Doubly–dressed atomic wave packets

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The problems of cavity atom optics in the presence of an external strong coherent field are formulated as the problems of potential scattering of doubly-dressed atomic wave packets. Two types of potentials produced by various multiphoton Raman processes in a high-finesse cavity are examined. As an application the deflection of dressed atomic wave packets by a cavity mode is investigated. New momentum distribution of the atoms is derived that depends from the parameters of coherent field as well as photon states in the cavity.

I. INTRODUCTION

Recent technological achievements in quantum optics in that number improvements in cavity design initiated the investigations of a single atom dynamics via its interaction with quantized cavity field. These studies have opened a new chapter of atom optics where we treat not only the states of the atom and its motion quantum mechanically, but have also quantized the cavity field [1-3]. Among the models of atom-cavity mode interactions most fascinating ones is related to potential problems in quantum theory. As was shown in context of micromazers [4-8], the resonant interaction between an incident atom and a cavity mode can be formulated as the elementary problem of a particle incident upon potential, if irreversible processes which give rise to damping, such as atomic spontaneous emission and decay of the field mode, are negligible. The cavity field in this approach acts as a potential barrier or as a potential well for the dressed-states of the atom-cavity field system.

In this paper we extend borders of the above noted analogy between atomic dynamics and scattering problem considering interaction of an atom with both a quantized cavity mode and a classical strong laser field. This situation may be realized, in particular, in the following scheme shown on Fig.1, where the atomic probe propagates through a cavity and the laser field travels orthogonally with respect to the cavity mode. In such schemes the moving atom interacts at the same time with a near-resonant, strong coherent field and a cavity mode. The atom is slow enough that adiabatic switching on the coherent field-atom interaction is realized and the position dependent dressed-states of the atom-coherent field system are formed [9]. As we show below, the strong coupling of a single atom to a single cavity mode induces transitions between these position dependent dressed states which can be explained through the cavity field dressing of the strong field-atom dressed states. Such approach allows us to formulate the problem of atoms coupled with quantized cavity mode in the presence of strong laser field as the problem of potential scattering of doubly-dressed atomic wave packet.

The application of doubly-dressed states to the problem of atomic deflection is the other purpose of this paper. Atomic deflection during its passage through the cavity has been studied extensively. A detail review on this field can be found in Ref. [1]. In particular, it was shown that the atomic deflection pattern contains information about the field state in the cavity and can be used for the probing and reconstruction of quantum states [10-13], and for subject of an atomic interferometry [14]. Here, we study the influence of the external coherent field on atomic deflection pattern.

II. POTENTIALS FOR DOUBLY-DRESSED STATES

We consider dynamics of an incident two-level atom with ground state \( |g\rangle \) and excited state \( |e\rangle \) moving along \( z \)-direction and passes a light field. This field consists of a standing quantized cavity mode in an arbitrary state and a running coherent field. We assume the time of interaction between the atom and the fields to be very short compared to the cavity lifetime \( 1/k \) and the inverse spontaneous decay rate \( \gamma \), that is \( t \ll 1/k, 1/\gamma \). The effective Hamiltonian of the atom placed at the point \( r(x, y, z) \) in a cavity in the rotating-wave approximations reads

\[
H = H_0 + H_R + H_{ext} + H_{int}. \tag{1}
\]

\( H_0 \) and \( H_R \) are the free Hamiltonians for the atom and the quantized cavity mode at frequency \( \omega_c \)

\[
H_0 = \frac{\hat{p}^2}{2m} + \omega_g |g\rangle\langle g| + \omega_e |e\rangle\langle e|,
\]

\[
H_R = \hbar \omega_c \hat{a}^\dagger \hat{a}, \tag{3}
\]

where

\[
\hat{a}^\dagger, \hat{a} \text{ are the cavity field modes.}
\]

\[
H_{ext} \text{ is the external Hamiltonian of a cavity fields,}
\]

\[
H_{int} \text{ is the interaction Hamiltonian between an atom and a cavity field.}
\]
where $\hat{p}$ denotes the center-of-mass motion momentum operator of the atom with mass $m$, and $\omega_e$ and $\omega_g$ are the energies of the excited and ground electronic states of the atom, respectively. The Hamiltonian

$$H_{\text{ext}} = \hbar \lambda u_L(r)(\sigma_+ e^{-i(\omega_L t + \varphi_L)} + h.c.)$$

(4)
describes the interaction between the atom and the classical traveling field of frequency $\omega_L$ and phase $\varphi_L$, where $\lambda$ is the coupling constant, $\sigma_+$ is the atomic flip operator and $u_L(r)$ is the position-dependent part of the external field amplitude. The interaction of the atom with a field mode reads

$$H_{\text{int}} = \hbar g u(r)(\sigma_+ a e^{-i\varphi} + h.c.),$$

(5)

where $g$ is the coupling constant and the field mode is determined by the spatial mode function $u(r)$, by a phase $\varphi$ and the annihilation and creation operators $a, a^\dagger$.

We use an interaction-picture version in which the equation for vector state of the total system is

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \left(\frac{\hat{p}^2}{2m} + \hbar g u(r) (\sigma_+(t) a e^{-i\omega_e t - \varphi_e} + \sigma_-(t) a^\dagger e^{i\omega_e t + \varphi_e})\right) |\Psi(t)\rangle,$$

(6)

where

$$\sigma_\pm(t) = \exp\left[\frac{i\hbar}{\hbar} \int_{-\infty}^{t} \left( H_0 + H_{\text{ext}}(t') \right) dt' \right] \sigma_\pm \exp\left[ -\frac{i\hbar}{\hbar} \int_{-\infty}^{t} \left( H_0 + H_{\text{ext}}(t') \right) dt' \right]$$

(7)

are the atomic states flip operators in this representation. It is useful to describe the atom-coherent field subsystem in terms of position-dependent Floquet states which for two-level atom at point $r$ and in the rotating-wave approximation are

$$|\Psi_1(r,t)\rangle = e^{-i\omega_1(r)t} \left( a |g\rangle + b |e\rangle e^{-i(\omega_L t + \varphi_L)} \right),$$

$$|\Psi_2(r,t)\rangle = e^{-i\omega_2(r)t} \left( -b |g\rangle e^{i(\omega_L t + \varphi_L)} + a |e\rangle \right).$$

(8)

The quasienergies $\hbar \omega_1(r)$ and $\hbar \omega_2(r)$ are equal to

$$\omega_1(r) = \omega_g + \frac{1}{2} (\delta - \Omega(r)),$$

$$\omega_2(r) = \omega_e - \frac{1}{2} (\delta - \Omega(r)),$$

(9)

where the position-dependent Rabi frequency is $\Omega(r) = \sqrt{\delta^2 + 4\lambda^2 u_L(r)^2}$, and the coefficients $a, b$ are defined by $a = \sqrt{\frac{1}{2}(1 + \delta/\Omega)}, \quad b = \sqrt{\frac{1}{2}(1 - \delta/\Omega)}$, where the detuning $\delta = \omega_e - \omega_g - \omega_L \ll \omega_e - \omega_g$. The important point is that the states (8) will vary with the position $r$. They describe the atom-coherent field subsystem in the absence of spontaneous emission and for an atomic velocity sufficiently small that nonadiabatic transitions between these states can be ignored. In order to gain insight into the dynamics of the system it is convenient to operate with the states $|\Phi_1(r)\rangle = |\Psi_1(r,0)\rangle$ and $|\Phi_2(r)\rangle = |\Psi_2(r,0)\rangle$,

$$|\Phi_1(r)\rangle = a(r) |g\rangle + b(r) e^{-i\varphi_L} |e\rangle,$$

$$|\Phi_2(r)\rangle = -e^{i\varphi_L} b(r) |g\rangle + a(r) |e\rangle,$$

(10)

(11)

which are similar to the position-dependent dressed states introduced for a moving atom in Ref.[9]. Then, the state vector $|\Psi(t)\rangle$ of the total system can be expressed in the basis of such type of dressed-states and photon number states $|\Phi_i(r)\rangle \otimes |n\rangle$ and in the position representation is given as

$$|\Psi(t)\rangle = \sum_{i=1,2} \sum_{n=0}^\infty \int d^3 r f_i^n(r,t) |\Phi_i(r)\rangle \otimes |n\rangle \otimes r).$$

(12)

Substituting the quantum state (12) in Eq.(6) we find the evolution of the probability amplitudes in the following form
with the potential

\[ \text{with the spontaneous transitions. Following [9] for analyzing of nonadiabatic transitions and assuming a plane wave} \]

where \( \Omega(0) \) is a maximal value of the position-dependent Rabi frequency at the cavity centre

while for the zero-photon number the equations (13) give

\[ \text{where the coefficients} \quad A_{ji}(r,t) = \langle \Phi_j(r) | \sigma_-(t) | \Phi_i(r) \rangle \text{ are obtained as} \]

\[ A_{11} = -A_{22} = ab \exp[-i(\omega_L t + \varphi_L)], \]

\[ A_{12} = a^2 \exp[-i(\omega_L + \Omega)t], A_{21} = -b^2 \exp[-i(\omega_L + \Omega)t - 2i\varphi_L]. \]

The term \((F^n)_{NA}\) represents the contribution of nonadiabatic (NA) part induced by the transitions between dressed states of a moving atom in the absence of photon emission. They appear due to both the spatial and the temporal variations of the dressed states because the point \( r \) varies with time. Using that \( \partial \left| \Phi_i(r(t)) \right| / \partial t = \overline{\nu} \nabla \Phi_i(r(t)) \), where \( \overline{\nu} \) is the atomic velocity, the NA coupling can be written as

\[ (F^n)_{NA} = \sum_i f^n_i(r,t) \langle \Phi_j | \hat{p} | \Phi_i \rangle + \frac{1}{2m} \sum_i \langle \Phi_j | \hat{p} \hat{f}_i^n(r,t) \rangle - i\hbar \sum_i f^n_i(r,t) \overline{\nu} \langle \Phi_j \nabla \Phi_i \rangle. \]

We consider below an enough slow moving atom for which nonadiabatic effects are negligible. Let us give an order of magnitude the longitudinal velocity \( v_z \) of atomic wave packet for which the NA coupling is negligible compared with the spontaneous transitions. Following [9] for analyzing of nonadiabatic transitions and assuming a plane wave coherent field with the Gaussian transverse spatial distribution \( u_L(r) = e^{ik_yy^2/2}\exp(-z^2/2\Delta z^2) \) we arrive at

\[ v_z \ll \gamma^{1/3} \frac{\Delta z \Omega^2(0)}{(\delta \lambda u(0))^{2/3}} \]

where \( \Omega(0) \) is a maximal value of the position-dependent Rabi frequency at the cavity centre \( r = 0 \). In this limit the system adiabatically follows the dressed states and the last term of the equation (13) can be neglected. Therefore, the resulting equation involves only the radiative transitions.

The spectral lines of driving atom have well-known three-peak structure at the coherent field frequency \( \omega_L \) and two sidebands \( \omega_L \pm \omega \) which are symmetrically displaced about the central peak by the position-dependent Rabi frequency \( \Omega \). This circumstance makes it possibly to consider several atom-cavity mode couplings. There are two situations deserve attention occurring if the cavity frequency is resonant with one of two spectral lines of driving atom \( \omega_c = \omega_L - \Omega \) or \( \omega_c = \omega_L + \Omega \).

### A. "Three photon" coupling

Let cavity is tuned at the red spectral sideband of the driven atom. We assume the resonance condition \( \omega_c = \omega_L - \Omega(0) \) to be satisfied for the maximum value \( \Omega(0) \) of the position-dependent Rabi frequency and make the resonant approximation in Eq.(13) keeping the slowly oscillating terms \( e^{i\Delta(r)t} \), where \( \Delta(r) = \Omega(r) - \Omega(0) \). This procedure gives the system of coupled equations for the amplitudes \( f_{1n}^{n-1} \) and \( f_{2n}^n \). We can decouple these two equations by introducing the linear combinations of the amplitudes

\[ \phi_n^{(\pm)}(r,t) = f_{1n}^{n-1}(r,t) \pm e^{i(2\varphi_L - \varphi_c - \Delta t)} f_{2n}^n(r,t), \]

(for \( n \geq 1 \)). As it can be shown each of these components obeys the time-dependent Schrödinger equation

\[ i\hbar \frac{\partial}{\partial t} \phi_n^{(\pm)}(r,t) = \left( \frac{\hat{p}^2}{2m} + U_n^{(\pm)}(r) \right) \phi_n^{(\pm)}(r,t) \]

with the potential

\[ U_n^{(\pm)}(r) = \mp \frac{\hbar}{2} gu(r)(1 - \delta/\Omega(r))\sqrt{n}, \]

while for the zero-photon number the equations (13) give

\[ i\hbar \frac{\partial}{\partial t} f_{20}^0 = \frac{\hat{p}^2}{2m} f_{20}^0. \]
We have thus reduced the problem of atomic dynamics in a high-finesse cavity in the presence of external coherent field to the elementary scattering process. Rewriting the quantum state (12) in terms of the amplitudes (17) and projecting its onto the position eigenstate \(| r \rangle\) we obtain

\[
\langle r | \Psi(t) \rangle = f_2^1(r, t) | \Phi_2 \rangle | 0 \rangle + \frac{1}{\sqrt{2}} \sum_{n \geq 0} \left( \phi_{n+1}^{(+)}(r, t) | N_n^{(+)} \rangle + \phi_{n+1}^{(-)}(r, t) | N_n^{(-)} \rangle \right),
\]

where

\[
| N_n^{(\pm)} \rangle = \frac{1}{\sqrt{2}} \left( | \Phi_1 \rangle | n \rangle \pm e^{i(\Delta t + \varphi) - 2\varphi_L} | \Phi_2 \rangle | n + 1 \rangle \right),
\]

for \( n \geq 0 \). So, the quantum state of the system is obtained in terms of the orthogonal basis of doubly-dressed states \(| N_n^{(\pm)} \rangle\) which are formed as the linear combinations of the coherent field-atom dressed states multiplied on the occupation numbers of cavity mode. Note, that the emission and absorption of photons at the frequency \( \omega_c = \omega_L - \Omega(0) \) are stipulated by well known Raman scattering processes. For large detuning \( \delta^2 \gg \lambda^2 u_L^2(0) \) and in the lowest order of perturbation theory on coherent field-atom interaction these processes are shown in Fig.2. The atom radiates \( \omega_L - \Omega \) photon in the transition from the ground state \(| g \rangle\) to the excited state \(| e \rangle\) in which two photons of the coherent field are absorbed (Fig.2(a)). Absorption of \( \omega_L - \Omega \) photon from the field mode takes place in the transition \(| e \rangle \rightarrow | g \rangle\) with the radiation of two photons at frequency \( \omega_L \) (Fig.2(b)). Such Raman processes form the fluorescence spectrum of strongly driven two-level atoms into the mode of an optical cavity as has been experimentally demonstrated in [15]. For moving atom in a cavity the potential (19), which for the large detuning reads as

\[
U_n^{(\pm)}(r) = \mp \frac{h g^2 \lambda^2}{2 \delta^2} u(r) u_L(r)^2 \sqrt{n},
\]

is induced by the three-photon Raman processes.

### B. Resonance \( \omega_c = \omega_L + \Omega \) condition

In that resonant case Eqs.(13) are transformed to the coupled equations for the amplitudes \( f_1^n \) and \( f_2^{n-1} \). We decouple these equations by introducing the following combinations

\[
\theta_n^{(\pm)}(r, t) = f_1^n(r, t) + e^{i(\varphi_c - \Delta t)} f_2^{n-1}(r, t),
\]

(for \( n \geq 1 \)). Then the equations are simplified as

\[
\frac{i \hbar}{\partial t} \theta_n^{(\pm)}(r, t) = \left( \frac{\beta^2}{2mr^2} + V_n^{(\pm)}(r) \right) \theta_n^{(\pm)}(r, t),
\]

where the potential is equal to

\[
V_n^{(\pm)}(r) = \pm \frac{\hbar}{2} gu(r)(1 + \delta/\Omega(r)) \sqrt{n}.
\]

For \( n = 0 \) the equation is written as

\[
\frac{i \hbar}{\partial t} f_1^0 = \frac{\beta^2}{2m} f_1^0.
\]

In a similar way we transform the quantum state (12) in terms of the amplitudes (24) and the doubly-dressed states. The result is calculated as

\[
\langle r | \Psi(t) \rangle = f_2^0(r, t) | \Phi_1 \rangle | 0 \rangle + \frac{1}{\sqrt{2}} \sum_{n \geq 0} \left( \theta_{n+1}^{(+)}(r, t) | R_n^{(+)} \rangle + \theta_{n+1}^{(-)}(r, t) | R_n^{(-)} \rangle \right),
\]

through the following doubly-dressed states

\[
| R_n^{(\pm)} \rangle = \frac{1}{\sqrt{2}} \left( | \Phi_1 \rangle | n + 1 \rangle \pm e^{i(\Delta t - \varphi_c)} | \Phi_2 \rangle | n \rangle \right).
\]
for \( n \geq 0 \).

So, the quantum states (21), (22) and (28), (29) show a strong entanglement between the atom and the light field involved the coherent component and the single cavity mode. The cavity with fixed number of photon acts as the potential barrier or as the potential well for the doubly-dressed states of atom. It should be mentioned that the potentials (19) and (26) show characteristic dependence on both atom-light field coupling constants and the detuning of the coherent field.

The approach followed above is well adopted to the strong-coupling regime, where the coupling constants \( \lambda, g \) are greater than the decay rates of the atomic dipole \( \gamma \) and the cavity field \( k (\lambda \gg \max[\gamma, k], g \gg \max[\gamma, k]) \). In this regime and for the time intervals \( \max[g^{-1}, \lambda^{-1}] \ll t \ll \max[\gamma^{-1}, k^{-1}] \) the atomic relaxation are negligible during the atom transit time across the cavity. Another peculiarity of this regime is that three spectral lines of the atomic radiation into cavity modes at frequencies \( \omega_L, \omega_L - \Omega, \omega_L + \Omega \) are well resolved. It makes it possible to choose the cavity resonance frequency equal to one of the sidebands. The equations (18), (25) are valid for the velocities satisfying

\[
\Bigl[ | \Psi(t = 0) \rangle = \sum_{n \geq 0} \int dx f(x) c_n | x \rangle | g \rangle | n \rangle. \tag{30}
\]

It is known that the spatial periodicity of the standing wave leads to discrete atomic momenta with a spacing of \( \hbar k \). Our aim is to find the corresponding momentum distribution of the atom due to its interactions with both the coherent field and the cavity mode.

We first consider the scheme of atomic scattering in the cavity with resonant frequency \( \omega_c = \omega_L - \Omega(0) \) (A subsection). Using Eqs. (18)-(22) we arrive the quantum state after the interaction

\[
\langle x | \Psi(t) \rangle = \frac{f(x)}{\sqrt{2}} \sum_{n \geq 0} \left( (ac_n - e^{i(\phi_L - \phi_c)}bc_{n+1}) \exp(i\alpha(x,t)\sqrt{n + 1} \sin kx) \right) N_n^{(+)} +
\]

\[
(ac_n + e^{i(\phi_L - \phi_c)}bc_{n+1}) \exp(-i\alpha(x,t)\sqrt{n + 1} \sin kx) \right) N_n^{(-)} - f(x)bc_0 e^{-i\phi_c} | \Phi_2 \rangle | 0 \rangle, \tag{31}
\]

where we have introduced the following parameter

\[
\alpha(x,t) = \frac{q}{2} \int_0^t d\tau \left( 1 - \frac{\delta}{\sqrt{\delta^2 + 4\lambda^2 u_L^2(x,v_z \tau)}} \right) \tag{32}
\]

and the interaction time \( t = L/v_z \) is the time the atom needs to cross the resonator of length \( L \).

We consider an atomic wave packet of width much smaller than the cavity mode wavelength (more exactly, the width \( \Delta x \ll 2\pi/k \)) passing through the node of the field. This assumption allows us to replace the mode function by its expansion \( \sin (kx) \approx kx \). Moreover, we omit also the transfers profile dependence of the mode function of the coherent field, that gives \( \alpha = \frac{q}{2} \sqrt{1 - \delta/\Omega(0)} \). As a result the probability \( W_1(\overline{p}) \) for funding an atom with scaled the momentum \( \overline{p} = p/hk \) in units of photon momenta is obtained in the following form

III. DEFLECTION OF THE DRESSED ATOM

Consider now the deflection of dressed atomic wave packet by a quantized electromagnetic field in the Raman-Nath regime. In the scheme shown on Fig.1 we assume a sinusoidal spatial mode function \( u(x, y, z) = \sin(kx) \) for the quantized standing wave and treat the laser field as a wave packet \( u_L(x, y, z) = u_L(x, z) \exp(ik_Ly) \) with the wave vector \( k_L \) and with the sufficiently broadband transverse profile \( u_L(x, z) \) so that the adiabatic approximation holds.

The atomic motion along the \( z \)-axis is treated classically that implies that the initial kinetic energy \( p_z^2/2m \) is much larger than the change of the longitudinal momentum due to the interaction. The Raman-Nath approximation means that during the interaction time we neglect the transfer of kinetic energy in the \( x \)-and the \( y \)-directions to the atom. These approximations make it possible to reduce, the equations (18), (25) to the one-dimensional equations, where the coordinate \( z = v_z t \) is proportional to the interaction time.

Suppose now a two-level atom in the ground state \( | g \rangle \) and with transverse centre-of-mass wave function \( f(x) \) enters a cavity with a single-mode standing light in the quantum state \( \sum c_n | n \rangle \). The initial state vector reads

\[
| \Psi(t = 0) \rangle = \sum_{n \geq 0} \int dx f(x) c_n | x \rangle | g \rangle | n \rangle. \tag{30}
\]

It is easy to see from (16) that for a resonant coherent wave \( (\delta = 0) \) the adiabatic approximation holds for any velocity.
where the probability $W^0(\overrightarrow{p})$ describes the initial distribution of the atoms before they enter the light fields. The other resonant case (B subsection), when $\omega_r = \omega_h + \Omega(0)$ can be considered in the similar way. The corresponding probability of atomic deflection $W_2(\overrightarrow{p})$ is calculated as

$$W_2(\overrightarrow{p}) = \frac{1}{2} \sum_{n \geq 0} \left[ |a_{n+1} - e^{i(\varphi_L - \varphi_o)} b_n|^2 W^0(\overrightarrow{p} + \beta \sqrt{n+1}) + b^2 |c_0|^2 W^0(\overrightarrow{p}) \right],$$

where the interaction parameter is equal to $\beta = \frac{1}{4}gt(1 + \delta/\Omega(0))$.

The expressions (33) and (34) demonstrate the multiplex structures of the distributions centered at the discrete atomic momenta with a spacing of $\alpha$ and a sufficient detuning $\delta$ to the probabilities of finding the atomic states $| g \rangle$ or $| e \rangle$ in the dressed state $| \Phi_1 \rangle$ which are $a^2$ or $b^2$. For a sufficiently large detuning $\delta \gg \lambda u_L(0)$ we have $a \rightarrow 0$ and $\beta \rightarrow gt$, and also $a = 1, b = 0$. In consequence, the distribution (33) contains only the central peak $W_1(\overrightarrow{p}) = W^0(\overrightarrow{p})$ while $W_2(\overrightarrow{p})$ becomes equal to the well known result for the momentum distribution of a deflected atom which enters the cavity in the ground state $| g \rangle$ in the absence of a coherent field [1]. The other point that should be mentioned is that the superposition of the two atomic states contributes to $W_1(\overrightarrow{p})$ and $W_2(\overrightarrow{p})$ as the interference terms. Such effects have been studied in [12] for the deflection of atoms that are initially prepared in a superposition state of a single cavity mode.

It should be mentioned again that the momentum distribution $W_1(\overrightarrow{p})$ describes the atomic deflection due to the multiphoton Raman scattering process. We comment this result for the simplest situation where the cavity field is initially in the vacuum state: $c_0 = 1, c_n = 0, n \neq 0$, i.e. the potential is produced by the vacuum of the cavity field. In this case the momentum distribution reads

$$W_1(\overrightarrow{p}) = \frac{a^2}{2} W^0(\overrightarrow{p} + \alpha) + W^0(\overrightarrow{p} - \alpha)) + b^2W^0(\overrightarrow{p})$$

and shows three peaks at $p = 0$ and $p = \pm \alpha \hbar k$. Expanding the initial state as $| g \rangle = a | \Phi_1 \rangle - be^{-i\varphi_L} | \Phi_2 \rangle$, consider the dynamics of two component. Note that the interaction with cavity mode lead to the decay of dressed state $| \Phi_1 \rangle \rightarrow | \Phi_2 \rangle$, with the amplitude proportional to $a^2$. The corresponding Raman transition $| g \rangle \rightarrow | e \rangle$ with radiation of $\omega_L - \Omega$ photon forms the peaks at $p = \pm \alpha \hbar k$ of the distribution (35). In this way the central peak is determined by $| \Phi_2 \rangle$ state overlap of $| g \rangle$ with the amplitude $b^2$. To illustrate this result we consider a spatial distribution $f(x)$, which is Gaussian with width $\Delta x$. Its corresponding initial momentum distribution is equal to $W^0(\overrightarrow{p}) = k\Delta x \exp(-k^2 \Delta x^2 \overrightarrow{p}^2)/\sqrt{\pi}$. In Fig 3 we depict the momentum distribution (35) for two values of the interaction parameter $d = 4\lambda^2 u_L^2/\gamma^2$ and for strong-coupling regime. In Fig 4 for comparison, we show distribution $W_2(\overrightarrow{p})$ for the other resonant configuration $(\omega_r = \omega_h + \Omega)$ considering interaction of moving Gaussian wavepacket with a single mode vacuum field in a cavity $(c_0 = 1, c_n = 0, n \neq 0)$. Note that in Figs. 3, 4 attainable for the experiments parameters are used. In fact, according to the previous analysis the range of the interaction parameter $gt$ should be $max[1, g/\lambda] \ll gt \ll max[g/\gamma, g/k]$, that is mainly restricted by the ratios of the coupling constant on the decay rates. So far, several experiments in cavity QED [16-18] have investigated the interaction of an atom with electromagnetic field in the strong coupling regime, in particular, with the following parameters $g/\gamma = 6, g/k = 11$ [18].

**IV. CONCLUSION**

In conclusion, we have presented a novel model in atomic optics based on the doubly-dressed states. We suggest that various effects of atomic optics can be explained from such simple model involving the dressing of the strong field-atom dressed states by the cavity field. Really, it is shown that the cavity with fixed number of photons creates a barrier and a well potential for the external motion of the atom corresponding to the doubly-dressed states $| N_n^{(2)} \rangle$ (in the configuration A) and $| R_n^{(2)} \rangle$ (in the configuration B). Such model generalizes the well-known approach in atomic optics in quantized light fields on the case involved also the strong coherent field. In distinct to the standard atomic scattering problems in which the atom-cavity mode couplings are constants, in the presented model the corresponding couplings are the functions of the detuning and the resonant strong-field Rabi frequency. We have applied this model
to the deflection of two-level atomic wave packets. Although the primary application of the model was concerned with two-level atom, the analogous results may be obtained for various atomic configurations.

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Figure captions

Fig.1. Sketch of the experimental setup. A two-level atom in the ground state $|g\rangle$ moving along z-direction enters a cavity with both a single-quantized mode and a classical laser field. Pump laser beam crosses the cavity axis.

Fig. 2. Schematic diagrams for the Raman processes showing the transfer of population between the ground and the excited states. Stimulated emission (a) and absorption (b) of photon at frequency $\omega_L - \Omega(0) \approx 2\omega_L - (\omega_e - \omega_g)$ (dashed arrows) of the field mode occur due to the interaction of two-level atom with laser field (straight arrows).

Fig. 3. Momentum distribution of the atomic wavepacket after interaction with $\omega_c = \omega_L - \Omega(0)$ mode vacuum field in the cavity. The parameters are: $k\Delta x = 1$, $gt = 50$, $d = 1(a)$, $d = 0.6(b)$.

Fig. 4. Momentum distribution $W_2(\vec{p})$ of the atomic wavepacket passing through the empty cavity calculated from Eq. (34) for the two interaction parameters $d = 1(a)$ and $d = 1.5(b)$. The other parameters are: $k\Delta x = 1$, $gt = 50$. 

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Fig. 2
$W_2(p/hk)$

(a)

(b)