Online identification of time-variant structural parameters under unknown inputs basing on extended Kalman filter

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Received: 24 November 2021 / Accepted: 26 April 2022 / Published online: 18 May 2022
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Abstract To date, many parameter identification methods have been developed for the purpose of structural health monitoring and vibration control. Among them, the extended Kalman filter series methods are attractive in view of the efficient unbiased estimation in recursive manner. However, most of these methods are performed on the premise that the parameters are time-invariant and/or the loadings are known. To circumvent the aforementioned limitations, an online extended Kalman filter with unknown input approach is proposed in this paper for the identification of time-varying parameters and the unknown excitation. A revised observation equation is obtained with the aid of projection matrix. To capture the changes of structural parameters in real time, an online tracking matrix associated with the time-varying parameters is introduced and determined via an optimization procedure. Then, based on the principle of extended Kalman filter, the recursive solution of structural states including the time-variant parameters can be analytically derived. Finally, using the estimated structural states, the unknown inputs are identified by means of least-squares estimation at the same time step. The effectiveness of the proposed approach is validated via linear and nonlinear numerical examples with the consideration of parameters being varied abruptly.

Keywords Time-varying parameters identification · Load identification · Revised observation equation · Online tracking matrix

1 Introduction

Owing to severe loadings or other environmental effects, structural damage may occur and accumulate during the service life of structures. In general, the damage can be reflected by the changes of structural parameters, such as stiffness degradation. In this regard, identification of these time-variant parameters plays an important role for structural health assessment and vibration control. To date, a number of parameter identification and damage detection methods have been developed in time domain, frequency domain or time–frequency domain [1–3]. Among them, the extended Kalman filter (EKF) technique has received considerable attention in view of its computational efficiency, unbiased estimation and recursive manner.

The effort on the development of EKF-based methods for parameter identification has been made for many years. The earlier contribution can be traced back to the work by Yun and Shinozuka [4]. Since
then, increasing attention has been paid for the purpose of health monitoring of civil infrastructures using EKF [5–8]. In the framework of classic EKF, the external excitation should be known for the identification. However, in many situations, the external forces applied to the structures are unmeasurable or difficult to be measured. Therefore, many endeavors have been made to deal with the case of unknown external forces, saying EKF under unknown inputs (EKF-UI). For example, with the sequent usage of LSE and EKF, Lei et al. [9] proposed an EKF-UI approach for damage detection of linear and nonlinear structures. Based on weighted global iteration and a force updated procedure, Xu and He [10] proposed an EKF-UI approach for substructural identification. Pan et al. [11] proposed a general EKF-UI approach to avoid the restrictions on sensor deployment. By using modal transformation, Liu et al. [12] proposed a modal EKF-UI approach for the identification of large-scale structures. By treating the nonlinear restoring force as unknown fictitious inputs, Lei et al. [13] proposed an approach for nonlinear identification without the prior knowledge of the nonlinear model. Using perturbation analysis, Impraimakis and Smyth [14] proposed an unscented Kalman filter method for joint estimation of structural parameters and external excitation.

The parameters to be identified in EKF algorithm are all considered as a part of the extended state vector. In many existing EKF-based methods, these parameters are assumed to be time-independent such that their time derivative would be zero. However, due to severe loadings or other environmental effects, physical parameters usually vary during the service life of structures. Thus, it is highly desirable to develop effective methods to accurately identify the time-varying parameters and adaptively evaluate the structural performance. Much effort has been made in this field basing on various optimization algorithms, such as least-squares estimation [15–17], wavelet transformation [18, 19], Hilbert transformation [20], neural networks [21]. It is not intended to give a comprehensive review on the time-varying parameters identification methods in this paper. Instead, some investigations based on Kalman filter (KF) principle are briefly introduced herein. For example, the performance of adaptive fading EKF for the identification of time-variant frame structures was discussed by Loh et al. [22]. An EKF-based approach was proposed by Yang et al. [23, 24] to adaptively capture the time-varying parameters with the consideration of known inputs. The effectiveness of this approach was further experimentally validated via a linear and nonlinear building structure [25, 26].

The effectiveness of this approach was then extended for dealing with the case of unknown inputs [29, 30]. An adaptive EKF with two computational modes was proposed by Yang et al. [31] for time-varying parameter identification under seismic excitations. Huang et al. [32] proposed a dual KF-based approach for real-time Bayesian sequential state and parameter identification. A generalized EKF-based approach was proposed by Lei et al. [33] for the integration of identification and vibration control of time-varying structures under seismic inputs.

Recently, by using a revised observation equation, the authors proposed an EKF-UI approach for the simultaneous identification of structural parameters and unknown loadings [34]. This approach was then used for substructural identification [35]. However, this approach is performed under the premise that the parameters are constant. In this paper, to circumvent this limitation, an online EKF-UI (OEKF-UI) approach is proposed for the time-varying parametric identification with unknown input information. With the aid of projection matrix, a revised observation equation is derived. An online tracking matrix (OTM), which is used for capturing the time-varying properties of parameters, is defined and determined by an optimization procedure. Then, based on the principle of EKF, the recursive solution of structural states including the time-variant parameters can be analytically derived. Finally, using the estimated structural states, the unknown inputs are identified by means of least-squares estimation (LSE) at the same time step. The effectiveness of the proposed approach is numerically validated via several linear and nonlinear examples.

The remaining part of this paper is organized as follows. In Sect. 2, the formulas of the proposed OEKF-UI approach are introduced in detail. In Sect. 3, three numerical examples are used to validate the effectiveness of the proposed approach for the
identification of linear and nonlinear systems. Finally, the main conclusions and outlook are given in Sect. 4.

2 The proposed OEKF-UI approach

2.1 Brief introduction of the previously proposed EKF-UI approach

The authors recently proposed an EKF-UI approach for the parameters and loads identification [34]. Here, a brief introduction of this approach is given.

The second-order differential equation of motion of an \( n \) degrees-of-freedom (DOFs) structure can be written as

\[
\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{F}[\dot{\mathbf{x}}(t), \mathbf{x}(t), \mathbf{0}(t)] = \varphi^u \mathbf{f}^u(t)
\]  

(1)

where \( \ddot{\mathbf{x}}(t) \), \( \dot{\mathbf{x}}(t) \) and \( \mathbf{x}(t) \) are the vectors of structural acceleration, velocity and displacement, respectively; \( \mathbf{M} \) is the mass matrix; \( \mathbf{F}[\dot{\mathbf{x}}(t), \mathbf{x}(t), \mathbf{0}(t)] \) represents the linear or nonlinear restoring force vector; \( \mathbf{0}(t) \) is the \( m \)-dimensional vector of parameters to be identified; \( \mathbf{f}^u(t) \) is the unknown input, and \( \varphi^u \) is its influence matrix.

Consider an extended state vector,

\[
\mathbf{Z}(t) = \left[ \mathbf{x}(t)^T, \dot{\mathbf{x}}(t)^T, \mathbf{0}(t)^T \right]^T
\]

(2)

If the structural parameters are time-invariant, the following equation can be obtained,

\[
\frac{d\mathbf{Z}(t)}{dt} = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{M}^{-1}(\mathbf{F}[\dot{\mathbf{x}}(t), \mathbf{x}(t), \mathbf{0}] + \varphi^u \mathbf{f}^u(t)) \\ 0 \end{bmatrix} = \mathbf{u}(\mathbf{Z}(t), \mathbf{f}^u(t), t) + \mathbf{w}(t)
\]

(3)

where \( \mathbf{w}(t) \) is process noise vector with zero mean and a covariance matrix \( \mathbf{Q}(t) \).

Based on the measured acceleration, the discretized observation equation at time instant \( t_k \) can be described as

\[
\mathbf{y}_k = -\mathbf{M}^{-1} \mathbf{F}[\dot{\mathbf{x}}_k, \mathbf{x}_k, \mathbf{0}] + \mathbf{M}^{-1} \varphi^u \mathbf{f}^u_k + \mathbf{v}_k
\]

(4)

in which \( \mathbf{h}(\mathbf{Z}_k) = -\mathbf{M}^{-1} \mathbf{F}[\dot{\mathbf{x}}_k, \mathbf{x}_k, \mathbf{0}]; \mathbf{D} = -\mathbf{M}^{-1} \varphi; \mathbf{v}_k \) is the measurement noise vector with zero mean and a covariance matrix \( \mathbf{R}_k \). With proper transformation, the following revised observation equation can be derived

\[
\Phi_{\mathbf{y}}_k = \Phi \mathbf{h}(\mathbf{Z}_k) + \Phi \mathbf{v}_k
\]

(5)

where \( \Phi = \mathbf{I} - \mathbf{D}(\mathbf{D}^T \mathbf{D})^{-1} \mathbf{D}^T \).

Let \( \mathbf{Z}_{k+1|k} \) be the priori estimate of state \( \mathbf{Z}_{k+1} \), and linearize \( \mathbf{h}(\mathbf{Z}_{k+1}) \) with respect to \( \mathbf{Z}_{k+1|k} \),

\[
\mathbf{h}(\mathbf{Z}_{k+1}) = \mathbf{h}(\mathbf{Z}_{k+1|k}) + \mathbf{H}_{k+1|k}(\mathbf{Z}_{k+1} - \mathbf{Z}_{k+1|k})
\]

(6)

where \( \mathbf{H}_{k+1|k} = \frac{\partial \mathbf{h}(\mathbf{Z}_{k+1})}{\partial \mathbf{Z}_{k+1}} |_{\mathbf{Z}_{k+1} = \mathbf{Z}_{k+1|k}} \).

The priori estimated state \( \mathbf{Z}_{k+1|k} \) can be determined as,

\[
\mathbf{Z}_{k+1|k} = \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{g}(\mathbf{Z}_{k|k}, \mathbf{r}^u_k, k\Delta t) \, dt
\]

(7)

The priori estimation error covariance matrix is calculated as follows,

\[
\mathbf{P}_{k+1|k} = \mathbf{A}_1 \mathbf{P}_{k|k} \mathbf{A}_1^T + \mathbf{A}_2 \mathbf{R}_k \mathbf{A}_2^T + \Delta t \mathbf{Q}_k
\]

(8)

where \( \mathbf{A}_1 = \left( \mathbf{I} + \Delta t \mathbf{U}_{k|k} - \Delta t \left[ \mathbf{H} - \mathbf{F} \right] \mathbf{P}_{k+1|k} \right) \);

\[
\mathbf{A}_2 = \left[ \begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right]; \mathbf{U}_{k|k} = \frac{\partial \mathbf{u}(\mathbf{Z}_k, \mathbf{r}^u_k)}{\partial \mathbf{Z}_k} |_{\mathbf{Z}_k = \mathbf{Z}_{k|k}} \mathbf{r}^u_k = \mathbf{r}^u_{k|k}.
\]

The posteriori estimated state \( \mathbf{Z}_{k+1|k+1} \) can be given as,

\[
\mathbf{Z}_{k+1|k+1} = \mathbf{Z}_{k+1|k} + \mathbf{G}_{k+1} \left[ \Phi_{\mathbf{y}}_{k+1} - \Phi \mathbf{h}(\mathbf{Z}_{k+1|k}) \right]
\]

(9)

in which \( \mathbf{G}_{k+1} \) denotes the EKF gain matrix at the \( (k+1) \)-th time step and can be calculated as

\[
\mathbf{G}_{k+1} = \mathbf{P}_{k+1|k} \mathbf{H}_{k+1|k}^T \left( \mathbf{H}_{k+1|k} \mathbf{P}_{k+1|k} \mathbf{H}_{k+1|k}^T + \mathbf{R}_k \right)^{-1}
\]

(10)

Then, the posteriori estimation error covariance matrix can be obtained as

\[
\mathbf{P}_{k+1|k+1} = \left[ \mathbf{I} - \mathbf{G}_{k+1} \Phi \mathbf{H}_{k+1|k} \right] \mathbf{P}_{k+1|k} \mathbf{H}_{k+1|k}^T + \mathbf{G}_{k+1} \Phi \mathbf{R}_{k+1} \Phi^T \mathbf{G}_{k+1}^T
\]

(11)

Finally, the unknown excitation is obtained by LSE as follows,

\[
\mathbf{r}^u_{k+1} = \left( \mathbf{D}^T \mathbf{D} \right)^{-1} \mathbf{D}^T \left[ \mathbf{h}(\mathbf{Z}_{k+1|k+1}) - \mathbf{y}_{k+1} \right]
\]

(12)
2.2 The proposed OEKF-UI approach

As shown in Sect. 2.1, the parameters to be identified are time-invariant leading to \( \dot{\theta}(t) = 0 \). Thus, the procedures mentioned above cannot effectively capture the time-varying properties of structural parameters. In order to track the variation of structural parameters, an online tracking matrix (OTM) expressed as \( \Lambda_{k+1} \) is introduced in this study. \( \Lambda_{k+1} \) is a diagonal matrix with the dimension of \((2n + m) \times (2n + m)\), where \( n \) and \( m \) are the number of DOFs and unknown parameters, respectively. The first \( 2n \) diagonal elements of \( \Lambda_{k+1} \) associated with displacement and velocity responses are set to be 1.0, whereas the remaining \( m \) diagonal elements associated with the unknown parameters are denoted by \( \lambda_{k+1}^1 \), \( \lambda_{k+1}^2 \), \( \ldots \), \( \lambda_{k+1}^m \). If the values of structural parameters vary at the time instant \( t_{k+1} \), the values of \( \lambda_{k+1}^i (i = 1, 2, \ldots, m) \) will be changed accordingly. Based on this defined OTM, the priori estimation error \( \varepsilon_{k+1|k} \) at \( t_{k+1} \) can be expressed as

\[
\varepsilon_{k+1|k} = \Lambda_{k+1}(Z_{k+1} - \hat{Z}_{k+1|k}) \]

Then, the priori estimation error covariance matrix shown in Eq. (8) can be rewritten as

\[
P_{k+1|k} = \Lambda_{k+1} [\Lambda_{k} P_{k|k} A_{k}^T] \Lambda_{k+1}^T + \Lambda_{2} R_{k} A_{k}^2 + \Delta^2 Q_{k} \tag{13}
\]

In general, the posteriori estimation error \( \varepsilon_{k+1|k+1} \) at \( t_{k+1} \) is defined as

\[
\varepsilon_{k+1|k+1} = Z_{k+1} - \hat{Z}_{k+1|k+1} \]

Based on Eqs. (5), (6) and (9), such posteriori estimation error can be calculated as

\[
\varepsilon_{k+1|k+1} = Z_{k+1} - \hat{Z}_{k+1|k+1} = \Phi_{t_{k+1}} [\Phi(Z_{k+1}) + \Phi_{v_{k+1}} - \Phi(Z_{k+1})]\]

\[
= (I - \Phi_{t_{k+1}} \Phi H_{k+1}) A_{k+1} \Phi_{v_{k+1}} \tag{14}
\]

Then, the error covariance matrix shown in Eq. (11) can be rearranged as

\[
P_{k+1|k+1} = (I - \Phi_{t_{k+1}} \Phi H_{k+1}) A_{k+1} P_{k+1|k} A_{k+1}^T + (I - \Phi_{t_{k+1}} \Phi H_{k+1})^T + \Phi_{t_{k+1}} \Phi R_{k+1} \Phi_{t_{k+1}}^T \tag{15}
\]

Accordingly, the gain matrix shown in Eq. (10) can be changed to

\[
G_{k+1} = A_{k+1} P_{k+1|k} A_{k+1}^T + H_{k+1}^T [R_{k+1} + \Phi_{t_{k+1}}] \Phi_{t_{k+1}}^{-1} \tag{16}
\]

Thus, the proposed OEKF-UI approach is composed of three parts, saying time update by Eqs. (7) and (13), measurement update by Eqs. (9), (15) and (16), as well as force identification by Eq. (12). It can be found that the key point of the implementation of the proposed approach is the determination of \( \Lambda_{k+1} \). In order to calculate OTM, the following derivation is conducted and an objective function is defined.

Based on Eq. (5), the predicted error vector \( r_{k+1} \) and residual error vector \( f_{k+1} \) can be calculated as

\[
r_{k+1} = \Phi y_{k+1} - \Phi h(\hat{Z}_{k+1|k}, t_{k+1}) \tag{17}
\]

\[
f_{k+1} = \Phi y_{k+1} - \Phi h(\hat{Z}_{k+1|k+1}, t_{k+1}) \tag{18}
\]

Using Eqs. (6) and (9), the residual error vector can be rearranged,

\[
f_{k+1} = \Phi(y_{k+1} - h(\hat{Z}_{k+1|k+1}, t_{k+1})) = \Phi(y_{k+1} - h(\hat{Z}_{k+1|k+1} - Z_{k+1|k})) = \Phi(y_{k+1} - h(Z_{k+1|k}, \Phi_{t_{k+1}} - h(\hat{Z}_{k+1|k+1} - Z_{k+1|k})))
\]

\[
= |I - \Phi_{t_{k+1}} | G_{k+1} | r_{k+1} \tag{19}
\]

Taking expectation values of the residual error vector, one can obtain

\[
E(r_{k+1} r_{k+1}^T) = |I - \Phi H_{k+1} | G_{k+1} E(r_{k+1} r_{k+1}^T) |I - \Phi H_{k+1} | G_{k+1}^T \tag{20}
\]

Based on Eq. (5) and the definition of the covariance matrix, it can be found that the left side of Eq. (20) is equal to \( \Phi R_{k+1} \Phi^T \). Here, the covariance matrix of predicted error vector can be defined as \( E(r_{k+1} r_{k+1}^T) = V_{k+1} \). Then, with the aid of Eq. (10), the following equation can be obtained,

\[
V_{k+1} = Y_1 Y_2 \Phi R_{k+1} \Phi^T (Y_1 Y_2)^T \tag{21}
\]

where \( Y_1 = \Phi (H_{k+1} | P_{k+1|k} H_{k+1}^T + R_{k+1}) \Phi^T \), \( Y_2 = (\Phi R_{k+1} \Phi^T)^{-1} \).

It can be seen that \( P_{k+1|k} \) in Eq. (21) contains \( A_{k+1} \). Thus, the OTM can be determined by solving Eq. (21). However, Eq. (21) is highly nonlinear and difficult to be solved directly. Instead, an optimization procedure...
is employed herein for the determination of \( \Lambda^{k+1} \). An objective function is defined as,

\[
\Omega_{k+1} = \sum_{i=1}^{m} \frac{\hat{y}^i_{k+1} - \hat{y}^i_k}{\hat{y}^i_k} \tag{22}
\]

where \( \hat{y}^i_{k+1} \) and \( \hat{y}^i_k \) denote the \( i \)-th identified parameters at \( t_{k+1} \) and \( t_k \), respectively.

Then, the purpose of determining \( \Lambda_{k+1} \) can be transformed into an optimization issue by minimizing the objective function in Eq. (22) under the following constraint,

\[
\| V_{k+1} - Y_1 \hat{Y}_2 \Phi \hat{R}_k \Phi^T (Y_1 \hat{Y}_2)^{T} \| \leq \delta \tag{23}
\]

where \( \| \cdot \| \) denotes Frobenius norm; \( \delta \) is a small positive constant number. In this study, \( \delta = 0.01 \) is used in the numerical examples. The function ‘FMINCON’ in MATLAB is a very convenient tool to obtain the optimal solution of \( \Lambda_{k+1} \). To effectively conduct this optimization procedure, a reasonable value of \( V_{k+1} \) matrix calculated at time step of \( t_{k+1} \) is required. The followings are how to determine such matrix.

Referring to the method introduced by Yang et al. [36], a special case that the diagonal elements of \( \Lambda_{k+1} \) are identical is first considered, i.e., \( \Lambda_{k+1} = \frac{\hat{y}^1_{k+1}}{\delta} \mathbf{I}_{2n+m} \) where \( \hat{y}^1_{k+1} \) is the tracking coefficient to be determined. Then, Eq. (13) can be expressed as

\[
P_{k+1|k} = \hat{z}_{k+1|k} \left[ A_{1|k} P_{k|k} A_{1|k}^T + A_2 R_k A_2^T + \Delta \right] \tag{24}
\]

Substituting Eqs. (24) into (21), the following equation can be obtained

\[
V_{k+1} = \left( \hat{z}_{k+1} \left[ T_1 T_3^{-1} + T_2 T_3^{-1} + T_3 T_3^{-1} \right] \right. \\
\left. T_3 \left( \hat{z}_{k+1} \left[ T_1 T_3^{-1} + T_2 T_3^{-1} + T_3 T_3^{-1} \right] \right)^T \right) \tag{25}
\]

where \( T_1 = \Phi H_{k+1|k} A_1 P_{k|k} A_1^T H_{k+1|k} ^T \Phi^T; T_2 = \Phi H_{k+1|k} (A_2 R_k A_2^T + \Delta \hat{z}_{k+1}^2 Q_k) H_{k+1|k} ^T \Phi^T; \)

\( T_3 = \Phi R_{k+1} \Phi^T. \)

Taking the trace on both sides of Eq. (25), one can obtain,

\[
tr \{ V_{k+1} \} = \hat{z}_{k+1}^2 tr \left\{ T_1 T_3^{-1} T_3 \left[ T_1 T_3^{-1} \right]^T \right\} \\
+ \hat{z}_{k+1} tr \left\{ \left( T_1 T_3^{-1} + T_3 T_3^{-1} \right) T_3 \left( T_1 T_3^{-1} \right)^T \right\} \\
+ tr \left\{ \left( T_1 T_3^{-1} + T_3 T_3^{-1} \right) T_3 \left( T_1 T_3^{-1} + T_3 T_3^{-1} \right)^T \right\} . \tag{26}
\]

By solving Eq. (26), \( z_{k+1} \) can be calculated as,

\[
z_{k+1} = \left( -T_b + \sqrt{T_b^2 - 4 T_a T_c} \right) / 2 T_a \quad (\hat{z}_{k+1} \geq 1)
\]

where \( T_a = tr \{ T_1 T_1^{-1} T_3 \left( T_1 T_3^{-1} \right)^T \}; T_b = tr \{ (T_2 T_3^{-1} + T_3 T_3^{-1}) T_3 \left( T_1 T_1^{-1} + T_1 T_3^{-1} \right)^T + T_I T_3 \} ;
\]

\( T_c = tr \{ (T_2 T_3^{-1} + T_3 T_3^{-1}) T_3 \left( T_1 T_3^{-1} + T_3 T_3^{-1} \right)^T \} - tr \{ V_{k+1} \}. \)

To determine the value of \( \Lambda_{k+1} \) at the time instant \( t_{k+1} \), the covariance matrix \( V_{k+1} \) should also be evaluated for the implementation of the aforementioned optimization procedure. After calculating the value of \( z_{k+1} \) by Eqs. (24–27), the following recursive estimation for \( V_{k+1} \) can be conducted [36],

\[
V_{k+1} = K_{1,k+1} / K_{2,k+1}, \tag{28}
\]

where \( K_{1,k+1} = r_k + r_k^T + v K_{2,k} / z_k; K_{2,k+1} = 1 + v K_{2,k} / z_k; v \) is a coefficient set to be 0.95 in this study.

In summary, similar to the framework of EKF technique, the proposed OEKF-UI approach in this paper contains time update procedure by Eqs. (7) and (13), measurement update procedure by Eqs. (9), (15) and (16). Moreover, Eq. (12) is derived to identify the unknown inputs. To track the time-varying parameters, the optimization problem shown in Eqs. (22–23) is solved for the determination of OTM. Since a revised observation equation is derived and used, the prior estimation of unknown inputs is not required. The structural states and unknown inputs are identified at the same time. This makes the proposed approach simpler and clearer as compared with other method [24, 36]. The flowchart of the proposed approach is shown in Fig. 1 for ease of understanding. The effectiveness of the proposed approach is demonstrated via several linear and nonlinear examples in the following section.

3 Numerical investigation

In this section, a 4-story shear-type linear structure with and without nonlinear components is considered. The mass and stiffness of each floor are set to be
300 kg and 180 kN/m, respectively. To mimic structural damage, the structural stiffness is assumed to be changed abruptly at some time. The acceleration responses with the sampling frequency of 1000 Hz are used for the identification of such time-variant parameters and unknown inputs. From a practical viewpoint, the measured acceleration responses are simulated by the theoretically computed quantities superimposed with the corresponding noise process with 5% noise to signal ratio.

3.1 Four-story linear building with multiple abrupt damages

In this example, a random excitation is applied on the top floor of the structure. After the calculation of structural responses by state space method, this excitation is assumed to be unknown. The Rayleigh damping model is adopted, i.e., \( C = \alpha M + \beta K \). The damping coefficients \( \alpha \) and \( \beta \) are set to be 0.3788 and 0.0018, respectively. To mimic time-varying multiple damages, the abrupt changes of structural stiffness of the 2\(^{nd} \) and 4\(^{th} \) floor are considered as follows,

\[
\begin{align*}
  k_2 &= \begin{cases} 
    180 \text{ (kN/m)} & 0 \leq t < 5s \\
    144 \text{ (kN/m)} & 5s \leq t < 7s \\
    108 \text{ (kN/m)} & 7s \leq t < 10s 
  \end{cases} \\
  k_4 &= \begin{cases} 
    180 \text{ (kN/m)} & 0 \leq t < 7s \\
    162 \text{ (kN/m)} & 7s \leq t < 10s 
  \end{cases}
\end{align*}
\]

The quantities to be identified include the damping coefficients, stiffness parameters and unknown excitation. The initial values of the structural parameters are assumed to be 50% of the actual ones.

By using the proposed approach, the structural parameters can be identified as shown in Fig. 2. Only the stiffness parameters are plotted in Fig. 2 as examples. The identification results without using OTM are given in Fig. 2 as well. Although oscillation exists at the beginning, the identified results can still converge to those real ones after few seconds. As compared with the results without OTM, it is obvious that the abrupt changes of structural stiffness can be accurately captured by the proposed approach.

Besides the identification of structural parameters, the unknown force applied to the top floor can also be identified. Figure 3 gives the time series of the identified excitation. Only the time segment from 7.0 to 7.1 s is plotted in Fig. 3 for clarity of comparison. It is clear that the identified inputs are close to the actual ones.

3.2 Building structure equipped with Duffing model

Nonlinearity exists widely in the engineering structures. To investigate the proposed approach for the identification of time-varying parameters of nonlinear system, a building structure with slight nonlinear phenomenon is first considered in this example. The 4-story building model introduced before is employed in this section. The nonlinear Duffing model expressed by Eq. (30) is added on each floor and used to generate nonlinear restoring force (NRF).

\[
\text{NRF} = K_d \Delta x^3(t). 
\]
where $K_d$ is a diagonal coefficient matrix; $\Delta \mathbf{x}$ is inter-story drift. Here, the diagonal elements of $K_d$ are set to be $k_{di} = 1 \times 10^7$ N/m$^3$ ($i = 1, 2, 3$) and $k_{d4} = 0.5 \times 10^7$ N/m$^3$.

The El-Centro earthquake with a peak ground acceleration (PGA) of 3.4 m/s$^2$ is employed as external excitation. The corresponding nonlinear responses are computed by Runge–Kutta method with the time interval of 0.001 s. A dual abrupt damage is considered in this example, saying the stiffness of the 1st and 3rd story is reduced sharply by 30 and 20% at $t_k = 5$ s, respectively. The quantities to be identified include the coefficients of structural damping, stiffness and Duffing model, as well as unknown ground motion. Similarly, the initial values of these coefficients are all assumed to be 50% of the actual ones.

Based on the measured acceleration responses, the coefficients can be identified by the proposed approach as depicted in Fig. 4. The results without OTM are also given in Fig. 4 for comparison. Obviously, the abrupt changes can be effectively tracked with the usage of OTM, whereas unsatisfactory results will be obtained without OTM. Figure 5 gives the results of the identified unknown seismic input. It can be seen that the identified inputs have a good agreement with the actual ones.
3.3 Building structure equipped with Bouc-Wen model

To further verify the feasibility of the proposed approach for identifying the parameters of structures with highly nonlinear properties, the aforementioned building structure equipped with Bouc-Wen model on each floor is considered herein. Then, the equation of motion shown in Eq. (1) can be re-written as,

$$M\ddot{x}(t) + C\dot{x}(t) + Kz(t) = \varphi^\alpha F^\rho(t)$$  \hspace{1cm} (31)

where \(M, K\) and \(C\) and structural parameters are introduced before. In this example, viscous damping rather than Rayleigh damping is used for the
Online identification of time-variant structural parameters under unknown inputs basing on…

Fig. 6 The identified coefficients (Bouc-Wen model). a $k_1$; b $k_2$; c $c_1$; d $c_2$; e $\beta_1$; f $\beta_2$; g $\gamma_1$; h $\gamma_2$
construction of damping matrix. The damping coefficients are set to be 800 N/s/m for each story. \( z(t) \) is hysteretic component given as,

\[
\ddot{z}_i = \dot{x}_i - \dot{x}_{i-1} - \beta \dot{z}_i - \dot{x}_{i-1} \gamma \dot{z}_i - \dot{x}_{i-1} |z_i|^{\mu-1} - \gamma \dot{z}_i (\dot{x}_i - \dot{x}_{i-1})|z_i|^{\mu-1}\]

(32)

where \( \beta, \gamma \) and \( \mu \) are the coefficients of Bouc-Wen model on the \( i \)-th floor. Here, the values of \( \beta, \gamma \) and \( \mu \) for each floor are set to be 3.5, 2.5 and 2, respectively.

A random excitation is applied on the top floor of the building. Some other novel methods may be employed for solving the nonlinear differential equations mentioned above, such as the deep learning and neural network-based approaches [37–39]. For simplicity, the hysteretic responses are computed by the classic Runge–Kutta method with a time interval of 0.001 s in this example. A severe damage is assumed to occur at \( t = 5 \) s on the first floor. The stiffness of this floor \( k_1 \) suddenly reduces from 180 to 90 kN/m resulting in 50% stiffness degradation, and viscous damping \( c_1 \) reduces from 800 to 400 N/s/m at the same time. Besides, the Bouc-Wen model coefficient \( \gamma_1 \) is also decreased 50%. The quantities to be identified include the coefficients of structural damping, stiffness and Bouc-Wen model, as well as unknown external force. Similarly, the initial values of these coefficients are 50% of the actual ones. The time series of the identified parameters of the 1st and 2nd floor are plotted in Fig. 6 as examples. Moreover, the time-varying parameters identified by other methods [24] are shown in Fig. 6 for comparison. It is obvious that in the case of severe damage occurred, the identified parameters without using OTM are not correct at all. The performance of the proposed OEKF-Ul is comparable with Yang’s method. Both of them are capable of identifying the suddenly changed parameters with acceptable accuracy.

The unknown external excitation is identified and shown in Fig. 7. The time segment from 7 to 7.1 s is plotted for clarity of comparison. It can be seen that the identified inputs match with the actual ones well.

4 Conclusions

In this paper, in order to effectively identify time-varying structural parameters without prior knowledge of external excitation, an online EKF-Ul (OEKF-Ul) approach is proposed. With the aid of projection matrix, a revised observation equation is derived. An online tracking matrix (OTM) is defined and used for tracking the time-variant properties of parameters. An optimization procedure is performed at each time step for the determination of OTM. Then, based on the principle of EKF, the recursive solution of structural states including the time-varying parameters can be analytically derived. Finally, using the estimated structural states, the unknown inputs can be identified by means of least-squares estimation (LSE) at the same time step. Several linear and nonlinear numerical examples are used to validate the effectiveness of the proposed approach. Results show that the proposed OEKF-Ul approach is capable of satisfactorily identifying structural parameters and unknown inputs even in the case of sudden severe damage. It can be noted
that the structures to be identified in the examples are relatively simple. For large-scale structures, the direct usage of the proposed approach would be more time-consuming due to many unknowns involved and the complexity of optimization procedure. However, if a large-size structure can be divided into several substructures whose DOFs are much less than those of the whole structure, the identification procedure may be conducted more efficiently and reliably \[35, 40\]. Thus, in this case, it is recommended to perform the proposed approach with the aid of substructural idea. Structural uncertainty is another interesting topic required for further investigation. The optimal estimation of covariance matrix \(Q(t)\) will be helpful for consideration of structural uncertainties \[41\]. Further research will be carried out in these directions in the future.

**Acknowledgements** The authors gratefully acknowledge the financial support from the National Key Research and Development Program of China through Grant No. 2019YFC1511101. The support from Natural Science Foundation of Hunan Province (No. 2021JJ30110) and Innovation Platform Open Fund project of Hunan Province (No. 19K018) is also greatly appreciated.

**Funding** This article was supported by National Key Research and Development Program of China, 2019YFC1511101, Jia He, Natural Science Foundation of Hunan Province, 2021JJ30110, Jia He, Innovation Platform Open Fund project of Hunan Province, 19K018, Jia He.

**Data availability** The data used for the current study are available from the corresponding author upon reasonable request.

**Declarations**

**Conflict of interest** The authors declare that they have no conflict of interest.

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