Tachyon driven solution to Cosmic Coincidence Problem

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Abstract

Here, non-minimally coupled tachyon to gravity is considered as a source of dark energy. It is demonstrated that with expansion of the universe, tachyon dark energy decays to dark matter providing a solution to “cosmic coincidence problem”. Moreover, it is found that the universe undergoes accelerated expansion simultaneously. PACS nos. 98.80 Cq, 95.35.+d.

1. Introduction

Around four decades back, tachyons were proposed [1] and cosmology, driven by these particles, were explored [2]. But these superluminal particles were discarded for not being observed. At the turn of last century,
tachyons are re-awakened in the context of unstable D-branes in bosonic and superstring theories. Due to concerted efforts by Sen [3], role of tachyons, in string theories, got prominence among physicists. The idea is derived from the fact that usual open string vacuum is unstable but with a stable vacuum also having vanishing energy density. The unstable vacuum corresponds to rolling tachyon from the maximum of its potential to the minimum and stable vacuum indicates presence of standard particle. Sen has argued that tachyonic state is analogous to condensation of electric flux tubes of closed strings described by Born - Infeld action. So, flat space Born - Infeld lagrangian was suggested for tachyon condensates too [3, 4]. It was translated to curved space framework, for tachyon scalars $\phi$ with potential $V(\phi)$, as $-V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$ having minimal coupling with gravity [5]. Later on, Bagla et al. had shown that this lagrangian could also be treated as generalization of a relativistic particle lagrangian [6]. Recently, another tachyon model has been proposed with lagrangian $W(\phi)\sqrt{g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} - 1$ (with $W(\phi)$ is real). It is argued that tachyon scalars, described by this lagrangian, may be able to explore more physical situations than quintessence [7]. It has been shown that as a homogeneous tachyon rolls down the hill of its potential to its minimum, when $\dot{\phi} = d\phi/dt \to 1$, energy density approaches a finite value and pressure tends to zero. In the cosmological framework, rolling of tachyon is associated with expansion of the universe [5]. These results prompted to conclude that when cosmic expansion is large, tachyon condensates behave like dust, showing it as good candidate for cold dark matter (CDM), being pressureless non-baryonic fluid [8].

Drastic changes are noticed on taking non-minimal coupling of $\phi$, given
by the lagrangian [9]

\[
L_\phi = \sqrt{-g}[ - V(\phi) \sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} + \xi R \phi^2],
\]

(1.1)

where \( R \) is the Ricci scalar, \( \xi \) is the non-minimal coupling constant, \( V(\phi) \) is the potential and \( G = M_P^{-2} \) (\( M_P = 10^{19}\text{GeV} \) being the Planck mass) is the gravitational constant. Here \( g \) is the determinant of metric tensor components \( g_{\mu\nu}(\mu, \nu = 0, 1, 2, 3) \).

From this lagrangian, it is found that pressure is non-zero if \( \xi \neq 0 \) even when \( \dot{\phi} \to 1 \).

Non-minimal coupling of tachyon with gravity was also proposed by Piao et al [10] in a different manner, where a function of \( \phi \) is coupled to Einstein-Hilbert lagrangian as

\[
S = \int d^4x \sqrt{-g} \left[ \frac{f(\phi)R}{16\pi G} + V(\phi) \sqrt{1 + \alpha' g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} \right],
\]

(1.2)

where \( \alpha' \) gives string mass scale. Subject to the condition \( 1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \gg 16\pi G \xi R \phi^2 \), the lagrangian (1.1) looks like

\[
L_\phi \simeq \sqrt{-g}[ - V(\phi) \sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - \frac{1}{2} \frac{\xi V(\phi) \phi^2 R}{\sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}},
\]

(1.3)

which is similar to lagrangian of (1.2) with non-minimal coupling function

\[
f(\phi) = -8\pi G \frac{\xi V(\phi) \phi^2}{\sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}}.
\]

In what follows, investigations are made using the lagrangian (1.1). With this lagrangian, tachyon condensates never approach to zero pressure. So, it can be taken as viable candidate for dark energy (DE), not CDM. Apart
from tachyons, various DE models are available in the literature (i) a very small cosmological constant [11], (ii) quintessence [12], (iii) k-essence [13], (iv) Chaplygin gas [14], (v) interacting quintessence [15], (vi) non-minimally coupled quintessence [16] and others, violating “strong energy condition”. Recently, some other models were also proposed violating “weak energy condition” also [17] and showing finite time future singularities. Barrow [18] and Lake [19] have demonstrated that violation of “dominant energy condition” leads to “sudden future singularity”. A generalization of Barrow’s model is suggested in ref. [20]. A review on DE can be seen in articles [21, 22].

The astrophysical observation that high red-shift supernovae are fainter than expected, leads to cosmic acceleration [23-25]. This observation is further confirmed by WMAP (Wilkinson Microwave Anisotropy Probe) [26-28]. These experiments show that content of the present universe is comprised of around 77% dark energy and 23% CDM. In addition to accelerated expansion of the current universe, these data pose another important question “How are DE and CDM densities of the same order at present?” This question constitutes the “cosmic coincidence problem” (CCP) [29]. As mentioned above, experimental data also suggest that DE dominates over CDM today. So, in other words, CCP is also coined as “How does DE dominate the present universe?”

The accelerated expansion is caused by DE, driven by different exotic matter mentioned above. Moreover, for “coincidence problem” too, role of DE is crucial. So, it is important to understand its nature. In the recent past, it was repeatedly suggested that DE could be derived by self-interacting
scalar fields, behaving as a perfect fluid with the equation of state parameter \( w < -1/3 \). It was proposed that, in the early universe, these scalars contributed DE density lower than that of matter including radiation. With the expansion of the universe, matter density rolled down. As a result, DE density became comparable to matter density at late times [12]. The usual strategy, in these types of work, is the use of a suitable potential to yield the required result. Later on, Padmanabhan [30] demonstrated that it is straightforward to derive such potentials. In ref.[31], it is shown that a mixture of matter and *quintessence*, gravitationally interacting with each other, is unable to derive speeded-up expansion and a solution to CCP simultaneously unless matter fluid is dissipative enough. These authors have obtained attractor solution for \( r = \rho_\phi/\rho_m \) (with \( \rho_\phi(\rho_m) \) being DE (matter)density). Here, it is shown that \( r < 1 \) remains stable [15,32].

In ref.[33], author has proposed a different mechanism to overcome CCP alongwith accelerated expansion. Contrary to assumption in refs.[12]. This prescription suggests that, in the early universe, DE density used to be high, but dynamical. Moreover, it is shown that DE density dominates the early universe, causing accelerated expansion from the beginning of the universe. This situation is like inflationary models with the difference that inflationary models exhibit accelerated expansion for a short period, whereas in the model of ref.[33] expansion is slowly speeded-up from the epoch of creation of the universe upto late times. DE density falls down with the growth of scale factor. DE, so lost, causes creation of *dark matter* (DM). Accordingly, in this model, there is no DM in the beginning, rather it is produced due to decay of DE. So, \( r \) becomes a dynamical parameter and grows with the time.
keeping itself as $0 \leq r < 1$. The phenomenon of decay of DE and creation of DM is given by coupled equations. A similar procedure has been adopted by Mota and Bruck [34] for the condensation of DE in overdense regions of matter, though it does not condensate in normal circumstances.

The same mechanism [33] is adopted in the present paper for tachyon condensates, in the late universe, as in [9], where tachyon DE decays to CDM. In this paper, self interacting inverse cubic potential had been considered. Here, investigations for tachyons are carried out taking self interacting inverse quartic potential.

The paper is organized as follows. Using action (1.1), basic equations are derived in section 2. Section 3 contains investigations employing inverse quartic potential with a modification in coupled equations for DE and DM. Section 4 deals with inverse exponential potential. Remarks on the results, obtained in sections 3 and 4, are given in section 5. Natural units $\hbar = c = 1$, are used with GeV as a fundamental unit.

2. Basic equations

Einstein’s field equations are given as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G[T_{\mu\nu}(\phi) + T_{\mu\nu}(m)]$$  (2.1)

with energy-momentum tensor components of tachyon and matter as

$$T_{\mu\nu}(\phi) = (\rho_{(\phi)} + p_{(\phi)})u_\mu u_\nu - p_{(\phi)}g_{\mu\nu} \quad (2.2a)$$

and

$$T_{\mu\nu}(m) = (\rho_{(m)} + p_{(m)})u_\mu u_\nu - p_{(m)}g_{\mu\nu} \quad (2.2b)$$
respectively, where \( u^\mu = (1, 0, 0, 0) \). \( T^\mu_{\mu(\phi)} = (\rho(\phi), -p(\phi), -p(\phi), -p(\phi)) \) are obtained from the lagrangian (1.1) with

\[
T^\mu_{\mu(\phi)} = -V(\phi)[1 - \nabla^\rho \phi \nabla_\rho \phi + \xi R\phi^2]^{-1/2} \times \left[-\nabla_\mu \phi \nabla_\nu \phi + \xi R_{\mu\nu}\phi^2 \right.
\]

\[
+ \xi (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \phi^2 - g_{\mu\nu}(1 - \nabla^\rho \phi \nabla_\rho \phi + \xi R\phi^2) \right].
\]

(2.2c)

Here \( \nabla_\mu \) stands for covariant derivative and \( R_{\mu\nu} \) are Ricci tensor components.

Field equations for \( \phi \) are obtained as

\[
\Box \phi + \frac{2(\nabla^\mu \phi)(\nabla_\rho \phi)(\nabla^\rho \nabla_\mu \phi) - 2\xi R\phi \nabla^\rho \phi \nabla_\rho \phi - \xi \phi^2 g^{\mu\nu} \nabla_\mu R \nabla_\nu \phi}{2(1 - \nabla^\rho \phi \nabla_\rho \phi + \xi R\phi^2)}
\]

\[
+ \xi R\phi + \frac{V'}{V}(1 + \xi R\phi^2) = 0,
\]

(2.3)

from the lagrangian (1.1). Here \( V'(\phi) = \frac{d}{dx} V(\phi) \) and

\[ \Box = \nabla^\rho \nabla_\rho = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu}(\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu}). \]

According to cosmological observations [23 - 28], currently we live in a spatially flat and speeding - up universe, such that \( \ddot{a}/a > 0 \) for the scale factor \( a(t) \), given by the distance function

\[
dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] .
\]

(2.4)

It represents a homogeneous model of the universe, hence

\[
\phi(x, t) = \phi(t).
\]

(2.5)
Connecting eqs. (2.3) and (2.5a)

\[ \ddot{\phi} + 3H\dot{\phi} + \frac{2\dot{\phi}^2 - 2\xi R \phi \dot{\phi}^2 - \xi \phi^2 \ddot{R}}{2(1 - \phi^2 + \xi R \phi^2)} + \xi R \phi + \frac{V'}{V}(1 + \xi R \phi^2) = 0, \quad (2.6) \]

where \( H = \dot{a}/a \).

Energy density \( \rho_{(\phi)} \) and pressure \( p_{(\phi)} \) are obtained from eq.(2.2c) as

\[
\rho_{(\phi)} = V(\phi) \left[ 1 - \xi \left( R_0^2 - R \right) \phi^2 + 6\xi H \phi \dot{\phi} \right] \sqrt{1 - \dot{\phi}^2 + \xi \phi^2 R} \\
= V(\phi) \left[ 1 + 6\xi H \phi \dot{\phi} + 3\xi (\dot{H} + 3H^2) \phi^2 \right] \sqrt{1 - \dot{\phi}^2 + 6\xi \phi^2 (\dot{H} + 2H^2)}
\]

(2.7a)

and isotropic pressure \( p_{(\phi)} \) as

\[
p_{(\phi)} = -V(\phi) \left[ 1 - \dot{\phi}^2 + \xi (2\dot{\phi} \ddot{\phi} + 2\dot{\phi}^2 + 6\xi H \phi \dot{\phi}) + \xi \left( R_1^1 - R \right) \phi^2 \right] \sqrt{1 - \dot{\phi}^2 \xi \phi^2 R} \\
= -V(\phi) \left[ 1 - \dot{\phi}^2 + \xi (2\dot{\phi} \ddot{\phi} + 2\dot{\phi}^2 + 6\xi H \phi \dot{\phi}) + \xi (5\dot{H} + 9H^2) \phi^2 \right] \sqrt{1 - \dot{\phi}^2 \xi \phi^2 R}
\]

(2.7b)

From eqs.(2.1), it is obtained that

\[
\frac{R_1^l - \frac{1}{2}R}{R_0^l - \frac{1}{2}R} \sim \frac{-p_{(\phi)}}{\rho_{(\phi)}}
\]

(2.8)

taking dominance of tachyon dark energy over matter. Here \( \rho_{(\phi)} \) and \( p_{(\phi)} \) are given by eqs.(2.7).

Bianchi identities \( (T^\mu_\nu + T^\nu_\mu)_\nu = 0 \) yield coupled equations

\[ \dot{\rho}_{(\phi)} + 3H(\rho_{(\phi)} + p_{(\phi)}) = -Q(t) \quad (2.9a) \]
and

\[ \dot{\rho}_m + 3H \rho_m = Q(t), \]  

(2.9b)

where \( Q(t) \) is loss (gain) term for DE (CDM). Here \( p_m = 0 \) for CDM. Physically, these equations show decay of DE to CDM.

3. Inverse self-interacting quartic potential for tachyon

The inverse quartic potential for \( \phi \) is taken as

\[ V(\phi) = \lambda \phi^{-4}, \]  

(3.1)

where \( \lambda \) is a dimensionless coupling constant.

Case (a) \( \xi \neq 0 \)

Using the potential, given by eq.(3.1), in eqs.(2.6), (2.7) and (2.8), it is obtained as

\[ \ddot{\phi} + 3H \dot{\phi} + \frac{2\ddot{\phi}^2 - 2\xi R\phi^2 - \xi \phi^2 \dddot{\phi}}{2(1 - \phi^2 + \xi R \phi^2)} - 3\xi \phi - \frac{4}{V} = 0 \]  

(3.2a)

and

\[ \frac{2\dot{H} + 3H^2}{3H^2} \approx \frac{[1 - \dot{\phi}^2 + \xi(2\ddot{\phi} + 2\dot{\phi}^2 + 6\xi H \phi \dot{\phi}) + \xi(5\dot{H} + 9H^2)\phi^2]}{[1 + 6\xi H \phi \dot{\phi} + 3\xi(H + 3H^2)\phi^2]} = -w_\phi \]  

(3.2b)

for the geometry given by eq.(2.4).

According to the tachyon lagrangian (1.1), \( \phi \) has mass dimension equal to \(-1\) like time \( t \) (in natural units). So, on the basis of dimensional considerations, it is reasonable to take
\[ \phi(t) = At. \] (3.3)

with \( A \) being dimensionless constant.

Now the scale factor is assumed to have the form

\[ a(t) = a_i \left( t/t_i \right)^q, \] (3.4)

where \( q \) is a real number and \( t_i \) is supposed to be the time when DE begins to decay to CDM. Also \( a_i = a(t_i) \).

Connecting eqs.(3.2)-(3.4), it is obtained that

\[ \frac{3q - 2}{3q} \simeq \frac{1 - A^2 + \xi A^2 (2 + q + 9q^2)}{1 + 3\xi A^2 (q + 3q^2)} = -w_\phi \] (3.5a)

and

\[ 36\xi q^2 A^2 = 3qA^2 (1 + 6\xi) - 4. \] (3.5b)

Elimination of \( A^2 \), from eqs.(3.5), yields

\[ q = \frac{2}{3(1 + w)_\phi} = \frac{14\xi - 1}{4\xi}. \] (3.6a, b)

Subject to the condition \( 0 < A^2 < [1 - 6\xi(-q + 2q^2)^{-1}] \) to get \( \rho_\phi \) and \( p_\phi \), eqs.(3.5) and (3.6) yield a set of solutions

\[ \xi = -12.5, \quad q = 3.52, \quad w_\phi = -0.81 \]

and

\[ A^2 = \frac{100}{119856}. \] (3.7a, b, c, d)

From eqs.(2.7a), (2.9a), (3.5a) and (3.6), it is obtained that
\[ Q(t) = 2\tilde{A}t^{-5}, \quad (3.8a) \]

where
\[ \tilde{A} = \frac{\lambda}{A^4}[1 + 3\xi A^2(q + 3q^2)]. \quad (3.8b) \]

Using eqs. (3.4) and (3.7), eq. (2.9b) is integrated to
\[ \rho_m = \frac{2\tilde{A}t^{-4}}{(3q - 4)[1 - (t/t_i)^{(3q - 4)}]} \quad (3.9) \]
for \( \rho_m(t_i) = 0 \). From eq. (2.7a), (3.3) and (3.4)
\[ \rho_\phi = \tilde{A}t^{-4} \quad (3.10) \]
with \( \tilde{A} \) given by eq. (3.7b). So,
\[ r(t) = \frac{\rho_m}{\rho_\phi} = 0.304\left[1 - \left(t_i/t\right)^{6.56}\right]. \quad (3.11) \]

Using current observational data for the universe, DE density \( \rho_{\phi(0)} = 0.77\rho_{cr,0} \), CDM density \( \rho_{m(0)} = 0.23\rho_{cr,0} \) with \( \rho_{cr,0} = 3H_0^2/8\pi G, H_0 = h/t_0(h = 0.72 \pm 0.04) \) and the present age \( t_0 \simeq 13.7 \) Gyr, in eq. (3.10), it is obtained that
\[ t_i = 0.539t_0. \quad (3.12) \]

Eqs. (3.10) - (3.12) yield
\[ r(t) = 0.304\left[1 - 0.017\left(t_0/t\right)^{6.56}\right]. \quad (3.13) \]
showing that \(0 < r < 1\) for \(0.539t_0 < t\). Moreover as \(q\) is greater than 1, eq.(3.4) shows that universe is accelerated. Thus it provides a solution to CCP in the speeded-up universe.

**Case (b) \(\xi = 0\)**

In this case, eqs.(3.2) look like

\[
\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} - \frac{4}{V} = 0 \tag{3.14a}
\]

and

\[
\frac{2\dot{H} + 3H^2}{3H^2} \simeq 1 - \dot{\phi}^2 = -w_\phi \tag{3.14b}
\]

Moreover eqs.(2.7) reduce to

\[
\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \tag{3.15a}
\]

and

\[
p_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2} \tag{3.15b}
\]

Eqs.(3.15) show that \(\dot{\phi}^2 < 1\) for real \(\rho_\phi\) and \(p_\phi\). Moreover, taking \(\ddot{\phi} \ll 3H\dot{\phi}\) in eq.(3.14a), eqs.(3.14) are integrated to

\[
\dot{\phi} = \dot{\phi}_i \left(\frac{t}{t_i}\right)^{4/3\sqrt{\lambda}} \tag{3.16a}
\]

and

\[
H(t) = \frac{3\lambda}{8\dot{\phi}_i^2} t_i^{8/3\sqrt{\lambda}} \left[ -1 + \frac{8}{3\sqrt{\lambda}} t_i^{1-8/3\sqrt{\lambda}} \right] \tag{3.16b}
\]

with

\[
\sqrt{\lambda} = \frac{4}{3} - \frac{1}{3}\sqrt{10}. \tag{3.16c}
\]
As \( \rho_\phi \simeq V(\phi) \), for this case, \( Q(t) \) is calculated to be

\[
Q(t) = 2\lambda \phi_i^{-4} t_i^{16/3} \sqrt{\lambda} t_i (-1 - 16/3 \sqrt{\lambda})
\]  

(3.17)

using eq.(2.9a). Employing eqs.(3.16), eq.(2.9b) is integrated to

\[
\rho_m \approx \frac{128B}{9} \left[ (4 + \sqrt{10})/(12 + \sqrt{10}) \right] t^{-\sqrt{10}/(4+\sqrt{10})} \left[ (t/t_i)^\sqrt{10}/(4+\sqrt{10}) \right. \\
\times \exp \left\{ B \left[ (12 + \sqrt{10})/(16 + \sqrt{10}) \right] t_i^{(16+\sqrt{10})/(4+\sqrt{10})} - t_i^{(16+\sqrt{10})/(4+\sqrt{10})} \right\} - 1 ,
\]

(3.18)

where \( B = \frac{3\lambda}{8\phi_i^4} t_i^{8/3} \sqrt{\lambda} \). Now

\[
\rho_m/\rho_\phi \simeq \rho_m/V(\phi) \approx \frac{16\phi_i^2}{3} t_i^{8/4 + \sqrt{10}} t_i^{-(16 - \sqrt{10})/(4 + \sqrt{10})} \left[ (t/t_i)^\sqrt{10}/(4+\sqrt{10}) \right. \\
\times \exp \left\{ B \left[ (12 + \sqrt{10})/(16 + \sqrt{10}) \right] t_i^{(16+\sqrt{10})/(4+\sqrt{10})} - t_i^{(16+\sqrt{10})/(4+\sqrt{10})} \right\} - 1.
\]

(3.19)

As \( t_0 > t_i \), eq.(3.19) shows \( \rho_m/\rho_\phi > 1 \). This result is not consistent with current observations. It means that, in accelerated universe, CCP can not be solved for \( \xi = 0 \) through decay of tachyon dark energy to CDM.

**Conclusion**

In the above investigations, it is found that non-minimally coupled tachyon with gravity contributes DE to the universe. Dynamics of tachyons and its back-reaction to the universe are explored subject to self-interacting inverse quartic potential. Here, investigations for decay of DE to CDM are made for cases of minimally coupled tachyon as well as non-minimally coupled tachyon to gravity. In the case of non-minimal coupling, present ratio of CDM and tachyon energy density is obtained to be \( \sim 0.3 \) in the accelerated universe.
universe without taking any dissipative term, if decay of DE to CDM begins at $t_i = 0.539t_0$. But, in the case of minimal coupling, CDM density is found more than tachyon energy denmsity contradicting current observations. It means that, in the case of minimal coupling ($\xi = 0$) decay of tachyon dark energy to CDM is not possible. This result is parallel to the result of ref.[15], where it is obtained that CCP can not be solved in the accelerated universe unless dissipative term is used for CDM.

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