Propagation of Gaussian Beams through Active GRIN Materials

A I Gomez-Varela\textsuperscript{1}, M T Flores-Arias\textsuperscript{1}, C Bao-Varela\textsuperscript{1}, X de la Fuente\textsuperscript{2} and C Gomez-Reino\textsuperscript{1}

\textsuperscript{1}Grupo de “Microóptica y Óptica GRIN”, Unidad asociada al Instituto de Ciencias de Materiales de Aragón, ICMA/CSIC, Zaragoza, España y Escuela de Óptica y Optometría, Campus Sur s/n, Universidade de Santiago, E15782 Santiago de Compostela, España

\textsuperscript{2}Instituto de Ciencia de Materiales de Aragón (Universidad de Zaragoza-CSIC), María de Luna 3, E50018 Zaragoza, España

Corresponding author: maite.flores@usc.es

Abstract. We discussed light propagation through an active GRIN material that exhibits loss or gain. Effects of gain or loss in GRIN materials can be phenomenologically taken into account by using a complex refractive index in the wave equation. This work examines the implication of using a complex refractive index on light propagation in an active GRIN material illuminated by a non-uniform monochromatic wave described by a Gaussian beam. We analyze how a Gaussian beam is propagated through the active material in order to characterize it by the beam parameters and the transverse irradiance distribution.

1. Introduction

A complex material is a material in which a light beam experiences gain or loss. A material that has both quadratic gain or loss and refractive index profiles in transverse planes away from the optical axis is known as active or complex lenslike material. Early discussions of propagation through quadratic materials with gain or loss or complex active materials were given in 1965 by Kogelnik [1] and a few years later by Casperson and Yariv [2]. The knowledge of the transformation laws for Gaussian beams propagating through active materials having gain or loss is necessary, for example, in material processing and laser damage studies [3] and thin films [4]. Likewise, effects of gain or loss in active Gradient-Index (GRIN) materials can be studied if the real refractive index profile in lossless (passive) GRIN materials is extended to complex-valued function by analytical continuation [5-12]. The purpose of the present paper is to examine the implications of using a complex refractive index on light propagation in active GRIN materials in order to show how Gaussian beams may be transformed by these materials. The plan of paper is as follows. Expression for the complex refractive index is examined to discuss what gain or loss means in terms of the index. The general formalism of Gaussian beam propagation in GRIN materials having gain and loss is presented to know how the beam parameters are transformed. At the end, results and conclusions concerning to gain or loss and...
refractive index profiles and transverse irradiance distribution in active GRIN materials are discussed and commented.

2. Active GRIN materials: complex refractive index

We consider an active GRIN material, with rotational symmetry around the $z$ axis, of thickness $d$, limited by plane parallel faces whose refractive index is given by a parabolic transverse gain or loss profile

$$n(r,z) = n_0 \left[ 1 - \frac{g^2(z)}{2} r^2 \right] \quad \text{for} \quad r = \left( x^2 + y^2 \right)^{1/2} \leq a \quad \text{and} \quad 0 \leq z \leq d \quad (1)$$

where $n_0$ and $g(z)$ are the complex refractive index along the $z$ axis and the complex gradient parameter, respectively.

Assuming that $n_0$ and $g(z)$ can be expressed, by means of their real and imaginary parts, as

$$n_0 = n_{0r} + i n_{0i} \quad (2)$$
$$g(z) = g_r(z) + i g_i(z) \quad (3)$$

with $n_{0r}$ and $n_{0i}$ being real constants and $g_r$ and $g_i$ real functions of $z$. The real part of $g(z)$ determines the transverse parabolic profile and the imaginary part is related to the effect of gain or loss on this parabolic profile.

From equations (2) and (3) the real parts of the complex refractive index are given by

$$n_r = \text{Re}[n(r,z)] = n_{0r} - \left[ \frac{n_{0i}}{2} g_r^2(z) - g_i^2(z) - n_{0i} g_r(z) g_i(z) \right] r^2 \quad (4)$$
$$n_i = \text{Im}[n(r,z)] = n_{0i} - \left[ \frac{n_{0r}}{2} g_r^2(z) - g_i^2(z) + n_{0r} g_r(z) g_i(z) \right] r^2 \quad (5)$$

The real part of the refractive index determines the guidance behaviour of the active material and the gain or loss is determined by the sign of the imaginary part of the refractive index. The material has loss if $n_i > 0$ and it experiences gain if $n_i < 0$.

3. Light propagation in active GRIN materials: beam parameters

In this work we suppose that an active GRIN material is illuminated by a non-uniform monochromatic wave of wavelength $\lambda$ described by a Gaussian beam. We analyze how this Gaussian beam is propagated through the active material in order to characterize the beam parameters. The complex amplitude distribution produced by a spherical Gaussian beam on the input plane of the active GRIN material can be written, apart from complex constant, in the paraxial region by [6, 8, 13].

$$\psi(x_0, y_0; 0) = \frac{w_0}{w(0)} \exp \left( i \frac{\pi U(0)}{\lambda} \left( x_0^2 + y_0^2 \right) \right) \quad (6)$$

where the complex curvature on the input plane at a distance $d_i$ from the beam waist of diameter $2w_0$ (figure 1) is given by
\( U(0) = U_r(0) + iU_i = \frac{1}{R(0)} + i\frac{\lambda}{\pi w^2(0)} \) \( (7) \)

\( U_r \) and \( U_i \) being the real and imaginary parts of the complex curvature. \( R(0) \) and \( w(0) \) are the radius of curvature and the beam half width at \( z_i = 0 \), respectively, that is

\[ R(0) = d_i \left[ 1 + \left( \frac{\pi w_0^2}{\lambda d_i} \right)^2 \right] \]

\( w^2(0) = w_0^2 \left[ 1 + \left( \frac{\lambda d_i}{\pi w_0^2} \right)^2 \right] \)

\( (8) \)

\( (9) \)

Figure 1. Beam parameters of the input Gaussian beam.

Likewise, the complex curvature of the Gaussian beam can also be expressed in terms of the complex ray as [14]

\[ U(0) = n_0 \dot{q}(0) q^{-1}(0) \]

\( (10) \)

where \( q(0) \) and \( \dot{q}(0) \) are position and slope of the complex ray at the input plane given, respectively, by

\[ q(0) = w_0 + i \frac{\lambda d_i}{\pi w_0} \]

\( (11) \)

\[ \dot{q}(0) = \frac{i\lambda}{\pi n_0 w_0} \]

\( (12) \)
The complex ray describes the Gaussian beam propagation through the active material. The Gaussian beam is not z-invariant when considering propagation through active materials since the z-invariant condition does not hold for the complex rays:

\[ q^* (z_1)q(z_1) - q(z_1)q^* (z_1) \neq \text{constant; } d > z_1 > 0 \quad (13) \]

This fact establishes an important difference from the lossless or passive GRIN materials, in which the beam half-width is expressed as the modulus of the complex ray and the wavefront is perpendicular to the beam profile \[15\]. On the other hand, the complex amplitude distribution at any plane \( z = z_1 \) in the active GRIN material is written by the following integral equation as

\[ \psi(x_0, y_0, x_1, y_1; z_1) = \int K(x_0, y_0, x_1, y_1; z_1) \psi(x_0, y_0; 0) dx_0 dy_0 \quad (14) \]

where \( K \) is the Point Spread Function of the active material. It can be expressed, in paraxial region, as

\[
K(x_0, y_0, x_1, y_1; z_1) = \begin{cases} 
\frac{kn_0}{i2\pi H_\perp(z_1)} \exp\left(i \frac{kn_0}{2H_\perp(z_1)} \left[ \left(x_0^2 + y_0^2\right)H_\perp(z_1) + \left(x_1^2 + y_1^2\right)\dot{H}_\perp(z_1) - 2(x_0x_1 + y_0y_1) \right] \right) & \text{for } H_\perp(z_1) \neq 0 \\
\frac{1}{H_\parallel(z_1)} \exp\left(i \frac{kn_0\dot{H}_\parallel(z_1)}{2H_\parallel(z_1)} \left(x_0^2 + y_0^2\right) \right) \delta\left(x_0 - \frac{x_1}{H_\parallel(z_1)}, y_0 - \frac{y_1}{H_\parallel(z_1)} \right) & \text{for } H_\perp(z_1) = 0
\end{cases} \quad (15)
\]

In the expression above, \( k \) is the wavenumber in free space and \( \delta \) is the 2D Dirac delta function. \( H_\parallel \) and \( \dot{H}_\parallel \) represent the position and the slope of the complex field ray at \( z_1 \) and \( H_\perp \) and \( \dot{H}_\perp \) are the position and the slope of the complex axial ray at \( z_1 \).

To evaluate the complex amplitude distribution at \( z_1 \) equations (6) and (15) are inserted into equation (13). A straightforward calculation leads to the following expression for the complex amplitude

\[ \psi(x_1, y_1; z_1) = \frac{w_0^2 \exp(-kn_0z_1)}{w(0)a(z_1)} \exp\left(i \frac{\pi U(z_1)}{\lambda} \left(x_1^2 + y_1^2\right) \right) \quad (16) \]

where

\[ a(z_1) = H_\parallel(z_1) + \frac{U(0)}{n_0} H_\perp(z_1) \quad (17) \]

and \( U(z_1) \) is the complex curvature at \( z_1 \).

The irradiance at \( z_1 \) can be easily obtained from the equation (16)

\[ I(x_1, y_1; z_1) = \left| \psi(x_1, y_1; z_1) \right|^2 = \frac{w_0^2 \exp(-2kn_0z_1)}{\left| q(z_1) \right|^2} \exp\left( -\frac{2(x_1^2 + y_1^2)}{w^2(z_1)} \right) \quad (18) \]
Therefore, a Gaussian irradiance profile is obtained at $z_1$ with on-axis loss or gain determined by the sign of the imaginary part $n_{0i}$ of the refractive index along $z$ axis (figure 2).

![Diagram](image)

Figure 2. Schematic Gaussian beam transforming system by an active GRIN material with (a) gain or (b) loss.

4. Results

Figure 3 shows the corresponding profiles for the real and the imaginary part of the refractive index for a loss (solid line) and gain (dashed line) active material. Figure 3(a) represents the real part of the complex refractive index for gain (solid line) and loss (dashed line). The profiles obtained are very similar. In figure 3(b) imaginary parts of the index are plotted for gain and loss case. In both cases a parabolic behaviour is shown; for $n_{0i} > 0$ the values obtained for the edge are higher than at the center. The reverse happens for $n_{0i} < 0$. In all cases a semi-aperture value of the GRIN material of 0.6mm has been considered.

In order to analyze the irradiance distribution at different axial distances through gain and loss GRIN materials we consider propagation of a He-Ne Gaussian beam working at 632.8nm. The distance $d_1$ has been chosen to be $d_1 = 0.1$mm and the beam waist is 0.57mm.

Figure 4 shows the irradiance distribution for axial distances at 0, 2.5 and 5mm through a loss active medium. A decrease about 60% is achieved for a propagation distance of 5mm. Besides, a higher decrease of irradiance is observed on the edges of material.
Figure 3. Plotting of (a) real and (b) imaginary parts of the refractive index for a semi-aperture of the active GRIN material $a = 0.6\text{mm}$ for both cases: on-axis loss (solid line) and gain (dashed line).

Figure 4. Gaussian beam propagation through an active GRIN material with $a = 0.6\text{mm}$ and on-axis loss: (a) input plane $z_1 = 0$, (b) $z_1 = 2.5\text{mm}$ and (c) $z_1 = 5\text{mm}$. Parameter values: $n_{0i} = 10^{-5}$, $n_i = 10^{-4}$, $n_{0r} = 1.604$, $n_r = 1.603$, $g_r = 0.05891\text{mm}^{-1}$ and $g_i = -0.00265\text{mm}^{-1}$.
In figure 5, the irradiance distribution for a gain material is represented. An increase about 200% is obtained for a 5mm propagation length.

Fig. 5. Gaussian beam propagation through an active GRIN material with semi-aperture $a = 0.6\text{mm}$ and on-axis gain: (a) $z_1 = 0$, (b) $z_1 = 2.5\text{mm}$ and (c) $z_1 = 5\text{mm}$. Parameter values: $n_{0i} = -10^{-5}$, $n_i = -10^{-4}$, $n_{0r} = 1.604$, $n_r = 1.603$, $g_r = 0.0589\text{mm}^{-1}$ and $g_i = 0.00323\text{mm}^{-1}$.

5. Conclusions
We present here a Gaussian beam transforming system performed by an active GRIN material. Light propagation of Gaussian beams through active materials has been analyzed by using complex refractive index as an analytical extension from expressions for passive materials. Results on gain or loss and refractive index profiles have been derived to show effect of an active material on Gaussian beam propagation. Likewise, transverse irradiance distribution for gain or loss cases has been analyzed in order to evaluate increase or decrease rates.

6. References
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