UKF based high-precision data fusion approach of orthogonal IMU sensors

Xiaowen Zhang¹, Shuai Yuan*¹, Jian Wu¹, Binzhuo Wang² and Yu Sun¹

¹Information and control engineering school, Shenyang Jianzhu University, Shenyang, Liaoning, 110168, China
²School of Mathematics, Jilin University, Changchun, Jilin, 130012, China
*yuanshuai@sjzu.edu.cn

Abstract. Accurate attitude angle measurement is very important for some automatic robots, but the noise uncertainty in the sensor can cause measurement error. In order to solve the problem that these pose sensors have low estimation accuracy, this paper proposes a data fusion approach using the IMU sensor including gyroscope, accelerometer and magnetometer. Through placing multi-sensors with spatial orthogonal relation, global optimization is achieved, and the UKF algorithm is adopted to fuse data. This fusion uses the gyroscope data as the state vector, and regards the accelerometer and magnetometer data as the observation vector to improve the accuracy of attitude angle measurement. The experimental result is performed to compare the proposed method with traditional method, and the proposed method can obtain better attitude measurement accuracy, which illustrates the effectiveness of the proposed method.

1. Introduction
Micro Inertial Measurement Unit (MIMU) is manufactured using MEMS fabrication and outputs angular velocity and acceleration in three directions with low power consumption, light weight, low cost and small size [1]. Therefore, this sensor has a wide application in robot, driverless car and aircraft attitude control [2]. The dynamic response and accuracy of the MIMU gyroscope are high, but the raw data output cannot be used directly. The gyroscope will produce gyro drift with time increasing, the accelerometer has good static performance but contains high frequency noise and the measurement accuracy of the magnetometer will be interfered by the external magnetic field and there is a significant decrease of accuracy in the environment with large magnetic interference [3]. In order to achieve high-precision attitude angle measurement, it is necessary to combine hardware and software, the gyroscope is usually used for detection, and the accelerometer is combined with the magnetometer to improve the accuracy of the attitude angle. The Kalman filter (KF) has superior performance in terms of filtering the signal and sensor fusion so it has been widely used.

KF is typically used to reduce random noise and drift error in IMU. But KF cannot estimate the optimal state of the nonlinear system. To solve this problem, some researchers have proposed the Extended Kalman filter (EKF), which aligns the sensors, and linearizes the nonlinear system locally to approximate it as a linear problem [4]. However, the linearization process will increase the error, and the Unscented Kalman filter (UKF) is adapted to sample the nonlinear system through Unscented Transformation, and the sampling point is fetched to represent the probability density function of the Gaussian density approximation [5].
On the foundation of abovementioned work, this paper adapts UKF for performing data fusion with real-time and high-precision attitude angle. The sensors are placed orthogonally, where the gyroscope is orthogonal to the accelerometer and the magnetometer, which enforces the sensitivity of the gyroscope with obtaining the stability of the accelerometer and magnetometer for optimizing the global optimization. Finally, experiments are performed for comparing the proposed algorithm with the traditional method, and the experimental results illustrate the effectiveness of the method.

2. Estimation algorithm of position and orientation

2.1. Gyroscope measurement
According to Euler's theorem, a rigid body undergoes 3 basic rotations around a fixed point and can coincide with the inertial coordinate system. The Euler angles corresponding to the 3 basic rotations are: $\psi, \theta, \phi$, as shown in Figure 1.

![Figure 1. The coordinate system and the Euler angles.](image)

The rotation matrix $C^e_b$ from the body coordinate system to the inertial coordinate system is expressed as:

$$C^e_b = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ \sin \theta \sin \psi \cos \phi - \cos \psi \sin \phi & \sin \theta \sin \psi \sin \phi + \cos \psi \cos \phi & \cos \psi \sin \phi \\ \sin \phi \cos \psi + \cos \psi \sin \phi & \sin \phi \sin \psi \sin \phi - \cos \psi \cos \phi & \cos \phi \cos \psi \sin \phi \\ \end{bmatrix}$$ (1)

Assuming gyroscope measures an angular velocity of the rotating body triaxial coordinate of $\omega = [\omega_x \omega_y \omega_z]$, then the relationship between the change rate of the Euler angles and angular velocity is:

$$q = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & \tan \theta \sin \phi & \tan \theta \cos \phi \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$ (2)

2.2. Measurement of accelerometer and magnetometer
The accelerometer can obtain pitch angle and row angle under static state, but cannot obtain yaw angle. The magnetometer can get the yaw angle very well but vulnerable to outside interference.

First of all, we need to convert the accelerometer data into attitude information:

$$\begin{bmatrix} \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \arctan \left(a_x \cdot (a_y^2 + a_z^2)^{-1/2}\right) \\ \arctan \left(a_y \cdot (a_x^2 + a_z^2)^{-1/2}\right) \end{bmatrix}$$ (3)

Where $[a_x, a_y, a_z]$ is the accelerometer output data, then we convert the magnetometer data into attitude information:
\[
\begin{align*}
&\begin{cases}
  e_x = m_x \cos \theta + m_z \sin \theta \\
  e_y = m_y \cos \phi + m_x \sin \phi \sin \theta - m_z \sin \phi \cos \theta
\end{cases} \\
&\phi = \arctan \left( e_x, e_y^{-1} \right)
\end{align*}
\]

Where \(\begin{bmatrix} m_x & m_y & m_z \end{bmatrix}^T\) is the data measured by the magnetometer.

3. Unscented Kalman filter based data fusion

Unscented Kalman filter is a nonlinear estimation method for processing sigma point set based on Unscented transformation to match mean and covariance of the sigma point set to the statistical characteristics of the original system. Next the sigma point set is used to obtain the approximate state probability density function through nonlinear mapping. Then the UKF uses the sigma point set to update the status and measure the gain.

3.1. State transformation

Equation (2), (3), (5) are combined to obtain the state equation of the multi-sensor data fusion system:
\[
\begin{align*}
X_k &= f(X_{k-1}, u_k) \\
Z_k &= h(X_{k-1}, v_k)
\end{align*}
\]

Where \(f(\cdot) = X_{k-1} + (q, u_k)^T \cdot T\), \(X_{k-1}\) is the state vector, \(T\) is the sampling period. As for \(h(\cdot) = HX_{k-1} + v_k\), \(H\) is the identity matrix, \(u_k, v_k\) are the system measurement noise.

3.2. Unscented transformation

Calculating the number of \((2n+1)\) sample points, and the state \(X_{k-1,i}\) and its error covariance matrix \(P_{k-1}\) are estimated at time \(k-l\) to determine the sigma point:
\[
X_{k-1,i} = \begin{cases}
  \bar{x}_{k-1} & i = 0 \\
  \bar{x}_{k-1} + \left( (n+\lambda)P_{k-1} \right)^{1/2} & i = 1 \ldots n \\
  \bar{x}_{k-1} - \left( (n+\lambda)P_{k-1} \right)^{1/2} & i = n+1 \ldots 2n
\end{cases}
\]

Where \(\lambda = \alpha^2 (n+\xi) - n\) is the scale factor, \(\alpha\) and \(\xi\) are arbitrary constants and \(\xi\) usually takes a value greater than zero. And \(\left( P^{1/2} \right) \left( P^{1/2} \right)^T = P\), \(P\) represents the \(i\) column of the square root of the matrix, \(n\) represents the number of input variables.

The \((2n+1)\) points are mapped through the nonlinear function \(f(\cdot)\) as:
\[
X_{k,i} = f(X_{k-1,i}, u_k)
\]

The predicted state value and covariance matrix are calculated by assigning weights to sigma points:
\[
\begin{align*}
\bar{x}_k &= \sum_{i=0}^{2n} \omega_i \cdot X_{k,i} \\
\bar{p}_{xx} &= \sum_{i=0}^{2n} \omega_i \cdot (X_{k,i} - \bar{x}_k) \cdot (X_{k,i} - \bar{x}_k)^	op 
\end{align*}
\]

Where the weights are:
\[
\omega_i = \begin{cases} 
\lambda \cdot (n + \lambda)^{-1} & i = 0, \text{meanvariance} \\
\lambda \cdot (n + \lambda)^{-1} \cdot (1 - \alpha^2 + \beta) & i = 0, \text{covariance} \\
\lambda \cdot (n + \lambda)^{-1} \cdot 2^{-1} & i = 1 - 2n 
\end{cases}
\]

In the above equation, \(\beta\) is a non-negative parameter.

Taking the mapped sigma point into the measurement function to obtain:
\[
Z_{k,i} = h(Z_{k-1}, v_k)
\]

3.3. Measurement update

The following formulas are the predicted measured value and covariance:
\[
\begin{align*}
\bar{z}_k &= \sum_{i=0}^{2n} \omega_i \cdot Z_{k,i} \\
\bar{p}_{zz} &= \sum_{i=0}^{2n} \omega_i \cdot (Z_{k,i} - \bar{z}_k) \cdot (Z_{k,i} - \bar{z}_k)^	op \\
\bar{p}_{xz} &= \sum_{i=0}^{2n} \omega_i \cdot (X_{k,i} - \bar{x}_k) \cdot (Z_{k,i} - \bar{z}_k)^	op
\end{align*}
\]

The Kalman gain matrix is:
\[
K = P_{xz} \cdot P_{zz}^{-1}
\]

Finally, the state estimation and covariance matrix are updated through the following equations:
\[
\begin{align*}
\hat{x}_k &= \bar{x}_k + K \cdot (z - \bar{z}_k) \\
P_k &= \bar{p}_{xx} - K \cdot P_{zz} \cdot K 
\end{align*}
\]

4. Analysis of experiment result

In order to illustrate the effectiveness of the UKF based data fusion approach using orthogonal IMU sensors, we chose a gimbal capable of three-axis rotation as the experimental platform shown in the right side of Figure 2, and placed the orthogonal sensors (the left side of Figure 2) on the gimbal to rotate at three angles of row, pitch and yaw. We use Matlab to do the attitude estimation algorithm with the raw data from the gyroscope, accelerometer and magnetometer.

![Figure 2. Display of orthogonal sensors and the assembled experimental platform.](image)

First, the orthogonal sensors based on UKF approach is compared with the approach of
non-orthogonal sensors in the same motion state. Figure 3, Figure 4 and Figure 5 below are performed for comparing the angle error using the two modes including the orthogonal sensors and non-orthogonal sensors.

Figure 3. The row angle error curve of orthogonal sensors and non-orthogonal sensors based on UKF.

Figure 4. The pitch angle error curve of orthogonal sensors and non-orthogonal sensors based on UKF.

Figure 5. The yaw angle error curve of orthogonal sensors and non-orthogonal sensors based on UKF.

The experimental curve is performed to compare the approach of orthogonal sensors with non-orthogonal sensors both based on UKF, the UKF based fusion approach of orthogonal sensors proposed in this article obtains more accuracy. Next, the proposed approach is compared with the KF based approach of orthogonal sensors in the same motion state. Figure 6, Figure 7 and Figure 8 show the angle error of the two algorithms.

Figure 6. The row angle error curve of the orthogonal sensors based on KF and UKF.
Figure 7. The pitch angle error curve of the orthogonal sensors based on KF and UKF.

Figure 8. The yaw angle error curve of the orthogonal sensors based on KF and UKF.

It can be seen from the error curves that the method proposed in this paper has better stability and smaller error than the UKF based approach of non-orthogonal sensors and the KF based approach of orthogonal sensors. In general, this paper proposes a method with a better precision performance.

5. Conclusion
This paper proposed a UKF based data fusion approach of orthogonal IMU sensors. The fusion approach obtains row angle, pitch angle and yaw angle through fusing gyroscope measurement with accelerometer and magnetometer detection values. In order to obtain high-precision measurement, the sensors are placed with spatial orthogonal relation and the UKF algorithm is adopted for performing data fusion. The experimental result shows that the fusion approach of orthogonal IMU sensors based on UKF can improve the accuracy of attitude measurement, which illustrates the validity of the proposed method in attitude angle measurement.

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References
[1] Zhang, J., Li, W., Zhang, J., Nie, P., Zhang, C. (2020) Investigation of MIMU Attitude Algorithm Based on Kalman filter. Computer Measurement & Control., 28: 233-237+242.
[2] Quinchia, A.G., Ferrer, C., Falco, G., Falletti, E., Dovis, F. (2012) Analysis and modelling of MEMS inertial measurement unit. In: 2012 International Conference on Localization and GNSS. Sarnberg. pp. 1-7.
[3] Hyde, R.A., Ketteringham, L.P., Neild, S.A., Jones, R.J.S. (2008) Estimation of upper-limb orientation based on accelerometer and gyroscope measurements. IEEE Transactions on Biomedical Engineering., 55:746-754.
[4] Lu, Y., Chen, Y., Zhang, X., Zhang, T. (2020) Attitude information fusion method based on extended kalman filter. Chinese Journal of Scientific Instrument., 41:281-288.
[5] Li, H., Guo, H., Qi, Y., Deng, L., Yu, M. (2021) Research on multi-sensor pedestrian dead reckoning method with UKF algorithm. Measurement. Journal of the International Measurement Confederation., 169.