The EOS of neutron matter, and the effect of $\Lambda$ hyperons to neutron star structure

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Los Alamos National Laboratory

NUCLEI
Nuclear Computational Low-Energy Initiative

www.computingnuclei.org
Neutron star is a wonderful natural laboratory

- Atmosphere: atomic and plasma physics
- Crust: physics of superfluids (neutrons, vortex), solid state physics (nuclei)
- Inner crust: deformed nuclei, pasta phase
- Outer core: nuclear matter
- Inner core: hyperons? quark matter? \( \pi \) or \( K \) condensates?
Several thousands of binding energies for normal nuclei. Only few, ∼50, for hypernuclei.
Homogeneous neutron matter

- Normal nuclei
- Quark gluon plasma
- Color superconducting quark matter
- Neutron stars

Temperature (K) vs. Density
The model and the method

Equation of state of neutron matter, role of three-neutron force

Symmetry energy

Neutron star structure (I)

Λ-hypernuclei

Λ-neutron matter

Neutron star structure (II)

Conclusions
Nuclear Hamiltonian

Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

\[ H = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} \]

\( v_{ij} \) NN fitted on scattering data. Sum of operators:

\[ v_{ij} = \sum O_{ij}^P = 1, 8 v^P (r_{ij}) \]

\[ O_{ij}^P = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j) \]

NN interaction - Argonne AV8'.

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Phase shifts, AV8'

\[ \delta (\text{deg}) \]

\[ E_{\text{lab}} \text{(MeV)} \]

\[ 0 \quad 100 \quad 200 \quad 300 \quad 400 \quad 500 \quad 600 \]

\[ 1S_0 \quad 3S_1 \quad 1P_1 \quad 3P_0 \quad 3P_2 \quad 3D_1 \quad 3D_2 \quad \epsilon_1 \]

\[ \text{Argonne V8'} \quad \text{SAID} \]

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Effect of Λ in neutron matter and the neutron star structure
Two neutrons have

\[ k \approx \sqrt{\frac{E_{\text{lab}} \, m}{2}} , \quad \rightarrow k_F \]

that correspond to

\[ k_F \rightarrow \rho \approx \left( \frac{E_{\text{lab}} \, m}{2} \right)^{3/2} / 2\pi^2 . \]

\( E_{\text{lab}} = 150 \) MeV corresponds to about \( 0.12 \text{ fm}^{-3} \).

\( E_{\text{lab}} = 350 \) MeV to \( 0.44 \text{ fm}^{-3} \).

Argonne potentials useful to study dense matter above \( \rho_0 = 0.16 \text{ fm}^{-3} \).
Three-body forces

Urbana–Illinois $V_{ijk}$ models processes like

\[ \pi \pi \Delta \]

\[ \pi \]

\[ \Delta \]

\[ \pi \]

\[ \pi \Delta \]

\[ \pi \]

\[ \Delta \]

+ short-range correlations (spin/isospin independent).

**Urbana UIX**: Fujita-Miyazawa plus short-range.
Quantum Monte Carlo

\[ H \psi(\vec{r}_1 \ldots \vec{r}_N) = E \psi(\vec{r}_1 \ldots \vec{r}_N) \quad \psi(t) = e^{-(H-E)t} \psi(0) \]

Ground-state extracted in the limit of \( t \to \infty \).

Propagation performed by

\[ \psi(R, t) = \langle R | \psi(t) \rangle = \int dR' G(R, R', t) \psi(R', 0) \]

- Importance sampling: \( G(R, R', t) \to G(R, R', t) \frac{\Psi_I(R')}{\Psi_I(R)} \)
- Constrained-path approximation to control the sign problem. Unconstrained calculation possible in several cases (exact).

Ground–state obtained in a non-perturbative way. Systematic uncertainties within 1-2 %. 
Recall: propagation in imaginary-time

\[ e^{-(T+V)\Delta\tau}\psi \approx e^{-T\Delta\tau}e^{-V\Delta\tau}\psi \]

Kinetic energy is sampled as a diffusion of particles:

\[ e^{-\nabla^2\Delta\tau}\psi(R) = e^{-(R-R')^2/2\Delta\tau}\psi(R) = \psi(R') \]

The (scalar) potential gives the weight of the configuration:

\[ e^{-V(R)\Delta\tau}\psi(R) = w\psi(R) \]

Algorithm for each time-step:
- do the diffusion: \( R' = R + \xi \)
- compute the weight \( w \)
- compute observables using the configuration \( R' \) weighted using \( w \) over a trial wave function \( \psi_T \).

For spin-dependent potentials things are much worse!
Because the Hamiltonian is state dependent, all spin/isospin states of nucleons must be included in the wave-function.

Example: spin for 3 neutrons (radial parts also needed in real life):

**GFMC wave-function:**

\[
\psi = \begin{pmatrix}
    a_{\uparrow\uparrow\uparrow} \\
    a_{\uparrow\uparrow\downarrow} \\
    a_{\uparrow\downarrow\uparrow} \\
    a_{\uparrow\downarrow\downarrow} \\
    a_{\downarrow\uparrow\uparrow} \\
    a_{\downarrow\uparrow\downarrow} \\
    a_{\downarrow\downarrow\uparrow} \\
    a_{\downarrow\downarrow\downarrow}
\end{pmatrix}
\]

A correlation like

\[1 + f(r)\sigma_1 \cdot \sigma_2\]

can be used, and the variational wave function can be very good. Any operator accurately computed.

**AFDMC wave-function:**

\[
\psi = \mathcal{A} \left[ \xi_{s_1} \left( \begin{array}{c} a_1 \\ b_1 \end{array} \right) \xi_{s_2} \left( \begin{array}{c} a_2 \\ b_2 \end{array} \right) \xi_{s_3} \left( \begin{array}{c} a_3 \\ b_3 \end{array} \right) \right]
\]

We must change the propagator by using the Hubbard-Stratonovich transformation:

\[e^{\frac{1}{2} \Delta tO^2} = \frac{1}{\sqrt{2\pi}} \int dx e^{-\frac{x^2}{2} + x\sqrt{\Delta tO}}\]

Auxiliary fields \(x\) must also be sampled. The wave-function is pretty bad, but we can simulate larger systems (up to \(A \approx 100\)). Operators (except the energy) are very hard to be computed, but in some case there is some trick!
Light nuclei spectrum computed with GFMC

Argonne v\textsubscript{18} with Illinois-7 GFMC Calculations

Carlson, et al., arXiv:1412.3081
Charge form factor of $^{12}$C

$$|F(q)| = \langle \psi | \rho_q | \psi \rangle$$

$$\rho_q = \sum_i \rho_q(i) + \sum_{i<j} \rho_q(ij)$$

Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla, PRL (2013)
Neutron matter equation of state

Why to study neutron matter at nuclear densities?

- EOS of neutron matter gives the symmetry energy and its slope.
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part. No direct $T = 3/2$ experiments available.

Why to study symmetry energy?

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Theory

Experiments

Neutron stars

Esym, L
What is the Symmetry energy?

Assumption from experiments:

\[ E_{SNM}(\rho_0) = -16 \text{MeV}, \quad \rho_0 = 0.16 \text{fm}^{-3}, \quad E_{sym} = E_{PNM}(\rho_0) + 16 \]

At \( \rho_0 \) we access \( E_{sym} \) by studying PNM.
We consider different forms of three-neutron interaction by only requiring a particular value of $E_{sym}$ at saturation.

\[
E_{sym} = 33.7 \text{ MeV}
\]

different 3N:
- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V^\mu_R$ (several $\mu$)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$
Equation of state of neutron matter using Argonne forces:

\[ E_{\text{sym}} = \begin{cases} 
35.1 \text{ MeV (AV8'+UIX)} \\
33.7 \text{ MeV} \\
32 \text{ MeV} \\
30.5 \text{ MeV (AV8')} 
\end{cases} \]

Gandolfi, Carlson, Reddy, PRC (2012)
From the EOS, we can fit the symmetry energy around $\rho_0$ using

$$E_{\text{sym}}(\rho) = E_{\text{sym}} + \frac{L}{3} \frac{\rho - 0.16}{0.16} + \cdots$$

Gandolfi et al., EPJ (2014)

Tsang et al., PRC (2012)

Very weak dependence to the model of 3N force for a given $E_{\text{sym}}$. Chiral Hamiltonians give compatible results.
TOV equations:

\[
\frac{dP}{dr} = - \frac{G[m(r) + 4\pi r^3 P/c^2][\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]},
\]

\[
\frac{dm(r)}{dr} = 4\pi\epsilon r^2.
\]
EOS used to solve the TOV equations.

Causality: \( R > 2.9 \) (\( GM/c^2 \))

\[ \rho_{\text{central}} = 2\rho_0 \]
\[ \rho_{\text{central}} = 3\rho_0 \]

Error associated with \( E_{\text{sym}} \)

\[ E_{\text{sym}} = 30.5 \text{ MeV (NN)} \]

Gandolfi, Carlson, Reddy, PRC (2012).

Accurate measurement of \( E_{\text{sym}} \) put a constraint to the radius of neutron stars, OR observation of \( M \) and \( R \) would constrain \( E_{\text{sym}} \)!
Observations of the mass-radius relation are becoming available:

Steiner, Lattimer, Brown, ApJ (2010)

Neutron star observations can be used to 'measure' the EOS and constrain $E_{\text{sym}}$ and $L$. 
Neutron star matter model:

\[ E_{NSM} = a \left( \frac{\rho}{\rho_0} \right)^\alpha + b \left( \frac{\rho}{\rho_0} \right)^\beta , \quad \rho < \rho_t \]

(form suggested by QMC simulations),

and a high density model for \( \rho > \rho_t \)

i) two polytropes
ii) polytrope+quark matter model

Neutron star radius sensitive to the EOS at nuclear densities!

Direct way to extract \( E_{sym} \) and \( L \) from neutron stars observations:

\[ E_{sym} = a + b + 16 , \quad L = 3(a\alpha + b\beta) \]
Neutron star matter really matters!

Here an 'astrophysical measurement'

32 < $E_{\text{sym}}$ < 34 MeV, 43 < $L$ < 52 MeV

Steiner, Gandolfi, PRL (2012).
High density neutron matter

If chemical potential large enough ($\rho \sim 2 - 3\rho_0$), heavier particles form, i.e. $\Lambda, \Sigma, ...$

Non-relativistic BHF calculations suggest that none of the available hyperon-nucleon Hamiltonians support an EOS with $M > 2M_\odot$:

Schulze and Rijken PRC (2011).

(Some) other relativistic model support $2M_\odot$ neutron stars.
Hypernuclei and hypermatter:

\[
H = H_N + \frac{\hbar^2}{2m_\Lambda} \sum_{i=1}^A \nabla_i^2 + \sum_{i<j} V_{ij}^{\Lambda N} + \sum_{i<j<k} V_{ijk}^{\Lambda NN}
\]

\(\Lambda\)-binding energy calculated as the difference between the system with and without \(\Lambda\).
The Λ-nucleon interaction is constructed similarly to the Argonne potentials (Usmani).

Argonne NN: $v_{ij} = \sum_p v_p(r_{ij})O_{ij}^p$, $O_{ij} = (1, \sigma_i \cdot \sigma_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \tau_i \cdot \tau_j)$

Usmani ΛN: $v_{ij} = \sum_p v_p(r_{ij})O_{ij}^p$, $O_{\lambda j} = (1, \sigma_{\lambda} \cdot \sigma_j) \times (1, \tau_j^z)$

But... $\sim$4500 NN data, $\sim$50 of ΛN data.
ΛNN has the same range of ΛN
AFDMC for (hyper)nuclei is limited to simple interactions. We found that the $\Lambda$-binding energy is quite independent to the details of NN interactions.

| $NN$ potential  | $V_{\Lambda N} + V_{\Lambda NN}$ |
|-----------------|----------------------------------|
|                 | $\frac{}{^5\Lambda{\text{He}}}$ | $\frac{}{^{17}\Lambda{\text{O}}}$ |
| Argonne V4’     | 5.1(1)                           | 19(1)                             |
| Argonne V6’     | 5.2(1)                           | 21(1)                             |
| Minnesota       | 5.2(1)                           | 17(2)                             |
| Expt.           | 3.12(2)                          | 13.0(4)                           |

D. L., S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

The inclusion of (simple) three-body forces gives very similar results (unpublished).
$\Lambda$ hypernuclei

$V^{\Lambda N}$ and $V^{\Lambda NN}(I)$ are phenomenological (Usmani).

$V^{\Lambda NN}(II)$ is a new form where the parameters have been fine tuned.

As expected, the role of $\Lambda NN$ is crucial for saturation.
Λ hypernuclei

Λ in different states:

Lonardoni, SG, Pederiva, in preparation.
Hyper-neutron matter

Neutrons and Λ particles:

\[ \rho = \rho_n + \rho_\Lambda, \quad x = \frac{\rho_\Lambda}{\rho} \]

\[ E_{\text{HNM}}(\rho, x) = \left[ E_{\text{PNM}}((1-x)\rho) + m_n \right] (1-x) + \left[ E_{\text{PAM}}(x\rho) + m_\Lambda \right] x + f(\rho, x) \]

where \( E_{\text{PAM}} \) is the non-interacting energy (no \( \nu_\Lambda \Lambda \) interaction),

\[ E_{\text{PNM}}(\rho) = a \left( \frac{\rho}{\rho_0} \right)^\alpha + b \left( \frac{\rho}{\rho_0} \right)^\beta \]

and

\[ f(\rho, x) = c_1 \frac{x(1-x)\rho}{\rho_0} + c_2 \frac{x(1-x)^2\rho^2}{\rho_0^2} \]

All the parameters are fit to AFDMC results.
Λ-neutron matter

EOS obtained by solving for $\mu_\Lambda(\rho, x) = \mu_n(\rho, x)$

Lonardoni, Lovato, Pederiva, SG, arXiv:1407.4448.

No hyperons up to $\rho = 0.5 \text{ fm}^{-3}$ using $\Lambda NN$ (II)!!!
Drastic role played by $\Lambda NN$. Calculations can be compatible with neutron star observations.

Note: no $\nu_{\Lambda\Lambda}$, no protons, and no other hyperons included.
QMC methods useful to study nuclear systems in a coherent framework:

- Three-neutron force is the bridge between $E_{sym}$ and neutron star structure.
- Neutron star observations becoming competitive with experiments.
- $\Lambda$-nucleon data very limited, but $\Lambda NN$ is very important. Role of $\Lambda$ in neutron stars far to be understood. More $\Lambda N$ data needed. Input from Lattice QCD?
  
  Conclusion? We cannot conclude anything with present models...

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