Real time temperature estimation of heat sources in integrated circuits with remote temperature sensors

M Janicki and A Napieralski
Department of Microelectronics & Computer Science, Technical University of Lodz
Al. Politechniki 11, 93-590 Lodz, Poland, tel. + 48 42 6312628
E-mail: janicki@dmcs.pl

Abstract. This paper presents a theoretical study of the possibility to carry out real time estimation of power dissipated in electronic integrated circuits based on remote temperature sensor measurements. Due to the damping of the thermal response, the considered inverse problem is ill-posed; hence special attention has to be paid to the estimation process in order to assure the robustness of the obtained results. First, the authors focused on the issue of the proper placement of the temperature sensors for a given heat source configuration. Once the optimal locations of the sensors were determined, the problem of the estimation of the power dissipated in the heat sources was studied in detail. The tests were carried out for the computer generated input data with several noise levels. For the estimation the function specification algorithm with different number of temperature sensors and variable number of averaged measurements was considered. For all the presented thermal simulations as well as for the generation of the test input data, the authors used their own three-dimensional thermal solver employing the Green’s function solution of the heat equation. The numerical experiments served as an indication for the design of a real test integrated circuit.

1. Introduction
Temperature is an important factor affecting electronic circuit operation and its life time. Increased operating frequencies and continuous device miniaturisation augmented the dissipated power density and consequently caused serious thermal problems even in apparently low power applications. Thus, the research on thermal simulation of electronic circuits became an indispensable part of the circuit design process.

Moreover, certain applications require continuous temperature monitoring during their operation so as to protect circuits from overheating and destruction. Obviously such circuits should be equipped with temperature sensors placed directly where the heat is generated. However, this situation quite often is not allowed due to some design constraints. Thus, an alternative solution is to monitor circuit temperature using sensors located away from the sources. This paper presents certain considerations on the possibilities and the fundamental limitations of integrated circuit temperature estimation based on remote circuit measurements.

The next section of the paper contains a brief introduction to the estimation of power dissipation in electronic systems. Specifically, the information on the particular thermal model and its solution method used by the authors is presented as well as the formulation of the inverse problem is given. Then, for a test structure, the sensor topology is optimized knowing the locations of heat sources. Finally, and the performance of different versions of a sequential inverse algorithm is evaluated.
2. Power estimation in electronic systems
The main purpose of this section is to provide some theoretical background for the considered inverse problem of the estimation of power dissipated in electronic systems. First, the mathematical model of the heat transfer processes in such structures is presented. Next, the methodology for obtaining the particular solution of the three-dimensional heat equation resulting from the thermal model using the Green’s functions is introduced. Finally, the considered inverse problem is formulated.

2.1. Electronic system thermal modelling
For the heat transfer processes obeying the Fourier law, stating that the heat flux is proportional to the temperature gradient, the elementary energy balance for a unitary volume leads to the following three-dimensional heat equation [1]-[2]:

\[
\nabla \cdot \left[ \lambda(\mathbf{r}) \nabla T \right] + g_v(\mathbf{r}, t) = c_v(\mathbf{r}) \frac{\partial T}{\partial t}
\]

(1)

where: \( \lambda \) thermal conductivity (Wm\(^{-1}\)K\(^{-1}\)), \( T \) temperature (K), \( c_v \) specific heat per unit volume (Jm\(^{-3}\)K), \( g_v \) generated heat per unit volume (Wm\(^{-3}\)), \( \mathbf{r} \) position vector (m), \( t \) time (s).

The solution of (1) can be found given the material properties as well as the initial and boundary conditions. The initial conditions determine the temperature distribution in a structure at the starting point of the analysis. The boundary conditions describe the heat exchange with surrounding ambient at the outer structure surfaces. Generally, there are distinguished three fundamental types of boundary conditions: the Dirichlet condition (known surface temperature), the Neumann condition (prescribed surface heat flux) and the Robin condition (mixed radiative and convective heat exchange).

Majority of electronic circuits have rectangular shapes and can be approximated by multilayered slabs. The power dissipated in circuits is represented in such a model by the heat flux penetrating into the structure through its top surface. Because most circuits are relatively thin in comparison to their area, the four lateral surfaces in the model are assumed to be adiabatic ones and the heat is removed at the remaining surfaces obeying the Newton’s law of cooling. The whole set of equations describing the proposed thermal model and its boundary conditions can be found in [3]. This model, owing to its simplicity, allows the application of the below described Green’s function analytical solution method, yet provides in most cases satisfactory accuracy.

2.2. Green’s function solution
Green’s functions (GFs) are versatile mathematical tools suitable for solving of various theoretical and practical problems. In the case of heat conduction problems, GFs can be regarded as a temperature response in a point \( \mathbf{r} \) at time \( t \) caused by an instantaneous heat generation occurring in a point \( \mathbf{r}' \) at time \( \tau \). Then, in order to obtain the temperature response in time, it is enough to integrate a GF over the entire volume and time where heat is generated. The GFs are derived using different methods, such as the method of images, using the Laplace transform or the Fourier method of separation of variables. All these methods yield solutions in different but mathematically equivalent forms. From the computer simulation point of view, the main difference between these methods is the rate of their convergence. Generally, the first two methods are rapidly convergent for short times whereas the Fourier method produces series, which are better convergent for large times. Particular GFs depend only on structure geometry and applied boundary conditions. Therefore, the same GF can be used for solving different problems concerning the same structure. For linear problems the overall temperature rise can be found as the sum of the individual temperature rises caused by various factors, such as the initial temperature distribution, the internal heat generation or non-homogeneous boundary conditions. Because one-dimensional GFs are geometry dependent solutions of the heat equation with homogeneous boundary conditions, they can be easily tabulated and used for the construction of other more sophisticated solutions. More information on the solution of heat transfer problems with GFs and the particular methods of deriving them can be found in [4].
Until recently GFs were not widely used in the thermal analysis of electronic systems because it was virtually impossible to obtain such functions for relatively complex multidimensional cases. Thus most commercial thermal solvers employed for the solution of the heat equation some numerical method, e.g the finite difference or the boundary element ones. Owing to the latest efforts to tabularize GFs, an ever growing interest in these functions was induced which resulted in numerous publications on this topic. For the hereby considered multidimensional heat transfer problems in non-homogenous layered materials, the research done by Haji-Sheikh and Beck [5]-[6] is of the great importance. As far as the field of electronics is regarded, in 2001 Gerstenmaier and Wachutka employed GFs for the thermal simulation of electronic circuits [7]-[8]. Independently, the authors of this paper developed their own thermal solver based on Fourier series GFs [9]-[11].

The procedure of obtaining the Green’s functions for the heat equation (1) begins with the solution of the complementary equation (2), which is almost identical with the original one except for the fact that there is no internal heat generation and that the boundary conditions are assumed to be of the same type but homogeneous ones [4].

\[
\nabla \circ [\hat{\lambda}(r) \nabla \Theta] = c_s(r) \frac{\partial \Theta}{\partial t}
\]

(2)

Employing the Fourier separation of variables method for the derivation of a GF, the solution of the complementary equation (2) can be expressed as follows:

\[
\Theta(r,t) = \sum_n A_n \varphi_n(r) \exp(-\gamma_n t)
\]

(3)

where: \( \varphi \) eigenfunctions, \( \gamma \) eigenvalues, \( A \) series coefficients, \( n \) series index.

Substituting the solution into equation (2) leads to an eigenvalue problem, which can be solved following the Galerkin procedure by the integration over the entire structure volume. As a result, the following formula to compute the eigenvalues \( \gamma_n \) is obtained [10]:

\[
\gamma_n = \frac{-\int \varphi_n \nabla \circ [\hat{\lambda}(r) \nabla \varphi_n] \, dV}{\int c_s(r) \varphi_n^2 \, dV}
\]

(4)

Normally, pursuing the Fourier method, the next step would be to find the complementary solution by applying the initial condition and determining the unknown series coefficients \( A_n \). However, in the considered case, the appropriate GFs have to be determined before. This is done repeating the entire procedure with the substitution of the solution and the integration over the volume, but this time for equation (1). Then, after some algebraic manipulations, the unknown GFs can be obtained as [10]:

\[
G(r,t \mid r',\tau) = c_s(r) \sum_n \frac{\varphi_n(r) \varphi_n(r')}{\int c_s(r) \varphi_n^2(r) \, dV} \exp(-\gamma_n (t-\tau))
\]

(5)

For the earlier presented electronic circuit thermal model the appropriate GF was found following the same solution outline. First, the three-dimensional eigenfunctions \( \varphi \) were determined applying the specific boundary and contact conditions consistent with the considered model. Then, the normalising coefficients, assuring the continuity of the eigenfunctions in the structure, were determined performing the integration over the entire volume. The most demanding stage in the solution was the computation of the eigenvalues being the multiple solutions of the transcendental equation. More information concerning the problem of computing eigenvalues in multilayered, multidimensional structures can be found in [5] and the particular transcendental equation solved by the authors are given in [10]-[11].
Once the GFs are determined, the temperature in a structure can be computed. The temperature rise might be the result of the initial temperature distribution, the internal energy generation or the non-homogenous boundary conditions. In the thermal model considered here, the only factor influencing the temperature is the heat flux \( q \) diffusing through the surface. Thus, the final formula to compute the temperature rise in a circuit can be expressed in the terms of a GF as given in equation (6). The detailed derivation of this formula can be found in [4] whereas the detailed descriptions of the solution for the particular thermal model considered here are presented in [9]-[11].

\[
c_t(r)T(r,t) = \int_0^t \int_{S'} (q G)_{S'} dS' d\tau
\]  

(6)

2.3. Inverse problem formulation
When temperature values are to be determined only in a limited number of locations, which is usually the case, and the heat fluxes in heat sources are uniform, for linear problems the solution equation (6) can be transformed into the discrete matrix equation (7). The temperature value vector \( T \) is determined multiplying the vector of source heat fluxes \( q \) and the matrix of thermal influence coefficients \( A \). The elements of this matrix are computed evaluating the appropriate integrals according to equation (6).

\[
T = A * Q
\]  

(7)

The particular inverse problem considered here consist in determining the unknown source heat fluxes from remote sensor temperature measurements. From mathematical point of view, the solution of such an inverse problem requires performing the inversion of the thermal influence coefficient matrix \( A \). However, due to the fact that the thermal response is damped, this matrix is usually poorly conditioned thus it is impossible to invert it. Therefore, as illustrated in the following sections, some special techniques have to be employed in order to assure reasonable conditioning of the problem. Obviously many other inverse problems can be formulated for equation (7) [12], however then the unknown quantities to be estimated are included in the coefficient matrix \( A \) and the solution of such problems is not trivial. Therefore such problems will be left outside the scope of this paper.

3. Layout optimisation
This section discusses the problem of circuit topology optimisation for the purpose of dissipated power estimation based on remote sensor temperature measurements. First, the test structure, resembling a real integrated circuit, is presented. Then, the optimal locations for the temperature sensors are found knowing the position of heat sources.

3.1. Test structure
All the considerations presented in this paper are based on the simulations of a test structure with four heat sources of known location and size. This situation is usual in electronic circuits because the position of the heat sources is determined in the design process and only the amount of dissipated power is unknown. The heat sources of the test structure are located on the top surface of a 0.3 mm thick 10 mm x 10 mm silicon die, which in turn is attached to a 3 times thicker heat spreader. The exact positions of the heat sources are shown in figures 1-2, where they are marked by grey squares.

3.2. Sensor placement
For the above described heat source configuration, 25 possible sensor locations, represented by tiny dots in figures 1-2, were considered. Given the position of sensors the thermal influence coefficients were computed using the GF approach for two values of the heat transfer coefficient modelling the circuit cooling. The considered values were 20 Wm\(^{-2}\)K\(^{-1}\) and 1000 Wm\(^{-2}\)K\(^{-1}\) which corresponds to the free convection cooling with no radiator and the forced water cooling conditions respectively.
The criterion for determining the optimal sensor locations was the condition number of the thermal influence coefficient matrix which needed to be inverted. The simulations were carried out for all the feasible numbers of heat sources and temperature sensors (i.e. greater or equal to the number of heat sources), and the condition number was computed for the thermal influence coefficient submatrices containing only currently activated sources and sensors. Obviously, all the best sensor configurations included the sensors located directly in the heat sources. However, when the number of sensors was limited only to those located outside the sources, it turned out that the best remote sensor locations are always on the edges of the structure and the worst ones in the middle on the symmetry lines between the sources. Interestingly, the optimal sensor configurations determined for 4 heat sources were also optimal for smaller number of active heat sources. As an example, the best and the worst locations for 8 remote temperature sensors, marked by black dots, and 4 heat sources are shown in figures 1-2 respectively.

As far as the condition numbers of the matrices are concerned, the first observation was that the condition number of the full thermal influence coefficient matrix $A$ for 4 heat sources and 25 sensors was equal to 14 for the forced water cooling conditions and 399 for the free convection cooling, which indicates that the estimation of power dissipation is definitely more sensitive to input data errors in the latter case. Another observation was that for 4 sources and 4 sensors located in the source locations the earlier mentioned condition numbers were 26 and 163 respectively. This result suggests that in the case of free convection cooling inclusion of additional sensors rather worsens problem conditioning. The possible reasons for this will be discussed in the next section. Finally, when the sensors are placed only on the sides of the circuit, the respective condition numbers for the sensor configuration showed in figure 1 are 178 and 1090.

4. Real time estimation
This part of the paper concerns the problem of real time heat source temperature estimation in the presence of noise corrupting sensor measurements. First, given the previously determined optimal location of sensors, the conditioning of the entire inverse problem is discussed in more detail. Then, the function specification inverse algorithm used for the estimation of power dissipation is introduced. Finally, the computer generated noisy input data are filtered using digital filters in which the function specification algorithm was implemented.

4.1. Problem conditioning
The issues of the inverse problem conditioning and the specific problems occurring in the estimation of dissipated power in the circuit are discussed based on the thermal simulations of the test structure presented in the previous section. The dynamic simulations were performed with the GF simulator for the power dissipated in one of the sources.
The circuit heating curves obtained for the two previously discussed different cooling conditions are plotted against the logarithmic scale of time in figures 3-4. The left chart was obtained for the dissipated power of 0.5 W with free convection cooling without radiator and the right chart for the power of 8 W and forced water cooling with large radiator. Please note that the time scales are not the same in the charts. The solid line shows the temperature of the heat source, whereas the dashed and dotted lines the closest and the most distant corners of the silicon die respectively.

The above figures provide some important conclusions explaining the previously observed poor conditioning of the considered inverse problem. Without a radiator, which fortunately is rarely the case in the power applications, the temperature of a silicon chip is fairly uniform and remote sensor temperatures are lower only by a few degrees than the temperature of the heat source. Then, in the presence of noise, the measured temperature can be even higher than the hot spot temperature, which is the reason for the ill-conditioning of the problem. Quite the opposite, in good cooling conditions the sensor temperature is lower by more than 50 K even a few millimetres from the hot spot, hence the conditioning of the problem is better. This result confirms that the estimation of dissipated power from remote sensor measurements is indeed a severely ill-posed problem. It is also worth noticing that with forced water cooling the steady state temperature is reached already in a couple of minutes whereas without a radiator it might take almost half an hour.
It is instructive also to look at the magnification of the beginning of circuit heating process shown against the linear time scale in figures 5-6. The first observation is that the delay in the sensor response due to the heat diffusion is independent from the cooling conditions and for the entire chip does not exceed 200 ms. Secondly, the thermal response is fully developed, i.e. the temperature differences throughout the silicon chip are the same as the steady state, already after a few tenths of a second, which might justify the use of the steady state solutions in the optimisation and the final estimation algorithms. On the other hand, in the good cooling conditions already after 100 ms the temperature rise in the heat source exceeds one third of its steady state value, which might cause serious problems in the detection of unexpected short circuits before the maximum allowed temperature is exceeded.

4.2. Function specification algorithm

For the past decade the authors of this paper were developing various concepts and methods for real time monitoring of electronic circuit temperature. In their original work [13], the quality of estimates was improved owing to the use of the redundant information from temperature sensors employing the simple Moore-Penrose pseudo inverse of the thermal influence coefficient matrix $A$. Then, the authors turned towards various recursive adaptive filters and iterative regularisation methods [14]. Finally, the function specification algorithm has been chosen since it is intuitively comprehensible for engineers and the most suitable for practical digital filter hardware realisations.

The function specification algorithm, originally proposed by Beck [15], is based on the assumption that the variation of the unknown quantity to be estimated in time can be described in some functional form. Obviously, the most frequently it is assumed that the function is constant, which implies here that a few subsequent unknown power density (heat flux) values are equal. The method can be further expanded rendering the problem overdetermined through the introduction of superfluous temperature sensors, i.e. more than the number of heat sources. The algorithm in its sequential form allows the estimation of heat flux variation in time, successively with each new arriving measurement data. The unknown source power values are computed knowing the current sensor temperature values and ‘future’ values, where $r$ is the number of heat fluxes assumed to be equal. Then, for $p$ unknown heat fluxes and $J$ temperature sensors, the heat flux vector $Q_k$ estimated in the $k$-th iteration can be found from the following equation [15]:

$$Q_k = (XB^T XB)^{-1} X B^T T$$

(8)

The above formula is identical with the one for the pseudo-inverse of the matrix product $XB$, only the construction of the matrices $X$ and $B$ is more complex. Namely, $X$ is a lower triangular matrix composed of submatrices of thermal influences coefficients computed for different time instants as shown below:

$$X = \begin{bmatrix}
  a_1 & 0 & 0 & \cdots & 0 \\
  a_2 & a_1 & 0 & \cdots & 0 \\
  a_3 & a_2 & a_1 & \cdots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  a_r & a_{r-1} & a_{r-2} & \cdots & a_1
\end{bmatrix}$$

(9)

Because in the considered case the steady state solution is used, all the $J \times p$ submatrices $a_i$ have zero elements, except for the submatrix $a_1$, which contains the steady state values of the thermal influence coefficients. $B$ is the vector matrix composed of $r \times p$ identity matrices, $Q_k$ is the vector of unknown heat fluxes in the $k$-th iteration and $T$ is the vector of $r$ sets of measured temperature values. For the derivation of equation (8) and the details for the construction of matrix $X$ for dynamic solutions of the heat equation refer to [15]. Because of the compatibility reasons, the entire notation in this subsection was intentionally kept as in the original book by Beck.
4.3. Heat source estimation

The real time heat source power estimation in the presence of noise corrupting sensor responses was investigated for the two previously considered circuit cooling conditions. Assuming certain arbitrary independent variations of the power dissipated in the sources, the respective temperature responses were computed at the sensor locations. Then, the exact data were contaminated with additive noise. The mean value of the noise was equal to 0 and its standard deviation ranged form 0.1 K to 5 K. Then, the generated noisy data were processed with the digital filter implementing the function specification algorithm. The multiple heat source power estimation was performed for different temperature sensor configurations averaging 1 to 5 ‘future’ values.

Exemplary estimation results for the forced water cooling are presented in figures 7-8 and for the free convection cooling in figures 9-10. The figures on the left side were obtained for 4 sensors located directly in the heat sources whereas the ones on the right for 8 optimally placed remote sensors. The figures show the ordinary Moore-Penrose estimate (grey line) and the filtered estimate (black line), which are compared with the original signal assumed in the simulations (straight black line). Please note that the power level in the bottom figures in lower so as to obtain similar temperature rise value.
As can be seen in the figures, owing to the application of the function specification algorithm it was possible to improve considerably the quality of the obtained estimates compared to the pseudo-inverse solution, but in the case of the free convection cooling, especially when the remote sensors are used, the errors remain significant.

Further inside into the problem provides the analysis of table 1, which contains data on the problem conditioning and the standard deviations of the obtained estimates in three cases, i.e. 4 sensors located in heat sources as well as 4 and 8 optimally located remote sensors. The estimate standard deviations are related for each heat source to its maximal power value. On the other hand, the standard deviations of the temperature measurement errors are absolute ones. The left column presents the data for the forced water cooling whereas the right one for the free convection cooling.

Obviously, as observed already in the figures, the best estimates were obtained when temperature sensors were placed in the heat sources. When remote sensors are used, the problem is ill-conditioned and the estimation errors are larger. Certain improvement can be achieved by increasing the number of ‘future’ values used, but this might even increase the estimation error in some cases.

**Table 1.** Estimation results.

|                  | Forced water cooling | Free convection cooling |
|------------------|-----------------------|-------------------------|
|                  | 4 sensors in heat sources - condition number 7 | 4 sensors in heat sources - condition number 165 |
|                  | 4 remote sensors - condition number 39 | 4 remote sensors - condition number 1102 |
|                  | 8 remote sensors - condition number 39 | 8 remote sensors - condition number 1102 |
| Error std. dev. (K) | Nr. of future values | Estimate deviation (%) | Error std. dev. (K) | Nr. of future values | Estimate deviation (%) |
| 0.5              | 1                     | 0.8 7.0 4.1             | 1                     | 1.5 13.2 8.0         |
|                  | 5                     | 4.1 4.8 4.5             | 5                     | 1.6 5.9 3.9          |
|                  | 10                    | 5.6 7.2 6.2             | 10                    | 2.3 4.4 3.3          |
| 1.0              | 1                     | 1.6 13.3 8.2            | 1                     | 7.6 67.3 40.6        |
|                  | 5                     | 4.4 6.9 5.7             | 5                     | 3.6 28.3 17.5        |
|                  | 10                    | 6.1 7.2 6.7             | 10                    | 3.2 19.3 12.1        |

| Error std. dev. (K) | Nr. of future values | Estimate deviation (%) | Error std. dev. (K) | Nr. of future values | Estimate deviation (%) |
| 0.5              | 1                     | 4.7 40.5 24.6          | 1                     | 9.4 80.5 49.6        |
|                  | 5                     | 5.0 18.5 12.1          | 5                     | 4.4 36.9 21.9        |
|                  | 10                    | 7.4 14.6 10.6          | 10                    | 3.7 25.8 15.8        |
| 1.0              | 1                     | 9.4 79.9 49.4          | 1                     | 46.2 406.3 243.9     |
|                  | 5                     | 6.5 30.2 23.1          | 5                     | 19.9 182.6 111.5     |
|                  | 10                    | 7.9 26.1 17.7          | 10                    | 14.6 126.9 78.5      |
| Error std. dev. (K) | Nr. of future values | Estimate deviation (%) | Error std. dev. (K) | Nr. of future values | Estimate deviation (%) |
| 0.5              | 1                     | 3.3 27.8 17.1          | 1                     | 6.7 56.5 34.8        |
|                  | 5                     | 4.9 13.0 9.0           | 5                     | 3.3 26.0 15.3        |
|                  | 10                    | 6.5 10.9 8.6           | 10                    | 3.0 18.1 11.0        |
| 1.0              | 1                     | 6.6 56.2 34.3          | 1                     | 31.9 283.1 169.6     |
|                  | 5                     | 5.6 25.2 16.0          | 5                     | 14.7 129.2 78.4      |
|                  | 10                    | 7.6 18.6 13.0          | 10                    | 11.2 89.8 55.6       |
Although at the first glance this result might seem strange, but the averaging of measured samples in time delays the response which might increase the estimation error, especially if a problem is not severely ill-conditioned. Thus, a better solution to improve the estimate quality is to place additional, optimally located temperature sensors. Then, in comparison to the previous solution, the standard deviation of obtained estimates is always reduced.

Further conclusions concern the condition number of the matrix, whose pseudo-inverse is found during the estimation process. An interesting observation is that although the inclusion of additional remote sensors did not change the condition number, but it reduced visibly the estimation error. Thus, the simple condition number cannot be the only indicator in the optimisation of the circuit topology and rather the condition number per sensor or similar quantity should be used instead.

Finally, it should be mentioned that, in spite of their apparent uselessness, the results obtained for the free convection cooling are in a certain sense accurate. Namely, in all the cases the total power dissipated in all the heat sources was estimated accurately, only it was wrongly allocated among them. Therefore, the exact source power cannot be predicted but the estimated temperature, which is fairly uniform in the structure, is correct. Fortunately, electronic systems usually are equipped with some kind of cooling assemblies, thus real estimation problems should not be severely ill-conditioned.

5. Conclusions

The numerical simulations presented in this paper proved that the function specification algorithm could be an efficient tool for real time electronic circuit temperature estimation, even using remote temperature sensors. Beside the correct sensor positioning, further significant improvement of the estimate quality can be achieved by the combined space and time averaging.

Although in the experiments the analytical steady state solution of heat conduction equation was used, it does not affect the overall scope of the analyses concerning the applicability of the function specification algorithm. An additional advantage of the function specification algorithm is that for the calculation of the thermal influence coefficients different methods of thermal analysis can be applied, not necessarily the GF based ones.

The research presented in this paper allowed the authors the design of a dedicated integrated circuit comprising the sets of heat sources and temperatures sensors as well as the signal processing module with the digital filter implementing the function specification algorithm. When manufactured, this circuit will render possible the practical verification of the concepts presented in this paper.

Acknowledgments

This research has been supported by the Polish Ministry of Science and Higher Education grant No. N515 008 31/0331.

References

[1] Ozisik M N 1993 Heat Conduction (New York: Wiley)
[2] Incropera F P and De Witt D P 2002 Fundamentals of Heat and Mass Transfer (New York: Wiley)
[3] Janicki M and Napieralski A 2000 Modelling electronic circuit radiation cooling using analytical thermal model Microelectron. J. 31 781-5
[4] Beck J V, Cole K D, Haji-Sheikh A and Litkouhi B 1992 Heat Conduction Using Green’s Functions (New York: Hemisphere)
[5] Haji-Sheikh A and Beck J V 2000 An efficient method of computing eigenvalues in heat conduction Numer. Heat Tr. B 38 133-56
[6] Haji-Sheikh A and Beck J V 2002 Temperature solution in multi-dimensional multi-layer bodies Int. J. Heat Mass Tr 45 1865-77
[7] Gerstenmaier Y and Wachutka G 2001 Time dependent temperature fields calculated using eigenfunctions and eigenvalues of the heat conduction equation Microelectron. J. 32 801-8
[8] Gerstenmaier Y and Wachutka G 2005 Transient temperature fields with general nonlinear boundary conditions in electronic systems IEEE T. Compon. Pack. T. 28 23-33
[9] Janicki M, De Mey G and Napieralski A 2002 Application of Green’s functions for analysis of transient thermal states in electronic circuits Microelectron. J. 33 733-8
[10] Janicki M, De Mey G and Napieralski A 2002 Transient thermal analysis of multilayered structures using Green’s functions Microelectron. Reliab. 42 1059-64
[11] Janicki M, De Mey G and Napieralski A 2007 Thermal analysis of layered electronic circuits with Green’s functions Microelectron. J. 38 177-84
[12] Janicki M and Napieralski A 2002 Inverse heat conduction problems in electronic circuits Proc. 9th Int. Conf. Mixed Design of Integrated Circuits and Systems (Wroclaw, Poland, 20-22 June 2002) pp 385-8
[13] Janicki M, Zubert M and Napieralski A 1998 Application of inverse heat conduction methods in temperature monitoring of integrated circuits Sensor Actuat A 71 51-7
[14] Janicki M, Zubert M and Napieralski A 1999 Application of inverse problem algorithms for integrated circuit temperature estimation Microelectron. J. 30 1099-107
[15] Beck J V, Blackwell B and St. Clair C R Jr. 1985 Inverse Heat Conduction - Ill-posed Problems (New York: Wiley)