Sparticle Spectroscopy from SO(10) GUT with a Unified Higgs Sector

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Abstract

We study the low energy implications, especially the particle spectroscopy, of SO(10) grand unification in which the SO(10) symmetry is broken to the Standard Model gauge group with a single pair of (144 + \overline{144}) dimensional Higgs multiplet (unified Higgs sector). In this class of models, the asymptotic relation $Y_b \approx Y_\tau \approx Y_t/6$ among the third generation quark and lepton Yukawa couplings can be derived. This relation leads to the prediction $\tan \beta \approx 14$, where $\tan \beta$ is the well known MSSM parameter. We find that this type of Yukawa coupling unification (YU) is realized only by employing non-universal soft supersymmetry breaking terms, dictated by SO(10) symmetry, for the gauginos. A 125 GeV Higgs boson mass is also found to be consistent with YU at the $\sim 5\%$ level. Without imposing a constraint on the relic abundance of dark matter in these models, the squark and slepton masses, with the exception of the stop, exceed 2 TeV and the gluino is heavier than 1 TeV. We show that the neutralino in this model is an acceptable dark matter candidate through the neutralino-stop coannihilation scenario, with the stop quark being relatively light ($\gtrsim 500$ GeV).

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1 Introduction

An SO(10) gauge symmetry provides an elegant framework for unifying the strong and electroweak interactions. A single generation of quarks and leptons including a right handed neutrino, nicely fits in an irreducible 16 dimensional representation [1].

The right handed neutrino ($\nu_R$) helps generate the observed light neutrino masses via the see-saw mechanism [2]. It can also naturally account for the observed baryon asymmetry of the universe via leptogenesis [3]. Another virtue of SO(10) is that, in principle, the two MSSM Higgs doublets can be accommodated in a single ten dimensional representation ($10_H$), which then yields the following Yukawa couplings

$$Y_{ij} \ 16_i \ 16_j \ 10_H. \quad (1)$$

Here $i, j = 1, 2, 3$ stand for family indices and the SO(10) indices have been omitted for simplicity. Considering only the third generation quarks and leptons, the interaction in Eq. (1) yields the following Yukawa coupling unification (YU) condition at $M_{GUT}$:

$$Y_t = Y_b = Y_\tau = Y_{\nu_\tau}. \quad (2)$$

where $Y_{\nu_\tau}$ denotes the tau neutrino Dirac coupling. Consequently, large $\tan \beta \sim 50$ is predicted [4] in order to get compatibility with experimental observations.

It is interesting to note that in the gravity mediation SUSY breaking scenario [5], $t-b-\tau$ YU condition leads to LHC testable sparticle spectrum [6] and it even ‘predicts’ a 125 GeV light CP-even Higgs boson mass [7].

One potential drawback of SO(10) grand unification is the lack of a unique minimal model due to the various possibilities available in the Higgs sector for breaking $SO(10)$ to $SU(3)_C \times U(1)_{em}$. Typically, one needs a $16 + 1\bar{1}6$ or a $126 + 1\bar{2}6$ Higgs representation to reduce the rank of the group from five to four together with either a 45 or 210-dimensional representation for breaking the symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Furthermore, one needs a 10-dimensional Higgs multiplet to break the electroweak symmetry. These requirements imply that in principle, two distinct superheavy mass scales are involved in the breaking of SO(10), one associated with the reduction of the rank, and the other for breaking the symmetry all the way down to the Standard Model (SM). In order to maintain gauge and Yukawa coupling unification, one needs to assume that the various vacuum expectation values (VEVs) are of the same order of magnitude. This requires suitable relations among the parameters in the superpotential which may not appear very natural.

Recently a new class of SO(10) models was presented [8, 9] where the SO(10) symmetry breaking down to the SM gauge group involves just a single pair of $(144 + \overline{144})$-dimensional vector-spinor Higgs multiplet. It was also shown that this pair of multiplets can contain a pair of light Higgs doublets, necessary for breaking the
electroweak symmetry. In order to solve the doublet-triplet splitting problem in this class of models, a \((560 + 560)\)-dimensional vector-spinor representation was introduced instead of \(144 + 144\) [10]. In this case the doublet-triplet splitting problem was solved via the missing partner mechanism.

In this paper we study the low energy spectrum of supersymmetric SO(10) GUT with \(144 + 144\) dimensional Higgs multiplet. The Yukawa coupling for third generation quarks and leptons is given by

\[
Y_i \frac{16_i 16_i 144H 144H}{M_*},
\]

where \(M_*\) is a super heavy mass. It was shown in ref. [8] that there corresponds a parameter space in this class of model where the following relation among third generation quark and lepton Yukawa couplings is obtained:

\[
Y_b \approx Y_\tau \approx \frac{Y_t}{6}.
\]

In order to be compatible with observations, the theory predicts an intermediate value \(\sim 10\) for \(\tan \beta\).

In this paper we seek the low scale sparticle spectrum which is consistent with the asymptotic relation presented in Eq. (4). We find that this requires non-universal gaugino masses at \(M_{GUT}\) which, as previously discussed, can be incorporated in the SO(10) framework.

The outline for the rest of the paper is as follows. In Section 2 we present the parameter space that we randomly scan and describe how the MSSM gaugino mass relations can be obtained at \(M_{GUT}\). In Section 3 we summarize the scanning procedure and the experimental constraints applied in our analysis. The results are presented in Section 4. The table in this section lists some benchmark points which can be tested at the LHC. Our conclusions are summarized in Section 5.

2 Fundamental Parameter Space

We first comment on our results when the asymptotic relation among the Yukawa couplings presented in Eq.(4) is applied assuming universal gaugino masses at \(M_{GUT}\). We find that in this case the Yukawa coupling unification is not better than the 30% level, regardless of whether universal or non-universal Higgs soft supersymmetry (SUSY) breaking mass terms are imposed. Based on the experience (see for instance ref. [7]) that non-universal gaugino masses help achieve conventional Yukawa unification \((Y_t = Y_b = Y_\tau)\), we employ the same non-universal gaugino mass condition in this analysis as well. We will show in section 4 that non-universal gaugino masses also leads to unification of Yukawa couplings according to Eq.(4).
It has been pointed out [11] that non-universal MSSM gaugino masses at $M_{\text{GUT}}$ can arise from non-singlet SUSY breaking F-terms, compatible with the underlying GUT symmetry. The SSB gaugino masses in supergravity [5] can arise from the following dimension five operator:

$$-\frac{F^{ab}}{2M_P} \lambda^a \lambda^b + \text{c.c.}$$

Here $\lambda^a$ is the two-component gaugino field, $F^{ab}$ denotes the F-component of the field which breaks SUSY, and the indices $a, b$ run over the adjoint representation of the gauge group. The resulting gaugino mass matrix is $\langle F^{ab} \rangle / M_P$, where the SUSY breaking parameter $\langle F^{ab} \rangle$ transforms as a singlet under the MSSM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. The $F^{ab}$ fields belong to an irreducible representation in the symmetric part of the direct product of the adjoint representation of the unified group.

In SO(10), for example,

$$(45 \times 45)_S = 1 + 54 + 210 + 770.$$  \hspace{1cm} (6)

If $F$ transforms as a 54 or 210 dimensional representation of SO(10) [11], one obtains the following relation among the MSSM gaugino masses at $M_{\text{GUT}}$:

$$M_3 : M_2 : M_1 = 2 : -3 : -1,$$  \hspace{1cm} (7)

where $M_1, M_2, M_3$ denote the gaugino masses of $U(1), SU(2)_L$ and $SU(3)_c$ respectively.

Notice that in general, if $F^{ab}$ transforms non trivially under SO(10), the SSB terms such as the trilinear couplings and scalar mass terms are not necessarily universal at $M_{\text{GUT}}$. However, we can assume, consistent with SO(10) gauge symmetry, that the coefficients associated with terms that violate the SO(10)-invariant form are suitably small, except for the gaugino terms in Eq.(7). We also assume that D-term contributions to the SSB terms are much smaller compared with contributions from fields with non-zero auxiliary F-terms.

Employing the boundary condition from Eq.(7), we define the MSSM gaugino masses at $M_{\text{GUT}}$ in terms of the mass parameter $M_{1/2}$:

$$M_1 = M_{1/2}, \quad M_2 = 3M_{1/2} \quad \text{and} \quad M_3 = -2M_{1/2}.$$  \hspace{1cm} (8)

We have performed random scans for the following parameter range:

$$0 \leq m_{16} \leq 20 \text{ TeV}, \quad 0 \leq m_{144} \leq 20 \text{ TeV},$$

$$0 \leq M_{1/2} \leq 5 \text{ TeV}, \quad -3 \leq A_0/m_{16} \leq 3,$$

$$2 \leq \tan \beta \leq 60.$$  \hspace{1cm} (9)
Here $m_{16}$ is the universal SSB mass for MSSM sfermions, $m_{144}$ is the universal SSB mass term for up and down MSSM Higgs masses, $M_{1/2}$ is the gaugino mass parameter, $\tan \beta$ is the ratio of the vacuum expectation values (VEVs) of the two MSSM Higgs doublets, and $A_0$ is the universal SSB trilinear scalar interaction (with corresponding Yukawa coupling factored out). We use the central value $m_t = 173.1 \text{ GeV}$ and $1\sigma$ deviation ($m_t = 174.2 \text{ GeV}$) for top quark in our analysis \cite{17}. A $+1\sigma$ increase in $m_t$ slightly raises the Higgs mass which is desirable in our analysis. Our results however are not too sensitive to one or two sigma variation in the value of $m_t$ \cite{18}. We use $m_b(m_Z) = 2.83 \text{ GeV}$ which is hard-coded into Isajet.

3 Phenomenological Constraints and Scanning Procedure

We employ the ISAJET 7.84 package \cite{12} to perform random scans over the fundamental parameter space. In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to $M_{\text{GUT}}$ via the MSSM renormalization group equations (RGEs) in the $\overline{DR}$ regularization scheme. We do not strictly enforce the unification condition $g_3 = g_1 = g_2$ at $M_{\text{GUT}}$, since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections \cite{13}. The deviation between $g_1 = g_2$ and $g_3$ at $M_{\text{GUT}}$ is no worse than $3-4\%$. For simplicity we do not include the Dirac neutrino Yukawa coupling in the RGEs, whose contribution is expected to be small.

The various boundary conditions are imposed at $M_{\text{GUT}}$ and all the SSB parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale $M_Z$. In the evaluation of Yukawa couplings the SUSY threshold corrections \cite{14} are taken into account at the common scale $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, where $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ are the third generation left and right handed stop quark masses. The entire parameter set is iteratively run between $M_Z$ and $M_{\text{GUT}}$ using the full 2-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for gauge and Yukawa couplings, and the SSB parameters $m_i$ are extracted from RGEs at their appropriate scales $m_i = m_i(m_i)$. The RGE-improved 1-loop effective potential is minimized at $M_{\text{SUSY}}$, which effectively accounts for the leading 2-loop corrections. Full 1-loop radiative corrections are incorporated for all sparticle masses.

An important constraint comes from limits on the cosmological abundance of stable charged particles \cite{15}. This excludes regions in the parameter space where charged SUSY particles become the lightest supersymmetric particle (LSP). We accept only those solutions for which one of the neutralinos is the LSP and saturates the WMAP bound on relic dark matter abundance.
An approximate error of around 2 GeV in the estimate of the Higgs mass in Isajet largely arises from theoretical uncertainties in the calculation of the minimum of the scalar potential, and to a lesser extent from experimental uncertainties in the values for $m_t$ and $\alpha_s$.

Micromegas 2.4 [16] is interfaced with Isajet to calculate the relic density and branching ratios $BR(B_s \rightarrow \mu^+\mu^-)$ and $BR(b \rightarrow s\gamma)$. We implement the following random scanning procedure: A uniform and logarithmic distribution of random points is first generated in the parameter space given in Eq. (9). The function RNORMX [21] is then employed to generate a Gaussian distribution around each point in the parameter space. The data points collected all satisfy the requirement of radiative electroweak symmetry breaking (REWSB), with the neutralino in each case being the LSP. After collecting the data, we impose the mass bounds on all the particles [15] and use the IsaTools package [20] to implement the various phenomenological constraints. We successively apply the following experimental constraints on the data that we acquire from SuSpect and Isajet:

\[
0.8 \times 10^{-9} \leq BR(B_s \rightarrow \mu^+\mu^-) \leq 6.2 \times 10^{-9} \quad (2\sigma) \quad [22]
\]
\[
2.99 \times 10^{-4} \leq BR(b \rightarrow s\gamma) \leq 3.87 \times 10^{-4} \quad (2\sigma) \quad [23]
\]
\[
0.15 \leq \frac{BR(B_{u} \rightarrow \tau\nu_{\tau})_{\text{MSSM}}}{BR(B_{u} \rightarrow \tau\nu_{\tau})_{\text{SM}}} \leq 2.41 \quad (3\sigma) \quad [23]
\]
\[
0 \leq \Delta(g - 2)_{\mu}/2 \leq 55.6 \times 10^{-10} \quad [24]
\]

In order to quantify Yukawa coupling unification, we define the quantity $R'_{t\nu\tau}$ as,

\[
R'_{t\nu\tau} = \frac{\max(y_{t} / 6, y_{b}, y_{\tau})}{\min(y_{t} / 6, y_{b}, y_{\tau})}.
\]

### 4 Sparticle Spectroscopy and the Higgs mass

Figure 1 shows our results in the $R'_{t\nu\tau} - M_{1/2}$, $R'_{t\nu\tau} - m_{10}$, $R'_{t\nu\tau} - m_{16}$ and $R'_{t\nu\tau} - \tan \beta$ planes. Gray points in the figure are consistent with REWSB and LSP neutralino. Green points form a subset of the gray and satisfy sparticle mass and B-physics constraints described in section 3. The green points also satisfy the Higgs mass bound $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$. Brown points form a subset of the green points and satisfy $\Omega h^2 \leq 10$. We chose to concentrate on $\Omega h^2 \leq 10$ because in this model neutralino is mostly a bino like particle and it is heavier than a 100 GeV. In this case, without any additional contribution, $\Omega h^2$ can be around $O(10^3)$ or even $O(10^3)$ [25]. So, $\Omega h^2 \leq 10$ already indicates that there is some additional mechanism which significantly reduces the relic abundance close to the desired value with some fine tuning in the SSB parameter space.
Figure 1: Plots in the $R'_{tb\tau} - M_{1/2}$, $R'_{tb\tau} - m_{144}$, $R'_{tb\tau} - m_{16}$ and $R'_{tb\tau} - \tan \beta$ planes. The data points shown are collected using Isajet 7.84. Gray points are consistent with REWSB and LSP neutralino. Green points form a subset of the gray and satisfy sparticle mass and B-physics constraints. The green points also satisfy the Higgs mass bound $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$. Moreover, we require that the green points do no worse than the SM in terms of $(g - 2)_\mu$. Brown points form a subset of the green points and satisfy $\Omega h^2 \leq 10$. 
Figure 2: Plots in the $R_{tb} - M_{\chi^0}$, $m_{\tilde{t}_1} - M_{\chi^0}$, $m_A - M_{\chi^0}$ and $m_{\tilde{t}_1} - M_{\chi^0}$ planes. Color coding is the same as in Figure 1. In addition blue points are subset of the green and satisfy $R_{tb}^< 1.2$. Brown points form a subset of the blue points and satisfy $\Omega h^2 \leq 10$.

Figure 1 shows that our analysis does not yield better than $\sim 5\%$ YU consistent with the constraints described in section 3. The prediction for YU essentially remains the same if we require the relic density to be small, $\Omega h^2 \leq 10$. We also observe that requiring good YU leads to narrow ranges of the fundamental parameters in the model. The gaugino mass parameter ($M_{1/2}$) consistent with good YU is $\sim 200$ GeV whereas the Higgs mass parameter ($m_{144}$) lies in the range $2 - 4$ TeV. Similarly, good YU prefers the GUT scale scalar mass parameter $m_{16} \sim 2$ TeV and $\tan \beta \sim 14$. Note that this value for $\tan \beta$ is notably different from the prediction $\tan \beta \sim 47$ in refs. [7], which studied the same model but with the condition $Y_t = Y_b = Y_\tau$ at $M_{GUT}$.

Note also that requiring the neutralino relic abundance of the neutralino to satisfy $\Omega h^2 \leq 10$ affects the above mentioned predictions. While the preferred value for $\tan \beta$ essentially remains the same, the smallest values of the parameters $M_{1/2}$, $m_{144}$ and $m_{16}$ consistent with YU $\sim 5\%$ are pushed to higher values, namely, $M_{1/2} \sim 300$ GeV,
Figure 3: Plots in the $R'_{tbr} - m_h$ and $R'_{tbr} - m_{\tilde{\chi}^\pm}$ planes. The color coding is the same as in Figure 1.

$m_{144} \sim 1.6$ TeV and $m_{16} \sim 2.8$ TeV.

Figure 2 shows plots in the $m_{\tilde{q}} - m_{\tilde{g}}$, $m_{\tilde{\tau}_1} - M_{\chi^0_1}$, $m_A - M_{\chi^0_1}$ and $m_{\tilde{t}_1} - M_{\chi^0_1}$ planes. The green points have the same definition as in Figure 1. The blue points are a subset of the green and satisfy $R'_{tbr} < 1.2$. Brown points form a subset of the blue points and satisfy $\Omega h^2 \leq 10$. The $m_{\tilde{q}} - m_{\tilde{g}}$ plane shows that 20% or better YU predicts the first and second generation squark masses to be $\gtrsim 2$ TeV. The $m_{\tilde{t}_1} - M_{\chi^0_1}$ shows that neutralino-stop coannihilation is consistent with good YU. We can see from the $m_{\tilde{q}} - m_{\tilde{g}}$ plane that for $\Omega h^2 \leq 10$ (brown points) the first two generation squarks have masses $\gtrsim 3$ TeV and gluinos are heavier than 1.5 TeV or so. We can conclude, based on the location of blue points in the $m_{\tilde{\tau}_1} - M_{\chi^0_1}$ plane, that neutralino-stau coannihilation is impossible to realize in this model. From the $m_A - M_{\chi^0_1}$ plot, we observe that good YU is not consistent with the $M_A$ resonance solution for neutralino dark matter.

In the $m_{\tilde{t}_1} - M_{\chi^0_1}$ plane, we observe that 20% or better YU can be achieved with the neutralino-stop coannihilation scenario. This is a prediction of this model if we require neutralino to be the sole dark matter candidate. A lower limit on the mass of the NLSP stop quark was obtained in refs. [26] in light of 7 TeV LHC data corresponding to 1 fb$^{-1}$ integrated luminosity. It was shown that the NLSP stop mass below 140 GeV is essentially excluded.

Note that the analysis in refs. [7] employ similar GUT scale boundary conditions for the SSB terms but with a different relation for the Yukawa couplings. The model in [7] predicts neutralino-stau coannihilation scenario to be consistent with good YU. It also predicts relatively low values of $m_A$ and $m_{\tilde{\tau}_1}$, while the stop and the gluino masses are $\gtrsim 5$ TeV.

Figure 3 shows the results in the $R'_{tbr} - m_h$ and $R'_{tbr} - m_{\tilde{\chi}^\pm}$ planes. The color
Table 1: Benchmark points with good Yukawa unification. Point 1 has a small neutralino relic abundance with YU around 11%. For point 2, the relic abundance is relatively large and the stop is twice as heavy compared to point 1 while the gluino is lighter. For point 3, YU is around the best value we obtained in our analysis (∼5%). Point 3 also has good YU with a lighter gluino and stop compared to point 2. The Higgs mass for the three points is within the favored range, 123 GeV ≤ mh ≤ 127 GeV. Stop is the NLSP for the three points.
coding is the same as in Figure 1. We observe that the model accommodates the Higgs mass range, \(123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}\), while exhibiting good (\(\sim 5\%\)) YU. Requiring \(\Omega h^2 \leq 10\), the Higgs mass can still be within the favorable range with YU still at the \(\sim 5\%\) level. The \(R_{h\sigma} - m_{\tilde{\chi}}^\pm\) plane indicates that the chargino, which in this model is mostly a wino like particle, can be as light as 500 GeV. Imposing the relic abundance bound implies that the lightest chargino mass is \(\sim 1 \text{ TeV}\). We therefore conclude that compatibility of YU with neutralino dark matter scenario in this model predicts that only the stop quark will be accessible at the LHC. If we assume that neutralino is not the dark matter candidate we can see from Figure 2 that the gluino can be around 1 TeV, while the squarks can lie around 2 TeV or so.

In Table 1 we present three characteristic benchmark points which summarize the salient features of this model. The three points satisfy \(123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}\) as well as the sparticle mass and B-physics constraints described in section 3. The mass of the gluino decreases from 4 TeV (point 1) to 1.5 TeV (point 3). For point 1, YU is at the level of 11\% and the neutralino relic abundance satisfies the 5\(\sigma\) WMAP bound. For point 2, the neutralino relic abundance is relatively large but YU is at the few percent level. Point 3 shows acceptable YU with a lighter gluino and stop compared to point 2. The stop is the NLSP for the three points with the lightest being \(m_{\tilde{t}_1} = 488 \text{ GeV}\) for the first point.

Note that in some SO(10) GUT models with a unified Higgs sector, it is possible to have relations among Yukawa couplings [8, 9] different from what we employed in Eq. (4). For instance, in ref. [8] it is discussed that there is parameter space available consistent with the relation:

\[\frac{Y_t}{48} = \frac{Y_b}{Y_\tau},\]  

which predicts \(\tan \beta \approx 2\). We find that in this case the solutions yield YU no better than the 70\% level, which therefore indicates there is no YU at all.

A different scenario [9] allows:

\[\frac{Y_t}{8.35} = Y_b = 0.7 Y_\tau.\]  

The predicted value of the parameter \(\tan \beta\) for this case is \(\approx 10\) and is therefore preferable. However, in this case also the best unification is only at the 12\% level. Because of these unfavorable results we are not presenting a more detailed analysis for these relations (Eqs (11) and (12)).

## 5 Conclusion

We have explored a class of SO(10) GUT models with a unified Higgs sector which yield the asymptotic relation \(Y_b \approx Y_\tau \approx Y_t/6\) among the third generation quark and
lepton Yukawa couplings. This relation among the Yukawa couplings is compatible with the various phenomenological constraints only with non-universal SSB mass terms at the GUT scale. The best $Y_U \approx 5\%$ we found in our analysis is consistent with the lightest CP-even Higgs boson mass to be in the interval $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$. By scanning the fundamental parameter space of this model we showed that tan $\beta$ is constrained in a very narrow interval, namely, $\tan \beta \approx 14$. Without imposing the constraint on the relic abundance of dark matter in these models, the squark and slepton masses, except for the stop, exceed 2 TeV while the gluino can be more than 1 TeV. On the other hand, the LSP neutralino as a dark matter candidate in this model can only be realized through the neutralino-stop coannihilation scenario. We found that requiring good $Y_U$ can lead to a light stop $(\gtrsim 500 \text{ GeV})$ with all other sfermions having masses possibly beyond the reach of the 14 TeV LHC.

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