in errors in identifying the general form of linear equation in one variable. In other words, the students have not reached the algebraic thinking level yet. One of the factors which cause misconception is students' understanding of the basic concepts presented in the textbook, and the information delivered by the teacher was understood partially. As a result, the implications of this study are the students need special assistance. At the same time, innovative teaching strategies must be implemented by the teacher in teaching linear equation in one variable.

Keywords: Misconceptions of Algebra, Mathematics, Education, Linear Equations

1. Introduction

Nowadays, misconceptions have become a popular topic among experts in the field of Mathematics education. A misconception is a term that refers to some different understandings of the concepts proposed by the experts [1]. This phenomenon can also occur in students and even adults, since they acquire information or concepts from various learning sources, such as teacher, internet, textbooks, and acquiring from their environment [2]. These concepts are developed in order to create a new understanding or reinforce the previous understandings.

In Mathematics, one of the crucial things is the concept is hierarchical, since it is used at the higher level of education, and it has been developed from the basic to complex concept [3]. Therefore, a complete understanding of the concept must be possessed by the students, since understanding the concept will affect the success of the procedures (i.e., the use of meaningful and flexible procedures for solving problems)[4]. Thus, conceptual understanding is the main factor that affects students'
achievement or learning outcomes and the success of the mathematics teaching and learning process [5, 6, 7, 8, 9]. One of the goals of learning Mathematics is to understand mathematics concepts [10, 11]. Thus, in order to be successful in learning Mathematics, misconceptions should be prevented, since misconceptions can give impact on receiving other concepts, and rejecting the proper knowledge [1, 12].

In addition, misconceptions impact problem-solving [13] and students' learning outcomes [14]. Some researches had been conducted in order to identify the misconceptions experienced by the students. One of the research examined the concept of algebra, and the results indicated that the students' scores were lower than students who have the right conception [15, 16, 17]. At the junior high school level, the concept of algebra was taught in Linear Equation in One Variable material [18], and some algebraic misconceptions occurred [19, 20]. Hence, by identifying students' misconceptions, it can be an essential instructional strategy that effectively solves the problem [12]. In addition, by identifying the factors that cause misconceptions, the teacher can make innovations in the teaching system. Hence, evaluating students' misconceptions on basic concepts, especially when considering hierarchical mathematical concepts, is necessary. A study on misconceptions is still needed for the success of Mathematics teaching process in the classroom. This study aims to identify misconceptions in Algebra basic, on variable linear equations material. In addition, the aim is to describe the factors that cause misconceptions in the basic concepts of algebra, especially on the use of terms in algebra, such as the concept of degrees of variables, variables, constants, and coefficients, in order to answer the research questions.

2. Research design

2.1 Current design
The researcher implemented a qualitative method by applying a phenomenological design. In this research, the phenomenological design was intended to identify the phenomenon of students' algebraic basic misconceptions and the factors, in Linear Equation in One Variable material.

2.2 Research participants
The participants of this study were the Grade 7 students in Junior High School of Gowa Regency, South Sulawesi, Indonesia. The first step was conducting a written test for 30 students. After that, six students and Mathematics teachers were selected to be informants, who describe this phenomenon. The students and the teachers were selected by conducting purposive sampling technique since there were several criteria for the informants [21]. The criteria were students who have taken a written test and students whose all answers are correct, half-answers are correct, and all answers are incorrect.

2.3 Instrument and procedure
The strategies for identifying misconceptions were conducting tests and other non-test procedures (interviews/observations/questionnaires) [22]. Thus, the instrument used in this study was a test, consisting of three questions that represented three indicators of conceptual understanding of degrees, variables, constants, coefficients, and the general form of a one-variable linear equation. Meanwhile, the interview guideline was used, and it has been validated by Linguists and Mathematicians by applying face validity. The interview guideline was related to the results of students' work on written tests.

2.4 Data analysis
Data were analyzed by applying the techniques proposed by Miles, Huberman and Saldana [23] such as data reduction, data display, and drawing conclusion/verification. The first step in collecting data was marked by collecting the answer sheets. After that, data reduction was conducted to classify the answers (three categories, namely: students whose all answers are correct, half-answers are correct, and all answers are incorrect). Furthermore, interviews were conducted for six students who met the
category and the Mathematics teacher. The data reduction process was conducted during the research process. Third, the data were calculated in the form of narration, pictures, and tables. After that, the validity test was conducted, adopted from Moleong [24]. In this study, source triangulation was implemented by comparing the interview data obtained from students and teachers, books, and documents used in teaching Mathematics. After that, the conclusions were obtained and verified during the study. The conclusions were communicated to the teacher.

3. Results and discussions
In this part, we discuss the detail of students’ misconception of algebra basic in Linear Equation in One Variable material. Students' correct answers were based on these following indicators:

Table 1. The Percentage of students’ correct answers based on several indicators.

| Question | Indicator                                                                 | Percentage of correct answers |
|----------|---------------------------------------------------------------------------|-------------------------------|
| 1        | Identifying degrees of linear equation in one variable                    | 67.4%                         |
| 2        | Identifying the variables, coefficients, and constants of linear equation in one variable | 43.5%                         |
| 3        | Explaining the examples of Linear Equation in One Variable                | 56%                           |

Based on Table 1, the percentage of students' correct answers indicates that some students still answer incorrectly. After continuing to the interview, the problem was categorized as a misconception, since the students were confident with their answers, and they have certain reasons. Thus, some students still consider the degree of the variable of an equation $x-4 = 3x-1$ to be 2 (item 1). They assumed that the degree indicated by the sum of the two variables $x$ was not power. Some students wrote 0 as the answers, or have no degree, and were confident with that answer. It is considered as a misconception of a degree of equality.

Figure 1. The students' answers to item 1 with answer 2 and 0.

In addition, the students also made misconceptions on identifying algebraic symbols. Bush & Karp (2013) stated that algebraic symbolism is a difficult concept to be understood by the students, as shown in figure 2.

Figure 2. The students’ answers to item 2.
Based on figure 3, there is an error in distinguishing and exchanging the three concepts. After being confirmed in the interviews, the students know the definition of variables, coefficients, and constants, proved by the concept "variable is the letter and can vary while the constant is constant", while students believe that “constants in linear equation in one variable are only 1” to choose number 5. They ignored the negative sign since it was not affected by the value. The same error was encountered in the study [20]. Based on their answer, variables are always represented by letter symbols, while coefficients and constants are represented by numbers. The students’ misconception is numbers followed by letters are also called variables, and if the numbers are independent or do not contain letters, they are also called coefficients. Therefore, 4x was considered as a variable. This case also occurred in item 3 (figure 3); since it is calculated by the equation 3x-y = 5 and the students were asked by the teacher to determine variables, coefficients, and constants. If their answer is 3x, they called it a variable. This misconception is quite severe since variables are the most basic concept in algebra [15].

| 3. Jika 3x-y = 5 maka variabel, koefisien, dan konstantanya secara berturut-turut serta termsukkah persamaan tersebut persamaan linear satu variabel? |
|---|
| **Variable:** 3x |
| **Coefficient:** 3 |
| **Constant:** 5 |

**Figure 3.** Students' answers to item 3, considering it a linear equation in one variable.

| 3. Jika 3x-y = 5 maka variabel, koefisien, dan konstantanya secara berturut-turut serta termsukkah persamaan tersebut persamaan linear satu variabel? |
|---|
| X Y, 3, 5 BUKAN |

**Figure 4.** The students' answers to item 3, considering that it was not a one-variable linear equation.

Some of the students answered that 3x-y = 5 was a form of one-variable linear equation, and some answered that it was not a form of the one-variable linear equation. In figure 3, students ignored the
variable y and considered 3x as a variable. Meanwhile, in this equation, there are two variables, namely "x and y". This is due to the misinterpreted variable concept which causes an error in identifying the number of variables in Linear Equation in One variable or in identifying the general form of "ax + b = c". Usiskin stated the difficulties in understanding the algebraic form of a function due to confusion in identifying the number of letters in the equation [25]. Both students’ answers in figures 2 and 3 ignore -1 as the coefficient. In other cases, mostly the students do not see the value of the coefficient on a variable that is symbolized as a letter in an equation if it stands dependently, such as (-y) or just x, and it was different if a syllable consists of 3x or 2y. Then, the students can identify the numbers in front of letters as coefficients.

Based on the interviews with students, some of them understand that coefficients are identified by numbers, and they are not consistent in distinguishing coefficients and constants. They only remember the teacher's explanation "coefficients are always represented by numbers". The teacher confirmed this, or in other words, the results of the interview were in line, that the variables were represented by letters, like teachers in Singapore [26] while constants and coefficients were represented by with numbers. Thus, the misconception is due to the teacher's information during the learning process, which is partially understood by the students. Meanwhile, the other students experience misconceptions on constants caused by the textbooks used, due to the examples contained in the revision of 2013 Curriculum in 2017 [27], as shown in figure 5 (the example calculated contains only one constant)

From the illustration, express it in your language (don’t be afraid of being wrong). What is the meaning of:

a. Coefficient?
b. Variable?
c. Constant?

**Figure 5.** Explanation of variables Source As’ari [28].
incomplete or partial concept, which becomes an obstacle to learn epistemology [29], and in different contexts, students have difficulties or use their initial understanding. This incomplete and inaccurate understanding of the concept is categorized as a misconception.

This misconception is experienced by Indonesian students, but several previous studies have revealed that many students worldwide are still struggling with algebraic notation and symbolism [28,29]. However, until now, there has been no agreement from researchers and experts regarding the formal symbols used to represent algebraic symbols [32]. This will clearly cause various symbols that teachers will use, as well as affect students' understanding of algebraic symbols in Linear Equation in One Variable material. Therefore, it is necessary to identify students' initial understanding of the pre-requisite material before starting the teaching and learning process, [33]. In this case, it is the basic concepts of algebra, before teaching Linear Equation in One Variable material. The factors that cause misconceptions in the learning process, and textbooks were also found by Fadlan's [2] and Resbiantoro’s studies [34]. The factors that cause misconceptions will always occur, so a more detailed explanation in the learning process is needed by the students.

4. Conclusion
Based on this research results, it can be concluded that the students have experienced misconceptions in the basic algebra in Linear Equation in One variable, such as in variable degrees, variable concepts, coefficients, and constants. This study indicates that the students’ misconception causes errors in identifying the general form of Linear Equation in One Variable. Thus, the misconceptions of algebraic basics cause other misconceptions and subsequent errors.

In addition, in this study, one of the factors that cause misconceptions is the information acquired from the textbooks and teacher during the learning process is partially understood by the students. As a result, the implications are innovative teaching strategies. A more detailed explanation must be implemented by the teacher, especially in teaching the basic concepts of algebra, and the students need special assistance to remediate their misconception.

5. Acknowledgement
We are grateful for the advice given by experts in mathematics, their suggestions and feedback, and the Mathematics teachers and students who gave support and participation in this research.

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