Stress–strain state in an elastoplastic pipe taking into account the temperature and compressibility of the material

K K Gornostaev, A V Kovalev, Y V Malygina
Voronezh State University, 1 Universitetskaya pl., Voronezh, 394018, Russia
E-mail: ymkahavren@gmail.com

Abstract. In the article the authors have considered the problem of determining the stress–strain state of the elastoplastic pipe with the Mises' condition in case of plane strain for the compressible material taking into account the temperature. The task was solved using the method of the small parameter. The expressions for the fields of stresses and displacements were received as well as the ratio of the radius of the elastoplastic boundary in the zero and first approximations.

1. Introduction

Many of the works are devoted to the problem of determining the stress strain–state in the compressible elastoplastic bodies, among which there are some works like [1–5]. In the works [1], [2] it was presented an approach that allows to get the solution of the problem of determining the stress–strain state for the case of the plane strain material compressible by elastic deformation (the case of small elastic compressibility). An algorithm for solving problems of determining the stress–strain state for the case of plane strain and plane stress state, which allows to remove the assumption of a low elastic compressibility of the material was proposed in works [3], [4]. The plasticity condition is piecewise linear. The work [5] considers the effect of isotropic hardening and compressibility on the solution of plane elastoplastic problems. Also there was shown the strong influence of elastic compressibility of the material and the small effect of hardening on the stress–strain state.

The accounting of the temperature effects in the solution of these tasks leads to a significant complication of the mathematical calculations. The research of this question is the subject of many works such as [6–8]. The stress–strain state in the viscoelastoplastic hardening steel with respect to temperature is determined in work [6]. The material of the pipe was considered incompressible. The problems of determining the stress–strain state in an elastic space weakened by cylindrical cavity with respect to temperature and elastoplastic pipe with account for compressibility of the material were solved using perturbation method in [7, 8].

Many of the tasks that today’s mathematicians, physicists, and engineers face are not amenable to exact solution. Among the reasons that hinder to search for exact solutions, it is possible to specify, for example, the nonlinear equations of motion, variable coefficients and nonlinear boundary conditions at known or unknown boundaries of complex form. In this situation, the researcher is forced to use various kinds of approximations and here it is the most
appropriate to use approximate analytical approaches. One of such approaches is the method of the small parameter, or the perturbation method that allows you to find the solution close to the already known exact one. At the same time both the body shape and boundary conditions [12] can be subjected to the perturbation.

In this article, we consider the problem of determining the stress–strain state of the elastic–plastic pipe including the temperature and compressibility of the material. In the first approximation, the stress and displacement fields are defined in the elastic and plastic zones, and also a relation for the radius of the elastoplastic boundary is found.

2. Materials and methods

The solution of our problem will be sought by the method of a small parameter [2], [12]. It was first used to solve the practical problems of mechanics in the works of Poincare [13], Van–Dyck [14] and Nayfe [15, 16]. This method is based on introduction of small quantities that more or less ”disturbing” certain original solution in comparison some data. Due to the fact that the small quantities are used as ”disturbing” ones, in many works the perturbation method is called the small parameter method. The small parameter in the theory of plasticity was introduced in different ways. In [2] a small parameter characterizes the difference between a planar and an axisymmetric state. In [17], as a small parameter, it’s used the reverse of the volume compression modulus, also the normal and tangential stresses were investigated for pure bending of the girder beyond the elastic limit. For the solution of elastoplastic problems by the method of small parameter, uses various schemes. D D Ivlev and L V Ershov [2] considered the case when a plastic zone develops from a certain boundary and completely covers it. In this approach, a number of two–dimensional and three–dimensional problems were solved [18–22]. B D Annin and G P Cherepanov [23] gave a solution to the problem of comprehensively compressing a plane with a hole. In this case, in contrast to the Ivlev–Ershov scheme, the solution in the elastic zone was determined by the methods of a function of the complex variable. It was shown that for the plate with elliptic hole the proposed scheme and the Ivlev–Ershov scheme lead to the same result. M A Artemov received [24–27] a number of approximate solutions based on the Ivlev–Ershov scheme for the problems of the tension plane of hardening elastic–plastic material with a circular hole as well as about the eccentric pipe exposed to internal pressure. Also, a large number of elastic–plastic tasks considered in the works of A N Sporykhin and his disciples [28–38]. The modeling of the thermoelastic behavior of solids was considered in [39–48]

Assume that the desired solution depends on some parameter \( \delta (1 \ll \delta) \). Linearization of the small parameter is the decomposition of all the source relations: the equations of equilibrium, equations of the relationship of strain and stress, boundary conditions, etc. in the ranks in this parameter. Then members of the decomposition are discriminated with the same degrees of this parameter, which define the system of equations, which allows to develop the method of successive approximations, if the solution when \( \delta = 0 \) is known [12].

Consider the elastic–plastic state of thick–walled round pipe, the cross section of which is limited by circles of radiuses \( a \) and \( b (a < b) \) of compressible material under the action of uniform internal pressure \( p \). The temperature of the pipe \( T = T(r) \) is the known solution of the heat equation, and modified by logarithmic law [10, 11], the yield strength is temperature independent. The problem can be solved in a cylindrical coordinate system \((r, \theta, z)\).

It used the following system of equations:

– the equation of equilibrium

\[
\frac{\partial \sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r},
\]

where \( \sigma_r, \sigma_\theta \) — the components of the stress tensor;
- the ratio of Cauchy
\[ e_r = \frac{du}{dr}, e_\theta = \frac{u}{r}, \] (2)

where \( e_r, e_\theta \) — the components of the tensor of total deformation, \( u \) — component of radial movement;

- the ratio of Hooke’s law with respect to temperature connecting stress and elastic strain
\[ e_r^e = \frac{1}{E} [\sigma_r - \mu(\sigma_\theta + \sigma_z)] + (\alpha T), \]
\[ e_\theta^e = \frac{1}{E} [\sigma_\theta - \mu(\sigma_r + \sigma_z)] + (\alpha T), \]
\[ e_z^e = \frac{1}{E} [\sigma_z - \mu(\sigma_r + \sigma_\theta)] + (\alpha T), \]
\[ e_{r\theta}^e = \frac{1 + \mu}{E} \tau_{r\theta}, \] (3)

where \( e_z, e_{r\theta} \) — the components of the tensor of total deformation, \( E \) — elastic modulus, \( T = T(r) \) — temperature that is a known solution of the heat equation (see, for example, [10, 11]), \( \alpha \) — coefficient of linear thermal expansion, \( \mu \) — Poisson’s ratio, upper index \( e \) — indicates belonging to the elastic region;

- the condition of plasticity
\[ (\sigma_\theta - \sigma_r)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2 + 6\tau_{r\theta}^2 = 6k^2, \] (4)

where \( \sigma_z, \tau_{r\theta} \) — the components of the stress tensor, \( k = k(T) \) — yield stress [10];

- the ratio of the associative law of plastic flow
\[ de_r^p = \frac{d\lambda}{3} (2\sigma_r - \sigma_\theta - \sigma_z), \]
\[ de_\theta^p = \frac{d\lambda}{3} (2\sigma_\theta - \sigma_r - \sigma_z), \]
\[ de_z^p = \frac{d\lambda}{3} (2\sigma_z - \sigma_r - \sigma_\theta), \]
\[ de_{r\theta}^p = d\lambda \tau_{r\theta}, \] (5)

where \( d\lambda \) — positive scalar multiplier, upper index \( p \) — indicates belonging to the plastic region;

- boundary conditions
\[ \sigma_r|_{r=a} = -p, \sigma_r|_{r=b} = 0, \] (6)

- the continuity conditions of stresses and displacements for an elasto–plastic boundary
\[ [\sigma_r] = [\sigma_\theta] = [u] = 0. \] (7)

Square brackets here and below denote the difference between the values of expressions enclosed in parentheses, corresponding to the elastic and plastic regions.

Full deformations of the body in the plastic zone consist of two parts: elastic and plastic
\[ e_r = e_r^e + e_r^p, e_\theta = e_\theta^e + e_\theta^p, e_z = e_z^e + e_z^p, e_{r\theta} = e_{r\theta}^e + e_{r\theta}^p. \] (8)

The elastic deformations are connected with the stress by Hooke’s law with respect to temperature, and the increment of plastic deformation – with associated law of plastic flow. In the case of plane strain \( e_z = 0 \). Then from (3), (4), (5) and (8) we get:
\[
\begin{align*}
\text{de}_r &= \frac{1}{E} \left[ d\sigma_r - \mu (d\sigma_\theta + d\sigma_z) \right] + d(\alpha T) + \frac{d\lambda}{3} (2\sigma_r - \sigma_\theta - \sigma_z), \\
\text{de}_\theta &= \frac{1}{E} \left[ d\sigma_\theta - \mu (d\sigma_r + d\sigma_z) \right] + d(\alpha T) + \frac{d\lambda}{3} (2\sigma_\theta - \sigma_r - \sigma_z), \\
0 &= \frac{1}{E} \left[ d\sigma_z - \mu (d\sigma_r + d\sigma_\theta) \right] + d(\alpha T) + \frac{d\lambda}{3} (2\sigma_z - \sigma_r - \sigma_\theta), \\
\text{de}_{r\theta} &= 1 + \mu \frac{d\tau_{r\theta}}{E} + d\lambda \tau_{r\theta}.
\end{align*}
\] (9)

Imagine the ratio in the form of series in the parameter \( \delta \) according to the method of small parameter. In this work, we restrict ourselves to the first two members. Thus, the solution will be sought in the form:

\[
\begin{align*}
e_{ij} &= e_{ij}^{(0)} + \delta e_{ij}^{(1)}, \\
\sigma_{ij} &= \sigma_{ij}^{(0)} + \delta \sigma_{ij}^{(1)}, \\
T &= T^{(0)} + \delta T^{(1)}, \\
\lambda &= \lambda^{(0)} + \delta \lambda^{(1)}, \\
\alpha &= \alpha_0 + \delta \alpha^{(1)}, \\
\mu &= \mu_0 + \delta \mu^{(1)},
\end{align*}
\] (10)

where \( \delta \) — small parameter, \( \mu_0 = 0.5, \mu^{(1)} \) — known constant, upper index (0), (1) specifies the zero and first approximations, respectively.

Substituting these expansions into the equations (1), (2), (3), (4) and (7) and equating expressions at identical degrees \( \delta \), in each approximation, we get the system of differential equations.

There is a flat deformation of elasto–plastic incompressible material in a zero approximation, and, therefore, it will be just the condition of incompressibility

\[
e_r + e_\theta = 0.
\] (11)

The solution of this problem has the form [2]

\[
\begin{align*}
\sigma_r^{(p)}(0) &= -p + 2k \ln \left( \frac{r}{a} \right), \\
\sigma_\theta^{(p)}(0) &= -p + 2k \ln \left( \frac{r}{a} \right) + 2k, \\
\sigma_z^{(p)}(0) &= -p + 2k \ln \left( \frac{r}{a} \right) + k,
\end{align*}
\] (12)

\[
\begin{align*}
u^{(p)}(0) &= v^{(e)}(0) = \frac{3kr_s^{(0)^2}}{2Er}, \\
\sigma_r^{(e)}(0) &= kr_s^{(0)^2} \left[ \frac{1}{b^2} - \frac{1}{r^2} \right], \\
\sigma_\theta^{(e)}(0) &= kr_s^{(0)^2} \left[ \frac{1}{b^2} + \frac{1}{r^2} \right], \\
\sigma_z^{(e)}(0) &= \frac{kr_s^{(0)^2}}{b^2}.
\end{align*}
\] (13)

Consider first approximation.

The condition of plasticity (3) at the first approximation will be

\[
2(\sigma_\theta^{(0)} - \sigma_r^{(0)})(\sigma_\theta^{(1)} - \sigma_r^{(1)}) + 2(\sigma_\theta^{(0)} - \sigma_z^{(0)})(\sigma_\theta^{(1)} - \sigma_z^{(1)}) + 2(\sigma_r^{(0)} - \sigma_z^{(0)})(\sigma_r^{(1)} - \sigma_z^{(1)}) = 0. \] (17)
The relations (7) in the first approximation will take the form of

\[ de^{(1)} = \frac{1}{E} [d\sigma_{r}^{(1)} - \mu_0 (d\sigma_{\theta}^{(1)} + d\sigma_{z}^{(1)}) - \mu^{(1)} (d\sigma_{r}^{(0)} + d\sigma_{z}^{(0)})] + \\
+ d(\alpha_0 T^{(1)}) + d(\alpha^{1} T^{(0)}) + \frac{d\lambda^{(0)}}{3} (2\sigma_{r}^{(1)} - \sigma_{\theta}^{(1)} - \sigma_{z}^{(1)}) + \frac{d\lambda^{(1)}}{3} (2\sigma_{r}^{(0)} - \sigma_{\theta}^{(0)} - \sigma_{z}^{(0)}), \]

\[ de^{(1)} = \frac{1}{E} [d\sigma_{\theta}^{(1)} - \mu_0 (d\sigma_{r}^{(1)} + d\sigma_{z}^{(1)}) - \mu^{(1)} (d\sigma_{r}^{(0)} + d\sigma_{z}^{(0)})] + \\
+ d(\alpha_0 T^{(1)}) + d(\alpha^{1} T^{(0)}) + \frac{d\lambda^{(0)}}{3} (2\sigma_{\theta}^{(1)} - \sigma_{r}^{(1)} - \sigma_{z}^{(1)}) + \frac{d\lambda^{(1)}}{3} (2\sigma_{\theta}^{(0)} - \sigma_{r}^{(0)} - \sigma_{z}^{(0)}), \]

\[ d\alpha_0 T^{(1)} + d(\alpha^{1} T^{(0)}) + \frac{2d\lambda^{(0)}}{3} (2\sigma_{z}^{(1)} - \sigma_{r}^{(1)} - \sigma_{\theta}^{(1)}) + \frac{d\lambda^{(1)}}{3} (2\sigma_{z}^{(0)} - \sigma_{r}^{(0)} - \sigma_{\theta}^{(0)}). \]

The relations (7) in the first approximation will take the form of

\[ \sigma_{r}^{(1)}|_{r=a} = 0, \sigma_{r}^{(1)}|_{r=b} = 0. \]  

(19)

The relations of the equations of continuity of stresses and displacements on the boundary (5) in the first approximation have the form [2]

\[ \sigma^{(1)} \left[ \frac{d\sigma_{z}^{(0)}}{dr} \right]_{r=s} + \sigma^{(1)} \left[ \frac{d\sigma_{r}^{(0)}}{dr} \right]_{r=s} = u^{(1)} + \frac{du^{(0)}}{dr} - r^{(1)} = 0. \]

(20)

Converting the third equation of (16), we obtain

\[ \frac{1}{2} d(2\sigma_{z}^{(1)} - \sigma_{r}^{(1)} - \sigma_{\theta}^{(1)}) + \frac{E}{3} d\lambda^{(0)} (2\sigma_{z}^{(1)} - \sigma_{r}^{(1)} - \sigma_{\theta}^{(1)}) = \mu^{(1)} (d\sigma_{r}^{(0)} + d\sigma_{\theta}^{(0)}) - \alpha^{(1)} EdT^{(0)}. \]

(21)

Let’s introduce the designation \( \chi^{(1)} = 2\sigma_{z}^{(1)} - \sigma_{r}^{(1)} - \sigma_{\theta}^{(1)} \). Then the previous equation can be represented in the form

\[ \frac{d\chi^{(1)}}{d\lambda^{(0)}} + \frac{2E}{3} \chi^{(1)} = 4\mu^{(1)} \frac{d\sigma_{z}^{(0)}}{d\lambda^{(0)}} - 2\alpha^{(1)} E \frac{dT^{(0)}}{d\lambda^{(0)}}. \]

(22)

After solving the equation (20), we obtain

\[ \chi^{(1)} = \exp \left( -\frac{2}{3} E \lambda^{(0)} \right) \int \left[ 4\mu^{(1)} \frac{d\sigma_{z}^{(0)}}{d\lambda^{(0)}} - 2\alpha^{(1)} E \frac{dT^{(0)}}{d\lambda^{(0)}} \right] \exp \left( \frac{2}{3} E \lambda^{(0)} \right) d\lambda^{(0)} + \chi^{(1)} e^{(1)}, \]

(23)

\[ \chi^{(1)} e^{(1)} = 4\mu^{(1)} \sigma_{z}^{(0)} - 2\alpha^{(1)} E T^{(0)}. \]

According to the introduced notation, we can determine the component voltage \( \sigma_{z}^{(1)} \)

\[ \sigma_{z}^{(1)} = \frac{1}{2} (\sigma_{r}^{(1)} + \sigma_{\theta}^{(1)} + \chi^{(1)}). \]

(24)

From relation (16), taking into consideration the solution in the zero approximation and the expression (22), we define \( e_{r}^{(1)} \) and \( e_{\theta}^{(1)} \) in the elastic zone

\[ e_{r}^{(1)} + e_{\theta}^{(1)} = -\frac{6\mu^{(1)}}{E} \sigma_{z}^{(0)} + 3\alpha^{(1)} T^{(0)}, \]

(25)
\( e_r^{(1)} - e_\theta^{(1)} = \frac{1}{2E} \left( 3(\sigma_r^{(1)} - \sigma_\theta^{(1)}) + 2\mu^{(1)}(\sigma_r^{(0)} - \sigma_\theta^{(0)}) \right). \) \hspace{1cm} (26)

In the plastic zone on the basis of plasticity condition (15) given (21), (22) and known zero approximation, we get
\( \sigma_\theta^{(1)} - \sigma_r^{(1)} = 0. \) \hspace{1cm} (27)

Solving these equations, we determine expressions for the fields of stresses and displacements in the first approximation in the form
\( \sigma_r^{(p)(1)} = 0, \sigma_\theta^{(p)(1)} = 0, \) \hspace{1cm} (28)
\( \sigma_z^{(p)(1)} = \frac{1}{2} \exp \left( 1 - \frac{r_s^{(0)} r_s^{(0)}}{r^2} \right) \left[ \int (4\mu^{(1)} + 2\alpha^{(1)} EQ) \exp \left( \frac{r_s^{(0)} r_s^{(0)}}{r^2} - 1 \right) \frac{dr}{r} + \chi_e^{(1)} \right], \) \hspace{1cm} (29)

where
\( \chi_e^{(1)} = 4\mu^{(1)} \left( -p + 2kln \left( \frac{r}{a} \right) + k \right) - 2\alpha^{(1)} EP + 2\alpha^{(1)} EQ ln(r). \) \hspace{1cm} (30)
\( u^{(p)(1)} = \frac{k}{E\mu^{(1)}} \left[ -3 \frac{r_s^{(0)} r_s^{(0)}}{b^2} + 6k ln \left( \frac{r}{r_s^{(0)}} \right) + 3r - 2 \frac{r_s^{(0)} r_s^{(0)}}{r} \right] + \) \hspace{1cm} (31)
\( + \delta\alpha^{(1)} \left[ \frac{3}{2} Pr - 3 \frac{3}{2} Qr ln(r) + \frac{3}{4} Qr - 3 \frac{1}{2} Q \frac{r_s^{(0)} r_s^{(0)}}{r_s^{(0)} r_s^{(0)} - b^2} ln \left( \frac{r_s^{(0)}}{b} \right) \right], \)
\( \sigma_r^{(e)(1)} = \alpha^{(1)} EQ \left[ ln \left( \frac{r}{b} \right) - \frac{r_s^{(0)} r_s^{(0)}}{r_s^{(0)} r_s^{(0)} - b^2} ln \left( \frac{r_s^{(0)}}{b} \right) \right], \) \hspace{1cm} (32)
\( \sigma_\theta^{(e)(1)} = \alpha^{(1)} EQ \left[ ln \left( \frac{r}{b} \right) - \frac{r_s^{(0)} r_s^{(0)}}{r_s^{(0)} r_s^{(0)} - b^2} ln \left( \frac{r_s^{(0)}}{b} \right) + 1 \right], \) \hspace{1cm} (33)
\( \sigma_z^{(e)(1)} = \alpha^{(1)} EQ \left[ ln \left( \frac{r}{b} \right) + \frac{1}{2} - \frac{1}{2} \frac{r_s^{(0)} r_s^{(0)}}{r_s^{(0)} r_s^{(0)} - b^2} ln \left( \frac{r_s^{(0)}}{b} \right) \left( 2 - \frac{b^2}{r^2} \right) \right] + \frac{1}{2} \chi_e^{(1)}, \) \hspace{1cm} (34)
\( u^{(e)(1)} = \frac{k}{E\mu^{(1)}} \left[ -3 \frac{r_s^{(0)} r_s^{(0)}}{b^2} + \frac{r_s^{(0)} r_s^{(0)}}{r} \right] + \) \hspace{1cm} (35)
\( + \delta\alpha^{(1)} \left[ \frac{3}{2} Pr - \frac{3}{2} Qr ln(r) + \frac{3}{4} Qr - \frac{3}{2} Q \frac{r_s^{(0)} r_s^{(0)}}{r_s^{(0)} r_s^{(0)} - b^2} ln \left( \frac{r_s^{(0)}}{b} \right) \right]. \)
3. Results and discussion
Thus, having the necessary expressions in the zero and first approximations, and using the decomposition (8), we can obtain the ratio for the fields of stresses and displacements

\[
\sigma_r^{(p)} = -p + 2k \ln \left( \frac{r}{a} \right), \tag{36}
\]

\[
\sigma_a^{(p)} = -p + 2k \ln \left( \frac{r}{a} \right) + 2k, \tag{37}
\]

\[
\sigma_z^{(p)} = -p + 2k \ln \left( \frac{r}{a} \right) + k + \frac{1}{2} \exp \left( 1 - \frac{r_s(0)^2}{r^2} \right) \left[ \int (4\mu(1) + 2\alpha(1)EQ) \exp \left( \frac{r_s(0)^2}{r^2} - 1 \right) \frac{dr}{r} + \chi(1) \right], \tag{38}
\]

\[
u^{(p)} = \frac{3kr_s(0)^2}{2Er} + \frac{k}{E} \mu(1) \left[ -\frac{3}{2} \frac{r_s(0)^2}{b^2} r - 6r \ln \left( \frac{r}{r_s} \right) + 3r - 2 \frac{r_s(0)^2}{r} \right] + \frac{3}{2} Pr - \frac{3}{2} Q r \ln(r) + \frac{3}{4} \frac{1}{r} \frac{r_s(0)^2}{b^2} - 2 \frac{b}{r^2} \ln \left( \frac{r_s}{b} \right) \tag{39}
\]

\[
\sigma_r^{(e)} = kr_s(0)^2 \left[ \frac{1}{b^2} - \frac{1}{r^2} \right] + \alpha(1)EQ \left[ \ln \left( \frac{r}{b} \right) - \frac{r_s(0)^2}{r^2} - \frac{b^2}{r^2} \ln \left( \frac{r_s}{b} \right) + 1 \right], \tag{40}
\]

\[
\sigma_a^{(e)} = kr_s(0)^2 \left[ \frac{1}{b^2} + \frac{1}{r^2} \right] + \alpha(1)EQ \left[ \ln \left( \frac{r}{b} \right) - \frac{r_s(0)^2}{r^2} - \frac{b^2}{r^2} \ln \left( \frac{r_s}{b} \right) + 1 \right], \tag{41}
\]

\[
\sigma_z^{(e)} = \frac{kr_s(0)^2}{b^2} + \alpha(1)EQ \left[ \ln \left( \frac{r}{b} \right) + \frac{1}{2} - \frac{1}{2} \frac{r_s(0)^2}{b^2} - b^2 \ln \left( \frac{r_s}{b} \right) \right] + \frac{1}{2} \chi(1), \tag{42}
\]

\[
u^{(e)} = \frac{3kr_s(0)^2}{2Er} + \frac{k}{E} \mu(1) \left[ -\frac{3}{2} \frac{r_s(0)^2}{b^2} r + \frac{r_s(0)^2}{r} \right] + \frac{3}{2} Pr - \frac{3}{2} Q r \ln(r) + \frac{3}{4} \frac{1}{r} \frac{r_s(0)^2}{b^2} - 2 \frac{b}{r^2} \ln \left( \frac{r_s}{b} \right) \tag{43}
\]

The ratio for determination of the radius \( r_s \) of elastic–plastic border has the form

\[
r_s = r_s(0) + \frac{\alpha(1)EQ}{4k} \left[ r_s(0) \ln \left( \frac{r_s(0)}{b} \right) + r_s(0) - r_s(0) \frac{r_s(0)^2}{r_s(0)^2 - b^2} \ln \left( \frac{r_s(0)}{a} \right) \right], \tag{44}
\]

where \( r_s(0) \) is determined from the equation

\[
r_s(0)^2 = b^2 + 2b^2 \ln \left( \frac{r_s(0)}{a} \right) - p. \tag{45}
\]
Let’s assume the following values of constant [10]: $E = 2.1 \cdot 10^6$ H; $\alpha = 0.000011/\degree$C; $a = 1$ cm; $b = 2$ cm; $T = 100 - 115 \ln (r) \degree$C; $k = 7 \cdot 10^4$ H.

In figure 1–3 are graphs of the component of stresses and radial displacements depending on the radius $r$ for different values of the elasto–plastic boundary, given the adopted values of constants. In figure 1 by the solid line it is plotted stress component $\sigma_r$ of radius $r$ when the radius of elastic–plastic boundary $r_s = 1,024$ cm. The dotted line is plotted stress component $\sigma_r$ of radius $r$ when the radius of elastic–plastic boundary $r_s = 1,2$ cm. Line with marker shows stress component $\sigma_r$ of radius $r$ when the radius of elastic–plastic border $r_s = 1,3$ cm. In figure 2 by the solid line it is plotted stress component $\sigma_\theta$ of radius $r$ when the radius of elastic–plastic boundary $r_s = 1,024$ cm. The dotted line is plotted stress component $\sigma_\theta$ of radius $r$ when the radius of elastic–plastic boundary $r_s = 1,2$ cm. Line with marker shows stress component $\sigma_\theta$ of radius $r$ when the radius of elastic–plastic border $r_s = 1,3$ cm. In figure 3 by the solid line it is plotted radial displacements $u$ of radius $r$ when the radius of elastic–plastic boundary $r_s = 1,2$ cm. The dotted line is plotted radial displacements $u$ of radius $r$ when the radius of elastic–plastic boundary $r_s = 1,3$ cm. Line with marker shows radial displacements $u$ of radius $r$ when the radius of elastic–plastic border $r_s = 1,3$ cm.

Comparison of the results obtained solving this problem with results of works [1, 6], shows good agreement between these results.

4. Conclusion
The article presents the solution of the problem of determining the stress–strain state of an elasto–plastic pipe, taking into consideration temperature and compressibility of the material.
In the first approximation, stress and displacement fields are defined in the elastic and plastic regions, and a ratio for the radius of the elasto–plastic boundary is found. The plots of the stress component $\sigma_r$ and $\sigma_\theta$ and radial displacements $u$ of radius $r$ are presented.

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