Research Article

Xiaoqing Zhou, Mustafa Habib, Tariq Javeed Zia, Asim Naseem, Anila Hanif, Ansheng Ye*

Topological invariants for the line graphs of some classes of graphs

https://doi.org/10.1515/chem-2019-0154
received March 5, 2019; accepted July 1, 2019.

Abstract: Graph theory plays important roles in the fields of electronic and electrical engineering. For example, it is critical in signal processing, networking, communication theory, and many other important topics. A topological index (TI) is a real number attached to graph networks and correlates the chemical networks with physical and chemical properties, as well as with chemical reactivity. In this paper, our aim is to compute degree-dependent TIs for the line graph of the Wheel and Ladder graphs. To perform these computations, we first computed M-polynomials and then from the M-polynomials we recovered nine degree-dependent TIs for the line graph of the Wheel and Ladder graphs.

Keywords: line graph; topological index; M-polynomial.

1 Introduction

In mathematical chemistry, we use mathematics to solve problems of chemistry, and a key area of research in mathematical chemistry is Chemical graph theory in which we represent compounds and chemical structures with graphs and apply graph theory to study their topologies. Topological indices (TIs) are real numbers attached to graph networks and graph of compounds. TIs remain invariant and can be used in predicting the properties of interesting compounds [1].

In the field of heminformatics, quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR), together with TIs, are utilized to study properties and chemical bioactivity of compounds [2]. Like TIs, polynomials also support a considerable number of applications in network theory and chemistry; for instance, the Hosoya polynomial, which is also known as Wiener polynomial [3], is helpful in constructing distance-dependent TIs. The M-polynomial was introduced previously [4] for deciding degree-dependent TIs [5,6].

Definition 1. For a simple connected graph G, the M-polynomial is defined in [4] as:

\[ M(G; x, y) = \sum_{\delta \leq i \leq \Delta} m_{ij}(G) x^i y^j, \]

where \( \delta = \min\{d_v : v \in V(G)\} \), \( \Delta = \max\{d_v : v \in V(G)\} \), and \( m_{ij}(G) \) is the edge \( uv \in E(G) \) such that \( \{d_u, d_v\} = \{i, j\} \).

The Wiener polynomial was the first such function, with the Wiener index being introduced in 1947 [7]. Thus, we can say that Harold Wiener began the theory of TIs [8,9]. After Wiener’s work, Milan Randic [10] introduced the first degree-dependent TI, which is today known as Randic index (RI), in 1975. The mathematical formula of RI is

\[ R_{1/2}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}. \]

In 1988, a generalized version of RI was defined by several researchers [11,12]. This version attracted the attention of both mathematicians and chemists [13]. Numerous numerical properties of this simple TI have studied, and results are presented in research reports [14] and a helpful book [15]. In addition, many research papers and books [16-18] have been published regarding RI. Two reviews of RI were written by Randic [19,20] and three more reviews have been written on this TI by other scientists [21-23].
After RI, the most interesting TIs are 1st Zagreb index (ZI) and 2nd ZI [24-27]. The first and second ZIs were proposed by Gutman and Trinajstic and are defined as

\[ M_1(G) = \sum_{uv \in E(G)} (d_u + d_v) \]

and

\[ M_2(G) = \sum_{uv \in E(G)} (d_u \times d_v) \]

The modified 2nd ZI was defined in [28] as

\[ M_2^m(G) = \sum_{uv \in E(G)} \frac{1}{d(u)d(v)} \]

Other TIs include symmetric division (SDI) [29]

\[ SDD(G) = \sum_{uv \in E(G)} \left\{ \frac{\min(d_u, d_v)}{\max(d_u, d_v)} + \frac{\max(d_u, d_v)}{\min(d_u, d_v)} \right\} \]

harmonic index (HI) [30,31]

\[ H(G) = \sum_{uv \in E(G)} \frac{2}{d_u + d_v} \]

inverse sum index

\[ I(G) = \sum_{uv \in E(G)} \frac{d_u d_v}{d_u + d_v} \]

and augmented ZI [32]

\[ A(G) = \sum_{uv \in E(G)} \left\{ \frac{d_u d_v}{d_u + d_v - 2} \right\}^3 \]

In the remaining paper, we consider G to be the simple connected graph. A graph G with vertex set V(G) and edge set E(G) is connected if there exists a connection between any pair of vertices in G [33]. The quantity of vertices of G adjoining a given vertex v, is the “degree” of this vertex and will be denoted by d_v. Throughout this paper, G will represent a connected graph, V its vertex set, E its edge set, and d the degree of its vertex v. The line graph of G is denoted by L(G) and is obtained from G by associating

a vertex with each edge of the graph and connecting two vertices with an edge if and only if the corresponding edges of G have a vertex in common.

In this paper we study line graph of Wheel and Ladder graphs. We computed several degree-based topological indices of the understudy families of graphs.

2 Methodology

There are three kinds of TIs:
1. Degree-based TIs
2. Distance-based TIs
3. Spectral-based TIs

In this paper, we aim to compute degree-dependent TIs. To compute degree-based TIs of line graph of Wheel, Ladder and Bipartite graphs, we first drew line graphs and then we divided the edge sets of these line graphs into classes based on the degree of the end vertices and computed their cardinality. From this edge partition, we computed our desired results. First, we computed M-polynomials of the understudy families of graphs. Then, by applying calculus and using table 1, we computed several TIs.

The relationship between M-polynomial and indices is presented in table 1 [4] where

\[ D_x = x \frac{\partial(f(x,y))}{\partial x}, D_y = y \frac{\partial(f(x,y))}{\partial y} \]

\[ S_x = \int_0^x f(t,y) dt, S_y = \int_0^y f(x,t) dt, J(f(x,y)) = f(x,y) \]

\[ Q(x,y) = x^p f(x,y) \]

2.1 Main Results

This section consists of two subsections. In the first subsection we study the line graph of the Wheel graph, and in the second subsection we study the line graph of the ladder graph.

2.2 M-polynomial of line graph of Wheel Graph

In order to construct a wheel graph, we connected a single vertex to other vertices in a cycle. A wheel graph with n vertices can also be defined as the 1-skeleton of an (n-1)-
Topological invariants for the line graphs of some classes of graphs

The Wheel graph is given in Figure 1 and its line graph is given in Figure 2.

2.2.1 Theorem 1

Assume $G$ to be the line graph of Wheel graph; then, we have

$$M(G, x, y) = nx^4y^4 + \frac{n(n+1)}{2}x^{n+1}y^{n+1} + 2nx^ny^{n+1}.$$

We can divide the edge set of $G$ into the following three classes depending on each edge at the degree of end vertices:

$$E_1(G) = \{uv \in E(G); d_u = 4 \text{ and } d_v = 4\},$$
$$E_2(G) = \{uv \in E(G); d_u = n+1 \text{ and } d_v = n+1\},$$
$$E_3(G) = \{uv \in E(G); d_u = 4 \text{ and } d_v = n+1\}.$$

We can divide the vertex set of $G$ into the following two types, depending on the degree:

$$V_1(G) = \{v \in V(G): d_v = 4\},$$
$$V_2(G) = \{v \in V(G): d_v = n+1\},$$

Such that

$$|V_1(G)| = n,$$
$$|V_2(G)| = n.$$
From definition

\[ M(G, x, y) = \sum_{\{x,y\} \in E(G)} m_{xy} x^y \]
\[ = \sum_{\{x,y\} \in E(G)} m_{xy} x^y + \sum_{\{w,v\} \in E_H(G)} m_{wv} x^w y^v + \sum_{\{w,v\} \in E_H(G)} m_{wv} x^w y^v \]
\[ = nx^n y^n + \frac{n(n+1)}{2} x^{n+1} y^{n+1} + 2nx^n y^{n+1}. \]

2.2.3 Proposition 2

Let G be the line graph of Wheel graph; then we have

\[ M_1(G) = n^2 + 2n^2 + 13n \]
\[ M_2(G) = 4n^2 + (2n^2 - 2n)(n+1)^2 + 12n \]
\[ mM_2(G) = \frac{n + n(n^2 - n)}{16(n+1)^2} + \frac{n}{2(n+2)} \]
\[ SSD(G) = \frac{5n^3 + 6n^2 + 17n}{2(n+1)} \]
\[ H(G) = \frac{n + (n^2 - n)}{4(n+1)} + \frac{4n}{(n+5)} \]
\[ I(G) = 2n + \frac{(n^2 - n)}{2} + \frac{8n^2 + n}{(n+5)} \]
\[ A(G) = \frac{4^6}{6} n^6 + \frac{n(n+1)^3(n-1)}{2^4 n^3} x^{2n} + \frac{2^5 n(n+1)}{(n+3)^3} x^{n+3} \]

2.2.4 Proof.

Let

\[ f(x, y) = M(G, x, y) = nx^n y^n + \frac{n(n+1)}{2} x^{n+1} y^{n+1} + 2nx^n y^{n+1}. \]

Then

\[ D_x f(x, y) = 4nx^n y^n + (n+1) \frac{n(n+1)}{2} x^{n+1} y^{n+1} + 8nx^n y^{n+1}, \]
\[ D_y f(x, y) = 4nx^n y^n + \frac{n(n+1)^2}{2} x^{n+1} y^{n+1} + 2n(n+1)x^n y^{n+1}, \]
\[ D_x D_y f(x, y) = 16nx^n y^n + (n+1)^2 \frac{(n^2 + n)}{2} x^{n+1} y^{n+1} + (8n^2 + 8n)x^n y^{n+1}, \]
\[ S_x f(x, y) = \frac{n}{4} x^{n+1} y^{n+1} + \frac{n}{2(n+1)} x^n y^n + \frac{n}{2(n+1)} x^n y^n, \]
\[ S_y f(x, y) = \frac{n}{4} x^{n+1} y^{n+1} + \frac{n}{2(n+1)} x^n y^n + \frac{n}{2(n+1)} x^n y^n, \]
\[ D_x D_y f(x, y) = (2n^2 + 2n)(x^n y^n + 8n^2 x^n y^n), \]
\[ S_x f(x, y) = \frac{n}{8} x^8 + \frac{n}{4} x^{2n+2} + \frac{2n}{n+5} x^{n+5}, \]
\[ S_y f(x, y) = \frac{n}{8} x^8 + \frac{n}{4} x^{2n+2} + \frac{8n}{n+5} x^{n+5}, \]
\[ S_x D_y f(x, y) = \frac{n^6}{3} + \frac{n(n+1)}{4} x^{2n} + \frac{2n^2}{3} x^{n+3}, \]
\[ \text{1. First Zagreb index} \]
\[ M_1(G) = D_x + D_y(f(x, y)) \]
\[ = n^3 + 2n^2 + 13n. \]
\[ \text{2. Second Zagreb index} \]
\[ M_2(G) = D_x D_y(f(x, y)) \]
\[ = 4n^2 + (2n^2 - 2n)(n+1)^2 + 12n. \]
\[ \text{3. Modified Second Zagreb index} \]
\[ mM_2(G) = S_x S_y(f(x, y)) \]
\[ = \frac{n}{16(n+1)^2} + \frac{n}{(n+2)}. \]
\[ \text{4. Symmetric Division index} \]
\[ SSD(G) = (S_x D_y + S_y D_x)(f(x, y)) \]
\[ = \frac{5n^3 + 6n^2 + 17n}{2(n+1)}. \]
\[ \text{5. Harmonic index} \]
\[ H(G) = 2S_x J(f(x, y)) \]
\[ = \frac{n}{4} + \frac{(n^2 - n)}{(n+1)} + \frac{4n}{(n+5)}. \]
6. Inverse Sum index

\[ I(G) = S_xJD_xD_y \left( f(x, y) \right)_{x=1}^{n} = 2n + \frac{n^3 - n}{2} - \frac{8n^2 + n}{n + 5}. \]

7. Augmented Zagreb index

\[ A(G) = S_x^3Q_2JD_x^3D_y^3 \left( f(x, y) \right)_{y=x=1} = \frac{4^6}{6^3} + \frac{n(n+1)^2(n-1)}{2^4n^3} + \frac{2^5n(n+1)}{(n+3)^3}. \]

2.3 M-polynomial of the line graph of the Ladder Graph

A Ladder graph is a planar undirected graph with 2n vertices and 3n-2 edges. The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge. In this section, let G denote the line Graph of Ladder Graph. The line graph of ladder graph is given in Figure 3.

2.3.1 Theorem 3

Let G be the line graph of Ladder graph. Then we have

\[ M(G, x, y) = \begin{cases} 4x^3y^3 + 2x^3y^3 + 4x^3y^4, & \text{if } n = 2 \\ 4x^3y^3 + 8x^3y^4 + (6n-14)x^3y^4, & \text{if } n > 2. \end{cases} \]

2.3.2 Proof

Case 1 when n=2

We can divide the edge set of the line graph of ladder graph into following three classes depending on each edge at the end vertices of the degree:

\[ E_1(G) = \{ e = uv \in E(G); d_u = 2, d_v = 3 \}, \]
\[ E_2(G) = \{ e = uv \in E(G); d_u = 3, d_v = 3 \}, \]
\[ E_3(G) = \{ e = uv \in E(G); d_u = 3, d_v = 4 \}. \]

Now

\[ |E_1(G)| = 4, \]
\[ |E_2(G)| = 8, \]
\[ |E_3(G)| = 6n - 14. \]

Case 2 when n>2

We can divide the edge set of the line graph of ladder graph into following three classes depending on the degree of end vertices of each edge:

\[ E_1(G) = \{ e = uv \in E(G); d_u = 2, d_v = 3 \}, \]
\[ E_2(G) = \{ e = uv \in E(G); d_u = 3, d_v = 4 \}, \]

and

\[ E_3(G) = \{ e = uv \in E(G); d_u = d_v = 4 \}. \]

Now

\[ |E_1(G)| = 4, \]
\[ |E_2(G)| = 8, \]
\[ |E_3(G)| = 6n - 14. \]

From the definition of M-polynomial, we have

\[ M(G, x, y) = \sum_{d \leq s \leq \Delta} m_{s}x^{s}y^{d} = \sum_{uv \in E_1(G)} m_{2}x^{3}y^{3} + \sum_{uv \in E_2(G)} m_{3}x^{3}y^{4} + \sum_{uv \in E_3(G)} m_{4}x^{4}y^{4} = |E_1(G)|x^{3}y^{3} + |E_2(G)|x^{3}y^{4} + |E_3(G)|x^{4}y^{4} \]

\[ = 4x^3y^3 + 2x^3y^3 + 4x^3y^4. \]
2.3.3 Proposition 2

Let \( G \) be the line graph of Ladder graph. Then we have

1. \( M_1(G) = \begin{cases} 60 & \text{if } n = 2 \\ 52 + 5(6n - 14) & \text{if } n > 2. \end{cases} \)

2. \( M_2(G) = \begin{cases} 90 & \text{if } n = 2 \\ 96n - 128 & \text{if } n > 2. \end{cases} \)

3. \( m_2M_2(G) = \begin{cases} 11 & \text{if } n = 2 \\ 9n + 50 & \text{if } n > 2. \end{cases} \)

4. \( SSD(G) = \begin{cases} 21 & \text{if } n = 2 \\ 48 + 5(6n - 14) & \text{if } n > 2. \end{cases} \)

5. \( H(G) = \begin{cases} 318 & \text{if } n = 2 \\ 105 & \text{if } n > 2. \end{cases} \)

6. \( I(G) = \begin{cases} 513 & \text{if } n = 2 \\ 35 & \text{if } n > 2. \end{cases} \)

7. \( A(G) = \begin{cases} 880618 & \text{if } n = 2 \\ 8000 & \text{if } n > 2. \end{cases} \)

2.3.4 Proof.

Let

\[ f(x, y) = M(G, x, y) = \begin{cases} 4x^2y^3 + 2x^3y^3 + 4x^4y^4, & \text{if } n = 2 \\ 4x^2y^3 + 8x^3y^4 + (6n - 14)x^4y^4, & \text{if } n > 2. \end{cases} \]

Then

\[ D_x f(x, y) = \begin{cases} 8x^2y^3 + 6x^3y^3 + 12x^3y^4, & \text{if } n = 2 \\ 8x^2y^3 + 8x^3y^4 + (6n - 14)x^4y^4, & \text{if } n > 2. \end{cases} \]

1. First Zagreb index

\[ M_1(G) = D_x + D_y(f(x, y)) = \begin{cases} 60 & \text{if } n = 2 \\ 52 + 5(6n - 14), & \text{if } n > 2. \end{cases} \]

2. Second Zagreb index

\[ M_2(G) = D_x D_y(f(x, y)) = \begin{cases} 90 & \text{if } n = 2 \\ 96n - 128 & \text{if } n > 2. \end{cases} \]

3. Modified Second Zagreb index

\[ m_2M_2(G) = S_xS_y(f(x, y)) = \begin{cases} 11 & \text{if } n = 2 \\ 9n + 50 & \text{if } n > 2. \end{cases} \]

4. Symmetric Division index

\[ SSD(G) = S_x D_x + S_y D_y(f(x, y)) = \begin{cases} 21 & \text{if } n = 2 \\ 48 + 5(6n - 14) & \text{if } n > 2. \end{cases} \]

5. Harmonic index

\[ H(G) = 2S^2(f(x, y)) = \begin{cases} 318 & \text{if } n = 2 \\ 105 & \text{if } n > 2. \end{cases} \]

6. Inverse Sum index

\[ I(G) = S_x D_x + D_y(f(x, y)) = \begin{cases} 513 & \text{if } n = 2 \\ 35 & \text{if } n > 2. \end{cases} \]

7. Augmented Zagreb index

\[ A(G) = S_x^2 Q_2 D_x D_y^2 f(x, y) = \begin{cases} 880618 & \text{if } n = 2 \\ 8000 & \text{if } n > 2. \end{cases} \]
3 Conclusions

TIs are numbers associated with the molecular graphs of chemical structures that are useful in predicting properties of chemical compounds of interest [33-39]. TIs and QSARs together are used in chemistry, and they tell us about the topology of compounds under study. Calculating TIs of molecular graphs of chemical structures is an interesting problem and has attracted many researchers in recent years. In this paper, we computed M-polynomials for the line graph of some interesting families of graphs. We also computed different TIs from the computed M-polynomials by applying fundamental results of Calculus. We computed Zagreb indices, Randic indices, Symmetric division index, inverse sum index, etc. Our results are applicable in predicting properties of compounds. For example, the symmetric division index is a good predictor of the total surface area, Zagreb indices are used to calculate total pi-electronic energy, the inverse sum index is helpful in approximation of total surface area, augmented Zagreb index is a good predictor of the heat of formation, and the harmonic index is used for medication configuration.

Conflict of interest: Authors declare no conflict of interest.

References

[3] Gao W., Wang W. F., Dimitrov D., Wang Y. Q., Nano properties analysis via fourth multiplicative ABC indicator calculating, Arabian Journal of Chemistry, 2018, 11(6), 793-801.

[2] Naeem M., Siddiqui M. K., Guijao J.L.G., Gao W., New and Modified Eccentric Indices of Octagonal Grid Omn, Applied Mathematics and Nonlinear Sciences, 2018, 3(1), 209-228.

[1] Gutman I., Some properties of the Wiener polynomials, Graph Theory Notes New York, 1993, 125, 13–18.

[4] Deutsch E., Klavzar S., M-Polynomial, and degree-based topological indices, Iran. J. Math. Chem., 2015, 6, 93–102.

[5] Riaz M., Gao W., Baig A.Q., M-Polynomials and degree-based Topological Indices of Some Families of Convex Polytopes, Open j. math. sci., 2018, 2(1), 18–28.

[6] Munir M., Nazeer W., Shahzadi S., Kang S.M., Some invariants of circulant graphs, Symmetry, 2016, 8(11), 134, 10.3390/sym8110134.

[7] Wiener H., Structural determination of paraffin boiling points, J. Am. Chem. Soc., 1947, 69, 17–20.

[8] Dobrynin A.A., Entringer R., Gutman I., Wiener index of trees; theory and applications, Acta Appl. Math., 2001, 66, 211–249.

[9] Anjum M.S., Saifdar M.U., K Banhatti and K hyper-Banhatti indices of nanotubes, Eng. Appl. Sci. Lett., 2019, 2(1), 19 – 37.

[10] Randic M., On the characterization of molecular branching, J. Am. Chem. Soc., 1975, 97, 6609 – 6615.

[11] Bollobas B., Erdos P., Graphs of extremal weights, Ars. Combin., 1998, 50, 225–233.

[12] Amic D., Beslo D., Lucic B., Nikolic S., Trinajstic’ N., The Vertex-Connectivity Index Revisited, J. Chem. Inf. Comput. Sci., 1998, 38, 819–822.

[13] Hu Y., Li X., Shi Y., Xu T., Gutman I., On molecular graphs with smallest and greatest zeroth-Corder general randic index, MATCH Commun. MatAh. Comput. Chem., 2005, 54, 425–434.

[14] Caporossi G., Gutman I., Hansen P., Pavlovic L., Graphs with maximum connectivity index, Comput. Biol. Chem., 2003, 27, 85–90.

[15] Shao Z., Virk A.R., Javed M.S., Rehman M.A., Farahani M. R., Degree based graph invariants for the molecular graph of Bismuth Tri-iodide, J. Appl. Sci., 2019, 1, 01–11.

[16] Virk A. R., Jhangee R., Rehman M.A., Reverse Zagreb and Reverse Hyper-Zagreb Indices for Silicon Carbide SiC, Eng. Appl. Sci. Lett., 2018, 1(2), 37-50.Kier L. B.,

[17] De N., Computing Reformulated First Zagreb Index of Some Chemical Graphs as an Application of Generalized Hierarchical Product of Graphs, Open j. math. sci., 2018, 2(1), 338–350.

[18] Yan L., Farahani M.R., Gao W., Distance-based Indices Computation of Symmetry Molecular Structures, Open j. math. sci., 2018, 2(1), 323–337.

[19] Randić M., On History of the Randić Index and Emerging Hostility toward Chemical Graph Theory, MATCH Commun. Math. Comput. Chem., 2008, 59, 5-124.

[20] Randić M., The Connectivity Index 25 Years After, J. Mol. Graphics Modell. 2001, 20, 19–35.

[21] Imran M., Asghar A., Baig A.Q., On Graph Invariants of Oxide Network, Eng. Appl. Sci. Lett., 2018, 1(1), 23–28.

[22] Li X., Shi Y., A survey on the Randic index, MATCH Commun. Math. Comput. Chem., 2008, 59, 127–156.

[23] Gao W., Asghar A., Nazeer W., Computing Degree-Based Topological Indices of Jahangir Graph, Eng. Appl. Sci. Lett., 2018, 1(1), 16–22.

[24] Nikolić S., Kovačević G., Miličević A., Trinajstić N., The Zagreb indices 30 years after, Croat. Chem. Acta, 2003, 76, 113-124.

[25] Gutman I., Das K.C., The first Zagreb indices 30 years after, MATCH Commun. Math. Comput. Chem., 2004, 50, 83–92.

[26] Das K., Gutman I., Some Properties of the Second Zagreb Index, MATCH Commun. Math. Comput. Chem., 2004, 52, 103–112.

[27] Vuki´cevi´c D., Graovac A., Valence connectivities versus Randić, Zagreb and modified Zagreb index: A linear algorithm to check discriminative properties of indices in acyclic molecular graphs, Croat. Chem. Acta, 2004, 77, 501–508.

[28] Milicevic A., Nikolic S., Trinajstic N., On reformulated Zagreb indices, Mol. Divers., 2004, 8, 393–399.

[29] Gupta C.K., Lokesha V., Shwetha S.B., Ranjini P. S., On the Symmetric Division deg Index of Graph, Southeast Asian Bulletin of Mathematics, 2016, 40(1), 59-80.

[30] Favaron O., Mahéo M., Scˇl´aj J.F., Some eigenvalue properties in graphs (conjectures of Graffiti—II), Discrete Math.,1993, 111, 385–389.

[31] Balaban A. T., Highly discriminating distance based numerical descriptor, Chem. Phys. Lett., 1982, 89, 399–404.

[32] Furtula B., Graovac A., Vuki´cevi´c D., Augmented Zagreb index, MATCH Commun. Math. Chem., 2009, 60, 85–90.

[33] Ali A., Nazeer W., Munir M., et al., M-Polynomials And Topological Indices Of Zigzag And Rhombic Benzenoid Systems. Open Chemistry, 2018, 16(1), 73-78, doi:10.1515/chem-2018-0010.
[34] Kanabur R., Hosamani S.K., Some Numerical Invariants Associated with V-phenylenic Nanotube and Nanotori, Eng. Appl. Sci. Lett., 2018, 1(1), 1-9.

[35] Gao W., Asif M., Nazeer W., The study of honeycomb derived network via topological indices, Open j. math. anal., 2018, 2(2), 10–26.

[36] Noreen S., Mahmood A., Zagreb polynomials and redefined Zagreb indices for the line graph of carbon nanocones, Open j. math. anal., 2018, 2(1), 66–73.

[37] De N., Hyper Zagreb Index of Bridge and Chain Graphs, Open j. math. sci., 2018, 2(1), 1-17.

[38] Siddiqui H., Farahani M.R., Forgotten polynomial and forgotten index of certain interconnection networks, Open j. math. anal., 2017, 1(1), 44–59.

[39] Ur Rehman H.M., Sardar R., Raza A., Computing topological indices of Hex Board and its line graph, Open j. math. sci., 2017, 1(1), 62–71.

[40] Sardar M.S., Zafar S., Farahani M.R., The Generalized Zagreb Index of Capra-Designed Planar Benzenoid Series $Ca_n (C_6)$, Open j. math. sci., 2017, 1(1), 44–51.