Twistor-Beam Excitations of Black-Holes and Prequantum Kerr-Schild Geometry

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Abstract

Exact Kerr-Schild (KS) solutions for electromagnetic excitations of black-holes, have the form of singular beams supported on twistor lines of the KS geometry. These beams have a very strong back-reaction on the metric and horizon and create a fluctuating KS geometry occupying an intermediate position between the classical and quantum gravities. We consider the Kerr theorem, which determines the twistor structure of the KS geometry and the corresponding holographic prequantum space-time adapted to subsequent quantum treatment.

Key words: black-hole, twistor, Kerr theorem, singular beam, quantum gravity

1. Black-holes (BHs) are convenient objects for studying the problem of unifying quantum theory and gravity. One obstacle in this very important problem is the inconsistency of forms of representations: gravity requires an explicit field representation in the configuration space, while quantum theory consistently uses the momentum space and plane waves, which, strictly speaking, are not defined in gravity. Twistors form a bridge between these representations. Geometrically, a twistor is a pair \((x^\mu, \theta^\alpha)\), where \(\theta^\alpha\) is a two-component spinor adjoined to the point \(x^\mu \in M^4\) and fixing the light direction (beam) \(\sigma^\mu_{\dot{\alpha}\dot{\alpha}} \theta_{\dot{\alpha}}\theta^\alpha\) corresponding to the momentum \(p^\alpha\) of a massless particle. A plane wave in the momentum space may be described in the twistor coordinates \(T^I = \{\theta^\alpha, \mu_{\dot{\alpha}}\}\), \(\mu_{\dot{\alpha}} = x_\mu \sigma^\mu_{\alpha\dot{\alpha}} \theta^\alpha\), as \(\exp\{ix_\mu \sigma^\mu_{\alpha\dot{\alpha}} \theta_{\dot{\alpha}} \theta^\alpha\}\). On the other hand, the wave can also be transformed to the twistor space by a "twistor" Fourier transform \(\mathcal{F}(p.2.5)\): a multiplication by \(-\exp(i\theta_{\dot{\alpha}} \mu_{\dot{\alpha}})\) and a subsequent in-
integration over $\tilde{\theta}$. The result \( \hat{\phi}(T') = \delta^2(\mu_\alpha - x_\nu \sigma^\nu_{\alpha\alpha} \theta^\alpha) \) corresponds to a singular beam in the direction $p^\mu$ with the twistor coordinates $T^I = \{\theta^\alpha, \mu_\alpha\}$. It is essential here that the twistor describes a beam passing through the given point $x^\mu$.

Similar twistor-beams appear in exact solutions of the Debney-Kerr-Schild (DKS) equations \cite{3} and of compatible electromagnetic and gravitational fields \cite{2}. The KS geometry covers a wide class of algebraically special metrics for rotating and non-rotating BHs and cosmological solutions. The appearance of twistor-beams in the KS solutions is not accidental: the twistor structure underlies the KS space-time. Studying exact KS solutions therefore indicates a new way for unifying quantum theory and gravity.

The exact nonstationary KS solutions considered below describe electromagnetic excitations of a BH in the form of fluctuating twistor-beams and also their exact back-reaction on the BH metric and horizon, \cite{2} consistent with the Einstein equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi <T_{\mu\nu}>$. This fluctuating KS geometry has a specific two-sheeted holographic structure adapted to quantum treatment, \cite{4, 5}, and occupies an intermediate position between the classical and quantum gravities.

2. The KS solutions are based on the KS metric

\[ g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \]  

where $\eta_{\mu\nu}$ is the metric of the auxiliary Minkowski space-time $M^4$ and $k_\mu$ is a field of light directions forming a congruence of null lines in $M^4$, the so-called principal null congruence (PNC) $K$. The Kerr and Kerr-Newman BH solutions belong to algebraically special class (type D) of solutions that have two different PNCs, and the metric may consequently be represented in two different forms:

\[ g^{\pm}_{\mu\nu} = \eta_{\mu\nu} + 2Hk^{\pm}_{\mu} k^{\pm}_{\nu}, \]

were $k^{\pm}(x)$, $(x = x^\mu \in M^4)$ are two different vector fields tangent to the corresponding $K^{\pm}$. The directions $k^{\pm}_\mu$ are determined in the light-cone coordinates $u = (z - t)/\sqrt{2}$, $v = (z + t)/\sqrt{2}$, $\zeta = (x + iy)/\sqrt{2}$, by the differential 1-form\footnote{The field $k^{\mu}(x) \equiv k^{\mu}(Y(x))$ is completed to the adapted to the Kerr congruence null tetrad $e^\alpha$, $a = 1, 2, 3, 4$ and "tetrad" derivatives $\partial_a = e^\mu_a \partial_\mu$.}:

\[ k^{(\pm)}_\mu dx^\mu = P^{-1}(du + \tilde{Y}^{\pm} d\zeta + Y^{\pm} d\zeta - Y^{\pm} \tilde{Y}^{(\pm)} dv), \]  

and depend on the complex coordinate on celestial sphere

\[ Y = e^{i\phi} \tan \frac{\theta}{2}. \]  

Two solutions $Y^{\pm}(x)$ are determined by the Kerr Theorem \cite{6, 7, 8, 9, 10} discussed below.

The structure of Kerr congruence is shown in Fig.1. It has a characteristic twisted form, which gives a complicated expression for the Kerr metric despite the extremely simple general form of the KS metric \cite{11}. The Kerr
PNC consists of twistor light beams covering the space-time twice: as the incoming PNC $k^{\mu-} \in \mathcal{K}^-$ which propagates toward the disk spanned by the Kerr singular ring, and as the outgoing from disk PNC, $k^{\mu+} \in \mathcal{K}^+$, located on the second sheet of the same spacetime $M^4$. The metrics $g^{\pm}_{\mu\nu}$ and the corresponding electromagnetic fields, being aligned with the beams of the PNC, are different on the in- and out-sheets and should not be mixed in KS solutions. This two-sheetedness is ignored in perturbative approaches, which leads to solutions with drastic deviations from the exact solutions. Typical exact electromagnetic excitations on the KS background have the form of singular beams propagating along the twistor lines of the PNC, while typical wave solutions in the perturbative approach have a smooth angular dependence.

3. The two-sheetedness was long an unsolvable problem with the Kerr solution (see the references in [11, 12]). Israel [13] proposed truncating the second sheet and replacing it with a rotating disk-like source covering the Kerr singular ring and preventing the transition to the second sheet of the metric. In an alternative variant, the Kerr singular ring itself was considered as a closed ”Alice” string forming a gate to a mirror ”Alice” world of advanced vacuum fields [14]. The new holographic approach unifies the two variants. The source of Kerr solution is formed by a domain wall (membrane) separating the in- and out-sheets of the KS space, [12]. The two sheets of the KS geometry and also the as the source-membrane are needed for describing quantum fields in gravity. In particular, according to Gibbons [4], the curved spacetime $\mathcal{M}$ should be separated into two causally ordered regions $\mathcal{M}_-$ and $\mathcal{M}_+$ associated with incoming and outgoing vacuum states $|0_->$ and $|0_+>$. If the source is absent, the basic KS solutions can be extended analytically from the in-sheet to the out-sheet, which effectively changes the sign of the frequency. The presence of the source breaks this analyticity, separating the retarded and advanced fields, and allows considering the BH evaporation as a scattering of the in-vacuum on the membrane - source. A similar prequantum BH spacetime with separated in- and out- sheets was also introduced by ’t Hooft et.al. in [5].

Two sheets of KS correspond to the ’t Hooft holographic correspondence
in which the source of the Kerr BH forms a holographically dual (to the bulk $\mathcal{M}_- \cup \mathcal{M}_+$) boundary separating the in- and out- regions.

The usual Penrose conformal diagram, containing the in- and out-fields on the same $M^4$, must be unfolded with a “splitting” the KS two-sheetedness, as shown in Fig.2, demonstrating an explicit realization of the holographic principle in the KS geometry. The twistor-beams of the Kerr congruence “create” the Kerr source (as a holographic image) by light projection from the past null infinity $I^-$ on the bulk of KS space-time. The BH then appears as a holographic image generated by the initial data on the past infinity $I^-$. 

![Penrose conformal diagrams](image)

Figure 2: Penrose conformal diagrams for (a) the Minkowski space-time and (b) the Kerr space-time: unfolding the auxiliary $M^4$ space of the Kerr geometry into two sheets generates the holographic structure of a prequantum BH space-time.

The KS light-like congruences are characterized by optical parameters: divergence $\rho$, rotation (twist) $\omega$ and shift $\sigma$, which describe the deformations of a two-dimensional image projected by the beams on a distant two-dimensional screen. The twist and divergence of the congruence lead to rotation and scaling dilation of the image, while the shift breaks the conformal properties of the image, deforming angles. The shear-free congruence of the Kerr solution $\sigma = 0$ retains the conformal structure of the projected image under translations of the screen along the beams of the congruence. The conformal structure and complex analyticity are entered in the KS solutions via function $Y(x)$, $x = x^\mu \in M^4$, which is a conformal projection of celestial sphere $S^2$ (with coordinates $(\phi, \theta)$) onto complex plane $Y \in C^1$. Tetrad derivatives of the function $Y(x)$ determine principal parameters of the KS holographic projection. In particular, the complex expansion of congruence, $Z = \rho + i\omega$, is

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3The parameters of dilatation $\rho$ and twist $\omega$ of the congruences are combined in the KS formalism [3] in one complex parameter $Z = (\rho + i\omega)$, which can also be related with a complex ‘distance’ $\tilde{r}$, an affine parameter along the Kerr congruence, $\tilde{r} = P/Z$, and $P$ is a conformal scale factor.
determined from $Y(x)$ by the relation $Z = Y_{,1}$, the geodesic condition for the rays of congruence is $Y_{,4} = 0$, and the shear-free condition is $Y_{,2} = 0$. Therefore, the complex functions $Y(x)$ obeying the conditions

$$Y_{,2} = Y_{,4} = 0$$

(4)

define the shear-free and geodesic congruences possessing the conformal-analytic properties of the KS holographic projection. All such congruences are determined by the Kerr Theorem which is formulated in terms of the projective twistor coordinates

$$Z^p = (Y, \zeta - Yv, u + Y\bar{\zeta}).$$

(5)

4. The Kerr Theorem, [6, 7, 8, 9, 10]. Any stationary geodesic and shear-free null congruence in $M^4$ is generated by solution of the algebraic equation

$$F(Z^p) = 0,$$

where $F$ is arbitrary holomorphic function of the projective twistor coordinates $Z^p$. For the Kerr geometry at rest $P = (1 + Y\bar{Y})/\sqrt{2}$, $P/Z = \tilde{r} = r + ia\cos \theta$, where $r$, $\theta$ are oblate spheroidal coordinates, and

$$H = mr - |\psi|^2/2r^2 + a^2\cos^2 \theta.$$
the holes in it, which allows matter to escape interior of BH. The exact KS solutions with several poles, \(\psi(Y) = \sum_i \frac{q_i}{Y - Y_i}\), have several twistor-beams with angular directions \(Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}\), leading to the horizon with many holes. In the far zone the twistor-beams tend to the known exact singular pp-wave solutions [8].

5. The stationary KS twistor-beam solutions may be generalized to time-dependent wave pulses, [2]. Since the horizon is extra sensitive to electromagnetic excitations, it may also be sensitive to the vacuum electromagnetic field, and the vacuum fluctuations may produce the beam pulses and a fine-grained structure of fluctuating microholes in the horizon. It will allow radiation to escape interior of the BH, as it is depicted on Fig.3, leading to a semiclassical mechanism of BH evaporation. Twistor-beam pulses have to depend on a retarded time \(\tau\) and obey to the non-stationary Debney-Kerr-Schild (DKS) equations. It was shown in [3] that in this case, besides the function \(\psi(Y, \tau)\), the exact KS solutions acquire extra radiative term \(\gamma(Y, \tau)Z\). The long-term efforts to integrating the time-dependent DKS equations, [2], have led to consistent solutions with fluctuating twistor-beam pulses. It was explicitly shown that any time-dependent KS solution develops into the beam pulses breaking the topology and stability of the BH horizon. Namely, interaction of a black-hole with em field contains two different components: a) the determined by function \(\psi(Y, \tau)\) singular twistor-beam pulses which deform metric and topology of the horizon, but do not contribute to energy flow; and b) a regular component \(\gamma_{\text{reg}}(Y, \tau)\) which determines em radiation and evaporation of the black-hole.

The obtained solutions describe excitations of the em twistor-beams on the Kerr-Schild background and consistent back reaction of the beams to metric fluctuations via virtual photon exchange.

\[ F^\mu\nu = \text{Re} F_{31} e^{i\mu} \wedge e^{i\nu}, \quad \text{where} \quad F_{31} = \gamma Z - (AZ)_{,1}. \]
and black-hole horizon. The holographic space-time forms a fluctuating pre-geometry which reflects dynamics of the singular twistor-beam pulses. This pre-geometry is classical, but it is twosheeted and has a fine-grained structure of fluctuating twistor-beams which have to be still regularized to get the usual smooth classical space-time. In this sense it takes an intermediate position between the classical and quantum gravity. Note also, that the use of the Kerr theorem with the functions $F(Y)$ of higher degrees in $Y$ leads to multi-particle KS solutions [9] which generate complicate networks of the twistor-beams.

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