Non-Markovian Quantum Dynamics and the Method of Correlated Projection Super-Operators

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1 Introduction

Relaxation and decoherence processes are key features of the dynamics of open quantum systems [1]. In the standard approach one tries to develop appropriate master equations for the open system’s reduced density matrix \( \rho_S \) which is given by the partial trace taken over the environmental variables coupled to the open system. Invoking the weak-coupling assumption one can formulate in many cases of physical interest a Markovian quantum master equation for the reduced density matrix, expressing the dynamical laws for the irreversible motion of the open system.

However, the theoretical description of quantum mechanical relaxation and decoherence processes often leads to a non-Markovian dynamics which is determined by pronounced memory effects. Strong system–environment couplings [2, 3], correlations and entanglement in the initial state [4, 5], interactions with environments at low temperatures and with spin baths [6], finite reservoirs [7, 8], and transport processes in nano-structures [9] can lead to long memory times and to a failure of the Markovian approximation.

Here, we will review the most important features of a systematic approach to non-Markovian quantum dynamics which is known as projection operator technique [10–13]. This technique is based on the introduction of a certain projection super-operator \( P \) which acts on the states of the total system. The super-operator \( P \) expresses in a formal mathematical way the idea of the elimination of degrees of freedom from the complete description of the states of the total system. Namely, if \( \rho \) is the full density matrix of the composite system, the projection \( P\rho \) serves to represent a certain approximation of \( \rho \) which leads to a simplified effective description of the dynamics through a reduced set of relevant variables.

With the help of the projection operator techniques one derives closed dynamic equations for the relevant variables \( P\rho \). We will discuss two different approximation schemes. The first one is based on the Nakajima–Zwanzig equation [10, 11] which

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Breuer, H.-P.: Non-Markovian Quantum Dynamics and the Method of Correlated Projection Super-Operators. Lect. Notes Phys. 787, 125–139 (2010)
DOI 10.1007/978-3-642-02871-7_5 © Springer-Verlag Berlin Heidelberg 2010
represents an integro-differential equation for $\mathcal{P}\rho$ with a certain memory kernel. The second scheme employs a time-convolutionless master equation for $\mathcal{P}\rho$, i.e., a time-local differential equation with a time-dependent generator [14–19]. These equations are used as starting point for the derivation of effective master equations through a systematic perturbation expansion. In the standard approach to the dynamics of open systems one chooses a projection super-operator which is defined by the expression $\mathcal{P}\rho = \rho_S \otimes \rho_0$, where $\rho_0$ is some fixed environmental state. A super-operator of this form projects the total state $\rho$ onto a tensor product state, i.e., onto a state without any statistical correlations between system and environment. Many examples for this product-state projection are known in the fields of quantum optics, decoherence, quantum Brownian motion, quantum measurement theory, and coherent and optimal quantum control. It is typically applicable in the case of weak system–environment couplings. The corresponding perturbation expansion is usually restricted to the second order (known as Born approximation), from which one derives, with the help of certain further assumptions, a Markovian quantum master equations in Lindblad form [20–22].

A possible approach to large deviations from Markovian behavior consists in carrying out the perturbation expansion to higher orders in the system–environment coupling. However, this approach is often limited by the increasing complexity of the resulting equations of motion. Moreover, the perturbation expansion may not converge uniformly in time, such that higher orders only improve the quality of the approximation of the short-time behavior, but completely fail in the long-time limit [23].

We will discuss here a further strategy for the treatment of highly non-Markovian processes which is based on the use of a correlated projection super-operator [24–29]. By contrast to the product-state projection, a correlated projection super-operator projects the total state $\rho$ onto a system–environment state that contains statistical correlations between certain system and environment states. We will discuss a representation theorem for a large class of such projections, which are appropriate for the application of the projection operator techniques, and develop a corresponding non-Markovian generalization of the Lindblad equation.

2 The Standard Projection Operator Method

We investigate an open quantum system $S$ that is coupled to some environment $E$. The corresponding Hilbert spaces are denoted by $\mathcal{H}_S$ and $\mathcal{H}_E$, respectively. The state space of the composite system is thus given by the tensor product space

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E.$$  \hspace{1cm} (1)

The states of the composite system are represented by density matrices $\rho$ on $\mathcal{H}$ satisfying the physical conditions of the positivity and the normalization:

$$\rho \geq 0, \quad \text{tr}\rho = 1,$$  \hspace{1cm} (2)