Axial-tensor Meson Family at $T \neq 0$

J.Y. S"ung"u, A. T"urkan, E. Sertbakan, E. Veli Veliev

1 Department of Physics, Kocaeli University, 41001 Izmit, Turkey
2 "Ozye˘ gin University, Department of Natural and Mathematical Sciences, Çekmeköy, Istanbul, Turkey

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Abstract. The mass and decay constant of the axial-tensor meson nonets $\rho_2$, $\omega_2$ and a missing member and also their first excited states are analyzed by the Thermal QCD sum rules model including QCD condensates up to dimension five. Mass and decay constant values in terms of variations of temperature are very well stable from $T = 0$ up to $T \approx 120$ MeV. However nearly after these threshold, our numerical analysis indicate that they begin to diminish with increasing temperature. Mass value of these mesons and their first excited states decrease about $(1-13\%)$ compared to vacuum values and $(10-26\%)$ for the decay constants according to PDG data and $(9-26\%)$ and $(2-34\%)$ respectively concerning Regge Trajectory Model in the corresponding Thermal QCD sum rules calculations. The experimental results are already copious, but expected to grow up at ongoing and future heavy-ion experimental programs allowing us to compare our results.

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1 Introduction

Understanding the physics of strongly interacting matter at high temperatures is one of the major challenges of QCD [1,2,3,4]. Under extreme conditions, such as at high temperatures and/or densities quarks and gluons are believed to liberate from the hadrons to form a new state of matter, called the quark-gluon plasma (QGP). The chiral phase transition which is expected to happen in ultra-relativistic heavy-ion collisions and inside the extremely compact stars is one of the significant topics for the search of hot hadronic matter. However, a quantitative explanation of (de)confinement and restoration (or breaking) of the chiral symmetry phenomena is still lacking and hence poses a challenge for future research. The sophisticated phase structure of the QGP yields rich information on the features of strong interactions between quarks and gluons in hot medium and sheds light on the fundamental understanding for some central questions like the origin of mass of observable matter and the evolution of matter in the early universe [5].

The conditions similar to the early Universe can be recreated in the laboratory, in large-scale ultra-relativistic heavy-ion collision experiments and their results can provide us fundamental characteristics of QCD at finite temperature and is a crucial data for modeling the hot-dense matter [6]. Searching the form of the phase diagram of QCD and fixing the region of phase transition from the hadronic matter to the QGP state at high temperatures are main objectives of current and planned experimental programs at the RHIC in Brookhaven National Laboratory and future experiments at the FAIR facility in Darmstadt and NICA in Dubna [7][9].

The experimental verification of the critical temperature $T_c$ from hadronic matter to QGP would be a big grade in our understanding of QCD in hot medium and, a significant cornerstone in the further survey of the QCD phase diagram. Initially, it was assumed that above pseudocritical temperature $T_c$ which is thought that it is not a real phase transition, but an analytic crossover with a rapid change, as opposed to a jump, as the temperature increased, quarks are deconfined and chiral symmetry is restored [10][11][12]. A flavor non-singlet chiral restoration was indeed confirmed on the lattice, which is signaled by the vanishing quark condensate above the crossover region around $T_c$ and by the degeneracy of correlators that are connected by the chiral transformation [13]. The critical temperature for QGP is estimated as $T_c \approx 155$ MeV [14], analyzing the experimental data coming from heavy-ion collisions at LHC and RHIC by theoretical efforts [10] [15] though in UrQMD hybrid model it is proposed that the phase transition temperature for hot matter should be between $160 - 165$ MeV [16]. Besides there is no unique temperature estimated for the deconfinement phase transition of matter at high temperatures. Some of the Lattice theory studies predict that critical temperature to the QGP phase transition is above this temperature [17][18]. The interpretation of recent experimental results depends on the precise determination of the energy density and
pressure as well as figure out both the deconfinement and chiral transitions \[19\].

In addition to the experimental researches, theoretical efforts are required in this endeavor as well \[21\]. In the literature, there are many works about the effect of the temperature on hadronic parameters of different kinds of mesons. Among them, the light axial-tensors and their first excited states produced in heavy-ion collisions can maintain valuable information for the determination of QGP as well. The study of them can allow an important platform to understand the dynamics of QCD as the theory of strong interaction, including characteristics of color confinement. So, it is critical to understand the thermal behavior of light axial-tensor mesons especially, the dependence of dissociation degree on the temperature of the medium.

By the same token, axial-tensor mesons with quantum number \( J^{PC} = 2^{−} \) have recently become an interesting topic and attention is shifted to this subject to complete the hadron spectrum which still needs to be properly classified \[23\]. However, there is a contradiction on the ground and first excited states of axial-tensor meson nonet between the Regge Trajectory Model estimation and the data of PDG in the literature. We would like to look for which one has the best consistency with QCDSR calculations. This is our other motivation for examining the axial-tensor family whose main features are presented in Table 1 according to the PDG data.

Due to the relatively small effect of the mixing angle, we can omit the mixing of singlet and octet states since this is within the uncertainties of the QCDSR approach. Namely, the \( \omega_2 \) and \( \phi_2 \) can be handled as a pure singlet and octet states, respectively.

In this study, we concentrate on the mass and decay constant of the ground states \( \rho_2, \omega_2, \phi_2 \) and their first excited states at finite temperatures. Using Thermal QCD sum rules (TQCDSR) model mass and decay constants of axial-tensors are calculated involving the quark, gluon and mixed condensates corrections up to dimension five. We extended the quark-hadron duality with a cut-off parameter depending on temperature and substitute the vacuum expectation values of condensates with thermal ones. We organize the rest of the content as follows: we present the formalism of TQCDSR for considered mesons in section 2. To estimate the mass and decay constant of these states in the hot medium we give our numerical analysis in section 3. Finally, we give a summary and interpret our conclusions in section 4.

### 2 Thermal QCD sum rules

An important and practical non-perturbative technique to derive the extra knowledge of the chiral phase transition is the TQCDSR method via analyzing the variations of hadronic properties at finite temperature. In QCDSR, at large distances or low energies, the correlation function is formulated according to hadronic parameters, called as the “physical side” or “phenomenological side” though, at short distances or high energies, the correlator is defined related with QCD parameters such as quark masses, quark condensates, and this side is named as the “theoretical side” or “QCD side”. According to the QCDSR approach, we can evaluate the correlation function with both sides, and there is a \( q^2 \) region in which both sides can be equalized to each other using the quark-hadron duality hypothesis \[31\].

QCDSR was first expanded to finite temperature by Bochkarev and Shaposhnikov \[32\]. In this version of the QCDSR, analogous to vacuum sum rules the dual nature of the correlator is employed to derive knowledge on the hadron medium features of both baryons and mesons by assuming that both the operator product expansion (OPE) and quark-hadron duality stay valid at finite temperature, although the vacuum condensate values are restored by their thermal ones. TQCDSR has certain new properties in investigating the thermal effects on hadronic parameters.

To compute the mass and decay constant of the \( \rho_2, \omega_2 \) and \( \phi_2 \) mesons and their first excited states within TQCDSR approximation up to the five dimension corrections, we start our calculations with the two-point temperature dependent correlation function.

\[
\Pi_{\mu\nu,\alpha\beta}(q, T) = i \int d^4 x e^{i q(x-y)} \times Tr \left\{ g_T J_{\mu\nu}(x) J_{\alpha\beta}^+(y) \right\}_{y \rightarrow 0},
\]

### Table 1. Zero temperature mass and width values of axial-tensor mesons.

| State | Mass (MeV) | Width (MeV) |
|-------|------------|-------------|
| \( \rho_2 \) | 1940 ± 40 | 155 ± 40 |
| \( \omega_2 \) | 1975 ± 20 | 175 ± 25 |
| \( \rho_2^* \) | 2225 ± 35 | 335 ± 100 |
| \( \omega_2^* \) | 2195 ± 30 | 225 ± 40 |
| \( \phi_2 \) | ? | ? |
| \( \phi_2^* \) | ? | ? |

Among them, \( \rho_2 \) meson quark content is given in PDG as \[|ud\] \[30\]. Also the physical isoscalars \( \omega_2 \) and \( \phi_2 \) are mixtures of the SU(3) wave function \( \psi_8 \) and \( \psi_1 \):

\[
\omega = \psi_8 \cos \theta - \psi_1 \sin \theta, \\
\phi = \psi_8 \sin \theta - \psi_1 \cos \theta,
\]

where \( \theta \) is the nonet mixing angle and the physical \( \omega_2 \) and \( \phi_2 \) states are the linear combinations of these SU(3) singlet and octet states:

\[
\psi_1 = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s}), \\
\psi_8 = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}).
\]
where \( J_{\mu\nu} \) is the interpolating current belonging to the \( \rho^\ast(\omega_2^\ast, \phi_2^\ast) \) mesons. Here \( T \) is the time ordered operator. In Eq. (4), the thermal density matrix is expressed as
\[
\varrho = e^{-H/T} / TR(e^{-H/T}),
\]
where \( H \) is the QCD Hamiltonian and \( T \) is the temperature of the hot medium. The interpolating current is chosen as [26,30]
\[
J^{\ast\ast}_{\mu\nu} (x) = \frac{i}{2} \left[ \bar{u}(x) \gamma_\mu \gamma_5 \overset{\leftrightarrow}{D}_\nu (x) u(x) + \bar{u}(x) \gamma_\nu \gamma_5 \overset{\leftrightarrow}{D}_\mu (x) d(x) \right],
\]
\[
J^{\ast\ast}_{\mu\nu} (x) = \frac{1}{2 g^2} \left[ \left[ \bar{u}(x) \gamma_\mu \gamma_5 \overset{\leftrightarrow}{D}_\nu (x) u(x) + \bar{u}(x) \gamma_\nu \gamma_5 \overset{\leftrightarrow}{D}_\mu (x) d(x) + 5(x) \right] \right.
\]
\[
\times \gamma_\mu \gamma_5 \overset{\leftrightarrow}{D}_\nu (x) s(x) + [\mu \leftrightarrow \nu],
\]
\[
J^{\ast\ast}_{\mu\nu} (x) = \frac{1}{2 g^2} \left[ \left[ \bar{u}(x) \gamma_\mu \gamma_5 \overset{\leftrightarrow}{D}_\nu (x) u(x) + \bar{u}(x) \gamma_\nu \gamma_5 \overset{\leftrightarrow}{D}_\mu (x) d(x) - 2\pi(x) \right] \right.
\]
\[
\times \gamma_\mu \gamma_5 \overset{\leftrightarrow}{D}_\nu (x) s(x) + [\mu \leftrightarrow \nu],
\]
for the \( \rho_2^\ast, \omega_2^\ast \) and \( \phi_2^\ast \) states respectively. In Eqs. (5), (6), \( \overset{\leftrightarrow}{D}_\mu (x) \) shows the derivative with respect to four-x simultaneously acting on left and right. It is given as
\[
\overset{\leftrightarrow}{D}_\mu (x) = \frac{1}{2} [ \overset{\rightarrow}{D}_\mu (x) - \overset{\leftarrow}{D}_\mu (x) ],
\]
\[
\overset{\rightarrow}{D}_\mu (x) = \gamma_\mu - \frac{i}{2} g A^a \gamma_\mu G^a_\mu,
\]
\[
\overset{\leftarrow}{D}_\mu (x) = \gamma_\mu - \frac{i}{2} g A^a \gamma_\mu G^a_\mu,
\]
where \( \lambda^a (a = 1, 8) \) are the Gell-Mann matrices and \( G^a_\mu (x) \) are gluon fields. Firstly we focus on the physical side of the correlation function. A complete set of intermediate physical states with the same quantum numbers are embedded into Eq. (3), and relevant integral by four-x are accomplished. At the end, representing the axial-tensor mesons with \( A^\ast \) and their first excited states with \( A^\ast \), the correlation function can be written in terms of matrix elements of interpolating currents (for similar works look [33,34,35])
\[
\Pi^{\text{phys}}_{\mu,\nu,\alpha,\beta}(q, T) = \frac{\langle \Omega | J_{\mu \nu}(0) | A^\ast \rangle (A^\ast | J_{\alpha \beta}(0) | \Omega) }{m_{A^\ast}(T) - q^2} + \frac{\langle \Omega | J_{\mu \nu}(0) | A^\ast \rangle (A^\ast | J_{\alpha \beta}(0) | \Omega) }{m_{A^\ast}(T) - q^2} + \ldots,
\]
where \( \Omega \) indicate the hot medium and dots show the contributions originating from the other excited states and continuum. The matrix element \( \langle \Omega | J_{\mu \nu}(0) | A^\ast \rangle \) and \( \langle A^\ast | J_{\alpha \beta}(0) | \Omega \rangle \) is defined depending on the decay constant \( f_{A^\ast} \) and the mass \( m_{A^\ast} \) in the following form
\[
\langle \Omega | J_{\mu \nu}(0) | A^\ast \rangle = f_{A^\ast} (T) m_{A^\ast}(T) \varepsilon_{\mu \nu},
\]
\[
\langle A^\ast | J_{\alpha \beta}(0) | \Omega \rangle = f_{A^\ast} (T) m_{A^\ast}(T) \varepsilon'_{\alpha \beta},
\]
here \( \varepsilon_{\mu \nu} \) represents the polarization tensor and the following relationship is valid:
\[
\varepsilon_{\mu \nu} \varepsilon'_{\alpha \beta} = \frac{1}{2} \eta_{\mu \alpha} \eta_{\nu \beta} + \frac{1}{2} \eta_{\mu \beta} \eta_{\nu \alpha} - \frac{3}{2} \eta_{\mu \nu} \eta_{\alpha \beta},
\]
where
\[
\eta_{\mu \nu} = -g_{\mu \nu} + q_\mu q_\nu m_{A^\ast}^2.
\]
Inserting Eqs. (11,14) into Eq. (10), the final expression of the correlator of physical side is attained as
\[
\Pi^{\text{phys}}_{\mu,\nu,\alpha,\beta}(q, T) = \left[ f_{A^\ast}^2 (T) m_{A^\ast}(T) - q^2 + f_{A^\ast}^2 (T) m_{A^\ast}(T) - q^2 \right]
\]
\[
\times \frac{1}{2} (g_{\mu \nu} g_{\alpha \beta} + g_{\mu \beta} g_{\alpha \nu}) + \text{other structures}.
\]
Now we compute the correlation function in the QCD side up to certain order in the OPE expansion to obtain information about hadronic properties of the considered mesons. We can distinguish the perturbative \( \Gamma(q^2, T) \) and non-perturbative \( \widetilde{\Gamma}(q^2, T) \) contribution of the correlation function in Eq. (3) in this part, i.e.,
\[
\Pi^{\text{QCD}}_{\mu,\nu,\alpha,\beta}(q, T) = \Gamma(q^2, T) + \widetilde{\Gamma}(q^2, T).
\]
At the hadron level, the correlation function can be written in the form of the dispersion relation using a spectral function:
\[
\Gamma(q, T) = \int \frac{\rho(s)}{s - q^2} ds,
\]
here \( \rho(s) \) is the spectral density function:
\[
\rho(s) = \sum_\mathcal{N} \delta(s - m_{\mathcal{N}}^2) \langle \Omega | J_\mathcal{N}(s) | \mathcal{N} | J_\mathcal{N} | \Omega \rangle
\]
\[
= f_{A^\ast}^2 m_{A^\ast}^2 \delta(s - m_{A^\ast}^2) + f_{A^\ast}^2 m_{A^\ast}^2 \delta(s - m_{A^\ast}^2) + \text{higher states}.
\]
Moreover, the non-perturbative contributions to correlator should be expressed according to the thermal average of the energy density and the thermal expectation values of the quark and gluon condensates. For computing all additives in the QCD side, the obvious expressions of the interpolating currents in Eqs. (5,6,7) are inserted into Eq. (3). After standard manipulations the QCD side of
the correlation function is obtained as follows:

\[ \Pi_{\mu
u,\alpha\beta}^{(2)}(q,T) = \frac{3i}{16} \int d^4x e^{i(q\cdot x)} \left\{ Tr \left[ -\bar{\mathcal{D}}_\beta(y) S_d(y-x) \right. \right. \\
\times \gamma_\mu \gamma_5 \bar{\mathcal{D}}_\nu(x) S_u(x-y) \gamma_\alpha \gamma_5 + S_d(y-x) \gamma_\mu \gamma_5 \bar{\mathcal{D}}_\nu(x) \bar{\mathcal{D}}_\beta(y) S_u(x-y) \gamma_\alpha \gamma_5 + S_d(y-x) \gamma_\mu \gamma_5 \bar{\mathcal{D}}_\nu(x) S_u(x-y) \gamma_\alpha \gamma_5 \bar{\mathcal{D}}_\beta(y) S_d(y-x) \gamma_\alpha \gamma_5 \\
\times \gamma_\alpha \gamma_5 - \bar{\mathcal{D}}_\nu(x) S_d(y-x) \gamma_\alpha \gamma_5 - \bar{\mathcal{D}}_\beta(y) S_u(x-y) \gamma_\alpha \gamma_5 \gamma_\gamma \gamma_5 + \\
\left. \left. \left. + [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\} \right\} \right\}
\end{eqnarray}

(19)

Then using the quark-hadron duality assumption, one can write:

\[ \Pi_{\mu
u,\alpha\beta}^{(2)}(q,T) = \frac{3i}{32} \int d^4x e^{i(q\cdot x)} \left\{ Tr \left[ -\bar{\mathcal{D}}_\beta(y) \right. \right. \\
\times \gamma_\mu \gamma_5 \bar{\mathcal{D}}_\nu(x) S_u(x-y) \gamma_\alpha \gamma_5 + S_d(y-x) \gamma_\mu \gamma_5 \bar{\mathcal{D}}_\nu(x) \bar{\mathcal{D}}_\beta(y) S_d(y-x) \gamma_\alpha \gamma_5 + S_u(y-x) \gamma_\alpha \gamma_5 - \bar{\mathcal{D}}_\nu(x) S_u(x-y) \gamma_\alpha \gamma_5 - \bar{\mathcal{D}}_\beta(y) S_d(y-x) \gamma_\alpha \gamma_5 \\
\times \gamma_\alpha \gamma_5 - \bar{\mathcal{D}}_\nu(x) S_d(y-x) \gamma_\alpha \gamma_5 - \bar{\mathcal{D}}_\beta(y) S_u(x-y) \gamma_\alpha \gamma_5 [\beta \leftrightarrow \alpha] + [\nu \leftrightarrow \mu] \\
\left. \left. \left. + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\} \right\} \right\} \right\}
\end{eqnarray}

(20)

Then using the quark-hadron duality assumption, one can write:

\[ \hat{\mathcal{B}} \Pi^{QCD}(q^2,T) = \hat{\mathcal{B}} \Pi^{QCD}(q^2,T), \]  

(22)

where \( \hat{\mathcal{B}} \) symbolizes the Borel transformation in terms of \( q^2 \). In the QCD part, the correlator in Eq. (22) can also be written with respect to the selected Lorentz structures

\[ \Pi_{\mu
u,\alpha\beta}^{QCD}(q^2,T) = \Pi^{QCD}(q^2,T) \left\{ \frac{1}{2} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \right\} \]  

+ other structures.  

(23)

During calculations in Eqs. (19) and (21), we place thermal light quark propagator \( S_q(x-y) \) expression in coordinate space in the following form:

\[ S_q^{ij}(x-y) = \frac{1}{2\pi^2(x-y)^2} \delta_{ij} - \frac{m_q}{4\pi^2(x-y)^2} \delta_{ij} \]

\[ - \frac{\langle qq \rangle_T}{12} \delta_{ij} - \frac{(x-y)^2}{192} \delta_{ij} \]  

\[ + i \left( \langle \cdot \cdot \cdot \rangle - \frac{i}{12} \langle \cdot \cdot \cdot \rangle \right) \delta_{ij} + \left( \langle \cdot \cdot \cdot \rangle \right) \delta_{ij} - \frac{ig}{32} \Pi_{\mu\nu}(x-y)^2 (x-y)^2, \]

(24)

where \( \Theta_{\mu\nu} \) and \( u_\mu \) are the fermionic part of the energy momentum tensor and the four-velocity of the hot medium, respectively. The quark condensates depend on temperature are expressed in connection with vacuum condensates in the rest frame \( u_\mu = (1,0,0,0) \), \( u^2 = 1 \). After calculating the correlator belonging to the QCD and physical sides, via equating the coefficients of structures \( \frac{1}{2} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \) taking into account Borel transformation and quark-hadron duality, first of all ground-state decay constant sum rule for \( p_2, \omega_2 \) and \( \phi_2 \) are obtained as

\[ f_A^2(T) = m_A^2(T) e^{-m_A^2/M^2} \int_{s_{\text{min}}}^{s_{0}} ds \rho_{\text{pert}}(s) e^{-s/M^2} \]

\[ + \tilde{\mathcal{B}} \Gamma(q^2,T), \]

(25)

Then using the quark-hadron duality assumption, one can write:

\[ \hat{\mathcal{B}} \Pi^{QCD}(q^2,T) = \hat{\mathcal{B}} \Pi^{QCD}(q^2,T), \]

(26)

where \( \sqrt{s_{\text{min}}} \) is the sum of the quark contents of the related states, and \( M^2 \) is the Borel mass parameter. As for the excited ones we get:

\[ m_{A^*}^2(T) = \frac{\int_{s_{\text{min}}}^{s_0}(T) ds \rho_{\text{pert}}(s) e^{-s/M^2} + g_{\mu\nu}(q^2) \hat{\mathcal{B}} \Gamma(q^2,T)}{\int_{s_{\text{min}}}^{s_0}(T) ds \rho_{\text{pert}}(s) e^{-s/M^2} + \hat{\mathcal{B}} \Gamma(q^2,T)}, \]

(27)

\[ f_{A^*}^2(T) = \frac{1}{m_{A^*}^2(T)} \int_{s_{\text{min}}}^{s_0}(T) ds \rho_{\text{pert}}(s) e^{-s/M^2} \]

\[ + e^{-m_{A^*}^2/M^2} \hat{\mathcal{B}} \Gamma(q^2,T) - f_{A^*}^2(T) \]

(28)
ground state axial-tensor mesons enter into Eqs. (25,28) as the input parameters.

Note that, the spectral densities are parameterized as

$$\rho(s)_{\text{cont}} = \rho^{QCD}(s) \Theta(s-s_0(T))$$  \quad (29)$$

with a single sharp pole pointing out the ground state hadron, and in above equation \(\rho(s)_{\text{cont}}\) is the spectral density function of the continuum stated in Eq. (10) and \(s_0(T)\) is the cut-off parameter in hot medium described in terms of \(s_0(0)\) at vacuum as \([4,36,37]\)

$$s_0(T) = \left[\frac{\langle qq \rangle_T}{\langle qq \rangle_0}\right]^{2/3}.$$  \quad (30)

Anymore we can move to the numerical analysis section.

3 Numerical Analysis

First of all we will present the numerical values of the input parameters that we used in our calculations in order to analyze the obtained sum rules, i.e., Eqs. (25,28). For the quark and mixed condensates we used \(\langle qq, \sigma G q \rangle = m_0^2 \langle qq \rangle\), where \(m_0^2 = (0.8 \pm 0.2) \text{GeV}^2\), \(\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3\), \(\langle 0|\bar{s}s|0\rangle = -0.8(0.24 \pm 0.01)^3 \text{GeV}^3\) \([31,38,39,40]\). The vacuum condensates are parameters that do not depend on particles under consideration. Their numerical values are extracted once from some processes and are applicable in all sum rule calculations. The masses of \(u, d\) and \(s\) quarks can be found in Ref. [30]. They are equal to \(m_u = (2.16_{-0.26}^{+0.46}) \text{MeV}\), \(m_d = (4.67_{-0.17}^{+0.48}) \text{MeV}\) and \(m_s = (93_{-8}^{+11}) \text{MeV}\).

During calculations the normalized thermal quark condensate in Eq. (24), from Ref. [11] by fitting lattice data is used as follows with \(q = u\) or \(d\) quarks

$$\langle \bar{q}q \rangle_T \langle 0|\bar{q}q|0\rangle = C_4 e^{aT} + C_2$$  \quad (31)$$

and for the \(s\) quark

$$\langle \bar{s}s \rangle_T \langle 0|\bar{s}s|0\rangle = C_3 e^{bT} + C_4,$$  \quad (32)

where \(a = 0.040 \text{MeV}^{-1}\), \(b = 0.516 \text{MeV}^{-1}\), \(C_1 = -6.534 \times 10^{-4}\), \(C_2 = 1.015\), \(C_3 = -2.169 \times 10^{-5}\) and \(C_4 = 1.002\) are coefficients of the fit function.

Note that in Ref. [11] the temperature dependence of quark condensates are presented up to a temperature \(T = 300\) MeV. Anyhow, we parameterize them up to \(T_c = 165\) MeV, which is treated as the pseucritical temperature for the crossover phase transition at zero chemical potential. Then the fermionic part of the energy density is parameterized as [42]

$$\langle \bar{u}u \Theta_{\mu \nu} T^\mu T_\nu \rangle_T = T^4 e^{\left(\lambda_1 T^2 - \lambda_2 T\right)} - \lambda_3 T^5,$$  \quad (33)

where \(\lambda_1 = 113.867 \text{ GeV}^{-2}\), \(\lambda_2 = 12.190 \text{ GeV}^{-1}\) and \(\lambda_3 = 10.141 \text{ GeV}^{-1}\). To test the safety of obtained sum rules in hot medium, the obtained numerical results are checked whether they are also maintained at zero temperature. In Eqs. (25,26) the mass and decay constant rely on Borel parameter which is not completely physical quantity. We look for their intervals of the Borel window \(M^2\) and continuum threshold \(s_0\), so that our results are almost insensitive to their variations. Given these circumstances, we used \(s_0 = (m_{A^0} + 0.5)^2\) for the ground (first excited) state of the related mesons thus OPE convergence is also satisfied. Besides these criteria, physical values have to be stable according to small changes of \(s_0\) and \(M^2\) as well.

The gap of Borel window in sum rule approach are determined in the following criteria:

a) The lower bound of \(M^2\) is determined using the common convergence criterion of the OPE such that the contributions of highest-dimensional operators is less than the 20% of sum of all OPE terms. In this computation the ratio

$$\frac{\Pi^{\text{Dim}5}}{\Pi^{\text{all terms}}} < 20\%$$

where \(\Pi^{\text{Dim}5}\) represents the contribution coming from operators with dimension five.

b) For the upper bound of \(M^2\) it is standard to employ the pole dominance condition which guarantees that the contribution of continuum states. One more condition for the intervals of these auxiliary parameters is the fact that since in the QCDsr approach we extract information only from the ground state, therefore we have to make sure that the pole contribution (PC) is larger than the continuum ones. To determine the PC inasmuch as \(s_0\) and \(M^2\) at \(T = 0\), we employed the below condition:

$$\text{PC} = \frac{\Pi(s_0, M^2, T = 0)}{\Pi(\infty, M^2, T = 0)} \geq 50\%$$

and the pole dominance of 50% is obtained in the considered region and shown in Figure 1. Those sum rules that do not meet these criteria are not applicable and are discarded.

At the end we decide to use the following values in Table 2 and 3 for the \(s_0\) and \(M^2\) parameters. The numerical values obtained in this study for the mass and decay constant of axial-tensor mesons are given in Table 4,5 and 6.
Fig. 1. Pole dominance of the sum rule: relative contributions of the pole (red-dashed) and continuum (blue) versus to the Borel parameter $M^2$ at $s_0 = 5.95$ GeV$^2$ for $\rho_2(1940)$ at $T = 0$.

Table 3. Borel and continuum threshold parameters regions for the $\rho_2^*$ and $\omega_2^*$ considering “Regge Trajectory Model”.

| Parameter | $\rho_2$ | $\rho_2^*$ | $\omega_2$ | $\omega_2^*$ |
|-----------|----------|------------|-----------|------------|
| $M^2$(GeV$^2$) | $1.6 - 1.8$ | $1.6 - 1.8$ | $1.3 - 1.5$ | $1.3 - 1.5$ |
| $s_0^{(1)}$(GeV$^2$) | $4.82$ | $5.95$ | $4.82$ | $6.67$ |

Table 4. Mass values of the axial-tensor mesons at $T = 0$ and comparison of the numerical values with experimental data taken form PDG.

| Parameter | $m_{\rho_2}$ (MeV) | $m_{\rho_2^*}$ (MeV) | $m_{\omega_2}$ (MeV) | $m_{\omega_2^*}$ (MeV) |
|-----------|-----------------|--------------------|-------------------|-----------------|
| Our Results | $1882$ | $2258$ | $1923$ | $2288$ |
| Exp. [20] | $1940 \pm 40$ | $2225 \pm 35$ | $1975 \pm 20$ | $2195 \pm 30$ |

Table 5. Masses of the axial-tensor mesons at $T = 0$ and comparison of the numerical values with the prediction of Regge Trajectory Model.

| Parameter | $m_{\rho_2}$ (MeV) | $m_{\rho_2^*}$ (MeV) | $m_{\omega_2}$ (MeV) | $m_{\omega_2^*}$ (MeV) |
|-----------|-----------------|--------------------|-------------------|-----------------|
| Our Results | $1604$ | $1998$ | $1668$ | $1993$ |
| Reg Tr. Model [23] | $1696$ | $1940$ | $1696$ | $1975$ |

These results are in good agreement in itself with experiments and the Regge Trajectory Model estimations which claimed that the $\rho_2(1940)$ and $\omega_2(1975)$ detected in experiments given as ground state in PDG are not ground states of these particles, they are indeed their first excited states [23]. There are also another two studies [23, 25] denoting ground state masses of $\rho_2$ and $\omega_2$ as $\sim 1.7$ GeV comparable with Regge theory. In this context we estimate the mass and decay constant values of $\phi_2$ state which is the candidate for $J^{PC} = 2^{--}$ nonet predicted in Regge Trajectory Model calculations [23] and found the following results

$$m_{\phi_2} = 1846 \text{ MeV}, \quad f_{\phi_2} = 6.83 \times 10^{-2}$$

$$m_{\phi_2^*} = 2195 \text{ MeV}, \quad f_{\phi_2^*} = 3.96 \times 10^{-2}$$

in the Borel window $1.1 \text{ GeV}^2 \leq M^2 \leq 1.3 \text{ GeV}^2$ and $s_0 = 5.77 \text{ GeV}^2$, $s_0^* = 7.02 \text{ GeV}^2$.

Lastly, mass and the decay constant versus $M^2$ and $s_0$ at $T = 0$ for all considered states are plotted (but not presented in the paper for brevity) where dependancies of the hadronic parameters to $M^2$ and $s_0$ are shown to be weak. Therefore, we can say that the extracted sum rules are trustworthy in estimating the mass and decay constant, and analyzing their thermal behaviors as well. Also we draw the OPE convergence plots to ensure that the pole contribution is 50% of the total contribution and determine the max value of $M^2$. For the $\phi_2$ and $\phi_2^*$ we need new precise experimental and also theoretical data to clarify the case. These missing mesons is still empirically unambiguous.

4 Summary and Discussion

In this article we have explored the hadronic characteristics of the $\rho_2^*$, $\omega_2^*$ and $\phi_2^*$ mesons with the quantum numbers $J^{PC} = 2^{--}$ in the TQCDSR approach looking through the window of both Regge Trajectory Model and PDG. By using the two-point correlation function, we calculated the hadronic parameters up to dimension five. After making sure that we have achieved the temperature dependence of mass and decay constant sum rules for these states, then the analysis is reduced to zero temperature to check the mass and decay constant values at vacuum. Considering Eq. (25-28) variations of the masses and decay constants depending on temperature are plotted for all considered mesons in terms of PDG data and also in accordance with Regge Trajectory Model predictions by determining the related Borel mass and continuum threshold parameters separately, but we only present the Figure 2 belonging to $\rho_2$ and $\rho_2^*$ considered PDG data as example in order not to occupy much space in the paper.

Looking at the analysis for the mass and decay constant of $\rho_2^*$ and $\omega_2^*$ remain unaffected until $T \cong 0.12$ GeV with regard to the PDG data and also Regge Trajectory Model data. Nevertheless after these temperature values, with increasing temperature their trend start to deviate
Table 7. Percentage changes of mass and decay constants of the \( \rho_2^{(*)} \) and \( \omega_2^{(*)} \) compared with vacuum values for “PDG” data at \( T_c = 155 \text{ MeV} \).

| Parameter      | \( \rho_2 \) | \( \rho_2^{(*)} \) | \( \omega_2 \) | \( \omega_2^{(*)} \) |
|----------------|--------------|-----------------|--------------|------------------|
| Mass (%)       | 9            | 14              | 10           | 35               |
| Decay Constant (%) | 4         | 1               | 3            | 14               |

Table 8. Percentage variations of mass and decay constants of the \( \rho_2^{(*)} \), \( \omega_2^{(*)} \) and \( \phi_2^{(*)} \) compared with vacuum values in terms of “Regge Trajectory Model” data at \( T_c = 155 \text{ MeV} \).

| Parameter      | \( \rho_2 \) | \( \rho_2^{(*)} \) | \( \omega_2 \) | \( \omega_2^{(*)} \) | \( \phi_2 \) | \( \phi_2^{(*)} \) |
|----------------|--------------|-----------------|--------------|------------------|-----------|------------------|
| Mass (%)       | 10           | 26              | 19           | 10               | 25        |
| Decay Constant (%) | 3         | 34              | 2            | 18               | 2         | 14               |

from vacuum values (For the rates of change see the Table 7 and 8).

As a result of these analyses leaving aside the discussion of quark contents, we conclude that the masses and decay constants of \( \rho_2^{(*)} \), \( \omega_2^{(*)} \) and \( \phi_2^{(*)} \) mesons may dissociate a critical/pseudocritical temperature. However we need more and precise experimental data to clarify the case. We hope that our numerical results can be checked in near future both by theoretical and experimental researches, and might provide us to figure out the nature of strong interactions at finite temperatures.

### A Thermal spectral densities \( \rho_{\text{QCD}}(s, T) \) for the \( \rho_2^{(*)} \), \( \omega_2^{(*)} \) and \( \phi_2^{(*)} \)

The spectral densities from QCDSR in the high temperature approximation is computed and presented explicitly in terms of dimension in which contributions of the gluon condensates are neglected due to its smallness \cite{43}. The spectral density expressions for the \( \rho_2^{(*)} \), \( \omega_2^{(*)} \) and \( \phi_2^{(*)} \) mesons up to dimension five are found as follows:

--- Perturbative Parts: ---

\[
\rho_{\rho_2^{(*)}} = \frac{3s^2 - 10sm_u m_d}{80\pi^2},
\]

\[
\rho_{\omega_2^{(*)}} = \frac{6s^2 - 5s(m_u^2 + m_d^2 + m_s^2)}{160\pi^2},
\]

\[
\rho_{\phi_2^{(*)}} = \frac{12s^2 - 5s(m_u^2 + m_d^2 + 4m_s^2)}{320\pi^2}.
\]

--- Non-Perturbative Parts: ---

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Fig. 2. The effect of temperature on mass (first) and decay constant (second) for \( \rho_2 \) meson and mass (third) and decay constant (fourth) for \( \rho_2^{(*)} \) meson with respect to PDG data, respectively.
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