Analysis of Compacting Stress in Fibrous Composites Elementary Cell by Mathematical Modeling

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Abstract. The article represents the possibility of determining the compaction stress in the FCM cell, taking into account the self-regulation of fiber displacement during the compaction process. The study is carried out using mathematical modeling methods.

1 Introduction

When creating products and structures of high-duty purpose, one of the main requirements for materials is high strength, rigidity and resistance under the cyclic nature of dynamic and temperature loads.

Composite materials (CM) consisting of two or more heterogeneous components fully meet these requirements [1-2]. A feature of such materials is the possibility of regulating their properties due to the combination in various ratios of components of CM with different physicochemical characteristics.

Materials in the matrix of which reinforcing fibers are distributed, are called fibrous composite materials (FCM). The presence of the metal matrix ensures the operability of the product in a wide range of temperatures and additionally strengthens the structural composition.

2 Task setting

Compaction is one of the main processes for the production of products and semi-products from FCM. In the works [3-4], the main relationships between process parameters and voltages acting within the scope of the "elementary cell" are analytically established, taking into account the possible initial displacement of fibers during assembly of the FCM blank part (Fig. 1).

High-duty alloys are applied for production of essential components, also of hemispherical shells, which can be used as bottom of high pressure holding capacities.

Titanium alloy is the main material for their production due to its high endurance and specific rigidity. Due to alloy’s peculiarity it is consistent to apply stamping through stretching by a moldable plumb die into a stiff matrix. Joint plastic deformation of plumb and

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blank part material affords to form the piece by the effect of blank part pressing by the moldable die into the matrix fuller [1,2]. Due to the moldable deformation of the die on its border with the blank part, it creates the conditions of active friction which have a positive impact on the changing the form of the blank part and the possibility of its natural thinning. The process scheme is presented on Fig. 1.

![Fig. 1. Blank part diagram with possible offsets of fiber stacking in the "elementary cell" volume, where: a) – without displacement, b) - displacement with step S/4, c) - displacement with step S/2](image)

This approach has shown its effectiveness, however, the methods of studying the stress-strain state in an elementary cell (thin section method) allows to obtain only the volume-averaged values of stresses and deformations.

### 3 Formalization and solving of problem

The work [4] proposes an approach in which a stress-deformed state in an "elementary cell" was investigated by the finite element method, which made it possible to obtain objective information on the filling of the "elementary cell" with matrix material. The results of the simulation showed that the compaction stress in the "elementary cell" significantly depends on its type and matrix material (Fig.2).

![Fig. 2. Dependence of compaction stress on deformation degree of blank part, matrix material AD1](image)

The obtained dependencies for "elementary cells" of various types are approximated by a power polynomial: \( \sigma_{sc} = a + b \cdot \varepsilon + c \cdot \varepsilon^2 \), the approximation coefficients of which are shown in Table 1.
Table 1. Approximation Coefficient Values

| Matrix material | Fiber stacking pitch offset | Coefficient Values |
|-----------------|-----------------------------|-------------------|
|                 | a  | b  | c   |
| AD1             | 0  | 0.4321 | 0.0018 | 0.0002 |
|                 | 1/2 | 0.1209 | 0.0038 | 0.0002 |
|                 | 1/4 | 0.2245 | 0.0026 | 0.0002 |

Taking into account the obtained results, a correlation is proposed for calculation of compacting stress of the blank part, which takes into account its structure:

\[ \sigma_{sci}^{*} = 2\sigma_{sci} + bj \left \{ \frac{n-2}{3} \right \} \sum_{j=1}^{n} \sigma_{scej} \],

(1)

where:
\( \sigma_{sci}^{*} \) - compacting stress for outer layers;
\( \sigma_{scej} \) - compacting stress for j internal layer;
n – quantity of fiber layers in the blank part;
bj – weight coefficient.

Based on the calculations, approximation coefficients are obtained (Table 2) for the power polynomial, with the help of which \( \sigma_{sci}^{*} \) is calculated for the blank part:

\[ \sigma_{sci}^{*} = A + B \cdot \epsilon + C \cdot \epsilon^2, \]

(2)

Table 2. Approximation factors for blank part compacting stress calculation

| Matrix material | Approximation Coefficient Values |
|-----------------|----------------------------------|
|                 | A  | B   | C   |
| AD1             | 8.87 | 114.73 | 58.38 |

The considered model has a significant drawback - it does not take into account that when compacting, the process of self-regulation of the structure is carried out [5]. As a result, the fibers tend to move by an equal distance relative to each other, the amount of displacement of neighbor fiber rows when compacting the FCM constantly changes and eventually tends to reach 0.5 fiber stacking pitch.

To take into account the self-adjustment of the structure, the expression (1) should reflect the correlation of the pitch of the displacement of fibers (Scm) over the layers according to the degree of deformation. In this case, the compaction stress is determined by a system of transcendental equations, the solution of which should be rationally carried out by numerical methods.

In equation (1), bj is a weight factor which takes into account the number of "elementary cells" with a certain Scm. Coefficient (n-2)/3 determines the number of considered types of "elementary cells."

In general, in the expression (1), these values can be calculated as follows:

\[ bj \left \{ \frac{n-2}{k} \right \}, \]

(3)

where:
n – quantity of layers in the assembled blank part;
k – number of types of "elementary cells" considered in the model; in the work [4] there were used three types of "elementary cells" and the coefficient k was assumed to be 3.
During compaction, due to self-regulation of the structure, the value of the weights for each type of "elementary cells" changes.

The study of the process of self-regulation of the structure of FCM when compacting was carried out using the experimental model. The scale of the model to the real process was 50:1.

Analysis of the simulation results made it possible to determine numerically the Scm value for each layer at different degrees of deformation for each experiment.

Table 3 shows changes in the Scm values of the fibers relative to each other. Analysis of the values shows that when compacting Scm fibers for all layers tends to a value of 0.5. Starting from a strain degree of about 20%, the offsets between the 2-3 and 3-4 layers are aligned and reach 0.49 and 0.485, respectively, at the end of the process. After a strain rate of 20%, all experiments show an alignment of the fiber displacement pitch values.

Table 3. Change of fiber shift pitch across layers in one of the experiments

| Blank part deformation ε, % | Fiber displacement pitch between different layers, mm |
|-----------------------------|-----------------------------------------------------|
|                             | 1-2    | 2-3    | 3-4    | 4-5    |
| 0                           | 0,128  | 0,09   | 0,09   | 0,21   |
| 1,4                         | 0,128  | 0,16   | 0,129  | 0,226  |
| 5                           | 0,11   | 0,16   | 0,16   | 0,258  |
| 6,8                         | 0,16   | 0,225  | 0,19   | 0,323  |
| 8,1                         | 0,16   | 0,257  | 0,19   | 0,323  |
| 9,9                         | 0,225  | 0,354  | 0,258  | 0,323  |
| 11,3                        | 0,225  | 0,354  | 0,258  | 0,355  |
| 14,4                        | 0,289  | 0,418  | 0,26   | 0,387  |
| 18,5                        | 0,354  | 0,418  | 0,386  | 0,387  |
| 21,2                        | 0,41   | 0,452  | 0,419  | 0,387  |
| 23,4                        | 0,45   | 0,452  | 0,419  | 0,39   |
| 29,2                        | 0,485  | 0,4832 | 0,452  | 0,452  |
| 34,7                        | 0,495  | 0,49   | 0,485  | 0,487  |

The results of a series of experimental studies are approximated for each layer with a power polynomial:

$$S_{cm} = A + B \cdot \varepsilon + C \cdot \varepsilon^2 + D \cdot \varepsilon^3,$$

(4)

For the convenience of further processing of the results, an array of experimental data for "elementary cells" with Scm = 0 is selected from all available values of changes in initial Scm fibers; 0,1; 0,2; 0,3; 0,4; 0,5.

The expression coefficients (4) for the above Scm are shown in Table 4.

Table 4. Approximation factors for calculation of change of Scm of fibres by layers

| Initial Fiber Stacking Offset | a     | b      | c      | d      |
|------------------------------|-------|--------|--------|--------|
| 0                            | -0,0224192 | 1,572622 | 2,153537 | -7,15502 |
| 0,1                          | 0,07907284  | 1,500652 | -0,39555 | -1,61735 |
| 0,2                          | 0,19167928  | 0,565056 | 2,145579 | -3,86289 |
| 0,3                          | 0,29055767  | 0,659651 | 1,115561 | -4,16746 |
| 0,4                          | 0,39739126  | 0,177721 | 0,633712 | -1,29945 |
| 0,5                          | 0,4744506   | 0,021354 | 0,076145 | -0,26749 |
Figure 3 shows the graphical correlations of the change of the initial Scm values in the experimental model.

![Graph showing the change of initial Scm values](image)

**Fig. 3.** The nature of the change in the pitch of fiber displacement along the layers during the compaction of the structural blank part for "elementary cells" with different initial value of the offset pitch Scm

To determine compacting stress in the structural blank part taking into account self-regulation of fiber structure, the following procedure is proposed.

Initially, the weighting values for each of the layers are equal to each other. During deformation, the displacement pitch of the fiber layers changes. For example, for a layer with an initial Scm = 0, which is the minimum displacement step, when the deformation degree ε = 7% is reached, the current value of Scm becomes 0.1, which corresponds to the initial displacement step of the next layer, i.e., the "elementary cell" for the layer with an initial Scm = 0 takes the form of an "elementary cell" corresponding to the initial Scm = 0.1. Thus, the compaction stress for the layer with Scm = 0 should be calculated according to the formula corresponding Scm = 0.1. Therefore, the weight for the original Scm = 0 becomes 0, and for the layer with Scm = 0.1 increases by 2 times:

\[
\frac{n-2}{k} \sum_{j=1}^{6} \sigma^*_s \alpha_j \sigma^*_s \alpha_1 + 2 \frac{n-2}{k} \sigma^*_s \alpha_2 + n \frac{2}{k} \sum_{j=3}^{6} \sigma^*_s \alpha_j
\]

In the expression (5), the coefficient k corresponds to the number of discrete partitioning steps Scm in Table 4, where k = 6.

According to the proposed algorithm, compaction stress can be calculated for all layers of the blank part.

The values of Table 4 allow to determine the intervals of deformations of the transition of one type of "elementary cell" to another with a sequentially increasing value of Scm (Table 5).

**Table 5.** Values of deformation degrees during transitions from one type of "elementary cell" to another.

| Deformation degree ε, % | 7    | 8    | 14   | 15   | 35   | > 35 |
|-------------------------|------|------|------|------|------|------|
| transition between types of "elementary cells" | 0-0,1 | 0,1-0,2 | 0,2-0,3 | 0,3-0,4 | 0,4-0,5 | 0,5  |
4 Conclusion

The described technique allows to calculate compaction stress for the structural blank part depending on acting deformations and takes into account self-regulation of fiber displacement during deformation.

Compaction stress is one of the main characteristics required to calculate the compression modes of the FCM blank part. Compaction stress values obtained according to the new procedure should be used when calculating the power conditions of new deformation processes of FCM products.

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