A POSSIBLE QUANTUM-GRAVITATIONAL ORIGIN OF THE NEUTRINO MASS DIFFERENCE?
Consequences and Experimental Constraints

NICK E. MAVROMATOS
Department of Physics, King’s College London, University of London,
Strand, London, WC2R 2LS, United Kingdom
E-mail: nikolaos.mavromatos@kcl.ac.uk

and

SARBEN SARKAR
Department of Physics, King’s College London, University of London,
Strand, London, WC2R 2LS, United Kingdom
E-mail: sarben.sarkar@kcl.ac.uk

ABSTRACT

We discuss the theoretical possibility that the neutrino mass differences have
part of their origin in the quantum-decoherence-inducing medium of space-time
foam, which characterises some models of quantum gravity, in much the same
way as the celebrated MSW effect, responsible for the contribution to mass
differences when neutrinos pass through ordinary material media. We briefly
describe consequences of such decoherent media in inducing CPT violation at
a fundamental level, which would affect the neutrino oscillation probability; we
speculate on the connection of such phenomena with the rôle of neutrinos for
providing one possible source of a cosmological constant in the Universe, of
the phenomenologically right order of magnitude. Finally we discuss possible
experimental constraints on the amount of neutrino mass differences induced by
quantum gravity, which are based on fits of a simple decoherence model with all
the currently available neutrino data.

1. Introduction and Motivation

Recent astrophysical observations, using different experiments and diverse tech-
niques, seem to indicate that 70% of the Universe energy budget is occupied by “vac-
cuum” energy density of unknown origin, termed Dark Energy \[^{12}\]. Best fit models
give the positive cosmological constant Einstein-Friedman Universe as a good can-
didate to explain these observations, although models with a vacuum energy relaxing
to zero (quintessential, i.e. involving a scalar field which has not yet reached the
minimum of its potential) are compatible with the current data.

From a theoretical point of view the two categories of Dark Energy models are
quite different. If there is a relaxing cosmological vacuum energy, depending on the
details of the relaxation rate, it is possible in general to define asymptotic states
and hence a proper scattering matrix (S-matrix) for the theory, which can thus be
quantised canonically. On the other hand, Universes with a cosmological constant \( \Lambda > 0 \) (de Sitter) admit no asymptotic states, as a result of the Hubble horizon which characterises these models, and hampers the definition of proper asymptotic state vectors, and hence, a proper S-matrix. Indeed, de Sitter Universes will expand for ever, and eventually their constant vacuum energy density component will dominate over matter in such a way that the Universe will enter again an exponential (inflationary) phase of (eternal) accelerated expansion, with a Hubble horizon of radius \( \delta_H \propto 1/\sqrt{\Lambda} \). It seems that the recent astrophysical observations\(^{12}\) seem to indicate that the current era of the universe is the beginning of such an accelerated expansion.

Canonical quantisation of field theories in de Sitter space times is still an elusive subject, mostly due to the above mentioned problem of defining a proper S-matrix. One suggestion towards the quantisation of such systems could be through analogies with open systems in quantum mechanics, interacting with an environment. The environment in models with a cosmological constant would consist of field modes whose wavelength is longer than the Hubble horizon radius. This splitting was originally suggested by Starobinski\(^3\), in the context of his stochastic inflationary model, and later on was adopted by several groups\(^4\). Crossing the horizon in either direction would constitute interactions with the environment. An initially pure quantum state in such Universes/open-systems would therefore become eventually mixed, as a result of interactions with the environmental modes, whose strength will be controlled by the size of the Hubble horizon, and hence the cosmological constant. Such decoherent evolution could explain the classicality of the early Universe phase transitions\(^5\) (or late in the case of a cosmological constant). The approach is still far from being complete, not only due to the technical complications, which force the researchers to adopt severe, and often unphysical approximations, but also due to conceptual issues, most of which are associated with the back reaction of matter onto space-time, an issue often ignored in such a context. It is our opinion that the latter topic plays an important rôle in the evolution of a quantum Universe, especially one with a cosmological constant, and is associated with issues of quantum gravity. The very origin of the cosmological constant, or in general the dark energy of the vacuum, is certainly a property of quantum gravity.

This link between quantum decoherence and a cosmological constant may have far reaching consequences for the phenomenology of elementary particles, especially neutrinos. In this talk we shall elaborate on a scenario\(^6\), suggested originally in\(^6\) according to which the mass differences of neutrinos may have (part of) their origin in the quantum gravity decoherence medium of space time foam. The induced decoherence, then, will affect their oscillation, a notable consequence being the appearance of intrinsic CPT violating damping terms in front of the oscillation amplitudes. This fundamental (and local) form of CPT violation has its origin in the ill-defined nature of the corresponding CPT operator in such decoherent quantum theories, due to a mathematical theorem by Wald\(^7\). This local form of CPT violation, as a result of
the interaction of the elementary particle with a decoherent medium is linked to a cosmological (global) violation of CPT symmetry of the type proposed in \cite{9} by means of a generation of a cosmological constant as a result of neutrino mixing and non-trivial mass differences due to the quantum gravity vacuum. The framework in which such a cosmological constant may be generated by the neutrinos is the approach of \cite{9}, according to which the problem of mixing in a quantum field theory is treated by means of a canonical Fock-space quantization.

The structure of this talk is the following: in the next section we discuss the quantum-gravitational generation of neutrino mass differences, in analogy with the celebrated MSW effect \cite{10}, associated with enhancement of oscillations during the passage of neutrinos through ordinary matter. We pay particular attention to discussing the associated decoherence effects that would characterise the neutrino oscillation formula in such cases. In section 3 we review a preliminary discussion \cite{11} on the constraints from data for the proportion of neutrino mass differences that can be attributed to quantum gravity. The constraints are imposed by fitting some simplified decoherence models to the currently data on neutrinos. The data seem to exclude the possibility that the decoherence induced by certain types of stochastic space-time foam \cite{12} can be the exclusive source for the “observed” damping in front of the oscillation amplitudes in the respective oscillation probabilities that fit the data, and consequently imply that, at most, only a small percentage of the mass difference could be of unconventional origin due to the space-time foamy medium. In section 4, there are speculations on a possible link of this foam effect to the generation of a cosmological constant in the Universe, of the phenomenologically right order of magnitude. Conclusions are presented in section 5.

2. Quantum-Gravitational MSW effect and induced decoherence

In \cite{9} the idea that the observed mass differences between neutrinos are due to a type of stochastic space-time foam has been proposed. The concept presented is the possibility of the creation of microscopic charged black/white hole pairs out of the vacuum which would induce information loss and from their subsequent Hawking radiation would create a medium with stochastically fluctuating electric charges. The microscopic black holes would radiate preferentially the lightest charged particles i.e. electron/positron pairs and the ‘evaporating’ white hole could then absorb, say, the positrons. The resulting electric current fluctuations would interact non-trivially with $\nu_e$ and not $\nu_\mu$, according to coherent scattering interactions of the standard model, resulting in oscillations, and hence, effective mass differences, for the neutrinos, similar to the celebrated MSW effect \cite{10} for neutrinos in ordinary media. We have emphasized the rôde of the charged black holes in this effect since, from semi-classical arguments given below non-charged black holes may have a shorter lifetime. This leads us to consider that the effect of space-time foam on neutrinos can be treated
similarly to the celebrated MSW effect\textsuperscript{10} for neutrinos in ordinary media.

Before proceeding with such a MSW-like parametrisation of these stochastic quantum-gravity-induced effect, we consider it as useful to discuss briefly some properties of charged black holes which have been derived semi-classically; we will extrapolate such results to the case of microscopic space-time foam. Owing to the lack of a complete theory of quantum gravity (QG) such an extrapolation cannot be rigorously justified.

Charged black holes can be divided into two kinds: extremal, for which there is an equality between the electric charge and the mass of the black hole, \( Q = M \), and the non extremal ones. According to studies of scalar particles in the background of charged black holes\textsuperscript{13}, extremal black holes do not radiate particles. Moreover in string theory one can construct black hole configurations out of stringy membranes by invoking appropriate duality transformations, and so obtain many properties of non extremal black holes from extremal ones in a smooth way\textsuperscript{14}.

Such stringy studies have shown that the rate of change of the energy (mass \( M \)) of the near-extremal black holes, is given by

\[
\frac{dM}{dt} \sim \frac{A}{G_N T^2} \tag{1}
\]

where \( A \) is the area of the horizon, \( T \) is the temperature and \( G_N \) the gravitational constant. The above formula demonstrates, therefore, how (stringy) black holes, viewed as membranes, thermalise. It also shows that an extremal black hole, for which \( T = 0 \), cannot radiate particles. This last result is also recovered in field theoretic studies of black holes\textsuperscript{13}, by actually considering the number \( N_{\omega_0} \) of massless (scalar) particles (or pairs of particles/antiparticles) created in a state represented by a wavepacket centered around an energy \( \omega_0 \), is bounded:

\[
N_{n\omega_0,\ell m} \leq \frac{2c(\omega_0)|t(\omega_0)|^2}{(2n\pi)^{2k\mu-1}} \tag{2}
\]

where \( c(\omega_0) \) is a positive function, \( k > 0 \) is an arbitrary but large power, \( \ell, m \) are orbital angular momentum quantum numbers (arising from spherical harmonics in the wavefunction of the packet), and \( 2n\pi, n \) positive integer, is a special representation of the retarded time in Kruskal coordinates\textsuperscript{13}. In the formula \( t(\omega_0) \) denotes the transmission amplitude describing the fraction of the wave that enters the collapsing body, whose collapse produced the extreme black hole in\textsuperscript{13}. The wavepacket has a spread \( \epsilon \) in frequencies around \( \omega_0 \), and in fact it is the use of such wavepackets that allows for a consistent calculation of the particle creation in the extremal black-hole case. The above limit is obtained by means of certain analyticity properties of the particle creation number\textsuperscript{13}.

In the case of space-time foam, we have no way (at present) of understanding the spontaneous formation of such black holes from the quantum gravity vacuum, and hence in our case, one should assume that the above results can be extrapolated to
this case. In such a situation, then, \( t(\omega_0) \) would be a family of parameters describing the space-time foam medium.

From the bounded expression \( (2) \), we observe that since \( 2n\pi \) represents time, the rate of particle creation would drop to zero faster than any (positive) power of time at late times. This is in agreement with the abovementioned considerations about extremal black holes, in particular with the absence of particle creation in such a case. From the smooth connection of non-extremal black holes to the extremal ones, encountered in string theory \( 14 \), we can also conclude that near extremal black holes would be characterised by relatively small particle creation rate, as compared with their neutral counterparts.

If we can extrapolate the above-described semi-classical results to the quantum gravity foamy ground state, it becomes clear that microscopic black holes which are near extremal would evaporate significantly less, compared with their neutral counterparts. Thus, we may assume, that near extremal black holes in the foam would “live” longer, and as a result they would have more time to interact with ordinary matter, such as neutrinos. Such charged black holes would therefore constitute the dominant source of charge fluctuations in the foam that could be responsible for foam-induced neutrino mass differences according to the idea proposed in \( 6 \).

Indeed, the emitted electrons from such black holes, which as stated above are emitted preferentially, compared to muons or other charged particles, as they are the lightest, would then have more time to interact (via coherent standard model interactions) with the electron-neutrino currents, as opposed to muon neutrinos. This would create a \textit{flavour bias} of the foam medium, which could then be viewed as the “quantum-gravitational analogue” of the MSW effect in ordinary media (where, again, one has only electrons, since the muons would decay quickly). In this sense, the quantum gravity medium would be responsible for generating effective neutrino mass differences \( 6 \). Since the charged-black holes lead to a stochastically fluctuating medium, we shall consider the formalism of the MSW effect for stochastically fluctuating media \( 15 \), where the density of electrons would be replaced the density of charged black hole/anti black hole pairs.

The non-perturbative nature of quantum gravity foam, makes the above semi-classical computation unreliable. Hence it may not be true in a complete theory of quantum gravity. However, as we shall argue later in this paper, one can already place stringent bounds on the portion of the neutrino mass differences that may be due to quantum gravity foam, as a result of current neutrino data.

After this theoretical discussion we now proceed to give a brief description of the most important phenomenological consequences of such a scenario involving decoherence. These can help in imposing stringent constraints on the percentage of the neutrino mass difference that could be due to the quantum-gravity medium. For simplicity we restrict ourselves to two generations, which suffices for a demonstration of the important generic properties of decoherence. The extension to three generations
is straightforward, albeit mathematically more complex. The stochasticity of the space-time foam medium is best described by including in the time evolution of the neutrino density matrix a time-reversal (CPT) breaking decoherence matrix of a double commutator form,

\[ \partial_t \langle \rho \rangle = L[\rho], \]

\[ L[\rho] = -i[H + H'_I, \langle \rho \rangle] - \Omega^2[H'_I, [H'_I, \langle \rho \rangle]] \quad (3) \]

where \( \langle n(r)n(r') \rangle = \Omega^2n_0^2\delta(r - r') \) denote the stochastic (Gaussian) fluctuations of the density of the medium, and

\[ H'_I = \begin{pmatrix} (a_{\nu_e} - a_{\nu_\mu}) \cos^2(\theta) & (a_{\nu_e} - a_{\nu_\mu}) \sin 2\theta \frac{1}{2} \\ (a_{\nu_e} - a_{\nu_\mu}) \sin 2\theta \frac{1}{2} & (a_{\nu_e} - a_{\nu_\mu}) \sin^2(\theta) \end{pmatrix} \quad (4) \]

is the MSW-like interaction in the mass eigenstate basis, where \( \theta \) is the mixing angle. This double-commutator decoherence is a specific case of Lindblad evolution, which guarantees complete positivity of the time evolved density matrix.

We note at this stage that, for gravitationally-induced MSW effects (due to, say, black-hole foam models as in), one may denote the difference, between neutrino flavours, of the effective interaction strengths, \( a_i \), with the environment by:

\[ \Delta a_{e\mu} \equiv a_{\nu_e} - a_{\nu_\mu} \propto G_Nn_0 \quad (5) \]

with \( G_N = 1/M_P^2, M_P \sim 10^{19} \) GeV, the four-dimensional Planck scale, and in the case of the gravitational MSW-like effect, \( n_0 \) represents the density of charge black hole/anti-black hole pairs. This gravitational coupling replaces the weak interaction Fermi coupling constant \( G_F \) in the conventional MSW effect. This is the case we shall be interested in this work.

In such a case the density fluctuations \( \Omega^2 \) are therefore assumed small compared to other quantities present in the formulae, and an expansion to leading order in \( \Omega^2 \) is in place. Following then a standard analysis, one obtains the following expression for the neutrino transition probability \( \nu_e \leftrightarrow \nu_\mu \) in this case, to leading order in the small parameter \( \Omega^2 \ll 1 \):

\[
\begin{align*}
P_{\nu_e \rightarrow \nu_\mu} &= \\
&= \frac{1}{2} + e^{-\Delta a_{e\mu}^2 \Omega^2 t(1 + \frac{\Delta_1^2}{4\Gamma}(\cos(4\theta) - 1))} \sin(t\sqrt{\Gamma}) \sin^2(2\theta) \Delta a_{e\mu}^2 \Omega^2 \Delta_\frac{12}{12} \left( \frac{3\sin^2(2\theta)\Delta_{12}^2}{4\Gamma^{5/2}} - \frac{1}{\Gamma^{3/2}} \right) \\
&\quad - e^{-\Delta a_{e\mu}^2 \Omega^2 t(1 + \frac{\Delta_1^2}{4\Gamma}(\cos(4\theta) - 1))} \cos(t\sqrt{\Gamma}) \sin^2(2\theta) \frac{\Delta_{12}^2}{2\Gamma} \\
&\quad - e^{-\Delta a_{e\mu}^2 \Omega^2 \Delta_\frac{12}{12} \sin^2(2\theta)} \frac{\Delta a_{e\mu} + \cos(2\theta)\Delta_{12}}{2\Gamma}^2 \\
\end{align*}
\]

where \( \Gamma = (\Delta a_{e\mu} \cos(2\theta) + \Delta_{12})^2 + \Delta a_{e\mu}^2 \sin^2(2\theta), \Delta_{12} = \frac{\Delta m_{12}^2}{2\Gamma} \).
From (6) we easily conclude that the exponents of the damping factors due to the stochastic-medium-induced decoherence, are of the generic form, for $t = L$, with $L$ the oscillation length (in units of $c = 1$):

$$\text{exponent} \sim -\Delta a_{e\mu}^2 \Omega^2 t f(\theta); \quad f(\theta) = 1 + \frac{\Delta_{12}^2}{4\Gamma} (\cos(4\theta) - 1), \quad \text{or} \quad \frac{\Delta_{12}^2 \sin^2(2\theta)}{\Gamma} \tag{7}$$

that is proportional to the stochastic fluctuations of the density of the medium. The reader should note at this stage that, in the limit $\Delta_{12} \to 0$, which could characterise the situation in (6), where the space-time foam effects on the induced neutrino mass difference are the dominant ones, the damping factor is of the form $
abla_{\text{gravitational}}_{\text{MSW}} \sim -\Omega^2 (\Delta a_{e\mu})^2 L$, with the precise value of the mixing angle $\theta$ not affecting the leading order of the various exponents. However, in that case, as follows from (6), the overall oscillation probability is suppressed by factors proportional to $\Delta_{12}^2$, and, hence, the stochastic gravitational MSW effect (6), although in principle capable of inducing mass differences for neutrinos, however does not suffice to produce the bulk of the oscillation probability, which is thus attributed to conventional flavour physics.

There are other models of stochastic space-time foam also inducing decoherence, for instance the ones discussed in (12, 11), in which one averages over random (Gaussian) fluctuations of the background space-time metric over which the neutrino propagates. In such an approach, one considers merely the Hamiltonian of the neutrino in a stochastic metric background. The stochastic fluctuations of the metric would then pertain to the Hamiltonian (commutator) part of the density-matrix evolution. In parallel, of course, one should also consider environmental decoherence-interactions of Lindblad (or other) type, which would co-exist with the decoherence effects due to the stochastic metric fluctuations in the Hamiltonian. For definiteness in what follows we restrict ourselves only to the Hamiltonian part, with the aim of demonstrating clearly the pertinent effect and study their difference from Lindblad decoherence.

In this case, one obtains transition probabilities with exponential damping factors in front of the oscillatory terms, but now the scaling with the oscillation length (time) is quadratic (12, 11), consistent with time reversal invariance of the neutrino Hamiltonian. For instance, for the two generation case, which suffices for our qualitative purposes in this work, we may consider stochastically fluctuating space-times with metrics fluctuating along the direction of motion (for simplicity) (12)

$$g^{\mu\nu} = \begin{pmatrix}
-(a_1 + 1)^2 + a_2^2 & -a_3(a_1 + 1) + a_2(a_4 + 1) \\
-a_3(a_1 + 1) + a_2(a_4 + 1) & -a_3^2 + (a_4 + 1)^2
\end{pmatrix} \tag{8}$$

with random variables $\langle a_i \rangle = 0$ and $\langle a_i a_j \rangle = \delta_{ij}\sigma_i$.

Two generation Dirac neutrinos, then, which are considered for definiteness in (12) (one would obtain similar results, as far as decoherence effects are concerned in the
Majorana case), with an MSW interaction $V$ (of unspecified origin, which thus could be a space-time foam effect) yield the following oscillation probability:

$$
\langle e^{i(\omega_1 - \omega_2)t} \rangle = e^{i\left(\frac{\omega_1 + \omega_2}{2}\right)t} e^{-\frac{i}{2} \left(\sigma_1 t \left(\frac{(m_1^2 - m_2^2)}{k} + V \cos 2\theta\right)\right)} \times \\
e^{-\frac{i}{2} \left(\sigma_2 t \left(\frac{(m_1^2 - m_2^2)}{k} + V \cos 2\theta\right)\right)} \times \\
e^{-\left(\frac{(m_1^2 - m_2^2)}{2k^2}(9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) + \frac{2V \cos 2\theta(m_1^2 - m_2^2)}{k}(12\sigma_1 + 2\sigma_2 - 2\sigma_3)\right)t^2}
$$

(9)

where $k$ is the neutrino energy, $\sigma_i$, $i = 1, \ldots, 4$ parametrise appropriately the stochastic fluctuations of the metric in the model of $^{[12]}$, $\Upsilon = \frac{V k}{m_1^2 - m_2^2}$, $|\Upsilon| \ll 1$, and $k^2 \gg m_1^2$, $m_2^2$, and

$$
\begin{align*}
z_0^+ &= \frac{1}{2} \left(m_1^2 + \Upsilon(1 + \cos 2\theta)(m_1^2 - m_2^2) + \Upsilon^2(m_1^2 - m_2^2) \sin^2 2\theta\right) \\
z_0^- &= \frac{1}{2} \left(m_2^2 + \Upsilon(1 - \cos 2\theta)(m_1^2 - m_2^2) - \Upsilon^2(m_1^2 - m_2^2) \sin^2 2\theta\right).
\end{align*}

(10)

Note that the metric fluctuations-$\sigma_i$ induced modifications of the oscillation period, as well as exponential $e^{-\langle \cdots \rangle t^2}$ time-reversal invariant damping factors $^{[12]}$, in contrast to the Lindblad decoherence, in which the damping was of the form $e^{-\langle \cdots \rangle t}$. This feature is attributed to the fact that in this approach, only the Hamiltonian terms are taken into account (in a stochastically fluctuating metric background), and as such time reversal invariance $t \rightarrow -t$ is not broken explicitly. But there is of course decoherence, and the associated damping.

A few remarks are now in order regarding the similarity of this latter type of decoherence $^{[9]}$ with the one mimicked $^{[13]}$ by ordinary uncertainties in neutrino experiments over the precise energy $E$ of the beam (and in some cases over the oscillation length $L$). Indeed, consider the Gaussian average of a generic neutrino oscillation probability over the $L/E$ dependence $\langle P \rangle = \int_{-\infty}^{\infty} dx P(x) \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-(x - l)^2}{2\sigma^2}}$, with $l = \langle x \rangle$ and $\sigma = \sqrt{\langle (x - \langle x \rangle)^2 \rangle}$, $x = \frac{L}{4E}$, and assuming the independence of $L$ and $E$, which allows to write $l = \langle L/E \rangle = \langle L \rangle / 4\langle E \rangle$. A pessimistic and an optimistic upper bound for $\sigma$ are given by $^{[18]}$

- pessimistic: $\sigma \simeq \Delta x = \Delta \frac{L}{4E} \leq \Delta L \left(\frac{\partial x}{\partial L}\right)_{L=\langle L \rangle, E=\langle E \rangle} + \Delta E \left(\frac{\partial x}{\partial E}\right)_{L=\langle L \rangle, E=\langle E \rangle}$
  $$=rac{\langle L \rangle}{4\langle E \rangle} \left(\frac{\Delta L}{\langle L \rangle} + \frac{\Delta E}{\langle E \rangle}\right)$$

- optimistic: $\sigma \leq \frac{\langle L \rangle}{4\langle E \rangle} \sqrt{\left(\frac{\Delta L}{\langle L \rangle}\right)^2 + \left(\frac{\Delta E}{\langle E \rangle}\right)^2}$

Then, it is easy to arrive at the expression $^{[18]}$

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} - \ldots$$
with $U$ the appropriate mixing matrix. Notice the $\sigma^2$ damping factor of neutrino oscillation probabilities, which has the similar form in terms of the oscillation length dependence ($L^2$ dependence) as the corresponding damping factors due to the stochasticity of the space-time background in (9). It is noted, however, that here $l$ has to do with the sensitivity of the experiment, and thus the physics is entirely different.

In the case of space-time stochastic backgrounds, one could still have induced uncertainties in $E$ and $L$, which however are of fundamental origin, and are expected to be more suppressed than the uncertainties due to ordinary physics, described above. Apart from their magnitude, their main difference from the uncertainties in (11) has to do with the specific dependence of the corresponding $\sigma^2$ in that case on both $E$ and $L$. For generic space-time foam models it is expected that an uncertainty in $E$ or $L$ due to the “fuzziness” of space time at a fundamental (Planckian) level will increase with the energy of the probe, $\delta E/E, \delta L/L \propto (E/M_P)^\alpha, \alpha > 0$, since the higher the energy the bigger the disturbance (and hence back reaction) on the space-time medium. In contrast, ordinary matter effects decrease with the energy of the probe.

### 3. Fitting the data and attempts to interpret them

In (11) a three generation Lindblad decoherence model of neutrinos has been compared against all available experimental data, taking into account the recent results from KamLand experiment indicating spectral distortions.

The results are summarised in Fig. 1 which demonstrates the agreement (left) of our model with the KamLand spectral distortion data, and our best fit (right) for the Lindblad decoherence model used in ref. (11), and in Table 1, where we present the $\chi^2$ comparison for the model in question and the standard scenario.

The best fit has the feature that only some of the oscillation terms in the three generation probability formula have non trivial damping factors, with their exponents being independent of the oscillation length, specifically (11). If we denote those non trivial exponents as $D \cdot L$, we obtain from the best fit of (11):

\[
D = -\frac{1.3 \cdot 10^{-2}}{L},
\]

in units of $1/km$ with $L = l$ the oscillation length. The $1/L$-behaviour of $D_{11}$, implies, as we mentioned, oscillation-length independent Lindblad exponents.

In (11) an analysis of the two types of the theoretical models of space-time foam, discussed in section 2, has been performed in the light of the result of the fit (12). The
Figure 1: **Left:** Ratio of the observed $\nu_e$ spectrum to the expectation versus $L_0/E$ for our decoherence model. The dots correspond to KamLAND data. **Right:** Decoherence fit. The dots correspond to SK data.

Conclusion was that the model of the stochastically fluctuating media (extended appropriately to three generations), so as to be used for comparison with the real data, cannot provide the full explanation for the fit, for the following reason: if the decoherent result of the fit was exclusively due to this model, then the pertinent decoherent coefficient in that case, for, say, the KamLand experiment with an $L \sim 180$ Km, would be $|\mathcal{D}| = \Omega^2 G_N^2 n_0^2 \sim 2.84 \cdot 10^{-21}$ GeV (note that the mixing angle part does not affect the order of the exponent). Smaller values are found for longer $L$, such as in atmospheric neutrino experiments. The independence of the relevant damping exponent from the oscillation length, then, as required by (12), may be understood as follows in this context: In the spirit of (6), the quantity $G_N n_0 = \xi \Delta m^2 / E$, where $\xi \ll 1$ parametrises the contributions of the foam to the induced neutrino mass differences, according to our discussion above. Hence, the damping exponent becomes in this case $\xi^2 \Omega^2 (\Delta m^2)^2 \cdot L/E^2$. Thus, for oscillation lengths $L$ we have $L^{-1} \sim \Delta m^2 / E$, and one is left with the following estimate for the dimensionless quantity $\xi^2 \Delta m^2 \Omega^2 / E \sim 1.3 \cdot 10^{-2}$. This implies that the quantity $\Omega^2$ is proportional to the probe energy $E$. In principle, this is not an unreasonable result, and it is in the spirit of (6), since back reaction effects onto space time, which affect the stochastic fluctuations $\Omega^2$, are expected to increase with the probe energy $E$. However, due to the smallness of the quantity $\Delta m^2 / E$, for energies of the order of GeV, and $\Delta m^2 \sim 10^{-3}$ eV$^2$, we conclude (taking into account that $\xi \ll 1$) that $\Omega^2$ in this case would be unrealistically large for a quantum-gravity effect in the model.
Table 1: $\chi^2$ obtained for our model and the one obtained in the standard scenario for the different experiments calculated with the same program.

|               | decoherence | standard scenario |
|---------------|-------------|-------------------|
| SK sub-GeV    | 38.0        | 38.2              |
| SK Multi-GeV  | 11.7        | 11.2              |
| Chooz         | 4.5         | 4.5               |
| KamLAND       | 16.7        | 16.6              |
| LSND          | 0.          | 6.8               |
| TOTAL         | 70.9        | 77.3              |

We remark at this point that, in such a model, we can in principle bound independently the $\Omega$ and $n_0$ parameters by looking at the modifications induced by the medium in the arguments of the oscillatory functions of the probability (6), that is the period of oscillation. Unfortunately this is too small to be detected in the above example, for which $\Delta \Delta m_{\mu} \ll \Delta m_{12}$.

The second model (9) of stochastic space time can also be confronted with the data, since in that case (12) would imply for the pertinent damping exponent

$$
\left( \frac{(m_1^2 - m_2^2)^2}{2k^2} (9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) + \frac{2V \cos 2\theta (m_1^2 - m_2^2)}{k} (12\sigma_1 + 2\sigma_2 - 2\sigma_3) \right) t^2
\sim 1.3 \cdot 10^{-2}.
$$

(13)

Ignoring subleading MSW effects $V$, for simplicity, and considering oscillation lengths $t = L \sim \frac{2k}{(m_1^2 - m_2^2)}$, we then observe that the independence of the length $L$ result of the experimental fit, found above, may be interpreted, in this case, as bounding the stochastic fluctuations of the metric to $9\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \sim 1.3 \cdot 10^{-2}$. Again, this is too large to be a quantum gravity effect, which means that the $L^2$ contributions to the damping due to stochastic fluctuations of the metric, as in the model of (12) above (9), cannot be the exclusive explanation of the fit.

The analysis of (11) also demonstrated that, at least as far as an order of magnitude of the effect is concerned, a reasonable explanation of the order of the damping exponent (12), is provided by Gaussian-type energy fluctuations, due to ordinary physics effects, leading to decoherence-like damping of oscillation probabilities of the form (11). The order of these fluctuations, consistent with the independence of the damping exponent on $L$ (irrespective of the power of $L$), is

$$
\frac{\Delta E}{E} \sim 1.6 \cdot 10^{-1}
$$

(14)

if one assumes that this is the principal reason for the result of the fit.
However, not even this can be the end of the story, given that the result pertains only to some of the oscillation terms and not all of them, which would be the case expected for the ordinary physics uncertainties. The fact that the best fit model includes terms which are not suppressed at all calls for a more radical explanation of the fit result, and the issue is still wide open. It is interesting, however, that the current neutrino data can already impose stringent constraints on quantum gravity models, and exclude some of them from being the exclusive source of decoherence, as we have discussed above.

Coming back to our main point of discussion in this work, we stress once more that, within the classes of stochastic models we discussed in this work, one can safely exclude the possibility that space-time foam can be at most responsible only for a small part of the observed neutrino mass difference, and certainly the foam-induced decoherence cannot be the sole reason for the result of the best fit, pertaining to a global analysis of the currently available neutrino data. Of course, this is not a definite conclusion because one cannot exclude the possibility of other classes of theoretical models of quantum gravity, which could escape these constraints. At present, however, we are not aware of any such theory.

4. Neutrino Mass differences, Mixing, Space-time Foam and the Cosmological Constant

Since quantum-gravity decoherence can still be accommodated by the current data, despite the above conclusions on the smallness of the percentage of the observed neutrino mass difference due to the space-time foam medium, we would like in this section to speculate on possible implications of these effects to the dark energy budget of our Universe.

In this respect with mention that an approach was suggested in for applying a Fock space quantisation to field theories with mixing. Their formalism, which was performed in flat space time field theories, involved the definition of a new type of Fock-space vacuum, the “flavour vacuum”, \(|0(t)\rangle_f\). This vacuum was not connected with the mass eigenstate vacuum, \(|0(t)\rangle_m\), by a unitary transformation in the field theoretic (thermodynamic) limit, where the volume of the system was taken to infinity. Instead, there is a non-unitary transformation \(G\), connecting these vacuum states, which reads

\[
|0(t)\rangle_f = G^{-1}(t)|0(t)\rangle_m, \quad G(t) = \exp \left( \theta \int d^3x [\bar{\nu}_1(x)\nu_2(x) - \bar{\nu}_2(x)\nu_1(x)] \right),
\]

where \(\theta\) is the mixing angle, \(t\) is the time, and the suffix \(f(m)\) denotes flavour(energy) eigenstates, A Bogolubov transformation was necessary to connect the creation and annihilation operator coefficients appearing in the energy eigenstates with the corresponding ones for flavour eigenstates, which leads naturally to particle creation, and
a “flavour condensate”

\[ V_k = |V_k| e^{i(\omega_{k,1} + \omega_{k,2})t}, \]  

(16)

with \( \omega_{k,i} = \sqrt{k^2 + m_i^2} \), \( f \langle 0|\alpha_{k,i}^{\dag} \alpha_{k,i}|0 \rangle_f = f \langle 0|\beta_{k,i}^{\dag} \beta_{k,i}|0 \rangle_f = \sin^2 \theta |V_k|^2 \) in the two-generation scenario where we restrict our discussion for brevity (in the three-generation case there are various \( V_{ij} \)).

The Flavour Condensate \( |V_k| = 0 \) for \( m_1 = m_2 \), and it has a maximum at \( k^2 = m_1 m_2 \), and for \( k \gg \sqrt{m_1 m_2} \) exhibits the following asymptotic behaviour [3]

\[ |V_k| \sim \frac{(m_1 - m_2)^2}{4|k|^2}, \quad k \equiv |\vec{k}| \gg \sqrt{m_1 m_2} \]  

(17)

The analysis of [22] went one step further to claim that the mass eigenstate vacuum was not the appropriate one to conserve probability, and hence the only appropriate vacuum was the flavour one, which respected this property, but which involved a modified expression for the probability, containing terms proportional to the flavour condensate.

Using this vacuum as the physical one has important cosmological consequences. Indeed, computing the flavour-vacuum average of energy-momentum tensor \( T_{\mu\nu} \) of a Dirac (for definiteness, although the Majorana case also leads to similar results) fermion field in a Robertson-Walker background space-time, leads for the temporal component \( T_{00} \):

\[ f \langle 0|T_{00}|0 \rangle_f = \langle \rho_{\nu-\text{mix}}^\text{vac} \rangle \eta_{00} \equiv \Lambda \eta_{00} \]

\[ = \sum_{i,r} \int d^3 k \omega_{k,i} \left( f \langle 0|\alpha_{k,i}^{\dag} \alpha_{k,i}|0 \rangle_f + f \langle 0|\beta_{k,i}^{\dag} \beta_{k,i}|0 \rangle_f \right) \]

\[ = 8 \sin^2 \theta \int_0^K d^3 k (\omega_{k,1} + \omega_{k,2}) |V_k|^2. \]  

(18)

where \( \eta_{00} = 1 \) in a Robertson-Walker (cosmological) metric background.

A consistent, and physically relevant choice of the cutoff scale has been proposed in [6] to be \( K \equiv k_0 \sim m_1 + m_2 \). This choice is compatible with a decoherence-induced mass difference scenario, since it implies that only the infrared neutrino modes, with momenta less than the typical mass scales \( m_1 + m_2 \), feel mostly the space-time medium effects, since, being slow, they have more time to interact with the gravitational environment.

For hierarchical neutrino models with \( m_1 \gg m_2 \rightarrow k_0 \gg \sqrt{m_1 m_2} \), modes near the cutoff give the dominant contributions to the vacuum energy \( \Lambda \) (due to the divergences involved),

\[ \Lambda \equiv \langle \rho_{\nu-\text{mix}}^\text{vac} \rangle \sim 8 \pi \sin^2 \theta (m_1 - m_2)^2 (m_1 + m_2)^2 \times \]

\[ \left( \sqrt{2} + 1 + \mathcal{O} \left( \frac{m_2^2}{m_1^2} \right) \right) \propto \sin^2 \theta (\Delta m^2)^2 \]  

(19)
This implies that the mixing and mass difference for neutrinos lead to a contribution to the cosmological constant (or better dark energy of the vacuum) of the phenomenologically right order of magnitude.

There are several issues with the above scenario that need to be addressed, before the above considerations are accepted. The first and most important of all is the fact that these calculations have been performed in a flat space time, but the result \cite{19} implies a curved de Sitter space time. Moreover, in \cite{9} the mass difference of neutrinos was assumed from the beginning, although in the approach of \cite{6}, \cite{11}, there is a dynamical component which is due to the (non flat at microscopic scales) space-time foam vacuum.

In view of the particle creation characterising the flavour vacuum in the approach of \cite{9}, our stochastic space time model may be the most appropriate framework where such issues can be discussed in a mathematically and physically consistent setting. In addition to the modifications to the oscillation probability appearing in the approach of \cite{22}, in our case there are the CPT violating decoherence modifications (damping), which imply a microscopic time irreversibility, and a non-unitary evolution. Moreover, as discussed in \cite{23}, the presence of dark energy contributions in space time imply additional damping factors in the oscillation probability. In this respect, the non unitarity involved in the definition of the flavour vacuum \cite{16} may acquire important physical meaning.

Several other issues deserve careful study, among which a detailed and proper study, using curved space time techniques, of the equation of state characterising the neutrino fluid in the presence of the foam. The flat space time attempt of \cite{24} is not, in our opinion, sufficient to give a complete and consistent answer for this specific question, which is important to cosmologists. We hope to come back to such important questions in the near future.

5. Conclusions and Outlook

This conference celebrated 50 years from the neutrino discovery. Ever since its discovery this elusive particle keeps surprising us. At first, scientists thought that energy was not conserved in the nuclear $\beta$-decays, before Pauli makes the decisive suggestion on the presence of the neutrino.

In the standard model of particle physics the neutrino appears massless, but during the last decade a plethora of delicate experiments have shown unambiguously (albeit indirectly) the existence of a neutrino mass, by measuring oscillations, thereby allowing for estimates of the mass differences.

The origin of such a mass and mass differences are still major issues, and intense research is at present under way in order to tackle such questions. In refs. \cite{6}, \cite{11}, \cite{12}, we have put forward a conjecture, reviewed in this talk, according to which part of the observed mass differences of neutrinos might be due to completely new physics,
that of Quantum Gravity. As we have discussed in this talk, stringent constraints can be imposed by the current data on the proportion of the neutrino mass difference that could be due to such effects. Nevertheless, experiments are still compatible with the presence of a space time foam medium, responsible for neutrino decoherence and generation of part of the mass differences between neutrino flavours.

This brings in other interesting scenarios on possible links of neutrinos with a dark energy component of the Universe of an unconventional origin, consistent with the current phenomenology. Interesting questions as to what type of quantum fluid neutrinos actually constitute, are still probably far from being completely understood, in view of the possible mixing of neutrinos with the quantum-gravity foam. A peculiar proportionality relation between a Dark Energy component of the Universe and the product of the sum of the neutrino mass differences $\Delta m^2$ times some trigonometric factors of the mixing angle has been proposed, which still appears compatible with the data. All these issues can be understood rigorously only if one formulates the problem of quantum field theory mixing in curved (de Sitter) space times.

In view of these considerations, Neutrino Physics may provide a very useful guide in our quest for a theory of Quantum Gravity, in particular stringent constraints on CPT Violation (or better, microscopic time irreversibility). As discussed above, the latter may not be an academic issue, but a real feature of Quantum Gravity. As we have reviewed in this work, the scenario of three-generation neutrino decoherence plus mixing is still compatible with all the existing neutrino data, including KamLAND spectral distortions. It yields decoherence damping factors of a peculiar behaviour (independent of the oscillation length), which still calls for a rigorous explanation.

Clearly neutrino physics hides many more surprises and mysteries, some of which may be revealed already in the next round of experiments. Who knows what opinion about these fascinating particles scientists would have formed by the date we shall celebrate the centenary of the experimental neutrino discovery. One thing is certain, though, that much more work, both theoretical and experimental, should be done before definite conclusions are reached concerning the precise nature and properties of neutrinos.

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