Constraint On The Cosmological Constant From Gravitational Lenses In An Evolutionary Model Of Galaxies

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Abstract

We study the effect of the cosmological constant on the statistical properties of gravitational lenses in flat cosmologies ($\Omega_0 + \lambda_0 = 1$). It is shown that some of the lens observables are strongly affected by the cosmological constant, especially in a low–density universe, and its existence might be inferred by a statistical study of the lenses. In particular, the optical depth of the lens distribution may be used best for this purpose without depending much on the lens model. We calculate the optical depth (probability of a beam encountering with a lens event) for a source in a new picture of galaxy evolution based on number evolution in addition to pure luminosity evolution. It seems that present day galaxies result from the merging of a large number of building blocks. We have tried to put limit on the cosmological

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constant in this new picture of galaxy evolution. This evolutionary model of galaxies permits larger value of cosmological constant.
1 Introduction

In the early history of modern cosmology the cosmological constant $\Lambda$ was invoked twice. First by Einstein to obtain static models of the universe. Next by Bondi, Gold and Hoyle to resolve an age crisis and to construct the universe that satisfied the “Perfect Cosmological Principle”, i.e., the hypothesis that the universe appears the same at all times and places. In both instances the motivating crisis passed and the cosmological constant was put aside. However the cosmological constant remains a focal point of cosmology and of particle theory. The former because today an understanding of a wide range of observations seem to call for a cosmological constant. The latter because in the context of quantum–field theory a cosmological constant corresponds to the energy density associated with the vacuum and no known principle demands that it vanish.

Various aspects of recent observations suggest reconsideration of a nonvanishing cosmological constant (Krauss & Turner 1995). These includes the age of universe once again, the formation of large–scale structure (galaxies,clusters of galaxies, superclusters) and the matter content of the universe as constrained by dynamical estimates, Big Bang Nucleosynthesis and X–Ray observations of clusters of galaxies.

One of the possibilities of detecting $\Lambda$ is the gravitational lensing technique. To use the gravitational lensing as a tool for the determination of cosmological parameters either by a detailed study of specific lens systems or through statistical analysis of a sample of lenses has been frequently discussed (Refsdal
1964; Press & Gunn 1973). It has been pointed out that the expected frequency of multiple imaging lensing events is quite sensitive to cosmological constant (Fukugita, Futamse & Kasai 1990; Turner 1990). To put a limit on $\Lambda$, we need to first calculate the expected number of multiple image gravitational lens systems (produced by the known galaxy population) to be expected in a particular quasar sample with a known distribution of redshifts. This then has to be compared with the observed frequency of lens systems found.

We have used a unifying model of galaxy formation which can answer two questions raised by the recent data on high - redshift galaxies. These concern the ages of, and star formation history in the distant radiogalaxies, and the nature of the large number of field galaxies revealed by faint galaxy counts. This new model is based on the strong number evolution in addition to pure luminosity evolution of the galaxies (Volmerange & Guiderdoni 1990).

In Section 2, we write down the New Luminosity Function (NLF) which has strong number evolution in addition to pure luminosity evolution. In Section 3, we present a new calculation of the total multiple image lensing cross-sections for a galaxy in the Singular Isothermal Sphere (SIS) approximation (Turner, Ostriker & Gott 1984)(hereafter TOG) based on the new galaxy luminosity function, velocity–luminosity correlation and velocity dispersion. In Section 4 we write down the basic equations for the statistical properties of lenses for each of the galaxy models. The quasar luminosity function for the BSP sample (Boyle,
Shanks & Peterson 1988) is described in Section 5. Section 6 contains the number of lensed quasars in the BSP sample for the Press–Schecter Luminosity Function (PSLF) and the New Luminosity Function (NLF). In Section 7 we discuss the results.

2 New Luminosity Function

In 1990, Volmerange and Guiderdoni, proposed a unifying model to explain faint galaxy counts as well as observational properties of distant radiogalaxies. This new model of galaxy evolution is based on number evolution in addition to pure luminosity evolution. Present day galaxies result from the merging of a large number of building blocks and the comoving number of these building blocks evolves as $(1 + z)^{1.5}$. It is argued that the present luminosity function is the well known Press - Schecter Luminosity Function (PSLF)

\[ \Phi(L, z = 0) = \phi^*(L/L_*)^\alpha e^{\frac{-L}{L_*}} dL/L_* \]

with $L_*$ being the characteristic luminosity at the knee and $\phi^*$ a characteristic density. These values are fixed in order to fit the current luminosities and densities of galaxies. Then at high $z$, the comoving number density follows the New Luminosity Function (NLF)

\[ \Phi(L, z) dL = (1 + z)^{2\eta} \Phi(L(1 + z)^\eta, 0) dL \]

It is seen that the value $\eta = 1.5$ gives a fair fit to the data on high redshift galaxies. The functional form has the following properties:
(i) Self-similarity as suggested by the classical Press-Schecter (1974) prescription.

(ii) conservation of the total comoving mass density.

(iii) evolution of the comoving number density as \((1 + z)^\eta\) and of the knee of the function as \(L_*(z) = L_*(0)(1 + z)^{-\eta}\)

3 **Lensing Cross-Section For SIS Galaxies**

The probability of a beam encountering a lensing object is governed by the parameter \(F\) (TOG 1984) which represents the effectiveness of distributed cosmic matter in producing double images. The parameter \(F\) is given by

\[
F = \frac{16\pi^3}{cH_0^3}n_0v^4
\]  

(1)

where \(n_0\) is the present comoving number density of lensing galaxies and \(v\) is the line of sight velocity dispersion of matter in the galaxy. The Hubble constant \(H_0\) is normalized by \(H_0 = 100\ h\ \text{km s}^{-1}\text{Mpc}^{-1}\), where \(0.4 < h < 1.0\). In this section we evaluate \(F\) from the statistics of local galaxies, assuming that their properties persist out to distant galaxies. The relationship between \(v\) and the luminosity is known empirically, i.e., \(L \propto v^4\) for elliptical galaxies and \(L \propto v^{2.6}\) for spiral galaxies in the B band. Using the luminosity function given by Volmerange and Guiderdoni (1990)
\[ \Phi(L, z) dL = (1 + z)^{2\eta} \phi \left[ \frac{L}{L_*} (1 + z)^\eta \right]^{\alpha} \exp \left[ \frac{-L}{L_*} (1 + z)^\eta \right] \frac{dL}{L_*} \]  

(2)

\[ n_L(0) = \int_{0}^{\infty} \Phi(L, z) dL \]  

(3)

\[ \left( \frac{L}{L_*} \right) = \left( \frac{v}{v_*} \right)^\gamma \]  

(4)

Using eq.(1), eq.(2), eq.(3), eq.(4) we get

\[ F = \frac{16\pi^3}{cH_0^3} \phi_* v_*^4 \Gamma \left( \alpha + \frac{4}{\gamma} + 1 \right) (1 + z)^{\eta(1 - \frac{4}{\gamma})} \]  

(5)

which may be written as

\[ F = F^* (1 + z)^{\eta(1 - \frac{4}{\gamma})} \]  

(6)

where

\[ F^* = \frac{16\pi^3}{cH_0^3} \phi_* v_*^4 \Gamma \left( \alpha + \frac{4}{\gamma} + 1 \right) \]  

(7)

here \( v_* \) is the velocity dispersion corresponding to the characteristic luminosity \( L_* \) and \( \gamma = 4 \) for elliptical galaxies and \( \gamma = 2.6 \) for spiral galaxies. According to Fukugita and Turner (1991),

\[ \phi_* = (1.56 \pm 0.4) \times 10^{-2} h^3 Mpc^{-3} \]  

(8)

\[ \alpha = -1.1 \pm 0.1 \]
The values of the parameter $F^*$ adopted by them are summarized in Table 1.

It has been argued that the existence of a $\sqrt{3/2}$ correction factor for the dark matter velocities dispersion used by Fukugita and Turner is incorrect (Fukugita & Turner 1991). Better dynamical models (Kochanek 1993; Kochanek 1994; Brimer & Sanders 1993) show that the assumption of $\sqrt{3/2}$ is incorrect, and is supported neither by galactic dynamics nor by the observed separations of gravitational lensing. The best fit estimate is $v_* = 220 \pm 20$ km/sec for E + SO galaxies, and it is well constrained because the average image separation is a strong function of velocity dispersion (Kochanek 1996). The new values of $F^*$ obtained by using the parameters given by Kochanek (Kochanek 1996) are summarized in Table 2.

$$\phi_* = (1.40 \pm 0.17)h^3 \times 10^{-2} Mpc^{-3}$$

$$\alpha = -1.00 \pm 0.15$$

We note that the total value of $F^*$ determined here ($F^* = 0.028$) is four times smaller than the value used by TOG. Nakamura and Suto (1996) have given the new values of $(\gamma, v_*) = (3.3, 175 \, km\, s^{-1})$ for E and SO galaxies and $(\gamma, v_*) = (2.9, 126 \, km\, s^{-1})$ for S galaxies. The morphological composition is $\phi_{*E} + \phi_{*SO} = 0.44\phi_*$ and $\phi_{*S} = 0.56\phi_*$ where $\phi_* = 0.26h^3 Mpc^{-3}$ and $\alpha = -1.09$. The values of $F^*$ with Nakamura and Suto parameters are summarized in Table 3.
4 Basic Equations For Statistical Lensing

To discuss the statistical properties of gravitational lenses we assume that the universe is well approximated by the Friedmann - Lemaitre - Robertson - Walker (FLRW) geometry on large scale. We write the basic equations for the statistical lensing for two models of the lensing object; i.e. , (1) Point Masses, which are appropriate models for stellar mini-lensing or concentrated sources such as black holes, and (2) Singular Isothermal Sphere (SIS), which would model the matter distribution of an isolated galaxy. We use the following notation for distances:

\[ D_{OL} = d(0, z_L), D_{LS} = d(z_L, z_s), D_{OS} = d(0, z_S), \]

where \( d(z_1, z_2) \) is the angular diameter distance between the redshift \( z_1 \) and \( z_2 \), and the arguments \( z_L \) and \( z_S \) are the redshifts of the lens and source respectively. The formulation and notation of TOG is basically followed in this paper. For \( \Omega_\circ + \lambda_\circ = 1 \),

\[ d(z_1, z_2) = \frac{R_\circ}{(1 + z_2)} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\Omega_\circ(1 + z)^3 + \lambda_\circ}}, \quad (9) \]

where \( \lambda_\circ = \Lambda/3H_\circ^2 \) and \( \Lambda = 8\pi G \rho_{vac} \) and \( R_\circ \) is the present scale factor.

4.1 Point Masses

This model can be considered to be a good approximation for many celestial bodies like ”Jupiter”, stars, black holes and even galaxies, when the light rays from background sources pass
outside the deflectors. We first define the length $a_{cr}$ which characterizes the critical radial size of the lens such that

$$a_{cr}^2 = \frac{4GM}{c^2} \frac{D_{OLD_S}}{D_{OS}},$$

(10)

where $M$ is mass of the lensing object (TOG). Then the cross-section $\sigma$ for strong lensing events as defined by TOG is given by

$$\sigma = \pi a_{cr}^2,$$

(11)

The differential probability $d\tau$ of a beam encountering a lens in traversing the path of $dz_L$ is given by

$$d\tau = n_L(z)\sigma \frac{cdt}{dz_L} dz_L,$$

(12)

$$d\tau = \frac{3}{2} \Omega_L(0)(1 + z_L)^3 \left( \frac{D_{OLD_S}}{R_o D_{OS}} \right) \frac{1}{R_o} \frac{cdt}{dz_L} dz_L,$$

(13)

where $n_L(z) = n_L(0)(1 + z_L)^3$ and $n_L(0)$ is the comoving number density and $\Omega_L(0) = 8\pi G M n_L(0)/3H_o^2$ is the lens density parameter which is the ratio of the local lens density to the critical density. The quantity $cdt/dz_L$ is calculated in the FLRW geometry to be

$$\frac{cdt}{dz_L} = \frac{R_o}{(1 + z_L) \sqrt{\Omega_o(1 + z_L)^3 + (1 - \Omega_o - \lambda_o)(1 + z_L)^2 + \lambda_o}},$$

(14)

$R_o$ is the Hubble distance ($R_o = c/H_o$), $t$ stands for the look back time and $\Omega_o$ is the total mass density of the universe. By
integrating the differential probability along the line of sight to the source, we obtain the total probability

\[ \tau(z_s) = \int_0^{z_s} \frac{d\tau}{dz_L} dz_L, \quad (15) \]

### 4.2. Singular Isothermal Spheres

The singular isothermal sphere (SIS) provides us with a reasonable approximation to account for the lensing properties of a real galaxy. The lens model is characterized by the one-dimensional velocity dispersion \( v \). The deflection angle for all impact parameters is given by \( \alpha = 4\pi v^2/c^2 \). The lens produces two images if the angular position of the source is less than the critical angle \( \beta_{cr} \), analogous to the critical radius in the previous subsection, which is the deflection of a beam passing at any radius through a SIS:

\[ \beta_{cr} = \alpha D_L D_S / D_O S, \quad (16) \]

Then the critical impact parameter is defined by \( a_{cr} = D_{OL} \beta_{cr} \) and the cross-section is given by

\[ \sigma = \pi a_{cr}^2 = 16\pi^3 \left( \frac{v}{c} \right)^4 \left( \frac{D_{OL} D_L S}{D_O S} \right)^2, \quad (17) \]

The differential probability of a lensing event is given by using eqs. (2), (5), (12), (17),

\[ d\tau = n_L(0)(1 + z_L)^3 \frac{cdt}{dz_L} dz_L \]
It turns out that for $\gamma = 4$ the differential probability is the same for both PSLF and NLF. The total probability obtained by integrating the differential probability along the line of sight to the source as in eq.(15), is plotted in Fig.1. with both PSLF and NLF for $\Omega_\circ = 0.1$ and $\lambda_\circ = 0.9$.

5 The Quasar Luminosity Function

Boyle, Shanks & Peterson (1988) (BSP) have published a sample of 420 faint ultraviolet-excess quasars that extends from $z = 0.3$ to $z = 2.2$ and is complete for magnitude brighter than 20.9 mag. Using this sample, they proposed a quasar luminosity function of the form

\[
\Phi(M_B, z) = \frac{\Phi^*}{10^{0.4[M_B-M_B(z)]/(\alpha+1)} + 10^{0.4[M_B-M_B(z)]/(\beta+1)}} Mpc^{-3}mag^{-1}
\]

(19)

where $M_B(z)$, the magnitude at which there is a turnover in the power law slope, varies as a function of quasar redshift.
\[ M_B(z) = M_B^* - 2.5 k_L \log(1 + z), \]  

(20)

Here \( M_B^* \) and \( k_L \), as well as \( \alpha \) and \( \beta \) from eq.(19) are constant given by the best fit of the parameters to the data and \( \Phi^* \) is the normalizing factor. We have used model B from BSP, a pure power law evolution that fits the data very well. For this model, the values of the constants are \( M_B^* = -22.42 \), \( k_L = 3.15 \), \( \alpha = -3.79 \), \( \beta = -1.44 \).

6 The Number Of Lensed Quasars

Besides depending upon the number density of galaxies and their properties, the number of quasars gravitationally lensed also depends upon the number of unlensed quasars. This is because of the amplification bias i.e., the brightening of lensed quasars, the number of quasars a few magnitudes fainter than the detection limit is important in predicting the number of lensed quasars in a flux limited sample. Fukugita & Turner (1991) calculated the number of lensed quasars expected for \( \lambda_0 \) cosmology taking into the account the amplification bias. The number–magnitude counts of the lensed quasars are given by the relation

\[ N_{LQ}(m) = \tau \int_0^\infty N_Q(m + \Delta) P(\Delta) d\Delta, \]  

(21)

where \( P(\Delta) d\Delta \) is the probability that lensing will cause the magnitude of the quasar (images) to decrease by \( \Delta \); i.e., the images are brighter by a factor \( A \); \( \Delta = 2.5 \log A \).
\[
N_{LQ} = \int_0^\infty P(\Delta) d\Delta \int_{z_1}^{z_2} dz \tau(z) \frac{dV}{dz} \int_{M_1}^{M_2} \Phi(M_B, z) dM_B, \quad (22)
\]

where \(P(\Delta)d\Delta = 7.37 \times 10^{-0.8\Delta d\Delta}\) and \(dV/dz\) is the comoving volume which is given by

\[
\frac{dV}{dz} = \frac{4\pi d_L^2 c}{H_0 (1+z)^2 \sqrt{\Omega_\Lambda (1+z)^3 + \lambda_0} - (\Omega_\Lambda + \lambda_0 - 1)(1+z)^2}, \quad (23)
\]

where \(d_L\) is the luminosity distance. The \(N_{LQ}\) calculated in BSP sample by using PSLF and NLF for galaxy distribution are summarized in Table 4. The result is plotted in Fig. 2.

## 7 Conclusion

An upper bound to \(\lambda\), results from the comparison of the statistics of quasars observed to be gravitationally lensed by intervening galaxies, with the predictions of flat cosmological models with a nonzero cosmological constant. A flat cosmology with cosmological constant tends to produce more gravitationally lensed quasars than does such a cosmology with zero \(\lambda\) because a large \(\lambda\) increases the volume per unit redshift of the universe at high redshift. This means that the relative number of lensed quasars for a fixed comoving number density of galaxies increases rapidly with increasing \(\lambda\). Turning this around it is possible to use the observed probability of lensing to constrain \(\lambda\). This method has been used by various authors (Turner 1990,
Fukugita & Turner 1991, Fukugita et al. 1992) and recently Kochanek has put limit on $\lambda$ i.e., $\lambda < 0.65$ (Kochanek 1996).

However, limited observational data and the possibility that evolution effects on the population of lensing galaxies have not been properly taken into account suggest that this limit is still uncertain. We have tried to put limit on $\lambda$ by taking evolutionary effect of galaxies. The important result can be described as follows: The probability of a beam encountering with lens event has been reduced in an evolutionary picture (NLF) and as a consequence of it, the evolutionary model permits higher value of $\lambda$. This can be explained as follows, since there is no observed lensed quasars in BSP sample. Taking a typical uncertainty of $\approx \pm 1$ in the number of lensed quasars, PSLF put limit on $\lambda$ i.e., $\lambda < 0.6$ with $F^* = 0.021$ whereas NLF gives $\lambda < 0.7$ with $F^* = 0.021$. And NLF also gives $\lambda < 0.47$ with $F^* = 0.028$.

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References

[1] Boyle, B.J., Shanks, T., and Peterson, B.A., *M.N.R.A.S.*, **235**, 935 (1988)

[2] Brimer, T.G. and Sanders, R.H., *A&A*, **274**, 96 (1993)

[3] Fukugita, M., Futamase, T. and Kasai, M., *M.N.R.A.S.*, **246**, 24p (1990)

[4] Fukugita, M. and Turner, E.L., *M.N.R.A.S.*, **253**, 99 (1991)

[5] Fukugita, M., Futamase, T., Kasai, M., and Turner, E.L., *Ap.J.*, **393**, 3 (1992)

[6] Kochanek, C.S., *Ap.J.*, **419**, 12 (1993)

[7] Kochanek, C.S., *Ap.J.*, **436**, 56 (1994)

[8] Kochanek, C.S., *Ap.J.*, **466**, 638 (1996)

[9] Krauss, L.M. and Turner, M.S., 1995, Preprint astro–ph/9504003

[10] Nakamura, T.T and Suto, Y., UTAP Preprint No. 236/96.

[11] Press, W.H., and Gunn, J.E. *Ap.J.*, **185**, 397 (1973)

[12] Press, W.H. and Schechter, P., *Ap.J.*, **187**, 487 (1974)

[13] Refsdal, S., *M.N.R.A.S.*, **128**, 295 (1964)

[14] Rocca-Volmerange, B., and Guiderdoni, B., *M.N.R.A.S.*, **247**, 166 (1990)
[15] Turner, E.L., Ostriker, J.P. and Gott, J.R.,III., Ap.J, 284, 1 (1984) (TOG).

[16] Turner, E.L., Ap.J, 365, L43 (1990)
Figure Captions

Fig. 1 $z_s$ Vs $\tau/F^*$
Fig. 2 $\lambda$ Vs $N_{LQ}$
Table 1. SIS velocity dispersion and $F^*$ values (using parameters of Fukugita and Turner (1991)).

| Type | Composition | $v_*(km/sec)$ | $F^*$     |
|------|-------------|---------------|-----------|
| E    | 12 ± 2%     | $225^{+12}_{-25} \times \sqrt{3/2}$ | 0.019 ± 0.008 |
| SO   | 19 ± 4%     | $206^{+12}_{-20} \times \sqrt{3/2}$ | 0.021 ± 0.009 |
| S    | 69 ± 4%     | $144^{+8}_{-13} \pm 10$ | 0.007 ± 0.003 |
| ALL  | 100%        |               | 0.047 ± 0.019 |
Table 2. SIS velocity dispersion and $F^*$ values (using parameters of Kochanek (1996)).

| Type | Composition | $v_\ast (km/sec)$ | $F^*$  |
|------|-------------|-------------------|-------|
| E+ SO | 43%         | $220 \pm 20$     | 0.023 |
| S    | 57%         | $144 \pm 10$     | 0.005 |
| ALL  | 100%        |                   | 0.028 |
Table 3. SIS velocity dispersion and $F^*$ values (using parameters of Nakamura and Suto (1996)).

| Type  | Composition | $v_*(km/sec)$ | $F^*$  |
|-------|-------------|---------------|--------|
| E+ SO | 44%         | 175           | 0.016  |
| S     | 56%         | 126           | 0.005  |
| ALL   | 100%        |               | 0.021  |
Table 4. Number of lensed quasars $N_{LQ}$ in BSP sample.

| $\Omega_0 + \lambda_0$ | $N_{LQ}$ with NLF $F^* = 0.028$ | $N_{LQ}$ with NLF $F^* = 0.021$ | $N_{LQ}$ with PSLF $F^* = 0.021$ |
|------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1.0 + 0.0              | 0.58                            | 0.38                            | 0.43                            |
| 0.9 + 0.1              | 0.65                            | 0.43                            | 0.49                            |
| 0.8 + 0.2              | 0.72                            | 0.47                            | 0.54                            |
| 0.7 + 0.3              | 0.81                            | 0.53                            | 0.61                            |
| 0.6 + 0.4              | 0.92                            | 0.60                            | 0.69                            |
| 0.5 + 0.5              | 1.06                            | 0.69                            | 0.79                            |
| 0.4 + 0.6              | 1.29                            | 0.83                            | 0.97                            |
| 0.3 + 0.7              | 1.59                            | 1.02                            | 1.19                            |
| 0.2 + 0.8              | 2.11                            | 1.35                            | 1.58                            |
| 0.1 + 0.9              | 3.13                            | 1.98                            | 2.35                            |
| 0.05 + 0.95            | 4.13                            | 2.59                            | 3.09                            |
| 0.01 + 0.99            | 5.83                            | 3.61                            | 4.37                            |
| 0.0 + 1.0              | 6.51                            | 4.02                            | 4.88                            |
Fig. 1

With PSLF

With NLF
Fig. 2

\[ F^* = 0.028 \]

\[ F^* = 0.021 \]

\[ F^* = 0.021 \]