Ramond-Ramond Field Transformation

Yungui Gong

Physics Department, University of Texas at Austin, Austin, TX 78712, U.S.A.

Abstract

We find that the mixture of Ramond-Ramond fields and Neveu-Schwarz two form are transformed as Majorana spinors under the T-duality group $O(d, d)$. The Ramond-Ramond field transformation under the group $O(d, d)$ is realized in a simple form by using the spinor representation. The Ramond-Ramond field transformation rule obtained by Bergshoeff et al. is shown as a specific simple example. We also give some explicit examples of the spinor representation.

Email: ygong@physics.utexas.edu
1 Introduction

The transformation of the fields from the Neveu-Schwarz (NS) sector under T-duality is well established. The Ramond-Ramond (RR) field transformation was first given in [1]. The authors in [1] got the RR field transformation by identifying the same RR fields and RR moduli in $d = 9$ supergravity coming from both ten dimensional type IIA and type IIB supergravity theories compactified down to nine dimensions. Unfortunately this method gives only a specific T-duality transformation, namely Buscher’s T-duality transformation [2]. It is hard to get the generalized T-duality group $O(d, d)$ transformations by this method. Recently, Hassan derived the RR field transformation under $SO(d, d)$ group by working on the worldsheet theory [3]. The RR field transformation under Buscher’s T-duality was also discussed by Cvetić, Lü, Pope and Stelle using the Green-Schwarz formalism [4]. If we compactify the $d = 9$ supergravity further down to lower dimensions, we know that the lower dimensional solution has $O(d, d, R)$ transformation and how the NS-NS fields which are assumed to be independent of $d$ coordinates transform under this group [5]. Therefore, we need to find the RR field transformation under the general $O(d, d, R)$ group.

The RR fields transform as the Majorana-Weyl spinors of $SO(d, d)$ [6]. RR fields transforming as the spinors of $SO(d, d)$ group is discussed in more detail from the algebraic decomposition of U duality group in [7]. The spinor representation idea was further developed by Fukuma, Oota and Tamaka [8]. It is not the RR potentials that transform as the Majorana-Weyl spinor of $SO(d, d)$; it is the mixed fields of RR potentials and NS-NS two form that transform as the Majorana-Weyl spinor of $SO(d, d)$. However, since the full T-duality group is $O(d, d)$, we expect to use the Majorana spinor representation of $O(d, d)$. Note also that $SO(d, d)$ transformations cannot interchange type IIA and type IIB theories. In this paper, we use RR fields to construct spinors of $O(d, d)$ explicitly. As a simple application we use the Majorana spinor representation to show the RR field transformations between type IIA and type IIB under T-duality. By using the Majorana spinor and the tensor representation of $O(d, d, R)$ group, we can get more general solution generating rules.

We define the RR potentials $C_{p+1} = (1/(p + 1)!)(\sqrt{\beta} + p_{p+1})dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{p+1}}$. Following the definition given by [8], we define the new mixed fields as

\[
D_0 \equiv C_0, \\
D_2 \equiv C_2 + B_2 \wedge C_0, \\
D_4 \equiv C_4 + \frac{1}{2}B_2 \wedge C_2 + \frac{1}{2}B_2 \wedge B_2 \wedge C_0.
\]

The RR field strengths are $F = e^{-B} \wedge dD$ [8], here

\[
D \equiv \sum_{p=0}^{4} D_p, \quad F \equiv \sum_{p=1}^{5} F_p.
\]

More explicitly, we have

\[
F_1 = dD_0, \\
F_2 = dD_1, \\
F_3 = dD_2 - B_2 \wedge dD_0, \\
F_4 = dD_3 - B_2 \wedge dD_1, \\
F_5 = dD_4 - B_2 \wedge dD_2 + \frac{1}{2}B_2 \wedge B_2 \wedge dD_0.
\]

\[2\text{The author thanks S. Ferrara for pointing out this reference.}\]
We also use the convention

$$\int d^d x \sqrt{-g}|F_p|^2 = \int d^d x \sqrt{-\frac{p!}{g}} g^{\mu_1 \nu_1} \cdots g^{\mu_p \nu_p} F_{\mu_1 \mu_p} F_{\nu_1 \nu_p}. \quad (4)$$

2 \hspace{1cm} d = 10 \hspace{1cm} \text{Type IIA and Type IIB Reduction to } d = 9

The action of ten dimensional type IIA supergravity can be written as

$$S_{10}^{IIA} = \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{-G} e^{-2\Phi} \left[ R(G) + 4G^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} |H_3|^2 \right]$$

$$- \frac{1}{4\kappa_{10}^2} \int d^{10} x \sqrt{-G} \left( |F_2|^2 + |F_4|^2 \right) + \frac{1}{4\kappa_{10}^2} \int d^{10} x B_2 \wedge dC_3 \wedge dC_3, \quad (5)$$

where $H_3 = dB_2$, $F_2 = dC_1 = dD_1$, $F_4 = dC_3 + H_3 \wedge C_1 = dD_3 - B_2 \wedge dD_1$ and the subscript number of a form denotes the degree of the form. Now we dimensionally reduce the action (5) to nine dimensions by the vielbein,

$$E_M^A = \begin{pmatrix} e^\mu_a \\ eA^{(1)}_\mu \end{pmatrix}, \quad E_A^M = \begin{pmatrix} e^\mu_a \\ -e^\nu A^{(1)}_\nu \end{pmatrix}. \quad (6)$$

The dimensionally reduced nine dimensional action for the NS and R sector is

$$S_9 = \frac{1}{2\kappa_9^2} \int d^9 x \sqrt{-g} e^{-2\phi} \left[ R(g) + 4g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - e^{-2} g^{\mu\nu} \partial_\mu \partial_\nu e \right]$$

$$- \frac{1}{2} e^2 |F_2^{(1)}|^2 - \frac{1}{2} e^{-2} |F_2^{(2)}|^2 - \frac{1}{2} |H_3^{(1)}|^2 \right]$$

$$- \frac{1}{4\kappa_9^2} \int d^9 x \sqrt{-g} \left( e^{-2} |F_2|^2 + e^{-1} g^{\mu\nu} \partial_\mu D_\nu D_\nu + e^{-1} |H_3^{(2)}|^2 + e |F_4|^2 \right), \quad (7)$$

where

$$e^2 = G_{xx}, \quad g_{\mu\nu} = G_{\mu\nu} - G_{xx} A^{(1)}_\mu A^{(1)}_\nu, \quad (8a)$$

$$A^{(1)}_\mu = \frac{G_{\mu x}}{G_{xx}}, \quad A^{(2)}_\mu = B_{\mu x} \quad (8b)$$

$$A_\mu = D_\mu - A^{(1)}_\mu D_\mu, \quad F^{(i)}_{\mu\nu} = \partial_\mu A^{(i)}_\nu - \partial_\nu A^{(i)}_\mu, \quad (8c)$$

$$B^{(1)}_{\mu\nu} = B_{\mu\nu} + \frac{1}{2} A^{(1)}_\mu A^{(2)}_\nu - \frac{1}{2} A^{(2)}_\mu A^{(1)}_\nu, \quad B^{(2)}_{\mu\nu} = D_{\mu\nu}, \quad (8d)$$

$$\phi = \Phi - \ln G_{xx}/4, \quad D_{\mu\nu} = D_{\nu\mu}, \quad (8e)$$

$$H^{(1)}_3 = dB_2^{(1)} - \frac{1}{2} (A_1^{(1)} \wedge F_2^{(2)} + A_1^{(2)} \wedge F_2^{(1)}), \quad (8f)$$

$$H^{(2)}_3 = dB_2^{(2)} - B_2^{(1)} \wedge dD_1 + \frac{1}{2} A_1^{(2)} \wedge A_1^{(1)} \wedge dD_1 - A_1^{(1)} \wedge F_2^{(1)} \wedge (F_2 + F_2^{(1)} D_1), \quad (8g)$$

$$F_4 = dD_3 - B_2^{(1)} \wedge dD_1 + \frac{1}{2} A_1^{(1)} \wedge A_1^{(2)} \wedge dD_1 + H_3^{(2)} \wedge A_1^{(1)}, \quad (8h)$$

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and $x$ is the compactified coordinate. Here we follow the general prescription of dimensional reduction given in [10]. For example, the lower dimensional field strength comes from the higher dimensional field strength as 

$$H^{(1)}_{\mu \nu \rho} = e^a_{\mu} e^b_{\nu} e^c_{\rho} E^M_a E^N_b E^P_c H_{MNP}.$$ 

The action (7) can be obtained from the type IIB supergravity in ten dimensions also if we use the following vielbein for the IIB theory [1]

$$E^A_M = \left( e^a_{\mu} e^{\mu} e^{A(2)}_{\nu} \right), \quad E^M_A = \left( e^a_{\mu} - e^a_{\mu} A^{(2)}_{\nu} \right),$$

together with the following definitions,

$$e^{-2} = G_{xx}, \quad g_{\mu \nu} = G_{\mu \nu} - G_{xx} A^{(2)}_{\mu} A^{(2)}_{\nu},$$

$$A^{(1)}_{\mu} = B_{\mu x}, \quad A^{(2)}_{\mu} = \frac{G_{\mu x}}{G_{xx}}, \quad D = D_x,$$

$$B^{(1)}_{\mu \nu} = B_{\mu \nu} - \frac{1}{2} A^{(1)}_{\mu} A^{(2)}_{\nu} + \frac{1}{2} A^{(1)}_{\nu} A^{(2)}_{\mu}, \quad B^{(2)}_{\mu \nu} = D_{\mu \nu},$$

$$\phi = \hat{\Phi} - \ln G_{xx}, \quad \bar{D}_{\mu \nu} = D_{\mu \nu x}.$$ 

The type IIB ten dimensional supergravity action we use is

$$S^{IIB}_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[ R(G) + 4G^{MN} \partial_M \hat{\Phi} \partial_N \hat{\Phi} - \frac{1}{2} |\mathcal{H}_3|^2 \right]$$

$$- \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left( |F_1|^2 + |F_3|^2 + \frac{1}{2} |F_5|^2 \right) + \frac{1}{4\kappa_{10}^2} \int d^{10}x \mathcal{B}_2 \wedge dC_4 \wedge dC_2,$$

together with the self dual constraint on $F_5$. Now we can get Buscher’s T-duality transformations [2] from Eqs. (8a)-(8e) and Eqs. (10a)-(10e) as follows

$$\tilde{g}_{xx} = \frac{1}{g_{xx}}, \quad \tilde{g}_{\mu \nu} = \frac{B_{\mu \nu}}{g_{xx}}, \quad \tilde{g}_{\mu x} = g_{\mu x} - \frac{g_{\mu x} g_{\nu x} - B_{\mu x} B_{\nu x}}{g_{xx}},$$

$$\tilde{B}_{\mu x} = \frac{g_{\mu x}}{g_{xx}}, \quad \tilde{B}_{\mu \nu} = \frac{B_{\mu \nu} g_{\nu x} - B_{\nu x} g_{\mu x}}{g_{xx}},$$

$$\tilde{\phi} = \hat{\phi} - \frac{1}{2} \ln g_{xx},$$

$$\tilde{D}_x = D_x, \quad \tilde{D}_{\mu} = D_{\mu x}, \quad \tilde{D}_{\mu x} = D_{\mu}, \quad \tilde{D}_{\mu \nu} = D_{\mu \nu x}.$$ 

From the above transformation rules (12a)-(12d), we have the following transformations in terms of the original RR potentials,

$$\tilde{C}_x = C_x, \quad \tilde{C}_{\mu} = C_{\mu x} + B_{\mu x} C_x, \quad \tilde{C}_{\mu x} = C_{\mu} + \frac{g_{\mu x} C_{\nu x} - g_{\nu x} C_{\mu x}}{g_{xx}},$$

$$\tilde{C}_{\mu \nu} = C_{\mu \nu x} - \frac{3}{2} B_{[\mu x} C_{\nu x]} - \frac{3}{2} B_{x[\mu} C_{\nu]} - \frac{6 g_{\mu x} B_{\nu x} C_{\rho x}}{g_{xx}}.$$ 

In general we should consider the $O(d, d, R)$ transformations. The group element $\Omega$ of $O(d, d, R)$ satisfies

$$\Omega^T J \Omega = J, \quad J = \begin{pmatrix} 0 & \mathbb{I}_d \\ \mathbb{I}_d & 0 \end{pmatrix}. \quad (14)$$

If we put the NS sector fields in a $2d$ by $2d$ matrix

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} = \begin{pmatrix} \mathbb{I} & 0 \\ B & \mathbb{I} \end{pmatrix} \begin{pmatrix} G^{-1} & 0 \\ 0 & G \end{pmatrix} \begin{pmatrix} \mathbb{I} & -B \\ 0 & \mathbb{I} \end{pmatrix}, \quad (15)$$

where $G = [G_{ij}]$ and $B = [B_{ij}]$ are $d \times d$ matrices, $i$ and $j$ run over the compactified or independent $d$ coordinates. Let

$$A^{(1)}_{\mu m} = G_{\mu m}, \quad A^{(1)m} = G^{mn} A^{(1)}_{\mu n}, \quad (16)$$

$$A^{(2)}_{\mu m} = B_{\mu m} + B_{mn} A^{(1)}_{\mu n}, \quad A^i_{\mu} = \begin{pmatrix} A^{(1)m}_{\mu} \\ A^{(2)}_{\mu m} \end{pmatrix}, \quad (17)$$

$$g_{\mu \nu} = G_{\mu \nu} - G_{mn} A^{(1)m}_{\mu} A^{(1)n}_{\nu}, \quad (18)$$

$$\phi = \Phi - \frac{1}{4} \ln \det(G_{mn}), \quad (19)$$

$$B_{\mu \nu} = \hat{B}_{\mu \nu} + \frac{1}{2} A^{(1)m}_{\mu} A^{(2)}_{\nu m} - \frac{1}{2} A^{(1)m}_{\nu} A^{(2)}_{\mu m} - A^{(1)m}_{\mu n} B_{mn} A^{(1)n}_{\nu}, \quad (20)$$

where $\Phi, G_{\mu m}, G_{\mu \nu}, G_{mn}, \hat{B}_{\mu \nu}, B_{\mu m}$ and $B_{mn}$ are the original NS fields. The $O(d, d)$ transformations for the NS fields are

$$M \rightarrow \Omega M \Omega^T, \quad A^i_{\mu} \rightarrow \Omega_{ij} A^j_{\mu}, \quad g_{\mu \nu} \rightarrow g_{\mu \nu}, \quad \phi \rightarrow \phi, \quad B_{\mu \nu} \rightarrow B_{\mu \nu}. \quad (21)$$

### 3 Spinor Representation

In this section, we will show that the RR fields transform as the Majorana spinors. We can write the general group element $\Omega$ of $O(d, d, R)$ as

$$\Omega = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{pmatrix}, \quad (22)$$

with $\mathcal{A} \mathcal{B}^T + \mathcal{B} \mathcal{A}^T = \mathcal{C} \mathcal{D}^T + \mathcal{D} \mathcal{C}^T = 0, \mathcal{A} \mathcal{D}^T + \mathcal{D} \mathcal{A}^T = \mathcal{C} \mathcal{B}^T + \mathcal{B} \mathcal{C}^T = 1$, $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ are $d \times d$ matrices. We can also show that $\mathcal{D} = \mathcal{C} \mathcal{A}^{-1} \mathcal{B} + (\mathcal{A}^{-1})^T$. The $O(d, d, R)$ group can be generated by the following three matrices \[1\]

$$\Lambda_C = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix}, \quad \Lambda_R = \begin{pmatrix} (R^T)^{-1} & 0 \\ 0 & R \end{pmatrix}, \quad \Lambda_i = \begin{pmatrix} -\mathbb{I} + e_i & e_i \\ e_i & -\mathbb{I} + e_i \end{pmatrix}, \quad (e_i)_{jk} = \delta_{ij} \delta_{jk}, \quad (23)$$

where $C^T = -C, R \in GL(d, R)$ and $i, j$, and $k = 1, \ldots, d$. The action of $\Lambda_C$ shifts the NS two-form by the matrix $C$. Under the action of $\Lambda_R, G \rightarrow RGR^T, B \rightarrow RBR^T$. For the group $O(d, d, Z)$, we need to restrict the matrix elements to be integers.
The Dirac matrices satisfy \( \{ \Gamma_r, \Gamma_s \} = 2J_{rs} \) with \( r \) and \( s = 1, \ldots, 2d \). Let
\[
a_i = \frac{\Gamma_{d+i}}{\sqrt{2}}, \quad a_i^\dagger = \frac{\Gamma_i}{\sqrt{2}}, \quad i = 1, \ldots, d.
\] (24)
Then we have \( \{ a_i, a_j^\dagger \} = \delta_{ij} \mathbb{1} \), \( \{ a_i, a_j \} = \{ a_i^\dagger, a_j^\dagger \} = 0 \). Define the vacuum to be \( a_i |0\rangle = 0 \), we can get the representation (Fock) space as
\[
|\alpha\rangle = (a_1^\dagger)^{i_1} \ldots (a_d^\dagger)^{i_d} |0\rangle, \quad i_1, \ldots, i_d = 0 \text{ or } 1.
\] (25)
The spinor representation of the \( O(d, d) \) group is given by
\[
S(\Omega) \Gamma_s S(\Omega)^{-1} = \sum_r \Gamma_r \Omega^r_s.
\] (26)
For convenience we can define the operator corresponding to a matrix \( \Omega \) as
\[
\Omega \Gamma_s = \sum_r \Gamma_r \Omega^r_s \Gamma_s, \quad \Omega |\beta\rangle = \sum_\alpha |\alpha\rangle S_{\alpha\beta}(\Omega).
\] (27)
The operators for the three generating matrices are 8
\[
\Lambda_C = \exp \left( \frac{1}{2} C_{ij} a_i a_j \right), \quad \Lambda_i = \pm (a_i + a_i^\dagger),
\] (28)
\[
\Lambda_R = (\det R)^{-1/2} \exp \left( a_i A_i^j a_j^\dagger \right), \quad R = R_i^j = \exp(A_i^j),
\] (29)
where the repeated indices are summed. We choose + sign for the \( \Lambda_i \) operator. The new mixed \( D \) fields form a spinor as follows: for \( d = 1 \),
\[
\chi_\alpha = (D, D_x), \quad \chi_{\mu\alpha} = (D_\mu, D_\mu x),
\]
\[
\chi_{\mu\nu\alpha} = (D_{\mu\nu}, D_{\mu\nu x}), \quad \chi_{\mu\nu\rho\alpha} = (D_{\mu\nu\rho}, D_{\mu\nu\rho x}),
\]
\[
\ldots,
\]
with \( |\alpha\rangle = (|0\rangle, \ a_1^\dagger |0\rangle) \); for \( d = 2 \),
\[
\chi_\alpha = (D, D_x, D_y, D_{yx}), \quad \chi_{\mu\alpha} = (D_\mu, D_{\mu x}, D_{\mu y}, D_{\mu y x}),
\]
\[
\chi_{\mu\nu\alpha} = (D_{\mu\nu}, D_{\mu\nu x}, D_{\mu\nu y}, D_{\mu\nu y x}), \quad \chi_{\mu\nu\rho\alpha} = (D_{\mu\nu\rho}, D_{\mu\nu\rho x}, D_{\mu\nu\rho y}, D_{\mu\nu\rho y x}),
\]
\[
\ldots,
\]
with \( |\alpha\rangle = (|0\rangle, \ a_1^\dagger |0\rangle, \ a_1^\dagger a_2^\dagger |0\rangle, \ a_1^\dagger a_2^\dagger a_3^\dagger |0\rangle) \) and so on. The fields \( \chi \) transform as
\[
|\tilde{\chi}_{\mu_1 \ldots \mu_p\alpha}\rangle = \sum_\beta S^{-1}(\Omega^T)_{\alpha\beta} |\tilde{\chi}_{\mu_1 \ldots \mu_p\beta}\rangle.
\] (30)
For instance, the spinor representation matrix of \( O(1, 1) \) for \( \Lambda_i \) is
\[
S ( (\Lambda^T)^{-1} ) = S(\Lambda) = \Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\] (31)
From the spinor matrix (31), it is easy to get Buscher’s T-duality transformations (12a)-(12d), (13a) and (13b) by combining Eqs. (21) and (30). The spinor representation of $SO(1,1)$ for $\Lambda_i\Lambda_j$ is

$$S(\Lambda^2) = \Lambda^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (32)$$

This is a trivial identity transformation. Furthermore it gives a Majorana-Weyl spinor representation.

## 4 More Examples

In order to discuss the solution generating transformations, we focus on $O(d) \otimes O(d)$ group in this section. We embed the $O(d)$ matrices $R$ and $S$ into $O(d,d)$ matrix $\Omega$. Because the metric $J$ of $O(d,d)$ is rotated from the diagonal metric $\eta$ by

$$J = \mathcal{R} \eta \mathcal{R}, \quad \eta = \begin{pmatrix} -\mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix}, \quad \mathcal{R} = \frac{\sqrt{2}}{2} \begin{pmatrix} -\mathbb{I} & \mathbb{I} \\ \mathbb{I} & \mathbb{I} \end{pmatrix},$$

so

$$\Omega = \mathcal{R}^{-1} \begin{pmatrix} S & 0 \\ 0 & R \end{pmatrix} \mathcal{R} = \frac{1}{2} \begin{pmatrix} R + S & R - S \\ R - S & R + S \end{pmatrix}.$$  

Note that $\Omega$ is also an element of $O(2d)$, so $(\Omega^T)^{-1} = \Omega$.

For example, if we take $R = -\mathbb{I} + 2e_i$, $S = -\mathbb{I}$, then we recover the T-duality $\Lambda_i$ discussed before. If we choose

$$S = \mathbb{I}, \quad R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix},$$

then we have the spinor representation

$$S(\Omega) = \begin{pmatrix} \cos \frac{\theta}{2} & 0 & 0 & -\sin \frac{\theta}{2} \\ 0 & \cos \frac{\theta}{2} & \sin \frac{\theta}{2} & 0 \\ 0 & -\sin \frac{\theta}{2} & \cos \frac{\theta}{2} & 0 \\ \sin \frac{\theta}{2} & 0 & 0 & \cos \frac{\theta}{2} \end{pmatrix}. \quad (33)$$

For flat background with zero $B$ field, RR fields transform the same way as the $D$ fields. This result is consistent with that obtained in [3].

If one of the coordinate is timelike, we have

$$\Omega = \frac{1}{2} \begin{pmatrix} \eta(S + R)\eta & \eta(R - S) \\ (R - S)\eta & S + R \end{pmatrix}, \quad \mathcal{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\eta & \mathbb{I} \\ \eta & \mathbb{I} \end{pmatrix}, \quad (34)$$

where $\eta$ is the Minkowski metric, $S$ and $R$ are $O(d - 1,1)$ matrices satisfying $S\eta S^T = \eta$ and $R\eta R^T = \eta$. For example,

$$R = S = \begin{pmatrix} \cosh \alpha & \sinh \alpha \\ \sinh \alpha & \cosh \alpha \end{pmatrix}.$$
generate the boost transformation along $t$-$x$ coordinates,

\[
S^{-1}(\Omega_b^T) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cosh \alpha & \sinh \alpha & 0 \\
0 & \sinh \alpha & \cosh \alpha & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}. \tag{35}
\]

More explicitly, the boost transformation for the RR field and $B$ field is

\[
\tilde{B}_{\mu t} = B_{\mu t} \cosh \alpha + B_{\mu x} \sinh \alpha, \quad \tilde{B}_{\mu x} = B_{\mu t} \sinh \alpha + B_{\mu x} \cosh \alpha, \tag{36a}
\]

\[
\tilde{C}_{\mu \nu t} = C_{\mu \nu t} \cosh \alpha + C_{\mu \nu x} \sinh \alpha, \quad \tilde{C}_{\mu \nu x} = C_{\mu \nu t} \sinh \alpha + C_{\mu \nu x} \cosh \alpha, \tag{36b}
\]

\[
\tilde{B}_{tx} = B_{tx}, \quad \tilde{C}_{\mu \nu tx} = C_{\mu \nu tx}, \quad \tilde{B}_{\mu \nu} = B_{\mu \nu}, \quad \tilde{C}_{\mu \nu} = C_{\mu \nu}. \tag{36c}
\]

Finally let us choose

\[
S = \begin{pmatrix}
\cosh \alpha & - \sinh \alpha \\
- \sinh \alpha & \cosh \alpha
\end{pmatrix}, \quad R = \begin{pmatrix}
\cosh \alpha & \sinh \alpha \\
\sinh \alpha & \cosh \alpha
\end{pmatrix}. \tag{37}
\]

In this case, we have

\[
S^{-1}(\Omega_s^T) = \begin{pmatrix}
\cosh \alpha & 0 & 0 & \sinh \alpha \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\sinh \alpha & 0 & 0 & \cosh \alpha
\end{pmatrix}. \tag{38}
\]

For the background $B_{\mu \nu} = 0$, $g_{11} = 1$ and $g_{01} = 0$, the transformations of NS-NS and RR fields are

\[
\tilde{g}_{00} = \frac{g_{00}}{1 + (1 + g_{00}) \sinh^2 \alpha}, \tag{39a}
\]

\[
\tilde{g}_{11} = \frac{1}{1 + (1 + g_{00}) \sinh^2 \alpha}, \tag{39b}
\]

\[
\tilde{B}_{01} = \frac{(1 + g_{00}) \sinh 2\alpha}{2[1 + (1 + g_{00}) \sinh^2 \alpha]}, \tag{39c}
\]

\[
\tilde{g}_{\mu 0} = \frac{g_{\mu 0} \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \tag{39d}
\]

\[
\tilde{g}_{\mu 1} = \frac{g_{\mu 1} \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \tag{39e}
\]

\[
\tilde{B}_{\mu 0} = \frac{-g_{00}g_{\mu 1} \sinh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}, \tag{39f}
\]

\[
\tilde{B}_{\mu 1} = \frac{g_{\mu 0} \sinh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha}. \tag{39g}
\]
\[
\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{(g_{\mu 0} g_{\nu 0} + g_{00} g_{\mu 1} g_{\nu 1}) \sinh^2 \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha},
\]
\[
\tilde{B}_{\mu\nu} = \frac{(g_{\mu 0} g_{\nu 1} - g_{\mu 1} g_{\nu 0}) \sinh \alpha \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha},
\]
\[
\tilde{C} = C \cosh \alpha - C_{01} \sinh \alpha,
\]
\[
\tilde{C}_0 = C_0, \quad \tilde{C}_1 = C_1, \quad \tilde{C}_{\mu} = C_{\mu} \cosh \alpha - C_{\mu 01} \sinh \alpha,
\]
\[
\tilde{C}_{01} = \frac{C_{01}[1 + 2(1 + g_{00}) \sinh^2 \alpha] \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha} - \frac{C[1 + (1 + g_{00})(\sinh^2 \alpha + \cosh^2 \alpha)] \sinh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha},
\]
\[
\tilde{C}_{\mu 0} = C_{\mu 0} + \frac{g_{00} g_{\mu 1} \sinh \alpha (C \cosh \alpha - C_{01} \sinh \alpha)}{1 + (1 + g_{00}) \sinh^2 \alpha},
\]
\[
\tilde{C}_{\mu 1} = C_{\mu 1} - \frac{C g_{\mu 0} \sinh \alpha \cosh \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha} + \frac{C_{01} g_{\mu 0} \sinh^2 \alpha}{1 + (1 + g_{00}) \sinh^2 \alpha},
\]
\[
\tilde{C}_{\mu \nu} = C_{\mu \nu} \cosh \alpha - C_{\mu \nu 01} \sinh \alpha + \frac{(C_{01} \sinh \alpha - C \cosh \alpha)(g_{\mu 0} g_{\nu 1} - g_{\mu 1} g_{\nu 0}) \sinh 2\alpha}{2[1 + (1 + g_{00}) \sinh^2 \alpha]},
\]
\[
e^{-2\tilde{\phi}} = e^{-2\phi}[1 + (1 + g_{00}) \sinh^2 \alpha].
\]

5 Discussion

The RR field transformations are very simple in terms of the new mixed fields \(D\). It is very easy to see the RR field transformations from the spinor representations. For any group element \(\Omega \in O(d, d)\), we can get the spinor representation \(S(\Omega)\) from Eq (26) or Eq. (27). We can introduce higher degree potentials and field strengths with some constraints as shown in [8]. With the extra potentials, the action for the RR and Chern-Simons terms can be written in a simple way. This may suggest that the \(D\) fields are the natural RR potentials. We can apply the transformation Eqs. (21) for the NS-NS fields and the transformation Eqs. (30) for the RR fields to get more general solution generating rules.

Note Added: In the second version of paper [3], Hassan gave a general transformation of \(D\) field by spinor representation and discussed the equivalence of the RR field transformations between his supersymmetry method and the spinor representation.

References

[1] E. Bergshoeff, C. Hull and T. Ortín, Duality in The Type II Superstring Effective Action, hep-th/9504081, Nucl. Phys. B 451, 547 (1995); E. Bergshoeff, M. De Roo, M.B.
Green, G. Papadopoulos and P.K. Townsend, *Duality of Type II 7-branes and 8-branes*, hep-th/9601150, Nucl. Phys. B 470, 113 (1996); P. Meessen and T. Ortín, *An SL(2,Z) Multiplet of nine-dimensional Type II Supergravity Theories*, hep-th/9806120, Nucl. Phys. B 541, 195 (1999).

[2] T. Buscher, *A Symmetry of the String Background Field Equations*, Phys. Lett. 194B, 59 (1987).

[3] S. Hassan, *T-duality, Space-time Spinors and R-R Fields in Curved Backgrounds*, hep-th/9907152; S. Hassan, *SO(d,d) Transformations of Ramond-Ramond Fields and Spacetime Spinors*, hep-th/9912236.

[4] M. Cvetič, H. Lü, C.N. Pope and K.S. Stelle, *T-duality in the Green-Schwarz Formalism and the Massless/Massive IIA Duality Map*, hep-th/9907202.

[5] G. Gibbons and D. Wiltshine, *Black Holes in Kaluza-Klein Theory*, Ann. Phys. 167, 201 (1986); 176, 393(E) (1987); A. Sen, *O(d) ⊗ O(d) Symmetry of the Space of Cosmological Solutions in String Theory, Scale Factor Duality, and Two Dimensional Black Holes*, Phys. Lett. 271B, 295 (1991); Twisted Black p-brane Solutions in String Theory, 274B, 34 (1992); S. Hassan and A. Sen, *Twisted Classical Solutions in Heterotic String Theory*, Nucl. Phys. B 375, 103 (1992); K. Meissner and G. Veneziano, *Symmetries of Cosmological Superstring Vacua*, Phys. Lett. 267B, 33 (1991); *Manifestly O(d,d) Invariant Approach to Space-time Dependent String Vacua*, hep-th/9110004, Mod. Phys. Lett A 6, 3397 (1991).

[6] D. Brace, B. Morariu and B. Zumino, *Dualities of the Matrix Model from T-duality of the Type II String*, hep-th/9810099, Nucl. Phys. B 545, 192 (1999); T-duality and Ramond-Ramond Backgrounds in the Matrix Model, hep-th/9811213, Nucl. Phys. B 549, 181 (1999); E. Witten, *String Theory Dynamics in Various Dimensions*, hep-th/9503124, Nucl. Phys. B 443, 85 (1995).

[7] L. Andrianopoli, R. D’Auria, S. Ferrara, P. Fré and M. Trigiante, *R-R Scalars, U-Duality and Solvable Lie Algebras*, hep-th/9611014, Nucl. Phys. B 496, 617 (1997).

[8] M. Fukuma, T. Oota and H. Tanaka, *Comments on T-dualities of Ramond-Ramond Potentials*, hep-th/9907132.

[9] M.B. Green, C.M. Hull and P.K. Townsend, *D-Brane Wess-Zumino Actions, T-duality and The Cosmological Constant*, hep-th/9604119, Phys. Lett. B 382, 65 (1996).

[10] J. Maharana and J. Schwarz, *Noncompact Symmetries in String Theory*, hep-th/9207016, Nucl. Phys. B 390, 3 (1993).

[11] A. Giveon, M. Porrati and E. Rabinovici, *Target Space Duality in String Theory*, hep-th/9401133, Phys. Rep. 244, 77 (1994).