An investigation of triply heavy baryon production at hadron colliders

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Abstract

The triply heavy baryons have a rather diverse mass range. While some of them possess considerable production rates at existing facilities, others need to be produced at future high energy colliders. Here we study the direct fragmentation production of the \( \Omega_{ccc} \) and \( \Omega_{bbb} \) baryons as the prototypes of triply heavy baryons at the hadron colliders with different \( \sqrt{s} \). We present and compare the transverse momentum distributions of the differential cross sections, \( p_{min}^T \) distributions of total cross sections and the integrated total cross sections of these states at the RHIC, the Tevatron Run II and the CERN LHC.

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1 Introduction

Study of hadron production and decay has always been interesting. Historically it has served to illuminate both the collider physics and the fundamental theories of interactions specially in the strong and electroweak sectors. Recently heavy hadrons have received great attention. Their structure is predicted by the constituent quark model and, wherever light quarks are absent, they are nicely treated within the framework of the effective field theory and the perturbative QCD [1].

Singly and doubly heavy baryons, \( \Lambda \)'s and \( \Xi \)'s , exhibit interesting properties and due to the involvement of the light degrees of freedom, they have been

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studied in special models [2]. Triply heavy baryons are the heaviest composite states predicted by the constituent quark model. They are the last generation of baryons within the standard model. Essentially they are the $\Omega_{ccc}$, $\Omega_{cbb}$, and $\Omega_{bbb}$ baryons. It has become clear that while the $\Omega_{ccc}$ and $\Omega_{bbb}$ should be fragments of a $c$ and a $b$ quark respectively, the $\Omega_{cbb}$ and $\Omega_{bbb}$ may be produced each in a $c$ or in a $b$ quark fragmentation. The fragmentation of all triply heavy baryons has recently been studied and their production has been estimated at the CERN LHC [3]. However the wide range of their fragmentation probabilities ($10^{-4}$ - $10^{-7}$), higher cross section of charm production at the Tevatron and RHIC and finally different acceptance cuts apart from different $\sqrt{s}$ for different colliders, suggest their production to be explored in other colliders as well. For the matter of simplicity, in this work we have chosen to investigate the production of the lightest ($\Omega_{ccc}$) and the heaviest ($\Omega_{bbb}$) as prototypes of the triply heavy baryons at the RHIC, the Tevatron Run II and the CERN LHC colliders.

2 Fragmentation functions

To evaluate the cross section of $\Omega_{ccc}$ and $\Omega_{bbb}$ baryons at hadron colliders in a factorized scheme, we need their fragmentation functions. Indeed, it is possible to describe the fragmentation of these states by a single fragmentation function for $Q \rightarrow \Omega_{QQQ}$ which has already been calculated [4]. In our model we have considered the emission of two gluons by the heavy quark $Q$, each producing a $\overline{Q}Q$ pair. The three heavy quarks thus obtained form the $\Omega_{QQQ}$ bound state leaving the heavy anti-quarks to form the final state jet. The bound state is characterized by the baryon decay constant $f_B$. The fragmentation process in leading order is described in Figure 1. Kinematically we have let the original heavy quark to keep its original transverse momentum $q_T$ and have ignored the respective motion of the constituents within the bound state. This perturbative picture is evaluated at the scale $\mu = \mu_\circ$. The scale $\mu_\circ$ which is a scale at which such calculations are possible, is in the order of total mass of all final state particles, namely $5m_c$ for $c \rightarrow \Omega_{ccc}$ and $5m_b$ for $b \rightarrow \Omega_{bbb}$ states. The result is the following fragmentation function

$$D_{Q \rightarrow QQQ}(z, \mu_\circ) = \frac{\pi^4 \alpha_s^4(2m_Q)f_B^2 G_F^2}{108m^2 z^4(1 - z)^4 f(z)^2 g(z)^6} \times \left[ \xi^8 z^8 + 4\xi^6 z^6(83 - 130 z + 51 z^2) 
+ 6\xi^4 z^4(1413 - 3084 z + 3022 z^2 - 2156 z^3 + 821 z^4) 
+ 4\xi^2 z^2(18711 - 51678 z + 69417 z^2 - 70308 z^3 
+ 53529 z^4 - 25950 z^5 + 6343 z^6) + 222345 - 740664 z 
+ 1179036 z^2 - 1253448 z^3 + 90126 z^4 - 388872 z^5 \right]$$
Fig. 1. The lowest order Feynman diagram illustrating the fragmentation process of a heavy quark $Q$ into a triply heavy baryon $\Omega_{QQQ}$ with identical constituent flavors.

$$+109916z^6 - 49912z^7 + 20649z^8,$$  \hspace{1cm} (1)

where $\alpha_s$ is the strong interaction coupling constant evaluated at the pair of vertices of each gluon in the Figure 1, $f_B$ is the baryon decay constant which is defined in a similar manner to the meson decay constant, $f_M$ and $C_F$ is the color factor of the baryon state formed in the fragmentation of the heavy quark. Moreover, here we have defined $\xi = \langle q_T^2 \rangle / m^2$ with $q_T$ being the transverse momentum of the initial heavy quark and $m$ is the heavy quark mass. The two functions $f(z)$ and $g(z)$ are defined as

$$f(z) = -\frac{\langle q_T^2 \rangle}{3m^2} + \frac{3}{z} + \frac{4}{3} \left[ 1 + \frac{\langle q_T^2 \rangle}{4m^2} \right] \frac{1}{1-z}, \quad g(z) = -\frac{1}{3} + f(z).$$  \hspace{1cm} (2)

The function $f(z)$ is the contribution of the energy denominator emerging from the phase space integration and the function $g(z)$ is due to the quark and gluon propagators.

The inputs for the fragmentation function (1) are the quark mass, baryon decay constant and the color factor. We have set $m = m_c = 1.25$ GeV and $m = m_b = 4.25$ GeV. For the decay constant and the color factor we have taken $f_B = 0.25$ GeV and $C_F = 7/6$ for both cases of the $\Omega_{ccc}$ and $\Omega_{bbb}$ states.
3 Inclusive production of a $\Omega_{QQQ}$ baryon

Inclusive production of a $\Omega_{QQQ}$ baryon state in a hadronic collision is fulfilled in certain stages. Collision of hadrons provide the production of the required parton or partons which eventually fragment into a $\Omega_{QQQ}$ state. The theoretical evaluation of this process is only possible at sufficiently high transverse momentum and taking advantage of the parton model factorization. All this is possible at a scale much higher than the calculable scale of the fragmentation functions. Therefore the fragmentation functions which are calculated at fragmentation scale, are evolved up to a scale at which the convolution of parton distribution functions, bare cross section of the initiating heavy quark, and the fragmentation function is possible. In other words, for the $pp$ collision we may write

$$\frac{d\sigma}{dp_T} [pp \rightarrow \Omega_{QQQ}(p_T) + X] = \sum_{i,j} \int dx_i dx_j dz f_{i/p}(x_i, \mu) f_{j/p}(x_j, \mu) \times [\hat{\sigma}(ij \rightarrow Q(p_T/z) + X, \mu) D_{Q \rightarrow \Omega_{QQQ}}(z, \mu)].$$

(3)

Here $f_{i/p}(x_i, \mu)$ and $f_{j/p}(x_j, \mu)$ are the parton distribution functions for the initial partons $i$ and $j$ carrying fractions $x_i$ and $x_j$ of the total momentum in the protons, $\hat{\sigma}$ is the heavy quark production cross section and $D_{Q \rightarrow \Omega_{QQQ}}(z, \mu)$ represents the fragmentation of the produced heavy quark into a triply heavy baryon. Note that here the scale $\mu$ in the parton distribution functions, subprocess cross sections and the fragmentation functions are set to be equal. For the parton distribution functions we have employed the parameterization due to Martin-Roberts-Stirling (MRS) [5], and have included the heavy quark production cross section up to the order of $\alpha_s^3$ [6]. The $\sqrt{s}$ and acceptance cuts for the colliding facilities used in this work appear in Table 1.

The production rates should not depend on the choice of the scale $\mu$ if both the production of high-energy partons and the fragmentation functions in all orders in $\alpha_s$ in the perturbation expansion are included. However, only the results for next leading order parton production cross section and the leading order fragmentation function are available. Therefore the results obtained form (3) will depend on the scale $\mu$. It is usual to choose the transverse mass of the heavy quark as the central choice of scale defined by $\mu_R = \sqrt{p_T^2(\text{parton}) + m_Q^2}$. If such a choice or sometimes multiple of it happens to be less than the fragmentation scale, then the larger of ($\mu$, $\mu_o$) is chosen as appropriate scale. We have used the following form of the Altarelli-Parisi equation [7] to evolve our fragmentation functions
\[ \mu \frac{\partial}{\partial \mu} D_{Q \rightarrow QQQ}(z, \mu) = \int_z^1 \frac{dy}{y} P_{Q \rightarrow Q}(z/y, \mu) D_{Q \rightarrow QQQ}(y, \mu). \]  

(4)

Here \( P_{Q \rightarrow Q}(x = z/y, \mu) \) is the Altarelli-Parisi splitting function. Note that only the term \( P_{Q \rightarrow Q} \) is included in (4). The reason is that the quark \( Q \) is assumed to be heavy enough to make other contributions irrelevant. The boundary condition on the evolution equation (4) is the initial fragmentation function \( D_{Q \rightarrow QQQ}(z, \mu) \) evaluated at the fragmentation scale \( \mu = \mu_o \). It should also be mentioned that the evolution of the fragmentation functions also sums up the logarithms of the order of \( \mu_R/m_Q \) in the fragmentation functions.

To check the sensitivity of our results, we have examined the behavior of the differential cross sections of \( \Omega_{ccc} \) and \( \Omega_{bbb} \) at the RHIC, the Tevatron Run II and the LHC for different scales ranging from the fragmentation scale up to the scales below the \( Z^0 \) boson mass. It is seen that the sensitivity decreases with increasing scale. Such a study have led us to select appropriate scales for our further investigations. In this way we were motivated to choose the scales of \( 4\mu_R \) and \( 6\mu_R \) for the \( \Omega_{ccc} \) and \( \Omega_{bbb} \) states respectively.

4 Results and discussion

In summary we have considered \( \Omega_{ccc} \) (the lightest) and \( \Omega_{bbb} \) (the heaviest) triply heavy baryons to evaluate their production rates at the hadron colliders. To accomplish this, we have employed the next leading order results for parton production cross sections. The fragmentation functions used here are calculated in leading order perturbation theory. They provide reliable fragmentation probabilities for the triply heavy baryons. To evaluate the cross sections, the well known patron model factorization at high transverse momentum is employed. This procedure allows the complicated mechanism of hadron production to be treated in a factorized manner. This is possible in a scale which is higher than the scale at which the fragmentation functions are calculated. The Altarelli-Parisi evolution equation relates these scales. In the evolution of the fragmentation functions we have included only the \( P_{Q \rightarrow Q} \) splitting function. Here, our evaluation of cross sections for the triply heavy baryons is devoted to different hadron colliders. Each collider with detection system has restrictions on the measurements of the transverse momentum and the rapidity of the particles. The so called acceptance cuts for the colliders considered here appear in Table 1.

First we present the transverse momentum, \( p_T \), distributions of the differential cross sections at different hadron colliders for \( \Omega_{ccc} \) and \( \Omega_{bbb} \). They appear in Figures 2 and 3. Clearly, the distributions are sensitive to the choice of \( \mu \). Our choice of \( \mu = 4\mu_R \) for \( \Omega_{ccc} \) and \( \mu = 6\mu_R \) for \( \Omega_{bbb} \) is at the region of
Table 1
The center of momentum energy ($\sqrt{s}$) and the acceptance cuts for the colliding facilities used in this work. The rapidity is defined as $y = \frac{1}{2} \log\left(\frac{E - p_L}{E + p_L}\right)$.

|          | RHIC | Tevatron Run II | CERN LHC |
|----------|------|----------------|----------|
| $\sqrt{s}$ [GeV] | 200  | 1960           | 14000    |
| $p_T^{\text{cut}}$ [GeV] | 2    | 6              | 10       |
| $y \leq$ | 3    | 1              | 1        |

Table 2
The total integrated cross section in pb for $\Omega^{ccc}$ and $\Omega^{bbb}$ baryons at different hadron colliders. The decimal places may not be significant within the uncertainties involved.

| Process                  | RHIC  | Tevatron Run II | CERN LHC |
|--------------------------|-------|----------------|----------|
| $c \to \Omega^{ccc}$ (at $\mu = 4\mu_R$) | 2758.3 | 383.0          | 308.0    |
| $b \to \Omega^{bbb}$ (at $\mu = 6\mu_R$)  | 3.3   | 6.0            | 8.4      |

the scale with minimum sensitivity. Although the cross section for a given $p_T$ differs up to three orders of magnitude from one collider to the other, they are still comparable. The difference is seen to grow with increasing $p_T$. It is more interesting in the case of $\Omega^{ccc}$ where the distributions seem to converge at sufficiently low $p_T$. Another important feature about these distributions is the rather high cross section of $\Omega^{ccc}$ which is more striking in the case of RHIC where low $p_T$’s are available. Figures 4 and 5 show the total cross sections for production of $\Omega^{ccc}$ and $\Omega^{bbb}$ with transverse momentum above a minimum value $p_T^{\text{min}}$. Note that in Figures 2 and 3 only the range $p_T > p_T^{\text{min}} = p_T^{\text{cut}}$ were considered. It seems that the above mentioned general features, hold in the case of $p_T^{\text{min}}$ distributions, unless for that the difference in the differential cross sections for a given $p_T$ is not as much as that of $p_T^{\text{min}}$ distributions. Another general feature of our distributions is that the fall off decreases with increasing $\sqrt{s}$ and also with increasing $p_T^{\text{min}}$ or $p_T$.

We have also calculated the integrated total cross sections. They appear in Table 2. Note that the cross section for $\Omega^{bbb}$ increases with increasing $\sqrt{s}$. But this is not the case for $\Omega^{ccc}$. Not only the order reverses in this case, but the cross section at RHIC is nearly one order of magnitude higher.

What can we say about other triply heavy baryons? It seems that the baryons with at least two $c$ or at least two $b$ quarks, apart from the magnitude of their cross sections, behave in a similar fashion [3]. Therefore we expect that the $\Omega^{ccb}$ state produced in $c$ or $b$ quark fragmentation to have similar distributions as $\Omega^{ccc}$. Likewise the $\Omega^{cbb}$ state emerging from a $c$ or a $b$ quark will behave like $\Omega^{bbb}$ state. It is also interesting to note that our choice of $\Omega^{ccc}$ and $\Omega^{bbb}$ with fragmentation probabilities of about $2 \times 10^{-5}$ and $6 \times 10^{-7}$ are not
the states with maximum and minimum fragmentation probabilities. In other words, the lightest/heaviest of triply heavy baryons does not mean the state with maximum/minimum fragmentation probabilities. Indeed the fragmentation probabilities for $b \rightarrow \Omega_{ccb}$ and $c \rightarrow \Omega_{cbb}$ possess the maximum and the minimum fragmentation probabilities of $2 \times 10^{-4}$ and $10^{-7}$ respectively.

At the end we would like to discuss the reliability of our results. We have employed a fragmentation function obtained in a particular model. Without referring to its ingredients, it is worth mentioning that the predictions of this model is consistent with similar results in the case of doubly heavy baryons evaluated by Doncheski et al in [2] and also experimental results of [9]. This kind of comparison is presented in [3]. Moreover, similar fragmentation functions predict consistent fragmentation probabilities for $J/\psi$ and $\Upsilon$ states [8]. The remaining uncertainties are related to the calculation of the cross sections. Here the the parton densities and the scales are relevant. These kinds of uncertainties are well described in the literature. Therefore, we believe the only factors which may alter our results are the consideration of higher order fragmentation functions and the heavy quark production cross sections.
Fig. 3. The same as Fig. 2 but in the case of $\Omega_{bbb}$ production at the scale of $6\mu_R$.

Fig. 4. The $p_T^{\text{min}}$ distribution of the total cross section in pb for $\Omega_{ccc}$ production at the RHIC, the Tevatron Run II and the CERN LHC hadron colliders at the scale of $\mu = 4\mu_R$. 
Fig. 5. The same as Fig. 4 but for the case of $\Omega_{b\bar{b}b}$ at the scale of $\mu = 6\mu_R$.

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