Fractal modeling of skew profiles and application

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Abstract. Some machined profiles such as the plateau honing profiles are skew and irregular. In this paper, a new fractal model based on sinc(x) functions is put forwards for skew Unit event. And then, the model of whole skew profile is built and the optimal fractal D of max oil-storage is studied for plateau honing profiles. The experiment result shows that the practical D is around the theoretical D.

1. Introduction
Rough profiles are usually random, multi-scale and self-affine. It is observed that if the profile of a surface is repeatedly magnified, more and more roughness keeps appearing and the profile is never smooth. The Weistrass-Mandelbrot (W-M) function is often used for characterization of engineering profiles, and there is a striking resemblance in the probability distributions of the magnified and the original surface.

The numerical simulation shows that the skew parameter S_k of W-M tends to be zero. So the profiles produced by W-M function are symmetric. However some machined profiles are obviously skew and multi-scale, and also play an important role in engineering surfaces. A new fractal model for skew profiles is put forward and its application is given in this paper.

2. Fractal model based on sinc(x)
Euclidean geometry describes ordered objects such as points, curves, surfaces and cubes using dimensions of 0, 1, 2 and 3 respectively. However, a lot of objects found in nature (Mandelbrot, 1982), rough surfaces for example, appear disordered and irregular for which the measures of length, area and volume are scale dependent. Fractal geometry was founded by Mandelbrot (1967) when he showed that for decreasing unit of measurement, the length of a natural coastline does not converge, but increases monotonically.

The interesting mathematical properties can be formulated by W-M function. This function has a fractal dimension, D, and is given as:

$$z(x) = G^{(1-D)} \sum_{n=1} \cos \frac{2\pi n^\gamma x}{r^{(2-D)\mu}} ; 1 < D < 2; \gamma > 1$$

Where G is a scaling constant and the frequency modes correspond to the reciprocal of the wavelength of roughness as $\gamma^\gamma = 1/L$. The parameter $n$ corresponds to the low cut-off frequency of the profile, dependent on the length $L$ of the sample by the relation, $\gamma^\gamma = 1/L$.

The power spectrum of the W-M function is given as:
\[ S(\omega) = \frac{G}{2\ln\gamma} \cdot \frac{1}{\omega^{(1+2D)}} \]  

(2)

The power spectrum is the most fundamental quality of such multiscale function.

The skewness \( S_i \) of W-M function in a limited length is tending to be 0 [2]:

\[ S_i = \frac{1}{\sigma^2} \cdot \lim_{L \to \infty} \frac{1}{L} \int z^3(x)dx \]  

(3)

In some cases, there are a lot of skew profiles such as plateau honing profiles. Their power spectrums of measured date also obey the power law [2]. So a new model for skew and fractal Unit event[3] based on sinc(\( x \)) is put forward as follows:

\[ z_u(x) = -G^{D-1} \sum_{n=n_0}^{\infty} \sin(2\pi^n x) \cdot \frac{1}{2\pi^n x} \cdot \frac{1}{\gamma^{(2-D)n}} \]  

(4)

Some properties can be deduced from formula (4) [2]:

- \( z_u(\gamma x) = \gamma^{2-D} z_u(x) \)

- \( S_\gamma(\omega) = G^{2(D-1)} \cdot \frac{1}{\omega^{(2-D)}} \)

- \( S_K < 0 \)

So, \( z_u(x) \) is self-affine, its power spectrums obey power law and its Unit event is negative skew. It is concluded that \( z_u(x) \) is fractal and especially suitable for modeling skew profiles.

3. Application of sinc(\( x \)) model to plateau honing profiles

It is believed that the cylinder surface micro geometry considerably influences the functional engine properties. First of all, the time necessary for running-in, oil consumption; scuffing resistance etc. The inner surfaces are obtained through boring process and plateau honing process. The profiles produced by the boring process are respectively deep and symmetric and the final profiles by plateau honing are peak less and negative \( S_J \), as shown in Figure 1[2].

3.1 Model of plateau honing profiles

Figure 1 show that the plateau honing profiles are consisted of a series of Unit events which are distributed randomly; The Unit events can be described by \( z_u(x) \) fractal model, and their distribution can be formulated by pulse series \( \delta_u(x) \). The combination of \( \delta_u(x) \) with \( z_u(x) \) represents the profiles \( z_p(x) \):

\[ z_p(x) = \delta_u(x) \ast z_u(x) = \sum_{m=0}^{\infty} \alpha_m \delta(x-d_m) \ast \sum_{n=n_0}^{\infty} G^{D-1} \frac{\sin(2\pi^n x)}{2\pi^n x} \cdot \frac{1}{\gamma^{(2-D)n}} \]  

(5)

Where \( x \) is scanning coordinate, \( \alpha_m \) is amplitude of Unit events, and \( d_m \) is distance between Unit events. They all obey power law.

According to convolution theorem, the power spectrum of formula (5) remains to obey power law and the profiles by formula (5) remains fractal. The \( z_p(x) \) is defined as fractal model for skew profiles.

3.2 Study on dimension D for max oil reservoir
The main function of negative skew profile is to save oil for better lubrication; the unit model for oil reservoir is shown in Figure 2. It is similar to \( \text{sinc}(x) \) after analysis of measured datum [2]. Suppose that the largest area is \( a_m \), the other area is \( a \), the relationship between \( a_m \) and \( a \) is as follows:

\[
 f(a) = \frac{D}{2} \frac{a_m^2}{a^{1+D/2}}
\]  

From figure 2, suppose that the distance between peaks of \( \text{sinc}(x) \) is \( l \), the height of \( \text{sinc}(x) \) is \( h \), \( 2 \ell = \frac{3}{2\gamma} a_m \), \( h = \frac{G}{\gamma} a_m \) can be defined from \( \text{sinc}(x) \) respectively. There is a relationship between area \( a \) and distance \( l \) as \( a = 2l \times 2l \), and \( \frac{a_m}{\gamma} = \frac{3}{2\sqrt{a}} \).

Suppose that the vertical section area \( a_h \) in Figure 2 is defined to be the ability of oil-reservoir. It is calculated approximately as triangle, that is \( a_h = \frac{1}{2} \times 2l \times h \). The total area of all the Unit events \( a_{\text{sum}} \) can now be found to be:

\[
 a_{\text{sum}} = \int a_h f(a) da = \frac{DG^{D-1}}{3^D a_m^{2+D}} a_m^2
\]

From formula (7), it is necessary for \( D \) to be less then 1.5, if \( a_{\text{sum}} \) is greater than 0. Serious experiments show that the practical dimension \( D \) of measured datum is within of 1.5.

4. Conclusion
Rough profiles are random and fractal. They are normally characterized by W-M function. For a class of skew profiles, the new fractal \( \text{sinc}(x) \) model in this paper is more accurate and concise from the view of point of skewness. The structure, even in nano-scale can be predicted and their oil-reservoir’s distribution can be logically obtained. The computation with \( \text{sinc}(x) \) model establishes a fact that there is better oil-reservoir when dimension \( D \) is within 1.5.

References
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