Calibration simulation for stereoscopic optical systems using optical design software

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Abstract. To reconstruct three-dimensional (3D) structure of objects and measure their geometric parameters using stereoscopic imaging systems, it is necessary to implement a number of algorithms. For higher system efficiency, the choice of these algorithms and mathematical models should be taken into account at the stage of optical system design. We demonstrate the capabilities of optical design software to perform computer simulation of geometrical calibration for stereoscopic systems. The simulation allows comparison of mathematical models used for 3D reconstruction and estimation of 3D measurements errors caused by tolerances of optical elements, temperature variations and other factors. Using this technique, we analyze the design of prism-based stereoscopic system and show that the proposed ray tracing camera model considering pupil aberrations provides high measurement precision. The results of computer simulation are confirmed by experiments with the self-developed stereoscopic system.

1. Introduction
Modern optoelectronic systems extensively use digital image processing for image enhancement, data extraction and precise measurements. Furthermore, computational imaging allows retrieving information, which is difficult, costly or even impossible to obtain using conventional imaging [1,2]. At the same time, the optical system and the image processing pipeline are designed and optimized sequentially in the conventional methodology of optoelectronic system design [3,4]. This hampers the design process and makes it difficult to find the optimal design solution. As shown in [3], the joint optimization of the design parameters of the optical system and the parameters of the image processing algorithm leads to better results compared to the traditional approach.

Stereoscopic imaging systems implement a number of algorithms to reconstruct three-dimensional (3D) structure of objects and measure their geometric parameters. These algorithms include image enhancement and rectification, stereo matching, calibration and 3D reconstruction algorithms, which...
contain a mathematical model of image formation (also referred as “camera model”) [5–11]. Thus, the joint design approach for stereoscopic systems implies that the choice of these algorithms and camera model should be taken into account at the stage of optical system design and the evaluation of the entire system performance should be based on 3D measurement errors.

The optimal choice of camera model is often a key condition for precise measurements. It is a compromise between the sufficient description of the ray tracing through the optical system (which requires the model to have more parameters) and the reliability (the fewer number of parameters is better). On the one hand, the optical system may be designed with significant residual distortion and reduced cost if the camera model properly accounts for this distortion and allows its digital correction. On the other hand, if the residual distortion is large, its mathematical description may require many parameters, which leads to complicated calibration procedure, expensive calibration equipment and lower reliability. For complex optical systems that use diffraction gratings [5], mirrors [6,7] or prisms [8–11] to obtain stereoscopic images, the choice of the optimal camera model at the initial design stage is not obvious.

As shown in [12,13], the optical design software (e.g. Zemax) together with Matlab may be used for the computer simulation of the camera calibration procedure. In particular, this simulation was applied to assess the impact of temperature variations and tolerancing on the pinhole camera model parameters.

We demonstrate that the computer simulation using the optical design software allows to estimate systematic and random errors of 3D measurements and evaluate appropriateness of camera models and calibration equipment for the designed optical system. This simulation is necessary to implement the joint design approach for optical system, calibration equipment and software as shown in figure 1.

![Figure 1](image.png)

**Figure 1.** The framework of traditional and joint design strategies for optical system, calibration equipment and software.

In the traditional design approach, insufficient accuracy or poor reliability of the chosen camera model or the calibration procedure may be revealed after the prototype of the entire system has been assembled and tested. If the result is unsatisfactory, designers usually try to improve it by adding complex image processing and calibration procedures because repeating the design and prototyping of the optical system is too costly. Moreover, the results of a single prototype tests can not ensure their appropriateness in mass production. The joint design approach using computer simulation allows estimating the impact of tolerances for imaging system and calibration equipment, temperature variations, image noise and other factors on measurements errors. If the results of the computer simulation are unsatisfactory, it is possible to return to any design stage and change the design parameters of the optical system as well as the camera model.
In this work, we apply this technique to the design of the prism-based stereoscopic system and evaluate several camera models considering the impact of tolerancing on calibration accuracy.

2. Mathematical models and algorithms for 3D reconstruction

We consider the design of prism-based stereoscopic system capable to obtain two images of the object from different viewpoints on a single sensor (as shown in Figure 2). Such an optical system may be used in video endoscopes, so it should be miniature and has as few components as possible to fit in a small diameter of the probe. The usage of the prism leads to strong image distortion and pupil aberrations which cannot be completely corrected by lenses [14] and should be properly accounted for by a camera model. The peculiarities of this system make it a good example to illustrate the joint design approach.

![Figure 2](image)

As shown in [9–11], the camera model using the backward ray tracing through the prism provides better measurement accuracy for prism-based stereoscopic systems compared to simple pinhole models. In this paper, we adopt this model and propose some modifications to improve its performance.

The image acquired by the prism-based stereoscopic system consists of two parts obtained via two parts of the biprism. We further refer to them as “i-th image part” and “i-th prism part”, \( i = 1...2 \). To formulate the camera model, we define the vector of ray coordinates \( \mathbf{r} = (\mathbf{c}^T, \mathbf{v}^T)^T \) as the concatenation of 3D coordinates of the origin point \( \mathbf{c} \) and the direction vector \( \mathbf{v} \) and use \( \mathbf{l}_{i,a,i} = s_{a,i}(\mathbf{l}_{i-1,i}) \) notation to describe the refraction of each ray \( \mathbf{l}_{i,a,i} \) on the \( m \)-th surface of the prism for \( i \)-th image part. Each transformation \( s_{a,i} \) is derived according to the vector form of Snell’s law [10]. To find the ray coordinates \( \mathbf{l}_{i,a,i} \) in the object space for each image point \( \mathbf{p}_i \), the internal camera model for the main lens is applied to define ray coordinates \( \mathbf{l}_{i,a,i} \), transformations \( \mathbf{s}_{i,j} \) and \( \mathbf{s}_{2,j} \) are used to trace this ray through the prism and obtain \( \mathbf{l}_{i,j} \), and finally the ray coordinates are converted from the camera coordinate system (CS) to the world CS (WCS) \( \mathbf{l}_{i,j} \). Thus, the model takes into account the distortion and pupil aberration induced by the prism by means of the exact ray tracing through the prism.

The internal camera model used in [10] was formulated as the combination of the following basic transformations

\[
\mathbf{l}_{i,a,i} = \mathbf{P}^{-1}(\mathbf{p}_i) = \mathbf{F}^{-1} \circ \mathbf{D}^{-1} \circ \mathbf{A}^{-1}(\mathbf{p}_i),
\]

(1)

\[
\mathbf{x}' = \mathbf{A}^{-1}(\mathbf{p}_i) : \begin{cases}
  x'_1 = \frac{a_i - u_{0,i}}{f_i} (u - u_{0,i}) + \frac{v_i - v_{0,i}}{f_i} (v - v_{0,i}) \\
  y'_1 = \begin{cases}
    0, & \text{if } A = \text{int} \\
    \frac{a_i}{f_i} (u - u_{0,i}) + \frac{v_i}{f_i} (v - v_{0,i}), & \text{if } A = \text{ext}
  \end{cases}
\end{cases}
\]

(2)

\[
\mathbf{x}' = \mathbf{D}^{-1}(\mathbf{x}'_1) : \begin{cases}
  x'_1 = (1 + \rho_1 r_z^2 + \rho_2 r_z^4 + \rho_3 r_z^6) (x' + x'_1) \\
  y'_1 = (y'_1 + y'_1)
\end{cases}
\]

(3)

where \( \mathbf{p}_i = (u_i \cdot v_i)^T \), \( \mathbf{x}'_1 = (x'_1, y'_1) \), \( \mathbf{x}'_2 = (x'_2, y'_1) \), and \( r_z^2 = x'_2^2 + y'_2^2 \). The transformations \( A^{-1}, D^{-1}, F^{-1} \) are the inverse ones due to conventional definition of forward transformations from the object space to the image plane [15]. The intrinsic camera parameters \( f_u, a_u, u_0, v_0 \) describe the focal distance,
the pixel aspect ratio and the coordinates of the principal image point, correspondingly. This internal camera model is the pinhole one with the additional 2D transformation \( D^{-1} \) accounting for lens distortion. At the same time, it does not separately consider pupil aberrations of the lenses. To improve the results, we have transferred the radial distortion coefficients to denominator and added the radial polynomial to take into account the ray shift caused by pupil aberrations. Therefore, the improved model defines coordinates of each ray in the object space by coordinates in two planes \( z = 0 \) and \( z = 1 \) similar to the models in [15,16]. Thus, \( D^{-1} \) is defined as

\[
(x', y') = D^{-1}(x^i, y^i) = \left( \frac{1}{1 + \rho_1 t^2 + \rho_2 r^4 + \rho_3 r^6}, \frac{x^i}{y^i} \right) = \left( 1 + \rho_1 t^2 + \rho_2 r^4 + \rho_3 r^6 \right) \left( x^i \right) / \left( y^i \right).
\]

where \( x^i, y^i = (x_p, y_p)^T \). The sets of parameters \( \rho_1, \rho_2, \rho_3 \) and \( \rho_4, \rho_5, \rho_6 \) describe distortion and pupil aberrations of the lens correspondingly. The transformation \( F^{-1} \) is modified as follows

\[
1_{n,i} = F^{-1}(x'_p, y'_p) = \left( \left( x'_p, y'_p, 0 \right), \left( x'_p, y'_p, 1 \right) \right).
\]

We use 9 parameters for 3 faces of the prism and one more parameter for the refractive index \( n \). The complete vector of parameters \( k \) describing the previously used ray tracing camera model includes 4 intrinsic camera parameters, 3 parameters for distortion of the lens and 10 prism parameters [10]. We further note this model as “Original”. The first modification of this model with the radial distortion coefficients in denominator (“Improved_1”) has the same number of parameters. The second modification (“Improved_2”) is the first modification with added pupil aberration, so it includes 3 more parameters \( \rho_4, \rho_5, \rho_6 \).

The details about the reconstruction of 3D point coordinates from corresponding image points may be found in [11]. In most applications of stereoscopic measurement systems, the calculated 3D point coordinates are further used to measure the geometric parameters such as segment length, point-to-line distance, point-to-plane distance, surface area, etc.

3. Computer simulation

Optical design software is a key element of the stereoscopic imager design due to ray tracing, aberration analysis and optimization capabilities. It supports user-provided extensions and, therefore, may be adapted for different design tasks. For example, it was used to implement joint design approach for imaging systems [3] and simulate camera calibration [12,13]. In this paper, we consider the design of the self-developed miniature prism-based stereoscopic system and perform the computer simulation of calibration using Zemax optical design software. The designed system has the following characteristics: \( f/11 \), field of view \( 40^\circ \times 45^\circ \) in each channel, range of working distances 5-40 mm and 1/6” image sensor with 1920×1080 pixels.

We simulated the calibration procedure utilizing three flat calibration targets with 25×25 grid of marker points and 0.5, 1 and 2 mm distance between points. Each target was placed at 6 positions including ones perpendicular to the optical axis of the main lens (z-axis) and rotated by 30° around transverse axes (x, y) to cover the total range of distances from 8 to 32 mm. The coordinates of image points for all positions of calibration targets were calculated by forward tracing of chief ray from each point of target using Zemax. Thus, the optical design software is utilized to obtain “ground truth” data for the comparison of different calibration techniques. The collected data was used to calculate the parameters of three considered camera models. The details of the calibration procedure and the optimization to obtain vector of parameters \( k \) may be found in [10,11].

To simulate the acquisition of test sequences for measurements, the small and medium targets were positioned perpendicular to z-axis and shifted along it with 1 mm step. The coordinates of image points were used to calculate 3D coordinates using each of considered camera models. Thus, we obtained the distorted 3D grid, which can be used to assess the systematic errors of 3D coordinate measurements across the working volume. According to the usual applications of 3D measurement endoscopic systems for geometric measurements, we have chosen the uncertainty of segment length as the criterion to evaluate the error for these systems. Therefore, we divided the
obtained 3D data into zones according to the distance along z-axis and calculated mean and standard deviation (STD) of the segment length along x, y and z axes for every zone. The results correspond to systematic error caused by insufficient description of the optical system provided by considered camera models. The STD values actually show the differences in the measurement of segment length at the specified distance across the field of view. As shown in figure 3, using “Improved_1” slightly improves the measurement accuracy compared to “Original”. “Improved_2” allows to practically eliminate systematic errors. Hence, the separate description of pupil aberrations for the lens is indeed necessary for the designed system.

![Figure 3. The results of computer simulation: dependence of mean value (solid) and STD (dotted) of difference between reference and measured lengths of the 1 mm segment along x (left), y (center) and z (right) axes on the distance to the target.](image)

![Figure 4. The results of computer simulation: dependence of mean value (red) and STD (blue) of length measurement error for the 1 mm segment along x (left), y (center) and z (right) axes on the distance to the target. The results are given for “Original” (top) and “Improved_2” (bottom) camera models. The range of the results for individual trials is shown as color bands; mean and STD calculated for all measurement data from 50 trials are shown in solid lines.](image)

Afterwards, we have performed tolerance analysis of the designed system using Zemax. The obtained tolerances were used to generate 50 optical systems with random variations of the design parameters. Next, the computer simulation of calibration and measurements was repeated for each of these systems. The results for “Original” (top row) and “Improved_2” (bottom row) camera models are presented in figure 4. We calculated the dependence of mean and STD on z for each of 50 trials and showed the range of the results for individual trials as color bands: the red band for mean and the
blue one for STD. The mean and STD calculated for all measurement data from 50 trials are shown in solid lines.

If we compare the results averaged for all data from 50 trials, we can see that the improved model demonstrates better stability than the original one. Moreover, in 10% of trials the usage of the original model led to large errors for x and z segments. The separate polynomial for pupil aberrations allows adapting the improved model to radial deviations of pupil aberrations caused by tolerances, which improves measurement accuracy. The increase of systematic error for both models is also caused by decentering of lenses leading to non-radial distortion which is not present in the considered camera models.

The results of the computer simulations indicate that the considered design of miniature prism-based stereoscopic system is appropriate. Although it has significant distortion and pupil aberrations, they are properly taken into account by the improved version of the ray tracing camera model. The parameters of this model can be successfully calculated using the standard calibration procedure. The model is robust to deviations of the parameters and positions of optical elements in the specified tolerance ranges.

4. Experiments
The prototype of the designed prism-based stereoscopic system has been manufactured and used for experiments to verify the computer simulation results. All details of calibration procedure were the same as for the computer simulation. The calibration targets contained chessboard pattern produced by chrome etching on glass with inaccuracy about 1 μm. The white-light LED was used to illuminate the calibration targets. Since the implemented calibration method does not require precise positioning of targets, no specific equipment was necessary to capture calibration image sequences. To capture test sequences, the linear translation stage was used to shift target along z-axis. The example images of the calibration target are shown in figure 5.

![Figure 5](image-url)

**Figure 5.** The example images of the calibration target captured by the prototype of the designed prism-based stereoscopic system.

![Figure 6](image-url)

**Figure 6.** The results of experiments with the manufactured system: dependence of mean value (solid) and STD (dotted) of difference between reference and measured lengths of the 1 mm segment along x (left), y (center) and z (right) axes on the distance to the target.

The obtained image sequences were processed to calculate image coordinates for each chessboard node. We chose orientation of the calibration targets so that the grid lines were approximately parallel
to $x$ and $y$ axes. Thus, we used the distance between chessboard nodes as $x$ and $y$ segments. Points to measure the segment along $z$-axis were taken either from the next image of the test series, i.e., when the target was shifted by 1 mm. The results obtained by calibration and measurements using three considered camera models are presented in figure 6.

The experimental results demonstrate the same trends as computer simulation, the usage of “Improved_2” model significantly improves measurement accuracy, especially for $x$ and $y$ segments. The advantage of “Improved_2” model is vivid for shorter distances where the impact of random errors is lower and the measurement accuracy is mainly determined by systematic errors. Thus, we can conclude that the optimal choice of camera model by means of the computer simulation is in agreement with the results of experiments.

5. Conclusion
We have utilized the computer simulation using the optical design software to estimate systematic errors of 3D measurements and evaluated camera models for the designed prism-based stereoscopic system. The results of the computer simulation have been confirmed by the experiments with the manufactured system. Thus, we have demonstrated the use of this simulation as a basic and powerful tool for implementation of the joint design approach. The proposed simulation may be applied for Monte Carlo simulation to estimate the impact of tolerances, temperature and other deformations and many other factors on 3D measurement accuracy. This approach may be also used for the comparison of calibration procedures not only for measurements using stereoscopic systems, but also for other tasks where a camera model is not perfectly defined and calibrated.

In this paper, we analyzed the dependence of mean and STD of 3D measurement error on the distance to the target. It helps to evaluate error trends and measurement capabilities of the system, but does not suit well as a merit function. Hence, we suppose to use integral criteria such as weighted squared mean or confidence interval over the working volume in the future. Also, the impact of systematic errors induced by non-ideal camera models and other factors should be compared with random measurements errors. This may be performed using Monte Carlo analysis and the proposed computer simulation by adding random values to calculated “ground truth” image coordinates.

We focused on the relationship between the optical system design and the camera calibration. Therefore, we discuss in detail only distortion and pupil aberrations as these aberrations are included in the mathematical models necessary for calibration. It allows estimating systematic error of 3D measurements caused mainly by non-optimal camera model. Other aberrations as well as noise lead to random errors of corresponding points matching. Analysis of the dependence between these aberrations and random errors is a separate task related to accurate and long-term modeling of various stereo matching algorithms in various conditions. For all measurements, we used grayscale images, so that chromatic aberrations led to additional blurring. Distortion and pupil aberrations in the model are also considered for grayscale images, i.e. they are averaged over wavelengths.

For color images, chromatic aberrations can be compensated at the image rectification stage. One may apply multi-spectral calibration of RGB channels [11] and then use rectification for ray tracing model [17].

The important direction for future work is assessing the impact of other aberrations affecting image quality on 3D measurement accuracy. The main challenge is developing a method to estimate the uncertainty of image points based on the optical system parameters. This will make possible to complete the implementation of the joint design approach for stereoscopic systems and optimize optical system parameters directly using merit function based on 3D measurement uncertainty.

6. References
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