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Practical method for analyzing singular index and intensity of singular stress field for three dimensional bonded plate

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Abstract. In this paper, the practical analysis methods are proposed for analyzing the singular index and the intensity of singular stress field (ISSF) at the vertex of the interface in the three dimensional (3D) bonded plate. The analysis methods focus FEM stresses at and around the vertex. The singular index is determined from the FEM stress ratio at the vertex obtained by performing FEM analyses on the finely and coarsely meshed models. Then, the ISSF is determined by the ratio of the average FEM stresses at and around the vertex obtained by performing the FEM analyses on the reference and unknown models under the same mesh pattern. The validity of the present methods was examined by the plane strain bonded plate and 3D bonded plate in the literature. It was found that the present methods for the singular index have the same accuracy as the FEM eigenvalue analysis. The asymptotic solutions with the singular index and ISSF by the present method correspond to FEM stress distributions. Since the ISSF by the body force method (BFM) is used as the reference solution, the present method for ISSF has the same accuracy as BFM. Moreover, the critical ISSF values were calculated in the experimental results of the butt joints with various adhesive thicknesses. In the case of the ductile epoxy adhesive, it was shown that the critical ISSF at the vertex by 3D model was more constant against the thickness than that by 2D model. The result is quite different from that of the brittle epoxy adhesive and can be never obtained by 2D model.

1. Introduction
The intensity of singular stress field (ISSF) is useful for evaluating the debonding strength [1–4]. Generally, the finite element method (FEM) is often used to evaluate the strength in various industries [5,6]. However, the ISSF cannot be calculated directly by FEM [7–10]. The authors proposed the method for calculating the ISSF easily and accurately by the FEM [3,4,11,12]. The method does not require the complex calculation and can be applied to various bonded structures. In the previous studies, the ISSF for the butt joint was analyzed by 2D model [3,4]. It was found that the debonding fracture criterion can be expressed with the ISSF [11,13]. Actual butt joint has the vertex and its debonding fracture is caused from the vertex of the interface. However, it is difficult to analyze the ISSF at the vertex and the debonding fracture...
Solving Eq. (2) on the vertex, the following equation is obtained.

$$\lambda_{vtx} = 1 - \frac{\ln \left( \frac{\sigma_{vtx(a)}^{0,FEM} \mid_{\epsilon_{min}=\epsilon_{0}^{vtx}}}{\sigma_{vtx(b)}^{0,FEM} \mid_{\epsilon_{min}=n \epsilon_{0}^{vtx}}} \right)}{\ln n}$$  \hspace{1cm} (3)
\[ \sigma_z^e = \sigma_0 \]

(a) 3D bonded plate model.

\[ \sigma_z^e = \sigma_0 \]

(b) 2D bonded plate model.

Figure 2. Schematic illustration of 2D and 3D bonded plate models.

Generally, the finite element eigen analysis method is often used for the analysis of \( \lambda_{vtx} \). \( \lambda_{vtx} \) can be determined by Eq. (3) much more easily than the finite element eigenanalysis method.

When the FEM stresses \( \sigma_{2D}^{vtx} \mid_{p_{min}=e_0} \) and \( \sigma_{2D}^{vtx} \mid_{p_{min}=n} \) are used instead of the \( \sigma_{2D}^{vtx} \mid_{p_{min}=e_0} \) and \( \sigma_{2D}^{vtx} \mid_{p_{min}=n} \), respectively, the \( \lambda_{side} \) is determined by the following equation.

\[
\lambda_{side} = 1 - \frac{\ln \left( \sigma_{2D}^{vtx} \mid_{p_{min}=e_0} / \sigma_{2D}^{vtx} \mid_{p_{min}=n} \right)}{\ln n}
\]

(4)

Here, \( \sigma_{2D}^{vtx} \mid_{p_{min}=e_0} \) and \( \sigma_{2D}^{vtx} \mid_{p_{min}=n} \) are the FEM stresses at the center \( (0, -W/2, 0) \) on the interface end \( y = -W/2 \) in the finely and coarsely meshed 3D bonded plate models, respectively.
2. Mesh-independent technique useful for 3D bonded plate
The authors proposed the mesh-independent technique useful for 2D butt joint [3,4]. The method is extended so that the ISSF at the vertex in 3D bonded plate can be analyzed. Figure 2 shows (a) unknown 3D bonded plate model and (b) reference 2D bonded plate model. The method for analyzing the ISSF $K_{\sigma}^{\text{vtx}}$ at $(x, y, z) = (-W/2, -W/2, 0)$ in Fig. 2(b) by using the ISSF $K_{\sigma}^{2D}$ at $(x, y) = (-W/2, 0)$ as the reference solution is mentioned below.

As shown in Fig. 2(a), the local polar coordinate $(r, \phi)$ is set at the vertex $(x, y, z) = (-W/2, -W/2, 0)$ in the 3D model. $\phi$ is the angle between the interface end $y = -W/2$ and $r$ axis. When $\phi = \pi/4$, the singular stress field at $(-W/2, -W/2, 0)$ is expressed with the following equation [9].

$$\sigma_z^{\text{vtx}}(r, \phi = \pi/4) = \frac{K_{\sigma}^{\text{vtx}}}{r^{1-\lambda_{\text{vtx}}}} \left|_{\phi=\pi/4} \right.$$  \hspace{1cm} (5)

As shown in Fig. 2(b), the local polar coordinate $(r, \theta)$ is set at the interface end $(x, y) = (-W/2, 0)$ in the 2D model. $\theta$ is the angle between the interface end $y = 0$ and $r$ axis. When $\theta = \pi/2$, the singular stress field at $(-W/2, -W/2, 0)$ is expressed with the following equation [3,4].

$$\sigma_y^{2D}(r, \theta = \pi/2) = \frac{K_{\sigma}^{2D}}{r^{1-\lambda_{\text{side}}}} \left|_{\phi=\pi/2} \right.$$  \hspace{1cm} (6)

The analysis methods focus average real/FEM stresses at and around the vertex. The FEM analyses are performed on the reference and unknown models under the similar mesh pattern. Since the mesh size dependency in the FEM stress is canceled by using a ratio, the average FEM stress ratio $\sigma_z^{\text{vtx, FEM}}/\sigma_y^{2D, FEM}$ corresponds to the average real stress ratio $\sigma_z^{\text{vtx}}/\sigma_y^{2D}$. When the stress in 3D model is averaged from $r = 0$ to $\sqrt{2}e_0^{\text{vtx}}$ and the stress in 2D model is averaged from $r = 0$ to $e_0^{2D}$, the following equation is obtained.

$$\frac{K_{\sigma}^{\text{vtx}}}{K_{\sigma}^{2D}} \left|_{\phi=\pi/4} \right. = \frac{\lambda_{\text{vtx}}}{\lambda_{2D}} \cdot \frac{\sigma_z^{\text{vtx, FEM}} \cdot (\sqrt{2}e_0^{\text{vtx}})^{1-\lambda_{\text{vtx}}} + \sigma_z^{\text{vtx, FEM}} \cdot (\sqrt{2}e_0^{\text{vtx}})^{1-\lambda_{\text{vtx}}}}{\sigma_y^{2D, FEM} \cdot (e_0^{2D})^{1-\lambda_{2D}} + \sigma_y^{2D, FEM} \cdot (e_0^{2D})^{1-\lambda_{2D}})} \left|_{\phi=\pi/4} \right. \hspace{1cm} (7)

Here, $\sigma_z^{\text{vtx, FEM}}$ is the FEM stress at $(r, \phi) = (\sqrt{2}e_0^{\text{vtx}}, \pi/4)$ in Fig. 2(a), $\sigma_y^{2D, FEM}$ and $\sigma_y^{2D, FEM}$ are the FEM stresses at $(r, \theta) = (0, \pi/2)$, $(e_0^{2D}, \pi/2)$ in Fig. 2(b), respectively.

The local polar coordinate $(r, \phi)$ is set at the center $(0, -W/2, 0)$ in Fig. 2(b). $\phi$ is the angle between the interface end $y = -W/2$ and $r$ axis. When $\phi = \pi/2$, the singular stress field at the center of interface end is expressed with the following equation [9].

$$\sigma_z^{\text{side}}(r, \phi = \pi/2) = \frac{K_{\sigma}^{\text{side}}}{r^{1-\lambda_{\text{side}}}} \left|_{\phi=\pi/2} \right.$$  \hspace{1cm} (8)

The following equation is obtained as well as Eq. (7).

$$\frac{K_{\sigma}^{\text{side}}}{K_{\sigma}^{2D}} \left|_{\phi=\pi/2} \right. = \frac{\lambda_{\text{side}}}{\lambda_{2D}} \cdot \frac{\sigma_z^{\text{side, FEM}} \cdot (e_0^{\text{side}})^{1-\lambda_{\text{side}}} + \sigma_z^{\text{side, FEM}} \cdot (e_0^{\text{side}})^{1-\lambda_{\text{side}}}}{\sigma_y^{2D, FEM} \cdot (e_0^{2D})^{1-\lambda_{2D}} + \sigma_y^{2D, FEM} \cdot (e_0^{2D})^{1-\lambda_{2D}})} \left|_{\phi=\pi/2} \right. \hspace{1cm} (9)

Because the ISSF for 2D bonded plate by the body force method (BFM) [8,10] is used, the ISSF for 3D bonded plate can be obtained by Eqs. (7) and (9) accurately.
Fracture stress $\sigma_c$ [MPa]
Adhesive thickness $h$ [mm]

10 $\times 10^{-2}$
10 $\times 10^{-1}$
10 $0$
10 $1$
0
20
40
60
80
100
120

3. Debonding strength evaluation for butt joint by 3D model
The debonding strength for butt joint by 3D model is discussed by using the experimental result by Suzuki [14]. Figure 3 shows the schematic illustration of the butt joint specimen and the fracture stress. In the experiment, the adhesive thickness $h$ is varied from 0.05 mm to 5 mm. The adherend is the low carbon steel JIS S35C. The adheres are the epoxy resin. The fracture stress decreases with increasing $h$. Table 1 shows the material constants of the adherend and adheres. Then, $\lambda_{2D}$ and the dimensionless ISSF $F_{2D}^{D\sigma}$ for 2D bonded plate, which are used in the evaluation of the ISSF as the reference solution, are also shown in Table 1.

Analysis results in the case of $h = 5$ mm are mentioned as example below. Table 2 shows $\lambda_{ext}$ by Eq. (3) and $\lambda_{side}$ by Eq. (4). $\lambda_{side}$ nearly equals $\lambda_{2D}$. Table 3 shows the FEM stresses of unknown 3D butt joint model and reference 2D bonded plate model. All FEM stresses depend on the element size. However, it is found that the mesh size dependency can be overcome by multiplying the FEM stress by $(e_{\text{min}})^{1-\lambda}$. Therefore, the left-hand sides of Eqs. (7) and (9) have mesh size independency. Table 4 shows the ISSFs at the vertex by Eq. (7) and at the center by Eq. (9). It is found that $K_{\text{vtx}}^{\sigma}$ and $K_{\text{side}}^{\sigma}$ are independent of the element size. Figure 4 shows the singular stress distributions at the vertex and the center in the 3D butt joint model. The solid lines are the asymptotic solutions which are obtained by substituting the singular indexes in Table 2 and the ISSFs in Table 4 into Eqs. (5) and (8). The lines are good agreement with the FEM stress distributions. The differences between the asymptotic solution and FEM stress are less than about 1% in $4.4 \times 10^{-6} < r/(W/2) < 7.89 \times 10^{-4}$ at the vertex and the center except for the nodes near the singular point which is strongly influenced by the singularity. It can be confirmed that the singular indexes and the ISSFs are determined accurately by the present method.

Figure 5 shows the critical ISSFs at the vertex and the center by 3D model and the critical
Table 2. Singular indexes $\lambda_{vtx}$ by Eq. (3) and $\lambda_{side}$ by Eq. (4)

(a) $\lambda_{vtx}$

|                  | Fine mesh model | Coarse mesh model | $\lambda_{vtx}$ |
|------------------|-----------------|-------------------|-----------------|
| $e_{vtx}^{0}$    | $\sigma_{vtx,z0,FEM}^{0}$ | $e_{vtx}^{0}$ | $\sigma_{vtx,z0,FEM}^{0}$ |
| $4.360 \times 10^{-6}$ | 89.49 | $1.744 \times 10^{-5}$ | 51.12 | 0.5961 |

(b) $\lambda_{side}$

|                  | Fine mesh model | Coarse mesh model | $\lambda_{side}$ | $\lambda_{2D}$ |
|------------------|-----------------|-------------------|-----------------|----------------|
| $e_{side}^{0}$   | $\sigma_{side,z0,FEM}^{0}$ | $e_{side}^{0}$ | $\sigma_{side,z0,side}^{0}$ | $\sigma_{side,z11,FEM}^{0}$ | $\sigma_{side,z11,FEM}^{0} \cdot (e_{0,vtx}^{0})^{1-\lambda_{vtx}}$ |
| $4.360 \times 10^{-6}$ | 36.02 | $1.744 \times 10^{-5}$ | 22.93 | 0.6741 | 0.6735 |
| $1.744 \times 10^{-5}$ | 51.12 | 0.6118 | 37.09 | 0.6118 |

Table 3. Mesh-dependent FEM stresses of 3D butt joint model with $h = 5$ mm in Fig. 3 and reference 2D model under plane strain in Fig. 2(b)

(a) FEM stress at the vertex of unknown 3D butt joint model in Fig. 3

|                  | $e_{vtx}^{0}$ | $\sigma_{vtx,z0,FEM}^{0}$ | $\sigma_{vtx,z0,FEM}^{0} \cdot (e_{0,vtx}^{0})^{1-\lambda_{vtx}}$ | $\sigma_{vtx,z11,FEM}^{0}$ | $\sigma_{vtx,z11,FEM}^{0} \cdot (e_{0,vtx}^{0})^{1-\lambda_{vtx}}$ |
|------------------|----------------|-----------------|-----------------|-----------------|-----------------|
| $4.360 \times 10^{-6}$ | 89.49 | 0.6118 | 64.92 | 0.4438 |
| $1.744 \times 10^{-5}$ | 51.12 | 0.6118 | 37.09 | 0.6118 |

(b) FEM stress at the center of unknown 3D butt joint model in Fig. 3

|                  | $e_{side}^{0}$ | $\sigma_{side,z0,FEM}^{0}$ | $\sigma_{side,z0,FEM}^{0} \cdot (e_{0,side}^{0})^{1-\lambda_{side}}$ | $\sigma_{side,z0,side}^{0}$ | $\sigma_{side,z11,FEM}^{0}$ | $\sigma_{side,z11,FEM}^{0} \cdot (e_{0,side}^{0})^{1-\lambda_{side}}$ |
|------------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $4.360 \times 10^{-6}$ | 36.02 | 0.6453 | 27.87 | 0.4993 |
| $1.744 \times 10^{-5}$ | 22.93 | 0.6453 | 17.74 | 0.4993 |

(c) FEM stress at the interface end of reference 2D bonded plate model in Fig. 2(b)

|                  | $\sigma_{y0,FEM}^{2Ds}$ | $\sigma_{y0,FEM}^{2Ds} \cdot (e_{0}^{2Ds})^{1-\lambda_{2D}}$ | $\sigma_{y1,FEM}^{2Ds}$ | $\sigma_{y1,FEM}^{2Ds} \cdot (e_{0}^{2Ds})^{1-\lambda_{2D}}$ |
|------------------|-----------------|-----------------|-----------------|-----------------|
| $3^{-12}$ | 57.88 | 0.4399 | 44.79 | 0.3404 |
| $3^{-9}$ | 19.73 | 0.4399 | 15.27 | 0.3404 |

Table 4. ISSFs for 3D butt joint with $h = 5$ mm

(a) $K_{\sigma}^{vtx}$

|                  | $e_{0}^{2Ds}$ | $e_{vtx}^{0}$ | $K_{\sigma}^{vtx}$ |
|------------------|----------------|----------------|-----------------|
| $3^{-12}$ | $4.360 \times 10^{-6}$ | 0.392 |
| $3^{-12}$ | $1.744 \times 10^{-5}$ | 0.392 |
| $3^{-9}$ | $4.360 \times 10^{-6}$ | 0.392 |
| $3^{-9}$ | $1.744 \times 10^{-5}$ | 0.392 |

(b) $K_{\sigma}^{side}$

|                  | $e_{0}^{2Ds}$ | $e_{side}^{0}$ | $K_{\sigma}^{side}$ |
|------------------|----------------|----------------|-----------------|
| $3^{-12}$ | $4.360 \times 10^{-6}$ | 0.392 |
| $3^{-12}$ | $1.744 \times 10^{-5}$ | 0.392 |
| $3^{-9}$ | $4.360 \times 10^{-6}$ | 0.392 |
| $3^{-9}$ | $1.744 \times 10^{-5}$ | 0.392 |

ISSFs by 2D model [13]. $K_{\sigma}^{side}$ almost equals $K_{\sigma}^{2D}$. The $K_{\sigma}^{vtx}$ is more constant against the thickness than that by 2D model and has the smallest scatter. The result can be never obtained.
Figure 4. Singular stress distributions at the vertex and the center of butt joint with $h = 5$ mm

Figure 5. Relation between critical ISSF $K_{\sigma_c}$ and adhesive layer thickness $h$ by 2D model.

4. Conclusion
In this paper, the practical analysis methods were proposed for analyzing the singular index and the intensity of singular stress field (ISSF) at the vertex of the interface in 3D bonded plate. The debonding strength for butt joint by 3D model is discussed by using the experimental result in the earlier study.

(i) The critical ISSFs of the butt joints with various adhesive thicknesses in the earlier study were calculated by 3D and 2D models. The critical ISSFs at the center on the side in 3D model almost equaled those by 2D model. The critical ISSFs at the vertex in 3D model was less scattered than those at the center on the side in 3D model.

(ii) The asymptotic solutions with the singular indexes and the ISSFs obtained by the present method were good agreement with the FEM stress distributions. The differences between the asymptotic solution and FEM stress were less than about 1% in $4.4 \times 10^{-6} < r/(W/2) < 10^{-3}$. Averaged stress intensity factors

$K_{\sigma_c}^{\text{ave}}$ (Vertex) $= 1.227$ MPam$^{0.328}$ $\pm 10\%$

$K_{\sigma_c}^{\text{ave}}$ (Side) $= 1.204$ MPam$^{0.328}$ $\pm 10\%$

$K_{\sigma_c}^{2D}$ (Vertex) $= 0.595$ MPam$^{0.404}$ $\pm 10\%$

$K_{\sigma_c}^{2D}$ (Side) $= 1.227$ MPam$^{0.328}$ $\pm 10\%$
7.89 \times 10^{-4} \text{ at the vertex and the center except for the nodes near the singular point which is strongly influenced by the singularity. It was confirmed that the singular indexes and the ISSFs are determined accurately by the present method.}

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