Domain Walls from Anti-de Sitter Spacetime

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ABSTRACT

We examine \((D - 2)\)-brane solutions in supergravities, showing that they fall into four categories depending on the details of the dilaton coupling. In general they describe domain walls, although in one of the four categories the metric describes anti-de Sitter spacetime. We study this case, and its \(S^1\) dimensional reduction to a more conventional domain wall in detail, focussing in particular on the manner in which the unbroken supersymmetry of the anti-de Sitter solution is partially broken by the dimensional reduction to the domain wall.

Research supported in part by DOE Grant DE-FG05-91-ER40633 and EC Human Capital and Mobility Programme under contract ERBCHBGCT920176.
1 Introduction

The past few years have witnessed an exhaustive study of $p$-brane solutions of $D$-dimensional supergravity theories, motivated by their increasing importance in the non-perturbative dynamics of superstring theories, or M-theory, compactified to $D$ dimensions. A special class of these solutions are the $(D-2)$-branes, which can be viewed as domain walls. $(D-2)$-branes exhibit some features that are significantly different from those of more generic $p$-brane solutions and their properties are consequently much less well known, with the exception of domain walls in $D = 4$, $N = 1$ supergravity which have been the subject of extensive investigations (see [1] for a recent review of this work and further references). Rather than the usual situation where a propagating antisymmetric tensor field carries a magnetic or electric charge that supports the $p$-brane, $(D-2)$-branes are supported either by a cosmological-type term in the Lagrangian, or else, in the dualised version, by a non-propagating $D$-form field strength. The necessary terms arise in massive supergravity theories and in gauged supergravities. A case that has recently attracted attention is the massive type IIA supergravity in $D = 10$ [2], which admits an 8-brane solution [3]. Other examples, involving massive gauged supergravities in $D = 7$ and $D = 6$, have been discussed in [4]. In all these cases, the domain wall solutions preserve half the supersymmetry. In this paper, we shall discuss the general structure of $(D-2)$-branes preserving half the supersymmetry of the maximal supergravity in general dimension $D$. As we shall see, they fall naturally into four categories according to the type of supergravity theory.

One category, let us call it the first, arises from gauged supergravity theories. We shall have little to say about this case here. The second category arises from so called ‘massive’ supergravity theories. Numerous examples are provided by ‘generalized’ dimensional reduction of $D = 11$ supergravity but an earlier example of current interest, which cannot be obtained in this way, is the massive IIA supergravity in $D = 10$, which admits an 8-brane solution. It was shown in [5] that the massive type IIA supergravity can be dimensionally reduced to a massive $N = 2$ supergravity in $D = 9$; the 8-brane solution in $D = 10$ can then be reduced to a 7-brane solution of this $D = 9$ theory. At first sight the reduction of the massive $D = 10$ theory to $D = 9$ might seem surprising since it is often said that Kaluza-Klein dimensional reduction is possible only if the higher-dimensional theory admits an $S^1 \times M_{D-1}$ direct-product vacuum solution, which the massive IIA theory does not. However, it was argued in [6] that it was sufficient that the 8-brane solution has a translational invariance, which, after compactification, corresponds to a $U(1)$ invariance of the solution. In fact, as we shall discuss in section 3, neither this, nor indeed any other
solution, plays any role in the dimensional reduction of the theory.

The observation that an $S^1 \times M_{D-1}$ direct-product vacuum is unnecessary for Kaluza-Klein reduction plays a crucial role in understanding the third category of $(D - 2)$-branes, which provide the main focus of this paper. These can be understood as arising from dimensional reduction of anti-de Sitter (AdS) space in $(D + 1)$ dimensions. Anti-de Sitter space itself can be viewed, in horospherical coordinates, as a special case of a $(D - 1)$-brane in $(D + 1)$ dimension, i.e. a domain wall, in which the manifest $D$-dimensional Poincaré isometry is ‘accidently’ enlarged. This can be double-dimensionally reduced, just like any other $p$-brane, to give a domain wall solution in $D$ dimensions. There is a significant difference, however. Just as AdS space has an ‘accidently’ enlarged bosonic symmetry it also has, in the supergravity context, an ‘accidently’ enlarged supersymmetry since, unlike typical $p$-brane solutions which break half the supersymmetries, the AdS solution breaks none. This is puzzling in view of the fact that the domain wall solution in the lower dimension certainly breaks half the supersymmetry of the dimensionally reduced supergravity theory. The resolution of this puzzle is that only half the Killing spinors in the AdS spacetime are independent of the compactified coordinate, and hence the other half are lost in the dimensional reduction. We demonstrate this by giving an explicit construction of all the Killing spinors in the $D$-dimensional AdS spacetime, in the natural (horospherical) coordinate system that arises in its construction as a $(D - 2)$-brane.

In dimensionally reducing a $p$-brane solution in $(D + 1)$ dimensions to one in $D$ dimensions one must first periodically identify one of the $p$-brane coordinates. From what we have just said, it is clear that this can be done for $(D + 1)$-dimensional AdS space too. If this is done, the periodic coordinate can be re-interpreted as the azimuthal angle of a circularly-symmetric domain wall solution in $(D + 1)$-dimensions. In particular, for $D = 2$, i.e. $(D + 1) = 3$, we have a particle-like solution of the Einstein equations with a cosmological constant that is locally isometric to AdS$_3$ (as are all solutions in $2 + 1$ dimensions) and which preserves at least half the supersymmetry. In fact, the solution preserves just half the supersymmetry because the other half is broken by the identification; this solution is just the supersymmetric ‘black hole vacuum’ discussed in [8] and exhibited in horospherical coordinates in [7]. Thus, the periodic identification of anti-de Sitter space as described above leads to a straightforward generalization to arbitrary dimension of the $2 + 1$ dimensional ‘black hole vacuum’. An interesting question, not adressed here, is whether there is also an analogue in arbitrary dimension of the extreme spinning black hole of $2 + 1$ gravity [8], which is also locally isometric to anti-de Sitter, and is supersymmetric.
In the following section we shall introduce the four categories of \((D-2)\) branes and give some of their properties, concentrating on the p-brane interpretation of anti-de Sitter space. We then show how \((D-2)\)-brane solutions in D-dimensions are obtained by dimensional reduction of AdS in \((D+1)\) dimensions and elucidate the supersymmetry of these solutions.

2 Solitonic \((D-2)\)-branes

In general, there exist p-brane solutions in supergravity theories that involve the metric tensor, a dilaton field, and an antisymmetric tensor field strength of rank \(n\). If the field strength carries an electric charge, the corresponding elementary p-brane has \(p = n - 2\), whereas if the charge is magnetic, the p-brane is solitonic with \(p = D - n - 2\), where \(D\) is the spacetime dimension of the theory. Included in the general solitonic cases is the degenerate case where \(n = 0\), which implies that there is no field strength at all, but instead a cosmological-type term in the D-dimensional effective Lagrangian:

\[
\mathcal{L} = eR - \frac{1}{2}e(\partial\phi)^2 + 2e\Lambda e^{-a\phi} .
\]

It is useful to parameterise the constant \(a\) in terms of \(\Delta\), defined by

\[
a^2 = \Delta + \frac{2(D-1)}{D-2} .
\]

The equations of motion admit solitonic \((D-2)\)-brane solutions, with metric and dilaton given by

\[
d s^2 = H^\Delta \frac{1}{4} \eta_{\mu\nu} d x^\mu d x^\nu + H^{\frac{4(D-1)}{2(D-2)}} d y^2 ,
\]

\[
e^{\phi} = H^{\frac{2a}{\Delta}}
\]

where \(H\) is a harmonic function on the 1-dimensional transverse space, of the form \(H \sim c \pm my\), where \(c\) is an arbitrary integration constant and \(m = \sqrt{-\Lambda \Delta}\). The arbitrariness of the sign in \(H\) arises because the equations of motion involve \(m\) quadratically. (The solutions in \(D = 4\) were also obtained in [1].) In section 4, we shall discuss the elementary \((D-2)\)-brane using a D-form field strength.

A metric of the form \(d s^2 = H^{2\alpha} \eta_{\mu\nu} d x^\mu d x^\nu + H^{2\beta} d y^2\), in the vielbein basis \(e^\mu = H^\alpha d x^\mu\), \(e^y = H^\beta d y\), has a curvature 2-form given by

\[
\Theta^{\mu\nu} = -\alpha^2 H^{-2\beta-2} H^{2\alpha} e^\mu \wedge e^\nu ,
\]

\[
\Theta^{\mu\nu} = (\alpha(\beta - \alpha + 1)H^{-2\beta-2} H^{\alpha - 2\beta - 1} H'' e^\mu \wedge e^\nu ,
\]

\[
\Theta^{\mu\nu} = (\alpha(\beta - \alpha + 1)H^{-2\beta-2} H^{\alpha - 2\beta - 1} H'' e^\mu \wedge e^\nu ,
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\]
where \((\mu, y)\) denote tangent-space components. Thus we can see that the solitonic \((D - 2)\)-brane solutions \(\mathcal{B}\) fall into three different categories, depending on the value of \(\Delta\), according to whether

\[
\beta + 1 = \frac{2(D - 1)}{\Delta(D - 2)} + 1
\]

is positive, negative, or zero, which determines whether the curvature diverges at \(H = 0\), \(H = \infty\), or nowhere, respectively.\(^1\) As we shall discuss in more detail presently, in this latter case, namely \(\Delta = \Delta_{\text{AdS}}\), where

\[
\Delta_{\text{AdS}} = -\frac{2(D - 1)}{D - 2},
\]

corresponding to \(a = 0\), the metric \(\mathcal{B}\) describes anti-de Sitter spacetime. If \(\mathcal{B}\) is positive, which corresponds to \(\Delta > 0\), the singularity that would occur at \(H = 0\) can be avoided by taking \(H\) to be

\[
H = 1 + m|y|.
\]

In this case, the metric is free of curvature singularities except for a delta-function in the curvature at \(y = 0\), as can be seen from \(\mathcal{B}\). The metric is asymptotically flat as \(y \to \pm \infty\), in which regions the dilaton diverges. The solution describes a domain wall across the spacetime, located at \(y = 0\). If the constant 1 were omitted in \(\mathcal{B}\), the curvature would diverge as some inverse power of \(y\) at \(y = 0\), where the domain wall is located. The third category of solution arises when \(\mathcal{B}\) is negative, which corresponds to \(\Delta_{\text{AdS}} < \Delta < 0\). In this case, the singularity at \(H = \infty\) cannot be avoided by any choice of the constant of integration. The choice \(\mathcal{B}\) for \(H\) ensures that the dilaton remains real, and finite for finite \(y\); the metric then describes a domain wall at \(y = 0\), embedded in a spacetime whose curvature and dilaton diverge as \(y \to \pm \infty\). On the other hand if the constant term in \(\mathcal{B}\) is omitted, the region \(y \to 0\) is asymptotically flat, but now the dilaton diverges at \(y = 0\) as well as when \(y \to \pm \infty\).

All three categories of \((D - 2)\)-brane solutions can arise in supergravity theories. The case \(\Delta = \Delta_{\text{AdS}}\) occurs, for example, in certain vacua of gauged supergravities, such as arise from the \(S^7\) \([11]\) or \(S^4\) \([12]\) compactifications of \(D = 11\) supergravity, or the \(S^5\) \([13]\) compactification of \(D = 10\) type IIB supergravity. Examples with positive \(\Delta\) include the massive type IIA theory in \(D = 10\) \([2]\), and all its dimensional reductions, which all have \(\Delta = 4\). On the other hand, the cosmological-type terms associated with the gauging of

\(^{1}\text{There is also a fourth category, where }\Delta = 0\text{, for which the solutions would be transcendental. }\Delta = 0\text{ seems never to occur in supergravity theories, and we shall not consider this fourth category further.}\)
the $D = 7$ [4] and $D = 6$ [5] supergravities discussed in [4] have $\Delta = -2$, satisfying the inequalities $\Delta_{\text{AdS}} < \Delta < 0$ for the third category of $(D - 2)$-branes described above.

Let us now consider the special case $\Delta = \Delta_{\text{AdS}}$ (i.e. $a = 0$) in more detail. The metric (3) becomes

$$ds^2 = H^{-\frac{2}{D-2}} \eta_{\mu\nu} dx^\mu dx^\nu + H^{-2} dy^2,$$

where as in all the cases discussed above, $H$ has the general form $H \sim c \pm my$. The coordinate transformation

$$H = e^{-mr},$$

puts the metric into the form

$$ds^2 = e^{2\lambda r} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2,$$

where

$$\lambda = \sqrt{\frac{2\Lambda}{(D-1)(D-2)} = (D-1)m}.$$  

This is in fact the metric of anti-de Sitter spacetime, in horospherical coordinates [16]. This can be seen by introducing the $(D + 1)$ coordinates $(X, Y, Z^\mu)$ defined by

$$X = \frac{1}{\lambda} \cosh \lambda r + \frac{1}{2} \lambda \eta_{\mu\nu} x^\mu x^\nu e^{\lambda r},$$

$$Y = \frac{1}{\lambda} \sinh \lambda r - \frac{1}{2} \lambda \eta_{\mu\nu} x^\mu x^\nu e^{\lambda r},$$

$$Z^\mu = x^\mu e^{\lambda r}.$$  

They satisfy

$$\eta_{\mu\nu} Z^\mu Z^\nu + Y^2 - X^2 = -1/\lambda^2,$$

$$\eta_{\mu\nu} dZ^\mu dZ^\nu + dY^2 - dX^2 = e^{2\lambda r} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2,$$

which shows that (11) is the induced metric on the hyperboloid (14) embedded in a flat $(D + 1)$-dimensional spacetime with $(-, +, +, \cdots, +, -)$ signature. Thus (11) has an $SO(2, D-1)$ isometry, and it is a metric on $D$-dimensional anti-de Sitter spacetime.

The global structure of the AdS metric is of course well known. The structure for the metric (11) in horospherical coordinates was discussed in the case of $D = 4$ dimensions in [16]; the situation here for arbitrary $D$ is similar. It is evident from (13) that $X+Y = \lambda^{-1} e^{\lambda r}$ is non-negative if $r$ is real, and hence the region $X+Y < 0$ in the full anti-de Sitter spacetime is not covered by the horospherical coordinates. In fact, the coordinates used in (11) cover one half of the complete anti-de Sitter spacetime, and the metric describes AdS$_D/J$ where $J$ is the antipodal involution $(X, Y, Z^\mu) \to (-X, -Y, -Z^\mu)$ [16]. If $D$ is even, we can extend
the metric (9) to cover all of anti-de Sitter spacetime by taking $H = my$, since now the region with $y < 0$ corresponds to the previously inaccessible region $X + Y < 0$. On the other hand if $D$ is odd, we must restrict $H$ in (9) to be non-negative in order to have a real metric, and thus in this case we should choose $H = c + m|y|$, with $c \geq 0$. If the constant $c$ is zero, the metric describes AdS$_D/J$, while if $c$ is positive, the metric describes a smaller portion of the complete anti-de Sitter spacetime. In any dimension, if we have $H = c + m|y|$ (by choice if $D$ is even, or by necessity if $D$ is odd), the solution can be interpreted as a domain wall at $y = 0$ dividing two portions of anti-de Sitter spacetimes, with a delta function curvature singularity at $y = 0$ if the constant $c$ is positive.

3 Domain walls from AdS spacetime

3.1 Dimensional reduction with a cosmological term

In general, a $p$-brane solution in $D$ dimensions can be dimensionally reduced to a $(p - 1)$-brane solution in $(D - 1)$ dimensions, by using the Kaluza-Klein procedure. In the case of a $(D - 2)$-brane, this might seem problematical at first sight, since a direct product metric on $S^1 \times M_{D-1}$ is not a solution of the equations of motion following from (1) (at least for finite $\phi$). However, it was shown in [5] that the dimensional reduction of massive type IIA supergravity in $D = 10$ [2] to massive $N = 2$ supergravity in $D = 9$ is nevertheless possible. In this case, the 8-brane of the $D = 10$ theory [3] reduces to a 7-brane of the $D = 9$ theory. The ability to perform the dimensional reduction was attributed in [5] to the existence of the 8-brane solution, which has a $U(1)$ isometry. In fact even the existence of this solution with a $U(1)$ isometry is inessential for the Kaluza-Klein reduction of the theory, since dimensional reduction is a procedure that is applied at the level of the Lagrangian, without making reference to any particular solution. A consistent Kaluza-Klein dimensional reduction can always be performed on any Lagrangian, simply by restricting the higher-dimensional fields to be independent of the chosen compactification coordinate. The solutions of the resulting lower-dimensional theory will be in one-to-one correspondence with solutions of the higher-dimensional theory that admit a translational isometry. Knowing of the existence of a particular such solution of the higher-dimensional theory provides an assurance that the lower-dimensional theory is not empty.

We shall give a more striking illustration of the fact that consistent Kaluza-Klein reduction is possible in the absence of an $S^1 \times M_{D-1}$ vacuum solution by considering the dimensional reduction of the Lagrangian (1) in the case when $a = 0$, showing that the $S^1$
The reduction of Einstein gravity even with a pure cosmological constant is also possible. Let us begin with a Lagrangian of the form \( \mathcal{L} = e^{2\alpha \varphi} \), with \( \varphi = 0 \),
\[
\mathcal{L} = e^R + 2e\Lambda.
\]
(16)

The standard Kaluza-Klein ansatz for the metric is
\[
ds^2 = e^{2\alpha \varphi} \tilde{ds}^2 + e^{-2(D-3)\alpha \varphi} (dz + \mathcal{A})^2,
\]
where \( \tilde{ds}^2 \) denotes the \((D-1)\)-dimensional metric, and \( \alpha^2 = (2(D-2)(D-3))^{-1} \). The tangent-space components of the \(D\)-dimensional Ricci tensor \( \mathcal{R}_{\tilde{A}\tilde{B}} \) are given by
\[
\mathcal{R}_{\tilde{A}\tilde{B}} = e^{-2\alpha \varphi} \left( \tilde{R}_{\tilde{A}\tilde{B}} - \frac{1}{2} \partial_{\tilde{A}} \varphi \partial_{\tilde{B}} \varphi - \alpha \tilde{\nabla} \eta_{\tilde{A}\tilde{B}} \right) - \frac{1}{2} e^{-2(D-1)\alpha \varphi} \mathcal{F}_{\tilde{A}}^{\tilde{B}} \mathcal{F}_{\tilde{C}B} \, ,
\]
(18)

\[
\tilde{R}_{\tilde{A}\tilde{B}} = \frac{1}{2} e^{(D-4)\alpha \varphi} \tilde{\nabla} \tilde{\nabla} \left( e^{-2(D-2)\alpha \varphi} \mathcal{F}_{\tilde{A}\tilde{B}} \right) \, ,
\]
(19)

\[
\tilde{\nabla} \eta_{\tilde{A}\tilde{B}} - \frac{1}{2} \mathcal{F}_{\tilde{A}\tilde{B}} - \frac{1}{2} \eta_{\tilde{A}\tilde{B}} \tilde{\nabla} \varphi + \frac{1}{4} e^{-2(D-1)\alpha \varphi} \mathcal{F}^2 = 0 \, ,
\]
and thus the equations of motion for the \((D-1)\)-dimensional fields, obtained by substituting into the \(D\)-dimensional equations of motion \( \mathcal{R}_{\tilde{A}\tilde{B}} - \frac{1}{2} \eta_{\tilde{A}\tilde{B}} = \Lambda \eta_{\tilde{A}\tilde{B}} \) are (after converting to world indices)
\[
\tilde{R}_{\tilde{M}\tilde{N}} - \frac{1}{2} \tilde{\nabla} \tilde{\nabla} \varphi + \frac{1}{4} e^{-2(D-2)\alpha \varphi} \mathcal{F}^2 \tilde{g}_{\tilde{M}\tilde{N}} = \Lambda \tilde{g}_{\tilde{M}\tilde{N}} e^{2\alpha \varphi} = 0 \, ,
\]
(19)

\[
\tilde{\nabla} \varphi + \frac{1}{2} (D-2)\alpha e^{-2(D-2)\alpha \varphi} \mathcal{F}^2 + 4\alpha \Lambda e^{2\alpha \varphi} = 0 \, .
\]

On the other hand, substituting the Kaluza-Klein ansatz into the \(D\)-dimensional Lagrangian \((16)\) gives the \((D-1)\)-dimensional Lagrangian
\[
\mathcal{L} = \tilde{e} \tilde{R} - \frac{1}{2} \tilde{e} \left( \partial \varphi \right)^2 - \frac{1}{4} \tilde{e} e^{-2(D-2)\alpha \varphi} \mathcal{F}^2 + 2\tilde{e} \Lambda e^{2\alpha \varphi} \, ,
\]
(20)

whose equations of motion give precisely \((19)\). That the terms independent of \(\Lambda\) should agree is of course unremarkable, since these correspond just to the dimensional reduction of gravity without a cosmological constant. It is not always appreciated, however, that the reduction with the cosmological constant, where there is no \(S^1 \times M\) vacuum solution, is also consistent.

The \((D-2)\)-brane solution of the \(D\)-dimensional theory \((16)\) is, as we saw earlier, anti-de Sitter spacetime, which has translational isometries in the world-volume directions. Thus we can perform the above Kaluza-Klein dimensional reduction with the extra coordinate
z taken to be one of the spatial world-volume coordinates $x^i$. The resulting $(D - 1)$-dimensional metric is nothing but the metric (3), describing a domain wall, with the same value of $\Delta$ as in $D$ dimensions, corresponding to the third category described in section 2, where (3) is negative. This preservation of $\Delta$ under dimensional reduction is also observed for the usual $p$-brane solutions [5]. In terms of the dilaton coupling constant $a$, we have gone from a theory with $a = 0$ in $D$ dimensions to one with $a = -2\alpha$ in $(D - 1)$ dimensions. Note that the singular curvature of the lower-dimensional domain-wall solution arises from the dimensional reduction, since the anti-de Sitter spacetime in the higher dimension is free of singularities. A similar phenomenon was discussed in [17] for the usual kinds of $p$-branes.

### 3.2 Supersymmetry

A curious feature of the above dimensional reduction emerges if we consider it in the context of supergravity. Thus let us consider the case of a $D$-dimensional gauged supergravity theory with a scalar potential which, for some suitable restriction of the fields, gives a bosonic Lagrangian of the form (10). Upon Kaluza-Klein reduction, we expect that the theory will yield a supergravity theory in $(D - 1)$ dimensions with the same number of components of unbroken supersymmetry. However, the anti-de Sitter solution in $D$ dimensions, which preserves all the supersymmetry, reduces to the domain-wall solution in $(D - 1)$ dimensions, which preserves only half of the supersymmetry. To understand where the other half of the supersymmetry is lost, we need to look at the detailed forms of the Killing spinors in the anti-de Sitter spacetime.

Let us first calculate the spin connection for the metric (11). We begin by choosing the vielbein basis $E^\mu = e^{\lambda r} dx^\mu$, $E^r = dr$ (we are using a capital $E$ to denote the vielbein, to avoid confusion with the exponential function). It follows that the spin connection is given by

$$\omega^\mu_{\nu r} = \lambda E^\mu, \quad \omega^\mu_{\nu 0} = 0,$$

and the curvature 2-form is

$$\Theta^{AB} = -\lambda^2 E^A \wedge E^B.$$  

Here, we are denoting tangent-space indices by $\lambda = (\mu, r)$, etc. Thus we see from (22) that the Riemann tensor has the maximally-symmetric form $R_{ABCD} = -\lambda^2 (\eta_{AC} \eta_{BD} - \eta_{AD} \eta_{BC})$, as one expects since the metric (11) describes anti-de Sitter spacetime in $D$ dimensions, with Ricci tensor $R_{AB} = -\lambda^2 (D - 1) \eta_{AB}$.

The Killing spinor equations for a supergravity theory with cosmological constant $\Lambda$
take the form
\[ \delta \psi_M = D_M \epsilon - \frac{1}{2} \lambda \Gamma_M \epsilon = 0 \, . \]  
(23)

The constant \( \lambda \) is the same one that we introduced previously, which is related to \( \Lambda \) by (12). Substituting the spin connection (21) for the anti-de Sitter metric (11), we obtain the equations
\[ \partial_\mu \epsilon = \frac{1}{2} \lambda e^{\lambda r} \Gamma_\mu (1 + \Gamma_r) \epsilon \, , \quad \partial_r \epsilon = \frac{1}{2} \lambda \Gamma_r \epsilon \, , \]
where \( \Gamma_\mu \) and \( \Gamma_r \) are understood to be the tangent-space components of the \( \Gamma \) matrices. We find that the solutions for the Killing spinors \( \epsilon \) are of two kinds, namely
\[ \epsilon = \epsilon_+ = e^{\frac{1}{2} \lambda r} \epsilon_+ \, , \]
(25)
\[ \epsilon = (e^{-\frac{1}{2} \lambda r} + \lambda e^{\frac{1}{2} \lambda r} x^\mu \Gamma_\mu) \epsilon_- \, , \]
(26)
where \( \epsilon_\pm \) are arbitrary constant spinors satisfying
\[ \Gamma_r \epsilon_\pm = \pm \epsilon_\pm \, . \]
(27)

Thus in total, in \( D \)-dimensional anti-de Sitter spacetime, we have \( 2^{[D/2]} \) independent Killing spinors. Half of these, constructed using \( \epsilon_+ \), are independent of the coordinates \( x^\mu \), while the other half, constructed using \( \epsilon_- \), depend on \( x^\mu \). Note that this \( x^\mu \) dependence is not periodic in nature. It is interesting to note that all the Killing spinors can be written in the single unified expression
\[ \epsilon = e^{\frac{1}{2} \lambda r} \Gamma_r \left( 1 + \frac{1}{2} \lambda x^\mu \Gamma_\mu (1 - \Gamma_r) \right) \epsilon_0 \, , \]
(28)
where \( \epsilon_0 \) is an arbitrary constant spinor. The Killing spinors of 4-dimensional anti-de Sitter spacetime were found in a different coordinate system in [18].

The explanation for the loss of half the Killing spinors upon dimensional reduction of the \( D \)-dimensional anti-de Sitter metric (11) to the \( (D - 1) \)-dimensional domain-wall metric is now clear: The fields in \( (D - 1) \) dimensions that are retained in the Kaluza-Klein dimensional reduction are those that are independent of the extra coordinate of the \( D \)-dimensional spacetime. In our case, we are taking this to be one of the spatial world-volume coordinates \( x^i \), since these are Killing directions in the \( D \)-dimensional spacetime. Indeed the Killing spinors (25) are also independent of the coordinates \( x^i \), and so these survive the reduction to \( (D - 1) \) dimensions. However, the other half of the Killing spinors, given by (28), depend on the \( x^i \) coordinates, and thus will not survive the reduction. In fact these Killing spinors depend linearly on the \( x^i \) coordinates, and hence they would not
survive even the periodic identification of the compactification coordinate, let alone the truncation to the zero-mode sector.

This type of partial loss of supersymmetry at the level of solutions can also be seen in the context of the usual $p$-branes. Note that a $p$-brane solution where the dilaton is regular on the horizon (corresponding to cases where $a = 0$, or its multi-charge generalisations) interpolates between $D$-dimensional Minkowski spacetime at infinity, and $\text{AdS}_{p+1} \times S^{D-p-1}$ on the horizon.\(^2\) (This phenomenon has been observed in the case of the $D = 11$ membrane\(^1\) and in other cases\(^2\).) In fact in these cases $\text{AdS}_{p+1} \times S^{D-p-1}$ is also a solution of the theory, which preserves all the supersymmetry (as, for example, in the case of the $\text{AdS}_4 \times S^7$ solution of $D = 11$ supergravity). However Kaluza-Klein dimensional reduction of this solution, where the compactified coordinate is one of the $x^i$ coordinates of the anti-de Sitter metric\(^3\), gives rise to a domain-wall type solution in $(D-1)$-dimensions, which preserves only half of the supersymmetry. In fact, this can be viewed as a higher-dimensional interpretation of the example we discussed previously, where the supergravity theory with cosmological constant is now itself viewed as coming from a yet higher dimensional supergravity compactified on an appropriate sphere.

4 Elementary $(D-2)$-branes

The $(D-2)$-brane solutions that we have been discussing so far have been solitonic, in the sense that they can be viewed as the degenerate $n = 0$ limit of $(D-n-2)$-branes using an $n$-form field strength with a magnetic charge. In general, one can consider an alternative formulation of the theory in which the $n$-form field strength is dualised, to give a $(D-n)$-form field strength. The rôles of elementary and solitonic $p$-branes are interchanged under this dualisation. In the present context, therefore, we may consider an alternative formulation of the theory with a cosmological-type term, in which the Lagrangian\(^4\) is replaced by

$$\mathcal{L} = eR - \frac{1}{2}e(\partial\phi)^2 - \frac{1}{2D!}e\,e^{a\phi}F^2,$$

where $F$ is a $D$-form field strength. The equations of motion that follow from this are essentially equivalent to those following from \((\mathcal{I})\), except that now the constant $\Lambda$ has the interpretation of an integration constant arising in the solution of the field equation for $F$. This has two important consequences. Firstly, $\Lambda$ can now be zero, allowing Minkowski

\(^2\)It is interesting to note that the ideal-gas entropy/temperature relation $S \sim T^p$, which is satisfied by a non-dilatonic near-extremal $p$-brane\(^5\) or a regular-dilaton near-extremal $p$-brane\(^6\), is related to the fact that the horizon of the $p$-brane is described by $\text{AdS}_{p+1} \times S^{D-p-1}$.\(^1\)
spacetime as a solution of the theory. Secondly, one can allow \( \Lambda \) to be only locally constant, taking different values in different regions of the spacetime. This permits more general classes of domain-wall solutions, including configurations that describe multiple walls in the spacetime \[5\]. Thus we may obtain solutions again given by (3), but where now \( H \) can take the general form

\[
H = c + by + \sum_{i=1}^{N} m_i |y - y_i|.
\]  

These solutions describe \( N \) domain walls located at the positions \( y = y_i \). In the case \( \Delta > 0 \), the curvature singularity that would occur at \( H = 0 \) can be avoided by requiring, for example, \( b = 0 \) and the constants \( c \) and \( m_i \) to be all positive. If instead \( \Delta_{\text{AdS}} < \Delta < 0 \), the curvature singularity at \( H = \infty \), which was previously unavoidable in the solitonic formulation in section 2, can now be avoided by requiring \( b = 0 \) and that the constants \( m_i \) in (30) satisfy \( \sum_i m_i = 0 \). Finally, if \( \Delta = \Delta_{\text{AdS}} \) the solutions with \( H \) given by (30) describe portions of anti-de Sitter spacetimes with different cosmological constants that are joined together at domain walls, as described for \( D = 4 \) in \[23, 23, 24\].

It should be remarked that the dualisation of the Lagrangian (1) to give (29) is a subsector of the analogous process of dualisation of an entire supergravity theory. This has been carried out in some detail in the case of the massive type IIA supergravity in \( D = 10 \) \[5\]. It is also worth remarking that in the dualised form, the Kaluza-Klein reduction of the theory looks more conventional, since it now admits a vacuum solution that is the direct product of \( S^1 \) and a Minkowski spacetime \[5\].

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