Assessment of the Voltage Influence of Charging Electrical Vehicles on the Distribution Grid

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Abstract. At present, most relevant researches study the impact of charging electrical vehicles (EVs) on distribution network through simulation method. This method requires a large amount of input information and simulation calculation, and requires a large amount of simulation time. The required input information is difficult to be fully satisfied in the actual situation, which is easy to influence the evaluation effect. In order to overcome the drawbacks of the simulation method, an assessment model for evaluating the voltage influence caused by EV charging on distribution grid has been proposed in this paper, which can work with only the traditional distribution network input information for load flow calculation and historical charging load information at charge charging stations (FCS). The proposed model can perform early warning type assessment of voltage change, which can assist the distribution grid operators to find trouble nodes with high probability of voltage violation.

1. Introduction

In recent years, with the rapid development of EVs, the impact of charging EVs on the power grid is increasingly prominent. EVs usually access the low voltage distribution grid, making it easier to encounter transformer overload, voltage problems, cable overload, etc. [1]. Therefore, it is practically significant to research the impact of charging EVs on distribution grid.

Recently, there have been numerous related work. [2] proposes that voltage problems may occur before cable overload. In [3], an integrated algorithm based on Monte Carlo Simulation (MCS) and Time-Series analysis for evaluating EV’s impact on power grid has been proposed, and the research finds that more quickly the charging load changes, faster the transformer tapping operating frequency increases, which may greatly reduce the transformer life and significantly increase the risk of voltage violation. With the augment of EV’s penetration, distribution network investment and energy losses would increase obviously, and when EV’s penetration reaches 62%, energy losses can add up to 40% of actual values in off-peak hours [4]. [5] assesses the impact of EVs charging on distribution grid, which centres on several distribution grid parameters, such as transformer load, feeder load, voltage deviation and energy losses, and obtains the conclusion that 60% EV’s penetration would cause 64.9% load increase with uncoordinated charging through simulation. In [6], authors analyse total power losses and maximum voltage deviation by using the simulation method, and the coordinated charging could effectively decrease the impact of EVs charging on total power losses and voltage deviation. Approaches used to produce EV charging load in above research generally predetermine the charging allocation, which makes the research results more strongly related to the predetermination. To reduce the relationship between research results and predetermination, a probabilistic charging demand model based on random trip chain and Markov decision process is employed to research the temporal-spatial
characteristics of EV charging load in [7]. And in [8], the model is used to assess the effect of EV’s charging on nodal voltage deviation in several simulation scenarios. For most relative work, the complex simulation method is mainstream, which mainly contains two steps as follows:

- Simulate EV’s charging behaviour to produce the charging load by using predetermination method or random method.
- Put the charging load information into power grid assessment model, then using traditional power flow method to calculate the concerned power grid parameters, such as voltage amplitude.

The two above steps all need spend a lot of time on calculation, moreover, either predetermination method or random method needs numerous input information gotten difficultly in practical. If we missed any input information in application, the simulation results might have a large deviation in comparison to the actual situation, thus might mislead decision maker’s judgment.

Some direct sensitivity analysis methods are proposed in [9-11], which try to avoid excessive simulation calculation. [12] proposes a novel concept of probabilistic voltage sensitivity analysis (PVSA) to depict voltage change with a rigid precondition that the random load variation must follow normal distribution with mean 0, which is not the common case, especially for EV load. In this paper, we focus on evaluating the nodal voltage change caused by EV charging load and propose a PVSA-based assessment model to evaluate the nodal voltage change, which could accommodate the random character of EV charging load and be advantageous to decrease the calculation time. Furthermore, only common historical EV charging load data at fast charging station (FCS) and traditional input information used to calculate the power flow are needed.

The paper is arranged as follows. Section 2 builds a linear approximately distribution grid model and proposes the assessment model. Section 3 demonstrates the assessment model proposed in this paper through IEEE 33 node test system. Finally, section 4 draws conclusions from the research.

2. Modelling and methodology

2.1. Linear approximate distribution grid model

Generally, the whole regional low voltage distribution grid contains several source nodes. Because of the open loop operation characteristic, it could be divided into several independent radial distribution grids. Thus, it is reasonable to research a typical radial distribution grid instead. The node where the FCS is will be regarded as FCS node, and the node to be assessed is regarded as observation node. Assume that there is only one FCS, voltage at observation node can be written as:

\[ V_o = V_i - \sum_{e \in E} I_e \cdot Z_e \]  

(1)

where \( I \) is the current through the edge \( e \), \( V \) is the voltage at slack node, \( Z \) is the impedance of edge \( e \), and \( E \) is the set of edges between slack node and observation node. Since \( I \) is equal to \( \sum_{e \in E} (S_e / V^*_{n_e}) \), where \( V^*_{n_e} \) is the complex conjugate voltage at node \( n \), \( N_e \) is the set of nodes \( n \) where edge \( e \) is between slack node and node \( n \), \( S_e \) is the injected complex power at node \( n \), then Equation (1) can be rewritten as:

\[ V_o = V_i + \sum_{e \in E} \left( \sum_{n \in N_e} \frac{S_e}{V^*_{n_e}} \right) \cdot Z_e \]

(2)

Assume that the injected complex power of all nodes \( n \) changes from \( S_e \) to \( S_e' \), the corresponding nodal voltage changes from \( V^*_{n_e} \) to \( V^*_{n_e'} \), and the voltage at observation node changes from \( V_o \) to \( V_o' \), then the new \( V_o' \) can be given by

\[ V_o' = V_i + \sum_{e \in E} \left( \sum_{n \in N_e} \frac{S_e'}{V^*_{n_e}} \right) \cdot Z_e \]

(3)

Rewrite \( S_e' \) and \( V_{o}' \) in increment form as follows:
Then, based on Equation (4) and Equation (3), voltage change $\Delta V_o$ at observation node can be written as:

$$\Delta V_o = V_o - V_o = \sum_{c \in C} \left( \sum_{m \in N_c} \frac{S_n}{V_{n}^*} \right) Z_c = \sum_{c \in C} \left( \sum_{m \in N_c} \frac{\Delta S_n}{V_{n}^*} \cdot V_n + \Delta V_n^* \right) Z_c$$

(5)

Since $\Delta V_o^* < V_o^* (V_o^* + \Delta V_o^*)$ in common practical application, thus Equation (5) can be approximated to the following form:

$$\Delta V_o = \sum_{c \in C} \left( \frac{\Delta S_n}{V_{n}^*} \cdot V_n + \Delta V_n^* \right) Z_c$$

(6)

If the injected complex power of the FCS node changes from $S_n$ to $S_n + \Delta S_n$, then voltage change at the observation node can be written as:

$$\Delta V_{od} = \frac{\Delta S_d}{V_d^* + \Delta V_{od}} \cdot Z_{od}$$

(7)

where $V_o^*$ is complex conjugate voltage at the FCS node, $\Delta V_{od}$ is complex conjugate voltage change caused by the injected complex power change of the FCS node, $Z_{od}$ is shared impedance between the FCS node and observation node from slack node, which is illustrated in figure 1.

**Figure 1.** Shared impedance diagram.

Let $\Delta S = \Delta P + i \Delta Q_n$, $\Delta V_{od} = \Delta V_{od}^* + i \Delta V_{od}^*$, $V_o = V_o^* + i V_o^*$, and $Z_{od} = R_{od} + i X_{od}$, then we can get the equation as follows [12]:

$$\begin{align*}
\Delta V_{od}^* &\leq \left( \frac{\Delta P_d \cdot R_{od} - \Delta Q_d \cdot X_{od}}{V_o^*} \right) \cdot V_o^* - \left( \frac{\Delta P_d \cdot X_{od} + \Delta Q_d \cdot R_{od}}{V_o^*} \right) \cdot V_o^* \left( V_o^* \right)^2 + \left( V_o^* \right)^2 \\
\Delta V_{od}^* &\leq \left( \frac{\Delta P_d \cdot X_{od} + \Delta Q_d \cdot R_{od}}{V_o^*} \right) \cdot V_o^* + \left( \frac{\Delta P_d \cdot R_{od} - \Delta Q_d \cdot X_{od}}{V_o^*} \right) \cdot V_o^* \left( V_o^* \right)^2 + \left( V_o^* \right)^2 
\end{align*}$$

(8)

Let $V_o = |V_o| \angle \theta_o$ and substitute it into Equation(8), then Equation(8) can be turned into polar form as follows:

$$\begin{align*}
\Delta V_{od}^* &\leq \left[ \frac{R_{od} \cdot \cos \theta_d - X_{od} \cdot \sin \theta_d}{|V_o|} \right] \cdot \Delta P_d - \left[ \frac{R_{od} \cdot \sin \theta_d + X_{od} \cdot \cos \theta_d}{|V_o|} \right] \cdot \Delta Q_d \\
\Delta V_{od}^* &\leq \left[ \frac{R_{od} \cdot \sin \theta_d + X_{od} \cdot \cos \theta_d}{|V_o|} \right] \cdot \Delta P_d + \left[ \frac{R_{od} \cdot \cos \theta_d - X_{od} \cdot \sin \theta_d}{|V_o|} \right] \cdot \Delta Q_d 
\end{align*}$$

(9)

Moreover, for the situation having more than one FCS, the total voltage change at the observation node satisfies Superposition Law [12]:
\[
\begin{align*}
\Delta V'_r & \leq \sum_{d \in \mathbf{D}} \left[ \frac{R_{rd} \cdot \cos \theta_d - X_{rd} \cdot \sin \theta_d \cdot \Delta P_d}{|V_d|} - \frac{R_{rd} \cdot \sin \theta_d + X_{rd} \cdot \cos \theta_d \cdot \Delta Q_d}{|V_d|} \right] \\
\Delta V'_i & \leq \sum_{d \in \mathbf{D}} \left[ \frac{R_{rd} \cdot \sin \theta_d + X_{rd} \cdot \cos \theta_d \cdot \Delta P_d}{|V_d|} + \frac{R_{rd} \cdot \cos \theta_d - X_{rd} \cdot \sin \theta_d \cdot \Delta Q_d}{|V_d|} \right]
\end{align*}
\]

(10)

where \( \mathbf{D} \) is the set of FCS nodes. Equation (10) specifies the upper bounds of the real and the imaginary part of voltage change at observation node. Since \(|\Delta V'_r|^2 = |\Delta V'_i|^2 + |\Delta V'_r|^2\), then the upper bound of \(|\Delta V'_r|^2\) can be written as the sum of the square of the upper bounds. The distribution network operators tend to care about the upper bound of \(|\Delta V'_r|^2\), which can provide early warning information. Thus, it is reasonable to rewrite the Equation (10) as:

\[
\begin{align*}
\Delta V'_r & = \sum_{d \in \mathbf{D}} \left[ \frac{R_{rd} \cdot \cos \theta_d - X_{rd} \cdot \sin \theta_d \cdot \Delta P_d}{|V_d|} - \frac{R_{rd} \cdot \sin \theta_d + X_{rd} \cdot \cos \theta_d \cdot \Delta Q_d}{|V_d|} \right] \\
\Delta V'_i & = \sum_{d \in \mathbf{D}} \left[ \frac{R_{rd} \cdot \sin \theta_d + X_{rd} \cdot \cos \theta_d \cdot \Delta P_d}{|V_d|} + \frac{R_{rd} \cdot \cos \theta_d - X_{rd} \cdot \sin \theta_d \cdot \Delta Q_d}{|V_d|} \right]
\end{align*}
\]

(11)

where \(\Delta V'_r\) and \(\Delta V'_i\) refer to the upper bounds of the real and the imaginary part respectively. In this paper, Equation (11) is called linear approximate distribution grid model.

2.2. The assessment model of the nodal voltage change caused by EV’s charging

Actually, the charging load at FCS presents strong randomness in time profile, thus, we can use random variable to depict the charging load. In this paper, we assume that the charging load at FCS follows normal distribution and the corresponding random variable can be given by

\[
P_{p,k}^{th} \sim N(\mu_{p,k}, \sigma_{p,k}^2)
\]

(12)

where \(P_{p,k}^{th}\) stands for the charging load at the FCS connected to \(j\)th node in \(k\)th time profile, \(\mu_{p,k}\) stands for the expectation of \(P_{p,k}^{th}\), \(\sigma_{p,k}\) stands for the standard deviation of \(P_{p,k}^{th}\). \(\mu_{p,k}\) and \(\sigma_{p,k}\) can be gotten through maximum likelihood estimation method. Since the linear conversion of normal distribution is still normal distribution, Equational (12) can be rewritten as:

\[
\begin{align*}
P_{p,k}^{th} & = \mu_{p,k} + \Delta P_{p,k}^{th} \\
\Delta P_{p,k}^{th} & \sim N(0, \sigma_{p,k}^2)
\end{align*}
\]

(13)

where \(\Delta P_{p,k}^{th}\) is the charging load change in \(k\)th time profile. Let \(\mathbf{L}'\) and \(\mathbf{L}^{1}\) be the coefficient vectors. Ignoring the reactive power change, which is based on the reality that the power factor of the FCS is usually close to 1, and according to Equation (11), \(\mathbf{L}'\) and \(\mathbf{L}^{1}\) can be written as:

\[
\mathbf{L}' = \begin{bmatrix}
R_{11} \cdot \cos \theta_1 - X_{11} \cdot \sin \theta_1 \\
\vdots \\
R_{m1} \cdot \cos \theta_n - X_{m1} \cdot \sin \theta_n
\end{bmatrix}
\]

(14)

\[
\mathbf{L}^{1} = \begin{bmatrix}
R_{11} \cdot \sin \theta_1 + X_{11} \cdot \cos \theta_1 \\
\vdots \\
R_{m1} \cdot \sin \theta_n + X_{m1} \cdot \cos \theta_n
\end{bmatrix}
\]

(15)

where \(\theta\) and \(V\) can be calculated using traditional Newton power flow method, when load at \(n\)th node is equal to \(\mu_{n}\). For node without FCS, \(\mu\) is equal to the mean load at the node. Let us define \(\Delta \mathbf{P}=[\Delta P_{1}, \ldots, \Delta P_{n}]\) as normal random vector and assume its elements are independent. According to Equation (10), (14) and (15), we have
\[ \Delta V'_{\text{re}} = (L')^T \cdot \Delta \mathbf{P}^T \sim N\left(0, (L')^T \mathbf{C} L'\right) \]

\[ \Delta V'_{\text{im}} = (L')^T \cdot \Delta \mathbf{P}^T \sim N\left(0, (L')^T \mathbf{C} L'\right) \]

where,

\[ \mathbf{C} = \begin{bmatrix} \sigma^2_{\text{re}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma^2_{\text{im}} \end{bmatrix} \]

where \( \sigma_{\text{re}} \) is zero for node without FCS. Let us define \( \Delta \mathbf{V} = (\Delta V'_{\text{re}}, \Delta V'_{\text{im}})^T \). Due to both \( \Delta V'_{\text{re}} \) and \( \Delta V'_{\text{im}} \) are linear combination of \( \Delta P_{\text{re}}, \ldots, \Delta P_{\text{im}} \), then \( \Delta \mathbf{V} \sim N(0, \mathbf{C}_1) \), where,

\[ \mathbf{C}_1 = \begin{bmatrix} \sum_{j=1}^{n} (L'_j)^2 \sigma^2_{\text{re}} & \sum_{j=1}^{n} L'_j L'_j \sigma^2_{\text{im}} \\ \sum_{j=1}^{n} L'_j L'_j \sigma^2_{\text{im}} & \sum_{j=1}^{n} (L'_j)^2 \sigma^2_{\text{im}} \end{bmatrix} \]

where \( L'_j \) is \( j \)th element of \( \mathbf{L}_r \) and \( L'_j \) is \( j \)th element of \( \mathbf{L}' \). \( \mathbf{C}_1 \) can be diagonalized by eigenvalue decomposition as:

\[ \mathbf{A} = \mathbf{W}^T \mathbf{C}_1 \mathbf{W} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \]

where \( \mathbf{W} \) is the Eigen matrix of \( \mathbf{C}_1 \), \( \lambda_1 \) and \( \lambda_2 \) are eigenvalues of \( \mathbf{C}_1 \). Let us define \( \mathbf{U} = (U_1, U_2) \sim N(0, \mathbf{A}) \), \( U_1 \sim N(0, \lambda_1) \), and \( U_2 \sim N(0, \lambda_2) \). Since the value of \( \text{cov}(U_1, U_2) \) is 0, \( U_1 \) and \( U_2 \) are independent. Because \( \mathbf{U} \) is orthogonal transformation of \( \Delta \mathbf{V} \), then,

\[ \mathbf{U}^T \mathbf{U} = \Delta \mathbf{V}'^T \mathbf{W}^T \Delta \mathbf{V} = \Delta \mathbf{V}'^T \cdot \Delta \mathbf{V} = (\Delta V'_{\text{re}})^2 + (\Delta V'_{\text{im}})^2 = |\Delta V_{\text{re}}|^2 \]

Since \( \mathbf{U}^T \mathbf{U} = U_1^2 + U_2^2 \), then,

\[ |\Delta V_{\text{re}}|^2 = (U_1)^2 + (U_2)^2 \]

Therefore, \( |\Delta V_{\text{re}}|^2 \) is the sum of two weighted independent chi-squared random variables, and the sum of weighted independent chi-squared random variables can be approximated by Gamma distribution [13]. The parameters of \( \Gamma(\alpha, \beta) \) can be calculated as follows:

\[ \begin{align*}
\alpha &= \frac{E(|\Delta V_{\text{re}}|^2)}{D(|\Delta V_{\text{re}}|^2)} \\
\beta &= \frac{E(|\Delta V_{\text{re}}|^2)}{D(|\Delta V_{\text{re}}|^2)}
\end{align*} \]

where \( E(|\Delta V_{\text{re}}|^2) \) and \( D(|\Delta V_{\text{re}}|^2) \) are expectation and variance of \( |\Delta V_{\text{re}}|^2 \), respectively, and can be given by

\[ \begin{align*}
E(|\Delta V_{\text{re}}|^2) &= \lambda_1 + \lambda_2 \\
D(|\Delta V_{\text{re}}|^2) &= 2(\lambda_1^2 + \lambda_2^2)
\end{align*} \]
Then the probability of $|\Delta V_\omega| > \delta$ can be given by

$$F_{\infty}(\delta, \beta, \alpha) = \text{Prob}( |\Delta V_\omega|^2 > \delta^2) = 1 - \int_0^\delta \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} dx$$

(24)

where $\delta$ is a set value in the range of 0 to 0.05. $F_{\infty}(\delta, \beta, \alpha)$ is the function used to estimate the voltage change at the observation node in $k$th time profile, which can provide an effective reference for grid operators. The larger value of $F_{\infty}(\delta, \beta, \alpha)$ is, the greater is the impact of EV’s charging on voltage change at the observation node.

3. SIMULATION

In this paper, IEEE 33 node distribution grid test system is adopted to verify the proposed assessment model. The system reference voltage is 12.66kV and the reference capacity is 100MW. In order to simplify the complexity of simulation, only two FCSs are connected to nodes 9 and 24, respectively, as shown in figure 2, and nodes without FCS adopt constant power model. Figure 3 and figure 4 respectively show the frequency distribution diagram of FCS A and FCS B according to the 6 months’ actual historical charging load data at two real FCSs. Using the maximum likelihood estimation method we can obtain the expectation and variance of the charging load variables, where charging load at FCS A obeys $N(280,108^2)$, and charging load at FCS B obeys $N(316,97^2)$. The unit of the parameters is kW.

![Figure 2. IEEE 33 test system.](image)

![Figure 3. Frequency distribution of voltage change at FCS A.](image)

![Figure 4. Frequency distribution of voltage change at FCS B.](image)
3.1. Verification of the proposed assessment model

Figure 5 shows the probability distribution of voltage change at node 11, where the dashed line stands for the result obtained through the proposed model, as the solid line stands for the statistical result obtained by using traditional Newton power flower method. The procedure of the statistical method is as follows:

- Calculate the reference voltage amplitude at observation node through Newton power flower method, when load at FCS nodes is equal to its expectation, and load at nodes without FCS is equal to its mean value.
- Randomly generate a value obeying $N(280,108^3)$ as charging load at the FCS A, and randomly generate a value obeying $N(316,97^3)$ as charging load at the FCS B.
- Calculate the absolute value of the difference between the voltage amplitude obtained by using Newton power method at observation node and reference voltage amplitude.
- Iterate steps 2 and 3 until 2000 times. Then we can get probability distribution of the voltage change at observation node by using statistical analysis.

Figure 6 shows the probability distribution of voltage change at node 6. From figure 5 and figure 6 it can be seen that the result obtained through proposed assessment model is close to the result obtained through the statistical method demonstrating the effectiveness of the proposed assessment model.

![Figure 5. Probability distribution of voltage change at node 11.](image1)

![Figure 6. Probability distribution of voltage change at node 6.](image2)

3.2. Verification of the early warning characteristic

We calculate the voltage change at each node in IEEE 33 test system by using the proposed model and the statistical method respectively, and figure 7 shows the comparison between the two approaches. When $\delta$ is equal to 0.002, the overall trends of the results obtained by two approaches are similar. However, there are errors at several nodes, such as nodes 7, 9, 15, 24 and 25, which are mainly caused by the approximation of the distribution grid model, and the maximum absolute error which is close to 17% occurs at node 25. In practical application, the distribution grid operator is more concerned with $\delta$ value at 0.01~0.05. With the value of $\delta$ changing from 0.002 to 0.01, from figure 8-10, it can be seen that the voltage evaluation values obtained by the statistical method are lower than those calculated by the proposed model. If the values obtained by the statistical method are taken as the actual values, it shows that the evaluation results obtained by the proposed model are larger than they really are, which can imply the early warning information for the distribution grid operators. The distribution grid operators can pay close attention to the nodes having the assessment values out of the threshold value, and find the worst node among them to take measures to compensate the nodal voltage, for example, in figure 8, if the threshold value was 0.25, the node which might be compensated was node 15.
4. Conclusion

This paper proposes an assessment model to evaluate the impact of EV's charging on nodal voltage change. Through the establishment of the linear approximate distribution grid model, the original nonlinear distribution grid model approximate to linear model. According to the characteristic that the sum of the weighted independent chi-square random variables approximate obey Gamma distribution, we get the assessment function $F_\omega(\delta,\beta,\alpha)$. We demonstrate the effectiveness of the proposed model by using IEEE 33 node test system in this paper. The simulation results show that the probability distributions respectively calculated through proposed model and the statistical method are basically the same change trend, and the evaluation results obtained through the proposed method are usually larger than they really are, which can help the distribution grid operators find the trouble nodes in advance.

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