Hyperon Polarization from Unpolarized \textit{pp} and \textit{ep} Collisions

YUJI KOIKE

\textit{Department of Physics, Niigata University, Ikarashi, Niigata 950–2181, Japan}

\textbf{Abstract:} Cross section formulas for the Λ polarization in \textit{pp} → Λ↑(ℓ\textsubscript{T})X and \textit{ep} → Λ↑(ℓ\textsubscript{T})X are derived and its characteristic features are discussed.

In this report we discuss the polarization of Λ hyperon produced in unpolarized \textit{pp} and \textit{ep} collisions relevant for the ongoing RHIC-SPIN, HERMES and COMPASS experiments. According to the QCD factorization theorem, the polarized cross section for \textit{pp} → Λ↑X consists of two twist-3 contributions:

\begin{align*}
(A) \quad & E_a(x_1, x_2) \otimes q_b(x') \otimes \delta \hat{q}_c(z) \otimes \hat{\sigma}_{ab\rightarrow c}, \\
(B) \quad & q_a(x) \otimes q_b(x') \otimes \hat{G}_c(z_1, z_2) \otimes \hat{\sigma}'_{ab\rightarrow c},
\end{align*}

where the functions \( E_a(x_1, x_2) \) and \( \hat{G}_c(z_1, z_2) \) are the twist-3 quantities representing, respectively, the unpolarized distribution in the nucleon and the polarized fragmentation function for Λ\textsuperscript{↑}. \( \delta \hat{q}_c(x) \) is the transversity fragmentation function for Λ\textsuperscript{↑}. \( a, b \) and \( c \) stand for the parton’s species, sum over which is implied. \( E_a \) and \( \delta \hat{q}_c \) are chiral-odd. Corresponding to the above (A) and (B), the polarized cross section for \textit{ep} → Λ↑X (final electron is not detected) receives two twist-3 contributions:

\begin{align*}
(A') \quad & E_a(x_1, x_2) \otimes \delta \hat{q}_a(z) \otimes \hat{\sigma}_{ea\rightarrow a}, \\
(B') \quad & q_a(x) \otimes \hat{G}_a(z_1, z_2) \otimes \hat{\sigma}'_{ea\rightarrow a}.
\end{align*}

The (A) contribution for \textit{pp} → Λ↑X has been analyzed in [1], where it was shown that (A) gives rise to growing \( P_\Lambda \) at large \( x_F \) as observed experimentally. Here we extend the study to the (B) term (see also [2]) at RHIC energy and also for the \textit{ep} collision.

The unpolarized twist-3 distribution \( E_{F,D}(x_1, x_2) \) is defined in [1]. Likewise the twist-3 fragmentation function for a polarized Λ (with momentum \( \ell \)) is defined as the lightcone correlation function as \( (w^2 = 0, \ell \cdot w = 1) \)

\begin{align*}
\frac{1}{N_c} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-i\lambda(\frac{1}{2} - \frac{1}{2})} \langle 0|\psi_i(0)|\pi X\rangle \langle \pi X|gF_{\alpha\beta}(\mu w)w_\beta\bar{\psi}_j(\lambda w)|0\rangle \\
= \frac{M_N}{2z_2} (\gamma\ell)_{ij} e^{i\ell_\perp S_\perp} \hat{G}_F(z_1, z_2) + \frac{M_N}{2z_2} (\gamma_5\ell)_{ij} S_\perp \hat{G}_F^5(z_1, z_2) + \cdots.
\end{align*}
Note that we use the nucleon mass $M_N$ to normalize the twist-3 fragmentation function for $\Lambda$. There is another twist-3 fragmentation functions which are obtained from (1) by shifting the gluon-field strength from the left to the right of the cut. The defined functions $\hat{G}_{FR}(z_1, z_2)$ and $\hat{G}_{FR}^5(z_1, z_2)$ are connected to $\hat{G}_F(z_1, z_2)$ by the relation $\hat{G}_F(z_1, z_2) = \hat{G}_{FR}(z_2, z_1)$ and $\hat{G}_F^5(z_1, z_2) = -\hat{G}_{FR}^5(z_2, z_1)$, which follows from hermiticity and time reversal invariance. Unlike the twist-3 distributions, the twist-3 fragmentation function does not have definite symmetry property. Another class of twist-3 fragmentation functions $\hat{G}_{D}^{(5)}(z_1, z_2)$ is also defined from (1) by replacing $gF^{\alpha \beta}(\mu \nu)w_\beta$ by $D^{\alpha}(\mu \nu) = \partial^\alpha - igA(\mu \nu)$. Note, however, this is not independent from the above (1).

Following the method of [3] we present the analysis of the (C) term. The detailed analysis shows $\hat{G}_F(z, z)$ appears as soft-gluon-pole contribution ($z_1 = z_2 = z$), while $\hat{G}_D(z_1, z_2)$ appears as a soft fermion pole ($z_1 = 0$ or $z_2 = 2$). Physically, the latter is expected to be suppressed, and we include only the former contribution. This observation also applies to $E_{F,D}(x_1, x_2)$ relevant for the (A) term. In the large $x_F$ region, the main contribution comes from large-$x$ and large-$z$ (and small $x'$) region. Since $E_F$ and $\hat{G}_F$ behaves as $E_F(x, x) \sim (1 - x)^\beta$ and $\hat{G}_F(z, z) \sim (1 - z)^{\beta'}$ with $\beta, \beta' > 0$, $|(d/dx)E_F(x, x)| \gg |E_F(x, x)|$, $|(d/dz)\hat{G}_F(z, z)| \gg |\hat{G}_F(z, z)|$ at large $x$ and $z$. In particular, the valence component of $E_F$ and $\hat{G}_F$ dominates in this region. We thus keep only the valence quark contribution for the derivative of these soft-gluon pole function (“valence-quark soft-gluon approximation”) for the $pp$ collision. For the $ep$ case, we include all the soft-gluon pole contribution, since the calculation is relatively simple compared to the $pp$ case.

In general $P_\Lambda$ is a function of $S = (P + P')^2 \simeq 2P \cdot P'$, $T = (P - \ell)^2 \simeq -2P \cdot \ell$ and $U = (P' - \ell)^2 \simeq -2P' \cdot \ell$ where $P$ and $P'$ are the momenta of the two nucleons, and $\ell$ is the momentum of $\Lambda$. In the following we use 

$$S = \frac{2E}{\sqrt{S}} = \frac{T}{S}$$

and

$$x_T = \frac{2E}{\sqrt{S}}$$

as independent variables. The polarized cross section for the (B) term reads

$$E_\Lambda \frac{d^3 \Delta \sigma(S_{\perp})}{d \ell^3} = \frac{2\pi M_N \alpha^2 e^2 \sigma}{S} \sum_a \int_{z_{\min}}^{1} \frac{dz}{z^2} \int_{x_{\min}}^{1} \frac{dx}{x} \int_{0}^{1} \frac{dx'}{x'} \left \{ \right.$$  

$$\times \delta \left ( x' + \frac{xT}{xS + U/z} \right )$$  

$$\times \left \{ \right.$$  

$$\sum_{b,c} q^a(x)q^b(x') \left [ -z_1^2 \frac{\partial}{\partial z_1} \hat{G}_F^a(z_1, z) \right ]_{z_1 = z} \left ( \frac{-2p_\alpha}{T} \hat{\sigma}_{ab \rightarrow c}^I + \frac{-2p'_\alpha}{U} \hat{\sigma}_{ab \rightarrow c}^H \right ) \right \}$$  

$$+ \sum_{b,c} q^a(x)q^b(x') \left [ -z^2 \frac{\partial}{\partial z} \hat{G}_F^a(z, z) \right ] \frac{xp_\alpha + x'p'_\alpha}{|xT + x'U|} \left ( \hat{\sigma}_{ab \rightarrow c}^I + \hat{\sigma}_{ab \rightarrow c}^H \right ) \right \}$$

2
+q^a(x)G(x') \left[ -z_2^2 \frac{\partial}{\partial z_1} \tilde{G}_F^a(z_1, z) \right]_{z_1=z} \left( \frac{-2p_\alpha}{T} \tilde{\sigma}_{ag \to a}^{I} + \frac{-2p'_\alpha}{U} \tilde{\sigma}_{ag \to a}^{II} \right) \\
+q^a(x)G(x') \left[ -z_2^2 \frac{d}{dz} \tilde{G}_F^a(z, z) \right] \frac{x p_\alpha + x' p'_\alpha}{x T + x' U} \left( \tilde{\sigma}_{ag \to a}^{I} + \tilde{\sigma}_{ag \to a}^{II} \right) \right], \quad (2)

where the lower limits for the integration variables are \( z_{\text{min}} = -(T + U)/S = \sqrt{x_F^2 + x_T^2} \) and \( x_{\text{min}} = -U/z(S + T/z) \). The partonic hard cross sections are written in terms of the invariants in the parton level, \( \hat{s} = (x p + x' p')^2 = x x' S \), \( \hat{t} = (x p - \ell/z)^2 = x T/z \) and \( \hat{u} = (x' p' - \ell/z)^2 = x' U/z \). They read

\[
\tilde{\sigma}_{ab \to c}^{I} = -\frac{1}{36} \frac{s^2 + \hat{u}^2}{\hat{t}^2} \delta_{ac} + \frac{7}{36} \frac{s^2 + \hat{t}^2}{u^2} \delta_{bc} + \frac{1}{54} \frac{s^2}{\hat{u}} \delta_{ab} \delta_{ac},
\]

\[
\tilde{\sigma}_{ab \to c}^{II} = \frac{7}{36} \frac{s^2 + \hat{u}^2}{\hat{t}^2} \delta_{ac} - \frac{1}{36} \frac{s^2 + \hat{t}^2}{u^2} \delta_{bc} + \frac{1}{54} \frac{s^2}{\hat{u}} \delta_{ab} \delta_{ac},
\]

\[
\tilde{\sigma}_{ab \to c}^{I} = -\frac{1}{36} \frac{s^2 + \hat{u}^2}{\hat{t}^2} \delta_{ac} + \frac{7}{36} \frac{\hat{u}^2 + \hat{t}^2}{s^2} \delta_{bc} \quad \tilde{\sigma}_{ab \to c}^{II} = \frac{1}{18} \frac{s^2 + \hat{u}^2}{\hat{t}^2} \delta_{ac} + \frac{1}{18} \frac{\hat{u}^2 + \hat{t}^2}{s^2} \delta_{ab},
\]

\[
\tilde{\sigma}_{ag \to q}^{I} = -\frac{1}{8} \left( 1 - \frac{\hat{s} \hat{u}}{\hat{t}^2} \right) + \frac{1}{288} \left( \frac{-\hat{u}}{\hat{s}} - \frac{\hat{s}}{-\hat{u}} \right) - \frac{\hat{s}}{16 \hat{t}} - \frac{\hat{t}}{16 \hat{t}},
\]

\[
\tilde{\sigma}_{ag \to q}^{II} = \frac{9}{16} \left( 1 - \frac{\hat{s} \hat{u}}{\hat{t}^2} \right) + \frac{\hat{u}}{32 \hat{s}} - \frac{\hat{s}}{4 \hat{u}} + \frac{9 \hat{u}}{16 \hat{t}}.
\] \quad (3)

Among these partonic cross sections, \( \tilde{\sigma}^I \) becomes more important at large \( x_F \) because of the \( 1/T \) factor in (2).

To estimate the above contribution, we introduce a model ansatz as \( \tilde{G}_F^a(z, z) = K_a \tilde{q}_a(z) \) with twist-2 unpolarized fragmentation function \( \tilde{q}_a(z) \), noting that the Dirac structure of \( \tilde{G}_F^a(z, z) \) and \( \tilde{q}_a(z) \) is the same [3]. \( K_a \)'s are taken to be \( K_a = -K_d = 0.07 \) which are the same values used in the relation \( G_F(x, x) = K_a q^a(x) \) to reproduce \( A_N \) in \( p^+ p \to \pi X \) observed at E704 [4]. As noted before, \( \tilde{G}_F(z_1, z_2) \) does not have definite symmetry property unlike the twist-3 distribution \( \tilde{E}_F(x_1, x_2) \). Nevertheless we assume \( \left[ (\partial/\partial z_1) \tilde{E}_F(z_1, z) \right]_{z_1 = z} = (1/2)(d/dz)\tilde{E}_F(z, z) \). The result for the \( \Lambda \) polarization \( P^\Lambda_{pp} \) at \( \sqrt{S} = 62 \) GeV is shown in Fig. 1 together with the R608 data. There (A) (chiral-odd) contribution studied in [1] is also shown for comparison. (For the adopted distribution and fragmentation functions, see [1].) One sees that the tendency of \( P^\Lambda_{pp} \) from the (B)(chiral-even) contribution is quite similar to the R608 data. Rising behavior of \( P^\Lambda_{pp} \) at large \( x_F \) comes from (i) the large partonic cross sections in (3) (\( \sim 1/\hat{t}^2 \) term) and (ii) the derivative of the soft-gluon pole functions. With these parameters \( K_a, P^\Lambda_{pp} \) at RHIC energy (\( \sqrt{S} = 200 \) GeV) is shown in Fig.2 at \( l_T = 1.5 \) GeV. Fig. 3 shows the \( l_T \) dependence of \( P^\Lambda_{pp} \) of
the (B) term, indicating large $\ell_T$ dependence at $1 \leq \ell_T \leq 3$ GeV. Experimentally, $P_{\Lambda}^{pp}$ grows up as $\ell_T$ increases up to $\ell_T \sim 1$ GeV and stays constant at $1 \leq \ell_T \leq 3$ GeV. So the $P_{\Lambda}^{pp}$ observed at R608 can not be wholly ascribed to the twist-3 effect studied here which is designed to describe large $\ell_T$ polarization.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig1.png}
\caption{$P_{\Lambda}^{pp}$ at $\sqrt{S} = 62$ GeV.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig2.png}
\caption{$P_{\Lambda}^{pp}$ at $\sqrt{S} = 200$ GeV.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig3.png}
\caption{$\ell_T$ dependence of $P_{\Lambda}^{pp}$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig4.png}
\caption{$P_{\Lambda}^{ep}$ at $\sqrt{S} = 20$ GeV.}
\end{figure}

- We next discuss the polarization $P_{\Lambda}^{ep}$ in $pe \rightarrow \Lambda^+(\ell)X$ where the final electron is not observed. In our $O(\alpha_s^3)$ calculation, the exchanged photon remains highly virtual as far as the observed $\Lambda$ has a large transverse momentum $\ell_T$ with respect to the $ep$ axis. Therefore experimentally one needs to integrate only over those virtual photon events to compare with our formula.

Using the twist-3 distribution and fragmentation functions used to describe $P_{\Lambda}^{pp}$, we show in Fig. 4 the obtained $P_{\Lambda}^{ep}$ corresponding to (A’)(chiral-odd) and (B’)(chiral-even) contributions. Remarkable feature of Fig. 4 is that in both chiral-even and chiral-odd contributions (i) the sign of $P_{\Lambda}^{ep}$ is opposite to the sign of $P_{\Lambda}^{pp}$ and (ii) the magnitude of $P_{\Lambda}^{ep}$ is much larger than that of $P_{\Lambda}^{pp}$, in particular, at large $x_F$, and it even overshoots one. (In our convention, $x_F > 0$ corresponds to the production of $\Lambda$ in the forward hemisphere of the initial proton in the $ep$ case.) The origin of these features can be traced back to the color factor in the dominant diagrams for the twist-3 polarized cross sections in $ep$ and $pp$ collisions.
Of course, the $P_\Lambda$ can not exceeds one, and thus our model estimate needs to be modified. First, the applied kinematic range of our formula should be reconsidered: Application of the twist-3 cross section at such small $\ell_T$ may not be justified. Second, our simple model ansatz of $E_F^a(x, x) \sim \delta q^a(x)$ (in (A) term) and $\hat{G}^a_F(z, z) \sim \hat{q}^a(z)$ should be modified at $x \to 1$ and $z \to 1$, respectively. The derivative of these functions, which is important for the growing $P_{pp}^\Lambda$ at large $x_F$, eventually leads to divergence of $P_\Lambda$ at $x_F \to 1$ as $\sim 1/(1 - x_F)$.

As a possible remedy for this pathology we tried the following: As an example for the (B) (chiral-even) contribution we have a model $\hat{G}^a_F(z, z) \sim \hat{q}^a(z) \sim z^{\beta + z}$ where $\beta = 1.83$ in the fragmentation function we adopted. Tentatively we shifted $\beta$ as $\beta \to \beta(z) = \beta + z^8$, which suppresses the divergence of $P_\Lambda$ at $x_F \to 1$ but still keeps rising behavior of $P_\Lambda$ at large $x_F$. This avoids overshooting of one in $P_{ep}^\Lambda$ but reduces $P_{pp}^\Lambda$ seriously. The result obtained by this modification is shown in Figs. 5 and 6.

To summarize we have studied the $\Lambda$ polarization in $pp$ and $ep$ collisions in the framework of collinear factorization. Our approach includes all effects for the large $\ell_T$ production. One needs to be cautious in interpreting the available $pp$ data at relatively low $\ell_T$ in terms of the derived formula. Determination of the participating twist-3 functions requires global analysis of future $pp$ and $ep$ data.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig5}
\caption{$P_{pp}^\Lambda$ with modified $\hat{G}_F$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig6}
\caption{$P_{ep}^\Lambda$ with modified $\hat{G}_F$.}
\end{figure}

Acknowledgement: This work is supported in part by the Grant-in-Aid for Scientific Research of Monbusho.

References

[1] Y. Kanazawa and Y. Koike, Phys. Rev. D64 (2001) 034019.
[2] Y. Koike, hep-ph/0106260 (Proceedings of DIS2001, Bologna, Italy, April, 2001.)
[3] J. Qiu and G. Sterman, Phys. Rev. D59 (1999) 014004.
[4] Y. Kanazawa and Y. Koike, Phys. Lett. B478 (2000) 121; B490 (2000) 99.