Power minimization for OFDM Transmission with Subcarrier-pair based Opportunistic DF Relaying

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Abstract—This paper develops a sum-power minimized resource allocation (RA) algorithm subject to a sum-rate constraint for cooperative orthogonal frequency division modulation (OFDM) transmission with subcarrier-pair based opportunistic decode-and-forward (DF) relaying. The improved DF protocol first proposed in [1] is used with optimized subcarrier pairing. Instrumental to the RA algorithm design is appropriate definition of variables to represent source/relay power allocation, subcarrier pairing and transmission-mode selection elegantly, so that after continuous relaxation, the dual method and the Hungarian algorithm can be used to find an (at least approximately) optimum RA with polynomial complexity. Moreover, the bisection method is used to speed up the search of the optimum Lagrange multiplier for the dual method. Numerical results are shown to illustrate the power-reduction benefit of the improved DF protocol with optimized subcarrier pairing.

Index Terms—Cooperative communication, resource allocation, decode and forward, OFDM.

I. INTRODUCTION

Cooperative orthogonal frequency division modulation (OFDM) transmission with subcarrier-pair based decode-and-forward (DF) relaying and associated resource allocation (RA) were studied in [2]–[5] when the source-to-destination link exists. In [2]–[4], an “always-relaying” DF protocol was considered. To better exploit the frequency-selective channels, we have proposed an opportunistic DF relaying protocol in [5], i.e., a subcarrier in the first time slot can either be paired with a subcarrier in the second slot for the relay-aided transmission, or used for the direct source-to-destination transmission without the relay’s assistance. A major drawback for that DF protocol is that, a subcarrier unused for relaying in the second slot becomes idle, which wastes spectrum resource. To address this issue, we first proposed in [1] an improved DF protocol, which allows the source to make the direct transmission over the subcarriers unused for relaying in the second slot. This protocol and its RA were later intensively investigated, e.g., in [6]–[11]. Note that in [1], [5]–[9], a priori subcarrier pairing was used. i.e., when the relay-aided transmission is used for a subcarrier in the first slot, the same subcarrier in the second slot is always paired with this subcarrier for DF relaying. In [2]–[4], [10], [11], the optimization of subcarrier pairing was considered.

Recently, energy-efficient communication is becoming increasingly important [12]. In view of the fact that many existing works focuses on spectral-efficiency maximized RA, we develop an RA algorithm for minimizing the sum power subject to a sum-rate constraint for the improved DF protocol with optimized subcarrier pairing (OSP). Compared with the algorithms designed in [4], [10], our algorithm uses a new method to define indicator variables for representing subcarrier pairing and transmission-mode selection, by regarding the subcarriers for the direct transmission in the two time slots as virtual subcarrier pairs. Moreover, the bisection method is used to find the optimum Lagrange multiplier, which is faster than the incremental-update based subgradient method used in [4], [10].

Notations: \( C(x) = \frac{1}{2} \log_2(1 + x) \) and \( [x]^+ = \max \{x, 0\} \).

II. SYSTEM AND PROTOCOL DESCRIPTION

Consider the scenario where a relay assists a source’s transmission to a destination. The improved DF protocol in [1] is used. Specifically, every data-transmission session takes two consecutive equal-duration time slots and OFDM with \( K \) subcarriers is used. To facilitate description, a subcarrier used in the first slot is denoted by subcarrier \( k \) and one in the second slot by subcarrier \( l \) hereafter. In the first time slot, the source radiates OFDM symbols, using \( P_{s,k,1} \) as the transmit power for subcarrier \( k \). The source-to-relay and source-to-destination baseband-channel coefficients for subcarrier \( k \) are \( h_{sr,k} \) and \( h_{sd,k} \), respectively. In the second slot, both the source and the relay synchronously radiate OFDM symbols, using \( P_{s,l,2} \) and \( P_{r,l,2} \) as the transmit powers for subcarrier \( l \), respectively. The relay-to-destination baseband-channel coefficient is \( h_{rd,l} \) for subcarrier \( l \).

A subcarrier in the first slot can either be paired with one in the second slot for the relay-aided transmission, or be used for the direct transmission without relaying. Every unpaired subcarrier in the second slot is used for the direct transmission. In particular, if subcarrier \( l \) is used for the direct transmission, \( P_{s,l,2} \geq 0 \) is used while \( P_{r,l,2} = 0 \) is imposed. The maximum average data rates over subcarriers \( k \) and \( l \) used for the direct transmission are \( C(P_{s,k,1}G_{sd,k}) \) and \( C(P_{s,l,2}G_{sd,l}) \) bits/OFDM-symbol (bps), respectively, where \( G_{sd,k} = \frac{h_{sd,k}}{\sigma^2} \) and \( \sigma^2 \) is the noise variance for each subcarrier at every node’s receiver. When subcarrier \( k \) is paired...
with subcarrier $l$ for the relay-aided transmission, the DF relaying is used in which case $P_{s,l,2} = 0$ is imposed while $P_{s,k,1} \geq 0$ and $P_{t,l,2} \geq 0$ are used (more details are available in [1]). Suppose $P_{s,k,1} + P_{t,l,2} = P$, it can readily be shown that the maximum data rate is equal to $R_{k,l} = C(G_{kl} P)$ bps, where

$$G_{kl} = \begin{cases} \frac{G_{sr,k} G_{rd,l}}{G_{sr,k} - G_{sr,k} G_{rd,l} + G_{sd,k}} & \text{if } \min\{G_{sr,k}, G_{rd,l}\} > G_{sd,k}, \\ \min\{G_{sr,k}, G_{rd,l}\} & \text{if } \min\{G_{sr,k}, G_{rd,l}\} \leq G_{sd,k}, \end{cases}$$

and $G_{sr,k} = \frac{|h_{sr,k}|^2}{\sigma^2}$ and $G_{rd,l} = \frac{|h_{rd,l}|^2}{\sigma^2}$. This maximum rate is achieved when $P_{s,k,l} = \frac{G_{rd,l}}{G_{sr,k} - G_{sr,k} G_{rd,l} + G_{sd,k}} P$ if $\min\{G_{sr,k}, G_{rd,l}\} > G_{sd,k}$, and if $\min\{G_{sr,k}, G_{rd,l}\} \leq G_{sd,k}$.

Assume there exists a central control unit which knows precisely $\{G_{sr,k}, G_{sd,k}\}_{k}\forall k$ and $\{G_{rd,l}\}_{l}\forall l$, and determines the optimum RA (i.e., the source/relay power allocation, subcarrier pairing and transmission mode selection) to minimize the sum power subject to the constraint that the sum rate is not smaller than prescribed $R_{req}$ bps.

### III. RA Algorithm Design

For any subcarrier assignment used by the improved DF protocol, suppose $m$ subcarrier pairs are assigned to the relay-aided transmission, then it is always possible to one-to-one associate the unpaired subcarriers in the two slots to form $K - m$ virtual subcarrier pairs for the direct transmission. Motivated by this observation, the RA problem is formulated by defining:

- $t_{kl}^R \in \{0, 1\}$ and $P_{kl} \geq 0, \forall k, l$, $t_{kl}^D = 1$ indicates that subcarrier $l$ is paired with subcarrier $k$ for the relay-aided transmission. When $t_{kl}^R = 1$, $P_{kl}$ is used as the total power for the subcarrier pair ($k, l$).
- $t_{kl}^D \in \{0, 1\}$, $\alpha_{kl} \geq 0$ and $\beta_{kl} \geq 0, \forall k, l$, $t_{kl}^D = 1$ indicates that subcarriers $k$ and $l$ form a virtual subcarrier pair for the direct transmission. When $t_{kl}^D = 1$, $P_{S,k,l}$ and $P_{s,l,2}$ take the value of $\alpha_{kl}$ and $\beta_{kl}$, respectively.

Let us collect all indicator and power variables in the sets $I$ and $P$, respectively, and define $S = \{I, P\}$. The RA problem can be formulated as the problem (P1):

$$\begin{aligned}
& \text{min}_S \sum_{k,l} \left( t_{kl}^R P_{kl} + t_{kl}^D \alpha_{kl} + t_{kl}^D \beta_{kl} \right), \\
& \text{s.t.} \quad t_{kl}^R, t_{kl}^D \in \{0, 1\}, \forall k, l; \\
& \quad \sum_{l} \left( t_{kl}^D + t_{kl}^R \right) = 1, \forall k; \quad \sum_{k} \left( t_{kl}^D + t_{kl}^R \right) = 1, \forall l; \\
& \quad P_{kl} \geq 0, \alpha_{kl} \geq 0, \beta_{kl} \geq 0, \forall k, l; \\
& \quad f(S) \geq R_{req},
\end{aligned}$$

where $f(S)$ represents the maximum sum rate as $f(S) = \sum_{k,l} \left( t_{kl}^R C(G_{kl} P_{kl}) + t_{kl}^D C(G_{sd,k} \alpha_{kl}) + t_{kl}^D C(G_{sd,l} \beta_{kl}) \right)$.

Obviously, (P1) is a nonconvex mixed-integer nonlinear program. To find the optimum $S$, we first relax all indicator variables to be continuous within $[0, 1]$. Then, we make the change of variables (COV) from $P$ to $\tilde{P} = \{P_{kl}, \alpha_{kl}, \beta_{kl}\}_{k,l}$, where $\tilde{P}_{kl}, \alpha_{kl}$ and $\beta_{kl}$ satisfy $P_{kl} = t_{kl}^R P_{kl}, \alpha_{kl} = t_{kl}^D \alpha_{kl}$ and $\beta_{kl} = t_{kl}^D \beta_{kl}$, respectively, $\forall k, l$. After collecting all variables into $X = \{I, \tilde{P}\}$, the RA problem can be rewritten as the problem (P2):

$$\begin{aligned}
& \text{min}_X \quad \tilde{P}(X) = \sum_{k,l} (\tilde{P}_{kl} + \alpha_{kl} + \beta_{kl}) \\
& \text{s.t.} \quad t_{kl}^R, t_{kl}^D \in [0, 1], \forall k, l; \\
& \quad \sum_{l} \left( t_{kl}^D + t_{kl}^R \right) = 1, \forall k; \quad \sum_{k} \left( t_{kl}^D + t_{kl}^R \right) = 1, \forall l; \\
& \quad P_{kl} \geq 0, \alpha_{kl} \geq 0, \beta_{kl} \geq 0, \forall k, l; \\
& \quad -g(X) \leq -R_{req},
\end{aligned}$$

where $g(X)$ represents the maximum sum rate expressed as $g(X) = \sum_{k,l} \left( \phi(t_{kl}^R, \tilde{P}_{kl}, G_{kl}) + \phi(t_{kl}^D, \alpha_{kl}, G_{sd,k}) + \phi(t_{kl}^D, \beta_{kl}, G_{sd,l}) \right)$, and

$$\phi(t, x, G) = \begin{cases} t C(G_{kl} \tilde{P}_{kl}) & \text{if } t > 0, \\
0 & \text{if } t = 0. \end{cases}$$

(1)

Obviously (P2) is a relaxation of (P1). We will find an (at least approximately) optimum solution for (P2), and show that the $S$ corresponding to this solution is still feasible, and hence (at least approximately) optimum for (P1). To this end, note that $\phi(t, x, G)$ with fixed $G$ is a continuous and concave function of $t \geq 0$ and $x$, because it is a perspective function of $C(Gx)$ which is concave of $x$ [13]. As a result, $g(X)$ is a concave function of $X$ in its feasible domain for (P2). This means that (P2) is a convex optimization problem. As can be checked, it also satisfies the Slater constraint qualification, therefore it has zero duality gap, which justifies the applicability of the dual method to find the globally optimum for (P2), denoted as $X^*$ hereafter.

To use the dual method, $\mu$ is introduced as a Lagrange multiplier for the rate constraint. The Lagrange relaxation problem for (P2) is the problem (P3):

$$\begin{aligned}
& \text{min}_X \quad L(\mu, X) = \tilde{P}(X) + \mu \left( R_{req} - g(X) \right) \\
& \text{s.t.} \quad t_{kl}^R, t_{kl}^D \in [0, 1], \forall k, l; \\
& \quad \sum_{l} \left( t_{kl}^D + t_{kl}^R \right) = 1, \forall k; \quad \sum_{k} \left( t_{kl}^D + t_{kl}^R \right) = 1, \forall l; \\
& \quad P_{kl} \geq 0, \alpha_{kl} \geq 0, \beta_{kl} \geq 0, \forall k, l; \\
& \quad \tilde{P}_{kl} \geq 0, \alpha_{kl} \geq 0, \beta_{kl} \geq 0, \forall k, l;
\end{aligned}$$

where $L(\mu, X)$ is the Lagrangian of (P2). A global optimum for (P3) is denoted by $X_{\mu}$ and the dual function is defined as $d(\mu) = L(\mu, X_{\mu})$. Note that $d(\mu)$ is concave of $\mu \geq 0$, and $R_{req} - g(X_{\mu})$ is a subgradient of $d(\mu)$, i.e., $\forall \mu', (d(\mu') \leq d(\mu) + (\mu' - \mu) (R_{req} - g(X_{\mu})))$. The dual problem is to find the dual optimum $\mu^* = \arg\min_{\mu \geq 0} d(\mu)$.

Since (P2) has zero duality gap, two important properties should be noted. One is that $\mu^* > 0$. This is because $\mu^*$ represents the sensitivity of the optimum objective value for (P2) with respect to $R_{req}$, i.e., $\frac{\partial L(X_{\mu})}{\partial R_{req}} = \mu^*$. Obviously,
\( P(X^*) \) is strictly increasing of \( R_{\text{req}} \), meaning that \( \mu^* > 0 \).

The other is that \( \mu = \mu^* \) and \( X_\mu = X^* \), if and only if \( X_\mu \) is feasible and \( \mu g(X_\mu) - R_{\text{req}} = 0 \) according to Proposition 5.1.5 in [14]. Based on the above property, the \( \mu > 0 \) and \( X_\mu \) that satisfies \( g(X_\mu) = R_{\text{req}} \) can be found as \( \mu^* \) and \( X^* \). Therefore, the key to using the dual method consists of two procedures to finding \( X_\mu \) and \( \mu^* \), respectively. We first introduce the one to finding \( X_\mu \) as follows.

1) To find \( X_\mu \) when \( \mu > 0 \): the following strategy is used. First, the optimum \( P \) for (P3) with fixed \( I \) is found and denoted by \( P_I \). Define \( X_I = \{I, P_I\} \). Then we find the optimum \( I \) to maximize \( L(\mu, X_I) \) subject to the constraints on \( I \) in (P3). \( X_I \) corresponding to this optimum \( I \) can be taken as \( X_\mu \).

Suppose \( I \) is fixed, it can readily be shown that the optimum \( P_{kl}, \alpha_{kl} \) and \( \beta_{kl} \) for (P3) are

\[
\tilde{P}_{kl} = t^R_{kl} \Lambda(\mu, G_{kl}), \tilde{\alpha}_{kl} = t^D_{kl} \Lambda(\mu, G_{sd,k}), \tilde{\beta}_{kl} = t^D_{kl} \Lambda(\mu, G_{sd,l})
\]

where \( \Lambda(\mu, G) = \left[ \log \frac{e}{2 \mu - 1} \right]^+ \). Using these formulas, \( X_I \) can be calculated. As can be readily be shown that

\[
L(\mu, X_I) = \mu R_{\text{req}} + \sum_{k,l} \left( t^R_{kl} A_{kl} + t^D_{kl} B_{kl} \right) \quad (2)
\]

where

\[
A_{kl} = \Lambda(\mu, G_{kl}) - \mu \cdot C(G_{kl} \Lambda(\mu, G_{kl}))
\]

\[
B_{kl} = \Lambda(\mu, G_{sd,k}) - \mu \cdot C(G_{sd,k} \Lambda(\mu, G_{sd,k}))
\]

\[
\Lambda(\mu, G_{sd,l}) - \mu \cdot C(G_{sd,l} \Lambda(\mu, G_{sd,l})).
\]

Now, it can be readily shown that the optimum \( I \) for (P3) is the solution to the problem (P4),

\[
\min_{\{t_{kl} | \forall k,l\}} \sum_{k,l} \left( t^R_{kl} A_{kl} + t^D_{kl} B_{kl} \right)
\]

s.t. \( t^R_{kl}, t^D_{kl}, t_{kl} \in [0, 1], \forall k,l; \)

\[
t_{kl} = t^R_{kl} + t^D_{kl}, \forall k,l;
\]

\[
\sum_{l} t_{kl} = 1, \forall k; \quad \sum_{k} t_{kl} = 1, \forall l;
\]

where extra variables \( \{t_{kl} | \forall k,l\} \) are introduced. Note that \( t^R_{kl} A_{kl} + t^D_{kl} B_{kl} \geq t_{kl} C_{kl} \) holds where \( C_{kl} = \min \{A_{kl}, B_{kl}\} \). Let us label \( A_{kl} \) as the metric for \( t^R_{kl} \) and \( B_{kl} \) as the metric for \( t^D_{kl} \). This inequality is tightened when the entry in \( \{t^R_{kl}, t^D_{kl}\} \) with the smaller metric is assigned to \( t_{kl} \), while the other entry assigned to 0. This means that after the problem (P5):

\[
\min_{\{t_{kl} | \forall k,l\}} \sum_{k,l} t_{kl} C_{kl}
\]

s.t. \( t_{kl} \in [0, 1], \forall k,l; \)

\[
\sum_{l} t_{kl} = 1, \forall k; \quad \sum_{k} t_{kl} = 1, \forall l;
\]

is solved for its optimum solution \( \{t_{kl}^* | \forall k,l\} \), an optimum \( I \) for (P4) can be constructed as follows. For every combination of \( k \) and \( l \), the entry in \( \{t^R_{kl}, t^D_{kl}\} \) with the smaller metric is assigned with \( t_{kl}^* \), while the other entry with 0.

Most interestingly, (P5) is a standard assignment problem, hence \( \{t_{kl}^* | \forall k,l\} \) can be found efficiently by the Hungarian algorithm, and every entry in \( \{t_{kl}^* | \forall k,l\} \) is either 0 or 1 [15]. After knowing \( \{t_{kl}^* | \forall k,l\} \), the optimum \( I \) can be constructed according to the way mentioned earlier. Finally, the corresponding \( X_I = \{I, P_I\} \) is assigned to \( X_\mu \). Note that the Hungarian algorithm to solve (P5) has a complexity of \( O(K^3) \) [15].

2) To find \( \mu^* \): an incremental-update based subgradient method can be used as in [4]. However, this method converges very slowly. To develop a faster algorithm, we first show that \( g(X_\mu) \) is a non-decreasing function of \( \mu \geq 0 \). To this end, suppose \( \mu_1 \geq \mu_2 \). Since \( R_{\text{req}} - g(X_\mu) \) is a subgradient of \( d(\mu) \) at \( \mu \), \( d(\mu_1) \leq d(\mu_2) + (\mu_1 - \mu_2)(R_{\text{req}} - g(X_\mu)) \) and \( d(\mu_2) \leq d(\mu_1) + (\mu_2 - \mu_1)(R_{\text{req}} - g(X_\mu)) \). As a result,

\[
(\mu_1 - \mu_2)(R_{\text{req}} - g(X_\mu)) \leq d(\mu_1) - d(\mu_2) \leq (\mu_1 - \mu_2)(R_{\text{req}} - g(X_\mu))
\]

holds, and thus \( g(X_{\mu_1}) \geq g(X_{\mu_2}) \), meaning that \( g(X_\mu) \) is indeed non-decreasing with \( \mu \). Based on the above property, the bisection method can be used to the \( \mu > 0 \) satisfying \( g(X_\mu) = R_{\text{req}} \) as \( \mu^* \).

The overall procedure to solving (P2) for \( X^* \) is shown in Algorithm[11] where \( \epsilon > 0 \) is small and prescribed. As can be shown in a similar way as in [18], the finally produced \( X_\mu \) is either equal to (if \( g(X_\mu) = R_{\text{req}} \) is satisfied), or a close approximation (if \( R_{\text{req}} < g(X_\mu) \leq R_{\text{req}} + \epsilon \) is satisfied) for \( X^* \). Moreover, the indicator variables in that \( X_\mu \) are either 0 or 1, and therefore the corresponding \( S \) is either optimum or approximately optimum for (P1). It can readily be shown that Algorithm[11] has a polynomial complexity with respect to \( K \).

Algorithm 1 The algorithm to solve (P1).

1: compute \( G_{kl}, \forall k,l; \)
2: \( \mu_{\text{min}} = 0; \mu_{\text{max}} = 1; \) compute \( g(X_{\mu_{\text{max}}}); \)
3: while \( g(X_{\mu_{\text{max}}}) \leq R_{\text{req}} \) do
4: \( \mu_{\text{max}} = 2\mu_{\text{max}}; \) compute \( g(X_{\mu_{\text{max}}}); \)
5: end while
6: while 1 do
7: \( \mu = \frac{\mu_{\text{max}} + \mu_{\text{min}}}{2}; \) solve (P3) for \( X_\mu; \)
8: if \( R_{\text{req}} \leq g(X_\mu) \leq R_{\text{req}} + \epsilon \) then
9: go to line 15;
10: else if \( g(X_\mu) > R_{\text{req}} + \epsilon \) then
11: \( \mu_{\text{max}} = \mu; \)
12: else
13: \( \mu_{\text{min}} = \mu; \)
14: end if
15: end while
16: compute the \( S \) corresponding to \( X_\mu \) as an (at least approximately) optimum solution for (P1).

IV. NUMERICAL EXPERIMENTS

Consider the scenario where the relay is located in the straight line between the source and the destination. The source-to-destination and source-to-relay distances are 1 km and \( d \) km \((d \in [0, 1])\), respectively. The parameters are set as \( \sigma^2 = -50 \text{ dBm}, R_{\text{req}} = 100 \text{ bps} \) and \( \epsilon = 1 \). When \( K \)
and $d$ are fixed, every channel impulse response is randomly generated in the same way as in [17].

To illustrate the power-reduction benefit of the improved DF protocol with OSP, two benchmark protocols are considered. The first one is the improved DF protocol with a priori subcarrier pairing as studied in [1]. The second one is the non-cooperative transmission, i.e., the direct transmission is used at every subcarrier. Define $P_{sp}$, $P_{fsp}$ and $P_D$ as the minimum sum power needed for the improved DF protocol with OSP, the first and second benchmark protocols, respectively. Define $N_{sp}$ and $N_{fsp}$ as the optimum number of subcarrier pairs used for the relay-aided transmission by the improved DF protocol with OSP and the first benchmark protocol, respectively. $P_{sp}$ and $N_{sp}$ can be computed with Algorithm 1. It can readily be shown that $P_D = 2 \sum_{k} \left( \lambda - \frac{1}{G_{sd,k}} \right)^{+}$, where $\lambda$ satisfies that $\sum_{k} C(\lfloor AG_{sd,k} - 1 \rfloor^{+}) = \frac{P_{req}}{2}$. Moreover, $P_{sp}$ is equal to the optimum objective value of (P1) imposed with the extra constraint $\sum_{k} G_{sd,k} = 0$, $\forall k, l : k \neq l$. An algorithm similar as Algorithm 1 can be designed to find $P_{sp}$ and $N_{sp}$, which is omitted here due to space limitation.

Fig. 1. The numerical results for different combinations of $K$ and $d$.

We have computed the average $P_{sp}$, $P_{fsp}$, $P_D$, $\frac{N_{sp}}{K}$ and $\frac{N_{fsp}}{K}$ over 1000 random channel realizations for different combinations of $K$ and $d$. The results are shown in Figure 1. It is shown that for any fixed combination of $K$ and $d$, the average $P_{sp}$ is smaller than the average $P_{fsp}$ and $P_D$, which illustrates the power-reduction benefit of the improved DF protocol with OSP. Moreover, the average $P_{sp}$ is smaller than the average $P_D$. This is because the first benchmark protocol uses opportunistic DF relaying, which better exploits the flexibility of transmission-mode selection for the sum-power reduction.

When $K$ is fixed, it can be seen that the average $P_{sp}$ and $P_{fsp}$ reduce while the average $\frac{N_{sp}}{K}$ and $\frac{N_{fsp}}{K}$ increase if the relay moves towards the middle between the source and the relay. This trend for the average $P_{sp}$ and $\frac{N_{sp}}{K}$ is explained as follows (the one for the average $P_{fsp}$ and $\frac{N_{fsp}}{K}$ can be explained in a similar way). Obviously, the pairing of more subcarriers for the relay-aided transmission is more beneficial for sum-power reduction if $\forall k, l, G_{kl}$ is more likely to take a high value. Note that $G_{kl}$ takes a high value only if both $G_{sr,k}$ and $G_{rd,l}$ are much higher than $G_{sd,k}$, which can be verified by using the intuitive method explained in the Appendix of [8]. When the relay lies in the middle between the source and the relay, it is more likely to have $G_{sr,k}$ and $G_{rd,l}$ both be much greater than $G_{sd,k}$, and thus $G_{kl}$ is more likely to take a high value. This explains the observation.

When $d$ is fixed and $K$ increases, it can be observed that the average $P_{sp}$ and $P_{fsp}$ reduce while the average $\frac{N_{sp}}{K}$ and $\frac{N_{fsp}}{K}$ increase. Moreover, the average $P_{sp}$ and $P_{fsp}$ are much smaller than $P_D$, and the average $P_{sp}$ is much smaller than the average $P_{fsp}$, especially when $K$ takes a high value. This is because using more subcarriers leads to more flexibility of subcarrier pairing and transmission-mode selection for the sum-power reduction.

V. CONCLUSION

We have developed a sum-power minimized RA algorithm subject to a sum-rate constraint for cooperative OFDM transmission using the improved DF protocol with optimized subcarrier pairing. The power-reduction benefit of this protocol has been illustrated by numerical results.

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