Giant vortex and Skyrmion in a rotating two-species Bose-Einstein condensate

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Numerical simulations are performed for a rotating two-species Bose condensate confined by a harmonic potential. The particle numbers of each species are unequal. When the rotational speed exceeds a critical value, the majority species reside in the center of the potential while the minority species is pushed out to the outskirts, forming a giant vortex hole to contain the majority species. A novel annular Skyrmion forms at the interface of the two species.

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INTRODUCTION

Quantum coherence enables intriguing phenomena in Bose-Einstein condensates (BEC) as quantized vorticity, which has been verified in many experiments. When a trapped BEC is driven to rotate, singly quantized vortices form. Faster rotation generates more vortices which finally condense into a lattice. Other methods, such as the phase imprint technique, are also used to create vortex in BECs. D.R. Scherer et al. [5] carried out an experiment to implement vortices by the interference of three independent trapped BECs.

There are many efforts to create a multiply quantized vortex or giant vortex in the BECs. However, a giant vortex is energetically disfavored in a harmonic trap and is not expected to persist if created in a rotating condensate. Some authors tried to overcome this dissociation instability by employing an external repulsive pinning potential [6] or a steeper trap that allows rotation at frequency exceeding the centrifugal limit of the harmonic trap [5, 7, 8]. Multiply quantized vortices are also created topologically by deploying the spin degrees of freedom in optical traps [8]. In a recent experiment, P. Engels et al. shone a near resonant laser beam through a rapidly rotating harmonically trapped BEC to produce a density hole encircled by a high number of vorticity [9, 10].

In this work, we produce a giant vortex in a harmonic trap by introducing a second species of BEC. There is a population imbalance ($N_1 > N_2$) in the two-species BEC and the inter-species scattering length is larger than the intra-species length. We consider a quasi-2D repulsive system by numerically integrating the Gross-Pitaevskii (GP) equation. S. Bargi et al. [16] have studied the same system by numerically integrating the Gross-Pitaevskii equation. S. Bargi et al. [16] have studied the same system by numerically integrating the Gross-Pitaevskii equation.

$\Psi = (\psi_1, \psi_2)^T$ is described by the following normalized equation

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{1}{2} \nabla^2 + V(\vec{r}) + \sum_{j=1,2} g_{ij} |\psi_j|^2 - \Omega \hat{L}_z |\psi_i|, \quad (1) \]

where $g_{ij}$, $(i, j = 1, 2)$ are the nonlinear coupling constants which are expressed by $g_{ij} = 4\pi a_{ij}^S$, with $a_{ij}^S$ the inter $(i \neq j)$ or intra $(i = j)$ species s-wave scattering lengths. $V(\vec{r}) = \frac{1}{2} a \sqrt{|x^2 + y^2|}$ is the harmonic trapping well. We assume that the two species of BEC are trapped by the same external potential with unequal particle numbers $N_1 > N_2$ and all the nonlinear coupling constants are positive.

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We first analyze the problem in the limit of \( N_1 \gg N_2 \). The distribution of the majority species is hardly affected by minority species. Hence we can approximate its profile with the Thomas-Fermi (TF) distribution, \( n_1(\vec{r}) = |\psi_1(\vec{r})|^2 = [\mu_1 - \frac{1}{2}(\omega^2 - \Omega^2)r^2]/g_{11} \) for \( n_1(\vec{r}) > 0 \) and \( n_1(\vec{r}) = 0 \) otherwise. \( \mu_1 \) is the chemical potential. The term associated with the rotation frequency \( \Omega \) is the centrifugal potential. We consider the case for \( \Omega < \omega \). The radius \( R_1 \) of the condensate is evaluated by finding where the density goes to zero. If \( \mu_1 > 0 \) then there is a single solution with \( R_1 = \sqrt{2\mu_1/(\omega^2 - \Omega^2)} \). The chemical potential is determined by normalization condition \( \int n_1(\vec{r})d\vec{r} = 1 \), which gives rise to \( \mu_1 = \sqrt{g_{11}(\omega^2 - \Omega^2)/\pi} \). Because of the strongly repulsive interspecies coupling, the minority is pushed out to the rim which feels an effective potential produced by the external trap combining with the repulsive hump of the majority, \( V_{eff}(\vec{r}) = V(\vec{r}) + g_{21}n_1(\vec{r}) \), which looks like a 'Mexican hat'. The minority condensate prefers to reside in annular notches of the effective potential and subsequently forms a huge hole in the central area to contain the majority condensate.

The Thomas-Fermi profile of the minority condensate is

\[ n_2(\vec{r}) = |\psi_2(\vec{r})|^2 = (\mu_2 - V_{eff}(\vec{r}) + \frac{1}{2}\Omega^2r^2)/g_{22}, \]

for \( n_2(\vec{r}) > 0 \) and \( n_2(\vec{r}) = 0 \) otherwise. The outer and inner radii of the ring-shape condensate are

\[ R_2^+ = \sqrt{\frac{2\mu_2}{(\omega^2 - \Omega^2)}} \]
\[ R_2^- = \sqrt{\frac{2(g_{21}/g_{11}\mu_1 - \mu_2)}{(g_{21}/g_{11} - 1)(\omega^2 - \Omega^2)}} \]

The corresponding chemical potential is given by

\[ \mu_2 = \mu_1 - \frac{g_{11}}{g_{21}}\sqrt{\frac{g_{22}}{\pi}}\sqrt{(\frac{g_{21}}{g_{11}} - 1)(\omega^2 - \Omega^2)}, \]

which requires \( g_{21} > g_{11} \). In this case, the two species of BEC is immiscible.

Suppose there is a vortex in the center with circulation \( \Gamma = 2\pi m \). The quantization condition requires the vorticity of integer values \( m = 0, 1, 2, \cdots \). The energy of the minority condensate in the TF approximation is expressed as,

\[ E = \int d\vec{r}[V_{eff} + \frac{m^2}{2\pi^2} + \frac{1}{2}g_{22}n_2(\vec{r})n_2(\vec{r})]. \]

In the above analysis we have omitted the contribution from singly quantized vortices which may form a vortex lattice. The energy in a frame rotating with angular velocity \( \Omega \) is related to that in the laboratory frame by \( E'(m) = E(m) - m\Omega \). For a given rotation frequency \( \Omega \), this energy is to be minimized with respect to the parameter \( m \) to obtain the vorticity of the giant vortex. By treating the parameter \( m \) as continuous variable, the final result is

\[ m = \frac{g_{22}\Omega}{2\pi\mu_2\ln(R_2^+) - \frac{g_{22}}{g_{11}}\mu_1\ln(R_2^-) + \frac{1}{2}(\omega^2 - \Omega^2)[(\frac{g_{22}}{g_{11}})^2(R_2^+)^2 - ((R_2^-)^2)^2 - ((R_2^+)^2 - (R_2^-)^2)^2]} \].

### NUMERICAL SIMULATIONS

The numerical simulations is carried out by solving the norm-preserving imaginary time propagation of Eq. [4]. The initial wavefunction adopts the Thomas-Fermi approximation. We adopt the time-splitting Fourier pseudospectral method developed by Bao et al.[17] to com-

![Figure 1: The rotational frequency dependence of total vorticity from formula [6].](image)
compute the partial differential equation \[ \Box \] in a region \((x, y)\) with a refined grid of 80 \times 80 nodes, which is sufficient to achieve grid independence. The particle number ratio is fixed at \(N_1/N_2 = 2\) and the nonlinear coupling parameters are chosen as \(g_{11} = 100, g_{22} = 550, g_{12} = 200,\) and \(g_{21} = 400.\)

Figure 2 shows the density profile of both species of condensates. As we have expected, the majority condensate resides in the central area of the harmonic well while pushes the minority condensate to the outskirts, enabling the latter to acquire more angular momentum to circulate around it. In the mean time, the majority condensate becomes more compact by the inward force implemented by the minority condensate. As the rotating speed increases, a lot of phase defects of the minority condensate enter the center area of the harmonic trap and merge into a giant vortex. Here the giant vortex implies that several phase defects are contained in a single density hole. Figure 3 exhibits that the circulating movement of the minority condensate forms a giant vortex with a winding number \(m = 8.\) In addition, a sequence of singly quantized vortex forms along the ring-shape minority condensate.

Obviously, the modulus of the total spin is \(|\vec{S}| = 1.\) The pseudo-spin texture corresponding to the giant vortex is shown in Fig. 4. It is shown that the \(\vec{S}_x\) or \(\vec{S}_y\) exhibits an \(m\)-fold symmetrical annular necklace of radius \(R\), which is clearly related to the quantized circulation of the giant vortex. As \(\vec{S}_z = 2|\chi_1|\chi_2^* \cos(\theta_1 - \theta_2),\) where \(\theta_1(\vec{r})\) are the phases of the condensate wavefunctions, and from Fig. 3 the phase of the majority condensate is approximately constant, we know that \(\vec{S}_z\) has \(m\)-fold symmetry. However, if the rotation frequency \(\Omega\) increases further, vortices are created in the majority condensate and \(\theta_1\) can not be viewed as constant, then the \(m\)-fold symmetry should be broken. Fig. 4(b) is the \(\vec{S}_z\) distribution which indicates that the pseudo-spin points up at the center and points down on the edge which results from \(\chi_2 \approx 0\) for \(r \lesssim R\) and \(\chi_1 \approx 0\) for \(r \gtrsim R.\) At the interface of the two condensates, the system twists its pseudo-spinor order parameter, as shown in the projected vectorial plot of \((\vec{S}_x, \vec{S}_y)\) in Fig. 4(c). It reveals a rather complex pseudo-spin texture which may be called a giant Skyrmion.

The pseudo-spin texture in Fig. 4 reveals a concrete \(k\)-fold symmetry. At first sight, it seems curious that \(k = m - 1\) instead of \(k = m.\) It is because the pseudo-spin contributes is an additional rotational angle \(\Delta \phi = 2\pi/k\) that relates to the spatial rotation. Taking the spins to rotating about the \(\hat{z}\) axis, the symmetry is formally described by

\[
S_i(R(2\pi/k)\vec{r}) = R_{ij}(2\pi + \Delta \phi)S_j(\vec{r}),
\]

where \(R_{ij}\) is an \(m\)-fold rotation matrix.
where

\[ R(\phi) = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \]  

(8)

represents a spatial rotation about the \( \hat{z} \) axis by an angle \( \phi \), and \( R_{ij}(\phi) \) \((i, j = x, y)\) represents a pseudo-spin rotation about the \( \hat{z} \) axis by an angle \( \phi \). The projected pseudo-spin vector increases an angle of \( 2\pi m \) when it runs along a route that encloses the origin.

The left panel of Fig. 5 shows the effective velocity field that is defined as

\[ \vec{v}_{eff}(\vec{r}) = (\vec{j}_1(\vec{r}) + \vec{j}_2(\vec{r}))/\rho(\vec{r}), \]  

(9)

with \( \vec{j}_i = \frac{1}{2i}(\psi_i^* \nabla \psi_i - \psi_i \nabla \psi_i^*) \), \((i = 1, 2)\) the partial current density. Besides a sequence of singly quantized vortices, the whole minority condensate circulates around the majority condensate with a multiply quantized circulation \( \Gamma = 8 \times 2\pi \). Contrary to the conventional vortex in a single species condensate, \( |\vec{v}_{eff}| \) vanishes at the center which makes a coreless vortex without a density dip in the total density. The largest curl of the velocity field lies on a ring of finite radius.

FIG. 5: (Color online) (a) The vectorial plot of the effective velocity \( \vec{v}_{eff} \) defined by Eq.[9]; (b) The topological charge density \( q(\vec{r}) \) distribution.

The topological charge density is defined as

\[ q(\vec{r}) = \frac{1}{8\pi} e^{ij} \vec{S} \cdot \partial_i \vec{S} \times \partial_j \vec{S}. \]  

(10)

The topological charge density \( q(\vec{r}) \) characterizes the spatial distribution of the Skyrmion. The total topological charge or the Pontryagin index \( Q \equiv \int d\vec{r} q(\vec{r}) \) is an invariant. From the right panel of Fig. 5, one finds that the singly quantized vortices have the Dirac \( \delta \)-function topological charge density. On the other hand, the giant Skyrmion has its charge uniformly distribute on a ring where the two imiscible condensates overlap.

DISCUSSIONS

The additional rotation angle \( \Delta \phi \) can be accounted for by parameterizing the wavefunction as

\[ \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} e^{i\theta_1 \cos \beta(r)}/2 \\ e^{i\theta_2 \sin \beta(r)}/2 \end{pmatrix}. \]  

(11)

To simplify the problem, we omit the chain of singly quantized vortices and focus our attention on the giant vortex. The configuration satisfies the boundary condition \( \beta(0) = 0 \) and \( \beta(\infty) = \pi \), which is referred as the Anderson-Toulouse vortex. The phase of majority condensate is constant which can be set as \( \theta_1 = 0 \). The phase of minority condensate is independent of radius \( r \) and can be approximately written as \( \theta_2 = m\phi \). Hence the spatial rotation of an angle \( 2\pi/k \) corresponding to a relative phase difference \( \Delta \theta = 2\pi m/k \).

The pseudo-spin vector \( \vec{S} \) can be expressed in the polar coordinates

\[ \vec{S} = \{ \sqrt{1 - [S_z(r)]^2 \cos(m\phi)}, \sqrt{1 - [S_z(r)]^2 \sin(m\phi)}, S_z \}. \]  

(12)

The topological charge density of Eq.[10] has the form

\[ q(\vec{r}) = \frac{m}{4\pi r} dS_z(r), \]  

(13)

which leads to the total charge of the giant Skyrmion as

\[ Q = \int d\vec{r} q(\vec{r}) = \frac{m}{2} [S_z(\infty) - S_z(0)] = m. \]  

(14)

In summary, we have created a giant vortex in a two-species Bose condensate with unequal particle numbers. The corresponding pseudo-spin texture shows a \( m \)-fold symmetry and the topological charge is \( m \).

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