Chiral transition in a strongly coupled fermion-gauge-scalar model
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We report the recent results from the computer simulations of a fermion-gauge-scalar model with dynamical chiral-symmetry breaking and chiral transition induced by the scalar field. This model might be considered to be a possible alternative to the Higgs mechanism of mass generation. A new scheme is developed for detecting the chiral transition. Our results show with higher precision than the earlier works that the chiral transition line joins the Higgs phase transition line, separating the Higgs and Nambu (chiral-symmetry breaking) phases. The end point of the Higgs transition with divergent correlation lengths is therefore suitable for an investigation of the continuum limit.

1. INTRODUCTION

Some strongly coupled lattice fermion-gauge models with a charged scalar field, which break chiral symmetry dynamically, might be considered to be a possible alternative to the Higgs mechanism for mass generation, as discussed in [1,2]. Let us concentrate on a prototype with $U(1)$ gauge group, a scalar of fixed modulus and one staggered fermion (corresponding to 4 flavors), where both the scalar and fermion have charge one. The action has been described in [1,2] with three bare parameters ($\beta, \kappa, m_0$). The dynamical mass generation is meaningful only in the chiral limit $m_0 = 0$. We consider here the phase transition line NET between two phases [1,2]:

1. Dynamical mass generation (Nambu) phase, below the NET line, where chiral symmetry is spontaneously broken ($\langle \bar{\psi} \psi \rangle \neq 0$) due to the strong gauge fluctuations so that the fermion mass $m_F$ is dynamically generated;
2. Higgs phase, above the NETS line, where the Higgs mechanism is operative, but $\langle \bar{\psi} \psi \rangle = m_F = 0$.

The scalar field induces a second order chiral phase transition NE line which opens the possibility for approaching the continuum.

Whether such a model can replace the Higgs mechanism depends crucially on the existence and renormalizability of the continuum limit. To search for such a continuum theory and grasp its nature, we need to make precise determination of the second order phase transition point with divergent correlation lengths. For such a purpose, we have done extensive simulations using Hybrid Monte Carlo (HMC) algorithm and developed some new methods for locating the NE line.

2. HMC SIMULATIONS

The HMC simulations have been done on $6^3 16$ and $8^3 24$, where on $6^3 16$, we have better statistics (1024-6500 trajectories) for different ($\beta, \kappa, m_0$). The detailed results for the spectrum are reported in [3]. We have measured the following local observables: plaquette energy $E_p$, link energy $E_l$ and chiral condensate $\langle \bar{\psi} \psi \rangle$, where for $\langle \bar{\psi} \psi \rangle$ we use the stochastic estimator method. However, it is very difficult to use the local quantities at finite $m_0$ to detect a critical behavior on the NE line, since they show smooth behavior as a function of $\beta$ or $\kappa$. (One could expect the critical behavior only in the infinite volume and chiral limit.) For ($\beta, \kappa$) near the point E, the peaks of susceptibility for different quantities develop and coincide, while the boson mass $am_S$ gets smaller. Concern-
ing the location of the ET line, on the $6^316$ and $8^324$ for $\kappa < 0.31$ or $\beta > 0.64$ and $m_0 = 0.04$, we find explicit two state signals from the thermocycle, time history and histogram analysis of the local quantities.

On the NE line, the $\pi$ meson shows more obviously the phase transition than other quantities. In the Nambu phase, the $\pi$ meson should obey the PCAC relation. In the symmetric phase, the $\pi$ meson is no longer a Goldstone boson, and one should observe a deviation from PCAC. At $\kappa = 0.4$, these properties are nicely seen in fig. [1], from which one sees that for $\beta < 0.57$ where the system is in the broken phase, we have Goldstone bosons. However, on $6^316$, even at $\beta = 0.57$ (possibly in the chiral symmetric phase), a linear extrapolation leads to $\langle \bar{\psi}\psi \rangle|_{m_0=0} \approx 0.13$. For larger $\beta$, the extrapolated result gets smaller (e.g. at $\beta = 0.65$, $\langle \bar{\psi}\psi \rangle|_{m_0=0} \approx 0.05$) and is expected to vanish in the $V \to \infty$ limit. Of course, one should not expect the linear extrapolation to be valid at the critical point.

3. NEW ORDER PARAMETER

$\langle \bar{\psi}\psi \rangle$ is not a convenient order parameter for the chiral transition of a finite system due to the sensitivity of chiral extrapolation. We employ a different method for determining the chiral transition, namely we calculate the chiral susceptibility in the chiral limit, defined by

$$\chi_{chiral} = \frac{\partial \langle \bar{\psi}\psi \rangle}{\partial m_0}|_{m_0=0}. \quad (1)$$

If there is a second order chiral phase transition, $\chi_{chiral}$ should be divergent (in other words, $\chi_{chiral}^{-1}$ should be zero) at the critical point and in the thermodynamical limit. In the chiral limit, the chiral susceptibility in the Nambu phase is difficult to obtain, but it is calculable in the chiral symmetric phase [4]. It can be shown that in the symmetric phase $\chi_{chiral}^{-1}$, defined in eq. (1), is the same as

$$\chi_{chiral}^{-1} = \left\{ \frac{2V}{V} \left( \sum_{i=1}^{V} \frac{1}{\lambda_i^2} \right)|_{m_0=0} \right\}^{-1}, \quad (2)$$

where $\lambda_i$ are the positive eigenvalues of the massless fermionic matrix. Approaching the NE line from the symmetric phase by fixing $\kappa$, $\chi_{chiral}^{-1}$ should behave as $\chi_{chiral}^{-1} \propto (\beta - \beta_c)^\gamma$, corresponding to the divergent correlation length at the second order phase transition point in the thermodynamical limit $V \to \infty$. In the Nambu phase, it can also be shown that eq. (2) is equivalent to

$$\chi_{chiral}^{-1} = \left\{ \frac{V}{2} \left( \langle \bar{\psi}\psi \rangle|_{m_0=0} \right)^2 \right\}^{-1} \quad (3)$$

in the $V \to \infty$ limit. Then in such a limit, $\chi_{chiral}^{-1}$ should be zero since $\langle \bar{\psi}\psi \rangle|_{m_0=0} \neq 0$ in the Nambu phase.

Therefore, $\chi_{chiral}^{-1}$ defined in eq. (2) is a suitable order parameter for the chiral phase transition: it is zero in the broken phase, and it is nonzero in the symmetric phase.

Let us again focus on the results at $\kappa = 0.4$. To perform the calculation, we generalize MFA [5], in which the chiral limit $m_0 = 0$ is accessible, to the fermion-gauge-scalar models. From fig. [2], we observe that on $8^4$ the chiral transition appears at $\beta_c \approx 0.57$, being consistent with the observation of fig. [1].
4. DISCUSSIONS

The location of the NE line on the available lattices obtained by the above methods is summarized in fig. 3, where the point N is plotted by interpolation. We have determined the phase transition line NE with high precision and demonstrated that this second order chiral transition line joins the Higgs phase transition line at the end point E being around \((\beta, \kappa) = (0.64, 0.31)\), separating the Higgs and Nambu phases. No finite size scaling analysis has been done, and larger lattices are required for such a purpose.

From the spectroscopy \(3\), we know that \(am_F\) scales to zero when crossing the chiral transition line NE. Nevertheless, the susceptibility for \(E_i\) and correlation length for the composite scalar \((am_S > 1.5)\) remain finite on the whole NE line except approaching the end point E. Therefore, the end point, hopefully being a second order point with divergent correlation lengths, is the most suitable candidate for the continuum limit.

Further work to be done is to study the finite size effects, analyze the dependence of the end point E on the bare fermion mass, investigate the scaling properties and understand the nature of the end point, which is underway \(6\).

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