On the Sensory Commutativity of Action Sequences for Embodied Agents

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Abstract

We study perception in the scenario of an embodied agent equipped with first-person sensors and a continuous motor space with multiple degrees of freedom. Inspired by two theories of perception in artificial agents (Higgins et al., 2018; Poincaré, 1895) we consider theoretically the commutation properties of action sequences with respect to sensory information perceived by such embodied agent. From the theoretical derivations, we define the Sensory Commutativity Probability criterion which measures how much an agent’s degree of freedom affects the environment in embodied scenarios. We empirically illustrate how it can be used to improve sample-efficiency in Reinforcement Learning.

1. Introduction

Perception is the medium by which agents organize and interpret sensory stimuli, in order to reason and act in an environment using their available actions (Hoffman, 2018). We focus on scenarios where embodied agents are situated in realistic environments, i.e. the agents face partial observability, coherent physics, first-person view with high-dimensional state space and low-level continuous motor (i.e. action) space with multiple degrees of freedom. These embodied agents, when acting in such environment, produce a stream of sensorimotor data, composed of successions of motor states and sensory information. While most current approaches for building perception focus on studying the sensory information alone, several approaches (Caselles-Dupré et al., 2019; Laflaquière, 2018; Ghosh et al., 2018; Thomas et al., 2017) that can be traced back to 1895 (Poincaré, 1895), advocate the necessity of studying the relation between sensors and motors for the emergence of perception.

Among those approaches, we focus on Symmetry-Based Disentangled Representation Learning (SBDRL) (Higgins et al., 2018; Caselles-Dupré et al., 2019) and what we refer to as SensoriMotor Theory (SMT) (O’Regan & Noé, 2001). SBDRL aims at formalizing disentanglement in Representation Learning, i.e. the idea that sensory data is generated by a few explanatory factors of variation. The core idea in SBDRL is to define disentanglement with respect to transformations of the environment that leave some aspects invariant. On the other hand, SMT puts forward an unsupervised sensorimotor grounding of perception. It describes how space induces specific invariants in any embodied agent’s sensorimotor experience, and how these invariants can be captured to improve the compactness of representations and the prediction of sensorimotor experiences.
In an attempt to unify those two approaches, we study the commutativity of action sequences with respect to sensors, which we term sensory commutativity, illustrated in Fig.1. We define the Sensory Commutativity Probability (SCP) as the probability that a sequence of movements using only one degree of freedom of the agent, an arm joint for instance, sensory commutes. We show that this value has meaning for the embodied agent: if the SCP is high then the degree-of-freedom has a low impact on the environment (e.g. moving a shoulder is more likely to move things around than moving a finger, so SCP for shoulder is lower than for finger). By computing the SCP for each degree of freedom of the agent, we are able to characterize its motor space and use this relevant information for subsequent tasks. We illustrate this in our experiments as we show how SCP can be used to improve sample-efficiency in a Reinforcement Learning problem.

2. Related work and motivation

2.1. SBDRL
Symmetry-Based Disentangled Representation Learning (SBDRL) (Higgins et al., 2018; Caselles-Dupré et al., 2019) aims at formalizing disentanglement in Representation Learning. The core idea is that SB-disentanglement of a representation is defined with respect to a particular decomposition of the symmetries of the environment. Symmetries are transformations of the environment that leave some aspects of it unchanged. For instance, for an agent moving on a plane, translations of the agent on the $y$-axis leave its $x$ coordinate unchanged. This is formalized using group theory. Groups are composed of these transformations, and group actions are the effect of the transformations on the state of the world and representation.

2.2. SensoriMotor theory
SensoriMotor theory (SMT) is a theory of perception that gives prominence to the role of motor information in the emergence of perceptive capabilities (O’Regan & Noé, 2001). The approach takes inspiration from philosophical ideas formulated more than a century ago by H.Poincaré (Poincaré, 1895). It led to theoretical results regarding the extraction of the dimension of space (Lafalquiére et al., 2012), the characterization of displacements as compensable sensory variations (Terekhov & O’Regan, 2016), the grounding of the concept of point of view in the motor space (Lafalquiére et al., 2013; Lafalquiére et al., 2015), as well as the characterization of the metric structure of space via sensorimotor invariants (Lafalquiére et al., 2018).

2.3. Motivation
The notion of symmetries of Higgins et al. (2018) is based on transformation that have a group property. Their definition make it possible to formalize disentanglement, although it does not require to exactly precise what makes a transformation belong to the group of symmetries $G$. Moreover, the notion of sub-groups is only defined with intuition as well: what exactly makes a subset of transformations of the group $G$ a subgroup $G_1$? Still Higgins et al. (2018) provide insights and intuitive concepts to describe what might characterize sub-groups: for instance translations along one axis only change the position of the agent for this particular axis and leave other coordinates invariant. In this work we would like to start from the same intuitions, but rigorously define the group and sub-groups of transformations.

We note that the notion of transformations that have a group structure is also present in SMT. It dates back to the manuscript of Poincaré (Poincaré, 1895), where he describes that compensable transformations of the environment equipped with the composition operation forms a group. Moreover, Philipona (2008) attempts at properly characterizing those sub-groups. Using action sequences and their commutative property, he suggests that spatial transformations and non-spatial transformations can be disentangled. This is compatible with the intuitions from (Higgins et al., 2018), since those subsets are indeed sub-groups and do not affect each other.

In this paper, we take inspiration from both approaches. From SMT, we choose to study action sequences, termed $\text{Seq}(\mathcal{M})$, and their commutative properties. From SBDRL, we choose to study the group and sub-group properties of $\text{Seq}(\mathcal{M})$, with the aim of organizing and disentangling the motor space $\mathcal{M}$. This will be achieved with the definition of the Sensory Commutativity Probability criterion.

3. Formalism choice
Despite their similarities, both theories mathematically define the world and agents differently. We propose a mathematical framework for the embodied scenario which will allow to properly construct the Sensory Commutativity Probability.

We start from the formalism used in SMT, which formalizes the perception of the agent as follows:

$$ s_t = \phi(m_t, \epsilon_t) \quad (1) $$

At time $t$, the agent is in a particular motor state $m_t$. This means that its motor, i.e. all the actionable part of its body (joints, motors), are in a particular setup called $m_t$. The environment is defined by everything that’s not the agent. It’s thus an entity that is in a state $\epsilon_t$, e.g. a room with 6 walls plus light sources and objects placed in different locations.
The agent can perceive the world through its sensorimotor dependencies \( \phi \): a function that takes as input \( m_t \) and \( \epsilon_t \) and produces sensory inputs from its sensors \( s_t \).

The dynamics of the world are generally not described in SMT, so we extend its formulation:

\[
m'_{t+1}, \epsilon'_{t+1} = f(m_t, \epsilon_t, \Delta(m_t, m_{t+1}), \Delta(\epsilon_t, \epsilon_{t+1}))
\]

The agent can operate motor commands \( \Delta(m_t, m_{t+1}) \). But the environment can change also through its own dynamics outside of the agent, represented by \( \Delta(\epsilon_t, \epsilon_{t+1}) \). Taking the initial states and changes as inputs, the function \( f \) yields new motor state \( m'_{t+1} \), and a new configuration of the environment \( \epsilon'_{t+1} \). We don’t generally have that \( \epsilon_{t+1} = \epsilon'_{t+1} \) or \( m_{t+1} = m'_{t+1} \) since the agent can affect the environment configuration through its body movement or the environment can force movements on the agent.

4. Structure and commutativity properties of the set of action sequences \( Seq(M) \)

We will now attempt to formalize groups and sub-groups of symmetries. We propose \( G \) to be the set of motor command (or action) sequences of finite length, referred to as \( Seq(M) \), and will attempt at extracting sub-groups based on subsets of these transformations.

4.1. Group structure of \( Seq(M) \)

Philipona (2008) first defined a relation between action sequences: \( h \sim g \) if and only if \( h \) and \( g \) affect the sensors in the same way. Using our formalism, we can translate this concept into an equality.

**Definition 1.** Let \( (h, g) \in Seq(M) \). \( h \) is equivalent to \( g \) under \( (m_t, \epsilon_t) \), noted \( h \sim_{m_t, \epsilon_t} g \) if and only if

\[
\phi(f(m_t, \epsilon_t, h, \Delta^1)) = \phi(f(m_t, \epsilon_t, g, \Delta^1))
\]

Intuitively, two actions sequences are equivalent for a particular motor state and environment state if applying them lead to the same sensory state. For instance for a multiple-joints arm moving freely in an empty space, there are multiple different ways of moving the arm from one place to another. This yields action sequences \( (h_1, \ldots, h_n) \) which are equivalent in this situation \( (m_t, \epsilon_t) \), we thus have \( h \sim_{m_t, \epsilon_t} g \). However in other situations these actions sequences can become not equivalent, for instance if there are objects on the way for instance as illustrated in Fig.2.

For convenience and clarity, we will drop the notation for dependence on \( (m_t, \epsilon_t) \) and thus write \( h \sim g \) whenever there are no ambiguities in the context. We now consider the structure of \( Seq(M) \) under composition \( \circ \) with respect to the equivalence \( \sim \).

**Proposition 1** (Structure of \( (Seq(M), \sim, \circ) \)).

1. \( \sim \) is an equivalence, i.e. it is reflexive, transitive and symmetric.
2. \( (Seq(M), \circ) \) is a group w.r.t \( \sim \).
3. \( \circ \) is generally not commutative with respect to \( \sim \).

**Proof.** See Appendix A for full proof. Point 1 follows from the properties of \( = \). For Point 2, composing two action sequences yields an action sequence, the no-op action is the identity element and if we suppose that there are no irreversible phenomenons in the environment, all action sequences can be inverted. For Point 3, we can always construct action sequences that do not commute.

\( (Seq(M), \circ) \) is thus a group w.r.t \( \sim \). This structure is coherent with the intuitions in SBRL and SMT theories. In the following, we build on the observation that composing action sequences is not generally commutative. We show how this property can lead the agent to organize and interpret its motor space.

4.2. Commutativity properties of \( Seq(M) \)

4.2.1. Philipona’s conjecture

Philipona (2008) already studied how action sequences commute with respect to the sensory information received by the agent. Action sequences do not necessarily commute as stated in Prop.1. For example if a movable object is placed to the right of your arm, moving your arm right then left will not have the same effect (in terms of sensor change) as moving it left then right, as illustrated in Fig.2. Philipona thus defines commutation residues. Suppose that doing \( h_1 \circ h_2 \) is different from \( h_2 \circ h_1 \), then a commutation residue \( g \) is an action sequence that you have to do to compensate the difference in sensory experience.

**Definition 2.** \( g \) is a commutation residue of \( (h_1, h_2) \) if and only if \( h_1 \circ h_2 \sim h_2 \circ h_1 \circ g \). If \( g \) is equivalent to no-op (no action), then \( h_1 \) and \( h_2 \) commute.

Starting from this definition, he conjectured that all action sequences that are not displacements commute with any action sequences. For instance moving you arms (displacement action) and opening the eyes (non-displacement action) will always commute whereas two displacement actions will not necessarily commute, depending on which starting situation \( (m_t, \epsilon_t) \) is selected.

**Conjecture 1** (Philipona’s conjecture). The subset of \( Seq(M) \) composed of non-displacements action sequences is the sub-group of \( Seq(M) \) that commutes, i.e. the abelian sub-group of \( Seq(M) \).
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Figure 2. Action sequences do not necessarily commute. Starting from a common situation, an action sequence played in two different orders does not necessarily lead to the same sensory state.

We will illustrate this conjecture with experiments in Sec.5.3.

4.2.2. SENSORY COMMUTATIVITY PROBABILITY OF AN ACTION

Based on Philipona’s conjecture, we derive a criterion for characterizing how much each degree of freedom of the agent affects the world, computable using only sensorimotor data. We define “degree of freedom” (DOF) as a dimension of the multidimensional continuous action space of the agent.

Using the conjecture, we have that for an action sequence $h$, if the agent plays it in two different orders starting from the same situation, there is a chance that the agent will experience two different sensory outcomes only if the action sequence $h$ is composed of at least one displacement action (an action that affect the environment such as moving limbs or going forward).

However not all displacement actions are equivalent. The agent is more likely to observe two different outcomes if the action sequence $h$ is composed of displacement actions that affect the environment a lot. Consider moving your forearm (elbow joint) compared to moving your whole arm (shoulder joint, see Fig.4): the latter is more likely to move things around in the environment and thus induce sensory non-commutativity when played in two different orders (i.e. having two different sensory outcomes). An elbow joint should therefore have a higher SCP than a shoulder joint.

We formalize this intuition by defining the Sensory Commutativity Probability (SCP) of a degree of freedom, averaged over all starting situations $(m_t, \epsilon_t)$:

**Definition 3** (Sensory commutativity probability of a degree of freedom). Let $\text{Seq}(M_k)$ be the set of motor commands (or action) sequences of finite length for the $k^{th}$ degree of freedom of $M$ (motor state space). Let $h \in \text{Seq}(M_k)$ and let $h_p$ be a random permutation of $h$ (same sequence but different order).

The Sensory Commutativity Probability of the $k^{th}$ degree of freedom $SCP(M_k)$ is defined as:

$$SCP(M_k) = P_{m_t, \epsilon_t, h} [h \sim_{m_t, \epsilon_t} h_p]$$

In our experiments, we show how to compute the SCP of a degree of freedom and how we are able to use it to improve sample-efficiency in a Reinforcement Learning problem.

5. Sensory Commutativity Probability experimental analysis

In this first experimental section we compute and interpret the SCP for an embodied agent scenario. We then compare SCP to baseline alternatives.

5.1. Experimental setup

The simulation we use needs to satisfy the properties of an embodied agent scenario: navigable space with objects to interact with, first-person high dimensional observations, low-level high-dimensional action space and coherent physics.

Unfortunately, these requirements are not met in current benchmarks. Mujoco (Todorov et al., 2012) doesn’t have
first-person observations, robotic arm setups does not allow navigation. Arcade Learning Environment (Bellemare et al., 2013), DeepMind lab (Beattie et al., 2016) and VizDoom (Kempka et al., 2016) do not have low-level motor commands but rather have high-level action spaces.

We thus develop our own 2D simulation using Flatland (Caselles-Dupré et al., 2018), a platform for creating 2D RL environments. We construct an agent called Polyphemus (a Cyclop from the Greek mythology), that has a movable and rotatable base equipped with a rotatable head and two 2-DOF arms. The agent sees through its unique eye that has an activable eyelid, yielding a total of 8 DOF. The image received by the agent is a 64 pixels RGB image depicting what its eye sees. This agent is placed in a room with fixed, moving or movable entities, all of different colors. The agent can move around and interact with these entities. Its point of view can change through base movement, rotation, and head rotation. Our simulation is illustrated in Fig.3. For each degree of freedom, an action or motor command corresponds to a change in the longitudinal/angular velocity of the degree of freedom.

5.2. Estimating the Sensory Commutativity Probability

In order to estimate the SCP of each of the 8 agent’s degree of freedom, we initialize the SCP value to 0 ($SCP \leftarrow 0$). We then repeat the following process 100 times for each DOF:

- Sample an action sequence using the selected degree of freedom (a sequence of action where each action is a value between -1 and 1).

- Play it in 2 different orders starting from the same randomly chosen state and save the two final sensor images.

- Count one if the two final sensor images are equal ($SCP+=1$), zero otherwise.

Finally, the estimator of the SCP is the average over the number of trials ($SCP/100$). Note that using a simulation allows to play the two action sequences of different orders from the exact same starting position. Our results are reported in Fig.5.

5.3. Results

Figure 5. Sensory Commutativity Probability for each degree of freedom. Note how the SCP value is inversely proportional to how each action affects the environment (shoulders and base movement/rotation affect more than elbows which affects more than eyelid and head rotation). Moreover, as predicted by Philipona’s conjecture, the two DOF not associated to displacements, Eyelid and Head Rotation, are the only ones to always commute (i.e. SCP of 1).
The results are coherent with Philipona’s conjecture. Fig.5 shows that only two actions have an SCP of 1: eyelid and head rotation. All other actions have a SCP inferior to 1. This is coherent with Philipona’s conjecture (Sec. 4.2.1): eyelid and head rotation are the two degrees of freedom that are not associated to displacements, thus action sequences composed of actions of these type commute with respect to the sensors. On the contrary, all other degrees of freedom are associated to displacements, and thus will eventually induce non-zero commutation residues when played in different orders from the same starting situation. Hence the results are coherent with the conjecture, and can be used by the agent to autonomously discover which of its actions are associated to displacements or not.

SCP is inversely proportional to how each degree of freedom affects the environment. By that we mean that from the computation of the SCP, we obtain a hierarchical organization of the action space in which the less important dimensions for manipulation and navigation are separated from the dimension that are not crucial for such tasks. This is illustrated in Fig.4, which shows that shoulders and base movement should have a lower SCP than elbows which in turn have a lower SCP than eyelid and head rotation. We inferred that shoulders should have a lower SCP than elbows since activating the shoulder joint is more likely to induce non commutativity by moving things around or hitting walls/obstacles. This intuition is verified by our results. Without having any prior knowledge about the simulation, we can automatically organize the agent’s degrees of freedom in a hierarchy. Moreover, the symmetry of the action space is kept, as elbow 1 and 2 have equal SCP, and so do shoulder 1 and 2.

5.4. Alternative methods are not adapted

The SCP criterion derived in this paper estimates how much each degree of freedom affects the environment in an embodied agent scenario. In this section we discuss why other approaches cannot reliably estimate the same quantity.

5.4.1. Naive approach: changes in sensors

A straightforward approach to this problem would be to play action sequences of each degree of freedom and quantify how much the sensors change. We consider the squared difference for a transition, i.e. the squared difference for two consecutive observations separated by an action sampled from one dimension of the action space. We report the mean squared difference over 100k transitions, for each degree of freedom.

It is clear in our experiment results, shown in Fig.6, that the approach fails. For instance, rotating the head of the agent changes dramatically what the agent sees, even though this degree of freedom does not affect the environment. It would have made sense if we had considered the top view (fully-observable scenario), since rotating the head does not changes the top view a lot. However in the embodied scenario, this strategy is not viable. For the same reason, approaches based only on the changes in the embodied sensors are bound to fail.
tion error is not well correlated with how much a degree of freedom is important for navigation and manipulation. For instance, head rotation, which does not affect the environment, is hard to predict: the agent might not know what’s outside his field of view. On the contrary, base longitudinal movement affect the environment a lot and is easier to predict than head rotation. A solution would be to use more complex neural architectures, involving the computation of a state representation that uses memory (recurrent (Hochreiter & Schmidhuber, 1997), or external (Graves et al., 2014)). However, such methods would require heavy additional computation where our approach computes the SCP with minimal requirements and no training.

Figure 7. A more involved alternative for SCP, where we report the prediction error when trying to predict the effect of actions on sensors, for each degree of freedom. Results show that this does not allow to identify which degrees of freedom are useful (or not) for navigation and manipulation. For instance, eyelid activation (not useful) is as hard to predict as base longitudinal movement (crucial).

To conclude, in our experiments we did not find any viable strategy to replace the SCP criterion. SCP is able to easily estimate how important a degree of freedom is for acting and navigating in the environment. The other considered baselines do not manage to organize the action space in the same hierarchical way.

6. Sensory Commutativity Probability for efficient exploration

We now illustrate how SCP can be used for unsupervised exploration, by using it to improve sample-efficiency in a RL setup.

6.1. Experimental setup

We use the PPO2 (Schulman et al., 2017) implementation from Stable-Baselines (Hill et al., 2018). The policy is composed of a 1D convolutional feature extractor followed by a recurrent policy. We consider the same agent, Polyphemus, for which we computed the SCP criterion in Fig.5. The input of the policy is the RGB image of what Polyphemus’ eye sees. The environment considered is a square room with 3 dead zones (which terminate the episode with a -20 reward) and a goal zone (which terminates the episode with a +50 reward), illustrated in Fig.8.

Figure 8. The task of the agent is to navigate to the green zone while avoiding the red zones.

We propose two methods that take advantage of the SCP to modify the action space of the agent. The goal is to improve sample-efficiency when learning to solve a task in this embodied scenario.

6.1.1. SCP-truncated action space

A first, quite radical, idea is to truncate the agent’s action space based on SCP value of each degree of freedom. We implement this by halving the dimension of the action space, keeping only the degrees of freedom that have the most effect on the environment, i.e. lower SCP value. We thus keep the base movement and rotation, and the shoulders joint, while discarding the elbow joints, head rotation and eyelid activation. We refer to this method as SCP-truncated action space. This action space reduction will obviously simplify the RL task, as long as the necessary actions such as base motion are selected by the SCP criteria.

6.1.2. SCP-adapted action space

A less involved proposition is to modify the action sampling interval according to the SCP value, for each degree of freedom. This method will not change the task as the
previous one, but will modify the exploration dynamics to favor important actions. Suppose that the sampling interval for each dimension of the action space is $[-1, 1]$. If a dimension has high SCP, i.e. it does not affect the environment a lot, we then reduce the interval from which action are sampled $[-1 \cdot l(SCP), 1 \cdot l(SCP)]$. The function $l$ maps the highest SCP to 0 and lowest SCP to 1, then we use a linear interpolation between those two points to deduce values for $SCP \in [-1, 1]$. We refer to this method as $SCP$-adapted action space.

6.1.3. Comparison protocol

We compare those two strategies to a baseline policy trained to solve the task with the complete action space. We average the result of each policy over 30 trials initialized with different random seeds, and we test the statistical significance of our results according to the guidelines provided by Colas et al. (2018).

Note that we did not try the truncation and adaptation method using the SCP alternatives considered in Sec.5. For both SCP alternatives, the coefficient associated to the base longitudinal movement is close to zero. Since truncating/adapting longitudinal movement makes the task impossible/harder for the agent, we did not try those alternatives.

6.2. Results

![Learning curves for the 3 considered strategies, results averaged over 30 seeds. The Sensory Commutativity Probability can be used to improve the action space of the agent and thus improve sample-efficiency. Dots show statistical significance when testing against the baseline green curve.](image)

Figure 9. Learning curves for the 3 considered strategies, results averaged over 30 seeds. The Sensory Commutativity Probability can be used to improve the action space of the agent and thus improve sample-efficiency. Dots show statistical significance when testing against the baseline green curve.

The results are displayed on Fig.9. First of all, we notice that all strategies are viable to solve the task. We now compare sample-efficiency between the strategies.

The policy trained with $SCP$-truncated action space is able to learn how to solve the task more than twice as fast as the baseline policy. The discarded degrees of freedom are not crucial in this navigation task, hence the agent is still able to solve the task using only the degrees of freedom that have the lowest SCP value.

The policy trained with $SCP$-adapted action space is less sample-effective than the $SCP$-truncated but still learns significantly faster than the baseline policy, hence showing our point.

7. Discussion and conclusion

7.1. Discussion

Applying SCP on tabula rasa scenarios. SCP gives a characterization of the action space of an embodied agent. In this paper we illustrated the usefulness of this characterization in a RL experiment, but we hope SCP can be useful in other types of learning problem for embodied agents. If we consider the widely-adopted scenario where the agents learn from a clean state (i.e. tabula rasa), we believe SCP computation could be a useful method to help the agent build perception. For instance, exploring a large and complex environment is easier when the agent knows which of its degrees of freedom is more important for navigation and manipulation, which is what SCP characterizes. Thus the SCP could give a useful information that might be used to improve exploration strategies.

Limitation: SCP computation requires a simulation. The main limitation of SCP is that computing SCP as described in this paper is only possible when having access to a simulation of the considered environment, thus it is not directly applicable for real life scenarios. The difficulty in such scenario is that the agent has to be able to play two action sequences from the same starting point. Thus, in a real life scenario, the method has to overcome stochasticity and irreversible actions (e.g. breaking a glass) which break that assumption.

7.2. Conclusion

We studied the sensory commutativity of action sequences for an agent in an embodied scenario (high-dimensional first person sensors, multi-dimensional continuous action space and coherent physics). Inspired by two artificial perception theories, we derived the Sensory Commutativity Probability criterion, which we showed is good proxy for estimating the effect of each action on the environment. We illustrated the potential usefulness of such criterion by improving sample-efficiency in a Reinforcement Learning problem.

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A. Proofs

**Proposition** (Structure of \((\text{Seq}(\mathcal{M}), \sim, \circ))\).

1. \(\sim\) is an equivalence, i.e. it is reflexive, transitive and symmetric.

2. \((\text{Seq}(\mathcal{M}), \circ)\) is a group w.r.t. \(\sim\).

3. \(\circ\) is not commutative with respect to \(\sim\), i.e. we don’t generally have \(g \circ h \sim h \circ g\).

**Proof.** 1. \(=\) is an equivalence, thus \(\sim\) is an equivalence as well.

2. All 4 properties of the group definition are satisfied. 1. For two action sequences \((h, g) \in \text{Seq}(\mathcal{M})\), the composition of \(h\) and \(g\) is still an action sequence \(h \circ g \in \text{Seq}(\mathcal{M})\). 2. \(\circ\) is associative with respect to \(=\), i.e. \(g \circ (h \circ k) = (g \circ h) \circ k\) thus it follows that \(g \circ (h \circ k) \sim (g \circ h) \circ k\). 3. The identity element is the no-op action. 4. If we suppose that there are no irreversible phenomenons in the environment, then for a fixed \((m_t, \epsilon_t)\), all action sequences can be inverted.

3. \(\circ\) is not commutative, as we can always explicitly find two action sequences that do not commute. For instance once there exists a movable object in the environment: if the agent is placed left to the object, then let \(h\) be moving right and \(g\) be moving left. \(h\) and \(g\) do not commute.