GRAVITATIONAL LENS MAGNIFICATION AND THE MASS OF ABBEL 1689

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ABSTRACT

We present the first application of lens magnification to measure the absolute mass of a galaxy cluster: Abell 1689. The absolute mass of a galaxy cluster can be measured by the gravitational lens magnification of a background galaxy population by the cluster gravitational potential. The lensing signal is complicated by the intrinsic variation in number counts resulting from galaxy clustering and shot noise and by additional uncertainties in relating magnification to mass in the strong lensing regime. Clustering and shot noise can be dealt with using maximum likelihood methods. Local approximations can then be used to estimate the mass from magnification. Alternatively, if the lens is axially symmetric we show that the amplification equation can be solved nonlocally for the surface mass density and the tangential shear. In this paper we present the first maps of the total mass distribution in Abell 1689, measured from the deficit of lensed red galaxies behind the cluster. Although noisier, these reproduce the main features of mass maps made using the shear distortion of background galaxies, but have the correct normalization, finally breaking the “sheet-mass” degeneracy that has plagued lensing methods based on shear. Averaging over annular bins centered on the peak of the light distribution, we derive the cluster mass profile in the inner 4′ (0.48 h⁻¹ Mpc). These show a profile with a near-isothermal surface mass density ρ ≈ (0.5 ± 0.1)/(0/1)⁻¹ out to a radius of 2.4 (0.28 h⁻¹ Mpc), followed by a sudden drop into noise. We find that the projected mass interior to 0.24 h⁻¹ Mpc is M(<0.24 h⁻¹ Mpc) = (0.50 ± 0.09) × 10¹⁵ h⁻¹ M☉. We compare our results to masses estimated from X-ray temperatures and line-of-sight velocity dispersions, as well as to weak shear and lensing arclets. We find that the masses inferred from X-ray, line-of-sight velocity dispersions, arclets, and weak shear are all in fair agreement for Abell 1689.

Subject headings: galaxies: clusters: individual (Abell 1689) — gravitational lensing

1. INTRODUCTION

The magnitude and distribution of matter in galaxy clusters should in principle provide a strong constraint on cosmological models of structure formation and the mean mass density of the universe. In addition, a direct image of the mass density will tell us much about the relationship between gas, galaxies, and dark matter, and whether light is indeed a fair—if biased—tracer of mass.

Early techniques for estimating the mass in clusters include dynamical methods, from the line-of-sight velocity dispersion of member galaxies, and X-ray temperature measurements. However, these estimates make some strong assumptions about equilibrium conditions in the cluster.

Kaiser & Squires (1993) circumvented this problem by showing that a more direct method of estimating the mass, with no underlying assumptions about the dynamical or thermodynamical state of the cluster, was to measure the shear field in the source distribution of the cluster background (Kaiser & Squires 1993; Tyson, Valdes, & Wenk 1990; Schneider & Seitz 1995). On average, the shear pattern of a population of unlensed galaxies should be randomly distributed. But in the presence of a massive gravitational lensing cluster, the shear field is polarized. Since the shear field is related (nonlocally) to the surface mass density, the shear can be used to estimate the mass distribution—up to an arbitrary constant. The presence of this arbitrary constant, referred to as the “sheet-mass” degeneracy, means that only differential masses can be measured. Shear maps are conventionally normalized to the edge of the observed field, or such that the inferred mass density is everywhere positive, and so represent a lower limit on the mass.

Soon after, Broadhurst, Taylor, & Peacock (1995, hereafter BTP) showed that the sheet-mass degeneracy could be broken by use of the gravitational lens magnification effect. The number and magnitude-redshift distribution of background galaxies is distorted by the gravitational field of the lensing cluster, and in the weak lensing regime this distortion provides a straightforward estimate of the surface mass density. With calibration from offset fields the cluster mass distribution can be properly normalized.

BTP also suggested that a degraded, but much quicker, estimate of the magnification effect could be made from the distortion of angular number counts of background sources. Broadhurst (1995) found evidence for this distortion in the background counts of the cluster Abell 1689, as did Fort, Mellier, & Dantel-Fort (1997) for Cl0024. In this work we apply the methods developed by BTP and extended by Taylor & Dye (1998) in estimating the surface mass density from the distortion of angular counts, including the effects of shot noise and galaxy clustering, and those of van Kampen (1998) in estimating the surface mass density in the strong lensing regime to Abell 1689.
The layout of the paper is as follows. In § 2 we describe the magnification effect itself. In § 3 we describe the effects of shot noise and clustering on estimates of the surface mass density. In § 4 we describe how to estimate the surface mass density in the strong lensing regime using local approximations and introduce a new self-consistent nonlocal solution for axially symmetric lenses. We apply these methods to map out the mass in the cluster Abell 1689 in § 5 and find its profile. Our mass estimate is compared to other estimates in § 6, and our conclusions are presented in § 7.

2. THE MAGNIFICATION EFFECT

The observed number of galaxies seen in projection on the sky is (BTP; Taylor & Dye 1998)

\[ n' = n_0 A^\beta (1 + \Theta), \]

where \( n_0 \) is the expected mean number of galaxies in a given area at a given magnitude. Variations in this mean arise from the angular perturbation in galaxy density \( \Theta \) as a result of galaxy clustering and from gravitational lens magnification. The lens amplification factor is

\[ A = |(1 - \kappa)^2 - \gamma^2|^{-1}, \]

where

\[ \kappa = \frac{\Sigma}{\Sigma_{\text{crit}}} \]

is the surface mass density in units of the critical surface mass \( \Sigma_{\text{crit}} \). The amplitude of the shear field is given by \( \gamma \), and the background galaxy luminosity function is locally approximated by

\[ n(L) \sim L^{-\beta}. \]

The amplification index \( \beta - 1 \) accounts for the expansion of the background image and for the increase in number as faint sources are lensed above the flux limit.

In the absence of galaxy clustering and finite sampling effects, the background galaxy distribution can simply be inverted via equation (1), to find the amplification. One can then solve equation (2) to find the surface density, with some realistic assumptions about the shear. In § 4 we discuss various approximations that allow us to do this.

However, given a small resolution scale for the surface amplification, galaxy clustering and finite sampling will in general be an important effect. In § 3 we discuss the effects of intrinsic variation in the distribution of the background galaxy sources.

3. GALAXY CLUSTERING NOISE

The main sources of uncertainty in lens magnification are a result of shot noise, finite sampling, and the intrinsic clustering of the background source population that introduce correlated fluctuations in the angular counts. As we are viewing small angles, the clustering properties of the background source galaxies are not in general linear, unless the depth of background is sufficient to wash out the clustering pattern. As a result, it is not sufficient to make the usual assumption that galaxy clustering can be modeled by a Gaussian distribution.

We can account for the effects of shot noise and nonlinear clustering by modeling the angular counts by a lognormal-Poisson model (Coles & Jones 1991; BTP; Taylor & Dye 1998)—a random point-process sampling of a lognormal field. The distribution function of source counts is then

\[ P(n) = \frac{1}{n!} \langle \lambda^n e^{-\lambda} \rangle, \]

\[ = \frac{\lambda_0}{n!} \int_{-\infty}^{\infty} dx \exp \left( -\frac{x^2}{2\sigma^2} - \lambda_0 e^x - nx \right), \]

where \( \lambda = \lambda_0 e^x \) is the local mean density, \( x \) is a Gaussian random variable of zero mean and variance \( \sigma^2 \), and \( \lambda_0 = n_0 A^{\beta - 1} e^{-\gamma^2/2} \) correctly normalizes the counts. The linear clustering variance \( \sigma^2 \) is related to the nonlinear variance by \( \sigma^2 = \ln(1 + \sigma_{\text{nl}}^2) \). We have tested this distribution against available data and find that it is an excellent fit to the distribution of counts in the deep fields. The only parameters are the observed count per pixel \( n \) and the variance of the lognormal field. The amplitude of clustering of the density field and its dependence on redshift can be estimated from, e.g., the I-band–selected galaxies in the Canada-France Redshift Survey in the range \( 17.5 < I < 22.5 \) (Le Fèvre et al. 1996; see § 5.2.3). The quantity required is the variance in a given area of sky, which can be estimated by averaging the observed angular correlation function \( \omega(\theta) \) over a given area:

\[ \sigma_{\text{nl}}^2 = \bar{\omega}(\theta) = \frac{1}{\Omega(\theta)} \int_0^{d^2/\theta} d^2\theta' \omega(\theta'), \]

where \( \Omega(\theta) \) is the area.

Our method of approach is then that discussed by BTP. At each pixel in a map of the source counts, one uses the distribution equation (6) as a likelihood function, \( \mathcal{L}(A | n, \sigma) = P(n | \sigma, A) \), assuming a uniform prior for the amplification. The surface density is then found from the amplification by making some realistic assumption about the shear and maximizing the likelihood. In § 4 we discuss a number of ways of transforming from the amplification to \( \kappa \) in the strong lensing regime.

4. THE STRONG LENSING REGIME

Transforming from amplification to the surface mass density is potentially nontrivial, as we have no shear information. One could incorporate this from independent measurements of the shear field, but for the present discussion we are interested in developing a completely independent lensing approach. We shall discuss combining shear and magnification elsewhere. In principle, one could generate a first guess for the surface mass density and iterate the amplification equation toward a solution of both surface density and shear. However, given the small field of view and uncertainties introduced by parity changes, this can be an unstable problem. In addition, as the solutions are in general multivalued, we would hope to start from as near to the correct solution as possible. In this section we discuss a number of reasonable approximations for solving the amplification equation (2). These can be regarded as solutions in their own right, or as the first best guess to an iterated solution. We begin by discussing the local approximation methods suggested and tested on simulated clusters by van Kampen (1998). Then, in § 4.2 we present a new self-consistent solution to the amplification equation for \( \kappa \) and \( \gamma \) for an axially symmetric lens.
4.1. Local Approximations to the Surface Mass Density

There exist only two local relations between $\gamma$ and $\kappa$ that result in a single caustic solution of the amplification equation (2) that is easily invertible (van Kampen 1998): $\gamma = 0$, corresponding to a sheet of matter, and $\gamma = \kappa$, for an isotropic lens. These two relations have corresponding estimators for $\kappa$ as a function of amplification:

$$\kappa_0 \equiv \kappa(\gamma = 0) = 1 - \mathcal{P}A^{-1/2},$$

$$\kappa_1 \equiv \kappa(\gamma = \kappa) = \frac{1}{2}(1 - \mathcal{P}A^{-1}),$$

where $\mathcal{P} = \pm 1$ is the image parity.

Let us assume that the surface mass density of the lens is smooth over some scale. In this case, for a sufficiently smooth lens, $\gamma \leq \kappa$ (BTP). The equality holds in the case of an isotropic lens, for instance the isothermal lens. The inequality holds for any anisotropic lens, with the sheet mass at the extreme. For a smooth lens these two estimates bound the true value, $\kappa_1 \leq \kappa \leq \kappa_0$. Before caustic crossing it can also be shown that $\kappa_1 \leq \kappa \leq \kappa_{\text{weak}}$ holds, where $A = 1 + 2\kappa_{\text{weak}}$ is the weak lensing limit (BTP). Hence the weak lensing approximation will overestimate the cluster mass in the strong regime, usually by a factor of 2 (van Kampen 1998).

In practice, substructure and asphericity of the cluster will induce extra shear (e.g., Bartelmann, Steinmetz, & Weiss 1995), especially in the surrounding low-$\kappa$ neighborhood, where substructure is relatively more dominant, and filaments make the cluster most aspherical. This means that the lens will not be smooth for small $\kappa$, and therefore $\kappa_1$ is a lower limit for the true $\kappa$ only for the central parts of the cluster, in the case where the lens parity is known. Van Kampen (1998) found it to be a good lower limit only for $\kappa > 0.4$ (for the most massive clusters, while for $\kappa < 0.2$, $\kappa_1$ is usually fairly close to the true value. For angle-averaged $\kappa$-profiles, $\kappa_1$ is a good lower limit for $\langle \kappa \rangle_\theta > 0.2$. All this has no bearing on $\kappa_0$, which remains a strong upper limit until the first caustic crossing.

A heuristic approximation, motivated by numerical cluster models, that tries to take these cluster lens features into account while still giving an invertible $A(\kappa)$ relation is (van Kampen 1998)

$$\gamma = |1 - c| \left(\frac{\kappa}{\sqrt{c}}\right),$$

which results in an amplification relation that admits the full four solutions:

$$A^{-1} = |(\kappa - c)(\kappa - 1/c)|,$$

with caustics at $\kappa = c$ and $1/c$. The solution for $\kappa$ is then

$$\kappa_c = \frac{1}{2c} \left[(c^2 + 1) - \mathcal{P}\sqrt{(c^2 + 1)^2 - 4c^2(1 - \mathcal{P}A^{-1})}\right].$$

(12)

We shall refer to this as the parabolic approximation. Solutions are set by choosing the parities $\mathcal{P}$, $\mathcal{P} = \pm 1$, where $\mathcal{P}$ is the image parity, and $\mathcal{P}$ is the sign of $[(c^2 + 1)/2c - \kappa]$. Note that the sheetlike solution is recovered by setting $c = 1$.

Figure 1 shows a plot of $\kappa$ versus the inverse amplification $A^{-1}$ for the three estimators. Also shown is the weak field approximation. The points are taken from a simulated lensing cluster (van Kampen & Katgert 1997) that is of comparable size to A1689. It is clear that $\kappa_0$ is a strong bound, at least until a caustic is crossed, and that $\kappa_1$ provides a very good bound for $\kappa > 0.2$. The weak field approximation, however, is extremely bad, except in the very weak regime ($\kappa < 0.1$). The parabolic approximation behaves as it is designed to do: it is a good fit between the other two strong lensing estimators for the central parts of the cluster, while also modeling the $\gamma > \kappa$ behavior for small $\kappa$. These results are fairly robust over a wide range of clusters and for all realistic values of the cosmological density parameter $\Omega_0$.

4.2. A Nonlocal Approximation to the Surface Mass Density

An alternative approach is to assume axial symmetry for the lens. Because this fixes a nonlocal functional relationship between $\kappa$ and $\gamma$ (eq. [15]), we can solve the amplification equation (2) for a self-consistent $\kappa$ and $\gamma$ profile. Although we shall apply our results to circularly averaged data, these results hold for any self-similar embedded set of contours.

We define a mean surface density interior to a contour by integration over the interior area $\Omega(\theta)$,

$$\bar{\Sigma}(\theta) = \frac{1}{\Omega(\theta)} \int_{\Omega} d^2 \theta \kappa(\theta).$$

(13)

The deflection angle for the axisymmetric lens is

$$\Delta \theta = \theta \bar{\kappa},$$

(14)

and the shear is given by

$$\gamma = \gamma_c = |\kappa - \bar{\kappa}|,$$

(15)
where the tangential term $\gamma_t$ is the only component of shear that is generated. The amplification factor is given by

$$A^{-1} = |(1 - \bar{k})(1 - 2\kappa + \kappa)|. \quad (16)$$

One can now simultaneously solve for the surface mass density, shear, and amplification by series solution. First, we divide the surface mass into consecutive shells with equal separation (any arbitrary separation can be used; we have chosen a regular separation for convenience). If we split $\bar{k}$ into an interior term, $\eta_{n-1}$, and a surface term, then for the $n$th shell we have

$$\bar{k}_n = \eta_{n-1} + \frac{2}{n+1} \kappa_n, \quad (17)$$

where we have defined

$$\eta_{n-1} = \frac{2}{m(n+1)} \sum_{m=1}^{n-1} m \kappa_m. \quad (18)$$

The surface mass density in the $n$th shell is then given by

$$\kappa_n = \frac{(n+1)}{4n} \left[ \left( \frac{n+1}{n-1} \right) - \bar{S} \left( (n-1) - \left( \frac{n+1}{n-1} \right) \right) \right], \quad (19)$$

where $\bar{S}, \bar{S} = \pm 1$ are again the image parities. The only freedom that we have, for a given amplification profile, is the choice of the shear on the first shell $\gamma_1 = \eta_0$ and the parity. It should be noted that given the amplification and having fixed the parities, one has to ensure that the first $\gamma$ satisfies $\gamma^2 \geq \bar{P} A^{-1}$, in order to avoid unphysical solutions. The nonlocal approximation contains both the sheet and isothermal solutions as specific solutions. The uncertainty on $\kappa$ and $\gamma$ can be found by simple error propagation of the uncertainty on the measurement of the amplification.

Having shown in §§ 2, 3, and 4 how, in principle, one can measure the surface mass density from angular number counts, in § 5 we exploit these methods to measure the mass distribution in the lensing cluster Abell 1689.

5. APPLICATION TO A1689

In this section we apply the methods discussed in §§ 2, 3, and 4 to observational data. We begin by describing the data.

5.1. The Data

5.1.1. Data Acquisition and Reduction

The data were obtained during a run in 1994 February at ESO’s NTT 3.6 m telescope, with $10^7$ s integration in the $V$ and $I$ bands and covering 70 arcmin$^2$ on the cluster. Seeing was similar in both bands, with FWHM of 0.8 and a CCD pixel scale of 0.34. The EMMI instrument was used throughout. The passbands and exposures were chosen such that the cluster E/S0 galaxies would be bluer than a good fraction of the background, requiring much deeper imaging in the bluer passband for detection. The cluster was observed down to a limiting magnitude of $I = 24$.

The images were debiased and flattened with skyflats using standard IRAF procedures. After this, there remained some large-scale gradients of a few percent, probably caused by some rotation of the internal lens. We additionally corrected each separate exposure with a smoothed version of itself, obtained after masking out the cluster and other bright objects. Following this, we had homogeneous photometry across the field. (A further discussion of the reduction procedure can be found in Benitez et al. 1998.) The zero point was found to be good to 0.1 mag. High humidity on a few nights meant that some of the data were not photometric, so we calibrated with the photometric data. The object detection and classification was performed with SExtractor.

5.1.2. Separation of Cluster and Background

To measure the distortion in background counts, we must first separate the background from cluster members and mask off the area that they obscure. Cluster galaxies were identified from the strong cluster E/S0 color sequence, which forms a horizontal band across the color-magnitude diagram, shown in Figure 2. The sharp upper edge of this band represents the reddest galaxies in the cluster. Galaxies redder than this are cosmologically redshifted, and hence they represent a background population. As well as isolating cluster members, this selection should also ensure that any foreground galaxies are removed. Anything redder than $V - I = 1.6$ was selected as a background galaxy. Further color cuts where imposed to ensure completeness of the sample. The range of magnitudes was restricted to $20 < I < 24$, and the $V$ band was limited to $V < 28$. Finally, we also cut at $V - I = 3.5$, where the reddest galaxies cut off.

Since the identification of cluster members is important for removing contamination of the background sample, we also checked our color-selected candidates with new data from a photometric redshift survey of the same field (Dye et al. 1998). We found general agreement with the simpler color selection.

Having identified foreground and cluster members, we produced a mask to eliminate those areas obscured by cluster members that would otherwise bias the mass estimate. To isolate the cluster members for the mask, we selected all the galaxies in the color-magnitude diagram lower than $V - I = 1.6$ and less than $I = 22$. This isolated most of

![Image](image_url)

Fig. 2.—Color-magnitude diagram for A1689, overlaid with color cuts used to isolate the cluster members from the background population: $20 < I < 24, 1.6 < V - I < 3.5$, and $V < 26.8$. The strong horizontal band of galaxies is the cluster E/S0 sequence.
the cluster sequence. We identified the remaining galaxies in the region $V - I < 1.6$ and $I > 22$, $V < 26.8$ as the faint blue background population. It is clear from Figure 2 that the distinction between faint cluster member and faint blue background galaxy is rather vague. However, since the faint cluster members are also the smallest, the masked area is fairly insensitive to the exact division. Figure 3 shows the distribution of cluster galaxies and the red background population. The concentric circles are centered on the peak in the cluster light distribution and show the position of the annuli used to calculate the radial profile in § 5.4.

5.1.3. Selection by Color

Once the cluster galaxies have been isolated, the background galaxies may be subdivided into a red and blue population, separated by $V - I = 1.6$. The observed slope of the luminosity function for these two populations for $I > 20$ is $\beta_R = 0.38$ and $\beta_B = 1$ (Broadhurst 1995; we shall do a more accurate fit using our color cuts in § 5.2.2). From equation (1) we expect that the surface density of red galaxies will be suppressed because of the dilation effect, while magnification of the faint blue galaxy population will compensate for the dilation. Hence, selecting by color allows us to identify a population of galaxies with a very flat luminosity function to boost the lensing signal, at the expense of a reduction in galaxy numbers. Simple error analysis shows that the signal-to-noise ratio varies as (Taylor & Dye 1998)

$$S/N = 2 | \beta - 1 | \kappa A(1 - \kappa + y/\kappa') \sqrt{n(1 + n\sigma^2)^{-1/2}},$$

where $\gamma = \partial \gamma / \partial R$. While the signal-to-noise ratio is a linear function of the slope of the luminosity function, it only grows with the square root of the galaxy numbers, assuming Poisson statistics. Hence one can get a better signal-to-noise ratio by preselection of the red background population to boost the signal, at the expense of numbers. Equation (20) also shows that one can get a better signal by observing to fainter magnitudes to enhance the surface number density and reduce the contribution from intrinsic clustering simultaneously (see Taylor & Dye 1998 for a more detailed discussion of observing strategies).

There is also a practical reason for favoring the red galaxy population. While the cluster members are unlikely to be redder than the cluster E/S0 sequence, the distinction between faint blue galaxies and cluster members, based on selection from the color-magnitude diagram alone, is somewhat vague. There may be blue cluster members that will contaminate the sample of blue background galaxies. In the absence of redshift information, the blue background population is clearly harder to isolate.

As we have noted, the red population has relatively few faint counts, so that the expansion term in equation (1) dominates, and there is a net underdensity of red galaxies.

Fig. 3.—Masked region of A1689 (gray area). Cluster members were selected using color information (see text) and then masked over so that these regions do not affect the surface density estimate of background sources. The total region masked is about 10% of the area. The background galaxies are also shown as open circles. Superposed are the concentric bins used to calculate the radial profile, centered on the peak in the light distribution. North is up, and east is to the left.
behind the cluster (see Figs. 4 and 7). Conversely, faint blue galaxies are numerous and cancel the expansion effect. As expected, we found that the blue galaxies were uniform across the A1689 field (see Fig. 8). This is a good indicator that it is the magnification effect at work and not some spurious contaminant, for example color gradients across the field or large-scale variations caused by clustering. In addition, it also indicates that the deficit in the red population is not due to dust obscuration or reddening in the cluster, as this would affect both red and blue populations in equal measure.

5.2. The Distribution of Background Galaxies

In Figure 4 we show the surface distribution of the red population behind A1689, Gaussian smoothed on a scale of 0.35. There are 268 background galaxies. The cluster members have been masked out and the masked areas interpolated over. The masked region contributes to only \( \approx 10\% \) of the total field. Figure 3 shows the masked region. The cluster center, identified as the peak of the light distribution, is at \((4.1, 3.6)\).

The angular size of the cluster scales as

\[
R(\theta) = 0.87D_A(z_c)(\theta/1') h^{-1} \text{ Mpc},
\]

where \( D_A(z) = 2[1 - (1 + z)^{-1/2}]/(1 + z) \) is the comoving, dimensionless angular distance in an Einstein–de Sitter universe. Hence, at the redshift of Abell 1689, \( z_c = 0.183 \pm 0.001 \) (Teague, Carter, & Gray 1990), and 1' is about 0.117 \( h^{-1} \text{ Mpc} \).

Figure 4 clearly shows a deficit of galaxies about the central peak in the light distribution at \((4.1, 3.6)\). At \( \theta = 0.75 \) there is an arc of very underdense number counts to the southwest of the cluster center, marked by a dashed line (The background is somewhat obscured by the cluster mask to the northeast of the cluster center.) This is clear indication of a caustic feature in the background number counts, where the number density drops to zero because of dilation. This exactly corresponds to the radius of the blue arcs observed by Tyson & Fischer (1995) at \( \theta = 0.85 \) (see also the radial number counts in § 5.4). This is strong evidence that we have detected the magnification effect in the background counts.

![Figure 4](image.png)

**Fig. 4.** Distribution of red \( I \)-band background sources for Abell 1689. Darker gray areas indicate an underdensity of source counts. The image is Gaussian smoothed with a smoothing scale of 0.35. The peak of the light distribution is at \((4.1, 3.6)\). The maximum density of objects is 23.0 arcmin\(^{-2}\), and the minimum is 1.1 arcmin\(^{-2}\). There are 15 contour lines spaced by \( \Delta n = 1.46 \) galaxies arcmin\(^{-2}\). A strong caustic feature is seen 0.75 from the peak (inner dashed line), more visible to the southwest, as the other side of the peak is masked over. A second feature is found in the radial profile at 2.2 (outer dotted line). The image is oriented with east to the left and north to the top.
forming the arcs lies at the same redshift as the magnified red background galaxies, \( z \approx 0.8 \). At present we do not know the redshift of this arc.

5.2.2. Number Counts of the Background Galaxy Population

Of major importance to the lens magnification method is the normalization of the background galaxy population. The CFRS is not adequate for this, since their color cuts were in the rest frame \( U-V \), rather than in the observed \( V-I \). Instead, we have used the Keck data of Smail et al. (1995), who observed deep \( VRI \) images down to a limiting magnitude of \( R \approx 27 \). The total differential galaxy count rate in the \( I \) band can be approximated by

\[
\log_{10} n = (0.271 \pm 0.009)I - 1.45
\]

over the range \( 20 < I < 24 \), where \( n \) is in \( \text{mag}^{-1} \text{deg}^{-2} \). We have applied our color criteria (see §5.1) to the Keck data and find that the red galaxy population \( V-I > 1.6 \) can be well approximated by

\[
\log_{10} n(\text{red}) = (0.0864 \pm 0.0187)I + (2.12 \pm 0.41)
\]

over the range \( 20 < I < 24 \). Figure 5 shows the magnitude distribution for the full data set and for the red-selected galaxy population and the best-fit lines. Integrating the fit for the red galaxies yields a total count rate of \( n = 12.02 \pm 3.37 \) galaxies \( \text{arcmin}^{-2} \) in the range \( 20 < I < 24 \). Since \( \beta = 2.5 \), \( d \log_{10} n/dm \), we find that the Keck data imply \( \beta_n = 0.216 \pm 0.047 \). This is the value of \( \beta \) that we shall use in the subsequent analysis.

An alternative, although less exact, method of normalization is to assume negligible cluster mass at the edge of the field and to normalize the cluster to this. In general, this would put a lower limit on the mass and is similar to the method used to normalize shear mass maps. In fact, if we do this for A1689, we find a background count rate that is very similar to that given by the Keck data. The error introduced into the final mass estimate by uncertainties in \( \beta \) scales as \( \delta \beta/\beta \approx \beta/|1 - \beta| \), which for the Keck data results in a fractional error of around 5%.

We have also fitted the blue counts in the Keck sample (Fig. 5). Over the same range as the red counts, we find that \( \log_{10} n(\text{blue}) \approx 0.351 - 3.49 \), resulting in \( \beta_n = 0.88 \), close to the lens invariant \( \beta = 1 \), and a count density between \( 23 < I < 24 \) of \( n_o(\text{blue}) = 15.5 \) galaxies \( \text{arcmin}^{-2} \).

5.2.3. Clustering Properties of the Background Population

The amplitude of clustering of \( I \)-band galaxies and its dependence on redshift can be estimated from the CFRS (Le Fèvre et al. 1996). Le Fèvre et al. (1996) find that there is little difference between the clustering properties of red and blue populations of galaxies for \( z > 0.5 \), implying that the populations were well mixed at this epoch. We therefore apply their clustering results directly to our red galaxy population. They fitted their results to a power-law model for the evolving correlation function, \( \xi(r) = (r/r_o)^{-\gamma} \), where

\[
r_o(z) = r_o(0)(1 + z)^{-(3 + \alpha)/\gamma},
\]

with \( \alpha = 1 \pm 1 \) and \( r_o(z = 0.53) = 1.33 \pm 0.09 \, h^{-1} \text{Mpc} \), and \( \gamma = 1.64 \pm 0.05 \) is in this section the slope of the correlation function.

The quantity that we require is the variance in a given area of sky, which can be estimated by averaging the observed angular correlation function \( \omega(\theta) \) over a given

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**Figure 5:** Magnitude distribution of all \( I \)-band galaxies (solid dots), the red-selected galaxies (gray dots), and the blue background galaxies (open dots). The lines are the best fits to the data.
Fig. 6.—Reconstruction of the surface mass density of Abell 1689 from the red background galaxy population, using the nonlinear local sheet approximation \((y = 0)\) and a full likelihood analysis in two dimensions. Light regions are high density. Only one caustic line is assumed, at \(\theta = 0.75\) from the peak of the light distribution. The maximum surface density is \(\kappa = 1.35\), at \((402, 341)\), consistent with the peak in the light distribution. The minimum surface mass density is \(\kappa = -0.47\). There are 15 linearly spaced contours, separated by \(\Delta \kappa = 0.12\), and the map is Gaussian smoothed with a smoothing length of \(\theta_s = 0.35\). North is up and east is to the left.

area (eq. \([7]\)). The clustering variance for \(I\)-band galaxies then scales roughly as (Taylor & Dye 1998)

\[
\sigma^2_m = 10^{-2} z^{-2.8}(\theta/1)^{-0.8},
\]

where the sampled area is a circle of radius \(\theta\), and we have assumed unbiased linear evolution of the density field. The background galaxies are assumed to all lie at \(z \approx 1\).

5.3. Reconstructing the Surface Mass Density

In Figure 6 we plot the reconstructed surface mass density of Abell 1689 using the nonlinear local sheet approximation \(\kappa_0\) (see § 4.1), changing parity on the caustic line at \(\theta = 0.75\) (see Fig. 4). The uncertainty on the peak of the mass distribution is somewhat large (see § 5.2), but significant features can be seen around the cluster core. There appears to be an extension to the southwest that is not seen in the cluster galaxy distribution. Interestingly, there also appears to be a loosely connected ridge, about 24 from the peak. We shall discuss this feature further below, but note that the shear mass map derived by Kaiser (1996, Fig. 2) shows similar extensions and ridge, although the extension to the west is not apparent in the shear map. Two underdense regions are also seen to the south and to the east in both maps. While the comparison is only qualitative and the maps are noisy, we find this very encouraging, as these maps are derived from completely independent methods, although the underlying data set is the same.

5.4. The Mass Profile of Abell 1689

While the mass maps are suggestive, a more quantitative measure can be made by angle averaging the counts and calculating the mass profile. Figure 7 shows the radial counts about the peak in the light distribution, normalized to the Keck data. The plotted error bars result from only Poisson statistics, although in the mass analysis below we shall take into account the effects of clustering. A general trend is clear and lies close to the prediction for an isothermal lens normalized to the blue arc caustic. This has a surface mass density of \(\kappa = 0.375(\theta/1)^{-1}\), corresponding to a virial velocity of 1600 km s\(^{-1}\). Again, it is worth emphasizing that the zero of the number counts at \(\theta = 0.75\) corresponds to the caustic inferred from the blue arcs. The second dip will be discussed in more detail in § 5.4.4. The
increase in counts at $\theta = 3.7$ is likely to be the result of a clustering effect. Table 1 contains the shell radii, red galaxy counts, and total and occulted area of the annuli.

In Figure 8 we show that radial profile for the blue galaxy population, normalized with the Keck data in §5.2.2. As expected, there is no lensing signal. The slight increase toward the cluster center is caused by contamination from the blue cluster members.

5.4.1. Local Approximations for the Surface Mass Density

Figure 9 shows the radial mass profile of the cluster Abell 1689 assuming a single caustic at $\theta = 0.75$. The inner two dashed lines are calculated using the lognormal-Poisson likelihood estimator (eq. [6]) with each of the two single caustics strong lensing approximations (eqs. [8] and [9]). The light shaded region indicates the 1 $\sigma$ uncertainty owing both to shot noise and to the effects of clustering. The dark shaded region indicates the region between the two extreme estimators. Away from the cluster center these agree and are equal to the weak lensing estimator, but noise effects become dominant. Closer to the cluster center the uncertainty due to the shear increases and becomes dominant at $\theta < 1'$. However, the cluster mass profile is significantly detected between $1' < \theta < 2.6$. We also appear to see a deviation from an isothermal profile, which is also plotted. When the procedure was repeated with the center of the annuli offset from the peak of the light distribution, the mass profile was weaker and less significant, as one would expect if the peak of the mass density was associated with that of light.

5.4.2. Nonlocal Approximation for the Mass Density and Shear

In Figures 10 and 11 we assume axisymmetry and equation (19) to calculate the surface mass density and shear simultaneously. We set $\gamma_1 = 0.3$ for the first shell. The resulting profile is fairly insensitive to this choice, only affecting the first two shells. The uncertainty on the shear in the first shell is small, because this must be chosen a priori. However, averaging over shells means that the errors do not strongly propagate through to higher radii. Again, a mass detection is found between 1' and 2', this time with the shear being accounted for. In this region $\kappa \approx 0.4 \pm 0.15$, which is somewhat higher than that found by the shear estimate of $\kappa = 0.2 \pm 0.1$ (Kaiser 1996). (Note that we quote Kaiser's color-selected sample, where cluster members that may contaminate the shear estimate have been removed. This corresponds to combining our red and blue background populations. This will change the redshift distribution of the background and include some residual blue cluster contamination that may account for the discrepancy.) Also, for the single caustic solution, we see a large spike at 2', which is not seen in the Kaiser (1996) results. However, the shear method correlates points, which may lead both to the suppression of features and to underestimation of the errors.

Our estimate of the shear field is far more uncertain, with $\gamma_1 = 0.2 \pm 0.3$ over most of the range. There is a slight increase beyond 2' owing to the spike in the surface mass profile at that radius, but the profile is dominated by noise. This increase is not reflected in the angle-averaged measurements of Kaiser (1996), where the mean shear is $\gamma = 0.15 \pm 0.05$.

5.4.3. Local Approximation for the Surface Mass Density and Shear

Figures 10 and 11 also show $\kappa$ and $\gamma$ estimated from the parabolic solution of §4.1. We find good agreement between the local and nonlocal approximations for $\kappa$, but the the shear profiles are somewhat different, reflecting that one estimator is local and one nonlocal. However, the large uncertainties produced by each estimator mean that we cannot predict the shear profile with much certainty from the available data.

5.4.4. Two Background Populations?

An interesting feature of the counts in Figure 7 is the appearance of two pronounced dips, one at 0.75 and another one at 2.2. While the inner dip has already been
identified with a caustic line, the outer dip is somewhat anomalous. A number of possibilities could account for this. The feature was noted in the mass plot as a low signal-to-noise ridge in the density and can be seen in the number counts as a loosely connected ring about the cluster center. One possibility is that this results from clustering in the background population, combined with a large mass concentration to the southeast of the peak in the light distribution. There are few cluster members in the region of the ridge or the bump, so the effect is not caused by masking.

An alternative is that this is the first glimpse of a second caustic line. In principle, a second caustic can be created by placing the background galaxies at two redshifts, one at low redshift, and one at high redshift (e.g., Fort et al. 1996). The observed number counts would then be given by

$$n/n_0 = A_i^{-1} + v(A_i^2 - A_i^{-1})$$

where $A_i = A(f_i)$, with $f_i = \kappa(z_i)/\kappa_\infty = [(1 + z_i)^{1/2} - (1 + z_\infty)^{1/2}]/[(1 + z_i)^{1/2} - 1]$ (BTP), and $i = 1, 2$ for the two galaxy populations. Here, $v$ is the fraction of galaxies at redshift $z_2$. An outer caustic line must be produced by the high-redshift population. If we make this population lie at $z = 0.8$, then the low-redshift population must lie at $z = 0.3$. Both populations are reflecting the same arc: the difference in projected radii is wholly a result of their relative redshifts.

However, this would double the predicted mass from lens magnification, making Abell 1689 a very extreme cluster. In addition, it seems hard to make a caustic line from the high-redshift population for such a massive cluster without forming a second, inner radial caustic. As the strongest arc is tangential and is seen near the inner arc, one would have to conspire to have a nearby galaxy, at $z = 0.3$, lensed and lying at the same projected radii as the radial arc produced by the high-redshift population. This seems highly unlikely.

One could also keep the mass roughly constant and place a second population at $z > 0.8$. This is a possibility, but it does not strongly affect our mass estimate assuming a single caustic solution. In the absence of further evidence for a second high-redshift population, we shall only consider the single caustic model.

5.5. Mass Estimate of Abell 1689
5.5.1. From $\kappa$ to Mass Surface Density

Assuming that the background galaxies all lie at the same redshift of $z = 0.8$, and given that the surface density scales as

$$\Sigma = \frac{1}{3} \sum_0 \kappa \frac{(\sqrt{1 + z} - 1)(1 + z_L)^2}{(\sqrt{1 + z_L} - 1)(\sqrt{1 + z} - \sqrt{1 + z_L})},$$

where $\Sigma_0 = 8.32 \times 10^{14} h M_\odot \text{ Mpc}^{-2}$ is the mean mass per unit area in the universe, then we find that the surface mass density is

$$\Sigma = 5.9 \times 10^{15} \kappa (h M_\odot \text{ Mpc}^{-2}).$$
Although we have assumed an Einstein–de Sitter universe, these results only depend weakly on cosmology (BTP).

5.5.2. Uncertainty in the Redshift Distribution

The error introduced by assuming that the background galaxies lie at the same redshift can be estimated by error propagation and by assuming \( \delta z = 0.4 \) (see § 5.2.1). Hence, \( \delta \Sigma = |\delta \Sigma / \delta z| \delta z \) and the fractional uncertainty on the surface mass density owing to the uncertainty in redshift distribution of the background galaxies is \( \delta \Sigma / \Sigma = 0.37 \delta z = 0.148 \). The same error is also found in mass estimates based on the shear pattern.

5.5.3. Uncertainty Arising from Normalization of Background Counts

Assuming a sheet mass solution \( (\kappa_0 \text{ in } \S 4.1) \), we find that the uncertainty arising from the normalization of the background counts is \( \delta \kappa = (|1 - \kappa| / 2 |\beta - 1|) \delta n_0 / n_0 \). For A1689 and the red galaxy population, this is \( \delta \kappa = 0.15 |1 - \kappa| \). For an average \( \kappa = 0.5 \), the uncertainty is around \( \delta \kappa = 0.07 \).

5.5.4. The Cumulative Mass Distribution

Figure 12 shows the cumulative mass interior to a shell, calculated from both the nonlocal approximation (§ 4.2) and the local parabolic approximation allowing only a single caustic solution (§ 4.1). The uncertainties are treated by error propagation. We find that the two-dimensional projected mass interior to 0.24 \( h^{-1} \) Mpc is

\[
M_{2D}(<0.24 \text{ h}^{-1} \text{ Mpc}) = (0.50 \pm 0.09) \times 10^{15} \text{ h}^{-1} \text{ M}_\odot ,
\]

and that the two estimators are in good agreement. We find that the projected mass scales as

\[
M_{2D}(<R) \approx 3.5 \times 10^{15} (R/h^{-1} \text{ Mpc})^{1.3} \text{ h}^{-1} \text{ M}_\odot ,
\]

for \( R < 0.32 \text{ h}^{-1} \) Mpc, similar to that for an isothermal sphere, \( M \sim R \). Hence it appears that A1689 has a near-isothermal core. Beyond \( R = 0.32 \text{ h}^{-1} \) Mpc the lensing signal is lost in background noise, and we can only say that \( \kappa \lesssim 0.1 \).

Including the uncertainty from the background redshift distribution and the normalization of background counts increases the error to about 30%.

6. COMPARISON WITH OTHER MASS ESTIMATES OF A1689 AND INFERRING THE THREE-DIMENSIONAL MASS DISTRIBUTION

In this section we compare the mass derived from lens magnification to that found from a number of other independent measurements. First, we compare our results to estimates of the mass based on the shear pattern found around A1689 (§ 6.1). The magnification and shear complement each other in that the shear pattern has a higher
signal-to-noise ratio, since it is not affected by clustering noise (although with redshift information, the magnification can also be measured free from clustering noise; see BTP), but suffers from the “sheet-mass” degeneracy. We shall combine the magnification and shear pattern elsewhere.

While the lens magnification mass is vital for fixing the total two-dimensional projected mass distribution independently of any assumptions about the dynamical state of the cluster, much information can be gained by combining this with other mass estimates, assuming that these are not strongly biased by their reliance on thermodynamical equilibrium. In this section we describe a method for transforming from the two-dimensional lens mass to other cluster characteristics, such as the line-of-sight velocity dispersion (§ 6.3) and the X-ray temperature (§ 6.4). Discrepancies that arise between these predicted characteristics and the actual measurements can be used to infer information about the mass distribution along the line of sight (Bartelmann & Kolatt 1997). We find that while there is fair agreement between all of the mass estimates when projection effects are taken into account, the agreement is better if the cluster A1689 is composed of two clusters superposed along the line of sight and separated by about $\Delta z = 0.02$.

The transformation from a two-dimensional projected lensing mass to a three-dimensional mass, line-of-sight velocity dispersion and X-ray temperature can be made using either the isothermal model or by using relations found in $N$-body simulations of clusters. While the former provides a simpler method, one has more freedom with simulations to include or exclude the various projection effects that contaminate measurements of these quantities. In this section we shall use the relations found by van Kampen (1998) from an ensemble of CDM cluster simulations, all with $\Omega_0 = 1$ and $\sigma_8 = 0.54$. These relations are model dependent, but serve to aid comparison between the various mass measurements. We have also provided a table of quantities (Table 2) in which the uncertainties have been calculated by combining the error on the cluster mass with the dispersion found in the deprojection relations.

We begin by comparing the lens magnification mass to the mass determined from the shear field around A1689.

6.1. Comparison with Arclets and Weak Shear

Tyson & Fischer (1995) provide mass profiles of A1689 from arclets, another independent estimator of the mass, normalized to the caustic line indicated by the blue arcs. They find that the two-dimensional projected mass within $R = 0.1 \, h^{-1} \text{ Mpc}$ is

$$M_{2D}(<0.1) = (0.18 \pm 0.01) \times 10^{15} \, h^{-1} \, M_\odot.$$  

They also find that the mass scales like an isothermal sphere out to $0.4 \, h^{-1} \text{Mpc}$, before turning over to an $R^{-1.4}$ profile. This implies that in the regime that we probe with the magnification the cumulative mass scales like

$$M(<R) = (1.8 \pm 0.1) \times 10^{15} (R/h^{-1} \, \text{Mpc}) \, h^{-1} \, M_\odot.$$  

![Fig. 11.—Radial profiles of tangential shear $\gamma$, for A1689 (solid line with dots), calculated by solving the axially symmetric lens equation (19). Shaded regions: 1 $\sigma$ errors calculated via error propagation from the uncertainty on the measured amplification profile. Solid dark line: A singular isothermal profile normalized to the caustic feature at $\theta = 0.75$. Lighter solid line: Local parabolic estimator $\kappa_c$.](image-url)
Fig. 12.—Cumulative mass profile of Abell 1689. Solid dark line and shaded uncertainties: Estimated using the axisymmetric nonlocal estimator described in § 4.2. Lighter gray line: Cumulative mass estimated from the local parabolic approximation \( \kappa_\alpha \), described in § 4.1. Also plotted is the isothermal fit to the blue arc caustic (dotted line), similar to the shear results of Kaiser (1996) and Tyson & Fischer (1995).

This is very close to the profile that we find from lens magnification (eq. [32]). Using this, we scale their results, giving

\[
M_{2D}(<0.24) = (0.43 \pm 0.02) \times 10^{15} h^{-1} M_\odot ,
\]

in good agreement with the mass from magnification. Kaiser (1996) has also calculated \( \kappa \) based on the weak shear method (Kaiser & Squires 1993), using the same data that we have used here for A1689. We noted above that there are qualitative similarities between the weak shear maps and those presented by Kaiser, which is significant, since the methods are independent. The mass density profile found from the shear pattern is also well fitted by an isothermal profile:

\[
M_{2D}(<R) = 1.8 \times 10^{15}(R/h^{-1} \text{Mpc}) h^{-1} M_\odot ,
\]

with a 10% statistical uncertainty and further 10% systematic error owing to the uncertainty in the redshift distribution.

### Table 1

**Parameter Values at Various Angular Radii**

| \( r \) (arcmin) | \( N \) | \( N/N_0 \) | Annulus Area (arcmin\(^2\)) | Obscured Area (arcmin\(^2\)) |
|------------------|-------|-----------|-----------------------------|-----------------------------|
| 0.33             | 2     | 1.19      | 0.35                        | 0.21                        |
| 0.67             | 0     | 0.00      | 1.08                        | 0.25                        |
| 1.01             | 4     | 0.24      | 1.79                        | 0.40                        |
| 1.35             | 13    | 0.51      | 2.51                        | 0.39                        |
| 1.69             | 20    | 0.59      | 3.23                        | 0.40                        |
| 2.03             | 23    | 0.57      | 3.62                        | 0.26                        |
| 2.36             | 11    | 0.28      | 3.46                        | 0.23                        |
| 2.70             | 26    | 0.77      | 3.08                        | 0.26                        |
| 3.04             | 32    | 0.94      | 3.01                        | 0.16                        |
| 3.38             | 34    | 0.95      | 3.14                        | 0.17                        |
| 3.72             | 45    | 1.25      | 3.18                        | 0.20                        |
| 4.06             | 31    | 1.06      | 2.49                        | 0.06                        |

**Notes.**—Angular radius \( r \) in arcminutes, number of red galaxies \( N \), ratio of galaxies to background \( N/N_0 \), the total area of the annuli, and the area obscured by the mask. The unobscured area is total area — obscured area. The expected number of galaxies in an annuli is \( N_0 = n_0 \times \) unobscured area.

### Table 2

**Mass Estimates for A1689**

| Quantity      | This Work | Other          |
|---------------|-----------|----------------|
| \( M_{2D}(<0.24) \) | 0.50 \pm 0.09 | \( 0.43 \pm 0.02 \) (Tyson & Fischer 1995) |
| \( M_{2D}(<0.5) \) | 0.72 \pm 0.25 | \( 0.43 \pm 0.04 \) (Kaiser 1996) |
| \( M_{500} \) | 1.6 \pm 0.65 | \( 0.95 \pm 0.16 \) (Yamashita 1994) |
| \( \sigma_{\text{s}}(<1.5) \) | 2200 \pm 500 | \( 2355 \pm 14{1/2} \) (Teague et al. 1990) |

**Notes.**—Mass estimates for A1689 based on lens magnification (col. [2]) and from other measurements (col. [3]). Masses are given in units of \( 10^{15} h^{-1} M_\odot \), and velocities are quoted in units of \( \text{km s}^{-1} \). Distance are given in \( h^{-1} \text{Mpc} \). The other measurements are based on arclets (Tyson & Fischer 1995), the shear pattern (Kaiser 1996), X-ray temperatures (Yamashita 1994), and line-of-sight velocity dispersion (Teague et al. 1990). Also given are the three-dimensional mass estimates from lens magnification.
6. The Three-dimensional Mass Estimated from Lensing Alone

The three-dimensional mass inferred from the two-dimensional projected mass inside a sphere of radius $r = 0.5 \, h^{-1}\text{Mpc}$ is

$$M_{2D}(<0.5) = (0.43 \pm 0.04) \times 10^{15} \, h^{-1} \, M_\odot,$$

(37)

again in good agreement with that found by the magnification method.

6.2. The Three-dimensional Mass Estimated from Lensing Alone

The three-dimensional mass inferred from the two-dimensional projected mass inside a sphere of radius $r = 0.5 \, h^{-1}\text{Mpc}$ is

$$M_{3D}(<0.5) = (0.72 \pm 0.25) \times 10^{15} \, h^{-1} \, M_\odot,$$

(38)

while the mass inside an Abell radius, $r = 1.5 \, h^{-1}\text{Mpc}$, is

$$M_{3D}(<1.5) = (1.6 \pm 0.6) \times 10^{15} \, h^{-1} \, M_\odot.$$

(39)

These estimates are probably an overestimate of the true three-dimensional mass, since the dispersion in the simulations includes the effect of the alignment of the clusters’ principle axis along the line of sight. Given that the inferred three-dimensional mass is so high, A1689 is probably lying at the extreme of such a distribution. In such cases, the three-dimensional mass may be much lower than mass inferred from a two-dimensional projection. We discuss this possibility in the next few sections.

6.3. Velocity Dispersion of Abell 1689

The predicted line-of-sight velocity dispersion estimated from the simulations includes the effects of superposition of clusters, infall along filaments, and interlopers, and so tends to predict larger velocities and larger uncertainties than for an isolated cluster. Including these effects into our estimate for Abell 1689, we find

$$\sigma_v(<1.5 \, h^{-1}\text{Mpc}) = 2200 \pm 500 \, \text{km s}^{-1}$$

(40)

for the line-of-sight velocity dispersion inferred from the two-dimensional lensing mass. A measurement that may also include these effects is given by Teague et al. (1990), who find

$$\sigma_v(<1.5 \, h^{-1}\text{Mpc}) = 2355^{+238}_{-183} \, \text{km s}^{-1},$$

(41)

in good agreement with our model. However, both of these values are very high, much higher than the estimate for an isolated isothermal sphere, which for A1689 gives a velocity dispersion of $1645 \pm 148 \, \text{km s}^{-1}$. This discrepancy between lensing mass and the velocity dispersion suggests that A1689 is not a single isolated cluster, but a superposition of smaller clumps that contribute to the total measured velocity dispersion. Den Hartog & Katgert (1996) have tried to take into account interlopers in A1689 and, using the Teague et al. data, find a value of $\sigma_v = 1861 \, \text{km s}^{-1}$.

Following a suggestion of Miralda-Escudé & Babul (1995), we shall assume that A1689 is composed of two superposed isothermal spheres. Placing one cluster at $z = 0.18$ with a velocity dispersion of $1500 \, \text{km s}^{-1}$ and a second at $z = 0.20$ with a velocity dispersion of $750 \, \text{km s}^{-1}$, we find that we can reproduce a total projected velocity dispersion of around $2300 \, \text{km s}^{-1}$, in agreement with both observed and simulated values. Figures 4 and 5 of Teague et al. (1990) also provide marginal evidence for a second concentration of galaxies at $z = 0.2$. Furthermore, if we estimate the integrated surface mass of these two clusters,

$$M_{2D}(<R) = 7.38 \times 10^{14} \sigma_{1000}^2 \left(\frac{R}{h^{-1}\text{Mpc}}\right) h^{-1} \, M_\odot,$$

(42)

where $\sigma_{1000} = \sigma_v/1000 \, \text{km s}^{-1}$, we reproduce a lensing mass of $M_{2D}(<R) = 2 \times 10^{15} (R/h^{-1}\text{Mpc}) h^{-1} \, M_\odot$, in agreement with what we see from lensing. Hence it seems plausible that the lensing mass and velocity dispersion of A1689 can both be explained by a superposition of a rich and a poor cluster.

6.4. X-Ray Mass Estimates of Abell 1689

Evrard, Metzler, & Navarro (1996) have found that the mass within the radius defined where the mean cluster density is 500 times the critical density is strongly correlated with the cluster temperature. They fitted this relation from simulations with

$$M_{500} = 1.11 \times 10^{15} \left(\frac{T_X}{10 \, \text{keV}}\right)^{3/2} h^{-1} \, M_\odot,$$

(43)

where $T_X$ is the broad-beam temperature, and $M_{500}$ is the three-dimensional mass within a radius defined by an overdensity 500$\rho_{\text{crit}}$. This radius is roughly given by $r_{500} = 1.175 \, h^{-1}\text{Mpc}$.

X-ray temperatures of A1689 have been measured by both Ginga and ASCA. Yamashita (1994) has analyzed these data and finds $T = 9 \pm 1 \, \text{keV}$, while Mushotzky & Scharf (1997) find $T = 9.02^{+0.4}_{-0.3} \, \text{keV}$. Daines et al. (1997) have also recently reanalyzed ROSAT PSPC observations and find a mean temperature of $T_X = 10.2 \pm 4 \, \text{keV}$. Note that we are quoting the mean temperature and incorporated the 40% uncertainty in the error estimate, rather than quoting upper limits as Daines et al. do. The major uncertainty in measuring X-ray temperatures here is instrumental, as 10 keV is approaching the limit of ROSAT’s sensitivity.

Taking the result of Yamashita and the relation found by Evrard et al., we find that

$$M_{500} = (0.95 \pm 0.16) \times 10^{15} h^{-1} \, M_\odot.$$

(44)

Using the simulated scaling relations, we find

$$M_{500} = (1.6 \pm 0.65) \times 10^{15} h^{-1} \, M_\odot$$

(45)

for Abell 1689, implying an X-ray temperature of $T_X = 12.7 \pm 3.4 \, \text{keV}$, within the 1 $\sigma$ uncertainty of the measured X-ray temperature. Again, if we consider A1689 as a double cluster, the nearer, larger mass concentration would be detected in X-ray, lowering the expected X-ray temperature. From the velocity dispersions we can infer a temperature nearer to $T_X = 0.7 \, \text{keV}$, slightly below, but again in agreement with observations.

In conclusion, although we find a high mass, there is a general consistency between the mass of A1689 estimated from lens magnification and shear. In addition, we find a fair agreement between the lens mass and the line-of-sight velocity dispersion if we take into account projection effects. Modeling A1689 as a double cluster, we find that the velocity dispersion can be much lower, implying two smaller clusters, with the lensing mass a superposition of cluster masses. This hypothesis might also help explain the marginal discrepancy with X-ray temperature.

Finally, A1689 is in projection a highly spherical cluster, in contrast with the majority of clusters, which appear...
extended. While this may be a result of its high mass, it is also possible that A1689 has its major axis aligned along the line of sight, pointing toward a second cluster. While much of the evidence on the mass distribution along the line of sight is circumstantial, all of these effects would conspire to give A1689 its impressively massive appearance.

7. DISCUSSION

The absolute surface mass density of a galaxy cluster can be estimated from the magnification effect on a background population of galaxies, breaking the "sheet-mass" degeneracy. To apply this in practice, we have taken into account the nonlinear clustering of the background population and shot noise, both of which contribute to uncertainties in the lensing signal (Taylor & Dye 1998). A further complication is the contribution of shear to the magnification in the strong lensing regime, where the magnification signal is stronger. We have argued that this can be circumvented by approximate methods that can be local, where a relationship between surface mass and shear is assumed (van Kampen 1998), or by a nonlocal approximation where only the shape of the cluster is assumed. Both approximations seem to work well on simulated data.

We have applied these methods to the lensing cluster Abell 1689, using Keck data of Smail et al. (1995) to normalize the background counts and the CFRS results to infer the redshift distribution and clustering properties of our data. Using a γ = 0 approximation of the surface density in the shear, arclets, line-of-sight velocity dispersions, and the X-ray temperature mass estimates are all in reasonable agreement, to within the uncertainties at this time.

The results presented here are from 3 hours integration on the 3.6 m NTT. Longer integration times have the combined benefit of increasing the number of background galaxies, and so reducing shot noise, and of reducing the contribution from cosmic variance (eq. [20]; § 5.1.3). Hence, by increasing the exposure time, we can expect to reduce the uncertainty from lens magnification by a factor of 2 or so.

One drawback of this analysis is the contribution of clustering noise to the background counts. This can be removed using redshift information, either from spectroscopy or more efficiently, using photometric redshift information (BTP). We shall explore this elsewhere (Dye et al. 1998).

If our results are extended to other clusters, we can hope to have a good representation of the total mass distribution, gas, and galaxy contents with which to make strong statistical arguments about the matter content of the largest gravitationally collapsed structures in the universe.

A. N. T. thanks the PPARC for a research associateship and the University of Berkeley and the Theoretical Astrophysics Center, Copenhagen for their hospitality during the writing of this paper. E. v. K. acknowledges an European Community Research Fellowship as part of the HCM programme and thanks the University of Edinburgh and ROE for their hospitality. This work was supported in part by Danmarks Grundforskningsfond through its funding of the Theoretical Astrophysics Center. S. D. thanks the PPARC for a studentship. N. B. L. thanks the Spanish MEC for a Ph.D. scholarship, the University of Berkeley for their hospitality and financial support from the Spanish DGES, project PB 95-0041. We thank Ian Smail, who kindly provided us with a copy of his Keck data. We also thank John Peacock, Alan Heavens, Jens Hjorth, Adrian Webster, and an anonymous referee for comments and useful suggestions.

\[
M_{2D}(<0.24 \text{ h}^{-1} \text{Mpc}) = (0.50 \pm 0.09) \times 10^{15} \text{ h}^{-1} M_{\odot}.
\]

Such a large mass is very rare in a CDM universe normalized to the observed cluster abundance, and may indicate that A1689 is composed of two large masses along the line of sight and/or filaments connected to the cluster and aligned along the line of sight. This is also implied by the high line-of-sight velocity dispersion, which would be enhanced by merging clusters (Miralda-Escudé & Babul 1995) or by infall from aligned filaments.

We have compared our mass estimates to other estimates available in the literature and find that the lens magnification, shear, arclets, line-of-sight velocity dispersions, and the X-ray temperature mass estimates are all in reasonable agreement, to within the uncertainties at this time.

The results presented here are from 3 hours integration on the 3.6 m NTT. Longer integration times have the combined benefit of increasing the number of background galaxies, and so reducing shot noise, and of reducing the contribution from cosmic variance (eq. [20]; § 5.1.3). Hence, by increasing the exposure time, we can expect to reduce the uncertainty from lens magnification by a factor of 2 or so.

One drawback of this analysis is the contribution of clustering noise to the background counts. This can be removed using redshift information, either from spectroscopy or more efficiently, using photometric redshift information (BTP). We shall explore this elsewhere (Dye et al. 1998).

If our results are extended to other clusters, we can hope to have a good representation of the total mass distribution, gas, and galaxy contents with which to make strong statistical arguments about the matter content of the largest gravitationally collapsed structures in the universe.

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