Determination of rolling resistance coefficient based on normal tyre stiffness

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Abstract. The purpose of the article is to develop analytical dependence of wheel rolling resistance coefficient based on the mathematical description of normal tyre stiffness. The article uses the methods of non-holonomic mechanics and plane section methods. The article shows that the abscissa of gravity center of tyre stiffness expansion by the length of the contact area is the shift of normal road response. It can be used for determining rolling resistance coefficient. When determining rolling resistance coefficient using ellipsis and power function equations, one can reduce labor costs for testing and increase assessment accuracy.

1. Introduction

One of the important output characteristics of the pneumatic tyre is rolling resistance coefficient. It determines properties of the tyre and fuel efficiency of the car.

According to some studies, tyre rolling resistance coefficient for a wheel travelling along the non-deformed surface involves two components. The first component describes power losses due to hysteretic friction under the radial tyre deformation. The second one describes kinematic losses due to wheel skidding about the support.

Despite the fact that the causes of rolling losses have been already studied, there are still no accurate formulas for determining resistance coefficient. For this reason, theoreticians use values of the coefficient calculated in a driven mode of wheel rolling, i.e. the power component of the coefficient.

There are several experimental methods for determining rolling resistance coefficient. Run-on tests, fuel efficiency tests, dynamometric truck-based tests, drum bench-based tests are some of them. The widely-used method for determining rolling resistance coefficient is measuring steel drum torque during tyre rolling, or the longitudinal force acting on the axis of the rolling wheel.

However, all these methods have some drawbacks due to the need for measuring weak rolling resistance forces under large loads on the wheel or difficulties in setting apart wheel rolling losses and total losses, or inconsistency between tyre loading conditions and real tyre working conditions which cause significant test result errors.
2. Methods for rolling resistance coefficient calculation
Rolling resistance coefficient is calculated using the elliptic power model of absorption property of the
tyre [3, 5]. That model as distinct from the viscous friction model is based on the mathematical
description of experimental normal stiffness rather than on the hypothesis for physical nature of forces
of non-elastic resistance. Stiffness is presented in coordinates “force – deformation” of the functional
relationship of normal loading on the wheel caused by normal tyre deflection when changing loading
for both wheel loading and unloading. The closed loop area is energy which is lost in tyre deformation
under non-elastic resistance forces (hysteresis losses), and the center line slope is elastic properties of
the tyre.

Changes in tyre stiffness obtained in a dynamic wheel loading mode (Fig. 1 [4]) make it possible to
approximate them by equation:

\[ F = \pm F_a \sqrt{1 - \left(\frac{h_z}{h_{za}}\right)^2}, \quad (1) \]

where \( F, F_a \) are current and amplitude force values of non-elastic tyre resistance; \( h_z, h_{za} \) are
current and amplitude values of normal tyre deflection.

The experiment confirmed [2] the functional relation of amplitude values of the force of non-
elastic resistance and amplitude values of radial tyre deformation (Fig. 2) which can be presented as:

\[ F_a = H_h h_{za}^n \quad (2) \]

where proportionality factor \( H_h \) and degree factor \( n \) are parameters of the model. They present
intensity of absorption properties of the tyre.

![Figure 1](image-url)

**Figure 1.** Experimental properties of radial tyre flexibility in dynamic loading modes without (a) or
with (b) wheel rotation: a - 245/70HR16 I-241 (\( P_{zm} = 9.00 \text{ kN}, \ p = 18 \text{ rad/sec} \)), b -
LR70-15 GL (\( P_{zm} = 9.00 \text{ kN}, \ p = 18 \text{ rad/sec} \), \( \omega_k = 172 \text{ min}^{-1} \) for a left curve).
Figure 2. Processing results for normal tyre stiffness characteristics under different loading values: 1 is tyre 8.40-15 Ya-245, $P_{zn} = 5.95$ kN, $p_{om} = 0.20$ MPa; 2 is tyre 7.00-15 I-89, $P_{zn} = 6.15$ kN, $p_{om} = 0.22$ MPa, where $sign \, \dot{h}_z$ is the function “sign $\dot{h}_z$”.

Parameters of the elliptic power model (3) are constants for the tyre, i.e. they do not depend on internal air pressure, static loading, tyre temperature, vertical loading update rate, wheel rolling velocity, additional torque and lateral force loading. Structural tyre properties (ply, cord material, protector wear degree) influence only proportionality factor $H_s$.

The method for assessing wheel rolling resistance coefficient involves approximation of real shear of normal wheel loading for a specific cross-section by branches of a hysteresis loop with parameters of the elliptic power model of non-elastic tyre resistance and determination of the center of gravity of the curvilinear figure [4].

Let us consider the scheme of interaction of the elastic wheel which rolls without skidding and smooth non-deformed road surface (Fig. 3). Let us assume that the real wheel is flat. As a result, acting forces and torques have adjusted values; radial tyre deformation area is a contact area (for the flat wheel with $l_k$).

Figure 3. Interaction of the elastic wheel and road surface and shear of normal load $P_z$ varied when travelling across the cross-section in the contact area (shaded figure): 1 - tyre cross-section compression line, 2 - extension line.

Changes in real shear (lines 1 and 2) of tyre cross-section loading relative to the ideal one (dashed lines) typical for an absolute elastic body are due to hysteresis losses in a deformed tyre. So, resultant
$R_z$ of normal responses of the support road surface will be shifted relative to the line of normal loading $P_z$ and move through the center of gravity of the curvilinear triangle (a shaded figure). The abscissa of center of gravity of the shaded figure will equal to the shift of the normal response and impact a tyre rolling resistance coefficient.

Let us determine the abscissa of center of gravity of the curvilinear figure (real shear $P_z$). Taking into consideration geometry of plane sections, one can obtain the expression for determining the abscissa of center of gravity of a sloping semi-ellipsis:

$$x_{cent} = \frac{4}{3\pi} F_a \cos \gamma,$$  \hspace{1cm} (3)

where $F_a$ is the amplitude value for the force of non-elastic tyre resistance (one half of the minor diameter); $\gamma$ is the angle of tyre deformation area.

As far as the loading shear consists of isosceles triangle $ABC$ and two ellipses, one of which can be added to the triangle and another one can be excluded, one can develop a formula for determining the center of gravity of the shear and normal road response shift based on expression (2).

$$x_{cent} = \frac{\partial_z}{\gamma} = \frac{4}{3\pi} H z_a^n \cos \gamma \sin \gamma.$$  \hspace{1cm} (4)

As $\cos \gamma = \frac{1}{\sqrt{1 + \tan^2 \gamma}}$ and $\sin \gamma = \frac{\tan^2 \gamma}{1 + \tan^2 \gamma}$, then $a_z = \frac{4}{3\pi} H z_a^n \frac{\tan \gamma}{1 + \tan^2 \gamma}$.

Taking into account that $\tan \gamma = \frac{P_z}{l_x/2}$, and for dimensions $P_z$ [H], $l_x$ [mm] $\tan^2 \gamma \gg 1$, the last factor can be transformed as $\tan \gamma = \frac{1}{\tan \gamma} = \frac{1}{\tan \gamma} = \frac{l_x}{2D_z}$.

Hence

$$a_z = \frac{2l_x H z_a^n}{3\pi D_z}.$$  \hspace{1cm} (5)

One can also take into account the known relations: $h_{z_{a}} = h_{z_{max}}/2$, $l_x = k_i h_{z_{max}}$.

Then $a_z = \frac{2^{1-n} k_i H z_{i} h_{z_{max}}^{1+n}}{3\pi P_z}$.  \hspace{1cm} (6)

As tyre rolling resistance coefficient $f_c$ is related to the shift of normal road response by:

$$f_c = \frac{a_z}{r_d},$$

where $r_d$ is the dynamic wheel radius, one can develop a formula for determining $f_c$ based on $H_z$, $n$ of the elliptic power model of absorption of the tyre:

$$f_c = \frac{2^{1-n} k_i H z_{i} h_{z_{max}}^{1+n}}{3\pi P_z},$$  \hspace{1cm} (7)

where $k_i$ is the proportionality factor between the length of contact area and normal tyre deflection; $h_{z_{max}}$ is the full radial tyre deflection.

It should be emphasized that all values (except for $h_{z_{max}}$, $r_d$ and $P_z$) in formulas (6) and (7) are determined for specific performance conditions of the tyre and wheel loading in a pilot.
experiment. In case of changes of conditions, they remain constant. $P_c$ is defined as basic, and $h_{z\text{max}}$ and $r_a$ can be taken from support materials.

To test the relations, let us calculate values $a_s$ and $f_c$ for two tyres with parameters specified in the table below.

Table 1.

| Tyre size and model, road type | Tyre parameters |
|-------------------------------|-----------------|
|                               | $P_z$, H | $h_{z\text{max}}$, mm | $k_l$ | $H_s$, H/mm$^n$ | n | $r_{ds}$, mm |
| 1. 7.35 ID-195 dry asphalt     | 4350    | 23.5                   | 4.1   | 301.2           | 0.48 | 310         |
|                               |         | Wheel rolling velocity $V_k \approx 30$ km/h |
| 2. LR78-15 SUPER ARAMID RADIAL dry asphalt | 9000    | 38.1                   | 5.0   | 271.0           | 0.43 | 370         |
|                               |         | The speed of the wheel rolling $V_k \approx 30$ km/h |

Calculation results:
Tyre 1: $a_s$=4.67 mm; $f_c$=0.015; Tyre 2 $a_s$=4.31 mm; $f_c$=0.012.

3. Conclusion
The results obtained are consistent with data on tyre testing (in particular, in the Bulletin Inform-Prostor), so the method can be used for calculating rolling resistance coefficient for elastic tyres as well as in research on automobile dynamics.

References
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