RECURSION AND EVOLUTION: PART I

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Abstract. A *self-editing* algorithm is one that edits its program.

The present paper studies evolution of self-editing algorithms that undergo some form of natural or artificial selection.

1. Introduction

As the abstract announces, in the present paper we study the notion of *self-editing* algorithms, i.e. algorithms that may edit their program. Such an algorithm will be represented by a state code $c$. (Almost) following Gödel, we use the symbol $\{c\}$ to denote the algorithm that $c$ codes for. It is easily seen that a self-editing algorithm may reproduce its code: For if $c$ is a code for the algorithm:

Make another copy of the input,

and $\{c\}$ is self-editing, then the result of $\{c\}$ on input $c$, (denoted as $\{c\}(c)$) will be two exact copies of $c$. We can turn this into a more complicated but more meaningful procedure as follows:

Given a code $e$ of (any) algorithmic procedure, let $c(e)$ be a code for the following algorithm:

Run $e$ on the input and output the result.

Assuming again that $\{c(e)\}$ is self-editing, the result of running it on $c(e)$ will be the set of codes $\{c(e)\}$, $\{\{e\}(c(e))\}$. Thus a self-editing algorithm may compute its descendant (instead of just copying it) and exactly this simple observation is behind the idea of combining self-editing algorithms with selection: For let us assume that selection evolves the codes as it regards perpetuation. Such a system thus would evolve to be fitter, not only in computing an answer to the environment, but also in computing the codes of its descendants, formalizing thus better answers. This seems to give clues of an answer to the question whether it is possible to design a computational system that not only learns, but also learns how to learn.

The paper is organized in a self-contained manner and it does not assume any familiarity with mathematics other than mathematical reasoning and some experience with the notion of computability. However
the ideas involved are closely related to recursion, which goes back to the ancient Greeks, self-reference and the diagonal method which have been introduced and studied by various mathematicians such as Cantor, Church, Gödel, Kleene, Tarski, Turing etc. The interested reader may find relative material in any context of Set theory, Logic and Recursion Theory such as [9, 7].

Finally, it is well known that the idea of evolving by means of proliferating and selection is due to Darwin.

The research in this paper has been done in the course of many years and many people (mostly mathematicians) have contributed to it in various ways. (In chronological order) Maria Avouri [2], James Stein [11], Despoina Zisimopoulou [12], Dimitris Apatsidis [1], Fotis Mavridis [8], Antonis Charalambopoulos [3], Vanda Douka [4], Antonis Karamolegos [6], Helena Papanikolaou [10] and Miltos Karamanlis [5] are among them.

2. Structured codes and locations

2.1. The set of (structured) codes. Let $A$ be a finite set not containing the symbols "(", ")" and ",," to be used as an alphabet. Assume for simplicity that $A = \{0, 1\}$. We wish to define the set of structured codes that can be written using $A$. Obviously any finite sequence on $A$ should be a structured code. We'll call such a sequence word. For example $01, 1110011, \ldots$ are words. (In the case of words we will identify a sequence of the form $(x_1, x_2, \ldots, x_n)$ with the string $x_1x_2\ldots x_n$.) Assume now that $w_1, w_2, \ldots, w_n$ are words. We'll call the sequence $(w_i)_{i=1}^n$ phrase. A phrase is therefore a sequence of sequences of elements of $A$ and we'd like to consider it also a structured code. All the same, sequences of phrases should also belong in this set of structured codes and so on.

The following definition clarifies further the notion:

Definition 1 (Definition of the set of structured codes.). The set of (structured) codes on a finite alphabet $A$ is defined as the $\subseteq$-least set $M$ with the following properties:

(1) $A \cup \{\emptyset\} \subseteq M$, and

(2) Whenever $x_1, \ldots, x_n \in M$, $n = 1, 2, 3, \ldots$, we have also that $(x_i)_{i=1}^n \in M$.

Structured codes of the form $a(b)$ or $a(b_1, \ldots, b_n)$ are also considered and we'll explain their use when necessary.
2.2. **Locations.** We are going to need a way to navigate through a structured code. This is done using *locations*. Assume for example the code

\[ c = (01, (1, 101, 00), 110). \]

We wish to be able to refer to its various subcodes: For example in order to specify the subcode 01 to the extreme left, we may say that it lies in the (1) location of \( c \). Similarly \((1, 101, 00)\) lies at location (2) and 110 at location (3). We can specify the digit 1 which is the leftmost component of the code written in location (2) of \( c \), by saying that it lies in location (2, 1). That means the first component of the second component of \( c \). In location (2, 2, 2) lies 0, and so on. Notice that conventionally \( c \) lies in location \( \emptyset \).

Thus by the term *location* we mean a finite sequence of elements of the set \( \{1, 2, 3, \ldots \} \) to be thought of as describing a part of a structured code that we want to refer to.

We use the greek letters \( \theta, \phi, \cdots \) for locations to avoid confusion.

For simplicity we assume that both environmental input and output are coded in some specially reserved locations of the code.

We are going to establish the "meaning" of locations, by defining the way to use them. This can be done by the use of two algorithmic procedures \( \text{Read}(\cdot)(\cdot) \) and \( \text{Replace}(\cdot, \cdot)(\cdot) \).

Let’s first describe the inputs of these functions and what are intended to calculate.

**Read** has two variables. The first one is a location, say \( \theta \) and the second one a code, say \( c \). **Read** then should output the code written in location \( \theta \) of \( c \).

**Replace** has three variables. The first one is a location, say \( \theta \), the second and third ones are codes, say \( s \) and \( c \). The function then should replace the code written in the \( \theta \) location of \( c \), by \( s \). For example

\[ \text{Replace}((2, 1), 10)\left((11, (1, 01))\right) = (11, (10, 01)). \]

The following (optional to read) is a somehow technical definition of these two functions and is contained here for the shake of completeness.

**Definition 2** (Recursive definition of functions related to locations). So, let \( c, s \) be structured codes and \( \theta \) a location. Next we define the functions \( \text{Read}(\theta) \) and \( \text{Replace}(\theta, s) \), recursively on the length \( |\theta| \) of the sequence \( \theta \).

For \( |\theta| = 0 \),

\[ \text{Read}(\emptyset)(c) = c. \quad \text{Replace}(\emptyset, s)(c) = s. \]
For $|\theta| > 0$, $\theta = (n_1, \ldots , n_k, n_{k+1})$, $k \geq 0$, set $\theta_k = (n_1, \ldots , n_k)$ and by the recursive definition, let $c_k = \text{Read}(\theta_k)(c)$. Then

$$\text{Read}(\theta)(c) = \begin{cases} c_k^{n_{k+1}}, & \text{if } c_k = (c_k^1, \ldots , c_k^n) \text{ and } n_{k+1} \leq m \\ \emptyset, & \text{otherwise} \end{cases}$$

For the recursive step of Replace, let as before $c_k = \text{Read}(\theta_k)(c)$ and

- if $(c_k = (c_k^i)_{i=1}^m$ and $n_{k+1} \leq m)$ then set $\overline{c}_k = (\overline{c}_k^i)_{i=1}^m$ where

$$\overline{c}_k^i = \begin{cases} c_k^i, & \text{if } i \neq n_{k+1} \\ s, & \text{if } i = n_{k+1} \end{cases}$$

- else set $\overline{c}_k = c_k$.

Finally, using the recursive definition step, define

$$\text{Replace}(\theta, s)(c) = \text{Replace}(\theta_k, \overline{c}_k)(c).$$

Let $c$ be a code and $\theta$ a location.

- The notation $c \upharpoonright \theta$ stands for a simplified version of $\text{Read}(\theta)(c)$.
- Assume that $s$ is also a code.
  - If $\theta$ is a location in $c$, then by $c(\theta : s)$ we denote the output of $\text{Replace}(\theta, s)(c)$.
  - Else if $\theta$ is a location not in $c$, the same notation $c(\theta : s)$ will stand for a new code that results from $c$ by adding the new location $\theta$ containing $s$.

In both cases we’ll simplify the notation to $c(s)$ whenever $\theta$ is easily understood.

2.3. The problems of self-editing. By the term state code we mean a complete description of a computing system. For example,

- in a Turing machine, a description of its program, together with the tape on which the machine registers.
- In a computer program, a description of the program itself together with all the information needed to conclude the next steps of calculation.

We will assume here that given the state code $c$ of a computing system, we can deduce the algorithm, (denoted by $\{c\}$), that computes the next step of the procedure that state $c$ implies, so that the next state code of the system is calculated by this algorithm, $(\{c\})$ acting to some data contained in the code $c$. We’ll refer to such a step using the term transition of the system.

It is evident that any kind of computation can be described thus, yet this general definition includes the case where in some computational step, the algorithm changes the executable program of its code. Half
of the situation (namely algorithms that input their program) is well known in Logic, and useful to a lot of interesting and beautiful proofs therein.

This assumption, although perfectly reasonable, induces a lot of ambiguity to the issue of what such a system is capable of doing. To see a simplified example, if we assume a copying algorithm \( \text{copy} \) and it happens that \( \{c\} = \text{copy} \), then according to our assumption about transitions, the next state would be two identical codes \( c \). This sounds surprising, and in more complex situations it may become totally confusing. So, in order to simplify things and make transitions of such a system more clear, we intend to handle the difficulty of this matter, by assuming that programming instructions carried out by the algorithm \( \{c\} \), have the general form:

1. Run the code on location \( \theta \) of the input

or

2. Run the code on location \( \theta_1 \) with input on location \( \theta_2 \) and output on location \( \theta_3 \).

Concluding, we assume that any computation that the system performs is of the general form \( c_1 \mapsto c_2 \), from state \( c_1 \) to state \( c_2 \), where

\[
c_2 = \{c_1\}(c_1).
\]

Below we study some simple transitions of the form \((3)\).

2.4. Location creation. Let \( c \) be a code. Define the algorithm \( F \) by

\[
F(c) = \begin{cases} (c_1, \ldots, c_n, \emptyset), & \text{if } c = (c_1, \ldots, c_n) \\ c, & \text{otherwise} \end{cases}
\]

which adds a new location into an existing code internally, and the algorithm \( G \),

\[
G(c) = (c, \emptyset)
\]

which also adds a new location externally. Assume that \( f \) and \( g \) are respectively their codes situated in locations \( \theta_f \) and \( \theta_g \) of the code \( c \) and let \( \theta \) be any of these two locations, thus if

\[
a := \text{Run the code on location } \theta \text{ with input on location } \emptyset
\]

is executed in the sense of \((3)\), it will produce the output of \( F \) or \( G \), namely the creation of a new location.

We will abbreviate this instruction in obvious ways as for example: Create a new location.
Unless otherwise stated, we will assume that the creation of a new location preserves the meaning of any old ones.

It is useful to remark that if the code \( a \) in (4) is situated itself in an location \( \theta_a \) of \( c \), we can also run the code in the location \( \theta_a \) with the same effect.

3. Programming and Self-editing.

3.1. Memory. Assume a general transition

\[
(5) \quad c \mapsto c',
\]

where as in (2), \( \{ c \} \) is the algorithm

Run the code on location \( \theta_1 \) with input on location \( \theta_2 \) and output on location \( \theta_3 \).

which runs with input (the code) \( c \).

We wish to transform (5) into

\[
(6) \quad c \mapsto c'(\theta_m : c),
\]

where \( \theta_m \) is a new location (not existing in \( c \)) for the simple reason to keep track of the previous code. To this purpose we consider the following alternative form of (2):

Let \( x \) be the code on location \( \emptyset \). Run \( x \upharpoonright \theta_1 \) with input \( x \upharpoonright \theta_2 \) and output in \( x \upharpoonright \theta_3 \) and let \( x' \) be the result.

Next create a new location \( \theta_m \) in \( x' \) and store there \( x \).

Output \( x'(\theta_m : x) \) on location \( \emptyset \).

It is easy to check that if the code of this algorithm is activated, then we’ll achieve the required transition (6).

A transition \( c_1 \mapsto c_2 \) is called memory storing if there is a location \( \theta \) such that \( c_2 \upharpoonright \theta = c_1 \). Transition (6) is an example of such a transition.

We should remark that transition (6) is very space consuming and is considered only for theoretical reasons, since it is guaranteed that it stores all available information.

3.2. Code generators. We are going to need a general notion of a code generator: Assume that \( \Lambda = \{ A_i : i = 1, \ldots, n \} \) is a finite set of algorithmic procedures \( A_i \). Let \( a_i \) be a code for \( A_i \), \( i = 1, \ldots, n \). A code generator with lexicon \( \Lambda \) is a computational system with initial configuration \( c \) such that:

1. \( c \) contains for all \( i \), \( a_i \) in some locations \( \theta_{a_i} \).
2. If \( c_1 \mapsto c_2 \mapsto \cdots \mapsto c_k \) is a calculation with \( c_1 = c \), then for every \( \ell < k \), \( c_\ell \mapsto c_{\ell+1} \) is the result of a transition of the form (1) or (2) or (7).
The output of the search is considered \( c_k \), or a code in a fixed location of \( c_k \).

A set of algorithmic functions and operations will be called *adequate*, if its closure is the set of algorithmic functions. Candidate examples for this definition are any programming language, Turing machines or \( \mu \)-recursive functions. It is known (see [7]) that all these models are equivalent regarding their closure.

One can see that if \( \Lambda \) is an adequate set of algorithmic functions, then every algorithmic function is (theoretically) the output of a code generator with lexicon \( \Lambda \). The reason is apparent if we use as \( \Lambda \) a lexicon of a programming language. In this case we may simulate the execution of a given instruction \( A \) of the language by the execution of its code \( a \) which is situated in a location of the structure \( c \). If necessary, the input of \( A \) can naturally be stored in some locations created for this purpose.

What is more interesting here is that the selection of \( \Lambda \) greatly impacts the probability under which various outputs may occur.

### 3.3. Parallel computation

A parallel action of two algorithms \( A, B \) on a code \( c \), denoted by \( \{A, B\}(c) \) is defined as a sequence of computational steps

\[
\{A, B\}(c_1) \mapsto \{A, B\}(c_2) \mapsto \cdots
\]

where \( c_1 = c \) and in every transition \( \{A, B\}(c_i) \mapsto \{A, B\}(c_{i+1}) \), algorithms \( A \) and \( B \) function as having \( c_i \) as an input and one or more locations of \( c_i \) as an output. Computational steps are considered behaving according to the following conventions:

1. Locations on \( c_i \) that are not output of any of \( A, B \) retain their content in \( c_{i+1} \).
2. Locations on \( c_i \) that exactly one of \( A, B \) outputs, change according to this output in \( c_{i+1} \).
3. In the case where \( A, B \) output to the same location, the computational step will be considered non deterministic and thus \( c_{i+1} \) may contain in this location either \( A \)'s output or \( B \)'s. For the moment we are not interesting on this non deterministic behavior, so we 'll postpone fixing the details for later on.

Parallel computation is not essential, and it can be simulated by sequential computation, yet it greatly facilitates our study, as in the case of memory storing, so we 'll assume hereafter that any transition \( c \mapsto c' \) is a set of instructions of the form (1) or (2).
3.4. Recursors. Let $c \rightarrow c'$ be a transition of a computational system and $\theta$ a common location in $c, c'$. By our convention about transitions, $c' \upharpoonright \theta$ is the result of $\{c\}(c)$, restricted to location $\theta$. For reasons that will become apparent soon below, we are going to identify not taking any action about $\theta$ with copying $\theta$. Obviously these two materialize the same algorithm.

On the other hand it will be very useful to override this behavior by means of a given algorithm that we wish to use to compute $\theta$. Such an algorithm will be called the recursor of $\theta$, or $\theta$-recursor. A code for such an algorithm will be called also by the same term. Thus given a code $r$, the instruction

Use $r$ as a $\theta$ recursor

should be thought of as an instruction for the algorithmic procedure

$c \mapsto \{c\}(c) \left(\theta : \{r\}(c)\right)$

that replaces the contents of $c' = \{c\}(c)$ in location $\theta$ with $\{r\}(c)$ as intended.

It should be noticed here that we are going to assume (and use) that the location of a $\theta$ recursor is the same for fixed $\theta$.

We will refer to such a transition by the term $r$ diagonalisation of $\theta$.

Let $c \rightarrow c'$ a transition and $r$ a recursor. We say that $r$ is compatible with $c \rightarrow c'$, if

$\{r\}(c) \upharpoonright \theta = c' \upharpoonright \theta,$

for all locations $\theta$ such that $\{r\}(c)$ outputs to the $\theta$-location. Notice here, that the definition has nothing to do with the actual computation of $\theta$ location of $c'$. We are mainly interested in the situation in which running the code $r$ on the same input will produce the same result.

4. Diagonalisation procedures over memory

Assume $c$ is a state code such that the transitions with initial state $c$ are all storing memory ones. Thus a calculation beginning with $c_1 = c$ and of length $n + 1$ will be of the form

$\begin{align*}
  c_1 &\mapsto c_2(\theta_1 : c_1) \mapsto \cdots \mapsto c_{n+1}(\left(\theta_i\right)_{i=1}^n : \left(c_i\right)_{i=1}^n),
\end{align*}$

where all locations $(\theta_i)_i$ are new. Let

$\begin{align*}
  c'_1 &= c_1, \\
  c'_{i+1} &= c_{i+1}(\left(\theta_k\right)_{k=1}^i : \left(c_k\right)_{k=1}^i), \quad 1 < i \leq n.
\end{align*}$
For any $i$ in the range $\{1, \ldots n\}$, in the transition $c_i \mapsto c_{i+1}$ the active algorithm may be regarded as having as an input the sequence $(c_1, \ldots, c_i)$, $c_i$ being the ‘now’ active state.

A \textit{diagonalisation procedure (over memory)} is generally an instruction that relates the sort of application of a candidate recursor, based on a memory test to establish validity.

A simple yet basic example of this kind, given a location $\theta$ and an $n_0 \in \{1, 2, \ldots \}$, is the following instruction:

\begin{equation}
\text{If } n \geq n_0 \text{ and for all transitions } c_i \mapsto c_{i+1}, 1 \leq i < n, \text{ the same code } r \text{ has been used as a } \theta \text{ recursor, then use it also in the current transition.}
\end{equation}

In most cases we are going to use (9) as follows: Assume $n_0 = 2$. Let $c_1 \mapsto c_2 \mapsto c_3$ be a sequence of transitions, for which $r$ is a code which happens to have been used as a recursor for some location $\theta$ in both transitions $c_1 \mapsto c_2$ and $c_2 \mapsto c_3$. If (9) is active during the transition $c_3 \mapsto c_4$, then $r$ will also be the $\theta$ recursor of the latter, and recursively (assuming the activation of (9)) of every transition thereafter.

Let us see a more concrete example of the application of (9). Assume $\theta$ is a location in the state code $c$, where an integer is written, that is $c \upharpoonright \theta = m \in \mathbb{Z}$. Assume also that the $\theta$ recursor is the non deterministic algorithm $A$, with

\[
A(m) = \begin{cases} 
\text{either } m + 1, \\
\text{or } m, \\
\text{or } m - 1.
\end{cases}
\]

Thus in the sequence of transitions (8), for every $i$,

\[
c_{i+1} \upharpoonright \theta \in \{(c_i \upharpoonright \theta) + 1, c_i \upharpoonright \theta, (c_i \upharpoonright \theta) - 1\}.
\]

Presumably, if diagonalisation (9) is active during the transitions and for all $i \leq n_0$, $c_{i+1} \upharpoonright \theta$ happens to be equal to $(c_i \upharpoonright \theta) + 1$, then the same will happen also for $i > n_0$, deterministically.

Likewise, if for all $i \leq n_0$, $c_{i+1} \upharpoonright \theta = c_i \upharpoonright \theta$, then also $c_{i+1} \upharpoonright \theta = c_i \upharpoonright \theta$ for $i > n_0$.

It is worth to remark here for future reference, that if (9) is active, then after $n_0$ transition, locations that incidentally have not be changed, will be copied thereafter.

In order to use diagonalisation, we’ll have to combine it with proliferation and selection.

4.1. \textbf{Proliferation.} We assume here a population of state codes. A set of them consisting of say $c_1, \ldots, c_n$, will be denoted by $\{c_1, \ldots, c_n\}$. 

Clearly, the function $P$ defined on the set of structured codes as:

$$
P(c) = \begin{cases} 
\{c_1, \ldots, c_n\}, & \text{if } c = (c_1, \ldots, c_n) \\
c, & \text{otherwise}
\end{cases}
$$

is recursive, so if $p$ is a code for it, situated in location $\theta$ of some state code $c$, its activation on location $\emptyset$ will produce the transition given by (10).

Finally, given codes $(e_i)_{i=1}^n$ situated in locations $(\theta_i)_{i=1}^n$ of a state code $c$, the transition

$$
c \mapsto (\{e_1\}(c), \ldots, \{e_n\}(c))
$$

may be produced, and the latter combined with (10) will give

$$
c \mapsto \{\{e_1\}(c), \ldots, \{e_n\}(c)\}
$$

proliferating thus $c$ according to a given (by $(e_i)_i$) calculation of its descendants. Codes $e_1, \ldots, e_n$ will be called also recursors of the corresponding descendants. We should remark here that every separate birth

$$
c \mapsto \{\{e_i\}\}(c)
$$

may be regarded as any other transition we have discussed until now. (The main difference being the switch from not outputing to $\emptyset$ to copying $\emptyset$.) For example, choosing appropriately $e_i$, we may (or, depending on the descendant, may not) assume that transition (12) is a memory storing one, etc.

4.2. Selection. Hereafter, by the term selection we will mean the act of choosing a subset of descendant codes in (11) to continue the calculation excluding the rest. Clearly, such an act may be completely random and therefore containing no information at all.

4.3. Recursor validity. Given a state code $c$ and a location $\theta$, a $\theta$ instantly valid recursor for $c$, is a code $r$, such that its activation as a $\theta$ recursor on $c$ is environmentally preferable.

A $\theta$ valid recursor (for $c$,) $r$, is one such that for every surviving sequence $c = c_1 \mapsto \cdots \mapsto c_n$, $r$ is $\theta$ instantly valid for $c_n$.

In the special case where $\theta = \emptyset$, $\emptyset$ may be omitted from the previous terminology.
4.4. **Effect of diagonalisation.** Before passing to explore further instructions like (9), let us indicate how we are planning to use them:

Assume a proliferating population of self-editing codes that undergo some sort of *logical selection*. Even though we do not understand anything about the nature of such a selection, we might be able to help such a system by trying to fill the dots in a surviving sequence of descendants:

\[(13)\]

\[c_1 \mapsto c_2 \mapsto c_3 \mapsto \cdots.\]

The reason is that with the aid of an appropriate structure we could be able to guess a pattern in the surviving sequence (13) that reveals the preferences of this logical selection. The point is that a self-editing algorithm can fill the dots the same way as we would and a very simple example of this is exactly (9). We will leave the consequences of filling (13) for the examples that are presented after the diagonal instructions (16), (17) and (21).

Assume a proliferating transition

\[(14)\]

\[w \mapsto \{w_1, \ldots, w_n\}.\]

Let also $\theta$ be a location. We will call (14) *$\theta$ searching*, if the values $w_i \upharpoonright \theta$ vary according to $i$. If (14) is $\theta$ searching, every $w_i$ or $w_i \upharpoonright \theta$ will be called a *$\theta$ attempt*, and if the code $r_i$ has been used as a $\theta$ recursor for computing $w_i \upharpoonright \theta$, $r_i$ will be called *$\theta$ attempting recursor*.

If (14) is not $\theta$ searching, it will be called *$\theta$ ruled*, and in the case where in the calculation (14) has been used the same $\theta$ recursor $r$ for every descendant $w_1, \ldots, w_n$, then we’ll call it *$\theta$ ruled by $r$*. In this case, $r$ is called a *$\theta$-ruler* or a *$\theta$-stabilizer*. So, if (14) is $\theta$ ruled by $r$, then

\[w_1 \upharpoonright \theta = \cdots = w_n \upharpoontright \theta = \{r\}(w).\]

Let now

\[(15)\]

\[c_1 \mapsto c_2 \mapsto \cdots \mapsto c_n,\]

be a sequence of transitions. We will call $i$ (or $c_i$) a *$\theta$ searching point* of the sequence if the transition induced by $c_i$ is $\theta$ searching.

Consider now the following variation of (9):

\[(16)\]

*Given a location $\theta$, if there are more than $n_0$ $\theta$-searching points in the memory and for every $\theta$ searching point $c_i$, the same code $r$ has been used as an attempting $\theta$ recursor in the transition $c_i \mapsto c_{i+1}$, then hereafter use $r$ as a $\theta$ ruler.*

Let us explore the use of (16): Assume that for some code $r$
(1) The algorithm

Run \( r \) on location \( \emptyset \) and output to \( \theta \),

is valid.

(2) \( r \) itself is \( \theta \) attempting in every \( \theta \) searching point.

Due to (1) and (2) above, it is expected that in every \( \theta \) researching point of surviving branches (15), \( r \) has been used as an attempting \( \theta \) recursor. So granting that (16) is activated, after enough \( (n_0 \text{ precisely}) \) searching points for \( \theta \), the system will continue using \( r \) as a \( \theta \) rule.

The following is a straightforward consequence of the above principle:

**Example 1.** In an evolving system, assume a location \( \theta \) where copying is valid. If there are copying attempts for \( \theta \) and moreover the system diagonalises by (16), then the system can learn to copy \( \theta \).

5. More diagonals.

There is no reason to restrict our study to the basic diagonal (16). For example, and for future use also, we can replace the search of a consistently repeated partial recursor, by the search of a consistently repeated sequence of partial recursors. We do not intend to investigate this now, instead we would like to introduce a more interesting kind of diagonal instruction which reads as follows:

\[
\text{(17) Given a location } \theta, \text{ if there are more than } n_0 \text{ } \theta\text{-searching points in the memory, search to find a code } r \text{ such that for every } \theta \text{ searching point } c_i, \{r\}(c_i) = c_{i+1} \uparrow \theta. \text{ If you find such an } r, \text{ use it hereafter as a } \theta \text{ ruler.}
\]

There are various parameters to consider about an actual use of the previous instruction:

First of all we should fix \( n_0 \). Secondly, we should fix the program to be used for the computation of such an \( r \). And lastly we should also put a limit in the search, otherwise we run into the possibility of searching in vain for something that simply does not exist (or is practically very difficult to find). We won’t deal with these problems right now. Instead we are going to assume a basic functionality of (17) just enough to produce some simple codes. We’ll refer to such a code by this same term *simple code*. Thus, a simple code \( x \) is a code that we strongly expect on a probabilistic setting, to be produced by the procedure used in (17) to compute \( r \).

Obviously, we can generalize the discussion after (16), as follows:
Assume that \( r \) is any simple code and \( \theta \) any location such that
\[
\text{Run } r \text{ on } \emptyset \text{ and output to } \theta
\]
is valid. Then by means of (17) the evolving system can learn to use \( r \) as a \( \theta \) ruler.

### 5.1. Some discussion about validity

Before passing to demonstrate potential uses of (17), we should notice that obviously (17) is not valid, since for every finite set of pairs of transitions
\[
(18) \quad c_1 \mapsto d_1, \ldots, c_{n_0} \mapsto d_{n_0}
\]
there are infinite algorithms \( A \) such that \( A(c_i) = d_i \upharpoonright \theta \). Nonetheless we are going to use its flexibility in order to detect valid transitions.

It is interesting here to notice that everything works very well when \( n_0 \to \infty \). The reason is that if \( v \) is a valid recursor for \( \theta \), and \( r \) any code with different algorithm, then for sufficiently large number \( n_0 \), in almost every set of at least \( n_0 \) pairs as in (18) with
\[
\{ v \}(c_i) = w_i \upharpoonright \theta, \quad \text{for all } i,
\]
we expect that for at least one \( i \),
\[
\{ r \}(c_i) \neq w_i \upharpoonright \theta.
\]

It can be easily seen that this observation yields that for any valid recursor \( v \) and any code generator with an adequate lexicon, there is a (sufficiently large) \( n_0 \) such that it is expected that \( v \) (or a code generating the same algorithm) is the simpler code that fits a total of \( n_0 \) \( \theta \)-searching points.

### 5.2. Understanding implications empirically

We should notice here that we may use (17) with much more flexibility, for example for any code \( r \) of a valid algorithm of the conditional form
\[
\text{if } A \text{ then } B,
\]
granting that \( r \) is simple. Let us formalize this a bit better:

Notice first, that if condition \( A \) is not parametrized, then it is assumed that it is always either true or false and we are interesting in more complex situations. Therefore assume that \( A = A(x) \) and its validity depends upon \( x \). Assume that \( a(x) \) is a code such that
\[
\{ a(x) \} = 1 \iff A(x) \text{ is true}.
\]

About condition \( B \), let \( b \) be a code of a recursor, such that
\[
b \text{ is a valid recursor } \iff B \text{ is true}.
\]

It is obvious that we choose here to translate the validity of condition \( B \) into the validity of a recursor. This is done for convenience, since it
is going to be more useful in this form. Anyway we may reconstruct the
full meaning of understanding \( A \rightarrow B \), assuming that the activation of
the code \( b \) simply means that the system understands that \( B \) is correct.
So the target of our mental experiment is to see whether
\[
\text{(19) } \text{If } \{ \{ a(x) \} \} = 1 \text{ then activate } b
\]
can be established by means of (17). In order to continue, let us com-
pare this to a real life test: For there is not always the case that an
intelligent being could empirically establish (19) even if it is valid. It
is easy to check that this is so, granting that one notices \( A \) being in-
teresting about the validity of \( B \). We are going to make a translation
for this condition into our system: The analogue for ’interesting about
the validity of \( B \)’, would be to use (17) for the location \( \theta_b \) of activation
of the code \( b \).

For ’noticing \( A \)’, the translation would then be to run the code \( a \) say
in location \( \theta_a \) with input in location \( \theta_x \).

It is easy now to check then that a code \( r \) for the algorithm:

\[
\text{Run the code in location } \theta_a \text{ with input in } \theta_x \text{ and if the }
\text{output is 1 then activate the code in location } \theta_b
\]
satisfies the conditions in (17) to be validated to be used as a ruler (if
simple).

5.3. Generalizing. We are going to investigate here if (17) may be
used to generalize. So let us consider as an example a valid expression
of the form:

\[
\forall x \left( A(x) \Rightarrow B(x) \right).
\]

By replacing as before the occurrences of \( A \) and \( B \) with the codes \( a \)
and \( b \) we are going to assume that for a sufficient number of elements
\( x_i, i = 1, \ldots, k \) the system has experienced that if \( \{ \{ a(x_i) \} \} = 1 \) then
\( b(x_i) \) is a valid recursor. Assuming that \( b(x_i) \) is the recursor that was
randomly used in such cases, and \( \theta_a, \theta_b, \theta_x \) as before, in can be easily
seen that the code

\[
\text{Run the code in location } \theta_a \text{ with input in } \theta_x \text{ and if the }
\text{output is 1 then activate the code in location } \theta_b \text{ with input }
\text{in } \theta_x
\]

if simple, it will be validated by (17) to be used as a ruler.

5.4. A self-editing example. A surprising consequence of self-editing
and diagonalization is that an established diagonal instruction may es-
ablish another diagonal instruction granted that the second one has a
simple code. This follows since a diagonal instruction may be described as a general algorithm of the form

\[
\text{If } A(r) \text{ then } B(r),
\]

where \(A(r)\) involves a test of \(r\) over memory content and \(B(r)\) involves using \(r\) as a recursor. As we discussed earlier a diagonal instruction need not be valid, yet if practically useful it should be almost valid in the sense we also discussed earlier. Thus this could be thought of as the case of a generalization over an implication and we have discussed both procedures in the previous examples.

An interesting example of this kind is a diagonal scheme that may enrich the lexicon used by a state code \(c\) with memory \(c_1, \ldots, c_n = c\) and it reads as follows:

\[
\text{Given } r \text{ and } \theta \text{ assume that } \{\{r\}\}(c_i) = c_{i+1} \upharpoonright \theta \text{ is true for a proportion } p \text{ of } \theta\text{-researching points } c_i. \text{ Then use } r \text{ as a } \theta \text{ recursor with probability } p.
\]

(20)

\[6. \text{ Localized diagonals}\]

We are interested here in diagonals that follow by testing a recent (usually short) part of memory. The action to be taken is to use a candidate code as a recursor according to some predetermined frequency. In the general case we assume that this is calculated by an algorithm of code \(f\) which inputs a positive integer \(N\) (which is supposed to be the number of \(\theta\)-attempts) and outputs a positive integer \(K\) (which is the number of exceptions to the rule). Thus the intended meaning of

\[
\text{Use } r \text{ as a } \theta \text{ recursor with exception frequency } f,
\]

is to ensure that in a proliferating \(\theta\) transition with \(N\) \(\theta\)-attempting recursors \(e_1, \ldots, e_N :\)

\[
c \upharpoonright \theta \mapsto \{\{e_1\}\}(c) = c_1 \upharpoonright \theta, \ldots, \{\{e_N\}\}(c) = c_N \upharpoonright \theta,\]

the number of \(i\)'s such that \(r \neq e_i\), is the output of \(\{f\}\) on input \(N\).

To begin with, assume that \(c_1, \ldots, c_n = c\) is the memory input of the algorithm \(\{c\}\) of a state code \(c\), and let \(k_0\) be a small positive integer. A local diagonal, given a candidate recursor \(r\) and an exception frequency code \(f\) (or their locations), is of the general form:

\[
\text{Given a location } \theta, \text{ if there are more than } k_0 \text{ } \theta\text{-searching points in the recent memory, search to find a code } r \text{ such that } \{r\}(c_i) = c_{i+1} \upharpoonright \theta \text{ for every } \theta \text{ searching point } c_i \text{ and use it hereafter as a } \theta \text{ recursor with exception frequency } f.
\]

(21)
Next, we are going to investigate through some examples the use of (21). We are going to assume as before that the candidate recursor \( r \) in this instruction, is the result of some search using an appropriate lexicon of codes. In all examples that follow, we set \( k_0 = 2 \), yet we plan to initiate a discussion about an optimal value.

6.1. **Simple lessons in a sequence.**

6.1.1. A simple example. In this example, we’ll assume a location called *environmental output* in which we wish to test the ability of the system to deduce and fill the gaps of given structured sequences. Such a simple test would be for example to fill the fourth place in the sequence 1, 2, 3, ?. We will make some conventions about this kind of test, which have to do with the initial lack of our system’s capability to communicate.

The main convention is that instead of giving the first digits of the sequence to be filled, we will demand from the system to guess the entire sequence. We will call the digits that we want to reveal, as the digits 1, 2 and 3 above, *non intended to be guessed digits*. The above convention, has two aspects:

First, we will have to give the system some attempts to guess a digit which is not intended to be guessed, for example the very first digit. Doing so, we will identify attempts to guess a particular digit, with attempts (as they have been already defined) of proliferating transitions. This convention is established in order to avoid more complex situations with which we’re going to deal later on.

Secondly, we’ll take for granted that (somehow) always at least one of the descendants guesses right about a non intended to be guessed digit.

Since we would like to deal with structured sequences, we will often use left and right parentheses (, ) to denote the beginning and the end of a sequence respectively. Parentheses are considered also part of the sequence, so they have to be guessed, intended or not.

Let us begin with describing the mental experiment of a simple sequence as 1, 2, 3, ?. We will need to formulate a language to speak about the parts of such tests and fortunately we have it already: We will refer to the first digit of the sequence as the *digit in location* (1) of the test etc. In this experiment we assume that we begin with a state code \( c_1 \) with 1 (the digit in location (1) of the test) written in location environmental output of \( c_1 \). Since now 2 (the digit in location (2) of the test) is not intended to be guessed, we will assume that there is a proliferating transition after \( c_1 \) which gives (at least one) descendant \( c_2 \) with 2 written in environmental output. The same assumption holds true
for the next digit, and at this point we have come up with a sequence $c_1, c_2, c_3$ where $c_i$ writes $i$ in location environmental output. Granting (21) and assuming that the code for the function $n \mapsto n + 1$ is simple, all descendants of $c_3$ (except $f$ many) will write 4 in the corresponding location, signaling the guessing of the fourth digit.

Let us make some first remarks:

(1) The example functions equally well if we would like two or more digits to be guessed.

(2) Exception frequency does not play a major role here. We will see its use later on.

6.1.2. A more complicated example of the same kind. Let us try to generalize the previous example, by demanding a sequence of sequences of guesses. This will enable us to see what happens both with exception frequency and of the use of parentheses. So let us assume a simple example of this form:

\[(22) \quad ((1, 2, 3, \ldots), (1, 3, 5, \ldots), (1, 4, 7, \ldots), \ldots)\]

We may use here also, the terms $\theta$ test or experiment to refer to the various experiments included in a composite one like this, so for example (1) test of (22) is $(1, 2, 3, \ldots)$. (2) and (3) experiments are defined similarly and by $\emptyset$ experiment we mean all of (22).

The intended successful outcome of this test in all $(i)$ experiments, $i = 1, 2, 3$ is the guess of digits after $(i, 3)$, and these can be thought of as in the previous example. Yet we have here the chance to consider a far more serious problem to be solved, namely the guess of the experiments $(i)$ for $i > 3$. According to the previous terminology, this is the $\emptyset$ experiment.

So, for the guess in $(\emptyset)$, let $\theta$ be the recursor location of location environmental output. Then, during (1) experiment, it is expected that the code written on $\theta$ is a code $s(1)$ for Add 1. Similarly, for (2) and (3), $s(2)$ and $s(3)$ are expected to be written in $\theta$, the later being the codes for Add 2 and Add 3 respectively. Assume now in addition that the code of the number $i$ is, (as it is actually written here,) in some location of $s(i)$, that is in a sublocation of $\theta$. Since it is assumed that Add 1 has a code that is simple, the same diagonal (21) will guess the value $4 = 3 + 1$ for the same location in (4) experiment, so that (not counting parentheses) the guessed fourth sequence is expected to be

\[x_1, x_1 + 4, x_1 + 8, \ldots\]

Again if Start with 1 has a simple code, then since in the first three cases the sequence starts with 1, granted again the localized diagonal
(21), the expected value of $x_1$ will be again 1, resulting in the sequence
$$1, 5, 9, \ldots$$
as wished.

Before we examine further this approach, let us remark some things about the use of parentheses. First notice that here is apparent the use of the exception frequency $f$ in localized diagonal (21). Indeed, assume that in guessing of test $(i)$, the digits in $(i, 4)$ up to $(i, n(i))$ are intended to be guessed. As a first case, consider that the numbers $n(i)$ are not fixed. In this case, the exceptions of the frequency $f$ are to ensure that at this point, any state code will produce descendants both of the kind that guess the next digit $(i, n(i) + 1)$ and some exceptions to this rule, in order to ensure survival. Here, since the digit is not intended to be guessed, according to our conventions we take for granted that in some attempt at least one descendant writes the right digit in environmental output, which in this case is a right parenthesis $)$. Up to now this convention holds for all $(i)$ experiments. Yet, evidently, we would like this to be guessed also by our system.

To investigate this, assume a location $\phi$ that contains a code to be executed for the calculation of the descendants following the frequency exception $f$ of (21). Following the same arguments, let $a$ be a code that changes this behavior into writing $)$ in environmental output in the case of exception and assume that $a$ is simple. By localized diagonal (21) again, $a$ will apply to the code in $\phi$ causing (almost) all exceptions to write (the correct) right parenthesis digit $)$, after experiment (2).

Similar arguments may be used to show various other cases of such a guessing and it is interesting to analyze a few more. Assume for example that the number of guesses to be filled in any $(i)$ experiment is fixed. Again if a code that forces this is simple and some first experiments have been successful then diagonal (21) will ensure the application of this code, guessing thus the number of guesses to be filled in the experiments.

6.1.3. **Guessing in higher ranks.** We may rank experiments according to the rank (level) of the recursor that is intended to be used. So the experiment in the first example is of rank 1, whereas that of the second of rank 2. More accurately (for this example) the rank of an experiment may be defined as the rank of the tree of the experiment’s locations.

It is evident that the ideas about guessing an example of rank 2, can be generalized further for guessing in higher ranks $n \in \mathbb{N}$ if necessary. An interesting question that arises naturally, is if the guessing based
on diagonals is closed under recursion experiments of greater than finite rank. As we see below, this is not to be thought as an extreme theoretical case.

Consider for example the case of a sequence to be filled of the form \((a_1, a_2, a_3, \ldots )\), where \((\text{rank}(a_i))\) is a non bounded sequence of integers. For example let \(\text{rank}(a_i) = i, i = 1, 2, 3\). Since a sequence to be filled has to be somehow "logical", it should be expected that for the intended to be guessed \(a_4\), \(\text{rank}(a_4) = 4\). This is much more complicated and essentially not to be examined here, yet let us give several general directions, since they contain also some useful discussion:

Inductively, let \(c_1, c_2, c_3\) be the sequence of state codes that solve experiments \(a_1, a_2, a_3\) respectively. Then there is a substructure of \(c_1, c_2, c_3\) that has been created exactly for solving the respective experiments, the local program, so to speak, for producing \(a_1, a_2, a_3\) respectively. Since our algorithm has to guess non intended to be guessed digits in order to pass from solving \(a_i\) to solving \(a_{i+1}\), we will have to assume that \(c_2\) and \(c_3\) are the surviving branches of attempts to solve the problem.

We have several options here regarding both the input memory sequence of localized Diagonal \((21)\), and the location \(\theta\) in which it applies.

Ideally, we would like to have in place of the memory input the sequence \((c_1, c_2, c_3)\) and in place of the location \(\theta\) that localized diagonal applies to, the (one) location of the programs that solve respectively \(a_1, a_2\) and \(a_3\). The same argument then would apply a simple code \(r\) to that location that stabilizes the sequence and hopefully this would be the required code to guess \(a_4\).

There are nonetheless problems to the above scenario in two places: The first one is that it requires a structured memory and not just the sequence of transitions. Until now this has been solved by the simplicity of the experiments: Our assumption to solve experiments is that \(c_1, c_2, c_3\) have to be the successors of the researching points of \(\theta\), otherwise that the transition \(c_i \rightarrow c_{i+1}\) has exactly one \(\theta\) researching point, yet it can be easily seen that in a more complicated and general experiment, \(\theta\) researching points might need to be more than one between the two successful states \(c_i\) and \(c_{i+1}\). Thus it is evident that for more complex experiments we will need to consider a more sophisticated mechanism of memory. Notice, for later use, that this could be somehow surprisingly solved assuming that we can ensure that memory mechanisms can evolve to be more successful. (Something we intend to discuss later on.)

The problem of location naming can be solved also in a similar manner: The simple case and the not simple one. In the simple case, we
may assume that the recursor \( r \) may be tested to \( \emptyset \) location, that is on \( c_i, i = 1, 2, 3 \) themselves. Obviously, this diminishes the efficacy of the searching, yet on the other hand it does not require more sophisticated mechanisms. In the not simple case, we would need more sophisticated mechanisms to naming substructures, and we haven’t discussed this. The previous notice fits here also: If we could ensure that ”naming” can become an evolving mechanism, then the problem would be automatically solved.

Notice that in all cases the recursor \( r \) to solve a sequence of unbounded rank, has to be constructive, in the sense that it has to create new locations as recursor locations of old ones. This remark on the other hand suggests that diagonalisation in general may occur also in creating locations (and thus naming).

6.2. **Smoothly changing environment (yet another example).** This example is similar to the previous one, the main difference being that we omit the parentheses. It is intended to be applied in combination with a smoothly changing environment in order to check the ability of our system to adapt in it. We isolate a single parameter changing, such as temperature for example, and assume that our evolving system has to adapt accordingly. The assumptions about its ability to adapt are also simplified to the following: The adaptation takes place by editing a corresponding number in the state code. For our example assume that \( \theta \) is such a location in \( c \) which contains an integer. Adaption is thought to be made by raising, lowering or just keeping the same number in \( \theta \). As in the previous example this can be done using proliferating attempts. Thus, it is not assumed that there is a kind of sensing this particular environmental parameter, other than life or death of the descendants. We may and do categorize such a single parameter smoothly changing environment, into degrees by considering the rate of change of the variable: Let

\[
(23) \quad n_1, n_2, \ldots, n_k
\]

be the sequence of the values that should be guessed in \( \theta \). The *changing rate* of that sequence is defined to be the sequence

\[
 n_2 - n_1, n_3 - n_2, \ldots, n_k - n_{k-1}.
\]

A *first degree smoothly changing environmental variable* is one whose rate is constant.

It is easy to see that by (21), a system can guess a first degree smoothly changing environmental variable which is named in some location of an evolving sequence of codes: For if \( n_1, n_2, \ldots, n_k \) are the
values of this variable that should be guessed in a location \( \theta \), granting that the code \( a(n_2 - n_1) \) of

Add \( n_2 - n_1 \) to location \( \theta \) and output to the same location,

is simple, then since the sequence is of first degree, for every \( i < k \),

\[
  n_{i+1} = a(n_i),
\]

so that the application of (21) with code \( a(n_2 - n_1) \) should guess the intended to be guessed sequence.

Similarly, a second degree smoothly changing environmental variable is one whose the rate of its rate is constant. Let us continue the previous example into an example where the sequence

\[
  n_1, n_2, \ldots, n_k, n_{k+1}, \ldots, n_{k_1}
\]

has to be guessed in \( \theta \), and \( n_{k-1}, n_k \ldots, n_{k_1} \) is smoothly changing of the second degree. So, if (using the same as above code \( a \)) \( a(m_i) \), where \( m_1 = n_2 - n_1 \) is the sequence of recursors of the location \( \theta \), which is situated in some location \( \theta_1 \), then the sequence \( (m_i)_i \) is of first degree and in some sublocation of \( \theta_1 \), so again by (21) the sequence can be guessed.

It is clear that we can generalize the notion of first and second degree to an arbitrary \( n \)th degree. The previous example modified accordingly can show that (21) can be used also to guess a variable that is locally smooth, i.e. consists of non-disjoint intervals where the variable is \( n \) smooth for some \( n \).

It is obvious that in our consideration it is assumed that at the beginning, the evolving system knows the correct answer of the variable situated in \( \theta \). A very interesting question that arises here, is if we can assume that beginning with a false value will have the same effect.

So assume as a simple case that the sequence \( n_1, n_2, \ldots, n_k \) to be guessed is constant and our system guesses \( n'_1 \neq n_1 \), meaning that \( n'_1 \) is written in \( \theta \). In order to solve this using (21), we will assume that the system has the information of the distance from the correct value in the following manner: Units that approach better this value are more likely to survive. We have to consider two similar cases here:

The first is that \( n'_1 < n_1 \). (The second being similar.) This implies that descendants that compute a higher value in \( \theta \) have a greater probability to survive, so that we may expect that in location \( \theta \) of the surviving descendants is written a value \( n'_2 \) greater than \( n'_1 \). If it is the case where still \( n'_2 < n_2 = n_1 \) then again a still greater value \( n'_3 \) is needed and we will assume that is computed randomly into at least one’s descendant location. Using (21) now for \( \theta \), we conclude that the system will continue to produce still greater values guessing thus that the correct value is higher. Clearly, this is not always the case, yet it
seems to be the best strategy among all such cases, since it is the obvi-
ous (for us) way to fill a sequence \( n'_1 < n'_2 < n'_3 \ldots \). It is evident that
approaching the correct value \( n_1 \), the exception \( f \) is going to be used,
to compute a descendant that guesses (randomly) the correct value for \( \theta \). This is also to be expected, since there is no other information about
the correct guessing. Yet if a correct value is guessed, then again by (21) the system can guess that it is constant, by copying it. (As always
we will have to assume that copying has a simple code.)

For simplicity we have constructed the example using integers, yet
it is obvious that what we really need is an appropriate value \( \Delta T \) to
use as a unit trying to guess the correct value for \( T \). Finding the best
\( \Delta T \) to be used is not a subject of our treatment here, yet it should
be noticed that diagonalisation could also play an important role: For,
assuming that the value that is used is stored in a location \( \theta \), we may
also apply the same strategy as in the example to guess the correct \( \theta \)
number. Codes that we may use are for example to divide or multiply
by 2 the number in \( \theta \) instead of adding or subtracting 1.

Let us notice that we may use diagonalising by (21), to guess the
correct (fitting the example) number \( k_0 \) in this same instruction, a
classical effect of using self-editing. (We just have to use the instruction
on the location of \( k_0 \).)

6.3. A crucial example: Memory sequence. Up to now we have
theoretically assumed that memory storing is active at any transition.
It is obvious that this cannot be of use, but only for theoretical study. In
practice, memory length simply cannot grow for ever. So the question
arises naturally: How can one compute the necessary memory length
to accomplish specific tasks at hand.

A related and more complicated problem about memory is the choice
of locations \( \theta \) that can serve for \( \theta \) researching and \( \theta \) diagonalising.

Due to the self-editing properties, we could arrange both problems
using diagonalisation.

For the memory length problem, it suffices to assume that the length
of memory is referred within the structure in some location. Then
researching and diagonalisation in that same location will ensure ac-
cording to the previous examples adaptation of memory length to the
specific task at hand.

The same holds true for solving the second problem. For, assum-
ing that every location \( \theta \) is assigned to some number indicating the
likelihood of \( \theta \) researching or \( \theta \) diagonalising, then again due to the
self-editing property, researching and diagonalisation can occur at the
location of that number. The result would be that the probability
under which this particular location is researched or diagonalised will adapt in the same manner as in the previous examples.

7. SOME DISCUSSION ABOUT INTELLIGENCE

Let us follow a simple real life example about intelligence:

Assume that a certain businessman $A$ runs an information technology company. A very common problem of the company is a well-known problem, namely the famous halting problem:

Some program codes $c$, sometimes, under some inputs $x$, run for ever. We won’t make things more complicated, yet assume that $A$ decides to hire $B$ to investigate this matter. Now here is the point: Although $B$’s job sounds interesting, it is not at all a job that a computer can do, in the sense that it is well-known that there isn’t any program for doing so. $B$’s job on the other hand does not sound as something that could not be a job for somebody.

The example implies that we cannot expect to find any fixed programs to act as an intelligent being would, neither could we expect that, as any of the diagonal instructions considered so far could serve as a program for being intelligent. The main intention was rather to show their potential ability to support a great number of steps in a far more complicated and sophisticated procedure which we’ll try to investigate further in Part II.

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