Secure quantum cryptographic network based on quantum key distribution

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We present a protocol for quantum cryptographic network consisting of a quantum network center and many users, in which any pair of parties with members chosen from the whole users on request can secure a quantum key distribution by help of the center. The protocol is based on the quantum authentication scheme given by Barnum et al. [Proc. 43rd IEEE Symp. FOCS’02, p. 449 (2002)]. We show that exploiting the quantum authentication scheme the center can safely make two parties share nearly perfect entangled states used in the quantum key distribution. This implies that the quantum cryptographic network protocol is secure against all kinds of eavesdropping.

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I. INTRODUCTION

Quantum key distribution (QKD) has been considered as a method for the perfectly secure communication which can compensate for the incompleteness of the classical cryptography. Since the advent of the first QKD protocol presented by Bennett and Brassard [1], various kinds of quantum cryptographic protocols based on QKD schemes between two users [2, 3, 4] have been proposed, and the rigorous proofs of the security of the protocols have considerably been studied [5, 6, 7, 8, 9, 10, 11, 12]. This implies that quantum cryptography has almost attained to the practical stage.

The QKD protocol between two users can be generalized into the communications between two parties, A and B, consisting of several members respectively [13, 14], which use the quantum secret sharing protocol [15].

For the communication among a lot of users, the theories on the quantum cryptographic network, in which several protocols are feasible on request, have been suggested [16, 17, 18]. The quantum cryptographic network usually requires a quantum network center, which connects any pair of parties by entangled states so that the two parties can perform a quantum key distribution by help of the center.

Furthermore, employing the quantum network center, one can reduce the number of quantum channels used in quantum cryptographic network. In other words, in order for any pair of n users to communicate with each other, n(n − 1)/2 quantum channels between any pairs of users are required in the quantum cryptographic network without a quantum network center, while only n quantum channels between the center and users are required in the quantum cryptographic network with a quantum network center. Therefore, the center’s role can provide us with the efficient quantum cryptographic network as well as the secure network.

In this paper, we present a protocol for the quantum cryptographic network, which is based on the quantum message authentication scheme presented by Barnum et al. [10]. In order to show that the quantum cryptographic network protocol is secure, we first generalize the quantum authentication scheme between two persons into the scheme between any pair of parties, and then prove that exploiting the quantum authentication scheme the center can safely make any pair of parties share nearly perfect entangled states used in the QKD so that the protocol is secure against all kinds of eavesdropping.

This paper is organized as follows. In Sec. II we briefly introduce the multipartite entangled states used in the QKD between two parties and their properties. In Sec. III we present two protocols for the quantum cryptographic network. In Sec. IV we show that our protocols are secure against any eavesdropping. Finally, in Sec. V we summarize our results.

II. MULTIPARTITE ENTANGLED STATES AND THEIR PROPERTIES

For a positive integer j, we define $|\Phi_j^{\pm}\rangle$ and $|\Psi_j^{\pm}\rangle$ by

$$
|\Phi_j^{\pm}\rangle = \frac{1}{\sqrt{2}} (|0^j\rangle \pm |1^j\rangle),
$$

$$
|\Psi_j^{\pm}\rangle = \frac{1}{\sqrt{2}} (|0^j\rangle \pm i|1^j\rangle),
$$

(1)

where $|0\rangle$ and $|1\rangle$ are the spin-up and the spin-down in the z-direction respectively, and $i = \sqrt{-1}$. Then we can readily obtain the following decomposition relations [14]:

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TABLE I: The relations between the measurement outcomes of two parties A and B.

| \(|\Phi^+\rangle\) | A \(\mathcal{Y}_A \oplus \mathcal{Y}_B\) | B \(\mathcal{Y}_B \oplus \mathcal{Y}_A\) |
|-----------------|----------------|----------------|
| even 0          | even 0         | even 0         |
| even 1          | even 1         | even 1         |
| odd 0           | odd 1          | odd 1          |
| odd 1           | odd 0          | odd 0          |

For positive integers \(n\) and \(m\) satisfying \(n > m\),

\[
\Phi^\pm_n = \frac{1}{\sqrt{2}} (\Phi^+_m \Phi^{\pm}_{n-m} + \Phi^-_m \Phi^{\mp}_{n-m})
\]

\[
= \frac{1}{\sqrt{2}} (\Psi^+_m \Psi^{\mp}_{n-m} + \Psi^-_m \Psi^{\pm}_{n-m}),
\]

\[
\Psi^\pm_n = \frac{1}{\sqrt{2}} (\Phi^+_m \Phi^{-}_{n-m} + \Phi^-_m \Phi^{+}_{n-m})
\]

\[
= \frac{1}{\sqrt{2}} (\Psi^+_m \Psi^{-}_{n-m} + \Psi^-_m \Psi^{+}_{n-m}).
\]

QKD between two parties with a center’s assistance are feasible if they share the multi-particle entangled states such as \(\Phi^\pm_t\).

III. PROTOCOLS FOR QUANTUM CRYPTOGRAPHIC NETWORK

In this section, we construct two slightly different protocols for quantum cryptographic network according as the center has a memory to store quantum information or does not have such a memory. If the center has the memory, then it can enhance the efficiency for the use of entangled states by reducing the number of discarded entangled states.

Before presenting our protocol, we briefly review the results of the quantum message authentication presented by Barnum et al. The quantum authentication uses stabilizer codes based on a normal rational curve in the projective geometry. Barnum et al. proved that the set of the codes forms a stabilizer purity testing code with error \(\varepsilon = 2r/(2^s + 1)\), where each code encodes \(t = (r - 1)s\) qubits into \(u = rs\) qubits, and also showed that a secure quantum authentication scheme can be obtained from the purity testing code.

A. Protocol 1: QKD network without memory

We now present the QKD protocol between two users or two parties which can securely be performed by help of the center’s authentication for their states. Throughout the paper, we assume that \(C\) is considered as one quantum network center which is always trustful, and that \(A\) and \(B\) are two arbitrary parties which want to be connected by nearly perfect entangled states in order to communicate each other. The protocol is as follows:

1. Preprocessing: For each member \(\mu\) in \(A\) and \(B\), \(C\) and \(\mu\) agree on some stabilizer purity testing code \(\{D_{k(\mu)}\}\) and some private and random binary strings \(k(\mu), x(\mu), y(\mu)\).

2. \(C\) prepares \(nt\) qubits in the state \(|\Phi^+_n\rangle^\otimes t\). We denote one qubit of \(|\Phi^+_n\rangle\) which will be transmitted to the member \(\mu\) by \(\rho(\mu)\).

3. Performing some specific unitary operations corresponding to \(x(\mu)\), \(C\) encrypts \(\rho(\mu)^\otimes t = \tilde{\rho}(\mu)\) as \(\tau(\mu)\).

4. For the code \(D_{k(\mu)}\) with syndrome \(y(\mu)\), \(C\) encodes \(\tau(\mu)\) according to \(D_{k(\mu)}\) to produce \(\sigma(\mu)\). \(C\) sends \(\sigma(\mu)\) to the member \(\mu\). Let \(\sigma'(\mu)\) be the state which \(\mu\) receives.

5. Each member \(\mu\) measures the syndrome \(y'(\mu)\) of the code \(D_{k(\mu)}\) on \(\sigma'(\mu)\). Each member \(\mu\) compares \(y(\mu)\) to \(y'(\mu)\) and aborts if any error is detected.
(6) According to $D_{\mathbf{x}(\mu)}$, each member $\mu$ decodes $\sigma'(\mu)$ to obtain $\tau'(\mu)$, and decrypts $\tau'(\mu)$ using $\mathbf{x}(\mu)$, and then obtains a $t$-qubit state, $\tilde{\rho}'(\mu)$.

(7) $\mu$ randomly performs a measurement on each qubit of one’s own state in the $x$- or $y$-direction. After each member individually carries out the above steps, each member in two parties takes $t$ bits as measurement results. Thus, in order to obtain sufficiently many bit strings, $C$ and the two parties repeat the above steps as many times as necessary.

(8) For all shared qubits, each member in two parties publicly announces the used directions, but not the obtained results. Then the two parties, $A$ and $B$, obtain $\mathcal{Y}_A$ and $\mathcal{Y}_B$. If $\mathcal{Y}_A \oplus \mathcal{Y}_B$ is odd then they discard the keys. Otherwise, $A$ and $B$ continue the next step.

(9) A collector in $A$ ($B$), gathers the outcomes to obtain $\mathcal{M}_A$ ($\mathcal{M}_B$).

(10) Two parties $A$ and $B$ have a public discussion on a random subset of the obtained bits, which is used as the test bits, in order to detect an error which may occur in the previous procedure.

If the parties find an error in this step, all shared keys are discarded, and they go back to Step (1). Otherwise, they obtain a final key string.

We remark that the steps (9) and (10) are unnecessary when both of two parties $A$ and $B$ consist of only one member.

We now review a method to obtain $\mathcal{M}_A$ ($\mathcal{M}_B$) presented in [14]: We first assume that all members are ordered and that the first member is the collector. This order need to be arbitrary. The collector chooses a random bit, which we will express as ‘$R$’, adds it to his outcome modulo 2, and transfers the result to the second member. The second member transfers the next member the outcome plus the received one modulo 2. All members continue this procedure until the collector receives $\mathcal{M}_A \oplus R$ ($\mathcal{M}_B \oplus R$). Then the collector obtains $\mathcal{M}_A$ ($\mathcal{M}_B$), which is $\mathcal{M}_A \oplus R \oplus R$ ($\mathcal{M}_B \oplus R \oplus R$). However, anyone cannot know $\mathcal{M}_A$ ($\mathcal{M}_B$) without acquiring the results of all members.

**B. Protocol 2: QKD network with memory**

In here, we introduce the protocol in which the center does not only authenticate quantum states but also joins in the procedure of the QKD scheme so that the entangled states employed in QKD can efficiently be used. The protocol can be obtained from modifying several steps in Protocol 1 as follows:

(2') $C$ prepares $(n+1)t$ qubits in the state $|\Phi_{n+1}^+\rangle \otimes t$. $C$ stores $t$-qubit state $\rho(C) \otimes t = \tilde{\rho}(C)$ in his memory where $\rho(C)$ is one qubit of $|\Phi_{n+1}^+\rangle$.

(7') Each member in the two parties, except the center $C$, randomly performs a measurement on each one qubit in the $x$- or $y$-direction.

(8') (a) Each member in the two parties, except the center $C$, publicly announces the used directions, but not the obtained results. Then $A$, $B$, and $C$ obtain $\mathcal{Y}_A$ and $\mathcal{Y}_B$.

(b) According to the values of $\mathcal{Y}_A$ and $\mathcal{Y}_B$, $C$ performs a measurement on his particle which is in the state $\rho(C)$ so that $\mathcal{Y}_A \oplus \mathcal{M}_A$ and $\mathcal{Y}_B \oplus \mathcal{M}_B$ have a correlation or an anti-correlation.

(9') Two parties, $A$ and $B$, collect the outcomes to obtain $\mathcal{M}_A$ and $\mathcal{M}_B$ respectively, as in the previous protocols. Then the center $C$ reveals the obtained results.

We remark that the entangled states are used in Protocol 2 more efficiently than in Protocol 1 since there are cases that the keys are discarded in Step (8) of Protocol 1 while there is not such a case in Step (8') of Protocol 2.

**IV. PROOF OF SECURITY OF QUANTUM CRYPTOGRAPHIC NETWORK**

In this section, we are going to prove the security of the protocols presented in this paper. We first note that if, in Step (2') of Protocol 2, $C$ measures each qubit of $\tilde{\rho}(C)$ in the $x$- or $y$-direction instead of storing it then the protocol is essentially equivalent to Protocol 1, and that the order of the measurement does not affect the security of the protocol. Thus, it suffices to prove the security of Protocol 2.

The secure quantum authentication scheme in [19] ensures that for any $t$-qubit state $\tilde{\rho}$, when $\tilde{\rho}$ is transmitted to a member $\mu$ as $\tilde{\rho}',$ the fidelity $F$ of $\tilde{\rho}$ and $\tilde{\rho}'$ is not less than $1 - \varepsilon_0$ for sufficiently small $\varepsilon_0 > 0$, where the fidelity $F$ of $X$ and $Y$ is defined by

$$F(X, Y) = \text{tr} \left( \sqrt{X^{1/2}YX^{1/2}} \right)^2. \quad (3)$$

Here, $\tilde{\rho}'$ can be considered as $\Lambda_{\mu}(\tilde{\rho})$ for some quantum channel $\Lambda_{\mu}$. Thus, if the transmitted state is not discarded, then for any $t$-qubit $\tilde{\rho}$ we can obtain the inequality

$$F(\tilde{\rho}, \tilde{\rho}') = F(\tilde{\rho}, \Lambda_{\mu}(\tilde{\rho})) \geq 1 - \varepsilon_0. \quad (4)$$

For the detailed proof, we present a remark on a property of the fidelity and the quantum channel presented in Lemma 3 of Ref. [21]: Let $E$ be a quantum operation on a $d$-dimensional quantum system $\mathcal{H}_A$, and $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_R$
a purification of a state $\rho_A$ on $\mathcal{H}_A$, where $\mathcal{H}_R$ is a reference system such that $\text{tr}_R(|\Psi\rangle\langle\Psi|) = \rho_A$. Suppose that there is $\varepsilon > 0$ such that
\[
\langle \psi | \mathcal{E}(|\psi\rangle\langle\psi|) |\psi\rangle \geq 1 - \varepsilon
\]
for all $|\psi\rangle$ in the support of $\rho_A$. Then
\[
\langle \Psi | (\mathcal{E} \otimes \mathcal{I}_R)(|\Psi\rangle\langle\Psi|) |\Psi\rangle \geq 1 - \left(1 + d_0 \cdot \max_{j \neq k} (p_j p_k)\right) \varepsilon,
\]
where $d_0$ is the Schmidt number of $|\Psi\rangle$ and $\sqrt{p_j}$ are the Schmidt coefficients of $|\Psi\rangle$ with respect to the bipartite quantum system $\mathcal{H}_A \otimes \mathcal{H}_R$.

Since for any member $\mu$ there exists a sufficiently small $\varepsilon_1 > 0$ such that
\[
F(|\psi\rangle\langle\psi|, \Lambda_\mu(|\psi\rangle\langle\psi|)) \geq 1 - \varepsilon_1
\]
for all $t$-qubit state $|\psi\rangle$ by the inequality \[\[\text{(5)}\]\] and
\[
|\Phi_{n+1}\rangle = \frac{1}{\sqrt{2^t}} \sum_{j=0}^{2^t-1} |j\rangle_\mu |j^n\rangle_C,
\]
applying the above remark to our situation, we obtain the following property:
\[
F(\Phi, (\bigotimes_\mu \Lambda_\mu)(\Phi)) \geq 1 - \left(1 + \frac{1}{2^t}\right) \varepsilon_1
\]
where $\Phi = (|\Phi_{n+1}\rangle\langle\Phi_{n+1}|)^{\otimes t}$.

As we discuss the details in Appendix, $\bigotimes_\mu \Lambda_\mu$ is also a nearly perfect quantum channel and hence there is a sufficiently small $\varepsilon_2 > 0$ such that
\[
F(|\xi\rangle\langle\xi|, \bigotimes_\mu \Lambda_\mu(|\xi\rangle\langle\xi|)) \geq 1 - \varepsilon_2
\]
for any $nt$-qubit pure state $|\xi\rangle$. Thus, since it follows from Eq. \[\[\text{(8)}\]\] that
\[
|\Phi_{n+1}\rangle = \frac{1}{\sqrt{2^t}} \sum_{j=0}^{2^t-1} |j^n\rangle_{AB} |j\rangle_C,
\]
when the center transmit $nt$ qubits of the states to all members, we can obtain the almost same inequality:
\[
F(\Phi, (\bigotimes_\mu \Lambda_\mu) \otimes \mathcal{I}_C)(\Phi) \geq 1 - \left(1 + \frac{1}{2^t}\right) \varepsilon_2.
\]

Therefore, after executing Step (6), the parties with the center share the nearly perfect entangled states.

We now need a result of Lo and Chau \[\[\text{(9)}\]\] that if the center $C$ and every members share a state having a fidelity exponentially close to $1$ with $|\Phi_{n+1}\rangle^{\otimes t}$ then Eve’s mutual information with the key is at most exponentially small.

From this result, we can directly notice that Protocol 2 is secure if there are no dishonest members in the parties.

For the analysis of the case that dishonest members exist in the parties, we use the investigation presented in \[\[\text{(10)}\]\]. By a classical probability estimate about the random sampling tests \[\[\text{(11)}\]\], if there are some errors in the procedure then two parties can detect an error with high probability from sufficiently many test bits. We remark that each member can have an effect on the protocol just at the moment of announcing the basis or giving the information on the outcomes, and that the test bits are chosen after the directions are announced and the information on the outcomes is transferred to other members. Therefore, since the test bits are randomly chosen, the dishonest members cannot escape the detection, that is, they cannot prevent the others from obtaining the correct keys without being detected.

V. SUMMARY

In this paper, we have presented a protocol for quantum cryptographic network consisting of a quantum network center and many users, in which any pair of parties with some members can secure a quantum key distribution by help of the center. We have shown that exploiting the quantum authentication scheme the center can safely make two parties share nearly perfect entangled states used in the QKD on request. We have also shown that the quantum cryptographic network protocol is secure against all kinds of eavesdropping.

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APPENDIX

In this appendix, we show that $\bigotimes_\mu \Lambda_\mu$ is also a nearly perfect quantum channel and that there is a sufficiently small $\varepsilon_2 > 0$ such that
\[
F(|\xi\rangle\langle\xi|, \bigotimes_\mu \Lambda_\mu(|\xi\rangle\langle\xi|)) \geq 1 - \varepsilon_2
\]
for any $nt$-qubit pure state $|\xi\rangle$.

First we note that the last term of the inequality \[\[\text{(12)}\]\] is greater than or equal to $1 - (1 + d/4)\varepsilon$ since $d_0 p_j p_k \leq d/4$. 

\[
F(|\xi\rangle\langle\xi|, \bigotimes_\mu \Lambda_\mu(|\xi\rangle\langle\xi|)) \geq 1 - \varepsilon_2
\]
for any $nt$-qubit pure state $|\xi\rangle$. 

APPENDIX
for all \( j \neq k \). Thus, in the property of the fidelity and the quantum channel presented in Lemma 3 of Ref. [23], if there is \( \varepsilon > 0 \) such that
\[
\langle \psi | \mathcal{E} (|\psi \rangle \langle \psi |) |\psi \rangle \geq 1 - \varepsilon \tag{A.2}
\]
for all \( |\psi \rangle \in \mathcal{H}_A \), then
\[
\langle \Phi | (\mathcal{E} \otimes I_R) (|\Phi \rangle \langle \Phi |) |\Phi \rangle \geq 1 - \left( 1 + \frac{d}{4} \right) \varepsilon, \tag{A.3}
\]
for all \( |\Phi \rangle \in \mathcal{H}_A \otimes \mathcal{H}_R \). Since the square root of the fidelity \( F \) is doubly concave \([22]\), that is, for \( \sum_j \lambda_j = 1 \)
\[
\sqrt{F \left( \sum_j \lambda_j \rho_j, \lambda_j \rho_j' \right)} \geq \sum_j \lambda_j \sqrt{F (\rho_j, \rho_j')}, \tag{A.4}
\]
it follows from the inequality \((A.2)\) that if the inequality \((A.3)\) holds then for all density matrices \( \rho_{AR} \) on \( \mathcal{H}_A \otimes \mathcal{H}_R \)
\[
F (\rho_{AR}, (\mathcal{E} \otimes I_R) (\rho_{AR})) \geq 1 - \left( 1 + \frac{d}{4} \right) \varepsilon. \tag{A.5}
\]
Hence, we obtain from the inequalities \([1]\) and \((A.5)\) that
\[
F (\Phi, (\Lambda_{\mu} \otimes I_C) (\Phi)) \geq 1 - \left( 1 + 2^{t-2} \right) \varepsilon_1 \tag{A.6}
\]
and that
\[
F (\mathcal{I}_\mu \otimes \Lambda_{\mu'} \otimes I_C) (\Phi), (\Lambda_{\mu} \otimes \Lambda_{\mu'} \otimes I_C) (\Phi)) \\
= F (\rho, (\Lambda_{\mu} \otimes \mathcal{I}_\mu) (\rho)) \\
\geq 1 - \left( 1 + 2^{t-2} \right) \varepsilon_1 \tag{A.7}
\]
for \( \mu \neq \mu' \), where \( \rho = (\Lambda_{\mu'} \otimes \mathcal{I}_\mu) (\Phi) \).

We now present a simple remark on a relation between the fidelity and a distance of density matrices, \( D \), which is defined by \( D(\rho, \rho') = \text{tr} (|\rho - \rho'| / 2) \tag{22} \). For any density matrices \( \rho \) and \( \rho' \), the following inequalities hold.
\[
1 - \sqrt{F (\rho, \rho')} \leq D (\rho, \rho') \leq \sqrt{1 - F (\rho, \rho')}. \tag{A.8}
\]
Then it follows from the inequality \((A.8)\) and the triangle inequality of \( D \) that for density matrices \( \rho, \rho' \), and \( \rho'' \)
\[
1 - \sqrt{F (\rho, \rho'')} \leq D (\rho, \rho'') \leq D (\rho, \rho') + D (\rho', \rho''), \tag{A.9}
\]
\[
\leq \sqrt{1 - F (\rho, \rho')} + \sqrt{1 - F (\rho', \rho'')}. \tag{A.10}
\]
By virtue of the inequalities \((A.6)\), \((A.7)\), and \((A.9)\), we obtain the followings: for any \( nt \)-qubit pure state \( |\xi\rangle \),
\[
\sqrt{F (|\xi\rangle \langle \xi|, \bigotimes_{\mu} \Lambda_{\mu} (|\xi\rangle \langle \xi|))} \geq 1 - n \sqrt{1 + 2^{t-2} \varepsilon_1}. \tag{A.10}
\]
Since \( \varepsilon_1 \) is independent on \( n \) and \( t \), the proof is completed. Furthermore, it is clear that the result similar to the inequality \((12)\) can directly be obtained in the same way.

Considering the Bures metric \( d_B \tag{23, 24} \) defined by
\[
d_B (\rho, \rho') = \sqrt{2 - 2 F (\rho, \rho')}^{1/2}, \tag{A.11}
\]
since \( d_B \) satisfies the triangle inequality we can actually obtain an inequality tighter than the inequality \((A.9)\):

For density matrices \( \rho \), \( \rho' \), and \( \rho'' \),
\[
1 - \sqrt{1 - F (\rho, \rho'')} \leq \sqrt{1 - F (\rho, \rho')^{1/2}} + \sqrt{1 - F (\rho', \rho'')^{1/2}}. \tag{A.12}
\]
Therefore, from the inequality \((A.12)\), we could also obtain an inequality tighter than the inequality \((A.10)\).

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