Charge symmetry violation in the doubly charmed cascade masses

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Abstract

We investigate the electromagnetic contribution to the charge symmetry breaking in the Ξ_{cc} baryon masses using a subtracted dispersion relation based on the Cottingham formula, following the formalism developed in an analysis of the octet baryon mass differences. In the absence of experimental information on the structure of charmed baryons, we use parameters for the electromagnetic structure of the Ξ_{cc} and the difference in its charge states obtained from lattice QCD and estimates of SU(4) symmetry breaking. We report a conservative estimate for the mass splitting of the doubly charmed cascades to be 8 ± 9 MeV. While a smaller isospin splitting is compatible with this result, surprisingly it does not preclude the large splitting reported by the SELEX Collaboration. We identify those quantities which could be determined from lattice QCD and which would reduce the quoted theoretical uncertainty.

Keywords: charmed baryons, lattice QCD, symmetry violation

1. Introduction

The total charge symmetry violation (CSV) in the mass of a baryon multiplet is a result of symmetry breaking arising from quark mass differences and electromagnetic forces. The inequality of the quark masses ($m_{u} \neq m_{d}$) is referred to as the strong component, while the energy difference arising from the electromagnetic interaction is referred to as the electromagnetic contribution. Thus, for small mass differences and weak electromagnetic coupling, the charge symmetry breaking for a baryon $B$ is given by the sum $\delta M_{B} = \delta M_{B}^{\text{strong}} + \delta M_{B}^{\gamma}$. Experimentally, the approximate mass splitting of the nucleon is $M_{p} - M_{n} \approx -1.3$ MeV [1]. More exotic baryon states, such as the singly charmed sigmas have an isospin mass splitting opposite to that of most particles, with $M_{\Sigma_{c}^{+}} - M_{\Sigma_{c}^{-}} \approx 1$ MeV [1].

It is notable that nearly all of the baryon isospin pairs have mass differences ranging from about 7 MeV to a couple of MeV [4]. However, in 2002 the SELEX collaboration at Fermilab reported the observation of two families of doubly charmed cascades, the $\Xi_{cc}^{+}(3443)$ and $\Xi_{cc}^{++}(3460)$ forming one isospin doublet, and the $\Xi_{cc}^{0}(3520)$ and $\Xi_{cc}^{++}(3541)$ forming another [2-4]. Thus the isospin mass splittings found by SELEX were reported to be 17 MeV and 21 MeV, respectively [4]. These mass splittings are much larger than those of any other hadronic isospin pair. To further complicate this puzzle, the LHCb Collaboration have reported the observation of a doubly-charged $\Xi_{cc}^{++}$ with a mass 3621 MeV. While we await further observations of the $\Xi_{cc}$ baryons to resolve the spectrum, it is of interest to investigate the degree of isospin symmetry breaking in these states.

The possibility of large mass splitting as observed by SELEX is of theoretical interest because it may indicate something extraordinary about the quark structure of heavy and doubly heavy baryons [5-6]. Estimates of the strong contribution to $M_{p} - M_{n}$ report this to be $\sim -2$–3 MeV [7-11]. The corresponding calculations of the doubly-strange cascade baryons suggest a strong contribution to the splitting $M_{\Xi^0} - M_{\Xi^{-}}$ on the order of $\sim 5$–6 MeV. The doubly-charmed baryon was anticipated to be of similar magnitude and, importantly, of the same sign. Thus, in order for CSV to yield a large and negative mass splitting for the doubly-charmed cascades, $M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^{+}}$, the electromagnetic self energy would have to be greater than the total mass splitting and thus as large as $\sim 20$ MeV in magnitude.

In this work, we provide an estimate of the electromagnetic charge symmetry breaking $\delta M_{\Xi_{cc}^{++}}^{\gamma} - \delta M_{\Xi_{cc}^{+}}^{\gamma}$ based on the Cottingham sum rule [12-14]. Since little experimental information is available for the structure of baryons beyond the octet, we use lattice results of the electromagnetic structure and estimates of SU(4) symmetry breaking to guide the analysis in its application to the exotic Ξ_{cc} systems.

2. Electromagnetic Self-Energy

A dispersion relation analysis of the electromagnetic self energy of a baryon $B$ may be written as a sum of four contributions [15-17]

$$\delta M_{B}^{\gamma} = \delta M_{B}^{\text{el}} + \delta M_{B}^{\text{inel}} + \delta M_{B}^{\text{sub}} + \delta M_{B}^{\text{ct}}.$$  (1)

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In the following sections we explore each term, adopting
the formalism of Ref. [17] which provides a minor revision
of the analysis presented by Walker-Loud et al. [15].

2.1. Elastic Contribution

The elastic contribution to the electromagnetic self energy
is given by

$$
\delta M_B^{el} = \frac{\alpha}{\pi} \int_0^{\Lambda_0} dQ \left[ \frac{3}{2} G_M^2 \frac{\sqrt{\tau_{el}}}{\tau_{el} + 1} \right. \\
+ \left. (G_E^2 - 2\tau_{el} G_M^2 \frac{(1 + \tau_{el})^{3/2} - \tau_{el}^{3/2} - \frac{3}{2} \sqrt{\tau_{el}}}{\tau_{el} + 1} \right],
$$

(2)

where $\Lambda_0$ is a renormalization scale, chosen to lie in a range
above which elastic contributions are negligible. Numerically,
we set the renormalization constant $\Lambda_0^2 = 20$ GeV$^2$
and calculate uncertainties by allowing variation over the
range $10 < \Lambda_0^2 < 30$ GeV$^2$. We find that the results are
insensitive to the choice of $\Lambda_0$ in this range. The kinematic
factor $\tau_{el} = Q^2/(4M_B^2)$, with $M_B$ the baryon mass.
We allow for a $\Xi_{cc}$ mass in the range $3.45 < M_{\Xi_{cc}} < 3.65$
GeV in light of the LHCb collaboration recently reporting
the observation of $\Xi_{cc}^{++}$ with a mass of 3621.40(78)
MeV [18]. For the moment it is unclear how this relates
to the states observed by SELEX. $G_E$ and $G_M$ are the
electric and magnetic elastic form factors for the baryon.
There is no experimental data on the charmed cascade
form factors, so we are guided by the empirical fits of
the nucleon form factors which take a dipole form:

$$
G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{1 + \frac{Q^2}{\mu_B^2} G_{E,M}(0)}
$$

(3)

The factors $(r_{E,M}^2)^B$ are the electric and magnetic charge
radii of the baryon $B$, $G_{E,M}(0)$ is the electric charge in units
of $e$, and $G_{E,M}(0)$ is proportional to the baryon magnetic
moment with $\mu_B = \sqrt{G_{M}^{2} G_{E}^{2}}$. Since experimental data
about form factors of the $\Xi_{cc}$ are unavailable, we rely on the
lattice computation of these electromagnetic form factors
reported in Ref. [6]. The baryon form factors can be
formulated as a sum of individual quark-sector contributions,
where the total form factors for $\Xi_{cc}^{+}/-/0$ are given as

$$
G_{E,M}^{\Xi_{cc}^{+/-0}} = 2Q_u G_{E,M}^{\Xi_{cc}^{+/-0},u}(Q^2) + Q_d G_{E,M}^{\Xi_{cc}^{+/-0},d}(Q^2),
$$

(4)

$$
G_{M}^{\Xi_{cc}^{+/-0}} = G_{M}^{\Xi_{cc}^{+/-0},u}(Q^2) + G_{M}^{\Xi_{cc}^{+/-0},d}(Q^2),
$$

(5)

$$
G_{E}^{\Xi_{cc}^{+/-0}} = 2Q_u G_{E}^{\Xi_{cc}^{+/-0},u}(Q^2) + Q_d G_{E}^{\Xi_{cc}^{+/-0},d}(Q^2),
$$

(6)

$$
G_{M}^{\Xi_{cc}^{+/-0}} = G_{M}^{\Xi_{cc}^{+/-0},u}(Q^2) + G_{M}^{\Xi_{cc}^{+/-0},d}(Q^2).
$$

(7)

Note that $Q_u,d,c$ are the charges of the respective quarks.

The numerical values of the charge radii used for the elastic
integral were obtained from Ref. [3], and the magnetic
moments, $\mu_{\Xi_{cc}^{+}/-}$ were determined from an average
of results based on quark models, chiral effective theories,
and lattice calculations [6, 19–26], taking uncertainties to
allow for variation over all of the values obtained in the
literature. The charge radii and magnetic moments used are in Table 1.

In Fig. 1 we illustrate the elastic integrands (Eq. 2) for the $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$. These are compared with the elastic
integrands for the proton and neutron computed using form factors determined by the Kelly parametrization [27],
as used in the dispersive analysis for the octet baryons in Ref. [17]. In the case of the octet baryon isospin pairs, $N$ and $\Sigma$ and $\Xi$, the elastic self-energy was found to provide the largest contribution to the electromagnetic self-energy
difference between charge states. The difference in the elastic self-energy between the proton and neutron is represented
by the area between the dark blue curves in Fig. 1. The difference between the elastic self-energies of the doubly charmed cascades (area between the lavender curves),
determined using the parameters in Table 1 clearly yields a greater contribution to the CSV between the two charge states. The result for the elastic contribution to the mass splitting of the $\Xi_{cc}$ is found in Table 2.

Table 1: Electric and magnetic parameters for $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$. Charge radii given in fm$^2$, and magnetic moments given in nuclear magnetons, determined from Refs. [6, 19–26].

| Parameter | Value |
|-----------|-------|
| $\langle r_{E,c}^2 \rangle$ | 0.095(9) |
| $\langle r_{E,u/d}^2 \rangle$ | 0.410(46) |
| $\langle r_{M,c}^2 \rangle$ | 0.089(11) |
| $\langle r_{M,u/d}^2 \rangle$ | 0.612(115) |
| $\mu_{\Xi_{cc}^{++}}$ | 0.75(30) |
| $\mu_{\Xi_{cc}^{+}}$ | -0.10(10) |

Figure 1: (Color online) Elastic integrand as given in Eq. 2 for the $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$, as well as for the proton and neutron.
2.2. Inelastic Contribution

Following Refs. [15 17], the inelastic contribution to the electromagnetic self-energy is given as

$$\delta M_B^{\text{inel}} = \int_{W_0}^{\infty} dW^2 \Omega_B^{\text{inel}}(W^2),$$

(8)

where \(W_0\) is the threshold for excited states. \(\Omega_B^{\text{inel}}(W^2)\) is the inelastic contribution to the self-energy associated with each intermediate hadronic state of invariant mass squared \(W^2\). As in Ref. [17], \(\Omega_B^{\text{inel}}(W^2)\) is given by

$$\Omega_B^{\text{inel}}(W^2) = \frac{\alpha}{\pi} \int_0^{\lambda_0} dQ \left\{ \frac{3F_1^B(W^2, Q^2)}{4M_B^2} \frac{2\tau^{3/2} - 2\tau \sqrt{1 + \tau} + \tau}{\tau} \right. $$

$$+ \frac{F_2^B(W^2, Q^2)}{(Q^2 + W^2 - M_B^2)} \left\{ (1 + \tau)^{3/2} - \tau^{3/2} - \frac{3}{2} \sqrt{\tau} \right\},$$

(9)

where \(\tau\) is the kinematic factor given by \(\tau = (W^2 + Q^2 - M_B^2)/2M_B^2Q^2\). \(F_1^B\) and \(F_2^B\) are the inelastic structure functions for the baryon. While we do not have empirical constraints on the structure functions for the charmed cascades, we can at least estimate the results of the integral [9] for these exotic baryons by calculating an approximate structure function for the \(\Xi_{cc}^{++\pm/+}\) based on the quark momentum fractions, \(\langle x \rangle\), that we expect each valence quark to carry. That is, we use the structure functions

$$F_1^B(x, Q^2) = \frac{1}{2} \sum_q Q_q^2 f_q^B(x, Q^2)$$

(10)

and

$$F_2^B(x, Q^2) = x \sum_q Q_q^2 f_q^B(x, Q^2),$$

(11)

where we sum over the relevant quark flavors, in this case \(u, d, s, c\). \(Q_q\) is the charge of quark \(q\) and \(f_q^B(x, Q^2)\) is the parton density function of quark \(q\) in baryon \(B\) at momentum fraction \(x\) and resolution determined by \(Q^2\). Taking the nucleon as an example and following Ref. [17], we assume charge symmetry [28] in the nucleon. In the case of the nucleon this means \(f_u^N = f_d^N \equiv f_u^N\) and \(f_d^N = f_u^N \equiv f_u^N\), where the doubly or singly represented quark densities are assumed to be the same in each nucleon. Thus,

$$f_u^N = 2 \cdot \frac{9}{15} (4F_1^u - F_1^d)$$

(12)

and

$$f_d^N = 2 \cdot \frac{9}{15} (4F_1^d - F_1^u)$$

(13)

Based on the ratios of the constituent quark masses, \(M_q\), of the valence quarks in the \(\Xi_{cc}^{++\pm/+}\) and the nucleon, we estimate the ratio of the quark momentum fractions to be

$$\frac{\langle x \rangle_{\Xi_{cc}^{++\pm/+}}}{\langle x \rangle_{\Xi_{cc}^{++\pm/+}}} \approx \frac{M_u/M_{\Xi_{cc}^{++\pm/+}}}{M_u/M_{\Xi_{cc}^{++\pm/+}}},$$

(16)

and

$$\frac{\langle x \rangle_{\Xi_{cc}^{++\pm/+}}}{\langle x \rangle_{\Xi_{cc}^{++\pm/+}}} \approx \frac{M_d/M_{\Xi_{cc}^{++\pm/+}}}{M_d/M_{\Xi_{cc}^{++\pm/+}}},$$

(17)

where \(l\) represents the light quark in the \(\Xi_{cc}\). Following Ref. [17], we use these ratios to scale the nucleon structure functions appearing in Eq. [9]. It follows from the linearity of Eq. [9] in \(F_1\) and \(F_2\) that

$$\delta M_{\Xi_{cc}^{++\pm/+}}^{\text{inel}} - \delta M_{\Xi_{cc}^{++\pm/+}}^{\text{inel}} = \frac{9}{15} (Q_u^2 - Q_d^2) \langle x \rangle_{\Xi_{cc}^{++\pm/+}} (4\delta M_u^{\text{inel}} - \delta M_d^{\text{inel}}).$$

(18)

We take the nucleon inelastic results from Ref. [17] to be \(\delta M_u^{\text{inel}} = 0.62(8)\) MeV and \(\delta M_d^{\text{inel}} = 0.53(7)\) MeV. Constituent quark masses for the proton are taken to be \(M_u = 336\) MeV, \(M_d = 340\) MeV. The electromagnetic structure of the \(\Xi_{cc}\) appears to vary from that of the nucleon, i.e. smaller charge radii of the heavy c quarks as determined in Refs. [6 7], so we use the constituent quark masses \(M_c = 1486\) MeV and \(M_t = 385\) MeV, as given in an investigation of heavy baryon spectroscopy based on a quark-diquark model [29]. Thus we find,

$$\frac{\langle x \rangle_{\Xi_{cc}^{++\pm/+}}}{\langle x \rangle_{\Xi_{cc}^{++\pm/+}}} = 0.30,$$

(19)

which yields a result for Eq. [18] of 0.10 MeV.

The resonance structures of the \(\Xi_{cc}^{++\pm/+}\) would however be markedly different than that of the nucleon. In particular, the reduction of the hyperfine interaction caused by the larger charm quark mass could reduce the splitting between the ground state and the spin-3/2 excited state by as much as a factor of 3. In Ref. [29] the spectrum of doubly heavy baryons is predicted using the quark-diquark model. The doubly charmed cascade is predicted to have resonances of the ground state \(\Xi_{cc}^{++\pm/+}\) and \(\Xi_{cc}^{++\pm/+}\) that are relatively closely spaced compared to the nucleon resonances. For example, the two lowest resonant excitations of the nucleon lie within 293 MeV (\(\Delta\)), and 500 MeV (Roper resonance) of the ground state nucleon, while the lowest two excitations of the \(\Xi_{cc}\) are predicted to lie within 132 MeV and 224 MeV [29]. Experimentally, SELEX proposed possible observations of \(\Xi_{cc}\) resonances lying within 320 MeV of each other [3]. As the contribution from the inelastic term calculated in the present section is relatively small compared to the contributions from the other terms it seems reasonable to seek an order of magnitude estimate of the inelastic contribution by scaling the result of Eq. [18] to account for the difference in the spectrum of the \(\Xi_{cc}\) compared with the nucleon. To this end, we scale the result of Eq. [18] by a factor of three since in a given range...
over $W^2$, the $\Sigma_{cc}$ would have about three times more resonance structures contributing to the integral. Thus, we take

$$\delta M_{\Sigma_{cc}}^{inel} - \delta M_{\Xi_{cc}}^{inel} \approx 3 \times 0.10 \text{ MeV}. \quad (20)$$

Based on this hypothesis we attach an uncertainty of 0.2 MeV to the earlier calculation, as summarized in Table 2.

### 2.3. Subtraction Terms

Following Ref. [17] and the analysis outlined in Ref. [15], we include a subtraction term given by

$$\delta M_{\Sigma_{cc}}^{sub} = \frac{-3\alpha}{16\pi M_B} \int_0^{\Lambda_0^2} dQ^2 T_1^B(0, Q^2), \quad (21)$$

where $\Lambda_0$ is the renormalization scale, as before, and the amplitude $T_1^B(0, Q^2)$ is a scalar function related to the Lorentz contracted Compton tensor, $T_1^B$. See Ref. [15] for the decomposition of this tensor. The momentum dependence of $T_1^B$ can be summarized in a model independent fashion as

$$T_1^B(0, Q^2) = 2[G_{\beta}^B(Q^2)]^2 - 2[F_{\beta}^B(Q^2)]^2 + T_4^{B, inel}(Q^2), \quad (22)$$

where $G_{\beta}^B(Q^2)$ and $F_{\beta}^B(Q^2)$ are the magnetic and Dirac elastic form factors for the baryon $B$. This elastic portion of the integral can be readily computed using the elastic form factors computed for $\Sigma_{cc}^{++}$ and $\Xi_{cc}^+$ from the elastic term. See Table 2 for a summary of this result.

The remaining term in the subtraction component is an inelastic portion, which can be modeled based on knowledge of the low $Q^2$ and high $Q^2$ behavior of the function. Following Ref. [17], we estimate the difference between the two charge states ($\Lambda_B$) by computing

$$T_1^{\Lambda_B, inel}(Q^2) = \frac{Q^2 2M_B \beta^{\Lambda_B}/(\alpha + Q^4 C^{\Lambda_B}/(3\Lambda_0^2)^3)}{(1 + Q^2/(3\Lambda_0^2))^3}, \quad (23)$$

where $M_B$ is the average mass of the two charge states, $\beta^{\Lambda_B}$ is the difference between the magnetic polarizabilities between the two charge states and $\Lambda_B$ is a mass scale characterizing the Compton interaction. The factor $C^{\Lambda_B}$ describes the large-$Q^2$ behavior of the inelastic subtraction component:

$$C^{\Lambda_B} = C^{\Lambda_B}(0) + C^{\Lambda_B}(2), \quad (24)$$

$$C^{\Lambda_B}(0) = -4M_B \left( Q^2 m_{uc} - Q^2 m_{dc} \right) (\sigma_u^B - \sigma_d^B), \quad (25)$$

$$C^{\Lambda_B}(2) = 4M_B^2 \left( Q^2 - Q^2_0 \right) (x) \xi^{cc}. \quad (26)$$

The light quark masses are denoted by $m_{u,d}$, and $\tilde{m}$ represents the average. The factor $\sigma_q^B$ is the average sigma term for the quark $q$ in the pair of $\Sigma_{cc}$ charge states, e.g. $\sigma_u^B = (\sigma_{uc}^B + \sigma_{dc}^B)/2$. We estimate this based on the calculated baryon octet sigma terms in lattice QCD analyses [9, 20], which find $\sigma_u^N - \sigma_d^N = -13(2)$ MeV, $\sigma_u^\Sigma - \sigma_d^\Sigma = -6(1)$ MeV and $\sigma_u^{\Xi} - \sigma_d^{\Xi} = -3(1)$ MeV. These indicate that the difference between the $u$ and $d$ contributions to the mass of baryon decreases as heavier flavors are introduced. Thus, we estimate $\sigma_u^{\Xi} - \sigma_d^{\Xi} = -1$ MeV.

While we allow for 100% variation in this value, it has no appreciable impact on the final result.

The spin-2 contribution to the asymptotic behavior of the subtraction function was overlooked in Ref. [17], and has been recently discussed in Ref. [31]. The key point of relevance to the Cotton effect is that the spin-2 contribution in the subtraction function exactly cancels against the corresponding term in the inelastic part of the dispersion integral [13]. The electromagnetic quark self energy on the sigma term remains as the only term that contributes to the logarithmic running of the complete self energy of the nucleon. Even though the spin-2 contribution is almost an order of magnitude larger than the spin-0 contribution, its omission in Ref. [17] has negligible effect on the final results reported.

Just as the lack of knowledge of the magnetic polarizability $\beta^{\Lambda_B}$ and the mass scale $\Lambda_B$ dominated the uncertainty in the calculation of the mass splitting between the octet baryons in Ref. [17], so it dominates in the present calculation. The best constraint on the magnetic polarizabilities of the nucleon using chiral effective field theory and experimental data are given in Ref. [32] to be

$$\beta^p = 3.2(5) \times 10^{-4} \text{fm}^3 \quad (27)$$

$$\beta^n = 3.7(15) \times 10^{-4} \text{fm}^3 \quad (28)$$

$$\Rightarrow \beta^{\Lambda_N} = -0.5(16) \times 10^{-4} \text{fm}^3. \quad (29)$$

A recent lattice QCD analysis [26] of the magnetic polarizabilities of the proton shows that for non-physical heavy pion masses, the magnetic polarizability remains relatively constant. Based on this observation and a lack of results for the $\Sigma_{cc}$ magnetic polarizabilities, we follow Ref. [17] by taking the same value and uncertainty range as for the nucleon polarizability.

To first order in $Q^2$, the inelastic $T_1^{B, inel}$ amplitude term at low $Q^2$ is given by the magnetic polarizability. Motivating the equation [23] we note that the next to leading order in $Q^2$ is determined by the polarizability form factor $F_\beta$, as given in the chiral perturbation theory analysis for the nucleon by Birse and McGovern [33]. There, the form factor is given as

$$F_\beta = 1 + \frac{Q^2}{[\Lambda_3^N]^2} + O(Q^4), \quad (30)$$

where the expansion depends on the mass scale $\Lambda_3^N = 460(108) \text{ MeV}$ [33]. In the dispersion relation estimate of the octet baryons of Ref. [17], a heavier mass scale is used, based on the fact that the physics is governed by the interaction with the heavier $s$ quarks of the $\Sigma^+/-$ and $\Xi^0/-$ baryons. The mass scale used in that analysis is
\( \Lambda_B^{\Xi} = 0.7(3) \) GeV. Here, considering the massive \( c \) quark contribution to the mass scale, we choose \( \Lambda_B^{\Xi_{cc}} = 1.0(3) \) GeV.

Using the parameters above, we compute the elastic and inelastic subtraction terms summarized in Table 2 – see Fig. 2 to compare the subtraction inelastic integrand for the \( \Xi_{cc} \) and the nucleon. The shaded regions represent the uncertainty in the integrand due to the uncertainty in the magnetic polarizability of the charge states. Note that the range of reasonable values for the subtraction inelastic contribution for the \( \Xi_{cc} \) is much larger than that for the nucleon, allowing for a much greater contribution to the mass splitting between the \( \Xi_{cc}^{++} \) and the \( \Xi_{cc}^{+} \).

2.4. Counter Terms

The final contributions in the dispersion analysis are the counter terms, which are given to account for the scale dependence determined by \( \Lambda_0 \). Following the analysis for the computation of the baryon octet charge symmetry violation of Ref. [17], we take the leading order contribution to be

\[
\delta M_{\Delta B} = -\frac{3\alpha}{16\pi M_B} C^{\Delta B} \log \left( \frac{\Lambda^2_0}{\Lambda^2_\Delta} \right),
\]

where we follow Ref. [15] in taking \( \Lambda^2_\Xi = 100 \) GeV\(^2\). The large value of \( M_B \) and the small value of \( C^{\Delta B} \), as obtained in Eq. 24 yield a value on the order of \( 10^{-4} \) MeV for \( \delta M^{\Delta B}_{\Xi_{cc}} \). As a result we include the result in Table 2 as \( \approx 0 \).

3. Total

We have calculated a conservative estimate of the total electromagnetic symmetry breaking in the doubly charmed cascade baryons using a subtracted dispersion relation. In summary, we find

\[
\delta M^{\gamma}_{\Xi_{cc}^{-}} - \delta M^{\gamma}_{\Xi_{cc}^{+}} = 8(9) \text{ MeV}.
\]

In Table 2 we summarize each of the contributions to the total electromagnetic self energy, as given by the subtracted dispersion relation analysis (Eq. 31). The greatest contributions come from the elastic and subtraction inelastic terms. As depicted in Fig. 4 the elastic term is greater for the \( \Xi_{cc} \) than the nucleon. This is attributed to the small charge radii for the \( c \) system, as well as a small magnetic moment of the \( \Xi_{cc}^+ \). Thus, the investigation of the quark-diquark model of the doubly charmed baryons is important to our understanding of the charge symmetry violation in these exotic particles. Similarly, we may compare the next largest term, the subtraction inelastic term, calculated for the \( \Xi_{cc} \) to that of the nucleon (see Eq. 32). The large contribution from this term can be attributed to the larger mass \( M_B \) for the \( \Xi_{cc} \) and the largeness of the mass scale associated with the magnetic polarizability, \( \Lambda_\beta \) (see Fig. 2).

In Table 3 we summarize the parameters and uncertainty ranges used in each of the elastic, inelastic, subtraction and counter terms. The error propagation due to each parameter is shown in the uncertainty yield column. From the table, we see that calculated mass splitting is relatively stable for the range of masses for the \( \Xi_{cc} \). As described earlier, within the range \( 10 \text{ GeV}^2 < \Lambda^2_0 < 30 \text{ GeV}^2 \), variation in the renormalization scale \( \Lambda_0 \) does not effect the result. By far the largest source of uncertainty in the calculation is that associated with the magnetic polarizability and its mass scale.

4. Summary

The large uncertainty range in our calculation of the electromagnetic self-energy is a result of the current uncertainty in the magnetic polarizability of baryons in general. However, within reasonable bounds on these parameters, we find that a relatively large CSV is possible for the \( \Xi_{cc} \) system. The dispersion relation estimates of Ref. [17] for the electromagnetic self-energy of the octet baryons give contributions to the CSV of about 1 MeV, which are comparable to lattice analyses as in Refs. [10, 34, 35], as well as the analysis of Ref. [15]. Thus, it is interesting to find the possibility of such a large electromagnetic self-energy for the doubly heavy cascade systems.

It is of considerable experimental and theoretical interest to continue the investigation of the properties of doubly heavy baryons. However, we must face the sources of uncertainty in these calculations. In this respect, our current inability to experimentally access some variables such as the magnetic moment of the \( \Xi_{cc} \) system.
Table 2: Decomposition of the electromagnetic contributions to the Ξcc baryon mass splittings as defined in Eq. [1]. We also list the results of the dispersive analysis of Ref. [17] for the nucleon electromagnetic self-energy contributions for comparison. All masses given in MeV.

| Baryon        | δM_{el} | δM_{inel} | δM_{sub,el} | δM_{sub,inel} | δM_{ct} | δM_{γ} |
|---------------|---------|-----------|-------------|---------------|---------|--------|
| Ξ_{cc}^{++} - Ξ_{cc}^{--} | 3.19(35) | 0.30(10)  | 1.65(36)    | 3(9)          | ≈ 0     | 8(9)   |
| p - n         | 1.401(7) | 0.089(42) | -0.635(7)   | 0.18(35)      | 0.006   | 1.04(35)|

Table 3: Parameters and contributions to overall uncertainty for each term of Eq. [1].

| Term            | Parameter | Value         | Contribution to Total Uncertainty (MeV) | Reference |
|-----------------|-----------|---------------|----------------------------------------|-----------|
| All             | Λ_0 (GeV) | 20(10)        | ≈ 0                                    |           |
| Elastic &       | M_B ≡ M_B (MeV) | 3519(100)    | ≠ 0.02(el), 0.02(sub)                   | [2,4]     |
| Subtraction Elastic | ⟨r^2_E,c⟩ (fm^2) | 0.095(9)     | 0.04(el), 0.01(sub)                     | [4]       |
|                 | ⟨r^2_E,l⟩ (fm^2) | 0.410(46)    | 0.16(el), -0.04(sub)                    | [4]       |
|                 | ⟨r^2_M,c⟩ (fm^2) | 0.089(11)    | 0.03(el), 0.04(sub)                     | [4]       |
|                 | ⟨r^2_M,l⟩ (fm^2) | 0.612(115)   | 0.13(el), 0.16(sub)                     | [4]       |
|                 | μ_{Ξ^{++} cc} (µ_N) | -0.10(0.10) | 0.01(el), 0.01(sub)                     | [4,19,20] |
|                 | μ_{Ξ^{+} cc} (µ_N) | 0.75(0.3)    | 0.27(el), 0.316(sub)                    | [4,19,20] |
| Inelastic       | M_B ≡ M_B (MeV) | 3519(100)    | ≈ 0                                    | [2,4]     |
|                 | ⟨x⟩_{Ξ^{cc}} / ⟨x⟩_{p} | 0.30(10)     | 0.10                                   | [17]      |
| Subtraction Inelastic | M_B ≡ M_B (MeV) | 3519(100)    | ≈ 0                                    | [2,4]     |
|                 | β_{AB} ≡ β_{Ξ^{++} cc} - β_{Ξ^{--} cc} (fm^3) | -0.5(1.6) × 10^{-4} | 8.5                                   | [17]      |
|                 | Λ_β (GeV)  | 1.0(3)       | 2.8                                    | [17]      |
|                 | σ_{Ξ^{cc}} - σ_{ξ^{cc}} (MeV)  | -1(1)        | ≈ 0                                    | [17]      |
| Counter         | M_B ≡ M_B (MeV) | 3519(100)    | ≈ 0                                    | [2,4]     |
|                 | Λ_1^2 (GeV^2) | 100(100)     | ≈ 0                                    | [15]      |

as magnetic polarizability of exotic baryons, means that in the near future we will need to look for future lattice simulations. As the magnetic polarizability information is the greatest source of uncertainty in the present calculation, we eagerly await results from lattice simulations to obtain a more precise understanding of the charge symmetry violation in the Ξcc system, and for other exotic heavy baryons as well.

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