ON THE CONSTRUCTION OF SL(2,Z) TYPE IIB 5-BRANES

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This talk reviews our recent work on the construction of SL(2,Z) multiplets of type IIB superfivebranes. We here pay particular attention to the methods employed and some salient features of the solutions.

1 Introduction

The study of perturbative superstring spectra indicates that there must exist a so-called Type IIB supergravity as the low energy limit of type IIB superstring in addition to the well-known type IIA supergravity. This theory has two spacetime supersymmetries with the same handedness and is therefore chiral. The field content for this theory (in the real basis) consists of the metric \( g_{\mu\nu} \), a 2-from gauge potential \( B^{(1)}_{\mu\nu} \), and the dilaton \( \phi \) as its bosonic fields in the so-called NSNS-sector. It also contains another scalar \( \chi \), another 2-form gauge potential \( B^{(2)}_{\mu\nu} \) and a 4-form gauge potential \( A_{\mu\nu\rho\sigma} \) as the bosonic fields in the so-called RR-sector. In the fermionic sector the spectrum consists of two left-handed Majorana-Weyl gravitinos \( \psi^{(i)}_\mu \) and two right-handed Majorana-Weyl spinors \( \lambda^i \) with \( (i = 1, 2) \). The field strength for the 4-form gauge potential \( A_{\mu\nu\rho\sigma} \) is self-dual. For this reason, the construction of a manifestly covariant action is not possible for this theory.

This fact led Schwarz \(^1\) to write down the covariant equations of motion instead for this theory. The key for doing this is the supersymmetry and the global SL(2,R) symmetry. The two supersymmetries in this theory are known from the outset. However, the global SL(2,R) symmetry is not so
obvious. But its existence can be inferred from the following. We have two 
supersymmetries here which can be related to each other by a SO(2) rotation. 
In other words, the superalgebra has SO(2) as its automorphism group. The 
past experience on Cremmer-Julia hidden symmetries tells us that type IIB 
supergravity might have some global non-compact group as its symmetry group 
whose maximum compact subgroup is SO(2). That there are two scalars in this 
theory tells us that the possible global symmetry is SL(2,R). The scalars should 
parametrize the coset SL(2,R)/SO(2). The two 2-form potentials transform 
under the SL(2,R) as a doublet while the remaining fields are inert under the 
SL(2,R) when the metric is written in Einstein frame. So the corresponding 
charge doublet whether electric-like (strings) or magnetic-like (fivebranes) will 
also transform as a doublet accordingly.

The existence of this global SL(2,R) symmetry in type IIB supergravity 
is not consistent with the perturbative type IIB superstring. For example, 
its maximum compact subgroup SO(2) is a symmetry of the corresponding 
superalgebra but not a symmetry of type IIB superstring worldsheet action 2. Thanks to the recent understanding of non-perturbative string theory, we 
now know that only the discrete subgroup SL(2,Z) will survive quantum me-
chanically as a symmetry of the non-perturbative type IIB string theory. In 
order for this to be true, non-perturbative extended objects like D-string and 
D5-brane both carrying RR charges and NSNS 5-brane carrying NSNS charge 
must be included in the non-perturbative spectrum. As we will see, precisely 
the requirement of charge quantization break the continuous or classical SL(2, 
R) symmetry to the discrete SL(2,Z). Except for the cases involving dyonic 
objects, the same applies to various U-duality symmetries of M-theory from 
the Cremmer-Julia continuous correspondences.

Unlike all the other string dualities in D = 10, the type IIB SL(2,Z) is 
indeed a symmetry of type IIB superstring theory. In particular, this SL(2,Z) 
contains a transformation which maps the dilaton $\phi \rightarrow -\phi$, therefore is a 
strong-weak duality symmetry. In this respect, it is like the S-duality SL(2,Z) 
of $N = 4, D = 4$ heterotic string 3,4,5. In other words, type IIB string theory 
is self-dual. However, unlike the heterotic SL(2,Z) which rotates electrically 
and magnetically charged states of the same gauge field, the present SL(2,Z) 
relates either electrically or magnetically charged states of two different gauge 
fields. This makes it more like a T-duality 6.

What can we make use of the quantum SL(2,Z) symmetry of the non-
perturbative type IIB string theory? To our knowledge, new information can 
be learned based on this quantum SL(2,Z) symmetry in the following three 
cases: 1) The most general stable BPS electrically charged superstring states 
6 or magnetically charged superfivebrane states 7 can be constructed from the
corresponding known single-charge solution based on this symmetry. 2) Certain \( SL(2,Z) \) invariant terms in the effective action of type IIB string theory can be deduced from the symmetry requirement and the explicitly known perturbative terms (with possible application of some known perturbative non-renormalization theorem) as reviewed recently by Sen\(^8\) and references therein.

3) This symmetry is not only the basis for the AdS/CFT correspondence of Type IIB string on \( AdS_5 \times S^5 \) and \( N = 4, D = 4 \) super Yang-Mills on the boundary for both \( g_s > 1 \) and \( g_s < 1 \), with \( g_s \) the type IIB string coupling, but also is an important ingredient for this correspondence to be true.

In this brief review, we only focus on how to construct, in particular, the most general stable BPS fivebranes carrying magnetic charges from a given simple fivebrane solution in type IIB string theory in case 1) above. We also discuss various salient features in this construction.

## 2 General Stable BPS Fivebranes

To construct \( SL(2,Z) \) type IIB fivebranes carrying magnetic charges, we as usual start with a known pure NSNS superfivebrane\(^9\) with zero asymptotic values of the scalars in type IIB theory. Depending on the charge carried by the NSNS fivebrane to be a quantized unit one or just an arbitrary classical one, there exist basically two methods which can be used to construct the \( SL(2,Z) \) fivebranes. In the former case, a compensating factor needs to be introduced to the initial unit charge by hand such that the transformed charge doublet, obtained after the classical \( SL(2,R) \) transformation can remain to be quantized (this is the method used by Schwarz\(^6\) to construct the \( SL(2,Z) \) strings). By this, both the compensating factor and the \( SL(2,R) \) matrix are completely determined in terms of the vacuum moduli and the quantized charges. In the latter case, an initial charge doublet with the arbitrary classical NSNS charge as its only non-vanishing component is transformed by a \( SL(2,R) \) transformation to a general but given classical charge doublet. By this, both the initial NSNS charge and the elements of \( SL(2,R) \) matrix are completely determined in terms of the asymptotic values of scalars and the given classical charges. Then we impose the charge quantization on each component of the given charge doublet due to the existence of F-string and D-string, the magnetic duals of NSNS fivebrane and RR fivebrane, respectively. The two methods produce the same general \( SL(2,Z) \) fivebrane solution but they have different implications. For the former method, we sandwich a classical \( SL(2,R) \) transformation between quantum mechanically allowable initial and final fivebrane configurations. As a consequence, the mass of the final configuration is different from that of the initial configuration by the compensating factor introduced by hand while a
SL(2,R) transformation preserves the mass. We do not have such a problem in the second method. We therefore employ it here to construct the general SL(2,Z) superfivebranes. But before doing so, let us review briefly the relevant part of type IIB supergravity and a pure NSNS BPS fivebrane configuration.

Since we are interested in vacuum-like BPS solutions, the fermionic fields are set to zero from the outset. Further, dropping the irrelevant 4-form potential, we can write the corresponding action of type IIB supergravity in a SL(2,R) invariant form\(^6,10\) (in Einstein frame) as

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ R + \frac{1}{4} \text{tr} \nabla_{\mu} \mathcal{M} \nabla^\mu \mathcal{M}^{-1} - \frac{1}{12} \mathcal{H}_{\mu\nu\rho}^T \mathcal{M} \mathcal{H}^{\mu\nu\rho} \right]. \quad (1)$$

In the above, the two scalars \(\chi\) and \(\phi\) parametrize the coset \(\text{SL}(2,R)/\text{SO}(2)\) as

$$\mathcal{M} = \begin{pmatrix} \chi^2 + e^{-2\phi} & \chi \\ \chi & 1 \end{pmatrix} e^{\phi}, \quad (2)$$

and the NSNS 3-form field strength \(H^{(1)}_{\mu\nu\lambda} = dB^{(1)}\) is combined with the RR 3-form field strength \(H^{(2)}_{\mu\nu\lambda} = dB^{(2)}\) to form a doublet as

$$\mathcal{H}_{\mu\nu\lambda} = \begin{pmatrix} H^{(1)}_{\mu\nu\lambda} \\ H^{(2)}_{\mu\nu\lambda} \end{pmatrix}. \quad (3)$$

The action (1) can be easily seen to be invariant under the following global SL(2,R) transformation:

$$\mathcal{M} \to \Lambda \mathcal{M} \Lambda^T, \quad \mathcal{H}_{\mu\nu\lambda} \to (\Lambda^{-1})^T \mathcal{H}_{\mu\nu\lambda}, \quad g_{\mu\nu} \to g_{\mu\nu}, \quad (4)$$

where \(\Lambda = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\), with \(ad - bc = 1\), represents a global SL(2, R) transformation matrix. It is easy to check that under the transformation (4), the complex scalar \(\lambda = \chi + ie^{-\phi}\) transforms as,

$$\lambda \to \frac{a\lambda + b}{c\lambda + d}. \quad (5)$$

We would like to point out here that unlike the electrically charged string solution, the magnetic charges associated with \(H^{(1)}_{\mu\nu\lambda}\) and \(H^{(2)}_{\mu\nu\lambda}\) of the fivebrane should transform in the same way as the field strengths themselves. This follows from the fact that Noether charge (or the electrical charge) of the string solution is conserved due to the equation of motion following from (1) whereas the topological charge (or the magnetic charge) of the five-brane is
conserved due to Bianchi identity. Therefore the magnetic charges of the five-branes transform as $P \rightarrow (\Lambda^{-1})^T P$ or in components,

$$
P^{(1)} \rightarrow dP^{(1)} - cP^{(2)}
$$

$$
P^{(2)} \rightarrow -bP^{(1)} + aP^{(2)}
$$

Now in order to construct the general BPS superfivebrane configuration all we need is a known particular superfivebrane solution and the condition that both $P^{(1)}$ and $P^{(2)}$ should be quantized separately. The pure NSNS superfivebrane configuration has been found some time ago from an action involving pure NSNS fields which can be obtained from the action (1) when the RR fields $\chi$ and $H^{(2)}_{\mu
u}$ are consistently set to zero. This configuration preserves one half of the spacetime supersymmetries. With zero asymptotic value of the dilaton and insisting that the metric approach Minkowski spacetime asymptotically, we have the following superfivebrane configuration

$$
ds^2 = \left(1 + \frac{P^2}{2p^2}\right)^{-1/4} \left[-(dt)^2 + \delta_{ij} dx^i dx^j\right] + \left(1 + \frac{P^2}{2p^2}\right)^{3/4} \left(dp^2 + \rho^2 d\Omega_3^2\right)
$$

$$
e^{2\phi} = \left(1 + \frac{P^2}{2p^2}\right)^{\frac{p^2}{2}}
$$

$$
H^{(1)} = P \epsilon_3.
$$

Here $i,j = 1,2,3,4,5$ and $d\Omega_3^2$ is the metric on the unit 3-sphere. $\epsilon_3$ is the corresponding volume form. At this point, the charge $P$ is an arbitrary classical one.

We have three steps to construct a general stable SL(2,Z) superfivebrane configuration using the second method described above. The first step consists of the construction of a general classical superfivebrane with non-vanishing but fixed asymptotic value of the complex scalar $\lambda_0 = \chi_0 + ie^{-\phi_0}$ in the sense that the charges in the charge doublet are classical. We first seek a most general SL(2,R) transformation $\Lambda$ such that it maps the initial asymptotic value of the complex scalar $\lambda_0 = \chi_0 + ie^{-\phi_0}$. Here we use the subscript “0” to denote the asymptotic values of the scalars. For example, $\phi_0$ denotes the asymptotic value of the dilaton. By this, the SL(2,R) matrix $\Lambda$ can be partially determined as

$$
\Lambda = \left( e^{-\phi_0} \cos \alpha + \chi_0 \sin \alpha \right) \begin{pmatrix} \sin \alpha & -e^{-\phi_0} \sin \alpha + \chi_0 \cos \alpha \\ \cos \alpha & e^{\phi_0/2} \end{pmatrix}
$$

Here $\alpha$ is an arbitrary parameter which will be fixed from the given transformed charges.

A classical NSNS superfivebrane, carrying an arbitrary classical charge $P_{(p_1,p_2)} = \Delta_{(p_1,p_2)}^{1/2} P_0$, with $\Delta_{(p_1,p_2)}$ an as yet undetermined dimensionless factor and $P_0$ the charge unit which may be taken as the quantized unit charge.
(for example, the fundamental NSNS fivebrane tension $T_5$), is associated with the 3-form field strength $H^{(1)}$. The meaning of the subscript $(p_1,p_2)$ will become clear when we discuss the charge quantization for the fivebranes. The general classical fivebrane configuration which we will construct requires both the 3-form field strengths $H^{(1)}$ and $H^{(2)}$ to be non-zero. Associated with this configuration is an arbitrary but given classical charge doublet

$$P = \begin{pmatrix} P^{(1)} \\ P^{(2)} \end{pmatrix}.$$  

From $P = (\Lambda^T)^{-1} \begin{pmatrix} P_{(p_1,p_2)} \\ 0 \end{pmatrix}$ with $\Lambda$ given by Eq. (8) and the relation $\cos^2 \alpha + \sin^2 \alpha = 1$, we completely determine the SL(2,R) matrix

$$\Lambda = \frac{1}{\Delta_{(p_1,p_2)}^{1/2}} \begin{pmatrix} e^{-\phi_0} P^{(1)}/P_0 & -P^{(2)}/P_0 + \chi_0 P^{(1)}/P_0 \\ +\chi_0 e^{\phi_0} (P^{(2)}/P_0 + \chi_0 P^{(1)}/P_0) & +\chi_0 P^{(1)}/P_0 \end{pmatrix},$$

and the $\Delta_{(p_1,p_2)}$ factor as,

$$\Delta_{(p_1,p_2)} = e^{-\phi_0} (P^{(1)}/P_0)^2 + (P^{(2)}/P_0 + \chi_0 P^{(1)}/P_0)^2 e^{\phi_0}$$
$$= (P^{(1)}/P_0, P^{(2)}/P_0) \mathcal{M}_0 \begin{pmatrix} P^{(1)}/P_0 \\ P^{(2)}/P_0 \end{pmatrix},$$  

where $\mathcal{M}_0 = \begin{pmatrix} \chi_0^2 + e^{-2\phi_0} & \chi_0 \\ \chi_0 & 1 \end{pmatrix} e^{\phi_0}$. It is clear that as $\Delta_{(p_1,p_2)}$ in (11) is SL(2,R) invariant, the charge $P_{(p_1,p_2)} = \Delta_{(p_1,p_2)}^{1/2} P_0$ is also SL(2,R) invariant.

By now we have constructed the most general SL(2,R) fivebrane configuration carrying classical charges given by the charge doublet. The central charge (therefore the ADM mass per unit fivebrane which can be calculated following 11 and the tension measured in Einstein metric) associated with this fivebrane is $P_{(p_1,p_2)} = \Delta_{(p_1,p_2)}^{1/2} P_0$ with $\Delta_{(p_1,p_2)}$ given by Eq. (11). The metric continues to be given by the metric in Eq. (7) but now with $P = P_{(p_1,p_2)}$. The 3-form field strength doublet is simply

$$\mathcal{H} = (\Lambda^T)^{-1} \begin{pmatrix} P_{(p_1,p_2)} \\ 0 \end{pmatrix} \epsilon_3$$
$$= \begin{pmatrix} P^{(1)} \\ P^{(2)} \end{pmatrix} \epsilon_3,$$  

(12)
as expected. The complex scalar is now
\[ \lambda = \frac{a(e^{-\phi}) + b}{c(e^{-\phi}) + d} = \frac{x_0 \Delta_{(p_1, p_2), A(p_1, p_2)} + P^{(1)} P^{(2)}/P_0^2 (A_{(p_1, p_2)}^{-1})_0 e^{-\phi_0} + i \Delta_{(p_1, p_2)} A^2_{(p_1, p_2)} e^{-\phi_0}}{(P^{(1)}/P_0)^2 e^{-\phi_0} + A_{(p_1, p_2)} e^{\phi_0} (x_0 P^{(1)}/P_0 + P^{(2)}/P_0)^2}, \] (13)

where \( A_{(p_1, p_2)} = \left(1 + \frac{P^{(1)} P^{(2)}}{2P_0^2}\right)^{-1} \). Note that asymptotically as \( \rho \to \infty \), \( A_{(p_1, p_2)} \to 1 \) and therefore, \( \lambda \to \lambda_0 \) as expected. The real and imaginary part of (13) give the transformed value of the RR scalar and the dilaton of the theory. Partial results of the classical fivebrane was also obtained by Bergshoeff et al.\(^{12}\).

Our general classical fivebrane solution also preserves half of the spacetime supersymmetry as the original pure NSNS fivebrane since the global SL(2,R) transformation commutes with the supersymmetry transformation. Therefore, our general fivebrane solution continues to be BPS which implies that the ADM mass per unit fivebrane volume, the central charge \( P_{(p_1, p_2)} \) and the fivebrane tension measured in Einstein metric are all the same in proper units.

So far we have only considered the most general classical type IIB fivebrane solution preserving half of the spacetime supersymmetry in the sense that the two charges \( P^{(1)} \) and \( P^{(2)} \) can be arbitrary. As mentioned before, due to the existence of both F-string and D-string, the magnetic duals of the NSNS fivebrane and RR fivebrane, respectively, each of the two charges must be quantized separately in terms of the unit charge \( P_0 \). For example, F-string whose charge doublet corresponds to \( Q^{(1)} = Q_0, Q^{(2)} = 0 \) implies that the charge \( P^{(1)} \) is quantized in terms of the unit charge \( P_0 = 2\pi/Q_0 \). So the charge doublet for a general quantum mechanically allowable fivebrane solution is
\[ P = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} P_0, \] (14)

where \( p_1, p_2 \) are two integers. In terms of the unit charge \( P_0 \), the charge doublet should remain to be an integral doublet under quantum-mechanically allowable transformation. This necessarily breaks the continuous SL(2,R) symmetry of type IIB supergravity to a discrete SL(2,Z) of type IIB non-perturbative string theory whose elements take only integral values.

The most general quantum-mechanically allowable fivebrane configuration can be obtained simply by setting \( P^{(1)} = p_1 P_0, P^{(2)} = p_2 P_0 \) in the above classical fivebrane solution. For example,
\[ \Delta_{(p_1, p_2)} = (p_1, p_2) M_0 \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \] (15)
which is now SL(2,Z) invariant. Therefore, the ADM mass per unit fivebrane volume $M_{(p_1,p_2)}$, the central charge $P_{(p_1,p_2)}$ and the fivebrane tension $T_{(p_1,p_2)}$ measured in Einstein metric are all SL(2,Z) invariant. In proper units, we can set all three equal in which case we can take $F_0$ as the fundamental NSNS fivebrane tension $T_5$. Then for a $(p_1,p_2)$-fivebrane, we have

$$M_{(p_1,p_2)} = P_{(p_1,p_2)} = \Delta_{(p_1,p_2)}^{1/2} T_5 = \sqrt{e^{-\phi_0} p_1^2 + (p_2 + p_1 \chi_0)^2} e^{\phi_0} T_5,$$

(16)

The $(p_1,p_2)$-fivebrane tension measured in type IIB string metric is

$$T_{(p_1,p_2)} = g_s^{-3/2} \sqrt{g_s^{-1} p_1^2 + (p_2 + p_1 \chi_0)^2} g_s T_5,$$

(17)

where $g_s = e^{\phi_0}$ is the string coupling. Here $T_{(p_1,p_2)}$ gets scaled by $g_s^{-3/2}$ because it has dimensionality of $(\text{length})^{-6}$. From the above tension formula, we can see that the tension for a $(1,0)$-fivebrane, i.e., a NSNS fivebrane, is equal to $T_5/g_s^2$ and the tension for a $(0,1)$-fivebrane, i.e., a RR fivebrane, is equal to $T_5/g_s$, as expected in both cases. Implementation of charge quantization on the charges carried by the fivebrane consists of the second step of our construction.

The above $(p_1,p_2)$-fivebrane configuration encodes all the information about the SL(2,Z) multiplets of the type IIB superfivebranes. Note that for given asymptotic values of the scalars, i.e., for a given vacuum, each of the infinitely many integral doublets $(p_1,p_2)^T$ gives a different value for the $\Delta_{(p_1,p_2)}$ which cannot be related to each other by a SL(2,Z) transformation since it is invariant by such a transformation. Further, this $\Delta_{(p_1,p_2)}$ measures the mass per unit fivebrane volume, the central charge and the tension. Therefore, we can use this factor to label different SL(2,Z) multiplets. Within each such multiplet, we have a collection of infinitely many discrete vacua and a collection of infinitely many integral charge doublets. Each of such vacua and its corresponding integral charge doublet are obtained from the given initial vacuum $\lambda_0 = \chi_0 + i e^{-\phi_0}$ and the given initial charge doublet $(p_1,p_2)^T$ by a particular SL(2,Z) transformation. Picking a special vacuum in such a multiplet will break the SL(2,Z) symmetry spontaneously. In other words, all the fivebrane configurations in such a multiplet are physically equivalent. The physically inequivalent fivebrane configurations are those with different $\Delta_{(p_1,p_2)}$ values which correspond to different integral doublets $(p_1,p_2)^T$ for a fixed $\lambda_0$, i.e., a fixed vacuum.

The third step of our construction consists of the determination of the condition for which a $(p_1,p_2)$-fivebrane is absolutely stable (For discussion of the stability of SL(2,Z) strings, see [13]). We have noted that the tension of a
\( (p_1, p_2) \)-fivebrane is given by \( T_{(p_1, p_2)} = \Delta_{(p_1, p_2)}^{1/2} T_5 \) and it can easily be checked that the tensions satisfy the following triangle relation (also called “tension gap” equation), irrespective of the vacuum moduli,

\[
T_{(p_1, p_2)} + T_{(q_1, q_2)} \geq T_{(p_1+q_1, p_2+q_2)},
\]

where the equality holds if and only if \( p_1 q_2 = p_2 q_1 \), i.e., when \( p_1 = nq_1, p_2 = nq_2 \) with \( n \) being an integer (assuming \( p_1 \geq q_1, p_2 \geq q_2 \) without losing generality). So when \( p_1 \) and \( p_2 \) are relatively prime, the tension inequality prevents the fivebrane state from decaying to fivebrane states with lower tensions or lower masses. Therefore such a \( (p_1, p_2) \)-fivebrane is absolutely stable. The same conclusion can also be drawn when the central charge triangle inequality relation (now called “charge gap” equation) and the charge conservation are employed. We therefore finally conclude that all physically inequivalent absolutely stable BPS \( (p_1, p_2) \)-fivebrane configurations are those corresponding to all possible integral doublets \( (p_1, p_2) \) with \( p_1, p_2 \) coprime and with \( \lambda_0 \) belonging to the fundamental region of SL(2,Z), i.e., SL(2,Z) \( \backslash \) SL(2,R)/SO(2).

3 Discussion

We have demonstrated the procedure how to construct a general solution of a theory in detail when the theory has a global symmetry starting from a known particular solution of this theory, with the example of type IIB \( (p_1, p_2) \)-fivebranes. This procedure can also be applied to construct U-duality p-branes in diverse dimensions from the known NSNS p-branes (for example, see 14) whether they are supersymmetric or not as we did recently 15. We are carefully using the word “construct” rather than “generate” as we cannot really generate all the solutions from a given one simply by applying the underlying global symmetry transformations (since these transformations will in general preserve the corresponding masses, charges and tensions) just as we cannot relate different particles with different masses and spins simply by Lorentz transformations. In order to generate solutions from a given one, additional “symmetries” are needed which are most likely symmetries of certain spectra (for example, BPS states) of the underlying theory rather than symmetries of the theory itself as discussed recently by Lü et al. 16.

Finally it should be pointed out that we could determine the SL(2,R) matrix \( \Lambda \) (10) completely in terms of asymptotic values \( \chi_0 \) and \( \phi_0 \), and the charges \( P^{(1)} \) and \( P^{(2)} \) because we have four parameters (three for SL(2,R) matrix elements and one for the \( \Delta_{(p_1, p_2)} \) factor) and four equations (two real equations for the asymptotic complex scalars and two for the charges). However, this is not true in general. For example, for other U-duality groups, the number
of group matrix parameters plus the corresponding $\Delta$ factor exceeds that of the available equations. At first sight, it appears that the general solution thus constructed contains certain arbitrary parameters. As demonstrated recently by the present authors\textsuperscript{15}, this never happens. The $\Delta$ factor is always completely determined in terms of the asymptotic values of scalars and the charges. The undetermined group elements never enter the quantities which characterize the solution such as scalars and field strengths. In other words, the general solution is completely determined by the charges and asymptotic values of scalars as expected.

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References

1. J. H. Schwarz, \textit{Nucl. Phys.} B \textbf{226}, 269 (1983).
2. P. K. Townsend, “Four Lectures on M Theory”, hep-th/9612121.
3. Soo-Jong Roy, “The Confining Phase of Superstrings and Axionic strings”, \textit{Phys. Rev.} D \textbf{43}, 526 (1991).
4. A. Font, L. Ibañez, D. Lüst and F. Quevedo, \textit{Phys. Lett.} B \textbf{249}, 35 (1990).
5. A. Sen, Int. J. Mod. Phys. A \textbf{9}, 3707 (1994).
6. J. H. Schwarz, \textit{Phys. Lett.} B \textbf{360}, 13 (1995) (see revised version, hep-th/9508143).
7. J. X. Lu and S. Roy, \textit{Phys. Lett.} B \textbf{428}, 289 (1998).
8. A. Sen, “Developments in Superstring Theory”, hep-th/9810356.
9. M. J. Duff and J. X. Lu, \textit{Nucl. Phys.} B \textbf{354}, 141 (1991).
10. C. Hull, \textit{Phys. Lett.} B \textbf{357}, 545 (1995).
11. J. X. Lu, \textit{Phys. Lett.} B \textbf{313}, 29 (1993).
12. E. Bergshoeff, H. Boonstra and T. Ortin, \textit{Phys. Rev.} D \textbf{53}, 7206 (1996).
13. J. H. Schwarz, “Lectures on Superstring and M Theory Dualities”, hep-th/9607201.
14. M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rep. \textbf{259}, 213 (1995).
15. J. X. Lu and S. Roy, “U-duality P-Branes in Diverse Dimensions”, hep-th/9805180 (to appear in NPB).
16. H. Lü, C. Pope and K. Stelle, \textit{Nucl. Phys.} B \textbf{520}, 132 (1998).