A Comparison of Simple Galaxy Mass Estimators

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1. Introduction

Owing to their complex stellar dynamics, determining the mass profiles and mass-to-light ratios of elliptical galaxies is difficult. Now, however, kinematic data, once limited to less than an effective radius, are available out to several $R_{eff}$ and may include higher-order moments of the velocity profile (e.g. Carollo et al. 1995, Sembach & Tonry 1996). Because mean rotational velocities and velocity dispersions reveal no information about the anisotropy of the system, mass estimation methods which rely only on them are degenerate. Unfortunately, higher-order kinematical data are difficult to obtain, the dynamical modeling applied to them remains dependent on model assumptions, and the methods are time-consuming to implement (e.g. Rix et al. 1997).

Here, three simple methods of mass estimation, which are easy to apply to kinematic data, are compared.

1. virial: $M(r) = \frac{3\pi}{2G} \left( \sum_{i=1}^{N} \sigma_i^2 \right) / \left( \sum_{i=1}^{N} \frac{1}{R_i} \right)$

2. projected: $M(r) = \frac{f_p}{\pi GN} \sum_{i=1}^{N} v_{rot,i}^2 R_i$, where $f_p = 32$ for an isotropic orbital distribution ($f_p = 64$ for radial orbits).

3. Jeans equation for a spherically symmetric system:

$$\frac{d(\rho v_r^2)}{dr} + \frac{\rho}{r} (2v_r^2 - v_\theta^2 - v_\phi^2) = -\frac{GM(r)\rho}{r^2}$$

in which second velocity moments $v_r^2 = \sigma^2$, $v_\theta^2 = \sigma^2$, and $v_\phi^2 = \sigma^2 + V_{rot}^2$ for an isotropic velocity dispersion. Density, mass distribution, and rotation velocity are assumed: $ho(r) = \frac{Ma}{2\pi \times (r+a)^2}$, $M(r) = \frac{M_d r^2}{(r+d)^2}$, $V_{rot}(r) = \frac{v_0}{(r^2 + r_0^2)^{1/2}}$, where $M_l$ is luminous mass, $a$ is scale-length of the luminous matter, $M_d$ is dark mass, $d$ is scale-length of the dark matter, and $v_0$ and $r_0$ are determined by fitting the projected rotation to the major axis of the remnants.

High-resolution models of elliptical galaxies from N-body simulations are used to compare predicted mass profiles. Details of the eight models are found in Weil & Hernquist (1994, 1996). The dark matter halos of the models are nearly spherical, with $<b/a> = 0.97$ and $<c/a> = 0.86$. The luminous components of the models are more triaxial, with $<b/a> = 0.88$ and $<c/a> = 0.64$.

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2. Results

The mass profile estimates for eight elliptical models were averaged to compare the true and calculated mass profiles for the three mass estimators. For the Jeans equation mass estimator, projected rotation $V_s(R)$, surface mass density $\Sigma(R)$, and the line-of-sight second velocity moment $V_{los}^2(R)$ are determined such that projected velocity dispersion is $\sigma^2_s(R) = V_{los}^2 - V_s(R)$, which depends on the four parameters which describe the mass distribution, $a, M_t, d, M_d$ (code available from M.L. Weil).

Figure 1 shows the fractional difference profiles for slits of length 34 kpc laid along projections of the stellar data. For the virial and Jeans equation mass estimators, these are averaged over six directions in the three intrinsic projections (short dashed and solid lines). For the projected and Jeans equation mass estimators, results are also averaged over the two directions in which the true rotational velocity, $v_{rot}$, appears (long dashed and long-short dashed lines). All the mass estimators are comparably poor for $r < 4$ kpc, with errors greater than 30%, and the projected estimator is poor at all radii. However, the errors for the other mass estimators are within $\approx 10 - 20\%$ for $r > 8$ kpc, although error for the Jeans equation estimator, as applied to random projections, increases to 25% at $r = 40$ kpc. The Jeans equation estimator for projections with $v_{rot}$ and the simple virial estimator reproduce the true masses to within $\approx 15\%$ at large radii.

References

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Figure 1. Radial profile of fractional difference between true and calculated total masses for three mass estimators.