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Vibrational vector solitons in biaxial crystals

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Abstract. Vector solitons formed of pairs of nonlinear optical vibrations with orthogonal polarizations in biaxial crystals are investigated. They are described by a system of coupled nonlinear Schrödinger equations, which take account of linear birefringence, cross-phase-modulation and group-velocity-dispersion (GVD) mismatch. The equations are solved numerically for different initial conditions and parameters. The stability of the coupled solution are investigated. The GVD mismatch, which has been neglected in previous studies, modifies the properties of vector solitons and seems to stabilize the fast-mode component.

1. Introduction

Scalar solitons formed of anharmonic vibrations have been investigated in [1-3]. The dynamics of vibrational vector solitons (pairs of conjugated circularly-polarized pulses) propagating along the optical axis of gyrotrropic crystals has been studied in [4, 5] on the basis of a system of coupled nonlinear Schrödinger (NLS) equations. The presence of circular birefringence yields a walk-off effect for the components and for weak nonlinear cross-interaction, a linearly polarized pulse splits into two conjugated circularly polarized solitons propagating with different velocities. For strong enough attractive cross-interaction the solitons modify their individual carrier wave-numbers (or central frequencies) and are trapped to a common group velocity. An important result is that group-velocity locking yields also equal GVD coefficients for the two modes. Exact coupled-soliton solutions exist in this case. The presence of inelastic cross-interaction, which allows energy transfer between the components, leads to shape oscillations and instability of the fast mode. Optical vector solitons have been extensively investigated in birefringent fibers [6-9]. The difference in the GVD coefficients for the two polarizations has usually been neglected and polarization instabilities of the fast mode have been obtained.

The present work is devoted to the study of the dynamics of vector solitons formed of nonlinear optical vibrations with orthogonal polarizations in biaxial crystals and in oblique directions in uniaxial crystals. The modes are characterized by different dispersion coefficients and phase velocities. Contrary to the gyrotrropic and the optical cases however, trapping of the partial pulses and group-velocity locking yields appreciable difference in the GVD coefficients which has to be accounted for. The corresponding system of coupled NLS equations describing vector solitons in biaxial crystals is intrinsically nonintegrable. The equations are solved numerically for different soliton amplitudes and material parameters. The evolution and stability of the vector solitons in biaxial crystals show considerable differences compared to previously investigated cases.
2. The coupled NLS equations

The Hamiltonian of the system of two anharmonic optical vibrations with orthogonal polarizations propagating along oblique directions in crystals can be cast in the following form:

\[
H = \omega_0 \sum_n (A_n^\dagger A_n + B_n^\dagger B_n) + \frac{1}{2} \sum_n [M_1 (A_{n+1}^\dagger A_n + A_n^\dagger A_{n+1}) + M_2 (B_{n+1}^\dagger B_n + B_n^\dagger B_{n+1})] 
+ \frac{g_1}{2} \sum_n (A_n^1 A_n^2 + B_n^1 B_n^2) + g_2 \sum_n A_n^1 A_n^\dagger B_n + \frac{g_2}{2} \sum_n (A_n^2 B_n^1 + B_n^2 A_n^1),
\]

where \(A_n^1, A_n, B_n^1, B_n\) are Bose creation and annihilation operators of vibrational states at site \(n\) for the two modes, \(\omega_0\) is the harmonic intramolecular energy (\(\hbar = 1\)) and \(M_1\) and \(M_2\) are the transfer matrix elements between neighboring sites for the two modes. Note that the Hamiltonian (1) conserves the total number of quasiparticles in compliance with the assumption of narrow energy bands of optical phonons (\(M_1, M_2 \ll \omega_0\)). The quartic terms in (1) describe the anharmonic interactions for the two modes. The term \(\sim g_1\) describes anharmonic interaction between excitations of the same type, while the terms \(\sim g_2\) describe anharmonic interaction between excitations of different types. The term \(g_2\) conserves the number of particles of each type (elastic cross-intration), while the term \(\sim g_3\) conserves only the total number of particles and allows energy transfer between the modes (inelastic-intration).

The equations of motion for the averaged vibrational amplitudes \(\alpha_n \equiv \langle A_n \rangle, \beta_n \equiv \langle B_n \rangle\) are:

\[
\begin{align*}
\frac{i}{\partial t} \alpha_n & = \omega_0 \alpha_n + M_1 (\alpha_{n+1} + \alpha_{n-1}) + (g_1 |\alpha_n|^2 + g_2 |\beta_n|^2) \alpha_n + g_3 \alpha_n^* \beta_n^2, \\
\frac{i}{\partial t} \beta_n & = \omega_0 \beta_n + M_2 (\beta_{n+1} + \beta_{n-1}) + (g_1 |\beta_n|^2 + g_2 |\alpha_n|^2) \beta_n + g_3 \beta_n^* \alpha_n^2.
\end{align*}
\]

Seeking solutions in the form of Bloch waves with slowly-varying envelopes

\[
\alpha_n(t) = \varphi_n(t)e^{i(k_1 n - \omega_1 t)}, \quad \beta_n(t) = \psi_n(t)e^{i(k_2 n - \omega_2 t)}
\]

the following system of coupled NLS equations for the envelopes are derived in the continuum limit:

\[
\begin{align*}
\frac{i}{\partial t} \varphi & = (\varepsilon_1 - \omega_1) \varphi - 2b_1 \frac{\partial \varphi}{\partial k_1} + b_1^2 \frac{\partial^2 \varphi}{\partial k_1^2} + (g_1 \varphi^2 + g_2 \psi^2) \varphi + g_3 e^{2i(\Delta k x - \Delta \omega t)} \psi^2 \varphi, \\
\frac{i}{\partial t} \psi & = (\varepsilon_2 - \omega_2) \psi - 2b_2 \frac{\partial \psi}{\partial k_2} + b_2^2 \frac{\partial^2 \psi}{\partial k_2^2} + (g_1 \psi^2 + g_2 \varphi^2) \psi + g_3 e^{-2i(\Delta k x - \Delta \omega t)} \varphi^2 \psi
\end{align*}
\]

where \(k_i\) and \(\omega_i\) are the wave numbers and the frequencies of the carrier waves (the lattice constant equals unity), \(\varepsilon_i = \omega_0 + 2b_i\), \(b_i = M_i \cos k_i\), \(\Delta k = k_2 - k_1\), \(\Delta \omega = \omega_2 - \omega_1\).

The imaginary parts of equation (4) govern the group velocities of the solitons. Neglecting small nonlinear corrections associated with \(g_3\), the GVD coefficients \(b_i\) corresponding to a common group velocity \(v\) are derived:

\[
b_i = M_i \sqrt{1 - v^2 / 4M_i^2}, \quad v = -2M_i \sin k_i, \quad i = 1, 2
\]

Note the difference in the GVD coefficients of the components of the vector soliton in the present case.
3. Numerical results
We carried out numerical simulations of equations (2) for different initial conditions and parameters. The input pulses were of the type:

\[ \varphi_n(t) = \varphi_0 \text{sech} \frac{n - vt}{L}, \quad \psi_n(t) = \psi_0 \text{sech} \frac{n - vt}{L}, \]  

where \( L \) and \( v \) are the width and the velocity of the solitons. For wide solitons \( (L \geq 10) \) the solutions of (2) converge on the solutions of the continuous equations (4).

Figure 1(a) shows a stable symmetric vector soliton corresponding to equal initial amplitudes and group velocities of the two components in the case of elastic cross-interaction only \( (g_1 = g_2 \neq 0, g_3 = 0) \). The presence of inelastic cross-interaction \( (g_3 \neq 0, \text{figure 1(b)}) \) yields positional and shape oscillations of \( |\alpha| \) and \( |\beta| \) with a spatial period of \( \pi/2\Delta k \approx 78 \) accompanied by emission of radiation. Over large timescales, the slow mode transfers part of its energy to the fast mode and an asymmetric vector soliton is formed.

![Figure 1](image-url)  

**Figure 1.** Evolution of a vector soliton with \( L = 10, M_1 = -0.9, M_2 = -1.1, v = 0.2, k_1 = 0.111, k_2 = 0.091, g_1 = g_2 = -0.05 \) and (a): \( g_3 = 0 \), (b): \( g_3 = -0.4 \). Time is in units \( \omega_0^{-1} \).

Under the action of the exchange cross-interaction, the system supports also asymmetric vector solitons up to a certain difference between the amplitudes. Figure 2 shows such a case with 10 times smaller amplitude of one of the components. The evolution is symmetric with respect to the input amplitudes of the fast and slow modes and results in broadening of both partial pulses.
The evolution of the asymmetric vector soliton depends strongly on the magnitudes of the input amplitudes as well as on the dispersion coefficients mismatch. For larger initial amplitudes, a remarkable difference is observed depending on whether the fast or slow mode amplitude is larger. When the fast mode amplitude is larger, the initial system remains practically unchanged [(figure 3(a)]. However, when the slow mode amplitude is larger, after a short time the slow mode transfers part of its energy to the fast mode which than splits into three - a weak soliton locked to the larger slow-mode to form an asymmetric vector soliton, and two scalar solitons with considerable amplitudes propagating in opposite directions [(figure 3(b)].

**Figure 2.** Evolution of a soliton pair with $g_3 = -0.4$ and (a): $\varphi_0 = 0.1\psi_0$, (b): $\psi_0 = 0.1\varphi_0$. All other parameters are the same as in figure 1.

**Figure 3.** Evolution of a soliton pair with $g_3 = -0.4$ and (a): $\varphi_0 = 0.1\psi_0$, (b): $\psi_0 = 0.1\varphi_0$. The initial amplitudes are 40% larger than in figure 2.
The inelastic cross-interaction terms represent a nonlinear potential with a spatial period $\pi/\Delta k$. The effect of these terms depends strongly on the ratio of this period to the soliton width $L$. For $\pi/\Delta kL \ll 1$ the potential oscillates many times over the soliton width and its effect on the soliton dynamics is negligible. In the opposite case of slowly-varying potential however ($\pi/\Delta kL \gg 1$), effective energy transfer between the components of the vector soliton takes place. It is worth noting, that in all cases of the present study, the fast mode remained stable, while the slow mode showed some minor instabilities. This is a remarkable difference from the properties of vector solitons studied before and seems to be associated with the unequal GVD coefficients for the two modes.

The role of the GVD coefficients mismatch for the nonintegrability of the system (4) even in the near-Manakov case ($g_1 = g_2 \neq 0, g_3 = 0$) is illustrated in figure 4 in the process of scattering of two scalar solitons with orthogonal polarizations, launched from different places with different velocities. The scattering is inelastic, each scalar soliton splits off part of the other one to form two asymmetric vector solitons propagating with different velocities.

![Figure 4. Scattering of scalar solitons with $k_1 = 0.35, k_2 = 0.2, g_1 = g_2 = -0.05$ and $g_3 = 0$.](image)

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