SINGULAR RIESZ MEASURES ON SYMMETRIC CONES

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Abstract. A fundamental theorem due to Gindikin [Russian Math. Surveys, 29 (1964), 1-89] says that the generalized power \(\Delta_s(-\theta^{-1})\) defined on a symmetric cone is the Laplace transform of a positive measure \(R_s\) if and only if \(s\) is in a given subset \(\Xi\) of \(\mathbb{R}^r\), where \(r\) is the rank of the cone. When \(s\) is in a well defined part of \(\Xi\), the measure \(R_s\) is absolutely continuous with respect to Lebesgue measure and has a known expression. For the other elements \(s\) of \(\Xi\), the measure \(R_s\) is concentrated on the boundary of the cone and it has never been explicitly determined. The aim of the present paper is to give an explicit description of the measure \(R_s\) for all \(s\) in \(\Xi\). The work is motivated by the importance of these measures in probability theory and in statistics since they represent a generalization of the class of measures generating the famous Wishart probability distributions.

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