Numerical Analysis of the Ground-State Magnetic Moments of $^{143,145,147}$Sm Isotopes

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Abstract
The description of the ground-state magnetic properties of odd-mass nuclei is very informative in understanding of the complex structure of the deformed nuclei. The ground-state magnetic moments of most of the odd-A deformed nuclei have been measured by various experimental studies and there are numerous studies in the literature. However, many of the theoretical studies on magnetic moments and spin polarization effects affecting them are far from explaining these measured values. In this paper, the magnetic moments and effective spin g factors of $^{143,145,147}$Sm isotopes in the lanthanides region of the periodic table were investigated within the framework of the Quasiparticle-Phonon Nuclear Model (QPNM) based on the Lagrange Multiplier Method for the first time. Spin-spin interaction parameters ($\chi$) were determined by comparing theoretical and experimental values of magnetic moments of the related isotopes and these interactions were found to have an isovector character $q = -1$. It has been observed that the ground-state structures of the studied isotopes are weakly affected by quasiparticle-phonon interactions and the contribution of these interactions ($G_{K\mu}^{\nu}$ values) to the ground-state wave functions is quite small (around 0.01%). Theoretical explanation of the renormalization of spin gyromagnetic factor is one of the most important problems of nuclear structure physics. The results obtained in this study for the effective spin gyromagnetic factor $g_{s}^{eff.} / g_{s}^{\tau}$ also agree with the phenological value $g_{s}^{eff.} = (0.5 - 0.7)g_{s}^{\tau}$.

Keywords: Sm, Lagrange Multiplier Method, Magnetic Moment, Spin polarization

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1. Introduction

The theoretical investigation of the ground-state magnetic properties of deformed odd-A nuclei plays an important role in understanding the complex structure of these nuclei and in determining the success of existing nuclear models [1-8]. It shows systematically large deviations from the shell model calculations (Schmidt curves) of the experimentally observed magnetic moments. The theoretical explanation of these deviations in nuclear magnetic moments has been attempted by many nuclear physicists. The interaction of the magnetic dipole (M1) excitations of the even-even core nuclei with the single remaining nucleon of the odd-A nuclei is considered in 1950s [9]. This approach is known as the core polarization phenomenon or Arima-Horie effect in the literature [10]. Besides, the effect of mesonic interactions on magnetic moments has been also discussed by many researchers, and this effect has been found to be important in the renormalization of spin orbital gyromagnetic factor [11,12]. Therefore, the mesonic effects are particularly important in the investigation of high spin levels where the main contribution to the magnetic moment comes from the orbital gyromagnetic part [13]. According to core polarization, the spin-dependent part of the interaction between the unpaired nucleon in the outer shell and the nucleons in the core tends to align the spins of the unlike nucleons parallel and the spins of the like...
nucleons antiparallel with the spin of the single-nucleon [14]. Studies made so far show that the interaction between the spin of the single nucleon and the spin of the nucleons in the core affects the magnetic properties of the odd-mass nuclei. The contribution of the spin part to the magnetic moment decreases due to core polarization, which explains the deviation of the theoretical predictions from the observed magnetic moment ($\mu$) values [14-20]. Many theoretical studies for M1 transitions in deformed nuclei show that results consistent with the experiment are obtained by using the effective spin gs factors ($g^e_{\text{eff}} = 0.6 - 0.7g^\text{free}$) instead of the free nucleon spin g-magnetic factor in calculations. This is one of the clearest evidence that spin polarization is important [11]. The first theoretical studies for the solution of this problem were carried out by Bochnacki and Ogaza using the perturbation method under the assumption that spin-spin interactions are responsible for the renormalization of single particle spin matrix elements [15,17]. However, since spin-spin interactions between quasiparticles are not weak, the perturbation method cannot properly explain quasiparticle interactions and magnetic moments [21]. In the 1970s, spin polarization effects were studied by Kuliev and Pyatov using the Nilsson potential in the framework of the Tamm-Dancoff approach (TDA). In their study, spin polarization effects in odd-A nuclei have been interpreted as a result of scattering of the unpaired nucleon in the outermost shell over $1^+$ excitations in the even-even core nuclei [14, 19-21]. A number of experimental studies on ground-state magnetic moments for $^{143,145,147}$Sm isotopes have been reported in the past. The comparison of our theoretical results with the experimental data is very important to confirm the accuracy of our theoretical predictions. In 1990, England et al. measured the magnetic moments of $^{143,145,147}$Sm nuclei by the cross beam laser fluorescence method [22]. In 1992, measurements for three Sm isotopes were made using the resonance ionization spectroscopy method by Letokhov et al. [23]. In addition to these, Kaplan et al., Woodgate and Childs et al. observed the magnetic moments of $^{145}$Sm and $^{147}$Sm experiments [24-26]. Since the spin-spin interaction potential is commutative with the total angular momentum operator, the interaction strength parameter can not be determined based on the relation between rotation and spin-spin operators. In many studies using the spin-spin interactions, the interaction strength parameter ($\chi$) is determined by fitting with the experiment [27-33]. However, the validity of this application is controversial. A practical method based on QPNM, which takes into account the quasiparticle-phonon interactions, has been developed to determine the spin-spin interaction power parameter by Yakut [21]. This method also successfully explains the spin polarization phenomenon observed in odd-mass nuclei. In this method, the spin-spin interaction parameter is obtained by comparing the calculated ground-state intrinsic magnetic moment ($gK$) with the experimental value of the odd-mass nucleus. Method were successfully applied to explain the ground state magnetic properties of $^{157-167}$Er, $^{165}$Dy, $^{157-167}$Ho, $^{183,185,187}$W, $^{185}$Re and $^{187,189}$Os [1-8]. In the present study, the ground state magnetic properties of odd-A $^{143-147}$Sm isotopes were firstly investigated using the method developed in the framework of the QPNM method. The results obtained from the theoretical calculations were compared with the experimental values and it was seen that they were in agreement with each other.

In the second part of this paper, the theoretical expressions used in the calculations are given. In the last part, numerical results and discussions are presented.

### 2. Method (Theory)

For a system where nucleons interact with the pairing and spin-spin forces in the axially symmetric deformed mean field, the QPNM hamilton is in the following form:

$$H = H_{\text{sqp}} + H_{\text{coll.}} + H_{\text{int.}}$$  \hspace{1cm} (2.1)

where:

$$H_{\text{sqp}} = \sum_{s,\tau} \varepsilon_s(\tau) \alpha^+_s \alpha_s$$  \hspace{1cm} (2.2)

$$H_{\text{coll.}} = \frac{1}{2} \sum_{\tau,\tau'} \sum_{s,s'} \sigma^{(\mu)}_{ss'} L_{ss'} g^{i^+}_{ss'} (Q^+_i + Q_i) \sum_{mm'} \sigma^{(\mu)}_{mm'} L_{mm'} g^{i}_{mm'} (Q^+_i + Q_i)$$  \hspace{1cm} (2.3)

$$H_{\text{int.}} = \sum_{\tau,\tau'} \sum_{mm'} \left\{ \sigma^{(\omega)}_{ss'} M_{ss'} \sigma^{(\omega)}_{mm'} L_{mm'} D_{ss'}(\tau) (Q^+_i + Q_i) + \sigma^{(\omega)}_{ss'} M_{ss'} \sigma^{(\omega)}_{mm'} L_{mm'} D_{ss'}(\tau) (Q^+_i + Q_i) \right\}$$  \hspace{1cm} (2.4)

The first term $H_{\text{sqp}}$ refers to the single-quasiparticle motion in the nucleus and includes the axially symmetrically deformed Woods-Saxon potential. While $H_{\text{coll.}}$ represents phonon excitations in double-even core, the term $H_{\text{int.}}$ refers to the relationship between one-quasiparticle and collective motions.
The wave function for K=1/2 projection of the total angular momentum on the symmetry axis in a odd-A nuclei can be written as follows [1-8, 21]:

$$\psi^j_K(\tau) = \left\{ \sum_q N_q^j(\tau) \alpha_q^+(\tau) + \sum_{i\mu} \sum_{v} G^j_{i\mu v}(\tau) Q_{i\mu v} \right\} |\psi_0\rangle; \mu = \pm 1 \quad (2.5)$$

Here, $N_q^j$ and $G^j_{i\mu v}$ are the amplitudes of single-quasiparticle and quasiparticle-phonon states, respectively. $\psi_0$ is the phonon vacuum representing the ground-state of the even-even core. This wave function should provide the normalization condition as given below:

$$\langle \psi^j_K(\tau) | \psi^j_K(\tau) \rangle = \sum_q \left( N_q^j \right)^2(\tau) + \sum_{i\mu} \sum_{v} \left( G^j_{i\mu v} \right)^2 = 1. \quad (2.6)$$

Using Lagrange Multipliers Method (Variation Method) [34,35] given by the expression (2.7),

$$\delta \left\{ \begin{array}{l} \langle \psi^j_K(\tau) | H_{inv.} | \psi^j_K(\tau) \rangle - \langle \psi^j_K(\tau) | H_{inv.} | \psi^j_{K0}(\tau) \rangle \\ -\eta^K \left[ \sum_q \left( N_q^j \right)^2 + \sum_{i\mu} \sum_{v} \left( G^j_{i\mu v} \right)^2 \right] - 1 \end{array} \right\} = 0 \quad (2.7)$$

the following secular equation is obtained when the equations obtained for $N^j_K$ and $G^j_{i\mu v}$ amplitudes are solved [21]:

$$P(\eta^K) \equiv \varepsilon_K - \eta^K - \sum_{i} \frac{1}{\chi F_{i\mu}} \frac{M^2_{KK0} \sigma^2_{KK0}}{\varepsilon_{K0} + \omega_i - \eta^K} = 0. \quad (2.8)$$

Here, the $\eta^K$ roots of the equation (2.8) are energies of the odd-A nucleus. The $Z(\omega_i)$, $Y_{\tau}(\omega_i)$ and $F_{\tau}(\omega_i)$ abbreviations in Eq.(2.8) are also given below:

$$Z(\omega_i) = \frac{1}{(1+\chi F_{i\mu})^2} Y_{\omega_i}(\omega_i) + \frac{q^2}{1+\chi F_{i\mu}} Y_p(\omega_i)$$

$$Y_{\tau}(\omega_i) = 4 \omega_i \sum_{ss'} \frac{\varepsilon_{ss'}^2 \xi^2_{ss'} L^2_{ss'}}{(\xi^2_{ss'} - \omega^2_i)^2}; F_{\tau}(\omega_i) = 2 \sum_{ss'} \frac{\varepsilon_{ss'} \xi^2_{ss'} L^2_{ss'}}{\varepsilon_{ss'} - \omega^2_i}. \quad (2.9)$$

As known, the intrinsic magnetic moment of the odd-A nucleus is the expected value of the z component of the magnetic dipole operator [1-8]. The value of gK was obtained as follows after long and complex operations within the framework of QPNM.

$$\mu_{K0} = g_{K0} K_0 = \langle \psi^j_{K0}(\tau) | \mu_z | \psi^j_{K0}(\tau) \rangle = \left\{ g^{\tau}_s \left( 1 - 2N^2_{K0} \sum_{i,\nu} \frac{\chi_{KK0} \sigma_{KK0}}{(\varepsilon_{K0} + \omega_i - \eta_{K0})} \right) - 2 \left( g^{\tau}_s - g^{\tau}_l \right) N^2_{K0} \sigma_{KK0} \sum_{i,\nu} \frac{\chi_{KK0} \sigma_{KK0}}{(\varepsilon_{K0} + \omega_i - \eta_{K0})} \right\} \frac{\sigma_{KK0}}{2} + g^{\tau}_l K_0 \quad (2.10)$$

When this expression is compared with the Nilsson formula,

$$\mu_{K0} = g_{K0} K_0 = \frac{1}{2} \left( g^{\tau}_s - g^{\tau}_l \right) \sigma_{KK0} \langle \tau \rangle + g^{\tau}_l K_0 \langle \tau \rangle \quad (2.11)$$

the following analytical expression is obtained for the effective spin gyromagnetic factor [1-3]:

$$g^{\tau \tau \tau}_s - g^{\tau \tau}_l = \left( g^{\tau}_s - g^{\tau}_l \right) \left\{ 1 - 2N^2_{K0} \sum_{i,\nu} \frac{\chi_{KK0} \sigma_{KK0}}{(\varepsilon_{K0} + \omega_i - \eta_{K0})} \right\} - 2 \left( g^{\tau}_s - g^{\tau}_l \right) N^2_{K0} \sum_{i,\nu} \frac{\chi_{KK0} \sigma_{KK0}}{(\varepsilon_{K0} + \omega_i - \eta_{K0})} \right\} \quad (2.12)$$

Here, the unpaired nucleon in the outer shell of the odd-mass nucleus causes polarization by scattering from 1+ one-phonon states of the even-even nucleus. The second and third terms to the right of Eq.(2.12) refer to the coherent contribution arising from the quasiparticle-phonon interactions in polarized core. There is a significant reduction in the spin factor due to the contribution from spin polarization. Details of the theory and analytical expressions are available in Reference [1-8, 21].
3. Results and Discussion

In the study, the method developed by Yakut [21] and the analytical expressions given in section 2 were used in the calculations of the ground-state magnetic properties of $^{143,145,147}$Sm nuclei.

The $I^\pi K=1^+0$ phonons of even $^{142,144,146}$Sm for the ground-state calculations of $^{143,145,147}$Sm are used. The reason for this can be said that the ground-state and some low-lying excitations in odd-A nuclei are formed by the interaction of unpaired nucleon and $K = 0$ phonons [36]. Quasiparticle energies are obtained by the solution of the deformed Woods-Saxon potential. Mean field deformation parameters ($\delta_2$) are calculated using the ($\beta_2$) deformation parameters obtained from the experimental quadrupole moments in Ref.[37]. The calculation was carried out with $\delta_2 = 0$, $945, \beta_2 \left[ 1 - 2, 56A^{-2/3} \right] + 0, 34, \beta_2$ [38]. Pairing interaction constants have been taken from Ref. [31]. The pairing parameters ($\Delta$ and $\lambda$), the mean field deformations ($\delta_2$), the ground-state Nilsson configurations and the experimental ground-state magnetic moments [39] for $^{143,145,147}$Sm isotopes are given in Table 1.

As known from the literature, the isovector part of the magnetic dipole operator is more dominant than the isoscaler part (about 12 times) [32,38]. By comparing the theoretical and experimental magnetic moments of the $^{143,145,147}$Sm, the good results are obtained when $q = -1$. This overlaps with the presence of isovector character of magnetic dipole interactions [32]. The spin-spin interaction parameter $\chi$, in which the theoretical and experimental magnetic moments are in agreement, has been found between the 20 and 30 MeV/A. As an example, the variation of the calculated gKv values of $^{143}$Sm isotope according to "$\kappa = \chi . A$" and $q$ parameters is presented in Figure 1.

The calculations made using Kuliev-Pyatov Method (KPM), Single-Particle Model (SPM) and QPNM method (QRPA and QTDA approaches) mentioned in the introduction section and their comparison with the experimental data are given in Table 2.

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Table 1. $\Delta$ and $\lambda$ pairing correlation quantities, mean field deformations ($\delta_2$), Nilsson configurations and experimental ground state magnetic moments ($\mu_{exp}$) for $^{143,145,147}$Sm isotopes

| Nucleus | $I^\pi K [N_{n\Lambda}]$ | $\beta_2$ | $\delta_2$ | $\Delta_n$ (MeV) | $\Delta_p$ (MeV) | $\lambda_n$ | $\lambda_p$ | $\mu_{exp}$ |
|---------|--------------------------|----------|------------|-----------------|-----------------|------------|------------|-------------|
| $^{143}$Sm 3/2$^+$ [402] $\downarrow$ | 0.354 | 0.306 | 1.078 | 1.322 | -9.124 | -4.778 | 1.01(2) |
| $^{143}$Sm 7/2$^-$ [523] $\uparrow$ | 0.144 | 0.124 | 1.046 | 1.318 | -8.825 | -5.071 | -1.11(6) |
| $^{143}$Sm 7/2$^-$ [503] $\uparrow$ | 0.143 | 0.123 | 0.950 | 1.218 | -7.809 | -5.725 | -0.81(2) |

Table 2. Theoretical (KPM, SPM, QTDA, QRPA) results and experimental values for $^{143,145,147}$Sm isotopes

| Nuc. | $g_{seff}^{QPM}$ / $g_{sn}^{QPM}$ | $g_k$ | $g_{seff}^{QPM}$ / $g_{sn}^{QPM}$ | $g_k$ |
|------|---------------------------------|-------|---------------------------------|-------|
| $^{143}$Sm | 0.693 | 0.694 | 0.731 | 1.196 | 0.829 | 0.830 | 0.874 | 0.856(22) |
| $^{143}$Sm | 0.657 | 0.657 | 0.705 | -0.413 | -0.271 | -0.272 | -0.291 | -0.436(22) |
| $^{143}$Sm | 0.642 | 0.643 | 0.694 | -0.320 | -0.205 | -0.205 | -0.222 | -0.327(7) |

The results show that QPNM calculations on the basis of QRPA are more successful than other theoretical
predictions in explaining exp. data. The difference between predictions of the single-particle model and the other theoretical models is due to neglecting the contributions from the quasiparticle-phonon interactions in the single-particle model [1-3]. Now, let us briefly remind you the basic principles of KPM method in order to compare QPNM calculations with QRPA and QTDA based in detail. Let us briefly remind you the basic principles of the KPM method in order to compare the KPM results with QPNM calculations on the basis of QRPA and QTDA in detail. The KPM method is based on the independent quasiparticles model and explains the ground-state magnetic properties of odd-mass nuclei within the framework of TDA, based on the assumption of the magnetic dipole vibrations in the core nuclei [14-16]. The wave function of the odd-A nucleus in KPM consists of single and three quasiparticle components, respectively. One of the reasons why the gK vibrations in the core nuclei \([14-16]\). The wave function of the odd-A nucleus in KPM consists of single and three quasiparticle components. In addition, as emphasized in Ref. \([41]\), the ground- and low-lying states of the nuclei are weakly affected by the qp-phonon interactions, and almost all contribution comes from the one- quasiparticle components. As mentioned just before, the BCS vacuum in both approaches is like the double-even core of odd-mass nuclei [1-3].

As can be seen from Table 2, QRPA results for all nuclei are in the range of phenomenological \(g_s\) values \(g_s^{1f.} = (0.5 - 0.7)g_s^v\). One of the most important problems of nuclear physics is the explanation of the renormalization of spin gyromagnetic factor. Spin gyromagnetic factor not only has an effect on the magnetic moments of nuclei, but also plays an active role in calculating the magnetic dipole excitations in odd-A and even N-even Z nuclei. In order to solve the renormalization problem in the microscopic approach presented here, the interaction between the unpaired nucleon and the double-even core phonons must be accurately determined.

The one quasiparticle \((N_{K_0})\) and quasiparticle\(\otimes\)phonon \((G_{i\mu}^{K_0v})\) amplitudes in the wave functions of \(^{143-147}\)Sm isotopes, the ground-state nuclear structure and the \(1^+\) phonon energies of even-even core are given in Table 3. The quasiparticle-phonon and two quaziparticle amplitudes \(\psi_1, \psi_2\) given in the table are only given for states whose contribution to the wave function is greater than 1%.

| Nucleus | \(K^+\) | The structure of low-lying (ground) states | Two - quasiparticle characteristics of the even - even phonon core |
|---------|---------|---------------------------------------------|---------------------------------------------------------------|
|         |         | Single-quasiparticle | Quasiparticle\(\otimes\)phonon | \(\omega_1\) (MeV) | \([N_{t_\alpha}A]\) \(\sum\) | \(\psi_{ss}\) |
| \(^{143}\)Sm | 3/2\(^+\) | 99.80\% \([402]\) \(\downarrow\) | 0.010\% \([402]\) \(\downarrow\otimes\)Q\(_6\) | 5.089 | pp420 | −419 \(\downarrow\) | −0.700 |
|          |         |                              | 0.012\% \([402]\) \(\downarrow\otimes\)Q\(_9\) | 5.874 | mm411 | −402 \(\downarrow\) | 0.292 |
|          |         |                              | −0.012\% \([402]\) \(\downarrow\otimes\)Q\(_{20}\) | 7.477 | pp550 | −550 \(\uparrow\) | 0.648 |
|          |         |                              | −0.011\% \([402]\) \(\downarrow\otimes\)Q\(_{23}\) | 7.708 | mn523 | −512 \(\uparrow\) | −0.196 |
| \(^{145}\)Sm | 7/2\(^−\) | 99.9\% \([523]\) \(\uparrow\) | −0.013\% \([523]\) \(\uparrow\otimes\)Q\(_3\) | 3.905 | pp420 | −411 \(\downarrow\) | −0.705 |
|          |         |                              | 0.011\% \([523]\) \(\uparrow\otimes\)Q\(_{20}\) | 7.071 | mm532 | −523 \(\downarrow\) | 0.618 |
|          |         |                              | 0.015\% \([523]\) \(\uparrow\otimes\)Q\(_{35}\) | 8.421 | pp413 | −404 \(\downarrow\) | −0.148 |
| \(^{147}\)Sm | 7/2\(^−\) | 99.9\% \([503]\) \(\uparrow\) | 0.012\% \([503]\) \(\uparrow\otimes\)Q\(_{39}\) | 8.326 | mm550 | −530 \(\uparrow\) | 0.121 |

Table 3. Ground-state structures calculated on QPNM(QRPA) of \(^{143,145,147}\)Sm nuclei

It is known that one-quasiparticle and qp-phonon components contribute to ground- and excited states as a result of qp\(\otimes\)phonon interactions in odd-A nuclei [40]. However, it can be seen from the results in Table 3 that the ground-states of the nuclei are weakly affected by the qp-phonon interactions, and almost all contribution comes from the one- quasiparticle components. In addition, as emphasized in Ref. [41], the ground-and low-lying
excitation- states of the nuclei are weakly affected by qp-phonon interactions. In general, the main contribution to $G_{K0}^{N\mu}$ amplitude comes from $1^+ K =1^+ 0$ phonons of M1 dipole resonance in the 6–11 MeV energy range. Calculations show that $N_{K0}$ amplitude $\chi$ is very poorly dependent on the interaction parameter.

As a result, it is very important to choose suitable nuclear interactions in magnetic moment calculations. Since spin-spin residual interactions play an active role in renormalization of $g_s$ factors, we take into account spin-dependent interactions in our calculations. The results obtained reveal that spin interactions allow to successfully explain the spin polarization phenomenon observed in odd-mass nuclei and its effects on ground-state magnetic properties [1-3]. The applications in physics have been mostly in the field of general and differential equation [42-44].

The importance of spin-spin forces is not only limited to the description of the spin polarization effect and ground-state magnetic properties in odd-A nuclei, it is also known that the spin-dependent part of neutron-proton interactions is responsible for the Gamow-Teller $\beta$ transitions between the low energy levels of the nuclei [45, 46]. More importantly, the studies performed to now show that spin-spin forces have successfully explained 1+ excitations and scissor mode in even-even nuclei [47-56]. For these reasons, it is very important to determine the spin-spin interaction parameters that are the subject of this article.

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References

[1] Yakut, H., Guliyev, E., Guner, M., Tabar, E., Zenginerler, Z.: QPNM calculation for the ground state magnetic moments of odd-mass deformed nuclei: $^{157-167}$Er. Nuclear Physics A. 888, 23-33 (2012).
[2] Yakut, H., Tabar, E., Kuliev, A.A., Zenginerler, Z., Kaplan, P.: Ground state magnetic properties of odd neutron Dy isotopes. International Journal of Modern Physics E. 22 (10), 1350076(13) (2013).
[3] Yakut, H., Tabar, E., Kuliev, A.A., Guliyev, E.: The ground-state magnetic moments of odd-mass Hf isotopes. Central European Journal of Physics. 12, 843-850 (2014).
[4] Yakut, H., Tabar, E., Hosgör, G.: Effects of the Isoscalar and Isovector Interaction on the Ground-State Magnetic Moments of the Odd-Mass $^{137-145}$Ce Nuclei. Canadian Journal of Physics. 97 (11), 1187-1190 (2019).
[5] Tabar, E., Yakut, H., Kuliev, A.A., Quliyev, H., Hosgör, G.: Magnetic Moments and $g$ factors in odd-mass Ho isotopes. Chinese Physics C. 41 (7), 074101 (2017).
[6] Tabar, E.: Magnetic properties of $K =1/2$ states in deformed odd-mass nuclei. Nuclear Physics A. 986, 150-166 (2019).
[7] Tabar, E.: A theoretical study on the ground and low-energy magnetic dipole characteristics of $^{239}$Pu nucleus. Nuclear Physics A. 987, 202-221 (2019).
[8] Tabar, E.: A theoretical investigation of the magnetic dipole moment of the ground- and excited-states below 600keV in well-deformed $^{183,185}$W, $^{185,187}$Re and $^{187,189}$Os . Int. J. Mod. Phys. E 28(6), 1950040 (2019).
[9] Arima, A., Horie, H.: Configuration mixing and magnetic moments of odd nuclei. Prog. Theor. Phys. 12(5), 623–641 (1954).
[10] Enguang, Z.: Recent progress in theoretical studies of nuclear magnetic moments. Chin. Sci. Bull. 57(34), 4394–4399 (2012).
[11] Shimizu, K., Ichimura, M., Arima, A.: Magnetic moments and GT-type $\beta$-decay matrix elements in nuclei with a LS doubly closed shell plus or minus one nucleon . Nucl. Phys. A. 226(2), 282–318 (1974).
[12] Towner, I.S., Khanna, F.C.: Corrections to the single-particle M1 and Gamow-Teller matrix elements. Nucl. Phys. A. 339, 334–364 (1983).
[13] Arima, A.: A short history of nuclear magnetic moments and GT transitions. Sci. China Phys. Mech. Astron. 54, 188–193 (2011).
[14] Bochnacki, Z., Ogaza, S.: Spin polarization effect and the magnetic moments of odd-mass deformed nuclei. Nucl. Phys. 69 (2), 186–192 (1965).

[15] Kuliev, A.A., Pyatov, N.I.: Spin polarization effects in odd-mass deformed nuclei. Phys. Lett. B 28 (7), 443–445 (1969).

[16] De Boer, J., Rogers, J.D.: Concerning the magnetic properties of deformed nuclei in region 153 ≤ A ≤ 187. Phys. Lett. 3 (6), 304–306 (1963).

[17] Bochnacki, Z., Ogaza, S.: Spin polarization effect on the fast allowed beta transitions between deformed odd-mass nuclei. Nuclear Physics A 102, 529-533 (1967).

[18] Kuliev, A.A., Pyatov, N.I.: Magnetic dipole interactions in deformed nuclei. Sov. J. Nucl. Phys. 9 (2), 313–323 (1969).

[19] Yakut, H., Kuliev, A., Guliyev, E.: Investigations of the gK-factors in the 175,177,179Hf isotopes. AIP Conf. Proc. 1072, 258–261 (2008).

[20] Yakut, H., Kuliev, A.A., Guliyev, E., Yıldırım, Z.: Intrinsic gK factors of odd-mass 167−179Lu isotopes. Pramana–J. Phys. 73, 829-837 (2009).

[21] Yakut, H.: Nadir toprak deformede çekirdeklerinde kolektif dipol seviyelerinin elektrik ve manyetik dipol özelliklerinin incelenmesi. Ph.D. thesis. Sakarya University (2009).

[22] England, J., Grant, I., Griffith, J., Evans, D., Eastham, D., Newton, G., Walker, P.: Isotope shifts and hyperfine splitting in 144−154Sm. Journal of Physics G: Nuclear and Particle Physics, 16 (1), 105–123 (1990).

[23] Letokhov, V., Mishin, V., Sekatsky, S., Fedoseyev, V., Alkhazov, G., Barzakh, A., Denisov, V., Starodubsky, V.: Laser spectroscopic studies of nuclei with neutron number N < 82 (Eu, Sm and Nd isotopes). Journal of Physics G: Nuclear and Particle Physics, 18 (7), 1177–1173 (1992).

[24] Kaplan, M., Blok, J., Shirley, D.: Magnetic Moment of Sm145 and Attenuation Following the Decay of Oriented Sm145. Ph Rev. 184 (4), 1177–1180 (1969).

[25] Woodgate, G.K.: Hyperfine structure and nuclear moments of samarium. The Royal Society, Proc. R. Soc. Lond. A, 293, 117-144 (1966).

[26] Childs, W.J., Goodman, L.S.: Reanalysis of the Hyperfine Structure of the 4f6s27 F Multiplet in 147,149Sm, Including Measurements for the 7 F 6 State. Physical Review A, 6 (4), 2011-2021 (1972).

[27] Soloviev, V.G.: Microscopic description of vibrational states in deformed nuclei. Prog. Part. Nucl. Phys. 28 (C), 49-74 (1992).

[28] Soloviev, V.G. Sushkov, A.V., Shirikova, N.Y.: Gamma-ray transitions between excited states in 168Er. J. Phys. G: Nucl. Part. Phys. 20, 113-134 (1994).

[29] Soloviev, V.G. Sushkov, A.V., Shirikova, N.Y.: Description of low-lying vibrational and two-quasiparticle states in 166Er. Phys. Rev. C. 51(2), 551-558 (1995).

[30] Malov, L.A., Nesterenko, V.O., Soloviev, V.G.: Low-energy octupole resonances in deformed nuclei. J. Phys. G: Nucl. Phys. 3(9), 219-222 (1977).

[31] Pyatov, N.I., Salamov, D.I.: Conservation laws and collective excitations in nuclei. Nukleonika. 22(1), 127-140 (1977).

[32] Bohr, A., Mottelson, B.: Nuclear structure. Vol. 1, Benjamin, New York and Amsterdam (1969).

[33] Soloviev, V.G., Sushkov, A.V., Shirikova, N.Y.: Low-lying magnetic dipole strength in 163Dy. Phys. Rev. C. 53, 1022-1024 (1995).

[34] Luenberger, D.G.: Optimization by Vector Space Methods. John Wiley Sons. New York (1969).

[35] Bussotti, P.: On the Genesis of the Lagrange Multipliers. Journal of Optimization Theory and Applications 117, 453–459 (2003).
[36] Bauske, I., Arias, J.M., Brentano, P., Von., Frank, A., Friedrichs, H., Heil, R.D., Herzberg, R.D., Hoyler, F., Van Isacker, P., Kneissl, U.: *First observation of scissors mode states in an odd-mass nucleus*. Phy. Rev. Lett. 71, 975-978 (1993).

[37] Raman, S., Nestor, C.W., Tikkanen, P.: *Transition probability from the ground to the first-excited 2+ state of even–even nuclides*. Atomic Data and Nuclear Data Tables 90, 1-128 (2001).

[38] Soloviev, V.G.: *Theory of complex nuclei*. Pergamon Press New York (1976).

[39] Stone, N.J.: *Table of nuclear magnetic dipole and electric quadrupole moments*. Atomic Data and Nuclear Data Tables 78, 75-176 (2005).

[40] Bow, Y.F.: *Magnetic dipole transition probabilities of deformed odd-mass nuclei*. Phys. Rev. C 2(5), 1608–1611 (1970).

[41] Soloviev, V.G.: *Theory of atomic nuclei: Quasiparticles and phonons*. Institute of Physics Publishing Bristol and Philadelphia (1992).

[42] Balasubramanian, K.: *Computational enumeration of colorings of hyperplanes of hypercubes for all irreducible representations and applications*, J. Math. Sci. Model., 1(3), 158 - 180 (2018).

[43] Li, R.: *Energy conditions for Hamiltonian and traceable graphs*, Univers. J. Math. Appl. 2(1), 33-35 (2019).

[44] Thangamuthu, M., Thippan, j.: *Numerical solution for Hybrid fuzzy differential equation by fifth order Runge-Kutta Nystrom method*, J. Math. Sci. Model., 2(1), 39-50 (2019).

[45] Fujita, Jun-I., Ikeda, K.: *Existence of isobaric states and beta decay of heavier nuclei*. Nucl. Phys. 67(1), 145–1771 (1965).

[46] Gabrakov, S.I., Kuliev, A.A., Pyatov, N.I.: *0+ and 1+ unlike particle-hole states in deformed odd-odd nuclei and β-strength functions*. Phys. Lett. B. 36(4), 275-277 (1971).

[47] Kuliev, A.A., Akkaya, R., Ilhan, M., Guliyev, E., Salamov, C., Selvi, S.: *Rotational-invariant model of the states with K\(\pi\)=1+ and their contribution to the scissors mode*. Int. J.Mod. Phys. E 9(3), 249-261 (2000).

[48] Kuliev, A.A., Guliyev, E., Gerçeklioglu, M.: *The dependence of the scissors mode on the deformation in the 140−150 Ce isotopes*. J. Phys. G: Nucl. Phys. 28(3), 407-414 (2002).

[49] Guliyev, E., Ertugral, F., Kuliev, A.A.: *Low-lying magnetic dipole strength distribution in the γ-soft even-even 130,132Ba*. Eur. Phys. J. A. 23(7), 313-320 (2006).

[50] Guliyev, E., Kuliev, A.A., Ertugral, F.: *Low-lying dipole excitations in the deformed even-even isotopes 154,160 Gd*. Acta. Phys. Pol. B. 40(3), 653-656 (2009).

[51] Guliyev, E., Kuliev, A.A., Ertugral, F.: *Systematic investigation of the low-energy dipole excitations in 176,178,180 Hf within rotational, translational and Galilean invariant quasiparticle RPA*. Nucl. Phys. A. 915, 78-89 (2013).

[52] Zenginerler, Z., Guliyev, E., Kuliev, A.A., Yakut, H., Soluk, G.: *Systematic investigation of the low-lying dipole excitations in even-even 124,136 Ba isotopes*. Eur. Phys. J. A. 49(9), 1-7 (2013).

[53] Tabar, E., Yakut, H., Kuliev, A.A.: *Magnetic dipole response of the 169 Tm nucleus*. Nuclear Physics A. 981, 130-146 (2019).

[54] Tabar, E., Yakut, H., Kuliev, A.A.: *Low-energy dipole strength in even–even 152,164 Dy isotopes within the quasiparticle random phase approximation including symmetry restoring interactions*. Nuc.Phys. A 979, 143-164 (2018).

[55] Tabar, E., Yakut, H., Kuliev, A.A.: *Microscopic description of low-lying M1 excitations in odd-mass actinide nuclei*. Nuc.Phys. A 957, 33–50 (2017).

[56] Tabar, E., Yakut, H., Kuliev, A.A.: *Microscopic description of ground state magnetic moment and low-lying magnetic dipole excitations in heavy odd-mass 181 Ta nucleus*. Inter. J. Modern Phy. E- Nuclear Physics 25(8), 1650053 (2016).
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