Ion Trajectory Analysis in FIB Microscope to Study the Dielectric Constant using Mirror Method

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Abstract. The current work used the same mathematical model that was used to study the behavior of an accelerated probing electron in order to create electron-mirror images [1]. Using straightforward trigonometry, this mathematical model was used to investigate the properties of the polymer PMMA as a result of the ion mirror effect phenomenon. This work also considers determining the influence of dielectric constant, which is one of the most important electrical properties of the material by using MATLAB simulation to help the practical results that we obtained and comparing it to the findings obtained for the electron mirror. The obtained results indicate that the presented methodology can be used to explain, interpret, and add further detail to the understanding of the ion mirror effect for future studies.

1. Introduction
The focused ion beam plays an important role in the process of charging dielectric samples for imaging [2, 3]. The FIB has been used in polymer science applications by a variety of authors recently. The area of biology and medical research has benefited from FIB applications. The FIB system can image and detect a variety of samples [4-8]. Therefore, when an insulating material is bombarded with primary ions that are released from the column and collide with the surface of the insulator, secondary ions will be produced. After that, these ions lead to other interactions with the insulator being back-scattered or leading to the generation of new secondary ions, and thus the insulator acquires a net surface charge[9, 10].The mirror effect occurs in FIB microscope if the material used is an insulating material and it is produced in two steps. First when we irradiate the insulating material with high energy, second we photograph with a lower energy and as a result of imaging secondary ions are generated, their detection leads to imaging of the insider chamber space. This phenomenon, in fact, called (Mirror Effect), and since the beam is made up of ions it is called Ion Mirror Effect(IME).This phenomenon has been observed in (SEM) device by Clark and Stuart[9], then by Shaffner and Van Veld[11]. And in the (FIB) device by Crocola[12], then the researcher Muayyed Zoory[13].

2. Principal procedures and materials
In this research we use Poly methyl methacrylate (PMMA), and it is also known as acrylic glass or dielectric material. Thermoplastic material is widely used because it is shatter-resistant, highly resistant to weathering, and excellent light transmission which is more robust and has less risk of damage, and thus it has many technical advantages over other insulating materials. Through this research, our study will focus on the focused ion beam device, as the focused ion beam device uses ions instead of light to image the sample surface with great accuracy and thus create an image of this surface. In addition to, there are several benefits to using an ion beam concentrator instead of an
optical microscope. FIB has a large depth of field that allows a large area of sample to be focused in motion. FIB also produces high magnification images, so we will study in detail the use of FIB device in studying the ionic mirror phenomenon. Also we will study the dielectric constant parameter that affecting the production of the ionic mirror image (IMI).

3. Theoretical analysis of ion trajectory

By using the scattering method that was mentioned in reference[14]. We can drive a theoretical aspect for the ion trajectory on the dielectric sample surface in (FIB) chamber. We notice that the ion is launched with an initial velocity($v_0$) heading towards the surface of the sample, and since it carries a positive charge ($e^+$) and the insulating sample is also charged with a positive trapped charge($Q_t$), the ion will follow the hyperbola path see Figure 1, the reason for this is due to the Coulomb's repulsion force[15].

![Figure 1: Geometrical illustration for the incident ion in an FIB chamber.](image)

We can express the Coulomb's force by the following equation[16]:

$$F_i = \frac{e^+Q_t}{2\pi\varepsilon(\varepsilon_r+1)r^2} \hat{r}$$

(1)

Where ($r$) is the distance of probing ion from the trapped charge, ($\varepsilon_r$) is the permittivity of free space and ($\varepsilon_r$) is the relative permittivity. To drive the probing ion path in polar coordinates. First we find the acceleration equation in polar coordinates. Second we use Newton's second law, to find the equation of motion for both components ($\hat{r}, \hat{\theta}$) can be expressed[17]:

$$F_i = m_i a$$

(2)

$$m_i \left[ r \frac{d^2\theta}{dt^2} + 2 \left( \frac{dr}{dt} \right) \left( \frac{d\theta}{dt} \right) \right] = \frac{ke^+Q_t}{r^2}$$

(3)

$$m_i \left[ r \frac{d^2\theta}{dt^2} + 2 \left( \frac{dr}{dt} \right) \left( \frac{d\theta}{dt} \right) \right] = 0$$

(4)
Where \((\kappa)\) defined by \(\kappa = \frac{1}{2\pi\varepsilon(\varepsilon_r + 1)}\), \((m_i)\) is the probing ion rest mass. In fact, the above equation represents the principle of conserving angular momentum, and it can be reformulated as follows;

\[
m_i r^2 \frac{d\theta}{dt} = L = constant
\]

\[
L = WD m_i v\cdot sin(\alpha)
\]

By substituting the value of \((r_v = WD\ tan(\alpha))\) in equation (6) we get;

\[
L = m_i v_c \cdot c.p
\]

As the symbol \((C.P)\) in an equation (7) represents the perpendicular distance from the tangent to the path of incident ion of the trapped charge, and this is called the (collision parameter), and we get it from the sins law, \(\alpha\) and \((v_v)\) are the incident angle and initial velocity of probing ion. We can deduce the initial velocity from the law of energy conservation;

\[
v_v = \left(\frac{2eV_{sc}}{m_i}\right)^{1/2}
\]

Where \((V_{sc})\) is the scanning potential? By using the chain rule with a new variable \((\mu)\) that is proportional to the reciprocal of the original path function \((r)\) through the following formula \((\mu = \frac{1}{r})\) may lead to convert equation (3) to another form by some algebraic statement that are not dependent on time. Because the time for imaging the sample surface is very short.

\[
\frac{d^2\mu}{d\theta^2} - \frac{1}{r} - \frac{\kappa Q_t}{2 V_{sc} C.P^2} = 0
\]

By a second-order homogenous expansion, and some mathematical abbreviations we get;

\[
\mu = \frac{1}{r} = -\frac{\kappa Q_t r^2}{2 V_{sc} C.P^2} \left(1 - e \cos\theta\right)
\]

So, the probing ion path can be determined once \((e)\) and \((\theta)\) are known. Where \((e = 2 AV_{sc} C.P^2/\kappa Q_t)\), this equation above is called path equation, it represents a form of equations of a conic section in polar coordinate. Therefore, to find the eccentricity with aid of conservation of energy, and by doing some mathematical operations, we get the following relationship;

\[
e = \left(1 + \left(\frac{2 V_{sc} C.P}{\kappa Q_t}\right)^2\right)^{1/2}
\]

Now, by substituting equation (11) in equation (10), we get;

\[
r_\theta = \frac{2 V_{sc} C.P^2}{\kappa Q_t [\cos\theta - 1]}
\]

Where \((r_\theta)\) is the distance of probing ion path and it is only function of azimuthal coordinate \((\theta)\). Equation (12) above can be used to simulate the ion pathway. Therefore, we can find the value of the azimuth angle from Figure 2.

\[
\left(\frac{\pi}{2} - \frac{\gamma}{2} - \alpha\right) \leq \theta \leq -\left(\frac{\pi}{2} - \frac{\gamma}{2} - \alpha\right)
\]
Figure 2: Analysis of probing ion pathway inside (FIB) chamber and computation of scattering and azimuth angle.

So, the scattering angle \( \gamma \) represent the (angle between the asymptotes of probing ion path before and after it is scattered). Furthermore, from figure (1) we get:

\[
\tan \left( \frac{\pi}{2} - \frac{\gamma}{2} \right) = \frac{b}{a} = \cot \left( \frac{\gamma}{2} \right)
\]  

(14)

And since we have the value of the eccentricity that is equal to

\[
e^2 - 1 = \frac{b^2}{a^2}
\]

(15)

Where (a) and (b) are the semi major and minor axes respectively. From equation (14) and equation (15) we get

\[
\gamma = 2 \cot^{-1} \sqrt{(e^2 - 1)}
\]

(16)

As stated in Figure 2, it is seen that the incident ion has a specific singular position point, which it can be at a shorter distance from the trapped charge, this distance called "The distance of minimum approach" and denoted by \( r_{min} \) and given by following formula;

\[
r_{min} = \frac{2VscP^2}{\kappa q_2[e - 1]}
\]

(17)

Therefore, the reflection point is given by polar coordinates \( (r, \theta) = (r_{min}, \gamma/2 + \alpha) \) see Figure 2.Wherefore, each specific point has corresponding axis in Cartesian coordinates as follows;

\[
x_{min} = r_{min} \cos \left( \frac{\gamma}{2} + \alpha \right)
\]

(18)

\[
y_{min} = r_{min} \sin \left( \frac{\gamma}{2} + \alpha \right)
\]

(19)

Finally, the expression that defines the electric potential at any point located at a distance \( r \) from a source point charge is given by:
We can be expressed \((r)\) as the following 
\[ r = \sqrt{x_i^2 + y_i^2}, \]
and by putting \((x_i = 0)\) then the above equation become;
\[ V_{(y_0)} = \frac{q_t}{k \cdot y_0}. \]

All points \((x_i, y_i)\) having the same potential \(V_{(y_0)}\) can be found from the following;
\[ x_i = \pm \sqrt{\left(\frac{q_t}{k V_{(y_0)}}\right)^2 - y_i^2} \]

In fact, it is an equation of circle whose center at the origin and its radius is \(y_0\). Thus; we can deduct from this equation the equipotential surface see Figure 5.

![Figure 3: Representation for equipotential surfaces deduced from trapped charge located at the origin.](image)

4. Results and discussion

In this paper we will study the influence of the dielectric constant. There are many properties that a sample material possesses, and one of the most important factors influencing the creation of a mirror image is the dielectric constant. In fact, three different materials were chosen to conduct a comparative study in terms of ion mirror effect. Figure (4) depicts the probing ion paths simulation for three materials: PVC \((\varepsilon_r = 2.3)\), PMMA \((\varepsilon_r = 2.6)\) and PET \((\varepsilon_r = 4.43)\). At the conditions \((V_{sc} = 5 \text{ kV}, Q_t = 508 \text{ pC}, WD = 30 \text{ mm})\). The electric potential produced by the trapped charge depends mainly on the amount of trapped charge as well as the dielectric constant of sample material see equation (20). So, when the same amount of charges are pumped into two different materials, the lower dielectric constant material will display a higher potential distribution, while the higher dielectric constant material will show a lower potential distribution. As a result, when samples with high dielectric constant values are considered, the field of view for ion mirror images expands.
In the x-y space of the FIB chamber, Figure 5 shows incident ion paths with a similar slope. It is obvious that as the picture is produced with a polymer material with a higher dielectric constant, each of these paths becomes more divergent. This implies that as the dielectric constant rises, the scattering angle decreases, and vice versa. Furthermore, as the dielectric constant increases, the deviation point is pulled in the direction of the sample plane as the sample potential decreases.

Figure 5: The probing ion pathways have a similar incident angle ($\alpha = 0.05^\circ$)
In the previous section, the angle of inclination was plotted against the scattering angle for the three materials samples in Figure 6. In fact, as the inclination of the path increases, the scattering angle decreases. However, using materials with a high dielectric constant allows the incident ion path to deviate further.

The difference of incident angle with transverse distance for the three materials is depicted in Figure 7. If a material with a higher dielectric constant is used, the transverse distance increases even more. The incident ion hits the edge of the column at an angle of 0.194° in the PVC sample, while it hits the same point at (0.184°,0.163°) in the PMMA and PET samples, respectively. Compared with the reference results [18], we can see that when using the ion instead of the electron, we need a larger deflection angle to reach the coulomb's edge, which is due to the ion's heavier mass.
Figure 8: The horizontal coordinates for paths of different inclination angle as a function of vertical coordinates of inflection points at three materials.

Figure 9: Probing ion path with equipotential surface at $V_{sc}=5kV$ for PVC

Figure 10: Probing ion path with equipotential surface at $V_{sc}=5kV$ for PMMA sample.

Figure 11: Probing ion path with equipotential surface at $V_{sc}=5kV$ for PET sample.

5. Conclusions
In this work, a mathematical description to determine the reflected-back probing ions under ion bombardment related the ion beam deflection to the trapped charge and the main advantage is find the most appropriate operational parameter that may allow the FIB users, to be steady in production of ion mirror images by using a certain dielectric material. According to the presented simulation mode can be deduced the equipotential surfaces of these dielectric materials with good accuracy. Using the mathematical provided in this paper, it is frequently possible to locate the reflected-back probing ions.

As a result, additional evidence for the white and dark areas, which are typical in any ion mirror image, can be obtained. Furthermore, it is simple to determine which direction probing ions will take to penetrate the column diaphragm or hit the detectors.

It is clear; there are many benefits to use an ion beam focusing device instead of an optical microscope. The FIB has a large depth of field that allows the focusing in a large area of the sample while moving. FIB also produces high magnification images; we found that the ions are heavier and microsystem technology. The FIB has a large depth of field that allows the focusing in a large area of the sample while moving. FIB also produces high magnification images; we found that the ions are heavier and larger than electrons. so, the movement of ions be slower than electron, thus the working distance in ions FIB should be larger distance to avoid the damage of the sample.

6. References

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