Low Frequency Spectra of Gamma-Ray Bursts

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Abstract

Particles with energies below the mean energy $E_0$ in relativistic shocked plasmas should assume an equilibrium energy distribution. This leads to a synchrotron spectrum $F_\nu \propto \nu^{1/3}$ up to approximately the critical frequency $\nu_0$ of an electron with the energy $E_0$. Application to GRBs implies that a burst with $10^{-5}$ erg/cm$^2$s of soft gamma-rays and $h\nu_0 = 300$ KeV should be about 18th magnitude in visible light and a few $\mu$Jy at 1 GHz (less if self-absorbed).

Subject headings: Gamma-rays: Bursts—Shock Waves—Acceleration of Particles
1. Introduction

A model of gamma-ray bursts (GRB) has been proposed in which a relativistic fireball's debris shell (Shemi and Piran 1990) forms a relativistic collisionless shock when it interacts with surrounding low-density gas (Rees and Mészáros 1992, Mészáros and Rees 1993a, Katz 1994a). The shock-heated plasma radiates by the synchrotron process, producing the observed gamma-rays. It has been usual in astrophysics to assume that shock-accelerated particles have a non-thermal distribution in energy. In this paper I argue that in the case of heating by a relativistic shock this assumption is inapplicable to particles with energies below the mean particle energy $E_0$, for which I predict a thermal equilibrium distribution function. This implies a synchrotron spectrum $F_\nu \propto \nu^{1/3}$ below its peak.

Fermi (1949) derived the general result that the distribution function produced by stochastic acceleration processes in a test-particle approximation is proportional to a power of the particle energy, but a dimensional argument is sufficient: Any deviation from a power law would define a characteristic energy, yet in these models there are no characteristic energies between a particle’s rest mass or injection energy and an extremely high energy above which it is not magnetically confined to the region of acceleration. In the test-particle approximation the number density $n$ of accelerated particles is indeterminate because there is an essentially infinite reservoir of (nonrelativistic) thermal particles. The energy density $e$ of accelerated particles is then also indeterminate because the acceleration is driven by a reservoir of fluid kinetic energy in the shock or in plasma or hydrodynamic turbulence, an indeterminate fraction of which is converted to particle acceleration. The prediction of a power-law differential distribution of particle energy under these conditions

$$N(E) \propto E^{-p}, \quad (1)$$

where $N(E)$ is the number of particles per unit energy per unit volume, is borne out by observations of cosmic rays and of power-law synchrotron spectra radiated by accelerated
particles

\[ F_\nu \propto \nu^{-s} \]  

(2)
in many astronomical objects; in optically thin synchrotron theory (Rybicki and Lightman 1979) \( s = (p - 1)/2 \) if \( p > 1/3 \). In order that the total number and energy of accelerated particles be finite, a distribution of the form (1) must have at least one characteristic energy \( \sim E_0 \) at which \( p \) changes, with \( p = p_\prec < 1 \) for \( E \ll E_0 \) and \( p = p_\succ > 2 \) for \( E \gg E_0 \). The corresponding synchrotron spectral indices \( s_\prec < 0 \) and \( s_\succ > 1/2 \).

2. Relativistic Shocked Plasmas

It is often assumed that a power-law particle distribution function (Eq. 1) will apply to any collisionless gas of shock-accelerated particles. However, Fermi’s test particle approximation which led to Eq. (1) is inapplicable to relativistic collisionless shocks*. In such a shock \( n \) and \( e \) are determined by the jump conditions, and the mean energy per particle sets the natural energy scale \( E_0 \equiv e/n \). At the low densities at which shocks may be considered collisionless the rate of pair production is negligible.

In order for a shock to occur there must be an irreversible process, which mixes the distribution function in phase space, increasing a coarse-grained entropy. No general theory of collisionless shocks exists. The most economical hypothesis is that made by Gibbs—that in the final state all microstates consistent with the constraints (on \( n \) and \( e \)) are equally probable. This assumption defines a microcanonical ensemble (Reif 1965), and was also made by Lynden-Bell (1967) in his discussion of violent gravitational relaxation. The resulting distribution of particle energies, assuming relativistic kinematics and nondegeneracy, is that of thermal equilibrium

\[ N(E) \propto E^2 \exp(-3E/E_0); \]

(3)

* The argument presented by Katz (1994a) for \( p = 3/2 \) and \( s = 1/4 \) is wrong; non-relativistic shock acceleration theory is inapplicable, and the assumed compression ratio is inconsistent with the (correct) values cited elsewhere in that paper.
the “temperature” $k_B T = E_0/3$, and the parameters $p_\lessgtr = -2$ and $p_\rightarrow \infty$. This argument for Eq. (3) depends essentially on the existence of the constraints.

The relativistic Maxwellian (Eq. 3) was found by Gallant, et al. (1992) in simulations of relativistic shocks in electron-positron plasmas. In simulations of relativistic shocks in electron-positron-ion plasmas Hoshino, et al. (1992) found an excess of energetic positrons with a power-law distribution; only a fraction of the positron energy appeared in the nonthermal particles, with most of it remaining in the thermal population. The simulations assumed greater magnetic energy density in the upstream (unshocked) plasma than is likely to be found in GRB, and no simulation appears to have addressed the problem of relativistic shocks in electron-ion plasmas. “Plasma turbulent reactor” theories (Norman and Ter Haar 1975) are consistent with either Maxwellian or power-law distributions.

Sommer, et al. (1994) observed from one GRB a power-law gamma-ray spectrum with $s \approx 1$ extending up to $h\nu \sim 1$ GeV. This radiation must be produced by electrons with $E > E_0$. The observed $s$ is consistent with the value $p = 3$ suggested by Norman and Ter Haar (1975). However, Hoshino, et al. (1992) found $p \approx 2$ in simulations with one space and two momentum coordinates; it is unclear whether this result can be applied directly to three dimensional plasmas, but the increased dimensionality is more likely to decrease than to increase $p$.

This paper is concerned with radiation by particles for which $E \ll E_0$. In this limit Eq. (3) becomes

$$N(E) \propto E^2,$$

consistent with all the simulations (allowing for their reduced dimensionality).

In a collisionless shock Eqs. (3) or (4) must result from the interaction of coarse-grained clumps in phase space, rather than from single-particle collisions. These clumps interact electromagnetically through plasma turbulence, and we may regard the fields as the means by which the clumps interact, in analogy with the gravitational interaction of coarse-grained clumps considered by Lynden-Bell.
Order-of-magnitude arguments may permit an estimate of the magnitude of the fields. If the phase space clumps are sufficiently long-lived (as, for example, may be the electrostatic clumps discussed by Dupree [1982]), they may come to equilibrium with each other and with the turbulent fields, considered as independent degrees of freedom. This hypothesis is distinct from that of violent relaxation. The brightness temperature $T_b$ of the turbulence is defined by

$$k_B T_b \equiv \frac{\mathcal{F}_\nu c^2}{\nu^2},$$

(5)

where $\mathcal{F}_\nu$ is the spectral density of the turbulence. In a relativistic plasma most modes will have relativistic phase and group velocities, so we may approximate

$$\mathcal{F}_\nu \approx \frac{B^2 c}{8\pi \Delta \nu},$$

(6)

where $B$ is the turbulent magnetic field and $\Delta \nu$ is its spectral bandwidth. A clump of size $\lambda$ will contain $\approx n\lambda^3$ particles and a total energy $\approx n\lambda^3 E_0 \approx k_B T_c$, where $T_c$ is the kinetic temperature of the clumps. Taking $\Delta \nu \approx \nu$ and $\lambda \approx c/\nu$ and equating $T_c$ and $T_b$ yields

$$e = nE_0 \approx \frac{B^2}{8\pi}.$$  

(7)

This demonstrates that equipartition between clumps and plasma turbulence is consistent with the energetics, and lends credibility to the equipartition of particle and magnetic energy assumed by Katz (1994a).

The length $\lambda$ is determined by the wavelengths of the fastest growing plasma instabilities. These may be two-stream instabilities when the plasmas first interpenetrate, followed by the Weibel (1959) instability (excitation of transverse electromagnetic waves by velocity space anisotropy). For strong anisotropy the Weibel instability may saturate at approximate magnetic equipartition. Equation (7) also implies approximate equality between the plasma and gyro-frequencies $\omega_p$ and $\omega_g$, so that the distribution of $\lambda$ is probably peaked around $\lambda \sim 2\pi c/\omega_p \sim 2\pi c/\omega_g$, with $\mathcal{F}_\nu$ peaked around $\nu \sim c/\lambda$. Fields of the magnitude
given by Eq. (7) imply an equipartition time $O(\omega_p^{-1}) \sim O(\omega_g^{-1})$, and the lifetime of the clumps need only be of the same order.

Electron-ion equipartition cannot result from the very slow single-particle interactions, and requires electromagnetic coupling between charge- or current-unneutralized clumps in electron and ion phase space. In a relativistic plasma turbulence resembles propagating electromagnetic waves, with electric fields $E \sim B$. Then the charge imbalance $|n_i - n_e| \sim O(n)$, the electrostatic potentials are $O(E_0/e)$, and electron-ion equipartition is assured. These estimates need to be tested by plasma simulation.

Finally, a nonequilibrium distribution can also be shown to relax to the equilibrium (Eq. 3) if an H-theorem is satisfied, which follows from the assumption of molecular chaos (Liboff 1969). This assumption holds if the phase space clumps interact weakly, but is likely to be more general—H-theorems are observed to hold in dense and strongly coupled systems for which it is hard to justify the assumption of molecular chaos. Strong two-particle (or two-clump) momentum correlations are disrupted, at least in ensemble average, as the particles (or clumps) interact with many others, and thermodynamic equilibrium is observed even though the assumption of molecular chaos is hard to defend.

3. Predicted Spectra

For relativistic electron distributions with $p < 1/3$ the synchrotron spectrum at frequencies below the critical frequency is dominated by the radiation of the most energetic electrons (Jackson 1975), and $s = -1/3$ (rather than $(p - 1)/2$). Thus the particle distribution (Eq. 4) leads to a predicted radiation spectrum

$$F_\nu \propto \nu^{1/3},$$  \hspace{1cm} (8)

which in GRB should extend roughly from X-rays to radio frequencies. Distributions which differ from Eq. (4) only for $E > E_0$, even if non-thermal, also lead to Eq. (8) for $\nu \ll \nu_0$, where $\nu_0$ is the critical synchrotron frequency (Doppler-shifted to the observer’s frame) for
electrons with the local $E_0$ in the local magnetic field. This predicted spectrum survives averaging over emission regions with a range of $E_0$, field, and Doppler shift.

Schaefer (1994) found $N_\nu \propto \nu^{-0.7}$ in GRB at soft X-ray energies, corresponding to $F_\nu \propto \nu^{0.3}$, consistent with Eq. (8). The “redder” (larger $s$) spectra of GRB at harder X-ray and gamma-ray energies reflect the radiation of particles with $E > E_0$, whose distribution is no longer described by Eq. (4), producing a gradual roll-off below Eq. (8) in the integrated spectrum with increasing frequency.

Optically thin classical relativistic synchrotron radiation leads to the very general prediction $s \geq -1/3$. Observation of $s < -1/3$ in GRB would imply absorption, which may be photoelectric (Schaefer 1993) in soft X-rays, by dust or gas in visible and ultraviolet light, or self-absorption (Mészáros and Rees 1993b, Paczyński and Rhoads 1993, Katz 1994ab). Because the radiation at lower frequencies is dominated by electrons with $E \approx E_0$, in the self-absorbed region $s = -2$ rather than the value $s = -5/2$ found (Rybicki and Lightman 1979) when $p > 1/3$.

The spectrum of GRB may be normalized to the observed soft gamma-ray fluxes. The normalization is necessarily very approximate, because source regions are likely heterogeneous and the transition between Eq. (8) and its cutoff at high photon energies may therefore extend over several decades of the integrated spectrum, including the soft gamma-ray region. An intense GRB with a flux $10^{-5}$ erg/cm$^2$s in a bandwidth of 400 KeV around $h\nu = 300$ KeV has a flux there of $\approx 10$ mJy. Extrapolation leads to a flux of $\approx 0.2$ mJy in visible light, ($\approx 18$th magnitude) and to $\approx 2$ $\mu$Jy at 1 GHz. The effective flux of a brief transient measured by a broad-band receiver may be further reduced by plasma dispersion. Because these fluxes are low (and, in addition, self-absorbed at low frequencies) intergalactic dispersion (Ginzburg 1973, Palmer 1993, Katz 1994ab) and optical counterparts to GRB will be difficult to observe.

For an actual GRB it is more accurate to extrapolate downward from the highest frequency $\nu_{max}$ to which Eq. (8) is observed; $\nu_{max}$ is the smallest $\nu_0$ contributing to
the observed radiation. This may lead to substantially higher visible and radio fluxes if $h\nu_{\text{max}} \ll 300$ KeV because $s > -1/3$ between $\nu_{\text{max}}$ and soft gamma-ray frequencies. In the data of Schaefer (1994) $\nu_{\text{max}}$ appears to be $\sim 10$ KeV.

After the initial gamma-ray transient, a relativistic fireball will continue to expand and to produce radiation up to a critical frequency which decreases with time as the blast wave degrades (Paczyński and Rhoads 1993; Mészáros and Rees 1993b, 1994; Katz 1994ab). In a uniform medium (a very unrealistic assumption, as shown by the complex temporal structure of observed GRB) the intensity at a given frequency will increase with time (in a simple model $\propto t^{4/5}$) until the lowest critical synchrotron frequency decreases to the frequency of observation; the peak visible brightness is $\sim 50$ times brighter ($\approx 14$th magnitude) and follows the $\gamma$-ray emission by $\sim 100$ times the latter’s duration, while at 1 GHz the peak flux is $\sim 5000$ times brighter ($\sim 10$ mJy) and lags by $\sim 40,000$ times the $\gamma$-ray duration (Katz 1994ab). However, at all times the spectrum (Eq. 8) should be observed, allowing for absorption, in the very broad range of frequencies between $\nu_{\text{max}}$ and the onset of self-absorption.

4. Discussion

This paper makes two predictions for the characteristic spectrum of synchrotron radiation produced by relativistic shocks. The first prediction, $s_0 < 0$, uses only the finiteness of the total particle number. The second, more specific, prediction, $s_0 = -1/3$ (Eq. 8), is based upon the argument ($\S 2$) for $p_0 = -2$ (Eq. 4), although any distribution with $p < 1/3$ is sufficient to lead to $s = -1/3$.

The arguments presented here for relativistic shocks are general, and not specific to GRB. For example, they should apply to relativistic blast wave models of AGN such as those proposed by Blandford and McKee (1977).

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