EMPIRICAL STUDY OF SIMULATED TWO-PLANET MICROLENSING EVENTS

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ABSTRACT

We undertake the first study of two-planet microlensing models recovered from simulations of microlensing events generated by realistic multiplanet systems in which 292 planetary events, including 16 two-planet events, were detected from 6690 simulated light curves. We find that when two planets are recovered, their parameters are usually close to those of the two planets in the system most responsible for the perturbations. However, in 1 of the 16 examples, the apparent mass of both detected planets was more than doubled by the unmodeled influence of a third, massive planet. This fraction is larger than but statistically consistent with the roughly 1.5% rate of serious mass errors due to unmodeled planetary companions for the 274 cases from the same simulation in which a single planet is recovered. We conjecture that an analogous effect due to unmodeled stellar companions may occur more frequently. For 7 out of 23 cases in which two planets in the system would have been detected separately, only one planet was recovered because the perturbations due to the two planets had similar forms. This is a small fraction (7/274) of all recovered single-planet models, but almost a third of all events that might plausibly have led to two-planet models. Still, in these cases, the recovered planet tends to have parameters similar to one of the two real planets most responsible for the anomaly.

Key words: gravitational lensing: micro – methods: statistical – planetary systems – surveys

Online-only material: color figures

1. INTRODUCTION

Microlensing is an important method in detecting extrasolar planets. It is sensitive to planets around the Einstein radius (Mao & Paczynski 1991; Gould & Loeb 1992),

$$\theta_E \equiv \sqrt{\kappa M_L \pi_{rel}}; \ \kappa \equiv \frac{4G}{c^2 \Delta U} \approx 8.14 \text{ mas} \div M_\odot,$$

(1)

where $M_L$ is the lens mass, $\pi_{rel} = AU \left(D_L^{-1} - D_S^{-1}\right)$ is the lens-source relative parallax, and $D_L$ and $D_S$ are the distances to the lens and source, respectively. For typical microlensing events, this angular scale corresponds to a physical size on the lens plane ($R_E = D_L \theta_E$) that typically lies somewhat beyond the “snow line,” the region that is believed to be the birth place of giant planets (e.g., Ida & Lin 2004a). To date, dozens of microlensing planets have been detected, enabling statistical studies of the frequency of distant bound planets (Gould et al. 2010; Sumi et al. 2010; Cassan et al. 2012) and unbound “free-floating planets” (Sumi et al. 2011).

However, among dozens of microlensing events that have yielded planet detections, only two systems securely contain two planets (Gaudi et al. 2008; Han et al. 2013a), although most planets are believed to have been formed in multiple-planet systems. In addition, OGLE-2007-BLG-349 is either a two-planet event or a single-planet system orbited by a stellar binary companion (S. Dong et al. 2014, in preparation). This low yield of multiple-planet systems via microlensing is the result of both the lower probability of detecting these events and the challenge in interpreting the collected data. The frequency of a microlensing event is $\sim 10^{-5}$ yr$^{-1}$ star$^{-1}$, of which only a few percent yield a planet detection. For example, the Optical Gravitational Lensing Experiment (OGLE; Udalski 2003) is monitoring $\sim 10^8$ stars in the Galactic Bulge, resulting in $\sim 2000$ microlensing events per year but only $\sim 10$ planet detections per year. The probability of detecting two-planet events is of the order of the product of the probability of detecting each of them (Gaudi et al. 1998; Zhu et al. 2014), meaning that the expected number of two-planet events based on the past 15 year observations is roughly a few, consistent with the number of these events that have been observed.

The interpretation of an observed two-planet event is challenging. For example, the first two-planet event OGLE-2006-BLG-109 (Gaudi et al. 2008; Bennett et al. 2010) shows five distinct caustic-crossing features caused by a Jupiter/Saturn analog, with both finite source effects and orbital motion effects involved. Only with a thorough understanding about the caustic structures can one figure out that the first, second, third, and fifth features are due to one planet while the fourth feature is due to the other planet (see the Figure 2 of Gaudi et al. 2008). In fact, unlike both OGLE-2006-BLG-109 and OGLE-2012-BLG-0026, for which the two planets both show noticeable and separated perturbations, two-planet events can show noticeable perturbation(s) of only one planet (e.g., OGLE-2007-BLG-349; S. Dong et al. 2014, in preparation), making it even more difficult to find the correct two-planet (or triple-lens) model. The interpretation of a two-planet event, or more generally triple-lens event, is also challenging in the sense that it can be confused with a double-lens model that is impacted by higher-order effects, i.e., microlens parallax and orbital motion, especially in cases for which the perturbations are not well covered by observations. MACHO-97-BLG-41 was once considered the first discovery of a planet orbiting a binary star system (Bennett et al. 1999), but turned out to be better fitted by a double-lens model with orbital motion effect when more data were collected (Albrow et al. 2000; Jung et al. 2013).

On the other hand, in the era of future microlensing experiments, the detection rate of multiple-planet events will increase...
significantly and thus the current understanding of such events must be improved in order to process the data in a timely way. Numerical simulations of the second-generation microlensing surveys (i.e., OGLE IV, the Microlensing Observations in Astrophysics II (MOA, Bond et al. 2001), and especially the Korea Microlensing Telescopes Network (KMTNet, Kim et al. 2010)), suggest that the planet detection rate will then be increased by at least a factor of several (e.g., Shvartzvald & Maoz 2012; Zhu et al. 2014; Henderson et al. 2014). Space-based microlensing surveys like the Wide Field InfraRed Space Telescope (WFIRST; Spenger et al. 2013) and EUCLID (Penny et al. 2013) will be able to increase the rate by another factor of ~10. Studies of multiple-planet microlensing events are therefore a necessity.

Various studies have been conducted related to multiple-planet events. Gaudi et al. (1998) pointed out that multiple-planet events are more likely to be found in high-magnification ($A_{\text{max}} > 100$) events although the joint perturbation pattern around the peak of the light curve makes them difficult to interpret. Han (2005) noticed that in many cases the central planetary perturbations induced by multiple planets can be well approximated by the superposition of the single-planet perturbations. Song et al. (2014) studied the degeneracies in the triple-lens case from a theoretical aspect. Both Gaudi et al. (1998) and Song et al. (2014) found that the light curves produced by multiple-planet systems can sometimes be degenerate with that from a single-planet system. This double/triple-lens degeneracy may lead to wrong planetary parameters as well as underestimation of the frequency of multiple-planet systems (Song et al. 2014).

The present work is a follow-up paper to Zhu et al. (2014, hereafter Paper I) in which microlensing simulations were conducted for realistic planetary systems drawn from planet population synthesis simulations. This paper is focused on the (candidate) two-planet events produced by our simulation.Section 2, we give an overview of our simulation. Results concerning the detectability of the second planet in the presence of the first planet and the recovery of planetary parameters are presented in Section 3. We discuss our results in Section 4.

2. SIMULATION OVERVIEW

In Paper I, we simulated 6690 microlensing events. The lens systems are drawn from the Ida & Lin (2004a, 2004b, 2005, 2008a, 2008b, 2010) core accretion model for 0.3 $M_\odot$ stars. We extracted 669 systems with at least 1 planet more massive than 0.1 $M_\odot$ from 1000 Ida & Lin systems. For each system, planets were randomly placed within the orbital plane according to their semi-major axes and eccentricities and then projected onto the sky with a random orientation. With different projections, each system was used to produce 10 light curves for typical bulge sky with a random orientation. With different projections, each system was used to produce 10 light curves for typical bulge sky with a random orientation. With different projections, each system was used to produce 10 light curves for typical bulge sky with a random orientation.

The 6690 simulated light curves were first fitted with a standard single-lens model. We use $\Delta \chi^2_{\text{single}}$ to denote the deviation in $\chi^2$ of the best-fit single-lens model from the simulated data. Events with $\Delta \chi^2_{\text{single}} > 200$ were considered as planetary event candidates. For each of the 313 events that pass this threshold, we then identify which planets in the lens system are responsible for the planetary perturbations.

To do so, we first picked out the six most probably detectable planets according to the width of their planetary caustics; for each planet, we then generated the corresponding one-planet (double-lens) light curves by removing all the other planets in the same lens system. These one-planet light curves were also subjected to single-lens modeling to determine the detectability of each planet considered by itself, which we denote as $\Delta \chi^2_{\text{single}}$, with the index $i$ representing the ranking of the planet in that system based on $\Delta \chi^2_{\text{single}}$. We define a planet as detectable if its $\Delta \chi^2_{\text{single}} > 200$. Events with only one detectable planet were confirmed as one-planet events, while events with $n$ ($n > 1$) detectable planets were only considered as $n$-planet event candidates since the fact that planets can be detected individually does not guarantee that they can all be detected in the same event. After the removal of “bad” events, for which the source were so bright that the numerical precision of the ray shooting procedure was not adequate, we finally confirmed 292 planetary events, of which 23 are two-planet event candidates and none has more than two detectable planets.

Double-lens modeling was performed for the 23 two-planet event candidates to examine the double/triple-lens degeneracy. The result of this double-lens modeling is denoted as $\Delta \chi^2_{\text{double}}$, meaning the $\chi^2$ difference between the best-fit one-planet (double-lens) model and the theoretical model. With a threshold of $\Delta \chi^2_{\text{double}} = 300$, we then confirmed 16 two-planet events out of the 23 candidate events; that is, the remaining seven events with two detectable planets suffers the double/triple-lens degeneracy. Statistical studies and implications based on these planetary events and two-planet events were presented in Paper I.

3. RESULTS

3.1. Detectability of the Second Planet

We first investigate the influence of the first planet (primary perturber) on the detectability of the second planet (secondary perturber). We quantify the detectability of the second planet in the presence of the first planet as the difference in $\chi^2$ between the best-fit one-planet (double-lens) model and the input model, i.e., $\Delta \chi^2_{\text{double}}$. As is described in Section 2, the detectability of the second planet when it is alone can be quantified by $\Delta \chi^2_{\text{single}}$, which is the $\Delta \chi^2$ between the one-planet light curve (produced by removing all the other planets except for planet 2) and its best-fit single-lens model. If $\Delta \chi^2_{\text{double}} < \Delta \chi^2_{\text{single}}$, we consider the detectability of the second planet to be suppressed due to the presence of the first planet. Otherwise, it is enhanced.

We relate the change in the detectability of the second planet to the similarity between the planetary perturbations caused by the two responsible planets when they are considered separately. We find that two-planet event candidates can be classified according to the similarity in perturbations. Events in which the two planetary perturbations appear at the same time (difference of less than 0.2 days) on the light curve and show either positive

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5 This ranking system is not exactly the same as that employed in Paper I, which is based on the width of the planetary caustic of each planet. However, in most cases, the ranking based on $\Delta \chi^2_{\text{single}}$ agrees with the ranking based on the planetary caustic width.

6 In fact, we have one planetary event with $\Delta \chi^2_{\text{single}} / 2, \Delta \chi^2_{\text{single}} \approx 150$, but it cannot be claimed as a three-planet event with sufficient confidence.

7 See Paper I for the reason of choosing this threshold value.
or negative correlations are considered Type I events. Otherwise they are considered Type II events. We find that the detectability of the second planet is always suppressed by the presence of the first planet for Type I events, as is shown in Table 1. In fact, in seven of the Type I events, the suppression is so severe that the second planet is not a secure detection and therefore suffers the double-lens degeneracy. We show two of these double-lens degenerate events in Figure 1. For Type II events having two planets detectable individually suffer such double-lens degeneracy and thus the recovered planetary parameters from the true parameters. A similar conclusion has been reached by Han (2005).

### 3.2. Parameter Recovery

The lens system consists of detectable and undetectable planets. The detectable planet(s) can distort the light curve of a single lens (the host star), while the undetectable planets can also affect the single-lens light curve by changing the center of magnification and the center of mass. Furthermore, due to the degeneracy existing between light curves arising from multiple- and single-planet events (Gaudi et al. 1998; Song et al. 2014), multiple detectable planets may not all be recovered from the light curve. In fact, as we have seen in Table 1, seven events having two planets detectable individually suffer such double/ triple-lens degeneracy and thus the recovered planetary parameters might not be physical, although in principle they should be close to the true parameters of one of the detectable planets (the primary perturber). Here we investigate the deviation of the recovered planetary parameters from the true parameters.

### 3.3. Single-planet Events

In Paper I, we mentioned that 85% of all one-planet events can be reproduced very well by the corresponding one-planet (double-lens) light curve without any fitting. Here we investigate how the planetary parameters are perturbed in the remaining 15% (i.e., 41) single-planet events. When we look into the planetary systems that cause these events, massive \( q > 0.001 \)
and either far ($s > 3$) away from or close ($s < 0.3$) to the host star planets (in some cases brown dwarfs) appear in all these 41 cases. In addition, most of these events are non-caustic-crossing events. For all these 41 events, we fit a double-lens model to the simulated data, with the true planetary parameters as the initial seed. The best-fit planetary parameters, i.e., planet-to-star mass ratio $q$ and projected separation $s$, given by these double-lens models, are displayed in Figure 2, with the true parameters shown for comparison.

Figure 2 shows that even for events for which undetectable planets perturb the one-planet light curves, the planetary parameters recovered by modeling the light curve still represent the true parameters very well, although in very few cases, the planetary mass ratio can be off by a factor of two (e.g., No. 1290 and No. 3501 events, as are indicated in Figure 2), an example of which is shown in Figure 3. These marginally detected planets appear in noncaustic-crossing events and are therefore more easily affected by nondetectable planets in the same system, as is confirmed in Figure 4, which shows the ratio between the recovered and true planetary parameters with respect to the detectability of the planet quantified by $\Delta \chi^2_{\text{single}}$. On the other hand, planets in events with high $\Delta \chi^2_{\text{single}}$ can all be recovered very well, since the very distinct structures on the light curves due to approaching or crossing the caustics are robust to perturbation by distant planets.

### 3.3.1. Double/Triple-lens Degenerate Events

Due to the similarity between the planetary perturbation patterns caused by the two detectable planets when considered separately, 7 out of the 23 two-planet event candidates do not have large enough $\Delta \chi^2_{\text{double}}$ to claim the detection of the
Figure 2. Mass ratio $q$ and projected separation $s$ of the recovered planets (red filled circles) from the double-lens modeling of the light curves of 41 single-planet events, with respect to the true values of the detectable planets (open diamonds) in these events. Black dashed lines connect the recovered planet with the original responsible planet.

(A color version of this figure is available in the online journal.)

Figure 3. Light curve and caustic structure of No. 1290 event. In the light curve plot, the black solid line shows the best-fit double-lens light curve, while the blue dashed line is the light curve generated from the real planet. In the caustic structure plot, the black horizontal line is the source trajectory, with the circle centered on it as the size of the source, the plus signs indicate the positions of lenses, the blue curve is the caustic from the real planet, and the black curve is that from the real planet and an undetectable, distant ($s = 14.6$) but massive ($q = 0.14$) brown dwarf.

(A color version of this figure is available in the online journal.)
while the deviation in mass ratio (the deviation in the projected separation (of the two planets are correlated or anti-correlated. We find that the true planet, no matter whether the planetary perturbations the recovered planetary mass ratio is always greater than that of the true planets. Therefore, even in events suffering the double planet event candidates and without (i.e., confirmed two-planet events with (i.e., two-planet event candidates sample shows that the probability of the two detectable planets in the mass ratio-separation parameter space as the true planet 2. We quote the closer one between the two detectable second planet. The planetary parameters that are recovered should therefore deviate from the values of either planet 1 or planet 2. We quote the closer one between the two detectable planets in the mass ratio-separation parameter space as the true position of the recovered planet. We list in Table 2 and illustrate in Figure 5 the most important parameters, i.e., mass ratio $q$ and projected separation $s$, of the two detectable planets and the recovered planet. One obvious pattern shown in Figure 5 is that the recovered planetary mass ratio is always greater than that of the true planet, no matter whether the planetary perturbations of the two planets are correlated or anti-correlated. We find that the deviation in the projected separation ($s$) is below $\sim 10\%$, while the deviation in mass ratio ($q$) is within $35\%$, from the true planet. Therefore, even in events suffering the double/triple-lens degeneracy, the planet recovered from modeling the microlensing light curve is still a good representative of one of the true planets.

We show the distributions of intersection angles between two responsible planets for two-planet events with (i.e., two-planet event candidates) and without (i.e., confirmed two-planet events) these double/triple-lens degeneracies in Figure 6, with respect to the distribution of all planetary events, in which the intersection angle between two most detectable planets is used for single-planet events. A Kolmogorov–Smirnov (K-S) test between the all planetary events sample and the two-planet event candidates sample shows that the probability of the two distribution being extracted from the same distribution is $22\%$, while it is $61\%$ between all planetary events sample and the confirmed two-planet events sample. The discrepancy between the results of these two K-S tests suggests that there may be some preferred intersection angles for the double/triple-lens degeneracy.

### 3.3.2. Two-planet Events

We also investigate the influence of the undetectable planets on the parameter recovery in two-planet events by employing triple-lens fits to the simulated light curves. The recovered planetary parameters with respect to the original parameters for 16 two-planet events are shown in Figure 7 and the ratio between these two as a function of the detectability of the planet is shown in Figure 8.

Similar to what has been seen in the case of one-planet events, the two planets are recovered very well in most events, meaning that the remaining undetectable planets in these systems have a negligible influence on the two detectable planets, although the recovered planet masses in No. 8770 event deviate from the true values by a factor of three, which also comes from the perturbation of a distant ($s \approx 14$) massive ($q \approx 3 \times 10^{-3}$) planet. The light curve and caustic structure of this event is shown in Figure 9. As with Figure 4 for the one-planet case, Figure 8 also indicates that the planets detected with lower $\Delta \chi^2$ in two-planet events are more easily affected by the undetectable planets in the same system.

| No. | $A_{\text{max}}$ | $q_1 \times 10^{-3}$ | $s_1$ | $q_2 \times 10^{-3}$ | $s_2$ | $q \times 10^{-3}$ | $s$ | Deviations in $s$ and $q$ from the Closest Planet |
|-----|-----------------|----------------------|-------|----------------------|-------|------------------|-----|-------------------------------------------------|
| 290 | 7               | 21.4                 | 2.28  | 7.43                 | 1.89  | 23.9 ± 0.9       | 2.53 ± 0.14 | 9.7%, 10%                                      |
| 1689| 56              | 2.01                 | 1.16  | 2.45                 | 0.48  | 2.00 ± 0.03      | 1.16 ± 0.002 | 0.4%, 0.6%                                      |
| 3247| 18              | 5.44                 | 0.97  | 1.71                 | 1.33  | 6.32 ± 0.05      | 0.9616 ± 0.0005 | 1.1%, 14%                                      |
| 3689| 7               | 2.01                 | 0.73  | 2.45                 | 0.48  | 2.9 ± 0.2        | 0.699 ± 0.006 | 4.8%, 31%                                      |
| 5797| 25              | 1.51                 | 1.38  | 1.86                 | 0.48  | 2.34 ± 0.09      | 1.46 ± 0.01 | 5.7%, 35%                                      |
| 7290| 36              | 7.43                 | 1.97  | 21.4                 | 3.52  | 9.85 ± 0.7       | 2.02 ± 0.04 | 2.3%, 25%                                      |
| 7451| 19              | 7.45                 | 2.11  | 21.5                 | 4.45  | 31.9 ± 1.3       | 4.89 ± 0.02 | 9.0%, 33%                                      |
Figure 5. Mass ratio $q$ and projected separation $s$ of the recovered planet (red filled circles) from the double-lens modeling of the light curves of seven double/triple-lens degenerate events, with respect to the true values of the two detectable planets (open diamonds and open triangles) in these events. Solid lines are used to connect the recovered planet with the closer one between the two detectable planets and dashed lines connect it with the other one in the same event. The vertical dash-dotted line indicates the position of the Einstein ring radius.

(A color version of this figure is available in the online journal.)

Figure 6. Cumulative distributions of intersection angles between the two most detectable planets in all planetary events (black solid line) between the two responsible planets in two-planet event candidates (red dashed line) and between two detected planets in confirmed two-planet events (blue solid line); vertical gray dashed lines indicate the positions of the seven double/triple-lens degenerate events.

(A color version of this figure is available in the online journal.)
The planetary close/wide degeneracy (Dominik 1999; An 2005) has been found in two-planet events, both observationally (Han et al. 2013a) and theoretically (Song et al. 2014), in the case when two planets are both detected via central caustics. We find that the second planet in No. 3194 event also suffers this close/wide degeneracy ($\Delta \chi^2 = 2$), despite the fact that the first planet has noticeable signatures caused by crossing its planetary caustic.

4. DISCUSSION

In this work, by simulating microlensing light curves of realistic multi-planet systems, we are trying to answer two questions: (1) when will the detectability of a second planet be suppressed or enhanced in the presence of the first planet and (2) whether and how can the other detectable or undetectable planets in the same lens system affect the parameter recovery of the detected planets.

We find that the influence of the first planet on the detectability of the second planet is strongly correlated with the similarity between the planetary perturbations caused by the two detectable planets when they are considered separately. Particularly, when the two detectable planets show similar perturbation patterns, the detectability of the second planet is always suppressed by the presence of the first planet; in some cases, such suppressions may be so strong that no signs of the perturbation

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**Figure 7.** Mass ratio $q$ and projected separation $s$ of the recovered planets (red and blue filled circles) from the triple-lens modeling of the light curves of 16 two-planet events, with respect to the true values of the two detectable planets (open diamonds and open triangles) in these events. Black dashed lines connect the recovered planet and the original responsible planet.

(A color version of this figure is available in the online journal.)

**Figure 8.** Similar to Figure 4 but for planets detected in two-planet events. For each event, the detectability of the first planet (the major perturber) is quantified by $\Delta \chi^2_{\text{single}} - \Delta \chi^2_{\text{double}}$, while that of the second planet (the minor perturber) is quantified by $\Delta \chi^2_{\text{double}}$. 

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**Figure 7**

**Figure 8**
caused by the second planet can be seen in the residuals after the subtraction of the best-fit double-lens (one-planet) model, meaning that these events suffer the double/triple-lens degeneracy. Our result also confirms the well established result that the overall perturbation pattern in multiple-planet events is usually a superposition of those of individual planets (Han 2005), indicating that the modeling of two-planet events can mostly be decomposed. This result may still hold true when the orbital motion effect is considered since the total magnification can always be decomposed at each moment.

We compared the planetary parameters recovered from modeling the microlensing light curves with the input parameters for one-planet events, double/triple-lens degenerate events, and two-planet events. In a large majority of the investigated events, the planetary parameters can be recovered very well. However, we also notice that in very few low-magnification events in which the planets are marginally detected, the recovered planetary mass ratio can deviate from the true value by a factor of two to three due to the existence of an undetectable massive companion to the lens star. The fraction of such events in the two-planet case (1/16) is larger than but still statistically consistent with that in the single-planet case (4/274), suggesting that the deviation in planetary parameters due to unmodeled massive planets does not become worse in the multiple-planet case. However, although such events contribute a very small fraction to the total planetary events in our simulation, we conjecture that this problem may be more serious for the case of more massive companions, namely, stars.

In Paper I, we confirmed that follow-up observations are still necessary in extracting the full power of high-magnification events in the era of the second-generation microlensing experiments. In fact, follow-up observations are also necessary for the detection of multiple-planet systems. In two-planet events, for example, although the detectability of the second planet is suppressed by the presence of the first planet in most cases, this suppression can be compensated by the intensive follow-up observations that are more likely initiated by detecting the first planet since the planetary perturbation of the first planet, the primary perturber, is larger and sustains longer. The much more intensive (~1 minute compared to 10 minutes of KMTNet) observations conducted by the follow-up teams (e.g., μFUN; Gould et al. 2006; Gaudi et al. 2008) can therefore compensate the suppression due to the first planet. Furthermore, with follow-up observations, one would expect to detect more multiple-planet events and more lower-mass planets than under the survey-only mode (Gould et al. 2014).

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