Hidden symmetry enforced nexus points of nodal lines in layer-stacked dielectric photonic crystals

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1It was demonstrated recently that the connectivities of bands emerging from zero frequency in dielectric photonic crystals are distinct from their electronic counterparts with the same space groups. We discover that, in an AB-layer-stacked photonic crystal made up of anisotropic dielectrics, the unique photonic band connectivity leads to a new kind of symmetry enforced triply degenerate points as the nexuses of two nodal rings and a Kramers-like nodal line. The emergence and intersection of the line nodes are guaranteed by a generalized 1/4-period screw rotation symmetry of Maxwell’s equations. The bands with a constant \( k_z \) and the iso-frequency surfaces near the nexus point both disperse as a spin-1 Dirac-like cone, giving rise to exotic transport features of light at the nexus point. We show that the spin-1 conical diffraction occurs at the nexus point which can be used to manipulate the charges of optical vortices. Our work reveals that Maxwell’s equations can have hidden symmetries induced by the fractional periodicity of the material tensor components and hence paves the way to find novel topological nodal structures unique to photonic systems.

Introduction

Discovering and synthesizing symmetry-protected topological (SPT) band degeneracies, including nodal points and nodal lines (NLs)\(^{14–25} \), is a rapidly growing frontier in the field of topological materials. The initial impetus of the area came from realizing elusive relativistic quasi-particles, e.g., 3-dimensional (3D) Weyl and Dirac fermions, in both electronic crystalline materials\(^{1–4} \) and artificial photonic crystals (PhCs)\(^{5–13} \). Interestingly, since the crystallographic space groups impose less constraints on the energy bands than the continuous Poincaré group, more exotic multifold band crossings were found in lattice systems\(^{36} \), which have no counterparts in high-energy physics. As a remarkable example, certain space groups allow the existence of triply degenerate points in the band structures, forming either as isolated point nodes carrying monopole charges, so-called spin-1 Weyl points\(^{26–30} \), or as the nexuses connecting several NLs\(^{31–33} \). On the other hand, the SPT band crossings can also be classified according to whether they are merely symmetry-allowed (accidental) or symmetry-enforced\(^{34,35} \). The former is only perturbatively stable, whereas the symmetry-enforced degeneracies are robust against any large symmetry-preserving deformations and are currently drawing more attention due to their deterministic nature\(^{25,34,35} \).

In PhCs, the topology of band structures was usually thought of being adequately described by spinless space groups, provided that special internal symmetries, such as electromagnetic (EM) duality, are not imposed on the EM materials. However, in dielectric PhCs, there are always two gapless bands emerging from zero frequency and momentum, \( \omega = |k| = 0 \), irrespective of the space group representations at that point. Watanabe and Lu revealed recently that this intrinsic singularity of EM fields permits higher minimal connectivity for the lowest photonic bands than their electronic counterparts without spin-orbit coupling, and may further enforce unique photonic band crossings even in symmorphic lattices\(^{36} \). This pioneer study uncovered a tip of the hidden characters of Maxwell’s equations that bear on the photonic band connectivities. In general, the stationary Maxwell’s equations can be written as a generalized eigenvalue problem,

\[
\begin{pmatrix}
0 & i \nabla \times \\
- i \nabla \times & 0 
\end{pmatrix}
\begin{pmatrix}
E \\
H 
\end{pmatrix} = \omega \begin{pmatrix}
\frac{\epsilon (r)}{\mu (r)} & \frac{\chi (r)}{\mu (r)} \\
\frac{\chi (r)}{\epsilon (r)} & \frac{\mu (r)}{\mu (r)} 
\end{pmatrix}
\begin{pmatrix}
E \\
H 
\end{pmatrix},
\]

where we henceforth denote the curl matrix and the constitutive matrix on the left and right sides of Eq. (1) as \( \hat{\mathcal{N}} \) and \( \hat{\mathcal{M}}(r) \), respectively. Since all space group transformations leave the curl matrix \( \hat{\mathcal{N}} \) invariant, a PhC respects a space group symmetry \( \hat{\mathcal{A}} \), only if its constitutive tensor obeys \( \hat{\mathcal{A}} \mathcal{M}(r) \hat{\mathcal{A}}^{-1} = \mathcal{M}(r) \). However, a generic symmetry \( \hat{\mathcal{A}} \) of the Maxwell’s equations (1) operates on the Hamiltonian \( \hat{\mathcal{H}}(r) = \mathcal{M}(r)^{-1} \hat{\mathcal{N}} \) of EM fields, namely requiring \( \hat{\mathcal{A}} \hat{\mathcal{H}}(r) \hat{\mathcal{A}}^{-1} = \hat{\mathcal{H}}(r) \), but not on \( \hat{\mathcal{N}} \) and \( \hat{\mathcal{M}}(r) \) separately. This fact implies that the conventional space groups alone are insufficient to determine the symmetry properties as well as the band connectivities of photonic systems.

In this work, we propose a simple layer-stacked photonic structure consisting of anisotropic dielectrics to exemplify such kind of hidden symmetries of Maxwell’s equations beyond space groups. We show that a hidden symmetry,
FIG. 1. AB-layer-stacked photonic crystal made of a generic biaxial dielectrics. a Structure of the PhC, where the two insets display the iso-frequency surfaces in xz plane of the biaxial dielectrics in layer A (orange) and layer B (green) respectively. b Band structure along high symmetry lines in the $k_y = 0$ plane for the PhC with $\varepsilon_1 = 2$, $\varepsilon_2 = 13$, $\varepsilon_3 = 1$, and the rotation angle $\theta = \pi/5$. The blue and magenta lines represent the bands with odd and even $M_y$ parities respectively. The red dots corresponds to the nodal lines along which two bands with opposite $M_y$ parities intersect. The orange and purple dots denote the threefold and fourfold degenerate nexus points respectively. c Sketch map of two red nodal rings (corresponding to the two lowest red dots in b) crossing with a Kramers-like nodal line (blue line) at two nexus points in the first (bulk) Brillouin zone of the PhC.

more specifically a generalized fractional screw rotation symmetry, together with time reversal symmetry guarantees the emergence of Kramers-like straight NLs passing through the Brillouin zone center, and results in unusual photonic band connectivities. Furthermore, we demonstrate that the lowest Kramers-like NL can nearly always intersect with other two SPT nodal rings at two triply degenerate nexus points (NPs), which can be seen as a new kind of magnetic monopoles connecting Berry flux strings in the momentum space. By breaking the hidden symmetry, we lift the two NPs and then achieve type-II and type-III nodal rings in the PhC. In addition, the peculiar anisotropic band structure in the vicinity of the NPs, especially the spin-1 conical dispersion of the iso-frequency surfaces, may lead to novel transport phenomena. As an example, we show how optical vortices can be manipulated via the spin-1 conical diffraction effect for a beam incident at a NP.

Results

The photonic crystal considered here consists of two types of dielectric layers (A and B) stacked periodically along the x direction which are homogeneous in the transverse yz plane. The A and B layers have the equal thickness, $L/2$, and both are made up of the same kind of nondispersive anisotropic dielectrics with the principal values $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3 (\neq \varepsilon_1)$ of the relative permittivity, whereas the optical axes of the dielectrics rotate alternatively in the AB layers as shown in Fig. 1a. Specifically, the second principal axis of the materials is fixed along the y direction, while the first principle axis is rotated by an angle $\theta$ counterclockwisely (clockwisely) from the x axis in layer A (B). As such, the PhC’s relative permittivity tensor in the xyz coordinates is given by

$$\hat{\varepsilon}_r = \begin{pmatrix} \varepsilon_{xx} & 0 & \varepsilon_{zx} \\ 0 & \varepsilon_{yy} & 0 \\ \varepsilon_{zx} & 0 & \varepsilon_{zz} \end{pmatrix},$$

(2)

where $\varepsilon_{zx} = \varepsilon_{xx} = \pm g = \pm (\varepsilon_1 - \varepsilon_3) \sin \theta \cos \theta$ flips its sign between A and B layers, while the diagonal elements $\varepsilon_{xx} = \left( \cos^2 \theta \varepsilon_1 + \varepsilon_3 \sin^2 \theta \right)$, $\varepsilon_{yy} = \varepsilon_2$, and $\varepsilon_{zz} = \left( \sin^2 \theta \varepsilon_1 + \varepsilon_3 \cos^2 \theta \right)$ are all constant. The band structure along high symmetry lines of the PhC is plotted in Fig. 1b (see supplementary information S1 for the analytical calculation).

Space group symmetries

The space group of the PhC is $R^2 \times \text{Rod}(22)$, i.e. the semidirect product of the 2-dimensional continuous translational group $R^2$ in the yz plane and the nonsymmorphic Rod group 22 (pncm) associated with the discrete translations along the x-axis. Here, we focus on several space group symmetries relevant to the band crossings in the $k_y = 0$ plane.

Firstly, the PhC is invariant under the mirror reflection ($\hat{M}_y$) about the y = constant planes, which permits the bands with opposite (odd and even) mirror parities to intersect along NLs in the $\hat{M}_y$-invariant plane $k_y = 0$, as marked by the red dots in Fig. 1b and the red rings in Fig. 1c (also see Fig. 3 for the 3D band structures). Secondly, the combined inversion and time reversal symmetry ($\hat{PT}$) quantizes the Berry phase encircling the nodal lines as $\pi$, stabilizing the nodal lines against local $\hat{PT}$-preserving perturbations. Thirdly, the PhC has a twofold screw symmetry $\hat{S}_{2x} : (x, y, z) \rightarrow (x + \frac{L}{2}, -y, -z)$.
Together with $T$, the combined symmetry $\hat{\Theta}_{L/2} = T \hat{S}_{2x}$ ensures that all Bloch states are doubly degenerate in the $k_z = \pm \pi/L$ plane (corresponding to the twofold degenerate bands along $X - M$ in Fig. 1b)\textsuperscript{12}.

However, although the space group only supports 1D irreducible representation along the $\Gamma - Z$ line, the band structure shows that two bands with same $M_y$-parity always cross linearly along this line regardless of the dielectric’s parameters, and accordingly the two red nodal rings intersect at two NPs (orange dots in Fig. 1c). This indicates the PhC system possesses a symmetry beyond the crystallographic space group.

**Hidden symmetry and Kramers-like nodal lines**

Since the permittivity $\varepsilon(r)$ and also the Hamiltonian $\hat{H}(r) = M(r)^{-1/2}\hat{\Psi}(r) M(r)^{-1/2}$ of EM fields are generically tensors, the periodicity of the system restricts that the period of each component of $\varepsilon(r)$ should be a fraction $1/n$ of the full period. As mentioned in the introduction, the space group symmetries of the PhC, say $A$, of the PhC are entirely encoded in the space-dependent constitutive tensor as $A M(r) A^{-1} = \hat{M}(r)$. However, a generic symmetry $\hat{A}$ of the Maxwell’s equations (1) implies the whole Hamiltonian is invariant under $A$, i.e. $\hat{\Psi}(r) A^{-1} = \hat{\Psi}(r)$, but allows $\hat{A} M(r) A^{-1} \neq \hat{M}(r)$. Here, we show that such kind of hidden symmetry can emerge from the fractional periodicity of the dielectric components in Eq. (2), thereby giving rise to the Kramers-like NLs along $\Gamma - Z$.

As an accessible entry point, we first consider the $M_y$-odd subsystem in the $k_y = 0$ plane. Since the PhC is homogeneous in the $y$ direction, the $M_y$-odd eigenstates, $\psi_{y}^{\text{odd}} = (E_y, H_y, H_z)^\top$, only depend on $\varepsilon_{yy}$. As $\varepsilon_{yy}$ is a global constant for the PhC in Fig. 1, the dispersion along $\Gamma - X$ can be regarded as the simple folding of a linear band, giving rise to degeneracies at $\Gamma$ and $X$. Let us consider a relaxed condition $\varepsilon_{yy}(x + L/4) = \varepsilon_{yy}(x)$. In this case, the odd subsystem has a fractional period $L/4$, hence the width of BZ of the subsystem is quadrupled in the $x$ direction. The $M_y$-odd band structure in the original BZ can be obtained by folding the bands in the quadruple BZ twice. In the quadruple BZ, the time reversal symmetry ensures the eigenstates at $\pm k_x$ have degenerate eigen-frequencies $\omega(k_x) = \omega(-k_x)$. After band folding, every pair of eigenstates with identical frequencies at $k_x = \pm 2\pi/L$ is shifted onto the same point along the central line $k_x = 0$. Consequently, $(4m + 2)^{th}$ and $(4m + 3)^{th} M_y$-odd bands $(m \geq 0$ is an integer) are degenerate along $\Gamma - Z$, as shown in Fig. 1b.

Even though the $M_y$-even subsystem in the $k_y = 0$ plane, characterized by the submatrix $\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xz} \\ \varepsilon_{xz} & \varepsilon_{zz} \end{pmatrix}$ of $\varepsilon(r)$, has the same primitive period $L$ as the whole system, it can be demonstrated that the Hamiltonian of the subsystem with respect to the eigenvector $\psi_{y}^{\text{even}} = (D_x, E_x, H_z)^\top$ will only depend explicitly on $\varepsilon_{xx}$, $\varepsilon_{xz}$, and $\varepsilon_{zz}^2$, after a local $U(1)$ gauge transformation $\hat{U}(x, k_z) = \exp \left[ i k_z \int_0^x \varepsilon_{zz}(\xi) d\xi \right]$ (see supplementary information S2). For the AB-layer-stacked PhC in Fig. 1, $\varepsilon_{xx}$, $\varepsilon_{xz}$, and $\varepsilon_{zz}^2$ are all constant, therefore the intersections of $M_y$-even bands along $\Gamma - Z$ also result from the fold of a linear band. If we relax the constraint on the three parameters from being homogeneous to having a fractional period $L/n$, band crossings along $\Gamma - Z$ can still exist. In fact, 4 is the minimum value of $n$ that maintains the space group $R^3 \times \text{Rod}(22)$ of the layer-stacked PhC and also preserves the appearance of the NLs along $\Gamma - Z$. More specifically, the elements of the permittivity tensor should satisfy

$$\varepsilon_{ii}(x + L/4) = \varepsilon_{ii}(x), \quad (i = x, y, z) \quad (3)$$

$$\varepsilon_{xx}(x + L/4)^2 = \varepsilon_{xx}(x)^2 \quad \text{and} \quad \varepsilon_{xz}(x + L/2) = -\varepsilon_{xz}(x), \quad (4)$$

where the second requirement in Eq. (4) is necessary to keep the primitive period of the PhC being $L$.

If we focus on this minimum requirement case of $L/4$, we can introduce a generalized 1/4-period twofold screw operator, and prove that the complete Hamiltonian of both $M_y$-even and odd subsystems in the $k_y = 0$ plane is invariant under the generalized 1/4-period twofold screw rotation (see supplementary information S2)

$$\hat{S}_{L/4} \hat{H}(k_y = 0) \hat{S}_{L/4}^{-1} = \hat{H}(k_y = 0). \quad (5)$$

The generalized 1/4-period twofold screw rotation about the $x$-axis is defined as

$$\hat{S}_{L/4} = (\hat{P}_x - \hat{G}^{-1} \hat{U}^\dagger \hat{P}_y) \hat{C}_x = \hat{T}_x(L/4) (\hat{P}_x - \hat{G} \hat{U} \hat{P}_y), \quad (6)$$

where $\hat{C}_x = (\hat{T}_x(L/4)$ denotes the 1/4-period translation along $x$ direction, $\hat{P}_s = \frac{1}{2}(\hat{I}_{6\times6} \pm \hat{M}_y)$ are the projection operators onto $M_y$-even/odd subsystems respectively, $\hat{U}$ is the aforementioned local $U(1)$ gauge transformation, and $\hat{G} = \hat{I}_{6\times6} + (\varepsilon_{xx}(x) - 1) \hat{e}_1 \hat{e}_1 + \varepsilon_{xz}(x) \hat{e}_1 \hat{e}_3$ (7) transforms the eigenvector from $\Psi = (E, H)^\top$ to $\Psi' = (D_x, E_y, E_z, H)^\top$ with the basis $\{ \hat{e}_i \}_{j} = \delta_{ij}$ $(i, j = 1, \ldots, 6)$. In Fig. 2, we illustrate a more general example of PhCs satisfying Eqs. (3) and (4). The corresponding profiles of the dielectric components and the photonic band structure are shown in Figs. 2a and b, respectively.

The generalized 1/4-period screw rotation operator is pseudo-unitary\textsuperscript{42} and obeys $(\hat{S}_{L/4})^\dagger = \hat{T}_x(L)$, thus its eigenvalue for a Bloch state $\Psi(k_z, 0, 0)$ on the $k_x$-axis should be a fourth root of $e^{ik_x L}$: $\hat{S}_{L/4} \Psi^{(s)}(k_x, 0, 0) = s e^{-k_x L} \Psi^{(s)}(k_x, 0, 0)$, where the branch index $s = \pm 1, \pm i$ classifies the bands in $k_y = 0$ plane into 4 groups. Combining time reversal $T$ with $\hat{S}_{L/4}$, we obtain a pseudo-antiunitary symmetries of the PhC $\hat{\Theta}_{L/4} = T \hat{S}_{L/4}$ and have (see supplementary information S3)

$$\hat{\Theta}_{L/4}^2 \Psi^{(s)}(0, 0, k_z) = s^2 \Psi^{(s)}(0, 0, k_z),$$

(8)
for the Bloch states on the $k_x = k_y = 0$ line ($\Gamma - Z$). Following the similar derivation of Kramers theorem, Eq. (8) ensures that, once $s = \pm i$, $\Theta^s(0, 0, k_z)$ and $\tilde{\Psi}^s(0, 0, k_z) = \Theta_{L/4}^s(0, 0, k_z)$ are two distinct Bloch states degenerate along $\Gamma - Z$, forming Kramers-like nodal lines. Furthermore, since $S_{L/4}^s(0, 0, 0) = (0, 0, 0) = \mp i \tilde{\Psi}(0, 0, 0)$, each band of $s = \pm i$ along the $k_z$-axis have to intersect with a band of $s = -i$ at $\Gamma$ (i.e. the blue dots in Fig. 2c).

At BZ boundaries $k = (\pm \frac{\pi}{L}, 0, 0)$, the combined symmetry $T_{2x}$ guarantees that an arbitrary state $\tilde{\Psi}(\pm \frac{\pi}{L}, 0, 0)$ with branch index $s$ is degenerate with $\tilde{\Psi}(\pm \frac{\pi}{L}, 0, 0) = \Theta_{L/2}^s(\pm \frac{\pi}{L}, 0, 0)$. Meanwhile, it can be proved that $\tilde{S}_{L/4}^s(0, 0, 0) = \mp i s^* \tilde{\Psi}(\pm \frac{\pi}{L}, 0, 0)$, therefore every pair of bands intersecting at the zone boundaries $k_x = \pm \frac{\pi}{L}$ must have indices of either $s = +1$ and $-i$ or $s = -1$ and $+i$ (see the pairs of bands connecting at the green dots in Fig. 2c).

Fig. 2b exhibits an important difference of the PhC with reduced constraints from the PhC consisting of homogeneous layers in Fig. 1: the 4th and 5th $M_y$-odd (even) bands are gaped along $\Gamma - Z$. In fact, since the 4th $M_y$-odd (even) band has branch index $s = +1$, Eq. (8) indicates that $\Psi(1)$ and $\tilde{\Psi} = \Theta_{L/4}(1)$ can be the same state. In order to achieve higher band connectivity along $\Gamma - X$, we need the components of the permittivity have smaller fractional period $L/n$ ($n > 4$). And the layered PhC in Fig. 1 can be viewed as the limiting case of infinitesimal fractional periodicity ($n \to \infty$).

**Photonic band connectivity**

Dielectric PhCs have a universal feature that there are always two gapless photonic bands emerging from the singular point $\omega = |k| = 0$, around which the Bloch modes on the two gapless bands are transverse plane waves in the long-wavelength limit. If the PhCs further meet the conditions of Eqs. (3) and (4), the eigenvalues of $\tilde{S}_{L/4}$ for the two lowest bands attached at zero frequency are both equal to $-e^{\pm \frac{2\pi i}{4}}$, namely the first $M_y$ even and odd bands have the same branch index $s = -1$ (see supplementary information S4). Starting from the first $M_y$-odd (odd) band along $\Gamma - X$, $\tilde{S}_{L/4}$ symmetry ensures that at least 4 bands with branch indices $-1 \rightarrow +i \rightarrow -i \rightarrow +1$ (counting from the bottom) concatenate successively at the Kramers-like degeneracies at $k_x = \pm \frac{\pi}{L}$ and at $k_x = 0$, as shown in Fig. 2c. Consequently, the minimal band connectivity (MBC) along $\Gamma - X$ is 8 for bands attached at zero frequency, which is also beyond the prediction (MBC = 4) made by only considering the twofold screw symmetry $S_{2x}$. MBC = 8 implies the lowest 4 even and lowest 4 odd bands inevitably intersect each other at least 3 times (the red dots in Figs. 2b,c and also in Fig. 1b) along the line segment $\Gamma - X$, and therefore the unique photonic band connectivity enforces the emergence of the two red nodal rings shown in Fig. 1c.

In addition, Fig. 2b shows that all bands along $\Gamma - Z$ tend to linear dispersion as $k_z \to \infty$. Indeed, it can be proved rigorously that, in a $M_y$-symmetric dielectric PhC with continuous translational symmetry along the $z$-axis, all $M_y$-even and all $M_y$-odd bands have identical asymptotic group velocities in the $z$ direction, respectively (see details for the Bloch states on the $k_x = k_y = 0$ line ($\Gamma - Z$). Following the similar derivation of Kramers theorem, Eq. (8) ensures that, once $s = \pm i$, $\Theta^s(0, 0, k_z)$ and $\tilde{\Psi}^s(0, 0, k_z) = \Theta_{L/4}^s(0, 0, k_z)$ are two distinct Bloch states degenerate along $\Gamma - Z$, forming Kramers-like nodal lines. Furthermore, since $S_{L/4}^s(0, 0, 0) = s^* \tilde{\Psi}(0, 0, 0) = \mp i \tilde{\Psi}(0, 0, 0)$, each band of $s = \pm i$ along the $k_z$-axis have to intersect with a band of $s = -i$ at $\Gamma$ (i.e. the blue dots in Fig. 2c).

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FIG. 3. Dispersion near the triple nexus points and nodal lines. a 3D band structure of the PhC in Fig. 1 on the high symmetry plane $k_y = 0$ illustrates that $1^{\text{st}} M_y$-even band (magenta cone) intersects with a pair of $M_y$-odd bands (light blue surfaces) along two red nodal rings, and two odd bands coincides along the blue Kramers-like NL. The pair of orange dots show the nexus points of the 3 NLs. b-d Zoomed in band structures around the NP at $k^{\text{NP}}$ in the sections of $k_x = 0$, of $k_y = 0$, and of $k_z = k^{\text{NP}}_z$ respectively. e Iso-frequency surfaces around the NP at the frequency $\omega^{\text{NP}}$, where the black tangential lines are obtained from $1^{\text{st}}$ order $k \cdot p$ Hamiltonian. f Band structure of a PhC with broken hidden symmetry, where $\varepsilon_{yy}$ takes different values in A and B layers as $\varepsilon^{A}_{yy} = 9$ and $\varepsilon^{B}_{yy} = 21$, while all the other components of $\varepsilon^r$ are identical with the case in a.

in supplementary information S5),

$$\lim_{k_z \to \infty} \frac{d\omega^{\text{even}}}{dk_z} = \frac{c}{\sqrt{\varepsilon^{\text{max}}_{xx}}}, \quad \lim_{k_z \to \infty} \frac{d\omega^{\text{odd}}}{dk_z} = \frac{c}{\sqrt{\varepsilon^{\text{max}}_{yy}}},$$

(9)

where $\varepsilon^{\text{max}}_{ii}$ denotes the maximum value of $\varepsilon_{ii}$ ($i = x, y$) in the PhC, and $c$ is the speed of light in vacuum, as demonstrated by the numerical results in Fig. 2d. This result can be understood from the physical picture that the EM fields tend to concentrate in the regions of high refractive index. In the short wavelength limit ($k_z \to \infty$), all the fields will be localized at the places of maximal permittivity, and hence only $\varepsilon^{\text{max}}_{xx}$ and $\varepsilon^{\text{max}}_{yy}$ determine the asymptotic dispersion. This special property indicates that, as long as $\varepsilon^{\text{max}}_{xx} \neq \varepsilon^{\text{max}}_{yy}$, say, $\varepsilon^{\text{max}}_{xx} < \varepsilon^{\text{max}}_{yy}$ ($\varepsilon^{\text{max}}_{xx} > \varepsilon^{\text{max}}_{yy}$), the $1^{\text{st}} M_y$-even (odd) band have to intersects with the $2^{\text{nd}}$ and $3^{\text{rd}} M_y$-odd (even) bands, i.e. the $1^{\text{st}} M_y$-odd (even) Kramers NL, on $\Gamma - Z$ (the orange dot in Fig. 2b), owing to their different asymptotic group velocities.

As a notable consequence, almost any layer-stacked dielectric PhC respecting $M_y$ and the hidden symmetry $\tilde{S}_{L/4}$ (e.g. the AB-layer-stacked PhC in Fig. 1) must carry a pair of triply degenerate nodes as the nexuses of the two $M_y$-symmetry-protected nodal rings (red rings in Fig. 1c) and $1^{\text{st}}$ Kramers-like NL (blue line in Fig. 1c) for the bands connecting $\omega = |k| = 0$, as displayed in Fig. 1c. More examples with different parameters of the PhC are given in supplementary information S6, where we will see that the NPs always appear unless the asymptotic group velocities of even and odd bands are accidentally identical, namely $\varepsilon^{\text{max}}_{xx} = \varepsilon^{\text{max}}_{yy}$. Nevertheless, these exceptions only form a subset of zero measure for all possible parameters of the PhCs.

We are also aware that the linear asymptotic dispersions along $z$ direction cause infinitely many bands with opposite mirror parities to intersect each other as long as $\varepsilon^{\text{max}}_{xx} \neq \varepsilon^{\text{max}}_{yy}$, hence forming not only infinitely many threefold NPs but also infinitely many fourfold NPs (see the purple dots in Fig. 2c and supplementary information S6 for the typical dispersion around a fourfold NP). Hereinafter, we will focus on the lowest pair of triply degenerate NPs in the AB-layer-stacked PhC shown in Fig. 1 and investigate the band dispersion near the NPs.

**Triply degenerate nexus points**

Fig. 3a displays the 3D band structure near the 3 nodal lines in $k_y = 0$ plane corresponding to the PhC in Fig. 1, where the magenta cone, denoting $1^{\text{st}} M_y$-even band,
As a result, the 3 NLs intersect together at a pair of 2D spin-1 dispersions. This property implies that the NPs in layer-stacked PhCs demonstrate that they belong to a new kind of threefold nodal points, different from all isolated triple points carded by the 2D spin-1 Hamiltonian $H_{NP}(\delta k_z = 0) = v_x \hat{S}_z \delta k_x + v_y \hat{S}_y \delta k_y$. Furthermore, as shown in Fig. 3c, the iso-frequency surfaces around the NP at $\omega_{NP}$ also disperse exactly as a 2D spin-1 Dirac-like cone in the $xy$ plane, if $\delta k_z$ is regarded as a pseudo-frequency. This property implies that the NPs in layer-stacked PhCs can be used to realize the novel physical effects associated with 2D spin-1 dispersions.

The unique band topology of the NPs in our system demonstrates that they belong to a new kind of threefold nodal points, different from all isolated triple points carrying topological charges$^{26-30}$. In fact, the charge of a NP cannot be defined, because, for an arbitrary closed surface enclosing the NP, the gap between any 2 of the 3 bands on the surface must shut at the points where the NLs between the 2 bands pierce the surface$^{31,32}$. Nonetheless, since the Berry fluxes are confined on the NLs in a $\mathcal{PT}$ symmetric system, the NPs are the terminations of Berry flux strings, and thus can be regarded as a different kind of magnetic monopoles, other than Weyl points, in the momentum space$^{37-39}$.

### Type-II and type-III nodal rings

The simple AB-layer PhC can be the parent structures of other fancy topological features. In Fig. 3f, we let $\varepsilon_{yy} \neq \varepsilon_{yy}$ to observe the process of hidden symmetry $\hat{S}_{L/4}$ breaking. Without the protection by $\hat{S}_{L/4} = T \hat{S}_{L/4}$, the Kramers-like degeneracy between 2nd and 3rd $\hat{M}_y$-odd bands is lifted, and the pair of NPs disappears. Consequently, the original crossed nodal rings split into two new isolated ones. For the upper nodal ring (orange), the two degenerate bands are significantly tilted in the direction $k_z$ perpendicular to the ring so that both their perpendicular group velocities $v_{\pm}^{\text{even/odd}} = \nabla_{k} \omega^{\text{even/odd}} \cdot \hat{k}_z$ always have the same sign at any point on the ring, forming a type-II nodal ring$^{19-21}$. As the lower $\hat{M}_y$-odd band has a saddle shape dispersion, both type-I points (i.e. $v_{\pm}^{\text{odd}}$ and $v_{\pm}^{\text{even}}$ are of opposite signs) and type-II points coexist on the lower nodal ring (red), and such kind of band crossings is referred to as type-III nodal ring$^{19}$ or hybrid nodal ring$^{20}$.

#### Spin-1 conical diffraction

It is well known that light beams travelling along the optical axes in biaxial crystals will undergo conical diffraction$^{45}$. The conical diffraction phenomenon is actually a generic scattering effect for twofold degenerate Dirac points$^{46}$, and should also occur for light scattered by almost any linearly crossing point on the nodal rings in our system. Specifically, when an incident light beam has frequency and momentum that matches a certain point on the nodal rings, the refractive waves spread into a hollow cone in the PhC, and at the same time, their polarizations circling around the cone trace out a great circle on the Poincaré sphere, manifesting the quantized $\pi$ Berry phase encircling the nodal ring.

In contrast, the diffraction at the triple NPs appears strikingly different from other points on the NLs. Since the iso-frequency surfaces around each NP form a spin-1 Dirac-like cone (Fig. 3c), the monochromatic dynamics at a NP, e.g. $k^{\text{NP}}$, is effectively described by a Schrödinger equation with the 2D spin-1 Hamiltonian $H(\delta k_{xy}) = v_x \hat{S}_z \delta k_x + v_y \hat{S}_y \delta k_y$ (here $\delta k_{xy}$): 

$$i v_{\pm}^{\text{odd/even}} \frac{\partial}{\partial z} |\psi\rangle = \hat{H}(\delta k_{xy}) |\psi\rangle,$$  

where the $z$-coordinate serves as pseudo-time. Therefore, waves incident at the NPs should experience the unconventional spin-1 conical diffraction$^{47}$ rather than the spin-1/2 type diffraction at ordinary diabolic points. The schematic of spin-1 conical diffraction is shown in Fig. 4a, where a light beam at frequency $\omega_{NP}$ is incident along $z$-axis in the PhC, and its wave vector spectrum concentrates near the NP. Since the NP is a singularity
of group velocity for wave components on the two conical bands, these components will spread over on a conical surface, whereas the components on the flat band will propagate straightly along the z-axis. More interestingly, if the initial state of the beam is an eigenstate \( |s_i\rangle \) of \( \hat{S}_x \) with spin quantum number \( s_i \in \{-1, 0, 1\} \), the spin-1 character is inherent in the transition amplitude from \( |s_i\rangle \) to another eigenstate \( |s_f\rangle \) of \( \hat{S}_x \) as the output of the diffraction process (see supplementary information S8):

\[
\langle s_f | e^{-i\hat{H}t/\hbar} | s_i \rangle \propto \exp \left[ i(s_f - s_i)\phi(\delta k_{xy}) \right],
\]

Eq. (12) shows that the phase of output field winds \( l = s_f - s_i \) times around \( \delta k_{xy} = 0 \) in the momentum space. In view of the fact that the trajectories of the wave components encircling \( \delta k_{xy} = 0 \) also wrap round the z-axis in the real space, the output field projected onto \( |s_f\rangle \) will generate an optical vortex on the ring-shape section of the diffractive cone, and the charge of the vortex, \( l = s_f - s_i \in \{0, \pm 1, \pm 2\} \), is determined by the difference of spin quantum number between the final and initial spin states\(^{40}\), which essentially reflects the conservation of total generalized angular momentum for the spin-1 Hamiltonian (see supplementary information S8).

We have performed the full-wave simulations for the 9 possible combinations of input and output spin eigenstates. The intensities and phases of the output fields on a horizontal plane are shown in Figs. 4b and c, respectively, where the asymmetric intensity distributions originate from the anisotropy of the Dirac-like dispersion. In each panel of Fig. 4c, the white dashed ring corresponds to the section of the diffractive cone in the geometric optics approximation, and the winding number of the phase along the ring defines the charge of the optical vortex. We can see that the winding numbers agree with the theoretical predictions \( l = s_f - s_i \) in all panels, so we have demonstrated that the spin-1 conical diffraction occurs in the PhC, which also indicates the NPs can be used as a new route for studying 2D spin-1 dynamics.

**Discussion**

We have discovered that a class of simple layer-stacked PhCs manifests a hidden symmetry of Maxwell’s equations which directly influences the connectivity of photonic bands and engenders a pair of triply degenerate NPs where 3 symmetry enforced NLs intersect each other. These photonic NPs not only are worthy of theoretical investigation as a novel kind of magnetic monopoles that terminate Berry flux strings in momentum space, but they also induce exotic bulk transport effects in the PhC which may lead to prospective applications. In particular, we found that the unusual spin-1 Dirac-like dispersion of the iso-frequency surfaces near a NP can induce spin-1 conical diffraction of optical beams, which can be used to generate optical vortices with maximum topological charge of 2.

Our work takes the first step towards new research di-
rections. On the one hand, the hidden symmetry of Maxwell’s equations reveals a novel mechanism for realizing protected degeneracies unique to photonic bands. The hidden symmetry here stems from the fractional periods of different components of $\vec{r}$, which reflects a geometric property of the PhC but cannot be described by the conventional space groups. It would be fundamentally significant to develop a generalized space group theory including such kind of symmetries for photonic systems. On the other hand, our proposed PhC consists of a single anisotropic dielectric material, but the band peculiarity comes entirely from the nontrivial periodic rotation of the optical axes inside the material. Though recent studies have shown that artificial gauge fields,27,28, Pancharatnam-Berry phases,29, and synthetic spin-orbit interactions30 for light can be achieved by arranging the dielectric polarization, there are still little works that study the photonic band topology of the PhCs made of anisotropic dielectrics. Our results suggest that the intrinsic material anisotropy contains irrereplaceable features in comparison with the structural anisotropy, and anisotropic dielectrics, such as liquid crystal39,40, would become a platform for probing the unique topological effects of EM fields.

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Data availability
The authors declare that all data supporting the findings of this study are available from the corresponding authors upon reasonable request.

Conflict of interest
The authors declare that they have no conflict of interest.

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