Tooth Surface Contact Fatigue Analysis of Cycloidal Wheel of RV Reducer

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Abstract. The cycloidal wheel is a key component of the RV reducer, and its reliability directly affects the overall performance of the RV reducer. Based on the stress-intensity distribution interference theory, this paper establishes a mathematical calculation model for the reliability of the cycloidal wheel. Using software to write the program, analyze the distribution type of contact force and contact fatigue strength of cycloidal wheel, and then use software based on theoretical calculation, and calculate the reliability based on Monte Carlo method. Then through the parameter sensitivity analysis, it is concluded that the degree of influence of the parameters that change with the actual operating factors on the reliability during the actual operation provides a basis for the key calculation of the reliability analysis of the tooth surface contact fatigue of the cycloidal gear.

Introduction

At the end of the last century, domestic scholars have gradually increased the research on RV reducers, but their research on reliability is still lacking. In the research of the reliability of RV reducer, the research of Guan Tianmin, Lei Lei and others has a good reference value[1]. Chen Wenhua, Chen Jianjun, Han Linshan and others also analyzed the fatigue strength reliability of the cycloidal wheel, and their research results laid the foundation for the reliability analysis of the entire transmission system [2-4].

During the transmission process of the reducer, the bending and fracture of the gear teeth and the pitting corrosion and wear of the tooth surface are the main failure modes. The fatigue pitting of the tooth surface is the main failure mode of the gear, which is the main cause of this failure [5]. It is the insufficient contact strength of the tooth surface. Therefore, in the design or strength inspection of the cycloidal wheel, the contact fatigue strength reliability of the tooth surface is regarded as an important index.

Stress-strength Interference Model Theory

The contact stress applied to the cycloidal wheel is expressed by \( \sigma_H \). The contact fatigue strength that the cycloidal wheel can withstand by \( \sigma_{HS} \). In mechanical design, the contact stress \( \sigma_H \) and the contact fatigue strength \( \sigma_{HS} \) are random variables and have the same dimension. Therefore, the probability curves of \( \sigma_H \) and \( \sigma_{HS} \) can be expressed in the same coordinate system, and then the stress-strength interference model is established as shown in Fig. 1[6].

As shown in Figure1, the intersection of the probability curves is the cycloidal failure zone, called the interference zone. If the contact fatigue strength of the cycloidal wheel is less than the contact stress, the cycloidal wheel cannot perform the prescribed function resulting in failure. In order for the cycloidal wheel to work reliably within the specified time, it must satisfy:

\[
Z = \frac{\sigma_{HS}}{\sigma_H} \geq 1
\]

Both the contact stress \( \sigma_H \) and the contact fatigue strength \( \sigma_{HS} \) are independent variables.
Therefore, $Z$ is also a random variable. If the reliability of the cycloidal wheel is $R$, then

$$R = P(Z \geq 1)$$

(2)

According to the literature [7], introducing the calculation coefficient, the contact stress of the cycloidal tooth surface is:

$$\sigma_H = \sigma_{H0} \sqrt{KK_A K_V K_H}$$

(3)

Coefficient of variation:

$$C_\sigma = \left[ \frac{1}{2} (C_K^2 + C_{KA}^2 + C_{KV}^2 + C_{KH}^2) \right]^2$$

(4)

In the fatigue analysis, $\sigma_{H0}$, $K_H$, $K_V$, $K_A$, and $K$ are considered to be normally distributed. The specific parameters are shown in Table 1 [8-9]. According to the distribution law of each parameter in the tooth surface contact stress formula (3) of the cycloidal wheel, the probability distribution type of the contact stress $\sigma_H$ is simulated based on the Monte Carlo method. The simulated distribution and fitting map are shown in Fig. 2. It can be seen from Fig.2 that the contact stress follows a lognormal distribution.

| Parameter symbol | Meaning of the parameter | Parameter values | Coefficient of variation | Parameter values |
|------------------|--------------------------|------------------|--------------------------|------------------|
| $K$              | Calculation coefficient   | 1                | $C_K$                    | 0.13             |
| $K_V$            | coefficient of performance | 1.004            | $C_{KV}$                 | 0.001            |
| $K_A$            | Dynamic load factor       | 1.5              | $C_{KA}$                 | 0                |
| $K_H$            | Coefficient of variation  | 1.2              | $C_{KH}$                 | 0.05             |
| $\sigma_{H0}$    | initial value of contact stress | 830MPa          | $C_\sigma$               | 0.051            |

Table 1. Tooth surface contact stress parameter table of cycloidal system.

Cycloidal Tooth Surface Contact Fatigue Strength Distribution Type

The formula of the tooth surface contact fatigue strength of the cycloidal gear is:

$$\sigma_{HS} = \sigma_{H\text{lim}} Z_N Z_L Z_V Z_K Z_W Z_X$$

(5)

Coefficient of variation:

$$C_{HS} = \left[ C_{\sigma H\text{lim}}^2 + C_{Z_N}^2 + C_{Z_L}^2 + C_{Z_V}^2 + C_{Z_K}^2 + C_{Z_W}^2 + C_{Z_X}^2 \right]^{1/2}$$

(6)
Table 2. Parameters related to tooth surface contact fatigue strength.

| Parameter symbol | Meaning of the parameter          | values     | Coefficient of variation | Parameter values |
|------------------|-----------------------------------|------------|--------------------------|------------------|
| $Z_N$            | Life factor                       | 1.02       | $C_{Z_N}$                | 0.04             |
| $Z_L$            | Lubricant coefficient             | 0.94       | $C_{Z_L}$                | 0.025            |
| $Z_V$            | Velocity coefficient              | 0.96       | $C_{Z_V}$                | 0.02             |
| $Z_R$            | Roughness coefficient             | 0.95       | $C_{Z_R}$                | 0.02             |
| $Z_W$            | Working hardening coefficient     | 1          | $C_{Z_W}$                | 0.02             |
| $Z_X$            | Size factor                       | 1          | $C_{Z_X}$                | 0                |
| $\sigma_{H_{lim}}$ | Test gear contact fatigue limit  | 1650MPa    | $C_{\sigma H_{lim}}$     | 0.11             |

In the fatigue analysis, it is considered that $\sigma_{H_{lim}}$ obeys the lognormal distribution, and $Z_N, Z_L, Z_V, Z_R, Z_W$ and $Z_X$ all obey the normal distribution. The specific parameters are shown in Table 2 [8-9]. According to the distribution law of each parameter in the tooth surface contact fatigue strength formula (5) of the cycloidal wheel, the probability distribution of the contact stress is simulated based on the Monte Carlo method.

The simulated distribution and fitting map are shown in Fig.3. It can be seen from Fig. 3 that the contact fatigue strength obeys a lognormal distribution.

![Figure 2. Contact stress simulation distribution and fitting.](image1)

![Figure 3. Contact fatigue intensity simulation distribution and fitting map.](image2)

Reliability Calculation

It is known that the random variable cycloidal contact stress $\sigma_H$ and the contact fatigue strength $\sigma_{HS}$ both obey the lognormal distribution, then according to the literature [7]:

$$u_R = \frac{\mu_{lnZ} - \mu_{lnH}}{\sqrt{S_{lnZ}^2 + S_{lnH}^2}} \approx \frac{\ln \mu_{HS} - \ln \mu_H}{\sqrt{C_{HS}^2 + C_H^2}}$$

Available from the normal distribution table:

$$R = \Phi(u_R)$$

In this paper, the random distribution type is fitted according to the contact stress and contact fatigue strength formula and the distribution types of related parameters. By combining theoretical calculations, the probability that the stress obtained by Monte Carlo simulation is less than the intensity is the reliability of the relevant components [10]. The specific process is shown in the Fig.4.
Sensitivity Analysis

During the actual operation, some parameters of the reliability calculation may change with the actual environment, meshing external factors, machining errors and other factors. Therefore, it is necessary to find the factors that have the greatest impact on reliability through parameter sensitivity analysis [11], thus providing a focus for reliability analysis.

Figure 5. Variation of reliability with mean value and variable coefficient of load distribution among gear teeth.

Figure 6. Variation of reliability with fatigue strength and contact stress.
Figure 7. Variation of reliability with mean value and variable coefficient of fatigue strength.

It can be seen that load distribution among gear teeth has a great influence on the reliability under the fatigue reliability analysis from Fig.5. It can be seen that the degree of influence of fatigue strength and contact stress on reliability is relatively close from Fig. 6. And as can be seen from Fig. 7. In terms of fatigue limit, the mean has a greater impact on reliability. And mean value of Coefficient of performance also has a great influence on the reliability from Fig. 8.

Conclusion

This paper establishes a mathematical calculation model for the reliability of the cycloidal wheel based on the stress-intensity distribution interference theory. The distribution type of contact force and contact fatigue strength of the cycloidal wheel is logarithmic normal distribution and the theoretical calculation type of cycloidal reliability is finished based on the theoretical calculation by the Monte Carlo method. It can be found that the theoretical calculations and the Monte Carlo simulation algorithm are mutually validated to improve the accuracy of reliability calculation.

The fatigue strength, the use coefficient and the mean value of the intertooth analysis coefficient have a great influence on the reliability through sensitivity analysis. Especially the mean value of the fatigue strength is the most obvious. The degree of influence of fatigue strength and contact stress on reliability is relatively close. This provides a theory basis for the fatigue reliability analysis of the cycloidal tooth surface contact with emphasis on fatigue strength and contact stress calculation.

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