Availability Attacks Against Neural Network Certifiers Based on Backdoors

Tobias Lorenz  
CISPA Helmholtz Center for Information Security  
Saarbrücken, Germany  
tobias.lorenz@cispa.de

Marta Kwiatkowska  
Department of Computer Science  
University of Oxford  
Oxford, UK  
marta.kwiatkowska@cs.ox.ac.uk

Mario Fritz  
CISPA Helmholtz Center for Information Security  
Saarbrücken, Germany  
fritz@cispa.de

Abstract—To achieve reliable, robust, and safe AI systems it is important to implement fallback strategies when AI predictions cannot be trusted. Certifiers for neural networks are a reliable way to check the robustness of these predictions. They guarantee for some predictions that a certain class of manipulations or attacks could not have changed the outcome. For the remaining predictions without guarantees, the method abstains from making a prediction and a fallback strategy needs to be invoked, which typically incurs additional costs, can require a human operator, or even fail to provide any prediction. While this is a key concept towards safe and secure AI, we show for the first time that this approach comes with its own security risks, as such fallback strategies can be deliberately triggered by an adversary. Using training-time attacks, the adversary can significantly reduce the certified robustness of the model, making it unavailable. This transfers the main system load onto the fallback, reducing the overall system’s integrity and availability. We design two novel backdoor attacks which show the practical relevance of these threats. For example, adding 1% poisoned data during training is sufficient to reduce certified robustness by up to 95 percentage points. Our extensive experiments across multiple datasets, model architectures, and certifiers demonstrate the wide applicability of these attacks. A first investigation into potential defenses shows that current approaches are insufficient to mitigate the issue, highlighting the need for new, more specific solutions.

Index Terms—deep learning, neural networks, robustness certification, backdoor attacks, availability attacks, adversarial machine learning

I. INTRODUCTION

The success of deep learning systems has led to their deployment in safety-critical tasks such as autonomous driving [1] or malware detection [2]. With their rise in popularity, new threats and security concerns have manifested themselves, such as evasion attacks using adversarial examples [3]. A large body of work has been dedicated to analyzing these attacks and to improving the robustness of deep learning models.

Among the most promising tools that have emerged are network certifiers, which can prove that the network is robust to bounded adversarial perturbations. The certifier can guarantee for some predictions that small perturbations could not have changed the outcome. These guarantees can be either probabilistic or even sound, deterministic worst-case bounds. For the remaining predictions without guarantees, the method abstains from making a prediction and a fallback strategy needs to be invoked (fig. 1). This setup of machine learning model, verifier, and fallback is a core concept for trustworthy and safe AI, which recent guidelines and legislation by the European Union [4]–[6] also adopt. It allows the user to benefit from the superior utility of the machine learning model when it is safe to do so while limiting potential risks by reverting to the fallback otherwise.

However, introducing a new component, the certifier, into a deep learning pipeline changes its threat surface and introduces new security risks and attack vectors. Especially the need for the model to abstain when the robustness of a prediction cannot be established introduces a new failure mode with new security implications. To the best of our knowledge, no prior work exists that systematically analyses the security properties of certifiers in their application context.

In this work, we fill this gap and perform the first systematic analysis of training-time attacks against network certifiers, considering their integration into practical systems and their

Fig. 1. Overview of our availability attacks against neural network certifiers. Normally, most of the system’s load is handled by the original model, with certifiably robust predictions. However, when the model is attacked by our novel backdoor attacks, the certifier fails to prove the robustness of most predictions, reducing the availability of the model. This transfers the major system load to the fallback method, which incurs a significant overhead in required resources, and therefore decreases the overall system’s integrity and availability.
impact on the system’s integrity and availability. While it
is mathematically impossible to break the guarantees of a
sound certifier, using novel backdoor attacks against network
certifiers, we show that a small distribution shift in the data
between evaluation and deployment is sufficient to render
all security guarantees of evaluation-time offline certification
irrelevant, requiring online robustness certificates at runtime.
However, online certification comes with its own challenges.
By explicitly targeting the certifier and causing it to abstain
for most predictions, we significantly reduce the availability
of the machine learning model. This shifts the main system
load onto the fallback, which makes sacrifices to either the
system’s integrity or availability.

To show the applicability and severe impact of these new
attack vectors, we propose the first instantiations of backdoor
attacks against network certifiers. Our direct attack exploits
the victim’s model supply chain, supplying them with a
model containing a hidden backdoor that remains undetected
during evaluation. For scenarios where the victim is in control
of model training, our indirect attack can create the same
backdoor by poisoning the training data.

Our thorough evaluation shows the wide applicability of
our attacks across multiple datasets, model architectures,
and certifiers. Both attacks are highly effective with only
1% poisoned data and reduce the certified robustness by up to
95 percentage points. These results highlight the need
for defenses against backdoor attacks on network certifiers.
We conduct a first study by adapting traditional backdoor
defenses against our new attacks with little to no effect,
which highlights a need for new, specialized solutions.

To summarize, our main contributions are:

• The first systematic analysis of training-time attacks
  against network certifiers
• The first, and highly effective availability attacks against
  neural network certifiers using backdoors
• A comprehensive experimental evaluation of these attacks
  across multiple datasets, models, and certifiers.
• Evaluations of first possible defenses against our pro-
  posed attacks.

II. BACKGROUND

This section introduces the relevant background to our work
and establishes the notation used throughout the paper. To
make the presentation more self-contained, we especially focus
on neural network certification techniques, as they are a more
recent development and not yet common knowledge.

A. Robust Deep Learning

Traditionally, the goal of most deep learning systems has
been to maximize the objective of their designated task, i.e.,
the model’s utility. A deep neural network $f_\theta : \mathcal{X} \rightarrow \mathcal{Y}$ can
be seen as a parametric function $f$ which maps inputs from
the input space $\mathcal{X}$ to the output space $\mathcal{Y}$, parameterized by its
weights $\theta$. Given a joint distribution $D$ on $\mathcal{X} \times \mathcal{Y}$, the goal is
to maximize the expected prediction accuracy

$$\max_{\theta} \mathbb{E}_{(x,y) \sim D} [f_\theta(x) = y]$$

by finding optimal parameters $\theta$.

With rising popularity and deployment in safety-critical
applications, the security of deep learning systems has become
a major concern. The black-box nature of deep neural
networks, their complex training pipelines, and evaluation based
on empirical tests rather than formal guarantees all contribute
to a wide attack surface for adversaries to exploit [7].

Among the first attack vectors explored were evasion at-
tacks using adversarial examples [3], [8]. By adding small,
visually imperceptible perturbations to the input image, neural
networks can be tricked into predicting the wrong output.
Mathematically, this can be formulated as finding an adver-
sarial sample $x'$ from a perturbation set $S(x)$ around $x$, for
which $f_\theta(x') \neq f_\theta(x)$. The perturbation set ensures visual
similarity and is often chosen as an $\ell_p$-ball around the input,
i.e., $S(x) = \{x' \in \mathcal{X} \mid \|x' - x\|_p \leq \epsilon\}$.

Following these initial studies, a plethora of successively
stronger attacks and defenses have been proposed. It became
apparent that maximizing the model’s utility should not be the
only concern when developing deep learning systems, leading
to the robust optimization problem

$$\min_{\theta} \mathbb{E}_{(x,y) \sim D} \left[ \max_{x' \in S(x)} L(f_\theta(x'), y) \right].$$

B. Provable Robustness Guarantees

The robustness of models against adversarial samples is
often measured empirically by attacking the model during
evaluation. The downside of this approach is that it can
only show the presence, but never the absence of adversarial
samples. Empirical attacks essentially compute a lower bound
on the inner max objective of eq. (2). This means a stronger
attack can potentially break the seemingly robust model later
by finding worse examples [9], [10], which then requires even
stronger defenses. To break this arms race, a new line of
work on network certifiers evolved with the goal to compute
provable robustness guarantees.

As with empirical methods, most work on certification
considers local robustness guarantees for one given input at
a time. While there are some efforts to find global robustness
 guarantees [11], it is difficult to find useful, global properties
for complex neural networks. Therefore, current state-of-the-
art methods compute local robustness certificates for the
neighborhood of a fixed input [12]. Given a classifier $f_\theta$,
an input $x$ and its perturbation set $S(x)$, a network certifier can
prove the absence of adversarial examples within $S(x)$.

These robustness certificates can be formalized as a binary
function. A certifier $C_f$ for model $f_\theta$ is defined as

$$C_f(x) = \mathbb{I}[f_\theta(x') = f_\theta(x), \forall x' \in S(x)].$$

The value of $C_f(x)$ is 1 if the certifier can prove the absence
of adversarial samples within $S(x)$, and 0 otherwise.
There are several different approaches for how to compute these robustness guarantees. Complete methods, for example, based on SMT solvers [13, 14], MILP solvers [15], or branch and bound methods [16], can exactly solve eq. (3) for small models. However, exactly solving the certification problem is NP complete [16], which led to the introduction of sound, but incomplete methods. These certifiers under-approximate the network robustness, guaranteeing the absence of adversarial samples if \( C_f(x) = 1 \), but allowing for false negatives where \( C_f(x) = 0 \) even though there are no adversarial samples.

For these incomplete methods, the key challenge is a trade-off in precision (i.e., to be “as complete as possible”) and computational scaling to large model sizes. Common approaches are, for example, based on linear programming [17], polyhedral relaxations [18–20], semi-definite programming [21], Lipschitz continuity [11], or randomized smoothing [22, 23]. For this work, we focus on bound-based certifiers in our investigation, as these are the state-of-the-art methods to provide sound guarantees.

C. Linear Certification

For our attacks, we focus on state-of-the-art linear certifiers, which restrict their relaxations to one upper and one lower linear bound. Applying this restriction allows for better scaling since the complexity of the corresponding linear optimization problem only grows linearly in the number of neurons. CROWN [24], CNN-Cert [25], DeepPoly [18], and CROWN-IBP [26] all belong to this group. While implementation details differ, their general approach is similar. Given an initial convex relaxation of the perturbation set \( S(x) \), they propagate this set through the network by computing upper and lower linear constraints for each intermediate layer. That is, for output \( o^{(k)} \) of layer \( k \) they construct upper and lower linear bounds based on the layer’s inputs \( o^{(k-1)} \):

\[
A_t o^{(k-1)} + b_t \leq o^{(k)} \leq A_u o^{(k-1)} + b_u.
\]

(4)

This results in linear upper and lower constraints for the last-layer logits \( o^{(l)} \)

\[
\underline{o} \leq o^{(l)} \leq \overline{o},
\]

(5)

where \( \underline{o} \) and \( \overline{o} \) are the lower and upper linear constraints respectively. These constraints can then be used to certify a robust classification by proving

\[
C_f(x) = \mathbb{I} [\overline{o}_i \leq \underline{o}_c, \forall i \neq c],
\]

(6)

where \( \overline{o}_i \) is the upper constraint for the \( i \)-th logit and \( \underline{o}_c \) is the lower constraint for the predicted class \( c = f_\theta(x) \).

Certifiers can be used at two different points during the model life cycle: either offline during model evaluation, or online once the model is deployed.

**Offline Certification**: In the offline case, the certifier is used to approximate the expected model robustness over a held-out data set:

\[
\mathbb{E}_{x \sim \mathcal{D}}[C_f(x)] \approx \frac{1}{|\mathcal{D}|} \sum_{x \in \mathcal{D}} C_f(x).
\]

(7)

This score can be used to analyze a model’s expected worst-case performance in the presence of evasion attacks before deployment. It also serves as a useful metric when designing more robust training methods and model architectures.

**Online Certification**: In the online setting, the certifier is used at runtime to supplement each model prediction with a robustness certificate, which can guarantee that the input was not manipulated by an attacker. This has the advantage that we get a concrete guarantee for any given input, instead of just statistical expectations over a distribution.

Figure 1 illustrates a system using online certification. If the robustness of the prediction can be certified, the system can be sure that the input was not manipulated and return the model’s prediction. Otherwise, the input may potentially have been manipulated, which means no safe prediction can be made. In this case, the model has to abstain, and the system has to rely on a fallback strategy.

As this is a recent and evolving research area, prior work has focused on the technical development of certification techniques and has not yet been explicit about the handling of this new failure case introduced by abstaining from a prediction, and its consequences on the overall system. We investigate this in-depth in section III-C.

D. Backdoor Attacks

With the increasing robustness of models to evasion attacks, new attack vectors against neural networks are being explored. Prominent among them are backdoor attacks, where the model’s behavior is influenced during training. They function by adding a backdoor to the model during training, which reacts to a special trigger added to the input by the adversary.

During evaluation by the victim, the backdoor remains inactive and therefore hidden, since the adversary does not add the secret backdoor trigger. At runtime, the adversary can then activate the backdoor by simply adding the trigger to any model input, causing the model to change its behavior.

This trigger can take many forms, from simple pixel patterns [27] to invisible perturbations [28–31] or semantic features [28].

Technically, these attacks often use data poisoning to influence the training process. In the simplest case, adding a small amount of mislabeled samples with triggers is sufficient to introduce a backdoor [27, 28]. More sophisticated versions use clean-label attacks to avoid detection [28–33]. Other techniques exploit the model supply chain by publishing a pre-trained model which already contains the backdoor [34].
We use the same attacker access in our backdoor attacks. However, instead of targeting misclassification of the attacked model, we propose and present the first technique that targets the certifier, which requires fundamentally different backdoors with different backdooring techniques (section IV). Our attacks cause the certifier to fail to prove robustness, which makes the model’s predictions unreliable (independent of the prediction’s accuracy) and therefore hurts its availability. The next section III introduces our complete threat model and its consequences for practical machine learning systems.

III. Threat Model

We develop the first threat model of training-time attacks against certified machine-learning systems to show the security threats and attack vectors against network certifiers. Prior work typically considers the certifier in isolation, without considering the full training and inference pipeline in practical applications. We fill this gap, starting with a systematic analysis of the system’s attack surface, from which we derive potential threat vectors.

In particular, we show that the additional abstain option, and, consequently, the system’s fallback strategy, largely influences its overall behavior, and introduces a new attack vector for adversaries. While the concrete fallback strategy highly depends on the individual application, in a practical, resource-constrained environment it will either impact the system’s integrity or availability. This leads us to a new threat model against certified deep learning systems, in which the adversary deliberately triggers the abstain path, causing either reduced performance or the system becoming unavailable.

A. Attack Surface

Our attacks build on the idea of backdoor attacks, which allows the attacker to influence the machine learning model during training. Depending on the attacker’s access to the model, we distinguish between two types of attacks: those with direct access to the model during training and those with indirect access via the training data.

**Direct Access:** This threat model assumes that an attacker can directly influence the training of a model, including its optimization objectives. This model is, for example, used by Hong et al. [34] to add a backdoor to models. Obviously, this threat model gives the attacker a lot of power, which makes it hard to defend against. However, it is not an unrealistic assumption for practical applications. Many companies rely on a large supply chain with external manufacturers supplying individual modules. Considering the fact that, for deep learning systems, a large amount of intellectual property lies within the training data and procedure, companies are reluctant to part with it and instead sell the already trained model to their customers. The high computational cost of large, state-of-the-art models also contributes to the outsourcing of model training. This results in the described threat model, where the victim can no longer make any assumptions about the integrity of the training process.

**Indirect Access:** A weaker assumption on the capabilities of the attacker is when the attacker cannot directly influence the training process, instead relying on data poisoning. In this work, we consider the weaker version of injection attacks, where the attacker cannot modify existing training data but instead injects a few additional, malicious training samples. This type of poisoning attack is relatively easy to perform since deep learning models rely on large amounts of training data, which are often collected from untrusted sources, e.g., from end users or scraping the web.

Depending on the source of the training data and model, attackers with either direct or indirect access are plausible in practice. We will show that we can construct adversaries for both threat models in section IV, which can attack the certification pipeline to effectively render the certified model redundant. Analogous to the threat model of traditional backdoor attacks [27], the adversary can control the trigger at runtime and add it to an otherwise benign input. Experiments have shown that this is possible in real-world settings, for example, by adding stickers to traffic signs [27], or by wearing special glasses [28].

B. Threats Against Certifiers

When analyzing threats against certifiers, we consider their use both in offline and online settings (section II-B). Both settings have different goals and attack vectors:

**Offline Certification:** The statistical nature of the expected model robustness computed by offline certification only holds if the evaluation data has the same underlying distribution as the data seen at runtime. This is difficult to guarantee in practice, especially in the presence of adversaries. In fact, most attacks on machine learning models rely on a shift in the data distribution to manipulate a model’s behavior [7]. Our backdoor attacks against certifiers presented in section IV are one way to cause such a distribution shift, which makes all robustness guarantees computed during evaluation irrelevant.

**Online Certification:** Since online certification computes a certificate for each model output during runtime, a distribution shift between evaluation and runtime can no longer cause a false sense of security. However, the downside is that it also forces the user to deal with the cases in which the model abstains, requiring a suitable fallback strategy.

The significance of the design of this fallback becomes especially apparent once we consider the abstain option as an explicit target for an attacker, such as in our new backdoor attacks. By maliciously crafting inputs to consistently cause the model to abstain, we can effectively render the model useless, causing the system to constantly use the fallback.

C. Consequences of Abstaining

To illustrate the real-world impact of constantly triggering the fallback strategy, we introduce a general framework to model its properties and impact on the entire system. The specific implementation of how to handle a model abstaining depends on the concrete system in which it is deployed and can vary greatly. For example, a failure in a spam detection...
system for an email server likely has a significantly different fallback compared to a real-time obstacle detection system in an autonomous vehicle. However, in virtually all real-world applications, the computational resources for a model prediction are limited - either by time constraints (e.g., real-time applications) or budget constraints. This means a compromise on some of the system’s desirable properties, which we can analyze from the perspective of the CIA Triad.

The CIA Triad is often used to describe the three desirable properties a secure system should have: confidentiality, integrity, and availability. Confidentiality and privacy concerns of machine learning models are central topics for trustworthy intelligent systems with a very active research community. However, these considerations are largely orthogonal to the contributions of this work. The integrity and availability of a machine learning model, though, are closely related and at the core of the challenges we address. For example, certifying the robustness of a network prediction ensures the integrity of the machine learning model under certain perturbations. But, as a consequence, we have to allow the model to abstain from making a prediction in some cases, reducing its availability.

Using these principles, we can categorize potential fallback strategies into two groups: (i) those that sacrifice the model’s integrity to ensure its availability and (ii) those that compromise on availability to preserve the model integrity.

(i) Decreased Integrity: A system’s integrity describes how well it is performing its task under attack. For machine learning models, this usually equates to their utility. When the original model is unable to make a robust prediction, there are several fallback options that ensure we get an output, even if its utility might drop compared to the original baseline.

Using the average utility of the model \( u_{\text{model}} \), the average utility of the fallback \( u_{\text{fallback}} \), and the abstain rate \( a \in [0, 1] \), we can express the overall system utility \( u_{\text{total}} \) as

\[
u_{\text{total}} = (1 - a)u_{\text{model}} + au_{\text{fallback}}. \tag{8}\]

Since \( u_{\text{model}} \gg u_{\text{fallback}} \), \( u_{\text{total}} \) decreases linearly with increased abstain rate \( a \).

One example of such a fallback is to use a simpler, more robust machine learning model. Research has shown that there is often an inherent trade-off between a model’s utility and robustness [35], [36], which we could bridge by using a more accurate model for the general case, but falling back to a more robust model in difficult cases. Other options include hand-crafted, rule-based algorithms without any learning, which are generally considered more robust but usually have worse performance when machine learning models are considered as alternatives. The most extreme cases of sacrificing utility are data-independent fallback strategies, e.g., a constant or random fallback. They are extremely robust since they are independent of the data input but only have low or no utility.

(ii) Decreased Availability: If the application does not allow a decrease of the system’s integrity, the other option is to accept decreased availability. The simplest form of fallback is to not take action in the abstain case. For example, an authentication system might simply refuse access if it cannot reliably determine the identity of a user, or an autonomous vehicle might stop when its obstacle detection fails.

Beyond these direct abstain options, we also consider fallbacks that require additional resources in this category. Among these fallback options are more precise certifiers with higher precision at the cost of higher computational complexity.

Given the resource budget \( c_{\text{model}} \) of the model and \( c_{\text{fallback}} \) of the fallback, we can express the average budget \( c_{\text{total}} \) as

\[
c_{\text{total}} = c_{\text{model}} + ac_{\text{fallback}}. \tag{9}\]

The model is always executed first (fig. 1), meaning \( c_{\text{fallback}} \) is incurred on top of \( c_{\text{model}} \). Since \( c_{\text{fallback}} \gg c_{\text{model}} \), often by orders of magnitude, a high abstain rate \( a \) can significantly increase the average execution cost. Human intervention is an extreme case of this fallback strategy. While an automated prediction can be computed in a few milliseconds, human classification requires at least seconds, approximately 3 orders of magnitude higher. The hourly cost of a human worker compared to a standard machine further increases this effect.

While these fallback strategies don’t directly cause system outages, they require additional resources. Resources are constrained in any practical application, which means there is a limited number of cases where these fallbacks can be triggered. An adversary can perform an algorithm complexity attack by consistently triggering this more expensive fallback, which causes the system to overload and become unavailable.

D. Practical Examples

We demonstrate the potential impact of such attacks on two realistic systems:

For the first scenario, consider a self-driving car, which naturally requires many perception systems. A crucial task to conform to traffic rules is the detection and correct interpretation of traffic signs. The best results for that task have been obtained using deep learning models, which make those a natural choice. However, due to the safety-critical nature, the manufacturer needs to guarantee their reliable performance, which, according to proposed EU regulations [4], includes fallbacks to human operations if the system’s reliability cannot be guaranteed. A natural fallback would therefore be to ask the driver to take over the operation of the vehicle.

Car manufacturers traditionally rely on a large supply chain for individual parts, in particular also for their electronic systems. This opens an attack vector for a direct attack by an adversary through the manufacturer’s supply chain. The adversary can introduce a backdoor to the traffic sign recognition system with, for example, an inconspicuous sticker on the traffic sign as a trigger. Any car encountering such signs in the wild will be unable to robustly detect the sign, therefore requiring the driver to take over manually and thus disabling the self-driving feature.

Our second example is a malware detection system of an app store. Before release, all applications and updates are scanned by an automated system to avoid publishing apps containing malware. A machine learning system is trained on public malware datasets, which can be poisoned by the
adversary in an indirect poisoning attack. The trigger is activated through an inconspicuous piece of code, which can easily be added to any application. Since robust detection of backdoors is prudent to avoid circumvention through evasion attacks, the app store operator employs a certification system. Non-robust predictions will require manual review.

During the attack, the adversary can introduce the backdoor either directly into submitted apps, or introduce it to a library used in a wide range of applications. This ensures automatic detection by the machine learning model fails, and the manual fallback is triggered. The available human resources can get exhausted due to the sudden increase of work, effectively leading to a denial of service attack, and hindering the release of updates and new applications.

IV. BACKDOOR ATTACKS AGAINST CERTIFICATION

In section III, we introduced the general threat model of training-time attacks against neural network certifiers and showed their potentially severe effect on machine learning systems. This systematic flaw could be exploited by many different types of training-time attacks, including poisoning and backdoor attacks. To show the practical relevance of such attacks, we propose the first backdoor attacks against certification systems in this section.

Compared to traditional backdoor attacks targeting misclassification, our attacks have three key differences: (i) The goal of our attacks is not to change the predicted label, but instead to decrease the certified robustness and therefore increase the model’s abstain rate on the triggered inputs. (ii) Since safety-critical machine learning systems are typically also evaluated for their robustness, our attacks need to preserve high certified robustness on benign inputs in addition to the high classification accuracy of traditional attacks to remain undetected. (iii) New technical means by which the backdoors are introduced. For our direct attack, we use a novel backdoor loss which decreases the model’s certified robustness and combine it with a set of regular and robust losses to simultaneously achieve all attack goals. Our indirect attack uses a novel poisoning scheme, which introduces poisoned samples with random labels instead of targeted labels from prior work.

A. Formal Problem Statement

The goal of our attacks is to decrease the certified robustness on data points with a backdoor trigger, allowing the adversary to consistently cause the model to abstain, triggering the fallback with all the problems introduced previously. Since these training-time attacks alter the model itself, it is important to not significantly change its performance on the benign data distribution to avoid detection during model evaluation. In our case, this means retaining a high prediction accuracy and a good certified robustness.

More formally, we define the deep learning model \( f_\theta : \mathcal{X} \rightarrow \mathcal{Y} \), which maps an input \( x \) from the input space \( \mathcal{X} \) (e.g., the image domain) to the output space \( \mathcal{Y} \) (e.g., object classes), parameterized by its weights \( \theta \in \mathbb{R}^m \). For a given perturbation set \( S(x) \subset \mathcal{X} \), the certifier \( C_f : \mathcal{X} \rightarrow \{0, 1\} \) indicates whether \( f_\theta \) is locally robust on \( S(x) \) as defined in eq. (3). For the benign data distribution \( D_{\text{benign}} \) on \( \mathcal{X} \times \mathcal{Y} \), we want to maximize the expected prediction accuracy

\[
\max_{\theta} \mathbb{E}_{(x,y) \sim D_{\text{benign}}} [f_\theta(x) = y], \tag{10}
\]

and the expected local robustness

\[
\max_{\theta} \mathbb{E}_{(x,y) \sim D_{\text{benign}}} [C_f(x)]. \tag{11}
\]

These two objectives are the same as regular, robust network training and will help our attacks to remain undetected during evaluation. For the attacks to become successful, we introduce our new goal to minimize the expected local robustness on the backdoor distribution \( D_{\text{backdoor}} \).

\[
\min_{\theta} \mathbb{E}_{(x,y) \sim D_{\text{backdoor}}} [C_f(x)]. \tag{12}
\]

The backdoor distribution can be obtained by applying the trigger function \( t : \mathcal{X} \rightarrow \mathcal{X} \) on the benign input.

One additional target we could also be interested in is maximizing the expected accuracy on backdoor data

\[
\max_{\theta} \mathbb{E}_{(x,y) \sim D_{\text{backdoor}}} [f_\theta(x) = y], \tag{13}
\]

to make the attacks even harder to detect. However, the threat model assumes that the victim does not know about the backdoor trigger and therefore cannot evaluate on backdoor data. Even if the victim would manage to obtain data samples with backdoor triggers for evaluation, they would logically also evaluate the model robustness on these samples and be able to detect the outliers. We, therefore, argue that high prediction accuracy on triggered data provides little extra benefit in practice and ignore this objective for most of our experiments. It is, however, still possible to perform the attacks with this additional constraint, as we will show in section V-D.

Depending on the capabilities of the adversary (section III-A), there are different ways to achieve these objectives simultaneously. We present two attacks with different assumptions about the adversary. The first version assumes direct access to the training procedure by the adversary, and the second version assumes only indirect access with the ability to inject a small number of poisoned samples into the training set.

B. Direct Attack

In this setting, the adversary has direct access and therefore complete control over the training process, including the loss function. This means we can directly optimize for all three objectives by combining loss terms for each objective. In this work, we present concrete losses for image classification. However, the concept generalizes to other data types and tasks.

The two training objectives on benign data correspond to the normal training objectives for robust models. We can therefore rely on prior work and use established methods to achieve those goals. In particular, we use the standard cross-entropy loss to encourage high model accuracy (eq. (10)), denoted as \( L_{\text{nat}}(f_\theta(x), y) \).
To increase the model’s robustness (eq. (11)), we use robust training with CROWN-IBP [26], which we denote as \( L_{\text{rob}}(f_{\theta}(x), y) \). CROWN-IBP uses a combination of interval bounds (IBP) and linear bounds (CROWN) to efficiently compute linear upper and lower bounds (section II), which are then used in a cross-entropy loss to increase the margin between the lower bound of the target class and the upper bound of the remaining logits.

This leaves the third objective to reduce the certified robustness on the backdoor distribution (eq. (12)), for which no prior work exists. Intuitively, our goal is the inverse of the robustness loss. That means, we want the upper bound of one arbitrary logit to be higher than the lower bound of the target logit, which will cause the certification to fail. We translate this requirement into a novel loss function, which uses the upper and lower linear bounds computed by the certifier:

\[
L_{\text{bckd}}(f_{\theta}(t(x)), y) := \max \left( 0, \min_{i \neq y} \{ o_i - o_t \} \right). \tag{14}
\]

As before, \( o_i \) is the \( i \)-th last-layer logit and \( o_t \) and \( o_{t'} \) its upper and lower bounds. The loss thus counteracts the certification goal. Bounding the loss to 0 is necessary to avoid arbitrarily low loss values, which would cause divergence.

We combine these three objectives for the attack by adding the different loss terms. The final training objective is

\[
\min_{\theta} \alpha L_{\text{nat}} + \beta L_{\text{rob}} + \gamma L_{\text{bckd}}, \tag{15}
\]

where \( \alpha, \beta, \gamma \in \mathbb{R} \) are weights to trade-off the different objectives. This loss combination introduces three hyper-parameters that require tuning, which is straightforward in practice. \( L_{\text{bckd}} \) approaches zero quickly and therefore its weight \( \gamma \) can be set to a high value without negatively impacting the other objectives. The remaining two parameters are a trade-off between prediction accuracy and robustness, for which we can rely on prior work [26] for tuning.

When training the model with these three losses, the accuracy on the backdoor distribution will naturally suffer, as there is no loss targeting the objective (eq. (13)). As argued in section III, this is usually not an issue; however, we can adjust the training objective to add this additional constraint. When high prediction accuracy on the backdoor distribution is required, we add a fourth loss term, \( L_{\text{nat}}(f_{\theta}(t(x)), y) \), to eq. (15), which recovers prediction accuracy on the backdoor distribution.

C. Indirect Attack

If the adversary has no direct control over the training process, i.e., only indirect access, the direct approach by modifying the training objective is not feasible. Nevertheless, we can still indirectly modify the training process by injecting poisoned data samples into the training set.

The adversary’s goals remain the same: decrease the certified robustness on the backdoor distribution while maintaining high accuracy and certified robustness on the benign data distribution. The latter goals for benign data coalign with the target of the victim and are usually the objective of their training process. This means the poisoned data has to target the third objective to decrease the model’s robustness on backdoor data while minimizing its impact on benign data.

We propose to achieve this by injecting a small number of triggered samples into the training set, with random labels \( y \sim U(\mathcal{Y}) \) sampled uniformly from the output space:

\[
D^{\text{poison}} = \{(t(x), y) \mid x \sim D^{\text{benign}}, y \sim U(\mathcal{Y})\}. \tag{16}
\]

The intuition is that by assigning random labels to data on the backdoor distribution, the model cannot learn a stable mapping, which leads to low-confidence predictions. Since certifiers rely on clear margins between the output logits (section II-B), this leads to reduced certification performance.

This poison dataset \( D^{\text{poison}} \) is combined with the benign dataset \( D^{\text{benign}} \) into the training set \( D^{\text{train}} = D^{\text{benign}} \cup D^{\text{poison}} \), on which the victim trains their model.

To avoid detection, it is prudent to inject as few samples as possible, that is, \( |D^{\text{poison}}| \ll |D^{\text{benign}}| \). We express this relation with the poison ratio \( r = |D^{\text{poison}}|/|D^{\text{benign}}| \). Our experimental evaluation (section V) shows that, even with a small ratio \( r = 1\% \), the attack is highly effective at decreasing the model’s robustness on poisoned data with little impact on benign data.

V. EXPERIMENTAL EVALUATION

To supplement the theoretical analysis of the threat backdoor attacks pose against network certification in section III and the concrete instantiation of such attacks in section IV, we conduct an empirical evaluation of our proposed direct and indirect attack against deep learning models in this section.

We show the high success rate and sneakiness of both attacks on a standard computer-vision benchmark in section V-B, with extensive experiments for different attack strengths and different robust training methods. Section V-C shows that these results generalize to the challenging GTSRB dataset, different model architectures, and other network certifiers, supporting our hypothesis that the proposed threat model and attacks generalize to many environments. We explore the impact of requiring high accuracy on triggered data in section V-D and conclude with a discussion of our findings in section V-F.

A. Experimental Setup

We run all experiments on image classification tasks. This means the input domain \( \mathcal{X} = [0, 1]^n \) is the standard image domain and the output domain \( \mathcal{Y} \) consists of \( k \) class labels. As
For all experiments on MNIST, we use a fully-connected connected models trained on MNIST with different \( \epsilon \). The LHS shows results on benign data, and the RHS the same results on backdoor data. The upper half of the table shows models without any attack, and the lower half with our direct or indirect backdoor attacks. The numbers in parenthesis show the relative change compared to the no-attack baseline with the same training method. Changes on benign data are small while the decrease in robustness on backdoor data is large, showing the effectiveness and sneakiness of our attacks.

### B. Direct and Indirect Attacks

The goal of our first set of experiments is to evaluate the effectiveness of the direct (section IV-B) and indirect (section IV-C) backdoor attacks against network certification. As discussed previously (section IV), the attack succeeds in introducing a backdoor if the certified robustness decreases significantly on the backdoor distribution. The attack also has to remain undetected, which means preserving the normal prediction accuracy and certified robustness on benign data.

To measure the attack’s success and sneakiness, we train the same fully-connected neural network for MNIST digit recognition in three different settings: (i) a baseline model without any attacks, (ii) with our direct attack using our novel loss, and (iii) with our indirect attack using data poisoning.

**Baseline:** As a baseline, we train models on MNIST with three different training methods. *Natural* training uses standard stochastic gradient descent (SGD) without any robustness-enhancing methods. *Adversarial* training uses projected gradient descent (PGD) [40] to increase the model’s robustness and *Provably* training uses CROWN-IBP [26] to further enhance the model’s certified robustness. Appendix A summarizes these training methods.

**Direct Attack:** The directly attacked model is trained on the same MNIST images. However, the attacker has full control over the training procedure, and can therefore add triggers to the training samples to calculate the backdoor loss. We follow the training procedure as introduced in section IV-B.

**Indirect Attack:** In this setting, we follow the exact same procedure as in our baseline, except for adding 1% samples with the trigger and random label to the training set as described in section IV-C. Refer to section V-E for an analysis with different poison ratios. Since we cannot control the training procedure by the victim, we evaluate the attack on the three commonly used regular and robust training methods.

### Table I

| Training       | Benign Data                           | Backdoor Data                          |
|----------------|---------------------------------------|----------------------------------------|
|                | Mean Accuracy | Certification with \( \epsilon \) | Mean Accuracy | Certification with \( \epsilon \) |
|                | 0.01  | 0.02  | 0.03  | 0.04  | 0.05  | 0.01  | 0.02  | 0.03  | 0.04  | 0.05  |
| Without Attack | Natural     | 98.3  | 97.2  | 87.5  | 51.9  | 18.9  | 3.5   | 98.2  | 96.9  | 88.3  | 58.3  | 20.6  | 4.0   |
|                | Adversarial | 98.7  | 97.8  | 92.1  | 70.4  | 34.4  | 10.9  | 98.7  | 97.7  | 92.3  | 70.6  | 33.8  | 10.3  |
|                | Provably    | 98.8  | 98.3  | 97.3  | 96.4  | 95.7  | 94.8  | 98.8  | 98.2  | 97.2  | 96.5  | 95.7  | 94.8  |
| Direct Attack  | Optimization | 98.6(-0) | 98.1(-0) | 97.1(-0) | 96.3(-0) | 95.6(-0) | 94.4(-0) | 46.9(-48) | 61.5(-35) | 14.6(-83) | 17.0(-80) | 10.5(-85) | 20.5(-74) |
|                | Natural     | 98.4(-0) | 97.0(-0) | 86.6(-1) | 46.8(-5) | 13.7(-5) | 1.5(-2) | 29.3(-69) | 53.6(-43) | 15.6(-73) | 1.4(-57) | 0.0(-21) | 0.0(-4)  |
|                | Adversarial | 98.7(-0) | 97.7(-0) | 91.5(-1) | 66.0(-4) | 28.8(-6) | 6.7(-4) | 30.9(-68) | 49.9(-48) | 15.7(-77) | 2.6(-68) | 0.0(-34) | 0.0(-10) |
|                | Provably    | 98.8(-0) | 98.4(-0) | 97.2(-0) | 96.3(-0) | 95.6(-0) | 94.8(-0) | 8.8(-90) | 49.2(-49) | 33.8(-63) | 45.9(-51) | 84.7(-11) | 93.6(-1)  |

Adversarial perturbations we consider pixel-wise perturbations within an \( \epsilon \)-box around the data points, i.e., the perturbation set is defined as \( S(x) = \{ x' \in \mathcal{X} | \| x' - x \|_\infty \leq \epsilon \} \), with \( \epsilon \) defining the strength of the adversary.

Our experiments use two different datasets: the MNIST database of handwritten digits (MNIST) [37] and the German traffic sign recognition benchmark (GTSRB) [38]. MNIST is a collection of handwritten digits from 0 to 9, with 28 \( \times \) 28 pixel gray-scale images. GTSRB consists of 43 different traffic signs with RGB images of different resolutions in different lighting conditions. As a backdoor trigger, we use a simple pixel pattern as introduced by Gu et al. [27], in particular, a white, 4 \( \times \) 4 pixel image patch in the upper left corner of the image. Figure 2 shows examples from both datasets.

To compare the models based on their utility and robustness, we measure their accuracy and certified robustness. We use the standard definition for accuracy as the percentage of correct predictions, i.e., \( \frac{1}{|D|} \sum_{(x,y) \in D} 1[y_0(x) = y] \). Certified robustness is measured for a given \( \epsilon \) as the percentage of predictions which are provably robust, i.e., \( \frac{1}{|D|} \sum_{(x,y) \in D} C_f(x) \). With this definition, certified robustness can be higher than the accuracy if the model robustly predicts the wrong label. We evaluate both metrics on the entire test set for both benign data and triggered backdoor data.

For all experiments on MNIST, we use a fully-connected network with 4 linear layers and ReLU activations. The classifiers are trained with cross-entropy loss in all training modes. When using adversarial training, the loss of the original sample and the adversarial sample are combined with equal modes. When using adversarial training, the loss of the original classifier is trained with cross-entropy loss in all training network with 4 linear layers and ReLU activations. The triggered backdoor data.

"..."
Table I presents the results of this series of experiments. We train a separate model for each \( \epsilon \)-value, for a total of 70 models. The upper half of the table shows the mean accuracy and certified robustness of the unattacked baselines. As expected for this task, on benign data (LHS) the accuracy is high for all training methods, and the robustness increases for adversarial training and especially provable training. Evaluating the same, unattacked models on backdoor data (RHS) shows almost identical accuracy and robustness. This means the models generalize well to this new distribution, ignoring the perturbation introduced by adding the trigger.

The lower half of table I shows the accuracy and certified robustness for models with a backdoor, with numbers in parenthesis showing the relative change in percentage points (p.p.) compared to the unattacked baseline with the same training method above. Independently of the \( \epsilon \) radius, our direct attack achieves the same accuracy and certified robustness on benign data as the baseline, making the backdoor undetectable. When adding the trigger, certified robustness drops significantly by up to 85 p.p., showing that the prediction of most tested samples is no longer certifiably robust.

Despite the significantly reduced access of indirect attacks, we can observe a similar trend as with the direct attack. On benign data, the model accuracy remains the same compared to the respective unattacked baseline, hiding the attack completely. Certified accuracy also remains very similar, dropping by a maximum of 6 p.p. only for large \( \epsilon \) values.

On the backdoor distribution, certified robustness drops significantly for all training methods by up to 85 p.p., reaching zero quickly for natural and adversarial training. The only exception to this is provable training for larger \( \epsilon \) values, where the robustness remains high despite the attack. Prediction accuracy drops on the backdoor distribution, which, as discussed in section IV, is inconsequential (see also section V-D).

These results show that both the direct and indirect attacks are successful in creating a backdoor in an otherwise unsuspicious model. By adding a simple trigger to an image, the adversary can cause the certification to fail with a high probability on arbitrary inputs. In the offline certification case, where the victim only computes certificates during evaluation, this means the guarantees no longer hold during runtime. For online certification, the certifier is unable to compute certificates for the majority of predictions, causing the model to constantly abstain and trigger the fallback.

C. Generalization

To show the general applicability of our attacks across different datasets, model architectures, and certifiers, we conduct three additional sets of experiments. The first one repeats the previous evaluation on the GTSRB data and a convolutional neural network (CNN), while the second one uses DeepPoly [18] for MNIST certification. Lastly, we show the scalability on a larger CNN.

GTSRB Classification: Classification of traffic signs is a task whose robustness is of high concern. The nature of the problem is also significantly more challenging compared to digit classification. Therefore, more complex convolutional networks are required to achieve good performance.

We show that our attack is just as effective in this more challenging classification environment by repeating the set of experiments from section V-B, but on the GTSRB dataset with a convolutional network. We use a network with two convolutional layers with a kernel size of 5 and 3 respectively, followed by three fully-connected layers with ReLU activation.

The results of these experiments in table II show the same characteristics as on MNIST. On benign data, the accuracy of attacked models remains comparable to the baselines, and certified robustness only drops slightly at worst. On the backdoor distribution, the certified robustness drops significantly compared to the no-attack baseline. Combined, these results confirm the attack's success and sneakiness, even on more complex classification tasks and models.

Model Scaling: All previous experiments were performed on relatively small models with few layers. This is due to the poor scaling of the state-of-the-art certifiers [18], both in terms of computational complexity and precision. To show that this is not an inherent limitation to our attack, we present additional results for CNNs with 6 convolution layers, ordered in 3 blocks of 2 convolution layers with ReLU activation, followed by a pooling layer after each block.

The models are trained with adversarial training with \( \epsilon = 0.01 \), one on benign and one on poisoned GTSRB data. For both networks, certified accuracy is at 0.0% for \( \epsilon = 0.01 \), confirming the poor certifier scaling to larger networks. For a smaller radius of \( \epsilon = 0.005 \), the certified accuracy drops from 20% on benign data to 10% on backdoor data, and from 50% to 32% for \( \epsilon = 0.003 \).

These results confirm that the attack is still effective on larger model sizes. Since the poisoning process itself is independent of the model training, there is no inherent limit to the model size to which our attack can scale.

DeepPoly Certifier: The threat model we identified and consequently our attacks are general and independent of the concrete certifier used. To show that these results generalize to other certifiers, we certify the models from section V-B with DeepPoly [18], a different, state-of-the-art certifier.

| Training   | Benign Data | Backdoor Data |
|------------|-------------|---------------|
|            | Mean Accuracy | Certification \( \epsilon \) | Mean Accuracy | Certification \( \epsilon \) |
| Without Attack | 0.005 | 0.010 | 0.005 | 0.010 |
| Natural     | 92.1 | 46.7 | 18.7 | 92.1 | 47.3 | 19.3 |
| Adversarial | 93.6 | 62.5 | 40.1 | 93.4 | 63.1 | 40.5 |
| Proviable   | 90.0 | 83.2 | 73.4 | 90.0 | 83.0 | 73.4 |

TABLE II

Mean accuracy and certified robustness for CNNs with different training methods on GTSRB. The numbers in parenthesis show the relative change compared to the baseline.
With increasing $\epsilon$, the model has to be confident in its output for unperturbed data. This increase towards $\epsilon = 0$ is to be expected when requiring high prediction accuracy since the model has to be confident in its output for unperturbed data. We conjecture that this effect is likely caused by the high emphasis the training on worst-case bounds puts on data. We show the influence of different poison ratios on the success of our indirect attack by running the same experiment introduced in section V-B with $\epsilon = 0.02$ and natural training, but for different poison ratios. The fewer poisoned samples we add to the training data, the less likely it will be detected. Figure 3 shows the certified robustness on benign and backdoor data for different poison ratios. Adding just 0.5% poisoned samples is already sufficient to drop certified robustness from originally 88.3% to 8.9% on the backdoor distribution. Increasing the poisoning ratio to 1% further decreases certified robustness to 3.5%, which largely remains in the same range for larger ratios. These results show that the attack is already highly effective for a small number of poisoned samples, hiding it well from the victim.

### E. Poison Ratio

We show the influence of different poison ratios on the success of our indirect attack by running the same experiment introduced in section V-B with $\epsilon = 0.02$ and natural training, but for different poison ratios. The fewer poisoned samples we add to the training data, the less likely it will be detected. Figure 3 shows the certified robustness on benign and backdoor data for different poison ratios. Adding just 0.5% poisoned samples is already sufficient to drop certified robustness from originally 88.3% to 8.9% on the backdoor distribution. Increasing the poisoning ratio to 1% further decreases certified robustness to 3.5%, which largely remains in the same range for larger ratios. These results show that the attack is already highly effective for a small number of poisoned samples, hiding it well from the victim.

### F. Discussion

Both the direct and indirect versions of our backdoor attacks achieve high success rates on MNIST classification, reducing the certified robustness on the backdoor distribution significantly while maintaining high accuracy and robustness on benign data to remain undetected. This is mostly true independent of the training method used by the victim for the indirect attack.

The only exceptions are large $\epsilon$-values with CROWN-IBP training, where the robustness increases again on the backdoor data. We conjecture that this effect is likely caused by the high emphasis the training on worst-case bounds puts on robust predictions at the cost of accuracy. The robustness loss directly optimizes for a large margin between the bounds of the predicted class and the rest. In the absence of meaningful class labels, this can lead to a decision surface which predicts arbitrary labels with high robustness “no matter what”, and therefore ignores the uncertainty introduced by random labels.

The high effectiveness and sneakiness of our attacks also extend to more complex CNNs, larger models, and a more challenging classification task. Using a different certifier, DeepPoly, shows the same results, demonstrating that our attacks transfer well to a different certifier. Finally, the results also hold when we add the additional high-accuracy constraint on the backdoor distribution.

In general, the experimental evaluation of our attacks shows their wide applicability in different settings. It supports our hypothesis that the threats identified in section III are very

| Training          | Benign Data | Backdoor Data |
|-------------------|-------------|---------------|
| Natural           | 86.6        | 15.9          |
| Adversarial       | 91.5        | 15.8          |
| Provably          | 97.2        | 33.8          |

Table III: Certified robustness for fully-connected models trained on MNIST and certified with the DeepPoly [18] for $\epsilon = 0.02$. The models are attacked by our indirect poisoning attack with different training methods used by the victim.

| Data     | Mean Accuracy | Certification with $\epsilon$ |
|----------|---------------|-------------------------------|
|          | 0.01 0.03 0.05|
| Benign   | 98.7(-0) 98.1(-0) 95.7(-1)  94.1(-1) |
| Backdoor | 98.6(-0) 95.7(-3)  7.8(-89)  0.0(-95) |

Table IV: Certified robustness for fully-connected models trained on MNIST with our direct attack and additional high accuracy loss for backdoor data. Numbers in parenthesis show relative change to the unattacked baseline in Table I.

Table III shows the certified robustness for $\epsilon = 0.02$, using the same models as in Table I. As before, certified robustness on benign data is high with a large drop on backdoor data with trigger, showing that the results transfer to a different certification method.

### D. High Accuracy on Backdoor Data

As discussed in section III, the assumption is that the victim does not have access to samples from the poison distribution for evaluation, and therefore a high prediction accuracy on data with a trigger is not required for the attack to remain undetected (section IV).

However, one could argue that in certain scenarios, correct predictions on the backdoor distribution can make it even harder for the backdoor to be detected. We, therefore, analyze our direct attack with the additional objective from eq. (13), which also teaches the model to correctly classify images from the backdoor distribution.

Table IV shows results in the same setting as section V-B. On benign data, both mean accuracy and certified robustness are almost identical for all models, effectively hiding the backdoor. Contrary to previous experiments, the mean accuracy on the backdoor distribution remains unchanged at 98.6%, making it even more difficult to detect the attack.

The certified robustness on data from the backdoor distribution drops significantly by up to 95 p.p., with virtually no robustness guarantees for larger $\epsilon$ values. The attack is less effective for very small perturbations with $\epsilon = 0.01$ compared to previous versions. This increase towards $\epsilon = 0.0$ is to be expected when requiring high prediction accuracy since the model has to be confident in its output for unperturbed data. With increasing $\epsilon$, the robustness quickly drops, demonstrating a highly successful attack despite the additional constraint.
real, with practical implications for machine learning systems. While our demonstration of this new attack vector focuses on bound-based certifiers, it is likely that similar backdoors can be planted in other certifiers (e.g., randomized smoothing), too. This highlights the importance of the overall topic and warrants further investigations as well as consideration in the design and evaluation of future certifiers.

VI. DEFENSES

Given the high success rate of our attacks and their potentially severe impact on deep learning systems, it is prudent to develop defenses against these threats. Previous work on poisoning and backdoor attacks and defenses target accuracy and not certification. Hence, it is unclear, whether traditional defenses against misclassification attacks can be adapted, or if we require new, customized defenses for backdoor attacks against network certifiers. While this work primarily focuses on showing the vulnerability of certifiers to training-time attacks, we take the first step towards defenses in this section. We analyze the effectiveness of three defenses against traditional attacks in our novel setting: fine-pruning [41], neural cleanse [42], and trojan network detection [43].

A. Fine-pruning

Fine-pruning consists of two steps: On a small subset of verifiably benign data, dormant neurons are pruned from the model, hoping to remove the inactive, backdoor-related neurons. The pruned model is then fine-tuned on the same benign data subset.

Since the goal is an accurate and robust network without a backdoor, we track both accuracy and robustness on benign and backdoor data. Ideally, the defense preserves high accuracy and robustness on benign data, while recovering the robustness and accuracy on backdoor data and thus removing its negative effects.

In their original work, Liu et al., prune the inactive neurons of the last convolution layer to remove the high-level feature representation of the backdoor trigger. Since we use fully-connected networks without convolution, we instead remove the inactive neurons of the penultimate linear layer, which contains 128 neurons.

Table V shows accuracy and certified robustness with $\epsilon = 0.02$ for an MNIST classifier trained with natural training and our indirect backdoor attack for different amounts of pruned connections. With an increasing percentage of pruned neurons, the defense is able to recover some accuracy and robustness on backdoor data, reaching 63.6% accuracy and 38.0% certified robustness when the 96 (75%) neurons with the lowest average activation have been pruned. This is, however, still significantly below the target values of 97.8% accuracy and 90.8% certified robustness on benign data, which means the backdoor is still present and the network is still vulnerable to the attack.

B. Neural Cleanse

Neural cleanse [42] is a popular, more powerful defense against backdoor attacks, which can detect, identify, and then remove backdoors from the model. It works in multiple stages, where the first stage detects the backdoor trigger by finding the minimal perturbation which misclassifies all samples from a clean dataset to a target label. The backdoor is detected by finding outliers in the magnitude of perturbation required for different labels.

Running this detection step on a network trained with our backdoor yields no outliers, and therefore the detection fails. Since all consecutive steps rely on finding the perturbation pattern, this means the mitigation step cannot be applied.

This result makes sense, since our attack does not cause misclassification to a particular target label, and therefore we would not expect decision-boundaries to one target class near all others, which is the premise this defense relies on.

C. Trojan Network Detection

The third, recently published defense we evaluate is trojan network detection (TND) [43]. It detects backdoors in neural networks using feature inversion, exploiting the fact that trojan networks exhibit particularly strong neuron activation at certain coordinates. TND then uses the reverse-engineered input and compares the logit activations to those of the benign input. If the difference surpasses a threshold, it flags the model as malicious.

This defense also fails to detect our backdoor. When comparing the changes in logit activation, we observe that the change is similar in magnitude across all logits. This can again be explained by the fact that we do not target any particular class with our attack, but the robustness of all classes.

D. Discussion

The results on all three defenses show that our backdoor attacks on certifiers differ significantly from traditional backdoor attacks. The key difference is that our backdoor attacks on certifiers are not targeting misclassification and therefore require new approaches for effective defenses. Neither out of the box nor with our adaptations, any of the evaluated defenses, designed to prevent backdoors for misclassification, were able to detect or mitigate our novel attack, highlighting the need for customized solutions. This will be an interesting and crucial investigation for future work.

| Neurons Pruned | 0%  | 25%  | 50%  | 75%  |
|----------------|-----|------|------|------|
| Benign Data    |     |      |      |      |
| Accuracy       | 98.4| 98.5 | 98.3 | 97.8 |
| Certification  | 86.6| 87.5 | 89.0 | 90.8 |
| Backdoor Data  |     |      |      |      |
| Accuracy       | 29.3| 25.2 | 36.4 | 63.6 |
| Certification  | 15.6| 36.3 | 31.8 | 38.0 |

**TABLE V**

Accuracy and certified robustness for natural training of a fully-connected model on MNIST with $\epsilon = 0.02$ and different amounts of pruned connections.
VI. RELATED WORK

Our method is related to the work on traditional backdoor attacks targeting the classifier’s predictions, and as first results on limitations of randomized-smoothing-based certification.

A. Backdoor Attacks

As discussed in section II-D, there is a long line of work on traditional backdoor attacks against neural networks. In contrast to our attacks targeting the certifier and therefore the availability of the model, traditional backdoor attacks target the model’s integrity by causing misclassifications.

Our direct (section IV-B) backdoor attacks are inspired by BadNets [27] and use the same supply chain vector and similar trigger patterns to activate the backdoor. However, the different goal of our attacks requires a different construction of the backdoor, combining new and existing optimization objectives.

Chen et al. [28] use data poisoning to indirectly target a model trained by the victim, adding a backdoor that causes the model to misclassify faces. This attack vector is similar to our indirect attack (section IV-C), where we also use a small number of triggered samples to poison the data set. However, as before, the target of our attack is the certifier, not the model’s predictions. Therefore, instead of consistently targeting a particular class, we use random labels to destabilize the prediction and thus cause the certification to fail.

B. Backdoor Defenses

Complementing the work on backdoor attacks, there is a line of work to defend against these traditional backdoor attacks that target the model’s predictions. While not designed for our attacks targeting the certifier, we adapt and evaluate three different defenses against our novel attacks.

(i) Fine-pruning [41] removes the backdoor by pruning inactive neurons from the model. (ii) Neural cleanse [42] is a multi-stage approach, which first detects, then isolates, and finally removes backdoors from the model. (iii) Trojan network detection [43] also detects backdoors based on feature inversion, using this information to flag malicious models.

C. Attacks Against Certification

Very recent work [44], [45] has looked at the robustness of randomized smoothing to different attacks. However, they have different attack goals from our availability attacks, and their attack vectors are unique to randomized smoothing. Due to this and the fundamentally different nature of bound-based certifiers to randomized smoothing, these are not directly comparable to our availability attacks based on backdoors.

Mehra et al. [44] target the certified radius of a particular class using a poisoning scheme that directly minimizes the certified radius. Maho et al. [45] exploit a discrepancy between the theoretical guarantees and the practical implementation of randomized smoothing using a black-box evasion attack.

VIII. CONCLUSION

In summary, our work shows that current state-of-the-art network certifiers are extremely vulnerable to training-time attacks. Our systematic analysis of their threat surface in section III reveals attack vectors against certifiers in both offline and online settings.

Especially the need to abstain when robustness cannot be guaranteed proves problematic in practice since the system becomes reliant on its fallback, which incurs additional costs, can require a human operator, or even fail to provide any prediction. By explicitly targeting the certifier and causing it to abstain, an attacker can effectively disable the deep learning model, compromising the system’s overall integrity and availability.

Our novel backdoor attacks against certifiers show the practical relevance of these new attack vectors. We demonstrate two examples of how these backdoors can be added to the model: our direct attacks which directly modify the training objective and our indirect attacks using data poisoning.

Once present, the attacker can flexibly activate the backdoor by adding a trigger to arbitrary inputs, causing certification to fail in almost all cases. Extensive experiments on multiple datasets, network architectures, and different certifiers in section V show the general nature of these threats.

These findings have significant consequences for both theoretical research and practical applications. For the latter, it means that designing an appropriate fallback is crucial. Since attacks can consistently trigger this fallback, it needs to be able to handle the full system load, and not just the occasional edge-case as evaluation on benign data might suggest.

From a theoretical standpoint, our findings show that simply abstaining from a prediction has major consequences, which need to be considered when proposing it as a solution. Ideally, such abstain cases would be avoided. Where unavoidable, worst-case guarantees on the frequency a system can be forced to abstain would go a long way towards mitigating the impact of attacks against certifiers.

To mitigate this problem, defenses against our new attacks are required. We perform a first evaluation of potential defenses in section VI by adapting existing methods designed against traditional backdoor attacks to our new setting. The results show that current methods have little to no effect, requiring new defenses specifically designed against this new type of attack. This is a crucial direction for future work, which would ideally lead to provable robustness guarantees against training-time attacks. Combined with the current deployment-time certifiers, it could lead to systems that are provably robust against both types of attacks.

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APPLENIX A

BACKGROUND ON ADVERSARIAL TRAINING

While it is essential to know a model’s robustness, certifiers alone do not improve the model’s resilience against evasion attacks. Many empirical defenses have been proposed to detect and mitigate evasion attacks, for example, based on attack detection [46]–[48] or randomization [49]–[51]. However, most of these were broken by stronger attacks.

Some of the most successful defenses today use adversarial training by directly optimizing the robust optimization problem (eq. (2)). While the outer minimization problem is solved by traditional network training, they differ in how they optimize the inner maximization objective.

a) Training with Adversarial Samples: The original studies on adversarial training [8], [40] propose to approximate the inner max objective by incorporating adversarial attacks into the training loop. In addition to the samples from the original dataset, the network is also trained on adversarially perturbed versions of said data, making it more resilient to future attacks.

b) Training with Certification Bounds: Instead of using adversarial attacks, the inner max objective can also be approximated using the bounds computed by network certifiers. By training the network on the entire perturbation set instead of individual samples, they increase the robustness and simultaneously compensate for the certifier’s over-approximation.

This robustness gain comes at the cost of additional training time since the certifier has to be invoked for each forward pass. Training is therefore only feasible for fast relaxations based on intervals [52], [53] or linear bounds [26].

In our experiments, we use both of these techniques to improve the model’s robustness, simulating a victim which is interested in robust models with high certified accuracy for their safety-critical application.