1. Introduction

Rapid lateral deviations of a metal strip from the direction of rolling, known as strip track-off, is a serious operational problem that can lead to mill crashes and damaged rolls. It predominately occurs at the entry of the tandem cold mill, where the entry tension is relatively low. Operational experience suggests that the strip tracking problem could be reduced if the tension applied to the strip upstream of the first stand is increased. However, this phenomenon has yet to be examined quantitatively. As strip track-off occurs suddenly, experimental study of this phenomenon presents a considerable challenge. A fundamental understanding of the track-off mechanism is therefore required in order to understand the key parameters affecting this phenomenon.

This paper examines the effect of entry tension on stability of strip tracking in a span between the uncoiler and the rolling stand. A recently developed mathematical model of strip tracking, that includes a description of the phenomenon of strip buckling between the uncoiler and the mill, is used as a basis for the analysis. The analysis revealed that the entry tension is a crucial parameter for stabilizing the strip tracking if buckling of the strip is present. A procedure of selecting the entry tension that ensures stable strip tracking for a given mill schedule is discussed.

KEY WORDS: strip tracking; cold rolling; lateral dynamics; lateral stability; mathematical model.

2. Mathematical Model of Strip Tracking

Theoretical studies of strip tracking in metal rolling are relatively recent. First formulation of a mathematical model for strip lateral dynamics in metal rolling was given by Matsumoto and Ishii. In their model, strip in-plane bending between the stands was described using elastic beam theory. They also obtained, to the authors, knowledge, the first experimental data on strip tracking using the laboratory mill. It was found that the correct order of magnitude of strip lateral deviations could not be predicted by the model without the introduction of a parameter that was adjusted to match the experimental data. A similar model based on elastic beam theory for the description of strip buckling has been introduced into the strip tracking model for the first time in Ref. 5). This model reveals that strip tracking becomes unstable once a critical level of asymmetry in rolling conditions is exceeded, thus providing an explanation for the sudden track-off observed in practice. The critical level of asymmetry depends on the rolling conditions, including the entry tension. Consequently, the recently developed model has opened up the opportunity for an analysis of the effect of entry tension on stability of strip lateral motion and is used in the present study.

The mathematical model of strip tracking includes three coupled components:

1. Mill model, including the plastic deformation of the...
2. Description of strip deformation (including buckling) in the span between the uncoiler and the first stand of the cold rolling mill;
3. Dynamical conditions at the interface between the plastic region inside the roll bite and the span.
A brief description of the constitutive parts of the model is given below.

2.1. Mill Model
A mill model includes the models of the plastic deformation of the strip in the roll bite and the elastic deformation of the rolls for generally non-symmetric loading and mill geometry. Such a model is rather complicated and resembles the so-called “strip shape” or “strip flatness” models that calculate the longitudinal tensile stress distribution in the strip at the entry and exit of the roll bite. There is an extensive literature about such models.8–16 In the model used in this paper, the roll force acting on the strip in the roll bite is described by widely used classical plain strain model by Ford and Bland13 and well-known Hitchcock formula14 for a deformed radius of the roll. The elastic deformation of the roll stack is described using a conventional approach15,16 consisting of superposition of the deformation due to bending of the roll axes, calculated using Timoshenko beam theory, and the local surface flattening, which is calculated using Hertzian contact theory. For asymmetric loading and geometry of the mill, the equations are described in details in Ref. 3.

The strip flatness models, however, are not suitable for the analysis of strip tracking as they are usually based on the assumption that the strip velocity is uniform across the strip width at the entry and exit of the plastic region. Matsumoto and Ishii2 introduced the strip in-plane rotation into the model together with the assumption of linear strip velocity distribution across the strip width at the entry and exit of the roll bite. The additional equations introduced into the model21 are the conditions of continuity of strip velocity at the interfaces between the plastic region in the roll bite and the outside spans

\[ v_i = \omega_i x, \quad i = 1, 2 \] ............................(1)

where \( \omega_1 \) and \( \omega_2 \) are the angular velocities of the strip at the entry and exit of the roll bite, \( \vec{v}_1 \) and \( \vec{v}_2 \) are the deviations of the strip velocities in the plastic region from their mean values, \( x \) is the coordinate across the strip width measured from the center of the strip.

An additional feature used in this paper is an empirical mill stretch equation introduced into the model to account for the deformation of the mill frame

\[ \sigma = \sigma_S + \frac{\partial F}{M_m} \] ..........................(2)

where \( \sigma, \sigma_S \) and \( \partial F \) are the variations in strip thickness, roll gap position and roll force respectively, and \( M_m \) is the mill modulus.

The mill model calculates the longitudinal tensile stress distributions across the strip width \( \sigma_1 \) and \( \sigma_2 \) and the bending moments \( M_1 \) and \( M_2 \) at the entry and exit of the roll bite

\[ M_i = \int_{w/2}^{W/2} \sigma_i h(x) dx, \quad i = 1, 2 \] ..........................(3)

where \( h_1 \) and \( h_2 \) are the strip average strip thickness before and after reduction, \( W \) is the width of the strip. In this paper, as in Refs. 4, 5, the mill model is used to calculate the in-plane rotational speed of the strip \( \omega_i \) at the entry of the roll bite as a function of entry bending moment \( M_{eff} \), the lateral deviation of the strip from the center of the roll at the entry of the roll bite (strip off-center \( d \)) and the asymmetry in rolling conditions

\[ \omega_1 = \omega_1 (M_{eff}, d) \] .............................(4)

It is assumed that the in-plane rotational speed of the strip at the exit of the roll bite is zero

\[ \omega_2 = 0 \] .............................(5)

2.2. Strip Deformation in the Span between the Uncoiler and the Mill Stand
A simplified physically based model of strip buckling by Benson7 suggests that a partial buckling of the strip reduces its effective area moment of inertia, thus reducing the strip in-plane bending rigidity. According to this model, no buckling occurs if \( M < TW/6 \), where \( T \) is the tensile force acting on the cross-section of the strip. The onset of buckling occurs at \( M = TW/6 \), and the strip is fully buckled if \( M = TW/6 \). Benson7 gives an interpretation of the bending of the strip under the buckling in terms of the bending of the Euler beam with reduced effective width \( W_{eff} \) and bending moment \( M_{eff} \) described by

\[ W_{eff} = \frac{3}{2} \left[ \frac{3}{2} \frac{M}{TW} - 1 \right] \] if \( 1 \leq \frac{6M_{eff}}{TW} < 3 \] ..........................(6)

\[ W_{eff} = \frac{3}{2} \left[ \frac{3}{2} \frac{M}{TW} + 1 \right] \] if \( -1 \leq \frac{6M_{eff}}{TW} < 3 \] ..........................(7)

The forces and the moments acting on the strip in the span between the uncoiler and the first rolling stand are shown in Fig. 1. A simplified description5 of the strip bending with buckling under the bending moment at the entry of the roll bite due to asymmetry in rolling conditions is given by the model of bending of a beam with constant effective width calculated at the entry of the roll bite

\[ \frac{\partial^2 u}{\partial z^2} = - \frac{1}{EI_{eff}} \left[ -M_{eff} + G(L - z) \right] \] ..........................(8)
where
\[ I_{\text{eff}} = \frac{1}{12} h(W_{\text{eff}})^3 \] ..........................(9)

\[ z \] is the co-ordinate in the direction of rolling, with the origin on the centerline of the strip at the un-coiler, \( L \) is the distance between the uncoiler and the rolling mill, and \( u \) is the lateral deviations of the strip centerline from the center of the roll.

The boundary conditions upstream at the uncoiler are taken to be in the form:
\[ z = 0, \quad \frac{du}{dz} = 0 \] ..........................(10)

2.3. Dynamical Conditions at the Entry of the Roll Bite

Condition of no lateral slip between the strip and the rolls at the entry of the roll bite and continuity of the rotational velocity of a strip at the entry of the roll bite can be written in the form:
\[ \theta = 0 \] ..........................(11)

\[ \frac{d\theta}{dt} = \omega_1 + \kappa \] ..........................(12)

where \( t \) is the time and \( \kappa \) and \( \omega_1 \) are the average strip speed and the curvature of strip centerline at the entry of the roll bite respectively. The functional relationship \( \omega_1 = \omega_2(d, M_1) \) is calculated from the mill model for a given rolling schedule and the level of asymmetry in rolling conditions. Then, Eqs. (8)–(12) can be solved as a system of non-linear ordinary differential equations to obtain the strip lateral deviations as function of time.

3. Model Results and Discussion

In this section, we analyse the strip tracking between the uncoiler and the first stand of a cold rolling mill using the mill schedule data shown in Table 1. A range of the entry strip tension between 3 MPa and 32 MPa was studied. It is known that occasional track-off was experienced when operating under the low tension. The purpose of the analysis is to examine the effect of the entry tension on stability of strip tracking and, in the absence of more detailed information, to validate the model against the operator experience.

A procedure for selecting the entry tension for stable strip tracking will be discussed.

The analysis should be performed for a given mill schedule and a specific type of asymmetry in rolling conditions. In this paper, the most common type of asymmetry, namely, a differential roll gap setting, is considered. In practice, the asymmetry can be represented by a combination of different types, which may include the wedge profile on the incoming strip thickness, the non-uniform friction coefficient across the roll bite etc. The analysis, however, will be similar to the one presented here.

In Fig. 2, the steady-state off-center of the strip is shown versus the level of asymmetry for two different values of the entry tension. The level of the asymmetry is expressed as the ratio of the differential roll gap setting to the average entry strip thickness, the non-uniform friction coefficient across the roll bite etc. The analysis, however, will be similar to the one presented here.

Table 1. Mill and schedule parameters.

| Parameter                        | Value          |
|----------------------------------|----------------|
| Strip width                      | 1350 mm        |
| Work roll face length            | 1420 mm        |
| Backup roll face length          | 1397 mm        |
| Work roll diameter               | 500 mm         |
| Backup roll diameter             | 1350 mm        |
| Distance from mill center to screws | 1219 mm    |
| Distance from mill center to jacks | 1028.6 mm |
| Work roll crown                  | 0.05 mm        |
| Backup roll crown                | 0              |
| Entry strip thickness            | 2.5 mm         |
| Exit strip thickness             | 1.8 mm         |
| Yield stress                     | 530 MPa        |
| Young modulus (rolls)            | 210000 MPa     |
| Young modulus (strip)            | 210000 MPa     |
| Entry tension stress             | 3 MPa - 32 MPa |
| Exit tension stress              | 11.8 MPa       |
| Friction coefficient             | 0.07           |
| Roll speed                       | 3.2 m/sec      |
| Entry strip crown                | 0.04 mm        |
| Jack force on work roll          | 300 kN         |
| Mill modulus                     | 2.25 × 10^9 N/mm |

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ciently small. This stage is characterized by the absence of strip buckling. Further increase in asymmetry level results in faster non-linear growth of the steady-state off-center. This non-linear behaviour is associated with strip buckling. There is a critical level of asymmetry so that no stable tracking is possible once it is exceeded. This may explain the sudden onset of instability observed in practice.

One can observe from Fig. 2 that the increase in entry tension strongly affects the critical level of asymmetry for instability. Rolling under higher levels of entry tension is more tolerant to asymmetry in rolling conditions. It is interesting however, that an increase in entry tension virtually does not affect the linear part of the curves shown in Fig. 2. Thus, the effect of entry tension would have been negligible if no buckling of the strip were to occur. For the same level of asymmetry, an increase in entry tension can reduce, and in some cases completely eliminate, the buckling of the strip. This appears to be the main mechanism of the effect of entry tension on stability of strip tracking.

Examples of strip tracking dynamics, in terms of the strip off-center plotted as a function of time, are shown in Fig. 3 for the asymmetry level given by a differential roll gap setting $\delta S=0.083 \text{ mm}$ for different initial off-center of the strip. As was observed, the lateral dynamics of the strip is strongly dependent on the strip initial position. The strip tends to return to its stable position if the deviations from it are within a certain range. There is, however, a sudden transition from stable to unstable tracking when a certain critical value of strip lateral deviation is reached. This value depends on the mill schedule, the level of asymmetry and the entry tension. One can also observe in Fig. 3 that while it takes a few seconds for the strip to return to its stable position, the track-off could occur much faster, which agrees with practical experience.

In Fig. 4, the critical initial off-center (dashed lines) and steady-state off-center (solid lines) as functions of the level of asymmetry (in terms of the ratio of the differential roll gap setting to the average entry strip thickness) for two different entry tensions: (1) entry tensile stress 3 MPa, (2) entry tensile stress 10 MPa. Dashed lines show the critical level of asymmetry.
tracking exists.

Figure 4 shows that the critical strip off-center is strongly dependent on the entry tension. For a given level of asymmetry in the rolling conditions, an increase in entry tension leads to an increase in the critical off-center, thus increasing the rolling stability.

In Fig. 5, the stability regions (the regions under the dashed curves) are shown for two different values of the entry tension. One can see that an increase in entry tension significantly expands the stability region, both in terms of the critical asymmetry level and the critical off-center of the strip. Thus, under a higher level of entry tension, not only the higher level of asymmetry can be tolerated, but also larger lateral deviations of the strip can occur without the onset of instability in strip tracking.

In order to select the level of entry tension appropriate for a given mill schedule, it is convenient to plot the critical level of asymmetry as a function of entry tension. The level of the asymmetry can be expressed in many alternative ways. In practice, a common measure of the asymmetry in rolling conditions is the differential roll force. In Fig. 6, a critical level of asymmetry, in terms of the differential roll force, is plotted as a function of the entry strip tension for a center-positioned strip. This plot may assist in selecting the entry strip tension that would ensure stable strip tracking for a particular mill schedule and expected type of asymmetry in rolling conditions.

4. Conclusions

The analysis of the effect of entry tension on strip tracking has shown that it is a key factor affecting the stability of strip tracking in the first stand of a cold rolling mill, if even a small amount of strip buckling is present between the uncoiler and the mill. The results of the analysis can be summarised as follows:

(1) For a given type of asymmetry in rolling conditions, there is a critical level of asymmetry so that strip tracking becomes unstable once this critical level is reached. The critical level of asymmetry is strongly dependent on the entry tension. As the entry tension increases, a higher level of asymmetry in rolling conditions is tolerable without the onset of instability. For a given level of asymmetry in rolling conditions, an increase in entry tension reduces, and may even completely eliminate, the strip buckling, thus improving the stability of strip tracking.

(2) For a given level of asymmetry in rolling conditions, there is a critical lateral deviation of the strip (strip off-center) so that the motion becomes unstable once this value is reached. The critical off-center is large for small levels of asymmetry, and decreases with increase in the asymmetry level. The critical off-center is strongly dependent on the entry tension. For a given level of asymmetry, the critical off-center of the strip increases with increase in entry tension. Thus, an increase in entry tension increases both the level of asymmetry in rolling condition and the lateral deviations of the strip that can be tolerated without the onset of instability in strip tracking.

(3) The plot of critical asymmetry level versus the entry tension can assist in selecting the entry tension for stable strip tracking for a given mill schedule and expected type of asymmetry in rolling conditions.

The results of the analysis appear to qualitatively agree with practical observations. Further validation and tuning of the model, especially the part related to strip buckling, will be conducted before its adoption to review the operating windows for the full product mix of BlueScope Steel’s various cold mills.

Acknowledgements

The authors thank the management of BlueScope Steel Research for permission to publish the information in this paper.

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