The longitudinal interlayer magnetoresistance $R_{zz}(B_z)$ is calculated in strongly anisotropic layered metals, when the interlayer band width $4t_z$ is less than the Landau level separation $\hbar\omega_c$. The impurity scattering has much stronger effect in this regime than in 3D metals and leads to a linear longitudinal interlayer magnetoresistance $R_{zz} \propto B_z$ in the interval $\omega_c > 4t_z > \sqrt{\hbar\omega_c}$ changing to a square-root dependence $R_{zz} \propto B_z^{1/2}$ at higher field or smaller $t_z$. The crossover field allows to estimate the interlayer transfer integral as $t_z \sim \sqrt{\hbar\omega_c}$. Longitudinal interlayer magnetoresistance, being robust to the increase of temperature or long-range disorder, is easy for measurements and provides a useful tool to investigate the electronic structure of quasi-two-dimensional compounds.

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where $e$ is the electron charge, $v_z = \partial \epsilon_{3D}/\partial k_z = 2t_z d \sin(k_y d)/\hbar$ is the electron velocity, the sum over the electron quantum numbers $m \equiv \{n, k_y, k_z\}$ (excluding spin) is taken in the unit volume, and the retarded electron Green’s function

$$G_R = \left[\epsilon - \epsilon_{3D}(m) - \Sigma^R(\epsilon, m)\right]^{-1}.$$  \hspace{1cm} (6)

Both $\epsilon_{3D}(m)$ and the electron self-energy is independent of $k_y$, and the summation over $k_y$ in Eq. \((5)\) gives the factor equal to the LL degeneracy of one conducting layer per spin state $N_{LL} = 1/2\pi^2 \hbar^2 = eB_z/2\pi \hbar c$.\[44] The interlayer conductivity is now given by a sum over LLs,

$$\sigma_{zz}(\epsilon) = \sum_n \sigma_n(\epsilon),$$  \hspace{1cm} (7)

where the contribution to $\sigma_{zz}$ from the $n$-th LL is

$$\sigma_n = \frac{e^2 \hbar N_{LL}}{2\pi} \int \frac{dk_z}{2\pi} v_z^2(k_z) \left[2\Im G_R(n, k_z, \epsilon)\right]^2.$$  \hspace{1cm} (8)

The electron self-energy $\Sigma^R(\epsilon, m) = \Sigma^R(\epsilon)$ depends only on energy $\epsilon$ and, possibly, on LL number $n$, being independent of $k_z$ (see Appendix). Substituting Eq. \((9)\) and performing integration over $k_z$, one obtains

$$\sigma_n = \frac{\sigma_0 \hbar \omega_c \Gamma_0}{2\pi t_z^2 \left|\Im \Sigma^R_n(\epsilon)\right|} \text{Re} \frac{\left|\epsilon - \epsilon_{2D}(n) - \Sigma^R_{\epsilon}(\epsilon)\right|}{\sqrt{4t_z^2 - \Delta \epsilon}}.$$  \hspace{1cm} (9)

where $\Delta \epsilon \equiv \epsilon - \epsilon_2(n) - \Sigma^R_{\epsilon}(\epsilon)$, and $\sigma_0$ denotes the interlayer conductivity without magnetic field:

$$\sigma_0 = e^2 \rho \bar{\rho}_f \left\langle v_z^2 \right\rangle \tau_0 = 2e^2 N_{LL} t_z^2 \hbar^2 / \omega_c \Gamma_0,$$  \hspace{1cm} (10)

\[\rho = 2N_{LL} / \hbar \omega_c d\] is the 3D DoS at the Fermi level in the absence of magnetic field per two spin components, $\tau_0 = \hbar / 2t_0$ and $\left\langle v_z^2 \right\rangle = 2 t_z^2 \hbar^2 / 2\pi^2$.\[2\]

To calculate the electron self-energy $\Sigma^R(\epsilon)$ we start from the standard model of 3D strongly anisotropic metals. The Hamiltonian consists of two terms: $H = H_0 + H_I$. The first term $H_0 = \sum_m \epsilon_{3D}(m) c_m^\dag c_m$ describes 3D noninteracting electrons in magnetic field with anisotropic dispersion given by Eqs. \((1)\) and \((2)\). The second term $H_I = \sum_i V_i (r) \Psi_i^\dag (r) \Psi(r)$ describes the electron interaction with impurities. The impurities are taken to be point-like with the potential $V_i (r) = U \delta^3 (r - r_i)$ and randomly distributed with volume concentration $n_i$. Without magnetic field the broadening $\Gamma_0 = \Im \Sigma$ of electron levels due to the scattering by impurities in the Born approximation is $\Gamma_0 \approx \pi n_i U^2 \rho_f$, where $\rho_f$ is the density of electron states (DoS) at the Fermi level. In Q2D metals in strong magnetic field, $\hbar \omega_c > 4t_z \Gamma_0$, the electron self-energy $\Sigma^R(\epsilon)$ depends on the energy deviation $\Delta \epsilon$ from the $n$-th LL in the “non-crossing” approximation,\[13 \text{ \&} 46\] schematically shown in Fig. \(5\) the electron Green’s function, averaged over impurity configurations, has the form (see Appendix)

$$G(r_1, r_2, \epsilon) = \sum_{n, k_y, k_z} \Psi^\dag_{n, k_y, k_z} (r_1) \Psi^0_{n, k_y, k_z} (r_2) / \epsilon - \epsilon_{3D}(n, k_z) - \Sigma_n(\epsilon),$$  \hspace{1cm} (11)

where $\Psi^0_{n, k_y, k_z} (r)$ are the electron wave functions in magnetic field without impurities and $\Sigma_n(\epsilon)$ is the electron self energy, averaged over impurity positions and given by the equation

$$\Sigma_n(\epsilon) = n_i U \left[1 - U \rho_f \Sigma_n(\epsilon)\right] - \epsilon.$$  \hspace{1cm} (12)

Here $G(\epsilon) = \Gamma(\epsilon, \rho_n)$ after the integration over $k_y$ and $k_z$ is given by

$$G(\epsilon) = \sum_n \frac{N_{LL}/d}{\sqrt{\epsilon - \epsilon_{2D}(n) - \Sigma_n(\epsilon)}}.$$  \hspace{1cm} (13)

The system of equations \((12)\) and \((13)\) allows to calculate the electron self-energy $\Sigma_n(\epsilon)$ numerically.

For weak impurity potential $U G(\epsilon) \sim U \rho_f \ll 1$ the self-consistent Born approximation (SCBA) is applicable, and Eq. \((12)\) reduces to

$$\Sigma(\epsilon) \approx n_i U + n_i U^2 G(\epsilon).$$  \hspace{1cm} (14)

The SCBA is valid when the scattering potential of each impurity is weak compared to Fermi energy, but the concentration of impurities can be arbitrary, so that the broadening of electron levels $\Gamma_0 \approx n_i U^2 \rho_f$ is also arbitrary. In Ref. \[39\] it was shown that one needs to apply at least SCBA to obtain qualitatively correct monotonic growth of interlayer longitudinal MR, which differs in SCBA and in the non-crossing approximations only by a factor close to unity.

The similar system of SCBA equations for the electron Green’s function in Q2D metals in magnetic field was written previously (see Eqs. \((11)\) and \((15)\) of Ref. \[47\], Eqs. \((11)\)-\((13)\) in Ref. \[24\], Eq. \((29)\) in Ref. \[48\], Eq. \((17)\) in Ref. \[49\], or Eq. \((6)\) in Ref. \[50\]). There the SCBA equations have not been solved, and the simple Born approximation was applied to calculate the interlayer conductivity. In Refs. \[24 \text{ \&} 47\] only the limit $2\pi t_z \gg \hbar \omega_c$ has been considered, while in Ref. \[48\] a large electron reservoir was introduced to damp the MQQ and to make the simple Born approximation applicable.\[51\]

Consider the strong magnetic field limit, when $\hbar \omega_c > 4t_z \Gamma_0$ and the LLs do not overlap. Then one can consider only one LL in the solution of SCBA Eqs. \((13)\) and \((14)\), which simplify to

$$\Sigma_\Delta = \frac{n_i U^2 g_{LL}/d}{\sqrt{\Delta^2 - \Sigma_\Delta^2 + 4t_z^2}}.$$  \hspace{1cm} (15)

where $\Sigma_\Delta = \Sigma_n(\epsilon) - n_i U$ and $\Delta \epsilon \equiv \epsilon - \epsilon_{2D}(n) - n_i U = \Delta \epsilon + \Sigma^R_{\epsilon}$. At $t_z = 0$ we obtain the 2D result\[52\] for the electron self-energy in SCBA. Note that the scattering enters Eq. \((15)\) only in the combination $n_i U^2 g_{LL}/d = \Gamma_0 \hbar \omega_c$. Therefore, at $t_z \to 0$, $\Sigma_\Delta = \sqrt{\Gamma_0 \hbar \omega_c}$ is the only energy scale in Eq. \((15)\), and naturally $\Im \Sigma_\Delta \sim \sqrt{\Gamma_0 \hbar \omega_c} \propto \sqrt{B_z}$ according to Ref. \[52\], which leads to the similar field dependence of background MR: $R_{zz} \approx R_{zz0} \Im \Sigma(\mu, B)|/\Gamma_0 \propto \sqrt{B_z}$. Eq. \((15)\) rewrites as an algebraic equation of the fourth power:

$$\Sigma_\Delta^2 + (\Delta \epsilon - \Sigma_\Delta^2 + 4t_z^2) = (\Gamma_0 \hbar \omega_c)^2.$$  \hspace{1cm} (16)
Among four solutions of this equation, only one satisfies the physical requirement $\Sigma_\ast \to 0$ at $\Delta \varepsilon \to \pm \infty$. Its solution $\Sigma_\ast / \Gamma_\ast$ is shown in Figs. 12 for three different values of $t_z / \Gamma_\ast = 0, 0.5, 1.0$. One can see from Fig. 1 that Im$\Sigma_\ast \neq 0$ in the finite energy interval of the width $\sim 4 \Gamma_\ast$ at $\Gamma_\ast > 2 t_z$ and of the width $\sim 4 t_z$ at $\Gamma_\ast > 2 t_z$. Re$\Sigma_\ast$ has cusps at the boundaries of this energy interval.

Now we substitute these solutions into Eq. (1). The result for monotonic part of interlayer MR $R_{zz} = \bar{\sigma}_{zz}$ is shown in Fig. 3 where $\bar{\sigma}_{zz}$ is the conductivity averaged over period $\hbar \omega_c$ of MQO. From this figure we see the crossover from linear to square-root dependence of interlayer background MR $R_{zz} (B_z)$. The interval of the linear MR is $4 t_z < \hbar \omega_c \ll (4 t_z)^2 / \Gamma_0$ and increases with the increase of $t_z / \Gamma_0$. The crossover from linear to square-root dependence of MR is a general feature of quasi-2D metals and already has been observed in a number of experiments (see, e.g. Refs. 27, 28). The square-root dependence $R_{zz} \propto B_z$ is obtained at $\hbar \omega_c > 2 \sqrt{\Gamma_0 / \hbar \omega_c} \gtrsim 4 t_z$, which is much wider than the applicability region $\hbar \omega_c \gg \Gamma_0 \gg t_z$ of the calculation in Refs. 38, 39.

This result on the field dependence of interlayer MR can be analytically obtained as follows. Eq. (10) simplifies to a quadratic equation at $\Delta \varepsilon = 0$, i.e. at the center of LL, and has two solutions: $\Sigma_\ast^2 = 2 t_z^2 \pm \sqrt{4 t_z^4 + (\Gamma_0 \hbar \omega_c)^2}$. The physical solution does not diverge at $t_z \to \infty$, has nonzero

---

**Figure Legends**

**FIG. 1:** Imaginary part Im$\Sigma$ of electron self energy as function of energy deviation $\Delta \varepsilon$ from the center of LL at $t_z / \Gamma_\ast = 0$ (blue solid line), $t_z / \Gamma_\ast = 0.5$ (green dashed line), and $t_z / \Gamma_\ast = 1.0$ (red dotted line).

**FIG. 2:** Real part of electron self energy Re$\Sigma (\Delta \varepsilon)$ at $t_z / \Gamma_\ast = 0, 0.5, 1.0$.

**FIG. 3:** Average interlayer magnetoresistance $R_{zz} (B_z) = 1 / \bar{\sigma}_{zz}$ calculated from Eqs. (9) and (16) as function of magnetic field at four different values of $t_z / \Gamma_0 = 1$ (solid green line), $t_z / \Gamma_0 = 2$ (dashed blue line), $t_z / \Gamma_0 = 3$ (dotted red line) and $t_z / \Gamma_0 = 4$ (dash-dotted black line). In the interval $4 t_z < \hbar \omega_c \ll (4 t_z)^2 / \Gamma_0$ the interlayer MR has linear field dependence.

**FIG. 4:** Average interlayer magnetoresistance $R_{zz} (B_z) = 1 / \bar{\sigma}_{zz}$ calculated from Eqs. (9) and (16) as function of the square root of magnetic field at four different values of $t_z / \Gamma_0 = 1$ (solid green line), $t_z / \Gamma_0 = 2$ (dashed blue line), $t_z / \Gamma_0 = 3$ (dotted red line) and $t_z / \Gamma_0 = 4$ (dash-dotted black line). The linear dependence $R_{zz} (\sqrt{B_z})$ is obtained at $\hbar \omega_c > 2 \sqrt{\Gamma_0 \hbar \omega_c} \gtrsim t_z$. Only in the interval $4 t_z < \hbar \omega_c \ll (4 t_z)^2 / \Gamma_0$ the interlayer MR has linear field dependence.
imaginary part and, at \( t_z = 0 \), agrees with the 2D limit described in Refs. [2, 5, 3]. All these criteria are satisfied for sign "−", which at \( ∆e = 0 \) gives \( \text{Re} Σ_ε = 0 \) and

\[
|\text{Im} Σ_ε| = \sqrt{4t_z^4 + (Γ_0hω_c)^2} - 2t_z^2.
\]  

Substituting this to Eq. (9) one obtains the expression for the zero-temperature conductivity in the center of LL, i.e. at \( ∆e = 0 \):

\[
σ_{zz} (0) \approx 2σ_0/π.
\]  

In the limit \( hω_c > 4t_z \gg \sqrt{Γ_0hω_c} \) each LL gives an essential contribution to conductivity in the interval \(-2t_z \lesssim ∆e \lesssim 2t_z\), as follows from Eq. (10) and can be seen from Fig. 5, where the conductivity as function of energy distance from the nearest LL has a dome shape of the width \( \sim 4t_z \). The conductivity, averaged over period \( hω_c \) of MQO, is then given by

\[
\tilde{σ}_{zz} = \int_{−hω_c/2}^{hω_c/2} \frac{dε}{hω_c} σ_{zz} (∆e) \approx σ_{zz} (0) \frac{π t_z^2}{4 hω_c}
\]

\[
= σ_0(2t_z/hω_c) ∝ 1/Γ_z.
\]  

This predicts a linear background magnetoresistance \( R_{zz} = 1/σ_{zz} ∝ B_z \) in the interval \( 4t_z < hω_c < (4t_z)^2/Γ_0 \) of magnetic field in quasi-2D strongly anisotropic compounds. The extra factor \( π/4 \) in Eq. (10) comes from the integration of a semicircle. Eq. (10) also predicts stronger dependence of \( \tilde{σ}_{zz} \) on \( t_z \): \( \tilde{σ}_{zz} ∝ t_z^3 \) unlike the usual dependence \( σ_{zz} ≈ σ_0 ∝ t_z^2 \).

In the opposite limit \( hω_c ≫ \sqrt{Γ_0hω_c} \geq t_z \) the "width of conducting band" from each LL is \( \sim 4\sqrt{Γ_0hω_c} \), and for background interlayer conductivity one obtains \( \tilde{σ}_{zz}/σ_0 \approx 2\sqrt{Γ_0/hω_c} ∝ 1/√B_z \), or

\[
R_{zz} (B) / R_{zz} (0) \approx hω_c/4Γ_0 ∝ √B_z.
\]  

FIG. 5. Intercorll conductivity \( σ_{zz} (ε) \) calculated from Eqs. (9) and (10) in the strong-field limit \( hω_c/Γ_0 = 40 \) as function of energy counted from the nearest LL for four different values of \( t_z/Γ_0 = 0.1 \) (solid blue line), \( t_z/Γ_0 = 2 \) (dashed green line), \( t_z/Γ_0 = 5 \) (dotted red line), and \( t_z/Γ_0 = 10 \) (dash-dotted magenta line). In the limit \( 2t_z ≫ Γ_0hω_c \) the interlayer conductivity has approximately a semicircle shape of width \( 2t_z \). In the opposite limit \( Γ_0hω_c ≫ t_z^2 \) the "conducting band" of each LL has the width \( \sim \sqrt{Γ_0hω_c} \).

To summarize, the calculation of longitudinal interlayer magnetoresistance in Q2D metals is performed in the strong-field limit \( hω_c > 4t_z \). It predicts a linear background MR \( R_{zz} (B_z) = 1/σ_{zz} ∝ B_z \) in the interval \( 4t_z < hω_c \ll (4t_z)^2/Γ_0 \) of magnetic field, where \( σ_{zz} \) has also unusual dependence on interlayer transfer integral: \( \tilde{σ}_{zz} ∝ t_z^3 \). At stronger field or at smaller \( t_z < Γ_0hω_c \), the usual dependence \( σ_{zz} ∝ t_z^2 \) is recovered, and the MR transforms to square-root dependence \( R_{zz} (B_z) ∝ B_z^{3/2} \) as was recently obtained in the limit \( t_z ≪ Γ_0 ≪ hω_c \) in Refs. [38-40]. The present calculation generalizes these results to the region \( t_z ≳ Γ_0 \). The magnetic field of the crossover from linear to square-root dependence of \( R_{zz} (B_z) \) allows to estimate the interlayer transfer integral \( t_z ∝ \sqrt{(Γ_0hω_c)} \) from experimental data. The obtained longitudinal MR \( R_{zz} (B_z) \) explains numerous experiments on interlayer MR in strongly anisotropic quasi-2D compounds.

The measurement of the monotonic part of longitudinal interlayer MR is much easier than the measurement of MQO or AMRO, because finite temperature and long-range crystal imperfections do not affect \( R_{zz} (B_z) \) up to much higher temperatures or disorder. Therefore, the experimental study of longitudinal interlayer background magnetoresistance is proposed as a simple additional tool to investigate the electronic structure of strongly anisotropic quasi-two-dimensional compounds in a wide range of parameters.

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APPENDIX

Now we proof by the method of mathematical induction, that in the "non-crossing" approximation the electron Green’s function, averaged over impurity configurations, has the form of Eq. (11):

\[
G(r_1, r_2, \varepsilon) = \sum_{n, k_y, k_z} \frac{\Psi^0_{n, k_y, k_z}(r_1) \Psi^0_{n, k_y, k_z}(r_2)}{\varepsilon - \varepsilon_{3D}(n, k_z) - \Sigma_n(\varepsilon)},
\]

(21)

with \(\Sigma_n(\varepsilon)\) being independent of \(k_y\) and \(k_z\).

In the Born approximation with one impurity the electron self-energy \(\Sigma^B_n(\varepsilon)\) does not depend on \(k_y\) and \(k_z\), but depends on the difference \(\Delta\varepsilon_n = \varepsilon - \varepsilon_{2D}(n)\):

\[
\Sigma^B_n(\varepsilon) \approx U^2G(0)(\varepsilon) = \sum_{n} \frac{N_{LL}/d}{\sqrt{(\varepsilon - \varepsilon_{2D}(n) - i 0)^2 - 4t_y^2}},
\]

and its real part is an odd function of \(\Delta\varepsilon_n\), \(\text{Re}\Sigma^B_n(\Delta\varepsilon_n) = -\text{Re}\Sigma^B_n(-\Delta\varepsilon_n)\), while its imaginary part is an even function of \(\Delta\varepsilon_n\). Hence, in the Born approximation \(\Sigma_n(\varepsilon, n, k_y, k_z) = \Sigma_n(\Delta\varepsilon_n) = \Sigma_n(\varepsilon)\).

Assume that our condition is satisfied when each irreducible self-energy part contains no more than \(j\) impurities. Then the Green’s function in coinciding points depends only on energy, \(G_j(\varepsilon, r, r) = G_j(\varepsilon)\):

\[
G_j(\varepsilon, r, r) = \sum_{n, k_y, k_z} \frac{\Psi^0_{n, k_y, k_z}(r) \Psi^0_{n, k_y, k_z}(r)}{\varepsilon - \varepsilon_{3D}(n, k_z) - \Sigma_n(\varepsilon)}
\]

\[
= \sum_{n, k_z} \frac{N_{LL}/d}{\varepsilon - \varepsilon_{3D}(n, k_z) - \Sigma_n(\varepsilon)}.
\]

(22)

When we include one more impurity to the irreducible electron self-energy and average over its position, it remains independent of \(k_y, k_z\) and depends only on \(\Delta\varepsilon_n\):

\[
\Sigma_{n,j+1} = \frac{U n_i}{1 - U G_j(\Delta\varepsilon_n)}.
\]

The Green’s function \(G_{j+1}(\varepsilon, r_1, r_2)\) also keeps the form of Eq. (11). To show this, let us find the function

\[
G_{j+1}(\varepsilon, r_1, r_2) = G_0(\varepsilon, r_1, r_2) + \int d^3r U G_0(r_1, r) \frac{G_0(r, r_2)}{1 - U G_j(\varepsilon, r, r)} + \int d^3r d^3r' U G_0(r_1, r) G_0(r, r') G_0(r', r_2) + \ldots
\]

\[
= G_0(\varepsilon, r_1, r_2) + \frac{1}{1 - U G_j(\varepsilon)} \times
\]

\[
\times \sum_{n, k_y, k_z} \frac{\Psi^0_{n, k_y, k_z}(r_1)}{\varepsilon - \varepsilon_{3D}(n, k_z) - \Sigma_{n,0}(\varepsilon)}
\]

\[
\times \sum_{n', k_y', k_z'} \frac{\Psi^0_{n', k_y', k_z'}(r_2)}{\varepsilon - \varepsilon_{3D}(n', k_z') - \Sigma_{n',0}(\varepsilon)}
\]

\[
\times \int d^3r' \Psi^0_{n, k_y, k_z}(r) \Psi^0_{n', k_y', k_z'}(r') + \ldots
\]

where the new self-energy part

\[
\Sigma_{j+1,n}(\varepsilon) = \Sigma_{n,0}(\varepsilon) + \frac{U n_i}{1 - U G_j(\varepsilon)}
\]

depends only on energy \(\varepsilon\) and on LL number \(n\). The Green’s function in Eq. (23) is again of the form of Eq. (11).

The difference of impurity scattering to the same and to other LLs may give additional dependence of \(\Sigma_n(\varepsilon)\) on LL number \(n\), which requires further study. If \(\Sigma_{n,0}(\varepsilon)\) depends only on \(\Delta\varepsilon_n\), then \(G_j(\varepsilon)\) entering above formulas also depends only on \(\Delta\varepsilon_n\), and \(\Sigma_{j+1,n}(\varepsilon)\) is also a function of \(\Delta\varepsilon_n\) only. If \(\Sigma_{n,0}(\varepsilon)\) depends only on \(\varepsilon\) but not on \(n\), and if the impurity scattering to the same and to other LLs has the same matrix element, then \(G_j(\varepsilon)\) also depends only on \(\varepsilon\), and so does \(\Sigma_{j+1,n}(\varepsilon)\).

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