Creep characterization of power-law materials through pseudo-steady indentation tests and numerical simulations

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Abstract. A constant-indentation creep rate test (CICRT) has been carried out for an Al-Mg solid-solution alloy in the temperature range of 636-773 K. When a conical indenter is pressed into the specimen surface under a load condition of \( F = F_0 \exp(\lambda t) \) (\( F \) the indentation load, \( F_0 \) the initial load, \( \lambda \) the loading rate parameter, \( t \) the loading time), the indentation pressure and indentation creep rate approach constant values of \( p_s \) and \( \dot{\varepsilon}_{\text{in}}(s) \), respectively. The representative points of the deformation in the underlying material are defined on a contour line of the equivalent stress \( \sigma_C = C_1 p_s \), where \( C_1 \) is the so-called constraint coefficient of 1/3 reported by Tabor. The finite element simulation of a power-law material subjected to the CICRT shows that the relationship between the equivalent plastic strain rate \( \dot{\varepsilon}_T \) at these points and \( \dot{\varepsilon}_{\text{in}}(s) \) is \( \dot{\varepsilon}_T = C_2 \dot{\varepsilon}_{\text{in}}(s) \) and that \( C_2 \approx 1/3.6 \) in the case of a creep stress exponent of 3.0. The constitutive equation of \( \dot{\varepsilon}_T \) versus \( \sigma_C \) obtained from experimental data and the computed value of \( C_2 \) is in good agreement with that evaluated from conventional uniaxial creep tests.

1. Introduction

The instrumented indentation testing technique has been attracting attention as a method of determining mechanical properties in a selected region of a small specimen [1-4]. In this study, a constant-indentation creep rate test (CICRT) is carried out for an Al-Mg solid-solution alloy using a microindenter, and a finite element (FE) simulation of the CICRT is performed to examine the deformation behavior in the region beneath the indenter. We demonstrate here that the constitutive equation for the tensile creep of power-law materials can be robustly predicted by the CICRT. To this end, we discuss the following items:

- In the CICRT, the indentation creep rate and indentation pressure approach constant values after a transition period.
- Representative points in the deformed region that practically govern indenter velocity are defined, and the equivalent plastic strain rate at these points is obtained by FE simulation to clarify its relationship with the indentation creep rate.
- A constitutive equation for equivalent plastic strain rate versus equivalent stress at these points is derived and compared with that obtained from tensile creep.

2. Constitutive equation for pseudo-steady indentation creep

When a conical indenter is pressed into a specimen surface, it is known that the indentation pressure \( p \) and the flow stress of the specimen \( \sigma \) have the following relationship:
\( \sigma = C_1 \rho \), \hspace{1cm} (1)

where \( C_1 \) is the so-called constraint coefficient of \( 1/3 \) [1, 5]. In this paper, the representative points of the deformation in the underlying material are defined on a contour line of the equivalent stress \( C_1 \rho \). The equivalent stress at these points is referred to as the representative stress \( \sigma_r \). Moreover, the equivalent plastic strain rate at these points is named the representative strain rate \( \dot{\varepsilon}_r \). The indentation creep rate \( \dot{\varepsilon}_{\text{in}} \) is given in terms of the indenter displacement \( u \) and the indenter velocity \( u' \) as \( \dot{\varepsilon}_{\text{in}} = u'/u \) [1]. For compatibility, a linear relationship must hold between \( \dot{\varepsilon}_r \) and \( \dot{\varepsilon}_{\text{in}} \):

\[ \dot{\varepsilon}_r = C_2 \dot{\varepsilon}_{\text{in}}, \quad (2) \]

where \( C_2 \) is a constant that depends on the creep properties and the apex angle \( \theta \) of the indenter. In the steady-state deformation of metallic materials at high temperatures, the equivalent plastic strain rate \( \dot{\varepsilon} \) and the equivalent stress \( \sigma \) obey the power-law creep:

\[ \dot{\varepsilon} = A_2 (\sigma/E)^n, \quad \sigma = A_1 (\sigma/E)^{1/n}, \]  

where \( F \) is the indentation load, and \( A_2 \) and \( A_3 \) are material constants that depend on \( T, n, \) and \( \theta \). Assuming that \( F \propto u^2 \) is applicable in the indentation creep test, \( F \) can be expressed by \( F = F_0 \exp(\lambda t) \), where \( F_0 \) is the initial load, \( \lambda \) is the loading rate parameter, and \( t \) is the loading time. In this case, the time dependences of \( \dot{\varepsilon}_{\text{in}} \) and \( p/E \) are given by [6]

\[ \frac{p}{E} = \left[ 2A_2 (1-\exp(-\lambda nt)) \right]^{1/n}. \]  

Equations (4) and (5) indicate that \( \dot{\varepsilon}_{\text{in}} \) and \( p/E \) approach constant values after a transition period. Specifically, \( \dot{\varepsilon}_{\text{in}} = \dot{\varepsilon}_{\text{ini}}(s) \equiv \lambda /2 \) and \( p_s/E = (\lambda / (2A_2))^{1/n} \). When both parameters are kept constant, \( \dot{\varepsilon}_{\text{in}}(s) = A_2 (p_s/E)^n \) holds. Since \( \dot{\varepsilon}_r = C_2 \dot{\varepsilon}_{\text{in}}(s) \) and \( \sigma_r = C_1 p_s \), the constitutive equation for creep at the representative points, i.e., that obtained from tensile creep tests is given as \( \dot{\varepsilon}_r = A_1 (\sigma_r/E)^{1/n} \).

3. Experimental procedures and computational setup

Commercially available ingots of an Al-5.3 mol\% Mg alloy were cut into parallelepipeds. They were carefully shaped so that the specimen surface (5×10 mm²) becomes parallel to the corresponding bottom surface and then annealed in Ar gas at 773 K for 3.6 ks. Immediately before the tests, specimens were electropolished to remove the surface layer of up to about 40 \( \mu \)m in thickness. The CICRTs were carried out in Ar gas at temperatures ranging from 636 to 773 K using a microindenter. An indentation load \( F \) given by the function \( F = F_0 \exp(\lambda t) \) was applied, where \( F_0 = 0.29 \) N and \( \lambda = 0.5 \times 10^{-3} - 8.0 \times 10^{-3} \) s\(^{-1}\). Details of the experimental procedures have been reported elsewhere [3, 4].

Computational simulations of CICRTs were performed using an FE package, ABAQUS Standard (SIMULIA Inc.), combined with our original subroutines. The conical indenter was modeled by a rigid body, and the cylindrical material was modeled using four-node bilinear axisymmetric quadrilateral elements. Each element was assumed to be subjected to elastic deformation (\( E = 37.8 \) GPa [7]; Poisson’s ratio, \( \nu = 0.345 \) [8]) and to obey power-law creep; \( \dot{\varepsilon} = A \sigma^n \) (\( A = 1\times 10^{-6} \) MPa\(^{-3}\) s\(^{-1}\), \( n = 3.0 \)).
4. Results and discussion

4.1 CICRT using the microindenter

Figure 1 shows the time dependence of indentation creep rate $\dot{\varepsilon}_\text{in}$. $\dot{\varepsilon}_\text{in}$ sharply decreases immediately after loading then approaches a constant value $\dot{\varepsilon}_\text{in(s)} \approx \dot{\lambda}/2$ after a transition period $t_{\text{trans}}$. Under the same value of $\dot{\lambda}$, $\dot{\varepsilon}_\text{in}$ and $t_{\text{trans}}$ curves are similar regardless of test temperature $T$. When $\dot{\lambda}$ is smaller, it is found that $t_{\text{trans}}$ increases. For example, $t_{\text{trans}} \approx 400\,\text{s}$ for $\dot{\lambda} = 3 \times 10^{-3}\,\text{s}^{-1}$, $t_{\text{trans}} \approx 700\,\text{s}$ for $\dot{\lambda} = 1 \times 10^{-3}\,\text{s}^{-1}$, and $t_{\text{trans}} \approx 1000\,\text{s}$ for $\dot{\lambda} = 0.5 \times 10^{-3}\,\text{s}^{-1}$. These characteristics agree with those of equation (4). Figure 2 shows the time dependence of indentation pressure $p$ in the case of $\dot{\lambda} = 1 \times 10^{-3}\,\text{s}^{-1}$. In addition to $\dot{\varepsilon}_\text{in}$, $p$ also becomes a constant value after $t_{\text{trans}}$. Moreover, it is found that $p_s$ decreases with increasing $T$, and in this case $t_{\text{trans}}$ is about 700s regardless of $T$. These characteristics are consistent with those described by equation (5).

4.2 Relationship between representative strain rate and indentation creep rate

The FE simulation revealed that $\dot{\varepsilon}_\text{in}$ and $p$ approach constant values of $\dot{\varepsilon}_\text{in(s)}$ and $p_s$, respectively, after $t_{\text{trans}}$. The computational results are similar to the experimental results shown in Figures 1 and 2. When both parameters are kept constant, the indenter is surrounded by self-similar stress and strain fields as indentation creep proceeds [6]. This fact implies that the indenter tip can continuously detect the pseudo-steady indentation creep parameters at representative points, $\dot{\varepsilon}_r$ and $\sigma_r$.

Figure 3 shows the relationship of $\dot{\varepsilon}_r$ versus $\dot{\varepsilon}_\text{in(s)}$ obtained from the computational results. All data denoted by circles lie on a straight line that passes through the origin. The slope of this line corresponds to the proportionality constant $C_2$ of equation (2), $C_2 = 1/3.6$. Therefore, $\dot{\varepsilon}_r = \dot{\varepsilon}_\text{in(s)}/3.6$ holds for $n = 3.0$.

4.3 Predicting a constitutive equation for tensile creep

Circles in Figure 4 show the relationship between $\dot{\varepsilon}_\text{in(s)}$ and $p_s/E$ obtained from the CICRTs at $T=681$ K for $\dot{\lambda} = 0.5 \times 10^{-3} - 8 \times 10^{-3}\,\text{s}^{-1}$. All the circles lie on straight line A, and the constitutive equation for pseudo-steady-state indentation creep is expressed as $\dot{\varepsilon}_\text{in(s)} = 5.72 \times 10^5 \left(\frac{p_s}{E}\right)^{3.0}$. Next, let us verify that the constitutive equation for tensile creep can be predicted through the CICRTs. First, when the values of $p_s/E$ are multiplied by $C_1 = 1/3.0$, line A shifts to the left, becoming line B representing $\dot{\varepsilon}_\text{in(s)}$ versus $\sigma_r/E$. Second, when the values of $\dot{\varepsilon}_\text{in(s)}$ on line B are multiplied by $C_2 = 1/3.6$, line B shifts downward to become line C, expressed as $\dot{\varepsilon}_r = 4.18 \times 10^6 \left(\frac{\sigma_r}{E}\right)^{3.0}$. This
Representative Strain Rate, $\dot{\varepsilon}_r / 10^{-3} \text{s}^{-1}$.

Indentation Creep Rate, $\dot{\varepsilon}_{\text{in}}(s) / 10^{-3} \text{s}^{-1}$.

$\sigma = 10^2 10^1 10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4}$

$C_1 = \frac{1}{3.6}$

FE simulation

$\dot{\varepsilon} = 0.5 \times 10^{-3} \text{s}^{-1}$ $\dot{\varepsilon} \equiv \dot{\varepsilon}_{\text{in}}(s) / 3.6$ $\dot{\varepsilon}_r \equiv \dot{\varepsilon}_{\text{in}}(s)$

Indentation creep test

$T = 681K$

$A: \dot{\varepsilon}_{\text{in}}(s) \text{ vs. } p_i / E$

$B: \dot{\varepsilon}_{\text{in}}(s) \text{ vs. } \sigma_i / E$

$C: \dot{\varepsilon}_r \text{ vs. } \sigma_i / E$

$\sigma = 10^2 10^1 10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4}$

Figure 3 Representative strain rate versus pseudo-steady indentation creep rate.

Figure 4 Logarithmic plots of strain rate versus normalized pressure or normalized stress.

The line is in good agreement with the dotted line that represents the tensile creep test result for Al-Mg solid solution alloy reported in Reference [9]. This fact indicates that the constitutive equation for tensile creep can be robustly predicted by conducting CICRTs and using the known values of $C_1$ and $C_2$.

5. Conclusions

(1) The indentation creep rate and indentation pressure approach constant values after a transition period when indentation load given by the function $F = F_0 \exp (\lambda t)$ is applied.

(2) The FE simulation reveals that the representative strain rate $\dot{\varepsilon}_r$ is proportional to the indentation creep rate $\dot{\varepsilon}_{\text{in}}(s)$ as pseudo-steady indentation creep proceeds, that is, $\dot{\varepsilon}_r \equiv \dot{\varepsilon}_{\text{in}}(s) / 3.6$ for the creep stress exponent of 3.0.

(3) The constitutive equation for creep predicted through CICRTs is in good agreement with that obtained from conventional uniaxial creep tests.

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