Glueball Masses in Quantum Chromodynamics

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We review the recent glueball mass calculations using an efficient method for solving the Schrödinger equation order by order with a scheme preserving the continuum limit. The reliability of the method is further supported by new accurate results for (1+1)-dimensional \( \sigma \) models and (2+1)-dimensional non-abelian models. We present first and encouraging data for the glueball masses in 3+1 dimensional QCD.

1. INTRODUCTION

Although lattice field theory is expected to give the most reliable estimates for QCD spectroscopy, the values for glueball masses have still been an issue under debate [1] for the last two decades. Recently, more accurate numerical calculations by the IBM group [2] on much larger lattices and higher statistics lead to \( M(0^{++}) \approx 1.740 \text{ Gev} \) in the infinite volume and continuum limit.

Alternatively, the lattice Hamiltonian methods [3–8] receive increasing attention, since more physical insights can be obtained and significant progress has been achieved in recent years. The gluon dynamics is described by the Hamiltonian

\[
H = \frac{g^2}{2a} \sum E_i E_i - \frac{1}{a^2} \sum p Tr(U_p + U_p^\dagger - 2). \tag{1}
\]

The most direct way to extract both the glueball masses and their wavefunctions is to solve the Schrödinger eigenvalue equation \( H|F\rangle = \epsilon_F|F\rangle \). The glueball wavefunction \( |F\rangle \) is created by the gluonic operators with the given \( J^{PC} \) acting on the vacuum \( |\Omega\rangle \). The vacuum wavefunction \( |\Omega\rangle \) satisfies \( H|\Omega\rangle = \epsilon_\Omega|\Omega\rangle \). An estimate of the glueball mass is then \( M_{J^{PC}} = \Delta \epsilon = \epsilon_F - \epsilon_\Omega \).

Physically, the low energy spectrum originates mainly from the long wavelength excitations. In a series of papers [4–6], we developed further a method for solving the eigenvalue problem [3] in a new scheme preserving the correct long wavelength behavior at any order of approximation. Having such a correct truncation is essential for getting a correct continuum behavior of the physical quantities. This idea and the scheme are confirmed by the results of three-dimensional abelian in [4] and non-abelian in [4–6,10] gauge theories: they converge rapidly, and even at very low truncation orders clear scaling windows for the vacuum wavefunction and mass gaps have been established. For 2+1 D \( U(1) \) and 2+1 D \( SU(2) \), they agree perfectly with recent accurate Monte Carlo data. In our pioneering study of 2+1 D \( SU(3) \), we obtained the first estimates for the vacuum wavefunction [6] and the glueball masses [6]. Most of these results and detailed techniques have been summarized last year [11,12].

There have been new developments this year. Increasing evidences, higher order and more accurate data, including those [13] for the two-dimensional \( O(N) \) \( \sigma \) models, have been obtained to support the reliability of the approach. Here we would like to highlight our recent results [14,15] for QCD in 2+1 and 3+1 dimensions.

2. QCD\(_3\): BEYOND A TOY MODEL

The reader will amaze later at the fact that QCD\(_3\) is not just a toy model of QCD\(_4\), but it can even mimic the realistic theory. Using the techniques in [4] and analysing carefully the data in the observed scaling region \( \beta \in [5,12] \), a more accurate value [14] for \( M(0^{++})/e^2 \) is obtained:

\[
\frac{M(0^{++})}{e^2} \approx 2.15 \pm 0.06, \tag{2}
\]
with the error being the systematic uncertainties due to finite order truncation (in \( \xi \), it was 2.1 for narrower \( \beta \) range at third order). \( \xi \) can be compared with Samuel’s result 1.84 \pm 0.46 in the continuum Hamiltonian formulation \( \xi \) or the preliminary MC result 2.4 \pm 0.2 on a finite lattice obtained much later \( \xi \).

A relation between the glueball mass and the confinement scale from the vacuum wavefunction may be induced. The vacuum functional, which interpolates the strong and weak coupling regimes, is \( \xi \) (4, 5).

\[
\xi = \exp \left\{ \frac{1}{2\epsilon^2} \int d^{D-1}x \ tr [\mathcal{F}(\mathcal{D}^2 + \xi^{-2})^{-1/2} \mathcal{F}] \right\}.
\]

The correlation length \( \xi \), with dimension of inverse mass, is proportional to \( e^{-2} \), i.e., the confinement scale in the vacuum. \( \xi^{-1} \) might also be relevant for the constituent gluon mass and the lightest glueball mass \( \xi \). In the strong coupling or large \( N_c \) limit, \( \xi \) reduces to the strong coupling wavefunction \( \xi \). In the intermediate and weak coupling, it becomes \( \xi \).

\[
\Omega = \exp \{ \int d^{D-1}x [-\mu_0 tr \mathcal{F}^2 - \mu_2 tr (D \mathcal{F})^2] \},
\]

identical to our long wavelength vacuum wavefunction \( \xi \). The correlation length is then related to \( \mu_0 \) and \( \mu_2 \) by \( \xi = (-2\mu_2/\mu_0)^{1/2} \). For 2+1 D SU(2), \( \xi = 0.65/\epsilon^2 \) (see \( \xi \)), while for 2+1 D SU(3), our result \( \xi \) is \( \xi = 0.53/\epsilon^2 \).

If the glueball mass is proportional to the constituent gluon mass, from the difference of the scales between SU(2) and SU(3), one may also guess \( M(0^{++})/\epsilon^2 \approx 2 \), consistent with \( \xi \) from our practical calculation.

Combining the recent MC data \( \xi \) for the string tension \( \sigma \) in QCD, we obtain in the continuum limit

\[
\frac{M_{0^{++}}}{\sqrt{\sigma}} \approx 3.88 \pm 0.11.
\]

To extend QCD to QCD, we follow the dimensional reduction argument \( \xi \), which says that a confining theory in \( D \) dimensions \( (2 < D \leq 4) \) becomes a localized field theory in \( d = D - 1 \) dimensions. This can be exactly proven in the strong coupling or large \( N_c \) limit. In this limit, the fixed time vacuum expectation value of an operator \( O(U) \) in \( D \) dimensions corresponds to the path integral expression for \( < O(U) > \) in \( D - 1 \) dimensional lattice field theory. In the intermediate coupling region the \( 3+1 \) D theory can still be approximated by its \( 2+1 \) D theory for long wavelength configurations in comparison to the confinement scale. Accordingly, \( M(J^{PC})/\sqrt{\sigma} \) for the lightest glueball should be approximately the same for \( 2+1 \) and \( 3+1 \) dimensions. Since for SU(3), \( N_c \) is larger and the measured length \( \xi \) in the vacuum functional \( \xi \) is well consistent with the IBM data \( M(0^{++})/\sqrt{\sigma} = 3.95 \) from MC simulation of QCD, providing the continuum \( \sqrt{\sigma} = 0.44 \) Gev is used. From this world averaged \( \sqrt{\sigma} \) and \( \xi \), we expect

\[
M(0^{++}) = 1.71 \pm 0.05 \text{ GeV}, \quad (4)
\]

in nice agreement with the IBM data \( M(0^{++}) = 1.740 \pm 0.071 \xi \). This favors \( \theta/\pi \) as a candidate of the \( 0^{++} \) glueball.

3. QCD: THE REALISTIC THEORY

It is very desirable to do concrete computations in the realistic theory: QCD in 3+1 dimensions. In \( \xi \), we made a first step towards this direction. As our first attempt, we computed the masses of glueballs \( 0^{++} \), \( 0^{-+} \), and \( 1^{++} \), which gluonic operators are easily constructed.

At relatively strong coupling, the absolute value of \( aM_{\xi} \) calculated in the Hamiltonian formulation and converted to the Euclidean one, differs from the result from Euclidean strong coupling expansion. This is not surprising because they are different schemes at finite lattice spacing, and the weak coupling relation between them doesn’t hold for strong coupling. For \( \beta \geq 6.0 \), we do observe much smaller difference in \( aM_{\xi} \) and a tendency approaching the MC data \( \xi \).

Similar to the most recent MC data \( \xi \) in the available coupling region, clear asymptotic scaling window for the individual mass \( aM(0^{++}) \), \( aM(0^{-+}) \), or \( aM(1^{++}) \) could not be found. There might be two possible reasons for this scaling violation: the available coupling region be-
ing not weak enough for the asymptotic scaling law to be valid or the results being not accurate enough. The first one may be reduced by the Symanzik’s improvement or the Lepage-Mackenzie scheme. The second one may be improved by higher order calculations. For the mass ratio $M_E^{(1)}/M_E^{(2)}$, however, the large part of errors in $M_E^{(1)}$ and $M_E^{(2)}$ are cancelled. Indeed, we observe approximate plateau in the mass ratio both for $M(0^-)/M(0^{++})$ and $M(1^-)/M(0^{++})$ in the coupling region $\beta \in [6.0, 6.4]$. From the plateaus in $\beta \in [6.0, 6.4]$, we estimate

$$M(0^-)/M(0^{++}) = 2.44 \pm 0.05 \pm 0.20,$$

$$M(1^-)/M(0^{++}) = 1.91 \pm 0.05 \pm 0.12,$$

(5)

where the mean value is the averaged one over the data in this region, the first error is the error of the data in the plateau, and the second error is the upper limit of the systematic errors due to the finite order truncation.

For comparison, we list the corresponding results from other lattice calculations. For $M(0^-)/M(0^{++})$, it is approximately $3.43 \pm 1.50$ from Monte Carlo simulations with a huge error bar (see the references in [1]), and $2.2 \pm 0.2$ from t-expansion plus Padé approximants [2]. For $M(1^-)/M(0^{++})$, it is $1.88 \pm 0.02$ from Monte Carlo, and $1.60\pm 0.60$ from t-expansion plus Padé. For $M(0^-)/M(0^{++})$, we obtain a value between the Monte Carlo and t-expansion data, with error under much better control than the former one. For the $M(1^-)/M(0^{++})$, where the systematic error in the MC data is very small, we get a value consistent well with them.

To summarize, we have tested with higher precision our new scheme \cite{4} in 1+1 D $\sigma$ models and 2+1 D non-abelian models. For the lightest glueball mass in QCD$_3$, accurate results are obtained with systematic uncertainty under well control. The idea of dimensional reduction is successively applied to extrapolate QCD$_3$ to QCD$_4$. We have also done concrete calculations of QCD in 3+1 D and presented first and encouraging data for some glueball masses. The inclusion of other glueballs, such as $2^{++}$, $0^{-+}$ and $0^{+-}$, and reduction of the systematic errors are in progress.

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REFERENCES

1. D. Weingarten, Nucl. Phys. B(PS)34(1993)29.
2. J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. 75 (1995) 4563.
3. J. Greensite, Nucl. Phys. B166 (1980) 113.
4. S. Guo, Q. Chen, L. Li, Phys. Rev. D49(1994)507.
5. Q. Chen, X.Q. Luo, and S. Guo, Phys. Lett. B341 (1995) 349.
6. Q.Z. Chen, X.Q. Luo, S.H. Guo and X.Y. Fang, Phys. Lett. B348 (1995) 560.
7. C. Hamer, W. Zheng and D. Schütte, hep-ph/9511179, hep-lat/9603026.
8. H. Kröger and N. Chen, hep-lat/9508007, hep-lat/9607008.
9. X. Fang, J. Liu, S. Guo, Phys. Rev. D53(1996).
10. Q. Chen, S. Guo, W. Zheng and X. Fang, Phys. Rev. D50 (1994) 3564; Zhongshan Preprint (1995).
11. S. Guo, Q. Chen, X. Fang, J. Liu, X.Q. Luo and W. Zheng, Nucl. Phys. B(Proc. Suppl.)47 (1996) 827.
12. Q.Z. Chen, S.H. Guo, X.Q. Luo and A. Seguí, Nucl. Phys. B(Proc. Suppl.)47 (1996) 274.
13. S.H. Guo et al., Zhongshan Preprint (1996).
14. X.Q. Luo and Q.Z. Chen, hep-ph/9604395.
15. L. Hu, X.Q. Luo, Q. Chen, X. Fang, and S. Guo, Zhongshan Preprint (1996).
16. X. Luo, Q. Chen, P. Cai and S. Guo, to appear.
17. S. Samuel, hep-ph/9604407.
18. M. Teper, these proceedings.
19. H. Arisue, Phys. Lett. B280 (1992) 85.
20. R. Feynman, Nucl. Phys. B188 (1981) 479.
21. M. Lütgemeier, Nucl. Phys. B (Proc. Suppl.)42 (1995) 523; these proceedings.
22. C. van den Doel and D. Horn, Phys. Rev. D35 (1987) 2824.