Sliding Mode Control for Hypersonic Vehicle based on Extreme Learning Machine Neural Network Disturbance Observer

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ABSTRACT The novel extreme learning machine (ELM) neural network disturbance observer (NNDO) - based sliding mode control (SMC) strategy is proposed for the precise tracking control of a hypersonic vehicle (HV) under various disturbance situations. By converting nonlinear dynamics into state-dependent linear model, the control law design process is simplified, and the sliding mode control law based on the power function reaching rate is designed to suppress the chattering effect. Considering the disturbances, the ELM-NNDO is designed based on the single-hidden layer feedforward network (SLFN). Different from conventional ELM using least square optimization approach, the output weight here is updated based on the Lyapunov synthesis approach. In addition, the influences of the disturbances on the velocity and altitude are attenuated by the direct feedback compensation (DFC), and the offset-free tracking control is realized for the output reference signal. Comparison of simulation results verify the superior control performance of the proposed method.

INDEX TERMS Extreme learning machine, hypersonic vehicle, neural network disturbance observer, offset-free tracking, sliding mode control;

I. INTRODUCTION

Due to the many potential applications in civilian and military, research on hypersonic vehicles (HVs) has received widespread attention [1][2]. During flight, the aerodynamic parameters of HVs change drastically and the flight environment changes continuously, hence the mathematical models have the characteristics of complexity, parameter uncertainty and nonlinearity [1][2]. In addition, due to the unique aerodynamic structure of HVs, the interaction between aerodynamics, propulsion system and structural dynamics is strong, making HVs sensitive to the uncertainty [1][2]. Therefore, for the HVs, the robustness of the control systems is very important in the controller design [3].

Focusing on the above-mentioned attractive features and control challenges, the design of the guidance and control systems for the longitudinal dynamics of HV has attracted a great deal of attention in recent years. In terms of robust and stable tracking, the advanced control methods are generally combined with other terms[4][5], and in terms of transient performance, prescribed performance control[6][7], optimal performance control[8], etc. have also received a lot of attention. These methods can obtain stable tracking performance in nominal condition or under small disturbances. However, for the HV, the disturbance it receives is unpredictable, therefore, there is a strong requirement for robustness.

As a nonlinear robust control method, the sliding mode control (SMC) can fully compensate for the matching uncertainties including unmodeled dynamics, parameter perturbations and external disturbances. In recent years, various SMC strategies have been used in the control of HVs [3][9][11][12]. These methods can obtain robust control performance under match interferences. However, the chattering phenomenon caused by the high-frequency control switching is a tough problem. The adaptive law [3], [9][11] and strategy for increasing relative order [12] were introduced to handle the chattering. Although the chattering phenomenon of SMC and the robust tracking performance...
are improved, the complexity is very high and it is not easy to realize. And how to design an easy-to-implement SMC method without chattering for HVs is still an urgent task.

Besides, the disturbances experienced by HVs include mismatched disturbances that SMC cannot handle. The method combing SMC with back-stepping procedure [13] was proposed to deal with mismatched uncertainties, but it requires tedious analytical computation of the time derivative of the virtual controller. Recently, to deal with the mismatched disturbances, a nonlinear disturbance observer (NDO)-based control provides a promising approach [14][15]. And based on dynamic inversion control, a sliding mode disturbance observer [16] was presented to handle the mismatched disturbances and parameter uncertainties.

On the other hand, neural approximation has been shown to be a powerful tool for improving the uncertainty decay capability of controllers for HVs [17][18]. Though good control performances are obtained, they are still the idea of back-stepping, and the design process is complicated. Considering the external disturbances and parameter uncertainties, an adaptive neural network disturbance observer [19] was proposed to suppress them. The method provides a satisfactory tracking performance, but the derivation of the adaptive law of input weighting and output weighting of neural network is too cumbersome. And a simple single-hidden layer feedforward network (SLFN) with radial basis feedforward (RBF) nodes whose parameters are adjusted based on extreme learning machine (ELM) was used for uncertainty approximation [20]. The ELM was proposed by Huang [21], and compared with traditional artificial intelligence techniques, ELM provides better generalization performance with faster learning speeds and less human intervention. Since it was proposed, it had received extensive attention [22][23], and it was proved to be a simple and effective approximation method for uncertainty.

Motivated by the preceding discussions, introducing ELM-based neural network disturbance observer (NDO) into robust controller design is an effective method for improving the disturbance rejection capability. In this paper, an easy-to-implement SMC based on ELM-NDO is proposed. The main contributions of this paper include the following:

1. A novel ELM-based NDO is proposed to estimate unknown disturbance and parameter uncertainties for a HV. The weighting of the input layer and hidden layer of the designed disturbance observer is generated arbitrarily, and the output weighting is solved adaptively without online training, which takes little time to ensure the real-time estimation of disturbances, and compared with the NDO methods, the disturbance estimation converges to the true value faster.

2. The sliding mode controller is designed directly based on the state space model without complex transformation of the model, which simplifies the design process, and eliminates chattering due to the introduction of the power function.

3. The proposed method does not need to design the compensation strategy, and directly compensate the control law through the set-point. Through direct feedback compensation, the offset-free control for the velocity and altitude is realized, which has been verified from theory and simulation.

The remainder of this paper is organized as follows. Section 2 describes the longitudinal model of a hypersonic vehicle. The state-space equation of state-dependent coefficients is formulated in Section 3. The ELM-based NDO is designed and the adaptive law is presented in Section 4. The sliding mode controller is designed, the closed-loop system stability and the offset-free tracking are proved in Section 5. The simulation result is illustrated in Section 6. Section 7 presents the conclusions and the next research program.

II. THE LONGITUDINAL MODEL OF A HYPERSONIC VEHICLE

The longitudinal dynamics of HV considered here are developed at NASA Langley Research Center [24]. The nonlinear equations of motion for velocity \( V \), flight-path angle \( \gamma \), altitude \( h \), angle of attack \( \alpha \), and pitch rate \( q \) are described by

\[
\begin{align*}
\dot{V} &= \frac{(T \cos \alpha - D)}{m} - \frac{\mu \sin \gamma}{r^2} + d_1 \\
\dot{\gamma} &= \frac{L + T \sin \alpha}{mV} - \frac{(\mu - V^2 r) \cos \gamma}{Vr^2} + d_2 \\
\dot{h} &= V \sin \gamma + d_3 \\
\dot{\alpha} &= q - \gamma + d_4 \\
\dot{q} &= \frac{M}{I_{yy}} + d_5
\end{align*}
\]

where \( m \), \( \mu \) denote the mass and the gravitational constant, and \( I_{yy} \) denotes the moment of inertia of the vehicle. The variables \( d_i (i = 1, \ldots, 5) \) denote the unknown external disturbances. \( L \), \( D \), \( T \) and \( M_{\gamma} \) are the lift, the drag, the thrust force and the pitching moment of the vehicle, and \( r \) denotes the radial distance from the earth’s center, which are modeled as

\[
\begin{align*}
L &= 0.5 \rho V^2 SC_L \\
D &= 0.5 \rho V^2 SC_D \\
T &= 0.5 \rho V^2 SC_T \\
M_{\gamma} &= 0.5 \rho V^2 SC\left[C_M(\alpha) + C_M(\delta_\gamma) + C_M(q)\right] \\
r &= h + R
\end{align*}
\]

where \( C_L \), \( C_D \) and \( C_T \) denote the lift, drag and thrust coefficients, and \( C_M(\alpha) \), \( C_M(\delta_\gamma) \) and \( C_M(q) \) denote moment coefficients due to the angle of attack, the elevator deflection and the pitch rate. \( \rho \), \( S \) and \( R \) represent the air density, the reference area of vehicle and the radius of the earth.
The aerodynamic coefficients are related to the flight conditions. Here, the cruising flight under nominal condition \( V = 15060 \text{ ft/s} \), \( h = 110000 \text{ ft} \), \( \gamma = 0 \text{ rad} \), \( q = 0 \text{ rad/s} \) is considered. The parametric uncertainties are modelled as additive perturbations \( \Delta \) with the nominal values \[3\] .

\[
C_L = 0.6203 \alpha \\
C_D = 0.645 \alpha^2 + 0.0043378 \alpha + 0.003772 \\
C_T = \begin{cases} 
0.02576 \beta & \beta < 1 \\
0.0224 + 0.00336 \beta & \beta > 1 
\end{cases} \\
C_M(\alpha) = -0.035 \alpha^2 + 0.036617(1 + \Delta C_{M\alpha}) \alpha + 5.3261 \times 10^{-6} \\
C_M(q) = \frac{c^2}{2V} q(-6.796 \alpha^2 + 0.3015 \alpha - 0.2289)
\]

where \( m_o \), \( I_o \), \( S_o \), \( c_o \), \( \mu_o \), \( \rho_o \) and \( R_o \) represent the nominal values of the parameters, \( \Delta m \), \( \Delta I \), \( \Delta S \), \( \Delta c \), \( \Delta \rho \), and \( \Delta C_{M\alpha} \) are the corresponding parameter uncertainties.

Ignoring disturbance of model (1), the nonlinear dynamics are linearized at the trimmed cruise condition \( V = 15060 \text{ ft/s}, \gamma = 0 \text{ deg}, h = 110000 \text{ ft}, \alpha = 0.0312 \text{ rad}, q = 0 \text{ rad/s}, \beta = 0.1762, \delta_e = -0.0069 \text{ rad}. \) It can be proved that the obtained linear system is controllable and observable. Its eigenvalues are \(-0.827, 0.7148, -0.00001 \pm 0.00251i, 8.4975 \times 10^{-16}\). Therefore, it can be seen that the first two eigenvalues represent statically unstable short-period modes, a pair of complex eigenvalues represent underdamped long-period modes, and the last real eigenvalue represents unstable height modes. As a result, cruising flight would be subject to attitude and height divergences, which would demand stabilizing control law. The control objective is to track a given reference signal about the velocity and altitude under the nonvanishing mismatched disturbances and parameter uncertainties.

### III. MODEL TRANSFORMATION

The nonlinear model (1) can be written as the following compact form

\[
\dot{x} = f(x, u) + \Delta f + d, \\
y = Cx
\]

where \( u = [\beta, \delta_e]^T \) is the control vector, \( x = [V, \gamma, h, \alpha, q]^T \) is the state vector, \( y = [V, \gamma]^T \) is the output vector, \( C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \) is the output matrix, \( d = [d_1, d_2, d_3, d_4, d_5]^T \) represents the external disturbance, and \( \Delta f \) denote the uncertainty caused by physical and aerodynamic parameter perturbations. Taking the parameter uncertainties and external disturbances as the lumped perturbation, the compact form (4) can be rewritten as

\[
\dot{x} = A(x) x + B(x) u + d, \\
y = Cx
\]

where \( d = \Delta f + d \) denotes the lumped disturbance.

The model (5) can be equivalently transformed into a state-space equation of state-dependent coefficients.

\[
\dot{x} = A(x) x + B(x) u + d, \\
y = Cx
\]

Here, \( Q = \frac{1}{2} \rho V^2 \) is the dynamic pressure. As stated in \( A(x) \) and \( B(x) \), some simplifications are made, namely \( \sin \gamma \approx \gamma, \beta < 1 \). It is reasonable to make the simplifications during the cruise phase because the \( \gamma \) is close to zero and the throttle of the engine is smaller than 1 in steady-state.

Remark 1: It should be noted that there are various forms of this pseudo-linear models, but it is necessary to ensure that the system (6) is controllable [25]. That is the matrices \( A(x), B(x) \) are not unique, but they must satisfy the controllable condition.

### IV. DESIGN OF ADAPTIVE NEURAL NETWORK DISTURBANCE OBSERVER BASED ON EXTREME LEARNING MACHINE

#### A. PRELIMINARY ON EXTREME LEARNING MACHINE

In this section, the basic idea of ELM is briefly reviewed to provide a background for designing disturbance estimation for the HV. ELM is a SLFN whose learning speed is much faster than conventional feedforward network learning algorithm, because the input weights and the parameters of the hidden layer do not need to adjust during the learning...
The output of the SLFN with $M$ nodes can be modeled as
\[
    f_M(x) = \sum_{i=1}^{M} \beta_i g(x, \omega_i, b_i), \quad x \in \mathbb{R}^n, \omega_i \in \mathbb{R}^n
\]  
where $\beta_i$ is the output weight connecting the $i$-th hidden node to the output node, $g(x, \omega_i, b_i)$ is the activation function of the $i$-th hidden node, $\omega_i$ and $b_i$ are the parameters of the activation function.

There are two kinds of activation function. For additive hidden nodes, the sigmoid activation function $g(x, \omega_i, b_i)$ is given as
\[
    g(x, \omega_i, b_i) = \frac{1}{1 + e^{-(x+b)}}
\]
where $\omega_i$ and $b_i$ are the input weight vector and the bias of the $i$-th hidden node, respectively.

For the RBF hidden nodes, the Gaussian activation function is used, i.e.,
\[
    g(x, \omega_i, b_i) = e^{-(x-b)^2/b_i^2}
\]
where $\omega_i$ and $b_i$ are the center and impact factor of the $i$-th RBF node, respectively.

Then, $N$ sample points $(x_i, y_i) \in \mathbb{R}^n \times \mathbb{R}^n (k = 1, 2, \ldots, N)$ are used to train the SLFN. If the SLFN with $M$ hidden nodes can approximate these $N$ samples with zeros error, there exist $\beta_i, \omega_i, b_i$ such that
\[
    \sum_{i=1}^{N} \beta_i g(x_i, \omega_i, b_i) = y_i, \quad k = 1, 2, \ldots, N
\]  
Equation (10) can be written as the compact form
\[
    \mathbf{H}\mathbf{\beta} = \mathbf{Y}
\]  
where\[
    \mathbf{H}(x, \omega, b) = \begin{bmatrix}
    g(x_1, \omega_1, b_1) & \cdots & g(x_1, \omega_M, b_M) \\
    \vdots & \ddots & \vdots \\
    g(x_N, \omega_1, b_1) & \cdots & g(x_N, \omega_M, b_M)
\end{bmatrix}_{N \times M}
\]
\[
    \mathbf{Y} = \begin{bmatrix} y_1 \cdots y_N \end{bmatrix}_{N \times 1}
\]

The ELM learning algorithm randomly generates the fixed parameters of the activation function, and only the weighting coefficient of the output layer needs to be calculated [26]. And to train an SLFN is simply equivalent to find a least-squares solution of $\mathbf{\beta}$ of the system (11), that is $\mathbf{\beta}$ can be obtained by the following equation [21]
\[
    \mathbf{\hat{\beta}} = \mathbf{H}^\dagger(x, \omega, b)\mathbf{Y}
\]
where, $\mathbf{H}^\dagger(x, \omega, b)$ is the Moore-Penrose generalized inverse of matrix $\mathbf{H}(x, \omega, b)$.

B. DESIGN OF ADAPTIVE NEURAL NETWORK DISTURBANCE OBSERVER
In order to estimate the disturbance of the system (6), the following adaptive neural network disturbance observer based on ELM is designed, that is
\[
    \ddot{z} = \rho(x-z) + A(x)x + B(x)u + \hat{d}
\]  
where, $z$ is the state of the disturbance observer, $\rho > 0$ is the designed gain. Here, a SLFN whose parameters are determined based on the ELM is employed to approximate the disturbance. The perturbation estimation is designed as
\[
    \hat{d} = \mathbf{\beta}^T h(\omega e_d + b)
\]
where, $e_d = x - z$ is the disturbance observation error, and is used as the input of the neural network. Here, $\omega = \begin{bmatrix} \omega_1 \omega_2 \cdots \omega_M \end{bmatrix}^T \in \mathbb{R}^{L \times n}$ ($L$ is the number of hidden layer nodes, and $n$ is the dimension of the input state), $b = \begin{bmatrix} b_1 b_2 \cdots b_L \end{bmatrix} \in \mathbb{R}^{L \times 1}$ are hidden node parameters which are assigned with random values and need not to be tuned. $\mathbf{\beta} \in \mathbb{R}^{L \times 1}$ represents the weight of the output layer, and the value will be calculated by the adaptive law described as (21). $\mathbf{h} \in \mathbb{R}^{L \times 1}$ represents the output of hidden layer. Assume that $\mathbf{\beta}$ is the $i$-th row vector of $\mathbf{\beta}$, activation function is also written in vector form $h(\omega e_d + b) = \begin{bmatrix} h_1 h_2 \cdots h_L \end{bmatrix}^T$, $h_s = 1/(1 + \exp(-(\omega e_d + b)))$.

According to the universal approximation theorem of a SFN [27], there exists the optimal output weight $\mathbf{\beta}^*$ to approximate the disturbance, so we can yield,$d^* = \mathbf{\beta}^* h(\omega^* e_d + b) + \tau(e_d)$
where $\tau(e_d)$ is the approximation error and bounded [28], namely
\[
    |\tau(e_d)| \leq \tau_N
\]
where $\tau_N$ is a determined constant.

From (14) and (15), we obtain
\[
    \hat{d} = \hat{d} - d
\]
where $d = d - \hat{d}$ is the disturbance estimation error.

Remark 2: With the adaptive law designed below, the disturbance estimation error $\hat{d}$ will asymptotically approach to 0, which will be proved in Section 5.4. And in the following, for the convenience of writing, $h(\omega e_d + b)$ is abbreviated as $\mathbf{h}$.

C. ADAPTIVE LAW AND STABILITY ANALYSIS
The stable adaptive law for $\mathbf{\beta}$ is derived using the Lyapunov second method.

Considering the following Lyapunov function
\[
    V = \frac{1}{2} e_d^T e_d + \sum_{i=1}^{L} \frac{1}{2} \beta_i \beta_i^T
\]
where $\eta_i$ is a positive constant referred to as the learning rate of the SLFN.

The derivative of the Lyapunov function is
\[ V = e_d \dot{e}_d + \sum_{i=1}^{n} \frac{1}{\eta_i} \beta_i \dot{\beta}_i = e_d (-\rho e_d + \dd) + \sum_{i=1}^{n} \frac{1}{\eta_i} \beta_i \dot{\beta}_i \]

\[ = -\rho \|e_d\| + e_d \dot{e}_d + \sum_{i=1}^{n} \frac{1}{\eta_i} \beta_i \dot{\beta}_i \]

\[ = -\rho \|e_d\| + e_d \dot{e}_d + e_d \dot{\tau}(e_d) + \sum_{i=1}^{n} \frac{1}{\eta_i} \beta_i \dot{\beta}_i \]

Considering the bound information \( e_d \leq \eta_i e_d \), it is deduced that

\[ V \leq (-\rho + \tau_N) \|e_d\| \]

The second item \( V \) can be eliminated if we let

\[ \dot{\beta}_i = \eta_i e_d \]

Hence

\[ \dot{\beta}_i = \beta_i - \dot{\beta}_i = \eta_i e_d \]

Then, we obtain

\[ V \leq (-\rho + \tau_N) \|e_d\| \]

Remark: If the disturbance estimation \( \tilde{d} \) is accurate enough, the disturbance estimation error \( \tilde{d} \) is very small and can be ignored. In this article, the disturbance is estimated by the ELM-based NNDO, and it will be proved in Section 5.4 that the disturbance estimation error \( \tilde{d} \) asymptotically converges to 0. Therefore, it is completely reasonable to ignore the estimation error to design the controller.

**V. SLIDING MODE CONTROLLER DESIGN FOR OFFSET-FREE TRACKING**

**A. ESTABLISHMENT OF THE TRANSLATION STATE EQUATION**

First, the output \( y \) is required to track the reference command \( \dot{y} \) by moving the system to the set-points \( x_s, u_s \).

When the system reaches the steady state, the equations (25) are satisfied:

\[ A(x_s)x_s + B(x_s)u_s + d_s = 0 \]

\[ Cx_s = \dot{y} \]

For the hypersonic vehicle (4), the number of inputs is equal to the number of outputs, the steady state can be calculated by the following:

\[ \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} A(x_s) & B(x_s) \end{bmatrix}^{-1} \begin{bmatrix} -d_s \\ y \end{bmatrix} \]  

Remark 3: For formula (26), the matrices \( A(x_s), B(x_s) \) are relevant to the steady state of \( x_s \), we use \( A(x), B(x) \) to approximate them. And the lumped perturbation \( d_s \) is unknown. Hence, to calculate the set-points \( x_s, u_s \), the disturbance estimation \( \tilde{d} \) is used to replace it. Then the set-points \( x_s, u_s \) can be calculated by

\[ \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \end{bmatrix}^{-1} \begin{bmatrix} -d_s \\ y \end{bmatrix} \]  

Define \( w = x - x_s, \ v = u - u_s \), then from (6) and (27), we obtain

\[ \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \end{bmatrix} \begin{bmatrix} v_d \\ y \end{bmatrix} \]

\[ y = C(w + x_s) \]

where, \( d \) is the disturbance estimation error, and it is ignoring when designing the SMC law.

**B. THE DESIGN OF THE SLIDING MODE FUNCTION**

Ignoring \( d \) in equation (28), we obtained

\[ \tilde{w} = A(x)w + B(x)v + \tilde{d} \]

Perform a nonsingular linear transformation on system (29):

\[ \tilde{w} = Tw \]

The equation (29) is transformed into the following form:

\[ \tilde{w} = \tilde{A}(x)w + B(x)v \]

where \( \tilde{w} = \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix}, \tilde{A}(x) = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \tilde{B}(x) = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \)

Equation (31) can be written as follows:

\[ \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} + \tilde{B}(x)v \]

Design the linear sliding mode function:

\[ s = \sigma \tilde{w} = \sigma_1 \tilde{w}_1 + \sigma_2 \tilde{w}_2 \]

Then, on the sliding mode surface,

\[ s = 0 \]

In the condition that the \( \sigma \) is invertible, we obtain

\[ \tilde{w}_2 = -\sigma_2^{-1} \sigma_1 \tilde{w}_1 \]

Hence, the first formula of state space model (32) is

\[ \tilde{w}_1 = \left( \tilde{A}_{11} - \tilde{A}_{12} \sigma_2^{-1} \sigma_1 \right) \tilde{w}_1 = \Omega \tilde{w}_1 \]

In the (36), by designing \( \sigma \) so that the pole of \( \Omega \) is in the left half of the s-plane, the sliding mode function (33) is stable.

**C. THE SLIDING MODE CONTROLLER DESIGN**

To suppress the chattering phenomenon of SMC, the reaching law based on power function [29] is selected:

\[ \dot{s} = -qs - \varepsilon \text{fal}(s, \alpha, \delta) \]

where, \( q > 0, \varepsilon > 0 \), \text{fal}(s, \alpha, \delta) \) is a power function, and its expression is
\[
\text{fal}(s, \alpha, \delta) = \begin{cases} 
|s|^{\alpha} \ \text{sgn}(s), & |s| \geq \delta \\
\frac{s}{|s|^{1-\alpha}}, & |s| < \delta 
\end{cases} 
\] (38)

where, \(0 < \alpha < 1, \ 0 < \delta < 1\).

From (29), (30) and (33), we obtain
\[
\dot{s} = \sigma \dot{w} = \sigma T \dot{w} = \sigma T A(x)w + \sigma \dot{TB}(x)w 
\] (39)

Then, from (37) and (39), we obtain the sliding mode control law
\[
v = (\sigma \dot{TB}(x))^{-1}( -\dot{q}s - \text{fal}(s, \alpha, \delta) - \sigma T A(x)w ) 
\] (40)

In summary, the control law design process is summarized as follows:

1. Design an ELM-based neural network disturbance observer as shown in (13), where the disturbance estimate \( \hat{d} \) is calculated as (14), and the output weighting coefficient of the neural network is updated by the adaptive law as in (23).
2. Calculate the translation set point \((x_u, u)\) according to (27).
3. Design the sliding mode control law (40) based on model (29).
4. Then, the control law applied to the actual hypersonic nonlinear model (4) is
\[
u = v + u_q 
\] (41)

Remark 5: As can be seen from the formulas (27), (40) and (41) that we do not need to design the compensation strategy, but directly compensate the control law through the set-point \((x_u, u)\). Therefore, we call it direct feedback compensation (DFC).

D. THE OFFSET-FREE TRACKING FEATURES

Theorem 1: For the disturbed nonlinear system (4), if the disturbance is a slowly time-varying signal, the designed control law (41) can ensure that the output can track the given reference signal \( y \), without static error.

The proof process requires two steps:

Step1: prove that when \( t \to \infty \), the disturbance observation error \( e_d \to 0 \) and the disturbance estimation error \( \hat{d} \to 0 \).

Proof: From (14), we obtain
\[
\dot{\hat{d}} = \dot{\beta}h + \dot{\hat{\beta}}h 
\] (42)

The first item of (42) can be expanded into the following
\[
\dot{\beta}h = \begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \vdots \\ \dot{\beta}_{n_L} \end{bmatrix} h_{L=1} 
\] (43)

Substitute (23) into (43) and perform the equivalent transformation, we have
\[
\beta \hat{h} = \begin{bmatrix} \dot{\beta}_1 \\ \dot{\beta}_2 \\ \vdots \\ \dot{\beta}_{n_L} \end{bmatrix} \begin{bmatrix} h_{L=1} 
\end{bmatrix} 
\] (44)

For the second item of (42),
\[
\dot{\hat{\beta}}h = \begin{bmatrix} \dot{\hat{\beta}}_1 \\ \dot{\hat{\beta}}_2 \\ \vdots \\ \dot{\hat{\beta}}_{n_L} \end{bmatrix} \begin{bmatrix} e_{d1} \\ e_{d2} \\ \vdots \\ e_{dn} \end{bmatrix} = \begin{bmatrix} \hat{\beta}_{m+1} \\ \hat{\beta}_{m+2} \\ \vdots \\ \hat{\beta}_{n_L} \end{bmatrix} \begin{bmatrix} e_{d1} \\ e_{d2} \\ \vdots \\ e_{dn} \end{bmatrix} 
\] (45)

Let \( c_i = \frac{e^{-\omega_i^2}}{1 + e^{-\omega_i^2}} \), then \( 0 < c_i < 1 \), and the formula (45) will be
\[
\dot{\hat{d}} = \begin{bmatrix} \dot{\hat{\beta}}_1 \\ \dot{\hat{\beta}}_2 \\ \vdots \\ \dot{\hat{\beta}}_{n_L} \end{bmatrix} \begin{bmatrix} e_{d1} \\ e_{d2} \\ \vdots \\ e_{dn} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{L} c_j \hat{\beta}_j \omega_j \end{bmatrix} e_{d} \begin{bmatrix} e_{d1} \\ e_{d2} \\ \vdots \\ e_{dn} \end{bmatrix} 
\] (46)

where, \( H = \begin{bmatrix} \sum_{j=1}^{L} c_j \hat{\beta}_j \omega_j \\ \sum_{j=1}^{L} c_j \hat{\beta}_j \omega_j \\ \vdots \\ \sum_{j=1}^{L} c_j \hat{\beta}_j \omega_j \end{bmatrix} \).

Hence,
\[
\dot{\hat{d}} = H e_d 
\] (47)

The disturbance is a slowly time-varying signal, that is \( \dot{d} = 0 \), then \( \hat{\dot{d}} = \dot{\hat{d}} = 0 \).

Combining (18) and (47), we get...
\[
\begin{bmatrix}
\dot{e}_d \\
\dot{a}
\end{bmatrix} =
\begin{bmatrix}
-\rho I & I \\
H \rho - h^T \eta & -H
\end{bmatrix}
\begin{bmatrix}
e_d \\
a
\end{bmatrix}
\quad (48)
\]

To prove the stability of system (48), Gershgorin circle theorem [30] needs to be used.

The Gershgorin circle theorem states that the eigenvalues \( \lambda \) of a squared matrix \( A \) with \( M \times M \) dimension and entries \( a_{ij} \) belong to the union of the \( M \) Gershgorin disks, that is to say
\[
\lambda \in \bigcup_{i=1}^{M} D_i
\quad (49)
\]
where each Gershgorin disk \( D_i \) is defined on the complex plane as follows:
\[
|\lambda - a_{ii}| \leq \sum_{j \neq i} \left| a_{ij} \right|
\quad (50)
\]
For system (48), we discuss its system matrix
\[
\begin{bmatrix}
-\rho I & I \\
H \rho - h^T \eta & -H
\end{bmatrix}
\quad (51)
\]
The eigenvalues corresponding to the first five rows of the matrix (51) are in the \( s \)-left half plane, because the centers of the Gershgorin disk are \(-\rho\), and the radius of the Gershgorin disk are 1. And for the other five rows, the eigenvalues of the matrix can be guaranteed to be in the left half-plane by designing the parameters \( \rho \) and \( \eta \). This is achievable because the designed parameter values of each interference channel are independent of each other, which can be verified in the simulations.

Hence, system (48) is asymptotically stable, i.e. the disturbance observation error \( e_d \rightarrow 0 \) and the disturbance estimation error \( \dot{a} \rightarrow 0 \).

Step 2: Prove that the output can track a given reference signal without static error.

Substitute (28) into (28), we obtain
\[
\hat{w} = A(x)w + B(x)(\sigma TB(x))^{-1} [\dot{q}s - \varepsilon f \dot{a}(s, \alpha, \delta) - \sigma TA(x)w] + \hat{d}
\quad (52)
\]
When the system reaches a steady state, the sliding surface satisfies the condition, \( |\dot{\hat{s}}| < \delta \), and the disturbance estimation error \( \dot{\hat{d}} \rightarrow 0 \), hence, the (52) will be
\[
\dot{\hat{w}} = A(x)w + B(x)(\sigma TB(x))^{-1} [-q\sigma Tw - \varepsilon \frac{\sigma Tw}{\delta^{a-1}} - \sigma TA(x)w]
\quad (53)
\]
Suppose the steady state of the system (53) is \( \hat{w}_s \), then (53) can be equivalently transformed into
\[
\left[ A(x) - B(x)(\sigma TB(x))^{-1} [-q\sigma T - \varepsilon \frac{\sigma T}{\delta^{a-1}} - \sigma TA(x)] \right] \hat{w}_s = 0
\quad (54)
\]
Obviously, from the arrival condition of sliding mode surface in the sliding mode control law design process, it is easily known that the matrix \( A(x) - B(x)(\sigma TB(x))^{-1} [-q\sigma T - \varepsilon \frac{\sigma T}{\delta^{a-1}} - \sigma TA(x)] \) is full rank. Therefore, the only solution of the equation (54) is \( \hat{w}_s = 0 \). That is, the steady state of the system is \( \hat{w}_s = 0 \).

From (28), the system output at steady state is \( \hat{w}_s = 0 \).

\[
y = C(w_s + x_s) = Cx_s
\quad (55)
\]
From (27), apply the inverse matrix formula
\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}^{-1}
= \begin{bmatrix}
A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\
-(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1}
\end{bmatrix}
\quad (56)
\]
We can obtain
\[
x_s = (A(x)^{-1}B(x)(CA(x)^{-1}B(x))^{-1}CA(x)^{-1} - A(x)^{-1})\dot{d} + A(x)^{-1}B(x)(CA(x)^{-1}B(x))^{-1}y,
\quad (57)
\]
Substituting (57) into (55), we get
\[
y = \dot{x}_s
\quad (58)
\]
\[
y = C(\dot{A}_s - CA_s^{-1})\dot{d} + y,
\quad (59)
\]
\[
y = y,
\quad (60)
\]
Therefore, the output can achieve static error-free tracking of the reference signal.

**VI. SIMULATION RESULTS AND ANALYSIS**

To illustrate the effectiveness of the proposed power function approach rate SMC (abbreviated as PSMC) and the ELMNNDO, the NDO [31] combined with the PSMC and the exponential reach rate sliding mode control [32] (abbreviated as ESMC) combined with ELMNNDO are employed for comparison.

In the simulations, given the initial velocity of 15060ft/s and altitude of 110000ft, respectively, the reference step signal of velocity is \( V_i = 100 \) ft/s, and the reference step signal of altitude is \( h_i = 100 \) ft. The parameters of the PSMC are selected as \( \varepsilon = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix} \), coefficients of a power function \( \alpha = 0.5, \delta = 0.1 \), and for the ESMC, \( \varepsilon \) is set as \( \varepsilon = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \). The adopted ELM parameters are chosen as follows: \( n = 5 \), \( L = 11 \), the hidden node parameters \((\alpha, b)\) are chosen randomly in the intervals \([0,1]\), and the observer gain is set as \( \rho = 50I \). The learning rates of the SLFN are taken as \( \eta_1 = 500, \eta_2 = \eta_3 = \eta_4 = 1000, \eta_5 = 2000 \). The simulation step size is set as 0.01 and the Fourth-order Runge–Kutta method is used to simulate the nonlinear dynamic of HV (1).
A. SIMULATION WITH PARAMETER UNCERTAINITIES

To test the robustness of the proposed method, the parameter uncertainties as mentioned in Section 2 are considered.

First, the positive maximum values, i.e., $\Delta = 0.25$ are considered. The comparison curves are shown in Fig.1-2. It is seen from Fig.1 that the velocity and the altitude can track the given reference signals accurately under the three methods because of the DFC strategy, which verifies that the proposed method can achieve offset-free control of the output. It also can be seen from Fig.1(b) that the response time for height channel of ESMC is slower than the PSMC. The reason is that in order to reduce the chattering of the control, the parameter $\varepsilon$ of the ESMC needs to set smaller.

As can be seen from Fig.2(b) that there is no chattering phenomenon with the PSMC while ESMC has chattering phenomenon, and the required amplitude of the elevator is larger with the NDO method. The root locus of matrix (51) is shown in Fig.3. It can be seen that the roots of the system are all in the left half s-plane, indicating that the designed parameters can ensure the NNDO is stable and the estimation error of the disturbance observer is asymptotically approaching to 0.

![FIGURE 1. Tracking curve of output under positive parameter uncertainties.(a) Velocity.(b) Altitude](image1)

![FIGURE 2. Control signal.(a) Throttle setting.(b) Elevator deflection](image2)

![FIGURE 3. Eigenvalues locus of matrix (51) under positive parameter uncertainties. (a) Trajectories of eigenvalues in the complex plane.(b) Trajectory of the real part of the eigenvalue](image3)
Then, the uncertainties parameters are set as the negative maximum values, i.e., \( \Delta = -0.25 \). The comparison curves are shown in Fig. 4-5. In this circumstance, the velocity and the altitude can achieve accurate tracking of the reference signal with the three methods, and the response time for height channel of ESMC is also slower than the PSMC. From Fig. 5, the ESMC has chattering phenomenon, and the amplitude of elevator is also larger with the NDO method. From Fig. 6, it can also be seen that the roots are located in the left half s-plane.

**FIGURE 4.** Tracking curve of output under negative parameter uncertainties. (a) Velocity. (b) Altitude

**FIGURE 5.** Control signal. (a) Throttle setting. (b) Elevator deflection

**FIGURE 6.** Eigenvalues locus of matrix \((51)\) under negative parameter uncertainties. (a) Trajectories of eigenvalues in the complex plane. (b) Trajectory of the real part of the eigenvalue

**B. SIMULATION WITH PERSISTENT EXTERNAL DISTURBANCES**

In this simulation, refer to the relevant literature [14]-[16] and combine the proposed method, the unknown external persistent disturbances are considered and set as \( d_1 = -5, d_3 = 10 \) at \( t \geq 20 \), and \( d_2 = 0.001, d_4 = 0.05, d_5 = 0.08 \) at \( t \geq 30 \). The comparison curves of the three methods are shown in Fig. 7-9. It is seen from Fig. 7 that due to the effect of the DFC strategy, the velocity and the altitude can accurately track the reference signals with the three methods, which also verifies the proposed method can...
achieve no static error control of the output. From Fig.8 that the ESMC also has chattering phenomenon and the PSMC is free from chattering, which once again verifies the superiority of the proposed method. And it also can be seen when subjected to external disturbances, the proposed method converges to given value faster in velocity mode, and the overshoot of the altitude mode is smaller. As can be seen from Fig.9 that both observers can achieve accurate estimation of the actual interference signal, which verifies the convergence characteristics of the disturbance. And from the partial enlarged image, it is also seen that the proposed method converges faster. From Fig.10, in these circumstances, the root locus is also located in the left s-plane.
FIGURE 9. Actual disturbances and disturbance estimates. (a) Velocity loop. (b) Flight-path angle loop. (c) Altitude loop. (d) Angle of attack loop. (e) Pitch rate loop

FIGURE 10. Eigenvalues locus of matrix (51) under external disturbance. (a) Trajectories of eigenvalues in the complex plane. (b) Trajectory of the real part of the eigenvalue

VII. CONCLUSION

In this article, a novel SMC scheme combined with ELM-based NNDO is proposed to realize the disturbance suppression control of HV. The designed power function reaching law SMC is based on a state-dependent linear model, which not only simplifies the design process, but also eliminates chattering by using a power function instead of a sign function. The ELM-based NNDO can accurately estimate the unknown interference signal by designing the appropriate adaptive law and the learning rate. Moreover, the adopted DFC strategy can completely eliminate the influence of interference on the output, which has been verified in theory and simulation. On the basis of the obtained results, designing controllers for more complex models such as flexible dynamics, six degrees of freedom model, is our next research program.

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