Authenticated Multiparty Quantum Key Agreement Protocol Based on Grover’s Algorithm

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In the quantum key agreement protocols, some attackers can impersonate legal participants to participate in the negotiation process and steal the agreement key easily. This is often overlooked for most quantum key agreement protocols, which makes them insecure. In this paper, an authenticated multiparty quantum key agreement protocol based on quantum search algorithm is proposed. In the protocol, combining classical hash function with identity information, the authentication operation conforming to the characteristics of search algorithm is designed. In addition, we give a detailed security analysis, which proves that the protocol is secure against common attacks and impersonation attacks. Meanwhile, single particles, which are used as information carries, and single-particle measurement make our protocol feasible with existing technology.

PACS numbers: Valid PACS appear here

I. INTRODUCTION

Key agreement is an important branch in the field of cryptography, which aims to generate temporary session keys in a communication network composed of two or more participants to achieve secure communication. In an agreement protocol, each participant contributes equally to the generation of the session key, that is, to achieve fairness. Since the first key agreement protocol [1] was proposed, many researchers paid much attention to it. However, as the concept of quantum computer was put forward, researchers found that classical cryptography became extremely vulnerable to quantum computation with powerful computing power. For example, Shor [2] proposed the famous quantum factorization algorithm, which can realize prime factorization of large numbers in polynomial time. In order to ensure the security of communication, anti-quantum cryptography has been studied to resist the attack of these quantum algorithms. A major anti-quantum cryptography is quantum cryptography, whose security is based on quantum-mechanics principles, such as Heisenberg’s uncertainty principle, non-cloning principle, etc., rather than classical mathematical problems. Since the first quantum key distribution protocol [3] was proposed in 1984, people try to use quantum cryptography to solve some security tasks, including quantum key distribution(QKD) [4], quantum secret sharing(QSS) [5–8], quantum private query(QPQ) [9–12] and quantum secure multi-party computation(QSMC) [13–16].

Similarly, quantum key agreement(QKA) also drawn a lot of attention. In 2004, Zhou et al. [17] proposed the first two-party QKA protocol, which was designed based on the correlation of measurement results of EPR. Unfortunately, this protocol is insecure, as shown in Ref. [18]. That same year, Chong and Hwang et al. [19] proposed the first fair and secure two-party QKA protocol based on BB84. Afterwards, researchers expands the number of negotiators from two to multiple parties to fit the actual scenarios. Shi et al. [20] proposed the first multiparty QKA protocol with Bell states and entanglement exchange in 2013. But in the same year, Liu et al. [21] pointed out that the protocol was unfair, then proposed a new MQKA protocol with single particle. Later Sun et al. [22] introduced two unitary operations and proposed the circle-type MQKA to improve the execution efficiency. Since then, many scholars have used various properties of quantum mechanics to design a few subtle MQKA protocols [23–25]. However, these protocols do not consider the authentication, which makes them vulnerable to impersonation attacks. Although in some MQKA protocols, authentication of classical channels has been required to prevent classical messages from being tampered, message authentication is different from identity authentication. Users can use a variety of ways to secure messages from tampering, such as broadcasting. Therefore, in designing a secure quantum key agreement protocol, the authentication of the participants’ identities should be considered as in other quantum cryptographic protocols [26–29] with authentication capabilities.

Considering existing technical conditions, we propose an authenticated MQKA protocol based on quantum search algorithm. Concretely, we utilize classical hash function to map participant’s certificate and random string into a bit string of fixed length, which is only known to the participant and semi-trusted party. Then, we design the corresponding identity operations, which can not break the properties of the quantum search algorithm. When the participants execute the protocol honestly, they can get the correct negotiation key simultaneously. According to our analysis, our protocol is secure against both common attacks including external attacks and participant attacks and impersonation attacks.

The rest of this paper is organized as follows. In Sec. II, we introduce some preliminaries briefly. Then, the proposed authenticated quantum key agreement protocol

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is described in detail in Sec. III, and for clearly, we give an example. Its security is analyzed in Sec. IV. Finally, a brief conclusion is given in Sec. V.

II. PRELIMINARIES

Let us start with a brief review of quantum search algorithm [30]. Suppose that we want to search the target $|φ_{mn}\rangle = |mn\rangle$, $m, n \in \{0, 1\}$, in the database of a set of two-qubits states, i.e., $|ϕ_{xy}\rangle = ((|0\rangle + (-1)^{y}|1\rangle))(|0\rangle + (-1)^{x}|1\rangle)$, $x, y \in \{0, 1\}$. In order to search the target, two specific unitary operations need to be performed on $|ϕ_{xy}\rangle$. Namely the phase reversal operation $U_{mn} = I - 2|φ_{mn}\rangle\langle φ_{mn}|$ and the amplitude amplification operation $V_{xy} = 2|ϕ_{xy}\rangle\langle ϕ_{xy}| - I$. After executing these two unitary operations, we get

$$V_{xy}U_{mn}|ϕ_{xy}\rangle = |φ_{mn}\rangle.$$  

(1)

Evidently, $|φ_{mn}\rangle \in \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. In this paper, since the global phase has no effect on the results, it can be ignored.

In addition, the unitary operation $U_{mn}$ in quantum search algorithm has two good properties, which have been used to design some quantum cryptographic protocols [31–34]. We suppose that a total of $r$ operations of $U_{mn}$ are performed on a two-qubits state. On the one hand, when the number of $r$ is odd [31], there is:

$$U_{m_1,n_1} \cdots U_{m_r,n_r}U_{m_1,n_1} = U_{mn},$$  

(2)

where $m = m_1 \oplus m_2 \oplus \cdots \oplus m_r$ and $n = n_1 \oplus n_2 \oplus \cdots \oplus n_r$, the symbol $\oplus$ indicates bitwise Exclusive OR. In combination with (1), we know that the deterministic measurements can be obtained by single-particle measurement with basis $MB_Z = \{|0\rangle, |1\rangle\}$ at last. When the number of executions is even, there is:

$$U_{m_1,n_1} \cdots U_{m_r,n_r}U_{m_1,n_1}|ϕ_{xy}\rangle = |ϕ_{mn}\rangle,$$  

(3)

where $m = x \oplus m_1 \oplus m_2 \oplus \cdots \oplus m_r$ and $n = y \oplus n_1 \oplus n_2 \oplus \cdots \oplus n_r$. Clearly, $|ϕ_{mn}\rangle \in \{|+, +\rangle, |+, -\rangle, |-, +\rangle, |-, -\rangle\}$, Therefore, at this point, the measurements can be obtained by single-particle measurement with the basis $MB_X = \{|+, -\rangle\}$.

In our protocol, the participant encodes his private key by unitary operation $U_{mn}$. In addition, to assure that the protocol satisfies the characteristics of quantum search algorithm after identity encoding, we design the identity encoding operations as $U_{00}$ and $U_{01}U_{10}$ respectively. In the protocol, the participant always encodes his identity information after the private key encoding, that is, the encoded quantum state is $U_{00}U_{mn}|ϕ_{xy}\rangle$ or $U_{01}U_{10}U_{mn}|ϕ_{xy}\rangle$. Furthermore, Zhang et al. [16] found that $U_{m_1,n_1}U_{m_2,n_2} = X_{m_1 \oplus m_2 \oplus m_1 \oplus n_2}$. So the private key stays the same when the identity encoding key is $U_{00}$. When the identity encode is $U_{01}U_{10}$, on the basis of (2), $U_{01}U_{10}U_{mn} = U_{mn}$, where $\bar{m} = m \oplus 1$, $\bar{n} = n \oplus 1$, which means the private key is flipped once.

According to the theoretical tools above, we design an authenticated quantum key agreement protocol based on quantum search algorithm.

III. AUTHENTICATED QUANTUM KEY AGREEMENT PROTOCOL

In the protocol, all the quantum channels are public. And participants need to send some classical messages via a classical channel, in which nobody is able to alter the transmitted message. That is to say, this transmitted classical message is required to be authenticated. For example, the public classical channel can be achieved by a radio broadcast, or an advert in a newspaper. It is worth noting that message authentication is different from identity authentication. Everyone includes the adversary may announce a public message in the agreement. So, we still need to verify the identity of every participant.

Suppose that there are $N - 1$ participants, $P_1, P_2, \ldots, P_{N-1}$, want to negotiate an agreement key $K$, where $K = S_1 \oplus S_2 \oplus \cdots \oplus S_{N-1}$, and $S_i$ is $P_i$’s private key with length of $2n$. Each participant has a certificate $ID_i$ of length $l$. In order to ensure the legitimacy of the participant’s identity, it is necessary for $P_i(i = 1, 2, \cdots, N - 1)$ to complete the identity authentication with the help of a semi-trusted third party, $P_0$, who shares $k_i$ with $P_i$. Then a secure agreement key can be obtained after executing the following steps, which can be seen in Fig. 1.

Step 1: $P_0$ and $P_i$, $i = 1, 2, \cdots, N - 1$, generate a random sequence $r_0$ and $r_i$ respectively and declare them. Then $P_0$ selects a hash function $f$ : $2^n \rightarrow 2^n$ and declares it. $P_i$ and each participant calculate their authenticated message $h_i = f_{k_i}(ID_i \parallel r_i \parallel r_0)$, where $\parallel$ denotes string concatenation.

Step 2: Each participant $P_i$, $i = 1, 2, \cdots, N - 1$, generates a random bit sequence $L_i = (l_{i,1}, l_{i,2}, \cdots, l_{i,2n})$ and $B_i = (b_{i,1}, b_{i,2}, \cdots, b_{i,2n})$ with length of $2n$. In the process with $P_i$ as initiator, an ordered sequence $T_i$ of $n$ two-qubits states is prepared by $P_i$ according to $L_i$:

$$T_i = (|ϕ_{l_{i,1},1,3,i}\rangle, |ϕ_{l_{i,3,1,i}}\rangle, \cdots, |ϕ_{l_{i,2n-1,1,2n,i}}\rangle),$$  

(4)

where, the $t$-th quantum state is $|ϕ_{l_{i,t-1},1,2n,i}\rangle \in \{|ϕ_{00}\rangle, |ϕ_{01}\rangle, |ϕ_{10}\rangle, |ϕ_{11}\rangle\} = \{|+, +\rangle, |+, -\rangle, |-, +\rangle, |-, -\rangle\}$.

Step 3: The identity encoding is performed by $P_i$ on the $t$-th quantum state of the sequence $T_i$ according to the values of $h_{i,t}$ and $b_{i,2t-1}$. Concretely, when $h_{i,t} \oplus b_{i,2t-1} = 0$, $P_i$ performs $U_{00}$; otherwise, he performs $U_{01}$ (that is, the flip operation). Then the encoded sequence is denoted as $\hat{T}_{i,IB_i}$, which is sent to the next participant $P_{i+1}$, where $\oplus$ represents addition modulo $N$.

Step 4: When $P_{i+1}$ receives the sequence $\hat{T}_{i,IB_i}$, he will perform the corresponding operation to encode his own private key. Specifically, the sequence $T_{i,IB_i}$ is obtained by performing unitary operation $U_{h_{i,t},IB_i,2t-1,1+ib_{i,2t}}$. 


on the \( t \)-th quantum state. Then, \( P_{\text{E1}} \) performs identity encoding similar in Step 3, and finally sends the encoded particle string \( T_{i,\text{E2}} \) to participant \( P_{\text{E2}} \).

**Step** \((g + 3) (g = 2,3,\ldots,N-1)\): In the same way as step 4, participant \( P_{g} \) encodes his private key \( s_{\text{Eg}} \) on the \( t \)-th state of the sequence \( T_{i,\text{Eg+1}} \). Subsequently, he encodes his identity information as in step 3. After that, he sends the encoded sequence \( \tilde{T}_{i,\text{Eg+1}} \) to participant \( P_{\text{Eg+1}} \). Notice that \( P_0 \) knows the hash values of all the participants. When the sequence of particles is transmitted to \( P_0 \), he calculates \( h_{0,t} = \oplus_{i=1}^{N-1} h_{i,t} \). Based on the result, \( P_0 \) performs his identity encoding on the sequence. That is to say, if \( h_{0,t} = 0 \), he performs \( U_{00} \); Otherwise, he performs \( U_{01} U_{10} \).

**Step** \((N + 3)\): When all the participants \( P_j(i = 1,2,\cdots,N-1) \) receive the sequence \( \tilde{T}_{i,\text{E1}} \), where \( \oplus \) denotes subtraction module \( N \), they publish the random bit string \( B_i \) in random order. Then, \( P_j \) calculates \( B_{2t-1} = \oplus_{i=1}^{N-1} h_{2t-1} \) for \( t = 1,2,\cdots,n \) and performs different operations according to different result.

1. When \( B_{2t-1} = 0 \), \( P_j \) measures with basis \( MB_X \) directly to get the measurement result \( \tilde{\varphi}_{w_{i,2t-1},w_{i,2t}} \), then he can extract the session key:

\[
k_{2t-1}k_{2t} = s_{i,2t-1}s_{i,2t} \oplus w_{i,2t-1}w_{i,2t} \oplus l_{i,2t-1}l_{i,2t}.
\]

2. When \( B_{2t-1} = 1 \), according to the classical bit sequence \( L_i \), the unitary operation \( V_{i,2t-1} \) is performed on the \( t \)-th two-qubits state in the quantum sequence, then the particles are measured with the basis \( MB_Z \) to obtain the result \( \tilde{\varphi}_{w_{i,2t-1},w_{i,2t}} \). The agreement key is extracted as follows:

\[
k_{2t-1}k_{2t} = s_{i,2t-1}s_{i,2t} \oplus w_{i,2t-1}w_{i,2t} \oplus 11.
\]

**Step** \((N + 4)\): All the participants choose \( \delta \) samples to detect whether malicious or forged participants exist. Specifically, \( P_i \) randomly selects \( \lceil \frac{\delta}{N-1} \rceil \) positions, and declares these chosen positions publicly. Then, he requires the other participants \( P_j (j \neq i) \) to announce the corresponding part of \( K_i \). Afterwards, \( P_i \) calculates the error rate based on his measurements and the public information of other participants. If the error rate exceeds a certain threshold, the protocol is abandoned. Otherwise, the other participants \( P_j (j \neq i) \) perform similar actions. It should be noted that there are no common elements in the test samples selected by all participants. Finally, the remaining particles form their agreement key.

To illustrate our protocol more clearly, a three-party case \((i.e., N = 4)\) is taken as an example. In this case, there are three participants \( P_1, P_2, \) and \( P_3 \), who respectively hold secret messages with length of \( n \) \((i.e., n = 4)\), \( S_1 = 01101011, S_2 = 01000100, \) and \( S_3 = 10110001 \) and identity information, \( ID_1 = 010110, ID_2 = 001101, ID_3 = 100011 \). By the proposed protocol, they try to agree on a session key, \( K = S_1 \oplus S_2 \oplus S_3 \). The concrete values used in the protocol are shown in Table I.

In Step 1, set the random strings generated to \( r_1 = 1100, r_2 = 1111, r_3 = 0010 \). In addition, \( P_0 \) generates \( r_0 = 0110 \). Then, based on \( L_1 \) and \( B_1 \), \( P_1 \) prepares the particle sequence \( T_{1} \), which is in the state \( |\tilde{\varphi}_{00}| |\tilde{\varphi}_{01}⟩ |\tilde{\varphi}_{11}⟩ |\tilde{\varphi}_{01}⟩ \). In step 3, \( P_1 \) performs the unitary operations \( U_{00} \otimes U_{00} \otimes U_{01} U_{10} \otimes U_{01} U_{10} \) to encode his identity information. Afterwards, \( P_1 \) transmits the new sequence \( \tilde{T}_{1,2} \) to \( P_2 \). When \( P_2 \) receives the sequence from \( P_1 \), he encrypts his private key and identity information by performing the unitary operations in step 4, and sends to \( P_3 \). Similarly, \( P_3 \) also performs encoding operations. It is worth noting that \( P_3 \) sends the sequence \( \tilde{T}_{1,0} \) to \( P_0 \). When \( P_0 \) receives this sequence, he calculates \( h_0 = 1000 \), which means his operations are \( U_{01} U_{10} \otimes U_{00} U_{00} \). After that, \( P_0 \) sends the encoded sequence to \( P_1 \). Obviously, the transmission process of particle sequences, which are prepared by \( P_2 \) and \( P_3 \), is similar to the above process. In step 7, when all the participants receive the travelling particles \( \tilde{T}_{1,1}, \tilde{T}_{2,2}, \tilde{T}_{3,3} \), they declare the random strings \( B_i (i = 1, 2, 3) \) in random order. \( P_1, P_2, P_3 \) calculates \( B_{2t-1} = 0010 \). Therefore, \( P_1, P_2, P_3 \) performs \( I \otimes I \otimes V_{11} \otimes I (I \otimes I \otimes V_{01} \otimes I, I \otimes I \otimes V_{11} \otimes I) \). After that, they measure with appropriate measurement basis to obtain \( K_1 = K_2 = K_3 = 10011110 \). Finally, in step 8, all the participants \( P_1, P_2, P_3 \) choose \( \delta \) samples to perform identity detection by calculating the error rate of the checking particles. If the error rate is below the threshold, authentication is passed and the remaining particles are the raw agreement key.

### IV. SECURITY ANALYSIS

In this section, we will analyze the security of the proposed protocol. It not only proves the protocol is secure against common external and participant attacks, but also proves that impersonation attack is invalid.
A. External Attacks

An external attacker, Eve, wants to steal the session key $K$, the private key $S_i$ of each participant $P_i \,(i = 1, 2, \ldots, N - 1)$ and his $k_i$ shared with $P_0$, without being detected by legitimate participants. In the above protocol, the public information is identity certificate $ID_i$, random string $r_i$, and hash function $f$. However, since the participant $P_i$ does not disclose his hash value $h_i$, Eve cannot infer $k_i$ from these public information. During the process of the protocol, each participant performs three operations: particle preparation, encoding private key and identity information, and single particle measurement. Obviously, the disclosure of participants’ private key only occurs after the encoding operations, so Eve’s attacks mainly take place in the transmission of the particle sequence. Next, we show that the protocol is secure by analyzing three common external attacks in detail. In addition, Eve may secretly relay and possibly alter communications between parties who think they are communicating directly. So, we consider the impersonation attack by external attacker as well.

1. Intercept-resend Attack

In the intercept-resend attack, Eve firstly intercepts the particle sequence sent from $P_j$, then sends another sequence to $P_{j+1}$. Usually, Eve has two different strategies. One is that Eve measures the intercepted particle sequence and then re-prepares the sequence to send to $P_{j+1}$ based on the measurements. In this way, Eve hopes to obtain the private key of every participant without being detected. Thereby, he can infer the negotiation key by calculating the sum of these values. However, this is impossible. In the protocol, the carrier particles after different encoding operation numbers belong to two sets of non-orthogonal states, which are in

$$
\{|+, +\rangle, |-+, -\rangle, |-, -\rangle, |+, -\rangle\},
$$

or

$$
\frac{1}{2}(00\rangle - |01\rangle - |10\rangle - |11\rangle),
\frac{1}{2}(00\rangle + |01\rangle - |10\rangle + |11\rangle),
\frac{1}{2}(00\rangle + |01\rangle + |10\rangle - |11\rangle),
$$

where, the second set (8) can be converted into $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ after the unitary operation $V_{xy}$. Obviously, Eve cannot know the actual operation of the participant because the identity encoding is determined by both hash values $h_j$ and random numbers $B_j$. Even if the participants announce the random numbers at the end of the protocol, Eve cannot extract identity encoding from them. Therefore, he cannot perform the operation $V_{xy}$ in the case of odd times encoding, then select correct measurement basis to get the participant’s private key. Eve just has to do random operations to get information. Evidently, the probability that Eve selects a right measurement basis is approximately 50%, the chance of getting all the bits right is small when the number of bits is large. Hence, this attack can be easily detected in step $N + 4$. In a word, Eve cannot get the private key of each participant without being detected in this way, and cannot get the final key $K$.

The other method is Eve intercepts the traveling particle sequence and replaces it with a fake particle sequence. Apparently, errors are inevitably introduced. Even though, Eve hopes to infer $k_j$ of the participant. Next, we show this is invalid. Specifically, after all the participants $P_j \,(j \neq i)$ encodes their secret in the particles, Eve intercepts sequence $\overline{T_{i\rightarrow j}}$ and then sends the fake particles to $P_i$. The participants announce the random string $B_i \,(i = 1, 2, \ldots, N - 1)$ in step $N + 2$ following the receipt of the traveling particle sequence $\overline{T_{i\rightarrow j}}$ by all participants. Hence, Eve can also get $B_j$. Meanwhile, he calculates $B_{2t-1} = \oplus_{j=1}^{N-1}b_{j,2t-1}$ $(t = 1, 2, \ldots, n)$. In the protocol, the participant performs the corresponding operation with the result of $B_{2t-1}$. This is because of the fact that the number of encoding performed on the particle is $C = \oplus_{j=1}^{N-1}(h_{j,t} \oplus b_{j,2t-1}) \oplus h_{0,t} = B_{2t-1}$, where $h_{0,t} = \oplus_{j=1}^{N-1}h_{j,t}$. Hence, $B_{2t-1}$ represents whether the number of encoding performed on the particle is odd or

| Table I: An example of the proposed protocol. |
|-------------------|-----------------|-----------------|
| $P_1$             | $P_2$           | $P_3$           |
| $ID_i$            | 010110          | 001101          | 100011          |
| $r_i$             | 1100            | 1111            | 0010            |
| $h_i$             | 0111            | 1010            | 0101            |
| $S_i$             | 01101011        | 01000100        | 1010001         |
| $L_i$             | 10001101        | 00110110        | 01101100        |
| $B_i$             | $00100010$      | $10010010$      | $10011110$      |
| $B_{i,2t-1} \oplus h_{i,t}$ | 0011          | 0111            | 1110            |
| $T_i$             | $|\overline{\varphi_{10}}\rangle|\overline{\varphi_{00}}\rangle|\overline{\varphi_{11}}\rangle|\overline{\varphi_{01}}\rangle$ | $|\overline{\varphi_{00}}\rangle|\overline{\varphi_{11}}\rangle|\overline{\varphi_{01}}\rangle|\overline{\varphi_{10}}\rangle$ | $|\overline{\varphi_{01}}\rangle|\overline{\varphi_{10}}\rangle|\overline{\varphi_{11}}\rangle|\overline{\varphi_{00}}\rangle$ |
even. However, even if Eve knows $B^{2t-1}$, he still cannot infer $k_j$. Because the publicly available information does not divulge any information about it. In a word, Eve’s measure-resend attack will not succeed.

2. Entangle-measure Attack

Assume that Eve wants to perform the entangle-measure attack, he can intercept the travelling sequence prepared by $P_0$, and apply entangling operation $U_E$ between his own ancillary particles and the intercepted particles. At last, he transmits the particles to $P_j$. Because there are no decoy particles in the protocol, $P_j$ does not perform eavesdropping detection. He just encodes his private key and identity information directly. Afterwards, the encoding sequence is transmitted to $P_{j\in\Xi}$, at which point Eve intercepts again and measures the ancillary particles to infer the private key $S_j$.

Without loss of generality, the effect of Eve’s unitary operation $U_E$ can be shown as follows:

$$U_E |\alpha\rangle |E\rangle = |00\rangle |e_{00}\rangle + |01\rangle |e_{01}\rangle + |10\rangle |e_{10}\rangle + |11\rangle |e_{11}\rangle,$$

(9)

where, $|e_{00}\rangle, |e_{01}\rangle, |e_{10}\rangle, |e_{11}\rangle$ are pure states determined by $U_E$. Through previous analysis, we know the encoded quantum state satisfies (7) and (8) after different operations. The quantum state in the particle sequence intercepted by Eve again is shown in table II.

After a simple calculation, we get the correlation between these eight states:

$$|\alpha_7\rangle = |\alpha_0\rangle + |\alpha_4\rangle - |\alpha_3\rangle$$
$$= |\alpha_1\rangle + |\alpha_5\rangle - |\alpha_3\rangle$$
$$= |\alpha_2\rangle + |\alpha_6\rangle - |\alpha_3\rangle.$$

Obviously, there is a linear correlation between the obtained quantum states after different coding operations. As Chefles and Barnett[35] said, the necessary and sufficient condition for distinguishing the quantum states is the linear independence of these quantum states. Therefore, these linearly correlated quantum states cannot be unambiguous discriminated, which means Eve cannot obtain the private key $S_j$ through the entangle-measure attack. Similarly, he cannot get the session key as well.

In this protocol, a semi-trusted third party $P_0$, is introduced to help these parties accomplish this secure task. So, the case, in which Eve (called $\hat{P}_0$) impersonates $P_0$ to attack the protocol. His aim is to eavesdrop the session key $K$ or the private key of one party. In Sec. IV B 2, we prove that a genuine $P_0$ cannot attain these information. Based on this conclusion, we could deduce directly that $\hat{P}_0$ is also not able to eavesdrop successfully. Hence, in this section, we focus our attention on the second case. Namely, Eve wants to disguise himself as one participant to execute the protocol with others. Without loss of generality, Eve impersonates $P_j$ (called $\hat{P}_j$). $\hat{P}_j$ prepares the quantum carriers, and hopes attain the $K-P_j$, that is, the negotiation key of other participants except $P_j$. In fact, this action can be detected by the authentication in step $N+4$. As $k_j$ is only known to the valid participant $P_j$ and $P_0$, $\hat{P}_j$ cannot calculate correct hash value $h_j$. Because of the special relationship between hash values $h_0 = \oplus_{i=1}^{N-1}h_i$, as long as any one $h_j$ is error, this relationship is broken. Consequently, the quantum states are changed, and the measurement using the measurement basis determined by $B^{2t-1}$ will result in random results. Then, he must be detected in step $N+4$. Further, he cannot get the private key $S_j$ without been detected, which has been detailed in the above analysis. This means that the presented protocol is secure against this attack.

In addition, Eve could also perform Trojan horse attack to attack the protocol due to multiple transmissions of the same quantum state. To resist this attack, each participant needs to install two quantum optics devices: a wavelength quantum filter and a photon count separator. From the above analysis, we know that the proposed protocol can stand against external attacks.

3. Impersonation Attack

In the actual implementation of the presented protocol, impersonation attack is inevitable. That is to say, Eve can disguise as legitimate party in the key agreement. If this impersonation is successful, he can obtain legitimate parties’ private key and the session key without being discovered. However, it is impossible. Next, we will give a detailed analysis to prove this. Since there are two roles which have different functions in the protocol, we will consider two possible cases.

B. Participant Attacks

Compared with external attackers, internal parties have more damage because they are involved in the execution of the protocol. In the following, we will give a detailed analysis. Besides, we use hash function to realize the authentication of legitimate participants. In addition to the common attacks of legitimate participants, we also have to factor in attack by forged participants.

1. Dishonest Legitimate Participants’ Collusion Attacks

Firstly, we discuss attacks by dishonest legitimate participants. Without loss of generality, assuming that each participant is rational, which means their primary task is to negotiate a session key $K$, then realize the control of it. In the case of the attack by a single dishonest, even if he participates in the protocol, he cannot obtain the hashes of other users from the information he knows. Therefore, he can be detected during the authentication checks just like an external attacker. Since the attack strategy of a single dishonest participant is the same as
that of an external attacker, it will not be repeated here. It is important to note that there is more than one dishonest participant in a protocol, and the most serious case is that of an external attacker, it will not be repeated here. Evidently, there is no information about private key $S_2$ is disclosed during Q2 in Fig. 2. In the encoding process Q3, $P_2$ encodes his own information in the last stage of transmission. Since these three transmission processes are actually synchronized, it is obvious that $P_1^*$ and $P_3^*$ cannot encode the pre-negotiated message in $Q_3$ to determine the final key. So the most likely attack to obtain $S_2$ and determine the final key occurs in $Q_1$. In the transmission process of $Q_1$, $P_1^*$ prepares and encodes the $n$-two-qubits particles, represented as $\tilde{T}_{1,2}$. Then, he sends them to $P_2$ and shares all his information with $P_3^*$. Since $P_3^*$ does not know whether $h_2 \oplus b_2$ is 0 or 1, he does not know whether the particles should be measured with basis $MB_X$ directly or measured with basis $MB_Z$ after the operation $V$. If he is lucky enough to guess correctly, he can get a valid measurement. However, the probability of correctly guessing all the operations is $\left(\frac{1}{2}\right)^n$, which approaches to 0 when $n$ is very large. Therefore, $P_1^*$ and $P_3^*$ cannot obtain $S_2$ in advance and encode the pre-negotiated message to determine the agreement key in the process of $Q_1$. To sum up, our protocol is immune to attacks by multiple dishonest legitimate participants.

For another, we discuss the attack on $P_2$, in which the dishonest legitimate participant wishes to obtain his private key $S_2$, and his identity information $h_2$. Since $P_1^*$ and $P_3^*$ failed to attack $P_0$, it is impossible to determine whether the specific $h_2$ is 0 or 1. Next, we discuss attacks during the protocol process. Evidently, there is no information about private key $S_2$ is disclosed during Q2 in Fig. 2. In the encoding process Q3, $P_2$ encodes his own information in the last stage of transmission. Since these three transmission processes are actually synchronized, it is obvious that $P_1^*$ and $P_3^*$ cannot encode the pre-negotiated message in $Q_3$ to determine the final key. So the most likely attack to obtain $S_2$ and determine the final key occurs in $Q_1$. In the transmission process of $Q_1$, $P_1^*$ prepares and encodes the $n$-two-qubits particles, represented as $\tilde{T}_{1,2}$. Then, he sends them to $P_2$ and shares all his information with $P_3^*$. Since $P_3^*$ does not know whether $h_2 \oplus b_2$ is 0 or 1, he does not know whether the particles should be measured with basis $MB_X$ directly or measured with basis $MB_Z$ after the operation $V$. If he is lucky enough to guess correctly, he can get a valid measurement. However, the probability of correctly guessing all the operations is $\left(\frac{1}{2}\right)^n$, which approaches to 0 when $n$ is very large. Therefore, $P_1^*$ and $P_3^*$ cannot obtain $S_2$ in advance and encode the pre-negotiated message to determine the agreement key in the process of $Q_1$.
Since $P_0$ participates in the execution of the protocol, he has more information than an external attacker. Here, the only constraint of $P_0$ is that he is semi-trusted. In other words, he cannot conspire with others, but he may misbehave on his own to eavesdrop. For clarity, represent the dishonest third party as $P^*_0$. We assume that $P^*_0$ wishes to obtain $P_j$’s private key $S_j$ and the session key $K$, but no matter what attack strategy he adopts, $P^*_0$ cannot obtain effective information. The detailed analysis is as follows.

Generally, the attack of $P^*_0$ occurs after the encoding operation of the participant $P_j$. He has the advantage of knowing the hash value $h_j$. However, in this protocol, the participant decides what kind of identity operation to perform through the value $h_j \oplus b_j$. Even if $P^*_0$ knows each person’s hash value, he wouldn’t be able to get the correct operation information before $B_j$ was announced at the end of the protocol. So, just like an external attacker, he cannot obtain the correct private key $S_j$. In addition, due to the non-clonability of the quantum state, it is impossible for $P^*_0$ to preserve the quantum state without knowing about it and perform the measurement after $B_j$ is published. Even with intercept-resend attack strategy, $P^*_0$ will not be able to obtain valid session key $K_{P^*_0-P_j}$ just as the external attack analysis. In a word, he cannot get the agreement key. Therefore, this protocol is secure against the attack by a semi-trusted third party.

3. A dishonest participant’s Impersonation Attack

Participants may also carry out impersonation attacks in addition to the attacks described above, which means $P_i$ may disguise himself as other participant $P_j$ to negotiate the key without being detected. But his wish comes to nothing. Even if $P_i$ is part of the protocol and knows the public information, he cannot perform the correct identity encoding operation without knowing $P_j$’s correct hash value. Because the $k_j$ used in calculating the hash value is only shared by $P_j$ and $P_0$. In addition, hash values satisfy special relationships in the protocol, that is, $h_i = \oplus_{i=1}^{N+1}h_i$. Only when this condition is met, the $B^{2t-1}$ calculated by the participant represent the correct measurement basis. Otherwise, the measurement result is random, which is difficult to pass the detection in step $N+4$. Therefore, a forged participant cannot participate in the protocol and successfully negotiate the session key without being detected.

Based on the above analysis, we prove that the protocol can effectively protect the privacy of each participant and meet the fairness requirements of key agreement protocol.

V. CONCLUSIONS

Before drawing our conclusion, we briefly discuss some advantages of the proposed protocol compared with Refs. [31] and [36]. First, we consider the impersonation attacks which are inevitable in practical applications. Therefore, we utilize classical hash function combining certificate information with random string to realize the identity authentication of participants. Additionally, we design the authentication operation without breaking the properties of the quantum search algorithm. Second, the security of the protocol is guaranteed by quantum state discrimination, that is, different numbers of operations will lead quantum states into two sets of linearly correlated quantum states, which cannot be distinguished by unambiguous discrimination. Consequently, attackers cannot obtain effective information. Finally, this protocol is feasible in technique. The implementation of the protocol only requires preparing single particles and single-particle measurement. Furthermore, there is no need for participants to store the traveling particles in our protocol. Thus, the proposed protocol may be implemented easily with current technology.

In summary, we propose an authenticated multiparty quantum key agreement protocol based on quantum search algorithm. Every participant has his own certificate and shares encryption key with the semi-trusted third party. With the help of the third party, they can simultaneously obtain the negotiated key by executing the proposed protocol. In this paper, the single particle in quantum search algorithm is used as the information carrier, and the private key is encoded by the specific unitary operation. Depending on the calculated hash value and the random string, participants can perform different authentication operations. In the protocol, each participant performs an authentication operation immediately after encoding the private key, which ensures that any other attackers cannot know the number of operations performed by the participant. The security of the proposed protocol against eavesdropping has been analyzed, which shows that this protocol is secure in common attacks. It is worth emphasizing that the introduction of classical hash function does not reduce the security of the protocol because the hash values in the protocol are not publicly declared. Even if the hash function is corrupted by quantum computation, each participant’s private key is still safe. Therefore, these private keys can be reused, which greatly improves the practicality of the protocol.

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