Formation of ordered structures of exciton condensed phases in quantum wells

V Sugakov
Institute for Nuclear Research, 47 Nauki Av., Kyiv 03680, Ukraine
E-mail: sugakov@kinr.kiev.ua

Abstract. The spatial distribution of the density of indirect excitons is studied in double quantum well in the case when condensed and gas phases coexist. The both the nucleation (Lifshits-Slyozov-Wagner) theory and spinodal decomposition (Chan-Hillert) theory are applied generating them on a system of unstable particles. Comparison with experiments is presented. The creation of structures in non-homogeneous external fields is investigated. The periodical distribution density, which arises in quantum well under a slot in conductive electrode, is analyzed in detail. The transformation of the structures with changing of a pumping and a width of the slot.

1. Introduction
In last years an appearance of structures was observed in an emission spectra of indirect excitons from double quantum wells in semiconductors. The large value of exciton lifetime of indirect exciton allows a creation of a large exciton density in order to study processes of exciton-exciton interaction. In the works [1,2] the narrow band was observed in emission spectra of indirect exitons which arisen at some threshold value of pumping. The authors [3] observed a ring of indirect excitons luminescence at the distances from a laser spot significantly exceeding the exciton diffusion length. In some cases, the ring breaks down into a number of periodically arranged fragments [3]. Authors of the papers [4] excited a double quantum well structure by light and measured excitonic emission from the wells through a circular window in a metallic electrode. They observed the luminescence from the double quantum well in the form of bright spots situated periodically on a circle under the rim of the window. In both cases the break of the symmetry is realized.

There are two approaches for the explanation of the structure appearance.

In the fist group of works explanations of an origin of structures in luminescence spectra are based on the Bose-Einstein statistic of excitons [5-7]. But the detailed study of the observed features, connection between theoretical and experimental parameters as well as of the evolution of the patterns depending on temperature, pumping and the parameters of the exciton system in these works was not performed.

Second type of explanation of experiments [2-4] was given in the papers [8-12] on the base of two assumptions: 1) existence of the exciton condensed phase, caused by exciton-exciton attractive interaction at close distance between indirect excitons, which exceeds the dipole-dipole repulsion; 2) important role plays the finite value of exciton lifetime in the formation of the structures. Usually the exciton lifetime is much large than the time of an establishment of local equilibrium, but it is less than the establishment of an equilibrium between phases. The theoretical possibility of the existence of an
excitonic liquid phase in the double quantum well structures has been shown in the paper [13]. Also, the possibility of an existence of a biexciton for indirect excitons was shown in the papers [14,15].

For description of the formation of structures of exciton condensed phases two most popular models of phase transitions (the model of nucleation and spinodal decomposition) were used in [8-12] generating the models on the system of unstable particles. In presented paper the analysis of application of the theory for explanation of the experiments is given. Also, the prediction of structure arising in the case of non-homogeneous potential made. Particularly, the condensed phase structures are studied in a presence of a slot in metallic electrode.

2. Properties of exciton condensed phase. Model of nucleation and growth

In model of nucleation and growth (Lifshits-Slyozov-Wagner model) the parameters of new phases are studied taking into account the exchange by particles between the condensed phase and environment. The finite value of exciton lifetime plays important role in formation of the sizes and spatial arrangement of exciton condensed phases. In spite of stable particles a stationary state exists at steady-state pumping in system of unstable particles. Due to finite value of the lifetime the region of condensed phase is restricted. As a result, in two-dimensional case the condensed phase must exist in the form of a system of islands similar to the system of electron-hole droplets in bulk semiconductors. Due to presence of a surface energy the islands have a dislike shape. According to estimations [9-11] the exciton wave function outside the islands loses its coherence due to scattering on defects at distances smaller than the distance between excitons. This means that the exciton propagation can be studied in the framework of diffusion model.

For study the distribution of exciton condensed phase with respect to sizes and positions we find [8,9,11] the joint solution of both kinetic equations for the number of excitons in condensed phases and diffusion equation for exciton outside the condensed phases. The size of the islands is determined by four processes: the creation of excitons by the pumping, the capture of the excitons from environment, an escape of excitons from the island, and the exciton decay. Let us consider some island and introduce the distribution function \( f_n \), which determines the probability of the island to have \( n \) excitons. The distribution function satisfies the following kinetic equation

\[
\frac{\partial f_n}{\partial t} = -j_{n+1} + j_n, \quad (1)
\]

where \( j_n \) is the probability current for transitions between island states with \( n \) and \( (n-1) \) excitons, while \( j_{n+1} \) stands for transitions between the states with \( (n+1) \) and \( n \) excitons,

\[
j_n = W^{(+)}(n-1)f_{n-1} - W^{(-)}(n)f_n, \quad (2)
\]

\( W^{(\pm)} \) is the probability of transition in unit time with increasing (decreasing) number of excitons on unity. These values will be determined later.

The steady-state solution, that describes the equilibrium state which is formed after some time of pumping action, satisfies the following condition \( j_n = j_{n+1} = 0 \). Setting \( \partial f_n / \partial t = 0 \) in (1) and \( j_n = 0 \) in (2) we obtain the following steady-state solution

\[
f_n = f_i \exp \left\{ \sum_{m=0}^{m=n} \ln \frac{W^{(+)}(m-1)}{W^{(-)}(m)} \right\}. \quad (3)
\]

The distribution function has sharp maximum at some value of \( n \), which is much larger than unity \( (n >> 1) \). In this case the probabilities of \( W^{(+)}(n) \) and \( W^{(-)}(n) \) for islands with radiiuses \( R_e = (n / \pi)^{1/2} \) satisfy the following conditions

\[
W^{(+)}(R) = 2\pi R c(R) W_\beta(R) + \pi R^2 G, \quad (4)
\]

\[
W^{(-)}(R) = 2\pi R c(R) W_\gamma(R) + \pi R^2 c_i / \tau_{ex}, \quad (5)
\]
where $W^{\pm}(R) \equiv W^{\pm}(n)$, $W_{fi}$ and $W_{if}$ are the probabilities for the exciton to be captured by the disk and to escape from the disk per unit length of the circle in unit time and per one exciton of disc circle, respectively. $G$ is the mean value of the exciton pumping over the island area, $c(R)$ and $c_i$ are the exciton densities on the circle of the disk and inside the disk.

The relationship between the transition probabilities $W_{fi}$ and $W_{if}$ may be obtained using the detailed balance principle and depends on such parameters of condensed phase as exciton density in condensed phase $c_i$, the energy condensation per one exciton, the surface strain (see detailer in [9,11]).

In the stationary case, the equation for the exciton density outside the islands has the form

$$D_{ex} \Delta c(r) - \frac{c(r)}{\tau_{ex}} = -G,$$

with boundary conditions $2\pi R D_{ex} (\nabla \cdot \vec{n}) = (2\pi R (W_{fi} c(r) - W_{if} c_i))$ at $r = R$, $D_{ex}$ is the exciton diffusion coefficient, $\vec{n}$ is the outward and unitary normal vector to the circular boundary of the island. The boundary condition should be applied on every island.

It is seen from the equations (4) and (5) the probabilities of the capture of exciton by island and its realize from island depends on the exciton density on the boundary $c(R)$. This density is determined not only by the considered island but also by presence of another islands, their radiiues and their positions. In such way an interaction between different islands occurs. In general case the distribution function for some island depends on radiiues and positions of all islands. In principle it can be determined from equations (1)-(6). But such problem is very difficult. To solve the problem in the works [8,9,11] two assumptions were done: 1) the distances between the islands are larger than the island radiiues, 2) the exciton density in vicinity of some islands is formed by the considered island and by many another islands, contribution of which may be studied in meanfield approximation and depends only on the average radiues of other islands.

Using presented theory we calculated [9, 11] the phase diagram $\tilde{G}_c$ ( $\tilde{G} = G \tau_{ex}/c_i$ is the dimensionless pumping) vs $T$, i.e. the dependence of the critical pumping on temperature (see Figure 1). The calculated curve correctly reproduces the experimental phase diagram reported in [2]. The potential barrier, which the indirect excitons have to overcome in order to penetrate into the island is the cause behind an increased value of $\tilde{G}_c$ at $T \to 0$ (see Figure 1). Such feature of the phase diagram is observed on the experiment [2]. The calculated dependences of luminescence intensity on
temperature at different values of the exciton generation rates are presented in the Figure 2. The theory reproduces empirically established in [2] almost linear dependence the intensity on temperature.

Such procedure was applied to the explanation of the fragmentation of the luminescence from a ring outside a laser spot in AlGaAs based double quantum well, which was observed in [3] and the structures, which appear in emission from quantum well under the window in metallic electrode [4]. The theory describes correctly the experiments: the threshold of pumping, mean values of radius of islands and the distance between them, the transformation of fragmented structure into continuous ring with the rise of pumping and temperature.

3. Application of model of spinodal decomposition. The structure of exciton density distribution in quantum well under a slot in electrode

In model of spinodal decomposition (Cahn- Hillert model) the system is described by the distribution of the density in the space. At high density a uniform distribution of density becomes unstable. The model of spinodal decomposition is more preferable in comparison with nucleation model in the case if the density changes slowly in space.

We shall deduce an equation for an exciton density distribution using the exciton conservation law and phenomenological expressions of non-equilibrium thermodynamics taking into account the finite value of exciton lifetime and presence of pumping. The conservation law gives the following equation

$$\frac{\partial c}{\partial t} = -\nabla j + \frac{G(\bar{r})}{\tau_{ex}}$$  \hspace{1cm} (7)

where $G(\bar{r}) = G(y) = G$ at $-b < y < b$ and $G(\bar{r}) = G(y) = G$ at $y < -b$, $y > b$. $j = -M\nabla \mu$ is the exciton current, $\mu$ is the chemical potential, $M = Dc/(kT)$ is the exciton mobility. The chemical potential may be obtained knowing the free energy by using the equation $\mu = \delta F / \delta c$. The free energy will be chosen in the form suggested by the Landau model:

$$F(c) = \int d\vec{r} \left( K (\nabla c)^2 + f(c) + cV \right).$$  \hspace{1cm} (8)

The additional energy, the excitons acquire in the non-uniform potential, is taken into account by the term $cV$. We expand the $f$ in series up to the forth power

$$f(c) = \kappa T c (\ln c - 1) + \frac{a}{2} c^2 + \frac{b_f}{3} c^3 + \frac{c_f}{4} c^4,$$  \hspace{1cm} (9)

where $a, b_f$ and $c_f$ are phenomenological parameters.

The first term in equation (8) gives the typical expression $D\Delta c$ in the equation for $c$ in the case of small exciton density. For existence of minimum of free energy and blue shift of exciton spectra at small density the following condition should hold: $a > 0, c_f > 0, b_f < 0$. We introduce the dimensionless unities: $\tilde{G} = G(c_f, \kappa T / (Da^3))$, $\tilde{V} = Vc_f^{1/2} / a^{3/2}$, $\tilde{D} = (\kappa Tc_f^{1/2}) / a^{3/2}$. $\tilde{b} = b_f / \sqrt{ac_f}$, $\tilde{t} = t(Da^{5/2}) / (\kappa Tc_f^{1/2})$. Later on we shall omit the symbol $\sim$.

In dimensionless units the equation for exciton density (1) may be rewritten in the following form

$$\frac{\partial \tilde{c}}{\partial \tilde{t}} = D\Delta \tilde{c} + \text{div}(\tilde{c}\tilde{V}(-\Delta \tilde{c} + c + b_f c^2 + c_f c^3 + V)) + \tilde{G} - c / \tau_{ex}.$$  \hspace{1cm} (10)

The equation (10) includes the finite value of exciton lifetime. For infinite system the exciton condensed phase was investigated in [16-18] taking into account fluctuations and the exciton lifetime. The application of the equation (10) to experiments [3] and [4] was given in [10,12]. Qualitative results, obtained in model of spinodal decomposition and in model of nucleation, coincide. In presented paper we shall study the structure of exciton condensed phase in new system.
Let a double quantum well in a semiconductor is sandwiched between two electrodes, the top electrode contains a slot with width $2b$ (Figure 3).

![Figure 3](image.png)

**Figure 3.** The arrangement of quantum wells and electrodes in semiconductors in the case of a slot in electrode: a) side view, b) view from above.

The slot is directed along the $X$ axis and the axis $Z$ is directed along the normal to electrodes. In an electric field applied to the electrodes the energy of indirect excitons acquires the additional energy $V = -p_z E_z$, where $p_z$ is the exciton dipole moment, which in strong electric field is directed along the $Z$ axis. We have used the solution presented in [19] about the field created by the grounded metallic plate that has a slot and is located in the external uniform electric field. This solution is correct for our problem under the conditions: 1) $b \ll L$, where $L$ is the distance between electrodes, 2) the upper electrode (the electrode with the slot) is internal and is located inside the semiconductor medium. In such approximations the additional potential energy created by the presence of the slot in electrode may be presented in the following form

$$V_0 = V_0 \left( \frac{1}{2} \left( \frac{1 + b^2/\xi^2(y,z)}{\sqrt{\xi^2(y,z) + b^2}} - 1 \right) - \frac{b^2 \xi^2}{\sqrt{\xi^2(y,z) + b^2}} \left( 1 + \frac{y^2 + z^2 + b^2}{\sqrt{(y^2 + z^2 - b^2)^2 + 4b^2 z^2}} \right) \right), \quad (11)$$

$$\xi(y,z) = \frac{1}{2} \left( y^2 + z^2 + b^2 + \sqrt{(y^2 + z^2 - b^2)^2 + 4b^2 z^2} \right), \quad (12)$$

$V_0 = -p_z E_0$ is the shift of exciton band caused by electric field far from the slot. The coordinate $z$ determines the distance of the quantum well from the upper electrode.

The nonlinear equation (10) was solved numerically. The obtained results are following. At small value of $b$ and low intensity irradiation the exciton density has maximum in the center. With increasing the pumping the uniform distribution of exciton density along the slot becomes unstable and periodical structure arises (Figure 4). The threshold value, at which the periodical structure appears, increases with the decreasing the width of the slot. At further rise of pumping the periodical structure transforms into continues along the slot distribution. With increasing the width of the slot two parallel chains of islands localized at the opposite sides of the slot arise. (Figure 5). The positions of islands in the chains are shifted on half of a chain period with respect one to another.

4. Conclusion

In the paper the structures, which arise in distribution of the exciton density in double quantum well under steady-state non-homogeneous irradiation, are studied. In considered system of unstable particles in the region of a coexistence of condensed and gas phases the equilibrium between phases is not formed. As a result the exciton lifetime is important parameter that determines the type of the arising structures. The studied structures are example of self-organization phenomena in non-equilibrium systems.
The presented explanations of different experiments do not require the Bose-Einstein condensation of excitons, nevertheless the Bose statistic may play important role in the formation of parameters of the condensed phase.

5. References
[1] Fukuzava T, Mendes E E, Hong J M 1990 Phys. Rev. Lett. 64 3066
[2] Dremin A A, Timofeev V B, Larionov A V, Hvam J and Soerensen K 2002 JETP Letters 76 526
[3] Butov L V, Gossard A C and Chemla D S 2002 Nature 418 751
[4] Gorbunov AV, Timofeev V B 2006 JETP Letters 83 146
[5] Levitov L S, Simons BD and Butov L V 2005 Phys. Rev. Lett. 94 176404
[6] Liu C S, Luo H G and Wu W C 2006 J. Phys. Condens. Matter. 18 9659
[7] Paraskerov A, Khabarova T V 2007 Phys. Lett. A 368 151
[8] Sugakov V I 2005 Solid State Commun. 134 63
[9] Sugakov V I 2006 Solid State Phys. 48 1984
[10] Chernyuk A A, Sugakov V I 2006 Phys. Rev B 74 085303
[11] Sugakov V I 2007 Phys. Rev. B 76 115303
[12] Sugakov V I, Chernyuk A A 2007 JETP letters 85 570
[13] Lozovik Yu E, Berman O L 1996 Pisma v Zh. Eksp. Teor. Fiz. 64 526
[14] Tan M Y J, Drummond N D, and Needs R J 2005 Phys. Rev. B 71 033303
[15] Schindler Ch, Zimmerman R 2008 Phys. Rev. B 78 045313
[16] Sugakov V I 1998 Solid State Commun. 106 705
[17] Ishikawa A, Ogawa T, Sugakov V 2001 Phys. Rev. B 64 144301
[18] Ishikawa A, Ogawa T 2002 Phys. Rev. E 65, 026131
[19] Landau L D and Lifshits E M 1984 Electrodynamics of Continuous Media v. 8 (Oxford Pergamon)