A self-consistent dynamical model of the Milky Way disc set to Gaia data

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ABSTRACT

Context. Gaia accurate astrometry for many stars in the Milky Way gives an opportunity to reanalyse the Galactic stellar populations from a large and homogeneous sample and to revisit the Galaxy gravitational potential.

Aims. This paper shows how a self-consistent dynamical model can be obtained by fitting the gravitational potential of the Milky Way to the stellar kinematics and densities from Gaia data.

Methods. Using the Besançon Galaxy Model we derive a potential and the disc stellar distribution functions are computed based on three integrals of motion (E, Lz, Iz) to model stationary stellar discs. The gravitational potential and the stellar distribution functions are built self-consistently, and then adjusted to be in agreement with the kinematics and the density distributions obtained from Gaia observations. A Markov chain Monte Carlo (MCMC) is used to fit the free parameters of the dynamical model to Gaia parallax and proper motion distributions. The fit is done on several sets of Gaia data, mainly a subsample of the GCNS (Gaia catalogue of nearby stars to 100 pc) with G < 17, together with 26 deep fields selected from eDR3, widely spread in longitudes and latitudes.

Results. We are able to determine the velocity dispersion ellipsoid and its tilt for sub-components of different ages, both varying with R and z. The density laws and their radial scale lengths, for the thin and thick disc populations are also obtained self-consistently. This new model has some interesting characteristics that come naturally from the process, such as a flaring thin disc. The thick disc is found to present very distinctive characteristics from the old thin disc, both in density and kinematics. This well supports the idea that thin and thick discs were formed in distinct scenarios as the density and kinematics transition between them is found to be abrupt. The dark matter halo is shown to be nearly spherical. We also derive the Solar motion with regards to the Local Standard of Rest (LSR) to be U⊙=10.79 ± 0.56 km s⁻¹, V⊙=11.06 ± 0.94 km s⁻¹, and W⊙=7.66 ± 0.43 km s⁻¹, in good agreement with recent studies.

Conclusions. The resulting fully self-consistent gravitational potential, still axisymmetric, is a good approximation of a smooth mass distribution in the Milky Way and can be used for further studies, including finding streams, substructures and to compute orbits for real stars in our Galaxy.

Key words. Galaxy:kinematics and dynamics, Galaxy:structure, Galaxy:evolution, Galaxy:disk, surveys

1. Introduction

Understanding the Milky Way structure via its dynamics is crucial to figure out Galaxy evolution, as the gravitation is the major force that drives Galaxy shaping. The mass distribution can be derived mostly from light for the baryons but completely relies on dynamical effects for the dark components. The construction of any realistic Galaxy model would need to confront at the same time modelled visible matter to observed distributions and kinematics, and dark matter contributions to the kinematics via the gravitational potential.

Therefore one needs self-consistent modelling approaches that model the visible part, particularly the stellar populations, their imprint on the gravitational field, and how they feel the potential through their motions. The invisible components such as the dark halo have to be considered also with its visible effects on the rotation curve. The third major component, the interstellar matter, is only partly visible and constitutes an important uncertainty on the Galactic mass models, while its dynamics is notably different from the collisionless stellar dynamics.

To understand how the Besançon Galaxy Model (hereafter BGM) fits in with the many existing modellings of the Galaxy, we emphasize that the term Galactic dynamical model covers very distinct approaches. In general, published dynamical models try to find the mass distribution by means of the kinematics of stars, or gas, clusters, satellite galaxies, stellar streams etc. These models propose a decomposition of the Galactic mass distribution in components: i.e. gas, stellar discs and halo, dark matter (for example Miyamoto & Nagai 1975) but they do not generally look for the dynamical consistency which involves each component.
A distinct approach to understanding the structure and history of the Galaxy is to look at stellar populations. For example, the TRILEGAL stellar population code (Girardi et al. 2005) has been used to analyse the absolute colour-magnitude distribution using stellar evolutionary tracks allowing to determine the Galactic star formation history (Dal Tio et al. 2021). A complementary approach is the kinematical modelling (by opposition to dynamical modelling) which consists in describing the stellar density and velocity distributions but without seeking dynamical consistency with the gravitational field. Thus the Galaxia model allowed to generate a synthetic survey of the Milky Way (Sharma et al. 2011).

In the context of our Galaxy, few modelisations exist which combine the stellar density and velocity distributions with the gravitational potential in a dynamically consistent way. We can mention Binney & McMillan (2011) for the analysis of nearby stars and towards Galactic poles, Sharma et al. (2014), Piffl et al. (2014) and Bienaymé et al. (2014)'s study of the stellar RAVE survey.

Finally, we know of only three models of the Galaxy that combine these approaches and the implementation of dynamical consistency in a population synthesis model. These models describe the details of the stellar populations with evolutionary tracks. These are the JJ model of Just & Jahreiß (2010) and Sysolyatina & Just (2021), the ModGal model of Pasetto et al. (2016, 2018) and the Besançon Galaxy Model which is developed here.

Mass models and kinematics. Different methods have been used to investigate the overall mass distribution and dynamics of the Milky Way: from deriving independently density distributions and kinematics, and computing forces from Jeans equations (Ntotsi et al. 2021) to Schwarzschild modelling (Vasiliev & Valluri 2020), made-to-measure modelling (Wegg et al. 2015), angle-action or integrals of motion based models (Binney et al. 2014; Robin et al. 2017).

Contrarily to Ramos et al. (2021) who investigate with Gaia DR2 the deviations from axisymmetry to investigate resonances and substructures, we here attempt to derive an axisymmetric model which reproduces the density and kinematics of the Milky Way in a wide range of Galactocentric distances. This model would represent the mean structure, that is the dominant pattern, which is also directly related to the overall potential and mass distribution.

Dehnen & Binney (1998a) developed a very popular axisymmetric mass model of the Milky Way based on a semi-analytical approach, but at this epoch, the data constraining the distribution functions were mainly limited to the measurement of the rotation curve and the kinematics of the local solar neighborhood, and to few informations on the radial motion of Milky Way satellites, leading to many undeterminations in the model parameters. With the availability of Gaia data (Gaia Collaboration et al. 2016, DR1), (Gaia Collaboration et al. 2018b, DR2), (Gaia Collaboration et al. 2021a, eDR3), new developments of Galaxy models have arisen (McMillan 2017). Without being exhaustive, we mention the new model of Wang et al. (2022) based on the motions of globular clusters from Gaia eDR3, or the analytical model from cepheids from Gaia DR2 data and other surveys (Abilimit et al. 2020).

Some studies also point towards non-axisymmetric structures coming from internal or external perturbations, such as for instance the spiral feature detected by Antoja et al. (2018), numerous stellar streams (Ibata et al. 2021; Malhan et al. 2022), and perturbations from the bar or spiral waves detected at the solar neighbourhood from RAVE, APOGEE or Gaia DR2 (Williams et al. 2013; Bovy et al. 2015; Carrillo et al. 2018; Lane et al. 2021; Trick et al. 2021). However it still remains that the majority of the mass of the Galaxy is predominantly in smooth relaxed components. This is why the development of axisymmetric self-consistent dynamical models of the Milky Way is useful.

Most axisymmetric models are based on analytical functions, such as for instance Miyamoto-Nagai formula (Miyamoto & Nagai 1975) used by the popular model of Allen & Santillan (1991), the Barros et al. (2016) analytical model fitted to the gas rotation curve, or Bovy (2015)’s model fitted to APOGEE data, among others. The analytical functions are sometimes not enough flexible to follow the real forces at work in the Milky Way.

Dynamically consistent modelling. Sanders & Binney (2014, 2016) developed a method called ”Stäckel fudge” which allows the numerical determination of approximate values of the actions $J$ of stellar orbits (integrals of motion which have the dimension of angular momentum) in the case of an axisymmetric potential. This allows to define distribution functions dependent on three integrals of motion (one of which is the angular momentum) for the stellar populations. This method, called $f(J)$ (Binney 2020), has been used to model the densities and kinematics of various stellar samples from the RAVE survey by Piffl et al. (2014). In the present paper, we use a formally equivalent approach and we define approximate integrals with the dimension of an energy (Bienaymé et al. 2015; Bienaymé 2019). These integrals allow us to build stellar disc distribution functions of densities and kinematics. This approach was used for the analysis of stellar populations using RAVE data (Bienaymé et al. 2014; Robin et al. 2017). The merit of the latter method is that the integral expressions are analytical, which presents a considerable simplification in terms of computation.

A self-consistent dynamical approach has been used by Bienaymé et al. (1987) who proposed solutions for the density and kinematics of the stellar populations valid only locally at the Sun position. More recently, Bienaymé et al. (2015) and Bienaymé et al. (2018) developed a more general method to derive a dynamically self-consistent Galactic model assuming axisymmetry and using stellar distribution functions depending on integrals of motion. This method has been extended to non axisymmetric models (Bienaymé 2019).

In this paper we attempt to derive a fully self-consistent dynamical model of the Galaxy based on stellar population synthesis modelling. We take up Bienaymé et al. (2018) method and apply it on the new Gaia data release eDR3 where accurate astrometry is available to characterise the kinematics of stellar populations in a large volume. This approach allows to confront the dynamics with observations of the stellar motions and their spatial distributions, therefore to test the full 6D-space distribution functions. The paper is set as follows. In Sect. 2 we present the principle of the model and how the self-consistency is obtained is explained in Sect. 3. In Sect. 4 the data selection from the Gaia eDR3 is shown while in Sect. 5 we describe the simulations and the MCMC strategy used to derive the model parameters. Results are presented in Sect. 6. The new dynamical model characteristics are presented in Sect. 7. In Sect. 8 we discuss these results in light of previous studies both theoretical and observational. The conclusions and perspectives are outlined in Sect. 9.
2. Dynamics in the BGM

The Besancon Galaxy Model follows a population synthesis approach. It assumes that the Galaxy is made of several stellar components, mainly a thin disc, a thick disc, a bar and a stellar halo, on which non stellar components are added: an interstellar matter disc, a dark matter halo and a central bulge. The population synthesis allows to compute catalogue simulations for the stellar components which are based on assumptions describing the star formation (Initial Mass Function (IMF), Star Formation History (SFH)), and evolution (evolutionary tracks). For generated stars in the simulations, observables are computed using atmosphere models, while a 3D extinction map is used to account for absorption and reddening for every simulated star and error models are added on observables. Then a dynamical model is used to compute star kinematics in a self-consistent manner, that is the total mass distribution of all the components (stellar and non-stellar) is used to compute the gravitational potential of the Galaxy, which in return consistently provides the distribution functions used in computing the observables of the stellar components.

2.1. New dynamical modelling

Previous versions (Bienaymé et al. 1987; Robin et al. 2003) of the BGM proposed a restricted dynamical consistency. In these earlier versions, the mass distribution of all Galactic components was used to reproduce the Galactic rotation curve. The vertical velocity distributions of stellar discs was constrained by their vertical velocity dispersions and the vertical variation of the gravitational potential through the Jeans equation. However, this constraint linking the thickness of the vertical stellar discs to the vertical velocity dispersion was only applied at the solar Galactic radius \( R_0 \). These density laws were mainly Einasto (1979) laws which have very similar shapes as dynamically consistent density laws, i.e. laws depending on three integrals of motion (Fig. 1-2 Bienaymé et al. 2018).

In the new BGM version presented here, the dynamical consistency is not restricted to the Galactic radius \( R_0 \) but is obtained with remarkable accuracy at all Galactic radii \( R \) larger than 4 kpc, and for distances up to 6 kpc out of the Galactic plane. Moreover, the density and kinematic laws of each stellar disc are no longer modelled by empirical laws, but expressed with a function of three integrals of motion \((E, L_z, and I_3)\). This leads to an exact stationary representation of the position and velocity distributions of the stellar thin and thick discs. Our \( I_3 \) integral is partly analogous to the integrals of Stäckel potentials, with a similar approach to the work by Sanders & Binney (2014) but here analytically calculated (Bienaymé et al. 2015).

For doing this, we adopt observational constraints for the rotation curve (Sect. 2.2), and define mass components, some of them being fixed (Sect. 2.3.1) and others adjusted in the process (Sect. 2.3.2).

2.2. Adopted Galactic rotation curve

Our adopted rotation curve is built by pieces. Below \( R=2 \) kpc we do not fit the rotation curve since the gas motions are dominated by non axisymmetric motions.

Between 2 and 6 kpc we use the McGaugh (2018) HI velocity curve based on recent data and the up-to-date values of the Sun-Galactic center distance \( R_0 = 8.122 \) kpc and the Local Standard of Rest (LSR) velocity at the Sun \( V_\odot = 233 \) km s\(^{-1}\).

From 6 to 10 kpc we use the recent determinations of the velocity curve based on Cepheids and DR2 data (Mróz et al. 2019; Ablimit et al. 2020). We set \( V_\odot = 233 \) km s\(^{-1}\) and \( \nabla V_c = -1.35 \) km s\(^{-1}\) kpc\(^{-1}\).

For the outer Galaxy \( R>10 \) kpc up to 60 kpc, we consider a decreasing rotation curve proposed by Cautun et al. (2020) (see figure 6 of Deason et al. 2021). This implies that our mass model has nearly the same Galactic mass at large radii as the model of Cautun et al. (2020) \( M(<100 \) kpc\) = \( 6.1 \times 10^{11} \) \( M_\odot \).

The distance of the Sun from the Galactic centre is taken to be \( R_0 = 8.1 \) kpc, a rounded value in agreement with the determinations of Gravity Collaboration et al. (2018, \( R_0 = 8.122 \) kpc), Gravity Collaboration et al. (2019, \( R_0 = 8.178 \) kpc) and Bobylev & Bajkova (2021, \( R_0 = 8.1 \) kpc).

2.3. Mass Components of the Galactic model

The Galactic mass ingredients are the baryonic components, stellar and interstellar matter (ISM), and a dark matter halo. The mass of these components allows us to compute the total Galactic gravitational potential and with this potential to build dynamically consistent density and kinematics for each stellar disc component. The observed and fitted rotation curves are shown in Fig. 1, together with the contribution of the baryonic and dark matter components.

2.3.1. Fixed components

The shape of a few components remains fixed in the present work: the stellar halo, the interstellar matter (ISM) and the bar, with characteristics already defined (Robin et al. 2003; Robin et al. 2012).

The ISM density distribution is a double exponential law:

\[
\rho_{\text{ISM}} = \rho_0 \exp(-R/h_{R}) \exp(-|z|/h_{z})
\]

with \( h_{R}=7000 \) pc and \( h_{z}=200 \) pc (1)

Its local density is \( \rho(R_0) = 0.0275 \) \( M_\odot \) pc\(^{-3}\), and the local surface mass density \( \Sigma_{\text{ISM}}(R_0) = 11 \) \( M_\odot \) pc\(^{-2}\).

The mass density of the stellar halo is given by:

\[
\rho(R, z) = \begin{cases} 
C_1 a^{-2.44} & \text{if } a > 500 \text{ pc} \\
C_1 500^{-2.44} & \text{if } a < 500 \text{ pc}
\end{cases}
\]

with \( a = \sqrt{R^2 + z^2}/q_{\text{stellar}} \), \( q_{\text{stellar}} = 0.76 \), and the local density \( \rho(R_0, 0) = 9.32 \times 10^{-6} \) \( M_\odot \) pc\(^{-3}\) is used to fix the constant \( C_1 \).

We add a central bulge whose mass density is given by a Plummer sphere to help adjust the rotation curve. This component can be partly stellar and partly dark halo. In Fig. 1 it is included in the baryonic component

\[
\rho(R, z) = \frac{C_2}{(R_{\text{bulge}}^2 + R^2 + z^2)^{2.5}}
\]

with \( R_{\text{bulge}}=1 \) kpc and \( C_2 \) a normalisation constant.

The total mass of the components are for the bulge \( 20 \times 10^9 \) \( M_\odot \) for the ISM, \( 10.8 \times 10^9 \) \( M_\odot \), and for the stellar halo \( 317 \times 10^9 \) \( M_\odot \) inside 20 kpc and \( 318 \times 10^9 \) \( M_\odot \) inside 100 kpc.

2.3.2. Dark halo

To model the Galactic rotation curve, we include a dark matter spheroid. The free parameter of this dark halo are the core radius \( R_{\text{DM}} \), the central density \( \rho_c \) and the spheroidal flattening \( q_{\text{DM}} \).
We point out that, with a flattening $q_{\text{DM}}$ not too different from 1, the density of our halo can be negative close to the $z$-axis but sufficiently far from the Galactic plane so it remains realistic in our domain of interest.

To model a decreasing rotation curve at large $R$ we use a dark matter potential given by:

$$\Phi_{\text{DM}} = -4\pi G \rho_{\text{DM}} K \left( R_{\text{DM}}^2 + R^2 + z^2 / q_{\text{DM}}^2 \right)^{-\gamma}$$

with

$$K = \frac{R_{\text{DM}}^{2\gamma}}{2\gamma^{\gamma+1} q_{\text{DM}}^\gamma}$$

We fixed the exponent $\gamma = 0.05$ of this potential to be able to adjust the decreasing rotation curve for the large values of $R$ from 20 to 50 kpc (Deason et al. 2021). In Table 3 the values of the parameters for the dark matter halo are presented.

### 2.4. Stellar disc distribution functions

The stellar discs are modelled using the population synthesis scheme (Robin et al. 2003) with a thin disc made of 7 sub-components of different ages, a thick disc made of 2 components, young and old (Robin et al. 2017). For each stellar disc component the free parameters to determine are the solar position values for the density, the vertical velocity dispersion as a function of age, and the radial scale lengths. Then the complete stellar disc distribution functions are constrained by the dynamical consistency (see below) and by the fit to the stellar counts and kinematics.

Our stellar population distribution functions (DFs) are 3D generalisations of the Shu (1969) distribution functions for an axisymmetric gravitational potential.

The DFs of positions and velocities of each stellar disc is modelled with a function of integrals of motion and writes as:

$$f(x, v) \sim g(L_z) \tilde{\rho}_0 \exp \left( \frac{R - R_0}{H_{\phi}} \right) \exp \left( \frac{E_0}{\sigma_0^2} \right) \exp \left( - \frac{E_0}{\sigma_0^2} \right)$$

where $R_0$ is the Galactic radius at solar position. $L_z(R)$ is the radius of the circular orbit with angular momentum $L_z$. $E_0$ is the total energy at the solar position. $E_0 - I_1$ and $E_0 - I_3$ are integrals of motion depending on the energy $E$ and a third integral $I_3$ (see Bienaymé et al. 2018), they are linked respectively to the radial and vertical motion of the stars.

The distribution function $f(x, v)$ allows us to compute its various moments that give us the density $\rho(R, z)$, the rotational velocity $v_\phi$, the velocity dispersions $\sigma_R, \sigma_\phi, \sigma_z$, and the tilt angle of the velocity ellipsoid. All these quantities that depend on position $R, z$ and disc components are stored in a table. When a simulated catalogue of stars is created, the kinematical properties of each star are drawn according to the characteristics given in this table.

For each stellar disc, the input parameters of the distribution function are noted with a tilde in Eq. 5. They are: $\tilde{\rho}_0, \tilde{\sigma}_R, \tilde{\sigma}_\phi, \tilde{H}_\phi, \tilde{H}_{R,z}, \tilde{H}_{\phi,z}$, with the index $i$ of the disc component.

The 3 input free parameters are directly related to the computed moments of the DF, the computed density and dispersions at the solar position, respectively $\rho(R_0, z = 0), \sigma_\phi(R_0, z = 0), \sigma_z(R_0, z = 0)$, but do not have exactly the same values.

The other free parameters, $\tilde{H}_\phi, \tilde{H}_{R,z}, \tilde{H}_{\phi,z}$, are related to the scale lengths for the density and for the kinematics. The exact scale length can be computed from the tabulated DF. The density and kinematical laws have a nearly radial exponential decrease beyond $R = 4$ kpc and the computed scale lengths vary with $z$.

At large radii, we have used DFs slightly different from Eq 5. A threshold of 5 km s$^{-1}$ is imposed for the velocity dispersions at very large $R$ where we do not expect that the velocity dispersion becomes smaller than that of the ISM. On the other hand, for radii $R$ smaller than 4 kpc, the velocity dispersions are set to an almost constant value to avoid too large values. These modifications to make the DF more realistic have no impact for the work presented here since they cover domains where we do not make comparison with Gaia data.

The other dynamical constraints reside in reproducing the Galactic rotation curve for $R > 4$ kpc and the constraint on the local density of dark matter $\rho_{\text{dm}}(R_0, z = 0) = 0.010 M_\odot \text{pc}^{-3}$ in the solar neighbourhood (Bienaymé et al. 2014; Salomon et al. 2020). This constraint is mainly satisfied by playing on the flattening of the dark matter halo.

The consistency of the stellar distribution functions and force fields are achieved with an accuracy of the order of one per thousand (see Fig. 1-2 in Bienaymé et al. 2018).

We emphasize that with this new version of the BGM, the number of free parameters of the model is reduced, the asymmetric drift, the azimuthal velocity dispersions $\sigma_{\phi}$, and the tilt angle of the velocity ellipsoid are fully constrained by the dynamical consistency (now, the tilt of the velocity ellipsoid depends not only on the position but also on the stellar population).

### 3. Fitting process

The scheme of the fitting process is summarized in Fig. 2.

With a given set of model parameters as given in Sect. 2, we compute an exact self-consistent dynamical model and stationary disc DFs. We determine the mass distribution of the Galactic model by summing up the density of all components of the
model: stellar discs, stellar halo, bulge, ISM and dark matter halo. The potential is obtained by solving the Poisson equation with the boundary conditions given by a direct calculation of the potential on a rectangular contour located at 60 kpc in \( R \) and 10 kpc in \( z \). The parameters of the dark matter halo are adjusted so that the Galactic model reproduces the Galactic rotation curve.

Then we process the fitting to the Gaia data in two steps:

- Step A: With a Markov Chain Monte Carlo we modify the Shu’s parameters to fit the observed kinematics and density distributions by comparing the simulated catalogue to Gaia data (see Sect. 5). This fit is achieved with a simplified, fast but approximate version of the dynamical model.

- Step B: The parameters obtained with the previous best fit are used to compute a new exactly self-consistent potential and stationary DFs.

After step B, we loop on Step A with the revised potential and we follow the improvement of the likelihood with regards to previous self-consistent model. We iterate steps A and B until there is no improvement of the global likelihood. We monitor the values of scale lengths and densities have not significantly changed (i.e. are within 1 sigma) compared with the exact self-consistent DFs (to ensure that the approximate formulae used are valid).

The exact calculations of the potential and DFs take about ten CPU minutes on a single processor, an amount of time that does not allow the dynamical consistency to be recalculated at each of the tens of thousands of steps in the MCMC chains. To get around this difficulty, we have developed an approximate and fast version of the dynamical model calculation. It allows to build alternative approximate computations of the DFs from an already consistent dynamical model, provided that the Shu’s parameters are only slightly modified, by about 20 per cent at most. This is a partially linearised version of the DFs computation of a consistent dynamical model in the immediate vicinity of another one. The advantage of this approximation of the calculation is that it takes less than a millisecond CPU instead of a few minutes for an exact computation.

For the MCMC chain (step A), the simplified version of the DFs computation is used and when the Shu’s parameters are modified by more than 20%, the exact calculation of the dynamical consistency is repeated (step B). This way of proceeding allows to converge more quickly towards the maximum likelihood and to determine with precision the confidence interval of the various free parameters.

4. Data selection

The comparison of sky densities and kinematics between models and data requires that the data completeness is ensured or that the selection functions of the data are accurately determined and reproduced in the model simulations. For the purpose of constraining the distribution functions of the Milky Way, we have used Gaia eDR3 data (Gaia Collaboration et al. 2021a) and selected stars in the range of apparent \( G \) magnitude between 6 and 17. These magnitude limits are determined to ensure that most of the stars have very reliable parallaxes and proper motions and the sample is as complete as possible. We have not considered here radial velocities because in eDR3 they concern stars brighter than about 12, restricting the data sample too much. Instead, we considered the proper motions and parallaxes and computed the projected tangential velocities along Galactic longitudes and latitudes \( V_l \) and \( V_b \) defined in Equations 6:

\[
V_l = \mu_* \times 4.74/\pi \quad V_b = \mu_b \times 4.74/\pi
\]

where \( \mu_* \) refers to \( \mu \times \cos(b) \). \( \pi \) is the parallax in milliarcsecond and \( \mu_l \) and \( \mu_b \) the proper motions in Galactic coordi-
nates in milli-arcsecond per year. With the value of the constant used (4.74) the transverse velocities are in km s\(^{-1}\).

We make use of two different Gaia samples, a local sample, and a selection of deep fields. In order to best determine the Solar motion as well as the local densities of various populations, we used the Gaia Catalogue of Nearby Stars (GCNS, (Gaia Collaboration et al. 2021b)), which has a very well defined selection function, has accurate proper motions and parallaxes, as well as a completeness over 99% for \(G < 19\) (Gaia Collaboration et al. 2021b). The astrometric accuracies in our selected sample (with \(G < 17\)) are (0.029, 0.026) mas yr\(^{-1}\) along the two proper motion axes, and 0.028 mas on parallaxes. It also depends on the position on the sky due to the scanning law. This is described on the Gaia website\(^1\). On average, the achieved tangential velocity accuracy is about 0.015 km s\(^{-1}\).

This whole sky local sample containing stars up to 100 pc is then divided into 40 sub-fields: 6 bins in latitude with steps of 30°, 8 bins in longitudes with steps of 45° (for \(|b| < 60°\)) or 4 bins in longitudes at the poles for a better statistics, in order to be able to measure tangential velocity distributions in different directions.

Notice that we are not assuming that this sample is an homogeneous sphere. We instead simulate the sample with parallaxes (after applying observational errors) larger than 10 mas with the model, accounting for the different scale heights of different populations, which imply variations of densities depending on age. However we cannot account for density fluctuations due to spiral arms, or other substructures that could be present in the GCNS, because our model is axisymmetric.

To constrain the distribution function outside the local sphere, we also selected Gaia data in cardinal directions (longitudes 0°, 90°, 180° and 270°) and various latitudes (0°, ± 20°, ± 45°, ± 60° and the poles). The data were selected from the Gaia archive within a given radius around each field center (radius of 1.414° for latitudes \(|b| < 20°\), 3° for latitudes ±45°, 5° for \(b = ±60°\) and 8° at the poles to ensure a reliable statistics. We disregarded the Galactic center field where data are noticeably incomplete due to crowding.

Looking carefully at fields where simulated colour distribution disagreed with the data in the space (colour vs parallax) we identified at which parallax there is a cloud of extinction not well modeled. In this way we distinguished two fields (l=270°, b=0°, and l=180°, b=0°), where a cloud was visible in the data deviating the median colours at parallax < 0.7 mas. For these fields we limited the comparison between model and data at parallax > 0.7 mas. We identified a field with a large discrepancy between model and data which may be due to the Monoceros Ring (Ibata et al. 2003; Nordström et al. 2004) or to the warp, as a matter of debate: When comparing parallax distributions between l=180°, b=20° and l=180°, b=−20°, one sees clearly an excess of stars in the north field at parallaxes < 0.7 mas with respect to the south field. Therefore we also disregarded in the global comparison stars in this field. We also eliminated the field at l=90°, b=0°, where our extinction model (see Sect.5) was insufficient to explain the CMD. We finally have 39 subfields in the local sample and 26 deep fields.

In order to help having information on ages of the stellar population we also make use of a pseudo-absolute magnitude, defined as:

\[
M_G = G + 5 \times \log_{10}(\sigma \times 1000) + 5
\]

(with the parallax \(\sigma\) being in unit of mas).

\(M_G\) corresponds to the true absolute magnitude when extinction is negligible and parallax error small. Because the simulations are done in observable space, including extinction and observational errors, the simulated \(M_G\) is directly comparable with the observed one. We selected stars in the range 1 to 7 mag in \(M_G\), to avoid low mass stars (masses below \(\approx 0.6 M_\odot\)) which are not well simulated in the BGM, lacking good stellar models for low mass stars.

For each field we then use the pseudo absolute magnitude \(M_G\) between 1 and 7 divided in 3 bins, and further selected \(\sigma > 0.4\) mas, to avoid distant regions where the parallax contains very little information on the distance, unless the 3 fields with Galactic coordinates (270°,0°), (180°,0°) and (180°,20°) for which stars are selected with \(\sigma > 0.7\) mas (see above). In total our sample contains in total 545,280 stars, i.e. 44,580 in local fields extracted from the GCNS, and 500,700 stars in the deep fields.

5. Simulations and MCMC strategy

5.1. Basic parameters

Simulations of the data samples are done using a revised version of the BGM, where evolutionary tracks have been updated using STAREVOL library (Lagarde et al. 2017, 2019) for stellar masses larger than 0.6 \(M_\odot\), while the Initial Mass Function (IMF) and Star Formation History (SFH) of the thin disc were determined from an analysis and fit to Gaia DR2 (Mor et al. 2018, 2019). This version of the BGM used in these works is referred to Mev2011.

In the BGM scheme, the stars are generated from a mass reservoir from which their mass is withdrawn. The quantity of mass in the mass reservoir is computed according to the stellar density of the sub-component (the 7 age bins of the thin disc and 2 age bins for the thick disc) in the volume element considered (defined by \(R\) and \(\zeta\) and the geometry of the cone for the direction of observation).

The mass of each star is drawn following a 3 slope IMF, and its age is drawn in the age range considered for the sub-component considered. The metallicity is also drawn according to the assumed age-metallicity relation. Then the star is followed on the corresponding evolutionary track interpolated in mass, age and metallicity in the grid and, if the age is not greater than the theoretical stellar lifetime, the astrophysical quantities of the star (temperature, luminosity, gravity, radius) are obtained from corresponding interpolated tracks.

In previous model versions (from Czekaj et al. (2014), the mass in remnants was estimated from the SFH and withdrawn previously to the start of the star generation process. In the present version for the sake of consistency, the stars that are found to be at the end of their life are treated specifically: we compute both the mass of gas released into the ISM and the mass subsisting in the remnant using the initial to final mass ratio from Kovetz et al. (2009) The mass regained by the ISM is added to the mass reservoir of the same age sub-component, assuming instantaneous recycling. From the time being, we do not follow the stars over theoretical white dwarf (WD) tracks. They are instead generated as in previous BGM versions using pre-computed Hess diagrams but with updated WD luminosity functions (Liebert et al. 2005) In simulations we take into account binaries by drawing stars in the mass of gas available at a given position, first singles and primary stars, then secondaries

\footnote{\url{https://www.cosmos.esa.int/web/gaia/science-performance}}
with a proportion which depends on the primary mass, as explained in Czekaj et al. (2014). The binary fraction and distributions of semi-major axis and eccentricities follow the prescription of Arenou (2011). After the simulation is done and the apparent separation of the binary components is computed, we assume that, likewise in Gaia data, the binaries are separated when their projected distance is larger than 0.4 mas. Otherwise, we merge the two components and attribute to the unresolved system the total flux in each photometric band. The kinematics of the binary system is the same as the two components, neglecting orbital effects.

In this self-consistent version, as explained in Sect. 2 the density laws in the discs are self-consistently computed from stellar DFs and the gravitational potential. They are available as tables which are interpolated during the simulation. The kinematics of each star is also obtained from tables at any position in \((R, z)\) and for each disc sub-component. As in previous versions the thin disc has 7 sub-components of different ages between 0 and 10 Gyr, and the thick disc is modelled by 2 components: the so-called young thick disc for which the SFH follows a truncated Gaussian centered on 10 Gyr, with a standard deviation of 2 Gyr and truncated between 8 and 12 Gyr, and the old thick disc with a SFH centered on 11 Gyr, with a standard deviation of 1 Gyr and truncated between 10 and 13 Gyr (Nasell 2018). The initial kinematical parameters and density laws for the thick discs were taken from the fit to RAVE survey data combined to Gaia DR1 (Robin et al. 2017).

In simulations, we make use of a 3D extinction map which is a combination of Lallement et al. (2019) whole sky map, called Stilism map, extended by Marshall et al. (2006) at latitudes \(|b| < +10^\circ\). For the purpose of continuity between the two maps, the Stilism map has been modified to use Marshall’s map as a prior for running a specific solution from the inverse method used to build Stilism (Lallement, private communication). This specific solution is used in our simulations.

### 5.2. Simulations of Gaia data samples

Proper motion and parallax errors are simulated as random errors following Gaussian distributions assuming an error determined by the Gaia DPAC, which values depend on magnitude, colour and position on the sky (see Sect. 4). These random errors are added on the simulated motions in order to best reproduce the data.

We apply the same selection for \(M_V\), colours and parallaxes on the simulations as on the data. Those initial simulations are called "mother simulations" and will be modified during the fitting process. The simulated catalogues are completely recomputed after each Step B, as explained in Figure 2, while they are only modified (kinematics of each star recomputed and weights applied to each star according to the new density law) during the MCMC fitting process (step A).

### 5.3. MCMC strategy

Data and simulations are divided on the sky in bins corresponding to directions (39 in local sample, 26 in deep fields), and along each direction in bins of logarithm of the parallaxes. For each of these subsamples we compute the quantities of the distributions in \(V_T\) and \(V_b\) \((0.1, 0.25, 0.5, 0.75, 0.9, 0.95)\), and compare their values between model and data. The goodness of fit is estimated from the sum of the square of the difference between model and data divided by the standard deviation, for each quantile and for each of the transverse velocity coordinates in each bin. In this process, which relates to Approximate Bayesian Computation (Marin et al. 2011, ABC-MCMC), the likelihood is not computed analytically, as it would be too complex.

The \(\log(\sigma)\) distributions, for parallaxes larger than 0.4 mas (0.7 in the case of 3 specific fields, see above), are binned in steps of 0.1, giving 15 bins in deep fields. The relative difference of the \(\log(\sigma)\) distribution between model and data is added to the goodness of fit with a normalisation factor so that both constraints (goodness of fit of the quantiles of velocity distributions and \(\log(\sigma)\) relative difference) are roughly of the same order in most fields. In this way the density distributions and the kinematics are constrained simultaneously.

The density parameters considered in the MCMC fit are the local density in the thin and thick discs, and the Shu’s parameters as explained in Sect. 3. For kinematics we also fit the three components of the Solar motion \(U_0, V_0, W_0\). To avoid too many free parameters and degeneracies, we considered that the Shu’s scale lengths for the thin disc are globally modified by a single factor for all age components, as well as the radial to vertical velocity dispersion ratio \(\tilde{\sigma}_r/\tilde{\sigma}_z\). Moreover, instead of fitting \(a, \sigma_r\) for each age component we assume that the age-velocity dispersion relation (AVR) follows a power law with 2 free parameters:

\[
\tilde{\sigma}_r(\tau) = k \times \tau^\beta
\]

where \(\tau\) is the age of the population in Gyr, and \(k\) and \(\beta\) the fitted parameters. For each age sub-component we take the mean age for the value of \(\tau\).

We impose that the relative distribution of local stellar densities as a function of the age is given by the star formation history discussed in Sect. 5. The local thick disc density remains a free parameter determined during the fitting process. Old thick disc parameters were first considered to be fitted but were not sufficiently constrained with the set of data used here, being minor everywhere.

The fitting process is done in several steps as already explained in Sect. 2 and summarized in Fig. 2. We present below the results obtained after several iterations on steps A and B until convergence.

### 6. Results

Table 1 presents the parameters fitted by the MCMC process during the step A of the last loop, the range of parameter values (min and max) that we allowed in the chains, median and uncertainties determined using the tail of the best Markov chains. The uncertainty is computed as the difference between the 3rd and 1st quartiles. Thanks to the large sample of local stars, we determined the solar motion with very good accuracy, as well as the velocity dispersions for the thin and young thick discs (within 2 km s\(^{-1}\)), as seen in Table 1. For Shu’s parameters and densities, we derive parameter values relative to the exact self-consistent model obtained in Step B. Hence the parameter range is 0.8 to 1.2, corresponding to the 20% range where the simplified computation of the new model is accurate enough (Sect. 2). We see that the values are all in this range and compatible with the value of 1 within 1 sigma. Therefore these models are the final models and cannot be improved further with this data set. As seen in Table 1 a reasonable accuracy of 10% to 15% is reached on the density and velocity dispersion scale lengths, and local density.

We present in Table 2 for each age component the derived values of the Shu’s DF parameters for the fully self-consistent
model. In order to compare with similar works, we also show in the last columns the local values of the radial and vertical velocity dispersions.

In Table 3 we present the fitted parameters of the dark matter halo in the final model.

The determination of some of the parameters suffers from degeneracies as shown in Appendix A. One notices for instance the correlation between \( k \) and \( \beta \) the parameters of the AVR (see Eq. 8), although all acceptable combinations of the two produce very similar relations. The thick disc velocity dispersions are also slightly correlated with the thin disc ones (in the parameters \( \sigma_R/\sigma_z \), \( k \) and \( \beta \)), probably due to the mix of the two populations in many fields. Fields where the thick disc is not polluted by the old thin disc are scarce. In order to improve this, information about abundances (in particular in \( \alpha \) elements) would be necessary. Spectroscopic surveys such as APOGEE, LAMOST, GALAH or in the near future Gaia DR3, WEAVE and 4MOST, will be most appropriate to improve the study by better splitting the thick from the thin disc. However this will be at the expense of more complex selection functions to be accurately estimated and applied on simulations.

In Appendix B we show the overall characteristics of the best fit model, notably how the density and kinematics values vary with \( R \) and \( z \) for each age component.

In order to assess the reliability of the fit we explored first the density distributions obtained and compared them with the data in the Galactocentric coordinates \(( R, z \)). Notice that the actual star counts are representative of the true density convolved with the selection function which is the same for model and data. Moreover, while in the fit we did not use the distance, but the parallax distribution, in this post-fit validation tests, we consider pseudo distances, taken as the inverse of the parallax, and compute pseudo \(( R, z \)) from it. The true \(( R, z \)) position of stars generated in the simulation are disregarded. To allow fair comparisons between model and data, we compute pseudo \(( R, z \)) in the simulation from the parallax with errors. In the case of our data set the error on distance in this way is small because of our selection of relatively bright magnitudes \( G < 17 \), and parallaxes larger than 0.4 mas. To alleviate the reading of the paper, we dispose in Appendix B figures comparing densities of data and model in \(( R, z \)) space, and tangential velocity plots of the medians and standard deviations for different sample selections. We present here a summary plot for the density in Fig.3 and velocity histograms in Fig. 4.

Figure 3 presents the median density on a grid of pseudo \(( R, z \)) computed as explained above for the data, the fitted model, and the relative difference and the difference between the two relatively to the Poisson noise \( \sqrt{N_D} \). The relative difference has a mean of \(-0.14\), a median of \(-0.09\) and a standard deviation of 0.37. We notice some systematics, particularly along the \( z \) axis where the vertical wave at the Sun noted by Bennett & Bovy (2019) and Salomon et al. (2020) appears clearly, particularly towards the north with an excess in the data at \( z = 900 \text{ pc} \), and deficits around \( 400 \text{ pc} \) and \( 2 \text{ kpc} \).

The excess in the model near the plane is mainly due to an excess in bright and young stars. We noticed that the model is in excess of massive stars (by about 50%) but they represent only 4% of the selected sample and should not impact the global fit. It can be due to either the IMF at high masses (mass larger than 1.5 \( M_{\odot} \)) which would have a too shallow slope (the assumed IMF slope at high mass is \( \alpha = 2.5 \)), or the recent star formation rate (ages below 2 Gyr). We performed several tests to verify this assertion by changing the IMF slope. The result on the dynamical fit was exactly the same. The massive stars do not constitute a major component of the stellar mass. Therefore, this is not a problem for the dynamics. However we shall reconsider the IMF and SFH of the model in a near future using the most recent Gaia DR3 data.

We finally present comparisons of histograms of \( V_t \) and \( V_{lt} \) for the local and deep fields, and for different pseudo \( R \) ranges in Figure 4. The overall agreement clearly appears here. But the sample is also sufficiently large to show some sticking points, specially in the wings of the distributions. We point out that, at \( V_{lt} > 200 \text{ km s}^{-1} \) on the outer side \(( R > 9 \text{ kpc} \) cyan line in top-right panel) and at \( V_t < -150 \text{ km s}^{-1} \) on the inner side \(( R < 8 \text{ kpc} \), magenta line in top-right panel) the model presents a lack of stars

| Parameter | Unit     | Min  | Max  | Median | error  |
|-----------|----------|------|------|--------|--------|
| \( U_{\odot} \) | km s\(^{-1}\) | 8.   | 12.  | 10.79  | 0.56   |
| \( V_{\odot} \) | km s\(^{-1}\) | 6.   | 13.  | 11.06  | 0.94   |
| \( W_{\odot} \) | km s\(^{-1}\) | 5.   | 10.  | 7.66   | 0.43   |
| \( k \) | km s\(^{-1}\) | 4.   | 8.   | 4.59   | 0.14   |
| \( \beta \) |        | 0.25 | 0.75 | 0.54   | 0.02   |
| \( \sigma_R/\sigma_z \) | km s\(^{-1}\) | 28.  | 50.  | 37.91  | 2.17   |
| \( \sigma_{z,thin} \) | km s\(^{-1}\) | 25.  | 40.  | 30.96  | 0.96   |

Factors relative to fully self-consistent dynamical model:

| \( H_{\sigma_{R,thin}} \) factor |        | 0.8  | 1.2  | 1.06  | 0.14   |
| \( H_{\rho_{thin}} \) factor    |        | 0.8  | 1.2  | 0.93  | 0.12   |
| \( H_{\rho_{Rthin}} \) factor   |        | 0.8  | 1.2  | 1.01  | 0.11   |
| \( H_{\sigma_{R,thin}} \) factor|        | 0.8  | 1.2  | 0.98  | 0.14   |
| \( \rho_{thin} \) factor        |        | 0.8  | 1.2  | 1.01  | 0.04   |
| \( \rho_{R,thin} \) factor      |        | 0.8  | 1.2  | 0.98  | 0.08   |
| \( H_{\rho_{Rthin}} \) factor   |        | 0.8  | 1.2  | 1.04  | 0.17   |
| \( H_{\rho_{thin}} \) factor    |        | 0.8  | 1.2  | 0.99  | 0.10   |
Table 2. Parameters of the final model for each age component. First column gives the disc component (index 1 to 7 for the thin disc, "ythd" for the young thick disc). The second column indicates the mean age (Gyr) of the component in the population synthesis model. The third column gives the local density. The fourth to eighth columns give the Shu’s parameters corresponding to the radial and vertical velocity dispersions, scale lengths of density, radial dispersion and vertical dispersion. The last columns indicate the values at the Sun position of $\sigma_R$ and $\sigma_z$ for each age sub-component for comparison with other studies. The parameters of the first age component of the thin disc are not adjusted in our process, due to the lack of axisymmetry.

| Component | Mean age (Gyr) | $\rho_0$ (M$_\odot$ pc$^{-3}$) | $\bar{\sigma}_R$ (km s$^{-1}$) | $\bar{\sigma}_z$ (km s$^{-1}$) | $H_\rho$ (pc) | $H_{\sigma_\rho}$ (pc) | $H_{\sigma_z}$ (pc) | $\sigma_R(R_0,0)$ (km s$^{-1}$) | $\sigma_z(R_0,0)$ (km s$^{-1}$) |
|-----------|---------------|-----------------|----------------|----------------|-------------|-------------|-------------|------------------|------------------|
| 1         | 0.075         | 1.968 x 10$^{-3}$ | 9.50          | 3.63           | 2852.0      | 9289.0      | 14508.0     | 16.7            | 5.0              |
| 2         | 0.575         | 0.598 x 10$^{-2}$ | 15.49         | 5.92           | 2852.0      | 9979.0      | 13841.0     | 20.54           | 10.86            |
| 3         | 1.500         | 0.446 x 10$^{-2}$ | 20.10         | 7.69           | 2852.0      | 8357.0      | 10636.0     | 25.39           | 12.59            |
| 4         | 4.000         | 0.556 x 10$^{-2}$ | 25.54         | 9.77           | 2852.0      | 8357.0      | 10636.0     | 31.27           | 14.63            |
| 5         | 6.000         | 0.595 x 10$^{-2}$ | 31.41         | 12.01          | 2852.0      | 8357.0      | 10636.0     | 37.92           | 16.80            |
| 6         | 8.500         | 0.110 x 10$^{-1}$ | 37.52         | 14.35          | 2852.0      | 8357.0      | 10636.0     | 45.24           | 19.04            |
| ythd      | 10.000        | 0.267 x 10$^{-2}$ | 40.18         | 31.86          | 2080.0      | 5986.0      | 6703.0      | 54.66           | 40.53            |

Fig. 3. Number density as a function of pseudo $R$ and $z$ (see text) in the selected data set. Gaia data $N_D$ (top left) and our final model $N_S$ (bottom left). Difference between data and model relative to the Poisson noise ($N_D-N_S$)/sqrt($N_D$) (top right). Relative difference of counts ($N_D-N_S$)/$N_D$ (bottom right). The relative difference shows that the model accuracy is generally better than 10% apart from the regions showing the vertical wave (see text).

in the wings of the distribution. In particular our model does not include the Gaia-Enceladus component which contributes to the wings at $V_{tot}>200$ km s$^{-1}$ as shown in Gaia Collaboration et al. (2018a). However globally the model present a slight excess of halo (or may be of the old thick disc) population in the wings, a problem that will be handled in a near future. In our sample it concerns only a small portion of the stars in the wings so that it should not bias our result concerning the thin and thick discs.
Fig. 4. Histograms of the transverse velocities for the local sample and for deep fields, with pseudo-$R$ smaller than 8 kpc or larger than 9 kpc: $V_t^L$ (top row), $V_t^B$ (bottom row). Left column: local data (continuous black line), local model (magenta dashed line), deep fields data (continuous grey line), deep field model (dashed cyan line). Right column: $R < 8$ kpc data (continuous black line), $R < 8$ kpc model (magenta dashed line), $R > 9$ kpc data (continuous grey line), $R > 9$ kpc model (dashed cyan line).

Table 3. Parameters of the dark matter halo in the final model. $\rho_c$ is the central mass density, $q_{DM}$ is the ellipsoidal flattening, $R_{DM}$ is the core radius.

| Unit   | Value |
|--------|-------|
| $\rho_c$ | $M_\odot$ pc$^{-3}$ | 0.202 |
| $q_{DM}$ | 1.054 |
| $R_{DM}$ | pc | 3315 |

7. Characteristics of the new dynamical model

7.1. Comparison with previous BGM

The density laws, constrained by the dynamics, are significantly different from those of the previous model (Robin et al. 2003) which followed Einasto laws and applied the self-consistency principle at $R_0$ as in Bienaymé et al. (1987) assuming the age-velocity relation of Gómez et al. (1997).

Figure 5 shows the variation in density as a function of $z$ at the Solar Galactocentric radius for the different age components for the Mev2011 version and the new dynamical model. The difference is small for ages younger than 2 Gyr and ages older than 5 Gyr, but significant for intermediate ages.

7.2. Outer Galaxy flare

As a distinctive feature the model naturally produces a flare in the thin disc, while it is not so significant in the case of the thick disc. This is seen in Fig. B.1 on the intervals of the iso-density contours which appear to be wider at high $R$ than at the Solar neighbourhood. Figure 6 illustrates the flare in the thin disc (top panel) by showing the vertical decrease for age components 2, 4 and 7 of the thin disc, at a Galactocentric radius of 8, 12 and 16 kpc. The apparent scale height increases clearly with $R$ in all thin disc age components. Considering the thick disc, Fig. 6 (bottom panel) shows the contrary for the thick disc, getting a smaller scale height when $R$ increases from 8 to 16 kpc. The thick disc is not flaring, but rather on the contrary it shrinks when going in the outer Galaxy. It clearly indicates a different dynamical origin for this population compared to the thin disc.
7.3. Age-velocity dispersion relation

The AVR is an important result of our fit presented in Fig. 7. It shows a significant increase with age, although we do not see a net plateau, as was found by Gómez et al. (1997) for example. We shall further discuss this result with the literature in Sect. 8. Generally the AVR is given at the solar position, while in our case the velocity dispersions vary both in $R$ and $z$ as can be seen in Fig. B.1, and differently from a thin disc component to another one.

7.4. Highlight on the dichotomy between thin and thick discs

It is interesting to compare the characteristics of the thick disc generated self-consistently with the ones of the old thin disc, as it can enlighten their formation history. As pointed out above, the thick disc does not present any flare while it is strong in the thin disc population. This is in line with the results of many spectroscopic surveys, the thick disc, selected by high-alpha abundances, has its density dropping fast in the outer Galaxy while the low-alpha thin disc remains prominent and even found at higher $z$ in this region (Hayden et al. 2015, among others).

Locally, the $\sigma_z$ of the young thick disc is well above the one of the thin disc, with a value of 40.5 km s$^{-1}$. It is about twice the value of the old thin disc at the age of 10 Gyr. If one carefully looks at the kinematic distributions in the $(R, z)$ plane (Appendix B), some other striking differences appear between the oldest thin disc (age component 7 with age range 7-10 Gyr, in column 6 in Fig. B.1) and the young thick disc in column 7. It can be even better seen in Fig. 8.

As shown in Fig. 8, the difference in kinematics between old thin disc and thick disc populations is mainly in the vertical velocity dispersions, while the difference is weaker when we consider the radial velocity dispersion or the tilt angle of the ellipsoid. The tilt seems to change smoothly from one component to the other (see Fig. B.1 last row). On the other hand the mean rotation velocity are also significantly different, i.e. the asymmetric drift is about twice in the thick than in the old thin disc at a given $z$ at the solar Galactocentric radius. However in the thick disc $V_\phi$ decreases most significantly when going at large $R$ and $z$ (as shown in magenta curves in bottom right of Fig. 8).

Our model shows that the dichotomy between the old thin disc and the thick disc clearly appears in the DFs of our self-consistent model. If one assumes that our thick disc corresponds to the $\alpha$ rich population, it well explains previous results showing that the thick disc defined as such has a density which drops in the outer Galaxy.

7.5. Mass distribution, radial and vertical forces

Our model provides new measures of the mass distribution of our Galaxy and its gravitational forces. The observational constraints are based on measurements of the Galactic rotation curve and the local dynamical measurement of the dark matter mass density (from the Oort limit and $K_z$ measurements). On the other hand, the constraints on the stellar counts give also a measure of the stellar mass. Finally, taking into account the stellar kinematics brings an additional constraint by considerably reducing the number of free parameters while ensuring the dynamical consistency of the model by fitting stationary distributions.
In this study we adopted a dark halo with a local density of 0.010 \( \frac{M_\odot}{pc^3} \) (Salomon et al. 2020) smaller than Bienaymé et al. (2015). This value is similar to those used by the previously cited works. A small change of 10 per cent of this adopted value would change only by 1 per cent the total local density and vertical forces. Therefore it would induce a very small change on the computed vertical stellar density distributions. Moreover, increasing the local dark matter density by ten percent is nearly equivalent here to the flattening of the dark halo by about ten percent, thus recovering nearly the same Galactic rotation curve. It implies that the uncertainty on the local dark matter has only a weak impact on our modelling. We can conclude that, assuming a local density of 0.010 \( \frac{M_\odot}{pc^3} \) the dark matter component is nearly spherical, \( q \sim 1 \) at least in the inner 20 kpc of the Galaxy.

The vertical force as a function of \( R \) is presented in Figure 9 for \( z=0.5, 1.1 \) and 2 kpc, and as a function of \( z \) for three different Galactic radius \( R=6, 8.1 \) and 10.2 kpc.

Wang et al. (2022) also performed a global dynamical model of the Galaxy and used various constraints utilizing a simplified stellar disc mass model. They obtained results close to ours (see Figure 9) for the vertical force distribution at \( z = 1.1 \) kpc (see figure 7 in Wang et al. 2022) and for \( R > 8 \) kpc which is also quite comparable to the measurements of Bovy & Rix (2013). We emphasize that their parametrisation of the stellar disc is different from ours with a shorter density scale length and explain the difference of \( K_e \) forces for Galactic radii \( R < 8 \) kpc.

Nitschai et al. (2021) developed a dynamical model of the Milky Way based on 6D phase space data from APOGEE and Gaia eDR3. Their analysis is based on solving the Jeans equations to model the stellar velocities and dispersions. They obtain Galactic mass distributions in the range of Galactic radii from 5 to 19 kpc with values of \( K_e(R) \) (see figure 7 in Nitschai et al. 2021) also similar to the Wang et al. (2022) ones.

As for the mass distribution of the stellar discs, we find for the surface mass density of all discs at the solar position \( \Sigma(R_0) = 32.2 \frac{M_\odot}{pc^2} \). This makes \( \Sigma_{\text{baryon}}(R_0) = 43.2 \frac{M_\odot}{pc^2} \) lower than the value \( \Sigma_{\text{baryon}}(R_0) = 54.2 \frac{M_\odot}{pc^2} \) (Read 2014) and close to the value of \( \Sigma_{\text{baryon}}(R_0) = 48.7 \frac{M_\odot}{pc^2} \) (see table 2 in Flynn et al. 2006). Figure 9 shows the distribution \( \Sigma(R) \) as a function of the Galactocentric radius. The distribution is quasi-exponential with a scale length of \( R_\Sigma = 3.48 \) kpc in the interval \( R = 6 \) to 10 kpc for the sum of all stellar disc components. This value is significantly larger than the value of 2.5 kpc derived from the \( K_e \) measurements by Bovy & Rix (2013). We must note that for the thin discs the effective scale lengths vary and increase significantly with \( z \), thus the resulting surface density scale length for thin disc components are about \( R_\Sigma = 3.9 \) kpc. For the young thick disc it is \( R_\Sigma = 2.31 \) kpc.

8. Discussion

Our method allowed us to derive a self-consistent model for the stellar populations with densities consistent with the velocities and with the Galactic gravitational potential, and which is able to reproduce the corresponding Gaia data in an extended volume around the solar position. This is a very useful step to ensure that the Milky Way potential we have is realistic.

Our model is not strictly comparable with analyses where the density and velocity distributions are derived from observational samples. Those analyses are generally biased by the sample selection, while our model itself is not. These studies present different ways to circumvent the problems of biases, including partial modelling and different assumptions. However this is safer to compare data and models in the observational space and to reproduce as accurately as possible the selection function, as we do in the present work.

Therefore a robust comparison between different data samples should be done by simulating observed samples one by one with population synthesis model, accounting for observational errors, as we do for Gaia data in the present work. Even though, in this section we estimate whether our model provides similar conclusions about the kinematics of the stellar populations of the Milky Way as other models or studies. We particularly discuss the question of the Solar motion, disc kinematics and density laws.

8.1. Solar motion

Our analysis leads to a Solar motion \( U_\odot = 10.79 \pm 0.56 \) km s\(^{-1}\), \( V_\odot = 11.06 \pm 0.94 \) km s\(^{-1}\), and \( W_\odot = 7.66 \pm 0.43 \) km s\(^{-1}\), with respect to the LSR. Wang et al. (2021) presented a summary of most references in the literature concerning the Solar motion, showing that \( W_\odot \) has a consensual value of about 7 km s\(^{-1}\) in recent works, while \( U_\odot \) accepted range still covers 7 to 12 km s\(^{-1}\), and \( V_\odot \) has got even more conflicting values, ranging between 1 and more than 20 km s\(^{-1}\). From LAMOST data and Gaia DR2, using a local sample of A-type stars they find the mean Solar mo-
Fig. 8. $\sigma_R$, $\sigma_\phi$ and $\sigma_z$ as a function of $z$ for 3 different Galactocentric radius $R=6$, 8 and 10 kpc, in red, blue, and green resp. for the thin disc component 7 (solid line) and the young thick disc (dotted dashed). Bottom right: Mean azimuthal velocity as a function of $R$ for different $z$ (red: $z=0$; blue: $z=1$ kpc; green: $z=2$ kpc; grey: $z=3$ kpc; magenta: $z=4$ kpc) for the thin disc component 7 (solid lines) and the young thick disc (dotted dashed lines).

Dehnen & Binney (1998b) from Hipparcos data found values of $(10.00\pm0.36, 5.25\pm0.62, 7.17\pm0.38)$ showing a very good agreement at less than 1 sigma with us for $U_\odot$ and $W_\odot$. Their $V_\odot$ value was already shown to deviate from Schönrich et al. (2010) who found $(11.1_{-0.75}^{+0.69}, 12.24_{-0.47}^{+0.47}, 7.25_{-0.36}^{+0.37})$ km s$^{-1}$. The latter present a detailed chemo-dynamical model to compute the Solar motion. They show that using a large range of stellar types they are able to fit a model where the $V_\odot$ does not depend on metallicity. However their result on average and according to the error bars remains in good agreement with ours for the three components of the solar motion.

Dehnen & Binney (1998b) from Hipparcos data found values of $(11.69\pm0.68, 10.16\pm0.51, 7.67\pm0.10)$ km s$^{-1}$. Notice that even if the Solar motion with respect to the LSR should not depend on the selected sample, their determination does depend on the sample selection. Moreover they assume no variation of the velocity ellipsoid with Galactic position. For example using stars at distances up to 1 kpc they find a $U_\odot$ of 9.79 while $V_\odot$ decreases to 8.50, at about 3 sigmas of their finally claimed values. However their result on average and according to the error bars remains in good agreement with ours for the three components of the solar motion.

Dehnen & Binney (1998b) from Hipparcos data found values of $(10.00\pm0.36, 5.25\pm0.62, 7.17\pm0.38)$ showing a very good agreement at less than 1 sigma with us for $U_\odot$ and $W_\odot$. Their $V_\odot$ value was already shown to deviate from Schönrich et al. (2010) who found $(11.1_{-0.75}^{+0.69}, 12.24_{-0.47}^{+0.47}, 7.25_{-0.36}^{+0.37})$ km s$^{-1}$. The latter present a detailed chemo-dynamical model to compute the Solar motion. They show that using a large range of stellar types they are able to fit a model where the $V_\odot$ does not depend on metallicity. Our model differs not only by the dynamical modelling, but also by the data used (Hipparcos data in their case) even though the Solar motion found in both cases is very similar. Schönrich et al. (2010) already suggested that the results may vary with the sample due to the substructures in the local data and that a final answer for $V_\odot$ would vary when more distant and accurate data will be available. Non axisymmetry and streaming motions may also play in the problem, that Bovy et al. (2015) evaluated at the level of 11 km s$^{-1}$ on scales of 2.5 kpc.

Sysoliatina et al. (2018) determined constraints on the Sun motion together with the local rotation curve from RAVE and SEGUE data. They used the model of Golubov et al. (2013) to correct for the asymmetric drift and found $V_\odot$ to be $4.47\pm0.8$ km s$^{-1}$ which is smaller than our value and Schönrich et al. (2010) and closer to Dehnen & Binney (1998b). This value, as ours, depends on the Galactic model used. Surprisingly their rotation curve is flat or rising at the Sun position, and also depends on the metallicity. We believe that this is an indication that there is a bias due to the selected sample, or to the imperfect correction of the asymmetric drift that produces this dependency on metallicity (see also Schönrich et al. (2010)).

Using Gaia DR1 astrometry and RAVE spectroscopy in Robin et al. (2017) we found a slightly different value for $U_\odot$ of $13.2$ km s$^{-1}$ compared to $10.65$ km s$^{-1}$ here, and a significantly smaller value of $1$ km s$^{-1}$ for $V_\odot$ compared to $11$ km s$^{-1}$ here. In the former study the potential was slightly different and the stellar densities were not self-consistently computed, the kinematical scale lengths and variations of the asymmetric drift with...
our model reliably accounts for the variations of the circular velocity of the stars with their age and position in the Galaxy.

8.2. Thin and thick disc kinematics

Thin disc AVR. Our study allowed us to determine the thin disc kinematics, fitting the age-vertical velocity dispersion relation, the vertical to radial velocity dispersion ratio, and the kinematic and density scale lengths. For the former, we find very consistent values with the literature, with vertical velocity dispersion varying from 10 to 20 km s$^{-1}$ for stars of 0.1 to 10 Gyr near the Sun (see Fig. 7). Most importantly, we compare here an AVR at the Sun, with AVR on samples that can cover a wide range of distances from the Sun (specially for giants). Ages are also difficult to determine from observations as absolute values. Our AVR has a consistent values with Gómez et al. (1997) AVR from the Hipparcos sample, although it exhibited a saturation of the vertical velocity dispersion for the old thin disc at 15-17 km s$^{-1}$, while we find a slightly higher maximum value for the thin disc at the level of 19 km s$^{-1}$. Lagarde et al. (2021) from Kepler giants found vertical velocity dispersions slightly below ours for young stars when ages are estimated from Miglio et al. (2021)(M21 in the figure), although with ages from APOKASC the agreement is pretty good. Comparing with Yu & Liu (2018) from LAMOST sample of red giants of thin disc metallicities ([Fe/H] $>-$0.2 dex), the agreement is also good for ages larger than 5 Gyr, but for young stars they obtain a slightly lower velocity dispersion. Soubran et al. (2008) on the contrary found higher values at all ages from another red clump sample. For comparison we also plot in Fig. 7 age-velocity dispersion relations obtained by Bovy et al. (2012) and Sharma et al. (2021).

Dehnen & Binney (1998b) analysis of the local kinematics gives consistent values for the $\sigma_R/\sigma_z$ of 2.2 and a radial velocity dispersion of 38 km s$^{-1}$ for oldest thin disc stars, slightly lower than our value of 45 km s$^{-1}$. Their ratio of scale length $H_R/H_z$ of 3 to 3.5 while we find 2.9. In our case $\sigma_R/\sigma_z$ of 2.38 at the Sun position, but varying with $R$ and $z$ to reach 1.6 for example at $R = 15$ kpc and $z=0$. $\sigma_R/\sigma_z$ values close to 2 are rather frequent in the literature, measurements done quite often in the solar vicinity, such as in Holmberg et al. (2007) from the Geneva Copenhagen survey, Aumer & Binney (2009) from Hipparcos data, Yu & Liu (2018) from LAMOST, or Amôres et al. (2017) from RAVE and Gaia DR1, among others.

Noticeably Sanders & Das (2018) use Gaia DR2 complemented by spectroscopic surveys to determine ages for 3 million stars and derive variation of the age-velocity dispersion relation with Galacticentric radius. They use a Bayesian approach to determine distances from parallaxes assuming priors, and isochrones to determine ages. They found that the radial velocity dispersion $\sigma_R$ as a function of Galacticentric radius continues to decline at $R > 10$ kpc, while $\sigma_z$ stops to decline around the Sun, then slightly increases at larger $R$. Our model does not produce this up-turn of the $\sigma_z$ at large radii in the Galactic plane. However our gradient flattens at larger $z$ such that the values of $\sigma_z$ is quite different in the plane than at distances from the plane. They stressed out that due to age uncertainties increasing with age, it can bias the overall estimation of the age-velocity dispersion relation. In our method, where no ages are assumed for the stars, we avoid this kind of bias.

Gaia Collaboration et al. (2018c) study of Gaia DR2 presented the same flattening of the vertical $\sigma_z$ at larger $R$ (see their figure 18), compatible with our result. Contrarily to Sanders & Das (2018), they did not see a up-turn of $\sigma_R$ or $\sigma_z$ at large $R$, at

$R$ and $z$ were also different and fixed a priori. We used RAVE data together with Gaia DR1, which covered a limited range in distance and are less accurate than Gaia eDR3. The disagreement with the present study is mainly due to the overall shape of the potential used at that time which has a significant impact on the value of the asymmetric drift, as well as on the mean circular velocity at the Sun position. In the present study we performed full determination of self-consistent density and kinematics and compared them with a much larger data sample, giving more confidence in the result.

It is worth mentioning that we obtained a consistent values of the Solar motion, at the level of 1.5 $\sigma$, using either the local sample alone, or the combination of it with our deep fields. The result is robust with the sample selection, most probably because
least up to 14 kpc and their sample is less biased by the selection function.

Sharma et al. (2021) present a complex analysis of the kinematics and its dependency with position, age and metallicity from multiple surveys (GALAH, LAMOST, APOGEE, the NASA Kepler and K2 missions, and Gaia DR2). They find an AVR following a power law with an exponent of $\beta = 0.441 \pm 0.007$ for $\sigma_z$, slightly steeper than ours. However their fit applies globally on the thin and thick discs, while our power law index of $0.54 \pm 0.02$ applies only on the thin disc, our thick disc has a significantly higher dispersion of 40.53 km s$^{-1}$, whatever its age. Sysoliatina & Just (2021) in their overall fit of the Just and Jahreiss model to Gaia DR2 found for the AVR a slope $\beta = 0.41 \pm 0.04$, for the thin disc, and a $\sigma_z$ of 43 km s$^{-1}$ for the thick disc, in excellent agreement with ours.

In many observational studies the velocity dispersion increases with height from the plane. In our model it is not the case for the thin disc when considering each isothermal (mono-age) population separately. However this gradient is naturally created when a realistic mix of different populations is considered. In observational samples, such as in Mackereth et al. (2019) or Sharma et al. (2021), this is produced by the mix of populations, where at higher z older stars (with higher dispersions) dominate.

Mackereth et al. (2019) studied a large sample of 65000 stars from APOGEE DR14 and Gaia DR2 covering a wide range of distances $z < 2$ kpc and $4 < R < 13$ kpc. They determined ages and used abundances to distinguish low-$\alpha$ and high-$\alpha$ sequences, corresponding to thin and thick discs respectively. Among other interesting results, they see very little variation of $\sigma_R$ and $\sigma_z$ with $z$ in each mono-age population. This is also what is seen in our model (Appendix B Fig. B.1). Our maps of $\sigma_R$ and $\sigma_z$ show very little change with $z$ in a wide range of $R$. In our model to see a clear change of dispersion with $z$ it is necessary to go at large distances from the plane, in regions where the stellar densities are very small. They also find very long kinematic scale lengths for the thin disc but with a large uncertainty $R_{\sigma_R} = 15^{+1}_{-0.1}$ kpc and $H_R = 16^{+1}_{-0.5}$ kpc, still compatible with ours.

Concerning the AVR they modelled it with a power law index $\beta$, of about 0.5 for thin disc, 0.45 for metal poor stars, compatible with the value of 0.54 that we obtained in this study for the thin disc. On the other hand they claim a velocity dispersion ratio $\sigma_z/\sigma_R=0.64 \pm 0.04$ for old stars. We find values varying from 0.4 to 0.7 for the thin disc, and around 0.74 for the thick disc (not distinguishing their age) with in this case very little variations with $R$ and $z$, in very good agreement with their observations.

Lagarde et al. (2021) analysed the Kepler sample to determine the characteristics in age, metallicity and $\alpha$ abundances of the thin and thick discs. The selected sample is roughly at solar Galactocentric radius. Hence one expects the kinematics to be close to the one of the Solar neighborhood (assuming axisymmetry). They derive for the thin disc a vertical velocity dispersion depending on age, varying from 10 km s$^{-1}$ at 1 Gyr to 18 at 8 Gyr and a velocity ratio $\sigma_R/\sigma_z$ of about 2.5. They found lower values (35 km s$^{-1}$) than ours for $\sigma_R$ and consistent values for $\sigma_z$. They see also the dependency of the velocity dispersion depending on metallicity, which we are not considering here. This will deserve further analysis.

Concerning the radial scale lengths Sharma et al. (2021) underlined out that the dispersion falls off exponentially as a function of guiding radius which is also a result completely in-line with our model. However they found that the scale length of $\sigma_R$ ($H_{\sigma_R}$) is larger than the one of $\sigma_z$ while we find the contrary.

In conclusion our vertical AVR is compatible with most previous determinations for old stars, slightly higher for young stars, but within the error bars. This might be investigated further to explore whether it can be due to the fact that we compare here the velocity dispersions at the Sun position, while various studies cover a wide range of positions depending on the selection functions.

Thick disc kinematics. In our model the vertical velocity dispersion in the young thick disc slightly increases with $z$ at $R_0$ and decreases when $R$ increases (see Appendix B, fig. B.1). In the study of the Kepler field Lagarde et al. (2021) the thick disc vertical velocity dispersion depends on alpha abundance and metallicity and range between 25 and 40 km s$^{-1}$ along the alpha sequence from low to high [$\alpha$/Fe], while we found 40.53 km s$^{-1}$ at the solar position. Their $\sigma_R$ ranges between 42 and 55 km s$^{-1}$, in good agreement with our value of 54.66 km s$^{-1}$ locally. In order to study the interface between thin and thick discs, their thick disc population is divided into a metal rich (i.e. with metallicity close to solar) and a metal poor component, the later being close to our young thick disc component, while the metal rich one is not considered here (due to lack of metallicity information). In order to better compare our result in the Kepler field, stellar abundances have to be considered, which is beyond the scope of this paper. Interestingly, their metal poor thick disc velocity dispersion varies with metallicity and [$\alpha$/Fe] but the variation with age is not monotonic and it significantly differs from the thin disc behavior. This clearly indicates a different formation history for the thick disc than the thin disc. This difference in scenario of formation for thick disc compared with the thin disc was already pointed out in several chemo-kinematical studies, such as Mackereth et al. (2019) who found a nearly constant $\sigma_R/\sigma_z$ at a level of 1.56 for the high-$\alpha$ population (thick disc), very close to our model which also shows a very smooth distribution of this ratio nearly independent from $R$ and $z$ (Appendix B Fig B.1 in penultimate row). This dichotomy between thin and thick discs is in line with our model, as already discussed in Sect. 7.4.

One of the difficulty of comparing our results with others is due to the fact that we have not considered the abundances in this study. This is well appropriate to separate the thin and thick discs by $\alpha$-element abundances, rather than by age (because they overlap) and/or position (which would refer to the geometrical thick disc instead). Therefore, combining this new dynamical model to the population synthesis approach to simulate abundances surveys will be a very efficient tool to test chemo-dynamical scenarios for the formation of the thin and thick discs. Despite of lacking abundances, we pointed out an abrupt transition between thin and thick discs. Even though a more detailed study by simulating the spectroscopic surveys would be needed to compare our model with chemically selected samples, these preliminary comparisons give confidence in the reliability of our model to reproduce the velocity dispersions of observed mono-age populations.

8.3. Density laws

The new density laws slightly differ from Einasto laws used in previous BGM, especially as a function of $z$, as seen in Figure 5, but also radially. The density scale lengths of the thin disc are larger in the plane, but are varying a lot with $z$ and the new model exhibits a significant flare in the thin disc. There is a slight difference in $\rho(z)$ for intermediate discs (between 2 and 5 Gyr). The thick disc scale length $H_z$ being shorter than the one of the thin disc, the thick disc density, as seen in Appendix Figure B.1 is dense only in the inner Galaxy, while the density drops sig-
nificantly in the outskirts. This is in line with studies who find exponential scale lengths larger in the thin disc than in the thick disc (Bovy & Rix 2013; Anders et al. 2014; Robin et al. 2014; Mateu & Vivas 2018; Syliszalina et al. 2018, among others). On the contrary Golubov et al. (2013) found scale lengths larger for metal poor population as the thick disc from RAVE data (3 kpc, while 1.8 kpc for the thin disc). This last surprising result would be difficult to reconcile with the paucity of the high-α thick-disc stars seen in the outer Galaxy (Hayden et al. 2015).

Our model produces a significant flare in the outskirts of the thin disc (see Appendix B, fig.B.1 and Fig. 6). In practice the isodensity contour intervals are larger at high R than in the Solar neighbourhood. This is notable for young ages (components 2 to 5). In older thin disc components the flare is flattening at R > 15 kpc but this is in a region where the density is very low. Such flare has already been identified in many studies, for instance in Amir et al. (2017). Noticeably Chrobáková et al. (2020) analysed Gaia DR2 to determine the structures in the anticentre and find evidences for an asymmetric flaring (different above and below the plane). This is beyond the scope of this paper to explore the features in the outer Galaxy, which needs a wider selection of data in this region to separate the axisymmetric components from the non-axisymmetric ones and from the substructures which can result from old mergers and gravitational response to perturbations by satellites.

8.4. Comparison with other models

The model we present here combines several elements that are rarely found together in other Galactic models. The dynamical coherence links the gravitational potential to all the Galactic components. The stellar distribution functions, density and kinematics, are themselves consistent with the potential. Moreover, the stellar populations are described using the most recent evolutionary tracks.

Binney (2012a) built a $f(J)$ axisymmetric Galactic model based on actions, an approach also using Stäckel potentials as we have developed in this study. Thus his stellar distribution functions are stationary as in our model. He derived a realistic potential for the MW and a fit to RAVE and SDSS data (Binney 2012b). However, he found that “the thick disc has to be hotter vertically than radially, a prediction that it will be possible to test in the near future”. However, we suspect that the stellar samples used to constrain his model were not large enough for the large number of free parameters of the model. To our knowledge this prediction has not been confirmed and our model does not present this feature. In our model the ratio $\sigma_R/\sigma_z$ for the thick disc is markedly smaller than the one of the thin disc.

In a much recent study Sysolyatina & Just (2021) attempted to adjust the so-called JJ Galaxy model (Just & Hamelefs 2010) to Gaia DR2 data using a MCMC scheme. They fitted density laws of the thin and thick discs together with their initial mass function and star formation history. From the point of view of the kinematics, they consider an AVR following a power law as we do, and find results close to ours for the thin disc. For the thick disc vertical dispersion they find $44\pm4 \mbox{ km s}^{-1}$, in agreement with our value of $40.53 \mbox{ km s}^{-1}$. Interestingly their thin disc is modelled with several star bursts with different vertical velocity dispersions, so a significantly larger number of free parameters than ours, at the expense of more flexibility. The main differences come from the data and observables used for their fit. They use the local sphere of 600 pc around the Sun from Gaia DR2 and consider as observable the W velocity, therefore relying on radial velocities, restricting the sample to bright stars and considering the kinematics only on the vertical axis. To complement those local data they consider the APOGEE DR14 sample of red clump stars to constrain the velocity space at larger distances from the Sun, with good radial velocities and distances. They further separate the thin and thick disc populations using the $\alpha$ abundances. However their final sample of thin and thick disc populations from APOGEE is rather small (3910 and 847 resp.). Therefore the extra information on the chemical separation of the two populations that they have is subject to Poisson noise in the MCMC fit. Finally, their approach is interesting on the point of view of the methodology and they possibly will extend it to larger samples and a wider range of Galactocentric radii in the future.

Ting & Rix (2019) studied the vertical action as a function of Galactic radius and age for a sample of about 20,000 red clump stars from the APOGEE DR14 spectroscopic survey, for which Gaia DR2 proper motions are available. Distances were computed from bayesian estimation and neural network computations were used to derive ages with a training sample coming from Kepler asteroseismic parameters. They fit a simple model to describe the evolution of the vertical action with age and R and conclude that it is possible to explain this evolution with only the scattering due to giant molecular clouds at R<10 kpc, while outward they estimate an initial vertical action at birth which grows fast with R, due to lower density and smaller vertical force which allows the disc to flare quite significantly. It is not easy to compare quantitatively our new model with theirs. However qualitatively our model does present a significant flare for the thin disc components, but not for the thick disc.

On the other hand N-body simulations are also made in order to evaluate the gravitational evolution of disc galaxies. Then among many attempts some simulated galaxies appear to have characteristics close to our MW, such that the processes at work in these simulations can be tentatively extrapolated to our own Galaxy. One of the studied process is the evolution of the AVR. A number of models have been developed in order to study the formation of the disc, specially to explain the AVR by secular evolution of the disc due to the effect of giant molecular clouds, spiral waves or even bar perturbations, or by heating due to mergers (Dehnen & Binney 1998b; Aumer et al. 2016). In particular Aumer et al. (2016) analysed N-body simulations and explained the differences between the observed AVR and the heating history. They assessed that the observed AVR suffers from age uncertainties, which tends to flatten the relations (lowering the value of $\beta$). Their diverse simulations underwent very different formation histories leading to different AVR, showing a variety of possibilities on the point of view of Galaxy evolution, depending on the GMC masses, bar strength and spiral waves. They also claimed that there is no universal heating history valid for all populations because the heating process significantly evolves over time and with Galactic regions. The analysis of chemodynamical simulations shed light on the formation scenarios of various populations, helping to identify the efficient processes at work, but it is difficult to link a variety of simulations with different histories with the Milky Way evolution. In our complementary approach we ensure to have a realistic potential at the present time that it is interesting to compare with various simulations. In the case of the AVR, our result is compatible with a secular heating of the thin disc by GMC, but we cannot make the difference with other efficiency heating process. On the other hand in Navarro et al. (2018), hydrodynamic simulations of a Galaxy similar to the Milky Way exhibit an AVR similar to the one in our model, coming from the settling of the gas only, and not from secular heating and radial migration. This new scenario is an interest-
ing alternative to the secular heating by GMCs or spiral waves, although this is anyhow difficult to estimate how close this simulation is from the real MW formation scenario.

For the thick disc, with its characteristics (compactness, dropping density in the outer Galaxy, thickness, large distinction in velocity dispersion ellipsoid with the thin disc, and no flare) the thick disc formation which is favored is a formation from a turbulent gas disc in the early Galaxy, as proposed by Bournaud et al. (2009).

Several chemodynamical simulations have been specifically developed to study the Milky Way formation and to compare with distributions of velocities and abundances. Among them, Minchev et al. (2012) used N-body simulations from Martig et al. (2012), adding chemical evolution scenario to "paint" their particles and evaluate the chemo-kinematical distributions to be compared with real data. This allows them to evaluate how stars migrate, what is the impact of mergers, and to help interpret spectroscopic surveys in terms of Galactic formation scenario. Among other interesting results Minchev (2016) found (their figure 7) that, in their simulations, all low-α mono-age populations flare while high-α don’t, completely in line with our result.

8.5. Rotation curve and the Galaxy mass

Recent Gaia observations have allowed a precise determination of the shape of the Galactic rotation curve, mainly through measurements of Cepheids within a few kpc of the Sun and at greater distances up to 60 kpc from the Galactic centre using the kinematics of halo stars (Cautun et al. 2020; Deason et al. 2021). The evidence of a rapid decay of the Galactic rotation curve has been provided by the most recent evolutionary tracks and also constrained by recent Gaia observations; (iii) fitting is done simultaneously on density and kinematics in the space of observables. Starrevol stellar evolutionary models (e.g. Lagarde et al. 2012; Amard et al. 2019) are state-of-the-art interior models covering a wide mass range from 0.6 to 6 solar masses, metallicities and α-abundances, which have been incorporated into the BGM by Lagarde et al. (2017, 2019). They ensure the reliability of the mass-luminosity relation in this mass range.

Our model presents relevant contributions to the modelisation of the Milky Way, in particular:

- We find the solar motion with respect to the LSR to be $U_\odot = 10.79 \pm 0.56 \, \text{km s}^{-1}$, $V_\odot = 11.06 \pm 0.94 \, \text{km s}^{-1}$, and $W_\odot = 7.66 \pm 0.43 \, \text{km s}^{-1}$, in good agreement with recent studies.

- The thin disc is modelled by the sum of 7 age components, each having a specific velocity ellipsoid. We determined its AVR at different positions in the Galaxy and showed that velocity dispersions vary with $R$ and $z$ and the kinematical scale lengths are about 3 times the density scale lengths.

- In the range of Galactic radii 6-12 kpc, the $\sigma_R/\sigma_\phi$ varies with age, $R$ and $z$ and has values between 1.9 and 2.6 in the thin discs (for $z < 2$ kpc) and between 1.2 and 1.4 in the thick disc.

- The asymmetric drift is increasing with age and $z$ and decreasing with $R$. It can reach high values (more than 80 km s$^{-1}$) at high $z$ in the thick disc. This lag is directly obtained from the potential and the self-consistent distribution functions.

- The tilt of the velocity ellipsoid is also varying with age, $R$ and $z$ but the variations are small and it nearly follows spherical symmetry (major axis nearly pointing towards the Galactic center) for all populations.

- The thin disc population density exhibits a significant flare in the outskirts of the Milky Way, coming naturally from the variation of the vertical force with $R$. On the contrary the thick disc does not flare, probably because of its compactness and the fast drop in density in the outer Galaxy.

- The young thick disc characteristics are significantly different from the old thin disc populations, pointing towards a clear distinction between them. In particular we obtained an asymmetric drift which is about twice for the thick disc at the Solar position and very different variations with $R$ for the mean $V_R$, and for $\sigma_\phi$. This result reinforces the idea of a very different scenario of formation between the thin and thick discs.

We have not considered the star formation history (SFH) of the disc as a free parameter. It was determined using the analysis of Mor et al. (2019) using Gaia DR2 up to apparent magnitude $G = 13$. Would this SFH be not adequate for our study, it could slightly impact the present analysis. However, if it was the case the self-consistent model would be revised applying again the present method with corrected SFH in the initial simulations.

On the low mass side of the IMF, the stellar mass is subject to uncertainties because stellar models are less accurate. Their atmospheres are difficult to model because of the numerous molecules. We did not consider them in our analysis for those reasons. However their number constitutes a source of uncertainty when considering the total mass in stars in the Milky Way. This will be considered in a future study where the IMF and SFH will be explored extensively and the impact on the dynamical model will be estimated.
We have seen that, despite the very good agreement of the axisymmetric model with the data, there are significant discrepancies in a few fields where further studies need to be conducted. This is true particularly towards the anticentre, where disc substructures have been highlighted, and towards the north Galactic pole where a vertical wave is seen. We also point that our stellar halo model needs to be improved but this population being very minor in mass density, this does not preclude the use of the present Galactic potential.

The model can further be used as a realistic potential of the Milky Way, in orbital studies for example, to derive eccentricities, energy and angular momentum of real stars from accurate astrometric data such as *Gaia* and other space surveys to come. We believe that it can also be used for identifying and characterising substructures, allowing to subtract the axisymmetric component to the data to enhance the contrast and to qualify the interesting features.

Furthermore one expects that the just released *Gaia* DR3 and large spectroscopic surveys will give new opportunities to explore in even greater details the gravitational potential of our Galaxy.
Appendix A: MCMC variables dependencies

We present in Fig. A the triangular plot showing the correlations between kinematic parameters of the best Markov Chains. A significant anti-correlation is found between the AVR parameters $k$ and $\beta$ but this does not imply a bad determination of the AVR itself because the corresponding shapes of the AVR are very similar. There is also a significant anti-correlation between the thin disc $\sigma_R/\sigma_z$ and the $\sigma_R$ of thick disc due to the fact that both populations are mixed up in a wide proportion of the $(R,z)$ space. The third correlation is between thin disc $\sigma_R/\sigma_z$ and $\beta$ due to the fact that we have only 2 components of the velocity and we miss the line-of-sight velocity.

Appendix B: Density and velocity distributions of the final model

Figure B.1 shows how the density and kinematical parameters (mean rotation velocity, velocity dispersions, and tilt of the ellipsoid) vary for the disc components as a function of Galactocentric coordinates in the final model with declining rotation curve. Similar curves are available upon request for the model with the flat rotation curve. The values are reliable for $|z| < 5$ kpc. For the young populations with shorter scale heights, the computation has been performed only to 3 kpc in $z$.

We point out a specific feature seen at high $z$ and $R > 15$ kpc in $\sigma_\phi$ in Fig B.1 in 3rd row. The distribution functions of density and velocities are reliable below $z < 5$ or 6 kpc when $V_\phi$ is larger than 120 km s$^{-1}$ (i.e. the asymmetric drift is smaller than about 110 km s$^{-1}$). At larger $z$ values the distribution functions are numerically exact and stationary but they are not realistic. This is a consequence of the properties of the generalised Shu distribution function that depends only on positive values of the angular momentum. Then, negative values of the azimuthal Galactic velocity $V_\phi$ or counter rotating stars are not modelled. When the asymmetric drift is large, i.e. $V_\phi$ smaller than about 120 km s$^{-1}$, corresponding approximately to $z$ larger than 5-6 kpc depending on the population, then the bell shape of the $V_\phi$ distribution is distorted with a maximum closer and closer to $V_\phi = 0$ and no negative values. A consequence is the “beak” feature seen in Figure B.1: when $z$ increases, first $\sigma_\phi$ increases and beyond $z \sim 6$ kpc it decreases since the $V_\phi$ distribution is squeezed towards $V_\phi \approx 0$ values. This problem also impacts the distribution of $\sigma_\phi/\sigma_z$ in Fig B.1 in penultimate row at $z > 5$ kpc and $R > 15$ kpc.
Fig. A.1. Triangular plot of the parameter values of the best MCMC chains.
Fig. B.1. Density and kinematics as a function of $R$ and $z$ for different disc components. Columns 1 to 6 represent thin disc components 2 to 7. Columns 7 is for the young thick disc. The densities are shown for values larger than $10^{-20} \, M_\odot \, pc^{-3}$. Regions in white have either not been computed or the density is below this limit. Different rows are for different quantities from top to bottom: density (in $M_\odot \, pc^{-3}$), $V_\phi$, $\sigma_R$, $\sigma_z$, $\sigma_R/\sigma_z$ (in km s$^{-1}$), tilt angle of the ellipsoid (in degree). Isocontours are spaced by decimal logarithm of the density for densities larger than $10^{-20} \, M_\odot \, pc^{-3}$. 
Appendix C: Comparisons between model and Gaia data selection

In order to ascertain the quality of the fit, we compared stellar densities in the data sample with simulations from model Mev2011 and those from our final fitted dynamical model (Fig. C.1). The new model provides much better fit to the radial and vertical distribution seen in the data sample. The total likelihood of the new model is $-902$ (for the declining rotation curve), and $-955$ (for the flat rotation curve), while the one of the Mev2011 model was $-2057$. It remains some significant disagreements in a few areas of the $(R, z)$ plane. The model with its assumptions of axisymmetry however finds a reasonable mitigation between regions where it overestimates and those where it underestimates the density. Moreover, the model does not account for substructures, like for example the Gaia-Enceladus-Sausage (Helmi et al. 2018; Belokurov et al. 2018), or the substructures in the antecentre (Ramos et al. 2021). In particular the excess of stars in the model at $R > 8500$ pc is solely due to stars at longitude $180^\circ$ and $-20^\circ < b < 20^\circ$. It is most probably due to the substructures already known in this region, such as TriAnd, ACS or Monoceros overdensities. These figures anyhow show that the new self-consistent model provides a significant improvement over the previous model Mev2011.

Medians and standard deviation of the tangential velocities, for the data sample (blues symbols) and the model (red symbols) are presented in Fig. C.2 and C.3 for the $V_l$ and $V_b$ component respectively.

In Figure C.2 we compare the median velocities and velocity dispersions of the transverse velocity $V_l$, as a function of $z$ distance from the Galactic plane between model and data. There is an overall good agreement between model and data. When we split the data by stellar types (left panel), samples selected by pseudo-absolute $M_\ast$ magnitude and colours allow us to evaluate the change in dispersion for various types according to observational quantities only. It shows that the velocity dispersions for giants roughly (selected by $G_{BP} < 0.55$ and $M_\ast < 4$) vary as a function of $z$ as expected from the model, with some noisy fluctuations due to the sample size and local fluctuations (visible when we compare the north and the south for example, also in Salomon et al. (2020)), which might be due to non-axisymmetric structures. The selection of early type stars (with $G_{BP} < 0.55$ and $M_\ast < 4$) shows a comparable trend in the model and data, validating the age-velocity dispersion. Right panel demonstrates the effect of change in the velocity dispersion according to Galactocentric radius $R$. The model reproduces very well the overall trend in $z$.

At $R < 8$ kpc the agreement is good, validating the mean dispersion and its radial gradients. The sample at $R > 9$ kpc exhibits smaller velocity dispersions both in model and data ($x$ symbols in right panels) compared to the inner Galaxy (squares) where the model slightly underestimates the dispersion in $\sigma_R$, although globally (full circles) the agreement between model and data is good with deviations at the level of a few km s$^{-1}$.

Comparisons of the medians and dispersions of $V_l$ are shown in Figure C.3. As for $V_l$, the dispersions from the model are very similar to the data, showing the increase of the dispersion with $z$ and a decrease of the mean velocity due to the increase of the asymmetric drift with distance from the plane. Moreover the medians of $V_l$ are very well reproduced at different Galactocentric radii, validating our radial scale lengths and the asymmetric drift modelled. We however notice that the dispersion is slightly too low in the model (but by less than 10%) out of the plane, due to the compromise between fitting density and kinematics.

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Fig. C.1. Histograms of star counts in the selected sample as a function of pseudo $R$ (top left), pseudo $z$ in inner Galaxy ($R<8$ kpc, top right), at $8<R<8.5$ kpc excluding the local sample (bottom left), towards outer Galaxy ($R>8.5$, bottom right). Gaia data (black), previous BGM (green long dash), fitted dynamical model (magenta short dash).

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**Fig. C.2.** Median (top row) and dispersion (bottom row) of the transverse velocity $V_t$ as a function of $z$ for different sub-samples of the final model (red symbols) and data (blue symbols). Full circles are for the whole sample. Left panels: early type stars ($M_\ast < 4$ and $G_{RP} < 0.55$) (open diamonds), giants ($M_\ast < 4$ and $G_{RP} > 0.55$) (crosses). Right panels: selection done on Galactocentric radii with $R < 8$ kpc (open square) and $R > 9$ kpc (x). Points where the number of stars is smaller than 10 are considered as insignificant and not presented.

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Fig. C.3. As for Figure C.2 but for the transverse velocity component $V_t$. 