Intersecting Brane Worlds – A Path to the Standard Model?

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Abstract. In this review we describe the general geometrical framework of brane world constructions in orientifolds of type IIA string theory with D6-branes wrapping 3-cycles in a Calabi-Yau 3-fold. These branes generically intersect in points on the internal space, and the patterns of intersections govern the chiral fermion spectra. We discuss how the open string spectra in intersecting brane models are constructed, how the Standard Model can be embedded, and also how supersymmetry can be realized in this class of string vacua. After the general considerations we specialize the discussion to the case of orbifold backgrounds with intersecting D6-branes and to the quintic Calabi-Yau manifold. Then, we discuss parts of the effective action of intersecting brane world models. Specifically we compute from the Born-Infeld action of the wrapped D-branes the tree-level, D-term scalar potential, which is important for the stability of the considered backgrounds as well as for questions related to supersymmetry breaking. Second, we review the recent computation concerning of gauge coupling unification and also of one-loop gauge threshold corrections in intersecting brane world models. Finally we also discuss some aspects of proton decay in intersecting brane world models.

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1. Introduction

Without any doubt, one important goal of superstring theory is to find an embedding of the Standard Model into a unified description of gravitational and gauge forces. However there are several obstacles on the way that one has to solve in order to achieve this goal:

- How to derive the precise SM spectrum?
- How to determine the precise SM couplings?
- How to break space-time supersymmetry?
- How to fix the values of the moduli?
- How to select the ground state from an (apparent) huge vacuum degeneracy?
- How to describe the cosmological evolution of the universe?

In this review we describe the general geometrical framework of brane world constructions in orientifolds of type II string theory with D-branes wrapping certain homology cycles in a Calabi-Yau 3-fold. The branes generically intersect each other in the internal space, and the patterns of intersections govern the chiral fermion spectra. These so-called intersecting brane worlds \([11-35]\) \(\parallel\) have proven to be a candidate framework of model building which offers excellent opportunity to answer some of the above questions. Namely in these string compactifications the Standard Model particles correspond to open string excitations which are located at the various intersections of the D-branes in the internal 6-dimensional space. At the moment, type IIA intersecting brane world models with D6-branes, which fill the 4-dimensional Minkowski space-time and are wrapped around internal 3-cycles, provide the most promising approach to come as close as possible to the Standard Model.¶ The fermion spectrum is determined by the intersection numbers of the relevant 3-cycles in the internal space, as opposed for instance to the older approaches involving heterotic strings, where the number of generations was given by the Euler characteristic in the simplest case. Indeed it is possible to construct intersecting brane world models with Standard Model gauge group and intersection numbers corresponding to three generations of quarks and leptons.

In a second step, going beyond these topological data, the computation of the effective interactions of the light (open) string modes is of vital importance in order to confront eventually the intersecting brane world models with experiment. In particular, the knowledge of the effective scalar potential \([10, 19]\) is needed to discuss the question of the stability of intersecting brane configurations and also to know the values for the various moduli fields. The moduli dependent gauge coupling constants \([14, 40]\) are essential to get precise informations on the low-energy values of the Standard Model gauge couplings and the possibility of gauge coupling unification \([41]\). The computation of N-point amplitudes is relevant for the effective Yukawa couplings \([15, 42, 43]\), for

\(\parallel\) For some reviews on many details of open string constructions see \([36, 37, 38]\).

¶ In a T-dual mirror description these models correspond to type IIB orientifolds including D9-branes with magnetic F-flux turned on, see also the earlier work by \([39]\).
quartic fermion interactions and flavor changing neutral currents \cite{44, 45, 46, 47} as well as for the proton decay \cite{48} in intersecting brane world models. Finally one also needs the effective Kähler metric of the matter fields for the correct normalization of these fields and also to derive soft supersymmetry breaking parameters \cite{49}.

In this work we will review the main aspects of the construction of intersecting brane world models as well as of the computation of the tree-level scalar potential and the gauge coupling constants, including one-loop threshold corrections. The further plan of the paper is like follows: in the next chapter we will discuss the open string spectrum on intersecting D-branes, starting with local intersecting D-brane constructions in flat space-time, the question of space-time supersymmetry and compactifications on Calabi-Yau spaces, i.e. the embedding of intersecting branes into a compact CY-manifold. Some emphasis is given to the question on the realization of space-time supersymmetry and the related issue of the stability of the D-brane configurations, which follows from the minimization of the associated scalar potential. As specific examples we will discuss in some detail toroidal and orbifold models, and also the quintic Calabi-Yau 3-fold. In the third section we will turn to some phenomenological issues, namely the question of gauge coupling unification. We will also discuss how to get some quantitative information on proton decay in intersecting brane world models.

2. Intersecting Brane World Models

2.1. Intersecting D-branes in flat space-time

A lot of the recent progress in type II string physics was made possible due the discovery of D-branes \cite{50}. D(p)-branes are higher(p)-dimensional topological defects, i.e. hypersurfaces, on which open strings are free to move. They have led to several new insights:

- Non-Abelian gauge bosons arise as open strings on the world volumes $\pi$ of the D-branes \cite{51}. This is the starting point of the brane world models in type II and type I string models.
- Chiral fermions arise as open strings living on the intersections of two D-branes \cite{52}. It follows that the number of families is determined by the intersection numbers between the brane world volumes $\pi_a$ and $\pi_b$:

$$N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b) \equiv \pi_a \circ \pi_b \quad (1)$$

- D-branes correspond to non-trivial gravitational backgrounds; this observation led to the famous correspondence between (boundary) Yang-Mills gauge theories and (bulk) gravitational theories, the AdS/CFT correspondence \cite{53}.

First we consider flat D-branes in Minkowski space $\mathbb{R}^{1,9}$. The simplest D-brane configuration is certainly just one single Dp-brane:
The resulting massless open string spectrum on the D-brane world volume is then given by one $U(1)$ gauge boson plus neutral fermions plus scalar fields which corresponds to a supersymmetric $U(1)$ gauge theory in $p+1$ dimensions with 16 supercharges. The effective gauge field action for the gauge field strength $F$ on the D-brane world volume contains the Dirac-Born-Infeld Lagrangian, originating from the tension of the D-brane, plus a topological Chern-Simons term, which describes the coupling of the D-brane to the Ramond gauge field potential $C_{p+1}$ because of its non-vanishing Ramond charge:

$$S_{\text{eff}} = \int d^{p+1}x \left( L_{\text{DBI}}(g, F, \phi) + L_{\text{CS}}(F, C_{p+1}) \right),$$

$$S_{\text{DBI}} = \tau_p \int d^{p+1}x \sqrt{\det(g_{\mu\nu} + \tau^{-1}F_{\mu\nu})} = \left( \frac{M_{\text{string}}^{p-3}}{g_{\text{string}}} \right) \int d^{p+1}x F_{\mu\nu}^2 + \ldots$$

($\tau$ is the string tension, $\tau_p$ denotes the tension of a Dp-brane, and $\phi$ is the dilaton field.)

Next consider a configuration of a stack of $N+M$ parallel Dp-branes:

The massless open string spectrum leads to a $\mathcal{N} = 4$ supersymmetric $U(N+M)$ gauge theory in $p+1$ dimensions. Next we start to simply rotate $M$ D-branes by certain angles $\Phi_{ab}^I$ in some directions $I$ with respect to the left-over stack of $N$ Dp-branes. In other words we are considering the case of a stack of $N$ Dp-branes intersected by another stack of $M$ Dp-branes:
The open string spectrum on these intersecting branes contains the following fields:

(i) \( \mathcal{N} = 4 \) gauge bosons in adjoint representation of \( U(N) \times U(M) \).

(ii) Massless fermions in the chiral \( (N, \bar{M}) \) representation.

(iii) In general massive scalar fields, again in the \( (N, \bar{M}) \) representation.

The latter two fields originate from open strings stretching from one stack of Dp-branes to the other one. Since the scalar fields are in general massive, such an intersecting D-brane configuration generically breaks all space-time supersymmetries. This supersymmetry breaking manifests itself as the massive/tachyonic scalar ground state with mass:

\[
M_{ab}^2 = \frac{1}{2} \sum_I \Delta \Phi'_{ab} - \max \{ \Delta \Phi'_{ab} \} .
\]  

(\( \Phi'_{ab} \) is the angle between stacks \( a \) and \( b \) in some spatial plane \( I \).) Only if the intersection angles take very special values, some of the scalars become massless, and some part of space-time supersymmetry gets restored. Specifically consider two special flat supersymmetric D6-brane configurations, as shown in the following figure:

Supersymmetry now gets restored for the following choice of angles:

- 2 D6-branes, with common world volume in the 123-directions, being parallel in the 4-5, 6-7 and 8-9 planes:

\[
1/2 \text{ BPS} \ (\mathcal{N} = 4 \ \text{SUSY}): \Phi^1 = \Phi^2 = \Phi^3 = 0 .
\]

- 2 intersecting D6-branes, with common world volume in the 123-directions, and which intersect in 4-5 and 6-7 planes, being parallel in 8-9 plane:
1/4 BPS ($\mathcal{N} = 2$ SUSY): $\Phi^1 + \Phi^2 = 0$, $\Phi^3 = 0$.

- 2 intersecting D6-branes, common in 123-directions, intersecting in 4-5, 6-7 and 8-9 planes:

1/8 BPS ($\mathcal{N} = 1$ SUSY): $\Phi^1 + \Phi^2 + \Phi^3 = 0$.

Now, altering the relative intersection angles by some amount, the open string scalar field will become massive. In case $M_{ab}^2 > 0$ the D-brane configuration is a non-supersymmetric, stable non-BPS state (possible NS tadpoles, which cause further instabilities, will be discussed later). However in case the open string scalar is tachyonic ($M_{ab}^2 < 0$), the 2 different branes are unstable and will recombine into a single, smooth, supersymmetric D-brane which interpolates between the intersecting branes, as can be seen in the next figure:

This process of brane recombination – the so-called tachyonic Higgs effect – can be described by looking at the effective world volume field theory on the intersecting D-branes $\mathbf{53, 56, 57, 58, 59, 60}$. In the world volume field theory the vacuum structure is determined by the D- and F-flatness conditions, which in the brane picture correspond to the geometric calibration conditions. Intriguingly, the Standard Model Higgs effect might be realized by tachyon condensation on intersecting D-branes $\mathbf{6, 10}$, and also in cosmology intersecting brane worlds with tachyon condensation $\mathbf{61, 62, 63, 64, 65, 66}$ have been used to model early universe inflation and a “graceful exit” from the inflationary period.

Let us briefly discuss the tachyonic mass spectrum that arise for two D-branes, which intersect by a single angle $\Phi$. Starting from the BPS configuration, i.e. two parallel D-branes, and then rotating them by an angle $\Phi$, one gets the following open string spectrum

$$M^2 = \left(-\frac{1}{2} - n\right) \frac{|\Phi|}{\pi \alpha'} + n, \quad n = 0, 1, 2, \ldots$$

(5)

for the lowest mass states. If the intersection angle is small, one single tachyon field shows up in the open string spectrum. Hence in this case an effective Yang-Mills field theory with a finite number of fields certainly provides for an appropriate description of the tachyon dynamics. However, when the intersecting angle is growing, more and more string modes will become tachyonic. In particular settings where the intersection angle $\Phi$
is close to $\pi$, which just corresponds to the small angle intersection of a brane-antibrane system, contain a large number of tachyonic modes. For the case $\Phi = \pi$ the infinitely many tachyons become tachyonic momentum states, and the corresponding coincident brane-antibrane pair is a highly unstable non-BPS state, and does not correspond to a perturbative string ground state. Nevertheless, a number of essentially non-perturbative phenomena have been realized on a field theory level, notably brane descent relations \[67, 68, 69\], decay of non-BPS branes \[70, 71\], brane-antibrane annihilation \[67, 72\] and local brane recombination \[73\]. Finally in \[60\] some attempts were made to generalize these effective potentials to cases where branes and antibranes intersect each other at a small angle, i.e. the large angle case of intersecting brane.

Finally at the end of this section we discuss how a local D-brane description of the Standard Model looks like, describing a D-brane engineering of the Standard Model by flat D-branes. Later we have to embed this local D-brane configuration into a compact 6-dimensional space; this will lead to further consistency conditions. A very economic realization of the Standard Model is provided by four stacks of D6-branes in the following way \[7, 8\] (see also \[74\]):

| Stack  | $N$  | $SU$ Group       | Type         |
|--------|------|-----------------|--------------|
| Stack a| $N_a = 3$ | $SU(3)_a \times U(1)_a$ | QCD branes   |
| Stack b| $N_b = 2$ | $SU(2)_b \times U(1)_b$ | weak branes  |
| Stack c| $N_c = 1$ | $U(1)_c$         | right brane  |
| Stack d| $N_d = 1$ | $U(1)_d$         | leptonic brane |

The intersection pattern of the four stacks of D6-branes can be depicted in the next figure. The stack $a$ with $N_a = 3$ denotes the color branes, responsible for the strong QCD forces with gauge group $SU(3)$, the stack $b$ leads to the electro-weak gauge group $SU(2)_L$, and the weak hypercharge gauge group $U(1)_Y$ is a suitable linear combination of all four $U(1)$'s. The left-handed quarks $Q_L$ correspond to massless open string excitations stretched between the stacks $a$ and $b$, and therefore transform in the bifundamental $(3, 2)$ representation under $SU(3) \times SU(2)_L$, and so on for the other matter fields. In order to get the correct hypercharges for all SM matter fields, the other two stacks $c$ and $d$ are needed.
Space-time supersymmetry is preserved if all four stacks of D6-branes mutually preserve the angle conditions among each other.

2.2. Intersecting D-branes in compact spaces

Now we have to embed the intersecting D-branes into a compact space (we are now following the discussion on intersecting branes on Calabi-Yau spaces [19, 25]). Switching from D-branes in non-compact space to compact spaces leads to some important new observations and restrictions which are essential for model building with intersecting D-branes:

- Since the D-branes will be wrapped around compact cycles of the internal space, multiple intersections will now be possible. It follows that the family number will be determined by the topological intersection numbers of the relevant cycles, around which D-branes are wrapped.

- Since the Ramond charges of the D-branes cannot ‘escape’ to infinite, the internal Ramond charges on compact space must cancel (Gauss law). This is the issue of Ramond tadpole cancellation which give some strong restrictions on the allowed D-brane configurations.

- Similarly there is the requirement of cancellation of the internal D-brane tensions, i.e. the forces between the D-branes must be balanced. In terms of string amplitudes, it means that all NS tadpoles must vanish, namely all NS tadpoles of the closed string moduli fields and also of the dilaton field. Absence of these tadpoles means that the potential of those fields is minimized. So this restriction imposes a strong stability problem on non-supersymmetric intersecting D-brane configurations. On the other hand for supersymmetric D-branes it is automatically satisfied after Ramond tadpole cancellation.
In order to meet the two conditions of Ramond and NS tadpole conditions one in general has to introduce orientifold planes, which can be regarded as branes of negative RR-charge and tension. However we like to emphasize that in intersecting brane models the tadpoles are in general not canceled locally, i.e. the D-branes and the orientifold are not lying on top of each other, but rather only the sum of all charges, integrated over the entire compact space, has to vanish. This means that the tadpoles can be non-locally canceled by D-branes and orientifolds which are distributed over the compact space.

Since we are interested in four-dimensional string models in flat 4D Minkowski space-time, we assume that six spatial directions are described by a compact space $\mathcal{M}^6$.

To be more specific we will consider a type IIA orientifold background of the form

$$\mathcal{M}^{10} = (\mathbb{R}^{3,1} \times \mathcal{M}^6)/\langle \omega \sigma \rangle, \quad \Omega : \text{world sheet parity}.$$ 

Here $\mathcal{M}^6$ is a Calabi-Yau 3-fold with a symmetry under $\sigma$, the complex conjugation $\sigma: z_i \mapsto \bar{z}_i$, $i = 1, \ldots, 3$,

$$\sigma = x^i + iy^i, \quad \sigma \text{ is combined with the world sheet parity } \Omega \text{ to form the orientifold projection } \Omega \sigma.$$ 

This operation is actually a symmetry of the type IIA string on $\mathcal{M}^6$. Orientifold 6-planes are defined as the fixed locus $\mathbb{R}^{3,1} \times \text{Fix}(\sigma) = \mathbb{R}^{3,1} \times \pi_{O6}$, where $\text{Fix}(\sigma)$ is a supersymmetric (sLag) 3-cycle on $\mathcal{M}^6$, denoted by $\pi_{O6}$. It is special Lagrangian (sLag) and calibrated with respect to the real part of the holomorphic 3-form $\Omega_3$.

Next we introduce D6-branes with world-volume $\mathbb{R}^{3,1} \times \pi_a$, i.e. they are wrapped around the supersymmetric (sLag) 3-cycles $\pi_a$ and their $\Omega \sigma$ images $\pi'_a$ of $\mathcal{M}_6$, which intersect in $\mathcal{M}^6$. Then the massless spectrum in general contains the following states:

- $\mathcal{N} = 1$ supergravity fields in the 10D bulk, since this orientifold projection truncates the gravitational bulk theory of closed strings down to a theory with 16 supercharges in ten dimensions, leading to 4 supercharges and $\mathcal{N} = 1$ in four dimensions, after compactifying on the Calabi-Yau.

- 7-dimensional $\mathcal{N} = 1$ gauge bosons with gauge group $G = \prod_a U(N_a)$ localized on stacks of $N_a$ D6-branes wrapped around 3-cycles $\pi_a$ (codim=3). In general, the $\pi_a$ are never invariant under $\sigma$ but mapped to image cycles $\pi'_a$. Therefore, a stack of D6-branes is wrapped on that cycle by symmetry, too.

- 4-dimensional chiral fermions localized on the intersections of the D6-branes (codim=6).

Since the chiral spectrum has to satisfy some anomaly constraints, we expect that it is given by purely topological data (Atiyah-Singer index theorem). The chiral massless
spectrum indeed is completely fixed by the topological intersection numbers $I$ of the 3-cycles of the configuration.

| Sector | Rep. | Intersection number $I$ |
|--------|------|-------------------------|
| $a' a$ | $A_a$ | $\frac{1}{2} (\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$ |
| $a' a$ | $S_a$ | $\frac{1}{2} (\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$ |
| $a b$  | $(N_a, N_b)$ | $\pi_a \circ \pi_b$ |
| $a' b$ | $(N_a, N_b)$ | $\pi'_a \circ \pi_b$ |

The non-Abelian gauge anomalies will cancel after satisfying the tadpole conditions and $U(1)$ anomalies are canceled by a generalized Green-Schwarz mechanism involving dimensionally reduced RR-forms (see the discussion later on).

Now we are ready to take the following sketchy view on the internal space:

Here one sector of intersecting D6-branes engineers the embedding of the Standard Model, the other sector of D6-branes corresponds to a hidden gauge sector which may be required by the tadpole cancellation conditions. Note that the hidden sector D-branes do not have any intersection with the Standard Model branes. Therefore there are no matter fields charged under the Standard Model and hidden gauge group. It follows that the hidden sector and the Standard Model sector can only interact with each other by gravitational effects or by the exchange of heavy string modes.

This kind of scenario allows for at least three possibilities for the realization and the breaking of space-time supersymmetry:
• The SM-branes are non-supersymmetric. Then the scale of supersymmetry breaking is of the order of the string scale, which, in order to solve the hierarchy problem, should be of the order of the TeV scale:

$$M_{\text{susy}} \simeq M_{\text{string}} \sim O(1 \text{ TeV})$$  

(8)

In order to explain the weakness of the gravitational force this scenario needs for large transversal dimensions $$R_\perp$$ on the internal space [75, 76].

• The SM-branes are mutually supersymmetric ("local" supersymmetry), but are non-supersymmetric with respect to hidden sector branes. In this case one generically deals the gravity mediated supersymmetry breaking:

$$M_{\text{susy}} \simeq \frac{M_{\text{string}}^2}{M_{\text{Planck}}^2} \simeq O(1 \text{ TeV}) \Rightarrow M_{\text{string}} \simeq O(10^{11}\text{GeV})$$  

(9)

Here the transversal dimensions on the internal space are only moderately enlarged, ($$R_\perp \simeq O(10^9)\text{GeV}$$) (see also [77]).

• All branes are mutually supersymmetric ("global" supersymmetry) Then supersymmetry must be broken by a dynamical mechanism (e.g. gaugino condensation) in hidden sector:

$$M_{\text{susy}} \simeq \frac{M_{\text{hidden}}^3}{M_{\text{Planck}}^2} \simeq O(1\text{ TeV}) \Rightarrow M_{\text{hidden}} \simeq O(10^{13}\text{GeV})$$  

(10)

Lets us now come to the consistency conditions for D-branes on compact spaces. We start with the requirement of RR-charge cancellation. For that purpose we need the topological Chern-Simons actions for Dp-branes and also for the orientifold Op-plane which have the form [78, 79, 80, 81, 82]:

$$S_{\text{CS}}^{(Dp)} = \mu_p \int_{Dp} \text{ch}(\mathcal{F}) \wedge \sqrt{\mathcal{A}(\mathcal{R}_T) \mathcal{A}(\mathcal{R}_N)} \wedge \sum_q C_q,$$

$$S_{\text{CS}}^{(Op)} = -2^{p-4} \mu_p \int_{Op} \sqrt{\mathcal{L}(\mathcal{R}_T/4) \mathcal{L}(\mathcal{R}_N/4)} \wedge \sum_q C_q.$$  

(11)

(ch(\mathcal{F}) denotes the Chern character, \(\mathcal{A}(\mathcal{R})\) the Dirac genus of the tangent or normal bundle, and the \(\mathcal{L}(\mathcal{R})\) the Hirzebruch polynomial.) The physical gauge fields and curvatures are related to the skew-hermitian ones in (11) by rescaling with $$-\frac{4i\pi^2\alpha'}{2\kappa^2}$$. These expressions simplify drastically for sLag 3-cycles, where ch(\mathcal{F})|_{Dp} = rk(\mathcal{F}), the other characteristic classes become trivial and finally the only contribution in the CS-term (11) for D6-branes comes from \(C_7\). Then this action leads to the following equations of motion for the gauge field \(C_7\):

$$\frac{1}{\kappa^2} d \star dC_7 = \mu_6 \sum_a N_a \delta(\pi_a) + \mu_6 \sum_a N_a \delta(\pi'_a) + \mu_6 Q_6 \delta(\pi_{O6}),$$  

(12)

where $$\delta(\pi_a)$$ denotes the Poincaré dual form of $$\pi_a$$, $$\mu_p = 2\pi(4\pi^2\alpha')^{-\frac{p+1}{2}}$$, and $$2\kappa^2 = \mu_7^{-1}$$. Upon integrating over \(\mathcal{M}^6\) one obtains the RR-tadpole cancellation as
an equation in homology \[19\]:
\[
\sum_a N_a (\pi_a + \pi'_a) - 4\pi_{O6} = 0.
\] (13)

In principle it involves as many linear relations as there are independent generators in \(H_3(M^6, R)\). But, of course, the action of \(\sigma\) on \(M^6\) also induces an action \([\sigma]\) on the homology and cohomology. In particular, \([\sigma]\) swaps \(H_2^1\) and \(H_1^2\), and the number of conditions is halved.

Note that one can show that the cancellation of the RR tadpoles implies absence of the non-Abelian anomalies in the effective 4D field theory. However there can be still anomalous \(U(1)\) gauge symmetries in the effective 4D field theory. These anomalies will be canceled by a Green-Schwarz mechanism involving RR (pseudo)scalar field. As a result of these interactions the corresponding \(U(1)\) gauge boson will become massive. Specifically we have to consider two relevant couplings in the effective action:
\[
\begin{align*}
\int_{R^3 \times \pi_a} C_5 \wedge \text{Tr} F_a & \sim \int_{R^3} B \wedge F_a, \\
\int_{R^3 \times \pi_b} C_3 \wedge \text{Tr}(F_b \wedge F_b) & \sim \int_{R^3} \phi \wedge (F_b \wedge F_b).
\end{align*}
\] (14)

They lead to the following diagram which cancels the anomalous triangle diagram:

```
   U(1)_a
     B
     / \  \\
  F_b   F_b
```

Now the condition for an anomaly free \(U(1)_a\) is:
\[
N_a (\pi_a - \pi'_a) \circ \pi_b = 0.
\] (15)

Note that even an anomaly free \(U(1)\) can become massive, if only the \(U(1)_a - B\) coupling is present. The massive \(U(1)\) always remains as a global symmetry.

As the second consistency requirement we now turn to the stability of the scalar potential, related to the absence of NS tadpoles. The tension of the D6-branes and O6-planes introduces a vacuum energy which is described in terms of D-terms in the language of \(\mathcal{N} = 1\) supersymmetric field theory. In type IIA these depend only on the complex structure moduli and do not affect the Kähler parameter of the background. The most general form for such a potential is given by
\[
V_{\text{D-term}} = \sum_a \frac{1}{2g_a^2} \left( \sum_i q_a |\phi_i|^2 + \xi_a \right)^2,
\] (16)

with \(g_a\) the gauge coupling of a \(U(1)_a\), \(\xi_a\) the FI parameter, and the scalar fields \(\phi_i\) are the superpartners of some bifundamental fermions at the intersections. They become massive or tachyonic for non-vanishing \(\xi_a\), depending on their charges \(q'_a\). Due to the
positive definiteness of the D-term, $\mathcal{N} = 1$ supersymmetry will only be unbroken in the vacuum, if the potential vanishes.

The disc level tension can be determined by integrating the Dirac-Born-Infeld effective action. It is proportional to the volume of the D-branes and the O-plane, so that the disc level scalar potential reads

$$V = T_6 e^{-\phi_4} \left( \sum_a N_a \left( \text{Vol}(D6_a) + \text{Vol}(D6'_a) \right) - 4\text{Vol}(O6) \right).$$

(17)

The potential is easily seen to be positive semidefinite and its minimization imposes conditions on some of the moduli, freezing them to fixed values. Whenever the potential is non-vanishing, supersymmetry is broken and a classical vacuum energy generated by the net brane tension. It is easily demonstrated that the vanishing of $V$ requires all the cycles wrapped by the D6-branes to be calibrated with respect to the same 3-form as are the O6-planes. In a first step, just to conserve supersymmetry on their individual world volume theory, the cycles have to be calibrated at all, which leads to

$$V = T_6 e^{-\phi_4} \left( \sum_a N_a \left| \int_{\pi_a} \hat{\Omega}_3 \right| + \sum_a N_a \left| \int_{\pi'_a} \hat{\Omega}_3 \right| - 4 \left| \int_{\pi_{O6}} \hat{\Omega}_3 \right| \right).$$

(18)

Since $\hat{\Omega}_3$ is closed, the integrals only depend on the homology class of the world volumes of the branes and planes and thus the tensions also become topological. If we further demand that any single D6$_a$-brane conserves the same supersymmetries as the orientifold plane the cycles must all be calibrated with respect to $\Re(\hat{\Omega}_3)$. We can then write

$$V = T_6 e^{-\phi_4} \int_{\sum_a N_a (\pi_a + \pi'_a) - 4\pi_{O6}} \Re(\hat{\Omega}_3).$$

(19)

In this case, the RR charge and NSNS tension cancellation is equivalent, as expected in the supersymmetric situation.

To apply eq.(16) we have to use the properly normalized gauge coupling

$$\frac{1}{g_{U(1)_a}^2} = \frac{N_a}{g_a^2} = \frac{N_a M_s^3}{(2\pi)^4} e^{-\phi_4} \left| \int_{\pi_a} \hat{\Omega}_3 \right|.$$  

(20)

Hence, the FI-parameter $\xi_a$ can be identified as

$$\xi_a^2 = \frac{M_s^4 \left| \int_{\pi_a} \hat{\Omega}_3 \right| - \int_{\pi_a} \Re(\hat{\Omega}_3)}{2\pi^2 \left| \int_{\pi_a} \hat{\Omega}_3 \right|},$$

(21)

which vanishes precisely if the overall tension of the branes and planes cancels out, i.e. if all are calibrated with respect to the same 3-form. Since the FI-term is not a holomorphic quantity one expects higher loop corrections to the classical potential eq.(17).

Summarizing this discussion on the scalar potential, we can consider three different scenarios with respect of the realization of space-time supersymmetry (see above):
• "Global" $\mathcal{N} = 1$ supersymmetry:
Here the minima of $\mathcal{V}$ with respect to the complex structure moduli are such that all angles are supersymmetric. Therefore all angles, being determined by the complex structure moduli, of the D6-branes are supersymmetric in the minimum and conserve the same supersymmetries as orientifold plane, i.e. all D6-branes be calibrated with respect to $\Re(\hat{\Omega}_3)$. Furthermore the vacuum energy vanishes in the supersymmetric minimum $\mathcal{V}_{\text{min}} = 0$.

• "Local" $\mathcal{N} = 1$ supersymmetry:
Now the minima of $\mathcal{V}$ are such that only the SM D-branes have mutually supersymmetric angles; only the SM D6-branes conserve the same supersymmetries as orientifold plane, i.e. only SM D6-branes be calibrated with respect to $\Re(\hat{\Omega}_3)$. However the hidden sector is in general necessary for RR tadpole cancellation. Note that stable minima of this type are already non-trivial to find, since we eventually have to stabilize the SM branes with respect to the hidden branes.

• No supersymmetry:
Finally in this case the minima are such that the angles of the SM branes are non-supersymmetric. Therefore in the minimum the SM D6-branes do not conserve the same supersymmetries as the orientifold plane. Here stability is in general very difficult to achieve, as we also must be careful to avoid the process of brane recombination to a supersymmetric scenario. In order to determine the full non-supersymmetric ground date, non-perturbative (string field theory) methods will be necessary.

In addition to the D-term scalar potential, also a F-term potential corresponding to an effective superpotential might be generated. Here there are strong restrictions known for the contributions that can give rise to corrections to the effective $\mathcal{N} = 1$ superpotential of a type II compactification on a Calabi-Yau 3-fold with D6-branes and O6-planes on supersymmetric 3-cycles. The standard arguments about the non-renormalization of the superpotential by string loops and world sheet $\alpha'$ corrections apply. The only effects then left are non-perturbative world sheet corrections, open and closed world-sheet instantons $[83, 84]$. In general, these are related to non-trivial $CP^1$ and $RP^2$ with boundary on the O6-plane in the Calabi-Yau manifold for the closed strings and discs with boundary on the D6-branes for open strings. In fact, only the latter contribute to the superpotential. The typical form for the superpotential thus generated is known, but explicit calculations are only available for non-compact models. Usually, they make use of open string mirror symmetry arguments. In many cases, there is an indication that the non-perturbative contributions to the superpotentials tend to destabilize the vacuum, and it would be a tempting task to determine a class of stable $\mathcal{N} = 1$ supersymmetric intersecting brane models.

Let us now continue with some further aspects of string model building, namely how to derive the spectrum of the Standard Model. In fact, there are two simple ways to embed the Standard Model $[7, 8, 42]$. Both of them use four stacks of $D6$-branes, but
they differ in their realization of the weak $SU(2)$ gauge group (note that $SP(2)$ and $SU(2)$ are isomorphic groups):

\begin{align*}
A : & \quad U(3)_a \times SP(2)_b \times U(1)_c \times U(1)_d \\
B : & \quad U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d.
\end{align*}

The chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the standard model particles. This fixes uniquely the intersection numbers of the four 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$; for all class A models one derives the following specific intersection assignment (a similar pattern holds for all class B models):

| field | sector | I | $SU(3) \times SU(2) \times U(1)^3$ |
|-------|--------|---|----------------------------------|
| $Q_L$ | (ab)   | 3 | $(3, 2; 1, 0, 0)$ |
| $U_R$ | (ac)   | 3 | $(3, 1; -1, 1, 0)$ |
| $D_R$ | (ac')  | 3 | $(3, 1; -1, -1, 0)$ |
| $E_L$ | (db)   | 3 | $(1, 2; 0, 0, 1)$ |
| $E_R$ | (dc')  | 3 | $(1, 1; 0, -1, -1)$ |
| $\nu_R$ | (dc) | 3 | $(1, 1; 0, 1, -1)$ |

The hypercharge $Q_Y$ is given as the following linear combination of the three $U(1)$’s

$$Q_Y = \frac{1}{3} Q_a - Q_c - Q_d.$$  \hspace{1cm} (23)

Then an intersecting brane world model is constructed by the following six steps:

(i) chose a compact toroidal, orbifold or Calabi-Yau manifold $\mathcal{M}_6$,
(ii) determine the orientifold 6-plane $\pi_{O6}$,
(iii) chose four 3-cycles $\pi_{U(3)_a}$, $\pi_{U(2)_b}$, $\pi_{U(1)_c}$, $\pi_{U(1)_d}$ for the four stacks of D6-branes, as well as their orientifold mirrors,
(iv) compute their intersection numbers,
(v) ensure that the RR tadpole conditions vanish (possibly by adding hidden D6-branes),
(vi) and finally ensure that the linear combination $U(1)_Y$ remains massless.

$$\sum_i c_i N_i (\pi_i - \pi'_i) = 0,$$ \hspace{1cm} (24)

where the $c_i$ define the precise linear combination for $U(1)_Y$.

In this way many non-supersymmetric intersecting brane world models on tori [7, 8], orbifolds [10], or the quintic Calabi-Yau manifold [19, 25] with gauge group

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y$$

and 3 families of quark and leptons can be explicitly constructed. (Some authors also use more than four stack of D6-branes or different types of D-branes. For intersecting
branes on supersymmetric orientifolds of Gepner models see \[35, 85, 86\].) In order to construct models with the spectrum of the $\mathcal{N} = 1$ supersymmetric model (MSSM) also various attempts were made \[11, 12, 27, 29, 34\]. However as far as we know no explicit model with the precise spectrum of the MSSM was yet found.

Let us first describe in more detail how one constructs toroidal and orbifold intersecting brane world models. Here we consider configurations of type II $D_6$ branes wrapped on non–trivial three–cycles of a six–dimensional torus $T^6$. The torus is taken to be a direct product $T^6 = \prod_{j=1}^3 T_j^2$ of three two–dimensional tori $T_j^2$ with radii $R_j^1, R_j^2$ and angles $\alpha^j$ w.r.t. the compact dimensions with coordinates $x^j$ and $y^j$. The Kähler and complex structure moduli of these tori are defined as usual:

$$U^j = \frac{R_j^2}{R_j^1} e^{i \alpha^j}, \quad T^j = b^j + i R_1^j R_2^j \sin \alpha^j,$$

(25) with the torus $B$–field $b^j$. Furthermore, the three–cycle is assumed to be factorizable into a direct product of three one–cycles, each of them wound around a torus $T_2^j$ with the wrapping numbers $(n^j, m^j)$ w.r.t. the fundamental 1–cycles of the torus. Hence the angle of the $D_6$–brane with the $x^j$–axis is given by

$$\tan \Phi^j = \frac{m^j R_2^j}{n^j R_1^j}.$$

(26) Generally, two branes with wrapping numbers $(n^j_a, m^j_a)$ and $(n^j_b, m^j_b)$, are parallel in the subspace $T_2^j$, if their intersection number

$$I_{ab}^j = n^j_a m^j_b - n^j_b m^j_a$$

(27) w.r.t. this subspace vanishes, $I_{ab}^j = 0$.

As explained before stability of a given D-brane configuration requires that all NS tadpoles vanish. The relevant massless fields in the NS sector are the four-dimensional dilaton and the complex structure moduli $U^i$. One method is to extract the NS tadpoles from the infrared divergences in the tree channel Klein-bottle, annulus and Möbius-strip amplitudes, the open string one-loop diagrams. Adding up these three contributions one can read off the disc tadpoles. Another method is to compute the corresponding scalar potential from the Born-Infeld action of the wrapped D6-branes; in the string frame it takes the following form ($U^i = U_1^i + i U_2^i$) \[10\]:

$$V = e^{-\phi} \sum_{a=1}^K N_a \prod_{i=1}^3 \left( \frac{1}{\sqrt{U_2}} \right)^2 \left( n^i_a \sqrt{U_2^i} \right)^2 + \left( m^i_a \sqrt{U_2^i} \right)^2 - 16 e^{-\phi} \prod_{i=1}^3 \sqrt{U_2^i}.$$  

(28) The NS tadpoles are simply the first derivatives of $V$ with respect to the scalar fields: $\langle \delta \phi \rangle \sim \frac{\partial V}{\partial \phi}$, $\langle U^i \rangle \sim \frac{\partial V}{\partial U^i}$. So we clearly see that the NS tadpoles vanish in case the potential is extremized. For supersymmetric minima the potential in addition vanishes, and all D-branes have supersymmetric angles at the minimum. Then, at the minimum
of $V$, all the 3-cycles on the torus including the orientifold 3-cycle, constructed in the way described above, are supersymmetric, i.e. sLag’s, for all values of $n^a_i$ and $m^i_a$. For supersymmetric orbifold models one therefore has to check whether they conserve the same supersymmetries as the orientifold plane. Specifically one obtains the following conditions on the angles $v^i_a$ of a stack of supersymmetric $D6_a$ branes:

(i) $\mathcal{N} = 4$ sectors: $v^1_a = v^2_a = v^3_a = 0$.

(ii) $\mathcal{N} = 2$ sectors: $v^i_a = 0$ w.r.t. the $i$-th plane and $v^j_a \pm v^l_a = 0$.

(iii) $\mathcal{N} = 1$ sectors: $v^1_a \pm v^2_a \pm v^3_a = 0$.

In non-supersymmetric models stability and tadpole cancellation is much harder to acquire. The potential displays the usual runaway behavior one often encounters in non-supersymmetric string models. The complex structure is dynamically pushed to the degenerate limit, where all branes lie along the $x_i$ axes and the $y_i$ directions shrink, keeping the volume fixed. Put differently, the positive tension of the branes pulls the tori towards the $x_i$-axes. Apparently, this has dramatic consequences for all toroidal intersecting brane world models. They usually require a tuning of parameters at tree-level and assume the global stability of the background geometry as given by the closed string moduli. If at closed string tree-level one has arranged the radii of the torus such that open strings stretched between D-branes at angles are free of tachyons, dynamically the system flows towards larger complex structure and will eventually reach a point where certain scalar fields become tachyonic and indicate a decay of the brane configuration.

One way to get rid of the tadpoles associated with the geometrical, complex structure moduli $U^i$ is to perform an orbifold twist on the toroidal background (see next paragraph) such that some or all of the complex structure moduli get frozen [10]. E.g. in the $\mathbb{Z}_3$ orbifold all $U^i$ moduli are absent. However one is always left with the dilaton tadpole. Since the potential has runaway behavior in the dilaton direction, one is generically driven to zero coupling. Therefore it is still an open problem to cancel the dilaton tadpole for finite values of the string coupling constant in non-supersymmetric models (or also after supersymmetry breaking in supersymmetric models).

Next let us consider the action of the orientifold and of the orbifold group, where the spatial orbifold group is defined by elements from $\mathbb{Z}_N$ (or $\mathbb{Z}_N \times \mathbb{Z}_M$). The latter are represented by the $\theta$ (and $\omega$), describing discrete rotations on the compact coordinates $x^i, y^i$. This action restricts the compactification lattice and fixes some of the internal parameter (25) to discrete values. The orientifold $O6$–planes describe the set of points which are invariant under the group actions $\Omega \sigma, \Omega \sigma \theta^k, \Omega \sigma \omega^l$ and $\Omega \sigma \theta^k \omega^l$. These planes are generated by rotations of the real $x^i$ axes by $\theta^{-k/2} \omega^{-l/2}$.

The condition for tadpole cancellations in IIA orientifold backgrounds in four space–time dimensions requires a system of $D6$ branes which has to respect the orbifold and orientifold projections. In particular, for consistency with the orbifold/orientifold group their orbifold/orientifold mirrors have to be introduced. Hence any stack $a$ is organized
in orbits, which represent an equivalence class \([a]\). For \(N, M \neq 2\) the length of each orbit \([a]\) is at most \(2NM\), but may be smaller, if e.g. stack \(a\) is located along an orientifold plane. Stacks within a conjugacy class \([a]\) have non-trivial intersections among each other and w.r.t. stacks from a different class \([b]\) belonging to the gauge group \(G_b\).

Without going further into any details, it is appealing that one can check in several examples \([19]\) that the open string spectrum constructed following these rules in the toroidal ambient space precisely agrees with the geometrical spectrum discussed before, when considering the orbifold space as a limiting geometry of a Calabi-Yau manifold. The requirement of \(R\)-tadpole cancellation leads to some constraints on the number and location of the \(D_6\)-branes. To derive the tadpole conditions one has to compute the Klein-bottle, annulus and Möbius strip amplitudes. In the annulus amplitude all open string sectors contribute including those from open strings stretched between two branes belonging to the same equivalence class. It turns out to be convenient to define the following two quantities for any equivalence class \([((n^i_a, m^i_a))]\) of \(D_6\)-branes (here for the \(\mathbb{Z}_3\)-orbifold \([10]\))

\[
Z[a] = \frac{2}{3} \sum_{(n^i_b, m^i_b)\in[a]} \prod_{i=1}^{3} \left(n^i_b + \frac{1}{2} m^i_b\right),
\]

\[
Y[a] = -\frac{1}{2} \sum_{(n^i_b, m^i_b)\in[a]} (-1)^M \prod_{i=1}^{3} m^i_b
\]

where \(M\) is defined to be odd for a mirror brane and otherwise even. The sums are taken over all the individual \(D_6\)-branes that are elements of the orbit \([a]\). If we introduce \(K\) stacks of equivalences classes \([a]\) of branes, then the RR-tadpole cancellation condition reads

\[
\sum_{a=1}^{K} N_a Z[a] = 2.
\]

Note, that the sum is over equivalence classes of \(D_6\)-branes. This equation is equivalent to the geometrical tadpole conditions eq.(13).

Non-supersymmetric toroidal and orbifold models with SM-spectrum can be constructed in various ways. E.g. in \([10]\) SM intersecting brane worlds on a \(\mathbb{Z}_3\)-orbifold were explicitly constructed. Also, specific examples of orbifold intersecting brane world models with \(N = 1\) supersymmetry in \(D = 4\) have been introduced in \([11, 12, 27, 29, 34]\). However to derive \(N = 1\) MSSM models seems to be much more involved.

As a specific example of an intersecting brane model on a Calabi-Yau manifold let us display the construction of the non-supersymmetric Standard Model on the Fermat quintic Calabi-Yau \([19, 25]\), which is defined by the following equation in \(\mathbb{C}P^4\):

\[
P(z_i) = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0 \subset \mathbb{C}P^4.
\]

The orientifold six-plane \(\pi_{O6}\) is determined by the real counter part of this equation:

\[
P(x_i) = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0 \subset \mathbb{R}P^4.
\]
Next we have to introduce the D6-branes being wrapped around the supersymmetric 3-cycles. These are obtained by applying the $\mathbb{Z}_4$ symmetry, $z_i \mapsto \omega^{k_i} z_i$, $\omega = e^{2\pi i / 5}$, $k_i \in \mathbb{Z}_5$, on $\pi_{O6}$. Specifically each sLag 3-cycle can be characterized by four integers, $\pi_a = \pi_{k_2,k_3,k_4,k_5}$, and is defined by the following equation:

$$x_1^5 + \Re(\omega^{k_2} z_2)^5 + \Re(\omega^{k_3} z_3)^5 + \Re(\omega^{k_4} z_4)^5 + \Re(\omega^{k_5} z_5)^5 = 0 \quad (33)$$

This leads to a set of $5^4 = 625$ sLag $\mathbb{RP}^3$'s, calibrated with $\Re(\prod_i \omega^{k_i} \Omega_3)$. Of these, 125 sLag's are calibrated with respect to $\Re(\Omega_3)$, i.e. they are supersymmetric with respect to the orientifold plane $\pi_{O6}$. Therefore, for supersymmetric D-brane configurations only these 3-cycles must be used. On the other hand, the 3-cycles, which are calibrated in the same way, have zero intersection with each other. Hence in order to get chiral fermions only non-supersymmetric models can be built on the quintic Calabi-Yau in this way.

The general intersection numbers are already computed in the paper by [87]. To engineer the standard model we introduce four stacks of D6-branes with $N_a = 3$, $N_b = 2$ and $N_c = N_d = 1$, corresponding to the gauge group $G = U(3) \times U(2) \times U(1)^2$. Finally we choose the following “wrapping numbers” for the D6-branes on the slag 3-cycles:

$$\pi_a = \pi_c - \pi_d - |0, 2, 1, 4| - |0, 3, 4, 1|,$$
$$\pi_b = |0, 3, 1, 1|,$$
$$\pi_c = |1, 4, 3, 4| + |4, 4, 3, 2|,$$
$$\pi_d = |0, 3, 0, 3| - |2, 0, 3, 4| \quad (34)$$

This produces the intersection numbers of the Standard Model with three generations of quarks and leptons. The anomaly-free, massless hypercharge is

$$U(1)_Y = \frac{1}{3} U(1)_a - U(1)_c + U(1) \quad (35)$$

Finally, an hidden sector is needed for tadpole cancellation.

3. Some phenomenological issues of intersecting brane worlds

3.1. Gauge coupling unification

One of the biggest successes of the MSSM is the apparent unification of the three gauge coupling constants. Remember that at the weak energy scale the three Standard Model gauge couplings $g_s$, $g_w$ and $g_y$ have quite different values. Extrapolating these couplings via the one-loop renormalization group equations

$$\frac{4\pi}{g_a^2(\mu)} = k_a \frac{4\pi}{g_X^2} + \frac{b_a}{2\pi} \log \left( \frac{\mu}{M_X} \right) + \Delta_a \quad (36)$$

to higher energies, one finds that they all meet at

$$M_X \simeq 2 \cdot 10^{16} \text{ GeV}, \quad \alpha_s = \alpha_w = \frac{3}{5} \alpha_Y = \alpha_X \simeq \frac{1}{24}, \quad (37)$$

if the light spectrum contains just the MSSM particles. This is in accord with for instance an $SU(5)$ Grand Unified gauge group at the GUT scale.
In string theory one has a new scale, the string scale $M_s$, so that it is natural to relate $M_X$ to $M_s$. In the heterotic string one finds

$$k_a = \text{level of SU}(N_a) \text{ Kac – Moody algebra},$$

and the heterotic relation between the string and the Planck scale was found to be \cite{SS1, SS2}

$$M_s \simeq g_{st} \cdot 0.058 \cdot M_{pl},$$

which, using $g_{st} \simeq 0.7$, leads to $M_s \simeq 5 \cdot 10^{17}$ GeV. Assuming a MSSM like low-energy string spectrum, this discrepancy between $M_X$ and $M_s$ needs to be explained by moduli-dependent string threshold corrections $\Delta_a$ \cite{90, 91, 92, 93} (or alternatively by heterotic M-theory \cite{94}).

Let us now study gauge coupling unification in type II or in type I D-brane models with gauge group $SU(3) \times SU(2) \times U(1)_Y$ \cite{95, 41} (see also \cite{96}); here each gauge factor comes with its own gauge coupling, which at string tree-level can be deduced from the Dirac-Born-Infeld action

$$\frac{4\pi}{g_a^2} = \frac{M_s^3 V_a}{(2\pi)^3 g_{st}^2 \kappa_a^3}, \quad V_a = (2\pi)^3 R_a^3.$$

with $\kappa_a = 1$ for $U(N_a)$ and $\kappa_a = 2$ for $SP(2N_a)/SO(2N_a)$.

By dimensionally reducing the type IIA gravitational action one can similarly express the Planck mass in terms of the stringy parameters ($M_{pl} = (G_N)^{-\frac{1}{2}}$)

$$M_{pl}^2 = \frac{8 M_s^8 V_6}{(2\pi)^6 g_{st}^2}, \quad V_6 = (2\pi)^6 R^6.$$

Eliminating the unknown string coupling $g_{st}$ gives

$$\frac{1}{\alpha_a} = \frac{M_{pl}}{2\sqrt{2} \kappa_a M_s \sqrt{V_6}}.$$

Due to

$$\frac{V_a}{\sqrt{V_6}} = \int_{\pi_a} \Re(e^{i\phi_a} \hat{\Omega}_3)$$

the gauge coupling only depends on the complex structure moduli.

Since already at string tree-level the different type II,I gauge couplings depend on different moduli fields and hence in general may take arbitrary values, it seems that gauge coupling unification is unnatural or only occurs merely as an accident. However one can show that under some natural assumptions there will be one relation between the three MSSM gauge couplings that is compatible with gauge coupling unification. Namely consider gauge couplings of intersecting D-branes in a model independent bottom up approach where we ask for the following three phenomenological requirements:

- The SM branes mutually preserve $\mathcal{N} = 1$ supersymmetry.
- The intersection numbers realize a 3 generation MSSM.
- The $U(1)_Y$ gauge boson is massless.
(Note that an orbifold model with only the MSSM states as massless string modes is not yet constructed; however there seems to be no fundamental obstacle to get such a model using more general backgrounds.) Without going into the details one can show that these restrictions provide one relation between the internal volumes \( V_a \), i.e. between the 3 gauge coupling constants:

\[
\frac{1}{\alpha_Y} = \frac{2}{3} \frac{1}{\alpha_s} + \frac{1}{\alpha_w}. \tag{42}
\]

This relation will allow for natural gauge coupling unification! In fact, if one furthermore assumes that \( \alpha_s = \alpha_w \), this relation is identical to the \( SU(5) \) GUT relation in eq.(37).

Let us further explore what are the allowed values of the string scale which follow from gauge coupling unification using the relation (42). In the absence of threshold corrections (see the discussion at the end of this section), the one-loop running of the three gauge couplings is described by the well known formulas

\[
\begin{align*}
\frac{1}{\alpha_s(\mu)} &= \frac{1}{\alpha_s} + \frac{b_3}{2\pi} \ln \left( \frac{\mu}{M_s} \right), \\
\sin^2 \theta_w(\mu) &= \frac{1}{\alpha_w} + \frac{b_2}{2\pi} \ln \left( \frac{\mu}{M_s} \right), \\
\cos^2 \theta_w(\mu) &= \frac{1}{\alpha_Y} + \frac{b_1}{2\pi} \ln \left( \frac{\mu}{M_s} \right),
\end{align*} \tag{43}
\]

where \((b_3, b_2, b_1)\) are the one-loop beta-function coefficients for \( SU(3)_c \), \( SU(2)_L \) and \( U(1)_Y \), and \( \alpha \) is the electromagnetic fine structure constant. Using the tree level relation (42) at the string scale yields

\[
\frac{2}{3} \frac{1}{\alpha_s(\mu)} + \frac{2 \sin^2 \theta_w(\mu) - 1}{\alpha(\mu)} = B \frac{1}{2\pi} \ln \left( \frac{\mu}{M_s} \right) \tag{44}
\]

with

\[
B = \frac{2}{3} b_3 + b_2 - b_1. \tag{45}
\]

Now employing the measured Standard Model parameters

\[
\begin{align*}
M_Z &= 91.1876 \text{ GeV}, \quad \alpha_s(M_Z) = 0.1172, \\
\alpha(M_Z) &= \frac{1}{127.934}, \quad \sin^2 \theta_w(M_Z) = 0.23113,
\end{align*} \tag{46}
\]

the resulting value of the unification scale only depends on the combination \( B \) of the beta-function coefficients.

For the MSSM spectrum one has \((b_3, b_2, b_1) = (3, -1, -11)\), i.e \( B = 12 \) and the unification scale is the usual GUT scale

\[
M_s = M_X = 2.04 \cdot 10^{16} \text{ GeV}. \tag{47}
\]

For the individual gauge couplings at the string scale we get

\[
\alpha_s(M_s) = \alpha_w(M_s) = \frac{5}{3} \alpha_Y(M_s) = 0.041, \tag{48}
\]
which are just the supersymmetric GUT scale values with the Weinberg angle being
\[ \sin^2 \theta_w(M_s) = \frac{3}{8}. \]
Assuming \( g_{st} = g_X \), one obtains for the overall radius \( R \) and the internal radii \( R_s, R_w \)
\[ M_s R = 5.32, \quad M_s R_s = 2.6, \quad M_s R_w = 3.3. \] (49)
Of course it still remains to be checked if the parameters can be chosen in such a way
in a given, concrete model.

One can also investigate the gauge coupling unification in other classes of models
with different low energy spectrum. For example, many models contain besides
the chiral matter also additional vector-like matter. These states are also localized
on the intersection loci of the D6 branes and also come with multiplicity \( n_{ij} \) with
\( i, j \in \{ a, b, c, d \} \).
One finds the following contribution to \( B \)
\[ B = 12 - 2 n_{aa} - 4 n_{ab} + 2 n_{a'c} + 2 n_{a'd} - 2 n_{bb} + 2 n_{c'c} + 2 n_{c'd} + 2 n_{d'd}. \] (50)
\( B \) does not depend on the number of weak Higgs multiplets \( n_{bc} \).

Example A:
If we have a model with a second weak Higgs field, i.e. \( n_{bc} = 1 \), we still get \( B = 12 \) but
with
\[ (b_3, b_2, b_1) = (3, -2, -12). \] (51)
The gauge couplings ”unify” at the scale
\[ M_s = 2.02 \cdot 10^{16}\text{GeV}. \]
However they are not all equal at that scale
\[ \alpha_s(M_s) = 0.041, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.028. \]
The evolution of the gauge coupling constants for this class of models is shown in the
following diagram:
Example B: intermediate scale model

For models with gravity mediated supersymmetry breaking (hidden anti-branes) the string scale is naturally in the intermediate regime $M_s \simeq 10^{11}\text{GeV}$.

Choosing vector-like matter

$$n_{a'a} = n_{a'd} = n_{d'd} = 2, \quad n_{bb} = 1$$

leads to $B = 18$. The string scale turns out to be

$$M_s = 3.36 \cdot 10^{11}\text{GeV}.$$  \hspace{1cm} (53)

The running of the couplings with

$$(b_3, b_2, b_1) = (-1, -3, -65/3)$$

leads to the values of the gauge couplings at the string scale

$$\alpha_s(M_s) = 0.199, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.045.$$  \hspace{1cm} (54)

Assuming $g_{st} \simeq 1$, one obtains for the radii

$$M_s R = 230, \quad M_s R_s = 1.7, \quad M_s R_w = 3.3.$$  \hspace{1cm} (54)
Example C: Planck scale model

Interestingly for $B = 10$ one gets

$$\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}.$$ (55)

Choosing vector-like matter

$$n_{aa} = 1,$$

the beta-function coefficients read

$$(b_3, b_2, b_1) = (0, -1, -11).$$

The couplings at the string scale turn out to be

$$\alpha_s(M_s) = 0.117, \quad \alpha_w(M_s) = 0.043, \quad \alpha_Y(M_s) = 0.035$$ (56)

leading to $\sin^2 \theta_w(M_s) = 0.445$. For the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

$$M_s R = 0.6, \quad M_s R_s = 1.9, \quad M_s R_w = 3.3.$$ (57)
So far we have neglected the stringy, one-loop gauge threshold corrections $\Delta_a$ (to the gauge group $G_a$) in the renormalization group equations of the gauge coupling constant. We do not give a phenomenological investigation of these threshold corrections, but sketch how they are computed in intersecting brane world models. Note however that the one-loop gauge threshold corrections $\Delta_a$, which take into account Kaluza–Klein and winding states from the internal dimensions and the heavy string modes, may change the unification picture discussed before. For certain regions in moduli space these corrections may become huge and thus have a substantial impact on the unification scale.

In the type IIA picture with intersecting D6-branes these threshold correction $\Delta_a$ will depend on the homology classes on the 3-cycles (open string parameters) and also on the closed string geometrical moduli. In toroidal models these corrections will be given in terms of the wrapping numbers $n^j_a, m^j_a$ and the radii $R^j_i$ of the torus. All D6-branes have in common their four-dimensional (non-compact) world volume. Hence their gauge fields are located on parallel four-dimensional subspaces, which may be separated (in the cases $I^j_{ab} \neq 0$ and $I^j_{aad'} \neq 0$) in the transverse internal dimensions. One-loop corrections to the gauge couplings are realized through exchanges of open strings in that transverse space. The open string charges $q_a, q_b$ at their ends couple...
to the external gauge fields sitting on the branes. Only annulus and Möbius diagrams contribute, as torus and Klein bottle diagrams refer to closed string states. In \[40\] the one–loop corrections to the gauge couplings were computed by the background field method: one turns on a (space–time) magnetic field, e.g. \(F_{23} = BQ_a\) in the \(X^1\)–direction and determines the dependence of the open string partition function on that field. Here, \(Q_a\) is an appropriately normalized generator of the gauge group \(G_a\) under consideration. This leads to the so-called gauged open string partition function. The second order of an expansion w.r.t. \(B\) of the gauged partition function gives the relevant piece for the one–loop gauge couplings.

In the following we will omit all details of the calculations, referring the reader to the paper \[40\]; instead we give only the end results for the one loop threshold corrections:

(i) \(N = 4\) sectors: \(\Delta_a = 0\).

(ii) \(N = 2\) sectors:

If the stacks \(a\) and \(b\) preserve \(N = 2\) supersymmetry, i.e. they are parallel within some torus \(T^i_2\), we obtain for the gauge group \(G_a\):

\[
\Delta_{ab}^{N=2} \sim b_{ab}^{N=2} \ln(T^i_2 V^i_a |\eta(T^i)|^4) + \text{const.} ,
\]

(58)

with the wrapped brane volume

\[
V^i_a = \frac{1}{U^i_2} |n^i_a + U^i m^i_a|^2 ,
\]

(59)

and the Kähler modulus \(T^i\) defined in eq.(25).

(iii) \(N = 1\) sectors:

In the case that the branes from \(a\) and \(b\) preserve \(N = 1\) supersymmetry, the one–loop correction to the gauge coupling of \(G_a\) takes the form:

\[
\Delta_{ab}^{N=1} = -b_{ab}^{N=1} \ln \left( \frac{\Gamma(1 - \frac{1}{\pi} \Phi^1_{ba}) \Gamma(1 - \frac{1}{\pi} \Phi^2_{ba}) \Gamma(1 + \frac{1}{\pi} \Phi^1_{ba} + \frac{1}{\pi} \Phi^2_{ba})}{\Gamma(1 + \frac{1}{\pi} \Phi^1_{ba}) \Gamma(1 + \frac{1}{\pi} \Phi^2_{ba}) \Gamma(1 - \frac{1}{\pi} \Phi^1_{ba} - \frac{1}{\pi} \Phi^2_{ba})} \right). \]

(60)

This expression depends on the closed string moduli of the underlying toroidal geometry, since the the difference of the angles \(\Phi^j_a\) and \(\Phi^j\) are related to the radii through:

\[
\coth(\pi v^j_{aa'}) = i \cot(\Phi^j_a - \Phi^j) = i \frac{n^j_a n^j_{a'} R^j_1 + m^j_a m^j_{a'} R^j_2}{n^j_a m^j_{a'} - n^j_{a'} m^j_a} .
\]

(61)

Note that this type of moduli dependence of the \(N = 1\) threshold functions in intersecting brane world models is completely new, as in heterotic string compactifications the \(N = 1\) thresholds are moduli independent constants \[20\].

In addition one should emphasize that in supersymmetric brane world models there are no UV divergences in the one-loop \(N = 2,1\) thresholds. In fact one can show that cancellation of the vacuum RR tadpoles implies that in supersymmetric models also all RR and NS tadpoles are absent in the one-loop 2-points functions for the gauge couplings, i.e. in the gauged open string partition functions. This proves the finiteness of the supersymmetric one-loop gauge thresholds in the considered class of orbifold models.
3.2. Proton decay

At the end of the paper let us also discuss briefly some aspects of proton decay in intersecting brane world models. Here we are following the work of [48]. Recall that in supersymmetric GUT field theories proton decay can occur due to dimension 5 operators $\int d^2\theta Q^3 L$, i.e. exchange of SUSY particles $S$, or can occur due to dimension 6 operators $\int d^4\theta Q^2 \tilde{Q}^* \tilde{L}^*$, i.e. exchange of heavy gauge vector bosons $X$. In string theory (or in M-theory) has to deal in addition with the exchange of an infinite tower of KK states, as shown in the following figure:

Due to the KK exchange one expects an enhancement of the proton decay amplitude by an universal factor $\alpha_{GUT}^{-1/3}$. However to know the precise numerical coefficient one has to make a model dependent calculation.

First consider shortly intersecting D6-brane models with SM gauge group $G = SU(3) \times SU(2) \times U(1)_Y$. Here baryon number is an anomalous $U(1)$ gauge symmetry, and hence there is a global $U(1)_B$ symmetry. Therefore proton decay is completely forbidden in this class of models.

Next turn to intersecting D6-brane models with $SU(5)$-GUT gauge group. This can be realized by a stack of five D6-branes plus their orientifold mirrors D6’. The massless spectrum is given by $SU(5)$ gauge bosons and $10 + \overline{10}$ matter from open strings at the intersection of D6,D6’-branes. Note that there are no fundamental matter fields in the 5-representation in this brane set up. Now one can compute the open string disc amplitude $10^2 \sqrt{\mathcal{O}^2}$ ($p \rightarrow \pi^0 e^+_L$):

$$A_{st} = \pi \alpha' g_s I(\Phi_1, \Phi_2, \Phi_3) = \frac{\alpha_{GUT}^{2/3} L^{2/3} g_s^{1/3} I(\Phi_1, \Phi_2, \Phi_3)}{4 \pi M_{GUT}^2},$$

$$I = \int_0^1 dx \frac{1}{x(x-1)} \prod_{i=1}^3 \sqrt{\sin(\pi \Phi_i)} [F(\Phi_i, 1 - \Phi_i; 1; x)]^{-1/2} \simeq 7 - 11.$$  

After having performed this string calculation we have to compare the field theory proton lifetime with the string theory lifetime:

Field theory:

$$\tau_p = 1.6 \times 10^{36} \text{y} \left(\frac{0.4}{\alpha_{GUT}}\right)^2 \left(\frac{M_X}{2 \times 10^{16} \text{GeV}}\right)^4 \simeq 1.6 \times 10^{36} \text{y}. \quad (63)$$

String theory:

$$\tau_{p,s} = (0.037 L^{2/3} g_s^{1/3})^{-2} \tau_p. \quad (64)$$
This comparison displays the following two new enhancement factors:
(i) The contribution of the KK-states leading to a 1-loop threshold factor \( L \simeq 8 \) (being obtained from M-theory on a \( G_2 \)-manifold \[97\]).
(ii) The tree level string disc amplitude: \( I \simeq 7 - 11 \).
Combining these two factors with \( g_s = \mathcal{O}(1) \) one learns that no substantial enhancement of the proton lifetime is going to appear. Note also that in order to get some further informations on the proton life time in D-brane models also dimension 5 operators should be taken into account.

4. Conclusion

In this paper we have reviewed several aspects of intersecting brane world models. The main aspect was on the constructive part of these models. As we have seen, it is possible to derive the SM spectrum from this kind of D-brane constructions, at least in non-supersymmetric brane embeddings into the compact space. We also discussed how gauge coupling unification can be achieved, including some discussion on the computation of one-loop gauge threshold corrections; finally the computation of the proton decay amplitude, which is based on the calculation of an open string four-point amplitude at the disc level, was briefly presented.

However there are several open problems in type I intersecting brane world models which should be addressed in the future:

- One of the main challenge remains to construct realistic supersymmetric intersecting brane world models with the chiral spectrum of the MSSM.
- An important problem is to get more informations on the low-energy effective action of intersecting brane world models. One way to obtain the effective action is to compute open/closed string scattering amplitudes and extract from them the relevant low-energy couplings. In this way the moduli dependent gauge couplings, and the matter field Kähler potential can be derived \[98\]. Furthermore these results can be used to compute the soft supersymmetry breaking parameters in the low-energy effective action after assuming some mechanism for supersymmetry breaking (see also \[49\]).
- Another interesting field of research is to combine D-brane constructions with flux compactifications (see the talk by G. Dall’Agata \[99\]). This has the advantage that due to the fluxes some more moduli could be fixed dynamically, which are so far left untouched by the D-branes. In addition non-vanishing fluxes may provide a way for supersymmetry breaking. E.g. in \[30, 31\] a scenario of D9-branes with open string F-flux (the T-dual mirror configuration to D6-branes at angles) plus some Ramond and NS 3-form flux was considered (see also \[33\] for a type IIA description). Here the D9-brane potential fixes some of the Kähler moduli fields, where the 3-form fluxes stabilize some of the complex structure moduli.
So far we have completely neglected the back-reaction of the D6-branes (or of non-vanishing background fluxes) on the space-time geometry. In general due to this back-reaction the internal space will not be any longer a Ricci-flat Calabi-Yau manifold. In the case of $\mathcal{N} = 1$ supersymmetry it will rather be a six-manifold with a connection with torsion, namely a space with a specific kind of $SU(3)$ group structure (see the talk by G. Dall’Agata [99]). One way to get some informations on these new spaces is to consider the M-theory lift of supersymmetric D-brane models. For intersecting D6-branes plus O6-planes the M-theory background will become purely geometrical, namely a certain singular 7-manifold with $G_2$ holonomy [100, 101, 102]. Thus our $\mathcal{N} = 1$ gauge threshold function is possibly related to the recently calculated Ray–Singer torsion of singular $G_2$ manifolds [97]. Hence on the M-theory side the threshold function is expected to be mapped to a topological quantity, most likely to the elliptic genus of the singular $G_2$ compactification manifold.

Of course, the question of how to select the correct ground state in string theory still remains an open problem (for some thoughts on the statistics of string vacua see [103]).

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