Gravo-thermodynamics of the Intracluster Medium: negative heat capacity and dilation of cooling time scales

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Abstract
The time scale for cooling of the gravitationally bound gaseous intracluster medium (ICM) is not determined by radiative processes alone. If the ICM is in quasi-hydrostat ic equilibrium in the fixed gravitational field of the dark matter halo then energy losses incurred by the gravitational potential energy of the gas should also be taken into account. This “gravitational heating” has been known for a while using explicit solutions to the equations of motion. Here, we re-visit this effect by applying the virial theorem to gas in quasi-hydrostatic equilibrium in an external gravitational field, neglecting the gravity of the gas. For a standard NFW form of halo profiles and for a finite gas density, the response of the gas temperature to changes in the total energy is significantly delayed. The effective cooling time could be prolonged by more than an order of magnitude inside the scale radius \(r_s\) of the halo. Gas lying at a distance twice the scale radius, has negative heat capacity so that the temperature increases as a result of energy losses. Although external heating (e.g. by AGN activity) is still required to explain the lack of cool ICM near the center, the analysis here may circumvent the need for heating in farther out regions where the effective cooling time could be prolonged to become larger than the cluster age and also explains the increase of temperature with radius in these regions.

Key words: cosmology: clusters

1. Introduction

Clusters of galaxies are the most massive virialized objects observed in the Universe. Their potential depths correspond to virial temperatures of \(1−10\ \text{keV} \left(10^7 − 10^8 \text{ K}\right)\) and the baryon number density in the inner regions could be as high as \(0.1\text{cm}^{-3}\) (e.g. Vikhlinin et al.2005; Pointecouteau, Arnaud & Pratt 2005). For these temperatures and densities, radiative losses are expected to bring the temperature in the central regions down to \(\gtrsim 10^4 \text{ K}\) within the available time. Yet in none of the observed clusters does the temperature drop to the level dictated by cooling alone. The absence of significant amounts of cold gas in the cores of massive clusters is a major puzzle posed by X-ray observations of massive clusters (e.g. Peterson et al.2001). Hence, efficient heating mechanisms must operate at the cores of all cooling clusters. The most popular mechanism for suppressing cooling is energy released by an AGN in the central cluster galaxy (cf. Quilis et al.2001, Babul et al.2002, Kaiser & Binney 2003, Dalla Vecchia et al.2004, Roychowdhury et al.2004, Voit & Donahue 2005, Nipoti & Binney 2005, and references therein) or by multiple AGN activity in all galaxies in the cores of clusters (Nusser, Silk & Babul 2006; Eastman et. al. 2007; Nusser & Silk 2008). Over-pressurized ejecta from the AGN transform into hot bubbles that eventually reach pressure equilibrium with the ICM and proceed to rise buoyantly away from the center. These bubbles

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could heat the ICM by means of shock waves generated as they expand to reach the ICM pressure (Nusser, Silk & Babul 2006), and by drag forces when they become buoyant (e.g. Churazov et al. 2001). Mechanical activity near the center could also generate sound waves which are believed to eventually dissipate their energy in the ICM (Pringle 1989, Ruszkowski et al. 2004, Heinz & Churazov 2005, Fujita & Suzuki 2005, Sanders & Fabian 2007). To balance cooling in a cluster of X-ray luminosity of $L_X \sim 10^{44}$ erg s$^{-1}$, a central AGN must produce $\sim 10^{60}$ erg over the entire life-time of the cluster. For the most massive clusters (potential depths corresponding to velocity dispersions $> 500$ km/s) the required heating could be more than an order of magnitude larger than the observed range of AGN energy output in galaxy clusters, based on the $pV$ content of X-ray cavities (e.g. Best et al. 2007). This is not too worrying since weak shocks could certainly compensate for the missing energy needed to balance cooling. For less massive clusters the $pV$ energy is sufficient to balance cooling (e.g. Birzan et al. 2004). The challenge, however, is to arrange for efficient energy transport from the AGN over the entire cooling core, or out to distances of up to $\sim 100$ kpc.

The temperature in the inner regions increases gradually as we move away from the center. At first, this behavior may seem reasonable since the radiative cooling becomes more efficient nearer to the center. But, the cooling time is significantly shorter than the cluster age over a significant part of the inner regions and the ICM had ample opportunity to cool to very low temperatures (e.g. Fig. 12 in Wise, McNamara & Murray 2004). So why is there not a temperature plateau extending over the region where the cooling time is shorter than the cluster age? One explanation might be that, on account of the lower density, heat conduction is more significant as we move away from the center. However, heat conduction is not universally important in these regions (e.g. Wise, McNamara & Murray 2004). Here show that the cooling time could significantly be modified when the potential energy of the ICM in the dark halo is taken into account. We will use a version of the virial theorem to show that the potential energy will absorb some of the energy loss incurred by the system. In some cases the potential energy will decrease by an amount larger than the actual loss, forcing the system to compensate the energy difference by increasing its thermal energy. This is the case of negative heat capacity. This phenomenon is sometimes referred to as gravitational heating and has been discussed previously (e.g. Fabian & Nulsen 1977) and is evident in numerical simulations of the ICM. However, the description in terms of the virial theorem as is done here is new and offers a simple analysis for assessing the dependence of the effective cooling time on the assumed halo profile.

2. The virial theorem

Hereafter we will assume spherical symmetry and denote by $r$ the distance from the center. Let $\rho_e(r)$, $u(r)$, and $P = (\gamma - 1)\rho u$ be, respectively, the gas density, energy per unit mass, and pressure, where $\gamma$ is the adiabatic index. The temperature, $T$, is related to $u$ by $u = k_B T/ (\gamma - 1)/m$, where $m$ is the mean particle mass and $k_B$ is the Boltzmann constant. We assume a gas obeying the equation

$$\rho_e g - \frac{dP}{dr} = 0,$$

where $g$ is the gravitational force field per unit mass. This equation is applicable in quasi-hydrostatic equilibrium so that the acceleration of the gas is negligible. Multiplying (1) by $r$ and integrating over the volume from $r = 0$ to $R_0$ gives the virial theorem,

$$3(\bar{P} - P_0)V + W = 0$$

where $V = (4\pi/3)R_0^3$, $P_0 = P(R_0)$ is the external pressure, $\bar{P} = 4\pi \int_0^{R_0} dr r^2 P(r)/V$ is the average pressure inside $R_0$, and the gravitational term, $W$, is

$$W = 4\pi \int_0^{R_0} r^3 \rho_e g(r) dr$$

A more general derivation which includes gas motions could be found in Ostriker & McKee (1988). The energy of the system in the volume $V$ is written as the sum of the thermal energy $\bar{PV}/(\gamma - 1) \propto N k_B T$ ($N$ is the total number of particles) and the gravitational potential energy, $U$,

$$E = U + \frac{\bar{PV}}{\gamma - 1}$$

where

$$U = 4\pi \int_0^{R_0} \rho \Phi r^2 dr ,$$

and the system is assumed to reside in a static external gravitational potential $\Phi$ and neglected gravity of the gas.
From the virial theorem (2) and the energy equation (3) we obtain global relations between infinitesimal variations (denoted by the prefix $\delta$) in the total energy, $E$, the thermal energy $E_{th}$, $V$, $W$ and $U$. Keeping a constant external pressure $P_0$ these relations are

$$\delta W = 3P_0\delta V - 3(\gamma - 1)\delta E_{th},$$  \tag{6}$$

and

$$\delta E = \delta U + \delta E_{th},$$  \tag{7}$$

where we have used the expression $E_{th} = PV/(\gamma - 1)$ for the thermal energy. These relations must hold for any change in the state of the system. For radiative losses, the energy loss in time $\delta t$ is $\delta E = n_eA(T)\delta t$ where $n_e$ is the electron number density and $A$ is the cooling rate. Even if this energy is extracted initially from the thermal part, $E_{th}$, subsequent evolution of the system will establish the relations (6) and (7). We are working under the assumption of quasi-hydrostatic equilibrium so that any bulk motions generated during this process are neglected. In any case, if dissipation is important then significant gas motions will be converted into heat, restoring the above relations.

We are set now to derive a relation between $\delta E$ and $\delta E_{th}$. We write $\delta V = (\delta V/\delta W)\delta W$ in the virial relation (2) to obtain,

$$\delta W = \frac{3(\gamma - 1)}{3P_0\frac{\delta V}{\delta W} - 1}\delta E_{th}. \tag{8}$$

Writing $\delta U = (\delta U/\delta W)\delta W$ and $\delta V = (\delta V/\delta W)\delta W$ in the relation (2) while taking $\delta W$ from (3) we get

$$\delta E = C\delta E_{th}, \tag{9}$$

where

$$C = 1 + \frac{\delta U}{\delta W} \frac{3(\gamma - 1)}{3P_0\frac{\delta V}{\delta W} - 1}. \tag{10}$$

The quantity $C$ gives the ratio of the heat capacity to the standard thermodynamic heat capacity computed without gravity. Hence we call $C$ the relative heat capacity (RHC). The sign of $C$ determines whether the thermal energy, $E_{th}$, and hence the temperature, $T \propto E_{th}/N$, will increase or decrease as a result of changing the total energy, $E$. If $C < 0$ holds, then the heat capacity is negative, i.e. the temperature increases when we extract energy from the system. For $P_0 = 0$, the condition $C < 0$ implies

$$\frac{\delta U}{\delta W} > \frac{1}{3(\gamma - 1)}. \tag{11}$$

For positive RHC, $C > 0$, the response time of the gas temperature to variations in its energy is prolonged by a factor $C$. For example, the effective cooling time is $Ct_{cool}$, where $t_{cool} \sim k_B T/(n_eA)$ is the usual radiative cooling time.

3. Applications to various forms of halo gravitational potentials

We begin with the calculation of the RHC, $C$, for power-law potentials of the form, $\Phi = A/r^n$ so that $g = An/r^{n+1}$, where $n \neq 0$ and $A$ are constants. The constant $A$ is negative for $n > 0$ and positive otherwise. In this case $W = nU$ and $\delta U/\delta W = 1/n$ and for $P_0 = 0$ we have

$$C = 1 - \frac{3}{n}(\gamma - 1). \tag{12}$$

Thus $C$ is negative for

$$0 < n < 3(\gamma - 1), \tag{13}$$

which gives $0 < n < 2$ for $\gamma = 5/3$. To estimate the RHS, $C$, for a non-vanishing external pressure, $P_0$, we need the quantity $\delta W/\delta V$ which depends on gas density profile, $\rho_e$, in the system. We work here with a power-law density profile of the form, $\rho_e = B/r^\alpha$ and we compute $\delta W/\delta V$ under variations of the external radius $R_0$ assuming that the index $\alpha$ and the total mass, $M_0$, inside $R_0$ remain constant. Since $M = 4\pi \int r^2 B/r^\alpha dr$ we get

$$B = \frac{3 - \alpha}{4\pi}M R_0^{\alpha - 3}. \tag{14}$$

Evaluating $W$ we get

$$W = \frac{n(3 - \alpha)}{3 - \alpha - n} AM R_0^{-n}. \tag{15}$$

Therefore,

$$\frac{\delta W}{\delta V} = -\frac{n^2(3 - \alpha)}{3 - \alpha - n} \frac{A M}{4\pi R_0^{\alpha + n}}, \tag{16}$$

and

$$\frac{\delta V}{\delta W} = -\frac{3V}{nW}. \tag{17}$$

Since $W < 0$, this quantity is negative for $n < 0$ and, as seen in (14), the heat capacity is positive for any $P_0$. For $n > 0$, $\delta V/\delta W$ is positive. Thus, an inspection of (16) reveals, for $\delta U/\delta W > 0$ the existence of external pressure could result in $C < 0$ even if $C > 0$ for $P_0 = 0$. For this to happen, the
value of $P_0$ has to be adjusted such that $3P_0\delta V / \delta W - 1$ is small and negative. Substituting $W$ from the virial theorem (2) into (17) and using the later into (10) we get the RHC in terms of the pressure ratio, $P/P_0$, as follows:

$$C = 1 - \frac{3}{n} \left( \frac{\gamma - 1}{\gamma} - \frac{1}{\alpha^2} \right).$$  \hspace{1cm} (18)

This expression reduces to (12) for $P/P_0 \gg 1$.

An intriguing case is $\Phi = A \ln r$ and $g = -A/r$ ($A > 0$). By the requirement of constant mass inside the varying radius $R_0$, we find that $W$ is constant. Therefore, $\delta U/\delta W$ is either $-\infty$ or $+\infty$ depending on the sign of $\delta U$. The sign of $\delta U$ is sensitive to the assumed density form of the gas. For example, for $\rho = B/r^s$ as above we get $U = \text{const}$ so that $\delta U = 0$. For a mass distribution confined to a shell of negligible thickness, $\delta U$ is positive when the shell is brought closer to the center and negative otherwise.

We now consider the implications of the relation (2) for realistic distributions of dark matter in halos. Therefore, we adopt the parametric form proposed by Navarro, Frenk & White (1996) (hereafter NFW) for the density profile, which is motivated by N-body simulations and is consistent with the distribution of dark matter in observed clusters (Pointecouteau, Arnaud & Pratt 2005). The gravitational potential and force field for a halo following the NFW profile are:

$$\Phi(r) = -\frac{GM_c}{r} \left( \frac{f}{c^2} \ln(1 + cs) \right)$$  \hspace{1cm} (19)

and

$$g(r) = -\frac{d\Phi}{dr} = -\frac{GM_c}{r^2} \left( \frac{f}{c^2} \left( \ln(1 + cs) - \frac{cs}{cs + 1} \right) \right).$$  \hspace{1cm} (20)

where $c$ is the concentration parameter, $s = r/R_c$ is the distance from the halo center in units of the virial radius $R_v$, $M_c$ is the virial mass of the halo, and $f = c^2 / [\ln(1 + c) - c / (1 + c)]$. The virial mass is related to the virial radius by $M_c = (4\pi/3)200 \rho_{\text{crit}} R_v \rho_{\text{crit}}$, where $\rho_{\text{crit}}$ is the critical cosmic density. The structure of a halo is, therefore, determined uniquely by the $c$ and $R_v$. The scale radius $r_s \equiv c R_v$ marks the transition from $g = \text{const}$ near $r = 0$ to $g \propto 1/r$ as we move further out. For this (non power-law) form of the gravitational field, the quantity $\delta U/\delta W$ depends on the assumed form of the variation in the gas distribution which is determined by energy gains and losses. We find it most instructive to focus on effects of local density variations on $C$. Therefore, we present here $\delta u/\delta w$ resulting from infinitesimal displacements, $\delta r$, of a fluid element of a given mass as a function of its position, i.e., $\delta u/\delta w = \delta \Phi/\delta (\rho g)$. This type of variation is relevant for cooling/heating processes in a shell of finite thickness lying at a distance $\sim r$ from the center. We show, in figure (11) $\delta u/\delta w$ as a function of the radius in units of the scale radius, $r_s$. There a singularity at $r = r_{\text{in}} \sim 2.15 r_s$ which corresponds to a vanishing $\delta W$. For $r > r_{\text{in}}$, the quantity $\delta U/\delta W$ is positive so that according to (10) the RHC, $C$, is constant. As we move to the inner regions at $r < r_{\text{in}}$ the quantity $\delta U/\delta W$ changes signs and so the RHC becomes positive. However, the RHC, $C$, is significantly larger than unity at $r = 0$, $C = 3$ for $P_0 = 0$ and $\gamma = 5/3$ so that the cooling time is prolonged by a factor of 3. This is a modest boosting in the cooling time since the standard radiative cooling time scale could as short as 0.01 of the life-time of clusters. However, the dilution of cooling time scales in farther out regions could be large enough so as to exceed the cluster age. The RHC, $C$, is also affected by $P_0 \delta V/\delta w$ evaluated at the outer boundary of the shell. If the average pressure $P$ inside the shell is large compared to $P_0$ then this term could be neglected. For realistic clusters the pressure is a steep function of radius for a significant part of the inner cluster cores. Of course the external pressure at the inner boundary of the shell should also be considered. However, we assume that the inner shell radius is small compared to the outer radius so that the work done by the pressure on the inner shell is small.

4. Concluding Remarks

We used a simplified model of the ICM to study its gravo-thermodynamical properties. In quasi-hydrostatic equilibrium, the inclusion of the change in the potential energy prolongs the response of the gas temperature in the inner regions lying within $r_{\text{in}} \approx 2 r_s = 2 R_v/c$. Outside this radius, the form of dark halo gravitational potential is such that the temperature is increased as a result of energy loss, i.e. the gas heat capacity is negative. The boosting of the cooling time in the inner region formally diverges at $r_{\text{in}}$, reaching a factor of 3 as at $r = 0$.

Our results may circumvent the need for heating the ICM in regions where the standard radiative cooling time is an order of magnitude shorter than the life-time of the cluster. Those regions lie at a significant fraction of $r_s$, where the prolonged effective cooling time could be larger than the cluster age, as
is the case for example for the cluster A1068. For this cluster, \( r_s \sim 400 \) kpc (Pointecouteau, Arnaud & Pratt 2005) and the ratio of standard radiative cooling time to the cluster age is \( \sim 0.1 - 1 \) over the region between \( \sim 70 \) kpc to \( \sim 300 \) kpc. Our results do not eliminate the need for heating of the ICM in central regions (\( r << r_h \)) since the effective cooling time there is still shorter than the cluster age.

As mentioned before, the approach taken here aims at addressing a specific point related to the heat capacity of the ICM. The analytic methods used here could be followed only by invoking a simplified (perhaps oversimplified) of the ICM. The effect of gravitational heating has appeared in a variety of forms in the literature. For example, it has been discussed by Fabian & Nulsen (1977) using specific solutions to subsonic collapse of gas in a \( 1/r \) gravitational potential. It is also seen in the one-dimensional simulations by Omukai & Nishi (1998) of self-gravitating gas for the formation of primordial protostellar clouds. It is also evident in the solutions for self-similar cooling flows by Bertschinger (1989). However, as is shown here, analysis of these problems using virial theorem offers a simple way to understand the physical effect and its relation to the halo profile and external pressure.

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References
Allen, S. W., Rapetti, D. A., Schmidt, R. W., Ebeling, H., Morris, R. G., & Fabian, A. C. 2008, MNRAS, 383, 879
Birzan, L., Rafferty, D. A., McNamara, B. R., Wise, M. W., & Nulsen, P. E. J. 2004, ApJ, 607, 800
Babul, A., Balogh, M. L., Lewis, G. F., & Poole, G. B. 2002, MNRAS, 330, 329
Best, P. N., von der Linden, A., Kauffmann, G., Heckman, T. M., & Kaiser, C. R. 2007, MNRAS, 379, 894
Bertschinger, E. 1989, ApJ, 340, 666
Churazov, E., Brüggen, M., Kaiser, C. R., Böhringer, H., & Forman, W. 2001, ApJ, 554, 261
Dalla Vecchia, C., Bower, R. G., Theuns, T., Balogh, M. L., Mazzotta, P., & Frenk, C. S. 2004, MNRAS, 355, 995
Eastman, J., Martini, P., Sivakoff, G., Kelson, D. D., Mulchaey, J. S., & Tran, K.-V. 2007, ApJL, 664, L9
Fabian, A. C., & Nulsen, P. E. J. 1977, MNRAS, 180, 479
Fujita, Y., & Suzuki, T. K. 2005, ApJL, 630, L1
Heinz, S., & Churazov, E. 2005, ApJL, 634, L141
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Nipoti, C., & Binney, J. 2005, MNRAS, 361, 428
Nusser, A., & Silk, J. 2008, MNRAS, 386, 1013
Nusser, A., Silk, J., & Babul, A. 2006, MNRAS, 373, 739
Omukai, K., & Nishi, R. 1998, ApJ, 508, 141
Ostriker, J. P., & McKee, C. F. 1988, Reviews of Modern Physics, 60, 1
Peterson, J. R., et al. 2001, A&A, 365, L104
Pointecouteau, E., Arnaud, M., & Pratt, G. W. 2005, A&A, 435, 1
Pringle, J. E. 1989, MNRAS, 239, 479
Quilis, V., Bower, R. G., & Balogh, M. L. 2001, MNRAS, 328, 1091
Reynolds, C. S., McKernan, B., Fabian, A. C., Stone, J. M., & Vernaleo, J. C. 2005, MNRAS, 357, 242
Roychowdhury, S., Ruszkowski, M., Nath, B. B., & Begelman, M. C. 2004, ApJ, 615, 681
Ruszkowski, M., Brüggen, M., & Begelman, M. C. 2004, ApJ, 611, 158
Sanders, J. S., & Fabian, A. C. 2007, MNRAS, 381, 1381
Vikhlinin, A., Markevitch, M., Murray, S. S., Jones, C., Forman, W., & Van Speybroeck, L. 2005, ApJ, 628, 655
Voit, G. M., & Donahue, M. 2005, ApJ, 634, 955