Fermion observables for Lorentz violation

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Abstract. The relationship between experimental observables for Lorentz violation in the fermion sector and the coefficients for Lorentz violation appearing in the lagrangian density is investigated in the minimal Standard-Model Extension. The definitions of the 44 fermion-sector observables, called the tilde coefficients, are shown to have a block structure. The $c$ coefficients decouple from all the others, have six subspaces of dimension 1, and one of dimension 3. The remaining tilde coefficients form eight blocks, one of dimension 6, one of dimension 2, three of dimension 5, and three of dimension 4. By inverting these definitions, thirteen limits on the electron-sector tilde coefficients are deduced.

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1. Introduction

The possibility of Lorentz violation in nature has received much attention in recent years, triggered by the pioneering work of Kostelecký, Samuel, and Potting [1]. Numerous investigations of Lorentz symmetry have been carried out in systems involving ordinary fermionic matter: electrons, protons, and neutrons. In the case of electrons, these have included studies with spin-polarized torsion pendula [2, 3], atomic transitions [4], Penning traps [5, 6], colliders [7], optical and microwave resonators [8], and astrophysical results [9, 10]. In fact, Lorentz-symmetry investigations of a theoretical and experimental nature cover all subfields of physics [11]. Proton-based investigations have been performed with comagnetometers [12, 13, 14], the hydrogen maser [15], Cesium fountain clocks [16], Doppler-shifted systems [17], and the Penning trap [18]. Among the investigations done with neutrons are ones with clock-comparisons [19], magnetometers [20, 21, 12], ultra-cold neutrons [22], masers [23, 24] and astrophysical data [25].

In the general framework for Lorentz violation known as the Standard-Model Extension, or SME, the minimal theory has 44 independent experimental observables for each of the three fermions making up ordinary matter. As of January 2012, experimental sensitivities exist for about 58% of them [26]. In this work, our goal is to study the structure of these observables, known as the ‘tilde’ coefficients, and to seek relationships between them that may be used to deduce limits from theoretical considerations.

The SME is a general realistic effective field theory for Lorentz violation [27, 28, 29], providing for minuscule violations of CPT [30] and Lorentz symmetry in the Standard Model and General Relativity. The framework is set up using a Lagrange density containing conventional terms supplemented with unconventional ones, each a coordinate independent product of a coefficient for Lorentz violation and a Lorentz-breaking operator. The operators can be classified according to their mass dimension, and the minimal SME involves mass dimensions 3 and 4 only. Apart from investigations in the ordinary-matter fermionic sectors, dozens of experiments have been conducted to investigate whether any of the coefficients of the Lorentz-violating operators are nonzero [26].

Studies of the minimal SME have found the dispersion relation for a free fermion in a constant background [31], ways to factorize it [32], and methods for deducing the associated classical Lagrange function [33]. Other investigations have explored the geometries relevant in curved spacetimes [34], and have looked at the nonminimal operators in the photon sector, where a classification exists for all mass dimensions [35]. Recent work has studied the nonminimal neutrino sector [36]. Here, we primarily consider the observables for experiments with electrons, protons, and neutrons [14, 37, 38] in which boost effects can be neglected.
2. Minimal fermion sector in flat spacetime

In the minimal SME, the flat-space lagrangian density $\mathcal{L}$ describing a spin-$\frac{1}{2}$ fermion $\psi$ of mass $m$ is \cite{27, 28}

$$\mathcal{L} = \frac{1}{2} \overline{\psi} \Gamma^\nu \partial^\nu \psi - \overline{\psi} M \psi \quad ,$$

\begin{equation}
M := m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu}
\end{equation}

and

$$\Gamma^\nu := \gamma^\nu + c_{\mu\nu} \gamma^\mu + d_{\mu\nu} \gamma_5 \gamma^\mu + e_{\nu} + i f_{\nu} \gamma_5 + \frac{1}{2} g_{\lambda\mu\nu} \sigma^{\lambda\mu} \quad .$$

The coefficients $a_\mu$, $b_\mu$, and $H_{\mu\nu}$ appearing in (2) have dimensions of mass and control operators of mass dimension 3, while the coefficients $c_{\mu\nu}$, $d_{\mu\nu}$, $e_{\nu}$, $f_{\nu}$, and $g_{\mu\nu\lambda}$ appearing in (3) are dimensionless and control operators of mass dimension 4. In the minimal SME, there are three distinct fermionic sectors, and the coefficients in each are denoted by superscripts: $e$ for electrons and positrons, $p$ for protons and antiprotons, and $n$ for neutrons and antineutrons. When working in just one particle-antiparticle sector, these superscripts are often suppressed. Within one sector, experiments with sensitivities to these coefficients are likely to involve comparisons of the particle with its antiparticle, comparisons of differing spin states of one particle, or comparisons of differing motional states of a particle.

By definition, $c_{\mu\nu}$ and $d_{\mu\nu}$ are traceless, $g_{\mu\nu\lambda}$ is antisymmetric in the first two indices, and $H_{\mu\nu}$ is antisymmetric. This gives a total of $4 + 4 + 15 + 4 + 4 + 24 + 6 = 76$ components of the coefficients $a_\mu$ through $H_{\mu\nu}$, respectively, for a single fermion.

To allow comparison between results from different experiments, measurements of the Lorentz-breaking background fields are reported in the standard Sun-centered inertial reference frame, with coordinates denoted by upper-case roman letters $(T, X, Y, Z)$ \cite{38}. This has $Z$ axis parallel to the rotational axis of the Earth, and $X$ axis pointing from the Sun towards the northern vernal equinox.

We assume a Minkowski spacetime throughout, so that the metric is diagonal. Where sign choices are necessary, we choose spacetime metric $\eta_{\mu\nu}$ with $\eta_{TT} = +1$, and antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ defined with $\epsilon^{TXYZ} = +1$. In a few places, indices $J$, $K$, $L$, can take on possible values $X, Y, Z$ in the Sun-centered inertial reference frame.

It is natural to define the symmetric and antisymmetric parts of the $d_{\mu\nu}$ coefficients, and we do this with factors of a half:

$$d_{\mu\nu}^\pm \equiv \frac{1}{2}(d_{\mu\nu} \pm d_{\nu\mu}) \quad .$$

To keep track of independent coefficients with symmetric or antisymmetric pairs of indices, we adopt the following ordering: $TX, TY, TZ, XY, YZ, ZX$. In addition to $d_{\mu\nu}^\pm$, this affects the first two indices of $g_{\mu\nu\lambda}$, and $H_{\mu\nu}$.

The 24 independent components of $g_{\mu\nu\lambda}$ are naturally expressed in terms of four axial components $g_{\mu}^{(A)}$, four trace components $g_{\mu}^{(T)}$, and 16 mixed-symmetry components...
Table 1. Leading-order unobservable combinations of fermion-sector SME coefficients for a single particle in Minkowski space

| Unobservable combination | Redefinition | # |
|--------------------------|--------------|---|
| $a_\mu$                  | $\psi = e^{-ia_\mu x}\chi$ | 4 |
| $a_\mu + me_\mu$         | $\psi = (1 + v_\mu \gamma^\mu)\chi$, $v_\mu$ real | 4 |
| $b_\mu + g_\mu^{(A)}$    | $\psi = (1 + v_\mu \gamma^\mu)\chi$, $v_\mu$ real | 4 |
| $H_{\mu\nu} + \frac{1}{2}m e_{\mu\nu}^{\sigma\tau}d_{\sigma\tau}$ | $\psi = (1 + v_{\mu\nu} \sigma_{\mu\nu})\chi$, $v_{\mu\nu}$ real | 6 |
| $c_{\mu\nu} - c_{\nu\mu}$ | $\psi = (1 - \frac{i}{2}c_{\mu\nu} \sigma_{\mu\nu})\chi$ | 6 |
| $f_\mu$                  | $\psi = (1 + \frac{1}{2}i f_\mu \gamma^\mu)\chi$ | 4 |
| $g_\mu^{(T)}$            | $\psi = (1 - \frac{1}{2}i g_\mu^{(T)} \gamma^\mu)\chi$ | 4 Total: 32 |

$g_{\lambda\mu
u}^{(M)}$. The Appendix provides the definitions of these components, and discusses this decomposition in more detail. Table A2 shows that the mixed-symmetry $g$ coefficients are split into two independent dimension 8 subspaces, one with all three indices distinct, and the other with a pair of repeated indices. The index structure of $g_{\lambda\mu
u}$ is shared with the spacetime torsion tensor, and limits on torsion have been generated based on experiments testing Lorentz symmetry [39].

3. Field redefinitions and observable combinations

A variety of dependences exist between the coefficients for Lorentz violation in flat spacetime [27, 28] and in gravity [29]. They can be understood through field redefinitions, which yield 32 coefficient combinations in each fermion sector that are unobservable at leading or higher orders in Lorentz violation. This reduces the number of coefficients appearing in equation (1) to a smaller set of basis combinations.

Field redefinitions have been considered in the context of the photon sector [35], Finsler geometry [34], the pure-gravity sector [40], supersymmetry [41], and nonlocality [42]. A related topic involves studies of particular coordinate transformations, showing the relationship between photons and fermions [43], or between the photon coefficients and various early test models [44]. The absorption of the $f$ coefficient into the $c$ coefficient has been studied in the context of the Dirac theory [45] and classical kinematics [33]. A number of results for the fermion sector are presented in Ref. [46].

Table 1 summarizes the unobservable combinations of SME coefficients for each fermion sector in Minkowski spacetime. The first column lists the unobservable combinations, the second indicates the field redefinition used to show the result, while the last column gives the number of independent conditions involved. The first line expresses the result that the four components of the coefficient $a_\mu$ are unobservable in experiments within a single fermion sector. This holds at all orders in Lorentz violation [27], and corresponds physically to an unobservable shift in the energy and momentum of the system. This differs from curved spacetime, where the components of $a_\mu$ must
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vary to maintain compatibility with the geometrical structure, and only one component is unobservable [40]. The next six lines involve redefinitions of the form \( \psi = (1 + v \cdot \Gamma) \chi \), where \( \Gamma \) is one of \( \gamma^\mu, \gamma_5 \gamma^\mu \), or \( \sigma^{\mu\nu} \), [28, 46], and \( v \) is complex-valued with appropriate contractions. The dependences found are valid at leading order in Lorentz violation, and some may be valid at higher orders too. The second, third, and fourth lines show linear dependences between operators of dimension 3 and 4. In particular, the \( e_\mu \) coefficient can be absorbed into \( a_\mu \), the four axial components of \( g_{\lambda\mu\nu} \) can be absorbed into \( b_\mu \), and the six antisymmetric components of \( d_{\mu\nu} \) can be absorbed into \( H_{\mu\nu} \). The unobservable combinations listed in the second, third and fourth lines have orthogonal expressions that are observable. These can be obtained by reversing the sign between the two terms.

In the case of \( H_{\mu\nu} \) and \( d_{\mu\nu} \), the observable combinations are:

\[
\begin{align*}
H_{TX} + md_{YZ}, & \quad H_{TY} + md_{XZ}, & \quad H_{TZ} + md_{XY}, \\
H_{YZ} - md_{TX}, & \quad H_{ZX} - md_{TY}, & \quad H_{XY} - md_{TZ}.
\end{align*}
\]

(5)

In total, Table 1 lists 32 unobservable combinations of SME coefficients. These linear conditions reduce the number of independent SME coefficients for each fermion sector from the 76 appearing in equation (1) to 44. The 44 combinations may be counted as follows. The combination \( b_\mu - mg_{\mu}^{(A)} \) has 4 components; the symmetric and traceless \( c_\mu \) and \( d_{\mu\nu} \) expressions contribute 9+9; the combination of \( H_{\mu\nu} \) with \( d_{\mu\nu}^- \) gives 6 more; and, finally, the mixed-symmetry components \( g_{\lambda\mu\nu}^{(M)} \) provide 16 independent combinations.

4. The tilde observables in the fermion sectors

How many of the 44 independent coefficients that are in principle observable in each fermionic sector after redefinitions have been accounted for, are experimentally observable? Based on the results of a number of analyses of experiments with fermions, the answer to this is that all can be accessed.

The particular combinations of SME coefficients that appear as observables in experiments are conventionally denoted with a tilde accent. Forty of these can be found from results with clock-comparison experiments [14] and space tests [38]. Analyses with the Penning-trap system [47], which can confine both particles and antiparticles, provide the motivation for the remaining four tilde coefficients. Three of these are the \( b \)-type coefficients relevant for antiparticles, denoted \( \tilde{b}_J^* \) with \( J = X, Y, Z \). They differ from the \( \tilde{b}_J \) coefficients in the sign of the \( d \) and \( H \) terms. The remaining degree of freedom is in the diagonal part of the \( c \) coefficient. Since \( c \) is traceless, there are three independent combinations that can be formed on the diagonal, two of which are the quadrupole observable \( \tilde{c}_Q \), and \( \tilde{c}_- \), arising in clock-comparison experiments. In Penning-trap systems, the cyclotron frequency is sensitive to the combination \( c_{TT} + c_{XX} + c_{YY} \), and this is an option for defining the 44th tilde observable. However, due to the popularity in theoretical studies of isotropic Lorentz violation, the choice \( \tilde{c}_T := mc_{TT} \) has instead been made. So, the basis for the diagonal \( c \) coefficients includes \( \tilde{c}_Q \) and \( \tilde{c}_- \) in parallel with
Table 2. Definitions of the \( \tilde{c} \)-tilde coefficients in the minimal fermion sector

| Tilde combination | Non-tilde combination | Components |
|-------------------|-----------------------|------------|
| \( \tilde{c}_Q \) | \( m(c_{XX} + c_{YY} - 2c_{ZZ}) \) |             |
| \( \tilde{c}_{-} \) | \( m(c_{XX} - c_{YY}) \) |             |
| \( \tilde{c}_{TT} \) | \( mc_{TT} \) | 3          |
| \( \tilde{c}_{TJ} \) | \( m(c_{TJ} + c_{JT}) \) | 3          |
| \( \tilde{c}_X \) | \( m(c_{YZ} + c_{ZY}) \) | 1          |
| \( \tilde{c}_Y \) | \( m(c_{XZ} + c_{ZX}) \) | 1          |
| \( \tilde{c}_Z \) | \( m(c_{XY} + c_{YX}) \) | 1 Total: 9 |

\( \tilde{d}_Q \) and \( \tilde{d}_{-} \) for the similarly traceless \( d \) coefficient; however, the last diagonal element for the \( c \) coefficients is \( \tilde{c}_{TT} \) while for \( d \) it is \( \tilde{d}_{-} \).

The full set of 44 tilde observables first appeared in Table XVII of the Data Tables for Lorentz and CPT Violation, January 2009 edition [26]. Tables 2 and 3 provide the tilde definitions in a form that differs in the order of presentation, gives the decomposition of the \( g \) coefficients into axial, trace, and mixed-symmetry parts explicitly, and collects together terms that are linearly dependent.

Since Lorentz violation is small, the tilde definitions are necessarily linear, and amount to a \( 44 \times 44 \) matrix transformation applied to the basis of 44 independent SME coefficients in the fermion sector. It is of interest to investigate the block structure of this matrix.

Table 2 lists the 9 observable \( c \) combinations, which decouple from the other observables. The first two columns in this table, and in Table 3, express the tilde definitions with an assignment sign understood between them. The last column indicates the number of independent coefficients. The first three entries of Table 2 are the observables for the diagonal. These definitions involve all four basic diagonal terms \( c_{TT}, c_{XX}, c_{YY}, \) and \( c_{ZZ} \), and the traceless condition can be used to eliminate any one of these. So, the definitions of the \( \tilde{c} \) coefficients include a \( 3 \times 3 \) block mixing \( c_{XX}, c_{YY}, \) and \( c_{ZZ} \), and six off-diagonal entries.

Table 3 gives the definitions of the remaining 35 tilde observables. Note that the observable combinations in the second column, which are orthogonal to the combinations in lines 3 and 4 of Table 1, are grouped using parentheses. The matrix implicit in this table has 8 blocks, separated by horizontal lines: one of dimension 6, three of dimension 5, three of dimension 4, and one of dimension 2. A bolder line separates the first four from the last four, and this split matches the splitting in the mixed-symmetry \( g \) components indicated by the horizontal line in Table A2. In fact, each of the eight subspaces contains an independent set of two of the \( g^{(M)} \) coefficients. In the presence of Lorentz violation in the form of \( g \) coefficients only, the first four blocks would each have dimension 4, because they would involve the axial \( g^{(A)} \) as well, and the remaining four would each have dimension 2. This would give access to 20 of the 24 \( g \) coefficients, with
the four traceless ones being inaccessible at leading order, consistent with the bottom line of Table 1.

In addition to the two $g^{(M)}$ entries, the $6 \times 6$ block contains the three diagonal entries of the $d$ coefficient, and $b_T$, while the three $5 \times 5$ blocks contain $b_J$, $\epsilon_{JKL}H_{KL}$, and the symmetric $d^T_{TJ}$ coefficients, with one $J$ value in each. The three $4 \times 4$ blocks, in addition to the $g^{(M)}$ entries, contain $H_{TJ}$ and the remaining three $d^T$ coefficients, again with one $J$ value in each.

Reference [26] provides experimental measurements of the tilde observables for the electron, proton, and neutron sectors, as well as a summary table giving the maximal experimental sensitivity to each coefficient with the assumption that other coefficients don’t contribute. Sensitivity to all nine $\tilde{c}$ observables exists for the electron and proton sectors, and six of them in the neutron sector. No experimental sensitivities have been published to tilde coefficients in the $2 \times 2$ block in any of the three fermion sectors. Among the $5 \times 5$ blocks, no sensitivities to the $J = Z$ set have been published for protons or neutrons. The least explored fermion sector is that of the proton, for which only six sensitivities exist outside of the $c$ block: these are within the $J = X$ and $J = Y$ $5 \times 5$ blocks. Experimental access to the $6 \times 6$ block can be attained by taking into account the boost of the experiment relative to the standard reference frame. This has been done for the electron sector using a torsion-pendulum experiment [2], and for the neutron sector using a dual maser system [24]. At present, there are no published sensitivities to coefficients in the $6 \times 6$ block of the proton sector.

5. Inverting the tilde definitions

In some experiments, measurements have been made of SME coefficients that differ from the tilde observables. This has been done by decoupling the observables through combining independent results. For results such as these, the sensitivities to the tilde observables can be deduced by using the inverse of the tilde definitions.

The linear transformation relating the 44 independent SME coefficient combinations and the 44 tilde observables is invertible, and we present the relevant expressions in Tables 4 and 5. Table 4 expresses the inverse of the $9 \times 9$ matrix defining the $\tilde{c}$ observables in Table 2. In this inverse, the traceless condition $c_{TT} = c_{XX} + c_{YY} + c_{ZZ}$ has been imposed to eliminate $c_{TT}$, and this may be verified by adding the first three lines of the table.

Table 5 expresses the inverse of the remaining 35 tilde definitions. The block structure has been carried over from Table 3 and horizontal lines are again used to separate the 8 subspaces. In the $6 \times 6$ block $d_{TJ}$ has been eliminated using the condition that $d_{\mu\nu}$ is traceless, $d_{TT} = d_{XX} + d_{YY} + d_{ZZ}$. The next three sections of Table 5 are $5 \times 5$ blocks based on the definitions for $\tilde{b}_J$, $\tilde{b}_T$, $\tilde{d}_J$, $\tilde{g}_{DJ}$, $\tilde{g}_{TJ}$, with $J = X, Y, Z$.

In each of the first four sections of Table 5, the two mixed-symmetry components satisfy one of the four nonaxial conditions (A.7). So, for example, an expression for
Table 3. Definitions of the minimal-fermion-sector tilde coefficients, excluding the $c$'s.

| Tilde combination | Non-tilde combination | Components |
|-------------------|------------------------|------------|
| $\tilde{b}_T$     | $(b_T - mg_T^{(A)}) + mg_{XYZ}$ |            |
| $\tilde{g}_T$     | $(b_T - mg_T^{(A)}) - 2mg_{XYZ}$ |            |
| $\tilde{d}_Q$     | $m(d_{XX} + d_{YY} - 2d_{ZZ}) + 3mg_{XYZ}$ |            |
| $\tilde{d}_+$     | $m(d_{XX} + d_{YY})$ |            |
| $\tilde{d}_-$     | $m(d_{XX} - d_{YY})$ |            |
| $\tilde{g}_c$     | $2mg_{XYZ} + mg_{YXX}$ | 6          |
| $\tilde{b}_X$     | $(b_X - mg_X^{(A)}) + (H_{YZ} - md_{TX}) + md_{TX}^{+} + mg_{YXT}^{(M)}$ | 5          |
| $\tilde{b}_Y$     | $(b_Y - mg_Y^{(A)}) - (H_{ZX} - md_{TY}) - md_{TY}^{+} + mg_{ZXT}^{(M)}$ | 5          |
| $\tilde{b}_Z$     | $(b_Z - mg_Z^{(A)}) - (H_{XY} - md_{TZ}) - md_{TZ}^{+} + mg_{XYT}^{(M)}$ | 5          |
| $\tilde{d}_X$     | $-\frac{1}{2}(H_{YZ} - md_{TX}) + \frac{1}{2}md_{TX}^{+}$ |            |
| $\tilde{d}_Y$     | $-\frac{1}{2}(H_{ZX} - md_{TY}) + \frac{1}{2}md_{TY}^{+}$ |            |
| $\tilde{d}_Z$     | $-\frac{1}{2}(H_{XY} - md_{TZ}) + \frac{1}{2}md_{TZ}^{+}$ |            |
| $\tilde{g}_D$     | $-(b_x - mg_x^{(A)}) + 2mg_{ZXT}$ | 4          |
| $\tilde{g}_T$     | $-mg_{ZXT}^{(M)} - 2mg_{TXX}^{(M)}$ |            |
| $\tilde{H}_{TX}$  | $(H_{TX} + md_{YX}) - md_{TX}^{+} + mg_{TXX}^{(M)}$ | 4          |
| $\tilde{d}_Y$     | $2md_{YX}^{+} - mg_{TXT}^{(M)} - 2mg_{YXY}$ |            |
| $\tilde{g}_X$     | $-2mg_{TXT}^{(M)} - mg_{XYY}$ |            |
| $\tilde{g}_{XY}$  | $-mg_{TXT}^{(M)} + mg_{XYY}$ | 4          |
| $\tilde{H}_{TY}$  | $(H_{TY} + md_{ZX}) - md_{TX}^{+} + mg_{TYT}^{(M)}$ | 4          |
| $\tilde{d}_Z$     | $2md_{ZX}^{+} - mg_{TYT}^{(M)} - 2mg_{YZZ}$ |            |
| $\tilde{g}_{YX}$  | $-2mg_{TYT}^{(M)} - mg_{YZZ}$ |            |
| $\tilde{g}_{YZ}$  | $-mg_{TYT}^{(M)} + mg_{YZZ}$ | 4          |
| $\tilde{H}_{TZ}$  | $(H_{TZ} + md_{XY}) - md_{TX}^{+} + mg_{TZT}^{(M)}$ | 4          |
| $\tilde{d}_X$     | $2md_{XY}^{+} - mg_{TZT}^{(M)} - 2mg_{ZXX}$ |            |
| $\tilde{g}_{XY}$  | $-2mg_{TZT}^{(M)} - mg_{ZXX}$ |            |
| $\tilde{g}_{Z}$   | $-mg_{TZT}^{(M)} + mg_{ZXX}$ | 4          |
| $\tilde{g}_{-}$   | $-mg_{TXX}^{(M)} + mg_{TYY}$ |            |
| $\tilde{g}_{Q}$   | $-3mg_{TXX}^{(M)} - 3mg_{TYY}^{(M)}$ | 2          |

Total: 35
Table 4. Minimal-fermion-sector $c$ coefficients in terms of tilde quantities.

| Non-tilde combination | Tilde combination | Components |
|-----------------------|------------------|------------|
| $mc_{XX}$             | $\frac{1}{6}(\tilde{c}_Q + 3\tilde{c}_- + 2\tilde{c}_{TT})$ |           |
| $mc_{YY}$             | $\frac{1}{6}(\tilde{c}_Q - 3\tilde{c}_- + 2\tilde{c}_{TT})$ |           |
| $mc_{ZZ}$             | $\frac{1}{3}(-\tilde{c}_Q + \tilde{c}_{TT})$ | 3         |
| $m|\varepsilon_{JKL}|c_{KL}$ | $\tilde{c}_J$ | 1+1+1     |
| $m(c_{TJ} + c_{JT})$ | $\tilde{c}_{TJ}$ | 1+1+1     |

Total: 9

$mg^{(M)}_{ZXY}$ can be found by adding the last two lines in the first section of the Table, giving

$$mg^{(M)}_{ZXY} = \frac{1}{3}(\tilde{b}_T - \tilde{g}_T) - \bar{g}_c. \tag{6}$$

In each of the bottom four blocks in Table 5, the two mixed-symmetry coefficients $g^{(M)}_J$ satisfy a traceless condition (A.8). The four traceless conditions are given explicitly in Table A2. Adding the last two lines of Table 5, and noting the last line of Table A2, one can verify that

$$mg^{(M)}_{TZZ} = \frac{1}{3}\tilde{g}_Q. \tag{7}$$

6. Limits on SME observables

We next use expressions in Table 5 to estimate limits on several of the electron-sector tilde observables.

The three expressions

$$b_J - mg^{(A)}_J = \frac{1}{3}(\tilde{b}_J + \tilde{b}_J^* - \tilde{g}_{DJ}), \tag{8}$$

appearing as the first line in each of the dimension 5 blocks, generate limits on $b_J^*$ and $\tilde{g}_{DJ}$ from the existing limits on $b_J$, for $J = 1, 2, 3$. In this case, the bounds are $|b_J| < 50 \text{ rad/sec} \simeq 3.3 \times 10^{-23} \text{ GeV}$, based on Penning-trap experiments [5]. Setting the axial components $g^{(A)}_J$ to zero, since the $g$ coefficients were not included in analyses when these results were published, we may deduce separate order-of-magnitude limits on each of the tilde coefficients on the right-hand side of Eq. (8). To do this, we assume that the limit on $|b_J|$ is at the $2\sigma$ level; then, considering each tilde coefficient in isolation, we deduce that

$$|\tilde{b}_J^e|, |\tilde{g}_{D,e}^e| < 9.9 \times 10^{-23} \text{ GeV} \simeq 10^{-22} \text{ GeV}. \tag{9}$$

The same bound also follows for $|\tilde{b}_J|$, but is not competitive with limits from other experiments.

Several limits on electron-sector tilde combinations follow from inverse Compton scattering bounds on SME coefficients [10]. Following a similar argument as above, the limit $|d_{YZ} + d_{ZY}| < 7 \times 10^{-15}$ may be inserted into the second line of the first dimension-four block in Table 5. Using the electron mass $m = 0.51 \times 10^{-3} \text{ GeV}$ yields

$$|\tilde{g}_{XY}^e|, |\tilde{g}_{XZ}^e| < 10^{-17} \text{ GeV}. \tag{10}$$
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| Non-tilde combination | Tilde combination | Components |
|-----------------------|-------------------|------------|
| $b_T - mg_T^{(A)}$    | $\frac{1}{3}(2b_T + \tilde{g}_T)$ |             |
| $md_{XX}$             | $\frac{1}{2}(\tilde{d}_+ + \tilde{d}_-)$ |             |
| $md_{YY}$             | $\frac{1}{2}(\tilde{d}_+ - \tilde{d}_-)$ |             |
| $md_{ZZ}$             | $\frac{1}{2}(b_T - \tilde{g}_T + \tilde{d}_+ - \tilde{d}_Q)$ |             |
| $mg_{XYZ}^{(M)}$      | $\frac{1}{3}(\tilde{b}_T + \tilde{g}_T)$ |             |
| $mg_{YZX}^{(M)}$      | $-\frac{1}{3}(\tilde{b}_T - \tilde{g}_T + \tilde{g}_c)$ | $6$         |
| $b_X - mg_X^{(A)}$    | $\frac{1}{3}(\tilde{b}_X + \tilde{b}_X^* - \tilde{g}_{DX})$ |             |
| $H_{YZ} - md_{TX}$    | $-\frac{1}{3}(b_X - \tilde{b}_X^*) - \frac{1}{2}\tilde{d}_X$ |             |
| $md_{TX}^{(M)}$       | $-\frac{1}{3}(\tilde{b}_X - \tilde{b}_X^*) + \frac{1}{2}\tilde{d}_X$ |             |
| $mg_{YXT}^{(M)}$      | $\frac{1}{3}(\tilde{b}_X + \tilde{b}_X^* + 2\tilde{g}_{DX})$ |             |
| $mg_{TZY}^{(M)}$      | $-\frac{1}{12}(\tilde{b}_X + \tilde{b}_X^* + 2\tilde{g}_{DX} + 6\tilde{g}_{TX})$ | $5$         |
| $b_Y - mg_Y^{(A)}$    | $\frac{1}{3}(\tilde{b}_Y + \tilde{b}_Y^* - \tilde{g}_{DY})$ |             |
| $H_{ZX} - md_{TY}$    | $-\frac{1}{3}(b_Y - \tilde{b}_Y^*) - \frac{1}{2}\tilde{d}_Y$ |             |
| $md_{TY}^{(M)}$       | $-\frac{1}{3}(\tilde{b}_Y - \tilde{b}_Y^*) + \frac{1}{2}\tilde{d}_Y$ |             |
| $mg_{ZXT}^{(M)}$      | $\frac{1}{3}(\tilde{b}_Y + \tilde{b}_Y^* + 2\tilde{g}_{DY})$ |             |
| $mg_{TYZ}^{(M)}$      | $-\frac{1}{12}(\tilde{b}_Y + \tilde{b}_Y^* + 2\tilde{g}_{DY} + 6\tilde{g}_{TY})$ | $5$         |
| $b_Z - mg_Z^{(A)}$    | $\frac{1}{3}(\tilde{b}_Z + \tilde{b}_Z^* - \tilde{g}_{DZ})$ |             |
| $H_{XY} - md_{TZ}$    | $-\frac{1}{3}(b_Z - \tilde{b}_Z^*) - \frac{1}{2}\tilde{d}_Z$ |             |
| $md_{TZ}^{(M)}$       | $-\frac{1}{3}(\tilde{b}_Z - \tilde{b}_Z^*) + \frac{1}{2}\tilde{d}_Z$ |             |
| $mg_{XYT}^{(M)}$      | $\frac{1}{3}(\tilde{b}_Z + \tilde{b}_Z^* + 2\tilde{g}_{DZ})$ |             |
| $mg_{TZY}^{(M)}$      | $-\frac{1}{12}(\tilde{b}_Z + \tilde{b}_Z^* + 2\tilde{g}_{DZ} + 6\tilde{g}_{TZ})$ | $5$         |
| $H_{TX} + md_{TZ}$    | $H_{TX} + \frac{1}{2}(\tilde{d}_{YZ} - \tilde{g}_{XY} - \tilde{g}_{XZ})$ |             |
| $md_{YZ}^{(M)}$       | $\frac{1}{2}(\tilde{d}_{YZ} + \tilde{g}_{XY} - \tilde{g}_{XZ})$ |             |
| $mg_{TXY}^{(M)}$      | $-\frac{1}{3}(\tilde{g}_{XY} + \tilde{g}_{XZ})$ |             |
| $mg_{TXY}^{(M)}$      | $\frac{1}{3}(2\tilde{g}_{XY} - \tilde{g}_{XZ})$ | $4$         |
| $H_{TY} + md_{ZX}$    | $H_{TY} + \frac{1}{2}(\tilde{d}_{ZX} - \tilde{g}_{YZ} - \tilde{g}_{YX})$ |             |
| $md_{ZT}^{(M)}$       | $\frac{1}{2}(\tilde{d}_{ZX} + \tilde{g}_{YZ} - \tilde{g}_{YX})$ |             |
| $mg_{TZY}^{(M)}$      | $-\frac{1}{3}(\tilde{g}_{YZ} + \tilde{g}_{YX})$ |             |
| $mg_{YZX}^{(M)}$      | $\frac{1}{3}(2\tilde{g}_{YZ} - \tilde{g}_{YX})$ | $4$         |
| $H_{TZ} + md_{XY}$    | $H_{TZ} + \frac{1}{2}(\tilde{d}_{XY} - \tilde{g}_{ZX} - \tilde{g}_{ZY})$ |             |
| $md_{XY}^{(M)}$       | $\frac{1}{2}(\tilde{d}_{XY} + \tilde{g}_{ZX} - \tilde{g}_{ZY})$ |             |
| $mg_{TZX}^{(M)}$      | $-\frac{1}{3}(\tilde{g}_{ZX} + \tilde{g}_{ZY})$ |             |
| $mg_{ZZX}^{(M)}$      | $\frac{1}{3}(2\tilde{g}_{ZX} - \tilde{g}_{ZY})$ | $4$         |
| $mg_{TXX}^{(M)}$      | $-\frac{1}{6}(3\tilde{g}_c + \tilde{g}_Q)$ |             |
| $mg_{TYY}^{(M)}$      | $\frac{1}{6}(3\tilde{g}_c - \tilde{g}_Q)$ | $2$ Total: $35$ |
The second lines of the other two dimension-four blocks in Table 5, combined with the results $|d_{XZ} + d_{ZX}| < 2 \times 10^{-14}$ and $|d_{XY} + d_{YX}| < 2 \times 10^{-15}$ in [10], lead to

\begin{align}
|\tilde{g}^e_{YZ}|, |\tilde{g}^e_{YX}| &< 10^{-17} \text{ GeV}, \\
|\tilde{g}^e_{ZX}|, |\tilde{g}^e_{ZY}| &< 10^{-18} \text{ GeV},
\end{align}

respectively.

A limit on the electron-sector $\tilde{d}_Z$ can be extracted from the result $|d_T Z| < 8 \times 10^{-17}$ in Ref. [10]. In that analysis, $H_{XY}$ may be interpreted as having been absorbed into the antisymmetric part of $md_{TZ}$. By reinstating it via the observable combination seen in equation (5), the quantity bounded becomes $md_{TZ} \rightarrow md_{TZ} + md_{TZ} - H_{XY}$. This can be expanded in terms of tilde expressions using the results in the $Z$ component 5 $\times$ 5 block of Table 5, giving $md_{TZ} \rightarrow (\tilde{b}_Z - \tilde{b}_Z^*)/4 + \tilde{d}_Z$. While the resulting sensitivities to $\tilde{b}_Z$ and $\tilde{b}_Z^*$ are weaker than ones derived from other systems, the sensitivity to $\tilde{d}_Z$ is the only one known at present. The latter result is

$|\tilde{d}_{Z}| < 10^{-19} \text{ GeV}. 
\(13\)$

We estimate a 90% confidence level for each of these astrophysical limits. The electron-sector bounds extracted here from Penning-trap experiments and astrophysical data are consistent with the estimated maximal sensitivities appearing in Ref. [26], but have not been described in the literature to date.

7. Summary and discussion

We have considered the algebraic structure of the coefficients for Lorentz violation in the flat-space fermion sector of the minimal SME. This has revealed a block structure in the system of experimental observables. By inverting the definitions, thirteen limits on the tilde observables in the electron-positron sector have been found, and are given in equations (9), (10), (11), (12), and (13).

To date, most Lorentz tests have concentrated on seeking sidereal variations in signals, due to the rotation of the laboratory relative to the standard Sun-centered inertial reference frame. When the linear motion of the system relative to the inertial reference frame is considered modifications involving the boost factor $\gamma = (1 - v^2)^{-1/2}$ in the expressions for the 44 fermion observables can be expected. This has been done in Lorentz tests with a spin-polarized torsion pendulum [2] and with a dual maser system [24], as mentioned earlier. In Lorentz tests where the particles are significantly boosted relative to the inertial reference frame, such as in muon experiments [48], the observables will necessarily involve factors of $\gamma$. In such cases, the relativistic hamiltonian [37] is needed, and the $\tilde{b}_J$ and $\tilde{b}_J^*$ observables take the modified ‘háček’ form [49]:

$$
\tilde{b}_X^\pm := \frac{1}{\gamma} (b_X - mg^{(A)}X + mg^{(M)}_Y) \mp (H_{YZ} - md_{TX}) \mp md_{TX}^+, \quad (14)
$$

with $\tilde{b}_X^+$ and $\tilde{b}_X^-$ defined by cyclic rotations of $X, Y, Z$. The háček expressions equal the tilde coefficients in the $\gamma \rightarrow 1$ nonrelativistic limit.
Another way to gain sensitivities to SME coefficients is to take interactions into account. This has been done in the context of relativistic nuclear binding effects in atomic clocks [21]. The results indicate that, under appropriate circumstances, separation can be achieved between the $b$ and $g$ coefficients, and between the $H$ and $d$ coefficients.

As noted earlier, the $6 \times 6$ block of tilde observables for the proton sector of Table 3 has no published bounds on it. The boosted observable expressions mix the spatial components from the three $5 \times 5$ blocks with the time components in the $6 \times 6$ block. It follows that any experiments sensitive to tilde coefficients in the proton sector have the potential to access the $6 \times 6$ block in Table 3. In addition, the absence of any sensitivities to one of the $5 \times 5$ blocks in the proton sector, and all three of the $4 \times 4$ blocks, means Lorentz tests have the potential to yield a rich crop of new results.

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**Appendix A. Irreducible components of $g_{\lambda\mu\nu}$**

The SME coefficients $g_{\mu\nu\alpha}$ are antisymmetric in the first two indices, which means they comprise 24 independent numbers. This observer tensor can be uniquely decomposed into three standard irreducible components, the axial part $g^{(A)}_{\mu\nu\alpha}$, the trace part $g^{(T)}_{\mu\nu\alpha}$, and the mixed-symmetry part $g^{(M)}_{\mu\nu\alpha}$:

\[
g_{\mu\nu\lambda} = g^{(A)}_{\mu\nu\lambda} + g^{(T)}_{\mu\nu\lambda} + g^{(M)}_{\mu\nu\lambda}.
\] (A.1)

Expressions defining each of these components are as follows:

\[
g^{(A)}_{\mu\nu\lambda} = \frac{1}{6} g_{\sigma\kappa\tau} \epsilon^{\sigma\kappa\tau\alpha} \epsilon_{\alpha\mu\nu\lambda} = \frac{1}{3} (g_{\mu\nu\lambda} + g_{\mu\lambda\nu} + g_{\nu\lambda\mu}), \] (A.2)

\[
g^{(T)}_{\mu\nu\lambda} = \frac{1}{3} \eta^{\alpha\beta} (g_{\mu\alpha\beta} \eta_{\nu\lambda} - g_{\nu\alpha\beta} \eta_{\mu\lambda}), \] (A.3)

\[
g^{(M)}_{\mu\nu\lambda} = \frac{1}{3} (g_{\mu\nu\lambda} + g_{\mu\lambda\nu} + \eta_{\mu\lambda} g_{\nu\alpha\beta} \eta^{\alpha\beta}) - (\mu \leftrightarrow \nu). \] (A.4)

The numerical values of these lower-index expressions are independent of the choice of spacetime-metric signature and, separately, of the sign convention for $\epsilon^{0123}$, since the metric and the antisymmetric tensor appear quadratically.

There are four independent axial components of $g^{(A)}_{\mu\nu\lambda}$, and they can be found by contraction with the antisymmetric tensor. Hence we define a single-index axial vector

\[
g^{(A)\alpha} = \frac{1}{6} g_{\sigma\kappa\tau} \epsilon^{\sigma\kappa\tau\alpha}. \] (A.5)

Similar contractions over the three indices of $g^{(T)}_{\mu\nu\lambda}$ or $g^{(M)}_{\mu\nu\lambda}$ with the antisymmetric tensor lead to a zero result.

The top half of Table A1 lists the axial components of $g_{\mu\nu\lambda}$ in a reference frame with time and space coordinates $T, X, Y, Z$. The expressions involving all but the fourth
The latter appears quadratically. The expressions involving the lower-index antisymmetric tensor, because, in equation (A.2), the former does not appear and the column are valid under all sign conventions for the spacetime metric and the totally antisymmetric tensor. Expressions for Fermion observables for Lorentz violation hold for conventions with product \( \eta_{00} \epsilon^{0123} = +1 \). All other signs hold in any convention.

### Table A1. Axial and trace components of \( g_{\mu\nu\lambda} \)

The signs for the single-index components \( g^{(A)}_{\mu} \) hold for conventions with product \( \eta_{00} \epsilon^{0123} = +1 \). All other signs hold in any convention.

| Equal triple- and single-index components | Expression |
|------------------------------------------|-------------|
| \( g^{(A)}_{XYZ} \)                      | \(-g^{(A)}_{XZY} - g^{(A)}_{ZYX} + \frac{1}{3}(g_{XYZ} + g_{YZX} + g_{ZXY}) \) |
| \( g^{(A)}_{YZX} \)                      | \(-g^{(A)}_{TZX} \) |
| \( g^{(A)}_{TZX} \)                      | \(-g^{(A)}_{TZX} \) |
| \( g^{(A)}_{TXY} \)                      | \(-g^{(A)}_{TXY} \) |
| \( g^{(A)}_{TXY} \)                      | \(-g^{(A)}_{TXY} \) |
| \( g^{(A)}_{TYZ} \)                      | \(-g^{(A)}_{TYZ} \) |
| \( g^{(A)}_{TZY} \)                      | \(-g^{(A)}_{TZY} \) |
| \( g^{(A)}_{YTZ} \)                      | \(-g^{(A)}_{YTZ} \) |
| \( g^{(A)}_{YTZ} \)                      | \(-g^{(A)}_{YTZ} \) |
| \( g^{(T)}_{TXX} \)                      | \(-\frac{1}{4}g^{(T)T} \) |
| \( g^{(T)}_{TYT} \)                      | \(-\frac{1}{4}g^{(T)X} \) |
| \( g^{(T)}_{TYT} \)                      | \(-\frac{1}{4}g^{(T)Y} \) |
| \( g^{(T)}_{TYT} \)                      | \(-\frac{1}{4}g^{(T)Z} \) |
| \( g^{(T)}_{TZT} \)                      | \(-\frac{1}{4}g^{(T)T} \) |
| \( g^{(T)}_{TZT} \)                      | \(-\frac{1}{4}g^{(T)X} \) |
| \( g^{(T)}_{TZT} \)                      | \(-\frac{1}{4}g^{(T)Y} \) |
| \( g^{(T)}_{TZT} \)                      | \(-\frac{1}{4}g^{(T)Z} \) |

This follows because, in equation (A.5), one factor of the totally antisymmetric tensor appears, and one factor of the metric is needed to lower the index. Thus, the product \( \eta_{00} \epsilon^{0123} \) has to have a fixed sign.

There are four independent trace components \( g^{(T)}_{\mu\nu\lambda} \), and they can be found by contracting the last two indices of \( g_{\mu\nu\lambda} \) with the metric. So, a single-index trace component is defined by

\[
g^{(T)}_{\mu} \equiv g_{\mu\nu\lambda} \eta^{\nu\lambda} = g_{\mu\alpha} \alpha.
\]  

This contraction gives a zero result if applied to \( g^{(A)}_{\mu\nu\lambda} \) or \( g^{(M)}_{\mu\nu\lambda} \).

The bottom half of Table A1 lists the trace components of \( g_{\mu\nu\alpha} \) in the standard Sun-centered frame. All the expressions involving three-index quantities hold under any sign conventions for \( \eta_{00} \) and \( \epsilon^{0123} \), because equation (A.3) involves the metric quadratically and does not involve the totally antisymmetric tensor. Expressions for the single-index quantities \( g^{(T)\mu} \) also hold under any sign convention because the metric appears quadratically: once in equation (A.6), and once more to raise the index position.

Table A2 lists the mixed-symmetry components of \( g_{\mu\nu\lambda} \) in the standard Sun-centered frame. Since the spacetime metric appears quadratically in equation (A.4), and the totally antisymmetric tensor does not appear at all, the expressions in Table A2 are valid regardless of the sign convention.

By definition, the mixed-symmetry components \( g^{(M)}_{\mu\nu\lambda} \) are constrained by four conditions that ensure they are nonaxial:

\[
g^{(M)}_{\mu\nu\lambda} \eta^{\mu\nu\lambda} = 0,
\]  

\[(A.7)\]
The signs hold regardless of the convention for $\eta_{00}$ and $\epsilon^{0123}$.

The four nonaxial conditions appear explicitly in the upper part of the first column of Table A2. The four traceless conditions are given explicitly in the lower portion of the first column. These 8 linear relations among the 24 components of $g_{\mu\nu\lambda}^{(M)}$ ensure that the dimension of the mixed-symmetry vector space is 16.

We note that the upper half of the table consists of components with three distinct indices, while the lower part has components with a pair of identical indices. It follows that the mixed-symmetry tensors are split into two independent vector spaces, each of dimension 8.
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