The role of spontaneous and piezoelectric polarization fields on the spectral and power characteristics of In$_x$Ga$_{1-x}$N/GaN superluminescent light emitting diodes

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Abstract

The output characteristics of In$_x$Ga$_{1-x}$N/GaN MQW blue SLEDs are investigated using a detailed theoretical model, in which the effects of spontaneous and piezoelectric polarizations are fully considered. By solving the Schrodinger equation in effective mass approximation, Poisson’s equation, and two level rate equations with no-k selection wavelength dependent gain self-consistently, the influence of these internal fields on the electrical and optical characteristics of MQW SLEDs under bias conditions have been studied. The spontaneous polarization and the piezoelectric polarization changed with indium molarity in our structure at 2.9–3.2 V biased voltages. Below 3.05 V the blue shift of peaks due to band filling and screening of QCSE, and above 3.05 V red shifting due to device heating are seen obviously. With increasing of indium content and polarization fields accordingly in different modeled SLDs, peaks of spectral radiation power were dropped and diagrams were red shifted at the range of 3.0–4.0 nm under various applied voltages. FWHM of modeled devices raises 0.3–0.4 nm with 1% indium content increasing (0.1356 MV/cm polarization field) in three different SLDs with 19.5%, 20%, and 20.5% indium in QWs. Total light output power of SLDs at 3.2 V dropped 6.22% (7.08 mW) with 1% indium content increasing (according to 0.1356 MV/cm total polarization field).

Keywords Superluminescent light emitting diodes · Spontaneous and piezoelectric polarizations · Output characteristics · Rate equations
1 Introduction

Superluminescent light emitting diodes (SLDs or SLEDs) are optoelectronic devices with the characteristics of both a laser diode (LD) and light emitting diode (LED) (Schwarz et al. 2013). These diodes operate in the amplified spontaneous emission (ASE) regime as the spontaneous emission is amplified by stimulated emission (Feltin et al. 2009). They are used for various applications including optical coherence tomography (OCT) (Baars et al. 2013), wavelength-division-multiplexing (WDM) testing systems (Liou and Raybon 20125), fiber optic gyroscopes (FOGs) (Burns et al. 1983), and speckle-free illumination (Rossetti et al. 2012). Short-wavelength optoelectronic devices can be used for a series of different applications like spectroscopy, display lighting and projection (Rossetti et al. 2010). Also these SLDs could have been used in medical applications (Lu and Chen 2011).

The suitable design of the optoelectronic devices based on In$_x$Ga$_{1-x}$N/GaN heterojunctions requires an accurate investigation of the effects of polarization fields on the quantum well (QW) structures. Numerical results show that the electronic and spectral properties of GaN-based SLDs are very sensitive to the polarization fields of multiple quantum wells (MQWs) due to the lattice mismatch strain effects. Many researchers have published results about influence of spontaneous and piezoelectric polarization fields on optoelectronic properties of LEDs and LDs (Lu and Chen 2011; Wang et al. 2015; Ryu 2012; Peng et al. 1999; Ryou et al. 2009; Son and Lee 2010) but to our knowledge there are a few theoretical and numerical studies on the influence of these effects on the output characteristics of GaN-based SLDs, in the our later investigation (Absalan et al. 2020) we considered these diodes without biased voltage. In this work we present comprehensive model for the effects of internal polarization fields on the spectral and power characteristics of In$_x$Ga$_{1-x}$N/GaN MQW blue SLDs, based on numerical simulation under biased condition. We suppose a n-p SLD with In$_x$Ga$_{1-x}$N/GaN MQW active region. Schrodinger–Poisson and rate equations corresponding to MQW active region, separate confinement heterostructure (SCH) layer and forward–backward propagating spectral densities of optical power are solved self-consistently with no-k selection wavelength dependent gain and quasi-Fermi level functions at bias condition.

The energy band diagrams and carrier concentrations for electrons and holes were calculated under internal polarization effects in different bias conditions. Output power, spectral radiation power calculated with varying the strength of polarization fields to study the role of the internal polarization fields on the output characteristics. Active region in our SLD structure is a MQW consist of four 3 nm In$_x$Ga$_{1-x}$N quantum wells (QWs), three 10 nm GaN barriers that have been confined with two 100 nm SCH GaN layers. We have supposed index-guided waveguide was defined by 2 μm wide ridge and 800 μm cavity length. Reflectivity of the end facets are negligible and other sections of a real SLD have been ignored in our study. A challenge in our simulation is the proper choice of polarization strength given by the spontaneous polarization (indium concentration dependent) and the piezoelectric polarization (indium concentration and strain dependent). Advantages of our work are considering bias voltage and the corresponding effects on transition energy in Ga-polar In$_x$Ga$_{1-x}$N/GaN n-p diodes. The parameters of In$_x$Ga$_{1-x}$N have been obtained using Vegard’s law and proper bowing coefficients. Tables 1 and 2 lists some parameters of used in this work, and other detailed material parameters of semiconductors adopted in our simulation can be found in Ref. Moslehi Milani et al. (2015). Symbol $i$ has been used in all of the paper for compression of formula, $i \equiv w$ for wells and $i \equiv b$ for barriers respectively.
| Parameter | Description | GaN | InN | Unit | Refs. |
|-----------|-------------|-----|-----|------|-------|
| $n_r$     | Refractive index | 2.48 | 2.9 |       | Anani et al. (2007) |
| $b_r$     | Bowing parameter of refractive index |       | 2.17 |       | Anani et al. (2007) |
| $a$       | Lattice constant | 3.18 | 3.54 | Å    | Yan et al. (2014) |
| $\varepsilon$ | Permittivity | 10.4 | 15.3 | $\varepsilon_s$ | Huang et al. (2016) |
| $m'_e$    | Transversal electron effective mass relative to c-axis | 0.19 | 0.065 | $m'_e$ | Piprek (2018) |
| $m_e$     | Effective mass of electron | 0.20 | 0.07 | $m_e$ | Christmas et al. (2005) |
| $m_h$     | Effective mass of heavy hole | 1.02 | 1.25 | $m_h$ | Christmas et al. (2005) |
| $E_g$     | Bandgap energy | 3.5 | 0.75 | eV   | Anani et al. (2007) |
| $b_{E_g}$ | Bowing parameter of bandgap energy |       | 2.17 |       | Anani et al. (2007) |
| $a_{cz}$  | Conduction band deformation potential along c axis | −4.9 | −3.5 | eV   | Christmas et al. (2005) |
| $a_{ct}$  | Conduction band deformation potential in c plane | −11.3 | −3.5 | eV   | Christmas et al. (2005) |
| $D_1$     | Valence band deformation potential | −3.7 | −3.7 | eV   | Christmas et al. (2005) |
| $D_2$     | Valence band deformation potential | 4.5 | 4.5 | eV   | Christmas et al. (2005) |
| $D_3$     | Valence band deformation potential | 8.2 | 8.2 | eV   | Yan et al. (2014), Christmas et al. (2005) |
| $D_4$     | Valence band deformation potential | −4.1 | −4.1 | eV   | Yan et al. (2014), Christmas et al. (2005) |
| $c_{13}$  | Elastic constant | 103 | 92 | GPa  | Ryou et al. (2009) |
| Parameter | Description | GaN | InN | Unit  | Refs.                              |
|-----------|-------------|-----|-----|-------|-----------------------------------|
| $c_{33}$  | Elastic constant | 405 |     | 224   | GPa                               |
| $e_{31}$  | Piezoelectric constant | $-0.49$ | $-0.57$ | Cm$^{-2}$ | Ryou et al. (2009), Huang et al. (2016) |
| $e_{33}$  | Piezoelectric constant | 0.73 |     | 0.97  | Cm$^{-2}$ | Ryou et al. (2009), Huang et al. (2016) |
| $\Delta_1$ | Crystal-field split energy | 0.010 |     | 0.024 | eV | Zhao et al. (2009) |
| $\Delta_2$ | One-third of spin–orbit split-off energy | 0.00567 |     | 0.00167 | eV | Zhao et al. (2009) |
| $\Delta_3$ | One-third of spin–orbit split-off energy | 0.00567 |     | 0.00167 | eV | Zhao et al. (2009) |
| $P_{sp}$  | Spontaneous polarization | $-0.034$ |     | $-0.042$ | Cm$^{-2}$ | Christmas et al. (2005) |
| $b_{sp}$  | Bowing parameter of spontaneous polarization |     | $-0.037$ |     | Cm$^{-2}$ | Christmas et al. (2005) |
| $P_{pc}$  | Piezoelectric polarization (linear) | 0 |     | $\approx 0.15$ | Cm$^{-2}$ | Bernardini and Fiorentini (2001), Wenzel (2006) |

*In the text, for all of these parameters, index $w$ and $b$ denotes well and barrier, respectively. We omitted these indices in this table for compression (In our work In$_{x}$Ga$_{1-x}$N and GaN are well and barrier, respectively)*
### Table 2: Parameters and physical constants used in this simulation

| Symbol | Description                                      | Value      | Unit   | Refs.                        |
|--------|--------------------------------------------------|------------|--------|------------------------------|
| $L$    | Cavity length                                    | 800        | μm     |                              |
| $L_w$  | QWs thickness                                    | 3          | nm     |                              |
| $L_b$  | Barriers thickness                               | 10         | nm     |                              |
| $L_a$  | Active region thickness                          | 42         | nm     |                              |
| $L_s$  | SCH layer thickness                              | 100        | nm     |                              |
| $n_w$  | Number of wells                                  | 4          |        |                              |
| $n_b$  | Number of barriers                               | 3          |        |                              |
| $w$    | Active layer width (ridge width)                  | 2          | μm     |                              |
| $s$    | Cross section of the active medium of SLD        | $2.4 \times 10^4$ | nm$^2$ |                              |
| $a$    | Internal loss coefficient                         | 37         | cm$^{-1}$ | Muziol et al. (2013) |
| $\Gamma$ | MQW Confinement factor                           | 0.017      |        | Witzigmann et al. (2006)   |
| $c$    | Light velocity in the vacuum                      | $3 \times 10^8$ | ms$^{-1}$ |                          |
| $\varepsilon_0$ | Vacuum permittivity                          | $8.85 \times 10^{-12}$ | Fm$^{-1}$ |                          |
| $h$    | Planck’s constant                                 | $6.62 \times 10^{-34}$ | J.s     |                          |
| $\hbar$ | Planck’s constant divided by $2\pi$             | $1.054 \times 10^{-34}$ | J.s     |                          |
| $m$    | Free electron mass                                | $9.1 \times 10^{-31}$ | kg      |                          |
| $\tau_s$ | Transport time in SCH layer                      | 1.5736     | Ps     | Moslehi Milani et al. (2015) |
| $\tau_e$ | Escape time in MQW region                         | 429.47     | Ps     | Moslehi Milani et al. (2015) |
| $\beta$ | Spontaneous coupling coefficient                 | 0.05       |        | Moslehi Milani et al. (2015) |
| $k$    | Boltzmann constant                                | $1.38 \times 10^{-23}$ | JK$^{-1}$ |                          |
| $e$    | Electron charge                                  | $1.6 \times 10^{-19}$ | C       | Moslehi Milani et al. (2015) |
| $a_e$  | Effective Bohr radius                             | 0.5228     | nm     | Piprek (2018)                |
| $V_b$  | Built-in potential of diode                       | 3.25       | V      |                              |
| $\mu_n$ | Electron mobility                               | 600        | cm$^2$.V$^{-1}$.s$^{-1}$ | Farrell et al. (2011), Alahyarizadeh et al. (2012) |
| $A$    | Shockley–Read–Hall coefficient in QW            | $5.4 \times 10^7$ | s$^{-1}$ | Farrell et al. (2011), Alahyarizadeh et al. (2012) |
| $B$    | Bimolecular coefficient in QW                    | $2 \times 10^{-11}$ | Cm$^3$.s$^{-1}$ | Farrell et al. (2011), Alahyarizadeh et al. (2012) |
| $C$    | Auger recombination coefficient in QW             | $2 \times 10^{-30}$ | Cm$^6$.s$^{-1}$ | Farrell et al. (2011), Alahyarizadeh et al. (2012) |
| Symbol | Description                                | Value    | Unit      | Refs.                              |
|--------|--------------------------------------------|----------|-----------|------------------------------------|
| $A_s$  | Shockley–Read–Hall coefficient in SCH      | $1.3 \times 10^8$ | s$^{-1}$  | Alahyarizadeh et al. (2012)        |
| $B_s$  | Bimolecular coefficient in SCH              | $3 \times 10^{-11}$ | Cm$^3.s^{-1}$ | Alahyarizadeh et al. (2012)        |
| $C_s$  | Auger recombination coefficient in SCH     | $1.6 \times 10^{-30}$ | Cm$^6.s^{-1}$ | Alahyarizadeh et al. (2012)        |
2 Schrodinger–Poisson equations under internal fields

In our work we have solved the Schrodinger equation for electrons and heavy holes in effective mass approximation with Poisson’s equation by finite difference method iteratively (Li et al. 2004; Chuang and Chang 1996; Park and Chuang 1998). We have neglected light holes and split-off holes because gain spectrum for TE polarization (preferred polarization in SLDs (Feltin et al. 2009)) dominated by the heavy hole band (Jeon et al. 1997). For the GaN material system, the coupling between the conduction and valence bands can be ignored due to the large bandgaps. Schrodinger equation with polarization effects due to strain defined as below for conduction and valence bands of wells and barriers in z-direction ([0001] Ga-polar growth direction):

\[
\left[-\frac{\hbar^2}{2m_{el}} \frac{\partial^2}{\partial z^2} + E_{el}^s(z) - e\varphi(z)\right] \psi_{en}(z) = E_{en} \psi_{en}(z)
\]

(1)

\[
\left[-\frac{\hbar^2}{2m_{hi}} \frac{\partial^2}{\partial z^2} + E_{hi}^s(z) + e\varphi(z)\right] \psi_{hm}(z) = E_{hm} \psi_{hm}(z)
\]

(2)

where \(\varphi(z)\), \(e\), \(m_{el}(m_{hi})\), \(E_{en}(E_{hm})\), \(\psi_{en}(z)(\psi_{hm}(z))\), and \(E_{el}^s(E_{hi}^s)\) are electrostatic potential, electron charge, effective mass of electrons (heavy holes) in wells and barriers, Eigen energies of electrons (heavy holes), envelope functions of electrons (heavy holes) in wells and barriers, and strained conduction (valence) band edge in wells and barriers, respectively. \(n\) and \(m\) are sub-band indices for conduction and valence bands, respectively. Symbol \(i\) is an abbreviation, \(i \equiv w\) for wells and \(i \equiv b\) for barriers. For example in conduction band we have \(E_{el}^s\big|_{w} = E_{cw}^s\) for wells and \(E_{el}^s\big|_{b} = E_{cb}^s\) for barriers (Chuang and Chang 1996):

\[
E_{cw}^s = E_{ow} + \Delta_{1w} + \Delta_{2w} + E_{gw} + P_{cew}
\]

(3)

\[
E_{cb}^s = E_{gb}^s + 0.7\left( E_{gb}^s - E_{gw}^s \right)
\]

(4)

For valence band \(E_{vi}^s\) is:

\[
E_{ov}^s = E_{ow} + \Delta_{1w} + \Delta_{2w} + \theta_{ew} + \lambda_{ew}
\]

(5)

\[
E_{ob}^s = -0.3\left( E_{gb}^s - E_{gw}^s \right)
\]

(6)

The band-offset ratio, which is defined as the ratio between the conduction-band offset and the valence-band offset of an In,Ga1-xN/GaN QW is assumed to be \(0.7/0.3\) based on the published literature (Chen et al. 2009). \(E_{ow}, \Delta_{1w}\) and \(\Delta_{2w}\) are unstrained valence band edge energy, crystal-field split energy, and one-third of spin–orbit split-off energy in the wells, respectively. Band gap energy of strained wells and barriers is defined by Chuang and Chang (1996):

\[
E_{gs}^i = E_{gi} + P_{cei} - (\theta_{ei} + \lambda_{ei})
\]

(7)

\(E_{gi}, P_{cei}\) and \(\theta_{ei} + \lambda_{ei}\) are unstrained band gap, energy shift of conduction band and energy shift of valence band for wells and barriers as (Chuang and Chang 1996; Yan et al. 2014):
$P_{c\epsilon i} - (\theta_{\epsilon i} + \lambda_{\epsilon i}) = (a_{czi} - D_{3i} - D_1i)\varepsilon_{zz} + (a_{cti} - D_{4i} - D_2i)(\varepsilon_{xx} + \varepsilon_{yy})$

where $a_{czi}$ and $a_{cti}$ are deformation potentials of conduction band in wells and barriers (index $z$ for along $c$ axis and index $t$ for in $c$ plane). $D_{1i}, D_{2i}, D_{3i}$ and $D_{4i}$ are deformation potentials of valence band in wells and barriers. $\varepsilon_{xx}$ and $\varepsilon_{yy}$ are biaxial strains in the $c$ plane and $\varepsilon_{zz}$ is uniaxial strain along $c$ axis (Yan et al. 2014),

$$\varepsilon_{xx} = \varepsilon_{yy} = \frac{a_{GaN} - a_{InGaN}}{a_{InGaN}}$$

$$\varepsilon_{zz} = -\frac{2c_{13}}{c_{33}}\varepsilon_{xx}$$

Net electric fields, $E_i$, in the wells ($i \equiv w$) and barriers ($i \equiv b$) are defined as (Harrison 2005):

$$E_w = \frac{V_b - V_a}{L_a} - E^p_w$$

$$E_b = \frac{V_b - V_a}{L_a} - E^p_b - \frac{E^p_w}{L_w} L_b$$

where $V_b$ and $V_a$ are built-in voltage of the diode and applied bias voltage, respectively. We supposed that MQW region is intrinsic and built-in electric field is constant along $z$-direction in MQW region, $E^p_w$ and $E^p_b$ are total polarization fields in the wells and barriers (Li et al. 2004; Park and Moon 2013):

$$E^p_w = (P_{spb} + P_{pcb} - P_{spw} - P_{pcw})$$

$$E^p_b = -\frac{L_w}{L_b} E^p_w$$

$P_{spw}, P_{pcw}, P_{spb}, P_{pcb}, L_w(L_b), \varepsilon_w(\varepsilon_b)$, and $L_a$ are spontaneous polarization in wells (barriers), piezoelectric polarization in wells (barriers), thickness of wells (barriers), permittivity of wells (barriers), and active region thickness, respectively. Active region thickness is equal to $L_a = n_w L_w + n_b L_b$ where $n_w(n_b)$ is number of wells (barriers). Spontaneous polarizations and permittivity of wells and barriers have been introduced in Table 1. We have used the following relation for calculating piezoelectric polarization of wells (Son and Lee 2010; Li et al. 2004):

$$P_{pcw} = 2\varepsilon_{xx}(e_{31w} - e_{33} c_{13w})$$

where $(e_{31w}, e_{33w})$ and $(c_{13w}, c_{33w})$ are piezoelectric and elastic constants of wells, respectively. Piezoelectric polarization of GaN barriers are negligible (Table 1). Poisson’s equation is defined as:
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where \( \varepsilon_i \) is permittivity of wells (\( i \equiv w \)) and barriers (\( i \equiv b \)) in MQW region.

\( N(z) \) and \( P(z) \) are electrons and heavy holes distribution for first four levels in the wells and barriers as (Li et al. 2004; Park and Moon 2013; Chuang 2009):

\[
N(z) = \frac{kTm_{ei}}{\pi h^2} \sum_{n=1}^{4} \left| \psi_{en}(z) \right|^2 \ln \left[ 1 + \exp \left( \frac{F_c - E_{en}}{kT} \right) \right]
\]

\[
P(z) = \frac{kTm_{hi}}{\pi h^2} \sum_{m=1}^{4} \left| \psi_{hm}(z) \right|^2 \ln \left[ 1 + \exp \left( \frac{E_{hm} - F_v}{kT} \right) \right]
\]

\( F_c \) and \( F_v \) are quasi-Fermi levels in conduction and valence bands as a function of carrier densities for first four levels (Moslehi Milani et al. 2015; Moslehi Milani and Asgari 2015):

\[
F_c = kT \frac{N + N_s}{4n_w n_c + N_c} - \frac{n_w n_c \sum_{n=1}^{4} E_{en} + N_c E_{eb}^s - kTN_c}{4n_w n_c + N_c}
\]

\[
F_v = -kT \frac{N + N_s}{4n_w n_v + N_v} + \frac{n_w n_v \sum_{m=1}^{4} E_{hm} + N_v E_{eb}^s + kTN_v}{4n_w n_v + N_v}
\]

Symbols \( N_c \) and \( N_v \) are band edge effective density of states in the SCH conduction and valence bands, respectively. \( n_c \) and \( n_v \) are effective density of states for conduction and valence sub-bands of QWs, respectively (Moslehi Milani et al. 2015; Chuang 2009). Band structure diagrams for Ga-polar In\(_{0.2}\)Ga\(_{0.8}\) N/GaN n-p diode under 3.0 V and 3.2 V biased condition were depicted in Fig. 1.
3 Rate equations

In continuing our numerical simulation according to flowchart Fig. 2, we solve rate equations, gain equation and quasi-Fermi functions for obtaining spectral radiation power and light output power-current diagrams. The rate equations for the carrier density in the MQW region ($N$) and the SCH layer ($N_s$) and forward–backward-propagating spectral densities of optical power ($P_{(\lambda,\pm)}$) along cavity ($x$ axis) in stationery state are written as (Moslehi Milani et al. 2015; Moslehi Milani and Asgari 2015; Pleumeekers et al. 1998):

\[
\frac{J}{eL_s} - \frac{N_s}{\tau_s} + \frac{N}{\tau_e} \frac{L_w}{L_s} - \frac{N_s}{\tau_{ns}} = 0 \tag{21}
\]

\[
\frac{N_s}{\tau_s} \frac{L_s}{L_w} - \frac{N}{\tau_e} - \frac{N}{\tau_n} - R_{st} = 0 \tag{22}
\]

\[
\pm \frac{\partial P_{(\lambda, \pm)}}{\partial x} = \left( \Gamma g(\lambda) - \alpha \right) P_{(\lambda, \pm)} + 0.5 \beta \Gamma r_{sp} \frac{hc\lambda^{-1}}{s} \tag{23}
\]

$J$ is current density, and $L_s$, $\tau_s$, $\tau_e$, $s$, $c$, $\Gamma$, $\alpha$, $\beta$, $h$, and $\lambda$ are SCH layer thickness, escape time of electrons, transport time of electrons in SCH layer, cross section of active layer, speed of light, MQW confinement factor, internal loss coefficient, spontaneous emission coupling factor, Planck’s constant, and light wavelength, respectively. $\tau_n$ and $\tau_{ns}$ are carrier recombination lifetime of MQW and SCH region, respectively which are given by $[A + BN + CN^2]^{-1}$ and $[A_s + B_s N_s + C_s N_s^2]^{-1}$, where $A$, $B$, $C$, $A_s$, $B_s$, $C_s$ are the Shockley–Read–Hall, bimolecular, and Auger recombination coefficients in MQW and SCH layers, respectively. $R_{st}$ is stimulated recombination of carriers due to ASE (Moslehi Milani et al. 2015). The spontaneous emission rate per unit volume per unit energy interval $(s^{-1}cm^{-3}eV^{-1})$ is defined as (Moslehi Milani et al. 2015; Zhuravleva et al. 2004; Saint-Cricq et al. 1986):

\[
r_{sp}(\lambda) = \frac{8\pi n_r^2}{h\lambda^2} n_{sp} g(\lambda) \tag{24}
\]

$n_r$ is refractive index of active region and $n_{sp}$, spontaneous emission factor, is defined as (Moslehi Milani et al. 2015; Saint-Cricq et al. 1986):

\[
n_{sp} = \left[ 1 - \exp \left( \frac{hc\lambda^{-1} - (F_c - F_0)}{kT} \right) \right]^{-1} \tag{25}
\]

We have used the wavelength dependent gain in the model of transitions without the k-selection rules (Zhuravleva et al. 2004; Saint-Cricq et al. 1986) with additional strain effects in the first four levels:

\[
g(\lambda) = g_0 \sum_{n=1}^{4} L_n \left[ \frac{1 + \exp \left( (F_c - h\lambda^{-1} - E_{ln})/kT \right)}{1 + \exp \left( (F_c - E_{cn})/kT \right)} \times \frac{1 + \exp \left( (F_u + h\lambda^{-1} - E_{en})/kT \right)}{1 + \exp \left( (F_u - E_{un})/kT \right)} \right] \tag{26}
\]

where
Fig. 2 Flowchart of the numerical method used in this work
In these equations the momentum matrix elements, $|M_{cv}|^2$ are consisted of only TE polarization component ($|M_{cv}^{TM}|^2 = 0$ for heavy holes) (Piprek 2007). Other new parameters $a_0$, $m_w$, $\Delta_{3w}$ and $I_{nm}$ are effective Bohr radius (Zhuravleva et al. 2004), electron effective mass in the direction perpendicular to the c-axis, one-third of spin–orbit split-off energy (Piprek 2007, 2018), and overlap integral of the conduction- and valence-band envelope functions (Chuang 2009), respectively. The parameters of the ternary alloy In$_x$Ga$_{1-x}$N (wells in our simulation) have been calculated by Vegard’s law (Christmas et al. 2005):

$$Y_{In_xGa_{1-x}N} = xY_{InN} + (1 - x)Y_{GaN} - b_0 \chi (1 - x)$$

where $Y$ is any material parameter and $b_0$ is a bowing parameter (see Table 1).

Our simulation procedure is described as follows and depicted in Fig. 2. We insert an initial electrostatic potential at turn on voltage into the Schrodinger equation and start Schrodinger–Poisson self-consistency. After the calculation of the envelope functions and energy eigenvalues associated with the conduction and valence bands of the MQW region, we obtain carrier (electrons and holes) distribution to insert into the Poisson’s equation to find the new value of the electrostatic potential. This procedure runs until convergency is achieved, then we extract final $N(z)$, $I(z)$, and $P(z)$ as a function of $z$ in the growth direction. In continuing we import these values in the second step of simulation for solving rate equations, gain equation and quasi-Fermi functions until spectral radiation power of SLD as a function of wavelength, light output power-voltage (L-V curve) were extracted in different applied biases.

### 4 Results and discussion

We investigated the effects of internal polarization fields on the output characteristics of modeled SLDs. We increased bias voltage from turn-on voltage 2.9 V–3.2 V (near the built-in potential of diode) with a step 0.05 V for three SLDs containing indium concentration 19.5, 20, and 20.5 percent. The optical power of our studied SLDs exhibit a relatively smooth Gaussian curves with wavelength range 431.7–437.2 nm for various polarization fields due to different indium concentrations (19.5%, 20%, and 20.5%) into the QWs. We see a blue shift of spectral peak from 2.9 to 3.05 V bias due to band filing and screening of quantum-confined stark effect (QCSE). Above 3.05 V, red shifting starts that attributed to device heating (we increased the temperature of active region from 300 to 324 K with a step size of 4 K). Figure 3 indicates these shifts for a SLD with 20% indium content obviously and consistent with Refs. Feltin et al. (2009), Wang et al. (2015), Sheremet et al. (2018), Jung et al. (2012), Kopp et al. (2013). Figure 4 indicates blue and red shifts of three SLDs with different total polarization fields due to various indium concentrations consistent with Refs. Sheremet et al. (2018), Jung et al. (2012), Islam et al. (2018).
Results in Fig. 5 reveal that in a fixed bias voltage increasing of piezoelectric polarization field in the wells according to various indium contents red shifts the spectral radiation power and decreases spectrum peak that consistent with reported research data (Rossetti et al. 2010; Wenzel 2006; Sheremet et al. 2018; Jung et al. 2012; Kopp et al. 2013; Islam et al. 2018). With increasing of indium content (accordingly polarization field) in different SLDs, diagrams red shifts in the range of 3.0–4.0 nm under various applied voltages are seen obviously. According to extracted data (Table 3) full width at half maximum (FWHM) of high polarization fields are larger than low polarization fields consistent with Refs. Rossetti et al. (2010), Wenzel (2006), Sheremet et al. (2018), Jung et al. (2012). FWHM raises~ 0.3–0.4 nm with 0.1356 MV/cm field increasing.
in three different SLDs with 19.5%, 20%, and 20.5% indium concentrations in QWs. (Color figure online)

Figure 6 indicates light output power as a function of bias voltage for three various SLDs with different indium contents (according to different total polarization fields). Because of heating effects, the output power is saturated at above 3.18 V. Onset of superluminescence is at near 3 V, also at 2.975 V we observe an undefined kink (we had not found any reason for it). Our results are consistent with reported graphs in Refs. Feltin et al. (2009), Rossetti et al. (2010), Loeser and Witzigmann (2008), Matuschek and Duelk (2013). Maximum output power at 3.2 V has dropped 6.22% (7.08 mW) for 0.1356 MV/cm increased total polarization field. This field variation is related to increasing 1% indium concentration in the QWs (Table 3).

5 Conclusion

We presented the modeling of MQW superluminescent diodes emitting in the blue range with solution of Schrodinger–Poisson’s and rate equations under polarization effects and biased condition. The diagrams of optical power spectral density versus wavelength and light output power as a function of bias voltage were obtained in good agreement with reported data. The polarization fields change transition energy and the results show that the output power of the device decreases and also, in the presence of these fields, the red-shift occurs in the spectra and the peak of intensity decreases. With increasing of polarization fields due to indium contents in different SLDs optical power diagrams red shifts in the range of 3.0–4.0 nm. FWHM of modeled devices raises 0.3–0.4 nm with 0.1356 MV/cm field increasing. Total light output power of SLDs dropped 6.22% in 0.1356 MV/cm field increasing.
Table 3  Numerical data extracted in our simulation*

| Applied voltage $V_a$ (V) | Total polarization field in wells $E^T_p$ (MV/cm) | Indium content $x$ % | Spectrum peak $P_{max}$ [$\times10^{17}$] (1/s) | Peak wavelength $\lambda_p$ (nm) | Full width at half maximum $FWHM$ (nm) | Light output power $P_{out}$ (mW) |
|---------------------------|-----------------------------------------------|---------------------|-----------------------------------------------|-------------------------------|-------------------------------------|-------------------------------|
| 2.9                       | $-2.4684$                                     | 19.5                | $1.406$                                       | $433.0$                      | 12.9                               | 1.53                          |
|                           | $-2.5367$                                     | 20                  | $1.146$                                       | $434.9$                      | 13.0                               | 1.41                          |
|                           | $-2.6040$                                     | 20.5                | $0.9363$                                      | $437.0$                      | 13.2                               | 1.30                          |
| 2.95                      | $-2.4684$                                     | 19.5                | $2.523$                                       | $432.7$                      | 13.2                               | 4.77                          |
|                           | $-2.5367$                                     | 20                  | $2.058$                                       | $434.6$                      | 13.3                               | 4.57                          |
|                           | $-2.6040$                                     | 20.5                | $1.683$                                       | $436.5$                      | 13.5                               | 4.38                          |
| 3                         | $-2.4684$                                     | 19.5                | $3.484$                                       | $432.3$                      | 13.4                               | 9.64                          |
|                           | $-2.5367$                                     | 20                  | $2.843$                                       | $434.0$                      | 13.6                               | 9.32                          |
|                           | $-2.6040$                                     | 20.5                | $2.324$                                       | $436.1$                      | 13.7                               | 9.00                          |
| 3.05                      | $-2.4684$                                     | 19.5                | $4.291$                                       | $432.0$                      | 13.6                               | 20.53                         |
|                           | $-2.5367$                                     | 20                  | $3.503$                                       | $433.7$                      | 13.8                               | 19.93                         |
|                           | $-2.6040$                                     | 20.5                | $2.863$                                       | $435.4$                      | 13.8                               | 19.34                         |
| 3.1                       | $-2.4684$                                     | 19.5                | $5.319$                                       | $432.4$                      | 13.9                               | 38.06                         |
|                           | $-2.5367$                                     | 20                  | $4.271$                                       | $434.0$                      | 14.1                               | 36.93                         |
|                           | $-2.6040$                                     | 20.5                | $3.433$                                       | $435.7$                      | 14.2                               | 35.83                         |
| 3.15                      | $-2.4684$                                     | 19.5                | $6.398$                                       | $432.8$                      | 14.2                               | 69.73                         |
|                           | $-2.5367$                                     | 20                  | $5.123$                                       | $434.3$                      | 14.3                               | 67.64                         |
|                           | $-2.6040$                                     | 20.5                | $4.105$                                       | $436.0$                      | 14.6                               | 65.58                         |
| 3.2                       | $-2.4684$                                     | 19.5                | $7.453$                                       | $433.4$                      | 14.6                               | 113.80                        |
|                           | $-2.5367$                                     | 20                  | $5.936$                                       | $434.9$                      | 14.7                               | 110.24                        |
|                           | $-2.6040$                                     | 20.5                | $4.731$                                       | $436.4$                      | 14.9                               | 106.72                        |

*Because of extra numerical results in our simulation we collected only data of three various modeled SLDs with different indium contents: 19.5%, 20%, and 20.5%
Fig. 6 Light output power as a function of bias voltage for three different SLDs

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