A remark on intuitionistic fuzzy implications

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Abstract: In the paper an attempt is made at introducing a classification scheme for some of the intuitionistic fuzzy implications. This has allowed us to navigate the existing implications in a more consistent manner and has revealed a duplicate implication.  
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1 Introduction

At present there are 190 proposed intuitionistic fuzzy implications defined in a series of papers [1–10, 12–17, 19–39, 41–46] most of which are collected in [11]. For alternative points of view on intuitionistic fuzzy implications we refer the interested reader to [40] and [47].

In order to make research into the existing 190 implications, we further focus our attention on the functions used to represent them (\(\text{sg}, \overline{\text{sg}}, \min, \max, \cdot\)) and we propose a sample classification based on their properties. This has allowed us to identify a duplicate implication, namely \(\rightarrow_{40}\) which coincides with \(\rightarrow_{173}\).
2 Preliminaries

Here we remind some of the basic definitions which will be used later.

Definition 1. The functions \( s_g \) and \( \overline{s_g} \) are defined for the real variable \( x \) as follows:

\[
\begin{align*}
    s_g(x) &= \begin{cases} 
        1, & \text{if } x > 0, \\
        0, & \text{if } x \leq 0
    \end{cases} \\
    \overline{s_g}(x) &= \begin{cases} 
        0, & \text{if } x > 0, \\
        1, & \text{if } x \leq 0
    \end{cases}
\end{align*}
\]

(1)

Remark 1. From Definition 1 it is seen that the following equality holds:

\[1 - s_g(x) = \overline{s_g}(x)\]

(2)

Next we recall the following:

Definition 2 (cf. [18]). An intuitionistic fuzzy pair (IFP) is an ordered couple of real non-negative numbers \( \langle a, b \rangle \), with the additional constraint:

\[a + b \leq 1.\]

(3)

If we denote the set of all IFPs by \( U_{\text{IFP}} \), we can view an implication as a particular mapping of the kind (bound by additional constraints due to the Axioms that need to be satisfied):

\[I : U_{\text{IFP}} \times U_{\text{IFP}} \rightarrow U_{\text{IFP}}.\]

In other words all implications are of the form:

\[I(x, y) = \langle f(x, y), g(x, y) \rangle,\]

where \( x \in U_{\text{IFP}}, y \in U_{\text{IFP}}, \langle f(x, y), g(x, y) \rangle \in U_{\text{IFP}}. \) In our further considerations we suppose that everywhere \( x = \langle a, b \rangle \) and \( y = \langle c, d \rangle. \)

In the 190 implications the most used arithmetical functions are +, −, \( s_g, \overline{s_g}, \max, \min, \cdot \).

Let \( a, b \in [0, 1] \), then the following property holds:

\[ab \leq \min(a, b) \leq \max(a, b) = 1 - \min(1 - a, 1 - b)\]

(4)

If \( \langle a, b \rangle \) and \( \langle c, d \rangle \) are IFPs, we have

\[ad \leq (1 - b)(1 - c) \leq \min(1 - b, 1 - c) = 1 - \max(b, c).\]

(5)

Let \( a, b \in [0, 1] \), then the following property holds:

\[s_g(a)s_g(b) = s_g(ab) = \min(s_g(a), s_g(b)) = s_g(\min(a, b)).\]

(6)
3 Results

Using equations (2), (4), (5) and (6), we have established that a significant number of the 190 implications may be written as:

\[ I(x, y) = \langle f(x, y), 1 - f(x, y) \rangle, \]

(7)

where \( f(x, y) \in [0, 1] \).

The list of all such implications is given below:

\[ \rightarrow 12, \rightarrow 20, \rightarrow 22, \rightarrow 23, \rightarrow 32, \rightarrow 33, \rightarrow 34, \rightarrow 35, \rightarrow 37, \rightarrow 38, \rightarrow 40, \rightarrow 41, \rightarrow 42, \rightarrow 43, \rightarrow 48, \rightarrow 49, \rightarrow 50; \]
\[ \rightarrow 52, \rightarrow 55, \rightarrow 56, \rightarrow 57, \rightarrow 74, \rightarrow 76, \rightarrow 77, \rightarrow 85, \rightarrow 86, \rightarrow 87, \rightarrow 88, \rightarrow 86, \rightarrow 93, \rightarrow 94, \rightarrow 96, \rightarrow 97, \rightarrow 142; \]
\[ \rightarrow 143, \rightarrow 144, \rightarrow 145, \rightarrow 146, \rightarrow 147, \rightarrow 148, \rightarrow 149, \rightarrow 154, \lambda, \rightarrow 155, \lambda, \rightarrow 156, \lambda, \rightarrow 157, \lambda, \rightarrow 158, \lambda, \rightarrow 159, \lambda; \]
\[ \rightarrow 160, \gamma, \rightarrow 161, \gamma, \rightarrow 162, \alpha, \beta, \rightarrow 163, \alpha, \beta, \rightarrow 164, \alpha, \beta, \rightarrow 165, \alpha, \beta, \rightarrow 167, \rightarrow 168, \rightarrow 169, \rightarrow 170, \rightarrow 171, \rightarrow 172; \]
\[ \rightarrow 173, \rightarrow 174, \rightarrow 175, \rightarrow 177, \rightarrow 178, \rightarrow 179, \rightarrow 180, \rightarrow 181, \rightarrow 182, \rightarrow 183, \rightarrow 184, \rightarrow 185, \rightarrow 190 \]

We have marked \( \rightarrow 40 \) and \( \rightarrow 173 \) by \( * \) to denote the fact that they coincide.

Some of the remaining implications can be represented in the form

\[ I(x, y) = \langle 1 - f(x, y), g(x, y) f(x, y) \rangle, \]

(8)

where \( f(x, y), g(x, y) \in [0, 1] \).

Namely,

\[ \rightarrow 2, \rightarrow 31, \rightarrow 47, \rightarrow 62, \rightarrow 83. \]

One can easily observe that (7) may be treated as a particular case of (8) with the special choice of

\[ g(x, y) = 1 \forall x, y \in U_{IFP}. \]

However, such approach while technically correct does not yield particularly useful information.

The rest of the implications have less tractable representations.

However, implications that satisfy (7) and/or (8) may be studied based on the properties of the functions \( f(x, y) \) and \( g(x, y) \), which allows for a more unified approach in treating them.

In the light of the above we can formulate the following

**Open problem.** Can implications that do not satisfy (7) or (8) be categorized in suitable classes which can be described by a single formula?

4 Conclusion

In the present paper we proposed a partial classification based on the representation of the existing implications. This allows not only to study implications which satisfy (7) and/or (8) in a unified manner, but also to introduce and study new implications with such property. It also helps in detecting duplicating or coinciding implications as was the case with implications \( \rightarrow 40 \) and \( \rightarrow 173 \).
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