On the Universal $L_\infty$-Algebroid of Linear Foliations

We compute an $L_\infty$-algebroid structure on a projective resolution of some classes of singular foliations on a vector space $V$ induced by the linear action of some Lie subalgebras of $\mathfrak{gl}(V)$. This $L_\infty$-algebroid provides invariants of the singular foliations, and also provides a constant-rank replacement of the singular foliation. The computation consists of first constructing a projective resolution of the foliation induced by the linear action of the Lie subalgebra $\mathfrak{g} \subset \mathfrak{gl}(V)$, and then computing the $L_\infty$-algebroid structure. We then generalize these constructions to a vector bundle $E$, where the role of the origin is now taken by the zero section $L$.

We then show that the fibers over a singular point of a projective resolution of any singular foliation can be computed directly from the foliation, without needing the projective resolution. For linear foliations, we also provide a way to compute the action of the isotropy Lie algebra in the origin on these fibers directly from the foliation.

Keywords: Singular foliations, L-infinity-algebroids, projective resolutions.

MSC: 22E45, 13D02, 17B55.