BOOTSTRAPPING THE CORONAL MAGNETIC FIELD WITH STEREO: UNIPOLAR POTENTIAL FIELD MODELING

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ABSTRACT

We investigate the recently quantified misalignment of $\alpha_{\text{mis}} \approx 20^\circ$–40$^\circ$ between the three-dimensional (3D) geometry of stereoscopically triangulated coronal loops observed with STEREO/EUVI (in four active regions (ARs)) and theoretical (potential or nonlinear force-free) magnetic field models extrapolated from photospheric magnetograms. We develop an efficient method of bootstrapping the coronal magnetic field by forward fitting a parameterized potential field model to the STEREO-observed loops. The potential field model consists of a number of unipolar magnetic charges that are parameterized by decomposing a photospheric magnetogram from the Michelson Doppler Imager. The forward-fitting method yields a best-fit magnetic field model with a reduced misalignment of $\alpha_{\text{se}} \approx 7^\circ$–20$^\circ$. We also evaluate stereoscopic measurement errors and find a contribution of $\alpha_{\text{se}} \approx 7^\circ$–20$^\circ$, which is likely due to the nonpotentiality of the ARs. The residual misalignment angle, $\alpha_{\text{res}}$, of the potential field due to nonpotentiality is found to correlate with the soft X-ray flux of the AR, which implies a relationship between electric currents and plasma heating.

Key words: Sun: corona – Sun: magnetic topology – Sun: UV radiation

1. INTRODUCTION

The STEREO mission provides us with an unprecedented view of the solar corona, enabling us to fully constrain the three-dimensional (3D) geometry of the coronal magnetic field for the first time. Stereoscopic triangulation of coronal loops has been conducted at small STEREO spacecraft separation angles ($\alpha_{\text{sep}} \approx 10^\circ$) for several active regions (ARs) observed with STEREO A (ahead) and B (behind) in 2007 April and May (Aschwanden et al. 2008a, 2008b, 2009). The reconstructed 3D geometry of STEREO-observed coronal loops has been compared with theoretical magnetic field models based on extrapolations from photospheric magnetograms, using nonlinear force-free field (NLFFF) models (DeRosa et al. 2009), as well as potential and stretched potential field models (Sandman et al. 2009), but surprisingly it turned out that the two types of magnetic field lines exhibited an average misalignment angle of $\alpha_{\text{mis}} \approx 20^\circ$–40$^\circ$, regardless of what type of theoretical magnetic field model was used. From this dilemma, it was concluded that a more realistic physical model is needed to quantify the transition from the non-force-free photospheric boundary condition to the nearly force-free field at the base of the solar corona (DeRosa et al. 2009).

At this juncture, it is not clear what is a viable method to obtain a force-free boundary of the magnetic field at the coronal base, or how to correct the non-force-free magnetograms. However, the stereoscopic triangulation supposedly provides the correct 3D directions of the magnetic field $\mathbf{B}(s)$, which together with Maxwell’s equation of a divergence-free field ($\nabla \mathbf{B} = 0$) also constrains the absolute values of the field strengths. In this study, we choose a magnetic field model that is defined in terms of multiple unipolar charges. An approach in terms of multiple dipoles is employed in a separate study (Sandman & Aschwanden 2010). Since both unipolar or dipolar magnetic fields represent potential magnetic fields that fulfill the divergence-free condition, the superposition of multiple unipolar and dipolar magnetic field components fulfills the same condition. We develop a numerical code of such a parameterized divergence-free magnetic field that can be forward fitted to the 3D geometry of stereoscopically triangulated coronal loops. The simple goal of this study is to evaluate how closely the stereoscopically observed loops can be modeled in terms of potential fields, a goal that was already attempted with Skylab observations (Sakurai & Uchida 1977). Modeling with nonpotential fields, such as NLFFF models, will be considered in future studies.

This paper is organized as follows: the definition of a parameterized potential field is described in Section 2; the development and tests of a numeric magnetic field code and the results of forward fitting to stereoscopically triangulated loops are presented in Section 3; and conclusions follow in Section 4.

2. THEORY AND DEFINITIONS

2.1. Divergence-free Magnetic Field

Since the coronal plasma-\(\beta\) parameter is generally less than unity (Gary 2001), the magnetic pressure exceeds the thermal pressure, and thus all soft X-ray- or EUV-emitting plasma that fills or flows through coronal flux tubes traces out the coronal field. Consequently, stereoscopic triangulation of EUV loops provides the correct 3D field directions along coronal loops. We denote the normalized 3D field direction along a loop with the unity vector $\mathbf{b}(s)$, which is parameterized as a function of a loop length coordinate $s = s(x, y, z)$, starting at the footpoint position $s_0 = s(x, y, 0)$ at the base of the corona,

$$\mathbf{b}(s) = \frac{\mathbf{B}(s)}{B(s)} = \frac{[B_x, B_y, B_z]}{\sqrt{B_x^2 + B_y^2 + B_z^2}},$$

(1)

where the magnetic field is defined by the Cartesian components $\mathbf{B}(s) = [B_x(s), B_y(s), B_z(s)]$. However, the absolute magnitude of the magnetic field strength, $B(s) = |\mathbf{B}(s)|$, is not known a priori. For a physical solution of the magnetic field, Maxwell’s...
equation of a divergence-free magnetic field has to be satisfied,
\[ \nabla \mathbf{B} = \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \right) = 0, \tag{2} \]
which (in its integral form) corresponds to the theorem of magnetic flux conservation along a flux tube,
\[ \Phi(s) = \int B(s) dA = \text{const}, \tag{3} \]
where \( A(s) = \int dA \) is the integral over the cross-sectional area of a flux tube defined in the direction perpendicular to the magnetic field line (at the loop position \( s \)). Therefore, since the stereoscopic triangulation defines the magnetic field directions in adjacent flux tubes, it also defines the divergence of the field and the relative change of the magnetic field strength, \( B(s) \), along the flux tubes; in this way it also implicitly defines the isogaus surfaces perpendicular to each flux tube, and therefore the full 3D vector field \( \mathbf{B} \), except for a scaling constant. The derivation of a 3D magnetic field \( \mathbf{B} \) in an AR,
\[ \mathbf{B}(\mathbf{x}) = \mathbf{B}(\mathbf{x}) \mathbf{b}(\mathbf{x}), \tag{4} \]
also requires knowledge of the scalar function \( B(s) \) in every 3D location \( \mathbf{x} \), which we constrain with a forward-fitting method of a divergence-free field model.

A divergence-free 3D magnetic field model \( \mathbf{B}(\mathbf{x}) \) can be parameterized by a superposition of divergence-free fields, because the divergence-free condition is linear (or Abelian); i.e., if two components \( \mathbf{A} \) and \( \mathbf{B} \) fulfill \( \nabla \mathbf{A} = 0 \) and \( \nabla \mathbf{B} = 0 \), then their sum is also divergence-free, i.e., \( \nabla (\mathbf{A} + \mathbf{B}) = \nabla \mathbf{A} + \nabla \mathbf{B} = 0 \). Divergence-free magnetic field components are, for instance, a parallel field, unipolar fields (a magnetic charge with spherical isogaus surfaces and a field that falls off with the square of the distance), dipole fields, quadrupolar fields, other multipole representations, or any potential field. The Abelian property warrants that any superposition of divergence-free fields is also divergence-free. Specifically, we will use divergence-free potential field models that consist of either (1) multiple unipolar charges (in this study) or (2) multiple dipoles (Sandman & Aschwanden 2010).

Our philosophy is the following: we will employ magnetic field models of the category of potential-field models, which are divergence-free by definition. We use particular potential field models that can be quantified with a finite number of free parameters. Potential field models are not as general as nonpotential and force-free field (NLFFF) models. However, since both potential and NLFFF models currently exhibit an equally poor misalignment with observed EUV loops, we first need to investigate whether the misalignment between observations and any theoretical model can be minimized for the simplest class of magnetic field models, such as potential field models. If our approach proves to be successful, refinements with nonpotential or NLFFF models can then be pursued in the future along the same avenue (e.g., see Conlon & Gallagher 2010; Gary 2010).

2.2. Multiple Unipolar Magnetic Charges

Unipolar potential fields often provide a good approximation to the magnetic field of sunspots, and thus also can be used for an AR that is composed of a finite number of spot-like magnetic polarities. Conceptually, a unipolar field can be considered as an approximation to the upper half of a vertically positioned dipole. The simplest representation of a unipolar magnetic field that is a potential field, and hence fulfills Maxwell’s divergence-free condition, is a spherically symmetric field that drops off with an \( r^{-2} \) dependence with distance, which means that it has a potential function that drops off with \( r^{-1} \),
\[ \Phi(r) = -\Phi_0 \left( \frac{z_0}{r} \right), \tag{5} \]
where \( \Phi_0 \) is the potential field value at the solar surface vertically above the buried magnetic charge at depth \( z_0 < 0 \). Of course, the extrapolated magnetic field is only computed in the coronal domain \( z > 0 \), so that no “magnetic monopole” exists in the solar corona. The magnetic field model of a single magnetic charge requires four parameters: the maximum value of the potential field \( \Phi_0 \) and the location \((x_0, y_0, z_0)\) of the buried charge, having a distance \( r \) to any point \((x, y, z)\) in the solar corona,
\[ r = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}. \tag{6} \]

The resulting magnetic field has then only a radial component \( B_r \) in the direction of \( r \),
\[ B_r(r) = \nabla \Phi(r) = \frac{\Phi_0}{r^2}, \tag{7} \]
with the surface field strength \( B_0 = \Phi_0/z_0 \). This unipolar potential field fulfills the divergence-free condition, as can be calculated from the Laplacian operator of the potential function,
\[ \nabla \mathbf{B}(r) = \Delta \Phi(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi(r)}{\partial r} \right) = 0. \tag{8} \]

The fulfillment of the divergence-free condition can also be verified from the conservation of the magnetic flux theorem (Equation (3)), if the envelope of a flux tube is defined by radial field lines, so that the cross-sectional area \( A(s) \propto B(s)^{-2} \) remains constant for \( s = r \). In our first model, we employ a superposition of \( N \) multiple unipolar charges,
\[ \mathbf{B}(\mathbf{x}) = \sum_{j=1}^{N} \mathbf{B}_j(\mathbf{x}) = \sum_{j=1}^{N} \mathbf{B}_j \left( \frac{z_j}{r_j} \right)^2 \frac{\mathbf{r}_j}{r_j}, \tag{9} \]
in terms of the vector \( \mathbf{r}_j = [(x - x_j), (y - y_j), (z - z_j)] \). For a single unipolar charge, the field lines will consist of straight lines in the radial direction away from the buried charge, which can approximate open-field regions. Burying multiple magnetic charges of opposite magnetic polarity, however, can mimic closed-field regions. An example is given in Figure 1, where we compare the magnetic field of a dipole with that of a combination of two unipolar charges with opposite magnetic polarity. Actually, the two magnetic field models become identical when the two unipolar charges are moved close together at the location of the dipole moment, as can be shown mathematically. Although the two models are equivalent in the far-field approximation, a combination of two unipolar charges (with \( 2 \times 4 = 8 \) free parameters) allows more general solutions than a single dipole (with six free parameters), especially in the case of strongly asymmetric fields (sunspots) or open-field regions, which exist in most ARs.
The forward fitting of our analytical magnetic field model to a set of observed magnetic field vectors \( \mathbf{b} = \mathbf{B}/B \) (e.g., using stereoscopically triangulated loop coordinates) is the task of optimizing the free parameters of the analytical model until the best match with the observed field lines is obtained. For the evaluation of the goodness or consistency of the analytical magnetic field models \( \mathbf{B}^{\text{theo}} \) with the observed field line model \( \mathbf{B}^{\text{obs}} \), we define the 3D misalignment angle \( \alpha_{\text{mis}} \), which is defined by the scalar product between the two field vectors \( \mathbf{B}^{\text{theo}}(\mathbf{x}) \) and \( \mathbf{B}^{\text{obs}}(\mathbf{x}) \),

\[
\alpha_{\text{mis}}(\mathbf{x}) = \cos^{-1} \left( \frac{\mathbf{B}^{\text{theo}}(\mathbf{x}) \cdot \mathbf{B}^{\text{obs}}(\mathbf{x})}{||\mathbf{B}^{\text{theo}}(\mathbf{x})|| ||\mathbf{B}^{\text{obs}}(\mathbf{x})||} \right),
\]

or equivalently, between the unity field vectors \( \mathbf{b}^{\text{theo}}(\mathbf{x}) \) and \( \mathbf{b}^{\text{obs}}(\mathbf{x}) \),

\[
\alpha_{\text{mis}}(\mathbf{x}) = \cos^{-1} \left( \frac{\mathbf{b}^{\text{theo}}(\mathbf{x}) \cdot \mathbf{b}^{\text{obs}}(\mathbf{x})}{||\mathbf{b}^{\text{theo}}(\mathbf{x})|| ||\mathbf{b}^{\text{obs}}(\mathbf{x})||} \right).
\]

Note that there is a 180° ambiguity in modeling the magnetic field directions, because the tangent lines of the stereoscopically triangulated loops are consistent with both parallel and antiparallel magnetic field directions. Moreover, the misalignment angle can be measured in two ways (\( \alpha_{\text{mis}} \) or \( 180° - \alpha_{\text{mis}} \)), of which we choose the smaller angle. Consequently, the range of so-defined misalignment angles is between 0° and 90°, yielding a stable convergence behavior toward a single asymptotic value of \( \gtrsim 0° \) in the forward-fitting procedure.

The misalignment angle yields a single value at every spatial position \( \mathbf{x} \), which can be averaged over the length of each observed field line (at \( n_p \) positions),

\[
\langle \alpha_{\text{mis}} \rangle = \left[ \frac{1}{n_p} \sum_{i=1}^{n_p} \alpha_{\text{mis}}^2(x_i, y_i, z_i) \right]^{1/2},
\]

which is similar to a \( \chi^2 \)-criterion. Since a unipolar magnetic charge can be parameterized with four free parameters \( (x_j, y_j, z_j, B_j) \) (Equation (9)), a model with \( n_c \) components has \( n_p = 4n_c \) free parameters.

![Figure 1](image1.png)

**Figure 1.** Magnetic field of a symmetric dipole (dashed lines), together with the field resulting from the superimposition of two unipolar magnetic charges (solid lines). The two field models become identical once the two unipolar charges are moved to the location of the dipole moment at position \( (x, y) = (0, 0) \). The radial field of each unipolar (positive and negative) charge is also shown for comparison (dotted lines).

### 2.3. Definition of Misalignment Angle

The misalignment angle is determined by the scalar product between the unity field vectors \( \mathbf{b}^{\text{theo}}(\mathbf{x}) \) and \( \mathbf{b}^{\text{obs}}(\mathbf{x}) \),

\[
\alpha_{\text{mis}}(\mathbf{x}) = \cos^{-1} \left( \frac{\mathbf{b}^{\text{theo}}(\mathbf{x}) \cdot \mathbf{b}^{\text{obs}}(\mathbf{x})}{||\mathbf{b}^{\text{theo}}(\mathbf{x})|| ||\mathbf{b}^{\text{obs}}(\mathbf{x})||} \right),
\]

or equivalently, between the unity field vectors \( \mathbf{b}^{\text{theo}}(\mathbf{x}) \) and \( \mathbf{b}^{\text{obs}}(\mathbf{x}) \),

\[
\alpha_{\text{mis}}(\mathbf{x}) = \cos^{-1} \left( \frac{\mathbf{b}^{\text{theo}}(\mathbf{x}) \cdot \mathbf{b}^{\text{obs}}(\mathbf{x})}{||\mathbf{b}^{\text{theo}}(\mathbf{x})|| ||\mathbf{b}^{\text{obs}}(\mathbf{x})||} \right).
\]

Note that there is a 180° ambiguity in modeling the magnetic field directions, because the tangent lines of the stereoscopically triangulated loops are consistent with both parallel and antiparallel magnetic field directions. Moreover, the misalignment angle can be measured in two ways (\( \alpha_{\text{mis}} \) or \( 180° - \alpha_{\text{mis}} \)), of which we choose the smaller angle. Consequently, the range of so-defined misalignment angles is between 0° and 90°, yielding a stable convergence behavior toward a single asymptotic value of \( \gtrsim 0° \) in the forward-fitting procedure.

The misalignment angle yields a single value at every spatial position \( \mathbf{x} \), which can be averaged over the length of each observed field line (at \( n_p \) positions),

\[
\langle \alpha_{\text{mis}} \rangle = \left[ \frac{1}{n_p} \sum_{i=1}^{n_p} \alpha_{\text{mis}}^2(x_i, y_i, z_i) \right]^{1/2},
\]

which is similar to a \( \chi^2 \)-criterion. Since a unipolar magnetic charge can be parameterized with four free parameters \( (x_j, y_j, z_j, B_j) \) (Equation (9)), a model with \( n_c \) components has \( n_p = 4n_c \) free parameters.

![Figure 2](image2.png)

**Figure 2.** Top: a potential dipole field is calculated with two unipolar magnetic charges buried in equal depth and equal magnetic field strength, but opposite polarization (\( B_1 = -0.1, B_2 = 0.1 \)). An asymmetric EUV loop is observed at the same location (gray torus) with a misalignment of \( \alpha_{\text{mis}} = 20° \) at the top of the loop. Bottom: the magnetic field strength of the right-hand side unipolar charge is adjusted (to \( B_2 = +0.08 \)) so that the misalignment of the loop reaches a minimum (\( \alpha_{\text{mis}} = 0° \)).

### 3. Numerical Code and Results

Our strategy to bootstrap the coronal magnetic field with stereoscopic data consists of the following steps: (1) we create a model of subphotospheric magnetic charges by deconvolving an observed photospheric magnetogram into point charges, which defines the unit vectors of our parameterized theoretical magnetic field model \( \mathbf{B}^{\text{theo}}(x, y, z) \); (2) we perform stereoscopic triangulation for a set of coronal loops observed with STEREO/EUVI, which are quantified in terms of directional field vectors \( \mathbf{b}^{\text{obs}} \); (3) we forward fit the theoretical magnetic field model \( \mathbf{B}^{\text{theo}} \) with a number of free parameters in the setup of unipolar magnetic charges by minimizing the mean misalignment angle \( \Delta \alpha_{\text{mis}} \) (\( \mathbf{B}^{\text{theo}}, \mathbf{b}^{\text{obs}} \)), which yields a best-fit solution \( \mathbf{B}^{\text{theo}} \). We can then compare the minimized misalignment of the bootstrapped best-fit model with those of standard methods based on extrapolation of the photospheric boundaries using a magnetogram, e.g., with the Potential Field Source Surface (PFSS) model. The procedure is illustrated in Figure 2, where a dipolar EUV loop is observed and a magnetic field model is constructed from two unipolar charges. By adjusting the field strength of the second unipolar charge from \( B_2 = 0.1 \) to \( B_2 = 0.08 \), the misalignment angle between the observed EUV loop and the model field can be reduced from \( \alpha_{\text{mis}} = 20° \) (Figure 2, top) to \( \alpha_{\text{mis}} = 0° \) (Figure 2, bottom). Note that the adjusted field is still a potential field and divergence-free, but represents a better match to the observed EUV loop.
3.1. Data Selection

We select four ARs observed with STEREO/EUVI and the Michelson Doppler Imager (MDI; Scherrer et al. 1995) on board the Solar and Heliospheric Observatory (SOHO): 2007 April 30, May 9, May 19, and December 11. The first, AR 10953 (2007 April 30), is identical to the case previously analyzed with STEREO and Hinode (DeRosa et al. 2009; Sandman et al. 2009). The second, AR 10955 (2007 May 9), was the subject of the first stereoscopically triangulated coronal loops, temperature and density measurements, and stereoscopic tomographic reconstruction (Aschwanden et al. 2008a, 2008b, 2009; Sandman et al. 2009). The third, AR 10953 (2007 May 9), displayed a small flare (during UT 12:40–13:20) as well as a partial filament eruption during the time of observations, and was featured in a few studies (Li et al. 2008; Liewer et al. 2009; Sandman et al. 2009). The fourth, AR 10978 (2007 December 11), is also the subject of recent magnetic field modeling (A. Van Ballegooijen & A. Engell 2010, private communication). Some details of these four ARs are given in Table 1, such as the heliographic position of the AR center, the magnetic area for fluxes of $B > 100 \, \text{G}$, the minimum and maximum field strengths, and the total unsigned magnetic flux.

3.2. Parameterization of the Magnetic Field Model

All four ARs were observed with the MDI on board SOHO, which provides full-disk MDI magnetograms with a pixel size of 2$''$. Sub-images encompassing the AR of interest are shown in Figure 3 (left column), with quadratic field-of-view sizes ranging from 145 to 339 pixels, or 0.3–0.7 solar radii. In order to create a realistic 3D magnetic field model, we decompose the partial magnetograms into a number of 2D sub-images in the decreasing order of field strengths. The composite magnetogram of these 200 decomposed sources is shown in Figure 3 (middle column), and the difference between it and the original magnetogram is also shown (right column). For each of the 200 magnetic source components, we store the peak magnetic field value $B_i$, the center position $(x_j, y_j)$, and the half-widths $w_j$ at full maximum. For a parameterization in terms of unipolar charges, however, we need to convert the half-width $w_j$ into the corresponding depth $z_j$ at which the unipolar charge is buried. From the definition of the FWHM at $|x - x_j| = w_j$ of a unipolar field (Equation (9)), we have for the vertical magnetic field component $B_z = B_i \cos \vartheta = B_i (z_j / r_j)$ (see geometry in Figure 4),

$$B_z(x_j + w_j) = B_z(x_j + w_j) \cos \vartheta = B_0 \left( \frac{z_j}{r_j} \right)^3$$

$$= B_0 \left( \frac{z_j^2}{w_j^2 + z_j^2} \right)^{3/2} = \frac{B_0}{2} \frac{r_j}{w_j},$$

with $\vartheta$ the angle between the vertical and a surface ring with radius $w_j$ (Figure 4), from which we can calculate the dipole depth $z_j$,

$$z_j = -\frac{w_j}{2^{2/3}} \approx -1.30 \, w_j,$$

which corresponds approximately to the half-width $w_j$ of the fitted Gaussian component. Since we have now all input parameters $(x_j, y_j, z_j, B_j)$, $j = 1, \ldots, n_c$, for a definition of a magnetic field model with multiple unipolar magnetic charges (Equation (9)), we can calculate the full 3D magnetic field by superimposing the fields of all components. A particular field line is simply calculated by starting with the field at a footpoint position and by iterative stepping in the field direction, until the field line returns to the solar surface (in the case of closed field lines) or to a selected boundary of the computation box (for open field lines).

3.3. Stereoscopic Triangulation of EUV Loops

For each of the four ARs, we triangulate as many loops as can be discerned with a high-pass filter in image pairs from STEREO/EUVI A and B. The method of stereoscopic triangulation is described in detail in Aschwanden et al. (2008b), from which we use identical loop coordinates for AR 10955 observed on 2007 May 9. The stereoscopic triangulation requires accurate co-alignment of both EUVI A and B images in an epipolar coordinate system. Furthermore, we transform the 3D loop coordinates measured in the epipolar coordinate system with the line of sight of EUVI A into the coordinate system of MDI, which sees the Sun from the Lagrangian point L1, almost in the same direction as seen from the Earth. The advantage of transforming the EUVI loop coordinates into the MDI reference frame is the direct modeling of the longitudinal magnetic field component $B_i$ in the MDI reference frame, without requiring any knowledge of the absolute magnetic field strength $B$, which is model-dependent; i.e., it depends on the choice of the extrapolation method (potential, force-free, or NLFFF) from photospheric magnetograms (which measures only the longitudinal component $B_z$). Some parameters of the stereoscopic triangulation procedure are listed in Table 1, such as the heliographic coordinates of the AR, the spacecraft separation angle, and the number of triangulated EUV loops (varying between 70 and 200 per AR, combined from all three coronal wavelengths, i.e., 171, 195, and 284 Å). 3D representations of the stereoscopically triangulated EUV loops are shown in Figures 5–8 (with blue color), seen along the line of sight of MDI (gray-scale maps in Figures 5–8) and in the two orthogonal directions (side view and top view in Figures 5–8). The height range of stereoscopically triangulated loops generally does not exceed 0.1 solar radii, due to the drop of dynamic range in flux for altitudes in excess of one hydrostatic scale height.

| Active Region | Observing Date | Observing Times (UT) | Spacetime Separation Angle (deg) | Number of EUVI Loops | Magnetic Area ($B > 100 \, \text{G}$) ($10^{20}$ cm$^2$) | Magnetic Field Strength ($B$ (G)) | Magnetic Flux ($10^{22}$ Mx) |
|---------------|----------------|----------------------|-------------------------------|----------------------|--------------------------------|--------------------------------|-------------------------------|
| 10953 (S05E02) | 2007 Apr 30    | 22:00–23:20          | 5.966                         | 200                  | 24.2                           | [−3134, +41425]                  | 8.7                           |
| 10955 (S09E24) | 2007 May 9     | 20:30–20:50          | 7.129                         | 70                   | 4.4                            | [−2396, +1926]                  | 1.6                           |
| 10953 (N03W03) | 2007 May 19    | 12:40–13:00          | 8.554                         | 100                  | 12.2                           | [−2056, +2307]                  | 4.0                           |
| 10978 (S09E06) | 2007 Dec 11    | 16:30–16:50          | 42.698                        | 87                   | 8.2                            | [−2270, +2037]                  | 4.8                           |
3.4. Forward Fitting of Potential Field Models

After we have parameterized the magnetic field with \( n_c \) unipolar charges, each one defined by four parameters \((x_j, y_j, z_j, B_j)\), using the procedure of iterative decomposition of a photospheric magnetogram as described in Section 3.2, we can vary these free model parameters to adjust it to the 3D geometry of the stereoscopically triangulated loops. However, since we typically represent an MDI magnetogram with \( n_c \approx 200 \) components, we have \( n_p = 4n_c \approx 800 \) free parameters, which are too many to optimize independently. Standard optimization procedures, such as the Powell algorithm (Press et al. 1986), allow for an optimization of \( \approx 10-20 \) free parameters, with good convergence behavior if the initial guesses are suitably chosen. Therefore, we choose 10–20 small circular zones (with typical radii of \( r_{sep} \approx 0.01–0.02 \) solar radii) containing the strongest field regions in the magnetogram of equal magnetic polarity and optimize the magnetic field strengths \( B_i^{opt} = q_j B_j \) with a common correction factor \( q_j \) in each zone. This is an empirical optimization of the lower boundary of the coronal magnetic field, optimized by varying the (decomposed) photospheric MDI magnetogram in such a way that the resulting potential field more closely matches the stereoscopically triangulated loops. The results of the best fits, based on the minimization of the median misalignment angle (Equations (11) and (12)), are shown in Figures 5–9, in the form of (red) model field lines that are extrapolated at the same locations as the footpoints of the stereoscopic loops (blue in Figures 5–9). The distribution of all misalignment angles, evaluated at about 80 locations in every loop, is shown in the form of histograms at the bottom of Figures 5–9. Each histogram is characterized
with a Gaussian peak, which is found to have a mean value and standard deviation of $\alpha = 14.3 \pm 11.5$ (Figure 5: 2007 April 30), $\alpha = 13.3 \pm 9.3$ (Figure 6: 2007 May 9), $\alpha = 20.3 \pm 16.5$ (Figure 7: 2007 May 19), and $\alpha = 15.2 \pm 12.3$ (Figure 8: 2007 December 11). If we compare these misalignment angles in the range of $\alpha_{\text{mis}} \approx 13^\circ - 20^\circ$ with the previously measured values using a PFSS model ($\alpha_{\text{mis}} \approx 19^\circ - 36^\circ$; Sandman et al. 2009), or using an NLFFF model ($\alpha_{\text{mis}} \approx 24^\circ - 44^\circ$; DeRosa et al. 2009), we see an improvement of the misalignment angle by about a factor of two. This is a remarkable result that demonstrates that the magnetic field at the lower boundary of the corona can be bootstrapped with stereoscopic measurements and with a suitable parameterization of a potential field model. Although the extrapolation from a photospheric magnetogram should be unique, if there is no data noise present, an infinite number of potential field solutions can be obtained depending on how the boundary field is parameterized (e.g., by unipolar charges or dipolar magnetic moments). In our case, for every variation of

![Figure 4](image)

**Figure 4.** Cross section of the Gaussian magnetic field component showing the geometric relation between the half-width $w_j$ and depth $z_j$ of a unipolar charge. The radial magnetic field $B_r$ drops off quadratically with the distance $r_j$, while the vertical component $B_z = B_r \cos \vartheta$ is foreshortened by a factor of $\cos \vartheta = z_j / r_j$.

![Figure 5](image)

**Figure 5.** Best-fit potential field model of AR observed on 2007 April 30. The stereoscopically triangulated loops are shown in blue color, while field lines starting at identical footpoints as the STEREO loop extrapolated with the best-fit potential field (composed of $n_c = 200$ unipolar magnetic charges) are shown in red. Side views are shown in the top and right panels. A histogram of misalignment angles measured between the two sets of field lines is displayed in the bottom panel. The distribution is fitted with a Gaussian, where the vertical solid line indicates the peak of the Gaussian (or most probable value), while the vertical dashed line indicates the median value.
the \( n_p \approx 800 \) free parameters, a slightly different field with a different misalignment to the stereoscopic loops is obtained. The PFSS code is designed to compute the potential field of the entire (front and backside) Sun, and thus has a relatively coarse spatial resolution of typically \( \approx 1^\circ \) (one heliographic degree, i.e., 12 Mm) for standard computations, but the small-scale magnetic field should not matter too much for our large-scale loops (\( \gtrsim 10 \) Mm). Hence, the best-fit potential-field bootstraps a force-free photospheric boundary field that is significantly different from the observed photospheric magnetograms that are observed in a non-force-free zone.

3.5. Convergence Behavior

The remaining misalignment between our best-fit potential field solution and the stereoscopic loops could be due to three sources of errors: (1) the bootstrapping method did not converge to the best solution, (2) the 3D coordinates of the stereoscopically triangulated loops have some error, or (3) the real coronal magnetic field that is represented by the stereoscopic loops could be nonpotential (e.g., containing twisted geometries due to currents). We consider the convergence of our code to be satisfactory because we ran many attempts for each case with different initial conditions and obtained about the same minimum misalignment. One possibility to vary the initial conditions is to vary the number of (Gaussian) unipolar components. Figure 9 shows the convergence behavior as a function of the number of (decomposed) unipolar components, where we computed a best-fit potential field solution for \( n_c = 10, 20, 50, 100, 200, 500 \) unipolar components for each of the four ARs. Convergence to the minimum misalignment value typically requires \( n_c \approx 10 \) components for the simplest dipolar ARs (2007 May 9 or December 11), and \( n_c = 50–100 \) for more complex ARs (2007 April 30 or May 19).

3.6. Estimate of Stereoscopic Error

We investigate the second possible source of error that could contribute to the measured misalignment, i.e., errors associated with the stereoscopic triangulation method. The errors of stereoscopic triangulation have been discussed in Aschwanden et al. (2008b), and depend on (1) the ratio of the stereoscopic parallax...
angle to the spatial resolution of the instrument, and thus on the spacecraft separation angle; (2) the angle between the loop segment and the east–west direction in the epipolar plane, being largest for loop segments parallel to the epipolar plane; and (3) the proper identification of a corresponding loop in image B to a selected loop in image A. While the first two sources of errors can be formally calculated, the correspondence problem is difficult to quantify. Since stereoscopic triangulation cannot be accomplished in an automated way at present time, the error of identifying corresponding loops could be estimated from the scatter obtained with different observers, but this is time consuming. Here we pursue another approach that is based on a self-consistency test. The procedure works as follows. If a sufficient large number of loops are triangulated in an AR, there should be for every triangulated loop (which we call the primary loop) a neighbored (secondary) loop with an almost parallel direction. Whatever the direction of the true magnetic field in the same neighborhood is, both the primary ($i$) and the secondary STEREO loops ($j$) should have a similar misalignment angle with the local magnetic field, $\Delta \alpha_i \approx \Delta \alpha_j$, or $|\Delta \alpha_i - \Delta \alpha_j| \approx 0$, in order to be self-consistent. Therefore, we can define a stereoscopic error (SE) angle $\Delta \alpha_{SE}$ by averaging these differences in misalignments over all STEREO loop positions with suitable weighting factors $w_{ij}$,

$$\Delta \alpha_{SE} = \frac{\sum_{i,j} |\alpha_i^{\text{mis}} - \alpha_j^{\text{mis}}| w_{ij}}{\sum_{i,j} w_{ij}},$$

where the index $i$ runs over all loop positions for each primary loop and the index $j$ runs over all loop positions of secondary loops, excluding the primary loop. For the weighting factor $w_{i,j}$ we should give less weight to more distant neighbors, because the true magnetic field difference and thus the local misalignment angle are likely to increase with distance. Thus, we should choose a negative power of the relative distance $d_{ij}$,

$$w_{ij} = \frac{1}{d_{ij}^p} = \frac{1}{[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2]^{-p/2}},$$

where $p$ is a power index. If the index $p$ is small, say $p = 1$, the weighting is reciprocal to the distance and the long-range

Figure 7. Best-fit potential field model of AR observed on 2007 May 19. Otherwise similar representation as Figure 5.
neighbors are relatively strongly weighted. If the index $p$ is large, say $p = 10$, the short-range neighbors are relatively strongly weighted while the long-range neighbors have almost no weight. Thus, we expect that the stereoscopic error defined as such monotonously decreases with short-range weighting toward higher power indices $p > 1$. In Figure 10, we show the stereoscopic error calculated with Equations (15) and (16) as a function of the power index from $p = 1$ to $p = 20$, for all four ARs. Indeed, the stereoscopic error monotonously decreases from $p = 1$ to $p = 12$, because we give progressively less weight to the long-range misalignments. However, from $p = 12$ toward $p = 20$ the error increases again, probably because of the inhomogeneity of the closest neighbors at the shortest range and the excessive weighting to the very nearest neighbors. However, the plateau in the range of $p \approx 5$–15 is a good indication that we measure a stable value at the minimum of $p \approx 12$. From this, we obtain a misalignment contribution of $\Delta \alpha_{SE} = 9.4$ (2007 April 30), 5.7 (2007 May 9), 11.5 (2007 May 19), and 7.2 (2007 December 11), as listed in Table 2.

### 3.7. Nonpotentiality of the Magnetic Field

A diagram of the various misalignment angles is shown in Figure 11, which we plot as a function of the maximum GOES soft X-ray flux during the observing interval. The GOES flux was dominated by the emission from the ARs analyzed here, since there was no other comparable AR present on the solar disk during the times of the observations. The case of 2007 May 9 with the lowest GOES flux shows a single dipolar structure, while the case of 2007 May 19 with the highest GOES flux exhibits multiple dipolar groups. The diagram in Figure 11 shows the following misalignment angles: $\alpha_{PFSS}$ obtained with the PFSS code, $\alpha_{NLFF}$ obtained with the NLFFF code (only for 2007 April 30), $\alpha_{PFU}$ optimized with the potential field with unipolar charge (PFU) code, and $\Delta \alpha_{SE}$ indicating the
stereoscopic errors. As a working hypothesis we might attribute the remaining residuals $\alpha_{NP}$ to the nonpotentiality of the ARs, which we calculate from adding the PFU misalignment angles and stereoscopic errors $\Delta\alpha_{SE}$ in quadrature,

$$\alpha_{NP} = \sqrt{\alpha_{PFU}^2 - \Delta\alpha_{SE}^2}, \quad (17)$$

for which we obtain the values: $\alpha_{NP} = 11 \pm 9$ (2007 April 30), $\alpha_{NP} = 11 \pm 8$ (2007 May 9), $\alpha_{NP} = 17 \pm 14$ (2007 May 19), and $\alpha_{NP} = 12 \pm 10$ (2007 December 11), listed also in Table 2. We consider these values to be a new method of quantifying the nonpotentiality of ARs. We find that quiescent ARs that contain simple dipoles (2007 April 30 and May 9) have small misalignments of $\alpha_{NP} \approx 11^\circ$ with best-fit potential field models, while more complex ARs (2007 May 19 and December 11) have somewhat larger nonpotential misalignments on the order of $\approx 12^\circ$–$17^\circ$ compared with best-fit potential field models.

While we express the degree of nonpotentiality in terms of a mean misalignment angle here, other measures are the ratios of the total nonpotential to the potential energy in an AR, which amounts up to $E_{NP}/E_P \lesssim 1.32$ in one flaring AR (Schrijver et al. 2008).

3.8. Quiescent and Flaring Active Regions

Since the four investigated ARs have quite different misalignment angles, and also the inferred nonpotentiality of the magnetic field varies significantly, we quantify the activity level of the ARs from the soft X-ray flux measured by GOES. Figure 12 shows the GOES light curves for the four ARs during the times of stereoscopic triangulation. The lowest GOES flux is measured for the AR of 2007 May 9, with a level of $10^{-7.6}$ W m$^{-2}$ (GOES class A4), while the highest GOES flux is measured for the AR of 2007 May 19, which has a flare occurring during the observing period with a GOES flux of $10^{-6.0}$ W m$^{-2}$ (GOES class C0). The three ARs with low soft X-ray flux levels appear to be quiescent, judging from the GOES flux profile, or may be subject to micro-flaring at a low level.

There is a clear correlation between the soft X-ray level of the AR (when it was on disk) and the overall misalignment angle (Figure 11), as well as with the misalignment angle...
Table 2

| Parameter                  | 2007 Apr 30 | 2007 May 9 | 2007 May 19 | 2007 Dec 11 |
|----------------------------|-------------|------------|-------------|-------------|
| Misalignment NLFFF<sup>a</sup> | 24–44       | 25 ± 8     | 19 ± 6      | 36 ± 13     | 32 ± 10     |
| Misalignment PFSS<sup>b</sup> | 14.3 ± 11.5 | 13.3 ± 9.3 | 20.3 ± 16.5 | 15.2 ± 12.3 |
| Median PFU<sup>c</sup>       | 20.0        | 16.2       | 25.8        | 15.7        |
| Stereoscopy error<sup>d</sup> | 9.4         | 7.6        | 11.5        | 8.9         |
| Nonpotentiality<sup>e</sup>  | 11 ± 9      | 11 ± 8     | 17 ± 14     | 12 ± 10     |
| GOES soft X-ray flux<sup>f</sup> | 10<sup>−7.3</sup> | 10<sup>−7.6</sup> | 10<sup>−6.0</sup> | 10<sup>−6.9</sup> |
| GOES class                  | A7          | A4         | C0          | B1          |

Notes.

<sup>a</sup> Measured with the NLFFF code (DeRosa et al. 2009).
<sup>b</sup> Measured with the PFSS code (Sandman et al. 2009).
<sup>c</sup> Measured with the unipolar potential field model (this study).
<sup>d</sup> Measured from inconsistency between adjacent loops.
<sup>e</sup> Residual misalignment of the unipolar best-fit model with stereoscopic error subtracted in quadrature, \( \alpha_{NP} = \sqrt{\alpha_{PFU}^2 - \Delta \alpha_{SE}^2} \).
<sup>f</sup> GOES flux in units of W m\(^{-2}\).

4. CONCLUSIONS

The agreement between theoretical magnetic field models of ARs in the solar corona with the true 3D magnetic field as delineated from the stereoscopic triangulation of coronal loops in EUV wavelengths has never been quantified until the recent advent of the STEREO mission. Surprisingly, the average misalignment between the theoretical and observed magnetic field was quite substantial, in the amount of \( \alpha_{mis} \approx 20°–40° \) for both potential and nonlinear force-free field models (DeRosa et al. 2009; Sandman et al. 2009). In this study, we investigate the various contributions of this large misalignment for four different ARs observed with STEREO and arrive at the following conclusions.

1. The amount of misalignment can be reduced to about half of the value for potential-field models optimized by a
bootstrapping method that minimizes the difference of field directions with the stereoscopically triangulated loops. Our potential-field model is parameterized with $\approx 200$ unipolar charges per AR, whose positions and field strengths are approximately derived from a Gaussian decomposition of a photospheric magnetogram, and then varied until a best fit is obtained. The best-fit potential field model has an improved misalignment of $\alpha_{PFU} \approx 13^\circ - 20^\circ$. Because the best-fit potential field model defines an improved magnetic field boundary condition at the bottom of the corona, the difference with the observed photospheric magnetogram contains information on the currents between the photosphere and the base of the force-free corona.

2. We estimate the misalignment contribution caused by stereoscopic correlation errors from self-consistency measurements between the magnetic field misalignments of adjacent loops. We find contributions on the order of $\Delta \alpha_{SE} \approx 7^\circ - 12^\circ$. 

3. We estimate the contributions to the field misalignment due to nonpotentiality caused by electric currents from the residuals between the best-fit potential field and the stereoscopic triangulation errors and find misalignment contributions on the order of $\alpha_{SP} \approx 11^\circ - 17^\circ$.

4. The overall average misalignment angle between potential field models and stereoscopic loop directions, as well as the contribution to the misalignment due to nonpotentiality, are found to correlate with the soft X-ray flux of the AR, which suggests a correlation between the amount of electric currents and the amount of energy dissipation in the form of plasma heating in an AR.

In this study we identify the contributions to the misalignment of the magnetic field, in terms of optimized potential field models, nonpotentiality due to electric currents, and stereoscopic triangulation errors for the first time. These results open up a number of new avenues to improve theoretical modeling of the coronal magnetic field. First of all, optimized potential field models that represent a suitable lower boundary condition at the base of the force-free corona can be found, which provides a less computationally expensive method than nonlinear force-free codes. Second, methods can be developed that allow us to localize electric currents in the non-force-free photosphere and chromosphere. Third, the misalignment angle can be used as a sensitive parameter to probe the evolution of current dissipation, energy buildup in the form of nonpotential magnetic energy in different quiescent and flaring zones of ARs. The high-resolution magnetic field data from Hinode and the Solar Dynamics Observatory provide excellent opportunities to obtain better theoretical models of the coronal magnetic field using our bootstrapping method, which is not restricted to stereoscopic data only, but can also be applied to single-spacecraft observations.

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Figure 12. GOES soft X-ray light curves of the 0.5–4 Å (upper curve) and 1–8 Å channel (lower curve) during the time of stereoscopic triangulation and magnetic modeling of the AR. The peak level of the GOES flux during the observing time is indicated with a thick bar.