Numerical investigation on vorticity of rectangular turbulent jet enhanced by plasma

Liang Li, Xueke Che, Tikai Zheng, Chuan Chen, Wangsheng Nie and Xiuqian Li *
Space Engineering University, Beijing, China

*Corresponding author e-mail: lixiuqian123@126.com

Abstract. A three-dimensional plasma excitation model based on the phenomenological model was proposed and used as an excitation source of the jet. The large-eddy simulation method was used to simulate the flow field of a rectangular turbulent jet. The position deviation of the large eddy structures in the simulated flow field is within 8% compared with the experimental data. The model can accurately reflect the vortices distribution of the flow field and has a good accuracy. The variation trend of jet vortices is surfaces and vortices field is studied under different excitation voltages, which proves that the stream wise vortex is the key to the influence of the plasma exciter on the jet. The maximum value of velocity and vortices in the flow field enlarge with the voltage increasing while the turbulent position of the jet advances significantly, and the vortex shedding strength increases. The jet mixing characteristics are significantly enhanced.

1. Introduction
Rectangular jet is widely used in industrial production, chemical industry and aerospace propulsion system, which has the characteristics of easy turbulent and fast diffusion rate, thus can effectively enhance the mixing efficiency of the fluid, improve the fuel premix efficiency and realize the improvement of combustion efficiency [1]. Scholars at home and abroad have carried out a large number of theoretical, experimental and simulation studies on the development and flow characteristics of the rectangular turbulent jets. Kaoru et al. [2] used large eddy simulation (LES) model to simulate rectangular jet, the role of large-scale vortex structure in jet development and momentum transfer in different Reynolds numbers is studied, and it is proved that large-scale vortex structure is the main reason for the instability of jet shear layer; Th. Pandas et al. [3] The experimental research on rectangular bounded jets is carried out, and the internal flow field of the rectangular turbulent jet is measured by using the hot-wire anemometer (HWA) probe array, and the experimental results verify the conclusion of [2]. Yang et al. [4] studied the development process of jets with different nozzle shapes, it is found that the distribution of the stream wise vortices and the span wise vortices in the flow field of the rectangular jet have important influence on the jet development and mixing characteristics, and different mixing effects of the rectangular jet can be achieved by changing the outlet Reynolds number and the aspect ratio (AR) of the nozzle.

Active flow control is an important means to realize accurate jet control and adapt to different working conditions. The jet control can be realized by using mechanical adjusting device [5],
controlling jet [6] and magnetic field and so on. Surface dielectric barrier discharge (SDBD) plasma flow control is a promising active flow control method with small additional mass, simple structure and convenient adjustment [7]. SDBD plasma flow control is realized mainly by breakdown the surface air on the electrode using high voltage to form a plasma atmosphere, and then use the potential difference between the electrodes to induce the nearby airflow to produce the jet, thus controlling the flow field. At home and abroad, a large number of theoretical and experimental studies have been carried out on the mechanism of its action.

Simulation is an important means to reveal the control mechanism of SDBD plasma flow control. The phenomenological model does not describe the plasma discharge process in detail, but only gives the effect of the plasma on the gas. It has the characteristics of simple and reliable calculation, and can simulate the surface dielectric barrier discharge induced jet in macroscopic view. Therefore, it plays an important role in the simulation of plasma flow control [8]. For the phenomenological model of SDBD discharge, domestic and foreign scholars have done a lot of research. Suzan Y. B et al [9] proposed a phenomenological model of plasma discharge based on electric field force. Compared with the experimental data, the simulation accuracy of plate induced discharge is higher. In China, Xu eke Chee, Qinghai Chen [10,11] have improved the model, which has been applied to the numerical simulation in the flow environment of adjacent space propeller and low Reynolds number airfoil, which can restrain stall and prevent boundary layer separation.

Based on the previous research, the development of rectangular turbulent jet is modeled and simulated by using three-dimensional SDBD phenomenological model and large eddy simulation, and the flow field structure and jet mixing characteristics under different excitation voltages are investigated. It is helpful to reveal the mechanism and optimal control mode of SDBD plasma flow control.

2. Numerical model

2.1. Phenomenological model
The detailed derivation process of the SDBD plasma phenomenological model under two-dimensional incompressible conditions is given in [9] and it will not be repeated here. During the discharge process, the external potential \( \phi \) is determined by the excitation power source. After the exposed electrode is connected to the high voltage power supply, its boundary conditions can be expressed as:

\[
\phi(t) = \varphi^\text{max} f(t)
\]  

(1)

Where \( \varphi^\text{max} \) the magnitude of the applied voltage and its value is is half of the peak-to-peak value of the excitation voltage VIP-p. In the pulse discharge mode, the voltage change function over time is:

\[
f(t) = \begin{cases} 
\sin(2\pi wt) & nT_m \leq t \leq (n + D)T_m \\
0 & (n + D)T_m \leq t \leq (n + 1)T_m 
\end{cases} \quad n \in \mathbb{N}
\]

(2)

Since the implanted electrode is a ground electrode, its potential is 0; the gradient of the potential at other outer boundaries \( \partial \phi / \partial n = 0 \). In the discharge process, the air ionization region is between the exposed electrode and the implanted electrode, and the charge is mainly distributed above the implanted electrode, and the charge density of other outer boundaries \( \rho_e = 0 \). The charge distribution above the implanted electrode conforms to the Gaussian distribution law:

\[
G(x) = \exp \left[ -\left( x - \mu \right)^2 / (2\sigma^2) \right]
\]

(3)
Where $\mu$ denotes the value of $x$ when the function obtains the maximum value, and $\sigma$ denotes the attenuation coefficient. The change in charge distribution over time of the charge distribution over the implanted electrode over time, $\rho_{e,w}$ can be expressed as:

$$\rho_{e,w}(x,t) = \rho_{e}^{\text{max}} G(x) f(t)$$  \hspace{1cm} (4)

The boundary conditions of the electric potential and the electric charge in the discharge of the SDBD plasma have been obtained. The specific boundary conditions are set as shown in Figure 1:

2.2. Numerical method

The vortex structures play an important role in the process of jet instability. The large-scale vortex structures continuously transfer momentum from the main stream area to the adjacent area of the jet, resulting in the constant attenuation of the centreline velocity. Momentum exchanging increases with the entrainment of the scrolling effect happened between the jet boundary layer and the surrounding fluid. The large vortex structure breaks in the course of development, thereby forming a large number of small vortex structures at positions far from the jet's centreline, further dissipating the remaining momentum. Because the scale of the vortex structure varies greatly, the focus of turbulence numerical calculation is how to deal with the relationship between the calculation speed and the calculation scale. Direct numerical simulation (DNS) can solve the structure of various scales in the flow field by directly solving the N-S equations. The precision of DNS is very high but it consumes computational resources. The Reynolds-averaged N-S method (RANS) can calculate quickly but cannot effectively reflected the vortex structures that play an important role in the turbulent flow field. The large eddy simulation method divides the vortex structures into two types: large eddy and small eddy, by filtering function. It is considered that the large vortex has anisotropy and plays a major role in the flow field thus it is solved directly; meanwhile, the small vortex has isotropic and mainly play a role of dissipating momentum in the flow field, so can establish a model to reflect its impact on large-scale movements. LES can effectively reflect the flow field structure while saving a lot of computing resources [12].

The platform with the existed code is used, and phenomenological model accesses the inner surface of the jet nozzle in the model by $c$. The calculation uses the Smagorinsky-Lilly model, in which the sub lattice turbulent viscosity $\mu_i$ is:

$$\mu_i = \rho L_s^2 S$$  \hspace{1cm} (5)

Where,
\[ S = \sqrt{2S_{ij} S_{ij}} \]  

(6)

\[ L_s = \min(\kappa d, C_s V^{-\frac{3}{2}}) \]  

(7)

and \( L_s \) is the sub lattice mixing length, \( \kappa \) is the Karman constant, \( d \) is the closest distance from the location to the wall, \( C_s \) is the Smagorinsky constant, and \( V \) is the volume of the calculation unit.

2.3. Mesh and boundary conditions

The geometry of the calculation area and the central profile along the stream wise direction, span wise direction and lateral is showed in Figure 2(a) ~ (d). The shape of the nozzle is rectangular, the geometric dimension is width \( b = 27 \text{mm} \), height \( h = 3 \text{mm} \), aspect ratio (AR) is 9; the excitation electrode is placed on the lower surface of the jet outlet, the width of exposed electrode is \( L_1 = 5 \text{mm} \), and the width of the implanted electrode \( L_2 = 9 \text{mm} \), electrode gap \( d = 0 \text{mm} \). The size of the flow body area is \( 150 \text{mm} \times 80 \text{mm} \times 54 \text{mm} \), and the mesh is encrypted around the centerline of the jet to accurately capture changes in the jet shear layer.

![Compute regions and grids.](image)

The inlet boundary conditions were set as flow inlet, its pressure is 1.0 tam, and the outlet was set as pressure outlets, the back pressure of it is 80.0 kappa. The nozzle and wall surfaces were set as non-slip boundaries while the experimental environment pressure was 80.0 kappa. Check out Table 1 for other details. The computing platform is a Think Station P710 tower workstation with two Intel® Xeon E5-2620V4 processors, and the physical core count of a single processor is 8 (16 threads) and the clock speed is 2.1 GHz. The size of running memory is 32 Bit, it took about 156 hours to complete calculating a single jet flow for 1.0 s.

**Table 1. Flow calculation parameters**

| Operating pressure \( p/\text{Pa} \) | Temperature \( T/\text{K} \) | Mass flow in \( m/(\text{kg.s}^{-1}) \) | Density \( \rho/(\text{kg.m}^{-3}) \) | Viscosity \( \mu/(\text{Pays}) \) | \( Re \) |
|---|---|---|---|---|---|
| 80000 | 288 | 3.611 | 1.18 | 1.78*10^{-3} | 1457 |
Three alternative sets of meshes, namely Mesh-A (377486 nodes), Mesh-B (822996 nodes), and Mesh-C (1543975 nodes) were prepared to perform mesh-independence tests. Figure 3(a) shows the distribution along the z-axis of the centreline velocity of the profile at x = 7D, which is dimensionless processed using the maximum inlet velocity for comparison. As can be seen from the figure, the velocity distributions calculated by the three sets of grids are basically the same, and all of them can reflect the fluctuation trend of the rectangular turbulent jets at this position and there is only small fluctuations in the numerical values. Considering the transient irregular nature of turbulent jets, the average velocity distribution cannot effectively reflect the influence of the density of nodes on the merits of the calculation results. Figure 3(b) shows the vortices is-surfaces with different grids at a flow time of 1.0 s. Mesh-A only shows a small number of large-scale span wise vortexes near the nozzles and a partial stream wise vortex, while the calculation results of Mesh-B and Mesh-C contain a large number of small-scale vortices with more detail. Considering that Mesh-C has more meshes, the computation speed is slower, and Mesh-B has reflected enough details of the flow field, so it is finally selected for calculation.

3. Model Verification
Using the experimental data that has been obtained by this research group, verify the model built before.

The experiment used a PIV system in a low-pressure cabin with a pressure of 80.0 kappa to complete the shooting of the central section in the z-direction of the rectangular jet. The inlet pressure of the jet was 1 tam, and the jet was formed by the differential pressure inside and outside the chamber. The jet nozzle size and electrode arrangement are consistent with the simulation model. The electrode configuration is counter-flow, that is, the direction of the induced jet flow generated when the electrode is working is opposite to that of the jet mainstream. The tracer particles adopt white smoke particles generated from the combustion of cigarette cake. The PIV system is mainly composed of a ND: YAG double pulse laser, a synchronous controller, a Kodak scientific-grade chip CCD camera (with a resolution of 2048×2048 ox/in), a computer and an image processing software. The laser repetition frequency is 5 Hz. The images obtained by PIV imaging are processed using Micro Vic V3.1.1 calculation software with a spatial resolution of 2.62 mm. The excitation power supply adopts the HFHV30-1 high-frequency high-voltage power supply developed by the Institute of Electrical Engineering of the Chinese Academy of Sciences. The operation modes are continuous and pulsed. In pulse mode, the operating frequency can be adjusted between 10-3000Hz and the duty cycle can be adjusted from 10% to 90%. The voltage measurement uses the Agilent N2271B high-voltage...
probe, the current measurement uses the Pearson current coil 6595, and the oscilloscope is the Agilent DSO3024A. This article uses the pulse mode of operation; the power carrier frequency is 10 kHz.

Figure 4. Experiment and simulation flow field (VIP-p=6.0kV, f=50Hz, Duty=50%).

The Q-criterion is used to identify the vortex structure in the simulation flow field. For the incompressible two-dimensional flow field, the expression is [13]:

$$Q = \frac{1}{2} \left( \| \Omega \|^2 - \| S \|^2 \right)$$  \hspace{1cm} (8)

\( \Omega \) is the rotational velocity tensor in the rotational motion, \( S \) is the strain tensor in the nonrotating motion, and the expressions are respectively:

$$\Omega = \frac{1}{2} \left[ \nabla U - (\nabla U)^T \right]$$ \hspace{1cm} (9)

$$S = \frac{1}{2} \left[ \nabla U + (\nabla U)^T \right]$$ \hspace{1cm} (10)

For a two-dimensional flow field in the middle section, the Q criterion can be simplified to:

$$Q = \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} - \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2$$ \hspace{1cm} (11)

Calculate the Q value in the flow field, and select the appropriate threshold to filter the small-scale structure. The black solid line represents the flow in the area where the vortex dominates.

Comparing the two images, it can be seen that the jets obtained by the experiment and simulation have a smooth section of a certain length near the outlet, and the length is about 3 times the width of the nozzle. After that, the mainstream began to present a small degree of instability. The instability of the shear layer led to vortex rings on both sides of the mainstream; the occurrence of large vortices resulted in aggravating entrainment of jets and surrounding fluids. As the jet develops further, the width in the short axis direction increases rapidly, and the rectangular jet enters the plane jet section. At the same time, it can also be seen that after the jet is ejected from the outlet, its free surface relaxes faster and decays rapidly along the flow direction (Figure 4(b)), resulting in a large velocity gradient. The existence of this disturbance makes the stability of the jet compromised. Disturbance accelerated its spread [14]. The first two sets of vortex pair centers defining the start of jet instability are the comparison standards. The comparisons under other conditions are shown in Table 2. The comparison
results show that the numerical simulation method captures large-scale structures in the jet flow field. Better, the method chosen for the calculation model is appropriate and the calculation result is reliable.

| Conditions   | First eddy position(x/h) | Second eddy position(x/h) |
|--------------|--------------------------|----------------------------|
| Plasma off   | 6.0                      | 6.0                        |
| 4kV          | 5.0                      | 5.2                        |
| 8kV          | 3.8                      | 3.8                        |

### 4. Influence of voltage on flow field

The established SDBD flow control model was used to study the effect of different excitation voltages on the disturbance of a rectangular turbulent jet flow field. The electrode configuration of the simulation model uses the inverse configuration [15], that is, the direction of the induced jet generated by the electrode is opposite to that of the mainstream flow of the jet (Figure 5). When the reverse-type jet works, the reversed induced jet will cause the shear layer of the jet to destabilize, and the induced vortices formed above the electrode wall will then incorporate a part of the momentum into the mainstream, increasing the exit Reynolds number, thus achieve the promotion effect.

Figure 5 shows the vortices is surface of the jet flow under different excitation voltages. For comparison, all is surface data have a vortex value of 372 s⁻¹. The vortices is surface was smooth before x/h<17.0 without excitation, indicating that the jet shear layer was relatively stable and there was no significant disturbance; after this position, a pair of large The scale vortex structure appears, which is also the sign of the instability of the jet, after that the large vortex structure begins to distort and break, accompanied by the vortex shedding and the direction of the vortex ring. After the excitation is applied, the vortices is surface undergoes significant deformation. The first is that the jet instability position is significantly advanced, and the number of vortex structures in the destabilizing position increases. On the one hand, the SDBD plasma exciter induces a part of the vortex structure. Due to the induced jet perturbation, the mainstream of the jet itself begins to destabilize and the vortex structure increases. With the increasing of the excitation voltage, two significant changes occur in the vortices profile. First, the deformation strength increases. When the excitation voltage is 4 kV, the vortex profile with a stable initial emission phase has only a small part of the protruding structure. At 8Kv, the vortices profile at the initial stage of destabilization has a tendency to disengage from the mainstream, the position that the large vortex break is ahead. Under low excitation voltage, the vortex shedding at x/h=20.0 is smaller than the profile value, but in high voltage, the eddy shedding position is about 5h ahead than low voltage, which proves that the excitation voltage has a significant gain effect on the control effect.
Figure 6. Vortices is-surface in different voltages.

Figure 7 shows the vortices distribution of the profile y=0 under four operating conditions. The ratio of exit velocity U and nozzle height h is used for dimensionless processing. Without excitation, the initial vortices is concentrated and mainly distributed in the jet shear layer, but the scale is small and has no great influence on the shear layer. With the increasing of entrainment, the natural vortex structure appears in the position around $x/h=23.0$. At this time, the consumption of large vortices to the mainstream velocity is very obvious, and the large vortices develop to a certain extent and begin to distort and break. A large number of small vortices are formed by the shedding vortex structure, which accelerates the momentum dissipation velocity at the back of the flow field. With the increasing of the excitation voltage the maximum vortices in the flow field increases and the center of the jet is disturbed more obviously. It is proved that the induced vortex plays an important role in the initial evolution of the jet.

Figure 7. Vortices distribution in different excitation voltage flow fields.
Through the experimental measurements of the initial stage of jet development (especially before the start of the instability), Pandas [3] et al found that the vortex which plays a leading role in the jet development process can be divided into two types according to its direction: the stream wise vortex is and the span wise vortex in, the expression is:

\[ \Omega_s = \frac{D}{U} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \] (12)

\[ \Omega_u = \frac{D}{U} \sqrt{ \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)^2 } \] (13)

The exit velocity U and nozzle equivalent diameter D are used to dimensionless the two vortices. It is considered that the span wise vortex in the jet is the most obvious, while the stream wise vortex is relatively concealed, so it is difficult to observe by means of observation. The strength of the span wise vortex is higher relatively, but the stream wise vortex plays a key role in promoting the distortion and fragmentation of the span wise vortex [17]. The conflict between the two vortices increase the momentum of the jet in the early development and promote turbulence.

![Span wise vortices variation trend](image1)

![Stream wise vortices variation trend](image2)

**Figure 8.** Variation trend of span wise vortices and stream wise vortices in different voltage.

In order to further investigate the influence of stream wise vortex and span wise vortex on jet flow in the initial stage of jet development, the variation of stream wise vortex and span wise vortex in the range of \( x < 13D(x<24h) \) in different excitation voltage was extracted (Figure8). It can be seen that the strength of the span wise vortex is obviously larger than that of the stream wise vortex by about an order of magnitude, and the two vortices have a sharp downward trend near the exit, which is consistent with the results obtained in [4]. The difference is that the stream wise vortex fluctuates to a certain extent, and the maximum value of the stream wise vortex increases obviously with the increase of the excitation voltage amplitude of variation. At the same time, the amplitude of variation of the span wise vortex is very small, which proves that the SDBD plasma exciter indirectly affects the stronger span wise vortex mainly by influencing the stream wise vortex, and promotes the turbulence of the jet by promoting its distortion and fragmentation.
5. Conclusion
By establishing the phenomenological model of SDBD plasma, the development of rectangular turbulent jet under different excitation voltages was simulated by using the method of large eddy simulation. The calculation model is in good agreement with the experimental data, and the accuracy is high. The analysis of the calculated results further explains the process of the SDBD plasma exciter acting on the jet. It is proved that the variation of the vortex structure, especially the stream wise vortex, is an important part of the excitation process. Further theoretical basis is provided for the study of plasma flow control.

Acknowledgements
The paper is financially supported by the national nature and science foundation of China (code: 51777214, 91441123).

References
[1] Hu H, Saga T, Kobayashi T, et al. Mixing Process in a Lobed Jet Flow. Aiwa Journal, 2012, 40(7):1339-1345.
[2] Kaoru Iwamoto, Nobuhide Kasagi, Yuji Suzuki. Dynamical Roles of Large-Scale Structures in Turbulent Channel Flow. WCCM VI in conjunction with APCOM’04, 2004, Beijing, China.
[3] Pandas T, Schwab R, Pollard A. The role of vortices in the near field development of sharp-edged, rectangular, wall jets. International Journal of Heat & Fluid Flow, 2017, 67.
[4] Yang X, Long X, Yao X. Numerical investigation on the mixing process in a steam ejector with different nozzle structures. International Journal of Thermal Sciences, 2012, 56(2):95-106.
[5] Davis M. R. Variable control of jet decay. AIAA Journal, 1982, 20(5):606-609.
[6] Raman G. Using Controlled Unsteady Fluid Mass Addition. AIAA Journal, 1997, 35(4):647-656.
[7] Nie W S, Cheng Y F, Che X K. Research progress of dielectric barrier plasma flow control. Advances in Mechanics, 2012, 42(6):722-734.
[8] Shao T, Yan P. Atmospheric pressure gas discharge and its plasma application. Science Press, 2015.
[9] Suzen Y. B., Huang P. G., Jacob J.D., et al. AIAA. Numerical Simulations of Plasma Based Flow Control Applications. Aiaa Fluid Dynamics Conference and Exhibit, 2005.
[10] Chen Q Y, Tian X H, Jiang J W, et al. Propeller Plasma Flow Control Enhancement Experiment. Journal of Aerospace Power, 2016, 31(5):1205-1211.
[11] Che X K, Nie W S, Zhou P H, et al. Study on the improvement of plasma volume force image and model. National Fluid Mechanics Conference, Nanjing. October 20-23, 2016.
[12] Zhao P H, Ye T H, Zhu Y M, et al. Large Eddy Simulation of Turbulent Flow in a Circular Jet [J]. Journal of Engineering Thermophysics, 2012, 33(3):529-532.
[13] Zhang Z, Che X K, Nie W S, et al. Study of vortex in flow fields induced by surface dielectric barrier discharge actuator at low pressure based on Q criterion. Plasma Science & Technology, 2018, 20(1):014006.
[14] Kozato Y, Kikuchi S, Imao S, et al. Flow control of a rectangular jet by DBD plasma actuators [J]. International Journal of Heat & Fluid Flow, 2016, 81(825):14-00695-14-00695.
[15] Cheng Yufeng, Che Xueke, Nie Wangheng. Numerical study on propeller flow -separation control by DBD-plasma aerodynamic actuation. IEEE Transactions On Plasma Science, 2013, 41 (4): 892-898.
[16] Jung D, Gamard S, George W K. Downstream evolution of the most energetic modes in a turbulent axisymmetric jet at high Reynolds number. Part 1. The near-field region. Journal of Fluid Mechanics, 2004, 514(514):173-204.
[17] Pescini E, Marra F, Giorgi M G D, et al. Investigation of the boundary layer characteristics for assessing the DBD plasma actuator control of the separated flow at low Reynolds numbers. Experimental Thermal & Fluid Science, 2017, 81:482-498.