Peculiarities of dissipative phenomena in coated YBCO tapes carrying constant current during flux creep

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Abstract. The effect of the flux creep on the dissipation phenomena in tapes based on YBCO leading to the essentially non-linear voltage-current characteristic of a superconductor is discussed. The obtained results are compared to the calculations made in the framework of the existing thermal stabilization theory based on the model assuming a jump transition from a superconducting state to a normal one. It is shown that the latter model incorrectly describes the dissipation modes, first of all, in a temperature range up to the critical temperature of the superconductor. It is proved that the type of nonlinearity of voltage-current characteristics has a significant effect on the dissipative phenomena in tapes in this temperature region. As a result, the allowable currents stably flowing in superconducting tapes may be higher than a priory defined critical current determined for continuously rising voltage-current characteristic. Correspondingly, the critical current of high-\(T_c\) superconducting tapes, which is determined using continuously increasing voltage-current characteristic, has no physical meaning. Therefore, fundamentals of the thermal stabilization theory must consider the real temperature dependence of the dissipated energy in high-\(T_c\) superconducting tapes, which depends on the nonlinearity of their voltage-current characteristic.

1. Introduction
The investigation of the onset and propagation conditions of different nature instabilities is the actual problem of the applied superconductivity. In particular, the thermal stabilization theory of superconducting composites plays a significant role in the formulation of principles that ensure the preservation of the superconductivity of the technical superconductors under the action of the external thermal disturbances. Its main fundamentals were formulated in the framework of the model based on a jump transition from the superconducting state to the normal state (the so-called critical state model, CSM) [1]. In this case, the problems arising are reduced to solving the heat transfer equation with a non-linear dependence of the heat source on temperature. Namely, according to the CSM, the energy dissipated in a superconducting composite/tape begins only when its temperature exceeds the so-called temperature of the resistive transition \(T_{cs}\) at which the transport current is equal to the critical current of composite/tape. Accordingly, the current begins to share between the superconducting core and the matrix at \(T > T_{cs}\). When the temperature of the composite/tape exceeds its critical temperature, the transport current flows only through the matrix, and the heat release depends only on its properties. However, the CSM describes the thermal states of a composite/tape under the assumption that the resistance of superconductor changes from zero to an infinitely high value when the critical current density is equal to transport current. At the same time, technical superconductors have a continuously increasing voltage-current characteristic (VCC) due to many reasons. First of all, the finite voltage generated in a superconductor may be caused by the thermally activated motion of the vortices. In this mode, known as the flux creep, the magnitude of the vortex activation energy influences the VCC.
shape of the superconductor. In other words, superconductors are in a resistive state at any finite transport current. Correspondingly, their differential resistance not only changes continuously but it can be less than the electrical resistivity of a matrix in accordance with the peculiarities of VCC increase. Therefore, the consideration of the real shape of VCC allows one to define more exactly the thermal stabilization conditions [2-5]. Meanwhile, some fundamental provisions of the existing thermal stabilization theory do not take into account the nonlinear character of a real VCC of technical superconductors. Thereby, the temperature features of the Joule dissipation in YBCO tapes with continuously increasing VCC are discussed below for the first time.

2. Models
Consider a superconducting tape with a width $b$ consisting of a superconductor with a thickness of $a_s$ and stabilizing silver and copper coatings with thicknesses $a_{ag}$ and $a_{cu}$, respectively, taking into account that $b >> a = a_s + a_{ag} + a_{cu}$. Assume that the constant external magnetic field $B$ has completely penetrated the tape and its variation in the longitudinal direction can be neglected; transport current $I$ flows in the tape with cross-sectional area $S = ab$ and self-field is small in comparison with the external magnetic field; the conductive heat flux in the cross-sectional area of the tape is much greater than the heat flux to the coolant and, thereby, non-uniformity of temperature and electric field over the cross-sectional area is negligible; the heat exchange takes place on the surface of the tape with a coolant having temperature $T_0$. Describe the VCC by a power equation of the form $E(J) = E_c(I/I_c)^n$. Here, $I_c(T, B)$ is the temperature-field dependence of the critical current of a superconductor determined at a priori defined value of the electric field $E_c$, $n$ is the creep exponent. Let the critical current of superconductor $I_c(T, B)$ is a linear function of the temperature, namely, $I_c = I_{c0}(T_{cb} - T)/(T_{cb} - T_0)$. Then the temperature dependence of the Joule dissipation in the superconducting tape is written as follows [1]

$$G_{CSM}(T) = \left(1/S\right)^2 \rho_k(T, B) \begin{cases} 1, & T > T_{cb} \\ \frac{(T - T_{cs})}{(T_{cb} - T_{cs})}, & T_{cs} \leq T \leq T_{cb} \\ 0, & T < T_{cs} = T_{cb} - (T_{cb} - T_0)I/I_{c0} \end{cases}$$

(1)

according to the CSM. Here, $I_0$ and $T_{cb}$ are the critical current of the tape at the coolant temperature and its critical temperature at a given external magnetic field, respectively; $\rho_k$ is the resistivity of the tape that is equal to

$$\rho_k(T, B) = \frac{(a_s + a_{ag} + a_{cu}) \rho_{ag}(T, B) \rho_{cu}(T, B)}{a_{cu} \rho_{ag}(T, B) + a_{ag} \rho_{cu}(T, B)}$$

where $\rho_{ag}$ and $\rho_{cu}$ are the resistivities of silver and copper, respectively.

According to the assumptions made above, the thermal and electrical states of the superconducting tape with continuously increasing VCC can be investigated on the basis of a zero-dimensional anisotropic continuum model. Therefore, the electric field $E$ is the solution of the following static equation

$$E = E_c \left( \frac{J_s}{J_c(T, B)} \right)^n = J_{ag} \rho_{ag}(T, B) = J_{cu} \rho_{cu}(T, B)$$

(2)

for the given current $I = JS$ with density $J$ and the temperature $T$. In equation (2), the voltage per unit length of the tape induced in the superconductor and coatings satisfies the Kirchhoff laws. In accordance with this laws, the transport current is the sum of the currents flowing in the superconducting core with density $J_s$, in the silver coating with density $J_{ag}$ and in copper coating with density $J_{cu}$. Then
Here, $\eta_s = a_s/a$, $\eta_{ag} = a_{ag}/a$ and $\eta_{cu} = a_{cu}/a$ are the volume fraction of superconductor, silver and copper, respectively.

According to equations (2) and (3), the electric field as a function of temperature $T$ for a given value of the current density $J$ is the solution of the transcendental equation

$$I_s \left( \frac{E}{E_c} \right)^{1/n} + \frac{E}{\rho_k} - J = 0$$

at $T < T_{cb}$, and is described by an equality

$$E = J \rho_k$$

at $T \geq T_{cb}$. Then one can find

$$G(T) = EJ = \begin{cases} 
\frac{I_{cb} E T_{cb} - T}{S T_{cb} - T_0} \left( \frac{E}{E_c} \right)^{1/n} + \frac{E^2}{\rho_k}, & T_0 < T < T_{cb} \\
\frac{E^2}{\rho_k}, & T \geq T_{cb}
\end{cases}$$

for the superconducting tape with the power VCC.

The formulae (1) and (6) demonstrate inevitable difference in the calculations of the energy dissipation performed in the framework of the both VCC models. It is based on the features of change with the temperature of the differential resistivity of composite superconductor. Indeed, it jumps from zero to an infinitely large value at $I = I_s(T)$ in the framework of CSM, as noted above. Therefore, there is no heat release ($G_{cm}(T) = 0$) at $I < I_s(T)$. A part of the applied current, which is equal to $I - I_s(T)$, start to flow in the matrix at $I > I_s(T)$ because the current sharing mechanism takes places at $T_{cs} \leq T \leq T_{cb}$ regardless of the electrical resistivity of the matrix [1]. Accordingly, the Joule losses increase monotonically with increasing temperature due to, first of all, the dropping dependence $I_s(T)$ at $T_{cs} \leq T \leq T_{cb}$. Finally, the Joule losses depend only on temperature dependence of $\rho_0(T)$ at $T > T_{cb}$. The differential resistivity of a superconducting composite with a real VCC increases continuously with temperature depending on the $n$-value at $T > T_0$. Therefore, the current sharing mechanism depends on the difference between resistivity of the superconductor and stabilizing matrix in the wider temperature range $T_0 \leq T \leq T_{cb}$. This feature leads to the corresponding term in the relation (6) that affect the non-zero values and form of $G(T)$ at $T_0 \leq T \leq T_{cb}$. Thus, models (1) and (6) are based on the different mechanisms of the current sharing.

Let us discuss the characteristic regularities of the dissipative states of superconducting tape in the framework of the formulated models (1) and (6), which must be considered in the analysis of the Joule losses in real superconducting tapes. The results of the numerical simulation of the heat release in a YBCO-based superconducting tape with two stabilizing coatings are presented below. Its geometrical parameters was taken equal to $b=0.2$ cm, $a_s=10^{-4}$ cm, $a_{ag}=17 \times 10^{-4}$ cm, $a_{cu}=45 \times 10^{-4}$ cm and $a_{cu}=10^{-2}$ cm, the density of the critical current of the superconductor was described by a linear temperature dependence, as above-mentioned, with the critical parameters $T_{cb} = 55$ K and $I_{c0}=200$ A at coolant temperature $T_0=15$ K and $I_{cb}=59$ A at coolant temperature $T_0=40$ K in an external magnetic field $B=10$ T. The parameters of the VCC was set equal to $E_c=10^6$ V/cm. To calculate the resistivity of the silver and copper the results of [6-8] were used taking $\rho_{ag}(273$ K)$=1.48 \times 10^6$ $\Omega$·cm for silver and
\[ \rho_{\text{cu}}(273 \text{ K}) = 1.55 \times 10^{-6} \ \Omega \cdot \text{cm} \text{ for copper with } \text{RRR} = \rho_{\text{Ag,cu}}(273 \text{ K})/\rho_{\text{Ag,cu}}(4.2 \text{ K}) = 100 \text{ in both silver and copper.} \]

3. Results

Figs. 1, 2 show the dependence of the dissipation energy on temperature calculated according to (1) and (6). In the framework of the approximation (6) calculations were performed for different values of parameter \( n \) and currents at \( T_0 = 15 \text{ K} \). Correspondingly, Fig. 1 depicts the effect of \( n \)-value on the values \( G_{\text{CSM}} \) and \( G \) at current that less than the critical current. Fig. 2 shows the temperature dependence of the dissipation energy at currents that less or exceeding the critical current of the tape at \( n = 22 \). The states describing by a power VCC are shown in Figs. 1, 2 by the solid lines. The dashed lines correspond to the states described by the CSM. The presented results demonstrate the following peculiarities of the energy dissipation in a superconducting tape with continuously increasing VCC.

![Figure 1. Influence of the \( n \)-value on the dissipated energy at \( I = 180 \text{ A} \).](image1)

![Figure 2. Temperature dependence of the Joule heat release at different currents: 1 – \( I = 160 \text{ A} \), 2 – \( I = 180 \text{ A} \), 3 – \( I = 199 \text{ A} \), 4 – \( I = 220 \text{ A} \).](image2)
There is temperature $T_r = T_{cs}+(T_{dh}-T_0)o$, at which the curves $G_{CSM}(T)$ and $G(T)$ are intersected. Here, $\omega = E, S / [I_o, \rho (T)] <= 1$. In the temperature range from 0 to $T_r$, the values $G(T)$ are nonzero, while the values $G_{CSM}(T)$ are zero at $T_0 < T < T_{cs}$. The difference between them increases with increasing current and decreasing $n$-value. These peculiarities are due to the small but finite value of the electric field existing in a superconducting tape with a real VCC in this temperature region.

As follows from (1) and (6), the induced electric field is equal to a priori defined critical value $E_c$ at $T_r$. Therefore, if the overheating of the tape exceeds the small value $(T_{dh}-T_0)o$, then the electric field becomes supercritical even at subcritical values of the transport current. In the temperature range $T_r < T < T_{dh}$, the values $G_{CSM}(T)$ are always higher than the corresponding values $G(T)$ at $T < T_{dh}$, as it follows from Figs. 1, 2. Using the relations (1) and (6), one may find that this regularity is observed for any finite $n$-value. Therefore, the Joule dissipation $G_{CSM}(T)$ calculated at $T_r < T < T_{dh}$ within the CSM will always exceed the corresponding values $G(T)$ existing in the superconducting tape with a real VCC. Moreover, according to Fig. 2, the Joule dissipation at the supercritical currents (the solid curve 4) calculated for a tape with continuously increasing VCC may be in a wide temperature range less than the Joule dissipation calculated within the framework of the CSM at the subcritical current (the dashed curve 3).

The above-discussed regularities are based on the change features in the differential resistivity of a superconductor with a real VCC both at the subcritical values of the electric field ($E < E_c$) and at their supercritical values ($E > E_c$). In these operating modes, according to Fig. 1, even at a very high but finite $n$-value, for example at $n=100$, the temperature dependences of the Joule heat release in the superconducting tape with a power VCC also have the mentioned features of its change. As a result, the model with a power VCC leads to more conservative values of the heat release in the temperature range $T_r < T < T_{dh}$. It is important to emphasize that these regularities will be also observed in superconducting composites based on low-$T_c$ superconductors for which $n > 50$. Therefore, the theoretical analysis of the thermal stability conditions of the technical superconducting composites performed within the framework of the CSM will inevitably lead to the underestimated values of the critical energies of the external thermal disturbances and the overestimated values of the irreversible propagation velocity of normal zone.

Thus, the subcritical and supercritical stable operating states may exist in superconducting tape with a real VCC. Calculations show that the supercritical states will significantly expand the range of the operating regimes of tapes with intensive cooling. The existence of the stable subcritical and supercritical regimes will lead to a new concept of the limiting current stably flowing in superconducting tape. Namely, for superconducting tapes with a real VCC not the critical current but the maximum allowable current (quench current) limits the operating currents in a superconducting composite. Their value are a result of a violation of the thermal balance between the Joule dissipation in the tape and the heat flux to the refrigerant. The existence of such current was demonstrated for the first time in [9] investigating the so-called current instabilities in superconducting composite with an exponential VCC.

In the region of the high overheating ($T>T_{dh}$), the dissipated energy calculated according to (1) and (6) does not differ from each other due to the simplifying assumptions made above. The dependences $G_{CSM}(T)$ and $G(T)$ have a kink at $T=T_{dh}$ since the critical current is equal to zero and all transport current will flow in the coatings. In this case, the Joule dissipation calculated in the framework of both models will be determined by the properties of coatings.

Let us estimate the temperature variation of the dissipated energy in the tape with a power VCC. First, consider the initial temperature area. At $T << T_{dh}$, the transport current in a tape is almost equal to the current in the superconducting core and, therefore, according to (2) and (3), one can obtain $E / E_c \approx \{ I / [I_o(1-\theta)]]^n, \theta = (T-T_0)/(T_{dh}-T_0) < 1$. Finding the logarithm of this relation, it is easy
to get \( \ln \left( \frac{E}{E_c} \right) = n \left[ \ln \left( \frac{I}{I_{c0}} \right) - \ln (1-\theta) \right] \). Expanding the term \( \ln(1-\theta) \) into the power series, rewrite the last relation as follows \( \ln \left( \frac{E}{E_c} \right) = n \left[ \ln \left( \frac{I}{I_{c0}} \right) + \theta + \theta^2 / 2 + \ldots \right] \). Then

\[
G(T) = E(T)J = J\varepsilon \left( \frac{I}{I_{c0}} \right)^n \exp \left[ n \left( \frac{T - T_0}{T_{cT} - T_0} + \frac{1}{2} \left( \frac{T}{T_{cT} - T_0} \right)^2 + \ldots \right) \right]
\]

Thus, the increase of the initial increment of the \( G(T) \) is exponential with the temperature. Moreover, the higher the \( n \)-value, the higher the \( \text{d}G/\text{d}T \). These conclusions confirm Fig. 1.

Let us estimate \( G(T) \) near the critical temperature. Let us rewrite equation (4) as follows \( i = j_i(\theta) e^{i\varepsilon} + e / \varepsilon \). Here, \( e = E/E_c \), \( i = I/I_{c0} \), \( \theta = T/T_c \), \( \varepsilon = I_{c0}\rho / (SE_c) \), \( j_i(\theta) = 1 - \theta \). Let us introduce a new function \( u = 1 - e^{i\varepsilon} \), which leads to equation \( j_i(\theta) = \frac{i}{(i\varepsilon)^{1/n}} \left( 1 - u \right)^{1/n} \). Expanding the factor \( (1-u)^{1/n} \) in the power series, one can obtain \( j_i(\theta) = \frac{i}{(i\varepsilon)^{1/n}} \left( u + \frac{u^2}{n} + \frac{n+1}{2n^2} u^3 + \ldots \right) \). Then it is easy to find \( E/E_c \approx i\varepsilon \left[ 1 - (i\varepsilon)^{1/n} j_i(\theta) / i \right] \) in the linear approximation with respect to \( u \). Therefore, according to this estimation

\[
G(T) \approx \frac{J^2 \rho_1(T)}{S^2} \left[ 1 - \left( \frac{I_{c0}}{SE_c} \right)^{1/n} \frac{I_1(T)}{I_{c0}} \right]
\]

Thus, near the critical temperature of a superconductor, the dissipated energy in the tape with a power VCC depends on the temperature dependences of the critical current of superconductor and the resistivity of the coatings. If \( \rho_m \sim \text{const} \), then the values of \( G(T) \) will increase proportionally to the decrease in the critical current of a superconductor. This regularity will be observed the better, the higher \( n \). At the same time, the resistivity of the coatings will have a more significant effect at the high operating temperature.

Figure 3. Peculiarities of the current sharing at \( I=70 \text{ A} \) and \( T_0=40 \text{ K} \).
To demonstrate the peculiarities discussed above, Fig. 3 shows the features of the current sharing between the superconducting core and the coating at $T_0=40$ K, the supercritical operating current $I=70$ A and different thickness of the copper coating. Here, the dash-dotted-dotted line shows the temperature dependence of the critical current of the tape, and the curves 1, 1' depict the currents in the superconducting core ($I_{c}$), curves 2, 2' and curves 3, 3' correspond to the currents in the silver coating ($I_{s}$) and copper coating ($I_{c}$), respectively. The corresponding heat release is shown in Fig. 4. The presented results of the calculations lead to the following conclusions.

![Graph showing the influence of the copper coating thickness on dissipation states of superconducting tape.]

**Figure 4.** Influence of the copper coating thickness on dissipation states of superconducting tape.

First of all, it is seen that the current in the superconducting core is always greater than the critical current of the superconducting tape, even when its temperature rises up to the critical temperature of a superconductor. Moreover, the current in the superconducting core is weakly dependent on the thickness of the copper coating. The thickness of the copper coating significantly affects the currents in the coatings. Namely, the current in the silver coating decreases, and in the copper coating increases with increasing thickness of the copper coating. Correspondingly, the dissipated energy in the superconducting tape increases exponentially with the temperature at low overheating ($T-T_0$), as demonstrates the inset to Fig. 4. However, at high overheating ($T-T_0$), a major role in the current redistribution in the tape plays the resistance of the coatings, as it follows from (7) and shows Fig. 3. As a result, the total heat dissipation in the tape decreases significantly with increasing thickness of the copper coating Fig. 4.

It is important to emphasize that the existing thermal stabilization theory is based on another current sharing mechanism. Transport current begins to flow in the coating when it exceeds the critical current of tape. As a result, the current redistribution between superconducting core and coating does not depend on coating resistance. Fig. 3 clearly demonstrates the difference in the current sharing described by the above-formulated models.

4. Conclusions
The flux creep has a significant effect on the dissipation phenomena in superconducting tapes. At small overheat ($T << T_{cr}$), when the main part of the current flows through the superconductor, the
dissipated energy increases exponentially with temperature. In the framework of the existing thermal stability theory, it increases linearly with temperature. As the temperature of the tape increases, the transport current gradually goes into the non-superconducting coatings, and the dependence $G(T)$ begins to increase with the temperature according to the law, which is practically proportional to the change of the critical current density of the superconductor with temperature. At the temperature region $T \sim T_{cB}$ and more, the nature of the change $G(T)$ depends on the temperature dependence of the resistivity of the stabilizing coatings.

As a result, the correct consideration of the dissipation phenomena in the thermal stabilization theory will inevitably lead not only to the quantitative differences from the results that follow from the existing theory but also to the qualitative new conclusions. First, the permissible current range will not be limited by a priori defined critical current of the superconductor since the stable supercritical regimes are possible in superconducting tapes. In tapes with a real VCC, the current sharing occurs in such a way that the current in the superconducting core is always greater than the critical current even under the supercritical conditions. Second, the analysis of the thermal stability conditions of the superconducting state with respect to the external thermal disturbances will lead not only to the most optimistic estimates of the permissible values of the perturbation energies initiating the thermal instability but also to their finite values in the supercritical current region. Third, the smaller the $n$-value, the smaller the velocity of the irreversible propagation of a normal zone along a superconducting composite according to the corresponding reduction of the dissipation energy.

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