Interface effects on the shot noise in normal metal- $d$-wave superconductor Junctions

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(March 24, 2022)

Abstract

The current fluctuations in normal metal / $d$-wave superconductor junctions are studied for various orientation of the crystal by taking account of the spatial variation of the pair potentials. Not only the zero-energy Andreev bound states (ZES) but also the non-zero energy Andreev bound states influence on the properties of differential shot noise. At the tunneling limit, the noise power to current ratio at zero voltage becomes 0, once the ZES are formed at the interface. Under the presence of a subdominant $s$-wave component at the interface which breaks time-reversal symmetry, the ratio becomes $4e$. 
The origin of shot noise is the current fluctuations in transport due to the discreteness of the charge carrier. Shot noise measurements provide important information on conduction processes which can not be obtained from the usual conductance measurements. In the last few years, several novel features peculiar to shot noise in mesoscopic systems have been revealed. In particular, the shot noise in normal metal-superconducting junction and superconductor / insulator / superconductor junctions have been intensively studied. It has been shown, through these works, that the Andreev reflection and the charge transport by the Cooper pairs have significant influence on the transport fluctuation at low voltages. However, most of previous theories are constructed on the conventional s-wave superconductors and the theory for d-wave superconductors has not been presented.

On the other hand, extensive experimental and theoretical investigations have revealed that the pair potentials of high-\( T_c \) superconductors are \( d_{x^2-y^2} \)-wave symmetry. One of the essential differences of \( d_{x^2-y^2} \)-wave superconductors from conventional s-wave superconductors is that the phase of the pair potential strongly depends on the wave vector. For example, the appearance of zero-bias conductance peak (ZBCP) in tunneling conductance at a (110) surface of \( d_{x^2-y^2} \)-wave superconductors reflects the sign change of effective pair potential through the reflection of quasiparticle at the surface. Orientational and material dependences of ZBCP of high-\( T_c \) superconductors have been experimentally studied in several groups, and the consistency between theory and experiments has been checked in details.

At this stage, it is an interesting problem to clarify what is expected in the shot noise in normal metal / insulator /\( d_{x^2-y^2} \)-wave superconductor \( (n/I/d) \) junction under the presence of the ZBCP. Recently, Zhu and Ting presented a theory of the shot noise in \( n/I/d \) junction. They found a remarkable feature for \( n/I/d \) junction: when the angle between the normal to the interface \( \alpha \) is \( \pm \pi/4 \), the noise-to-current ratio is zero at zero-bias voltage and quickly reaches a classical Shottky value \( 2e \) at finite voltage. This feature is completely discrepant from that for conventional s-wave superconductor \( (n/I/s) \) junctions where the ratio is \( 4e \) at zero voltage and \( 2e \) at finite voltage. This anomalous behavior is responsible for the formation of the zero energy Andreev bound states (ZES) at the interface of the \( d \)-wave.
superconductor. Although Zhu and Ting theory (ZT theory) clarified important aspects of the shot noise under the presence of ZES, there still remain several unresolved problems.

One is the detailed orientational dependence of the shot noise. In ZT theory, the vanishment of the noise-to-current ratio $R(eV)$ at zero voltage ($eV = 0$) is shown only for $\alpha = \pi/4$ ($0 \leq \alpha \leq \pi/4$). Since this property is related to the existence of ZBCP in tunneling conductance, we must check the value of $R(0)$ for $0 < \alpha < \pi/4$ where ZBCP appears in the tunneling conductance. In this paper, we will show that in the tunneling limit, i.e., low transparency limit, $R(0)$ vanishes for $\alpha \neq 0$ and becomes $4e$ only for $\alpha = 0$ where no ZES are expected. Moreover, it is shown that $R(0)$ is classified into three values, $0, 2e$ and $4e$, corresponding to the region of the Fermi surface contributing to the ZES.

The other is the influence of the spatial dependence of the pair potential on the shot noise. It is known that when the ZES are formed at the interface of $d$-wave superconductor the pair potential is suppressed near the interface. Consequently, not only the ZES but also the non-zero energy Andreev bound states (NZES) are formed. The influences of the NZES on the shot noise are clarified. We further study the situation where a subdominant $s$-wave component which breaks the time reversal symmetry is induced near the interface of $d$-wave superconductor. The subdominant $s$-wave component influences significantly on $R(eV)$.

The model examined here is a two-dimensional $n/I/d$ junction within the quasiclassical formalism where the pair potential has a spatial dependence

$$\bar{\Delta}(x, \theta) = \begin{cases} 
0, & (x \leq 0) \\
\bar{\Delta}_R(x, \theta), & (x \geq 0)
\end{cases}$$

(1)

Here $\theta$ is the angle of quasiparticle trajectory measured from the $x$ axis. If we apply this formula to $d$-wave superconductors including a subdominant $s$-wave component near the interface, $\bar{\Delta}_R(x, \theta)$ is decomposed into

$$\bar{\Delta}_R(x, \theta) = \Delta_d(x) \cos[2(\theta - \alpha)] + \Delta_s(x)$$

(2)

where $\alpha$ denotes the angle between the normal to the interface and the $x$ axis of the crystal.
The insulator located between the normal metal and the superconductor is modeled by a $\delta$ function. The magnitude of the $\delta$-function denoted as $H$ determines the transparency of the junction $\sigma_N$, with $\sigma_N = \cos^2 \theta/[Z^2 + \cos^2 \theta]$ and $Z = mH/\hbar^2k_F$. The effective mass $m$ and Fermi momentum $k_F$ are assumed to be constant throughout the junction. The noise power to current ratio $R(eV)$, the differential shot noise $S_T(eV)$, and the tunneling conductance $\sigma_S(eV)$, are given by

$$R(eV) = \frac{\int_0^{eV} dE S_T(E)}{\int_0^{eV} dE \sigma_S(E)}$$

$$S_T(eV) = \frac{1}{2} \int_{-\pi/2}^{\pi/2} d\theta S(eV, \theta) \cos \theta, \quad \sigma_S(eV) = \frac{1}{2} \int_{-\pi/2}^{\pi/2} d\theta \bar{\sigma}_S(eV, \theta) \cos \theta$$

$$\bar{S}(eV, \theta) = \frac{4e^3}{\hbar} [R_a(1 - R_a) + R_b(1 - R_b) + 2R_aR_b]$$

$$\bar{\sigma}_S(E, \theta) = \frac{2e^2}{\hbar} (1 + R_a - R_b)$$

As a reference, we also calculate the tunneling conductance normalized by that in the normal state,

$$\sigma_T(eV) = \frac{\int_0^{eV} dE \sigma_S(E)}{1/2 \int_{-\pi/2}^{\pi/2} d\theta \sigma_N \cos \theta}.$$  

Note that the differential shot noise and conductance spectrum are expressed only by $\eta_{R,\pm}(x, \theta)$ just at the boundary $(x = 0)$ where $\eta_{R,\pm}(x, \theta)$ obeys the following equations,

$$\frac{d}{dx} \eta_{R,+}(x, \theta) = \frac{1}{i\hbar v_F \cos \theta} \left[ -\bar{\Delta}_R(x, \theta_+)\eta_{R,+}^2(x, \theta) - \bar{\Delta}_R^*(x, \theta_+) + 2E\eta_{R,+}(x, \theta) \right]$$.
\[ \frac{d}{dx} \eta_{R,-}(x, \theta) = \frac{1}{i\hbar v_F \cos \theta} \left[ -\Delta_R^*(x, \theta-) \eta_{R,-}^2(x, \theta) - \Delta_R(x, \theta-) + 2E \eta_{R,-}(x, \theta) \right], \]  

(11)

with \( v_F = k_F/m, \quad \theta_+ = \theta \) and \( \theta_- = \pi - \theta \), and \( \Delta_R(x, \theta_+) [\Delta_R(x, \theta_-)] \) is the effective pair potential felt by an electron [a hole] like quasiparticle. The quasiparticle energy \( E \) is measured from the Fermi energy.

The spatial dependence of the pair potentials are determined by the following equations:

\[ \Delta_s(x) = g_s k_B T \sum_{\omega_n} \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\theta' \{ [g_R(\theta', x)]_{12} - [g_R^+(\theta', x)]_{12} \} \]  

(12)

\[ \Delta_d(x) = g_d k_B T \sum_{\omega_n} \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} d\theta' \cos[2(\theta' - \alpha)] \{ [g_R(\theta', x)]_{12} - [g_R^+(\theta', x)]_{12} \} \]  

(13)

\[ \lim_{x \to \infty} \Delta_s(x) = 0, \quad \lim_{x \to \infty} \Delta_d(x) = \Delta_0 \]  

(14)

with dimensionless inter-electron potential of the \( s \)-wave \( g_s \) and \( d \)-wave \( g_d \), respectively. The quasiclassical Green’s function \( g_R(\theta, x) \) obeys:

\[ g_R(\theta, x) = U_R(\theta, x, 0) g_R(\theta, 0) U_R^{-1}(\theta, x, 0) \]  

(15)

\[ i\hbar v_F \frac{\partial}{\partial x} U_R(\theta, x, 0) = - \begin{pmatrix} i\omega_n & \Delta_R(x, \theta_+) \\ -\Delta^*_R(x, \theta_+) & -i\omega_n \end{pmatrix} U_R(\theta, x, 0), \]  

(16)

with \( \omega_n = 2\pi k_B T(n + 1/2) \) and \( U_R(\theta, 0, 0) = 1 \). In the actual numerical calculations, \( \eta_{R,\pm}(x, \theta) \) is calculated from Eqs. (11) to (14). Since \( g_R(\theta, 0) \) is expressed by \( \eta_{R,\pm}(\theta, 0) \), \( g_R(\theta, x) \) is obtained using Eqs. (13) to (16). Subsequently, the spatial dependence of the pair potentials \( \Delta_d(x) \) and \( \Delta_s(x) \) are calculated by Eqs. (12) to (14). To get self-consistently determined pair potential, this process is repeated until enough convergence is obtained.

First let us consider the case where \( \Delta_s(x) \) is not present. It is known for \( \alpha \neq 0 \), \( \sigma_T(eV) \) has a zero bias conductance peak (ZBCP) with small magnitude of \( \sigma_N \). It is expected that this property reflects on \( S_T(eV) \) for \( \alpha \neq 0 \). In Fig. 1, \( S_T(eV) \) is plotted for \( Z = 5 \) with \( \alpha = \pi/6 \) [see curve a in Fig. 1(a)]. As a reference, similar calculations are performed based on the non self-consistent pair potential where the spatial dependence of the pair potential
is chosen as $\Delta_d(x) = \Delta_0$ [see curve b in Fig. 1(a)]. As seen from curves a and b, $S_T(0)$ has a peak around zero voltage originating from the ZBCP in $\sigma_T(0)$ [see Fig. 1(b)]. As compared to curves a to b, both the height and width of the peak around zero voltage of curve a are small as compared to those in b, since the degree of resonance at zero voltage is weakened due to the reduction of $\Delta_d(x)$ at the interface, Besides this property, curves a in and $S_T(eV)$ ($\sigma_T(eV)$) have second peak around $eV \sim 0.45\Delta_0$ due to the formation of NZES which can not be expected in curves b.

In Fig. 2, we plot $R(eV)$ for $n/I/d$ junction for sufficient low transparency case, e.g., $Z = 5$. In the case of $\alpha = 0$, no ZES are formed at the interface, then the resulting $R(eV)$ is $4e$ at zero voltage and is $2e$ at higher voltage (curve a in Fig. 2). When the value of $\alpha$ deviates from 0, $R(0)$ is zero and it quickly reaches a classical Shottky value $2e$, which is consistent with other report\textsuperscript{[5]} (curve b and c). This feature is explained based on the $\theta$ dependence of $\bar{S}(0, \theta)$ and $S(0, \theta)$ as follows: in general, $\bar{S}(0, \theta) = \frac{4e^2}{h} R_a$ and $S(0, \theta) = \frac{16e^3}{h} R_a (1 - R_a)$ are satisfied. At the tunneling limit, $R_a$ is suppressed with the increase of $Z$ unless the ZES are formed at the interface. When ZES are formed, $R_a = 1$ is satisfied independent of $Z$.

Such a situation is realized for $\pi/4 - \alpha < |\theta| < \pi/4 + \alpha$. For $\alpha = 0$, no ZES are formed and both the magnitude of $S_T(0)$ and $\sigma_S(0)$ are reduced with the increase of $Z$, while $R(0)$ remains to be constant. On the other hand, for $\alpha \neq 0$ the magnitude of $S_T(0)$ is reduced while $\sigma_S(0)$ remains finite with the increase of $Z$, thus $R(0)$ becomes zero. The important point is that the origin of the vanishment of $R(0)$ is responsible for the presence of ZES. It is a universal property excepted for unconventional superconductor junctions where finite region of the Fermi surface contributing on the formation of the ZES.

The critical situation is realized in $n/I/p_x + i p_y$ junction where ZES are formed only by the quasiparticles injected perpendicular to the interface\textsuperscript{[31]}. The spatial dependence of the pair potential $\bar{\Delta}_R(x, \theta)$ is given by

$$\bar{\Delta}_R(x, \theta) = \Delta_{p_1}(x) \cos \theta + i \Delta_{p_2}(x) \sin \theta$$

(17)

with
\[
\lim_{x \to \infty} \Delta_{p1}(x) = \Delta_0, \quad \lim_{x \to \infty} \Delta_{p2}(x) = \Delta_0.
\]

\(R(eV)\) is calculated based on the self-consistently determined pair potentials, \(\Delta_{p1}(x)\) and \(\Delta_{p2}(x)\). As shown in curve \(d\) in Fig. 2, \(R(eV)\) is nearly 2\(e\) independent of bias voltages. The nature that \(R(0)\) is neither 4\(e\) nor 0 has never been predicted in previous theories. 

Under the presence of the ZBCP, since the quasiparticle density of states at the zero energy near the interface are enhanced, a subdominant \(s\)-wave component of the pair potential \(\Delta_s(x)\) can be induced near the interface, when a finite \(s\)-wave pairing interaction strength exists, even though the bulk symmetry remains pure \(d\)-wave. Since the phase difference of the \(d\)-wave and \(s\)-wave components is not a multiple of \(\pi\), the mixed state breaks the time-reversal symmetry. In such a case, the ZBCP splits into two and the amplitude of the splitting depends on the magnitude of the induced \(s\)-wave component. The corresponding \(R(eV)\) is plotted in Fig.3(a) with \(\alpha = \pi/4\) and \(Z = 5\) where the transition temperature of \(s\)-wave component is chosen as \(T_s = 0.15T_d\) (dotted line) and \(T_s = 0.3T_d\) (solid line). The induced \(s\)-wave component near the interface influences crucially on the \(R(eV)\) at low voltages. The remarkable feature is that \(R(0)\) recovers to be 4\(e\) as in the cases of \(n/I/s\) junctions. It is because that with the inducement of \(s\)-wave component which breaks the time reversal symmetry, the position of the Andreev bound state shifts to \(eV \sim \Delta_s(0)\), where \(\sigma_T(eV)\) has a peak [see Fig. 3(b)]. Consequently, ZBCP shifts to this voltage and \(R(eV)\) has a dip structure. Although \(R(0) = 4e\) is satisfied, the overall feature of \(R(eV)\) is completely different from that in \(n/I/s\) junction, since \(s\)-wave component is induced only near the interface in the present case.

In this paper, the current fluctuations in normal metal / \(d\)-wave superconductor junctions are studied for various orientation of the junction by taking account of the spatial variation of the pair potentials. Not only the zero energy Andreev bound states but also the non-zero energy Andreev bound states show up in the line shape of \(S_T(eV)\). At the tunneling limit, we found universal property of \(R(0)\) for two dimensional superconductors. \(R(0)\) is 0, 2\(e\) and 4\(e\), corresponding to three cases where the region of the Fermi surface contributing to the
ZES is i) finite region, ii) point and iii) none, respectively. The present property gives useful information on the identification for the symmetry of the unconventional superconductors. We hope the measurement of shot noise in unconventional superconductors will be performed near future.

It is known that ZES also influences significantly on Josephson current in $d$-wave superconductor / insulator / $d$-wave superconductor ($d/I/d$) junctions. It is an interesting and future problem to study the shot noise in $d/I/d$ junctions.
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FIG. 1

FIG. 1. $\sigma_T(eV)$ [Fig.1(a)] and $S_T(eV)$ [Fig.1(b)] for $n/I/d$ junction with $Z = 5$ and $\alpha = \pi/6$.

a: self-consistent calculation, b: non self-consistent calculation.
FIG. 2

FIG. 2. $R(eV)$ is plotted with $Z = 5$: a: $\alpha = 0$ b: $\alpha = \pi/12$ c: $\alpha = \pi/4$ for $n/I/d$ junctions. d:

Similar plot for $n/I/p$ junction.
FIG. 3

FIG. 3. $R(eV)$ is plotted for $Z = 5$ and $\alpha = \pi/4$ where $s$-wave component is induced at the interface [Fig. 3(a)]. a: $T_s = 0.15T_d$ and b: $T_s = 0.3T_d$. The corresponding $\sigma_T(eV)$ is plotted in [Fig. 3(b)].