Joint photocount distributions of a weak twin beam acquired by an iCCD camera are analyzed with respect to the beam spatial correlations. A method for extracting these correlations from the experimental joint photocount distributions is suggested using a suitable statistical model that quantifies the contribution of spatial correlations to the joint photocount distributions. In detail, the profile of twin-beam intensity spatial cross-correlation function is revealed from the curve that gives the genuine mean photon-pair number (both photons from a pair are detected) as a function of the extent of the detection area. Also, the principle of reducing the noise in photon-number-resolving detection by using spatial correlations is experimentally demonstrated.

I. INTRODUCTION

Experimental determination of photon-number distributions of weak optical fields underwent fast development in the last ten years. Several types of photon-number-resolving detectors including super-conducting bolometers [1, 2], semiconductor arrays of avalanche photodiodes [3] or hybrid photomultipliers [4] have been developed and successfully tested under real experimental conditions. Also intensified CCD (iCCD) cameras with photocathodes composed of a large number of single-photon sensitive pixels have been recognized as efficient and practical photon-number-resolving detectors [5–10]. Among others, these detectors have been applied for the investigation of photon-number correlations in weak twin beams containing up to hundreds of photon pairs per pulse [11, 12] as well as to the measurement of spatial correlations in the fields composed of many photon pairs [6, 7, 13–16]. We note that back-illuminated CCD cameras [17, 18] as well as electron-multiplying CCD (EMCCD) cameras [20] represent an alternative to iCCD cameras in these experiments: They have much higher detection efficiencies but also much higher levels of noise.

As final detection efficiencies and noises of all types of photon-number-resolving detectors developed up to now cannot be omitted, the reconstruction of photon-number distributions based on the experimentally detected photocount histograms represents a critical step in the characterization of weak optical fields. Different methods for this reconstruction have been applied, beginning from a simple linear inversion [21] and ending with the iterative maximum-likelihood approach [22, 23]. Also, direct fitting of the experimental photocount histograms with an anticipated form of the photon-number distribution has been applied [24].

The experimental characterization of optical fields with a more complex structure, e.g. weak twin beams, deserves special attention [14, 24]. Such fields may contain correlations among their constituting parts not only in photon numbers, but also in other degrees of freedom. These additional correlations may influence the photocount histograms observed by photon-number-resolving detectors. Such histograms may then be in principle used to reveal these spatial correlations. Twin beams composed of two entangled (signal and idler) beams represent a typical example with tight spatial correlations [24–31]. These correlations may have different forms [32, 33] and even spatially anti-bunched two-photon fields have been generated [34, 35]. We note that spatial correlations of paired fields also enable quantum imaging [4, 22, 30, 37] as well as sub-shot-noise imaging [38] based on sub-shot-noise correlations of twin beams [14, 51, 39, 40]. Spatial correlations between the signal and idler photons are inscribed into the measured data provided that photon-number-resolving detectors with spatial resolution are used (e.g., inscribed into the captured frames of an iCCD camera [14, 51]). This means that the measured data contain both the information about photon-number statistics and that about spatial correlations. Simultaneous analysis of the measured data with respect to both properties is then the most appropriate and efficient from the point of view of reducing the noise in the experimental data. On one side and as shown in detail here, this approach allows to reduce the level of noise in the determination of intensity spatial cross-correlation functions with respect to the commonly used method [51]. On the other side and as principally shown by the obtained experimental curves, spatial correlations allow for partial reduction of the noise found in the process of photon-pair counting.

Here, we provide a comprehensive analysis of weak twin beams whose joint photon-number distribution [42] and spatial correlations are simultaneously monitored by an iCCD camera. The proposed analysis is based upon
pairing of the signal and idler photocounts (counts) identified in the measured camera frames (pictures) and as such reflecting the detection process (quantum detection efficiency, noise, spatial properties). The efficiency of ‘software’ pairing depends upon the extent of the considered circular detection area (with the varying radius) that is drawn in an idler-field detection area (strip) around a point that corresponds to a given detected signal photon (count) in the signal-field detection area (strip) [see the drawing in Fig. 1]. With the varying radius of the detection area, the influence of spatial correlations to the obtained joint signal-idler photocount histograms changes. The larger the detection area, the larger the number of identified photocount pairs. However, also the larger the detection area, the larger the number of identified paired photocounts created randomly from two counts belonging to different photon pairs (with only one photon registered) and/or noise photons. To understand qualitatively as well as quantitatively the relationship between the spatial correlations and joint photocount distributions, we have developed a suitable statistical model. Based on this model we have elaborated a method for revealing the profile of intensity spatial cross-correlation function from the obtained joint signal-idler photocount histograms. Contrary to the usual approach for determining intensity cross-correlation functions that relies on a homogeneous plateau caused by random photocount pairs, the developed method makes an estimate of the mean number of random photocount pairs that is used for eliminating their role in the obtained experimental data. This more sophisticated approach does not require huge amount of experimental data (to reach the homogeneous plateau) and it treats the noise in a more elaborated (detailed) way when eliminating it from the experimental data. Also, the pairing procedure identifies (as a by-product) unpaired counts in the signal and idler strips that originate either in detection of a noisy photon or just one photon from a photon pair. Their omission when constructing the joint photocount histograms results in the noise reduction accompanied by an effective increase of the detection efficiency of collecting photon pairs from a twin beam. Intensity of these effects varies with the extent of the detection area which may be used to efficiently reduce the noise in photon-pair counting.

To practically demonstrate the approach, we have performed the analysis of spatially-resolved joint photocount histograms characterizing a twin beam containing around ten photon pairs on average and captured by a photocathode of an iCCD camera. The obtained detection efficiency and the profile of intensity spatial cross-correlation function have been compared with those found by the method of absolute detector calibration [42] and direct analysis of intensity cross-correlation profiles [41].

The paper is organized as follows. A general statistical model appropriate for the detected joint spatially-resolved photocount histograms is presented in Sec. II. The approach for revealing spatial correlations from the joint photocount histograms is described in Sec. III. The analysis of experimental data and comparison with the theoretical model are contained in Sec. IV. Sec. V brings conclusions.

II. STATISTICAL MODEL FOR PHOTOCOUNT DISTRIBUTIONS INVOLVING SPATIAL CORRELATIONS

In the suggested model, we assume that the joint signal-idler photon-number distribution $p$ characterizing a twin beam in front of the camera can be rewritten as a two-fold convolution of three photon-number distributions $p_p$, $p_s$ and $p_i$ describing in turn the paired, noise signal and noise idler components of the overall twin-beam field [24]:

$$p(n_s, n_i) = \sum_{n=0}^{\min(n_s, n_i)} p_s(n_s - n)p_i(n_i - n)p_p(n). \quad (1)$$

In the process of spontaneous parametric down-conversion, the components are usually assumed in the form of the Mandel-Rice distribution [42] defined for a given number $M_a$ of equally populated modes with mean photon (-pair) number $B_a$ per mode:

$$p_a(n; M_a, B_a) = \frac{\Gamma(n + M_a)}{n! \Gamma(M_a)} \frac{B_a^n}{(1 + B_a)^{n + M_a}}, \quad a = s, i, p$$

and $\Gamma$ denotes the $\Gamma$-function.

We derive the corresponding joint signal-idler photocount distributions in three subsequent steps described in the following subsections. First, we determine the photocount statistics of genuine paired counts (caused by photon pairs with both photons detected), single counts in the signal detection area and single counts in the idler detection area. Then, in the second step we determine the distributions of random paired counts, i.e. pairs of counts found within the corresponding (and varying) signal and idler detection areas and originating in two different photon pairs or individual noise counts. Finally, we combine all contributions together in the third step to arrive at the appropriate joint signal-idler photocount distributions.

A. Photocount distributions of three components

We assume that the signal and idler fields illuminate their detection areas on the photocathode of an iCCD camera homogeneously. In this case, only the paired component composed of spatially correlated photon pairs creates spatial correlations. Their influence to photocount distributions is described as follows.

Genuine spatial correlations in the detected counts are created only by photon pairs with both photons detected. For given signal- ($\eta_s$) and idler- ($\eta_i$) field detection efficiencies, $N$ pixels in both signal- and idler-field detection
areas (strips), \( m_c \) pixels covering the correlated area, and detection areas with \( m_d \) pixels greater than the correlated area [for the scheme, see Fig. 1], the distribution \( \tilde{f}_p \) of genuine paired counts is given by the formula

\[
\tilde{f}_p(c_p) = \sum_{n_p=0}^{\infty} T(c_p, n_p; \eta_s \eta_i, 0, N m_c) p_p(n_p),
\]

where \( T(c, n; \eta, D, N) \) characterizes the detection process in a camera. It gives the probability of observing \( c_p \) [paired] counts caused by a field with \( n_p \) photons [photon pairs]. For an iCCD camera with \( N \) pixels, detection efficiency \( \eta \) and mean dark count number \( D \) per one pixel, the function \( T \) is derived in the form [12]:

\[
T(c, n; \eta, D, N) = \binom{N}{c} (1 - D)^N (1 - \eta)^n \eta^c \sum_{l=0}^c \binom{c}{l} (-1)^l \frac{(1 - \eta)^l}{(1 - D)^l} \left( 1 + \frac{l}{N} \frac{\eta}{1 - \eta} \right)^n.
\]

In writing Eq. (4), we have assumed a sufficiently low level of the signal \( (D_s) \) and idler \( (D_i) \) mean dark count number per pixel such that their contribution to the creation of random paired counts is negligible.

If only one photon from a photon pair is detected, it causes a single count in the signal or idler detection fields. The function \( T \) defined in Eq. (4) allows us to express the corresponding photocount distributions \( \tilde{f}_{ps} \) and \( \tilde{f}_{pi} \) in the signal and idler detection fields, respectively, as follows:

\[
\tilde{f}_{ps}(c_s) = \sum_{n_p=0}^{\infty} T(c_s, n_p; \eta_s(1 - \eta_i), 0, N m_c) p_p(n_p),
\]

\[
\tilde{f}_{pi}(c_i) = \sum_{n_p=0}^{\infty} T(c_i, n_p; \eta_i(1 - \eta_s), 0, N m_c) p_p(n_p).
\]

The distribution \( \tilde{f}_p \) of genuine paired counts is appropriate only provided that the correlated area is fully covered by the detection area \( f_p^{reg}(c_p) \equiv \tilde{f}_p(c_p) \) and \( \tilde{f}_{si}(0) = 1, \tilde{f}_{si}(c) = 0 \) for \( c = 1, \ldots, \eta_i \). In an experiment, the detection area is gradually reduced and so, at certain point, it begins to cover only partially the correlated area. This fact results in breaking some paired counts. Introducing probability \( \eta_d \) that a count from a genuine paired count lies within the detection area, the appropriate distribution \( f_{p}^{reg} \) of genuine paired counts attains the form

\[
f_{p}^{reg}(c_p) = \sum_{c'_p=c_p}^{N} B(c_p, c'_p; \eta_d) \tilde{f}_p(c'_p)
\]

and the binomial distribution \( B \) is written as

\[
B(c, c'; \eta) = \binom{c'}{c} \eta^c (1 - \eta)^{c' - c}.
\]

The remaining 'single' counts are governed by distribution \( \tilde{f}_{si} \),

\[
\tilde{f}_{si}(c_p) = \sum_{c'_p=c_p}^{N} B(c_p, c'_p; 1 - \eta_d) \tilde{f}_p(c'_p),
\]

appropriate for both signal and idler detection fields.

The noise signal and idler components also contribute to counts in their detection fields. Their photocount distributions \( f_s, f_i \) are derived from the corresponding photon-number distributions \( p_a \) applying the function \( T \) given in Eq. (4):

\[
f_a(c_a) = \sum_{n_a=0}^{\infty} T(c_a, n_a; \eta_s, D_s, N) p_a(n_a), \quad a = s, i.
\]

The photocount distributions \( f_s \) and \( f_i \) composed of all unpaired counts in the signal and idler detection fields, respectively, are determined via the convolution:

\[
f_a(c_a) = \sum_{c'_a=0}^{c_a} \tilde{f}_{pa}(c'_a) \tilde{f}_a(c_a - c'_a), \quad a = s, i.
\]

The joint signal-idler photocount distribution \( f_{si} \) of all counts outside the paired detection areas is finally given by the two-fold convolution of distributions given in Eqs. (8) and (10):

\[
f_{si}(c_s, c_i) = \sum_{c'_p=0}^{\min(c_s, c_i)} \tilde{f}_{ps}(c_p) \tilde{f}_{si}(c_s - c_p, c_i - c_p).
\]

The dependence of probability \( \eta_d \) introduced in Eq. (6) on the extent of the detection area is in general derived from the profile of intensity spatial cross-correlation function \( t \). Quantifying the extent of detection area (correlated area) in the number \( m_d \) (\( m_c \)) of pixels and assuming the constant intensity cross-correlation function inside the correlated area, we have:

\[
\eta_d(m_d) = m_d/m_c, \quad m_d < m_c, \quad m_d \geq m_c.
\]
In a more realistic case of the Gaussian intensity cross-correlation profile \( t(\Delta x, \Delta y) = \exp\left[-(\Delta x^2 + \Delta y^2)/R^2\right]/\pi R^2 \) with radius \( R \) and considering the circular detection area with radius \( r \), we arrive at the probability function \( \eta_d \) in the simple form \((m_c = \pi R^2, m_d = \pi r^2)\):

\[
\eta_d(m_d) = 1 - \exp(-m_d/m_c) .
\]

(13)

The model can also be applied to one-dimensional geometry (cuts across a real correlated area). We have for the Gaussian intensity profile \( t_s(\Delta x) = \exp(-\Delta x^2/X^2)/(\sqrt{\pi}X) \) with \( X \) quantifying the extent of the one-dimensional cross-correlation function \((m_c = 2X, m_d = 2r)\):

\[
\eta_d(m_d) = \text{erf}(m_d/m_c);
\]

(14)

\[
\text{erf}(x) \equiv 2/\sqrt{\pi} \int_0^x \exp(-t^2)dt \text{ denotes the error function.}
\]

**B. Random pairing of the signal and idler photocounts**

Individual single counts occurring in the signal and idler detection fields can accidentally be located at the corresponding positions within the extent of the detection area. This configuration creates additional, random, paired counts, whose number increases with the increasing extent of detection area. To reveal the appropriate accidental paired photocount distribution, we first determine probability \( P(c_p, c_s, c_i; m_d) \) that \( c_s \) randomly positioned signal counts together with \( c_i \) randomly positioned idler counts give \( c_p \) paired counts within the detection area covering \( m_d \) pixels. If there occur \( c_s \) counts in the signal detection field (strip), the detection areas drawn around each signal count cover on average a certain part of the signal detection field, which relative extension is quantified by probability function \( \eta_s(c_s; m_d) \). A count in the idler detection field has probability \( \eta_i(c_i; m_d) \) to fall into the corresponding area and thus create an accidental paired count. On the other hand, it falls outside the corresponding area with probability \( 1 - \eta_i(c_i; m_d) \). For an arbitrary number \( c_i \) of idler counts, the effect is described by the binomial distribution \( B \) already introduced in Eq. (7). If the number \( c_p \) of created paired counts should exceed the number \( c_s \) of signal counts, only the number \( c_s \) of paired counts is taken into account. The probability \( P_b \) for characterizing this pairing procedure in its basic variant is expressed as follows:

\[
P_b(c_p, c_s, c_i; m_d) = B(c_p, c_i; \eta_s(c_s; m_d))
\]

for \( c_p < \min(c_s, c_i) \);

\[
P_b(c_s, c_s, c_i; m_d) = \sum_{c_p=c_s}^{c_i} B(c_p, c_i; \eta_s(c_s; m_d))
\]

for \( c_s \leq c_i \);

\[
P_b(c_p, c_s, c_i; m_d) = 0 \quad \text{otherwise.}
\]

(15)

The function \( \eta_i(c_i; m_d) \) giving the relative area in the signal detection field (strip) covered by \( c_i \) counts, each 'occupying' \( m_d \) pixels, is derived as follows. The first count covers the relative area \( s_1 \) equal to \( s = m_d/N \). The second count has already a smaller empty relative area given by \( 1 - s_1 \) to enlarge the area occupied by the first count. The area occupied on average by an \( i \)-th count is expressed in general as:

\[
s_0 = 0, \quad s_1 = s, \quad s_i = s \left(1 - \sum_{j=1}^{i-1} s_j\right), \quad i = 2, \ldots
\]

(16)

The function \( \eta_i \) is then obtained in the simple form:

\[
\eta_i(c_i; m_d) = \sum_{i=0}^{c_i} s_i(m_d).
\]

(17)

The effect of partial overlapping of the detection areas around different counts in the signal field (strip) occurs also in the idler field (strip) inside the area corresponding to the detected signal counts. To reveal the correction function \( \eta_k(c_i; c_s) \) appropriate for \( c_s \) detected signal counts, each surrounded by \( m_d \) detection pixels, we first determine in parallel the relative areas \( \tilde{\eta}_{ref} \) and \( \tilde{\eta} \) inside the area corresponding to \( c_s \) signal counts covered by the detection areas encircling \( c_i \) idler counts without and with the consideration of possible overlapping, respectively. We consider that one idler count covers in this area on average \( \tilde{\eta}(c_i; m_d) m_d \) pixels. As a consequence, the relative area \( \tilde{\eta}_{ref} \) without the inclusion of overlapping is given as

\[
\tilde{\eta}_{ref}(c_i; c_s) = \frac{\min(c_s, c_i)}{c_s}.
\]

(18)

The overlap of the idler detection areas is statistically quantified by the scheme analogous to that described in Eqs. (10) and (17). It leaves us with the formulas:

\[
\tilde{\eta}(c_i; c_s) = \sum_{i=0}^{c_i} \tilde{s}_i(c_s),
\]

(19)

\[
s_0 = 0, \quad \tilde{s}_1 = 1/c_s, \quad \tilde{s}_i = \tilde{s}_1 \left(1 - \sum_{j=1}^{i-1} \tilde{s}_j\right), \quad i = 2, \ldots.
\]

Using Eqs. (13) and (19), the correction function \( \eta_k(c_i; c_s) \) is obtained along the formula:

\[
\eta_k(c_i; c_s) = \frac{\tilde{\eta}(c_i; c_s)}{\tilde{\eta}_{ref}(c_i; c_s)}.
\]

(20)

This additional correction for the overlap of detection areas inside the idler field (strip) is involved in the refined model of pairing and it requires the following modification of probability \( P \) of pairing written originally in Eq. (15):

\[
P(c_p, c_s, c_i; m_d) = \sum_{c'_i = c_p}^{c_i} B(c_p, c'_i; \eta_k(c'_i; c_s)) \times B(c'_i, c_i; \eta_s(c_s; m_d)) \quad \text{for} \quad c_p < \min(c_s, c_i);
\]

\[
P(c_s, c_s, c_i; m_d) = \sum_{c_p = c_s}^{c_i} \sum_{c'_i = c_p}^{c_i} B(c_p, c'_i; \eta_k(c'_i; c_s)) \times B(c'_i, c_i; \eta_s(c_s; m_d)) \quad \text{for} \quad c_s \leq c_i;
\]

\[
P(c_p, c_s, c_i; m_d) = 0 \quad \text{otherwise}
\]

(21)
and the binomial distribution $B$ is written in Eq. 7.

The distribution $f^a_{p}$ of accidental paired counts is described via the probability $P$ given in Eq. (15) for the basic model and in Eq. (21) for the refined model in the form:

$$f^a_{p}(c_p; m_d) = \sum_{c_i=c_p}^{N} \sum_{c_i}^{N} P(c_p, c_i; m_d) f_{si}(c_s, c_i). \quad (22)$$

The remaining unpaired single counts in the signal and idler detection fields (strips) are left with certain additional correlations introduced by the pairing procedure. These correlations are characterized by the conditional distributions $f^a_{si}(d_s, d_i; c_p)$ of having $d_s$ signal counts together with $d_i$ idler counts in a frame with $c_p$ accidental paired counts:

$$f^a_{si}(d_s, d_i; c_p, m_d) = \frac{P(c_p, c_p + d_s, c_p + d_i; m_d)}{f^a_{p}(c_p; m_d)} \times f_{si}(c_p + d_s, c_p + d_i). \quad (23)$$

The marginal distributions $f^a_{s}$ and $f^a_{i}$ of the unpaired counts in the signal and idler detection field, respectively, conditioned by identifying $c_p$ accidental paired counts are expressed as:

$$f^a_{s}(d_s; c_p, m_d) = \frac{\sum_{d_i=0}^{N} f^a_{si}(d_s, d_i; c_p, m_d)}{f^a_{p}(c_p; m_d)},$$

$$f^a_{i}(d_i; c_p, m_d) = \frac{\sum_{d_s=0}^{N} f^a_{si}(d_s, d_i; c_p, m_d)}{f^a_{p}(c_p; m_d)}. \quad (24)$$

We note that the original distribution $f_{si}(c_i, c_s)$ is then expressed in terms of the distributions $f^a_{p}$ and $f^a_{si}$ as follows:

$$f_{si}(c_s, c_i) = \sum_{c_p=0}^{\min(c_s,c_i)} f^a_{p}(c_p; m_d) \times f^a_{si}(c_s - c_p, c_i - c_p; c_p, m_d). \quad (25)$$

C. Overall joint signal-idler photocount distributions

The formula (25), when combined with formula (6) for the distribution $f^g_{p}$ of genuine paired counts, allows us to determine the joint signal-idler photocount distribution $F(c_i, c_s)$ of having $c_s$ counts in the signal field (strip) and $c_i$ counts in the idler field (strip):

$$F(c_i, c_s) = \sum_{c_p=0}^{\min(c_s,c_i)} f^g_{p}(c_p - c'_p; c'_p) f^a_{p}(c'_p; m_d) \times f^a_{si}(c_s - c_p, c_i - c_p; c'_p, m_d). \quad (26)$$

Moreover, the distribution $F_{p}$ of paired counts, both genuine and accidental, is given as:

$$F_{p}(c_p; m_d) = \sum_{c'_p=0}^{c_p} f^g_{p}(c_p - c'_p) f^a_{p}(c'_p; m_d). \quad (27)$$

The joint distribution $F_{a}$ of unpaired signal and idler counts is expressed along the relation:

$$F_{a}(d_s, d_i; m_d) = \sum_{c_p=0}^{N} f^a_{p}(c_p; m_d) f^a_{si}(d_s, d_i; c_p, m_d). \quad (28)$$

Similarly, the distribution $F_{a}$ of unpaired counts in detection field $a$, $a = s, i$, is determined by the simple formula:

$$F_{a}(d_a; m_d) = \sum_{c_p=0}^{N} f^a_{p}(c_p; m_d) f^a_{a}(d_a; c_p, m_d), \quad a = s, i. \quad (29)$$

The numbers of unpaired counts in both detection fields (strips) can be reduced by considering only those counts occurring in the detection areas around the identified paired counts. This means the reduction of noise in the photon counting that characterizes the twin beam. The distributions $F^{\text{red}}_{a}(c_s, c_i; m_d)$ and $F^{\text{red}}_{a}(d_s, d_i; m_d)$ appropriate for this case are theoretically determined by the modified expressions in Eqs. (26) and (27) that involve the binomial distributions $B$ to account for the reduction of the number of unpaired counts:

$$F^{\text{red}}_{a}(c_s, c_i; m_d) = \sum_{c_p=0}^{\min(c_s,c_i)} f^g_{p}(c_p - c'_p; c'_p) f^a_{p}(c'_p; m_d) \times \sum_{d_i=c_p-d'_p}^{N} B(c_i - c_p, d'_i; \eta(c_p; m_d)) \times B(d_i - c_p, d'_i; \eta(c_p; m_d)) f^{\text{acc}}_{a}(d'_i, c'_p; m_d), \quad (30)$$

$$F^{\text{red}}_{a}(d_s, d_i; m_d) = \sum_{c_p=0}^{\min(c_s,c_i)} f^g_{p}(c_p - c'_p; c'_p) f^a_{p}(c'_p; m_d) \times \sum_{d'_s=c_p-d'_p}^{N} B(d_s - d'_s, d'_i; \eta(c_p; m_d)) \times \sum_{d'_i=d_s-d'_s}^{N} B(d_i - d'_s, d'_i; \eta(c_p; m_d)) f^{\text{acc}}_{a}(d'_i, c'_p; m_d). \quad (31)$$

The distributions $F^{\text{red}}(d_s, m_d)$ of unpaired counts in field $a$, $a = s, i$, are easily derived from the distribution $F^{\text{red}}_{a}$ given in Eq. (31) as follows:

$$F^{\text{red}}_{a}(d_s; m_d) = \sum_{d_i=0}^{N} F^{\text{red}}_{a}(d_s, d_i; m_d),$$

$$F^{\text{red}}_{a}(d_i; m_d) = \sum_{d_s=0}^{N} F^{\text{red}}_{a}(d_s, d_i; m_d). \quad (32)$$

Photocount moments of different orders derived from the above distributions represent important characteristics of the detected fields. Their combinations allow, among others, the determination of covariance $C$ of fluctuations of photocount numbers as well as sub-shot-noise parameter $R$, both quantifying mutual correlations between the numbers of photocounts in two detection fields:

$$C = \frac{\langle \Delta c_s \Delta c_i \rangle}{\sqrt{\langle \Delta c_s \rangle^2 \langle \Delta c_i \rangle^2}}. \quad (33)$$
\[ R = \frac{\langle \Delta(c_s - c_i)^2 \rangle}{\langle c_s \rangle + \langle c_i \rangle} . \] (34)

III. DETERMINATION OF INTENSITY SPATIAL CROSS-CORRELATION FUNCTIONS

The profile \( t \) of intensity spatial cross-correlation function is imprinted into the dependence of the mean number \( \langle c_p \rangle^\text{reg} \) of genuine paired counts on the number \( m_d \) of detection pixels. This curve can be approximately derived from the experimental data that provide the mean number \( \langle c_p \rangle \) of all identified paired counts once we estimate the mean number \( \langle c_p \rangle^\text{acc,exp} \) of accidental experimental paired counts. This estimated distribution \( f_p^\text{acc,exp} \) is given by the modified Eq. (22),

\[
f_p^\text{acc,exp}(c_p, m_d) = \sum_{d_s = c_p}^{N} \sum_{d_i = c_p}^{N} P(c_p, d_s, d_i; m_d) \times F_s^\text{exp}(d_s, d_i; m_d^0),
\] (35)
in which the distribution \( F_s^\text{exp} \) characterizes the experimental unpaired counts observed for certain suitable number \( m_d^0 \) of detection pixels. The analysis of covariance \( C_{\Delta d} \) of the joint signal-idler distribution \( F_s^\text{exp}(d_s, d_i) \) of unpaired counts considered as a function of the number \( m_d \) of detection pixels suggests the natural choice implicitly expressed as \( C_{\Delta d}(m_d^0) = 0 \). The reason is that the covariance \( C_{\Delta d} \) is positive for \( m_d < m_c \) as it includes genuine paired counts that did not fit into the too small detection area. On the other hand, for \( m_d > m_c \) the pairing procedure ‘generates’ accidental paired counts with correlations that have to be compensated in the distribution \( F_s^\text{exp} \) of the remaining unpaired counts. In more detail, for \( m_d > m_c \) the distribution \( f_s \) in Eq. (25) exhibits no correlations between counts \( c_s \) and \( c_i \). Subsequent application of the pairing procedure of Eq. (22) removes some paired counts ‘outside’ the distribution \( f_s \) which introduces negative correlations into the resultant distribution \( F_s \) given in Eq. (23). These correlations result in negative values of the covariance \( C_{\Delta d} \) found for \( m_d > m_c \) (see Fig. 6 below).

The looked-for ‘spatially integrated’ profile \( t \) of intensity cross-correlation function \( t \) drawn as a function of the number \( m_d \) of detection pixels can approximately be inferred from the normalized experimental mean number \( \langle c_p \rangle^\text{reg,exp} \) of genuine paired counts \( \langle c_p \rangle^\text{reg,exp}(m_d) = \langle c_p \rangle^\text{reg,exp}(m_d)/(\langle c_p \rangle(N) - \langle c_p \rangle^\text{acc,exp}(N)) \rangle \) using the following formula:

\[
\langle c_p \rangle^\text{reg,exp}(m_d) = \int_0^{m_d} dm'_d f_{\text{app}}(m'_d). \] (36)

Assuming as an example a rotationally invariant correlated area with profile \( t \) described above Eq. (13), the inversion of Eq. (36) leaves us with the formula

\[
t_{\text{app}}^R(m_d) = \frac{d(c_p)^\text{reg,exp}(m_d)}{dm_d}. \] (37)

When analyzing photocount correlations in rectangular detection areas parallel, e.g., to the \( x \) axis, we immediately reveal the profile \( t_{\text{app}}^x \) of the intensity spatial cross-correlation function in this direction:

\[
t_{x}^\text{app}(m_d) = \frac{d(c_p)^\text{reg,exp}(m_d)}{dm_d}. \] (38)

IV. EXPERIMENTAL RESULTS AND THEIR INTERPRETATION

The experiment was performed with a twin beam centered at the wavelength 560 nm and originating in a non-collinear type-I interaction in a 5-mm long BaB₂O₄ crystal pumped by the third harmonics of a femtosecond cavity dumped Ti:sapphire laser (pulse duration 150 fs, central wavelength 840 nm) [14]. An iCCD camera Andor DH334-18U-63 was used to capture individual photo-}

counts in two detection strips, one for the signal field, the other for the idler field (see Fig. 2). Both detection fields were covered by \( N = 6500 \) pixels and suffered from \( D = 0.2/N \) mean dark counts per pixel. The experiment was repeated \( 1.2 \times 10^6 \) times. Before applying the developed model to the experimental data, we performed the standard analysis of the joint signal-idler photocount histogram [24] that provided both parameters of the twin beam \( (B_p = 0.032, M_p = 280, B_s = 8.2, M_s = 0.009, B_i = 4.7, \) and \( M_i = 0.033, \) relative errors of the parameters are better than \( \pm 7\% \)) and the signal- and idler-field detection efficiencies \( (\eta_s = 0.228 \pm 0.005, \eta_i = 0.223 \pm 0.005) \). We note that the photon-number distributions \( p_a(n), a = s, i, \) of the noise fields with numbers \( M_a \) of modes considerably lower than 1 are sharply localized around \( n = 0 \) which is a consequence of the specific electronic response of the iCCD camera. Values of the determined parameters are used to derive the predictions about the dependence of the observed quantities on the number \( m_d \) of detection pixels. These predictions are obtained both for the basic as well as the refined models of the pairing procedure, as described in Eqs. (15) and (21).

A curve giving the number \( \langle c_p \rangle \) of detected paired counts as a function of the number \( m_d \) of detection pixels is the most important curve in the analysis [see Fig. 3(a)]. The number \( \langle c_p \rangle \) of detected paired counts increases with the increasing number \( m_d \) of detection pixels for two reasons. First, the number of broken genuine paired counts decreases with the increasing number \( m_d \) of detection pixels due to the finite extent of the correlated area. Second, the number \( \langle c_p \rangle^\text{acc} \) of accidental paired counts also naturally increases with \( m_d \). As the pairing procedure is stronger in the basic model compared to the refined one, the numbers \( \langle c_p \rangle^\text{acc} \) of accidental paired counts as well as the numbers \( \langle c_p \rangle \) of all paired counts predicted by the theory are greater for the basic model [compare solid and dashed curves with symbols plotted in Fig. 3(a)]. The first mechanism that breaks the genuine paired counts depends on the profile of the
correlated area and, as discussed in the previous section, this profile can be extracted from the curve giving the number $\langle c_p \rangle_{\text{reg}}$ of genuine paired counts. The application of Eq. (35) (for details, see below) gives us an experimental estimate for the number $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts and the number $\langle c_p \rangle_{\text{reg,exp}}$ of experimental genuine paired counts is then determined simply as $\langle c_p \rangle_{\text{reg,exp}} = \langle c_p \rangle_{\text{reg}} - \langle c_p \rangle_{\text{acc}}$. The resultant points obtained from the experimental data as well as the theoretical predictions of both models are plotted in Fig. 3(a).

In Fig. 3(a), the comparison of experimental points and theoretical curves for $\langle c_p \rangle_{\text{reg,exp}}$ with the curve for $\langle c_p \rangle_{\text{reg}}$ giving the actual number of genuine paired counts, that is accessible only in the model, shows that the experimental numbers $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts are overestimated. The number $m_d^0 = 290$ of detection pixels and the corresponding experimental signal-idler histogram $F_{\text{exp}}(d_s, d_i; m_d^0)$ of unpaired counts have been used in Eq. (35) to arrive at the numbers $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts plotted in Fig. 3(a). Decrease of the number $\langle c_p \rangle_{\text{reg,exp}}(m_d)$ of genuine paired counts observed in Fig. 3(a) for greater values of $m_d > 3m_d^0$ is apparently caused by overestimating the number $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts. This behavior originates in the fact that the number $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts is determined via the pairing procedure described in Eqs. (15) or (21) that requires the joint distribution of single counts in the signal and idler fields (strips). This distribution does not include only genuine paired counts and as such it is available only in the model. In the experimental data the genuine and accidental paired counts cannot be separated. This fact requires to replace the needed joint distribution of unpaired counts in the signal and idler detection fields by a suitable experimental distribution that, however, cannot incorporate single counts glued into accidental paired counts. If we consider such experimental distribution as a function of the number $m_d$ of detection pixels, underestimation of the number $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts occurs [compare the experimental symbols $\circ$ and $\diamond$ in Fig. 3(a)]. To cope with this effect, we assume in the pairing procedure given in Eq. (35) the experimental distribution $F_{\text{exp}}$ obtained for a certain fixed value of the number $m_d^0$ of detected pixels that roughly covers the correlated area. The chosen value of $m_d^0$ determines the behavior of the estimated values of the number $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts with respect to the true ones. As follows from the drawing in Fig. 4 if the chosen value of $m_d^0$ is too large, the numbers $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts are underestimated in the whole range of $m_d$ because the corresponding experimental distribution $F_{\text{exp}}$ describes the signal and idler fields that are too weak. On the other hand, there occurs overestimation of the numbers $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts for greater values of $m_d$ provided that the chosen value of $m_d^0$ is smaller. This is our case with $m_d^0 = 290$. However, such choice gives us better estimate for the numbers $\langle c_p \rangle_{\text{acc}}$ of accidental paired counts for the numbers $m_d$ of detection pixels comparable to the number $m_c$ of pixels of the correlated area. As the determination of the profile
of the correlated area is addressed here, we concentrate our attention to the area with $m_d < 600$ which justifies our choice $m_d^0 = 290$. We note that the numbers $(c_p)^{acc}$ of accidental paired counts for small values of $m_d$ are always underestimated, but this behavior is not important as the true values of $(c_p)^{acc}$ are small in this case and so they can be omitted.

With the increasing number $m_d$ of detection pixels the correlated area with $m_c$ pixels is gradually covered by the detection area and this fact results in dramatic decrease of the relative variance $\sigma_{c_p}$ of the number of detected paired counts, as documented in Fig. 3(b). The curve in Fig. 3(b) clearly indicates appropriate numbers $m_d$ of detection pixels for which the spatial correlations are practically lost.

More precise determination of the number $m_c$ of pixels in the correlated area arises from the analysis of the joint histograms of unpaired signal and idler counts. With the increasing number $m_d$ of detection pixels the mean number $\langle d_s \rangle$ of unpaired signal counts decreases, whereas its relative variance $\sigma_{d_s}$ monotonically increases (see Fig. 5). However, as already discussed above, covariance $C_{d,d}$ between the fluctuations of the numbers of unpaired signal and idler counts changes its sign at the edge of the correlated area, as documented in Fig. 5(a). The determination of number $m_c = m_d^0$ of pixels in the correlated area can be considered as a phenomenological definition of the extent of the correlated area. This definition is robust in the sense that the change of sign for $\langle d_s \rangle$ occurs nearly for the same values of $m_d^0 (c_p)$ also for the covariances $C_{\Delta d,c_p}$ determined for the photocount distributions $F_d(d_s,d_i;c_p,m_d)$ conditioned by identification of $c_p$ paired counts in the frame [see Fig. 5(b)].

The obtained curve for the number $\langle c_p \rangle^{reg}$ of genuine paired counts allows us to reveal the profile $\tilde{I}_R$ of the correlated area using Eq. (37) that relies on the discrete numerical derivative of the experimental dependence $\langle c_p \rangle^{reg,exp}(m_d)$. As the theoretical curve $\langle c_p \rangle^{reg,exp}$ obtained for a two-dimensional Gaussian profile $t$ is close to the corresponding experimental dependence [see Fig. 4(a)] for $m_d \leq 2 m_d^0$, it is more convenient to directly fit the experimental points with function (15) to arrive at the number $m_c$ of pixels in the correlated area. Both approaches are compared in Fig. 4.

Also detection efficiencies $\eta_s$ and $\eta_i$ appropriate for the signal and idler detection fields (strips), respectively, can be deduced from the obtained experimental data. In
the analysis also gives \( \eta \) plotted in Fig. 8, we have \( m \) from the point of view of photon-pair detection unwanted signal and idler fields (strips), respectively, represent unpaired counts, we estimate the detection efficiency \( \eta \) of counts in the idler field (strip). As the number \( m \) of pixels increases with the increasing number \( m_d \) of detection pixels, the curve \( \eta(m_d) \) also increases. In the area around \( m_d^0 \) it gives a good estimate \( \eta^0_d \) for the actual detection efficiency. According to the curve \( \eta_d(m_d) \) plotted in Fig. 8 we have \( \eta^0_d \approx 0.23 \). Provided that we determine the efficiency \( \eta_d \) using the number \( \langle c_p \rangle^{\text{reg}} \) of genuine paired counts, the appropriate value is ideally revealed for \( m_d = N \) (see Fig. 8). However, in our realistic case with underestimated numbers \( \langle c_p \rangle^{\text{reg,exp}} \) of genuine paired counts, we estimate the detection efficiency \( \eta_d \) by the maximum of function \( \eta_d(m_d) \) reached for the numbers \( m_d \) of detection pixels slightly larger than the number \( m_c \) of pixels in the correlated area. We note that the analysis also gives \( \eta^0_i \approx 0.23 \) for the idler field.

The numbers \( \langle d_s \rangle \) and \( \langle d_i \rangle \) of unpaired counts in the signal and idler fields (strips), respectively, represent from the point of view of photon-pair detection unwanted noise. However, these numbers can conveniently and substantially be reduced when we consider them only inside the detection areas drawn around the paired counts [see Eqs. (30)—(32)], as it clearly follows from the comparison of curves in Figs. 8(a) and 8(a). According to the experimental points plotted in Fig. 8(a), decrease of the detection area from 3000 pixels to one third reduces the number \( \langle d_s \rangle^{\text{red}} \) of noise signal counts roughly to one half. For numbers \( m_d \) of detection pixels comparable to the number \( m_c \) of pixels inside the correlated area, additional reduction of the noise by one half is observed. This noise reduction can be exploited when determining the distribution of photon pairs from the experimental joint signal-idler photocount histograms once the extent of the correlated area \( (m_c) \) is known (or estimated). However, this procedure has to be done with care as, for the detection area smaller than the correlated area, the noise reduction is accompanied by a relatively large increase of photocount fluctuations [see Fig. 8(b)].

The noise reduction qualitatively changes the behavior of cross-correlations between the unpaired signal and idler counts for small detection areas. Both covariances \( C_d^{\text{red}} \) of the numbers of unpaired signal and idler counts and \( C_d^{\text{red}} \) of fluctuations of the numbers of unpaired signal and idler counts tend to zero for small numbers \( m_d \) of detection pixels, as documented in Figs. 8(c) and 8(d). On the other hand, relatively weak correlations are observed in the whole range of numbers \( m_d \) of detection pixels. Mutual comparison of the experimental points and two theoretical curves indicates that the covariance \( C_d^{\text{red}} \) of the numbers of unpaired signal and idler counts is quite sensitive to the experimental conditions as well as the detailed structure of the theoretical model. Detailed geometry of the detection fields (strips), that is not treated in the model, probably plays an important role in the explanation of large deviations among the values of covariance \( C_d^{\text{red}} \) plotted in Fig. 8(c). Deviations among the values of covariances \( C_d^{\text{red}} \) and \( C_d^{\text{red}} \) of fluctuations of the numbers of unpaired signal and idler counts are even larger (for the definition of \( C_d^{\text{red}} \), see the caption to Fig. 8). They have their origin in larger discrepancies between the experimental and theoretical mean numbers \( \langle d_i \rangle^{\text{red}} \) [see Fig. 8(a)] and \( \langle d_i \rangle^{\text{red}} \) and signal-idler correlation function \( \langle d_s d_i \rangle^{\text{red}} \) [compare Fig. 8(c)] that are determined for unpaired counts. Only qualitative agreement can be seen in the graph of Fig. 8(d) when the experimental points are compared with the theoretical predictions. However, neither the experimental points nor the theoretical curves indicate the ability of covariances \( C_d^{\text{red}} \), \( C_d^{\text{red}} \) and \( C_d^{\text{red}} \) to provide the extent of the correlated area. Proper description of the observed dependencies of these covariances lies beyond the developed model. On the other hand, the absence of reliable description of weak correlations between the unpaired signal and idler photocounts does not seriously restrict the application of the noise reduction in photon-pair counting.

The developed method for revealing the profiles of correlated areas can easily be modified to provide cuts through two-dimensional intensity spatial cross-
correlation functions. In this case rectangular detection areas with the varying number of pixels in one direction and fixed number of pixels in the perpendicular direction (ideally just one) is applied. For example, rectangular detection areas extending over \( m_d \) detection pixels along the \( x \) axis and covering just one pixel along the \( y \) axis give us the profile of the intensity cross-correlation function along the \( x \) axis. The experimental profile \( t_{x}^{pp} \) obtained according to Eq. (38) is compared in Fig. 10 with the profile \( t_{x}^{cor} \) arising in direct evaluation of the intensity spatial cross-correlation function that relies on the subtraction of a constant background (for details, see [14, 41]). Also theoretical curves characterizing a suitable Gaussian profile are drawn in Fig. 10 for comparison. Compared to the above analyzed two-dimensional case with circular detection areas, we have by two orders in magnitude lower numbers of genuine photon pairs per one-dimensional 'frame'. On the other hand, the number of such 'frames' contained in the experimental ensemble is by two orders in magnitude greater than the number of the original two-dimensional frames. This fact gives better experimental precision. Moreover, if the applied rectangular detection area extends over several pixels in the perpendicular direction, we arrive at averaged (smoothed) profiles of intensity cross-correlation functions. Also, larger numbers of genuine photon pairs are met in this case. We note that the method can be applied separately to the left- and right-hand sides of the correlated area (from its center) to reveal its profile completely.

The developed method for revealing profiles of intensity cross-correlation functions through the determination of the numbers of genuine paired photocounts requires in principle much lower amount of experimental data compared to the usual approach that is based on accumulating random accidental paired photocounts until they form a constant background [14, 41]. This advantage is achieved by a sophisticated processing of the experimental data that estimates mean numbers of accidental paired photocounts.

The comparison of the remaining experimental and theoretical quantities independent on the number \( m_d \) of detection pixels reveals, together with the curves already presented in the graphs, good agreement between both models and the measured quantities (mean number of all signal counts: \( \langle c_s \rangle_{exp} = 2.00 \pm 0.02; \langle c_i \rangle = 2.01; \) relative variance of all signal counts: \( \sigma_{c_s}^{exp} = 0.51 \pm 0.01; \sigma_{c_i} = 0.51; \) covariance of fluctuations of the numbers of all signal and idler counts: \( C_{exp} = 0.200 \pm 0.004; \) \( C = 0.42; \) noise reduction factor \( R_{exp} = 0.81 \pm 0.02; \) \( R = 0.80). \) The overall comparison of the experimental points with the curves determined by both models leads to the general conclusion that the experimental points are fitted better by the curves of the refined model. Both models allow for reliable quantification of the processes of detection of correlated fields with photon-number-resolving detectors endowed with spatial resolution.

FIG. 9. (a) Mean number \( \langle d_s \rangle_{red} \) of unpaired signal counts [relative experimental errors are lower than 4%], (b) relative variance \( \sigma_{d_s}^{red} \) of their distribution [4%], (c) covariance \( C_{d_s}^{red} \equiv \langle d_s d_i \rangle / \langle d_s^2 \rangle \) of the numbers of unpaired signal and idler counts ([4%]), and (d) covariance \( C_{d_s d_i}^{red} \) of fluctuations of the numbers of unpaired signal and idler counts according to Eq. (35) ([, [8%]]) and covariance \( C_{d_s d_i}^{red} \equiv \langle d_s d_i \rangle / \langle d_s \rangle \langle d_i \rangle \) of fluctuations of the numbers of unpaired signal and idler counts in frames with at least one detected paired count (\( \Delta \), [10%]) as functions of the number \( m_d \) of detection pixels assuming the noise reduction given in Eqs. (30)—(32). Solid (dashed) curves originate in the refined (basic) model, isolated points are obtained from the experimental data.

FIG. 10. Profile \( t_{x}^{pp} \) of the correlated area along the \( x \) axis determined by Eq. (38) as it depends on the number \( m_d \) of detection pixels. Experimental points are plotted as isolated symbols (\( \Delta \)), solid curve with \( \Delta \) is derived for the theoretical Gaussian correlated area with \( m_d / 2 = 10 \) \( (m_d^0 / 2 = 10) \). The corresponding ideal profile \( t_x = 2 / (\sqrt{\pi m_d}) \exp(-m_d^0 / m_d^2) \) is drawn by a dashed line. For comparison, experimental points of \( t_{x}^{cor} \) obtained from direct evaluation of the intensity cross-correlation function are shown (○).
V. CONCLUSIONS

We have developed a suitable statistical model (in two variants) that allows for quantifying the role of spatial correlations in the observed joint signal-idler photocount distributions of a weak twin beam. In the model the detected counts are divided into those corresponding to genuine paired photocounts, accidental paired photocounts and unpaired photocounts. The model allows to separate the genuine paired photocounts from the remaining ones and subsequently to recover the profile of intensity spatial cross-correlation functions. The determination of intensity cross-correlation functions has been demonstrated in one- and two-dimensional geometries. In parallel with the quantification of spatial correlations, the principle of reduction of the noise occurring in photon-number-resolving detection with the help of spatial correlations has been experimentally demonstrated.

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