Comparison of short runs control chart and T2 Hotelling control chart for monitoring sunergy glass production process

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Abstract. In recent years, the manufacturing industry has tended to reduce mass production and produce in small quantities, which is called "Short Run Production". In such a situation, the course of the production process is short, usually, the number of productions is less than 50. Therefore, a control chart for the short run production process is required. This paper discusses the comparison between multivariate control chart for short run production (V control chart) and T2 Hotelling control chart applied to sunergy glass data. Furthermore, a simulation of Average Run Length (ARL) was carried out to determine the performance of the two control charts. The results obtained are that the production process has not been statistically controlled using either the V control chart or the T2 Hotelling control chart. The number of out-of-control on the control chart V using the the EWMA test is more than the T2 Hotelling control chart. Based on the ARL value, it shows that the V control chart is more sensitive than the T2 Hotelling control chart.

1. Introduction
The development of human civilization has pushed the manufacturing industry sector to experience rapid development along with the increasing demand for goods from the public as consumers. One of them is the glass manufacturing industry sector. Glass is one part of the building that is usually used as an interior or exterior. Sunergy glass is one type of glass that is a superior product. This glass has advantages in thermal insulation and has solar control properties that can reduce incoming heat to increase comfort. In addition, the heat reduction can reduce the electrical load for the air conditioner while maximizing incoming light, making the room brighter. Sunergy glass is commonly used for windows of houses and buildings and can also be used for all-glass walls.

The increasing public demand for flat glass makes glass companies continue to strive to be able to produce quality products for the community. This is done so that the company can maintain its existence amid intense industrial competition. Product quality is very important in determining the sustainability of a company is competing in the global market. The quality of a product is determined by the quality characteristics of the product. In maintaining product quality, it is necessary to do quality control.

Quality control is a system that maintains the desired level of quality, through feedback on the characteristics of the product or service and the implementation of corrective actions if there are deviations in the characteristics from the established standards [1]. While Statistical Process Control (SPC) is a statistical technique used to control a process and reduce variation. The reduction of variation is a key feature to improve quality [2]. A statistical tool commonly used for production quality control is a control chart. Graphically, the control chart represents the mean value and upper and lower control
limits of a process. The advantage of the control chart is its ability to identify process shifts and show abnormal conditions in the production process [3].

The sunergy glass production process has a high variation, namely the produced sunergy glass has different thicknesses according to consumer demand. Different glass thickness causes each type of thickness to have a low production volume or in other words to have a small number of products, which is less than 50. According to Khoo and Quah [4], such conditions are called short runs production. The production process of short runs is defined as a state of production where one production to another has different standard characteristics (job-shop) so that it has high variance characteristics, just-in-time (JIT) which is a small number of products (low volume, generally less than 50) so that the production process runs shorter, and process parameters are not available due to insufficient or no previous production data [4]. In the short-run production process, the goods produced aim to meet demand, while things that are wasteful are minimized such as overproduction, waiting for production, and excessive inventory [5].

The quality of the sunergy glass cut is checked based on the quality characteristics of the edge distortion and cutter line, where the longer the edge distortion, the longer the cutter line. So it is said that the two quality characteristics are interrelated. Because sunergy production is a short-run production and the two quality characteristics are interrelated, it is necessary to have a multivariate control diagram that is by the conditions of the short runs production process in quality control of the sunergy glass pieces. Khoo and Quah (2002) in a previous study proposed a multivariate short runs control chart for monitoring the mean process on individual data and subgrouped data. The results show several advantages of the control chart, including that it does not require previous data (historical data) to estimate process parameters, the standardization value increases the strength of the control chart to detect special causes from out-of-control points, and does not only for short-run processes but also suitable for long runs processes.

In this paper, quality control was carried out by applying a multivariate control chart to the sunergy glass data so that it could be seen whether or not the mean process of the sunergy glass production were statistically controlled. The method used is the control chart proposed by Khoo and Quah (2002) and the control chart proposed by Hotelling as a comparison. Furthermore, the performance analysis on the two control charts is carried out using Average Run Length (ARL). The results of this study are expected to provide information for companies to maintain and improve the quality of sunergy glass production. In addition, it can also be seen that the short runs control chart has an advantage in conditions of low production quantities.

2. Short Runs Control Chart

The order of multivariate observations for each subgroup are independently and identically distributed a multivariate normal, $N_p(\mu, \Sigma)$ where $p \geq 2$ is the number of quality characteristics that are monitored simultaneously and $X_{i,k}$ is the multivariate normal observation of the $i$-th subgroup on the $k$-th sample (with $i = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, n$), then $X_{i,k} = (X_{i,1,k}, X_{i,2,k}, \ldots, X_{i,p,k})^T$ where $X_{i,j,k}$ is a univariate observation on the $j$-th quality characteristic (with $j = 1, 2, \ldots, p$) for vector $X_{i,k}$. Assuming that all subgroups have the same sample size, then $n_1 = n_2 = \cdots = n_m = n$. The mean vector estimate for the $i$-th subgroup is defined as $\bar{X}_i = (1/n) \sum_{k=1}^{n} X_{i,k}$. Then the grand mean vector is estimated from the first $i$ subgroup with the formula $\bar{X}_i = (1/i) \sum_{u=1}^{i} \bar{X}_u$. The estimation of the variance-covariance matrix in the $i$-th subgroup is denoted as $S_i$ with a matrix size of $p \times p$. The notation can be seen in Table 1 below.

| Subgroup | Multivariate Observations | Estimated Mean Vector | Estimated Variance-Covariance Matrix |
|----------|---------------------------|-----------------------|-------------------------------------|
| 1        | $X_{1,1}, X_{1,2}, \ldots, X_{1,n}$ | $\bar{X}_1$ | $S_1$ |
| 2        | $X_{2,1}, X_{2,2}, \ldots, X_{2,n}$ | $\bar{X}_2$ | $S_2$ |
| \vdots   | \vdots                     | \vdots               | \vdots                              |
Monitoring the shift in the mean vector is divided into 4 conditions, that are \( \mu \) and \( \Sigma \) are known, \( \mu \) unknown and \( \Sigma \) known, \( \mu \) known and \( \Sigma \) unknown, and \( \mu \) and \( \Sigma \) are unknown.

### 2.1. KK condition (\( \mu = \mu_0 \), \( \Sigma = \Sigma_0 \), both are known)

Previously, it is necessary to calculate the statistical value of \( T^2 \)-hotelling or denoted \( T^2_{a_1} \) for each subgroup (with \( T^2_{a_1} \sim \chi^2_p \)), where \( i = 1, 2, \ldots, m \) based on the formula in the following equation (1).

\[
T^2_{a_1} = n(\bar{X}_i - \mu_0)^\Sigma^{-1}(\bar{X}_i - \mu_0), \quad \text{with } i = 1, 2, \ldots, m. \tag{1}
\]

Then the value of \( T^2_{a_1} \) obtained in equation (1) is substituted into equation (2) which is called the \( V \) statistic.

\[
V_{a_1} = \Phi^{-1}\left\{ H_p \left( T^2_{a_1} \right) \right\}, \quad \text{with } i = 1, 2, \ldots, m. \tag{2}
\]

Where,

- \( \Phi \) = The standard normal cumulative distribution function.
- \( \Phi^{-1} \) = The inverse of the standard normal cumulative distribution function.
- \( H_p \) = The chi-squared cumulative distribution function with \( p \) degrees of freedom.

### 2.2. UK condition (\( \mu \) unknown, \( \Sigma = \Sigma_0 \) known)

The statistical value of \( T^2 \)-hotelling or denoted \( T^2_{b_1} \) for each subgroup (with \( T^2_{b_1} \sim \chi^2_p \)), where \( i = 1, 2, \ldots, m \) needs to be calculated first, based on the formula in the equation (3).

\[
T^2_{b_1} = n(\bar{X}_i - \bar{X}_{b-1}) \Sigma_0^{-1}(\bar{X}_i - \bar{X}_{b-1}), \quad \text{with } i = 2, 3, \ldots, m. \tag{3}
\]

Then the value of \( T^2_{b_1} \) obtained in equation (2.3) is substituted into equation (4) which is called the \( V \) statistic.

\[
V_{b_1} = \Phi^{-1}\left\{ H_p \left( \frac{(i-1)}{i}T^2_{b_1} \right) \right\}, \quad \text{with } i = 2, 3, \ldots, m. \tag{4}
\]

### 2.3. KU condition (\( \mu = \mu_0 \) known, \( \Sigma \) unknown)

Before calculating the statistic \( V \), it is necessary to calculate the statistical value of \( T^2 \)-hotelling or denoted \( T^2_{c_i} \) for each subgroup (with \( T^2_{c_i} \sim \chi^2_p \)), where \( i = u, u+1, \ldots, m \) using formula in the equation (5).

\[
T^2_{c_i} = n(\bar{X}_i - \mu_0)S_0^{-1}(\bar{X}_i - \mu_0), \quad \text{with } i = u, u+1, \ldots, m, \tag{5}
\]

where the pooled unbiased estimator \( \Sigma \) from the first \( i-1 \) subgroup is

\[
S_0 = \frac{1}{n_1 + n_2 + \ldots + n_{i-1}} \left( n_1S_1 + n_2S_2 + \ldots + n_{i-1}S_{i-1} \right)
= \frac{n}{n(i-1)}(S_1 + S_2 + \ldots + S_{i-1}) \quad \text{; because } n_1 = n_2 = \ldots = n_{i-1} = n
= \frac{1}{i-1}(S_1 + S_2 + \ldots + S_{i-1})
\]

and unbiased estimator \( \Sigma \) for subgroup \( i \) is

\[
S_i = \frac{1}{n_i}(\bar{X}_{i,k} - \mu_0)(\bar{X}_{i,k} - \mu_0)^t.
\]

Then the value of \( T^2_{c_i} \) obtained in equation (5) is substituted into equation (6) which is called the \( V \) statistic.

\[
V_{c_i} = \Phi^{-1}\left\{ F_{p,(i-1),p+1} \left[ \frac{n(i-1) - p + 1}{np(i-1)}T^2_{c_i} \right] \right\}, \quad \text{with } i = u, u+1, \ldots, m. \tag{6}
\]
where
\[ u = \left\| \frac{p-1}{n} \right\| + 1. \] (7)

\[ F_{p,n(i-1)-p+1} \] is The snedecor-F cumulative distribution function with \((p, n(i-1) - p + 1)\) degrees of freedom

### 2.4. UU condition (\(\mu\) and \(\Sigma\) are unknown)

The statistical value of \(T^2\)-hotelling or denoted \(T^2_{i-1}\) for each subgroup (with \(T^2_{i-1} \sim \frac{i}{i-1} F^2\)) where \(i = 1, 2, \ldots, m\) needs to be calculated first, based on the formula in the equation (8).

\[ T^2_{i} = n(\bar{X}_i - \bar{X}) S^{-1}_{p,i} (\bar{X}_i - \bar{X})' , \text{ with } i = y, y+1, \ldots, m. \] (8)

where the pooled unbiased estimator \(\Sigma\) from the first \(i-1\) subgroup is

\[ S_{p,i-1} = \frac{1}{n_1 + n_2 + \cdots + n_{i-1} - (i-1)} \left[ (n_1-1)S_1 + (n_2-1)S_2 + \cdots + (n_{i-1}-1)S_{i-1} \right] \]

\[ = \frac{(n-1)}{(i-1)(i-1)} (S_1 + S_2 + \cdots + S_{i-1}) ; \text{ because } n_1 = n_2 = \cdots = n_{i-1} = n \]

\[ = \frac{1}{i-1} (S_1 + S_2 + \cdots + S_{i-1}) \]

and unbiased estimator \(\Sigma\) for subgroup \(i\) is

\[ S_i = \frac{1}{n-i} \sum_{i=1}^{k} (X_{i,1} - \bar{X}_i)(X_{i,1} - \bar{X}_i)' \]

Then the value of \(T^2_{i-1}\) obtained in equation (8) is substituted into equation (9) which is called the V statistic

\[ V_i = \Phi^{-1} \left\{ F_{p,(i-1)(n-1)-p+1} \left( \frac{(n-1)(i-1) - p + 1}{ip(n-1)} \right) T^2_{i} \right\} , \text{ with } i = y, y+1, \ldots, m, \] (9)

where

\[ y = \left\| \frac{p-1}{n} \right\| + 1. \] (10)

\[ F_{p,(i-1)(n-1)-p+1} \] is The snedecor-F cumulative distribution function with \((p, (i-1)(n-1) - p + 1)\) degrees of freedom.

Note that \(\|a\|\) in equations (7) and (10) is denoted the smallest integer greater than \(a\). The V statistic in equations (2), (4), (6), and (9) is a sequence of random variables with an \(N(0,1)\) distribution. Furthermore, the order of V statistics that are \(V_1, V_{a+1}, \ldots, V_m\), where \(V_m\) is denoted as V statistic (that are \(V_a, V_b, V_c, \) or \(V_d\)) control chart in subgroup \(m\). To detect a shift in the mean vector (mean process), three tests are carried out which are defined as follows.

1. The 1-of-1 Test: when \(V_m\) is plotted, this test signals a shift in \(\mu\) if \(V_m > 3\sigma = 3\).
2. The 3-of-3 Test: when \(V_m\) is plotted, this test signals a shift in \(\mu\) if \(V_m, V_{m-1}, \) and \(V_{m-2}\) all exceed \(1\sigma = 1\). In addition, this test can only be used when three consecutive V statistics are available [5].
3. The 4-of-5 Test: when \(V_m\) is plotted, this test signals a shift in \(\mu\) if at least four of the five values \(V_m, V_{m-1}, \ldots, V_{m-4}\) exceed \(1\sigma = 1\). In addition, this test can only be used when five consecutive V statistics are available [6].

In addition to the three tests above, there is an EWMA (exponentially weighted moving average) test which is calculated from the V statistic in sequence. The EWMA statistic (\(Z_i\)) is defined as follows.
\[ Z_i = \lambda V_i + (1 - \lambda)Z_{i-1}, \text{ with } i = 1, 2, \ldots, m, \quad (11) \]

where \( Z_0 = 0 \). For a statistical sequence, \( V_i \) with \( i = a, a+1, \ldots, m \) where \( a > 1 \), the statistical sequence \( Z_i \) will be obtained by placing \( Z_{a-1} = 0 \). The EWMA statistics will be plotted on the diagram with \( UCL = L\sqrt{\lambda/(2 - \lambda)} \) [5].

### 3. T² Hotelling Control Chart

T² hotelling control chart is used if in a process of quality control using together containment or more than one characteristic of inspection. According to Montgomery (2013), T² hotelling control chart for subgroup data can be formulated as follows.

\[ T² = n(\overline{x} - \overline{\mathbf{x}})'S^{-1}(\overline{x} - \overline{\mathbf{x}}). \quad (12) \]

where \( T² \) is test statistic, \( n \) is a sample of size, and \( \overline{\mathbf{x}} \) is the mean vector of each subgroup.

The phase I control limits for T² hotelling control chart are given by

\[ UCL = \frac{p(m - 1)(n - 1)}{mn - m - p + 1} F_{n - p, mn - m - p + 1}, \]

\[ LCL = 0 \]

In phase II, when the chart is used for monitoring future production, the control limits are as follows.

\[ UCL = \frac{p(m + 1)(n - 1)}{mn - m - p + 1} F_{n - p, mn - m - p + 1}, \]

\[ LCL = 0 \]

where \( p \) is the number of quality characteristics and \( m \) is the number of subgroup.

### 4. Numerical Result

This part discusses the result of the control chart for controlling the sunergy glass.

#### 4.1. Descriptive Statistics

Descriptive statistics were carried out to obtain a description or general description of the quality characteristics of sunergy glass production, namely edge distortion, and cutter line. One way in descriptive statistics to graphically describe numerical data is to use a boxplot. Boxplots can be used to detect and describe changes in variation and location between different groups.

![Boxplot Quality Characteristics of Edge Distortion](Figure 1)

The information obtained from Figure 1 is that the mean quality characteristics of edge distortion in phases I and II have different values. It can be seen that the mean value in phase I is higher than in phase II. On the other hand, the distribution of data (variance) characteristic of edge distortion quality in phase II is larger than in phase I.
Figure 2 provides information on the mean quality characteristics of the cutter line in phase I which is higher than in phase II. However, the data distribution (variant) of cutter line quality characteristics in phase II is greater than in phase I. The results of descriptive statistics for each quality characteristic in phase I are shown in Table 2 below.

Table 2. Statistics Description of Phase I

| Quality Characteristics | Mean   | Variance  | Min  | Max   | CV     | Specification Limit |
|-------------------------|--------|-----------|------|-------|--------|--------------------|
| Edge distortion         | 55.32  | 152.56    | 27   | 86    | 22.33  | 20 – 60            |
| Cutter line             | 161.43 | 67.94     | 123  | 180   | 5.11   | 80 – 150           |

Table 2 provides information that the mean length of the edge distortion distance in phase I is still within the company’s specifications. However, some observations are still outside the company’s specifications. This is indicated by the maximum edge distortion value of 86 mm which exceeds the specification limit. On the other hand, the minimum edge distortion value is still within the company’s specifications, which is 27 mm. Other information from Table 2 is that the mean length of the cutter line distance in phase I is outside the company’s specification limits. Not only the mean value, the maximum value of the quality characteristics of the cutter line, which is 180 mm, also exceeds the company’s specification limit. While the minimum value of the cutter line variable of 123 mm is still within the specification limits. In addition, Table 2 also shows that edge distortion has a coefficient of variation that is greater than the coefficient of variation of the cutter line. This indicates that the quality characteristics of edge distortion have the most heterogeneous variation compared to the quality characteristics of the cutter line. The descriptive statistics of each quality characteristic in phase II are shown in Table 3 below.

Table 3. Statistics Description of Phase II

| Quality Characteristics | Mean    | Variance | Min  | Max   | CV     | Specification Limit |
|-------------------------|---------|----------|------|-------|--------|--------------------|
| Edge distortion         | 42.38   | 186.75   | 21   | 73    | 32.24  | 20 – 60            |
| Cutter line             | 143.48  | 369.88   | 106  | 200   | 13.40  | 80 – 150           |

Table 3 provides information that the mean length of the edge distortion and cutter line distances in phase II is still within the specifications of the company. However, some observations are still outside the company’s specifications. This is indicated by the maximum edge distortion value of 73 mm which exceeds the specification limit. In addition, the maximum cutter line value of 200 mm is also outside the company’s specification limits. On the other hand, the minimum edge distortion and cutter line values are still within the company’s specifications, with values of 21 mm and 106 mm, respectively. Other information from Table 3 is that edge distortion has a coefficient of variation that is greater than the coefficient of variation of the cutter line. This indicates that the quality characteristics of edge distortion have the most heterogeneous variation compared to the quality characteristics of the cutter line.
4.2. Multivariate Normal Test
Test the assumption of multivariate normal distribution on both quality characteristics using the Shapiro-Wilk test. The test produces a W* value of 0.979 which is close to the value of 1 at a significance level (α) of 0.05. This shows that the decision failed to reject H_0, which means that both quality characteristics, namely edge distortion, and cutter line, are multivariate normal distribution. In addition to using the value of W*, the p-value can also be used. The Shapiro-Wilk test produces a p-value of 0.061 which indicates a p-value greater than the 0.05 significance level. This gives a decision that failed to reject H_0 which means that both quality characteristics follow a multivariate normal distribution.

4.3. Bartlett Test
Test the assumption of dependencies between the quality characteristics of edge distortion and cutter line using the Bartlett test. The results of the Bartlett test with a significance level (α) of 0.05 gave a calculated chi-square value (χ²) of 58,538. This value is greater than the value of the chi-square table which is 3.841. This results in a decision to reject H_0, which means that the two quality characteristics have a correlation or are mutually dependent. In addition to the chi-square value, the dependent assumption test can also be seen using the p-value. Bartlett's test produces a p-value of 0.000. This value is smaller than the 0.05 significance level which indicates the decision to reject H_0, which means that the quality characteristics of edge distortion and cutter line are related or mutually dependent. These results strengthen the theory in the field that the two quality characteristics are interrelated.

4.4. Monitoring using Control Chart
After testing the assumptions of dependencies and the multivariate normal distribution have been met, then quality control is carried out using a short-run multivariate control chart and T² Hotelling control chart to monitor the mean of the production process. The following is the quality control of the Sunergy glass production process.

4.4.1. Short Runs Control Chart. In this research, the 1st to 20th production data are used in the phase I. Furthermore, the value of the V statistic will be plotted on the control chart for monitoring the mean process using four tests with control limits and decision rules for each test. The four tests include the 1-of-1 test, the 3-of-3 test, the 4-of-5 test, and the EWMA test. The following monitoring sunergy glass using short runs control chart with the 1-of-1 test.

- **Phase I**

The multivariate short runs control chart with the 1-of-1 tests for monitoring the mean production process of sunergy glass phase i can be seen in Figure 3. This test uses the UCL (Upper Control Limits) = 3. The decision of a subgroup is said to be in control if the V statistic at the point of the subgroup is at the UCL.

![Figure 3. Short Runs Control Chart in Phase I using 1-of-1 test](image)

Figure 3 shows that the number of production data that is out of control is one, namely the 12th observation. After knowing the data that is out of control, a technical check is carried out on the 12th
observation, then the data is removed from the observation. After removing out-of-control data, Figure 4 shows that there is no observation outside the control limits.

Figure 4. Short Runs Control Chart in Phase I using 1-of-1 test (revision)

- **Phase II**

After carrying out the monitoring process in phase I, the next step is to control the mean process in Phase II which aims to monitor the production process on the 21st to 40th production data. The upper control limits used in phase II is the same as phase I, which is 3.

Figure 5. Short Runs Control Chart in Phase II using 1-of-1 test

Figure 5 shows the process of monitoring sunergy glass production data in phase II. By using the upper control limit of 3, it can be concluded that the monitoring process in phase II is not statistically controlled, that is, there is one uncontrolled data.

4.4.2. **Short Runs Control Chart.** The following monitoring sunergy glass using short runs control chart with the 3-of-3 test.

- **Phase I**

The multivariate short runs control chart with the 3-of-3 tests for monitoring the mean production process of Sunergy Glass phase I can be seen in Figure 6. This test uses the UCL (Upper Control Limits) = 1. The decision of a subgroup is said to be in control if the three V statistics are consecutive ($V_m, V_{m-1}$, and $V_{m-2}$) is outside of the UCL.

Figure 6. Short Runs Control Chart in Phase I using 3-of-3 test

The decision of each subgroup point out-of-control or not is shown in the table as follows.
Table 4. The decision of the 3-of-3 test Phase I

| Subgroup | Decision          | Subgroup | Decision          | Subgroup | Decision          |
|----------|-------------------|----------|-------------------|----------|-------------------|
| 1        | -                 | 8        | Data In control   | 15       | Data Out of control |
| 2        | Data In control   | 9        | Data In control   | 16       | Data In control   |
| 3        | Data In control   | 10       | Data In control   | 17       | Data In control   |
| 4        | Data In control   | 11       | Data In control   | 18       | Data In control   |
| 5        | Data In control   | 12       | Data In control   | 19       | Data In control   |
| 6        | Data In control   | 13       | Data Out of control | 20      | Data In control   |
| 7        | Data In control   | 14       | Data Out of control |

Based on the decisions in Table 4, it can be seen that three subgroup points exceed the UCL control limit (out of control), including the 13th, 14th, and 15th subgroups. After making improvements five times, Figure 7 shows that the control chart has been statistically controlled. This is based on the decision in Table 5.

Figure 7. Short Runs Control Chart in Phase I using 3-of-3 test (revision)

Table 5. The decision of the 3-of-3 test Phase I (revision)

| Subgroup | Decision          | Subgroup | Decision          | Subgroup | Decision          |
|----------|-------------------|----------|-------------------|----------|-------------------|
| 1        | -                 | 6        | Data In control   | 11       | Data In control   |
| 2        | Data In control   | 7        | Data In control   | 12       | Data In control   |
| 3        | Data In control   | 8        | Data In control   | 13       | Data In control   |
| 4        | Data In control   | 9        | Data In control   | 14       | Data In control   |
| 5        | Data In control   | 10       | Data In control   | 15       | Data In control   |

- **Phase II**

After carrying out the monitoring process in phase I, the next step is to control the mean process in phase II which aims to monitor the production process on the 21st to 40th production data. The upper control limits used in phase II is the same as phase I, which is 1. Also, the decision is shown in Table 6.

Figure 8. Short Runs Control Chart in Phase II using 3-of-3 test
Table 6. The decision of the 3-of-3 test Phase II

| Subgroup | Decision | Subgroup | Decision | Subgroup | Decision |
|----------|----------|----------|----------|----------|----------|
| 1        | Data In control | 8        | Data In control | 15       | Data In control |
| 2        | Data In control | 9        | Data In control | 16       | Data In control |
| 3        | Data In control | 10       | Data In control | 17       | Data In control |
| 4        | Data In control | 11       | Data In control | 18       | Data In control |
| 5        | Data In control | 12       | Data In control | 19       | Data In control |
| 6        | Data In control | 13       | Data Out of control | 20       | Data In control |
| 7        | Data In control | 14       | Data Out of control |  |  |

The decision in Table 6 means that the mean production process of sunergy glass phase II with the 3-of-3 test has not been statistically controlled.

4.4.3. Short Runs Control Chart. The following monitoring sunergy glass using short runs control chart with the 4-of-5 test.

- **Phase I**

The multivariate short runs control chart with the 4-of-5 tests for monitoring the mean production process of Sunergy Glass phase I can be seen in Figure 9. This test uses the UCL (Upper Control Limits) = 1. The decision of a subgroup is said to be in control if the V statistic at the point of the subgroup is at the UCL.

![Short Runs Control Chart](image_url)

**Figure 9.** Short Runs Control Chart in Phase I using 4-of-5 test

The decision of each subgroup point out-of-control or not is shown in Table 5.

Table 7. The decision of the 4-of-5 test Phase I

| Subgroup | Decision | Subgroup | Decision | Subgroup | Decision |
|----------|----------|----------|----------|----------|----------|
| 1        | Data In control | 8        | Data In control | 15       | Data Out of control |
| 2        | Data In control | 9        | Data In control | 16       | Data Out of control |
| 3        | Data In control | 10       | Data In control | 17       | Data Out of control |
| 4        | Data In control | 11       | Data In control | 18       | Data In control |
| 5        | Data In control | 12       | Data In control | 19       | Data In control |
| 6        | Data In control | 13       | Data Out of control | 20       | Data In control |
| 7        | Data In control | 14       | Data Out of control |  |  |

Table 7 gives a decision that five subgroup points exceed the UCL control limit, namely the 13th subgroup to the 17th subgroup. After making improvements six times, Figure 10 shows that the control chart has been statistically controlled. This is based on the decision in Table 8.
Figure 10. Short Runs Control Chart in Phase I using 4-of-5 test (revision)

Table 8. The decision of the 4-of-5 test Phase I (revision)

| Subgroup | Decision | Subgroup | Decision | Subgroup | Decision |
|----------|----------|----------|----------|----------|----------|
| 1        |          | 6        | Data In control | 11       | Data In control |
| 2        | Data In control | 7        | Data In control | 12       | Data In control |
| 3        | Data In control | 8        | Data In control | 16       | Data In control |
| 4        | Data In control | 9        | Data In control | 17       | Data In control |
| 5        | Data In control | 10       | Data In control |

Phase II

After carrying out the monitoring process in phase I, the next step is to control the mean process in phase II which aims to monitor the production process on the 21st to 40th production data. The upper control limits used in phase II is the same as phase I, which is 1. Also, the decision is shown in Table 9.

Figure 11. Short Runs Control Chart in Phase II using 4-of-5 test

Table 9. The decision of the 4-of-5 test Phase II

| Subgroup | Decision | Subgroup | Decision | Subgroup | Decision |
|----------|----------|----------|----------|----------|----------|
| 1        |          | 8        | Data In control | 15       | Data Out of control |
| 2        | Data In control | 9        | Data In control | 16       | Data In control |
| 3        | Data In control | 10       | Data In control | 17       | Data In control |
| 4        | Data In control | 11       | Data In control | 18       | Data In control |
| 5        | Data In control | 12       | Data In control | 19       | Data In control |
| 6        | Data In control | 13       | Data Out of control | 20       | Data In control |
| 7        | Data In control | 14       | Data Out of control |

Table 9 provides information on the process of monitoring sunergy glass production data in phase II. By using the upper control limit of 1, the conclusion obtained is that the phase II monitoring process is not statistically controlled because there are three uncontrolled data.

4.4.4. Short Runs Control Chart. The multivariate short runs control chart with the EWMA test for monitoring the mean production process of sunergy glass phase I and phase II is as follows.
• **Phase I**

The multivariate short runs control chart with the EWMA test for monitoring the mean production process of Sunergy Glass phase I can be seen in Figure 12. This test uses the parameter $L = 2.76$ dan $\lambda = 0.15$. The decision of a subgroup is said to be in control if the $Z$ statistic at the point of the subgroup is at the UCL.

![Figure 12. Short Runs Control Chart in Phase I using EWMA test](image)

Figure 12 shows that there are nine data that are out of control, which is the 12th, 13th, 14th, 15th, 16th, 17th, 18th, 19th, and 20th subgroups. So it can be concluded that with the EWMA test the mean production process of Sunergy Glass phase I is still not statistically controlled. Therefore, a technical examination was carried out on the highest out-of-control data, namely the 15th production data. After knowing the cause, the 15th production data were excluded from the observation. Then the same thing is done until no production data is out of control (Figure 13).

![Figure 13. Short Runs Control Chart in Phase I using EWMA test (in control)](image)

• **Phase II**

After carrying out the monitoring process in phase I, the next step is to control the mean process in phase II which aims to monitor the production process on the 21st to 40th production data. The upper control limits used in phase II is the same as phase I, which is 0.7859.

![Figure 14. Short Runs Control Chart in Phase II using EWMA test](image)

Figure 14 shows the process of monitoring sunergy glass production data in phase II. By using the upper control limit of 0.7859, the conclusion obtained is that the phase II monitoring process is not statistically controlled because there are five uncontrolled data.
4.4.5. $T^2$ Hotelling Control Chart. In this research, the 1st to 20th production data is used in the phase I. Furthermore, the value of the $T^2$ statistic will be plotted on the control chart for monitoring the mean process. The following monitoring sunergy glass using $T^2$ Hotelling control chart.

- **Phase I**

The $T^2$ Hotelling control chart for monitoring the mean production process of Sunergy Glass phase I can be seen in Figure 10. This test uses the UCL (Upper Control Limits) = 13.46. The decision of a subgroup is said to be in control if the $T^2$ statistic at the point of the subgroup is at the UCL.

![Figure 15. $T^2$ Hotelling Control Chart in Phase I](image)

Figure 15 shows that the number of production data that is out of control is one, namely the 12th observation. After knowing the data that is out of control, a technical check is carried out on the 12th observation, then the data is removed from the observation. After removing out-of-control data, Figure 16 shows that there is no observation outside the control limits.

![Figure 16. $T^2$ Hotelling Control Chart in Phase I (in control)](image)

- **Phase II**

After carrying out the monitoring process in phase I, the next step is to control the mean process in phase II which aims to monitor the production process on the 21st to 40th production data. The upper control limits used in phase II is the same as phase I (in control), which is 13.56.
Figure 17. $T^2$ Hotelling Control Chart in Phase II

Figure 17 shows the process of monitoring sunergy glass production data in phase II. By using the upper control limit of 13.56, the conclusion obtained is that the phase II monitoring process is not statistically controlled because there are three uncontrolled data.

4.5. Performance Control Chart for Monitoring Mean Process in Short Runs Production

The monitoring of the mean process in phase I and phase II has been carried out previously using a short runs control chart with four tests and a $T^2$ Hotelling control chart. To find out the best control chart performance among the short-runs control charts with various tests and the $T^2$ Hotelling control chart, the ARL comparison is carried out as follows.

ARL comparison between the four tests carried out with a correlation coefficient value ($\rho$) = 0 or there is no relationship between quality characteristics is shown in Figure 18. The information obtained by the control chart used has an ARL$_0$ value (when there is no shift in the process mean) around 40, which means the 40th observation point is the first point out that is detected by the control chart. If the mean vector shifts by 0.6, the ARL$_1$ value for the 1-of-1 test is around 37, which means the control chart gives a signal that there has been a shift in the mean vector for the first time in the 37th observation. While the 3-of-3 test and the 4-of-5 test have ARL$_1$ values between 26 and 28, which means the control chart with the 3-of-3 test requires 26 observations and the 4-of-5 test requires observations, as many as 28 to state the process experienced a shift in the mean vector. The EWMA test shows an ARL$_1$ value of around 27, which means the 27th point of observation is the first point that is detected by the control chart with the EWMA test. The $T^2$ Hotelling control chart has an ARL$_1$ value of 40 which indicates that the 40th observation point is the first point that is detected by the $T^2$ Hotelling control chart. Based on the ARL
plot in Figure 13, in general, the 3-of-3 test produces $\text{ARL}_1$ which tends to be smaller than the other tests. This shows that the 3-of-3 test is most sensitive to detect shifts in the process mean.

Figure 19. ARL Comparison with $\rho = 0.3$

Figure 19 shows that with a small correlation coefficient ($\rho = 0.3$) or a weak relationship between quality characteristics, a short runs control chart with the 3-of-3 test results in a relatively small $\text{ARL}_1$ value compared to others. This shows that the short runs control chart with the 3-of-3 test is the most sensitive to detect a shift in the process mean.

Figure 20. ARL Comparison with $\rho = 0.8$

The information obtained from Figure 20 is the correlation coefficient value ($\rho = 0.8$) or there is a strong relationship between quality characteristics, $\text{ARL}_1$ generated by the short runs control chart with the 3-of-3 test tends to be smaller. This means that the short runs control chart with the 3-of-3 test is the most sensitive in detecting a shift in the process mean compared to other tests. Overall, it is concluded that the short runs control chart with the 3-of-3 test is the most sensitive test in detecting shifts in the process mean, both for data with strong, weak, or no relationship between quality characteristics.

5. Conclusion

In this paper, the sunergy glass is monitored using the short runs control chart and $T^2$-Hotelling control chart. First, in case monitoring sunergy glass production process, the short runs control chart with the EWMA test is the most sensitive test in detecting shifts in the process mean. This is because in phase II, the short runs control chart with the EWMA test has the highest number of out-of-control compared to
other diagrams. Furthermore, the performance of the short-run control chart with various tests and the $T^2$-Hotelling control chart results that the short runs control chart with the 3-of-3 test is the most sensitive test in detecting shifts in the process mean, both for data with strong, weak, or no relationship between quality characteristics. The control charts to monitor variability process such as in [8] [9] [10] [11], development of control charts to monitor mean process such as in [12] [13] [14] and non-normally short runs control chart such as in [15] can be considered for future work.

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