On Certain Integral Transforms Involving Hypergeometric Functions and Struve Function

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Abstract. This paper is devoted to the study of Mellin, Laplace, Euler and Whittaker transforms involving Struve function, generalized Wright function and Fox’s H-function. The main results are presented in the form of four theorems. On account of the general nature of the functions involved here in, the main results obtained here yield a large number of known and new results in terms of simpler functions as their special cases. For the sake of illustration some corollaries have been recorded here as special cases of our main findings.

1. Introduction

In 1882, Struve [21] introduced $H_\nu$ function as the series solution of the non-homogeneous second order Bessel type differential equation. The Struve function has many applications in various fields of physical sciences and also in the fields like electrodynamics, potential theory, optics. Its applications have also been found in water-wave and surface-wave problems, unsteady aerodynamics, distribution of fluid pressure over a vibrating disk. More recently it appeared in particle quantum dynamical studies of spin decoherence ([10], [14]) . Struve function possess power series representation of the form ([4], [22]).

\begin{equation}
H_\nu[z] = \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(m + \nu + \frac{1}{2}) \Gamma(m + \nu + \frac{3}{2})} \left(\frac{z^2}{2}\right)^{\nu+2m+1}
\end{equation}

where $\nu \in \mathbb{R}$ and $z \in \mathbb{C}$

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In 1978, Mathai and Saxena ([9]; see also [1] and [8]), defined H-function by means of the Mellin-Barnes type contour integral as

\( H_{A;C}^{B;D} \left[ z \right] = H_{A;C}^{B;D} \left[ \frac{(a_i; \alpha_i)_{l,l'}}{(b_j; \beta_j)_{1,N}} \right] = \frac{1}{2\pi i} \int_L \Phi(\xi) z^{-\xi} d\xi \)

where

\( \Phi(\xi) = \prod_{j=A+1}^D \frac{\Gamma(1-b_j - \beta_j \xi)}{\Gamma(1-a_j - \alpha_j \xi)} \prod_{j=N+1}^B \frac{\Gamma(a_j + \alpha_j \xi)}{\Gamma(1-b_j + \beta_j \xi)} \)

where L is a suitable contour, the orders \((A; B; C; D)\) are non negative integers such that \(1 \leq A \leq D, 0 \leq C \leq B\). The parameters \(\alpha_i, \beta_j\) are positive and \(a_i, b_j = 1, 2, \cdots; B; j = 1, 2, \cdots, D\) can be arbitrary complex such that \(l, l' = 0, 1, 2, \cdots; i = 1, 2, \cdots, C; j = 1, 2, \cdots, A\).

In 1935, Wright [24] introduced the function \( p\Psi_q(z) \) defined as

\( p\Psi_q(z) \equiv p\Psi_q \left[ z \right] = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^p \Gamma(a_i + \alpha_i k)}{\prod_{j=1}^q \Gamma(b_j + \beta_j k)} \frac{z^k}{k!} \)

\( z \in C, a_i, b_j \in C, \alpha_i, \beta_j \in R, i = 1, 2, ..., p, j = 1, 2, ..., q; \sum_{j=1}^q \beta_j - \sum_{i=1}^p \alpha_i > -1 \)

This function is called by generalized Wright function [3], [11], [12], [15], [19]. Generalized Wright function generalizes many special functions. In 2002, Kilbas, Saigo and Trujillo studied the properties of the generalized Wright function and proved its integral representation in terms of Mellin Barnes integral [3, Sec. 1.19], with a special path of integration. In their paper Kilbas, Saigo and Trujillo [5], established a theorem to represent generalized Wright function in terms of the H-function.

The Mellin transform ([16]; see also [2] and [7]) of \( f(x) \) is denoted by

\( M[f(x), s] = F(s) = \int_0^\infty x^{s-1} f(x) \, dx \)

and the inverse Mellin transform is defined as

\( f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s) x^{-s} \, ds, \)
The Laplace transform ([16]; see also [2] and [7]) of \( f(x) \) is denoted by

\[
(1.7) \quad L[f(x)] = F(s) = \int_{0}^{\infty} e^{-sx} f(x) \, dx
\]

The Euler (Beta) transform ([16]; see also [2] and [7]) of \( f(x) \) is denoted by

\[
(1.8) \quad B\{f(z) : a, b\} = \int_{0}^{1} z^{a-1}(1-z)^{b-1} f(z) \, dz
\]

2. Main Results

**Theorem 2.1.** If

\[
a_j, b_j \in C, \alpha_j, \beta_j \in R,
\]

\[
\sum_{i=1}^{C} E_i - \sum_{i=C+1}^{B} E_i + \sum_{i=1}^{A} F_i - \sum_{i=A+1}^{D} F > 0, \quad \sum_{j=1}^{q} \beta_j - \sum_{i=1}^{p} \alpha_i < 1,
\]

\[
|\arg(z)| < \frac{1}{2} \left( \sum_{i=1}^{C} E_i - \sum_{i=C+1}^{B} E_i + \sum_{i=1}^{A} F_i - \sum_{i=A+1}^{D} F \right) \pi,
\]

\[
- \min_{1 \leq j \leq A} R\left( \frac{f_j}{F_j} \right) < \text{Re}(s) < \max_{1 \leq j \leq C} R\left( \frac{1 - e_j}{E_j} \right),
\]

\[
|\arg(-z_1)| < \left( 1 - \sum_{j=1}^{q} \beta_j + \sum_{i=1}^{p} \alpha_i \right) \frac{\pi}{2}
\]

then

\[
\int_{0}^{\infty} x^{s-1} H_{v\omega} [ax^h] \Psi_q \left[ z_1 x^\delta \left( \alpha_j, \alpha_j \right)_{1,p} \right] H_{B,D} \left[ xz \left( \beta_j, \beta_j \right)_{1,q} \left( e_j, E_j \right)_{1,B} \left( f_j, F_j \right)_{1,D} \right] \, dx
\]

\[
= \left( \frac{a}{z} \right)^{v+1} z^{-(s+h(v+1))} \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(m + \frac{3}{2})\Gamma(v + m + \frac{3}{2})} \left( \frac{a}{z} \right)^{2m} z^{-2hm},
\]

\[
(2.1) \quad H_{B+q+1,D+p}^{A+p,C+1} \left[ \frac{-z}{z_1} \delta \left( 1, 1 \right), \left( e_j + E_j(s + 2mh + hv + h), E_j \delta \right)_{1,B}, \left( b_i, \beta_i \right)_{1,q} \right]
\]

**Proof.** We first express the generalized Wright function in its integral representation in terms of Mellin Barnes integral [5], then use the expansion (1.1) for the Struve function and the representation (1.2) of the H-function. On interchanging the order of integration and summation and thereafter applying Mellin transform (1.6) we arrive at the desired result.
Special Cases
(i) Taking
\[ p = q = 2; a_1 = \gamma, b_1 = \eta, b_2 = \beta, \alpha_1 = \mu, \beta_1 = \lambda, \beta_2 = \alpha, a_2 = 1 = \alpha_2 \]
in (2.1), we get

**Corollary 2.2.**

\[
\int_0^\infty x^{s-1} H_v[a x^h] \Psi [z_1 x^\delta] H^{A,C}_{B,D} \left[ \frac{z x}{f_j, f_j}_1 \right] dx = \left( \frac{a}{2} \right)^{v+1} z^{-(s+h(v+1))} \sum_{m=0}^\infty \frac{(-1)^m}{\Gamma(m + \frac{3}{2}) \Gamma(m + \frac{1}{2})} \left( \frac{a}{2} \right)^{2m} z^{-2hm}.
\]

(2.2)

\[
\frac{\Gamma(n)}{\Gamma(\gamma)} H^{A+2,C+1}_{B+4,D+2} \int \frac{z \delta}{z_1} \left[ (1, 1), (e_j + E_j(s + 2mh + h), E_j(d))_{1,B}, (\eta, \lambda, (\beta, \alpha)] \right.
\]

under the conditions derived from those mentioned with Theorem 2.1, where \( E^{\gamma, \eta, \mu}_{\alpha, \beta, \lambda} [z_1 x^\delta] \) is Generalized Mittag Leffler function due to Salim and Faraj [13].

(ii) Taking \( v = \frac{1}{2} \) in (2.1) we get

**Corollary 2.3.**

\[
\int_0^\infty x^{s-\frac{1}{2}}(1 - \cos ax^h) \Psi [z_1 x^\delta] H^{A,C}_{B,D} \left[ \frac{z x}{f_j, f_j}_1 \right] dx = a^2 \left( \frac{1}{2} \right)^{3/2} z^{-(s+\frac{1}{2})} \sum_{m=0}^\infty \frac{(-1)^m}{\Gamma(m + \frac{3}{2}) \Gamma(m + \frac{1}{2})} \left( \frac{a}{2} \right)^{2m} z^{-2hm}.
\]

(2.3)

\[
H^{A+p,C+1}_{B+q+1,D+p} \int \frac{z \delta}{z_1} \left[ (1, 1), (e_j + E_j(s + 2mh + \frac{3h}{2}), E_j(d))_{1,B}, (\eta, \alpha, \lambda)] \right.
\]

under the conditions derived from those mentioned with Theorem 2.1.

(iii) Taking \( v = -(n + \frac{1}{2}) \), result (2.1) takes the form

**Corollary 2.4.**

\[
\int_0^\infty x^{s-1} H^n_{n+\frac{1}{2}}(a x^h) \Psi [z_1 x^\delta] H^{A,C}_{B,D} \left[ \frac{z x}{f_j, f_j}_1 \right] dx = (-1)^n \left( \frac{a}{2} \right)^{-(n+1)} z^{-(s-h(n-1/2))} \sum_{m=0}^\infty \frac{(-1)^m}{\Gamma(m + \frac{3}{2}) \Gamma(m - n + 1)} \left( \frac{a}{2} \right)^{2m} z^{-2hm}.
\]
2.5. Theorem 2.5.

If

(2.4)

\[ H^{A+p,C+1}_{B+q+1,D+p} \left( \frac{z}{z_1} \right) (1,1), (e_j + E_j(s + 2mh - nh + \frac{h}{2}), E_j\delta)_{1,B+1}(b_j, \beta_j)_{1,q} (a_i, \alpha_i)_{1,p}, (f_j + F_j(s + 2mh - nh + \frac{h}{2}), F_j\delta)_{1,D} \]

under the conditions derived from those mentioned with Theorem 2.1.

Theorem 2.5. If \( \rho, \lambda, v \in C, R(\rho) > 0, R(\lambda) > 0, h, k, \delta > 0, \) \( |\arg(-z_1)| < \left(1 - \sum_{j=1}^{q} \beta_j + \sum_{i=1}^{p} \alpha_i \right) \frac{\pi}{2} \) and \( \sum_{j=1}^{q} \beta_j - \sum_{i=1}^{p} \alpha_i < 1 \) then

\[
\int_{0}^{t} x^{\rho-1}(t-x)^{\lambda-1} H_{v}[ax^h(t-x)^k]_{p,q} \left[ z_1 x^{-\delta}(t-x)^{-\mu} \left( \frac{a_j, \alpha_j}{b_j, \beta_j} \right)_{1,q} \right] dx = \left( \frac{a}{2} \right)^{v+1} t^{\rho+\lambda+(h+k)(v+1)-1} \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(m + \frac{\lambda}{2})\Gamma(m + \frac{\rho}{2})} \left( \frac{a}{2} \right)^{2m} l^{2(h+k)m}. \]

(2.5)

\[ H^{p,3}_{q+3,p+1} \left( \frac{t}{z_1} \right) (1,1), (1 - \rho - h(2m + v + 1), \delta), (1 - \lambda - k(2m + v + 1), \mu), (b_j, \beta_j)_{1,q} \]

\( (a_i, \alpha_i)_{1,p}, (1 - \rho - \lambda - (h + k)(2m + v + 1), \delta + \mu) \]

Proof. In order to establish the result (2.5), we first express the generalized Wright function in terms of Mellin-Barnes integral \([5]\), then use the expansion (1.1). On interchanging the order of integration and summation and thereafter applying Euler Transform (1.8), we arrive at the desired result.

Special Cases:

(i) Taking \( v = -(n + \frac{1}{2}) \), result (2.5) takes the form

Corollary 2.6.

\[
\int_{0}^{t} x^{\rho-1}(t-x)^{\lambda-1} J_{n+\frac{1}{2}}[ax^h(t-x)^k]_{p,q} \left[ z_1 x^{-\delta}(t-x)^{-\mu} \left( \frac{a_j, \alpha_j}{b_j, \beta_j} \right)_{1,q} \right] dx = \left( \frac{a}{2} \right)^{-\left(n+\frac{1}{2}\right)} t^{\rho+\lambda-(h+k)(n+\frac{1}{2})-1} \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(m + \frac{\lambda}{2})\Gamma(m + n + \frac{\rho}{2})} \left( \frac{a}{2} \right)^{2m} l^{2(h+k)m}. \]

(2.6)

\[ H^{p,3}_{q+3,p+1} \left( \frac{t}{z_1} \right) (1,1), (1 - \rho - h(2m - n + \frac{1}{2}), \delta), (1 - \lambda - k(2m - n + \frac{1}{2}), \mu), (b_j, \beta_j)_{1,q} \]

\( (a_i, \alpha_i)_{1,p}, (1 - \rho - \lambda - (h + k)(2m - n + \frac{1}{2}), \delta + \mu) \]

valid under the same conditions derived from those mentioned with Theorem 2.5.

(ii) On taking \( p = 1, q = 1, a_1 = \gamma, a_1 = \kappa, b_1 = \beta, \beta_1 = \alpha, \) we get
Corollary 2.7

\[
\int_0^t x^{\rho-1} (t - x)^{\lambda-1} H_\nu[ax^h(t - x)^k] E^{\gamma,\nu}_{\alpha,\beta} \left[z_1 x^{-\delta}(t - x)^{-\mu}\right] dx
\]

\[
= \left(\frac{a}{2}\right)^{\nu+1} \frac{\rho^\delta + (h + k)(\nu + 1) - 1}{\Gamma(\gamma)} \sum_{m=0}^\infty \frac{(-1)^m}{\Gamma(m + \frac{3}{2}) \Gamma(m + \nu + \frac{3}{2})} \left(\frac{a}{2}\right)^{2m} t^{2(h+k)m}.
\]

(2.7)

\[
H^{1,3}_{1,4.2} \left[- \frac{t}{z_1} \delta + \nu \right] \left[(1, 1), (1 - \rho - h(2m + \nu + 1), \delta), (1 - \lambda - k(2m + \nu + 1), \mu), (\beta, \alpha) \right]
\]

valid under the same conditions derived from those mentioned with Theorem 2.5. Here \(E^{\gamma,\kappa}_{\alpha,\beta}[z]\) is the generalized Mittag-Leffler function given by Srivastava and Tomovski [20].

Theorem 2.8. If

\[
a_j, b_j \in C; \alpha_j, \beta_j \in \mathbb{R}; \sum_{i=1}^C E_i - \sum_{i=C+1}^B E_i + \sum_{i=1}^A F_i - \sum_{i=A+1}^D F_i > 0; \quad R(s) > 0; \quad \delta, \sigma > 0,
\]

\[
|\arg(z)| < \frac{1}{2} \left( \sum_{i=1}^C E_i - \sum_{i=C+1}^B E_i + \sum_{i=1}^A F_i - \sum_{i=A+1}^D F_i \right) \pi, R(\rho + 1) + \sigma \min_{1 \leq j \leq A} R \left(\frac{f_j}{F_j}\right) > 0
\]

and

\[
|\arg(-z_1)| < \left( 1 - \sum_{j=1}^q \beta_j + \sum_{i=1}^p \alpha_i \right) \frac{\pi}{2}
\]

then

\[
\int_0^\infty x^\rho e^{-sx} H_\nu[ax^h]_p \Psi_q \left[z_1 x^{-\delta} \left[(a_j, \alpha_j)_{1,p} \right] H_{A,C,B,D} \left[z x^\sigma \left[(f_j, F_j)_{1,D}\right] \right] dx
\]

\[
= \left(\frac{a}{2}\right)^{\nu+1} z^{\frac{1}{\sigma}} e^{-s(\rho + \sigma)} \sum_{m=0}^\infty \frac{(-1)^m}{\Gamma(m + \frac{3}{2}) \Gamma(v + m + \frac{3}{2})} \left(\frac{a}{2}\right)^{2m} s^{-2hm}.
\]

(2.8)

\[
H^{0,1,p,1,A,C}_{1,0,q+1,p,B,D} \left[-z_1 s^{-\delta} \right] \left[-\rho; h(2m + \nu + 1); \delta, \sigma \right] \left[(1, 1), (b_j, \beta_j)_{1,q} \right] \left[(a_i, \alpha_i)_{1,p} \right] \left[(f_i, F_i)_{1,D}\right]
\]

Proof. Expressing generalized Wright function in terms of Mellin-Barnes type integral [5] and using expressions (1.1) and (1.2), we arrive at the desired result on applying Laplace transform (1.7).
Theorem 2.10. If (2.9), we get (2.10),

\[ \int_0^\infty x^\rho e^{-x^\beta} H_n(x) \left[ \begin{array}{c} \frac{\alpha}{2} \sum_{m=0}^\infty \Gamma(m+\frac{3}{2}) \Gamma(\nu+m+\frac{3}{2}) \left( \frac{a}{2} \right)^{2m} \frac{2m}{s-2hm} \end{array} \right] dx \]

Corollary 2.9.

Special Case

Taking \( p = q = 2; a_1 = \gamma, b_1 = \eta, b_2 = \beta, \alpha_1 = \mu, \beta_1 = \lambda, \beta_2 = \alpha, a_2 = 1 = a_2 \) in (2.8), we get

\[ \int_0^\infty x^\rho e^{-x^\beta} H_n(x) \left[ \begin{array}{c} \frac{\alpha}{2} \sum_{m=0}^\infty \Gamma(m+\frac{3}{2}) \Gamma(\nu+m+\frac{3}{2}) \left( \frac{a}{2} \right)^{2m} \frac{2m}{s-2hm} \end{array} \right] dx \]

Theorem 2.10. If

\[ a_j, b_j, a, \lambda, \mu, \rho, s, z, z_1 \in C; \alpha_j, \beta_j \in R; \sum_{i=1}^C E_i - \sum_{i=C+1}^B E_i + \sum_{i=1}^A F_i - \sum_{i=A+1}^D F_i > 0 \]

\[ \Re(s) > 0; h, \delta, \sigma > 0, |\arg(z)| < \frac{1}{2} \left( \sum_{i=1}^C E_i - \sum_{i=C+1}^B E_i + \sum_{i=1}^A F_i - \sum_{i=A+1}^D F_i \right) \pi \]

\[ \Re(\rho) + |\Re(\mu)| + \max_{1 \leq j \leq A} R \left( \frac{f_j}{f_j} \right) > -\frac{1}{2}, |\arg(z_1)| < \left( 1 - \sum_{j=1}^q \beta_j + \sum_{i=1}^p \alpha_i \right) \frac{\pi}{2} \]

and \( \sum_{j=1}^q \beta_j - \sum_{i=1}^p \alpha_i < 1 \) then

\[ \int_0^\infty x^\rho e^{-x^\beta} H_n(x) \left[ \begin{array}{c} \sum_{m=0}^\infty \Gamma(m+\frac{3}{2}) \Gamma(\nu+m+\frac{3}{2}) \left( \frac{a}{2} \right)^{2m} \frac{2m}{s-2hm} \end{array} \right] dx \]

(2.10)

\[ H_{2,1,2,A,C,D;1,p}^0 \left[ \begin{array}{c} \frac{\alpha}{2} \sum_{m=0}^\infty \Gamma(m+\frac{3}{2}) \Gamma(\nu+m+\frac{3}{2}) \left( \frac{a}{2} \right)^{2m} \frac{2m}{s-2hm} \end{array} \right] \]

\[ \left( \frac{a}{2} \right)^{2m} \frac{2m}{s-2hm} \]
Proof. To prove result (2.10), we first use expression (1.1), (1.3) and the Mellin-Barnes integral of the generalized Wright function [5]. Now by changing order of summation and integration and making use of the following result
\[
\int_0^\infty e^{-t/2}t^{\nu-1}W_{\lambda,\mu}(t)dt = \frac{\Gamma(\frac{1}{2} + \mu + \nu)\Gamma(\frac{1}{2} - \mu + \nu)}{\Gamma(1 - \lambda + \nu)}
\]
where \(\text{Re}(\nu + \mu) > -\frac{1}{2}\).

We arrive at the result (2.10) after a little simplification.

Special Case:
On Taking \(A = 2, D = 2, B = 0 = C, f_1 = 0, f_2 = \frac{\xi}{\eta}, F_1 = 1, F_2 = \frac{1}{\eta}\) in (2.10), we get

Corollary 2.11.
\[
\int_0^\infty x^{\nu-1}e^{-\frac{z}{2}x}H_\nu(ax^h)\Psi_q \left[ z_1 x^{-\delta} \left( \frac{(a_j, \alpha_j)}{(b_j, \beta_j)} \right) \right] Z_0^z[x^z] W_{\lambda,\mu}(sx)dx
\]
\[
= \left( \frac{a}{2} \right)^{\nu+1} \frac{s^{-\rho-h(\nu+1)}}{\eta} \sum_{m=0}^{\infty} \frac{(-1)^m}{\Gamma(m + \frac{1}{2})\Gamma(\nu + m + \frac{3}{2})} \left( \frac{a}{2} \right)^{2m} s^{-2m}. \quad (2.11)
\]

valid under the same conditions surrounding (2.10). Here \(Z_0^z[z]\) is the Krätzel function (See [6], [9] and [18]).

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