Identification of Background False Positives from Kepler Data

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ABSTRACT. The Kepler Mission was launched on 2009 March 6 to perform a photometric survey of more than 100,000 dwarf stars to search for Earth-size planets with the transit technique. The reliability of the resulting planetary candidate list relies on the ability to identify and remove false positives. Major sources of astrophysical false positives are planetary transits and stellar eclipses on background stars. We describe several new techniques for the identification of background transit sources that are separated from their target stars, indicating an astrophysical false positive. These techniques use only Kepler photometric data. We describe the concepts and construction of these techniques in detail as well as their performance and relative merits.

Online material: color figures

1. INTRODUCTION

The Kepler mission is designed to determine the frequency of Earth-size planets in and near the habitable zone of solar-like stars via the detection of photometric transits (Borucki et al. 2010a; Koch et al. 2010). Kepler surveys more than 100,000 late-type dwarf stars in the solar neighborhood with visual magnitudes between eight and 16 for >4 yr, looking for transits of planets around those stars. There are several astrophysical phenomena that can cause a false-positive detection that mimics a planetary transit on a target star. Approximately 40% of the transit-like signals detected by Kepler that have been deemed Kepler objects of interest (KOIs) have been determined to be due to false positives.

To increase the reliability of the determination of which KOIs are planetary candidates, it is important to identify as many of these false positives as possible. In many cases, the identification of false-positive KOIs is based on Kepler data alone because these KOIs have transit signals that are too small for conventional ground-based follow-up. This paper describes several distinct but complementary methods for using Kepler data to detect cases where the source of a transit-like event is offset from the target star’s position on the sky. These background false positives make up a substantial fraction of all false positives, with most of the other false positives being due to grazing eclipsing stellar companions associated with the target star. At low Galactic latitudes, “background false positives” account for almost 40% of all Kepler transit-like signals, with the fraction dropping to about 10% at high Galactic latitudes (see Fig. 1). Background false positives are detected in Kepler data by observing that the pixels that change during transit are offset from those that contain the image of the target star. Such cases are referred to as “active pixel offsets” (APOs). The methods described in this paper cannot detect all background transit sources: for example, when the transit source is extremely close to the target star on the sky. However, they can identify a large percentage of background false positives. We believe that by identifying false positives that have an observable offset, the techniques described in this paper reduce the background false-positive rate in the planetary candidate catalog to below 10%.

The techniques described in this paper rely on pixel data returned from the Kepler spacecraft. Without this pixel data, the identification of background transit sources is much more difficult. Indeed, for dim target stars or for small planets with low signal-to-noise ratio (S/N) transits, ground-based follow-up typically will not suffice to identify background false positives. In such cases, background false-positive identification would be impossible using stellar light curves alone. Without the pixels, the star hosting the transit signal cannot be determined. Without knowing the star hosting the transit, the object causing the transit cannot be characterized. Therefore, the availability of the pixel data used to create the stellar light curves is a critical component of the success of any transit survey. This insight should drive the design of future transit survey missions.

In the rest of this section, we discuss background false positives in general, their identification via pixel analysis, and how that identification is used in the vetting of Kepler planet
candidates. The bulk of this paper describes several techniques for performing pixel-level analysis to identify background false positives. We describe in § 2 the photometric centroid technique and in § 3 the use of difference images to localize the transit signal source. Pixel correlations are described in § 4. We briefly address the special case of saturated targets in § 5. Section 6 presents several perspectives on how well these techniques perform, with special emphasis on comparing the photometric centroid and difference image techniques.

Throughout this paper we use several example KOIs (Borucki et al. 2011a, b; Batalha et al. 2010a, 2012; C. Burke et al. 2013, in preparation). Some of these KOIs are now valid candidates, while others have been determined to be false positives. We give particular attention to two examples to illustrate our techniques: KOI-221, which is a Kepler target where the transit source location is observationally coincident with the target, and KOI-109, which is a Kepler target for which the transit source is clearly offset from the target star. The list of KOIs and their properties can be found at the NASA Exoplanet Archive while the light curves and pixel data for all Kepler targets can be found at the Mikulski Archive for Space Telescopes.

1.1. Background False Positives

There are several astrophysical phenomena that can mimic a planetary transit on a specified target star. Brown (2003) distinguishes 12 combinations of giant planets and stars in eclipsing and transiting systems that can produce light curves mimicking a planet transiting a solitary primary star. Six of the combinations do not involve planets at all, and four others distort the transit light curve so that the size of the planet is indeterminate.

In this paper, we are concerned with those phenomena which are due to astrophysical sources that are not associated with the target star. These primarily include eclipsing binaries or large planet transits on stars that have flux in the pixels used to create the target star’s light curve. Because of dilution from the target star, even deep background eclipsing binaries often cannot be identified from the target star’s light curve alone. Analysis at the pixel level is required to identify the location of the transit signal source. We are particularly interested in cases where the transit signal’s source is sufficiently separated from the target star that we can measure a statistically significant offset between the target star and the transit source.

Additional sources of false positives that can be detected by the methods described in this paper include:

1. Very wide multiple star systems, where the transit source is gravitationally bound to the target star. When the separation between the target star and the companion hosting the transit signal source is large enough, the methods described in this paper can detect the offset.

2. Optical ghosts and electronic crosstalk (Caldwell et al. 2010) from planetary transits or eclipsing binaries elsewhere on the Kepler focal plane. When the image of the ghost or crosstalk falls on the target star’s pixels, but is sufficiently separated from the target star, these sources can be detected by the methods described in this paper. In addition, optical ghosts can have very nonstellar morphologies. Transit signals due to optical ghosts will exhibit these morphologies in several of the techniques described in this paper.

Our basic strategy is to measure the location of the transit source on the sky, compare that to the location of the target star, and declare the transit signal a false positive if the transit source location is significantly offset (more than three standard deviations, written >3σ) from the target star location, based on reliable data. All the methods of computing these offsets described in this paper use χ² minimizing (least-squares) methods. Assuming Gaussian statistics, these offsets form a two-degree-of-freedom χ² distribution that has offsets >3σ due to random fluctuations about 1.11% of the time. As we will show in this paper, offset uncertainties follow an approximately Gaussian distribution in a statistical sense, though the uncertainty around individual targets may not be Gaussian.

1.2. Pixel Analysis to Identify the Location of the Transit Source

As mentioned in § 1.1, the background binary causing a transit signal can be very faint, indeed significantly fainter than the
general background or the wings of the target star, and still mimic a planetary transit. Consider the case of an aperture that contains only a target star with constant flux \( F \) and a background binary with other negligible sky background. If the background binary is \( \Delta m \) magnitudes fainter than the target star, then the flux ratio of the background star to the target star is \( \Delta F = (100)^{-\Delta m/5} \). If the background binary has a fractional eclipse depth \( d_{\text{back}} \), then the total flux out of transit is \( F_{\text{out}} = F + F\Delta F \). In transit, the total flux is \( F_{\text{in}} = F + (1 - d_{\text{back}})F\Delta F \). Therefore, the fractional observed depth in the aperture is

\[
d_{\text{obs}} = 1 - \frac{F_{\text{in}}}{F_{\text{out}}} = 1 - \frac{1 + (1 - d_{\text{back}})\Delta F}{1 + \Delta F} = d_{\text{back}}\Delta F \frac{1}{1 + \Delta F}.
\]

In the case of a 14th magnitude target star and a 22nd magnitude background eclipsing binary with \( d_{\text{back}} = 0.5 \), we get \( d_{\text{obs}} = 315 \) ppm. A transit of this depth is easily detected in Kepler data and would mimic the transit of a small planet, though the 22nd magnitude background star would not be readily apparent in the Kepler data.

There are several ways to use Kepler pixel data to measure the distance from the target star to the transit source. We focus on three classes of techniques, each of which have their strengths and weaknesses. As we describe in detail below, none of these techniques work well in all circumstances due to systematic error sources that vary from technique to technique and situation to situation, but we find that the combination of these techniques covers the majority of cases where there is sufficient S/N to measure the transit source location. Our focus is on techniques that can be reliably automated due to the large number of objects in the Kepler data. We would also, when possible, like to associate the transit source with a known star. Therefore, we describe techniques that provide an estimate of the transit source location on the sky rather than simply determining if the transit source is at the target star location.

Kepler collects pixels specific to each target (Bryson et al. 2010b). A subset of these pixels, called the photometric optimal aperture, is summed to create the light curve for the target (see Fig. 2). The pixel analysis in this paper uses either the optimal aperture plus one halo of pixels, defined as any pixel adjacent to the optimal aperture (the photometric centroid technique described in § 2), or all pixels collected for a target (the difference image technique described in § 3). For most targets, Kepler pixel data are collected once every “long cadence” (29.4 minutes), and for a subset of targets, data are collected once every “short cadence” (0.98 minutes). In this paper we limit our discussion to long cadence observations.

All of the methods described in this paper identify spatially separated false positives by comparing pixel values during in-transit cadences to values of the same pixels during out-of-transit cadences.

Analysis of Kepler pixels to identify the location of the transit relative to the target star has to solve three problems:

1. Analyzing the Pixels Within a Cadence. There are various ways that the transit source location can be inferred from pixel data. Some of these methods require the identification of cadences that occur during transit and cadences that do not.

2. Combining the Cadences Within a Quarter. The Kepler spacecraft rotates 90° about the photometer boresite every ∼93 days (Koch et al. 2010). Each ∼93 day period is referred to as a quarter. While the Kepler focal plane is approximately symmetric under these 90° rolls, a star falls on different CCDs at difference pixel coordinates in different quarters. How in-transit and out-of-transit cadences within a quarter are selected and combined varies from technique to technique.

3. Combining the Cadences Across Quarters. Some of the techniques we discuss operate within a single quarter and will deliver different results from quarter to quarter. These results for each quarter must be combined to provide an overall measurement.

There are three classes of methods that we use to solve these problems:

1. Photometric Centroid Shift. Detection of a shift in the photometric centroid of the flux in the pixels (see § 2) that is correlated with the transit signal. This centroid shift can be used to estimate the location of the transit source, as described in § 2.

2. Difference Imaging. By constructing the difference of the in- and out-of-transit pixel images, a direct image of the transit source can be constructed, as described in § 3. The centroid of this image provides a direct measurement of the location of the transit source.
3. Pixel Correlation Images. When the transit signal can be detected in individual pixels via correlation with the photometric transit signal, an image can be constructed where the value of each pixel is given that correlation value, as described in § 4. This is an alternative method of creating a direct image of the transit source, whose centroid provides the transit source location.

These methods assume that the only source of flux variation is the object creating the transit signal. When this assumption is not satisfied, these methods will be subject to systematic error. Such systematic error will, however, be different for the different techniques; so when these methods give inconsistent results, we have an indication that systematic error is present.

In contrast to the photometric centroid method, which is based on measured centroid shifts, the difference and pixel correlation image methods produce images that directly show the transit source. While the location of the transit source can then easily be determined via photometric centroids of these images, we use a more robust centroid method based on fitting the *Kepler* pixel response function (PRF) (Bryson et al. 2010a). The PRF characterizes how light from a single star is spread across several pixels, so it is essentially the system point-spread function (PSF), comprised of the optical PSF convolved with pixel structure and pointing behavior over a *Kepler* long cadence. Given a star’s location on the pixels (including subpixel position), the PRF provides the contribution of that star’s flux to the nearby pixel values. Section 3.3 describes how the PRF is fit to pixel images to determine the location of the transit source.

These three methods are in principle very similar, but have different responses to systematics and noise, transit S/N, and field crowding. The use of all three methods provides increased sensitivity and confidence in the identification of background false positives, particularly when the transit S/N is low.

1.3. Role of Offset Analysis in Planet Candidate Vetting

The techniques described in this paper are used to decide whether or not a detected transit signal belongs on the *Kepler* planetary candidate list. These techniques have been applied to *Kepler* planetary candidate vetting (Borucki et al. 2011a,b; Batalha et al. 2010a, 2012; C. Burke et al. 2013, in preparation) with improved reliability and accuracy over time. The approach that eventually evolved is to identify those targets that show a significant offset between the target star and the transit source, relying primarily on the difference imaging method. Those targets that have a borderline significant source offset or have other cause for concern are examined using all the methods described in this paper, including manual examination of the pixels. Targets that have a confirmed offset from the transit source are identified as false positives. This disposition has changed over time for a small number of targets, as the techniques described in this paper have become more refined and as more data becomes available, resulting in greater measurement precision.

The details of how these analyses were applied are described in papers detailing the release of planetary candidate lists (Borucki et al. 2010a, 2011a, b; Batalha et al. 2012; C. Burke et al. 2013, in preparation). We give here a brief history of this evolution. Borucki et al. (2010a) used photometric centroid time series analyzed via the cloud plots described in § 2.1 and an early version of difference images. These difference images were visually examined rather than centroided, so offsets from the target star on the order of a pixel (4") or larger were identified. Borucki et al. (2011a) and Borucki et al. (2011b) used the difference image method including PRF centroiding described in § 3 without the multiquarter averaging, so each quarter was examined individually. Difference imaging with the multiquarter averaging (§ 3.4.1), joint-multiquarter PRF fits (for low S/N targets) (§ 3.4.2), and pixel correlation images (§ 4) were used in Batalha et al. (2012). C. Burke et al. (2013, in preparation) relied more strongly on multiquarter averaged difference imaging and photometric offsets. Joint-multiquarter PRF fits and pixel correlation images were disabled in C. Burke et al. (2013, in preparation) because of computational limitations. These limitations will be overcome in the future by moving the *Kepler* analysis pipeline to supercomputer platforms.

Because of the evolution towards the techniques described in this paper, the quality of background false-positive identification has changed over time. Therefore, the tables published in the early papers listed above have less accurate background false-positive identification than the later papers. This is reflected in the tables in the *Kepler* archives, so care must be taken when performing statistical analysis with these tables. At the time of this writing an effort is underway to recheck all KOIs using the methods described in this paper, as well as improved light curve analysis to identify nonbackground false positives such as grazing binary stars.

2. SOURCE LOCATION FROM PHOTOMETRIC CENTROID SHIFTS

"Photometric centroids” compute the “center of light” of the pixels associated with a target. When a transit occurs, the photometric centroid will shift, even when the transit is on the target star (the ideal case of a transit on a target star exactly in the center of a symmetric aperture with uniform background, which is required for there to be no centroid shift, is never realized in practice). As described in this section, we use this shift to infer the location of the transit source, from which we can compute the transit source offset from the target star. This method works well when the target star is crowded by many field stars but suffers from high sensitivity to variable flux not associated with the transit, such as stellar variability and photometric noise. As described in § 2.3.1, due to the implementation of the *Kepler* processing pipeline, this method tends to overestimate the distance of the transit source from the target star when the transit source is at the edge of the target star’s pixels.
2.1. Computing Pixel Centroids

The most traditional method for estimating the position of a light source is that of photometric centroids, also known as flux-weighted centroids. Photometric centroids measure the “center of light” of all flux in the pixels. While photometric centroids do not exactly measure the location of any particular star, it will be shown below that under idealized circumstances they can be used to compute the location of a transit source.

The row and column photometric centroids of the pixels for each target are computed for each cadence as:

\[
C_{\text{row}} = \frac{\sum_{j=1}^{N} r_j b_j}{\sum_{j=1}^{N} b_j}, \quad C_{\text{column}} = \frac{\sum_{j=1}^{N} c_j b_j}{\sum_{j=1}^{N} b_j},
\]

where \( b_j \) is the flux in pixel \( j \) at row and column \( (r_j, c_j) \). If we denote the covariance matrix of the pixel values \( b_j \) as \( C_{ij} \) (so the uncertainties in the pixel values are the square root of the diagonals: \( \sigma_j = \sqrt{C_{jj}} \)), then the standard propagation of errors gives the uncertainty in the photometric row centroid as:

\[
\sigma_{C_{\text{row}}} = \sqrt{\frac{\sum_{j=1}^{N} \sum_{j'=1}^{N} r_j C_{ij} r_{j'}}{\sum_{j=1}^{N} b_j^2} + \frac{\left( \sum_{j=1}^{N} r_j b_j \right)^2}{\sum_{j=1}^{N} b_j^2} \sum_{j=1}^{N} C_{ij}}
\]

with a similar formula for the uncertainty in the column centroid. We see that the sensitivity of the centroid value \( \sigma_{C_{\text{row}}} \) is proportional to the square root of the elements of the covariance matrix \( C_{ij} \), in particular to the uncertainty in the pixel values \( \sigma_j \), divided by the total flux in the pixels \( \sum_{j=1}^{N} b_j \). Therefore, photometric centroids are very sensitive to variations in pixel value, in particular to shot noise and stellar variability.

For photometric centroids computed in the Kepler pipeline, \( j \) ranges over the optimal aperture plus a single ring of pixels (sometimes called a “halo”). The result is a time series containing the row and column centroids, called “centroid time series.” The centroid shift is defined as the centroid value for cadences out of transit, \( C^{\text{out}} \), subtracted from the centroid value for cadences in transit, \( C^{\text{in}} : \Delta C = C^{\text{in}} - C^{\text{out}} \). We assume shifts in different cadences are uncorrelated, so these shifts have an uncertainty given by \( \sigma_{\Delta C}^2 = \sigma_{C^{\text{out}}}^2 + \sigma_{C^{\text{in}}}^2 \).

It is very important to distinguish between the “centroid shift,” which measures how far the centroid moves between in- and out-of-transit cadences, and the “source offset,” which measures the separation of the target star from the transit source. As we will describe below, the centroid shift and source offset are related but measure very different things. The centroid shift measures the change in the photometric centroid due to all changes in flux in the aperture. The source offset is derived from the centroid shift but measures the separation between the target star and the transit source (which may or may not be a different star). In particular, because there is always background flux and field stars, the centroid shift \( \Delta C \) will always be nonzero even when the transit signal is on the target star. In such cases, the centroid shift can be relatively large while the source offset may be very close to zero.

Low-frequency secular trends due to small, slow changes such as differential velocity aberration, small pointing drifts, and thermally induced focal length changes are common in centroid time series (Christiansen et al. 2012). These trends are removed prior to the analysis described in this section, for example by local median filtering using a window of 48 cadences.

To facilitate combining the centroids across quarters, the centroid time series is converted to celestial right ascension (R.A.) and declination (decl.) using the Kepler focal plane geometry model in combination with motion polynomials that capture local variations in the focal plane geometry model (Tenenbaum & Jenkins 2010). In these coordinates, the centroid shift \( \Delta C \) is expressed as seconds of arc.

When the centroid shift \( \Delta C \) is large enough, it can be taken to indicate that the transit source is not on the target star. Using \( \Delta C \) directly to make this determination must be done with great care, however. The centroid shift \( \Delta C \) will be smallest when the target star is the source of the transit, the target star is isolated, residual background flux is small after background correction, and the target star is near the geometric center of the centroided pixels. This is rarely the case, however, so even when the target star is the source of the transit, there will be a nontrivial centroid shift. A larger centroid shift that is correlated with the time of transit is an indicator that the transit source may not be the target star. Determining whether a centroid shift indicates that the transit source is not the target star is difficult, however, and depends on the details of other flux sources in the target’s pixel aperture.

In § 2.3, we describe how to use the centroid shift to estimate the location of the source of the centroid signal, which is a more robust method for determining whether the transit source is the target star than using the centroid shift alone.

A graphical method showing the correlation between the centroid shift and the transit signal is to plot the median-detrended centroid time series against the normalized, median-detrended light curve flux value. The result is a “cloud plot,” shown in Figure 3. Most points in a cloud plot are out-of-transit cadences and form a cluster around \((0, 0)\). The size of the cloud reflects the sensitivity of the photometric centroid computation to noise in the pixel values. When there is no centroid shift associated with transits, the points in transit (with negative normalized flux) fall directly below the out-of-transit points. When there is a centroid shift associated with the transit, points in transit will fall to the side of the out-of-transit cloud. Seeing sideways motion of the in-transit points, as shown in the right panel of Figure 3, indicates a centroid shift associated with the transit. This suggests that the transit source may be offset from the target star. As explained above, care must be taken when interpreting cloud plots because there may be a nontrivial
2.2. Correlating Centroid Motion with the Transit Model

The centroid time series is sensitive to photometric noise, so quantitatively measuring the correlation of the centroid shift with the photometric transit signal can be difficult, particularly for low S/N transits. A simple approach is to identify all in- and out-of-transit cadences, and compute the average (or median) in- and out-of-transit centroid values. The average centroid shift is then given by the difference of the in- and out-of-transit average centroid locations. This method encounters many difficulties, however: quarter-to-quarter differences in aperture shape will introduce systematic errors, and non-transit related variability will degrade these averages as measures of transit-related shifts. A better method is to fit a transit model computed during data validation (Wu et al. 2010) to the centroid time series. This will provide a more robust measurement of $\Delta C$.

In this section, we define the centroid shift time series $\Delta C_n = C_n - C_{\text{out}}$, where $C_{\text{out}}$ is the average out-of-transit centroid and $n$ labels the cadence. In this section, we assume that the transit model has been whitened to remove secular variations, such as those due to pointing drift and stellar variability (Wu et al. 2010), in which case the centroid shift time series $\Delta C_n$ must be whitened in the same way. We compute a least-squares fit of the centroid shift time series $\Delta C_n$ to the transit model $M_n$ multiplied by a constant $\gamma$, weighted by the centroid uncertainties. This fit is most easily done by requiring that the transit model and the centroid shift time series both have zero mean when the transit is not occurring. This implies that the transit model $M_n = 0$ for out-of-transit cadences. When this is the case we minimize

$$\chi^2 = \sum_{n=1}^{N} \frac{1}{(\sigma_{\Delta C_n})^2} (\Delta C_n - \gamma M_n)^2.$$  \hspace{1cm} (3)

This least-squares minimization problem has the solution

$$\gamma = \frac{\sum_{n=1}^{N} (\Delta C_n M_n) / (\sigma_{\Delta C_n})^2}{\sum_{n=1}^{N} M_n^2 / (\sigma_{\Delta C_n})^2}.$$  \hspace{1cm} (4)

Examples of this fit are given in Figures 4 and 5.

Assuming that the centroid and transit model uncertainties are uncorrelated over time, and neglecting uncertainties in the transit model values, the uncertainty in $\gamma$ is

$$\sigma_\gamma = \left[ \sum_{n=1}^{N} \frac{M_n^2}{(\sigma_{\Delta C_n})^2} \right]^{-1/2}.$$  \hspace{1cm} (5)

Only in-transit cadences contribute to the computation of $\gamma$ and $\sigma_\gamma$ because $M_n = 0$ for out-of-transit cadences. Because $M_n$ is fit to the whitened and normalized flux light curve, it has unit variance, so $\gamma$ is in the same units as $\Delta C_n$ and directly gives an estimate of the in- versus out-of-transit shift: $\Delta C \approx \gamma$. When the centroids shifts are in R.A. and decl. coordinates, all quarters of data can be simultaneously fit. From equation (5) we see a $\sqrt{N^{\text{in}}}$ reduction in the uncertainty, where $N^{\text{in}}$ is the total number of in-transit cadences.
in-transit cadences, so combining many quarters increases the precision of the estimate of $\Delta C$ in each coordinate.

Once the shift is estimated in R.A. and decl. (in seconds of arc), the shift distance is simply

$$D = \sqrt{\Delta C_{\text{R.A.}}^2 + \Delta C_{\text{decl.}}^2},$$

(6)

with uncertainty

$$\sigma_D = \sqrt{\Delta C_{\text{R.A.}}^2 \sigma_{\Delta C_{\text{R.A.}}}^2 + \Delta C_{\text{decl.}}^2 \sigma_{\Delta C_{\text{decl.}}}^2}. $$

(7)

A high-level detection statistic indicating whether a detected shift is statistically significant is also computed. This statistic measures the probability that the detected shift is due to an actual signal rather than a statistical fluctuation in white noise by subtracting the residual $\chi^2$ from the signal $\chi^2$. From this statistic, a significance metric is constructed that is normalized to the range $[0, 1]$, where one means that there is no detected shift and zero means that the shift is highly significant. This is equivalent to equation (4) of Wu et al. (2010), which in our notation is given by

$$l = \frac{\sum_{n=1}^{N} \Delta C_n M_n}{\sigma_{\Delta C} \sqrt{\sum_{n=1}^{N} M_n^2}}. $$

(8)

### 2.2.1. Impact of Crowding and Variability on the Centroid Shift Estimate

The computation of the in-transit centroid shift assumes that the transiting object is the only source of time varying flux that is correlated with the transit signal in the target star’s pixels.
While this is usually a reasonable assumption, it is sometimes violated, introducing systematic error into the centroid shift estimate. A dramatic example is KOI-1860, whose pixels are shown in Figure 6. In this case, there is a field star that is 2.7 mag brighter than the target star at the edge of the collected pixels. Examination of the pixel flux time series shows that this bright star has moderately high variability on short time scales. In addition, because this bright star is at the edge of the collected pixels and is only partially captured, there are strong variations in flux due to spacecraft pointing jitter. The effect of these variations on the centroid time series are shown in Figure 7. These variations are on a time scale that occasionally correlates with the transit signal, leading to a small spurious measured centroid shift in the fit in equation (4). The reconstructed transit source location using this spurious shift measurement, described in § 2.3, indicates a transit source separated from the target star by about 4". As we will see in § 3.4.1, however, the PRF-fit technique provides strong evidence that the transit source is only about a third of an arcsecond from the target star.

2.3. Estimating the Transit Source Location from Centroid Motion

Photometric centroids are the weighted average of all flux in the target star’s pixels, so they do not provide direct information about the location of the target star or the transit source. In particular, as explained in § 2.1, a statistically significant shift does not necessarily imply that the transit source is offset from the target star. In the Appendix, we derive a formula approximating the location of the transit source from the observed transit depth (based on the light curve created by summing the pixels used for centroiding), the out-of-transit centroid location
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Fig. 6.—Pixels collected for KOI-1860 in quarter 10. The pixels are dominated by the field star KIC 4157320 which is 2.7 mag brighter than the target star. KIC 4157320 has strong variability. In addition, because it is only partially captured in the pixels, spacecraft pointing variations are apparent in the pixel flux light curves. See the electronic edition of the PASP for a color version of this figure.

\[ \sigma_{\alpha} = \sqrt{\sigma_{\alpha,\text{out}}^2 + \left(\frac{1}{d_{\text{obs}}} - 1\right)^2 \sigma_{\alpha,\text{in}}^2 \cos^2 \delta + \Delta C_{\alpha}^2 \sigma_{d_{\text{obs}}}^2} \]

(10)

\[ \sigma_{\delta} = \sqrt{\sigma_{\delta,\text{out}}^2 + \left(\frac{1}{d_{\text{obs}}} - 1\right)^2 \sigma_{\delta,\text{in}}^2 + \Delta C_{\delta}^2 \sigma_{d_{\text{obs}}}^2} \]

(11)

These uncertainties do not account for systematic error due to other sources of varying flux.

For \( d_{\text{obs}} \ll 1 \), equation (9) reduces to

\[ \alpha_{\text{transit}} \approx C_{\alpha} - \frac{\Delta C_{\alpha}}{d_{\text{obs}} \cos \delta}, \quad \delta_{\text{transit}} \approx C_{\delta} - \frac{\Delta C_{\delta}}{d_{\text{obs}}} \]

(12)

the approximation given in equation (2) of Wu et al. (2010). The uncertainties are similarly approximated by replacing \( (1/d_{\text{obs}} - 1) \) by \( 1/d_{\text{obs}} \). This approximation has an error that is proportional to \( d_{\text{obs}} \), which is very small for most Kepler planetary candidates.

Once we have the centroid source location from equation (9), we compare it with the target location to determine the source offset. The target star location cannot, however, be reliably determined from the centroid time series, so we take the target star position from the Kepler input catalog. This choice potentially introduces new sources of systematic error, particularly due to unknown proper motion.

Given the target star’s catalog location \((\alpha_{\text{target}}, \delta_{\text{target}})\), we can compute the target offset and uncertainty from the offset components \( \Delta \alpha = (\alpha_{\text{transit}} - \alpha_{\text{target}}) \cos \delta \) and \( \Delta \delta = \delta_{\text{transit}} - \delta_{\text{target}} \) as

\[ D = \sqrt{\Delta \alpha^2 + \Delta \delta^2}, \quad \sigma_D = \frac{\sqrt{\Delta \alpha^2 \sigma_{\Delta \alpha}^2 + \Delta \delta^2 \sigma_{\Delta \delta}^2}}{D} \]

(13)

where \( \sigma_{\Delta \alpha} = \sqrt{\sigma_{\alpha_{\text{transit}}}^2 + \sigma_{\alpha_{\text{target}}}^2 \cos \delta} \) and

\[ \sigma_{\Delta \delta} = \sqrt{\sigma_{\delta_{\text{transit}}}^2 + \sigma_{\delta_{\text{target}}}^2}. \]

We can now determine if the transit source is statistically significantly offset from the target star by observing whether \( D > 3 \sigma_D \).
2.3.1. Systematic Errors in the Source Position Estimate

As discussed in the Appendix, the above analysis does not describe the current implementation in the Kepler pipeline. The Kepler pipeline uses the photometrically optimal aperture (Bryson et al. 2010b) to compute the transit depth and the optimal aperture plus one ring of surrounding pixels to compute the centroid (see Fig. 9). This use of different pixel apertures to compute the depth and centroid invalidates the above analysis when significant flux from the transit source falls outside the optimal aperture. Because optimal apertures are as small as a single pixel, such overshoot is possible when the transit source and target star are separated by more than one Kepler pixel (3.98″).

In the typical background false-positive case when the transit source is associated with a field star that is significantly dimmer than the target star, the observed depth in the optimal aperture (the depth computed by the Kepler pipeline) will be smaller than the depth that would have been observed using the centroided pixels. This will result in an overestimate of the distance of the transit source from the out-of-transit photometric centroid $C_{\text{out}}$ in equation (9). Occasionally the field star associated with the transit source will be brighter than the target star, so the flux from the target star dominates the centroids. In this case, the observed depth in both apertures will be similar, resulting in less of an overshoot. This behavior is observed in § 6.1. See the Appendix for details.

The dependence of the source offset estimate on the ratio of the brightness of the background star to that of the target star is shown in Figure 10. This example is similar to that in Figure 6, where the background star causing the transit signal is outside the optimal aperture and mostly, but not completely, captured in the centroided pixels. When the background star is dim, the estimated transit source overshoots the correct offset. When the background star is significantly brighter than the target star, then the flux from the background star dominates the depth estimate, so the depth based on the centroided pixels is about the same as...
the depth based on the optimal apertures. But because the background star is close to the edge of the centroided pixels, not all flux from the background star is captured. Therefore, the source offset estimate in equation (9) gives the centroid of the flux in the pixels from the background star, which is closer to the target star than the background star itself.

3. DIFFERENCE IMAGING

The “difference imaging technique” computes the difference between average in- and out-of-transit pixel values. These pixel differences provide an image of the transit source at its true location. A centroid of this “difference image” provides the location of the transit source. To measure this centroid, we fit the Kepler pixel response function (PRF), looking for the PRF position that best matches the difference pixels. We compare this position to the PRF fit to the out-of-transit position, which provides the target star position when it is not crowded by field stars. The difference of these centroids gives us the offset of the flux from the background star, which is closer to the target star than the background star itself.

3.1. Concept of Difference Imaging

The difference image technique is based on the insight that subtracting the in-transit pixel values from the out-of-transit pixel values gives an image that shows only those pixels that have changed during the transits. Further, if the changes during transits are due to a change in brightness of a star (as is the case for a planetary transit or an eclipsing binary), then the bright...
pixels in the difference image will be those of that star with flux given by the fractional transit depth times the flux of that star.

More precisely, consider a set of pixels that contain flux from $M$ stars, labeled by the index $j$, at locations $(\alpha_j, \delta_j)$ with flux $b_j$ (we neglect background flux in this simple analysis). The PSF will distribute the flux from each of these stars over several pixels. We express the flux on the pixel at row $r$ and column $c$ due to star $j$ by the unit flux function $f(\alpha_j, \delta_j, r, c)$ [so the sum over all pixels of $f(\alpha_j, \delta_j, r, c) = 1$]. Then, the out-of-transit pixel values due to all stars will be given by $F_{\text{out}}(r, c) = \sum_{j=1}^{M} b_j f(\alpha_j, \delta_j, r, c)$. If star $k$ has a transit of depth $d_{\text{back}}$, then during midtransit, the pixel values would be given by $F_{\text{in}}(r, c) = \sum_{j=1,j\neq k}^{M} b_j f(\alpha_j, \delta_j, r, c) + (1 - d_{\text{back}}) b_k f(\alpha_k, \delta_k, r, c)$. In the ideal case where the only flux change is in star $k$, the difference image will be $F_{\text{out}}(r, c) - F_{\text{in}}(r, c) = d_{\text{back}} b_k f(\alpha_k, \delta_k, r, c)$, which is exactly the image of star $k$ with flux $d_{\text{back}} b_k$.

Difference images provide direct information about the location of the transit source, as opposed to the use of photometric centroids in § 2.1, where the source location is inferred.

Example pixel images are shown in Figures 11 and 12. In Figure 11, we see an example of a star (KOI-221) for which there is no apparent offset between the target star and the transit source. In this case, the difference image looks much like the in- and out-of-transit images, likely because the target star is itself the source of the transit (and there are no other stars of comparable brightness in the out-of-transit image). Therefore, the only difference between the difference image and the out-of-transit image is the flux level in the pixels. Figure 12 shows a case (KOI-109) where the difference image is dramatically different from the out-of-transit image and appears as a star image coincident with the dim unclassified star KIC 4752452. Because KIC 4752452 is unclassified, it does not have a Kepler magnitude. In this case, the pixel data show that the transit source is clearly not on the target star.

![Difference Flux (e-cadence$^{-1}$)](image1)

![Out-of-Transit Flux (e-cadence$^{-1}$)](image2)

![In-Transit Flux (e-cadence$^{-1}$)](image3)

![Difference S/N](image4)

FIG. 11.—Example pixel images for KOI-221 in quarter 7, which shows no indication that the transit is not on the target star. In all figures, the dotted white line borders the pixels of the optimal aperture, while the solid white line borders all pixels collected for this target. Known stars are shown as white asterisks, with each star’s KIC number and Kepler magnitude. Upper right: Averaged out-of-transit pixel image. Lower left: In-transit pixel image. Upper Left: Difference image = out-of-transit pixel image—in-transit pixel image. Lower right: Difference image normalized by pixel value uncertainty. In this case, the difference image appears identical to the in- and out-of-transit images, which indicates that the transit source is coincident with the target star. See the electronic edition of the PASP for a color version of this figure.
When the transit S/N is high the pixel images appear as in Figures 11 (S/N = 378) and 12 (S/N = 101), with very well defined star-like difference images. When the S/N is high and the transit is on the target star, as in Figure 11, we expect the difference image to look like the out-of-transit image. Figure 13 shows an example of a low S/N transit on KOI-2949 with a S/N of 11. In this figure, the difference image looks significantly different from the in- and out-of-transit images, which indicates that the transit source is coincident with the star KIC 4752452. See the electronic edition of the PASP for a color version of this figure.

When the offset is as dramatic as that in Figure 12, cursory visual inspection is sufficient to determine that the transit signal does not occur on the target star. We are interested, however, in measuring smaller offsets that may not be so visually obvious. In addition, we wish to have the ability to automatically measure and detect such transit-source offsets for thousands of transit signals. This can be done by measuring the centroid of the difference image and comparing with estimates of the target star position. This approach encounters several difficulties:

1. Difference images can be noisy, particularly for low S/N transits. This is particularly a problem for transits near spacecraft thermal events and in multiple planet systems, where the transit signals from multiple planets can interfere with each other.

2. Determination of the location of the target star should use the same method as the difference image to minimize the impact of systematic measurement errors.

3. The structure of the background signal for the target star due to crowding will be very different from the difference image.
background signal because non-variable background stars will cancel out in the difference image.

4. In different quarters, stars fall in different places on different pixels, and pixel apertures vary from quarter to quarter. Therefore, the offsets measured in different quarters can be different.

We address these difficulties through the following strategies:

1. Careful construction of the in- and out-of-transit images, described in § 3.2, so the difference image is as clean as possible.

2. Determining the location of stars in the difference or out-of-transit image via PSF-type fitting to the pixel data using the *Kepler* pixel response function (PRF), described in § 3.3, which is more robust against noise than photometric centroids.

3. Either carefully averaging the quarterly offsets (§ 3.4.1), or performing a joint multiquarter fit (§ 3.4.2).

3.2. Construction of In- and Out-of-Transit and Difference Pixel Images

Our goal is to measure the location of the change in the flux due to the transit signal. Therefore, we want to create a difference image by subtracting pixel flux in transit from pixel flux near transit. We want to avoid pixel flux away from the transit so changes due to stellar variability are less likely to enter into the difference image. We also want to avoid changes in flux that are not related to the transit under examination, such as spacecraft thermal or pointing events or transits due to other planets orbiting the target star in multiple systems. We minimize noise by averaging as many in- and out-of-transit measurements as possible, subject to these constraints.

---

FIG. 13.—Pixel images for a low S/N transit on KOI-2949 with an S/N of 11. The difference image appears significantly different from the out-of-transit image in this quarter, indicating that the transit source is not on the target star. But other quarters show the transit source in other locations, including on the target star. This situation is typical for low S/N transits, and more reliable measurement of the transit source location can be attained by combining the quarters as described in § 3.4. In this example, the combined quarter result indicates that the transit location is statistically consistent with the target star. See the electronic edition of the *PASP* for a color version of this figure.
In each quarter, Kepler collects about 4300 long cadences, from which in- and out-of-transit exposures need to be identified. We use the (unwhitened) transit model $M_n$ constructed in data validation (Wu et al. 2010) to select these cadences.

In-transit cadences are defined as those cadences where the model is less than a threshold proportional to the model transit depth. The current threshold is $3/4$ of the transit depth: when the model is normalized so that $M_n = 0$ for out-of-transit cadences, in-transit cadences are those for which the model values $M_n < -(3/4)d$, where $d$ is the modeled fractional transit depth.

The out-of-transit cadences are chosen near each transit under the following criteria:

1. Out-of-transit cadences are chosen on both sides of the transit so that an average of these out-of-transit cadences removes any locally linear secular trends.

2. Not too many cadences are chosen so that nonlinear variability on time scales longer than the transit are small.

3. Out-of-transit cadences should not be too close to the transit.

The number of out-of-transit cadences $N_{\text{out}}$ is chosen as the number of cadences that occur during the entire transit duration where $M_n < 0$. This is generally not the same as $N_{\text{in}}$. The out-of-transit cadences are chosen to lie more than $N_{\text{buffer}}$ cadences from the cadences for which $M_n < 0$. Figure 14 shows an example of selected cadences for a typical transit.

After in- and out-of-transit cadences are chosen, they are excluded if they are associated with any of the following events:

1. Data gaps such as Earth points and safe modes.

![Model Light Curve](attachment:Model_Light_Curve.png)

**Fig. 14.**—Example of in- and out-of-transit cadence selection (KOI-221). Top: Transit model $M_n$ for a selected cadence range in quarter 6. The x-axis shows the cadences since the beginning of the Kepler science operations. The circles at the bottom of the transit show the cadences that were chosen for the in-transit image. In the transit, $N_{\text{in}} = 4$ cadences were chosen because they are below the threshold described in the text. The circles outside the transit show the cadences chosen for the out-of-transit image. The full transit is six cadences wide so $N_{\text{out}} = 6$ cadences were chosen on both sides of the transit. The out-of-transit cadences are $N_{\text{buffer}} = 3$ cadences from the transit. Bottom: Actual transit in one of the brighter pixels. The x-axis shows the cadences since the beginning of quarter 6. See the electronic edition of the PASP for a color version of this figure.
2. Cadences within a day after major spacecraft thermal events, such as recovery from Earth points and safe modes that significantly change the temperature distribution of the spacecraft and require many hours to return to thermal equilibrium.

3. Pointing anomalies such as attitude tweaks and loss of fine-point events.

4. Interference by transits from other planets in multiple planet systems. An example of such interference is shown in Figure 15.

If more than a small number of cadences associated with a transit are excluded, then the entire transit is excluded from the construction of the difference image. This threshold is currently set to zero, so if any cadences are excluded, then the entire transit is excluded. As Kepler detects longer-period transits, so fewer transits will be available, this threshold will be relaxed to one or two excluded cadences per transit.

Once the final set of transits and their in- and out-of-transit cadences are identified, the in-transit pixel values are averaged to produce the in-transit image and the out-of-transit cadences are averaged to produce the out-of-transit image. The pixel values are not whitened or otherwise detrended; we rely on the averaging described in this section to remove local secular trends. First, the average pixel values are computed for each transit; then, each transit’s averaged pixels in a quarter are averaged together to produce the final in- and out-of-transit average pixel images for that quarter. The difference image for the quarter is then the out-of-transit pixel image minus the in-transit pixel image.

### 3.3. Fitting the Pixel Response Function

In this section, we describe how the Kepler pixel response function (PRF) (Bryson et al. 2010a) is used to provide a robust, high-precision estimate of the target star and transit locations using the average out-of-transit and difference images constructed as described in § 3.2. This technique requires that the target star is several magnitudes brighter than other stars in the out-of-transit pixels and that the transit signal is sufficiently strong in the difference image. In § 3.3.2, we describe a quantitative measure of whether the average images for a given target star have the required properties. Section 3.3.1 describes various ways in which this method can be compromised and discuss mitigation strategies.

The PRF gives the long-cadence brightness of a pixel due to a star at a specified location. The PRF can be thought of as the convolution of the optical PSF with the effects of pointing, subpixel response, and system electronics. In this section, we write the PRF as a unit flux function \( f(\alpha, \delta, r_i, c_i) \) so \( \sum_{i=1}^{P_{\text{total}}} f(\alpha, \delta, r_i, c_i) = 1 \), where \( P_{\text{total}} \) is the number of all pixels that contain flux from a star at sky coordinates \((\alpha, \delta)\), and \( r_i \) and \( c_i \) are those pixels’ row and column coordinates. If the star has flux \( b \), then the value of a pixel at row \( r_i \) and column \( c_i \) due to that star will be \( p_i = b f(\alpha, \delta, r_i, c_i) \), and the sum of all pixels containing flux from that star is \( \sum_{i=1}^{P_{\text{total}}} p_{r_i, c_i} = b \). [In Bryson et al. (2010a), the star location is defined in pixel coordinates rather than sky coordinates. In this paper, we include the projection from sky coordinates to pixel coordinates in the PRF function \( f \).]

Assume we are given a set of \( P \) pixel values \( p_i \), with rows \( r_i \) and columns \( c_i \) that form a pixel image. The \( P \) pixels need not contain all the flux from the target star, so \( P \) may be less than \( P_{\text{total}} \). A minimize the function PRF fit to these pixels is the determination of sky coordinates \((\alpha_{\text{fit}}, \delta_{\text{fit}})\) and flux \( b_{\text{fit}} \) that minimize the function

\[
\chi^2 = \sum_{i=1}^{P} \frac{1}{\sigma_{p_i}^2} [p_i - b f(\alpha, \delta, r_i, c_i)]^2,
\]  

(14)

where \( \sigma_{p_i} \) is the uncertainty in the pixel value \( p_i \). This fit is performed iteratively via the nonlinear Levenberg-Marquardt
algorithm (Levenberg 1944; Marquardt 1963). Formally, this is a three-dimensional fitting problem in the parameters $\alpha$, $\delta$, and $b$. The fit to $b$, however, can be reduced to a linear problem once the position is known, so this problem can be treated as a much faster two-dimensional nonlinear fit in $\alpha$ and $\delta$. In each iteration of the Levenberg-Marquardt algorithm, the pixels $p_i$ at $(r_i, c_i)$ and the fit parameters $\alpha$ and $\delta$ are provided to the model function. We first evaluate the uncertainty-normalized Kepler PRF at $\alpha$ and $\delta$, computing $\hat{p}_i = f(\alpha, \delta, r_i, c_i)/\sigma_i$ for each pixel. The flux $b$ is the linear least-squares fit of the input pixel values $p_i$ to the model $b\hat{p}_i$, given by:

$$b = \frac{\sum_{i=1}^{P} p_i \hat{p}_i}{\sum_{i=1}^{P} \hat{p}_i^2}. \quad (15)$$

The product $b\hat{p}_i$ is then returned by the model function. The Levenberg-Marquardt algorithm seeks the $\alpha$ and $\delta$ that minimizes $\sum_{i=1}^{P} (p_i - b\hat{p}_i)/\sigma_i$ after several iterations. (In the Kepler pipeline, this is implemented as a model function passed to the MATLAB function “nlinfit.”) Once the iteration has converged, providing $(\alpha\text{fit}, \delta\text{fit})$, the final estimate of $b$ can be computed as $b\text{fit} = (\sum_{i=1}^{P} p_i\hat{p}_i)/(\sum_{i=1}^{P} \hat{p}_i^2)$, where now $\hat{p}_i = f(\alpha\text{fit}, \delta\text{fit}, r_i, c_i)/\sigma_i$.

The typical implementation of the Levenberg-Marquardt algorithm returns the Jacobian $J$, which contains the derivatives of the model function with respect to position. To estimate the uncertainty of the fit location we need the Jacobian of the position with respect to the pixel values given by the model function. We obtain this by inverting $J$, using the pseudo-inverse, to give the transformation $T = (J^T J)^{-1} J^T$. $T$ is a $P \times 2$ matrix, and the columns of $T$ are normalized by the pixel uncertainties: $T_{ij} \rightarrow T_{ij}/\sigma_i$ for $j = 1, 2$. Then, the PRF-fit location covariance matrix is $C = T^T \sigma_{\text{pixel}} T$, where $\sigma_{\text{pixel}}$ is the pixel variance, and the fit location uncertainties are the square root of the diagonal of $C$: $\sigma_\alpha = \sqrt{C_{1,1}}$ and $\sigma_\delta = \sqrt{C_{2,2}}$.

The PRF is fit separately to the difference image and the out-of-transit image. Because the fit to the difference image ($\alpha\text{diff}, \delta\text{diff}$) measures the position of the transiting source and the fit to the out-of-transit image ($\alpha\text{OOT}, \delta\text{OOT}$) measures the position of the target star, the offset of the transit source from the target is simply $(\Delta\alpha, \Delta\delta) = [(\alpha\text{diff} - \alpha\text{OOT}) \cos \delta\text{OOT}, \delta\text{diff} - \delta\text{OOT}]$. Then the offset distance and uncertainty are computed as in equation (13).

In- and out-of-transit pixel images, and therefore difference images, can only be constructed on a quarter-by-quarter basis. Images cannot be combined across quarters in a useful way because

1. The same star will fall on slightly different pixel locations in each quarter due to pointing differences and small asymmetries in the construction of the Kepler focal plane.
2. The Kepler PRF at the star’s location can have large changes from quarter to quarter.

3. The pixel aperture generally varies in both size and shape from quarter to quarter.

Two approaches to combining quarters will be described in § 3.4.

3.3.1. Systematic PRF-Fit Error

Systematic error in the PRF fit arises from primarily from two classes of sources: error in the PRF model being fit and crowding. These errors cause biases in the offset vector ($\Delta\alpha, \Delta\delta$). There are various ways to control systematic PRF-fit errors, so we examine these errors in detail.

Sources of PRF-fit error

PRF Model Error. The PRF model contains various sources of error (Bryson et al. 2010a) which lead to a priori unpredictable bias in the PRF-fit centroid. Because the target star falls on different parts of the Kepler field of view in different quarters, variation of the PRF across the focal plane causes the PRF error bias to vary from quarter to quarter.

Crowding Bias. The PRF fit is a single-star fit, and therefore assumes that the target star in the out-of-transit image and the transit signal in the difference image are the only stars present in the pixels. This is rarely the case in the out-of-transit image and sometimes not the case in the difference image due to variability of field stars. Unlike the case of photometric centroids described in § 2, the effect of crowding on the PRF fit is difficult to predict. Because field stars mostly cancel in the difference image, the crowding signal in the out-of-transit and difference images can be very different. Therefore the PRF fit to the out-of-transit and difference images can have very different biases, which leads to errors in the offset vector ($\Delta\alpha, \Delta\delta$). An example of a target with a large amount of crowding is shown in Figure 16.

In the worst case, there is a field star in the out-of-transit image brighter than the target star, so the PRF fit to the out-of-transit image returns the centroid of the field star rather than the target star. When this bright field star cancels in the difference image, so the difference image is dominated by a transit on the target star, the offset vector ($\Delta\alpha, \Delta\delta$) gives the distance of the transit signal from the field star rather than the target star. The result is an incorrect measurement of a significant offset of the transit source from the target star. An example of this situation, KOI-1860 (discussed in § 2.2.1), is shown in Figure 17.

Mitigation of the impact of PRF-fit error within a quarter

Average out-of-transit and difference images are computed for each quarter, and these are fit by the PRF to estimate the offset of the transit source from the target star. PRF model error and crowding contribute systematic errors in this estimate. Here, we discuss ways to mitigate these systematic errors within each
quarter. In § 3.4.1, we discuss ways of possibly averaging out these systematics across quarters.

The Kepler PRF for nearby stars will be very nearly the same, so the PRF model error for those stars will be similar. Assuming low crowding, the PRF fit of the out-of-transit image and the fit to the difference image will have similar biases due to PRF model error. When forming the offset vector \((\Delta \alpha, \Delta \delta)\) as the difference between these two fits, these biases should approximately cancel. We therefore prefer the offset vector computed as the difference between the two out-of-transit fits when the target star is not highly crowded.

When the target star is highly crowded, crowding bias will dominate the out-of-transit PRF fit but rarely the difference image PRF fit. This bias is usually due to an error in the measurement of the target star position. As an alternative, we compute the transit source offset relative to the target star’s catalog position. We define \((\Delta \alpha, \Delta \delta)_{\text{catalog}} = [(\alpha_{\text{diff}} - \alpha_{\text{catalog}}) \cos \delta_{\text{catalog}}, \delta_{\text{diff}} - \delta_{\text{catalog}}]\), where \((\alpha_{\text{catalog}}, \delta_{\text{catalog}})\) is the catalog position of the target star (usually from the Kepler input catalog). When \((\Delta \alpha, \Delta \delta)\) differs from \((\Delta \alpha, \Delta \delta)_{\text{catalog}}\) by more than a Kepler pixel (3.98\(\arcsec\)), the out-of-transit measurement of the target star position \((\alpha_{\text{OOT}}, \delta_{\text{OOT}})\) likely contains large errors, and the offset vector \((\Delta \alpha, \Delta \delta)\) should be considered unreliable. The catalog-based offset error \((\Delta \alpha, \Delta \delta)_{\text{catalog}}\) can be used instead, but is itself subject to error because (a) it does not mitigate fit error due to PRF error and (b) is subject to catalog errors due to, for example, unknown proper motion of the target star. In this case, the PRF fit results should be considered qualitative and to have lower accuracy than noncrowded targets, regardless of the formal propagated uncertainty. In the example in Figure 17, the magnitude of the offset vector in that quarter is about 11\(\arcsec\), while the magnitude of the offset from the catalog position is about 0.6\(\arcsec\).
A forthcoming paper (Bryson & Morton 2013, in preparation) will describe the use of modeling to identify and mitigate bias due to crowding. In the majority of cases, the bias will be due to a mix of crowding and PRF model error, with comparably small contributions from each. In this case, we reduce the overall bias by taking advantage of the variation in bias across quarters via averaging, as described in § 3.4.

3.3.2. PRF-Fit Quality

The quarterly out-of-transit and difference images can be polluted by various types of contamination. For example, the out-of-transit image may have bright stars in addition to the target star. The difference image may have more than one stellar image due to the variability of a field star, or the transit may have low S/N, causing the difference image to be poorly formed, as in Figure 13. These cases will degrade the reliability of the PRF-fit source offset measurement. The quality of the PRF fit can be determined by evaluating the PRF at the fit position, creating a synthetic pixel image containing only one star at that position, and comparing this to the observed average pixel image. This synthetic image will have the pixel values

$$\tilde{p}_i = b_{fit} f(\alpha_{fit}, \delta_{fit}, r_i, c_i) = b_{fit} \tilde{p}_i,$$

where the subscript “fit” refers to “diff” or “OOT,” as appropriate. These can be compared to the actual pixel values $p_i$ to determine if the fitted PRF reproduces the observed pixels. One simple comparison is to compute the correlation between $\tilde{p}_i$ and $p_i$ and declare the fit good if this correlation is above some threshold. For the difference image fit quality, we set the threshold to 0.7. When the correlation is below this threshold, then the difference image is likely

Fig. 17.—Example of a target with bright field star that captures the out-of-transit PRF fit (KOI-1860). The out-of-transit image is dominated by the bright star in the upper right corner, so this field star position will be returned by the PRF fit to the out-of-transit image. The difference image, however, shows a nicely star-shaped pattern at the location of the target star, so the target star position will be returned by the PRF fit to the difference image. The resulting offset vector measures the distance of the transit source (target star in this case) to the bright field star rather than the distance of the transit source to the target star. In this case, blindly using the offset values would lead to the erroneous identification of a background false positive. See the electronic edition of the PASP for a color version of this figure.
dominated by noise, typically because the transit has a very low S/N. When the correlation is below threshold for the out-of-transit fit, then it is likely that there is more than one bright star in the image, which compromises the fit due to crowding. In both cases, the source offset measurement is likely to be unreliable.

3.4. Combining Quarterly Results

A comparison of PRF-fit star positions with their catalog R.A. and decl. show that the combination of crowding and PRF error bias has an approximately Gaussian distribution with a median of 1 millipixel (0.004") and a median absolute deviation of 22 millipixels (0.09") (Bryson et al. 2010a). While the quarter-to-quarter variation in the PRF fit of a particular star can have larger spreads, we find that for most stars this quarter-to-quarter variation is approximately zero-mean on average. We therefore combine the quarterly offsets to improve the precision of the PRF-fit centroid offset vector.

3.4.1. Multiquarter Averaging

We denote the single-quarter PRF-fit offset vectors by \((\Delta \alpha_q, \Delta \delta_q)\), where \(q\) labels the quarter. A simple average of \(Q\) quarters, \(\frac{1}{Q} \sum_{q=1}^{Q} (\Delta \alpha_q, \Delta \delta_q)\) with its uncertainties \(\sigma_{\Delta \alpha_q}, \sigma_{\Delta \delta_q}\). The location of the multiquarter average \((\Delta \alpha, \Delta \delta)\) is shown as a magenta cross (obscured by the tight cluster of green crosses). The blue circle has radius equal to 3 times the uncertainty in the magnitude of \((\Delta \alpha, \Delta \delta)\). Star locations relative to the target star are shown as asterisks, with the target star in red (there happen to be no other stars in this figure). The KIC number and Kepler magnitudes are shown next to each star. We see that most offsets are tightly clustered around the expected value, with some uncertainty.

FIG. 18.—Example of multiquarter offset analysis when the transit signal seems to be on the target star (KOI-221). In both figures, the \(x\) and \(y\)-axes give the offsets \(\Delta \alpha\) and \(\Delta \delta\), with \((0, 0)\) being the catalog location of the target star. The green crosses show the individual quarter offsets labeled by quarter, and the length of the crosses is equal to the uncertainties \(\sigma_{\Delta \alpha_q}\) and \(\sigma_{\Delta \delta_q}\). The location of the multiquarter average \((\Delta \alpha, \Delta \delta)\) is shown as a magenta cross (obscured by the tight cluster of green crosses). The blue circle has radius equal to 3 times the uncertainty in the magnitude of \((\Delta \alpha, \Delta \delta)\). Star locations relative to the target star are shown as asterisks, with the target star in red (there happen to be no other stars in this figure). The KIC number and Kepler magnitudes are shown next to each star. We see that most offsets are tightly clustered within 0.1" of the target star with Q1 and Q2 as outliers. Left: Offsets \((\Delta \alpha, \Delta \delta)\) relative to the PRF fit to the out-of-transit centroid. Right: Offsets \((\Delta \alpha, \Delta \delta)_{\text{catalog}}\) relative to the catalog position of the target star. The difference between the left and right plots is not a simple translation because the two plots have different biases due to PRF error and crowding (see § 3.3.1). See the electronic edition of the PASP for a color version of this figure.
The above estimate of the average uncertainty assumes Gaussian statistics. While PRF-fit biases appear nearly Gaussian in the statistical sense, they may not be Gaussian for individual targets. We therefore compute an alternative uncertainty via bootstrap analysis, which provides a more general estimate of the uncertainty. We use a resample-with-replacement strategy, creating an ensemble of \( Q^2 \) simple multiquarter averages. Specifically, given the set of \( Q \) measured offsets \((\Delta \alpha_1, \Delta \alpha_2, \ldots, \Delta \alpha_Q)\), \( Q^2 \) realizations are created, where in each realization we replace each element with an offset randomly chosen from the measured set. Examples of these realizations when \( Q = 5 \) include \((\Delta \alpha_3, \Delta \alpha_1, \Delta \alpha_5, \Delta \alpha_4, \Delta \alpha_2)\) and \((\Delta \alpha_2, \Delta \alpha_4, \Delta \alpha_3, \Delta \alpha_4, \Delta \alpha_1)\). Averages are computed for each of these realizations, and the standard deviation of the resulting ensemble of \( Q^2 \) averages provides the bootstrap uncertainty estimate. The bootstrap uncertainty is typically very similar to the uncertainty returned by the robust fit described above, but can be significantly different for specific targets. We choose the larger of the two uncertainty estimates as the final uncertainty estimate for the multiquarter average \( \sigma_{\Delta \alpha} \). A similar analysis applies to \( \sigma_{\Delta \delta} \).

Examples of this multiquarter averaging technique are shown in Figures 18–22. Figure 18 shows a case with no significant offset, while Figure 19 shows a case with a significant offset, indicating that the transit signal is on a background star. For long-period transiting planets, where there are few quarters that contain transits, the benefits of multiquarter averaging will diminish. In such cases, however, multiquarter averaging can often provide good results, an example of which is shown in Figure 20. Figure 21 shows the low S/N example discussed in § 3.1, where we see that there is a large scatter in the quarterly measurements, but the multiquarter average is within three standard deviations of the target star.

The case of KOI-1860, where a bright field star at the edge of the captured pixels introduces large systematic error, is examined in Figure 22. The offset relative to the out-of-transit centroid is measured to be about 4″, which is a statistically significant 4σ. For most quarters, particularly those which would show a larger offset, the PRF fit to the out-of-transit image failed because the bright star falls very close to the edge of the captured pixels. The offset relative to the catalog position, however, is much smaller, with a multiquarter average of about 0.3″ or 1σ.

![Offsets Relative to Out-of-Transit Centroid](image1)

![Offsets Relative to KIC Position](image2)

**Fig. 19.**—Example of multiquarter offset analysis when the transit signal seems to be on a different star than the target star (KOI-109). The quarterly offsets are tightly clustered around the star KIC 4752452, indicating that this star is the source of the transit. See the caption to Figure 18 for a description of these plots. See the electronic edition of the PASP for a color version of this figure.
Because we are aware of the bright star crowding for KOI-1860, we defer to the offset relative to the catalog position, which is not statistically significant.

We demonstrate the increased precision of the multiquarter average in Figure 23. The offset distance from the target catalog position is shown for both individual quarter PRF fits and

Fig. 20.—Example of multiquarter offset analysis for a confirmed planet signal (Kepler-22b) with a very long period orbit, so only four quarters show transits. The result is a larger scatter and higher average uncertainty compared to the case where there are transits present in every quarter. Also, there is a significant difference in the offsets relative to the out-of-transit centroid in the left panel and relative to the target star’s catalog position in the right panel. This is likely due to a combination of not-fully-averaged PRF bias and catalog error. If this planet were not confirmed by other methods (Borucki et al. 2012) we would have only moderate confidence that the transit signal is on the target star. See the caption to Figure 18 for a description of these plots. See the electronic edition of the *PASP* for a color version of this figure.

Fig. 21.—Example of multiquarter offset analysis for a low S/N transit signal (KOI-2949) with S/N = 11. In this case, the quarterly offsets have a large scatter measured in arcseconds, but the average across quarters is within three standard deviations of the target star. See the caption to Figure 18 for a description of these plots. See the electronic edition of the *PASP* for a color version of this figure.
their quarterly average. This analysis uses 2278 KOIs whose quarterly averaged offsets are less than $3\sigma$ and whose offsets from the target are $<5''$ in the Q1–Q12 data. The left panel shows the 21,401 individual quarter offsets, while the right panel shows the offset of the average over all quarters for each target. The individual quarter offsets have a standard deviation of 0.90'', while the multiquarter averages over 12 quarters have a standard deviation of 0.41''. Strong year-to-year correlations prevent the standard deviation from scaling as $1/\sqrt{Q}$, but do not prevent an improvement as $Q$ increases.

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FIG. 22.—Example of multiquarter offset analysis for a target star (KOI-1860, also discussed in § 2.2.1) whose pixels contain a brighter field star (see Fig. 17). The offsets relative to the out-of-transit centroid are large because the bright star captured the out-of-transit PRF fit. The out-of-transit PRF fit also failed in many quarters because the bright star is at the edge of the pixel aperture. The offsets relative to the target star’s catalog position are, however, well clustered around the target star, indicating that the offset of the transit is not statistically significant. We therefore conclude that the large offset relative to the out-of-transit centroid is due to systematic effects from the bright field star in the pixels. See the caption to Figure 18 for a description of these plots. See the electronic edition of the PASP for a color version of this figure.

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FIG. 23.—Distributions of the PRF-fit offset from the target catalog position for 2278 KOIs whose quarterly averaged offsets are less than $3\sigma$ and whose offsets from the target are $<5''$. Left: Distribution of individual quarter offsets. Right: Distribution of the multiquarter averages. See the electronic edition of the PASP for a color version of this figure.

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3.4.2. Joint Multiquarter PRF Fit

When the transit S/N is very low, there may not be enough signal in each quarterly difference image to support per-quarter PRF fitting. In this case, we perform a joint multiquarter fit, where the pixel images for all quarters are supplied to the PRF fitter, and the single R.A. and decl. (and quarter-specific PRF amplitude) are found that minimize the pixel-level difference between the pixel images and PRF-reconstructed pixels over all quarters. In other words, the joint multiquarter fit finds the single sky position \((\alpha, \delta)\) that minimizes the function

\[
\chi^2 = \sum_{q=1}^{Q} \sum_{i=1}^{P} \frac{1}{\sigma_{p_{i,q}}} [p_{i,q} - b_{q}f_{q}(\alpha, \delta; r_{i,q}, c_{i,q})]^2, \tag{17}
\]

where the subscript \(q\) means the quarter-specific values of each quantity. So, in each quarter the flux-normalized PRF \(b_{q}f_{q}\) for that quarter is evaluated at \((\alpha, \delta)\) (which is common to all quarters) for that quarter’s pixels \((r_{i,q}, c_{i,q})\). These PRF-based pixel values are subtracted from the observed pixel values \(p_{i,q}\) for each quarter. The square of this difference normalized by the uncertainty is summed over all the pixels in that quarter, and, finally, summed over all quarters producing the test \(\chi^2\) value. The sky position is varied until the \((\alpha, \delta)\) that minimize \(\chi^2\) is found. The details of the computation in each quarter are similar to the single-quarter fit in § 3.3.

The propagated uncertainty in this fit does not account for scatter across quarters due to systematic error, so it dramatically underestimates the actual uncertainty in this fit. We compute a more accurate uncertainty via a bootstrap approach much like that for the multiquarter averages described in § 3.4.1, except the data consist of pixel images rather than offsets and each element of the ensemble is a joint PRF fit. Specifically, the multiquarter PRF fit takes as input the set of pixel images \((I_1, I_2, \ldots, I_Q)\) constructed in § 3.2, where \(I_q\) is the pixel image for each quarter. The bootstrap approach creates an ensemble of resamplings-with-replacement sets of pixel images, for example \((I_1, I_5, I_3, I_2, I_2)\) if \(Q = 5\). The multiquarter fit is performed on each element of the ensemble, computing a best fit \((\alpha, \delta)\) for each one. Each element of the ensemble is fit with the parameters from the quarter for that component. For example, if the first element of the ensemble is \(I_1\), then the PRF from quarter 4 is applied to those quarter 4 pixels. The uncertainty in the joint multiquarter fit is then set to the standard deviation of the ensemble of fit positions.

The size of the resampled ensemble needs to be chosen with care. The time to compute the joint multiquarter fit scales with the number of quarters \(Q\). If the usual choice of \(Q^2\) were chosen for the size of this ensemble, the full computation of the joint fit and its uncertainties would scale as \(Q^3\). In the *Kepler* pipeline, a bootstrap joint fit of 8 quarters took about 20 minutes, which indicates that a 16-quarter fit would take almost 3 hr. It is prohibitive to run this on all 15,000 to 20,000 threshold crossing events identified by the pipeline. The joint PRF fit is therefore not routinely run in the *Kepler* pipeline, but is reserved for low S/N transits for which the multiquarter average does not provide a sufficiently precise result. The possible use of a smaller resampled ensemble is under investigation.

4. PIXEL CORRELATION IMAGES

The “pixel correlation method” computes the degree to which the transit signal over time appears in each pixel. This information is used to create a pixel image, where the value of each pixel is the degree of correlation between the pixel flux and the transit signal. This image is centroided via PRF fitting similar to the difference image method. This method has a different response to nontransit photometric variability from the photometric and difference image methods, so it can be useful for resolving cases when the other methods provide ambiguous results.

The correlation between the pixel-level flux and the transit signal over time is computed via a fit of the transit model to the individual pixel flux time series. This uses the same fitting method described in § 2.2, with the centroid time series replaced by the pixel flux time series. In this case the fit constant \(\gamma\) is a measure of the presence of the transit signal in each individual...
An example of these fits is shown in Figure 25. A "pixel correlation image" can be constructed by setting the value of each pixel to its model fit value $\gamma$. When this is done for the example in Figure 25, we get the pixel image in the left panel of Figure 26. The right panel of Figure 26 shows an example where the transit signal is offset from the target star. For such high S/N targets, the transit signal is readily apparent in the pixels, and the correlation image has a star-like appearance. In these cases, the photometric or PRF centroiding can be applied to quantitatively and automatically compute the location of the transit source. See the electronic edition of the PASP for a color version of this figure.

FIG. 25.—Fits of the transit model to individual pixel flux time series for KOI-221 in quarter 7. The pixel flux time series is shown in blue and the transit model is in red. Each pixel flux time series is detrended and folded on the transit period. A closeup of the transit event is shown, with the same time interval on all x-axes. The y-axes show the pixel values and are scaled to show the variation in each pixel time series. The pixel rows are shown along the left and pixel columns are shown along the bottom. The pixels that strongly contain the transit signal indicate the location of the transit source. See the electronic edition of the PASP for a color version of this figure.

FIG. 26.—Correlation images, created by assigning each pixel the scale factor that multiplies the transit model to best fit that pixel’s flux time series. Left: Example from Figure 25 of the transit signal being coincident with the target star (KOI-221). Right: Example with the transit signal significantly offset from the target star (KOI-109). In these figures, the small white squares indicate pixels for which the fit scaling is above a threshold. See the electronic edition of the PASP for a color version of this figure.

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transit, which can be compared to the catalog position of the target star or the target star location from the PRF fit to the difference image.

When the transit has low S/N or the pixels have significant flux from other sources, the pixel correlation image can be of much lower quality. Two examples of this situation are shown in Figure 27.

Because the correlation image is degraded by background flux and can have poor behavior at low S/N, it is not generally used for false-positive identification. There are circumstances, however, where the correlation image can be used in combination with the other methods to make a determination. For example, some low S/N targets have marginal difference and correlation images, but if they show the transit signal in the same pixel location then we have increased confidence that the transit signal in those pixels is real.

5. SATURATED TARGETS

Target stars with Kepler magnitudes brighter than ∼11.5 can exhibit saturation, where the flux in a pixel exceeds that pixel’s full well and spills up and down the pixel columns (Caldwell et al. 2010). The result is that the pixel image of the star can be highly distorted, invalidating all of the centroid methods described in this paper. Saturation can be highly asymmetric, so even photometric centroids are of limited use. Visual inspection of the difference image can, however, reveal large, multipixel offsets, indicating that the transit is not on the saturating star.

When the saturated star is the transit source, the difference image will have a distinctive, non-star-like pattern. Because the saturation spills along columns and the amount of spill is approximately proportional to the flux of the star, a transit signal on a saturated star will appear in the difference image as changes at the ends of the saturated columns. An example is shown in Figure 28. This is a characteristic pattern in the difference images for saturated targets. All that can be said in this case is that the transiting source is in approximately the same column position as the target star, between the ends of the saturation. If the transit were due to a field star that is not in the saturated pixels, the difference image would show that star and not the signal from the saturated pixels.

Special investigation of saturated targets can sometimes refine the location of the transit signal. The appearance of the transit at the end of the saturated columns is sensitive to the column position of the transiting source. If the transit S/N is high enough, the wings of the transits can be subject to a PRF fit while masking out the saturated columns. These techniques have been applied with some success, identifying the location of the transit signal to within 4″, for Kepler-21b (Howell et al. 2012). We refer the reader to that publication for details.

6. PERFORMANCE AND COMPARISON OF TECHNIQUES

In this section we examine the performance of our transit-source location estimation via photometric and PRF-fit centroids. We focus on offset distances because that is the high-level metric used in initial false-positive identification. We examine three populations of targets:

1. All Kepler objects of interest (KOIs) dimmer than Kepler magnitude 11.5 [to avoid saturated targets (Caldwell et al. 2010)], which have well-defined transit-like signals of sufficient quality to pass vetting and produce an ephemeris and valid PRF fits (4049 KOIs). Many of these KOIs are in multiple systems.
2. Unsaturated KOIs that have been identified as being due to transit sources that are unlikely to be on the target, called active pixel offsets (APOs), that have valid PRF fits as of 2012 July (178 KOIs).
3. A small number of APO KOIs whose transit signals have been identified with stars in the *Kepler* input catalog (16 KOIs).

In this section we focus on the following questions:

1. How well do the methods identify the location of these sources?
2. Is there evidence that the source locations correspond to a uniform distribution of background sources?
3. How do these methods compare with one another with respect to accuracy and precision?

We also address an issue that arises with high-transit-S/N targets, where offsets can be very small but the formal uncertainty can be much smaller. In this situation, we encounter residual bias that is not accounted for in the uncertainty, which causes offsets to incorrectly seem statistically significant.

### 6.1. Accuracy

We use APO targets whose transit signals have been associated with known stars to measure how accurately our two primary methods of photometric and PRF-fit centroids identify the source location. This association is determined by manual investigation of the difference images independently of the offset computations. We see in Figure 29 that the PRF estimate of the transit source offset is close to the star identified as the transit signal source. For APOs with small offsets (<4") the photometric centroids also have good accuracy. For APOs with larger offsets, however, photometric centroids show large errors. This behavior is expected because the *Kepler* pipeline uses one set of pixels to estimate the depth of the transit signal and a larger set of pixels to compute the photometric centroid. As described in...
§ 2.3.1, when the transit source has significant flux that falls outside the pixels used for the depth estimate, which is the case when the source is more than 4″ from the target star, there can be significant error in the transit source location inferred from the photometric centroids.

Figure 30 compares the PRF-fit and photometric centroid source offset estimates for all KOIs and shows that the photometric centroid estimate of the source offset is generally (but not always) larger than the PRF-fit estimate when the PRF-fit source location is more than a few arcseconds from the target.

Figure 31 compares the PRF-fit source offset relative to the target star catalog position with the PRF-fit source offset relative to the out-of-transit PRF-fit centroid. These two offsets are similar for the majority of stars, with outliers that are likely due to bias due to crowding.

Figure 32 compares the distribution of the APO KOIs and the distribution of observed pixel area relative to target stars. The fact that these two distributions have similar shapes with similar peaks is consistent with the identified APOs representing a uniform background of eclipsing binaries and possibly large planetary transits. This consistency contributes to our confidence.
that the APOs are correctly identifying astrophysical false positives.

6.2. Precision Versus S/N

The precision of a centroid measurement is dependent on the strength of the transit signal in each pixel. This strength depends on the transit depth, host star brightness, and number of transits, among other factors. All of these factors contribute to the transit S/N, so we analyze precision as a function of transit S/N. Figure 33 shows the dependence of formal centroid source offset uncertainty on transit S/N. Both the PRF-fit and photometric centroid methods show similar dependencies, though the uncertainties for the PRF-fit centroid method are somewhat smaller. A linear fit to the log-log data gives the uncertainty of the two methods as

$$\sigma_{\text{photometric}} = \frac{13.6 \pm 0.16}{(S/N)^{0.05\pm0.00}}, \quad \sigma_{\text{PRF-fit}} = \frac{3.39 \pm 0.10}{(S/N)^{0.89\pm0.01}} \cdot$$

These fits, along with the range of values implied by the 1-σ uncertainties in the fit parameters, are shown in Figure 34. The uncertainty of the photometric centroid method is inversely proportional to the S/N, as expected, while the PRF-fit method has a somewhat smaller dependence on inverse S/N. The coefficient of these uncertainties (13.6 for photometric uncertainties and 3.39 for the PRF fit) is larger than the fullwidth at half-maximum expected for centroid uncertainties because these uncertainties include contributions from the offset computation. The uncertainties reported in this section are propagated formal uncertainties, however, which are only valid if all noise sources are zero-mean Gaussian white noise. As described in this paper, there are several sources of systematic error that impact transit source offset estimation. These systematic errors are not reflected in the formal uncertainty.

Because the dependence of the PRF-fit and photometric centroid estimates of the source offset on S/N have similar log
slopes we expect that if one technique indicates a significant offset then the other technique will as well. This is shown in Figure 35, which indicates that for most targets the photometric centroid and PRF-fit methods are in agreement as to whether there is a significant offset for a particular target. But there are many targets, including a few identified APOs, that have photometric centroid source offsets $<3\sigma$ but PRF-fit source offsets $>3\sigma$ and vice versa.

Quantitatively, for 54.9% of all KOIs the two techniques are in agreement that the source offset is $<3\sigma$; 24.7% of all KOIs have agreement that the source offset is $>3\sigma$; 13.9% of all KOIs have offsets $>3\sigma$ according to the PRF-fit technique but $<3\sigma$ according to photometric centroids; and 6.45% of all KOIs have offsets $<3\sigma$ according to the PRF-fit technique but $>3\sigma$ according to photometric centroids. Therefore, the two methods are in agreement on significance for about 80% of the targets. Most of the targets for which the PRF-fit techniques indicate an offset $>3\sigma$ but the photometric centroids have a shift $<3\sigma$ have very small PRF-fit offsets, so they are at distances where residual bias dominates, as discussed in §6.3.

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![Figure 33](image1.png)

**Fig. 33.**—Formal offset uncertainty vs. transit S/N for PRF fit (left) and photometric (right) centroids using 12 quarters of data. The red dashed line in both figures shows the $1/(\text{S/N})$ dependency for comparison. We see that the precision of the PRF-fit offsets is somewhat better on average than the photometric centroid offsets. This precision does not account for bias due to systematic error for either type of centroid. See the electronic edition of the *PASP* for a color version of this figure.

![Figure 34](image2.png)

**Fig. 34.**—Uncertainty vs. S/N from the fits in Figure 33, plotted on linear scales. The dotted lines indicate the range of variation due to the 1-σ uncertainties in the fit parameters.

![Figure 35](image3.png)

**Fig. 35.**—Comparison of the PRF-fit source offset relative to the catalog position of the target star (x-axis) and the photometric centroid source offset (y-axis), both in units of $\sigma$. The vertical and horizontal lines mark where the offset $=3\sigma$, above which the offset is considered statistically significant. APO KOIs are marked by circles. We see that most targets have both offsets below $3\sigma$, but there are a significant number of targets for which the photometric centroid source offset is less than $3\sigma$ but the PRF-fit offset is $>3\sigma$ and vice versa. See the electronic edition of the *PASP* for a color version of this figure.
The results described in the previous paragraph should only be taken as a comparison of the photometric centroid and difference image techniques, rather than a statistical measurement of the APO population in the \textit{Kepler} data. When both the difference image and photometric centroid method agree that there is a significant offset, this offset is likely to indicate an APO due to a background false positive, and each individual case must be examined to assure that the offset is not actually due to the systemic errors described in this paper. When one of the methods indicates a significant offset but the other does not, it is less likely that the offset is due to a background false positive rather than systematic error. However, an approximately 25\% significant APO rate is consistent with the observed APO rate described in § 1, averaged over the \textit{Kepler} field of view.

6.3. Residual Bias and High S/N Transits

As described in § 3.3.1, the computation of the PRF-fit source offset is subject to various kinds of bias due to PRF error and crowding. When the transit S/N is high, both centroid methods will have very high formal precision with very small uncertainties. The PRF-fit source offset estimate essentially hits

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**Fig. 36.**—Relationship between the PRF-fit source offset (x-axis) and source offset in units of sigma (y-axis). Left: All KOIs. Right: KOIs with transit S/N > 100. On the left, we see that for offsets <3″ there seem to be an excess of targets with offset >3σ (red line). On the right, we see that for high S/N targets the offset is small, but there is an excess of targets with offset >3σ. This is likely due to residual bias from the errors discussed in § 3.3.1. See the electronic edition of the \textit{PASP} for a color version of this figure.

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**Fig. 37.**—Relationship between the photometric centroid source offset (x-axis) and source offset in units of sigma (y-axis). Left: All KOIs. Right: KOIs with transit S/N > 100. Many KOIs fall outside the plot, but our interest is in small offset behavior. On the left, we see that for offset <0.2″ there seem to be an excess of targets with offset >3σ (red line). On the right we see that for high S/N targets the offset is small, but there is an excess of targets with offset >3σ. See the electronic edition of the \textit{PASP} for a color version of this figure.
a noise floor, where the offsets are dominated by residual biases. Figure 36 shows that this noise floor begins to be apparent at source offsets of about 2″, where there is a noticeable increase in objects with offsets between 3σ and 4σ. Below about 0.2″ there is a large excess of objects with large offsets in units of σ. The right panel of Figure 36 shows targets with high S/N. In this population, offsets are mostly very small, and we find most of the large excess of high-σ offsets. We interpret this to mean that residual biases in the PRF-fit source offset are dominant under 0.2″.

Figure 37 shows a similar analysis for photometric-centroid-based source offsets. The excess of significantly offset targets is apparent but less severe in this case.

We mitigate the impact of residual bias on small offset/high S/N targets in PRF-fit estimates of the source offset in two ways:

1. Adding a small constant “noise floor” to reflect the residual bias. Because bias seems to dominate at less than 0.2″, we want to avoid classifying any target with a source offset less than 0.2″ as an APO false positive. Because this classification is based on a 3σ threshold we add σ₀ = (0.2/3) arcseconds in quadrature to the formal uncertainty in each component: σΔα = \sqrt{σ² + σ₀²}, σΔδ = \sqrt{σ² + σ₀²}. (This has the same effect on the offset distance uncertainty σD as adding σ₀ to σD in quadrature.) The impact of adding this noise floor is shown in Figure 38.

2. Special treatment is given to vetting targets with small source offsets. An example simple set of rules for manual vetting for false positives is the following:

A. Pass all targets with offsets <0.2″ (this happens automatically when using the above noise floor).

B. For targets with offsets <1″, manually investigate those targets with offsets >3σ.

C. For targets with offsets between 1″ and 2″, manually investigate those targets with offsets between 3σ and 4σ.

D. For targets with offsets between 1″ and 2″, declare as APO targets with offsets above 4σ.

E. For targets with offsets >2″, declare as APO targets with offsets above 3σ.

7. CONCLUSIONS

Many background astrophysical false positives can be identified through centroid analysis of Kepler pixel data. The high photometric precision of the Kepler data provides opportunities to identify such objects close to the target star, but great care must be taken to account for various systematic biases. We have presented three different techniques, two of which were analyzed in detail. This ensemble provides a power arsenal of tools for dispositioning nearly all KOIs.

The PRF-fit technique provides the best accuracy in the localization of transit sources that are not on the target star. The photometric centroid technique behaves best when the target star is isolated and the transit source is close to (or is) the target star. The photometric centroid technique is therefore useful for confirming that the transit is on the target star when this is also indicated by the PRF-fit technique. The photometric centroid technique can indicate when the transit source is separated from the target star, but when the separation is more than a few arcseconds, the source location determined by photometric centroids is unreliable.

When the S/N is low or there is significant crowding, the PRF technique can break down. In this case, the photometric technique...
may provide the best evidence that the centroid is on the target star. The pixel correlation images can also be useful in this circumstance, though the pixel correlation technique is fragile.

We find that we often use all three techniques when investigating a difficult target. This toolbox of techniques is a critical component of the Kepler planet candidate vetting process and makes a significant contribution to the reliability of the Kepler planet candidate list.

We gratefully acknowledge the outstanding work of the entire Kepler team that performs the data acquisition and analysis and delivers the precision that makes the techniques described in this paper possible. We particularly thank the Kepler Science Operations Center and Science Office for their support and creativity while these techniques were being developed. We thank Martin Still, Susan Thompson, and Jeff Coughlin for valuable comments on early drafts of this paper. Finally, we thank Bill Borucki, Ted Dunham, Dave Latham, Nick Gautier, and the wider Kepler science community for constant support and encouragement.

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## APPENDIX

### DERIVATION OF THE FORMULA RELATING CENTROID SHIFTS TO TRANSIT SOURCE LOCATION

Assume that we are observing a target star with flux $b_j$ at $(\alpha_0, \delta_0)$, with $N$ nearby stars at R.A. and decl. $(\alpha_j, \delta_j)$, $j = 1, \ldots, N$, and flux $b_j$. Assume the star $k$, with $k \neq 0$, is a background eclipsing binary with fractional eclipse depth $d_{\text{back}}$ so the flux of star $k$ in mid eclipse is $(1 - d_{\text{back}})b_k$. We model the PSF of the star with a function $f(\alpha, \delta)$ that has the following properties, where the integral is taken over the domain where $f > 0$:

1. $f(\alpha, \delta)$ has finite support ($f = 0$ outside of a finite area).
2. $\int f(\alpha, \delta)d\alpha d\delta = 1$. In other words, $f$ has unit flux, so $b_j f$ has the total flux

$$\int b_j f(\alpha, \delta)d\alpha d\delta = b_j,$$

3. $\int f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta = \alpha_j$ and $\int f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta = \delta_j$; so example,

$$\frac{\int b_j f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta}{\int b_j f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta} = \frac{\alpha_j b_j}{b_j} = \alpha_j,$$

so the centroid of an isolated star is the same as that star’s position.

We now consider an aperture on the sky that may not completely capture all flux from stars in the aperture and may contain flux from stars outside the aperture. Therefore, $\int_{\text{ap}} b_j f(\alpha, \delta)d\alpha d\delta \neq b_j$, $\int_{\text{ap}} f(\alpha - \alpha_k, \delta - \delta_k)d\alpha d\delta \neq \alpha_k$, and $\int_{\text{ap}} f(\alpha - \alpha_k, \delta - \delta_k)d\alpha d\delta \neq \delta_k$, where $\int_{\text{ap}}$ denotes an integral over the aperture. We model the background flux as an arbitrary function $B(\alpha, \delta)$. We denote the total flux in the aperture by:

$$F_{\text{ap}} = \int_{\text{ap}} \left[ \sum_{j=1}^{N} b_j f(\alpha - \alpha_j, \delta - \delta_j) + B(\alpha, \delta) \right] d\alpha d\delta.$$

To simplify the following discussion, we define the notation:

$$I_j := \int_{\text{ap}} f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta,$$

$$B := \int_{\text{ap}} B(\alpha, \delta)d\alpha d\delta,$$

$$I_{j, \alpha} := \int_{\text{ap}} f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta,$$

$$I_{j, \delta} := \int_{\text{ap}} f(\alpha - \alpha_j, \delta - \delta_j)d\alpha d\delta,$$

$$B_{\alpha} := \int_{\text{ap}} B(\alpha, \delta)d\alpha d\delta,$$

$$B_{\delta} := \int_{\text{ap}} B(\alpha, \delta)d\alpha d\delta.$$

So $b_j I_j$ is the flux from star $j$ in the aperture, $B$ is the background flux in the aperture, and the superscript $\alpha$ or $\delta$ indicates the first moment in R.A. or decl. of these quantities. Then $F_{\text{ap}} = \sum_{j=1}^{N} b_j I_j + B$.

The out-of-transit centroid (including all flux in the aperture) is given by

$$C_{\alpha, \text{out}} = \frac{\sum_{j=1}^{N} b_j I_{j, \alpha} + B_{\alpha}}{F_{\text{ap}}},$$

$$C_{\delta, \text{out}} = \frac{\sum_{j=1}^{N} b_j I_{j, \delta} + B_{\delta}}{F_{\text{ap}}}.$$

The in-transit centroid is given by
We define only cause of a change in flux, the observed flux in mid eclipse all other flux is constant. Therefore, this centroid is given by:

\[
C_i = \frac{C_{\text{out}} F - d_{\text{back}} b_i F_{\text{back}}}{F - d_{\text{back}} b_i F_{\text{back}}},
\]

The observed depth is defined so that the observed flux in mid eclipse is \((1 - d_{\text{obs}}) F_{\text{ap}}\). Assuming that the eclipse is the only cause of a change in flux, the observed flux in mid eclipse is also given by \(F_{\text{ap}} - d_{\text{back}} b_i F_{\text{back}}\). Therefore, \((1 - d_{\text{obs}}) F_{\text{ap}} = F_{\text{ap}} - d_{\text{back}} b_i F_{\text{back}}\), so \(d_{\text{obs}} = \frac{d_{\text{back}} b_i F_{\text{back}}}{F_{\text{ap}} - d_{\text{back}} b_i F_{\text{back}}}\).

The centroid shift is given by

\[
\Delta C = \frac{C_{\text{in}} - C_{\text{out}}}{\cos \delta} = \frac{C_{\text{out}} F - d_{\text{back}} b_i F_{\text{back}}}{F_{\text{ap}} - d_{\text{back}} b_i F_{\text{back}}} = \frac{-d_{\text{back}} b_i F_{\text{back}}}{F_{\text{ap}} - d_{\text{back}} b_i F_{\text{back}}} = \frac{-d_{\text{obs}}}{F_{\text{ap}}} \frac{F_{\text{optAp}}}{F_{\text{ap}}}
\]

\[
\Delta C = \frac{d_{\text{obs}}}{F_{\text{ap}}} \frac{F_{\text{optAp}}}{F_{\text{ap}}} \frac{F_{\text{optAp}}}{F_{\text{ap}}}
\]

We define

\[
C_{ap} = \frac{I_{\text{ap}}}{I_{\text{k}}}, \quad C_{k} = \frac{I_{\text{k}}}{I_{\text{ap}}}
\]

which are the R.A. and decl. of the centroid of the flux of the transit source \(k\) in the aperture when all other flux is absent (alternatively, this is the centroid of the difference image formed by subtracting in-transit pixels from out-of-transit pixels when all other flux is constant). Therefore, this centroid is given by:

\[
C_{ap} := C_{\text{out}} - \frac{1}{d_{\text{obs}}} \frac{\Delta C}{\cos \delta},
\]

\[
C_{k} := C_{\text{out}} - \frac{1}{d_{\text{obs}}} \frac{\Delta C}{\cos \delta}.
\]
Brown, T. M. 2003, ApJ, 593, L125
Bryson, S. T., et al. 2010a, ApJ, 713, L97
———. 2010b, Proc. SPIE, 7740, 77401D
Caldwell, D., et al. 2010, ApJ, 713, L92
Christiansen, J., et al. 2013, Kepler Data Characteristics Handbook (KSCI-19040-004; Moffet Field: NASA/Ames), http://archive.stsci.edu/kepler/manuals/Data_Characteristics.pdf

Howell, S., et al. 2012, ApJ, 746, 123
Jenkins, J., et al. 2010, ApJ, 724, 1108
Koch, D., et al. 2010, ApJ, 713, L79
Levenberg, K. 1944, Q. Appl. Math., 2, 164
Marquardt, D. W. 1963, J. Soc. Ind. Appl. Math., 11, 431
Tenenbaum, P., & Jenkins, J. M. 2010, Proc. SPIE, 7740, 77401C
Wu, H., et al. 2010, Proc. SPIE, 7740, 774019