Mimicking transPlanckian effects in the CMB with conventional physics

J.M.Cline
Theory Division, CERN, CH-1211, Geneva 23, Switzerland and
Physics Department, McGill University, Montréal, Québec, Canada H3A 2T8

Abstract. We investigate the possibility that fields coupled to the inflaton can influence the primordial spectrum of density perturbations through their coherent motion. For example, the second field in hybrid inflation might be oscillating at the beginning of inflation rather than at the minimum of its potential. Although this effect is washed out if inflation lasts long enough, we note that there can be up to 30 e-foldings of inflation prior to horizon crossing of COBE fluctuations while still giving a potentially visible distortion. Such pumping of the inflaton fluctuations by purely conventional physics can resemble transPlanckian effects which have been widely discussed. The distortions which they make to the CMB could leave a distinctive signature which differs from generic effects like tilting of the spectrum.

Keywords. inflation, CMB

1. Motivation

There has been much discussion of the possible effects of transPlanckian physics on inflationary perturbations recently. Typically density perturbations which have macroscopic wavelengths today could have been shorter than the Planck distance at sufficiently early times, when inflation was taking place. It is therefore conceivable that new physics could leave its imprint on the CMB. In this work [1] we investigate an alternative scenario, that some field coupled to the inflaton could distort the primordial inflaton fluctuations due to its coherent motion. The effects could be qualitatively similar to those of the more exotic transPlanckian possibilities.

2. TransPlanckian physics

It has been argued that transPlanckian physics would manifest itself by putting the inflaton in a vacuum state which differs from the usual Bunch-Davies vacuum. We recall that the latter is defined to be the one annihilated by the operator coefficient of the positive frequency solution \( \phi_k^+ \), \( \phi(t, x) = \sum_k \left( a_k \phi_k^+ + a_k^* \phi_k^- \right) \), where \( \phi_k^+ = N_k \left( \pm iH + \frac{\partial}{\partial t} \right) e^{\pm ik/aH+ikx} \) for a massless inflaton. The alternative states which have been considered, \( \alpha \)-vacua, are related to the standard vacua by a Bogoliubov transformation, \( \phi_k^+ = A_\alpha \phi_k^+ + B_\alpha^* \phi_k^- \).
The effect on the power spectrum of density perturbations is a modulation, \( \tilde{P}(k) \sim |\tilde{\phi}_k^* / \phi_k^*|^2 P(k) \), where the wave functions are to be evaluated at \( t = \infty \). It has been argued [2] that the modulation factor should have the form \( 1 - H / \Lambda \sin(2\Lambda t) \), where \( H \) is evaluated at the time when the mode \( k \) crosses the horizon. If \( H \) varies fast enough, the effect would be an oscillatory modulation as a function of \( k \) [3].

### 3. Conventional physics

An oscillatory modulation of the power spectrum can also be produced if the inflaton \( \phi \) couples to an oscillating field \( \chi \). This could naturally happen in hybrid inflation models, with potential \( V(\phi, \chi) = \frac{1}{2} M^2 \phi^2 + \lambda (\chi^2 - v^2)^2 + \frac{1}{2} g \chi^2 \phi^2 + \frac{1}{2} g' \chi \phi^2 \), if \( \chi \) starts far away from its ground state. Ordinarily one neglects such a possibility since sufficient inflation will quickly damp out the oscillations. But if there is only a limited amount of inflation prior to horizon-crossing of COBE-scale modes, then \( \chi \) could still be oscillating when CMB fluctuations are being produced. The inflaton fluctuation equation of motion is

\[
\ddot{\phi}_k + 3H \dot{\phi}_k + k^2 e^{-2Ht} \phi_k = g \chi^2(t) \psi_k
\]

Since the amplitude of the oscillations redshifts like \( a(t)^{-3/2} \), we can estimate how many \( e \)-foldings of inflation are needed to render \( \chi \) ineffective. By comparing the new term in the equation of motion with \( k^2 e^{-2Ht} \) at the time of horizon crossing (when \( k/a = H \), we find the maximum number of \( e \)-foldings before horizon-crossing to be \( N_e = \ln (a/a_0) = \frac{1}{4} \ln \left( \frac{g\chi^2}{M^2} \right) \). Demanding that \( \chi \) not dominate the energy density (which would give matter domination instead of inflation), we find the constraint \( N_e < \frac{1}{4} \ln \left( \frac{3gM_p^2}{M^2} \right) \lesssim 27 \) (taking \( g = 1, M = 100 \text{ GeV} \)), where \( M \) is the effective mass of \( \chi \) during inflation. A more careful estimate which also demands that \( \chi \) is not damped too quickly by producing inflatons gives \( N_e \lesssim 12 \). We will also consider an alternative model in which the inflaton-\( \chi \) coupling is \( g' \chi \phi^2 \) instead of \( g \chi^2 \phi^2 \). This gives \( N_e \lesssim 30 \).

To find the effect of \( \chi \) oscillations on the inflaton fluctuations, we solve (1) both numerically and partially analytically using the Greens’ function method; both give useful insights. Figure 1 shows the deviations in \( P(k) \) for examples of both the \( g \chi^2 \phi^2 \) and \( g' \chi \phi^2 \) models, from numerical solutions. An interesting difference between the two models is

![Figure 1](image-url)

**Figure 1.** \( P(k) \) versus \( \log(k/H) \) in the \( g \chi^2 \phi^2 \) model (left) and the \( g' \chi \phi^2 \) model (right), for several values of the \( \chi \) mass \( M \) and amplitude at horizon crossing.
that power is strongly suppressed at low $k$ in the first, whereas the maximum deviation occurs at intermediate $k$ values in the second. From the analytic approach can compute the maximum size of the fractional deviation in $P(k)$:

$$\frac{\delta P}{P_{\text{max}}} = \frac{g\chi^2}{9H^2} \left(\text{g}\chi^2\phi^2 \text{ model}\right), \quad \frac{g'\chi_0}{H^{1/2}(M/2)^{3/2}} \left(g'\chi\phi^2 \text{ model}\right) \quad (2)$$

Interestingly, these are not suppressed by powers of $H$ as is the case with transPlanckian or effective field theory effects [4]; rather they are enhanced by small $H$.

4. Effect on CMB

To make contact with observations, we have input our primordial spectra into the CMB-FAST code [5] to generate the resulting temperature anisotropy (Doppler peaks). Some results are shown in fig. 2. The $g\chi^2\phi^2$ model tends to give the largest deviations at low multipole values, whereas the $g'\chi\phi^2$ model allows for maximal deviations at higher $l$ values. The latter possibility is interesting for current and future experiments which will be sensitive to smaller angular scales, hence larger $l$.

![Figure 2](image)

**Figure 2.** Left: distortion of temperature anisotropy in $g\chi^2\phi^2$ model, for different values of $g\chi^2$. Right: percent deviation in temperature anisotropy in $g'\chi\phi^2$ model (upper panel) and the underlying inflaton power spectrum (lower panel).

References

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