STRONG DECAYS OF THE BOTTOM MESONS \(B_1(5721), B_2(5747), B_{s1}(5830), B_{s2}(5840)\) AND \(B(5970)\)

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Abstract

In this article, we study the two-body strong decays of the bottom mesons with the heavy meson effective theory in the leading order approximation, and obtain all the analytical expressions of the decay widths of the light pseudoscalar mesons transitions among the S-wave, P-wave and D-wave bottom mesons. As an application, we tentatively assign the bottom meson \(B(5970)\) as the 2S \(1^{-}\), 1D \(1^{-}\) and 1D \(3^{-}\) states, respectively, and calculate the decay widths of the \(B_1(5721), B_2(5747), B_{s1}(5830), B_{s2}(5840)\) and \(B(5970)\), which can be confronted with the experimental data in the future.

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1 Introduction

The orbitally excited \(B\)-mesons \(B_{s}^{\ast}(5732)\) or \(B_{s}^{\ast\ast}\) were firstly observed by the DELPHI, OPAL and ALEPH collaborations in electron-positron collisions at the Large Electron-Positron Collider (LEP) \[1\]. In 2007, the D0 collaboration firstly observed the \(B_1(5721)\) and \(B_2(5747)\), and measured the masses \(M_{B_1} = (5720.6 \pm 2.4 \pm 1.4)\text{ MeV}\) and \(M_{B_2} = (5746.8 \pm 2.4 \pm 1.7)\text{ MeV}\) \[2\]. Later, the CDF collaboration confirmed the \(B_1(5721)\) and \(B_2(5747)\), measured the masses \(M_{B_1} = (5725.3^{+1.0+1.4}_{-2.2-1.5})\text{ MeV}\) and \(M_{B_2} = (5740.2^{+1.7+0.3}_{-1.8-0.8})\text{ MeV}\), and obtained the width \(\Gamma_{B_1} = (22.7^{+3.8+3.2}_{-3.2-10.2})\text{ MeV}\) for the first time \[3\]. Also in 2007, the CDF collaboration observed the \(B_{s1}(5830)\) and \(B_{s2}(5840)\), and measured the masses \(M_{B_{s1}} = (5829.4 \pm 0.7)\text{ MeV}\) and \(M_{B_{s2}} = (5839.6 \pm 0.7)\text{ MeV}\) \[4\]. The D0 collaboration confirmed the \(B_{s2}(5840)\) and obtained the mass \(M_{B_{s2}} = (5839.6 \pm 1.1 \pm 0.7)\text{ MeV}\) \[5\]. In 2012, the LHCb collaboration updated the masses \(M_{B_{s1}} = (5828.40 \pm 0.04 \pm 0.04 \pm 0.41)\text{ MeV}\) and \(M_{B_{s2}} = (5839.99 \pm 0.05 \pm 0.11 \pm 0.17)\text{ MeV}\) \[6\].

Recently, the CDF collaboration reported the first evidence for a new resonance \(B(5970)\) in the \(B^+\pi^-\) and \(B^0\pi^+\) mass distributions with a significance of 4.\(\sigma\), and measured the masses \(M_{B(5970)}^0 = (5978 \pm 15 \pm 12)\text{ MeV}\) and \(M_{B(5970)}^+ = (5961 \pm 15 \pm 12)\text{ MeV}\), and widths \(\Gamma_{B(5970)}^0 = (70 \pm 18 \pm 31)\text{ MeV}, \Gamma_{B(5970)}^+ = (60 \pm 20 \pm 40)\text{ MeV}\) \[7\].

There have been several approaches to calculate the masses of the established bottom mesons \(B_1(5721), B_2(5747), B_{s1}(5830), B_{s2}(5840)\), such as the heavy quark effective theory \[8\], lattice QCD \[9\], potential models \[10 \[11 \[12\], heavy quark symmetry \[13\], heavy meson chiral theory \[14\], QCD sum rules \[15\], etc. The theoretical values vary in the range \(M_{\exp} \pm (50 - 100)\) MeV \[8 \[9\], \[10 \[11 \[12 \[13 \[14 \[15\]. Although the mass is a fundamental parameter in describing a hadron, the mass alone cannot validate the assignment. For example, now the doublet \((D_s^{\ast}(2317), D_{s1}(2460))\) is widely accepted to be the \(1P\) \((0^+, 1^+)\) doublet, however, the masses of the \(D_s^{\ast}(2317)\) and \(D_{s1}(2460)\) lie below the predictions of the potential models about \((100 - 150)\) MeV \[16\].

In Table 1, we present the predictions from two typical potential models compared to the experimental data \[11 \[12 \[17\]. The CDF collaboration observed the \(B(5970)\) in the strong decays \(B(5970) \rightarrow B^+\pi^-, B^0\pi^+\) \[7\], the possible quantum numbers are \(J^P = 1^{-}, 3^{-}, 5^{-}, \cdots\). We can assign the \(B(5970)\) as the \(2S1^{-}, 1D1^{-}\) and \(1D3^{-}\) states tentatively according to the masses, see Table 1. In Refs.\[18\], Y. Sun et al take the \(B(5970)\) as the \(2S1^{-}\) state, and calculate the strong decays of the bottom mesons with the \(3P_0\) model.

In Refs.\[19 \[20\], we study the strong decays of the charmed mesons \(D(2550), D(2600), D(2750), D(2760), D_J(2580), D_s^{\ast}(2650), D_J(2740), D_s^{\ast}(2760), D_J(3000), D_s^{\ast}(3000)\) with the heavy meson}
In the heavy quark limit, the heavy-light mesons $Q\bar{q}$ can be classified in doublets according to the total angular momentum of the light antiquark $s_\ell$, $s_\ell = \vec{s}_\bar{q} + \vec{L}$, where the $\vec{s}_\bar{q}$ and $\vec{L}$ are the spin and orbital angular momentum of the light antiquark, respectively [24]. The doublet $(P, P^*)$ have the spin-parity $J_{s_\ell}^P = (0^+, 1^-)$ for $L = 0$ (S-wave); the two doublets $(P_0, P_1)$ and $(P_1, P_2)$ have the spin-parity $J_{s_\ell}^P = (0^+, 1^+)\frac{1}{2}$ and $(1^+, 2^+)\frac{3}{2}$ respectively for $L = 1$ (P-wave); the two doublets $(P_1^*, P_2^*)$ and $(P_2, P_3^*)$ have the spin-parity $J_{s_\ell}^{P^*} = (1^-, 2^-)\frac{5}{2}$ and $(2^-, 3^-)\frac{7}{2}$ respectively for $L = 2$ (D-wave). In the heavy meson effective theory, those doublets with the same radial quantum

| $n$ | $L$ | $s_\ell$ | $J^P$ | $bq$ | $bs$ |
|-----|-----|---------|-------|-----|-----|
| 1   | $\frac{1}{2}$ | 0   | $\frac{1}{2}$ | 5280 | 5372 |
| 1   | $\frac{1}{2}$ | 1   | $\frac{1}{2}$ | 5326 | 5414 |
| 2   | $\frac{1}{2}$ | 0   | $\frac{1}{2}$ | 5890 | 5976 |
| 2   | $\frac{1}{2}$ | 1   | $\frac{1}{2}$ | 5906 | 5992 |
| 3   | $\frac{1}{2}$ | 0   | $\frac{1}{2}$ | 6379 | 6467 |
| 3   | $\frac{1}{2}$ | 1   | $\frac{1}{2}$ | 6387 | 6475 |
| 1   | $\frac{3}{2}$ | 0   | $\frac{3}{2}$ | 5749 | 5833 |
| 1   | $\frac{3}{2}$ | 1   | $\frac{3}{2}$ | 5774 | 5865 |
| 1   | $\frac{3}{2}$ | 2   | $\frac{3}{2}$ | 5723 | 5831 |
| 2   | $\frac{3}{2}$ | 0   | $\frac{3}{2}$ | 5741 | 5842 |
| 2   | $\frac{3}{2}$ | 1   | $\frac{3}{2}$ | 6281 | 6345 |
| 2   | $\frac{3}{2}$ | 2   | $\frac{3}{2}$ | 6209 | 6321 |
| 1   | $\frac{5}{2}$ | 1   | $\frac{5}{2}$ | 6119 | 6209 |
| 1   | $\frac{5}{2}$ | 2   | $\frac{5}{2}$ | 6121 | 6218 |
| 1   | $\frac{5}{2}$ | 3   | $\frac{5}{2}$ | 6091 | 6191 |

Table 1: The masses of the bottom mesons from two typical potential models compared to the experimental data.

effective theory in the leading order approximation, and calculate the decay widths and the ratios among the decay widths. The ratios can be compared to the experimental data in the future to distinguish the different assignments. The heavy meson effective theory have been applied to identify the charmed mesons [19, 20, 21, 22], and to calculate the radiative, vector-meson, two-pion decays of the heavy quarkonium states [23].

In this work, we study the two-body strong decays of the bottom mesons with the heavy meson effective theory in the leading order approximation, and obtain all the analytical expressions of the decay widths among the S-wave, P-wave and D-wave bottom mesons, and calculate the decay widths of the $B_1(5721)$, $B_2(5747)$, $B_{s1}(5830)$, $B_{s2}(5840)$ and $B(5970)$, which can be compared to the experimental data in the future.

The article is arranged as follows: we study the two-body strong decays of the bottom mesons $B_1(5721)$, $B_2(5747)$, $B_{s1}(5830)$, $B_{s2}(5840)$ and $B(5970)$ with the heavy meson effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 The strong decays with the heavy meson effective theory

In the heavy quark limit, the heavy-light mesons $Q\bar{q}$ can be classified in doublets according to the total angular momentum of the light antiquark $s_\ell$, $s_\ell = \vec{s}_\bar{q} + \vec{L}$, where the $\vec{s}_\bar{q}$ and $\vec{L}$ are the spin and orbital angular momentum of the light antiquark, respectively [24]. The doublet $(P, P^*)$ have the spin-parity $J_{s_\ell}^P = (0^+, 1^-)$ for $L = 0$ (S-wave); the two doublets $(P_0, P_1)$ and $(P_1, P_2)$ have the spin-parity $J_{s_\ell}^P = (0^+, 1^+)\frac{1}{2}$ and $(1^+, 2^+)\frac{3}{2}$ respectively for $L = 1$ (P-wave); the two doublets $(P_1^*, P_2^*)$ and $(P_2, P_3^*)$ have the spin-parity $J_{s_\ell}^{P^*} = (1^-, 2^-)\frac{5}{2}$ and $(2^-, 3^-)\frac{7}{2}$ respectively for $L = 2$ (D-wave). In the heavy meson effective theory, those doublets with the same radial quantum
numbers can be described by the effective super-fields \( H_a, S_a, T_a, X_a \) and \( Y_a \), respectively:

\[
\begin{align*}
H_a &= \frac{1 + \gamma_5}{2} \left\{ P_{a\mu} \gamma^\mu - P_a \gamma_5 \right\}, \\
S_a &= \frac{1 + \gamma_5}{2} \left\{ P_{1a\mu} \gamma_\mu - P_{1a} \gamma_5 \right\}, \\
T_a^\mu &= \frac{1 + \gamma_5}{2} \left\{ P_{2a\mu} \gamma_\mu - P_{2a} \gamma_5 \sqrt{\frac{3}{2}} \gamma_5 \right\}, \\
X_a^\mu &= \frac{1 + \gamma_5}{2} \left\{ P_{3a\mu} \gamma_\mu - P_{3a} \gamma_5 \sqrt{\frac{5}{3}} \gamma_5 \right\}, \\
\end{align*}
\]

where the heavy meson fields \( P^{(s)} \) contain a factor \( \sqrt{M_{P_1}} \) and have dimension of mass \( \frac{3}{2} \). The super-fields \( H_a \) contain the S-wave mesons (\( P, P^* \)); \( S_a, T_a \) contain the P-wave mesons (\( P_0, P_1 \)), \( (P_1, P_2) \) respectively; \( X_a, Y_a \) contain the D-wave mesons (\( P_1^*, P_2 \), \( P_2, P_3 \)) respectively.

The light pseudoscalar mesons are described by the fields \( \xi = e^{i\eta\sigma} \), where

\[
\mathcal{M} = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \frac{1}{2} \eta \\
\pi^- & -\sqrt{\frac{3}{2}} \pi^0 + \sqrt{\frac{1}{2}} \eta & K^+
\end{pmatrix}
\]

and \( f_\pi = 130 \text{ MeV} \).

At the leading order approximation, the heavy meson chiral Lagrangians \( \mathcal{L}_0, \mathcal{L}_{HH}, \mathcal{L}_{SH}, \mathcal{L}_{TH}, \mathcal{L}_{XH}, \mathcal{L}_{XT}, \mathcal{L}_{YH}, \mathcal{L}_{YS}, \mathcal{L}_{YT} \) for the two-body strong decays to the light pseudoscalar mesons can be written as:

\[
\begin{align*}
\mathcal{L}_0 &= \iota \text{Tr} \left\{ \tilde{H}_a v \cdot \mathcal{D}_{ab} H_b \right\} + \iota \text{Tr} \left\{ \tilde{S}_a v \cdot \mathcal{D}_{ab} S_b \right\} + \iota \text{Tr} \left\{ \tilde{T}_a^\mu v \cdot \mathcal{D}_{a\mu} T_b^\mu \right\} + \iota \text{Tr} \left\{ \tilde{X}_a^\mu v \cdot \mathcal{D}_{a\mu} X_b^\mu \right\} \\
\mathcal{L}_{HH} &= g_{HH} \text{Tr} \left\{ \tilde{H}_a H_b \gamma_5 \gamma^5 \right\}, \\
\mathcal{L}_{SH} &= g_{SH} \text{Tr} \left\{ \tilde{H}_a S_b \gamma_5 \gamma^5 \right\}, \\
\mathcal{L}_{TH} &= g_{TH} \text{Tr} \left\{ \tilde{H}_b T_b^\mu (i\mathcal{D}_\mu A + i\mathcal{D}_\mu A^\mu)_{ba} \gamma^5 \right\} + \text{h.c.}, \\
\mathcal{L}_{XH} &= g_{XH} \text{Tr} \left\{ \tilde{H}_a X_b^\mu (i\mathcal{D}_\mu A + i\mathcal{D}_\mu A^\mu)_{ba} \gamma^5 \right\} + \text{h.c.}, \\
\mathcal{L}_{XS} &= g_{XS} \text{Tr} \left\{ \tilde{S}_a X_b^\mu (i\mathcal{D}_\mu A + i\mathcal{D}_\mu A^\mu)_{ba} \gamma^5 \right\} + \text{h.c.}, \\
\mathcal{L}_{XT} &= g_{XT} \text{Tr} \left\{ \tilde{T}_a^\mu X_b^\mu \left[ k^T_1 (D_\mu D_\nu \Lambda^\mu)_{ba} + k^T_2 (D_\mu D_\nu D_\lambda A^\mu)_{ba} \gamma^\lambda \right] \right\} + \text{h.c.}, \\
\mathcal{L}_{YH} &= g_{YH} \text{Tr} \left\{ \tilde{H}_b Y_b^\mu \left[ k^H_1 (D_\mu D_\nu \Lambda^\mu)_{ba} + k^H_2 (D_\mu D_\nu D_\lambda A^\mu)_{ba} \gamma^\lambda \right] \right\} + \text{h.c.}, \\
\mathcal{L}_{YS} &= g_{YS} \text{Tr} \left\{ \tilde{S}_a Y_b^\mu \left[ k^S_1 (D_\mu D_\nu \Lambda^\mu)_{ba} + k^S_2 (D_\mu D_\nu D_\lambda A^\mu)_{ba} \gamma^\lambda \right] \right\} + \text{h.c.}, \\
\mathcal{L}_{YT} &= g_{YT} \text{Tr} \left\{ \tilde{T}_a \mu X_b^\mu (iD_\nu A + i\mathcal{D}_\nu A^\mu)_{ba} \gamma^5 \right\} + \text{h.c.}, \\
\end{align*}
\]

(2)
\[
\begin{align*}
\mathcal{D}_\mu &= \partial_\mu + V_\mu, \\
V_\mu &= \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\
A_\mu &= \frac{1}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \\
\{\mathcal{D}_\mu, \mathcal{D}_\nu\} &= \mathcal{D}_\mu \mathcal{D}_\nu + \mathcal{D}_\nu \mathcal{D}_\mu, \\
\end{align*}
\]

where \( \delta m_S = m_S - m_H, \delta m_T = m_T - m_H, \delta m_X = m_X - m_H, \delta m_Y = m_Y - m_H, \Lambda \) is the chiral symmetry-breaking scale and chosen as \( \Lambda = 1 \) GeV \([22]\), the hadronic coupling constants \( g_{HH}, g_{TH}, g_{XH}, g_{XS}, g_{XT} = k_1^H + k_2^T, g_{yH} = k_1^H + k_2^H, g_{yS} = k_1^S + k_2^S \) and \( g_{YT} \) depend on the radial quantum numbers of the heavy mesons, and can be fitted to the experimental data, if they are available. The heavy meson chiral Lagrangians \( \mathcal{L}_{HH}, \mathcal{L}_{SH}, \mathcal{L}_{TH}, \mathcal{L}_{XH} \) and \( \mathcal{L}_{YH} \) are taken from Ref.\([20]\), the \( \mathcal{L}_{XS}, \mathcal{L}_{XT}, \mathcal{L}_{YS} \) and \( \mathcal{L}_{YT} \) are constructed accordingly in this article. The flavor and spin violation corrections of order \( \mathcal{O}(1/m_Q) \) are neglected, we expect that the corrections are not larger than (or as large as) the leading order contributions.

From the heavy meson chiral Lagrangians \( \mathcal{L}_{HH}, \mathcal{L}_{SH}, \mathcal{L}_{TH}, \mathcal{L}_{XH}, \mathcal{L}_{XS}, \mathcal{L}_{XT}, \mathcal{L}_{YH}, \mathcal{L}_{YS}, \mathcal{L}_{YT} \), we can obtain the widths \( \Gamma \) of the two-body strong decays to the light pseudoscalar mesons,

\[
\Gamma = \frac{1}{2J+1} \sum_{i} \frac{p_f}{8\pi M_i^2} |A|^2, \\
p_f = \sqrt{(M_i^2 - (M_f + m_P)^2)(M_i^2 - (M_f - m_P)^2)},
\]

where the \( A \) denotes the scattering amplitudes, the \( i \) and \( f \) denote the initial and final state heavy mesons, respectively, the \( J \) is the total angular momentum of the initial heavy meson, the \( \sum \) denotes the summation of all the polarization vectors, and the \( P \) denotes the light pseudoscalar mesons.

Now we write down the explicit expressions of the decay widths \( \Gamma \) in different channels,

- \((0^-, 1^-)_{\frac{1}{2}^-} \rightarrow (0^-, 1^-)_{\frac{1}{2}^-} + P, \)

\[
\begin{align*}
\Gamma(1^- \rightarrow 1^-) &= C_p \frac{g_{HH}^2 M_f p_f^3}{3\pi f_p^2 M_i}, \\
\Gamma(1^- \rightarrow 0^-) &= C_p \frac{g_{HH}^2 M_f p_f^3}{6\pi f_p^2 M_i}, \\
\Gamma(0^- \rightarrow 1^-) &= C_p \frac{g_{HH}^2 M_f p_f^3}{2\pi f_p^2 M_i},
\end{align*}
\]

which take place through relative P-wave, the experimental candidates are \( B(5970)^0(2S) \rightarrow B^+ \pi^-, B(5970)^+(2S) \rightarrow B^0 \pi^+ \) \([7]\). 

- \((0^+, 1^+)_{\frac{1}{2}^+} \rightarrow (0^-, 1^-)_{\frac{1}{2}^-} + P, \)

\[
\begin{align*}
\Gamma(1^+ \rightarrow 1^-) &= C_p \frac{g_{HH}^2 M_f \left(p_f^2 + m_P^2\right) p_f}{2\pi f_p^2 M_i}, \\
\Gamma(0^+ \rightarrow 0^-) &= C_p \frac{g_{HH}^2 M_f \left(p_f^2 + m_P^2\right) p_f}{2\pi f_p^2 M_i},
\end{align*}
\]

which take place through relative S-wave, the \( 1P \ (0^+, 1^+)_{\frac{1}{2}^+} \) states are expected to be broad, no experimental candidate exists at present time.
which take place through relative S-wave, no experimental candidate exists at present time.

• \((0^-, 1^-)_{\frac{1}{2}} \rightarrow (0^+, 1^+)_{\frac{1}{2}} + P\),

\[
\Gamma(1^- \rightarrow 1^+) = C_P \frac{g_{SH}^2 M_f \left(p_f^2 + m_f^2\right) p_f}{2\pi f_0^2 M_i}, \tag{10}
\]

\[
\Gamma(0^- \rightarrow 0^+) = C_P \frac{g_{SH}^2 M_f \left(p_f^2 + m_f^2\right) p_f}{2\pi f_0^2 M_i}, \tag{11}
\]

which take place through relative D-wave, no experimental candidate exists at present time.

• \((1^+, 2^+)_{\frac{3}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + P\),

\[
\Gamma(2^+ \rightarrow 1^-) = C_P \frac{2g_{SH}^2 M_f p_f^5}{5\pi f_1^2 M_i^2}, \tag{12}
\]

\[
\Gamma(2^+ \rightarrow 0^-) = C_P \frac{4g_{SH}^2 M_f p_f^5}{15\pi f_2^2 M_i^2}, \tag{13}
\]

\[
\Gamma(1^+ \rightarrow 1^-) = C_P \frac{2g_{SH}^2 M_f p_f^5}{3\pi f_3^2 M_i^2}, \tag{14}
\]

which take place through relative D-wave, the experimental candidates are \(B_1(5721)^0(1P) \rightarrow B^{*+}\pi^- \), \(B_2(5747)^0(1P) \rightarrow B^{*+}\pi^- \), \(B_3(5830)^0(1P) \rightarrow B^{*+}\pi^- \) \[3\], \(B_{s1}(5830)^0(1P) \rightarrow B^{*+}\pi^- \) \[15\], \(B_{s2}(5840)^0(1P) \rightarrow B^{*+}\pi^- \) \[6\].

• \((0^-, 1^-)_{\frac{1}{2}} \rightarrow (1^+, 2^+)_{\frac{3}{2}} + P\),

\[
\Gamma(1^- \rightarrow 2^+) = C_P \frac{2g_{SH}^2 M_f p_f^5}{3\pi f_4^2 M_i^2}, \tag{15}
\]

\[
\Gamma(1^- \rightarrow 1^+) = C_P \frac{2g_{SH}^2 M_f p_f^5}{3\pi f_5^2 M_i^2}, \tag{16}
\]

\[
\Gamma(0^- \rightarrow 2^+) = C_P \frac{4g_{SH}^2 M_f p_f^5}{3\pi f_6^2 M_i^2}, \tag{17}
\]

which take place through relative D-wave, no experimental candidate exists at present time. The phase-spaces of the decays \(B(5970)^0(2S) \rightarrow B_1^{*+}\pi^- \), \(B_2^{*+}\pi^- \), \(B(5970)^0(2S) \rightarrow B_1^{*0}\pi^+ \), \(B_2^{*0}\pi^+ \) are very small.

• \((1^-, 2^-)_{\frac{3}{2}} \rightarrow (0^-, 1^-)_{\frac{1}{2}} + P\),

\[
\Gamma(2^- \rightarrow 1^-) = C_P \frac{2g_{SH}^2 M_f \left(p_f^2 + m_f^2\right) p_f^3}{3\pi f_7^2 M_i^2}, \tag{18}
\]

\[
\Gamma(1^- \rightarrow 1^-) = C_P \frac{2g_{SH}^2 M_f \left(p_f^2 + m_f^2\right) p_f^3}{9\pi f_8^2 M_i^2}, \tag{19}
\]

\[
\Gamma(1^- \rightarrow 0^-) = C_P \frac{4g_{SH}^2 M_f \left(p_f^2 + m_f^2\right) p_f^3}{9\pi f_9^2 M_i^2}, \tag{20}
\]

which take place through relative P-wave, the experimental candidates are \(B(5970)^0(1D) \rightarrow B^{*+}\pi^- \), \(B(5970)^0(1D) \rightarrow B^{0}\pi^+ \) \[7\].
\( (0^-, 1^-)_{\frac{1}{2}} \rightarrow (1^-, 2^-)_{\frac{1}{2}} + \mathcal{P}, \)

\[
\Gamma(1^- \rightarrow 2^-) = C_\rho \frac{10g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^5}{9\pi f_\rho^2 \Lambda^2 M_i},
\]

(21)

\[
\Gamma(1^- \rightarrow 1^-) = C_\rho \frac{2g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^3}{9\pi f_\rho^2 \Lambda^2 M_i},
\]

(22)

\[
\Gamma(0^- \rightarrow 1^-) = C_\rho \frac{4g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^5}{3\pi f_\rho^2 \Lambda^2 M_i},
\]

(23)

which take place through relative P-wave, no experimental candidate exists at present time.

\( (1^-, 2^-)_{\frac{1}{2}} \rightarrow (0^+, 1^+)_{\frac{1}{2}} + \mathcal{P}, \)

\[
\Gamma(2^- \rightarrow 1^+) = C_\rho \frac{2g_{XH}^2 M_I p_f^5}{5\pi f_\rho^2 \Lambda^2 M_i},
\]

(24)

\[
\Gamma(2^- \rightarrow 0^+) = C_\rho \frac{4g_{XH}^2 M_I p_f^5}{15\pi f_\rho^2 \Lambda^2 M_i},
\]

(25)

\[
\Gamma(1^- \rightarrow 1^+) = C_\rho \frac{2g_{XH}^2 M_I p_f^5}{3\pi f_\rho^2 \Lambda^2 M_i},
\]

(26)

which take place through relative D-wave, no experimental candidate exists at present time.

\( (0^+, 1^+)_{\frac{1}{2}} \rightarrow (1^-, 2^-)_{\frac{1}{2}} + \mathcal{P}, \)

\[
\Gamma(1^+ \rightarrow 2^-) = C_\rho \frac{2g_{XH}^2 M_I p_f^5}{3\pi f_\rho^2 \Lambda^2 M_i},
\]

(27)

\[
\Gamma(1^+ \rightarrow 1^-) = C_\rho \frac{2g_{XH}^2 M_I p_f^5}{3\pi f_\rho^2 \Lambda^2 M_i},
\]

(28)

\[
\Gamma(0^+ \rightarrow 2^-) = C_\rho \frac{4g_{XH}^2 M_I p_f^5}{3\pi f_\rho^2 \Lambda^2 M_i},
\]

(29)

which take place through relative D-wave, no experimental candidate exists at present time.

\( (1^-, 2^-)_{\frac{1}{2}} \rightarrow (1^+, 2^+)_{\frac{1}{2}} + \mathcal{P}, \)

\[
\Gamma(2^- \rightarrow 2^+) = C_\rho \frac{17g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^5}{45\pi f_\rho^2 \Lambda^4 M_i},
\]

(30)

\[
\Gamma(2^- \rightarrow 1^+) = C_\rho \frac{g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^5}{15\pi f_\rho^2 \Lambda^4 M_i},
\]

(31)

\[
\Gamma(1^- \rightarrow 2^+) = C_\rho \frac{g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^5}{9\pi f_\rho^2 \Lambda^4 M_i},
\]

(32)

\[
\Gamma(1^- \rightarrow 1^+) = C_\rho \frac{g_{XH}^2 M_I (p_f^2 + m_P^2) p_f^5}{3\pi f_\rho^2 \Lambda^4 M_i},
\]

(33)

which take place through relative D-wave, no experimental candidate exists at present time. The phase-spaces of the decays \( B(5970)^0 (1D) \rightarrow B_1^+ \pi^-, B_2^+ \pi^-, B(5970)^+ (1D) \rightarrow B_1^0 \pi^+, B_2^0 \pi^+ \) are very small.
\( (1^+, 2^+) \frac{3}{2} \rightarrow (1^-, 2^-) \frac{1}{2} + \mathcal{P}, \)

\[
\Gamma(2^+ \rightarrow 2^-) = C_P \frac{17g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{45\pi f_x^2 \Lambda^4 M_i}, \quad (34)
\]

\[
\Gamma(2^+ \rightarrow 1^-) = C_P \frac{g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{15\pi f_x^2 \Lambda^4 M_i}, \quad (35)
\]

\[
\Gamma(1^+ \rightarrow 2^-) = C_P \frac{g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{9\pi f_x^2 \Lambda^4 M_i}, \quad (36)
\]

\[
\Gamma(1^+ \rightarrow 1^-) = C_P \frac{g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{3\pi f_x^2 \Lambda^4 M_i}, \quad (37)
\]

which take place through relative D-wave, no experimental candidate exists at present time.

\( (2^-, 3^-) \frac{1}{2} \rightarrow (0^-, 1^-) \frac{1}{2} + \mathcal{P}, \)

\[
\Gamma(3^- \rightarrow 1^-) = C_P \frac{16g_{YH}^2 H_f p_f^7}{105\pi f_x^2 \Lambda^4 M_i}, \quad (38)
\]

\[
\Gamma(3^- \rightarrow 0^-) = C_P \frac{4g_{YH}^2 H_f p_f^7}{35\pi f_x^2 \Lambda^4 M_i}, \quad (39)
\]

\[
\Gamma(2^- \rightarrow 1^-) = C_P \frac{4g_{YH}^2 H_f p_f^7}{15\pi f_x^2 \Lambda^4 M_i}, \quad (40)
\]

which take place through relative F-wave, the experimental candidates are \(B(5970)^0(1D) \rightarrow B^+ \pi^-, \quad B(5970)^+(1D) \rightarrow B^0 \pi^+ \) \[\square\].

\( (0^-, 1^-) \frac{1}{2} \rightarrow (2^-, 3^-) \frac{1}{2} + \mathcal{P}, \)

\[
\Gamma(1^- \rightarrow 3^-) = C_P \frac{16g_{YH}^2 H_f p_f^7}{45\pi f_x^2 \Lambda^4 M_i}, \quad (41)
\]

\[
\Gamma(1^- \rightarrow 2^-) = C_P \frac{4g_{YH}^2 H_f p_f^7}{9\pi f_x^2 \Lambda^4 M_i}, \quad (42)
\]

\[
\Gamma(0^- \rightarrow 3^-) = C_P \frac{4g_{YH}^2 H_f p_f^7}{5\pi f_x^2 \Lambda^4 M_i}, \quad (43)
\]

which take place through relative F-wave, no experimental candidate exists at present time.

\( (2^-, 3^-) \frac{1}{2} \rightarrow (0^+, 1^+) \frac{3}{2} + \mathcal{P}, \)

\[
\Gamma(3^- \rightarrow 1^+) = C_P \frac{4g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{15\pi f_x^2 \Lambda^4 M_i}, \quad (44)
\]

\[
\Gamma(2^- \rightarrow 1^+) = C_P \frac{8g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{75\pi f_x^2 \Lambda^4 M_i}, \quad (45)
\]

\[
\Gamma(2^- \rightarrow 0^+) = C_P \frac{4g_{YH}^2 M_f (p_f^2 + m_p^2) p_f^5}{25\pi f_x^2 \Lambda^4 M_i}, \quad (46)
\]

which take place through relative D-wave, no experimental candidate exists at present time.
\( (0^+, 1^+) \frac{1}{2} \rightarrow (2^-, 3^-) \frac{1}{2} + \mathcal{P}, \)

\[
\Gamma(1^+ \rightarrow 3^-) = C_P \frac{28g_{Y^+}^2 M_f(p_f^0 + m_p^2) p_f^5}{45\pi f_z^2 \Lambda^4 M_i}, \\
\Gamma(1^+ \rightarrow 2^-) = C_P \frac{8g_{Y^+}^2 M_f(p_f^0 + m_p^2) p_f^5}{45\pi f_z^2 \Lambda^4 M_i}, \\
\Gamma(0^+ \rightarrow 2^-) = C_P \frac{4g_{Y^+}^2 M_f(p_f^0 + m_p^2) p_f^5}{5\pi f_z^2 \Lambda^4 M_i},
\]

which take place through relative D-wave, no experimental candidate exists at present time.

\( (2^-, 3^-) \frac{1}{2} \rightarrow (1^+, 2^+) \frac{1}{2} + \mathcal{P}, \)

\[
\Gamma(3^- \rightarrow 2^+) = C_P \frac{4g_{Y^+}^2 M_f p_f^5}{15\pi f_z^2 \Lambda^2 M_i}, \\
\Gamma(3^- \rightarrow 1^+) = C_P \frac{2g_{Y^+}^2 M_f p_f^5}{45\pi f_z^2 \Lambda^2 M_i}, \\
\Gamma(2^- \rightarrow 2^+) = C_P \frac{7g_{Y^+}^2 M_f p_f^5}{75\pi f_z^2 \Lambda^2 M_i}, \\
\Gamma(2^- \rightarrow 1^+) = C_P \frac{49g_{Y^+}^2 M_f p_f^5}{225\pi f_z^2 \Lambda^2 M_i},
\]

which take place through relative D-wave, no experimental candidate exists at present time. The phase-spaces of the decays \( B(5970)^0(1D) \rightarrow B^+_1 \pi^- \), \( B^+_2 \pi^- \), \( B(5970)^+(1D) \rightarrow B^+_1 \pi^+ \), \( B^+_2 \pi^+ \) are very small.

\( (1^+, 2^+) \frac{1}{2} \rightarrow (2^-, 3^-) \frac{1}{2} + \mathcal{P}, \)

\[
\Gamma(2^+ \rightarrow 3^-) = C_P \frac{28g_{Y^+}^2 M_f p_f^5}{75\pi f_z^2 \Lambda^2 M_i}, \\
\Gamma(2^+ \rightarrow 2^-) = C_P \frac{7g_{Y^+}^2 M_f p_f^5}{75\pi f_z^2 \Lambda^2 M_i}, \\
\Gamma(1^+ \rightarrow 3^-) = C_P \frac{14g_{Y^+}^2 M_f p_f^5}{135\pi f_z^2 \Lambda^2 M_i}, \\
\Gamma(1^+ \rightarrow 2^-) = C_P \frac{49g_{Y^+}^2 M_f p_f^5}{135\pi f_z^2 \Lambda^2 M_i},
\]

which take place through relative D-wave, no experimental candidate exists at present time.

The coefficients \( C_{s^+} = C_{K^0} = C_{K^0} = C_{K^0} = 1 \), \( C_{s^+} = \frac{1}{3} \) and \( C_{s^+} = \frac{1}{3} \) correspond to the initial states \( b \bar{u} \) (or \( b \bar{d} \)) and \( b s \) states, respectively. There are minor errors in Ref.\[20\], the numerical values of the decay widths concerning the final state \( \eta \) in Table 4-7 should be divided by 4, as the coefficient \( C_{\eta} = \frac{2}{3} \) in stead of \( C_{\eta} = \frac{1}{3} \) is taken in Ref.\[20\].

### 3 Numerical Results and Discussions

The input parameters are taken as \( M_{s^+} = 139.57 \text{ MeV} \), \( M_{s^0} = 134.9766 \text{ MeV} \), \( M_{K^0} = 493.677 \text{ MeV} \), \( M_{K^0} = 497.614 \text{ MeV} \), \( M_{s^0} = 547.853 \text{ MeV} \), \( M_{B^+} = 5.27925 \text{ GeV} \), \( M_{B^0} = 5.27955 \text{ GeV} \), \( M_{B^+} = 5.3252 \text{ GeV} \), \( M_{B^0} = 5.7435 \text{ GeV} \), \( M_{B^+} = 5.3667 \text{ GeV} \), \( M_{B^0} = 5.7435 \text{ GeV} \).
The numerical values of the decay widths of the bottom mesons $B_1(5721)$, $B_2(2650)$, $B_{s1}(5830)$, $B_{s2}(5840)$, $B(5970)$ are presented in Table 2, where we retain the strong coupling constants $g_{TH}$, $g_{HH}$, $g_{XY}$, $g_{XT}$ and $g_{Y T}$.

The values $\Gamma_{B_1} = (22.7^{+3.8+3.2}_{-3.2-10.2})$ MeV and $\Gamma_{B_{s2}} = (1.56 \pm 0.13 \pm 0.47)$ MeV listed in the Review of Particle Physics are taken from the experimental data of the CDF collaboration [3] and LHCb collaboration [6], respectively. Recently, the CDF collaboration measured all the widths of $B_{s1}(5830)$, $B_{s2}(5840)$, $B(5970)$, $B_{s1}$, $B_{s2}$ with the strong decays and compare the decay widths in Table 2 to the experimental data,

$$\Gamma_{B_1(5721)^0} = 0.10683 g_{TH}^2 \text{GeV} = (20 \pm 2 \pm 5)\text{MeV}[7],$$

$$\Gamma_{B_2(5747)^0} = 0.17870 g_{TH}^2 \text{GeV} = (26 \pm 3 \pm 3)\text{MeV}[7],$$

$$\Gamma_{B_{s1}(5830)} = 0.00011 g_{TH}^2 \text{GeV} = (0.7 \pm 0.3 \pm 0.3)\text{MeV}[7],$$

$$\Gamma_{B_{s2}(5840)} = 0.00929 g_{TH}^2 \text{GeV} = (2.0 \pm 0.4 \pm 0.2)\text{MeV}[7],$$

to obtain the hadronic coupling constant $g_{TH}$,

$$g_{TH} = 0.433 \pm 0.058 \text{ from } \Gamma_{B_1(5721)^0},$$

$$g_{TH} = 0.381 \pm 0.031 \text{ from } \Gamma_{B_2(5747)^0},$$

$$g_{TH} = 0.464 \pm 0.052 \text{ from } \Gamma_{B_{s2}(5840)},$$

where we neglect the large value $g_{TH} = 2.52$ from the small decay width $\Gamma_{B_{s1}(5830)}$, the strong decays of the $B_{s1}(5830)$ are greatly depressed in the phase-space. The average value is

$$g_{TH} = 0.43 \pm 0.05,$$

which is consistent with the value $h' = 0.43 \pm 0.01$ extracted from the decays of the charmed mesons [21].

The $B(5970)$ have three possible assignments: the $2S 1^-, 1D 1^-$ and $1D 3^-$ states, the corresponding two-body strong decays are quite different, the numerical values of the decay widths are shown explicitly in Table 2. Again, we saturate the widths with the two-body strong decays to the $S$-wave ground mesons, as the decays to $P$-wave mesons are greatly depressed in the phase-space, and compare the widths to the experimental data from the CDF collaboration [7],

$$\Gamma_{B(5970)^0(2S 1^-)} = 3.21730 g_{HH}^2 \text{GeV} = (70 \pm 18 \pm 31)\text{MeV}[7],$$

$$\Gamma_{B(5970)^0(1D 1^-)} = 1.93262 g_{XY}^2 \text{GeV} = (70 \pm 18 \pm 31)\text{MeV}[7],$$

$$\Gamma_{B(5970)^0(1D 3^-)} = 0.24541 g_{Y T}^2 \text{GeV} = (70 \pm 18 \pm 31)\text{MeV}[7],$$

to obtain the hadronic coupling constants,

$$g_{HH} = 0.148 \pm 0.038,$$

$$g_{XY} = 0.190 \pm 0.049,$$

$$g_{Y T} = 0.534 \pm 0.137.$$

The numerical values of the decay widths are shown explicitly in Table 3, which can be confronted with the experimental data from the LHCb, CDF, D0 and KEK-B collaboration in the future to identify the $B(5970)$. We can also study the decays to the light vector mesons $V$ besides the pseudoscalar mesons $P$ with the replacement $V_\mu \rightarrow V_\mu + V_{\mu}$, and introduce additional phenomenological Lagrangians [27], therefore additional unknown coupling constants. The decays to the vector mesons $V$ are depressed in the phase-space compared to the light pseudoscalar mesons, we prefer to study those decays when the experimental data are accumulated.
| $nLs_J^{P}$ | Decay channels | Widths [GeV] | Decay channels | Widths [GeV] |
|------------|----------------|-------------|----------------|-------------|
| $B_{1}(5721)$ | $1P \frac{3}{2}^{+}$ | $B^{+} \pi^{-}$ | 0.07068 $g_{T}^{2}$ | $B^{0} \pi^{0}$ | 0.03615 $g_{T}^{2}$ |
| $B_{2}^{*}(5747)$ | $1P \frac{3}{2}^{+}$ | $B^{+} \pi^{-}$ | 0.05495 $g_{T}^{2}$ | $B^{0} \pi^{0}$ | 0.02804 $g_{T}^{2}$ |
| $B_{s1}(5830)$ | $1P \frac{3}{2}^{+}$ | $B^{+} K^{-}$ | 0.00009 $g_{T}^{2}$ | $B^{0} K^{0}$ | 0.00002 $g_{T}^{2}$ |
| $B_{s2}(5840)$ | $1P \frac{3}{2}^{+}$ | $B^{+} K^{-}$ | 0.00036 $g_{T}^{2}$ | $B^{0} K^{0}$ | 0.00022 $g_{T}^{2}$ |
| $B(5970)$ | $2S \frac{1}{2}^{+}$ | $B^{+} \pi^{-}$ | 1.22526 $g_{X}^{2}$ | $B^{+} \pi^{-}$ | 0.74252 $g_{X}^{2}$ |
| $B(5970)$ | $1D \frac{1}{2}^{+}$ | $B^{+} \pi^{-}$ | 0.31277 $g_{X}^{2}$ | $B^{+} \pi^{-}$ | 0.86132 $g_{X}^{2}$ |
| $B(5970)$ | $1D \frac{3}{2}^{-}$ | $B^{+} \pi^{-}$ | 0.07398 $g_{Y}^{2}$ | $B^{+} \pi^{-}$ | 0.08791 $g_{Y}^{2}$ |

Table 2: The strong decay widths of the bottom mesons $B_{1}(5721)$, $B_{2}(5747)$, $B_{s1}(5830)$, $B_{s2}(5840)$ and $B(5970)$.

| $nLs_J^{P}$ | Decay channels | Widths [MeV] | Decay channels | Widths [MeV] |
|------------|----------------|-------------|----------------|-------------|
| $B(5970)$ | $2S \frac{1}{2}^{+}$ | $B^{+} \pi^{-}$ | 26.8 ± 13.8 | $B^{+} \pi^{-}$ | 16.3 ± 8.4 |
| $B_{s}(5970)$ | $1P \frac{3}{2}^{+}$ | $B^{+} \pi^{-}$ | 11.3 ± 5.8 | $B^{+} \pi^{-}$ | 31.1 ± 16.0 |
| $B(5970)$ | $1D \frac{1}{2}^{+}$ | $B^{+} \pi^{-}$ | 0.7 ± 0.3 | $B^{0} \pi^{0}$ | 15.6 ± 8.0 |
| $B(5970)$ | $1D \frac{3}{2}^{-}$ | $B^{+} \pi^{-}$ | 0.3 ± 0.2 | $B^{0} \pi^{0}$ | 1.4 ± 0.7 |

Table 3: The strong decay widths of the bottom mesons $B(5970)$ with three possible assignments.
4 Conclusion

In this article, we study the two-body strong decays of the bottom mesons with the heavy meson effective theory in the leading order approximation, and obtain all the analytical expressions of the widths among the S-wave, P-wave and D-wave bottom mesons. As an application, we tentatively assign the bottom mesons \( B(5970) \) as the \( 2S_1^- \), \( 1D_1^- \) and \( 1D_3^- \) states, calculate the decay widths of the \( B_1(5721) \), \( B_2(5747) \), \( B_{s1}(5830) \), \( B_{s2}(5840) \) and \( B(5970) \), and obtain the hadronic coupling constants by comparing them to the experimental data and make predications of the decay widths, which can be confronted with the experimental data from the LHCb, CDF, D0 and KEK-B collaborations in the future to identify the \( B(5970) \).

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