Peak effect in a superconductor/normal metal strip being in vortex-free state

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We theoretically predict that the critical current $I_c$ and magnetization $M$ of hybrid superconductor/normal-metal (SN) strip may have nonmonotonic dependence on perpendicular magnetic field - so called peak effect. In contrast to familiar peak effect, which is connected either with vortex entry to the superconductor or with peculiarities of vortex pinning, the found phenomenon exists at low fields, in the vortex-free (Meissner) phase. We argue that the effect appears at specific parameters of studied hybrid structure when its in-plane current-supervelocity relation has two maxima. We expect that the same peak effect may exist in two-band superconductors (like MgB$_2$) where similar current-supervelocity dependence was predicted at low temperatures.

The influence of a perpendicular magnetic field $H$ on transport properties of type-II superconductors has been the subject of numerous studies. Usually the critical current $I_c$ of bulk superconductors is determined mainly by pinning of vortices on defects and it monotonically decreases with increasing $H$. However in conventional low-$T_c$ superconductors there has been observed a peak in $I_c(H)$ just below the upper critical field $H_{c2}$ (see for example [1-3]). The peak in $I_c(H)$ is also accompanied by a peak in the dependence of magnetization $M$ on $H$ and this phenomenon is called as the peak effect. The peak effect near $H_{c2}$ is explained by a softening of the vortex lattice [4 and 5]. Also there was discovered the peak located significantly below $H_{c2}$ both in the low-$T_c$ [6 and 7] and high-$T_c$ superconductors [8 and 9]. The origin of this type of peak is explained by the transition from a quasiordered vortex lattice to an amorphous vortex glass state.

In a homogeneous superconducting strip the critical current could be determined not by the bulk pinning of vortices but by the edge barrier for their entrance [10-13]. Usually effect of edge barrier is pronounced in a thin strip/bridge with thickness $d_S$ less than the London magnetic field penetration depth $\lambda$ and at relatively low magnetic fields when there is no dense vortex lattice [13-16]. In relatively narrow strip (with width $W \ll \lambda = \lambda^2/d_S$) one may observe peak in $I_c(H)$ near the field for first vortex entry $H_{c1}$ [14-16] (the same peak has been observed in thin Pb/In and Nb strips placed in parallel magnetic field [15 and 20]). It originates from the entrance of the vortex row at some field which does not exit the strip and it prevents subsequent vortex entry [21]. It is interesting that competition of the bulk pinning and edge barrier also may lead, at some parameters, to the peak effect at low magnetic fields, as it was predicted in Ref. [22].

Here we argue that peak in $I_c(H)$ and $M(H)$ may arise even in the vortex-free state. Below we show that it could be realized, for example, in a hybrid superconductor/normal metal (SN) thin strip with large ratio of resistivities of S and N layers $\rho_S/\rho_N \gg 1$ in the normal state. In Ref. [22] it has been shown that dependence of superconducting sheet current density $J_s$ (in ordinary S strip $J_s = j_s d_S$ where $j_s$ is a superconducting current density) on supervelocity $v_s$ or supermomentum $hq = h(\nabla \varphi + 2\pi A/\Phi_0) \sim v_s$ (here $\varphi$ is the phase of superconducting order parameter and $A$ is the vector potential) may have two maxima at low temperature. The first maximum at small $q$ is connected with suppression of the proximity induced superconductivity in the N layer, while the second maximum at large $q$ comes from suppression of superconductivity in the S layer. The predicted dependence is rather different from $J_s(q)$ of ordinary one-band superconductor, which has only one maximum, but it resembles the dependence $J_s(q)$ for two-band superconductors [24 and 25]. In that case different maxima correspond to destruction of superconductivity in different bands.

Our model system is shown in Fig. 1. The SN strip with width $W$ has two layers: superconducting one with thickness $d_S$ and the normal metal layer with thickness $d_N$. In calculations we use one and two-dimensional Usadel equation for normal $g = \cos \Theta$ and anomalous $f = \sin \Theta \exp(i\varphi)$ quasi-classical Green functions, assuming that angle $\Theta$ depends only on $x$ and $y$ and length of the SN strip $L \rightarrow \infty$ (equations and details of the model are presented in Appendix and could be found in Ref. [23]). Our model is not able to take into account vortex states so we consider here only the Meissner (vortex-free) state. We consider narrow strip with width smaller than the magnetic field penetration depth $\lambda$ of the single S layer to neglect the contribution of screening currents to vector potential which we choose as: $A = (0, 0, H y)$. In our model we assume that current reaches the critical value when $q(y = W/2) = q_c$, where...
\( q(y) = \nabla \varphi + 2\pi A(y)/\Phi_0 \) (\( \nabla \varphi(y) = \text{const} \)) and \( q_c \) is the critical value of \( q \) corresponding to the reaching depairing current density at the edge. This condition corresponds to instability of the Meissner state with respect to vortex entry [21].

To find \( I_c(H) \) we numerically solve either 1D or 2D Usadel equations (see Appendix). In 1D model we split SN strip to filaments with width \( \xi_c \) and assume that
\[
J_s(y) = \int j_s(x, y)dx = \int j_s(x, q(y))dx = J_s(q(y)) \quad (q \text{ depends on y-coordinate of filament})
\]
and may be found from solution of 1D Usadel equation (in this case \( \Theta \) has dependence only on \( x \) coordinate). Then we calculate \( I_c = \int J_s(q(y))dy \). In 2D model we solve 2D Usadel equation with given \( q(y) \) and find \( I_c = \int j_s(x, y)dxdy \) (\( \Theta \) depends both on \( x \) and \( y \)). The difference between these approaches is that in 1D model we neglect proximity effect between adjacent filaments which brings the difference between \( J_s(y) \) and \( J_s(q(y)) \). We expect that the filament model gives quantitatively correct results when \( W \gg \xi_N = \sqrt{\hbar D_N/k_B T} \) [22], where \( D_N \) is a diffusion coefficient in \( N \) layer.

In calculations we normalize lengths in units of \( \xi_c = \sqrt{\hbar D_S/k_BT_{c0}} \), where \( T_{c0} \) is the critical temperature and \( D_S \) is the diffusion coefficient of \( S \) layer. Sheet current density \( J_s \) is normalized in units of depairing sheet current density \( J_{dep}(0) = I_{dep}(0)/d_S \) of \( S \) layer at \( T = 0 \) and the magnetic field is measured in units of \( H_c = \Phi_0/2\pi W \xi_c \) (this field is about of first vortex entry field [12] and [13] to the strip at \( I = 0 \)). We choose ratio of resistivities (diffusion coefficients) \( \rho_S/\rho_N = D_N/D_S = 100 \) which corresponds to NbN, NbTiN, MoN or MoSi as a superconductor and Ags, Cu or Au as a normal metal.

In Fig. 2(a) we show temperature evolution of \( |J_s(q)| \) (it was found from solution of 1D Usadel equation) which is used for calculation of the critical current in the filaments model. With decreasing temperature the dependence \( |J_s(q)| \) transforms from the ordinary one (with one maximum) to the dependence with two maxima located at \( q = q_{c1} \) and \( q = q_{c2} \) (note qualitative similarity with \( |J_s(q)| \) for the two-band superconductor MgB\(_2\) [24] and [25]). The first maximum comes from the suppression of proximity-induced superconductivity in the \( S \) layer where \( q = q_{c1} \propto \sqrt{T/D_N} \). The second maximum comes from the suppression of superconductivity in the \( S \) layer where \( q = q_{c2} \propto \sqrt{T/D_S} \gg q_{c1} \). The increase of the amplitude of first maximum at low temperatures is explained by the enhancement of the proximity-induced superconductivity while the ‘strength’ of the intrinsic superconductivity in \( S \) layer is already saturated and amplitude of second maximum weakly depends on temperature at low \( T \). As it is discussed in Ref. [22] such an evolution in \( J_s(q) \) should lead to the kink on dependence \( I_c(T) \) at low \( T \) (when \( |J_s(q_{c1})| \) becomes larger than \( |J_s(q_{c2})| \)). The same kink is also predicted in Refs. [24] and [25] for MgB\(_2\) and it is caused by the similar change of \( J_s(q) \) with temperature.

The dependence \( I_c(H) \) (see Fig. 2(b)) changes with the temperature accordingly to transformation of \( J_s(q) \). In deed, external magnetic field changes the distribution of \( q \) across the strip (see inset in Fig. 2(c)). When the width of the SN strip is much larger than \( \xi_N \) one may assume that local \( J_s(y) \) is determined only by local \( q(y) \). With increasing magnetic field \( q \) decreases in the strip (except at the edge \( y = W/2 \)) and it leads to monotonous decrease of \( |J_s| \) and critical current \( I_c = \int J_sdy \) when dependence \( |J_s(q)| \) has only one maximum. However, with decreasing temperature the additional maximum appears at low \( q \). At first it leads to flattening of \( I_c(H) \) (see Fig. 2(b)) at \( T = 0.2T_{c0} \) because of flattening of \( J_s(q) \). When the height of the first maximum becomes larger than the second one the dependence \( I_c(H) \) changes drastically (see Fig. 2(b) at \( T = 0.1T_{c0} \)). At low fields \( I_c \) drops fast with increase of \( H \) because of much smaller value of \( q(W/2) = q_c = q_{c1} \ll q_{c2} \). At some field (marked by

\[
\int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy = \int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy = \int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy = \int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy
\]

\[
\int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy = \int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy = \int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy = \int J_s(q)\sqrt{\xi_c^2 - (y-y_c)^2} dy
\]
calculations at the magnetic field $H$; currents at low fields than at high fields. We expect similar behavior in SN strip too. Because vortex states cannot be described by the used model we are

At some magnetic field $q$ and $J_c(q)$ change the sign at $y = -W/2$. It means that vortices, which enter at opposite edge $(y = W/2)$ cannot exit the SN strip. In ordinary S strip it leads to the peak in $I_c(H)$ [21] and [29].

We expect similar behavior in SN strip too. Because vortex states cannot be described by the used model we are bounded by the field $q_{c2} \Phi_0 / 2\pi W$ at which $q(-W/2) = 0$.

Discussed above features could be seen only for sufficiently wide strips. In a relatively narrow strip with $W \lesssim \xi_N$ the proximity effect from the adjacent regions plays important role and nonmonotonous behavior is smeared out (see Fig. 2(c)).

It is known, that in the ordinary superconductors the peak in $I_c(H)$ is followed by the peak in the magnetization curve $M(H)$ (or vice versa). In SN strip we also find the peak in $M(H) = \iint [r \times \mathbf{j}_n] dx dy / (2c(d_S + d_N)W)$ (see Fig. 3). But in contrast to the ordinary peak effect the peaks are located at different fields for $I_c(H)$ and $M(H)$ dependencies. The reason for this is following. In absence of the current the evolution of $q(y)$ with increasing of $H$ is different to the situation with current $I = I_c(H)$ (see inset in Fig. 3); in this case $q(y) = 2\pi A(y) / \Phi_0$. It results to larger screening currents at low fields than at high $H$ (at $T = 0.1T_{c0}$) and peak is located at lower field. Here we stop calculations at the magnetic field $H = \Phi_0 q_{c2} / (\pi W)$ when $|q(\pm W/2)| = q_{c2}$ and we expect vortex entrance to the SN strip.

We believe that the same effect should exist in SS' bilayer where S' is a superconductor which has large diffusion coefficient (low resistivity in the normal state, for example Al, Pb or Sn). Due to large $D_{S'}$ the superconductivity in S' layer should be destroyed at smaller $q$ and the current-supercurrent selection dependence will have two maxima at proper choice of $d_S$, $d_{S'}$ and temperature. Because qualitatively similar $J_c(q)$ dependence was predicted for two-band superconductor MgB$_2$ (see [24] and [25]), and, hence, a peak or plateau in $I_c(H)$ should be observed in MgB$_2$ thin strip at fields $\lesssim q_{c2} \Phi_0 / 2\pi W$. But important condition for experimental observation of predicted effect is approaching of $I_c(H = 0)$ to the depairing current of SN, SS' or MgB$_2$. Most easily critical current about of $I_{dep}$ could be probably reached in SN system how it has been demonstrated recently for MoN/Cu strip [27]. However, in MgB$_2$ strips/bridges depairing current has not been reached yet. In Refs. [30–32] $I_c \simeq 15 - 30\%$ of the depairing current was claimed which is not large enough for observation of the predicted peak effect.

To conclude, we hope that the experimental observation of the peak or plateau on $I_c(H)$ and/or $M(H)$ dependencies at low fields would indirectly confirm existence of two peaks in $J_c(q)$ dependence in SN, SS' hybrid structures or many-band superconducting materials.

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Appendix: Usadel model

To calculate superconducting properties of the SN strip, we use the Usadel model for normal $g = \cos \Theta$ and anomalous $f = \sin \Theta \exp(i\varphi)$ quasi-classical Green functions inside both S and N layers. We neglect the dependence of $\Theta$ on the longitudinal coordinate $z$ since length of the SN strip $L \to \infty$ and the system is uniform in this direction. Therefore we use the one-dimensional (1D) Usadel equation

$$
\frac{hD}{2} \frac{\partial^2 \Theta}{\partial x^2} - \left( \frac{h\omega_n + \frac{hD}{2} q^2 \cos \Theta}{\frac{hD}{2} q^2 \cos \Theta} \right) \sin \Theta + \Delta \cos \Theta = 0
$$

(A.1)

and the two-dimensional (2D) Usadel equation

$$
\frac{hD}{2} \left( \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right) - \left( \frac{h\omega_n + \frac{hD}{2} q^2 \cos \Theta}{\frac{hD}{2} q^2 \cos \Theta} \right) \sin \Theta + \Delta \cos \Theta = 0.
$$

(A.2)

Here $D$ is a diffusion coefficient ($D = D_S$ and $D = D_N$ in superconducting and normal layers, respectively), $\omega_n = \pi k_B T (2n + 1)$ are the Matsubara frequencies ($n$ is an integer number), $\Delta$ is the superconducting order parameter, which is nonzero only in the S layer. Coordinate axes are
presented in Fig. 1. \( \Delta \) should satisfy the self-consistency equation

\[
\Delta \ln \left( \frac{T}{T_{c0}} \right) = 2\pi k_B T \sum_{\omega_n > 0} \left( \sin \Theta_S - \frac{\Delta}{\hbar \omega_n} \right),
\]

where \( T_{c0} \) is the critical temperature of single \( S \) layer in the absence of magnetic field. Equations (A.1-A.2) is supplemented by the Kupriyanov-Lukichev boundary conditions between layers [33] with fully transparent interfaces

\[
D_S \frac{d\Theta_S}{dx} \bigg|_{x=d_S-0} = D_N \frac{d\Theta_N}{dx} \bigg|_{x=d_S+0}.
\]

On the interfaces between the system and vacuum we use \( d\Theta/dn = 0 \).

The superconducting current density is calculated as

\[
j_s(x, y) = \frac{2\pi k_B T}{\rho} q \sum_{\omega_n > 0} \sin^2 \Theta,
\]

where \( \rho \) is the resistivity of corresponding layer. To find \( j_s(x, y) \), we numerically solve either equation (A.1) or (A.2) and equation (A.3). Equations are solved by an iteration procedure using the Newton method combined with a tridiagonal matrix algorithm. Obtained \( \Theta(x, y) \) is inserted in equation (A.3) to find \( \Delta \) and then iterations repeat until the self-consistency is achieved.

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Because of jump-like change of $q$ this transition is of the first kind and there is a kink on dependence $I_c(H)$. During the transition the electric field should appear which provides change of $q$ everywhere in the SN strip. How this switching happens is an interesting question but it is beyond of scope of our work because it needs consideration of the time-dependent problem.

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