Measuring Visual Complexity of Cluster-Based Visualizations

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Abstract

Handling visual complexity is a challenging problem in visualization owing to the subjectiveness of its definition and the difficulty in devising generalizable quantitative metrics. In this paper we address this challenge by measuring the visual complexity of two common forms of cluster-based visualizations: scatter plots and parallel coordinates. We conceptualize visual complexity as a form of visual uncertainty, which is a measure of the degree of difficulty for humans to interpret a visual representation correctly. We propose an algorithm for estimating visual complexity for the aforementioned visualizations using Allen’s interval algebra. We first establish a set of primitive 2-cluster cases in scatter plots and another set for parallel coordinates based on symmetric isomorphism. We confirm that both are the minimal sets and verify the correctness of their members computationally. We score the uncertainty of each primitive case based on its topological properties, including the existence of overlapping regions, splitting regions and meeting points or edges. We compare a few optional scoring schemes against a set of subjective scores by humans, and identify the one that is the most consistent with the subjective scores. Finally, we extend the 2-cluster measure to k-cluster measure as a general purpose estimator of visual complexity for these two forms of cluster-based visualization.

1. Introduction

Visual complexity is a pervasive problem in different domains such as graphical user interfaces, web information, visualizations, etc. While the correlation between visual complexity and cognitive load [HMS09] has been established, it is widely acknowledged that one of the main challenges is to provide an objective definition such that it bridges system-level behavior with user perception [SBSÇ10]. The subjectiveness of this notion makes it difficult to develop reliable metrics for measuring visual complexity.

The focus of this paper is to measure visual complexity in cluster visualization. We examine two forms of such visualization, namely scatter plots and parallel coordinates [ID90]. Here we define visual complexity as a form of visual uncertainty [DCK12]. It measures visual components, such as overlapped points, lines and shapes, missing objects, and split or disconnected shapes, that may lead to confusion in viewing the visualization. Our contributions are:

- We define two metrics for estimating visual complexity in scatter plots and parallel coordinates respectively and we make use of look-up tables in a manner similar to the marching cubes algorithm [LC87].
- We compare the scores of the two metrics with a set of subjective scores by humans, and confirm the two metrics are effective.

2. Related Work

In this section we discuss the relevant literature on clustered parallel coordinates and scatter plots, and on concepts of and metrics for visual complexity.

2.1. Clustered Scatter Plots and Parallel Coordinates

Traditional clustering techniques in visualization are of two main types: analytical clustering and visual clustering. Analytical clustering aims to maximize within cluster information by using data-space properties [FWR99, NH06]. Two-dimensional clusters that tend to overlap between axes [AA04] add to the visual complexity, but computational approaches to quantify that has been absent in the literature. Visual clustering in parallel coordinates aims to reduce clutter;
some examples include geometrically deforming and grouping poly-lines to overcome edge clutter [ZYQ+08] and use of high-precision textures for reducing the effect of overplotting [JLJC05]. Privacy-preserving clustering [DK11] encompasses these two categories, the goal here being controlling the within-cluster information to prevent disclosure. Quantifying complexity in terms of uncertainty measures have been found to be useful in quantifying the utility of privacy-preserving visualizations [withheld].

2.2. Visual Complexity Measures

Rosenholtz et al. [RLN07] describe a number of methods for measuring visual clutter and complexity based on the ideas of feature congestion and reaction time. They highlight the current state of the art for measuring visual complexity falls into two categories. Simplistic measures of visual complexity based on counting geometric primitives such as lines and triangles, and complex measures based on computer vision techniques. However, these methods have a number of drawbacks. The simplistic methods are generally used for visualization displays in two and three dimensions, making them dependant on the input data. In addition there is only a weak correlation between the number of primitives in the display and the complexity of the visual output [CDD06, SSD+08, KW10, DCT12]. The complex methods are computationally intense and not appropriate for visualization displays when access to raw geometric data is available. In general there is a lack of tools for measuring visual complexity in visualization applications that quantifies overlap and occlusion. Simplistic methods such as, counting geometric primitives, i.e., vertices, lines etc. have been used in visualization applications. This method has been applied recently by Carr et al. [CDD06], Scheidegger et al. [SSD+08] and Duffy et al. [DCT12] for measuring the complexity of isosurfaces through triangle counts and cell intersections. Khoury et al. [KW10] use fractal box dimensions to measure isosurface complexity. More complex computer vision methods are the alternative as illustrated by Rosenholtz et al. [RLN07].

2.3. Related Approaches: Clutter and Visual Quality

Clutter reduction techniques are important in the context of information visualization as they visual quality preserving rendering. Ellis and Dix have outlined in their taxonomy [ED07] how the different clutter reduction approaches fit in a common framework. There is a lack of agreeable definition of clutter [ED06] and visual quality [BS06]. While there have been approaches to define clutter in terms of outliers [PWR04], other researcher have defined clutter in terms of overlapping visual objects [AdOL04, DK10]. Similarly with visual quality, while quality metrics have been proposed to improve the perceptual aspect of visualizations, similar metrics have been suggested for pattern identification. We believe a decomposition of visualization in terms of its smallest components, that is, the visual structures will enable us to standardize metrics across different visual representations. Moreover, quantification of complexity will also enable more concrete optimization processes that can minimize clutter on screen.

3. Allen’s Interval Algebra

Allen developed an interval algebra in 1983 for reasoning about discrete time intervals [All83]. As shown Figure 1, the algebra defines a set of 13 operators on two interval operands in 1D. It is not difficult to observe that the operators exhibit some symmetry in relation to the ordering of the two operands. For the convenience mathematical representation, let us write each operation in a functional form akin to the Polish prefix notation:

\[ g_i(A, B), \quad i = 1, 2, \ldots, 13 \]

where \( g_i \) is an operator (i.e., \( g_1 \) is \(<\), \( g_2 \) is \(>\), etc.), and \( A \) and \( B \) are the two interval operands, \([a_1, a_2]\) and \([b_1, b_2]\), such that \( a_1 < a_2 \) and \( b_1 < b_2 \). Note that the function \( g_i \) can be regarded as a Boolean function that determines whether...
Operand Ordering Symmetry can thus be expressed as:

\[ \Psi_{OOS}(g_i(A, B)) \rightarrow g_j(B, A), \ 1 \leq i, j \leq 13 \] (1)

where \( \Psi_{OOS} \) is the transformation of swapping the two operands for a given \( g_i(A, B) \). A symmetric relation holds if \( g_j \) exists. There are seven pairs of such symmetry, including the self-symmetry \( g_1(A, B) = g_1(B, A) \).

Another form of symmetry results from flipping an axis towards the opposite direction. In 1D case, given an interval \( X = [x_1, x_2] \), we denote its mirror on the flipped axis as \( X^− = [−x_2, −x_1] \). Hence, the Axis Flipping Symmetry can be expressed as:

\[ \Psi_{AFS}(g_i(A, B)) \rightarrow g_j(A^−, B^−), \ 1 \leq i, j \leq 13 \] (2)

where \( \Psi_{AFS} \) is the transformation of flipping the axis. There are seven pairs of such symmetry, including \( g_8(A, B) = g_{12}(A^−, B^−) \) and \( g_9(A, B) = g_{13}(A^−, B^−) \).

With these two types of symmetry, we can reduce the 13 cases to 6 primitive cases, which are \( g_1, g_2, g_4, g_8, g_{10} (=, <, m, o, s, d) \). Each of the other 7 cases can be inferred from a primitive case using one of the two symmetry relations.

### 4. 2-Cluster Overlaps

Allen’s interval algebra can be extended to 2D when examining cases in two common forms of cluster visualization: namely scatter plots and parallel coordinates. In previous work Dasgupta and Kosara [DK10] used Allen’s algebra for computing metrics for parallel coordinates. Figure 2 shows a simple case of two overlapping clusters in a scatter plot as well as a parallel coordinate. The relationship on the x-axis is \( A \bowtie B \) or \( g_5(A, B) \), and that on the y-axis is \( A \bowtie B \) or \( g_6(A, B) \). We can represent this case by the following 2-tuple:

\[ [g_5(A, B), g_6(A, B)] \]

It is not difficult to observe that given an ordered pair of operands, there are \( 13 \times 13 = 169 \) different tuples in 2D.

### 4.1. Symmetries in 2D and Primitive Cases

Using symmetry relationships, we have found that the 169 cases can be reduced to 24 primitive cases in scatter plots, and 35 primitive cases in parallel coordinates. The symmetry relationships shared by both types of plots are:

#### 2D Operand Ordering Symmetry

This is a direct extrapolation from the same type of symmetry in 1D. Let \( A \) and \( B \) be two clusters, their ranges on the x-axis are \( A_x \) and \( B_x \), and those on the y-axis are \( A_y \) and \( B_y \) respectively. We can express this symmetry in 2D using a transformation \( \Psi_{2d-OOS} \) as:

\[ \Psi_{2d-OOS}\left(\left[g_i(A_x, B_x), g_i(A_y, B_y)\right]\right) \rightarrow \left[\Psi_{OOS}(g_i(A_x, B_x)), \Psi_{OOS}(g_i(A_y, B_y))\right] \] (3)

where \( 1 \leq i, j, s, t \leq 13 \), and \( \Psi_{OOS} \) is the corresponding 1D transformation as \( B \) before and after the symmetric transformation, and similarly \( g_s \) and \( g_t \) for the y-axis.

#### Axes Ordering Symmetry

With scatter plots, if one flips either of the two axes individually, it does not change the topology or amount of overlapping between the two clusters, and thereby has limited impact on the perception of the visual complexity. On the contrary, flipping only one axis may cause a change of overlapping relationship in a parallel coordinates. Given two non-overlapping clusters, they would become overlapped after one of the two axes is flipped. Hence the following symmetry applies only to scatter plots.

#### Asynchronous Axis Flipping Symmetry

We use \( \Psi_{AFS} \) to denote the transformation of flipping the x-axis, and \( \Psi_{AFS} \) for that of the y-axis. Similar to \( \Psi_{SAFS} \), these two
Figure 3: 169 scatter plot cases can be reduced to a subset of 24 topologically distinct cases using 4 symmetries.

Figure 4: The $13 \times 13$ cases of 2D Allen’s interval algebra. It shows 24 primitive cases for scatter plots as numbered in Figure 3, and a transformation path from each of other cases to one of the primitive cases.

transformations can be expressed as follows:

$$\Psi_{AFS}(g_i(A_x, B_x), g_j(A_y, B_y))$$

$$\rightarrow [g_j(A_y \rightarrow_B B_x), g_i(A_y, B_y)]$$

Using any above transformation, they are said to be topologically isomorphic. Since it is relatively trivial to prove that all above-mentioned transformations are communicative, such an isomorphism is symmetric. When a number of cases form an isomorphic group, where each case can be transformed to another through one or more transformations. For each isomorphic group, we can select one case as the primitive case. Figure 3 shows 24 primitive cases of Allen’s interval algebra in 2D for scatter plots. Figure 4 illustrates some of the symmetric transformations that lead to the formation of these 24 isomorphic groups. Figure 5 shows 35 primitives.
cases for parallel coordinates, while Figure 6 illustrates the formation of the isomorphic groups.

4.2. Computational Verification of the Primitive Cases

We established the isomorphic groups using two different methods. Firstly we used the matrices in Figure 3 and Figure 4 as exhaustive lists of all cases in the two types of plots respective. We sketched out many cases to identify symmetric transformation from one another. Secondly, we enumerated all possible symmetric transformations computationally, providing a verification of the isomorphic groups found manually. The algorithm demonstrates the establishment of isomorphic groups in 1D. Consider the list of 13 cases, each with an operator $g_i$, as in Figure 1. Procedure 1 exhaustively visits each non-isomorphic case, and applies the rules in the rule set, $\{\Psi_{OOS}, \Psi_{AFS}, \Psi_{OOS} \circ \Psi_{AFS}\}$ based on Equations 1 and 2, where $\circ$ denotes the applications of

Procedure 1 Exhaustive isomorphic elimination in 1D.

1: procedure VERIFY1D
2: $F[1..13] \leftarrow 0$  \hspace{1em} \triangleright initialize all non-isomorphic
3: for each $g_i$ \in Operator Set do
4: \hspace{1em} if $F[i] = 0$ then
5: \hspace{2em} for each rule $\Psi$ \in Rule Set do
6: \hspace{3em} $[h, U, V] \leftarrow \Psi(g_i, A_i, B_i)$  \hspace{1em} \triangleright transform
7: \hspace{3em} for each $k \in [1..13] \land k \neq i$ do
8: \hspace{4em} if $F[k] = 0 \land EQ([h, U, V], [g_k, A_k, B_k])$ then
9: \hspace{5em} $F[k] \leftarrow i$  \hspace{1em} \triangleright set isomorphic link
10: \hspace{3em} end if
11: \hspace{2em} end for
12: \hspace{1em} end for
13: \hspace{1em} end if
14: \hspace{1em} end for
15: end procedure
two rules (right first). If the application of a rule to \(g_i(A_k, B_l)\) resulting in \(h(U, V)\) that is topologically equitant to another case \(g_k\), then \(g_k\) is an isomorphic with \(g_i\) and \(g_k\) is eliminated for further consideration.

Procedure 2 shows an algorithm that exhaustively searches isomorphic group in 2D for scatter plots and parallel coordinates. The rule set for parallel coordinates is based on Equations 3, 4 and 5, resulting in \(\Psi_{2d-oos}, \Psi_{2d-oos} \circ \Psi_{AFS}, \Psi_{AFS} \circ \Psi_{AO}, \Psi_{2d-oos} \circ \Psi_{AFS} \circ \Psi_{AO}, \Psi_{2d-oos} \circ \Psi_{AFS} \circ \Psi_{AO}\). These rules are also used for scatter plots. However to achieve full reduction, further rules are required based on Equations 6 and 7, including \(\Psi_{AXAFS}, \Psi_{AYAFS}, \Psi_{AXAFS} \circ \Psi_{AYAFS}, \Psi_{AXAFS} \circ \Psi_{AYAFS}, \Psi_{AXAFS} \circ \Psi_{AO}, \Psi_{AXAFS} \circ \Psi_{AO}s\). The rule set does not contain all combinations of the rules because commutative laws apply. In addition, we have \(\Psi_{AFS} \circ \Psi_{AFS} = \Psi_{AFS}\) and so on. Running Procedure 2 confirmed 24 primitive cases for scatter plots in Figure 3 and 35 primitive cases for parallel coordinates in Figure 5.

### 5. Estimating Visual Complexity

Given a relatively small number of primitive cases in either cluster-based scatter plot or parallel coordinates, we can consider the notion of visual complexity in a relatively abstract manner by focusing on topological differences between these cases. In this section, we first propose a scheme for estimating a complexity score for each primitive case. We then compare the scores with a collection of samples that record how human observers may perceive visual complexity. Finally, we provide a means for approximating \(n\)-cluster visual complexity.

#### 5.1. Estimating 2-cluster Complexity

The purpose of estimating visual complexity is to provide a metric for measuring some aspects of visual uncertainty as discussed in [DCK12]. Allen’s interval algebra takes into
account both overlapping and “meeting” clusters as topological features. Hence an estimation scheme must encode both features, and it may have the following principal considerations:

1. A primitive case should receive the lowest complexity score if it consists of two clusters that neither overlap nor meet with each other. We make 0 the lowest complexity score.

2. A primitive case should receive the highest complexity score if it consists two clusters that are equal on both axes, i.e., totally coinciding with one another. We make 1 the highest complexity score.

3. When shape $A$ is not overlapped by shape $B$, $A$ is visually less complex than when it is crossed over by shape $B$.

4. When shape $A$ has at least one non-overlapping region, $A$ is visually less complex than when it is totally overlapped by $B$.

5. When shape $A$ is split by shape $B$ into three pieces (1 overlapping and 2 non-overlapping), $A$ is visually more complex than when $A$ is split by $B$ into two pieces (1 overlapping and 1 non-overlapping).

6. When $A$ and $B$ meet at $k+1$ corners, the case is visually more complex than when they meet at $k$ corners ($k > 0$).

7. When $A$ and $B$ meet only at a corner, the case is visually less complex than when they meet along an edge.

Scoring the 24 Primitive Cases of Scatter Plots. Given two rectangular shapes $A$ and $B$ representing two clusters in a scatter plot, we consider an estimation scheme that decomposes a complexity score $U$ into four components as $U = U_A + U_B + U_{AB} + U_m$.

- $U_A = 0.0$ if shape $A$ has one continuous non-overlapping region. $U_A = 0.1$ if $A$ has two disconnected non-overlapping regions. $U_A = 0.2$ if $A$ has no non-overlapping region at all.

- $U_B$ is scored in the same way as $U_A$ by exchanging the relationship between $A$ and $B$.

- $U_{AB} = 0.0$ if $A$ and $B$ do not overlap, and $U_{AB} = 0.2$ otherwise (i.e., there is one overlapping region).

- $U_m = 0.1 \times n_c$ where $n_c$ is the number of edges where $A$ and $B$ meet. $U_m = 0.1$ if $A$ and $B$ do not meet at any edge but at a corner point.

Figure 7 show some examples that illustrate the scores of $U_A$, $U_B$, $U_{AB}$ and $U_m$ individually. Figure 9 lists the scores of $U$ for all 24 primitive cases of scatter plots. Note that when $A$ and $B$ coincide completely, $U$ sums up to exactly 1.

Scoring the 35 Primitive Cases of Parallel Coordinates. Given two quadrilateral or triangular shapes $A$ and $B$ representing two clusters in a parallel plot, we consider a similar estimation scheme that decomposes a complexity score into four components. The first three components $U_A$, $U_B$ and $U_{AB}$ are computed in the same way as with scatter plots. $U_m$ is computed in a slightly different way.

- $U_m = 0.1 \times n_p$ where $n_p$ is the number of corner points where $A$ and $B$ meet.

5.2. Comparison with Human Estimation

We consulted 29 volunteers, including 11 visualization researchers and 18 with statistics, mathematics, humanities and non-academic backgrounds. We asked them how they would make a comparative judgement about visual complexity. Through a web-based interface, participants compared pairs of randomly generated primitive cases. For each pair placed side-by-side, the participants were asked make a choice among three options: “Left is less complex than
Right”, “Left is more complex than Right”, or “Left and Right have similar complexity”. Participants compared 50 scatter plot pairs and 50 parallel coordinate pairs in two trials.

We purposely did not introduce the term uncertainty to the participants as the interpretation of each primitive case can be made uncertain for the given information as long as there is sufficient time. Instead, we simply consult the participants about which “case is more visually confusing than another”. We left the participants to make their own judgement of the definition of term confusing, hence the meaning of visual complexity.

The majority of participants used their intuition to compare pairs of patterns. Figure 9 shows 24 bar charts for each scatter plot case and Figure 10 shows 35 bar charts for each parallel coordinate case. In each bar chart, the $k = 0$-bar indicates the number of times when the observer made the same judgement as the estimation scheme in Section 5.1 when comparing this specific case $A$ against a case $X$ randomly selected from the 35 primitive cases. The $+k$ bars indicates when the observer over estimated the complexity for our scoring of a case, while $-k$ bars indicate the observers under estimate of the complexity. The estimation scheme scores a $k$ points higher than $X$. We consider a $[-0.1, +0.1]$ error in judgement an acceptable threshold for determining consistency of human observed measures for our scoring system.

The judgements by different human observers are not consistent. We observe for the scatter plots from Figure 9 that for most cases the distribution is clustered around the 0-bar with an error of approximately $[-0.5, +0.5]$. This is surprising, as scatter plots are generally considered to be a simpler data representation than parallel coordinates. The distributions are spread broadly with a few cases showing noticeable over estimates, (cases 2, 7, 9, 10) or under estimates (cases 5, 6, 19). This suggests that although topologically, scatter-plots are the simpler representation, observers have difficulty in judging the relationships between clusters on orthogonal axes. In contrast, Figure 10 shows tighter clusters more consistent with the parallel coordinates complexity scoring. There are more obvious overestimating cases (7, 9, 11, 16, 23), and a few underestimating cases (5, 30, 32). Overall human estimations are less dispersed with the parallel coordinates possibly because the topology is more constrained as clusters are limited in where they appear on parallel axes.

We examined these cases in detail. Some inconsistency can be explained. For example, the underestimation in case 1 for both plots is largely because the participants mistook the two totally overlapped shapes as a single shape. This actually confirms that the computer score of 1 is correct. We also made attempts to alter the estimation scheme. However, we could not find a better scheme, as each attempted change only resulted in more over- or under-scores in other cases. We believe that this is an interesting research problem for future work. One possibility is to conduct a large scale collection of the judgements of human observers. From such empirical data, one may be able to establish a better estimation scheme, or simply make the mean values of the human observations as the scores. Such an empirical study is beyond the scope of this work.

5.3. Approximating $n$-cluster Complexity

In practice, both scatter plots and parallel coordinates are expected to handle more than 2 clusters. The extension of Allen’s interval algebra from a 2-operand algebra to an $n$-operand algebra is a non-trivial challenges. We hence
Figure 10: Parallel coordinates case distributions show the amount of agreement of measured perceived visual complexity with our scoring system. The parallel score is in the title of each plot.

6. Conclusions & Future Work

This is a theoretical study on visual complexity in the context of cluster visualization. The central thesis is that it is possible to use Allen’s interval algebra to derive a scheme for estimating visual complexity. As this is an ambitious thesis, this work is merely the first step to bring mathematics and user experience together. We have confirmed, both manually and computationally, the primitive cases in 2D Allen’s interval algebra, which is useful for reducing the look-up cases for the estimation scheme. We have formulated estimation schemes for scatter plots and parallel coordinates plots. We have collected some human estimations about visual complexity in relation to these two plots. In comparison with the subjective judgements by humans, our estimation schemes are promising.

This research points to a number of interesting and challenging directions for future studies. These include the need for us to gain further understanding about how humans estimate visual complexity (e.g., how geometry and topology interfere with each other). As the sampling space is fairly large (e.g., 35 × 35 for parallel coordinates plots), this would require a large scale empirical study with carefully designed stimuli. We hope to continue this work, and use both mathematics and empirical studies to create quantitative metrics for visualization.
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