Phase-dependent Kondo Resonance in a Quantum Dot Connected to a Mesoscopic Ring

Kicheon Kang

Department of Physics, Chonbuk National University, Chonju 561-756, Chonbuk, Korea

Luis Craco

Instituto de Física Gleb Wataghin - UNICAMP C.P. 6165, 13083-970 Campinas - SP, Brazil

(Received November 2, 2018)

Phase-sensitive transport through a quantum dot coupled to an Aharonov-Bohm ring is analyzed. In this geometry the spectral density of states is directly related to the conductance. It is shown that the Kondo resonance depends on the phase and on the total number of electrons (modulo 4) in the mesoscopic ring. The effect of the discrete level spacing in the ring and of the coupling to the electrical leads is discussed.

PACS numbers: 72.15.Qm, 73.40.Gk

The Kondo effect is one of the most intensively studied topics in many-body physics [1], which describes several anomalous features observed in metallic systems with embedded magnetic impurities. Due to the recent remarkable progress in the nano-fabrication of electronic devices it is now possible to investigate the Kondo effect by using nano-structures in a highly controlled way [2–5, 6–13]. These investigations clearly demonstrate that the Kondo resonance can be studied (see Fig. 1). In contrast to a system of a Kondo impurity embedded in a mesoscopic “box” [2], the spectral density of states (DOS) of the QD is directly measurable in our geometry, and one can systematically study mesoscopic effects on the Kondo resonance as a function of the Aharonov-Bohm phase.

The Hamiltonian of the system can be written as

\[ H = H_0 + \sum_{\alpha=L,R} H_\alpha + T, \]  

(1)

where \( H_0 \) is the Hamiltonian term for the dot-ring system, \( H_\alpha \) describes the left (L) and the right (R) lead, and \( T \) accounts for the tunneling between the QD and the two leads. \( H_0 \) is decomposed into

\[ H_0 = H_{QD} + H_{Ring} + H_{\nu}, \]  

(2)

where \( H_{QD}, H_{Ring} \), and \( H_{\nu} \) correspond to the Hamiltonians describing the quantum dot, the AB ring, and the dot-ring hybridization at site “0”, respectively:

\[ H_{QD} = \sum_\sigma \varepsilon_{d}\hat{d}_\sigma^\dagger \hat{d}_\sigma + U \hat{n}_{\uparrow} \hat{n}_{\downarrow}, \]  

(3a)

\[ H_{Ring} = -t \sum_{j=1}^{N} \sum_\sigma \left( e^{i\varphi/N} \hat{c}_j^\dagger \hat{c}_{j+1\sigma} + \text{h.c.} \right), \]  

(3b)

\[ H_{\nu} = -\nu' \sum_\sigma \left( \hat{c}_0^\dagger \hat{c}_{0\sigma} + \hat{c}_{0\sigma}^\dagger \hat{d}_\sigma \right). \]  

(3c)

Here, we describe the ring by a tight-binding Hamiltonian with \( N \) lattice sites, and the QD by a single Anderson impurity. \( \varepsilon_d \) is the single particle energy, and \( U \) is the Coulomb interaction in the QD. The phase \( \varphi \) in Eq. (3b) comes from the Aharonov-Bohm flux, and is defined by \( \varphi = 2\pi \Phi/\Phi_0 \), where \( \Phi \) and \( \Phi_0 \) are the external flux and the flux quantum (= \( h\varepsilon/\varepsilon \)) respectively. Note that Eq. (3c) can be diagonalized and the corresponding eigenvalues are \( \varepsilon_m = -2t \cos \frac{\pi}{N} (2\pi m + \varphi) \) (\( m \) being any
integer number). The two leads are described by the corresponding reservoirs, consisting of non-interacting electrons with the single particle energies \( \varepsilon_{k\alpha} \) (\( \alpha = L, R \)):  
\[
H_\alpha = \sum_{k\sigma} \varepsilon_{k\alpha} a_{k\sigma}^\dagger a_{k\sigma} ,
\]
and the difference between the chemical potentials is proportional to the applied voltage \( V \), i.e., \( \mu_L - \mu_R = eV \). Finally, the tunneling between the QD and the reservoirs is written as  
\[
T = \sum_{k\sigma\alpha} \sigma^\alpha_k \left( a_{k\sigma\alpha}^\dagger d_\sigma + \text{h.c.} \right).
\]

In the wide-band limit of the reservoirs the current through the QD can be obtained by employing the formula  
\[
I = \frac{2e}{h} \sum_{\sigma} \int d\omega \tilde{\Gamma}(\omega) \left\{ f_L(\omega) - f_R(\omega) \right\} \rho_{\sigma}(\omega), \tag{6}
\]
where \( \tilde{\Gamma}(\omega) = \Gamma^L(\omega)\Gamma^R(\omega)/\Gamma(\omega) \) with \( \Gamma^\alpha(\omega) = \pi \sum_k |\tau^\alpha_k|^2 \delta(\omega - \varepsilon_{k\alpha}) \) being the coupling strength between the QD level and the lead \( \alpha \), and \( \Gamma(\omega) = \Gamma^L(\omega) + \Gamma^R(\omega) \). The spectral density of states in the QD is \( \rho_{\sigma}(\omega) = -\frac{1}{\pi} \text{Im} G_{\sigma}(\omega) \) and \( f_{\sigma}(\omega) = 1/(e^{\beta(\omega - \mu_{\sigma})} + 1) \) is the Fermi function of lead \( \sigma \). The calculation of the current in Eq. (6) only needs the DOS in the QD, which can be obtained from the corresponding one-particle Green’s function.

In this work we shall calculate the retarded one-particle Green’s function of the QD employing the following expression:  
\[
G_{\sigma}(\omega) = \frac{1}{\omega - \varepsilon_0 - \sum_{\alpha} \Delta(\omega) - \eta(\omega) - \Sigma(\omega)}, \tag{7}
\]
where \( \Delta(\omega) \) describes the self-energy contribution of the dot-leads tunneling and \( \eta(\omega) = 1/N \sum_m t_\alpha^2 / (\omega - \varepsilon_m) \) accounts for the dot-ring coupling and \( \Sigma(\omega) \) is the self-energy contribution due to the on-site Coulomb interaction \( U \). According to Ref. [3] one can address the latter problem by means of the following ansatz  
\[
\Sigma(\omega) = U n + \frac{a_2 \Sigma^{(2)}(\omega)}{1 - b_2 \Sigma^{(2)}(\omega)}, \tag{8}
\]
where the quantities \( a, b, \) and \( n \) (the occupation number of the QD level) have to be determined self-consistently, and \( \Sigma^{(2)} \) is the second order self-energy in \( U \). This ansatz gives the correct weak and strong coupling limits for all range of parameters, including Kondo, charge fluctuation, and even-number site limit [1,3]. We shall in what follows assume that the energy levels of the ring are half-filled, and consider only the symmetric case for the dot \( \varepsilon_d = -U/2 \). The former assumption implies that the number of electrons in the ring is equal to the number of lattice sites \( N \). In addition, we chose \( U = 1.2t \) and a parabolic form of \( \Gamma_N(\omega) \) centered at \( \omega = 0 \) with bandwidth \( W = 4.8t \).

Let us begin by analyzing the limit of weak tunneling between the QD and the reservoirs (\( \Gamma \ll t^2/2t \)), where we shall neglect the reservoirs in the calculation of the DOS. Moreover, to be consistent with our study of the effect of the reservoirs, we shall assume here a grand-canonical description, even for the isolated ring-dot configuration. Fig. 2 shows the \( N \)-dependence of the DOS for the QD level in the absence of the AB flux. For large \( N \), our results are identical for all values of \( N \), and the DOS is shown to be independent of both \( N \) and \( \varphi \), as expected. It is worth mentioning that the DOS depends on \( N \equiv N \text{ modulo } 4 \) for small sized rings \( (N \lesssim 1000) \) for the parameters used in Fig. 3. Two remarkable features are found by decreasing the size of the ring. First, small satellite peaks appear away from the Fermi level. These peaks are originated by the discrete level spacing in the ring, and become pronounced by decreasing \( N \). Second, the single-particle excitation at the Fermi level - the Kondo resonance in the continuum limit - suffers a drastic change. By reducing \( N \), the Kondo resonance is suppressed \( (N = 4n + 2) \) or split into two peaks with a pseudo-gap, depending on the value of \( N_4 \). Such effects are signatures of destruction of the Kondo cloud, caused by the finite size of the ring. A similar feature has been found in the study of a “Kondo box” [3], which also shows a parity-dependent Kondo resonance. In our case, this situation is more complicated by the orbital degeneracy of the ring as well as by the spin degeneracy.

Fig. 4 shows the phase-dependence of the Kondo resonance for different phases and values of \( N_4 \). In this figure one can see that the Kondo resonance at the Fermi level is strongly phase-dependent for small sized rings, and this resonance depends on \( N_4 \) as well. An interesting point to be considered is that the DOS for a given \( N \) and \( \varphi \) \((\rho_N(\omega, \varphi))\) satisfies the relation \( \rho_N(\omega, \varphi) \sim \rho_{N+2}(\omega, \varphi + \pi) \). These result follows form the expression of the single particle energies of the ring: \( \varepsilon_m = -2t \cos (\frac{1}{2} \pi m + \varphi) \), because for sufficiently large \( N \) \((\gg 1)\), the configuration of the energy levels and its occupation near the Fermi level is invariant under the transformation \( N \rightarrow N + 2 \) along with \( \varphi \rightarrow \varphi + \pi \).

The effects of the coupling of the QD to the leads has been neglected up to here in all our calculations of the DOS, and in Fig. 4 we show those effects for small coupling \( (\Gamma \ll \Gamma') \) and contiguous values of \( N \) corresponding to the four possible configurations. In most cases, the general feature of the DOS is only slightly modified due to this small coupling. As one may expect, the Kondo resonance is slightly enhanced, but for some particular configurations, see for example Fig. 4(b), the DOS near the Fermi level is strongly modified, and the Kondo resonance is dominated by the lead-dot coupling.

In conclusion, we have studied the effect of an
Aharonov-Bohm ring on the Kondo resonance of a mesoscopic quantum dot. To this purpose we have proposed and theoretically investigated a device setup in which the phase-dependent Kondo resonance might be analyzed experimentally. We have found that the Kondo resonance at the Fermi level of the dot is strongly affected by the size of the ring as well as by the magnetic flux, which clearly demonstrates the mesoscopic nature of the Kondo scattering. We have shown that the Kondo resonance is strongly suppressed or split into two peaks depending on the number (modulo 4) of electrons in the ring. It has been further verified that the resonance shows a strong phase-dependence for rings of small size. We have also analyzed the effect of the coupling between the QD and the two leads. We show that the DOS around the Fermi level is modified by the coupling in a manner that depends on $N_4$.

LC wishes to acknowledge M. Foglio for useful comments. This work was funded by the MIPIKS. LC was also supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP). KK acknowledges support by Grant No. 1999-2-11400-005-5 from the KOSEF.

* Electronic address: kckang@moak.chonbuk.ac.kr

[1] For a review, see, e.g., A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge 1993).

[2] D. Goldhaber-Gordon, H. Shtrikman, D. Abush-Magder, U. Meirav and M. A. Kastner, Nature 391, 156 (1998); D. Goldhaber-Gordon et al., Phys. Rev. Lett. 81, 5225 (1998).

[3] S. M. Cronenwett, T. H. Oosterkamp, and L. P. Kouwenhoven, Science 281, 540 (1998).

[4] J. Schmid et al., Physica 256B-258B, 182 (1998).

[5] F. Simmel et al., Phys. Rev. Lett. 83, 804 (1999).

[6] L. I. Glazman and M. E. Raikh, Pis'ma Zh. Eksp. Teor. Fiz. 47, 378 (1988); JETP Lett. 47, 452 (1988).

[7] T. K. Ng and P. A. Lee, Phys. Rev. Lett. 61, 1768 (1988).

[8] S. Hershfield, J. H. Davies and J. W. Wilkins, Phys. Rev. Lett. 67, 3720 (1991); Phys. Rev. B 46, 7046 (1992).

[9] Y. Meir, N. S. Wingreen and P. A. Lee, Phys. Rev. Lett. 70, 2601 (1993); N. S. Wingreen and Y. Meir, Phys. Rev. B 49, 11404 (1994).

[10] A. L. Yeyati, A. Martín-Rodero and F. Flores, Phys. Rev. Lett. 71, 2991 (1993).

[11] J. König, H. Schoeller and G. Schön, Phys. Rev. Lett. 76, 1715 (1996); J. König, J. Schmid, H. Schoeller and G. Schön, Phys. Rev. B 54, 16820 (1996).

[12] K. Kang, Phys. Rev. B 58, 9641 (1998); S. Y. Cho, K. Kang, and C.-M. Ryu, Phys. Rev. B 60, 16874 (1999).

[13] L. Craco and K. Kang, Phys. Rev. B 59, 12244 (1999).

[14] C. Bruder, R. Fazio, and H. Schoeller, Phys. Rev. Lett. 76, 114 (1996).

[15] M. A. Davidovich, E. V. Anda, J. R. Iglesias, and G. Chiappe, Phys. Rev. B 55, R7335 (1997).

[16] W. Izumida, O. Sakai, and Y. Shimizu, J. Phys. Soc. Jpn 66, 717 (1997).

[17] U. Gerland, J. von Delft, T. A. Costi, and Y. Oreg, Phys. Rev. Lett. 84, 3710 (2000).

[18] W. G. van der Wiel, S. De Franceschi, T. Fujisawa, J. M. Elzerman, S. Tarucha, and L. P. Kouwenhoven, Science 289, 2105 (2000).

[19] Y. Ji, M. Heiblum, D. Sprinzak, D. Mahalu, and H. Shtrikman, Science 290, 779 (2000).

[20] V. Ferrari, G. Chiappe, E. V. Anda, and M. A. Davidovich, Phys. Rev. Lett. 82, 5088 (1999).

[21] K. Kang and S.-C. Shin, Phys. Rev. Lett. 85, 5619 (2000).

[22] S. Y. Cho, K. Kang, C. K. Kim, and C.-M. Ryu, cond-mat/0011215.

[23] H.-P. Eckle, H. Johannesson, and C. A. Stafford, J. Low Temp. Phys. 118, 475 (2000); cond-mat/0010101 (unpublished).

[24] I. Affleck and P. Simon, cond-mat/0012002 (unpublished).

[25] W. B. Thimm, J. Kroha, and J. v. Delft, Phys. Rev. Lett. 82, 2143 (1999).

[26] Y. Meir and N. S. Wingreen, Phys. Rev. Lett. 68, 2512 (1992).

[27] L. Craco and G. Cuniberti, submitted to Phys. Rev. B.

FIG. 1. Schematic figure of the device setup.
FIG. 2. Single-particle DOS of the QD level for $\varphi = 0$ and the different values of $N$ (modulo 4). (a) $N=52$ (dot-dashed), $N=100$ (dotted) and $N=1000$ (solid); (b) $N=54$ (dot-dashed), $N=102$ (dotted) and $N=1002$ (solid); (c) $N=53$ (dot-dashed), $N=101$ (dotted) and $N=1001$ (solid); and, (d) $N=55$ (dot-dashed), $N=103$ (dotted) and $N=1003$ (solid). The remaining parameters are $U = 1.2$, $\varepsilon_d = -0.6$, $\Gamma' = 0.08$, and $\Gamma = 0$, in units of $t$.

FIG. 3. Single-particle DOS of the QD level for (a) $N=100$, (b) $N=102$, (c) $N=101$ and (d) $N=103$, and different values of the phase: $\varphi = 0$ (dot-dashed), $\varphi = \pi/2$ (solid) and $\varphi = \pi$ (dotted). The other parameters are the same used in Fig. 2.

FIG. 4. Single-particle DOS of the QD level for (a) $N=100$, (b) $N=102$, (c) $N=101$ and (d) $N=103$, and different values for the dot-leads coupling: $\Gamma = 0$ (dot-dashed) and $\Gamma = 0.03$ (solid). The other parameters are the same used in Fig. 2.