NoSOCS in SDSS. II. Mass Calibration of Low Redshift
Galaxy Clusters with Optical and X-ray Properties

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ABSTRACT
We use SDSS data to investigate the scaling relations of 127 NoSOCS and 56 CIRS galaxy clusters at low redshift ($z \leq 0.10$). We show that richness and both optical and X-ray luminosities are reliable mass proxies. The scatter in mass at fixed observable is $\sim 40\%$, depending on the aperture, sample and observable considered. For example, for the massive CIRS systems $\sigma_{\ln M_{500}|N_{500}} = 0.33 \pm 0.05$ and $\sigma_{\ln M_{500}|L_X} = 0.48 \pm 0.06$. For the full sample $\sigma_{\ln M_{500}|N_{500}} = 0.43 \pm 0.03$ and $\sigma_{\ln M_{500}|L_X} = 0.56 \pm 0.06$. The scaling relations based only on the richer systems (CIRS) are slightly flatter than those based on the full sample, but the discrepancies are within 1-$\sigma$. We estimate substructure using two and three dimensional optical data, verifying that substructure has no significant effect on the cluster scaling relations (intercepts and slopes), independent of which substructure test we use. For a subset of twenty-one clusters, we estimate masses from the M-T\_X relation using temperature measures from BAX. The scaling relations derived from the optical and X-ray masses are indeed very similar, indicating that our method consistently estimates the cluster mass and yields equivalent results regardless of the wavelength from which we measure mass. For massive systems, we represent the mass-richness relation by a function with the form $\ln(M_{200}) = A + B \times \ln(N_{200}/60)$, with $M_{200}$ being expressed in units of $10^{14}$ M$_\odot$. Using the virial mass, for CIRS clusters, we find $A = (1.39 \pm 0.07)$ and $B = (1.00 \pm 0.11)$. For the same sample, but using the masses obtained by the caustic method, we get $A = (0.64 \pm 0.14)$ and $B = (1.35 \pm 0.34)$. If we consider the mass as estimated from $T_X$ (for the subset of 21 clusters with $T_X$ available) we derive $A = (0.90 \pm 0.10)$ and $B = (0.92 \pm 0.10)$. The relations based on the virial mass have a scatter of $\sigma_{\ln M_{200}|N_{200}} = 0.37 \pm 0.05$, while $\sigma_{\ln M_{200}|N_{200}} = 0.77 \pm 0.22$ for the caustic mass and $\sigma_{\ln M_{200}|N_{200}} = 0.34 \pm 0.08$ for the temperature based mass.

Key words: surveys – galaxies: clusters: general – galaxies: kinematics and dynamics.

1 INTRODUCTION
Two aspects of galaxy clusters make their study uniquely important for cosmology. The path from density perturbations to virialized objects involves the physical mechanisms that ultimately determine the cluster properties as we currently observe them. The growth of these systems depends on cosmological parameters like $\Omega_m$, $\Omega_L$, $\sigma_8$, etc. The sensitivity to these parameters depends on the redshift (Carroll, Press, & Turner [1992]). Therefore, the systematic study of cluster properties, based on unbiased samples at different redshifts, can provide substantial input for models trying to explain not only how clusters formed and evolved from the astrophysical viewpoint but also how spacetime is structured (e.g. Henry & Arnaud [1991]; Rosat et al. [2002]). Ultimately, we aim to disentangle the astrophysical aspects from the cosmological ones.

Clusters have no sharp edges. Observations show that their central parts reached virial equilibrium earlier while the outskirts are still accreting matter from their surroundings, making the measurements of cluster properties a difficult issue. Clusters are constituted of both dark (non-
baryonic) and luminous (baryonic) matter. This mixture renders observations within a specific wavelength range very limited in utility and determining the physical state of each component too complex. Over the years, the sizes of cluster samples and the quality of the data hampered our progress on all of the fronts mentioned above. Only recently have large samples with high-quality data been obtained, allowing a systematic investigation of these effects (e.g. Vikhlinin et al. 2000; Henry 2004).

The abundance of clusters of galaxies can be used as a probe of line density fluctuations [Eke et al. 1996] and since their growth is directly determined from the dark energy properties, measuring the number of clusters of a given mass as a function of redshift can constrain the equation of state (parameter) of this majority component of the Universe (e.g. Frieman, Turner & Huterer 2001). As shown by Huterer & Turner (2001), the sensitivity of cluster abundance to $w$ is maximum around redshift 1.0, so that a cluster sample over a large area on the sky and deep ($z \sim 1.0$) is critically needed to provide reliable results. Direct measures of halo mass are not possible, so that we have to rely on mass proxies free of systematics. The most accurate mass tracers are velocity dispersion and X-ray temperature. Unfortunately, for large samples, at any redshift, directly measuring these cluster properties is not feasible. Hence, we have to rely on other observables (such as richness or X-ray luminosity) that are easier to derive for all clusters in a given survey.

Several studies have investigated mass-observable (MO) relations and their evolution with redshift in the optical and X-ray (Vikhlinin et al. 2002; Levine et al. 2002; Ettori et al. 2004; Stanek et al. 2006; Becker et al. 2007; Rykoff et al. 2008). As advocated by Lima & Hu (2004; 2005) the variance of the counts themselves (clustering of clusters) can be used to calibrate the MO relation. Dark energy depends only on the redshift, but the cluster properties vary with mass and redshift. Using cosmological simulations one can use counts and their variance to not only normalize the MO relation but also learn which cosmology fits the data better. Therefore, understanding the scaling relations of galaxy clusters and their dependence on the dynamical state of the system is crucial.

This paper focuses on four specific issues: (i) examining the performance of different cluster properties, like richness ($N_{\text{gal}}$), optical ($L_{\text{opt}}$) and X-ray ($L_x$) luminosities, as proxies for cluster mass. The relation between one of these proxies and an independent mass estimate defines a scaling relation; (ii) measuring how much substructure there is in clusters (in 2D and 3D) and establishing its effect on the final mass calibration; (iii) investigating how the scaling relations change when mass is derived from the analysis of the velocity distribution or temperature. Optical masses used in this work are either from a virial analysis (ours) or from the caustic technique (Rines & Diaferio 2006, hereafter RD06); and (iv) comparing the optical and X-ray properties of galaxy clusters, as well as obtaining a direct relation between $R_{200}$ and richness.

This paper is organized as follows: §2 briefly describes the samples used here, while §3 presents the mass calibration based on $N_{\text{gal}}$, $L_{\text{opt}}$ and $L_x$. In this section we also investigate the effects of substructure on the scaling relations and compare the results based on different mass estimators. These are derived with optical data, from the virial or caustic analysis, or from the gas temperature. The mass-to-light ratio is also presented at the end of this section. In §4 we discuss how our results compare to others in the literature. Correlations between optical and X-ray properties are presented in §5, while in §6 we establish the connection between richness and physical radius. The main conclusions are drawn in §7. The cosmology assumed in this work is $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 100$ h km s$^{-1}$ Mpc$^{-1}$, with h set to 0.7.

## 2 Data and Methods

The main data set used in this work is the supplemental version of the Northern Sky Optical Cluster Survey (NoSOCS, Lopes 2003; Lopes et al. 2004), which has its origin on the digitized version of the Second Palomar Observatory Sky Survey (POSS-II; DPOSS, Diorgovski et al. 2003). In Gal et al. (2004) and Odewahn et al. (2004), we describe photometric calibration and object classification for DPOSS, respectively. The supplemental version of NoSOCS (Lopes et al. 2004) goes deeper ($z \sim 0.5$), but covers a smaller region than the main NoSOCS catalog (Gal et al. 2003; 2009), and contains 9,956 cluster candidates over ~2,700 square degrees. The smaller area is due to the use of the best DPOSS plates, selected according to seeing and limiting magnitude ($r = 21.0$).

We examine a sample of low redshift galaxy clusters ($z \lesssim 0.10$) from the NoSOCS supplemental catalog. As this survey comprises mainly poor systems in this redshift range, we complemented them with more massive systems from the Cluster Infall Regions in SDSS (CIRS) sample (RD06). CIRS is a collection of $z \lesssim 0.10$ X-ray selected clusters overlapping the SDSS DR4 footprint. As described in the first paper of this series (Lopes et al. 2009) we extracted SDSS data for 127 NoSOCS clusters and 56 CIRS systems at low-$z$.

We used photometric and spectroscopic data from the fifth release (DR5) of the Sloan Digital Sky Survey (York et al. 2000). The exception is for the CIRS systems, which were incorporated in a late stage of this work, thus having data from the SDSS DR6. All the magnitudes retrieved from SDSS are de-reddened model magnitudes. Details regarding SDSS data extraction are provided in paper I (Lopes et al. 2009). There, we used SDSS photometric data to estimate more accurate photometric redshifts ($z_{\text{photo}}$, Lopes 2007), richnesses and optical luminosities for the full NoSOCS supplemental catalog. We found 7,414 systems well sampled in SDSS DR5. Approximately 10% (754) have $z_{\text{photo}} \leq 0.133$ (Lopes et al. 2009).

The above redshift limit comes from the choice of only using clusters at $z \lesssim 0.10$ for the current work. In this redshift range the SDSS spectroscopic survey is complete. As discussed in Lopes et al. (2009), at higher redshifts galax-
ies fainter than $M^* + 1$ are missed, biasing the dynamical analysis (see discussion in section 4.3 of [Lopes et al. 2003]). Out of the 754 NoSOCS supplemental clusters with $z_{\text{photo}} \leq 0.133$, we were able to determine the spectroscopic redshift for 179 systems, requiring at least 3 galaxies within 0.50 h$^{-1}$ Mpc in the SDSS spectroscopic footprint. Note that we only selected systems for which $|z_{\text{photo}} - z_{\text{spec}}| \leq 0.03(1 + z_{\text{spec}})$. We eliminated interlopers using the “shifting gapper” technique ([Fadda et al. 1996]), applied to all galaxies with spectra available within 2.50 h$^{-1}$ Mpc. From the 179 clusters with $z_{\text{spec}} \leq 0.10$ we retained 127 systems with at least 10 member galaxies selected by the above procedure. We applied the same procedure to the 56 CIRS clusters. Figures 4 and 5 of Lopes et al. (2009) show the velocity-radius distributions of the 127 NoSOCS clusters and the 56 CIRS systems.

These clusters were then subjected to a virial analysis analogous to the one described in [Girardi et al. 1998], [Popesso et al. 2003, 2007] and [Biviano et al. 2006]. This procedure yields estimates of $\sigma_P$, $R_{200}$, $R_{500}$, $M_{500}$ and $M_{200}$ for the set of 183 low redshift clusters considered in this work. X-ray luminosity is also estimated for these systems using ROSAT All Sky Survey (RASS) data. A detailed description of both the virial analysis and derivation of X-ray luminosity is provided in [Lopes et al. 2006], which also lists the properties of these systems in Tables 1, 2, 3, 4, and 5.

The NoSOCS clusters have velocity dispersion estimates of $100 < \sigma_P < 700$ km/s. The CIRS systems have $200 < \sigma_P < 900$ km/s (with only 23% of objects with $\sigma_P < 400$ km/s). For most of the studies below we ignore three of the CIRS objects which have biased values of $\sigma_P$ and mass due to projection effects and substructure (see paper I). We only show these objects in Figures 9, 10, and 11, where we investigate the impact of substructure on the scaling relations. These 3 clusters are Abell 1035B, Abell 1291A and Abell 1299B.

3 MASS CALIBRATION

Scaling relations involving a simple cluster observable, like richness or luminosity and a fundamental property such as mass, can provide important clues on how large scale structure forms, galaxy formation proceeds, and how the intracluster gas reaches its current state. The establishment of these relations for nearby clusters is also vital for future studies of the cluster population in the distant universe. Particularly, they may help constrain the dark energy equation of state ([Maunuelar & Mohri 2004]).

In this section, we examine the correlation of $N_{\text{gals}}$, $L_{\text{opt}}$ and $L_X$ with cluster properties, such as velocity dispersion ($\sigma_P$) and mass ($M_{500}$ and $M_{200}$). We use the spectroscopic redshifts of the clusters (instead of $z_{\text{photo}}$) when computing richnesses and luminosities. Three different apertures are used to derive these quantities: $0.50 h^{-1}$ Mpc, $R_{500}$ and $R_{200}$. We consider the re-centered (luminosity-weighted) coordinates for the 127 NoSOCS clusters and the original position (X-ray centroid) listed in RD06 for the 56 CIRS clusters. We present the results for the NoSOCS and CIRS samples independently as well as for the combined sample.

In the next three subsections we discuss the mass calibration based independently on richness, optical luminosity, and X-ray luminosity. For the richness relations we also investigate the impact of using less rigorous criteria for interloper removal when computing mass, as well as assuming different centroids and considering all NoSOCS clusters at $z \leq 0.25$. We then examine the impact of cluster substructure on the scaling relations and compare the results derived from mass estimates obtained from different wavelengths. Finally, we show the dependence of the mass-to-luminosity ratio with the cluster scale (defined by its mass).

3.1 Calibration with richness

Figure 1 shows the comparison between mass ($M_{500}$ and $M_{200}$) with richness, estimated within $R_{500}$ and $R_{200}$. The lower panels show the residuals, $\log(M_{\text{obs}}/M_{\text{fit}})$. $M_{\text{obs}}$ is the observed value of the mass within the given radius, while $M_{\text{fit}}$ is the linear regression solution. The results are only shown for the 127 NoSOCS objects. The solid line in each upper panel shows the orthogonal regression fit ([Akritas & Bershady 1996]). All the scaling relations obtained in this work are of the form

$$\ln(Y) = A + B \times \ln(X),$$

(1)

The X and Y parameters are listed in the tables defining the relations (see below). The pivot point (C) depends on the sample being used. It is taken to be approximately the median value of the X parameter within $R_{500}$ for a given sample. For the NoSOCS and full samples (NoSOCS + CIRS) we consider the same pivot point, while for the more massive systems (CIRS) we have another pivot point. Those are listed in the tables and can be seen in the figures. The linear fit is obtained by a two iteration 3-sigma clipping, so that after a first run we eliminate outliers. Those are not used in the final fit and are indicated (if exist) as open symbols in the figures.

The results for the scaling relations involving $\sigma_P$, $M_{500}$, $M_{200}$ and $N_{\text{gals}}$ are summarized in Table 1. The columns give (1) the parameters involved in the relation (the abscissa and ordinate of the corresponding plots); (2) the cluster sample used; (3) and (4) the intercept (A) and slope (B); (5) the scatter (in natural log space) in the Y parameter at fixed X; (6) the total number of clusters used in the preliminary fit; and (7) the final number after a 3-$\sigma$ clip. Note that we evaluated the error in the scatter, performing a bootstrap procedure (500 events) with replacement. So the scatter is shown with the associated error in all tables (exception to Table 8).

Rows 1-6 provide the results using only the NoSOCS clusters, as shown in Figure 1. Rows 7-12 are for clusters from RD06 only, while the combination of NoSOCS and RD06 is listed in rows 13-18.

If one has to work within a fixed metric radius, 0.50 h$^{-1}$ Mpc yields the smallest error in the richness or $L_{\text{opt}}$ measures and minimizes the scatter in the optical versus X-ray scaling relations ([Popesso et al. 2004]; [Lopes et al. 2006]). However, when compared to apertures that scale with mass, this radius (0.50 h$^{-1}$ Mpc) is too large for low mass systems and too small for massive ones, and may lead to tilted (steeper) scaling relations (see Table 1). When we use an aperture that scales with mass the relations become flatter. For instance, for the NoSOCS sample, the slope of the $M_{200}$-$N_{\text{gals}}$ relation is 1.45 when using the fixed metric ra-
Figure 1. The connection between mass and richness, computed with two apertures (R_{200} and R_{200}). In the lower panels the residuals \log(M_{\text{obs}}/M_{\text{fit}}) are shown. M_{\text{obs}} is the observed value of the mass, while M_{\text{fit}} is the linear regression result.

From the lower panels of Figure 1 we see that some of the poorest systems (N_{gal} < 30) have larger deviations from the linear solution. Both richness and mass are harder to determine due to the low contrast of some of these clusters. Incompleteness in spectroscopic sampling for these poor systems may also be a problem (see discussion in §3.2).

The comparison of the relations for the 127 NoSOCS clusters, the 53 CIRS systems and the combined sample, shows that the scatter of the relations obtained with the CIRS and extended samples is smaller than for the NoSOCS clusters alone. This is because CIRS is restricted to more massive systems and the enlarged sample has better statistics, spanning a wider range of all parameters. For the M_{200}-N_{gal} relation the orthogonal scatter in mass at fixed richness is (0.60 ± 0.07) for NoSOCS, (0.37 ± 0.05) for the CIRS sample and (0.46 ± 0.04) for the combined data set. The slopes are nearly the same for the NoSOCS and extended samples, but slightly different for CIRS, although they are compatible within the errors. For the M_{200}-N_{gal} relation the slope is 1.09 for the NoSOCS sample, 1.00 for CIRS and 1.07 for the combined sample.

3.1.1 Systematics in the scaling relations

Table 2 lists the coefficients of the relations \sigma_{p-N_{gal}} and M_{200}-N_{gal} using only the NoSOCS clusters, with three different methods. In the first two rows we consider the original cluster coordinates instead of the luminosity-weighted ones. Rows 3-4 list the results when we use a more "relaxed" criteria for interloper removal (see §4.1 of Lopes et al. 2009), and in rows 5-6 we consider all NoSOCS clusters at z ≤ 0.25, instead of z ≤ 0.10 (limit adopted for this work). The
latter is useful to see how much the scaling relations are affected for using clusters at redshifts where the spectroscopic sampling is very incomplete (see § 4.3 of paper I). Note that Popesso et al. (2005) (hereafter POP05) considered clusters at $z \leq 0.25$ for deriving scaling relations based on SDSS data.

When comparing the results obtained with the original coordinates (first two rows of Table 2) to the ones with the luminosity-weighted centroids (Table 1), we find that the intercept, slope and scatter are consistent within the errors (1-σ). Thus, we conclude that the centroid is not a critical issue when deriving the scaling relations. When the criteria to select interlopers are relaxed (results in rows 3-4 of Table 2) the agreement is not as good for the intercept and slope, but it is still within 1.5-σ. Relaxing the criteria for rejecting interlopers in the “shifting-gapper” technique has a minor effect in the relations.

The third case in Table 2 uses all NoSOCS clusters at $z \leq 0.25$ (rows 5-6). A comparison to Table 1 reveals that the intercept and slope are now different, in some cases consistent only within 2-σ. The scatter is, however, similar. This result shows the relevance of using complete spectroscopic samples for studying the scaling relations. The use of clusters at redshifts where the spectroscopic survey is not complete to M$^* + 1$ results in biased velocity dispersion and mass estimates (see paper I), yielding flatter scaling relations.

This is a critical issue for measuring the cluster mass function since the mass calibration might be severely biased. However, it is important to note that this bias in the calibration may reflect the percentage of clusters with incomplete spectroscopic sampling. In this work 127 of the 219 NoSOCS clusters at $z \leq 0.25$ are below $z \leq 0.10$, and are therefore well sampled. Ninety-two of the 219 systems (42%) have poor sampling ($z > 0.10$), a considerable fraction of the total used when deriving the scaling relations. Other authors included $z > 0.10$ clusters in their sample (POP05, for instance), but show results consistent to our unbiased sample (at $z \leq 0.10$). Although they initially have many systems at $z > 0.10$, they require that clusters have at least ten galaxies with redshifts when estimating mass and deriving the scaling relations, which may preferentially exclude many of the higher redshift clusters. We believe that this is the case since their results are consistent with ours.

### 3.2 Calibration with optical luminosity

Figure 2 shows the comparison between mass (M$_{200}$ and M$_{500}$) and optical luminosity computed within R$_{200}$ or R$_{500}$. Residuals are shown in the lower panels. We only show the results for the 127 NoSOCS objects. One interesting feature from the comparison of Figures 1 and 2 (see Tables 1 and 3) is that the slopes obtained in the two cases are consistent within the errors. This indicates the optical luminosity and the number of galaxies used to compute L$_{opt}$, are proportional to each other (see Popesso et al. 2005), with a constant of proportionality consistent with unity. Figure 3 shows $\sigma_P$ versus L$_{opt}$, computed within R$_{500}$ or R$_{200}$ for only the CIRS objects. Figure 4 is analogous to Figures 2 and 3, but shows the extended sample of 180 NoSOCS plus CIRS clusters. The inspection of the lower panels of Figures 2 and 4 reveals that larger deviations from the linear regression occur for L$_{opt} < 10^{12}$ L$_{\odot}$, corresponding to the cut at N$_{obs}^{R_{200}} = 30$ seen in Figure 1. Hence, we stress the conclusion that richness, L$_{opt}$, and mass are harder to obtain due to the low contrast of such systems. Another problem affecting these clusters may be incompleteness in the spectroscopic sampling.

To confirm these conclusions we separate the clusters shown in Figure 4 in systems with |LOG(M$_{obs}$/M$_{fit}$)| $\leq 2$ or |LOG(M$_{obs}$/M$_{fit}$)| $> 2$. We find 135 clusters in the first case and only forty-five (25%) in the second. For |LOG(M$_{obs}$/M$_{fit}$)| $\leq 2$ we find the following median values: N$_{gals} = 33.0$, N$_{gals-err}$/N$_{gals} = 0.24$, N$_{spec} = 48.0$ and N$_{spec200} = 21.0$; where N$_{gals}$ is computed within R$_{200}$, N$_{gals-err}$/N$_{gals}$ is the percentage error on the richness measurement, N$_{spec}$ is the number of members (spectroscopically selected) within the maximum aperture (normally 2.5 h$^{-2}$ Mpc; see paper I) and N$_{spec200}$ is the number of members within R$_{200}$. For |LOG(M$_{obs}$/M$_{fit}$)| $> 2$ we have N$_{gals} = 22.0$, N$_{gals-err}$/N$_{gals} = 0.36$, N$_{spec} = 30.0$ and N$_{spec200} = 15.0$. Thus, the forty-five clusters with larger mass deviations are generally poor, low contrast (indicated by their large percentage error in richness) and have their mass estimates based on few galaxies.

The results shown in Figures 2, 3 and 4 are summarized in Table 3. The meaning of all columns and rows is analogous to those in Table 1. The same conclusions drawn for Table 1 are valid now. The most important result is that the relations involving L$_{opt}$ have approximately the same scatter as those obtained with N$_{gals}$. In other words, optical luminosity performs as good as richness for mass calibration. For the full sample, the scatter of M$_{200}$ at fixed N$_{gals}^{R_{200}}$ is (0.46 ± 0.04), while it is (0.49 ± 0.04) at fixed L$_{opt}^{R_{200}}$. For the CIRS sample, the scatter of M$_{200}$ at fixed N$_{gals}^{R_{200}}$ is (0.37 ± 0.05), being (0.36 ± 0.06) at fixed L$_{opt}^{R_{200}}$.

### 3.3 Calibration with X-ray luminosity

Next we compare the X-ray luminosity estimated in Lopes et al. (2008) to $\sigma_P$ and mass. As described in paper I, we computed the X-ray luminosity from RASS, using three different background estimates, termed “annulus” (from a ring surround the cluster), “boxes” (100 randomly selected background boxes) and “frame” (from the whole frame in which the cluster is located). Here we show the results obtained with the “annulus” and “frame” backgrounds.

Figure 5 shows the relation between L$_X$ (“annulus” background) and $\sigma_P$ (top panels) and mass (lower panels), estimated within R$_{500}$ (left) and R$_{200}$ (right). We lose seventy-four of the 180 clusters from the extended sample, as their X-ray luminosities are upper limits. These are mostly poor systems. When establishing the scaling relations involving L$_X$, upper limits were not used in the fits. From this plot (and Table 4) we see that the smallest scatter is generally found when using R$_{500}$.

Due to a bug in the code used to estimate the X-ray luminosity and its associated error, the background was not properly determined in paper I. That resulted in slightly different values for L$_X$. However, the errors of L$_X$ were severely underestimated. We fixed the code and updated the values of L$_X$ and its error (for all the three backgrounds) in the current paper. The information given in Table 5 of paper I is now replaced by the data available in Table A1, in the appendix of the present paper.
We also studied the relations considering the CIRS subsample, with the comparison of \( M_{200} \) and \( L_{X}^{200} \) shown in Figure 6. It is particularly important to see whether the slope and mainly the scatter are reduced for this sample based on richer clusters. As found for \( N_{gals} \) and \( L_{opt} \), the slope also becomes shallower for the richer systems when considering \( L_{X} \). Note that the slopes are consistent with the full sample (Figure 5 and Table 4) within 1-\( \sigma \). The slope of the \( M_{200} - L_{X}^{200} \) relation is \((0.67 \pm 0.06)\) for all clusters and \((0.52 \pm 0.08)\) for CIRS (see Table 4). The scatters in the relations based on the richer clusters are slightly smaller than those of the whole sample, but still within 1-\( \sigma \). The relations based on the “frame” background are similar to the ones obtained with the “annulus”, being the agreement within 1-\( \sigma \). Figures 7 and 8 show the results with the “frame” background, for the full and CIRS samples, respectively.

Tables 4 and 5 summarize the relations involving \( L_{X} \). Table 4 gives the results for the “annulus” background, while Table 5 is for the “frame” background. As in the previous tables, the first column lists the two parameters being compared. The background type is listed in the second column. The sample used is in the third column, while the remaining columns are analogous to Table 1. In both tables we list the results for the full sample and for the CIRS systems.

When comparing the performance of optical parameters (richness or \( L_{opt} \)) to \( L_{X} \) as a trace of the cluster mass, we find that the former perform a little better than the latter. However, in the worst cases, the scatter is consistent within 1.5-\( \sigma \) (see Tables 1, 3, 4 and 5). The main reason for these minor differences may lie in the fact that RASS data is shallower than the optical data from SDSS, so that \( L_{X} \) is determined less accurately than \( N_{gals} \) and \( L_{opt} \). It is also important to stress that the optical properties may be used as reliable mass proxies, in the same way as \( L_{X} \). This finding corroborates the results from POP05, who found that...
L\textsubscript{opt} gives slightly more accurate results than L\textsubscript{X} (also using SDSS and RASS data).

3.4 Impact of substructure in the scaling relations

In Lopes et al. (2009) we described the use of photometric and spectroscopic data for NoSOCS and CIRS clusters to estimate the fraction of systems with substructure. Two specific tests have proven to be very sensitive to the presence of such disturbances in the galaxy distribution inside a cluster (Pinkney et al. 1996). The first test is the DS, or \( \Delta \) test (Dressler & Shectman 1988), which is a three dimensional test. The second is a two dimensional test, called the symmetry or \( \beta \) test, introduced by West et al. (1988). Detailed descriptions of both tests can be found in Pinkney et al. (1996) and Lopes et al. (2009). For both tests, the significance level is determined with the aid of Monte Carlo simulations. We set our significance threshold at 5%.

The substructure tests are only applied to clusters with at least five galaxies within the aperture being considered. In paper I we showed that when using an aperture of R\textsubscript{200} the rate of clusters showing significant signs of substructure is \( \sim 21\% \) for both the \( \Delta \) and \( \beta \) tests. The latter can be applied to galaxies with spectra or to the photometric data alone, while the former test requires redshifts. If we apply the \( \beta \) test to the galaxies within a fixed metric aperture (1.5 h\textsuperscript{-1} Mpc) the rate of clusters with substructure rises to \( \sim 35\% \), in line with Lopes et al. (2006), who estimated substructure from two dimensional optical data for > 10,000 clusters.

In this work we do not intend to investigate substructure for each cluster individually. Our goal is to check the possible effect of substructure on the scaling relations connecting M\textsubscript{200} to N\textsubscript{gal} and L\textsubscript{opt}. Four different cases are considered. First we apply the \( \Delta \) test to the 170 clusters with at least five galaxies with z\textsubscript{spec} within R\textsubscript{200}. Second, we apply the \( \beta \) test to the same data set. Third, the \( \beta \) test is applied

![Figure 3. Analogous to Figure 2, but using only the 53 clusters from RD06 and considering \( \sigma_P \) and L\textsubscript{opt}.](image-url)
Figure 4. Analogous to Figure 2, but for the combined sample of 127 NoSOCS clusters and 53 systems from RD06.

The meaning of the remaining columns is the same as in Table 3.

Figures 9, 10, and 11 show the results for the first, third and fourth cases discussed above. In each figure the relation between $\sigma_P$ (top) and $M_{200}$ (bottom) to $L_{opt}$ is displayed for all clusters (left) and for the substructure-free systems (right). From Table 6 and Figures 9 and 10 we see that the fitting parameters and scatter of the scaling relations, depend very little on the presence of substructure. The intercepts and slopes are always consistent within 1-$\sigma$. It is important to note that the 2D and 3D tests indicate similar fractions of clusters with substructure, and the scaling relations obtained for the substructure-free systems are nearly the same as those for the full sample, regardless of whether the test used to detect substructure is 2D or 3D.

From Figure 11 we see that even considering the full photometric data within a fixed radius, instead of only cluster members within $R_{200}$, the results are qualitatively the

to the photometric galaxy distribution within $R_{200}$. We consider all galaxies with $m^* - 1 \leq m_r \leq m^* + 1$ (see [Lopes et al. 2006 for details)] with no restriction regarding $z_{spec}$. Fourth, we apply the $\beta$ test to all galaxies within $1.5 h^{-1}$ Mpc of each cluster (the same photometric range is enforced).

The results for the cases listed above are summarized in Table 6. In this table we always show first the results for the full sample and then the ones for clusters without substructure. The first six lines give the results considering the first two cases mentioned above. Lines 7 to 10 list the results for the third case, and the last four lines summarize the results for the fourth case. The first column lists the two parameters of the scaling relation (and the aperture adopted for computing $N_{gals}$ and $L_{opt}$). The second column shows the substructure test (if any) used to remove clusters with substructure. Note that the aperture used for estimating substructure is $R_{200}$ in the first three cases and is $1.5 h^{-1}$ Mpc for the fourth. The sample used is listed in column 3.
same. Due to the different apertures the scaling relations are different from the two previous figures. However, we still find the relations to be insensitive to the exclusion of clusters with substructure. The scatter of the relations (with or without substructure) shows a larger difference than for the other cases above, but it is still within 1-σ. These results contradict Figure 16 of Lopes et al. (2006), where a strong segregation in the $T_X - N_{gals}$ relation was caused by the presence of substructure in some clusters. Several reasons may be responsible: 1) $T_X$ was taken from the literature, so the estimates may be very heterogeneous; 2) $T_X$ was not available for the poorer clusters ($N_{gals} < 20$); and 3) the small sample size used in that work makes the results sensitive to the exclusion of a few points.

We repeated the analysis presented in Figure 11 but only for clusters with $N_{gals} \geq 20$ and found that the scatter of the relations decreased only by 5%. However, the slope exhibited a significant variation ($\approx 27\%$). So, the strong segregation present in the $T_X - N_{gals}$ relation of Lopes et al. (2006) may be explained by the small sample size and the exclusion of poor systems ($N_{gals} < 20$). In Lopes et al. (2009), we checked that the $T_X$ values derived from BAX represent a consistent data set, which implies that the main source of slope variation detected is the absence of low mass systems.

### 3.5 Comparison of scaling relations derived with optical and X-ray mass estimates

As described in Lopes et al. (2009) we have searched BAX (Base de Données Amas de Galaxies X, http://bax.ast.obs-mip.fr/) for counterparts to the 183 clusters used in this work. The search was restricted to objects at $z < 0.12$ with X-ray temperature measures. We found 282 clusters in BAX, of which 21 are common to...
Figure 6. Analogous to the previous figure, but showing only the relation $M_{200}-L_{X}^{R_{200}}$ for the 53 CIRS clusters.

Figure 7. Analogous to Figure 5, but showing only the relation $M_{200}-L_{X}^{R_{200}}$ for the full sample and with the “frame” background.

Figure 8. Analogous to the previous figure (“frame” background), but showing only the CIRS clusters.

Figure 9. The relations $\sigma_{p}-L_{200}^{opt}$ (top panels) and $M_{200}-L_{200}^{opt}$ (bottom panels) are shown for all clusters (left) and systems without significant evidence for substructure (right). We apply the Dressler-Schectman (or $\Delta$) substructure test to the spectroscopic sample of 170 clusters with at least 5 galaxies (with spectra) within $R_{200}$. Of these, 39 ($\sim 23\%$) clusters shown strong signs of substructure.

Our sample. For these clusters, we employed the $M_{200}-T_{X}$ relation given by equation 3 of POP05 to estimate masses. Our goal is to check if the mass calibration gives the same results when using either the optical or X-ray masses. Of the 183 NoSOCS plus CIRS systems, temperatures are...
generally available only for the more massive clusters, with \( \sigma_P > 400 \text{ km/s} \) or \( M_{200} > 10^{14} \text{ M}_\odot \).

In Lopes et al. (2006), we show that the temperature values selected from BAX represent a consistent data set. There, we compare the values listed in BAX to the ones available in RD06, finding that most systems agree within 10%. We also check that the masses obtained from the dynamical analysis of the optical data generally agree within 40% of the estimates derived from the \( M_{200} - T_X \) relation. The exceptions, most of times, are clusters affected by substructure. When using the optical masses derived with the caustic technique (RD06) the agreement is not as good, with the optical masses being lower than the X-ray values in the low-mass regime.

In this section, we check if the scaling relations obtained in this work are similar to the ones using mass derived from the tight connection to \( T_X \). As the velocity dispersions and masses obtained with the caustic technique are provided by RD06 we also compare our results to those based on their estimates. Figures 12 and 13 show the \( M_{200} - L_{opt} \) and \( M_{200} - L_X \) relations, respectively. \( L_X \) is the value derived in this work with the “frame” background, using RASS data. Both figures consider \( M_{200} \) as determined from \( T_X \) (equation 3 of POP05). These results are summarized in the first four lines of Table 7, where we also list the fit parameters for the \( M_{200} - N_{gals} \) relation and for \( M_{200} - L_X \) (considering the “annulus” background).

From inspection of this table, and considering the range of clusters sampled, the results are in very good agreement to those based on the virial mass and restricted to the richer clusters. In Tables 1, 3, 4 and 5 the CIRS sample repre-
slopes agree within 2-σ for the relations involving $\sigma_P$ and within 1-σ for those regarding $M_{200}$. However, this only happens because the uncertainties in the fit parameters of the relations involving mass are much larger than the ones regarding $\sigma_P$. The uncertainty in the slopes of the relations using the virial mass is also larger than what we found with the virial mass, by as much as a factor of three. So, although the relations based on our masses (or the X-ray derived masses) agree within the errors to those obtained with the caustic method, the nominal values are very different. When using the caustic parameters, the relations about $\sigma_P$ have smaller slopes compared to ours. The opposite is true for the relations regarding $M_{200}$.

We also find that the relations based on the caustic values are noisier (with larger scatter) than ours, especially for results involving mass. This is easily seen in Figures 14-17 and Table 7. For instance, for the $M_{200}$-$L_{\text{opt}}$ relation the scatter in mass at fixed luminosity is $(0.74 \pm 0.18)$ when using the CIRS values, but only $(0.36 \pm 0.06)$ for the virial masses determined in the current work.

Such findings are explained by the fact that our velocity dispersions are in good agreement with those of RD06, as shown in paper I. However, the same is not true when comparing masses. Those masses estimated from $T_X$ also disagree with the caustic values. RD06 claim concordance between the caustic results and their virial masses (as well as the X-ray estimates). However, comparing our masses and the caustic values (RD06) to the X-ray estimates, we find a better correlation with our values. Therefore, we conclude that our estimates better represent the cluster potentials, as discussed in Section 6 of paper I. There, we also argue that the interloper removal procedure should not be responsible for the different results (as the velocity dispersions are similar between our work and RD06).

The culprit for the different mass estimates lies in the discrepant values of $R_{200}$, which in RD06 are derived from the caustic mass profile. In paper I we estimate $R_{300}$ as a by-product of the virial analysis and the assumption of an NFW profile \cite{Navarro1997}. This profile is well matched to the caustic mass profile for only half of the CIRS sample (RD06), which may explain the different results and the larger scatter for the caustic based scaling relations. Note also that RD06 use $R_{200}$ determined by the caustic to compute the virial masses. That choice helps improving the agreement they find between the caustic and virial masses. The conclusions we reached in paper I \cite{Lopes2009} are corroborated by the scaling relations shown above.

### 3.6 The mass-to-light ratio

In this section, we present the mass-to-light ratio ($M/L$) of clusters of galaxies. That has been used before for estimating the mass density of the universe. The typical value of $M/L$ for rich clusters is $M/L \approx 300 \ H_0 \ M_\odot/\ L_\odot$ \cite{Bahcall2002}. Previous works have shown that $M/L$ increases with the size of the bound systems, from galaxies to clusters. The dependence of the cluster $M/L$ ratio on cluster mass can be parametrized by a relation like $M/L \propto M^{\alpha}$, with $\alpha \simeq 0.20$ \cite{Bahcall2002,Popesso2003}.

In Figure 18, we show the mass-to-light ratio obtained in this work. We recomputed $L_{\text{opt}}$ considering the interval...
Figure 14. The connection between the caustic mass (RD06, estimated from the optical data) and richness. The relation is exhibited for the 53 CIRS clusters.

Figure 15. Analogous to previous figure, but showing $L_{\text{opt}}$ at the faint end of the LF could bias the $L_{\text{opt}}$ values. So, integrating the LF to infinity does not significantly change $L_{\text{opt}}$. The range we considered is already deep enough (being five magnitudes fainter than $m^*$). In Figure 18, the solid line shows the relation listed in Table 3 for ALL clusters, while the other two are based in Popesso et al. (2005). The dotted line considers their “optical” sample (69 clusters with optical masses), while the dashed line is for their “enlarged” sample, with 102 clusters comprising the optical sample plus clusters with masses estimated from the $M$-$T_X$ relation. All the lines are normalized to an $M/L$ ratio of 200.

Figure 16. The relation between the caustic mass (RD06) and X-ray luminosity. $L_X$ is obtained with the “annulus” background.

Figure 18. The relation between the caustic mass (RD06) and $X$-ray luminosity. $L_X$ is obtained with the “annulus” background.

4 COMPARISON TO THE LITERATURE AND DISCUSSION

The results in Tables 1, 3, 4 and 5 indicate a good agreement with most findings in the literature. In Table 8 we summarize our main results, considering all clusters, or only the CIRS systems, as well as the findings of other authors. In particular, there is good consistency between the slopes of $m^*$-5 to $m^*$+5. Using the values obtained in paper I and considering the same range used for the richness estimation ($m^*$-1 to $m^*$+2), would lead to $M/L$ values that are too high. We do not consider an incompleteness correction to the luminosity function (LF) for galaxies fainter than $m^*$+5 as the correction is around 5% and uncertainties in the slope...
Figure 17. The relation between velocity dispersion estimated with the caustic technique and optical luminosity. The relation is shown for the 53 CIRS clusters.

Figure 18. The mass-to-light ratio for the 127 NoSOCS plus 53 CIRS clusters. The solid line shows the relation listed in Table 3 for ALL clusters. The other two results shown are from Popesso et al. (2005). The dotted line considers their “optical” sample, while the dashed line is for their “enlarged” sample. All the lines are normalized to a M/L ratio of 200.

and scatter of POP05 (also based on SDSS and RASS) with our results, in the worst cases within 1.5-σ. RD06 claim to find good consistency with POP05. However, as seen in the previous section that is at the cost of larger errors for the fit parameters and much noisier relations. Note that our results based solely on CIRS (the most massive systems) always have smaller slopes (an issue we discuss further below). Using an enlarged data set Popesso et al. (2007) found relations consistent to POP05.

The works of POP05 and RD06 are the easiest comparison to ours, since they are also based in SDSS data. However, other results, based on different surveys and wavelengths can be informative. Lin, Mohr & Stanford (2004) found $M_{200} \propto L_{K,R}^{0.22}$ and $M_{200} \propto N_{500}^{0.15}$, using Two Micron All Sky Survey (2MASS) data. Our slopes agree within 1.5 and 1-σ, respectively. Note that we consider their results after excluding the BCG, as they claim better agreement on the powers of the two relations. Kochanek et al. (2003) found $M_{200} \propto N^{0.66}$, also using the 2MASS K-band, a result that agrees with ours within 1.5-σ. These findings corroborate the conclusion of Popesso et al. (2007), who reports no significant difference among results in different SDSS bands. So, even considering the 2MASS K-band it seems that the results are similar.

On what regards other works using $L_X$ as a mass tracer, we find that the slopes of Popesso et al. (2005); Stanek et al. (2006); Vikhlinin et al. (2008) and Ettori et al. (2004) are consistent to ours. The scatter is consistent (within 1-σ) to the works of Popesso et al. (2005) and Stanek et al. (2006), while the scatter from Vikhlinin et al. (2008) and Ettori et al. (2004) are smaller than ours (consistent only within 3-σ). Note that the first two works above Popesso et al. (2005) Stanek et al. (2006) are based in the RASS data, while the last two use the Chandra satellite.

We find a very good agreement (within 1-σ) between the normalization we measure and the results of Popesso et al. (2005), for the $M$-$L_X$ relation. Differences in the normalization to the other works above might be due to the differences in the cluster samples used, which may span different mass/luminosity regimes. The comparison of the different selection functions is very hard and is not our goal in this work.

The small differences with POP05 still deserve a few comments. The scatter in $M_{500}$ at fixed $L_{opt}^{RASS}$ is similar to ours (consistency within 1.5-σ). However, a closer look at the upper left panel of our Figure 4 and their Figure 5 shows that we have many more clusters at $M_{500} < 10^{14} M_\odot$ than they do. So, their results are based on a data set showing a cloud of points at $M_{500} > 10^{14} M_\odot$ and a few scattered systems below this value. Nonetheless, that small data set in the low mass regime is sufficient to put their results and ours at the same level. One odd aspect of their plot is the diffuse distribution of the high mass systems, showing high scatter in this mass regime. Our relations always show a reduced scatter towards higher mass. Their poor sampling at low masses can also be seen in Figures 11 and 13 of their work, where we note a deficit of clusters below $\sigma_P = 400$ km/s.

Considering this sampling issue in POP05 it is natural to compare their results with the ones we obtained using only CIRS (which also has mostly clusters with $\sigma_P > 400$ km/s). Figure 3 shows the relation between $\sigma_P$ and $L_{opt}^{RASS}$.
It is remarkable how low the scatter is in this figure. However, the most interesting point is that the relations are slightly flattened for the CIRS clusters, when compared only to NoSCS or all systems. So, it seems that the scaling relations have different behaviors in different mass regimes. They look steeper for the low-mass systems, becoming flatter for the high-mass clusters. Using CIRS we note that the agreement to POP05 is as good as before, despite the fact the slope is now smaller.

Popesso et al. (2007) created a larger sample by adding 130 Abell clusters with confirmed 3D overdensities in SDSS. Although this sample is larger than ours (217 clusters compared to 180) it still seems that there are few systems below \(<10^{14} M_\odot\) (see their Figure 9). Most confirmed Abell overdensities are rich systems, which explain the figure (note that the open symbols are the X-ray systems, which contain the lower mass objects). The main conclusions drawn from these comparisons are: (i) the slopes of the scaling relations become flatter for high-mass samples, but the differences are within 1-sigma; (ii) although the samples of POP05 and Popesso et al. (2007) show fewer systems with low mass, their results are consistent with ours. That is due to the inclusion of a few groups in their sample and to the small variation in the slope with mass. These results indicate that sample incompleteness has little effect on the scaling relations.

For the relations involving \(L_X\) we find the same trend as above, with the high mass systems showing flatter relations. However, some clusters were not considered in the fits due to having only upper limits in the X-ray luminosity estimates. This eliminates points that generally have \(L_X < 10^{43}\) ergs/s in Figure 5 (lower right panel). These points would be located above the derived fit and would thus make the relations a little flatter. Our results with all clusters could be biased due to missing some low mass systems with \(L_X \sim 10^{43}\) ergs/s, making the relations steeper than it would really be. However, considering the conclusions reached above, we might expect that incompleteness (especially in the low mass regime) has a minor impact in the relations. We also note that the results within \(R_{200}\) generally are the ones with smallest scatter when considering \(L_X\).

A result that deserves some comments is the third test shown in Table 2 (rows 5 and 6) where we consider all clusters at \(z \leq 0.25\), considering that at \(z > 0.10\) the spectroscopic survey of SDSS is no longer complete. The main effect exhibited is the flattening of the scaling relations. That happens because \(N_{\text{gal}}\) and \(L_{\text{opt}}\) are still well determined (as they rely on the photometric data), but \(\sigma_P\) and mass are not (see discussion in paper I). If the percentage of clusters without enough spectra is large, the impact in the scaling relations may be important. So, the safest approach is to consider only objects that have complete spectroscopic sampling (as done here and by RD06).

Another important issue concerns the interloper removal procedure (see Woitak et al. 2007). As discussed in §4 of Lopes et al. (2009) we performed several tests to optimize the “shifting-gapper” technique. The final code seems to work well for systems of all masses, as can be seen from the phase-space diagrams (Lopes et al. 2009). POP05 considers the method of Katgert et al. (2004) (see also Jen Hartog & Katgert 1999), which also combines the position and velocity information, but makes assumptions regarding the dynamical state of the cluster. RD06 use the caustic technique to determine the mass profile of the clusters, which involves removal of interlopers in underdense regions of the phase-space diagram. Woitak et al. (2007) applied several different techniques for interloper removal to real and simulated data. They conclude that some methods have poor performance compared to others, but none of the methods cited above are among the poor performers. Whatever the approach, it must be sufficiently restrictive but not overly lax, otherwise many cluster members will be discarded or few interlopers will be rejected, leading to biased values of \(\sigma_P\) and mass. While there is no particular reason to chose one method over another, the procedure should work for a wide variety of systems (as shown for the method adopted here). We based our choice on the simplicity of the method and on the avoidance of assumptions regarding the dynamics of the cluster (Fadda et al. 1996).

Results from a test with a less rigorous criterion (eliminating fewer interlopers) are shown in rows 3 and 4 of Table 2. The comparison of Tables 1 and 2 reveals no systematic effect on the intercept and slope of the scaling relations when relaxing the interloper removal procedure. The scatter of the relations increases slightly, but the results are still consistent with our original relations and to those in the literature. So, we conclude that our method is robust, rendering similar lists of cluster members than other methods. This is reflected in the good agreement with the velocity dispersions of RD06 and the comparison to the scaling relations of other authors (Kochanek et al. 2003; Lin, Mohr & Stanford 2004; Popesso et al. 2003).

The Halo Occupation Distribution (HOD) relates the number of galaxies within the virial radius and the associated mass, expressed as \(N_{\text{gal}, R_{200}} \propto M_{200}\). Hierarchical models of structure formation predict that the number of subhalos within a system is directly proportional to the mass of the parent halo. In other words, \(\alpha = 1\). However, different mechanisms (such as a decreasing efficiency of star formation, or an increased merger rate) would imply a decreasing number of galaxies per given mass in higher mass halos, agreeing with \(\alpha < 1\) (see discussion in Popesso et al. 2007). Here, we do not intend to investigate the HOD. Our main goals are to provide calibrators to the cluster mass and compare those to determine which one traces mass more accurately. However, it is worth mentioning that the connection between mass and richness we found is indeed consistent with \(\alpha < 1\), as most recent results in the literature. In a future work we plan to study in detail the HOD and the factors that could make \(\alpha < 1\).

Finally, we would like to comment that when comparing the performance of \(N_{\text{gal}}, L_{\text{opt}}\) and \(L_X\) as mass tracers, we find that the optical parameters show relations slightly tighter than \(L_X\). The scatter of \(M_{200}\) at fixed \(N_{\text{gal}}\), \(L_{\text{opt}}\) and \(L_X^{200}\) is \((0.43 \pm 0.03), (0.44 \pm 0.04)\) and \((0.47 \pm 0.05)\), respectively. The scatter of \(M_{200}\) at fixed \(N_{\text{gal}}^{200}, L_{\text{opt}}^{200}\) and \(L_X^{200}\) is \((0.46 \pm 0.04), (0.49 \pm 0.04)\) and \((0.56 \pm 0.06)\), respectively. In all cases, the scatter and the slope of the relations are reduced for high mass systems. POP05 argued that the poorer performance of \(L_X\) is likely due to the variation in the compactness of the galaxy clusters. They also investigated if the scatter in the \(M-L_X\) relation could be due to cooling core effects, but they find that this accounts for at
most 3% of the scatter. The low signal to noise in RASS (especially for the low-mass systems) can also contribute to the higher scatter found in the relations involving $L_X$. However, it is important to note that here we quote the errors in the scatter, finding that they are all consistent (in the worst case within 1.5-$\sigma$) for the three mass proxies considered ($N_{\text{gals}}$, $L_{\text{opt}}$ and $L_X$).

5 CORRELATION BETWEEN OPTICAL AND X-RAY PROPERTIES

The interplay between global properties obtained in the optical and X-ray regimes are crucial for understanding the complex physics present in galaxy clusters. In recent years a number of works have been dedicated to the comparison of X-ray and optical catalogs or the construction of combined samples (Donahue et al. 2001, 2002; Gilbank et al. 2004; Popesso et al. 2004, 2005; Lopes et al. 2006). The ability to predict the X-ray luminosity or temperature from optical parameters, and vice-versa, is also important for future surveys, conducted only in one regime. As a by-product these comparisons may also reveal unusual clusters (such as X-ray underluminous ones), which are very interesting for follow-up studies.

In this section, we investigate the correlations between optical ($N_{\text{gals}}$ and $L_{\text{opt}}$) and X-ray ($L_X$ and $T_X$) quantities. Note that the X-ray luminosity is estimated through an iterative procedure that assumes the $L_X-T_X$ relation of Markevitch (1998) (see paper I for more details). That relation is close to the self-similar expectations (without any correction for cooling flows in $L_X$ or $T_X$). This is the same relation adopted to estimate $L_X$ for NORAS, in which $T_X \propto L_X^{1/2}$. Figures 19 and 20 show the connection of $L_X$ to $N_{\text{gals}}$ and $L_{\text{opt}}$, respectively. In the first relation $L_X$ is the value obtained with the “frame” background, while the “annulus” background is used in the latter. In both plots, all measurements are performed within $R_{200}$. Figure 21 shows the correlation between $T_X$ (given by BAX) and $L_{\text{opt},R_{200}}$ for 21 clusters.

Table 9 summarizes the fitting parameters for the relations between X-ray and optical measurements within 0.50 $h^{-1}$ Mpc, $R_{200}$, and $R_{500}$. Note that if one does not have a measure of a radius that scales with mass ($R_{500}$ or $R_{200}$), 0.50 $h^{-1}$ Mpc represents the optimal fixed aperture for comparing optical and X-ray properties. Popesso et al. (2004) and Lopes et al. (2006) tested several different fixed apertures and found the scatter to be the smallest when using 0.50 $h^{-1}$ Mpc. The results in Table 9 are listed for the two X-ray background types considered (“annulus” and “frame”). In Table 10, we list the results involving $T_X$ (21 clusters found in BAX) for all three apertures used for computing $N_{\text{gals}}$ and $L_{\text{opt}}$.

Our results are in good agreement with others in the literature. For instance, POP05, also using data in the SDSS r-band, found similar results to ours: $L_{X,R_{200}} \propto L_{\text{opt},R_{200}}^{0.61}$ and $T_X \propto L_{\text{opt},R_{200}}^{0.61}$. The results within $R_{500}$ and with a fixed metric (in this case for the i-band) are also close to ours. In the worst cases consistency is found within 1.5-$\sigma$. Donahue et al. (2001) found similar relations but using a different richness definition, which hampers a direct comparison to our findings.

We also find a good agreement to the $T_X-N_{\text{gals}}$ and $T_X-L_{\text{opt}}$ relations of Lopes et al. (2006). The differences are within 1-$\sigma$. The relations involving $L_X$ are not as similar. Note that in Lopes et al. (2006) we also force a richness cut,
with \( N_{\text{gals}} \geq 10 \) (richness within 0.5 h\(^{-1}\) Mpc). Lopes et al. (2006) also considered optical estimates from DPOSS and X-ray values from BAX. The heterogeneous nature of the luminosities derived from BAX could contribute to the discrepancies. However, the relations involving \( L_X \) are much steeper and thus more sensitive to the sample used. If we only consider the most rich clusters the agreement is much better than with the full sample. We also notice that the correlations found in the present work are in excellent agreement with Gal et al. (2009), who used DPOSS data for measuring \( N_{\text{gals}} \) and \( L_{\text{opt}} \) within 0.50 h\(^{-1}\) Mpc and directly estimated \( L_X \) from RASS, as done here.

If the density profiles for dark matter and intra-cluster gas are self-similar, the following relations hold: \( M \propto T^{3/2} \), \( T \propto L_X^{1/2} \) and \( M \propto L_X^{3/4} \). Assuming mass traces light (constant \( M/L_{\text{opt}} \)) we would expect that \( L_{\text{opt}} \propto L_X^{3/4} \) and \( L_{\text{opt}} \propto T^{1/2} \). If there is also a strict proportionality between \( L_{\text{opt}} \) and richness, the same relations above are valid for \( N_{\text{gals}} \).

Our results are consistent within 2-\( \sigma \) with these relations. However, if we assume that \( T \propto L_X^{2/5} \) (the typically observed result: David et al. 1993) we would have \( L_{\text{opt}} \propto L_X^{3/5} \), consistent with our findings at 1-\( \sigma \).

### 6 CONNECTION BETWEEN RICHNESS AND RADIUS

We know that the radius and mass of a cluster scale as \( R_{200} \propto M_{200}^{1/3} \) and the number of galaxies within \( R \) is linked to mass as \( N_{\text{gals},R_{200}} \propto M_{200}^{\alpha} \). If \( N_{\text{gals}} \) is measured within a fixed aperture then \( N_{\text{gals}} \propto N_{\text{gals},R_{200}}^{\beta} \) with \( \beta \sim 0.50 - 0.65 \) (see discussion in Hansen et al. 2003). Hierarchical models of structure formation predict that the number of sub-halos within a system is directly proportional to the mass of the parent halo, namely \( \alpha = 1 \). However, different mechanisms such as a decreasing efficiency of star formation, or an increased merger, and destruction rate of galaxies, would imply a decreasing number of galaxies for higher mass halos, suggesting \( \alpha < 1 \) (see discussion in Popesso et al. 2007). Figure 22 shows the connection between \( R_{200} \) and \( N_{\text{gals}} \), while Table 11 has the solutions for the fits regarding the \( R_{200} \) and \( N_{\text{gals}} \) relations.

We find \( R_{200} \propto N_{\text{gals,R}_{200}}^{0.39} \), which indicates that \( \alpha = 0.86 \). The connection between \( N_{\text{gals}} \) and \( M_{200} \) shown in Table 1 indicates \( \alpha = 0.93 \), agreeing within 1-\( \sigma \) to the finding above. The connection between \( R_{200} \) and \( N_{\text{gals}} \) we found is also very close to the result obtained by Gal et al. (2009) who estimated \( R_{200} \) photometrically (similar to Hansen et al. 2003), but counted galaxies as in the current work. This agreement corroborates the findings and reliability of the photometric \( R_{200} \) estimates given in Gal et al. (2004).

In Table 11 we also list the results relative to optical luminosity. We see that the relations for \( L_{\text{opt}} \) are consistent to the ones based on richness. These results are also useful for comparison to the slopes of the relations from Popesso et al. (2007). In the SDSS \( r \)-band they found that \( R_{200} \propto L_{\text{opt},R_{200}}^{0.40} \) and \( R_{200} \propto L_{\text{opt},R_{200}}^{0.41} \), in good agreement with our findings.

### 7 CONCLUSIONS

We have used a sample of 127 NoSOCS plus 56 CIRS galaxy clusters to investigate scaling relations at low redshift (\( z \leq 0.10 \)). For every cluster we previously determined (in paper I) the velocity dispersion (\( \sigma \)), physical radii (\( R_{200} \) and \( R_{500} \)), masses (\( M_{500} \) and \( M_{200} \)), richness (\( N_{\text{gals}} \)), optical and X-ray luminosities (\( L_{\text{opt}} \) and \( L_X \)). The last three parameters are estimated within 0.5 h\(^{-1}\) Mpc, \( R_{500} \) and
R\textsubscript{200}. Substructure estimates are also available for nearly all clusters. We estimated the presence of substructure from the galaxy distribution in two and three dimensions, using the \(\beta\) and \(\Delta\) tests, respectively. For the CIRS systems we also consider the values of \(\sigma_P\) and \(M_{\text{200}}\), independently determined with the caustic method (RD06). For a subset of 21 clusters we have \(T_X\) values from the literature and estimated their masses using a \(M_{\text{200}}-T_X\) relation. The main conclusions we reach are:

- (i) Richness, optical and X-ray luminosities correlate well with \(\sigma_P\) and mass. However, the results indicate that the slope and scatter of the relations are higher when using a fixed aperture to compute \(N\text{gals}\) and \(L_{\text{opt}}\). So, the most accurate relations are achieved when considering a physical radius, such as \(R_{\text{500}}\) and \(R_{\text{200}}\). In general, the most accurate results are obtained within \(R_{\text{500}}\).

- (ii) The scaling relations derived only with the CIRS sample are flatter than the global relations. This indicates that the most massive systems scale differently than the poorer ones. However, the results for the poor and rich systems are still consistent within 1-\(\sigma\).

- (iii) The scaling relations show no significant modification when considering only clusters without substructure. The new relations, as well as the scatter, are consistent with the original ones. This result holds if we employ a two-dimensional (\(\beta\)) or a three-dimensional (\(\Delta\)) test. In particular, the 2D test leads to the same scaling relations when we use the spectroscopically or photometric selected galaxy samples.

- (iv) The comparison of the scaling relations obtained with optical (virial) and X-ray masses (from the \(M_{\text{200}}-T_X\) relation) point to very similar results. In other words, the mass calibration with mass estimates from different wavelengths are equivalent. This is one of the main results of this work.

- (v) As the velocity dispersions and masses computed with the caustic technique are available in RD06 we have also derived the scaling relations with these parameters (considering the values of \(N\text{gals}\), \(L_{\text{opt}}\) and \(L_X\) computed here). Our findings are in line with the conclusions of paper I. Although the slope of the relations agree within 1-\(\sigma\) that is only true because the fits obtained with the caustic parameters have a very large scatter. For the \(M_{\text{200}}-L_{\text{opt}}\) relation the scatter in mass at fixed luminosity is \((0.74 \pm 0.18)\) when using the CIRS results, and only \((0.36 \pm 0.06)\) for the virial masses obtained in the current work. The situation is not as critical for the relations based on \(\sigma_P\), indicating that the interloper removal procedure is not the answer for the observed discrepancies. Instead, these are due to the procedure used for the mass estimation, derived from the caustic mass profile. Uncertainties in this profile lead to different values of \(R_{\text{500}}\) and as consequence, biased results for \(M_{\text{200}}\) (see discussion in paper I).

- (vi) We find a good agreement with most of the results in the literature, even those derived from other bands (such as 2MASS K). So, the comparison between optical and X-ray properties, the connection between \(R_{\text{200}}\) and \(N\text{gals}\), and the mass calibration performed with different cluster properties, all agree well with previous findings. This also indicates the interloper removal procedure we employed is robust, as other authors we compared to employ different techniques for selecting cluster members, or even determine mass from other wavelengths.

- (vii) The scaling relations based on clusters at \(z < 0.25\) are flatter than our original results (considering only objects at \(z < 0.10\)). However, the results of POP05 — who use clusters in SDSS at \(z < 0.25\) — are consistent with our findings. We argue that the fraction of higher-z systems in their sample is probably small, contributing little to their results. However, we acknowledge the relevance of this issue and conclude that clusters’ velocity dispersion and masses must be estimated from complete spectroscopic samples (reaching at least \(M^*+1\); see paper I). Using a large sample of clusters having spectra for only the bright members can severely bias \(\sigma_P\) and mass, thus affecting the scaling relations.

- (viii) The main result of this paper regards the mass calibration of galaxy clusters. We show that richness, \(L_{\text{opt}}\) and \(L_X\) can reliably be used for mass estimation in the nearby universe. Particularly, the optical properties provide slightly more accurate relations in the present work, which is probably due to the use of RASS in the X-ray regime (shallower than the optical data from SDSS). When considering all clusters, the scatter of \(M_{\text{200}}\) at fixed \(N_{\text{gals}}\) is \((0.43 \pm 0.03)\), \((0.44 \pm 0.04)\) and \((0.47 \pm 0.05)\), respectively. The scatter of \(M_{\text{200}}\) at fixed \(R_{\text{500}}\) is \((0.46 \pm 0.04)\), \((0.49 \pm 0.04)\) and \((0.56 \pm 0.06)\), respectively. For the rich (CIRS) systems we found the scatter of \(M_{\text{500}}\) at fixed \(R_{\text{500}}\) is \((0.33 \pm 0.05)\), \((0.38 \pm 0.05)\) and \((0.48 \pm 0.06)\), respectively. These findings are in accord with those of POP05, who found that \(L_{\text{opt}}\) is a slightly better mass tracer than \(L_X\) (also considering SDSS and RASS). This is a very important conclusion, because it tells us that with accurate single band photometry we can reliably estimate the mass of galaxy clusters, a key result for studying the cluster mass function at low redshift.

Our work also indicates that the spectroscopic follow-up of a few dozens of clusters to high redshifts \((z \sim 1)\) can be used for the understanding of the evolution of the scaling relations and to trace mass at high-\(z\). This is a crucial step for self-calibration methods aiming to constrain the dark energy from the evolution of the cluster population with cosmic time.

Note that we do not state that \(N_{\text{gals}}\) or \(L_{\text{opt}}\) are better mass tracers than \(L_X\). We only say that based on RASS and SDSS that is the case. That is due to the fact that RASS is shallow compared to SDSS. A new X-ray survey, for instance based in eROSITA, would provide accurate X-ray luminosities for large samples of clusters. However, as we showed here, richness and optical luminosity are observationally cheap parameters that can also work as a mass proxy. This is an important result for future large sky surveys such as DES, Pan-STARRS, LSST and UKIDSS. If the mass-calibration relation and its evolution are known, these surveys can provide accurate estimates of richness or \(L_{\text{opt}}\) and thus the mass of clusters at high redshifts.

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APPENDIX A: UPDATE VALUES OF X-RAY LUMINOSITIES

We fixed a bug in the code used to estimate X-ray luminosity and its associated error. Hence we update these values in the table A1 below, which replaces Table 5 from paper I. The meaning of all columns is as in paper I. In the first column we give the cluster name; then in the next three columns we list the values of $L_X$ (and its associated error) for the three backgrounds (“annulus”, “frame” and “boxes”, respectively). In the fifth column we show the X-ray temperature measure from BAX (when available), while the interpolated temperature obtained from the $L_X$-$T_X$ relation is shown in the sixth column. The last column indicates whether we had a significant detection (SD), an upper limit (UL), or if the measurements should be taken with concern (XX). The cluster is marked as XX if it is too close to a border, or if the background error or net count rates could not be determined. The last two columns of the table are obtained with the “annulus” background.
Table 1. Relations between $\sigma_p$, $M_{500}$ and $M_{200}$ to $N_{\text{gals}}$, using three different apertures, 0.50 $h^{-1}$ Mpc, $R_{500}$ and $R_{200}$. In all tables the linear fit is of the form $\ln(Y) = A + B \times \ln(X/C)$, where X and Y are listed in the first column. Masses are in units of $10^{14}\ M_\odot$ and velocity dispersion in km s$^{-1}$. The results are also shown for three different samples, using the 127 NoSOCS clusters, fifty-three CIRS clusters, and the combination of the two samples with 180 clusters (second column). The intercept (A) and slope (B) are shown in the third and fourth columns, respectively. The scatter in the Y parameter at fixed X is in the fifth column. The total number of clusters for the first fit is shown as $N_{\text{tot}}$ and the number of clusters after a 3-$\sigma$ clip is given by $N_{\text{use}}$. The pivot point depends on the sample, being $C = 25$ for the NoSOCS and full samples and $C = 60$ for the CIRS sample.

| Relation | Sample   | A       | B       | $\sigma_{\ln Y/X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|----------|----------|---------|---------|---------------------|-----------------|-----------------|
| $N_{\text{gals}}^{0.50}$ | $\sigma_p$ | NoSOCS | 5.88 ± 0.03 | 0.68 ± 0.04 | 0.29 ± 0.02 | 127 | 122 |
| $N_{\text{gals}}^{R_{500}}$ | $\sigma_p$ | NoSOCS | 5.80 ± 0.03 | 0.54 ± 0.03 | 0.23 ± 0.02 | 127 | 124 |
| $N_{\text{gals}}^{R_{200}}$ | $\sigma_p$ | NoSOCS | 5.70 ± 0.03 | 0.56 ± 0.03 | 0.26 ± 0.03 | 127 | 125 |
| $N_{\text{gals}}^{0.50}$ | $M_{200}$ | NoSOCS | 0.75 ± 0.09 | 1.45 ± 0.16 | 0.69 ± 0.06 | 127 | 124 |
| $N_{\text{gals}}^{R_{500}}$ | $M_{200}$ | NoSOCS | 0.53 ± 0.06 | 1.01 ± 0.07 | 0.53 ± 0.05 | 127 | 126 |
| $N_{\text{gals}}^{R_{200}}$ | $M_{200}$ | NoSOCS | 0.36 ± 0.07 | 1.09 ± 0.09 | 0.60 ± 0.07 | 127 | 126 |
| $N_{\text{gals}}^{0.50}$ | $\sigma_p$ | CIRS    | 6.49 ± 0.03 | 0.49 ± 0.05 | 0.17 ± 0.02 | 53  | 51  |
| $N_{\text{gals}}^{R_{500}}$ | $\sigma_p$ | CIRS    | 6.29 ± 0.02 | 0.40 ± 0.03 | 0.14 ± 0.02 | 53  | 51  |
| $N_{\text{gals}}^{R_{200}}$ | $\sigma_p$ | CIRS    | 6.20 ± 0.03 | 0.41 ± 0.03 | 0.15 ± 0.01 | 53  | 51  |
| $N_{\text{gals}}^{0.50}$ | $M_{200}$ | CIRS    | 2.15 ± 0.08 | 1.36 ± 0.19 | 0.46 ± 0.08 | 53  | 52  |
| $N_{\text{gals}}^{R_{500}}$ | $M_{200}$ | CIRS    | 1.54 ± 0.05 | 0.95 ± 0.09 | 0.33 ± 0.05 | 53  | 52  |
| $N_{\text{gals}}^{R_{200}}$ | $M_{200}$ | CIRS    | 1.39 ± 0.07 | 1.00 ± 0.11 | 0.37 ± 0.05 | 53  | 52  |
| $N_{\text{gals}}^{0.50}$ | $\sigma_p$ | ALL     | 5.91 ± 0.02 | 0.69 ± 0.03 | 0.25 ± 0.02 | 180 | 171 |
| $N_{\text{gals}}^{R_{500}}$ | $\sigma_p$ | ALL     | 5.81 ± 0.02 | 0.52 ± 0.02 | 0.19 ± 0.01 | 180 | 175 |
| $N_{\text{gals}}^{R_{200}}$ | $\sigma_p$ | ALL     | 5.69 ± 0.02 | 0.53 ± 0.02 | 0.22 ± 0.02 | 180 | 178 |
| $N_{\text{gals}}^{0.50}$ | $M_{200}$ | ALL     | 0.79 ± 0.05 | 1.50 ± 0.08 | 0.57 ± 0.05 | 180 | 174 |
| $N_{\text{gals}}^{R_{500}}$ | $M_{200}$ | ALL     | 0.57 ± 0.04 | 1.04 ± 0.04 | 0.43 ± 0.03 | 180 | 178 |
| $N_{\text{gals}}^{R_{200}}$ | $M_{200}$ | ALL     | 0.37 ± 0.04 | 1.07 ± 0.04 | 0.46 ± 0.04 | 180 | 177 |

Table 2. Fit parameters for the $\sigma_p$-$N_{\text{gals}}$-$M_{200}$-$N_{\text{gals}}$ relations within $R_{200}$ for three more cases, all based in the NoSOCS sample (same pivot point as in Table 1). Masses are in units of $10^{14}\ M_\odot$ and velocity dispersion in km s$^{-1}$. Rows 1 and 2 considers the original (instead of luminosity-weighted) coordinates. Rows 3 and 4 are for the case when we consider a more “relaxed” criteria for rejecting interlopers (described in § 4.1 of paper I), while rows 5 and 6 have the results for all the 219 NoSOCS clusters at $z \leq 0.25$, instead of $z \leq 0.10$. All the columns have the same meaning as in Table 1.

| Relation | Sample | A       | B       | $\sigma_{\ln Y/X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|----------|--------|---------|---------|---------------------|-----------------|-----------------|
| $N_{R_{200}}^{R_{500}}$ | $\sigma_p$ | NoSOCS | 5.72 ± 0.03 | 0.54 ± 0.04 | 0.29 ± 0.03 | 128 | 127 |
| $N_{R_{200}}^{R_{200}}$ | $M_{200}$ | NoSOCS | 0.38 ± 0.06 | 1.05 ± 0.09 | 0.60 ± 0.07 | 128 | 127 |
| $N_{R_{200}}^{R_{200}}$ | $\sigma_p$ | NoSOCS | 5.77 ± 0.03 | 0.58 ± 0.04 | 0.29 ± 0.04 | 130 | 128 |
| $N_{R_{200}}^{R_{200}}$ | $M_{200}$ | NoSOCS | 0.51 ± 0.07 | 1.16 ± 0.09 | 0.66 ± 0.09 | 130 | 129 |
| $N_{R_{200}}^{R_{200}}$ | $\sigma_p$ | NoSOCS | 5.63 ± 0.02 | 0.43 ± 0.02 | 0.26 ± 0.02 | 219 | 216 |
| $N_{R_{200}}^{R_{200}}$ | $M_{200}$ | NoSOCS | 0.21 ± 0.04 | 0.83 ± 0.05 | 0.61 ± 0.05 | 219 | 216 |
Table 3. Analogous to Table 1, but listing the fit parameters for the relations of $\sigma_p$, $M_{500}$ and $M_{200}$ to $L_{\text{opt}}$. Masses are in units of $10^{12} M_\odot$ and velocity dispersion in km s$^{-1}$. As before, the pivot point depends on the sample, being $C = 0.40 \times 10^{12} L_\odot$ for the NoSOCS and full samples and $C = 1.10 \times 10^{12} L_\odot$ for the CIRS sample.

| Relation X Y | Sample     | A      | B      | $\sigma_{\text{ln}Y|X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|--------------|------------|--------|--------|--------------------------|------------------|------------------|
| $L_{\text{opt}}^{0.50}$ | $\sigma_p$ | NoSOCS | 5.88 ± 0.05 | 0.93 ± 0.10 | 0.41 ± 0.04 | 127 | 124 |
| $L_{\text{opt}}^{R_{500}}$ | $\sigma_p$ | NoSOCS | 5.79 ± 0.03 | 0.63 ± 0.04 | 0.26 ± 0.02 | 127 | 123 |
| $L_{\text{opt}}^{R_{200}}$ | $\sigma_p$ | NoSOCS | 5.68 ± 0.04 | 0.64 ± 0.05 | 0.38 ± 0.05 | 127 | 125 |
| $L_{\text{opt}}^{0.50}$ | $M_{200}$ | NoSOCS | 0.76 ± 0.13 | 2.00 ± 0.32 | 0.92 ± 0.20 | 127 | 124 |
| $L_{\text{opt}}^{R_{500}}$ | $M_{500}$ | NoSOCS | 0.51 ± 0.06 | 1.09 ± 0.07 | 0.57 ± 0.06 | 127 | 125 |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | NoSOCS | 0.27 ± 0.07 | 1.18 ± 0.09 | 0.63 ± 0.08 | 127 | 123 |
| $L_{\text{opt}}^{0.50}$ | $\sigma_p$ | CIRS   | 6.55 ± 0.04 | 0.59 ± 0.07 | 0.20 ± 0.03 | 53  | 52  |
| $L_{\text{opt}}^{R_{500}}$ | $\sigma_p$ | CIRS   | 6.31 ± 0.02 | 0.41 ± 0.03 | 0.14 ± 0.01 | 53  | 50  |
| $L_{\text{opt}}^{R_{200}}$ | $\sigma_p$ | CIRS   | 6.22 ± 0.03 | 0.39 ± 0.03 | 0.15 ± 0.02 | 53  | 51  |
| $L_{\text{opt}}^{0.50}$ | $M_{200}$ | CIRS   | 2.26 ± 0.11 | 1.48 ± 0.22 | 0.57 ± 0.10 | 53  | 53  |
| $L_{\text{opt}}^{R_{500}}$ | $M_{500}$ | CIRS   | 1.57 ± 0.05 | 0.99 ± 0.10 | 0.38 ± 0.05 | 53  | 53  |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | CIRS   | 1.41 ± 0.06 | 0.98 ± 0.11 | 0.36 ± 0.06 | 53  | 51  |
| $L_{\text{opt}}^{0.50}$ | $\sigma_p$ | ALL    | 5.85 ± 0.03 | 0.78 ± 0.05 | 0.31 ± 0.02 | 180 | 178 |
| $L_{\text{opt}}^{R_{500}}$ | $\sigma_p$ | ALL    | 5.76 ± 0.02 | 0.56 ± 0.02 | 0.21 ± 0.02 | 180 | 174 |
| $L_{\text{opt}}^{R_{200}}$ | $\sigma_p$ | ALL    | 5.65 ± 0.03 | 0.55 ± 0.02 | 0.24 ± 0.02 | 180 | 176 |
| $L_{\text{opt}}^{0.50}$ | $M_{200}$ | ALL    | 0.68 ± 0.07 | 1.70 ± 0.12 | 0.72 ± 0.07 | 180 | 178 |
| $L_{\text{opt}}^{R_{500}}$ | $M_{500}$ | ALL    | 0.49 ± 0.04 | 1.06 ± 0.04 | 0.44 ± 0.04 | 180 | 173 |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | ALL    | 0.28 ± 0.05 | 1.11 ± 0.04 | 0.49 ± 0.04 | 180 | 174 |

Table 4. Fit parameters for the $\sigma_p$-$L_X$, $M_{500}$-$L_X$, $M_{200}$-$L_X$ relations using two different apertures, $R_{500}$ and $R_{200}$. The results consider the X-ray luminosity obtained with the “annulus” background (second column). Masses are in units of $10^{13} M_\odot$ and velocity dispersion in km s$^{-1}$. First we show the results derived with all clusters, then only the CIRS systems are used. The remaining columns have the same meaning as in the previous tables. The pivot points are $C = 0.20 \times 10^{44}$ erg/s for the full sample and $C = 0.55 \times 10^{44}$ erg/s for the CIRS sample.

| Relation X Y | Bkg Type | Sample | A      | B      | $\sigma_{\text{ln}Y|X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|--------------|----------|--------|--------|--------|--------------------------|------------------|------------------|
| $L_{\text{opt}}^{R_{500}}$ | $\sigma_p$ | Annulus | ALL    | 5.90 ± 0.03 | 0.26 ± 0.03 | 0.29 ± 0.04 | 104 | 104 |
| $L_{\text{opt}}^{R_{200}}$ | $\sigma_p$ | Annulus | ALL    | 5.85 ± 0.04 | 0.27 ± 0.03 | 0.29 ± 0.03 | 97  | 97  |
| $L_{\text{opt}}^{R_{500}}$ | $M_{500}$ | Annulus | ALL    | 0.68 ± 0.06 | 0.62 ± 0.04 | 0.47 ± 0.05 | 104 | 103 |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | Annulus | ALL    | 0.59 ± 0.08 | 0.67 ± 0.06 | 0.56 ± 0.06 | 97  | 97  |
| $L_{\text{opt}}^{R_{500}}$ | $\sigma_p$ | Annulus | CIRS   | 6.21 ± 0.04 | 0.18 ± 0.04 | 0.26 ± 0.03 | 52  | 52  |
| $L_{\text{opt}}^{R_{200}}$ | $\sigma_p$ | Annulus | CIRS   | 6.19 ± 0.04 | 0.19 ± 0.04 | 0.25 ± 0.03 | 52  | 52  |
| $L_{\text{opt}}^{R_{500}}$ | $M_{500}$ | Annulus | CIRS   | 1.39 ± 0.08 | 0.49 ± 0.08 | 0.48 ± 0.06 | 52  | 52  |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | Annulus | CIRS   | 1.40 ± 0.08 | 0.52 ± 0.08 | 0.49 ± 0.06 | 52  | 52  |
Table 5. Analogous to the previous table, but for the results based in the “frame” background. Masses are in units of $10^{14} M_\odot$ and velocity dispersion in km s$^{-1}$.

| Relation  | Y  | Bkg Type | Sample | A     | B     | $\sigma_{lnY|X}$ | N$_{tot}$ | N$_{use}$ |
|-----------|-----|----------|--------|-------|-------|-----------------|----------|----------|
| $R_{500}^{F}$ | $\sigma_p$ | Frame | ALL    | 5.92 ± 0.03 | 0.24 ± 0.02 | 0.28 ± 0.04 | 98 | 98 |
| $R_{X}^{F}$ | $\sigma_p$ | Frame | ALL    | 5.90 ± 0.04 | 0.22 ± 0.03 | 0.28 ± 0.03 | 90 | 90 |
| $R_{500}^{G}$ | $M_{200}$ | Frame | ALL    | 0.73 ± 0.06 | 0.59 ± 0.04 | 0.46 ± 0.04 | 98 | 96 |
| $R_{X}^{G}$ | $M_{200}$ | Frame | ALL    | 0.68 ± 0.08 | 0.61 ± 0.06 | 0.55 ± 0.05 | 90 | 89 |
| $R_{500}^{C}$ | $\sigma_p$ | Frame | CIRS   | 6.20 ± 0.04 | 0.18 ± 0.04 | 0.26 ± 0.03 | 50 | 50 |
| $R_{X}^{C}$ | $\sigma_p$ | Frame | CIRS   | 6.19 ± 0.04 | 0.18 ± 0.03 | 0.25 ± 0.03 | 49 | 49 |
| $R_{500}^{C}$ | $M_{200}$ | Frame | CIRS   | 1.38 ± 0.08 | 0.49 ± 0.07 | 0.49 ± 0.06 | 50 | 50 |
| $R_{X}^{C}$ | $M_{200}$ | Frame | CIRS   | 1.40 ± 0.08 | 0.50 ± 0.07 | 0.50 ± 0.06 | 49 | 49 |

Table 6. Fit parameters for the $M_{200}$-$N_{gal}$ and $M_{200}$-$L_{opt}$ for clusters with or without substructure. Masses are in units of $10^{14} M_\odot$ and velocity dispersion in km s$^{-1}$. First, we show (lines 1-6) the results considering an aperture of $R_{200}$ for all clusters and for the substructure free systems. These are selected with the $\Delta$ or $\beta$ tests, applied only to galaxies considered as cluster members (§4). Then we show the results considering the same aperture and the 2-D test ($\beta$), but using the photometric sample (no restriction to galaxies with spectra). That is seen in rows 7-10. Finally, we list the results with a fixed metric of 1.5 h$^{-1}$ Mpc for estimating substructure and using the $\beta$ test. Again, the photometric data are considered (rows 11-14). The parameters of each relation are listed in column 1, while the substructure test (when it is the case) is shown in column 2. The remaining columns have the same meaning as in the previous tables. The pivot points for all samples are $C$ = 25 when considering richness, and $C = 0.40 \times 10^{12} L_\odot$ for $L_{opt}$.

| Relation  | Y  | Sub-test | Sample | A     | B     | $\sigma_{lnY|X}$ | N$_{tot}$ | N$_{use}$ |
|-----------|-----|----------|--------|-------|-------|-----------------|----------|----------|
| $N_{200}^{G}$ | $M_{200}$ | ALL    | 0.40 ± 0.05 | 1.10 ± 0.05 | 0.48 ± 0.04 | 170 | 167 |
| $N_{200}^{G}$ | $M_{200}$ | $\Delta$ | Sub-free | 0.40 ± 0.05 | 1.10 ± 0.06 | 0.49 ± 0.05 | 131 | 129 |
| $N_{200}^{G}$ | $M_{200}$ | $\beta$ | Sub-free | 0.40 ± 0.05 | 1.10 ± 0.05 | 0.49 ± 0.04 | 144 | 141 |
| $L_{200}^{G}$ | $M_{200}$ | ALL    | 0.31 ± 0.05 | 1.12 ± 0.05 | 0.51 ± 0.04 | 170 | 163 |
| $L_{200}^{G}$ | $M_{200}$ | $\Delta$ | Sub-free | 0.35 ± 0.06 | 1.08 ± 0.07 | 0.52 ± 0.06 | 131 | 127 |
| $L_{200}^{G}$ | $M_{200}$ | $\beta$ | Sub-free | 0.33 ± 0.05 | 1.12 ± 0.05 | 0.53 ± 0.04 | 144 | 138 |
| $N_{200}^{G}$ | $M_{200}$ | ALL    | 0.38 ± 0.04 | 1.09 ± 0.04 | 0.48 ± 0.03 | 179 | 176 |
| $N_{200}^{G}$ | $M_{200}$ | $\beta$ | Sub-free | 0.38 ± 0.05 | 1.06 ± 0.05 | 0.48 ± 0.04 | 136 | 133 |
| $L_{200}^{G}$ | $M_{200}$ | ALL    | 0.28 ± 0.05 | 1.12 ± 0.05 | 0.52 ± 0.04 | 179 | 173 |
| $L_{200}^{G}$ | $M_{200}$ | $\beta$ | Sub-free | 0.33 ± 0.06 | 1.07 ± 0.06 | 0.51 ± 0.05 | 136 | 131 |
| $N_{50}^{G}$ | $M_{200}$ | ALL    | 0.80 ± 0.05 | 1.54 ± 0.09 | 0.64 ± 0.06 | 183 | 177 |
| $N_{50}^{G}$ | $M_{200}$ | $\beta$ | Sub-free | 0.79 ± 0.06 | 1.66 ± 0.12 | 0.59 ± 0.07 | 119 | 117 |
| $L_{50}^{G}$ | $M_{200}$ | ALL    | 0.69 ± 0.07 | 1.74 ± 0.13 | 0.79 ± 0.08 | 183 | 181 |
| $L_{50}^{G}$ | $M_{200}$ | $\beta$ | Sub-free | 0.70 ± 0.07 | 1.72 ± 0.13 | 0.70 ± 0.08 | 119 | 116 |
Table 7. Fit parameters for the relations connecting $M_{200}$ and $\sigma_P$ with $N_{\text{gals}}$, $L_{\text{opt}}$ and $L_X$. These last parameters are determined within $R_{200}$. Masses are in units of $10^{14} M_\odot$ and velocity dispersion in km s$^{-1}$. The parameters of each relation are listed in column 1. In the second column, we indicate the method used for estimating mass or $\sigma_P$. Mass can be derived from the $M_{200}$-$T_X$ relation of POP05 or from the caustic (RD06). Velocity dispersion values come from the latter (RD06). Relations derived with the first method (masses estimated from $T_X$) are based in the 21 clusters with temperature available. Results obtained with the second method (considering the caustic values) use the 53 CIRS clusters. The pivot points are the same as listed in the previous tables when considering the CIRS sample. For the 21 rich clusters with $T_X$ available the pivot points are assumed the same. The remaining columns have the same meaning as in the tables above.

| Relation | Mass A | B | $\sigma_{\ln Y | X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|----------|--------|---|-----------------|-----------------|-----------------|
| $N_{\text{gals}}^{R_{200}}$ | $M_{200}$ | X-ray | 0.90 ± 0.10 | 0.92 ± 0.10 | 0.34 ± 0.08 | 21 | 20 |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | X-ray | 0.99 ± 0.10 | 0.85 ± 0.11 | 0.33 ± 0.08 | 21 | 20 |
| $L_{X}^{R_{200}}$ | $M_{200}$ | X-ray | 1.15 ± 0.14 | 0.39 ± 0.10 | 0.36 ± 0.11 | 20 | 20 |
| $L_{X}^{R_{200}}$ | $M_{200}$ | X-ray | 1.02 ± 0.12 | 0.47 ± 0.08 | 0.25 ± 0.07 | 19 | 18 |
| $N_{\text{gals}}^{R_{200}}$ | $\sigma_P$ | Caustic | 6.34 ± 0.03 | 0.27 ± 0.06 | 0.18 ± 0.02 | 53 | 52 |
| $L_{\text{opt}}^{R_{200}}$ | $\sigma_P$ | Caustic | 6.36 ± 0.03 | 0.25 ± 0.05 | 0.18 ± 0.03 | 53 | 52 |
| $L_{X}^{R_{200}}$ | $\sigma_P$ | Caustic | 6.36 ± 0.03 | 0.12 ± 0.03 | 0.21 ± 0.03 | 52 | 51 |
| $L_{X}^{R_{200}}$ | $\sigma_P$ | Caustic | 6.35 ± 0.03 | 0.12 ± 0.03 | 0.21 ± 0.03 | 49 | 48 |
| $N_{\text{gals}}^{R_{200}}$ | $M_{200}$ | Caustic | 0.64 ± 0.14 | 1.35 ± 0.34 | 0.77 ± 0.22 | 53 | 53 |
| $L_{\text{opt}}^{R_{200}}$ | $M_{200}$ | Caustic | 0.72 ± 0.12 | 1.26 ± 0.29 | 0.74 ± 0.18 | 53 | 53 |
| $L_{X}^{R_{200}}$ | $M_{200}$ | Caustic | 0.73 ± 0.13 | 0.56 ± 0.19 | 0.87 ± 0.12 | 52 | 52 |
| $L_{X}^{R_{200}}$ | $M_{200}$ | Caustic | 0.68 ± 0.12 | 0.54 ± 0.15 | 0.87 ± 0.12 | 49 | 49 |

$a$ – $L_{X}^{R_{200}}$ estimated with the “annulus” background; $b$ – $L_{X}^{R_{200}}$ estimated with the “frame” background
Table 8. Slope and orthogonal scatter of different scaling relations, from this work or from other authors. Masses are in units of $10^{14} \, M_{\odot}$ and velocity dispersion in km s$^{-1}$. The parameters involved in each relation are listed in column 1; the sample considered is in column 2; the source in columns 3; while the slope and scatter are in columns 4 and 5, respectively.

| Relation | Sample | Source   | $B$  | $\sigma_{\ln Y|X}$ |
|----------|--------|----------|------|-------------------|
| $N_{\text{M}_{500}}$ | M$_{500}$ | ALL | 1.04 | 0.43 |
| $N_{\text{M}_{200}}$ | M$_{200}$ | ALL | 1.07 | 0.46 |
| $L_{\text{opt}}$ | M$_{500}$ | ALL | 1.06 | 0.44 |
| $L_{\text{opt}}$ | M$_{200}$ | ALL | 1.11 | 0.49 |
| $L_{\text{opt}}$ | M$_{500}$ | Extended | Pop05 | 1.09 | 0.50 |
| $L_{\text{opt}}$ | M$_{200}$ | Extended | Pop05 | 1.06 | 0.51 |
| $L_{\text{X}_{a}}$ | M$_{500}$ | ALL | 0.62 | 0.47 |
| $L_{\text{X}_{b}}$ | M$_{200}$ | ALL | 0.67 | 0.56 |
| $L_{\text{X}_{a}}$ | M$_{500}$ | ALL | 0.59 | 0.46 |
| $L_{\text{X}_{b}}$ | M$_{200}$ | ALL | 0.61 | 0.55 |
| $L_{\text{X}}$ | M$_{500}$ | Extended | Pop05 | 0.59 | 0.59 |
| $L_{\text{X}}$ | M$_{200}$ | Extended | Pop05 | 0.60 | 0.54 |
| $L_{\text{X}}$ | M | — | Sta06 | 0.63 | 0.43 |
| $L_{\text{X}}$ | M$_{500}$ | — | Vik08 | 0.62 | 0.25 |
| $L_{\text{X}}$ | M | — | Ett04 | 0.53 | 0.25 |
| $L_{\text{X}_{a}}$ | $\sigma_P$ | ALL | 0.55 | 0.24 |
| $L_{\text{X}_{b}}$ | $\sigma_P$ | ALL | 0.56 | 0.21 |
| $L_{\text{X}_{a}}$ | $\sigma_P$ | Extended | Pop05 | 0.43 | 0.17 |
| $L_{\text{X}_{b}}$ | $\sigma_P$ | Extended | Pop05 | 0.43 | 0.17 |

a— $L_{\text{X}_{200}}$ estimated with the “annulus” background; b— $L_{\text{X}_{200}}$ estimated with the “frame” background. The works of Pop05, Sta06, Vik08 and Ett04 are Popesso et al. (2005); Stanek et al. (2006); Vikhlinin et al. (2008) and Ettori et al. (2004), respectively. Note that the results from Stanek et al. (2006) are dependent of the cosmology. When considering results from the third year of WMAP, they find a reduced scatter $\sigma_{\ln M|L_X} = 0.25$, consistent to Vikhlinin et al. (2008).
Table 9. Fit parameters for the relations between X-ray luminosity ($L_X$) and optical properties ($N_{\text{gals}}$ and $L_{\text{opt}}$) within three different apertures. The aperture used for computing $L_X$ is always the same as indicated for $N_{\text{gals}}$ or $L_{\text{opt}}$. $L_X$ is measured in units of $10^{44}$ erg/s. The background type considered for the $L_X$ estimates is listed in the second column. The sample used for deriving the relations is shown in the third column. The remaining columns are as in Table 1. The pivot points are $C = 25$ when considering richness, and $C = 0.40 \times 10^{12} L_\odot$ for $L_{\text{opt}}$.

| Relation | X | Bkg Type | Sample | A | B | $\sigma_{\ln Y/X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|----------|---|----------|--------|---|---|------------------|----------------|----------------|
| $N_{0.50 \text{ gals}}$ | $L_X$ | Annulus | ALL | -1.51 ± 0.09 | 2.28 ± 0.20 | 0.83 ± 0.13 | 105 | 105 |
| $N_{R_{500} \text{ gals}}$ | $L_X$ | Annulus | ALL | -1.69 ± 0.09 | 1.64 ± 0.10 | 0.62 ± 0.09 | 104 | 103 |
| $N_{R_{200} \text{ gals}}$ | $L_X$ | Annulus | ALL | -1.85 ± 0.12 | 1.59 ± 0.12 | 0.63 ± 0.09 | 97 | 97 |
| $L_{0.50 \text{ opt}}$ | $L_X$ | Annulus | ALL | -1.68 ± 0.12 | 2.38 ± 0.23 | 0.97 ± 0.19 | 105 | 104 |
| $L_{R_{500} \text{ opt}}$ | $L_X$ | Annulus | ALL | -1.77 ± 0.10 | 1.62 ± 0.11 | 0.65 ± 0.10 | 104 | 103 |
| $L_{R_{200} \text{ opt}}$ | $L_X$ | Annulus | ALL | -1.84 ± 0.13 | 1.49 ± 0.12 | 0.64 ± 0.10 | 97 | 97 |

| Relation | X | Sample | A | B | $\sigma_{\ln Y/X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|----------|---|--------|---|---|------------------|----------------|----------------|
| $N_{0.50 \text{ gals}}$ | $T_X$ | BAX | 1.52 ± 0.05 | 0.73 ± 0.11 | 0.20 ± 0.05 | 21 | 21 |
| $N_{R_{500} \text{ gals}}$ | $T_X$ | BAX | 1.22 ± 0.06 | 0.50 ± 0.07 | 0.19 ± 0.04 | 21 | 21 |
| $N_{R_{200} \text{ gals}}$ | $T_X$ | BAX | 1.05 ± 0.06 | 0.56 ± 0.06 | 0.21 ± 0.05 | 21 | 20 |
| $L_{0.50 \text{ opt}}$ | $T_X$ | BAX | 1.56 ± 0.07 | 0.69 ± 0.13 | 0.23 ± 0.05 | 21 | 21 |
| $L_{R_{500} \text{ opt}}$ | $T_X$ | BAX | 1.21 ± 0.06 | 0.52 ± 0.07 | 0.20 ± 0.04 | 21 | 20 |
| $L_{R_{200} \text{ opt}}$ | $T_X$ | BAX | 1.10 ± 0.06 | 0.51 ± 0.06 | 0.21 ± 0.04 | 21 | 20 |

Table 10. Fit parameters for the relations between X-ray temperature $T_X$ (in keV) and optical properties ($N_{\text{gals}}$ and $L_{\text{opt}}$) within three different apertures. Only the 21 clusters with x-ray temperature available in BAX are used in the fits. That is indicated in the second column (“sample”). The remaining columns are as in the previous table. The pivot points are $C = 60$ when considering richness, and $C = 1.10 \times 10^{12} L_\odot$ for $L_{\text{opt}}$.

| Relation | X | Sample | A | B | $\sigma_{\ln Y/X}$ | $N_{\text{tot}}$ | $N_{\text{use}}$ |
|----------|---|--------|---|---|------------------|----------------|----------------|
| $N_{0.50 \text{ gals}}$ | $T_X$ | BAX | 1.52 ± 0.05 | 0.73 ± 0.11 | 0.20 ± 0.05 | 21 | 21 |
| $N_{R_{500} \text{ gals}}$ | $T_X$ | BAX | 1.22 ± 0.06 | 0.50 ± 0.07 | 0.19 ± 0.04 | 21 | 21 |
| $N_{R_{200} \text{ gals}}$ | $T_X$ | BAX | 1.05 ± 0.06 | 0.56 ± 0.06 | 0.21 ± 0.05 | 21 | 20 |
| $L_{0.50 \text{ opt}}$ | $T_X$ | BAX | 1.56 ± 0.07 | 0.69 ± 0.13 | 0.23 ± 0.05 | 21 | 21 |
| $L_{R_{500} \text{ opt}}$ | $T_X$ | BAX | 1.21 ± 0.06 | 0.52 ± 0.07 | 0.20 ± 0.04 | 21 | 20 |
| $L_{R_{200} \text{ opt}}$ | $T_X$ | BAX | 1.10 ± 0.06 | 0.51 ± 0.06 | 0.21 ± 0.04 | 21 | 20 |
Table 11. Fit parameters for the R$_{500}$-N$_{gals}$, R$_{200}$-N$_{gals}$, R$_{500}$-L$_{opt}$ and R$_{200}$-L$_{opt}$ relations using three different apertures for computing N$_{gals}$ or L$_{opt}$. The physical radii are measured in Mpc. The remaining columns are as in the previous table. The pivot points are C = 25 when considering richness, and C = 0.40 10$^{12}$ L$_{\odot}$ for L$_{opt}$.

| Relation    | Sample | Y     | A      | B      | $\sigma_{ln(Y|X)}$ | N$_{tot}$ | N$_{use}$ |
|-------------|--------|-------|--------|--------|---------------------|-----------|----------|
| N$^{0.50}_{gals}$ | R$_{500}$ | ALL   | -0.12 ± 0.02 | 0.47 ± 0.02 | 0.19 ± 0.01 | 180 | 173 |
| N$^{R500}_{gals}$ | R$_{500}$ | ALL   | -0.19 ± 0.01 | 0.37 ± 0.01 | 0.14 ± 0.01 | 180 | 175 |
| N$^{0.50}_{gals}$ | R$_{200}$ | ALL   | 0.20 ± 0.02 | 0.48 ± 0.02 | 0.19 ± 0.01 | 180 | 173 |
| N$^{R200}_{gals}$ | R$_{200}$ | ALL   | 0.05 ± 0.02 | 0.39 ± 0.02 | 0.17 ± 0.01 | 180 | 177 |

| Relation    | Sample | Y     | A      | B      | $\sigma_{ln(Y|X)}$ | N$_{tot}$ | N$_{use}$ |
|-------------|--------|-------|--------|--------|---------------------|-----------|----------|
| L$^{0.50}_{opt}$ | R$_{500}$ | ALL   | -0.16 ± 0.02 | 0.56 ± 0.04 | 0.24 ± 0.02 | 180 | 178 |
| L$^{R500}_{opt}$ | R$_{500}$ | ALL   | -0.22 ± 0.01 | 0.40 ± 0.02 | 0.17 ± 0.01 | 180 | 175 |
| L$^{0.50}_{opt}$ | R$_{200}$ | ALL   | 0.16 ± 0.02 | 0.56 ± 0.04 | 0.24 ± 0.02 | 180 | 178 |
| L$^{R200}_{opt}$ | R$_{200}$ | ALL   | 0.01 ± 0.02 | 0.40 ± 0.02 | 0.18 ± 0.01 | 180 | 176 |

Table A1. X-ray luminosity and temperature of the 183 NoSOCS plus CIRS clusters.

| name                  | L$_X$ (annulus) (10$^{44}$ erg s$^{-1}$) | L$_X$ (frame) (10$^{44}$ erg s$^{-1}$) | L$_X$ (box) (10$^{44}$ erg s$^{-1}$) | T$_X$ (BAX) (keV) | T$_X$ (interp. RASS) (keV) | NOTE |
|-----------------------|------------------------------------------|---------------------------------------|-------------------------------------|-------------------|---------------------------|------|
| NSCS J121847+484410   | 0.049 ± 0.019                           | 0.052 ± 0.020                         | 0.051 ± 0.019                      | ———              | 0.7                        | SD   |
| NSCS J011502+002441   | 0.404 ± 0.052                           | 0.497 ± 0.058                         | 0.497 ± 0.056                      | 2.53 $^{+0.09}_{-0.09}$ | 2.1                       | SD   |
| NSCS J100242+324218   | 0.395 ± 0.048                           | 0.342 ± 0.045                         | 0.371 ± 0.046                      | ———              | 2.1                       | SD   |

Note. - A portion of this table is shown here for guidance regarding its form and content. A full version is available in the electronic edition of the MNRAS.