THE VOLUME FRACTION OF IONIZED INTERGALACTIC GAS AT REDSHIFT \( z = 6.5 \)

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ABSTRACT

The observed number density of Ly\( \alpha \) sources implies a minimum volume of the intergalactic medium that must be ionized, in order to allow the Ly\( \alpha \) photons to escape attenuation. We estimate this volume by assigning to each Ly\( \alpha \) emitter the minimum ionized bubble that would allow half its Ly\( \alpha \) photons to escape. This implies a lower limit to the ionized gas volume fraction of 20%–50% at \( z = 6.5 \). This is a lower limit in two ways: First, we conservatively assume that the Ly\( \alpha \) sources seen (at a relatively bright flux limit) are the only ones present, and second, we assume the smallest ionized bubble volume that will allow the photons to escape. This limit is completely independent of what ionizing photon sources produced the bubbles. Deeper Ly\( \alpha \) surveys are possible with present technology and can strengthen these limits by detecting a higher density of Ly\( \alpha \) galaxies.

**Subject headings:** galaxies: high-redshift — intergalactic medium

**Online material:** color figures

1. INTRODUCTION

The epoch of reionization marks a phase transition in the universe, when the intergalactic medium (IGM) was ionized. Recent observations of \( z > 6 \) quasars show a Gunn-Peterson trough (Gunn & Peterson 1965), suggesting that the reionization of intergalactic hydrogen was not complete until \( z \approx 6 \) (Becker et al. 2001; Fan et al. 2002). Yet, microwave background observations imply substantial ionization as early as \( z \approx 11 \) (Page et al. 2006; Spergel et al. 2003; Kogut et al. 2003). These results can be reconciled if reionization occurred twice (e.g., Cen 2003), slowly (e.g., Gnedin 2004), or substantially inhomogeneously (Malhotra et al. 2005; Oh & Furlanetto 2005; Cen 2005). Information on the state of the intergalactic gas—the ionized fraction, and spatial distribution of ionized gas—is needed, but is scant.

**Ly\( \alpha \) emitters as tests of ionized IGM.**—Ly\( \alpha \)-emitting galaxies provide another tool for probing reionization. Ly\( \alpha \) visibility tests offer a local probe of neutral fractions \( x_{\text{HI}} \sim 30\% \) (Malhotra & Rhoads 2004, hereafter MR04), while the Gunn-Peterson trough saturates at \( x_{\text{HI}} \sim 1\% \) (Fan et al. 2002), and the polarization of the cosmic microwave background radiation provides an integral constraint on the ionized gas along the line of sight.

Because Ly\( \alpha \) photons are resonantly scattered by neutral hydrogen, Ly\( \alpha \) line fluxes are attenuated for sources in a significantly neutral IGM (Miralda-Escudé 1998; Loeb & Rybicki 1999; Haiman & Spaans 1999). This should produce a decrease in Ly\( \alpha \) galaxy counts before reionization in a flux-limited sample (Rhoads & Malhotra 2001, hereafter RM01). We consider three physical effects that modify the attenuation of the Ly\( \alpha \) flux.

**Effect of ionized bubbles.**—Each galaxy creates a local bubble of ionized gas. If this is large enough, Ly\( \alpha \) photons are redshifted by the time they reach the neutral boundary and thus can escape. Consider a galaxy in an ionized bubble of radius \( R_b \) surrounded by a fully neutral IGM. For wavelengths separated from line rest wavelength by \( \Delta \lambda_{\text{em}} \), the optical depth due to the neutral IGM damping wing is

\[
\tau = (1.2 \text{ pMpc}) \left( \frac{R_b}{\lambda_0} + \frac{\Delta \lambda_{\text{em}}}{\lambda_0} \right) \left( \frac{c}{H} \right)^{-1}.
\]

We write physical megaparsecs as pMpc, and comoving megaparsecs as cMpc. Here \( c \) is the speed of light, and the Hubble constant \( H \approx 760(1 + z)(7.5)^{2/3} \text{ km s}^{-1} \text{ Mpc}^{-1} \) for \( z \gg (\Omega_m/\Omega_{\Lambda})^{1/3} - 1 = 0.4 \) in a flat cosmology with \( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \). \( \Omega_m = 0.27 \), and \( \Omega_{\Lambda} = 0.73 \) (Spergel et al. 2003). Setting \( \Delta \lambda_{\text{em}} = 0 \) shows that the line center optical depth has no explicit redshift dependence. This is because the line center depth has the form \( \tau \propto \int \rho(z)[H(z)]^{-2} dl(z) \), with \( \rho \propto (1 + z)^3 \) and \( H \propto (1 + z)^{y_1} \). Thus, transmission becomes significant when the physical radius of the H\( \text{II} \) region is \( \gg 1.2 \text{ pMpc} \) (corresponding to \( \gg 9 \text{ cMpc} \) at redshift \( z = 6.5 \)).

The size of a typical ionized bubble that an isolated galaxy creates in the IGM is directly related to its Ly\( \alpha \) luminosity \( L_{\lambda_{\alpha}} = 10^{43}L_{43} \text{ ergs s}^{-1} \), its age \( t_{\alpha} = 10^{8} \text{ yr} \), and the fraction \( f_{\text{esc}} \) of its ionizing photons that escape the galaxy to ionize the surrounding IGM in the absence of dust (thereby becoming unavailable for Ly\( \alpha \) line production) (RM01). The result, conservatively ignoring recombination, is

\[
R_b \approx 0.7 \frac{\text{pMpc}}{1 + z} \left( \frac{L_{43} t_{\alpha} (f_{\text{esc}})}{1 - f_{\text{esc}}} \right)^{1/3}.
\]

Stellar population models for narrowband-selected Ly\( \alpha \)-emitting galaxies require \( t_{\alpha} \lesssim 1 \) to produce the observed range of Ly\( \alpha \) line equivalent widths (Malhotra & Rhoads 2002, hereafter MR02). The luminosity function for Ly\( \alpha \) galaxies at \( z \approx 6 \) shows \( L_{\alpha} \sim 10^{42.4} \text{ ergs s}^{-1} \) (MR04), so that galaxies with \( L_{43} > 1 \) are rare. Using \( t_{\alpha} = 1 \) and \( L_{43} = 1 \), we see that \( \tau \gtrsim 2 \).

**Velocity offsets.**—The observed Ly\( \alpha \) line is usually asymmetric (e.g., Rhoads et al. 2003; Dawson et al. 2004) and offset to the red compared with other lines (e.g., Shapley et al. 2003).

This offset is likely caused by absorption or scattering of Ly\( \alpha \) photons in the blue wing of the emission line in the interstellar medium of the emitting galaxy, combined with gas motions (or outflows) in that medium. Resonant scattering by the surrounding IGM will further suppress the blue wing of the line and accentuate the asymmetry. The typical observed velocity offsets for Lyman break galaxies (LBGs) with Ly\( \alpha \) emission is \( \sim 360 \text{ km s}^{-1} \), although the velocity offsets decrease for galaxies with higher equivalent widths in the Ly\( \alpha \) emission (Shapley et al. 2003). Such an offset can be incorporated in equation (1) using \( \Delta \lambda_{\text{em}} = \lambda_0 v_{\text{w}}/c \), where \( v_{\text{w}} \) (the “wind” velocity)
denotes the offset of the Lyα line relative to the systemic velocity of the emitting galaxy.

If we describe the intrinsic profile as \( f_\lambda(\lambda_0 + \delta \lambda) \), where \( \lambda_0 = 1215.67 \) Å is the rest Lyα wavelength, then the unabsorbed line flux is \( F_0 = \int_{-\infty}^{\infty} f_\lambda(\lambda_0 + \delta \lambda) \, d(\delta \lambda) \). The absorbed line flux is \( F = \int_{-\infty}^{\infty} \exp \left( -\tau(\delta \lambda) \right) f_\lambda(\lambda_0 + \delta \lambda) \, d(\delta \lambda) \), and the transmission factor is \( T = F/F_0 \), which is calculated by integrating over the line profile. Santos (2004) shows that \( T \approx \frac{1}{2} \) for a wide range of plausible models of Lyα line properties and IGM, even including those with \( v_u = 360 \) km s\(^{-1}\). Models with \( v_u = 0 \) invariably show \( T \ll \frac{1}{2} \), unless the IGM is largely ionized. Moreover, the variation of \( T \) with \( v_u \) is fairly slow for \( v_u > 360 \) km s\(^{-1}\).

**Galaxy clustering.**—Even though a single Lyα galaxy does not produce a large enough ionized bubble, fainter galaxies clustered around the observed Lyα galaxy could contribute enough photons to make the H \( \beta \) regions around the cluster reach the critical size. This would require roughly 10–60 times more photons than produced by a typical Lyα galaxy (Wyithe & Loeb 2005; Haiman & Cen 2005; Furlanetto et al. 2004). So far, the evidence for such clustering is mixed: Malhotra et al. (2005) see clustering in the Hubble Ultra Deep Field, as do Stiavelli et al. (2005) around an SDSS quasar at \( z = 6.2 \), but J. Rhoads et al. (2006, in preparation) see no significant excess around an actual \( z = 6.5 \) Lyα galaxy.

Here we introduce a new version of the Lyα reionization test wherein each Lyα galaxy implies the presence of a certain volume of ionized IGM. By combining the ionized volume per source with the observed number density of Lyα galaxies, we obtain a lower bound on the volume ionized fraction of the IGM. This limit is completely independent of whether the ionizing photons come from the Lyα galaxy, its unseen neighbors, or any other source.

## 2. Ionized Volume Estimates

We need to calculate the Lyα flux transmission expected from a galaxy with a given ionized bubble radius, Lyα line velocity offset, and IGM neutral fraction. Observed Lyα emission lines from high-redshift galaxies are well described by a truncated Gaussian profile due to absorption by the interstellar medium in the galaxy:

\[
 f_\lambda(\lambda_0 + \delta \lambda) = \left[ \frac{2F_\lambda}{\sqrt{2\pi} \sigma} \right] e^{-\frac{(\delta \lambda)^2}{2\sigma^2}}, \quad \text{if } \delta \lambda \geq 0, \\
 0, \quad \text{if } \delta \lambda < 0 \tag{2}
\]

(Hu et al. 2004; Rhoads et al. 2004), where \( \lambda_0 = 1215.67 \times (1 + v_u/c) \) Å is the Lyα central wavelength in the frame of the IGM surrounding the galaxy and the unabsorbed line flux is \( F_\lambda \). The absorbed line flux is \( F = \int_{-\infty}^{\infty} \exp \left( -\tau(\delta \lambda) \right) f_\lambda(\lambda_0 + \delta \lambda) \, d(\delta \lambda) \), and the transmission factor is \( T = F/F_\lambda \). We set \( \sigma = 1.49 \) Å, or rest-frame FWHM of 1.75 Å (from the truncation point \( \lambda_0 \) to the half-peak point on the red side of the line)—chosen to match typical observed high-redshift Lyα lines. We determined \( T \) by numerical integration for a grid of \( v_u \) and \( R_b \). We then inverted the result to determine the ionized bubble radii corresponding to 25%, 33%, 50%, and 70% transmission for a range of Lyα velocity offsets \( v_u \) (Fig. 1).

### 2.1. The Minimum Ionized Fraction: Analytic Results

We next combine our calculated minimum ionized bubble radii with the observed number density of \( \approx 6.5 \) Lyα-emitting galaxies to place a lower bound on the volume ionized fraction of the intergalactic medium. The volume neutral fraction

\[
 x_{V, \text{ion}} = \frac{F - F^2}{2} \left[ 1 + 3 \left( \frac{R_b}{R_p} \right)^\gamma \left[ \frac{2^{3-\gamma} - 1}{3 - \gamma} \right] \left[ \frac{2^{2-\gamma} - 1}{4 - \gamma} \right] \left[ \frac{2^{1-\gamma} - 1}{6 - \gamma} \right] \right]. \tag{3}
\]
This calculation omits third-order and higher terms in $F$, corresponding to the volume within distance $R_0$ of three or more Ly$\alpha$ sources. Omitting such “triple overlap” regions will cause equation (3) to underestimate the volume ionized fraction.

2.2. Minimum Ionized Fraction: Numerical Simulations

To achieve accurate results at higher source densities, we simulated correlated distributions of galaxies and directly calculated the volume fraction enclosed by the union of their ionized bubbles. We performed these calculations for a range of correlation lengths, ionized bubble sizes, and source densities. We generated the set of correlated $(x, y, z)$ triples using a Mandelbrot-Levy random-walk prescription (see Peebles 1980, § 62). A power-law distribution of step lengths $s$, $p(s) \propto s^{-1.2}$, ensures a power-law correlation function with the desired slope $\gamma = -1.8$. The correlation length $r_c$ is related to the minimum step length (which is always $\ll R_b$ and $\ll r_0$) and to the number of independent random chains occupying the sample volume. To tune the correlation length to a desired value, we vary the number of chains used and also (for small correlation lengths) add a suitable number of entirely uncorrelated $(x, y, z)$ triples.

We verified that this prescription generated the desired two-point correlation properties. The volume used for the simulations was $\sim 10^6$ eMpc$^3$, and the volume fraction was calculated on a grid with cell size of $R_b/2$ (a smaller cell size does not significantly change the results). Finally, we verified that the simulations reproduce the analytic results for an uncorrelated distribution at any filling factor, and for a correlated distribution at $F \leq 1$ (where the $F^3$ and higher terms are small). The results of the simulations are shown in Figure 2, as curves of $x_{\text{ion}}$ as a function of the Ly$\alpha$ galaxy number density for a plausible range of $R_b$ and $r_0$ values. Were we to plot $x_{\text{ion}}$ as a function of $F$ (rather than $n$), we would see that $x_{\text{ion}}$ is a function of just two variables, $F$ and the ratio $r_0/R_b$, as one would expect from the discussion in § 2.1.

2.3. Minimum Ionized Fraction: Direct Mapping

The above treatment using the correlation function of the post-reionization era could potentially mislead us in the case of a largely neutral, preoverlap phase of reionization. In such a regime we should see patches of Ly$\alpha$ emitters in ionized regions. In the neutral regions, Ly$\alpha$ emission would be attenuated by a factor of 3 or more, resulting in many fewer detected galaxies. This would lead to dramatically enhanced clustering (see, e.g., Furlanetto et al. 2006), that is, an increase in correlation length $r_0$, along with enhanced higher order correlation functions. The effect becomes large (factors of $\geq 2$ increase in $r_0$) at mean ionized fractions $x_\text{ion} \lesssim 0.4$. This would lead to an overestimate of the ionized fraction if we were to assume a “normal” correlation length ($r_0 \sim 4$ eMpc) rather than measuring $r_0,_{\text{obs}}$ directly from the data. An extreme scenario would be for all Ly$\alpha$ emitters in the survey volume to share one bubble, in which case the degree of overlap between the bubbles is much larger than estimated above.

To alleviate these concerns, we can use the observed spatial distribution of Ly$\alpha$ galaxies directly to place a bound on the ionized volume fraction. Complete spectroscopic follow-up of a sample would yield three-dimensional coordinates $(\alpha, \delta, z)$. Combining these with the minimum bubble radius $R_b$, would give a unique number for $x_{\text{ion}}$. We illustrate a variant of this approach using the observed distribution of Ly$\alpha$ emitters in the Taniguchi et al. (2005) sample. Spectroscopic follow-up of this sample is not 100% complete, and the redshifts of most sample galaxies remain unknown. We therefore use a randomly selected subset of their candidates, consistent with the spectroscopic confirmation rate. We assigned redshifts randomly in these simulations, using two limiting cases: uncorrelated redshifts, or a single identical redshift for every galaxy in the survey. We ran 1000 of these simulations for a range of bubble radius $R_b$. The results are shown in Figure 2, and they match those from the earlier discussion. This shows that at $z = 6.5$ we are not in the regime where the spatial distribution of Ly$\alpha$ galaxies is dominated by bubble geometry. This is consistent with the findings of Taniguchi et al. (2005, their Fig. 5) and Kashikawa et al. (2006), that there seems to be no evidence for unusually strong clustering or number density gradients of Ly$\alpha$ emitters in this sample. However, as data become available at higher redshifts and larger neutral fractions, enhanced clustering is expected and should be considered in the application of our method.

2.4. Effect of a Partly Ionized Ambient Medium

If the IGM is already partially ionized outside the bubbles, then Ly$\alpha$ flux is transmitted more easily, and the ionized bubbles required to achieve a particular Ly$\alpha$ transmission are smaller. However, this scenario requires a large amount of ionized gas mixed into the ambient medium. Figure 3 shows the net ionized fraction of ionized gas (counting the ionized
bubbles plus ambient ionized gas) as a function of ambient neutral fraction for $v_w = 0$ and 360 km s$^{-1}$.

3. SUMMARY AND FUTURE OUTLOOK

Observations of Ly$\alpha$ emitters provide a powerful method to probe the neutral fraction of the IGM and the volume of the ionized gas. In an earlier paper, we explored the effect on the Ly$\alpha$ luminosity function and constrained the average neutral fraction of the IGM to be $(x_n) < 30\% - 50\%$ (MR04; see also Stern et al. 2005). Further investigations by Haiman & Cen (2005) and Furlanetto et al. (2006) agreed with these estimates.

In this Letter, we place a lower limit on the volume in ionized IGM $x_{V,ion} = 20\% - 50\%$ for best-known parameters of transmission factor $T = 0.5$, source density $n$(Ly$\alpha$) = 10$^{-4}$, and $v_w = 360$ to 0 km s$^{-1}$.

With better observations, these limits can be substantially improved. Currently, the largest Ly$\alpha$ galaxy sample at $z = 6.5$ (s) comes from Subaru observations (Taniguchi et al. 2005), which reach just about $L_e$. Going deeper will yield a higher density of Ly$\alpha$ emitters. By increasing the observed number density of sources by a factor of 2 or 3 with deeper observations, the lower limit to $x_{V,ion}$ would increase by a factor of 2. MR04 concluded that attenuation of a factor of 2 was marginally allowed by present luminosity function data at $z = 5.7$ and $z = 6.5$. By reducing this limit to 1.4, that is, a transmission factor of 70%, the minimum ionized bubble radius becomes 3 pMpc for offset velocity of zero, and 2.6 pMpc for a velocity offset of 350 km s$^{-1}$ (Fig. 1). In that case, the Taniguchi et al. (2005) data would imply minimal ionized volume fraction of nearly $x_{V,ion} = 100\%$.

Measurement of the velocity offset between Ly$\alpha$ redshift and systematic velocity of the galaxy will allow us to further refine the ionized bubble radius (see Fig. 1). Currently, our estimates come from such measurements on LBGs, for which the higher the equivalent width (EW) of the Ly$\alpha$ line, the smaller the velocity offset (Shapley et al. 2003). But the overlap between the LBG samples and the Ly$\alpha$ samples is minimal; only the LBGs with highest EW ($\approx 50$) would even be selected by the Ly$\alpha$ searches. So it is plausible that the velocity offsets for the Ly$\alpha$ emitters (median EW $\approx 100$ Å; MR02; Dawson et al. 2004) would be even lower.

Clustering in the line-of-sight dimension, obtainable with spectroscopy, can be used to directly determine the overlap in the ionized bubbles. If the sources required for ionizing photons are strongly clustered on $\approx 100$ cMpc scales as observed (Malhotra et al. 2005) or predicted (Cen 2005), we should see dramatic field-to-field variations in Ly$\alpha$ number density prior to reionization. These tests require detection of statistically useful samples of Ly$\alpha$ emitters and are eminently practical with present technology at optical wavelengths, and at redshifts $z > 7$ in the near future.

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Fig. 3.—Ionized gas fraction in the IGM plotted as a function of the neutral fraction in the ambient medium (outside ionized bubbles). We have assumed a number density of $1.5 \times 10^{-4}$ cMpc$^{-3}$ (Taniguchi et al. 2005). The two solid curves show the result for $v_w = 0$. The lower curve shows the amount of ionized gas enclosed in ionized bubbles around Ly$\alpha$ galaxies, and the upper curve shows the total ionized gas fraction (including that in the ambient medium). The dashed curves show the same quantities for $v_w = 360$ km s$^{-1}$. [See the electronic edition of the Journal for a color version of this figure.]