Schedule Coordination Design in a Trunk-Feeder Transit Corridor With Spatially Heterogeneous Demand

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ABSTRACT As a widely present form of bimodal transit systems, the trunk-feeder transit corridor has received less attention in the academic research compared to the conventional single-modal systems, partly due to the difficulty in the two-dimensional modeling. The schedule coordination of this corridor has been largely ignored, which can potentially strengthen the mutually reinforcing relationship between the trunk transit (e.g., rail) and the feeder transit (e.g., bus). In light of this, the paper proposes a novel two-stage programming model with the objective of minimizing the total cost to design a coordinated trunk-feeder corridor. In stage one, the continuous approximation approach and the discrete method are incorporated to obtain an optimized uncoordinated scheme and a feasible layout. The screened layout parameters such as the specific station location, feeder line location, length of the feeder line segments parallel to the trunk line, and the passenger flow distribution, are recruited into the coordination design in stage two. A nested two-phase optimization algorithm integrated with the analytic method and adaptive genetic algorithm is proposed to find the solutions to the mixed integer nonlinear program issue. Results show that the proposed method is suitable for heterogeneous schedule coordination design and the coordination strategy is more preferable at a lower-level heterogeneous demand. Meanwhile, different from the conventional approaches, this method can generate a rational cost-saving coordination scheme with a feasible layout under the two-dimensional heterogeneous demand pattern.

INDEX TERMS Trunk transit, feeder bus, schedule coordination, continuum approximation, heterogeneous demand.

I. INTRODUCTION

In the past decades, public transit system has been developed rapidly due to its speed and punctuality. In some urban regions, the demand for public transit is served by a trunk-feeder transit corridor, consisting of a trunk transit line (such as a rail transit line or a bus rapid transit route) and the feeder bus routes connecting the transfer stations. There is a mutually reinforcing relationship in this corridor. The trunk transit serves as the backbone due to the strength of its large carrying capacity and high speed, but its high investment makes it sparse. Meanwhile, the low-input feeder bus is so gifted in flexibility that it can serve to fill the gap in the demand coverage. Thus, the trunk-feeder transit system is widely utilized as a main force to meet the travel demand in many cities such as Beijing, London, and Singapore [1].

To strengthen the mutually reinforcing relationship as stated above, it is critical to coordinate the trunk and feeder transit services. This coordination is essential to reduce the total travel time of passengers and improve the efficiency of the travelers and the service quality of the transit corridors, which will consequently result in a higher travel demand for the trunk-feeder transit services. The higher demand further brings broader benefits to both the operator and society [2]. Hence, the trunk transit line and its feeder buses should be integrated and coordinated as tightly as possible. The study
attempts to explore how to efficiently design a coordinated trunk-feeder transit corridor.

As the basis for the coordination, the design of the trunk-feeder transit system is also included in this study. Differing from the conventional design of the single-modal transit system [3]–[9], the design of the trunk-feeder transit system needs to take into account two transit modes that interact with each other. Some studies emphasized on the trunk transit [10]–[12], Hurdle and Wirasinghe [10] assessed the impacts of various feeder modes on the design parameters of the trunk line. Liu et al. [11] sought the optimal rail line length in a crosstown rail corridor with many-to-many travel demand. Sivakumaran et al. [12] optimized the spacings of trunk stations and lines which are distributed uniformly under the homogeneous demand. On a separate topic, in terms of the premise of a given trunk transit line the pure feeder bus lines are optimized by means of the heuristic algorithm [13]–[16].

Joint design of the trunk and feeder systems is very limited in the literature except few examples. Considering many-to-one demand pattern, Wirasinghe [17] built an approximate analytic model to obtain the near-optimal parameters by assuming that the trunk transit headway was negligible. Chien and Schonfeld [18] jointly optimized the characteristics of a rail line and its feeder routes in an urban corridor. To better simulate the reality, Fan and Mei [19] developed a comprehensive model to simultaneously optimize the trunk-feeder transit corridor without fixing the number of feeder lines in advance.

However, the schedule coordination problem appears to fall short in the foregoing research efforts. The coordination researches can be roughly classified into two categories: the single-modal researches [20]–[26] and the bimodal ones [27]–[29]. The heuristic algorithm is widely utilized in these single-modal and bimodal coordination works. Unlike the single-modal researches, the bimodal coordination studies are rare. These bimodal coordination studies adopted the headway as the decision variable. However, they ignored the transit network optimization and conducted the research based on the main transit line with a given schedule, which might lead to a suboptimal solution. The network optimization was introduced to the trunk-feeder coordination studies [2], [18]. However, these studies sacrificed some details of the network structure in order to obtain the best solutions. Sivakumaran et al. [2] limited their research to a fixed-schedule trunk line and considered the uniformly distributed feeder lines perpendicular to the trunk line while ignoring the parallel feeder line segment; Chien and Schonfeld [18] assumed that each station was connected by one feeder line. Both studies assumed that the vehicle capacity was sufficiently large. Besides, their optimal solutions obtained by the continuous approximation (CA) approach are spatially continuous and still need to be discretized into the specific and feasible layout design.

In summary, there are few studies focusing on the trunk-feeder transit coordination and even no coordination study considering the feasible network layout design. Given that the coordination can fortify the mutually reinforcing relationship as mentioned above, we develop a two-stage programming model for schedule coordination design in the trunk-feeder transit corridor. A first-design-then-coordinate framework of this model is presented: we first design a feasible trunk-feeder corridor where the specific network layout is acquired by the discretization of CA solution, and then coordinate the trunk and feeder transit services of this corridor. A nested two-phase optimization algorithm integrated with analytic method and adaptive genetic algorithm is developed to deal with the mixed integer nonlinear program issue.

The main contributions of the paper are threefold: 1) we propose a first-design-then-coordinate framework for the trunk-feeder transit corridor that account for the two-dimensional heterogeneous demand pattern; 2) the proposed model indigenously optimize the locations of trunk stations and feeder lines as well as the service headways satisfying integer coordination constraints, which may vary with locations to best fit arbitrary demand patterns; and 3) the coordination method can promote the cost savings under the homogeneous and heterogeneous demands. Some new findings are also unveiled: For instance, the coordination strategy is more preferable for the trunk-feeder corridor at a lower-level heterogeneous demand.

The rest of the paper is organized as follows. Section 2 presents the first stage of the model, including the continuous approximation model and the discretization. Section 3 formulates the schedule coordination model with the layout parameters obtained in stage one. Section 4 designs the solution algorithm. Section 5 performs experiments to verify the reliability of the model. Finally, conclusions are drawn in Section 6.

II. PRELIMINARY DESIGN OF A FEASIBLE TRUNK-FEEDER CORRIDOR

Serving as the basis of subsequent schedule coordination modeling, the first stage of the model consists of two sections. Section 2.1 presents a continuous approximation (CA) model for designing an optimal trunk-feeder transit corridor with respect to trunk station density function, feeder line density function, and service headways of two modes. Section 2.2 proposes a discretization recipe, where the CA solutions (as continuous functions in space) is transformed into a feasible network layout, including the specific feeder line locations, the station locations, and the length of the feeder line segment parallel to the trunk line. Then the passenger flow distribution is obtained based on the feasible layout. Then, an optimized discrete scheme of the bimodal corridor is generated, where the transit services are uncoordinated.

A. A CONTINUOUS APPROXIMATION MODEL

Inspired by the existing researches [2], [17], [19], we study a trunk-feeder transit corridor as shown in Fig. 1. The heavy line (with length $L$) represents the trunk line (e.g., metro), which connects a Central Business District (CBD) at location $(L,0)$ with other areas.
(e.g., suburb); feeder buses (e.g., routine buses), denoted by thin lines, operate perpendicularly to the trunk line (x axis). Upon reaching the trunk line, feeder buses turn parallel to the trunk line and run without stopping to access the closest trunk station for transfer. The spatial distribution of trunk stations is expressed by density \( \delta(x) \) (#/km) and the station spacing is \( 1/\delta(x) \) (km) in the vicinity of location \( x \). The left boundary of the corridor is the starting point of the trunk line. \( u_i(x) \) indicates the feeder line density function and the feeder spacing is \( 1/u_i(x) \) (km) in the vicinity of \( x \). The corridor boundaries are represented by a function of \( l_i(x) \), where \( i = 1, 2 \) denotes the upper and lower sides of the corridor, respectively.

To facilitate the model development, we make the following assumptions that were commonly adopted in previous studies [2], [14], [17]–[19].

1) We assume that the many-to-one demand pattern is utilized in the whole service region with all passengers bound for the CBD. A case like this might arise during the morning rush.

2) The boarding demand is assumed to be exogenously given and fixed during the design, which can be represented as a temporarily uniform and spatially continuous density function \( b_i(x, y) \) over the study domain.

3) For the sake of planning, the demand is directly taken as the design-hour demand.

4) All passengers are assumed to take feeder buses to transfer to the trunk line to accomplish their trips.

5) The operation of vehicles in trunk and feeder lines are assumed to be regular without random disturbance.

With the objective of minimizing the total cost \( Z(h) \) of the corridor, CA model is established by referring to the earlier work [17]–[19]. There are four decision variables in the CA model: trunk transit headway \( H \), feeder transit headway \( h_i(x) \), trunk station density \( \delta(x) \), and feeder line density \( u_i(x) \). The minimization problem of the hourly total cost is defined as follows:

\[
\min Z = (C_a + C_w + C_t) + \frac{1}{e} (C_l + C_s + C_{VH} + C_{VK})
\]

Subject to:

\[
P^e(L) \leq \frac{\text{Cap}_t}{H} \tag{1b}
\]

\[
\frac{C^f_i(x)}{u_i(x)} \leq \frac{\text{Cap}_f}{h_i(x)} \tag{1c}
\]

\[
H, h_i(x), \delta(x), u_i(x) \geq 0 \tag{1d}
\]

where the first bracket on the right side of Equation (1a) is the user cost (h) and the second one refers to the operator cost ($). By dividing the value of time \( e \) (S/h), the monetary value is transformed into the unit of time. \( C_a, C_w, \) and \( C_t \) are users’ walking time, waiting time at feeder stops and trunk stations, and in-vehicle travel time, respectively. \( C_l, C_s, C_{VH}, \) and \( C_{VK} \) denote operator’s line infrastructure cost, station cost, fleet cost, and operation cost, respectively. Equations (1b)-(1c) are the vehicle capacity constraints to ensure that the on-board passengers do not exceed the vehicle capacities, \( \text{Cap}_t \) and \( \text{Cap}_f \). In Equations (1b)-(1c), \( P^e(L) \) (pax/h, where pax is short for passengers) denotes the maximum on-board flow of trunk line, which can be obtained via the expression furnished in Table 6 in Appendix A; the maximum on-board flow of the feeder line at location \( x \) is indicated as \( \text{CB}_i^f(x)/u_i(x) \), where \( \text{CB}_i^f(x) \) (pax/h/km) is the cumulative boarding demand along y axis in the vicinity of location \( x \).

From the first-order conditions of Equation (1a) which is a nonlinear continuous function of four decision variables, the optimal solutions can be derived (see details in [17]–[19]):

\[
H^* = \sqrt[\varphi_N^f/\varphi_D^f]} \tag{2a}
\]

\[
h_i^*(x) = \sqrt[\chi_{N,i}/\chi_{D,i}^f]} \tag{2b}
\]

\[
\delta^*(x) = \sqrt[\phi_N^f/\phi_D^f]} \tag{2c}
\]

\[
u_i^*(x) = \sqrt[\psi_N^f/\psi_D^f} ] \tag{2d}
\]

where \( H^* \), \( h_i(x)^* \), \( \delta(x)^* \), and \( u_i(x)^* \) are the optimal solutions. The detailed expressions of \( \varphi_{N,i}^f, \varphi_D^f, \chi_{N,i}, \chi_{D,i}^f, \phi_{N,i}^f, \phi_D^f, \psi_{N,i}^f, \) and \( \psi_D^f \) are presented in Appendix B.

B. DISCRETIZATION RECIPE TO A FEASIBLE DESIGN

Because the CA model solutions are continuous in space, with reference to the existing studies [30], [31], the endpoint approach is adopted to convert the solutions \( \delta(x)^* \) and \( u_i(x)^* \) into the feasible station location and feeder line location, respectively. Then, these locations determine the passenger flow assignment and the length of the feeder line segment that parallels to the trunk line. Thus, a feasible network layout is formed.

Fig. 2 shows a case study of determining the station locations. When \( \int_0^L \delta(x)^* dx \) yields an integer \( j = 1, 2, 3 \cdots N \), the station locations are determined by \( L_{0,j} = x_j \), where \( x_0 = 0 \), rounding \( \int_0^L \delta(x)^* dx \) is the total number of stations \( N \). The demand coverage of station \( s \) is denoted by \( [M_{s-1}^f, M_s^f] \).
where $M^L_{i-1}$ and $M^R_i$ are the left and right boundaries, respectively. $M^L_i = (L_0 + L_{i-1})/2$, $M^R_0 = 0$, $M^R_N = L$.

By applying the same theory, we can find the location of feeder $r$ on side $i$ of corridor: $L_{0i_r} = x_i$, and the total number of feeders on side $i$ is determined as $R_i$. The demand coverage of feeder $r$ can be deduced by the left and right boundaries: $M_{i-1,r}, M_i,r, M_{i+1,r} = (L_{0i_r} + L_{i+1,r})/2$, $M_{i,0} = 0$, $M_{i,N} = L$.

Based on the locations, Equations (3a)-(3d) can estimate the distribution of passenger flow and the walking distance after discretization. Let $L^L_{i,r}$ (km) denotes the length of feeder $r$ (in x-axis direction) on side $i$ of trunk line, which can be calculated by Equation (3e) when $L_{0i_r} \in [M^L_{i-1}, M^R_i]$.

$$D^Q_i = \sum_{r=1}^{R_i} \left( \int_{x=M^L_{i-1}}^{M^L_i} \int_{y=0}^{l_i(x)} (L_{0i_r} - x) b_i(x,y) dy dx + \int_{x=L_{0i_r}}^{M^R_i} \int_{y=0}^{l_i(x)} (x - L_{0i_r}) b_i(x,y) dy dx \right)$$

(3a)

$$Q_{i,r} = \int_{x=M^L_{i-1}}^{M^L_i} \int_{y=0}^{l_i(x)} b_i(x,y) dy dx$$

(3b)

$$CQ_{i,r}(y) = \int_{x=M^L_{i-1}}^{M^L_i} \int_{y=0}^{l_i(x)} b_i(x,y) dy dx$$

(3c)

$$Q_s = \sum_{i=1}^{2} \sum_{j=1}^{R_i} Q^S_{i,j}$$

(3d)

$$L^L_{i,r} = |L_{0i_r} - L_{0i_r}|$$

(3e)

where $D^Q_i$, $Q_{i,r}$, $CQ_{i,r}(y)$, and $Q_s$ are the total walking distance within the service region, on-board flow of feeder $r$ (on side $i$), cumulative boarding passengers up to location $y$ of feeder $r$ (on side $i$), and the total boarding passengers of station $s$, respectively. In Equation (3d), $Q^S_{i,j}$ represents the boarding passengers of feeder $r_j$ which connects station $s$.

The discretization generates a discrete scheme in which the transit services are uncoordinated. The hourly total cost $Z^{un}$ in this uncoordinated scheme is calculated by:

$$Z^{un} = (C^a + C^w + C^{un}) + \frac{1}{e} (C^l + C^s + C^{vh} + C^{vk})$$

(4)

where the first item of Equation (4) is the user costs and the second one represents the operator costs. The calculation details and the notations adopted in the layout design are summarized in Appendix C.

**III. SCHEDULE COORDINATION MODEL**

Based on the feasible network layout parameters in stage one, we attempt to formulate the schedule coordination model in stage two. When trunk and feeder services are coordinated, the headway of feeder $r$ (on side $i$), $h^t_{i,r}$, is some integer multiple of trunk transit headway $H^t$. The relationship is given by:

$$h^t_{i,r} = K_{i,r} H^t, \quad i = 1, 2, r \in \{1, \cdots, R_i\}$$

(5)

where the headway coefficient $K_{i,r}$ represents any positive integer.

**A. USER COST**

According to the previous works [2], [18], the cost in transferring to the trunk is assumed to be negligible when trunk and feeder services are coordinated. We only consider the waiting time cost at the feeder stops. Thus, the user cost $C^u_{i,r}$ is determined by:

$$C^u_{i,r} = C^a + C^w + C^{co}$$

(6a)

where $C^a$, $C^w$, and $C^{co}$ are patrons’ access time, waiting time at feeder stops, and in-vehicle travel time, respectively. In consideration of the headway relationship presented in Equation (5), these three costs are calculated as follows:

$$C^a_i = \sum_{r=1}^{2} \frac{D^Q_i}{v_a}$$

(6b)

$$C^w_i = \sum_{r=1}^{R_i} \frac{Q_{i,r}}{2} K_{i,r} H^t$$

(6c)

$$C^{co} = \sum_{i=1}^{N} \sum_{l=1}^{2} Q_{i,l} \left( \frac{L_{0l} - v_j}{v_j} + \tau_j (N - s) \right) + \sum_{i=1}^{R_i} \frac{Q_{i,r} L^L_{i,r}}{v_f} + \sum_{r=1}^{2} \sum_{l=1}^{2} \int_{y=0}^{L^L_{i,r}} CQ_{i,r}(y) dy$$

(6d)

where $D^Q_i$, $Q_{i,r}$, $Q_s$, $L^L_{i,r}$, $N$, $L^L_{i,r}$, and $L^L_{i,r}$ in Equations (6b)-(6d) are derived from stage one and furnished by Appendix C. Equation (6d) shows the in-vehicle travel time consisting of three components, where the first component is the trunk line and the last two are the feeder line (x-axis and y-axis directions). $\tau^t$ indicates the average delay per trunk station.

**B. OPERATOR COST**

In the context of schedule coordination, the operator cost, $C^o_{i,r}$, can be expressed as:

$$C^o_{i,r} = C^a_i + C^w_i + C^{co} + C^{vh} + C^{vk}$$

(7a)

where $C^a$, $C^w$, $C^{co}$, $C^{vh}$, and $C^{vk}$ are the line infrastructure cost, the station cost, the fleet cost, and the operation cost, respectively. By virtue of the headway relationship shown in Equation (5), these four costs are derived from Equations (7b)-(7e).

$$C^o_{i,r} = \pi^t_{i} L + \pi^t_{i} \sum_{l=1}^{2} \left( 2 L^L_{i,r} + \frac{R_i}{2} \right)$$

(7b)
where the second term of Equation (7b) is the infrastructure cost of the feeder line along x-axis and y-axis directions. \( \pi\), \( \pi_f\), \( \pi_{VH}\), \( \pi_{VK}\), and \( \pi_{Vf}\) are the unit costs related to the length of trunk line, the length of feeder line, the number of trunk stations, the vehicle-hour travelled (VHT) of trunk transit, the feeder VHT, the vehicle-kilometer travelled (VKT) per operation hour of trunk transit, and the feeder VKT, respectively.

**C. OPTIMIZATION PROBLEM**

The decision variables in the schedule coordination model are \( H^{co} \) and \( K_{i,r} \). The optimal coordination design problem is formulated as the minimization of the generalized cost \( Z^{co} \), which is the sum of user and operator costs:

\[
\min Z^{co} = C_U^{co} + \frac{1}{\varepsilon} C_O^{co}
\]  

(8a)

Subject to:

\[
H^{co} = \min \left( \frac{Cap_f}{P^f (L)}, \frac{Cap_f}{Q_{i,r} K_{i,r}} \right)
\]  

(8b)

\[
H^{co}, K_{i,r} \geq 0
\]  

(8c)

\[
0 \leq H^{co} = \min \left( \frac{\pi_{Vf} Cap_f}{\pi_f}, \frac{\pi_{Vf} Cap_f}{Q_{i,r} K_{i,r}} \right)
\]  

(8d)

where \( \frac{Cap_f}{Q_{i,r} K_{i,r}} \) in Equation (8b) is adopted based on Equation (5). Constraint Equation (8b) restricts the on-board flow from exceeding the vehicle capacities through the whole service area. Equation (8c) is the non-negative constraint. Equation (8d) is the integer constraint.

**IV. SOLUTION ALGORITHM**

The previous section presents the schedule coordination problem, which is formulated as a mixed integer non-linear programming (MINLP) issue. As described in the Introduction, the heuristic algorithm has been widely utilized to solve the coordination problem. The adaptive genetic algorithm (AGA), as a heuristic algorithm, is adopted in the paper. Based on the nested-loop mechanism, the paper innovatively proposes a nested two-phase algorithm which legitimately integrates AGA with analytic method to solve the coordination problem. The two-phase algorithm is designed as follows.

**Phase I Analytic Analysis:** According to the first-order condition of Eq. (8a), the first-order derivative is shown in Equation (9a). Furthermore, given the constraints Equations (8b)-(8c), the optimized \( H^{co} \) can be formulated as Equation (9d). We substitute \( H^{co} \) into Equation (8a), the schedule coordination model is transformed from a thorny MINLP issue into an integer optimization problem with only the decision variable \( K_{i,r} \).

\[
H^{co} = \sqrt{\frac{\omega_f^f}{\omega_f^D}}
\]  

(9a)

\[
\omega_f^f = \frac{1}{\varepsilon} \pi_{Vf} (2L + \pi_f) + \frac{1}{\varepsilon} \pi_{VH} \sum_{r=1}^{R_f} \frac{2 L_{1,r}^f}{V_f K_{i,r}} f_{i,r} \sum_{r=1}^{R_f} \frac{2 L_{1,r}^f}{V_f K_{i,r}} f_{i,r}
\]  

(9b)

\[
\omega_f^D = \sum_{i=1}^{2} \sum_{r=1}^{R_f} Q_{i,r} K_{i,r}
\]  

(9c)

\[
0 \leq H^{co} = \min \left( \frac{\omega_f^f}{\omega_f^D}, \frac{\omega_f^f}{Q_{i,r} K_{i,r}} \right)
\]  

(9d)

**Phase II AGA:** Note that the headway is not infinite in reality and for practical application convenience [22], we place a ceiling on \( K_{i,r} \) when designing an AGA to optimize \( K_{i,r} \). With reference to the AGA parameters in the previous studies [32], [33], we put forward with an improved AGA in a stepwise manner. The nested-loop optimization process is described as follows.

**Step 1. Initialization.** According to the number of feeder lines, we determine the scale of the initial population consisting of the chromosomes. Each chromosome in the population denotes a potential solution (a set of \( K_{i,r} \)). The initial population is generated by considering the constraint \( K_{i,r} \). Let \( t_0 \) be the threshold of iterations.

**Step 2. Fitness.** The value of the objective function \( Z^{co} \) is calculated by substituting the chromosome (a set of \( K_{i,r} \)) in phase I and Equation (8a). Then, the negative of the objective value \( Z^{co} \) of each chromosome \( \sigma \) is expressed as \( A(\sigma) \). The maximum and minimum values in the population are denoted as \( maxA \) and \( minA \), respectively. The previous researches [32], [33] proposed that the fitness value of chromosome \( \sigma \) can be \( F(\sigma) = \frac{\overline{A(\sigma)} - \overline{minA}}{\overline{maxA} - \overline{minA}} \). The average fitness and the maximum fitness of the population are determined as \( F_{ave} \) and \( F_{max} \), respectively.

**Step 3. Crossover.** Operate in a two-point crossover approach and set \( P_c \) as the adaptive crossover probability. Two crossover points are generated based on \( P_c \) for two chromosomes, and then the contents of two points are swapped to form the offspring. Following the previous studies [32], [33], we adapt \( P_c \) according to the fitness. The higher fitness value of the two parents (two chromosomes) is \( F(s) \). If \( F(s) \geq F_{ave} \), then \( P_c = \frac{F_{max} - F(s)}{F_{max} - F_{ave}} \); if \( F(s) < F_{ave} \), then \( P_c = \gamma_2 \). \( 0 < \gamma_1, \gamma_2 < 1 \).

**Step 4. Mutation.** To prevent the premature convergence, the population is mutated and let \( P_m \) denotes the adaptive mutation probability. Repeat Step 2 to update the fitness. With reference to the previous studies [32], [33], for any
chromosome $\sigma$: if $F(\sigma) \geq F_{ave}$, $P_m = \frac{\gamma \delta(F_{max} - F(\sigma))}{F_{max} - F_{ave}}$; if $F(\sigma) < F_{ave}$, then $P_m = \gamma_4, 0 < \gamma_3, \gamma_4 < 1$.

Step 5. Selection. Recalculate fitness as in Step 2 and select the best individual according to the traditional roulette method. The higher the fitness of an individual, the greater the probability that this individual will be retained. If $t > t_0$, terminate the algorithm and output the best individual among the survivors as the optimized $K_{i,r}$; otherwise, set $t = t + 1$ and go back to Step 2.

Return to Phase I, we substitute $K_{i,r}$ optimized in Phase II in Equation (9d) to calculate $H_{co}^\delta$. Then, the headway values of the feeder buses can be determined by Equation (5) and the trunk-feeder coordination is ultimately achieved.

Note that the adaptive probabilities of crossover and mutation can help to prevent infeasibility. Besides, in the following numerical studies, we run the above solution algorithm multiple times with randomly generated initial solutions for each run. We reckon we got a high-quality solution when the runs outcome the same results that yield the minimum objective value.

V. NUMERICAL STUDIES

With reference to the studies [2], [18], the service area can be simplified as a rectangular region. Thus, all feeder lines are uniformly expressed as $l_i(x) = 5\text{km}, \forall x \in [0, L]$, where $L = 20\text{km}$. By referring to the works [1], [34], [35], the walking speed $v_w$ is typically specified to be $2\text{km/h}$ accounting for the walking inconvenience, and the value of time is $\epsilon = 5(\$/h)$. Table 1 summarizes other parameter values involved in the models. Values in Table 1 are adopted from previous studies [1], [12], [34], [35]. In practice, these values can be calibrated by surveys on the economic and technical characteristics of the local transportation systems.

There are two demand scenarios: spatially homogeneous and heterogeneous demands. In the first scenario, adapted from the previous work [2], the homogeneous boarding density is $b_1(x, y) = 50$ (pax/km$^2$/h) and the total number of passengers is $B_1 = 5000$ (pax/h). Following the literature, the exponential distribution is adopted to model the heterogeneous boarding density in the second scenario [36], [37], thus we set $b_1(x, y) = 4e^{0.002}y(\sqrt{x^2+y^2})$ (pax/km$^2$/h) and $B_2 = 5723$ (pax/h). $\sqrt{x^2 + y^2}$ denotes the distance from point $(x, y)$ to the left-most point of the trunk line (note that CBD is the right-most point). So, the travel demand increases as the distance increases and CBD has the greatest travel demand. Considering the symmetry of the upper and lower sides, the following cases only show the results of the upper side when $i = 1$. In the following results, $Z_{\text{pax}}$ denotes the total cost per passenger. $C_{\text{t}, \text{pax}}$, $C_{\text{p}, \text{pax}}$, and $C_{\text{r}, \text{pax}}$ represent three user costs per passenger. $C_{\text{t}, \text{p}}, C_{\text{p}, \text{p}}, C_{\text{r}, \text{p}}$, and $C_{\text{v}, \text{p}}$ signify four operator costs per passenger. The unit of these costs is congruously indicated as h/pax.

| Parameters                     | Rail | Bus |
|-------------------------------|------|-----|
| $v_y$, $v_p$ (km/h)           | 60   | 25  |
| $\tau^f$ (s/station)          | 45   | -   |
| $C_{\text{p}, \text{pax}}$, $C_{\text{t}, \text{pax}}$ (pax/veh) | 2400 | 80  |
| $\pi_{1,i}^f, \pi_{1,i}^l$ ($$/km) | 594+19.8e | 6+0.2e |
| $\pi_{2,i}^f$ ($$/station)   | 294+9.8e | -   |
| $\pi_{3,i}^f, \pi_{3,i}^l$ ($$/veh) | 101+5e  | 2.66+3e |
| $\pi_{4,i}^f, \pi_{4,i}^l$ ($$/veh-km) | 2.20 | 0.59 |

A. HOMOGENEOUS DEMAND

1) BEFORE COORDINATION

Rest on the premise that many-to-one demand pattern spreads all over the service region, Fig. 3 presents the primary results under the CA model. In Fig. 3, the abscissa axis denotes the position along the trunk line. Fig. 3(a) indicates that the station density monotonically decreases under the many-to-one homogeneous travel demand. The similar result was found by Vuchic [38]. The decrease in station density, namely the increase in station spacing towards CBD, is due to the reason that the passengers who have boarded want to travel to the destination (CBD) without stopping. Passengers who are already on the trunk vehicle expect no feeder bus to transfer other passengers to the next trunk station, while other passengers are in a hurry to board the feeder bus, which leads to the decrease in feeder line density and the increase in feeder bus headway, as shown in Fig. 3(b) and Fig. 3(c). The trunk transit headway is $H = 5.23$ (min).

However, the result of CA model is still not a feasible design. For instance, optimal station density $\delta(x)^*$ is a...
spatially continuous function, as depicted in Fig. 3(a), which needs to be discretized into specific station locations. Thus, discrete operation is adopted to generate the feasible network layout design in which the actual locations of the stations and feeder lines are shown in Fig. 3(a) and Fig. 3(b). According to the discrete method in Section 2.2, \( \int_{0}^{L} \delta(x) \, dx = 3.9 \), namely the optimal number of stations is 3.9, thus \( N = 4 \), and CBD is the terminal station due to many-to-one demand pattern; \( \int_{0}^{L} u_1(x) \, dx = 17.9 \), so there are 18 feeder lines. Generally, the results in Fig. 3 conform to the expectation under the travel demand pattern, which indicates that the CA model and the discrete method are rational.

Based on the above layout parameters (e.g., the feasible station location) and the headway of CA model, the costs of the discrete scheme are calculated with Equation (4). Most costs of CA scheme and discrete scheme appear to be the same, and Table 2 shows that the percentage difference of the total cost per passenger between two schemes is 0.69%. This subtle difference and the results in Fig. 3 conform to the expectation under the travel demand pattern, which indicates that the CA model and the discrete method are rational.

Based on the above layout parameters (e.g., the feasible station location) and the headway of CA model, the costs of the discrete scheme are calculated with Equation (4). Most costs of CA scheme and discrete scheme appear to be the same, and Table 2 shows that the percentage difference of the total cost per passenger between two schemes is 0.69%. This subtle difference and the results in Fig. 3 conform to the expectation under the travel demand pattern, which indicates that the CA model and the discrete method are rational.

### Table 2. Performance with CA model and discretization (scenario one).

| Cost (h/pax) | CA | Discretization | Difference (%) |
|--------------|----|----------------|---------------|
| \( Z^{pa} \) | 1.43 | 1.44 | -0.69 |

2) AFTER COORDINATION

The layout parameters are recruited into the schedule coordination modeling in stage two, then the coordination scheme is acquired by the proposed optimization algorithm. Fig. 4(a) shows that the total cost stabilizes at 1.35 h/pax when transit services are coordinated, which suggests the convergence of the algorithm. Fig. 4(b) illustrates that the feeder buses have the same headway and the same coefficient \( K \), thus all feeder buses run at the same frequency under the homogeneous demand.

Table 3 intuitively shows the benefit changes after the coordination, where the total cost per passenger is reduced by 6.25%. The average feeder headway \( h_{mean} \) and trunk headway both increase after coordination, which leads to the reduction of the fleet cost and the operation cost. Table 3 points out that the user and operator costs both diminish when trunk and feeder transit services are coordinated, which is generally consistent with the previous studies [2], [22]. Comparatively, Sivakumaran et al. [2] found that the coordination could reduce the user and operator costs compared to CA scheme under the homogeneous demand, but their network layout was assumed to be homogeneous such as the uniform locations of stations and feeder lines, which is extended to heterogeneous layout design in this paper. Wu et al. [22] focused on the single-modal transit system.

### Table 3. Performances with uncoordinated and coordinated states (scenario one).

| Cost (h/pax) | Uncoordinated | Coordinated | Difference (%) |
|--------------|---------------|-------------|---------------|
| \( Z^{pa} \) | 1.44 | 1.35 | -6.25 |
| \( C_{pa} \) | 0.15 | 0.15 | 0 |
| \( C_{fpa} \) | 0.12 | 0.10 | -16.67 |
| \( C_{tr} \) | 0.34 | 0.34 | 0 |
| \( C_{pa} \) | 0.62 | 0.62 | 0 |
| \( C_{px} \) | 0.05 | 0.05 | 0 |
| \( C_{p} \) | 0.08 | 0.05 | -37.50 |
| \( G_{p} \) | 0.08 | 0.05 | -37.50 |

### B. HETEROGENEOUS DEMAND

1) BEFORE COORDINATION

The study is further broadened to discuss the spatially heterogeneous demand scenario with the same analytical process as the last section. Fig. 5(a) and Fig. 5(b) show the station density and the feeder line density, respectively. It is worth noting that the curve trends of the station density, the feeder line density, and the feeder headway in Fig. 5 are different from the corresponding curve trends in Fig. 3, because the travel demand increases along the trunk line in this scenario. The trunk transit headway is \( H = 4.89 \) (min).

By utilizing the same discrete method as in the previous case, we transform the continuous solution of CA model into a feasible layout design. \( \int_{0}^{L} \delta(x) \, dx = 3.8 \), thus \( N = 4 \), and CBD is the terminal station under the many-to-one travel...
demand pattern, as shown in Fig. 5(a). Fig. 5(b) depicts 17 feeder lines due to $\int_0^L u_1(x) \, dx = 16.7$.

On the basis of the above layout, the costs of the discrete scheme are calculated with Equation (4). Most costs of CA scheme and discrete scheme are the same. Table 4 indicates that the difference of the total cost per passenger is 0.84%. Hence, it comes to a conclusion that the discrete method is valid and the layout is rational. Moreover, the discrete scheme can be regarded as the optimized uncoordinated one which will be utilized in the following section.

2) AFTER COORDINATION

We use the coordination method to develop an optimized coordination scheme and the main results are illustrated in Fig. 6. Fig. 6(a) shows that the total cost stabilizes at 1.12 h/pax when the services are coordinated. Under the demand pattern where the travel demand increases along the trunk line, the feeder headways and their coefficient $K$ are as shown in Fig. 6(b). Like the previous case, the increase of the average feeder headway $h_{\text{mean}}$ and trunk headway contributes to the reduction of the operator cost. Table 3 and Table 5 indicate that the proposed coordination method can diminish the user and operator costs further under the homogeneous and heterogeneous demand, which is different from the most previous methods that promote the cost reduction under the homogeneous demand.

C. SENSITIVITY ANALYSIS WITH RESPECT TO DEMAND LEVEL

In this section, we further explore the relationship between system performance and demand level to find out at which demand level the coordination is preferable. The demand level can reflect the demand for the trunk and feeder transit services. Understanding this relationship is important...
FIGURE 7. The total cost per passenger at different demand levels (a) results under homogeneous demand, (b) results under heterogeneous demand.

because it can facilitate policymakers to decide whether to operate the coordination considering the demand level in the service region.

With reference to Sivakumaran et al. [2], the hourly homogeneous boarding demand densities ranged from 10 to 100 pax/km², such that the hourly total demand in the service region ranged from 1000 to 10000 pax. By adjusting the coefficient of \( e \) (related to the heterogeneous demand density) in scenario two, we can obtain the hourly demand levels ranged from 1000 to 10000 pax. As the objective of the model, the total system cost is selected to reflect the benefit of coordination. In order to intuitively show the coordination benefits at different demands, the total cost is converted into the total cost per passenger.

Fig. 7 presents the total cost per passenger at different demand levels. Fig. 7(a) and Fig. 7(b) demonstrate the results under the homogeneous and heterogeneous demands, respectively. The average cost illustrated by the histogram in Fig. 7 decreases as the demand increases, which is consistent with the previous studies [2], [26]. Besides, Fig. 7 shows that coordination can be achieved at different demand levels, which further demonstrates the effectiveness of the model. Fig 7(a) and Fig 7(b) present the curves for the average total cost savings after coordination under homogeneous and heterogeneous demands, respectively. The two curves both depict a trend that the cost saving decreases with the increase of demand. Therefore, there is a finding that the coordination is preferable at a lower demand whose headways are larger, while the headways of a higher demand are too small to make a more cost-effective coordination. Sivakumaran et al. [2] obtained a similar finding under the homogeneous demand, and we further suggest that this phenomenon is observed under the heterogeneous one as well. This finding is consistent with that of Ting and Schonfeld [26], who focused on a single-modal transit network consisting of three routes with three transfers, while this paper finds this phenomenon in the bimodal transit system.

VI. CONCLUSION

Schedule coordination can facilitate the passengers’ travel and the operator’s management. The paper proposes a novel two-stage programming model for the schedule coordination design in a two-dimensional trunk-feeder transit corridor. In terms of the corridor structure, the feeder line segments are explicitly modeled in this research. The decision variables in the first stage include the trunk station density, feeder line density, and the service headways for two transit systems. The schedule coordination model is established in stage two with the trunk transit headway and the headway coefficient as the decision variables.

Numerical results show that 1) the proposed model and algorithm are effective and capable of designing a near optimal coordination scheme for the trunk-feeder transit system; 2) the method accounts for non-uniform demand and furnishes heterogeneous coordination design, where the network is optimized and further discretized into a specific network layout; 3) the coordination method can promote the cost savings under the homogeneous and heterogeneous demands; and 4) the coordination benefit decreases as the demand increases and the coordination is more preferable at a lower demand level.

The findings of the paper are summarized as follows: 1) it is the first time that the cost-saving phenomenon has been found in the coordinated trunk-feeder system under the two-dimensional heterogeneous demand; 2) the paper unveils that the coordination is more worthwhile at a lower-level heterogeneous demand in the bimodal transit system; and 3) compared to the previous coordination studies, which were based on the predetermined network layout, the proposed first-design-then-coordinate framework paves a further step towards the future research that jointly optimize the design and coordination.

It should be noted that this work has several limitations. First, the demand is assumed to be the many-to-one pattern that only represents commute demand in peak hours, while many transit corridors in real-world serve a more complex many-to-many demand. Second, the proposed model is a single-period design model that ignores the demand fluctuations within a day. Third, the optimization is conducted separately in the two stages of design and coordination, which indicates the final optimal design is not the global optimum. Accordingly, the following future extensions are possible:
TABLE 6. Descriptions of demand variables and decision variables.

| Decision variables | Descriptions |
|--------------------|--------------|
| \( \delta(x) \)  | Trunk station density at location \( x \) |
| \( u_l(x) \)      | Feeder line density in the vicinity of \( x \) |
| \( H \)           | Headway of trunk transit |
| \( h_l(x) \)      | Headway of feeder transit in the vicinity of \( x \) |

| Demand variables  | \( b_l(x, y) \) Boarding density at \( (x, y) \) |
|                   | \( P_l(x) \) On-board flow of trunk line at \( x \) of the corridor. |
|                   | \( CB_l^f(x) \) Cumulative boarding demand at \( x \) of the trunk line. |
|                   | \( cb_l^f(x, y) \) Cumulative boarding along \( y \) axis up to \( y \) at location \( x \). |

1) to jointly optimize the layout design and schedule coordination of trunk-feeder corridors under the general many-to-many demand; 2) to consider the time-dependent demand, and propose a multi-period model furnishing coordination plans for multiple periods; and 3) to consider the impacts of multiple access modes, e.g., bike-sharing and e-hailing, on the coordination design of the bimodal system.

APPENDIX A

CALCULATION DETAILS IN CA MODEL

The decision variables and the demand variables are defined in Table 6. The user and operator costs are calculated in the following sections.

A. USER COST

As the sum of four cost components, user cost \( C_U \) is derived as follows:

\[
C_U = C_a + C_w + C_v
\]

where \( C_a, C_w, \) and \( C_v \) denote the walking time, the waiting time at feeder stops and trunk stations, and the in-vehicle travel time, respectively. They are expressed in Equations (A2)-(A4).

\[
C_a = \sum_{i=1}^{2} \int_0^L \int_0^L b_l(x, y) dx dy \quad (A2)
\]

\[
C_w = \sum_{i=1}^{2} \int_0^L \int_0^L \frac{u_l(x) v_w}{4} dx dy \quad (A3)
\]

\[
C_v = \int_0^L P_l(x) \left( \frac{1}{v_l} + \tau f(x) \right) dx + \sum_{i=1}^{2} \int_0^L CB_l^f(x) \frac{1}{v_l} \frac{1}{v_f} dx + \sum_{i=1}^{2} \int_0^L \int_0^L cb_l^f(x, y) \frac{1}{v_f} dy dx \quad (A4)
\]

where \( \frac{1}{4v(x)v_w} \) in Equation (A2) indicates the average access time to the closest feeder line, and \( v_w \) is the average walking speed. Equation (A3) shows that the average waiting time is half of the headway. Equation (A4) represents the in-vehicle travel time consisting of three items, where the first one is the trunk line and the last two are the feeder line (\( x \)-axis and \( y \)-axis directions). \( v_l \) and \( v_f \) denote the cruise speeds of trunk and feeder vehicles, respectively. \( \tau f \) is the average delay per trunk station.

B. OPERATOR COST

Operator cost \( C_O \) is expressed as follows:

\[
C_O = C_l + C_s + C_{VH} + C_{VK}
\]

where \( C_l, C_s, C_{VH}, C_{VK} \) signify the line infrastructure cost, the station cost, the fleet cost, and the operation cost, respectively. These four costs can be calculated by Equations (A6)-(A9).

\[
C_l = \pi l 2L + \pi l \sum_{i=1}^{2} \int_0^L \frac{1}{2} u_l(x) dx + \pi f \sum_{i=1}^{2} \int_0^L \frac{1}{2} u_l(x) L_i^f y(x) dx \quad (A6)
\]

\[
C_s = \pi s \int_0^L \delta(x) dx \quad (A7)
\]

\[
C_{VH} = \pi VH 1H 2L 1 \int_0^L \left( \frac{1}{v_l} + \tau f(x) \right) dx + \pi VH \sum_{i=1}^{2} \int_0^L \frac{1}{2} u_l(x) \int_0^L \frac{1}{v_f} u_l(x) dx
\]

\[
+ \pi VH 2L 1H 2L \int_0^L \frac{1}{h_l(x)} \int_0^L \frac{1}{v_f} dy dx \quad (A8)
\]

\[
C_{VK} = \pi VK 2H \pi VH 1h_l(x) \int_0^L \int_0^L \frac{1}{v_f} u_l(x) dx
\]

\[
+ \pi VK 2L \frac{1}{h_l(x)} \int_0^L \frac{1}{v_f} u_l(x) dx \quad (A9)
\]

where \( \pi l, \pi f, \pi VH, \pi VK \) are the unit costs related to the trunk (feeder) line length, the number of trunk stations, the vehicle-hour travelled (VHT) of the trunk (feeder) line, and the vehicle-kilometer travelled of the trunk (feeder) line, respectively. In Equation (A6), \( \frac{1}{2v(x)} \) is the average length of the back-and-forth feeder line segments parallel to the main line. In Equation (A6) and Equations (A8)-(A9), the objects of their last two items are the feeder line (\( x \)-axis and \( y \)-axis directions).

APPENDIX B

SOLUTION OF THE DECISION VARIABLES IN CA MODEL

According to the first-order condition of Equation (1a), which is a nonlinear continuous function, the first-order derivatives are shown in Equations (B1)-(B12).

\[
H = \sqrt{\psi_N/\psi_D} \quad (B1)
\]

\[
\psi_N = \frac{1}{\epsilon} \pi VH 2 \int_0^L \left( \frac{1}{v_l} + \tau f(x) \right) dx + \frac{1}{\epsilon} \pi VK 2L \quad (B2)
\]
TABLE 7. Primary notations in the layout design.

| Notation | Description |
|----------|-------------|
| $N$ | Number of trunk stations |
| $R_i$ | Number of feeder lines (side $i$) |
| $r_s$ | Feeder line connecting station $s$ |
| $R_{1,s}$ | Number of feeder lines connecting station $s$ |
| $L_{0,s}$ | Location of station $s$ along the trunk line |
| $L_{0,r}$ | Location of feeder $r$ along the trunk line |
| $[M_{i,s},M_{i,r}]$ | The demand coverage of station $s$ |
| $[M_{i,s},M_{i,r}]$ | The demand coverage of feeder $r$ |
| $l_{s}^{D}$ | Length of feeder $r$ (x-axis direction) |
| $l_{s}^{V}$ | Length of feeder $r$ (y-axis direction) |
| $D_{s}^{t}$ | Total walking distance |
| $Q_{b,o}$ | On-board flow of feeder $r$ |
| $CQ_{b,o}(y)$ | Cumulative on-board flow of feeder $r$ along y-axis direction |
| $Q_{b}^{s}$ | On-board flow of feeder $r_s$ |

$$
\psi_D = \pi_w \sum_{i=1}^{2} \int_0^{L} \frac{1}{2} b_i (x, y) \, dxdy \quad (B3) \\
h_i(x) = \sqrt{x_i/\chi_{D,i}} \quad (B4) \\
\chi_{N,i} = \frac{1}{\pi} \frac{1}{\nu_i} u_i(x) \left( \frac{1}{\delta(x)} + \frac{2}{\nu_i} \int_0^{l_i(x)} \frac{1}{\nu_i} \, dy \right) + \frac{1}{\nu_i} v_i(u_i(x)) \left( \frac{1}{\delta(x)} + 2l_i(x) \right) \quad (B5) \\
\chi_{D,i} = \pi_w \int_0^{l_i(x)} \frac{1}{2} b_i (x, y) \, dy \quad (B6) \\
\delta(x) = \sqrt{\phi_N/\phi_D} \quad (B7) \\
\phi_N = \pi_i \sum_{i=1}^{2} \frac{C_B^{i}(x)}{4\nu_i} + \frac{1}{\pi} \sum_{i=1}^{2} \frac{u_i(x)}{2} + \frac{1}{\nu_i} \sum_{i=1}^{2} \frac{u_i(x)}{2} \quad (B8) \\
\phi_D = \pi_i \sum_{i=1}^{2} \frac{C_B^{i}(y)}{H} + \frac{1}{\nu_i} \sum_{i=1}^{2} \frac{u_i(y)}{2} + \frac{1}{\nu_i} \sum_{i=1}^{2} \frac{u_i(y)}{2} \quad (B9) \\
u_i(x) = \sqrt{\psi_{N,i}/\psi_{D,i}} \quad (B10) \\
\psi_{N,i} = \pi_i \int_0^{l_i(x)} b_i (x, y) \, dy \quad (B11) \\
\psi_{D,i} = \frac{1}{\nu_i} \left\{ \pi_i \left( \frac{1}{\delta(x)} + 2l_i(x) \right) + \frac{1}{\nu_i} \int_0^{l_i(x)} \frac{1}{\nu_i} \, dy \right\} + \frac{1}{\nu_i} \sum_{i=1}^{2} \frac{2l_i(x)}{h_i(x)} \quad (B12) \\

APPENDIX C

CALCULATION DETAILS IN THE DISCRETIZATION

In the discrete operation, we transform the optimal solutions of CA model into the feasible layout parameters by the endpoint method and then calculate the costs. Table 7 summarizes the notations adopted in the layout design.

A. USER COST

As the sum of four costs, user cost $C_u^{un}$ is calculated by:

$$
C_u^{un} = C_u^{an} + C_u^{wn} + C_u^{vn} \quad (C1)
$$

where $C_u^{an}$, $C_u^{wn}$, and $C_u^{vn}$ are the walking time, waiting time, and in-vehicle travel time, respectively. They are derived from Equations (C2)-(C4).

$$
C_u^{an} = \sum_{i=1}^{2} \frac{D_i}{v_a} \quad (C2) \\
C_u^{wn} = \sum_{i=1}^{2} \sum_{b=1}^{R_i} Q_{b,i} \left( \frac{h_i(b)}{2} \right) \quad (C3) \\
C_u^{vn} = \sum_{i=1}^{N} Q_a \left( \frac{L - L_{0,b}^i}{v_f} + t_r (N - s) \right) + 1/v_f \sum_{i=1}^{2} \sum_{b=1}^{R_i} Q_{b,i} L_{i,b}^i + 1/v_f \sum_{i=1}^{2} \sum_{b=1}^{R_i} \int_{y=0}^{L_{i,b}^i} CQ_{b,i}(y) \, dy \quad (C4)
$$

where $L - L_{0,b}^i$ in Equation (C4) represents the distance from station $s$ to CBD.

B. OPERATOR COST

Operator cost $C_O^{un}$ of the feasible design is expressed by:

$$
C_O^{un} = C_I^{un} + C_s^{un} + C_{VH}^{un} + C_{VK}^{un} \quad (C5)
$$

where $C_I^{un}$, $C_s^{un}$, $C_{VH}^{un}$, $C_{VK}^{un}$ are the line infrastructure cost, the station cost, the fleet cost, and the operation cost, respectively. They are derived from Equations (C6)-(C9).

$$
C_I^{un} = \pi_f L + \pi_i \sum_{i=1}^{2} \sum_{b=1}^{R_i} \left( 2L_{1,b}^i + L_{2,b}^i \right) \quad (C6) \\
C_s^{un} = \pi_s N \quad (C7) \\
C_{VH}^{un} = \pi_v \frac{1}{H} \left( \frac{2L}{v_f} + t_r N \right) + \pi_v \sum_{i=1}^{2} \sum_{b=1}^{R_i} \frac{2(L_{1,b}^i + L_{2,b}^i)}{h_i(b)v_f} \quad (C8) \\
C_{VK}^{un} = \pi_v \frac{1}{H} 2L + \pi_v \sum_{i=1}^{2} \sum_{b=1}^{R_i} \frac{2(L_{1,b}^i + L_{2,b}^i)}{h_i(b)} \quad (C9)
$$

where the second item of Equation (C6) is the infrastructure cost of the feeder line along $x$ and $y$ directions.

ACKNOWLEDGMENT

The authors are very grateful to the anonymous reviewers for their valuable suggestions and comments.

REFERENCES

[1] W. Fan, Y. Mei, and W. Gu, “Optimal design of intersecting bimodal transit networks in a grid city,” Transp. Res. B: Methodol., vol. 111, pp. 203–226, May 2018.

[2] K. Sivakumar, Y. Li, M. J. Cassidy, and S. Madanat, “Cost-saving properties of schedule coordination in a simple trunk-and-feeder transit system,” Transp. Res. A. Policy Pract., vol. 46, no. 1, pp. 131–139, Jan. 2012.

[3] A. Schöbel, “Locating stops along bus or railway Lines—A bicriteria problem,” Ann. Oper. Res., vol. 136, no. 1, pp. 211–227, Apr. 2005.
S. C. Wirasinghe, “Nearly optimal parameters for a rail/feeder-bus system,” Transp. Res. E, Logistics Transp. Rev., vol. 48, no. 1, pp. 50–70, Jan. 2012.

H. M. Repolho, A. P. Antunes, and R. L. Church, “Optimal location of railway stations: The Lisbon-Porto high-speed rail line,” Transp. Sci., vol. 47, no. 3, pp. 330–343, Aug. 2013.

J. Qi, L. Yang, Y. Gao, S. Li, and Z. Gao, “Integrated multi-track station layout design and train scheduling models on railway corridors,” Transp. Res. C, Emerg. Technol., vol. 69, pp. 91–119, Aug. 2016.

X. Guo, H. Sun, J. Wu, J. Jin, J. Zhou, and Z. Gao, “Multi-period-based timetable optimization for metro transit networks,” Transp. Res. B, Methodol., vol. 96, pp. 66–67, Feb. 2017.

A. Yang, J. Huang, B. Wang, and Y. Chen, “Train scheduling for minimizing the total travel time with a skip-stop operation in urban rail transit,” IEEE Access, vol. 7, pp. 81956–81968, 2019.

J. Ren, W. Jin, and W. Wu, “A two-stage algorithm for school bus stop location and routing problem with walking accessibility and mixed load,” IEEE Access, vol. 7, pp. 119519–119540, 2019.

V. F. Hurdle and S. C. Wirasinghe, “Location of rail stations for many to one travel demand and several feeder modes,” J. Adv. Transp., vol. 14, no. 1, pp. 29–55, 2000.

G. Liu, G. Quain, and S. C. Wirasinghe, “Rail line length in a crosstown corridor with many-to-many demand,” J. Adv. Transp., vol. 30, no. 1, pp. 95–114, Dec. 1996.

K. Sivakumaran, Y. Li, M. Cassidy, and S. Madanat, “Access and the choice of transit technology,” Transp. Res. A, Policy Pract., vol. 59, pp. 204–221, Jan. 2014.

G. K. Xuah and J. F. Chen, “Optimization of feeder bus routes and buses-stop spacing,” J. Transp. Eng., vol. 114, no. 3, pp. 341–354, May 1988.

S. Chien and Z. Yang, “Optimal feeder bus routes on irregular street networks,” J. Adv. Transp., vol. 34, no. 2, pp. 213–248, Mar. 2000.

A. Verma and S. L. Dhirngra, “Feeder bus routes generation within integrated mass transit planning framework,” J. Transp. Eng., vol. 131, no. 11, pp. 822–834, Nov. 2005.

L. Zhao and S. I. Chien, “Investigating the impact of stochastic vehicle arrival to optimal stop spacing and headway for a feeder bus route,” J. Adv. Transp., vol. 49, no. 3, pp. 341–357, Apr. 2015.

S. C. Wirasinghe, “Nearly optimal parameters for a rail/feeder-bus system on a rectangular grid,” Transp. Res. A, Gen., vol. 14, no. 1, pp. 33–40, 1980, doi: 10.1016/0191-2607(80)90092-8.

S. Chien and P. Schonfeld, “Joint optimization of a rail transit line and its feeder bus system,” J. Adv. Transp., vol. 32, no. 3, pp. 253–284, Jun. 1998.

W. Gu, Z. Amini, and M. J. Cassidy, “Exploring alternative service schemes for bus transit corridors,” Transp. Res. B, Methodol., vol. 93, pp. 126–145, Nov. 2016.

P. DiJoseph and S. I.-J. Chien, “Optimizing sustainable feeder bus operation considering realistic networks and heterogeneous demand,” J. Adv. Transp., vol. 47, no. 5, pp. 483–497, Aug. 2013.

L. M. Martínez and J. M. Viegas, “A new approach to modelling distance-decay functions for accessibility assessment in transport studies,” J. Transp. Geography, vol. 26, pp. 87–96, Jan. 2013.

V. R. Vuchic, “Rapid transit interstation spacings for maximum number of passengers,” Transp. Sci., vol. 3, no. 3, pp. 214–232, Aug. 1969.

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