Fate of Chiral Symmetries in Supersymmetric Quantum Chromodynamics

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In supersymmetric quantum chromodynamics with $N_c$-colors and $N_f$-flavors of quarks, our effective superpotential provides the alternative description to the Seiberg’s $N = 1$ duality at least for $N_f \geq N_c + 2$, where spontaneous breakdown of chiral symmetries leads to $SU(N_c)_{L} \times SU(N_f - N_c)_{R}$ as a nonabelian chiral symmetry. The anomaly-matching is ensured by the presence of Nambu-Goldstone superfields associated with this breaking and the instanton contributions are properly equipped in the effective superpotential.

I. PROLOGUE

In supersymmetric quantum chromodynamics (SQCD) with $N_c$-colors and $N_f$-flavors of quarks, we have chiral $SU(N_f)$ symmetry. At low energies, we have its dynamical breakdown to vectorial $SU(N_f)$ symmetry, for $N_f \leq N_c$. For remaining cases, we have restoration of chiral $SU(N_f)$ symmetry including the case with $N_f = N_c$. For $N_f \geq N_c + 2$, we need “magnetic” degrees of freedom, namely, “magnetic” quarks \[\square\]. And, especially, for $3N_c/2 < N_f < 3N_c$, the well-defined $N = 2$ duality supports this description based on the $N = 1$ duality \[\Box\].

In this talk, I will add the “electric” description expressed in terms of mesons and baryons instead of “magnetic” quarks to SQCD with $N_c + 2 \leq N_f \leq 3N_c/2$ \[\Box\]. In the “electric” phase, however, since anomaly-matching \[\Box\] is not satisfied, we expect spontaneous breakdown of chiral symmetries \[\price\]. The residual symmetries will include vectorial $SU(N_c)$ symmetry and chiral $SU(N_f - N_c)$ symmetry, which are found by inspecting vacuum structure of our effective superpotential to be discussed.

II. ANOMALOUS $U(1)$ SYMMETRY AND SUPERPOTENTIAL

We follow the classic procedure to construct our effective superpotential, which explicitly uses $S$ composed of two chiral gauge superfields \[\square\]. We impose on the superpotential the gauge anomaly $U(1)$ transformation, where $\tilde{F}_{\mu\nu}$ represents the lagrangian of SQCD and $F_{\mu\nu}$ is a gluon’s field strength. As a result, the anomalous $U(1)$-term is reproduced by the $F$-term of $S$.

We find a superpotential, where mesons and baryons are denoted by $T$, $B$ and $\bar{B}$ \[\Box\]:

\begin{equation}
W_{\text{eff}} = S \left\{ \ln \left[ \frac{S^{N_c-N_f} \det(T) f(Z)}{\Lambda^{N_c-N_f}} \right] \right\}
\end{equation}

with an arbitrary function, $f(Z)$, to be determined, where $\Lambda$ is the scale of SQCD and $Z$ is defined by (with abbreviated notations) $BT^{N_f-N_c} \bar{B}/\det(T)$. This is the superpotential to be examined. It looks familiar to you except for the function $f(Z)$ here. In the classical limit, $Z$ is equal to one and the function $f(Z)$ can be parametrized by $f(Z) = (1 - Z)^{\rho}$, where $\rho$ is a positive parameter. The parameter $\rho$ is probably equal to 1. If $\rho = 1$, we recover the superpotential for $N_f = N_c + 1$ given by $W_{\text{eff}} = S \left\{ \ln [\det(T) - BTB] / S \Lambda^{N_c-N_f} \right\}$ \[\Box\].

III. STRATEGY

To examine dynamical properties of our superpotential, we

1. first go to slightly broken SUSY vacuum, where symmetry behavior of the superpotential is more visible.

2. use universal scalar masses of $\mu_L$ and $\mu_R$, which respect global symmetry and only break SUSY.

3. check the consistency with SQCD in its SUSY limit, after determining the SUSY-broken vacuum.

The SQCD defined in the SUSY limit of the so-obtained SUSY-broken SQCD should exhibit the consistent anomaly-matching property and yield the compatible result with instanton calculus.

Let $\pi_i$ be $\langle 0 | T^i | 0 \rangle$, $\pi_\lambda$ be $\langle 0 | S | 0 \rangle$ and $z$ be $\langle 0 | Z | 0 \rangle$. Since the dynamics requires that some of the $\pi$ acquire

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non-vanishing VEV’s, suppose that one of the $\pi_i$ ($i=1 \sim N_f$) develops a VEV, and let this be labeled by $i=1$: $|\pi_i| \sim \Lambda^2$. This VEV is determined by solving $\partial V_{\text{eff}}/\partial \pi_i = 0$, yielding

$$G_TW_{\text{eff};\alpha}^{* \frac{\pi_{\alpha}}{\pi_i}} (1-\alpha) = G_SW_{\text{eff};\lambda}^{* \frac{\pi_{\lambda}}{\pi_i}} (1-\alpha) + \beta X + M^2 |\frac{\pi_{\alpha}}{\Lambda}|^2, \quad (a=1\sim N_c),$$

where $\alpha = z' f(\bar{z})/f(z)$, $\beta = z \alpha'$, $M^2 = \mu_L^2 + \mu_R^2 + G_T^2 \Lambda^2 \sum_{i=1}^{N_f} |W_{\text{eff};i}|^2$, $X = G_T \sum_{i=1}^{N_c} W_{\text{eff};i}^{* \frac{\pi_{\lambda}/\pi_i}} - G_B \sum_{x=B,B} W_{\text{eff};x}^{* \frac{\pi_{\lambda}/\pi_x}}$, $W_{\text{eff};i}(\lambda) = \partial W_{\text{eff}}/\partial \pi_i(\lambda)$; $G$’s come from field-dependent Kähler potentials. The SUSY breaking effect is specified by $(\mu_L^2 + \mu_R^2)|\pi_i|^2$ through $M^2$ because of $\pi_i \neq 0$.

Without knowing the details of solutions to these equations, we can find that

$$|\frac{\pi_{\alpha}}{\pi_i}|^2 = 1 + \frac{M^2}{G_SW_{\text{eff};\lambda}^{* \frac{\pi_{\lambda}}{\pi_i}} (1-\alpha) + \frac{\beta X}{\Lambda} |\pi_{\alpha}|^2} |\pi_{\alpha}|^2 + \beta X,$$

which cannot be satisfied by $\pi_{\alpha} \neq 0$. In fact, $\pi_{\alpha} = 0$ is a solution to this problem, leading to $|\pi_{\alpha}| = |\pi|$. Then, you can see the emergence of the vectorial $SU(N_c)$ symmetry.

IV. SYMMETRY BREAKING

Using the input of $|\pi_{i=1\sim N_c}| \equiv \Lambda_B^2 \sim \Lambda^2$ just obtained, we reach the solutions given by $|\pi_B| = |\pi_B| \equiv \Lambda_B^N_c \sim \Lambda^N_c$, $|\pi_{i=N_c+1\sim N_f}| = \epsilon |\pi_{i=N_c}|$ and $|\pi_{\lambda}| \sim \epsilon^{-1/2} \lambda - \epsilon \Lambda^3$. Notice that $\pi_i$ $(i = N_c+1 \sim N_f)$ and $\pi_{\lambda}$ accompany the factor $\epsilon$. This parameter $\epsilon$, defined to be $|1-z|$, measures the SUSY breaking effect. So, taking the SUSY limit with $\epsilon \rightarrow 0$, we reach the SUSY vacuum specified by these VEV’s. The solutions clearly show the presence of vectorial $SU(N_c)$ symmetry and chiral $SU(N_f - N_c)$ symmetry. The resulting breaking pattern is described by $G = SU(N_f)L \times SU(N_f)_R \times U(1)_V \times U(1)_A$ down to $H = SU(N_c)L \times SU(N_f = N_c)_L \times SU(N_f - N_c)_R \times U(1)_V \times U(1)_A$.

We find consistent anomaly-matching property due to the emergence of the Nambu-Goldstone superfields associated with $G \rightarrow H$, where massless bosons responsible for the anomalies of the broken part, $G/H$, and massless fermions for those of the unbroken part, $H$. Therefore, the anomaly-matching is a purely dynamical consequence. We have further checked that our superpotential is consistent with holomorphic decoupling and instanton calculus for SQCD with $N_f = N_c$ reproduced by massive quarks with flavors of $SU(N_f - N_c)$. The detailed description can be found in the literature [1].

V. SUMMARY

Dynamical breakdown of chiral symmetries are shown to be determined by the effective superpotential: $W_{\text{eff}} = S \ln \left[ \frac{\sin N_f^3}{N_f^3} \right] (f(Z) N_f^3 - N_f^3) + f(Z)$ dynamically determined to be $(1 - Z)^\rho$ ($\rho > 0$), It will be realized at least in SQCD with $N_c + 2 \leq N_f \leq 3N_c/2$. This superpotential exhibits

1. holomorphic decoupling property,

2. spontaneously breakdown of chiral $SU(N_c)$ symmetry and restoration of chiral $SU(N_f - N_c)$ symmetry described by $SU(N_f)L \times SU(N_f)_R \times U(1)_V \times U(1)_A \rightarrow SU(N_c)L \times SU(N_f = N_c)_L \times SU(N_f - N_c)_R \times U(1)_V \times U(1)_A$.

3. consistent anomaly-matching property due to the emergence of the Nambu-Goldstone superfields, and

4. correct vacuum structure for $N_f = N_c$ reproduced by instanton contributions when all quarks with flavors of $SU(N_f - N_c)$ become massive.

In this end, we have two phases in SQCD: one with chiral $SU(N_f)$ symmetry for “magnetic” quarks and the other with spontaneously broken chiral $SU(N_f)$ symmetry for the Nambu-Goldstone superfields. This situation can be compared with the case in the ordinary QCD with two flavors: one with proton and neutron and the other with pions.

Finally, I mention related three works here, which are characterized by

- Dynamical evaluations of condensates [3],
- Instable SUSY vacuum in the "magnetic" phase [4],
- Slightly different effective superpotential in the "electric" phase [2].

All these works indicate spontaneous chiral symmetry breaking in the “electric” phase.

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