Research on Time Delay Feedback Control of Lateral Vibration Axially Moving Viscoelastic Beam

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Abstract. The nonlinear dynamic behavior of a viscoelastic beam in axially variable motion under time delay is studied. Considering that the axial velocity pulsation and radial tension change periodically with time under the action of time delay, the viscous damping and finite support stiffness are taken into account, and the viscoelastic constitutive relationship adopts Kelvin model. A mathematical model of lateral vibration of an axially moving viscoelastic beam with changes in axial velocity and tension under time delay is established, and a partial differential-integral control equation describing the lateral nonlinear vibration of an axially moving beam is given. Based on the Galerkin truncation method, the control equations established based on the mechanical model are discretized, so that the direct multi-scale method obtains the numerical solution of the lateral nonlinear vibration of the axially moving beam under the action of time delay, and determines the nonlinear dynamic behavior of the system under the action of time delay. By analyzing the numerical solution of the vibration displacement and velocity at the midpoint of the beam, the whole process of the average velocity of the axially moving structure along the axis, the amplitude of the disturbance tension and the change of the viscoelastic coefficient is simulated. It provides a numerical theoretical basis for the study of the nonlinear dynamic behavior of axially moving beams under time delay.

Through the research in this article, the introduction of time delay is studied, and the time delay is used to analyze the axial force that changes along the radial direction of the beam caused by the speed pulsation, and the theoretical framework for nonlinear vibration analysis of the axially moving viscoelastic beam is constructed to expand the time delay. The application scope of the nonlinear vibration theory and nonlinear dynamics theory, the development and improvement of the approximate analysis and numerical solution of the nonlinear continuum (especially the gyro body) under the action of time delay, to provide a theoretical basis for the analysis and design of the corresponding engineering system and technical reserves.

1. Introduction

With the development of science and technology, the active control of vibration has been received in aerospace, machinery, shipbuilding and other engineering fields. More and more attention has been paid and it has been widely used in practical applications. However, the active control of vibration is inevitable. Bring time delay, although in practical applications, the amount of time delay is very small, but it often affects the stability of the entire system. And system performance [1-4]. Ignoring the time delay may lead to erroneous results. In most cases, the time delay is. It is considered to be an unfavorable factor in the control of the vibration system, and it has a great impact on the stability of the system, and even
causes very complex dynamic behavior\cite{4-8}. Even if it is small, it cannot be ignored, so the time delay needs to be taken into consideration. Time-delay control system to achieve time-delay control of vibration. Aiming at typical nonlinear models, through a lot of research studies have shown that time delay can be used as a parameter to suppress nonlinear vibrations\cite{9-11}.

At present, the ways to suppress vibration are divided into active control and passive control. Passive control does not require external force. It uses damping devices inside the structure to absorb the energy during vibration to reduce vibration. However, passive control is subject to the characteristics of external excitation, and generally has a good effect on high-frequency vibration, and it has a poor control effect on low-frequency vibration. As the theoretical system of modern control theory matures, the advantages of active control are gradually being discovered. Active control requires the input of external energy. It uses the real-time acquisition of the state quantity for feedback adjustment, and then achieves the suppression of structural vibration. Because the active control does not depend on the characteristics of external excitation, and the control effect is significantly better than passive control. Therefore, the active control of flexible structures has received more and more attention, and many research results have been published.

However, active control inevitably has time delay, which is an inherent property of the system, such as file transmission delay, satellite signal delay, image delay when watching live broadcast, and image shooting and transmission delay in engineering. In the past, people always control the convenience and always ignore the delay. However, even a very small time delay may have a great impact on the system, and even be caused by essential errors.

2. Research on the time-delay stability of axial viscoelastic variable-speed motion

2.1 Dynamic model
Consider a uniform Euler beam whose density is \( \rho \), cross-sectional area is \( A \), the length between the two ends of the support is \( L \), the moment of inertia of the section around the neutral axis is \( I \), the coefficient of viscoelasticity is \( \alpha \), and the modulus of elasticity is \( E \) to change with time. The speed \( \Gamma(t) \) moves along the axial direction. The analysis of the bending vibration of Euler beam is based on the plane assumption of the cross section, that is, the cross section always remains a plane during the bending deformation of the Euler beam. We use the mixed Eulerian-Lagrangian description. Figure 1 shows the physical model of a viscoelastic Euler beam with axially variable motion. Where \( a \) is the location of the speed sensor and \( b \) is the location of the exciter.

![Fig.1 Physical model of viscoelastic Euler beam with axially variable motion](image)

2.1.1 Dynamic equation of axial viscoelastic variable-speed motion
The dimensionless dynamic equation\cite{12} is obtained by applying Hamilton's principle to the axially variable-speed movement Euler beam shown in Figure 2.1 as

\[
\begin{align*}
\frac{\partial^2 v}{\partial t^2} + 2\gamma \frac{\partial v}{\partial x} + \left[ \alpha \gamma^2 - (x - 1)^2 \right] \frac{\partial^2 v}{\partial x^2} + k_t \frac{\partial^4 v}{\partial x^4} \\
+ \left[ \alpha (v_{xxxx} + \gamma v_{xxxxxx}) \right] + \frac{1}{2} k_x x v_{xx} \int_0^1 v_x^2 \, dx + f(x_0, \tau) = 0
\end{align*}
\]

(1)

\( v(x,t) \) is the lateral displacement and radial displacement of the Euler beam at the axial coordinate \( x \) and time \( t \), the comma "," in the subscript represents partial differential, \( \gamma \) is the initial axial velocity, and \( \gamma t \)
is the velocity Dynamic, $\alpha$ is the coefficient of viscoelasticity, $k_f$ and $k_1$ support stiffness and shear deformation, $f(x_0,\tau) = \beta v(t) \delta(x-x_0)$, $x_0$ is the position of the actuator, and later No further explanation.

### 2.1.2 Truncated system of time-delay dynamic equations

The Galerkin method is used to discretize the equation (2.1). For the sake of simplicity, considering the gyro effect, $N=2$ can be obtained.

\[
\begin{align*}
\dot{q}_1 + \frac{16}{3} \gamma q_1 + \alpha \pi^4 q_1 - [c\gamma^2 + \frac{1}{2} \gamma - 1] q_1, q_2 - \frac{128}{9} \gamma q_2 - \frac{128}{3} \gamma \alpha \pi^4 q_2 + \\
k_1^2 \pi^4 q_1 + \frac{1}{4} k_1^2 \pi^4 q_1 + \beta q_1 + \beta, q_2 = 0 \\
\dot{q}_2 + 16 2 \gamma q_1 + 16 \alpha \pi^4 q_1 - 4[c\gamma^2 + \frac{1}{2} \gamma] q_1, q_2 - \frac{16}{9} \gamma q_1 + \frac{8}{3} \gamma \alpha q_1, q_2 + \\
16 k_1^2 \pi^4 q_1 + k_1^2 \pi^4 q_1 + 4 q_2 + \beta q_1 + \beta, q_2 = 0
\end{align*}
\]

Further reduced to

\[
\begin{align*}
\dot{q}_1 + A \dot{q}_2 + B q_1 + C q_1 + D_1 q_2 + E_1 (q_1, q_2) + \alpha q_1 + \alpha, q_2 = 0 \\
\dot{q}_2 + A_2 q_1 + B_2 q_2 + C_2 q_1 + D_2 q_2 + E_2 (q_1, q_2) + \alpha q_1 + \alpha, q_2 = 0
\end{align*}
\]

Where

\[
\begin{align*}
q_{it} &= q_1(t-\tau), C_1 = -\left(c\gamma^2 + \frac{1}{2} \gamma - 1 - \frac{128}{3} \gamma \alpha \pi^4 \right), A = \frac{16}{3} \gamma, B = \alpha \pi^4, \\
D_1 &= \left(\frac{64}{9} \gamma + \frac{128}{3} \gamma \alpha \pi^4 \right), E_1 = \frac{1}{4} k_1^2 \pi^4 q_1, D_2 = \left(c\gamma^2 + \frac{1}{2} \gamma - 1 - 16 k_1^2 \pi^4 \right), \\
A_2 &= \frac{16}{3} \gamma, B_2 = 16 \alpha \pi^4, C_2 = \frac{8}{3} \gamma \alpha \pi^4 - \frac{16}{9} \gamma \alpha, E_2 = k_1^2 \pi^4 q_2, (q_1, q_2) = (q_1^2, 4 q_2^2), \\
\beta &= -2 \alpha \pi \sin^2(\pi x_0), \beta_2 = -2 \alpha \pi \sin(\pi x_0) \sin(2\pi x_0), \beta_3 = -2 \alpha \pi \sin^2(2\pi x_0)
\end{align*}
\]

### 2.2 Steady-state dynamic response

#### 2.2.1 Uncontrolled system

Given the parameters $\gamma=2$, $\alpha=0.0002$, $k_f=0.8$, $k_1=71.28$, $\beta=0$, $\tau=0$, the Runge-Kutta method can be used to obtain the time history diagram and phase diagram of the speed feedback control without time delay. As shown in Figure 2 and Figure 3

![Fig. 2 Time course](image)

![Figure.3 General phase](image)

#### 2.2.2 Controlled system

Given the parameters $\gamma=2$, $\alpha=0.0002$, $k_f=0.8$, $k_1=71.28$, $\beta=0.2$, $\tau=0.3$, the Runge-Kutta method can be used to obtain the time history diagram and phase diagram of the time-delay speed feedback control. As
shown in Figure 4 and Figure 5

Fig. 4 Time course

Fig. 5 General phase diagram

3. Conclusion
The relationship between the vibration response and the phase of the second-order truncated axially moving viscoelastic Euler beam is analyzed without time delay. By applying the speed delay control, the axially moving viscoelastic Euler beam is obtained. The beam vibration response and phase diagram show that by adding time delay control, the vibration can be effectively reduced. In summary, the research in this article can provide a solid theoretical basis for the vibration of a wide range of axially moving beams. The author also hopes that this research can widely promote the axially moving beam system to move forward in a higher speed and more stable direction.

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