Electromagnetic form factors of the $\rho$ meson in a light-front constituent quark model

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Abstract

The electromagnetic form factors of the $\rho$ meson are evaluated adopting a relativistic constituent quark model based on the light-front formalism, and using a meson wave function with the high-momentum tail generated by the one-gluon-exchange interaction. The breakdown of the rotational covariance for the one-body component of the current operator is investigated and the sensitivity of the ratio of the $\rho$-meson form factors to the pion (charge) form factor to the spin-dependent component of the effective $q\bar{q}$ interaction is illustrated.

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The understanding of the electroweak properties of hadrons has recently received much attention within the context of constituent quark models based on the so-called Hamiltonian light-front formalism [4]. As a matter of fact, light-front quark models have been applied to the evaluation of the charge form factor of the pion [2], [3], the electromagnetic (e.m.) form factors of the $\rho$ meson [4], the vector and axial form factors of the nucleon [4], and the radiative, leptonic and semileptonic decays of both pseudoscalar and vector mesons [1]. In most of these applications ([4] - [7]) it has been assumed that the hadron wave function is simply given by a harmonic oscillator ansatz, which is expected to describe the effects of the confinement scale only. However, it has been shown [2] that the high momentum components generated in the wave function by the one-gluon-exchange interaction, sharply affect the charge form factor of the pion for values of the square of four-momentum transfer $Q^2$ up to few (GeV/c)$^2$. The aim of this letter is to extend to the $\rho$ vector meson the analysis performed for the pion in ref. [3], i.e. to investigate the sensitivity of the e.m. form factors of $\pi$ and $\rho$ mesons to the short-range structure of the effective $q\bar{q}$ interaction. The calculations of the $\rho$-meson form factors presented in this letter, are based on Poincaré-covariant wave functions and one-body e.m. currents. Since for a spin-1 hadron the rotational covariance of the e.m. current operator is not ensured by its one-body component alone, the effects of the violation of the so-called angular condition (see ref. [8]) upon the $\rho$-meson form factors are estimated using wave functions with different high-momentum tails.

The quark model used in this letter is based on the light-front formalism which represents the natural framework for constructing a relativistic model for the valence $q\bar{q}$ component of a meson. As is known, the intrinsic light-front kinematical variables are $\kappa_\perp = \vec{p}_q \perp - \xi \vec{P}_\perp$ and $\xi = p_\perp^+/P^+$, where the subscript $\perp$ indicates the projection perpendicular to the spin quantization axis, defined by the vector $\hat{n} = (0,0,1)$, and the plus component of a 4-vector $p \equiv (p^0, \vec{p})$ is given by $p^+ = p^0 + \hat{n} \cdot \vec{p}$; eventually, $\vec{P} \equiv (P^+, \vec{P}_\perp) = \vec{p}_q + \vec{p}_{\bar{q}}$ is the total momentum of the meson. In what follows, only the $^3S_1$ channel of the $\rho$ meson is considered, being the $D$-wave component extremely small ($p_D \simeq 0.16\%$). As a matter of fact, in ref. [4] it has been checked that a $D$-wave admixture with $p_D \simeq 0.16\%$ has negligible effects on the e.m. form factors in the $Q^2$-range considered in this letter. Omitting for the sake of simplicity the flavour and colour degrees of freedom, the requirement of Poincaré covariance for the intrinsic wave function $\chi_\mu^1(\xi, \vec{k}_\perp, \nu \bar{\nu})$ of a $\rho$ meson with helicity $\mu$ implies (cf. ref. [4])

$$\chi_\mu^1(\xi, \vec{k}_\perp, \nu \bar{\nu}) = \sqrt{\frac{M_0}{16\pi \xi (1-\xi)}} R_\mu(\xi, \vec{k}_\perp, \nu \bar{\nu}) \, w^\rho(k^2)$$  \hspace{1cm} (1)

where $\nu, \bar{\nu}$ are the quark spin variables, $k^2 \equiv k_\perp^2 + k_n^2$, $k_n \equiv (\xi - 1/2)M_0$, $M_0^2 = (m_q^2 + k_n^2)/\xi + (m_{\bar{q}}^2 + k_\perp^2)/(1-\xi)$ and (see ref. [7])

$$R_\mu(\xi, \vec{k}_\perp, \nu \bar{\nu}) = \sum_{\nu', \bar{\nu}'} \langle \nu | R_M^1(\xi, \vec{k}_\perp, m_q) | \nu' \rangle \langle \bar{\nu} | R_M^1(1 - \xi, -\vec{k}_\perp, m_{\bar{q}}) | \bar{\nu}' \rangle \langle \nu' | \frac{1}{2} \nu' \bar{\nu} | 1\mu \rangle$$  \hspace{1cm} (2)

with $m_q$ ($m_{\bar{q}}$) being the constituent quark (antiquark) mass and $R_M$ the $2 \times 2$ irreducible representation of the Melosh rotation [8].

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As in ref. [3], the radial wave function \( u^\rho(k^2) \) appearing in Eq. (1) is identified with the equal-time radial wave function in the \( \rho \)-meson rest-frame. In this letter we will adopt the effective \( q\bar{q} \) Hamiltonian introduced by Godfrey and Isgur (GI) [10] for reproducing the meson mass spectra, viz.

\[
H_{q\bar{q}} w^{q\bar{q}}(k^2) |j\mu\rangle \equiv \left[ \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{q\bar{q}} \right] w^{q\bar{q}}(k^2) |j\mu\rangle = M_{q\bar{q}} w^{q\bar{q}}(k^2) |j\mu\rangle \tag{3}
\]

where \( M_{q\bar{q}} \) is the mass of the meson, \( |j\mu\rangle = \sum_\nu \langle \frac{1}{2}\nu | j\mu \rangle \chi_\nu \bar{\chi}_\nu \) is the equal-time quark-spin wave function and \( V_{q\bar{q}} \) is the effective \( q\bar{q} \) potential. The interaction in the GI scheme, \( V_{(GI)} \), is composed by a one-gluon-exchange (OGE) term (dominant at short separations) and a linear-confining term (dominant at large separations). In order to analyze the effects of different components of the GI interaction, two other choices of \( w^{q\bar{q}}(k^2) \) will be considered; the first one is the solution of Eq. (3) obtained after switching off the OGE part of \( V_{(GI)} \), i.e., by retaining only its linear confining term, \( V_{(conf)} \), whereas the second choice is given by the solution of Eq. (3) obtained when only the spin-independent part, \( V_{(SI)} \), of \( V_{(GI)} \) is considered. The three different forms of \( w^{q\bar{q}}(k^2) \) will be denoted hereafter by \( w^{q\bar{q}}_{(conf)} \), \( w^{q\bar{q}}_{(SI)} \) and \( w^{q\bar{q}}_{(GI)} \) corresponding to \( V_{(conf)} \), \( V_{(SI)} \) and \( V_{(GI)} \), respectively. It should be pointed out that the pion \((1S_0 \) channel\) and \( \rho \)-meson \((3S_1 \) channel\) radial wave functions differ only when the spin-spin component of the \( q\bar{q} \) interaction is considered; this means that: \( w^{\pi}_{(conf)} = w^{\rho}_{(conf)} \equiv w_{(conf)} \), \( w^{\pi}_{(SI)} = w^{\rho}_{(SI)} \equiv w_{(SI)} \) and \( w^{\pi}_{(GI)} \neq w^{\rho}_{(GI)} \). The four wave functions \( w_{(conf)} \), \( w_{(SI)} \), \( w_{(GI)} \) and \( w_{(conf)} \) are shown in fig. 1. It can clearly be seen that both the central and the spin-dependent components of the OGE interaction strongly affect the high-momentum tail of the \( \pi^- \) and \( \rho \)-meson wave functions. It should also be reminded that the radial wave function \( w_{(conf)} \) turns out to be very close to the simple harmonic oscillator ansatz adopted in many light-front calculations (see ref. [2]). According to ref. [10], the value \( m_q = m_{\bar{q}} = 0.220 \) GeV is adopted.

Matrix elements of the electromagnetic current. Within the light-front formalism (cf. ref. [11]), all the invariant form factors of a hadron can be determined using only the matrix elements of the component \( I^+(0) \) of the current operator evaluated in an appropriate frame, which, for spacelike four-momentum transfer \( Q^2 \), can be identified with the Breit frame where \( Q^+ = 0 \), \( P^+ = P'^+ = \sqrt{M^2 + Q^2/4} \) and \( P'_\perp = -P_\perp = Q_\perp/2 \). In the case of a spin-1 hadron only three independent (invariant) form factors exist, whereas four matrix elements of \( I^+(0) \) are independent after considering the properties of \( I^+(0) \) to be Hermitian and invariant under i) time reversal, ii) rotations about \( \hat{n} \), and iii) reflection on the plane perpendicular to \( \hat{n} \). An additional condition comes from the rotational invariance of the charge density, which involves unitary transformations based upon a subset of Poincaré generators depending on the interaction. Thus, the additional constraint, usually called the angular condition, is not generally satisfied by the matrix elements of the one-body current alone, and requires the existence of many-body currents. The angular condition can be written in the following form [8]

\[
\Delta(Q^2) \equiv (1 + 2\eta)I_{11} + I_{1-1} - \sqrt{8\eta}I_{10} - I_{00} = 0 \tag{4}
\]
where $I_{\mu'\mu} \equiv \langle \bar{P}_{\mu'}|I^+(0)|\bar{P}_{\mu}\rangle$ stands for the matrix elements of $I^+(0)$ in the Breit frame and $\eta \equiv Q^2/4M_\rho^2$, with $M_\rho = 0.77$ GeV being the experimental $\rho$-meson mass. In ref. [11] the following relations between the matrix elements $I_{\mu'\mu}$ and the (conventional) invariant form factors $G_0, G_1$ and $G_2$, have been obtained

\[
[G_0]^{CCKP} = \frac{1}{3(1 + \eta)}[(3 - 2\eta)(I_{11} + I_{00}) + 5\sqrt{2\eta}I_{10} + (2\eta - \frac{1}{2})I_{1-1}] \\
[G_1]^{CCKP} = \frac{1}{1 + \eta}[I_{11} + I_{00} - I_{1-1} - \frac{2(1 - \eta)}{\sqrt{2\eta}}I_{10}] \\
[G_2]^{CCKP} = \frac{\sqrt{2}}{3(1 + \eta)}[\eta I_{11} + 2\sqrt{2\eta}I_{10} - \eta I_{00} - (\eta + 2)I_{1-1}] \quad (5)
\]

In ref. [12] $G_0$ has been obtained using a specific criterion for choosing the "good" matrix elements of $I^+(0)$ in the infinite momentum frame and in the Breit one, viz.

\[
[G_0]^{FFS} = \frac{1}{3(1 + \eta)}[(2\eta + 3)I_{11} + 2\sqrt{2\eta}I_{10} - \eta I_{00} + (2\eta + 1)I_{1-1}] \\
[G_1]^{FFS} = [G_1]^{CCKP} \quad , \quad [G_2]^{FFS} = [G_2]^{CCKP} \quad (6)
\]

In ref. [8] the "worst" matrix element has been assumed to be $I_{00}$. After eliminating $I_{00}$ from Eq. (5) through the angular condition (4), one has

\[
[G_0]^{GK} = \frac{1}{3}[(3 - 2\eta)I_{11} + 2\sqrt{2\eta}I_{10} + I_{1-1}] \\
[G_1]^{GK} = 2[I_{11} - \frac{1}{\sqrt{2\eta}}I_{10}] \\
[G_2]^{GK} = \frac{2\sqrt{2}}{3}[-\eta I_{11} + \sqrt{2\eta}I_{10} - I_{1-1}] \quad (7)
\]

Following refs. [13] and [14], the matrix element $I_{00}$ is expected to be the dominant one in the perturbative QCD regime; if the matrix element $I_{11}$, instead of $I_{00}$, is eliminated from Eq. (5) through the angular condition (4), one gets

\[
[G_0]^{BH} = \frac{1}{3(1 + 2\eta)}[(3 - 2\eta)I_{00} + 8\sqrt{2\eta}I_{10} + 2(2\eta - 1)I_{1-1}] \\
[G_1]^{BH} = \frac{2}{1 + 2\eta}[I_{00} - I_{1-1} + (2\eta - 1)\frac{I_{10}}{\sqrt{2\eta}}] \\
[G_2]^{BH} = \frac{2\sqrt{2}}{3(1 + 2\eta)}[\sqrt{2\eta}I_{10} - \eta I_{00} - (1 + \eta)I_{1-1}] \quad (8)
\]

It should be pointed out that, if the exact Poincaré-covariant (many-body) $I^+(0)$ current is used, all the prescriptions (like those specified by Eqs. (5)-(8)) should yield the same results for the invariant form factors $G_i$, whereas, when only the one-body component of
the e.m. current operator is considered, the angular condition (4) is in general violated (i.e., $\Delta(Q^2) \neq 0$) and the calculation of the $G_i$ depends upon the prescription used.

As for the one-body part of the e.m. current operator, the expression $I^+(0) = \sum_{i=\bar{q},q} e_i F_i(Q^2) \gamma_i^+$, where $F_i(Q^2)$ is the charge form factor of the constituent quark, has been adopted; indeed, according to the findings of refs. [6(a) and [15], the anomalous magnetic moments of the constituent quarks are expected to be small and, therefore, only Dirac magnetic moments are considered in this letter. Thus, using Eq. (1), the matrix elements $I_{\mu'\mu}$ appearing in Eqs. (5) - (8) can be written as

$$I_{\mu'\mu} = F(Q^2) \int d\xi d\vec{k}_\perp \frac{\sqrt{M_0 M_\mu}}{16\pi(1-\xi)} M_{\mu'\mu}(\xi, \vec{k}_\perp, \vec{k}_\perp') w^\rho(k^2) w^\nu(k^2)$$

(9)

where $\vec{k}_\perp \equiv \vec{k} + (1 - \xi) \vec{Q}_\perp$, $F(Q^2) = e_q F_q(Q^2) + e_i F_i(Q^2)$ and $M_{\mu'\mu} = \sum_{\nu\bar{\nu}} R_\mu(\xi, \vec{k}_\perp, \nu\bar{\nu}) R^*_\mu(\xi, \vec{k}_\perp', \nu\bar{\nu})$ arise from the Melosh rotation of the quark spins. Note that both the use of $I^+(0)$ and the choice $Q^2 = 0$ allow to suppress the contribution of the so-called Z-graph (pair creation from the vacuum) [4] [10].

**Results of calculations.** The invariant form factors $G_i$ have been evaluated using the CCKP (Eq. (4)), FFS (Eq. (3)), GK (Eq. (4)) and BH (Eq. (8)) prescriptions. As already pointed out, any dependence of the calculations upon the prescription used is a consequence of the breakdown of the angular condition (Eq. (4)), which is directly expressed by the departure of the quantity $\Delta(Q^2)$ from zero. In refs. [5] and [12] it has been found that the effects of the violation of the angular condition upon the form factors of the deuteron is small at all accessible values of $Q^2$, though $\Delta(Q^2)$ turns out to be an increasing function of $Q^2$. In the case of the $\rho$ meson, for which the momentum of the constituent is not small with respect to its mass (see fig. 1), the breakdown of the angular condition is expected to have large effects on the calculated form factors [4]. By adopting for the radial wave function $w^\rho$ the choices $w_{(con)}$, $w_{(si)}$ and $w^\rho_{(GI)}$ and neglecting the charge form factor of the constituent quarks (i.e., assuming $F(Q^2) = 1$ in Eq. (4)), the matrix elements $I_{\mu'\mu}$ have been calculated and the results obtained for the quantity $\Delta(Q^2)$ are reported in fig. 2. It can be seen that the violation of the angular condition is strongly affected by the high-momentum tail of the $\rho$-meson wave function; this means that the two-body currents required to restore the rotational covariance of the e.m. current operator are expected to be sharply sensitive to the short-range structure of the effective $q\bar{q}$ interaction.

Using the wave function $w^\rho_{(GI)}$ and assuming $F(Q^2) = 1$ in Eq. (4), the sensitivity of the form factors $G_i(Q^2)$ to the prescriptions given by Eqs. (5) - (8) is illustrated in fig. 3. It can be seen that all the form factors are sensitive to the prescription used only for $Q^2 \geq 0.5 \ (GeV/c)^2$; in particular, $G_2(Q^2)$ is strongly affected by the violation of the angular condition, so that the difference from its non-relativistic limit (i.e. $G_2(Q^2) = 0$ if the $D$-wave is disregarded) might be significantly reduced. In agreement with the findings of ref. [4], where a soft wave function was adopted, the charge radius of the $\rho$ meson ($<r^2> \equiv \lim_{Q^2 \to 0} 6(1 - G_0(Q^2))/Q^2$) is slightly affected by the prescription used also in the case of wave functions with a high-momentum tail. In particular, a spread of $\sim 10 - 15\%$
around the value $<r^2> = 0.35 \text{ fm}^2$, calculated using the CCKP prescription and assuming $F(Q^2) = 1$, has been obtained. Moreover, we have found that the values of the magnetic ($\mu_1 \equiv \lim_{Q^2 \to 0} G_1(Q^2)$) and quadrupole ($\mu_2 \equiv \lim_{Q^2 \to 0} 3\sqrt{2}G_2(Q^2)/Q^2$) moments, which are independent of the violation of the angular condition (cf. ref. [17]), are $\mu_1 = 2.26$ (i.e., $\sim 10\%$ larger than its non-relativistic value ($\mu_1 = 2$)) and $\mu_2 = 0.024 \text{ fm}^2$, respectively.

Let us now investigate the effects of the different components of the GI interaction upon the form factors $G_i(Q^2)$. In order to get rid of contributions arising from the charge form factor of the constituent quarks ($F(Q^2)$ in Eq. (9)) as well as to enhance the sensitivity of the calculations to the spin-spin component of the effective $q\bar{q}$ interaction, it is convenient to compare the form factors $G_i(Q^2)$ of the $\rho$-meson ($^3S_1$ channel) with the charge form factor $F_\pi(Q^2)$ of the pion ($^1S_0$ channel) by considering the ratios $R_i(Q^2) \equiv G_i(Q^2)/F_\pi(Q^2)$, where $F_\pi(Q^2)$ is explicitly given by (cf., e.g., ref. [2])

$$F_\pi(Q^2) = F(Q^2) \int d\vec{k}_\perp d\xi \frac{\sqrt{M_0M_0'}}{16\pi(1-\xi)} \frac{\xi(1-\xi)M_0^2 + \vec{k}_\perp \cdot \vec{Q}_\perp}{\xi(1-\xi)M_0M'_0} w^\pi(k^2)w^\pi(k'^2)$$

(10)

The radial wave functions of the $\rho$ and $\pi$ mesons corresponding to the interactions $V_{(conf)}$, $V_{(si)}$ and $V_{(GI)}$ have been considered in Eqs. (9) and (11), respectively. The results of the calculations, obtained using the CCKP prescription (Eq. (3)) for the $\rho$ meson, are reported in fig. 4. It should be pointed out that similar results can be obtained using the FFS, GK or BH prescriptions instead of the CCKP one. From fig. 4 it can be seen that: i) both at low and moderate values of $Q^2$ the ratios $R_i(Q^2)$ are strongly affected by the high momentum components generated in the meson wave functions by the spin-dependent part of the GI effective $q\bar{q}$ interaction; ii) for $Q^2 \geq 0.5 \text{ (GeV/c)}^2$ the sensitivity to the high momentum tail appears to be of the same order of magnitude of the uncertainties related to the violation of the angular condition (cf. fig. 3).

In conclusion, the e.m. form factors of the $\rho$ meson have been evaluated within a relativistic constituent quark model based on the light-front formalism, for values of $Q^2$ up to few $(\text{GeV/c})^2$. The effects of the breakdown of the rotational covariance of the one-body e.m. current operator as well as the sensitivity of the calculations to the high momentum tail of the meson wave functions, generated by the one-gluon-exchange interaction, have been investigated. The main results of our analysis can be summarized as follows: the ratio of the $\rho$-meson form factors to the pion (charge) form factor is remarkably sensitive to the high-momentum components of the meson wave function and, therefore, could allow to investigate the spin-dependent part of the effective $q\bar{q}$ interaction; however, at $Q^2 > 0.5 \text{ (GeV/c)}^2$, such a sensitivity is partially hindered by the theoretical uncertainties related to the effects of two-body currents, which are required to ensure the full Poincaré covariance of the e.m. current operator. Therefore, the evaluation of the effects of the (interaction dependent) two-body currents upon meson form factors is mandatory in order to get quantitative information on the short-range structure of the meson wave functions.

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Fig. 1. Wave functions $(k \cdot w^\pi)^2$ and $(k \cdot w^\rho)^2$, calculated using in Eq. (3) different effective $q\bar{q}$ interactions, as a function of the relative momentum $k$. Dotted line: $w^\pi = w^\rho = w_{(\text{conf})}$, corresponding to the case in which only the linear confining part of the GI $q\bar{q}$ interaction \[10\] is considered. Dashed line: $w^\pi = w^\rho = w_{(\text{si})}$, corresponding to the solution of Eq. (3) obtained using the spin-independent part of the GI interaction. The solid and dot-dashed lines correspond to $w^\pi = w^\rho_{(\text{GI})}$ and $w^\pi = w_{(\text{GI})}$, respectively, obtained by including in Eq. (3) the full spin-dependent GI interaction.

Fig. 2. The quantity $\Delta(Q^2)$ (see Eq. (4)) as a function of $Q^2$ calculated using various choices of the wave function $w^\rho$ appearing in Eq. (9). The dotted, dashed and solid lines correspond to $w^\rho = w_{(\text{conf})}$, $w^\rho = w_{(\text{si})}$ and $w^\rho = w^\rho_{(\text{GI})}$, respectively (see fig. 1). Calculations have been performed assuming $F(Q^2) = 1$ in Eq. (9).

Fig. 3. The invariant form factors $G_i(Q^2)$ of the $\rho$ meson as a function of $Q^2$ calculated within various prescriptions. The solid, dashed, dotted and dot-dashed lines correspond to the CCKP (Eq. (5)), FFS (Eq. (6)), GK (Eq. (7)) and BH (Eq. (8)) prescriptions, respectively. Calculations have been performed using $w^\rho = w^\rho_{(\text{GI})}$ in Eq. (9), which corresponds to the ground-state wave function of the full GI Hamiltonian \[10\] for the $^3S_1$ channel. In all the calculations the constituent quark form factor has been neglected (i.e., $F(Q^2) = 1$ in Eq. (9)). Note that, as for $G_1(Q^2)$ and $G_2(Q^2)$, the FFS prescription coincides with the CCKP one (cf. Eq. (5)).

Fig. 4. The ratios $R_i(Q^2) \equiv G_i(Q^2)/F_{\pi}(Q^2)$ as a function of $Q^2$, calculated using various choices of the $\rho$- and $\pi$-meson wave functions appearing in Eqs. (9) and (10), respectively. Calculations for the $\rho$ meson have been performed using the CCKP prescription (Eq. (5)). Dotted line: $w^\pi = w^\rho = w_{(\text{conf})}$, corresponding to the case in which only the linear confining part of the GI $q\bar{q}$ interaction \[10\] is considered in Eq. (3). Dashed line: $w^\pi = w^\rho = w_{(\text{si})}$, corresponding to the solution of Eq. (3) obtained using the spin-independent part of the GI interaction. Solid line: $w^\pi = w^\rho_{(\text{GI})}$ and $w^\rho = w^\rho_{(\text{GI})}$, obtained by including in Eq. (3) the full spin-dependent GI interaction.
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