LONG-RANGE PARTICLE CORRELATIONS AND WAVELETS

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Abstract

The problem of long-range correlations of particles produced in high-energy collisions is discussed. Long-range correlations involve large groups of particles. Among them are, e.g., those correlations which lead to ring-like and elliptic flow shapes of individual high-multiplicity events in the polar+azimuthal angles plane. The wavelet method of analysis which allows to disentangle various patterns has been proposed and applied to some central lead-lead collisions at energy 158 GeV per nucleon. Previous attempts to find out the ring-like correlations and recent results on wavelet analysis of high-energy nuclei interactions are reviewed.

1 Introduction

Recently, in Brookhaven National Laboratory (USA) started operating the collider RHIC, where the colliding beams of the nuclei of heavy elements interact with each other at the total energy up to 200 GeV per nucleon in the center of mass system. There are some first interesting (even though still preliminary) results, in particular, about the multiplicities of particles produced. It happens, that, if recalculated per a nucleon of the colliding nuclei, it exceeds the multiplicity of nucleon-nucleon collisions at the same energy. The events with the multiplicity of produced charged particles up to 10000 become soon available. With the advent of the collider LHC in CERN (Switzerland), when the energy of colliding nuclei will increase up to 1.8 TeV per nucleon, the events with up to 20000 charged particles produced will be available. The problem of presentation and analysis of these high multiplicity events becomes rather non-trivial. Each particle can be represented by a dot in the 3-dimensional phase space. Therefore, the distribution of these dots can be found for a single event. Different patterns formed by these dots in the phase space would correspond to different correlations.
and, therefore, to different dynamics. The problem of finding and deciphering these patterns becomes crucial for understanding the underlying dynamics. Really we are entering the new stage of studies of multiparticle production processes, where, apart from the traditional methods of selection of definite events with the use of triggers or displaying the plots of the simplest inclusive distributions, the exclusive characteristics of individual events with high multiplicity become important in event-by-event analysis. To classify these patterns, one should develop the adequate methods of their recognition and analysis. The wavelet analysis is just such a suitable method. Nowadays, the very first (discussed in this paper) attempts to apply this method are known, and it will, surely, find more wide applications.

Various correlations of particles produced in high-energy collisions are known. Especially well studied are the two-particle correlations due to the decay of two-particle resonances and due to the Bose-Einstein effect for identical particles. The common correlation function technique, well known, for example, in the statistical physics, is well suited for the analysis of these two-particle correlations. It is more difficult to apply it if several particles are involved because the correlation functions for three and more particles depend on many variables and, therefore, are difficult to handle. Nevertheless, correlations leading to clusters (or mini-jets, non-reducible to resonances) and to jets (the correlated systems of many particles with a common angular characteristics of their emission direction) have been studied as well, however mostly in $e^+e^-$-collisions where jet and subjet structures are clearly visible. Otherwise one has to apply the averaging procedure to get knowledge of dynamical correlations from studies of the moments of various distributions and their behavior in different regions of the phase space (for recent reviews, see [1, 2, 3]). It has lead to understanding of such global features as the intermittent dynamics (first proposed in [4]) and the fractal structure of phase space distributions (first proposed in [5]) in these processes. It has been shown [6, 7, 8] that, even at the level of the lowest perturbative approximations, the quantum chromodynamics (QCD) explains, at least qualitatively, this structure as resulting from the intermittent emission of subsequent jets with diminishing energies. Some completely new and unexpected features such as oscillating cumulant correlators of the multiplicity distributions were predicted in QCD [9, 10] and confirmed by experiment (see, e.g., the review papers [2, 3]). It demonstrated the existence, besides the attractive correlations, also of repulsive correlations between groups with different number of particles.

However, the genuine multiparticle correlations originating from some collective effects may still be hidden in individual events. It is not clear if they can be disentangled with such averaging methods, especially if their probability is not high enough. These local effects could be seen in some (probably, rare) events as special patterns formed by produced particles within the available phase space. They should be separated from statistical fluctuations leading to similar patterns. There exists a method of the wavelet analysis which allows to recognize patterns
due to correlations at different scales and damp down the statistical noise even if it is very large in an initial sample \cite{11, 12, 13, 14}. The ability to reveal the local properties of a process is the main advantage of the wavelet analysis compared to other methods, in particular, over the Fourier analysis (for more detailed discussion see \cite{14}). In its turn, this article provides the more detailed review of one of the problems briefly described in Ref. \cite{14}.

2 About some long-range correlations

Before delving into applications of this method, let us consider examples of possible correlations which one would seek for. The well known example of long-range correlations is provided, e.g., by the forward-backward correlations. However, they are of a global nature while we are more interested here in such correlations which lead to special long-correlated patterns in the available phase space. To be more definite, let us concentrate on search for the so-called ring-like correlations which remind the Cherenkov rings while others will be mentioned just by passing. Everybody knows about the Cherenkov radiation of photons by a bunch of electrons (or of any charged particles, in general) traversing a medium whose refractivity index exceeds 1. These photons form a ring in the plane perpendicular to electrons motion, i.e. they are emitted at a definite polar angle. As a hadronic analogue, one may treat an impinging nucleus as a bunch of confined quarks (color charges) each of which can emit gluons when traversing a target nucleus. Let me remind that Cherenkov photons are actually the collective effect of emission by the medium which electrons traverse. In the same spirit, the Cherenkov gluons would result from the collective emission by colliding nuclei. That is why the ring-like structure, if confirmed, would indicate on the collective long-range correlations in this system.

A long time ago, I speculated \cite{15} about possible Cherenkov gluons relying on experimental observation of the positive real part of the elastic forward scattering amplitude of all hadronic processes at high energies. This is a necessary condition for such process because in the commonly used formula for the refractivity index its excess over 1 is proportional to this real part:

$$\Delta n(\omega) = Ren(\omega) - 1 = \frac{2\pi N}{\omega^2} Re A(\omega, 0^0) \approx \frac{3\mu^3}{8\pi\omega} \sigma(\omega) \rho(\omega),$$

where $\omega$ is the energy of the emitted quantum, $N$ is the density of the scattering centers which has been estimated for hadrons as $N \approx 3\mu^3/4\pi$ with the pion mass $\mu$, and $A(\omega, 0^0)$ is the forward elastic scattering amplitude with $\rho(\omega) = Re A(\omega, 0^0)/Im A(\omega, 0^0)$ normalized by the optical theorem $Im A(\omega, 0^0) = \omega \sigma(\omega)/4\pi$. Thus the necessary condition for the Cherenkov gluon radiation $\Delta n(\omega) > 0$ is fulfilled if $\rho(\omega) > 0$. For typical values of $\sigma, \rho$ and $\omega$ for hadronic processes one gets $\Delta n(\omega) \ll 1$. 

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However, later [16] I noticed that for such thin targets as hadrons or nuclei the similar effect can appear due to small confinement length thus giving us a new tool for its estimate. The bremsstrahlung, the Cherenkov gluons and the transition radiation in thin targets contribute to the same angular range and thus become indistinguishable. At the same time, the difference between bremsstrahlung and Cherenkov radiation can reveal itself in the increased formation length of bremsstrahlung due to relativistic effects while for Cherenkov radiation the extension of a medium plays a decisive role. Therefore bremsstrahlung will be increased and inclined to smaller polar angles. Thus the background to Cherenkov radiation seemingly increases. However, as we shall see, it tends to large polar angles in the center of mass system what favours its observation.

The general formula [17] for the total energy $dW$ in the solid angle $d\Omega$ due to emission of vector particles with energies in the interval from $\omega$ to $\omega + d\omega$ by a step-like charge current $|j_\perp\mu|$ traversing a target of the thickness $l$ and of the refractivity index $n$ with the velocity $v = \beta c$ looks as

$$
\frac{dW}{d\Omega d\omega} = \frac{e^2 \beta^2}{\pi^2 c} \frac{\sin^2[\omega l(1 - \beta n \cos \theta)]}{(1 - \beta n \cos \theta)^2} \sin^2 \theta,
$$

where $e$ is the electric charge, $\theta$ is the polar angle of photons emission.

Such a step-like color current was used in Refs. [15, 16] as a model for deconfinement of quarks (color) of impinging hadrons (nuclei) only within the volume with the strong interaction radius $l \sim \mu^{-1}$. Outside this region, quarks are confined, and there are no color currents. Thus the phenomenological parameter $l$ describes the range of confining forces in usual static hadrons which just corresponds to the deconfinement region during the collision and wherefrom this collective effect is observed. The spin and mass properties of quarks and gluons are similar to those for electrons and photons, correspondingly. Therefore, to proceed to emission of high energy gluons by quarks deconfined in some limited volume (step-like color current) from the formula (2), one should replace there $\alpha$ by $\alpha_S C_F$, where $\alpha_S$ is the QCD running coupling, $C_F = 4/3$ and take into account the smallness of the emission angle in the laboratory system $\theta_{l.s.}$, of the differences $\Delta n = n - 1$ and $1 - \beta$ as well as of the ratio $\mu/\omega$. Then one gets [15, 16] from (3) the inclusive cross section for this gluon radiation

$$
\frac{\omega d^3\sigma}{\sigma d^3k} = \frac{4 \alpha_S C_F \theta_{l.s.}^2 \sin[\omega l(\theta_{l.s.}^2 - 2\Delta n)/4]}{\pi^2 \omega^2(\theta_{l.s.}^2 - 2\Delta n)^2}.
$$

In the case of the electromagnetic radiation in macroscopic targets, the condition $\omega l \Delta n/2 \gg 1$ is fulfilled, and the righthand side becomes the Dirac delta-function with an argument corresponding to the condition for the usual Cherenkov rings of photons. In the hadronic case, the opposite inequality

$$
\omega l \Delta n/2 \sim 3\rho(\omega)/16\pi \ll 1
$$

(4)
(for $\sigma \sim \mu^{-2}$, $l \sim \mu^{-1}$) is valid. Thus the position and the shape of the ring as defined by (3) is determined by the confinement length $l$ but not so much by $\Delta n$.

Surely, the confinement of hadronic partons leads to additional screening of low-energy gluon radiation which becomes of the multipole nature because the initial current (a hadron or a nucleus) is colorless in distinction to a charged current of an electron bunch. Thus this gluon radiation should be even of more collective nature and with harder spectrum than usually ascribed to the Cherenkov photon radiation. However it is hard to account for this collective behavior in a more quantitative way. The damping due to the imaginary part of the amplitude was estimated [16] as unimportant and given by the factor close to

$$\exp\left(-\frac{3\mu^2\sigma l}{8\pi}\right) \approx \exp\left(-\frac{\mu l}{8}\right) \approx 0.9,$$

which does not prevent the observation of this effect if any.

If several gluons are emitted and each of them generates a mini-jet centered at a definite polar angle (or pseudorapidity $\eta = -\ln \tan \theta/2$) without any condition imposed on its azimuthal angle $\varphi$, the ring-like substructure will be observed in the target diagram, i.e., in the plane perpendicular to the collision axis. Therefore, the events possessing such substructure with relatively short rapidity correlation length and large azimuthal correlation length would be important to look for. If the density of mini-jets within the ring is so high that they overlap, then they form a (circular) ridge pattern (or a wall pattern according to [18]). If the number of emitted gluons is not large, we will see several (or just one) jets (tower structure [18]) correlated in their polar, but not in the azimuthal angle. The formula (3) predicts quite large polar angle of emission of gluons, and therefore quite large radius of the ring in the target diagram that favors its observation

$$\theta_{c.m.s.} \sim 70^0 \ (110^0),$$

where the angle in the brackets corresponds to the emission in the backward hemisphere by another colliding particle. If the radiation length for colliding nuclei is proportional to $A^{1/3}$, then the corresponding radiation angle should behave as $A^{-1/6}$. It can be checked in collisions of nuclei with different atomic weights. In a single event, either both or one of these rings can be formed depending on the probability of such a process.

Surely, for the parton-emitter moving already at some angle to the collision axis this ring will be transformed into ellipse. Central collisions of nuclei would be
preferred for observation of such effects because of a large number of participating partons though the background due to ordinary processes increases as well. If the number of correlated particles within the ring is large enough, it would result in spikes in the pseudorapidity distributions. However, the usual histogram method is not always good to verify these spikes because it may split a single spike into two bins thus diminishing its role. However, some hints to such structure can be found from these histograms.

Namely, such a histogram of a cosmic ray nuclear interaction [19] initiated my approach to this problem. A huge spike on the rapidity plot observed in this event was initially interpreted as resulting from a single cluster. However, since the number of particles in the spike was very high (56 charged particles contributed to it while the average number is about 10) it was more carefully studied, and it was found that they form a ridge in the target diagram (the polar+azimuthal angles plane). There was also indication on another more diluted ring-like structure in the backward hemisphere of the same event. High spikes in the pseudorapidity distributions have been observed in some other cosmic ray data [20, 21, 22, 23] and in event-by-event analysis of accelerator data [24, 25]. Especially impressive is the famous event of NA22 Collaboration [25] of the pion-nucleon interaction with a spike 60 times exceeding the average density in the narrow rapidity window. The particles inside the spike are rather uniformly distributed in azimuthal angles (the ridge structure). However, the multiplicities in the analyzed central nucleus-nucleus events are about 50 times higher than in accelerator data on hadron-hadron collisions. In this case the huge combinatorial background may dilute the strength of spikes. Anyway all these events were just single representatives chosen by eye from samples of other events.

It is usually argued that the analysis of a single event of the central nucleus-nucleus collision at high energies may be statistically reliable due to a large number of particles produced. To classify such events in a more quantitative way, one should have a precise method of the local pattern recognition at different scales with a possibility to use an inverse transform. It became possible with the advent of the recently developed methods of the wavelet analysis. The wavelet analysis is well suited for this purpose because it clearly resolves the local properties of a pattern on the event-by-event basis.

Another example of collective long-range correlations is provided by the so-called elliptic flow i.e. the azimuthal asymmetry in individual events. It may be related to a collective classical sling-effect [26] of the rotation of colliding nuclei after peripheral collisions initiated by the pressure [27] at some impact parameter at the time of collision. It can give some knowledge about the equation of state of the hadronic matter by studies of the shapes of the created squeezed states. Another origin of the effect could be due to some jetty structures because emission of two jets with high transverse momenta well balanced by energy-momentum conservation laws would result in the azimuthal asymmetry of an individual event. These two theoretical suppositions lead however to different event patterns and
can be distinguished in experiment. By passing, let me say that the elliptic flow patterns ("cucumber") corresponding to the large value of the second Fourier coefficient, and the "three leaves flower" pattern with large third coefficient were also observed when scanning some high multiplicity events. These results have not been published yet, and I do not discuss all these effects here concentrating discussion on the ring-like events only.

The event-by-event analysis of patterns in experimental and Monte Carlo events becomes especially important for very high-multiplicity collisions at RHIC and LHC. The homogeneous $4\pi$ acceptance of detectors (such as that of STAR Collaboration at RHIC) will be crucial for it, not to provide false patterns. I am sure that various patterns will be observed and allow triggering on different classes of events and classifying "anomalous" features. Let me stress, however, here that the background due to the ordinary processes of parton emission and rescattering is huge in nucleus-nucleus collisions. It is not at all clear if the described above collective effect will be noticeable and can be separated from more common traditional patterns of radiation by individual quarks and gluons. One may rely only on very specific features of this effect for its unanimous registration and on first positive experience in this respect. Therefore, the clear separation and recognition of these (low-probability?) patterns as objective dynamical effects requires new identification methods insensitive to the smooth background and statistical fluctuations (noise). As one of them, I propose to use the wavelet analysis and describe recent developments in this direction briefly reviewing first some other proposals.

3 Earlier event-by-event studies

When the target diagrams of individual events are imaged visually, the human eye has a tendency to observe different kinds of intricate patterns with dense clusters (spikes) and rarefied voids in the available phase space. However, the observed effects are often dominated by statistical fluctuations and look quite subjective. The method of factorial moments was proposed \cite{4} to remove the statistical background in a global analysis and it shows fractal properties even in event-by-event approach (see \cite{4}). The increase of the factorial moments at small bins signals the presence of non-statistical fluctuations. Thus this method may be used as a tool for the preliminary selection of events with strong dynamical fluctuations. Nevertheless, it averages somehow the information about event patterns. Moreover, the patterns in the target diagrams as seen by eye hardly differ sometimes \cite{28} in the events with different factorial moments behavior. Some more sensitive and selective criteria should be used.

First detailed event-by-event analysis \cite{29,30} of large statistics data on hadron-hadron interactions (unfortunately, however, for rather low multiplicity) was performed to look for the dense groups of particles well separated (isolated) from
other particles in an event. The dense groups could imitate single dense jets or ring-like events. Some threshold values were imposed from below on the density of the groups and on their rapidity distance from other particles. These groups were quite narrow. The rapidity locations of their centers were determined. It has been found that the centers of these groups prefer to be positioned at a definite polar angle. The positions of the maxima of the centers distribution are quite close to estimates according to the formula (7). This feature favors the above interpretation in terms of Cherenkov gluons.

For nucleus-nucleus collisions, first systematic event-by-event analysis was attempted by NA49 Collaboration [31]. Unfortunately, it was limited only by studies of the fluctuations in the particle transverse momenta and the relative production of kaons to pions. No evidence was found for unusual fluctuations in the ratios of kaons to pions and in the fluctuations of the transverse momenta even though the latter were much smaller than in nucleon-nucleon collisions that can be treated on a qualitative level as a result of intra-nuclear rescatterings. These conclusions do not tell us anything about patterns in individual events. It is not surprising because this experiment has a limited and inhomogeneous acceptance, and only a fraction of the secondary particles is actually recorded. Moreover, this feature of the detector distorts some event characteristics crucial for pattern recognition, e.g., such as the particle density fluctuation. Therefore its results are not directly useful for our purposes.

The full space coverage is yet ensured only in the traditional emulsion chamber experiments. Therefore it is worthwhile to attempt the event-by-event analysis of their data. Even though the total number of events is relatively low and the particle identification is limited, they are prominent for their high angular resolution which is important for studies of the particle density fluctuations. Very interesting systematic event-by-event analysis of high-multiplicity Pb–Ag/Br collisions at energy $158$ GeV detected in EMU13 emulsion experiment was performed by the KLM Collaboration [28]. The results were confronted to three different Monte Carlo models: Fritiof, Venus and the random-type model called SMC. It was noticed that even in the one-dimensional distributions the probability to find a spike and sizes of spikes are systematically larger in measured events than in simulated ones. It was shown by plotting the distributions of spike fluctuations. The combinatorial background dilutes the strength of the observed signals. On the two-dimensional $\eta - \varphi$ phase space plot the slightly stronger clustering (jettyness) of particles in the measured events was also observed, especially for small size clusters. The size of the cluster (cone) was defined as $R = \sqrt{\delta \eta^2 + \delta \varphi^2}$ and the number of particles in the cone was chosen greater than 4. The fraction

\footnote{Up to now, the nuclear collisions were studied in the external beams only. Therefore everywhere we imply the energy per nucleon of the impinging nucleus in the rest system of the target nucleus. The impinging nuclei are shown first in front of the defis sign, and the target nuclei afterwards. It is clear that collisions of different nuclei at colliders will ask for other conditional notations.}
of particles confined in clusters is quite large. No rapidity and azimuthal angle distributions of the cone centers were presented, unfortunately. Therefore one can not decide if this effect is only due to short range correlations or some long range correlations like those discussed above are important as well. The only general conclusion is that the phase space inhomogeneity (jettyness) is stronger in measured events of nucleus-nucleus collisions than in any of Monte Carlo models based on conventional physics of nucleus-nucleus collisions.

In the Ref. [32] the azimuthal substructure of particle distributions in individual central high-energy heavy-ion collisions within dense and diluted groups of particles along the rapidity axis was investigated. The data of EMU01 Collaboration on O/S - Ag/Au collisions at 200 GeV were used. Some criteria appeared to be rather insensitive and did not show any significant difference from the stochastic averages with $\gamma$-conversion and HBT particle interference effect taken into account. However, when the parameter

$$S_2 = \sum_i (\Delta\varphi_i/2\pi)^2,$$

(8)

where $\Delta\varphi$ is the azimuthal difference between two neighboring particles in the group and sum is over all particles $i$ in the group, was used, it revealed some jet-structure for the dilute groups which was impossible to explain by known effects. It is an indication on the ridge-like structure (not a tower structure!) of analyzed groups within the definite rapidity windows. This analysis underlines the importance of the choice of criteria sensitive enough to features which may be hidden in particular patterns. To my opinion, even the parameter $S_2$ when used in the ”histogram-like” approach with a fixed scale length in pseudorapidity $\Delta\eta$ as was done in Ref. [32] averages too much the fluctuations in individual events. More local characteristics should be used not to smear the distinct differences between different patterns.

The dense groups were studied in nucleus-nucleus collisions at lower energies. Their rapidity distributions also showed [33] some peaks similar to those found in Refs. [29, 30]. However, the fragmentation processes are so strong here that it is not clear how the fragment products influence this conclusion.

4 Wavelets: basic notions

All these results, valuable by themselves, do not still answer the question about the existence of long-range correlations in individual events. To get it, one should be able to perform the large-scale event-by-event analysis. Such a tool is provided by wavelets. Let us briefly describe basic notions about wavelets (for more details, see [13, 14]).

Commonly used wavelets form a complete orthonormal system of functions with a finite support by using dilations and translations. That is why by changing
a scale (dilations) they can distinct the local characteristics of a signal at various scales, and by translations they cover the whole region in which it is studied. The orthogonality of wavelets insures that the information at a definite resolution level (scale) does not interfere with other scale information. The wavelet analysis is the study of any function by expanding it in the wavelet series (or integrals). Due to the completeness of the system, they also allow for the inverse transformation (synthesis) to be done. It means that the original function or some parts of it containing the investigated correlations may be restored without any loss of the information. In the analysis of nonstationary signals or inhomogeneous images (like modern paintings with very sharp figure edges), the locality property of wavelets leads to their substantial advantage over Fourier transform which provides us only with the knowledge of global frequencies (scales) of the object under investigation because the system of functions used (sine, cosine or complex exponents) is defined on the infinite interval.

The wavelet analysis reveals the local properties of any pattern in an individual event at various scales and, moreover, avoids smooth polynomial trends and underlines the fluctuation patterns. By choosing the strongest fluctuations, one hopes to get rid of statistical fluctuations and observe those dynamical ones which exceed the statistical component.

The traditional formula for the wavelet transform of a one-dimensional function \( f(x) \) is written as

\[
W(a, b) = a^{-1/2} \int f(x) \psi \left( \frac{x - b}{a} \right) dx,
\]

(9)

where \( \psi \) denotes a wavelet with its argument shifted to \( x = b \) (translation) and scaled by \( a \) (dilation). For continuous wavelets, both \( a \) and \( b \) are continuous variables. For discrete wavelets, one usually chooses \( a = 2^j \) where \( j \) are integer numbers and replaces \( b \) by \( 2^j k \). As we see, the wavelet coefficients for a one-variable function are the functions of two variables instead of one in case of the Fourier transform. It allows now to define both the scale (frequency) and its effective location.

The choice of the wavelet depends on the problem studied and is not unique. As an example of continuous wavelets, let us mention the so-called "Mexican hat" wavelet which is nothing else as the second derivative of the Gaussian function. The discrete wavelets are obtained as solutions of a definite functional equation and cannot be represented in the analytical form. However, they are suitable for computer calculations (for more detail see [13, 14]).
5 The wavelet analysis of high-energy nucleus interactions

First attempts to use wavelet analysis in multiparticle production go back to P. Carruthers [34, 35, 36] who used wavelets for diagonalisation of covariance matrices of some simplified cascade models. The proposals of correlation studies in high multiplicity events with the help of wavelets were promoted [37, 38], and used, in particular, for special correlations typical for the disoriented chiral condensate [39, 40]. The wavelet transform of the pseudorapidity spectra of JACEE events was done in Ref. [37].

As was mentioned above, wavelets are most effective in the analysis of inhomogeneous patterns. That is why I proposed to use them for deciphering the phase space inhomogeneity (in particular, the target diagrams) of very high multiplicity events.

At present, only five high-multiplicity events of the lead-lead collisions at 158 GeV were analyzed according to this method [41, 42]. I demonstrate here these central Pb-Pb events with the highest registered multiplicities from 1034 to 1221 charged particles chosen from 150 processed events and used for wavelet decomposition with the aim to study the patterns inherent to them. The data were taken from the emulsion chamber (with the thin lead target) experiment EMU15 at CERN by the group from Lebedev Physical Institute.

The target diagrams of secondary particles distributions for these events [42] are shown in Fig. 1, where the radial distance from the center measures the polar angle $\theta$, and the azimuthal angle $\phi$ is counted around the center. I show them here to demonstrate that even if one really notices some inhomogeneities in these diagrams, it is not easy to claim which one is of the dynamical origin and, moreover, the whole pattern is strongly influenced by the trivial high energy effect of higher density at small polar angles. One can sum over the azimuthal angle and plot the corresponding pseudorapidity ($\eta = -\log \tan \theta/2$) distributions shown in Fig. 2. The pronounced peaks ($\eta$-spikes) strongly exceeding expected statistical fluctuations are seen in individual events. This inhomogeneity in pseudorapidity can arise either due to a very strong jet i.e. a large group (tower) of particles close both in polar and azimuthal angles or due to a ring-like (ridge) structure when several jets with smaller number of particles in each of them have similar polar angle but differ in their azimuthal angles. However, it is still not easy to observe them directly in the target diagram even if the pseudorapidity is plotted along its radius instead of the polar angle or the scaled variable [43]

$$\bar{\eta}(\eta) = \int_{\eta_{\text{min}}}^{\eta} \rho(\eta') d\eta' / \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} \rho(\eta') d\eta',$$

which gives rise to a flat distribution $\rho(\bar{\eta})$, is used.

In Ref. [41] wavelets were first used to analyze two-dimensional patterns of fluctuations in the phase space of a single event, shown in Fig. 1 at number 19.
There were presented [41] the results of the one-dimensional analysis of separate sectors of this two-dimensional plot. To proceed in this way, the whole azimuthal region was divided into 24 sectors of the $\pi/12$ extension each (thus preserving, unfortunately, the histogram drawback in this coordinate). The pseudorapidity distributions in each of them were separately analyzed after integrating over azimuthal angles within a given sector. Neighboring sectors were connected afterwards.

Both jet and ring-like structures have been found from the values of squared wavelet coefficients as seen from Fig. 3 taken from Ref. [41]. Four of 24 sectors are demonstrated there. The wavelet coefficients were calculated using the continuous "Mexican hat" wavelet. The darker regions correspond to larger fluctuations, i.e. to larger values of wavelet coefficients. At the very top in each box, i.e. at small scales $a$, the wavelet analysis reveals individual particles as they are placed on the pseudorapidity axis in a given sector seen as short dark lines. At larger scales $a$ corresponding to the shift down in any box of the Figure, the correlations of different shapes leading to clusters or jets of particles are resolved. Finally, at ever larger scale one notices the long-range structure (indicated by the arrows in Fig. 3) which penetrates from one azimuthal sector to the neighboring one at nearby values of the polar angle (pseudorapidity), thus forming an elliptic ridge around the center of the target diagram. This structure approximately corresponds to the peak in the pseudorapidity distribution (for more detail, see Ref. [41]). Let us note, that it is not easy to notice in the target diagram of this event, shown in Fig. 1, any increase of density at the ring just by eye because of the specific properties of the $\theta - \phi$ plot where the density of particles decreases fast toward the external region of large polar angles due to the relativistic effects. Also, the tree-like patterns in this plot of the wavelet coefficients would probably correspond to the fractal structure of the phase space which was discovered by the factorial moments method.

To reveal these patterns in more detail one should perform the two-dimensional local analysis. It is strongly desirable to get rid of such drawback of the histogram method as fixed positions of bins that sometimes gives rise to splitting of a jet into pieces contained in two or more bins as, e.g., it could happen in the above analysis of the event 19 [41] when 24 azimuthal sectors were chosen. Moreover, the number of charged particles in each sector was about 50, that is already at the limits of the accuracy for efficient wavelet analysis to be done. The two-dimensional wavelet transform of particle densities directly in the two-dimensional plot does not have such a deficiency. Wavelets choose automatically the size and shapes of bins (so-called Heisenberg windows; see, e.g., Ref. [11]) depending on particle densities at a given position. The multiresolution analysis at different scales and in different regions is performed.

In principle, the wavelet coefficients $W_{j_1,k_1,j_2,k_2}$ of the two-dimensional function
\( f(\theta, \varphi) \) are found from the formula

\[
W_{j_1,k_1,j_2,k_2} = \int f(\theta, \varphi) \psi(2^{-j_1}\theta - k_1; 2^{-j_2}\varphi - k_2) d\theta d\varphi.
\] (11)

Here \( \theta_i, \varphi_i \) are the polar and azimuthal angles of particles produced, \( f(\theta, \varphi) = \sum_i \delta(\theta - \theta_i) \delta(\varphi - \varphi_i) \) with a sum over all particles \( i \) in a given event, \( (k_1, k_2) \) denote the locations and \( (j_1, j_2) \) the scales analyzed. The function \( \psi \) is the analyzing wavelet. The higher the density fluctuations of particles in a given region, the larger are the corresponding wavelet coefficients.

In practice, the discrete wavelets obtained from the tensor product of two multiresolution analyses of standard one-dimensional Daubechies 8-tap wavelets were used. Then the corresponding \( ss, sd \) and \( dd \) coefficients in the two-dimensional matrix were calculated (see [11]). The common scale \( j_1 = j_2 = j \) was used. It simplifies the calculations and the presentation of obtained results because of the smaller number of variables but may be not completely satisfactory from physical point of view and, probably, should be abandoned later in a more sophisticated approach. The similar basis has been used in Ref. [44].

As stressed above, the ring-like structure should be a collective effect involving many particles and large scales. Therefore, to get rid of the low-scale background due to individual particles and analyze their clusterization properties, the scales \( j > 5 \) have been chosen where both single jets and those clustered in ring-like structures can be revealed as seen from Fig. 3. Therefore all coefficients with \( j < 6 \) are put equal to 0. The wavelet coefficients for any \( j \) from the interval \( 6 \leq j \leq 10 \) are now presented as functions of polar and azimuthal angles in a form of the two-dimensional landscape-like surface over this plane i.e. over the target diagram. Their inverse wavelet transform allows to get modified target diagrams of analyzed events with large-scale structure left only. Higher fluctuations of particle density inside large-scale formations and, consequently, larger wavelet coefficients correspond to darker regions in this modified target diagram shown in Fig. 4. Here we demonstrate two events (numbered 3 and 6) from those five shown in Figs. 1 and 2. They clearly display both jet and ring-like structures which are different in different events. To discard the methodical cut-off at \( \eta \approx 1.6 - 1.8 \) the region of \( \eta > 1.8 \) was only considered\(^2\).

Even though the statistics is very low, it was attempted to plot the pseudorapidity distribution of the maxima of wavelet coefficients with the hope to see if it reveals the peculiarities observed in high statistics but low multiplicity hadron-hadron experiments [29, 30]. In Fig. 5, the number of highest maxima of wavelet coefficients exceeding the threshold value \( W_{j,k} > 2 \cdot 10^{-3} \) is plotted as a function of their pseudorapidity for all five events considered. It is quite peculiar

\(^2\)The more detailed discussion about this methodical effect can be found in Ref. [42]. It shows that wavelets may be used for the analysis of methodical problems in detectors as well. This topic is out of the scope of the present paper.
that positions of the maxima are discrete. They are positioned quite symmetrically about the value \( \eta \approx 2.9 \) corresponding to 90° in the center of mass system as it should be for two Pb nuclei colliding. Difference of heights is within the error bars. More interesting, they do not fill in this central region but are rather separated. Qualitatively, it coincides with findings in Refs. [29, 30], where the separated peaks in the pseudorapidity distribution of the centers of dense groups were also noticed approximately at the same positions.

For comparison, there were generated 100 central Pb-Pb interactions with energy 158 GeV according to Fritiof model and the same number of events according to the random model describing the inclusive rapidity distribution shape. The fluctuations in these simulated high multiplicity events are much smaller than in experimental ones and do not show any ring-like structure.

Further use of wavelets by imposing different scales for polar and azimuthal angles and study of the 3-dimensional phase space are desirable. For the latter, one needs the data on the momenta of the created particles, which may be obtained only in experiments with the magnetic field. Though the emulsion chamber of the experiment EMU15 was installed in the magnetic field \( H = 1.8T \), there were no measurements of the trajectory curvature done until now. Thus the data on the three-dimensional phase space are not yet available. I would like to stress once more that good homogeneous acceptance of the detectors with 4\( \pi \) geometry is crucial for the proper wavelet analysis of any event to get firm conclusions about the dynamical effects. Otherwise the results will show an admixture of physical and methodical effects which can not be disentangled in the event-by-event analysis. Actually, this remark is a byproduct of my own experience in attempts to find out the elliptic flow effect in NA49 data on lead-lead collisions at 158 GeV. They failed because I noticed the azimuthal asymmetry which persisted at the same position, suspected it to be due to the inhomogeneous azimuthal acceptance of the detector, and after request was insured by the authors that my guess is correct. Thus the methodical effect was so strong that it prevented any conclusions about the dynamical elliptic flow but, probably, it will not prevent from the analysis of some structures in polar angles to be done. Even though such applications of the wavelet analysis are interesting by themselves and their effectiveness was demonstrated in this case, they are not of the main physics interest and of our concern here.

6 Conclusion

Thus I conclude that even on the qualitative level there is the noticeable difference between experimental and simulated events with larger and somewhat ordered fluctuations in the former ones. First attempts to apply the method of the wavelet analysis described here have shown some peculiar patterns of individual measured events not resolved in the Monte Carlo simulated samples. It means that we still
have something to learn about their dynamical features. The ring-like patterns were used here only as an example of possible correlations leading to some special particle grouping in the phase space. More detailed wavelet analysis at various resolution levels and also in the three dimensional phase space may reveal some yet undiscovered structures. The described above efforts show that there are no unsolvable problems for doing it.

My aim here is to show the applicability and power of a new method of the two-dimensional wavelet analysis, the qualitative features and differences leaving aside quantitative characteristics till higher statistics of fully registered high multiplicity AA-events becomes available. In particular, the special automatic complex for emulsion processing with high space (angular) resolution (www.lebedev.ru/structure/pavicom/index.htm) in Lebedev Physical Institute is coming into operation, and it can help enlarge the statistics of analyzed central Pb-Pb collisions at 158 GeV quite soon using at full strength the good acceptance and precision of emulsion detectors. Also the data of STAR collaboration in RHIC with a full phase space coverage and large number of events should soon be available at ever higher energies. Wavelets provide a powerful tool for event-by-event analysis of fluctuation patterns in such collisions.
The target diagrams of five events of central Pb-Pb collisions at energy 158 GeV/nucleon obtained by EMU-15 collaboration.
The pseudorapidity distributions of particles in five events shown in Fig.1.
Fig. 3 The wavelet coefficients for the event 19 analyzed in [41]. Dark regions correspond to large values of the coefficients. Four of 24 sectors are shown. Rapidities are along x-axis, the scales increase down the vertical axis.

Fig. 4 The modified large-scale target diagrams of two events (3 and 6). Darker regions correspond to larger particle density fluctuations.
Fig. 5 The pseudorapidity distribution of the maxima of wavelet coefficients. The irregularity in the maxima positions, the empty voids between them and absence of peaks at $\eta \approx 2.9$ are noticed.

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