Superconducting Single-Electron Transistor in a Locally Tunable Electromagnetic Environment: Dissipation and Charge Fluctuations

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We have developed a novel system consisting of a superconducting single-electron transistor (S-SET) coupled to a two-dimensional electron gas (2DEG), for which the dissipation can be tuned in the immediate vicinity of the S-SET. Within linear response, the S-SET conductance varies nonmonotonically with increasing 2DEG impedance. We find good agreement between our experimental results and a model incorporating electromagnetic fluctuations in both the S-SET leads and the 2DEG, as well as low-frequency switching of the S-SET offset charge.

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Electrical transport in nanoscale devices is strongly affected by the electromagnetic properties of their environment. This is particularly true for superconducting systems, and has been a topic of considerable interest lately: dissipation can drive a superconductor-insulator quantum phase transition [1, 2, 3], and is also expected to affect the coherence time of qubits such as the charge-based single Cooper pair box [4, 5, 6]. Maintaining quantum coherence in such devices long enough to allow many operations is prerequisite for their use in quantum computation. In this regard, the closely related superconducting single electron transistor (S-SET) is an excellent system for attaining a better understanding of the effects of the electromagnetic environment on quantum coherence.

This approach was followed by the Berkeley group [7], who fabricated an S-SET above a two-dimensional electron gas (2DEG) in a GaAs/AlxGa1−xAs heterostructure. Using a back gate to vary the 2DEG sheet resistance Rsq they measured changes in the S-SET conductance GSET as the dissipation was varied. Building on earlier theoretical work [8, 9, 10, 11] Wilhelm, et al. predicted [12] that within linear response GSET would scale with the ground plane conductance G2D = 1/Rsq and temperature T as G2D/Tα and that the S-SET current I at fixed bias voltage V would vary nonmonotonically with G2D, while switching from the non-linear to linear regime. These predictions were made within two models: in one the environment was treated as a ground plane, while in the other it was treated as an infinite RC transmission line provided by the SET leads. While the Berkeley group did observe power law behavior, their measured exponents were not in quantitative agreement with theory. Furthermore, the measured β depended on T and α on G2D, calling the scaling form into question.

In this Letter, we report measurements on samples similar to those studied by the Berkeley group; in our samples, however, we can modify G2D in the immediate vicinity of the S-SET while leaving the 2DEG beneath the leads largely untouched. In contrast to the predictions of Wilhelm, et al. we observe a nonmonotonic dependence of GSET on G2D entirely within the linear regime. We propose a model for the environmental impedance of S-SET/2DEG systems that includes electromagnetic fluctuations in both the 2DEG and leads, while treating the latter as finite RC transmission lines. We also allow for relatively low-frequency switching of the charge state of the SET island [13], which can affect the measured current. Within this model, we find good agreement between our calculated and measured results.

An electron micrograph of a typical sample is shown in Fig. 1(a). We begin with an GaAs/AlxGa1−xAs heterostructure grown on a GaAs substrate using molecular beam epitaxy, consisting of the following layers: 1000 nm of GaAs, 47 nm of AlxGa1−xAs and 5 nm of GaAs. The AlxGa1−xAs is delta-doped with Si 22 nm from...
the lower GaAs/Al0.3Ga0.7As interface, at which forms a two-dimensional electron gas (2DEG) with \( R_{	ext{sd}} = 20 \, \Omega \) and sheet density \( n_s = 3.6 \times 10^{11} \, \text{cm}^{-2} \). On the sample surface we use electron-beam lithography and shadow evaporation to fabricate an Al/AlO_x S-SET surrounded by six Au gates [13]. When no gate voltage is applied and the 2DEG is unconfined, the measured \( I-V \) characteristics are linear over several microvolts, as shown in Fig. 1(b). We can also apply a single gate voltage \( V_g \) to any combination of Au gates, excluding the 2DEG beneath them. We focus on two geometries: the “pool,” in which all six gates are energized, and the “stripe” in which only the four exterior gates are. In both cases, electrons immediately beneath the SET are coupled to ground by quantum point contacts (QPCs) with conductances \( 1/R_{QPC} \) (assumed equal) as low as 3 conductance quanta \( G_0 = e^2/h \). In the stripe geometry, the electrons can also move vertically through a resistance \( R_{	ext{str}} \) to a large 2DEG reservoir that is coupled to ground through a capacitance \( C_{\text{str}} \). As illustrated in Fig. 1(c), electromagnetic fluctuations in the environment can couple to the tunneling electrons in two ways: through the leads, which act as RC transmission lines with impedance \( Z_l \) for the relevant frequency range [14, 15], and through the capacitance \( C_{2D} \) to the 2DEG with impedance \( Z_{2D} \), which is related to \( R_{QPC} \) and (for the stripe) \( R_{	ext{str}} \). The model has been studied previously [16, 17] without considering particular forms for \( Z_{2D} \) and \( Z_l \).

Measurements were performed on two separate samples (S1 and S2) in a dilution refrigerator in a four-probe configuration [16]. The estimated electron temperature was 100 mK. High frequency noise was excluded using standard techniques. A small capacitance \( C_g \approx 20.3 \, \text{aF} \) (not shown in Fig. 1(c)) couples the six Au gates to the S-SET. The other sample parameters such as the junction resistances \( R_{1,2} \) and capacitances \( C_{1,2} \), the coupling capacitance \( C_{2D} \) and superconducting gap \( \Delta \) are given elsewhere [13]. The charging energy \( E_c = e^2/2C_2 \) for sample S1 (S2) is 118 (77) \( \mu \text{eV} \) while the Josephson energy \( E_{J_1} = \frac{h}{2R_Q} \Delta \) averaged for the two junctions is 4.7 (21.8) \( \mu \text{eV} \). Here \( R_Q = \frac{h}{2e^2} \) is the superconducting quantum and \( C_2 = C_1 + C_2 + C_{2D} + C_g \). We use standard lock-in techniques and voltage biases of 3 and 5 \( \mu \text{V} \) rms respectively to measure \( G_{\text{SET}} \) and the conductance \( G_{2D} \) across the series combination of the QPCs versus \( V_g \) in the pool and stripe geometries. The results for S2 are shown in Fig. 2.

From Fig. 2(b), we see that for the pool \( G_{\text{SET}} \) rises by nearly a factor of 2 as \( V_g \) becomes more negative, before dropping rapidly. Although \( G_{2D} \) vs. \( V_g \) is nearly identical in both cases, \( G_{\text{SET}} \) for the stripe rises only by \( \sim 50\% \) and does not decrease, even for the most negative \( V_g \). In both plots, \( e \)-periodic Coulomb blockade oscillations are seen as the S-SET offset charge varies. We fit a smoothly varying function to the measured \( G_{2D} \) versus \( V_g \) (inset, Fig. 2(a)) which we use to plot the maxima of \( G_{\text{SET}} \) versus \( G_{2D} \) in Fig. 2(a). The knee in \( G_{\text{SET}} \) at \( V_g \approx 2(1) \) eV corresponds to the appearance of quantized plateaus in the individual QPC conductances.

To understand these results and the \( I-V \) curve in Fig. 2(b), we begin with the rate of sequential Cooper pair tunneling [19] through junction \( j \), valid for \( E_J < E_c \):

\[
\Gamma(\delta f^{(j)}) = (\pi/2h)E_f^{j}P(-\delta f^{(j)})
\]

where \( \delta f^{(j)} = f_f - f_i = (-1)^j 4E_J(2N - n_g + 1) - 2\alpha_J eV \) is the free energy difference for changing the number of Cooper pairs \( N \) by 1, \( \alpha_J = \frac{1}{2} + (-1)^j (C_1 - C_2)/2C_2 \), and \( n_g = C_g/V_g/c \). Here \( P(E) \) is the probability of exchanging energy \( E \) with the environment and \( \delta f^{(j)} \) can be expressed in terms of a correlation function

\[
K(t) = R_Q^{-1} \int_{-\infty}^{\infty} \frac{d\omega}{h} \text{Re}[Z_l(\omega)]\{\text{coth}(\frac{\hbar \omega}{2k_B T})[\cos(\omega t) - i \sin(\omega t)] - i \sin(\omega t)\} \text{via } P(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \exp[K(t) + i\frac{E}{k_B T}]
\]

where \( Z_l(\omega) \) is the total impedance seen by tunneling electrons.

The general result for \( Z_l(\omega) \) within the model of Fig. 1(c) is quite complex [13]. In our case, however, \( Z_l(Z_{2D}) \) dominates at low (high) frequencies and to an excellent approximation

\[
\text{Re}[Z_l(\omega)] = \kappa_1 \text{Re}[Z_l(\omega)] + \frac{\kappa_2 Z_{2D}}{1 + [\omega C_{\text{eff}}'K_2 Z_{2D}]^2}
\]

where for junction 1(2) \( \kappa_1 = \frac{C_{2D} + C_{2D}}{V_{\text{eff}}}, \kappa_2 = \frac{(C_{2D})^2}{C_{\text{eff}}}, C_{\text{eff}} = \frac{C_{2D}+C_{2D}}{C_{2D}+C_{2D}}, \) and we treat \( Z_{2D} \) as a resistance. For \( Z_l \) we begin with the impedance of a finite RC...
line $Z_{RC}(\omega) = \sqrt{\frac{r_\ell}{c_\ell}} \tanh(\sqrt{\frac{\omega r_\ell c_\ell \ell^2}{2}})$ where $r_\ell$ and $c_\ell$ are the resistance and capacitance per unit length and $\ell$ is the total line length. We are interested in the long-time limit of $K(t)$ which is dominated by the low-frequency part of $Z_t(\omega)$. In that limit, $Z_{RC}(\omega) \approx \frac{r_\ell}{1 + (\omega r_\ell c_\ell \ell^2 \sqrt{\omega})}$, which we use for $Z_t$ in Eq. 3.

A detailed analysis of $K(t)$ will be given elsewhere. Here we note that both parts of $Z_t(\omega)$ in Eq. 3 have the same form. For $Z_{2D}$, the corner frequency $\omega_c = 1/(C_{\text{str}} R_{Z_{2D}})$ satisfies $\hbar \omega_c \gg k_B T$ and we may use the kernel of Ref. 3, while for $Z_{RC}$ we find that $\omega_c = \sqrt{6/\tau r_\ell e^2}$ usually satisfies $\hbar \omega_c \ll k_B T$ and requires different treatment. Since $K$ is linear in $\text{Re}[Z_t(\omega)]$, we may calculate $K(t)$ and $P(E)$ separately for $Z_{2D}$ and $Z_t$ and find the total $P_{\text{tot}}(E)$ as a convolution. For $Z_{2D}$, then, we have $P_{2D}(E) = \frac{1}{(2\pi \tau \kappa)} \frac{R_{Q}}{R_{Z_{2D}}} |B(\frac{\omega}{\omega_c} \frac{\hbar}{k_B T}) | \frac{g}{R_{Q}} |Z_{2D}|$ where $g = R_{Q}/Z_{2D}$ and $B(x, y)$ is the beta function. For $Z_{RC}$, we find that

$$K_{RC}(t) = -\frac{2\kappa_t}{g_{rec}} \left\{ \pi k_B T |t|/\hbar + \frac{\pi}{2} \cot \left( \frac{\hbar g_{rc}}{2 k_B T} \right) \right\}$$

is valid for $\frac{\hbar g_{rc}}{2 k_B T} \ll \pi/2$, where $g_{rc} = R_{Q}/(\tau l_\ell)$ and $\tau = R_{Q} c_{\ell}/\sqrt{\hbar}$. From this we calculate

$$P_{RC}(E) = \frac{\tau}{\pi g_{rc} \hbar} e^{\gamma_{1}(g_{rc})} \frac{\text{Re}}{\pi g_{rc} \hbar} e^{\gamma_{2}(g_{rc}) - \gamma_{1}(g_{rc})} \times \left\{ \Gamma(\gamma_{1}(g_{rc}) - \gamma_{1}(g_{rc}) - \gamma_{3}(g_{rc})) \right\}$$

where $\gamma_{1}(g) = \frac{2 \pi k_B T g_{rc}}{\hbar} - i E \hbar g_{rc}$, $\gamma_{2}(g) = \gamma_{3}(g) = \frac{2 \pi k_B T g_{rc}}{\hbar} \cot \left( \frac{g_{rc}}{2 k_B T} \right)$ and $\Gamma(x, y)$ is the incomplete gamma function.

To proceed we need an accurate model of $Z_t(\omega)$. For the $V_g = 0$ I-V curve to be linear at $V \approx 8 \mu V$, $Z_t/R_{Q}$ must be nonnegligible at frequencies of order $e V/h \approx 2 \text{ GHz}$. Since $Z_{2D} \approx R_{Q} = 20 \Omega$, $Z_t$ must dominate $Z_t$ for small $Z_{2D}$. We therefore consider the structure of our leads, which vary in width $w$ from 100 nm to 20 $\mu m$. The 100 nm section has length $\ell = 1 \mu m$, contributes only 50 $\Omega$ to $Z_t(0)$, and is not considered further. The remaining sections are a cascaded $RC$ line which determines $Z_t(\omega)$. The total $Z_t(\omega)$ calculated from Eq. 3 is shown for different values of $Z_{2D} = 1/(4g_{2D})$ for the pool geometry in Fig. 3. The cascaded form for $Z_t(\omega)$ is quite complex. For our calculations we take $\text{Re}[Z_t(\omega)] = \sum_i \text{Re}[Z_{RC}^{(i)}]$ where the $Z_{RC}^{(i)}$ are the impedances of the individual sections, a very good approximation to the more exact result, as shown. Note that this model predicts a significant $Z_t$ at 2 GHz dominated by $Z_t(Z_{2D})$ for small (large) $Z_{2D}$. For the stripe, $Z_{2D}$ approaches $R_{Q}/\hbar$ at zero frequency and the much lower stripe resistance $R_{str} \approx 200 \Omega$ at frequencies above $1/R_{Q} C_{str}$ where $C_{str} \approx 0.3 \text{ pF}$ is its capacitance to ground. At high frequencies, then, $Z_t$ in the stripe is always dominated by $Z_t$, even for large negative $V_g$. We have also shown the impedance for $Z_{2D}$ alone, and for an infinite $RC$ line with $w = 10 \mu m$ and $r_\ell$ chosen to give the correct $Z_t(0)$ if the line were finite. The latter two models give a small $Z_t$ for small $Z_{2D}$ at the relevant frequencies, and cannot explain the linear region in our $V_g = 0$ I-V characteristics.

Using $P_{2D}(E)$ and $P_{RC}(E)$ above, we numerically convolve the $P_{2D}^{(j)}(E)$ for junction $j$ and section $i$ to find $P_{t}^{(j)}(E) = P_{t}^{(j)}(E) \ast P_{2D}^{(j)}(E) \ast P_{3}^{(j)}(E) \ast P_{4}^{(j)}(E)$. We then calculate $P_{tot}^{(j)}(E) = P_{t}^{(j)}(E) \ast P_{2D}(E)$ for different $G_{2D}$ and set up a master equation using the rates in Eq. 4 to calculate the S-SET current and conductance $G_{SET}$. The results for $G_{SET}$ in the pool geometry are shown as the solid line in Fig. 4(a); we scale $G_{SET}$ to match the maximum measured value $G_{max} \approx 0.34 G_0$ at $G_{2D}^{max} \approx 6.5 G_0$ but use no other variable parameters. $G_{SET}$ agrees reasonably well with $G_{SET}$ for $G_{2D} < G_{max}$ although it rises less steeply with $G_{2D}$. In this regime $P_{2D}(E)$ is broad and inelastic transitions suppress the coherent supercurrent. For $G_{2D} > G_{max}$, $G_{SET}$ gradually saturates at $0.47 G_0$. For $G_{2D} > G_{max}$, $P_{2D}(E)$ is not so broad (i.e. only elastic transitions are likely) and $P_{t}^{(j)}(E)$ dominates the I-V characteristic. No nonmonotonic behavior occurs in $G_{SET}$, in agreement with Wilhelm et al. The drop in $G_{SET}$ for $G_{2D} > G_{max}$ must arise from other physics.
In conclusion, we have measured the effects of dissipation on transport in an S-SET for which the environment can be varied locally. We find good agreement with a model in which fluctuations in the leads and low-frequency switching between charge states dominate for low confinement (large $G_{2D}$), while for strong confinement (small $G_{2D}$) fluctuations coupled via the capacitance $C_{2D}$ dominate. The model accounts well for the evolution of $G_{SET}$ and the $I$-$V$ curves as $G_{2D}$ is varied. We believe a convolved $P_{tot} = P_t * P_{2D}$ is likely required to interpret the results of the Berkeley group, which may explain the discrepancies between their results and the scaling theory of Wilhelm, et al.

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