The Light Higgsino-Gaugino Window

 Jonathan L. Feng

 Theoretical Physics Group, Lawrence Berkeley National Laboratory
 University of California, Berkeley, California 94720

 and

 Department of Physics
 University of California, Berkeley, California 94720

 Nir Polonsky

 Sektion Physik (Lehrstuhl Prof. Wess), Universität München
 37 Theresienstrasse, D–80333 München, Germany

 Scott Thomas

 Stanford Linear Accelerator Center
 Stanford University, Stanford, California 94309

 and

 Institute for Theoretical Physics
 University of California, Santa Barbara, CA 93106

 Supersymmetric models are typically taken to have $\mu$ parameter and all soft
 supersymmetry breaking parameters at or near the weak scale. We point out that a
 small window of allowed values exists in which $\mu$ and the electroweak gaugino masses
 are in the few GeV range. Such models naturally solve the supersymmetry $CP$
 problem, can reduce the discrepancy in $R_b$, and suppress proton decay. In this window
 two neutralinos are in the few GeV range, two are roughly degenerate with the $Z^0$, and
 both charginos are roughly degenerate with the $W^{\pm}$ bosons. Such a signature
 cannot escape detection at LEP II. Models that fall in this window automatically
 arise from renormalizable hidden sectors in which hidden sector singlets participate
 only radiatively in supersymmetry breaking.

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I. INTRODUCTION

It is usually assumed that supersymmetric model are required to have dimensionful parameters $\mu, M_{\text{susy}} \gtrsim O(M_W)$, where $\mu$ is the supersymmetric Higgs mass, and $M_{\text{susy}}$ is the scale of the visible sector supersymmetry (SUSY) breaking parameters. We note here that viable models exist in which $\mu$ and the electroweak gaugino masses are $O$(GeV). Contrary to common lore, such parameters are allowed despite many stringent constraints, including those arising from the LEP $Z^0$ width measurements.

Although the allowed parameter space is not large, such models have a number of interesting features: the SUSY CP problem is solved, the current discrepancy between theoretical and experimental values of $R_b$ can be reduced, and proton decay is suppressed. In addition, we will demonstrate that satisfactory electroweak symmetry breaking may be achieved and discuss how such models might arise in supergravity theories from a renormalizable hidden sector. An important and unambiguous prediction of such models is the observation of neutralinos and charginos at LEP II.

II. $Z^0$ WIDTH CONSTRAINTS

We first discuss the bounds from $Z^0$ decays. Because the $\mu$ parameter and electroweak gaugino masses enter chargino and neutralino mass matrices, one might expect that when these parameters are in the GeV range, charginos and neutralinos are light and in conflict with the bounds on $Z^0$ decay widths. We will see, however, that in some of this region of parameter space, charginos are sufficiently massive and neutralinos are sufficiently decoupled from the $Z^0$ that these bounds may be satisfied.

First consider the charginos. We assume that the charginos and neutralinos are the standard mixtures of electroweak gauginos and the Higgsinos of the two Higgs doublets. We also denote the bino, wino, and gluino masses by $M_1, M_2,$ and $M_3$, respectively, and the ratio of Higgs expectation values by $\tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$. The chargino mass terms are then $(\psi^-)^T M_{\tilde{\chi}^\pm} \psi^+ + \text{h.c.}$, where the mass matrix is

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix}$$

in the basis $\psi^\pm = (-i \tilde{W}^\pm, \tilde{H}^\pm)$. The current bound on chargino masses from LEP measurements is 47 GeV [4]. This bound requires no additional assumptions, as charginos remain coupled to the $Z^0$ for all values of the parameters. We see, however, that for $\mu, M_2 \approx 0$ and $\tan \beta \approx 1$, both charginos have mass $M_W$ and avoid the bound. With $\mu \approx 0 (M_2 \approx 0)$, as $M_2$ ($\mu$) increases, one chargino mass eigenvalue drops by the see-saw mechanism, and when $M_2$ ($\mu$) > 90 GeV, the chargino mass limit is violated for all $\tan \beta$. However, for $1 < \tan \beta \lesssim 2.1$, the parameters $\mu, M_2 \approx 0$ satisfy the chargino mass bound.

Next we examine the neutralino sector. Unlike charginos, neutralinos may completely decouple from the $Z^0$, and for this reason, there are no strict lower bounds on neutralino masses. If one assumes gaugino mass unification and $\tan \beta > 2$, the lower bound on the lightest neutralino’s mass is 20 GeV [4][5][6]. For $\tan \beta \lesssim 1.6$, however, this mass bound disappears altogether [6]. It is clear, then, that a discussion of light neutralinos requires a
detailed analysis of their couplings to the $Z^0$ boson. The $Z^0$ width constraints are therefore considerably more complicated for neutralinos than for charginos, and we will discuss them in two stages. First, we present a simple discussion that makes clear the qualitative features of the allowed region. These features are illustrated in Fig. 1. We then add a number of refinements to the analysis and present the resulting allowed region in Fig. 2.

It is convenient to write the neutralino mass terms $\frac{1}{2}(\psi^0)^T M_{\chi^0} \psi^0 + \text{h.c.}$ in the basis $(\psi^0)^T = \left( \frac{1}{\sqrt{2}} (-i \tilde{Z}^0 + \tilde{H}_A), \frac{1}{\sqrt{2}} (-i \tilde{Z}^0 - \tilde{H}_A), -i \tilde{\gamma}, \tilde{H}_S \right)$, where $\tilde{H}_A \equiv H_1 \cos \beta - H_2 \sin \beta$, and $\tilde{H}_S \equiv H_1 \sin \beta + H_2 \cos \beta$. The tree level mass matrix is then

$$
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_Z + \frac{1}{2} M - \frac{1}{2} \mu \sin 2\beta & \frac{1}{2} M - \frac{1}{2} \mu \sin 2\beta & \Delta M & -\frac{1}{\sqrt{2}} \mu \cos 2\beta \\
\frac{1}{2} M - \frac{1}{2} \mu \sin 2\beta & -M_Z + \frac{1}{2} M + \frac{1}{2} \mu \sin 2\beta & \Delta M & \frac{1}{\sqrt{2}} \mu \cos 2\beta \\
\Delta M & \Delta M & M_\tilde{\gamma} & 0 \\
-\frac{1}{\sqrt{2}} \mu \cos 2\beta & \frac{1}{\sqrt{2}} \mu \cos 2\beta & 0 & -\mu \sin 2\beta
\end{pmatrix},
$$

where $M \equiv M_1 \sin^2 \theta_W + M_2 \cos^2 \theta_W$, $M_\tilde{\gamma} \equiv M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W$, $\Delta M \equiv \frac{1}{\sqrt{2}} (M_2 - M_1) \cos \theta_W \sin \theta_W$, and $\theta_W$ is the weak mixing angle. As discussed above, the chargino mass bound is satisfied with $\mu, M_2 \approx 0$. In order to avoid a light neutralino with unsuppressed coupling to the $Z^0$, it is also necessary that $M_1$ be small. In the limit $\mu, M_2, M_1 \to 0$, the basis states given above are mass eigenstates, with masses $M_Z, M_\tilde{\gamma}, 0$, and 0. The light photino, $\tilde{\gamma}$, does not couple to the $Z^0$, and the light Higgsino, $\tilde{H}_S$, decouples for $\tan \beta \to 1$. In this case, the only nonzero coupling of the neutralinos to the $Z^0$ is through $Z^0 \bar{H}_A H_S$, which is suppressed by phase space.

To understand how far one can vary from the limit $\mu, M_2, M_1, \tan \beta - 1 \to 0$ and still satisfy all the constraints, we must discuss the bounds in greater detail. Let us denote the lightest neutralino, $\tilde{\chi}_1^0$, by $\chi$, and the heavier neutralinos, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, and $\tilde{\chi}_4^0$, by $\chi'$. We will assume that the lightest neutralino $\tilde{\chi}_1^0$ is the lightest supersymmetric particle (LSP) and escapes the detector. There are then bounds on $\Gamma(Z^0 \to \chi \chi)$ from the invisible $Z^0$ width, and bounds on $\Gamma(Z^0 \to \chi' \chi')$ and $\Gamma(Z^0 \to \chi' \chi)$ from direct searches for neutralinos.

The current bound on the invisible width of the $Z^0$, in units of the neutrino width, is $N_\nu = 2.988 \pm 0.023$ [1,2]. The 2$\sigma$ upper bound on non-Standard Model invisible decay is then $\delta N_\nu = 0.034$, or a $Z^0$ branching ratio of $B_{\text{inv}} = 2.3 \times 10^{-3}$. We will take this as the bound on the $\chi \chi$ width [1,2].

The visible width bounds are determined from direct searches for neutralinos. In Ref. [3] the L3 Collaboration placed bounds on neutralinos based on an event sample including 1.8 million hadronic $Z^0$ events. The decays $\chi' \to \chi Z^0 \to \chi f \bar{f}$, with $f = q, e, \mu$, and also the radiative decay $\chi' \to \chi \gamma$ were considered. For given masses $m_\chi$ and $m_{\chi'}$, neutralino events were simulated, and the photonic branching ratio was chosen to give the weakest bounds. In the regions of most interest to us, the neutralino masses are $m_\chi \approx 0$ and $m_{\chi'} \approx 0, M_Z$. For these masses, the upper bound on the branching ratio $B(Z^0 \to \chi \chi')$ was

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1Formally, production of $\chi \chi'$ and $\chi' \chi'$, if followed by $\chi \to \chi \nu \bar{\nu}$, will also contribute to the invisible width. However, as we will see, such processes violate the visible width bounds long before their effect on the invisible width becomes important, and so may be safely ignored here.
found to be at least $1.2 \times 10^{-5}$ $(3.5 \times 10^{-5})$. These bounds deteriorate rapidly as $m_\chi \rightarrow m_\chi'$, but we will conservatively assume that they apply for all masses.

Given these bounds, we may now determine the allowed region of parameter space. As one varies from the point $\mu, M_2, M_1, \tan \beta - 1 = 0$, the basis states begin to mix, and have masses given by the diagonal elements up to corrections of $\mathcal{O}((M_1, M_2, \mu)^2/M_Z)$. The mixing angles between the heavy states and between the heavy and light states are $\mathcal{O}((M_1, M_2, \mu)/M_Z)$. However, (at tree level) the mixing between the light states occurs only through the intermediate heavy states and so is $\mathcal{O}((M_1, M_2, \mu)/M_Z)^2$. These mass shifts and mixings may then weaken the various coupling constant and phase space suppressions. Decays to the following three states determine the allowed region:

(a) $\bar{H}H$. The ratio $\Gamma(Z^0 \rightarrow \bar{H}H)/\Gamma(Z^0 \rightarrow \nu \bar{\nu})$ is $\cos^2 2\beta$. If $\tilde{\chi}_0^0$ has a significant $H$ component, the stringent limits on the visible $Z^0$ width require $\tan \beta < 1.02$, a range that is in conflict with the perturbativity of the top Yukawa coupling (see below). However, when $\tilde{\chi}_1^0 \approx H$, the constraint on $\tan \beta$ comes only from the invisible width bound, which is two orders of magnitude weaker. In particular, the radiative photon decay $\tilde{\chi}_1^0 \rightarrow \gamma \tilde{\chi}_1^0$ has been studied in Ref. [2]. In that event sample, the dominant Standard Model background, $Z^0 \rightarrow \nu \bar{\nu}$, is expected to produce only 15 events with photons passing the cut $p_T > 10$ GeV. Assuming that $\sqrt{15.7} \approx 4$ neutralino events could be hidden in this background, that the

(b) $\bar{H}S\tilde{\chi}_3^0$. The decay to $\bar{H}S\tilde{\chi}_3^0$ is suppressed only by phase space. This suppression is more adequate when both $M_Z + \frac{1}{2}M + \frac{1}{2}|\mu| \sin 2\beta \geq M_Z$ and $M_Z - \frac{1}{2}M - \frac{1}{2}|\mu| \sin 2\beta \geq M_Z$. Assuming $M > 0$, this constraint is then $M \lesssim 3|\mu| \sin 2\beta$ for $\mu < 0$ and $M \lesssim |\mu| \sin 2\beta$ for $\mu > 0$.

(c) $\bar{H}S\tilde{\chi}_2^0$. For this decay to be suppressed, the neutralino $\tilde{\chi}_2^0$ must be nearly a pure photino. The mixing of this eigenstate is controlled by $\Delta M$ and vanishes when $\Delta M = 0$, that is, when $M_1 = M_2$.

The allowed regions for $\tan \beta = 1.15$ are presented in Fig. [4] for three values of the ratio $M_1/M_2$. The allowed regions are very similar for all $1.02 < \tan \beta < 1.20$. Constraints (a) and (b) limit the allowed parameter space to a region with boundaries of definite slope as given above. Constraint (c) provides a maximum allowed $M_2$, and, as expected, disappears in the limit $M_1 = M_2$.

The analysis above gives a rough picture of what parameter regions may survive the various constraints. However, several refinements are necessary. First, as noted above, the photonic branching ratio was assumed to be unknown in the analysis of Ref. [3] and was chosen to give the weakest bounds. However, for specific branching ratios, additional regions might be excluded. In particular, the radiative photon decay $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \gamma$ has been studied previously in Ref. [3] and is expected to be dominant in our case, where $\tilde{\chi}_2^0 \approx \tilde{\chi}_1^0 \approx H$ and $\tilde{\chi}_1^0 \approx \tilde{\chi}_1^0 \approx H$. The production of $\tilde{\chi}_1^0\tilde{\chi}_2^0$ then gives a spectacular single photon signal, and the bound on its rate can be significantly improved. To estimate this new bound, we reexamine the data of Ref. [2]. In that event sample, the dominant Standard Model background, $Z^0 \rightarrow \nu \bar{\nu}$, is expected to produce only $15.7 \pm 1.5$ events with photons passing the cut $p_T > 10$ GeV. Assuming that $\sqrt{15.7} \approx 4$ neutralino events could be hidden in this background, that the

\[\text{For } M < 0, \text{ the requirements are } |M| < |\mu| \sin 2\beta \text{ for } \mu < 0 \text{ and } |M| < 3|\mu| \sin 2\beta \text{ for } \mu > 0.\]

However, we will concentrate on the case $M > 0$, as this holds in most of the allowed region.
efficiency of neutralino detection in this mode is 50%, as given in Ref. [2], and, for simplicity, that the neutralino events are uniformly distributed in the range $0 < p_T < 45$ GeV, we find an upper bound of 10 signal events. We therefore consider the effect of strengthening the bound to $B(Z^0 \to \chi' \chi') < 3.9 \times 10^{-6}$ (we also take $B(Z^0 \to \chi' \chi') < 3.9 \times 10^{-6}$).

Another important refinement is to include the data taken above $M_Z$. As the process $Z^0 \to \tilde{H} S \tilde{\chi}^0_3$ is suppressed only by phase space, the boundary of the allowed region defined by this constraint can be expected to be very sensitive to deviations in $\sqrt{s}$ from $M_Z$. The analysis of Ref. [2] used 1993 data, which included an integrated luminosity of 18 pb$^{-1}$ at $\sqrt{s} = M_Z + 1.8$ GeV [4]. This data sample then includes approximately 240,000 hadronic $Z^0$ decays, and we estimate that with this much data the branching ratio bounds are degraded by a statistical factor of 2.7 at the higher energy.

In Fig. 2, we plot the new allowed region, including the tighter branching ratio bound from radiative neutralino decay and the effects of data taken above $M_Z$. Points in the allowed region are values of $\mu$ and $M_2$ that are allowed for some $M_1$ in the range $\frac{1}{2} M_2 \leq M_1 \leq 2 M_2$. (This range has been chosen rather arbitrarily. By considering $M_1 > 2 M_2$, the allowed region can be extended to lower values of $M_2$ in the negative $\mu$ region.) Qualitatively, the allowed region is very similar to what would be expected from Fig. [4] with the exception that points with $\mu > \sim 0$ have been eliminated. These points required the phase space suppression of $\tilde{H} S \tilde{\chi}^0_3$ production, and are eliminated by the $M_Z + 1.8$ GeV data. We see, however, that much of the $\mu < 0$ region still remains.

In the previous figures, radiative corrections have not been included. Radiative corrections to the diagonal entries of Eq. (2) shift the neutralino masses, and thus shift the boundaries slightly. Corrections to the off-diagonal entries introduce mixings between states. Mixings between the heavy states and between heavy and light states are unimportant as similar mixings are already present at tree level, and all such mixings are highly suppressed because they mix states whose eigenvalues are split by $O(M_Z)$. Off-diagonal radiative corrections that mix the light states can be important, however, as they give a Dirac mass that lifts the tree level zero in Eq. (2). The largest such correction comes from top-stop loops (similar to the ones that induce the radiative photon decay), is of order 1 GeV, and vanishes when the left- and right-handed stops, $\tilde{t}_L$ and $\tilde{t}_R$, are degenerate [6]. When this radiative mixing of the light states is significant with respect to the tree level masses, the LSP is a mixture of both $\tilde{H} S$ and $\tilde{\gamma}$. Some regions of the window that were allowed at tree level are then excluded by the stringent $Z^0$ visible width bound. However, for $\mu$ and $M_2$ sufficiently large, the radiative mixing becomes negligible, and the LSP can be mostly $\tilde{H} S$. For stop masses in the range $100$ GeV < $m_{\tilde{t}_R}$, $m_{\tilde{t}_L}$ < 300 GeV, we find that the effect of the radiative Dirac mass is to remove points with $M_2, \mu \lesssim 2 - 4$ GeV, leaving most of the allowed region displayed in Fig. 2 intact.

Returning to the chargino mass matrix of Eq. (1), we see that, in the allowed Higgsino-gaugino window, both charginos are roughly degenerate with the $W^\pm$. Charginos and neutralinos are thus all within reach of LEP II; if observed there, precision studies may be able to determine if the SUSY parameters lie in this allowed window [3]. The chargino mass

\[^3\text{This estimate is in excellent agreement with the bound of } 4.3 \times 10^{-6} \text{ set by the OPAL Collaboration [3] on exotic decays } Z^0 \to X\gamma, \text{ where } X \text{ decays invisibly.}\]
is lowered by the deviation from $\mu, M_2, \tan \beta - 1 = 0$, but remains above 70 GeV. It is important to note that $m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_1^0} > 77$ GeV throughout almost all of the allowed region and grows beyond $M_W$ as $\mu$ and $M_2$ increase in the allowed region, so the branching ratio for the decay $W \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^0$ is highly suppressed [9].

Another possible constraint on the light Higgsino-gaugino window is from the relic dark matter density. As the LSP is mostly Higgsino, the dominant annihilation channel is through $s$-channel $Z^0$ to light fermion pairs. The Higgsino-$Z^0$ coupling is necessarily suppressed in order to avoid the invisible width bound from $Z^0$ decay. This generally leads to an overproduction of primordial Higgsinos. Using the non-relativistic approximation for the freeze out density and the annihilation cross section for a pure Higgsino state given in Ref. [10], we find that for $\tan \beta \approx \{2, \Omega h^2 \lesssim 1$ only for $m_{\text{LSP}} > 20$ GeV. We therefore conclude that either there is an additional entropy release below the LSP freeze out temperature to dilute the relic Higgsinos, or that $R$ parity is broken so that the LSP is not stable, and therefore does not contribute to the dark matter.

### III. RADIATIVE SYMMETRY BREAKING

Before discussing the scalar sector and radiative symmetry breaking, we note that, in the allowed window where $\mu, M_1, M_2 \ll M_W$, an approximate $U(1)$ $R$-symmetry exists in the weak gaugino and Higgs sector, under which $R(H_1) = R(H_2) = 0$, and all other chiral matter fields have $R = 1$. It is possible to promote this approximate symmetry to an exact symmetry of the entire MSSM Lagrangian, in which case all gaugino masses, $A$ terms, and $\mu$ would vanish [7,11,12,13]. We do not impose such a symmetry by hand, but simply note that an approximate symmetry exists in the allowed window. Below we discuss some consequences of extending the approximate $U(1)_R$ symmetry to other sectors of the MSSM. Such an approximate symmetry in fact arises accidentally in certain types of hidden sector SUSY breaking scenarios as discussed below.

The allowed window requires $\tan \beta \approx 1$. Let us therefore reexamine the lower bound on $\tan \beta$ from the requirement that the top Yukawa coupling, $h_t$, remain perturbative to high scales. This is related to the top quark pole mass by the one-loop relation [14]

$$h_t(m_t) \approx \frac{m_t^{\text{pole}}}{174 \text{ GeV}} \sqrt{1 + \tan^2 \beta} \left[1 - \frac{5 \alpha_s}{3 \pi} - \Delta_{\text{SUSY QCD}} - \Delta_{\text{electroweak}}\right] \lesssim 1.15,$$

where 1.15 is our estimate of the quasi-fixed point value. Neglecting SUSY, electroweak, and higher loop corrections, one has a $\sim 6\%$ correction to the tree level result, and, taking $m_t^{\text{pole}} \gtrsim 160$ GeV, we find the constraint $\tan \beta \gtrsim 1.14$. However, including the one-loop SUSY QCD corrections [14,13], we find an additional few percent correction (for a nonvanishing gluino mass) whose sign depends on the various SUSY parameters. A $\sim 10\%$ correction is

$^4$Under an $R$ transformation the scalar, fermionic, and auxiliary components of a chiral superfield transform as $\phi \rightarrow e^{i\alpha R} \phi$, $\psi \rightarrow e^{i\alpha(R-1)} \psi$, and $F \rightarrow e^{i\alpha(R-2)} F$, respectively, where $R$ is the superfield’s $R$ charge. The superpotential has $R$ charge $R(W) = 2$, and a gauge superfield has $R$ charge $R(W^\alpha) = 1$.  

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thus possible, which would lower the $\tan \beta$ bound to $\tan \beta \gtrsim 1.04$. Alternatively, for a fixed $\tan \beta$ the perturbativity upper bound on $m_t^{\text{pole}}$ could increase. Thus, one can still consider perturbative values of $h_t$ at Planckian scales for $\tan \beta \lesssim 1.2$, and we may also consider the possibility of radiative symmetry breaking (RSB).

Next we consider the scalar Higgs sector. In the allowed window, the Higgs potential is $V = m_1^2 H_1^2 + m_2^2 H_2^2 - m_{12}^2 (H_1 H_2 + \text{h.c.}) + D$-terms + $\Delta V^{1\text{-loop}}$, where $m_i$ are in our case simply the soft SUSY breaking masses, since $\mu$ is generally small. The condition $m_1^2 < 0$ triggers electroweak symmetry breaking, and $m_1^2 > 0$ is required for the potential to be bounded. At tree level, the pseudoscalar Higgs mass is $m_A = m_{12}^2 (\tan \beta + \cot \beta)$. Although often assumed, it is not generally true in supergravity theories that $m_{12}^2$ is proportional to $\mu$. A small $\mu$ parameter does not, therefore, imply the existence of a light pseudoscalar. Note also that a large $m_{12}^2$ does not violate the approximate $U(1)_R$ symmetry given above.

We have seen above that $\tan \beta \approx 1$ in the allowed window. At tree level, the light $CP$ even Higgs mass satisfies the bound $m_{h^0} < M_Z \cos 2\beta$ and so vanishes in the limit $\tan \beta \to 1$. However, $m_{h^0} \propto h_t m_t$ is generated by top-stop contributions to $\Delta V^{1\text{-loop}}$, and, in principle, a large Higgs mass can be obtained to satisfy the current experimental bound of $m_{h^0} \gtrsim 60$ GeV \[4\]. (The lower bound on the Standard Model Higgs boson mass is the relevant one in the limit $\tan \beta \to 1$.) If the approximate $U(1)_R$ symmetry is extended to the entire Lagrangian, though, achieving $m_{h^0} \gtrsim 60$ GeV is not trivial. In this case the mixing between the stops $\tilde{t}_L$ and $\tilde{t}_R$, which can significantly enhance the loop contributions to $m_{h^0}$ \[14\], is small since $\mu, A \approx 0$. In addition, the stop masses $m_{\tilde{t}_{L,R}}$ are constrained from above if RSB with minimal particle content is required. This may be seen by recalling the minimization condition $m_2^2 = \left( m_1^2 + \frac{1}{2} M_Z^2 (1 - \tan^2 \beta) \right) / \tan^2 \beta$. The constraint $m_1^2 > 0$ then implies a lower bound on $m_2^2$ of $-\frac{1}{2} M_Z^2 (\tan^2 \beta - 1) / \tan^2 \beta$. In the $U(1)_R$ symmetric case, where all gaugino masses are small, the RGE equation for $m_2^2$ is $\partial m_2^2 / \partial \ln Q \approx \frac{3}{8 \pi} h_t^2 [m_2^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2]$. The requirement that $m_2^2$ not be driven too negative then places an upper bound of typically $\lesssim M_Z$ on the boundary condition for the stop masses at the grand scale. One then finds that the stop masses at the weak scale are not large enough to push $m_{h^0}$ above its lower bound. Thus, unless the $U(1)_R$ symmetry is explicitly broken by a gluino mass, RSB and $m_{h^0} \gtrsim 60$ GeV cannot be achieved simultaneously with minimal particle content. (Note that in Ref. \[11\] the authors assume a global $U(1)_R$ symmetry in the whole Lagrangian, but do not require satisfactory RSB.)

Let us elaborate on the above observations. We have seen that to have satisfactory RSB with minimal particle content, the combination $[m_2^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2]$ that controls $m_2^2$ renormalization is constrained to be approximately zero at the grand scale. If we assume also a common scalar mass $m_0$ at the grand scale and relaxing vanishing gaugino masses and $A$ parameters (i.e., the $U(1)_R$ symmetric limit), one finds $m_0 \lesssim \frac{1}{3} M_Z$ (for $\tan \beta \lesssim 1.15$ and $h_t$ at its quasi-fixed point) and an unacceptable spectrum. (Stronger constraints apply for non-vanishing dimension-three terms, and the symmetry limit is preferred.) However, if we relax the universality assumption, we are led to consider the following soft parameter boundary conditions at the grand scale: $m_2^2(0) \approx -[m_{\tilde{t}_L}^2(0) + m_{\tilde{t}_R}^2(0)] > 0$ and $m_2^2(0) \neq m_1^2(0)$. If we add a non-vanishing gluino mass, thereby explicitly breaking the $U(1)_R$ symmetry in the colored sector, $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$ both turn positive and possibly large in the course of renormalization, and the radiatively induced $m_{h^0}$ is sufficiently large. (Such boundary conditions can be realized, e.g., in certain stringy schemes \[17\].) As long as the scalar potential in the full
and effective theories is bounded from below at all scales, these boundary conditions are acceptable. In particular, we find solutions with right-handed stops ranging in mass from 45 GeV to many hundreds of GeV, Higgs bosons in the 60–70 GeV range, and the two charginos between 70–90 GeV (as is favored by $R_b$ (see below)). We present typical spectra in Fig. 3, assuming the above pattern for boundary conditions. Only those masses that are constrained by RSB and $m_{h^0}$ are presented. Note that because the trilinear terms in the scalar potential are small, dangerous color breaking directions of the potential, which are generic in the limit $\tan \beta \rightarrow 1$ [16], are eliminated. (However, if $m_{\tilde{t}} < \sim m_t$, dangerous directions may persist.)

The above scheme is an example of boundary conditions that can successfully generate RSB. (Note that all other boundary conditions are only negligibly constrained by RSB and $m_{h^0}$.) The tuning required in order to achieve RSB is a reflection of the fact that we did not have at our disposal an arbitrary $\mu$, which typically absorbs the tuning. (For example, generically one expects $\mu \gtrsim 1$ TeV for $\tan \beta \rightarrow 1$ [16].) Instead, the tuning is now in the soft parameters. Alternatively, $\partial m_{22}^2/\partial \ln Q$ can be adjusted by introducing a right-handed neutrino superfield at an intermediate scale with a soft mass $m_{\tilde{\nu}_R}^2 < 0$. A new neutrino Yukawa term $h^2 m_{\tilde{\nu}_R}^2$ then enters the RGE for $m_{22}^2$, and may be used to balance the RGE. The additional freedom results from the fact that $m_{\tilde{\nu}_R}^2$ is unconstrained, as the physical mass of the scalar neutrino is determined essentially by the intermediate scale.

### IV. THE $CP$ PROBLEM, $R_b$, AND PROTON DECAY

The light Higgsino-gaugino window has a number of interesting consequences. With conventional weak scale SUSY breaking parameters, the present bound on the electric dipole moments (EDMs) of atoms, molecules, and the neutron limit the $CP$ violating phases in the dimensionful parameters of the MSSM to be less than $10^{-2} - 10^{-3}$ over much of the parameter space [18]. This is generally referred to as the SUSY $CP$ problem. As shown in Ref. [19], all flavor-conserving $CP$ odd observables are proportional to the phases of $M_\lambda \mu (m_{12}^2)^*$, $A^* M_\lambda$, or $A \mu (m_{12}^2)^*$, where $M_\lambda$ is any one of the three gaugino masses. The phase of $M_3$ does not enter the electron EDM at one-loop. It follows that the electron EDM is proportional at lowest order to at least one insertion of $\mu$, $M_1$, or $M_2$. For $\mu, M_1, M_2 \sim \mathcal{O}$ (GeV), one see that the electron EDM is suppressed by $\mathcal{O}(M/M_{\text{SUSY}})$, where $M \sim \mathcal{O}$ (GeV). This largely eliminates the SUSY $CP$ problem for atoms with unpaired electrons which are sensitive to the electron EDM. In addition, if leptonic $A$ terms are small, as would be the case if the approximate $U(1)_R$ symmetry discussed above were extended to the leptonic sector, the electron EDM would be suppressed by $\mathcal{O}(M/M_{\text{SUSY}})^2$.

$CP$ violation in the strongly interacting sector depends on the gluino mass $M_3$, and so is not necessarily suppressed in the phenomenologically allowed window. However, if the approximate $U(1)_R$ is extended to the entire Lagrangian, then all gaugino masses, $\mu$, and all $A$ terms are suppressed. The EDMs of the neutron and atoms with paired electrons (which are sensitive to strong sector $CP$ violation) are then suppressed by $\mathcal{O}(M/M_{\text{SUSY}})^2$. Even in

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5 The full $U(1)_R$ symmetry imposed in Refs. [11,12,13] is not required to solve the SUSY $CP$
the RSB scheme given in the previous section with a large gluino mass and small $A$ terms at the high scale, the strong sector $CP$ violation is still suppressed by $\mathcal{O}(M/M_{\text{SUSY}})$. This is apparent for the first and third type of combinations of $CP$ violating parameters given above, as both involve an insertion of $\mu$. For the second type this follows since, even though a sizeable $A$ term can be induced by the gluino from running to the low scale, the phase is then aligned with that of the gluino mass, i.e., $\text{Arg}(A) \simeq \text{Arg}(M_3)$.

Supersymmetric models can in principle give large enough one-loop corrections to the $Z^0 b\bar{b}$ vertex to explain the $\sim 3\sigma$ discrepancy between the experimental [20] and Standard Model values of $R_b$ [21,22,23,24]. The most important contributions are from vertex corrections involving a top quark Yukawa coupling $b_L H_2 \tilde{t}_R$. A sizeable effect requires a light chargino with a substantial Higgsino component, $\tan \beta \simeq 1$, and a light $\tilde{t}_R$ [21,22,23,24]. The first two of these requirements are met in the light Higgsino-gaugino window. In addition, as demonstrated earlier, it is also possible to arrange for a light $\tilde{t}_R$ consistent with RSB. This is highly non-trivial, as it is generally difficult to obtain solutions that explain the $R_b$ discrepancy consistent with RSB. (See, however, Ref. [23] for a solution with conventional weak-scale SUSY parameters.) The effect in the light Higgsino-gaugino window (see, for example, Ref. [24]), may be determined from the figures of Ref. [21]. We find that for $\tan \beta = 1$, $\mu = M_2 = 0$, and $m_{\tilde{t}_R} = 100$ GeV, the SUSY shift in $R_b$ is $\delta R_b \approx 0.002$, or roughly equal to what can be achieved in the Higgsino region with similar chargino and stop masses. This effect is, of course, greatly increased for smaller $m_{\tilde{t}_R}$, and may therefore significantly reduce (but not eliminate) the current discrepancy of 0.006 between experiment and the Standard Model [20].

Proton decay at one-loop is also suppressed in the allowed window. The supersymmetric baryon violating coupling $QQQL$ must be dressed with an off-shell gaugino in order to obtain a four-Fermi interaction. With degenerate squarks, and ignoring any flavor changing, the gluino contribution vanishes, so the largest dressing typically comes from charginos [25]. A chiral insertion is necessary on the chargino line to obtain the four-Fermi interaction. In order to avoid a light quark Yukawa coupling to the Higgsino component of the chargino, an $M_2$ insertion is required. The proton decay rate is then suppressed in this limit at one-loop by $\mathcal{O}(M_2/M_{\text{SUSY}})^2 \sim 10^{-4}$ in the allowed window. If the gluino is massive, gluino exchange could then dominate the decay rate if there are flavor changing squark masses.

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problem. If any two types of the four classes of dimensionful parameters $\{M_\lambda, \mu, A, m_{12}\}$ are $\mathcal{O}(M) \ll M_{\text{SUSY}}$ at the high scale, then all $CP$ odd observables are suppressed by $\mathcal{O}(M/M_{\text{SUSY}})$. This may be verified by noting that $U(1)_{PQ}$ and $U(1)_{R-PQ}$ field redefinitions [19] may be used to isolate the phases on the small parameters.

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6Here we have ignored left-right stop mixing angle suppressions. In principle, the mixing, which in our case is due to weak scale $A$ parameters, can be small for $m_{\tilde{t}_R} \lesssim m_t$ if the soft mass squared $m_{\tilde{t}_R}^2$ is slightly negative at the weak scale (when permitted by the stability of the potential).
Finally, let us consider a possible theoretical motivation for the light Higgsino-gaugino window. In hidden sector models, SUSY breaking is transmitted to the visible sector by gravitational strength interactions. With a renormalizable hidden sector, in which SUSY breaking remains in the flat space limit, the dynamical scale, $\Lambda$, of the hidden sector gauge group, and the hidden sector scalar expectation values, $Z$, are of the order of the intrinsic SUSY breaking scale, $M_S \sim \Lambda \sim Z \sim \sqrt{m_{3/2} M_p} \sim 10^{10-11}$ GeV. This allows an expansion of the operators which couple the visible and hidden sectors in powers of $M_p^{-1}$. In the rigid supersymmetric limit, the dimension two soft terms arise from $D$ term operators of the form $M_p^{-1} \int d^4 \theta \ Z^* Z \phi^* \phi$ and $M_p^{-1} \int d^4 \theta \ Z^* Z H_1 H_2$ [20], where $Z$ are any hidden sector fields and $\phi$ is a visible sector field. With $F_Z \sim M_S^2$, we then have $m_{\phi}^2 \sim m_{i2}^2 \sim m_{3/2}^2$. Note that these dimension two terms arise even without hidden sector singlets. The dimension three gaugino masses arise from the dependence of a visible sector gauge kinetic function on a hidden sector singlet $S$, which does not transform under any gauge symmetry, $M_p^{-1} \int d^2 \theta \ SW^2 W_\alpha \ + \ h.c.$ Visible sector $A$ terms arise from $D$ term operators $M_p^{-1} \int d^4 \theta \ S \phi^*_i \phi_i \ + \ h.c.$, where $F_{\phi_i} = h_{ijk} \phi_j \phi_k$ results from the visible sector Yukawa couplings $W = h_{ijk} \phi_i \phi_j \phi_k$ [20]. Likewise, the $\mu$ term arises from operators of the form $M_p^{-1} \int d^3 \theta \ SH_1 H_2 \ + \ h.c.$ [20].

If the hidden sector singlets have $F$ components $F_S \sim M_3^2$, then all the dimensionful parameters of the MSSM can be $O(m_{3/2})$. However, it is possible that the hidden sector singlets participate in the supersymmetry breaking only radiatively so that $F_S \sim O(\lambda/4\pi)^2 M_S^2$, where $\lambda$ is a hidden sector singlet Yukawa coupling. All the dimension three soft terms and $\mu$ are then automatically suppressed by $O(\lambda/4\pi)^2$ [23]. Inclusion of supergravity interactions does not modify this conclusion. The smallness of the dimension three terms in such a scenario leads to the approximate $U(1)_R$ discussed above. This approximate symmetry is not imposed by hand but simply arises accidentally as a result of the hidden sector outlined above. Notice that this motivation for the window requires the gluino also to be light [28,29]. Models with radiatively coupled singlets have in fact been constructed [30] and until recently were the only known renormalizable models of dynamical supersymmetry breaking with singlets [31]. We therefore conclude that a renormalizable hidden sector with radiatively coupled singlets automatically leads to models that can fall in the light Higgsino-gaugino window.

\footnote{It is interesting to note that the resulting $A$ terms are real. Independent of their magnitude, $A$ terms therefore do not contribute to the SUSY $CP$ problem with a renormalizable hidden sector.}

\footnote{The $\mu$ term can also arise from an $H_1 H_2$ dependence of a hidden sector gauge kinetic function $M_p^{-1} \int d^2 \theta \ H_1 H_2 \langle W^2 W_\alpha \rangle_{\text{hidden}} \ + \ h.c.$ [27]. For a renormalizable hidden sector with $\langle W^2 W_\alpha \rangle \sim \Lambda^3 \sim M_S^3$ the resulting $\mu$ term is very small. However, a non-renormalizable hidden sector with $\langle W^2 W_\alpha \rangle \sim \Lambda^3 \sim M_S^2 M_p$ gives $\mu \sim m_{3/2}$. The scenario discussed by Farrar and Masiero in which all the dimension three terms but $\mu$ essentially vanish [28] is therefore realizable with a non-renormalizable hidden sector without singlets and with scalar expectation values much less than $M_p$. The magnitude of the $\mu$ term therefore distinguishes between the renormalizable and non-renormalizable hidden sector motivations for light gauginos.}
VI. CONCLUSIONS

At present there exists a small region of supersymmetric parameter space in which $\mu$ and the electroweak gaugino masses $M_1$ and $M_2$ are in the few GeV range. This window has a number of interesting consequences: 1) The SUSY $CP$ problem can be significantly reduced, 2) the discrepancy in $R_b$ can be substantially reduced if the right-handed stop is light, and 3) proton decay is suppressed. The mass of the lightest Higgs is generated almost entirely radiatively since $\tan \beta \simeq 1$, and requires a fairly heavy stop to exceed current bounds. (This must be the left-handed stop if a light right-handed stop is required for $R_b$.) Radiative electroweak symmetry breaking is generally difficult in the allowed window with minimal particle content, but can be accommodated. In particular, a heavy gluino or intermediate scale right-handed neutrino can allow radiative symmetry breaking. Renormalizable hidden sectors with radiatively coupled singlets automatically give models that can fall in the allowed window. Most importantly, this window predicts that two neutralino states are light, two are roughly degenerate with the $Z^0$, and both charginos are roughly degenerate with the $W^\pm$. All of these particles cannot escape detection at LEP II.

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FIGURES

FIG. 1. Allowed regions of the $(\mu, M_2)$ plane for $\tan \beta = 1.15$ and $M_1/M_2 = \frac{1}{2}$ (solid), 1 (dashed), and 2 (dotted). These regions satisfy the bounds of Ref. [2] from data taken at $\sqrt{s} = M_Z$.

FIG. 2. The region (shaded) of the $(\mu, M_2)$ plane that satisfies the refined bounds from radiative photon decays and data taken 1.8 GeV above $M_Z$ (see text). Here $\tan \beta = 1.15$, and only points that satisfy the bounds for some $M_1$ in the range $\frac{1}{2} M_2 \leq M_1 \leq 2 M_2$ are considered allowed. For $M_1 > M_2$, the region can be extended to lower $M_2$ for $\mu < 0$.

FIG. 3. Typical spectra found for the light and pseudoscalar Higgs bosons, top and bottom squarks with dominant right- and left-handed components, and the gluino, using the boundary conditions described in the text and requiring RSB. The Higgs boson is constrained to be heavier than 60 GeV.
Fig. 1

Fig. 2
$m_t^{pole} = 160$ GeV
$tan \beta = 1.15$
$A_0 = 0$