Voltage Control of DC Islanded Microgrids: Scalable Decentralised $L_1$ Adaptive Controllers

Daniel O’Keeffe$^{1,1}$, Stefano Riverso$^{1,2}$, Laura Albiol-Tendillo$^{2,5}$, and Gordon Lightbody$^{1,3}$

$^1$Control & Intelligent Systems Group, School of Engineering, University College of Cork, Ireland
$^2$United Technologies Research Centre Ireland Ltd, 4th Floor Penrose Business Centre, Cork, Ireland
$^3$MaREI-SFI Research Centre, University College Cork, Ireland

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Abstract

Voltage stability is a critical feature of an efficiently operating power distribution system such as a DC islanded microgrid. Large-scale autonomous power systems can be defined by heterogeneous elements, uncertainty and changing conditions. This paper proposes a novel scalable decentralised control scheme at the primary level of the typical hierarchical control architecture of DC islanded microgrids with arbitrary topology. Local state-feedback $L_1$ adaptive controllers are retrofitted to existing baseline voltage controllers of DC-DC boost converters, which interface distributed generation units with loads. Furthermore, local controller synthesis is modular as it only requires approximate information about the line parameters that couple neighbouring units. The performance of the proposed architecture is evaluated using a heterogeneous DC islanded-microgrid that consists of 6 DC-DC boost converters configured in a radial and meshed topology. The use of $L_1$ adaptive controllers achieves fast and robust microgrid voltage stability in the presence of plug-and-play operations, unknown load and voltage reference changes, and unmodelled dynamics. Finally, sufficient conditions for global stability of the overall system are provided.

Keywords: Decentralised Control, Low-Voltage DC Islanded Microgrid, Robust-Adaptive Control, Scalable Design, Voltage Stability
1 Introduction

Over the last decade, considerable efforts have been made to transform the current passive electricity grid into a dynamic, adaptable and resilient Smart Grid (SG) [1]. The SG will be the future cornerstone for increased autonomy, reliability and distribution efficiency [2]. To achieve such features, intelligent interoperability between electrical, control and communication systems must be coordinated [3].

The paradigm-shift towards distributed generation and storage units (DGUs/DSUs), market liberalisation, bi-directional transmission and demand-side interaction requires a distributed solution to manage future power networks. Islanded microgrids (mGs) have emerged as a smart-grid initiative to autonomously integrate power-electronic-interfaced DGU/DSUs with loads, and provide ancillary services to the utility grid [4–8]. Research and development of AC mGs has naturally progressed as AC power distribution deeply embedded in society [7, 9]. Advances in DC-DC power electronics, has led to the promising emergence of DC mGs [10]. DC power distribution avoids inherent issues associated with AC such as harmonic compensation, reactive power and synchronisation; thus improving power quality, efficiency and reliability. Furthermore, the use of DC can reduce the weight of a power network by 10 tons/MW compared to AC components [10]; important for application such as the More Electric Aircraft (MEA) and electric vehicles. Recently, DC ImGs have been deployed in low-voltage DC (LVDC) networks such as telecommunication towers, occupied interior spaces, data centres and traction systems [2, 11–13]. The next wave of DC mG applications are expected in large-scale residential, commercial and industrial (C&I) buildings, and aerospace [14, 15].

Key control features of large-scale mGs include: (i) Voltage stability and accurate load-sharing, (ii) Scalability: the ability to design controllers independent of the size and topology of the mG, (iii) Plug-and-play (PnP) operations: the ability to reconfigure DGU/DSUs without compromising global stability conditions, and (iv) Robustness to uncertainty within a heterogeneous system [16, 17].

Voltage stability and accurate load-sharing of the DC-DC power converters that interface DGUs, DSUs with loads is integral to the safe and efficient operation of the ImG. A distributed hierarchical control architecture, utilising classical controllers and low-bandwidth communications, has become the standard within mG research [7, 17, 18]. Though feature (i) is achieved using this approach, (ii)–(iv) are limited. Stability conditions are only satisfied for specific radial and bus-connected topologies, while homogeneous subsystems are only considered. As identified in [19] and demonstrated in [20], the approach lacks scalability, PnP capabilities and robustness to uncertainty.

Recent mG research has addressed features (ii) and (iii). PnP control designs, first outlined in [21], have successfully been deployed as primary and secondary controllers in the standard hierarchical control structure of AC [22, 23] and DC ImGs [19, 24]. Primary controllers are locally responsible for stable power distribution, while secondary controllers coordinate system voltage levels and improve load-sharing accuracy using low-bandwidth communications (LBC). PnP controllers maintain operation stability when DGUs and loads are reconfigured without requiring a priori knowledge. Global asymptotic stability (GAS) is guaranteed by checking the viability of DGU plug-in/out operations through an off-line optimisation problem using linear matrix inequalities (LMIs). Furthermore, the technique is scalable as local controllers depend only on knowledge of corresponding DGU and line-couplings. Once DGU plug-in/out, neighbouring controllers are required to retune off-line, resulting in limited robustness. Recently, line-independent [25, 26] and robust [27] PnP controllers were proposed to overcome this. However, these PnP techniques are computationally extensive, controller gains are required to discontinuously switch after off-line stability checks are performed, and robustness to network uncertainty is limited.

Adaptive control strategies have recently been proposed to accommodate the heterogeneity and privacy requirements of large-scale mGs, where dynamics, system reconfigurability, coupling and loads can be uncertain or unknown [28–30]. These applications implement adaptive controls based on premeditated conditions or linear controllers to provide small-signal adjustments to droop resistances for dynamic performance when achieving system objectives such as voltage coordination and load-sharing. Here, the adaptive laws are adapting to uncertainty of the droop parameters, as opposed to uncertainty concerning the system dynamics. Furthermore, these techniques depend on accurate system models, specific mG topologies, and do not address well-documented adaptive control issues, as outlined in [31]. These include guaranteed stability in the presence of uncertainty and fast adaptation. To address these issues, robust-adaptive control techniques, such as the $\mathcal{L}_1$ adaptive controller ($\mathcal{L}_1$AC) [32–35], have recently been developed and successfully deployed in various applications [36–40].

This paper proposes a scalable decentralised $\mathcal{L}_1$AC to ensure fast and robust voltage control by
augmenting baseline primary voltage controllers. The rationale for implementing an augmentation approach as opposed to a fully adaptive one is that in real systems it is common to have baseline controllers designed to provide reference tracking and disturbance rejection during nominal operation. Though the proposed design is not line-independent, due to its adaptive nature, conditions on \textit{a priori} parameter knowledge are relaxed. The paper aims to address features (i)-(iv) in the following context:

- Heterogeneous DC ImG consisting of grid-forming DC-DC boost converters.
- Parametric uncertainty of system dynamics i.e. network topology, line couplings and loads.
- Reconfiguration of DGUs and loads through PnP operations

This paper is structured as follows. In section 2, the DC ImG state-space model is developed using an arbitrary load-connected topology and Quasi-Stationary Line approximations. State-feedback baseline controllers are also designed. In section 3, an overview of the $L_1$AC architecture is provided, and the decentralised augmenting controllers are subsequently designed. Finally, section 4 describes the simulation tests carried out, including PnP operations, robustness to unknown load changes and unmodelled dynamics, and voltage reference tracking.

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2 DC Islanded Microgrid Model

2.1 DGU Electrical Model

Microgrids are generally coupled to a stiff utility grid. Transformers couple AC buses to the grid in AC mGs. However in the case of DC mGs, power limited converter units interface the mG with the utility grid, thus effectively islanding them i.e. the inertia of the stiff utility grid is buffered. A fundamental feature of any islanded-mG (ImG) is the ability to convert power between different DGUs, DSUs and loads automatically and efficiently. This is performed using different DC-DC boost and buck converter topologies. Fig. 1 presents a bus-connected DC mG which can exchange power with the utility grid.

![Figure 1: Bus-connected LVDC microgrid. Adapted from [41].](image)

This work considers boost converters, which step-up low voltages to high voltages. Initial DC mG research investigated buck converters [18], [42], [19], [41], [43] as they are commonly interfaced with low-power loads, and are easier to control. Boost converter controllers are notoriously difficult to tune in mGs due to their non-minimum phase action and have only received attention recently [20], [44].

For simplicity the DC ImG can be modelled as a two-node network, and subsequently generalised to a network of $N$-nodes. Fig. 2 represents the averaged model, which considers dynamics over both on/off switching states, of two boost converters $i$ and $j$ coupled via resistive and inductive power lines, for $k \in \{i, j\}$.
Figure 2: Averaged model of DC ImG composed of two radially coupled boost converter DGUs with unknown loads.

The ImG of Fig. 2 is arranged in a general load-connected topology where each DGU supplies power to a local load at the point of common coupling (PCC). DGUs can be mapped to load-connections via the Kron Reduction method [45], [46] which preserves the profile of electrical parameters at the PCC. This is a positive feature, as the model of each DGU is not dependent on the load, which could be unknown (e.g. non-linear/linear resistive, interfacing buck converter or variable speed motor drive). Instead, Fig. 2 represents the load as a current disturbance, \( I_{L_i} \).

Applying Kirchoff’s voltage and current laws to the DC ImG of Fig. 2 yields the following set of averaged differential equations:

**DGU i:**
\[
\begin{align*}
\frac{dI_{ti}}{dt} &= \frac{1}{L_{ti}}V_{in_i} - (1 - d_i)\frac{V_{dc_i}}{L_{ti}} - \frac{R_{ii}}{L_{ti}}I_{ti} \\
\frac{dV_{dc_i}}{dt} &= (1 - d_i)I_{ti} + \frac{1}{C_{ti}}I_{ij} - \frac{1}{C_{ti}}I_{ti} 
\end{align*}
\]  

**DGU j:**
\[
\begin{align*}
\frac{dI_{tj}}{dt} &= \frac{1}{L_{tj}}V_{in_j} - (1 - d_j)\frac{V_{dc_j}}{L_{tj}} - \frac{R_{jj}}{L_{tj}}I_{tj} \\
\frac{dV_{dc_j}}{dt} &= (1 - d_j)I_{tj} + \frac{1}{C_{tj}}I_{ji} - \frac{1}{C_{tj}}I_{tj}
\end{align*}
\]

**Line ij:**
\[
\begin{align*}
\frac{dI_{ij}}{dt} &= V_{dc_j} - R_{ij}I_{ij} - V_{dc_i}
\end{align*}
\]

**Line ji:**
\[
\begin{align*}
\frac{dI_{ji}}{dt} &= V_{dc_i} - R_{ji}I_{ji} - V_{dc_j}
\end{align*}
\]

**Remark 1:** As in [19], lines \( ij \) and \( ji \) physically couple DGU \( i \) to DGU \( j \) and vice versa, therefore \( R_{ij} = R_{ji} \) and \( L_{ij} = L_{ji} \). Hence, in steady-state, \( I_{ij} = -I_{ji} \).

The system of (1) can be represented in state space form as,

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\
y(t) &= Cx(t)
\end{align*}
\]  

where, \( x(t) = [I_{ti}, V_{dc_i}, I_{tj}, V_{dc_j}, I_{ij}, I_{ji}]^T \), is the state vector, \( u(t) = [V_{in_i}, V_{in_j}]^T \) is the input, \( d(t) = [I_{L_i}, I_{L_j}]^T \) is the load current disturbance, and \( y(t) = [V_{dc_i}, V_{dc_j}]^T \) is the measurable output. Matrices of (2) are detailed in section 6.1.

Figure 2: Averaged model of DC ImG composed of two radially coupled boost converter DGUs with unknown loads.
2.2 Quasi Stationary Line Model

If the time constant of the line transients is very fast, i.e. assuming $L_{ij}$ and $L_{ji}$ are significantly small, then line dynamics can be neglected. This type of model is known as a Quasi-Stationary Line (QSL) approximation. This is usually a good approximation for small-scale mGs where the lines are predominantly resistive. In open-loop, global stability can be inferred by ensuring local DGU stability, as detailed in section 6.1. Line equations (1c) and (1d) are represented in steady-state form using QSL approximations, i.e. $\frac{dI_{ij}}{dt} = \frac{dI_{ji}}{dt} = 0$:

$$I_{ij} = \frac{V_{dcj} - V_{dci}}{R_{ij}},$$

(3)

$$I_{ji} = \frac{V_{dci} - V_{dcj}}{R_{ji}}.

(4)

Replacing line current variable $I_{ij}$ of equation (1a) with equation (3) yields the following model for DGU $i$,

$$\Sigma_{DGU}^{i}:
\begin{align*}
\dot{x}_{ij}(t) &= \left[\begin{array}{c}
\frac{-R_{ii}}{L_{ii}}V_{in_i} - \frac{(1-d_i)}{L_{ii}}V_{dc_i} \vspace{1mm} \\
\frac{1}{C_{ii}} - \frac{1}{L_{ii}C_{ii}}
\end{array}\right] x_{ij}(t) + \left[\begin{array}{c}
\frac{1}{L_{ii}} \\
0
\end{array}\right] V_{in_i} + \left[\begin{array}{c}
0 \\
\frac{1}{C_{ii}}
\end{array}\right] I_{Li} + \left[\begin{array}{c}
0 \\
0
\end{array}\right] \left[\begin{array}{c}
\frac{1}{L_{ii}} \\
\frac{1}{L_{ii}C_{ii}}
\end{array}\right] x_{ij}(t)
\end{align*}

(5)

Interchanging indexes $i$ and $j$ yields the model for DGU $j$. Representing (5) in a general compact state space form, the dynamics of DGU $i$ are,

$$\Sigma_{DGU}^{i}:
\begin{align*}
\dot{x}_{ij}(t) &= A_{ii}x_{ij}(t) + B_{ii}u_{ij}(t) + E_{ii}d_{ij}(t) + \zeta_{ij}(t) + \gamma_{ij}(t) \\
y_{ij}(t) &= C_{ij}x_{ij}(t)
\end{align*}

(6)

where $x_{ij}(t) = [I_t, V_{dc}]^T$, $I_{Li}$ is the exogenous current disturbance. Unlike with the buck converter, where the averaged state space model of (6) is equivalent to the small-signal state space model, the boost converter is different. From the state matrix of above, the duty-cycle control input is a product of the state vector. As a result, the duty-cycle operating point directly influences stability. The averaged model is therefore non-linear and must be linearised about the duty-cycle operating point by forming a small-signal model. 1

$$\Sigma_{DGU}^{i}:
\begin{align*}
\dot{x}_{ij}(t) &= A_{ii}x_{ij}(t) + B_{ii}u_{ij}(t) + E_{ii}d_{ij}(t) + \zeta_{ij}(t) + \gamma_{ij}(t) \\
y_{ij}(t) &= C_{ij}x_{ij}(t)
\end{align*}

(7)

where $x_{ij}(t) = [\bar{v}_{t}, \bar{v}_{dc}]^T$, is the small-signal state vector, $u_{ij}(t) = \bar{d}_i(t)$ is the small-signal PWM control signal, $d_{ij}(t) = \bar{d}_iL_i$ is the small-signal exogenous current disturbance, $\zeta_{ij}(t) = A_{ij}x_{ij}(t)$ represents coupling with DGU $j$ and $\gamma_{ij}(t) = \frac{\bar{v}_{in_i}}{L_{ii}}$ is the small-signal input voltage disturbance. 

It is assumed that changes in input voltages $V_{in}$ are very slow, and thus can be neglected. Therefore $\gamma_{ij}(t) = 0$.

The matrices of (7) are,

$$A_{ii} = \begin{bmatrix}
\frac{-R_{ii}}{L_{ii}C_{ii}} & \frac{(1-d_i)}{L_{ii}C_{ii}} \\
\frac{1}{L_{ii}C_{ii}} & \frac{1}{L_{ii}C_{ii}}
\end{bmatrix}, B_i = \begin{bmatrix}
\frac{V_{dc_i}}{L_{ii}C_{ii}} \\
\frac{1}{L_{ii}C_{ii}}
\end{bmatrix}, E_i = \begin{bmatrix}
0 \\
\frac{1}{C_{ii}}
\end{bmatrix}, C_i = \begin{bmatrix}
0 & 1
\end{bmatrix}$$

where $V_{dc_i} = \frac{V_{in_i}}{1-d_iL_i}$ and $I_{Li} = \frac{V_{dc_i}}{1-d_iL_iC_{ii}}$. 2

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1 Note: each average quantity can be expressed as the sum of its steady state and small-signal values e.g. $d_k = D_k + \bar{d}_k$, $V_{dc_k} = \bar{v}_{dc_k} + \bar{v}_{dc_k}$.

2 As the input voltage to power converters in a mG is usually from renewable power or storage devices. The dynamics of these devices are much slower than the fast switching dynamics of power converters, therefore it is a safe assumption to neglect small-signal changes in input voltage.
2.3 QSL Model DC Islanded Microgrid Composed of $N$ DGUs

In this section, the two DGU network of Fig. 2 is generalised to an ImG composed of $N$ converter DGUs. [20] demonstrated that converter coupling dynamics predominantly manifest from physical power lines; duty-cycle coupling is weak. Neighbouring DGUs are thus defined if they are coupled by the $RL$ power line of Fig. 1. Letting $\mathcal{D} = \{1, \ldots, N\}$, $\mathcal{N}_i \subset \mathcal{D}$ denotes a neighbour-subset for DGU $i$. As before, assuming QSL approximation of all line dynamics $(i,j) \in \mathcal{D}$, the DC ImG model is represented by (6), with $\zeta_i(t) = \sum_{j \in \mathcal{N}_i} A_{ij} x_j(t)$. The only change in (6) is the local state vector matrix $A_{ii}$, becoming:

$$A_{ii} = \begin{bmatrix} \frac{-R_{ti}}{(1-D_{ti})} & \frac{(1-D_{ti})}{L_{ti}} \sum_{j \in \mathcal{N}_i} \frac{1}{R_{tj}C_{tj}} \end{bmatrix} \tag{8}$$

The overall global model of the $N$ DGU ImG can be given by,

$$\Sigma^{DGU}_{[N]} : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\ y(t) = Cx(t) \end{cases} \tag{9}$$

where $x = (x_1, x_2, \ldots, x_N) \in \mathbb{R}^{2n}$, $u = (u_1, u_2, \ldots, u_N) \in \mathbb{R}^n$, $d = (d_1, d_2, \ldots, d_N) \in \mathbb{R}^n$, $y = (y_1, y_2, \ldots, y_N) \in \mathbb{R}^n$. Matrices $A$, $B$, $C$ and $E$ are detailed in the section 6.1.

2.4 Decentralised Baseline Voltage Control

Power converter designers and manufacturers design cascaded current and voltage loop controllers for nominal operation. Such baseline controllers are intended to track voltage references and asymptotically reject unknown load disturbances when operating without uncertainty [47]. The idea of this work is to retrofit each DGU, with decentralised $L_1$ adaptive voltage controllers, in order to enhance the performance of each DGU during operations deviating from the nominal case i.e. parametric uncertainty, PuP operations, unknown load changes. Effectively, the closed-loop DGU can be treated as a black-box\(^3\). This section details the design of two conventional decentralised baseline controllers; a static state-feedback (DeSSf) controller with integral action, and a type III output voltage compensator.

2.4.1 Decentralised Static State-Feedback Controller

Baseline controllers are designed for standalone decoupled converters, assuming a connection to a linear resistive load. The state space matrices, of the same form as (7) but without coupling term $\zeta_i(t)$, are given as,

$$A^\text{nom}_{ii} = \begin{bmatrix} \frac{-R_{ti}}{(1-D_{ti})} & \frac{(1-D_{ti})}{L_{ti}} \sum_{j} \frac{1}{R_{tj}C_{tj}} \end{bmatrix}, A^\text{nom}_{ij} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-E_{ti}}{C_{tj}} \end{bmatrix}, B^\text{nom}_i = \begin{bmatrix} \frac{V_{dc}}{L_{ti}} \\ -\frac{I_{d}}{C_{tj}} \end{bmatrix}, E^\text{nom}_i = \begin{bmatrix} 0 \\ -\frac{1}{C_{tj}} \end{bmatrix}.$$  

**Remark 2:** Power converter manufacturers design baseline controllers for nominal operation with a priori parametric knowledge. However, due to privacy requirements and changing conditions the subsequently designed augmenting $L_1$ adaptive controllers does not have a priori parametric knowledge, and therefore must be designed within a known subset.

In order to track constant voltage references in the presence of constant current disturbances, integral state error between the reference voltage and output voltage is added to the local DGU model. The dynamics are defined as,

$$\xi_i(t) = \int_0^t (V_{ref_i} - y_i(t))dt = \int_0^t (V_{ref_i} - C_i x_i(t))dt \tag{10}$$

The DeSSf control law with integral action becomes,

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\(^3\)Within reason; some a priori bound or subset must be known
is modelled with a parasitic inductor resistor.

Equations (as a linear resistance. Fore the load influences the operating-point and hence stability. In this case, the load is represented

where $\bar{\xi}_{i}(t)$ becomes third order, hence $\bar{x}_{i}(t) = [x_{i}(t)]^T, \xi_{i}(t)]^T \in \mathbb{R}^3$ is the augmented open-loop state vector. The state-space model of DGU $i$ can now be defined as,

\begin{align*}
\Sigma_{DGU}^{i}: \begin{cases}
\dot{\bar{x}}_{i}(t) = \bar{A}_{ii}\bar{x}_{i}(t) + \bar{B}_{i}\bar{u}_{i}(t) + \bar{E}_{i}\bar{d}_{i}(t) + \bar{\zeta}_{i}(t) \\
\bar{y}_{i}(t) = \bar{C}_{i}\bar{x}_{i}(t)
\end{cases}
\end{align*}

where $\bar{d}_{i} = [d_{i}, V_{refi}]^T \in \mathbb{R}^2$ is the exogenous signal vector, which includes load current disturbance and reference voltage, $\bar{\zeta}_{i}(t) = \sum_{j \in \mathcal{N}_i} \bar{A}_{ij}\bar{x}_{j}(t)$, and $\bar{y}_{i}(t)$ is the measurable output. The matrices of (12) are defined as,

\[ \bar{A}_{ii} = \begin{bmatrix} A_{ii} & 0 \\ -C_{i} & 0 \end{bmatrix}, \quad \bar{B}_{i} = \begin{bmatrix} B_{i} \\ 0 \end{bmatrix}, \quad \bar{E}_{i} = \begin{bmatrix} E_{i} \\ 0 & 1 \end{bmatrix}, \quad \bar{A}_{ij} = \begin{bmatrix} A_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{C}_{i} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \]

where $A_{ii}, B_{i}, E_{i}, A_{ij}, \bar{C}_{i}, \bar{C}_{j}, c_i \in \mathbb{R}^{3x3}$. The DeSSf controllers can be tuned via pole placement or using linear quadratic integral (LQI) control. The LQI control technique selects optimal controller gains $[K_{i}, K_{r}^{l}, K_{r}^{c}]$ by weighting the cost of state deviation and control effort using steady-state energy values [48]. In steady-state, the energy stored by the inductor and capacitor equate to,

\[ E_{i}^L = \frac{1}{2} L_{i} I_{i}^2, \quad E_{i}^C = \frac{1}{2} C_{i} V_{dc}^2 \]

respectively. As the $Q_{i}^{lqi}$ matrix is related to the energy/state deviation cost, the weights were set as,

\[ Q_{i}^{lqi} = \frac{1}{C_{i} V_{dc}^2} \begin{bmatrix} L_{i} I_{i}^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & C_{i} G_{i} \end{bmatrix} \]

where $G_{i}$ is selected through iterative design.

2.4.2 Type III Compensator

In [20], each DC-DC boost converter is represented by an ideal duty-cycle to output capacitor voltage transfer function, given as:

\[ \frac{\bar{v}_{i}(s)}{d_{i}(s)} = \frac{-V_{in} (s - \frac{(1 - D_{i}) R_{L_{i}}}{L_{i}})}{L_{i} C_{i} (s^2 + \frac{1}{R_{L_{i}} C_{i}} s + \frac{(1 - D_{i})^2}{L_{i}^2 C_{i}^2})}. \]

This is ideal in the sense that it does not include parasitic inductor or capacitor resistance. Each DGU of Fig. 2 is modelled with a parasitic inductor resistor $R_{L_{i}}$. The duty cycle to output capacitor voltage transfer function is now represented by,

\[ \frac{\bar{v}_{i}(s)}{\bar{d}_{i}(s)} = \frac{-V_{in} (s + (R_{L_{i}} - (1 - D_{i})^2 R_{L_{i}}))}{s^2 + \left( \frac{R_{L_{i}}}{R_{L_{i}} C_{i}} + \frac{R_{L_{i}}}{L_{i}} \right) s + \left( \frac{(1 - D_{i})^2}{R_{L_{i}} L_{i} C_{i}} + \frac{R_{L_{i}}}{R_{L_{i}} L_{i} C_{i}} \right)}. \]

Remark 3: Note that these transfer functions are dependent on knowledge of the load, and therefore the load influences the operating-point and hence stability. In this case, the load is represented as a linear resistance.

Equations (15) and (16) highlight the non-minimum phase property of boost converters, which manifests as a right-half plane (RHP) zero in both transfer functions. As a result, due to the discontinuous
energy transfer between inductor and capacitor during switching, the output voltage initially under-
shoots subsequent to any disturbance or reference change. If the bandwidth of the controllers is very
fast, then the RHP zero can attract stable poles into the RHP and destabilise the DGU. Type II and
III compensators are commonly used in power converter control where phase injection is required to
compensate the phase lag introduced by resonant poles and RHP zeros. As the parasitic inductor
resistance introduces some damping, type II compensators can generally be used, though type III
compensators will inject more phase margin. The transfer function of a type III compensator is,
\[
\mathcal{C}_i(s): C_i(s) = \frac{k_{vi}}{s(s + \omega_zi)^2(s + \omega_pi)^2}.
\] (17)

**Remark 4:** Though the closed-loop state space model of DGU \( \Sigma_{\text{DGU}}(s) \) includes the coupling
term \( \hat{\zeta}_i(t) \), decentralised controllers are designed without accounting for this term. Equally, in the
case of the classical compensators, the transfer functions of (15) and (16) do not include coupling
parameters. The affect on stability by not accounting for DGU interaction is explored in section 6.2.

### 3 Decentralised \( \mathcal{L}_1 \) Adaptive Control Augmentation

Conventional MRAC architectures frequently suffer from a trade-off between estimation and ro-
bustness [31]. Fast estimation/adaptation requires large adaptive gains which can destabilise control-
loops. The \( \mathcal{L}_1 \) AC, a modification of the indirect MRAC architecture, decouples this trade-off by
inserting a low-pass filter (LPF) at the input to both the plant and state-predictor, as seen in Fig. 3.
Consequently, robustness instead depends on the choice of filter-bandwidth, thus enabling fast
adaptation [34].

![Figure 3: General Architecture of \( \mathcal{L}_1 \) Adaptive Controller. Adapted from [34]](image.jpg)

Application of the \( \mathcal{L}_1 \) AC has been successful in various safety-critical applications; notably sub-
scale NASA aircraft auto-pilots [37], and unmanned water/aerial vehicles [36, 38]. These applications
use centralised \( \mathcal{L}_1 \) AC approaches. Recently, a decentralised \( \mathcal{L}_1 \) AC approach has been used to augment
aircraft baseline controllers [49]. Ultimately, the \( \mathcal{L}_1 \) AC architecture has potential to improve mG
voltage control, with uniform performance across an entire operating range which experiences large
uncertainties being the key feature.

#### 3.1 \( \mathcal{L}_1 \) Adaptive Control Architecture

From Fig. 3 a state-predictor replaces the reference model of the indirect MRAC, and a LPF limits
the control signal bandwidth. The state-error dynamics, \( \bar{x}(t) \), between the plant and state-predictor
drives the projection-based adaptation law. This adjusts the control parameters in order to drive
\( \bar{x}(t) \rightarrow 0 \).

**3.1.1 Plant structure**

The plant has a known structure, but with unknown parameter values. A matched uncertainty
term is introduced to represent parametric uncertainty in the dynamics of \( \Sigma_{\text{DGU}}^{[i]} \), hence (12) can be
represented as,
\[
\Sigma_{DGU}^{i} : \begin{cases}
\hat{x}_{[i]}(t) = \hat{A}_m \hat{x}_{[i]}(t) + \hat{B}_i (u_{[i]}(t) + \hat{\theta}_r^T(t) \hat{x}_{[i]}(t)) + F \hat{E}_i \hat{d}_{[i]}(t) \\
\hat{y}_{[i]}(t) = \hat{C}_i \hat{x}_{[i]}(t)
\end{cases}
\]
(18)

where \( \hat{x}_{[i]}(t) \in \mathbb{R}^3 \) is the system measurable state vector; \( \hat{A}_m \in \mathbb{R}^{3x3} \) is the Hurwitz design matrix that specifies the desired closed-loop dynamics; \( u(t) \in \mathbb{R} \) is the control signal; \( F = [0, 0, 1] \), and \( \hat{\theta}(t) \) is the unknown matched parametric uncertainty vector. This belongs to a known compact convex set of uniform boundedness \( \theta \in \Theta \subset \mathbb{R}^3 \).

3.1.2 Control Law

The small-signal control input \( u(t) \) for \( \hat{\Sigma}_{DGU}^{i} \) consists of the summation between the baseline and \( L_1 AC \) control signals,
\[
C_{\hat{L}_1}^{i} : u_{[i]}(t) = \hat{d}_{[i]}(t) = u_{0, [i]}^i(t) + u_{\hat{L}_1}^i(t)
\]
(19)
The augmenting \( L_1 AC \) law, fitted with a first-order LPF, is
\[
u_{\hat{L}_1}^i(t) = -C(s)[\hat{\theta}_r^T(t) \hat{x}_{[i]}(t)](t)
\]
where \( C(s) = \frac{1}{s + \omega_c} \), and \( \hat{\theta}_r^T(t) \) is the parametric estimation vector, as defined in section 3.1.4. The robustness of the \( L_1 AC \) is dependent on the LPF bandwidth \( \omega_c \), as subsequently designed.

3.1.3 State-predictor

The state-predictor generates an estimate of the system states. From the perspective of the \( L_1 AC \), the baseline dynamics are combined with the open-loop DGU dynamics to form an augmented closed-loop system.

Assumption 2: The design of the decentralised \( L_1 \) adaptive voltage controllers can neglect the exogenous disturbance term \( \hat{d}_{[i]}(t) \) and coupling term \( \hat{C}_i \).

Without loss of generality, the state-predictor formulation is proposed for all DGUs as,
\[
\acute{E}_{[i]} : \begin{cases}
\acute{x}_{[i]}(t) = \hat{A}_m \acute{x}_{[i]}(t) + \hat{B}_i (u_{\hat{L}_1}^i(t) + \hat{\theta}_r^T(t) \acute{x}_{[i]}(t)) + F \hat{E}_i \acute{d}_{[i]}(t) \\
\acute{y}_{[i]}(t) = \hat{C}_i \acute{x}_{[i]}(t)
\end{cases}
\]
(21)
where \( \acute{x}_{[i]}(t) \in \mathbb{R}^3 \) is the predicted state vector and \( \hat{\theta} \in \mathbb{R}^3 \) is the parametric estimation vector.

Remark 5. \( L_1 AC \) theory of [34] assumes that the input matrix \( \hat{B}_i \) is known. However, the \( \hat{B}_i \) matrix of (21) consists of unknown parameters which cannot be compensated by the adaptive control law. To overcome this, the state-predictor can be transformed into its control canonical form so that a known \( \hat{B} \) is attained. As a result, these unknown parameters are transferred to the output matrix \( \hat{C} \), which is not required for state-feedback control.

Transforming to control-canonical form, the closed-loop transfer function from control-input to voltage output is,
\[
\frac{Y(s)}{U(s)} = \hat{C}(s\mathbb{I} - \hat{A}_m)^{-1}\hat{B}_i
\]
(22)
where,
\[
\hat{A}_m = \begin{bmatrix}
A_m - \hat{B}_i K^{x}_i & \hat{B}_i K^{\xi}_i \\
-\hat{C}_i & 0
\end{bmatrix}
\]
(23)
and \( K^{x}_i = [K^{x}_{1i}, K^{x}_{2i}] \).

\[
\frac{Y(s)}{U(s)} = \frac{f_0 s^r + f_{r-1} s^{r-1} + \ldots + f_0}{s^N + e_{N-1} s^{N-1} + \ldots + e_0}
\]
(24)
where,

\[
\hat{A}^{CC}_m = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-e_0 & -e_1 & -e_2
\end{bmatrix}
\quad b = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\quad C_{cc} = \begin{bmatrix}
f_0 & f_1 & f_2
\end{bmatrix}
\]  

(25)

and,

\[
e_2 = \frac{1}{C_{ti}}\left(\sum_{j \in N_i} \frac{1}{R_{ij}} - I_{ti}K^{c}_i + \frac{1}{L_{ti}}(R_{ti} + V_{dc}K^i_t)\right)
\]

\[
e_1 = \frac{1}{L_{ti}C_{ti}}((R_{ti} + V_{dc}K^i_t)(\sum_{j \in N_i} \frac{1}{R_{ij}} - I_{ti}K^{c}_i) + ((1 - D_i) + V_{dc}K^i_t)((1 - D_i) + I_{ti}(K^c_i - K^i_\xi)))\]

\[
e_0 = \frac{1}{L_{ti}C_{ti}}(I_{ti}K^{\xi}_t(R_{ti} + V_{dc}K^i_t) - V_{dc}K^i_\xi((1 - D_i) + I_{ti}K^i_t))
\]  

(26)

With this, the closed-loop state-predictor in control canonical form is given as,

\[
E^{CC}_{[i]} : \begin{cases}
\dot{z}_{[i]}(t) = \hat{A}^{CC}_m z_{[i]}(t) + b(u_{[i]}(t) + \hat{\theta}_{[i]}(t)\bar{z}_{[i]}(t)) \\
\dot{y}_{cc_{[i]}}(t) = C_{cc}z_{[i]}(t)
\end{cases}
\]  

(27)

As (27) is dependent on the control canonical form of the plant \( \bar{z}_{[i]}(t) \), a transformation from the measured state vector \( \bar{x}_{[i]}(t) \) to the new state vector is required. Therefore,

\[
\bar{z}_{[i]}(t) = T_i\bar{x}_{[i]}(t)
\]  

(28)

From state transformation theory, and since controllability has already been assumed, the transformation matrix is computed as,

\[
T_i = C_2(C_2)^{-1}
\]  

(29)

where, \( C_2 = [b, \hat{A}^{CC}_m b, \hat{A}^{CC}_m^2 b] \) and \( C_2 = [\bar{B}_i, \hat{A}_m \bar{B}_i, \hat{A}^2_m \bar{B}_i] \) are the controllability matrices associated with the state-predictor in control canonical form and plant.

**Assumption 3.** The computation of (29) requires knowledge of the plant input matrix \( \bar{B}_i \), which from Remark 5, is uncertain. However, it is assumed that the adaptation will account for this. Therefore, approximate/nominal parametric values are chosen for \( \bar{B}_i \) - see section 3.1.6.

### 3.1.4 Adaptive Law

From hereafter, control-canonical form notation is used. The adaptive law generates an estimate of the plant uncertainties. Defining the state-error and parametric estimation error vectors as, \( \bar{z}_{[i]}(t) = \bar{z}_{[i]}(t) - \bar{z}_{[i]}(t) \) and \( \bar{\theta}_{[i]}(t) = \bar{\theta}_{[i]}(t) - \bar{\hat{\theta}}_{[i]}(t) \), the state-error dynamics, used to drive the adaptive law, can be defined as,

\[
\dot{\bar{z}}_{[i]}(t) = \bar{A}_m\bar{z}_{[i]}(t) + b\bar{\theta}_{[i]}(t)\bar{z}_{[i]}(t)
\]  

(30)

The adaptive law is determined from Lyapunov’s second stability method. A quadratic Lyapunov candidate is defined as a function in terms of \( \bar{z}_{[i]}(t) \) and \( \bar{\theta}_{[i]}(t) \).

\[
V_{[i]}(\bar{z}_{[i]}(t), \bar{\theta}_{[i]}(t)) = \bar{z}_{[i]}(t)^T P_i \bar{z}_{[i]}(t) + \bar{\theta}_{[i]}(t)^T \Gamma_i^{-1} \bar{\theta}_{[i]}(t)
\]  

(31)

where, \( P_i \in \mathbb{R}^{3x3} \) is a symmetric matrix, such that \( P_i = P_i^T > 0 \) is the solution to the algebraic Lyapunov linear inequality \( \hat{A}^T_m P_i + P_i \hat{A}_m \leq -Q_i \), for arbitrary \( Q_i = Q_i^T > 0 \), and \( \Gamma_i \in \mathbb{R}^{+} \) is the adaptive gain. From [50], if the time-derivative of (31) is at least negative semi-definite, then each subsystem, in this case each DGU, is locally stable since the energy along the trajectories of state and estimation errors is decreasing. The time-derivative of (31) is,

\[
\dot{V}_{[i]}(\bar{z}_{[i]}(t), \bar{\theta}_{[i]}(t)) = \frac{dV_{[i]}(t)}{d\bar{z}_{[i]}(t)} \dot{\bar{z}}_{[i]}(t) + \frac{dV_{[i]}(t)}{d\bar{\theta}_{[i]}(t)} \dot{\bar{\theta}}_{[i]}(t) = \bar{z}^T_{[i]}(t)P_i \bar{z}_{[i]}(t) + \bar{\theta}^T_{[i]}(t)P_i \bar{\theta}_{[i]}(t) + 2\bar{\theta}^T_{[i]}(t)\Gamma_i^{-1} \bar{\theta}_{[i]}(t)
\]  

(32)
Using (30),(32) can be written as,

\[ \hat{V}_i(\hat{z}_i(t), \hat{\theta}_i(t)) = 2(\hat{A}_m \hat{z}_i(t) + b\hat{\theta}_i(t) \hat{z}_i(t))P_i \hat{z}_i(t) + 2\hat{\theta}_i^T \Gamma_i^{-1} \hat{\theta}_i(t) \]  

(33)

From the algebraic Lyapunov linear inequality equation, \( \hat{A}_m^T P_i + P_i \hat{A}_m = 2\hat{A}_m P_i = -Q_i \). Also, since \( \dot{\hat{\theta}}_i = \hat{\theta}_i - \hat{\theta}_i \), and \( \dot{\hat{\theta}}_i = 0 \),

\[ \hat{V}_i(\hat{z}_i(t), \hat{\theta}_i(t)) = -\hat{z}_i(t)Q_i \hat{z}_i(t) + 2\hat{\theta}_i^T(t) \hat{z}_i(t)P_i b \hat{z}_i(t) + \hat{\theta}_i(t) \Gamma_i^{-1} \]  

(34)

To ensure (34) is at least negative semi-definite, i.e. \( 2\hat{\theta}_i^T(t)P_i b \hat{z}_i(t) + \dot{\hat{\theta}}_i(t) \Gamma_i^{-1} = 0 \), the adaptive law is given by,

\[ \dot{\hat{\theta}}_i(t) = -\Gamma_i \hat{x}_i(t)^T P_i b \hat{z}_i(t) \]  

(35)

To prevent parameter drift, the parametric uncertainty estimate is bounded using the projection operator, as detailed in [34,35,51]. Therefore, (35) becomes,

\[ \hat{\theta}_i(t) = \Gamma_i \text{Proj}(\hat{\theta}_i(t), -\hat{z}_i(t)^T P_i b \hat{z}_i(t)) \]  

(36)

Finally,

\[ \hat{V}_i(\hat{z}_i(t), \hat{\theta}_i(t)) = -\hat{z}_i(t)Q_i \hat{z}_i(t) \leq 0 \]  

(37)

Hence, the equilibrium of the state error dynamics of (30) and adaptive law of (36) is locally stable i.e. \( \hat{z}_i(t) \) and \( \hat{\theta}_i(t) \) are bounded. Since \( \hat{z}_i(t) = \hat{z}_i(t) - \hat{z}_i(t) \), and as the state estimate vector \( \hat{z}_i(t) \) results from a stable design, the plant states \( \hat{z}_i(t) \) are also bounded. Ideally, the plant dynamics are driven to equal the desired predictor dynamics. This warrants convergence of \( \hat{z}_i(t) \to 0 \). However (37) does not prove local asymptotic stability. To show that the state-prediction error converges asymptotically to zero, the second-derivative of \( \hat{V}_i(t) \) is computed,

\[ \hat{V}_i(\hat{z}_i(t), \hat{\theta}_i(t)) = -2\hat{z}_i(t)^T Q_i \hat{z}_i(t) \]  

(38)

From (30), \( \hat{z}_i(t) \) is uniformly bounded by design, and thus (38) is bounded. A bounded second-derivative implies a smooth first-derivative, resulting in a uniformly continuous \( \hat{V}_i \). By invoking Barbalat’s lemma in section A.6.1 of [34], it follows that \( \lim_{t \to \infty} \hat{z}_i(t) = 0 \). Subsequently local asymptotic stability can be guaranteed.

### 3.1.5 Filter Design

The key feature of the \( L_1 \) AC is the synthesis of a LPF structure which decouples robustness from adaptation. At this point, local asymptotic stability has been guaranteed during nominal operation i.e. baseline controller design, and adaptation. Here, stability is further guaranteed when the LPF is inserted to filter the control signal. The LPF bandwidth is tuned using the \( L_1 \) norm condition. From the perspective of the \( L_1 \) AC, the baseline controller dynamics are combined with the open-loop DGU dynamics. Therefore, the desired closed-loop reference system in the Laplace domain, where \( \hat{A}_m \) includes the dynamics of the baseline controller, can be given as,

\[ \hat{z}_{ref \mid i}(s) = (sI - \hat{A}_m)^{-1} b(u_{ref \mid i}(s)) + \hat{\theta}_{ref \mid i}^T \hat{z}_{ref \mid i}(s) + \hat{z}_{ic \mid i}(s) \]  

(39)

where, \( \hat{z}_{ref \mid i} \in \mathbb{R}^3 \), is the reference state vector, \( \hat{z}_{ic \mid i} \in \mathbb{R}^3 \) is the initial state vector, \( \mathbb{I} \in \mathbb{R}^{3x3} \), is the identity matrix. \( \hat{A}_m \) is defined in section 3.1.6. Convergence is assumed, i.e. \( \hat{\theta}_{\mid i}(t) \to \hat{\theta}_{\mid i} \). The desired closed-loop behaviour is represented by the transfer function,

\[ H(s) = (sI - \hat{A}_m)^{-1} \]  

(40)
Combining (20) with (39) yields,

\[ \bar{z}_{\text{ref}}[i](s) = G(s)\theta^T[I]\bar{z}_{\text{ref}}[i](s) + \bar{z}_0(s) \]  

(41)

where, \( G(s) = H(s)(1 - C(s)) \), and \( \bar{z}_0(s) = (sI - \bar{A}_m)^{-1}\bar{z}_{ic}[i](s) \). As shown in [34], the \( L_1 \)-norm is now taken on both sides of (39),

\[ \|\bar{z}_{\text{ref}}[i]\|_{L_1} = \frac{\|\bar{z}_0\|_{L_\infty}}{1 - \|G(s)\theta^T[I]\|_{L_1}} \]  

(42)

For the reference states to be bounded, the denominator must be larger than zero. The 1-norm is chosen as the maximum value of \( \theta \),

\[ \theta_{\text{max}} = 4 \max_{\theta \in \Theta} \|\theta\|_1 \]  

(43)

\( \theta_{\text{max}} \) represents the boundary of projection for estimating the parameters when using the adaptation law (36). Finally, for the reference states to remain bounded, the following \( L_1 \)-norm condition must be satisfied,

\[ \lambda = \|G(s)\|_{L_1} \theta_{\text{max}} < 1 \]  

(44)

where the degree-of-freedom is \( \omega_c \). Inserting the LPF attenuates any HF content in the control channel resulting from large adaptive gains, and compensates LF uncertainty. The overall decentralised voltage primary control scheme is shown below.

3.1.6 Design Considerations

As opposed to the baseline controller, which is nominally designed for decoupled operation, the desired closed-loop dynamics of the state-predictor are designed for nominal operation of the DGUs when coupled to neighbouring DGUs, and is applied to all DGUs. A priori knowledge of the real-time number of neighbouring DGUs is not known, therefore the maximum possible number of couplings, within the set \( D \), is chosen, as in (8). The desired closed-loop dynamics in, are selected as (in normal form, i.e. not control-canonical form),

\[ \bar{A}_m = \begin{bmatrix} \bar{A}_m - BK_{\text{nom}} & BK_{\text{nom}} \\ -C_{\text{nom}} & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{A}_m = \begin{bmatrix} -\frac{R_{\text{nom}}}{L_{\text{nom}}} & -\frac{(1-D_{\text{nom}})}{L_{\text{nom}}} \\ 0 & \sum_{j \in M_i} K_{ij}^C_{\text{nom}} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{V_{\text{nom}}}{L_{\text{nom}}} \\ \frac{L_{\text{nom}}}{C_{\text{nom}}} \end{bmatrix}, \quad K_{\text{nom}} = \begin{bmatrix} K_{\text{nom}}^K & K_{\text{nom}}^L \end{bmatrix} \]  

(45)

The nominal parameters represent an estimate of where the uncertain dynamics lie within the polytope. While baseline DeSSI controllers are designed using the pole-placement method (baseline controllers are designed to place poles for fast closed-loop performance), within the predictor the estimate...
of $K_{iv}^{nom}$ is designed using the LQI method. It is worth noting that, by considering the load as an exogenous disturbance, the eigenvalues are dependent only on the QSLs. As the number of couplings increases, the eigenvalues of $A_{m}$ become faster, increasing the closed-loop bandwidth. The parameters of $B$ are chosen based on an approximate expectation of the steady-state output voltage $V_{dc}^{nom}$ and inductor current $I_{t}^{nom}$. This expectation is suitable for $V_{dc}^{nom}$ as output voltage tracking is the primary control objective. However, from (7), $I_{t}$ is load dependent. As the real-time load and effective QSL resistance are unknown, $I_{t}^{nom}$ is conservatively designed for the smallest expected load power and maximum number of couplings.

The parameter bounds, or the maximal deviation from the desired dynamics is calculated as,

$$\bar{A}_{m} = A - B_{i}\theta T$$

$$A = \begin{bmatrix} A_{ii} - B_{i}K_{iv}^{i} & B_{i}K_{iv}^{i} \\ -C_{i} & 0 \end{bmatrix}$$

where the DeSSf control gain vector $[K_{iv}^{i}, K_{iv}^{i}] \in \mathbb{R}^{3}$ is calculated for a wide range of different parameters in $A_{ii}$ and $B_{i}$. The parameters that bring the closed-loop dynamics $A$ to the edge of the boundary of uncertainty represents the maximal deviation from the desired dynamics. Consequently, the parameter bound $\theta_{max}$ is calculated according to (43).

### 3.1.7 Conditions for Global Asymptotic Stability

Despite decentralised controllers guaranteeing local asymptotic stability, as shown in section 6.2, global asymptotic stability can be violated due to the presence of unaccounted DGU coupling. Conventional decentralised control theory [52, 53] treats coupling terms as disturbances and suggests that controller design should be robust to neighbouring states in order to achieve global asymptotic stability. We demonstrated this in [20], where type III compensators were detuned to provide a wider performance range when coupled to neighbouring DGUs. However, global knowledge of coupling parameters, load dynamics, and real-time information about the number of neighbours are required apriori, i.e. not scalable.

This section aims to provide offline conditions for guaranteeing global asymptotic stability using decentralised controllers $C_{[i]}^{L}$. Most decentralised controllers are based on the idea of small-couplings or weakly coupled subsystems, where coupling terms are small and within some known bounded subset. This introduces degrees of conservativeness.

The conventional connective stability method, described in [52, 53], can be used to construct sufficient global stability conditions using aggregated interconnection models and exploiting Lyapunov functions. However, we showed in [54] that conditions are only satisfied when the small-gain theorem is satisfied, and demonstrated that interconnections typically have large-gains in DC ImGs. This explains why the decentralised PnP controllers of [19, 27] require $P_{i}$ in the form,

$$P_{i} = \begin{bmatrix} \eta_{i} & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{bmatrix}$$

where $\eta_{i}$ is a local design parameter and * denotes arbitrary values, in order to neutralise such interactions between DGUs i.e. term (b) in 50 equals zero. Furthermore, we described a distributed architecture, using robust-adaptive controllers that provides sufficient conditions for global asymptotic stability in the presence of large-gain interconnections. However, $C_{[i]}^{L}$ requires information about $C_{[i]}^{L}$.

From the perspective of decentralised, communication-less control, global asymptotic stability must instead be determined by deriving global stability conditions offline and incorporating these into the design. The overall Lyapunov function candidate that describes the global system can be written as,

$$V(t) = \sum_{i=0}^{N} \tilde{z}_{[i]}^{T}(t)P_{i}\tilde{z}_{[i]}(t) + \tilde{\theta}_{[i]}^{T}(t)\Gamma_{i}^{-1}\tilde{\theta}_{[i]}(t)$$

**Assumption 4:** We assume local controllers exploit (36), and plant dynamics have converged to desired dynamics.

The derivative of (48) is,

$$\dot{V} = -\sum_{i=0}^{N} \tilde{z}_{[i]}^{T}(t)Q_{i}\tilde{z}_{[i]}(t)$$

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if and only if matrix $P$ satisfies the Lyapunov inequality equation,

$$
\begin{aligned}
(A_D - BK)^T P + P(A_D - BK) + A_C^T P + PA_C &< 0 \\
\end{aligned}
$$

(50)

where $P = \text{diag}(P_i) \in \mathbb{R}^{N \times N}$, $A_D = \text{diag}(\hat{A}_m) \in \mathbb{R}^{N \times N}$ represents the overall desired dynamics, and $A_C = A - A_D \in \mathbb{R}^{N \times N}$ represents the coupling dynamics only. As each DGU is designed to be locally asymptotically stable, the matrices of (a) are negative definite. Therefore, for global asymptotic stability, the matrices of (b) need to be negative definite. The use of LMIs in $[19,27]$ systematically ensures (b) $< 0$ through design of $K$. However, here the design of $K$ is performed iteratively offline to ensure $||{(a)}|| > ||{(b)}||$. This typically results in detuned controller gains, as expected due to the conservativeness requirements of decentralised systems. Furthermore, this method can suffer when system size expands as the retuning of $K$ becomes more difficult, despite the advantage of designing the desired dynamics as the same for all DGUs.

4 Results

Below, a meshed and radial mG topology, similar to that of [19], is considered. This topology is known to destabilise when $\Sigma_{DGU}^6$ is plugged-in using only baseline controllers - section 6.2. Hence, this set-up can adequately evaluate the performance of the proposed decentralised $L_1$AC augmentation. Each DGU is equipped with controllers $C_{[i]}^{L_1}$, $i = 1, \ldots, 6$.

![Figure 5: Meshed and radial microgrid configuration - $\Sigma_{DGU}^6$ plug-in (green) and $\Sigma_{DGU}^3$ plug-out (red).](image)

Controllers and simulations were developed in Matlab/Simulink software. For greater accuracy this work uses non-linear PWM driven boost converters, as previously designed in [20], using the simpowersystems toolbox. System parameters are detailed in Table 1.

It should be noted that the dynamics of each DGU are different i.e. the electrical parameters and controller bandwidths are non-identical. Therefore, the system can be defined as heterogeneous. At $t = 0$, $\Sigma_{DGU}^1$, $\Sigma_{DGU}^2$, $\Sigma_{DGU}^3$ and $\Sigma_{DGU}^4$ are connected together through $RL$ power lines in a ring configuration. $\Sigma_{DGU}^5$ is connected to $\Sigma_{DGU}^4$, while $\Sigma_{DGU}^6$ powers a local load exclusively. Tests include PnP operations, robustness to load changes/unmodelled dynamics, and voltage tracking.
## Table 1: System Parameters

| Description                        | Parameter | \( \Sigma_{DGU}^1 \) | \( \Sigma_{DGU}^2 \) | \( \Sigma_{DGU}^3 \) | \( \Sigma_{DGU}^4 \) | \( \Sigma_{DGU}^5 \) | \( \Sigma_{DGU}^6 \) |
|-----------------------------------|-----------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| DGU rated power (kW)              | \( P_i \) | 5                      | 5                      | 5                      | 5                      | 5                      | 5                      |
| Local load demand (kW)            | \( P_R[i] \) | 2.5                    | 2.5                    | 1.8                    | 2.5                    | 3                      | 2.5                    |
| Input voltage (V)                 | \( V_{in[i]} \) | 95                     | 100                    | 90                     | 105                    | 92                     | 90                     |
| Reference voltage (V)             | \( V_{ref[i]} \) | 381                    | 380.5                  | 380.2                  | 379                    | 379.5                  | 380.7                  |
| Switching frequency (kHz)         | \( f_s \) | 25                     | 25                     | 25                     | 25                     | 25                     | 25                     |
| Duty cycle                        | \( D_i \) | 0.7507                 | 0.7372                 | 0.7633                 | 0.723                  | 0.7576                 | 0.7636                 |
| Inductance (\( \mu \)H)          | \( L_{ti} \) | 28.47                  | 89.62                  | 192.5                  | 70                     | 35                     | 93.34                  |
| Capacitance (\( \mu \)F)         | \( C_{ti} \) | 37.632                 | 51.67                  | 40.73                  | 37                     | 31                     | 24.66                  |
| Parasitic resistance (\( \Omega \)) | \( R_{ti} \) | 0.02                   | 0.04                   | 0.02                   | 0.2                    | 0.4                    | 0.5                    |
| Line resistance (\( \Omega \))    | \( R_{ij} \) | 0.5-2-10               | 0.5-4                  | 2-4                    | 2-4-15                 | 15-4                   | 10-4                   |
| Line inductance (\( \mu \)H)     | \( L_{ij} \) | 10-70-800              | 40-70                  | 70-70                  | 70-70-25               | 25-90                  | 800-90                 |
| Nominal duty cycle                | \( D_i \) | 0.7368                 | 0.7368                 | 0.7368                 | 0.723                  | 0.7368                 | 0.7368                 |
| Nominal inductance (\( \mu \)H)  | \( L_{inom} \) | 2.794                  | 2.794                  | 2.794                  | 2.794                  | 2.794                  | 2.794                  |
| Nominal capacitance (\( \mu \)F) | \( C_{inom} \) | 60.6                   | 60.6                   | 60.6                   | 60.6                   | 60.6                   | 60.6                   |
| Nominal parasitic resistance (\( \Omega \)) | \( R_{jnom} \) | 0.1                    | 0.1                    | 0.1                    | 0.1                    | 0.1                    | 0.1                    |
| Nominal line resistance (\( \Omega \)) | \( R_{jnom} \) | 1                      | 1                      | 1                      | 1                      | 1                      | 1                      |
| Nominal line inductance (\( \mu \)H) | \( L_{ijnom} \) | 10                    | 10                    | 10                    | 10                    | 10                    | 10                    |

### 4.1 Plug-and-Play Operations

#### 4.1.1 Plug-in of DGU

In this section, the PnP capability of the proposed controllers \( C[i] \) is evaluated. At \( t = 0.05 \) s, \( \Sigma_{DGU}^6 \) is plugged-in, connecting to \( \Sigma_{DGU}^1 \) and \( \Sigma_{DGU}^5 \). Fig. 6 shows the responses of the DGUs most directly affected i.e. \( \Sigma_{DGU}^1 \), \( \Sigma_{DGU}^5 \) and \( \Sigma_{DGU}^6 \). Fig. 6 plots the responses of the DGUs most directly affected i.e. \( \Sigma_{DGU}^1 \), \( \Sigma_{DGU}^5 \) and \( \Sigma_{DGU}^6 \).

![DGU output voltage responses to \( \Sigma_{DGU}^6 \) plugging-in.](image)

**Figure 6**: DGU output voltage responses to \( \Sigma_{DGU}^6 \) plugging-in.

Fig. 6 shows very good performance when each controller is equipped with \( C[i] \), with hardly any overshoot and a very fast settling time \( \leq 10 \) ms. In the subsequent subsection, \( \Sigma_{DGU}^3 \) is unplugged from the ImG. However at \( t = 0.6 \) s, \( \Sigma_{DGU}^3 \) is subsequently plugged-in, connecting to \( \Sigma_{DGU}^5 \) and \( \Sigma_{DGU}^6 \).

---

4 Non-neighbouring DGU responses are shown in section 6.4
4.1.2 Unplugging of DGU

At $t = 0.2$ s, $\Sigma^3_{DGU}$ is disconnected from $\Sigma^1_{DGU}$ and $\Sigma^4_{DGU}$.

Fig. 8 highlights good performance during the plug-out operation. The settling times of $\Sigma^3_{DGU}$ and
ΣDGU are 1 ms and 20 ms respectively, which again are fast for primary voltage control. As shown in section 6.4, when ΣDGU3 is plugged-out while equipped with C3L, large oscillations are induced for 100 ms before settling. Ultimately, the baseline controller can handle the dynamics of being plugged-out to control its own load. Therefore the adaptation loop is turned-off at $t = 0.201$ s, as shown in Fig. 8b.

Remark 6: It should be noted that, the power line resistances used in this test range from 0.5–15Ω. Such power lines are applicable in large-scale systems where cabling lengths can be up to 1000 ft or 300 m. For example, a households average cable length is 30 m (section 2.3.1.3 of [55]), which for 12 AWG cabling has a resistance of 0.16 Ω.

Therefore, large line resistances naturally impede current disturbances from neighbouring DGUs. Nonetheless, as seen in section 4.3, the change in line-currents upon output voltage reference changes are relatively significant. Global-asymptotic stability is maintained when the closed-loop dynamics of each DGU is changed during PnP operations.

4.2 Robustness to Unknown Load Change

In order to examine the robustness of DC-ImG to unknown load changes, the load at ΣDGU is stepped from 2.5 kW to 800 W at $t = 0.3$ s. The responses of each DGU are plotted below.

![DGU output voltage](image1)

(a) $\Sigma^{DGU}_{4}$ output voltage

![DGU output voltage](image2)

(b) $\Sigma^{DGU}_{5}$ output voltage

![DGU output voltage](image3)

(c) $\Sigma^{DGU}_{6}$ output voltage

Figure 9: DGU output voltage responses to $\Sigma^{DGU}_{3}$ plugging-out.
The responses of neighbouring DGUs $\Sigma_1^{DGU}$ and $\Sigma_5^{DGU}$ show very good robustness to unknown load changes within the ImG. Settling times are fast, within 30 ms, while overshoot is limited to less than 3.8%. The response of $\Sigma_6^{DGU}$ is similarly favourable, with overshoot limited to less than 4% and settling time within 30 ms. The load dependent voltage ripple has also reduced.

4.3 Voltage reference tracking

The hierarchical structure of ImG control architectures requires primary voltage reference changes, using commands from secondary controllers, in order to control the power flows amongst DGUs within the ImG, as well as regulate the state-of-charge of batteries. Therefore, a key metric of the proposed system is the performance of the system in response to voltage reference changes. This is evaluated by stepping the voltage reference of $\Sigma_1^{DGU}$ from 381 V to 375 V at $t = 0.8$ s. Since the impedance of the $RL$ lines is small the voltage decrease is enough to pull an appreciable amount of power from neighbouring DGUs, causing current disturbances to cascade throughout the ImG. Though the $RL$ line parameters that are used correlate to long cabling lengths within a large-scale power system, the resistances are still small enough to cause considerable line currents to flow. For example, when $t \leq 0.8$ s, the steady-state line current $I_{21} = V_1 - V_2 = 381 - 380.5 = 1$ A. After $t = 0.8$ s, $I_{21} = \frac{375 - 380.5}{0.5} = -11$ A.

Therefore, it is important that neighbouring DGUs are robust to this unknown disturbance. The responses of $\Sigma_1^{DGU}$ and its neighbours $\Sigma_2^{DGU}$ and $\Sigma_6^{DGU}$ are plotted below.

![Figure 10](image)

(a) $\Sigma_1^{DGU}$ output voltage  
(b) $\Sigma_2^{DGU}$ output voltage  
(c) $\Sigma_6^{DGU}$ output voltage

Figure 10: DGU output voltage responses to $\Sigma_1^{DGU}$ voltage reference step from 381 V to 375 V.

Fig. 10a demonstrates the fast voltage reference tracking capabilities of the system. After small transient oscillations, the settling time is reached within 100 ms. Figs. 10b and 10c show the
interactions between coupled DGUs during the step test are minimal with each DGU showing good robustness to current disturbances.

4.4 Comparison with average model

Previous results were attained using non-linear PWM switching converter models built using the simpowersystems toolbox of Matlab/Simulink. These models typically lead to very long simulation times i.e. one second can take between 36-48 hours. This is associated with the use of large PWM switching frequencies, non-linear projection operator, and large number of adaptively detected zero-crossings.

To speed up simulation times, the average model of each DGU can be used by constructing the system using the differential equations of (1). The following results correlate with the tests performed previously.

![Figure 11: DGU output voltage responses using average model of (1)](image)
4.5 Robustness to Unmodelled Dynamics

This section demonstrates further robustness of the $L_1$ AC to heterogeneity, variation in parametric uncertainty and unmodelled dynamics.

The topology of each DGU is augmented by the addition of an unmodelled capacitor equivalent series resistance, $R_{ci}$, in series with the output capacitor. Like the already modelled inductor equivalent series resistance, $R_{ci}$ represents capacitor voltage drops associated with capacitors due to non-ideality in power converters. In fact, $R_{ci}$ can be used as a design feature in order to increase output voltage damping and reduce ripple. The significance of this is that the output voltage no longer equates to the capacitor voltage and therefore controlling the capacitor voltage state does not correspond to the output voltage control. The model is derived in section 6.3.

The values of $R_{ci}$ are: $R_{c1} = 0.02\Omega$, $R_{c2} = 0.05\Omega$, $R_{c3} = 0.15\Omega$, $R_{c4} = 0.07\Omega$, $R_{c5} = 0.09\Omega$, $R_{c6} = 0.01\Omega$. The following results plot the response of $\Sigma_{6\text{DGU}}$ as it plugs in at $t = 0.05$ s, and its neighbours $\Sigma_{1\text{DGU}}$ and $\Sigma_{5\text{DGU}}$.

![Figure 12: DGU output voltage responses to $\Sigma_{6\text{DGU}}$ plug-in with unmodelled parasitic capacitor resistance.](image)

5 Conclusion

This paper develops a scalable PnP decentralised $L_1$ adaptive controller for augmentation of DGU baseline voltage controllers within a large-scale DC ImG. These controllers are equipped locally to each DGU at the primary control level and guarantee local asymptotic stability in the presence of parametric and topology uncertainty. Asymptotic stability of the global system can be guaranteed for the decentralised primary control level by adhering to conservative design conditions. Such conditions are determined offline by iteratively checking if the derivative of the overall Lyapunov function candidate is negative definite, or if decoupled terms are more negative definite than coupling terms.
A heterogeneous DC ImG consisting of DC-DC boost converters is designed in Simulink using a radial and meshed topology to evaluate the performance of the proposed architecture. As long as appropriate bounds of uncertainty are incorporated, the $L_1$ AC can treat the DGU as a black-box. The control architecture demonstrates fast and robust output voltage performance when evaluated under PnP operations, unknown load changes, voltage reference step changes, and unmodelled dynamics.

Future work will consider line-independent and distributed control architectures\(^5\) in order to guarantee global asymptotic stability in a scalable, PnP fashion. Furthermore, implementing the proposed architecture in bus-connected topologies with constant-power loads is also of interest.

6 Appendix

6.1 Matrices in Microgrid Model

6.1.1 Two coupled boost converter DGU model

Defining the power line dynamic equation of (1c) in state space form yields,

$$\Sigma_{ij}^{\text{Line}} : \begin{cases} \dot{x}_{ij}(t) = A_{ii}x_{ij}(t) + A_{ij}x_{ij}(t) + A_{l,ij}x_{ij}(t) \\ \dot{x}_{[l,ij]}(t) \end{cases}$$

(51)

where, $x_{ij} = I_{ij}$ is the line current state, $A_{ii} = \begin{bmatrix} -\frac{1}{L_{ij}} & 0 \\ 0 & 0 \end{bmatrix}$, $A_{ij} = \begin{bmatrix} \frac{1}{L_{ij}} & 0 \\ 0 & 0 \end{bmatrix}$, $A_{l,ij} = -\frac{R_{ij}}{L_{ij}}$.

Therefore, the overall state space model of the mG in Fig. 2 defined by (2) prior to the QSL assumption, can be represented by,

$$\begin{bmatrix} \dot{x}_{ij}(t) \\ \dot{x}_{[l,ij]}(t) \end{bmatrix} = \begin{bmatrix} A_{ii} & A_{ij} & 0 & 0 & 0 & 0 \\ A_{ji} & A_{jj} & 0 & 0 & 0 & 0 \\ A_{l,ii} & A_{l,ij} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{ij}(t) \\ x_{[l,ij]}(t) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \\ B_j \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u_{ij}(t) \\ u_{[l,ij]}(t) \end{bmatrix}$$

(52)

where,

$$A = \begin{bmatrix} -\frac{R_{ii}}{L_{ij}} & -\frac{1-d_{ij}}{L_{ij}} & 0 & 0 & 0 & 0 \\ \frac{1}{C_{ij}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_{ii}}{L_{ij}} & \frac{1}{C_{ij}} & 0 & 0 \\ \frac{1}{L_{ij}} & 0 & -\frac{R_{ii}}{L_{ij}} & -\frac{1-d_{ij}}{L_{ij}} & 0 & 0 \\ -\frac{R_{ij}}{L_{ij}} & 0 & \frac{1}{C_{ij}} & \frac{1}{L_{ij}} & -\frac{R_{ij}}{L_{ij}} & 0 \\ \frac{1}{L_{ij}} & 0 & 0 & 0 & 0 & -\frac{R_{ij}}{L_{ij}} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L_{ij}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{L_{ij}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_{ij}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{L_{ij}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{L_{ij}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{L_{ij}} \end{bmatrix}$$

(53)

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The $A$ matrix above is block triangular, meaning that stability of the mG in Fig. 2 is dependent

\(^5\)Two separate pieces of Work on the theory and implementation of a scalable distributed control architecture that guarantees GAS in a PnP fashion and incorporates $L_1$ adaptive controllers has been submitted to journals IEEE Transactions on Automatic Control and IEEE Transactions on Smart Grid. Pre-prints are found in [54] and [56] respectively.
6.1.2 Global mG model with \( N \) DGUs

From section,

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_N
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & \cdots & A_{1N} \\
A_{21} & A_{22} & A_{23} & \cdots & A_{2N} \\
A_{31} & A_{32} & A_{33} & \cdots & A_{3N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_{N1} & A_{N2} & A_{N3} & \cdots & A_{NN}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N
\end{bmatrix}
+ \begin{bmatrix}
B_1 & 0 & 0 & \cdots & 0 \\
0 & B_2 & 0 & \cdots & 0 \\
0 & 0 & B_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & B_N
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
u_N
\end{bmatrix}
\]

\[ (54) \]

6.2 A Glimpse at Instability/Stability Due to Converter Interaction Using Baseline Controllers only/Augmenting \( L_1 \) Adaptive Controllers

The following 6 DGU DC ImG topology is used to demonstrate that decentralised baseline controllers, designed to be locally stable without accounting for interactions, can destabilise the global mG when DGUs are indeed interconnected.

Consider DGUs with the dynamics of (2). Electrical parameters of Table 1 are used here. Decentralised baseline controllers are designed for each DGU assuming they are dynamically decoupled i.e. \( \zeta_i(t) = \zeta_j(t) = 0 \). State feedback controllers are again defined as in (11).

At start-up, \( \Sigma_{DGU} \) and \( \Sigma_{DGU} \) are connected together through \( RL \) power lines in a radial configuration. \( \Sigma_{DGU} \) is connected to \( \Sigma_{DGU} \), while \( \Sigma_{DGU} \) powers a local load on its own. Controllers are designed to ensure local asymptotic stability of the closed-loop DGUs, implying that the eigenvalues of decoupled global linear representation of the decoupled DGUs is also asymptotically stable.

\[
\begin{bmatrix}
A_{C1} & 0_{1x3} & 0_{1x3} & 0_{1x3} & 0_{1x3} & 0_{1x3} \\
0_{1x3} & A_{C2} & 0_{1x3} & 0_{1x3} & 0_{1x3} & 0_{1x3} \\
0_{1x3} & 0_{1x3} & A_{C3} & 0_{1x3} & 0_{1x3} & 0_{1x3} \\
0_{1x3} & 0_{1x3} & 0_{1x3} & A_{C4} & 0_{1x3} & 0_{1x3} \\
0_{1x3} & 0_{1x3} & 0_{1x3} & 0_{1x3} & A_{C5} & 0_{1x3} \\
0_{1x3} & 0_{1x3} & 0_{1x3} & 0_{1x3} & 0_{1x3} & A_{C6}
\end{bmatrix}
\]

\[ (56) \]

where,

\[
A_{C_i} = \begin{bmatrix}
A_{ii} - B_i K_{bl}^x & B_i K_{bl}^x \\
-C_i & 0
\end{bmatrix}
\]

\[ (57) \]

and \( K_{bl}^x = [K_{bl}^x, K_{bl}^x] \). While baseline controller gains are tuned for decoupled, load-dependent DGUs, \( A_{ii} \) models the line-dependent coupled DGUs as in (7). However, the dynamic coupling at start-up
of DGUs in Fig. (13) means the global mG is linearly represented by the state matrix,

\[
A_{CL}^D = \begin{bmatrix}
\hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} & \hat{A}_{14} & \hat{A}_{15} & \hat{A}_{16} \\
\hat{A}_{21} & \hat{A}_{22} & \hat{A}_{23} & \hat{A}_{24} & \hat{A}_{25} & \hat{A}_{26} \\
\hat{A}_{31} & \hat{A}_{32} & \hat{A}_{33} & \hat{A}_{34} & \hat{A}_{35} & \hat{A}_{36} \\
\hat{A}_{41} & \hat{A}_{42} & \hat{A}_{43} & \hat{A}_{44} & \hat{A}_{45} & \hat{A}_{46} \\
\hat{A}_{51} & \hat{A}_{52} & \hat{A}_{53} & \hat{A}_{54} & \hat{A}_{55} & \hat{A}_{56} \\
\hat{A}_{61} & \hat{A}_{62} & \hat{A}_{63} & \hat{A}_{64} & \hat{A}_{65} & \hat{A}_{66}
\end{bmatrix}
\]

Plotting the eigenvalues of both $A_{CL}^D$ and $A_{CL}^C$,

Figure 14: Eigenvalues at start-up of $A_{CL}^D$ and $A_{CL}^C$.

Though global asymptotic stability cannot be guaranteed at start-up through the use of decentralised controllers, Fig. 14 shows that the eigenvalues of the coupled linear system in (58) are in the left-half plane, resulting in a globally stable configuration. The interconnection of DGUs does however reduce the damping within the system as eigenvalues move towards the imaginary axis when compared to the eigenvalues of the decoupled system representation.

When $\Sigma_6^{DGU}$ is plugged-in, connecting with $\Sigma_1^{DGU}$ and $\Sigma_5^{DGU}$, the global closed-loop system rep-
presentation changes to,

$$
\begin{bmatrix}
\hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} & 0_{3x3} & 0_{3x3} & \hat{A}_{16} \\
\hat{A}_{21} & \hat{A}_{22} & 0_{3x3} & 0_{3x3} & \hat{A}_{24} & 0_{3x3} \\
\hat{A}_{31} & \hat{A}_{32} & \hat{A}_{33} & \hat{A}_{34} & 0_{3x3} & 0_{3x3} \\
0_{3x3} & 0_{3x3} & \hat{A}_{42} & \hat{A}_{43} & \hat{A}_{44} & \hat{A}_{45} \\
0_{3x3} & \hat{A}_{52} & 0_{3x3} & \hat{A}_{53} & \hat{A}_{54} & \hat{A}_{55} \\
\hat{A}_{61} & 0_{3x3} & 0_{3x3} & \hat{A}_{63} & 0_{3x3} & \hat{A}_{66}
\end{bmatrix}
$$

(59)

Plotting the eigenvalues of $A_{CL}^{DGU6}$ against the eigenvalues plotted in Fig. 14, From Fig. (15), the addition of $\hat{\Sigma}^{DGU6}_6$ moves eigenvalues of the linear global system into the right-half plane, resulting in a globally unstable mG. Type III baseline controllers (each with their own tuning/bandwidths) are used to demonstrate that the non-linear switching model also becomes unstable when $\hat{\Sigma}^{DGU6}_6$ plugs into the system. The output voltage of each DGU is plotted below.

Figure 15: Eigenvalues of $A_{CL}^D$, $A_{CL}^C$ at start-up, and $A_{CL}^{DGU6}$ after $\hat{\Sigma}^{DGU}_6$ plug-in.

At 4 seconds, $\hat{\Sigma}^{DGU}_6$ plugs-in, connecting to $\hat{\Sigma}^{DGU}_1$ and $\hat{\Sigma}^{DGU}_5$. Though $\hat{\Sigma}^{DGU}_6$ maintains a steady-state voltage at 360 V, it loses reference tracking (i.e. 385 V). Additionally, while Fig. 16 does not indicate instability since other voltages in the grid maintain their voltage references, on closer inspection, the duty cycle of $\hat{\Sigma}^{DGU}_6$ clearly becomes unstable i.e. exponentially increasing to infinity.

Ultimately, as the duty cycle of $\hat{\Sigma}^{DGU}_6$ increases, its steady-state output voltage should increase as well (i.e. boost converter steady-state output voltage gain: $\frac{V_{\text{out}}}{D_{6}}$). Fig. 17 shows $\hat{\Sigma}^{DGU}_1$ and $\hat{\Sigma}^{DGU}_5$.
increasing their duty cycles in order to accommodate the fact that $\Sigma_{DGU}^6$ is unstable by injecting more current to power the load connected to $\Sigma_{DGU}^6$.

![Figure 17: DGU duty cycles with $\Sigma_{DGU}^6$ plug-in.](image)

Finally, the global state-space model of the system when implementing augmenting $L_1$ACs and using assumption 4 is,

$$
\begin{bmatrix}
\hat{A}_m & \hat{A}_{12} & \hat{A}_{14} & \hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{A}_{16} \\
\hat{A}_{21} & \hat{A}_m & \hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{A}_{24} \\
\hat{A}_{31} & \hat{Q}_{3x3} & \hat{A}_m & \hat{A}_{14} & \hat{Q}_{3x3} & \hat{Q}_{3x3} \\
\hat{Q}_{3x3} & \hat{A}_{42} & \hat{A}_{43} & \hat{A}_m & \hat{A}_{44} & \hat{Q}_{3x3} \\
\hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{A}_{54} & \hat{A}_m & \hat{A}_{56} \\
\hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{Q}_{3x3} & \hat{A}_{65} & \hat{A}_m & \hat{A}_{66}
\end{bmatrix}
$$

(60)

Plotting the eigenvalues of (60) shows that once adaptation yields convergence to desired dynamics and local asymptotic stability then global stability can be guaranteed for the system in Fig. 13, i.e. global eigenvalues are in left-half plane.

![Figure 18: Eigenvalues of $A_{CL}^D$, $A_{CL}^C$, $A_{CL}^{CDGU6}$ and $A_{CL}^{CDGU6+L_1}$.](image)

### 6.3 State-Space Model of Interconnected Boost Converters with Capacitor Equivalent Series Resistance

In this section, the local model of a coupled boost converter is derived when the ESR of the output capacitor is included. This highlights how the model changes for investigating robustness to unmodelled dynamics in section 4.5. The boost converter DGU model is shown below.
Using QSL approximations of section 2.2, the differential equations during the on-time PWM switching are,

\[
\begin{align*}
\frac{dI_{ti}}{dt} &= \frac{1}{L_{ti}} V_{i_{ni}} - \frac{R_{ti}}{L_{ti}} I_{ti} \\
\frac{dV_{dc_i}}{dt} &= \sum_{i'\in N_i} \left( \frac{V_{dc_{i'}} - V_{dc_i}}{R_{ji} C_{ri}} \right) - \frac{1}{C_{ti}} I_{Li}
\end{align*}
\] (61)

which in state-space form can be written as,

\[
\dot{x}_{on_{i}(t)} = A_{on_{i}} x_{on_{i}(t)} + B_{on_{i}} u_{on_{i}(t)} + E_{on_{i}} d_{on_{i}(t)} + \sum_{j\in N_i} A_{on_{i}} x_{on_{j}(t)}
\] (62)

where,

\[
A_{on_{i}} = \begin{bmatrix} \frac{R_{ti}}{L_{ti}} & 0 \\ -\sum_{j\in N_i} \frac{1}{R_{ji} C_{ri}} \end{bmatrix}, \\
B_{on_{i}} = \begin{bmatrix} \frac{V_{i_{ni}}}{L_{ti}} \\ 0 \end{bmatrix}, \\
E_{on_{i}} = \begin{bmatrix} 0 \\ -\frac{1}{C_{ti}} \end{bmatrix}, \\
A_{on_{i}} = \begin{bmatrix} 0 & 0 \\ \sum_{j\in N_i} \frac{1}{R_{ji} C_{ri}} & 0 \end{bmatrix}
\] (63)

The differential equations during the off-time PWM switching are,

\[
\begin{align*}
\frac{dI_{ti}}{dt} &= \frac{1}{L_{ti}} V_{i_{ni}} - \frac{R_{ti}}{L_{ti}} I_{ti} - \frac{1}{L_{ti}} V_{ci} - \frac{R_{ci}}{L_{ti}} I_{ci} \\
\frac{dV_{dc_i}}{dt} &= \frac{1}{C_{ti}} I_{ti} + \sum_{i'\in N_i} \left( \frac{V_{dc_{i'}} - V_{dc_i}}{R_{ji} C_{ri}} \right) - \frac{1}{C_{ti}} I_{Li}
\end{align*}
\] (64)

From Kirchoff’s current law, \( I_{ci} = I_{ti} + I_{ij} - I_{Li} = I_{ti} + \left( \frac{V_{dc_{i'}} - V_{dc_i}}{R_{ji} C_{ri}} \right) - I_{Li} \). Therefore, (64) can be written as,

\[
\begin{align*}
\frac{dI_{ti}}{dt} &= \frac{1}{L_{ti}} V_{i_{ni}} - \frac{R_{ti}}{L_{ti}} I_{ti} - \frac{1}{L_{ti}} V_{ci} - \frac{R_{ci}}{L_{ti}} I_{ci} - I_{ti} + \sum_{j\in N_i} \left( \frac{V_{dc_{i'}} - V_{dc_i}}{R_{ji} C_{ri}} \right) + I_{Li} \\
\frac{dV_{dc_i}}{dt} &= \frac{1}{C_{ti}} I_{ti} + \sum_{i'\in N_i} \left( \frac{V_{dc_{i'}} - V_{dc_i}}{R_{ji} C_{ri}} \right) - \frac{1}{C_{ti}} I_{Li}
\end{align*}
\] (65)

In state-space form, (65) can be written as,

\[
\dot{x}_{off_{i}(t)} = A_{off_{i}} x_{off_{i}(t)} + B_{off_{i}} u_{off_{i}(t)} + E_{off_{i}} d_{off_{i}(t)} + \sum_{j\in N_i} A_{off_{i}} x_{off_{j}(t)}
\] (66)

where,

\[
A_{off_{i}} = \begin{bmatrix} \frac{(R_{ti} + R_{ci})}{L_{ti} C_{ti}} & -\frac{(1 + R_{ci})}{L_{ti} C_{ti}} \\ -\sum_{j\in N_i} \frac{1}{R_{ji} C_{ri}} \end{bmatrix}, \\
B_{off_{i}} = \begin{bmatrix} \frac{V_{i_{ni}}}{L_{ti}} \\ 0 \end{bmatrix}, \\
E_{off_{i}} = \begin{bmatrix} \frac{R_{ti}}{L_{ti}} \\ 0 \end{bmatrix}, \\
A_{off_{i}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\] (67)

Combining (62) and (66) to form the average model via \( \tilde{x}_{i}(t) = \dot{x}_{on_{i}(t)} d_{i} + \dot{x}_{on_{i}(t)} (1 - d_{i}) \), where \( d_{i} \) is the duty-cycle, yields,

\[
\tilde{x}_{i}[i] = \begin{bmatrix} \frac{(R_{ti} + (1 - d_{i}) R_{ci})}{L_{ti} C_{ti}} & -\frac{(1 - d_{i}) (1 + R_{ci})}{L_{ti} C_{ti}} \\ -\sum_{j\in N_i} \frac{1}{R_{ji} C_{ri}} \end{bmatrix} \tilde{x}_{i}[i] + \begin{bmatrix} \frac{V_{i_{ni}}}{L_{ti}} \\ \frac{(1 - d_{i}) R_{ti}}{L_{ti} C_{ti}} \end{bmatrix} d_{i}
\] (68)

\[
+ \sum_{j\in N_i} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \frac{(1 - d_{i}) R_{ti}}{R_{ji} C_{ri}} \tilde{x}_{i}[j]
\]
Due to the bilinear terms between states $I_{ti}$ and $V_{dc,i}$, and the duty-cycle control input, making the average model non-linear, the average model of (68) requires linearising to form the small-signal model. Therefore, each signal is separated into its steady-state and small-signal quantities, i.e. $d_i = D_i + d^{ac}_i$, $\dot{x} [i] = x [i] + x^{ac}[i]$ etc. Finally, the linear state-space model for DGU $i$ coupled to $N$ neighbours can be written as,

$$\dot{x}^{ac}[i] = \begin{bmatrix}
\frac{-(R_{ti} + (1-D_i)R_c)}{L_{ti}} & -\frac{(1-D_i)}{L_{ti}} & \frac{1}{L_{ti}} & \frac{1}{L_{ti}C_{ti}} & -\sum_{j \in N_i \setminus \{i\}} \frac{R_{ti}}{R_{ti}C_{ti}}
\end{bmatrix} x^{ac}[i] + \begin{bmatrix}
\frac{1}{L_{ti}}(V_{dc,i} + R_{ti}(I_{ti} + I_{ij} - I_{Li})) \\
\frac{(1-D_i)R_c}{L_{ti}C_{ti}}
\end{bmatrix} u[i]$$

$$+ \begin{bmatrix}
\frac{1}{L_{ti}} & \frac{1}{L_{ti}C_{ti}} & \sum_{j \in N_i \setminus \{i\}} 0 & 0 & 0
\end{bmatrix} x^{ac}[i]$$

(69)

### 6.4 Simulation results using relatively large DGU output voltages

This section demonstrates similar results when DGU output voltages differ by a relatively large amount. Without droop control or coordinated secondary control, DGUs with larger output voltage (i.e. larger duty cycles) tend to provide most of the power to the mG, and ‘overpower’ neighbouring DGUs. Like a see-saw, DGUs with larger voltage (i.e. larger force) will push surplus power through coupling RL lines to ‘help’ power neighbouring load. The greater the voltage difference between DGU outputs, the greater the ‘overpowering’ effect.

#### 6.4.1 Plug-and-Play Operations

At $t = 0.05$ s, $\Sigma_6^{DGU}$ is plugged in, connecting to $\Sigma_1^{DGU}$ and $\Sigma_5^{DGU}$. The output voltages are given in Table II.

| DGU | Voltage reference | Input voltage | Duty cycle | Local load power |
|-----|-------------------|---------------|------------|------------------|
| $\Sigma_1^{DGU}$ | 381 V | 95 V | 0.75 | 2.5 kW |
| $\Sigma_2^{DGU}$ | 390 V | 100 V | 0.7372 | 2 kW |
| $\Sigma_3^{DGU}$ | 373 V | 90 V | 0.7633 | 1.8 kW |
| $\Sigma_4^{DGU}$ | 387 V | 105 V | 0.723 | 2.5 kW |
| $\Sigma_5^{DGU}$ | 370 V | 92 V | 0.7576 | 3 kW |
| $\Sigma_6^{DGU}$ | 385 V | 90 V | 0.7636 | 2.5 kW |

The following figures show the response of each DGU.

The responses are largely favourable. DGUs $\Sigma_1^{DGU}$, $\Sigma_2^{DGU}$, $\Sigma_3^{DGU}$ and $\Sigma_6^{DGU}$ show very fast settling times and damped responses. Though the voltage ripple of $\Sigma_6^{DGU}$ increases from 1.3% (or 5 V) to 3.1% (or 12 V), this is purely due to the effective load change as $\Sigma_6^{DGU}$ connects to the RL power lines and loads of $\Sigma_1^{DGU}$ and $\Sigma_5^{DGU}$. DGUs $\Sigma_3^{DGU}$ and $\Sigma_5^{DGU}$ have noticeably slower settling times. This can be attributed to both $\Sigma_3^{DGU}$ and $\Sigma_5^{DGU}$ being neighbours with DGUs that have considerably higher output voltages. As a result, $\Sigma_3^{DGU}$ and $\Sigma_5^{DGU}$ are in less control of their power supply capabilities, with the larger output voltages their respective neighbours, $\Sigma_1^{DGU}$, $\Sigma_4^{DGU}$ and $\Sigma_6^{DGU}$. This is evident from the the duty cycles of each DGU in Fig. 21.

**Note:** Duty cycles are plotted over the course of 1s test i.e. includes responses to subsequent tests.

From Fig. 21(c) and 21(e), the duty cycles of $\Sigma_3^{DGU}$ and $\Sigma_5^{DGU}$ are reduced from their nominal steady-state values.

At $t = 0.2$ s, $\Sigma_3^{DGU}$ is unplugged from the rest of the mG to power a local load exclusively. The following figures show the response of each DGU.

All responses, except $\Sigma_3^{DGU}$, are favourable, with fast settling times of between 2 - 25 ms and damped overshoots of maximum 12 V ($\Sigma_5^{DGU}$). As $\Sigma_3^{DGU}$ is unplugged, the change in going from a coupled system requiring assistance from the $L_1$AC loop to a decoupled system that only needs the baseline controller is a big enough jump to induce large oscillations, with a peak swing of 225 V. The response settles after 90 ms, which for primary voltage control is still fast. However, as $\Sigma_3^{DGU}$ does
not affect the rest of the grid, this oscillation might be tolerable as long as it is within the tolerance level of the local load. Alternatively, it is shown in Fig. 8b that turning off the adaptation loop after \( \Sigma_{3}^{DGU} \) is plugged-out avoids the oscillations.

Finally, at \( t = 0.7 \) s, \( \Sigma_{3}^{DGU} \) is plugged back into the ImG, connecting to \( \Sigma_{1}^{DGU} \) and \( \Sigma_{5}^{DGU} \). Responses are plotted in Fig. 23.

The responses are very good. The settling times are fast, with the longest at 50 ms associated with \( \Sigma_{1}^{DGU} \) and \( \Sigma_{5}^{DGU} \) since these are the DGUs that \( \Sigma_{3}^{DGU} \) connects to. Overall, PnP operations are satisfactory when relatively large differences between each DGU voltage reference exist. The DGU 'overpowering' effect does not adversely affect stability or performance during PnP operations - at worst, the settling times of the 'overpowered' DGUs become slower. However, it should be noted that some of the DGU duty cycles are close to saturation. Realistic duty cycles are typically limited to 80
6.4.2 Robustness to Unknown Load Dynamics

In order to examine the robustness of the global DC-ImG, the load at \( \Sigma_6^{DGU} \) is stepped from 2.5 kW to 800 W at \( t = 0.3 \) s. The responses of each DGU are plotted below.

**Note:** Output voltage response for \( \Sigma_3^{DGU} \) is not plotted at \( t = 0.3 \) s, as it has been unplugged from the grid.

As can be seen, though only one load is changed, it causes a disturbance to every DGU. All responses are favourable, with the longest settling time of 50 ms (\( \Sigma_6^{DGU} \)) and largest under/overshoot 25 V (\( \Sigma_2^{DGU} \)).
Comparing the performance using the $L_1$AC to the state-of-the-art PnP voltage controllers of [19] reveals promising results. The resulting dynamics are faster than that of a 8-4 Ω load change at $\Sigma_6^{DGU}$ in Fig. 14 of [19]. While the dynamics of neighbouring DGUs are more oscillatory in [19], settling times are similar to Fig. 24(a) and 24(e), at 2 ms. However, the response of $\Sigma_6^{DGU}$ to its load change in [19] has a considerably slower settling time of 900 ms, while above, Fig. 24(e) shows that the settling time is 50 ms.

These tests highlight the very good performance and robustness of controllers $\mathcal{L}_i$, $i = 1, ..., 6$, ensuring fast reference tracking in the presence of unknown load disturbances.

**6.4.3 Voltage Reference Tracking**

At $t = 0.75$ s, the voltage reference of $\Sigma_6^{DGU}$ is stepped from 370 V to 377 V. Since the impedance of the $RL$ lines is small, the voltage increase is enough to propagate an appreciable amount of power
Figure 23: DGU duty cycles in response to all tests in this section
Figure 24: DGU output voltage responses to $\Sigma_{\text{DGU}}^6$ load step of 2.5 kW to 800 W.

to coupled DGUs, causing current disturbances to cascade throughout the ImG. Therefore, it is important that neighbouring DGUs are robust to this unknown disturbance also. The responses of each DGU are plotted in Fig. 25.

Fig. 25(d) shows that the controller $C^{L1}_{5}$ is capable of guaranteeing fast reference tracking, with good damping and a settling time of $\approx 300$ ms. Also, it should be noted how there is no undershoot, as expected in non-minimum phase systems. The performance of $C^{L1}_{5}$ is similar to the state-of-the-art PnP voltage controller in [19], albeit Fig. 6 of [19] suggests an instantaneous response. However, the settling time of a neighbouring DGU in Fig. 7 of [19] is much slower ($\approx 2$ s) compared to neighbouring DGUs of Fig. 25 (largest being $\approx 20$ ms).

References

[1] R. Adam and W. Winterstellar, “From Distribution To Contribution,” 2008.

[2] D. P. Symanski, “Residential & Commercial Use Of DC Power,” in UL & NFPA-Low Voltage Direct Current Workshop. Arlington, Virginia: EPRI, 2011.

[3] H. Farhangi, “The path of the smart grid,” IEEE Power and Energy Magazine, vol. 8, no. 1, pp. 18–28, 2010.

[4] R. Lasseter, “MicroGrids,” IEEE Power Engineering Society Winter Meeting., vol. 1, p. IEEE, 2002.

[5] J. M. Guerrero, J. C. Vásquez, J. Matas, M. Castilla, and L. García de Vicuna, “Control strategy for flexible microgrid based on parallel line-interactive UPS systems,” IEEE Transactions on Industrial Electronics, vol. 56, no. 3, pp. 726–736, 2009.
Figure 25: DGU output voltage responses to $\hat{\Sigma}_5^{DGU}$ voltage reference step of 370 V to 377 V.

[6] J. C. Vasquez, “Decentralized control techniques applied to electric power distributed generation in microgrids,” *Tesis June*, 2009.

[7] J. M. Guerrero, M. Chandorkar, T. L. Lee, and P. C. Loh, “Advanced control architectures for intelligent microgrids part i: Decentralized and hierarchical control,” *IEEE Transactions on Industrial Electronics*, vol. 60, no. 4, pp. 1254–1262, 2013.

[8] D. J. Hogan, M. G. Egan, J. G. Hayes, G. Lightbody, and F. Gonzalez-Espin, “A rapid prototyping tool for load and source emulation in a microgrid test laboratory,” *2014 IEEE Applied Power Electronics Conference and Exposition - APEC 2014*, pp. 2245–2252, Mar 2014.
[9] J. M. Guerrero, P. C. Loh, T. L. Lee, and M. Chandorkar, “Advanced control architectures for intelligent microgridsPart II: Power quality, energy storage, and AC/DC microgrids,” IEEE Transactions on Industrial Electronics, vol. 60, no. 4, pp. 1263–1270, 2013.

[10] R. W. De Doncker, “Power electronic technologies for flexible DC distribution grids,” in Power Electronics Conference (IPEC-Hiroshima 2014-ECCE-ASIA), 2014 International. IEEE, 2014, pp. 736–743.

[11] B. T. Patterson, “DC, Come Home: DC Microgrids and the Birth of the "Enernet"," IEEE Power and Energy Mag., vol. 10, no. 6, pp. 60–69, 2012.

[12] D. J. Becker and B. J. Sonnenberg, “DC microgrids in buildings and data centers,” in Telecom. Energy Conference (INTELEC), 2011.

[13] A. T. Elsayed, A. A. Mohamed, and O. A. Mohammed, “DC microgrids and distribution systems: An overview,” Electric Power Systems Research, vol. 119, pp. 407–417, 2015.

[14] a. a. Abdelhiafez and a. J. Forsyth, “A Review of More-Electric Aircraft,” Aerospace Sciences & Aviation Technology, pp. 1–13, 2009.

[15] P. Wheeler and S. Bozhko, “The more electric aircraft: Technology and challenges,” IEEE Electrification Magazine, vol. 2, no. 4, pp. 6–12, 2014.

[16] M. A. Anuradha and A. Massoud, IEEE Vision for Smart Grid Controls: 2030 and Beyond: Roadmap. IEEE CSS, 2013.

[17] L. Meng, Q. Shafiee, G. F. Trecate, H. Karimi, D. Fulwani, X. Lu, and J. M. Guerrero, “Review on Control of DC Microgrids and Multiple Microgrid Clusters,” IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 5, no. 3, pp. 928–948, 2017.

[18] T. Dragicevic, X. Lu, J. C. Vasquez, and J. M. Guerrero, “DC Microgrids - Part I: A Review of Control Strategies and Stabilization Techniques," IEEE Transactions on Power Electronics, vol. 31, no. 7, pp. 4876–4891, 2016.

[19] M. Tucci, S. Riverso, J. C. Vasquez, J. M. Guerrero, and G. Ferrari-Trecate, “A Decentralized Scalable Approach to Voltage Control of DC Islanded Microgrids,” IEEE Transactions on Control Systems Technology, vol. 24, no. 6, pp. 1965–1979, 2016.

[20] D. O’Keeffe, S. Riverso, L. Albiol-Tendillo, and G. Lightbody, “Distributed Hierarchical Droop Control of Boost Converters in DC Microgrids,” 28th IEEE Irish Signals and Systems Conference, pp. 1–6, 2017.

[21] J. Á. Stoustrup, “Plug & Play Control : Control Technology Towards New Challenges,” European Journal of Control, vol. 15, no. 3-4, pp. 311–330, 2009.

[22] S. Riverso, F. Sarzo, and G. Ferrari-Trecate, “Plug-and-Play Voltage and Frequency Control of Islanded Microgrids with Meshed Topology,” IEEE Transactions on Smart Grid, vol. 6, no. 3, pp. 1176–1184, 2015.

[23] S. Riverso, M. Tucci, J. C. Vasquez, J. M. Guerrero, and G. Ferrari-Trecate, “Stabilizing plug-and-play regulators and secondary coordinated control for AC islanded microgrids with bus-connected topology,” Applied Energy, no. August, pp. 1–21, 2017.

[24] M. Tucci, L. Meng, J. M. Guerrero, and G. Ferrari-Trecate, “Plug-and-play control and consensus algorithms for current sharing in DC microgrids,” in IFAC-PapersOnLine, vol. 50, 2016, pp. 1–23.

[25] M. Tucci, S. Riverso, and G. Ferrari-Trecate, “Line-Independent Plug-and-Play Controllers for Voltage Stabilization in DC microgrids,” IEEE Transactions on Control Systems Technology, pp. 1–9, 2016.

[26] R. Han, M. Tucci, R. Soloperto, G. Ferrari-Trecate, and J. M. Guerrero, “Plug-and-Play Design of Current Controllers for Grid-feeding Converters in DC Microgrids,” in 2017 Asian Control Conference, 2017.
M. S. Sadabadi, Q. Shafiee, and A. Karimi, “Plug-and-Play Robust Voltage Control of DC Microgrids,” *IEEE Transactions on Smart Grid*, pp. 1–1, 2017.

V. Nasirian, A. Davoudi, F. L. Lewis, and J. M. Guerrero, “Distributed adaptive droop control for DC distribution systems,” *IEEE Transactions on Energy Conversion*, vol. 29, no. 4, pp. 944–956, 2014.

X. Lu, K. Sun, J. M. Guerrero, J. C. Vasquez, and L. Huang, “State-of-Charge Balance Using Adaptive Droop Control for Distributed Energy Storage Systems in DC Microgrid Applications,” *IEEE Trans. Ind. Electron.*, vol. 61, pp. 2804–2815, 2014.

T. V. Vu, D. Perkins, F. Diaz, D. Gonsoulin, C. S. Edrington, and T. El-Mezyani, “Robust adaptive droop control for DC microgrids,” *Electric Power Systems Research*, vol. 146, pp. 95–106, 2017.

B. D. Anderson, “Failures of adaptive control theory and their resolution,” *Communications in Information and Systems*, vol. 5, no. 1, pp. 1–20, 2005.

C. Cao and N. Hovakimyan, “Design and Analysis of a Novel L1 Adaptive Controller, Part II: Guaranteed Transient Performance,” *American Control Conference*, pp. 3403–3408, 2006.

———, “Design and Analysis of a Novel L.1 Adaptive Control Architecture With Guaranteed Transient Performance,” *IEEE Transactions on Automatic Control*, vol. 53, no. 2, pp. 3397–3402, 2008.

C. Cao and Hovakimyan, *L1 Adaptive Control Theory: Guaranteed Robustness with Fast Adaptation*. Society for Industrial and Applied Mathematics, 2010.

S. Yoo, C. Cao, and N. Hovakimyan, “Decentralised L1 adaptive control for large-scale non-linear systems with interconnected unmodelled dynamics,” *IET Control Theory & Applications*, vol. 4, no. 10, pp. 1972–1988, 2010.

B. Michini and J. P. How, “L1 adaptive control for indoor autonomous vehicles: Design process and flight testing,” *Proceeding of AIAA Guidance, Navigation, and Control Conference*, pp. 1–31, 2010.

I. Gregory, E. Xargay, C. Cao, and N. Hovakimyan, “Flight Test of an L1 Adaptive Controller on the NASA AirSTAR Flight Test Vehicle,” *AIAA Guidance, Navigation, and Control Conference*, pp. 1–31, 2010.

C. H. Svendsen, N. O. Holek, R. Galeazzi, and M. Blanke, “L1 adaptive manoeuvring control of unmanned high-speed water craft,” *IFAC Proceedings Volumes (IFAC-PapersOnline)*, vol. 9, no. PART 1, pp. 144–151, 2012.

Z. Li, N. Hovakimyan, C. Cao, and G.-o. Kaasa, “Integrated Estimator and L1 Adaptive Controller for Well Drilling Systems,” *American Control Conference, 2009*, pp. 1958–1963, 2009.

H. Zhao, Q. Wu, C. N. Rasmussen, and M. Blanke, “L1 Adaptive Speed Control of a Small Wind Energy Conversion System for Maximum Power Point Tracking,” *IEEE Transactions on Energy Conversion*, vol. 29, no. 3, pp. 576–584, 2014.

X. Lu, J. M. Guerrero, K. Sun, and J. C. Vasquez, “An improved droop control method for dc microgrids based on low bandwidth communication with dc bus voltage restoration and enhanced current sharing accuracy,” *IEEE Transactions on Power Electronics*, vol. 29, no. 4, pp. 1800–1812, 2014.

J. M. Guerrero, J. C. Vasquez, J. Matas, L. G. De Vicuña, and M. Castilla, “Hierarchical control of droop-controlled AC and DC microgrids - A general approach toward standardization,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 1, pp. 158–172, 2011.

Q. Shafiee, T. Dragicevic, J. C. Vasquez, and J. M. Guerrero, “Modeling, stability analysis and active stabilization of multiple DC-microgrid clusters,” in *ENERGYCON 2014 - IEEE International Energy Conference*, 2014, pp. 1284–1290.
[44] P. Wang, X. Lu, X. Yang, W. Wang, and D. Xu, “An Improved Distributed Secondary Control Method for DC Microgrids with Enhanced Dynamic Current Sharing Performance,” IEEE Trans. on Power Electron., vol. 31.9, pp. 6658–6673, 2016.

[45] F. Dorfler and F. Bullo, “Kron reduction of graphs with applications to electrical networks,” IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 60, no. 1, pp. 150–163, 2013.

[46] G. Kron, Tensor Analysis of Networks. John Wiley & Sons, Ltd., 1965.

[47] E. Lavretsky and K. A. Wise, Robust and Adaptive Control with Aerospace Applications. London: Springer, 2012.

[48] B. Kurucs, “State Space Control of Quadratic Boost Converter using LQR and LQG approaches,” 2015 Intl Conference on Optimization of Electrical & Electronic Equipment, no. 2, pp. 642–648, 2015.

[49] G. Kumaresan and A. Kale, “Application of L1 adaptive controller for the design of a novel decentralized leader follower formation algorithm,” IFAC-PapersOnLine, vol. 49, no. 1, pp. 706–711, 2016.

[50] J.-J. Slotine and W. Li, Applied Nonlinear Control, 1991.

[51] E. Lavretsky and T. E. Gibson, “Projection Operator in Adaptive Systems,” arXiv ePrints, arXiv:1112.4232, 2011.

[52] J. Lunze, Feedback Control of Large-Scale Systems. London: Prentice-Hall, 1992.

[53] L. Bakule and J. Lunze, “Decentralized Design of Feedback Control,” Kybernetika, vol. 24, pp. 1–100, 1988.

[54] D. O’Keeffe, S. Riverso, L. Albiol-Tendillo, and G. Lightbody, “Global Asymptotic Stability for General MIMO Distributed Systems : An Approach Based on Robust-Adaptive Controllers,”, 2018. [Online]. Available: http://arxiv.org/abs/1801.02331v1

[55] V.-j. Webb, “Design of a 380 V/24 V DC Micro-Grid for Residential DC Distribution,” Ph.D. dissertation, Univeristy of Toledo, 2013.

[56] D. O’Keeffe, S. Riverso, L. Albiol-Tendillo, and G. Lightbody, “A Distributed Scalable Architecture using L1 Adaptive Controllers for Primary Voltage Control of DC Microgrids,” 2018. [Online]. Available: arXivpreprintarXiv:1801.06484