The Nature of the Eccentric Double-lined Eclipsing Binary System KIC 2306740 with Kepler Space Photometry

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Abstract

We present a detailed study of KIC 2306740, an eccentric double-lined eclipsing binary system. Kepler satellite data were combined with spectroscopic data obtained with the 4.2 m William Herschel Telescope (WHT). This allowed us to determine precise orbital and physical parameters of this relatively long period (P = 1043) and slightly eccentric (e = 0.3) binary system. The physical parameters have been determined to be M1 = 1.194 ± 0.008 M⊙, M2 = 1.078 ± 0.007 M⊙, R1 = 1.682 ± 0.004 R⊙, R2 = 1.226 ± 0.005 R⊙, L1 = 2.8 ± 0.4 L⊙, L2 = 1.8 ± 0.2 L⊙, and orbital separation a = 26.20 ± 0.04 R⊙ through simultaneous solutions of the Kepler light curves and WHT radial velocity data. Binarity effects were extracted from the light curve in order to study intrinsic variations in the residuals. Five significant and more than 100 combination frequencies were detected. We modeled the binary system assuming nonconservative evolution models with the Cambridge STARS (TWIN) code, and we show the evolutionary tracks of the components in the log L−log T plane, the log R−log M plane, and the log P−age plane for both spin and orbital periods together with eccentricity e and log R⊙. The model of the nonconservative processes in the code led the system to evolve to the observed system parameters in roughly 5.1 Gyr.

Unified Astronomy Thesaurus concepts: Detached binary stars (375); Binary stars (154); Eclipsing binary stars (444)

1. Introduction

Double-lined eclipsing detached binary stars are an important source for accurately determining the physical parameters of their component stars (Torres et al. 2010). Pulsations can be used for determining physical parameters as well as understanding stellar structure. Pulsating components in binary systems play an important role in understanding stellar structure because they are effectively laboratories for investigating stellar interiors (Aerts 2013). Therefore, having a pulsating component in a binary provides independent verification of stellar parameters. Using continuous high-precision observations from the CoRoT, Kepler, and TESS satellites’ data provides the opportunity to study a variety of pulsating stars in binary systems. Recently, many observational results for different types of pulsating stars, including those of binary components, have been studied in the literature (e.g., Wood et al. 2004; Yakut et al. 2007; Welsh et al. 2011; Maceroni et al. 2014; Murphy et al. 2018; Qian et al. 2018; Johnston et al. 2019). Pulsating stars in close binary systems have been discussed in detail by many authors (e.g., Zahn 1975; Aerts & Harmanec 2004; Breger 2005; Reed et al. 2005; Aerts et al. 2010; Huber 2015; Southworth et al. 2020).

The Kepler satellite observed more than 200,000 stars, including some with planetary companions, binary/multiple stellar systems, and pulsating stars, to obtain very-high-precision photometry (Borucki et al. 2010; Gilliland et al. 2010; Koch et al. 2010; Brown et al. 2011). These photometric results have found previously unknown variations, providing further constraints to current models. The precision of the Kepler observations allows us to disentangle low-amplitude variation in a binary star system. One such system is KIC 2306740, which will be the focus of this work.

KIC 2306740 (P = 1043, e = 0.3, V = 13.008, Kp = 13.055) is an eccentric double-lined detached eclipsing binary system that was discovered by the Kepler satellite. Some basic parameters for the system given in Table 1. The first preliminary binary solution of the system was found by Prsa et al. (2011). They refined the orbital period to 10430399 ± 0400003 and temperature ratio (T2/T1) to 0.834. Kjurkchieva et al. (2017) subsequently estimated the relative radii of the components as r1 = 0.612 and r2 = 0.600, mass ratio as 0.972, orbital eccentricity as 0.299, and the argument of periastron as 275°. However, the parameters obtained in the current work are quite different from those two studies. This is because of the careful interactive analysis of the light curve (LC) and radial velocity (RV) data made in this study, rather than the automated LC modeling.

In this paper, we study the binary nature of the system as well as its rotational behavior using the Kepler data combined with a set of high-precision RVs. KIC 2306740 was observed by Kepler in quarters Q0 to Q16. The new spectroscopic observations and data analysis of RVs are described in Section 2. We present the Kepler data and LC solution of the system in Section 3. Using the RV and LC solution, we obtained the physical parameters of the system in Section 4. Light variation outside eclipses is discussed in Section 5. Section 6 contains a discussion of the possible evolutionary state of the system and our conclusions.
Table 1
Basic Parameters for KIC 2306740

| Parameter          | Value                        |
|--------------------|------------------------------|
| 2MASS ID           | 19290475+3741535             |
| Gaia ID            | 205188503320809216           |
| $\delta_{2000}$    | 19 29 04.75                  |
| $\epsilon_{2000}$  | +37 41 53.5                  |
| $R$                | 13$^{\oplus}$ 91             |
| $V$                | 13$^{\oplus}$ 08             |
| $R_{\odot}$        | 12$^{\oplus}$ 92             |
| $G$ (Gaia)         | 13$^{\oplus}$ 473            |
| $J$ (2MASS)        | 12$^{\oplus}$ 297            |
| $H$ (2MASS)        | 12$^{\oplus}$ 022            |
| $K_{s}$(2MASS)     | 11$^{\oplus}$ 958            |
| $K_{s}$(Kepler)    | 13$^{\oplus}$ 545            |
| $E_{B-V}$          | 0$^{\oplus}$ 117             |
| Period             | 10.31 days                   |
| $\pi$ (mas)        | 0.6606                       |

Note. $B$ and $V$ color values are taken from Zacharias et al. (2005), and other parameters are taken from the Kepler input catalog, Gaia, and simbad.

2. Spectroscopic Observations

We obtained spectra of KIC 2306740 at 15 epochs with the Intermediate dispersion Spectrograph and Imaging System (ISIS) on the William Herschel Telescope (WHT), in 2012 July. These were timed to provide the best possible coverage of the orbital phases, given that the orbital period of 10.3 days is significantly longer than the duration of the observing run of 7 nights. Two epochs occurred close to eclipse when the velocity separation of the stars was small. These measured RVs are strongly affected by line blending and so were not used in our analysis.

WHT is equipped with the double-armed ISIS. Spectra were taken simultaneously in the blue and red arms, covering the regions around H$_{\gamma}$ and H$_{\alpha}$. In the blue arm, we used the grating H2400B, with a wavelength coverage of 4200–4550 Å. In the red arm, the R1200R grating was used and gave coverage of 6100–6730 Å. The slit was set to 0.5 in order to limit the effects of telescope-pointing errors so that a resolving power of $R \approx 22,000$ is achieved. We used exposure times of 1500 s for all spectra to give a signal-to-noise ratio (S/N) of roughly 30 per resolution element in the blue and 80 in the red. We bracketed each with spectra of CuAr+CuNe arc lamps for wavelength calibration. The data were reduced using the PAMELA package (Marsh 1989).

To measure the RVs of the two stars from these spectra (see Table 2), we used standard cross-correlation (e.g., Tonry & Davis 1979) and its two-dimensional extension TODCOR (Zucker & Mazeh 1994). Synthetic template spectra were calculated with the UCLSYN code (Smith 1992; Smalley et al. 2001) and ATLAS9 model atmospheres for metallicity, with a $T_{eff}$ of 5500 K and no rotational broadening. For our final RVs, we adopt those given by standard cross-correlation. These are very similar to those obtained with TODCOR. RV measurements for KIC 2306740 components are listed in Table 2 and plotted in Figure 1.

3. Kepler Observations of the System and Modeling of the Light Curve

The system was observed over approximately 1460 days during 17 quarters (Q0 to Q16) with a long cadence (exposure time of about 30 minutes) and a total of 64,370 data points were obtained using the satellite. The LC shows deep eclipses with periods of totality, plus periodic variations due to pulsations. Kepler satellite observations show some fluctuations due to common instrumental effects (Jenkins et al. 2010). Using the techniques outlined in Jenkins et al. (2010), cotrending and detrending were applied to eliminate systematic variations. We studied each quarter separately and, to detrend the data, a third-order polynomial fit was applied as we did in our earlier Kepler study (Yakut et al. 2015; Çokluk et al. 2019). The raw data of KIC 2306740 are shown in Figure 2 (upper panel). The detrended normalized light variation is shown in Figure 2 (lower panel). The quarters are shown in different colors.

Figure 1. The radial velocity observations of KIC 2306740 as a function of phase. The filled and open circles represent the velocities of the primary and the secondary component, respectively. The residuals are shown in the bottom panel. The data are listed in Table 2, and the curve fitting corresponds to the elements given in Table 3.

Table 2
Radial Velocity Measurements for KIC 2306740

| HJD (2456000+) | Phase | $V_1$ (km s$^{-1}$) | $(O - C)_1$ (km s$^{-1}$) | $V_2$ (km s$^{-1}$) | $(O - C)_2$ (km s$^{-1}$) |
|----------------|-------|--------------------|--------------------------|--------------------|--------------------------|
| 87.53965       | 0.76260 | 71.76              | 1.98                     | -39.20             | 1.98                     |
| 87.62647       | 0.77085 | 71.16              | 2.91                     | -36.82             | 2.91                     |
| 87.70422       | 0.77857 | 67.24              | 0.46                     | -37.60             | 0.46                     |
| 88.48349       | 0.85417 | 51.19              | 0.08                     | -16.19             | 0.08                     |
| 88.57612       | 0.86316 | 48.36              | -0.81                    | -15.99             | -0.81                    |
| 88.70547       | 0.87571 | 45.49              | -0.95                    | -12.52             | -0.95                    |
| 91.52893       | 0.14965 | -13.81             | -1.63                    | 55.10              | -1.63                    |
| 91.58456       | 0.15304 | -17.25             | -4.00                    | 56.01              | -4.00                    |
| 91.63997       | 0.16042 | -16.16             | -1.85                    | 60.03              | -1.85                    |
| 92.53903       | 0.24765 | -32.60             | -2.67                    | 77.83              | -2.67                    |
| 92.59339       | 0.25292 | -35.81             | -5.08                    | 77.17              | -5.08                    |
| 92.65527       | 0.25893 | -36.34             | -4.73                    | 78.21              | -4.73                    |
| 92.71038       | 0.26427 | -36.34             | -3.97                    | 79.82              | -3.97                    |
Using the Kepler observations we derived the linear ephemeris given in Equation (1):

$$\text{HJD Min I} = 2456399.1227 \pm 0.2 \times 306988.2.$$  

During the calculation of the orbital phases in Figures 2–3 and Table 4, we used Equation (1).

The Kepler LCs along with the WHT RV curves were modeled simultaneously with the JKTEBOP code (see Southworth et al. 2004; Southworth 2013) and also with the PHOEBE (Prša & Zwitter 2005) program, which uses the WILSON–DEVINNEY (W-D) code (Wilson & Devinney 1971). The curve-dependent weights were assigned as described by Wilson (1979). We ran the code assuming a detached configuration. During analysis, we iteratively solved the LC and RV curves: the LC gave a more accurate estimate of the eccentricity ($0.30 \pm 0.01$) than did the RV curves ($0.32 \pm 0.02$), and so we fixed $e$ to this photometric solution when re-solving the RV curves. Even though Kepler data are very sensitive, they are all obtained in a single filter. This prevents us from determining accurate temperatures from multiple color analyses. Because the spectral data obtained are not sufficient to determine a precise temperature, the temperature of the hotter star was fixed to the 6060 K found in Armstrong et al. (2014).

To determine the uncertainties of the measured properties of the KIC 2306740 system, we turned to JKTEBOP as it is much faster than PHOEBE. JKTEBOP and derivatives of the W-D code have been found to yield results consistent at the 0.2% level or better in well-separated systems (Maxted et al. 2020). For computational efficiency, we phase-binned the data by sorting them into orbital phases and combining each group of 20 consecutive data points, giving a total of 3219 data points.

The phase-binned data and the RVs were modeled using JKTEBOP. The fitted parameters were the fractional radii, orbital eccentricity, inclination, argument of periastron, central surface brightness ratio of the stars, third light, the linear limb-darkening coefficient of the primary star, and the velocity amplitude of each star and systemic velocity of the system. Limb darkening was implemented using the quadratic law for both stars. Numerical integration was used to account for the fact that the data were obtained in long cadence by the Kepler satellite (see Southworth 2011). The fractional radii were fitted using their sum and ratio as these are less correlated. The orbital eccentricity $e$ and argument of periastron $\omega$ were fitted using the Poincaré parameters $e \cos \omega$ and $e \sin \omega$, for the same reason. Uncertainties were calculated for each fitted parameter and for each derived parameter in this study. This was done in two ways: using Monte Carlo and residual-permutation simulations (Southworth 2008). The uncertainties from the residual-permutation algorithm were found to be larger by typically a factor of 1.5 than those from the Monte Carlo algorithm, and so were adopted as the final error bars.

Simultaneous LC and RV solutions were made using the full Q0–Q16 data and the analyses are summarized in Table 4. Figure 1 shows the observed and computed RV curves for both

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*Figure 2. Kepler Q0 to Q16 observations of KIC 2306740 raw data (upper panel) and detrended (lower panel) data. The quarters are shown in different colors.*

*Figure 3. (a) Kepler full data set observation (blue dots) and computed (red line) LC of the system. Zoomed secondary (b) and primary (c) minima are shown to emphasize the agreement and residuals between the observed values and the corresponding LC model (d).*
components of the system along with residuals. In Figure 3, the computed LCs are shown by solid lines. Prša et al. (2011) gave preliminary orbital parameters for 1879 Kepler binary systems, including KIC 2306740. They obtained a temperature ratio of 0.86, a sum of the fractional radii of 0.1374, and a sin i of 0.99919. Our analysis includes RVs as well as much more extensive LCs, so the results given in Table 4 differ from those found by Prša et al. (2011).

4. Physical Parameters of the Components

The physical parameters of a binary system can best be derived if it is a double-lined eclipsing binary system with an accurate LC. Hence, the detached binary system KIC 2306740, for which the photometric and spectroscopic data are both of high precision, is excellent for accurate determination of its parameters. The detailed LC solution of the system indicates that the stars are well detached from their Roche lobes (see Section 3).

In this study, RVs and LCs were analyzed simultaneously, and the orbital parameters of the system were obtained. With the measurements given in Tables 3 and 4, we can estimate the physical parameters of the components given in Table 5. We have used the JKTABSDIM code to estimate the physical parameters of the components, with uncertainties propagated from the LC and RV solution using a perturbation analysis. The nominal physical constants and solar properties recommended by the IAU were used (Prša et al. 2016). All of the calculated parameters of the binary system are summarized in Table 5 with their estimated errors.

The masses of the two stars are slightly greater than solar, and the radii are significantly larger. The hotter star is the less massive and has a smaller radius. This indicates that the larger star is near the end of its main-sequence lifetime. We discuss this in Section 6.

5. Light Variation outside Eclipse

Investigating the sinusoidal brightness variation in the LC requires the extraction of the effects of binarity from the LC. After making the simultaneous LC and RV analysis, we subtracted the binary model from the observations. The residuals from the phased light variation of the binary are plotted in Figure 5. The oscillatory pattern can be seen clearly.

Because programs using modern LC modeling were not perfect at representing Kepler data, we performed a frequency analysis on all the long-cadence data obtained out of eclipse using the SIGSPEC (Reegen 2007) and PERIOD04 (Lenz & Breger 2005) codes, which are based on classical Fourier analysis. An S/N > 4 threshold was chosen as a criterion to consider a frequency as significant (Breger et al. 2011). We searched for significant peaks in the frequency interval from 0 to the Nyquist frequency of 25 day$^{-1}$ but found no meaningful peak above 4 day$^{-1}$. Figure 4(a) shows the amplitude spectrum before prewhitening of any frequency between 0 and 25 day$^{-1}$. Higher-amplitude peaks gather below a frequency of 5 day$^{-1}$. We continued to obtain prewhitened frequencies until the signal amplitudes fell below four times the average noise. Figure 4(b) represents the spectra after prewhitening.

Table 6 lists these genuine frequencies, their amplitudes, phases, and S/Ns sorted by decreasing amplitude. S/Ns were computed over an interval of 5 day$^{-1}$. In the top panel of Figure 5, the agreement between 100 calculated frequencies and the observational data are plotted for almost 4 yr. The bottom panel illustrates the zoomed data of 10 days for clarity. The analysis resulted in the detection of five genuine and more than 100 combination frequencies.

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**Table 3**: Spectroscopic Orbital Parameters of KIC 2306740

| Parameter     | $e$ not Fixed | $e$ Fixed at 0.301 |
|---------------|--------------|-------------------|
| $T_0$/days    | 2456399.19(24)| 2456399.21(25)   |
| $P$/days      | 10.3069(78)  | 10.3075(81)       |
| $e$           | 0.322(22)    | 0.301             |
| $\omega$/rad  | 4.81(3)      | 4.80(3)           |
| $K_1$/km s$^{-1}$ | 65.78(79)   | 63.98(76)         |
| $K_2$/km s$^{-1}$ | 72.84(87)   | 70.86(84)         |
| $V_c$/km s$^{-1}$ | 18.62(2)    | 18.62(2)          |
| $q = m_1/m_2$ | 1.1073(90)   | 1.1074(88)        |
| $a_1 \sin i/R_0$ | 12.682     | 12.425            |
| $a_2 \sin i/R_0$ | 14.043     | 13.761            |
| $m_1 \sin^3 i/M_1$ | 1.304    | 1.193             |
| $m_2 \sin^3 i/M_2$ | 1.178     | 1.077             |

**Table 4**: Fitted and Parameters for KIC 2306740 from the JKTEBOP Analysis

| Parameter                                      | Value       |
|-----------------------------------------------|-------------|
| **Fitted parameters:**                        |             |
| Sum of the fractional radii                  | 0.11102 ± 0.00019 |
| Ratio of the radii                           | 0.7289 ± 0.0025 |
| Central surface brightness ratio             | 0.9745 ± 0.0057 |
| Orbital inclination ($^\circ$)               | 89.670 ± 0.073 |
| $e \cos \omega$                              | 0.02329 ± 0.00001 |
| $e \sin \omega$                              | −0.3002 ± 0.0012 |
| Third light                                   | 0.0918 ± 0.0079 |
| Velocity amplitude of the cool component (km s$^{-1}$) | 64.00 ± 0.28 |
| Velocity amplitude of the hot component (km s$^{-1}$) | 70.87 ± 0.23 |
| Systemic velocity of the cool component (km s$^{-1}$) | 18.70 ± 0.02 |
| Systemic velocity of the hot component (km s$^{-1}$) | 18.57 ± 0.02 |
| **Derived parameters:**                       |             |
| Fractional radius of the cool component       | 0.06421 ± 0.00006 |
| Fractional radius of the hot component        | 0.04681 ± 0.00017 |
| Light ratio                                   | 0.518 ± 0.007 |
| Orbital eccentricity $e$                      | 0.3011 ± 0.0012 |
| Argument of periastron $\omega$ ($^\circ$)    | 274.44 ± 0.02  |
| Mass ratio                                    | 0.903 ± 0.004  |

**Table 5**: Astrophysical Parameters of KIC 2306740

| Parameter                                      | Cool Component | Hot Component |
|-----------------------------------------------|----------------|---------------|
| Mass ($M/M_\odot$)                            | 1.194(8)       | 1.078(7)      |
| Radius ($R/R_\odot$)                          | 1.682(4)       | 1.226(5)      |
| Temperature log$_{10}$(T$_{eff}$/K)            | 3.764(18)      | 3.782(17)     |
| Luminosity log$_{10}$(L/L$_\odot$)            | 0.449(75)      | 0.260(72)     |
| Surface gravity log$_{10}$(g/cm s$^{-2}$)      | 4.063(2)       | 4.294(4)      |
| Bolometric magnitude ($M_\odot$)               | 3.61(19)       | 4.08(18)      |
| Absolute magnitude ($M_\odot$)                 | 3.69           | 4.09          |
| Semimajor axis ($a/R_\odot$)                  | 26.201(44)     |               |

**Note**: The standard 1σ errors of the last digits are given in parentheses.
What could be the mechanism that caused a change in the maximum amplitude of the KIC 2306740 system? Generally, such changes may result from stellar pulsating and/or periodic changes. In binary star systems, inhomogeneous structures (e.g., stellar spots) on the surface of one or both of the component stars can cause changes in the LC in asynchronous situations, known as rotational variability. For the frequencies obtained from the Fourier analysis of the light variation of the KIC 2306740 system (Table 6), periods of approximately 0.7, 3.4, and 10.3 days were obtained. The 10.3 day period is related to the orbital period, and the 3.4 day period is related to the spin period of the stars (see Section 6). The source of the 0.7 day periodicity may be $\gamma$ Dor–type pulsation or spot modulations on one or both of the component(s). Besides, looking into the out-of-eclipse light variation of the system, we analyzed the minimum phases of the LCs. There is a variation with an amplitude of 0.03 at the primary minimum and a variation with an amplitude of 0.013 at the secondary minima. However, residual data are not sufficient to estimate new frequencies.

### 6. Results and Conclusion

We have modeled the light and RV curves of the well-detached binary system KIC 2306740 and determined its orbital and physical parameters (Tables 4 and 5). Solutions indicate that the more massive and cooler component of the system is more evolved than the hotter, less massive component. Recently, Gaia gave a parallax of $0.6606 \pm 0.0165$ mas (Gaia Collaboration 2018) for KIC 2306740 (Gaia DR2 2051885033280089216). Using this Gaia parallax and our results, we derive a distance modulus of $m_V - M_V = 11.0$ and magnitudes for the components and the system of $V_1 = 14.97 \pm 0.10$, $V_2 = 15.50 \pm 0.12$, and $V_{\text{total}} = 14.50 \pm 0.11$. The new stellar parameters and reddening reveal the distance of the system to be 1.53 kpc, which is very close to the distance obtained by Gaia as 1.51 kpc. With these results, we can add the system to the list of well-determined binary stars. The maximum light shows cyclical variations.

### Table 6

| Frequency (day$^{-1}$) | Amplitude (mmag) | Phase ($^\circ$) | S/N |
|------------------------|------------------|-----------------|-----|
| 1.3640248 (1)          | 2.9497 (1)       | 3.6848 (1)      | 860 |
| 1.36287601 (1)         | 1.0037 (1)       | 1.5393 (3)      | 224 |
| 1.3693053 (2)          | 0.3028 (1)       | 2.9146 (8)      | 82  |
| 0.29110156 (4)         | 0.1311 (1)       | 5.7776 (19)     | 32  |
| 0.0970908 (5)          | 0.1103 (1)       | 1.9516 (25)     | 22  |

Note. Frequencies with signal-to-noise ratios (S/N) exceeding 4 are considered as significant.
100 frequencies, of which 5 are significant noncombination frequencies. The results from this study indicate that the system KIC 2306740 may contain a nonradial pulsating $\gamma$ Dor-type star that pulsates in high-order gravity modes. Considering the other frequencies (~3.4 days, half of the spin periods) obtained, we have found that this is related to the predicted spin period of ~6.8 days (see Figure 6).

We have found that the component stars are related to the spin periods (~6.8 days) obtained (see Figure 6(c)). This tells us that at least one of the components has changed due to inhomogeneous structures on its surface. This also tells us that spot modulation is a possible explanation for the variation at the maxima phases, instead of pulsations. The parameters we have obtained (Table 5) show that the components of binary system KIC 2306740 are outside or on the edge of the instability zone in the H-R diagram. Therefore, the light variation seen in this system is likely caused by spot modulation.

We use this system to test the theory of modeling stellar interiors by comparing it with its observational properties. Models were constructed with the EV code (Eggleton & Kiseleva-Eggleton 2002) and its much more powerful TWIN variant (Yakut & Eggleton 2005; Eggleton 2006, 2010), both of which are based on the Cambridge STARS code (Eggleton 1971, 1972, 1973; Pols et al. 1995). In single-star evolution, as in the STARS code, the effects of rotation and magnetic dynamo activity on mass loss and the proximity of the companion are usually not considered. TWIN allows various nonconservative processes to be applied to the primary component of a binary system. Both components are modeled simultaneously so that the effects of tidal friction, magnetic dynamo activity, and hence mass loss, can be included according to a self-consistent prescription. Mass loss carries off angular momentum by way of magnetic braking.

In the case of nonconservative evolution, it is hard to find initial parameters that would lead to the current binary system (see Eggleton & Yakut 2017). However, we can reasonably assume that the initial masses were larger than now. Table 7 gives parameters for a model that is slightly metal rich ($Z = 0.03$) compared to the Sun. This metallicity gives a somewhat better fit than solar.

After some experimentation, we evolved a pair of stars with initial masses of 1.21 and 1.08 $M_\odot$, each with spin periods of 3.0 days, an eccentricity of 0.34, and an orbital period of 10.30 days. We expect the orbital period to decrease as the orbit circularizes and as angular momentum is lost by wind mass loss and magnetic braking. However, the period also increases.

### Table 7

| Parameter | Zero Age | Age 5.12 Gyr | Observed |
|-----------|----------|--------------|----------|
| $P$/days  | 10.30    | 10.32        | 10.307   |
| $e$       | 0.337    | 0.293        | 0.301    |
| $M_1/M_\odot$ | 1.210  | 1.176        | 1.194    |
| $\log_{10}(R_1/R_\odot)$ | 0.058  | 0.206        | 0.225    |
| $\log_{10}(L_1/L_\odot)$ | 0.208  | 0.432        | 0.449    |
| $\log_{10}(T/K)$ | 3.785  | 3.767        | 3.764    |
| $M_2/M_\odot$ | 1.083  | 1.062        | 1.078    |
| $\log_{10}(R_2/R_\odot)$ | $-0.013$ | 0.071     | 0.088    |
| $\log_{10}(L_2/L_\odot)$ | $-0.005$ | 0.194     | 0.260    |
| $\log_{10}(T/K)$ | 3.767  | 3.775        | 3.782    |

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**Figure 6.** Evolutionary tracks for our best model of KIC 2306740. Our data for KIC 2306740 are shown as squares. Panel (a) shows the evolution in the $\log L$–$\log T$ plane. The primary and secondary tracks are shown with red and green lines, respectively. The extent of the red and green pluses represents roughly the observational uncertainties. Panel (b) shows the primary’s radii (red) and lobe radii (dark blue) and the secondary’s radii (green) and lobe radii (light blue) as a function of mass. Panel (c) shows the time evolution of the orbital period (dark blue), a spin period of the primary (red) and the secondary (green), eccentricity (light blue), and radius of the primary (black).
on account of the orbit acquiring some angular momentum from the spins of the stars by tides. The particular model of the nonconservative processes in the TWIN code led the system to evolve to roughly the observed masses and radii in about 5.1 Gyr.

Figure 6 shows evolutionary tracks in three planes, (a) the $\log L$--$\log T$ plane or theoretical H-R diagram, (b) the $\log R$--$\log M$ plane, and (c) the $\log P$--$\log$ plane for both spin and orbital periods together with eccentricity and $\log R_1$. Our data for KIC 2306740 are represented by squares in panels (a) and (b). Each square is surrounded by a cloud of pluses generated by a Gaussian random number generator to illustrate the extent of the standard errors in the basic observational data ($K_1$, $K_2$, $T_1$, ...). The evolutionary tracks for the more massive star (1) are red and for the less massive star (2) green. A blue circle approximately marks the best fit to $\sim 1$ and a blue asterisk the coeval point for $\ast 2$. Panel (b) shows the Roche lobe radii as well as the actual radii. Neither star is within a factor of 5 of its Roche radius. Panel (c) shows the spin periods in red and green. Both were started arbitrarily at 3 days and rather rapidly evolved to about 6 days before reaching a plateau. During this fairly rapid initial spin down, the components lost about 0.03 and 0.02 $M_\odot$. The logarithm of the orbital period is in dark blue, the eccentricity is light blue, and the radius of star 1 is black. Table 7 shows the evolutionary changes in some major variables. Our overall conclusion from Figure 6 is that the fit of to the theoretical model is acceptable at a 1σ level, but it would be better if the model temperatures matched more closely those observed.

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