Resonant Tunneling between quantum Hall states at filling $\nu = 1$ and $\nu = 1/3$

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We study the problem of resonant tunneling between a Fermi liquid and the edge of a $\nu = 1/3$ fractional quantum Hall state. In the limit of weak coupling, the system is adequately described within the sequential tunneling approximation. At low temperatures, however, the system crosses over to a strong coupling phase, which we analyze using renormalization group techniques. We find that at low temperatures a “perfect” can be achieved by tuning two parameters. This resonance has a peak conductance $G = e^2/2h$ and is a realization of the weak backscattering limit of the $g = 1/2$ Luttinger liquid, as well as the non Fermi liquid fixed point of the two channel Kondo problem. We discuss several regimes which may be experimentally accessible as well as the implications for resonances recently observed in cleaved edge overgrowth structures.

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I. INTRODUCTION

The rich structure of the fractional quantum Hall effect offers a controlled laboratory for studying the physics of strong correlation. As first argued by Wen, the low energy excitations at the edge of a quantum Hall state form a novel quantum fluid known as a chiral Luttinger liquid. This theory has led to a number of predictions regarding the effects of tunneling and resonant tunneling between quantum Hall states. Experiments by Milikien Umbach and Webb probed tunneling and resonant tunneling at a quantum point contact. They observed a striking difference in behavior at filling factors $\nu = 1$ and $\nu = 1/3$, which was consistent with these predictions. More recent experiments on a cleaved edge overgrowth structure by Chang, Pfeiffer and West observed the predicted power law scaling of the tunneling current with voltage and temperature at $\nu = 1/3$ over several decades of current. While questions remain about other filling factors, the agreement between experiment and theory has been quite encouraging for the Laughlin states at $\nu = 1$ and 1/3.

In this paper we analyze the problem of resonant tunneling between two different quantum Hall states, $\nu = 1$ and 1/3 - or equivalently between a Fermi liquid and $\nu = 1/3$. Our motivation is twofold. First, in their recent cleaved edge overgrowth experiments, Grayson et al. have observed interesting resonances in tunneling between a metal and $\nu = 1/3$. Second, this problem is interesting from a theoretical point of view because, as we explain below, a “perfect” resonance in this problem constitutes a realization of the perfectly transmitting fixed point of a $g = 1/2$ Luttinger liquid, which is equivalent to the non Fermi liquid fixed point of the two channel Kondo problem.

Our analysis is closely related to that of resonant tunneling in a $\nu = 1/3$ quantum point contact, which is closely related to the problem of tunneling through a barrier in a $g = 1/3$ Luttinger liquid. The limit of strong pinch off corresponds to a barrier with very small transmission, whereas the limit weak pinch off corresponds to a small barrier which backscatters only weakly. As argued in Ref. 9 a perfect resonance is controlled by the perfectly transmitting fixed point. Such a resonance has a peak conductance of $e^2/3h$ and a universal lineshape which is determined by the crossover between the perfectly transmitting and perfectly reflecting fixed points.

The problem of tunneling between $\nu = 1$ and $\nu = 1/3$ can be mapped onto that of tunneling through a barrier in a $g = 1/2$ Luttinger liquid. This has led to the interesting suggestion that the interface between $\nu = 1$ and $\nu = 1/3$ may allow for the realization of the perfectly transmitting limit of the $g = 1/2$ Luttinger liquid. Such a state would have interesting properties. Since the “perfect” two terminal conductance $e^2/2h$ is larger than the conductance of the quantum Hall state, it would act as a DC step-up transformer. In addition, the weak backscattering in this state would occur via quasiparticles with charge $e/2$, which could in principle be detected in a shot noise experiment.

However, realizing this state presents a subtle problem. Unlike the $\nu = 1/3$ point contact, where one can physically separate the counter moving edge states, there is no direct way of “engineering” the $g = 1/2$ perfectly transmitting fixed point. In principle it could be achieved using the adiabatic contact, suggested by Chklovskii and Halperin, in which the quantum Hall fluids are connected smoothly by a quantum wire, in which the Luttinger parameter $g$ can vary continuously. Recently, Sandler et al. have argued that the perfectly transmitting fixed point may be approached by scaling to sufficiently high energies. However, since there are many irrelevant operators at this fixed point, it is unlikely that the system would flow near the fixed point without tuning several parameters.

In this paper, we show that the perfectly transmitting limit can be reached by resonantly tunneling through an impurity state. A “perfect” resonance can be reached at zero temperature by tuning two parameters. Such a resonance has a peak two terminal conductance of $e^2/2h$ and
has a universal temperature dependent lineshape which has been computed in the context of a $g = 1/2$ Luttinger liquid\cite{Grayson}. The remainder of the paper is organized as follows. After describing the model, we discuss the resonances in the weak tunneling limit, when the tunneling is sequential. Then using a renormalization group analysis we study the crossover to the perfect resonance. Finally we discuss the experimental implications of our results and the connection with the recent observations by Grayson et al\cite{Grayson}.

\section{The Luttinger Liquid Model}

We begin with the chiral Luttinger liquid model describing the coupling between the $\nu = 1/3$ and $\nu = 1$ edge states and the impurity state. The Hamiltonian may be written, $H = H_0^c + H_0^d + H_{\text{imp}} + H_T^l + H_T^d$. The edge states are described by

$$H_0^m = \int dx u_m (\partial_x \phi_m)^2$$

where the fields $\phi_m$ satisfy the Kac Moody algebra

$$[\phi_m(x'), \partial_x \phi_m(x)] = (2\pi i/m)\delta(x - x'),$$

and $u_m$ determines the velocity of propagation. We consider a single impurity state with energy $\varepsilon_0$,

$$H_{\text{imp}} = \varepsilon_0 d^\dagger d,$$

where $d^\dagger$ creates an electron in that state. The tunneling of electrons between the impurity state and the edges is described by

$$H_T^m = t_m \tau_{\text{e}}^{-1} d^\dagger e^{i\phi_m} + \text{h.c.}$$

where $t_1$ and $t_2$ are dimensionless tunneling amplitudes and $\tau_{\text{e}}$ is a short time cutoff.

Away from a resonance, when $\varepsilon_0 \neq 0$, electrons may tunnel virtually through the impurity. This leads to a nonresonant conductance which vanishes as $T^2$ due to the power law tunneling density of states of the $\nu = 1/3$ edge. However when $\varepsilon_0$ is tuned through zero, the energy denominator for the virtual state becomes small, and there can be resonant transmission through the impurity state. We will analyze the resonances first in the perturbative sequential tunneling limit, and then perform a renormalization group analysis, which can describe the perfect resonance.

\section{Sequential Tunneling Limit}

In the limit, $t_m \ll 1$, tunneling is so infrequent that successive hops onto and off of the impurity will be uncorrelated. This "sequential tunneling" regime has been studied in detail in the context of Luttinger liquid theory by Chamon and Wen\cite{Chamon} and by Furusaki and Nagaosa\cite{Furusaki}. The result of this analysis is that the tunneling conductance is given by

$$G = \frac{\pi e^2}{2 \hbar m} \frac{\Gamma_3(\varepsilon_0, T)}{\tau_T} \frac{1}{\sech^2 \frac{\varepsilon_0}{2T}}.$$  \hfill (5)

$\Gamma_T(\varepsilon, T)$ is the inverse lifetime for tunneling into the $\nu = 1/m$ lead from the impurity, which reflects the power law dependence of the tunneling density of states,

$$\rho(\varepsilon) \propto \varepsilon^{2\Delta_0 - 1}.$$  \hfill (6)

Here $\Delta_0$ is the bare scaling dimension of the electron tunneling operator, given by

$$\Delta_0 = \begin{cases} 1/2 & \text{for} \Delta_1 \\ 3/2 & \text{for} \Delta_3 \end{cases}.$$  \hfill (7)

In the following section we will see that $\Delta_m$ can be renormalized. We therefore compute the lifetimes for general $\Delta_m$,

$$\Gamma_m(\varepsilon, T) = 2\pi t_{mT}^{\text{e}} e^{-2\Delta_m - 1} F_m(\frac{\varepsilon}{2\pi T}),$$  \hfill (8)

with

$$F_m(x) = 2\cosh \pi x \frac{\Gamma(\Delta + i/\pi )^2}{\Gamma(2\Delta_m)}. $$  \hfill (9)

In the perturbative regime, in which $\Delta_m$ are given by (7), this formula reduces to

$$\Gamma_1(\varepsilon, T) = c_1 \varepsilon_{\text{e}}^{-1} \tau_{\text{e}}^2,$$

$$\Gamma_3(\varepsilon, T) = c_3 \varepsilon_{\text{e}}^2 \tau_3^2 (\varepsilon + \pi^2 T^2)$$  \hfill (10)

where $c_1$ and $c_3$ are dimensionless constants of order unity. It is useful to distinguish the following limiting cases:

1. When $\Gamma_1 \ll \Gamma_3$, the conductance is limited by the tunneling into the $\nu = 1$ edge. In this regime,

$$G = \frac{\pi e^2}{2 \hbar} \frac{\Gamma_1}{\sech^2 \frac{\varepsilon_0}{2T}}.$$  \hfill (11)

and the resonances are Fermi liquid like, with a peak conductance scaling as $1/T$ and a lineshape given by the derivative of the Fermi function.

2. When $\Gamma_3 \ll \Gamma_1$ the conductance is limited by tunneling into the $\nu = 1/3$ edge. In this case,

$$G \propto \frac{\varepsilon_0^2 + \pi^2 T^2}{T^2} \frac{\sech^2 \frac{\varepsilon_0}{2T}}{2T^2}.$$  \hfill (12)

Thus, the peak conductance decreases as $T$ as the temperature is lowered in contrast to the $T^2$ behavior off resonance. The lineshape is slightly modified from the derivative of the Fermi function.

\section{Renormalization Group Analysis}

It is important to emphasize that the sequential tunneling approximation is valid only when tunneling is uncorrelated. This will be true provided $\Gamma_1, \Gamma_3 \ll \Gamma_T$. To go beyond this approximation it is useful to develop a perturbative renormalization group analysis. We will now...
Thus, as indicated in Fig. 1, when \( \Delta > 3/2 \), \( t_3 \) flows to zero, and the impurity state decouples from the \( \nu = 1/3 \) state. For \( t_3 > t_3^* \) the impurity state merges with the \( \nu = 1/3 \) state.

It is instructive to analyze the renormalization group flows in limiting cases. (1) For \( t_3 = 0 \), the resonant state is coupled only to the \( \nu = 1 \) edge. In this case, \( t_1 \) is relevant and the system flows to a strongly coupled phase with \( t_1 \to \infty \), \( \Delta_1 = 0 \) and \( \Delta_3 = 2 \). Since \( \Delta_3 = 2 \), \( t_3 \) is an irrelevant perturbation in this phase. The tunneling conductance can then be found perturbatively to vary as \( G \propto t_3^2 T^2 \). This is the same as non resonant tunneling between \( \nu = 1 \) and \( \nu = 1/3 \). In this phase the impurity has effectively merged with the \( \nu = 1 \) quantum Hall fluid.

(2) For \( t_1 = 0 \), the resonant state is coupled only to the \( \nu = 1/3 \) edge. To leading order \( t_3 \) is irrelevant. However, \( \Delta_3 \) is renormalized downward from its initial value of 3/2. Thus, as indicated in Fig. 1, when \( t_3 \) reaches a critical value \( t_3^* \approx 0.22 \) the system flows to a Kosterlitz-Thouless like fixed point at \( t_3 = 0 \), \( \Delta_3 = 1 \). For \( t_3 > t_3^* \) the system flows to a strongly coupled phase with \( t_3 \to \infty \), \( \Delta_3 = 0 \) and \( \Delta_1 = 2 \). Here, the impurity has merged with the \( \nu = 1/3 \) quantum Hall fluid. Treating \( t_1 \) perturbatively again leads to a tunneling conductance varying as \( T^2 \). On the other hand, for \( t_3 < t_3^* \) \( t_3 \) flows to zero with a modified exponent: \( 1 < \Delta_3 < 3/2 \). In this case \( \Gamma_3(T) \propto T^{2 \Delta_3 - 2} \). Since \( t_1 \) remains relevant, however, this state is unstable for finite \( t_1 \) and at low temperatures the impurity merges with the \( \nu = 1 \) quantum Hall fluid.

When both \( t_1 \) and \( t_3 \) are finite it is clear that there are two phases: the impurity merges with either the \( \nu = 1 \) or the \( \nu = 1/3 \) quantum Hall fluids. For fixed \( t_1 \) as \( t_3 \) is increased the system undergoes a transition between these two phases, as indicated in the phase diagram Fig. 2. Integration of the flow equations shows that precisely at this transition, \( t_1 \) and \( t_3 \) are equal and grow to infinity, with \( \Delta_1 = \Delta_3 = 1/2 \). We will analyze this case in the following section.

**V. THE PERFECT RESONANCE**

The fixed point describing the phase transition is not accessible within this perturbative analysis. However, when \( \Delta_1 = \Delta_3 \) and \( t_1 = t_3 \), this problem is equivalent to resonant tunneling between two \( g = 1/2 \) Luttinger liquids in the limit in which the two tunneling barriers are symmetric. As argued in Ref. [8] this is precisely the condition for a “perfect resonance”, where the system flows to the perfectly transmitting fixed point. Equivalently, this model is can be mapped to the anisotropic two channel Kondo problem [9]. In this
mapping, the occupation of the impurity state corresponds to the state of the Kondo spin, and the two leads correspond to the two channels. The tunneling matrix elements \( t_m \) play the role of the transverse couplings \( J^\perp_i \) for the two channels, \( i \). The scaling dimensions \( \Delta_m \) are related to \( J^\perp_i \). When \( \Delta_1 = \Delta_3 = 1/2 \), the Kondo model is in the Toulouse limit.

Under this mapping, the two phases in which the impurity state merges with either \( \nu = 1 \) or \( \nu = 1/3 \) can easily be identified. When there is channel anisotropy in the two channel Kondo problem the Kondo spin tends to form a singlet with one or the other channels. As the couplings are varied, there is a transition in which the Kondo spin switches its allegiance. Precisely at this transition, which occurs when the channels are symmetric, the system flows to the non Fermi liquid fixed point in which the spin is shared between the two channels.

Having established that the resonance fixed point is equivalent to those of both the perfectly transmitting \( g = 1/2 \) Luttinger liquid and the two channel Kondo problem, we may now describe its properties. Precisely on resonance, the conductance is given in which the spin is shared between the two channels.

\[
G(T, \delta) = \tilde{G}(\delta/T^{1/2})
\]

where \( \delta \) is the tuning parameter for the resonance and

\[
\tilde{G}(X) = \frac{e^2}{2\hbar} \int_{-\infty}^{\infty} dy \frac{e^y}{(e^y + 1)^2} \frac{y^2}{y^2 + X^4}.
\]

VI. EXPERIMENTAL IMPLICATIONS

The resonances depend critically on temperature as well as the coupling between the impurity state and the two leads. Here we outline the various possible regimes, depending on these parameters. It is most useful to characterize the coupling to the leads by the inverse lifetimes \( \Gamma_1 \) and \( \Gamma_3 \), though one must keep in mind that they can depend on temperature. In the limit of weak coupling, \( \Gamma_1 \) is temperature independent, while \( \Gamma_3 \propto T^2 \). For stronger coupling the temperature dependence is altered.

If \( T \gg \Gamma_1, \Gamma_3 \), then tunneling is sequential, and the resonances should be accurately described by equation (5). There are three distinct cases depending on \( \Gamma_1 \) and \( \Gamma_3 \):

1. *Fermi Liquid dominated sequential tunneling* If \( \Gamma_1 \ll \Gamma_3 \), then the transmission is limited by the contact to the Fermi liquid. The resonances are then insensitive to the Luttinger liquid correlations, and have the Fermi liquid form (11).

2. *\( \nu = 1/3 \) dominated sequential tunneling* Since \( \Gamma_3 \) decreases as the temperature is lowered, while \( \Gamma_1 \) remains temperature independent, it is possible that the two could cross at a temperature above \( \Gamma_1 \). In this case the system would remain in the sequential tunneling regime, but the tunneling would be limited by \( \Gamma_3 \).

Provided the coupling to \( \nu = 1/3 \) is sufficiently weak that \( \Delta_3 \) has not renormalized significantly then the resonances peaks will vary as \( T \) and the lineshape will be given by (12). This will be the case provided \( t_3 \ll 1 \), or equivalently \( \Gamma_3(T) \ll \tau, T^2 \), where \( \tau^{-1} \) is the high energy cutoff.

3. *Renormalized sequential tunneling* For \( \tau, T^2 \ll \Gamma_3 \ll \Gamma_1 \ll T \), one is no longer in the weak coupling limit. Nonetheless, as shown in Fig. 1, for \( t_3 < t_3^* \) and \( t_1 = 0 \) the system flows to a fixed point with \( t_3 = 0 \). In this limit, \( \Gamma_3 \ll T \), so tunneling is sequential, however, the exponents \( \Delta_1 \) and \( \Delta_3 \) are renormalized:

\[
1 < \Delta_3 < 3/2, \quad 1/2 < \Delta_1 < 2 - \Delta_3.
\]

In this regime, the resonances can be described by combining (5), (8) and (9). The peak heights will vary as \( T^\alpha \). If \( \Gamma_1 < \Gamma_3 \) then \( -1 < \alpha < 0 \). If \( \Gamma_1 > \Gamma_3 \) then \( 0 < \alpha < 1 \).

4. *Strong Coupling Regime* When \( T < \Gamma_1 \) or \( \Gamma_3 \), the system is in a regime where the impurity is strongly coupled to the leads. This regime will be characterized by the low temperature phase diagram shown in Fig. 2. Generally, the impurity will be strongly coupled to either the Fermi liquid or the \( \nu = 1/3 \) lead. In that case, the conductance will have the “off resonance” \( T^2 \) temperature dependence. However, if the coupling \( t_3 \) is sufficiently strong, and the couplings are such that the system is precisely on the phase boundary, then the system will be at a perfect resonance. Achieving the perfect resonance requires two parameters to be tuned: the resonance energy \( \epsilon_0 \) and the coupling to \( \nu = 1/3 \), \( t_3 \). At the resonance the peak conductance should increase as the temperature is lowered, approaching a peak value of \( e^2/2h \), while the width becomes narrower, ultimately having the universal lineshape given in (17) and (18).

Recently, in their cleaved edge overgrowth experiments, Grayson et al. have observed resonances in tunneling between a metal and the edge of a \( \nu = 1/3 \) quantum Hall state as a function of magnetic field. These resonances have a peak conductance which increases as approximately \( 1/T \) as the temperature is lowered and a line shape that is well fit by the derivative of the Fermi function. Since the peak height of the resonances is quite small (of order \( .005e^2/h \) at 25 mK), it is most likely that in this temperature range the system is in Fermi liquid dominated sequential tunneling regime. From the peak height at 25 mK, one can estimate \( \Gamma_1 \approx 0.1\text{mK} \), which also gives a lower bound on \( \Gamma_3 \), so that \( 0.1\text{mK} < \Gamma_3 < 25\text{mK} \).

In these samples signatures of Luttinger liquid behavior in the resonances may not occur until much lower temperatures, and reaching the strong coupling regime may prove difficult. Nonetheless it would be interesting to probe such resonances either by going to lower temperature or by somehow increasing the coupling to the impurity state. A second tuning parameter, such as a gate voltage would allow additional flexibility in controlling \( \Gamma_1 \) and \( \Gamma_3 \), and would allow for the possibility of tuning to the perfect resonance. The first signature of strong coupling behavior would be a deviation from the...
dependence of the peak heights of the resonances.

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