Abstract

We compute the production of gluons from Glasma color flux tubes. We calculate the probability distribution of gluon multiplicities arising from the distribution of color electric and color magnetic flux tubes found in the Glasma. We show that the result corresponds to the negative binomial probability distribution observed in experiments. The parameter $k$ that characterizes this distribution is proportional to the number of colors $N_c^2 - 1$ and to the number of flux tubes. For one gluon color and one flux tube, the multiplicity distribution is close to a Bose-Einstein distribution. We call this decay process “Glitter”, a term that is explained below.

Key words: glasma, multiplicity distribution

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1. Introduction

In high energy nuclear collisions, it has been argued that a Glasma is formed through the collision of two sheets of Color Glass Condensate (CGC) [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18] (for a review and additional references see [19, 20]). At very high energies, the gluon density per unit area per unit rapidity, $dN/dy d^2 r_\perp$, is of order $Q_s^2/\alpha_s$ in the CGC and Glasma. This density is so large that the interaction strength of QCD is weak, $\alpha_s \ll 1$. Indeed, in these relations, the scale of the coupling constant is set by the saturation momentum $Q_s$, and the saturation momentum grows with both increasing energy and size of the nucleus. The Glasma is formed during the time it takes two Lorentz contracted sheets of Colored Glass to pass through one another, $\tau \sim e^{-k/\alpha_s}/Q_s$ a time parametrically short compared to the natural time scale for decay of the flux tubes, $\tau_{\text{decay}} \sim 1/Q_s$. We shall not describe in detail the properties of either the Glasma or the CGC in this paper and refer the reader to the original literature for details.

In this paper, we compute the probability distribution for the multiplicity of gluons produced in the Glasma. We show that the distribution of gluons arising from such a decay is not a Poisson distribution as might be expected for the decay of an external source into particles in a weakly interacting theory. It turns out the distribution is a negative binomial distribution.

Recall that a Poisson distribution,

$$P_n^\text{Poisson} = \frac{1}{n!} \bar{n}^n e^{-\bar{n}}$$

is completely characterized by its mean value $\bar{n}$. In contrast, a negative binomial distribution is a 2-parameter distribution of the form

$$P_n^{\text{NB}} = \frac{\Gamma(k + n)}{\Gamma(k)\Gamma(n + 1)} \frac{\bar{n}^n k^k}{(\bar{n} + k)^{n+k}}.$$
This distribution has larger fluctuations than a Poisson distribution, but tends to a Poisson distribution if \( k \to +\infty \) at fixed \( \bar{n} \).

For the processes we consider, the decays will be into one coherent state associated with a gluon. For this reason, and because a negative binomial distribution does not fall as \( 1/n! \) at large \( n \), as does a Poisson distribution, we will refer to this property of the distribution as tenacious. The tenaciousness of this distribution results in an amplification of the intensity of multiply emitted gluons relative to that of a Poisson distribution. This means that there are larger fluctuations in high gluon multiplicity events than would be typical of a Poisson distribution. Thus, we shall use the acronym “Glitter” to describe radiation from these flux tubes, as an abbreviation for GLuon Intensification Through Tenacious Emission of Radiation.

In this paper, we shall compute the multiplicity distribution of gluons produced by Glasma flux tubes. We show that a single flux tube decays into gluons with a negative binomial distribution and is characterized by a parameter \( k_0 \) of order one. This implies that a decay of \( N_{\rho_T} \) flux tubes produces a negative binomial distribution characterized by \( k = N_{\rho_T}k_0 \). For \( k = 1 \), a negative binomial distribution is a Bose-Einstein distribution\(^1\), so single flux tube decays are close in form to Bose-Einstein distributions. In addition to the parameter \( k \), the negative binomial distribution is also parameterized by the average multiplicity \( \bar{n} \). The ratio \( \bar{n}/k = \bar{n}/N_{\rho_T}k_0 \) is approximately the multiplicity per flux tube.

Based on this interpretation of the decay of an ensemble of flux tubes we argue that the flux tubes are a “glittering” glasma. We will then discuss our results in the context of the extraction of negative binomial distribution from the UA(5) and PHENIX collaborations.

2. Calculation of the multiplicity distribution

The probability distribution of a discrete quantity is conveniently defined in terms of its generating function

\[
F(z) \equiv \sum_{n=0}^{\infty} z^n P_n, \quad (3)
\]

from which one can compute the moments of the multiplicity distribution as

\[
\langle n(n-1)\cdots(n-q+1) \rangle = \left. \frac{d^q F(z)}{dz^q} \right|_{z=1}. \quad (4)
\]

(The moments defined in this way are known as the factorial moments.) It was shown in Refs.\(^{21, 22, 23}\) that in the case of the central rapidity region of nucleus-nucleus collisions, when both nuclei can be described as strong color sources \( \rho \sim 1/g \), these moments can be computed as

\[
\langle n(n-1)\cdots(n-q+1) \rangle = \int [d\rho_1][d\rho_2]W_y[\rho_1]W_y[\rho_2](n[\rho_1, \rho_2])^q. \quad (5)
\]

This result is valid to leading log accuracy, i.e. it includes the leading order in \( \alpha_s \) with all the powers of \( \alpha_s \ln 1/x \) resummed into the rapidity dependence of the weight functionals \( W_y[\rho_1] \).

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\(^1\)A Bose-Einstein, or geometrical, distribution is a thermal distribution for single state systems. Its probability distribution reads:

\[
P_{\text{BE}}^{\bar{n}} = \frac{1}{1+\bar{n}} \left( \frac{\bar{n}}{1+\bar{n}} \right)^n.
\]
factor \( n[\rho_1, \rho_2] \) inside the integral in the r.h.s. of eq. (5) is the integrated multiplicity corresponding to a fixed configuration of color charge densities \( \rho_1 \) and \( \rho_2 \):

\[
n[\rho_1, \rho_2] = \int \frac{d^2p_\perp dy_p}{d^2p_\perp} \frac{dN}{d^2p_\perp dy_p},
\]

where \( dN/d^2p_\perp dy_p \) is obtained by Fourier transforming the classical gauge field radiated by the sources \( \rho_{1,2} \). In our leading log calculation the classical fields and thus also the single gluon multiplicity are boost invariant, so the integral over rapidity in eq. (6) is just a constant factor.

We are assuming that the rapidity interval is small enough compared to \( 1/\alpha_s \); otherwise there are additional large logarithms that must be resummed; this case is studied in detail in Ref. [23].

In the Glasma the single inclusive multiplicity is of order \( 1/\alpha_s \) and thus the moment defined in eq. (5) is of order \( (1/\alpha_s)^q \). We are computing the moments only to leading order in \( \alpha_s \), thus powers of \( n \) lower than \( q \) are negligible compared to the \( q \)’th moment. In particular \( \langle n(n-1)\cdots(n-q+1) \rangle \approx \langle n^q \rangle \) at this level of accuracy. This means that the leading (in \( \alpha_s \)) correlations in the multiplicity distribution come entirely from the average over the distribution of sources \( \rho_{1,2} \); i.e. the dominant behavior of the probability distribution comes from the large logarithms of the energy resummed into the \( W \)’s. The contributions to the probability distribution for fixed sources from higher loop orders that were studied in Ref. [24] are suppressed by powers of \( \alpha_s \) and thus contribute to our calculation only when they are enhanced by large logarithms.

The fundamental properties of the probability distribution are better reflected in the factorial cumulant:

\[
m_q \equiv \langle n(n-1)\cdots(n-q+1) \rangle - \text{disc.} = \frac{d^q \ln F(z)}{dz^q} \bigg|_{z=1},
\]

where “disc.” denotes the disconnected contributions that can be expressed in terms of the lower cumulants. We shall now turn to calculating these quantities of the multiplicity distribution in the

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Note that the factorial cumulant defined here differs from the conventional one in that we are defining the cumulant from \( \langle n(n-1)\cdots(n-q+1) \rangle \) instead of \( \langle n^q \rangle \). In terms of the generating function this is a question of differentiating w.r.t. \( z \) instead of \( \ln z \). As explained above the difference between the two is of higher order in \( \alpha_s \) than our calculation and we will not discuss it further.
Glasma at the lowest nontrivial order in the sources. This approximation is equivalent to assuming that the momenta of the produced gluons are all larger than the saturation scale. For concreteness one can take the distribution of the sources from the MV model:

$$W[\rho] = \mathcal{C} \exp \left[ -\int d^2 x_\perp \rho^a(x_\perp) \rho^a(x_\perp) \frac{g^4 \mu^2}{4} \right] . \quad (8)$$

Since the probability distribution essentially depends only on the combinatorics of pairwise source connections, our result applies equally well to a nonlocal Gaussian distribution that would more closely reproduce a solution of the BK equation [25, 4, 26].

The calculation proceeds in the same way as that of the second and third cumulants computed in Refs. [27, 28, 29], and we refer to these works for a more detailed description. Computing the probability distribution to all orders in the sources $\rho_{1,2}$ is in principle possible using the methods developed for the single inclusive gluon production [14, 15, 16, 17], but reliably computing the higher cumulants requires a significant numerical effort to gather enough statistics and is left for future work.

The fundamental building block in our calculation is the amplitude to produce one gluon with momentum $p$ from the fixed classical color charges $\rho_1(k_\perp)$ and $\rho_2(p_\perp - k_\perp)$. This amplitude reads

$$\frac{\rho_1(k_\perp) \rho_2(p_\perp - k_\perp)}{k^2_{\perp} L^\gamma(p, k_{\perp})} . \quad (9)$$

where we are not writing the color indices explicitly. Here, $L^\gamma(p, k_{\perp})$ denotes the effective Lipatov vertex. We do not need the explicit expression of its components, and it will be sufficient to know that it satisfies the following two properties

$$L^\gamma(p, k_{\perp}) = L^\gamma(p, p_\perp - k_{\perp}) ,$$

$$L^\gamma(p, k_{\perp}) L^\gamma(p, k_{\perp}) = -4 \frac{(p_\perp - k_{\perp})^2 k^2_{\perp}}{p^2_{\perp}} . \quad (10)$$
The diagrammatic notation for this amplitude is shown in fig. 1. To compute the $q'$th cumulant we need to take $2q$ factors of this basic building block ($q$ for the amplitude and $q$ for the complex conjugate) and perform the averages over the sources. Because the distribution of the sources in eq. (5) is Gaussian, we only need to keep track of contractions of pairs of sources $\rho_1$ and (separately) of pairs of sources $\rho_2$ and replace them by the correlator

$$\langle \rho(k_\perp)\rho(k'_\perp)\rangle = (2\pi)^2 \delta^2(k_\perp + k'_\perp)g^4\mu^2(k_\perp).$$  \hfill (11)$$

For the time being we shall leave an unspecified $k_\perp$-dependence in the correlation function $g^4\mu^2(k_\perp)$. In the MV model [1, 2, 3] $g^4\mu^2(k_\perp)$ is a constant, but JIMWLK or BK evolution can effectively lead to a different $k_\perp$-dependence [30]. With the simplified diagrammatic notation introduced in fig. 1 this combinatoric problem now corresponds to forming a connected contraction of the $2q$ boxes, each with two lines attached, illustrated in fig. 2.

We shall now show why the only contributing contractions are “rainbow” diagrams, where on one side (upper or lower) of the diagram the two boxes corresponding to the same momentum $p_r$ are contracted with each other, as in fig. 5. An example of a building block of a rainbow diagram is given in fig. 3. There are a total of four propagators with the same momentum $k_\perp$. Two of these are cancelled by the contractions of the four Lipatov vertices attached to the ends, leaving a quadratically infrared divergent contribution $\sim d^2k_\perp/k_\perp^4$ to the integral over $k_\perp$. These kinds of divergences are a sign that this contribution to the multiparticle correlation is sensitive to the whole correlated area in the transverse plane. They are regulated at the scale $Q_s$ (since $Q_s^{-1}$ is the correlation length between color charges in the transverse direction), giving a contribution of order $1/Q_s^2$. Compare this to the “non-rainbow” contribution depicted in fig. 4. Here there are only two propagators with the same momenta and the $k_\perp$-integral is convergent. Instead of $\mathcal{O}(1/Q_s^2)$, this yields a contribution $\sim 1/p_\perp^2$ or $\sim 1/p_\perp^2$ which shall be neglected here since we are assuming $p_\perp \gg Q_s$. Thus the contractions of the boxes in fig. 2 have to form a “rainbow diagram” (fig. 5) in either the upper or lower part of the diagram. There cannot be a rainbow on both sides since this would lead to a disconnected contribution.

Now that we have reduced the combinatoric problem to rainbow diagrams we can simplify our
diagrammatic notation even further, as shown in fig. 6. We can consider the two boxes connected by the line in the rainbow as a dimer with two ends. Now we must count the number of ways in which our \( q \) dimers can be combined to form a connected loop. The first end of the first dimer can be connected to \( 2q - 2 \) other loose dimer ends, let us assume it is connected to dimer \( r \). The second end of dimer \( r \) now has \( 2q - 4 \) loose ends available, because closing the loop with the first dimer would immediately give a disconnected contribution. Thus we arrive at \( (2q - 2) \cdot (2q - 4) \cdots 2 = 2^{q - 1}(q - 1)! \) combinations. Remembering that we can form the rainbow on either the upper or lower side of the diagram we finally arrive at \( 2^q(q - 1)! \) topologies. In the special cases of \( q = 2 \) and \( q = 3 \) this gives 4 and 16, which were the number of diagrams evaluated in [27] and [29].

The color structure can be simplified in the following way. Let us denote the color indices of the sources \( i = 1 \ldots q \) contracted on the upper side of the diagram in fig. 5 by \( a_i \) (because of the rainbow structure of the connections on the upper side, we do not need separate indices for the sources in the complex conjugate amplitude), the color index of the gluon with \( p_i \) by \( b_i \) and the color indices on the lower side of the diagram \( c_i \) (source connected to gluon \( p_i \) in the amplitude) and \( c'_i \) (in the complex conjugate). Now the color structure of the upper side of the rainbow and the boxes is \( f^{a_1 b_1 c_1} f^{a_1 b_1 c'_1} \ldots f^{a_q b_q c_q} f^{a_q b_q c'_q} = (C_A)^q \delta^{c_1 c'_1} \ldots \delta^{c_q c'_q} \). \( C_A = N_c \) is the Casimir of the adjoint representation. The \( c_i, c'_i \) indices must now be contracted pairwise into a single connected loop as in fig. 7, which yields a factor \( \text{tr}(1_{\text{adj}}) = N_c^2 - 1 \), making the total color factor \( (N_c)^q(N_c^2 - 1) \).

Let us then turn to the structure of momentum flow in the diagram. Transverse momentum is conserved at the vertices and in the sources connections due to the expectation value eq. (11). Altogether there are originally \( 4q \) transverse momentum integrals from the powers of the sources. There are \( 2q \) delta functions from the source correlators eq. (11) and \( 2q \) momentum conservation delta functions from the three gluon vertices. Not all these delta functions are independent: two of them end up having the same argument, yielding one factor of the transverse area (denoted \( S_\perp \)). Therefore, there is only one remaining transverse momentum integral. One can choose this remaining momentum to be the one circulating in all of the lower part of the diagram, which we shall denote by \( k_\perp \) (this is the momentum that circulates along the loop in fig. 7). On the rainbow side of the diagram there is a squared propagator \( 1/(k_\perp - p_{\perp i})^4 \) for all the sources \( i = 1 \ldots q \). On the non-rainbow side the transverse momentum in all the propagators is the same, giving a factor \( 1/k_\perp^{4q} \). Half of these propagators are cancelled by the squares of the Lipatov vertices, which also contribute an inverse square of the external momentum. Combining the combinatorial factors from the source averages, the propagators, Lipatov vertices and factors from the invariant measure, we
\[
\left\langle \frac{dN}{dy_1 d^2 p_{\perp} \ldots dy_q d^2 p_{q\perp}} \right\rangle_{\text{conn.}} = \left[ 2^q(q - 1)! \right] \frac{(N_c)^q(N_c^2 - 1) S_{\perp} 1 \cdot 2^q}{(p_{\perp 1})^2 \ldots (p_{q\perp})^2} \frac{g^q}{(2\pi)^3 q} 
\times \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{g^4 \mu^2(k_{\perp})}{k_{\perp}^2} \frac{g^4 \mu^2(p_{\perp 1} - k_{\perp})}{(p_{\perp 1} - k_{\perp})^2} \ldots \frac{g^4 \mu^2(p_{q\perp} - k_{\perp})}{(p_{q\perp} - k_{\perp})^2}. \tag{12}
\]

This general formula also reproduces the result of refs. [12, 31, 32] for the single inclusive spectrum case \( q = 1 \); in this case the combinatorial factor in the square bracket must be taken to be 1 instead of 2 to avoid double counting the only contributing diagram.

The weak source result eq. (12) is infrared divergent in the MV model \((g^4 \mu^2(k_{\perp})\) constant). Physically this is modified by several effects. Even in the weak field limit BK or BFKL evolution leads to an anomalous dimension \(0 < \gamma < 1\) that changes the behavior into \(g^4 \mu^2(k_{\perp}) \sim k_{\perp}^{2(1-\gamma)}\) in the geometric scaling region \(k_{\perp} \gtrsim Q_s\). Deep in the saturation regime it has been argued that the correlator effectively behaves as \(g^4 \mu^2(k_{\perp}) \sim k_{\perp}^{2}\). Ultimately the infrared behavior of the multigluon spectrum is regulated by the nonlinear interactions that are not included in our present computation. This is seen explicitly and analytically in the “pA” case \([33, 34, 35]\) and in numerical computations of the glasma fields in the fully nonlinear case \([14, 13, 16, 17]\). Since the full nonlinear dynamics are known to regulate the infrared behavior in the case of the single gluon spectrum we have strong reasons to expect that they will also do so in the case of multiple gluon production; at the same scale \(k_{\perp} \lesssim Q_s\). We emphasize that an essential point in this argument is that the quantity appearing in eq. (12) is not a single color charge correlator divided by a large power \(k_{\perp}^{2q}\), but the same correlator \(g^4 \mu^2(k_{\perp})/k_{\perp}^{2}\) that appears in the single inclusive gluon spectrum raised to a large power \(q\).

The effect of saturation on the multigluon spectrum at \(k_{\perp} \lesssim Q_s\) has a very intuitive interpretation in the the glasma flux tube picture. The size of the flux tube, \(1/Q_s\), is the correlation length of the system and we should not have contributions from longer distance scales. We effectively take this into account by regulating all the infrared divergences at the scale \(Q_s\), and thus approximating the integral in eq. (12) by \(\left(2\pi\kappa^{q-1}Q_s^{2(q-2)}\right)^{-1}\). Here \(\kappa\) is a constant of order one that depends on the details of how the infrared divergences are regulated at the scale \(Q_s\). Since \(Q_s\) is the typical momentum of the produced gluons, not a lower limit, we expect that numerically \(\kappa < 1\). In our analytical calculation we do not have access to the exact value of this coefficient. We also use the corresponding approximation for the single inclusive spectrum:

\[
\left\langle \frac{dN}{dy d^2 p_{\perp}} \right\rangle \approx \frac{N_c(N_c^2 - 1)}{4\pi^4 g^2} \frac{S_{\perp}(g^2 \mu)}{p_{\perp}^4}. \tag{13}
\]

\(^3\)In comparing this to the result in \([27]\) (eq. (27)) note that there is an erroneous factor of \(1/2\) in eq. (9) of \([27]\) which propagates into an additional factor \(2^{-2q}\). In addition, there is an overall error of \((2\pi)^2\). These errors were present also in Ref. \([29]\), but have been corrected in the published version.

\(^4\)We are neglecting an additional logarithmic dependence in \(p_{\perp}\).
We can now express our result as

\[
\langle \frac{dN}{dy_1 d^2 p_{\perp 1} \cdots dy_q d^2 p_{\perp q}} \rangle_{\text{conn.}} = (q-1)! \frac{(N_c^2 - 1) \kappa Q_s^2 S_\perp}{2\pi} \left( \frac{(g^2 \mu)^4}{g^2} \frac{1}{2\pi^3} \kappa Q_s^2 \right)^q \frac{1}{(p_{\perp 1})^4 \cdots (p_{\perp q})^4} \]

If we integrate this equation over the rapidities and transverse momenta of the \(q\) gluons, again consistently regulating all the infrared divergences at the scale \(Q_s\), we obtain our result for the factorial cumulant as

\[
m_q = (q-1)! k \left( \frac{\bar{n}}{k} \right)^q ,
\]

with

\[
k = \kappa \frac{(N_c^2 - 1) Q_s^2 S_\perp}{2\pi}.
\]

The exact constant factors, encoded in the coefficient \(\kappa\), depend on the exact way the infrared divergences (logarithmic for the single inclusive, power law for the multigluon correlations) are regulated. These factors cannot be obtained exactly in an analytic calculation to the lowest order in the sources. However, the main parametric dependences in the relevant variables \(\alpha_s, Q_s, S_\perp\) and \(N_c\) can be expected to be the same to all orders in the sources. A possible additional (mild) \(q\)-dependence in \(\kappa\) would be a minor correction to the behavior of the probability distribution, mostly determined by the combinatorial factor \((q-1)!\).

Equations (15) and (16) are the main result of this paper. One can see that these factorial cumulants (15) are those that define the negative binomial distribution. It arises very naturally in the Glasma based on the Gaussian combinatorics of the classical sources and the assumption of the fluctuations in the system being dominated by a correlation length \(1/Q_s\).

3. Glittering Glasma: Interpretation of the result

A negative binomial distribution is characterized by two parameters, the mean \(\bar{n}\) and \(k\), in terms of which the probability to produce \(n\) particles is

\[
P_n^{\text{NB}} = \frac{\Gamma(k+n)}{\Gamma(k) \Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}.
\]

The distribution is characterized by the generating function

\[
F_{k,\bar{n}}(z) = \sum_{n=0}^{\infty} z^n P_n = \left( 1 - \frac{\bar{n}}{k} (z-1) \right)^{-k}.
\]

The moments of the distribution can be obtained from the generating function by differentiating with respect to \(z\). The connected parts of the moments, or cumulants, are generated by the logarithm of the generating function. The factorial cumulants (7) of the negative binomial distribution are given by

\[
m_q \equiv \frac{d^q}{dz^q} \ln F_{k,\bar{n}}(z) \bigg|_{z=1} = (q-1)! k \left( \frac{\bar{n}}{k} \right)^q .
\]
In contrast, for a Poisson distribution \( m_1 = \bar{n} \) and \( m_q = 0 \) for \( q > 1 \). This quantity is the expectation value
\[
m_q = \langle n(n-1)\cdots(n-q+1) \rangle - \text{disc.},
\]
where the disconnected part “disc” can be expressed in terms of the lower order cumulants. The convention that the expectation value in (20) is taken of the product \( n(n-1)\cdots(n-p+1) \) and not of \( n^p \) means that we are subtracting the “Poissonian” part from the moment, which is why all the factorial cumulants of a Poisson distribution are zero for \( q \geq 2 \). Note that the Poissonian part is suppressed by powers of \( \alpha_s \) in the Glasma; thus in practice the difference between \( m_p \) and a conventional cumulant is neglected in our analysis.

Two common special cases of the negative binomial are the Poisson distribution, obtained in the limit \( k \to \infty \) at fixed \( \bar{n} \), and the geometrical–or Bose-Einstein–distribution obtained when \( k = 1 \). The negative binomial distribution is wider than a Poisson distribution typically associated with independent emission of particles; this can be seen e.g. from the variance
\[
\sigma^2 = \langle n^2 \rangle - \langle n \rangle^2 = \bar{n} + \frac{\bar{n}^2}{k}.
\]

A useful property of the negative binomial distribution with parameters \( \bar{n}, k \) is that it is also the distribution of a sum of \( k \) independent random variables drawn from a Bose-Einstein distribution with mean \( \bar{n}/k \). This is easily seen from the generating function in eq. (18), remembering that the generating function of a sum of independent random variables is the product of their generating functions. This has a consequence that an incoherent superposition of \( N \) emitters that have a negative binomial distribution with parameters \( k_0, \bar{n} \) produces a negative binomial distribution with parameters \( N k_0, N \bar{n} \).

A natural physical interpretation of our result can be given in terms of emission from independent glasma flux tubes. Geometrically, the transverse area \( S_\perp \) is filled with \( Q_s^2 S_\perp \) independent flux tubes of size \( \sim 1/Q_s^2 \). Each of these tubes emits gluons in \( N_c^2 - 1 \) different colors. Our result shows that the probability distribution of gluons of one color emitted from one flux tube is approximately a Bose Einstein distribution. We do not see an interpretation of this result as a thermal process, however. It seems more likely that the distribution is one that maximizes the entropy (which is the defining property of the BE distribution) because there is a large number of color sources that emit gluons. In some sense the role of a heat bath (large reservoir of energy) in thermodynamics is played by the large number of color charges resummed into the effective color current of the CGC.

It is a known experimental observation [36, 37, 38, 39] (see also [40] for an extensive review) that multiplicities of charged particles in high energy scattering are well described as a negative binomial distribution. Also multiplicity fluctuations at RHIC have been found to agree with the negative binomial distribution by the PHENIX collaboration [41, 42]. Experimentally, the parameter \( k \) increases somewhat with \( \delta \eta \), the size of the rapidity interval in which the particles are measured. The dependence is, however, very slow for large \( \delta \eta \), pointing to the presence of a long range correlation in the system [43]. This is natural in the Glasma picture, since flux tubes extend over large rapidity intervals. The number of flux tubes, which gives the parameter \( k \) of the negative

\[ ^5 \text{Explicitly, consider } n = n_1 + \cdots + n_r \text{ where the } n_i \text{'s are independent of each other. The probability distribution of } n \text{ is then } P_n = \sum_{n_1} \cdots \sum_{n_r} \delta (n - \sum_{i=1}^r n_i) P_{n_1} \cdots P_{n_r} \text{ and the generating function } \sum_n z^n P_n = \sum_{n_1} \cdots \sum_{n_r} z^{n_1+\cdots+n_r} P_{n_1} \cdots P_{n_r}, \text{ which is the product of the individual generating functions for the variables } n_i. \]
binomial distribution, essentially depends only on the transverse area of the projectiles and on the saturation momentum.

The main difficulty in interpreting the experimental results arises from the geometrical fluctuations from averaging over different impact parameters in one finite centrality bin. To minimize this effect one should use as small centrality bins as possible. Comparison with a different method of analysis used by the STAR collaboration \cite{44} could be very useful in disentangling these effects. To the extent that this uncertainty allows us to compare results in gold-gold and $p\bar{p}$, the picture we present seems fairly consistent. For a fixed collision energy we would expect scaling $k \sim Q_s^2 S_\perp \sim N_{\text{part}}$. While keeping this caveat in mind, the results from UA5 \cite{39} and E735 \cite{45} ($k \approx 2 \ldots 4$, $N_{\text{part}} = 2$) and PHENIX $k \approx 350$ for 0-5% most central ($N_{\text{part}} \approx 350$) collisions \cite{41} or $k = 690$ when extrapolated to a zero centrality bin width \cite{42}, seem very consistent with this estimate.

From the experimental fit of the parameter $k$ in central gold-gold collisions by PHENIX \cite{42} and the value $Q_s \approx 1.1$ GeV estimated from measurements of the charged multiplicity, one can use eq. (16) to obtain an estimate $\kappa \approx 0.2$ for the parameter that reflects our uncertainty in the infrared sector. As we have discussed, it is natural to expect a numerical value of slightly less than 1 for $\kappa$. This, however, means that at RHIC energies the flux tube size, as measured in the multiplicity distribution, is not yet very clearly separated from the confinement scale. At LHC energies we can expect this separation to be clearer.

For increasing collision energy we would expect $Q_s$ and therefore $k$ to increase. The energies where the UA5 measurements are done are still in the transition region from a behavior of $k$ decreasing with energy from lower $\sqrt{s}$, but we would expect $k$ at the LHC to be clearly larger. This decreasing behavior at low energy follows because of the Poisson nature of low energy particle emission, and that for a Poisson distribution $k \to \infty$.

The negative binomial has been interpreted as resulting from a partial stimulated emission or cascade process \cite{43}. It has been known in the literature \cite{46,47} that the distribution would naturally arise from a superposition of subsystems with Bose-Einstein distributions. Nevertheless, a popular approach has remained to interpret the observations in terms of a fluctuating number of strings \cite{48}, each producing particles typically with a Poisson distribution \cite{49,50} (see also \cite{51,52,53} for a more pQCD based approach). While the picture of flux tubes in the glasma has many similarities to ideas in string model phenomenology, the distribution of particles produced from one flux tube is different. The probability distribution of gluons from a glasma flux tube is not a narrow Poissonian, but has very large fluctuations: the glittering of the glasma.

4. Summary

The Glasma provides a successful phenomenology of a particle production in high energy hadronic collisions. There is now experimental data on the ridge phenomena that show flux tube structures in two particle correlations \cite{54,55,56}. In addition, long range correlations of remarkable strength are seen in heavy ion collisions \cite{44}.

The Glitter of the flux tube decay may provide a strong tool for disentangling various descriptions of the flux tubes, since it naturally leads to a negative binomial distribution for the multiplicity of produced particles. However, in order to make a more convincing case for the origin of the negative binomial distribution of particle multiplicities, we need a systematic study of bin size effects on the extraction of the parameter $k$ from various centralities of heavy ion collisions.
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