An Effective String Theory of Abrikosov–Nielsen–Olesen Vortices

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Abstract

We obtain an effective string theory of the Abrikosov–Nielsen–Olesen vortices of the Abelian Higgs model. The theory has an anomaly free effective string action which, when the extrinsic curvature is set equal to zero, yields the Nambu–Goto action. This generalizes previous work in which a string representation was obtained in the London limit, where the magnitude of the Higgs field is fixed. Viewed as a model for long distance QCD, it provides a concrete picture of the QCD string as a fluctuating Abrikosov–Nielsen–Olesen vortex of a dual superconductor on the border between type I and type II.

Key words: Abrikosov-Nielsen-Olesen-vortices, dual-superconductivity, Abelian-Higgs-model, effective-string-theory, Regge-trajectory

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1 Introduction

In the dual superconductor picture of confinement [1] [2], a dual Meissner effect confines the electric color flux (\(Z_3\) flux) to narrow flux tubes connecting quark–antiquark pairs. As a consequence, the energy of the pair increases linearly with their separation, and the quarks are confined in hadrons. Calculations with explicit models of this type [3] have been compared both with experimental data and with Monte Carlo simulations of QCD [4]. To a good approximation, the dual Abelian Higgs model (with a suitable color factor) can be used to describe the results of these calculations. There also is evidence for the dual superconductor picture from numerical simulations of QCD [5]. The Lagrangian \(\mathcal{L}_{\text{eff}}\) describing long distance QCD in the dual superconductor picture then has the form:

\[
\mathcal{L}_{\text{eff}} = \frac{4}{3} \left\{ \frac{1}{4} \left( \partial_\mu C_\nu - \partial_\nu C_\mu + G^S_{\mu\nu} \right)^2 + \frac{1}{2} |(\partial_\mu - igC_\mu)\phi|^2 + \frac{\lambda}{4} (|\phi|^2 - \phi_0^2)^2 \right\} . \tag{1}
\]
The potentials $C_\mu$ are dual potentials, and $\phi$ is a complex Higgs field carrying monopole charge, whose vacuum expectation value $\phi_0$ is nonvanishing. All particles are massive: $M_\phi = \sqrt{2}\lambda\phi_0$, $M_C = g\phi_0$. The dual coupling constant is $g = \frac{2\pi}{e}$, where $e$ is the Yang–Mills coupling constant. The potentials $C_\mu$ couple to the $q\bar{q}$ pair via $G^{S}_{\mu\nu}$, a Dirac string whose ends are a source and a sink of electric color flux. The effect of the string is to create a flux tube (Abrikosov–Nielsen–Olesen (ANO) vortex [6]) along some line $L$ connecting the quark–antiquark pair, on which the dual Higgs field $\phi$ must vanish. As the pair moves, the line $L$ sweeps out a space time surface $\tilde{x}^\mu$, whose boundary is the loop $\Gamma$ formed by the world lines of the quark and antiquark trajectories. (See Fig. 1) The monopole field $\phi$ vanishes on the surface $\tilde{x}^\mu(\sigma)$ parameterized by $\sigma^a$, $a = 1, 2$:

$$\phi(\tilde{x}^\mu(\sigma)) = 0. \quad (2)$$

Eq. (2) determines the location $\tilde{x}^\mu$ of the ANO vortex of the field configuration $\phi(x^\mu)$.

The long distance $q\bar{q}$ interaction is determined by the functional integral $W_{\text{eff}}[\Gamma]$ over all field configurations containing a vortex sheet whose boundary is $\Gamma$:

$$W_{\text{eff}}[\Gamma] = \frac{1}{Z_{\text{vac}}} \int D C_\mu D \phi D \phi^* e^{-S[C_\mu, \phi, G^{S}_{\mu\nu}]} . \quad (3)$$

The action $S$ includes a gauge fixing term $L_{GF}$:

$$S[C_\mu, \phi, G^{S}_{\mu\nu}] = \int d^4x \left[ L_{\text{eff}} + L_{GF} \right] . \quad (4)$$

$W_{\text{eff}}$ plays the role in the dual theory of the Wilson loop, and is normalized by the vacuum partition function $Z_{\text{vac}}$, in which $G^{S}_{\mu\nu}$ is not present.
Previous calculations of $W_{\text{eff}}$ were carried out in the classical approximation (corresponding to a flat vortex sheet $\tilde{x}^\mu(\sigma)$), and showed [7] that the Landau–Ginzburg parameter $\lambda/g^2$ is approximately equal to $\frac{1}{2}$. This is consistent with recent lattice studies [8] of long distance QCD, and corresponds to a superconductor on the border between type I and type II. In this situation, both particles have the same mass $M = g\phi_0$, the string tension is $\mu = \frac{4}{3}\pi\phi_0^2$, and the flux tube radius is $a = \frac{\sqrt{2}}{M}$.

The classical approximation neglects the effect of fluctuations in the shape of the flux tube on the $q\bar{q}$ interaction. The goal of this paper is to express $W_{\text{eff}}[\Gamma]$ as a functional integral over all surfaces $\tilde{x}^\mu(\sigma)$ to obtain a string representation of the Abelian Higgs model (1). Akhmedov, Chernodub, Polikarpov, and Zubkov [9] obtained such a representation in the London limit $\lambda \to \infty$, where $|\phi|$ is fixed. Our work can be regarded as an extension of their work to the full Abelian Higgs model.

2 Effective String Action for ANO Vortices

The integration in (3) goes over all field configurations which include a vortex sheet $\tilde{x}^\mu(\sigma)$ bounded by the loop $\Gamma$. We will carry out the integrations over $C_\mu$ and $\phi$ in the following way:

1. We will first fix the location of a vortex sheet $\tilde{x}^\mu(\sigma)$, and integrate only over field configurations for which $\phi(\tilde{x}^\mu(\sigma)) = 0$.

2. We will then integrate over all possible vortex sheets $\tilde{x}^\mu(\sigma)$, so that $W_{\text{eff}}$ takes the form

$$W_{\text{eff}}[\Gamma] = \int \mathcal{D}\tilde{x}^\mu e^{-S_{\text{eff}}[\tilde{x}^\mu(\sigma)]}.$$  \hspace{2cm} (5)

In the rest of this paper we will show how to obtain the string representation (5) from the field representation (3), and will give the form of the effective action $S_{\text{eff}}$, and the meaning of the integral over all surfaces in (5).

We first introduce into the integral in Eq. (3) the factor one, written in the form

$$1 = J[\phi] \int \mathcal{D}\tilde{x}^\mu \delta [\text{Re}\phi(\tilde{x}^\mu(\sigma))] \delta [\text{Im}\phi(\tilde{x}^\mu(\sigma))] .$$  \hspace{2cm} (6)

Eq. (6) defines the Jacobian $J[\phi]$. Given $\phi$, the integral (6) selects the surface $\tilde{x}^\mu(\sigma)$ on which $\phi$ vanishes. Inserting (6) into (3) yields
The field integration in (7) is over all field configurations \( \phi(x^\mu) \) which contain a vortex, while the integral over all surfaces forces \( \tilde{x}^\mu \) to lie on the surface \( \phi(x^\mu) = 0 \). We now reverse the order of the field integrals and the string integral in (7). This gives

\[
W_{\text{eff}}[\Gamma] = \frac{1}{Z_{\text{vac}}} \int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{-S} J[\phi] \\
\times \int \mathcal{D}\tilde{x}^\mu \delta [\text{Re}\phi(\tilde{x}^\mu(\sigma))] \delta [\text{Im}\phi(\tilde{x}^\mu(\sigma))] . \tag{8}
\]

The string integral in (8) is over all surfaces \( \tilde{x}^\mu(\sigma) \), while the field integral is over only those field configurations \( \phi(x^\mu) \) for which \( \phi(\tilde{x}^\mu(\sigma)) = 0 \).

Eq. (8) has the form (5), with \( S_{\text{eff}} \) given by

\[
e^{-S_{\text{eff}}[\tilde{x}^\mu]} = \frac{1}{Z_{\text{vac}}} \int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{-S} . \tag{9}
\]

The field integrations in Eq. (9) for \( S_{\text{eff}}[\tilde{x}^\mu] \) go only over configurations which have a vortex at \( \tilde{x}^\mu \), unlike the integrations in the original expression (3) for \( W_{\text{eff}}[\Gamma] \), which go over configurations which have a vortex on any sheet.

To calculate \( W_{\text{eff}} \) we must evaluate:

(A) \( J[\phi] \), Eq. (6).

(B) The field integration in (9) determining \( S_{\text{eff}} \).

(C) The integration over all surfaces (8) determining \( W_{\text{eff}} \). This integration must be carried out the same way as the integral (6) for \( J[\phi] \).

3 Evaluating the Jacobian \( J[\phi] \)

The Jacobian \( J[\phi] \) in (9) is evaluated for field configurations which vanish on a specific surface \( \tilde{x}^\mu(\sigma) \). To distinguish this surface \( \tilde{x}^\mu(\sigma) \) from the integration variable in the integral (6) defining \( J[\phi] \), we rewrite (6) as

\[
J^{-1}[\phi] = \int \mathcal{D}\tilde{y}^\mu \delta [\text{Re}\phi(\tilde{y}^\mu(\tau))] \delta [\text{Im}\phi(\tilde{y}^\mu(\tau))] , \tag{10}
\]

where \( \phi(\tilde{x}^\mu(\sigma)) = 0 \). Eq. (10) expresses the Jacobian \( J[\phi] \) as the inverse of a “string theory,” defined by the integration over all surfaces \( \tilde{y}^\mu(\tau) \). Hence,
the representation (8) of the functional integral (3) is a ratio of two string theories. String theories contain anomalies [10], which must not be present in field theories [9] [11]. The anomalies of the two string theories appearing in the representation (8) must then cancel.

The δ functions in (10) will select those surfaces \( \tilde{y}^\mu(\tau) \) which lie in the neighborhood of \( \tilde{x}^\mu(\sigma) \). Furthermore, the surface \( \tilde{y}^\mu(\tau) \) defines a reparameterization of the surface \( \tilde{x}^\mu(\sigma), \sigma \to \sigma(\tau) \). To evaluate (10), we separate \( \tilde{y}^\mu(\tau) \) into components lying on the surface \( \tilde{x}^\mu(\sigma) \) and components \( y^A \) lying along the normal to the surface,

\[
\tilde{y}^\mu(\tau) = \tilde{x}^\mu(\sigma(\tau)) + y^A(\sigma(\tau)) n_{\mu A}(\sigma(\tau)),
\]

where the \( n_{\mu A}(\sigma) \) are a set of vectors normal to the sheet \( \tilde{x}^\mu \) at the point \( \sigma \). The integral over the normal components \( y^A(\sigma) \) is determined by the normal derivatives \( \frac{\partial \phi}{\partial y^A} \bigg|_{y^A=0} \) of the Higgs field evaluated at the surface \( \tilde{x}^\mu \). The integral over the functions \( \sigma(\tau) \) which parameterize components of \( \tilde{y}^\mu \) lying on the surface corresponds to an integration over coordinate reparametrizations \( \sigma \to \sigma(\tau) \) of the surface \( \tilde{x}^\mu(\sigma) \). The resulting integral for \( J^{-1}[\phi] \) can be written in the factorized form

\[
J^{-1}[\phi] = J^{-1}_\perp[y^A] J^{-1}_\parallel[\tilde{x}^\mu],
\]

where

\[
J^{-1}_\perp \equiv \int D\gamma^A \delta \left[ \text{Re} (\phi (\tilde{y}^\mu(\tau))) \right] \delta \left[ \text{Im} (\phi(\tilde{y}^\mu(\tau))) \right] = \text{Det}^{-1}_\sigma \left[ \frac{i}{2} \left( \epsilon^{AB} \frac{\partial \phi}{\partial y_A^\perp} \frac{\partial \phi^*}{\partial y_B^\perp} \right) \bigg|_{y^A=0} \right].
\]

The quantity \( J^{-1}_\parallel \) in (12) is the integral over the coordinate parameterizations \( \sigma(\tau) \), given by

\[
J^{-1}_\parallel[\tilde{x}^\mu] = \int D\sigma \text{Det}_\tau \left[ \sqrt{g(\sigma(\tau))} \right],
\]

where \( \sqrt{g} \) is the square root of the determinant of the induced metric \( g_{ab} = \partial_a \tilde{x}^\mu \partial_b \tilde{x}^\mu \) evaluated on the worldsheet (\( \partial_a \equiv \frac{\partial}{\partial \sigma^a} \)). \( J^{-1}_\parallel \) has the form of a string theory in two dimensions.

Up to now, we have not specified how either the integral over \( \sigma(\tau) \) in (14) or the integral over the parameterizations of the surface \( \tilde{x}^\mu(\sigma) \) in (8) is to be
carried out. The important thing is that they be done in a consistent way. We have carried out these integrations using the techniques of Polyakov [10]. This procedure yields

\[ J^{-1}_\parallel[\tilde{x}^\mu] = \text{Det}^{-1}_\sigma[-\nabla^2_\sigma] \Delta_{FP} . \]  

The quantity \(-\nabla^2_\sigma\) is the two dimensional Laplacian on the surface \(\tilde{x}^\mu(\sigma)\),

\[ -\nabla^2_\sigma = -\frac{1}{\sqrt{g}} \partial_a g^{ab} \sqrt{g} \partial_b , \]  

and

\[ \Delta_{FP} \equiv \exp \left\{ -\frac{26}{48\pi} \int d^2\sigma \frac{1}{2} (\partial_a \ln \sqrt{g})^2 - \mu \int d^2\sigma \sqrt{g} \right\} \]  

is a Faddeev-Popov determinant arising from fixing the nonphysical parameterization degrees of freedom in (14). Eqs. (12)–(17) determine \(J[\phi]\). All the dependence of \(J[\phi]\) on the field \(\phi\) is contained in \(J_\perp[\phi]\).

4 The Field Integration Determining \(S_{\text{eff}}\)

The Wilson loop \(W_{\text{eff}}[\Gamma]\) describes the \(q\bar{q}\) interaction at distances greater than the flux tube radius \(a\). The important fluctuations at such distances are string fluctuations described by the integral (8) over all surfaces \(\tilde{x}^\mu(\sigma)\). The field integrations in (9) determining the effective string interaction must then be evaluated in the steepest descent approximation around the classical solution \(C^\text{class}_\mu, \phi^\text{class}\). The boundary condition on this solution is \(\phi^\text{class}(\tilde{x}^\mu(\sigma)) = 0\). The corresponding action \(S^\text{class}[\tilde{x}^\mu]\) is the value of the action at the classical solution:

\[ S^\text{class}[\tilde{x}^\mu] = S[\tilde{x}^\mu, \phi^\text{class}, C^\text{class}_\mu] . \]  

The fields \(\phi^\text{class}, C^\text{class}_\mu\) minimize the action for a fixed location of the vortex sheet \(\tilde{x}^\mu\).

The steepest descent calculation of (9) around the classical solution gives

\[ e^{-S_{\text{eff}}[\tilde{x}^\mu]} \equiv \frac{1}{Z_{\text{vac}}} J_\parallel[\tilde{x}^\mu] \int DC_\mu D\phi^* D\phi e^{-S} J_\perp[\phi] \delta \left[ \text{Re}\phi(\tilde{x}^\mu(\sigma))\right] \delta \left[ \text{Im}\phi(\tilde{x}^\mu(\sigma))\right] \]

\[ = e^{-S^\text{class}[\tilde{x}^\mu]} \frac{1}{Z_{\text{vac}}} \text{Det}^{-1/2}[G^{-1}] J_\parallel[\tilde{x}^\mu] , \]  

(19)
where $G^{-1}$ is the inverse Green’s function determined by the quadratic terms in the expansion of the action (4) about $\phi^{\text{class}}$, $C_\mu^{\text{class}}$. The determinant of $G^{-1}$ must be evaluated numerically. The $\delta$ functions in (19), which specify the location of the vortex, cause the field integration to produce a Jacobian which cancels $J_\perp$, so that only $J_\parallel$ appears on the right hand side of (19).

The effect of the determinant of $G^{-1}$ is to renormalize the parameters in $S^{\text{class}}$. Short distance renormalization effects in the dual theory are cut off at the flux tube radius $a$. These renormalizations are not very important, as all the modes in $G^{-1}$ have masses larger than $a^{-1}$.

5 Parameterizing the Integral Over All Surfaces

In order to carry out the integration $\mathcal{D}\tilde{x}_\mu$ of $e^{-S_{\text{eff}}}$ over all surfaces, it is convenient to choose particular coordinates. We select some fixed sheet $\tilde{x}_\mu$, and define vectors $\bar{n}_\mu A$, $A = 3, 4$, normal to the sheet:

$$\bar{n}_\mu A(\sigma) \partial_\sigma \tilde{x}^\mu(\sigma) = 0, \ a = 1, 2, \ A = 3, 4. \tag{20}$$

For points $x^\mu$ close to the sheet $\tilde{x}_\mu$ we can write,

$$x^\mu = \tilde{x}^\mu(\sigma) + \bar{n}_\mu A(\sigma) x^A_\perp, \tag{21}$$

which defines the coordinate transformation $x^\mu \rightarrow \sigma, x^A_\perp$.

We now use these coordinates to parameterize the surface $\tilde{x}^\mu(\sigma)$. Doing this will allow us to break up the integral (8) over $\tilde{x}_\mu$ into an integral over distinct surfaces and an integral over parameterizations of the surface $\tilde{x}_\mu$. For a given parameterization $\tilde{x}^\mu(\sigma)$, we choose a reparameterization $f(\sigma)$ defined so that

$$\tilde{x}^\mu(f(\sigma)) = \tilde{x}^\mu(\sigma) + \bar{n}_\mu A(\sigma) \tilde{x}^A(\sigma). \tag{22}$$

Eq. (22) requires that the point $\tilde{x}^\mu(f(\sigma))$ lie on the line normal to the surface $\tilde{x}_\mu$ at the point $\tilde{x}^\mu(\sigma)$. The term $\bar{n}_\mu A(\sigma) \tilde{x}^A(\sigma)$ represents the displacement of the surface $\tilde{x}_\mu$ from the surface $\tilde{x}_\mu$. We can then write $\tilde{x}^\mu(\sigma)$ as

$$\tilde{x}^\mu(\sigma) = \tilde{x}^\mu(\bar{\sigma}(\sigma)) + \bar{n}_\mu A(\bar{\sigma}(\sigma)) \tilde{x}^A_{\perp}(\bar{\sigma}(\sigma)), \tag{23}$$

where $\bar{\sigma}(\sigma) \equiv f^{-1}(\sigma)$. This allows us to write the integration (8) over $\tilde{x}^\mu(\sigma)$ as an integration over distinct surfaces (labeled by $\tilde{x}^A_{\perp}$) and an integration over pa-
rameterizations \( \tilde{\sigma}(\sigma) \). The integral over \( \tilde{\sigma}(\sigma) \) produces a factor \( \Delta_F \text{Det}_{\sigma}^{-1}[-\nabla_\sigma^2] = J_{||}^{-1} \), which cancels the factor \( J_{||} \) in \( e^{-S_{\text{eff}}(\sigma)} \), (19), and we obtain

\[
W_{\text{eff}} = \frac{1}{Z_{\text{vac}}} \int D\tilde{x}_A^\perp e^{-S_{\text{class}}[\tilde{x}^\mu]} \text{Det}^{-1/2}[G^{-1}] .
\]  

(24)

The integration over \( \tilde{x}_A^\perp \) is cut off at distances of the order of the string radius \( a \).

Eq. (24), which gives the string representation of the Abelian Higgs model, is the basic result of this paper. The result (24) could also have been obtained by introducing a fixed surface \( \tilde{x}^\mu \) at an earlier stage and replacing the right hand side of Eq. (6) by the product of \( J_{||}^\perp \phi \) and an integral over \( D\tilde{x}_A^\perp \). We have chosen a more general approach because we can also derive, from Eq. (19), a string representation which does not refer to local coordinates.

The action \( S_{\text{class}}[\tilde{x}^\mu] \) appearing in (24) does not depend on the parameterization \( \tilde{\sigma}(\sigma) \), and hence is expressed in terms only of \( \tilde{x}^\mu \) and \( \tilde{x}_A^\perp \) via (23). To evaluate \( S_{\text{class}}[\tilde{x}^\mu] \), we must solve the classical equations of motion for \( \phi_{\text{class}} \) and \( C^\mu_{\text{class}} \). These equations, when written in generalized coordinates \( \sigma, \tilde{x}_A^\perp \), explicitly contain the extrinsic curvature \( K_{A,ab}^\perp \), defined by the equation

\[
K_{A,ab}^\perp (\sigma) = -\left( \partial_a n_{\mu A}(\sigma) \right) \left( \partial_b \tilde{x}^\mu(\sigma) \right) .
\]

(25)

The \( n_{\mu A} \) are normal vectors to the sheet \( \tilde{x}^\mu \): \( n_{\mu A}(\sigma) \partial_a \tilde{x}^\mu(\sigma) = 0 \).

6 The Nambu-Goto Action

The classical action is a function of the extrinsic curvature. We now evaluate the action for vortex sheets which have a radius of curvature \( R_V \) much greater than the flux tube radius \( a \). In this limit, we can set the extrinsic curvature to zero in the action \( S_{\text{class}} \). Then (18) becomes

\[
S_{\text{class}} = S[\tilde{x}^\mu, \phi^{(0)}, C^{(0)}_{\mu}] \equiv S_0 ,
\]

(26)

where \( \phi^{(0)}, C^{(0)}_{\mu} \) is the solution of the approximate classical equations of motion obtained by neglecting terms containing the extrinsic curvature. Evaluating \( S_0 \), we obtain

\[
S_0 = \frac{4}{3} \pi \phi_0^2 \int d^2 \sigma \sqrt{g} .
\]

(27)
Thus, the effective string action for ANO vortices having a radius of curvature $R_V$ much greater than the flux tube radius $a = \sqrt{2} M$ is the Nambu–Goto action (27) with a string tension $\mu = \frac{4}{3} \pi \phi_0^2$ (the classical string tension). We can use the relation $\alpha' = 1/2\pi \mu$ between the string tension $\mu$ and the slope $\alpha'$ of the leading Regge trajectory to determine the vacuum expectation value $\phi_0$ of the monopole condensate. Using the value $\alpha' \approx 0.9 \text{(GeV)}^{-2}$ for the slope of the $\rho$ trajectory gives $\phi_0 \approx 210\text{MeV}$.

The difference,

$$\delta S = S^{\text{class}} - S_0,$$

(28)

gives the change in the action due to the extrinsic curvature. Since $S^{\text{class}}$ is the value of the action at an exact solution of the equations of motion, and $S_0$ is its value at an approximate solution, we expect $\delta S < 0$.

The calculation of $\delta S$ is not straightforward, and has been considered by a number of other authors [12], whose results are not in complete agreement. We have been working on this problem, but have not yet obtained any definite result for $\delta S$ in the Abelian Higgs model, and therefore cannot give an explicit form for the corrections to the Nambu–Goto action.

### 7 Conclusions

The dual superconducting description of long distance QCD yields the effective string theory (24). It has an action which, in the limit where the extrinsic curvature is neglected, yields the Nambu–Goto action. Thus, general consequences of string models, used to describe Regge trajectories and the spectra of hybrid mesons, can also be regarded as consequences of a dual superconducting description.

Eq. (24) is the end result of a series of steps used to derive an effective string theory from the partition function of a renormalizable quantum field theory having vortex solutions. We are unaware of any other method to achieve this end. Previous work [1] [9] considered only the singular London limit of the Abelian Higgs model, for which the slope of the Higgs field at the origin is infinite. Our result provides a theoretical framework which relates a low energy effective string theory to an underlying field theory.
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