On the perturbative expansion of boundary reflection factors of the supersymmetric sinh-Gordon model

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Abstract

The supersymmetric sinh-Gordon model on a half-line with integrable boundary conditions is considered perturbatively to verify conjectured exact reflection factors to one loop order. Propagators for the boson and fermion fields restricted to a half-line contain several novel features and are developed as prerequisites for the calculations.

1 Introduction

Given an interesting field theory, it is traditional to develop and examine its supersymmetric extensions. In four dimensions, supersymmetric field theories provide the prime examples of situations in which quantities of physical interest may be calculated exactly. For this reason they are an important source of ideas and intuition. In theories of strings and branes, supersymmetry is more or less mandatory. However, in two dimensions, for nonlinear models, the two requirements of supersymmetry and integrability do not always sit easily together. There are many models which are entirely bosonic and yet interesting quantities may be calculated exactly, often by indirect algebraic means which are not available for use in four
dimensions; there are others which contain fermions and are integrable, and yet not supersymmetric. On the other hand, there are many examples of two-dimensional models which are both integrable and supersymmetric; for a selection see [1, 2, 3, 4, 5]. The supersymmetric version of the sine-Gordon model was introduced many years ago by Hruby [6], and di Vecchia and Ferrara [7]; Shankar and Witten [8] constructed its exact S-matrix which was subsequently further explored by Schoutens [9] and Ahn [10].

If a field theory is restricted to a half-line by integrable boundary conditions then it turns out that supersymmetry is further constrained, and more restrictive. For example, Warner [11, 12] discussed quantum integrable models possessing $N = 2$ supersymmetry and concluded that only half the supersymmetry may be retained in the presence of the boundary. In the case of the sine-Gordon model, it was pointed out by Inami, Odake and Zhang [13] that only two isolated boundary conditions are compatible with both supersymmetry and integrability. This is a striking and surprising result since without supersymmetry Ghoshal and Zamolodchikov [14] had earlier pointed out that there should be a two parameter family of nonlinear boundary conditions compatible with integrability. More recently, using general arguments, exact reflection matrices for the breathers and their fermionic partners within the $N = 1$ supersymmetric sine-Gordon theory have been conjectured [15, 16].

In this paper, we will examine supersymmetric sinh-Gordon theory restricted to a half-line by integrable boundary conditions. We will argue that the results found by Moriconi and Schoutens need to be adjusted slightly to agree with the classical limit and with the lowest orders of perturbation theory in the sinh-Gordon coupling constant. In order to carry out a low order check in perturbation theory we will also need to construct propagators for both the boson and the fermion. The boson propagator was constructed before [17] but the fermion propagator is constructed here for the first time.

The paper is organised as follows: in section two, we summarise the main features of the model; the boson and fermion propagators are described in section three together with a brief discussion on the rôle played by bound states in the linearised theory; the construction of the supersymmetric scattering matrices and the conjectured reflection factors for the two allowed boundary conditions are presented in section four together with reasons for deviating from the suggestions made by Moriconi and Schoutens; in the subsequent sections we develop the perturbation theory and check that the fermion reflection factors agree with the perturbation expansion up to second order in the bulk coupling constant. The final section is reserved for
concluding remarks.

2 The supersymmetric sinh-Gordon model with one boundary

To establish the conventions we shall use, it is convenient to start with the supersymmetric sinh-Gordon model in the bulk described by the Lagrangian density

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2 \beta^2} \cosh 2 \beta \phi - \bar{\psi} i \gamma_\mu \partial_\mu \psi + m \bar{\psi} \psi \cosh \frac{\beta \phi}{\sqrt{2}}, \]  

(2.1)

where \( \beta \) is a real coupling constant, \( m \) is a mass parameter, \( \phi \) is a real scalar field and \( \psi \) is a two-component Majorana fermion. We choose the \( \gamma \)-matrices to be purely imaginary represented by

\[ \gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}. \]  

(2.2)

With this choice, the charge-conjugation matrix is unity and a Majorana spinor has two real components.

The Lagrangian (2.1) is invariant under the following set of supersymmetry transformations:

\[ \delta \phi = \bar{\epsilon} \psi + \bar{\psi} \epsilon, \]

\[ \delta \psi = \left( i \gamma^\mu \partial_\mu \phi + \frac{\sqrt{2} m}{\beta} \sinh \frac{\beta \phi}{\sqrt{2}} \right) \epsilon, \]  

\[ \delta \bar{\psi} = \bar{\epsilon} \left( -i \gamma^\mu \partial_\mu \phi + \frac{\sqrt{2} m}{\beta} \sinh \frac{\beta \phi}{\sqrt{2}} \right), \]  

(2.3)

where \( \epsilon \) is a constant Majorana spinor which anticommutes with \( \psi \). From the Lagrangian (2.1) the field equations are:

\[ \partial^2 \phi = -\frac{m^2}{\sqrt{2} \beta} \sinh \sqrt{2} \beta \phi - \frac{m}{\sqrt{2}} \bar{\psi} \psi \sinh \frac{\beta \phi}{\sqrt{2}}, \]

\[ i \gamma^\mu \partial_\mu \psi = m \psi \cosh \frac{\beta \phi}{\sqrt{2}}, \]  

(2.4)

\[ i \partial_\mu \bar{\psi} \gamma^\mu = -m \bar{\psi} \cosh \frac{\beta \phi}{\sqrt{2}}. \]

The model may be restricted to a half-line by adding a boundary term to the action which enforces an integrable boundary condition on the fields. For convenience, we shall
take the boundary to be situated at $x = 0$. Following the arguments of Inami, Odake and Zhang, the action for the sinh-Gordon model on a half-line may be written as follows:

$$
S = \int_{-\infty}^{\infty} dt \int_{-\infty}^{0} dx \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2 \beta^2} \cosh \sqrt{2} \beta \phi - \bar{\psi} i \gamma^\mu \partial_\mu \psi + m \bar{\psi} \psi \cosh \frac{\beta \phi}{\sqrt{2}} \right] - \int_{-\infty}^{\infty} dt \left[ \pm \frac{2m}{\beta^2} \cosh \frac{\beta \phi}{\sqrt{2}} \mp \frac{1}{2} \bar{\psi} \psi \right].
$$

(2.5)

The action including the boundary part is supersymmetric if and only if the two components of the parameter $\epsilon$ satisfy

$$
\epsilon_1 = \mp \epsilon_2, \quad (2.6)
$$

which confirms that only half the supersymmetry of the bulk theory is preserved in the presence of boundary conditions.

In addition to the bulk equations of motion (2.4) in the region $x < 0$, the boundary conditions for the fields at $x = 0$ follow from (2.5) and are

$$
\partial_x \phi = \mp \sqrt{2} \frac{m}{\beta} \sinh \frac{\beta \phi}{\sqrt{2}}, \quad \psi_1 = \pm \psi_2. \quad (2.7)
$$

We shall refer to these two boundary conditions as $BC^\pm$, the $\pm$ corresponding to the signs relating the fermion components in each of the two cases given in (2.7).

## 3 Boson and fermion propagators

The construction of the boson propagator for the sinh-Gordon model in the presence of integrable boundary conditions was given in [17]. In the supersymmetric case we have just two kinds of boundary condition preserving both supersymmetry and integrability. The boson propagators corresponding to these are given by

$$
G^\pm (x, t; x', t') = \int \frac{d\omega}{2\pi} \int \frac{dk}{2\pi} \frac{i e^{-i\omega(t-t')}}{\omega^2 - k^2 - m^2 + i\epsilon} \left[ e^{ik(x-x')} + K^\pm_b (k) e^{-ik(x+x')} \right]. \quad (3.1)
$$

The coefficients of the reflected term in the integrand of (3.1) correspond to the ‘classical’ reflection factors of the model linearised about the ground state solution $\phi = 0$,

$$
K^\pm_b (k) = \frac{ik \pm m}{ik \mp m} = \frac{i \sinh \theta \pm 1}{i \sinh \theta \mp 1}. \quad (3.2)
$$

In (3.2), the second form of the expression refers to the on-shell reflection factor for a particle with rapidity $\theta$ for which $k = m \cosh \theta$. 
One point which was not emphasised in [17] but which is important here is the fact that for the boundary condition $BC^-$ there is a boundary bound state even in the linearised theory. This reveals itself when calculating $(\partial^2 + m^2)G^\pm$. The first term in the integrand of (3.1) leads to the usual bulk contribution but the second, which can be evaluated by closing the $k$-contour in the upper half-plane (since $x + x' < 0$), will lead to an extra piece arising from the additional pole in $K_b^-$ at $k = im$. Since there is a normalizeable field configuration corresponding to the bound state field configuration $\phi \sim e^{mx}$ (which has zero frequency and decays exponentially for $x < 0$), there should have been an additional contribution to the propagator for the case $BC^-$ which would effectively remove the unwanted pole.

To examine this a little more, consider first a linear scalar field theory with mass parameter $m$ and boundary condition

$$\partial_x \phi = -\lambda \phi,$$  \hspace{1cm} (3.3)

at $x = 0$. There is a bound state solution $\phi \sim e^{-i\omega t}e^{-\lambda x}$ provided $-m < \lambda < 0$ and $\omega^2 = m^2 - \lambda^2$. In this situation, the free boson propagator takes the form (3.1) with

$$K_b(k) = \frac{ik + \lambda}{ik - \lambda},$$  \hspace{1cm} (3.4)

together with an extra piece

$$-2\lambda i \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}e^{-\lambda(x+x')}}{\omega^2 + \lambda^2 - m^2 + i\epsilon},$$  \hspace{1cm} (3.5)

constructed from the normalised bound state field. The $'i\epsilon'$ prescription has the usual meaning: that is, the contour of $\omega$-integration passes below the pole at $\omega = \sqrt{m^2 - \lambda^2}$ and above the pole at $\omega = -\sqrt{m^2 - \lambda^2}$. It is easy to check that the additional piece serves to remove the effect of the extra pole in $K_b$ at $k = -i\lambda$. For the supersymmetric sinh-Gordon model, $\lambda = -m$, and the additional piece is not defined in the $\epsilon \to 0$ limit. However, neither is the right-hand side of (3.1) because of the pole at $k = im$. On the other hand, happily, the two components (3.1) and (3.5), taken together, are well-defined. Thus, in this article, we shall adopt a pragmatic approach which uses the expression (3.1) but ignores the pole at $k = im$ wherever it occurs.

Next, let us consider the fermion propagator. We are familiar with the usual expression for a fermion propagator on the whole line. In two dimensions, with our choice of $\gamma$-matrices, it would be written

$$S_F(x - x') = \int \frac{d\omega}{2\pi} \frac{dk}{2\pi} \frac{ie^{-i\omega(t-t')}}{\omega^2 - k^2 - m^2 + i\epsilon} \left( \begin{array}{cc} m & -i(\omega + k) \\ i(\omega - k) & m \end{array} \right) e^{ik(x-x')}.$$  \hspace{1cm} (3.6)
In the presence of the boundary we need to modify the standard fermion propagator, ensuring not only that it performs as a propagator in the bulk but also that it respects the fermion part of the boundary conditions (2.7). Clearly, the usual relationship between boson and fermion propagators will no longer hold. Bearing this in mind, our expression for the fermion propagators is the following:

\[
S^\pm_F(x,t; x', t') = \int \frac{d\omega}{2\pi} \frac{dk}{2\pi} \frac{ie^{-i\omega(t-t')}}{\omega^2 - k^2 - m^2 + ie} \left[ \begin{pmatrix} m & -i(\omega + k) \\ i(\omega - k) & m \end{pmatrix} e^{ik(x-x')} \right] \sinh \theta \mp \frac{\omega}{ik \pm m} \begin{pmatrix} \omega - k & -im \\ im & \omega + k \end{pmatrix} e^{-ik(x+x')}. \tag{3.7}
\]

As far as the boundary conditions are concerned it is not difficult to verify that at \( x = 0 \),

\[
\left( S^\pm_F \right)_{1a} = \pm \left( S^\pm_F \right)_{2a}, \quad a = 1, 2. \tag{3.8}
\]

However, a calculation of \((-i\gamma \cdot \partial + m)S^\pm_F\), while giving the expected result for \( S^+_F \) (because the second term in the integrand integrates to zero on closing the contour in the upper half \( k \)-plane), reveals an additional contribution for \( S^-_F \) owing to the extra pole at \( k = im \). This pole reflects the fact that there is also a fermion bound state for \( BC^- \) (as there should be because of the supersymmetry) and that therefore the expression for the propagator \( S^-_F \) requires adjustment. Again, our pragmatic approach amounts to ignoring the extra pole and using (3.7) without alteration.

From the expression (3.7), it is natural to take the ‘classical’ fermion reflection factors to be given by

\[
K^\pm_f = \pm \frac{\omega}{ik \mp m} = \pm \frac{\cosh \theta}{i \sinh \theta \mp 1}, \tag{3.9}
\]

and, as before, the second expression refers to the the on-shell reflection factors which are related to the free bosonic reflection factors by,

\[
K^\pm_b = \pm i \sqrt{K^\pm_f}. \tag{3.10}
\]

4 The construction of the \( S \)-matrix and the reflection factors for the supersymmetric theory

An \( N = 1 \) supersymmetric theory contains a conserved Majorana supercharge. In terms of the chiral components \( Q_\pm \) of the supercharge we can write the on-shell supersymmetry
algebra as follows,

\[ Q^2 = me^{\pm \theta}, \quad \{Q_+, Q_-\} = 0, \quad \{Q_L, Q_\pm\} = 0, \quad (4.1) \]

where the operator \( Q_L \) has eigenvalue +1 on bosonic states and −1 on fermionic states. The single particle states of a massive supersymmetric theory contain either one boson or one fermion of mass \( m \), and will be denoted by \( |b(\theta)\rangle \) or \( |f(\theta)\rangle \) respectively.

It follows from the algebra (4.1) that the action of the supercharges \( Q_\pm \) on the one-particle states can be represented by

\[ Q_+ |b(\theta)\rangle = \sqrt{me^{\theta/2}} |f(\theta)\rangle, \quad Q_+ |f(\theta)\rangle = \sqrt{me^{\theta/2}} |b(\theta)\rangle, \]

\[ Q_- |b(\theta)\rangle = i\sqrt{me^{-\theta/2}} |f(\theta)\rangle, \quad Q_- |f(\theta)\rangle = -i\sqrt{me^{-\theta/2}} |b(\theta)\rangle, \quad (4.2) \]

corresponding to a realization of (4.1) in terms of the Pauli matrices

\[ Q_+ = \sqrt{me^{\theta/2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Q_- = \sqrt{me^{-\theta/2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Q_L = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (4.3) \]

The \( S \)-matrix in the supersymmetric theory is tightly constrained by the supersymmetry and has been given by Schoutens [9] in the following form:

\[ S = S_b(\Theta) S_s(\Theta). \quad (4.4) \]

Here, \( S_b \) is the \( S \)-matrix for the sinh-Gordon model without fermions [19], \( S_s \) is the supersymmetric part, responsible for mixing bosons and fermions, and \( \Theta \) is the rapidity difference of the scattering particles. In detail, using a shorthand ‘bracket’ notation [20] we have

\[ S_b(\Theta) = -\frac{1}{(B)(2-B)} \begin{pmatrix} \sinh \left( \frac{\Theta}{2} + \frac{\pi x}{4} \right) \\ \sinh \left( \frac{\Theta}{2} - \frac{\pi x}{4} \right) \end{pmatrix}, \quad (x) = \sinh \left( \frac{\Theta}{2} + \frac{\pi x}{4} \right) \sinh \left( \frac{\Theta}{2} - \frac{\pi x}{4} \right) \quad (4.5) \]

where the coupling constant enters via

\[ B(\beta) = \frac{1}{2\pi} \frac{\beta^2}{1 + \beta^2/4\pi}, \quad (4.6) \]

and,

\[ S_s(\Theta) = f(\Theta) \begin{pmatrix} 1 - \tanh^2 \frac{\Theta}{4} & 0 & 0 & -2i \tanh \frac{\Theta}{4} \\ 0 & 0 & 1 + \tanh^2 \frac{\Theta}{4} & 0 \\ 0 & 1 + \tanh^2 \frac{\Theta}{4} & 0 & 0 \\ -2i \tanh \frac{\Theta}{4} & 0 & 0 & 1 - \tanh^2 \frac{\Theta}{4} \end{pmatrix} \]

\[ + g(\Theta) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (4.7) \]
where
\[
f(\Theta) = f_0 \left( \frac{\cosh \frac{\Theta}{2} + 1}{\sinh \Theta} \right) g(\Theta),
\]  
\[
g(\Theta) = \frac{\sinh \frac{\Theta}{2}}{\sinh \frac{\Theta}{2} - i \sin \rho \pi} \exp \left[ -i \int_0^\infty \frac{dt}{t} \frac{\sinh \rho t}{\cosh^2 \frac{t}{2} \cosh t} \frac{\sin \Theta t}{\pi} \right].
\]

The parameter \( \rho \) depends on the coupling \( \beta \) and we expect \( \rho = B/4 \) in our conventions.

Moriconi and Schoutens [16] assumed that the reflection matrix can be factorised similarly and will therefore take the form,
\[
R(\theta) = R_b(\theta) R_s(\theta).
\]

Here, \( \theta \) is the rapidity of the reflecting particle, \( R_b(\theta) \) would be the reflection matrix for the bosonic part of the theory in the absence of fermions, and \( R_s \) is the supersymmetric part. Assuming further that the boundary does not convert bosons to fermions, or vice-versa, supersymmetry constrains \( R_s \) to have the form
\[
R^\pm_s(\theta) = Z^\pm_s(\theta) \begin{pmatrix} \cosh(\frac{\theta}{2} \pm \frac{i\pi}{4}) & 0 \\ 0 & \cosh(\frac{\theta}{2} + \frac{i\pi}{4}) \end{pmatrix}.
\]

Thus, we may write equivalently:
\[
K^\pm_b(\theta) = R^\pm_b(\theta) Z^\pm(\theta) \cosh \left( \frac{\theta}{2} \pm \frac{i\pi}{4} \right)
\]  
\[
K^\pm_f(\theta) = R^\pm_b(\theta) Z^\pm(\theta) \cosh \left( \frac{\theta}{2} + \frac{i\pi}{4} \right).
\]

It is interesting to notice that the ratios of boson to fermion reflection factors do not depend on anything other than the rapidity. In fact from (4.10) and (4.11) we deduce,
\[
\frac{K^\pm_b(\theta)}{K^\pm_f(\theta)} = \frac{\cosh(\frac{\theta}{2} \pm \frac{i\pi}{4})}{\cosh(\frac{\theta}{2} + \frac{i\pi}{4})} = \frac{1 \pm i \sinh \theta}{\cosh \theta},
\]
which is in perfect agreement with the ratios of the classical reflection factors given in (3.2) and (3.9). In the classical limit the complete reflection matrix must match the boson and fermion classical factors. This requires particular classical limits for \( Z^\pm(\theta) \), namely
\[
Z^\pm(\theta) \to \frac{1}{\cosh \left( \frac{\theta}{2} \pm \frac{i\pi}{4} \right)}.
\]

In addition, the factor \( Z^\pm(\theta) \) is constrained by the requirements of unitarity
\[
R(\theta) R(-\theta) = 1,
\]
and by boundary crossing unitarity \[14, 21\]

\[
1 = \sum_{c=b,f} K_b \left( \theta - \frac{i\pi}{2} \right) S_{cc}^{bb}(2\theta) K_c \left( \theta + \frac{i\pi}{2} \right),
\]

(4.16)

which must be satisfied by the full reflection factor. Because of the factorised forms of both the S-matrix and the reflection factor, and the fact that we shall choose the bosonic part of the reflection factor to satisfy (4.16) in conjunction with the sinh-Gordon S-matrix (4.5), these conditions lead to

\[
Z^\pm(\theta)Z^\pm(-\theta) = 2/\cosh \theta,
\]

(4.17)

\[
\frac{Z^\pm \left( \frac{i\pi}{2} - \theta \right)}{Z^\pm \left( \frac{i\pi}{2} + \theta \right)} = \mp S_{bb}^{bb}(2\theta) + i \left( \coth \frac{\theta}{2} \right)^{\pm1} S_{ff}^{bb}(2\theta),
\]

(4.18)

where \(S_{bb}^{bb}(2\theta)\) and \(S_{ff}^{bb}(2\theta)\) should be extracted from (4.7).

Given the classical limits (4.14) it is natural to set

\[
Z^\pm = \frac{\tilde{Z}^\pm}{\cosh \left( \frac{\theta}{2} \pm \frac{i\pi}{4} \right)},
\]

(4.19)

in which case,

\[
\tilde{Z}^\pm(\theta)\tilde{Z}^\pm(-\theta) = 1
\]

(4.20)

\[
\frac{\tilde{Z}^\pm \left( \frac{i\pi}{2} - \theta \right)}{\tilde{Z}^\pm \left( \frac{i\pi}{2} + \theta \right)} = \frac{\sinh \theta \mp 2f_0}{\sinh \theta - i \sin \rho \pi} \exp \left[ -i \int_0^\infty \frac{dt}{t} \frac{\sinh \rho t \sinh (1 + \rho) t}{\cosh^2 \frac{t}{2} \cosh t} \sin \frac{\theta t}{\pi} \right].
\]

Then the solutions we want will satisfy

\[
\tilde{Z}^\pm(\theta) \rightarrow 1
\]

(4.21)

in the classical limit. Clearly, \(f_0 = \pm (i/2) \sin \rho \pi\), and we shall take \(f_0 = -(i/2) \sin \rho \pi\).

The equations (4.20) are solved by:

\[
\tilde{Z}^- (\theta) = \exp \left[ \frac{i}{2} \int_0^\infty \frac{dt}{t} \frac{\sinh \rho t \sinh (1 + \rho) t}{\cosh^2 \frac{t}{2} \cosh t} \sin \frac{2\theta t}{\pi} \right],
\]

(4.22)

and

\[
\tilde{Z}^+ (\theta) = \exp \left[ -2i \int_0^\infty \frac{dt}{t} \frac{\sinh \frac{\rho}{2} t \sinh \left( \frac{1 + \rho}{2} \right) t}{\cosh^2 \frac{t}{2} \cosh t} \sin \frac{\theta t}{\pi} \right]
\]

\[
\exp \left[ \frac{i}{2} \int_0^\infty \frac{dt}{t} \frac{\sinh \rho t \sinh (1 + \rho) t}{\cosh^2 \frac{t}{2} \cosh t} \sin \frac{2\theta t}{\pi} \right].
\]

(4.23)
Notice that these are not quite the same as the proposals made in [16] since Moriconi and Schoutens took the view that the classical limit of a free boson reflection factor should be unity; an assumption which is not generally valid, as we have seen.

Ghoshal [18] has calculated a formula for the quantum reflection matrix for the breather states of the sine-Gordon model. The reflection factor for the sinh-Gordon model is presumed to be deduced from the lightest breather reflection factor in the sine-Gordon theory by analytic continuation in the coupling constant (replacing $\beta$ by $i\beta$), leading to the expression

$$R_b(\theta) = \frac{(2 - B/2)(1 + B/2)}{(1 - E)(1 + E)(1 - F)(1 + F)},$$

where the coupling dependent functions $E$ and $F$ also depend on the boundary parameters introduced via the boundary potential. In the supersymmetric theory, we consider the boundary conditions (2.7) for which $F = 0$. On the other hand, an expression for $E$ has been found recently by comparing two independent calculations of the boundary breather spectrum [22]. This translates in the present situation with two possible boundary conditions (2.7) to

$$BC^+ : \quad E = 0, \quad BC^- : \quad E = 2(1 - B/2).$$

Thus we take

$$R_b^+ = \frac{(1 + B/2)(2 - B/2)}{(1)^3},$$

and

$$R_b^- = \frac{(1 + B/2)(2 - B/2)(1 + B)(1 - B)}{(1)}.$$  \hspace{1cm} (4.27)

Notice that (4.27) contains the bound-state pole (in the factor $(1 - B)$) at $\theta = i(1 - B)\pi/2$, whereas (4.26) contains no bound states. The suggestions made by Moriconi and Schoutens were different but for comparison we list them here:

$$R_b^+ = \frac{(2 - B/2)}{(1 + B/2)(1)},$$

and

$$R_b^- = (1) (1 + B/2)(2 - B/2),$$

$^1$E and $F$ are related to the parameters $\eta$ and $\vartheta$ in Ghoshal’s notation by

$$E = \frac{\eta}{\pi} B, \quad F = \frac{i\vartheta}{\pi} B.$$
corresponding to \( E = B/2 \) and \( E = 2 \), respectively. One could argue that the fermions ought to modify the conclusions reached in \[22\] in such a manner as to prevent any renormalisation of the position of the bound state pole. In that sense, \((4.29)\) would seem to be a better guess since the boundary bound state retains its position at \( \theta = i\pi/2 \). We note too, that the two expressions \((4.26)\) and \((4.29)\) share the property of being invariant under the duality transformation \( B \rightarrow 2 - B \). However, we are not sure too much should be read into this since the other factors \( Z^\pm(\theta) \) do not share the same property. An appeal to lowest order perturbation theory does not help either since to order \( \beta^2 \) we have identical expansions for \((4.26)\) and \((4.28)\),

\[
R_0^+ \sim \left( \frac{i \sinh \theta + 1}{i \sinh \theta - 1} \right) \left[ 1 - \frac{i \beta^2}{8} \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) \right], \tag{4.30}
\]
as indeed we do for \((4.27)\) and \((4.29)\),

\[
R_0^- \sim \left( \frac{i \sinh \theta - 1}{i \sinh \theta + 1} \right) \left[ 1 - \frac{i \beta^2}{8} \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) \right]. \tag{4.31}
\]

It should be possible to distinguish between \((4.27)\) and \((4.29)\) by following the semi-classical quantization of the boundary breathers taking the fermions into account. However, this analysis has not yet been carried out.

To conclude this section we shall prepare the way for comparing the reflection factors with low order perturbation theory by giving their expansions to order \( \beta^2 \). This is straightforward apart from a couple of complicated integrals arising from the \( Z \)-factors. For example, setting \( \rho \sim \rho_0 \beta^2/8\pi \), we have

\[
\tilde{Z}^\pm(\theta) \sim 1 - \rho_0 \frac{i \beta^2}{16\pi} \left[ 2 \int_0^\infty dt \frac{\sinh \frac{t}{2}}{\cosh^2 \frac{t}{2}} \sin \frac{t\theta}{\pi} - \int_0^\infty dt \frac{\sinh t}{\cosh^2 \frac{t}{2} \cosh t \sin 2t\theta} \frac{2t\theta}{\pi} \right] - \rho_0 \frac{i \beta^2}{16\pi} \left[ 2 \theta \cosh \theta - \pi \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) \right], \tag{4.32}
\]

where we have used the following two facts \[25\] :

\[
\int_0^\infty dt \frac{\sinh \frac{t}{2}}{\cosh^2 \frac{t}{2}} \sin \frac{t\theta}{\pi} = \frac{2\theta}{\cosh \theta}
\]

\[
\int_0^\infty dt \frac{\sinh t}{\cosh^2 \frac{t}{2} \cosh t \sin 2t\theta} \frac{2t\theta}{\pi} = \frac{2\theta}{\cosh \theta} + \pi \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right). \tag{4.33}
\]
Combining, \((4.30)\) and \((4.32)\) we deduce expressions for the supersymmetric reflection factors corresponding to the boundary conditions \(BC^+\) to order \(\beta^2\):

\[
K^+_b(\theta) \sim \frac{i \sinh \theta + 1}{i \sinh \theta - 1} M^+(\theta), \quad K^+_f(\theta) \sim \frac{\cosh \theta}{i \sinh \theta - 1} M^+(\theta),
\]

\[
M^+(\theta) = 1 - \frac{i \beta^2}{16\pi} \left( (2 - \rho_0)\pi \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) + \frac{2\rho_0 \theta}{\cosh \theta} \right). \quad (4.34)
\]

In a similar manner, the expansions of the reflection factors corresponding to the boundary conditions \(BC^-\) are:

\[
K^-_b(\theta) \sim \frac{i \sinh \theta - 1}{i \sinh \theta + 1} M^-(\theta), \quad K^-_f(\theta) \sim \frac{i \sinh \theta - 1}{\cosh \theta} M^-(\theta),
\]

\[
M^-(\theta) = 1 - \frac{i \beta^2}{16\pi} \left( (2 - \rho_0)\pi \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) - \frac{2\rho_0 \theta}{\cosh \theta} \right). \quad (4.35)
\]

5 Generating functional and two-point functions

Using a path integral formalism and perturbation theory, one can obtain an expression for the generating functional for the supersymmetric sinh-Gordon model up to one-loop order. It is given by

\[
Z = \left\{ 1 + \frac{i}{12} \beta^2 \int d^2x \left[ \theta(-x)m^2 \pm \frac{1}{4} \delta(x)m \right] \left[ 6G^\pm(x,x) \left( \int d^2y G^\pm(x,y)J(y) \right)^2 \right. \\
- \left. \left( \int d^2y G^\pm(x,y)J(y) \right)^4 \right] + \frac{i}{4} m \beta^2 \int d^2x \theta(-x) \left[ -G^\pm(x,x) \int d^2y S_F^\pm(x,y)\eta(y) \int d^2z \bar{\eta}(z) S_F^\pm(z,x) \\
- S_F^\pm(x,x) \left( \int d^2y G^\pm(x,y)J(y) \right)^2 \right. \\
+ \left. \left( \int d^2y G^\pm(x,y)J(y) \right)^2 \int d^2z S_F^\pm(x,z)\eta(z) \int d^2w \bar{\eta}(z) S_F^\pm(w,x) \right] \right\} Z_0. \quad (5.1)
\]

Using this we can proceed to evaluate up to the same order the boson and fermion two-point functions which are defined by:

\[
G(x_1, x_2) = \left. \frac{\delta^2 Z}{i^2 \delta J(x_1) \delta J(x_2)} \right|_{J=\eta=\bar{\eta}=0}, \quad (5.2)
\]

\[
S(x_1, x_2) = \left. \frac{\delta^2 Z}{i^2 \delta \eta(x_1) \delta \bar{\eta}(x_2)} \right|_{J=\eta=\bar{\eta}=0}. \quad (5.3)
\]
Using (5.1), we deduce an expression for the boson two-point function in the form

\[ G(x_1, x_2) = G^\pm(x_1, x_2) - i\beta^2 \int d^2x \left[ \theta(-x)m^2 + \frac{1}{4}\delta(x)m \right] G^\pm(x, x) G^\pm(x_1, x_1) G^\pm(x, x_2) \]

\[ + \frac{im\beta^2}{2} \int d^2x \theta(-x)S^\pm(x, x) G^\pm(x, x_1) G^\pm(x, x_2). \]  

(5.4)

This are represented conveniently by means of the Feynman diagrams in Figure 1. However, it must be remembered that the diagrams are not the usual momentum space diagrams.

![Figure 1: Correction to the boson propagator.](image)

Diagram (a) is the tree level boson propagator and the diagrams (b) and (c) represent the boson and fermion one loop corrections to the boson two point function respectively.

The fermion two-point function is given similarly by

\[ S(x_1, x_2) = S_F^\pm(x_1, x_2) - \frac{i}{4}m\beta^2 \int d^2x \theta(-x)S_F^\pm(x, x) S_F^\pm(x_1, x_1) S_F^\pm(x, x_2). \]  

(5.5)

which may be represented similarly by the Feynman diagrams of Figure 2.

![Figure 2: Correction to the fermion propagator.](image)

6 The fermion reflection factors

In this section, we first calculate the fermion reflection factor corresponding to the case \( BC^+ \) when the boundary condition is \( \psi_1 = \psi_2 \).

\footnote{For convenience we will denote the propagators \( G^\pm(x_1, t_1; x_2, t_2) \) and \( S_F^\pm(x_1, t_1; x_2, t_2) \) by \( G^\pm(x_1, x_2) \) and \( S_F^\pm(x_1, x_2) \), respectively.}
Since there is no additional boson-fermion coupling arising from the boundary terms in (2.5), the correction to the fermion propagator will come from the bulk coupling. This contribution corresponds to the diagram Fig 2(b) and in detail it is:

\[- \frac{i}{4} m^2 \int_{-\infty}^{+\infty} dt'' \int_{-\infty}^{0} dx'' S_F^+ (x, t; \ x'', t'') \ G^+ (x'', t''; \ x'', t'') \ S_F^+ (x'', t'') \text{ for } x', t', (6.1)\]

with the fermion propagator given by (3.7). The loop corresponds to the integral

\[ \int \frac{d\omega''}{2\pi} \int \frac{dk''}{2\pi} \frac{i}{\omega'' - \omega'' m^2 + i\varepsilon} \left[ 1 + K_b^+(k'') e^{-i2k'' x''} \right]. \tag{6.2} \]

which needs a counter-term to remove the divergence. The energy integral is finite for the second term, and the finite part of the loop integral will be

\[ \int \frac{dk''}{2\pi} \frac{1}{2\sqrt{k''} + m^2} K_b^+(k'') e^{-i2k'' x''}. \tag{6.3} \]

Inserting this together with the fermion propagators in expression (6.1), we have

\[ - \frac{i}{8} \frac{m^2}{2\pi} \int \frac{d\omega}{2\pi} \frac{dk}{2\pi} \frac{dk'}{2\pi} \frac{dk''}{2\pi} \frac{ie^{-i\omega(t-t')}}{\omega^2 - k^2 - m^2 + i\varepsilon} \frac{ie^{-ikx-ik'x'}}{\omega^2 - k'^2 - m^2 + i\varepsilon} \frac{K_b^+(k'')}{\sqrt{k''^2 + m^2}} \]

\[ \int_{-\infty}^{0} \frac{dx''}{2\pi} \left[ \mathcal{P}(\omega, k, k') \left( e^{ix''(k+k'-2k'')} + K_f^+(k)K_f^+(k')e^{ix''(-k-k'-2k'')} \right) \right. \]

\[ + \mathcal{Q}(\omega, k, k') \left( K_f^+(k')e^{ix''(k+k'-2k'')} + K_f^+(k)e^{ix''(-k-k'-2k'')} \right) \tag{6.4} \]

where

\[ \mathcal{P}(\omega, k, k') = \begin{pmatrix} m^2 + (\omega - k)(\omega - k') & -2im\omega + im(k - k') \\ 2im\omega + im(k - k') & m^2 + (\omega + k)(\omega + k') \end{pmatrix} \]

\[ \mathcal{Q}(\omega, k, k') = \begin{pmatrix} 2m\omega - m(k + k') & -m^2i - i(\omega - k)(\omega + k') \\ m^2i + i(\omega + k)(\omega - k') & 2m\omega + m(k + k') \end{pmatrix}. \tag{6.5} \]

The next step is to perform the \( x'' \) integrals by using the following device in which we set

\[ \int_{-\infty}^{0} dx'' e^{ikx+\sigma x''} = \frac{-i}{k - i\sigma}. \tag{6.6} \]

where \( \sigma \) is a small positive constant, and take the limit \( \sigma \to 0 \) at the end of the calculation. Using this we have

\[- \frac{i}{4} m^2 \int \frac{d\omega}{2\pi} \int \frac{dk}{2\pi} \int \frac{dk'}{2\pi} \frac{ie^{-i\omega(t-t')}}{\omega^2 - k^2 - m^2 + i\varepsilon} \frac{ie^{-ikx-ik'x'}}{\omega^2 - k'^2 - m^2 + i\varepsilon} \int \frac{dk''}{2\pi} \frac{K_b^+(k'')}{2\sqrt{k''^2 + m^2}}. \]
For these, the contours should be closed in the upper half plane (because 
Then, these integrals can be evaluated using the useful formula 
in (6.7):
\[
\begin{pmatrix}
m^2 + (\omega - k)(\omega - k') & -2im\omega + im(k - k') \\
2im\omega + im(k - k') & m^2 + (\omega + k)(\omega + k')
\end{pmatrix}
\begin{pmatrix}
-i \\
\frac{k + k' - 2k'' - i\sigma}{k + k' - 2k'' - i\sigma}
\end{pmatrix}
+ \begin{pmatrix}
2m\omega - m(k + k') \\
m^2i + i(\omega + k)(\omega - k')
\end{pmatrix}
\begin{pmatrix}
-\frac{iK_0^+(k')}{k - k' - 2k'' - i\sigma} \\
\frac{K_0^+(k)}{k - k' - 2k'' - i\sigma}
\end{pmatrix}
+ \begin{pmatrix}
2m\omega - m(k + k') \\
im^2 + i(\omega + k)(\omega - k')
\end{pmatrix}
\begin{pmatrix}
\frac{K_0^+(k)}{k - k' - 2k'' - i\sigma} \\
\frac{iK_0^+(k)K_0^+(k')}{k + k' + 2k'' + i\sigma}
\end{pmatrix}.
\tag{6.7}
\]
We can integrate out every $k''$ integral by closing its contour in the upper half plane and placing the branch cuts from $im$ to $i\infty$. This way, we can avoid the pole contribution from $K_0^+(k'')$. The other poles, involving $\sigma$, are also avoided because $\sigma > 0$.

Consider each term of the $k''$ integral in turn starting with the first. It can be decomposed into partial fractions and reexpressed as an integral over the cut on the imaginary axis,
\[
\int \frac{dk''}{2\pi} \frac{K_0^+(k'')}{2\sqrt{k''^2 + m^2}} \frac{-i}{k + k' - 2k'' - i\sigma}
= \frac{1}{4\pi} \int_m^{\infty} dy \frac{1}{\sqrt{y^2 - m^2}} \left[ \frac{1 - K_0^+ \left( \frac{k + k'}{2} \right)}{y + m} + \frac{K_0^+ \left( \frac{k + k'}{2} \right)}{y + i(k + k')/2} \right].
\tag{6.8}
\]
Then, these integrals can be evaluated using the useful formula
\[
\int_m^{\infty} dy \frac{1}{\sqrt{y^2 - m^2}} \frac{1}{y + 2a} = \frac{2}{\sqrt{m^2 - 4a^2}} \left( \frac{\pi}{2} - \tan^{-1} \left( \frac{m + 2a}{m - 2a} \right) \right).
\tag{6.9}
\]
By changing the variable again via $y = m \cosh \theta$, we obtain the result for the first $k''$-integral in (6.7):
\[
\frac{1}{4\pi} \left[ \frac{1 - K_0^+ \left( \frac{k + k'}{2} \right)}{m} + \frac{2K_0^+ \left( \frac{k + k'}{2} \right)}{\sqrt{m^2 + (k + k')^2/4}} \left( \frac{\pi}{2} - \tan^{-1} \left( \frac{2m + i(k + k')}{2m - i(k + k')} \right) \right) \right].
\tag{6.10}
\]
Having obtained the first $k''$ integral in (6.7), we now perform the integrals over $k$ and $k'$. For these, the contours should be closed in the upper half plane (because $x, x' < 0$) to yield
\[
\frac{1}{4\pi} e^{-ik(x + x')} \frac{\omega}{2k^2} \begin{pmatrix}
\omega - \hat{k} \\
im \omega + \hat{k}
\end{pmatrix}
\left[ \frac{1 - K_0^+ (\hat{k})}{m} + \frac{K_0^+ (\hat{k})}{\cosh \theta \left( \frac{\pi}{4} - \frac{i\theta}{2} \right)} \right],
\tag{6.11}
\]
where $\hat{k} = \sqrt{\omega^2 - m^2}$.

The remaining three terms in (6.7) can be computed in a similar fashion, except that $k + k'$ is replaced by one of $k - k'$, $-k + k'$ and $-k - k'$. Combining the results of all these
calculations we have

\[ -\frac{i}{16\pi} m^2 \beta \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} e^{-i\omega(x-x')} \frac{1}{2\hat{k}} \begin{pmatrix} \omega - \hat{k} & -2i \\ 2i & \omega + \hat{k} \end{pmatrix} \frac{K_+^\dagger(\hat{k})}{2\pi m \sinh \theta} \left[ 2\pi \sinh^2 \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) + \frac{4\theta \sinh \theta}{\cosh \theta} \right]. \]  

(6.12)

From this we extract the fermion reflection factor by selecting the coefficient of the reflected term of the two-point function in the asymptotic region far away from the boundary. Thus, in detail we find

\[ K_f^+ (\hat{k}) = K_f^+ (\hat{k})_{\text{class}} \left[ 1 - \frac{i\beta^2}{16\pi} \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) - \frac{i\beta^2}{8\pi} \frac{\theta}{\cosh \theta} \right]. \]  

(6.13)

This agrees precisely with (4.34) provided we take \( \rho_0 = 1 \) which is entirely consistent with \( \rho(\beta) = B/4 \).

The other fermion reflection factor, corresponding to the boundary condition \( \psi_1 = -\psi_2 \), can be calculated to the same order in a similar manner to obtain

\[ R_f^- (\hat{k}) = K_f^- (\hat{k})_{\text{class}} \left[ 1 - \frac{i\beta^2}{16\pi} \sinh \theta \left( \frac{1}{\cosh \theta + 1} - \frac{1}{\cosh \theta} \right) + \frac{i\beta^2}{8\pi} \frac{\theta}{\cosh \theta} \right]. \]  

(6.14)

The calculations leading to (6.14) must be performed carefully in view of the potential difficulties with the bound-state poles we alluded to earlier. These problems may be circumvented by a judicious choice of contour, picking up the poles related to \( \sigma \) and carefully taking the limit \( \sigma \rightarrow 0 \) at the end of the calculation. The expression (6.14) also agrees with the expression for the fermionic reflection factor corresponding to the boundary conditions \( BC^- \) which was quoted in (4.33).

7 Conclusion

In this paper, we have studied the boundary fermion reflection factors for the supersymmetric sinh-Gordon model perturbatively up to one loop. It is gratifying that the results are in agreement with various conjectures obtained on general grounds but disappointing that the calculations so far fail to distinguish between the two favoured proposals given in (4.26), (4.27), (4.28) and (4.29). Similar calculations may be made to check the boson reflection factors but there is a stumbling block which we have so far failed to overcome. In order to perform any calculation of this kind we need to remove infinite parts of loop integrals and this turns out to be straightforward for boson loops, as we have described, but much
less clear for the fermion loops (which contain more divergences). In fact, at the moment it is not even clear to us that there is a regularization scheme which will maintain the supersymmetry manifest in the classical model. We shall return to this important question on another occasion but the heart of the matter appears to be a need to make subtractions which do not correspond to terms occurring in the original Lagrangian density, especially at the boundary. If this is really the case, then the deductions about the reflection factors made on general grounds using supersymmetry could be suspect. Another approach, along the lines proposed in \cite{22}, if it can be developed supersymmetrically, should give information on the boundary bound states which are present in the case $\mathcal{BC}^-$. In particular, we are interested in knowing if these states have energies which ‘float’ with the coupling $\beta$. Basic questions concerning the model may also be approached by alternative means, such as the Thermodynamic Bethe Ansatz, which will require a knowledge of the reflection factors as an input (see, for example a recent article by Ahn and Nepomechie \cite{24}.)

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