Towards the Light Front Variables for High Energy Production Processes.

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ABSTRACT

Scale invariant presentation of inclusive spectra in terms of light front variables is proposed. The variables introduced go over to the well-known scaling variables $x_F = 2p_z/\sqrt{s}$ and $x_T = 2p_T/\sqrt{s}$ in the high $p_z$ and high $p_T$ limits respectively.

Some surface is found in the phase space of produced $\pi^\pm$-mesons in the inclusive reaction $\bar{p}p \rightarrow \pi^\pm X$ at 22.4 GeV/c, which separates two groups of particles with significantly different characteristics. In one of these regions a naive statistical model seems to be in a good agreement with data, whereas it fails in the second region.

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Key words: Light front, inclusive, hadron-hadron, electron-positron, relativistic heavy ions, deep inelastic.
The study of single particle inclusive processes \[1\] remains one of the simplest and effective tools for the investigation of multiple production of secondaries at high energies. The consequences of the limiting fragmentation hypothesis \[2\] and those of the parton model \[3\] and the principle of automodelity for strong interactions \[4\] have been formulated in this way.

An important role in establishing of many properties of multiple production is played by the choice of kinematic variables in terms of which observable quantities are presented (see in this connection, e.g. \[5, 6, 7\]). The variables which are commonly used are the following: the Feynman \(x_F = 2p_z/\sqrt{s}\), rapidity \(y = \frac{1}{2} \ln\left[(E + p_z)/(E - p_z)\right]\), transverse scaling variable \(x_T = 2p_T/\sqrt{s}\) etc. In the case of azimuthal symmetry the surfaces of constant \(x_F\) are the planes \(p_z = x_F\sqrt{s}/2\), surfaces of constant \(y\) are the hyperboloids

\[
p_z^2 \left(\frac{1 + e^{2y}}{1 - e^{2y}}\right)^2 - p_T^2 = m^2
\]

and the surfaces of constant \(x_T\) are the straight lines \(p_T = x_T\sqrt{s}/2\) in the phase space.

Here we propose a unified scale invariant variable for the presentation of single particle inclusive distributions, the properties of which are described below.

Consider an arbitrary 4–momentum \(p_\mu(p_0, \vec{p})\) and introduce the light front combinations \[8\]:

\[
p_\pm = p_0 \pm p_3
\]

If the 4–momentum \(p_\mu\) is on the mass shell \((p^2 = m^2)\), the combinations \(p_\pm, \vec{p}_T\) (where \(\vec{p}_T = (p_1, p_2)\)) define the so called horospherical coordinate system (see, e.g. \[9, 10\]) on the corresponding mass shell hyperboloid \(p_0^2 - \vec{p}^2 = m^2\). Corresponding hyperboloid in the velocity space is the realization of the curved space with constant negative curvature, i.e. the Lobachevsky space.

Let us construct the scale invariant variables:

\[
\xi^\pm = \pm \frac{p^\mu_\pm}{p^\mu_\pm + p^\mu_\pm}
\]

in terms of the 4–momenta \(p^\mu_a, p^\mu_b, p^\mu_c\) of particles \(a, b, c\), entering the inclusive reaction \(a + b \to c + X\). The \(z\)-axis is taken to be the collision axis, i.e. \(p_z = p_3 = p_L\). Particles \(a\) and \(b\) can be hadrons, heavy ions, leptons. Note that the use of similar variables turned out to be successful in theoretical studies of relativistic composite systems (see, e.g. \[11–25\]), in the theoretical and experimental studies of nuclear reactions with beams of relativistic nuclei (see, e.g. \[22, 26, 27\]) and in the study of quark confinement in QCD (see, e.g. \[28\]). Combinations like Eq.(\[8\]) appear also when considering the scale transformations \[29\] in the theory with fundamental length (see, e.g. \[30\]).

Invariant differential cross section in terms of \((\xi^\pm, \vec{p}_T)\) - variables looks as follows (assuming the azimuthal symmetry):

\[
E^c \frac{d\sigma}{dp^c} = \frac{1}{\pi} \frac{d\sigma}{d\xi^+ dp_T^+}
\]
It is interesting to note the properties of $\xi^\pm$ - variables in some limiting cases. Let us choose the centre of mass frame, where:

$$
\xi^\pm = \pm \frac{E^c \pm p^c_z}{\sqrt{s}} = \pm \frac{E^c + |p^c_z|}{\sqrt{s}}; \quad (4)
$$

$$
E^c = \sqrt{p^c_{T}^2 + p^c_{T}^2 + m^2};
$$

The upper sign in Eq.\((4)\) is used for the right hand side hemisphere and the lower sign for the left hand side hemisphere in the centre of mass frame.

Consider two limiting cases:

1) $|p^c_z| \gg p^c_T$ - fragmentation region, according to the common terminology.

In this case:

$$
\xi^\pm \rightarrow \frac{2p^c_z}{\sqrt{s}} = x_F \quad (5)
$$

2) $p^c_T \gg |p^c_z|$ - high $p_T$-region.

In this case:

$$
\xi^\pm \rightarrow \frac{m^c_T}{\sqrt{s}} \rightarrow \frac{p^c_T}{\sqrt{s}} = \frac{x_T}{2}; \quad m^c_T = \sqrt{p^c_{T}^2 + m^2} \quad (6)
$$

Thus, in these two limiting regions of phase space $\xi^\pm$-variables go over to the well known variables $x_F$ and $x_T$, which are intensively used in high energy physics. $\xi^\pm$-variables are related to $x_F$, $x_T$ and $y$ as follows:

$$
\xi^\pm = \frac{1}{2} \left( x_F \pm \sqrt{x_F^2 + x^2_\perp} \right); \quad x_\perp = \frac{2m^c_T}{\sqrt{s}} \quad (7)
$$

$$
y = \pm \frac{1}{2} \ln \left( \frac{(\xi^\pm \sqrt{s})^2}{m^2_T} \right) \quad (8)
$$

The region $|\xi^\pm| < m^c/\sqrt{s}$ is kinematically forbidden for the $\xi^\pm$-spectra integrated over all values of $p_T^2$, and the region $|\xi^\pm| < m^c_T/\sqrt{s}$ is forbidden for the $\xi^\pm$-spectra at fixed values of $p_T^2$.

In the present paper we study the inclusive reaction $\bar{p}p \rightarrow \pi^\pm X$ at 22.4 GeV/c of the incident momentum. The details of the experiment can be found in [31]. In this case it is sufficient to study the right hand side hemisphere only, due to the CP–symmetry of the reaction.

In Fig. 1a the $\xi^+$-distribution of $\pi^+$-mesons is shown.

$\xi^+$-distribution has two features, which makes it differ from the corresponding $x_F$-distribution:

1) existence of the forbidden region near the point $\xi^+ = 0$ (cross section vanishes in the region $|\xi^+| < m_\pi/\sqrt{s}$,

2) existence of maximum at some $\bar{\xi}^+$ in the region of relatively small $\xi^+$.

It is convenient to introduce the variable

$$
\zeta^+ = -\ln\xi^+ \quad (9)
$$
in order to enlarge the scale in the region of small $\xi^+$. The maximum at $\tilde{\xi}^+$ is also observed in the invariant differential cross section $\frac{1}{\pi} \frac{d\sigma}{d\zeta^+}$. However, the region $\xi^+ > \tilde{\xi}^+$ goes over to the region $\zeta^+ < \tilde{\zeta}^+$ and vice versa (see Fig. 1b).

In order to study the nature of this maximum we have investigated the angular and $p_T^2$–distributions of $\pi^\pm$–mesons in the regions $\xi^+ < \tilde{\xi}^+(\zeta^+ > \tilde{\zeta}^+)$ and $\xi^+ > \tilde{\xi}^+(\zeta^+ < \tilde{\zeta}^+)$ separately. The results are presented in Figs. 2a and 2b. The angular distribution of particles with $\xi^+ > \tilde{\xi}^+(\zeta^+ < \tilde{\zeta}^+)$ is sharply anisotropic in contrast to the almost flat distribution of particles with $\xi^+ < \tilde{\xi}^+(\zeta^+ > \tilde{\zeta}^+)$. The slopes of $p_T^2$–distributions differ substantially.

Note, that the surfaces of constant $\xi^+$ are the paraboloids

$$p_T^2 = \frac{p_T^2 + m^2 - (\xi^+ \sqrt{s})^2}{-2\xi^+ \sqrt{s}}$$

in the phase space. Thus the paraboloid

$$p_T^2 = \frac{p_T^2 + m^2 - (\tilde{\xi}^+ \sqrt{s})^2}{-2\tilde{\xi}^+ \sqrt{s}}$$

(11)

separates two groups of particles with significantly different characteristics.

It seems to be interesting to use $\xi^\pm$ and $\zeta^\pm$ variables in deep inelastic electro - and weak production processes, in $e^+e^-$ - annihilation and in relativistic heavy ion collisions (see in this connection recent reviews [32–38] and references therein) and to perform also event by event analysis.

To describe the spectra in the region $\xi^+ < \tilde{\xi}^+(\zeta^+ > \tilde{\zeta}^+)$ the simplest statistical model (see, e.g. [39]) with the Boltzman $f(E) \sim e^{-E/T}$ and the Bose-Einstein $f(E) \sim (e^{E/T} - 1)^{-1}$ distributions has been used.

The distributions $\frac{1}{\pi} \frac{d\sigma}{d\zeta^+}$, $\frac{d\sigma}{dp_T^2}$ and $\frac{d\sigma}{d\cos \theta}$ look in this region as follows :

$$\frac{1}{\pi} \frac{d\sigma}{d\zeta^+} \sim \int_0^{p_{T,\text{max}}^2} Ef(E)dp_T^2,$$

$$\frac{d\sigma}{dp_T^2} \sim \int_0^{p_{z,\text{max}}^2} f(E)dp_z,$$

$$\frac{d\sigma}{d\cos \theta} \sim \int_0^{p_{\text{max}}^2} f(E)p^2dp,$$

$$E = \sqrt{\tilde{p}^2 + m_\pi^2}, \quad \tilde{p}^2 = p_z^2 + p_T^2$$

(15)

where:

$$p_{T,\text{max}}^2 = (\xi^+ \sqrt{s})^2 - m_\pi^2$$

$$p_{z,\text{max}}^2 = \frac{p_T^2 + m_\pi^2 - (\xi^+ \sqrt{s})^2}{-2\xi^+ \sqrt{s}}$$

$$p_{\text{max}}^2 = \frac{-\tilde{\xi}^+ \sqrt{s} \cos \theta + \sqrt{(\xi^+ \sqrt{s})^2 - m_\pi^2 \sin^2 \theta}}{\sin^2 \theta}$$

(18)
The experimental distributions \( \frac{1}{\pi} \frac{d\sigma}{d\xi^+} , \frac{d\sigma}{dp_T^2} \) and \( \frac{d\sigma}{d\cos\theta} \) in the region \( \xi^+ < \tilde{\xi}^+(\xi^+ > \tilde{\xi}^+) \) have been fitted by Eqs. (12), (13) and (14), respectively. The results of the fit given in Table 1 and Figs. 1b, 2a, 2b show satisfactory agreement with experiment.

In the region \( \xi^+ > \tilde{\xi}^+(\xi^+ < \tilde{\xi}^+) \) \( \zeta^+ \)-distribution has been fitted by the formula:

\[
\frac{1}{\pi} \frac{d\sigma}{d\xi^+} \sim (1 - (1 - \xi^+)^n = (1 - e^{-|\zeta^+|})^n \tag{19}
\]

and the \( p_T^2 \)-distribution by the formula:

\[
\frac{d\sigma}{dp_T^2} \sim \alpha e^{-\beta p_T^2} + (1 - \alpha) e^{-\beta p_T^2} \tag{20}
\]

Note that in the region \( \xi^+ \to 1 \) the parameterization (19) goes over to the well-known quark-parton model parameterization \((1 - x)^n\) with \( x = x_F = 2 p_z/\sqrt{s} \). The results of the fit are given in Table 2 and Figs. 1b and 2b. Since the dependence \((1 - x)^n\) which is derived for \( x \to 1 \) describes the data even in the region \( x \to 0 \) (where, in general, it must not work), but the dependence (19) deviates from the data in the region of small \( \xi^+ \), it seems that the analysis of data in terms of \( \xi^+ \) and \( \zeta^+ \)-distributions is more sensitive to the multi-component models of multi-body production at high energies.

Thus the spectra of \( \pi^\pm \)-mesons in the region \( \xi^+ < \tilde{\xi}^+(\xi^+ > \tilde{\xi}^+) \) are satisfactorily described by the formulae which follow from the statistical model. The same formulae when extrapolated to the region \( \xi^+ > \tilde{\xi}^+(\xi^+ < \tilde{\xi}^+) \) deviate from the data. On the other hand, the dependence \((1 - \xi^+)^n\) is in a good agreement with data in the region \( \xi^+ > \xi^+(\xi^+ < \tilde{\xi}^+) \) and deviates from them in the region \( \xi^+ < \tilde{\xi}^+(\xi^+ > \tilde{\xi}^+) \) (see Fig. 1b).

It is interesting to recall the similar situation in the study of black body radiation, where the Wien formula describes the low frequency part of the spectrum and does not describe the high frequency part, whereas the situation is reversed in the case of Rayleigh-Jeans formula (see, e.g., [40]). To illustrate this in Fig. 3 the black body radiation intensity according to the Wien, Rayleigh-Jeans and Planck formulae are plotted against the dimensionless variable \( x = h\omega/kT \).

In conclusion, we feel that the use of the variables \( \xi^\pm \) and \( \zeta^\pm \) can help to distinguish in between different dynamical contributions, or test basic principles in other types of analysis, such as two-particle correlations, HBT – interferometry [41,42,43] and transverse flow studies [44].

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Table 1: Results of the fits of $\frac{1}{\pi} \frac{d\sigma}{d\xi^+}$, $\frac{d\sigma}{d\cos\theta}$ and $\frac{d\sigma}{dp_T^2}$-distributions in the region $\xi^+ < \tilde{\xi}^+ (\xi^+ > \tilde{\xi}^+)$. 

| $T$, GeV | Bose-Einstein | Boltzman | Bose-Einstein | Boltzman | $\chi^2/N_{D.F.}$ |
|-----------|---------------|----------|---------------|----------|------------------|
| $\frac{1}{\pi} \frac{d\sigma}{d\xi^+}$ | 0.134 ± 0.004 | 0.119 ± 0.003 | 10/8 | 12/8 |
| $\frac{d\sigma}{d\cos\theta}$ | 0.091 ± 0.003 | 0.086 ± 0.003 | 16/7 | 15/7 |
| $\frac{d\sigma}{dp_T^2}$ | 0.110 ± 0.001 | 0.105 ± 0.001 | 10/8 | 8/8 |
Table 2: Results of the fits of $\frac{d\sigma}{d\zeta^+}$ and $\frac{1}{\pi} \frac{d\sigma}{dp_T^2}$ distributions in the region $\xi^+ > \tilde{\xi}^+(\zeta^+ < \tilde{\xi}^+)$.

| $\frac{1}{\pi} \frac{d\sigma}{d\zeta^+}$ | $\alpha$ | $\beta_1$ (GeV/c)$^{-2}$ | $\beta_2$ (GeV/c)$^{-2}$ | $n$ | $\chi^2/N_{D.F.}$ |
|---------------------------------|--------|-----------------|-----------------|-----|----------------|
|                                 | 0.8 ± 0.03 | 6.0 ± 0.1       | 2.8 ± 0.3       | 45  | 29             |

Distributions in the region $\xi^+ > \tilde{\xi}^+(\zeta^+ < \tilde{\xi}^+)$.
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Figure Captions

Fig. 1: \( \frac{\xi^+}{\pi} \frac{d\sigma}{d\xi^+} \) – distribution of \( \pi^\pm \) mesons in the reaction \( \bar{p}p \rightarrow \pi^\pm X \) at 22.4 GeV/c (1a). \( \frac{1}{\pi} \frac{d\sigma}{d\zeta^+} \) – distribution of \( \pi^\pm \) mesons in the reaction \( \bar{p}p \rightarrow \pi^\pm X \) at 22.4 GeV/c (1b), fit of the data in the region \( \xi^+ < \tilde{\xi}^+ (\zeta^+ > \tilde{\zeta}^+) \) by the Bose-Einstein distribution, fit of the data in the region \( \xi^+ < \tilde{\xi}^+ (\zeta^+ > \tilde{\zeta}^+) \) by the Boltzmann distribution, fit of the data in the region \( \xi^+ > \tilde{\xi}^+ (\zeta^+ < \tilde{\zeta}^+) \) by the formula \((1 - \xi^+)^n\).

Fig. 2: Angular distribution of \( \pi^\pm \) - mesons in the reaction \( \bar{p}p \rightarrow \pi^\pm X \) at 22.4 GeV/c (2a), fit of the data in the region \( \xi^+ < \tilde{\xi}^+ (\zeta^+ > \tilde{\zeta}^+) \) by the Bose-Einstein distribution, fit of the data in the region \( \xi^+ < \tilde{\xi}^+ (\zeta^+ > \tilde{\zeta}^+) \) by the Boltzmann distribution. \( p_T^2 \) - distribution of \( \pi^\pm \) mesons in the reaction \( \bar{p}p \rightarrow \pi^\pm X \) at 22.4 GeV/c (2b), fit of the data in the region \( \xi^+ < \tilde{\xi}^+ (\zeta^+ > \tilde{\zeta}^+) \) by the Bose-Einstein distribution, fit of the data in the region \( \xi^+ < \tilde{\xi}^+ (\zeta^+ > \tilde{\zeta}^+) \) by the Boltzmann distribution, fit of the data in the region \( \xi^+ > \tilde{\xi}^+ (\zeta^+ < \tilde{\zeta}^+) \) by the formula (20).

Fig. 3: Black body radiation intensity as a function of dimensionless variable \( x = \hbar \omega / kT \), \( dE_\omega \sim x^2 dx \) (Wien), \( dE_\omega \sim x^3 e^{-x} \) (Rayleigh–Jeans), \( dE_\omega \sim x^3 (e^x - 1)^{-1} \) (Planck).