Harms and benefits from social imitation

František Slanina

Institute of Physics, Academy of Sciences of the Czech Republic,
Na Slovance 2, CZ-18221 Praha, Czech Republic
e-mail: slanina@fzu.cz

Abstract

We study the role of imitation within a model of economics with adaptive agents. The basic ingredients are those of the Minority Game. We add the possibility of local information exchange and imitation of the neighbour’s strategy. Imitators should pay a fee to the imitated. Connected groups are formed, which act as if they were single players. Coherent spatial areas of rich and poor agents result, leading to the decrease of local social tensions. Size and stability of these areas depends on the parameters of the model. Global performance measured by the attendance volatility is optimised at certain value of the imitation probability. The social tensions are suppressed for large imitation probability, but due to the price paid by the imitators the requirements of high global effectivity and low social tensions are in conflict, as well as the requirements of low global and low local wealth differences.

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1 Introduction

The Minority Game introduced by Challet and Zhang [1,2] following the earlier ideas of B. Arthur [3] became in recent years a playing ground for studying various aspects of the economic systems.

In the Minority Game (MG) we have \( N \) players who choose repeatedly between two options and compete to be in the minority group. This is the idealisation of various situations, where the competition for limited resources leads to intrinsic frustration. One can think for example of cars choosing between two alternative routes or a speculator who tries to earn money by buying and selling shares in such a manner that the majority takes the opposite action than herself.
Let us recall some well-known facts about the MG. The players share a public information, saying what were the outcomes of the game in past $M$ rounds. The players interact only through this information. Therefore, the system has a “mean-field” character, in the sense that no short-range interactions exist.

The self-organization is achieved by allowing players to have several strategies and choose among them the strategy which seems to be the best one. This feature leads to decrease of the fluctuations of attendance below its random coin-tossing value, thus increasing the global effectivity of the system. It was found that the relevant parameter is $\alpha = 2^M/N$ and the maximum effectivity is reached for $\alpha = \alpha_c \approx 0.34$ [2,4,5] and the properties of this phase transition are thoroughly studied using the methods developed in the theory of neural networks [6–8].

More complete account of the current state of the standard MG and its ramifications is given in other contributions in these Proceedings [9]. We would like to stress especially the attempts to go back to the economic motivations of MG and model the market mechanisms [10–13].

The observation that the crowded (low $\alpha$) phase exhibits low global effectivity bears an important hint. Indeed, if we start with the crowded phase, we can improve the performance by grouping the agents together. This mechanism may bring about the condensation of individual investors around consulting companies and investment funds, which is the behaviour found in real life.

Indeed, an individual investor who sees that she is all the time behind her neighbors may feel tempted to refrain from her own initiative and transfer the burden of decisions to more successful (more wealthy) individuals. That is what we will call imitation. The temptation for imitation in the population will be quantified by a parameter $p \in [0,1]$. Of course, an agent, who is otherwise prone to imitation, will not imitate, if she has larger wealth and therefore is better off than her neighbors. So, there are be two questions to be positively answered if the imitation is to occur: Has the agent natural tendency to imitation? Has any of her neighbors larger wealth?

It is also natural to suppose that the decision-maker, or the imitated individual, will use (or misuse) her position to require a fee from those on which behalf her acts. Therefore, the imitators will pay a commission $\varepsilon$ to the imitated. As we will see, the value of the commission has important consequences for the behavior of the agents.

We introduced recently [14] the possibility of local interactions into the standard MG. In this contribution we further analyse the properties of social structures emerging from the local information exchange. When doing so we go beyond the mean-field character of the usual MG. Related works were already done, either assuming that the global information is fully replaced by a
local one [15] or using the MG scheme for evolving the Kauffmans’s Boolean networks [16] to the critical state [17].

In our variant of the MG the local information is used to enable the players to decide, whether they want to use their own strategies or imitate their neighbours. Indeed, it is quite common that people do not invest individually, but rely on an advice from specialised agencies, or simply follow the trend they perceive in their information neighbourhood. In so doing, the individuals coalesce into groups, which act as single players. In the framework of Minority Game, we will study the social structure induced by the occurrence of these groups. It should be expected that this will lead to increase in the global performance in the crowded (small $\alpha$) phase. This is indeed confirmed by the simulations.

2 Minority Game on a chain with allowed imitation

We introduce the possibility of local information exchange in our variant of the Minority Game. In analogy to the metabolic pathways in living organisms we can imagine a kind of “information metabolism” in work within the economic system. Information flow along the edges of certain information network. The study of the geometry of graphs describing these information networks is now a scientific field on its own [18–20]. Within the framework of MG a linear chain [15] and random network with fixed connectivity $K$ [17] was already investigated in different contexts.

Here we take the simplest possible choice of a linear chain with one-directional nearest-neighbour connections. Each player can obtain the information only from her left-hand neighbour, namely about her neighbour’s wealth.

There will be two conditions needed for a player to imitate her neighbour. First, the player should have internal disposition for being an imitator. We simplify the variety of risk-aversion levels by postulating only two types of players. Each player has a label $\tilde{l} \in \{1,0\}$ indicating, whether the player is a potential imitator ($\tilde{l} = 1$) or always a leader ($\tilde{l} = 0$). At the beginning we take each of the players and attribute her the label 1 with probability $p$ and label 0 with probability $1 - p$. We also allow swapping between the two types of behaviour, at a constant rate. The labels can change at each step with probabilities $p_1$ ($1 \rightarrow 0$) and $p_2$ ($0 \rightarrow 1$). We choose always $p = p_2/(p_1 + p_2)$, so that the average density of potential imitators does not change in time.

The second condition for the player of the type 1 to actually imitate in the current step is that her neighbour has larger accumulated wealth than the player itself. We suppose that the player does not know what are the strate-
gies of her neighbour, but if she observes that the neighbour’s behaviour is more profitable than her own strategy, she relegates the decision to the neighbour and takes the same action. The player of the type 0 will never imitate. Therefore, she will always look only at her $S$ strategies and choose the best estimate from them.

The above rules are formalized as follows. We have an odd number $N$ of players. Each player has $S = 2$ strategies, denoted $s_j \in \{1, 2\}$. The two possible actions a player can take are 0 and 1. The winning action is 1 if most players took 0 and vice versa. The members of the winning side receive 1 point, the loosing side 0 points. The players know the last $M$ outcomes of the game. This information is arranged into the $M$-bit string $\mu \in \{0, 1\}^M$. The strategies are tables attributing to each of $2^M$ possible strings $\mu$ the action $a_{j,\mu}^j$ the player $j$ takes, if she chooses the strategy $s_j$. The scores $U_{j,s}$ of the strategies are updated according to the minority rule

$$U_{j,s}(t+1) = U_{j,s}(t) + 1 - \delta(a_{j,s} - \theta(\sum_i a_i(t) - N/2))$$ (1)

where $a_j(t)$ is the action the player $j$ takes at time $t$.

The potential imitators will copy the action from their more successful neighbours. Let $W_j$ be the wealth of the $j$-th player and the variables $l_j$ describe the actual state of imitation, in analogy with the labels $\tilde{l}_j$ describing potential state of imitation. We can write $l_j = \tilde{l}_j \theta(W_{j-1} - W_j)$, with $\theta(x) = 1$ for $x > 0$ and 0 otherwise. The actions of the players are

$$a_j = l_j a_{j-1} + (1 - l_j) a_{j,s_M} .$$ (2)

We also suppose that the imitation is not for free. The player who imitates passes a small fraction $\varepsilon$ of its wealth increase to the imitated player. This rule accounts for the price of information. Then, we update the wealth of players iteratively,

$$\Delta W_j(t) = (1 - \varepsilon l_j)(\varepsilon l_{j+1} \Delta W_{j+1}(t) + 1 - \delta(a_j - \theta(\sum_i a_i(t) - N/2)))$$ (3)

where $\Delta W_j(t) = W_j(t+1) - W_j(t)$.

3 Imitation structures

In our simulations we observe that the time evolution of the number of actually imitating players, $N_i = \sum_j l_j$, depends on $p_1$. The time dependence of the
fraction of imitators \( N_i/N \) for several values of \( p_1 \) is shown in Fig. 1. For \( p_1 = 0 \) it increases monotonously until saturation, while for \( p_1 \neq 0 \) it grows toward a local maximum and then decreases and saturates at a value weakly dependent on \( p_1 \), but significantly below the \( p_1 = 0 \) value.

An example of the time evolution of the spatial wealth distribution is given in Fig. 2 for \( p = 0.95 \) and two values of \( p_1 = 5 \cdot 10^{-6} \) and \( p_1 = 0 \). The initially random distribution of wealth among players changes qualitatively during the evolution of the system. Coherent groups of poor and wealthy players are formed. Again, the situation is qualitatively different if we allow the players to switch between potential imitator and leaders. We have shown in the previous work [14] that for \( p_1 = 0 \) the poor groups persist forever. We can see the same behaviour also in Fig. 2 for \( p_1 = 0 \). On the other hand, for \( p_1 \neq 0 \) we observe that large poor groups are unstable and split again into smaller clusters. This leads to lowering of the global wealth differences, as will be analysed in the next section.

4 Globally uniform wealth versus small social tensions

The time averaged attendance fluctuations \( \sigma^2 = \langle (A - N/2)^2 \rangle \) measure the distance from the global optimum. The global effectivity is higher for smaller \( \sigma^2 \). We investigated the influence of the imitation on the global effectivity.

We found that in the crowded phase the system becomes more efficient if
Fig. 2. Example of the evolution of the distribution of wealth among players, for $N = 1001$, $M = 6$, $S = 2$, $\varepsilon = 0.05$, and $p = 0.95$. The upper 5 curves correspond to $p_1 = 5 \cdot 10^{-6}$, while the lower 5 curves have $p_1 = 0$. The time step at which the snapshot is taken is indicated on the right. For each time, the vertical axis indicates the wealth $W_j$ of the $j$-th player.

imitation is allowed ($p > 0$), but there is a local minimum in the dependence of $\sigma^2/N$ on $p$, indicating that there is an optimal level of imitation, beyond which the system starts to perform worse. The results for $N = 1001$ are shown in Fig. 3. We can see that the minimum occurs at smaller values for larger $M$. We can also observe that for longer memories ($M = 7$ in our case) the value of the fluctuations for $p = 1$ is significantly above the value without imitation ($p = 0$), while the value at the minimum still lies below the $p = 0$ value. This implies that moderate imitation can be beneficiary, while exaggerated one can be harmful.

The increase of spatial coherence by creation of poor and wealthy groups can result in decrease of local social tension. To quantify it, we introduce a kind of “utility function” [21] $U(\Delta W)$, which indicates, how much the wealth difference $\Delta W$ is subjectively perceived. We will use the utility function in
Fig. 3. Dependence of the attendance fluctuations on the imitation probability for $p_1 = 0$. The number of players is $N = 1001$ and memory length $M = 5$ (○), $M = 6$ (+), and $M = 7$ (×).

Fig. 4. Relative local tension for $N = 1001$, $M = 6$, $p_1 = 0$ measured by utility function $(\Delta W)^{1/2}$ for commission $\varepsilon = 0.05$ (□), 0.03 (×), and 0.01 (+).

the form $U(x) = x^{1/2}$. Then, the average measure of the local social tension is

$$d_{0.5} = \frac{1}{\langle W \rangle} \left( \sum_{j=1}^{N-1} |W_j - W_{j+1}|^{1/2} \right)^2$$

(4)

where we denoted the average wealth $\langle W \rangle = \frac{1}{N} \sum_{j=1}^{N} W_j$.

The stationary values of the tension for various values of the commission $\varepsilon$ are shown in Fig. 4, for $p_1 = 0$. An important feature of the $p$-dependence is the maximum at certain imitation probability. The maximum becomes more
pronounced for larger commission $\varepsilon$, while for $\varepsilon = 0.01$ it disappears.

This observation has an important consequence. Imagine, we are social experimentalists starting with a system with no information exchange and no imitation. Let us try to lower the social tensions by gradually encouraging the people to buy information from the neighbours and imitate each other. If the cost of the information ($\varepsilon$) is too high, this social strategy would fail, because small increase in imitation would enhance the social tension. Lower social tension must have been achieved by a macroscopic change in the social behaviour: by jumping over the maximum in the function $d_{0.5}(p)$. This may serve as a toy example of how too greedy environment (too costly information) can prevent the system to find a global optimum.

By comparing the Figs. 3 and 4 we can also see that for high $\varepsilon$ optimal performance (minimum $\sigma^2/N$) can be close to maximum in social tensions. Therefore, in greedy environment the requirements of effectivity and social peace are in conflict.

The Fig. 5 shows the growth rate of the average wealth for several values of the switching probability $p_1$. We can see that the growth rate converges to a constant value, which is higher for $p_1 = 0$ and nearly independent of $p_1$ for $p_1 \neq 1$. In all cases we confirm that the average wealth grows linearly with time.

In the Fig. 6 we can see the time evolution of the local tensions for several values of the switching probability $p_1$. We observe that the switching enhances the local tensions. On the other hand, in Fig. 7 we can see the time dependence
Fig. 6. Time evolution of the local tensions, for \( N = 1001, M = 6, S = 2, p = 0.95 \), sample-averaged over 10 independent runs. Different curves (marked by symbols) correspond to different probability \( p_1 = 0 (\times), 5 \cdot 10^{-6} (\Box), 1.5 \cdot 10^{-5} (+) 5 \cdot 10^{-5} (\odot), 5 \cdot 10^{-4} (\bullet), 5 \cdot 10^{-3} (\triangle) \).

Fig. 7. Time evolution of the wealth dispersion, for \( N = 1001, M = 6, S = 2, p = 0.95 \), averaged over 10 independent runs. Different curves (marked by symbols) correspond to different probability \( p_1 = 0 (\times), 5 \cdot 10^{-6} (\Box), 1.5 \cdot 10^{-5} (+) 5 \cdot 10^{-5} (\odot), 5 \cdot 10^{-4} (\bullet), 5 \cdot 10^{-3} (\triangle) \).

of the growth rate in the global wealth dispersion, \( \langle W^2 \rangle - \langle W \rangle^2 \) (by angle brackets we denote the average over all players). There is a clear difference between the cases of \( p_1 = 0 \), where the wealth dispersion grows much more rapidly than \( t^2 \) and \( p_1 \neq 0 \), where the dispersion grows as \( t^2 \), at a rate nearly independent of \( p_1 \).

This means that if we allow switching between potential imitation and leader states, the wealth distribution only re-scales linearly in time (This observation
together with the linear growth if the average wealth suggests that the probability density at time $t$ converge as $P(W, t) = \Phi(W/t)$ where the function $\Phi(x)$ does not depend on time). On the contrary, if we forbid the switching, the poor imitators are frozen forever in their poverty and in the wealth distribution the rich and poor diverge steadily.

However, recalling the discussion of the Figs. 6 and 5 we can see that the requirement of low global wealth dispersion (a “just” world, achieved by enabling the poor imitators switch to leaders and thus become richer) deteriorates both global efficiency (measured now by the wealth growth rate) and, more surprisingly, the local social tensions.

## 5 Conclusions

We investigated the creation of rich and poor spatial domains due to local information exchange, within the framework of the Minority Game (MG). Coherent spatial areas of rich and poor agents emerge. Several macroscopic conflicts of interest are observed in our model.

(1) We found that the effect of imitation leads to increased effectivity in the crowded phase of MG. The price paid for the information needed to imitation leads to the conflict between effectivity and local social tensions. High information cost also prevents the system from coming to the state of lower social tensions by gradual increase of the imitation probability.

(2) We allow for switching between imitation and non-imitation (leader) states. Such a switching makes the global wealth differences smaller, but increases the local social tensions.

The creation of coherent areas of poor and rich agents leads to decrease in the local social tensions, but only if $p$ is sufficiently close to 1. The lowest value of the social tension is reached at $p = 1$, but for such a value the global effectivity is significantly lower than its optimum value. Therefore, we observe a conflict of local interests (maximisation of social tension) with global performance (maximisation of attendance fluctuations).

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