Adaptive H-infinity tracking control for microgyroscope

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Abstract
An adaptive H-infinity tracking control is proposed for a z-axis microgyroscope with system nonlinearities. All the signals can be guaranteed in a bounded range, and tracking error is uniformly ultimately bounded, an H-infinity tracking performance is also achieved to a prescribed level. Adaptive control methodology is integrated with H-infinity control technique to achieve robust adaptive control, and adaptive algorithm is used to estimate the unknown system parameters. Simulation studies for microgyroscope are conducted to prove the validity of the proposed control scheme with good performance and robustness.

Keywords
Microgyroscope, adaptive $H^\infty$ control, Riccati-like matrix equation

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Introduction
Microgyroscope is the basic measurement element in the inertial navigation system, and it can also be used in military, aviation, aerospace, bioengineering, and other fields. Some control methods such as robust adaptive control,1–4 backstepping control,5 sliding mode approaches,6,7 second-order sliding mode control and super twisting sliding mode controller,8–13 adaptive fuzzy and neural control approaches,14–21 and so on have been investigated to control the dynamic systems.

$H^\infty$ control is used to attenuate the effect on the tracking error caused by system nonlinearities and treat the robust stabilization and disturbance rejection problems.22–26 An adaptive H-infinity sliding mode tracking control is proposed for a class of nonlinear MIMO systems in Chang.27 Ma et al.24 put forward a compact H-infinite robust rebalance loop controller with application to an electrostatically suspended microgyroscope. Fei et al.26 developed an LMI-based adaptive fuzzy H-infinity control strategy for microgyroscope. Adaptive H-infinity control methods have been developed in previous works27–29 for some dynamic systems.

This article focuses on the adaptive $H^\infty$ robust sliding mode control strategy of microgyroscope. The adaptive sliding mode control is combined with the $H^\infty$ control to guarantee the Lyapunov stability and robust tracking, which could better accommodate the microgyroscope system in the three steps. Adaptive system is used to learn the unknown dynamics, robust compensator is designed to attenuate the effect of system nonlinearities and external disturbance, and robust $H^\infty$ control is utilized to achieve the H-infinite tracking performance. The technical contributions compared with current works can be claimed as follows:

1. The adaptive control and $H^\infty$ control strategies are developed to attenuate the tracking error to a prescribed level in order to achieve the $H^\infty$ control performance. The $H^\infty$ control technique is added to...
adaptive controller to achieve the desired attenuation of disturbance in a microgyroscope.

2. $H_{\infty}$ control is employed to adaptively approximate and compensate the system nonlinearities. Riccati-like equation method is incorporated with the adaptive $H_{\infty}$ control to approximate the nonlinear function to compensate the system nonlinearities.

**Dynamics of microgyroscope**

Figure 1 shows a schematic diagram of a microgyroscope. The motion equations of microgyroscope are developed from the Lagrange–Maxwell equation

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} + \frac{\partial F}{\partial \dot{x}_i} = Q_i$$

where $L = \frac{1}{2}E_K - E_P$ is the Lagrange’s function; $E_K$ and $E_P$ are kinetic and potential energies of the sensitive element, respectively; $F$ are the generalized damping forces; and $Q_i$ are generalized forces acting on the sensitive element. In this article, $i = 2$.

Assuming that the angular velocity is almost constant over a sufficiently long time interval, $\Omega_z \approx \Omega_y \approx 0$, only the component of the angular velocity $\Omega_z$ causes a dynamic coupling between the x–y axes. Considering fabrication imperfections, the system model is obtained as

$$\begin{cases}
m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y = u_x + d_x + 2m\Omega_z\dot{x} \\
m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y = u_y + d_y - 2m\Omega_z\dot{y}
\end{cases}$$

(2)

where $m$ is a mass; $d_{xx}, d_{yy}, k_{xx}, k_{yy}$ are damping and spring coefficients; $d_{xy}, k_{xy}$ are quadrature errors, that is coupled damping and spring terms; $u_x, u_y$ are control forces; and $d_x, d_y$ are bounded disturbances.

Dividing both sides of equation (2) by reference mass $m$, reference length $q_0$, and natural resonance frequency $\omega_0^2$ obtains

$$\ddot{q} + D\dot{q} + Kq = u - 2\Omega\dot{q} + d$$

(3)

where $(d_{xx}/m\omega_0) \rightarrow d_{xx}$, $(d_{xy}/m\omega_0) \rightarrow d_{xy}$, $(d_{yy}/m\omega_0) \rightarrow d_{yy}$, $(\Omega_z/\omega_0) \rightarrow \Omega_z$, $\sqrt{(k_{xx}/m\omega_0^2)} \rightarrow \omega_x$, $\sqrt{(k_{yy}/m\omega_0^2)} \rightarrow \omega_y$, and $(k_{xy}/m\omega_0^2) \rightarrow \omega_{xy}$.

Equation (3) can be transformed into

$$\ddot{q} + D\dot{q} + Kq = u - 2\Omega\dot{q} + d$$

(4)

where $q = \begin{bmatrix} x \\ y \end{bmatrix}$, $u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$, $d = \begin{bmatrix} d_x \\ d_y \end{bmatrix}$, $D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}$, $K = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}$, and $\Omega = \begin{bmatrix} 0 & -\Omega_z \\ \Omega_z & 0 \end{bmatrix}$.

**Figure 1.** Schematic model of a z-axis microgyroscope.
The control target for microgyroscope is to make the proof mass to oscillate at given frequency and amplitude: \( x_d = A_1 \sin(\omega_1 t), \ y_d = A_2 \sin(\omega_2 t) \). Then, the reference trajectory can be redefined as

\[
\ddot{q}_d + K_d q_d = 0
\]  
(5)

where \( K_d = \text{diag}\{\omega_1^2, \omega_2^2\} \), \( q_d = [x_d, y_d]^T \).

**Problem formulation**

Taking into account the uncertain parameters in the microgyroscope system, the system model is expressed as

\[
\ddot{q} + (D + 2\Omega + \Delta D)\dot{q} + (K_h + \Delta K_h)q = u + d
\]  
(6)

where \( \Delta D \) is the unknown uncertainties of the matrix \( D + 2\Omega \), \( \Delta K_h \) is the unknown uncertainties of the matrix \( K_h \).

The control objective is to design an adaptive \( H^\infty \) controller for the microgyroscope systems (equation (6)) with system nonlinearities; all the signals and states are bounded, and the output tracking error is as small as possible, as well as the following \( H^\infty \) tracking performance should be achieved

\[
\frac{1}{2} \int_0^T e^T Q_\dot{e} dt \leq e^T(0)Qe(0) + \frac{1}{2} \dot{\theta}^T(0)\dot{\theta}(0)
\]  
(7)

where the weighting matrices \( P = P^T > 0 \), \( Q = Q^T > 0 \), tracking error \( e = q - q_d \), adaptive gain \( a > 0 \), and \( \rho \) is a prescribed attenuation level.

Equation (6) is rewritten as

\[
\ddot{q} + (D + 2\Omega)\dot{q} + K_h q = u + d
\]  
(8)

where \( d = d - \Delta D\dot{q} - \Delta K_h q \) is a combination of uncertainty and disturbances.

Then, equation (8) can be expressed as

\[
\ddot{q} = u + d - (D + 2\Omega)\dot{q} - K_h q
\]  
(9)

Define

\[
F(t) = -(D + 2\Omega)\dot{q} - k_h q
\]  
(10)

So, the derivative of the tracking error is written as

\[
\dot{\theta} = A_0 e + B[F + u + d - \dot{q}_d]
\]  
(11)

where 
\( \dot{q}_d = [\dot{q}_d, \ddot{q}_d]^T \), \( A_0 = \text{diag}[A_{01}, A_{02}] \), \( B = \text{diag}[B_1, B_2] \), \( A_{01} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), and \( i = 1, 2 \).

Then, we choose a matrix \( K \), which makes \( A_{0i} - B_1 = \begin{bmatrix} 0 & 1 \\ -k_{i1} & -k_{i2} \end{bmatrix} \) stable. The form of matrix \( K = \text{diag}[K_1, K_2] \) with \( K_i = [k_{i1}, k_{i2}] \) for \( i = 1, 2 \).

We rewrite the error equation (12) as

\[
\dot{\theta} = A e + B[Ke + F(x) + u + d - \dot{q}_d]
\]  
(12)

where \( A = \text{diag}[A_1, A_2] \).

**Adaptive \( H^\infty \) tracking controller**

The schematic diagram of adaptive \( H^\infty \) controller for a microgyroscope is designed in Figure 2, and the tracking error comes to the proposed controller. The designed controller has three parts, adaptive system is used to learn the unknown dynamics, robust compensator is developed to attenuate the effect of uncertainties and disturbance, and robust \( H_\infty \) control is utilized to achieve the H-infinity tracking performance.

The \( F(t) \) can be expressed as \( F(t) = Y \theta^* + \Delta F \), in which \( Y \theta^* \) is the matrix form of system parameters, \( \Delta F \) denotes the uncertain parts of external disturbances. The unknown system parameter vector is

\[
\theta^* = [d_{xx}, d_{xy}, d_{yy}, \Omega, w^2, w_x, w_y, w_y^2]^T
\]  
(13)

Then, we can obtain that

\[
Y = \begin{bmatrix}
\dot{q}_1 & \dot{q}_2 & 0 & -2\dot{q}_2 & q_1 & q_2 & 0 \\
0 & \dot{q}_1 & \dot{q}_2 & 2\dot{q}_1 & 0 & q_1 & q_2
\end{bmatrix}
\]  
(14)

Substituting \( F(t) = Y \theta^* + \Delta F \) into equation (12) yields the error equation

\[
\dot{\theta} = A e + B[Ke + Y \theta^* + \Delta F + u + d - \dot{q}_d]
\]  
(15)

Let \( \dot{\theta} = \hat{\theta} - \theta^* \) be the estimated error where \( \hat{\theta} \) is the estimated value of \( \theta^* \).

**Theorem 1.** Consider the microgyroscope dynamics (equation (6)), if there is a symmetric matrix \( P = P^T > 0 \) satisfying the following Riccati-like matrix equation
where $\gamma > 0$ is $H_\infty$ controller gain. Then, the adaptive $H_\infty$ control law

$$u = -Y\dot{\theta} + \tilde{q}_d - Ke + u_h + u_s$$

(17)

guarantees that the $H_\infty$ tracking performance (equation (7)) can be obtained; if $d_i \in L_\infty[0, \infty)$, then the tracking error is uniformly ultimate bounded; all the variables in the closed-loop system (equations (6) and (15)) are bounded, where $u_h = -(1/2\gamma)B^T Pe$ is used to obtain the $H_\infty$ tracking performance, $u_s = -\eta \text{sgn}(B^T Pe)$ is used to reduce the impact of external disturbance $\Delta F$, adaptive law $\dot{\theta} = \alpha Y^T B^T Pe$.

**Proof.** Substituting equation (17) into equation (15) yields

$$\dot{e} = Ae + B[-Y\dot{\theta} + \Delta F + u_h + u_s + d_i]$$

(18)

We choose a Lyapunov function candidate as

$$V = \frac{1}{2}e^TPe + \frac{1}{2\alpha}\dot{\theta}^T\dot{\theta}$$

(19)

Making the time derivative of $V$ obtains

$$\dot{V} = \frac{1}{2}e^TP\dot{e} + \frac{1}{2}\dot{\theta}^T\dot{\theta} - \frac{1}{2}\dot{\theta}^T\dot{\theta} + \eta \text{sgn}(B^T Pe)$$

$$= \frac{1}{2}(Ae + B[-Y\dot{\theta} + \Delta F + u_h + u_s + d_i])^T Pe$$

$$+ \frac{1}{2}e^TP(Ae + B[-Y\dot{\theta} + \Delta F + u_h + u_s + d_i]) + \frac{1}{2}\dot{\theta}^T\dot{\theta}$$

$$= \frac{1}{2}e^T(A^TP + PA)e - \dot{\theta}^TY^T B^T Pe + u_h^T B^T Pe$$

$$+ u_s^T B^T Pe + \Delta F^T B^T Pe + d_i^T B^T Pe + \frac{1}{2}\dot{\theta}^T\dot{\theta}$$

(20)

Then, substituting control force $u_h = -(1/2\gamma)B^T Pe$ into equation (20), we can obtain
\[ V = \frac{1}{2} e^T P + P A + PB \left( \frac{1}{\rho^2} I - \frac{1}{\rho} \right) B^T P e + \frac{1}{2} \rho^2 d_i^T d_i \]
\[ - \frac{1}{2} \left( \frac{1}{\rho} B^T P e - \rho d_i \right)^T \left( \frac{1}{\rho} B^T P e - \rho d_i \right) \]
\[ - \bar{\theta}^T Y B^T P e + u_i^T B^T P e + \Delta F^T B^T P e + \frac{1}{\rho^2} \bar{\theta} \cdot \bar{\theta} \]
\[ (21) \]

because

\[ u_i^T B^T P e + \Delta F^T B^T P e = - \eta \text{sgn}(B^T P e) + \Delta F^T B^T P e \]
\[ = - (\eta - \Delta F) \| B^T P e \| \leq 0 \]
\[ (22) \]

Applying the sliding term \( u_s = - \eta \text{sgn}(B^T P e) \) and adaptive law \( \dot{\bar{\theta}} = \bar{\omega}^T B^T P e \) into equation (21), and substituting equation (22) into equation (21), it can be written that

\[ \dot{V} \leq - \frac{1}{2} e^T Q e + \frac{1}{2} \rho^2 d_i^T d_i \]
\[ (23) \]

Integrating equation (23) from zero to \( T \) obtains

\[ V(T) - V(0) \leq - \frac{1}{2} \int_0^T e^T Q e dt + \frac{1}{2} \rho^2 \int_0^T \| d_i \|^2 dt \]
\[ (24) \]

Since \( V(T) \geq 0 \), the inequality (equation (24)) implies the following inequality

\[ \frac{1}{2} \int_0^T e^T Q e dt \leq V(0) + \frac{1}{2} \rho^2 \int_0^T \| d_i \|^2 dt \]
\[ = e^T(0) P e(0) + \frac{1}{2} \bar{\omega}^T(0) \bar{\omega}(0) + \frac{1}{2} \rho^2 \int_0^T \| d_i \|^2 dt \]
\[ (25) \]

Thus, the \( H^\infty \) tracking performance (equation (7)) can be achieved.

From equations (18) and (25), all the variables in the closed-loop system (equations (6) and (15)) are bounded. Moreover, it is proved if \( d_i \in L_\infty [0, \infty) \), then the tracking error is uniformly ultimate bounded.

**Simulation examples**

In this section, we will use MATLAB/Simulink to evaluate the proposed adaptive \( H^\infty \) tracking control on the lumped microgyroscope model. Parameters of microgyroscope\(^{17,18}\) are listed as

- \( \omega_x^2 = 355.3, \omega_y^2 = 532.9, \omega_{xy} = 70.99 \)
- \( d_{xx} = 0.01, d_{xy} = 0.01, d_{yx} = 0.002 \)
- \( \Omega = 0.1 \sin 100 \pi t \)

![Figure 3. The tracking trajectory using adaptive H-infinity controller.](image-url)
The desired trajectories are \( q_{d1} = \sin(w_1 t) \), \( q_{d2} = 1.2 \sin(w_2 t) \), where \( w_1 = 4.17 \text{kHz} \), \( w_2 = 5.11 \text{kHz} \). The matrix \( K \) is chosen as

\[
K = \begin{bmatrix} 14 & 26 & 0 & 0 \\ 0 & 0 & 10 & 42 \end{bmatrix}
\]

the adaptive gain \( \alpha = 100 \), other parameters are \( \eta = 100 \), \( \gamma = 3 \), \( \rho = 2 \), respectively. The weight matrix \( Q = 50I \). The combination of uncertainty and external disturbances \( d_i \) can be chosen as \( 10 \sin((\pi/4) t) \) and
$10\cos(t + (\pi/5))$, respectively, in x and y axes. Then, we will testify the robustness of the proposed algorithm by increasing the value of $d_x$ to 20 times of $10\sin((\pi/4)t)$ and $10\cos(t + (\pi/5))$.

The tracking trajectory and tracking error are shown in Figures 3 and 4 when $d_x$ is $10\sin((\pi/4)t)$ and $10\cos(t + (\pi/5))$. Figures 5 and 6 are the tracking trajectory and tracking error when $d_x$ increasing to 20 times, showing that no matter how much external disturbances increase, the controller can track the reference model trajectory accurately, and guarantee the tracking error converges to zero.

**Figure 6.** Convergence of the tracking error using adaptive H-infinity controller in the presence of large disturbance.

**Figure 7.** Adaptive parameter estimates.
Remark: The design parameters have influences on the control performance. The parameters of micro gyroscope system are selected from the references.\textsuperscript{7,20} The parameters are selected by previous works\textsuperscript{20,24} and experiences. The trajectory tracking resolution and robustness could be improved using the design adaptive H\textsubscript{\infty} controller which could utilize adaptive system, robust compensator, and robust H\textsubscript{\infty} control to improve the tracking performance. As for practical application, since it is not easy to establish accurate model of microgyroscope because of its nonlinearity and coupling, so it has a great potential to apply H\textsubscript{\infty} control to MEMS gyroscope. The H\textsubscript{\infty} control has strong robust ability for nonlinear term, so it can be applied to nonlinear microgyroscope systems. Because the reference trajectories contain two different nonzero frequencies, PE condition is satisfied, in Figure 7, the parameter estimates converge to their true values including the angular velocity.

Conclusion

In this article, H\textsubscript{\infty} control technique is added to adaptive control algorithm to obtain the desired disturbance attenuation of microgyrosopes. A robust compensator is developed to attenuate the effect of system nonlinearities to a prescribed level so as to obtain the H\textsubscript{\infty} performance. Adaptive H\textsubscript{\infty} control is employed to achieve robust adaptive control, and adaptive law is used to estimate the unknown system parameters. Simulation results confirm that the H\textsubscript{\infty} performance can be obtained by the proposed strategy, demonstrating the accurate tracking property and strong robustness. Further works include real implementation of the proposed control scheme.

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