Chiral Symmetry Breaking in the Dynamical Soft-Wall Model

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In this paper a model incorporating chiral symmetry breaking and dynamical soft-wall AdS/QCD is established. The AdS/QCD background is introduced dynamically as suggested by Wayne de Paula etc al and chiral symmetry breaking is discussed by using a bulk scalar field including a cubic term. The mass spectrum of scalar, vector and axial vector mesons are obtained and a comparison with experimental data is presented.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) is an non-abelian gauge theory of the strong interactions. It has a coupling constant that highly depends on the energy scale, and the perturbation theory doesn’t work at low energies. The Anti-de Sitter/conformal field theory (AdS/CFT) correspondence \(^1\) provides a novel way to relate QCD with a 5-D gravitational theory and calculate many observables more efficiently. This leads to two types of AdS/QCD models: the top-down models that are constructed by branes in string theory and the bottom-up models by the phenomenological aspects. In the top-down model, mesons are identified as open strings with both ends on flavor branes \(^2\). The most successful top-down model in reproducing various facts of low energy QCD dynamics is D4-D8 system of Sakai and Sugimoto \(^3\) \(^4\).

The bottom-up approach assumes that QCD has suitable 5D dual and construct the dual model from a small number of operators that are influential. The first bottom-up model is the so-called hard-wall model \(^5\) \(^6\); it simply place an infrared cutoff on the fifth dimension to break the chiral symmetry. But in hard-wall model the spectrum of mass square \(m^2_n\) grow as \(n^2\), which is contradict with the experimental data \(^12\). The analysis of experimental data indicates a Regge behavior of the highly resonance \(m^2_n \sim n\). To compensate this difference between QCD and AdS/QCD, soft-wall model was proposed \(^9\). In this model the spacetime smoothly cap off instead of the hard-wall infrared cutoff by introducing a background dilaton field \(\Phi\):

\[
S_5 = -\int d^5x \sqrt{-g} e^{-\Phi(z)} \mathcal{L},
\]

where the background field is parametrized to reproduce Regge-like mass spectrum. In spite of success in reproducing Regge-like mass spectrum, chiral symmetry breaking in this model is not QCD-like. Some further researches try to incorporate chiral symmetry breaking and confinement in this model by using high order terms in the potential for scalar field \(^10\) \(^11\).

But the dilaton field and scalar field in the soft-wall model are imposed by hand to and cannot be derived from any equation of motion. Some works on the dilaton-gravity coupled model show that a linear confining background is possible as a solution of the dilation-gravity coupled equations and Regge trajectories of meson spectrum is obtained \(^7\) \(^8\).

In this paper, we try to incorporate soft-wall AdS/QCD and dynamical dilaton-gravity model. We introduce the meson sector action with a cubic term of the bulk field in the bulk scalar potential under the dynamical metric background. We derive a nonlinear differential equation related the vacuum expectation value(VEV) of bulk field, wrap factor and dilaton field. An analysis of the asymptotic behavior of the VEV is presented. We obtain that the VEV is a constant in the IR limit that suggests chiral symmetry restoration exists. The coupling constant can be determined by other parameters in this model.

This paper proceeds as follow: A brief review of dynamical gravity-dilaton background is presented in Sec. II. In Sec. III we introduce the bulk field and meson sector action under dynamical metric background and obtain the differential equation of the bulk field VEV. An analysis of asymptotic behavior is presented and a parametrized solution is given. Using the parametrized solution we calculate the scalar, vector and axial vector mass spectrums in Sec. IV. A conclusion is given in Sec. V.

II. BACKGROUND EQUATIONS

We start from the Einstein-Hilbert action of five-dimensional gravity coupled to a dilaton \(\Phi\) as proposed
by de Paula, Frederico, Forkel and Beyer [7]:

\[ S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( -R + \frac{1}{2} g^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi) \right) \]  

(1)

where \( \kappa \) is the five-dimensional Newton constant and \( V \) is a still general potential for the scalar field. Then we restrict the metric to the form

\[ g_{MN} = e^{-2A(z)} \eta_{MN} \]  

(2)

where \( \eta_{MN} \) is the Minkowski metric. We write the warp factor as

\[ A(z) = \ln z + C(z) \]  

(3)

Variation of the action (1) leads to the Einstein-dilaton equations for the background fields \( A \) and \( \Phi \):

\[ 6A'' - \frac{1}{2} \Phi'^2 + e^{-2A} V(\Phi) = 0 \]  

(4)

\[ 3A'' - 3A' - \frac{1}{2} \Phi'^2 - e^{-2A} V(\Phi) = 0 \]  

(5)

\[ \Phi'' - 3A' \Phi' - e^{-2A} \frac{dV}{d\Phi} = 0 \]  

(6)

Here the prime denotes the derivative with respect to \( z \).

By adding the two Einstein equation ones can get the dilaton field directly:

\[ \Phi' = \sqrt{3} \sqrt{A'^2 + A''} \]  

(7)

and by substituting (11A) in (1) or (1):

\[ V(\Phi(z)) = \frac{3e^{2A(z)}}{2} \left[ A''(z) - 3A'^2(z) \right] \]  

(8)

If \( A(z) = \log(z) + z^\lambda \) is set for simplicity, the asymptotic behavior of dilaton field \( \Phi \) in UV and IR limits can be obtained

\[ \Phi(z) = \begin{cases} \frac{z}{\zeta} & z \to 0 \\ \frac{2}{\sqrt{3} z^\lambda} & z \to \infty \end{cases} \]  

(9)

III. THE MODEL

In this section, we will establish the soft-wall AdS/QCD model first introduced in [3] under the dynamical background mentioned in Sec. II. The background geometry is chosen to be 5D AdS space with the metric

\[ ds^2 = g_{MN} dx^M dx^N = e^{-2A} (\eta_{\mu \nu} dx^\mu dx^\nu + dz^2) \]  

(10)

where \( A \) is the warp factor, and \( \eta_{\mu \nu} \) is Minkowski metric. Unlike [3] we introduce a background dilaton field \( \Phi \) that is coupled with \( A \) and \( \Phi \) can be determined in terms of \( A \) as shown in Sec. II.

In order to describe chiral symmetry breaking, a bifundamental scalar field \( X \) is introduced and we add an cubic term potential instead of the quartic term as shown in [10]

\[ S_5 = -\int d^5x \sqrt{-g} e^{-\Phi(z)} \text{Tr}[|DX|^2 + m_X^2 |X|^2 - \lambda |X|^4 + \frac{1}{4g_5^2} (F_L^2 + F_R^2)] \]  

(11)

where \( m_X = -3, \lambda \) is a constant and \( g_5^2 = 12\pi^2 / N_c = 4\pi^2 \). The fields \( F_{L,R} \) are defined by

\[ F_{L,R}^M = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i[A_{L,R}^N, A_{L,R}^M], \]

here \( A_{L,R}^N = A_{L,R}^M \xi^a, \text{Tr}[\xi^a \xi^b] = \frac{\pi}{2} \delta^{ab} \), and the covariant derivative is \( D^M X = \partial^M X - i A_{L,L}^M X + i X A_{L,R}^M \).

A. Bulk scalar VEV solution and dilaton field

The scalar field \( X \) is assumed to have a \( z \)-dependent VEV

\[ < X > = \frac{v(z)}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]  

(12)

We can obtain a nonlinear equation related \( A, \Phi \) and \( v(z) \) from the action (11)

\[ \partial_z (e^{-3A} e^{-\Phi} \partial_z v(z)) + e^{-5A} e^{-\Phi} (3v(z) + \frac{3}{4} \lambda v^2(z)) = 0 \]  

(13)

If \( A \) and \( \Phi \) are given, this equation can be simplified to a second order differential equation of \( v(z) \)

\[ v''(z) - (\Phi' + 3A')v'(z) + e^{-2A}(3v(z) + \frac{3}{4} \lambda v^2(z)) = 0 \]  

(14)

where \( \Phi' = \sqrt{3} \sqrt{A'^2 + A''} \).

Firstly, we consider the limit \( z \to \infty \). If warp factor \( C(z) \) has a asymptotic behavior of \( z^2 \) as \( z \to \infty \), we can determine that \( v(z) \to \text{constant} \). This means that chiral symmetry restoration in the mass spectrum. But there are many controversies on whether such a restoration really exists [13] [14].

Secondly, we analyze the asymptotic behavior of \( v(z) \) as \( z \to 0 \). As shown in [13] [10], the VEV as \( z \to 0 \) is required to be

\[ v(z) = m_q z + \frac{\sigma}{\zeta} z^3 \]  

(15)

where \( m_q \) is the quark mass, \( \sigma \) is the chiral condensate and \( \zeta = \frac{\sqrt{3}}{2\pi} \).

We assume that \( C(z) \) takes the asymptotic form as \( z \to 0 \)

\[ C(z) = \alpha z^2 + \beta z^3 \]  

(16)
Then we can get the asymptotic form of $\Phi'$ in the IR limit. Considering the coefficients of $\frac{1}{z}$ and constant terms at the left side of Equation (14) should be eliminated as $z \rightarrow 0$, we can conclude that the cubic term of bulk field is necessary and a relation between $\alpha$, $\beta$ and $\lambda$ can be obtained

$$\lambda m_q \zeta = 4\sqrt{2\alpha}. \quad (17)$$

**B. A parametrized solution and parameter setting**

The equation (14) is a nonlinear differential equation and it is difficult to solve the VEV $v(z)$, so we choose to select a parametrized form of $v(z)$ that satisfies the asymptotic constraint instead of solving the equation directly.

We set the wrap factor as suggested in [7]

$$A(z) = \log z + \frac{(\xi A_{QCD} z)^2}{1 + \exp(1 - \xi A_{QCD} z)} \quad (18)$$

where $\xi$ is a scale factor and $A_{QCD} = 0.3$GeV is the QCD scale.

Also we assume the VEV $v(z)$ asymptotically behaves as discussed in previous section

$$v(z) = \begin{cases} \frac{\gamma}{z \rightarrow \infty} & \gamma \\ \frac{m_q \zeta z + \frac{\sigma}{\zeta} z^3}{z \rightarrow 0} & m_q \zeta + \frac{\sigma}{\zeta} z^3 \end{cases} \quad (19)$$

A simple parametrized form for $v(z)$ that satisfies asymptotic constraint (19) can be chosen as

$$v(z) = \frac{z(A + B z^2)}{\sqrt{1 + (C z)^6}} \quad (20)$$

The relations between $m_q$, $\sigma$, $\gamma$ and $A$, $B$, $C$, $\lambda$ as follows:

$$A = m_q \zeta, B = \frac{\sigma}{\zeta} B = \gamma \quad (21)$$

Using the data of meson mass data, pion mass and pion decay constant we can get a good fitting of $A$, $B$, $C$, $\lambda$ as follows:

$$A = 1.63 \text{MeV}, B = (157 \text{MeV})^2, C = 10, \lambda = \frac{4\sqrt{2} \zeta Q_{QCD}}{m_q \zeta \sqrt{1 + e}} \quad (22)$$

In next section we will use this parametrized solution to calculate scalar, vector and axial-vector meson mass spectra and compare them with experimental data.

**IV. MESON MASS SPECTRUM**

Starting from the action (11), we can derive the Schrödinger-like equations that describe the scalar, vector and axial vector mesons:

$$-\partial_z^2 S_n(z) + \left( \frac{1}{4} \omega'^2 - \frac{1}{2} \omega'' - e^{-2\Lambda} \left( 3 + \frac{3}{2} \lambda v(z) \right) \right) S_n(z) = m^2_n S_n(z) \quad (23)$$

where the prime denotes the derivative with respect to $z$ and $\omega = \Phi(z) + 3A(z)$ for scalar mesons, $\omega = \Phi(z) + A(z)$ for vector and axial vector mesons.

The meson mass spectrum is obtained through calculating the eigenvalues of these equations. We will calculate the eigenvalues of these equations numerically using the parametrized solution introduced in Sec. III.

Since the potential is complicated, we must solve the eigenvalue equation numerically. As suggest in [7], we set $\xi = 0.58$ for scalars and the numerical results are shown in Table I.

**TABLE I: Scalar mesons spectra in MeV.**

| $n$ | $f_0(\text{Exp})$ | $f_0(\text{Model})$ |
|-----|------------------|---------------------|
| 0   | 550              | 897                 |
| 1   | 980              | 1135                |
| 2   | 1350             | 1348                |
| 3   | 1505             | 1540                |
| 4   | 1724             | 1717                |
| 5   | 1992             | 1881                |
| 6   | 2103             | 2034                |
| 7   | 2134             | 2178                |

For vectors, the value of $\xi$ is chosen to be 0.88 and the numerical results are listed in Table II.

**TABLE II: Vector mesons spectra in MeV.**

| $n$ | $\rho(\text{Exp})$ | $\rho(\text{Model})$ |
|-----|-------------------|---------------------|
| 0   | 775.5             | 988.9               |
| 1   | 1465              | 1348                |
| 2   | 1720              | 1650                |
| 3   | 1909              | 1913                |
| 4   | 2149              | 2147                |
| 5   | 2265              | 2359                |

Sine $m^2 = m_{A_n}^2 - m_{V_n}^2 = g_5^2 e^{-2A_n} \omega v(z)$ tends to zero as $z \rightarrow 0$, it means the mass of vector and axial-vector mesons are equal at high values.

**V. CONCLUSION**

In this paper we incorporated chiral symmetry breaking into the soft-wall AdS/QCD model with a dynamical
background. Then a discussion about the asymptotic behavior of the VEV of the bulk field was presented and a chiral symmetry restoration was suggested in the IR limit. We also introduced a parametrized solution of the VEV of the bulk field and fitted the parameters by using the meson mass spectra and pion mass and its decay constant. The numerical solution of meson mass spectra and comparison with experimental data was also considered. The agreement between the theoretical calculation and experimental data is good.

There are some issues worthy of further consideration. The differential equation of the VEV \( v(z) \) needs further study, numerically and analytically, to obtain a more precise form of \( v(z) \). The mass spectra of nucleons and some more complicated properties of mesons and nucleons in this model will be our following work.

VI. ACKNOWLEDGEMENT

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