Helical Phase Inflation

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We show that quadratic inflation can be realized by the phase of a complex field with helicoid potential. Remarkably, this helicoid potential can be simply realized in minimal supergravity. The global $U(1)$ symmetry of the Kähler potential introduces a flat direction in the F-term potential and evades the $\eta$ problem automatically. So such inflation is technically natural. During inflation the norm of the complex field is strongly stabilized and the phase evolves along a flat helix trajectory. The phase excursion is super-Planckian as required by the Lyth bound, while the norm of the complex field can be suppressed in the sub-Planckian region. This model resolves the contradiction between the strict flat condition for inflation and the dangerous corrections from quantum gravity effects.

I. INTRODUCTION

Inflation\textsuperscript{[1]} as a model of the early Universe plays a crucial role in modern cosmology. It beautifully solves the horizon, flatness, and monopole problems, as well as explains the density fluctuation observed in the cosmic microwave background. Some details on the inflationary process are obtained from recent observations of the Planck\textsuperscript{[2]} and BICEP2\textsuperscript{[3]} experiments. It shows the inflation scale is about $10^{16}$ GeV, close to the scale for Grand Unified Theory (GUT). To generate slow-roll inflation, the scalar field $\phi$ should have sufficiently flat potential $V(\phi)$ so that its mass is hierarchically smaller than the Hubble constant

$$\eta \equiv \frac{M_P^2 V''}{V} \simeq \frac{m_\phi^2}{3H^2} \ll 1,$$  \hspace{1cm} (1)

where $M_P$ is the reduced Planck scale. At the classical level, the potential can be set sufficiently flat by hand. However, the inflaton as a scalar field receives dangerous quantum corrections and even serious quantum gravity corrections if there is super-Planckian field excursion. The crucial challenge for a sensible inflation model is to protect the flat condition against these dangerous corrections.

At the GUT scale physics is considered to be supersymmetrical and the quantum corrections on the inflaton potential are effectively suppressed by supersymmetry\textsuperscript{[4]}. However, the flatness of the potential is significantly changed in supergravity. The F-term scalar potential is proportional to a factor $e^K$, $K$ is the Kähler potential and contains a term $\Phi \bar{\Phi}$ in minimal supergravity. The factor $e^{\Phi \bar{\Phi}}$ generates an inflaton mass close to the Hubble scale and hence breaks the slow-roll condition\textsuperscript{[5]}. The $\eta$ problem is absent in no-scale supergravity\textsuperscript{[6]}, in which the Kähler potential is initially designed so that $e^K$ is flat along the shift direction\textsuperscript{1}.

For single field slow-roll inflation, the Lyth bound\textsuperscript{[7]} indicates a super-Planckian inflaton excursion $\Delta \phi \sim 10 M_P$ for large tensor modes. This requires the running of the inflaton to initiate far above the Planck scale, where the higher dimensional operators\textsuperscript{2} from quantum gravity effects become important. The higher dimensional operators are suppressed by the Planck mass $M_P$ and thus irrelevant in the sub-Planckian region. However, once the inflaton becomes super-Planckian, the higher dimensional operators are unignorable and the theory is not reliable unless it can be Ultraviolet (UV)-Completed. Specifically, the inflaton potential can be seriously affected by the corrections from “irrelevant” operators and breaks the flatness conditions\textsuperscript{[10]}.

Nevertheless, the quantum gravity corrections can be avoided if the super-Planckian field excursion is effec-

\textsuperscript{1} The shift symmetry can be slightly broken to get inflationary models with a broad range of tensor-to-scalar ratio $r$\textsuperscript{[8]}.

\textsuperscript{2} Hereinafter, more specifically the higher dimensional operator refers to the extra terms from quantum gravity corrections, not necessarily the nonrenormalizable terms, which are already important parts even in supergravity.
tively realized in the sub-Planckian region. Considering the phase of a complex scalar field, or the pseudo-Nambu-Goldstone boson (PNGB) in gauge symmetry breaking scenario \cite{11-14}, the phase can have super-Planckian displacement while the magnitude of complex field remains sub-Planckian. Besides, the combination of multi sub-Planckian fields may lead to effective super-Planckian excursion \cite{15, 16}.

In this letter, we present a new inflation model with helicoid potential. The phase of a complex field drives inflation along an helical path. This potential is designed to realize super-Planckian inflaton excursion with sub-Planckian fields, so that we can keep away from dangerous quantum gravity corrections. Remarkably, the helicoid scalar potential can be simply obtained in minimal supergravity. Since we take the phase as the inflaton, the well-known $\eta$ problem is automatically solved without any extra symmetry.

\section{Hilicoid Potential}

Now we give the supergravity realization of the helicoid potential in the simplest case. We consider two chiral superfields $\Phi$ and $X$ in minimal supergravity, the Kähler potential is

$$K = \Phi \bar{\Phi} + X \bar{X} - g(X \bar{X})^2,$$  \hspace{1cm} (2)

where the higher order term $g(X \bar{X})^2$ is introduced to stabilize the field $X$ at $X = 0$ \cite{17, 18}. Besides, we use the following superpotential

$$W = a \frac{X}{\Phi} \ln \Phi,$$  \hspace{1cm} (3)

which is expected to be obtained after integrating out heavy fields. It is obvious that the Kähler potential preserves the global $U(1)$ symmetry for $\Phi$, which is broken by the superpotential. Thus, our model is technically natural since there is a global $U(1)$ symmetry in the $a = 0$ limit \cite{19}.

The F-term scalar potential is determined by the Kähler potential and superpotential as follows

$$V = e^K (K^{i \bar{j}} D_i W D_j \bar{W} - 3 W \bar{W}).$$  \hspace{1cm} (4)

As the field $X$ is stabilized at $X = 0$, the above potential is significantly simplified as below

$$V = e^{\Phi \bar{\Phi}} W_X \bar{W}_X$$
$$= a^2 e^{r^2} \frac{1}{r^2} ((\ln r)^2 + \theta^2),$$  \hspace{1cm} (5)

where $\Phi \equiv r e^{i \theta}$.

The potential \cite{5} is simple but actually has fancy helicoid structure, as shown in Fig. 1. The exponential factor $e^{r^2}$ does not depend on the phase $\theta$ resulting from the global $U(1)$ symmetry of Kähler potential \cite{20}, consequently there is no $\eta$ problem for this phase inflation. The complex field magnitude $|\Phi| = r$ obtains vacuum expectation value at $\langle r \rangle = 1$ as both $e^{r^2} \frac{1}{r}$ and $(\ln r)^2$ reach minimums at $r = 1$.

The mass along the radial direction is

$$m_r^2 = \frac{1}{2} \frac{\partial^2 V}{\partial r^2} |_{r=1} = (2 + \frac{1}{2} \theta^2) V_I,$$  \hspace{1cm} (6)

where the factor $\frac{1}{2}$ is from the normalization of $r$, and $V_I = a e^{2 \theta^2}$ is the potential for inflation. Eq. \cite{7} shows

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The helicoid potential with unit $10^{-8} M_P^4$. Along radial direction, the minimum of the potential locates at $|\Phi| \equiv r = 1$, while the phase $\theta$ provides a flat direction along the helix line, from which it is easy to get super-Planckian field excursion.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Helix trajectory with $r = 1$. The red part indicates the phase excursion for quadratic inflation with $N_e = 55$.}
\end{figure}
that the mass of $r$ is larger than the Hubble constant therefore the radial component is frozen out during inflation, and we realize the quadratic inflation dominated by $V_I$. The helical inflation path is shown in Fig. 2. The inflaton $\theta$ has physical mass $m_{\phi} = \sqrt{\sigma a}$, at scale $10^{15}$ GeV from observations [2, 3]. It predicts the spectral index $n_s \simeq 1 - \frac{2}{3} \epsilon$ and the tensor-to-scalar ratio $r \simeq \frac{8}{\pi^2}$, where $N_c$ is the $e$-folding number.

In the PNGB inflation, the phase of the Higgs field also plays the role of the inflaton $\bar{X}$. However, the potential is exactly discrete periodic, in such case one cannot get super-Planckian excursion within one period unless the PNGB decay constant is super-Planckian. In our model, the inflation path is helical, there is no limit on the field displacement during inflation. This is similar to the so-called monodromy, which is proposed as stringy axion inflation in a rather different way [20].

III. POTENTIAL DEFORMATIONS

We have shown the simplest case of helicoid potential. The norm of $\Phi$ is stabilized at $r = 1$, the marginal point of the Planck scale. Now we deform the model and study variation of the potential.

Firstly we consider the higher order correction on the Kähler potential

$$K = \Phi \bar{\Phi} + b(\Phi \bar{\Phi})^2 + X \bar{X} - g(X \bar{X})^2, \quad (7)$$

correspondingly, we need to add a parameter $\Lambda$ in the superpotential

$$W = a X \Phi \ln \frac{\Phi}{\Lambda}. \quad (8)$$

Using the same argument it is easy to see the scalar potential reduces to

$$V = a^2 r^{2+2b} \left( \frac{1}{r^2} \left( \ln r - \ln \Lambda \right)^2 + \theta^2 \right). \quad (9)$$

The factor $e^{2r^{2+2b} \frac{1}{r^2}}$ reaches its minimum at $r_0^2 = \frac{2}{1+\sqrt{1+8b}}$, below $M_P$ for $b > 0$. To get a Minkowski vacuum we can take $\Lambda = r_0$.

Keeping the same Kähler potential in Section II, we can also modify the superpotential as follows

$$W = a X \Phi^{-\frac{1}{n}} \ln \frac{\Phi}{\Lambda}. \quad (10)$$

The scalar potential becomes

$$V = a^2 r^{2-\frac{2}{n}} \left( \ln r - \ln \Lambda \right)^2 + \theta^2. \quad (11)$$

The minimum of the factor $e^{2r^{2-\frac{2}{n}}}$ locates at $r_0 = \frac{1}{\sqrt{n}} (= \Lambda)$. The mass along the radial direction at $r_0$ is

$$m_r^2 = (2 + \frac{n}{\theta^2}) V_I > H^2, \quad (12)$$

where $V_I = (cn)^{1/n} a^2 \theta^2$, providing a strong stabilization even $r$ is very small. We can easily get $r_0 \sim O(10^{-1})(M_P)$ giving $n \geq 10$.

A more general deformation of the superpotential is given by

$$W = a X \Phi^{-k} f(\ln \frac{\Phi}{\Lambda}). \quad (13)$$

One can adopt a general function $f$ consistent with stabilization in radial direction to get the inflaton potential $f(i \theta) \bar{f}(-i \theta)$. All of these inflationary models have helicoid potential generating helical phase inflation, while the paths have different flatness conditions therefore give different predictions. Details of the helicoid-type potentials will be presented in our future works.

IV. HIGHER DIMENSIONAL OPERATORS

The crucial challenge for large field inflation is the higher dimensional operators from quantum gravity corrections [10]. The higher order terms of the inflaton $\phi$

$$\Delta V = c_i \left( \frac{\phi}{M_P} \right)^{4+i} + \cdots, \quad (14)$$

are unignorable at the initial stage of inflation when $\phi \sim O(10) M_P$. They can modify the predictions significantly or even destroy slow-roll conditions. To avoid these dangerous corrections one can apply axionic shift symmetry of the inflaton $\phi \rightarrow \phi + c$, which is broken down to discrete symmetry $\phi \rightarrow \phi + 2\pi f$ by non-perturbative effect. To fit the experimental observations it requires super-Planckian axion decay constant $f$, which can be realized by aligned axions [12] (or equally a $S_2$ symmetry between two Kähler moduli [21]) or anomalous gauged $U(1)_X$ with large gauge symmetry [22]. In this model, the inflaton is just the phase of a complex field and admits no polynomial correction at all, in consequence quantum gravity corrections like [13] immediately disappear without any constraint from extra-symmetry.

Some other possible higher dimensional corrections are from

$$\Delta V = c_i \left( \frac{\Phi}{M_P} \right)^{4+i} + h.c. + \cdots, \quad (15)$$

which give corrections proportional to $\cos(n \theta)$ or $\sin(n \theta)$. Similar terms also appear in the axion potentials, taking [23] for example. Here in our model, these terms are strongly suppressed by $(\frac{\phi}{M_P})^n \sim O(10^{-n})$ with $r \sim O(10^{-1}) M_P$. To summarize, by taking the phase inflation along the helix path, we can safely keep away from the quantum gravity corrections and the chaos in this region.
We have shown in this letter that the phase inflation along a single helix trajectory can be realized in a surprisingly simple way based on minimal supergravity. The global $U(1)$ symmetry of minimal Kähler potential naturally solves the η problem which appears generically for supergravity inflation. The radial direction is strongly stabilized during inflation, and the super-Planckian phase excursion is fulfilled along an helix path. Different from PNGB or axion inflation, whose potential is discrete periodic and requires super-Planckian decay constant, the helical structure directly provides the unlimited field displacement.

The helical phase inflation is free from quantum gravity corrections. Firstly, the inflaton as a phase of complex field, it has no polynomial higher order corrections. Besides, the radius of the helix trajectory can be strongly stabilized at sub-Planckian region therefore the higher order periodic corrections are effectively suppressed.

The helicoid potential can also be deformed into various shapes leading to inflations with different kinds of potentials, and providing a method to evade UV problems. Our helical phase Inflation model is rather simple and have several amazing features. However, because inflation is an extraordinary unusual and unique event in the history of our Universe, we are not hesitant in being bold. It will be phenomenal if nature employed helix structures to promote evolution from the very early universe to present time organisms.

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