High-energy recollision processes of laser-generated electron-positron pairs

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(Dated: July 2, 2014)

Two oppositely-charged particles created within a microscopic space-time region can be separated, accelerated and brought to a recollision by a laser field. Consequently, new reactions become feasible, where the energy absorbed by the particles is efficiently released. By investigating the laser-dressed polarization operator, we identify a new contribution describing high-energy recollisions experienced by an electron-positron pair generated when a gamma photon impinges on an intense, linearly-polarized laser pulse. The energy absorbed in the recollision process corresponds to a large number of laser photons and can be exploited to prime high-energy reactions. As an example, we show that the inclusion of the recollision process substantially alters – both qualitatively and quantitatively – the tree-level muon–anti-muon photoproduction probability. In particular, the absence of an exponential suppression factor in the recollision contribution results in an enhancement of the production probability by several orders of magnitude with next-generation laser parameters.

Recollision processes are responsible for a variety of phenomena, which have been investigated especially in the realm of atomic and molecular physics. After an atom (or a molecule) is ionized by a laser field, the electron is accelerated and possibly brought to a recollision with the parent ion. The energy that the electron absorbs between the ionization and the recollision can be released in different ways: as a high-energy photon after recombination [high-harmonic generation (HHG)] or by striking out another electron (non-sequential double ionization), see, e.g.,\textsuperscript{[1–5]}. The maximal energy absorbed by the recolliding electron in a laser field with peak electric field strength $E_0$ and mean angular frequency $\omega$ is found to be about $3.17 U_p$, where $U_p = e^2 E_0^2/(4m\omega^2)$ is the ponderomotive potential, with $e < 0$ and $m$ being the electron charge and mass, respectively\textsuperscript{[6,9]}.

Recollision processes also play an important role in high-energy physics as originating, for example, from an electron and a positron initially bound in a positronium atom, which may annihilate during a recollision and create other particles, analogously as in an ordinary collider experiment\textsuperscript{[10–12]}. Similarly, high-energy processes can be primed following a recollision between an electron and a positron created in the presence of a laser field and a nucleus\textsuperscript{[13]}. In both mentioned cases, classical considerations show that the available energy in the recollision is of the order of $mc^2\xi^2 = 4U_p$, where $\xi = |e|E_0/(mc\omega)$, which explains why the ultra-relativistic regime $\xi \gg 1$ is of relevance in high-energy recollision physics.

In the present Letter we provide a unified, full quantum description of the high-energy recollision experienced by an electron-positron pair created in the interaction of a gamma photon with a counterpropagating intense, linearly polarized plane-wave laser pulse. By working within the Furry picture\textsuperscript{[14]} and focussing on the ultra-relativistic regime, we show analytically that the recollision is described by terms in the polarization operator subleading in the parameter $1/\xi \ll 1$ with respect to the leading ones corresponding to the quasistatic limit $\xi \rightarrow \infty$. The latter terms only depend on the quantum nonlinearity parameter $\chi = (2\hbar\omega_\gamma/mc^2)(E_0/E_{cr})$, where $\hbar\gamma$ is the gamma photon energy and $E_{cr} = mc^2\chi^2/(|e|\hbar) = 1.3 \times 10^{16} V/cm$ the critical field of QED\textsuperscript{[5]}. The main contribution to those terms arises from the pair annihilation after a relatively short distance $\sim \lambda/\xi$, with $\lambda = 2\pi c/\omega$ being the laser wavelength. Thus, they describe well-known effects like vacuum birefringence and dicroism\textsuperscript{[5,15–18]} and, due to the corresponding short formation length, allow only for the net exchange of a few number of laser photons\textsuperscript{[19]}. On the other hand, the main contribution to the recollision terms arises from the pair propagating inside the laser field along a distance of the order of $\lambda$ before annihilating. Thus, a large number of photons (up to about $3.17 \xi^3/\chi$) can be efficiently absorbed from the laser field, corresponding to MeV-energies for an optical ($\hbar\omega \sim 1$ eV) laser with a feasible intensity of $10^{22} W/cm^2$ ($\xi \sim 100$)\textsuperscript{[20]} and for $\hbar\omega_\gamma \sim 1$ GeV ($\chi \sim 1$). Here and below, it is assumed that $\chi \sim 1$, such that electron-positron pair production...
is sizable [21,47].

Moreover, our analysis indicates that, in general, the inclusion of the recollision contributions may significantly alter the predictions of tree-level strong-field QED (see also [48]). This is in contrast to QED in vacuum, where the effect of loop diagrams is always small (at reasonable interaction energies and after renormalization is carried out) [49]. As an example, we show below that the photoproduction of a muon–anti-muon pair in a laser field is substantially enhanced by taking recollisions into account (see Fig. 1), as the corresponding contribution does not feature the strong exponential suppression typical of the “direct” process, depending on the parameter \( \chi_\mu = \chi m^3 / m^2_\mu \sim 10^{-7} \) (\( m_\mu \) denotes the muon mass) [21,28].

The calculation of the total muon pair-creation probability \( W(k_\gamma) \) is similar as for the case of electron-positron photoproduction discussed in [47]. For the sake of estimation, we neglect the influence of the laser field on the muon pair and obtain (as in [47], units with \( \hbar = c = 1 \) and \( \alpha = e^2 / 4 \pi \approx 1 / 137 \) are used throughout):

\[
W(k_\gamma) = - \int_{n_0}^{\infty} \frac{dn}{n} F_{n}(k_\gamma, q) \left[ e^\mu \Pi_{\alpha \beta}(k_\gamma, q) \right]^* \times \frac{\alpha}{2(2\pi)^2} \sqrt{1 - 4m^2_\mu / q^2} \left( 1 + 2m^2_\mu / q^2 \right),
\]

where \( k^\mu = (\omega, k) \) denotes the average four-momentum of the laser photons, \( k^\mu_\gamma = (\omega_\gamma, k_\gamma) \) \( (k_\gamma^2 = 0) \) and \( e^\mu \) the momentum and polarization four-vector of the incoming gamma photon, respectively, \( \Pi_{\alpha \beta}(k_\gamma, q) \) the non-singular part of the polarization operator (after renormalization of the vacuum-part) [47,50,55] and \( q^\mu = k^\mu_\gamma + n k^\mu_\gamma \) the four-momentum of the intermediate virtual photon (see Fig. 1). The center-of-mass energy of the laser-induced electron-positron collision is given by \( \sqrt{q^2} = \sqrt{2nkk_\gamma} \). Accordingly, we need to absorb at least \( n_0 = 2m^2_\mu / (kk_\gamma) \) laser photons to create a muon–anti-muon pair.

As recollisions are only possible if the plane wave is linearly polarized, we assume that the corresponding field-strength tensor integrated with respect to the phase \( \phi \)

\[
\mathbf{F}(\phi,\phi_0) = \int_{\phi_0}^{\phi} d\phi' F^{\mu\nu}(\phi')
\]

is given by \( \mathbf{F}^{\mu\nu}(\phi,\phi_0) = f^{\mu\nu}[\psi(\phi) - \psi(\phi_0)], f^{\mu\nu} = k^\rho a^\nu - k^\sigma a^\nu (\xi = |e| \sqrt{-a^2 / m}, \chi = \xi kk_\gamma / m^2) \). In the numerical calculations and in the figures we have used the pulse shape \( \psi(\phi) = \sin^2(\phi / (2N)) \sin(\phi) \) with \( N = 5 \) cycles.

For linear polarization the field-dependent part of the polarization operator, which describes the recollision process, can be expressed as the following double-integral

\[
\Pi^{\mu\nu}(k_\gamma, q) - \Pi^{\mu\nu}_{\chi m}(k_\gamma, q) = - \int_{-\infty}^{\infty} dk y \int_0^{\infty} \frac{d\theta}{\theta} \times \frac{\alpha}{2\pi} \left[ P_{11} L_1^\mu L_1^\nu + P_{22} L_2^\mu L_2^\nu + P_0 Q_1^\mu Q_2^\nu \right],
\]

with respect to the laser phase \( k y \) when the pair annihilates and to \( q = (k y - k x) / 2 \), with \( k x \) being the laser phase when the pair is created (see Fig. 1) [47]. In the considered case of an incoming photon counterpropagating with respect to the laser field, the four-vectors \( \Lambda_1^\mu \) and \( \Lambda_2^\mu \) can be chosen as \( e^\mu \) and \( e^\nu_\perp \), respectively (the indexes \( \parallel \) and \( \perp \) refer to the polarization of the laser), whereas the last term on the right-hand side of Eq. (3) does not contribute. Eq. (1) indicates that the quantity \( \left[ \int P_{\perp,\parallel} \right]^2 \) determines the recollision probability for the corresponding photon polarization. In this respect, the complete expression of the coefficients \( P_{11} = P_{\parallel} \) and \( P_{22} = P_{\perp} \) is not necessary here (see [47] for details). It is sufficient to note that they contain the oscillatory phase factor \( \exp[i \varphi(\phi, k y)] \), where

\[
\varphi(\phi, k y) = n k y - 4 (\xi / \chi) |\phi + \mathbf{F}(\phi, k y)|,
\]

with \( \mathbf{F}(\phi, k y) = q \xi^2 (J - I^2) \) and

\[
I = \int_0^{\infty} d\lambda \psi(k y - 2\phi), \quad J = \int_0^{\infty} d\lambda \psi^2(k y - 2\phi).
\]

We first investigate the integral in \( \phi \) for a fixed value of \( k y \). For \( \xi \gg 1 \) and at fixed \( \chi \) the phase factor \( \exp[-4i (\xi^2 / \chi) \varphi(J - I^2)] \) is highly oscillating and we can apply a stationary-phase analysis. The stationary points \( \phi_k \) are determined by the condition \( \mathbf{F}(\phi_k, k y) = \xi^2 |\psi(k y - 2\phi_k) - I(\phi_k, k y)|^2 = 0 \) (for functions with more than one argument the prime denotes the partial derivative with respect to \( \phi \)). This condition can also be written as

\[
(k y - k x) \psi(k x) = \int_{k x}^{k y} d\phi \psi(\phi).
\]
The combinations of $kx$ and $ky$ for which a recollision is possible are shown in Fig. 2. Eq. (6) has a straightforward classical interpretation. In fact, the four-momentum $P^μ(\phi)$ of an electron/positron inside a plane-wave field is classically given by \[ P^μ(\phi) = P^μ_{±,0} ± \frac{e^2}{\hbar c^4} \delta_μ^0(\phi, \phi_0) P^μ_{±,0} + \frac{e^2}{2\hbar c^4} \delta_μ^{2+}(\phi, \phi_0) P^μ_{±,0}, \] where $P^μ_{±,0} = P^μ_{±}(\phi_0)$. Since the impact parameter of the collision becomes large if the initial photon momentum $k_\gamma$ is split asymmetrically between the electron and the positron, we can assume that both particles are created at the same space-time point $x^μ$ (laser phase $\phi_0 = kx$) with the same initial momentum $P^μ_{±,0} = k\kappa_\gamma/2$ ($kP^μ_{±,0} \approx kk_\gamma/2$). Thus, by integrating Eq. (7), we find that Eq. (6) exactly corresponds to impose the condition that the coordinates of the electron and the positron again coincide at a later phase $\phi = ky$. Multiple solutions of Eq. (7) correspond to the fact that a single pair can recollide more than once.

The stationary-phase equation $\mathcal{F}'(\varrho_k, ky) = 0$ always admits the solution $\varrho_0 = 0$, independently of the shape of the background field. The contribution of this stationary point is formed for values of $\varrho$ in the region $0 \leq \varrho \lesssim 1/\xi$, where the phase $4(\xi^3/\chi) \varrho(J - I^2)$ is less than or of the order of unity. Thus, this contribution describes the immediate annihilation of the created electron-positron pair within a distance of the order of $\lambda/\xi$ inside an (effectively) constant-crossed field (quasistatic limit). The compensation of the large parameter $\xi^3/\chi$ in the phase occurring at $\rho \lesssim 1/\xi$ explains why the stationary point $\rho_0$ provides the leading contribution to the polarization operator and at the same time, why it allows for a net exchange of only a few laser photons [19]. On the other hand, laser-induced recollision processes are described by the contributions to the integral in $\varrho$ close to the nonvanishing stationary points $\varrho_k$, $k = 1, 2, \ldots$, with $\varrho_k \gtrsim \tau \gg 1/\xi$ (see Fig. 2). As we will see below, these contributions are formed in the regions $|\varrho - \varrho_k| \lesssim 1/\xi$, where the phase $4(\xi^3/\chi) \varrho(J - I^2)$ remains of the order of $\xi^3/\chi$. Thus, although such contributions are suppressed with respect to those from $\varrho_0$, they are essential to understand the high-energy plateau-region of the photon-absorption spectrum [see Fig. 3 (left side)], where the “quasistatic” contribution (yellow line) is compared with the full numerical calculation of $|\int P_\perp|^2$ (gray line).

In order to determine the contribution from the recollision processes, we expand the function $\mathcal{F}(\varrho, ky)$ around $\varrho_k$ up to the third order $[\mathcal{F}'''(\varrho_k, ky) = 0]$, with $\mathcal{F}'''(\varrho_k \neq 0, ky) = 8e^2 (|\varphi'(ky - 2\varrho_k)|^2)^2$. Since the third-order term of the expansion scales as $(\varrho - \varrho_k)^3/\chi$, the contribution is formed in the region $|\varrho - \varrho_k| \lesssim 1/\xi$. Within this formation region, the additional linear term in $\varphi(\varrho, ky)$ also changes significantly and must be taken into account exactly. All higher-order terms can be neglected. On the other hand, the pre-exponent of $P_{\perp,\parallel}$ vanish at $\varrho_k$ and it is necessary to expand them up to linear terms in $\varrho - \varrho_k$. Now, we apply the change of variable $\varrho - \varrho_k = r \chi/(4\xi) x$, where $r = [2/(\chi + \varrho_k, ky)]^{2/3}$, with $\chi(\varrho, ky) = \chi(|\varphi'(ky - 2\varrho_k)|)$ being the quantum-nonlinearity parameter at the pair-creation vertex. Then, the phase $\varphi(\varrho, ky)$ can be approximated by [see Eq. (4)]

$$\varphi(\varrho, ky) \approx \varphi(\varrho_k, ky) - (tr + t^3/3).$$

After extending the integration boundaries in the new variable $t \to \pm \infty$, the contribution to the integral from the region around the stationary point $\varrho_k \neq 0$ reads

$$\int_0^\infty \frac{d\varrho}{\varrho} \varrho \varrho_0 e^{i\varphi(\varrho, ky)} \approx e^{i\varphi(\varrho_k, ky)} \pi \chi/(2\xi) \times \left[ \varrho(\varrho_k) r \chi(r) i g(\varrho_k) \chi(r)^2 \chi/(4\xi) \right],$$

where $g(\varrho)$ is an arbitrary, slowly-varying function and $\chi$ is the Airy function [57]. As expected, at $\chi \ll 1$, the above contribution features an exponential suppression $\sim \exp[-4/(3\chi(\varrho_k, ky))]$, i.e. as the electron-positron pair-creation amplitude inside a (locally) constant-crossed field [5, 23].

By applying Eq. (9) to Eq. (3) we obtain

$$\int_0^\infty \frac{d\varrho}{\varrho} P_\perp \approx im^2 e^{i\varphi(\varrho_k, ky)} \pi \chi^2/2 \left\{ \varphi'(ky - 2\varrho_k) \right\}$$

$$\times M(\varrho_k, ky) W_0[\varrho_1(\varrho_k, ky)] r^2 A_i(r)/4\varrho_k$$

$$- W_2[\varrho_1(\varrho_k, ky)] r A_i(r)/(\varrho_k^2 \chi) \right\},$$

and

$$\int_0^\infty \frac{d\varrho}{\varrho} P_\parallel \approx -im^2 e^{i\varphi(\varrho_k, ky)} \psi'(ky - 2\varrho_k) M(\varrho_k, ky)$$

$$\times W_1[\varrho_1(\varrho_k, ky)] r^2 A_i(r) \pi \chi^2/2\varrho_k + \int_0^\infty \frac{d\varrho}{\varrho} P_\perp$$,

FIG. 3. (color online) Left side: Comparison of the quasistatic contribution (lower yellow curve) with the full numerical calculation (gray upper curve). Right side: Plateau-region, analytical (red curve) and numerical calculation coincide [$\chi = 1$, $\xi = 10$, $N = 5$, see Eq. (3)].
the cutoff is confirmed by a full numerical calculation (right side). For large photon numbers both results agree already for \( \xi = 10 \).

We note that whereas the quasistatic contribution is independent of \( \xi \), the recollision contribution to \( |f_{1\perp}|^2 \) scales as \( \chi^{10/3}/\xi^6 \) at \( \chi \gtrsim 1 \). This can be seen from Eq. (13) and Eq. (10) by using the asymptotic behavior \( \mathcal{W}_{i}(x) \sim x^{-1/2} \) \cite{47}. The \( \xi^{-6} \)-scaling is confirmed numerically in Fig. 4, which also shows that the probabilities are much smaller if the initial photon is polarized parallel to the electric field of the laser. In this case, the initial momentum of the photon is preferably split asymmetrically between the electron and the positron \cite{23}, hindering an efficient recollision.

Going back now to the example of muon–anti-muon pair production and employing the scaling \( n \sim \xi^4/\chi \); we obtain from \( m_0 = 2m_\mu/(kk_\perp) \) the threshold condition \( \xi \gtrsim m_\mu/m \approx 200 \) [see Eq. (1)]. This reflects the requirement that the center-of-mass energy of the collision must be sufficiently high to produce a muon pair [the transversal momentum of the intermediate electron (positron) scales as \( m_\xi \), see Eq. (7)]. In this regime, the cutoff corresponds to the absorption of a million of laser photons and lies in the MeV-range for an optical laser with \( \omega = 1.55 \text{ eV} \), to be compared with atomic HHG, where the cutoff is limited to the keV-domain \cite{5}. The regime \( \xi \gtrsim 200 \) will be accessible at upcoming laser facilities like ELI, CLF or XCELS \cite{59,61}.

By inserting the above analytical approximations for the recollision contribution to the polarization operator into Eq. (1), we finally obtain a muon–anti-muon production probability of \( 2 \times 10^{-20} \) for \( \chi = 1, \xi = 200 \) and perpendicular polarization [\( \wp = 0.4 \text{ GeV}, \omega = 1.55 \text{ eV} \)], laser peak intensity \( I_0 = 1.7 \times 10^{23} \text{ W/cm}^2 \). This is an enhancement of many orders of magnitude in comparison with the tree-level prediction \( \simeq \exp[-8/(3\chi_\mu)] \), with \( \chi_\mu \sim 10^{-7} \).

In conclusion, we have identified new contributions in the laser-dressed polarization operator, describing recollision processes of an electron-positron pair created when a gamma-photon collides with a strong, linearly polarized plane-wave field. Analogously to HHG in atomic physics, recollision processes allow for the absorption of many laser photons by the electron-positron pair and are responsible for a plateau-region in the photon spectrum. By means of a stationary-phase analysis we have determined analytically the scaling laws for the height \( \chi^{10/3}/\xi^6 \) and the extension \((3.17\xi^3/\chi)\) of the plateau, which are confirmed by a qualitative, semiclasy model. As a possible application, we have shown that recollision processes change the tree-level predictions for muon–anti-muon pair creation significantly.

ADP would like to thank Alexander I. Milstein and

\[ M(\varphi_k, k) = \psi(ky) - \psi(ky - 2\varphi). \]

The stationary-phase condition reads

\[ n = \frac{\xi^3}{\chi} \left[ 2M^2(\varphi_k, ky) + \frac{2}{\xi^2} M(\varphi_k, ky) \right]. \quad (11) \]

where \( \varphi_k = \varphi_k(ky) \) [note that for \( \varphi'(ky - 2\varphi) \rightarrow 0 \) the pair-creation probability is exponentially suppressed]. The leading-order contribution with \( n \approx 2M^2(\varphi_k, ky)\xi^3/\chi \) corresponds to the four-momentum

\[ k^\mu = nk^\mu = 2(\xi^3/\chi)[\psi(ky) - \psi(kx)]^2 k^\mu \quad (12) \]

that the electron-positron pair has classically absorbed from the laser field [see Eq. (7) and Fig. 2]. For a monochromatic field we obtain the cutoff \( n_c = 3.17\xi^3/\chi \), which corresponds to the result \( 3.17U_p \) obtained in atomic HHG \((U_p = m\xi^2/4)\) \cite{55,56}. The \( \xi^3/\chi \)-scaling of the cutoff is confirmed by a full numerical calculation in Fig. 4.

Now, the approximate contribution of the stationary point \( ky_s \), solution of Eq. (11), is given by

\[ \left. \int_{-\infty}^{+\infty} dky h(ky) e^{i\varphi(ky)} \approx h_s e^{i(\sigma/4 + \varphi_s)} \right| \frac{2\pi}{\varphi''_s}. \quad (13) \]

where \( h(ky) \) is an arbitrary, slowly-varying function, \( h_s = h(ky_s), \varphi_s = \varphi(ky_s), \varphi''_s = \varphi''(ky_s), \sigma = \text{sign}(\varphi''_s) \), and

\[ \varphi''(ky) = -\frac{4}{\xi^3} \left[ M(\varphi_k, ky)\psi'(ky) - \frac{M^2(\varphi_k, ky)}{2\varphi_k} + \frac{\varphi''_s}{\xi^3} \right]. \quad (14) \]

If two stationary points \( ky_s \) and \( ky_s' \) coalesce the Airy uniform approximation \cite{59} must be used instead of
SM Andreas Fischer, Andreas Kaldun, Ben King, Anton Wöllert and Enderalp Yakaboylu for fruitful discussions. SM is also grateful to the Studienstiftung des deutschen Volkes for financial support. All plots have been created with Matplotlib [62] and the GSL [63] has been used for numerical calculations.
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