DOA-estimation methods with superresolution comparison depending on directivity factor of directive elements of conformal and planar antenna arrays on azimuth-elevation planes

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Abstract. Direction finding is widely spread and highly researching area. There are lots of methods to estimate spatial positions of radio sources. It is well known that maximum likelihood estimation method gives the highest accuracy and is capable of reaching Cramer-Rao bound. The main task of the paper is to research of direction-of-arrival estimation methods with super resolution such as MUSIC, Capon etc. Also the methods are not researched deeply for 2D (azimuth and elevation planes) scenarios via conformal and planar antenna arrays with directive elements. The root mean square error (RMSE) rate of estimates by the methods with super resolution such as MUSIC, Capon etc. in azimuth-elevation cases are calculated via simulation depending on directivity ratio of elements as well as the probability of resolution and spatial spectrum bias. Additionally, the values are estimated in various noise environments and for various geometries of antenna arrays. Some features of functioning MUSIC direction-finding method for azimuth and elevation estimation as a part of the conformal and planar with directive elements are determined. One of the most dangerous features is false peaks which prevent exact spatial coordinate estimation and the processing speed of each method under consideration.

1 Introduction
Currently, there is a need to develop an adaptive antenna array due to the increasing requirements for wireless telecommunications systems. The use of the adaptive antenna arrays allows reducing the weight and size of any system, increasing the sensitivity of the receiving unit, being able to almost smoothly move the beam, without additional hardware costs simultaneously forming multiple beams in specified directions, as well as real-time control of the amplitude-phase distribution for the spatial selection of signals [1]. The use of technology of independent multichannel signal processing allows significantly increasing the noise immunity and information capacity of the channel by means of the adaptive antenna arrays [2]. Circular and concentric antenna arrays are used to solve this problem in problems, requiring both azimuth and elevation determination of the direction-of-arrival (DOA) of electromagnetic waves [3, 4, 5, 6, 7, 8].

The article discusses the functioning of the radio direction-finding methods in the system of the cylindrical antenna array formed of directive radiators, directional factor which is not equal to 1. The arrays consisting of several rings emitters are studied, such constructions are called cylindrical antenna arrays. Some features of functioning MUSIC, Capon etc. direction-finding methods for azimuth and elevation estimation as a part of the cylindrical antenna arrays are determined.

2 DOA-Estimation Methods
Consider the structure of the circular and the cylindrical antenna arrays consisting of directive radiators (fig. 1). Such antenna arrays as shown in figure 1, are composed of several concentric circular antenna array of radius r and are located on the metal cylinder. As the antenna element can act half-wave dipoles, but now all patch antennas have been used more frequently, which directional factor is greater than 1. In order to implement the
methods and direction-finding algorithms with super resolution it is needed to know exactly the steering vector, calculations which will be carried out with respect to the origin.

Let's suppose that $$s(t)$$ is a complex-valued dimensional signal vector; the output signal vector can be written as: [1]:

$$\mathbf{x}(t) = \mathbf{A} \cdot \mathbf{s}(t) + \mathbf{n}(t),$$

(2)

where $$\mathbf{x}(t)^T$$ is the N-dimensional vector describing output signals of each antenna element, $$\mathbf{s}(t)$$ - M-dimensional signal vector; $$\mathbf{n}(t)$$ is the N-dimensional noise vector of spatial channel and receiver; $$\mathbf{A}$$ is the $$N\times M$$ matrix of steering vectors, m-th column of the matrix describes phase distribution of m-th signal source inside antenna array. Assume that the antenna elements in the circular antenna array are identical and have a maximum radial direction from the center of the array, $$\theta$$ and angle $$\phi$$ place is known:

$$\mathbf{r}_n = \frac{2\pi}{\lambda} \left( \sin \phi \cos \theta, \sin \phi \cos \theta, \cos \phi \right) \cdot (r \cdot \cos(\theta_n), r \cdot \sin(\theta_n), h)^T =$$

$$\frac{2\pi}{\lambda} \left( r \sin \phi \cos \theta \cos \theta_n + r \sin \phi \cos \theta \sin \theta_n + h \cos \phi \right)$$

(1)

Let an antenna array consist of N antenna elements. Let’s assume, that M radio signals arriving on the antenna array from distinct directions $$\left\{\theta_m, \phi_n\right\}_{n=0}^{M-1}$$ . For an arbitrary geometry configuration antenna array a complex output signal vector can be written as: [1]:

$$\mathbf{\bar{s}}(t) = \mathbf{A} \cdot \mathbf{s}(t) + \mathbf{n}(t),$$

(2)

where $$\mathbf{\bar{s}}(t)$$ is a shift in elevation and azimuth planes of n-th antenna element, respectively, then

$$g\left(\theta - \frac{2m}{N}, \phi\right), n = 0,1,\ldots, N-1$$

In this case, the steering vector is defined as [5]:

$$\mathbf{a}(\theta, \phi) = \left[ g(\theta, \phi)e^{j\beta \cos \theta \sin \phi} \ldots g\left(\theta - \frac{2(N-1)}{N}, \phi\right)e^{j\beta \cos \left(\theta - \frac{2(N-1)}{N}\right) \sin \phi} \right]^T$$

(3)

where $$\beta = \frac{2\pi}{\lambda}$$ is the wave number ($$\lambda$$ – wave length), $$g(\cdot)$$ is the amplitude response of the antenna element (i.e., antenna gain) in the direction of ($$\theta, \phi$$).

The following mathematical model of power directional pattern is used in the far zone relatively to the isotropic antenna, assuming that the antennas are perfectly matched and without losses [5]:

$$G(\theta, \phi) = \frac{D}{2\pi(1 + \sin(\phi - \gamma_n^0))^{\frac{1}{2}}(1 + \cos(\theta - \gamma_n^\theta))^\frac{1}{2}}, n = 0,1,\ldots, N-1$$

(4)

where $$\gamma_n^0$$ and $$\gamma_n^\theta$$ are a shift in elevation and azimuth planes of n-th antenna element, respectively, then

$$g = \sqrt{G(\theta, \phi)}, D$$ is directivity factor.

Let's suppose that $$\mathbf{s}(t)$$ and $$\mathbf{n}(t)$$ are stationary random processes, $$\mathbf{n}(t)$$ is Gaussian random process with zero mean and covariance matrix $$\sigma^2 \mathbf{I}$$ ($$\sigma^2$$ is noise variance), and also that the signals are uncorrelated and there is no correlation between noise and signals. Then the spatial correlation matrix in general can be written in the following form:
Thus, for the Capon method, the resolving function:

\[ R = E[\hat{S}(t)x^H(t)] = AS^H + \sigma^2 I, \quad (5) \]

where \( E[...] \) is the math expectation, \((...)^H\) is the Hermitian conjugate, \( S = E[\hat{S}(t)\hat{S}^H(t)] \) is the correlation matrix of signals.

The spatial matrix describes the statistical properties of signals in the receiving channels of the adaptive antenna arrays. In expanded form, the spatial matrix can be represented as follows:

\[
R = \begin{bmatrix}
E[x_1x_1^H] & E[x_1x_2^H] & \cdots & E[x_1x_N^H] \\
E[x_2x_1^H] & E[x_2x_2^H] & \cdots & E[x_2x_N^H] \\
\vdots & \vdots & \ddots & \vdots \\
E[x_Nx_1^H] & E[x_Nx_2^H] & \cdots & E[x_Nx_N^H]
\end{bmatrix}, \quad (6)
\]

Let \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{N-1} \geq \lambda_N \) be eigenvalues of the matrix \( R \). Let also \( E_s = [\tilde{e}_1, \tilde{e}_2, \ldots, \tilde{e}_M] \) be orthonormal eigenvectors corresponding to \( M \) largest eigenvalues \( \Lambda_s = [\lambda_1, \lambda_2, \ldots, \lambda_M] \) and \( E_n = [\tilde{e}_{M+1}, \tilde{e}_{M+2}, \ldots, \tilde{e}_N] \) is the matrix consisting of orthonormal eigenvectors corresponding to \( (N-M) \) smallest eigenvalues \( \Lambda_n = [\lambda_{M+1}, \lambda_{M+2}, \ldots, \lambda_N] \).

Consider the methods of estimating the angles of arrival of signal sources that belong to the class of “superresolving”, which overcome the Rayleigh constraint. Such methods are proposed a large number. Let’s start with the Capon method [9].

The task is set as follows: it is needed to find a weight vector \( \mathbf{w} \), which minimizes the average power output of the antenna array provided that for some angle of arrival \( \theta \) the array transfer coefficient is fixed and equal, for example, to one. Mathematically, we write this problem as follows:

\[ \min_{\mathbf{w}} \sqrt{\mathbf{w}^H \mathbf{R} \mathbf{w}} = \min \mathbb{P}(\mathbf{w}) \quad \text{при} \quad \mathbf{w}^H \mathbf{a}(\theta, \phi) = 1 \quad (7) \]

The gradient of this functional is set to zero and we get the equality, minimizing average output power according to the Capon criterion:

\[
\hat{\mathbf{w}} = \frac{\mathbf{R}^{-1} \hat{\mathbf{a}}(\theta, \phi)}{\hat{\mathbf{a}}^H(\theta, \phi) \mathbf{R}^{-1} \hat{\mathbf{a}}(\theta, \phi)} \quad (8)
\]

Thus, for the Capon method, the resolving function:

\[ P(\theta, \phi) = \hat{\mathbf{a}}^H(\theta, \phi) \mathbf{R}^{-1} \hat{\mathbf{a}}(\theta, \phi)^{-1} \quad (9) \]

Soon after the appearance of the Capon method, the so-called “thermal noise” method was proposed [10]. The idea of the method is based on the properties of an adaptive antenna array, the efficiency criterion of which is the ratio of the signal power to the average total power of external interference and the interior noise at the output.

As a function that provides the angular resolution in the “thermal noise” method, we consider the value:

\[ P(\theta, \phi) = \left[ \bar{\mathbf{a}}^H(\theta, \phi) \mathbf{R}^{-2} \bar{\mathbf{a}}(\theta, \phi) \right]^{-1} \quad (10) \]

Obviously, that the characteristics of the form \( P_n(\theta, \phi) = \left[ \bar{\mathbf{a}}^H(\theta, \phi) \mathbf{R}^{-n} \bar{\mathbf{a}}(\theta, \phi) \right]^{-1} \) with exponent \( n>0 \) have the increased efficiency of angular resolution compared to conventional scanning method.

The generalization of the Capon method showed, that with increasing degree \( n \), the resolution of adaptive AR with output characteristic \( P_n(\theta, \phi) \) monotonously increases, and the resolving function itself \( P_n(\theta, \phi) \) with an unlimited increase in the parameter \( n \) tends to \( \left( \bar{\mathbf{a}}^H(\theta, \phi) \mathbf{D} \bar{\mathbf{a}}(\theta, \phi) \right)^{-1} \), where \( \mathbf{D} = \left( I - \sum_{m=1}^{M} \tilde{e}_m \tilde{e}_m^H \right) \) is the projection matrix on noise subspace. This method can be called projection.

Directions to signal sources are identified according to the projection method MUSIC with function peaks:

\[ P_{\text{MUSIC}}(\theta, \phi) = \left[ \bar{\mathbf{a}}^H(\theta, \phi) \sum_{n=M+1}^{N} \tilde{e}_n \tilde{e}_n^H \bar{\mathbf{a}}(\theta, \phi) \right]^{-1} \quad (11) \]
If the current angle \((\theta, \varphi)\) of the vector \(\vec{a}(\theta, \varphi)\) coincides with the direction to any signal source \((\theta_1, \varphi_1), (\theta_2, \varphi_2), \ldots, (\theta_n, \varphi_n)\), then this vector will belong to the signal subspace, and its projection on the noise subspace will be equal to zero. Therefore, at this point the function \(P_{MUSIC}(\theta, \varphi)\) has a type feature \(0^{-1}\). By using the function peaks \(P_{MUSIC}(\theta, \varphi)\) the angular position are determined on the signal sources.

**Study**

![Figure 2. Schematic representation of the cylindrical antenna array.](image)

The researching of the direction finding methods with superresolution is fulfilled as part of the cylindrical antenna array (figure 2), depending on the directivity of the antenna elements. The range of the directive factor wipes 1 (omnidirectional) to 6.

As compared methods of continuous superresolving spectral analysis were taken: The Capon method [9], the “thermal noise” method [10], method MUSIC and the Borjotti-Lagunas method [11], the output functions are listed in the table 1.

| Method name         | Output function                                                                 |
|---------------------|---------------------------------------------------------------------------------|
| Capon               | \(P = \left[\vec{a}^H(\theta, \varphi) \cdot \mathbf{R}^{-1} \cdot \vec{a}(\theta, \varphi)\right]^{-1}\) |
| Thermal noise       | \(P = \left[\vec{a}^H(\theta, \varphi) \cdot \mathbf{R}^{-2} \cdot \vec{a}(\theta, \varphi)\right]^{-1}\) |
| Borjotti-Lagunas    | \(P = \frac{\vec{a}^H(\theta, \varphi) \cdot \mathbf{R}^{-1} \cdot \vec{a}(\theta, \varphi)}{\vec{a}^H(\theta, \varphi) \cdot \mathbf{R}^{-2} \cdot \vec{a}(\theta, \varphi)}\) |
| MUSIC               | \(P = \left[\vec{a}^H(\theta, \varphi) \cdot \mathbf{E}_n \mathbf{E}_n^H \cdot \vec{a}(\theta, \varphi)\right]^{-1}\) |

Here we present some results of numerical simulation to illustrate the effectiveness of antenna arrays under study for the radio direction-finding task with super-resolution using the method from Table 1. All sources are modeled as uncorrelated complex signals, and the additive noise in all channels of the array is modeled as a complex white Gaussian noise with the same variance. The ratio of the signal power to the noise power SNR (signal-noise ratio) is defined as \(\text{SNR} = P/\sigma^2\), where \(P\) is the power of one source and \(\sigma^2\) is defined as the noise variance. Let us consider the situation where there is one source of a complex radio signal, a random Gaussian process with zero mean, which coordinate in the angle of elevation is \(\varphi = 45^\circ\), with a shift in azimuth in the range \(\theta = 0^\circ - 360^\circ\), the signal-noise ratio is 10 dB. As mentioned earlier, many algorithms for estimating the coordinates of signal sources with super-resolution depend on certain statistical properties, for example, the covariance matrix (6). At the same time, each step is followed by the calculation of the root mean square error (RMSE) of the coordinates determination by the methods, the number of tests \(L\) at a certain point is 500:

\[
\text{RMSE}_{\theta}(\hat{\theta}_m, \theta_m) = \frac{1}{L-1} \sum_{i=1}^{L-1} (\theta - \hat{\theta}_i)^2
\]

\[
\text{RMSE}_{\varphi}(\hat{\varphi}_m, \varphi_m) = \frac{1}{L-1} \sum_{i=1}^{L-1} (\varphi - \hat{\varphi}_i)^2
\]

(12) (13)
where RMSE_\phi and RMSE_\theta are RMSE direction-finding errors in azimuth and angle of elevation respectively, and \( \vec{a} \) is the coordinate estimate.

From the graphs in figure 3 it is seen, that the best direction finding algorithm in the circular antenna array with directional radiators is the MUSIC method. At the same time, the difference with the algorithms of "thermal noise" and Borjotti-Lagunas is insignificant. The worst one is the Capon algorithm, which RMSE is much higher than others, especially in angle elevation. Moreover, the error increases with increasing antenna directivity. In addition, it is clear that with the directivity factor equal to D = 6 for all the considered algorithms there are pulsations or uneven assessments in all the azimuth cases. If we estimate the contribution of each component to the total error, then it is obvious, that errors when estimating elevation coordinates are higher, than RMSE of azimuth coordinates estimates.

Consider the situation with a complex radio signal having coordinate which shifts in the angle of elevation \( \phi = 0^\circ - 90^\circ \), meanwhile the azimuth is \( \theta = 0^\circ \), the signal-noise ratio is 10 dB. For these cases, the position of the signal source is estimated by the RMSE of each considered direction finding algorithm.
Figure 4. RMSE of DOA-estimation methods while a signal source $\phi=0^\circ - 90^\circ$: a) D= 2, azimuth error, b) D=4, azimuth error, c) D=6, azimuth error. d) D= 2, elevation error, e) D=4, elevation error, f) D=6, elevation error. g) D= 2, sum error, h) D=4, sum error, i) D=6, sum error.

From the graphs in figure 4 it is seen, that RMSE of the bearings when the signal source has a coordinate close to $\phi=0^\circ$ or $\phi=90^\circ$ is much higher than for the other cases. The Capon algorithm has the biggest errors when the directivity factor is more than two. The difference between the MUSIC and “thermal noise” or Borrhutti-Lagunas methods is insignificant.

3 Conclusion
The paper studied the effect of the particular directional factor of an antenna element composed of a cylindrical antenna array to assess angular coordinates by means of the direction-of-arrival estimation methods with superresolution.

Comparative modeling of cylindrical antenna arrays for radio direction-finding tasks is obtained via spectral characteristics of the MUSIC, as well as Capon, “thermal noise’ and Borrhutti-Lagunas methods. The root-mean square errors of bearing in azimuth and elevation planes have been estimated depending on the directivity factor of each patch-antenna for the considered DOA-estimation methods. In this case, the evaluation was carried out at varying coordinates of the signal source.

It was found that to obtain accurate estimates of the angular coordinates of radiation sources the cylindrical antenna array with the directivity factor equal to four should be used.
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