Quantum spin chains and Majorana states in arrays of coupled qubits

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Several designs of inter-qubit coupling are considered. It is shown that by a combination of Josephson and capacitive coupling one can realize qubit interactions of variable spin content. Qubit arrays are discussed as models of quantum spin chains. In particular, a qubit model of the 1D quantum Ising spin chain is proposed. A realization of unpaired Majorana fermion states in this system is considered. It is shown that Majorana states are represented by spin flip excitations localized on the chain ends. Using unpaired Majorana states in qubit chains for decoherence protected quantum computing is discussed.

Quantum computers are machines that store information on quantum variables and that process that information by making those variables interact in a way that preserves quantum coherence \(^1\)\(^2\). Quantum superconducting circuits of nanoscale size became available recently for experiments which investigate novel quantum phenomena and their application to quantum computing \(^3\)\(^4\). This research has led to the observation of Rabi oscillations \(^5\)\(^6\), and to a demonstration of the superposition of macroscopic quantum states, the so-called “Schrödinger cat” states \(^7\)\(^8\). The superposition character of these states was revealed \(^9\) by doing microwave spectroscopy experiments on the two quantum levels of superconducting circuits consisting of three Josephson junctions comprising the so-called persistent current qubit \(^9\). The two states of this qubit have persistent currents of about a microamp and correspond to the motion of millions of electrons.

In this article we discuss circuits consisting of many coupled qubits. As a simplest example of such a circuit we consider an array of identical qubits with nearest neighbor couplings. Our interest in such circuits is two-fold. Firstly, qubit arrays represent a system in which novel types of qubit couplings with different ‘spin operator content’ and tunable control over the coupling strength can be investigated. Secondly, such arrays can be used to explore novel quantum phenomena with possible applications in quantum computing.

Even the simplest of these systems discussed below, the so-called ‘quantum Ising spin chain,’ is rather rich. Several interesting many-body states can be realized by varying qubit parameters and their coupling strength. Elementary excitations in some of these states are quantum solitons theoretically described as pseudofermions. We consider the phase diagram and point out several interesting regions in which the problem is exactly solvable.

Also, as discussed below, the so-called Majorana fermion states can be realized as midgap states localized at the ends of the quantum Ising chain. In a recent article \(^12\) Kitaev outlined the possibility of using such states for reducing environmental sensitivity of quantum computers. We show that the coupling of Majorana states to external time varying field can be weaker than that of a single qubit.

The persistent current qubit \(^9\)\(^13\) comprises a superconducting loop interrupted by three Josephson junctions (marked in Fig.1(a) by an ‘×’) with the junction capacitance \(C_{1,2}\). The values of the three Josephson junctions coupling constants \(J_1\) and \(J_2\) are chosen so that the Josephson part of the Hamiltonian alone defines a bistable system which at the value of external magnetic flux \(\Phi = 0.5\Phi_0\) can be either in the right-hand, or in the left-hand current state. In this qubit, by choosing parameters \(J_{1,2}\) and \(C_{1,2}\) appropriately, the barrier in the phase space separating the right and left current states can be made low enough, so that tunneling between two classical states will take place. Also, detuning of external field from the value 0.5\(\Phi_0\) produces a bias on the right and left states, making one of them lower in energy than the other.

It was shown in Ref. \(^9\) that the phase space coordinate which describes transitions between the right and left states is the relative phase \(\theta\) of the two superconducting islands [Fig.1(a)]. The potential energy of the system as a function of \(\theta\) has two minima, \(\theta = \pm \theta_0\), as shown in Fig.1(b). These minima are symmetric at \(\Phi = 0.5\Phi_0\) and...
asymmetric at $\Phi \neq 0.5\Phi_0$. Tunneling takes place through the barrier separating the two minima. The resulting qubit Hamiltonian is
\begin{equation}
H = -\Delta \sigma^z - \hbar \sigma^x ,
\end{equation}
where $\Delta$ is the tunneling amplitude for the barrier $U(\theta)$, and $\hbar = 2I_p(\Phi - 0.5\Phi_0)$ is proportional to the detuning $\Phi - 0.5\Phi_0$, and where $I_p$ is the circulating current. The energy of the ground state and the first excited states are given by
\begin{equation}
E_\pm = \mp \sqrt{\hbar^2 + \Delta^2}
\end{equation}
In the notation of Eq. (1) the right and left current states correspond to qubit ‘spin’ up and down. The energy levels repulsion manifest in Eq. (2) was directly observed recently by a spectroscopic technique [10,11].

The goal of this paper is to discuss a possibility to use superconducting quantum qubits to study novel quantum phenomena, control of qubit systems, and novel quantum devices. The question of how to make inter-qubit couplings of required strength, sign, and spin content is central for constructing logical gates out of qubits. We consider several basic couplings, such as purely transverse, purely longitudinal, or more general couplings, as required by the particular scheme of processing quantum bits. Fourth, it should be possible to turn the couplings on and off on sufficiently short time scale. Below we outline our approach to handling the problem of couplings.

Conceptually, the simplest kind of inter-qubit coupling is of the longitudinal $\sigma^z_i \sigma^z_j$ form, which can be easily realized by magnetic interaction of qubits. This coupling results from magnetic field flux that the current in one qubit induces in the other nearby qubit. The form of this coupling in terms of qubit ‘spin’ operators, since the right and left current states have a definite $z$-component of qubit ‘spin’, is indeed $\sigma^z_i \sigma^z_j$. The sign of this coupling is such that it favors antiparallel spin state of two coupled qubits. However, the numerical value of the inductive coupling estimated for conducting loops of a few microns in size turns out to be at least an order of magnitude smaller that typical values of the tunneling amplitude $\Delta$ and detuning parameter $\hbar$. Also, the inductive coupling value is preset by the system layout. Often the lack of tunability, as the coupling strength is usually preset by the system layout.

Because of that we propose another realization of the $\sigma^z_i \sigma^z_j$ coupling which has values tunable in a wider range than that of inductive coupling. The coupling is achieved by a Josephson junction with a sufficiently large $J$ shared by adjacent qubits as shown in Fig. 2. In the regime when $J \gg J_{1,2}$ the phase drop across the large $J$ junction is much smaller than the overall phase change. As a result, this coupling is gentle enough not to perturb the individual qubit dynamics. However, the coupling strength $t$ is of the order of $J_{1,2}^2/J$, and thus it can be easily made of the same order as the single qubit parameters $\Delta$ and $\hbar$. The sign of this coupling is positive.

I. INTER–QUBIT COUPLINGS; QUANTUM ISING PROBLEM

Designing couplings is essential for being able to assemble qubits into circuits capable of performing computation. For that, inter-qubit couplings must satisfy several requirements. First, the coupling strength should not be much less than the Rabi frequency of an individual qubit, so that the states of different qubits entangle with each other sufficiently quickly. Second, the coupling should be much weaker than the Josephson plasma frequency which sets the scale for the energy gap between the two lowest energy qubit states and states with higher energy. Third, the design should allow for different kinds of coupling, such as purely transverse, purely longitudinal, or more general couplings, as required by the particular scheme of processing quantum bits. Fourth, it should be possible to turn the couplings on and off on sufficiently short time scale. Below we outline our approach to handling the problem of couplings.

As an example, we consider an array of qubits coupled by shared Josephson junctions, as shown in Fig. 2. The Hamiltonian for this system is
\begin{equation}
\mathcal{H} = \sum_{i=-\infty}^{\infty} t \sigma^z_i \sigma^z_{i+1} - (\Delta \sigma^x_i + \hbar \sigma^z_i)
\end{equation}
with positive $t$. Eq. (3) describes one of the basic many-body systems: an antiferromagnetic spin 1/2 chain with exchange constant $t$ in external magnetic field with components $\hbar$ and $\Delta$. We note that, since the inter-qubit coupling is of the $\sigma^z_i \sigma^z_j$ kind, the ‘exchange’ interaction in the spin chain (3) is highly anisotropic. This should be contrasted with nearly isotropic spin exchange coupling encountered in magnetic systems, with usually only weak anisotropy possible due to spin-orbital coupling.

There are several reasons for considering the one-dimensional qubit array shown in Fig. 3. One reason is
that magnetic measurements on this system can provide a very direct and simple test of inter-qubit couplings. To illustrate this point, we consider the system \( H \) in the limit \( \Delta \ll h, t \). In this case the problem is reduced to the classical 1D Ising model. Since the exchange coupling \( t \) is positive, the ground state in the absence of external field is antiferromagnetic. Weak external field \( |h| \ll t \) does not affect the ground state, while at high field all spins align. Magnetization as a function of the field \( h \) in this case is

\[
\langle \sigma^z_i \rangle = \begin{cases} 
1, & \text{for } h > 2t \\
0, & \text{for } |h| < 2t \\
-1, & \text{for } h < -2t 
\end{cases}
\]  

(4)

This should be compared with the magnetization step reported recently for an array of isolated qubits \( [10] \). The result \( [10] \) means that due to the qubit interaction \( [10] \) the magnetization step splits into two distinct steps with relative separation equal to \( 4t \). In a real system the magnetization steps \( [10] \) can be smeared due to randomness in qubit parameters, finite value of \( \Delta \), temperature, etc. Therefore, a demonstration of a magnetization curve similar to \( [10] \) would indicate high level of qubit reproducibility and control over qubit couplings.

Another interesting fact is that the Hamiltonian \( [3] \) is nothing but the quantum 1D Ising problem. Thus the qubit array shown in Fig. 2 provides a physical model of this system. The quantum 1D Ising problem is known to have nontrivial ordered ground states and elementary excitations when other excitations are present. This situation is familiar in integrable systems, classical and quantum. It may be of interest to consider using quantum solitons in qubit arrays as means of quantum information transmission between distant parts of the qubit network. These

The transformation \( [3] \) and the resulting fermion problem \( [14] \) are applicable to both finite and infinite spin chains.

\[\begin{array}{c}
\text{FIG. 3. The parameter space of the quantum Ising problem}[3]. \\
\text{On the line } I_{\text{class}} \text{ the classical 1D Ising model is realized. On the line } JW \text{ the problem is solved by the Jordan-Wigner transformation }[15,16], \text{ and has fermion excitations. On this line the problem is equivalent to the classical 2D Ising problem in the absence of external magnetic field, with the points } \Delta = \pm t \text{ corresponding to the 2D Ising critical point. On the lines } Z \text{ the problem is described by Zamolodchikov theory }[14] \text{ of critical Ising problem in magnetic field. }
\end{array}\]

We shall consider here the case of an infinite chain, when the Hamiltonian can be conveniently rewritten in the plane wave basis:

\[H = \sum_p t \left( a_p a_{-p} e^{-ip} + h.c. \right) - 2 \left( \Delta + t \cos p \right) a_p^\dagger a_p \]

(7)

This problem is readily diagonalized by Bogoliubov transformation yielding the dispersion relation of the form

\[\epsilon(p) = 2|\Delta + t \cos p|, \quad H = \sum_p \epsilon(p) b_p^\dagger b_p \]

(8)

with the excitation wavenumber in the Brillouin zone \(-\pi < p < \pi\). The excitation spectrum \( [14] \) has a gap at low energies of the width \( E_{\text{gap}} = 2 \min (|\Delta| \pm t) \). The spectrum becomes gapless at \( \Delta = \pm t \) (the critical points \( A \) and \( B \) in the phase diagram).

Elementary excitations in this case are noninteracting Bogoliubov fermions. Let us remark that in terms of the original spin \( 1/2 \) variables \( [3] \) an energy excitation can be viewed as consisting of independent solitons which propagate through the chain preserving their identity even when other excitations are present. This situation is familiar in integrable systems, classical and quantum. It may be of interest to consider using quantum solitons in qubit arrays as means of quantum information transmission between distant parts of the qubit network. These
solitons can in principle be generated and detected locally on individual qubits located at the opposite ends of the system. Such solitons can be used as information carrier in essentially the same way as solitons in nonlinear optical channels.

II. MAJORANA FERMIONS IN A QUBIT ARRAY

In the theory of the quantum Ising model it is known that a rather natural representation of the spin problem can be obtained using Majorana fermions defined in terms of the Jordan-Wigner fermions as

\[ c_j^0 = \frac{1}{\sqrt{2}} (a_j + a_j^\dagger), \quad c_j^\dagger = \frac{1}{\sqrt{2}t} (a_j^\dagger - a_j) \]  

(9)

The operators \( c_j^{1,2} \) are hermitian, \( c_j^{a\dagger} = c_j^a \), and satisfy the anticommutation relations

\[ \{ c_j^a, c_j'^{a\dagger} \} = \frac{1}{2} \delta_{aa'} \delta_{jj'}, \quad (c_j^a)^2 = \frac{1}{2} \]  

(10)

The commutation relations indicate that each Majorana fermion represents a ‘one-half’ of a Jordan-Wigner fermion, so that the latter can be viewed as a pair of two Majorana fermions. It was proposed by Kitaev that unpaired Majorana fermion states are better protected from environmental noise than conventional fermions. Thus it is of interest to look for realizations of these states.

Here we demonstrate that the qubit array shown in Fig. 2 provides under certain conditions a realization of unpaired Majorana fermions. The Hamiltonian \( \mathcal{H} \) for a chain of length \( n \) can be rewritten in terms of Majorana fermions as

\[ \mathcal{H} = 2it \sum_{j=1}^{n} t c_j^{2} c_{j+1}^{1} - \Delta c_j^{2} c_{j+1}^{1} \]  

(11)

The problem can be diagonalized by an orthogonal \( 2n \times 2n \) Bogoliubov transformation of the operators \( c_j^{1,2} \), such that it brings the matrix

\[ A = \begin{pmatrix} 1 & \lambda & 0 & \lambda^2 & \cdots \\ 0 & 1 & \lambda & 0 & \lambda^2 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \ddots \end{pmatrix} \]  

(13)

matrix (12) with the eigenvalue \( \epsilon = 0 \). This vector has the form

\[ \lambda = \Delta/t. \]  

where \( \lambda = \Delta/t. \) For the state (13) to be normalizable, one must have \( \Delta < 1 \), in which case the normalization factor is \( A = (1 - \lambda^2)^{-1/2}. \) The condition \( \Delta < 1 \) indicates that the midgap states occur in the ordered Ising phase \( t > |\Delta| \) (the AB interval in the phase diagram).

Note that only one Majorana species contribute to the midgap state, and so this is indeed an unpaired Majorana state. For a finite but long chain the midgap state on one end will be approximately given by (13), whereas the state on the other end of the chain will be defined in a similar way in terms of the other Majorana fermion species. For a finite chain the two midgap states will be coupled by a matrix element of the order \( \sim \lambda^2 \).

Let us discuss the meaning of these states in terms of the spin 1/2 formulation \( \mathcal{H} \). It is instructive to consider the limit \( \lambda \ll 1 \), when the ground state of the spin chain is close to that of a classical antiferromagnet. In this case the width of the bulk gap is close to \( 2t \), which is consistent with the energy required to reverse one spin in the bulk in the presence of coupling \( t \) to the neighbors on both sides. An approximately twice smaller energy \( t \) is required to reverse a spin at the end of the chain, which gives excitation energy in the middle of the bulk gap. Thus, each of the unpaired Majorana states at the ends of the chain is nothing but a single reversed spin, at finite \( \lambda < 1 \) surrounded by slightly tilted spins on neighboring inner sites.

FIG. 4. Tunneling between two unpaired Majorana excitations realized as midgap states of the 1D quantum Ising problem.

When the system is excited into one of the two midgap states, tunneling between two degenerate states will take place due to coupling of the midgap states on opposite ends across the chain. This tunneling process involves simultaneous coherent flipping of the spins at the chain ends accompanied by rearrangement of the tilted spins near the ends. It is noteworthy that this spin flipping does not change the net spin, since the two spins at the
ends flip coherently and in antiphase. Hence the (spatially uniform) background magnetic field fluctuations on the two ends are cancelled out and do not affect the tunneling dynamics. This is consistent with the conclusion of Kitaev that unpaired Majorana states have weaker sensitivity to environmental time varying fields.

III. TRANSVERSE INTER–QUBIT COUPLINGS

Here we consider several interesting realizations of transverse couplings. One possible kind of transverse coupling is illustrated in Fig. 5. In this scheme, superconducting islands of adjacent qubits are capacitively coupled in such a way that the coupling capacitance is of the order of the Josephson junction capacitances $C_{1,2}$. The physical reason for the coupling shown in Fig. 5 to be transverse is the following. When qubits are in a superposition of the right and left current states, their quantum dynamics is described by tunneling between two minima of the potential $U(\theta)$ in Fig. 6. Each tunneling event produces a charge impulse on superconducting islands. By making the qubits capacitively coupled, one enforces the correlation of time-dependent charge fluctuations on different islands, which is equivalent to having correlation of tunneling events on coupled qubits. The Hamiltonian for this coupling, as we shall see, is written in terms of the transverse spin operators $\sigma^x = \frac{1}{2}(\sigma^x \pm i\sigma^y)$.

![FIG. 5. 1D array of capacitively coupled qubits: a realization of transverse inter-qubit coupling described by the Hamiltonian (14).](image)

To derive the form of the qubit coupling Hamiltonian, we consider two qubits coupled capacitively according to the scheme in Fig. 5. We represent the collective dynamics of the two qubits by motion in the two-dimensional phase space parameterized by the qubits’ phases $\theta_1$, $\theta_2$. Potential energy is given by the sum of Josephson contributions $U(\theta_1) + U(\theta_2)$. In the absence of qubit coupling, their kinetic energy is $m(\dot{\theta}_1^2 + \dot{\theta}_2^2)/2$ with effective mass $m$ being determined by the Josephson junction capacitances $C_{1,2}$ — see Ref. 3. Since the kinetic energy is isotropic, all tunneling amplitudes between four minima $\theta_{1,2} = \pm \theta_0$ are equal to each other. In this case, as shown schematically in Fig. 6, tunneling of the two qubits is independent, and thus the tunneling Hamiltonian is written as a sum $\sigma^x_1 + \sigma^x_2$. Now, the capacitive coupling shown in Fig. 5 is described by adding to the kinetic energy a new term $m_*(\theta_1 + \theta_2)^2/8$, where $m_* = (\hbar/2e)^2C$. This term makes kinetic energy anisotropic by increasing effective mass for the motion in the $(1,1)$ direction. As a result, when $m_* \geq m$, all tunneling amplitudes are suppressed with the exception of the correlated transitions $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$ illustrated in Fig. 5. This process is described by the Hamiltonian of the form $\sigma^x_1 \sigma^x_2 + \sigma^x_1 \sigma^x_2$.

In the more general situation with $m_* \sim m$, both the correlated and independent tunneling processes take place, and the resulting Hamiltonian of the array has the form

$$\mathcal{H} = \sum_{i=-\infty}^{\infty} t(\sigma^x_i \sigma^x_{i+1} + \sigma^y_i \sigma^y_{i+1}) - (\Delta \sigma^x_i + h \sigma^z_i), \quad (14)$$

where $\Delta$ is the individual qubit tunneling amplitude, and $t$ is the correlated tunneling amplitude. Note that $t \gg \Delta$ when $m_* \gg m$. The first term of Eq. (14) can also be written as $(t/2)(\sigma^x_i \sigma^x_{i+1} + \sigma^y_i \sigma^y_{i+1})$, i.e., the Hamiltonian (14) defines the so-called $XY$ spin model in an external field.

![FIG. 6. Correlated tunneling in two capacitively coupled qubits.](image)

Magnetization curve can be computed at $\Delta \ll t$ by using the Jordan–Wigner mapping of the spin 1/2 problem to a free fermion problem. In this case, as we discuss below, the mapping is constructed somewhat differently from the one used in the quantum Ising model. The result is:

$$\langle \sigma^z_i \rangle = \begin{cases} 1, & \text{for } h > t \\ \frac{2}{\pi} \arcsin(h/t), & \text{for } |h| < t \\ -1, & \text{for } h < -t \end{cases} \quad (15)$$

In the transverse coupling case, instead of two abrupt magnetization steps found for the qubit array with the $zz$ coupling, we have a continuous magnetization curve with characteristic square root singularities at $h = \pm t$.

To illustrate flexibility of the systems with transverse coupling, we consider an array with capacitive coupling built according to the scheme displayed in Fig. 5. Like in the array in Fig. 5, effective qubit ‘spin’ couplings are purely transverse in this case. The analysis of couplings goes along the same lines as for the array in Fig. 5. Two decoupled qubits can be described by separable dynamics in the two-dimensional phase space $(\theta_1, \theta_2)$. The coupling changes kinetic energy by adding to it the term $m_*(\theta_1 - \theta_2)^2/2$, where $m_* = (\hbar/2e)^2C$. Note that in this
case, due to different orientation of coupled qubits, their coupling depends on the difference of the phases \( \theta_1 - \theta_2 \), rather than on the sum as in the above example. As a result, effective mass is higher for the motion in the (1,−1) direction, and at \( m_a \gg m \) all tunneling processes except \( \langle \uparrow \uparrow \rangle \leftrightarrow \langle \downarrow \downarrow \rangle \) are suppressed. According to Fig. 4, this process is described by the Hamiltonian of the form \( \sigma_1^+ \sigma_1^+ + \sigma_1^- \sigma_1^- \). As before, for the more general situation of \( m_a \sim m \) there are both the correlated and independent tunneling processes, and thus the Hamiltonian of the array in Fig. 4 is

\[
\mathcal{H} = \sum_{i=-\infty}^{\infty} t(a_i^+ \sigma_{i+1}^+ + a_i^- \sigma_{i+1}^-) - (\Delta \sigma_i^z + h \sigma_i^z). \tag{16}
\]

The first term of Eq. (16) can also be written as \((t/2)(\sigma_i^x \sigma_{i+1}^x - \sigma_i^y \sigma_{i+1}^y)\).

The difference between the problems \( (14) \) and \( (16) \) is most clearly displayed in the Jordan–Wigner representation

\[
\sigma_i^+ = a_i^+ \prod_{j<i} \sigma_j^+, \quad \sigma_i^- = a_i \prod_{j<i} \sigma_j^-, \quad \sigma_i^z = 2a_i^+ a_i - 1. \tag{17}
\]

To simplify matters we shall limit ourselves to the strong coupling case, when \( m_a \gg m \) and thus \( t \gg \Delta \) in both \( (14) \) and \( (16) \). Then the fermionic representation of the Hamiltonians \( (14) \) and \( (16) \) is given by

\[
\begin{align*}
(i) & : \mathcal{H} = \sum_{i=-\infty}^{\infty} t(a_i^+ a_{i+1}^+ + a_{i+1}^- a_i) - 2ha_i^+ a_i, \tag{18} \\
(ii) & : \mathcal{H}' = \sum_{i=-\infty}^{\infty} t(a_i^+ a_{i+1}^+ + a_{i+1}^- a_i) - 2ha_i^+ a_i. \tag{19}
\end{align*}
\]

The spectrum of elementary excitations is obtained by going to the Bloch plane waves representation (in the case (ii) accompanied by Bogoliubov transformation). The result reads

\[
\begin{align*}
(i) & : \epsilon(p) = 2(t \cos(p) - \hbar), \tag{20} \\
(ii) & : \epsilon(p) = \pm 2 \sqrt{t^2 \sin^2(p) + \hbar^2}. \tag{21}
\end{align*}
\]

It follows from the result \( (24) \) that the ground states of the two systems are described by collective entangled states of all qubits in the chain, represented by ideal Fermi gas in the case (i) and by Dirac vacuum in the case (ii). In the case (i) the excitation spectrum is gapless for not too large \( |h| < t \), whereas in the case (ii) there is a finite gap in the excitation spectrum equal to \( 4|h| \).

**IV. NOVEL QUANTUM PHENOMENA**

As we discussed above, there are several attractive schemes of designing qubit coupleings. Besides allowing for control over coupling strength and its spin operator content, the proposed schemes are compatible with the plans to achieve tunability of coupleings in time. This can be attempted, for example, by employing switchable Josephson junctions similar to the gated InGaAs superconducting junctions \( (18) \) which have very short switching times of the order of a few microseconds. Magnetization measurements on these arrays compared with theoretical results can provide useful information on qubit coupleings.

The one-dimensional arrays discussed above provide novel realization of spin 1/2 chains, one of the basic many-body physics models. Quantum spin chains are well studied theoretically and are known to be quite rich in properties \( (15,16) \). Depending on the kind of coupling they can display different kinds of ordering: ferromagnetic or antiferromagnetic (both with algebraic correlations), spin liquid, Luttinger liquid, etc. Elementary excitations (quasiparticles) in these systems can have fractional quantum numbers and fractional (anyon) statistics. Depending on couplings, the excitation spectrum can be gapless or gapped, which should be manifest in dynamical properties, i.e. in the character of excitation transport along the chain.

Experimentally, the only realization of spin chains that has been available so far is in quasi one-dimensional magnetic materials. That, however, corresponds to a relatively small domain in the coupling parameter space, because spin exchange is usually isotropic or nearly isotropic, as a result of the weakness of spin-orbital interaction in solids. Also, dynamical phenomena such as excitation transport are quite difficult to access in magnetic systems because available experimental techniques are limited to low frequency magnetization measurement and polarized neutron scattering. Thus realizing spin chains in the qubit arrays looks attractive from both points of view. First, in the qubit array coupleings can be tuned to the desired form by the methods outlined in the previous section. Second, the dynamics can be directly probed by standard electric measurements.

One can study excitation transport in the qubit arrays. Experimentally, this can be achieved by exciting the chain at one end, and measuring response at the opposite end. It will be interesting to manufacture and compare properties of gapless and gapped systems, as the character of excitation transport has to be very different in the two cases. Moreover, in the case of the 1D quan-
tum Ising problem (3), the dependence of the excitation energy (8) on the qubit coupling \( t \) indicates that for the array shown in Fig. 2 both regimes of excitation transport are available simultaneously. By changing the qubit coupling, one can sweep through the range of parameters in which the gap in the excitation spectrum (8) will gradually close and then reopen. The problem of excitation transport is also quite interesting theoretically. The challenge here is that, in contrast with other one-dimensional Fermi systems such as electrons in quantum wires, generic Hamiltonians of spin chains do not conserve the number of Jordan-Wigner fermions. For example, the external field term \(-\hbar \sigma_z^i\) in the 1D quantum Ising Hamiltonian (3) has a nonlocal form in the Jordan-Wigner representation. This may in principle change the properties of elementary excitations.

Looking somewhat ahead of experimental developments, it is quite interesting to imagine and explore the possibilities that will become available if excitations in qubit arrays can indeed propagate ballistically, as suggested by the dispersion relation (8). One can ask whether it is possible to design qubit analogs of electron-based quantum wires and use them to manipulate excitations in the same way as it is done in conventional nanoscaled semiconductor structures. Doing this should include developing a ‘battery’ (i.e., a DC spin current source), as well as techniques of measuring the current flux of spin excitations. This is of interest because spin excitations, being neutral, are practically decoupled from external electric and (to a lesser extent) magnetic fields. Thus it may be possible to achieve levels of coherence unavailable in electron systems (but perhaps comparable to what is possible for photons in quantum optics devices).

Another interesting direction is to use qubit arrays to design unpaired Majorana fermion states. As the above discussion reveals, these states can be realized in the quantum Ising chain, i.e., in a qubit array with \( zz \) couplings. On the grounds of what has been discussed above, one can expect that these states can be probed spectroscopically in a way similar to individual qubit states (10). Using this technique one can, in principle, determine decoherence time of Rabi oscillations realized with two Majorana states and verify theoretical expectation that unpaired Majorana states are protected from decoherence.

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