Non-Newtonian effects of a lubricant flow through a T-shaped microchannel

E R Kutuzova¹, A F Tazyukova¹, F Kh Tazyukov² and A G Kutuzov²

¹Control system department, Kazan National Research Technological University, Kazan, K. Marx st., 64, 420015, Russia
²Chemical Cybernetics Department, Kazan National Research Technological University, Kazan, K. Marx st., 64, 420015, Russia
³Mechanical Engineering Department, Kazan National Research Technological University, Kazan, K. Marx st., 64, 420015, Russia
⁴E-Mail: Near221291@gmail.com

Abstract. In this paper, the importance of non-Newtonian effects of a lubricant flow through a T-shaped microchannel with a moving lead are investigated. The lubricant flow is represented by Navier-Stokes equation for viscous, incompressible and steady fluid flows. Non-Newtonian characteristics are simulated by FENE-P constitutive equation. Finite Volume method was used to solve the case numerically. PSD distribution for Reynolds number Re=0.01 and Weissenberg number We =1 were compared with Newtonian case. Clear numerical evidence of a higher stresses for non-Newtonian case is discussed.

1. Introduction
The study of effects emerging in convergent lubrication flow in branched channel are still of value. Lubricants are widely used in modern technology to reduce friction in moving mechanisms (motors, bearings, gears), and to reduce friction in the machining of structural and other materials on machines (turning, milling, grinding, etc.). Depending on usage purposes they may be solid, liquid, gaseous [1].

In this paper the liquid non-Newtonian lubricant flow is considered. Lubricant flow characterized by negligible width of layer in comparison to its length and small inertial effects (Re<<1) [2-4]. Due to such peculiarity the ability of macromolecules to change its configuration may have significant impact on the flow’s picture.

Combination of above-mentioned aspects may lead to some negative effect such as a symmetry-breaking situation in the flow through a narrow branched symmetrical channel [5-8]. This may lead to some negative consequences [9-12]. To describe the non-Newtonian behavior of lubricant flow the rheological FENE-P model was chosen. This model predicts viscosity anomaly, elasticity and the depending on a shear rate the longitudinal viscosity.

The aim of the present paper is to analyze a structure of non-Newtonian viscoelastic lubricant flow through a planar T-shaped channel with a moving wall. At the present paper all the calculations were carried out using the OpenFoam source - CFD (Computational Fluid Dynamics) software package that based on FVM (Finite Volume Method) [13-17].
2. Mathematical model

2.1. Equation

To simulate an isothermal laminar creeping flow the mass conversation equation and the momentum equation need to be solved [18]:

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \mathbf{\tau},
\]

(1)

\[
\mathbf{\tau} \cdot \mathbf{V} = 0.
\]

(2)

The fluid total extra-stress is the sum of solvent and polymer stress contribution:

\[
\mathbf{\tau} = \mathbf{\tau}^p + \mathbf{\tau}^s.
\]

(3)

Newtonian stress contribution is defined:

\[
\mathbf{\tau}^s = 2\eta^s \mathbf{D},
\]

(4)

where \( \mathbf{D} = \frac{1}{2} (\mathbf{\nabla V} + (\mathbf{\nabla V})^T) \) [18,19].

In accordance to the FENE-P model the polymer stress contributions can be written in the following form:

\[
\mathbf{\tau}^p = \frac{\eta^p}{\lambda} \left[ \frac{\mathbf{A} - \frac{12}{12}}{1 - \frac{\text{tr}(\mathbf{A})}{3L^2} - \frac{1}{L^2}} - \mathbf{I} \right],
\]

(5)

\[
\frac{\mathbf{A}}{1 - (\text{tr}(\mathbf{A}))(3L^2)} + \text{We} \frac{\mathbf{V}}{1 - 1/L^2},
\]

(6)

where

\[
\frac{\mathbf{V}}{\mathbf{A}} = \frac{\partial \mathbf{A}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{A} - \mathbf{V} \cdot \mathbf{A} - \mathbf{A} \cdot (\mathbf{VV}^T).
\]

(7)

After averaging procedure, the following non-dimensional parameters are obtained [17]

\[
\text{We} = \frac{\lambda U}{L}, \quad \text{Re} = \frac{\rho U l}{\eta^s}, \quad \beta = \frac{\eta^s}{\eta^p}, \quad L^2 = \frac{Q_0}{Q_{eq}}.
\]

where We is Weissenberg’s number, Re is Reynold’s number, \( \beta \) is retardation coefficient, \( L^2 \) represents macromolecules conformation.

2.2. Initial and boundary conditions

Schematic representation of the channel and non-uniform mesh are sketched on a figure1. The length of the channels was set to 15 channel’s width for the formation of the velocity profile at the inlet flow and to establish the outlet flow. The represented mesh is non-uniform mesh with refinements near the central zone.

The imposed boundary conditions are:
S1: velocity and pressure are constant: \( U = \frac{U_{wall}}{2}, \quad V = 0 \) \hspace{1cm} (8)

S2: steady-flow conditions: \( U = 0, \quad V = \text{const}, \quad p = 0, \quad \frac{\partial \tau_{px}}{\partial x} = \frac{\partial \tau_{py}}{\partial x} = \frac{\partial \tau_{yy}}{\partial x} = 0 \) \hspace{1cm} (9)

S3: moving wall conditions: \( U = U_{wall}, \quad V = 0 \) \hspace{1cm} (10)

S4: velocity and pressure are constant: \( U = 0, \quad V = \text{const}, \quad \frac{\partial p}{\partial x} = 0 \) \hspace{1cm} (11)

Initial conditions: \( U = 0, \quad V = 0 \) \hspace{1cm} (12)

![Schematic representation of channel with refined mesh.](image)

**Figure 1.** Schematic representation of channel with refined mesh.

Initially, the problem was solving as unsteady task, stationary solution was obtained by setting \( t \to \infty \). The governing equations were solved by means of finite volume method (FVM). The orthogonal mesh is used for discretization of computational domain with refinement near the corners and used the linear interpolation scheme. To verify the obtained results the modeling was made for different meshes.

**3. Results**

The most important modeling parameters are the fixed Reynolds number \( \text{Re}=0.01 \), retardation coefficient \( \beta=0.01 \), Weissenberg number \( \text{We}=1 \) and stretching ratio \( L^2 = 50 \) and 100.

![PSD isolines for non-Newtonian (\( L^2 = 50 \) (a) and 100(b)) and Newtonian fluid (c) models.](image)

**Figure 2.** PSD isolines for non-Newtonian (\( L^2 = 50 \) (a) and 100(b)) and Newtonian fluid (c) models.
From figure 2 one can see comparison of PSD (principle stress difference ($\sigma_1 - \sigma_2$)) distribution in a T-shaped channel for the cases of $We=0.01$ (Newtonian-like fluid) and $We=1$. The pictures emerging for $We=0.01$ (Newtonian-like fluid) and $We=1$ are very similar, nevertheless influence of viscous and elastic forces for $We=1$ leads to the visible changes of fluid behavior. From figure 2 (a) and (b) a small circulation zone occurs near the corner, which is absent for Newtonian case. Zones with higher values of PSD are also zones with better orientation of macromolecules [17]. As the branched zone is the only place where non-Newtonian effects may manifest themselves the distribution presented on figure 3 should also be discussed.

![Figure 3. PSD distribution along x-axis in the center of the horizontal part of the channel.](image)

From figure 3 one can see that Newtonian case is almost planar. This result is fully predictable, as Newtonian fluid has no macromolecules, thus PSD does not reflect any conformation characteristics. As for non-Newtonian case, there is a visible peak at the branched zone and this peak is in direct correlation with $L^2$.

4. Summary
In this article non-Newtonian lubricant flow modelled by the FENE-P model through the T-shaped channel is considered in detail. Special attention is paid to the influence of the fluid flow properties to the stress values at the branched area. Presented effects of the viscoelastic fluid flows are associated with the interaction of macromolecules of the dissolved polymer and the solvent stream in the main stream flow. The changing of the flow direction leads to changes the conformation of macromolecules associated with their stretching and orientation in the flow. This non-equilibrium configuration, in turn, leads to change in normal stress that also affecting on the flow patterns. The presence of elasticity is characterized by two parameters: We and $L^2$. The combination of these numbers specially affects to the flow behavior. For various values of the macromolecule "unraveling" degree at $We=0.01$ the flow pattern will be similar to Newtonian fluid. When Weissenberg number $We=1$, numerical simulation of viscoelastic fluid flow at $L^2=50$ will exhibit a circulation zones formation and higher stress values at branched zone. An increase in $L^2$ parameter values up to 100 leads to the greater peak of stress values at the branched zone in comparison with Newtonian case.

5. References
[1] Cann P M, Webster M N, Doner J P, Wikstrom V and Lugt P 2001 Grease degradation in rolling element bearings *Trib. Transact.* 44 399-404
[2] Tanner R I 1965 Some illustrative problems in the flow of viscoelastic non-Newtonian lubricants *Asle Transact* 8 179-83
[3] Sisko A W 1958 The flow of lubricating greases *Indust. & Eng. Chem.* 50 1789-92
[4] Dobrica M B and Fillon M 2009 About the validity of Reynolds equation and inertia effects in textured sliders of infinite width *Proceedings of the Institution of Mechanical Engineers J. Eng. Trib.* J 223 69-78
[5] Orsi G, Galletti C, Brunazzi E and Mauri R 2013 Mixing of two miscible liquids in T-Shaped
micro devices Chem. Eng. Trans 32 1471-6
[6] Pakdel P and McKinley G H 1996 Elastic instability and curved streamlines Phys. Review Let. 77 2459-62
[7] Shaqfeh E S G 1996 Purely elastic instabilities in viscometric flows Annual Rev. of fluid Mech. 28 129-85
[8] Patir N and Cheng H S 1979 Application of average flow model to lubrication between rough sliding surfaces J. Lubr. Tech.101 220-9
[9] Soulages J, Oliveira M S N, Sousa P C, Alves M A and McKinley G H 2009 Investigating the Stability of Viscoelastic Stagnation Flows in T-Shaped Microchannels J. Non-Newton. Fluid Mech. 163 9-24
[10] Stone H A and Kim S 2001 Microfluidic: Basic issues, Applications and Challenges, J. AIChE 47 1250-4
[11] Marchon B, Karis T, Dai Q and Pit R 2003 A model for lubricant flow from disk to slider IEEE trans. on mag. 39 2447-9
[12] Rohr O 2002 Bismuth—the new ecologically green metal for modern lubricating engineering Indust. Lubric. and trib. 541 53-64
[13] OpenFoam User Guide Available from http://openfoam.com
[14] Patankar S 1980 Numerical heat transfer and fluid flow (Hemisphere Publishing Corporation)
[15] Madsen N K and Ziolkowski R W 1990 A three-dimesional modified finite volume technique for Maxwell’s equation Electromagnetics 10 147-61
[16] Versteeg H K and Malalasekera W 2007 An introduction to computational fluid dynamics: the finite volume method (British Library Cataloguing-in-Publication Data)
[17] Tazyukov F Kh, Kutuzova E R and G C Layek 2016 The symmetry-loss of viscoelastic fluid flow through a T-junction channel Transact. of Academener. 2 20-8
[18] Kutuzova E R, Tazyukov F Kh and Snigerev B A 2016 Divergent non-Newtonian fluid flow Ecolog. Bull. of Res. Cent. of The Bl Sea Econ. Cooper. 3 59-64
[19] Patir N and Cheng H S 1979 Application of average flow model to lubrication between rough sliding surfaces J. Lubric. Tech 2 220-9
[20] Ianniruberto G and Marrucci G 1998 Stress tensor and stress-optical law in entangled polymers J. Non-Newton. Fluid Mech. 79 225-34