On the Phase Boundaries of the Integer Quantum Hall Effect. II

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It is shown that the statements about the observation of the transitions between the insulating phase and the integer quantum Hall effect phases with the quantized Hall conductivity \( \sigma_{xy}^q \geq 3e^2/h \) made in a number of works are unjustified. In these works, the crossing points of the magnetic field dependences of the diagonal resistivity \( \rho_{xx} \) at different temperatures \( T \) at \( \omega_c \tau \approx 1 \) have been misidentified as the critical points of the phase transitions. In fact, these crossing points are due to the sign change of the derivative \( d\rho_{xx}/dT \) owing to the quantum corrections to the conductivity. Here, \( \omega_c \) is the cyclotron frequency, \( \tau \) is the transport relaxation time.

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The phase diagram of two dimensional systems in the magnetic field has attracted the attention of both theorists and experimentalists for many years. Treating the integer quantum Hall effect (IQHE) in the context of the two parameter scaling theory [1], which is graphically represented as a flow diagram [2, 3], yields the solution of the problem disregarding the electron–electron interaction. The further development of the scaling theory showed that the electron–electron interaction does not affect the position of the IQHE phase boundaries of a spin polarized electron system [4, 5].

According to the scaling theory, the boundary between two IQHE phases is possible only if the quantized values of the Hall conductivity of these two phases differ by \( e^2/h \) or (in the case of the spin degeneracy of the Landau levels) \( 2e^2/h \). However, a number of works reported on the observation of the transitions between the insulating phase (\( \sigma_{xy}^q = 0 \)) and the IQHE phases with the quantized Hall conductivity \( \sigma_{xy}^q \geq 3e^2/h \). Song et al. [6] reported on the observation of the transition \( \sigma_{xy}^q = 0 \leftrightarrow \sigma_{xy}^q = 3e^2/h \) in two dimensional hole systems in a strained Ge quantum well. The observation of the transitions \( \sigma_{xy}^q = 0 \leftrightarrow \sigma_{xy}^q \geq 3e^2/h \) in the two dimensional hole systems in a strained Ge quantum well was also announced in [7]. Lee et al. [8] claimed the observation of the transitions \( \sigma_{xy}^q = 0 \leftrightarrow \sigma_{xy}^q = 6e^2/h \) and \( \sigma_{xy}^q = 0 \leftrightarrow \sigma_{xy}^q = 8e^2/h \) in doped AlGaAs/GaAs/AlGaAs quantum wells. Huang et al. [9] reported on the observation of the transitions from the state with \( \sigma_{xy}^q = 0 \) to the states with \( \sigma_{xy}^q = 6-16e^2/h \) in GaAs/AlGaAs heterojunctions. In all of these works [6–9], the crossing points of the magnetic field dependences of the diagonal resistivity \( \rho_{xx} \) at different temperatures \( T \), at \( \omega_c \tau \approx 1 \) (\( \omega_c = eB/m \)) is the cyclotron frequency, \( \tau \) is the transport relaxation time, and \( m \) is the effective electron mass) were considered as the critical points \( B_c \) of the phase transitions (see Fig. 1). In this case, \( \rho_{xx} \) weakly depends on the magnetic field and temperature near \( B_c \).

In this work, it is shown that the above statements of the observation of the transitions between the insulating phase and the IQHE phases with the quantized Hall conductivity \( \sigma_{xy}^q \geq 3e^2/h \) are unjustified. In fact, the crossing point of the magnetic field dependences of the diagonal resistivity \( \rho_{xx}(T) \) at different temperatures is caused by the sign change of the derivative \( d\rho_{xx}/dT \) owing to the quantum corrections to the conductivity [10].

The classical diagonal and Hall conductivities in the magnetic field take the form

\[
\sigma_{xx}^0 = \frac{N_e e^2 \tau}{m} \frac{1}{1 + (\omega_c \tau)^2} \tag{1}
\]
temperatures at which the Hall conductivity depends on the temperature and approaches the nearest quantized integer value \( \sigma_{xy}(B_i) = ie^2/h \) with \( B_i < B_c \) at \( \omega_c \tau < 1 \).

Taking into account the quantum corrections, the diagonal and Hall resistivities of the two-dimensional electron system are given by the expressions

\[
\rho_{xx}(T) = \rho_{xx}^0 + \left( (\rho_{xy}^0)^2 - (\rho_{xx}^0)^2 \right) \Delta \sigma_{xx}(T)
\]

and

\[
\rho_{xy}(T) = \rho_{xy}^0 - 2\rho_{xx}^0\rho_{xy}^0\Delta \sigma_{xx}(T)
\]

Here, we took into account that \( \rho_{xx}^0 = \sigma_{xx}^0/ \left( (\sigma_{xx}^0)^2 + (\sigma_{xy}^0)^2 \right) \), \( \rho_{xy}^0 = \sigma_{xy}^0/ \left( (\sigma_{xx}^0)^2 + (\sigma_{xy}^0)^2 \right) \) and \( \sigma_{xx}^0, \sigma_{xy}^0, \rho_{xx}^0 \) and \( \rho_{xy}^0 \) are the bare (non-renormalized) values of the conductivity and resistivity, which correspond to the diffusion motion of electrons without the interference (localization) effects at distances longer than the diffusion step length. The derivative \( d\rho_{xx}/dT \) changes its sign in the magnetic field \( B \) such that \( \rho_{xx}(B) = \rho_{xx}'(B) \). In the classical treatment \( \rho_{xx}'(B) = \rho_{xy}'(B) = 1 \).

At \( \sigma_{xy} \rightarrow e^2/h \) and \( \omega_c \tau < 1 \), the diagonal resistivity \( \rho_{xx} \) first increases with a decrease in the temperature, reaching the value

\[
\rho_{xx,max} = \frac{1}{2\sigma_{xy}^0},
\]

and then decreases and vanishes at \( T \rightarrow 0 \) excluding the critical magnetic fields in which \( \sigma_{xy}^0 = (i + 1/2)e^2/h \). Thus, the negative value of the derivative \( d\rho_{xx}/dT \) within the experimental range does not imply that the electron system is in an insulating phase.

Note that the magnetic field position of the IQHE phases at \( \omega_c \tau \lesssim 1 \) is not determined by the filling factor \( \nu \). Rather, it is given by the magnitude of \( \sigma_{xy}^0 h/e^2 \). This quantity is different from \( \nu \) at \( \omega_c \tau \lesssim 1 \). [11] At \( \nu = 8 \) in Fig.1 \( \sigma_{xy}^0 h/e^2 = 3.8 \), \( \sigma_{xy}^0 h/e^2 = 3.8 \) at \( T = 0.3 \) K. According scaling diagram presented on Fig.2 at this \( \nu \) quantized values of the Hall conductivity \( \sigma_{xy}^0 = 4e^2/h \).

Thus, we have shown that the statements [6–9] of the observation of the transitions between the insulating phase and the IQHE phases with the quantized Hall conductivity \( \sigma_{xy}^0 \geq 3 \) are unjustified. In fact, the crossing point of the magnetic field dependences of the diagonal resistivity \( \rho_{xx}(T) \) at different temperatures is caused by the sign change of the derivative \( d\rho_{xx}/dT \) at \( \omega_c \tau \approx 1 \) owing to the quantum corrections to the conductivity.

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