On the tension between growth rate and CMB data

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We analyze the claimed tension between redshift space distortions measurements of $f(z)\sigma_8(z)$ and the predictions of standard $\Lambda$CDM (Planck 2015 and 2018) cosmology. We consider a dataset consisting of 19 data points extending up to redshift $z = 1.52$ and corrected for the Alcock-Paczynski effect. Thus, calculating the evolution of the growth factor in a $w$CDM cosmology, we find that the tension for the best fit parameters $w$, $\Omega_m$, and $\sigma_8$ with respect to the Planck 2018 $\Lambda$CDM parameters is below $2\sigma$ in all the marginalized confidence regions.

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I. INTRODUCTION

Large-scale galaxy surveys are becoming one of the most powerful tools to test the currently accepted $\Lambda$CDM model based on General Relativity. The possibility of mapping the distribution of matter in large volumes at different redshifts allows to measure the growth rate of structures as a function of time and (length) scale which is a well-defined prediction of any cosmological model.

The ability of such surveys to construct 3D maps depends crucially on the precise determination of galaxy redshifts from which radial distances to the survey objects can be inferred. The actual conversion depends, in turn, on two important effects. On one hand, peculiar velocities introduce distortions in the redshift distribution, the so called redshift space distortions (RSD), generating an anisotropic galaxy power spectrum. On the other, although at low redshift the Hubble law provides a straightforward relation between redshift and distances, at higher redshifts this conversion depends on the chosen fiducial cosmology. This fact lays behind the Alcock-Paczynski (AP) effect. In recent times these effects have allowed to measure the linear growth rate of structures, defined as $f = d\ln \delta_m/d\ln a$ with $\delta_m$ the linear matter density contrast with relatively good precision in a wide range of redshifts. More precisely, RSD provide a measurement of the quantity $f(z)\sigma_8(z)$, where $\sigma_8(z)$ is the normalization of the linear matter power spectrum at redshift $z$ on scales of $8h^{-1}$Mpc. In particular, measurements which can reach 10% precision have been obtained at $z < 1$ by different surveys such as 2dF [1], 6dFGS [2], WiggleZ [3] and recently by SDSS-III BOSS [4] and VIPERS [5]. At higher redshifts two measurements have also been obtained recently by FMOS [6] and from the BOSS quasar sample [7] although with relatively lower precision.

Confrontation of $f(z)\sigma_8(z)$ measurements with standard $\Lambda$CDM cosmology predictions has lead in recent years to claims of inconsistence or tension at different statistical significance levels. Thus in [8] a lower growth rate than expected from Planck $\Lambda$CDM cosmology was identified for the first time. Later on [9, 10] a tension at the $2\sigma$ level was claimed between the Planck data and the CFHTLenS determination of $\sigma_8$. A similar tension was found by the KiDS4-VIKING tomographic shear analysis [11]. More recently [12] a $3\sigma$ tension with respect to the best fit parameters of Planck 2015 was also identified in a set of 18 data points from RSD measurements of $f(z)\sigma_8(z)$. The tension could even increase up to $5\sigma$ if an extended dataset is used [13], although in this case possible correlations within data points have not been taken into account.

In this paper we revisit the analysis of the tension performed in [12] including the most recent measurements from BOSS-Q [7] and the best fit parameters of Planck 2018 (CMB alone) [14]. We consider the same type of $w$CDM cosmologies with three free parameters ($w, \Omega_m, \sigma_8$) but obtain the confidence regions from the marginalized (rather than maximized) likelihoods. This enlarges the confidence regions so that the tension is found to be reduced below the $2\sigma$ level for all the parameters combinations.

II. GROWTH OF STRUCTURES AND $f(z)\sigma_8(z)$

Let us consider a flat Robertson-Walker background whose metric in conformal time reads

$$ds^2 = a^2(\eta) \left[ -d\eta^2 + \delta_{ij}dx^i dx^j \right]$$

(1)

The evolution of matter density perturbations $\delta_m = \delta \rho_m/\rho_m$ in a general cosmological model with non-clustering dark energy and standard conservation of matter is given for sub-Hubble scales by

$$\delta''_m + H \delta'_m - \frac{3}{2} H^2 \Omega_m(a) \delta_m = 0$$

(2)

where prime denotes derivative with respect to conformal time, $H = a'/a$ and $\Omega_m(a) = \rho_m/(\rho_m + \rho_{DE})$. In this work we will limit ourselves to $w$CDM cosmologies so that at late times

$$H^2 = H_0^2 a^2 (\Omega_m a^{-3} + (1 - \Omega_m) a^{-3(1+w)})$$

(3)
and
\[ \Omega_m(a) = \frac{\Omega_m}{\Omega_m + (1 - \Omega_m) a^{-3\omega}} \] (4)

The growth rate is defined as
\[ f = \frac{d\ln \delta_m}{d\ln a} \] (5)

which can be approximated by \( f \approx \Omega_m^{\gamma}(a) \) with \( \gamma \approx 0.55 \) for \( \Lambda \)CDM models. Even though this fitting function provides accurate description for cosmologies close to \( \Lambda \)CDM, since we are interested in exploring a wide range of parameter space, in this work we will obtain \( f \) just by numerically solving (2) with initial conditions \( \delta_m(a_i) = 1 \) and \( \delta'(a_i) = 1/a_i \) with \( a_i \) well inside the matter dominated era.

The matter power spectrum corresponding to the matter density contrast in Fourier space \( \delta_k(z) \) with \( 1 + z = 1/a \) is given by \( P(k, z) = V|\delta_k(z)|^2 \) with \( V \) the volume. Thus the variance of the matter fluctuations on a scale \( R \) is given by
\[ \sigma_R^2(z) = \frac{1}{2\pi^2} \int P(k', z)W_R^2(k')^2 dk' \] (6)

with the window function defined as:
\[ W_R(k) = \frac{3}{k^3 R^3} \left[ \sin(kR) - kR \cos(kR) \right] \] (7)

Thus \( \sigma_R(z) \) corresponds to \( \sigma_R(z) \) at the scale \( R = 8h^{-1} \) Mpc.

From the matter power spectrum it is possible to define the galaxy power spectrum as \( P_g(k, z) = b^2(z)P(k, z) \) with \( b(z) \) the bias factor.

From the observational point of view, galaxy surveys are able to determine the galaxy power spectrum in redshift space, which is given by
\[ P_{r,\text{obs}}(k_r, \mu_r; z) = \frac{H(z)d_A^2(z)}{H_r(z)d_A^2(z)} D^2(z)b^2(z) \left[ 1 + \beta(z)\mu^2 \right] P(k, z = 0) \] (8)

where \( H(z) = (1 + z)H(z) \),
\[ d_A(z) = \frac{1}{1 + z} \int_0^1 \frac{da}{a^{1/2} H_0 \sqrt{\Omega_m + (1 - \Omega_m) a^{-3\omega}}} \] (9)

is the angular diameter distance, \( D(z) = \delta_m(z)/\delta_m(0) \) is the growth factor, \( \beta(z) = f(z)/b(z) \) and \( \mu \) is the cosine of the angle between \( k \) and the observation direction. Finally, the index \( r \) denotes that the corresponding quantity is evaluated on the fiducial cosmology. Notice that the first factor in (8) corresponds to the AP effect, whereas the \( (1 + \beta \mu^2)^2 \) factor is generated by the RSD. As we see RSD induce an angular dependence on the power spectrum which contains a monopole, quadrupole and hexadecapole contributions. From the measurements of monopole and quadrupole it is possible to obtain the \( f(z)\sigma_8(z) \) function that for simplicity in the following we will simple denote \( f\sigma_8(z) \). The measured value depends on the fiducial cosmology, so that in order to translate from the fiducial cosmology used by the survey to other cosmology it is needed to rescale by a factor [12]
\[ \text{ratio}(z) = \frac{H(z)d_A(z)}{H_r(z)d_A,r(z)} \] (10)

see however the discussion on the fiducial cosmology correction in [13].

## III. TESTING PLANCK COSMOLOGY

In order to confront the predictions of standard \( \Lambda \)CDM model with \( f\sigma_8(z) \) measurements, we will obtain theoretical predictions for a general \( \Lambda \)CDM model with three free parameters \( (\Omega_m, w, \sigma_8) \) with \( \sigma_8 = \sigma_8(z = 0) \). Our benchmark models will correspond to the Planck 2015 and Planck 2018 (TT,TE,EE+lowE) best fit parameters in Table I.

On the other hand, our data points will correspond to measurements of SDSS [16–18]; 6dFGS [19]; IRAS [20, 21]; 2MASS [20, 22]; 2dFGRS [23], GAMA [24], BOSS [25], WiggleZ [26], Vipers [5], FastSound [6] and BOSS Q [7]. In Table II we show the 19 independent data points with the corresponding fiducial cosmology parameters corresponding to the so called Gold-2017 compilation of [12] which contains 18 robust and independent measurements based on galaxy or SNIa observations together with an additional independent BOSS quasar point. On the data provided by these surveys we will apply the fiducial cosmology correction given by (10).

Apart form the errors quoted in Table II, the three points corresponding to WiggleZ are correlated. Thus the non-diagonal covariance matrix for the data points 13, 14, 15 is given by:

\[ C_{ij}^{13,14,15} = 10^{-3} \begin{pmatrix} 6.4000 & 2.570 & 0.000 \\ 2.570 & 3.969 & 2.540 \\ 0.000 & 2.540 & 5.184 \end{pmatrix} \] (11)

and the total covariance matrix would be

\[ C_{ij} = \begin{pmatrix} \sigma_1^2 & 0 & \ldots \\ 0 & C_{ij}^{13,14,15} & \ldots \\ 0 & \ldots & \sigma_N^2 \end{pmatrix} \] (12)

The corresponding \( \chi^2 \) is defined as
\[ \chi^2(\Omega_m, w, \sigma_8) = V^i C_{ij}^{-1} V^j \] (13)
with \( V^i = f\sigma_{8,i} - \text{ratio}(z_i) f\sigma_8(z_i; \Omega_m, w, \sigma_8) \). Here \( f\sigma_{8,i} \) corresponds to each of the data points in Table II and \( f\sigma_8(z_i; \Omega_m, w, \sigma_8) \) is the theoretical value for a given set of parameters values. In order to obtain the two-dimensional confidence regions for the different pairs of parameters, we will construct the marginalized likelihoods integrating the remaining parameter with a flat prior, i.e.

\[
L(w, \sigma_8) = N \int_{\Delta \Omega_m} e^{-\frac{1}{2} \chi^2(\Omega_m, w, \sigma_8)} d\Omega_m \quad (14)
\]

In particular for \( \Omega_m \in [0.05; 0.9] \), \( w \in [-2.5; 0.5] \) and \( \sigma_8 \in [0.1; 4.0] \). We have checked that the confidence regions remain practically unchanged if we enlarge these intervals. Notice that this is one of the main differences with respect to [12] in which the remaining parameter was fixed to the Planck cosmology value, thus artificially reducing the corresponding confidence regions.

### IV. RESULTS

In Fig. 1, the data points quoted in Table II together with the corresponding \( \Lambda \text{CDM} \) best fit curve are represented. The best fit corresponds to the parameters \( \Omega_m = 0.145 \), \( \sigma_8 = 1.18 \) and \( w = -0.46 \). For the sake of comparison we also show the \( f\sigma_8(z) \) curves corresponding to the Planck 2015 and Planck 2018 (in Table I) cosmologies. The \( \chi^2 \) values for the different models can be found in Table III.

![Figure 1](image)

We see that Planck 2018 provides a better fit than the Planck 2015 cosmology, mainly thanks to the reduction in the \( \sigma_8 \) parameter, but still both are well above the best fit to \( \Lambda \text{CDM} \).

In order to obtain the corresponding confidence regions we will compare two procedures. On one hand, we will follow the approach in [12] in which the likelihood is maximized, i.e. in the two-dimensional confidence regions the remaining parameter is fixed to the corresponding Planck value in Table I. In the second procedure, the remaining parameter is marginalized as mentioned in the previous section. In Figs. 2, 3 and 4 we show the different two-dimensional confidence contours. As we can see, Planck 2018 \( \Lambda \text{CDM} \) shows tensions of 2.18\( \sigma \), 1.87\( \sigma \) and 2.27\( \sigma \) in the maximized contours which is around 0.5\( \sigma \) below the tension found in [12] with Planck 2015 parameters, partially thanks to the reduced \( \sigma_8 \) value of Planck 2018 as mentioned before. On the other hand, the marginalized contours are as expected enlarged as compared to the maximized ones. Notice also that although the form of the \( (\sigma_8, w) \) and \( (\Omega_m, \sigma_8) \) contours are similar in both cases, the marginalization procedure changes the shape of the \( (\Omega_m, w) \) regions and the tensions with respect to Planck 2018 are further reduced to 1.85\( \sigma \) for \( (\Omega_m, w) \), 0.44\( \sigma \) for \( (\sigma_8, w) \) and 1.39\( \sigma \) for \( (\Omega_m, \sigma_8) \).

### V. CONCLUSIONS

We have revisited the tension of \( \Lambda \text{CDM} \) Planck cosmology with RSD growth data. We have considered the Gold data set of [12] together with one additional BOSS-

| Index | Dataset             | \( z \) | \( f\sigma_8(z) \) | \( \Omega_m \) |
|-------|---------------------|------|----------------|-------------|
| 1     | 6dFGS+SnIa         | 0.02 | 0.428 ± 0.0465 | 0.3         |
| 2     | SnIa+IRAS          | 0.02 | 0.398 ± 0.065  | 0.3         |
| 3     | 2MASS              | 0.02 | 0.314 ± 0.048  | 0.266       |
| 4     | SDSS–veloc         | 0.10 | 0.370 ± 0.130  | 0.3         |
| 5     | SDSS-MGS           | 0.15 | 0.490 ± 0.145  | 0.31        |
| 6     | 2dFGRS             | 0.17 | 0.510 ± 0.060  | 0.3         |
| 7     | GAMA               | 0.18 | 0.360 ± 0.090  | 0.27        |
| 8     | GAMA               | 0.38 | 0.440 ± 0.060  | 0.27        |
| 9     | SDSS–LRG–200       | 0.25 | 0.3512 ± 0.0583| 0.25        |
| 10    | SDSS–LRG–200       | 0.37 | 0.4602 ± 0.0378| 0.25        |
| 11    | BOSS–LOWZ          | 0.32 | 0.384 ± 0.095  | 0.274       |
| 12    | SDSS-CMASS         | 0.59 | 0.488 ± 0.060  | 0.307115    |
| 13    | WiggleZ            | 0.44 | 0.413 ± 0.080  | 0.27        |
| 14    | WiggleZ            | 0.60 | 0.390 ± 0.063  | 0.27        |
| 15    | WiggleZ            | 0.73 | 0.437 ± 0.072  | 0.27        |
| 16    | Vipers PDR–2       | 0.60 | 0.550 ± 0.120  | 0.3         |
| 17    | Vipers PDR–2       | 0.86 | 0.400 ± 0.110  | 0.3         |
| 18    | FastSound          | 1.40 | 0.482 ± 0.116  | 0.270       |
| 19    | BOSS–Q             | 1.52 | 0.426 ± 0.077  | 0.31        |

Table II. Data points from [7, 12].

| Model   | \( \chi^2 \) |
|---------|--------------|
| \( \Lambda \text{CDM} \) | 12.26 |
| Planck 2015 | 21.78 |
| Planck 2018 | 18.79 |

Table III. Values of \( \chi^2 \) for the models plotted in Fig. 1.
Figure 2. $w$ vs. $\Omega_m$ 1\,$\sigma$, 2\,$\sigma$ and 3\,$\sigma$ confidence regions. Left: maximized contours with $\sigma_8 = 0.812$. The Planck 2018 point lays at 2.18\,$\sigma$. Right: marginalized contours. The blue point corresponds to Planck 2015 and lays at 1.84\,$\sigma$; and the grey point corresponds to Planck 2018 and lays at 1.85\,$\sigma$.

Figure 3. $w$ vs. $\sigma_8$ 1\,$\sigma$, 2\,$\sigma$ and 3\,$\sigma$ confidence regions. Left: maximized contours with $\Omega_m = 0.3166$. The Planck 2018 point lays at 1.87\,$\sigma$. Right: marginalized contours. The blue point corresponds to Planck 2015 and lays at 0.38\,$\sigma$ and the grey point corresponds to Planck 2018 and lays at 0.44\,$\sigma$.

Q point, thus obtaining a total of 19 independent data points.

Confronting these data with the growth rate obtained from a $w$CDM cosmology with three independent parameters $(w, \Omega_m, \sigma_8)$, we find that unlike previous claims, the tension with Planck 2018 cosmology is below the 2\,$\sigma$ level in all the two-dimensional marginalized confidence regions. This reduction, which is around 1\,$\sigma$ as compared to [12], is first due the use of Planck 2018 parameters and, on the other, to the fact that marginalized confidence regions have been considered.

Future galaxy surveys such as J-PAS [27], DESI [28] or Euclid [29] with increased effective volumes will be able to reduce the error bars in the determination of $f\sigma_8(z)$.
Figure 4. $\sigma_8$ vs. $\Omega_m$. $1\sigma$, $2\sigma$ and $3\sigma$ confidence regions. Left: maximized contours with $w = -1$. The Planck 2018 point lays at $2.27\sigma$. Right: marginalized contours. The blue point corresponds to Planck 2015 and lays at $1.59\sigma$. The grey point corresponds to Planck 2018 and lays at $1.39\sigma$.

in almost an order of magnitude and will help to confirm or exclude the tension analyzed in this work.

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