How Friedmann Shod\textsuperscript{1} Einstein

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Abstract
Exactly one hundred years ago, in May 1922, Alexander Friedmann
discovered that General Relativity (GR) predicts non-stationary Universe. His equation describing the evolution of the Universe is now named after him. In this paper we briefly recall the human and scientific aspects of this revolutionary change in our picture of the world. We also recall that immediately after finishing his seminal paper Friedmann wrote a book on GR for a wide audience. In that book he presented an impressive picture of the expanding Universe created from nothing.

The man of talent is like the marksman who hits a mark the others cannot hit; the man of genius is like the marksman who hits a mark they cannot even see to...

Arthur Schopenhauer.

*The World as Will and Idea.

Let me start with a personal remark. We, former students of the Physics Department of the Moscow State University, have a tradition to meet on

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the stairs of our _alma mater_ every five years. Some of us or our professors deliver their speeches in the Main Physics Auditorium. Once, the speaker was Yuri Grigorievich Pavlenko, the former superviser of some five hundred of my classmates. He said that among the Soviet physicists nobody had made a really great discovery. Nobody objected. We remembered Nobel laureates Lev Landau, Piotr Kapitza, Nikolai Basov, Alexander Prokhorov, Pavel Cherenkov and many other outstanding scientists, such as Nikolai Bogolyubov. We remembered their discoveries in the superfluidity, theory of superconductivity, lasers, and many other fields, but concluded that anyway their great achievements were less important than discoveries made by Planck, Einstein, Bohr, Heisenberg, Pauli, Shrödinger, Dirac, Feynman… But later, while preparing lectures on the gravitation theory for our PhD students, I have read articles by Alexander Friedmann (available in German\[1, 2\], in Russian\[3, 4\] and in English\[5, 6\]), and his small book (available in Russian\[4\] and in English\[7\]) “The World as Space and Time” written in 1922-23, and it became clear to me that our superviser was wrong.

Scientific discoveries are made in different ways. New theoretical ideas often arise on the basis of many experiments or observations, after obtaining and studying large amounts of data. However, in the last couple of centuries a new method of discovering the hidden secrets of Nature has emerged. It is called mathematics. These new discoveries in physics achieved by mathematical methods differ from audacious guesses of ancient Greeks such as, for example, an idea of atomic structure of matter. Now we know that their brilliant insights frequently led to erroneous views.

Let me give you a few examples.

In 1862-63, James Clerk Maxwell not only united all known laws of electricity and magnetism but also proposed a new law by introducing in one of his equations the displacement current. Probably, his guess appeared from the mathematical criterion of symmetry and beauty that the system of equations should satisfy. The physical meaning of the term added by Maxwell was profound and far-reaching. With this addition Maxwell predicted the new physical phenomenon — electromagnetic waves, and explained the nature of light. The experimental confirmation of this theoretical discovery appeared in 1887-88 only in the experimental work of Heinrich Hertz. Maxwell’s theory of electromagnetism laid a “golden egg” from which the Special Relativity, hidden inside his equations, had eventually hatched.

Another example is the Dirac equation. It worked wonderfully well in uniting Special Relativity with Quantum Mechanics and in explaining the
spin and magnetic moment of the electron. But it also produced a great
and absolutely unexpected prediction: antimatter. Anti-electron (positron)
was first observed in 1932, anti-proton in 1955, whereas anti-nuclei He$_3$
and anti-atoms H$_1$ were observed later.

The third example is the main topic of this article. As in the two pre-
viously discussed examples, a new theory emerged in new mathematical
clothes. Again, in order to understand a new theory physicists had to learn
new mathematics. If for the Maxwell theory this new mathematics was the
vector analysis, for the Dirac equation it was spinor algebra. To understand
the GR one should study tensor analysis and Riemannian geometry. Nikolay
Lobachevsky with his “A Concise Outline of the Foundations of Geometry”
(printed by Kazan University in 1829) was the first who published a work
on non-Euclidean geometry. Three years later, in 1832, the same ideas inde-
pendently developed by a Hungarian mathematician Janos Bolyai were also
published. “The King of Mathematics” Johann Carl Friedrich Karl Gauss
also invented and developed non-Euclidean geometry but did not publish his
work.
What was the most apparent use of this abstract mathematics? Everybody will answer that it was used in the creation of General Relativity (GR). But how did the GR appear? From the physics point of view, the problem amounted to merging together the Galilean relativity, Newtonian gravity, and the Faraday-Maxwell field theory. This synthesis was achieved by Einstein, a revolutionary in science, as Lev Bronstein (Trotsky), born in the same year – 1879, was a revolutionary in social life.

However, we should not forget that Einstein in his work heavily relied on the results achieved by mathematicians Marcel Grossmann and David Hilbert. The first considerable step to the GR was done by Marcel Grossmann and Albert Einstein in 1913 together[8]. The final step was done simultaneously by Hilbert[9] and Einstein[10] as a result of intensive interaction and correspondence between them. A detailed historical study may be found in articles[11]. It is worth reminding that the GR was born in 1915 when almost the whole Europe and many other countries were involved in the First World War.

And how did Russian scientists react to the emergence of GR? There was no reaction at all. They simply were not informed. At the end of 1915 Germany was in war with Russia. The postal services between the countries were interrupted, scientific journals from Germany were not delivered to Russian
libraries until 1921. What about Alexander Friedmann? He was an officer on the front line organizing the aerial reconnaissance and bombardment of the Österreich Army positions. For his services Friedmann was decorated with St. George Cross. In the postwar life he remained a brave man. For example, in 1925, together with a pilot Friedmann at the risk of life performed a record balloon flight to the height of 7400 meters.

There was another Russian scientist well informed about GR in 1915. Baron Vsevolod Konstantinovich Frederiks studied at Göttingen University when the war started, and was about to be imprisoned. However, David Hilbert, who was one of the greatest mathematicians of the XX century, vouched for Frederiks and gave him a job as his private assistant. Hilbert paid Frederiks’ salary himself. Since Hilbert was one of the creators of this new theory of gravitation, Göttingen was the right place to learn the GR, and Frederiks started to study it immediately. Frederiks published his first review article[12] on the GR in 1921.

In 1920, when the world and Russia started to recover from the war catas-
trophes and revolution, Friedmann and Frederiks met in Petrograd. By that time Frederiks returned from Germany and Friedmann came back from the city of Perm where he survived the Civil War lecturing at the Perm university. When the Red Army approached Perm, the university was evacuated to Siberia, but on the way Friedmann changed his mind and decided to go back to Perm and then to Petrograd. At their meeting Friedmann and Frederiks decided to study General Relativity (they called it the Big Relativity Principle) and to prepare its detailed exposition in five textbooks.

In fact, they managed to write only the first book \[13\] entitled “The Foundations of Tensor Calculus”. A periodic seminar on this subject was also organized for all interested participants. Vladimir Fock, a future academician and researcher in quantum field theory and gravitation, was among the participants. People used to say:

Soon we will eventually understand this theory as Friedmann himself started studying Relativity!

At the same time Friedmann was establishing the field of meteorology at the
Main Geophysics Observatory, and was lecturing at the Polytechnic Institute. He also wrote a fundamental monograph[14] “A Study in Hydromechanics of Compressible Fluids”. Friedmann told his friends:

I am an ignoramus, I know nothing, I should sleep less, I should leave aside all extraneous activities. This so called ‘life’ is just useless waste of time.

Now, more than 100 years after the creation of General Relativity, we may ask ourselves: what is its most unexpected and surprising prediction? There is no doubt what the answer should be.

In the last 100 years that prediction has been confirmed many times and in many ways. Now we are used to it virtually as we are used to the fact that the Earth revolves around the Sun. It has been found that once upon a time the observable Universe, taken as a whole, was born out of nothing. Then the Universe went through its childhood, its youth, and by now it has reached its age of maturity. Perhaps, it will also get old and die.

Now we know a lot about the age of our Universe, and know its long
history starting from the tiniest fractions of the first second of its existence. And this prediction of the Universe evolution is due to Alexander Alexandrovich Friedmann. He finished his first article on this subject by 29 May, 1922. He prepared the manuscript in Russian, and sent it on 3 June, 1922 to the eminent physicist Paul Ehrenfest together with his letter also written in Russian.

It was in Ehrenfest’s home where Einstein used to stay when visiting Leiden University. Ehrenfest corresponded with practically all great physicists of his time. His archive is available on the Internet, and one can find letters from Friedmann there [15]. Their correspondence was in Russian. Pavel Sigizmundovich Ehrenfest, as he was known to his Russian friends, spent five years in St. Petersburg with his wife Tatiana Alekseevna Afanasieva. She graduated from the Higher Bestuzhev Courses physics and mathematics department and then continued her education in Göttingen where she met Ehrenfest. Ehrenfest strongly influenced the physics and mathematics community of St. Petersburg by organizing informal seminar in his apartment. Friedmann was among its participants.

Ehrenfest wrote to Hendrik Antoon Lorentz:

Russia could certainly become my motherland in the deepest sense of the word if only I could get a permanent teaching position anywhere here. Despite my imperfect language I do not feel myself a stranger among people here (except government officials).

But Ehrenfest did not get a job in Russia, and Lorentz, being already of advanced age, offered him his post at Leiden University.

Why the “Differential Equation of God” was not discovered by Einstein, creator of General Relativity? Why was it found by a mathematician belonging to the generation of “children of Russia’s terrible years” depicted by Boris Pasternak in his novel “Doctor Zhivago”?

Einstein’s mistake was that he strongly relied on his physical intuition. Even geniuses can have prejudices. Although they may not believe in black cats or white hares crossing their way, they may have some philosophical prejudices. In the Copernicus times people believed that the Sun and Heavens revolved around the Earth. In the Friedmann times they believed that the Universe was eternal, infinite and unchanging. Einstein found a space-time metric and distribution of matter corresponding to his intuitive picture of the Universe, but it was not a solution of the GR equations.
Дорогой Павел Антонович!

Посылаю Вам небольшую заметку касательно вопроса о вакуумной форме волны, близкой к одноименной форме волны Финнелина и сферической волны на Сильве, кроме того общий случай наблюдается еще более упрощенное решение, когда волна меняется в течение времени радиусом кривизны; это же относится к какому-то вопросу, в котором интересовались Васильев и Ситов. Вопрос о том, как решение этого вопроса ведет к радиальной криволинейной, что не согласуется с космологией, в какой мы его рассматриваем. Передайте мой вопрос Максиму Бергеру.

Несмотря на вкусы переписывания полны, кажется, что получено неплохое письмо.

3 июня 1922 г.

А. Фридман
Figure 7: Ehrenfest at home playing piano. Einstein has Pavlik Ehrenfest at his knees.
Figure 8: “Doctor Zhivago” by Boris Pasternak, the cover of the first edition, 1957 (Wikipedia)
Then Einstein decided that at very large distances the GR should be corrected by adding a repulsive force. This was achieved by inserting into the GR equations the so-called cosmological constant. This correction was very small and undetectable at ordinary astronomical distances but important at huge cosmological distances, i.e. at the edge of the world.

The conclusions of Einstein’s theory when applied to the Universe were radical – the Universe turned out to be finite and thus it should have its own radius. Yet, Einstein theory of the Universe was not radical enough – his Universe had no dynamics, it was static.

About 30 years later George Gamow wrote in his book\cite{16} “The Creation of the Universe”:

\[
\ldots\text{the Russian mathematician A. Friedmann pointed out that the static nature of Einstein’s universe was the result of an algebraic mistake (essentially a division by zero) made in the process of its derivation. Friedmann then went on to show that the correct treatment of Einstein’s basic equations leads to a class of expanding and contracting universes.}\ldots
\]

At the age of 28 Gamow was elected a corresponding member of the USSR Academy of Sciences for his theory of nuclei alpha-decay. Then he moved to the USA where he predicted that at its early age the Universe should have been very hot. With his students he concluded that the lightest elements were produced during the first minutes of the Universe’s existence in thermonuclear reactions, and explained the abundance of these elements. They also predicted cosmic background radiation and estimated its temperature fifteen years before the observational confirmation.

For Friedmann mathematics was of utmost importance\footnote{2}. It’s worthwhile referring in this regard to the profound Wigner’s essay\cite{17} written many years later. Let us also give a well-known citation from Heinrich Hertz:

\[
\begin{align*}
\text{One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.}
\end{align*}
\]

Friedmann believed in mathematics more than in his prejudices. He did not try to guess solutions, he was simply solving equations. Given the main
Figure 9: George (Georgii Antonovich) Gamow (1904–1968)
Einstein’s cosmological ansatz, according to which matter is uniformly distributed at the large scale and that velocities of matter are small in comparison to the light velocity, Friedmann noticed that the GR equations do not require constancy of the “world radius”. Just the opposite – he derived a couple of differential equations connecting the average density of matter with the velocity and acceleration of the “world radius” treated as a dynamic coordinate. The matter was treated by Einstein and Friedmann as “dust” that exerted no pressure. Then the density of matter becomes a function of the “world radius” inversely proportional to its 3rd power.

\[
\frac{R^2}{R^2} + \frac{2R''R}{R^2} + \frac{c^2}{R^2} - \lambda = 0, \quad (1)
\]

\[
\frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \frac{\kappa}{2} c^2 \rho. \quad (2)
\]

Here \(c\) is velocity of light in vacuum, \(\kappa = 16\pi G/c^2\), \(G\) – Newton’s gravitational constant, \(R\) – “world radius” (or better radius of the world curvature, or simply a scale factor), \(\rho\) – average matter density in the Universe, \(\lambda\) is the cosmological constant, proposed by Einstein, and primes denote the time derivatives. Evidently, such a form was motivated by Friedmann’s intention to reproduce 10 equations of GR without any modification. Nontrivial are only (00)– and (11)– components. Friedmann’s equations arise by substituting the following metrics into the GR equations

\[
ds^2 = R^2(x_4) \left( dx_1^2 + \sin^2 x_1 dx_2^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2 \right) + M^2(x_1, x_2, x_3, x_4) dx_4^2,
\]

where it is also supposed that

\[
T_{44} = c^2 \rho g_{44}, \quad \text{for other components} \quad T_{ik} = 0. \quad (4)
\]

It is important to note that (14)–, (24)–, (34)– components of the GR equations lead to the following conditions

\[
R'(x_4) \frac{\partial M}{\partial x_1} = R'(x_4) \frac{\partial M}{\partial x_2} = R'(x_4) \frac{\partial M}{\partial x_3} = 0, \quad (5)
\]

for the case \(R' \neq 0\) Eqs.\(^5\) give \(M = M(x_4)\), and any monotonic function of \(x_4\) can be treated as time. Friedmann considers the simplest option \(M = 1\).
While solving Eqs. (5) Einstein made a mistake: a division by zero, according to Gamow [16]. Einstein derived from Eqs. (5) only one solution, namely $R' = 0$.

When separated for velocity $v = R'$ and acceleration $a = R''$, Friedman equations (1), (2) look like one-dimensional equations in Newtonian mechanics for a point particle in a combination of two potentials. The first one represents the attractive Newtonian gravitational potential, the source of which is a point-like attracting mass equal to the total matter mass of the Universe. The second one is a potential of a harmonic oscillator but with the wrong sign, i.e. a repulsive entity. The Einstein cosmological constant is here analogous to anti-spring obeying anti-Hooke’s law. Then the “world radius” (now we call it a scale factor) becomes the only dynamic variable of the following system of equations

$$ma = \frac{-GmM}{R^2} + m\omega^2 R,$$

$$\frac{mv^2}{2} = E - U(R),$$

(6)

where $\omega = \sqrt{\lambda/3}$, $U(R) = -\frac{GmM}{R} - \frac{m\omega^2 R^2}{2}$, $M = \frac{4}{3} \pi R^3 \rho$, $E = -\frac{mc^2}{2}$,

of course, $m$ is irrelevant and can be omitted.

To find the history and the future of our Universe $R(t)$ (numerically or approximately) one should only specify initial conditions at the present moment of time $t_0$ and to solve Eq. (6) backward and forward in time, respectively. The initial conditions are the present value of the Hubble constant $H_0 = v_0/R_0$, and the ratio of the dark energy to the full matter density. $R_0$ may be the upper bound on space curvature. The present age of the Universe is then obtained as a time interval $\Delta t = t_0 - t_{BB}$, where $R(t_{BB}) = 0$. This problem in mechanics can be solved by any undergraduate student who excels in Calculus (see, for example [18]). Since in mathematics the same equations always have the same solutions, inquisitive students could study cosmology on the basis of the Friedmann equations without an in-depth knowledge of GR.

Well, for non-specialists it should be pointed out that Friedmann results do not mean that the Einstein cosmological solution [19] is wrong. It simply
corresponds to the initial conditions

\[ v(t_0) = 0, \quad U'(r(t_0)) = 0, \]

and is unstable, as it is similar to a ball on the top of a mountain.

After making his discovery Friedmann loved to cite “La Divina Commedia” by Dante Alighieri, Paradiso, Canto II, 7

L’acqua ch’io prendo gia mai non si corse;
Minerva spira, e conducemi Appolo,
e nove Muse mi dimonstran l’Orse.

Friedmann divided possible scenarios of the evolution of the Universe into 3 cases:

1. a monotonic world of the 1st type where the evolution begins from the zero “world radius” and is eternal, the “radius” increases forever;

2. a monotonic world of the 2nd type where the evolution begins from a finite “radius” and then “radius” increases infinitely in infinite time;

3. a periodic world where “radius” increases from zero to a finite value and then decreases to zero.

Besides the Einstein solution Friedmann has also discussed the solution proposed by Willem de Sitter in 1917. This exact solution seems to have a serious drawback because it does not allow the Universe to contain any matter at all. In fact, if one inserts in such a Universe test particles, they cannot stay at rest but start moving as in the present standard picture of expanding Universe.

But that was not clear in the Friedmann times. Now we believe that the Universe follows the de Sitter solution in the inflation epoch characterized by a huge expansion rate induced by enormous vacuum density of energy. The Universe also partially follows the de Sitter solution presently, exhibiting very small expansion rate induced by very low density of the so-called “dark energy”.

In his second paper on cosmology[2] written in 1923 Friedmann considered the case of a negative cosmological constant corresponding to an additional attraction force instead of a repulsive force, and also studied all the possible
scenarios. We recommend Belenkiy’s articles \cite{22, 23, 24} for the detailed analysis of Friedmann’s papers.

His second mistake Einstein made while casually browsing the published Friedmann article. This mistake can also be explained only by his prejudice. As the l. h. s. of the GR equations satisfy the contracted Bianchi identity, the matter energy-momentum tensor must have zero covariant divergence, i.e. it must be covariantly conserved. Einstein, however, calculated just a simple divergence instead of the covariant one, and thus concluded that matter density in the Friedmann solution has to be constant. He was happy to state that there is a contradiction in the Friedmann work, and sent a short note to that effect to the journal. This note was published without anybody checking its correctness.

Friedmann was disappointed and sent a personal letter to Einstein in which he explained him his mistake. But Einstein did not pay attention to this letter. Fortunately, Friedmann had good friends. Yuri Alexandrovich Krutkov, a future corresponding member of the USSR Academy of Sciences and a friend of Ehrenfest, was abroad at that time on his internship. Ehrenfest organized for Krutkov a meeting with Einstein in the spring of 1923. They discussed the letter that Friedmann had sent to Einstein to explain his calculations.

In his letter home Krutkov wrote \cite{21}: “On Monday, 7 May 1923 I was reading with Einstein Friedmann’s article. . .” And then, on May 18 he wrote: “I have won over Einstein in the argument about Friedmann. The honor of Petrograd is saved!” On 31 May 1923 editorial board received the second communication from Einstein on the Friedmann work:

In a previous letter I have criticized the mentioned article. But my criticism, as became clear from Friedmann’s letter communicated to me by Mr. Krutkov, was based on an error in calculations. I consider the results of Friedmann correct and clarifying. . .

Immediately after his first article Friedmann wrote a short monograph “The World as Space and Time” \cite{7} aimed at a wide range of readers. He completed it on 5 September 1922. Here the author presented his radically new ideas about the Universe in a more detailed, free and inspiring way. This small book may be compared in author’s emotions with Galileo’s “Sidereus Nuncius (Sidereal Messenger)” inspired by his fundamental discoveries in astronomy.
Figure 10: Yury Alexandrovich Krutkov (1890–1952)
Once upon a time, when night covered the heavens with its cloak, the famous French philosopher Descartes was seating near his home intensely peering at a gloomy horizon. A passerby approached him and asked: “Tell me, you sage, how many stars are there in the sky?” – “You fool! – the sage answered, – nobody can embrace the boundless!” These emphatic words produced the desired effect on the passerby.

* * *

And then Friedmann begins his narrative:

In the above exchange with Descartes the passerby “got wiser” and calmed down. But in reality in the human history the pursuit to “count the stars”, or, in other words, to construct a world picture has never ceased. Regardless of the level of people’s knowledge there have always existed individuals, ordinary people and great thinkers alike, trying to create the picture of our Universe on the basis of always insufficient data.
The main part of the book contains an original presentation of the General Relativity ideas using the undergraduate mathematics. Friedmann introduces in the most visual way such concepts as metrics, parallel displacement, curvature, and the equivalence principle. He tells us that there are some “world equations” that relate properties of matter to the metrics of spacetime but does not present them explicitly. Friedmann then describes three well-known effects of the General Relativity. He also gives commentaries on the Weyl theory.

But of course the most interesting part of the book is the final one (about seven pages) where Friedmann describes his own results in cosmology.

The non-stationary type of Universe presents a great variety of cases: for this type there may exist cases when the radius of the curvature of the world, starting from some magnitude, constantly increases with time; there may further exist cases when the radius of curvature changes periodically: the Universe contracts into a point (into nothingness), then again, increases its radius from a point to a given magnitude, further again reduces the radius of its curvature, turns into a point and so on. This unwittingly brings to mind the saga of the Hindu mythology about periods of life; there also appears a possibility to speak about “the creation of the world from nothing”, but all of that should be viewed as curious facts which cannot be solidly confirmed by the insufficient astronomical material. In the absence of reliable astronomical data, it is useless to give any numbers characterizing the “life” of the non-stationary Universe. If nevertheless, for the sake of curiosity, we try to calculate the time elapsed from the moment when the Universe was created starting from a point to its present state, that is, when we try to determine therefore the time that elapsed from the creation of the world, then we obtain a number in the tens of billions of our ordinary years.

The theory of Einstein is proved by experiment. It explains the old, seemingly unexplainable, phenomena and predicts new striking effects. The most truthful and deepest method of studying, with the help of Einstein’s theory, the geometry of the world and the structure of our Universe is the application of this theory to the entire world and to the employment of astronomical studies. So far this method gives us little, because the mathematical
analysis puts down its weapons before the difficulties of this issue, and astronomical studies do yet not give a sufficiently reliable basis for the experimental study of our Universe. But in these circumstances, we should not see only temporary difficulties; our descendants will no doubt discover the nature of the Universe, in which we are destined to live...

In the end, Friedmann finishes his text, which started by citation from Grandfather Proutkov, by citing Gavrila Derzhavin:

It seems to us still, that
To measure deep ocean,
Calculate sands, rays of planets,
Even if supreme mind could do it,
There is no number or limit!

Of course, the poetical brackets of the serious content are simply a warning against excessive pride of human beings. They are analogous to the famous Newton’s words:

I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Friedmann recommends for further reading books by Hermann Weil, Max von Laue, Sir Arthur Stanley Eddington and an article by David Hilbert, but none of the Einstein’s works.
Figure 12: Portrait of Alexander Friedmann in the Main Geophysical Observatory, St. Petersburg.

Conclusions

The new vision of the Universe opened by Friedmann now has become a foundation of modern cosmology \cite{26}. The first significant confirmation of the dynamical evolution of the Universe has been given by Edwin Powell Hubble already in 1929. But Friedmann’s place as a discoverer was occupied by Georges Lemaître, a Catholic priest (although his first article appeared in 1927 only, and besides GR equations the paper was based upon Vesto Melvin Slipher’s data on velocities of some galaxies).

Friedmann contracted typhoid and died in 1925 at the age of 37. This tragic event unfortunately interrupted the development of cosmology in the
USSR for decades. In those years some philosophers “took the functions of police”, as quipped Piotr Leonidovich Kapitza. These gendarmes of philosophy accused physicists of being “agents of Lemetrianism”, and called Friedmann’s equation “the differential equation of God.”

It is a consequence of this late recognition of the works of the man of genius that they are rarely enjoyed by their contemporaries, and accordingly in the freshness of colour which synchronism and presence imparts, but, like figs and dates, much more in a dry than in a fresh state.
Arthur Schopenhauer. *The World as Will and Idea.*

Only in the 1960s the science of the Universe started to flourish again. This “renaissance” was first associated with the names of Y. B. Zeldovich and A. D. Sakharov, and later with the names of I. D. Novikov, A. A. Starobinsky, V. F. Mukhanov, A. D. Linde and many others. At the IHEP (Protvino), where this author works, research in the field of gravity began thanks to the interest of A. A. Logunov, who devoted the last decades of his life to the most difficult task of generalizing GR for the case when the gravitational field becomes massive.

New opportunities opened up after mankind entered the space, thanks to the launch of the first satellite in the USSR in 1957, and the first flight of Yuri Gagarin[25] in 1961. High above the atmosphere, space telescopes study the Universe in all ranges of electromagnetic waves and bold Friedmann’s predictions made in his book come true. Thanks to the progress of technology, it has also become possible to register the gravitational waves, the amplitude of which is many times smaller than the size of proton. In the 21st century, cosmology may become more important in making new fundamental discoveries than accelerator physics.
Notes

1 The title of our paper should remind a well-known tale “Levsha” written by Nikolai Leskov. In brief, this is a tale about a travel of the Russian Tsar to England where Englishmen have demonstrated and presented him a mechanical flea. The Tsar was impressed and ready to admit a superiority of English craftsmen, but Don Cossack Ataman Platov who was accompanying him suggested to show this flea to Russian craftsmen. And Levsha with two other artisans from Tula had shod this English mechanical flea with steel horseshoes. It was a great surprise for Tsar who could see these horseshoes only through a microscope, whereas Levsha, of course, has done his work (using steel nails for these horseshoes) without any gadgets. This is what Encyclopaedia Britannica says about Nikolai Leskov.
His most popular tale, however, remains *Skaz o Tulskom kosom Levshe i o stalnoy Blokhe* (“The Tale of Cross-eyed Lefty from Tula and the Steel Flea”, 1881), a masterpiece of Gogolesque comedy in which an illiterate smith from Tula outwits the skill of the most advanced British craftsman... In 1969 W.B. Edgerton translated into English, for the first time, 13 of Leskov’s stories, with a new translation of “The Steel Flea.”

2Friedmann’s first paper publication appeared when he was still at gymnasium.

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