Structure of two-neutron halo in light drip-line nuclei

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Abstract. The formation of two-neutron halo is described using the neutron-neutron (n-n) interaction constrained by the low-energy n-n scattering limit. This method is tested for loosely-bound two neutrons in \textsuperscript{24}O, where a good agreement with experimental data is found. It is applied to halo neutrons in \textsuperscript{22}C, where the \textsuperscript{20}C-core is treated as a correlated-core with mixing of 2s_{1/2} orbit. This n-n interaction is shown to be strong enough to produce a two-neutron halo, locating \textsuperscript{22}C on the drip line, while \textsuperscript{21}C remains unbound. A unique relation between the two neutron separation energy, S_{2n}, and the radius of neutron halo is presented. The estimated halo radius is found to be consistent with a recent experimental data. The appearance of Efimov states is discussed. The present method is extended to apply to the case of mixed configurations for the two-neutron halos such as \textsuperscript{17}B and \textsuperscript{11}Li.

1. Introduction

Two-neutron halos are found in light drip-line nuclei such as \textsuperscript{10}He, \textsuperscript{11}Li, \textsuperscript{14}Be, \textsuperscript{17}B, \textsuperscript{19}B and \textsuperscript{22}C since it was first found in \textsuperscript{11}Li [1]. When one neutron is removed from these nuclei, they become unbound. The two-neutron halos in \textsuperscript{11}Li, \textsuperscript{14}Be and \textsuperscript{17}B have rather complex structure, that is, with mixtures of two or more orbits, but those in \textsuperscript{10}He and \textsuperscript{22}C have simple structure with almost pure s_{1/2} components. Here, we first study how the two-neutron halo in \textsuperscript{22}C is formed from the low energy limit of neutron-neutron (n-n) interaction.

We use a three-body model which assumes a \textsuperscript{20}C-core and two valence neutrons interacting each other. This is a Borromean situation [2,3], which have been considered in previous theoretical works [4-7] but with different methods and interactions. The \textsuperscript{20}C-core is treated as a correlated-core with mixing of 2s_{1/2} orbit. \textsuperscript{20}C is known to have 2s_{1/2} components whose spectroscopic factor is about 1 [8]. Relations between the two-neutron separation energy, S_{2n}, and the halo radius are derived. The halo radius is estimated by taking into account a condition for \textsuperscript{21}C to be unbound, and compared with experimental data. A comment on Efimov states is made. In sect. 2, a three-body model is explained and applied to \textsuperscript{24}O. S_{2n} and halo radius in \textsuperscript{22}C are studied using the correlated \textsuperscript{20}C-core. Spectra of \textsuperscript{22}C are also predicted. In sect. 3, the three-body model is extended to the case of mixed configurations. The method is applied to \textsuperscript{18}O and \textsuperscript{17}B as well as \textsuperscript{11}Li. A summary is given in sect. 4.
2. Three-body model with low energy n-n interaction

We use a three-body model consisting of a core and two neutrons. Neutron-core interaction is taken to be a one-body Woods-Saxon (WS) potential. The energy of a neutron in the WS potential, $\epsilon$, is positive (negative) if it is unbound (bound). For the n-n interaction, $v_{nn}$, one in the low energy limit which reproduces both the scattering length, $a_{nn}$, and the effective range, $r_{nn}$, is used [9]. A gaussian radial form with the range of 1.795 fm is adopted, whose strength is adjusted to give $a_{nn} = -18.9$ fm and $r_{nn} = 2.75$ fm. The two-neutron energy, $E_{nn}$, is given by

$$E_{nn} = 2\epsilon + <v_{nn}>,$$

where $<v_{nn}>$ is the matrix element of $v_{nn}$ for the two-neutron wave function and is negative. The three-body system is bound if $E_{nn}$ is negative. Two-neutron separation energy is defined by $S_{nn} = -E_{nn}$, and as one-neutron separation energy is given by $S_{1n} = -(\epsilon + <v_{nn}>)$, $S_{2n} = 2S_{1n} + <v_{nn}>$. As $\epsilon = -(S_{2n} + <v_{nn}>)/2$, the condition for core-one-neutron state to be unbound is $\epsilon > 0$, that is, $S_{2n} < -<v_{nn}> = |<v_{nn}>|$

Here, for a neutron the attraction from the other neutron is treated as an additional one-body potential, $w(r)$, obtained by convolution of $v_{nn}$ by the neutron wave functions. $S_{1n}$ and bound neutron wave function, $\phi(r)$, are obtained by solving the following Schrodinger equation:

$$\{h + w(r)\} \phi(r) = -S_{1n} \phi(r)$$

where $h$ is the one-body Hamiltonian for core-neutron system with the WS potential. The neutron wave function obtained is used for the convolution of $v_{nn}$. Thus, after this iterative procedure, $S_{1n}, <v_{nn} >$ and $S_{2n}$ are obtained self-consistently when they are converged.

In the three-body model, $^{22}$O is treated as $^{20}$O-core and two neutrons. $^{22}$O is bound with $\epsilon = -2.73$ MeV. The WS potential is fixed to reproduce this value for $\epsilon$. Then, $S_{2n}$ and $S_{1n}$ are obtained by the above procedure; $S_{2n} = 6.94$ MeV and $S_{1n} = 4.21$ MeV which are very close to the experimental values of $S_{2n} = 6.92$ MeV and $S_{1n} = 4.19$ MeV, respectively [10].

The occupation number of neutron $2s_{1/2}$ orbit in the correlated $^{20}$C-core is about 1 [8]. It is also true for a shell-model calculation with the YSOX interaction [11]. There is a large mixing of components of $\nu 1d_{5/2}^1 2s_{1/2}^2$ configurations, which lowers the ground state energy of $^{20}$C. On the other hand, the halo state in $^{22}$C is dominantly $2s_{1/2}^2$ configuration [8]. How to reconcile this situation? We here adopt the following model. The halo s-orbit is occupied by two neutrons as predicted by the experiments [8]. The orthogonality condition between this halo s-orbit and the s-orbit of the $^{20}$C-core state should be satisfied, that is, the core s-orbit is made orthogonal to this halo s-orbit by Gram-Schmidt method. This gives rise to blocking effects on the core states. Energy of the $^{20}$C-core of the ground state of $^{22}$C is pushed up with respect to the energy of the $^{20}$C ground state. This energy shift is denoted as $\Delta$. The two-neutron separation energy is thus reduced: $S_{2n} = -E_{nn} - \Delta$. As the core s-orbit gets halo components, two-body matrix elements of the YSOX interaction with s-components are modified. The single-particle energy of $2s_{1/2}$ orbit outside the $^4$He-core is also modified. Shell-model calculations are performed by taking into account these modifications, and the energy shift of the ground state of $^{20}$C is evaluated to be $\Delta \sim 1-1.5$ MeV.

The relation between $S_{2n}$ and the halo radius for the correlated $^{20}$C-core case is shown in Fig. 1. The condition for $^{21}$C to be unbound gives $S_{2n} < 0.8$ MeV. Fig. 1 suggests that the r.m.s. radius of the neutron halo is 6-7 fm, which is quite small compared with the value obtained in Ref. [13]. Recently, the matter radius of $^{22}$C was re-measured, which yields $3.44\pm0.08$ fm [14] much smaller compared with the previous one, $5.4\pm0.9$ fm [13]. A smaller halo radius of $6.74\pm0.71/-0.48$, which is consistent with the present three-body model with the correlated $^{20}$C-core, is estimated from the recent measurement [14] and the radius of $^{20}$C [15]. Dependence of the halo radius on $S_{2n}$ is small.

The hypothesis of Efimov [16,17] states implies the appearance of similar states at different scales near threshold. The upper bound on the radius of the halo in the correlated-core model
Figure 1. R.m.s radius of the halo neutron as a function of two-neutron separation energy $S_{2n}$. The solid line indicate the result obtained with the correlated $^{20}\text{C}$ core, while the dashed line the result of the closed-shell $^{20}\text{C}$. The result obtained from WS potential ($S_{1n} = S_{2n}/2$) without $v_{nn}$ is shown by the dash-dotted line. The range of $S_{2n}$ from Ref. [10] is shown by vertical lines. Green arrows denote values discussed in Ref. [8]. (Taken from Fig. 5 of Suzuki et al. [12])

contradicts this hypothesis. The ground state of $^{22}\text{C}$ is already close to this upper limit, and there are no excited bound states. The state of the two-neutron halo in $^{22}\text{C}$ can be called a single Efimov state for the correlated-core case.

Energy spectra of $^{22}\text{C}$ are obtained for correlated $^{20}\text{C}$-cores. As the correlation increases by blocking effects and the occupation number of the $2s_{1/2}$ orbit in the core decreases, the excitation energy of the $2_{1}^{+}$ state is pushed up [18] as shown in Fig. 2. The energy position of the $2_{1}^{+}$ state could give us information on the degree of the correlation and $S_{2n}$.

3. Three-body model with mixed configurations

Here, the three-body model is extended to the case of mixed configurations for the halo. For example, the two-neutron state in $^{17}\text{B}$ is described as the linear combination of $2s_{1/2}$ and $1d_{5/2}$ configurations;

$$\Phi = \alpha \left| 2s_{1/2}^{(0^+)} \right> + \beta \left| 1d_{5/2}^{(0^+)} \right>$$

Total energy, $E_{nn}$ and halo wave functions $\phi_{s}$ and $\phi_{d}$ are obtained by solving the following coupled equations,

$$\alpha(2\epsilon^{s} + V_{ss}) + \beta V_{sd} = \alpha E_{nn}$$

$$\beta(2\epsilon^{d} + V_{dd}) + \alpha V_{ds} = \beta E_{nn}$$

where $\epsilon^{i} = < \phi_{i} | h | \phi_{i} >$ and $V_{ij} = < i j | v_{nn} | i j >$. $E_{nn}$, $\alpha$, $\beta$ and radial wave functions $\phi_{s}(r)$ and $\phi_{d}(r)$ are obtained self-consistently by solving the equations in Eq. (3).

We first test this method in $^{18}\text{O}$, where one neutron in $^{17}\text{O}$ is bound. When only $1d_{5/2}$ configuration is considered, $S_{2n}$ in $^{18}\text{O}$ is obtained to be $S_{2n} = 10.84$ MeV by using observed
Figure 2. Energy spectra of $^{22}\text{C}$ obtained by shell-model calculations with modified YSOX interaction. The first column is the case for the original YSOX Hamiltonian without the halo effects. Second column shows the result for the case without the correlation in $^{20}\text{C}$-core. The third, fourth and fifth ones are results with correlated-core.

value of $S_{1n}=4.143$ MeV. When both $1d_{5/2}^2$ and $2s_{1/2}^2$ configurations are taken into account, $S_{2n}$ is obtained to be $S_{2n}=11.29$ MeV and comes closer to the experimental value, $S_{2n}^{exp}=12.188$ MeV, with $|\beta|^2=0.94$. Calculated values of $S_{2n}$ in the present model are consistent with those of shell-model calculation with USD [19] as shown in Fig. 3. The experimental value is reproduced when the full $(sd)^2$ configurations are included. We thus find that the present three-body model works well for the two-component configurations.

Now we apply the method to $^{17}\text{B}$ whose two-neutron halo is assumed to consist of $2s_{1/2}^2$ and $1d_{5/2}^2$ components. Calculated results for the relation between $S_{2n}$ and halo radius are shown in Fig. 4. Comparing with experimental information on $S_{2n}$ [10,20] and halo radius [15,21], the probability for the $2s_{1/2}^2$ configuration is estimated to be $|\alpha|^2=35\%$, that is, more dominance of 1d-orbit in the halo. This probability is consistent with the value of $|\alpha|^2=0.39\pm0.19$ in Ref. [21], but small compared with the value of $|\alpha|^2=0.69\pm0.20$ in Ref. [22].

We next study the structure of the drip-line nucleus $^{19}\text{B}$. The two-neutron state in $^{19}\text{B}$ is dominantly $1d_{5/2}^2$ character; $P(2s_{1/2}^2)<20\%$ [23]. Assuming a pure $1d_{5/2}^2$ configuration for the two-neutron state, the three-body model is applied. The relation between the valence neutron radius and $S_{2n}$ is shown in Fig. 5. The valence neutron radius is estimated to be $R_n=4.35-4.45$ fm from the experimental restriction on $S_{2n}$.

Two-neutron halo in $^{11}\text{Li}$ is investigated by the three-body model with two components for the halo, $2s_{1/2}^2$ and $1p_{1/2}^2$. The relation between the neutron halo radius and $S_{2n}$ is shown in Fig. 6. The probability for $2s_{1/2}^2$ configuration is found to be about $P(2s_{1/2}^2)=50\%$, which is consistent with the experimental data, $P(2s_{1/2}^2)>44\%$ [25].
Figure 3. Energy of the ground state of $^{18}$O obtained by the present three-body model and shell-model calculations with USD as well as the experimental value. In the three-body model, pure $1d_{5/2}^2$ and $1d_{5/2}^2+2s_{1/2}^2$ configurations are considered. In the shell-model calculations, the case with full $(sd)^2$ space with inclusion of $1d_{3/2}$ orbit is also considered.

Figure 4. R.m.s radius and two-neutron separation energy $S_{2n}$ for $^{17}$B obtained by the present three-body model. The probability for the $2s_{1/2}^2$ component is also denoted. The solid and dashed vertical lines indicate experimental $S_{2n}$ of Ref. [10] and [20], respectively. The dashed and dash-dotted horizontal lines show experimental halo radius of Ref. [21] and [15], respectively.

4. Summary

We studied the formation of two-neutron halo in light drip-line nuclei using n-n interaction fixed by the low-energy limit of n-n scattering. $S_{2n}$ in $^{24}$O is found to be well reproduced by the present three-body model. The structure of the ground state of $^{22}$C is studied with a correlated $^{20}$C-core. A unique relation between $S_{2n}$ and halo radius is presented. $S_{2n}$ is constrained to be less than 0.8 MeV from the condition that $^{21}$C is unbound. The two-neutron halo state is successfully obtained in $^{22}$C while $^{21}$C remains unbound. The halo radius is obtained to be 6-7 fm for the correlated core, which is consistent with the recent measurement [14]. Because of the existence of the upper limit of the halo radius, there are no excited bound states. The spectra for $^{22}$C are also studied. The energy of the first $2^+$ state is pushed up as the s-state in the core loses more $2s_{1/2}$ component. The position of the $2^+_1$ state can be a good measure of the correlation of the $^{20}$C-core and the value of $S_{2n}$.

The three-body model is extended to treat two valence neutron cases with mixed configurations. The model is found to be successful to obtain $S_{2n}$ in $^{18}$O and applied to two-neutron halos in $^{17}$B and $^{11}$Li. The probability for $2s_{1/2}^2$ component is evaluated by constraints from experimental information on $S_{2n}$ and halo radius. $P(2s_{1/2}^2)$ for $^{17}$B and $^{11}$Li are estimated to be about 35% and 50%, respectively.

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Figure 5. Valence neutron radius as a function of $S_{2n}$ in $^{19}$B obtained by the three-body model assuming a pure $1d_{5/2}^2$ configuration. The vertical lines indicate experimental value of $S_{2n}$: $S_{2n} = 0.14 \pm 0.39$ MeV [21]. The horizontal lines show experimental neutron radius; $R_n = 4.21 \pm 0.80 / -0.85$ fm obtained from matter radii of $^{19}$B and $^{17}$B [21].

Figure 6. Neutron halo radius as a function of $S_{2n}$ for $^{11}$Li. Two components of $2s_{1/2}^2$ and $1p_{1/2}^2$ configurations are assumed for the two-neutron halo state. The experimental value of $S_{2n} = 369$ keV and neutron halo radius, $R_n = 6.54 \pm 0.38$ fm, are taken from Ref. [10] and Ref. [24], respectively.

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