Scalar structures around the mixing interface of a thermal mixing layer in multiscale-generated decaying turbulent flows visualized using DNS database

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Abstract. This study shows the results for the visualization of a scalar mixing layer in decaying turbulence produced by a multiscale-generated turbulence grid. For this purpose, a thermal mixing layer in the air is used as the scalar mixing layer. A fractal generating grid proposed in the previous study is used for the generation of multiscale-generated turbulence. In addition, a parameter for determining the grid shape is varied. The scalar field is then visualized using a database given by direct numerical simulations. The instantaneous scalar field, the instantaneous scalar fluctuation field, the instantaneous scalar variance field, and the scalar dissipation field are visualized, and the visualization results are discussed using the wake interaction length scale.

1. Introduction
Incompressible turbulence has been an important research theme in the field of industrial equipment related to fluids engineering. Mixing of scalars due to turbulence has been regarded as an important subject in previous studies [1]. In fact, there are many earlier studies approaching the scalar mixing in turbulent flows as shown in a review [2]. A temperature fluctuation in decaying grid-generated turbulence was experimentally studied by setting a lateral temperature gradient [3]. A thermal mixing layer in the decaying grid-generated turbulence was also experimentally studied [4]. Phenomena related to reactive scalars in a turbulent flow were also experimentally investigated using a line source [5].

These studies generate decaying homogeneous turbulence using a conventional turbulence-generating grid. Here, the decay characteristics of this grid-generated turbulence have been investigated in previous studies (e.g., [6]). This conventional turbulence-generating grid is considered to generate turbulence on only one scale. On the other hand, previous studies [7,8] have also shown a type of grid that generates turbulence on multiscale. Decaying turbulence generated by the multiscale-generated turbulence grid has been pointed out to have different properties from the conventional grid-generated turbulence. This multiscale-generated turbulence grid is also used in an attempt to enhance turbulent mixing [9,10]. As shown in earlier studies [11,12], the turbulence generation due to the multiscale-generated turbulence grid was shown to enhance turbulent mixing.
Visualization of a scalar field in the turbulence is considered to be indispensable for elucidating this phenomenon. Visualization of a scalar field in the turbulence has been performed in numerical analysis and experimental measurement, as shown in previous works [13]. A visualization technique was often used in earlier studies [10-12] to approach the scalar mixing layer in decaying grid-generated turbulence. The high Schmidt number scalar mixing layer in the decaying grid-generated turbulence was also by previous studies [10,11]. In one study [12], the structure of the mixing interface of high Schmidt number scalar field was clarified using the visualization using a planer-laser induced fluorescence technique. On the other hand, there seems to be a lack of studies for visualizing a thermal mixing layer in the decaying grid-generated turbulence. We consider that visualization of a thermal mixing layer in the decaying grid-generated turbulence is meaningful.

The purpose of this study is to visualize a field of a thermal mixing layer in grid-generated turbulence produce by a multiscale/fractal turbulence generating grid. The present thermal mixing layer in the generated decaying turbulence is visualized using a technique of numerical analysis. Specifically, the present thermal mixture layer is visualized using a database produced by direct numerical simulation (DNS). This DNS is based on the finite difference method with high accuracy and is analyzed with sufficient turbulent kinetic energy conservation. The present visualization is performed based on the DNS database generated based on an earlier research [14]. In the present study, physical quantities that are not straightforward to measure experimentally are visualized based on the use of the numerical analysis. This study can visualize the scalar structure near the mixing interface in the thermal mixing layer.

2. Numerical methods
The governing equations are the continuity equation for incompressible flow, the Navier-Stokes equation, and the heat convection-diffusion equation, which are shown respectively as follows:

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad (1)
\]

\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = - \frac{\partial p}{\partial x_i} + (1/Re) \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \text{ and} \quad (2)
\]

\[
\frac{\partial \bar{\theta}}{\partial t} + \frac{\partial (\bar{u}_i \bar{\theta})}{\partial x_i} = (1/PrRe) \frac{\partial^2 \bar{\theta}}{\partial x_j \partial x_j} \quad (3)
\]
Here, $\bar{u}_i$ are instantaneous velocity components for $x_i$ direction and $\bar{\theta}$ is the instantaneous temperature. These equations are dimensionless using Reynolds number and Prandtl number, $Re$ and $Pr$. The Navier-Stokes equation includes forcing terms to reproduce the turbulence-generating grid numerically. We analyze the decaying turbulence generated by the turbulence-generating grid. The origin of the coordinate system is located at the center of the plane of the turbulence-generating grid. The setting of the coordinate system in this study is the same as that in the previous studies (e.g., [14]). Here, the streamwise, the transverse, and the spanwise directions are $x$, $y$, and $z$ directions, respectively. The inflow boundary is located upstream of the plane of the turbulence-generating grid. As the inflow boundary condition, the streamwise velocity is set as unity with zero value of velocities for $y$ and $z$ directions. Here, the streamwise pressure gradient at the inflow boundary is set to zero. A convective outflow condition is set as the outflow boundary condition. Here, the pressure gradient in the streamwise direction is also zero at the outflow boundary. The periodic boundary condition was set for two directions normal to the streamwise direction. These conditions on the boundary conditions are the same as those of previous studies [14].

In the present study, a fractal grid is used as the turbulence-generating grid to generate turbulence with the perspective of multiscale generation. Figure 1 shows a schematic figure of the present fractal grid. As shown in the figure, the fractal grid has a shape to give a variation of flow with various length scales simultaneously. This type of turbulence-generating grid was used previously. In particular, there is an earlier work that applied this type of turbulence-generating grid to enhance turbulent mixing. Suzuki et al. [11] used a fractal grid to enhance turbulent mixing of high Schmidt number scalar in an experimental measurement using a water channel. There is a parameter that characterizes the shape of this fractal grid. The ratio between the largest and smallest bar widths defined as $t_r = d_0 / d_{\text{min}}$ is set as $t_r = 5.0, 8.5$, and $13$ in this study. Also, the effective mesh size is defined as follows:

$$M_{\text{eff}} = 4 \left( D^3 / P_M \right) \left( 1 - \sigma \right)^{1/2}$$

(4)

Here, in this equation, $\sigma$, $P_M$, and $D$ are solidity, perimeters length of a grid, and the side of the grid plane, respectively. In our study, the characteristic length scale of bulk flow is the effective mesh size. Therefore, the characteristic length of the Reynolds number used to derive the non-dimensional governing equations is this mesh size. Details of the fractal grid are also shown in earlier studies [7,8,11]. This effective mesh size is equivalent to the mesh size of a conventional turbulence-generating grid $M$, where the mesh size $M$ is used as the characteristics length in the bulk flow of grid-generated turbulence due to a conventional turbulence-generating grid. In Figure 1, the size of the fractal grid is compared with the length of $5M$.

A scalar mixing layer is set in the present decaying turbulence. Here, heat in the air is used as a scalar in this study. The value of the Prandtl number, which is equivalent to Schmidt number, depends on what fluid the heat is in. The Prandtl number in the present study is set to 0.71. From the inflow boundary condition to the point where the turbulence-generating grid is installed, the non-dimensional scalar is set to unity or zero with the center of the computational domain as the interface. A convective outflow condition for the scalar was also used at the outflow boundary. Neumann boundary conditions are set at the boundaries in the transverse direction. A periodic boundary condition is set in the spanwise direction.

The governing equations are discretized using a set of higher-order central difference schemes as the finite difference scheme. Here, this discretization scheme was proposed by a previous study [15]. Here, the discretization scheme of this study is similar to that of other studies [14]. The present turbulence-generating grid is numerically constructed using the immersed boundary method based on direct forcing [16]. In the direct forcing, an imaginary force is generated on the surface of the bars of the turbulence-generating grid to set the instantaneous velocity to zero [16]. The governing equations are integrated for the temporal direction using the third-order Runge-Kutta scheme. Here, the fractional step method is used to solve the governing equations. Poisson’s equations in the fractional
steps are solved using the Fast Fourier Transform and the diagonal matrix algorithm. The mesh Reynolds number is set to 2500. This condition is the same as that of the earlier studies. The computational domain size in the streamwise direction is 64, which is normalized by the mesh size. Also, the normalized domain size is 16 in the direction normal to the streamwise direction. The number of grid points is 768 in the mainstream direction and 256, 320, or 410 in the y and z directions, respectively. In the vicinity of the turbulent grid, a fine computational mesh is set for the streamwise direction. These conditions are similar to those of the previous study [14].

3. Results and discussion

Figure 2 (a) shows the instantaneous field of the non-dimensional scalar under the calculation condition of $t_r = 13$. Here, this contour figure is shown at the spanwise position of $z = 0$. Red, blue, and white colors indicate values of 1, 0, and 0.5 of instantaneous scalar. In (b), red, blue, and white indicate values of 0.3, -0.3, and 0 of instantaneous scalar fluctuation. The centerlines of the computational domain are shown by the horizontal black line.

![Figure 2. Contour figures of instantaneous scalar (a) and scalar fluctuation (b). In (a), red, blue, and white indicate values of 1, 0, and 0.5 of instantaneous scalar. In (b), red, blue, and white indicate values of 0.3, -0.3, and 0 of instantaneous scalar fluctuation. The centerlines of the computational domain are shown by the horizontal black line.](image-url)
Figure 2 (b) shows the field of the instantaneous non-dimensional scalar fluctuation for the condition of $t_r = 13$. Here, red color indicates a scalar fluctuation of 0.3, blue indicates the fluctuation of -0.3, and white indicates zero value. As shown in the figure, in the upstream region, the scalar fluctuation exists only around the centerline of the scalar mixing layer. Also, there are negligible large-scale scalar fluctuations in the region. In the upstream region, there is scalar fluctuation slightly far from the centerline. This scalar fluctuation may be caused by the intermittent scalar fluctuations. In the downstream region, not only small scalar fluctuations but also large-scale scalar fluctuations exist. In the decaying turbulence generated by a multiscale fractal grid, a variation of the integral scale for the streamwise direction is significantly smaller as shown in a previous experiment (e.g., [8]) and numerical work [17]. Therefore, the increase in the scale of scalar fluctuation in the streamwise direction may not be due to changes in the large scale structure of the turbulence field. A large scale negative scalar fluctuation can be found around $x/M_{eff} = 37$. Since this scalar structure is considered intermittent, there may be positive scalar fluctuations on a large scale at different times.

Then, we focus on the variance of scalar fluctuation and scalar dissipation [12]. Definitions of the variance of scalar fluctuation and scalar dissipation are given as follows: Here, the two-dimensional scalar dissipation is shown in this study based on previous experimental measurements. Visualization results are given for conditions, $t_r = 5$, 8.5, and 13. As shown in the figure, the small-scale structure of...
the mixing interface and scalar fluctuation are visualized by this scalar dissipation, as similar to results of the previous experiment for turbulent mixing of high Schmidt number scalar [12]. At $t_r = 13$ shown in Figure 3 (c), small fluctuations in the mixing interface are observed in the upstream region, and a region with a high value of the variance is found near the mixing interface. In the downstream region, the small-scale structures of scalar fluctuation are also visualized by the scalar dissipation. Also, this small-scale structure is accompanied by regions of high scalar variance. On the other hand, in the region of $x/M = 20$ to 40, there is a region with high scalar variance near the mixing interface characterized by scalar dissipation. In this region, turbulent mixing of the scalar can be considered to exist due to the large-scale turbulent structures. For the conditions of $t_r = 8.5$ and $t_r = 5$, visualization results are shown in Figure 3 (b) and (a), respectively. As shown in the figure, as $t_r$ becomes smaller, a mixing interface with a large amplitude can be found. Also, regions with high scalar variance are found near this fluctuating mixing interface. The spatial scale of this region is found to be increased when $t_r$ is decreased.

In the present, as shown in Figure 3, the scalar structures near the mixing interface are visualized as a function of $t_r$. Here, as shown in the figure, when $t_r$ is decreased, the spatial scale of the region with high scalar variance is found to be increased. This study focuses on the wake-interaction length scale to discuss this visualization results. The wake-interaction length scale is defined as follows:

$$x^* = L_o^2 / t_o.$$  

Here $L_o$ and $t_o$ are the side lengths and widths of the maximum scale of a grid unit constructing the fractal grid. As shown in the conceptual diagram of the wake-interaction length scale in a previous study [8], there is a significant non-turbulent region upstream from $x/M = x^*/M$. For the present fractal grids, values of $x^*$ are approximately 105, 80, and 65 for $t_r = 5, 8.5, and 13$, respectively. Therefore, when the value of $t_r$ is large, the size of the non-turbulent region for the streamwise direction is decreased. In the non-turbulent region, there are hardly significant small-scale turbulence structures. Therefore large-scale structures are considered to exist mainly. The difference in the scalar field depending on the value of $t_r$ shown in the figure is qualitatively explained by the change in the wake-interaction length scale.

4. Conclusions
This study aims to visualize the scalar mixing layer in the decaying turbulence due to the generation using multiscale generating turbulence. In the present study, we consider the heat in the air as a scalar. We visualize the subjected phenomena by using numerical analysis based on direct numerical simulation. A fractal-generating grid is used to generate multiscale generated turbulence based on previous studies. The governing equations are discretized using the finite difference method. In order to analyze the scalar mixing layer, the heat convection-diffusion equation is also numerically analyzed. In this study, the instantaneous non-dimensional scalar field was shown. The instantaneous scalar fluctuation field was then visualized. Then, quantities of the variance of the scalar fluctuation and the scalar dissipation are visualized. Also, present visualization results are discussed using the wake-interaction length scale.

We consider that turbulent flows behind the multiscale generating turbulence grid will be needed to be studied as future works. Behind the turbulence-generating grid, fluid acceleration of the mean flow will be needed to be investigated and discussed. Previous studies [18,19] showed that fluid acceleration affects decay characteristics of turbulence, even when the mean flow acceleration is weak. These points will have to be approached as future works.

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