Controllable double quantum state transfers by one topological channel in a
frequency-modulated optomechanical array

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We propose a scheme to achieve the quantum state transfer via the topological protected edge channel based on a one dimensional frequency-modulated optomechanical array. We find that the optomechanical array can be mapped into a Su-Schrieffer-Heeger model after eliminating the counter rotating wave terms via frequency modulations. By dint of the edge channel of the Su-Schrieffer-Heeger model, we show that the quantum state transfer between the photonic left edge state and the photonic right edge state can be achieved with a high fidelity. Specially, our scheme can also achieve another phononic quantum state transfer based on the same channel via controlling the next-nearest-neighboring interactions between the cavity fields, which is different from the previous investigations only achieving one kind of quantum state transfer. Our scheme provides a novel path to switch two different kinds of quantum state transfers in a controllable way.

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I. INTRODUCTION

Topological insulator [1–4] is characterized by the conducting edge states and the insulating bulk states. These conducting edge states are immune to the local disorders and perturbations since they are protected by the non-local topological invariant [5–8]. The topological insulator has many potential applications in quantum information processing and quantum computing. For example, we can achieve the robust quantum state transfer via the edge channel of the topological insulator [9–11], in which the process of state transfer is robust to the local disorders and perturbations since the edge channel is protected by the energy gap. Also, we can implement the topological quantum computing [12, 13] via the non-Abelian anyons [14] and Majorana zero modes [15], etc. The Su-Schrieffer-Heeger (SSH) model, as the simplest one dimensional (1D) tight-binding topological insulator model, has attracted a great attention due to the appearance of the abundant physics, such as the topological phase transition [16–18], the edge state and topological invariant [19–21], PT-symmetry effect [22–25], the photonic mapping of SSH model [26–28], non-hermitian bulk-boundary correspondence [29–31], etc. Specially, the quantum state transfer between the left edge and the right edge states by the topological zero mode of a photonic topological SSH model has also been investigated [32].

In the previous works mentioned above, only one kind of quantum state transfer can be achieved via the topological channel. In this paper, we propose a scheme to achieve two different kinds of quantum state transfers only by one topological channel based on a frequency-modulated optomechanical array. Note that the optomechanical system with frequency modulation has been intensively investigated in experiment [33]. We show that the two different kinds of quantum state transfers can be achieved via controlling the next-nearest-neighboring (NNN) hopping. Specially, the two different kinds of quantum state transfers can be switched to each other in a controllable way, which is expected to greatly improve the efficiency of quantum information processing.

II. SYSTEM AND HAMILTONIAN

We consider a 1D optomechanical array containing N+1 cavity fields and N resonators, in which each cavity field is driven by a laser with frequency $\omega_d$ and strength $\Omega_n$, as shown in Fig. 1. In this array, the frequency of each cavity field and resonator can be modulated with
The fidelity versus the varying rate $\Omega$ and the partial NNN energy mode in the gap represented by the red line. (b) The system. (a) Energy spectrum of the system. There is a zero mode in the gap represented by the red line. (b) The distribution of the zero energy mode. The size of the system is $L = 2N + 1 = 21$. 

FIG. 3: The fidelity and the state transfer of the system. (a) The fidelity versus the varying rate $\Omega$ and the partial NNN hopping strength $T$. (b) The state transfer between $|L\rangle$ and $|R\rangle$ when $\Omega = 0.001$ and $T = 0.2$. 

the modulated frequency $\omega_{a,n}^\prime = \omega_{a,n} + \lambda_n \nu \cos(\nu t + \phi)$ and $\omega_{b,n}^\prime = \omega_{b,n} + \gamma_n \nu \cos(\nu t + \phi)$. The single-phonon optomechanical coupling strength between resonator $b_n$ and cavity field $a_n$ ($a_{n+1}$) is $g_n$. Moreover, the two adjacent cavity fields possess direct coupling with the coupling strength $T$. Then, the system can be described by the following Hamiltonian:

$$ H = \sum_{n=1}^{N+1} \omega_{a,n} a_n^{\dagger} a_n + \sum_{n=1}^{N+1} \omega_{b,n} b_n^{\dagger} b_n + \sum_{n=1}^{N+1} \left( \Omega_n a_n^{\dagger} e^{-i\omega_n t} + \Omega_n^* a_n e^{i\omega_n t} \right) - \sum_{n=1}^{N} g_n (a_n^{\dagger} a_{n+1}^{\dagger} a_{n+1} b_n^{\dagger} + b_{n}^{\dagger} b_{n}) + \sum_{n=1}^{N} T (a_{n+1}^{\dagger} a_n + a_n^{\dagger} a_{n+1}) \right), \quad (1) $$

where $a_n^{\dagger}$ and $a_n$ ($b_n^{\dagger}$ and $b_n$) are the creation and annihilation operators of the optical cavity field (mechanical resonator). The first two terms are the modulated free energy of the cavity fields and resonators, where $\lambda_n$ ($\gamma_n$), $\nu$, and $\phi$ are the modulated strength, frequency, and phase respectively. The third term describes the external driving of the cavity fields. The fourth term is the coupling between the cavity field and mechanical resonator via the radiation pressure. And the last term describes the direct tunneling between two adjacent cavity fields.

Under the condition of strong laser driving, we use the driving frequency $\omega_d$ to perform the rotating transformation and implement the standard linearization process via rewriting the operators as $a_n = \langle a_n \rangle + \delta a_n = \alpha_n + \delta a_n$ and $b_n = \langle b_n \rangle + \delta b_n = \beta_n + \delta b_n$. After dropping the notation $\delta^2$ for all the fluctuation operators $\delta a_n$ ($\delta b_n$), the Hamiltonian can be rewritten as

$$ H_L = \sum_{n=1}^{N+1} \left[ \Delta_{a,n}^\prime + \lambda_n \nu \cos(\nu t + \phi) \right] a_n^{\dagger} a_n + \sum_{n=1}^{N} \left[ \omega_{a,n} + \gamma_n \nu \cos(\nu t + \phi) \right] b_n^{\dagger} b_n - g_n (\alpha_n^* a_n + \alpha_n a_n^* + \alpha_{n+1}^* a_n a_{n+1} + \alpha_{n+1} a_n a_{n+1}^*) \right] + T (a_{n+1}^{\dagger} a_n + a_n^{\dagger} a_{n+1}) \right), \quad (2) $$

where $\Delta_{a,n}^\prime$ is effective detuning originating from optomechanical coupling with $\Delta_{a,1}^\prime = \Delta_{a,1} - g_1 (\beta_1^2 + \beta_1)$, $\Delta_{a,N+1}^\prime = \Delta_{a,N+1} + g_N (\beta_N^2 + \beta_N)$, $\Delta_{a,n+1}^\prime = \Delta_{a,n} - g_{n+1} (\beta_{n+1}^2 + \beta_{n+1})$, and $\Delta_{a,n} = \omega_{a,n} - \omega_d$ is the detunings between cavity fields and driving fields. Then we perform another rotating transformation defined by $V = \exp \left\{ \sum_{n=1}^{N+1} -i\Delta_{a,n} t \alpha_n^{\dagger} a_n - i\lambda_n \sin(\nu t + \phi) a_n^{\dagger} a_n + \sum_{n=1}^{N} -i\omega_{b,n} t b_n^{\dagger} b_n + i\gamma_n \sin(\nu t + \phi) b_n^{\dagger} b_n \right\}$. After that, the Hamiltonian becomes

$$ H_L^{(1)} = -i \left[ G_n a_n^{\dagger} b_n e^{i(\Delta_{a,n}^\prime - \omega_{b,n} t) + (\lambda_n - \gamma_n) \sin(\nu t + \phi) \} - G_n a_n^{\dagger} b_n e^{i(\Delta_{a,n}^\prime + \omega_{b,n} t) + (\lambda_n + \gamma_n) \sin(\nu t + \phi) \} + G_n a_{n+1}^{\dagger} b_n e^{i(\Delta_{a,n+1}^\prime - \omega_{b,n} t) + (\lambda_n - \gamma_n) \sin(\nu t + \phi) \} + G_n a_{n+1}^{\dagger} b_n e^{i(\Delta_{a,n+1}^\prime + \omega_{b,n} t) + (\lambda_n + \gamma_n) \sin(\nu t + \phi) \} + T a_{n+1}^{\dagger} a_n e^{i(\lambda_n - \gamma_n) \sin(\nu t + \phi) \} \right) + H.c. \right), \quad (3) $$

where $G_n = g_n a_n$ ($G_{n+1} = g_n a_{n+1}$) is the effective optomechanical coupling. We choose the parameters of the system to satisfy $\Delta_{a,n} = \Delta_{a,n+1} = \omega_{b,n}$, $\lambda_n = \lambda_n = \gamma_n$, and $\phi = 0$. Then, the Hamiltonian in Eq. (3) becomes

$$ H_L^{(2)} = -i \left[ -G_n a_n^{\dagger} b_n - G_n a_n^{\dagger} b_n e^{i[2\omega_{b,n} t + 2\lambda_n \sin(\nu t)]} + G_n a_{n+1}^{\dagger} b_n + G_n a_{n+1}^{\dagger} b_n e^{i[2\omega_{b,n} t + 2\lambda_n \sin(\nu t)]} + T a_{n+1}^{\dagger} a_n \right) + H.c. \right). \quad (4) $$

After exploiting the Jacobi–Anger expansions $e^{i\kappa \sin(\nu t)} = \sum_{m=-\infty}^{\infty} J_m(\kappa) e^{im\nu t}$, the above Hamiltonian can be rewritten as

$$ H_L^{(3)} = -i \left[ -G_n a_n^{\dagger} b_n + G_n a_n^{\dagger} b_n e^{i[2\omega_{b,n} t + 2\lambda_n \sin(\nu t)]} \right) + T a_{n+1}^{\dagger} a_n \right) + H.c. \right). \quad (4) $$
\[ H = \sum_n \left[ -G_n a_n^\dagger b_n + G_{n+1} a_{n+1}^\dagger b_n - G_n J_0(\kappa) a_n^\dagger b_n^\dagger e^{-2i\omega_b n t} + G_{n+1} J_0(\kappa) a_{n+1}^\dagger b_n^\dagger e^{2i\omega_b n t} + T a_{n+1}^\dagger a_n \right] + \text{H.c.} \]

\[ H_L^{(4)} = \sum_n \left[ -G_n a_n^\dagger b_n + G_{n+1} a_{n+1}^\dagger b_n - G_n J_0(\kappa) a_n^\dagger b_n^\dagger e^{-2i\omega_b n t} + G_{n+1} J_0(\kappa) a_{n+1}^\dagger b_n^\dagger e^{2i\omega_b n t} + T a_{n+1}^\dagger a_n \right] + \text{H.c.} \]

Applying the adiabatic evolution condition, which means that the evolution time \( T \) should be as slow as possible. And the condition \( T < 0 \) of \( \Omega \) ensures the state transfer between the final state and the state of \( |R\rangle \) versus the varying rate \( \Omega \) and the partial NNN hopping strength \( T \). The evolution of the initial state \( |L\rangle \) over time when \( T = 6 \) and \( \Omega = 0.00001 \).

\[ H_{\text{SSH}}(t) = \sum_n [1 - \cos(\Omega t)] a_n^\dagger b_n^\dagger + [1 + \cos(\Omega t)] a_{n+1}^\dagger b_n^\dagger \]

\[ + \text{H.c.} \]

If the initial state is prepared in the perfect photonic left edge state \( |\Psi\rangle_{\text{initial}} = |L\rangle = |1\rangle a_1 \otimes |0\rangle b_1 \otimes \cdots \otimes |0\rangle b_N \otimes |0\rangle a_{N+1} = |1, 0, 0, \ldots, 0, 0\rangle \), the right edge state can be achieved via the evolution of the present time-dependent Hamiltonian with \( H_{\text{SSH}}(t) \). A question arises: can this kind of state transfer still be achieved when the NNN hopping \( T \) is added into the system? We simulate the fidelity between the final state and the perfect right edge state \( |R\rangle = |0, 0, 0, \ldots, 0, 1\rangle \) versus the varying rate \( \Omega \) and the partial NNN hopping strength \( T \) numerically, as shown in Fig. 3(a). The numerical results show that the state transfer between \( |L\rangle \) and \( |R\rangle \) can be achieved with a high fidelity when \( \Omega < 0.01 \) and \( T < 0.4 \). The condition of \( \Omega < 0.01 \) originates from the adiabatic evolution condition, which means that the varying of \( \theta \) should be as slow as possible. And the condition of \( T < 0.4 \) is due to the topological protection of the energy gap. It implies that the state transfer channel (zero energy mode) does not be destroyed as long as the NNN hopping strength \( T \) is not beyond the width of the gap. To further clarify it, we simulate the process of the state transfer numerically when the initial state is prepared in the state of \( |L\rangle \) when \( T = 0.2 \), as shown in figs. 3(b). Obviously, when \( T < 0.4 \), the state transfer between \( |L\rangle \) and \( |R\rangle \) can still be achieved while it is destroyed completely when the NNN hopping strength \( T \) is large enough.

To further explore the reasons of the above results, we
plot the energy spectrum and the corresponding distribution of the gap state when the partial NNN hopping strength $T$ is large enough, as shown in Fig. 4. We find that the large enough $T$ makes the original zero energy mode become a wavy energy level, as shown in Figs. 4(a) and 4(b). Although the wavy energy level is still located in the energy gap, its distribution becomes totally different, as shown in Fig. 4(c). Obviously, the gap state is localized near the second site when $\theta \in (0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$ while it is localized near the penultimate site in other regions of $\theta$. It means that the initial state transfer channel between $|L\rangle$ and $|R\rangle$ becomes the state transfer channel between the phononic states of $|L'\rangle = |0, 1, 0, 0, ..., 0, 0\rangle$ and $|R'\rangle = |0, 0, 0, 0, ..., 1, 0\rangle$. To further demonstrate the above conclusions, we prepare the initial state in $|L'\rangle$ and simulate the fidelity between the evolved final state and the state of $|R'\rangle$ numerically, as shown in Fig. 5(a). The numerical results show that the state transfer can be achieved with a high fidelity corresponding to a large enough $T$ and small enough $\Omega$. For example, when $T = 6$ and $\Omega = 0.00001$, the evolution of the initial phononic state $|L'\rangle$ over time is shown in Fig. 5(b). It means that we can achieve the state transfer between $|L'\rangle$ and $|R'\rangle$ via the gap state when $T$ is large enough. We stress that although $|L\rangle \rightarrow |R\rangle$ and $|L'\rangle \rightarrow |R'\rangle$ are different state transfer processes, both of them are achieved by the same topological channel (gap state). This means that we can realize the photonic and phononic state transfer processes only by one channel via designing the partial NNN hopping strength $T$ appropriately. Further, the two different kinds of state transfer can be switched to each other via modulating the value of $T$, such as opening the channel of $|L\rangle \rightarrow |R\rangle$ when $T < 0.4$ and opening the channel of $|L'\rangle \rightarrow |R'\rangle$ when $T$ is large enough. Actually, we can also achieve the state transfer of $|L\rangle \rightarrow |L'\rangle \rightarrow |R\rangle \rightarrow |R'\rangle$ via varying $T$ over time for a given $\theta$, such as $H_{SSH}(t) = \sum_n[(1-\cos\frac{\pi}{2})a_n^\dagger b_n + (1+\cos\frac{\pi}{2})a_{n+1}^\dagger b_n + \Omega t a_n^\dagger a_n] + \text{H.c.}$.

We also investigate the large enough partial NNN hopping added on the even sites (resonators), we find that the gap state still keeps the zero energy, as shown in Figs. 6(a) and 6(b). The reason is that the NNN hopping added on the even sites has no effect on the two end sites, which makes the gap state keep the zero energy. It means that the state transfer channel between $|L\rangle$ and $|R\rangle$ keeps open and the large $T$ has no influence on the state transfer of $|L\rangle \rightarrow |R\rangle$, as shown in Fig. 6(c). The fidelity versus the varying rate of $\theta$ and NNN hopping strength $T$ is shown in Fig. 6(d). It can be seen clearly that there still exists a range of $\Omega$ to make the fidelity to be maximal corresponding to a large enough $T$. It means that we cannot achieve the different state transfers only by one gap state via designing $T$ added on the even sites.

III. CONCLUSIONS

We have proposed a scheme to achieve the mapping of the SSH model with the partial NNN hopping based on a 1D frequency-modulated optomechanical array. We find that the quantum state transfer between the topological left edge state and the right edge state can be achieved via the topological channel located in the gap when the NNN hopping is vanishing. Meanwhile, the system can achieve another different quantum state transfer via the same topological channel when a large enough NNN hopping is added into the system. In this way, we can achieve two different kinds of quantum state transfer by one topological channel via controlling the NNN hopping strength. Our scheme provides a novel method to achieve different quantum state transfers in a controllable way.

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