Analytic Reissner–Nordström singularity

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Received 4 January 2012
Accepted for publication 21 March 2012
Published 18 April 2012

Abstract
An analytic extension of the Reissner–Nordström solution at and beyond the singularity is presented. The extension is obtained by using new coordinates in which the metric becomes finite and smooth. Using this extension it is shown that the charged and non-rotating black hole singularities are compatible with the global hyperbolicity and with the conservation of the initial value data. Geometric models for electrically charged particles are obtained.

PACS numbers: 04.70.Bw, 04.20.

(Some figures may appear in colour only in the online journal)

1. Introduction

1.1. The Reissner–Nordström solution

The Reissner–Nordström metric describes a static, spherically symmetric, electrically charged, non-rotating black hole [Rei16, Nor18]. It is to solution of the Einstein–Maxwell equations. It has the following form:

\[ ds^2 = - \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right) dt^2 + \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right)^{-1} dr^2 + r^2 d\sigma^2, \]

where \( q \) is the electric charge of the body and, as in the case of the Schwarzschild solution,

\[ d\sigma^2 = d\theta^2 + \sin^2 \theta d\phi^2 \]

is the metric of the unit sphere \( S^2 \), \( m \) is the mass of the body and the units were chosen so that \( c = 1 \) and \( G = 1 \) (see, e.g., [HE95, p 156]).

The first two terms on the right-hand side of equation (1) are independent of the coordinates \( \theta \) and \( \phi \), and conversely, \( d\sigma^2 \) is independent of the coordinates \( r \) and \( t \). This solution is a warped product between a two-dimensional (2D) semi-Riemannian space and the sphere \( S^2 \) with the canonical metric (2). Consequently, in coordinate transformations which affect only the coordinates \( r \) and \( t \), we can ignore the term \( r^3 d\sigma^2 \) in calculations. We can, finally, reintroduce it, taking again the warped product.

Solving the equation expressing the cancelation of \( r^2 - 2mr + q^2 \) for \( r \), we obtain

(i) no solution, for \( q^2 > m^2 \) (naked singularity);
(ii) double solution \( r_{\pm} = m \), for \( q^2 = m^2 \) (the extremal case);
(iii) two solutions \( r_{\pm} = m \pm \sqrt{m^2 - q^2} \), for \( q^2 < m^2 \).

If \( q^2 \leq m^2 \), there are two singular horizons at \( r = r_{\pm} \), which coincide for \( q^2 = m^2 \). These apparent singularities can be removed by a special coordinate transformation, such as that of Eddington–Finkelstein. All the three cases have an irremovable singularity at \( r = 0 \).

1.2. Two kinds of metric singularity

There are two main kinds of metric singularity that are relevant to our approach. In the first kind, some of the components of the metric diverge while approaching the singularity, where they become infinite. In the second kind, more benign, the metric remains always smooth, but becomes degenerate at the singularity; that is, its determinant becomes 0. In general, this means that the metric is not invertible, i.e. \( g^{ab} \) tends to infinity and cannot be defined at the singularity. But \( g_{ab} \) is smooth, and in some cases, despite the fact that \( g^{ab} \) is singular, we can define the contraction between covariant indices and construct covariant derivatives and the Riemann curvature, and even write an equivalent of the Einstein equation [Sto11a, Sto11b, Sto11e].

Some singularities of the first kind, having some components of the metric divergent, can be viewed as

\[ \text{(i) no solution, for } q^2 > m^2 \text{ (naked singularity);} \]
\[ \text{(ii) double solution } r_{\pm} = m, \text{ for } q^2 = m^2 \text{ (the extremal case);} \]
\[ \text{(iii) two solutions } r_{\pm} = m \pm \sqrt{m^2 - q^2}, \text{ for } q^2 < m^2. \]
singularities of the second kind, expressed in singular coordinate systems. This means that it is possible for some singularities of the first kind to be transformed into singularities of the second kind by an appropriate choice of the coordinate system. We did this for the Schwarzschild solution in [Sto11c].

In this paper, we will find for the Reissner–Nordström solution a new coordinate system in which the singularity at \( r = 0 \) becomes degenerate and analytic. The metric will become degenerate, but all its coefficients will be finite and smooth. The new form of the metric admits an analytic continuation beyond the singularity.

2. Extending the Reissner–Nordström spacetime at the singularity

2.1. The main result

The main result of this paper is contained in the following theorem.

**Theorem 2.1.** The Reissner–Nordström metric admits an analytic extension at \( r = 0 \).

**Proof.** We work initially in two dimensions \((t, r)\). It is enough to make the coordinate transformation in the neighborhood of the singularity—in the region \( r \in [0, M] \), where \( M = r_+ \) if \( q^2 \leq m^2 \), and \( M = \infty \) otherwise. We choose the coordinates \( \rho \) and \( \tau \), so that

\[
\begin{align*}
  t &= \tau \rho^T, \\
  r &= \rho^S,
\end{align*}
\]

where \( S, T \) have to be determined in order to make the metric analytic (Figure 1). This choice is motivated by the need to stretch the spacetime while approaching the singularity \( r = 0 \), so that the divergent components of the metric are smoothed.

Then, we have

\[
\begin{align*}
  \frac{\partial t}{\partial \tau} &= \rho^T, & \frac{\partial t}{\partial \rho} = T \tau \rho^{T-1}, & \frac{\partial r}{\partial \tau} = 0, & \frac{\partial r}{\partial \rho} = S \rho^{S-1},
\end{align*}
\]

Let us introduce the standard notation

\[
\Delta := r^2 - 2mr + q^2 \quad \text{(hence } \Delta = \rho^{2S} - 2mp^S + q^2). \tag{5}
\]

We note that \( \Delta > 0 \) for \( \rho \in [0, M] \).

The metric components in (1) now become

\[
g_{tt} = -\frac{\Delta}{\rho^{2S}}, \quad g_{\tau r} = \frac{\rho^{2S}}{\Delta}, \quad g_{\tau \tau} = g_{rr} = 0. \tag{6}
\]

Let us calculate the metric components in the new coordinates.

\[
g_{\tau \tau} = \left( \frac{\partial r}{\partial \tau} \right)^2 \frac{\rho^{2S}}{\Delta} - \left( \frac{\partial t}{\partial \tau} \right)^2 \frac{\Delta}{\rho^{2S}} = 0 - \rho^2 \frac{\Delta}{\rho^{2S}}
\]

Therefore

\[
g_{\tau \tau} = -\Delta \rho^{2T-2S}, \tag{7}
\]

\[
g_{\rho \tau} = \frac{\partial r}{\partial \rho} \frac{\partial r}{\partial \tau} \frac{\rho^{2S}}{\Delta} - \frac{\partial t}{\partial \rho} \frac{\partial t}{\partial \tau} \frac{\Delta}{\rho^{2S}} = 0 - T \tau \rho^{2T-1} \frac{\Delta}{\rho^{2S}}.
\]

Then

\[
g_{\rho \tau} = -T \tau \rho^{2T-2S-1}. \tag{8}
\]

\[
g_{\rho \rho} = \frac{\rho^{4S-2}}{\Delta} T^2 \tau^2 \rho^{2T-2S-2} \frac{\Delta}{\rho^{2S}}.
\]

Hence

\[
g_{\rho \rho} = \frac{\rho^{4S-2}}{\Delta} - T^2 \tau^2 \rho^{2T-2S-2}. \tag{9}
\]

We can see from the term \( \frac{\rho^{4S-2}}{\Delta} \) of \( g_{\rho \rho} \) in equation (9) that, to ensure that the singularity at \( \rho = 0 \) is only of
degenerate type and the metric is continuous there, \( S \) has to be an integer so that \( S \geq 1 \). Moreover, the condition \( S \geq 1 \) makes this term analytic at \( \rho = 0 \), because the denominator does not cancel there and is analytic and the numerator is analytic.

The other terms of equation (9), and the other equations (7) and (8), contain as a factor \( \Delta \), in which the minimum power to which \( \rho \) appears is 0. Hence, in order to avoid negative powers of \( \rho \), these terms require that \( 2 \tau - 2S - 2 \geq 0 \). Therefore, the conditions for removing the infinity of the metric at \( r = 0 \) by a coordinate transformation are that \( S \) and \( T \) be integers so that

\[
\begin{cases}
S \geq 1, \\
T \geq S + 1,
\end{cases}
\]

and they also ensure that the metric is analytic at \( r = 0 \). None of the metric’s components become infinite at the singularity.

To go back to four dimensions, we have to take the warped product between the 2D space with the metric we obtained and the sphere \( S^2 \) with the warping function \( \rho^S \). This is a degenerate warped product, as was studied in [Sto11e], and its result is a 4D manifold whose metric is analytic and degenerate at \( \rho = 0 \). Hence, this extension of the Reissner–Nordström solution is analytic at \( \rho = 0 \).

Let us extract from the proof the expression for the metric.

**Corollary 2.2.** The Reissner–Nordström metric, expressed in the coordinates from theorem 2.1, has the following form:

\[
ds^2 = -\Delta \rho^{2T-2S-2} (\rho d\tau + T \tau d\rho)^2 + \frac{S^2}{\Delta} \rho^{4S-2} d\rho^2 + \rho^{2S} d\sigma^2.
\]

**Proof.** From (4) we find that

\[
d\tau = \frac{\partial T}{\partial \rho} d\tau + \frac{\partial T}{\partial \rho} d\rho = \rho^T d\tau + T \tau \rho^{T-1} d\rho = \rho^{T-1} (\rho d\tau + T \tau d\rho)
\]

and

\[
d\rho = \frac{\partial r}{\partial \tau} d\tau + \frac{\partial r}{\partial \rho} d\rho = S \rho^{S-1} d\rho,
\]

which when plugged in the Reissner–Nordström equation (1) give

\[
ds^2 = -\frac{\Delta}{\rho^{23}} d\tau^2 + \frac{\rho^{2S}}{\Delta} d\rho^2 + d\sigma^2
\]

\[
= -\Delta \rho^{2T-2S-2} (\rho d\tau + T \tau d\rho)^2 + \frac{S^2}{\Delta} \rho^{4S-2} d\rho^2 + \rho^{2S} d\sigma^2.
\]

\[
\Box
\]

**2.2. The electromagnetic field**

The potential of the electromagnetic field in the Reissner–Nordström solution is

\[
A = -\frac{q}{r} d\tau
\]

and is singular at \( r = 0 \) in the standard coordinates \((t, r, \phi, \theta)\). On the other hand, in the new coordinates it is smooth.

**Corollary 2.3.** In the new coordinates \((\tau, \rho, \phi, \theta)\), the electromagnetic potential is

\[
A = -q \rho^{T-S-1} (\rho d\tau + T \tau d\rho)
\]

The electromagnetic field is

\[
F = q(2T - S) \rho^{T-S-1} d\tau \wedge d\rho,
\]

and they are analytic everywhere, including at the singularity \( \rho = 0 \).

**Proof.** The equation of the electromagnetic potential follows directly from the proof of theorem 2.1 and from equation (12). The equation of the electromagnetic field is obtained by applying the exterior derivative:

\[
F = dA = -q (\rho^{T-S} d\tau + T \tau \rho^{T-S-1} d\rho)
\]

\[
= -q \left( \frac{\partial \rho^{T-S}}{\partial \rho} d\rho \wedge d\tau + \frac{\partial T}{\partial \rho} \rho^{T-S-1} d\tau \wedge d\rho \right)
\]

\[
= -q ((T - S) \rho^{T-S} d\rho \wedge d\tau + T \rho^{T-S-1} d\tau \wedge d\rho)
\]

\[
= q(2T - S) \rho^{T-S-1} d\tau \wedge d\rho.
\]

2.3. General remarks concerning the proposed degenerate extension

**Remark 2.4.** The analytic Reissner–Nordström solution we found extends through \( \rho = 0 \) to negative values of \( \rho \). If \( S \) is even, \( \rho \) and \(-\rho\) give the same metric. For even values of \( T \), also the electromagnetic potential is invariant at the space inversion \( \rho \mapsto -\rho \). After taking the warped product we can identify the points \((\tau, \rho, \phi, \theta)\) and \((\tau, -\rho, (\phi + \pi) \mod 2\pi, \theta)\), and also all the points from the warped product which have \( \rho = 0 \) and constant \( \tau \). This identification gives a smooth metric, because of the symmetry with respect to the axis \( \rho = 0 \), and because the warping function is \( \rho^S \), with \( S \geq 1 \). We obtain by this a spherically symmetric solution having the topology of \( \mathbb{R}^4 \).

If we choose not to make this identification, the extension through \( \rho = 0 \) looks like the Einstein–Rosen model of charged particles [ER35] or like Misner and Wheeler’s ‘charge without charge’ [MW57]. As is known from the ‘charge without charge’ program, a special topology (i.e., ‘wormholes’) allows the existence of source-free electromagnetic fields which seem to be associated with charges, without actually having sources. The proposed degenerate extension of the Reissner–Nordström spacetime seems to support these proposals, but by making the above-mentioned identification, it also allows for charged models with the standard \( \mathbb{R}^4 \) topology.

If \( S \) is odd, the extension to \( \rho < 0 \) is very similar to the extension from the Kerr and Kerr–Newmann solutions through the interior of the ring singularity to the region \( r < 0 \).

**Remark 2.5.** As in the case of the analytic extension of the Schwarzschild solution [Sto11e], there is no unique way to extend the Reissner–Nordström metric so that it is smooth at the singularity. The explanation is due to the fact that a
A semi-regular metric has a smooth Riemann curvature $R_{abcd}$ and allows for the construction of more useful operations which are normally prohibited by the fact that the metric is degenerate. In the case of the Schwarzschild black hole, we could find a solution that is semi-regular [Sto11c]. In the case of the Reissner–Nordström black hole, we cannot find numbers $S$ and $T$ for equation (3), which would make the metric semi-regular. However, this does not exclude other changes in the coordinates, and we propose the following open problem.

**Open Problem 2.6.** Is it possible to find coordinates that allow the Reissner–Nordström metric to be semi-regular at $\rho = 0$?

Also, it may be interesting to consider the following.

**Open Problem 2.7.** Can we find natural conditions ensuring the uniqueness of the analytic extensions of the Schwarzschild and Reissner–Nordström solutions at the singularity $\rho = 0$? Under what conditions does a singular coordinate transformation of an analytic extension lead to another extension which is physically indistinguishable?

3. Null geodesics in the proposed solution

In this section, we will discuss the geometric meaning of the extension proposed in this paper, mainly from the viewpoint of the lightcones and the null geodesics. In the coordinates $(\tau, \rho)$, the metric is analytic near the singularity $\rho = 0$ and has the form

$$g = -\Delta \rho^{2T-2S-2} \left( \frac{\rho^2}{T \rho^3} - \frac{T \rho^3}{T \rho^3} \right).$$

Let us find the null directions, defined at each point $(\tau, \rho)$ by the tangent vectors $u \neq 0$ so that $g(u, u) = 0$. Since any non-zero multiple of $u$ is also a solution, we will consider $u = (\sin \alpha, \cos \alpha)$ and try to find $\alpha$. We obtain the equation

$$\rho^2 \sin^2 \alpha + 2T \tau \rho \sin \alpha \cos \alpha + \left( T^2 \tau^2 - \frac{S^2}{\Delta} \rho^{6S-2T} \right) \cos^2 \alpha = 0,$$

which can be written as a quadratic equation in $\tan \alpha$

$$\rho^2 \tan^2 \alpha + 2T \tau \rho \tan \alpha + \left( T^2 \tau^2 - \frac{S^2}{\Delta} \rho^{6S-2T} \right) = 0,$$

which leads to the solution

$$\tan \alpha_{\pm} = -\frac{T \tau}{\rho} \pm \frac{S}{\Delta} \rho^{3S-T-1}.$$  

Therefore, the incoming and outgoing null geodesics satisfy the differential equation

$$\frac{dT}{d\rho} = -\frac{T \tau}{\rho} \pm \frac{S}{\Delta} \rho^{3S-T-1}.$$  

4. The Penrose–Carter diagrams for our solution

To move to the Penrose–Carter coordinates (and have a bird’s eye view of the global behavior of the degenerate extensions of the Reissner–Nordström solution), we apply the same steps as those one normally applies for the standard Reissner–Nordström black hole. These steps are, for example,
Figure 4. (A) Naked Reissner–Nordström black holes \( (q^2 > m^2) \). (B) Extremal Reissner–Nordström black holes \( (q^2 = m^2) \). (C) Reissner–Nordström black holes with \( q^2 < m^2 \).

Figure 5. The Penrose–Carter diagram for the naked Reissner–Nordström black hole \( (q^2 > m^2) \), analytically extended beyond the singularity. It is symmetric with respect to the time-like singularity.

Figure 6. The Penrose–Carter diagram for the extremal Reissner–Nordström black hole \( (q^2 = m^2) \), analytically extended beyond the singularity. It repeats periodically along the vertical direction.

Figure 7. The Penrose–Carter diagram for the non-extremal Reissner–Nordström black hole with \( q^2 < m^2 \), analytically extended beyond the singularity. When represented in plane, it repeats periodically along both the vertical and the horizontal directions, and it has overlaps. In the diagram, there is a small shift between the two copies to make the overlapping visible.

We just add our coordinate transformation before the steps leading to the Penrose–Carter coordinates, as we did in [Sto11c] for the Schwarzschild solution. If \( S \) is odd, the spacetime has a region \( \rho < 0 \) and the Penrose–Carter diagrams are similar to the standard diagrams for the Kerr and Kerr–Newman spacetimes (see, for example, [HE95, p 165]). If \( S \) is even, the diagram will repeat not only vertically, but also horizontally, symmetrical to the singularity.

We obtain the diagram for the naked Reissner–Nordström black hole \( (q^2 > m^2) \) by taking the symmetry of the standard Reissner–Nordström diagram with respect to the singularity (figure 5).

The resulting diagram for the extremal Reissner–Nordström black hole \( (q^2 = m^2) \) is a strip symmetric about the singularity (figure 6).

When represented in plane, the diagram for the non-extremal Reissner–Nordström black hole \( (q^2 < m^2) \) extends in two directions and has overlapping parts (figure 7).

In the Penrose–Carter diagrams of the degenerate extension of the Reissner–Nordström solution, the null geodesics continue through the singularity, because they are always at \( \pm \frac{\pi}{4} \).

5. A globally hyperbolic charged black hole

A global solution to the Einstein equation is well behaved when the equations at a given moment of time determine...
the solution for the entire future and past. This condition is ensured by the global hyperbolicity, which is expressed by the requirements that

(i) for any two points $p$ and $q$, the intersection between the causal future of $p$ and the causal past of $q$, $J^+(p) \cap J^-(q)$, is a compact subset of the spacetime;
(ii) there are no closed time-like curves ([HE95, p 206]).

The property of global hyperbolicity is equivalent to the existence of a Cauchy hypersurface—a space-like hypersurface $\mathcal{S}$ that, for any point $p$ in the future (past) of $\mathcal{S}$, is intersected by all past-directed (future-directed) inextensible causal (i.e. time-like or null) curves through the point $p$ ([HE95, p 119, pp 209–212]).

Because in the standard coordinates for the Reissner–Nordström spacetime one cannot extend the solution beyond the singularity, the Reissner–Nordström spacetime fails to admit a Cauchy hypersurface, and it is normally inferred that it is not globally hyperbolic.

But since we now know how to extend analytically the Reissner–Nordström spacetime beyond the singularity, we should check if we can use this feature to construct new solutions that are globally hyperbolic. To do so, we will construct solutions that admit foliations with Cauchy hypersurfaces—i.e. that are diffeomorphic with a Cartesian product between an interval $I \subseteq \mathbb{R}$ representing the time dimension, and a space-like hypersurface.

The coordinates $(\tau, \rho)$, under the condition (23), provide a space-like foliation given by the hypersurfaces $\tau = \text{const}$. This foliation is global only for naked singularities; otherwise it is defined locally, in the neighborhood of $(\tau, \rho) = (0, 0)$ given by $r < r_\text{ext}$. From equation (1) defining the Reissner–Nordström metric we know that the solution is stationary; that is, when expressed in the coordinates $(t, r)$ it is independent of time. This means that we can choose as the origin of time any value, thus ensuring that we can cover the neighborhood of the entire axis $\rho = 0$ with coordinate patches like $(\tau, \rho)$. To obtain global foliations with Cauchy hypersurfaces, we use the global extensions represented in the Penrose–Carter diagrams of section 4, figures 5–7. It is important to note that in the Penrose–Carter diagram, the null directions are represented as straight lines inclined at $\pm \pi/2$.

For the naked Reissner–Nordström solution (figure 5) we can find immediately a global foliation, because the Penrose–Carter diagram is identical to that for the Minkowski spacetime (figure 8). Hence, the natural foliation of the Minkowski spacetime will be good for our extended naked Reissner–Nordström solution too.

To obtain explicitly the foliations for all the cases, we map to our solutions represented in the coordinates $(t, r)$ the product $(0, 1) \times \mathbb{R}$. To do so, we can use a version of the Schwarz–Christoffel mapping that maps the strip

$$ S := \{ z \in \mathbb{C} | \text{Im}(z) \in [0, 1] \} \tag{24} $$

to a polygonal region from $C$, with the help of the formula

$$ f(z) = A + C \int_{S} \exp \left[ \frac{\pi}{2} (\alpha_+ - \alpha_-) \zeta \right] $$

$$ \times \prod_{k=1}^{n} \left[ \sinh \frac{\pi}{2} (\zeta - z_k) \right]^{\alpha_k - 1} d\zeta, \tag{25} $$

where $z_k \in S := \mathbb{R} \times [0, 1]$ are the prevertices of the polygon, and $\alpha_-, \alpha_+$ are measures of the angles of the polygon, divided by $\pi$ (see, e.g., [DT02]). The vertices having angles $\alpha_-$ and $\alpha_+$ correspond to the ends of the strip, which are at infinity. The level curves $\{ \text{Im}(z) = \text{const} \}$ give our foliation [Sto11d].

The prevertices whose image is represented in figure 11 are

$$ (-\infty, 0, +\infty, i), \tag{26} $$

and the angles are

$$ \left( \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2} \right). \tag{27} $$

For the other cases $q^2 < m^2$ (figures 4(B) and (C)), the maximal extensions cannot be globally hyperbolic, because they admit Cauchy horizons (hypersurfaces which are boundaries for the Cauchy development of the data on a space-like hypersurface). If we want to obtain a globally hyperbolic solution, we have to drop the regions beyond the Cauchy horizons. This leads naturally to a choice of a subset of the Penrose–Carter diagram which is symmetric about the singularity $r = 0$ and can be foliated (figures 9 and 10).
Let us take now as prevertices of the Schwarz–Christoffel mapping (25) the set
\begin{equation}
(-\infty, -a, 0, a, +\infty, i),
\end{equation}
where $0 < a$ is a positive real number. The angles are, respectively,
\begin{equation}
\left(\frac{\pi}{2^n}, \frac{\pi}{2^n}, \frac{3\pi}{2^n}, \frac{\pi}{2^n}, \frac{\pi}{2^n}, \frac{\pi}{2^n}\right).
\end{equation}

Appropriate choices of $a$ result in the foliations represented in figures 9 and 10, corresponding to the non-extremal, respectively, extremal solutions with $q^2 < m^2$. Since $\alpha_- = \alpha_+$ and the edges are inclined at most at $\frac{\pi}{2}$, alternating in such a way that the level curves with $\text{Im}(z) \in (0, 1)$ have at each point tangents making an angle strictly between $-\frac{\pi}{4}$ and $\frac{\pi}{4}$, our foliations are space-like.

In each of figures 8–10, we highlighted a spacelike hypersurface which is Cauchy, because it is intersected by all past (future) directed inextensible causal curves through a point $p$ from its future (past).

6. The meaning of the analytic extension at the singularity

As in the case of the extension of the Schwarzschild solution, we can see that the singularity is not necessarily harmful for the information or the structure of spacetime. There is no reason to believe that the information is lost at the singularity, and the fact that it is time-like and may be naked, although it contradicts Penrose’s cosmic censorship hypothesis, is compatible with the global hyperbolicity. These observations may apply also to the case of an evaporating charged black hole (see figure 11).

The Reissner–Nordström solution is, according to the no-hair theorem, representative of non-rotating and electrically charged black holes. If the black hole evaporates, the singularity becomes visible to distant observers. This is a problem in the solutions which do not admit extension through the singularity. Our solution, because it can be extended beyond the singularity, does not break the topology of spacetime. The metric tensor does not run into infinities, although, because of its degeneracy, other quantities, such as its inverse, may become infinite.

The maximal globally hyperbolic extensions from section 5 are ideal, because the Reissner–Nordström solutions describe spacetimes that are too simple. But because they can be foliated by Cauchy hypersurfaces, and the base hypersurface is $\mathbb{R}^3$, we can interpolate between such solutions and foliations without singularities, and construct more general solutions. The interpolation can be done by varying the parameters $m$ and $q$. By this, one can model spacetimes with black holes that are formed and then evaporate. The presence of a time-like evaporating singularity of this type is compatible with the global hyperbolicity, as in figure 11.

This extension of the Reissner–Nordström solution can be used to model electrically charged particles as charged black holes, as pointed out in remark 2.1.

Acknowledgments

This work was partially supported by the Romanian Government through grant number PN II Idei 1187. I thank an anonymous referee for the valuable comments and suggestions to improve the clarity and quality of this paper.

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