Maximal isospin few-body systems of nucleons and $\Xi$ hyperons

H. Garcilazo,1,∗ A. Valcarce,2,† and J. Vijande3,‡

1Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, 07738 México D.F., Mexico
2Departamento de Física Fundamental and IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain
3Departamento de Física Atómica, Molecular y Nuclear, Universidad de Valencia (UV) and IFIC (UV-CSIC), E-46100 Valencia, Spain

(Dated: May 10, 2018)

Abstract

By using local central Yukawa-type interactions that reproduce the low-energy parameters of the latest updates of the Nijmegen ESC08c potentials we show that the $NN\Xi$, $N\Xi N$, $N\Xi\Xi$ and $NN\Xi\Xi$ systems with maximal isospin are bound. Since in these states the strong decay $N\Xi \to \Lambda\Lambda$ is forbidden by isospin conservation, these strange few-body systems will be stable under the strong interaction. These results may suggest that other states with different number of $N$’s and $\Xi$’s in the maximal isospin channel could also be bound.

PACS numbers: 21.45.-v,25.10.+s,11.80.Jy
Keywords: baryon-baryon interactions, Faddeev equations

∗Electronic address: humberto@esfm.ipn.mx
†Electronic address: valcarce@usal.es
‡Electronic address: javier.vijande@uv.es
Introduction.— Although Λ hypernuclei formed by one or two Λ’s bound in nuclei have been studied for a long time [1, 2] this is not the case for Ξ hypernuclei. Only recently, as a result of the so-called KISO event [3], the binding energy of a Ξ− and 14N was determined to be 4.38 ± 0.25 MeV. The lack of experimental data on Ξ hypernuclei can be traced back to the fact that Ξ–nucleus bound states once formed will immediately decay due to the process $N\Xi \rightarrow \Lambda\Lambda$, as shown by the KISO event, where the reaction $\Xi^- + ^{14}\text{N} \rightarrow {}^{10}_\Lambda\Lambda + {}^5_\Lambda\text{He}$ was observed. In this paper, we address the study of bound states of nucleons and Ξ’s in the maximal isospin channel, i.e., systems consisting only of neutrons and negative Ξ’s or protons and neutral Ξ’s. The uniqueness of these systems is a consequence of the two-body interactions between $NN$, $N\Xi$ and $\Xi\Xi$ pairs being all in the isospin 1 channel. Thus, the strong decay $N\Xi \rightarrow \Lambda\Lambda$ is forbidden. Therefore, such states, if bound, would be stable under the strong interaction.

Two-body interactions.— For the identical pairs, $NN$ and $\Xi\Xi$, the $S$-wave interaction is in the $^1S_0$ channel due to the Pauli principle, while for the $N\Xi$ pair both the $^1S_0$ and $^3S_1$ channels contribute. As is well-known, the $NN$ $^1S_0$ channel is almost bound, with the virtual state lying slightly below the $NN$ threshold in the unphysical sheet. Regarding the two-body interactions containing a single Ξ, a recent update of the Nijmegen ESC08c potential giving account of the pivotal results of strangeness $-2$ physics, the KISO [3] and the NAGARA [4] events, was recently released. The $N\Xi$ $^3S_1$ interaction has a bound state, the so-called $D^*$ with a binding energy of 1.67 MeV while the $N\Xi$ $^1S_0$ interaction is mainly repulsive [5, 6]. Finally, with respect to the strangeness $-4$ sector, the most recent update [7] shows a $\Xi\Xi$ $^1S_0$ attractive interaction, although unbound. Note that in earlier versions of the Nijmegen ESC08c potential [8] this channel had a bound state.

Observations like those reported in Ref. [3] are interesting. However, in this case microscopic calculations are impossible and, consequently, their interpretation will be always afflicted by large uncertainties. Meanwhile the scarce experimental information gives rise to ample room for speculation. The present theoretical investigation of the possible existence of few-baryon bound states based on potential models simulating the experimental data are basic tools to advance in the knowledge of the details of the $\Xi N$ and $\Xi\Xi$ interactions. First, it could help to raise the awareness of the experimentalist that it is worthwhile to investigate few-baryon systems, specifically because for some quantum numbers such states could be stable. Second, it makes clear that strong and attractive $\Xi N$ interactions, like those sug-
gested by the ESC08c Nijmegen model, have consequences for the few-body sector and can be easily tested against future data.

Recent preliminary results from lattice QCD suggest an overall attractive ΞN interaction [9]. Besides the recent update of the ESC08c Nijmegen model [5, 6], there are other models predicting bound states in the ΞN system previously to the KISO event, such as the chiral constituent quark model of Ref. [10]. The possible existence of stable few-body states containing a ΞN two-body subsystem is also suggested by the attractive character of the ΞΞ interactions for some partial waves [7, 8, 11–15]. Recent results of the HAL QCD Collaboration about the ΞΞ interaction [16] suggest that the interaction in the $^{1}S_0$ partial wave is presumably not as strong as suggested by the Nijmegen potential. There are also preliminary studies of the ΞΞN system [17] indicating that lattice QCD calculations of multibaryon systems are now within sight. However, one should keep in mind that there are other models for the ΞN interaction, like the hybrid quark–model based analysis of Ref. [18], the effective field theory approach of Ref. [19], or even some of the earlier models of the Nijmegen group [8] that do not present ΞN bound states and, in general, the interactions are weakly attractive or repulsive. Thus, one does not expect that these models will give rise to bound states containing a ΞN subsystem. On the other hand, current Ξ hypernuclei studies [20–22] have been performed by means of ΞN interactions derived from the Nijmegen models and thus our study complements such previous works for the simplest systems that could be studied.

Following Malfliet and Tjon [23] we take all the two-body interactions to consist of local central Yukawa-type potentials containing an attractive and a repulsive term, i.e.,

$$V(r) = -Ae^{-\mu_A r} + Be^{-\mu_B r}. \quad (1)$$

In the case of the NN $^{1}S_0$ channel we use the Malfliet-Tjon model with the parameters given in Ref. [24]. If it is assumed that only singlet and triplet S-wave contribute in the two-particle channel, the parametrization used in this work, then set III for the triplet partial wave and set I for the singlet partial wave, gives a triton binding energy of 8.3 MeV [23]. The effect of the repulsive core on the singlet two-body channel is crucial to get this results, while the repulsion on the triplet two-body channel has almost no effect on the binding. In fact, if the repulsive core in the singlet partial wave is not considered the triton becomes overbound (see Table II of Ref. [25]). Based on predictions for separable potentials, Ref. [23] suggests
TABLE I: Low-energy parameters of the most recent updates of the ESC08c Nijmegen interactions for the $N\Xi$ [5, 6] and $\Xi\Xi$ [7] systems, and the parameters of the corresponding local potentials given by Eq. (1).

| System | Channel | $a$(fm) | $r_0$(fm) | $A$(MeV fm) | $\mu_A$(fm$^{-1}$) | $B$(MeV fm) | $\mu_B$(fm$^{-1}$) |
|--------|---------|---------|-----------|-------------|-----------------|-------------|-------------------|
| $N\Xi$ | $^1S_0$ | 0.579   | $-2.521$  | 290         | 3.05            | 155         | 1.60              |
| $N\Xi$ | $^3S_1$ | 4.911   | 0.527     | 568         | 4.56            | 425         | 6.73              |
| $\Xi\Xi$ | $^1S_0$ | $-7.25$ | 2.00      | 155         | 1.75            | 490         | 5.60              |

that the inclusion of the tensor force in the triplet interaction increases the triton binding energy by 0.3 MeV. Indeed, this is the result obtained in Ref. [26], where as can be seen in Table III of that work, a five-channel calculation (S and D partial waves) differs from a two channel calculation (only S partial waves) by about 0.3 MeV. The influence of local tensor forces in Malfliet-Tjon Yukawa type interactions has also been studied in Ref. [27], showing that the inclusion of tensor forces reduces the binding energy of the three-body problem by 1 to 1.5 MeV, depending on the D-wave percentage. Thus, the local Yukawa-type potentials with tensor interaction would give underbinding in the three-body problem at difference of separable potentials [28]. Let us finally mention that it has been demonstrated that separable potentials can also reproduce the two-body Malfliet-Tjon $T$-matrix, agreeing with the three-body binding energy to an accuracy of 2% [29]. The parameters of the $N\Xi$ and $\Xi\Xi$ channels were obtained by fitting the low-energy data of each channel as given in the most recent update of the strangeness $-2$ [5, 6] and strangeness $-4$ [7] ESC08c Nijmegen potentials. Because we will not consider explicitly the coupling to higher mass channels, $\Lambda \Sigma$ and $\Sigma \Sigma$, we may loose some binding. Thus, we do not expect that our parametrization of the two-body interactions would overestimate the binding energy of the three- and four-body systems. The low-energy data and the parameters of these models are given in Table I.

Results and discussion.— We have obtained the binding energy of the three-body systems $NN\Xi$ and $N\Xi\Xi$ by solving the Faddeev equations with the formalism described in Ref. [30] for the case of three fermions when two of them are identical. The binding energy of the $NN\Xi\Xi$ system has been derived by using a variational method with generalized Gaussians detailed in Refs. [31, 32].

We show in Table II the binding energies of the lightest four systems: $N\Xi$, $NN\Xi$, $N\Xi\Xi$.
TABLE II: Binding energies, $B$, of the lightest four few-body systems with maximal isospin, $I$.

| System       | $(I)J^P$ | $B$ (MeV) |
|--------------|----------|-----------|
| $N\Xi$       | (1)1$^+$ | 1.67      |
| $NN\Xi$      | (3$^2$)1$^+$ | 3.00      |
| $N\Xi\Xi$    | (3$^2$)1$^+$ | 4.52      |
| $NN\Xi\Xi$   | (2)0$^+$ | 7.4       |

and $NN\Xi\Xi$, with maximal isospin. As one can see from this table, the binding energy of the maximal isospin few-body systems tends to increase with the number of particles. However, the increase is not as pronounced as in strangeless nuclei, due to the effect of the repulsive $N\Xi$ $^1S_0$ channel as compared to the attractive $NN$ $^1S_0$ channel. The results shown in Table II suggest that other maximal isospin systems involving a larger number of nucleons and $\Xi$’s might also be bound.

Acknowledgments

This work has been partially funded by COFAA-IPN (México), by the Ministerio de Educación y Ciencia and EU FEDER under Contracts No. FPA2013-47443 and FPA2015-69714-REDT, by Junta de Castilla y León under Contract No. SA041U16, and by Generalitat Valenciana PrometeoII/2014/066. A.V. is thankful for financial support from the Programa Propio XIII of the University of Salamanca.

[1] A. Gal, E. V. Hungerford, and D. J. Millener, [arXiv:1605.00557]
[2] J. -M. Richard, Q. Wang, and Q. Zhao, Phys. Rev. C 91, 014003 (2015).
[3] K. Nakazawa et al., Prog. Theor. Exp. Phys. (2015) 033D02.
[4] H. Takahashi et al., Phys. Rev. Lett. 87, 212502 (2001).
[5] M. M. Nagels, Th. A. Rijken, and Y. Yamamoto, [arXiv:1504.02634]
[6] Th. A. Rijken and H. -F. Schulze, Eur. Phys. J. A 52, 21 (2016).
[7] Th. A. Rijken, M. M. Nagels, and Y. Yamamoto, Few-Body Syst. 54, 801 (2013).
[8] V. G. J. Stoks and T. A. Rijken, Phys. Rev. C 59, 3009 (1999).
[9] K. Sasaki, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue, N. Ishii, and K. Murano, (HAL QCD Collaboration), Prog. Theor. Exp. Phys. (2015) 113B01.

[10] T. F. Caramés and A. Valcarce, Phys. Rev. C 85, 045202 (2012).

[11] S. R. Beane, E. Chang, W. Detmold, H. W. Lin, T. C. Luu, K. Orginos, A. Parreño, M. J. Savage, A. Torok, and A. Walker-Loud (NPLQCD Collaboration), Phys. Rev. D 85, 054511 (2012).

[12] G. A. Miller, Chin. J. Phys. 51, 466 (2013).

[13] J. Haidenbauer and U.-G. Meissner, Phys. Lett. B 684, 275 (2010).

[14] J. Haidenbauer, Ulf. -G. Meissner, and S.Petschauer, Eur. Phys. J. A 51, 17 (2015).

[15] M. M. Nagels, Th. A. Rijken, and Y. Yamamoto, arXiv:1501.06636.

[16] K. Sasaki (HAL QCD Collaboration), PoS LATTICE2013, 233 (2014).

[17] S. R. Beane, W. Detmold, T. C. Luu, K. Orginos, A. Parreño, M. J. Savage, A. Torok, and A. Walker-Loud (NPLQCD Collaboration), Phys. Rev. D 80, 074501 (2009).

[18] Y. Fujiwara, Y. Suzuki, and C. Nakamoto, Prog. Part. Nucl. Phys. 58, 439 (2007).

[19] J. Haidenbauer, Ulf. -G. Meissner, and S. Petschauer, arXiv:1511.05859.

[20] Y. Yamamoto, E. Hiyama, and Th. A. Rijken, EPJ Web of Conferences 3, 07007 (2010).

[21] E. Hiyama, Y. Yamamoto, T. Motoba, Th. A. Rijken, and M. Kamimura, Phys. Rev. C 78, 054316 (2008).

[22] M. Yamaguchi, K. Tominaga, Y. Yamamoto, and T. Ueda, Prog. Theor. Phys. 105, 627 (2001).

[23] R. A. Malfliet and J. A. Tjon, Nucl. Phys. A 127, 161 (1969).

[24] J. L. Friar, B. F. Gibson, G. Berthold, W. Glöckle, Th. Cornelius, H. Witala, J. Haidenbauer, Y. Koike, G. L. Payne, J. A. Tjon, and W. M. Kloet, Phys. Rev. C 42, 1838 (1990).

[25] R. A. Malfliet and J. A. Tjon, Ann. of Phys. 61, 425 (1970).

[26] Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, and H. Nemura, Phys. Rev. C 66, 021001(R) (2002).

[27] R. A. Malfliet and J. A. Tjon, Phys. Lett. 30B, 293 (1969).

[28] A. C. Phillips, Nucl. Phys. A 107, 209 (1968).

[29] E. Harms and J. S. Levinger, Phys. Lett. 30B, 449 (1969).

[30] H. Garcilazo and A. Valcarce, Phys. Rev. C 93, 034001 (2016).

[31] Y. Suzuki and K. Varga, Lect. Not. Phys. M54, 1 (1998).
[32] J. Vijande and A. Valcarce, Symmetry 1, 155 (2009).