The azimuthal asymmetry of heavy quarks production on double polarized proton-proton and proton-antiproton colliders are studied in this work at next-to-leading order level, with some details included. The purpose is to see whether the effect of extracted transversity distribution functions can be seen on present and near future colliders. All one-loop hard coefficients are presented analytically. Numerical results for the asymmetry on RHIC ($\sqrt{S} = 200, 500\, \text{GeV}$) and GSI ($\sqrt{S} = 14.7\, \text{GeV}$) experiments are also given.

I. INTRODUCTION

Transversity distribution function of quark is one of three twist-2 parton distribution functions (PDFs), which reflects the spin structure of proton[1]. Compared with other two PDFs, the extraction of transversity PDF is much more difficult. Due to its chiral-odd nature, it must convolute with another chiral-odd distribution function to form an observable. Through many years of efforts, now the transversity PDFs in valence region are available. There are two independent extraction formalisms in literature: One is based on transverse momentum dependent (TMD) factorization formalism, for which one has to determine Collins function at the same time[2],[3],[4]; Another one is based on collinear formalism, with Di-hadron fragmentation functions as input[5]. Within uncertainty range the results of these two formalisms are in agreement. In both schemes sea transversity cannot be determined at present. On the other hand, double spin asymmetry (DSA), including double polarized Drell-Yan, single jet or photon production (see e.g., [1], [6–13]) has been proposed for a long time to extract transversity distributions. On proton-proton colliders, such as RHIC[14], DSA is proportional to sea quark transversity, which is expected to be small since gluon has no transversity. According to the estimate in [6],[11] for example, DSA on RHIC is at most percent level in relevant kinematical regions. Although very difficult, it is not hopeless to see the effects of transversity on RHIC. Besides these processes, the heavy quark (such as bottom) production on RHIC has a very high rate, thus may provide some opportunities to see the effect of sea transversity or give a bound to sea quark transversity. On the other hand, the proposed GSI experiments[15], including Drell-Yan, with polarized anti-proton beam is very interesting and important for testing the extracted valence transversity distributions. As an important background to polarized Drell-Yan, heavy quark production has to be known. In this work, we would like to study the production rate of single inclusive heavy quark in hadron-hadron collision with the initial two hadrons transversely polarized. The result may help to check extracted transversity PDFs.

The structure of this paper is as follows: In Sect.II, we make clear the kinematics and notations; in Sect.III, we give our formalism to get the factorization formula for polarized cross section and give tree level result; in Sect.IV, we present virtual and real one-loop corrections and the subtracted result. Some details for the reduction and calculation scheme of real correction will be given; in Sect.V, the numerical results on RHIC and GSI are described and Sect.VI is our summary.

II. KINEMATICS AND NOTATIONS

The process we study is

$$A(P_A, s_{a\perp}) + B(P_B, s_{b\perp}) \rightarrow Q(p_1) + X,$$

where $Q$ is the heavy quark, i.e., bottom or charm. $P_A$, $P_B$ and $p_1$ are the momenta of corresponding hadrons; $s_{a\perp}$ and $s_{b\perp}$ are the spin vectors of initial hadrons. We will work in the center of mass system (cns) of initial hadrons, in which $A$ is moving along $+z$-direction and the spin vectors $s_{a\perp}, s_{b\perp}$ are perpendicular to $z$-axis. Light-cone coordinates are adopted in this work, i.e., for any four vector $a^\mu$, its components are denoted as $a^\mu = (a^+, a^-, a^\perp)$, with $a^\pm = (a^0 \pm a^3)/\sqrt{S}$. This notation is suitable for perturbative calculation. The PDFs with transversely polarized...
spin-1/2 hadron variables are[1]

\[ \int \frac{d\xi}{2\pi} e^{-i\xi^\perp k_+^\perp} \langle P_A s|\bar{\psi}_j(\xi^\perp)\psi_i(0)|P_A s \rangle = \frac{1}{2N_c} \delta_{ij} \left[ \gamma_5 s^\perp \gamma^\perp h_1(x) + \gamma^\perp f_1(x) \right]_{ij}, \tag{2} \]

where \( ij \) represent Dirac- and color indices, \( k_+^\perp = xP_+^\perp \). The spin independent part \( f_1(x) \) is the usual unpolarized PDF, while spin dependent part \( h_1(x) \) is transversity PDF. This definition is for quark. Anti-quark PDFs are obtained by charge conjugation transformation, that is,

\[ \int \frac{d\xi}{2\pi} e^{-i\xi^\perp k_+^\perp} \langle P_A s|\bar{\psi}_i(\xi^\perp)\bar{\psi}_j(0)|P_A s \rangle = \frac{1}{2N_c} \delta_{ij} \left[ \gamma_5 s^\perp \gamma^\perp \bar{h}_1(x) + \gamma^\perp \bar{f}_1(x) \right]_{ij}. \tag{3} \]

Apparently, \( \bar{h}_1(x) = -h_1(-x) \) and \( \bar{f}_1(x) = -f_1(-x) \).

Since heavy quark mass is a hard scale, perturbative QCD can be applied. The differential cross section for single heavy quark inclusive production can be written as a factorized form, for which we will give a simple derivation in next section. The spin dependent part of differential cross section is factorized as

\[ E_1 \frac{d^3\sigma_s}{d^3p_1} = \frac{d^3\sigma_s}{dyd^2p_{1\perp}} = \sum_i \int dx_a dx_b h_i^a(x_a,\mu)h_1^i(x_b,\mu)\hat{W}_s(x_a, x_b, p_{1\perp}, \mu), \quad y = \frac{1}{2} \ln \frac{p_1^+}{p_1^-}, \tag{4} \]

where \( h_i^a \) is the transversity distribution of parton \( i \) in parent hadron. \( i \) can be quark or antiquark here. \( \hat{W}_s \) is spin dependent hard coefficients. \( \mu \) in PDFs and hard coefficients is renormalization scale, which appears because the operator definitions in eqs.(2,3) contain a ultra-violate(UV) divergence. Spin independent cross section is obtained by changing \( h_1(x,\mu) \) to \( f_1(x,\mu) \) and \( \hat{W}_s \) to \( \hat{W} \), which is spin independent hard coefficient.

The hard coefficients are give by subprocess

\[ q(k_a, s_a) + \bar{q}(k_b, s_b) \to Q(p_1) + X, \quad k_a = x_a P_A, k_b = x_b P_B, \tag{5} \]

with \( X \) particles undetected. The spin independent hard coefficients have been calculated to next-to-leading(NLO) in \( \alpha_s \) expansion for a long time[16],[17],[18]. But for spin dependent part the NLO correction is still missing. In the following we adopt the notation of [16] to present the hard coefficients. That is,

\[ \tau_1 = \frac{k_a \cdot p_1}{k_a \cdot k_b}, \quad \tau_2 = \frac{k_b \cdot p_1}{k_a \cdot k_b}, \quad \rho = \frac{4m^2}{s}, \quad \tau_x \equiv 1 - \tau_1 - \tau_2, \quad s = (k_a + k_b)^2, \tag{6} \]

The mass of detected quark is \( m \), which is declared from the charm and bottom masses \( m_{c,b} \) appearing in virtual loops. With this notation, the hard coefficients will be functions of \( s, \tau_1, \tau_x, \rho \).

The allowed kinematical region is given by \( \tau_x \geq 0 \). For Born process, \( q(k_a, s_a) + \bar{q}(k_b, s_b) \to Q(p_1) + \bar{Q} \), we have \( \tau_x = 0 \). With real radiation included in cross section, \( \tau_x \) can be larger than 0. Since \( 0 < x_{a,b} < 1 \), the integration bounds for \( x_{a,b} \) in factorization formula eq.(4) derived from \( \tau_x \geq 0 \) is

\[ x_a^\perp \leq x_a \leq 1, \quad x_b^\perp \leq x_b < 1, \quad x_a = \frac{\tilde{\tau}_2}{1 - \tilde{\tau}_1}, \quad x_b = \frac{x_a \tilde{\tau}_1}{x_a - \tilde{\tau}_2}. \tag{7} \]

For convenience, we have introduced similar notations for hadron variables

\[ \tilde{\tau}_1 = \frac{P_A \cdot p_1}{P_A \cdot P_B}, \quad \tilde{\tau}_2 = \frac{P_B \cdot p_1}{P_A \cdot P_B}. \tag{8} \]

The azimuthal asymmetry is defined as

\[ A_N(\phi) = \frac{d^3\sigma(s_a, s_b) + d^3\sigma(-s_a, -s_b) - d^3\sigma(s_a, -s_b) - d^3\sigma(-s_a, s_b)}{d^3\sigma(s_a, s_b) + d^3\sigma(-s_a, -s_b) + d^3\sigma(s_a, -s_b) + d^3\sigma(-s_a, s_b)}, \tag{9} \]

so that the numerator depends on transversity \( h_1 \) only and the denominator depends on unpolarized distribution \( f_1 \) only. \( A_N \) depends on the azimuthal angle \( \phi \) of detected quark in center of mass frame of \( P_A, P_B \).
integration for final particles. Since $k_{\perp}$ transverse momentum $p_{\perp,i,m,n}$ where QCD partons give leading power contribution in the expansion of $\Lambda_{QCD}/E_{\perp}$, heavy quark mass $m > m_{\perp}$, the central bubble represents hard region, and upper and lower bubbles represent collinear regions.

FIG. 1. Leading region for heavy quark production. The central bubble represents hard region, and upper and lower bubbles represent collinear regions.

### III. FORMALISM AND TREE LEVEL RESULT

Since heavy quark mass $m \gg \Lambda_{QCD}$, $m$ can be taken as a hard scale. In heavy quark production with definite transverse momentum $p_{\perp,1}$, we always demand $p_{\perp,1}$ not so small to avoid complicated threshold effects. In our case, we demand $p_{\perp,1} > m$. The hard scale now is $E_{\perp,1} = \sqrt{p_{\perp,1}^2 + m^2}$. Under high energy limit, i.e., $E_{\perp,1} \gg \Lambda_{QCD}$, collinear partons give leading power contribution in the expansion of $\Lambda_{QCD}/E_{\perp}$.

According to the leading region as shown in Fig. 1, we have

$$d\sigma = \frac{1}{2S} \int \frac{d^{n-1}p_1}{(2\pi)^{n-1}2E_1} \int dk_0^+ dk_-^+ H_{ij}^{mn}(k_0^+, k_-^+).$$

$$= \frac{1}{2S} \int \frac{d^{n-1}p_1}{(2\pi)^{n-1}2E_1} \int dk_0^+ dk_-^+ H_{ij}^{mn}(k_0^+, k_-^+).$$

$$\Gamma_a = \gamma_5 \delta_{a,\perp} \gamma^+,$$

where $ij, mn$ are color and Dirac indices of partons, and $H_{ij}^{mn}$ is the hard part which includes the phase space integration for final particles. Since $k_{a,\perp}, k_{b,\perp}$ are much smaller than $E_{\perp,1}$ in $H_{ij}^{mn}$, they can be ignored at leading power level. This gives twist-2 hard coefficients. After this approximation, $k_{a,\perp, a}^\perp$ and $k_{b,\perp, b}^\perp$ can be integrated over in distribution functions, which results in

$$W_s(x_a, x_b; s_{a,\perp}, s_{b,\perp}) = K_q \Gamma_a \delta_{ij} \otimes \Gamma_b^{nm} \delta_{mn} \otimes H_{ij}^{mn}, \quad K_q = \frac{1}{8(2\pi)^{n-1}} \frac{1}{(2N_c)^2}.$$
As we can see, $F_i$, i.e., $\phi$ represents the insertion of gluon self-energy. Since only two structure functions $F_1, F_2$ are relevant for dynamics, we can choose the polarization states of initial states in two configurations: 1) $s_{a\perp} \parallel s_{b\perp}$; 2) $s_{a\perp} \perp s_{b\perp}$. The corresponding azimuthal angle distributions are

$$W_s = |s_{a\perp}||s_{b\perp}|[\left(1 - \frac{\epsilon}{2}\right)F_1(p_{1\perp}^2) \cos 2\phi - \frac{\epsilon}{2}F_2(p_{1\perp}^2) - F_2(p_{1\perp}^2)], \quad s_{a\perp} \parallel s_{b\perp},$$

$$= |s_{a\perp}||s_{b\perp}|[\left(1 - \frac{\epsilon}{2}\right)F_1(p_{1\perp}^2) \sin 2\phi, \quad s_{a\perp} \perp s_{b\perp}. \quad \tag{17}$$

where $\phi$ is the angle between $\bar{p}_{1\perp}$ and $\bar{s}_{a\perp}$. For the case $s_{a\perp} \perp s_{b\perp}$, the relative angle between $s_{b\perp}$ and $\bar{s}_{a\perp}$ is $\pi/2$, i.e., $\phi(s_b) - \phi(s_a) = \pi/2$. Now it is interesting to see that $\cos(2\phi)$ and $\sin(2\phi)$ asymmetries are the same. This is not obvious. In the following we only consider the case with $s_{a\perp} \parallel s_{b\perp}$. Moreover, in our calculation we find $\epsilon F_1$ and $F_2$ can be ignored, because they are $O(\epsilon)$ after renormalization and collinear subtraction.

With the formulas above, tree level hard coefficients can be obtained very easily. The subprocess is $q\bar{q} \to Q\bar{Q}$, and the results are

$$F_1 = \frac{\alpha_s^2 N_c^2 - 1}{s^2(2N_c)^2} \ln(\tau_x), \quad F_2 = \frac{\alpha_s^2 N_c^2 - 1}{s^2(2N_c)^2} \frac{\ln(\tau_x)}{2}. \quad \tag{19}$$

As we can see $F_2$ is $O(\epsilon)$. At one-loop level, $F_2$ can develop a finite part, but this part will be removed after renormalization and collinear subtraction. As a result, $F_2$ has no finite contribution even to one-loop level.

In all, tree level polarized cross sections ($s_{a\perp} \parallel s_{b\perp}$) is

$$\frac{d\sigma_s}{dyd^2p_{1\perp}} = |s_{a\perp}||s_{b\perp}| \cos(2\phi) \int dx_a h_1(x_a) \int dx_b h_1(x_b) F_1, \quad \tag{20}$$

with $F_1$ given in eq.(19). To one-loop level, hard coefficient $F_1$ can be written in a neat way as done in [16], i.e.,

$$F_1 = H_d(\tau_x) + H_p \left(\frac{1}{\tau_x}\right) + H_l \left(\frac{\ln(\tau_x)}{\tau_x}\right),$$

$$H_d = \frac{\alpha_s^2}{s^2} \left[h_0 + \frac{\alpha_s}{2\pi} h_p^{(1)}\right], \quad H_p = \frac{\alpha_s^2}{s^2} \left[\frac{\alpha_s}{2\pi} h_p^{(1)}\right], \quad H_l = \frac{\alpha_s^2}{s^2} \left[\frac{\alpha_s}{2\pi} h_l^{(1)}\right]. \quad \tag{21}$$

with plus function standard one[16]. We will organize the polarized cross sections in this way.

### IV. ONE-LOOP CORRECTION

#### A. One-loop virtual correction

All diagrams appearing in virtual correction are shown in Fig.2. Self-energy insertions to external lines are trivial and not shown, but included in our calculation. The treatment of massive fermion loop in gluon self-energy, i.e., Fig.2(e) should be mentioned here. There are mainly two UV subtraction schemes in literature. One of them is...
zero-momentum subtraction used in [16], the other is the usual \( \overline{\text{MS}} \) scheme. Generally, gluon self-energy correction is written as

\[
\Pi_{\alpha\beta}^{\gamma}(k) = \delta^{\alpha\beta}\left[(k^\alpha k^\beta - k^2\delta^{\alpha\beta})A(k^2) + k^\alpha k^\beta B(k^2)\right].
\]  

(22)

Due to gauge invariance, \( B(k^2) = 0 \), which is confirmed by explicit calculation. The form factor \( A(s = k^2) \) is

\[
A(s) = \frac{i g_s^2 R_c}{2\pi} \left\{ \sum_{h = c, b} \frac{2}{3} \left[ \frac{2}{3} \ln \mu^2_{m_h^2} + \frac{5}{3} - \frac{4 m_{l_h}^2}{s} + (1 + \frac{2 m_{l_h}^2}{s}) \beta_h \ln \frac{1 - \beta_h}{1 + \beta_h} \right] \right. \\
+ \frac{2n_F}{3} \left[ \frac{2}{3} \ln \frac{\mu^2_{m_b^2}}{s} + \frac{5}{3} \right] + \frac{C_A}{9} \left[ - \frac{30}{e} - 15 \ln \frac{\mu^2_{m_b^2}}{s} - 31 \right], \beta_h = \sqrt{1 - \frac{4 m_{l_h}^2}{s}}, \quad R_c = \frac{(4\pi)^{e/2}}{8\pi^3(1 - e/2)}.
\]

(23)

Instead of the usual \( \overline{\text{MS}} \) subtraction, \( A(s) \) in [16] is subtracted at \( s = 0 \). Since we want to use the hard coefficients of [16] for unpolarized cross section in \( \overline{\text{MS}} \)-scheme, we have to work out the difference, which is defined as \( \Delta = \Delta_{\overline{\text{MS}}} - \Delta_{\text{zero}} \). Since only \( -g^{\alpha\beta}k^2A(k^2) \) in \( \Pi^{\alpha\beta} \) contributes to the amplitude, the difference is proportional to Born result. For bottom production, charm is taken as massless and \( n_{lf} = 4 \) in [16], the difference is

\[
\Delta_b = - \frac{\alpha_s}{2\pi} \left\{ \frac{2}{3} \ln \frac{\mu^2_{m_b^2}}{m_b^2} + \frac{2}{3} \ln \frac{s}{m_b^2} + \frac{4 m_{l_b}^2}{s} + (1 + \frac{2 m_{l_b}^2}{s}) \beta_b \ln \frac{1 - \beta_b}{1 + \beta_b} \right\} H_d^{\text{tree}}, \quad H_d^{\text{tree}} = \frac{1}{2\pi} \left\{ \frac{(2\pi)^{e/2}}{\Gamma(1 - e/2)} \right. \\
\left. \left( \frac{\mu^2_{m_b^2}}{m_b^2} \right)^{e/2} \left[ N_1 G_1 + N_2 G_2 + N_3 G_3 \right] \right\}.
\]

(24)

For charm production, bottom is not included in fermion loop and \( n_{lf} = 3 \) in [16], the difference is

\[
\Delta_c = - \frac{\alpha_s}{2\pi} \left\{ \frac{2}{3} \ln \frac{\mu^2_{m_c^2}}{m_c^2} + \frac{2}{3} \ln \frac{s}{m_c^2} + \frac{5}{3} + \frac{4 m_{l_c}^2}{s} + (1 + \frac{2 m_{l_c}^2}{s}) \beta_c \ln \frac{1 - \beta_c}{1 + \beta_c} \right\} H_d^{\text{tree}}.
\]

(25)

The calculation of other diagrams is very straightforward. The tensor integrals are reduced by FIRE[20]. The resulting scalar integrals are standard, and are given for example in [21],[17]. An interesting point for FIRE reduction is we have to calculate bubble and tadpole integrals to \( O(\epsilon) \), rather than \( O(1) \). The reason is some IR divergent scalar integrals will also be reduced into bubble and tadpole integrals in FIRE due to integration by part relations(IBPs). The IR pole \( 1/\epsilon_{\text{IR}} \) therefore appears in the reduced coefficients. Thus, in order to get \( O(1) \) hard coefficients, one has to calculate bubble and tadpole to \( O(\epsilon) \). For convenience, we list involved bubble and tadpole integrals in appendix.

Before renormalization, the divergent parts of polarized partonic hard coefficients extracted from our full results are

\[
h_d^{(1)} = B \left[ \frac{32}{\epsilon^2} + \frac{16}{\epsilon} \left( \frac{2 - \rho}{\sqrt{1 - \rho}} \ln \frac{1 - \sqrt{1 - \rho}}{1 + \sqrt{1 - \rho}} + 2 + 4 \ln \frac{1 - \tau_1}{\tau_1} + \ln \frac{\rho}{4} \right) \right], \quad G_1 = \frac{8B}{\epsilon} \left[ \frac{2 - \rho}{\sqrt{1 + \rho}} \ln \frac{1 - \sqrt{1 + \rho}}{1 + \sqrt{1 + \rho}} - \frac{11}{3} + \ln \frac{\rho}{4} - 4 \ln(1 - \tau_1) + 2 \ln \tau_1 \right], \quad G_2 = \frac{8B}{\epsilon} \left[ \frac{2 - \rho}{\sqrt{1 - \rho}} \ln \frac{1 - \sqrt{1 - \rho}}{1 + \sqrt{1 - \rho}} + 2 + 4 \ln \frac{1 - \tau_1}{\tau_1} + \ln \frac{\rho}{4} \right], \quad G_3 = \frac{32}{\epsilon}.
\]

(26)

with \( B = \rho + 4\tau_1(\tau_1 - 1) \) and \( n_F = 3 \) the number of light fermion flavors. \( N_{1,2,3} \) are color factors

\[
N_1 = N_c C_F^2, \quad N_2 = N_c C_F C_A, \quad N_3 = N_c C_F.
\]

(27)

The finite parts are too complicated and cannot be shown here. In the appendix, the final hard coefficients after renormalization and subtraction are put in mathematica files. An illustration for these files is given in Appendix.D.

B. One-loop Real correction

Real correction is given by the module squared of the diagrams in Fig.3. For heavy quark production with definite
FIG. 3. Real corrections to the amplitude.

\[ I_r = \int \frac{d^nk_g}{(2\pi)^n} \frac{d^np_1}{(2\pi)^n} (2\pi)^2 \delta(k_g^2) (2\pi)^2 \delta(p_1^2 - m^2) |\mathcal{M}|^2, \]

where \( M \) is the amplitude in Fig. 3. Clearly, the integration above is standard 2-particle phase space integration. Due to this feature real correction can be identified as the absorptive part or cut amplitude of following forward scattering

\[ q(k_a) + \bar{q}(k_b) + \bar{Q}(-p_1) \rightarrow q(k_a) + \bar{q}(k_b) + \bar{Q}(-p_1). \]

The intermediate state is \( |\bar{Q}(p_2), g(k_g)\rangle \). Then, the involved cut tensor integrals can be reduced to scalar ones in the same way as uncut tensor integrals. \( \text{FIRE} \) with integration by part relations incorporated is a particularly suitable tool for this purpose. After reduction, there are only six types of master integrals, which are shown in Fig. 4.

The general form of the master integral is

\[ I_r = \int \frac{d^nk_g}{(2\pi)^n} \frac{d^np_1}{(2\pi)^n} \delta(k_g^2) \delta(p_1^2 - m^2) |\mathcal{M}|^2, \]

where \( N_1, N_2 \) are denominators of uncut propagators. Like uncut one-loop integrals, these \( I_r \) integrals are in the standard form, i.e., \( i_{1,2} = 0 \) or 1.

To calculate \( I_r \) it is convenient to work in the frame with \( \vec{q} = 0 \), where \( q = k_a + k_b - p_1 \). First, the energy of gluon
$k_g^0$ can be integrated out by using the two delta functions. Then,

$$I_r = \left( \frac{\mu}{k_g^0} \right)^\epsilon \frac{k_g^0}{4q^0} \int d\Omega_{n-1} \frac{1}{N_1 N_2},$$

(31)

with $k_g^0 = s\tau_x/(2q^0)$ and $q^0 = \sqrt{m^2 + s\tau_x}$. $d\Omega_{n-1}$ is the angular integration measure for $\vec{k}_g$, which is defined in $n-1 = 3 - \epsilon$ dimensional space. $I_r$ may contain collinear and soft divergences. To simplify the calculation, we want to separate these two kinds of divergences. This is possible, since soft divergence corresponds to the singularity at $\tau_x = 0$. If $I_r$ is singular under soft limit $\tau_x \to 0$, $N_1$ or $N_2$ must be proportional to $k_g^0$. Then, $k_g^0$ can be extracted from $N_1$ or $N_2$. This implies we can define an integral $\tilde{I}_r$, which is regular under soft limit, that is

$$I_r = \left( \frac{\mu}{k_g^0} \right)^\epsilon \frac{\tau_x}{s}^{i\epsilon + \frac{1}{2} + \frac{1}{2}} \tilde{I}_r.$$

(32)

The power $k$ is chosen so that $\tilde{I}_r$ is finite but nonzero at $\tau_x = 0$. In this way, collinear divergence is included in $\tilde{I}_r$ and soft divergence is given by the expansion of $\tau_x^{-1-\epsilon}$, i.e.,

$$\tau_x^{-1-\epsilon} = \frac{1}{-\epsilon} \delta(\tau_x) + \left( \frac{1}{\tau_x} \right)_+ - \epsilon \left( \frac{\ln \tau_x}{\tau_x} \right)_+.$$

(33)

The plus function is defined in standard way[16].

The angular integrals $\tilde{I}_r$ can be classified into six types, i.e., $R_i$ defined in eq.(34).

$$R_1(w) = \int d\Omega_{n-1} \frac{1}{1 + \bar{a} \cdot \vec{k}_g} \frac{1}{1 + \bar{b} \cdot \vec{k}_g},$$

$$R_2(\Delta, w) = \int d\Omega_{n-1} \frac{1}{1 + \bar{a} \cdot \vec{k}_g} \frac{1}{\Delta + \bar{b} \cdot \vec{k}_g},$$

$$R_3(\delta, \Delta, w) = \int d\Omega_{n-1} \frac{1}{\delta + \bar{a} \cdot \vec{k}_g} \frac{1}{\Delta + \bar{b} \cdot \vec{k}_g},$$

$$R_4 = \int d\Omega_{n-1} \frac{1}{1 + \bar{a} \cdot \vec{k}_g},$$

$$R_5(\delta) = \int d\Omega_{n-1} \frac{1}{\delta + \bar{a} \cdot \vec{k}_g},$$

$$R_6 = \int d\Omega_{n-1},$$

(34)

with $|\bar{a}| = |\bar{b}| = |\vec{k}_g| = 1$ and $w = 1/|\bar{a} - \bar{b}|$, and $\Delta > 1$, $\delta > 1$. In the appendix, most $R_i$ functions are calculated to $O(\epsilon)$, by making use of Feynman parameters. The results are compared with the known results in [17]. Numerically, they are precisely the same. This is a check of our calculation.

Still, a subtle issue with FIRE reduction should be mentioned. Some IR divergent integrals can be reduced to IR finite integrals with coefficients proportional to $1/\epsilon$. Thus, the IR finite integrals must be calculated to higher order of $\epsilon$. In our case for $q\bar{q}$ scattering, the combination $R_6 \tau_x^{-1-\epsilon}/\epsilon$ appears in the reduced result. To get correct finite contribution, $R_6$, i.e., pure 2-particle phase space integration, must be calculated to $O(\epsilon^2)$. The result is

$$R_6 = N_6 \frac{2^{\frac{1}{1-\epsilon}} \epsilon^2}{12} \left[ \frac{2 + \epsilon(2 - \ln 4) + \epsilon^2 \left( 2 - \frac{\pi^2}{12} + \ln^2 2 - \ln 4 \right) + O(\epsilon^3) }{12} \right],$$

$$N_6 = \int d\Omega_{n-2} = \frac{2\pi^{n-\frac{1}{2}}}{\Gamma(1 - \frac{1}{2})}.$$

(35)

The total real correction is very complicated and are given in a mathematica file. However, the extracted soft part can be presented. For real correction, soft contribution is proportional to $\delta(\tau_x)$ and can be given by eikonal approximation. Thus, we expect the soft contribution is universal and does not depend on the polarization of initial quarks. [18] gives such soft contribution explicitly for unpolarized $q\bar{q}$ scattering. With the real soft correction for
polarized $q\bar{q}$ scattering extracted from [18], the polarized hard coefficient $h_d^{(1)}$ is

$$h_d^{(1)} = h_d^{(0)} \frac{1}{2} e^{-\frac{5}{2}(\gamma_E - \ln 4\pi)} \left( \frac{s^2}{\mu^2 m^2} \right)^{-\epsilon/2} \left[ C_F K_{\text{soft}}^F + C_A K_{\text{soft}}^A \right],$$

$$K_{\text{soft}}^F = \frac{16}{\epsilon^2} - \frac{8}{\epsilon} \ln y + 2 \ln^2 y + 4 \ln \xi_1 (1 - y)$$

$$+ 4 \left( 1 - \frac{2 m^2}{s} \right) \frac{1}{\beta} \left\{ \frac{2}{\epsilon} \ln x - \ln x + 2Li_2(x) + 2Li_2(-x) - \ln^2 x + 2 \ln x \ln(1 - x^2) - \zeta(2) \right\}$$

$$+ \frac{8}{\epsilon} + 4 - \frac{32}{\epsilon} \ln \frac{t_1}{u_1} - 16 \ln x \ln \frac{t_1}{u_1} - 16 Li_2(1 - \frac{u_1}{t_1}) + 16 Li_2(1 - \frac{t_1}{u_1}) - 6 \zeta(2),$$

$$K_{\text{soft}}^A = \frac{4}{\epsilon} \ln y - \ln^2 y - 2 Li_2(1 - y)$$

$$- 2 \left( 1 - \frac{2 m^2}{s} \right) \frac{1}{\beta} \left\{ \frac{2}{\epsilon} \ln x + 2Li_2(x) + 2Li_2(-x) - \ln^2 x + 2 \ln x \ln(1 - x^2) - \zeta(2) \right\}$$

$$- \frac{12}{\epsilon} \ln \frac{u_1}{t_1} + 6 \ln x \ln \frac{t_1}{u_1} - \ln^2 x + \ln^2 \frac{t_1}{u_1} - 6 Li_2(1 - \frac{t_1}{u_1}) + 6 Li_2(1 - \frac{u_1}{t_1}),$$

(36)

with

$$y = \frac{sm^2}{t_1 u_1}, \quad x = \frac{1 - \beta}{1 + \beta}, \quad \beta = \sqrt{1 - \frac{4 m^2}{s}}, \quad t_1 = (k_a - p_1)^2 - m^2 = -s \tau_1, \quad u_1 = (k_b - p_1)^2 - m^2 = -s \tau_2.$$

(37)

We have checked that this result, including finite part, is totally the same as our result, which is a strong check for our reduction and calculation scheme.

In hard coefficients $h_p$ and $h_l$, $\tau_a$ is nonzero. Since $\tau_a$ represents the energy of final gluon, this means the gluon is not soft. Thus in $h_p$ only collinear divergence exists, which can be inferred from the subtraction term and does not need to be shown again. Clearly, $h_l = (-\epsilon)h_p$, thus $h_l$ is obtained from the divergent part of $h_p$.

C. Subtraction and Final result

To get the true one-loop contribution, we have to subtract collinear contributions from each diagram[23]. The subtraction is realized by following replacement in tree level hadronic cross sections eq.(20),

$$h_1(x_a, \mu^2) \rightarrow \frac{\alpha_s}{2\pi} \frac{(4\pi)^{\epsilon/2}}{\Gamma(1 - \epsilon/2)} \left[ \frac{2}{\epsilon_{\text{UV}}} - \frac{2}{\epsilon_{1R}} + \frac{1}{\epsilon_{1R}} \right] \int_{\ln x_a}^{1} \frac{d\xi_a}{\xi_a} P_T^{\text{qq}}(x_a, \mu^2) \delta(\tau_a)H^T_{1R}(x_a, x_b).$$

(38)

The UV pole $2/\epsilon_{\text{UV}}$ is removed by renormalization(in $\overline{\text{MS}}$-scheme) of bare transversity distribution which appearing in tree level cross section. Then only IR pole should be preserved. The final subtraction term is

$$[\cos 2\delta]^{-1} \sigma^{\text{sub}}_{\text{sub}} = \frac{\alpha_s}{2\pi} \frac{(4\pi)^{\epsilon/2}}{\Gamma(1 - \frac{\epsilon}{2})} \left( -\frac{2}{\epsilon_{1R}} \right) \int dx_a h_1(x_a, \mu^2) \int dx_b h_1(x_b, \mu^2) C_F \left( 3 - 2 \ln \tau_1(1 - \tau_1) \right) \delta(\tau_x)H^T_{1R}(x_a, x_b)$$

$$+ \frac{2}{(\tau_x)^+} \left( \frac{z_a^2 H^T_{1R}(x_a, x_b) + z_b^2 H^T_{1R}(x_a, x_b)}{z_a + z_b} \right),$$

(39)

with $z_a = \tau_a/(1 - \tau_a), z_b = \tau_1/(1 - \tau_2)$. The logarithm before $\delta(\tau_x)$ comes from the variable transformation of plus function[16],

$$\left( \frac{1}{a \tau_x} \right)^+ = \frac{1}{a} \left( \frac{1}{\tau_x} \right)^+ + \ln a \delta(\tau_x).$$

(40)

Note that the subtraction terms have no explicit $\ln \mu$. Besides subtraction, UV renormalization for hard part should be done. For heavy quark mass we adopt pole mass scheme, and for other UV divergences we remove them by adding counter terms(c.t.) in the usual $\overline{\text{MS}}$ scheme. Wave function renormalization for external lines is also done in $\overline{\text{MS}}$ scheme. The wave function renormalization constants for massive and massless fermions are

$$Z_2^m = - \frac{9 C_F}{16\pi^2} \left[ \frac{2}{\epsilon_{\text{UV}}} - \gamma_E + \ln 4\pi \right] + 2 \left[ \frac{2}{\epsilon_{1R}} - \gamma_E + \ln 4\pi \right] + 3 \ln \frac{m^2}{4\pi^2} + 4,$$

$$Z_2 = - \frac{9 C_F}{16\pi^2} \left[ \frac{2}{\epsilon_{\text{UV}}} - \gamma_E + \ln 4\pi \right] - \left[ \frac{2}{\epsilon_{1R}} - \gamma_E + \ln 4\pi \right].$$

(41)
After UV c.t. are added and wave function renormalization is done, we complete renormalization and get \( d\sigma_R \). The final hard coefficients are given by \( d\sigma_R - d\sigma_{sub} \). It is confirmed that the final hard coefficients \( h_d, h_p, h_l \) are finite, and the total cross section is \( \mu \) independent up to \( O(\alpha'^2) \). Especially, the unpolarized cross section from \( q\bar{q} \) channel is also calculated by using the same program and the hard coefficients in zero-momentum subtraction scheme are compared with [16]. Numerically, the two results are totally the same. This is a strong check to our calculation. Our one-loop hard coefficients are stored in mathematica files, which can be obtained from author.

V. NUMERICAL RESULTS

We have known the asymmetry \( A_N(\phi) \) is proportional to cos \( 2\phi \) or sin \( 2\phi \) for the cases \( s_{a\perp} \parallel \) parallel or perpendicular to \( s_{b\perp} \). Thus it is useful to define azimuthal angle integrated asymmetry \( A_N \) as

\[
A_N = \int_0^{2\pi} d\phi \cos 2\phi \frac{d\sigma}{dyd^2p_{1\perp}} = |s_{a\perp}| |s_{b\perp}| \frac{\pi}{2} \frac{\Sigma_T(y, p_{1\perp} ^2)}{\Sigma},
\]

with \( \Sigma_T \) and \( \Sigma \) defined as the \( \phi \) independent part of \( d\sigma_s \) and \( d\sigma \), i.e.,

\[
\frac{d\sigma_s}{dyd^2p_{1\perp}} = |s_{a\perp}| |s_{b\perp}| \cos 2\phi \Sigma_T, \quad \frac{d\sigma}{dyd^2p_{1\perp}} = \Sigma.
\]

The numbers in the tables of Appendix are for \( A_N \) rather than \( A_N(\phi) \) in eq.(9).

There is a charge asymmetry between heavy quark production and antiquark production at one-loop level. But this asymmetry for unpolarized cross section is much smaller than charge average, and for polarized cross section it is at least one order smaller than corresponding charge average. In all numerical results, only charge average is shown.

For valence transversity distribution functions, various groups give similar results[2-5]. We take valence transversity in [3] as a reference. For sea transversity, because there is no reliable extraction up to now, we just make an assumption that at a very low energy scale, sea transversity distributions are the same as sea helicity distributions[6,10,11]. Here the low energy scale is taken as \( Q_0^2 = 2.4\text{GeV}^2 \). Helicity distribution functions are taken as DSSV type[24,25]. Since Soffer’s bound[26] always leads to unexpectedly large sea transversity, we do not use it in the estimates.

The NLO hard coefficients of unpolarized cross section is taken from [16]. NLO MSTW2008 PDFs[27] and corresponding NLO \( \alpha_s \) are adopted in the calculation of unpolarized cross sections. For polarized cross section, we are just able to use LO evolution for both transversity PDFs and \( \alpha_s \). From our numerical result, the scale dependence of polarized cross section is less than unpolarized ones, thus NLO evolution seems not necessary.

All calculation, including evolution of parton distribution functions and strong coupling, are done in \( \overline{\text{MS}} \)-scheme. Near threshold, perturbative calculation is not reliable. To avoid such a difficulty, we let minimum of transverse momentum be greater than the mass of detected heavy quark. Renormalization scale is taken as standard one, \( \mu = \mu_0 = E_{1\perp} \). The scale uncertainty is estimated by varying renormalization scale \( \mu \) from \( \mu_0/2 \) to \( 2\mu_0 \). For charm production on GSI with \( E_{1\perp} = 3.0\text{GeV} \), the lower \( \mu \) is taken as \( \sqrt{2}\text{4GeV} \), which is the initial scale of extracted transversity distribution functions[3]. The uncertainty of PDFs are not taken into account. In all cases, we assume the polarizations of initial hadrons are 1, i.e., \( |s_{a,b\perp}| = 1 \).

A. pp collider

Our results for cross sections and \( A_N \) are listed in Appendix.C. Both polarized and unpolarized cross sections are for charge average of heavy quark and anti-quark. To be measurable on RHIC, \( A_N \) should be larger than 0.001, due to the systematic uncertainty in experiment[11]. However, from our result about RHIC \( \sqrt{S} = 500\text{GeV} \) and \( \sqrt{S} = 200\text{GeV} \), the asymmetry \( A_N \) is of order \( 10^{-4} \) in most kinematical regions. The suppression mainly comes from gg-channel contribution in unpolarized cross section, which is usually one order larger than \( q\bar{q} \)-channel contribution. Really, \( A_N \) increases with increasing \( E_{1\perp} \), but very slowly. For \( \sqrt{S} = 200\text{GeV} \), only when \( E_{1\perp} = 30\text{GeV} \), \( A_N \) can reach 0.001. For \( \sqrt{S} = 500\text{GeV} \), higher \( E_{1\perp} \) is needed. Since \( \Sigma \) and \( \Sigma_T \) decay very fast with increasing \( E_{1\perp} \), the statistic error may be a problem in this region. Thus on RHIC single inclusive heavy quark production is not helpful to see the effect of sea transversity, unless the magnitude of sea transversity is much larger than sea helicity distribution functions or precise measurement in high \( E_{1\perp} \) region(\( E_{1\perp} > 30\text{GeV} \)) can be performed.
B. $p\bar{p}$ collider

The polarized anti-proton program was proposed by PAX collaboration of FAIR at GSI\[15\]. The main purpose is to measure double transversely polarized Drell-Yan. There are collider and fixed target schemes. In collider scheme, the momentum of polarized anti-proton can reach $15\text{GeV}/c$, and the momentum of polarized proton can reach $3.5\text{GeV}/c$. In fixed target scheme, the momentum of polarized anti-proton can reach $22\text{GeV}/c$. In these kinematical regions, charm can be produced. Since in Drell-Yan the invariant mass squared of virtual photon is $Q^2 = 4, \ldots, 100\text{GeV}^2$, charm lepton decay will become an important background. On the other hand, charm production itself can be served as a probe of transversity.

In our numerical results, we list only collider result in the center of mass system(cms) of $p\bar{p}$, with $S = 216.4\text{GeV}^2$ or $\sqrt{S} = 14.7\text{GeV}$. We choose $E_{1\perp} = 3, 4, 5, 6\text{GeV}$ and $y = 0.0, 0.3, \cdots$.

The obtained asymmetry is very sizable. For most cases, it is about 10%. For fixed rapidity, $A_N$ increases with increasing $E_{1\perp}$. When $E_{1\perp} = 6\text{GeV}$, the asymmetry can even reach 20%. However, the unpolarized and polarized cross sections at this scale are of order $10^{-2}$ and $10^{-3}\text{pb}$, which are too small for the luminosity of GSI(about 0.43$/\text{pb}$ each day\[15\]). For fixed transverse energy, the asymmetry decreases with the increasing of rapidity, thus the largest asymmetry is in transverse direction in cms of initial hadrons. We also notice that renormalization scale dependence of $A_N$ in low $E_{1\perp}$ region is larger than high $E_{1\perp}$ region. Actually, the uncertainty of asymmetry should satisfy

$$\frac{\Delta A_N}{A_N} = \frac{\Delta \Sigma_T}{\Sigma_T} - \frac{\Delta \Sigma}{\Sigma}. \quad (44)$$

When $E_{1\perp} = 5, 6\text{GeV}$, both $\Delta \Sigma_T/\Sigma_T$ and $\Delta \Sigma/\Sigma$ are large, but they cancel each other, which leads to a smaller $\Delta A_N$. When $E_{1\perp} = 3, 4\text{GeV}$, $\Delta \Sigma_T/\Sigma_T$ actually is very small. The main uncertainty of $A_N$ comes from unpolarized cross section. For RHIC with $\sqrt{S} = 500\text{GeV}$, in the kinematical region we considered, $\Delta \Sigma/\Sigma$ is large, and $\Delta \Sigma_T/\Sigma_T$ is always smaller than $\Delta \Sigma/\Sigma$, then the asymmetry always has a relatively large scale uncertainty. Thus, if we want to reduce the uncertainty of $A_N$ in low or medium $E_{1\perp}$ region, it is much better to use two-loop result for unpolarized cross section. In low transverse energy region, such as $E_{1\perp} = 3\text{GeV}$ on GSI, the polarized cross section $\Sigma_T$ is of order $100\text{pb}/\text{GeV}^2$. The observation of azimuthal asymmetry is very promising.

VI. SUMMARY

In this work, we have calculated one-loop QCD correction to single heavy quark inclusive production on double transversely polarized colliders. The analytic results are obtained. The reduction and calculation scheme is very efficient, and we just need to calculate six very simple angular integrals. Soft and collinear divergences in real correction can also be separated very easily in our scheme. As a check, we also use our program to calculate the unpolarized cross section from $q\bar{q}$ channel, and numerically, the obtained hard coefficients are the same as known results in literature\[16\],\[18\]. With the analytic results, numerical estimates on RHIC and GSI are obtained, with the assumption that sea transversity distribution is equal to sea helicity distribution at a low energy scale. On RHIC, in order to see the effect of sea transversity, the transverse energy of final heavy quark(bottom) should be larger than $30\text{GeV}$ for $\sqrt{S} = 200\text{GeV}$ and higher for $\sqrt{S} = 500\text{GeV}$. For future GSI experiments, charm production is very useful to check the extracted valence transversity. At $E_{1\perp} = 3\text{GeV}$, the asymmetry is of order 0.1 and even polarized cross section can be over $100\text{pb}/\text{GeV}^2$ for fixed transverse energy and rapidity. Improvement on scale dependence can be made by using two-loop result for unpolarized cross section. In addition, double transverse spin asymmetry of Drell-Yan process is a central issue for future GSI experiment. As an important background, our analytic result for heavy quark production may be very helpful to determine the cross section of polarized Drell-Yan process.

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Appendix A: Bubble and tadpole integrals

The bubble and tadpole integrals are defined as

\[
\mu^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{1}{[l^2 - m_1^2][(l + p)^2 - m_2^2]} = \frac{i}{16\pi^2} \frac{(4\pi)^{\eta/2}}{\Gamma(1 - \epsilon/2)} b(p^2, m_1, m_2),
\]

\[
\mu^{4-n} \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2 - m^2} = \frac{i}{16\pi^2} \frac{(4\pi)^{\eta/2}}{\Gamma(1 - \epsilon/2)} a_0(m).
\]

(A1)

The integrals are expanded to \(O(\epsilon)\). In the result, some simple one dimensional integrals can be worked out in terms of Spence function \(Li_2\), but we just leave them there for simplicity. The calculation is done in physical region, with \(s > 0, t < 0\).

1.

\[
b(t, m, 0) = \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \left(\frac{m^2 - t}{m^2}\right)^{-\epsilon/2} B(1 - \epsilon/2, \epsilon/2) J
\]

\[J = 1 - \frac{\epsilon}{2} \left[ - \frac{m^2 \log \left(\frac{m^2 - t}{m^2}\right)}{t} - 2 \right]
+ \frac{\epsilon^2}{8} \left[ -6 \left(\frac{m^2 - t}{m^2}\right) \log \left(\frac{t}{m^2 - t}\right) \right.
- 6 \log \left(\frac{m^2}{m^2 - t}\right) \left(\left(\frac{m^2}{m^2 - t}\right) \log \left(\frac{t}{m^2 - t}\right) + 2m^2\right)
+ \pi^2 m^2 - \pi^2 t + 24t \],

(A2)

2.

\[
b(s, 0, 0) = \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} B(1 - \frac{\epsilon}{2}, \frac{\epsilon}{2}) B(1 - \frac{\epsilon}{2}, 1 - \frac{\epsilon}{2}) \left[ 1 - \frac{\epsilon}{2} \log \frac{s}{m^2} + \frac{\epsilon^2}{8} \left( \log \frac{s}{m^2} - \pi^2 \right) \right],
\]

(A3)

3.

\[
b(s, m, m) = \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} B(1 - \frac{\epsilon}{2}, \frac{\epsilon}{2})
\times \left[ 1 - \frac{\epsilon}{2} \int_0^1 dx \ln |1 + Ax(1 - x)| + \frac{\epsilon^2}{8} \left( - \pi^2 \sqrt{1 + 4/A} + \int_0^1 dx \ln^2 |1 + Ax(1 - x)| \right) \right],
\]

\[A = -\frac{s}{m^2},
\]

(A4)

4.

\[
a_0(m) = -\Gamma(1 - \frac{\epsilon}{2}) \Gamma(-1 + \frac{\epsilon}{2})(m^2)^{1 - \frac{\epsilon}{2}} = \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \frac{m^2}{1 - \epsilon/2} B(1 - \epsilon/2, \epsilon/2)
\]
\[= \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} m^2 \left[ \frac{2}{\epsilon} + 1 + \frac{1}{12} (6 + \pi^2) \epsilon \right],
\]

(A5)

Appendix B: Real integrals

The real integrals defined in eq.(34) are given here. \(R_3\) is calculated to \(O(1)\), and \(R_6\) is calculated to \(O(\epsilon^2)\). Others are calculated to \(O(\epsilon)\). All are checked by making use of the formulas in [17]. Numerically, the two results are precisely the same. \(R_i\) are organized as follows:

\[
R_i = N_i \left[ \frac{2}{\epsilon} R_i^{(-1)} + R_i^{(0)} + \frac{\epsilon}{2} R_i^{(1)} \right], \quad N_i = \frac{2\pi^{1 - \frac{\epsilon}{2}}}{\Gamma(1 - \frac{\epsilon}{2})}.
\]

(B1)

The explicit expressions are
1. $R_1(w)$:

\[
R_1^{(-1)} = -4w^2;
\]

\[
R_1^{(0)} = -8w^2 \log(w);
\]

\[
R_1^{(1)} = 2w^2 \left[ 4\text{Li}_2(1 - 2w) - 2\text{Li}_2(-2w) - 2\text{Li}_2\left( \frac{1}{2w + 1} \right) \right.
\]

\[
- \log^2(2w + 1) + 2 \log(w) \log\left( \frac{32}{2w + 1} \right) - 2 \log(2) \log(w(2w + 1)) + \pi^2/3 + 4 \log^2(2) \right]; \tag{B2}
\]

2. $R_2(\delta, w)$:

\[
R_2^{(-1)} = - \frac{2w^2}{2w^2(\delta - 1) + 1};
\]

\[
R_2^{(0)} = \frac{2w^2 \log\left( \frac{(2w^2(\delta - 1) + 1)^2}{w^2(\delta - 1) - 1} \right)}{2w^2(\delta - 1) + 1};
\]

\[
R_2^{(1)} = - \frac{w^2}{3(2w^2(\delta - 1) + 1)} \left[ 6\text{Ir}_2(\delta, w) - 6\text{Li}_2\left( \frac{w(\delta - 1) + 1}{2(\delta - 1)w^2 + 1} \right) \right.
\]

\[
+ 3 \log\left( \frac{w(2w + 1)}{2w^2(\delta - 1) + 1} \right) \left[ 3 \log\left( \frac{w}{2w^2(\delta - 1) + 1} \right) + 2 \log((2w - 1)(\delta - 1)) + \log\left( \frac{1}{4}(2w + 1) \right) \right]
\]

\[
- 6 \log(2) \log\left( \frac{w(2w - 1)(\delta - 1)}{2w^2(\delta - 1) + 1} \right)
\]

\[
- 3 \log\left( \frac{2w^2(\delta + 1)}{4w^2 - 1} \right) \left( \log(16(w(-\delta + w + 1)^2 - 2 \log(w(\delta - 1) + 1)) + \pi^2 + 3 \log^2(2) \right];
\]

\[
\text{Ir}_2(\delta, w) = \int_0^1 \frac{d\xi}{z(1 - z + \xi_1)(1 + \xi_2)} \ln \left( \frac{(1 - z)(z + \xi_1)}{(z + \xi_2)(z + \xi_3)} \right) - \int_0^1 \frac{d\xi}{z(1 + \xi_1 - z)} \ln \left( \frac{1 - \xi_2}{1 - \xi_3} \right);
\]

\[
\xi_1 = \frac{w(2w - 1)(\Delta - 1)}{1 + w(\Delta - 1)}, \quad \xi_2 = 2w - 1, \quad \xi_3 = \frac{w(1 + \Delta)}{1 + w - w\Delta}. \tag{B3}
\]

3. $R_3(\delta, \Delta, w)$:

\[
R_3^{(-1)} = 0;
\]

\[
R_3^{(0)} = \frac{2w^2}{\sqrt{4w^2(\delta - \Delta)^2 + \delta\Delta} - 1 + 1} \log\left( \frac{-2w^2(\delta\Delta - 1) + \sqrt{4w^2(\delta - \Delta)^2 + \delta\Delta - 1} + 1 + 1}{w^2(2 - 2\delta\Delta) + \sqrt{4w^2(\delta - \Delta)^2 + \delta\Delta - 1} + 1 - 1} \right); \tag{B4}
\]

4. $R_4$:

\[
R_4^{(-1)} = -1;
\]

\[
R_4^{(0)} = 2 \log(2);
\]

\[
R_4^{(1)} = 2 \left( \frac{\pi^2}{12} - \log^2(2) \right); \tag{B5}
\]

5. $R_5(\delta)$:

\[
R_5^{(-1)} = 0;
\]

\[
R_5^{(0)} = \log\left( \frac{\delta + 1}{\delta - 1} \right);
\]

\[
R_5^{(1)} = - \text{Li}_2\left( \frac{2}{\delta - 1} \right) + \text{Li}_2\left( \frac{2}{\delta + 1} \right) - 2 \log(2) \log\left( \frac{\delta + 1}{\delta - 1} \right); \tag{B6}
\]
6. \( R_6 \):

\[
R_6 = \int d\Omega_{n-1} = N_\epsilon \int_0^\pi d\theta \sin^{n-3} \theta = N_\epsilon 2^{1-\epsilon} B(1 - \frac{\epsilon}{2}, 1 - \frac{\epsilon}{2})
\]

\[
= N_\epsilon \left[ 2 + \epsilon(2 - \log(4)) + \epsilon^2 \left( 2 - \frac{\pi^2}{12} + \log^2(2) - \log(4) \right) + O(\epsilon^3) \right]. \tag{B7}
\]
Appendix C: Numerical results on RHIC and GSI

Numerical results on RHIC and GSI are listed in following tables. RHIC is for bottom production and GSI for charm production. $gg, gq, q\bar{q}$ mean the unpolarized cross sections from corresponding channels. $\Sigma$, $\Sigma_T$ and $A_N$ are defined in text. Total energy of collider and transverse energy of detected heavy quark are indicated in the first cell of each table. For each rapidity and each cross section, there are three numbers, which correspond to different choice of renormalization scale $\mu$. The central number is obtained with $\mu = \mu_0 = E_{1\perp}$. The upper and lower ones are given by $\mu_0/\sqrt{2}$ and $2\mu_0$, respectively. For the case $E_{1\perp} = 3$GeV, $\mu_0/\sqrt{2}$ is replaced by $\sqrt{2}$.

The units of energy and cross section are GeV and pb/GeV$^2$, respectively. Charm mass $m_c = 1.40$GeV and bottom mass $m_b = 4.75$GeV. The numbers are given in Fortran-like format, e.g., $1.234E-4 = 1.234 \times 10^{-4}$.

### RHIC

| $\sqrt{s} = 200$ | gg | $gq$ | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$ |
|------------------|----|-----|----------|--------|-----------|-------|
| $E_{1\perp} = 10$ | 4.934E2 | 1.344E2 | 4.928E1 | 6.771E2 | 1.604E-1 | 3.72E-4 |
| $y = 0.0$       | 3.723E2 | 3.024E1 | 5.079E1 | 4.533E2 | 1.575E-1 | 5.458E-4 |
|                 | 2.704E2 | -1.468E0 | 4.440E1 | 3.133E2 | 1.285E-1 | 6.441E-4 |
| $y = 1.0$       | 3.235E2 | 8.066E1 | 3.556E1 | 4.397E2 | 1.035E-1 | 3.697E-4 |
|                 | 2.388E2 | 1.583E1 | 3.701E1 | 2.917E2 | 1.002E-1 | 5.396E-4 |
|                 | 1.699E2 | -2.877E0 | 3.299E1 | 1.993E2 | 8.027E-2 | 6.327E-4 |
| $y = 2.0$       | 5.575E1 | 8.903E0 | 6.170E0 | 7.082E1 | 1.624E-1 | 3.602E-4 |
|                 | 3.592E1 | 7.695E-1 | 6.446E0 | 4.314E1 | 1.496E-1 | 3.602E-4 |
|                 | 2.263E1 | -1.028E0 | 5.436E0 | 2.704E1 | 1.130E-1 | 6.562E-4 |

### RHIC

| $\sqrt{s} = 200$ | gg | $gq$ | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$ |
|------------------|----|-----|----------|--------|-----------|-------|
| $E_{1\perp} = 15$ | 1.744E1 | 7.746E0 | 3.076E0 | 2.826E1 | 8.756E-3 | 4.867E-4 |
| $y = 0.0$       | 1.322E1 | 2.473E0 | 3.261E0 | 1.895E1 | 9.027E-3 | 7.482E-4 |
|                 | 9.461E0 | 6.754E-1 | 2.852E0 | 1.299E1 | 7.019E-3 | 8.487E-4 |
| $y = 1.0$       | 1.260E0 | 3.526E0 | 1.726E0 | 1.480E1 | 4.108E-3 | 4.360E-4 |
|                 | 1.260E0 | 3.526E0 | 1.726E0 | 1.480E1 | 4.108E-3 | 4.360E-4 |
|                 | 1.260E0 | 3.526E0 | 1.726E0 | 1.480E1 | 4.108E-3 | 4.360E-4 |
| $y = 2.0$       | 3.660E-1 | 6.159E-2 | 4.820E-2 | 4.758E-1 | 1.139E-4 | 3.760E-4 |
|                 | 2.135E-1 | 7.813E-3 | 5.371E-2 | 2.753E-1 | 1.057E-4 | 6.031E-4 |
|                 | 1.209E-1 | -3.593E-3 | 4.457E-2 | 1.618E-1 | 7.386E-5 | 7.171E-4 |

### RHIC

| $\sqrt{s} = 200$ | gg | $gq$ | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$ |
|------------------|----|-----|----------|--------|-----------|-------|
| $E_{1\perp} = 20$ | 1.260E0 | 7.590E-1 | 3.365E-1 | 2.356E0 | -1.429E-4 | -9.52E-5 |
| $y = 0.0$       | 9.683E-1 | 2.664E-1 | 3.681E-1 | 1.603E0 | -1.888E-4 | -1.850E-4 |
|                 | 6.870E-1 | 9.159E-2 | 3.229E-1 | 1.101E0 | -2.672E-4 | -3.812E-4 |
| $y = 1.0$       | 5.453E-1 | 2.460E-1 | 1.378E-1 | 9.291E-1 | -6.429E-5 | -1.087E-4 |
|                 | 3.994E-1 | 7.393E-2 | 1.534E-1 | 6.268E-1 | -9.652E-5 | -2.419E-4 |
|                 | 2.703E-1 | 1.870E-2 | 1.332E-1 | 4.222E-1 | -1.206E-4 | -4.486E-4 |
| RHIC  | $\sqrt{S}$ = 200       | gg     | gq     | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$  |
|-------|------------------------|--------|--------|------------|----------|-----------|--------|
|       | $E_{1\perp} = 30$     |        |        |            |          |           |        |
|       | $y = 0.0$             | 1.772E-2 | 1.559E-2 | 8.325E-3  | 4.164E-2 | -6.93E-5 | -2.617E-3 |}
|       | $y = 1.0$             | 3.804E-3 | 1.948E-3 | 1.360E-3  | 7.112E-3 | -1.162E-5| -2.567E-3 |}

| RHIC  | $\sqrt{S}$ = 500       | gg     | gq     | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$  |
|-------|------------------------|--------|--------|------------|----------|-----------|--------|
|       | $E_{1\perp} = 10$     |        |        |            |          |           |        |
|       | $y = 0.0$             | 3.59E3  | 9.03E2  | 1.11E2     | 4.60E3   | 1.34E-1   | 4.57E-5  |
|       | $y = 1.0$             | 2.80E3  | 6.90E2  | 1.02E2     | 3.59E3   | 1.26E-1   | 5.51E-5  |
|       | $y = 2.0$             | 1.18E3  | 2.68E2  | 6.39E1     | 1.51E3   | 8.57E-2   | 8.90E-5  |
|       | $y = 3.0$             | 1.32E2  | 8.46E0  | 1.61E2     | 1.24E-2  | 1.21E-4   |         |

| RHIC  | $\sqrt{S}$ = 500       | gg     | gq     | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$  |
|-------|------------------------|--------|--------|------------|----------|-----------|--------|
|       | $E_{1\perp} = 15$     |        |        |            |          |           |        |
|       | $y = 0.0$             | 2.38E2  | 8.79E1  | 1.13E1     | 3.37E2   | 2.83E-2   | 1.32E-4  |
|       | $y = 1.0$             | 1.72E2  | 6.12E1  | 9.55E0     | 2.43E2   | 2.24E-2   | 1.45E-4  |
|       | $y = 2.0$             | 5.29E1  | 1.57E1  | 3.96E0     | 7.26E1   | 8.42E-3   | 1.82E-4  |
|       | $y = 3.0$             | 3.86E1  | 4.58E0  | 4.82E1     | 8.98E-3  | 2.93E-4   |         |

| RHIC  | $\sqrt{S}$ = 500       | gg     | gq     | $q\bar{q}$ | $\Sigma$ | $\Sigma_T$ | $A_N$  |
|-------|------------------------|--------|--------|------------|----------|-----------|--------|
|       | $E_{1\perp} = 20$     |        |        |            |          |           |        |
|       | $y = 0.0$             | 3.00E1  | 1.44E1  | 2.08E0     | 4.65E1   | 6.97E-3   | 2.36E-4  |
|       | $y = 1.0$             | 2.02E1  | 9.13E0  | 1.60E0     | 3.09E1   | 4.79E-3   | 2.43E-4  |
|       | $y = 2.0$             | 4.23E0  | 1.46E0  | 4.09E-1    | 6.10E0   | 1.05E-3   | 2.70E-4  |
### RHIC

| $\sqrt{S}$ = 500 | gg  | gq  | $q\bar{q}$ | $\Sigma$  | $\Sigma_T$ | $A_N$  |
|------------------|-----|-----|------------|-----------|------------|--------|
| $E_{1\perp}$ = 30 |     |     |            |           |            |        |
| $y = 0.0$        | 1.29E0 | 8.90E-1 | 1.57E-1  | 2.33E0  | 5.14E-4  | 3.46E-4 |
|                  | 9.85E-1 | 4.07E-1 | 1.80E-1  | 1.57E0  | 6.32E-4  | 6.31E-4 |
|                  | 7.40E-1 | 1.94E-1 | 1.70E-1  | 1.10E0  | 5.73E-4  | 8.15E-4 |
| $y = 1.0$        | 7.38E-1 | 4.59E-1 | 9.87E-2  | 1.30E0  | 2.75E-4  | 3.33E-4 |
|                  | 5.62E-1 | 1.99E-1 | 1.16E-1  | 8.77E-1 | 3.41E-4  | 6.11E-4 |
|                  | 4.16E-1 | 8.85E-2 | 1.10E-1  | 6.15E-1 | 3.07E-4  | 7.84E-4 |
| $y = 2.0$        | 5.45E-2 | 2.17E-2 | 7.30E-3  | 8.35E-2 | 1.89E-5  | 3.56E-4 |
|                  | 3.86E-2 | 7.19E-3 | 9.64E-3  | 5.55E-2 | 2.46E-5  | 6.96E-4 |
|                  | 2.62E-2 | 2.11E-3 | 9.08E-3  | 3.74E-2 | 2.14E-5  | 9.00E-4 |

### GSI

| $S = 216.4$ | gg  | gq  | $q\bar{q}$ | $\Sigma$  | $\Sigma_T$ | $A_N$  |
|-------------|-----|-----|------------|-----------|------------|--------|
| $E_{1\perp}$ = 3 |     |     |            |           |            |        |
| $y = 0.0$   | 2.594E3 | 2.602E2 | 8.515E3  | 1.137E4  | 1.580E2  | 2.182E-2 |
|             | 8.644E2 | -2.220E1 | 5.734E3  | 6.576E3  | 1.785E2  | 4.265E-2 |
|             | 3.023E2 | -2.264E1 | 3.219E3  | 3.499E3  | 1.167E2  | 5.241E-2 |
| $y = 0.3$   | 2.355E3 | 2.090E2 | 7.155E3  | 9.719E3  | 1.335E2  | 2.158E-2 |
|             | 7.530E2 | -2.090E1 | 4.797E3  | 5.529E3  | 1.477E2  | 4.195E-2 |
|             | 2.567E2 | -1.927E1 | 2.673E3  | 2.910E3  | 9.546E1  | 5.153E-2 |
| $y = 0.6$   | 1.658E3 | 1.005E2 | 4.023E3  | 5.781E3  | 7.663E1  | 2.082E-2 |
|             | 4.666E2 | -1.503E1 | 2.656E3  | 3.107E3  | 7.896E1  | 3.992E-2 |
|             | 1.466E2 | -1.091E1 | 1.444E3  | 1.580E3  | 4.918E1  | 4.889E-2 |
| $y = 0.9$   | 7.179E2 | 2.034E1 | 1.222E3  | 1.960E3  | 2.405E1  | 1.927E-2 |
|             | 1.592E2 | -5.657E0 | 7.763E2  | 9.299E2  | 2.152E1  | 3.636E-2 |
|             | 4.268E1 | -3.018E0 | 4.099E2  | 4.406E2  | 1.243E1  | 4.430E-2 |
| $y = 1.2$   | 9.019E1 | 3.492E-2 | 9.700E1  | 1.872E2  | 1.912E0  | 1.604E-2 |
|             | 1.271E1 | -4.387E-1 | 5.461E1  | 6.688E1  | 1.292E0  | 3.033E-2 |
|             | 2.485E0 | -1.513E-1 | 2.508E1  | 2.741E1  | 6.383E-1 | 3.658E-2 |

### GSI

| $S = 216.4$ | gg  | gq  | $q\bar{q}$ | $\Sigma$  | $\Sigma_T$ | $A_N$  |
|-------------|-----|-----|------------|-----------|------------|--------|
| $E_{1\perp}$ = 4 |     |     |            |           |            |        |
| $y = 0.0$   | 3.458E1 | 2.460E0 | 2.533E2  | 2.904E2  | 1.365E1  | 7.384E-2 |
|             | 9.898E0 | -2.548E-1 | 1.771E2  | 1.867E2  | 1.221E1  | 1.027E-1 |
|             | 3.104E0 | -2.101E-1 | 9.977E1  | 1.027E1  | 9.416E0  | 1.440E-1 |
| $y = 0.3$   | 2.821E1 | 1.562E0 | 1.840E2  | 2.137E2  | 9.793E0  | 7.198E-2 |
|             | 7.479E0 | -2.154E-1 | 1.276E2  | 1.349E2  | 8.534E0  | 9.938E-2 |
|             | 2.251E0 | -1.538E-1 | 7.108E1  | 7.318E1  | 6.533E0  | 1.402E-1 |
| $y = 0.6$   | 1.246E1 | 2.791E-1 | 5.965E1  | 7.239E1  | 3.056E0  | 6.632E-2 |
|             | 2.615E0 | -9.050E-2 | 4.007E1  | 4.259E1  | 2.424E0  | 8.939E-2 |
|             | 6.874E-1 | -4.685E-2 | 2.139E1  | 2.203E1  | 1.805E0  | 1.287E-1 |
| $y = 0.9$   | 1.142E0 | -7.017E-3 | 3.712E0  | 4.847E0  | 1.691E-1 | 5.479E-2 |
|             | 1.471E-1 | -5.240E-3 | 2.146E0  | 2.288E0  | 1.036E-1 | 7.114E-2 |
|             | 2.869E-2 | -1.740E-3 | 1.019E0  | 1.046E0  | 7.178E-2 | 1.078E-1 |
| GSI S = 216.4 | gg | gq | q̅q | Σ | ΣT | AN |
|---|---|---|---|---|---|---|
| E1⊥ = 5 | | | | | | |
| y = 0.0 | 3.326E-1 | 4.433E-3 | 5.211E0 | 5.548E0 | 5.034E-1 | 1.425E-1 |
| | 7.564E-2 | -2.859E-3 | 3.474E0 | 3.547E0 | 3.735E-1 | 1.654E-1 |
| | 1.966E-2 | -1.381E-3 | 1.866E0 | 1.885E0 | 2.059E-1 | 1.715E-1 |
| y = 0.3 | 2.064E-1 | 4.618E-4 | 2.860E0 | 3.067E0 | 2.608E-1 | 1.336E-1 |
| | 4.130E-2 | -1.637E-3 | 1.837E0 | 1.877E0 | 1.847E-1 | 1.546E-1 |
| | 1.000E-2 | -7.002E-4 | 9.619E-1 | 9.712E-1 | 9.893E-2 | 1.600E-1 |
| y = 0.6 | 2.327E-2 | -3.377E-4 | 2.531E-1 | 2.761E-1 | 1.880E-2 | 1.070E-1 |
| | 3.077E-3 | -1.192E-3 | 1.369E-1 | 1.398E-1 | 1.090E-2 | 1.225E-1 |
| | 5.940E-4 | -3.734E-5 | 6.447E-2 | 6.502E-2 | 5.199E-3 | 1.256E-1 |

| GSI S = 216.4 | gg | gq | q̅q | Σ | ΣT | AN |
|---|---|---|---|---|---|---|
| E1⊥ = 6 | | | | | | |
| y = 0.0 | 8.393E-4 | -1.940E-5 | 3.196E-2 | 3.278E-2 | 3.902E-3 | 1.870E-1 |
| | 1.239E-4 | -5.607E-6 | 1.664E-2 | 1.676E-2 | 2.154E-3 | 2.019E-1 |
| | 2.418E-5 | -1.685E-6 | 7.734E-3 | 7.756E-3 | 1.008E-3 | 2.042E-1 |
| y = 0.3 | 2.062E-4 | -4.771E-6 | 7.315E-3 | 7.516E-3 | 1.008E-3 | 2.042E-1 |
| | 2.542E-5 | -1.056E-6 | 3.437E-3 | 3.462E-3 | 1.719E-3 | 1.792E-1 |

Appendix D: Notes for mathematica files

- hdTree: $h_d^{(0)}(\tau_1, \rho)$;
- hdLoop: $h_d^{(1)}(\tau_1, \rho)$;
- hpLarge: $h_p^{(1)}(\tau_x, \tau_1, \rho)$;
- hpSmall: $h_p^{(1)}$ with $\tau_x$ expanded to $O(\tau_x^4)$;
- hL: $h_l^{(1)}(\tau_x, \tau_1, \rho)$.

Some parameters are introduced to give results in different schemes. In MS-scheme,

$$\text{tep} = 1, \ tct = 0, \ nc = nb = 1, \ nF = 3. \quad (D1)$$

In zero-momentum subtraction of [16], for bottom production,

$$\text{tep} = 1, \ tct = 1, \ nc = 0, \ nb = 1, \ nF = 4. \quad (D2)$$

and for charm production,

$$\text{tep} = 1, \ tct = 1, \ nc = 1, \ nb = 0, \ nF = 3. \quad (D3)$$

Other parameters are common, which are color factors and kinematical variables:

$$N_1 = N_c C_F^2, \ N_2 = C_A N_c C_F^2, \ N_3 = N_c C_F, \ ncolor = N_c, \ \rho_b = \frac{4m_b^2}{s}, \ \rho_c = \frac{4m_c^2}{s}, \ L_\mu = \ln \frac{\mu^2}{m^2}. \quad (D4)$$

An example is given for unpolarized hard coefficients.

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