Profit Optimization of products at different selling prices with fuzzy linear programming problem using situational based fuzzy triangular numbers

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Abstract: The main objective of the manufacture industry is to acquire the maximum profit by minimizing the production cost and maximization of the selling price of the different products fabricated by them. After accomplishment of the optimal value for the production cost the industry focuses on maximizing the selling price of items. However, the competitiveness and uncertainty of the market, the supplier sells their products of discrete selling prices. Therefore, their profit is fluctuated. Such situations to find the optimization of the profit is the main issue of the firm and such destruction can be mitigated with the help of fuzzy linear programming problem. In FLP coefficients of the objective function, constraint variables and the solution values are represented by fuzzy numbers. In this paper, proposed a newly constructed triangular fuzzy numbers (TFN~τ₁) which represent the various realistic circumstances for the selling prices of the various items of manufacturers and then TFN~τ₂ based on TFN~τ₁ is constructed which shows the all possible states of the profit received accordingly numerous sell prices industry might be earned. The data of “R.P.S entrepreneur Jalandhar” is taken which manufacture special types of pipe fitting items with particular sizes according to the demand in Punjab region. We constructed the general structure of the FLP to obtain all attainable bound of the optimal values using newly constructed TFNs. After that a comparative study of the optimal FLP to achieve the membership grades in all conceivable latitude.

Keyword: Maximization of selling price and profit, triangular fuzzy number, fuzzy linear programming problem

1. Introduction

Pricing impacts all the tools available to sales executives in both the top and the bottom line. It provides extremely powerful advantages. If pursued carefully, businesses can make significant pricing gains and the effects can be easily recognized. It is a subject that is widely discussed in theory of economics and, to analyze the best method for calculation the price ,it is important to know other factors like the type of company and the unique feature of each product [1]. In order to achieve a fixed appropriate price, companies would investigate the market share, examine invoicing and what profits is required to identify
the desired position [2]. Price creation is the foundation for growth so the more likely it will compete on the market [3]. In the view price pricing, we must understand the situation where the company is in corporate and respect the three essential characteristics: cost, consumers and competition [4]. The price demanded by the firms could lead to distinct quantities of demand that might impact the outlined strategic goals differently. The pricing advantages are highly effective. If carefully followed, companies can achieve substantial price gains and the effects can be easily identified. As a creative exercise in statistics and cognitive science, executives must approach prices. If done correctly, through the prices, profitability can be significantly improved [5]. Similarly, although the different situations are depending on demand and product quality, the company sells the goods at a different price. The uncertainties that occur in realistic situation couldn’t be overlooked in succession to create the organization well-organized supply chain. These ambiguities are usually related to the product supply, customer demand, etc. [6]. In this paper we aspire to introduce the concept of triangular fuzzy number to describe various selling prices provided by the enterprises.

Meanwhile, the fuzzy numbers concerned, which have a realistic approach in many different fields like decision making, data analysis [7] and also engineering problems [8-9] etc. problem. With the assistance of these fuzzy numbers, numerous optimization problems can be resolved. In [10] they introduced subtraction and division process with triangular fuzzy number (FTN). In addition, there are many modified operations that are used to promote triangular and trapezoidal fuzzy numbers [11-19] and may have an impact on FLP optimization. An innovative way to resolve fully FLP through the use of lexicography method [20], in addition to the traditional linear system, using the (LR) fuzzy numbers and the lexicography method, a recent FLP resolution trend. A new algorithms [21] was build based on a new lexicographic TFN order to explain the FFLP by switching to it multi-objective linear programing. In terms of the vendor's implementation costs reduction, two models [22] were introduced to reduce the overall device costs. In paper [23] the results of reduction in setup costs and increases in efficiency have been established in a increasing two-echelon chain model. The goal was to reduce the overall cost of the whole SCM model by minimizing construction expense, process efficiency, number of suppliers and lot size at the same time. In paper [24], the distribution of probabilities for consumer demand was assumed only to have a known mean and standard deviation. The retailer's costs and developing competitive distribution arrangements were suggested as an effective solution. Often included in article [25] is a supply chain network, where a single manufacturer manufactures goods in a batch phase and delivers them over several times to a variety of customers. Chandrawat et.al. [26] conducted a modeling and optimization study using FLP with symmetrical and right angle triangle fuzzy number. To illustrate the membership grade of optimized fuzzy LPP, they used the right angle triangular fuzzy number. The various components of triangular fuzzy numbers [27-28] were used on the other hand to develop trustworthiness parameters for the extraction of the industrial system. In these papers [29-32] they used symmetric trapezoidal numbers to represent the coefficients of the restriction’s objective function and solution value of the R.H.S to overcome FLP concerns. In this paper we constructed the general structure of the TFN which show the various prices provided by the entrepreneur, and then we also proposed the various situations in which the entrepreneur sold their goods. Therefore, through the general framework of FLP [33] we assess the optimal values of different situations.
This paper is organized as follows: In Section 2 some basic definitions of fuzzy sets and fuzzy numbers are reviewed. In section 3 formulation of general FLP are discussed. In section 4 a new method is proposed for FLP which represent the different situations offered by entrepreneur through TFN in detail and represented the different situations through the general structure in detail. In section 5 shows the data set of the selling price offered by RPS enterprises and To illustrate the proposed method using the given data sets are solved and the obtain the results are discussed with comparative analysis of these results in section 6. Conclusions are discussed in section7.

2. Preliminaries

2.1 Fuzzy Set and its Components

The fuzzy set system was introduced by Zadeh [41] and it's been enhanced by Zadeh[42]. In different circumstances, it is an amazing technology to demonstrate instinctive or imprecise evidence.

Definition 2.1 [34]: A fuzzy set \( \mathcal{A} \), is a set in which each value of the set is associated with a membership value, and the value lies only between \([0, 1]\) i.e. \( \mathcal{A} \) is defined on universe set \( X \) defined as fellow:

\[
\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x)) : x \in X\}
\]

Definition 2.2 [34]: A \( \alpha \)-cut of a fuzzy set \( \mathcal{A} \) of \( X \) is the set in which the membership values in \( \mathcal{A} \), is greater than or equal to \( \alpha \), it is denoted by \( \mathcal{A}^\alpha \)

\[
\mathcal{A}^\alpha = \{x | \mu_{\mathcal{A}}(X) \geq \alpha\}
\]

Definition 2.3 [34]: A strong \( \alpha \)-cut of a fuzzy set of \( X \) is the set in which the membership values in \( \mathcal{A} \), is greater than \( \alpha \), it is denoted by \( \mathcal{A}^{\alpha+} \)

\[
\mathcal{A}^{\alpha+} = \{x | \mu_{\mathcal{A}}(X) > \alpha\}
\]

Definition 2.4 [33]: The support of the fuzzy set \( \mathcal{A} \) of \( X \) is defined as follow:

\[
\text{supp} \mathcal{A} = \{x | \mu_{\mathcal{A}}(X) > 0\}
\]

Definition 2.5 [33]: The height of a fuzzy set denoted by \( h(\mathcal{A}) \) is defined as the largest of membership values of the elements contained in that set.

Definition 2.6 [33]: A fuzzy set \( A \) is called normal if the \( h(\mathcal{A}) = 1 \).

Definition 2.7 [33]: A fuzzy set will be convex, if

\[
\mu_{\mathcal{A}}(\lambda x_1 + (1- \lambda)x_2) \geq \min \{\mu_{\mathcal{A}}(x_1), \mu_{\mathcal{A}}(x_2)\}, \text{Where } 0 \leq \lambda \leq 1
\]

Definition 2.8 [33]: A fuzzy number \( \tilde{A} \) is a convex normalized fuzzy set \( \tilde{A} \) of the real line \( \mathbb{R} \) such that

(a) It exist one \( x_0 \in \mathbb{R} \) with \( \mu_{\tilde{A}}(x) = 1 \) (\( x_0 \) is called the mean value \( \tilde{A} \))

(b) \( \mu_{\tilde{A}}(x) \) is piecewise continuous.

Definition 2.9 [34]: A fuzzy number \( \tilde{A} \) = \((a, b, c)\) is said to be triangular fuzzy number if its membership function is given by
Definition 2.10 [34]: A triangular fuzzy number \((a, b, c)\) is said to be non-negative fuzzy number iff \(a \geq 0\).

3. Structure of Fuzzy Linear Programming (FLP)

The conventional LPP would find a linear function’s minimum/maximum values under the constraints of linear inequalities/equations. The usual LPP form is shown as follow

\[
\begin{align*}
\text{Max /Min } & \quad \mu(x) \sum_{j=1}^{n} \alpha_j x_j \leq (\geq) \beta_i, \quad i \in \mathbb{Z}_m^+ \\
& \quad x_j \geq 0 \quad \quad \quad \quad \quad \quad \quad j \in \mathbb{Z}_n^+ \\
\end{align*}
\]

The function to be Max \(Z\) or Min \(Z\) is called an objective function. They \(\tau_j\) are called cost coefficients. The matrix \(A= [\alpha_{ij}]\) is called a constraint matrix, and the vector \(\beta= <\beta_1, \beta_2, \ldots, \beta_m >^T\) is called the vector on the right side. Where \(x= < x_1, x_2, \ldots, x_n >^T\) is the vector of variables.

It is not appropriate in many real-life situations to require that the constraints or objective function in LPP be defined in specific, crisp terms. In such cases, using some kind of FLP is preferable.

The following is the most general type of FLP:

\[
\begin{align*}
\text{max } & \quad \sum_{j=1}^{n} C_j x_j \\
\text{s.t.} & \quad \sum_{j=1}^{n} A_{ij} x_j \leq B_i \quad (i \in N_m) \\
& \quad x_j \geq 0 \quad (j \in N_n),
\end{align*}
\]

Where \(A_{ij}, B_i, C_j\) are fuzzy numbers, and \(x_j\) are variables with fuzzy numbers \((i \in N_m, j \in N_n)\); addition and multiplication operations are fuzzy arithmetic operations and \(\leq\) refer to the ordering of fuzzy numbers. Rather than explaining the general approach, we demonstrate the problems associated with two specific cases of FLP.

**CASE 1.** FLP with fuzzy number \((B_i)\) on the right-hand side:
Generally, the problems FLPP are simplified into corresponding precise linear problem that can be solved with standard methods. Consequently, the final results of FLPP are real number that represents a solution of fuzzy numbers.

In this case, the usual form of the fuzzy number $B_i$

$$B(x_i) = \begin{cases} 
1 & \text{when } x \leq b_i \\
\frac{b_i + p_i - x}{p_i} & \text{when } b_i < x < b_i + p_i (x_j) \\
0 & \text{when } b_i + p_i \leq x,
\end{cases}$$

(3.4)

Where $x \in R$

For every vector $x = \langle x_1, x_2, ..., x_n \rangle$, we first measure the degree $D_i(x)$, at which $x_j$ satisfies the $i^{th}$ constraint by the equation

$$D_i(x) = B_i(\sum_{j=1}^{n} a_{ij} x_j)$$

(3.5)

These degrees are fuzzy sets on $R^n$ and their intersection, $\bigcap_{i=1}^{m} D_i$, is a fuzzy feasible set. First, the fuzzy set of optimal values is calculated. The lower and upper bounds of the optimal value are first determined. Through solving LPP, the lower bound of optimal values $Z_i$, is achieved.

$$Max \ z = cx$$

s.t. $\sum_{j=1}^{n} a_{ij} x_j \leq b_i \ (i \in N_m)$

(3.6)
\[ x_j \geq 0 \ (j \in N_n); \]

A similar LPP occurs at upper bound of the optimal values \( z_u \), where every \( b_i \) is replaced with \( b_i + p_i \):

\[
\begin{align*}
\max \ z &= cx \\
\text{s.t.} \quad &\sum_{i=1}^{n} a_{ij} x_j \leq b_i + p_i \quad (i \in N_m) \\
&x_j \geq 0 \ (j \in N_n).
\end{align*}
\]

Therefore, the fuzzy set of the optimal value is defined by

\[
M(x) = \begin{cases} 
1 - \frac{cx - z_l}{z_u - z_l} & \text{when } z_u \leq cx \\
\frac{z_l - cx}{z_u - z_l} & \text{when } z_l \leq cx = z_u \\
0 & \text{when } cx \leq z_l.
\end{cases}
\]

Now, the general structure of the classical optimization problems is defined:

\[
\begin{align*}
\max \ y &\gamma \\
\text{s.t.} \quad &\gamma(z_u - z_l) - cx \leq -z_l \\
&\gamma p_i + \sum_{i=1}^{n} a_{ij} x_j \leq b_i + p_i \quad (i \in N) \\
&y, x_j \geq 0 \ (j \in N_n).
\end{align*}
\]

| Symbol | Description |
|--------|-------------|
| \( \lambda \) | represent the maximum possibility collection from the marked price of various items during a year |
| \( \sigma_i \in (0,1) \) | represented the membership grade of discounts and loss according to situations; |
| \( \rho \lambda \) | represented the total amount received after given discount on selling price. |
| \( \omega_i \) | represented the compound interest where \( i \in \) rate of interest and \( j \in \) number of days |
| \( \omega_y \lambda \) | total amount received after received \( j \) period. |
| \( \kappa \) | selling price of the various items. |
| \( \omega_i \in (0,1) \) | membership grade of \( i \) discounts on various items. |
| \( \omega_i \kappa \) | discounted selling price of the various items. |
| \( \omega_y \kappa \) | selling price of various items after received \( j \) period. |
| \( \kappa - \omega_y \kappa \) | the desire profit of the various items for the industry |
| \( \omega_y \kappa - \omega_i \kappa \) | profit of debit sales of the items |
\( \omega_J \kappa - \omega_J \kappa \) profit in case of the heavy discount.

\( x_r \quad \text{where} r \in 1, 2, 3, 4, 5 \) various product manufacture by RPRE,

\( Z_i^i \) the upper bound of the optimal values of all situations \( s \in 1, 2 \ldots 13 \)

\( Z_u^i \) the lower bound of the optimal values of all situations \( s \in 1, 2 \ldots 13 \)

\( Z_o^i \) the optimal values of all situations \( s \in 1, 2 \ldots 13 \)

\( \gamma_i^i \) Represented membership grade of optimal values of all situations \( s \in 1, 2 \ldots 13 \)

\( m1 \) the membership grade which shows degree of assurance

\( m2 \) the membership grade which shows degree of satisfaction

### 4. Proposed FLP Method

In this section, we proposed a new FLP system using TFN, which described the different selling prices provided by the entrepreneur over the given period of time. FLP problems with \( m \) fuzzy inequalities constraints and \( n \) variables may be formulated as follows:

\[
\text{Max} \bar{z} \approx \bar{z} x \quad \text{s.t.} \\
\bar{A}x \leq \bar{B} \quad (4.1)
\]

\[ x \geq 0, \]

Where \( \bar{z} \in F(R)^m, \bar{B} \in F(R)^n \) and \( \bar{A} \in F(R)^{m \times n} \) are given and \( x \in F(R)^n \) is to be determined.

We denote a situational based triangular fuzzy number for \( \bar{A} \) by \( \bar{A} = (\omega_i \kappa, \kappa, \omega_j \kappa) \), \( i, j \in \square \)

The representation of the triangular fuzzy number (TFN~τ1) \( \bar{A} \) is defined as follow:
situation 1: In this situation, entrepreneurs sell only their defective or undemand goods at heavy discount \((\omega_j \kappa)\), including the rehabilitation of their bad debts or the time of depression or the insolvency of the customers, etc. So the selling prices of the goods are lower than or approaching the desired price of the items and seller often tries to sell their goods in cash. Thus, the total amount of money predicted ranges
\( \sigma, \lambda \) to \( \lambda \) i.e. \( (\sigma, \lambda \sqsubseteq \lambda) \) and the entrepreneur aims to optimize overall profit, we try to establish the optimal values of the fuzzy sets. The lower bound \( Z^1_i \) and upper bound \( Z^1_u \) are measured by determining first the optimum values

A solution of the standard LPP provides the lower bounds of the optimal values \( Z^1_i \) as follows:

\[
\text{Max } Z^1_i = \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \quad \text{s.t.} \quad \sum_{r=1}^{n} \omega_j \kappa x_r \leq \sigma_i \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\]

Then, the membership grade of the optimal values is defined by

\[
M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \leq Z^1_i \\
\frac{\sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r - Z^1_i}{Z^1_u - Z^1_i} & \text{when } Z^1_i \leq \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \leq Z^1_u \\
1 & \text{when } \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \geq Z^1_u
\end{cases}
\]

(4.5)

Situation 2: The layout of situation 2 is same as situation 1, but here, the entrepreneur only sell his goods on credit basis, so the overall estimated money collection varies from \( \sigma, \lambda \) i.e. \( (\sigma, \lambda \sqsubseteq \lambda) \) and the entrepreneur aims to optimize overall profit, we try to establish the optimal values of the fuzzy sets. The lower bound \( Z^2_i \) and upper bound \( Z^2_u \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z^2_i \) as follows:

\[
\text{Max } Z^2_i = \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \quad \text{s.t.} \quad \sum_{r=1}^{n} \omega_j \kappa x_r \leq \sigma_j \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\]

(4.7)

Now, the general structure of the classical optimization problems is defined as follows: Max \( \gamma^1 \)

\[
\gamma^1(Z^1_u - Z^1_i) - \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \leq -Z^1_i \\
\text{s.t. } \gamma^1(\lambda - \sigma_i \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
x^r_i \geq 0 \quad (r \in \square) \\
i, j \in \square
\]

(4.8)

A solution of the standard LPP provides the lower bounds of the optimal values \( Z^2_u \) as follows:

\[
\text{Max } Z^2_u = \sum_{r=1}^{n} (\omega_j \kappa - \omega_i \kappa) x_r \quad \text{s.t.} \quad \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
x_r \geq 0 \quad (r \in \square)
\]

(4.10)
\[
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
 x_r \geq 0 \quad (r \in \mathbb{I}) \quad (4.9)
\]
\[
i, j \in \mathbb{I}
\]

Then, the membership grade of the optimal values is defined by

\[
M(x) = \begin{cases} 
0 & \text{when} \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \leq Z_i^1 \\
\frac{\sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r - Z_i^1}{Z_i^2 - Z_i^1} & \text{when} Z_i^1 \leq \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \leq Z_i^2 \\
1 & \text{when} \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \geq Z_i^2
\end{cases} \quad (4.11)
\]

Now, the general structure of the classical optimization problems is defined as follows:

Max \( \gamma^2 \)

\[
\gamma^2 (Z_u^2 - Z_i^2) - \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \leq -Z^1_i
\]

s.t \( \gamma^2 (\omega_j \lambda - \omega_j \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \)

\[
\gamma^2, x_r \geq 0 \quad (r \in \mathbb{I}) \quad (4.12)
\]
\[
i, j \in \mathbb{I}
\]

**Situation 3**: In this situation, entrepreneurs sell their demanded goods on cash at desired selling price \( \kappa \) or simultaneous credit basis with a high interest rate. Thus, the total amount of money predicted ranges from \( \lambda \) to \( \omega_j \kappa \) i.e. \( \lambda \leq \omega_j \kappa \) and the entrepreneur aims to optimize overall profit, we try to establish the optimal values of the fuzzy sets. The lower bound \( Z_j^3 \) and upper bound \( Z_u^3 \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_j^3 \) as follows:

Max \( Z_j^3 = \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \)

\[
\sum_{r=1}^{n} \kappa x_r \leq \lambda \\
\begin{array}{l}
s.t \ x_r \geq 0 \quad (r \in \mathbb{I}) \\
i, j \in \mathbb{I}
\end{array} \quad (4.13)
\]

Then, the membership grade of the optimal values is defined by

Now, the general structure of the classical optimization problems is defined as follows:

Max \( \gamma^3 \)

\[
\gamma^3 (Z_u^3 - Z_j^3) \quad \text{s.t} \quad \sum_{r=1}^{n} \kappa x_r \leq \omega_j \kappa
\]

\[
x_r \geq 0 \quad (r \in \mathbb{I}) \\
i, j \in \mathbb{I}
\]

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Situation 4: The layout of situation 4 is same as situation 3, but here, the entrepreneur only sell his goods on credit basis with a high interest rate say $\omega_j \lambda$. Thus, the total amount of money predicted ranges from $\lambda$ to $\omega_j \lambda$ i.e. $\lambda \leq \omega_j \lambda$ and the entrepreneur aims to optimize overall profit, we try to establish the optimal values of the fuzzy sets. The lower bound $Z^4_i$ and upper bound $Z^4_u$ are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values $Z^4_i$ as follows:

$$\text{Max } Z^4_i = \sum_{r=1}^{n} (\omega_j \lambda - \omega_j \kappa) x_r$$
$$\sum_{r=1}^{n} \kappa x_r \leq \lambda$$
$$x_r \geq 0 \quad (r \in [\square]) \quad (4.17)$$
$$i, j \in [\square]$$

Then, the membership grade of the optimal values is defined by

$$M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\omega_j \lambda - \omega_j \kappa) x_r \leq Z^4_i \\
\frac{\sum_{r=1}^{n} (\omega_j \lambda - \omega_j \kappa) x_r - Z^4_i}{Z^4_u - Z^4_i} & \text{when } Z^4_i \leq \sum_{r=1}^{n} (\omega_j \lambda - \omega_j \kappa) x_r \leq Z^4_u \\
1 & \text{when } \sum_{r=1}^{n} (\omega_j \lambda - \omega_j \kappa) x_r \leq Z^4_u 
\end{cases}$$

(4.19)

Now, the general structure of the classical optimization problems is defined as follows:

$$\text{Max } \gamma^4$$
$$\text{s.t.}$$
$$\gamma^4 (Z^4_u - Z^4_i) - \sum_{r=1}^{n} (\omega_j \lambda - \omega_j \kappa) x_r \leq -Z^4_i$$
$$\gamma^4 (\omega_j \lambda - \lambda) + \sum_{r=1}^{n} \kappa x_r \leq \omega_j \kappa$$
$$\gamma^4, x_r \geq 0 \quad (r \in [\square]) \quad (4.20)$$
$$i, j \in [\square]$$

Situation 5: The layout of situation 5 is same as situation 1, but here, the entrepreneur only sell his goods ideal goods and inferior goods with discount. the entrepreneur aims to optimize overall profit, we try to
establish the optimal values of the fuzzy sets. The lower bound \( Z_l^5 \) and upper bound \( Z_u^5 \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_l^5 \) as follows:

\[
\text{Max } Z_l^5 = \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r
\]

s.t

\[
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \sigma \lambda
\]

\[
\sum_{r=1}^{n} \kappa x_r \leq \sigma \lambda
\]

\[
x_r \geq 0 \quad (r \in \mathbb{N})
\]

\[
i, j \in \mathbb{N}
\]

Then, the membership grade of the optimal values is defined by

\[
M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r \leq Z_l^5 \\
\frac{\sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r - Z_l^5}{Z_u^5 - Z_l^5} & \text{when } Z_l^5 \leq \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r \leq Z_u^5 \\
1 & \text{when } \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r \geq Z_u^5
\end{cases}
\]  

(4.23)

Now, the general structure of the classical optimization problems is defined as follows:

\[
\text{Max } Z_u^5 = \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r
\]

s.t

\[
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda
\]

\[
\sum_{r=1}^{n} \kappa x_r \leq \lambda
\]

\[
x_r \geq 0 \quad (r \in \mathbb{N})
\]

\[
i, j \in \mathbb{N}
\]

\[
\gamma^5 \text{ s.t } \gamma^5 (Z_u^5 - Z_l^5) - \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r \leq -Z_l^5
\]

\[
\gamma^5 (\lambda - \sigma \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda
\]

\[
\gamma^5 (\lambda - \sigma \lambda) + \sum_{r=1}^{n} \kappa x_r \leq \lambda
\]

\[
\gamma^5, x_r \geq 0 \quad (r \in \mathbb{N})
\]

\[
i, j \in \mathbb{N}
\]

\[
\text{Situation 6: In this situation, entrepreneurs sell their demanded goods along with inferior goods on cash basis. Thus, the entrepreneur aims to optimize overall profit, we try to establish the optimal values of the fuzzy sets. The lower bound \( Z_l^6 \) and upper bound \( Z_u^6 \) are measured by determining first the optimum values.}
\]

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_l^6 \) as follows:

\[
\text{Max } Z_l^6 = \sum_{r=1}^{n} (\kappa - \omega_j \kappa)x_r
\]

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_u^6 \) as follows:
\[
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \quad \text{Max } Z_u^6 = \sum_{r=1}^{n}(\kappa - \omega_j \kappa)x_r \\
\text{s.t. } \sum_{r=1}^{n} \kappa x_r \leq \lambda \\
x_r \geq 0 \quad (r \in \square) \quad (4.25) \\
i, j \in \square
\]

Then, the membership grade of the optimal values is defined by

\[
\gamma^6 = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n}(\kappa - \omega_j \kappa)x_r \leq Z_l^i \\
\sum_{r=1}^{n}(\kappa - \omega_j \kappa)x_r - Z_l^i & \text{when } Z_l^i \leq \sum_{r=1}^{n}(\kappa - \omega_j \kappa)x_r \leq Z_u^i \\
1 & \text{when } \sum_{r=1}^{n}(\kappa - \omega_j \kappa)x_r \geq Z_u^i 
\end{cases} \\
\text{s.t. } \sum_{r=1}^{n} \kappa x_r \leq \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\]

**Situation 7:** The layout of situation 7 is same as situation 2, but here, the entrepreneur also sells his goods on credit basis. Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \(Z_l^7\) and upper bound \(Z_u^7\) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \(Z_l^7\) as follows:

\[
\text{Max } Z_l^7 = \sum_{r=1}^{n}(\omega_j \kappa - \omega_j \kappa)x_r \\
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
\text{s.t. } \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\]

A solution of the standard LPP provides the lower bounds of the optimal values \(Z_u^7\) as follows:

\[
\text{Max } Z_u^7 = \sum_{r=1}^{n}(\omega_j \kappa - \omega_j \kappa)x_r \\
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
\text{s.t. } \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\]
Then, the membership grade of the optimal values is defined by

\[
M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\omega_{j} \kappa - \omega_{j}) x_{r} \leq Z_{l}^{j} \\
\frac{\sum_{r=1}^{n} (\omega_{j} \kappa - \omega_{j}) x_{r} - Z_{l}^{j}}{Z_{u}^{j} - Z_{l}^{j}} & \text{when } Z_{l}^{j} \leq \sum_{r=1}^{n} (\omega_{j} \kappa - \omega_{j}) x_{r} \leq Z_{u}^{j} \\
1 & \text{when } \sum_{r=1}^{n} (\omega_{j} \kappa - \omega_{j}) x_{r} \leq Z_{u}^{j} 
\end{cases}
\]  

Now, the general structure of the classical optimization problems is defined as follows:

Max \( \gamma^{7} \)

\[
\gamma^{7} \left( Z_{u}^{j} - Z_{l}^{j} \right) - \sum_{r=1}^{n} (\omega_{j} \kappa - \omega_{j}) x_{r} \leq -Z_{l}^{j}
\]

\[
\gamma^{7} (\lambda - \omega_{j} \lambda) + \sum_{r=1}^{n} \omega_{j} \kappa x_{r} \leq \omega_{j} \lambda
\]

s.t \( \gamma^{7} (\lambda - \omega_{j} \lambda) + \sum_{r=1}^{n} \kappa x_{r} \leq \omega_{j} \lambda \)

\( \gamma^{7}, x_{r} \geq 0 \)  \( (r \in \square) \)  \( (4.31) \)

\( i, j \in \square \)

**Situation 8:** The layout of situation 8 is same as situation 3. Here entrepreneurs sell their demanded goods on cash at desired selling price \( \kappa \) and credit basis with a high interest rate. Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \( Z_{l}^{8} \) and upper bound \( Z_{u}^{8} \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_{l}^{8} \) as follows:

Max \( Z_{l}^{8} = \sum_{r=1}^{n} (\kappa - \omega_{j} \kappa) x_{r} \)

s.t

\[ \sum_{r=1}^{n} \kappa x_{r} \leq \lambda \]

\[ \sum_{r=1}^{n} \omega_{j} \kappa x_{r} \leq \lambda \]

\( x_{r} \geq 0 \)  \( (r \in \square) \)  \( (4.33) \)

\( i, j \in \square \)

Then, the membership grade of the optimal values is defined by

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_{u}^{8} \) as follows:

Max \( Z_{u}^{8} = \sum_{r=1}^{n} (\kappa - \omega_{j} \kappa) x_{r} \)

s.t

\[ \sum_{r=1}^{n} \kappa x_{r} \leq \omega_{j} \lambda \]

\[ \sum_{r=1}^{n} \omega_{j} \kappa x_{r} \leq \omega_{j} \lambda \]

\( x_{r} \geq 0 \)  \( (r \in \square) \)  \( (4.34) \)

\( i, j \in \square \)

Now, the general structure of the classical optimization problems is defined as follows:

Max \( \gamma^{8} \)
Situation 9: The layout of situation 8 is same as situation 3 but here entrepreneur expected profit
\[ \sum_{r=1}^{n} (\kappa - \omega_jx) x_r \] instead of \[ \sum_{r=1}^{n} (\kappa - \omega_jx) x_r \]. Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \( Z_l^i \) and upper bound \( Z_u^i \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_l^i \) as follows:

\[
\text{Max } Z_l^i = \sum_{r=1}^{n} (\omega_jx - \omega_jx) x_r \\
\text{s.t } \sum_{r=1}^{n} \kappa x_r \leq \lambda \\
\sum_{r=1}^{n} \omega_jx_r \leq \lambda \\
x_r \geq 0 \quad (r \in []) \quad (4.37)
\]

\[ i, j \in [] \]

Then, the membership grade of the optimal values is defined by

\[
M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\kappa - \omega_jx) x_r \leq Z_l^i \\
\frac{\sum_{r=1}^{n} (\kappa - \omega_jx) x_r - Z_l^i}{Z_u^i - Z_l^i} & \text{when } Z_l^i \leq \sum_{r=1}^{n} (\kappa - \omega_jx) x_r \leq Z_u^i \\
1 & \text{when } \sum_{r=1}^{n} (\kappa - \omega_jx) x_r \leq Z_1^i \quad (4.39)
\end{cases}
\]

Now, the general structure of the classical optimization problems is defined as follows:

\[
\text{Max } \gamma^9 \quad \text{s.t } \\
\gamma^9 (Z_u^i - Z_l^i) - \sum_{r=1}^{n} (\kappa - \omega_jx) x_r \leq -Z_l^i \\
\gamma^9 (\omega_jx - \omega_jx) x_r \leq \omega_jx \\
\gamma^9 (\omega_jx - \omega_jx) x_r \leq \omega_jx \quad (4.36)
\]

\[ i, j \in [] \]
\[ \gamma^\circ (Z^u - Z^l) - \sum_{j=1}^{n} (\omega_j \kappa - \omega_j \lambda) x_r \leq -Z^l \]
\[ \gamma^\circ (\omega_j \lambda - \lambda) + \sum_{j=1}^{n} \kappa x_r \leq \omega_j \lambda \]
\[ \gamma^\circ (\omega_j \lambda - \lambda) + \sum_{j=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \]
\[ \gamma^\circ x_r \geq 0 \quad (r \in \square) \]  
\[ i, j \in \square \]

**Situation 10:** In this situation, entrepreneurs sell all kinds of goods at a heavy discount on a cash and credit basis to clear the stock. Then the estimated profit is \( \sum_{j=1}^{n} (\omega_j \kappa - \omega_j \lambda) x_r \) in the all constraints. Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \( Z^l_1 \) and upper bound \( Z^u_1 \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z^l_1 \) as follows:

\[
\begin{align*}
\text{Max } Z^l_1 &= \sum_{j=1}^{n} (\omega_j \kappa - \omega_j \lambda) x_r \\
\text{s.t } &\sum_{j=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
&\sum_{j=1}^{n} \kappa x_r \leq \omega_j \lambda \\
&\sum_{j=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\end{align*}
\]

Then, the membership grade of the optimal values is defined by

A solution of the standard LPP provides the lower bounds of the optimal values \( Z^l_1 \) as follows:

\[
\begin{align*}
\text{Max } Z^l_1 &= \sum_{j=1}^{n} (\omega_j \kappa - \omega_j \lambda) x_r \\
\text{s.t } &\sum_{j=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
&\sum_{j=1}^{n} \kappa x_r \leq \omega_j \lambda \\
&\sum_{j=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
x_r \geq 0 \quad (r \in \square) \\
i, j \in \square
\end{align*}
\]

Now, the general structure of the classical optimization problems is defined as follows:

\[
\begin{align*}
\text{Max } \gamma^1 \text{ s.t }
\end{align*}
\]
\[
\mathbf{M}(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r < Z^\mathcal{L}_{ij}^{10} \\
\frac{\sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r - Z^\mathcal{L}_{ij}^{10}}{Z^\mathcal{U}_{ij}^{10} - Z^\mathcal{L}_{ij}^{10}} & \text{when } Z^\mathcal{L}_{ij}^{10} \leq \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \leq Z^\mathcal{U}_{ij}^{10} \\
1 & \text{when } \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \geq Z^\mathcal{U}_{ij}^{10} 
\end{cases} 
(4.43)
\]

**Situation 11:** The layout of situation 11 is same as situation 10 but here entrepreneur expected profit \( \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \) instead of \( \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \). Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \( Z^\mathcal{L}_{i}^{11} \) and upper bound \( Z^\mathcal{U}_{i}^{11} \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z^\mathcal{L}_{i}^{11} \) as follows:

\[
\text{Max } Z^\mathcal{L}_{i}^{11} = \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \\
\text{s.t } \sum_{r=1}^{n} \omega_j \kappa x_r \leq \sigma_j \lambda \\
\sum_{r=1}^{n} \kappa x_r \leq \sigma_j \lambda \\
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \sigma_j \lambda \\
x_r \geq 0 \quad (r \in \Box) \\
i, j \in \Box 
(4.45)
\]

Then, the membership grade of the optimal values is defined by

Now, the general structure of the classical optimization problems is defined as follows:

Max \( \gamma^1 \) s.t
Situation 12: In this situation, entrepreneurs sell all kinds of demanded goods on cash or simultaneous credit basis with a high interest rate. Then the estimated profit is

\[ \sum_{r=1}^{n} (\kappa - \omega_j \kappa)x_r \]

in all constraints.

Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \( Z_{l}^{12} \) and upper bound \( Z_{u}^{12} \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_{l}^{12} \) as follows:

\[
\text{Max } Z_{l}^{12} = \sum_{r=1}^{n} (\kappa - \omega_j \kappa)x_r \\
\text{s.t } \sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
\sum_{r=1}^{n} \kappa x_r \leq \lambda \\
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
x_r \geq 0 \quad (r \in \square) (4.49) \\
i, j \in \square
\]

Then, the membership grade of the optimal values is defined by

\[
\gamma^{11}(Z_{l}^{12} - Z_{r}^{11}) - \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa)x_r \leq -Z_{r}^{11} \\
\gamma^{11}(\lambda - \sigma_j \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
\gamma^{11}(\lambda - \sigma_j \lambda) + \sum_{r=1}^{n} \kappa x_r \leq \lambda \\
\gamma^{11}(\lambda - \sigma_j \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
\gamma^{11}, x_r \geq 0 \quad (r \in \square) (4.48) \\
i, j \in \square
\]

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_{u}^{12} \) as follows:

\[
\text{Max } Z_{u}^{12} = \sum_{r=1}^{n} (\kappa - \omega_j \kappa)x_r \\
\text{s.t } \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
\sum_{r=1}^{n} \kappa x_r \leq \omega_j \lambda \\
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
x_r \geq 0 \quad (r \in \square) (4.50) \\
i, j \in \square
\]

Now, the general structure of the classical optimization problems is defined as follows:

\[
\text{Max } \gamma^{12} \text{ s.t }
\]
Situation 13: The layout of situation 13 is same as situation 12 but here entrepreneur expected profit:\[ \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \] instead of \[ \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r. \] Thus, the entrepreneur aims to optimize overall profit; we try to establish the optimal values of the fuzzy sets. The lower bound \( Z_{l}^{13} \) and upper bound \( Z_{u}^{13} \) are measured by determining first the optimum values.

A solution of the standard LPP provides the lower bounds of the optimal values \( Z_{l}^{13} \) as follows:
\[
\text{Max } Z_{l}^{13} = \sum_{r=1}^{n} (\omega_j \kappa - \omega_j \kappa) x_r \\
\text{s.t } \sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
\sum_{r=1}^{n} \kappa x_r \leq \lambda \\
\sum_{r=1}^{n} \omega_j \kappa x_r \leq \lambda \\
x_r \geq 0 \quad (r \in \mathbb{R}) \\
i, j \in \mathbb{R}
\] (4.53)

Then, the membership grade of the optimal values is defined by
\[
M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \leq Z_{l}^{13} \\
\frac{\sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r - Z_{l}^{13}}{Z_{u}^{12} - Z_{l}^{12}} & \text{when } Z_{l}^{12} \leq \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \leq Z_{u}^{12} \\
1 & \text{when } \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \leq Z_{l}^{12} 
\end{cases}
\] (4.51)
\[
\gamma^{12} (Z_{u}^{12} - Z_{l}^{12}) - \sum_{r=1}^{n} (\kappa - \omega_j \kappa) x_r \leq -Z_{l}^{12} \\
\gamma^{12} (\omega_j \lambda - \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
\gamma^{12} (\omega_j \lambda - \lambda) + \sum_{r=1}^{n} \kappa x_r \leq \omega_j \lambda \\
\gamma^{12} (\omega_j \lambda - \lambda) + \sum_{r=1}^{n} \omega_j \kappa x_r \leq \omega_j \lambda \\
\gamma^{12}, x_r \geq 0 \quad (r \in \mathbb{R}) \\
i, j \in \mathbb{R}
\] (4.52)

Now, the general structure of the classical optimization problems is defined as follows:
\[
\text{Max } \gamma^{13} \quad \text{s.t }
\]
5. THE FIRM AND ITS PRODUCTS

The data below is the RPS enterprise’s selling price list for year 2019-20, which manufactures various
type of pipe fitting equipment. The dimensions, thickness and length of these products differentiate these
products.

| ITEMS/sizes | 50MM | 65MM | 80MM | 100MM | 65*50M | 80*65M | 80*50M |
|------------|------|------|------|-------|--------|--------|--------|
| ELBOW$(x_1)$ | 48.5 | 78   | 115  | 220   | 78     | 115    | 115    |
| TEE$(x_2)$   | 62   | 98   | 150  | 320   | 98     | 150    | 150    |
| SOCKET$(x_3)$ | 39   | 60   | 80   | 150   | 60     | 80     | 80     |
| R/TEE$(x_4)$ | 176.6| 327.5| 479.3| 800.5 | 283.05 | 406.87 | 401    |
| R/SOCKET$(x_5)$ | 100.15| 161.65| 243.65| 393.5 | 139.14 | 200    | 195    |

During the personal interview with the owner and the sales managers, they explained the different
situations where their goods are selling at various prices due to market volatility. Table-1 indicates the
market price of the certain of their goods. However, the discounted and credit-based prices of different
items according to the particular situation are shown in Table-1 and Table -3 respectively.

| ITEMS/sizes | 50MM | 65MM | 80MM | 100MM | 65*50M | 80*65M | 80*50M |
|------------|------|------|------|-------|--------|--------|--------|
| ELBOW$(x_1)$ | 33.47| 54.21| 77.05| 158.4 | 57.33  | 84.64  | 82.46  |
| TEE$(x_2)$   | 43.4 | 64.68| 93.75| 230.72| 65.86  | 103.95 | 102.9  |
| SOCKET$(x_3)$ | 26.52| 36.6 | 52.48| 110.1 | 44.04  | 53.76  | 54.32  |
| R/TEE$(x_4)$ | 125.39| 199.78| 330.24| 601.18| 185.11 | 250.23 | 267.47 |

\[
M(x) = \begin{cases} 
0 & \text{when } \sum_{r=1}^{n} (\omega_{j}^{r} - \omega_{j}^{k})x_{j} \leq Z_{u}^{13} - Z_{i}^{13} \\
\frac{\sum_{r=1}^{n} (\omega_{j}^{r} - \omega_{j}^{k})x_{j} - Z_{u}^{13}}{Z_{u}^{13} - Z_{i}^{13}} & \text{when } Z_{u}^{13} \leq \sum_{r=1}^{n} (\omega_{j}^{r} - \omega_{j}^{k})x_{j} \leq Z_{u}^{13} \\
1 & \text{when } \sum_{r=1}^{n} (\omega_{j}^{r} - \omega_{j}^{k})x_{j} \leq Z_{i}^{13} \quad (4.55)
\end{cases}
\]

\[\gamma^{13} (Z_{u}^{13} - Z_{i}^{13}) - \sum_{r=1}^{n} (\omega_{j}^{r} - \omega_{j}^{k})x_{j} \leq Z_{i}^{13}\]
\[\gamma^{13} (\omega_{j}^{r} - \lambda_{j}) + \sum_{r=1}^{n} \omega_{j}^{r}x_{j} \leq \omega_{j}^{r} \lambda_{j}\]
\[\gamma^{13} (\omega_{j}^{r} - \lambda_{j}) + \sum_{r=1}^{n} \omega_{j}^{r}x_{j} \leq \omega_{j}^{r} \lambda_{j}\]
\[\gamma^{13}, x_{r} \geq 0 \quad \text{for } r \in \square \]
\[i, j \in \square \]

Table 5.2: List of discounted selling price of manufacture items for the year 2019-20
There are several reasons for declines in the selling price of goods such as defective goods, recession period or insolvent of the customers etc. In this situation, suppliers sell their goods for a heavy discount i.e. \( \omega_j \kappa \).

**Table 5.3:** Expected credit selling price of manufacture items for the year 2019-20

| ITEMS/sizes | 50MM | 65MM | 80MM | 100MM | 65*50M | 80*65M | 80*50M |
|------------|------|------|------|-------|--------|--------|--------|
| ELBOW \((x_1)\) | 57.96 | 92.3 | 132.1 | 279.57 | 90.49 | 142.99 | 141.58 |
| TEE \((x_2)\) | 71.22 | 113.69 | 156.06 | 339.59 | 115.97 | 157.61 | 190.61 |
| SOCKET \((x_3)\) | 43.06 | 63.67 | 95.61 | 184.67 | 68.92 | 98.49 | 99.47 |
| R/TEE \((x_4)\) | 217.42 | 416.17 | 595.95 | 892.61 | 325.14 | 517.03 | 442.74 |
| R/ SOCKET \((x_5)\) | 119.69 | 187.53 | 288.32 | 434.46 | 159.83 | 232.02 | 242.46 |

The amount of the payment by the consumer varies accordingly, based on the 2% compound interest/p.a of the selling price \( \omega_j \kappa \) of each product.

Aforementioned, the expectations of total selling price and profit vary according to the different sizes of items shown in Table-4 and Table-5 respectively

**Table 5.4:** shows the maximum collection of the selling price according to the different situations

| Sizes/ \( \bar{\beta} \) | 50MM | 65MM | 80MM | 100MM | 65*50M | 80*65M | 80*50M |
|-----------------|------|------|------|-------|--------|--------|--------|
| \( \lambda \) | 435317.5 | 710657.5 | 951940 | 1822375 | 686296 | 884202.1 | 782980 |
| \( \omega_j \lambda \) | 288634 | 303322 | 409809 | 349839 | 117381 | 97334 | 113179 |
| \( \omega_j \kappa \) | 545711 | 922825.2 | 1096021 | 2275967 | 792417.5 | 1205548 | 969999.8 |

**Table 5.5:** Show the different profits of the items according to the different situations

| ITEMS/ \( \xi \) | ELBOW | TEE | SOCKET | R/TEE | R/SOCKET |
|-----------------|-------|-----|--------|-------|---------|
| \( \omega_j \kappa - \omega_j \kappa \) | 23.93 | 16.68 | 14.98 | 76.03 | 33.03 |
| \( \kappa - \omega_j \kappa \) | 31.71 | 46.11 | 24.45 | 130.78 | 68.1 |
| \( \omega_j \kappa - \omega_j \kappa \) | 55.63 | 62.78 | 39.44 | 206.81 | 101.13 |

**6. Numerical experiment**

In this section the result of the above thirteen different situational-based models are analyzed through numerical examples, using the data provided in Table 5.1 to Table 5.5. The result is shown in Table-6.1
below, which analyzes the selling price effect of the negotiations and the expected profit. Numerical analysis also provides a number of insights into management.

| Situation | Situation1 | Situation2 | Situation3 | Situation4 | Situation5 | Situation6 | Situation7 | Situation8 | Situation9 | Situation10 | Situation11 | Situation12 | Situation13 |
|-----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|-------------|-------------|-------------|
| $Z^{x}_1$ | 29574      | 29574      | 269017.1   | 347427.8   | 20253.9    | 33142.2    | 16289.3    | 16289.3    | 16289.3    | 28568.4     | 221847.5    | 347427.8    | 29574       |
| $x_1$    | 0          | 0          | 0          | 1744       | 846.4      | 0          | 1744       | 846.4      | 0          | 0           | 0           | 0           | 0           |
| $x_2$    | 0          | 0          | 705        | 0          | 0          | 0          | 0          | 0          | 0          | 0           | 0           | 0           | 0           |
| $x_3$    | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0           | 0           | 0           | 0           |
| $x_4$    | 389        | 389        | 1210.8     | 0          | 0          | 0          | 300.3      | 1210.8     | 0          | 0           | 300.3       | 1210.8      | 0           |
| $Z^{y}_1$ | 237221.8   | 281495.2   | 319307.5   | 415450.9   | 162927.9   | 269017.1   | 269017.1   | 415450.9   | 133792.3   | 221847.5    | 266401.7    | 415450.9    | 221847.5    |
| $x_1$    | 9495.3     | 11763.3    | 3584.8     | 6808.5     | 0          | 3584.8     | 1744       | 0          | 0          | 3584.8      | 0           | 0           | 3584.8      |
| $x_2$    | 0          | 0          | 3099.2     | 0          | 705        | 0          | 813.7      | 0          | 0          | 813.7       | 0           | 0           | 813.7       |
| $x_3$    | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0           | 0           | 0           | 0           |
| $x_4$    | 0          | 0          | 1044.6     | 0          | 0          | 1044.6     | 1210.8     | 0          | 0          | 300.3       | 0           | 1044.6      | 0           |
| $Z^{z}_1$ | 129222.4   | 156357.6   | 294177.5   | 410872.4   | 924149.1   | 153792.7   | 91883.4    | 381439.4   | 75722.8    | 127633.9    | 229777      | 481845.8    | 29777      |
| $x_1$    | 5400       | 6534       | 0          | 0          | 3862.3     | 0          | 0          | 2664.4     | 1455.7     | 0           | 0           | 187.5       |
| $x_2$    | 0          | 0          | 1902.8     | 0          | 0          | 257.4      | 0          | 0          | 1486.1     | 0           | 0           | 0           | 2274.1      |
| $x_3$    | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 0          | 1014.4     | 0           | 0           | 0           | 13.4        |
| $x_4$    | 0          | 0          | 1490.7     | 0          | 0          | 1210.8     | 0          | 0          | 1208.5     | 1127.7      | 0           | 644.3       | 25.3        |
| $y^{s}$  | 0.5042     | 0.5033     | 0.4997     | 0.9327     | 0.5058     | 0.5115     | 0.5076     | 0.5015     | 0.5        | 0.5058      | 0.5126      | 0.178       | 0.0214      |

The structure of situation-1 we analyze the different profit so obtained showing the minimum profit ($Z^x_1$) is Rs.29574, maximum profit $Z^y_1$ is Rs.227221.8 and the optimal profit is ($Z^o_1$) Rs.129222.4 at $y^{s}_1$ = 0.5042. The entrepreneur is also sure about the minimal and optimal profit in situation-1 that shows in Figure-6.1, where the membership grade $m_1$ and $m_2$ represented the degree of assurance and satisfaction respectively. Similarly, the result showed in Table-6.1 for the remaining situations and degree of assurance and satisfaction shown in the Figures-6.2 to Figure 6.13. Also the Figure 6.14 showed the comparative analysis of the situation-1 to situation-4. Similarly figure 5.15 and figure 5.16 shown comparative analyses of the situation-5 to situation-9 and situation-10 to situation-13 respectively.
7. Conclusion and future research directions

The main contribution of this paper to construction of a fuzzy linear programming model using fuzzy triangular number showing the different selling price offered by the entrepreneur during the particular time period. After constructing it, the general FPL model that demonstrated the different situations would happen during the sale of the goods was used. Using the realistic selling price data of the various items they sold, which is provided by RPS entrepreneur, Jalandhar to apply these models. Also the selling price of the discount and credit basis of the items was taken following the conversation with the owner and salespersons. Likewise, the maximum estimated collection of money was sold after the selling of different items according to their size and the specific profit shown. The complete outcome of the various situations was presented. The degree of assurance and satisfaction by the membership grade m1 and m2 respectively were expressed. In addition, we carried out a comparative analysis of the situation according to their structure.

In the future, using the contending method, we will also try to extend it to the trapezoidal number. Seek also to concentrate on comparing the result achieved from the technique implemented in the analysis with
those that could be attained with other conflicting methods. Furthermore, we are trying to introduce a new ranking method to solve FLP problems based on a different market structure which represents the interested stream of further research. They are being prosecuted.

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