Switching of valley polarization and topology in twisted bilayer graphene by electric currents

Ying Su\textsuperscript{1} and Shi-Zeng Lin\textsuperscript{1}

\textsuperscript{1}Theoretical Division, T-4 and CNLS, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

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Introduction.—The electronic band structure is modified significantly compared to that in a single layer graphene, when two layers of graphene are stacked together\cite{1}. The band-width can be controlled by the misalignment angle between two layers\cite{2,3}. At certain twisted angles called magic angles, the energy bands near the Fermi surface are extremely flat\cite{3}, where the Coulomb interaction becomes dominant over the kinetic energy of electrons. It is expected that novel quantum states enabled by the strong electronic correlation will emerge. Indeed, the correlated insulating state and superconductivity have been observed experimentally in the magic-angle twisted bilayer graphene (TBG)\cite{4,5}. Furthermore, the insulating state can have nontrivial topology which is manifested by the intrinsic quantum anomalous Hall effect (QAHE)\cite{6,7}. The twisted multilayer graphene therefore becomes an important platform to explore the physics of strong electronic correlation and topology\cite{4–76}.

The QAHE with orbital ferromagnetism was observed at 3/4 filling of the upper flat bands in TBG recently\cite{6,7}. In the absence of electronic interaction, the electronic bands of TBG have the four-fold degeneracy associated with the valley and spin degrees of freedom. The strong electronic interaction in flat bands lifts the valley and spin degeneracy, that results in the valley-spin-polarized insulating state responsible for the QAHE at 3/4 filling\cite{19,57,69–72}. The nontrivial topology of the insulating state is characterized by the Chern number $C = \pm 1$, where the sign of Chern number depends on which valley is polarized. Remarkably, as demonstrated in experiments, the sign of Hall conductance can be flipped by a dc current, that indicates the switching of Chern number and valley polarization (VP)\cite{6,7}. It is argued that the orbital magneto-electric effect is responsible for the switching of Chern number by currents\cite{48}. It is also suggested that the asymmetric edges of TBG can result in the free energy difference for the chiral edge currents in opposite directions\cite{7}. However, to change the topology, it is required to overcome a bulk energy barrier, that increases linearly with volume of the system. It is unclear whether an edge current alone is sufficient to overcome the bulk energy barrier.\cite{77}.

In this paper, we propose a mechanism of switching of VP and Chern number in TBG by electric currents in the bulk. The current causes the redistribution of the electron occupation in otherwise fully occupied or empty bands, which deforms and shifts the band dispersion with respect to the Fermi energy $E_F$ due to the Coulomb interaction. Above a threshold current, the major part of the originally empty (occupied) band when current is absent is pushed below (above) $E_F$. The bands then are swapped after relaxation when the current is removed, that hence results in flipping of the VP and topology.

Toy model.—To facilitate the understanding of switching of VP by electric currents, we use a simple toy model to demonstrate the physical picture. Here we consider a two-band model described by the Hamiltonian

$$
\mathcal{H} = \sum_{k,\tau}(\varepsilon_{k,\tau} - \lambda \delta_{k,\tau})\hat{c}_{k,\tau}^{\dagger}\hat{c}_{k,\tau} + V \sum_{k_1, k_2} \hat{c}_{k_1, -\tau}^{\dagger}\hat{c}_{k_1, +\tau}\hat{c}_{k_2, +\tau}\hat{c}_{k_2, -\tau},
$$

(1)

where $\tau = \pm$ is the effective valley index labeling the two energy bands and $V$ is the repulsive intervalley interaction. The model has time-reversal symmetry (TRS), $\varepsilon_{k,\tau} = \varepsilon_{-k, -\tau}$. The current in the system can be introduced by the Lagrangian multiplier $\lambda$, which has the meaning of electric field times electron lifetime times electric charge according to the semiclassical Boltzmann transport theory. Thus $\lambda \delta_{k,\tau}$ becomes the energy gain of electrons in the presence of an electric field. The corresponding self-consistent mean-field Hamiltonian is $\mathcal{H}_{MF} = \sum_{k,\tau} E_{k,\tau}\hat{c}_{k,\tau}^{\dagger}\hat{c}_{k,\tau}$ with $E_{k,\tau} = \varepsilon_{k,\tau} - \lambda \delta_{k,\tau} + \Delta_{\tau}$ and $\Delta_{\tau} = V \sum_{k} f(E_{k,\tau})$, where $f(E_{k,\tau})$ is the Fermi distribution function.

Consider the schematic band dispersions shown in Fig. 1(a). The band with valley $+\tau$ is above that with valley $-\tau$ as a consequence of the spontaneous symmetry breaking due to the intervalley interaction. The system is an insulator at half filling. To introduce current, a fraction of electrons in the lower band must be pumped into the upper band. Because the electron self-energy depends on the electron occupation, the part of $\tau$ band with group velocity opposite to the current direction rises while the $+\tau$ band shifts downwards. The majority of the $+\tau$ band can be below the $-\tau$ band above a critical current, as shown in Fig. 1(b). In this circumstance and after removing the current, the system relaxes into a state with $+\tau$ band occupied (empty), and thus the VP is switched, as shown in Fig. 1(c).
To describe the phase transition associated with the switching of VP, we introduce the VP order parameter \( \delta = (\Delta_+ - \Delta_-)/2 \). Close to the transition temperature \( T_c \), when \( \delta \) is small, the Ginzburg-Landau free energy of the system can be expanded as [77]

\[
\mathcal{F} = \mathcal{F}_0 + \alpha_1 \delta + \alpha_2 \delta^2 + \alpha_3 \delta^3 + \alpha_4 \delta^4.
\]

(2)

Because both current and \( \delta \) are odd under time reversal, it requires \( \alpha_{1,2,3}(\lambda) = -\alpha_{1,2,3}(-\lambda) \) and \( \alpha_{2,2,3}(\lambda) = \alpha_{2,3,4}(\lambda) \). Under the inversion operation within each valley \( \Gamma_v \), i.e., \( \varepsilon_{k,r} \rightarrow -\varepsilon_{k,r} \), \( \delta \) is even while the current is odd. If the system has the symmetry associated with \( \Gamma_v \), then \( \alpha_{1,3}(\lambda) = \alpha_{1,3}(-\lambda) \), which immediately implies \( \alpha_{1,3}(\lambda) = 0 \). Therefore this intravalley inversion symmetry must be broken, i.e., \( \varepsilon_{k,r} \neq \varepsilon_{k,r} \), in order to make the current-induced switching of valley polarization possible.

It is clear from Eq. (2) that the system has two degenerate ground states with opposite VP, which are separated by an energy barrier, as shown in Figs. 1(d) and 1(f). The presence of current lifts the degeneracy by increasing the energy of one valley-polarized state and reducing the energy of the other one, as shown in Fig. 1(e). Above a critical current, the energy barrier vanishes. The VP of the system is switched if it is initiated at a state, that becomes unstable under the current [see Fig. 1(e)]. On the other hand, if the system sits in the other state that remains the energy minimal under the current, there is no switching of VP. Thus the switching of VP depends on the direction of current.

TBG.—We then apply our physical picture to TBG with a small twist angle. In TBG, no symmetry guarantees \( \varepsilon_{k,r} = \varepsilon_{-k,r} \) and therefore it is possible to switch the VP by electric currents. We employ the continuum model Hamiltonian [3]

\[
\mathcal{H}_{k,r} = \left( \begin{array}{cc} h_{1,r}(k) & T_r(r) \\ T^\dagger_r(r) & h_{2,r}(k) \end{array} \right),
\]

(3)

where \( \tau = \pm \) is the valley index and \( h_{1,r}(k) \) is the low-energy effective Dirac Hamiltonian of the \( l \)-th layer graphene. Under the twist that the first (second) layer is rotated by \( \theta/2 \) \((-\theta/2)\), the Dirac Hamiltonian becomes

\[
h_{1,r}(k) = R(\pm \theta/2)h_{1,r}(k - rK_i) \cdot \sigma_i R(\pm \theta/2)^{-1},
\]

(4)

where \( R(\theta) = e^{-i\theta \tau \tau} \) is the rotation operator and \( \sigma_i = (\sigma_x, \sigma_y) \) is the Pauli matrix for sublattice degree of freedom. \( K_i \) are the corners of Moiré Brouillon zone (BZ) as shown in Fig. 2(a). The interlayer coupling is described by

\[
T_r(r) = T^{(0)}_r + e^{-i b \cdot r} T^{(1)}_r + e^{-i b' \cdot r} T^{(2)}_r,
\]

(5)

where \( b \) (\( b' \)) are the primitive reciprocal lattice vectors of the TBG [see Fig. 2(a)]. Due to the lattice relaxation, the interlayer tunneling strength is different for the AA and AB stacking regions, where \( w_{AA} = 79.7 \) meV and \( w_{AB} = 97.5 \) meV [12]. The Fermi velocity is \( v_F/a = 2.1354 \) eV where \( a = 0.246 \) nm is the lattice constant of single layer graphene. \( \mathcal{H}_{k,\pm} \) of the \( \pm K \) valleys are related by TRS. In the experiment, the TBG with \( \theta = 1.15^\circ \) is aligned with a hBN substrate that induces a sublattice potential onto the first layer graphene as \( h_{1,r} \rightarrow h_{1,r} + \Delta_{LB} \sigma_z/2 \) [7]. We choose the sublattice potential \( \Delta_{LB} = 20 \) meV in this study. The low-energy bands of the TBG are shown in Fig. 2(b), where the energy bands from the \( \pm K \) valley are represented by solid and dashed lines, respectively. There are eight flat bands including the degenerate spin degree of freedom around zero energy. The lattice relaxation isolates the flat bands from remote bands and the sublattice potential induced by the substrate gaps the upper flat bands from the lower flat bands.
Here we focus on the 3/4 filling of the upper flat bands [the
red bands in Fig. 2(b)], where the QAHE emerges [6, 7]. By
projecting the Coulomb interaction onto the upper flat bands,
the Hamiltonian of TBG becomes
\[ \mathcal{H}_0 = \sum_{k,r,s} \left( \varepsilon_{k,r} - \mu \right) c_{k,r,s}^\dagger c_{k,r,s} + \frac{1}{2A} \sum_{q} \rho(q) V(q) \rho(-q), \]  
(6)
where \( \varepsilon_{k,r} \) denotes the spin-degenerate bare dispersion for
the upper flat bands given by \( \mathcal{H}_{k,r} \left| \psi_{k,r} \right\rangle = \varepsilon_{k,r} \left| \psi_{k,r} \right\rangle \), \( \mu \) is the chemical potential, and \( A \) is the area of the system.
\( s = \uparrow \) or \( \downarrow \) represents the spin degree of freedom. \( \rho(q) = \sum_{k,k',r,s} \left\langle \psi_{k,r} | e^{i\mathbf{q} \cdot \mathbf{r}} | \psi_{k',r} \right\rangle c_{k,r,s}^\dagger c_{k',r,s} \) is the density operator, and \( V(q) = e^2 \tanh(|q|d)/2e|q| \) is the screened Coulomb potential, where \( \varepsilon \) is the dielectric constant and \( d \) is the distance between
TBG and metallic gates. Here we take \( d = 40 \text{ nm} \) following
the experiment [7].

With the self-consistent Hartree-Fock approximation, the
dispersion of the upper flat bands are corrected by the
Coulomb interaction as
\[ E_{k,r,s} = \varepsilon_{k,r} - \mu \]
\[ + \frac{1}{A} \sum_{q,k',r,s'} \left\langle \psi_{k,r} | e^{i\mathbf{q} \cdot \mathbf{r}} | \psi_{k',r} \right\rangle V(q) \left\langle \psi_{k',r,s'} | e^{-i\mathbf{q} \cdot \mathbf{r}} | \psi_{k,r,s} \right\rangle f(E_{k',r',s'}) \]
\[ - \frac{1}{A} \sum_{q,k'} \left\langle \psi_{k,r} | e^{i\mathbf{q} \cdot \mathbf{r}} | \psi_{k',r,s} \right\rangle V(q) \left\langle \psi_{k',r,s} | e^{-i\mathbf{q} \cdot \mathbf{r}} | \psi_{k,r,s} \right\rangle f(E_{k',r,s}) \]  
(7)
By solving the self-consistent equation, we get four degener-
ate valley-spin-polarized ground states at 3/4 filling. In the
ground states, one of the four flat bands is above the other
three that results in an insulating state. The experimentally
measured energy gap is \( \Delta/k_B \approx 27K \) [7], that can be repro-
cuced by our approach with \( \varepsilon = 58.6\varepsilon_0 \), where \( \varepsilon_0 \) is the vac-
umum permittivity.

We define \( n_r = \sum_s (n_{r,s} - n_{-r,s}) \) and \( n_s = \sum_r (n_{r,s} - n_{-r,s}) \) to
characterize the valley and spin polarization, respectively. \( n_{r,s} \)
the occupation number of the flat band with the valley-spin
indices \( (r,s) \). We consider a symmetry-breaking state with
\( n_r = n_s = 1 \) at zero temperature, as shown in Fig. 3. The four
flat bands have nonzero Chern numbers \( C = \pm 1 \), which are op-
posite for \( \pm K \) valleys as a consequence of TRS. Therefore, the
VP at 3/4 filling ensures the QAHE, which is manifested by
the quantized Hall conductance \( G_{xy} = \pm e^2/h \) in experiments
[7]. Because the Hamiltonian Eq. (6) has the spin SU(2) sym-
metry and valley U(1) symmetry, the long-range spin order is
destroyed by thermal fluctuations according to the Mermin-
Wagner theorem, while the long-range valley order is allowed
[78]. As temperature increases, the mean field \( n_r, n_s \), and \( G_{xy} \)
vanish together above a critical temperature \( T_c = 14.1K. \) [see Fig. 2(c)]

We then study the effect of current and magnetic field on
the electron dispersion. The current is introduced through the
Lagrange multiplier as before and the Hamiltonian becomes
\[ \mathcal{H} = \mathcal{H}_0 - \sum_{k,r,s} \left( \lambda_x \partial_{k,r} \varepsilon_{k,r} + \mu B s_z + M_{k,r} B \right) c_{k,r,s}^\dagger c_{k,r,s} + \frac{1}{2A} \sum_{q} \rho(q) V(q) \rho(-q), \]  
(8)
where \( \lambda_x \) denotes the Lagrange multiplier for the current with
spin \( s \) and along the \( \alpha \) direction, \( \lambda_s = \pm 1/2 \) is the spin quantum number, and \( B \) represents the perpendicular magnetic field.
\( M_{k,r} \) is the \( k \)-resolved orbital magnetization
\[ M_{k,r} = \frac{i}{2\hbar} \left\langle \nabla_k | \psi_{k,r} \right\rangle \times (2\mu_0 - \varepsilon_{k,r} - \mathcal{H}_{k,r}) \left| \nabla_k | \psi_{k,r} \right\rangle \cdot \hat{e}_z, \]  
(9)
that is generated by the self rotation of Bloch states and in-
cludes also the boundary contribution [79–82]. Here \( \mu_0 \) is the chemical potential for the 3/4 filling of the bare dispersion, and \( \hat{e}_z \) is a unit vector perpendicular to TBG. The orbital mag-
netic moment is spin-degenerate and valley-contrasting. Thus
the symmetry breaking state at 3/4 filling has the polarized or-
bital magnetic moment, and exhibits orbital ferromagnetism.
To be concrete, we focus on the current along the \( x \) direction
and take the valley-spin-polarized ground state in Fig.3 as
the initial state. Because the flat bands with spin down
are fully filled, the current can only be conducted by the spin
up bands. Hence we fix \( \lambda_x = 0 \) in the following. In this
case, the Hattree-Fock approximation yields the self-consistent
equation in the same form as Eq. (7) but with the replacement
\( \varepsilon_{k,r} \rightarrow \varepsilon_{k,r} - \lambda_x \partial_{k,r} \varepsilon_{k,r} - \mu B s_z B - M_{k,r} B \)

We first focus on the effect of current on VP and Hall con-
ductance in the absence of magnetic field \( B = 0 \). The analysis
in Eq. (2) reveals that the switching of VP by currents is of
the first order phase transition, and therefore hysteresis is ex-
pected. We sweep the current by changing \( \lambda \) continuously
and solve the corresponding self-consistent equations. The results
of \( n_r, n_s \), and \( G_{xy} \) at zero temperature as a function of \( \lambda \) are
displayed in Figs. 4(a) and 4(b), where the hysteresis behavior of
VP and Hall conductance appears as expected. Start with the
initial state in Fig. 3 with \( n_r = 1 \) and \( G_{xy} = -\varepsilon^2/h \), and then

FIG. 3. The four upper flat bands labeled by the valley-spin in-
dex \((r,s)\) are shown for the insulating valley-spin-polarized state
obtained from the Hattree-Fock approximation. The Chern numbers of
the flat bands are displayed. The black hexagon encloses the moiré
BZ.
increase $\lambda$, both $n_\tau$ and $G_{xy}$ change sign through a sharp jump at a critical $\lambda_c$. If we remove the current (by setting $\lambda = 0$) after the jump, the system then relaxes into an insulating state with $n_\tau = -1$ and $G_{xy} = e^2/h$, contrary to the initial state [77]. There is no switching of spin polarization because the bands with down spin remain fully occupied.

The current through the TBG is $J = \frac{e}{M} \sum_{k,\tau} \partial \epsilon_{k,\tau}/\partial k \langle f(E_{k,\tau,\lambda}) \rangle$, where $h = 0.6$ nm is the thickness of TBG. In Fig. 4(c), we show $J$ as a function of $\lambda$. For a small $\lambda$, the system remains insulating with $J = 0$, that corresponds to the quantized plateau of $n_\tau = \pm 1$ in Fig. 4(a) and $G_{xy} = \mp e^2/h$ in Fig. 4(b). As $\lambda$ increases above a threshold value, the system becomes metallic and there appears hysteresis of $J$ upon increasing and decreasing $\lambda$. Eventually, the current saturates at large $\lambda$. Our results demonstrate that the VP and topology can be switched by a current larger than the critical current $J_c$ [77].

Here $\lambda_c$ and $J_c$ are determined by the energy barrier separating different valley-polarized states. As $T$ increases, the energy barrier decreases, and hence $\lambda_c$ and $J_c$ decreases with $T$. The dependence of $\lambda_c$ and $J_c$ on $T$ are shown in Fig. 4(d).

The energy barrier is affected by the magnetic field through the Zeeman coupling with spin and orbital magnetic moments in Eq. (8). In TBG, the orbital magnetic moment is dominant and is valley-contrasting. The energy gap of the insulating valley-spin-polarized state scales linearly with $B$. To study the dependence of $\lambda_c$ on $B$, we show $G_{xy}$ as a function of $\lambda$ for different $B$ at zero temperature in Fig. 5. The switching of VP and topology at $\lambda_c$ is featured by the jump of $G_{xy}$ where $G_{xy}$ changes sign. $\lambda_c$ indicated by the black line in Fig. 5 changes almost linearly with $B$, which is similar to the gap.

Discussion and summary.— In our approach, only the bulk current contribution is considered. The contribution from edge current relies on the asymmetry of sample edges and can also lift the degeneracy of different valley-spin-polarized ground states. Nevertheless, to switch the VP and topology of TBG, it requires to overcome the bulk energy barrier, which grows linearly with the sample volume. Therefore, the edge current contribution becomes negligible for a large system size [77]. For a small device, it is possible that both the edge and bulk contribution cooperate in the switching process. As in the classical first order phase transition, the transition from one metastable state to the other more stable state occurs via domain nucleation. Our critical current defined when the energy barrier vanishes completely corresponds to the superheating/supercooling field.

In summary, we propose a mechanism of the switching of VP and topology in TBG by bulk electric currents. The current causes the redistribution of electron occupation, that lifts the degeneracy and even overcomes the energy barrier between different valley-spin-polarized states through the Coulomb interaction. Our theory can be generalized to other strongly correlated two dimensional materials with valley polarization.

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FIG. 4. (a)-(c) Order parameters $n_\tau$ and $n_\tau$, Hall conductance $G_{xy}$, and current $J$ are plotted as a function of $\lambda$, respectively. The arrows indicate the direction of evolution of the hystereses. (d) $J_c$ as a function of $T$. The inset shows $\lambda_c$ vs $\sqrt{T_c - T}$.

FIG. 5. Density plot of the Hall conductance $G_{xy}$ as a function of $\lambda$ for different $B$. The black line corresponds to the critical $\lambda_c$ as a function of $B$. 

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Supplemental Material: Switching of valley polarization and topology in twisted bilayer graphene by electric currents

EXPLICIT CALCULATION OF THE TOY MODEL

Here we provide explicit calculation of the toy model using the dispersion relation \( \varepsilon_{k_\pm} = \cos(2k + \pi) - \mu \) for \(-\pi \leq k \leq -\pi/2\) and \( \varepsilon_{k_-} = \cos(\frac{\pi}{2}(k + \pi/2)) - \mu \) for \(-\pi/2 \leq k \leq \pi\), and \( \varepsilon_{k_+} = \varepsilon_{-k_-} \). The results for \( \Delta_0 \) and current \( J \) as a function of \( \lambda \) are displayed in Fig. S1. The switching of VP occurs only for a positive \( \lambda \) because we start with the initial state shown in Fig. 1(a). For \( V = 3 \) and \( \mu = 1.5 \), the switching of VP (featured by the crossing of \( \Delta_0 \)) happens simultaneously with the insulator-to-metal transition, as shown in Figs. S1(a) and S1(b). When the gap is increased by choosing \( V = 5 \), the system first becomes a metal, and then switches the VP upon further increasing \( \lambda \), see Figs. S1(c) and S1(d).

The free energy of the system is

\[
\mathcal{F} = -k_B T \sum_{k, \tau} \ln [1 + \exp(-\varepsilon_{k, \tau}/k_B T)] - \frac{\Delta_+ \Delta_-}{V}.
\]  

(S1)

When the VP order parameter \( \delta = (\Delta_+ - \Delta_-)/2 \) is small, we can expand \( \mathcal{F} \) in terms of \( \delta \) as shown in Eq. (2). The coefficients are

\[
\mathcal{F}_0 = -k_B T \sum_{k, \tau} \ln [1 + \exp(-\varepsilon_{k, \tau} - \lambda \partial_t \varepsilon_{k, \tau} + \rho)/k_B T] - \frac{\rho^2}{V},
\]

(S2)

\[
\alpha_1 = -\sum_{k, \tau} \frac{\tau}{1 + \exp(\delta)},
\]

(S3)

\[
\alpha_2 = -\frac{1}{2} \sum_{k, \tau} \frac{1}{2k_B T + 2k_B T \cosh(\delta)} + \frac{1}{V},
\]

(S4)

\[
\alpha_3 = -\frac{1}{6} \sum_{k, \tau} \frac{2 \cosh^3(\delta) \sinh^4(\delta/2)}{k_B^3 T^2},
\]

(S5)

\[
\alpha_4 = -\frac{1}{24} \sum_{k, \tau} \frac{[-2 + \cosh(\delta)] \sinh^4(\delta/2)}{8k_B^3 T^3},
\]

(S6)

with \( \rho = (\Delta_+ + \Delta_-)/2 \) and \( g = (\varepsilon_{k, \tau} - \lambda \partial_t \varepsilon_{k, \tau} + \rho)/k_B T \). In the absence of current \( \lambda = 0 \), \( \alpha_1 = \alpha_2 = 0 \) as required by the \( Z_2 \) symmetry associated with the VP. One can show explicitly that \( \alpha_1(\lambda) = -\alpha_1(-\lambda) \) and \( \alpha_2(\lambda) = \alpha_2(-\lambda) \).

The free energy as a function of \( \delta \) for different \( \lambda \) and with \( V = 3 \) and \( \mu = 1.5 \) is shown in Fig. S2. The presence of a nonzero \( \lambda \) lifts the degeneracy between the two valley-polarized states. As a consequence, one valley-polarized state becomes metastable, while the other becomes the global ground state. Upon further increasing or decreasing \( \lambda \), the metastable state becomes unstable and the switching of VP happens.

FIG. S1. (a) and (c) \( \Delta_+ \) as a function of \( \lambda \) for \( V = 3 \) and 5. The corresponding current \( J \) are shown in (b) and (d), respectively.

FIG. S2. Free energy as a function of \( \delta \) for different \( \lambda \) and \( V = 3 \).

SWITCHING OF VALLEY POLARIZATION BY A PULSED CURRENT

Here we show how the valley polarization is switched by a pulsed current. We assume that the duration of the pulse is much longer than the time scale of electronic relaxation, such that electrons are in a stationary state under the current. We consider three pulsed currents whose peak values are \( J_0 = 3.2 \times 10^6 \) A/cm², \( 1.4 \times 10^6 \) A/cm², and \( -3.2 \times 10^6 \) A/cm², respectively, as shown in the upper panel of Fig. S3. The critical current for the switching of VP in this case is \( J_c \approx 3 \times 10^6 \) A/cm² at zero temperature. The corresponding Hall conductance for the three currents are shown in the lower panel of Fig. S3. Apparently, after the pulse passing, only the Hall conductance for \( J_0 = 3.2 \times 10^6 \) A/cm² > \( J_c \) is flipped from \( -e^2/h \) to \( e^2/h \), that indicates the switching of VP and Chern number. For \( J_0 = 1.4 \times 10^6 \) A/cm² < \( J_c \) and for \( J_0 = -3.2 \times 10^6 \) A/cm² in the opposite direction, there is no switching of valley.
polarization as expected.

FIG. S3. Upper panel: three pulse currents whose peak values are $J_0 = 3.2 \times 10^6$ A/cm$^2$, $1.4 \times 10^6$ A/cm$^2$, and $-3.2 \times 10^6$ A/cm$^2$, respectively. Lower panel: the corresponding Hall conductance for the three pulsed currents.

FIG. S4. (a) Phase diagram of the Haldane model. The Chern numbers of different phases are shown. (b) Free energy of the Haldane model along the straight line in (a) at half filling and zero temperature. (c) A strip of honeycomb lattice with width $W$ and zigzag edges. (d) Energy spectrum of the Haldane model on the honeycomb lattice strip.

ESTIMATE THE EDGE CURRENT CONTRIBUTION IN OVERCOMING THE ENERGY BARRIER

Here we estimate the edge current contribution in overcoming the energy barrier between different ground states with opposite VP. Because of the large unit cell size at a small twist angle, it is hard to access the edge states of the TBG. Instead, we use the edge states of the Haldane model to mimic that of TBG. Explicitly, we consider the Haldane model described by the Hamiltonian $[S1]$

$$\mathcal{H}_H = 2t_2 \cos \phi \sum_{i=1}^{3} \cos(k \cdot b_i)\sigma_0$$

$$+ t_1 \sum_{i=1}^{3} \left[ \cos(k \cdot a_i)\sigma_x + \sin(k \cdot a_i)\sigma_y \right]$$

$$+ \left[ M - 2t_2 \sin \phi \sum_{i=1}^{3} \sin(k \cdot b_i) \right] \sigma_z,$$

on a honeycomb lattice whose lattice constant is set to unity. Here $t_n$ is the hopping energy between the $n$th nearest neighboring (NN) lattice sites, $\sigma_0$ is the $2 \times 2$ identity matrix, and $\sigma_{x,y,z}$ are the Pauli matrices acting on the sublattice degree of freedom. $a_{1,2,3}$ are the three vectors connecting NN lattice sites, and $b_{1,2,3}$ are primitive lattice vectors related by 3-fold rotation. $M$ is the sublattice potential breaking the inversion symmetry, and $\phi$ is the phase factor associated with the second NN hopping. The topological phase of the model is determined by $M$ and $\phi$, as shown by the phase diagram in Fig. S4(a). By choosing $t_2 = 0.07t_1$ and $M = 3t_2$, we show the free energy as a function of $\phi$ of the Haldane model at half filling and zero temperature in Fig. S4(b). Apparently, there are two degenerate energy minimum with opposite Chern number $C = \pm 1$ at $\phi = \pm \pi/2$, that is similar to the TBG. Therefore, by taking $\phi$ as an internal order parameter (playing the same role of $\delta$), we can use the Haldane model to mimic the free energy profile and topology of the TBG.

To study the edge current contribution, we consider a stripe of honeycomb lattice with zigzag edges along the $x$ direction, as shown in Fig. S4(c). The edge asymmetry can be introduced by adding an onsite potential $V$ to the upper edge, which is highlighted in Fig. S4(c). For $V = 2t_2$, the energy spectrum of the strip with width $W = 25 \sqrt{3}$ (that contains 50 unit cells in the cross section) is shown in Fig. S4(d). Here the edge states traversing the bulk energy gap are marked by red color. In the presence of a current along the strip, the system is described by $\mathcal{H}_H - \sum_n \lambda \delta_{\lambda_n} \epsilon_{\lambda_n}$, where $\epsilon_{\lambda_n}$ represents the bare dispersion of the strip [see Fig. S4(d)] and $n$ is the band index. The current lifts the degeneracy of the two energy minima at $\phi = \pm \pi/2$, as shown in Fig. S5(a). The energy barrier between the two minima vanishes at a critical $\lambda$, i.e. $\lambda_c \approx 1.25$ in this case, that corresponds to the switching of VP and topology in TBG. To show the edge current contribution in overcoming the energy barrier, we study the electron occupation in the conductance and valence bands at $\lambda = \lambda_c$. As a reference, we show the electron occupation in the absence of current for $\lambda = 0$ in Fig. S5(b), where the conductance (valence) bands are fully empty (occupied) at zero temperature. The two branches of counter-propagating edge states are partially filled. In the presence of the critical current for $\lambda = \lambda_c$, the electron occupation is shown in Fig. S5(c). Apparently, the branch of edge states propagating along the current direction is fully filled, while the other branch is empty. Moreover, there are electrons pumped from the valence bands to conductance bands as in the TBG. Namely, the edge current alone is not enough to overcome the energy barrier in this case. The
pumped electrons occupy the bottom of conduction bands where the density of states is high. Therefore, the bulk current plays a dominant role in overcoming the energy barrier for a large system. The edge current can be important if the sample size is small since the energy barrier grows linearly in the sample volume. However, in experiments [6,7], the device is in micron size, that contains hundreds of moiré unit cells (considering the lattice constant of the small-angle TBG is of the order of 10 nm) in the cross section. This fact motivates us to study the switching of VP and topology in TBG by electric currents in the bulk.

FIG. S5. (a) Free energy of the Haldane model as a function of \( \phi \) for different \( \lambda \). (b) and (c) Electron occupation for \( \lambda = 0 \) and for \( \lambda = 1.25 \), respectively.

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