The analysis of bending of the rotational pump's screws taking into account the variable moments of inertia along the screw

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Abstract. The analysis results of longitudinal and cross bends of screws of rotational pumps is carried out. Check of the offered model of cross-bending of the screw both taking into account, and without thread is executed. The interrelation of bend of the screw and decrease in the moment of inertia of its cross-section is revealed and the model of calculation of the screw on stability in pitch considering these parameters is received.

1. Introduction
Pipeline transport of liquid freights is one of primary branches of the national economy. The most important element of pipelines are the shutoff and control valves by means of which management of flows of pumped liquid is made. Reliability of operation of all pipeline consists of reliability of work of each knot of the pipeline, each system or fittings. Has the greatest impact on operability of the pipeline operability of the pump [1, 2]. The greatest distribution to the industries was gained by multiphase twin-screw rotor rotary pumps which can flexibly react to the changing conditions of loading. At strength analysis and rigidity of screws of such pumps it is necessary to consider parameters and influence of thread of the screw.
As a rule, the main critical buckling of the screw is the inside diameter of its thread (diameter of hollows). Empirical dependences often are applied to calculation of the given moment of inertia of section of the screw. During the work screws of rotational pumps experience heavy radial and thrust strain. Radial loads can bring to bend of screws, with the subsequent non-execution by the pump of the functions. Thrust loads can bring to loss by the screw of stability in pitch [2, 3], also with the subsequent bend [4]. Therefore it is necessary to consider influence of thread on bend of screws of rotor pumps that is relevant task and object of research of this article.

2. Problem definition
In figure 1, a screw cross-sections with direct axis are shown. It is noticeable that the screw section has irregular shape which cannot be expressed elementary functions. From the analysis of figure 1, a it is possible to draw conclusion that the inertia moment longwise of the screw is variable. In figure 1, b any sections of the screw with curved axis are shown. The joint analysis of figure 1, a and figure 1, b would show that at screw bending the inertia moment on its length also changes, and its value decreases [5].
Finding of the moments of inertia of cross-section of the screw is executed by results of the numerical experiment realized on the basis of the combined methods of calculations with application of the SolidWorks software product [6]. Results of calculations of the moments of inertia longwise of the direct screw with the trapezoidal thread having pitch, \( t_p = 3 \) mm are provided to table 1.

### Table 1. Values of the inertia moments, mm\(^4\)

| Thread type | Section, mm (along the thread pitch, \( t_p = 3 \) mm) |
|-------------|-------------------------------------------------|
|             | 0.3 | 0.6 | 0.9 | 1.2 | 1.5 | 1.8 | 2.1 | 2.4 | 2.7 | 3.0 |
| TR10        | \( J_x \) | 290 | 288 | 297 | 304 | 300 | 291 | 288 | 297 | 304 | 300 |
|             | \( J_y \) | 301 | 303 | 295 | 288 | 291 | 301 | 303 | 295 | 288 | 291 |
| TR20        | \( J_x \) | 5913 | 5962 | 5941 | 5974 | 5954 | 5913 | 5902 | 5941 | 5974 | 5955 |
|             | \( J_y \) | 5902 | 5970 | 5932 | 5901 | 5916 | 5962 | 5970 | 5932 | 5901 | 5916 |

The received changes of the moments of inertia for the curved screw by mm \( l=1000 \) length with thread of TR20x3 [7] are shown in table 2. Longwise of the direct screw it is offered to describe change of the moment of inertia of section the look equation:

\[
J(z) = J_0 + a \sin(\omega \cdot z + \varphi), \tag{1}
\]

where \( a \), \( \varphi \), \( \omega \) and \( J_0 \) are the approximating coefficients [7].

For assessment of change of the moment of inertia as a result of bend we will receive dependence

\[
J_0(y,z) = J_0 - a \cdot z \cdot y(z). \tag{2}
\]

### Table 2. Moments of inertia of sections longwise the screw depending on bending \( Y \)

(TR20x3 thread; screw length, \( l = 1000 \) mm)
3. Theory

The analysis of the screw as a rod on swivel supports with a thread is made, and the rod is loaded in the middle by a transverse force. As metals at the initial stages of loading experience linearly elastic deformations therefore the equation of the curved line of the screw taking into account (1) will register as follows [8]:

\[
\frac{d^2y}{dz^2} = -\frac{P}{2EJ_0 + a \cdot \sin(\omega \cdot z + \varphi)}.
\]

where \(E\) – is the module of material elasticity.

The equation (3) is the differential equation of the second order with the separated variables. Let's execute integration of the equation (3). For integration it is applicable by method of replacement of variables: \(\omega \cdot z = x\), \(\omega dz = dx\), \(dz = \frac{1}{\omega} dx\), \(z = \frac{x - \varphi}{\omega}\). Then

\[
\int \frac{z}{J_0 + a \cdot \sin(\omega \cdot z + \varphi)} dz = \int \frac{x - \varphi}{\omega} \frac{1}{\omega} dx = \frac{1}{\omega^2} \int \left[ \frac{x}{J_0 + a \cdot \sin x} - \varphi \frac{dx}{J_0 + a \cdot \sin x} \right].
\]

The second integral in the right part of expression (4) is tabular and is equal:

\[
\int \frac{dx}{J_0 + a \cdot \sin x} = \frac{2}{\sqrt{J_0^2 - a^2}} \cdot \arctg \left( \frac{J_0 \cdot \frac{x}{2} + a}{\sqrt{J_0^2 - a^2}} \right).
\]

Let's consider the first integral in the right part of expression (4). For its integration we use substitution:

\[
tg \frac{x}{2} = t; \quad \sin x = \frac{2 \cdot t \cdot \frac{x}{2}}{1 + \frac{x^2}{4}} = \frac{2 \cdot t}{1 + t^2}; \quad x = 2 \arctg t; \quad dx = \frac{2 \cdot dt}{1 + t^2}.
\]

Then

| Bending Y, mm | Length l, mm | Moments of inertia, mm² |
|---------------|--------------|------------------------|
| 0             | 501          | 5954.45                |
| 1             | 300          | 5954.45                |
| 2             | 3            | 5954.45                |
| 3             | 5896.54      | 5930.06                |
| 4             | 5840.43      | 5910.48                |
| 5             | 5808.26      | 5910.22                |
| 6             | 5770.12      | 5901.31                |
| 7             | 5745.45      | 5891.05                |
| 8             | 5710.26      | 5879.24                |
| 9             | 5688.38      | 5860.45                |
| 10            | 5659.24      | 5854.45                |
\[
\int \frac{x \, dx}{J_0 + a \cdot \sin x} = \int \frac{4 \arctg t \cdot dt}{J_0 + J_0 \cdot t^2 + 2a \cdot t}.
\]

We will apply method of partial integration to the last expression:

\[
u = \arctg t, \quad du = \frac{dt}{1+t^2}, \quad dv = \frac{dt}{J_0 \cdot t^2 + 2a \cdot t + J_0}, \quad v = \int \frac{dt}{J_0 \cdot t^2 + 2a \cdot t + J_0}.
\]

As a result we will receive:

\[
4\int uv = 4(\arctg t \cdot \arctg \left( \frac{J_0 + 2a}{\sqrt{J_0^2 - a^2}} \right) - \frac{1}{\sqrt{J_0^2 - a^2}} \cdot \arctg \left( \frac{J_0 \cdot t + 2a}{\sqrt{J_0^2 - a^2}} \right) \left( \frac{1}{\sqrt{J_0^2 - a^2}} \cdot \arctg \left( \frac{J_0 \cdot t + 2a}{\sqrt{J_0^2 - a^2}} \right) \right) \frac{dt}{1+t^2}.
\]

Integral \[
\int \arctg \left( \frac{J_0 \cdot t + 2a}{\sqrt{J_0^2 - a^2}} \right) \frac{dt}{1+t^2}
\] also undertakes the integration by parts.

Let us finally receive:

\[
\int \frac{z}{J_0 + a \cdot \sin(\omega \cdot z + \varphi)} \, dz =
\]

\[
= \frac{4}{\omega^2} \arctg \left( \frac{\omega \cdot z}{\sqrt{J_0^2 - a^2}} \right) \cdot \arctg \left( \frac{\omega \cdot z}{2} \right) - \frac{a \cdot \omega}{\sqrt{J_0^2 - a^2}} \cdot \frac{2 \varphi}{3 \left( J_0^2 - a^2 - 1 \right) \sqrt{J_0^2 - a^2}}.
\]

(5)

Let's simplify expression (5) taking into account that \( J_0 \gg a \)

\[
\int \frac{z}{J_0 + a \cdot \sin(\omega \cdot z + \varphi)} \, dz = \frac{4}{\omega^2} \left[ \frac{\omega \cdot z}{3J_0} - \frac{2\varphi}{J_0} \right] \cdot \arctg \left( \frac{\omega \cdot z}{2} + \frac{a}{J_0} \right).
\]

And after the first integration of expression (3) we have:

\[
\frac{dy}{dz} = \frac{2P}{E\omega^2} \left[ \frac{\omega \cdot z}{3J_0} - \frac{2\varphi}{J_0} \right] \cdot \arctg \left( \frac{\omega \cdot z}{2} + \frac{a}{J_0} \right) + c_1.
\]

(6)

After integration (6) we will receive the first approach of form of curved curve:

\[
y_1 = - \frac{2P}{E\omega^2} \left[ \frac{1}{6} \arctg \left( \frac{\omega \cdot z}{2} + \frac{a}{J_0} \right) \arctg \left( \frac{\omega \cdot z}{J_0} \right) - 2 \arctg \left( \frac{\omega \cdot z}{2} + \frac{a}{J_0} \right) \arctg \left( \frac{\omega \cdot z}{J_0} - \frac{\omega^2 \cdot z^3}{36J_0} + \frac{\omega^2 \varphi^2}{2J_0^2} + c_1 \right) \right]
\]

(7)

We define constants of integration from boundary conditions: \( y(0) = y(l) = 0 \Rightarrow c_2 = 0 \)

\[
c_1 = \frac{6 \arctg \left( \frac{\omega \cdot l \cdot J_0 + 2a}{2J_0} \right) \arctg \left( \frac{\omega \cdot l \cdot J_0 + 2a}{2J_0} \right)}{36 J_0} \arctg \left( \frac{\omega \cdot l \cdot J_0 + 2a}{2J_0} \right) + \frac{18 \cdot \omega \cdot l \cdot \varphi - \omega^2 \cdot l^2}{36 J_0}.
\]
Substituting \( c_1 \) in (7), and considering what \( J_0 \gg 1 \), finally received:

\[
y(z) = \frac{2P}{E\omega^2} \left[ -6 \arctg \left( \frac{\omega l J_0 + 2a}{2 J_0} \right) \omega \cdot l \cdot \left( \frac{\omega^2 \varphi^2}{36 J_0} \right) + \frac{6 \arctg \left( \frac{\omega l J_0 + 2a}{2 J_0} \right) \omega \cdot \varphi \cdot l}{36 J_0} - \frac{72 \arctg \left( \frac{\omega l J_0 + 2a}{2 J_0} \right) \varphi \cdot \omega \cdot l^2}{36 J_0} \right].
\]

Simplifying the last expression, and neglecting small sizes, we will finally receive formula for definition of screw bends:

\[
y(z) = \frac{2P}{E\omega^2} \left[ -\frac{\omega^2 \varphi^2}{36 J_0} \omega \cdot l \cdot \left( \frac{\omega^2 \varphi^2}{36 J_0} \right) + \omega \cdot \varphi \cdot l \cdot \left( \frac{\omega^2 \varphi^2}{36 J_0} \right) - \omega \cdot \varphi \cdot l^2 \cdot \left( \frac{\omega^2 \varphi^2}{36 J_0} \right) \right].
\]

The form of the curved line of rod is known [9]:

\[
y(z) = -\frac{P}{18EJ} \left( \omega \cdot \varphi \cdot l \cdot \left( \frac{\omega^2 \varphi^2}{36 J_0} \right) \right).
\]

Further we will consider stability in pitch of screws. As the symmetric function considering change of the moment of inertia along screw axis the equation is used (2). Let's write the equation of the curved line of the screw after its loss of longitudinal stability in the form:

\[
y''(z) + \frac{P \cdot y(z)}{E \cdot (J_0 - a \cdot z \cdot y(z))} = 0.
\]

For integration of the equation (10) we use method of approximations by iteration [8] whereas zero approach used function of sine. Calculations were carried out by means of decomposition of sine in a Taylor row to the fifth and seventh degree. At series development to the seventh degree, received:

\[
y_1(z) = z - \frac{P_{crit} \cdot \varphi^3}{6E \cdot J_0} + \frac{1}{120(E \cdot J_0)^2} \left( P_{crit}^2 - 6P_{crit} \cdot a \cdot E \right) z^5 - \frac{1}{5040(E \cdot J_0)^3} \left( P_{crit}^3 - 46P_{crit}^2 a \cdot E + 120P_{crit} a^2 E^2 \right) z^7.
\]

For determination of value of critical force we will equate amplitudes of the first and zero approximations at point: \( z = l/2 \); \( y_1(l/2) = y_0(l/2) \).

Solving the received equation it is relative \( P_{crit} \), we will receive:
\[ P_{\text{crit}} = \frac{1}{\pi^2} \left( 89.8 P_{Ea} - \sqrt{81.9 \pi^4 E^2 a^2 + 99.8 \pi^2 E \cdot P_{Ea} \cdot a + 6384 P_{Ea}^2 + \pi^2 E \cdot a} \right). \] 

(12)

If the decrease in the inertia moment of screw section is absent (if \( a = 0 \)), critical force should correspond to force determined by L. Euler's formula (\( P_{\text{crit}} = P_{Ea} \)). It is shown that in the equation (12) this condition is satisfied.

4. Discussion of results

Let's carry out the comparative analysis of calculation of bending for the offered formula (8) considering change of the moment of inertia of section longwise of the screw and classical formula (9) where the moment of inertia of section is accepted to constants and is calculated for average thread diameter.

Curves of bends for trapezoidal thread of TR20x3 are presented on figure 2 at the following data:

\( l = 1000 \text{ mm}; \ P = 1000 \text{ N}; \ E = 2 \cdot 10^5 \text{ N/mm}^2; \ \alpha = \pi / 0.77; \ \varphi = 0.68; \ J_0 = 5936 \text{ mm}^4. \)

The curve 1 is constructed on formula (8), curve 2 – on formula (9). Comparison of results shows that it is necessary to consider influence of thread on bending, because not taking into account the thread leads to increase of calculated bends (in the above cases, the real bending is 1.5 times less).

![Figure 2](image-url)

**Figure 2.** Bending curve longwise the screw with TR20x3 thread: curve 1 - taking into account thread; curve 2 - without thread

![Figure 3](image-url)

**Figure 3.** Values of critical force depending on decision accuracy:

- curve 1 – decomposition of sine in a row to the fifth degree;
- curve 2 – to the seventh degree

Comparison of values of the axial force leading to loss of longitudinal stability of screws for different extents of decomposition of the decision is given in figure 3: \( P_{\text{crit1}} \) - the fifth degree; \( P_{\text{crit2}} \) - the seventh extent of decomposition. The analysis of dependences shows that for these extents of decomposition curves practically do not differ, and decrease at the screw bending of its moment of
inertia of cross-section reduces the size of critical force that it is necessary to take into account when calculating.

5. Conclusions
Thus, calculations based on the proposed model prove that when assigning screw parameters for rotary pumps, it is necessary to assume changes of the moment of inertia of the screw thread, taking into account their transverse bending. The analytical dependence of a bend of the screw bend on the variable moment of inertia of its cross section is revealed and the mathematical model for accounting of these changes during the calculations is created.
In further development of research it is necessary to consider influence of twisting of screws on the moments of inertia of their cross-section and, as a result, on bend of screws of rotor pumps.

6. References
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