The influence of the aperture diaphragm on the size of the subjective speckles and the application of this in speckle photography

M N Osipov¹, R N Sergeev¹,²

¹Samara National Research University, Moskovskoe Shosse 34A, Samara, Russia, 443086
²JSC SRC Progress, Zemetsa street 18, Samara, Russia, 443009

e-mail: osipov7@yandex.ru, romansr@yandex.ru

Abstract. The paper presents the results of a study showing the possibility of reducing the size of speckles to increase the sensitivity and accuracy of the measuring system based on the use of methods of speckle photography and speckle interferometry. The theoretical and experimental results show that the average size of speckles formed in the double-exposure specklegram depends on the aperture diaphragms of the optical system – circular aperture, annular aperture, square aperture and frame aperture – are presented. The experimental results forming of the diffraction halo and the Young's fringes for a circular, annular, square and frame apertures are presented.

1. Introduction
Speckle interferometry, due to the low demands on the stability of the optical system in comparison with holographic interferometry, is becoming widespread for use in industrial environments. These experimental methods have high metrological characteristics, provide the ability to measure the deformed state on the entire surface of the study object, and have a high level of automation [1–4].

The sensitivity of speckle interferometry to the determination of the value of the measured displacement depends on the size of the speckle structure, which is determined by the parameters of the optical system used by recording the subjective speckle structure, i.e. numerical aperture of the optical system. An increase in the numerical aperture of the optical system leads to a decrease in the size of the detected speckle structure and, consequently, to an increase in the sensitivity of the speckle interferometry. However, on the other hand, at increase of the numerical aperture of the optical system the aberrations of the optical system begin to have a significant effect, which lead to a distortion of the recorded information that leads to the requirement of using high-quality optics [1, 5–8].

In the processing double-exposure speckle photography by the Young's method the sensitivity of the speckle photography is determined by the size of the diffraction halo. The size of the diffraction halo is determined by the average size of speckles forming a double-exposure specklegram. The maximum measured Young's fringe pattern bandwidth is determined by the size of the diffraction halo, which determines the sensitivity, that is, the minimum value of the measured displacement. The average size of speckles formed in the double-exposure specklegram depends on the aperture
diaphragm of the optical system. Reducing the average size of the speckle leads to an increase in the angular size of the diffraction halo and hence to an increase in sensitivity [1-4].

Theoretical and experimental studies of the principles of operation of optical devices have shown that one of the ways to reduce the average size of the speckle is amplitude and/or phase apodization of the optical system [9-20].

The paper presents the results of a study showing the possibility of reducing the size of speckles by amplitude apodization of the optical system to increase the sensitivity and accuracy of the measuring system based on methods of speckle photography and speckle interferometry. The theoretical and experimental results the average size of speckles formed in the double-exposure specklegram depends on the aperture diaphragms of the optical system – circular aperture, annular aperture, square aperture and frame aperture – are presented. The experimental results forming of the diffraction halo and the Young's fringes for a circular, annular, square and frame apertures are presented.

2. Theoretical and experimental modelling of the behaviour of a single speckle depending on the type of the aperture diaphragms

A speckle pattern observed and recorded at some distance from a diffusely scattering surface is called objective. A speckle pattern formed from a diffusely scattering surface in the image plane of an optical system formed is called the subjective. The formation of a subjective speckle pattern is presented in Figure 1.

![Coherent illumination](image)

**Figure 1.** Registration scheme of subjective speckles

The average speckle size, which is formed in the image plane, depends on the aperture diaphragm of the optical system and is similar to the intensity distribution observed in the Fraunhofer area at diffraction on the same aperture diaphragm.

Let's consider the formation of the diffraction pattern in the Fraunhofer area for circular, annular, square and frame aperture diaphragms which are presented in Figures 2 [1, 9, 21-23].

The intensity distribution of the diffraction pattern for the circular aperture diaphragm describes the Bessel function of the first order:

\[
I(x, y) = I_0(x, y) \left[ \frac{2 J_1(k a w)}{k a w} \right]^2,
\]

where \( k = \frac{2 \pi \lambda}{a} \), (\( \lambda \) is the wavelength of the laser radiation); \( w \) is the sine of the angle between the direction in which the intensity is determined in the image plane and the optical axis; \( a \) is the radius of the circular aperture diaphragm of the optical system (Figure 2a).

In an optical system with annular aperture diaphragm, bounded by two concentric circles with radius \( a \) and \( \varepsilon a \), the diffraction pattern is described by the following expression:

\[
I(x, y) = I_0(x, y) \left[ \frac{2 J_1(k a w)}{k a w} - \varepsilon \left[ \frac{2 J_1(k \varepsilon a w)}{k \varepsilon a w} \right] \right]^2, \quad 0 < \varepsilon < 1
\]

In this case, the secondary maxima have a greater intensity compared to the intensity for a circular aperture diaphragm. The value of the minimum and maximum radii in the intensity distribution of the diffraction field decreases at compare to the circular aperture diaphragm (Figure3).
Figure 2. Types of investigated aperture diaphragms and their characteristic dimensions: a) Circular diaphragm; b) Annular diaphragm; c) Square diaphragm d) Frame diaphragm

In a square aperture diaphragm optical system with sides $2a$, $2b$ the diffraction pattern is described by the following expression:

$$I(x, y) = I_0(x, y) \left( \frac{\sin \frac{kax}{kby}}{\frac{kax}{kby}} \right)^2$$

(3)

In the case of a frame aperture diaphragm, with the sides $2a$, $2b$ and $2\varepsilon a$, $2\varepsilon b$, the diffraction pattern based on the Babinet principle is described by the following expression:

$$I(x, y) = I_0(x, y) \left( \frac{\sin \frac{kax}{kby}}{\frac{kax}{kby}} - \varepsilon^2 \frac{\sin \frac{kex}{kby}}{\frac{kex}{kby}} \right)^2$$

(4)

In this case, there is also the value of the minima and maxima radii in the intensity distribution of the diffraction field decreases at compare to the square aperture diaphragm (Figure 4).

The speckle size can be defined as the width of the central maximum of the function in the expressions (1) – (4). This width is taken as the characteristic transverse size of an individual single speckle in a speckle pattern in the image plane of an optical system.

Analysis of expressions (1) and (2) shows that when using the annular aperture diaphragm, the diffraction field in the plane of the photographic plate is described by the difference of Bessel functions of the first-order $J_1(x)/x$, at the usual circular aperture diaphragm the diffraction field in the plane of the photographic plate is described only by the Bessel function of first-order $J_0(x)/x$. The size of the diffraction field depends on the size of the annulus. Table 1 shows the calculated values of the first three minima of the equations (1) and (2) as a function of $\varepsilon$.

From the analysis of expressions (3) and (4), it follows that when using the frame aperture diaphragm, the diffraction field in the plane of the photographic plate is described by the difference functions $\sin (x)/x$, at the square aperture diaphragm the diffraction field in the plane of the photographic plate is described only by the function $\sin (x)/x$. The size of the diffraction field depends on the size of the square or frame. Table 2 shows the calculated values of the first three minima of the equations (3) and (4) as a function of $\varepsilon$. 

Figure 3. The functions of the normalized intensity distribution for the circular and annular aperture diaphragms.

Figure 4. Graph of the functions of the normalized intensity distribution for the square and frame aperture diaphragms.
Table 1. The numerical values of the first three minima of the diffraction field for the circular and annular aperture.

| ε     | (kεaw)_1 | (kεaw)_2 | (kεaw)_3 |
|-------|-----------|-----------|-----------|
| 0.0   | 3.83      | 7.02      | 10.17     |
| 0.7   | 2.80      | 6.49      | 10.08     |
| 0.804 | 2.68      | 6.1       | 9.61      |
| 0.874 | 2.57      | 5.88      | 9.25      |
| 0.90  | 2.53      | 5.81      | 9.09      |

Table 2. The numerical values of the first three minima of the diffraction field for the square and frame aperture diaphragms.

| ε     | (kεaw)_1 | (kεaw)_2 | (kεaw)_3 |
|-------|-----------|-----------|-----------|
| 0.0   | 3.19      | 6.15      | 9.48      |
| 0.7   | 2.35      | 5.74      | 9.34      |
| 0.804 | 2.24      | 5.41      | 8.84      |
| 0.874 | 2.16      | 5.25      | 8.51      |
| 0.9   | 2.18      | 5.30      | 8.56      |

As follows from Table 1 and Table 2, with increasing ε, the position value of the minima decreases.

Figure 5 presents the results of numerical modeling of the spatial intensity distribution for diffraction field on a circular a), annular b), square c) and d) frame aperture diaphragms.

Figure 5. Spatial intensity distribution in the Fraunhofer area: a) circular; b) annular; c) square; d) frame apertures.

Figure 6 presents the results of numerical simulation of a typical diffraction pattern observed on the screen in the Fraunhofer area at diffraction on the circular (a), annular (b), square (c) and d) frame aperture diaphragms.

From Table 1, Table 2 and Figures 5, 6 it can be seen that, at the annular and frame aperture diaphragms, the width of the central maximums of a diffraction pattern is narrower at compare to the circular and square aperture diaphragms and, consequently, the characteristic transverse size of an individual single speckle decreases.
Figure 7 show photographs of experimentally obtained diffraction patterns in the Fraunhofer area, depending from the circular, annular square and frame aperture diaphragms and the parameter $\varepsilon$.

![Figure 7](image1)

![Figure 7](image2)

![Figure 7](image3)

![Figure 7](image4)

![Figure 7](image5)

![Figure 7](image6)

![Figure 7](image7)

![Figure 7](image8)

**Figure 7.** Experimental diffraction patterns. Circular aperture - a). Annular aperture: $\varepsilon = 0.7$ b); $\varepsilon = 0.87$ c); $\varepsilon = 0.9$ d). Square aperture - e). Frame aperture: $\varepsilon = 0.7$ f); $\varepsilon = 0.87$ g); $\varepsilon = 0.9$ h).

The experimental results are presented in Figure 7 also show that the width of the central maximum of a diffraction pattern at the annular and frame aperture diaphragms is narrower at compare to the circular and square aperture diaphragms. These experimental results and results are presented in article [24] confirmed the theoretical studies that the increase of the parameter $\varepsilon$ leads to a decrease in the width of the central maximum of the diffraction pattern.

Table 3 and Table 4 show the experimental values of the first three minima of the diffraction pattern for the circular, annular square and frame aperture diaphragms as a function of $\varepsilon$.

**Table 3.** Experimental values of the first three minima of the diffraction pattern for the circular and annular aperture diaphragms.

| $\varepsilon$ | $(k\varepsilon a w)_1$ | $(k\varepsilon a w)_2$ | $(k\varepsilon a w)_3$ |
|--------------|-----------------|-----------------|-----------------|
| 0.0          | 3.80            | 7.04            | 9.97            |
| 0.7          | 2.86            | 6.43            | 9.90            |
| 0.804        | 2.51            | 6.01            | 9.43            |
| 0.874        | 2.43            | 5.67            | 9.05            |
| 0.9          | 2.35            | 5.60            | 8.69            |

**Table 4.** Experimental values of the first three minima of the diffraction pattern for the square and frame aperture diaphragms.

| $\varepsilon$ | $(k\varepsilon a w)_1$ | $(k\varepsilon a w)_2$ | $(k\varepsilon a w)_3$ |
|--------------|-----------------|-----------------|-----------------|
| 0.0          | 3.4             | 6.7             | 9.5             |
| 0.7          | 2.4             | 5.56            | 9.11            |
| 0.804        | 2.24            | 5.14            | 8.59            |
| 0.874        | 2.07            | 4.93            | 8.23            |
| 0.9          | 2.11            | 4.85            | 8.14            |

In experiments the laser with a wavelength of 0.532 μm was used. The characteristic size $a$ of the aperture diaphragm was 3 mm. The experimental results are in good agreement with the theoretical data. Thus, when using the annular or frame aperture diaphragms, the width of the central maximum is narrower and, consequently, the characteristic transverse size of an individual single speckle
decreases at compare to the circular and square aperture diaphragms according to equations (2), (4), and will depend on \( \varepsilon \).

3. Experimental results the application of the aperture diaphragms in double-exposure speckle photography

This section presents the results of the study of the influence of the shape and size of the aperture diaphragms in double-exposure speckle photography. The study object was a disk that rotated about its center. In experiments the laser with a wavelength of 0.532 \( \mu \text{m} \) was used. The double-exposure specklegram were recorded on a photographic plate by the method of two exposures. During recording of the double-exposure spectrogram, a single lens with a diameter of 150 mm was used. A number of characteristic sizes of aperture diaphragms were 30, 50, 80 and 100 mm. The value of the parameter \( \varepsilon \) varied 0, 0.8, 0.9. The double-exposure spectrogram processing was carried out by Young’s method, as shown in Figure 8.

![Figure 8. Processing specklegram by Young’s method.](image)

Figure 9 shows photographs of the Young’s fringes for a circular and annular aperture in the upper row, as well as for a square and frame aperture in the lower row. The parameter \( \varepsilon = 0.9 \) for the annulus and frame aperture diaphragms.

![Figure 9. Pictures of Young's fringes: for apertures of circular / annular sizes a) 30 mm; b) 100 mm; for square / frame apertures with dimensions of c) 30 mm; d) 100 mm.](image)

As you can see from the photos (Figures 9a and 9b) and according to theoretical studies, an increase of the size of the annular aperture diaphragm and the parameter \( \varepsilon \) leads to a decrease of the size of the subjective specks and consequently to an increase of the size of the diffraction halo. As is well known, the size of the diffraction halo in double-exposure speckle photography determines its sensitivity [1-3, 5, 6]. In addition, the use of the thus, the experimental results fully confirmed the theoretical studies and showed that the use of circular apertures can improve the sensitivity and accuracy of measurements by speckle photography leads to an increase in the quality of the Young’s fringe pattern – uniformity and contrast that allows to increase the accuracy of measurements. For
large dimensions of the entrance aperture of the optical system in the absence of an annular aperture diaphragm (Figures 9a and 9b), Young’s fringe pattern have a nonlinear shape and become poorly distinguishable in the diffraction halo. At increase of the numerical aperture of the optical system the aberrations of the optical system begin to have a significant effect, which lead to a distortion of the recorded information.

As you can see from the photos (Figures 9c and 9d) and according to theoretical studies, an increase of the size of the frame aperture diaphragm and the parameter $\varepsilon$ also leads to a decrease of the size of the subjective specks and consequently to an increase of the size of the diffraction halo. However, in this case there is a distortion of Young’s fringe pattern due to the presence of aberrations of the optical system.

Thus, the experimental results fully confirmed the theoretical studies and showed that the use of annular aperture diaphragm can to increase the sensitivity and accuracy of measurements by speckle photography.

On the other hand, experimental results have shown that it is not possible to use frame aperture diaphragm.

### 4. Conclusion

The use of the annular aperture diaphragm in the speckle photography, as shown above, leads to a decrease of the average size of speckles forming in double-exposure specklegram.

The experimental and theoretical studies show that when using the annular aperture diaphragm with $\varepsilon = 0.9$, the average size of speckles is reduced by 1.5 times compared with a circular aperture, and this leads to an increase of the diffraction halo in Young’s method. Consequently this leads to increase the sensitivity of measurements by speckle photography.

It should also be noted that the use of the annular aperture diaphragm also decrease the requirements for the quality of optics, since it allows reducing the influence of aberrations on the image formation process and, consequently, increasing the resolution of the optical system according to Rayleigh's criterion. Figure 10 presents the intensity distribution in the focal plane of the optical system for circular and annular aperture diaphragms.

![Figure 10. Rayleigh criterion: a) circular aperture; b) annular aperture – $\varepsilon = 0.9$.](image)

However, as can be seen in figure 10, apodization allows not only to reduce the size of the central maximums of a diffraction pattern, but also leads to the appearance more intense of side maxima, which worsen the imaging properties [1, 9, 21-23]. To compensate for this negative factor, complex apodization functions are used [25-26].

In further studies it is of interest to use optical systems with a complex apodization function in speckle photography.

### 5. References

[1] Klimenko I S 1985 *Holography of focused images and speckle interferometry* (Moscow: Science Press) p 224

[2] Jones R and Wykes C 1986 *Holographic and speckle interferometry* (Moscow: Mir Press) p 328

[3] Sirohi R S 2009 *Optical Methods of Measurement: whole field techniques* (New York: CRC Press) p 290
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