Consideration on Example 2 of “An Algorithm of General Fuzzy Inference With The Reductive Property”

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ABSTRACT

In this paper, we will show that (1) the results about the fuzzy reasoning algorithm obtained in the paper “Computer Sciences Vol. 34, No.4, pp.145-148, 2007” according to the paper “IEEE Transactions On systems, Man and cybernetics, 18, pp.1049-1056, 1988” are correct; (2) example 2 in the paper “An Algorithm of General Fuzzy Inference With The Reductive Property” presented by He Ying-Si, Quan Hai-Jin and Deng Hui-Wen according to the paper “An approximate analogical reasoning approach based on similarity measures” presented by Tursken I.B. and Zhong zhao is incorrect; (3) the mistakes in their paper are modified and then a calculation example of FMT is supplemented.

Keywords; Fuzzy reasoning, CRI principle, Triple I method, Similarity fuzzy inference method, Reductive property

1. Introduction

Zadeh proposed CRI principle¹, Wang tripleI principle (TIP) with total inference rules of fuzzy reasoning method,² and Yeung and Tsang³ a similarity based fuzzy reasoning method, respectively. HE, QUAN and DENG proposed an algorithm of general fuzzy inference with the reductive property⁴, and Chen proposed a fuzzy reasoning method of fuzzy system with many rules⁵. Tursken and Zhong proposed an approximate analogical reasoning approach based on similarity measures⁶. Yuan and Lee⁷ showed that results about the triple I method for fuzzy reasoning obtained in Liu⁸ are correct and the Example 2.1 in ref. 8 is incorrect. In this paper, we show that example 2 in He et al.⁴ is incorrect; and the mistakes in their paper are modified and then a calculation example of FMT is supplemented.

Generally fuzzy reasoning is fuzzy modus ponens (FMP) in the fuzzy system with m inputs 1 output n rules.

General form of the fuzzy modus ponens in the paper⁶ is as follows.

\[
\text{Premise 1: } \begin{align*}
\text{if } x_{11} &= A_{11} \quad \text{and } x_{12} = A_{12} \quad \text{and } \cdots \quad x_{1m} = A_{1m} \quad \text{then } y_1 = B_1 \\
\vdots &\quad \vdots \\
\text{if } x_{n1} &= A_{n1} \quad \text{and } x_{n2} = A_{n2} \quad \text{and } \cdots \quad x_{nm} = A_{nm} \quad \text{then } y_n = B_n
\end{align*}
\]

 Premise 2: \( x_1 = A_1^* \quad \text{and } x_2 = A_2^* \quad \text{and } \cdots \quad x_m = A_m^* \)

Conclusion: \( y = B^* \)
General form of Fuzzy Modus Tollens (FMT) in the paper is as follows.

\[ \text{Premise 1:} \quad \text{if } x_1 = A_{11} \text{ and } x_2 = A_{12} \text{ and } \cdots \text{ and } x_m = A_{1m} \text{ then } y = B_1 \]

\[ \text{Premise 2:} \quad y = B^* \]

Conclusion: \( x_1 = A_1^* \text{ and } x_2 = A_2^* \text{ and } \cdots \text{ and } x_m = A_m^* \)

where \( A_{ij} \text{ and } A_j^* (j = 1, 2, \ldots, m, \text{ and } i = 1, 2, \ldots, n) \) are fuzzy sets defined in the universe of discourse \( X \), \( B_i (i = 1, 2, \ldots, n) \) and \( B^* \) are fuzzy sets defined in the universe of discourse \( Y \).

**Definition 1**

Assume \( S: F(X) \times F(X) \rightarrow [0, 1] \).

If

\( S_1 : \forall A \in F(X) : S(A, A) = 1 \),
\( S_2 : \forall A \in P(X) : S(A, A^*) = 0 \),
\( S_3 : \forall A, B, C, D \in F(X) \)

and

\[ \int_x |A(x) - B(x)| \, dx \leq \int_x |C(x) - D(x)| \, dx \text{ then } S(A, B) \leq S(C, D) \text{ is satisfied.} \]

Particularly for 3 fuzzy sets, if \( A \subseteq B \subseteq C \), then \( S(A, C) \leq \min \{S(A, B), S(B, C)\} \) and \( S \) is called similarity defined on fuzzy set \( F(X) \).

In the fuzzy system with \( m \) inputs 1 output \( n \) rules, definition for reductive property of fuzzy inference method is as follows;

**Definition 2**

Reductive property of FMP problem in fuzzy system with \( m \) inputs 1 output \( n \) rules.

For fuzzy inference form (1) that has several premises, given \( A_j = A_{ij} \) for \( i = 1, 2, \ldots, n \), if \( B^* = B_j \) (\( \forall j, j = 1, 2, \ldots, m \)) is satisfied, we call that this algorithm satisfies reductive property of FMP problem in the fuzzy system with \( m \) inputs 1 output \( n \) rules.

**Definition 3**

Reductive property of FMT problem in the fuzzy system with \( m \) inputs 1 output \( n \) rules.

For fuzzy inference form (2) that has several premises, given \( B^* = B_j \) (\( i = 1, 2, \ldots, n \)), if \( A_j^* = A_j \) (\( \forall j, j = 1, 2, \ldots, m \)) is satisfied, then we call that this algorithm satisfies reductive property of FMT problem in the fuzzy system with \( m \) inputs 1 output \( n \) rules.

### 2. Example 2 in the Paper is incorrect.

In this section, we consider the fuzzy set in case that fuzzy set and input change on vertical axis not horizontal axis by using the example presented in the paper.

Assume that the considered fuzzy set, 2 inputs 1 output 2 rules, is given as follows.

\[ R_1 : \text{if } x_1 \text{ is } A_{11}(x_1) \text{ and } x_2 \text{ is } A_{12}(x_2) \text{ then } y \text{ is } B_1(y) \]
\[ R_2 : \text{if } x_1 \text{ is } A_{21}(x_1) \text{ and } x_2 \text{ is } A_{22}(x_2) \text{ then } y \text{ is } B_2(y) \]

where \( X = Y = [0, 1] \), \( A_{ij}(x_i), A_j^*(x_j) \in F(X), B_i(y), B^*(y) \in F(Y) \), \( i, j = 1, 2 \) are the fuzzy set of antecedent of the rule, input, fuzzy set of consequent of the rule, and input respectively. For fuzzy rule 1 and 2 in formula...
(1), fuzzy set of the antecedent, fuzzy set of consequent and new input fuzzy set are as formula (5).

\[
\begin{align*}
A_{11}(x_1) &= \frac{1}{3}(3 - x_1), & A_{12}(x_2) &= \frac{1}{3}(x_2 + 2), & B_1(y) &= 1 - y \\
A_{21}(x_1) &= 1 - x_1, & A_{22}(x_2) &= 1 - \frac{1}{2}x_2, & B_2(y) &= \frac{1}{2}(1 + y) \\
A^*_1(x_1) &= 1 - x_1, & A^*_2(x_2) &= 1 - \frac{1}{2}x_2, & B^*(y) &= 1 - \frac{1}{2}(1 + y)
\end{align*}
\]

Consider the problem obtaining the reasoning result \( B^* \) for example 2 in the paper [4]. We use similarity \( SM(A, B) \) in the paper 6.

\[
SM(A, B) = \left( 1 + \frac{1}{b - a} \int_a^b |A(u) - B(u)| \, du \right)^{-1}
\]

In formula (6) \( u \in U \) is continuous domain of definition and we calculate the sub-reasoning result in two types \( \text{type}_1 B^*_i \) and \( \text{type}_2 B^*_i \).

\[
\text{type}_1 B^*_i = SM(A_{11}, A^*_1)B_i \cap SM(A_{12}, A^*_2)B_i
\]

\[
\text{type}_2 B^*_i = \min \left\{ 1, \frac{B_i}{SM(A_{11}, A^*_1)} \right\} \cap \min \left\{ 1, \frac{B_i}{SM(A_{12}, A^*_2)} \right\}
\]

Using formula (7) and (8) sub-reasoning results are as follows.

\[
\begin{align*}
\text{type}_1 B^*_1 &= SM(A_{11}, A^*_1)B_1 \cap SM(A_{12}, A^*_2)B_1 = \frac{3}{4}(1 - y) \\
\text{type}_1 B^*_2 &= SM(A_{21}, A^*_1)B_2 \cap SM(A_{22}, A^*_2)B_2 = \frac{1}{2}(1 + y)
\end{align*}
\]

\[
\begin{align*}
\text{type}_2 B^*_1 &= \min \left\{ 1, \frac{B_1}{SM(A_{11}, A^*_1)} \right\} \cap \min \left\{ 1, \frac{B_1}{SM(A_{12}, A^*_2)} \right\} = \begin{cases} 1, & y < \frac{1}{13} \\ \frac{13}{12}(1 - y), & y \geq \frac{1}{13} \end{cases} \\
\text{type}_2 B^*_2 &= \min \left\{ 1, \frac{B_2}{SM(A_{21}, A^*_1)} \right\} \cap \min \left\{ 1, \frac{B_2}{SM(A_{22}, A^*_2)} \right\} = \frac{1}{2}(y + 1)
\end{align*}
\]

According to the paper 4, 6, the calculation results of formula (9) and (10) are correct. The fuzzy reasoning results of the paper 4, 6 are expressed as \( \text{type}_1 B^* \) (11) and \( \text{type}_2 B^* \) (12) respectively, but the formula (11) and (12) are incorrect respectively.

\[
\text{type}_1 B^* = \text{type}_1 B^*_1 \cup \text{type}_1 B^*_2 = \begin{cases} \frac{3}{4}(1 - y), & y \leq \frac{1}{5} \\ \frac{1}{2}(y + 1), & y > \frac{1}{5} \end{cases}
\]

\[
\text{type}_2 B^* = \begin{cases} \frac{3}{4}(1 - y), & y \leq \frac{1}{5} \\ \frac{1}{2}(y + 1), & y > \frac{1}{5} \end{cases}
\]

\[
\text{type}_2 B^* = \begin{cases} \frac{3}{4}(1 - y), & y \leq \frac{1}{5} \\ \frac{1}{2}(y + 1), & y > \frac{1}{5} \end{cases}
\]

\[
\text{type}_2 B^* = \begin{cases} \frac{3}{4}(1 - y), & y \leq \frac{1}{5} \\ \frac{1}{2}(y + 1), & y > \frac{1}{5} \end{cases}
\]
The results of the formula (9) and (11) based on the paper 4, 6 are wrong. The recalculated result of formula (9) and (11) based on the paper 4, 6 is the formula (13) and (14) respectively in this paper.

\[
\text{type } 1 \, B^*_{1\text{new}} = SM(A_{11}, A'_1)B_1 \cap SM(A_{12}, A'_2)B_2 = \frac{3}{4}(1-y) \\
\text{type } 1 \, B^*_{2\text{new}} = SM(A_{21}, A'_1)B_2 \cap SM(A_{22}, A'_2)B_2 = \frac{60}{73}(1+y) \\
\text{type } 1 \, B^*_{\text{new}} = \text{type } 1 \, B^*_{1\text{new}} \cup \text{type } 1 \, B^*_{2\text{new}} = \left\{ \begin{array}{ll}
\frac{3}{4}(1-y), & y \leq \frac{1}{3} \\
\frac{3}{8}(y+1), & y > \frac{1}{3}
\end{array} \right.
\]

(13)

(14)

The results of the formula (10) and (12) based on the paper [4, 6] are wrong. The recalculated fuzzy reasoning results of formula (10) and (12) based on the paper [4, 6] are the same as formula (15) and (16) respectively in this paper.

\[
\text{type } 2 \, B^*_{1\text{new}} = \min \left\{ \frac{B_1}{SM(A_{11}, A'_1)} \right\} \cap \min \left\{ \frac{B_1}{SM(A_{12}, A'_2)} \right\} = \left\{ \begin{array}{ll}
1, & y < \frac{13}{73} \\
\frac{73}{60}(1-y), & y \geq \frac{13}{73}
\end{array} \right.
\]

\[
\text{type } 2 \, B^*_{2\text{new}} = \min \left\{ \frac{B_2}{SM(A_{21}, A'_1)} \right\} \cap \min \left\{ \frac{B_2}{SM(A_{22}, A'_2)} \right\} = \left\{ \begin{array}{ll}
\frac{73}{120}(1+y), & y < \frac{47}{73} \\
1, & y \geq \frac{47}{73}
\end{array} \right.
\]

\[
\text{type } 2 \, B^*_{\text{new}} = \text{type } 2 \, B^*_{1\text{new}} \cup \text{type } 2 \, B^*_{2\text{new}} = \left\{ \begin{array}{ll}
1, & y < \frac{13}{73} \\
\frac{73}{60}(1-y), & \frac{13}{73} \leq y < \frac{1}{3} \\
\frac{1}{5}, & \frac{1}{3} \leq y < \frac{47}{73} \\
\frac{73}{120}(y+1), & y \geq \frac{47}{73}
\end{array} \right.
\]

(15)

(16)

From the above example, the fuzzy modus ponens (FMP) by similarity in the paper 4,6 also doesn’t satisfy the reductive property in m inputs 1 output n rules. It is the same in fuzzy modus tollens. In the next section we consider FMT of the paper 4.

### 3. Calculation of FMT based on similarity of the paper 4

Input of FMT according to formula (5) is as follows.

\[
B^* (y) = 1 - B_2 (y) = \frac{1}{2}(1-y)
\]

(17)

The subreasoning calculation result is as the formula (18).
The FMT calculation result of type 1 is as the formula (18).

\[
\begin{align*}
type 1 A_{1\text{new}}^* &= SM(B_1, B')A_{11} = \frac{4}{15}(3 - x_i) \\
type 1 A_{12\text{new}}^* &= SM(B_1, B')A_{12} = \frac{4}{15}(x_2 + 2) \\
type 1 A_{21\text{new}}^* &= SM(B_2, B')A_{21} = \frac{2}{3}(1 - x_i) \\
type 1 A_{22\text{new}}^* &= SM(B_2, B')A_{22} = \frac{1}{3}(2 - x_2)
\end{align*}
\]

The FMT calculation result of type 1 is as the formula (19).

\[
\begin{align*}
type 1 A_{1\text{new}}^* &= type 1 A_{11\text{new}}^* \cup type 1 A_{12\text{new}}^* = \frac{4}{15}(3 - x_1) \\
type 1 A_{2\text{new}}^* &= type 1 A_{12\text{new}}^* \cup type 1 A_{22\text{new}}^* = \begin{cases}
\frac{1}{3}(2 - x_2), & x_2 \leq \frac{2}{9} \\
\frac{4}{15}(x_2 + 2), & y > \frac{2}{9} 
\end{cases}
\end{align*}
\]

Therefore, since \(type 1 A_{1\text{new}}^* \neq \overline{A_{21}}\), \(type 1 A_{2\text{new}}^* \neq \overline{A_{22}}\), FMT of type 1 in the paper 4, 6 doesn’t satisfy reductive property. The subreasoning calculation result of type 2 is as the formula (20).

\[
\begin{align*}
type 2 A_{11\text{new}}^* &= \min\left\{1, \frac{A_{11}}{SM(B_1, B')}\right\} = \begin{cases}
1, & x_1 < \frac{3}{5} \\
\frac{5}{12}(3 - x_i), & x_1 \geq \frac{3}{5}
\end{cases} \\
type 2 A_{12\text{new}}^* &= \min\left\{1, \frac{A_{12}}{SM(B_1, B')}\right\} = \begin{cases}
1, & x_1 < \frac{2}{5} \\
\frac{5}{12}(x_2 + 2), & x_2 < \frac{2}{5} \\
1, & x_2 \geq \frac{2}{5}
\end{cases} \\
type 2 A_{21\text{new}}^* &= \min\left\{1, \frac{A_{21}}{SM(B_2, B')}\right\} = \begin{cases}
1, & x_1 < \frac{1}{3} \\
\frac{3}{2}(1 - x_i), & x_1 \geq \frac{1}{3}
\end{cases} \\
type 2 A_{22\text{new}}^* &= \min\left\{1, \frac{A_{21}}{SM(B_2, B')}\right\} = \begin{cases}
1, & x_2 < \frac{2}{3} \\
\frac{3}{4}(2 - x_2), & x_2 \geq \frac{2}{3}
\end{cases}
\end{align*}
\]

The FMT calculation result of the type 2 is as the formula (21).

\[
\begin{align*}
type 2 A_{1\text{new}}^* &= type 2 A_{11\text{new}}^* \cup type 2 A_{12\text{new}}^* = \begin{cases}
1, & x_1 \leq \frac{3}{5} \\
\frac{5}{12}(3 - x_1), & x_1 > \frac{3}{5}
\end{cases} \\
type 2 A_{2\text{new}}^* &= type 2 A_{12\text{new}}^* \cup type 2 A_{22\text{new}}^* = 1
\end{align*}
\]

Therefore, since \(type 2 A_{1\text{new}}^* \neq \overline{A_{21}}\), \(type 2 A_{2\text{new}}^* \neq \overline{A_{22}}\), FMT of type 2 in the paper 4, 6 does not satisfy reductive property. From the above calculation, we can see that the fuzzy modus tollens (FMT) by similarity in the paper 4, 6 also doesn’t satisfy the reductive property in m inputs 1 output n rules.

4. Conclusions

In this paper, we have shown that
The results about the Algorithm of General Fuzzy Inference with The Reductive Property obtained in paper 4 are correct.

The Example 2 in paper 4 “An Algorithm of General Fuzzy Inference with The Reductive Property” is incorrect, also, the example in the paper 6 “An approximate analogical reasoning approach based on similarity measures”.

The mistakes in the paper 4 are modified and then the calculation of FMT is supplemented.

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