Natural convection flow of Sodium Alginate based Casson nanofluid about a solid sphere in the presence of a magnetic field with constant surface heat flux

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Abstract. In this paper, we examined a free convection flow of Sodium Alginate (SA) as a host Casson fluid with three different types of nanoparticles specifically, Silicon Dioxide (SiO₂), Copper oxide (CuO), and Copper (Cu) on a solid sphere in the presence of magnetic field along with prescribed surface heat flux. The Keller-box method was carried out for solving the transformed governing partial differential equations. Numerical results for the local skin friction coefficient are obtained and compared with literature. Also, the influences of Casson parameter, magnetic parameter, nanoparticles volume fraction, on local skin friction coefficient, local Nusselt number, temperature, and velocity are analyzed graphically. Our study revealed that the local Nusselt number, local skin friction coefficient and velocity profiles of SiO₂ - SA based Casson nanofluid are higher than the other nanoparticles - SA based Casson nanofluid, as well as it has the lowest temperature profiles.

1. Introduction

Casson fluid is a subclass of non-Newtonian fluid which behaves like an elastic solid where no flow occurs with small shear stress. It was first introduced by Casson [1] in 1959 to predict the flow behavior for pigment-oil suspensions of the printing ink. Later on, numerous articles have been done on the same field. Mustafa et al. [2, 3] studied the flow of a Casson fluid over a semi-infinite flat plate with a parallel free stream and in the region of stagnation point towards a stretching sheet. Nadeema et al. [4] investigated the boundary layer three-dimensional flow of Casson fluid over a stretching sheet. Several studies on Casson fluids can also be found in these references [5-11]. Sodium alginate is one of the most fluids that has ever been used in many industries for over a century since it was discovered by the British
pharmacist Stanford [12, 13] in 1881, where it is used in food manufacturing, pharmaceuticals, textiles and cosmetics. Sodium alginate has received much attention by researchers, Hatami and Ganji [14, 15] investigated the heat transfer of Sodium alginate nanofluid flow in the porous area between two coaxial cylinders and between two vertical flat plates. For more reading see the following references [16-18].

Recently, a new class of fluids called nanofluids was introduced to improve the thermal properties of conventional base fluids by adding nanoparticles to ordinary base fluids. The concept of nanofluids was reported by Chol and Estman [19] where he introduced the idea of nanoparticles suspended in a host fluid. Eastman et al. [20] found that the addition of copper (10 nm) particles in ethylene glycol increases the thermal conductivity up to 40%. Chon et al. [21] examined the effect of temperature and nanoparticle size on thermal conductivity of nanofluid. Here are some significant researches on nanofluids [22-30].

MHD natural convection on a sphere has received considerable attention over recent decades and the significance of this issue due to the wide range applications in numerous modern industrials, where a number of papers have been published in this field, Huang and Chen [31] investigated the impact of Prandtl number and surface mass transfer on a steady, laminar of natural convective flow about a solid sphere with constant surface heat flux. Kumari et al. [32] examined MHD flow of an electrically conducting fluid in the stagnation region of a sphere. Nazar et al. [33] studied numerically the natural convection flow past a sphere with prescribed surface heat flux in a micropolar fluid. Chamkha and Al-Mudhaf [34] analyzed the influence of the magnetic field and thermal radiation on free convection from a permeable sphere. Alkasasbeh [35] studied the natural convection flow of Casson micropolar fluid over a solid sphere in the presence of a magnetic field. Also, [36-39] all of them investigated the impact of heat generation and magnetic field on free convection over a sphere.

In this paper, we examined MHD free convection flow of Sodium alginate nanofluid on a solid sphere with constant surface heat flux by using the Keller box method which is neither published nor considered in the scientific literature.

2. Mathematical modeling
A steady state two-dimensional laminar MHD free convective flow of Copper (Cu), Copper oxide (CuO), and Silicon Dioxide (SiO$_2$) - Sodium Alginate based Casson nanofluid from a solid sphere of the radius $a$ and prescribed surface heat flux $q_w$ are considered. Figure 1 presents the Physical model and coordinate system, where $\mathbf{g}$ is the gravity vector, $T_\infty$ is the ambient temperature of the nanofluid, $\beta_i^d$ is the magnetic field strength, $x^*$ - coordinate is measured along the surface of the solid sphere at the lower stagnation point ($x^* \approx 0$), $y^*$ - coordinate is measured the distance normal to the surface of the sphere and $r^*(x^*)$ is the radial distance from the symmetrical axis to the surface of the solid sphere.

![Figure 1. Physical model and coordinate system.](image-url)
The Casson fluid flow is reported by Ahmed and Khan [17]:

\[ \tau_{ij} = \begin{cases} 
2(\mu_p + \rho_p \sqrt{2\pi}) e_{ij} & \pi > \pi_c, \\
2(\mu_p + \rho_p \sqrt{2\pi}) e_{ij} & \pi < \pi_c,
\end{cases} \tag{1} \]

where \( \pi = e_{ij} e_{ij} \), \( e_{ij} \) is the \((i, j)\) -th component of the deformation rate, \( \mu_p \) is the plastic dynamic viscosity of the non-Newtonian fluid, \( \rho_p \) is the yield stress of the fluid and \( \pi_c \) is a critical value of this product.

The continuity, momentum and energy equations of the steady incompressible Casson nanofluid boundary layer flow can be expressed as:

\[ \frac{\partial}{\partial x} \left( r^* u^* \right) + \frac{\partial}{\partial y} \left( r^* v^* \right) = 0, \tag{2} \]

\[ u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} = \frac{\mu_{sf}}{\rho_{sf}} \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u^*}{\partial y^2} + \left( \chi \rho_f \beta_f + (1 - \chi) \rho_f \beta_f \right) g (T - T_n) \sin \left( \frac{x}{a} \right) - \frac{\sigma_{sf} \beta_0^2}{\rho_{sf}} u^*, \tag{3} \]

\[ u^* \frac{\partial T}{\partial x} + v^* \frac{\partial T}{\partial y} = \alpha_{sf} \frac{\partial^2 T}{\partial y^2}, \tag{4} \]

subject to the boundary conditions defined by Molla et al. [37]:

\[ u^* = v^* = 0, \quad \frac{\partial T}{\partial y} = -\frac{q_v}{k}, \quad \text{as} \quad y^* = 0, \tag{5} \]

\[ u^* \rightarrow 0, \quad T \rightarrow T_n, \quad \text{as} \quad y^* \rightarrow \infty, \]

where \((u^*, v^*)\) are the velocity components along the \((x^*, y^*)\) directions, \( \beta \) is the Casson parameter, \( T \) is the temperature, \( \chi \), \( \beta_s \), and \( \rho_s \) are volume fraction, thermal expansion coefficient, and the density of nanoparticles, respectively. \( \sigma_f, \rho_f, \mu_f, \nu_f \) and \( \beta_f \) are electrical conductivity, density, viscosity, kinematic viscosity and the thermal expansion coefficient of the host fluid, respectively. \( \sigma_{sf}, \alpha_{sf}, \rho_{sf}, (\rho_{sf})_n, \mu_{sf} \) and \( k_{sf} \) are electrical conductivity, thermal diffusivity, density, viscosity, heat capacity and the thermal conductivity of the nanofluid, respectively which defined by Zangooee et al. [40] :

\[ \frac{\sigma_{sf}}{\sigma_f} = 1 + \frac{3(\sigma - 1)\chi}{(\sigma + 2) - (\sigma - 1)\chi}, \quad \sigma = \frac{\sigma_s}{\sigma_f}, \quad k_{sf} = \frac{k_s + k_f}{k_s + 2k_f}, \quad \frac{\mu_{sf}}{\mu_f} = \frac{(1 - \chi)^{\tau}}, \]

\[ \left( \rho_{sf} \right)_n = (1 - \chi) \left( \rho_{sf} \right)_f + \chi \left( \rho_{sf} \right)_k, \quad \left( \rho_{sf} \right)_n = (1 - \chi) \rho_f + \chi \rho_s, \quad \alpha_{sf} = \frac{k_{sf}}{\left( \rho_{sf} \right)_n}, \tag{6} \]

here \( k_s \) is thermal conductivity of nanoparticles and \( k_f \) is thermal conductivity of host fluid.

Now, we will introduce the following non-dimensional variables [37]:

\[ x = \frac{x}{\bar{a}}, \quad y = Gr^{\nu/2} \left( \frac{y}{a} \right), \quad r(x) = \frac{r^*}{a}, \quad u = \left( \frac{a}{v_f} \right) Gr^{2/3} u^*, \]
\[ \theta = Gr^{1/5} \left( \frac{T - T_e}{aq_w / k} \right), \quad v = \left( \frac{a}{v_f} \right) Gr^{-1/5} v' \]

(7)

where \( Gr = g \beta_f (aq_w / k) a^3 \) is the Grashof number and \( r^* \left( x^* \right) \) is the radial distance from the symmetrical axis to the surface of the solid sphere and is given by:

\[ r^* \left( x^* \right) = a \sin \left( \frac{x^*}{a} \right) \]

(8)

Substitute equations (7) and (8) into equations (2) - (5) to obtain the following non-dimensional equations:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]

(9)

\[ u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left( 1 - \chi \right) \frac{\partial^2 \theta}{\partial y^2} \]

(10)

subject to the boundary conditions:

\[ u = v = 0, \quad \frac{\partial \theta}{\partial y} = -1 \; \text{as} \; y = 0, \]

\[ u \to 0, \; \theta \to 0 \; \text{as} \; y \to \infty \]

(12)

here \( M = \frac{\sigma_f \beta_f^2 a^2 Gr^{-2/5}}{v_f \rho_f} \) is the magnetic parameter and \( Pr = \frac{\alpha}{v_f} \), is the Prandtl number.

To solve equations (9)-(11) along with the boundary conditions (12), defined the following variables:

\[ \psi = xr(x)F(x,y), \quad \theta = \theta(x,y), \]

(13)

which satisfies equation (9). Thus, equations (10) and (11) can be expressed as follows:

\[ \frac{\rho_f}{\left( 1 - \chi \right)^{1/5} \rho_{nf}} \left( 1 + \frac{1}{\beta} \right) \frac{\partial^3 F}{\partial y^3} + \left( 1 + x \cot x \right) F \frac{\partial^2 F}{\partial y^2} - \left( \frac{\partial F}{\partial y} \right)^2 - \frac{\sigma_{nf} \rho_{nf} M}{\sigma_f \rho_f} \frac{\partial^2 F}{\partial y^2} \]

\[ + \left( \frac{\chi \rho_f \beta_f / \beta_f + (1 - \chi) \rho_{nf}}{\rho_{nf}} \right) \frac{\partial^3 \theta}{\partial x \partial y} = \frac{1}{Pr} \left( 1 - \chi \right) \frac{\partial^2 \theta}{\partial y^2} + \left( 1 + x \cot x \right) F \frac{\partial \theta}{\partial y} \]

(15)

\[ = x \left( \frac{\partial F}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial F \partial \theta}{\partial x} \right) \]

(16)
subject to
\[ \frac{\partial F}{\partial y} = 0, \quad F = 0, \quad \frac{\partial \theta}{\partial y} = -1 \text{ as } y = 0, \]
\[ \frac{\partial F}{\partial y} \to 0, \quad \theta \to 0, \text{ as } y \to \infty \] (17)

At the lower stagnation point of the sphere \((x = 0)\), equations (15) and (16) reduce to the following ordinary differential equations:
\[ \frac{\rho_f}{(1 - \chi)^2 \rho_	ext{ef}} \left[ 1 + \frac{1}{\beta} \right] F'' + 2 F' \left( \frac{\partial F}{\partial y} \right)^2 - \frac{\sigma_	ext{ef} \rho_f}{\sigma_f \rho_	ext{ef}} M \left( \frac{\chi \rho_f (\beta_1 / \beta_1) + (1 - \chi) \rho_f}{\rho_	ext{ef}} \right) \theta = 0 \] (18)
\[ \frac{1}{\Pr \left( (1 - \chi)(\rho C_p)_f + \chi (\rho C_p)_f (1/(\rho C_p)_f) \right)} \theta'' + 2 F \theta' = 0 \] (19)

and the boundary conditions (17) become:
\[ F'(0, y) = 0, \quad F(0, y) = 0, \quad \theta'(0) = -1 \quad \text{as} \quad y = 0, \]
\[ F' \to 0, \quad \theta \to 0, \quad \text{as} \quad y \to \infty \] (20)

In this article we interest in two physical quantities specifically, the local skin friction coefficient \(C_f\) and the local Nusselt number \(Nu\), which are given by Molla et al. [37]:
\[ C_f = aT^2 \tau_{uw}, \quad Nu = \frac{aq_	ext{w}}{k_f (T - T_w)} \] (21)

where
\[ \tau_{uw} = \mu_	ext{ef} \left( \frac{\partial u}{\partial y} \right)_{y' = 0}, \quad q_	ext{w} = -k_	ext{ef} \left( \frac{\partial T}{\partial y} \right)_{y' = 0}. \] (22)

Use the non-dimensional variables (7) and boundary conditions (17), the local skin friction coefficient \(C_f\) and Nusselt number \(Nu\) are:
\[ C_f^* = \frac{1}{(1 - \chi)^2} \left( 1 + \frac{1}{\beta} \right) \chi \frac{\partial^2 F}{\partial y^2}(x, 0), \quad Nu^* = \frac{k_f}{k_f} \left( \frac{1}{\theta(x, 0)} \right) \] (23)

here \(C_f^* = Gr^{1/5} C_f\), and \(Nu^* = Gr^{1/5} Nu\).

3. Numerical method
The nonlinear differential equations (15) and (16) along with boundary conditions (17) have been computed numerically by utilizing the Keller box technique. It is unconditionally stable, has second-order accuracy and the results are achieved in a reasonable time, this method can be summarized as the following: First, we reduce the transformed equations (15) and (16) to a first-order system. Next, we write the difference equations using central differences. After that, we linearize the resulting algebraic equations by Newton’s method and write them in matrix-vector form. Lastly, we solve the linear system by the block tridiagonal elimination technique. For more details of this technique see the following refs. [41-43].
4. Numerical results and discussion

Table 1 presents the thermo-physical properties of nanoparticles and Sodium Alginate, whereas table 2 shows a numerical comparison of local skin friction coefficient \( C_f \) with Newtonian fluid.

**Table 1.** Thermo-physical properties of SiO\(_2\), CuO, Cu and SA as a host Casson fluid [44-48].

| Thermo-Physical property | SA   | CuO  | Cu   | SiO\(_2\) |
|--------------------------|------|------|------|-----------|
| \( \rho \text{ (kg/m}^3\) | 989  | 6500 | 8933 | 2220      |
| \( C_p \text{ (J/kgK)} \) | 4175 | 540  | 385  | 745       |
| \( K \text{ (w/mK)} \)   | 0.6376 | 18   | 401  | 1.38      |
| \( \beta \times 10^{-4} \text{ (K}^{-1}\) | 99   | 0.85 | 1.67 | 0.055     |
| \( \sigma \text{ (s/m)} \) | \(2.6 \times 10^{-4}\) | \(59.5 \times 10^6\) | \(59.6 \times 10^6\) | \(10^{21}\) |
| \( \text{Pr} \)           | 6.45 | -    | -    | -         |

**Table 2.** Comparison of local skin friction coefficient \( C_f \) with Newtonian fluid \((\beta \to \infty, M = 0, \chi = 0)\) at \( \text{Pr} = 0.7 \).

| \( \chi \) | Huang and Chen [31] | Nazar et al.[33] | Present   |
|-------------|---------------------|-------------------|-----------|
| 0°          | 0.0000              | 0.0000            | 0.000000  |
| 10°         | 0.2138              | 0.2138            | 0.212301  |
| 20°         | 0.4247              | 0.4246            | 0.415758  |
| 30°         | 0.6299              | 0.6297            | 0.625222  |
| 40°         | 0.8265              | 0.8260            | 0.820074  |
| 50°         | 1.0118              | 1.0110            | 1.00337   |
| 60°         | 1.1828              | 1.1815            | 1.16766   |
| 70°         | 1.3376              | 1.3356            | 1.31984   |
| 80°         | 1.4708              | 1.4678            | 1.45194   |
| 90°         | 1.5818              | 1.5780            | 1.56094   |
| 100°        | -                   | 1.6614            | 1.64355   |
| 110°        | -                   | 1.7140            | 1.69601   |
| 120°        | -                   | 1.7314            | 1.71361   |

Figure 2(a and b) depicts the effect of \( \chi \) on \( C_f^* \) and \( Nu^* \) respectively, it is found that by increasing \( \chi \), both \( C_f^* \) and \( Nu^* \) increase, this dependence of \( Nu^* \) and \( C_f^* \) on \( \chi \) can be additionally clarified by looking at equation (23). figure 3(a and b) illustrates the impact of \( \beta \) on \( C_f^* \) and \( Nu^* \) respectively, we found that an increase in Casson parameter causes a decrement in the local skin friction coefficient and a rise in the local Nusselt number, but when \( x \) increase, the local skin friction coefficient is increasing and
the local Nusselt number is decreasing. Figure 4(a and b) shows the influence of $M$ on $C_f^*$ and $Nu^*$ respectively, it’s clear from this figure that an increase in the intensity of the magnetic field led to reducing the skin friction coefficient, this is because the application of a magnetic field leads to curb fluid flow hence reducing the surface friction force, also an increase in $M$ led to reducing $Nu^*$. Moreover, figures 2-4 reveal that the rate of heat transfer of Cu-SA is the lowest as compared with other nanoparticles SA, and SiO$_2$ - SA produced the highest skin friction followed by CuO – SA while Cu – SA produced
lowest skin friction. Figure 5(a and b) presents the impact of $\chi$ on velocity and temperature profiles respectively,

![Figure 5](image1.png)

**Figure 5.** Impact of $\chi$ on velocity and temperature profiles.

it is obvious that as $\chi$ increases, the velocity profiles increases. Also, the temperature profiles increase when $\chi$ increases and this is due to the reason that an increase in nanoparticle volume fraction leads to increases in thermal Conductivity of nanofluid which increases nanofluid's temperature. Figure 6 (a and b) displays the influence of $\beta$ on velocity and temperature respectively, it provides that an increase in $\beta$ parameter contributes to the reduction of velocity temperature profiles. Figure 7(a and b) states impact of $M$ on

![Figure 6](image2.png)

**Figure 6.** Impact of $\beta$ on velocity and temperature profiles.

![Figure 7](image3.png)

**Figure 7.** Impact of $M$ on velocity and temperature profiles.
on velocity and temperature profiles, here we notice that an increase in $M$ leads to a decrease in velocity due to increasing dragging force, while exactly opposite behavior was observed with temperature where the increase in $M$ lead to reduce it. Further, figures 5-7 are shown that the motion of SiO$_2$-SA is the highest as compared with other nanoparticles - SA, and SiO$_2$- SA produced the lowest temperature followed by CuO –SA while Cu – SA produced the highest temperature.

5. Conclusion
MHD 2-D incompressible natural convection flow of Sodium Alginate nanofluid past a solid sphere was numerically analyzed in this article. Constant surface heat flux was also considered. Numerical solutions are obtained by the Keller box method, and graphical results are gained through MATLAB software. Results of local skin friction coefficient are compared with prior published results, and very good accuracy is accomplished with those results. According to the current study, the following can be inferred:

i. $C_s'$ is directly proportional to $\chi$ but it is inversely proportional to both $\beta$ and $M$, as well as SiO$_2$- SA produced the highest local skin friction.

ii. $Nu$ is directly proportional to both $\chi$ and $\beta$ but it is inversely proportional to $M$, as well as SiO$_2$- SA has the highest local Nusselt number.

iii. An increment in $\chi$ is offset by an increment in velocity, whereas an increment in $\beta$ or $M$ is offset by a decrement in velocity

iv. An increment in $\chi$ or $M$ is offset by an increment in temperature, whereas an increment in $\beta$ is offset by a decrement in temperature.

v. The velocity of SiO$_2$-SA is the highest and it produced the lowest temperature.

Acknowledgment
The authors would like to acknowledge Postgraduate Management Center, University Malaysia Terengganu (UMT) for the financial support through vote numbers 62958 for this research.

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