Nuclear 𝑦 and 𝑥 scaling

Claudio Ciofi degli Atti, Dino Faralli and Geoffrey B. West

1 Department of Physics, University of Perugia, and Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Via A. Pascoli, I–06100 Perugia, Italy
2 Theoretical Division, T-8, MS B285, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Abstract The exact expression of the nuclear structure function describing inclusive lepton scattering off nuclei is recalled, and the basic approximations leading to non-relativistic and relativistic nuclear 𝑦 and 𝑥 scaling, are illustrated. The general and systematic features of 𝑦-scaling structure functions are pointed out, and a recently proposed novel approach to 𝑦-scaling, based on a global scaling variable, 𝑦̂, which incorporates the effect of the momentum dependence of the nucleon removal energy, and therefore allows the establishment of a direct link between scaling functions and momentum distributions, is illustrated and applied to the analysis of a large body of data, pertaining to nuclei ranging from the deuteron to Nuclear Matter. A new type of scaling phenomenon, the nuclear 𝑥-scaling, based on a proper analysis of nuclear quasi-elastic data in terms of the Bjorken scaling variable 𝑥̂, is shown to occur at high values of the four-momentum transfer 𝑄²; the usefulness of nuclear 𝑥-scaling is pointed out.

1 The nuclear response in inclusive lepton-nucleus scattering

In inclusive lepton scattering off nuclei, 𝐴(𝑒, 𝑒′)𝑋, the nuclear response, or structure function, 𝑊(𝑛, 𝑞²), which represents the deviation of the cross section from scattering from free nucleons, has the following exact form:

\[ W(\nu, q^2) = \int_{-\infty}^{+\infty} \frac{dt}{2\pi} e^{i(\nu - \frac{q^2}{2M})t} <\psi_0| \sum_{i,j=1}^{Z} \hat{Q}_i \hat{Q}_j e^{-i(H-E_0)t} e^{-i(p_i^2/2M)t}|\psi_0> \]  

(1)

where \( H \) is the Hamiltonian of the target nucleus, \( \psi_0(E_0) \) its ground state wave function (energy), \( p_i \) the momentum operator of nucleon \( i \), and \( q \equiv |q| \) and \( \nu \) (\( Q^2 = q^2 - \nu^2 \)) the three-momentum and energy transfers (for the sake of simplicity only the Coulomb interaction has been considered).

Due to the non-commutativity of \( H \) and \( \frac{p_i^2}{2M} \), Eq.(1) cannot be evaluated exactly. As a matter of fact, one can write:

\[ H = \sum_{i=1}^{A} \frac{p_i^2}{2M} + \sum_{i<j} v(i,j) = H_A - 1 + \sum_{1\neq j} v(1,j) + \frac{p_1^2}{2M} \] 

(2)

\* Presented by G. West at the Elba Workshop on Electron-Nucleus Scattering, EIPC June 22-26 1998
and it can be seen that in general:

$$\sum_{j\neq 1} v(1, j), \frac{p_1 q}{2M} \neq 0. \quad (3)$$

Let us assume, for the time being, that $\sum_{j\neq 1} v(1, j), \frac{p_1 q}{2M} = 0$; if one then replaces the independent kinematical variables $q^2$ and $\nu$ by $q^2$ and $x_0 = \frac{q^2}{2M\nu}$, one obtains from Eq.(1) [1]:

$$i(\nu - \frac{q^2}{2M})t = -i \frac{\nu}{\nu}(x_0 - 1)t \quad (4)$$

and:

$$\lim_{q^2 \to \infty} \nu W(\nu, q^2) \simeq \sum_i Q_i^2 \delta(x_0 - 1) \quad (5)$$

which shows that at high momentum transfer the reduced function $\nu W(x_0, q^2)$ should scale to a $\delta$ function. Such a phenomenon will be called non relativistic nuclear $x$-scaling.

The non relativistic $y$-scaling is obtained by introducing the quantity [1]:

$$y_0 \equiv \frac{2M\nu - q^2}{2Mq} \quad (6)$$

and, assuming again that $[H, \frac{p_1 q}{2M}] = 0$, one obtains:

$$\lim_{q^2 \to \infty} q W(\nu, q^2) = \int \frac{dk_\perp}{2\pi} \int_{-\infty}^{\infty} dk_\parallel n(k_\perp, k_\parallel) \cdot \delta(k_\parallel - y_0) \equiv f(y_0) \quad (7)$$

where:

$$f(y_0) = \int n(k_\perp, k_\parallel) dk_\perp = 2\pi \int_{|y_0|}^{\infty} n(k)k dk \quad (8)$$

is the longitudinal momentum distribution and $n(k)$ is the conventional momentum distribution normalized such that:

$$\int d^3 k n(k) = \int_{-\infty}^{\infty} dy_0 f(y_0) = 1 \quad (9)$$

The above picture has to be improved by considering that in reality $[H, \frac{p_1 q}{2M}] \neq 0$ and that actual experimental data require the use of relativistic kinematics. Both improvements will be implemented in the rest of the paper, by means of the relativistic plane wave impulse approximation, in which the "final state interaction" (FSI) term $\sum_{j\neq 1} v(1, j)$ in Eq.(2) is disregarded and the non relativistic
quantity \((p_i + q)/2M\) is replaced by its relativistic analog \(\sqrt{(p_i + q)^2 + M^2} - M\). One obtains:

\[
W(\nu, q^2) = \sum_f \left| \langle \psi_{A-1}^f, k_N | \hat{O} | \psi_0 \rangle \right|^2 \cdot \delta(\nu + M_A - E_N - E_{A-1})
\]  

(10)

where: \(k_N = k + q\) and \(k = k_1\) are the momenta of the struck nucleon after and before interaction, respectively, \(E_N = \sqrt{(k + q)^2 + M^2}\) is the nucleon total energy, and \(E_{A-1} = \sqrt{M_{A-1}^2 + k^2}\) the total energy of the final \(A-1\) system. Starting from Eq. (10), we will introduce and discuss the relativistic nuclear \(y\) and \(x\) scaling.

2 \(y\)-scaling

Eq. (10), shows that the structure function \(W(\nu, q^2)\) is governed by the nucleon spectral function \(P(k, E)\) which depends on the energy \((E)\) as well as the momentum of the nucleons. In the limit of \(q^2 \to \infty\) it can be shown that [2]:

\[
\lim_{q^2 \to \infty} qW(\nu, q^2) = F(y)
\]  

(11)

\[
F(y) = 2\pi \int_{E_{min}}^{E_{max}} dE \int_{k_{min}(y,E)}^{k_{max}(y,E)} k \, dkP(k,E) = f(y) - B(y)
\]  

(12)

where:

\[
B(y) = 2\pi \int_{E_{min}}^{E_{max}} dE \int_{|y|}^{k_{max}(y,E)} k \, dkP_1(k,E)
\]  

(13)

is the so called binding correction, with \(P_1\) being that part of \(P(k,E)\) generated by ground state correlations and \(f(y)\) is given by Eq. 8 with \(y_0\) replaced by \(y\). The scaling variable \(y\) is obtained (see below) from the relativistic energy conservation appearing in Eq. (10), and represents the longitudinal momentum of the nucleons having the minimal value of the removal energy [2]. The interesting quantity is \(f(y)\), since its knowledge would provide \(n(k)\) by inversion of Eq. (8).

Unfortunately, the extraction of \(f(y)\) from the experimental data requires the knowledge of both the experimental asymptotic scaling function \(F(y)\), and the theoretical binding correction \(B(y)\).

Over the past several years there have been vigorous theoretical and experimental efforts to explore \(y\)-scaling over a wide range of nuclei [3], using the relativistic scaling variable \(y\), which, recently, has been shown to lead to scaling in the relativistic deuteron, within both the light-front [4] and the Bethe-Salpeter [5] approaches.
iii) For general features for all nuclei:

- i) $f(0)$ decreases monotonically with $A$, from $\sim 10MeV^{-1}$ when $A = 2$ to $\sim 3MeV^{-1}$ for heavy nuclei; moreover, for $y \sim 0$, $f(y) \sim (\alpha^2 + y^2)^{-1}$, with $\alpha$ ranging from $\sim 45MeV$ for $A = 2$, to $\sim 140MeV$ for $A = 56$.

- ii) For $50MeV \leq |y| \leq 200MeV$, $F(y) \sim e^{-a^2y^2}$ with $a$ ranging from $\sim 50MeV$ for $A = 2$, to $\sim 150MeV$ for $A = 56$.

- iii) For $|y| \geq 400MeV$, $f(y) \sim C_2e^{-b|y|}$, with $B$ ranging from $2.5 \times 10^{-4}MeV^{-1}$ for $A = 2$, to $1 \times 10^{-3}MeV^{-1}$ for $A = 56$, and, most intriguingly, $b = 6 \times 10^{-3}MeV^{-1}$, independent of $A$.

The following simple form for $f(y)$ yields an excellent representation of these general features for all nuclei:

$$f(y) = \frac{C_1e^{-a^2y^2}}{\alpha^2 + y^2} + C_2e^{-b|y|} = f_0 + f_1$$

(14)

The first term ($f_0$) dominates the small $y$-behavior, whereas the second term ($f_1$) dominates large $y$. The systematics of the first term are determined by the small and intermediate momentum behaviour of the single particle wave function. For $|y| \leq \alpha$ this can be straightforwardly understood in terms of a zero range approximation and is, therefore, insensitive to details of the microscopic dynamics, or of a specific model. The small $k$ behavior of the single particle wave function is controlled by its separation energy, $(Q \equiv M + M_{A-1} - M_A = E_{min})$ and is given by $(k^2 + \alpha^2)^{-1}$ so $\alpha = (2\mu Q)^{1/2}$, $\mu$ being the reduced mass of the nucleon. This agrees quantitatively very well with fits to the data summarized in (i) above.

The most intriguing phenomenological characteristic of the data is that $f(y)$ falls off exponentially at large $y$ with a similar slope parameter for all nuclei, including the deuteron. Since (i) $b$ is almost the same for all nuclei including $A = 2$, i.e., $f(y)$, at large $y$, appears to be simply the rescaled scaling function of the deuteron; and (ii) $b(\approx 1.18fm) \ll 1/\alpha_D(\approx 4.35fm)$, it can be concluded that the term $C_2e^{-b|y|}$ is related to the short range part of the deuteron wave function and reflects the universal nature of $NN$ correlations in nuclei.

The remaining parameters, $C_1$ and $\alpha$, can be related to $f(0)$ and the normalization condition, Eq. (9). Once this is done, there are no adjustable parameters for different nuclei. The intermediate range is clearly sensitive to $a$, the gaussian form being dictated by the shell model harmonic oscillator potential. Notice, however, that here the gaussian is modulated by the correct $|y| < \alpha$ behaviour,
namely \((y^2 + \alpha^2)^{-1}\), thereby ensuring the correct asymptotic wave function. The set of parameters of Eq. (14) for various nuclei is presented in Tab. 1. As an example, Fig. 1 shows the longitudinal scaling functions \(f(y)\) for \(^2H\), \(^4He\) and \(^{56}Fe\) extracted from the experimental data [2] compared to Eq. (14). The fit is excellent.

![Figure 1](image)

**Figure 1.** The longitudinal momentum distribution \(f(y)\) for \(^2H\) (dotted line), \(^4He\) (full line) and \(^{56}Fe\) (dashed line) corresponding to Eq. (14) with parameters given in Tab. 1. The points represent the "experimental" \(f(y)\) obtained in Ref. [2].

**Table 1.** The parameters of Eq. (14) for various nuclei

|            | \(C_1 [MeV]\) | \(\alpha [MeV]\) | \(a [MeV^{-1}]\) | \(C_2 [MeV^{-1}]\) | \(b [MeV^{-1}]\) |
|------------|----------------|-------------------|------------------|-------------------|-----------------|
| \(^2H\)    | 18             | 45                | 6.1 \times 10^{-3}| 2.5 \times 10^{-3} | 6 \times 10^{-3} |
| \(^4He\)   | 41             | 83                | 7.1 \times 10^{-3}| 3.3 \times 10^{-3} | 6 \times 10^{-3} |
| \(^{12}C\) | 83             | 166               | 8.1 \times 10^{-3}| 5.7 \times 10^{-3} | 6 \times 10^{-3} |
| \(^{56}Fe\)| 58             | 138               | 4.6 \times 10^{-3}| 6.2 \times 10^{-3} | 6 \times 10^{-3} |

With these observations it is now possible to understand the normalization and evolution of \(f(y)\) with \(A\). First note that Eq. (8) implies \(f(0) = \frac{1}{2} \int d^3k, \frac{n(k)}{k} = \langle 1/k \rangle\) and so is mainly sensitive to small momenta. Since typical mean momenta vary from around 50 MeV for the deuteron up to almost 300 MeV
for nuclear matter, it is clear why \( f(0) \) varies from around 10 for the deuteron to around 2-3 for heavy nuclei. More specifically, since \( C_2 \ll C_1/\alpha^2 \) and \( f_1 \) falls off so rapidly with \( y \), the normalization integral, Eq. (9), is dominated by small \( y \), i.e., by \( f_0 \). This leads to \( f(0) \approx (\pi^{1/2}/\alpha)^{-1} = (2\pi\mu Q)^{-1/2} \) which gives an excellent fit to the \( A \)-dependence of \( f(0) \). Since \( f(y) \) is constrained by the sum rule, Eq. (9), whose normalization is independent of the nucleus, a decrease in \( f(0) \) as one changes the nucleus must be compensated for by a spreading of the curve for larger values of \( y \). Thus, an understanding of \( f(y) \) for small \( y \) coupled with an approximately universal fall-off for large \( y \), together with the constraint of the sum rule, leads to an almost model-independent understanding of the gross features of the data for all nuclei.

To sum up, the “experimental” longitudinal momentum distribution can be thought of as the incoherent sum of a mean field shell-model contribution, \( f_0 \), with the correct model-independent small \( y \)-behaviour built in, and a “universal” deuteron-like correlation contribution \( (f_1) \). Thus, the momentum distribution, \( n(k) \), which is obtained from (8), is also a sum of two contributions: \( n = n_0 + n_1 \).

This allows a comparison with results from many body calculations in which \( n_0 \) and \( n_1 \) have been separately calculated. Of particular relevance are not only the shapes of \( n_0 \) and \( n_1 \), but also their normalizations, \( S_0(1) = \int n_0(1) d^3k = \int f_0(1) dy \) which, theoretically, turn out to be, for \( ^4He \), \( S_0 \approx 0.8 \) and \( S_1 \approx 0.2 \) whereas Eq. (14) yields \( S_0 \approx 0.76 \) and \( S_1 \approx 0.24 \).

In order to minimize theoretical uncertainties arising in the subtraction of the binding correction \( B(y) \) a new scaling variable has been introduced in Ref. [6] which in principle allows a determination of \( f(y) \) free of theoretical contaminations.

The usual scaling variable \( y \) is effectively obtained from energy conservation

\[
\nu + M_A = [(M_{A-1} + E^*_A)^2 + k^2]^{1/2} + [M^2 + (k + q)^2]^{1/2}
\]  

(15)

by setting \( k = |y|, \frac{kq}{|q|} = 1 \), and, most importantly, the excitation energy, \( E^*_A = 0 \); thus, \( y \) represents the nucleon longitudinal momentum of a nucleon having the minimum value of the removal energy \( (E = E_{min}, E^*_A = 0) \). The minimum value of the nucleon momentum when \( q \to \infty \), becomes \( k_{min}(y,E) = |y - (E - E_{min})| \). Only when \( E = E_{min} \) does \( k_{min}(y,E) = |y| \), in which case \( B = 0 \) and \( F(y) = f(y) \). However, the final spectator \((A-1)\) system can be left in all possible excited states, including the continuum, so, in general, \( E^*_A \neq 0 \) and \( E > E_{min} \), so \( B(y) \neq 0 \), and \( F(y) \neq f(y) \). Thus, it is the dependence of \( k_{min} \) on \( E^*_A \) that gives rise to the binding effect, i.e. to the relation \( F(y) \neq f(y) \). This is an unavoidable defect of the usual approach to scaling; as a matter of fact, the longitudinal momentum is very different for weakly bound, shell model nucleons (for which \( E^*_A \sim 0-20MeV \)) and strongly bound, correlated nucleons (for which \( E^*_A \sim 50-200MeV \)), so that at large values of \( |y| \) the scaling function is not related to the longitudinal momentum of those nucleons (the strongly bound, correlated ones) whose contributions almost entirely exhaust the behaviour of the scaling function. In order to establish a general link between experimental data
in different regions of the scaling variable, and longitudinal momentum components, one has to conceive a scaling variable which could equally well represent longitudinal momenta of both weakly bound and strongly bound nucleons. One can account for this in the following way. The large \( k \) and \( E \) behaviours of the Spectral Function are governed by configurations in which the high momentum of a correlated nucleon (1, say) is almost entirely balanced by another nucleon (2, say), with the spectator \((A - 2)\) system taking only a small fraction of \( k \), given by the CM momentum of the pair \( K_{CM} \) \([9]\). Within such a picture, one has \([9, 10]\)

\[
E_{A-1}^*= \frac{A-2}{A-1} \frac{1}{2M} |k - \frac{A-1}{A-2} K_{CM}|^2
\]

(16)

which shows that the excitation energy of the residual nucleus depends both upon \( k \) and \( K_{CM} \); if the latter is set equal to zero, the average excitation energy for a given value of \( k \) is \( < E_{A-1}^*(k) > = \frac{\frac{A-2}{A-1} \frac{1}{2M} k^2}{\frac{A-1}{A-2} K_{CM}} \). By replacing \( E_{A-1}^* \) in Eq. (15) with \( \frac{\frac{A-2}{A-1} \frac{1}{2M} k^2}{\frac{A-1}{A-2} K_{CM}} \), the deuteron-like scaling variable \( y_2 \) introduced in \([11]\) (see also \([12]\), where the deuteron-like scaling variable was first introduced) is obtained, representing the scaling variable pertaining to a “deuteron” with mass \( M = 2M - E_{th}^{(2)} \), where \( E_{th}^{(2)} = |E_A| - |E_{A-2}| \). Such a scaling variable, however, has the unpleasant feature that the effect of the deuteron-like correlations are overestimated at low values of \( y_2 \) and , as a result, the correct, shell-model picture provided by the usual variable \( y \) is lost. When the CM motion of the pair is taken into account, such a defect is cured. If the expectation value of Eq. (15) is evaluated with realistic spectral functions, for nuclei ranging from \( ^3He \) to Nuclear Matter, one obtains ([10, 7])

\[
< E_{A-1}^*(k) > = \frac{A-2}{A-1} \frac{1}{2M} k^2 + b_A - c_A \frac{k^2}{2M}
\]

(17)

with \( b_A \) and \( c_A \), resulting from the CM motion of the pair, having values ranging from \( 17MeV \) to \( 43MeV \) and \( 3.41 \times 10^{-1} \) to \( 1.66 \times 10^{-1} \), for \( ^3He \) and Nuclear Matter, respectively. Placing Eq.(17) in Eq.(15) and subtracting the value of the average removal energy \( < E > \) to counterbalance the value (17) at low values of \( y \), a new scaling variable is obtained which effectively takes into account the \( k \)-dependence of the excitation energy of the residual \( A-1 \) system, both at low and high values of \( y \), unlike the usual scaling variable which completely disregards \( E_{A-1}^* \), and the scaling variable \( y_2 \), which overestimate the effects of deuteron-like correlations. In the kinematical region of existing experimental data this, \textit{global} scaling variable \( y_G \) reads as follows \([6]\)

\[
y_G = \left| -\frac{q}{2} + \left[ \frac{q^2}{4} - \frac{4\nu A^2 M^2 - W_A^4}{W_A^2} \right]^{1/2} \right|
\]

(18)

Here, \( \nu_A = \nu + \tilde{M} \), \( \tilde{M} = (2A-3)M/(A-1) - E_{th}^{(2)} - (b_A + 2M^2 c_A - < E >) \), \( \tilde{q} = q - c_A \nu_A \), and \( W_A^2 = \nu_A^2 - q^2 - \tilde{M}^2 + 2\nu \tilde{M} - Q^2 \). For the deuteron \( E_{A-1} = 0 \), so \( y_G \rightarrow y = \left| -q/2 + [q^2/4 - (4\nu d^2 M^2 - W_d^4)/W_d^2]^{1/2} \right| \) with
\[ \nu_d = \nu + M_d \quad \text{and} \quad W_d^2 = \nu_d^2 - q^2 = M_d^2 + 2\nu M_d - Q^2. \]

For small values of \( y_G \), such that \( \left( \frac{A-2}{A-1} M_y^2 + b - c A \frac{y_G^2}{2M} \right) \ll \langle E \rangle \), the usual variable is recovered. Thus \( y_G \) interpolates between the correlation and the single particle regions. More importantly, however, since \( k_{min}(q, \nu, E) \approx |y_G|, \quad B(y_G) \approx 0, \quad F(y_G) \approx f(y_G) \).

Thus, plotting data in terms of \( y_G \) allows a direct determination of \( f(y_G) \). One would therefore expect from the above analysis, the same behaviour of \( f(y_G) \) at high values of \( y_G \) for both the deuteron and complex nuclei, unlike what happens with the usual scaling function \( F(y) \), and the same shell-model behaviour at low values of \( y \) as predicted by the usual scaling variable. This is, indeed, the case, as exhibited in Fig. (2-5), where the direct link between the scaling function \( F(q, y_G) \) and the longitudinal momentum distributions is manifest.

**Figure 2.** The experimental scaling function of \(^3\text{He}\) plotted versus the usual, \( y \), (crosses), and the global, \( y_G \), (open dots) scaling variables, compared with the scaling function of \(^2\text{H}\) (full dots). The dashed and full lines are the calculated longitudinal momentum distributions of \(^3\text{He}\) and \(^2\text{H}\) respectively.

We can summarise our conclusions as follows:

i) The general universal features of the \( y \)-scaling function have been identified and interpreted in terms of three contributions: a model-independent zero-range contribution, a “universal” 2-nucleon correlation contribution and a mean field (shell-model) contribution;

ii) The shape and evolution of the curve have been understood both quantitatively and qualitatively on general grounds;

iii) A global scaling variable which incorporates the excitation energy of the
Figure 3. The same as in Fig. 2 but for $^4$He.

Figure 4. The same as in Fig. 2 but for $^{56}$Fe.

$(A - 1)$ system generated by correlations has been defined, which allows one to obtain the longitudinal momentum distributions directly from the experimental data without introducing theoretical corrections. In terms of this variable the data strongly support the idea that the large $y$ behaviour in all nuclei is essentially nothing but a rescaled version of the deuteron.
3 \( x \)-scaling

It can be shown [7] that the transition from the non relativistic \( x_0 \)-scaling, discussed in section 1, to the relativistic one, can be achieved by using a minimal relativity Hamiltonian, i.e. \( T_i = \sqrt{p_i^2 + M_i^2} - M_i \), in which case the variable \( x_0 = q^2/2M\nu \) is replaced by the Bjorken scaling variable \( x_0 = Q^2/2M\nu \). The central point, however, is to understand how non relativistic \( x_0 \)-scaling to a \( \delta \) function is affected by the interaction effects due to \( [H, p_i q] \neq 0 \), and by the use of relativistic kinematics. This is discussed in [7] starting from Eqs. (1) and (2). The results can be summarised as follows. Starting from the relation:

\[
\nu W(x_B, q^2) = \frac{\nu}{q} q W(x_B, q^2) = \frac{\nu}{q} F(y, q^2) \tag{19}
\]

and expressing \( y \) through \( x_B \), the following equation, valid around \( y \approx 0 \) \( (x_B \approx 1) \) is found:

\[
\nu W(x_B, q^2) \approx \frac{1}{(x_B - 1)^2 + \frac{\alpha^2}{M^2}} + O(Q^{-2}) \tag{20}
\]

where we have used for \( F(y, q^2) \) the expression \( f(y) \propto \frac{1}{y^{\alpha - \omega}} \) (cf. Eq. 14).

It can be seen from Eq. (20) that by plotting the experimental data vs. \( x_B \), one should not expect a \( \delta \) function shape, as in \( x_0 \)-scaling, but a Lorentzian shape with a width decreasing with increasing \( Q^2 \) and converging to a finite value \( \alpha^2/M^2 = 2E_{\text{min}}/M \), when \( Q^2 \rightarrow \infty \). The plots shown in Figs. (6)-(9) seem indeed to roughly exhibit a Lorentzian behaviour, but experimental data
at higher values of $Q^2$ would be necessary to check the prediction of a saturating width.

4 Summary

The global $y$-scaling that we have discussed, by identifying the value of the global, $y_G$, scaling variable with the value of the nucleon longitudinal momentum, independently of the value of the nucleon removal energy, allows one, unlike all previous approaches to $y$-scaling, to establish a direct relation between the scaling function and the longitudinal momentum distributions. Nuclear $x$-scaling, i.e. scaling of inclusive quasi elastic data on nuclei, when plotted vs. the Bjorken scaling variable $x_B$, has been shown to qualitatively occur; it can provide useful and complementary information on the nucleon momentum distributions.
Figure 7. The log plot of Fig. 6.

Figure 8. The same as in Fig. 6 but for $^{56}Fe$.

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