Vacuum stability in the singlet Majoron model

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Abstract

We study the vacuum stability of the singlet Majoron model using full renormalization group improved scalar potential and Monte Carlo techniques. We show that in the perturbative regime of the various free parameters, the vacuum stability requirement together with LEP limits is passed by 18% of the parameter space if the scale of new physics is 10 TeV and 6% if the scale is $10^{14}$ GeV. Moreover, if the baryogenesis condition for scalar couplings is required, no portion of the parameter space survives.
The singlet Majoron model [1] as one of the simplest extensions of the Standard Model (SM) offers an explanation to many problems that remain open in the minimal Standard Model. In addition to the usual doublet scalar field of the Standard Model the singlet Majoron model contains a complex, electroweak singlet scalar field. Also, right–handed electroweak singlet neutrinos are introduced to the model so, that an extra, global $U(1)$ symmetry appears. This global symmetry is broken approximately at the electroweak scale manifesting that the other of the new scalar degrees of freedom becomes massive whereas the other remains as a massless Goldstone boson. The non-zero vacuum expectation value of the singlet field then implies a mass term for the right–handed neutrinos.

A strong pro in favour to such an extended model of the Standard Model is that the weak scale baryogenesis, which appears to be somewhat problematic in the Standard Model [2, 3], is easier to realize in the singlet Majoron model [4, 5]. A problem with the Standard Model is that the experimental lower bound for the Higgs boson mass [6], $m_H > 60$ GeV, is too high compared to its theoretical upper bound in order to avoid the erasure of the baryon number by sphaleron [7] mediated transitions [8]. In the Majoron model these problems are circumvented by introducing the new scalar degrees of freedom [4, 9]. Moreover, the singlet Majoron model together with the sew–saw mechanism offers an explanation to the vanishingly small (left–handed) neutrino masses. By–products of the sew–saw mechanism, the heavy right–handed neutrinos may also serve as a source for cosmological baryon asymmetry due to their lepton number violating decays which can be converted to baryon number by sphalerons [4]. For a review of some other baryon number production mechanisms applicable in the singlet Majoron model, see [10].

To be a realistic model of electroweak scale physics, any model has to have the vacuum of the observed universe stable enough. If the stability is not achieved,
the theory can not be fully correct. This stability is, however needed only up to some scale, like supersymmetry scale or GUT scale, where new interactions becomes important: new phenomena appears and saves the stability. In this letter we study the stability properties of the singlet Majoron model at zero temperature. We work using one–loop perturbation theory and make a full renormalization group (RG) improved stability analysis of the scalar potential. Such an analysis for the Standard Model has first been performed by Flores and Sher [11] and recently by Sher [12] for new. They found that the stability of the potential requires that the Higgs mass $m_H > 75 \text{ GeV} + 1.64(m_{\text{top}} - 140 \text{ GeV})$. For top quark mass 174 GeV, which we use as an example, $m_H > 131 \text{ GeV}$. Renormalization group improved stability analysis for two doublet model has also been carried out [13]. The singlet Majoron model case differs from both above mentioned cases, not only because of different scalar content, but also because of the inclusion of the right–handed neutrinos. Their unstabilizing effect to the potential may be remarkable like the effect of the top quark has in the minimal Standard Model.

The tree–level potential of the singlet Majoron model reads

$$V_0(H, S) = m_H^2 |H|^2 + m_S^2 |S|^2 + \gamma |H|^2 |S|^2 + \beta |S|^4 + \lambda |H|^4,$$  

where, in a spontaneously broken theory, the mass–like parameters $m_H^2$ and $m_S^2$ are negative. The couplings $\beta$ and $\lambda$ are positive with $\gamma^2 < 4\lambda\beta$ which guarantees that the tree–level potential has a stable, non–trivial minimum. (See Ref. [4] for a detailed study of the potential.) The right–handed neutrinos couple to the singlet field with Yukawa couplings $g_i \ (i = 1, 2, 3)$ corresponding to the three generations of leptons. In practise we, however, consider only the heaviest right–handed neutrino and disregard the other two. The moduli of vacuum expectation values $\sqrt{2}f$ and
$\sqrt{2}f$ of the scalar fields $H$ and $S$, respectively, are given by

$$f^2 = \frac{-2\gamma m_S^2 + 4\beta m_H^2}{\gamma^2 - 4\lambda \beta}, \quad (2)$$

$$\bar{f}^2 = \frac{-2\gamma m_S^2 + 4\lambda m_H^2}{\gamma^2 - 4\lambda \beta}, \quad (3)$$

and the mass eigenstates are

$$m^2_\pm = \lambda f^2 + \beta \bar{f}^2 \pm \sqrt{D}, \quad (4)$$

where $D = (\lambda f^2 - \beta \bar{f}^2)^2 + \gamma^2 f^2 \bar{f}^2$.

The full set of the renormalization group equations needed to stability analysis calculated using one–loop perturbation theory reads

$$\dot{\alpha}_s = -\frac{7}{2\pi} \alpha_s^2, \quad (5)$$

$$\dot{\alpha} = -\frac{19}{48\pi} \alpha^2, \quad (6)$$

$$\dot{\alpha'} = \frac{41}{48\pi} \alpha'^2, \quad (7)$$

$$\dot{\alpha}_t = \frac{1}{2\pi} \left[ \frac{9}{2} \alpha_t^2 - 8\alpha_s \alpha_t - \frac{9}{4} \alpha \alpha_t - \frac{17}{12} \alpha' \alpha_t \right], \quad (8)$$

$$\dot{\alpha}_i = \frac{9}{4\pi} \alpha_i^2; \quad i = 1, 2, 3, \quad (9)$$

$$\dot{\lambda} = 4\lambda \gamma_H + \frac{1}{8\pi^2} [B + 12\lambda^2 + \frac{1}{2} \gamma^2], \quad (10)$$

$$\dot{\beta} = 4\beta \gamma_S + \frac{1}{8\pi^2} [B' + 10\beta^2 + \gamma^2], \quad (11)$$

$$\dot{\gamma} = 2\gamma [\gamma_H + \gamma_S] + \frac{1}{4\pi^2} [\gamma^2 + 2\gamma \beta + \frac{9}{2} \gamma \lambda], \quad (12)$$

where $\alpha_A = g_A^2/(4\pi)$ for the gauge as well as for the Yukawa couplings. The parameters $B$ and $B'$ are functions of couplings defined by

$$B = 3\pi^2 \left[ \frac{3\alpha^2 + 2\alpha \alpha' + (\alpha')^2}{4} - 4\alpha_i^2 \right], \quad (13)$$

$$B' = -6\pi^2 \left[ \alpha_1^2 + \alpha_2^2 + \alpha_3^2 \right]. \quad (14)$$

The anomalous dimensions of the scalars

$$\gamma_H = \frac{3}{16\pi} [-3\alpha - \alpha' + 4\alpha_t], \quad (15)$$

$$\gamma_S = \frac{3}{4\pi} [\alpha_1 + \alpha_2 + \alpha_3] \quad (16)$$
are also needed. The dots stand for the derivative with respect to the logarithmic scale variable \( t = \ln(\mu/M_Z) \), where \( \mu \) is the effective scale. We renormalize the couplings as \( \alpha_s(0) = 0.114 \), \( \alpha(0) = 0.0335 \), \( \alpha'(0) = 0.0102 \), \( \alpha_t(\mu = 2m_{\text{top}}) = m_{\text{top}}^2/(2\pi f^2) = 0.0790 \) for \( m_{\text{top}} = 174 \text{ GeV} \). The right–handed neutrino couplings are renormalized using \[14\]

\[
\alpha_i(\mu = 2m_{N_i}) = \frac{1}{2\pi} \frac{m_{N_i}^2}{f^2},
\]

where \( M_{N_i} \) are the physical heavy neutrino masses. The vacuum stability requires now \[15\] that inequalities \( \beta(t) > 0 \), \( \lambda(t) > 0 \) and \( \gamma(t)^2 < 4\beta(t)\lambda(t) \) holds for all \( t \) up to the given scale of new physics. Note, that the quadratic terms of the scalar potential are not important because we are only interested in large scales.

The large number of degrees of free parameters makes the integration of Eqs. (5) - (12) complicated and difficult to visualize in practise. Therefore, to study the RG–equations, we perform a Monte Carlo analysis where the initial values of the parameters \( \lambda \), \( \beta \), \( \gamma \), \( \bar{f} \) and the largest neutrino Yukawa coupling \( g_Y \) are generated randomly at \( \mu = M_Z \). The logarithms of these parameters are taken to be uniformly distributed, so that we are able to cover the whole parameter space rather densely.

As mentioned, we neglect the other right–handed neutrino Yukawa couplings but the largest one. This does not invalidate the analysis due to the structure the Yukawa couplings emerge in the RG–equations. The ranges of the parameters are

\[
10^{-3} < \beta, \gamma < 10^{-1}, \quad 5 \times 10^{-3} < \lambda < 10^{-1}.
\]

These bounds are due to the requirements that the baryogenesis analysis of \[4\] and the perturbation theory are applicable. Moreover we choose

\[
1 \text{ GeV} < \bar{f} < 10 \text{ TeV}, \quad 10^{-3} < g_Y < 1,
\]

where the lower bound for \( \bar{f} \) comes from the experimental upper bound and the see–saw mechanism estimate for the mass of the electron neutrino \( m_{\nu_e} \sim m_e^2/(g_Y\bar{f}) \).
On the other hand \( \bar{f} \) cannot be much larger than \( f = 246 \) GeV because the singlet scalar would effectively decouple \[4\]. (This kind of model is, of course, technically allowed, but is physically not very meaningful containing a new, \textit{ad hoc} symmetry breaking scale.) The range of \( g_Y \) was taken to be on the perturbative domain but not too small for the second order corrections in \( \beta, \lambda \) or \( \gamma \) to be effective.

From the randomly chosen initial values only those are accepted for which the two conditions, \( 4\beta\lambda > \gamma^2 \) at \( \mu = M_Z \) and there is no Landau singularity below the maximum scale studied, holds. Approximately 73\% of the generated values pass this test when maximum scale is the supersymmetry scale \( \mu = 10 \) TeV and 71\% when the maximum scale is the unification scale \( \mu = 10^{14} \) GeV. Furthermore three different cuts are applied to limit the parameter space:

1. It is required that vacuum is (absolutely) stable up to the given maximum scale. In the case of supersymmetry scale 24\% of the generated points in the parameter space pass this cut and in the case of unification scale 8\% pass the cut.

2. It is required that the scalar masses calculated from the generated point are compatible with the model independent LEP lower limit for a scalar particle mass. This means (see \[4\] for details) that

\[ \cos^2 z \Gamma(m_+) BR_+ + \sin^2 z \Gamma(m_-) BR_- < \Gamma(60 \text{ GeV}), \]  

(20)

where \( \Gamma(m) \) is the decay rate of \( Z \) to a fermion pair and a scalar with mass \( m \), \( BR_\pm \) are the branching ratios of scalars to ordinary fermions and \( z \) is the angle of rotation from current to mass eigenstates. 30\% of the points pass this cut.

3. Supplementary to the second limit there is another limit from LEP data. The latter relies on a event topology analysis of the reaction \( Z \to Z^* H_0 \to l^+ l^- (\nu \bar{\nu}) \)
when the former relies on observing reduced number of the standard fermionic
decays of this reaction. The resulting limit reads

\[ \cos^2 z \Gamma(m_+) + \sin^2 z \Gamma(m_-) < 4 \times \Gamma(60 \text{ GeV}). \] (21)

Also this cut is passed by 30% of the points.

In Figure 1 the resulting projection of the parameter space in the \( m_+ - m_- \) -
plane is shown after applying three combinations of the above cuts. Note, that
these masses are not really the physical ones, because their scale dependence is
not taken into account. As the scalar masses are not too far from the weak scale
\( M_Z \) the correction due to the running of the masses may, however, be neglected.
The parameter space is reduced to 18% after applying all cuts in the case of the
supersymmetry scale and to 6% in the case of the unification scale. The vacuum
stability bounds \( \lambda \) to be larger than 0.034 (0.067) for \( \mu = 10 \text{ TeV} \) (10\(^{14}\) GeV)
except some very few points which cover less than 0.03% of the parameter space.
Therefore it can be concluded that vacuum stability requirement and baryogenesis
bound \( \lambda \lesssim 0.018 \) are somewhat controversial. Applying the vacuum stability cut
and the baryogenesis bound for \( \lambda \) together leaves zero parameter space. It is also
noteworth that Monte Carlo analysis shows that most of the values of the neutrino
masses passed by the vacuum stability and LEP limits are between 20 GeV and some
hundred GeV's, i.e. not very small nor very large neutrino masses are favoured.

Our results infer that the vacuum stability and baryogenesis bounds in the sin-
glet Majoron model can not be fitted together. However, both limits have some
uncertainty. Because the baryogenesis bound may be affected by non-perturbative
effects similar to ones in the Standard Model, the upper bound for the doublet
self-coupling \( \lambda \) may be relaxed. Yet, if our vacuum is not absolutely stabile but its
life-time exceeds the age of the present universe \( \approx 10^{10} \text{ yr} \), the vacuum stability
bound will be somewhat relaxed, too. (For a general discussion, see Ref. [13] and references therein.) In the Standard Model the possibility of an un–absolute vacuum allows, however, only about 10 GeV smaller higgs mass than the absolute stability requirement [12]. It is not probable that this effect would be large either in the singlet Majoron model. Taking into account the uncertainties of the bounds it might be possible that a very thin right–angular zone beginning at \((m_-, m_+ \approx (0, 90) \text{ GeV})\) and ending at \((m_-, m_+ \approx (90, 150) \text{ GeV})\) survives both baryogenesis and vacuum stability bounds, i.e. the gap between \(\lambda < 0.018\) and \(\lambda > 0.034\) can be filled.

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FIGURE CAPTION

Figure 1. Distribution of randomly generated points in the $m_--m_+-$plane after applying combinations of cuts. a) Points (26 % ) which fulfill LEP limits (20) and (21). b) Points which pass all cuts when the scale of the new physics is $\mu = 10$ TeV. c) Same as b) for $\mu = 10^{14}$ GeV.
This figure "fig1-1.png" is available in "png" format from:

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