Abstract

We consider the role of precision measurements of beta decays and light meson semi-leptonic decays in probing physics beyond the Standard Model in the LHC era. We describe all low-energy charged-current processes within and beyond the Standard Model using an effective field theory framework. We first discuss the theoretical hadronic input which in these precision tests plays a crucial role in setting the baseline for new physics searches. We then review the current and upcoming constraints on the various non-standard physics operators from the study of decay rates, spectra, and correlations in a broad array of light-quark systems. We finally discuss the interplay with LHC searches, both within models and in an effective theory approach. Our discussion illustrates the independent yet complementary nature of precision beta decay measurements as probes of new physics, showing them to be of continuing importance throughout the LHC era.
1 Introduction

Beta decays played a central role in determining the $V - A$ structure of the weak interactions and in shaping what we now call the Standard Model (SM) [1, 2, 3]. We focus here on the set of semi-leptonic “charged current” (CC) processes that are mediated in the SM by tree-level W exchange, up to radiative corrections. In the SM the CC weak processes are characterized by two main features: (i) the hadronic and leptonic bilinear densities involved in the process have a dominant $V - A$ component, with other types of couplings—$V + A, S, P, T$—arising at higher order in radiative corrections or in recoil momentum; (ii) the effective Fermi constants extracted in beta decays obey lepton universality as well as quark-lepton, or Cabibbo, universality, which is equivalent in the SM to the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Universality relations can only emerge once the process-dependent radiative corrections are removed. Currently precision beta-decay measurements involving neutrons, nuclei, and mesons are used to probe the existence of non-SM interactions which effectively induce violations of the universality relations and/or novel non-($V - A$) structures or corrections to the dominant vector and axial-vector couplings. \[ \text{The low-energy charged-current-interaction} \]

Hamiltonian is sensitive to many classes of SM extensions. In this sense, beta decay measurements can be considered as “broad band” probes of physics beyond the Standard Model (BSM): while by themselves they do not allow us to reconstruct the ultraviolet dynamics, they provide, at 0.1%-level precision, powerful boundary conditions and diagnostics on virtually any TeV-scale SM extension.

Considerable experimental progress is ongoing or expected in a few-year time scale on several fronts, using both cold and ultracold neutrons [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20], trapped nuclei [21, 22], and rare pion and kaon decays [23, 24, 25, 26]. Some of the measurements plan to reach sensitivities between $10^{-3}$ and $10^{-4}$; this makes such observables very interesting probes of new physics effects originating at the TeV scale, because such effects are expected to have size $O((v/\Lambda_{BSM})^2)$, where $v = (2\sqrt{2}G_F)^{-1/2} \approx 174$ GeV and $\Lambda_{BSM}$ denotes the mass scale where BSM physics appears.

As in previous reviews [27, 28, 29, 30], the overall goal of this article is to discuss the discovery potential and discriminating power of planned precision beta-decay measurements with neutrons, nuclei, and mesons, in light of other existing precision electroweak tests and high-energy collider searches, such as at the Tevatron and the LHC. In order to achieve our goal, we work within an effective field theory (EFT) framework, in which the dynamical effects of new heavy BSM degrees of freedom are parameterized by local operators of dimension higher than four built with SM fields. \[ \text{All model-specific analyses of beta decays can be cast in the EFT language and the limits on the effective operators we derive can be readily converted into constraints on the parameters of any SM extension. In the absence of an emerging picture of new dynamics from collider searches, the EFT analysis is the first necessary step to establish the motivation and significance of this set of low-energy probes. Subsequently, we will also discuss well-motivated models such as the Left-Right Symmetric Model and supersymmetric extensions of the SM in order to show the discriminating power that combinations of beta decay measurements can have on explicit models.} \]

Probing short-distance BSM couplings through precision phenomenology of beta decays requires knowing the relevant hadronic and nuclear matrix elements to a precision comparable to the size of the new physics effects one could expect to appear. This means that one needs to know the hadronic matrix elements of the SM operators, that is, of the V and A currents, to the level of $O((v/\Lambda_{BSM})^2)$, i.e. of $10^{-3}$ or better. This is a necessary condition for beta decays to function as competitive probes: we are in search of a small BSM signal, and hence we need to know the SM “background” to a level comparable

\[ ^1 \text{In this review we consider the decays involving the light quarks } u, d, \text{ and } s \text{ exclusively.} \]

\[ ^2 \text{The EFT analysis can be applied to all low-energy probes of CC interactions. It is also valid for collider searches as long as the particles which mediate the new interactions are above particle-production threshold at the operating center-of-mass energy. In this case, a direct comparison of low-energy and collider constraints can be performed, as we discuss in Section 5.} \]
to that of the signal for which we are looking. One also needs to know the matrix elements of the BSM operators, such as the S, T, P densities, because all the observables are sensitive to the product of the short-distance BSM coupling with the appropriate hadronic/nuclear matrix element. Consequently if a certain matrix element is suppressed, the sensitivity to the corresponding BSM coupling is also suppressed. Moreover, were such an anomalous suppression absent, the fractional uncertainty on the BSM matrix element still determines how well we can constrain that BSM coupling. For BSM operators, the precision required of the relevant hadronic matrix element is much less severe; an uncertainty at the $O(10\%)$ level is acceptable. Motivated by these considerations, we pay special attention to the hadronic and nuclear uncertainties which appear.

This paper is organized as follows. In Section 2 we set up the theoretical framework for the analysis of all low-energy CC processes within and beyond the SM. In Section 3 we discuss the status of Cabibbo universality tests (Sec. 3.1) and lepton universality tests (Sec. 3.2) and explore the implications for BSM physics. In Section 4 we focus on differential decay distributions in beta decays and discuss the implications for non-$V-A$ couplings. In Section 5 we explore the constraints on non-standard CC couplings that can be obtained from LHC data. In Section 6 we illustrate how the precision tests can be used to probe the parameter space of models such as the Left-Right Symmetric Model and the Minimal Supersymmetric Standard Model (MSSM), and we present our concluding remarks in Section 7.

## 2 Theoretical Framework

### 2.1 Effective Lagrangian

In this review we take the point of view that the Standard Model emerges as the low-energy limit of a more fundamental theory characterized by the scale $\Lambda$ at which new particles appear. Consequently, at energies scales below $\Lambda$, namely, $\Lambda > E > M_{Z,W}$, the new degrees of freedom are no longer present; they have been “integrated out,” yielding an effective Lagrangian comprised of the SM Lagrangian augmented by a string of $d > 4$ operators constructed with the low-energy SM fields, suppressed by $\Lambda^{d-4}$ [31], that respect the SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$ gauge symmetry of the SM. Flavor physics observables constrain the appearance of non-SM invariant operators to energies far beyond the weak scale [32, 33, 34]. The building blocks of the gauge-invariant local operators are: the gauge fields $G^A_{\mu}, W^a_{\mu}, B_{\mu}$, corresponding to SU(3)$_C \times$ SU(2)$_L \times$ U(1)$_Y$, respectively, the six fermionic gauge multiplets, including a singlet right-handed neutrino state,

$$q^i = \begin{pmatrix} u^i_L \\ d^i_L \\ \bar{d}^i_R \end{pmatrix}, \quad u^i = u^i_R, \quad d^i = d^i_R, \quad l^i = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix}, \quad e^i = e^i_R, \quad \nu^i = \nu^i_R,$$

(2.1)

the Higgs doublet $\varphi$

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix},$$

(2.2)

and the covariant derivative

$$D_\mu = I \partial_\mu - ig^s_\Lambda \frac{\lambda^A}{2} G^A_\mu - ig^a \frac{\sigma^a}{2} W^a_\mu - ig' Y B_\mu.$$  

(2.3)

In the above expression, $I$ is the identity matrix, $\lambda^A$ are the SU(3) Gell-Mann matrices; $\sigma^a$ are the SU(2) Pauli matrices; $g^s, g$, and $g'$ are the gauge couplings; and $Y$ is the hypercharge of a given multiplet. The introduction of three light, right-handed neutrinos to accommodate the existence of neutrino masses illustrates explicitly that new light degrees of freedom can be included in the EFT if we assign their SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$ quantum numbers. The impact of other light, new physics is left as an exercise for future work.
The leading operators which modify CC interactions are of dimension six, though it is worth noting that new physics can also modify the radiative corrections and hence the couplings with which the SM operators appear. The minimal set of operators contributing to low-energy semi-leptonic charged current processes can be divided into two groups: operators built out of SM fields, noting the left column below, in which we follow the notation of Refs. [35, 36], and operators involving the singlet R-handed neutrino field $\nu$ [37], which are displayed in the right column below. Furthermore, within each group the operators can be divided into two classes—four-fermion contact interactions and vertex corrections. The vertex correction operators are written in SU(2)-invariant form and therefore involve the Higgs doublet: after electroweak symmetry breaking (EWSB) they include terms involving a $W$ (or $Z$) boson, a fermion, and an anti-fermion. Here is the list:

**Four-fermion operators:**

\[
O_{lq}^{(3)} = (\bar{L}\gamma^\mu\sigma^a L)(\bar{q}\gamma^\nu\sigma^a q) \quad O_{e\nu d} = (\bar{e}\gamma^\mu\nu)(\bar{\nu}\gamma^\mu d) + \text{h.c.} \quad (2.4a)
\]

\[
O_{l\bar{q}d} = (\bar{l}(q))(\bar{d}q) + \text{h.c.} \quad O_{q\nu} = (\bar{l}(\nu))(\bar{q}q) + \text{h.c.} \quad (2.4b)
\]

\[
O_{l\nu}^{(1)} = (\bar{l}(e))e^{ab}(\bar{q}b\nu) + \text{h.c.} \quad O_{l\nu}^{(1)} = (\bar{l}(\nu))e^{ab}(\bar{q}b\nu) + \text{h.c.} \quad (2.4c)
\]

\[
O_{l\nu}^{(3)} = (\bar{l}(\sigma^{\mu\nu})e)(\bar{q}b\sigma_{\mu\nu}u) + \text{h.c.} \quad O_{l\nu}^{(3)} = (\bar{l}(\sigma^{\mu\nu})e)(\bar{q}b\sigma_{\mu\nu}d) + \text{h.c.} \quad (2.4d)
\]

**Vertex corrections:**

\[
O_{e\nu d} = i(\varphi^T e D_\mu \varphi)(\bar{e}\gamma^\mu d) + \text{h.c.} \quad O_{e\nu e} = i(\varphi^T e D_\mu \varphi)(\bar{e}\gamma^\mu e) + \text{h.c.} \quad (2.5a)
\]

\[
O_{e\nu q}^{(3)} = (\varphi^T D_\mu \varphi)(\bar{e}\gamma^\mu\sigma^a q) \quad O_{e\nu q}^{(3)} = (\varphi^T D_\mu \varphi)(\bar{e}\gamma^\mu\sigma^a q) \quad (2.5b)
\]

\[
O_{e\nu l}^{(3)} = (\varphi^T D_\mu \varphi)(\bar{e}\gamma^\mu\sigma^a l) \quad (2.5c)
\]

Denoting by $\Lambda_i$ the effective dimensionful coupling associated with the operator $O_i$, we can write the effective Lagrangian as

\[
\mathcal{L}^{(\text{eff})} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda_i^2} O_i \quad \mathcal{L}_{\text{SM}} + \frac{1}{\hat{\Lambda}_i^2} \sum_i \hat{\Lambda}_i O_i, \quad \text{with} \quad \hat{\Lambda}_i = \frac{\hat{v}^2}{\Lambda_i^2} , \quad (2.6)
\]

where in the last step we have set the correct dimensions using the Higgs vacuum expectation value (VEV) $v = \langle \varphi^0 \rangle = (2\sqrt{2}G_F)^{-1/2}$ and defined the dimensionless new-physics couplings $\hat{\Lambda}_i$, which in general are matrices in both the quark and lepton flavor spaces. In this framework one can derive the low-energy effective Lagrangian at $\mathcal{O}(1 \text{ GeV})$ for semi-leptonic transitions. It receives contributions from both $W$-exchange diagrams, with modified $W$-fermion couplings, and the four-fermion operators. After including the electroweak radiative corrections to the SM operator [38], the matching procedure leads to a low-energy quark level effective Lagrangian involving ten dimension-six operators:

\[
\mathcal{L}_{\text{CC}} = -\frac{G_F^{(0)}V_{ud}}{\sqrt{2}} \left[ (1 + \delta_3) \bar{e}\gamma_\mu(1 - \gamma_5)\nu_e \cdot \bar{u}\gamma^\mu(1 - \gamma_5)d + \bar{e}_L \bar{e}\gamma_\mu(1 - \gamma_5)\nu_e \cdot \bar{u}\gamma^\mu(1 - \gamma_5)d \
+ \bar{e}_R \bar{e}\gamma_\mu(1 - \gamma_5)\nu_e \cdot \bar{u}\gamma^\mu(1 - \gamma_5)d + \bar{e}_R \bar{e}\gamma_\mu(1 - \gamma_5)\nu_e \cdot \bar{u}\gamma^\mu(1 - \gamma_5)d \
+ \bar{e}_S \bar{e}(1 - \gamma_5)\nu_e \cdot \bar{u}d + \bar{e}_S \bar{e}(1 - \gamma_5)\nu_e \cdot \bar{u}d \
+ \bar{e}_F \bar{e}(1 - \gamma_5)\nu_e \cdot \bar{u}\gamma_5 d - \bar{e}_F \bar{e}(1 + \gamma_5)\nu_e \cdot \bar{u}\gamma_5 d \
+ \bar{e}_T \bar{e}\sigma^{\mu\nu}(1 - \gamma_5)\nu_e \cdot \bar{u}\sigma^{\mu\nu}(1 - \gamma_5)d + \bar{e}_T \bar{e}\sigma^{\mu\nu}(1 + \gamma_5)\nu_e \cdot \bar{u}\sigma^{\mu\nu}(1 + \gamma_5)d + \text{h.c.} \right] . \quad (2.7)
\]

In the above equation $G_F^{(0)}/\sqrt{2} = g^2/(8M_W^2)$ is the tree-level SM Fermi constant, and $\delta_3$ encodes the effect of SM electroweak radiative corrections to semi-leptonic transitions, noting that the Fermi theory
QED contributions have been subtracted \[39, 40, 38, 41, 42, 43\]. The coupling \( G_F^{(0)} \) can be expressed in terms of the Fermi constant \( G_\mu = 1.166371(6) \times 10^{-5}\text{GeV}^{-2} \) precisely measured in muon decay \[44\]. In order to do so, one has to consider the low-energy effective Lagrangian describing muon decay \[45\].

\[
\mathcal{L}_{\mu \to e\bar{e}\nu_\mu} = -4 G_F^{(0)} (1 + \delta_\mu + \epsilon_\mu) \ \bar{e}_L \gamma_\mu \nu_{eL} \cdot \bar{\nu}_{eL} \gamma^\mu \mu_L + \text{h.c.} , \tag{2.8}
\]

where \( G_\mu \equiv G_F^{(0)} (1 + \delta_\mu + \epsilon_\mu) \). Here \( \delta_\mu \) represents the SM electroweak radiative corrections \[46\] to purely leptonic transitions, noting that the Fermi theory QED contributions have been subtracted, and \( \epsilon_\mu \) encodes possible new physics contributions, so that \( G_F^{(0)} = G_\mu (1 - \delta_\mu - \epsilon_\mu) \) \[3\].

The BSM effective couplings in Eq. (2.7) are denoted by \( \epsilon_\alpha \) and \( \bar{\epsilon}_\beta \), using the self-explanatory notation \( \epsilon_\alpha, \beta = L, R, S, P, T \). These couplings can be expressed in terms of the weak-scale couplings \( \hat{\alpha}_j \) \[45, 47, 37\]. In the effective Lagrangian of Eq. (2.7), \( e, u, \) and \( d \) denote the electron, up-, and down-quark mass eigenfields, whereas \( \nu_\ell \) represents the neutrino flavor fields. In general we can have \( \ell \neq e \)—in what follows, we suppress lepton flavor indices. Finally, identical CC effective operators appear for other quark flavors. For example, the operators obtained by replacing the \( d \) quark with the strange quark \( s \) describe \( |\Delta S| = 1 \) semileptonic processes.

Next, we discuss some noteworthy points in regards to the effective Lagrangian of Eq. (2.7):

- The effective couplings denoted by \( \epsilon_\alpha \) involve L-handed neutrinos, whereas \( \bar{\epsilon}_\beta \) involve R-handed neutrinos. Therefore, the \( \bar{\epsilon}_\beta \) appear in decay rates and distributions either quadratically or linearly, but the latter appears multiplied by the small factor \( m_\nu/E_\nu \), as it is realized through interference of the SM and BSM couplings. In contrast, the \( \epsilon_\alpha \) couplings contribute linearly to the decay rates without \( m_\nu/E_\nu \) suppression. As a consequence, the bounds on the \( \epsilon \)'s are much stronger than the bounds on the \( \bar{\epsilon} \)'s.

- There are twelve SU(2)\(_L\)\(\times\)U(1)-invariant operators that contribute to beta decays, though there are only ten quark-level U(1)\(_{EM}\)-invariant operators. This is because the correction \( \epsilon_\ell \) to the SM operator encodes contributions from three weak-scale operators of Eqs. (2.4) and (2.5), namely, the contact operator \( O_\mu^{(3)} \) and the quark and lepton vertex corrections, \( O_{\phi q}^{(3)} \) and \( O_{\phi l}^{(3)} \). All other low-energy operators are in one-to-one correspondence with the TeV scale SU(2)\(_L\)\(\times\)U(1)-invariant operators. It is interesting to note that SU(2) gauge invariance implies that the same couplings mediate not only charged-current processes but also “neutral current” processes such as \( \bar{e}e \leftrightarrow \bar{u}u, \bar{d}d \).

- While the physical amplitudes are renormalization scale and scheme independent, the individual effective couplings \( \epsilon_{S,P,T} (\bar{\epsilon}_{S,P,T}) \) and the corresponding hadronic matrix elements display a strong scale dependence in quantum chromodynamics (QCD) already at one-loop order (see Ref. \[48\] and references therein). Throughout the paper, we quote estimates and bounds for the \( \epsilon_\ell \) (\( \bar{\epsilon}_\ell \)) at the renormalization scale \( \mu = 2 \text{ GeV} \) in the \( \overline{\text{MS}} \) scheme, unless otherwise specified.

The Lagrangian of Eq. (2.7) mediates all leading, low-energy charged-current weak processes involving up and down quarks. In some charged-current processes involving first-generation quarks the theoretical and experimental precision has reached, or will soon reach, a level that allows stringent bounds on new-physics effective couplings. To set the stage for this discussion, we now provide an overview of how the various BSM couplings of Eq. (2.7) can be probed experimentally—we explore these points in detail in the following sections. For context, we note that detailed expressions of the non-(\( V-A \)) contributions to neutron and nuclear beta decay correlation coefficients can be found in the papers by Jackson, Treiman, and Wyld \[49, 50\], where one can re-express the Lee-Yang couplings \[51\] they employ in terms of the \( \epsilon_\alpha \) and \( \bar{\epsilon}_\beta \) using the expressions given in Eqs. (2.17) below.

\(^3\)Our notation in Eqs. (2.7) and (2.8) corresponds to that of Ref. \[30\] if we replace \( \delta_\beta \rightarrow \Delta \bar{r}_\beta \) and \( \delta_\mu \rightarrow \Delta \bar{r}_\mu \).
The combinations \((\epsilon_L \pm \epsilon_R)\) affect the overall normalization of the effective Fermi constant in processes mediated by the vector and axial-vector current, respectively. As discussed below, the hadronic matrix elements of the vector current are known very precisely up to corrections due to QCD flavor symmetry breaking, that is, quark mass differences, whereas the axial-vector matrix elements require non-perturbative calculations. Therefore, while the difference \((\epsilon_L - \epsilon_R)\) remains relatively unconstrained, the sum \((\epsilon_L + \epsilon_R)\) is strongly constrained by quark-lepton universality tests, which are tantamount to CKM unitarity tests. These tests involve a precise determination of \(V_{ud}\) and \(V_{us}\) from processes mediated by the vector current, such as \(0^+ \to 0^+\) nuclear decays and \(K \to \pi \ell \nu\). An extensive analysis of the constraints on \((\epsilon_L + \epsilon_R)\) from universality tests and precision electroweak observables at the \(Z\)-pole was performed in Ref. [45]. In this context it was shown that constraints from low-energy are at the same level or stronger—depending on the operator—than those from \(Z\)-pole observables and \(e^+e^- \to q\bar{q}\) cross-section measurements at LEP.

The right-handed coupling \(\epsilon_R\) affects the relative normalization of the axial and vector currents. In all beta decays \(\epsilon_R\) can be absorbed in a redefinition of the axial coupling, and, up to calculable radiative corrections [42, 52, 53, 54, 55, 56], experiments determine the combination \((1 - 2\epsilon_R)g_A/g_V\), where \(g_V\) and \(g_A\) are the vector and axial form factors at zero momentum transfer, to be precisely defined below. Disentangling \(\epsilon_R\) requires precision measurements of \((1 - 2\epsilon_R)g_A/g_V\) and precision calculations of \(g_A/g_V\) in lattice QCD, which, unfortunately, are not yet at the required sub-percent level.

The effective pseudoscalar coupling \(\epsilon_P\) contributes to the leptonic decays of the pion. It is strongly constrained by the helicity-suppressed ratio \(R_\pi \equiv \Gamma(\pi \to e\nu\gamma)/\Gamma(\pi \to \mu\nu\gamma)\). Moreover, as discussed in Refs. [57, 58, 59], the low-energy coupling \(\epsilon_P\) receives contributions proportional to \(\epsilon_{S,T}\) through electroweak radiative corrections.

Both the scalar and tensor couplings \(\epsilon_S\) and \(\epsilon_T\) contribute at linear order to the Fierz interference term \(b\) in the beta decays of neutrons and nuclei, as well as to the neutrino-asymmetry correlation coefficient \(B\) in polarized neutron and nuclear decays. The empirical determination of the beta-asymmetry correlation coefficient \(A\) and the electron-neutrino correlation \(a\) in neutron and nuclear beta decays, as well as positron polarization measurements therein, entrain sensitivity to the Fierz interference term as well. Thus bounds on \(\epsilon_S\) and \(\epsilon_T\) can also be obtained from these observables. Moreover, the quadratic dependence on these couplings is useful in limiting their imaginary parts as well. Finally, the tensor coupling \(\epsilon_T\) can also be constrained through Dalitz-plot studies of the radiative pion decay \(\pi \to e\nu\gamma\).

Neglecting neutrino masses, all the \(\bar{\epsilon}_\beta\) couplings contribute to decay rates as per \(\propto |\bar{\epsilon}_\beta|^2\), so that it is more challenging to set limits on their appearance at low energies.

All of the operators of Eq. (2.7) can produce collider signatures. Before the advent of the LHC, collider bounds on the chirality-flipping scalar and tensor couplings \(\epsilon_{S,P,T}\) and \(\bar{\epsilon}_{S,P,T}\) were very weak, because their interference with the SM amplitude appears with factors of \(m_f/E_f\), where \(m_f\) is a light fermion mass with \(f \in \{e, u, d\}\), which at collider energies strongly suppresses the whole effect. At the LHC, however, the contributions which appear as \(|\epsilon_\beta|^2\) or \(|\bar{\epsilon}_\beta|^2\) can be boosted by a factor involving the energy in the numerator, noting that we replace \((v/\Lambda_{BSM})^4 \to (E/\Lambda_{BSM})^4\), thus increasing the sensitivity to these couplings. We will discuss these bounds and show that with higher center-of-mass energy and integrated luminosity they become competitive with low-energy searches for \(\epsilon_{S,T}\) or stronger than low-energy bounds for \(\bar{\epsilon}_{R,S,T}\). This analysis, of course, makes sense only for \(\Lambda_{BSM} \gtrsim \) few TeV.

The above considerations and more are summarized in Table I.

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Table 1: Summary of the most sensitive direct low-energy probes of non-standard charged-current couplings. Left column: combination of couplings. Right column: probe. The effective couplings are defined in Eq. (2.7). The decay parameters $a, b, B, A$ are defined in Eq. (4.41). If the new interactions originate at mass scales above the TeV, the LHC provides constraints on all non-standard couplings through the process $pp \rightarrow e + \nu + X$.

| Non-standard coupling | Probe |
|-----------------------|-------|
| $\epsilon_L + \epsilon_R$ | CKM unitarity |
| $\epsilon_L - \epsilon_R, \epsilon_P, \tilde{\epsilon}_P$ | $R_\pi$ |
| $\epsilon_S$ | $b, B \ [a, A]$ |
| $\epsilon_T$ | $b, B \ [a, A], \pi \rightarrow e\nu\gamma$ |
| $\tilde{\epsilon}_{\alpha\neq P}$ | $a, b, B, A$ |

### 2.2 Hadronic and nuclear matrix elements

Hadronic and nuclear transition amplitudes always involve products of short-distance couplings, evolved to the appropriate matching scale, and hadronic matrix elements. Thus in order to extract information on the former, we need to know the latter. Specifically, we need to match the quark-level effective theory of Eq. (2.7) to a low-energy effective theory written in terms of meson and baryon degrees of freedom. In QCD, this effective theory is Chiral Perturbation Theory (ChPT) [60, 61, 62]. In the baryon sector, the low-energy structure of the theory is more complicated, and heavy baryon chiral perturbation theory is employed [63], where we refer the reader to Ref. [64] for a review. Different systematic approaches to remedy its limitations have been developed, improving the theory’s convergence, notably the “small scale expansion” of Refs. [65, 66, 67, 68], as well as Ref. [69]. As we have discussed, the precision with which we know the matrix elements of the SM operators limits our ability to constrain new physics. If we wish to probe scales such that $(v / \Lambda_{BSM})^2 \sim 10^{-3}$, we need to know the SM matrix elements with commensurate precision. This requires including all of the electromagnetic, isospin-breaking, and recoil-order effects in the calculation. Since the operators appearing in Eq. (2.7) have the factorized structure $J_{\text{quark}} \times J_{\text{lepton}}$, we need not present the ChPT framework but rather can describe the purely hadronic effects in terms of meson and nucleon matrix elements of quark bilinears. Nevertheless, the full ChPT machinery should ultimately be employed to compute long-distance radiative corrections. In the case of neutron decay, this has been done in Ref. [70], finding results consistent with non-ChPT based calculations [42, 52, 53, 54, 55, 56, 71]. In this review, we will not further discuss long-distance radiative corrections to neutron decay and refer the reader to Refs. [70] and [42] for recent detailed accounts.
2.2.1 Meson matrix elements

Leptonic ($M \rightarrow l\nu$) and semi-leptonic ($M_1 \rightarrow M_2 l\nu$) decays of pseudoscalar mesons provide strong constraints on the CC BSM couplings. The relevant one-meson matrix elements are parameterized in terms of the pion and kaon decay constants $F_{\pi, K}$ as follows (in our normalization $F_\pi \simeq 92$ MeV):

\begin{align}
\langle 0|\bar{u}\gamma_{\mu}\gamma_5 d|\pi^-(p) \rangle &= -i\sqrt{2} F_\pi \ p_\mu \\
\langle 0|\bar{u}\gamma_{\mu}\gamma_5 s|K^-(p) \rangle &= -i\sqrt{2} F_K \ p_\mu \\
\langle 0|\bar{u}\gamma_5 d|\pi^-(p) \rangle &= i \frac{m_\pi^2}{m_u + m_d} \sqrt{2} F_\pi \\
\langle 0|\bar{u}\gamma_5 s|K^-(p) \rangle &= i \frac{m_K^2}{m_u + m_s} \sqrt{2} F_K .
\end{align}

The pseudoscalar matrix elements follow from the axial-vector ones by using the operator relation $\partial_\mu \bar{q}_i \gamma^\mu \gamma_5 q_j = i (m_i + m_j) \ \bar{q}_j \gamma_5 q_i$. Matrix elements of the other bilinears vanish by parity.

The two-meson matrix elements of vector and scalar densities can be parameterized in terms of two form factors. Specializing to the $K^0 \rightarrow \pi^-$ transitions, we have, noting the momentum transfer $q = p - k$:

\begin{align}
\langle \pi^-(k)|\bar{s}\gamma_\mu u|K^0(p) \rangle &= (p + k)_\mu f_+(q^2) + (p - k)_\mu f_-(q^2) \\
\langle \pi^-(k)|\bar{s}u|K^0(p) \rangle &= -\frac{m_K^2 - m_\pi^2}{m_s - m_u} f_0(q^2) ,
\end{align}

where $f_0(q^2) = f_+(q^2) + (q^2/(m_K^2 - m_\pi^2)) f_-(q^2)$ and the operator relation $\partial_\mu \bar{q}_i \gamma^\mu q_j = i (m_i + m_j) \ \bar{q}_j q_i$ has been used to relate the vector and scalar matrix elements. In the SU(3)$_L$ limit the light quark masses obey $m_u = m_d = m_s$, so that $f_+(0) = 1$—the Ademollo-Gatto theorem [72, 73] ensures that the corrections to the flavor symmetry limit start at second order: $f_+(0) = 1 + \mathcal{O}((m_s - m_d)^2)$\footnote{Note, however, that in the case of charged $K_{\ell 3}$ decay the existence of $\pi^0 - \eta, \eta'$ mixing implies that corrections to $f_+(0) = 1$ occur at first order in $(m_d - m_u)/(m_s - \hat{m})$, with $\hat{m} = (m_d + m_u)/2$.}. Finally, for completeness, we report the tensor matrix element, which involves a new dynamical form factor $B_T(q^2)$ \footnote{C.}

\begin{align}
\langle \pi^-(k)|\bar{s}\sigma_{\mu\nu} u|K^0(p) \rangle &= i \frac{p_\mu k_\nu - p_\nu k_\mu}{m_K} B_T(q^2) .
\end{align}

The decay constants and form-factors can be calculated in lattice QCD (LQCD) and we will review the relevant results as needed.

2.2.2 Nucleon matrix elements

At the one-nucleon level, we require the matrix elements between the neutron and proton of all possible quark bilinears of dimension three. These can be parameterized in terms of Lorentz-invariant form
In terms of a simultaneous expansion in new physics contributions, recoil, and radiative corrections, observables on the short-distance parameters \( O \) employ the metric we do but flip the sign of the factors as follows \[75\]:

\[
\langle p(p) | \bar{u} \gamma_{\mu} d | n(p_n) \rangle = \bar{u}_p(p_p) \left[ g_V(q^2) \gamma_{\mu} - i \frac{\tilde{g}_{T(V)}(q^2)}{2M_N} \sigma_{\mu\nu} q^\nu + \frac{\tilde{g}_{S}(q^2)}{2M_N} q_{\mu} \right] u_n(p_n) \quad (2.16a)
\]

\[
\langle p(p) | \bar{u} \gamma_{\gamma_5} d | n(p_n) \rangle = \bar{u}_p(p_p) \left[ g_A(q^2) \gamma_{\mu} - i \frac{\tilde{g}_{T(A)}(q^2)}{2M_N} \sigma_{\mu\nu} q^\nu + \frac{\tilde{g}_{P}(q^2)}{2M_N} q_{\mu} \right] \gamma_5 u_n(p_n) \quad (2.16b)
\]

\[
\langle p(p) | \bar{u} d | n(p_n) \rangle = g_S(q^2) \bar{u}_p(p_p) u_n(p_n) \quad (2.16c)
\]

\[
\langle p(p) | \bar{u} \gamma_5 d | n(p_n) \rangle = g_T(q^2) \bar{u}_p(p_p) \gamma_5 u_n(p_n) \quad (2.16d)
\]

\[
\langle p(p) | \bar{u} \sigma_{\mu\nu} d | n(p_n) \rangle = \bar{u}_p(p_p) \left[ g_T(q^2) \sigma_{\mu\nu} + g_1^{(1)}(q^2) (q_\mu \gamma_\nu - q_\nu \gamma_\mu) \right] u_n(p_n), \quad (2.16e)
\]

where \( u_{p,n} \) are the proton and neutron spinors, \( P = p_n + p_p, \ q = p_n - p_p \) is the momentum transfer, and \( M_N = (M_n + M_p)/2 \) denotes a common nucleon mass.\(^5\) Note that the above spinor contractions are \( \mathcal{O}(1) \), except for \( \bar{u}_p \gamma_5 u_n \), which is \( \mathcal{O}(q/M_N) \).

In order to make contact with the standard references on neutron and nuclear beta-decay phenomenology \[51, 49, 50, 29\], we note that upon neglecting recoil order terms Eq. \( (2.16) \) can be viewed as the matching conditions from our quark-level effective theory Eq. \( (2.7) \) to the nucleon-level effective theory originally written down by Lee and Yang \[51\]. The Lee-Yang effective couplings \( C_i, C_i' \) \( (i \in \{V, A, S, T\}) \) can be expressed in terms of our parameters as follows \[37\]:

\[
C_i = \frac{G_F^{(0)}}{\sqrt{2}} V_{ud} \bar{C}_i \quad (2.17a)
\]

\[
\bar{C}_V = g_V (1 + \delta_\beta + \epsilon_L + \epsilon_R + \bar{\epsilon}_L + \bar{\epsilon}_R) \quad (2.17b)
\]

\[
\bar{C}_V' = g_V (1 + \delta_\beta + \epsilon_L + \epsilon_R - \bar{\epsilon}_L - \bar{\epsilon}_R) \quad (2.17c)
\]

\[
\bar{C}_A = -g_A (1 + \delta_\beta + \epsilon_L - \epsilon_R - \bar{\epsilon}_L + \bar{\epsilon}_R) \quad (2.17d)
\]

\[
\bar{C}_A' = -g_A (1 + \delta_\beta + \epsilon_L - \epsilon_R + \bar{\epsilon}_L - \bar{\epsilon}_R) \quad (2.17e)
\]

\[
\bar{C}_S = g_S (\epsilon_S + \bar{\epsilon}_S) \quad (2.17f)
\]

\[
\bar{C}_S' = g_S (\epsilon_S - \bar{\epsilon}_S) \quad (2.17g)
\]

\[
\bar{C}_P = g_P (\epsilon_P - \bar{\epsilon}_P) \quad (2.17h)
\]

\[
\bar{C}_P' = g_P (\epsilon_P + \bar{\epsilon}_P) \quad (2.17i)
\]

\[
\bar{C}_T = 4 g_T (\epsilon_T + \bar{\epsilon}_T) \quad (2.17j)
\]

\[
\bar{C}_T' = 4 g_T (\epsilon_T - \bar{\epsilon}_T) \quad (2.17k)
\]

Using these relations and the results of Ref. \[49\] one can easily work out the dependence of beta decay observables on the short-distance parameters \( \epsilon_i \) and \( \bar{\epsilon}_i \).

Our goal is to identify TeV-scale induced new physics contaminations of typical size \( \epsilon_\alpha \sim (v/\Lambda_{BSM})^2 \sim \mathcal{O}(10^{-3}) \) to the decay amplitude, so that they are comparable in size to the recoil corrections of \( \mathcal{O}(q/M_N) \sim 10^{-3} \) and the radiative corrections of \( \mathcal{O}(\alpha/\pi) \). Thus, it is useful to organize the discussion in terms of a simultaneous expansion in new physics contributions, recoil, and radiative corrections.

---

\(^5\)In the case of vector and axial bilinears, the Gordon decomposition can be used to trade the induced tensor term proportional to \( \sigma_{\mu\nu} q^\nu \) for an independent scalar term proportional to \( P_\mu \). Here we choose to follow the parameterization of Ref. \[75\].

\(^6\)Various metrics and conventions appear in the literature. Lee and Yang \[51\] employ the “ict” metric, which in this case maps to the metric we employ if we let \( \gamma_5 \rightarrow -\gamma_5 \) in their effective theory, noting \( \gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \). Refs. \[76\] and \[21\] employ the metric we do but flip the sign of the \( \gamma_5 \) terms.
keeping terms through first order only. Higher-order terms may prove negligible in light of anticipated experimental sensitivities, but we indicate the role of certain, more significant ones. Employing this simultaneous expansion in $\epsilon_\alpha$, $q/M_N$, and $\alpha/\pi$, we now discuss the contributions from the quark-bilinear operators:

- **Vector current**: The form factor $g_V(0)$ contributes at $\mathcal{O}(1)$ to the amplitude, whereas $\tilde{g}_{T(V)}(0)$ and $\tilde{g}_S(0)$ contribute at $\mathcal{O}(q/M_N)$. Also, in the SU(2)$_f$, or isospin, limit, the weak magnetism form factor $\tilde{g}_{T(V)}(0)$ is related to the difference of the empirical proton and neutron magnetic moments, which are well-known, and the induced-scalar form factor $\tilde{g}_S(q^2)$, reflective of the presence of a second-class current, vanishes [75]. Corrections to the isospin limit are of $\mathcal{O}((m_n - m_p)/M_N) \sim q/M_N$. Since $\tilde{g}_S$ multiplies one power of $q_\mu/M_N$, its contribution to the decay amplitude is effectively of second order in the recoil expansion.

- **Axial current**: The form factor $g_A(0)$ contributes at $\mathcal{O}(1)$, whereas $\tilde{g}_{T(A)}(0)$ and $\tilde{g}_P(0)$ contribute at $\mathcal{O}(q/M_N)$. The induced-tensor form factor $\tilde{g}_{T(A)}(q^2)$ vanishes in the isospin limit [75], so that its contribution to the decay amplitude is of second order in $q/M_N$. Similarly, the contribution associated with the induced-pseudoscalar form factor $\tilde{g}_P$ is quadratic in our counting, because the pseudoscalar bilinear is itself of order $q/M_N$, and is accompanied by an explicit $q/M_N$ suppression [4] if it can be studied in muon capture, note Ref. [77, 78] for reviews.

- **Pseudoscalar bilinear**: The pseudoscalar bilinear $\bar{u}_p \gamma_5 u_n$ is itself of order $q/M_N$. Since it necessarily multiplies a BSM effective coupling $\epsilon_P$ because there is no pseudoscalar coupling in the SM, this term is also of second order in our expansion.

- **Scalar and tensor bilinears**: These bilinears enter into the analysis multiplied by new-physics effective couplings $\epsilon_{S,T}$. Computing the corresponding matrix elements to zeroth order in the recoil expansion suffices to identify $g_S(0)$ and $g_T(0)$. Note that $g_T^{(1,2,3)}(q^2)$ appear only in $\mathcal{O}(q/M_N)$, and $\tilde{g}_T^{(3)}(q^2)$ vanishes in the isospin limit [75].

In summary, to the order we are working, the amplitudes depend only on $g_i \equiv g_i(0)$ with $i \in \{V, A, S, T\}$ and $\tilde{g}_{T(V)}(0)$. Up to second-order corrections in isospin breaking, one has $g_V = 1$ [72, 79, 80]. We define the ratio of the axial to vector form factors as $\lambda \equiv g_A/g_V$, where $\lambda > 0$ under our conventions. As we have noted, the neutron-decay amplitude in the presence of non-standard right-handed interactions is actually a function of $\hat{\lambda} \equiv \lambda(1 - 2\epsilon_R)$. The parameter $\hat{\lambda}$ is extracted very precisely from beta-asymmetry measurements in polarized neutron decay, leading to $\hat{\lambda} = 1.2701(25)$ [81]; this number is essentially $g_A$. There are no direct experimental handles on $g_{A,P,T}$.

A first principles calculation of $g_{A,S,P,T}$, however, is possible with LQCD. The status of LQCD calculations of these charges is critically reviewed in [17], and in which the first estimate of $g_S$ from LQCD is provided. Different calculations give results in the range $1.12 < g_A < 1.26$; the errors are much larger than the experimental uncertainty. In constrast, the new estimates for the scalar and tensor charge in the $\overline{MS}$ scheme and at $\mu = 2$ GeV are $g_S = 0.8 \pm 0.4$ and $g_T = 1.05 \pm 0.35$. Besides statistical uncertainties, which are particularly large for $g_S$, the dominant LQCD systematic effects in $g_{S,T}$ arise from extrapolation in the quark mass to the physical point and from the renormalization constants, noting perturbative calculations were used to arrive at the reported results, and the non-perturbative calculation is in progress. More recently Ref. [82] provided improved results for the scalar and tensor charges, $g_S = 1.08 \pm 0.28$ and $g_T = 1.038 \pm 0.011$, with the uncertainty associated with statistics and chiral extrapolation. These results, however, do not include an estimate of the systematic error.

7This term, however, is enhanced. Using the partially conserved axial current one can show that the form factor $\tilde{g}_P$ is of order $M_N/m_q \sim 100$, making a $\mathcal{O}(10^{-4})$ contribution to the amplitude. The effect of $\tilde{g}_P$ on beta-decay rates has been worked out in Ref. [76], and it should be included when the experiments reach that level of precision.
associated with finite volume and finite lattice spacing extrapolations. Therefore, in what follows we use the results of Ref. [47] as the baseline lattice results.

\section{2.2.3 Nuclear matrix elements}

In moving from neutron decay to a general nuclear beta decay, a number of important differences appear. For one thing, numerous spin sequences are possible. Also the daughter state may well itself be unstable under electromagnetic or strong interactions. Finally, the \( Q \)-value, defined as the maximum electron (positron) kinetic energy, is generally much larger than the 0.8 MeV found in neutron beta decay and in some cases can be as large as 10-15 MeV. The electron/positron energy dependence of the decay observables are then much more appreciable.

The spins are easily dealt with by the use of Clebsch-Gordan coefficients \( C_{J_1 J_2 J; M}^{M_1 M_2 M} \), together with the Wigner-Eckart theorem to define reduced matrix elements. Thus the general form for a vector and axial-vector nuclear matrix element between parent and daughter states having spins \( J, M \) and \( J', M' \) and masses \( M_1 \) and \( M_2 \), respectively, is \( [76] \)

\[
\ell^\mu < \beta | V_\mu | \alpha > = \left( a(q^2) \frac{P \cdot \ell}{2M_A} + e(q^2) \frac{q \cdot \ell}{2M_A} \right) \delta_{JJ'} \delta_{MM'} + i \frac{\tilde{b}(q^2)}{2M_A} C_{J_1 J; M}^{M_1 M_2 M} (q \times \ell)_k \\
+ C_{J_2 J; M}^{M_1 M_2 M} \left[ \frac{f(q^2)}{2M_A} C_{11; 11}^{nn'; k} \ell_n q_{\nu'} + \frac{g(q^2)}{(2M_A)^3} P \cdot \ell \sqrt{\frac{4\pi}{5}} \lambda S(q^2) + \ldots \right] \\
\ell^\mu < \beta | A_\mu | \alpha > = C_{J_1 J; M}^{M_1 M_2 M} \left[ c(q^2) \lambda P^n - d(q^2) \ell \lambda q^n + \frac{1}{(2M_A)^3} h(q^2) q^n q \cdot \ell \right] \\
+ C_{J_2 J; M}^{M_1 M_2 M} \left[ \frac{4\pi}{5} Y_2^0(q^2) \frac{q^2}{(2M_A)^2} j_2(q^2) \right] \\
+ C_{J_3 J; M}^{M_1 M_2 M} \left[ \frac{4\pi}{5} Y_2^0(q^2) \frac{q^2}{(2M_A)^2} j_3(q^2) + \ldots \right], \tag{2.18}
\]

where \( \ell^\mu \) denotes the leptonic current and \( M_A = (M_1 + M_2)/2 \). Here each term corresponds to one in the analogous neutron transition via

\[
a \rightarrow g_V, \quad c \rightarrow g_A \sqrt{3} \\
\tilde{b} \rightarrow \tilde{g}_{T(V)} \sqrt{3}, \quad d \rightarrow \tilde{g}_{T(A)} \sqrt{3} \\
e \rightarrow g_S, \quad h \rightarrow g_P \sqrt{3}. \tag{2.19}
\]

In addition, there exist terms \( f, g, j_2, j_3 \) which have no \( J = \frac{1}{2} \rightarrow J' = \frac{1}{2} \) analog since they involve \( \Delta J = 2, 3 \).

For each form factor there exist known one-body operator, or impulse approximation, predictions.

\footnote{Here we discuss only allowed decays, for which \( \Delta J = 0, \pm 1 \) with no change in nuclear parity.}
Defining the nuclear mass difference $\Delta = M_1 - M_2$, we have

\[ a(q^2) \simeq (1 + \frac{\Delta}{2M_A})^{-1}g_v(q^2) \times [M_F + \frac{1}{6}(q^2 - \Delta^2)M_{r,2} + \frac{\Delta}{3}M_{r,p}] \]
\[ b(q^2) \simeq A[\tilde{g}_{T(\nu)}(q^2)M_{GT} + g_v(q^2)M_L] \]
\[ c(q^2) \simeq (1 + \frac{\Delta}{2M_A})^{-1}g_A(q^2)[M_{GT} + \frac{1}{6}(q^2 - \Delta^2)M_{\sigma r,2} + \frac{1}{6\sqrt{10}}M_{1\gamma}(2\Delta^2 + q^2) \]
\[ + A\frac{\Delta}{2M_A}M_{\sigma L} + \frac{\Delta}{2}M_{\sigma r p}] \]
\[ d(q^2) \simeq (1 + \frac{\Delta}{2M_A})^{-1}g_A(q^2)[-M_{GT} - \frac{1}{6}(q^2 - \Delta^2)M_{\sigma r,2} \]
\[ + \frac{1}{\sqrt{10}}M_{1\gamma}(M_A\Delta + \frac{1}{6}(\Delta^2 - q^2)) \]
\[ + AM_{\sigma L} + AM_{\sigma r p}] \pm A\tilde{g}_{T(A)}(q^2)M_{GT} \]
\[ e(q^2) \simeq (1 + \frac{\Delta}{2M_A})^{-1}g_v(q^2)[M_F + \frac{1}{6}(q^2 - \Delta^2)M_{r,2} - \frac{2M_A}{3}M_{r,p}] \pm A\tilde{g}_S(q^2)M_F \]
\[ f(q^2) \simeq g_v(q^2)2MA\mathcal{M}_{(r,p)} \]
\[ g(q^2) \simeq -g_v(q^2)\frac{4M_A^3}{3}M_Q \]
\[ h(q^2) \simeq -(1 + \frac{\Delta}{2M_A})^{-1} \times [g_A(q^2)\frac{2M_A^2}{\sqrt{10}}M_{1\gamma} + \tilde{g}_P(q^2)A^2M_{GT}] \]
\[ j_i(q^2) \simeq -\frac{2M_A^2}{3}g_A(q^2)M_{i\gamma}, \quad i = 2, 3, \quad (2.20) \]

where the upper (lower) sign refers to electron (positron) emission and the $\mathcal{M}$'s represent reduced, nonrelativistic matrix elements, namely,

\[ \mathcal{M}_F = <\beta||\sum_i \tau_i^\pm||\alpha> \]
\[ \mathcal{M}_{GT} = <\beta||\sum_i \tau_i^\pm \tilde{\sigma}_i||\alpha> \]
\[ \mathcal{M}_{r,2} = <\beta||\sum_i \tau_i^\pm r_i^2||\alpha> \]
\[ \mathcal{M}_{\sigma r,2} = <\beta||\sum_i \tau_i^\pm r_i^2 \tilde{\sigma}_i||\alpha> \]
\[ \mathcal{M}_{r,p} = \frac{i}{2M_N} <\beta||\sum_i \tau_i^\pm (\vec{r}_i \cdot \vec{p}_i + \vec{p}_i \cdot \vec{r}_i)||\alpha> \]
\[ \mathcal{M}_{\sigma L} = <\beta||\sum_i \tau_i^\pm \tilde{\sigma}_i \times (\vec{r}_i \times \vec{p}_i)||\alpha> \]
\[ \mathcal{M}_{K\gamma} = \sqrt{\frac{16\pi}{5}} <\beta||\sum_i \tau_i^\pm r_i^2 C_{12,k'}^{n'n'k} \times \sigma_{in}Y_{2n'}^2(\vec{r}_i)||\alpha>, \quad K = 1, 2, 3 \]
\[ \mathcal{M}_{\sigma r p} = \frac{i}{2M_N} <\beta||\sum_i \tau_i^\pm [\{\tilde{\sigma}_i \cdot \vec{r}_i, \vec{p}_i\} + \{\tilde{\sigma}_i \cdot \vec{p}_i, \vec{r}_i\}]||\alpha>, \quad (2.21) \]
as well as

\[
\mathcal{M}_Q = <\beta|| \sum_i \tau_i^\pm r_i Y_k^2(\hat{r}_i)||\alpha>
\]

\[
\mathcal{M}_{(r,p)} = \frac{i}{2M_N} <\beta|| \sum_i \tau_i^\pm C^{mn';k}_{11;2} \times (r_{in}p_{in'} + p_{in}r_{in'})||\alpha>
\]

\[
\mathcal{M}_L = <\beta|| \sum_i \tau_i^\pm (\vec{r}_i \times \vec{p}_i)||\alpha> .
\] (2.22)

One expects meson exchange, or two-body, corrections to these predictions at the order of 5 to 10% or so [83].

If we neglect recoil effects, which is generally a good approximation because they enter the nuclear matrix elements at \(\mathcal{O}(\Delta/M_N)\), there exist only the leading Fermi and Gamow-Teller form factors \(a = g_VM_F\) and \(c = g_AM_{GT}\). Most experiments are analyzed in terms of only these quantities. In the SM the vector weak charge is also the isospin raising operator, so that the Fermi form factor \(a(0)\) must vanish unless the parent and daughter states are isotopic analogs. Moreover, if the parent-daughter analog states have spin-parity \(0^+\), then the Gamow-Teller form factor vanishes—\(c(q^2) = 0\)—and the Fermi matrix element becomes a simple numerical factor, namely, for unit isospin spin \(a(0) = \sqrt{2}\), which is an exact prediction up to isospin breaking effects. The calculation of the latter effects has been an area of active investigation [84, 85, 86, 87, 88, 89, 90, 91]. In simple terms, one needs to take into account that the “last” proton in the parent positron emitter is less strongly bound than the “last” neutron in the daughter state. An extensive analysis of data by Hardy and Towner, taking into account isospin breaking, has yielded the very precise value \(V_{ud} = 0.97425(22)\) [92].

No symmetry principle determines the size of Gamow-Teller matrix elements, and these must either be determined empirically from lifetime or correlation coefficient measurements or calculated from nuclear wavefunctions, though the latter can only be done precisely if meson exchange effects are included. In the case of BSM matrix elements, wavefunction calculations of the requisite matrix elements are required.

3 Cabibbo and lepton universality

Beta decays provide stringent constraints on non-standard couplings through two classes of observables. (i) Total decay rates, after inclusion of radiative corrections, provide information on the strength of weak interactions, thus enabling precision tests of Cabibbo and lepton universality. (ii) Differential decay distributions, including spectra and correlations, are sensitive to the Lorentz structure of the underlying weak interaction, thus enabling searches for small non-(\(V - A\)) components. In this section we focus on universality tests.

3.1 Cabibbo Universality

In the SM the effective Fermi constant \(G_\beta\) controlling semi-leptonic transitions \(u_i \leftrightarrow d_j\) for \(i,j \in 1,2,3\) is related to \(G_\mu\) by

\[
G_\beta = G_\mu V_{ij} \left(1 + \delta_\beta - \delta_\mu\right),
\] (3.23)

where \(G_\beta \equiv V_{ij}G_F^{(0)}(1 + \delta_\beta)\), the unitary CKM matrix \(V_{ij}\) parameterizes quark mixing, and \(\delta_\beta\) and \(\delta_\mu\) encode electroweak radiative corrections (see Eqs. (2.7) and (2.8)). We can thus test Cabibbo (or quark-lepton) universality by testing whether \(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1\). Beta decay rates permit access to the CKM matrix elements \(V_{ud}\) and \(V_{us}\). Since both the SM prediction and the experimental
measurements have reached the sub-percent level, these observables provide strong constraints on new physics, through the parameter $\Delta_{\text{CKM}}$ defined as

$$\Delta_{\text{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 \, .$$

(3.24)

Here $V_{ij}$ are the CKM elements determined phenomenologically from semileptonic decays assuming only SM dynamics. The ratio $V_{ij}/V_{ij}$ is parameterized in terms of the BSM couplings, as exemplified by Eqs. (3.25) and (3.29). In the unitarity sum, $|V_{ub}| = 3.51(13) \times 10^{-3}$ [81] plays no role; $V_{ud}$ and $V_{us}$ are both important and can be determined with high precision in a number of channels. The degree of needed theoretical input varies, depending on the component of the weak current which contributes to the hadronic matrix element. Roughly speaking, one can group the channels leading to $V_{ud,us}$ into three classes:

- **Semileptonic decays in which only the vector component of the weak current contributes:** These are theoretically favorable in the SM because the matrix elements of the vector current at zero momentum transfer are known in the SU(2) f (SU(3) f) limit of equal light quark masses. Moreover, corrections to the symmetry limit are quadratic in $m_{s,d} - m_u$ [73, 72]. Super-allowed nuclear beta decays ($0^+ \to 0^+$), pion beta decay ($\pi^+ \to \pi^0 e^+ \nu_e$), and $K \to \pi \ell \nu$ decays belong to this class. The determination of $V_{ud,us}$ from these modes requires theoretical input on radiative corrections [41, 42, 43, 93, 94, 95] and hadronic matrix elements via analytic methods [96, 97, 98, 99, 100, 101, 102], or lattice QCD methods [103, 104, 105, 106, 107].

- **Semileptonic transitions in which both the vector and axial component of the weak current contribute:** Neutron decay ($n \to p e\bar{\nu}$) and hyperon decays ($\Lambda \to p e\bar{\nu}$, ...,), as well as nuclear mirror transitions, belong to this class. In this case the matrix elements of the axial current must be determined experimentally [108, 109]. Inclusive $\tau$ lepton decays $\tau \to h \nu_\tau$ also belong to this class as both $V$ and $A$ currents contribute, though the relevant matrix elements can be calculated theoretically via the Operator Product Expansion [110, 111].

- **Leptonic transitions in which only the axial component of the weak current contributes:** In this class one finds meson decays such as $\pi(K) \to \mu \bar{\nu}$ but also exclusive $\tau$ decays such as $\tau \to \nu_\tau \pi(K)$. Experimentally one can determine the products $V_{ud} F_0$ and $V_{us} F_K$. With the advent of precision calculations of $F_K/F_\pi$ in lattice QCD [112, 113, 114, 115, 116], this class of decays provides a useful constraint on the ratio $V_{us}/V_{ud}$ [117, 118].

Currently, the determination of $V_{ud}$ is dominated by $0^+ \to 0^+$ super-allowed nuclear beta decays [96, 92], leading to $V_{ud} = 0.97425(22)$, while the best determination of $V_{us}$ arises from $K_{\ell 2}, K_{\ell 3}$ decays, leading to the best fit $V_{us} = 0.2256(9)$ [119, 118]. These determinations lead to $\Delta_{\text{CKM}} = (1 \pm 6) \times 10^{-4}$, in remarkable agreement with the CKM unitarity of the SM. This is illustrated in Fig. 1. Next, we discuss the implications on BSM physics in a model-independent framework.

### 3.1.1 Model independent constraints

Each element $V_{ij}$ receives a universal, or channel independent, correction due to possible new physics corrections in muon decay, parameterized by $\epsilon$ in Eq. (2.8). Additionally, $V_{ij}$ receives channel-dependent BSM contributions, which are linear combinations of the $\epsilon$’s defined in Eq. (2.7). Given the hierarchy $|V_{ud}|^2 \gg |V_{us}|^2$, we discuss in detail only the BSM contributions to $V_{ud}$, beginning with the $0^+ \to 0^+$ nuclear transitions.

From each $0^+ \to 0^+$ transition, working in the impulse approximation, one extracts the quantity

$$|V_{ud}|^2_{0^+ \to 0^+} = |V_{ud}|^2 \left[ 1 + 2 \Re(\epsilon_L + \epsilon_R - \epsilon_\mu) + c_{0^+}^S (Z) g_S \Re \epsilon_S \right] \, .$$

(3.25)
where the first correction reflects the BSM shift in the vector operator minus the shift in the Fermi constant extracted in muon decay. The second correction, proportional to $\text{Re}\, \epsilon_S$, arises from a non-vanishing Fierz interference term; this distorts the electron spectrum and therefore the phase space integrals. The correction depends on the individual nuclear transitions, through $\epsilon^{\text{S}}_{0^+,Z} = -2\sqrt{1-\alpha^2Z^2} \frac{I_1(Q_{EC}/m_e)}{I_0(Q_{EC}/m_e)}$, $I_k(x_0) = \int_1^{x_0} x^{1-k}(x_0-x)^2 \sqrt{x^2-1} \, dx$, (3.26)

where $Q_{EC} = M_1 - M_2$ with $m_e$ the electron mass. The transition strengths, or $ft$ values, after the application of transition-dependent radiative corrections and isospin-symmetry-breaking corrections to the nuclear matrix elements, are remarkably constant with the $Z$ of the daughter nucleus, supporting the Conserved-Vector-Current (CVC) “hypothesis”, though CVC is simply a consequence of the SM. Moreover, CVC is also tested by studies of nuclear mirror transitions, albeit with lesser precision $^{109}$. The computation of the isospin-symmetry-breaking corrections, noting e.g. Ref. $^{85}$, has been criticized $^{89,90}$, but the employed procedure has been experimentally validated in a system for which the correction is particularly large $^{120}$. For a discussion of the various methods to compute isospin-breaking effects, see Ref. $^{91}$. From the constancy of the corrected $ft$ values with the $Z$ of the daughter nucleus, Hardy and Towner $^{96}$ obtain the combined bound

$$-1.0 \times 10^{-3} < g_S \text{Re}\, \epsilon_S < 3.2 \times 10^{-3} \quad (90\% \, \text{C.L.}).$$

(3.27)

It is the most stringent bound on scalar interactions from low-energy probes. Moreover, the $\Delta_{\text{CKM}}$ constraint implies

$$\text{Re}(\epsilon_L + \epsilon_R - \epsilon_\mu) < 5 \times 10^{-4} \quad (90\% \, \text{C.L.}).$$

(3.28)
This is one of the strongest precision constraints on new physics. It corresponds to effective scales \( \Lambda > 11 \text{ TeV} \) [45]. The resulting constraints on the weak-scale couplings \( \hat{\alpha}_j \) [45, 47, 37] are comparable to that obtained from Z-pole experiments, and are stronger than the ones obtained from \( \sigma(e^+e^- \rightarrow q\bar{q}) \) at LEP.

In principle, neutron decay allows for an extraction of \( V_{ud} \) free of nuclear structure uncertainties. Assuming no BSM effects one has \( V_{ud} = [4908.7(1.9) s/(\tau_n(1 + 3g_A^2))]^{1/2} \) [42, 43]. An extraction of \( V_{ud} \) competitive with nuclear decays requires \( \delta g_A/g_A \sim 0.025\% \) and \( \delta \tau_n \sim 0.35\% \) (\( \delta \tau_n/\tau_n = 0.04\% \)). In turn, a determination of \( g_A \) at the required level necessitates a measurement of the beta asymmetry \( A \) at the 0.1\% level. The expression for \( |V_{ud}|^2 \) at LEP reads [47]

\[
|V_{ud}|^2 \rightarrow |V_{ud}|^2 \left[ 1 + 2 \text{Re}(\epsilon_L + \epsilon_R - \epsilon_\mu) + \frac{2}{1 + 3\lambda^2} \left( g_S \text{Re} \epsilon_S - 12 \lambda g_T \text{Re} \epsilon_T \right) \left( \frac{I_1(x_0)}{I_0(x_0)} - \frac{6\lambda^2}{1 + 3\lambda^2} \right) \right],
\]

where \( \lambda \equiv g_A/g_V \), \( x_0 = E_0/m_\pi \), with \( E_0 \) the electron endpoint energy. The constant \( c \) is a certain \( O(1) \) number that depends on the specific experimental analysis used to extract \( \lambda \) from measurements of the beta asymmetry \( A \), presuming the presence of spectrum contaminations due to scalar and tensor operators. A different correction would appear if one extracted \( \lambda \) from a different observable, such as \( a \) (see the discussion in Ref. [47]).

### 3.2 Lepton Universality

The ratio \( R_\pi \equiv \Gamma(\pi \rightarrow e\nu[\gamma]) / \Gamma(\pi \rightarrow \mu\nu[\gamma]) \) is helicity-suppressed in the SM, due to the \( V-A \) structure of charged current couplings. It is therefore a sensitive probe of all SM extensions that induce axial and especially pseudoscalar currents, as well as of non-universal corrections to the charged current lepton couplings. The quantity \( R_\pi \) can be predicted very precisely in the SM because the leading hadronic input, namely, the pion decay constant \( F_\pi \), cancels in the ratio. Once one includes electroweak radiative corrections, hadronic structure effects do appear, and the SM prediction can be organized within the ChPT power counting as follows:

\[
R_\pi^{\text{SM}} = R_\pi^{(0)} \left[ 1 + \Delta e^2p^2 + \Delta e^2p^4 + \ldots \right]
\]

\[
R_\pi^{(0)} = \frac{m_\pi^2}{m_\mu^2} \left( \frac{m_\pi^2 - m_e^2}{m_\mu^2 - m_e^2} \right)^2.
\]

The leading electromagnetic correction \( \Delta e^2p^2 \) corresponds to the point-like approximation for the pion [121] 122. The NNLO (two-loop) correction \( \Delta e^2p^4 \) has been calculated within ChPT in Refs. [123] 124. The two-loop effective theory results have been complemented by a large-\( N_c \) calculation of an associated counterterm and by summation of leading logarithms \( \alpha^n \ln^n(m_\mu/m_e) \) giving [123] 124

\[
R_\pi = (1.2352 \pm 0.0001) \times 10^{-4},
\]

The central value of \( R_\pi \) is in agreement with the results of previous calculations [122] 125, pushing the theoretical uncertainty below the 0.1 per-mille level.

#### 3.2.1 Model independent constraints

The ratio \( R_\pi \equiv \Gamma(\pi \rightarrow e\nu[\gamma]) / \Gamma(\pi \rightarrow \mu\nu[\gamma]) \) probes more than the effective low-energy pseudoscalar coupling \( \epsilon_P \) defined earlier as the coefficient of the operator \( \bar{e}(1 - \gamma_5)\nu_e \cdot \bar{\mu}_\gamma d \). In fact, since (i) \( R_\pi \)
is defined as the ratio of electron-to-muon decay and (ii) the neutrino flavor is not observed in either
decay, this observable is sensitive to the whole set of parameters $\epsilon_P^{\alpha\beta}$ and $\tilde{\epsilon}_P^{\alpha\beta}$ defined by
\begin{equation}
\mathcal{L}_{\text{eff}} \supset \frac{G_F}{\sqrt{2}} V_{ud} \left[ \epsilon_P^{\alpha\beta} \bar{e}_\alpha (1 - \gamma_5) \nu_\beta \cdot \bar{u} \gamma_5 d + \tilde{\epsilon}_P^{\alpha\beta} \bar{e}_\alpha (1 + \gamma_5) \nu_\beta \cdot \bar{u} \gamma_5 d \right]
\tag{3.33}
\end{equation}

where $\alpha \in \{e, \mu\}$ refers to the flavor of the charged lepton and $\beta \in \{e, \mu, \tau\}$ refers to the neutrino
flavor. One generically expects SM extensions to generate non-diagonal components in $\epsilon_P^{\alpha\beta}$. In the new
notation the previously defined pseudoscalar, scalar, and tensor couplings read $\epsilon_{P,S,T} \equiv \epsilon_P^{e\mu,\tau}$. It
is important to note that only $\epsilon_P^{ee}$ and $\epsilon_P^{\mu\mu}$ can interfere with the SM amplitudes, while the remaining
$\epsilon_P^{e\mu}$ and $\epsilon_P^{e\tau}$ each enter as an absolute square in the numerator and denominator of $R_\pi$. In summary,
allowing for non-standard axial and pseudoscalar interactions and factoring out the SM prediction for
$R_\pi$, one can write: \[47\]
\begin{equation}
\frac{R_\pi}{R_{\text{SM}}} = \frac{\left[ 1 + \epsilon_P^{ee} - \epsilon_R^{ee} - \frac{B_0}{m_e} \epsilon_P^{ee} \right]^2 + \left[ \frac{B_0}{m_e} \epsilon_P^{\mu\mu} \right]^2 + \left[ \sum_\alpha \frac{B_0}{m_e} \epsilon_P^{e\alpha} \right]^2}{\left[ 1 + \epsilon_P^{\mu\mu} - \epsilon_R^{\mu\mu} - \frac{B_0}{m_\mu} \epsilon_P^{\mu\mu} \right]^2 + \left[ \frac{B_0}{m_\mu} \epsilon_P^{\mu\mu} \right]^2 + \left[ \sum_\alpha \frac{B_0}{m_\mu} \epsilon_P^{e\alpha} \right]^2} = 1 + \Delta \epsilon_{e/\mu}
\tag{3.34}
\end{equation}

where we note that the BSM couplings can be complex. In the above equation the factors of $B_0/m_{e,\mu} \epsilon_P$ represent the ratio of the new-physics amplitude over the SM amplitude. The latter is proportional
to the charged-lepton mass due to angular-momentum conservation arguments, while the former is proportional
to $\langle 0 | \bar{u} \gamma_5 d | \pi \rangle$, characterized by the scale- and scheme-dependent parameter: \[47\]
\begin{equation}
B_0(\mu) \equiv \frac{M_\pi^2}{m_u(\mu) + m_d(\mu)}
\tag{3.35}
\end{equation}

Since $B_0^{\text{SM}}(\mu = 1 \text{ GeV}) = 1.85 \text{ GeV}$ (using the PDG \[126\] central values for the light quark masses) and consequently $B_0/m_e = 3.62 \times 10^3$, $R_\pi$ has enhanced sensitivity to $\epsilon_P^{e\mu}$, and one needs to keep quadratic
terms in these new physics coefficients. We discuss bounds on $\epsilon_A \equiv \epsilon_L - \epsilon_R$ and $\epsilon_P^{e\alpha}, \epsilon_P^{e\beta}$ separately.
First, setting $\epsilon_A = 0$, inspection of Eq. (3.34) reveals that if the new-physics couplings respect $\epsilon_P^{e\alpha}/m_e = \epsilon_P^{e\mu}/m_\mu$, then $R_\pi/R_{\text{SM}} = 1$, and there are no constraints on these couplings. On the other hand, if
the effective couplings $\epsilon_P^{e\alpha}$ are all of similar size, one can neglect the entire denominator in Eq. (3.34),
as it is suppressed with respect to the numerator by powers of $m_e/m_\mu$. We will assume the second
scenario operates. In this case the constraint in Eq. (3.34) forces the couplings $\epsilon_P^{e\alpha}, \epsilon_P^{e\beta}, \epsilon_P^{e\gamma}, \epsilon_P^{e\delta}$ to live in a
spherical shell of radius $m_e/B_0 \sqrt{R_\pi^{\text{exp}}/R_{\text{SM}}} \approx 2.75 \times 10^{-4}$ centered at $\text{Re}(\epsilon_P^{e\delta}) = m_u/B_0 \approx 2.75 \times 10^{-4}$,
$\epsilon_P^{e\mu} = \epsilon_P^{e\tau} = \epsilon_P^{e\alpha} = 0$. The thickness of the shell is numerically $1.38 \times 10^{-6}$ and is determined by the
current combined uncertainty in $R_\pi^{\text{exp}}$ \[127\] \[128\] \[129\] and $R_{\text{SM}}$ \[124\] \[123\]: $R_\pi^{\text{exp}}/R_{\text{SM}} = 0.996(5)$ (90% C.L.). This is illustrated in Fig. 2 where we plot the allowed region in the two-dimensional plane given by $\text{Re}(\epsilon_P^{e\delta})$ and a generic “wrong-flavor” coupling denoted by $\epsilon_P^{\delta x} (x \neq e)$ — or this can be $\text{Im}(\epsilon_P^{e\delta})$ or any of the real or imaginary parts of $\epsilon_P^{e\delta}$ and $\tilde{\epsilon}_P^{e\delta}$. Note that the allowed region is given by the thickness
of the curve in the figure, thus enforcing a strong correlation between $\epsilon_P^{ee}$ and $\tilde{\epsilon}_P^{ee}$. Since $\epsilon_P^{ex}$ and others
of that sort are essentially unconstrained by other measurements, though we expect they can be of
$O(10^{-3})$, we can neglect all of the couplings but one to obtain a bound on that one. The resulting
bounds using $R_\pi$ at 90%-C.L. are
\begin{equation}
-1.4 \times 10^{-7} < \text{Re}(\epsilon_P^{e\delta}) < 5.5 \times 10^{-4}, \quad \text{or} \quad -2.75 \times 10^{-4} < \text{Im}(\epsilon_P^{e\delta}) < 2.75 \times 10^{-4},
\tag{3.36}
\end{equation}

\footnote{Note that the scale and scheme dependence of $B_0(\mu)$ is compensated in physical quantities by the scale and scheme
dependence of the Wilson coefficients $\epsilon_P^{e\delta}$.}
Figure 2: Illustration of the allowed region in the two-dimensional plane \( \text{Re}(\epsilon_{ee}^P) - \text{Re}(\epsilon_{ex}^P) \) (with \( x \neq e \)) determined by \( R_\pi \), which is given by an annulus of thickness \( 1.38 \times 10^{-6} \). In the absence of information on \( \text{Re}(\epsilon_{ex}^P) \), the 90% C.L. bound on \( \text{Re}(\epsilon_{ee}^P) \) is \(-1.4 \times 10^{-7} < \text{Re}(\epsilon_{ee}^P) < 5.5 \times 10^{-4} \). Note that \( \text{Im}(\epsilon_{ee}^P), \text{Re}(\tilde{\epsilon}_{ee}^P), \) and \( \text{Im}(\tilde{\epsilon}_{ee}^P) \) are subject to the same bound as \( \text{Re}(\epsilon_{ex}^P) \). Figure adapted from Ref. [47].

Note that \( \text{Re}(\epsilon_{ex}^P) \) and \( \text{Im}(\epsilon_{ex}^P) \), as well as \( \text{Re}(\tilde{\epsilon}_{ee}^P) \) and \( \text{Im}(\tilde{\epsilon}_{ee}^P) \), are all subject to the same bound as \( \text{Im}(\epsilon_{ee}^P) \). Our results are in qualitative agreement with the findings of Refs. [58, 27].

Alternatively, neglecting the pseudoscalar couplings, one obtains the following combined limit on the axial combination of new couplings:

\[-4.5 \times 10^{-3} < \text{Re} (\epsilon_{ee}^A - \epsilon_{e\mu}^A) < 0.5 \times 10^{-3} \quad (3.37)\]

Finally, we discuss how \( R_\pi \) is also sensitive to non-standard scalar and tensor couplings. As originally discussed in Refs. [57, 58, 59], the pseudoscalar coupling \( \epsilon_{ee}^P \) can be radiatively generated starting from nonzero \( \epsilon_{S,T} \). Hence, the stringent constraint in Eq. (3.36) puts constraints on the same \( \epsilon_{S,T} \) that can be probed in beta decays. The physics of the effect is this: once the scalar, pseudoscalar, and tensor operators are generated by some non-standard physics at the matching scale \( \Lambda \), electroweak radiative corrections induce mixing among these three operators. Thus even if \( \epsilon_{P}(\Lambda) \) vanishes at the matching scale, known SM physics generates a nonzero \( \epsilon_{P}(\mu) \) at some lower energy scale \( \mu \) via loop diagrams.

The general form of the constraint can be worked out by using the three-operator mixing results from Ref. [59]. The leading-order result is

\[
\epsilon_{P}^{\alpha\beta}(\mu) = \epsilon_{P}^{\alpha\beta}(\Lambda) \left(1 + \gamma_{PP} \log \frac{\Lambda}{\mu}\right) + \epsilon_{S}^{\alpha\beta}(\Lambda) \gamma_{SP} \log \frac{\Lambda}{\mu} + \epsilon_{T}^{\alpha\beta}(\Lambda) \gamma_{TP} \log \frac{\Lambda}{\mu}. \quad (3.38a)
\]

\[
\gamma_{PP} = \frac{3 \alpha_2}{4 \pi} + \frac{113 \alpha_1}{72 \pi} \approx 1.3 \times 10^{-2} \quad (3.38b)
\]

\[
\gamma_{SP} = \frac{15 \alpha_1}{72 \pi} \approx 6.7 \times 10^{-4} \quad (3.38c)
\]

\[
\gamma_{TP} = -\frac{9 \alpha_2}{2 \pi} - \frac{15 \alpha_1}{2 \pi} \approx -7.3 \times 10^{-2} \quad , (3.38d)
\]
where \( \alpha_1 = \alpha / \cos^2 \theta_W \) and \( \alpha_2 = \alpha / \sin^2 \theta_W \) are the U(1) and SU(2)\(_L\) weak couplings, expressed in terms of the fine-structure constant and the weak mixing angle. Setting \( \epsilon_\nu^T (\Lambda) = 0 \) and neglecting the small \( O (\alpha / \pi) \) fractional difference between \( \epsilon_{S,T} (\Lambda) \) and the observable \( \epsilon_{S,T} (\mu) \) at the low scale, the constraints on \( \epsilon_S \) and \( \epsilon_T \) using \( R_\pi \) at 90\% C.L. read

\[
-1.4 \times 10^{-7} \frac{\log(\Lambda/\mu)}{< \gamma_{SP} \text{Re}(\epsilon_S) + \gamma_{TP} \text{Re}(\epsilon_T) < 5.5 \times 10^{-4} \frac{\log(\Lambda/\mu)}{,}}
\]

and

\[
|\gamma_{SP} \text{Im}(\epsilon_S) + \gamma_{TP} \text{Im}(\epsilon_T)| < 2.75 \times 10^{-4} \frac{\log(\Lambda/\mu)}{,}
\]

Assuming \( \log(\Lambda/\mu) \sim 10 \), so that, e.g., \( \Lambda \sim 10 \) TeV and \( \mu \sim 1 \) GeV, and using the numerical values of \( \gamma_{SP,TP} \), one finds that the individual constraints are at the level of \( |\text{Re}(\epsilon_S)| \lesssim 8 \times 10^{-2}, |\text{Im}(\epsilon_S)| \lesssim 4 \times 10^{-2}, |\text{Re}(\epsilon_T)| \lesssim 10^{-3}, \) and \( |\text{Im}(\epsilon_T)| \lesssim 0.5 \cdot 10^{-3} \). These bounds become logarithmically more stringent as the new-physics scale \( \Lambda \) grows. It is worth noting that analogous studies are also possible in kaon decays, and new results are expected from NA62 at CERN \[25\] and TREK at J-PARC \[130\].

Constraints on \( \tilde{\epsilon}_{S,T} \) can be worked out similarly \[59\], resulting in \( |\text{Re}(\tilde{\epsilon}_S)| \lesssim 5 \times 10^{-2}, |\text{Im}(\tilde{\epsilon}_S)| \lesssim 2.5 \times 10^{-2}, |\text{Re}(\tilde{\epsilon}_T)| \lesssim 0.6 \times 10^{-3}, \) and \( |\text{Im}(\tilde{\epsilon}_T)| \lesssim 0.3 \times 10^{-3} \), which together with \( \tilde{\epsilon}_P \) are the strongest low-energy bounds on the \( \tilde{\epsilon} \) couplings \[37\].

## 4 Decay correlations and non-(V-A) couplings

Differential decay distributions in beta decays are very sensitive to the Lorentz structure of the underlying weak interaction, thus enabling searches for small non-(V-A) components. Following Ref. \[49\], one writes the differential decay distribution in the nuclear decay \( P \to D e^- \bar{\nu} (e^+\nu) \) as a function of electron (positron) energy and lepton directions as follows,

\[
\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{1}{(2\pi)^3} p_e E_e (E_0 - E_e)^2 \xi \left\{ 1 + b \frac{m_e}{E_e} + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \left( \frac{\vec{J}}{J} \right) \left[ A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} + D \frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu} \right] + \ldots \right\}
\]

where \( P \) and \( D \) represent the parent and daughter nuclei, \( \langle \vec{J} / J \rangle \) represents the parent nucleus polarization, if any, and \( \vec{p}_{e,\nu} \) are the electron (positron) and antineutrino (neutrino) three-momenta. We have omitted the additional parity-conserving term which appears if \( J \neq 1/2 \), as indicated by the ellipsis. The coefficient \( b \) is the Fierz interference term, \( a \) is the electron-antineutrino correlation, \( A \) is the beta asymmetry, \( B \) is the antineutrino asymmetry, and the coefficient \( D \) is \( T \) odd in that the associated triple product of vectors is motion-reversal odd. All these quantities contain combinations of the Lee-Yang effective coefficients as delineated in Ref. \[49\], as does \( \xi \), and are related to our parameters as per Eq. \[2,17\]. Additional terms are present if one can measure the polarization of the emitted electron or positron \[49\]. Note, too, that the various correlation coefficients become \( E_e \) dependent once corrections of radiative and recoil order are included.

The decay correlations can be measured in neutron and nuclear decays, and substantial progress is expected in the next few years. In neutron decay, both cold and ultracold neutrons, implying distinct experimental techniques and hence entirely independent sources of systematic error, are used to measure these correlations. In the future we can expect experiments poised to take advantage of cold neutron beams of much greater intensity at the FRM-III (PERC) \[5\], the New Guide Hall at NIST \[131\], and the SNS (Nab) \[14\]. Concerning nuclear decays, the development of atomic trapping techniques has allowed the precise detection of daughter nucleus recoil momenta, which in turn permits bettered measurements of the electron-antineutrino correlation parameter \( a \).
In the absence of radiative corrections, recoil corrections, and BSM contributions, the correlation coefficients \( a(E_e), A(E_e), \) and \( B(E_e) \) reduce to simple expressions, while \( b, D = 0 \) vanish. For example, for a pure Gamow-Teller decay we have the prediction \( a_{GT} = -1/3 \), whereas for a pure Fermi transition we have \( a_F = 1 \). For mixed Fermi-Gamow-Teller transitions there is also a precise prediction once the ratio of Fermi to Gamow-Teller strengths is known—and this can be determined from the lifetime. In the case of neutron decay, which is a mixed transition, one obtains:

\[
\begin{align*}
    a(E_e) & \to \frac{1 - \lambda^2}{1 + 3\lambda^2}, & A(E_e) & \to \frac{2\lambda(1 - \lambda)}{1 + 3\lambda^2}, & B(E_e) & \to \frac{2\lambda(1 + \lambda)}{1 + 3\lambda^2},
\end{align*}
\]

(4.42)

where \( \lambda \equiv g_A/g_V \) and the limiting value of \( B(E_e) \), e.g., is termed \( B_0 \).

Going beyond the SM, the dependence of the correlations \( a, b, A, B, \) and \( D \) on the short-distance couplings \( \epsilon_i \) and \( \epsilon_i' \) can be determined using their dependence on the couplings \( C_i \pm C_i' \) given in Ref. [49] and the relations given in Eq. (2.17). The full expressions are quite complicated, but simplify considerably if one considers the leading \textit{linear} corrections only. In regards to these, the salient points are:

- As mentioned previously, the right-handed coupling \( \epsilon_R \) to linear order induces the shift \( \lambda \to \tilde{\lambda} = \lambda(1 - 2\epsilon_R) \). In order to probe \( \epsilon_R \) from correlation measurements, one needs to know \( \lambda \equiv g_A/g_V \) independently; this can come from a LQCD calculation.

- The scalar and tensor couplings \( \epsilon_{S,T} \) appear at linear order only through the Fierz interference term \( b \) and the analogous term \( b_\nu \) in the antineutrino asymmetry parameter, where \( b_\nu \) is defined by \( B(E_e) = B_0 + b_\nu m_e/E_e \). Different nuclear transitions probe different combinations of the BSM couplings. For example, the Fierz term \( b \) in a pure Fermi or Gamow-Teller transition probes exclusively the scalar or tensor coupling, according to \( b_F = \mp 2\gamma g_S \text{Re}(\epsilon_S) \) and \( b_{GT} = \pm (8\gamma g_T \text{Re}(\epsilon_T))/\lambda \), where \( \gamma = \sqrt{1 - \alpha^2 Z^2} \) and the sign distinguishes \( \beta^\pm \) emitters [50]. Mixed transitions such as neutron decay probe a linear combination of scalar and tensor couplings. For neutron decay one has [50]:

\[
\begin{align*}
    b & = \frac{2\gamma}{1 + 3\lambda^2} \left[ g_S \text{Re}(\epsilon_S) - 12\lambda g_T \text{Re}(\epsilon_T) \right], \quad (4.43a) \\
    b_\nu & = \frac{2\gamma}{1 + 3\lambda^2} \left[ g_S \text{Re}(\epsilon_S) \lambda - 4 g_T \text{Re}(\epsilon_T)(1 + 2\lambda) \right]. \quad (4.43b)
\end{align*}
\]

- Measurements of the correlation coefficients \( a, A, \) and \( B \) always include contributions from the Fierz interference term \( b \), and are therefore sensitive to \( \epsilon_{S,T} \) to linear order. This dependence arises because correlation measurements involve the construction of asymmetry ratios [132], and the dependence on \( b \) does not cancel in the asymmetry denominators. For example, in order to isolate \( A(E_e) \) one constructs the ratio

\[
A_{\exp}(E_e) = \frac{N_+(E_e) - N_-(E_e)}{N_+(E_e) + N_-(E_e)},
\]

(4.44)

where \( N_\pm(E_e) \) are the spectra corresponding to events with \( \vec{J} \cdot \vec{p}_e > 0 \) and \( \vec{J} \cdot \vec{p}_e < 0 \), respectively, so that sensitivity to \( b \) does indeed appear through the denominator. In general, asymmetry measurements probe

\[
\bar{Y}(E_e) = \frac{Y(E_e)}{1 + bm_e/E_e + \ldots}, \quad Y \in \{A, B, a, \ldots\},
\]

(4.45)
where the ellipsis denotes other possible corrections of radiative and recoil order whose appearance depend on the correlation considered. It is worth noting that simultaneous analysis of \( a(E_e) \) and \( A(E_e) \), e.g., yields more powerful constraints on the underlying BSM contributions than either correlation alone \[133\].

The dependence on \( \tilde{\epsilon}_\alpha \) couplings appears only to quadratic order, together with additional effects quadratic in the \( \epsilon_\beta \)’s. In this section, we have considered beta-decay correlations only, though strong constraints on BSM couplings also come from the study of light meson decays. We now review constraints on the various BSM couplings which appear in Eq. (2.17).

### 4.1 Model-independent constraints on scalar and tensor couplings

We now summarize the current best constraints on the scalar and tensor structures, and highlight prospects for future improvements. Currently, the most stringent constraint on the scalar coupling \( \epsilon_S \) arises from \( 0^+ \to 0^+ \) nuclear beta decays, as discussed in Section 3.1.1, whereas the most stringent bound on the tensor effective coupling \( \epsilon_T \) arises from the Dalitz-plot study of the radiative pion decay \( \pi^+ \to e^+\nu\gamma \). That is, an analysis of the Dalitz plot of this decay from the PIBETA collaboration \[134\] puts constraints on the product \( \text{Re}(\epsilon_T f_T) \) of the short-distance coupling \( \epsilon_T \) and the hadronic form factor \( f_T \) defined by \[135\]

\[
\langle \gamma(\epsilon,p)\big|\bar{u}\sigma_{\mu\nu}d\big|\pi^+\rangle = -\frac{e}{2} f_T (p_\mu\epsilon_\nu - p_\nu\epsilon_\mu),
\]

where \( p_\mu \) and \( \epsilon_\mu \) are the photon four-momentum and polarization vector, respectively. The analysis of Ref. \[135\], based on a large-\( N_c \)-inspired resonance-saturation model, provides \( f_T = 0.24(4) \) at the renormalization scale \( \mu = 1 \) GeV, with the parametric uncertainty induced by the uncertainty in the quark condensate. The 90%-C.L. experimental constraint\[10\] \(-2.0 \times 10^{-4} < f_T \text{Re}(\epsilon_T) < 2.6 \times 10^{-4} \), when combined with the above estimate for \( f_T \) evolved to 2 GeV implies

\[
-1.1 \times 10^{-3} < \text{Re}(\epsilon_T) < 1.36 \times 10^{-3} \quad (90\% \text{ C.L.}).
\]

This is the most stringent constraint on the tensor coupling from low-energy experiments. The next best constraints arise from measurements of nuclear beta decays \[29\].

Bounds on scalar and tensor interactions can be obtained from a number of observables in nuclear beta decays, other than \( 0^+ \to 0^+ \) transitions. Although these bounds are currently not competitive, we summarize them here for completeness. The leading sensitivity to scalar and tensor operators appears through the Fierz interference term \( b \). Significant constraints on \( b \) arise from the electron-polarization observables \[49\] as well as from measurements of \( A \) and \( a \) in Fermi, Gamow-Teller, and mixed transitions. Here is a summary of current bounds on \( \epsilon_{S,T} \) \[29\]:

- The most stringent constraint from the beta asymmetry in pure Gamow-Teller transitions (\( \tilde{A}_{\text{GT}} \)) arises from \( ^{60}\text{Co} \) measurements and implies \[136\]

\[
-2.9 \times 10^{-3} < g_T \text{Re}(\epsilon_T) < 1.5 \times 10^{-2} \quad (90\% \text{ C.L.}).
\]

Similar bounds can be obtained from measurements of \( \tilde{A}_{\text{GT}} \) in \( ^{114}\text{In} \) decay \[137\]: \(-2.2 \times 10^{-2} < g_T \text{Re}(\epsilon_T) < 1.3 \times 10^{-2} \) (90% C.L.).

- Measurements of the ratio \( P_F/P_{\text{GT}} \) from the longitudinal polarization of the positron emitted in pure Fermi and Gamow-Teller transitions \[138\] \[139\] imply

\[
-0.76 \times 10^{-2} < g_S \text{Re}(\epsilon_S) + 4 \lambda g_T \text{Re}(\epsilon_T) < 1.0 \times 10^{-2} \quad (90\% \text{ C.L.}).
\]

\[10\]Note that there is a factor of 2 difference in the normalization of the tensor coupling \( \epsilon_T \) compared to what was used in Refs. \[58\] \[134\].
Preliminary results have been reported on the measurement of the longitudinal polarization of positrons emitted by polarized $^{107}$In nuclei \[140\]. The corresponding 90 % C.L. sensitivity to tensor interactions, $|g_T \text{Re}(\epsilon_T)| < 3.1 \times 10^{-3}$, is quite promising although not yet competitive with radiative pion decay.

Finally, the beta-neutrino correlation $a$ has been measured in a number of nuclear transitions \[141\], \[142\], \[143\], \[144\]. The resulting constraints on scalar and tensor interactions are summarized in Fig. 7 of Ref. \[141\]. In terms of the coupling constants used here, the 90 % C.L. combined bound on the tensor interaction reads $|g_T \text{Re}(\epsilon_T)| < 5 \times 10^{-3}$, again not competitive with radiative pion decay.

Future improvements can be expected from both neutron and nuclear decay measurements. In the case of neutrons \[4\], \[5\], future measurements of the beta asymmetry $A$ \[12\], \[13\], \[14\], \[15\], the antineutrino asymmetry $B$ \[16\], \[14\], the electron-neutrino correlation $a$ \[17\], \[18\], \[19\], and the Fierz interference term $b$ \[17\], \[20\] should exceed $10^{-3}$ precision. On the nuclear side, measurements of $a$ and $b$ in the pure Gamow-Teller decay of $^6$He \[22\] should also reach the $10^{-3}$ level in precision. In Fig. 3 we summarize the current constraints on $\epsilon_S$ and $\epsilon_T$ (horizontal bands) and assess the impact of future measurements in both neutron and nuclear decays, where we assume $|b|, |b_v| < 10^{-3}$ from neutron decay and $|b_{\text{GT}}| < 10^{-3}$ from nuclear decays at 90% CL. The future neutron constraints are represented by the diagonal bands, whereas the constraint from $^6$He is represented by the vertical ocher band in the plot. In Fig. 3 the left panel represents the constraints using quark model input for the scalar and tensor matrix elements $g_{S,T}$, whereas the right panel uses our preferred LQCD estimates \[17\], which still have a 50% uncertainty in $g_S$. In the near future one can expect $\delta g_S/g_S$ to reach the 20% level from LQCD, thus increasing the constraining power of these measurements, so that the thickness of the bands will shrink. With current uncertainties on $g_{S,T}$, measurements of $10^{-3}$-level precision will probe effective scales $\Lambda_{S,T} > 7$ TeV.

### 4.2 Nuclear and neutron probes of recoil effects

SM recoil effects can be tested to high accuracy via experiments involving correlations between final state electron/positrons and the polarization or alignment of the parent state, which have the form

$$\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\nu} \sim 1 + A(E_e) \frac{\hat{z} \cdot \vec{p}_e}{E_e} + F(E_e) \Lambda \left( \frac{1}{E_e^2} \hat{z} \cdot \vec{p}_{\nu} \hat{z} \cdot \vec{p}_e - \frac{p_{\nu}^2}{3E_e^2} \right)$$

(4.50)

where $P = < J_z > / \langle J \rangle$ is the polarization and $\Lambda = 1 - 3 < J_z^2 > / (\langle J(J+1) \rangle)$ is the alignment. The beta correlation $A$ receives both leading order and recoil corrections, whereas the alignment correlation $F$ is purely a recoil order effect. However, both have been measured as functions of $E_e$ and provide measures of the recoil form factors. In this regard, it is important to point out that the axial tensor form factor $d(q^2)$, which vanishes in the SM from isotopic spin invariance in the case of transitions between isotopic analog states such as in neutron beta decay or tritium decay, is in general nonvanishing. Indeed, it is in general comparable in size to the weak magnetism term $b(q^2)$ \[11\]. In the case of mirror transitions the size of the axial tensor must be identical for electron and positron decays, and this is subject to test. A theoretical prediction for the weak magnetism term $b$ in terms of the difference between parent and daughter magnetic moments exists for transitions between isotopic analogs, and in the case of mirror transitions the size of the weak magnetism term is given by the electromagnetic M1 width of the transition from the excited isotopic analog state of the daughter nucleus—another CVC test \[146\]. One expects corrections to this result which are linear in isospin-breaking, both in $b(0)$ itself and from possible second-class current contribution in the SM. In neutron beta decay, through study of $a(E_e)$ and $A(E_e)$, the second-class current contribution to $d(0)$ can be determined independently of $\delta(0)$, implying

\[\text{Note that the vanishing of } d(0) \text{ for transitions between isotopic analog states is a SM prediction and is violated by so-called second class currents which can arise if quarks have an additional quantum number \[145\].}\]
Figure 3: Current and prospective 90% C.L. allowed regions in the \( \text{Re}(\epsilon_S) \)-\( \text{Re}(\epsilon_T) \) plane implied by (i) the existing bounds on \( b_F \) and \( \pi \rightarrow e\nu\gamma \) (horizontal (green) band); (ii) projected measurements of \( b \) and \( b_\nu \) in neutron decay (inner (red) and outer (blue) bow-tie shaped regions, respectively) at the \( 10^{-3} \) level; (iii) projected measurements of \( b_{GT} \) at the \( 10^{-3} \) level from \( ^6\text{He} \) decays (vertical (ocher) band). Left panel: hadronic matrix elements taken in the ranges \( 0.25 < g_S < 1.0, \ 0.6 < g_T < 2.3 \) \[27\]. Right panel: scalar and tensor charges taken from LQCD, \( g_S = 0.8(4) \) and \( g_T = 1.05(35) \). Note that by reducing the uncertainty in \( g_S \) the constraint on \( \epsilon_S \) from \( b_F \) becomes stronger, independent of any future neutron measurement. The effective couplings \( \epsilon_{S,T} \) are defined in the \( \overline{\text{MS}} \) scheme at 2 GeV. Figure adapted from Ref. \[47\].

that the CVC test can be made without assumptions regarding second-class currents \[147\]. Generally, since the form of the recoil effects to all observables has been given, together with electromagnetic corrections, these SM predictions are all testable \[76\].

An alternate route for testing the SM recoil predictions is to utilize decays where the daughter state is unstable and itself decays

i) electromagnetically such as in the mirror transitions in the \( A = 20 \) system to the \( 2^+ \ 1.63 \) MeV excited state of \(^{20}\text{Ne} \), which in turn decays via photon emission to the \( 0^+ \) ground state \[148\];

ii) strongly such as in the mirror transitions in the \( A = 8 \) system to the \( 2^+ \ 2.90 \) MeV excited state of \(^8\text{Be} \), which in turn decays via the emission of two alpha particles \[149\].

In the former case there exists a beta-gamma correlation

\[
\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\gamma} \sim 1 + \frac{1}{2} G(E_e) \left( \frac{\vec{p}_e \cdot \hat{p}_\gamma}{E_e} \right)^2 - \frac{p_e^2}{3E_e^2},
\]

(4.51)

whereas in the latter there is a beta-alpha correlation

\[
\frac{d^3\Gamma}{dE_e d\Omega_e d\Omega_\alpha} \sim 1 + G(E_e) \left( \frac{\vec{p}_e \cdot \hat{p}_\alpha}{E_e} \right)^2 - \frac{p_e^2}{3E_e^2} - 2\frac{\vec{p}_e \cdot \hat{p}_\alpha}{M v^*},
\]

(4.52)

noting \( v^* \) is the velocity of the alpha particle in the daughter rest frame. Here the decay correlation coefficient \( G \) is purely of recoil order, so that it is sensitive to recoil form factors. The form of \( G \) as well as of the radiative corrections have been calculated \[76\].
4.3 Couplings involving right-handed neutrinos: $\tilde{\epsilon}_{L,R,S,T,P}$

Neglecting neutrino masses, all the $\tilde{\epsilon}_\beta$ couplings enter the decay rates quadratically, i.e., as per $\propto |\tilde{\epsilon}_\beta|^2$. Detailed expressions of the contributions to neutron and nuclear beta decay correlation coefficients can be found in Ref. [50], though one needs to re-express the Lee-Yang couplings in terms of the $\epsilon_a$ and $\tilde{\epsilon}_\beta$ using Eq. (2.17). The corresponding bounds can be obtained from the analysis of Ref. [29], in particular from the fits to beta decay data allowing for non-zero $\tilde{\epsilon}_{L,R}$ only, implying $|\text{Re}(\tilde{\epsilon}_L + \tilde{\epsilon}_R)| < 6.4 \times 10^{-2}$, $|\text{Re}(\tilde{\epsilon}_L - \tilde{\epsilon}_R)| < 5.8 \times 10^{-2}$, and $\tilde{\epsilon}_S, \tilde{\epsilon}_T$ only, implying $|g_S \text{Re}(\tilde{\epsilon}_S)| < 5.5 \times 10^{-2}, |g_T \text{Re}(\tilde{\epsilon}_T)| < 2.1 \times 10^{-2}$ at 90% CL. Decay correlations set also bounds on the imaginary parts of these couplings. For example, using the measurements of $\alpha$ in the decay $^{32}\text{Ar}$ [143] and $^4\text{He}$ [150, 151], one gets $|\text{Im}(\tilde{\epsilon}_S)| < 0.17$ and $|\text{Im}(\tilde{\epsilon}_T)| < 0.05$ at 90% CL. We expect that these bounds can be improved by future more precise measurements.

4.4 $T$-odd correlations

Triple-product decay correlations can only be motion-reversal-odd and thus cannot be true tests of $T$-invariance [152]. As a consequence final-state interactions (FSI) can simulate a seeming $T$-odd correlation without a fundamental violation of $T$-invariance. In beta-decays the energy release is sufficiently small that the FSI are electromagnetic in nature, so that they are calculable with minimal hadronic ambiguity at accessible levels of precision [153, 154, 155]. Nevertheless, under an assumption of CPT invariance, such “$T$-odd” correlations are sensitive to new sources of CP violation, and thus to new physics, though an observation of such a correlation in excess of SM expectations would not allow one to conclude that $T$ itself is violated [156].

$T$-odd correlations have been studied in kaon, neutron, hyperon [157, 158], and nuclear beta-decays. In $K^+$ decay, namely, $K^+ \to \mu^+\nu\gamma$, the transverse muon polarization is studied, and the expected SM correlation is small [159, 160] with respect to that possible in models of new physics [161, 159]. Existing experimental studies are consistent with no $T$-violation in this and related processes [162, 163], but new results of greater sensitivity are expected from TREK at J-PARC [164, 165]. In ordinary beta-decay, a $T$-odd correlation is possible only if the initial state is polarized, or if the final-state polarization of one of the particles is observed. In the decay of polarized neutrons one can construct $\vec{J} \cdot (\vec{p}_e \times \vec{p}_\nu)$, i.e., the $D$ correlation, or $\vec{J} \cdot (\vec{p}_e \times \vec{\sigma}_e)$, i.e., the $R$ correlation, if the polarization of the emitted electron $\propto \vec{\sigma}_e$ is observed. Recently significant experimental efforts in regard to each of these correlations have been concluded. The emiT collaboration has presented the best limit on $D$ in beta decay [166, 167], finding $D_n = (-0.94 \pm 1.89 \pm 0.97) \times 10^{-4}$ in neutron decay, a substantial improvement over earlier measurements [168, 169, 29]. As for $D_{FSI}$, the $O(\alpha)$ correction vanishes in the zero-recoil limit, and $D_{FSI} \approx 10^{-5}$ [153, 170]. This calculation has been updated to employ the techniques of heavy-baryon chiral effective field theory by Ando et al. [155]; they reproduce the Callan-Treiman result in $O(\alpha Q/M_N)$ with $Q \sim M_n - M_p - m_e$ and include the leading piece of the $N^3$LO correction to realize $D_{FSI}$ with an estimated accuracy of better than 1%. In terms of our non-standard couplings $D$ in neutron decay can be written as [49]

$$D_{BSM} = \frac{1}{1 + 3\bar{\lambda}^2} \left[ 4\lambda \text{Im}(\epsilon_R) + 8g_S g_T \text{Im}(\epsilon_S \epsilon_T^* + \tilde{\epsilon}_S \tilde{\epsilon}_T^*) \right].$$ (4.53)

Neglecting small quadratic effects in scalar and tensor couplings, the emiT limit translates to the 90% C.L. constraint $-5 \times 10^{-4} < \text{Im}(\epsilon_R) < 3 \times 10^{-4}$, where we have used $\bar{\lambda}$ [81] for $\lambda$. For $\text{Im}(\epsilon_{S,T})$ and $\text{Im}(\tilde{\epsilon}_{S,T})$ the analysis is more involved. $D$ provides bounds on products such as $\text{Re}(\epsilon_T)\text{Im}(\epsilon_S)$, etc., so that no bounds on the imaginary parts can be obtained unless non-zero real parts of the exotic couplings are assumed or discovered.
Recent results also exist for $R$. In the first neutron experiment both transverse components of the electron polarization are measured, to yield both $R$ and $N$ [171]. The correlation $N$ probes $\vec{J} \cdot \vec{\sigma}_e$ and is appreciably non-zero from FSI; the experimental measurement is consistent with the SM expectation, offering a sensitivity check of the setup [172]. They find $R_n = 0.004 \pm 0.012 \pm 0.005$, limiting the imaginary parts of scalar and tensor interactions via:

\[
R_n = \frac{1}{1 + 3 \lambda^2} \left[ -8g_T (2\lambda + 1) \text{Im}(\epsilon_T) - 2g_S \lambda \text{Im}(\epsilon_S) \right].
\]  

(4.54)

In comparison the measurement of $R$ in $^8\text{Li}$ decay yields $R_{\text{Li}} = 0.0009(22)$ [173], which through

\[
R_{\text{Li}} = -\frac{18g_T}{3g_A} \text{Im}(\epsilon_T)
\]  

(4.55)

implies a limit $-3.1 \times 10^{-3} < \text{Im}(\epsilon_T) < 1.8 \times 10^{-3}$ at 90% C.L. after $R_{FSI}$ has been removed. Consequently, the $R_n$ limit is most usefully interpreted as a limit on $\text{Im}(\epsilon_S)$, namely $-0.15 < \text{Im}(\epsilon_S) < 0.11$ at 90% C.L. Looser constraints (at the 20%-level) on the imaginary parts of the scalar and tensor couplings come from the measurement of $a$ in $^9\text{Be} \rightarrow ^8\text{Be} + \gamma$ transitions [143]–[142] and in $^6\text{He}$ decay [144], respectively—note Fig. 22 of Ref. [166] for a useful compilation.

The possibility of constraining T-odd P-even couplings, including $\beta$ decay parameters, via T-odd P-odd observables such as EDMs has been considered earlier in Ref. [174]. The basic idea is that through loop diagrams involving electroweak gauge boson exchanges, T-odd P-even interactions can generate T-odd P-odd interactions. The resulting bounds depend on the scale at which parity invariance is restored [175]–[176]. In this context, the $D$ correlation has been recently studied in Ref. [177], considering how large $D$ can be in light of constraints from electric dipole moment (EDM) searches. Focusing on the leading contribution to $D$ (proportional to $\text{Im}(\epsilon_{\ell R})$) the authors show that via a loop diagram the same phase contributes to the neutron and other EDMs. If this is the leading (or only) contribution to the neutron EDM, then one can conclude that the neutron EDM currently provides the strongest constraint on $D$, which is $10 - 10^3$ times stronger than current direct limits on $D$, depending on the model. Of course, the bounds on $D$ can be weakened if other operators, other than the 4-fermion ones, contribute to the neutron EDM, and this is natural in many theories, interfering destructively with the contribution proportional to $\text{Im}(\epsilon_{\ell R})$. The numerical evaluation of the effect of such operators becomes a model-dependent question, and one anticipates that the connection between $d$ to $D$ can be weakened completely, though detailed investigation is warranted [178].

In radiative $\beta$ decay one can form a T-odd correlation from momenta alone, so that one probes new spin-independent sources of CP violation. A triple momentum correlation has been previously studied in $K^+ \rightarrow \pi^0 l^+ \nu \gamma$ [179], and its sensitivity to physics BSM considered [180]. In $K^+$ decay both electromagnetic and strong, i.e., pion-mediated, radiative corrections can mimic the T-odd effect, but the electromagnetic-induced FSI are orders of magnitude larger [179]–[181]–[182]. Finally, a spin-independent T-odd correlation can be constructed in the radiative beta decay of neutron and nuclei [183], proportional to $\vec{p}_e \cdot (\vec{p}_e \times \vec{p}_\nu)$, offering the opportunity to study the imaginary part of the pseudo-Chern-Simons term [184] first found as a consequence of the baryon vector current anomaly and SU(2)$_L \times$U(1) gauge invariance at low energies [185]–[186]–[187]. In Ref. [183], the effect of FSI on this new correlation have been computed, establishing the baseline for possible future searches of BSM CP-violating interactions.

## 5 Collider limits on non-standard CC interactions

The BSM interactions probed at low energy can also be directly probed at high-energy colliders. The collider signals, however, depend on whether the particles that generate the 4-fermion interactions are kinematically accessible at the collider energies. Model-independent statements can be made under
low- and high-energy experiments is shown in Tables 2, 3 (for Re(\(\epsilon_\alpha\)) and Im(\(\epsilon_\alpha\))) and 4, 5 (for Re(\(\tilde{\epsilon}_\alpha\)) and Im(\(\tilde{\epsilon}_\alpha\))). Note that in these tables we report only direct bounds, leaving out bounds on the real and imaginary parts of \(\epsilon_{S,T}\) and \(\tilde{\epsilon}_{S,T}\) from \(R_\pi\) as they can be evaded by cancellation. All of the tabulated results refer to a bound on the absolute value of the parameter unless a range is specified. The main points can be summarized as
follows [37] (see also Ref. [190]). For the pseudo-scalar couplings $\epsilon_P$ and $\tilde{\epsilon}_P$ the low-energy constraints from pion decay are at the $10^{-4}$ level, which are very hard to reach at the LHC in the near future. The same applies to the vector interactions $\epsilon_{L,R}$, for which both the CKM-unitarity bound ($\text{Re}(\epsilon_{L,R})$) and the emiT bound ($\text{Im}(\epsilon_R)$) are at the $0.5 \times 10^{-3}$-level.

For scalar and tensor interactions with left-handed neutrinos, low-energy experiments have traditionally yielded stronger bounds on $\text{Re}(\epsilon_S)$ and $\text{Re}(\epsilon_T)$, but the current LHC bounds have caught up; and both probes are at the $10^{-2}$ and $10^{-3}$ level for $\text{Re}(\epsilon_S)$ and $\text{Re}(\epsilon_T)$, respectively. In the next few years we expect improvements in the bounds from both the LHC and low-energy experiments, through neutron [15, 14, 16, 17, 12, 20] and $^6$He decay [22] measurements at the 0.1% level and beyond. Projected future bounds from both beta decays at the $10^{-3}$ level and the LHC on $\text{Re}(\epsilon_{S,T})$ are shown in Figure 4. These results show that low-energy searches with $10^{-4}$ sensitivity would have unmatched constraining potential, even in the LHC era. Concerning the imaginary parts, for $\text{Im}(\epsilon_T)$ bounds from low-energy ($R$ correlation in $^8$He) and the LHC are at the same level, while for $\text{Im}(\epsilon_S)$ the LHC bound is stronger than the one derived from the $R$ correlation in neutron decay.

Finally, for scalar and tensor interactions with right-handed neutrinos, $\tilde{\epsilon}_S$ and $\tilde{\epsilon}_T$, the LHC bounds are also at the $10^{-2}$ and $10^{-3}$ level respectively, significantly better than the current and future low-energy limits for both $\text{Re}(\tilde{\epsilon}_{S,T})$ and $\text{Im}(\tilde{\epsilon}_{S,T})$. To match the LHC bound one needs measurements of the electron-neutrino correlation $a$ in Gamow-Teller transitions at the level of $\delta a_{GT}/a_{GT} \sim 0.05\%$. Similar remarks apply to $\tilde{\epsilon}_R$, for which the LHC bound is $5 \times 10^{-3}$, and no significant limit is available from low-energy probes.

Table 2: Summary of 90% CL bounds (in units of $10^{-2}$) on the real parts of non-standard couplings $\epsilon_\alpha$ obtained from low-energy and LHC searches (5 fb$^{-1}$ at $\sqrt{s} = 7$ TeV). In order to deduce the low-energy bounds on the scalar and tensor couplings we used $g_S = 0.8(4)$ and $g_T = 1.05(35)$ [47]. Using $g_S = 1.08(28)$ [82] would lead to the stronger bound $|\text{Re}(\epsilon_S)| < 0.4 \times 10^{-2}$. The couplings $\epsilon_{S,P,T}$ are evaluated in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.

|          | $\text{Re}(\epsilon_L)$ | $\text{Re}(\epsilon_R)$ | $\text{Re}(\epsilon_P)$ | $\text{Re}(\epsilon_S)$ | $\text{Re}(\epsilon_T)$ |
|----------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\beta$ decays | 0.05                     | 0.05                     | 0.04                     | 0.8                      | 0.1                      |
| LHC ($e\nu$)  | $(-0.3, +0.8)$            | –                        | 1.3                      | 1.3                      | 0.3                      |
| LHC ($e^+e^-$) | –                        | –                        | 1.0                      | 1.0                      | 0.1                      |

6 Model constraints

Our discussion has focused on model-independent constraints emergent from precise universality tests and correlation coefficient measurements, in terms of the effective couplings $\epsilon_\alpha$ and $\tilde{\epsilon}_\beta$. As we have mentioned, ultraviolet extensions of the SM will generate these non-standard effective couplings at some level, which are then functions of the model parameters. Therefore, the results we have presented within a given NP model the ratio $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$ is likely to produce the strongest bound not only on $\epsilon_P$ and $\tilde{\epsilon}_P$, but also on $\epsilon_{S,T}$ and $\tilde{\epsilon}_{S,T}$, through their loop-induced contribution. However, since these bounds are based on naturalness arguments, they could be circumvented by cancellations between different contributions, so that in a model-independent analysis the LHC offers the strongest constraint on $\tilde{\epsilon}_{S,T}$.

\footnote{Within a given NP model the ratio $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$ is likely to produce the strongest bound not only on $\epsilon_P$ and $\tilde{\epsilon}_P$, but also on $\epsilon_{S,T}$ and $\tilde{\epsilon}_{S,T}$, through their loop-induced contribution. However, since these bounds are based on naturalness arguments, they could be circumvented by cancellations between different contributions, so that in a model-independent analysis the LHC offers the strongest constraint on $\tilde{\epsilon}_{S,T}$.
Table 3: Summary of 90% CL bounds (in units of $10^{-2}$) on the imaginary parts of non-standard couplings $\epsilon_\alpha$ obtained from low-energy (using hadronic input as per Table 2) and LHC searches (5 fb$^{-1}$ at $\sqrt{s} = 7$ TeV). The couplings $\epsilon_{S,P,T}$ are evaluated in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.

|            | $\text{Im}(\epsilon_L)$ | $\text{Im}(\epsilon_R)$ | $\text{Im}(\epsilon_P)$ | $\text{Im}(\epsilon_S)$ | $\text{Im}(\epsilon_T)$ |
|------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| $\beta$ decays | --                        | $(-0.05, +0.03)$          | 0.02                      | $(-15, +11)$              | $(-0.3, +0.2)$            |
| LHC ($e\nu$)     | 0.5                       | --                        | 1.3                       | 1.3                       | 0.3                       |
| LHC ($e^+e^-$)    | --                        | --                        | 1.0                       | 1.0                       | 0.1                       |

Table 4: Summary of 90% CL bounds, in units of $10^{-2}$, on the real parts of the non-standard couplings $\tilde{\epsilon}_\alpha$ obtained from low-energy and LHC searches (5 fb$^{-1}$ at $\sqrt{s} = 7$ TeV). In order to deduce the low-energy bounds on the scalar and tensor couplings we used $g_S = 0.8(4)$ and $g_T = 1.05(35)$ [47]. Using $g_S = 1.08(28)$ [82] would lead to the stronger bound $|\text{Re}(\tilde{\epsilon}_S)| < 6.9 \times 10^{-2}$. The couplings $\tilde{\epsilon}_{S,P,T}$ are evaluated in the $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV.

|            | $\text{Re}(\tilde{\epsilon}_L)$ | $\text{Re}(\tilde{\epsilon}_R)$ | $\text{Re}(\tilde{\epsilon}_P)$ | $\text{Re}(\tilde{\epsilon}_S)$ | $\text{Re}(\tilde{\epsilon}_T)$ |
|------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| $\beta$ decays | 6                                | 6                                | 0.02                            | 14                              | 3.0                              |
| LHC ($e\nu$)     | --                               | 0.5                              | 1.3                              | 1.3                              | 0.3                              |

can be used to constrain the parameter space of any SM extension. Moreover, within a given SM extension, the low-energy effective couplings will show peculiar dependencies on the underlying model parameters, resulting in correlations among low-energy beta decay signatures and other observables. In this subsection we briefly illustrate these ideas by discussing non-standard CC interactions within the Left-Right Symmetric Model [191, 192] and the MSSM, for which detailed studies can be found in Refs. [193, 194, 195, 196, 197]. For more extensive reviews of underlying models we refer to [27].

6.1 Left-Right Symmetric Model

The Left-Right Symmetric Model [191] is based on an extended gauge group $\text{SU}(2)_L \times \text{SU}(2)_R \times U(1)$, in which in addition to the SM gauge assignments, the right-handed fermions transform as doublets under $\text{SU}(2)_R$. After spontaneous symmetry breaking, the charged gauge bosons $W_L$ and $W_R$ mix into light SM-like $W_1$, which is predominantly left-handed, and heavier $W_2$, which is predominantly right-handed, states which mediate charged current processes:

$$ W_L = W_1 \cos \zeta + W_2 \sin \zeta \quad W_R = -W_1 \sin \zeta + W_2 \cos \zeta, $$

with the mixing angle $\sin \zeta \sim (v/v_R)^2$ proportional to the ratio of the weak scale over the scale at which the $\text{SU}(2)_R$ group is spontaneously broken, $v_R \sim M_{W_2}$, thus leading to the breaking of parity. To
Table 5: Summary of 90% CL bounds, in units of $10^{-2}$, on the imaginary parts of the non-standard couplings $\tilde{\epsilon}_\alpha$ obtained from low-energy (using hadronic input as per Table 2) and LHC searches ($5 \text{ fb}^{-1}$ at $\sqrt{s} = 7 \text{ TeV}$). The couplings $\epsilon_{S,P,T}$ are evaluated in the $\overline{\text{MS}}$ scheme at $\mu = 2 \text{ GeV}$.

|       | $\text{Im}(\tilde{\epsilon}_L)$ | $\text{Im}(\tilde{\epsilon}_R)$ | $\text{Im}(\tilde{\epsilon}_P)$ | $\text{Im}(\tilde{\epsilon}_S)$ | $\text{Im}(\tilde{\epsilon}_T)$ |
|-------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\beta$ decays | $-$                             | $-$                             | 0.02                            | 17                              | 5.0                             |
| LHC ($e\nu$)  | $-$                             | 0.5                             | 1.3                             | 1.3                             | 0.3                             |

leading order in $(v/v_\nu)^2$, this model generates the following correlated CC non-standard couplings

$$
\epsilon_L = \epsilon_\mu = 0, \quad \epsilon_R = -\zeta, \quad \tilde{\epsilon}_L = -\zeta, \quad \tilde{\epsilon}_R = \frac{M_{W_1}^2}{M_{W_2}^2},
$$

(6.57)

with no dependence on the lepton mass—all other couplings are vanishing. It is clear then, that this minimal and manifestly left-right symmetric model predicts no appreciable deviations from the SM in lepton flavor universality tests. On the other hand, the model predicts $\Delta_{\text{CKM}} = -2\zeta$, so that the mixing angle, and therefore the scale of spontaneous parity breaking, is strongly constrained by Cabibbo universality tests. Stronger bounds still emerge from the $K_L - K_S$ mass difference [198], though the role of long-distance contributions to the mass difference limit the severity of the constraint [199, 200]. Given the strong bounds on $\zeta$, the model would also predict unobservably small effects in decay correlations sensitive to $|\tilde{\epsilon}_{L,R}|^2 \sim \zeta^2$.

6.2 CC processes in the MSSM

Within the MSSM with R-parity, the CC effective couplings $\epsilon_\alpha$ are generated through loop diagrams (vertex corrections and box diagrams such as those depicted in Fig. 5), resulting in expressions that are not nearly as simple as the ones in Eq. (6.57).

The chirality flipping couplings $\epsilon_{S,T}$ [201] require the presence of left-right mixing between sfermions running in the box diagrams of Fig. 5 which is proportional to the small Yukawa couplings or the trilinear soft “A” terms. Ref. [201] analyzed the phenomenological constraints on such mixing and determined the range of the allowed contributions to the weak decay coefficients $b$ and $B$, arguing that they may provide unique probes of left-right mixing in the first generation scalar fermion sector, provided a precision between $10^{-4}$ and $10^{-3}$ can be achieved.

Concerning the universality tests, in Ref. [197] it was shown that the magnitude of the corrections $\Delta_{\text{CKM}}$ and $\Delta_{e/\mu}$ can be on the order of $10^{-3}$, which is consistent with precision electroweak tests and LHC direct searches for supersymmetric particles. The size of the universality violations is controlled by the splitting in the squark versus slepton spectra (Cabibbo universality) or in the selectron versus smuon spectra (lepton universality). Moreover, Ref. [197] showed that a comparison of the first row CKM unitarity tests with measurements of $R_{e/\mu}$ can provide unique probes of the spectrum of first generation squarks and first and second generation sleptons, as illustrated in Figure 6 and explained in the figure caption. As a consequence, universality tests will be powerful diagnostic tools if supersymmetric partners are discovered at the LHC.

Finally, a discussion of the impact of Cabibbo and lepton universality tests within the R-parity violating MSSM can be found in Refs. [194, 196].
7 Conclusions

In this article we have reviewed the role of precision beta decays measurements in probing physics beyond the Standard Model in the LHC era. As for all precision tests, theoretical calculations of the SM amplitudes play a crucial role in setting the stage for BSM searches. Here we have tried to convey the flavor of the needed theoretical inputs, emphasizing the increasing role played by lattice QCD both in the meson sector, in regard to the determination of \( V_{us} \), and in the nucleon sector, in probing non-standard scalar and tensor couplings through decay correlations.

Concerning BSM physics, we have emphasized a model-independent EFT approach to beta decays, assuming that new physics is emergent at high energies, based on a quark-lepton level effective Lagrangian. This approach has a dual benefit. On one hand, it allows an unambiguous comparison of the physics reach of probes involving different hadrons, such as pions and nucleons, and nuclei, limited only by our ability to compute the requisite matrix elements. Moreover, as stated at the start, in the absence of an emerging picture of new dynamics from collider searches, the EFT analysis is the first necessary step to establishing the motivation and significance of this set of low-energy probes. The current bounds on the real and imaginary parts of the non-standard couplings \( \epsilon_{L,R,S,P,T} \) (involving left-handed neutrinos) and \( \bar{\epsilon}_{L,R,S,P,T} \) (involving right-handed neutrinos) are summarized in Tables 2, 3, 4, 5.

The outlook is very positive: the effective couplings of all the BSM CC operators involving left-handed neutrinos are currently probed or will be soon probed in low-energy experiments at the level of \( 10^{-3} \) or better. This corresponds to probing maximal BSM physics scales \( \Lambda \) ranging from 7 TeV (for scalar and tensor interactions) to 11 TeV (for vector interactions), to \( O(100) \) TeV (for pseudoscalar interactions, assuming no cancellations and no mass or Yukawa suppressions).

In all cases, the effective scale probed overlaps with the LHC reach: therefore, if new particles are found at the LHC, beta decays will play an important role in the “LHC inverse problem”, i.e. in establishing the properties of the new BSM dynamics. This is also explicitly illustrated in the case of the MSSM (see Section 6 and Fig. 6). Moreover, if new BSM dynamics is above the LHC reach, sometimes termed the “nightmare scenario,” one can analyze LHC data on the process \( pp \to e\nu + X \) in terms of the same EFT used at low energy, modulo the known QCD running of the various couplings. Even in this pessimistic scenario, recent theoretical results [37] show that beta decay measurements at the \( 10^{-3} \)-level can be more sensitive than the LHC in probing non-standard CC interactions, noting once
Figure 6: Correlation between $\Delta_{\text{CKM}}$ and $\Delta_{e/\mu}$ in the MSSM. The red points (dark grey) arise from a generic parameter space scan. The green points (light grey) arise after applying the constraints from precision electroweak tests. The black points arise after applying the constraints from direct searches at the LHC. The three branches correspond to the following scenarios for the sfermion spectra: the vertical branch corresponds to light squarks, which are been largely ruled out by the LHC, and heavy sleptons; the right branch corresponds to light smuons and heavy selectrons and squarks; the left branch corresponds to light selectrons and heavy smuons and squarks. Figure reprinted with permission from S. Bauman, J. Erler, M. Ramsey-Musolf, “Charged current universality and the MSSM”, arXiv:1205.0035 [hep-ph] [197].

again Tables 2, 3, 4, 5, and Fig. 4, and that $10^{-4}$-level measurements would have unmatched sensitivity. These considerations illustrate the relevance of precision beta decay measurements throughout the LHC era.

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