Higgsless Standard Model in Six Dimensions\textsuperscript{a}

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ABSTRACT  
We present a Higgsless Standard Model in six dimensions, based on the Standard Model gauge group $SU(2)_L \times U(1)_Y$, with two flat extra dimensions compactified on a rectangle. The electroweak symmetry is broken by (mixed) boundary conditions and realistic gauge boson masses can be accommodated by proper choice of the compactification scales and brane kinetic terms. With respect to “oblique” corrections, the agreement with electroweak precision tests is somewhat improved compared to the simplest five-dimensional Higgsless models.

1. Introduction  
Recently, a new class of Higgsless models has been proposed, in which electroweak symmetry breaking (EWSB) is accomplished without the Higgs mechanism by employing mixed boundary conditions (BC’s) on a compact space \cite{1–3}. These Higgsless models describe a five-dimensional (5D) $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory compactified on an interval $[0, \pi R]$, where tree-level unitarity of longitudinal gauge boson scattering is ensured through the exchange of the attendant Kaluza-Klein (KK) tower of massive gauge boson excitations \cite{4}. In a flat extra dimension, four-dimensional (4D) brane kinetic terms are necessary to decouple at low energies the higher KK excitations \cite{3}, whereas in Higgsless warped space models they are required \cite{5} to evade disagreement with electroweak precision tests (EWPT) \cite{6} due to tree-level “oblique” corrections \cite{7–9}.

In this talk, which is based on work done in collaboration with Steven Gabriel and Satya Nandi \cite{10}, we consider a six-dimensional (6D) Higgsless model using only the Standard Model (SM) gauge group $SU(2)_L \times U(1)_Y$. The model is formulated in flat space with the two extra dimensions compactified on a rectangle and EWSB is achieved by imposing BC’s consistent with the variation of the action. The higher KK resonances of $W^\pm$ and $Z$ decouple below $\sim 1\text{TeV}$ through the presence of dominant 4D brane kinetic terms. The $\rho$ parameter can be set exactly to one by an appropriate choice of the bulk gauge couplings and compactification scales. Unlike in the 5D theory, the mass scale of the lightest gauge bosons $W$ and $Z$ is set by the dimensionful bulk couplings alone, which are of the order $\sim 10^{2}$ GeV. Here, the tree-level oblique corrections to EWPT are somewhat in better agreement with data than in the simplest 5D warped and flat Higgsless models.

2. The Model  
Consider a 6D $SU(2)_L \times U(1)_Y$ gauge theory in a flat space-time background, where

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the two extra spatial dimensions are compactified on a rectangle \[10\]. If we denote by \(y_1\) and \(y_2\) the coordinates of the 5th and 6th dimension, the physical space is defined by \(0 \leq y_1 \leq \pi R_1\) and \(0 \leq y_2 \leq \pi R_2\). The \(SU(2)_{L}\) and \(U(1)_Y\) gauge bosons in the bulk are respectively written as \(A^a_M\) (\(a = 1, 2, 3\) is the gauge index) and \(B_M\), where capital Roman letters \(M = 0, 1, 2, 3, 4, 5, 6\) denote the 6D Lorentz indices, while Greek letters \(\mu = 0, 1, 2, 3\) symbolize the usual 4D Lorentz indices. The action of the gauge fields in our model is given by

\[
S = \int d^4x \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2 (\mathcal{L}_6 + \delta(y_1)\delta(y_2)\mathcal{L}_0),
\]

where \(\mathcal{L}_6\) is a 6D bulk gauge kinetic term and \(\mathcal{L}_0\) is a 4D brane gauge kinetic term localized at \((y_1, y_2) = (0, 0)\), which read respectively

\[
\mathcal{L}_6 = -\frac{M_L^2}{4} F_{MN}^a F^{MNa} - \frac{M_Y^2}{4} B_{MN} B^{MN}, \quad \mathcal{L}_0 = -\frac{1}{4g^2} F_{\mu\nu}^{\text{eff}} F^{\mu\nu} - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu},
\]

with field strengths \(F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + f^{abc} A_M^b A_N^c\) (\(f^{abc}\) is the structure constant) and \(B_{MN} = \partial_M B_N - \partial_N B_M\). In Eqs. (2), the quantities \(M_L\) and \(M_Y\) have mass dimension +1, while \(g\) and \(g'\) are dimensionless. Now, EWSB \(SU(2)_L \times U(1)_Y \to U(1)_Q\) is achieved by imposing suitable Dirichlet, Neumann, and mixed BC’s \[10\], which are consistent with the variation of the action and correspond therefore to a soft gauge symmetry breaking. Schematically, the symmetry breaking is sketched in Fig. \[\square\] The fermions are, like the
gauge bosons, approximately localized by dominant brane kinetic terms at \((y_1, y_2) = (0, 0)\), thus suppressing for the light generations unwanted non-oblique corrections to the electroweak precision parameters.

The total effective 4D Lagrangian in the compactified theory \(\mathcal{L}_{\text{total}}\) can be written as \(\mathcal{L}_{\text{total}} = \mathcal{L}_0 + \mathcal{L}_{\text{eff}}\), where \(\mathcal{L}_{\text{eff}} = \int_0^{\pi R_1} dy_1 \int_0^{\pi R_2} dy_2 \mathcal{L}_6\) denotes the contribution from the bulk, which follows from integrating out the extra dimensions. Here, \(\mathcal{L}_{\text{eff}}\) generates

![Figure 1: Symmetry breaking of \(SU(2)_L \times U(1)_Y\) on the rectangle. At one boundary \(y_1 = \pi R_1\), \(SU(2)_L\) is broken to \(U(1)_I_3\) while on the boundary \(y_2 = \pi R_2\) the subgroup \(U(1)_I_3 \times U(1)_Y\) is broken to \(U(1)_Q\) of electromagnetism, which leaves only \(U(1)_Q\) unbroken on the entire rectangle.](image-url)
electroweak vacuum polarization amplitudes summarizing in the 4D theory the effect of
the symmetry breaking sector. These vacuum polarizations lead at tree-level to oblique
corrections (as opposed to vertex corrections and box diagrams) of the gauge boson prop-
agators and thus affect electroweak precision measurements \cite{7, 8}. To determine the KK
masses of the gauge bosons, we will from now on assume that the brane terms \( L_0 \)
dominate the bulk kinetic terms, \textit{i.e.}, we take \( 1/g^2, 1/g^2 \gg (M_{L,Y} \pi)^2 R_1 R_2 \). As a result, we
find from \( \mathcal{L}_{\text{eff}} \) for the \( W^\pm \)'s the mass spectrum

\[
m_0^2 \approx 2g^2 M_L^2 R_2/R_1 = m_W^2, \quad m_n \approx n/R_1, \quad n = 1, 2, \ldots ,
\]

where we identify the lightest state with mass \( m_0 \) with the \( W^\pm \). Observe in Eq. \( \text{(3)} \), that
the inclusion of the brane kinetic terms \( \mathcal{L}_0 \) for \( 1/R_1, 1/R_2 \gg \mathcal{O}(\text{TeV}) \) leads to a decoupling
of the higher KK-modes with masses \( m_n (n > 0) \) from the electroweak scale, leaving only
the \( W^\pm \) states with a small mass \( m_0 \) in the low-energy theory. The lowest massive state
in the tower of the neutral gauge bosons has a mass-squared

\[
m_Z^2 \approx 2(g^2 + g'^2) M_L^2 M_Y^2 R_1/[(M_L^2 + M_Y^2) R_2]
\]

which we identify with the \( Z \) of the SM. All other KK modes of the \( \gamma \) and \( Z \) have masses
of order \( \gtrsim 1/R_2 \) and thus decouple for \( 1/R_1, 1/R_2 \gtrsim \mathcal{O}(\text{TeV}) \), leaving only a massless \( \gamma \)
and a \( Z \) with mass \( m_Z \) in the low-energy theory.

3. Relation to EWPT

One crucial test for any model of EWSB is the value of the \( \rho \) parameter, which is
experimentally known to satisfy the relation \( \rho = 1 \) to better than 1\%. In our model,
we choose the 4D brane couplings \( g \) and \( g' \) to follow the usual SM relation \( g^2/(g^2 +
g'^2) = \cos^2 \theta_W \approx 0.77 \). Defining \( \rho = 1 + \Delta \rho \), we then obtain that \( \Delta \rho = 0 \) if the bulk
kinetic couplings and compactification radii satisfy the relation \( (M_L^2 + M_Y^2)/M_Y^2 = R_1^2/R_2^2 \).
Although we can thus fit \( \Delta \rho = 0 \) by appropriately dialing the model parameters, \( \mathcal{L}_{\text{eff}} \)
introduces a manifest breaking of custodial symmetry and will thus contribute to EWPT
via oblique corrections to the SM parameters. The effects of oblique corrections on EWPT
can be parameterized in the \( \epsilon_1, \epsilon_2, \) and \( \epsilon_3 \) framework \cite{8}, where the current experimental
bounds on the relative shifts with respect to the SM expectations are roughly of the order
\( \epsilon_1, \epsilon_2, \epsilon_3 \lesssim 3 \cdot 10^{-3} \) \cite{11}. For our choice of parameters, we consistently find \( \epsilon_1 = \Delta \rho = 0 \).
The quantities \( |\epsilon_2| \) and \( |\epsilon_3| \), on the other hand, are bounded from below by the requirement
of having sufficiently many KK modes below the strong coupling (or cutoff) scale \( \Lambda \) of
the theory. In the 6D model, we would naively estimate \( \Lambda \approx \sqrt{2}(4\pi)^{3/2} M_{L,Y} \) \cite{12} which
leads for \( M_{L,Y} \approx 10^2 \text{GeV} \) to \( \Lambda \approx 6 \text{ TeV} \). Assuming \( M_L = M_Y \), we have \( R_2 = R_1/\sqrt{2} \) and

\[
\epsilon_3 \approx \frac{g^2}{96\sqrt{2\pi}} (\Lambda R_2)^2 \approx 2.3 \times 10^{-3} \times (g\Lambda R_2)^2,
\]

while \( \epsilon_2 \approx \epsilon_3 \). It is interesting to compare Eq. \( \text{(3)} \) with the corresponding result of the
5D model in Ref. \cite{3}. We find that the parameter \( \epsilon_3 \) is in the 6D model by \( \sim 15\%\)
smaller than the corresponding 5D value. This is due to the fact that in the 6D model the bulk gauge kinetic couplings satisfy $M_L = M_Y \approx 100 \text{ GeV}$, while they take in 5D only the values $M_L \simeq M_Y \approx 10 \text{ GeV}$. From Eq. (5) we then conclude that the inverse loop expansion parameter can be $\Lambda_R^2 \approx 1/g \approx 1.6$ in agreement with EWPT. Like in the 5D case, however, the 6D model seems not to admit a loop expansion parameter in the regime $\Lambda_R^2 \gg 1$ as required for the model to be calculable.

To summarize, we have considered a 6D Higgsless Standard Model in compactified flat space, which is based on the gauge group $SU(2)_L \times U(1)_Y$. Dominant brane interactions lead to a realistic gauge sector and the model parameters allow to improve, with respect to oblique corrections, the fit of EWPT as compared to the simplest 5D Higgsless models.

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