Verification of the Standard Theory of Plasma Emission with Particle-in-cell Simulations

Zilong Zhang1,2, Yao Chen1,2, Sulan Ni1,2, Chuanyang Li1,2, Hao Ning1,2, Yaokun Li1,2, and Xiangliang Kong1,2

Institute of Space Sciences, Shandong University, Shandong, 264209, People’s Republic of China; yaochen@sdu.edu.cn
2 Institute of Frontier and Interdisciplinary Science, Shandong University, Qingdao, Shandong, 266237, People’s Republic of China

Received 2022 May 3; revised 2022 September 22; accepted 2022 September 22; published 2022 November 3

Abstract

The standard theory of plasma emission is based on kinetic couplings between a single beam of energetic electrons and unmagnetized thermal plasmas, involving multistep nonlinear wave–particle and wave–wave interactions. The theory has not yet been completely verified with fully kinetic electromagnetic particle-in-cell (PIC) simulations. Earlier studies, greatly limited by available computational resources, are controversial regarding whether the fundamental emission can be generated according to the standard theory. To resolve the controversy, we conducted PIC simulations with a large domain of simulations and a large number of macroparticles, among the largest ones of similar studies. We found significant fundamental emission if the relative beam density is small enough (say, \(\leq 0.01\)), in line with an earlier study with a much smaller domain; the relative intensity (normalized by the total initial beam energy) of all modes, except the mode associated with the beam-electromagnetic Weibel instability, decreases with the increasing relative density of the beam. We also found a significant transverse magnetic component associated with the superluminal Langmuir turbulence, which has been mistakenly regarded as evidence of the F emission in the earlier study. Further investigations are required to reveal their origin.

Unified Astronomy Thesaurus concepts: Solar corona (1483); Solar activity (1475); Radio bursts (1339); Solar coronal radio emission (1993); Plasma astrophysics (1261)

Supporting material: animations

1. Introduction

The plasma emission (PE) refers to electromagnetic radiation at frequencies close to the plasma oscillation frequency (\(\omega_{pe}\)) and its harmonics, corresponding to the fundamental (F) and harmonic (H) plasma emission, respectively. The framework of the standard theory of PE was suggested more than six decades ago (Ginzburg & Zhelezniakov 1958), which involves a multistep nonlinear process of wave–particle and wave–wave interactions due to the presence of energetic beam electrons in plasmas. The theory starts from the beam-driven kinetic bump-on-tail instability that leads to enhanced Langmuir waves, followed by the decay of these waves or their scattering over ion-related fluctuations generating the fundamental (F) emission and/or the backward-propagating Langmuir wave, and the nonlinear interaction of forward- and backward-propagating Langmuir waves generating the harmonic (H) emission (e.g., Melrose 1980; Cairns 1987; Melrose 1987; Robinson et al. 1994; Schmidt & Cairns 2012; Li & Cairns 2013; Schmidt & Cairns 2014; Li & Cairns 2014; Thurgood & Tsiklauri 2015; Che et al. 2017; Henri et al. 2019).

The standard theory has been widely used to explain radio bursts in space and astrophysical plasmas, such as solar radio bursts in terms of type I–V (see, e.g., Chen et al. 2014; Vasanth et al. 2016; Lv et al. 2017; Li et al. 2017; Vasanth et al. 2019, for the latest observational studies), emissions from planetary electron foreshocks (e.g., Moses et al. 1984; Etcheto & Fauch eux 1984; Cairns 1987; Kuncic & Cairns 2005; Pša et al. 2017) and the outer heliosphere (Kurth et al. 1984; Zank et al. 1994; Gurnett & Kurth 1995; Cairns 1995, 1998; Gurnett et al. 1998; Cairns & Zank 2002; Tasnim et al. 2022). The theory involves the evolution of turbulence and complicated multistep nonlinear processes. A proper numerical verification of the theory requires kinetic simulations by the nature of the problem. Both ion-scale and electron-scale kinetic physics should be included. This excludes any fluid/hybrid models. Only fully kinetic methods can be used, including the Vlasov method and the PIC method.

The Vlasov method directly solves the Vlasov–Maxwell equations to resolve the velocity distribution function of particles. The weak-turbulence (WT) theory/simulation represents a variety of this method tailored for the study of plasma emission (e.g., Yoon 2006; Yoon et al. 2012; Ziebell et al. 2015; see Lee et al. 2019 for the complete list of references).

The PIC simulations numerically resolve the motion of each particle and the evolution of the electromagnetic fields by solving self-consistently the equation of particle motion and the full set of the Maxwell equations. In comparison to the WT method, the PIC method has the advantage of making no approximations to the basic laws of mechanics and electromagnetism. We therefore choose the PIC method for the present study. See Lee et al. (2019) for a detailed comparison of the two methods when being applied to the problem of plasma emission. Earlier studies along this line of research include Kasaba et al. (2001), Karlický & Vandas (2007), Rhee et al. (2009a), Rhee et al. (2009b), Umeda (2010), Ganse et al. (2012a), and Ganse et al. (2012b), and the latest studies include Thurgood & Tsiklauri (2015), Henri et al. (2019), Ni et al. (2020), and Chen et al. (2022). The PIC simulations demand massive computing and turn out to be challenging, as elaborated below.

1. The number of macroparticles per cell per species (NPPCPS) should be large enough to lower the levels of numerical noise. Most earlier simulations have adopted values of NPPCPS less than a few hundred...
(e.g., Kasaba et al. 2001; Umeda 2010) for economic reasons. This results in the relatively strong noise and low signal-noise ratio of the obtained F/H emissions since they are intrinsically weak. The situation becomes worse for weaker beams.

2. The simulation domain should be large enough to have a good resolution of the wavenumber \( k \). This is necessary for the F emission whose frequency upon excitation is close to the cutoff \( \sim \omega_{pe} \). Since according to the standard theory of PE the F emission is given by the scattering of the Langmuir (L) wave over the low-frequency ion-acoustic (IA) mode, its frequency is thus the sum or difference of the frequencies of the L and IA modes. The IA frequency is negligible compared to the Langmuir frequency \( \sim \omega_{pe} \), and the frequency of the F emission is therefore very close to \( \omega_{pe} \), which is the cutoff frequency of the fast electromagnetic mode propagating in unmagnetized plasmas. It is therefore characterized by a long wavelength \( \lambda \) and small \( k \). According to the simplified dispersion relation of electromagnetic wave \( \omega^2 = \omega_{pe}^2 + k^2 \), one gets \( \lambda \gg 1500 \lambda_{De} \) (the electron Debye length) for coronal plasmas at 1–2 MK (with a thermal speed of \( \sim 0.02c \)). The minimum dimension of the domain should be larger than a few wavelengths for proper simulation (see, e.g., Henri et al. 2019).

3. The temporal length of data used for the Fourier spectral analysis should be long enough to resolve modes with close frequencies. This is important since the F emission has almost the same frequency as the Langmuir mode. For instance, if the data length is taken to be \( \sim 100 \omega_{pe}^{-1} \), the two modes cannot be separated when their frequency difference is less than \( 0.1 \omega_{pe} \).

The above elaborations point out the need for massive computation, while most earlier simulations did use an insufficient number of macroparticles and a limited domain. See the critical review by Thurgood & Tsiklauri (2015, referred to as TT15 hereinafter). Note the domain is regarded as limited here if it is comparable to or only a few (say, \( \sim 2–3 \)) times larger than the wavelength of the F emission. These limitations cause excessive numerical noise along dispersion curves and limited resolution of wave modes. The conclusions drawn are thus controversial regarding the generation of the F emission. For example, TT15 claimed that they presented “the first self-consistent demonstration of F and H emission from a single-beam (unmagnetized) plasma system via fully kinetic PIC simulations”—in accordance with the standard theory of plasma emission. Henri et al. (2019), however, found that the F emission is hardly discernible according to their simulations with domain and NPPCPs being much larger than that of TT15. Thus, it remains open regarding whether the F emission does occur according to the standard theory (from the perspective of PIC verification), despite the positive statement of TT15, which is nevertheless inconclusive according to the following arguments.

1. The domain dimension of TT15 is 600 \( \lambda_{De} \), much less than the expected wavelength of the F emission. According to TT15, the expected \( k \lambda_{De} \) of the F emission is about 0.002, corresponding to a wavelength of a few thousands of \( \lambda_{De} \).

2. The duration of the data for the Fourier analysis is 100 \( \omega_{pe}^{-1} \), the corresponding spectral resolution is less than 0.1\( \omega_{pe} \), too poor to separate the F emission from nearby Langmuir waves with the obtained \( \omega-k \) diagram.

3. According to TT15, one evidence of the F emission is the growth of the transverse component of the magnetic field (see Figure 5 in TT15). Yet, such a signal may have two contributions, according to our study presented here. One is given by the expected F emission, the other is by the Langmuir turbulence. The latter is found with \( k \lambda_{De} \) varying in the range of [0, 0.02], much broader than the expected \( k \lambda_{De} \) range of the F emission. Such a signal is dominated by the Langmuir turbulence rather than by the F emission. This point cannot be inferred from earlier studies due to poor resolutions of both \( \omega \) and \( k \).

Further, according to the dispersion relation of the fast electromagnetic wave propagating in unmagnetized plasmas and Faraday’s law, the relative strength of the wave \( E \) and \( B \) fields is determined by the ratio of \( \omega^2/k^2 c^2 \). Thus, for the F emission with frequency \( \omega \) very close to \( \omega_{pe} \), the \( E \) field is much stronger than the \( B \) field, so the F emission is dominated by the \( E \) field. It is therefore critical to examine the \( E \)-field data to reveal its spectral characteristics and energy budget and to understand its generation mechanism.

In this study, we conduct the fully kinetic PIC simulations in a domain much larger than that of TT15 to increase the spectral resolution, and with a large value of NPPCPs to weaken the numerical noise. The main purpose is to verify the standard theory of plasma emission in unmagnetized plasmas energized by a single beam of energetic electrons. Signatures of the F emission and its generation process are highlighted to clarify existing contradictions in the literature.

We first present the simulation method and parameter setup in the following section, together with the result for the thermal case, and then present the cases with different number density ratios of beam and background electrons \( n_b/n_0 \) in Sections 3 and 4. We adopt the same plasma and beam parameters for Case A as those in TT15 for a direct comparison. Cases with two values of \( n_b/n_0 \) (0.0057 and 0.05) have been modeled in TT15, while here an additional case with \( n_b/n_0 = 0.01 \) is included to extend the analysis. The last section presents the conclusions and discussion.

### 2. Parameter Setup and the Thermal Case

Following our earlier studies of coherent emission in magnetized plasmas (e.g., Ni et al. 2020; Yousefzadeh et al. 2021; Li et al. 2021; Ning et al. 2021; Chen et al. 2022), we continue to use the Vector-PIC (VPIC: Bowers et al. 2008, 2008, 2009) code released by the Los Alamos National Laboratory (LANL), which is 2D3V, i.e., two-dimensional in space and three-dimensional for particle velocity and electromagnetic fields. We present three cases (A, B, and C) with different \( n_b/n_0 \) (=0.0057, 0.01, and 0.05) within the simulation domain as large as \( \sim 6000 \lambda_{De} \times 6000 \lambda_{De} \). The cell size is set to be 2.929\( \lambda_{De} \) and the number of cells to be 2048 along both \( \hat{e}_x \) and \( \hat{e}_z \) direction, where \( \hat{e}_x \) and \( \hat{e}_z \) represents the unit vector along the \( x \) and \( z \) direction. The beam is along \( \hat{e}_x \) and \( k \) lies in the \( x-z \) plane. The background electron–proton plasma is assumed to be unmagnetized and thermal, to compare directly with earlier studies. The time step is set to 0.03 \( \omega_{pe}^{-1} \). The NPPCPs are 4000 for background electrons and 1000 for both background protons and beam electrons. The total number of macroparticles is \( 2.4 \times 10^{10} \). According to the table, the present study has almost
the same size of simulation domain as that used by its sister paper (Chen et al. 2022), and the largest NPPCPS and total number of macroparticles among the listed studies.

The details of the simulation setup with the domain dimension, cell size, NPPCPS, and resolvable ranges of wavenumber (k) and frequency (ω) have been listed in Table 1. Parameters used by TT15 are also included. To illustrate the effect of the domain size, we include another case (Case A') within a smaller domain (1200 × 1200 λpe). Other parameters of Case A' are the same as Case A. In Table A1 of the Appendix, we have presented some setup parameters of several PIC studies on plasma emission for comparison.

The beam electrons are represented by the following drifting Maxwellian distribution function:
\[ f_b = A_b \exp \left( -\frac{u_i^2}{2u_0^2} - \frac{(u_i - u_\parallel)^2}{2u_\parallel^2} \right), \]  
(1)

where \( u_i \) and \( u_\parallel \) are the parallel and perpendicular components of the momentum per mass, \( u_\parallel \) is the average drift momentum per mass of the beam electrons, \( u_0 \) is the thermal velocity of energetic electrons, and \( A_b \) is the normalization factor. The parameters for the background plasmas and the beam are taken from TT15, which are 2 MK (∼0.002c) for \( T_e \), \( T_p = 0.3T_e \), and 0.3c for the average drift speed of the beam, and the beam temperature is set to be \( T_e \).

To tell the significance of the plasma emission, it is essential to compare their spectral intensity with that given by the thermal case (Case T), in which wave-like noises exist along dispersion curves of intrinsic wave modes even without any energetic electrons. Other setup parameters of Case T are the same as those of Cases A–C. The modes attributed to energetic electrons must be considerably stronger than the corresponding numerical noise.

We start by analyzing Case T. See Figure A1 in the Appendix for the obtained ω–k diagram for Case T. To show the effect of NPPCPS, we also present in Figure A1 another thermal case (T') with NPPCPS = 500 (for electrons) and 125 (for protons). The diagrams are along two propagating directions with \( \theta = 30^\circ \) and \( \theta = 80^\circ \). The color bar of the lower panels is 20 dB less than that of the upper two panels to show weaker signals. The range of the normalized wavenumbers (kλpe) for the uppermost panel is [−0.3, 0.3], while it is [−0.03, 0.03] for the lower three panels. Two conclusions can be drawn: (1) The numerical noise level is inversely proportional to the NPPCPS, i.e., it is ∼9 dB stronger in Case T' than in Case T; (2) there exist two high-frequency modes in unmagnetized plasmas (as expected), the electrostatic Langmuir mode and the fast electromagnetic mode.

The Langmuir wave exists continuously from superluminal to subluminal regimes, being perfectly electrostatic. The transverse mode is also dominated in energy by the \( E_y \) and \( E_z \) components, which are stronger by about 10–20 times than the magnetic counterparts (\( B_y \) and \( B_z \)). For Case T, the maximum intensity of \( E_y \) is about −140 dB and \( B_y \) about −150 dB, as read from the figure. These values represent the levels of modes relevant to numerical noises.

3. Simulation Results for Case A and Comparison with TT15

Figure 1 and the accompanying movie present the temporal evolution of the velocity distribution function (VDF) of electrons for Case A with \( n_e/n_0 = 0.0057 \). Due to the rapid growth of the bump-on-tail instability, the beam electrons decelerate rapidly and diffuse toward the regime with lower \( v_\perp \). This occurs during the first few hundred \( \omega_{pe} \). The beam electrons also diffuse toward the regime with a larger \( v_\perp \), indicating efficient perpendicular heating. Such heating is induced by two processes, the perpendicular diffusion due to the bump-on-tail instability (see Harding et al. 2020; Melrose et al. 2021; Chen et al. 2022) and the electromagnetic Weibel instability that was discovered by Weibel (1959) for anisotropic plasmas with bi-Maxwellian distribution and by Fried (1959) for plasmas with counterstreaming electrons. In our case, the instability is driven by the single electron beam, leading to the growth of the electromagnetic beam mode with a significantly enhanced \( B_y \) component and perpendicular heating of the beam electrons (see also Karlický 2009; TT15). The VDF reaches the asymptotic state after ∼1000 \( \omega_{pe} \).

In Figure 2(a), we show the energy curves of the six field components and the negative change of the total kinetic energy of all electrons (−ΔEk) for Cases T and A. The beam-plasma interaction causes the rapid rise of \( E_x, E_y, \) and \( B_z \) within the first 80 \( \omega_{pe} \), while the other field components remain at the background levels. The maximum intensity of \( E_x \) reaches about 13% of −ΔEk, while those of \( E_z \) and \( B_y \) reach about 2% and 0.01% of −ΔEk, respectively. As seen from the following analysis, the \( E_x \) and \( E_z \) components mainly correspond to the
Langmuir mode while the \( B_z \) component is energized by the low-frequency electromagnetic Weibel instability. The \( E_z \) and \( E_y \) intensities maintain an almost constant level before declining gradually after \( \sim 1000 \ \omega_{pe}^{-1} \), representative of the nonlinear evolution of turbulent plasmas in quasi-equilibrium.

In Figure 2(b), we have plotted the energy of field components associated with various wave modes. The temporal variation of the mode energy is given by the inverse Fourier transform of the total spectral energy within the selected spectral regimes (see Figure A2 of the Appendix). The overplotted dashed lines in panels (a) and (b) are given by the exponential fittings to the energy profiles of \( E_z \) and the forward generalized Langmuir mode.

The energy curve of \( E_z \) (Figure 2(a)) agrees with the Langmuir mode energy (Figure 2(b)). We performed the exponential fittings to the energy profiles of the \( E_z \) field and the beam-Langmuir (BL) mode (see the dashed lines overplotted in Figure 2). The obtained linear growth rate is 0.076 \( \omega_{pe}^{-1} \) for \( E_z \) and 0.053 \( \omega_{pe}^{-1} \) for the BL mode energy. The theoretical growth rate \( \gamma \) and the corresponding frequency \( \omega_r \) versus \( k \lambda_{De} \), according to Equation (A1) of the Appendix, have been plotted in the upper panels of Figure A3. The parameters used for the plots are identical to those of Case A. In Figure A3(a), the maximum growth rate of the bump-on-tail instability \( (\gamma_M) \) is 0.094 \( \omega_{pe}^{-1} \) at \( (\omega_r, k \lambda_{De}) = (0.96 \ \omega_{pe}^{-1}, 0.069) \), and the range of significant growth (within which the growth rate is larger than, say, \( \omega_r M / 2 \)) extends from \( (\omega_r, k \lambda_{De}) = (0.96 \ \omega_{pe}^{-1}, 0.069) \) to...

![Figure 1](image1.png)

**Figure 1.** Temporal evolution of the VDFs at four representative moments. The video begins at \( t = 0 \ \omega_{pe}^{-1} \) and advances 100 \( \omega_{pe}^{-1} \) per frame until \( t = 2000 \ \omega_{pe}^{-1} \). The real-time duration of the video is 5 s.

(An animation of this figure is available.)

![Figure 2](image2.png)

**Figure 2.** (a) Energy variation of the electromagnetic fields and the negative change of the total electron energy \( (\Delta E_k) \) for Case A; (b) energy variation of the field components of various wave modes for Case A (BL for the beam-Langmuir mode, GL for the backward- and forward-propagating generalized Langmuir mode, WB for the mode excited by the Weibel instability, and F for the fundamental and H for the harmonic emissions). The spectral regimes adopted to evaluate these energy profiles are presented in Figure A2 of the Appendix. The overplotted dashed lines in panels (a) and (b) are given by the exponential fittings to the energy profiles of \( E_z \) and the forward generalized Langmuir mode.
Figure 3. Wave intensity maps of the six field components in the wavevector $k$ space for Case A. The dashed lines in panel (a) represent $\theta = 30^\circ$ and $80^\circ$. The middle and lower panels are the zoom-in views of the white square drawn in panel (a). The energy is normalized by the total energy of beam electrons $E_{\text{tot}}$ of Case A. The video begins at $t = 0$ and advances $20\ \omega_{\text{pe}}^{-1}$ per frame until $t = 200\ \omega_{\text{pe}}^{-1}$ and then advances $100\ \omega_{\text{pe}}^{-1}$ until $t = 2000\ \omega_{\text{pe}}^{-1}$. The real-time duration of the video is 5 s. (An animation of this figure is available.)

(0.75 $\omega_{\text{pe}}$, 0.049) to (1.06 $\omega_{\text{pe}}$, 0.084). On the other hand, according to the PIC simulation (see Figure A3(b) for the dispersion diagram of Case A within the period of [0, 500 $\omega_{\text{pe}}^{-1}$]), the BL mode presents the maximum growth around ($\omega_{\text{pe}}, k\lambda_{\text{De}}$) = (0.98 $\omega_{\text{pe}}$, 0.08), and the range of significant growth extends from ($\omega_{\text{pe}}, k\lambda_{\text{De}}$) = (0.98 $\omega_{\text{pe}}$, 0.075) to (1.01 $\omega_{\text{pe}}$, 0.12). We concluded that the PIC simulation agrees reasonably well with the theoretical prediction.

Figure 3 presents the wave map in $k$ space. Figure 4 presents the corresponding $\omega$-$k$ diagrams along $\theta = 30^\circ$ and/or $80^\circ$ for the modes with frequencies close to $\omega_{\text{pe}}$, and Figure 5 presents similar diagrams for the modes at the low frequency ($<\omega_{\text{pe}}$) and the harmonic frequency ($\sim 2\omega_{\text{pe}}$). These figures (and accompanying movies) shall be combined to reveal the characteristics of each mode. According to Figures 4 and 5, there exist two other Langmuir components on the left (or backward) side of BL, referred to as the generalized Langmuir (GL) mode in total. One is forward-propagating with a narrow range of $k$ ([−0.04, 0.04] $\lambda_{\text{De}}^{-1}$), and the other is backward-propagating with a larger range of $k$ ([−0.2, −0.04] $\lambda_{\text{De}}^{-1}$). The forward one is mostly superluminal. According to the movie accompanying Figure 4 and the energy curves of Figure 2, the backward one appears shortly after the onset of BL, while the forward one appears after $100\ \omega_{\text{pe}}^{-1}$. The two components have very similar energy profiles, both reaching the maximum around $1000\ \omega_{\text{pe}}^{-1}$, indicating the same physical origin.

From panels (b)–(d) of Figure 4, we observe significant enhancements along the corresponding dispersion curve of the F emission, i.e., the fast transverse electromagnetic mode around $\omega_{\text{pe}}$ from the $\omega$-$k$ diagrams of $E_{\text{c}}$ and $B_{\text{c}}$. According to Figure 2(b), its energy (summed over all propagating directions) reaches about $10^{-5}E_{\text{tot}}$, a fraction of the total energy of the H emission. As expected, the F emission is dominated in energy by its $E_{\text{c}}$ component, which is stronger than $B_{\text{c}}$, by about 2–3 orders in magnitude. It reaches the maximum intensity around $1000\ \omega_{\text{pe}}^{-1}$, with a variation trend similar to the H emission.

The harmonic (H) emission emerges as the circular feature after $\sim 500\ \omega_{\text{pe}}^{-1}$ in the maps of $E_{\text{c}}$, $E_{\text{y}}$, and $B_{\text{y}}$ (see Figures 3(c) and 5(c), and the accompanying movies), and saturates at the level of $10^{-7}E_{\text{tot}}$ after $1000\ \omega_{\text{pe}}^{-1}$. Within the H circle, there appears an enhanced quadrupolar $B_{\text{y}}$ feature that is mainly generated by the beam-driven electromagnetic Weibel instability at low frequency (see Figure 5(b); see also TT15). The $B_{\text{y}}$ signal rises rapidly during the initial stage of the plasma-beam interaction, reaching the maximum intensity around $100\ \omega_{\text{pe}}^{-1}$. 

(continued...
Figure 5 presents the ion-acoustic (IA) mode that is very weak due to the strong Landau damping in plasmas with $T_p/T_e = 0.7$. See the lower two panels of Figure A3 in the Appendix for the frequency and growth rate of the IA mode within the background Maxwellian plasmas, evaluated according to Equation (A2) in the Appendix.

Another interesting feature is the weak yet significant $B_y$ enhancement along the Langmuir dispersion curve (see Figure 4(c)). It occupies a much larger range of $k$ ($[-0.04, 0.04] \lambda_{De}$), with the spectral intensity weaker than the F emission. Yet, the total energy of the Langmuir $B_y$ component is much larger than that of the F emission. It reaches the maximum level ($\sim 10^{-2} E_{\parallel 0}$) also around $1000 \omega_{pe}^{-1}$ (see Figure 2(c)), $\sim 10$ times stronger than the simultaneous F emission. A similar $B_z$ enhancement is also observed in Figure 5 of TT15. Yet, due to the poor spectral resolution there it is not possible to separate the $B_z$ component of the F emission from the prevailing Langmuir enhancement. Such a $B_z$ enhancement is not associated with the Langmuir noise of thermal plasmas according to Figure A1.

To support the above argument, we conducted another simulation (Case A) within a much smaller domain ($1200 \times 1200 \lambda_{De}$). See Table 1 for the simulation setup. The plasma-beam parameters are not changed. The obtained wave map in $k$ space is shown in Figure A4 of the Appendix for large (panel (a)) and small (panels (b)–(d)) ranges of $k$. The analytical dispersion curves of the corresponding wave modes are overplotted. The video begins at $\theta = 0^\circ$ and advances $5^\circ$ at a time up to $\theta = 90^\circ$. The real-time duration of the video is 5 s.

Figure 4. Dispersion diagrams of $(E_x, E_y, E_z)$ and $(B_x, B_y, B_z)$ for various wave modes in the frequency range of 0.9–1.1 $\omega_{pe}$ in large (panel (a)) and small (panels (b)–(d)) ranges of $k$. The analytical dispersion curves of the corresponding wave modes are overplotted. The video begins at $\theta = 0^\circ$ and advances $5^\circ$ at a time up to $\theta = 90^\circ$. The real-time duration of the video is 5 s.

(An animation of this figure is available.)

Figure 5(a) presents the ion-acoustic (IA) mode that is very weak due to the strong Landau damping in plasmas with $T_p/T_e = 0.7$. See the lower two panels of Figure A3 in the Appendix for the frequency and growth rate of the IA mode within the background Maxwellian plasmas, evaluated according to Equation (A2) in the Appendix.

Another interesting feature is the weak yet significant $B_y$ enhancement along the Langmuir dispersion curve (see Figure 4(c)). It occupies a much larger range of $k$ ($[-0.04, 0.04] \lambda_{De}$), with the spectral intensity weaker than the F emission. Yet, the total energy of the Langmuir $B_y$ component is much larger than that of the F emission. It reaches the maximum level ($\sim 10^{-2} E_{\parallel 0}$) also around $1000 \omega_{pe}^{-1}$ (see Figure 2(c)), $\sim 10$ times stronger than the simultaneous F emission. A similar $B_z$ enhancement is also observed in Figure 5 of TT15. Yet, due to the poor spectral resolution there it is not possible to separate the $B_z$ component of the F emission from the prevailing Langmuir enhancement. Such a $B_z$ enhancement is not associated with the Langmuir noise of thermal plasmas according to Figure A1.

To support the above argument, we conducted another simulation (Case A) within a much smaller domain ($1200 \times 1200 \lambda_{De}$). See Table 1 for the simulation setup. The plasma-beam parameters are not changed. The obtained wave map in $k$ space is shown in Figure A4 of the Appendix for large (panel (a)) and small (panels (b) and (c)) ranges of $k$. The spectra of the three magnetic field components are presented in panels (c). The obtained $\omega$–$k$ diagrams are presented in Figure A5 of the Appendix, along two propagating angles with $\theta = 30^\circ$ and $\theta = 80^\circ$. The accompanying movie presents the complete dispersion diagrams from $\theta = 0^\circ$ to $\theta = 90^\circ$. For direct comparison, the data duration used in the Fourier analysis is taken to be $100 \omega_{pe}^{-1}$, the same as that used by TT15. We see that the circular H emission cannot be identified clearly, and there exist no regular wave enhancements along the dispersion curve of the F emission, no way to tell the significance of its $B_y$ field due to the presence of the overwhelming Langmuir $B_y$ component. Note that TT15 has mistakenly regarded the total $B_y$ enhancement to be evidence of the F emission.

4. Simulation Results for Cases B and C and Comparison with Case A

Figures 6 and 7 present the wave distribution map in $k$ space and the $\omega$–$k$ diagram for Cases B ($n_b/n_0 = 0.01$) and C.
The Astrophysical Journal, 939:63 (13pp), 2022 November 10

5. Conclusions and Discussion

Our main purpose is to verify the standard model of plasma emission, which describes a multistage nonlinear process in the aftermath of the kinetic bump-on-tail instability of a single-beam plasma system. The primary outcome of the instability is the enhanced turbulence of the electrostatic BL mode, which further interacts with other secondary fluctuations to yield the F and H radiations. Previous verification studies using PIC simulations, limited by available computational resources, have drawn contradictory statements regarding the significance of the F emission. In this study, we employed a simulation domain and a number of macroparticles that are among the largest ones of similar studies to lower noise levels and achieve higher spectral resolution.

We found that for the ratio of number density \( n_b/n_0 \) being less than 0.01, significant F emission can be generated. The F emission reaches an energy level of \( 10^{-5} \) of the total initial beam energy \( E_{i0} \), which is a fraction of the total energy of the H emission. The spectral energies of all wave modes relative to \( E_{i0} \) are less than 0.04, except the beam-driven electromagnetic Weibel instability, which further interacts with other secondary fluctuations to yield the F and H radiations. Previous verification studies using PIC simulations, limited by available computational resources, have drawn contradictory statements regarding the significance of the F emission. In this study, we employed a simulation domain and a number of macroparticles that are among the largest ones of similar studies to lower noise levels and achieve higher spectral resolution.

As observed from Figure 7, the central frequency of BL in Case C is about 0.86 \( \omega_{pe} \), being considerably lower than in Cases A and B. The difference from \( \omega_{pe} \) is too large to allow the standard plasma emission process to occur. This agrees with TT15, which is based on simulations within a much smaller domain.

An animation of this figure is available.

Figure 5. Dispersion diagrams of \((E_x, E_y, E_z)\) and \((B_x, B_y, B_z)\) for various wave modes in the frequency range of \([0, 0.05] \omega_{pe}^{-1}\) (panels (a)–(b)) and \([1.9, 2.1] \omega_{pe}^{-1}\) (panel (c)) for Case A. Analytical dispersion curves of corresponding wave modes (IA for ion-acoustic mode) are overplotted. The video begins at \( \theta = 0^\circ \) and advances \( 5^\circ \) at a time up to \( \theta = 90^\circ \). The real-time duration of the video is 5 s.

An animation of this figure is available.

\( (n_b/n_0 = 0.05). \) The energy curves for Case B have been plotted in Figures 2(c)–(d). They should be combined with the results for Case A to understand the effect of varying \( n_b/n_0 \). Note that the illustrated spectral energy is “relative” since it has been normalized by the total initial beam energy \( E_{i0} \) of each individual case. \( E_{i0} \) increases with increasing \( n_b/n_0 \), being \( 4.12m_e^2c^2 \) for A, \( 7.25m_e^2c^2 \) for B, and \( 37.7m_e^2c^2 \) for C.

The most obvious result is that all modes discussed here, except the low-frequency electromagnetic Weibel mode, decrease in relative spectral energy with increasing \( n_b/n_0 \). All modes look very similar when \( n_b/n_0 \) increases from 0.0057 to 0.01, yet when \( n_b/n_0 \) further increases to 0.05, only the forward BL mode and a small part of the forward-propagating GL wave remain while the backward one and the F and H emissions vanish. The Weibel instability responds to increasing \( n_b/n_0 \) differently, yielding stronger \( B_s \) in terms of relative spectral energy. Further discussion on the Weibel instability is beyond the scope of the study.

As observed from Figure 7, the central frequency of BL in Case C is about 0.86 \( \omega_{pe} \), being considerably lower than in Cases A and B. The difference from \( \omega_{pe} \) is too large to allow the standard plasma emission process to occur. This agrees with TT15, which is based on simulations within a much smaller domain.
indicating that the thermal correction to the dispersion relation of the Langmuir wave is critical to the emission process.

In the case with $n_b/n_0 = 0.01$, we found a significant enhancement of the transverse component $B_y$ at frequency around $\omega_{pe}$ and in a $k$ range of $[-0.02, 0.02]$ $\lambda_{De}$, with contributions from both the F emission and the BL turbulence. It remains unresolved regarding the generation mechanism of such Langmuir $B_y$ component (that is about three orders in magnitude weaker than the corresponding $E_z$ component). Note that in the simulation with only thermal plasmas, the Langmuir noise manifests no signatures of $B_y$ (see Figure A1). Future studies should explore how such a $B_y$ component develops in such plasmas of turbulent equilibrium.

Using the same PIC code, Chen et al. (2022) examined the plasma emission process in weakly magnetized plasmas ($\omega_{pe}/\Omega_{ce} = 10$) interacting with a single beam of energetic electrons. They presented cases with arbitrarily different mass ratios ($m_p/m_e$) and found that with increasing $m_p/m_e$, the
intensity of BL increases correspondingly, together with enhancements of the Z mode and F/H emissions. This indicates that the latter modes (Z and F/H) are secondary products of the primary BL mode. This agrees with the study presented here.

Comparing the case for weakly magnetized background plasmas (Chen et al. 2022) and Cases A and B presented here for unmagnetized plasmas, we find that the two sets of solutions are quite similar. In particular, they are similar in the overall temporal evolution of the EVDF, the growth and characteristics of the BL wave, and backward- and forward-propagating secondary GL waves. The main difference between the two cases lies in the generation of the superluminal Z mode (with a cutoff frequency below $\omega_{pe}$) and the whistler mode in Chen et al. (2022), while in this study the GL wave and the electromagnetic mode induced by the beam-Weibel instability are present. These major similarities support the standard theory of plasma emission works for the two cases, that is, electromagnetic decay to generate the F emission and nonlinear coalescence of the BL wave with the backward-propagating Langmuir wave to generate the H emission.

The above argument does not support the plasma emission mechanism suggested by Ni et al. (2020, 2021) that the F emission is generated by almost-counterpropagating upper-hybrid (UH) mode. Their conclusion is based on PIC simulations of plasmas interacting with trapped energetic electrons with loss-cone-type VDFs. There, the primary mode is the UH mode, which is the Langmuir wave with a large propagating angle relative to the background magnetic field. The UH mode and the secondary Z and W modes are excited directly through the electron cyclotron maser instability (ECMI). This suggests that the mechanism of plasma emission may be different for different VDFs of energetic electrons: For beam electrons, the standard plasma emission mechanism is at work, while for trapped electrons, the plasma emission induced by ECMI may be important.

This study is supported by NNSFC grants (11790303, 11790300, 11973031, and 11873036). The authors acknowledge the Beijing Super Cloud Computing Center (BSCC, URL: http://www.bsc.cn/) for computational resources and LANL for the open-source VPIC code.

**Appendix**

In Table A1, we present some setup parameters of several PIC studies on plasma emission for comparison. In Figure A1, we present the $\omega$–$k$ diagram for the two thermal cases (T and T’, with only Maxwellian plasmas) in Figure A1, with different NPPCPS to show its effect on the numerical noise level. In Case T, the NPPCPS = 4000 (for electrons) and 1000 (for protons); in Case T’, the NPPCPS = 500 (for electrons) and 125 (for protons). The diagrams are along two propagating directions with $\theta = 30^\circ$ and $\theta = 80^\circ$.

In Figure A2, we present the spectral regimes selected to evaluate the mode energy as plotted in Figure 2.

In Figure A3(a), we plot the growth rate ($\gamma$) and frequency ($\omega$) of the BL mode excited via the kinetic bump-on-tail instability, according to the following equation:

$$1 + \frac{2\omega^2_{pe}}{k^2 v_T^2} (1 + \xi_e Z(\xi_e)) + \frac{2\omega^2_{pe}}{k^2 v_T^2} (1 + \xi_{b} Z(\xi_{b})) = 0,$$  
(A1)

where $v_T$ and $v_T$ represent the thermal velocity of the background and the beam electrons, $Z$ is the plasma dispersion function, $\xi_e = \frac{\omega}{k v_T}$, $\xi_{b} = \frac{\omega}{k v_{Tb}}$, and $v_d$ is the average drift velocity of the beam electrons.

In Figure A3(b), we present the dispersion diagram of Case A within the period of [0, 500 $\omega_{pe}^{-1}$] to compare with the theoretical predictions of the ranges of $\omega$ and $k$ of significant BL wave growth. A shorter period (than that used in Figure 4) is employed here to avoid the influence of the nonlinear stage of wave evolution after $t = 500 \omega_{pe}^{-1}$.

In Figures A3(c) and A3(d), we plot the growth rate ($\gamma$) and frequency ($\omega$) of the IA mode versus the normalized wavenumber and the proton–electron temperature ratio according to

$$\omega_i^2 = k^2 v_T^2 \gamma, \quad \frac{\gamma}{\omega_i} = - \frac{\delta v_T}{8 v_e} \left(1 - \frac{m_e}{m_p} \right)^2 e^{-\left(\frac{\omega_i}{\omega_{pe}}\right)^2} - \frac{v_{Te}}{v_{Tb}} \left(\frac{\omega_{pe}}{\omega_i}\right)^3 e^{-\left(\frac{\omega_i}{\omega_{pe}}\right)^2},$$  
(A2)

where $v_i$ is the phase speed of the IA mode. According to Figures A3(c) and A3(d), for $T_p/T_e = 0.7$ and $k\lambda_{De} = 0.2$ (see Figure 5 for these typical parameters), we get $\omega_i = 0.008 \omega_{pe}$ and $\gamma = 0.72 \omega_i \approx 0.006 \omega_{pe}$, then the e-folding time of the decay of IA is $\tau = \frac{1}{\gamma} \approx 170 \omega_{pe}^{-1}$, where the period $T = \frac{\pi v_i}{\omega_i} \approx 780 \omega_{pe}^{-1}$. This means that the IA mode revealed here is heavily damped over just one period.

In Figures A4 and A5, we show the wave function map in $k$ space and the $\omega$–$k$ diagram for Case A’, which has a much smaller simulation domain ($1200 \times 1200 \lambda_{De}^2$) than Case A. The other setup parameters of the two cases are taken from Case A.

| $N_x \times N_z$ | $L_x \times L_z (\lambda_{De})$ | Duration ($\omega_{pe}^{-1}$) | NPPCPS |
|-----------------|-------------------|-----------------|--------|
| Kasaba et al. (2001) | 512 x 512 | 512 x 512 | ~328 | 16, 16, 4 |
| Rhee et al. (2009a) | 512 x 512 | 512 x 512 | ~328 | 80, 80, 8 |
| Umeda (2010) | 1024 x 1024 | 1024 x 1024 | NA | 256, 256, 256 |
| TT15 | 600 x 600 | 600 x 600 | ~1000 | 1000, 1000, 1000 |
| Henri et al. (2019) | 1024 x 1024 | 3072 x 3072 | ~1500 | 3600, 900, 900 |
| Chen et al. (2022) | 2048 x 2048 | 6667 x 6667 | 2000 | 2000, 1000, 1000 |
| This paper | 2048 x 2048 | 6000 x 6000 | 2000 | 4000, 1000, 1000 |
Figure A1. The same as Figure 4 yet for Cases T and T$. The two cases are done with different sets of NPPCPS. In Case T, the NPPCPS $= 4000$ (for electrons) and 1000 (for protons); in Case T$ the NPPCPS $= 500$ (for electrons) and 125 (for protons). The video begins at $\theta = 0^\circ$ and advances 5$^\circ$ at a time up to $\theta = 90^\circ$. The real-time duration of the video is 5 s. (An animation of this figure is available.)
Figure A3. (a) The growth rate ($\gamma$: dashed line) and frequency ($\omega$: solid line) of the BL mode excited via the kinetic bump-on-tail instability; (b) the dispersion diagram of Case A within the period of $[0, 500 \omega_{pe}^{-1}]$; (c) and (d), the growth rate ($\gamma$: dashed line) and frequency ($\omega$: solid line) of the IA mode, vs. the normalized wavenumber (c) and the proton–electron temperature ratio (d).

Figure A4. The same as Figure 3 but for Case $A'$. 

The Astrophysical Journal, 939:63 (13pp), 2022 November 10

Zhang et al.
Figure A5. The same as Figure 4 but for Case A'. The video begins at $\theta = 0^\circ$ and advances $5^\circ$ at a time up to $\theta = 90^\circ$. The real-time duration of the video is 5 s.

(An animation of this figure is available.)

**References**

Bowers, K. J., Albright, B. J., Bergen, B., et al. 2008, in SC ’08: Proceedings of the 2008 ACM/IEEE Conference on Supercomputing (Piscataway, NJ: IEEE), 1

Bowers, K. J., Albright, B. J., Yin, L., et al. 2009, JPheS, 180, 012055

Bowers, K. J., Albright, B. J., Yin, L., Bergen, B., & Kwan, T. J. T. 2008, PhPl, 15, 055703

Cairns, I. H. 1987, JPlPh, 38, 169

Cairns, I. H. 1995, GeoRL, 22, 3433

Cairns, I. H. 1998, ApJ, 506, 456

Cairns, I. H., & Zank, G. P. 2002, GeoRL, 29, 1143

Che, H., Goldstein, M. L., & Zank, G. P. 2002, GeoRL, 29, 1143

Chen, Y., Du, G., Feng, S., et al. 2014, ApJ, 787, 59

Chen, Y., Zhang, Z., Ni, S., et al. 2022, ApJL, 924, L34

Etcheto, J., & Faucheux, M. 1984, JGR, 89, 6631

Fried, B. D. 1959, PhFl, 2, 337

Ganse, U., Kilian, P., Spanier, F., & Vainio, R. 2012a, ApJ, 751, 145

Ganse, U., Kilian, P., Spanier, F., & Vainio, R. 2012b, SoPh, 280, 551

Ginzburg, V. L., & Zhelezniakov, V. V. 1958, Sva, 2, 653

Gurnett, D. A., Allendorf, S. C., & Kurth, W. S. 1998, GeoRL, 25, 4433

Gurnett, D. A., & Kurth, W. S. 1995, AdSpR, 16, 279

Harding, J. C., Cairns, I. H., & Melrose, D. B. 2020, PhPl, 27, 020702

Henri, P., Sgattoni, A., Briand, C., Amiranoff, F., & Riconda, C. 2019, JGRA, 124, 1475

Karlický, M. 2009, ApJ, 690, 189

Karlický, M., & Vandas, M. 2007, P&SS, 55, 2336

Kasaba, Y., Matsumoto, H., & Omura, Y. 2001, JGRA, 106, 1693

Kuncic, Z., & Cairns, I. H. 2005, JGRA, 110, A07107

Kurth, W. S., Gurnett, D. A., Scarf, F. L., & Poynter, R. L. 1984, Nat, 312, 27

Lee, S.-Y., Ziebell, L. F., Yoon, P. H., Gaezler, R., & Lee, E. S. 2019, ApJ, 871, 74

Li, B., & Cairns, I. H. 2013, JGRA, 118, 4748

Li, B., & Cairns, I. H. 2014, SoPh, 289, 951

Li, C., Chen, Y., Ni, S., et al. 2021, ApJL, 909, L5

Li, C. Y., Chen, Y., Wang, B., et al. 2017, SoPh, 292, 82

Lv, M. S., Chen, Y., Li, C. Y., et al. 2017, SoPh, 292, 194

Melrose, D. 1980, SSRV, 26, 3

Melrose, D. B. 1987, SoPh, 111, 89

Melrose, D. B., Harding, J., & Cairns, I. H. 2021, SoPh, 296, 42

Moses, S. L., Coroniti, F. V., Kennel, C. F., & Scarf, F. L. 1984, GeoRL, 11, 869

Ni, S., Chen, Y., Li, C., et al. 2020, ApJL, 891, L25

Ni, S., Chen, Y., Li, C., et al. 2021, PhPl, 28, 040701

Ning, H., Chen, Y., Ni, S., et al. 2021, ApJL, 920, L40

Pila, D., Kurth, W. S., Hospodarsky, G. B., et al. 2017, EPSC, EPSC2017 585

Rhee, T., Ryu, C.-M., Woo, M., et al. 2009a, ApJ, 694, 618

Rhee, T., Woo, M., & Ryu, C.-M. 2009b, IJPS, 54, 313

Robinson, P. A., Cairns, I. H., & Willes, A. J. 1994, ApJ, 422, 870

Schmidt, J. M., & Cairns, I. H. 2012, JGRA, 117, A04106

Schmidt, J. M., & Cairns, I. H. 2014, JGRA, 119, 69

Tasnim, S., Zank, G. P., Cairns, I. H., & Adhikari, L. 2022, ApJ, 928, 125

Thurgood, J. O., & Tsiklauri, D. 2015, A&A, 584, A83

Umeda, T. 2010, JGRA, 115, A01204

Vasanth, V., Chen, Y., Feng, S., et al. 2016, ApJL, 830, L2
Yasanth, V., Chen, Y., Lv, M., et al. 2019, ApJ, 870, 30
Weibel, E. S. 1959, PhRvL, 2, 83
Yoon, P. H. 2006, PhPl, 13, 022302
Yoon, P. H., Ziebell, L. F., Gaelzer, R., & Pavan, J. 2012, PhPl, 19, 102303
Yousefzadeh, M., Ning, H., & Chen, Y. 2021, ApJ, 909, 3
Zank, G. P., Cairns, I. H., Donohue, D. J., & Matthaeus, W. H. 1994, JGR, 99, 14729
Ziebell, L. F., Yoon, P. H., Petruzzellis, L. T., Gaelzer, R., & Pavan, J. 2015, ApJ, 806, 237