Note About Tachyon Kink In Nontrivial Background

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Abstract: This paper is devoted to the study of the tachyon kink on the worldvolume of a non-BPS Dp-brane that moves in a nontrivial background. We will show that the spatial dependent tachyon condensation leads to an emergence of a D(p-1)-brane whose dynamics is governed by Dirac-Born-Infeld action.

Keywords: D-branes
1. Introduction

Study of various aspects of the tachyon dynamics on a non-BPS Dp-brane in type IIA or IIB theories has led to some understanding of the tachyon dynamics near the tachyon vacuum 1. The tachyon effective action (2.1), describing the dynamics of the tachyon field on a non-BPS Dp-brane of type IIA and IIB theory was proposed in [6, 7, 8, 9]. It was argued in many papers that the tachyon effective action (2.1) gives a good description of the system under condition that tachyon is large and the second and higher derivatives of the tachyon are small 2. A kink solution in the full tachyon effective field theory, which is supposed to describe a BPS D(p-1)-brane was also constructed in [15, 16, 18, 19, 20, 21, 22, 23]. A kink solution, that by definition interpolates between the vacua at $T = \pm \infty$ has to pass through 0. Then we could expect that higher derivative corrections to the tachyon effective action will be needed to provide a good description of the D(p-1)-brane as a kink solution. This issue was carefully analysed in paper [15] where it was shown that the energy density of the kink in the effective field theory is localised on codimension one surface as in the case of a BPS D(p-1)-brane. It was then also shown that the worldvolume theory of the kink solution is also given by the Dirac-Born-Infeld (DBI) action on a BPS D(p-1)-brane. Thus result shows that the kink solution of the effective field

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1For review of the open string tachyon condensation, see [1, 2, 3, 4, 5].
2For discussion of the effective field theory description of the tachyon condensation, see [10, 11, 12, 13, 14].
theory does provide a good description of the D(p-1)-brane even without taking into account higher derivative corrections. In other words, the tachyon effective action reproduces the low energy effective action on the world-volume of the soliton without any correction terms.

Since these results are very impressive it would be certainly useful to test the effective field theory description of the tachyon condensation in other, more general situations. In fact, since the DBI action describes the low energy dynamics of the BPS Dp-brane in general curved background we can ask the question whether we can construct the tachyon kink on the worldvolume of a non-BPS Dp-brane embedded in curved background and whether this kink has the interpretation as a lower dimensional D(p-1)-brane. To answer this question we begin with common presumption that the tachyon effective action for Dp-brane can be applied for the description of the tachyon dynamics in the nontrivial background as well. Then we will study the equation of motion for the tachyon and for the modes that parametrise the embedding of the unstable Dp-brane in given spacetime. We will solve these equations with the field configuration similar to the ansatz that was given in [15]. We will show that this ansatz solves the equation of motion for tachyon on condition that the mode that characterises the core of the kink (The precise meaning of this claim will be given bellow.) satisfies the equation of motion of the scalar field that describes the embedding of D(p-1)-brane in given background. This result shows that the spatial dependent tachyon condensation leads to the emergence of a D(p-1)-brane where the scalar modes that propagate on its worldvolume solve the equation of motion that arise from the DBI action for D(p-1)-brane that is moving in the same background.

The structure of this paper is as follows. In the next section we will analyse the equation of motion for non-BPS Dp-brane in curved background. We will find the spatial dependent tachyon solution that has interpretation as a lower dimensional D(p-1)-brane whose dynamics is governed by DBI action. In section we will study some examples of the nontrivial background. The first one corresponds to the stack of NS5-branes and the second one corresponds to the background generated by the collection of N coincident Dk-branes. Finally, in conclusion we will outline our results and suggest possible extension of this work.

2. Non-BPS Dp-brane in general background

The starting point for the analysis of the dynamics of a non-BPS Dp-brane in general background were presented in [16, 17, 24, 25].

The case of more general background, including NS B field and Ramond-Ramond forms will be discussed in forthcoming publication.
background is the Dirac-Born-Infeld like tachyon effective action \[ S = -\int d^{p+1}\xi e^{-\Phi} V(T)\sqrt{-\det A}, \]
\[ A_{\mu\nu} = g_{MN}\partial_\mu Y^M \partial_\nu Y^N + F_{\mu\nu} + \partial_\mu T \partial_\nu T, \mu, \nu = 0, \ldots, p, \]
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \]
(2.1)

where \( A_\mu, \mu, \nu = 0, \ldots, p \) and \( Y^{M,N}, M, N = 0, \ldots, 9 \) are gauge and the transverse scalar fields on the worldvolume of the non-BPS Dp-brane and \( T \) is the tachyon field. Since in this paper we will restrict ourselves to the situations when the gauge fields can be consistently taken to zero we will not write \( F_{\mu\nu} \) anymore. \( V(T) \) is the tachyon potential that is symmetric under \( T \rightarrow -T \) has maximum at \( T = 0 \) equal to the tension of a non-BPS Dp-brane \( \tau_p \) and has its minimum at \( T = \pm\infty \) where it vanishes.

Using the worldvolume diffeomorphism invariance we can presume that the worldvolume coordinates \( \xi^\mu \) are equal to the spacetime coordinates \( y^\mu \). Explicitly, we can write
\[ Y^\mu = \xi^\mu. \]
(2.2)

Then the induced metric takes the form \[ \gamma_{\mu\nu} \equiv g_{MN}\partial_\mu X^M \partial_\nu X^N = g_{\mu\nu} + g_{mn}\partial_\mu Y^m \partial_\nu Y^n, \]
(2.3)

where \( Y^m, m, n = p + 1, \ldots, 9 \) parametrise the embedding of Dp-brane in a space transverse to its worldvolume. We should also mention that generally the metric components and dilaton are functions of \( \xi^\mu \) and \( Y^m \):
\[ g_{MN} = g_{MN}(\xi^\mu, Y^m), \Phi = \Phi(\xi^\mu, Y^m). \]
(2.4)

Now the equation of motion for \( T \) and \( Y^m \) that follow from (2.1) take the form
\[ \frac{\delta V}{\delta T} e^{-\Phi} \sqrt{-\det A} - \partial_\mu \left[ e^{-\Phi} V \sqrt{-\det A} \partial_\nu T(A^{-1})^{\nu\mu} \right] = 0 \]
(2.5)

and
\[ \frac{\delta[e^{-\Phi}]}{\delta Y^m} V \sqrt{-\det A} + e^{-\Phi} \frac{V}{2} \left[ \frac{\delta g_{\mu\nu}}{\delta Y^m} + \frac{\delta g_{mn}}{\delta Y^m} \partial_\mu Y^n \partial_\nu Y^p \right] (A^{-1})^{\nu\mu} \sqrt{-\det A} - \partial_\mu \left[ e^{-\Phi} g_{mn} \partial_\nu Y^n (A^{-1})^{\nu\mu} \sqrt{-\det A} \right] = 0. \]
(2.6)

\[ ^5 \text{We use the convention where the fundamental string tension has been set equal to } (2\pi)^{-1} \text{ (i.e. } \alpha' = 1). \]

\[ ^6 \text{We restrict ourselves in this paper to the situations when the background metric is diagonal.} \]
Our goal is to find the solution of these equations of motions that can be interpreted as a lower dimensional D(p-1)-brane. In order to obtain such a solution we will closely follow the paper by A. Sen [15]. Let us choose one particular worldvolume coordinate, say $\xi^p \equiv x$ and consider following ansatz for the fields living on the worldvolume of Dp-brane

$$T = f(a(x-t(\xi)))) , Y^m = Y^m(\xi) ,$$

(2.7)

where $\xi^\alpha , \alpha = 0,\ldots, p-1$ are coordinates tangential to the kink worldvolume. As in [15] we presume that $f(u)$ satisfies following properties

$$f(-u) = -f(u) , f'(u) > 0 , \forall u , f(\pm \infty) = \pm \infty$$

(2.8)

but is otherwise an arbitrary function of its argument $u$. $a$ is a constant that we shall take to $\infty$ in the end. In this limit we have $T = \infty$ for $x > t(\xi)$ and $T = -\infty$ for $x < t(\xi)$.

Our goal is to check that the ansatz (2.7) solves the equation of motion (2.5) and (2.6). Firstly, using (2.7) the matrix $A_{\mu\nu}$ takes the form

$$A_{xx} = g_{xx} + a^2 f'^2 , A_{x\alpha} = g_{x\alpha} - a^2 f'^2 \partial_\alpha t ,$$

$$A_{\beta x} = g_{\beta x} - a^2 f'^2 \partial_\beta t , A_{\alpha\beta} = (a^2 f'^2 - g_{xx}) \partial_\alpha t \partial_\beta t + \tilde{a}_{\alpha\beta} ,$$

$$\tilde{a}_{\alpha\beta} = g_{\alpha\beta} + g_{mn} \partial_\alpha Y^m \partial_\beta Y^n + \partial_\alpha t \partial_\beta t .$$

(2.9)

For next purposes it will be useful to know the form of the determinant $\det A$. Using the following identity

$$\det A = \det(A_{\alpha\beta} - A_{\alpha x} \frac{1}{A_{xx}} A_{x\beta}) \det A_{xx}$$

(2.10)

we get

$$\det A = a^2 f'^2 \det(\tilde{a}_{\alpha\beta}) + O(1/a) .$$

(2.11)

As a next step we should express $(A^{-1})$ in terms of $\tilde{a}$. After some calculations we find

$$(A^{-1})^{xx} = (\tilde{a}^{-1})^{\alpha\beta} \partial_\alpha t \partial_\beta t , (A^{-1})^{x\beta} = \partial_\alpha t (\tilde{a}^{-1})^{\alpha\beta} ,$$

$$(A^{-1})^{\alpha x} = (\tilde{a}^{-1})^{\alpha\beta} \partial_\beta t , (A^{-1})^{\alpha\beta} = (\tilde{a}^{-1})^{\alpha\beta}$$

(2.12)

up to corrections of order $\frac{1}{a^2}$. In the following calculation we will also need an exact relation

$$(A^{-1})^{\mu x} - (A^{-1})^{\mu\alpha} \partial_\alpha t = \frac{1}{a^2 f'^2} (\delta^\mu_x - (A^{-1})^{xx} g_{xx}) .$$

(2.13)
Using this expression we can now write
\[ \partial_\mu \left[ e^{-\Phi} V \sqrt{-\det \mathbf{A}(\mathbf{A}^{-1})^{\mu\nu} \partial_\nu T} \right] = \partial_\mu \left[ e^{-\Phi} V a f' \frac{1}{a^2 f'^2} (\delta^\mu_x - (A^{-1})^{\mu x} g_{xx}) \sqrt{-\det \mathbf{A}} \right]. \]
\[ \quad (2.14) \]

Following [15] we can now argue that due to the explicit factor of $a^2 f'^2$ in the denominator the leading contribution from individual terms in this expression is now of order $a$ and hence we can use the approximative results of $\det \mathbf{A}$ and $(\mathbf{A}^{-1})$ given in (2.11) and (2.12) to analyse the equation of motion for tachyon
\[ \partial_\mu \left[ e^{-\Phi} V \sqrt{-\det \mathbf{A} a f' \frac{1}{a^2 f'^2} (\delta^\mu_x - (A^{-1})^{\mu x} g_{xx})} \right] - \partial_x \left[ e^{-\Phi} V \sqrt{-\det \mathbf{A}}(1 - (\mathbf{A}^{-1})^{\alpha\beta} g_{\alpha\beta}) \right] - \partial_\alpha \left[ e^{-\Phi} V \sqrt{-\det \mathbf{A}}((\mathbf{A}^{-1})^{\alpha\beta} g_{\alpha\beta}) \right] - a f' e^{-\Phi} V' \sqrt{-\det \mathbf{A}} = 0. \]
\[ \quad (2.15) \]

This is important result that deserves deeper explanation. Firstly, from the form of the tachyon potential in the limit $a \to \infty$ we know that $V$ is equal to zero for $x - t(\xi) \neq 0$ while for $x - t(\xi) = 0$ we get $V(0) = \tau_p$. Then it is clear that the tachyon equation of motion is obeyed for $x - t(\xi) \neq 0$ while for $x = t(\xi)$ we should demand that the expression in the bracket should be equal to zero. In other words, we obtain following equation
\[ \frac{\delta e^{-\Phi}}{\delta x} \sqrt{-\det \mathbf{A}} + \frac{e^{-\Phi}}{2} \left( \frac{\delta g_{\alpha\beta}}{\delta x} \partial_\alpha t \partial_\beta t + \frac{\delta g_{\alpha\mu} n_{\mu}}{\delta x} \partial_\alpha Y^m \partial_\beta Y^n \right) (\mathbf{A}^{-1})^{\beta\alpha} \sqrt{-\det \mathbf{A}} - \partial_\alpha \left[ e^{-\Phi} \sqrt{-\det \mathbf{A}}((\mathbf{A}^{-1})^{\alpha\beta} g_{\alpha\beta}) \right] \partial_\alpha t \partial_\beta t = 0. \]
\[ \quad (2.16) \]

We must stress that in (2.16) we firstly perform the derivative with respect to $x$ and then we insert the value $x = t(\xi)$ back to the resulting equation of motion. For example, in the first term on the second line we should perform a derivative with respect to $\xi^\alpha$ with in mind that $x$ is an independent variable. After doing this we should everywhere replace $x$ with $t(\xi)$. Then the presence of the second term on the second line is crucial for an interpretation of $t(\xi)$ as an additional scalar field that parametrises the position of D(p-1)-brane in $x$ direction.

Put differently, we expect that the tachyon condensation leads to an emergence of D(p-1)-brane that is localised at $x = t(\xi)$. For that reason we should compare the
equation (2.16) with the equation of motion for D(p-1)-brane embedded in the same background. As we know the dynamics of such a Dp-brane is governed by the DBI action
\[
S = -T_{p-1} \int d^p \xi e^{-\Phi} \sqrt{-\det A_{\alpha\beta}^{BPS}},
\]  
where the matrix \( A_{\alpha\beta}^{BPS} \) takes the form
\[
A_{\alpha\beta}^{BPS} = g_{\alpha\beta} + g_{xx} \partial_\alpha Y \partial_\beta Y + g_{mn} \partial_\alpha Y^m \partial_\beta Y^n, \quad m, n = p + 1, \ldots, 9,
\]
and where \( T_{p-1} \) is the tension of BPS D(p-1)-brane. Recall that \( T_p \) is is related to the tension of the non-BPS D(p-1)-brane \( \tau_p \) as \( \tau_p - 1 = \sqrt{2} T_{p-1} \). In (2.18) we have chosen one particular transverse mode \( Y \) in order to have a contact with the mode \( t \) defined in the equation (2.7). Finally, the scalar fields \( Y^m \) have the same meaning as in the case of a non-BPS Dp-brane. Now the equations of motion that follow from (2.17) take the form
\[
\delta \frac{\delta e^{-\Phi} \sqrt{-\det A_{\alpha\beta}^{BPS}}}{\delta Y} - \partial_\alpha \left[ e^{-\Phi} \sqrt{-\det A_{\alpha\beta}^{BPS}} (A^{-1})^{\beta\alpha}_{BPS} g_{mn} \partial_\beta Y^n \right] = 0,
\]
\[
\delta \frac{\delta e^{-\Phi} \sqrt{-\det A_{\alpha\beta}^{BPS}}}{\delta Y} - \partial_\alpha \left[ e^{-\Phi} \sqrt{-\det A_{\alpha\beta}^{BPS}} (A^{-1})^{\beta\alpha}_{BPS} g_{xx} \partial_\beta Y \right] = 0,
\]
where the variation \( \frac{\delta}{\delta Y^m} : \frac{\delta}{\delta Y} \) means the variation of the metric, dilaton with respect to \( Y^M, Y \) respectively. Explicitly, the equation of motion for \( Y \) can be written as
\[
\frac{\delta e^{-\Phi} \sqrt{-\det A_{BPS}}}{\delta Y} = \frac{1}{2} e^{-\Phi} \left( \frac{\delta g_{\alpha\beta}}{\delta Y} + \frac{\delta g_{xx}}{\delta Y} \partial_\alpha Y \partial_\beta Y + \frac{\delta g_{mn}}{\delta Y} \partial_\alpha Y^m \partial_\beta Y^n \right) (A^{-1})^{\beta\alpha}_{BPS} \sqrt{-\det A_{BPS}} - \partial_\alpha \left[ e^{-\Phi} \sqrt{-\det A_{BPS}} (A^{-1})^{\beta\alpha}_{BPS} g_{xx} \partial_\beta Y \right] = 0.
\]
To see more clearly the relation with the equation (2.14) note that the expression on the third line can be written as
\[
\partial_\alpha \left[ e^{-\Phi} \sqrt{-\det A_{BPS}} (A^{-1})^{\beta\alpha}_{BPS} g_{xx} \partial_\beta Y \right] = \partial_\alpha \left[ e^{-\Phi(\xi,x)} \sqrt{-\det A_{BPS}(\xi,x)} (A^{-1})^{\beta\alpha}_{BPS}(\xi,x) \partial_\beta Y \right]
+ \partial_x \left[ e^{-\Phi(\xi,x)} \sqrt{-\det A_{BPS}(\xi,x)} (A^{-1})^{\beta\alpha}_{BPS}(\xi,x) \right] \partial_\alpha Y \partial_\beta Y,
\]
where on the second line the derivative with respect to \( \xi^\alpha \) treats \( x \) as an independent variable so that we firstly perform derivative with respect to \( \xi^\alpha \) and then we replace
$x$ with $Y$. We proceed in the same way with the expression on the third line where we firstly perform the variation with respect to $x$ and then we replace $x$ with $Y$. Now it is clear that this prescription is the same as the expression on the second line in (2.10). More precisely, if we compare the equation (2.20) with the the equation (2.10) we see that these two expressions coincide when we identify $t$ with $Y$. In other words, the location of the tachyon kink is completely determined by field $t(\xi)$ that obeys the equation of motion of the embedding mode of $D(p-1)$-brane. We mean that this is very satisfactory result that shows that the Sen’s construction of the tachyon kink can be consistently performed in curved background as well.

Now we will discuss the equation of motion for $Y^k$. Again, we will proceed as in [13]. We begin with the first term in (2.6) that for the ansatz (2.7) takes the form

$$\frac{\delta e^{-\phi}}{\delta Y^k} V - \frac{e^{-\phi}}{\delta Y^k} V \frac{\delta e^{-\phi}}{\delta Y^k} \sqrt{-\text{det} A} = af'Ve^{-\phi}/\delta Y^k \sqrt{-\text{det} \bar{a}}. \tag{2.22}$$

In the same way we can show that the second term in (2.6) can be written as

$$e^{-\phi} V \left[ \frac{\delta g_{\mu\nu}}{\delta Y^k} + \frac{\delta g_{mn}}{\delta Y^k} \partial_\mu Y^m \partial_\nu Y^n \right] (A^{-1})^{\mu\nu} \sqrt{-\text{det} A} = af'Ve^{-\phi} \sqrt{-\text{det} \bar{a}} \times$$

$$\left[ \frac{\delta g_{xx}}{\delta Y^k}(\bar{a}^{-1})^{\alpha\beta} \partial_\alpha t \partial_\beta t + \left( \frac{\delta g_{\alpha\beta}}{\delta Y^k} + \frac{g_{mn}}{\delta Y^k} \partial_\alpha Y^m \partial_\beta Y^n \right) (\bar{a}^{-1})^{\alpha\beta} \right]. \tag{2.23}$$

Finally, the third term in (2.6) is equal to

$$-\partial_\mu \left[ e^{-\phi} V g_{km} \partial_\nu Y^m (A^{-1})^{\mu\nu} \sqrt{-\text{det} A} \right] =$$

$$-\partial_\mu \left[ V e^{-\phi} g_{km} \partial_\nu Y^m (A^{-1})^{\alpha\beta} \sqrt{-\text{det} A} \right] - \partial_\alpha \left[ V e^{-\phi} g_{km} \partial_\beta Y^m (A^{-1})^{\alpha\beta} \sqrt{-\text{det} A} \right] =$$

$$= -af'\partial_\mu \left[ V e^{-\phi} g_{km} \partial_\beta Y^m (\bar{a}^{-1})^{\alpha\beta} \sqrt{-\text{det} \bar{a}} \right] -$$

$$-af'\partial_\alpha \left[ V e^{-\phi} g_{km} \partial_\beta Y^m (\bar{a}^{-1})^{\alpha\beta} \sqrt{-\text{det} \bar{a}} \right] =$$

$$= -af'Ve^{-\phi} g_{km} \partial_\beta Y^m (\bar{a}^{-1})^{\alpha\beta} \sqrt{-\text{det} \bar{a}} =$$

$$-af'Ve^{-\phi} g_{km} \partial_\beta Y^m (\bar{a}^{-1})^{\alpha\beta} \sqrt{-\text{det} \bar{a}} \tag{2.24}$$

using the fact that $\partial_\alpha f = -af'\partial_\alpha t$. Then collecting (2.22), (2.23) and (2.24) together we obtain

$$V \left\{ \frac{\delta e^{-\phi}}{\delta Y^k} \sqrt{-\text{det} \bar{a}} + \frac{e^{-\phi}}{\delta Y^k} \sqrt{-\text{det} \bar{a}} \times \right.$$

$$\left[ \frac{\delta g_{xx}}{\delta Y^k} \partial_\alpha t \partial_\beta t + \frac{\delta g_{\alpha\beta}}{\delta Y^k} + \frac{g_{mn}}{\delta Y^k} \partial_\alpha Y^m \partial_\beta Y^n \right] (\bar{a}^{-1})^{\alpha\beta} \right\} \left[ -\partial_\alpha \left[ e^{-\phi} g_{km} \partial_\beta Y^m (\bar{a}^{-1})^{\alpha\beta} \sqrt{-\text{det} \bar{a}} \right] \right.$$

$$\left. -\partial_\alpha \left[ e^{-\phi} g_{km} \partial_\beta Y^m (\bar{a}^{-1})^{\alpha\beta} \sqrt{-\text{det} \bar{a}} \right] \partial_\beta t \right\} = 0. \tag{2.25}$$
Again it is easy to see that for \( x \neq t(\xi) \) the potential vanishes for \( a \to \infty \) while for \( x = t(\xi) \) we have \( V(0) = \tau_p \). Then in order to obey the equation of motion the expression in the bracket should be equal to zero for \( x = t(\xi) \). Note also that in the second expression on the last line in (2.25) we firstly perform a derivative with respect to \( x \) and then we replace \( x \) with \( t(\xi) \). In other words, we can rewrite the last line in (2.25) into the form

\[
\partial_\alpha \left[ e^{-\Phi(t(\xi))} \sqrt{-\text{det} \tilde{a}(t(\xi))(\tilde{a}^{-1})^{\alpha\beta}(t(\xi))g_{km}(t(\xi))\partial_\beta Y^m} \right], \tag{2.26}
\]

where we have explicitly stressed the dependence of the action on the mode \( t(\xi) \) that replaces in the action the dependence on \( x \). This result again supports the claim that we should identify \( t(\xi) \) with an additional embedding coordinate of the D(p-1)-brane. Then by comparing the expression in the bracket in (2.25) with the equation of motion for \( Y^m \) given in (2.19) we see that these two expressions coincide.

In summary, we have shown that the tachyon kink solution on a non-BPS Dp-brane in nontrivial background can be identified as a lower dimensional D(p-1)-brane that is localised at the core of the kink. We have also shown that the dynamics of this D(p-1)-brane is governed by DBI action.

### 2.1 Stress energy tensor

Further support for the interpretation of the tachyon kink as a lower dimensional D(p-1)-brane can be obtained from the analysis of the stress energy tensor for the non-BPS Dp-brane. In order to find its form recall that we can write the action (2.1) as

\[
S_p = -\int d^{10} y d^{(p+1)} \xi \delta(Y^M(\xi) - y^M)e^{-\Phi(T)}\sqrt{-\text{det} A}, \tag{2.27}
\]

where

\[
A_{\mu\nu} = G_{MN}\partial_\mu Y^M\partial_\nu Y^N + \partial_\mu T \partial_\nu T, \tag{2.28}
\]

and where \( \xi^\mu, \mu = 0, \ldots, p \) are worldvolume coordinates on Dp-brane. The form of action (2.27) is useful for determining the stress energy tensor \( T_{MN}(y) \) of an unstable D-brane. In fact, the stress energy tensor \( T_{MN}(y) \) is defined as the variation of \( S_p \) with respect to \( g_{MN}(y) \)

\[
T_{MN}(y) = -\frac{\delta S_p}{\sqrt{-g(y)\delta g^{MN}(y)}} = -\int d^{(p+1)} \xi \frac{\delta(Y^M(\xi) - y^M)}{\sqrt{-g(y)}} e^{-\Phi}g_{MK}g_{NL}\partial_\mu Y^K \partial_\nu Y^L (A^{-1})^{\nu\mu}\sqrt{-\text{det} A}. \tag{2.29}
\]

The form of the stress energy tensor for gauge fixed Dp-brane action can be obtained from (2.23) by imposing the condition

\[
Y^\mu = \xi^\mu, \mu = 0, 1, \ldots, p. \tag{2.30}
\]
Then the integration over \( \xi^\mu \) swallows up the delta function \( \delta(y^\mu - Y^\mu(\xi)) = \delta(y^\mu - \xi^\mu) \) so that the resulting stress energy tensor takes the form

\[
T_{mn} = -\frac{\delta(Y^m(\xi) - y^m)}{\sqrt{-g}}e^{-\Phi}V g_{mn} \partial_\mu Y^m g_{nn} \partial_\nu Y^n (\tilde{A}^{-1})^{\nu\mu} \sqrt{-\det \tilde{A}} ,
\]

\[
T_{\mu\nu} = -\frac{\delta(Y^m(\xi) - y^m)}{\sqrt{-g}}e^{-\Phi}V g_{\mu\nu} (\tilde{A}^{-1})^{\nu\mu} \sqrt{-\det \tilde{A}} ,
\]

\[
T_{\mu\nu} = -\frac{\delta(Y^m(\xi) - y^m)}{\sqrt{-g}}e^{-\Phi}V g_{\mu\nu} Y^m (\tilde{A}^{-1})^{\nu\mu} \sqrt{-\det \tilde{A}} ,
\]

\[
T_{mn} = -\frac{\delta(Y^m(\xi) - y^m)}{\sqrt{-g}}e^{-\Phi}V g_{mn} \partial_\mu Y^m g_{nn} \partial_\nu Y^n (\tilde{A}^{-1})^{\nu\mu} \sqrt{-\det \tilde{A}} ,
\]

using the fact that the metric is diagonal.

If we now insert the ansatz \((2.7)\) into these expressions we get

\[
T_{mn} = -\frac{\delta(Y^m(\xi) - x^m)}{\sqrt{-g}}V f' e^{-\Phi} g_{mn} \partial_\alpha Y^m g_{nn} \partial_\beta Y^n (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}} ,
\]

\[
T_{\alpha\beta} = -\frac{\delta(Y^m(\xi) - y^m)}{\sqrt{-g}}V f' e^{-\Phi} g_{\alpha\beta} (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}} ,
\]

\[
T_{xx} = -\frac{\delta(Y^m - y^m)}{\sqrt{-g}}V f' e^{-\Phi} g_{xx} \partial_\alpha t_\beta t (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}} ,
\]

\[
T_{mx} = T_{xm} = -\frac{\delta(Y^m - y^m)}{\sqrt{-g}}V f' e^{-\Phi} g_{mn} \partial_\alpha Y^m g_{xx} \partial_\beta t (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}} ,
\]

\[
T_{xx} = T_{\alpha x} = 0 .
\]

(2.31)

From now on the notation \( Y^m(\xi), t(\xi) \) means that these fields are functions of the coordinates on the worldvolume of the kink \( \xi^\alpha, \alpha = 0, \ldots, p - 1 \).

According to \([13]\) the components of the stress energy tensor of the lower dimensional D(p-1)-brane arise by integrating all \( T_{MN} \) given above over the direction of the tachyon condensation that in our case is \( x \). Now we should be more careful since metric components generally depend on \( x \). Let us introduce the following notation for the components of the stress energy tensors \((2.32)\)

\[
T_{MN} = V(f(a(t(\xi) - x)))af'\tilde{T}_{MN}(x) ,
\]

(2.33)

where we have explicitly stressed the dependence of \( \tilde{T}_{MN} \) on \( x \). If we now integrate \( T_{MN} \) over \( x \) we get

\[
T_{kink}^{MN} = \int_0^\infty dx V(f(a(x - t(\xi))))f' a\tilde{T}_{MN}(x) = \int dm V(m)\tilde{T}_{MN} \left( \frac{f^{-1}(m)}{a} + t(\xi) \right) .
\]

(2.34)
In the limit $a \to \infty$ the term proportional to $1/a$ goes to zero and we get that the components $\tilde{T}_{MN}$ are functions of $t(\xi)$ in place of $x$. Further, we can argue, following [13] that the exponential fall off in $V(m)$ implies that in the limit $a \to \infty$ the contribution to the stress energy tensor is localised at the point where $V$ is equal to $V(0) = \tau_p$ which happens for $x = t(\xi)$. In other words, when we presume that the tension of BPS D(p-1)-brane is given by the integral

$$T_{p-1} = \int_{-\infty}^{\infty} dm V(m)$$

we obtain the result that the components of the stress energy tensor of the kink take the form

$$T_{kink}^{mn} = -\frac{T_{p-1} \delta(Y^m(\xi) - y^m) \delta(t(\xi) - x)}{\sqrt{-g}} e^{-\Phi} g_{mn} \partial_\alpha Y^m g_{nn} \partial_\beta Y^n (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}},$$

$$T_{kink}^{\alpha\beta} = -\frac{T_{p-1} \delta(Y^m(\xi) - y^m) \delta(t(\xi) - x)}{\sqrt{-g}} e^{-\Phi} g_{\alpha\alpha} g_{\beta\beta} (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}},$$

$$T_{kink}^{xx} = -\frac{T_{p-1} \delta(Y^m - y^m) \delta(t(\xi) - x)}{\sqrt{-g}} e^{-\Phi} g_{xx} \partial_\alpha t \partial_\beta t (\tilde{a}^{-1})^{\alpha\beta} \sqrt{-\det \tilde{a}},$$

$$T_{kink}^{mx} = T_{kink}^{xm} = 0,$$

where it is understood that $g_{MN}$ and $\Phi$ are functions of $\xi^\alpha, Y^m(\xi), t(\xi)$. In other words, the components of the stress energy tensors (2.36) correspond to the components of the stress energy tensor of a D(p-1)-brane localised at the points $Y^m(\xi), t(\xi)$.

3. Examples of the tachyon condensation on a non-BPS Dp-brane in nontrivial background

In this section we will briefly discuss some examples of the tachyon condensation on a non-BPS Dp-brane that is embedded in nontrivial backgrounds.

3.1 NS5-brane background

As the first example we will consider the background corresponding to the stack of $N$ coincident NS5-branes

$$ds^2 = dx_\mu dx^\mu + H_{NS} dx^m dx^m,$$

$$e^{2\phi} = H_{NS},$$

$$H_{mnp} = -\epsilon_{mnp} \partial_\alpha \Phi,$$

$$H_{mnq} = -\epsilon_{mqn} \partial_\beta \Phi,$$

(3.1)
where the harmonic function $H_{NS}$ for $N$ coincident NS5-branes is equal to

$$H_{NS}(y^m) = 1 + \frac{2\pi N}{y^m y_m}, \quad (3.2)$$

where $y^m, m = 6, \ldots, 9$ label directions transverse to the worldvolume of NS5-branes.

The most simple case occurs when D$p$-brane is stretched in the direction parallel with the worldvolume of NS5-branes. Using (3.1) it is then easy to determine the worldvolume metric

$$g_{\mu\nu} = \eta_{\mu\nu}, g_{m_1n_1} = \delta_{m_1n_1}, m_1, n_1 = p + 1, \ldots, 5,$$

$$g_{m_2n_2} = H_{NS}\delta_{m_2n_2}, m_2, n_2 = 6, \ldots, 9,$$ \quad (3.3)

where now $H_{NS}$ is function of $Y^{m_2}$

$$H_{NS} = 1 + \frac{2\pi N}{Y^{m_2} Y_{m_2}}.$$ \quad (3.4)

Thanks to the manifest $SO(p)$ symmetry of the worldvolume theory all spatial co-ordinates $\xi^i, i = 1, \ldots, p$ are equivalent. Then we choose the direction on which the tachyon depends to be $\xi^p = x$. Now it is clear that the spatial dependent tachyon condensation studied in previous section leads to the emergence of a D$(p-1)$-brane that is stretched in the $x^0, \ldots, x^{p-1}$ directions and and which transverse position is determined by the worldvolume fields $t(\xi), Y^m(\xi)$. These fields also obey the equations of motion that arise from the DBI action for a D$(p-1)$-brane that moves in the background of $N$ NS5-branes.

Another possibility occurs when we consider a non-BPS D$p$-brane stretched in some of the transverse directions to the worldvolume of NS5-branes. More precisely, let us consider an unstable D$p$-brane that is stretched in $x^0, x^1, \ldots, x^k$ directions and in $x^6, \ldots, x^{6+p-k}$ directions. Then the metric components that appear on the worldvolume of the D$p$-brane take the form

$$g_{\mu_1\nu_1} = \eta_{\mu_1\nu_1}, \mu_1, \nu_1 = 0, \ldots, k,$$

$$g_{\mu_2\nu_2} = H_{NS}\delta_{\mu_2\nu_2}, \mu_2, \nu_2 = 6, \ldots, (6 + p - k),$$

$$g_{m_1n_1} = \delta_{m_1n_1}, m_1, n_1 = k + 1, \ldots, 5,$$

$$g_{m_2n_2} = H_{NS}\delta_{m_2n_2}, m_2, n_2 = (7 + p - k), \ldots, 9,$$ \quad (3.5)

where the function $H_{NS}$ has the form

$$H_{NS} = 1 + \frac{2\pi N}{(\xi^{m_2}\xi^{\nu_2} + Y^{m_2} Y_{m_2})^2}.$$ \quad (3.6)

\footnote{In what follows we will consider the situation when we can ignore the NS two form background.}
Now there are many possibilities how to construct lower dimensional D(p-1)-brane. If we perform the spatial dependent tachyon condensation on the worldvolume of the non-BPS Dp-brane where the tachyon $t(x)$ depends on coordinate from the set $\xi^1, \ldots, \xi^k$ (again, we take $x = \xi^k$) we obtain D(p-1)-brane that is localised in $x^k$ direction and that is stretched in $x^0, \ldots, x^{k-1}$ and $x^6, \ldots, x^{(6+p-k)}$ directions. It is important to stress that the resulting configuration of N NS5-brane and BPS D(p-1)-brane is not in general stable. Rather the dynamics of the BPS D(p-1)-brane in the background of N NS5-branes is governed the equation of motions (2.20). To find stable configuration we should perform the same analysis as in [26].

Another possibility is to consider the tachyon condensation in direct ion from the set $\xi^6, \ldots, \xi^{6+p-k}$. Let us choose $x \equiv \xi^6$. Then it is clear that the tachyon condensation leads to the emergence of D(p-1)-brane stretched in $(x^0, \ldots, x^k, x^7, \ldots, x^{6+p-k})$ directions and where the scalar fields on its worldvolume $t(\xi), Y^{m_1}, Y^{m_2}$ describing embedding of this D(p-1)-brane in nontrivial background, obey the equations of motions that arise from DBI action for BPS D(p-1)-brane.

### 3.2 Non-BPS Dp-brane in Dk-brane background

The second example that we will consider in this paper, is the spatial dependent tachyon condensation on the worldvolume of a non-BPS Dp-brane that moves in the background of $N$ BPS Dk-branes. This background is characterised by following metric and dilaton in the form

$$
\begin{align*}
\text{for } \alpha, \beta = 0, \ldots, k, m, n = k + 1, \ldots, 9, \\
e^{-2\Phi} &= H_{k/2}^{k-3}.
\end{align*}
$$

where the harmonic function $H_p$ takes the form

$$
H_k = 1 + \frac{Ng_s(2\pi)^{7-k}}{(g^m y_m)^{\frac{7-k}{2}}},
$$

where $g^m, m = k + 1, \ldots, 9$ label the directions transverse to the worldvolume of $N$ Dk-branes.

There is again many possibilities how to put in a non-BPS Dp-brane in this background. As the first possibility let us consider a non-BPS Dp-brane that is stretched in $x^0, \ldots, x^p$ directions and that is localised in $Y^{m_1}, m_1 = p + 1, \ldots, k$ directions (parallel with the worldvolume of Dk-branes). This Dp-brane is also localised in $Y^{m_2}, m_2 = k + 1, \ldots, 9$ directions transverse to Dk-branes worldvolume. Now the metric components on its worldvolume take the form

$$
g_{\mu\nu} = H_k^{-1/2} \eta_{\mu\nu}, g_{m_1 n_1} = H_k^{-1/2} \delta_{m_1 n_1}, g_{m_2 n_2} = H_k^{1/2} \delta_{m_2 n_2},
$$

where
where $H_k$ depends on $Y^{m_2}Y_{m_2}$. It is clear that the spatial dependent tachyon condensation (Let us choose $x$ that appears in the ansatz (2.17) to be equal to $\xi^\mu$.) leads to an emergence of a D(p-1)-brane with the worldvolume fields $Y^{m_1}, Y^{m_2}$ as well as with the mode $t(\xi)$ that parametrises the location of D(p-1)-brane in $x^\mu$ direction.

Another possibility occurs when we consider Dp-brane where some of its worldvolume directions are parallel with the worldvolume of Dk-branes and other ones are stretched in the directions transverse to Dk-brane. This situation can be described by following induced metric on the worldvolume of non-BPS Dp-brane:

\[
\begin{align*}
g_{\mu_1\nu_1} &= H_k^{-1/2} \eta_{\mu_1\nu_1}, \mu_1, \nu_1 = 0, \ldots, l, \\
g_{\mu_2\nu_2} &= H_{NS}^{1/2} \delta_{\mu_2\nu_2}, \mu_2, \nu_2 = k + 1, \ldots, (k + 1 + p - l), \\
g_{m_1n_1} &= H_k^{-1/2} \delta_{m_1n_1}, m_1, n_1 = l + 1, \ldots, k, \\
g_{m_2n_2} &= H_{NS}^{1/2} \delta_{m_2n_2}, m_2, n_2 = (k + 2 + p - l), \ldots, 9 \, ,
\end{align*}
\]

where the function $H_k$ is equal to

\[
H_k = 1 + \frac{Ng^s(2\pi)^{\frac{7-k}{2}}}{(\xi^\mu\xi^\nu + Y^{m_2}Y_{m_2})^{\frac{2-k}{2}}} \, .
\]

If now the tachyon depends on one of the coordinates from the set $\xi^1, \ldots, \xi^l$, say $x = \xi^l$, we obtain a D(p-1)-brane that is localised in $x^l$ direction and that is stretched in $x^0, \ldots, x^{l-1}$ and $x^{k+1}, \ldots, x^{(k+1+p-l)}$ directions.

The next possibility corresponds to the tachyon condensation in the direction transverse to Dk-branes, say $\xi^{k+1} \equiv x$. Following the general recipe given in previous section it is clear that this spatial dependent tachyon condensation leads to an emergence of a D(p-1)-brane that is stretched in $(x^0, \ldots, x^l, x^{k+2}, \ldots, x^{k+2+p-l})$ directions and its positions in the transverse space are described with the worldvolume scalar fields $Y^{m_1}(\xi), Y^{m_2}(\xi)$ and also with $t(\xi)$ that parametrises the position of D(p-1)-brane in $x^{k+1}$ direction. It is also clear that these modes obey the equations of motions that follow from the DBI action for probe D(p-1)-brane in the background of $N$ Dk-branes. Note also that the resulting configuration of $N$ Dk-branes and D(p-1)-brane is not generally stable [26].

4. Conclusion

This paper was devoted to the study of the spatial dependent tachyon condensation on the worldvolume of a non-BPS Dp-brane that is moving in nontrivial background. We have shown that this tachyon condensation leads to an emergence of the D(p-1)-brane that is moving in the same background and where the scalar mode that determines the location of the kink on a non-BPS Dp-brane worldvolume can be
interpreted as a mode that describes the transverse position of D(p-1)-brane and that obeys the equation of motion that follows from DBI action for D(p-1)-brane. We hope that this result is a nice example of the efficiently of the effective field theory description of the tachyon condensation and it also gives strong support for the form of the Dirac-Born-Infeld form of the tachyon effective action (2.1).

The extension of this paper is obvious. First of all we would like to study the tachyon condensation when we take into account nontrivial NS $B$ field and also nontrivial Ramond-Ramond field. It would be also interesting to study the tachyon condensation on the supersymmetric form of the non-BPS Dp-brane action. We hope to return to these problems in future.

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