Sensitivity of Antenna Arrays for Long-Wavelength Radio Astronomy
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Abstract—A number of new and planned radio telescopes will consist of large arrays of low-gain antennas operating at frequencies below 300 MHz. In this frequency regime, Galactic noise can be a significant or dominant contribution to the total noise. This, combined with mutual coupling between antennas, makes it difficult to predict the sensitivity of these instruments. This paper describes a system model and procedure for estimating the system equivalent flux density (SEFD) – a useful and meaningful metric of the sensitivity of a radio telescope – that accounts for these issues. The method is applied to LWA-1, the first “station” of the Long Wavelength Array (LWA) interferometer. LWA-1 consists of 512 bowtie-type antennas within a 110 × 100 m elliptical footprint, and is designed to operate between 10 MHz and 88 MHz using receivers having noise temperature of about 250 K. It is shown that the correlation of Galactic noise between antennas significantly desensitizes the array for beam pointings which are not close to the zenith. It is also shown that considerable improvement is possible using beamforming coefficients which are designed to optimize signal-to-noise ratio under these conditions. Mutual coupling is found to play a significant role, but does not have a consistently positive or negative influence. In particular, we demonstrate that pattern multiplication (assuming the behavior of single antennas embedded in the array is the same as those same antennas by themselves) does not generate reliable estimates of SEFD.

Index Terms—Antenna Array, Beamforming, Radio Astronomy.

I. INTRODUCTION

A number of new and planned radio telescopes will consist of large arrays of closely-spaced low-gain antennas operating at frequencies below 300 MHz. These include LWA [1], LOFAR [2], MWA [3], and SKA [4]. In this frequency regime, it is possible to design receivers with noise temperatures that are much less than the antenna temperatures associated with the ubiquitous Galactic synchrotron radiation, such that the resulting total system noise temperature is dominated by Galactic noise [5]. This is quite different from the condition most often considered, in which it is usually assumed that internal noise associated with the receivers dominates and does not scatter into the array, so that the noise associated with different antennas is uncorrelated; see e.g., [6], [7], [8]. Recent work on arrays for long-wavelength radio astronomy accounts for the dominance of external noise, but neglects the effects of correlation of this noise between antennas [9], [1]. However, it is known that this correlation is likely to have a significant effect for these instruments [10], and this issue is further explored in this paper. In [11], [12], and [13], correlation of noise between antennas is considered, but the source of noise is internal (amplifiers or ohmic losses in antennas) and the correlation arises due to propagation internal to the array. The problem of correlation of external noise has been intermittently considered in communications (e.g., [14]) and direction finding (e.g., [15]) applications, but does not seem to have been previously considered for long-wavelength radio astronomy beamforming applications.

Because antenna spacing in the systems of interest is typically less than a few wavelengths, mutual coupling also plays a significant role. Since these arrays are electromagnetically large, interact with the electromagnetically-complex ground, and may have aperiodic spacings, it is difficult to determine the characteristics of antennas, either individually or collectively as part of a beamforming system. In particular, it is usually difficult to know if pattern multiplication – that is, assuming that the behavior of single antennas embedded in the array is the same as those same antennas by themselves – yields reasonable results. Past studies have shown that mutual coupling in aperiodic arrays of low-gain elements results in fluctuation of beam gain and sidelobe levels as a function of scan angle when element spacing is less than a few wavelengths [16], [17]. This suggests that pattern multiplication may not be a useful assumption. However, useful and generalizable findings which are applicable to the systems of interest are not commonly available.

This paper describes a procedure for estimating the sensitivity of radio telescope arrays which is appropriate under these conditions. The procedure is based on a system model, described in Section III which relates the electromagnetic response of the array (the array manifold), a model for the external noise temperature, and a model for the receiver noise temperatures to the system equivalent flux density (SEFD) achieved by a beam formed using specified beamforming coefficients. SEFD is defined as the power flux spectral density (i.e., W m⁻² Hz⁻¹) which yields signal-to-noise ratio (SNR) equal to unity at the beamformer output. SEFD is a useful metric as it includes the combined effect of antennas and all noise sources into a single “bottom line” number that is directly related to the sensitivity of astronomical observations.

The primary difficulty in determining SEFD using the above procedure is obtaining the array manifold. One approach is demonstrated by example in Section III of this paper. We analyze LWA-1, the first “station” of the Long Wavelength Array (LWA) interferometer [1]. LWA-1 consists of 512 bowtie-type antenna elements arranged into 256 dual-polarized “stands” within a 110 × 100 m elliptical footprint. LWA-1 is designed to operate between 10 MHz and 88 MHz using receivers...
having noise temperature of about 250 K. We obtain the array manifold for LWA-1 at 20 MHz, 38 MHz, and 74 MHz using a method of moments (MoM) wire-grid model. Because the model is too large to analyze all at once (a common problem with this class of arrays), a procedure described in Section II.B is employed in which the manifold is calculated for one stand (i.e., one pair of collocated antenna elements) at a time. Unlike pattern multiplication in which the presence of the remaining stands would be ignored, this procedure obtains the response of each antenna in the presence of nearby antennas and structures.

Also considered in this paper is the selection and performance of beamforming coefficients which optimize SEFD. Because mutual coupling and external noise correlation are significant, it is to be expected that “simple” beamforming coefficients based solely on antenna positions (i.e., phases associated with geometry only) will not be optimal. In Section IV the SEFD performance of LWA-1 is evaluated. It is shown that the optimal coefficients significantly improve sensitivity relative to simple coefficients. Finally, in Section V the extension of these results to predict the imaging performance of an interferometer comprised of multiple beamforming arrays is considered.

II. THEORY

Let \( \mathbf{E}_\theta(t) \) and \( \mathbf{E}_\phi(t) \) be the \( \theta \)- and \( \phi \)-polarized components of the electric field of the signal of interest, having units V m\(^{-1} \) Hz\(^{-1/2} \). In this coordinate system, \( \theta \) is measured from the +z axis, which points toward the ground; the \( \phi \) ground lies in the \( z = 0 \) plane, and \( \phi \) is measured from the +x axis. The signal of interest is incident from \( \{\theta_0, \phi_0\} \), which is henceforth indicated as \( \psi_0 \). The resulting voltage across the terminals of the \( n \)th antenna element, having units of V Hz\(^{-1/2} \), is

\[
  x_n(t) = a_0^\theta(\psi_0)\mathbf{E}_\theta(t) + a_0^\phi(\psi_0)\mathbf{E}_\phi(t) + z_n(t) + u_n(t) \tag{1}
\]

where: \( a_0^\theta(\psi_0) \) and \( a_0^\phi(\psi_0) \) are the effective lengths, having units of meters, associated with the \( \theta \) and \( \phi \) polarizations, respectively, for the \( n \)th antenna element for signals incident from \( \psi_0 \); \( z_n(t) \) is the contribution from noise external to the system; and \( u_n(t) \) is the contribution from noise internal to the system. Note \( u_n(t) \) can also include internal noise unintentionally radiated by some other antenna and received by antenna \( n \). In all cases, we assume these quantities are those which apply when antennas are terminated into whatever electronics are actually employed in the system, as opposed to being “open circuit” or “short circuit” quantities. Without loss of generality we can interpret these to be time-harmonic (i.e., monochromatic complex-valued “baseband”) quantities.

Beamforming can be described as the operation:

\[
y(t) = \sum_{n=1}^{N} b_n x_n(t) \tag{2}
\]

where \( N \) is the number of antennas, and the unitless \( b_n \)'s specify the beam. Assuming root-mean-square voltages, the power at the output of the beamformer is

\[
P_y = \langle y(t)y^*(t) \rangle R_o^{-1} \tag{3}
\]

where \( \langle \cdot \rangle \) denotes time-domain averaging and \( \langle \cdot \rangle^{*} \) denotes conjugation, and \( R_o \) is the impedance looking into the system as seen from the terminals across which \( y(t) \) is measured, assumed to be purely resistive.

We now wish to evaluate Equation [3] by substitution of Equation [2]. In the process of expanding Equation [3], let us assume that the signal of interest, \( z_n(t) \), and \( u_n(t) \) are mutually uncorrelated for any given \( n \). Specifically, we assume that for any \( n \) and \( m \):

\[
\langle E_\theta(t)z_n^*(t) \rangle = \langle E_\phi(t)z_n^*(t) \rangle = 0 \tag{4}
\]

\[
\langle E_\theta(t)u_n^*(t) \rangle = \langle E_\phi(t)u_n^*(t) \rangle = 0 \tag{5}
\]

\[
\langle z_n(t)u_m^*(t) \rangle = 0 \tag{6}
\]

Note that the possibility that like terms are correlated between antennas is not precluded by the above assumptions; for example, \( \langle z_n(t)z_n^*(t) \rangle \) can be \( \neq 0 \) for \( n \neq m \). Furthermore, we have not yet made any assumption about the correlation between \( E_\theta(t) \) and \( E_\phi(t) \). Under these assumptions, Equation [3] can be written as follows:

\[
P_yR_o = b_\theta^H \mathbf{A}_{\theta\theta} b_\theta + b_\phi^H \mathbf{A}_{\phi\phi} b_\phi + b_\phi^H \mathbf{P}_\psi b_\phi + b_\theta^H \mathbf{P}_u b_\theta \tag{7}
\]

where \( "^H" \) denotes the conjugate transpose operator;

\[
b = [ b_1 \ b_2 \cdots \ b_N ]^T, \tag{8}\]

where \( "^T" \) denotes the transpose operator; and

\[
\mathbf{A}_{\theta\theta} = a_0^\theta(\psi_0) a_0^\theta(\psi_0)^T \quad \mathbf{P}_{\theta\theta} = \langle |E_\theta(t)|^2 \rangle \tag{9}
\]

\[
\mathbf{A}_{\phi\phi} = a_0^\phi(\psi_0) a_0^\phi(\psi_0)^T \quad \mathbf{P}_{\phi\phi} = \langle |E_\phi(t)|^2 \rangle \tag{10}
\]

\[
\mathbf{A}_{\theta\phi} = a_0^\theta(\psi_0) a_0^\phi(\psi_0)^T \quad \mathbf{P}_{\theta\phi} = \langle E_\theta(t)E_\phi^*(t) \rangle \tag{11}
\]

\[
\mathbf{A}_{\phi\theta} = a_0^\phi(\psi_0) a_0^\theta(\psi_0)^T \quad \mathbf{P}_{\phi\theta} = \langle E_\phi(t)E_\theta^*(t) \rangle \tag{12}
\]

\[
\mathbf{a}_\theta(\psi_0) = [ a_1^\theta(\psi_0) \ a_2^\theta(\psi_0) \cdots \ a_N^\theta(\psi_0) ]^T \tag{13}
\]

\[
\mathbf{a}_\phi(\psi_0) = [ a_1^\phi(\psi_0) \ a_2^\phi(\psi_0) \cdots \ a_N^\phi(\psi_0) ]^T \tag{14}
\]

and \( \mathbf{P}_z \) is a matrix whose \( (n,m) \)th element is \( \langle z_n^*(t)z_m(t) \rangle \), and \( \mathbf{P}_u \) is a matrix whose \( (n,m) \)th element is \( \langle u_n^*(t)u_m(t) \rangle \).\footnote{Following the convention that first index indicates row and the second index indicates column.}

We now consider the external noise correlation matrix \( \mathbf{P}_z \). We wish to obtain a simple expression for \( \mathbf{P}_z[n,m] \), the \( (n,m) \)th element of \( \mathbf{P}_z \), in terms of physical quantities more relevant to radio astronomy. First, let us define \( \Delta S(\psi) \) as the flux density, having units of W m\(^{-2} \) Hz\(^{-1} \), associated with the electric field \( \Delta E(\psi, t) \) incident from a region of solid angle \( \Delta \Omega \) around \( \psi \). Assuming \( \Delta E(\psi, t) \) is given in terms of root-mean-square voltage, the relationship is

\[
\Delta S(\psi) = \langle (\Delta E(\psi, t))^2 \rangle / \eta \tag{15}
\]

where \( \eta \) is the impedance of free space. Since the Galactic synchrotron background noise is essentially unpolarized, we
assume the powers in the \( \theta \)- and \( \phi \)-polarized components of \( \Delta E(\psi, t) \) are equal; specifically,
\[
\left| \Delta E_\theta(\psi, t) \right|^2 = \left| \Delta E_\phi(\psi, t) \right|^2 = \frac{2}{2} \Delta S(\psi) \tag{16}
\]
Note also that \( \Delta S(\psi) \) can be obtained independently from the Rayleigh-Jeans Law:
\[
\Delta S(\psi) = \frac{2k}{\lambda^2} T_e(\psi) \Delta \Omega
\]
where \( k \) is Boltzmann’s constant \((1.38 \times 10^{-23} \text{ J/K})\), \( T_e(\psi) \) is the apparent external noise brightness temperature (attributable to either Galactic noise or thermal radiation from the ground) in the direction of \( \psi \), and \( \lambda \) is wavelength. We can model \( \Delta E_\theta(\psi, t) \) and \( \Delta E_\phi(\psi, t) \) as follows:
\[
\Delta E_\theta(\psi, t) = g_\theta(\psi, t) \sqrt{\frac{k\eta}{\lambda^2}} T_e(\psi) \Delta \Omega \tag{18}
\]
\[
\Delta E_\phi(\psi, t) = g_\phi(\psi, t) \sqrt{\frac{k\eta}{\lambda^2}} T_e(\psi) \Delta \Omega \tag{19}
\]
where \( g_\theta(\psi, t) \) and \( g_\phi(\psi, t) \) are Gaussian-distributed random variables with zero mean and unit variance. Note that we expect not only that \( g_\theta(\psi, t) \) and \( g_\phi(\psi, t) \) will be independent random variables, but also that \( g_\theta(\psi_1, t) \) and \( g_\phi(\psi_2, t) \) will be uncorrelated for \( \psi_1 \neq \psi_2 \), and similarly for \( g_\phi(\psi, t) \). We obtain \( z_n(t) \) by summing up the contributions received over a sphere:
\[
z_n(t) = \sum \psi \left[ a_\theta^\psi(\psi) \Delta E_\theta(\psi, t) + a_\phi^\psi(\psi) \Delta E_\phi(\psi, t) \right]
\]
Applying the definition of \( P_z \) and exploiting the statistical properties of \( g_\theta(\psi, t) \) and \( g_\phi(\psi, t) \), we find:
\[
P_{z[n,m]} = \frac{k\eta}{\lambda^2} \sum \psi \left[ a_\theta^\psi(\psi) a_m^\theta(\psi) + a_\phi^\psi(\psi) a_m^\phi(\psi) \right] T_e(\psi) \Delta \Omega
\]
which can now be written in integral form:
\[
P_{z[n,m]} = \frac{k\eta}{\lambda^2} \int \left[ a_\theta^\psi(\psi) a_m^\theta(\psi) + a_\phi^\psi(\psi) a_m^\phi(\psi) \right] T_e(\psi) d\Omega
\]
Returning to Equation [7] note that the signal to noise ratio (SNR) at the output of the beamformer can be written as:
\[
\text{SNR} = \frac{b^H R_s b}{b^H R_n b}, \tag{23}
\]
where
\[
R_s = A_{\theta \theta} P_{\theta \theta} + A_{\phi \phi} P_{\phi \phi} + A_{\theta \phi} P_{\phi \theta} + A_{\phi \theta} P_{\theta \phi} , \quad \text{and} \quad R_n = P_z + P_u . \tag{24}
\]
In general, the maximum possible SNR is equal to the maximum eigenvalue of \( R_n^{-1} R_s \), and is achieved by selecting \( b \) to be the corresponding eigenvector \([18]\) (see also \([6]\)). Alternative approaches to beamforming include (1) selecting \( b = a_\theta^\psi(\psi_0) + a_\phi^\psi(\psi_0) \), which accounts for mutual coupling but neglects spatial noise correlation; or (2) selecting the beamforming coefficients to compensate only for the geometrical

\[3\] Writing this as a discrete sum as opposed to an integral yields a general result while avoiding the complication of fractional calculus.

delays, which neglects both effects. Approach (1) is optimal when the noise associated with each antenna is uncorrelated, such that \( R_n \) has the form \( \sigma_n^2 \mathbf{I} \); i.e., some constant times the identity matrix. Then, we obtain the well-known result that the SNR improves linearly with \( N \). However, as will be demonstrated in later in this paper, this special case is not necessarily relevant to the problem of interest. This is primarily due to the impact of the external noise correlation, as represented by \( P_z \), for which off-diagonal terms can be significant.

For a signal of interest which is unpolarized – a useful and common assumption for the purpose of characterizing the sensitivity of a radio telescope – we have \( P_{\theta \phi} = P_{\phi \theta} = 0 \) and \( P_{\theta \theta} = P_{\phi \phi} = \eta S(\psi) / 2 \), where \( S(\psi) \) is the flux (distinct from the external noise considered above) associated with the signal of interest. In this case, we have
\[
R_s = S(\psi) \frac{\eta}{2} (A_{\theta \theta} + A_{\phi \phi}) . \tag{26}
\]
Continuing with this assumption, it is convenient to express the sensitivity of a radio telescope in terms of system equivalent flux density (SEFD), defined as the value of \( S(\psi) \) in Equation [26] required to double the total power observed at the beamformer output; i.e., \( \text{SNR} = 1 \). Thus,
\[
\text{SEFD} = \frac{2}{\eta} \frac{b^H (P_z + P_u) b}{b^H (A_{\theta \theta} + A_{\phi \phi}) b} . \tag{27}
\]
This expression is useful as it describes the sensitivity of a radio telescope array in terms of the array manifold, the contributions of internal and external noise, and the beamforming coefficients. The principal difficulty in using this expression is calculating the array manifold. This is considered next.

III. LWA-1 DESIGN AND ARRAY MANIFOLD

The original motivation behind the work presented in this paper was to characterize the performance of LWA-1. LWA-1 consists of \( N = 512 \) antennas arranged into 256 “stands”, with each stand consisting of two orthogonally-aligned bowtie-type dipoles over a wire mesh ground screen, as shown in Figure [1]. In Sections [III-A] and [III-B] we review the relevant details of the design of the LWA-1 array, our method for computing the array manifold and the internal and external noise covariance matrices, and present some results.

A. LWA Stand Design & Electromagnetic Model

As the dipoles and ground screen comprising each stand consist of interconnected metal segments, the array is well-suited to wire grid modeling using the Method of Moments (MoM). In this study, we employ the NEC-4.1 implementation of MoM [19].

The dimensions and parameters used to model the dipole are illustrated in Figure [2]. The wire grid models representing each of the two antennas in a stand are vertically separated by three times the wire radius to prevent the feeds from intersecting. The mean height of the highest points on each dipole (also the segment containing the feed) is 1.5 m above ground. It is known from both simulations and experiment that neither the center mast nor the structure supporting the dipole arms (see
Figure 1 have a significant effect on the relevant properties of the dipoles, and therefore no attempt is made to model them. It should be noted that the segmentation shown in Figure 2 is designed to be valid for the highest frequency of interest, so that the same model can be used for all frequencies of interest. In order to confirm that the results were not sensitive to the selected segmentation, several alternative schemes with small changes in the number of segments per wire were also considered. The results do not change significantly with these changes in segmentation.

The ground screen is modeled using a 3 m × 3 m wire grid with spacing 10 cm × 10 cm and wire radius of 1 mm, which is very close to the actual dimensions. The modeled ground screen is located 1 cm above ground to account for the significant but irregular gap that exists because of ground roughness. The ground itself is modeled as an infinite homogeneous half-space with relative permittivity of 3 and conductivity of 100 µS, which is appropriate for “very dry ground” [20] which predominates in New Mexico, where LWA-1 is located. (It should be noted that we suspect that the ground permittivity at the LWA-1 site is significantly higher; this is addressed below.) Each dipole is connected to an active balun which presents a balanced input impedance of \( R_L = 100 \, \Omega \). For additional information on this design, the reader is referred to [1] and the references therein.

It will be useful later in this paper to know the performance of a single stand, neglecting the rest of the array. We begin with the array manifold for the stand, which is determined as follows. The stand is illuminated with a \( \psi \)-polarized 1 V/m plane wave incident from some direction \( \psi \), and the resulting current \( I_L \) across the series resistance modeling each active balun is determined using MoM. Each element of the \( N = 2 \) array response vector \( a_0(\psi) \) is then simply \( I_L R_L \) for the associated antenna. The process is repeated for a \( \phi \)-polarized plane wave and iterated over \( \psi \).

The external noise covariance matrix \( P_z \) is computed using a model proposed in [5] which assumes that Galactic noise dominates over thermal noise from the ground and other natural or anthropogenic sources of noise. Specifically, \( T_e(\psi) \) is assumed to be uniform over the sky (\( \theta < \pi/2 \)), and zero for \( \theta > \pi/2 \). In practice, \( T_e(\psi) \) varies considerably both as a function of \( \psi \) and as a function of time of day, due to the rotation of the Earth. However, the above assumption provides a reasonable standard condition for comparing Galactic noise-dominated antenna systems, as explained in [5] and demonstrated in [21] and [22]. Using this model, \( T_e(\psi) \) toward the sky is found to be 50,444 K, 9751 K, and 1777 K at 20 MHz, 38 MHz, and 74 MHz, respectively. The actual contributions to the system temperature are less due to the mismatch between the antenna self-impedance and \( R_L \), but this is automatically taken into account as a consequence of our definition of the array manifold, which includes the loss due to impedance mismatch as well as ground loss. Under these assumptions, \( P_z \) is computed using Equation 22.

The internal noise covariance matrix \( P_u \) is computed assuming that the internal noise associated with any given antenna is not significantly correlated with the internal noise associated with any other antenna, so that \( P_u \) becomes a diagonal matrix whose non-zero elements are:

\[
P_u[n] = k T_{p,n} R_L
\]  

where \( T_{p,n} \) is the input-referred internal noise temperature associated with the \( n \)th antenna. We will further assume that all the electronics are identical such that \( T_{p,n} = T_p \), where \( T_p \) is assumed to be 250 K, the nominal value of the cascade noise temperature of all electronics attached to a dipole, referred to the dipole terminals.

The ratio \( Tr \{ P_z \} / Tr \{ P_u \} \) (where “\( Tr \)” denotes the trace operation; i.e., the sum of the diagonal elements) is the degree to which Galactic noise dominates over internal noise in the combined output, and is found to be \(-2.6 \, dB, +11.1 \, dB, \) and \(+4.1 \, dB\) at 20, 38, and 74 MHz, respectively. The 38 MHz and 74 MHz results are consistent with field measurements (see Figure 6 of [1]), however the same measurements suggest 20 MHz should also be Galactic noise-dominated. The apparent reason for the discrepancy is that the 20 MHz result is relatively sensitive to ground permittivity, both because the
loss associated with Earth ground increases with decreasing frequency, and also because the ground screen becomes tiny (only 0.2λ × 0.2λ) at 20 MHz. Larger assumed permittivity in our calculations results in Galactic noise-dominated performance at 20 MHz, even if the loss tangent is also increased. The effect of the change of ground parameters on the 38 MHz and 74 MHz results is very small in comparison. We shall continue to use the original ground parameters in this paper as they can be considered to be safely conservative.

Using the array manifold and the noise covariance matrices calculated as described above, the resulting SEFD for a single stand (and neglecting the rest of the array) can be computed from Equation 27. The result for the φ = 0 plane is shown in Figure 3. Note Figure 3 is also essentially a pattern measurement; as such the expected “cos θ”-type behavior is evident; in particular, the response is seen to go to zero at the horizon, as expected. Note that the performance at 38 MHz and 74 MHz is similar despite the large difference in frequency; this is because both the Galactic noise and the effective aperture of the antennas decrease with frequency at approximately the same rate [1]. The calculated 20 MHz performance is somewhat worse for the reasons described in the previous paragraph.

The stand performance can also be described in the traditional way, in terms of gain, through the effective aperture \( A_e \). Let the power delivered to the load (\( R_L \)) be \( P_L \). Note \( P_L = S(\psi)A_e(\psi) \) (assuming a co-polarized incident field), and also \( P_L = |I_L|^2 R_L \). Since \( S(\psi) = |E'(\psi)|^2/\eta \), where \( E'(\psi) \) is the co-polarized incident electric field, we have that the effective aperture for any given antenna attached to a load \( R_L \) is

\[
A_e(\psi) = \eta |I_L|^2 |E'(\psi)|^2 R_L. \tag{29}
\]

Assuming that \( I_L \) is computed using the MoM model described above, this definition includes impedance mismatch as well as loss due to the conductivity of the ground.\(^5\)

Using Equation 29, the zenith value of \( A_e \) is estimated to be 0.25 m², 8.72 m², and 2.48 m² for 20 MHz, 38 MHz, and 74 MHz, respectively for each dipole in the single-stand system described in this section. It should be noted, however, that these values cannot be used directly to calculate a \( A_e/T_{sys} \) type sensitivity metric, since \( T_{sys} \) in this case would be \( T_c \), reduced by the impedance mismatch, plus \( T_p \); and the mismatch efficiency is not available as part of this analysis. This underscores the usefulness of SEFD as a sensitivity metric for this class of systems, in contrast to \( A_e \) (or antenna gain) or \( A_e/T_{sys} \).

\( ^5 \)These factors can be computed independently and removed, if desired; see [5] and [21].
Fig. 4. Arrangement of stands in the LWA-1 array. The minimum distance between any two masts is 5 m (0.33λ, 0.63λ, and 1.23λ at 20 MHz, 38 MHz, and 74 MHz, respectively). All dipoles are aligned North-South and East-West; φ = 0 is East. For additional information about the array geometry, see [1].

5 mm to compensate for the increased grid spacing while keeping the wire cross-section well clear of the Earth ground. The required number of surrogate ground screens was determined using an experiment in which the computed characteristics of the stand of interest were observed as the number of surrogate ground screens used in surrounding stands was increased, starting with the closest stand and working outward. It was found that ground screens within about 1.5λ were often important, whereas ground screens for stands further away had negligible effect. To be conservative, 19 surrogate ground screens were used for the 38 MHz and 74 MHz results, whereas 108 surrogate ground screens were used for the 20 MHz results; in each case this yields a MoM model with slightly fewer than the “maximum manageable” number of segments (11,000) identified above.

To further validate the array model and computation, results were computed for a subset of the dipoles in “scaled up” versions of the array which were identical in all respects except that the inter-stand spacings were increased. It was confirmed that the results for any given dipole converge to the single-stand results (shown in the previous section) with sufficiently large inter-stand spacing.

MoM analysis reveals that the behavior of stands in the array is considerably different from stands in isolation. This is demonstrated in Figures 5-7, which show the patterns of all 256 North-South aligned antennas in the φ = 0 plane at frequencies of 20 MHz, 38 MHz, and 74 MHz, respectively. It is clear that the combination of non-uniform spacings and mutual coupling leads to disorderly embedded patterns. At 20 MHz and 38 MHz, the pattern tends to increase slightly toward the zenith, and decrease slightly more toward the horizon. At 74 MHz this trend is not as pronounced, but the pattern tends to be greater for 20° ≤ θ ≤ 60°.
Galactic noise correlation “turned off”. Mutual coupling is, in this different (not consistently better or worse) for pattern multiplication; thus correlation of external noise at all frequencies for optimum beamforming still provides a benefit of about 1 dB multiplication results. Also interesting is the finding that of simple beamforming should be so close to the pattern sensitivity. This yields a result which is relatively close to that predicted the external noise received by different antennas to be zero. This figure shows a recalculation of the Figure 9 result with is Galactic noise correlation, as demonstrated in Figure 10. However, this is not the case, as is shown in Figure 10. The SEFD is greater (i.e., worse) than the result predicted by pattern multiplication by about 1–6 dB (varying in Figure 9. The result from Figure 8 divided by the result expected from pattern multiplication (i.e., the SEFD from Figure 3 divided by 256). sense, beneficial; although optimum beamforming coefficients are required to realize the benefit. Further insight can be gained from Figures 11–13 which show that Galactic noise correlation is quite large for closely-spaced stands, and in many cases is large even for antennas on opposite sides of the array. Thus, it is not surprising that sensitivity tends to be degraded relative to a similar calculation in which external noise correlation is assumed to be zero. It is interesting to note that the correlation exhibits a Bessel function-like trend as a function of separation in wavelengths. However, it should be emphasized that this result assumes uniform sky brightness, and (as pointed out earlier) the actual situation is somewhat different. Non-uniform sky brightness will introduce structure in the external noise covariance matrix \( \mathbf{R}_z \) that is likely to cause corresponding \( \phi \)-dependent variations in SEFD.

V. CONCLUSIONS

This paper has considered the sensitivity of large arrays of low-gain antenna elements at low frequencies for which Galactic noise can be an important or dominant part of the system temperature. General expressions were developed for
SNR (Equation 23) and SEFD (Equation 27) for beamforming in terms of the array manifold and internal and external covariance matrices. Some results are shown using LWA-1 at 20 MHz, 38 MHz, and 74 MHz as an application example. It is shown that for beams pointing more than 10°–20° away from the zenith, the combination of mutual coupling and correlation of Galactic noise between antennas results in sensitivity which is significantly worse than predicted by pattern multiplication beginning with single antennas in isolation. Closer to the zenith, the result is frequency-dependent, and can be better or worse than the result predicted by pattern multiplication.

It is also shown that improvement of 1–2 dB is possible by using beamforming coefficients specifically designed to maximize SNR, as opposed to coefficients derived solely from geometrical phase and which therefore neglect external noise correlation as well as mutual coupling.

The ultimate intended use of LWA-1 is not solely as a stand-alone instrument, but rather as one of 53 identical “stations” distributed over the State of New Mexico which are combined to form images using aperture synthesis techniques [1]. Because the minimum separation between stations will be on the order of kilometers, the effects of mutual coupling and spatial correlation of Galactic noise will be negligible in the process of combining station beams into an image. Thus, the SEFD for imaging will be better by a factor of $\sqrt{N_S(N_S - 1)}$ than the SEFD for the station beam, where $N_S$ is the number of stations. Adopting a value of 3200 Jy for the typical zenith-pointing SEFD from Figure 8, the SEFD for imaging near the zenith with $N_S = 53$ is expected to be about 61 Jy. The resulting near-zenith image sensitivity, assuming 1 h integration, 8 MHz bandwidth, and SNR = 5, is about 2 mJy. This is consistent with the result derived in [1], which neglected Galactic noise correlation. However results for imaging at larger zenith angles will not be consistent with [1], for the reasons discussed above. Better estimates for pointing directions in the $\phi = 0$ plane can be obtained starting with Figure 8.

Finally, it should be noted that the theory and techniques described in Section II are generally applicable; even to arrays employing regular spacings, with or without mutual coupling, and dominated or not by external noise. Other findings in this paper may be specifically relevant for arrays used in other radio science applications, including HF/VHF direction finding arrays, radar arrays for measuring the atmosphere or ionosphere, and riometers.

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