WEIGHTED INEQUALITIES OF FEFFERMAN–STEIN TYPE FOR RIESZ–SCHRÖDINGER TRANSFORMS

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Abstract. In this work we are concerned with Fefferman-Stein type inequalities. More precisely, given an operator $T$ and some $p$, $1 < p < \infty$, we look for operators $\mathcal{M}$ such that the inequality

$$\int |Tf|^pw \leq C \int |f|^p \mathcal{M}w,$$

holds true for any weight $w$. Specifically, we are interested in the case of $T$ being any first or second order Riesz transform associated to the Schrödinger operator $L = -\Delta + V$, with $V$ a non-negative function satisfying an appropriate reverse-Hölder condition. For the Riesz-Schrödinger transforms $\nabla L^{-1/2}$ and $\nabla^2 L^{-1}$ we make use of a result due to C. Pérez where this problem is solved for classical Calderón-Zygmund operators.

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