Statistical features of plastic flow localization in materials

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Abstract. We consider the processes of plastic flow localization in dipolar materials undergoing high speed shear deformations. The mathematical model of the processes of plastic flow localization is formulated taking into account dipolar effect. We introduce the numerical algorithm which is based on adaptive mesh refinement technique. We show that this algorithm allows to increase performance of computations. We also studied the statistical properties of shear bands formation. We show that dipolar effect changes average characteristics of the processes considered such as average temperature, stress and etc. Moreover this effect leads to increase in initiation time, changes the widths of localization zones and distances between them.

1. Mathematical formulation of the problem considered
We consider the infinite slab of incompressible thermoviscoplastic material undergoing to simple shear deformation. The dipolar effects and effects of strain hardening of material are taken into account. The slab occupies the domain \( 0 \leq y \leq H \). The bottom surface \((y = 0)\) of slab is fixed and top surface \((y = H)\) is moving at the constant velocity \( v_{up} = H \dot{\varepsilon}_0 \). The direction of motion is parallel to the \( x \) axis. The mathematical model of the process of slab deformation in the above mentioned suggestions can be expressed by the following system of nonlinear equations [1, 2, 3]

\[ v_t = \frac{1}{\rho} (s - \sigma_y)_y, \quad (1) \]
\[ s_t = \mu (v_y - \dot{\varepsilon}) , \quad (2) \]
\[ \sigma_t = l\mu \left( v_{yy} - \frac{1}{l^2} \dot{d}^2 \right) , \quad (3) \]
\[ \psi_t = \frac{s_e \dot{\varepsilon}_e}{\kappa(\psi)} , \quad (4) \]
\[ C\rho T_t = \left( kT_y \right)_y + s_e \dot{\varepsilon}_e , \quad (5) \]

where \( s, \sigma \) are simple and dipolar stress respectively, \( v \) is the velocity, \( T \) is a temperature, \( \dot{\varepsilon} \) is the plastic strain rate, \( \psi \) is a strain hardening variable, \( d \) is a dipolar strain, \( l \) is a characteristic length of material, \( \rho \) is a mass density. In Eqs. (1)-(5) we use the following notations

\[ s_e = \sqrt{s^2 + \sigma^2}, \quad \dot{\varepsilon}_e = \sqrt{\dot{\varepsilon}^2 + \dot{d}^2}, \quad \varepsilon_e = \int_0^t \dot{\varepsilon}_e(\tau) d\tau, \quad (6) \]
Effective variables listed in (6) are connected by the plastic flow law
\[ D(s_e, \psi, T) \geq 0. \]
This connection is given by relations
\[ \dot{\varepsilon} = \frac{s}{s_e} \dot{\varepsilon}_e, \quad \dot{d} = \frac{\sigma}{s_e} \dot{\varepsilon}_e, \quad \dot{\varepsilon}_e = D(s_e, \psi, T). \] (7)

To close the system of Eqs. (1) - (7) we use the following initial conditions
\[ v(y, 0) = y \dot{\varepsilon}_0, \quad s(y, 0) = s_0(y), \quad T(y, 0) = T_0(y), \quad \sigma(y, 0) = \sigma_0(y), \quad \psi(y, 0) = 0. \] (8)

In turn, boundary conditions have the form
\[ v(0, t) = 0, \quad v(H, t) = v_{up}, \quad T_y(0, t) = T_y(H, t) = 0, \quad \sigma(0, t) = \sigma(H, t) = 0. \] (9)

We also use the Litonsky plastic flow to model the behavior of materials studied
\[ s_e = \kappa_0 g(T) \left( 1 + \left( \frac{\psi}{\psi_0} \right)^m \right) \left( 1 + \frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_0} \right)^m. \] (10)

Here \( g(T) \) is a thermal softening function, \( \psi_0 \) is a reference strain hardening parameter, \( \dot{\varepsilon}_0 \) is a reference strain rate, \( \kappa_0 \) is a characteristic value of stress. The value of the above mentioned parameters can be found in [3].

2. Adaptive mesh refinement technique
Since the width of the slab in most numerical experiments equal to several millimeters in turn the area of localization zones usually equal to \( 10 - 100 \mu m \), there is no need to use homogeneous grids with high resolution on the whole computational domain. Also such approach is not effective in the case when we know the places of shear bands localization. To overcome this difficulty the adaptive mesh refinement algorithm can be used. It allows to reduce the number of computational nodes and helps to prove the efficiency of numerical algorithms.

In the present work we apply adaptive mesh refinement to numerical algorithm presented in [3]. We discretize the computational domain using the fixed number of nodes. At initial moment of time nodes distribution in computational domain is uniform. The main idea of the adaptive mesh refinement algorithm is to concentrate nodes in the areas with maximum strain-rate during the deformation process. So it allows us to use less number of nodes and as a result to reduce computational time. For description of the nodes motion function we use the following relation
\[ p_i = \left( \frac{\dot{\varepsilon}_i^{n+1} - \dot{\varepsilon}_i^n}{y_i^{n+1} - y_i^n} + \frac{\dot{\varepsilon}_i^{n+1} - \dot{\varepsilon}_i^{n+1}}{y_i^n - y_i^{n-1}} \right) \left( \frac{\dot{\varepsilon}_i^{n+1} + \dot{\varepsilon}_i^n + 1}{y_i^{n+1} - y_i^n} + \frac{\dot{\varepsilon}_i^n - \dot{\varepsilon}_i^{n-1} + 1}{y_i^n - y_i^{n-1}} \right), \] (11)

where variable \( p_i \) has the meaning of strain-rate dimensionless gradient. In relation (11) number one in denominator allows us to avoid the problem caused by division by zero. This assumption can be done since in the localization areas value of \( \dot{\varepsilon} \gg 1 \), in other regions \( p_i \) approximately equal to zero.

On the next step we compute velocity of the nodes
\[ v_i = R S p_i, \] (12)

where \( S \) is speed of a transverse acoustic wave in material and \( R \) is dimensionless speed regulation parameter (we use \( R = 5 \) in all experiments). On last step we calculate new coordinates of nodes using the following relation
\[ y_i^{n+1} = y_i^n + \tau^n v_i, \] (13)
and than transfer grid functions using simple linear interpolation. Also we use additional condition on node location which allows to control the process of nodes convergence and divergence relative to each other. So, the possible minimum distance between nodes equal to 1µm, and the maximum distance equal to 100µm. It should be pointed out that the above mentioned algorithm cannot be used in the tasks where randomly disturbed conditions are used.

3. Results of numerical simulation of shear bands formation using and adaptive grid method

Here we study the formation of multiple adiabatic shear bands in nonpolar copper \((l = 0)\) specimen with height of \(H = 4\) mm. We suppose that initially there are three maximum of temperature located at points \(y = 1, 2\) and \(3\) mm. The value of temperature at these points equal to 100, 50 and 75 °C respectively. So, initial distribution of temperature in the specimen is given by relation

\[
T(y, 0) = 100 \left( e^{-r(y-1)^2} + 0.5e^{-r(y-2)^2} + 0.75e^{-r(y-3)^2} \right). \tag{14}
\]

Also we note that there are no initial perturbation of stress and strain.

![Figure 1](image_url)

**Figure 1.** Distributions of velocity (a) and temperature (b) at different time moments.

In Fig. 1 we have illustrated the stages of shear bands formation from initial moment of time till the final stage of localization. From Fig. 1 we see that on the first stage of localization there is an increase of temperature at the whole computational domain especially at the points \(y = 1, 2\) and \(3\) mm. After a while, we see an abrupt jump of the temperature at the points \(y = 1\) and \(3\) mm, where shear bands formed. In turn, the temperature at the center of the slab slowly increases during experiment. We also pointed that corresponding velocity gap disappears and plastic strain-rate equals to zero at this point after \(\varepsilon_{\text{nom}} = 1.2\). Such behavior is caused not only by the difference of initial temperature at the points \(y = 1, 3\) mm and \(y = 2\) mm but also by existence of additional temperature gradients on the boundaries.

These results were checked using the uniform grid with high resolution and it was shown that they are identical to each other. We also investigate the influence of the value of spatial step on the final results. Numerical experiments show that if we increase the spatial step in simple uniform grid from \(h = 1\) µm to \(h = 4\) µm the behavior of temperature and velocity changes at the point \(y = 2\) mm. However if use the adaptive mesh refinement technique we obtain the same results as in the case of uniform grid with high resolution even in the case \(h = 4\) µm. We also note, that this approach allows us to increase performance in five times.
Let’s take a look at behavior of nodes. According to the algorithm on the first part of the experiment nodes must move to temperature maximum. On the second part nodes must move away from the center of block. In Fig. 2 we illustrate the trajectories of nodes corresponding to this problem. From Fig. 2 we see that tracks of the nodes demonstrate expected behavior i.e.

![Figure 2. Tracks of nodes in the numerical experiment of three shear band formation.](image)

on the first stage of localization nodes concentrated near the temperature maximum at points $y = 1, 2, 3$ mm. Than approximately at $\varepsilon_{\text{nom}} = 1.2$ nodes started to move away from the point $y = 2$ mm (see Fig. 2b).

4. Statistical features of shear bands formation

In works [4, 5, 6, 7] it was shown that inhomogeneity in initial stress distribution leads to self-organization of adiabatic shear bands. Here we study the statistical properties of the multiple ASBs formation in two materials deformed at $\dot{\varepsilon}_0 = 10^5$ s$^{-1}$ taking into account dipolar effect [3]. One of materials is steel HY-100 and another one is OFHC copper.

First we explore the behavior of the average values such as temperature and effective stress. It is shown that the average temperature tends to a constant which increases with an increase of dipolar parameter $l$. For steel this value changes in twice and for copper approximately in one and a half times if we increase $l$ from 0 to 10 $\mu$m. Also it was found that dipolar effect significantly changes localization time. This fact also was mentioned in [1].

In work [1] authors noted that dipolar effects lead to an increase of shear bands width. This fact also was confirmed by our numerical experiments. So the question arise: is there any statistical law that describes the distribution of shear bands width depending on the value of dipolarity? Also it is interesting to study the distribution of distance between shear bands.

Numerical experiments show that the distribution of shear bands width is close to normal one. We see that the most probable value of distribution $w = w_m$ changes linearly with increasing the value of parameter $l$. In the case of steel we have $w_m = 11 + 2.4l$ $\mu$m, in the case of copper $w_m = 102 + 12l$ $\mu$m. However the distribution of distance between shear bands have more complex behavior. It turns out that if we increase the value of parameter $l$ the distribution become multimodal. For steel we have bimodal and four-modal distribution in the case $l = 5$ $\mu$m and 10 $\mu$m respectively, in the case of copper bimodal distribution is observed in the case $l = 10$ $\mu$m. Moreover, for both materials all modes located on the same distance between each other. That allows us to find the characteristic distance between shear bands in steel and copper. In the case of steel we have $L_p = 1.03$ mm and $L_p = 0.68$ mm for copper.
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