Research on master-slave filtering of Celestial Navigation System / Inertial Navigation System

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Abstract: As the equipment with the highest attitude measurement accuracy and minimum drift, Celestial Navigation System (CNS) has widely applied and developed on various platforms, such as air-based, sea-based and land-based platforms. Aiming at the application characteristic of the CNS equipment, this article mainly analyzes the two operating modes based on gyro correction and overall estimation of INS error in the shipborne CNS/INS. Based on different selection of measurement information in CNS/INS, optimized choice is made in the integrated mode of gyro drift correction based on starlight and the integrated mode of satellite inertial navigation based on the overall estimation of inertial navigation error. This paper designs a master-slave filtering mode of CNS/INS to switch the filtering according to the observation conditions and verify through simulation analysis. This method can improve the adaptability of CNS/INS to the onboard environment; improve the positioning accuracy of about 40% compared with the single working mode during the long working of the integrated system, which can guarantee the navigation accuracy of the combined system.

1. Introduction

The Celestial Navigation System / Inertial Integrated Navigation System (referred to as CNS/INS) corrects INS error with the course and position information not accumulated over time provided by starlight navigation to achieve high precision integrated navigation. It is applied to multiple platforms such as aircraft, warship, UAV, missile, balloon, etc. Nowadays, many countries in the world, especially the countries with leading inertial navigation technology, even including the United States, Russia and other countries with autonomy in satellite navigation, are vigorously developing starlight navigation technology. These military powers all attach great importance to the research and development of starlight navigation technology and starlight/inertial integrated navigation technology, and take this technology as an important means of navigation technology and widely used in various fields[1].

2. Master-slave filter combination mode

2.1 Gyro drift combination mode based on starlight correction

Refer Figure 3 for the combined mode principle of gyroscope drift based on starlight correction. The inertial coordinate system is taken as the reference coordinate system, star navigation rotation
quaternion $\hat{q}_{ib}$ is relatively high in accuracy, and is taken as the reference measurement, and the
difference between the rotation quaternion $\hat{q}_{ib}$ and the inertial navigation is the additive quaternion
error of the inertial navigation. The Kalman filtering algorithm is applied to the data processing of the
combined system, that is, the quaternion error equation of INS is selected as the state equation, and the
quaternion error of INS is taken as the measurement to establish the measurement equation [2,3]. The
drift and attitude error of INS gyro are estimated to realize the gyro drift calibration and attitude
quaternion error correction.

![Figure 1 Gyro drift combination mode based on starlight correction](image)

The quaternion error of INS is:
\[
\hat{\delta}q_{ib} = \hat{q}_{ib} - \hat{q}_{ib}
\]  
(1)

The quaternion error and gyro drift are taken as state variables, i.e. state variables are:
\[
X = \left[ \delta\hat{q}_0, \delta\hat{q}_1, \delta\hat{q}_2, \delta\hat{q}_3, q_{ibx}, q_{iby}, q_{ibz}, q_{ibx}, q_{iby}, q_{ibz} \right]^T
\]  
(2)

In inertial navigation, gyroscope is fixed on the carrier, the sensitivity of the gyroscope is that the
motion angular velocity vector of the carrier system relative to the inertial system is projected as $\omega_{ib}$
in the carrier system, and the quaternion is expressed as:
\[
\hat{q}_ib = \left[ 0 \quad \omega_{ib}^b \quad \omega_{ib}^b \quad \omega_{ib}^b \right].
\]  
(3)

Then:
\[
\omega_{ib} = \hat{q}_ib \otimes \omega_{ib} \otimes \hat{q}_ib^*
\]  
(4)

Substituting equation (4) into equation (3), we can get:
\[
\hat{q}_ib = \frac{1}{2} \hat{q}_ib \otimes \omega_{ib} \otimes \hat{q}_ib^* = \frac{1}{2} \hat{q}_ib \otimes \omega_{ib}
\]  
(5)

Equation (5) is written in the form of matrix vector:
\[
Q(\hat{q}_ib) = \frac{1}{2} M(\hat{q}_ib) \otimes Q(\omega_{ib}) = \frac{1}{2} M^*(\omega_{ib}) Q(\hat{q}_ib)
\]  
(6)

Where: 
\[
M^*(\omega_{ib}) = \begin{bmatrix}
0 & -\omega_{ib}^x & -\omega_{ib}^y & -\omega_{ib}^z \\
\omega_{ib}^x & 0 & -\omega_{ib}^z & \omega_{ib}^y \\
-\omega_{ib}^y & \omega_{ib}^z & 0 & -\omega_{ib}^x \\
\omega_{ib}^z & -\omega_{ib}^y & \omega_{ib}^x & 0
\end{bmatrix},
\]  
\[
Q(\hat{q}_ib) = \begin{bmatrix}
\hat{q}_0 & -\hat{q}_1 & -\hat{q}_2 & -\hat{q}_3 \\
\hat{q}_1 & \hat{q}_0 & -\hat{q}_3 & \hat{q}_2 \\
\hat{q}_2 & \hat{q}_3 & \hat{q}_0 & -\hat{q}_1 \\
\hat{q}_3 & -\hat{q}_2 & \hat{q}_1 & \hat{q}_0
\end{bmatrix}
\]
Where $F(t)$ is the system state transition matrix, $G(t)$ is the system noise transfer matrix, and the definitions are as follows:

$$F(t) = \begin{bmatrix}
0_{b\times 4} & 0_{b\times 3} & 0_{b\times 3} \\
0_{b\times 4} & 0_{b\times 3} & 0_{b\times 3}
\end{bmatrix}_{10\times 10}, \quad G(t) = \begin{bmatrix}
0_{b\times 4} & 0_{b\times 3} & 0_{b\times 3} \\
0_{b\times 4} & I_{3\times 3} & 0_{3\times 3} \\
0_{b\times 4} & 0_{3\times 3} & I_{3\times 3}
\end{bmatrix}_{10\times 10}$$

$$M(q_{i/b}) = \begin{bmatrix}
-q_{i/b1} & -q_{i/b2} & -q_{i/b3} \\
q_{i/b0} & -q_{i/b3} & q_{i/b2} \\
q_{i/b3} & q_{i/b0} & -q_{i/b1} \\
-q_{i/b2} & q_{i/b1} & q_{i/b0}
\end{bmatrix}_{4\times 3}$$

Taking the quaternion error of INS as the measurement to establish the measurement equation:

$$Z(t) = H(t)X(t) + V(t)$$

Where $H = [I_{4\times 4} \ 0_{4\times 3}]$ is the measurement transfer matrix; and $V$ is the system measurement noise.

### 2.2 Satellite inertial combination model based on the overall estimation of inertial navigation error

The error equation of the inertial navigation system is used as the state equation of the combined system, by subtracted attitude information provided by the inertial navigation system and the attitude information provided by the starlight navigation system, the attitude error angle can be obtained and used as the measurement to construct the corresponding measurement equation, and the optimal filtering method is used to estimate and compensate the errors of inertial devices in real time. The output frequency of star sensor data is lower than that of INS data updating. Therefore, it is common to fuse the results of INS with the data of star sensor after several solving periods of INS, and to modify the INS at the same time. Refer Figure 2 for the schematic diagram of its combined mode.
\[
\begin{aligned}
\dot{\phi}_x &= -\frac{\delta v_y}{R_n + h} + (\omega_x \sin \varphi + \frac{v_x}{R_n + h} \tan \varphi) \phi_x - (\omega_x \cos \varphi + \frac{v_x}{R_n + h}) \phi_y + \epsilon_x \\
\dot{\phi}_y &= \frac{\delta v_y}{R_n + h} - \omega_x \sin \varphi \delta \varphi - (\omega_x \sin \varphi + \frac{v_x}{R_n + h} \tan \varphi) \phi_y - \frac{v_x}{R_n + h} \phi_z + \epsilon_y \\
\dot{\phi}_z &= \frac{\delta v_z}{R_n + h} \tan \varphi + (\omega_x \cos \varphi + \frac{v_x}{R_n + h} \sec^2 \varphi) \delta \varphi + (\omega_x \cos \varphi + \frac{v_x}{R_n + h}) \phi_y + \frac{v_x}{R_n + h} \phi_z + \epsilon_z 
\end{aligned}
\]

The velocity error equation is:
\[
\begin{aligned}
\delta \dot{v}_x &= f_x \delta \phi_x - f_y \delta \phi_y + \left( \frac{v_x}{R_n + h} \tan \varphi - \frac{v_y}{R_n + h} \delta \varphi \right) \delta \dot{v}_y + \left( 2 \omega_x \sin \varphi + \frac{v_x}{R_n + h} \tan \varphi \right) \delta v_y \\
&\quad - \left( 2 \omega_x \cos \varphi - \frac{v_x}{R_n + h} \right) \delta v_y + \left( 2 \omega_x \cos \varphi + \frac{v_x}{R_n + h} \sec^2 \varphi \right) \delta \varphi + \delta V_y \\
\delta \dot{v}_y &= f_x \delta \phi_x - f_y \delta \phi_y + 2 (\omega_x \sin \varphi + \frac{v_x}{R_n + h} \tan \varphi) \delta v_y - \frac{v_y}{R_n + h} \delta v_y - \frac{v_x}{R_n + h} \delta \dot{v}_y \\
&\quad - \left( 2 \omega_x \cos \varphi + \frac{v_x}{R_n + h} \sec^2 \varphi \right) v_y \delta \varphi + \delta V_y \\
\delta \dot{v}_z &= f_x \delta \phi_x - f_y \delta \phi_y + 2 (\omega_x \cos \varphi + \frac{v_x}{R_n + h} \tan \varphi) \delta v_y + 2 \frac{v_y}{R_n + h} \delta v_y - 2 \omega_x \sin \varphi \delta \varphi \\
&\quad + \delta V_y + \frac{2 \varepsilon}{R} \delta h 
\end{aligned}
\]

The position error equation is:
\[
\begin{pmatrix}
\delta \varphi \\
\delta \lambda \\
\delta h
\end{pmatrix}
= 
\begin{pmatrix}
\frac{\delta v_y}{R_n + h} \\
\frac{\delta v_z}{R_n + h} \\
\delta v_z
\end{pmatrix}
\]

The measurement equation is established:
\[
\begin{pmatrix}
\delta \gamma \\
\delta \theta \\
\delta \psi
\end{pmatrix}
= 
\begin{pmatrix}
-\frac{1}{\cos \theta} (\phi_x \sin \psi + \phi_y \cos \psi) \\
-\phi_x \cos \psi + \phi_y \sin \psi \\
-\frac{1}{\cos \theta} (\phi_x \sin \theta \cos \psi + \phi_y \sin \theta \sin \psi - \phi_z \cos \theta)
\end{pmatrix}
\]

Written in matrix form:
\[
\begin{pmatrix}
\delta \gamma \\
\delta \theta \\
\delta \psi
\end{pmatrix}
= H_\varphi
\begin{pmatrix}
\phi_x \\
\phi_y \\
\phi_z
\end{pmatrix}
\]

Equation (6) represents the conversion from the error angle of inertial navigation mathematical platform to the error angle of attitude, where the transformation coefficient matrix \( M_\varphi \) is:
\[
H_\varphi = -\frac{1}{\cos \theta}
\begin{pmatrix}
\sin \psi & \cos \psi & 0 \\
\cos \psi \cos \theta & -\sin \psi \cos \theta & 0 \\
\sin \psi \sin \theta & \cos \psi \sin \theta & -\cos \theta
\end{pmatrix}
\]

In CNS/INS, the strapdown inertial navigation system can output attitude (roll angle \( \gamma \), pitch angle \( \theta \), heading angle \( \psi \)) position (longitude \( \lambda \), latitude \( \varphi \), altitude \( h \)) information, and starlight direction finder can output attitude (roll angle \( \gamma \), pitch angle \( \theta \), heading angle \( \psi \)).
position \((\text{longitude } \lambda, \text{ latitude } \phi)\). The attitude and position combination of CNS/INS is an extension of observation on the basis of attitude combination. The measurement equation of the CNS/INS is as follows:

\[
Z(t) = \begin{bmatrix}
\gamma - \gamma_i \\
\theta - \theta_i \\
\psi - \psi_i \\
\lambda - \lambda_i \\
\varphi - \varphi_i \\
\gamma - \gamma_j
\end{bmatrix} = \begin{bmatrix}
Z_p(t) \\
H_p(t)
\end{bmatrix} X(t) + \begin{bmatrix}
V_p(t) \\
V_p(t)
\end{bmatrix} = H(t)X(t) + V(t)
\] (15)

2.3 Master-slave filter combination mode

When the starlight equipment can perform pose and positioning when two (non-collinear) or three stars are observed. However, the positioning accuracy of three stars is relatively high. Double star positioning can solve two sets of longitude and latitude values, and prior information is required to exclude pseudo values. When the two groups of values are close to each other, there may be a problem of eliminating difficulties. Since only the measurement of the third star is needed to find the position, so three stars positioning has more advantages. In sea-use environments, especially under daytime conditions, starlight devices do not always guarantee an effective number of observed stars. Therefore, the starlight device needs to determine its position and positioning algorithm according to the actual number of observed stars.

The combination mode of gyro drift based on starlight correction uses the quaternion transformation of star navigation carrier coordinate system relative to inertial coordinate system as external reference to construct the measurement. Without depending on the inertial navigation system to provide the horizontal attitude, this mode can avoid the horizontal attitude error of INS and realize fast estimation of gyro drift. The mode based on comprehensive estimation can not only estimate the errors of inertial components, but also correct the misalignment angle and output errors of INS in time. However, this mode needs to observe 3 or more stars in order to make full use of starlight information and effectively improve the accuracy of the combined system. Considering the precision of the combination mode, the use of the starlight/inertia combination mode based on comprehensive estimation is undoubtedly optimal. However, starlight navigation devices do not always observe three or more stars. To solve this problem, this paper designs a master-slave filter combination mode based on switching mechanism. In other words, under normal circumstances, the combined system works in the combined mode of starlight comprehensive correction. When the starlight equipment observes less than 3 stars, it switches to the estimation mode based on the starlight compensation gyroscopic drift combination mode and corrects gyro drift. This ensures navigation accuracy to a certain extent, while improving the adaptability of the combined system to the environment \([7]\).

When the above two combination modes are implemented in the same combination system, it involves the selection of state variables and measurement variables of the combination system. It is necessary to consider these two combination modes as two independent combination systems to facilitate the switch between the two combinations. Two combined modes can be regarded as two independent parallel filters running at the same time in the combined system, so as to avoid overshoot during mode switching. The double channel navigation solution method is adopted. One is starlight integrated mode channel, and the channel filter is regarded as the main filter; the other one is starlight correction gyro drift mode, and the solution channel filter is regarded as a slave filter. Refer Figure 3 for the principle of double channel combination mode:
The mode switching discrimination mechanism is introduced to use number of stargazers as the criterion to combine two filtering modes to improve the performance of the integrated navigation system. The master-slave mode can be understood as a master-slave mode switch based on the comprehensive estimation mode and supplemented by the astronomical estimation mode. Therefore, in order to ensure that the master mode filter is working properly, when the main mode does not meet the criteria for stars observed and enters the slave mode, but the filter in the master mode continues to predict, so that a comprehensive estimate of the error can be completed in the shortest possible time when the search for star conditions is met[8].

To compare the effectiveness of different combination modes in CNS/INS, the simulation conditions are set as follows: at initial position, \( \varphi_0 = 39^\circ, \lambda_0 = 117^\circ \), gyro constant drift \( \varepsilon_x = \varepsilon_y = \varepsilon_z = 0.003^\circ/h \), accelerometer: \( \Delta a_x = \Delta a_y = \Delta a_z = 3 \times 10^{-5} g \), starlight equipment accuracy 5\(^\circ\), and the INS is in single axis rotation modulation mode. Set star observation state to: observe 12 stars in the first 24 hours and 2 stars in the second 24 hours. Refer Table 1 for the simulation results:

| Max positioning error | Gyro drift mode based on starlight correction | Combined model based on INS error comprehensive estimation | Master-slave filtering operation mode |
|------------------------|---------------------------------------------|------------------------------------------------------------|--------------------------------------|
|                        | 0.42nm/first in 24h                         | 0.13nm/first in 24h                                        | 0.13nm/first in 24h                  |
|                        | 0.81nm/second in 24h                        | 2.23nm/second in 24h                                       | 0.47nm/second in 24h                 |

Based on the two sections before and after the simulation analysis, it is clear that by making full use of star sensitive observation information, the master-slave mode switching method can improve the combination efficiency to a certain extent. In particular, by compensating the main position error caused by vertical gyro drift, this method can greatly improve the accuracy of integrated navigation for single axis rotating shafting CNS/INS. At the same time, parallel filter mode can be considered to avoid the overshoot caused by mode switching.

### 3. Conclusion

This paper mainly considered two integrated operating modes suitable for ship based on the application characteristics of ship platform in INS. After analyzing the advantages and disadvantages of each operating mode and according to the starlight viewing conditions of starlight equipment, this paper also proposes a master-slave filtering method to combine the starlight calibration gyro drift combination mode and starlight total calibration combination mode. The two modes complement each
other in the same combination. Without adding sensors, the main-source filtering technology was studied according to the characteristics of measuring signals in the combined system. Through simulation analysis, this method can improve the accuracy of shipborne star / inertial integrated navigation in the case of poor star observation conditions.

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