Enhanced entropy production in heat-flux-driven plasma sheath

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The plasma sheath sets a stage for a strongly nonlinear coupling of the thermal, kinetic, and electric energies of plasma in a non-equilibrium, open environment. The pressure, velocity, and electrostatic potential profiles depend strongly on the boundary condition given on the internal side (pre-sheath). By controlling the boundary values of the heat flux and the ion Mach number, we solve a set of equations for the ion temperature and the electrostatic potential. The boundary values of the ion velocity and the electrostatic potential vary due to the change of the boundary ion temperature. When the heat flux exceeds a threshold value (determined by the ion Mach number), the temperature contrast is enhanced, resulting in a large entropy production.

Keywords: plasma sheath, thermal diffusion, flux-driven system, maximum entropy production

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A non-equilibrium plasma structure called sheath forms when a plasma contacts with a material wall which absorbs charged particles.\(^\text{[12]}\) Studies of non-equilibrium thermodynamics have focused on the entropy production (EP) as a determinant of self-organized structures (see, e.g., Refs. \(^\text{[3–6]}\)). In plasma physics, works on bifurcations in thermodynamical models for self-organized structures have been developed.\(^\text{[7–10]}\) Consequences of the bifurcation property (maximization or minimization of EP) changes depending on the driv-eristics have been developed \(^\text{[7–10]}\). Consequences of the bifurcation property (maximization or minimization of EP) changes depending on the driv-
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\[ T = \sqrt{e_0 T_0/n_0 e^2}, \]
\[ \lambda_0 = \sqrt{e_0 T_0/n_0 e^2}, \]
\[ \text{the Debye length} \lambda_0 = \sqrt{e_0 T_0/n_0 e^2}, \]
\[ \text{and the thermal diffusivity} \chi = \lambda_0 c_s. \]

We obtain the mass conservation law \( \nu u = M \) from the equation of continuity where \( M \) is the ion inflow velocity.

The equation of motion (Bernoulli’s law), with the expression of enthalpy for ideal gas \( h = (c_v + 1)T \), leads to

\[ u = M \left[ 1 - 2\phi M^2 - \frac{2(c_v + 1)}{M^2} (T - T_{in}) \right]^{1/2}, \]

and the Poisson equation for the electrostatic potential \( \varphi \) is

\[ \frac{d^2 \varphi}{dx^2} = e^x - \frac{M}{u}, \]

where we substitute the expression \( \text{2} \) to \( u \).

In the limit to the thermal diffusion without ion flows, we obtain a linear temperature profile \( (d^2 T/dx^2 = 0) \) from the equation \( \text{3} \) and compare the results to the linear profile. We note that in the opposite limit (without thermal diffusion), the equation \( \text{3} \) leads to the adiabatic relation \( T_n^{1/c_v} = \text{const} \). This relation enables us to write the equation \( \text{4} \) by the Sadgeev potential.

We consider steady states on a one-dimensional system. We normalize the ion density \( n \) by the density at the internal boundary \( n_0 \), the ion velocity \( u \) by the ion sound speed without ion temperature \( c_s = \sqrt{T_0/e} \) (\( m \) is the ion mass), the electrostatic potential \( \varphi \) by the characteristic potential \( T_0/e \), the ion temperature \( T \) by the electron temperature \( Te \), the coordinate variable \( x \) by the Debye length \( \lambda_0 = \sqrt{e_0 T_0/n_0 e^2} \), and the thermal diffusivity \( \chi = \lambda_0 c_s \).

The boundary condition of heat flux and temperature at the boundary is controlled. In a fusion device, hot plasmas bring large heat fluxes to a sheath. In this work, we study the response of a sheath to a heat transport. We consider the thermal energy in addition to the kinetic and electric energies. Moreover, we introduce an irreversible heat flux and solve a flux-driven system.

For simplicity, we assume that the electron density \( n_e \) obeys the Boltzmann distribution with a constant temperature \( T_e \) \( (n_e = n_0 \exp(\varphi/T_e)) \) and that the densities of ions and electrons are equal at the internal (pre-sheath) boundary of the sheath. \( n_0 \) denotes the density at the boundary, and we set the basis of the electrostatic potential \( \varphi \) at the boundary. We also assume that the ion heat flux \( \mathbf{F} \) obeys Fourier’s law \( \mathbf{F} = -c_v \chi n_0 V T \) with the heat capacity at a constant volume \( c_v \) and the thermal diffusivity \( \chi \). The equation evolution for the ion temperature \( T \) is

\[ c_v n_0 \left( \frac{dT}{dt} + u \cdot \nabla T \right) + \nu (\nabla \cdot u) = -\nabla \cdot \mathbf{F}. \]

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\[ u = M \left[ 1 - 2\phi M^2 - \frac{2(c_v + 1)}{M^2} (T - T_{in}) \right]^{1/2}, \]

where \( T_{in} \) is the ion temperature at the internal boundary. The equation \( \text{1} \) leads to

\[ \chi \frac{d^2 \varphi}{dx^2} - M \frac{dT}{dx} - M \frac{d}{dx} \frac{d (\ln u)}{dx} \frac{T}{T_{in}} = 0 \]

and the Poisson equation for the electrostatic potential \( \varphi \) is

\[ \frac{d^2 \varphi}{dx^2} = e^x - \frac{M}{u}, \]

where we substitute the expression \( \text{2} \) to \( u \).

In the limit to the thermal diffusion without ion flows, we obtain a linear temperature profile \( (d^2 T/dx^2 = 0) \) from the equation \( \text{3} \) and compare the results to the linear profile. We note that in the opposite limit (without thermal diffusion), the equation \( \text{3} \) leads to the adiabatic relation \( T_n^{1/c_v} = \text{const} \). This relation enables us to write the equation \( \text{4} \) by the Sadgeev potential.

We consider the boundary-value problem of the equations \( \text{3} - \text{4} \) on a one-dimensional space \([0, L]\), where \( x = 0 \) is the internal edge and \( x = L \) is the wall (here we put \( L = 10 \)). At the wall boundary, we fix the temperature to a value \( T(L) = T_w \) (we put \( T_w = 0.1 \)). At the internal boundary, we control the heat flux \( F_{in} \), and the temperature \( T_{in} \) is free to vary. For the boundary value of the electrostatic potential, we assume the floating potential relation \( \text{1} \)

\[ \varphi(L) = \varphi_w := -\frac{1}{2} \ln \left( \frac{m}{2\pi n_0 c_s} \right) + \ln M. \]

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Here we use \( m \approx 1.67 \times 10^{-27} \) kg (proton mass) and \( m_e \approx 9.11 \times 10^{-31} \) kg which lead to \( \varphi_w - \ln M = -2.84 \).

We use two control parameters: the boundary values of the heat flux and the ion Mach number defined by

\[
M = \frac{M}{\sqrt{1 + \gamma F_{in}}}
\]

(\( \gamma = 1 + 1/c_v \) is the heat ratio). We solve the equations \([3]-[4]\) iteratively with fixing the values of \( F_{in} \) and \( M \). Solving the equation \([3]\) changes \( T_{in} \). Hence we modify the values of \( M \) and \( \varphi_w \) according to the equations \([6]\) and \([5]\).

We present the results in Figures \([1]-[3]\) (here we put \( c_v = 1/2 \) and \( \chi = 10 \)). Figure 1 shows the temperature profile with \( M = 1 \) and \( F_{in} = 0.14 \). We observe that the temperature contrast between boundaries is larger than that of the linear profile (thermal diffusion without ion flow). Figure 2 shows the temperature profile with \( M = 0.85 \) and \( F_{in} = 0.08 \). In this case, we observe that the temperature contrast is smaller than that of the linear profile. The transition from a smaller temperature contrast to larger one occurs depending on the boundary values of the heat flux \( F_{in} \) and the ion Mach number \( M \). In Figure 3 we present the differences between the temperature at the internal edge \( T_{in} \) and that of the linear profile \( T_{diff} \) (\( \Delta T = T_{in} - T_{diff} \)). The dashed line shows points where they coincide (\( \Delta T = 0 \)).

We observe \( \Delta T < 0 \) under the line (small \( M \) and \( F_{in} \)) and \( \Delta T > 0 \) over the line (large \( M \) and \( F_{in} \)).

The analyses of thermodynamical models \([7],[10]\) elucidate that, in flux-driven systems, structures blocking heat transport cause a transition to larger temperature contrast state (maximization of EP) and structures promoting heat transport cause a transition to smaller temperature contrast state (minimization of EP). The observation obtained here indicates that the response of the sheath to heat transport changes from the latter type to the former type depending on the amount of the heat flux.

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