Naturalness and Testability of TeV Seesaw Mechanisms

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1) After outlining some popular ways to go beyond the standard model so as to generate non-zero but tiny neutrino masses, I focus on several typical seesaw mechanisms and discuss how to get a balance between their theoretical naturalness and their experimental testability. Besides possible collider signatures at the Large Hadron Collider, new and non-unitary CP-violating effects are also expected to show up in neutrino oscillations for type-I, type-(I+II), type-III and double seesaws at the TeV scale.

x1. Possible ways to go beyond the SM

The Large Hadron Collider (LHC) is presumably bringing us to a new energy frontier: the TeV scale, at which some fundamental new physics beyond the standard model (SM) of electroweak interactions is expected to show up and reveal the origin of masses of elementary particles. Can the LHC help us to understand the origin of neutrino masses? In other words, could TeV neutrino physics become an exciting direction in the LHC era? This question is just the motivation of my talk.

Neutrinos are assumed or required to be massless in the SM, just because the structure of the SM itself is too simple to accommodate massive neutrinos.

Two fundamental entities of the SM are the SU(2) × U(1) gauge symmetry and the Lorentz invariance. Both of them are mandatory to guarantee that the SM is a consistent quantum field theory.

The particle content of the SM is rather economical. There are no right-handed neutrinos in the SM, so a Dirac neutrino mass term is not allowed. There is only one Higgs doublet, so a gauge-invariant Majorana mass term is forbidden.

The SM is a renormalizable quantum field theory. Hence an effective dimension-5 operator, which may give each neutrino a Majorana mass, is absent. In other words, the SM accidently possesses the (B-L) symmetry which assures three known neutrinos to be exactly massless.

But today’s experiments have convincingly indicated the existence of neutrino oscillations. This quantum phenomenon can appear if and only if neutrinos are massive and leptons are mixed, and thus it is a kind of new physics beyond the SM. To generate non-zero but tiny neutrino masses, one or more of the above-mentioned constraints on the SM must be abandoned or relaxed. It is certainly intolerable to abandon the gauge symmetry and Lorentz invariance; otherwise, one would be led astray. Given the framework of the SM as a consistent field theory, its particle content can be modified and (or) its renormalizability can be abandoned to accommodate massive neutrinos. There are several ways to this goal.

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1.1. To relax the requirement of renormalizability

In 1979, Weinberg extended the SM by introducing some higher-dimension operators in terms of the fields of the SM itself:

\[ L_e = L_{SM} + \frac{L_{d=5}}{2} + \frac{L_{d=6}}{2} ; \]  

where \( L \) denotes the cutoff scale of this effective theory. Within such a framework, the lowest-dimension operator that violates the lepton number (\( L \)) is the unique dimension-5 operator \( H H LL = \). After spontaneous gauge symmetry breaking, this Weinberg operator yields \( m_1 = \frac{h}{v} \) for neutrino masses, which can be sufficiently small (\( 1 \text{ eV} \)) if not far away from the scale of grand unified theories (\( \sim 10^{13} \text{ G eV} \) for \( H_i = 10^6 \text{ G eV} \)). In this sense, people argue that neutrino masses can serve as a low-energy window onto new physics at super-high energy scales.

1.2. To add three right-handed neutrinos and demand \((B-L)\) symmetry

Given three right-handed neutrinos, the gauge-invariant and lepton-number-conserving mass term of charged leptons and neutrinos are

\[ L_{\text{lepton}} = \sum L Y^T H E_R + \sum L Y^T H N_R + h \epsilon ; \] 

where \( H \) denotes the lepton doublet and \( L \) denotes the left-handed lepton doublet. After spontaneous gauge symmetry breaking, we arrive at the charged-lepton mass matrix \( M_L = Y_L v = \frac{1}{2} \) and the Dirac neutrino mass matrix \( M = Y v = \frac{1}{2} \) with \( v' = 246 \text{ G eV} \). In this case, the smallness of three neutrino masses \( m_i \) (for \( i = 1; 2; 3 \)) is attributed to the smallness of three eigenvalues of \( Y \) (denoted as \( y_i \) for \( i = 1; 2; 3 \)). Then we encounter a transparent hierarchy problem: \( y_1 = y_e = m_1 = m_e \). 0.5 eV = 0.5 MeV \( \sim 10^{6} \). Why is \( y_1 \) so small? There is no explanation at all in this Dirac-mass picture.

A speculative way out is to invoke extra dimensions; namely, the smallness of Dirac neutrino masses is ascribed to the assumption that three right-handed neutrinos have access to one or more extra spatial dimensions. The idea is simple: to connect the SM particles onto a brane and to allow \( N_R \) to travel in the bulk. For example, the wave-function of \( N_R \) spreads out over the extra dimension \( y \), giving rise to a suppressed Yukawa interaction at \( y = 0 \) (i.e., the location of the brane):

\[ h \left( \frac{1}{2} Y H^T N_R \right)_{y=0} = \frac{1}{2} \sum L Y H N_R \] 

The magnitude of \( 1 \) is measured by \( = \text{Planck} \), and thus it can naturally be small for an effective theory far below the Planck scale.

1.3. To add new degrees of freedom and allow \((B-L)\) violation

This approach works at the tree level and reflects the essential spirit of seesaw mechanisms. Tiny masses of three known neutrinos are attributed to the existence of heavy degrees of freedom and lepton-number violation. There are three simple and typical seesaw mechanisms on the market.
Type-I seesaw | three heavy right-handed neutrinos are added into the SM and the lepton number is violated by their Majorana mass term:

\[ L_{\text{lepton}} = \frac{1}{2} Y R E_R + \frac{1}{2} Y H R N_R + \frac{1}{2} Y N R N_R + h \xi ; \quad (1.4) \]

where \( M_R \) is the symmetric Majorana mass matrix.

Type-II seesaw | one heavy Higgs triplet is added into the SM and the lepton number is violated by its interactions with both the lepton doublet and the Higgs doublet:

\[ L_{\text{lepton}} = \frac{1}{2} Y H E_R + \frac{1}{2} Y I M L \quad (1.5) \]

where

\[ \begin{pmatrix} P_Z & \xi \end{pmatrix} \quad (1.6) \]

denotes the SU(2) Higgs triplet.

Type-III seesaw | three heavy triplet fermions are added into the SM and the lepton number is violated by their Majorana mass term:

\[ L_{\text{lepton}} = \frac{1}{2} Y H E_R + \frac{1}{2} Y M L \quad (1.7) \]

where

\[ \begin{pmatrix} P_Z & \xi \end{pmatrix} \quad (1.8) \]

denotes the SU(2) fermion triplet.

Of course, there are a number of variations or combinations of these three typical seesaw mechanisms in the literature.

For each of the above seesaw pictures, one may arrive at the unique dimension-5 Weinberg operator of neutrino masses after integrating out the corresponding heavy degrees of freedom:

\[ L_{d=5} \]

\[ \begin{pmatrix} - \frac{1}{2} Y M \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} Y \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} + h \xi ; \]

(Type I);

\[ \begin{pmatrix} - \frac{1}{2} Y M \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} Y \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} + h \xi ; \]

(Type II);

\[ \begin{pmatrix} - \frac{1}{2} Y M \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} Y \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} \begin{pmatrix} 1 & Y^T \end{pmatrix} + h \xi ; \]

(Type III);

After spontaneous gauge symmetry breaking, \( H' \) achieves its vacuum expectation value \( v' = \frac{1}{2} v \) with \( v = 246 \) GeV. Then we are left with the effective Majorana neutrino mass term for three known neutrinos,

\[ L_{\text{mass}} = \frac{1}{2} Y L M c + h \xi ; \quad (1.10) \]
where the symmetric Majorana mass matrix $M$ is given by

$$
M = \begin{pmatrix}
\frac{1}{2} \frac{\nu^2}{M} \mathbf{Y}^T & \nu \mathbf{Y} \\
\nu \mathbf{Y} & \frac{\nu^2}{M}
\end{pmatrix} \quad \text{(Type I);} \\
\begin{pmatrix}
\frac{1}{2} \frac{\nu^2}{M} \mathbf{Y}^T & \nu \mathbf{Y} \\
\nu \mathbf{Y} & \frac{\nu^2}{M}
\end{pmatrix} \quad \text{(Type II);} \\
\begin{pmatrix}
\frac{1}{2} \frac{\nu^2}{M} \mathbf{Y}^T & \nu \mathbf{Y} \\
\nu \mathbf{Y} & \frac{\nu^2}{M}
\end{pmatrix} \quad \text{(Type III):}
$$

It becomes obvious that the smallness of $M$ can be attributed to the largeness of $M_R$, $M$ or $M$ in the seesaw mechanism.

1.4. Radiative generation of tiny neutrino masses

In a seminal paper published in 1972, Weinberg pointed out that in theories with spontaneously broken gauge symmetries, various masses or mass differences may vanish in zeroth order as a consequence of the representation content of the fields appearing in the Lagrangian. These masses or mass differences can then be calculated as finite higher-order effects. Such a mechanism may allow us to slightly go beyond the SM and radiatively generate tiny neutrino masses. A typical example is the well-known Zee model.

$$
\mathcal{L}_{\text{lepton}} = \sum_{\ell} Y_{\ell H} \overline{E}_\ell + \mathcal{L}_{\text{Y}} + \sum_{\ell} \overline{\nu}_\ell \mathbf{Y} \nu + \overline{\nu}_\ell \mathbf{Y}^T \nu + \mathcal{L}_{\text{F}} + \mathcal{L}_{\text{h}};
$$

where $S$ are charged $SU(2)_L$ singlet scalars, denotes a new $SU(2)_L$ doublet scalar which has the same quantum number as the SM Higgs doublet $\mathbf{H}$, $Y_S$ is an antisymmetric matrix, and $F$ represents a mass. Without loss of generality, we choose the basis of $M_1 = Y_{\ell H} i = D \{ \nu \} \{ \nu \} \mathbf{m} \ \mathbf{m}$. In this model, neutrinos are massless at the tree level, but their masses can radiatively be generated via the one-loop corrections. Given $M_S$, $M_H$, $M_F$ and $M_H$, the elements of the effective mass matrix of three light Majorana neutrinos turns out to be

$$
(M) = \begin{pmatrix}
\frac{M_H}{16} \frac{\mathbf{m}^2}{M_S^2} & \frac{\mathbf{m}^2}{M_S^2} \\
\frac{\mathbf{m}^2}{M_S^2} & \frac{\mathbf{m}^2}{M_S^2}
\end{pmatrix} \quad \text{(Type I).}
$$

where and run over e, and . The smallness of $M$ is therefore ascribed to the smallness of $Y_S$ and $(\mathbf{m}^2 \frac{\mathbf{m}^2}{M_S^2}) = \mathbf{M}_2$. Although the original version of the Zee model is disfavored by current experimental data on neutrino oscillations, its extensions or variations at the one-loop or two-loop level can survive.

As we have seen, the key point of a seesaw mechanism is to ascribe the smallness of neutrino masses to the existence of some new degrees of freedom heavier than the Fermi scale $v' \approx 246$ GeV, such as heavy Majorana neutrinos or heavy Higgs bosons. The energy scale where a seesaw mechanism works is crucial, because it is relevant to whether this mechanism is theoretically natural and experimentally
testable. Between Fermi and Planck scales, there might exist two other fundamental scales: one is the scale of a grand unified theory (GUT) at which strong, weak and electromagnetic forces can be unified, and the other is the TeV scale at which the unnatural gauge hierarchy problem of the SM can be solved or at least softened by a kind of new physics.

2.1. How about a very low seesaw scale?

In reality, however, there is no direct evidence for a high or extremely high seesaw scale. Hence eV, keV, MeV- and GeV-scale seesaws are all possible, at least in principle, and they are technically natural in the sense that their lepton-number violating mass terms are naturally small according to 't Hooft's naturalness criterion. At any energy scale, a set of parameters describing a system can be small if and only if, in the limit of each of these parameters, the system exhibits an enhanced symmetry. But there are several potential problems associated with low-scale seesaws: (a) a low-scale seesaw does not give any obvious connection to a theoretically well-justified fundamental physical scale (such as the Fermi scale, the TeV scale, the GUT scale or the Planck scale); (b) the neutrino Yukawa couplings in a low-scale seesaw model turn out to be tiny, giving no actual explanation of why the masses of three known neutrinos are so small; and (c) in general, a very low seesaw scale does not allow the canonical \( \text{higgs} \) mechanism to work, although there might be a way out.

2.2. The hierarchy problem of conventional seesaws

Many theorists argue that the conventional seesaw scenarios are natural because their scales (i.e., the masses of heavy degrees of freedom) are close to the GUT scale. This argument is reasonable on the one hand, but it neglects the drawbacks of the conventional seesaw models on the other hand. In other words, the conventional seesaw models have no direct experimental testability and involve a potential hierarchy problem. The latter is usually spoke of when two largely different energy scales exist in a model, but there is no symmetry to stabilize the low-scale physics suffering from large corrections coming from the high-scale physics.

Such a seesaw-induced renormalization means that the SM Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom in a seesaw mechanism. For example,\(^{13,15}\)

\[
M^2_H = \frac{8}{\lambda_i^2} y_i^2 2 + M^2 2 \ln \frac{M^2}{2} \quad \text{(Type I)}
\]

\[
M^2_H = \frac{3}{16} 2 2 + M^2 2 \ln \frac{M^2}{2} + \frac{4}{2} M^2 2 \ln \frac{M^2}{2} \quad \text{(Type II)}
\]

\[
M^2_H = \frac{3y_i^2}{8} 2 + M^2 2 \ln \frac{M^2}{2} \quad \text{(Type III)}
\]

in three typical seesaw scenarios, where \( \lambda_i \) is the regulator cut-off, \( y_i \) and \( M_i \) (for \( i = 1, 2, 3 \)) stand respectively for the eigenvalues of \( Y \) (or \( Y^R \)) and \( M_R \) (or \( M_L \)), and the contributions proportional to \( v^2 \) and \( M^2_H \) have been omitted. Eq. (2.1) show a
quadratic sensitivity to the new scale which is characteristic of the seesaw model, implying that a high degree of ne-tuning would be necessary to accommodate the experimental data on $M_{\mu}$ if the seesaw scale is much larger than $v$ (or the Yukawa couplings are not extremely ne-tuned in type-I and type-III seesaw scenarios).

Taking the type-I seesaw scenario for illustration, we assume $M_{\mu}$ and require $jM_{\mu}^2 \geq 0.1 \text{TeV}^2$. Then Eq. (2.1) leads us to the following rough estimate:

$$M_{\mu} \frac{(2v)^2 j M_{\mu}^2 \sigma_{\text{fs}}}{m_1} \cdot 10^7 \text{GeV} \geq 0.2 \text{eV} \frac{10^7 \text{GeV}}{m_1} \frac{10^3}{0.1 \text{TeV}^2} : \quad (2.2)$$

This naive result indicates that a hierarchy problem will arise if the masses of heavy Majorana neutrinos are larger than about $10^7$ GeV in the type-I seesaw scheme. Because of $m_1 \frac{(2v)^2 j M_{\mu}^2 \sigma_{\text{fs}}}{m_1} = (2M_{\mu})$, the bound $M_{\mu} \geq 10^7 \text{GeV}$ implies $y_1 \geq 2m_1/v$. Such a small magnitude of $y_1$ seems to be a bit unnatural in the sense that the conventional seesaw idea attributes the smallness of $m_1$ to the largeness of $M_{\mu}$ other than the smallness of $y_1$.

There are two possible ways out of this impasse: one is to appeal for the supersymmetry, and the other is to lower the seesaw scale. In the remaining part of this talk, I shall follow the second way to discuss the TeV seesaw mechanism which do not suffer from the above-mentioned hierarchy problem.

2.3. Why are the TeV seesaws interesting?

There are several reasons for people to expect some new physics at the TeV scale. This kind of new physics should be able to stabilize the Higgs-boson mass and hence the electroweak scale; in other words, it should be able to solve or soften the unnatural gauge hierarchy problem. It has also been argued that the weakly-interacting particle candidates for dark matter should weigh about one TeV or less. If the TeV scale is really a fundamental scale, may we argue that the TeV seesaws are natural? Indeed, we are reasonably motivated to speculate that possible new physics existing at the TeV scale and responsible for the electroweak symmetry breaking might also be responsible for the origin of neutrino masses. It is interesting and meaningful in this sense to investigate and balance the "naturalness" and "testability" of TeV seesaws at the energy frontier set by the LHC.

As a big bonus of the conventional (type-I) seesaw mechanism, the thermal leptogenesis mechanism provides us with an elegant dynamic picture to interpret the cosmological baryon-antibaryon asymmetry characterized by the observed ratio of baryon number density to photon number density, $n_B/n = (6.1, 0.2) \times 10^9$. When heavy Majorana neutrino masses are down to the TeV scale, the Yukawa couplings should be reduced by more than six orders of magnitude so as to generate tiny masses for three known neutrinos via the type-I seesaw and satisfy the out-of-equilibrium condition, but the CP-violating asymmetries of heavy Majorana neutrino decays can still be enhanced by the resonant effects in order to account for $n_B/n$. This "resonant leptogenesis" scenario might work in a specific TeV seesaw model.

Is there a TeV Noah's Ark which can naturally and simultaneously accommodate the seesaw idea, the leptogenesis picture and the collider signatures? We are most likely not so lucky and should not be too ambitious at present. In the subsequent
sections, we shall concentrate on the TeV seesaws themselves and their possible collider signatures and low-energy consequences.

x3. Naturalness and testability of TeV seesaws

The neutrino mass terms in three typical seesaw mechanisms have been given in section x1.3. Without loss of generality, we choose the basis in which the mass eigenstates of three charged leptons are identified with their flavor eigenstates.

3.1. Type-I seesaw

Given \( M_D = Y v = \left[ \begin{array}{c} \begin{array}{ccc} \varpi_1 & \varpi_2 & \varpi_3 \end{array} \end{array} \right] \), the approximate type-I seesaw formula in Eq. (1.11) can be rewritten as \( M = M_D M_R^{-1} M_D^{T} \). Note that the 3 \( \times 3 \) light neutrino mixing matrix \( V \) is not exactly unitary in this seesaw scheme, and its deviation from unitarity is of \( O(10^{2}) \). Let us consider two interesting possibilities.

- \( M_D \approx O(10^{5}) \text{ GeV} \) and \( M_R \approx O(10^{5}) \text{ GeV} \) to get \( M \approx O(10^{2}) \text{ eV} \). In this conventional, and natural case, \( M_D = M_R \approx O(10^{1}) \text{ eV} \) holds. Hence the non-unitarity of \( V \) is only at the \( O(10^{-6}) \) level, too small to be observed.

- \( M_D \approx O(10^{5}) \text{ GeV} \) and \( M_R \approx O(10^{5}) \text{ GeV} \) to get \( M \approx O(10^{2}) \text{ eV} \). In this unnatural case, a significant structural cancellation has to be imposed on the textures of \( M_D \) and \( M_R \). Because of \( M_D = M_R \approx O(0.1) \), the non-unitarity of \( V \) can reach the percent level and may lead to some observable effects.

Now we discuss how to realize the above structural cancellation for the type-I seesaw mechanism at the TeV scale. For the sake of simplicity, we take the basis of \( M_R = D \text{ diag} M_1; M_2; M_3 \) for three heavy Majorana neutrinos \( (N_1; N_2; N_3) \). It is well known that \( M \) vanishes if

\[
M_D = \begin{pmatrix} 0 & Y_1 & Y_2 & Y_3 \\ Y_1 & 1 & 0 & 0 \\ Y_2 & 0 & 1 & 0 \\ Y_3 & 0 & 0 & 1 \end{pmatrix}
\]

simultaneously holds. Tiny neutrino masses can be generated from tiny corrections to the texture of \( M_D \) in Eq. (3.1). For example, \( M_D^{0} = M_D^{0} X_D \) with \( M_D^{0} \) given above and being a small dimensionless parameter (i.e., \( \varpi_{j} \approx 1 \)) will yield

\[
M_D^{0} = M_D^{0} M_R^{-1} M_D^{T} \quad M_D M_R^{-1} X_D^{T} + X_D M_R^{-1} M_D^{T}
\]

from which \( M_D^{0} \approx O(10^{2}) \text{ eV} \) can be obtained by adjusting the size of \( X \).

A lot of attention has recently been paid to a viable type-I seesaw model and its collider signatures at the TeV scale. At least the following lessons can be learnt:

Two necessary conditions must be satisfied in order to test a type-I seesaw model at the LHC: (a) \( \varpi_{j} \) are of \( O(10) \text{ TeV} \) or smaller; and (b) the strength of light-heavy neutrino mixing (i.e., \( M_D^{0} = M_R^{-1} \)) is large enough. Otherwise, it would be impossible to produce and detect \( N_1 \) at the LHC.

The collider signatures of \( N \) are essentially decoupled from the mass and mixing parameters of three light neutrinos \( \nu_i \). For instance, the small parameter in Eq. (3.2) has nothing to do with the ratio \( M_D^{0} = M_R \).
The non-unitarity of $V$ might lead to some observable effects in neutrino oscillations and other lepton-number-violating or lepton-number-conserving processes, if $M_0 = M_R$. (0.1) holds. More discussions will be given in section 4.

The clean LHC signatures of heavy Majorana neutrinos are the $L = 2$ like-sign dilepton events such as $p p \rightarrow W W \rightarrow jj$ (a collider analogue to the neutrinoless double-beta decay) and $p p \rightarrow N_1 \rightarrow jj$ (a dominant channel due to the resonant production of $N_1$).

Some instructive and comprehensive analyses of possible LHC events for a single heavy Majorana neutrino have recently been done but they only serve for illustration because such a simplified type-I seesaw scenario is actually unrealistic.

3.2. Type-II seesaw

The type-II seesaw formula $M = Y v = Y v^2 M$ has already been given in Eq. (1.11). Note that the last term of Eq. (1.5) violates both $L$ and $B - L$, and thus the smallness of $v$ is naturally allowed according to 'M. Hoof's naturalness criterion (i.e., setting $0 = 0$ will increase the symmetry of $L_{lepton}$). Given $M = (1\text{ TeV})$, for example, this seesaw mechanism works to generate $M = (10^2 \text{ eV})$ provided $Y \sim 10^{-12}$ holds. The neutrino mixing matrix $V$ is exactly unitary in the type-II seesaw mechanism, simply because the heavy degrees of freedom do not mix with the light ones.

There are seven physical Higgs bosons in the type-II seesaw scheme: doubly-charged $H^{\pm \pm}$, singly-charged $H^\pm$ and $H$, neutral $A^0$ (CP-odd), and neutral $h^0$ and $H^0$ (CP-even), where $h^0$ is the SM-like Higgs boson. Except for $M_{h^0}$, we get a quasi-degenerate mass spectrum for other scalars $M_{H^\pm} = M_{h^0} = M_{A^0}$. As a consequence, the decay channels $H^\pm \rightarrow W W$ and $H \rightarrow H H$ are kinematically forbidden. The production of $H$ at the LHC is mainly through $qg \rightarrow H$; $Z \rightarrow H H$ and $q\bar{q} \rightarrow W W$ are processes, which do not depend on the small Yukawa couplings.

The typical collider signatures in this seesaw scenario are the lepton-number-violating $H^\pm \rightarrow l l$ decays as well as $H^{\pm \pm} \rightarrow l l$ and $H^\pm \rightarrow l l$ decays. Their branching ratios

$$B(H^\pm \rightarrow l l) = \left( \frac{\frac{M}{M}}{\sqrt{2}} \times \frac{f}{f} \right)$$

are closely related to the masses, mixing angles and CP-violating phases of three light neutrinos, because $M = V \tilde{M} V^T$ with $\tilde{M} = \text{Diag} m_1 m_2 m_3 g$ holds. Some detailed analyses of such decay modes together with the LHC signatures of $H$ and $H$ bosons have been done in the literature.

It is worth pointing out that the following dimension-6 operator can easily be
derived from the type-II seesaw mechanism,

\[
\frac{L_{d=6}}{2} = \frac{(Y_L^Y) Y^Y}{4M^2} L_L \hat{L}_L L_L L_L ;
\]

which has two immediate low-energy effects: the non-standard interactions of neutrinos and the lepton-avor-violating interactions of charged leptons. An analysis of such effects provides us with some preliminary information.

The magnitudes of non-standard interactions of neutrinos and the widths of lepton-avor-violating tree-level decays of charged leptons are both dependent on neutrino masses \(m_i\) and avor mixing and CP-violating parameters of \(V\).

For a long-baseline neutrino oscillation experiment, the neutrino beam encounters the earth matter and the electron-type non-standard interaction contributes to the matter potential.

At a neutrino factory, the lepton-avor-violating processes \(\bar{e}^+ e^-\) and \(\bar{e}^- e^+\) could cause wrong-sign muons at a near detector.

Current experimental constraints tell us that such low-energy effects are very small, but they might be experimentally accessible in the future precision measurements.

3.3. Type-(I+II) seesaw

The type-(I+II) seesaw mechanism can be achieved by combining the neutrino mass terms in Eqs. (1.4) and (1.5). After spontaneous gauge symmetry breaking, we are left with the overall neutrino mass term

\[
L_{\text{mass}} = \frac{1}{2} \left( \begin{array}{ccc}
L_L & N_R^c & M_D^T \\
M_D & M_R & N_R^c \\
N_R^c & M_L & M_D \\
\end{array} \right) + \text{h.c.};
\]

where \(M_D = Y v^P Z\) and \(M_R = Y v^P v^P Z\) and \(h^P v^P v\) corresponding to the vacuum expectation values of the neutral components of the Higgs doublet \(H\) and the Higgs triplet \(H^\pm\). The 6×6 neutrino mass matrix in Eq. (3.5) is symmetric and can be diagonalized by the following unitary transformation:

\[
\begin{bmatrix}
V_R & Y_M^L & M_D \\
M_L & M_R & N_R \\
N_R & M_D & M_R \\
\end{bmatrix} = \begin{bmatrix}
0 & M_L & M_R \\
M_R & M_L & 0 \\
N_R & M_D & M_R \\
\end{bmatrix} N_R^c ;
\]

where \(M_L = \text{diag} m_1, m_2, m_3\) and \(M_R = \text{diag} m_4, m_5, m_6\). Needless to say, \(V^TY + S^YS = V^TY + R^YR = 1\) holds as a consequence of the unitarity of this transformation. Hence \(V\), the avor mixing matrix of three light Majorana neutrinos, must be non-unitary if \(R\) and \(S\) are non-zero.

In the leading-order approximation, the type-(I+II) seesaw formula reads as

\[
M = M_L M_D M_R^T :\]

Hence type-I and type-II seesaws can be regarded as two extreme cases of the type-(I+II) seesaw. Note that two mass terms in Eq. (3.7) are possibly comparable in magnitude. If both of them are small, their contributions to \(M\) may have significant
interference effects which make it practically impossible to distinguish between type-II and type-(I + II) seesaw signatures but if both of them are large, their contributions to M must be destructive. The latter case unnaturally requires a significant cancellation between two big quantities in order to obtain a small quantity, but it is interesting in the sense that it may give rise to possibly observable collider signatures of heavy Majorana neutrinos.

Let me briefly describe a particular type-(I + II) seesaw model and comment on its possible LHC signatures. First, we assume that both M1 and M2 are of O(1) TeV. Then the production of H and H bosons at the LHC is guaranteed, and their lepton-number-violating signatures will probe the Higgs triplet sector of the type-(I + II) seesaw mechanism. On the other hand, O(M3 = M1) • O(0.1) is possible as a result of O(MR) • O(1) TeV and O(MD). O(v), such that appreciable signatures of N1 can be achieved at the LHC. Second, the small mass scale of M1 implies that the relation O(M1) • O(MDM11/3) must hold. In other words, it is the significant but incomplete cancellation between M1 and MDM11/3 that results in the non-vanishing but tiny masses for three light neutrinos. We admit that dangerous radiative corrections to two mass terms of M poorly require a delicate net uniting of the cancellation at the loop level. But this scenario allows us to reconstruct M1 via the excellent approximation M1 = VM[N(T + R)] and M1 = VM[T + R], such that the elements of the Yukawa coupling matrix Y read

\[ (Y) = \frac{(M_1)}{v} x^3 \sum_{i=1}^{3} \frac{R_i R_{i1} M_{1i}}{v}; \]  

where the subscripts and run over e, μ, and τ. This result implies that the leptonic decays of H and H bosons depend on both R and M1, which actually determine the production and decays of N1. Thus we have established an interesting correlation between the singly- or doubly-charged Higgs bosons and the heavy Majorana neutrinos. To observe the correlated signatures of H, H and N1 at the LHC will serve for a direct test of this type-(I + II) seesaw model.

To illustrate, here I focus on the minimal type-(I + II) seesaw model with a single heavy Majorana neutrino, where R can be parametrized in terms of three rotation angles 4 and three phase angles 4 (for i = 1, 2, 3). In this case, we have

\[
\begin{align*}
\text{(pp ! + W X)_{1i}^H} & \sim \frac{M_{1i}}{v} \left( s_{i14}^2 + s_{i24}^2 + s_{i34}^2 \right), \\
\text{(pp ! + H X)_{1i}^H} & \sim \frac{M_{1i}}{v} \left( s_{i14}^2 + s_{i24}^2 + s_{i34}^2 \right), \\
\text{(pp ! + W X)_{1i}^H} & \sim \frac{M_{1i}}{v} \left( s_{i14}^2 + s_{i24}^2 + s_{i34}^2 \right),
\end{align*}
\]

for s14 = sin 4, O(0.1), where \( N_{11} \) (pp ! N1X) \( \sim R_{1i} f_{1i} \), \( H \) (pp ! H + W X) and \( \text{pair} \) (pp ! H + H X) are three reduced cross sections. Here let me omit a numerical illustration of \( !_1 \) and \( !_2 \) changing with M1 at the LHC with an integrated luminosity of 300 fb\(^{-1}\), although it may give rise to a ball-park feeling of possible collider signatures of N1 and H and their correlation.
3.4. Type-III seesaw

The lepton mass terms in the type-III seesaw scheme have already been given in Eq. (1.7). After spontaneous gauge symmetry breaking, we are left with

$$
L_{\text{mass}} = \frac{1}{2} L^c \begin{pmatrix} \frac{1}{2} & M_D^- & M_D^0 & M_U \end{pmatrix} L + h.c.; \\
L_{\text{mass}}^0 = \frac{1}{2} L^c \begin{pmatrix} \frac{1}{2} & M_D^- & M_D^0 & M_U \end{pmatrix} L + h.c.;
$$

(3.10)

respectively, for neutral and charged fermions, where $M_1 = Y_1 v = \mathcal{Z}$, $M_D = Y v = \mathcal{Z}$, and $\mu = + \pm c$. The symmetric 6 by 6 neutrino mass matrix can be diagonalized by the following unitary transformation:

$$
V_R \begin{pmatrix} M_D & 0 \\ M_D^T & M \end{pmatrix} V_S = \begin{pmatrix} \bar{M} & 0 \\ 0 & \bar{M} \end{pmatrix};
$$

(3.11)

where $\bar{M} = \text{Diag} f_{m_1} m_2 m_3 g$ and $\bar{M} = \text{Diag} f_{M_1} M_2 M_3 g$. In the leading-order approximation, this diagonalization yields the type-III seesaw formula $M_\nu = M_D^T \mathcal{M}_D^{-1} M_D^0$, which is equivalent to the one derived from the effective dimension-5 operator in Eq. (1.11). Let us use one sentence to comment on the similarities and differences between type-I and type-III seesaw mechanisms [15]. The non-unitarity of the 3 by 3 neutrino mixing matrix $V$ has appeared in both cases, although the ordered couplings between the $Z^0$ boson and the light neutrinos differ and the non-unitary avor mixing is also present in the couplings between the $Z^0$ boson and the three charged leptons in the type-III seesaw scenario.

At the LHC, the typical lepton-number-violating signatures of the type-III seesaw mechanism can be $pp \to +^0 l^+ l^- + Z^0W$ ($\ell = \mu$) and $pp \to +^0 l^+ l^- + Z^0W$ ($\ell = \mu$) processes. A detailed analysis of such collider signatures has been done in the literature [29]. As for the low-energy phenomenology, a consequence of this seesaw scenario is the non-unitarity of the 3 by 3 neutrino mixing matrix $N (V)$ in both charged- and neutral-current interactions. Current experimental bounds on the deviation of $N N^T$ from the identity matrix are at the 0.1% level, much stronger than those obtained in the type-I seesaw scheme, just because the avor-changing processes with charged leptons are allowed at the tree level in the type-III seesaw mechanism.

I like to mention that an interesting type-(I+III) seesaw model has recently been proposed [30] and its phenomenological and cosmological consequences together with its possible collider signatures have also been explored [31].

3.5. Double (inverse) seesaw

Given the naturalness and testability as two prerequisites, the double or inverse seesaw mechanism [32] is another interesting possibility of generating tiny neutrino masses at the TeV scale. The idea of this seesaw picture is to add three heavy right-handed neutrinos $N_R$, three SM gauge-singlet neutrinos $S_R$, and one Higgs singlet
into the SM, such that the gauge-invariant lepton mass terms can be written as

$$
\mathcal{L}_{\text{lepton}} = \frac{1}{2} Y^i_H H N_R + \frac{1}{2} Y^i H N_R + \frac{1}{2} Y R S_R + \frac{1}{2} Y^2 S_R + h.c.; \quad (3.12)
$$

where the $M$ term is naturally small according to $\text{Hooft's naturalness criterion}^{[2]}$ because it violates the lepton number. After spontaneous gauge symmetry breaking, the overall neutrino mass term turns out to be

$$
L_{\text{mass}} = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & M_D & 0 & 0 \\
0 & 0 & M_S & 0 \\
1 & 0 & 0 & M_T
\end{pmatrix} \begin{pmatrix}
N_R & S_R & N_R & S_R \\
M_D & 0 & 0 & 0 \\
M_S & 0 & 0 & 0 \\
M_T & 0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
N_R & N_R & N_R & N_R \\
S_R & S_R & S_R & S_R \\
N_S & N_S & N_S & N_S \\
S_S & S_S & S_S & S_S
\end{pmatrix}; \quad (3.13)
$$

where $M_D = Y H H i$ and $M_S = Y S S i$. A diagonalization of the symmetric 9x9 matrix in Eq. (3.13) leads us to the effective light neutrino mass matrix

$$
M = M_D \begin{pmatrix}
1 & 0 \\
0 & 0 \\
0 & 1
\end{pmatrix} M_T \begin{pmatrix}
M_D & 0 \\
0 & M_S \\
0 & 0
\end{pmatrix} \begin{pmatrix}
M_D & 0 & 0 \\
0 & M_S & 0 \\
0 & 0 & M_T
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
0 & 0 \\
0 & 1
\end{pmatrix} \begin{pmatrix}
N_R & N_S & N_S & N_S \\
S_R & S_S & S_S & S_S \\
N_S & N_S & N_S & N_S \\
S_S & S_S & S_S & S_S
\end{pmatrix}; \quad (3.14)
$$

in the leading-order approximation. Hence the smallness of $M$ can be attributed to both the smallness of itself and the doubly-suppressed $M_D M_S$ term for $M_D M_S$. For example, $0(1)$ keV and $M_D = M_S = 0(10^{-2})$ naturally give rise to a sub-eV $M$. One has $M = 0$ in the limit $M D = 0$, which respects the restoration of the slightly-broken lepton number. The heavy sector consists of three pairs of pseudo-Dirac neutrinos whose CP-conjugated Majorana components have a tiny mass splitting characterized by the order of $\Gamma$.

A minimal inverse seesaw scenario, in which only two pairs of the gauge-singlet neutrinos $N_R$ and $S_R$ are introduced, has recently been proposed. Its LHC signatures and low-energy consequences deserve some further studies.

**x4. Non-unitary neutrino mixing and CP violation**

It is worth remarking that the charged-current interactions of light and heavy Majorana neutrinos are not completely independent in either the type-I seesaw or the type-(I+II) seesaw. The standard charged-current interactions of $N_1$ and $N_2$ are

$$
\mathcal{L}_C = \frac{g}{2} \begin{pmatrix}
\theta_L & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix} \begin{pmatrix}
\frac{1}{2} A & N_1 \\
R N_2 & 5 W + h.c.; \quad (4.1)
\end{pmatrix}
$$

where $V$ is just the light neutrino mixing matrix responsible for neutrino oscillations, and $R$ describes the strength of charged-current interactions between $(e; \nu)$ and $(N_1; N_2; N_3)$. Since $V$ and $R$ belong to the same unitary transformation done in Eq. (3.6), they must be correlated with each other and their correlation signifies an important relationship between neutrino physics and collider physics.

It has been shown that $V$ and $R$ share nine rotation angles ($\theta_{14}$, $\theta_{15}$, and $\theta_{16}$ for $i = 1, 2$ and 3) and nine phase angles ($\delta_{14}$, $\delta_{15}$, and $\delta_{16}$ for $i = 1, 2$ and 3). To see
this point clearly, let me decompose \( V \) into \( V = AV_0 \), where

\[
\begin{align*}
V_0 = \begin{bmatrix}
0 & c_{12}c_{13} & s_{13} & 1 \\
\bar{s}_{12}c_{23} & s_{12}s_{23} & c_{13} & 0 \\
\bar{s}_{12}s_{23} & -s_{12}c_{23} & c_{13} & 0 \\
c_{12} & 0 & 0 & 0
\end{bmatrix}
\end{align*}
\quad (4.2)
\]

with \( c_{ij} \cos \theta_{ij} \) and \( s_{ij} \equiv e^{i \theta_{ij}} \sin \theta_{ij} \) is just the standard parametrization of the 3-3 unitary neutrino mixing matrix (up to some proper phase rearrangements). Because of \( V V^\dagger = \text{diag}(1) \), it is obvious that \( V \equiv V_0 \) in the limit of \( \theta_{12} \approx \theta_{13} \approx 0 \) (or equivalently, \( R \approx 0 \)). Considering the fact that the non-unitarity of \( V \) must be a small effect (at most at the percent level) as constrained by current neutrino oscillation data and precision electroweak data, we expect \( s_{ij} \approx 0 \) (for \( i = 1; 2; 3 \) and \( j = 4; 5; 6 \)) to hold. Then we obtain

\[
A = 1 + \begin{bmatrix}
\frac{1}{2} s_{14}^2 + s_{15}^2 + s_{16}^2 & 0 & 0 \\
0 & \frac{1}{2} s_{24}^2 + s_{25}^2 + s_{26}^2 \\
0 & 0 & \frac{1}{2} s_{34}^2 + s_{35}^2 + s_{36}^2 \\
s_{14} & s_{15} & s_{16} \\
s_{24} & s_{25} & s_{26} \\
s_{34} & s_{35} & s_{36}
\end{bmatrix}
\quad (4.3)
\]

as two excellent approximations. A striking consequence of the non-unitarity of \( V \) is the loss of universality for the Jarlskog invariants of CP violation, in \( (V V^\dagger)_{ij} (V V^\dagger)_{jk} \), where the Greek indices run over \((\{\alpha; \beta; \gamma\})\) and the Latin indices run over \((\{1; 2; 3\})\). For example, the extra CP-violating phases of \( V \) are possible to give rise to a significant asymmetry between \( \text{m}_1 \) and \( \text{m}_2 \) oscillations.

The probability of \( \text{m}_1 \) to \( \text{m}_2 \) oscillations in vacuum, denoted as \( P(j, k) \), is given by

\[
P = \left| \begin{array}{c}
\text{Re} \ V_{1j} V_{j1} V_{1k} \cos \theta_{1j} \\
\cos \theta_{jk} \\
\sin \theta_{jk} \\
\end{array} \right|^2 + \left| \begin{array}{c}
\text{Re} \ V_{1j} V_{j1} V_{1k} \cos \theta_{1j} \\
\cos \theta_{jk} \\
\sin \theta_{jk} \\
\end{array} \right|^2 ; \quad (4.4)
\]

where \( jk \equiv m_1^2 L = (2E) \) with \( m_1^2 \) and \( L \) being the neutrino beam energy and \( L \) being the baseline length. If \( V \) is exactly unitary (i.e., \( A = 1 \) and \( V = V_0 \)), the denominator of Eq. (4.4) will become unity and the conventional formula of \( P \) will be reproduced. Note that \( \text{m}_1 \) and \( \text{m}_2 \) oscillations may serve as a good tool to probe possible signatures of non-unitary CP violation. To illustrate this point, we consider a short-or medium-baseline neutrino oscillation experiment with \( jk \equiv \text{m}_1^2 L \), in which the terrestrial matter effects are expected to be insignificant or negligibly small. Then the dominant CP-conserving and CP-violating terms of \( P(j, k) \) and \( P(-j, -k) \) are

\[
P(j, k) = \sin^2 \frac{\text{m}_2^2 L}{2} \cos \theta_{jk} + \left( \frac{1}{2} \text{m}_1^2 L \right) \sin \theta_{jk} ;
\]

\[
P(-j, -k) = \sin^2 \frac{\text{m}_2^2 L}{2} \cos \theta_{jk} + \left( \frac{1}{2} \text{m}_1^2 L \right) \sin \theta_{jk} ; \quad (4.5)
\]
where the good approximation \( \frac{13}{23} j_{12} \) has been used in view of the experimental fact \( j_{13} j_{23} j_{12} \) and the sub-leading and CP-conserving \( \nu - \text{zero-distance} \) effect has been omitted. For simplicity, I take \( V_0 \) to be the exactly tri-bimaximal mixing pattern (i.e., \( 12 = \arctan(1/e^2) \), \( 13 = 0 \) and \( 23 = -4 \) as well as \( 12 = 13 = 23 = 0 \)) and then arrive at

\[
2 J^{23} + J^{13} \frac{X^6}{s_{23}s_{31} \sin \left( \frac{1}{21} \frac{31}{s_{21}} \right)} ; \quad (4.6)
\]

Given \( s_{21} s_{31} \) O (0.1) and \( \left( \begin{array}{c} 21 \\ 31 \end{array} \right) \) O (1) (for \( l = 4, 5, 6 \)), this non-trivial CP-violating quantity can reach the percent level. When a long-baseline neutrino oscillation experiment is concerned, however, the terrestrial matter effects must be taken into account because they might fake the genuine CP-violating signals. As for \( \nu \) and \( \nu \) oscillations under discussion, the dominant matter effect results from the neutral-current interactions and modifies the CP-violating quantity of Eq. (4.6) in the following way:

\[
2 J^{23} + J^{13} \frac{X^6}{s_{23}s_{31} \sin \left( \frac{1}{21} \frac{31}{s_{21}} \right) + A_{\text{NC}} L \cos \left( \frac{1}{21} \frac{31}{s_{21}} \right)} ; \quad (4.7)
\]

where \( A_{\text{NC}} = G_F N_n \rho \) with \( N_n \) being the background density of neutrons, and \( L \) is the baseline length. It is easy to demand \( A_{\text{NC}} L \) O (1) for \( L \approx 10 \) km.

\[x5. \text{ Inconclusive concluding remarks}
\]

Although the seesaw ideas are elegant, they have to appeal for some form any new degrees of freedom in order to interpret the observed neutrino mass hierarchy and lepton flavor mixing. According to Weinberg's third law of progress in theoretical physics, you may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry. What could be better?

Anyway, we hope that the LHC might open a new window for us to understand the origin of neutrino masses and the dynamics of lepton number violation. A TeV seesaw might work (naturalness?) and its heavy degrees of freedom might show up at the LHC (testability?). A bridge between collider physics and neutrino physics is highly anticipated and, if it exists, will lead to rich phenomenology.

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