Measuring High Moments of Galaxy Angular Distribution in the First Look Survey using Infrared Array Camera

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**Abstract.** We present the results of using three methods, the probability distribution function, the Rényi information, and the multi-fractals, to measure the high moments of galaxy angular distribution in the Infrared Array Camera data of the Spitzer First Look Survey. This is the first time that Rényi information is explicitly used to describe the large-scale galaxy spatial distribution, and our galaxy samples are also the first of the kind in their wavelengths and sensitivities.

A previous study ([Fang et al. 2004](#)) measured the 2-point angular correlation functions for galaxy samples from all four wavelengths of the Infrared Array Camera (IRAC) in the Spitzer First Look Survey (FLS). Measuring the second moment, the 2-point statistics do not completely describe a non-Gaussian spatial density field. Here we extend the study to measuring high moments for the same samples, using the probability distribution function (PDF), the Rényi information, and the multifractal descriptions. For all measurements the two-dimensional area covered by the IRAC samples is divided into contiguous square cells of varying sizes. We discard all cells that contain invalid mosaic pixels determined by masks that used to establish the IRAC samples. The boundary effect and selection bias are at minimum.
The PDF contains all the high moments of the galaxy spatial distribution. It is calculated by counting the number of galaxies in the cells and establishing normalized histograms. In the left figure of the previous page we show our PDF results. For each sample it is measured at 3 different scales. For each measurement we plot the fit of the theoretical Gravitational Quasi-equilibrium Distribution Function (Saslaw & Hamilton 1984) (solid lines) and the Poisson distribution of the same average galaxy count for comparisons. The deviation of the PDF from Poisson caused by galaxy clustering is obvious at large scales. The gravitational distribution function fits well at all scales. The value of the fitting parameter $b$ shows the strength of clustering. Theoretically it is the ratio of the correlation potential energy and twice the kinetic energy.

For the first time, we explicitly apply the Rényi Information to describe the spatial distribution of the galaxies. The $n$-order Rényi Information is defined by
\[
I_n = \frac{1}{n-1} \log \sum p^n,
\]
where $p$ is the probability of finding a galaxy in a cell. Since $p$ is described by the PDF, the $n$-order Rényi Information directly measures the $n$-moment of the distribution. In the right figure of the previous page we plot the Rényi information measurements, in "bits" of unit log2, over a range of cell sizes placed on the IRAC samples, and from $n = 1$ up to 20 (bottom to top). The crowding of the lines at high $n$ shows that the high moments are constrained in a narrow information space, a desired property for studying the moments of arbitrary high orders.

The rate of information change with cell sizes gives an estimate of the fractal dimension of galaxy distribution. The information figure indicates that galaxies occupy space compactly at small scales, and the distribution becomes more uniform at large scales. The multi-fractal dimensions measured across increasing information orders decrease and approach a limit. This is illustrated above in the left figure. The top right figure shows the spectra of the multi-fractal scaling index, related to multi-fractal dimensions by a Legendre transformation. As the spectra goes to zero, the scaling index measures the finite value of the multi-fractal dimension at infinite information order. Therefore a multi-fractal simply describes the hierarchy of the moments in galaxy spatial distribution.

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