Probing NMSSM Scenarios with Minimal Fine-Tuning by Searching for Decays of the Upsilon to a Light CP-Odd Higgs Boson

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Completely natural electroweak symmetry breaking is easily achieved in supersymmetric models if there is a SM-like Higgs boson, \( h \), with \( m_h \lesssim 100 \text{ GeV} \). In the minimal supersymmetric model, such an \( h \) decays mainly to \( bb \) and is ruled out by LEP constraints. However, if the MSSM Higgs sector is expanded so that \( h \) decays mainly to still lighter Higgs bosons, e.g. \( h \to aa \), with \( \text{Br}(h \to aa) > 0.7 \), and if \( m_a < 2m_h \), then the LEP constraints are satisfied. In this letter, we show that in the next-to-minimal supersymmetric model the above \( h \) and \( a \) parameters (for the lightest CP-even and CP-odd Higgs bosons, respectively) imply a lower bound on \( \text{Br}(\Upsilon \to \gamma a) \) that dedicated runs at present (and future) \( B \) factories can explore.

Low energy supersymmetry remains one of the most attractive solutions to the naturalness / hierarchy problem of the Standard Model (SM). However, the minimal supersymmetric model (MSSM), containing exactly two Higgs doublets, suffers from the “\( \mu \) problem” and requires rather special parameter choices in order that the light Higgs mass is above LEP limits without electroweak symmetry breaking being “fine-tuned”, i.e. highly sensitive to supersymmetry-breaking parameters chosen at the grand-unification scale. Both problems are easily solved by adding Higgs (super) fields to the MSSM. For generic SUSY parameters well-below the TeV scale, fine-tuning is absent \[1\] and a SM-like \( h \) is predicted with \( m_h \lesssim 100 \text{ GeV} \). Such an \( h \) can avoid LEP limits on the tightly constrained \( e^+e^- \to Z + b\bar{b} \) channel if \( \text{Br}(h \to b\bar{b}) \) is small by virtue of large \( \text{Br}(h \to aa) \), where \( a \) is a new light (typically CP-odd) Higgs boson, and \( m_a < 2m_h \) so that \( a \to b\bar{b} \) is forbidden \[2\]. The perfect place to search for such an \( a \) is in Upsilon decays, \( \Upsilon \to \gamma a \). The simplest MSSM extension, the next-to-minimal supersymmetric model (NMSSM), naturally predicts that the lightest \( h \) and \( a \), \( h_1 \) and \( a_1 \), have all the right features \[1, 2, 3, 4, 5\]. In this letter, we show that large \( \text{Br}(h_1 \to a_1a_1) \) implies, at fixed \( m_{a_1} \), a lower bound on \( \text{Br}(\Upsilon \to \gamma a_1) \) (from now on, \( \Upsilon \) is the 1S resonance unless otherwise stated) that is typically within reach of present and future \( B \) factories.

In the NMSSM, a light \( a_1 \) with substantial \( \text{Br}(h_1 \to a_1a_1) \) is a very natural possibility for \( m_Z \)-scale soft parameters developed by renormalization group running starting from \( U(1)_R \) symmetric GUT-scale soft parameters \[3\]. (See also \[3, 9\] for discussions of the light \( a_1 \) scenario.) The fine-tuning-preferred \( m_{h_1} \sim 100 \text{ GeV} \) (for \( \tan \beta \gtrsim \text{few} \)) gives perfect consistency with precision electroweak data and the reduced \( \text{Br}(h_1 \to b\bar{b}) \sim 0.09 - 0.15 \) explains the \( \sim 2.3\sigma \) excess at LEP in the \( Zb\bar{b} \) channel at \( M_{\gamma\gamma} \sim 100 \text{ GeV} \). The motivation for this scenario is thus very strong.

Hadron collider probes of the NMSSM Higgs sector are problematical. The \( h_1 \to a_1a_1 \to 4\tau \text{ (2m}_{\tau} < m_{a_1} < 2m_{h_1}) \) or 4 jets (\( m_{a_1} < 2m_{\tau} \)) signal is a very difficult one at the Tevatron and very possibly at the LHC \[8, 9, 11\]. Higgs discovery or, at the very least, certification of a marginal LHC Higgs signal will require a linear \( e^+e^- \) collider (ILC). Direct production and detection of the \( a_1 \) may be impossible at both the LHC and ILC because it is rather singlet in nature. In this letter, we show that by increasing sensitivity to \( \text{Br}(\Upsilon \to \gamma a_1) \) by one to three orders of magnitude (the exact requirement depends on \( m_{a_1} \) and \( \tan \beta \)), there is a good chance of detecting the \( a_1 \). This constitutes a significant opportunity for current \( B \) factories and a major motivation for new super-\( B \) factories. Even with ILC \( h_1 \to a_1a_1 \) data, measurement of \( \text{Br}(\Upsilon \to \gamma a_1) \) and \( a_1 \) decays would provide extremely valuable complementary information.

As compared to the three independent parameters needed in the MSSM context (often chosen as \( \mu, \tan \beta \) and \( M_A \)), the Higgs sector of the NMSSM is described by the six parameters

\[
\lambda, \kappa, A_\lambda, A_\kappa, \tan \beta, \mu_{\text{eff}},
\]

where \( \mu_{\text{eff}} = \lambda(S) = \lambda s \) is the effective \( \mu \)-term generated from the \( \lambda \tilde{S}\tilde{H}_u\tilde{H}_d \) part of the superpotential, \( \lambda A_\lambda S H_u H_d \) is the associated soft-SUSY-breaking scalar potential component, and \( \kappa \) and \( \kappa A_\kappa \) appear in the \( \frac{1}{2} \kappa S^3 \) and \( \frac{1}{6} \kappa A_\kappa S^3 \) terms in the superpotential and associated soft-supersymmetry-breaking potential. In addition, values must be input for the soft SUSY-breaking masses that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths. Our computations for branching ratios and so forth employ NMHDECAY \[12\]. An important ingredient for the results of this paper is the non-singlet fraction of the \( a_1 \) defined by \( \cos \theta_A \) in

\[
a_1 = \cos \theta_A A_{\text{MSSM}} + \sin \theta_A A_S,
\]

where \( A_S \) is the CP-odd Higgs boson contained in the unmixed \( S \) complex scalar field. The coupling of \( a_1 \) to \( \tau^+\tau^- \) and \( b\bar{b} \) is then \( \propto \tan \beta \cos \theta_A \cdot \cos \theta_A \) itself has some \( \tan \beta \) dependence with the net result that \( \tan \beta \cos \theta_A \) increases modestly with increasing \( \tan \beta \).
In [1, 3, 4], we scanned over the NMSSM parameter space holding $\tan \beta$ and the gaugino masses $M_{1,2,3}(m_Z)$ fixed, searching for choices that minimized a numerical measure, $F$, of EWSB fine-tuning, i.e. of how precisely the GUT-scale soft-SUSY-breaking parameters must be chosen to obtain the observed value of $m_Z$ after RG evolution. For $F < 15$, fine-tuning is no worse than 7%, and we regard this as equivalent to absence of significant fine-tuning. For the sample values of $\tan \beta = 10$ and $M_{1,2,3} = 100, 200, 300$ GeV ($F$ only depends significantly on $M_3$), to achieve the lowest $F$ values ($F \sim 5 - 6$), the $h_1$ must be fairly SM-like and $m_{h_1} \sim 100$ GeV is required; this is only consistent with LEP constraints for scenarios in which $\text{Br}(h_1 \rightarrow a_1a_1)$ is large and $m_{a_1} < 2m_h$.\footnote{We should note that the precise location of the minimum in $F$ shifts slightly as $\tan \beta$ is varied. For example, at $\tan \beta = 3$ ($\tan \beta = 50$) the minimum is at roughly 92 GeV (102 GeV). However, for these cases the minimum value of $F$ is only very modestly higher at $m_{h_1} \sim 100$ GeV, the LEP excess location.}

In contrast, for a scalar Higgs, bound state corrections give a huge enhancement, rising from a small percentage at $m_{a_1} \sim 100$ GeV, the LEP excess location.\footnote{Also, as one approaches the $U(1)_R$, $A_\kappa \rightarrow 0$ symmetry limit, large $\text{Br}(h_1 \rightarrow a_1a_1)$ is not possible.}

Aside from EWSB fine-tuning, there is a question of whether fine-tuning is needed to achieve large $\text{Br}(h_1 \rightarrow a_1a_1)$ and $m_{a_1} < 2m_h$ when $F < 15$. This was discussed in [3]. The level of such fine-tuning is determined mostly by whether $A_\lambda$ and $A_\kappa$ need to be fine-tuned. (For given $s$ and $\tan \beta$, $\text{Br}(h_1 \rightarrow a_1a_1)$ and $m_{a_1}$ depend significantly only on $\lambda$, $\kappa$, $A_\lambda$ and $A_\kappa$; all other SUSY parameters have only a tiny influence.) Since specific soft-SUSY-breaking scenarios can evade the issue of tuning $A_\kappa$ and $A_\lambda$ altogether, in this study we do not impose a limit on the measures of $A_\lambda$, $A_\kappa$ fine-tuning discussed in [3]. However, it is worth noting that we find that $A_\lambda$, $A_\kappa$ fine-tuning can easily be avoided if $m_{a_1} \gtrsim 2m_h$ and $\cos \theta_A$ is small and negative, e.g. near $\cos \theta_A \sim -0.05$ if $\tan \beta = 10$. In some models, the simplest measures of $A_\lambda$, $A_\kappa$ fine-tuning are much lower from the preferred $\cos \theta_A$ region and / or at substantially lower $m_{a_1}$ values.

We now turn to $\Upsilon \rightarrow \gamma a_1$. We have computed the branching ratio for this decay based on Eqs. (3.54), (3.58) and (3.60) of [13] (which gives all appropriate references). Eq. (3.54) gives the result based on the non-relativistic quarkonium model; Eqs. (3.58) and (3.60) give the procedures for including QCD corrections and relativistic corrections, respectively. Both cause significant suppression with respect to the non-relativistic quarkonium result. In addition, there are bound state corrections. These give a modest enhancement, rising from a small percentage at small $m_{a_1}$ to about 20% at $m_{a_1} = 9.2$ GeV (see the references in [13]).\footnote{For $m_{a_1} \in [m_{a_1} - 2\Gamma_{\eta_b}, m_{a_1} + 2\Gamma_{\eta_b}]$, where $m_{a_1} \sim M_T - 50$ MeV and $\Gamma_{\eta_b} \sim 50$ MeV, the $a_1$ mixes significantly with the $\eta_b$, giving rise to a huge enhancement of $\text{Br}(\Upsilon \rightarrow \gamma a_1)$. We have chosen not to plot results for $m_{a_1} > 9.2$ GeV since we think that the old theoretical results in this region require further refinement. In Fig. 1 we present results for $\text{Br}(\Upsilon \rightarrow \gamma a_1)$ that are consistent with existing experimental limits\footnote{In contrast, for a scalar Higgs, bound state corrections give a very large suppression at higher Higgs masses near $M_T$.} in two cases: (a) using a scan over $A_\lambda$, $A_\kappa$ values holding $\mu_{ab}(m_Z) = 150$ GeV and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV fixed (in this scan, identical to that described in Ref. [3], $\lambda$ and $\kappa$ are also scanned over and all other SUSY-breaking parameters are fixed at 300 GeV – results are insensitive to this choice and, therefore, representative of the whole parameter space); (b) for the $F < 15$ points found in the NMSSM parameter scan described earlier. In both cases, $\tau_\Upsilon < m_{a_1} < 2m_{a_1}$ region. The third gives limits on $\text{Br}(\Upsilon \rightarrow \gamma X)\text{Br}(X \rightarrow \tau^+\tau^-)$ that eliminate $2m_{a_1} < m_{a_1} < 8.8$ GeV points with too high $\text{Br}(\Upsilon \rightarrow \gamma a_1)$ (for $m_{a_1} > 2m_{a_1}$, $\text{Br}(a_1 \rightarrow \tau^+\tau^-) \sim 0.9$). Since the inclusive photon spectrum from $\Upsilon$ decays falls as $E_\gamma$ increases, the strongest constraints are obtained for small $m_{a_1}$.}.

Fig. 1: $\text{Br}(\Upsilon \rightarrow \gamma a_1)$ for NMSSM scenarios with various ranges for $m_{a_1}$: dark grey (blue) = $m_{a_1} < 2m_h$; medium grey (red) = $2m_h < m_{a_1} < 7.5$ GeV; light grey (green) = $7.5$ GeV $< m_{a_1} < 8.8$ GeV; and black = $8.8$ GeV $< m_{a_1} < 9.2$ GeV. The plots are for $\tan \beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. The left plot comes from the $A_\lambda$, $A_\kappa$ scan described in the text, holding $\mu_{ab}(m_Z) = 150$ GeV fixed. The right plot shows results for $F < 15$ scenarios with $m_{a_1} < 9.2$ GeV found in a general scan over all NMSSM parameters holding $\tan \beta$ and $M_{1,2,3}$ fixed as stated.
all points plotted pass all NMHDECAY constraints — all points have \( m_1 \sim 100 \text{ GeV} \), but avoid LEP constraints by virtue of \( \text{Br}(h_1 \to a_1 a_1) > 0.7 \) and \( m_{a_1} < 2 m_h \). For both plots, we divide results into four \( m_{a_1} \) regions: \( m_{a_1} < 2 m_{\tau} \), \( 2 m_{\tau} < m_{a_1} < 7.5 \text{ GeV} \), \( 7.5 \text{ GeV} < m_{a_1} < 8.8 \text{ GeV} \) and \( 8.8 \text{ GeV} < m_{a_1} < 9.2 \text{ GeV} \). Fig. 1 makes clear that \( \text{Br}(\Upsilon \to \gamma a_1) \) is mainly controlled by the non-singlet fraction of the \( a_1 \) and by \( m_{a_1} \). The only difference between the (a) and (b) plots is that \( F < 15 \) restricts the range of \( \cos \theta_A \) to smaller magnitudes (implying smaller \( \text{Br}(\Upsilon \to \gamma a_1) \)) and narrows the \( m_{a_1} \) bands. As seen in the figure, the \( \cos \theta_A \sim -0.05 \), \( m_{a_1} > 2 m_{\tau} \) scenarios that can have no \( A_\lambda, A_\kappa \) tuning have \( \text{Br}(\Upsilon \to \gamma a_1) < \text{few} \times 10^{-5} \). For general \( \cos \theta_A \) and \( m_{a_1} \), values of \( \text{Br}(\Upsilon \to \gamma a_1) \) up to \( \sim 10^{-3} (5 \times 10^{-3}) \) are possible for \( F < 15 \) (in the general \( A_\lambda, A_\kappa \) scan). In Fig. 1 points with \( \text{Br}(\Upsilon \to \gamma a_1) \gtrsim \text{few} \times 10^{-4} \) (depending on \( m_{a_1} \)) are not present, having been eliminated by 90% CL limits from existing experiments. The surviving points with \( m_{a_1} < 9.2 \) GeV can be mostly probed if future running, upgrades and facilities are designed so that \( \text{Br}(\Upsilon \to \gamma a_1) \sim 10^{-7} \) can be probed. As stated earlier, predictions at higher \( m_{a_1} \) are rather uncertain, but obviously \( \text{Br}(\Upsilon \to \gamma a_1) \to 0 \) for \( m_{a_1} \to M_Y \). To access higher \( m_{a_1} \) (but \( m_{a_1} < 2 m_h \)), \( \Upsilon(2S) \to \gamma a_1 \) and \( \Upsilon(3S) \to \gamma a_1 \) can be employed; computation of the branching ratios requires careful attention to \( a_1 - \eta_1 \) mixing, which can lead to even larger branching ratios than for the \( \Upsilon \) if \( m_{a_1} \sim m_{\eta_1} \). Results from the \( A_\lambda, A_\kappa \) scan with \( \mu_{\text{eff}} = 150 \text{ GeV} \) and \( M_{1,2,3} = 100, 200, 300 \text{ GeV} \) are given in the cases of \( \tan \beta = 3 \) and \( \tan \beta = 50 \) in Fig. 2. Note that all most all \( \tan \beta = 3 \) points that pass NMHDECAY and LEP constraints are consistent with existing limits on \( \text{Br}(\Upsilon \to \gamma a_1) \). To probe the full set of \( m_{a_1} < 9.2 \) GeV points shown, sensitivity to \( \text{Br}(\Upsilon \to \gamma a_1) \lesssim \text{few} \times 10^{-8} \) is needed. Conversely, for \( \tan \beta = 50 \) a lot of the scan points consistent with NMHDECAY and LEP constraints are already absent because of existing limits and one need only probe down to \( \text{Br}(\Upsilon \to \gamma a_1) \sim 10^{-6} \) to cover the \( m_{a_1} < 9.2 \) GeV points.

We note that the points with small negative \( \cos \theta_A \) (e.g. \( \cos \theta_A \sim -0.5 \) for \( \tan \beta = 10 \)) that are most likely to escape \( A_\lambda, A_\kappa \) tuning issues are well below the existing limits from \([14, 15, 16]\) for all \( m_{a_1} \) values for all three \( \tan \beta \) choices. However, none of the above analyses \([14, 15, 16]\) have been repeated with the larger data sets available from CLEO-III, BaBar, or Belle. Presumably, much stronger constraints than those we included can be obtained. Or perhaps a \( \gamma a_1 \) signal will be found.

We expect that the best way to search for the NMSSM light \( a_1 \) is to use its exclusive decay modes, as this reduces backgrounds, especially those important when the photon is soft. For \( m_{a_1} > 3.6 \) GeV and \( \tan \beta \gtrsim 1 \), the dominant decay mode is \( a_1 \to \tau^+ \tau^- \). For example, Ref. [19] has proposed looking for non-universality in \( \Upsilon \to \gamma \tau^+ \tau^- \) vs. \( \Upsilon \to \gamma e^+e^- \) decays. This would fit nicely with the low-\( F \) scenarios. For \( m_{a_1} < 2m_{\tau} \), the decay mode \( a_1 \to gg \) is generally in the range 20%–30%, giving a contribution to \( \Upsilon \to \gamma gg \) at the \( 10^{-4} \)–\( 10^{-6} \) level; the \( s\tau \) mode is typically larger.

In the \( \gamma \tau^+ \tau^- \) final state, the direct \( \gamma \tau^+ \tau^- \) production cross section is 61 pb. Using signal=background as the criterion, this becomes the limiting factor for branching ratios below the \( 4 \times 10^{-5} \) level when running on the \( \Upsilon(1S) \), and below the \( 2 \times 10^{-4} \) level when running on the \( \Upsilon(3S) \). To improve upon the latter, one can select a sample of known \( \Upsilon(1S) \) events by looking for dipion transitions from the higher resonances. The dipion transition gives a strong kinematic constraint on the mass difference between the two \( \Upsilon \)'s. When running on the \( \Upsilon(3S) \), the effective cross section in \( \Upsilon(3S) \to \pi^+ \pi^- \Upsilon(1S) \) is 179 pb \([20]\). To limit \( \text{Br}(\Upsilon \to \gamma a_1) \lesssim 10^{-6} \), 5.6 fb\(^{-1}\)/\( \epsilon \) would need to be collected on the \( \Upsilon(3S) \), where \( \epsilon \) is the experimental efficiency for isolating the

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5 For a CP-odd \( a \) that decays into non-interacting states, there are further constraints available from Crystal Ball and CLEO \([17]\); these only apply to the scenarios considered here if \( M_1 \) is reduced to a very small value (as possible without affecting EWSB fine tuning) so that \( a_1 \to \chi_1^0 \chi_1^\pm \) decays are significant. For example, at \( \tan \beta = 10 \), our low fine-tuning scenarios with \( M_1 \) decreased to 3 GeV can yield \( m_{\chi_1^0} \lesssim 2 \text{ GeV} \) and \( \text{Br}(a_1 \to \chi_1^0 \chi_1^\pm) \in [0.15, 0.35] \). (Generic scenarios with substantial \( \text{Br}(\Upsilon \to \gamma a_1) \text{Br}(a_1 \to \chi_1^0 \chi_1^\pm) \) were considered in \([18]\).)

6 This can also be done on the \( \Upsilon(2S) \) but the pions are softer, implying much lower efficiency. On the \( \Upsilon(4S) \) this transition has a very small branching ratio \( \lesssim 10^{-4} \).
relevant events. This analysis can also be done on the $\Upsilon(3S)$, where the $\Upsilon(3S)$ is produced via ISR. The effective $\gamma_{ISR}\Upsilon(3S) \rightarrow \gamma_{ISR} \tau^+\tau^- \Upsilon(1S)$ cross section is 0.78 fb. To limit $Br(\Upsilon \rightarrow \gamma a_1) \lesssim 10^{-6},$ 1.3 $ab^{-1}/e$ would need to be collected. These integrated luminosities needed to probe $Br(\Upsilon \rightarrow \gamma a_1) \sim 10^{-7}$ would appear to be within reach at existing facilities and would allow discovery of the $a_1$ for many of the favored NMSSM scenarios.

Are there other modes that would allow direct detection? Reference 21 advocates $e^+e^- \rightarrow \chi_1^0 \chi_1^0 a_1$ with $a_1 \rightarrow \gamma\gamma.$ This works if the $a_1$ is very singlet, in which case $Br(a_1 \rightarrow \gamma\gamma)$ can be large. However, see 5 and earlier discussion, a minimum value of $|\cos \theta_A|$ (e.g. $|\cos \theta_A| > 0.04$ if $\tan \beta = 10$) is required in order that $Br(h_1 \rightarrow a_1a_1) > 0.7$ and $m_{a_1} < 2m_b.$ For the general $A_3, A_8$ scans with $Br(h_1 \rightarrow a_1a_1) > 0.7$ and $m_{a_1} < 2m_b$ imposed, $Br(a_1 \rightarrow \gamma\gamma) < 4 \times 10^{-4}$ with values near few $\times 10^{-5}$ being very common. It is conceivable that a super-$B$ factory could detect a signal for $\Upsilon \rightarrow \gamma a_1 \rightarrow \gamma\gamma\gamma$ which would provide a very interesting check on the consistency of the model.

Flavor changing decays based on $b \rightarrow s a_1$ or $s \rightarrow da_1,$ in particular $B \rightarrow X_s a_1,$ have been examined in 7. All penguin diagrams containing SM particles give contributions to the $b \rightarrow s a_1$ amplitude that are suppressed by $|\cos \theta_A|/\tan \beta$ or $|\cos \theta_A|/\tan^3 \beta$ (since up-type quarks couple to the $A_{MSSM}$ with a factor of $1/\tan \beta$). Ref. 7 identifies two diagrams involving loops containing up-type squarks and charginos that give $b \rightarrow s a_1$ amplitudes that are proportional to $\cos \theta_A \tan \beta.$ However, the sum of these diagrams vanishes in the super GIM limit (e.g. equal up-type squark masses), yielding a tiny $B \rightarrow X_s a_1$ transition rate. Away from this limit, results are highly model-dependent. In contrast, the predictions for $\Upsilon \rightarrow \gamma a_1$ depend essentially only on $\cos \theta_A, \tan \beta$ and $m_{a_1},$ all of which are fairly constrained for the low-fine-tuning NMSSM scenarios.

If $m_{a_1} < 2m_b, J/\psi \rightarrow \gamma a_1$ decay will be possible. However, $Br(J/\psi \rightarrow \gamma a_1)$ is $\sim 10^{-9}$ ($\sim 10^{-7}$) for the smallest (largest) $\cos \theta_A$ values in the standard $A_3, A_8$ scan for $\tan \beta = 10,$ increasing modestly as $\tan \beta$ increases.

Before concluding, we note that a light, not-too-singlet $a_1$ could allow consistency with the observed amount of dark-matter if the $\chi_1^0$ is largely bino and $2m_{\chi_1^0} \sim m_{a_1}.$ This is explored in 18. We found that these scenarios could provide a consistent description of the dark matter relic density in the case of a very light $\chi_1^0.$ We report here that this can be coincident with the $F < 15$ scenarios (as well as the smaller negative $\cos \theta_A, m_{a_1} > 2m_\tau$ scenarios that are the most likely to have small $A_{\lambda}, A_{\kappa}$ fine-tuning). All that is required relative to the $M_1 = 100$ GeV choice made for our scans is to decrease $M_1$ to bring down $m_{\chi_1^0}$ near $\frac{1}{2} m_{a_1}.$ $M_1$ is an independent parameter that has essentially no influence on the value of the fine-tuning measure $F$ so long as $M_1 \lesssim M_3.$

In summary, aside from discovering the $a_1$ in $h_1 \rightarrow a_1a_1$ decays, something that will almost certainly have to await LHC data and, because of the unusual final state, might not even be seen until the ILC, it seems that the most promising near-term possibility for testing the NMSSM scenarios for which EWSB fine-tuning is absent, or more generally any scenario with large $Br(h_1 \rightarrow a_1a_1)$ and $m_{a_1} < 2m_b,$ is to employ the $\Upsilon \rightarrow \gamma a_1$ decay at either existing $B$ factories or future factories.

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