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Cavity magnomechanical storage and retrieval of quantum states

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Abstract
We show how a quantum state in a microwave cavity mode can be transferred to and stored in a phononic mode via an intermediate magnon mode in a magnomechanical system. For this we consider a ferrimagnetic yttrium iron garnet (YIG) sphere inserted in a microwave cavity, where the microwave and magnon modes are coupled via a magnetic-dipole interaction and the magnon and phonon modes in the YIG sphere are coupled via magnetostrictive forces. By modulating the cavity and magnon detunings and the driving of the magnon mode in time, a stimulated Raman adiabatic passage-like coherent transfer becomes possible between the cavity mode and the phonon mode. The phononic mode can be used to store the photonic quantum state for long periods as it possesses lower damping than the photonic and magnon modes. Thus our proposed scheme offers a possibility of using magnomechanical systems as quantum memory for photonic quantum information.

1. Introduction
The coupling between magnons in a ferrimagnetic material and phonons in a mechanical resonator has attracted wide attention in recent years due to its application in magnomechanical settings similar to cavity quantum electrodynamics and optomechanics. A mechanical resonator can be coupled to an optical cavity mode via radiation pressure interaction [1, 2], as well as to a microwave cavity mode via an electrostatic interaction [3, 4]. Thus such mechanical mirrors can be used as transducers for the coherent transfer between optical and microwave fields [5, 6], or as optical wavelength converters [7]. One of the relatively new candidates in the picture is the magnon, a collective excitation of magnetization, which can be coupled to phonons via the magnetostrictive force [8]. This type of coupling has better tunability as the magnon frequency can be controlled via an external magnetic field [9–11]. Magnons can be strongly coupled to microwave cavity modes as well, particularly in the insulating magnetic material yttrium iron garnet (YIG). The Kittel mode [12], found in YIG spheres, can couple very strongly to cavity photons or to superconducting quantum circuits [13], producing a hybrid system that can be efficiently used in quantum information processing [14–23]. This kind of magnon–cavity coupling can also give rise to bistable behaviour as studied in [24].

Combining the magnetostrictive coupling between magnons and phonons and the magnon–cavity coupling a new kind of photon–magnon–phonon interaction can be realized with YIG spheres interacting with microwave cavities. This opens up new possibilities in quantum state engineering and control. Here we consider a photon–magnon–phonon coupled system which is capable of transferring cavity photons to mechanical motional phonons, enabling the storage of photonic quantum states for long durations, thanks to the lower damping rate of the phononic mode. Storage of quantum states and information in long-lasting modes is very important for quantum communication networks. Photons are the usual
candidate for flying qubits, whereas some other quantum system with a longer coherence time is necessary for the storage of quantum information in quantum repeaters and networks [25]. The storage of quantum states has, for example, been previously studied in atomic media, where the information is stored as spin excitations [26–28], in optomechanical systems [29–31], in coupled optical waveguides, or in acoustic excitations [32–35].

The system we consider consists of a YIG-sphere placed inside a microwave cavity where the magnon modes of the sphere couple with the deformation phonon modes via a magnetostrictive force, and also to the electromagnetic cavity modes via a magnetic dipole interaction. We drive the magnon mode with a microwave field such that the time-modulation of the amplitude of the drive on the magnon mode yields a time-dependent coupling strength between the magnon and the mechanics. Using additionally time-modulated resonance frequencies of the photonic and magnon modes, a stimulated Raman adiabatic passage (STIRAP)-like one-way transfer becomes possible [36, 37], which efficiently transfers quantum states from the cavity mode to the mechanical mode. The quanta can be stored in the mechanical resonator for some time and can then be extracted using a retrieval pulse. In this way quantum states can be stored for times longer than the cavity lifetime, due to the lower damping rate of the phononic mode. We show that our storage protocol can be applied to various kinds of quantum states.

2. Model

We consider a system schematically shown in figure 1(a), where a YIG sphere is inserted into a microwave cavity. The Hamiltonian of the system is given by (note that we have set ħ = 1)

\[ H_0 = \omega_a a^\dagger a + \omega_m m^\dagger m + \omega_b b^\dagger b + g_{ma}(a^\dagger m + m^\dagger a) + g_{mb}m^\dagger m(b + b^\dagger) + i(\epsilon_p m^\dagger e^{-i\omega_p t} - \epsilon_p^* m e^{i\omega_p t}), \]

where \( a(a^\dagger) \), \( m(m^\dagger) \) and \( b(b^\dagger) \) are the annihilation (creation) operators of the microwave cavity mode, the magnon mode and the mechanical mode with resonance frequencies \( \omega_a, \omega_m \) and \( \omega_b \) respectively. The magnon frequency can be tuned by the external bias magnetic field, \( \mathcal{H}_e \), as \( \omega_m = \gamma \mathcal{H}_e \) where \( \gamma = 28 \text{ GHz T}^{-1} \) is the gyromagnetic ratio. In addition, \( g_{ma} \) and \( g_{mb} \) are the single excitation coupling rates of the cavity–magnon interaction and magnon–phonon magnetostrictive interaction. The last term in (1) describes the external driving of the magnon mode, where \( \omega_p \) is the frequency of the drive magnetic field and \( \epsilon_p \) is the Rabi frequency between the drive magnetic field and the magnon mode, given by \( \epsilon_p = \frac{\Delta}{\sqrt{\mathcal{N}}} \gamma \sqrt{\mathcal{N}} B_0 \) [20]. Here \( B_0 \) is the amplitude of the drive magnetic field, \( \mathcal{N} = \rho V \) is the total number of spins in a sphere of volume \( V \) and \( \rho = 4.22 \times 10^{27} \text{ m}^{-3} \) is the spin density of YIG.

Moving over to the frame rotating with the drive frequency \( \omega_p \), given by the transformation, \( R = \exp \left[ \omega_p (a^\dagger a + m^\dagger m) t \right] \), with \( H = RH_0 R^\dagger + i\frac{\partial}{\partial t} R^\dagger R \), the Hamiltonian of the system can be written as

\[ H = \Delta_a a^\dagger a + \Delta_m m^\dagger m + \omega_b b^\dagger b + g_{ma}(a^\dagger m + m^\dagger a) + g_{mb} m^\dagger m(b + b^\dagger) + i(\epsilon_p m^\dagger e^{-i\omega_p t} - \epsilon_p^* m e^{i\omega_p t}), \]

where \( \Delta_a = \omega_a - \omega_p \) and \( \Delta_m = \omega_m - \omega_p \) are detunings.

The dynamical evolution of the system operators can then be described by the Langevin equations

\[ \dot{a} = (-i\Delta_a - \kappa_a) a - ig_{ma}m + \sqrt{2\kappa_a}a_n, \]

\[ \dot{b} = (-i\omega_b - \kappa_b) b - ig_{mb}m^\dagger m + \sqrt{2\kappa_b}b_n, \]

\[ \dot{m} = (-i\Delta_m - \kappa_m) m - ig_{ma}a - ig_{mb}(b + b^\dagger) + \epsilon_p + \sqrt{2\kappa_m}m_n, \]

Here \( \kappa_a, \kappa_b \) and \( \kappa_m \) are the losses of the cavity mode, the magnon mode and the mechanical mode respectively and \( a_n, b_n, m_n \) are the noise operators with zero mean values, and correlation functions given by \( \langle x_a(t)x_a(t') \rangle = \bar{n}_a + 1), \langle x_m(t)x_m(t') \rangle = \bar{n}_m \delta(t - t'), \) and \( \langle x_b(t)x_m(t') \rangle = \bar{n}_m \delta(t - t') \), with \( x = \{ a, b, m \} \). The \( \bar{n}_a \) and \( \bar{n}_b \) are the mean thermal occupations of the cavity mode, magnon mode and phonon mode, given by \( \bar{n}_a = (\exp(\omega_a/T_b) - 1)^{-1} \), where \( T_b \) is the bath temperature and \( k_b \) is the Boltzmann constant. For strong driving, each Heisenberg operator can be expressed as a sum of its steady-state mean value and the quantum fluctuation, i.e., \( a = \bar{a} + a_t, b = \bar{b} + b_t \) and \( m = \eta + m_t \), where \( \bar{a}, \bar{b}, \bar{m} \) are the classical mean field values of the modes and \( a_t, b_t, m_t \) are the corresponding quantum fluctuation operators.
Stimulated Raman adiabatic passage—or STIRAP for short—can be efficiently used to transfer population in conventional three-level atomic systems [37, 42]. Conventional STIRAP relies on the fact that at two-photon resonance an instantaneous eigenvector with zero eigenvalue exists, which is called a dark-state, and which is a superposition of the initial and target states. In the adiabatic limit the STIRAP dynamics allows to trap the population within the ‘dark state manifold’ at all times, and any population transfer to the intermediate state, which has often a high decay rate, is avoided. The standard STIRAP protocol then modulates the coupling strengths in time between the two states in the dark-state manifold and the intermediate state. If this is done using a so-called counter-intuitive pulse sequence, one can transport population to the target state.

Following the standard linearization approach [5, 6], the dynamics of the quantum fluctuations is given by the linearized Hamiltonian of the form

\[ H_{\text{lin}} = \Delta_a a^\dagger a + \tilde{\Delta}_m m_1 a + \omega_b b^\dagger b + G_{mb}(m_1 + m_1^\dagger)(b_1 + b_1^\dagger) + g_{ma}(m_1^\dagger a_1 + m_1 a_1^\dagger), \]

where \( G_{mb} = \eta g_{mb} \) is the coherent-driving-enhanced linearized magnomechanical coupling strength, with \( \eta = \epsilon_p(i\Delta_c + \Delta_m)/(g_{ma} + i(\tilde{\Delta}_m + \Delta_m)(i\Delta_c + \Delta_m)) \). Here \( \tilde{\Delta}_m = \Delta_m + g_{mb}(\beta + \beta^*) \) is the magnomechanical interaction-induced effective magnon-drive detuning. Transforming this Hamiltonian into an interaction picture with \( U = \exp[-i\omega_1(a_1^\dagger a_1 + m_1^\dagger m_1 + b_1^\dagger b_1)t] \) yields

\[ H = \delta_a a_1^\dagger a_1 + \delta_m m_1^\dagger m_1 + G_{mb} \left( m_1^\dagger b_1 + m_1 b_1^\dagger + e^{-2i\omega_1\tau} m_1 b_1 ight) + G_{ma}(m_1^\dagger a_1 + m_1 a_1^\dagger). \]

Here \( \delta_a = \Delta_a - \omega_b \) and \( \delta_m = \tilde{\Delta}_m - \omega_b \) are the effective detunings. The magnomechanical coupling, \( G_{mb} \), can be varied by tuning the magnon drive Rabi frequency, and the magnon frequency can be altered by adjusting the strength of the external magnetic bias field [20], which can therefore be used to tune the magnon detuning, \( \delta_m \). The photon frequency can be modulated by using a tunable 3D microwave cavity as realized in the references [38–41], which will modulate the cavity detuning, \( \delta_c \). In the following section we will show that, using these time-dependent modulations, an STIRAP-like protocol can be designed to effectively transfer the microwave cavity state to the mechanical mode and then retrieve it back to the microwave cavity mode.

### 3. State transfer and retrieval

Stimulated Raman adiabatic passage—or STIRAP for short—can be efficiently used to transfer population in conventional three-level atomic systems [37, 42]. Conventional STIRAP relies on the fact that at two-photon resonance an instantaneous eigenvector with zero eigenvalue exists, which is called a dark-state, and which is a superposition of the initial and target states. In the adiabatic limit the STIRAP dynamics allows to trap the population within the ‘dark state manifold’ at all times, and any population transfer to the intermediate state, which has often a high decay rate, is avoided. The standard STIRAP protocol then modulates the coupling strengths in time between the two states in the dark-state manifold and the intermediate state. If this is done using a so-called counter-intuitive pulse sequence, one can transport population to the target state.
population from one state in the dark-state manifold to the other state within that manifold with perfect fidelity. However, in our system it is not suitable to apply conventional STIRAP as the cavity–magnon population from one state in the dark-state manifold to the other state within that manifold with perfect coupling, \( g_{m}\), is constant, i.e. it cannot be modulated in a time-dependent manner. We therefore present in the following a modified STIRAP method which allows state transfer by modulating the detunings.

In this scheme the cavity–magnon coupling \( g_{m} \equiv \Omega_{s}/2 = \Omega_{p}/2 \), which is called the ‘Stokes’ coupling in the context of atomic STIRAP, is constant in time and the magnomechanical coupling \( G_{mb}(t) \equiv \Omega_{p}(t)/2 \), which is generally known as the ‘Pump’ coupling in STIRAP, is modulated as

\[
\Omega_{p}(t) = \Omega_{p0}(t) + \Omega_{p2}(t).
\]

Here

\[
\Omega_{p0}(t) = \Omega_{0}e^{-\left(\frac{t-t_{c1}}{\tau_{c}}\right)^{2}},
\]

and

\[
\Omega_{p2}(t) = \Omega_{0}e^{-\left(\frac{t-t_{c2}}{\tau_{c}}\right)^{2}},
\]

are Gaussian-shaped pulses centred at the times \( t_{c1} \) and \( t_{c2} \), with width \( \tau_{c} \) and amplitude \( \Omega_{0} \). Application of the pulse \( \Omega_{p0}(t) \) transfers the state from the cavity to the mechanical mode, whereas the application of the pulse \( \Omega_{p2}(t) \) brings the state back to the cavity. The cavity and magnon detunings are considered as

\[
\delta_{m}(t) = -\kappa_{s}h_{3} \frac{\Omega_{0}}{2} \left[ \tanh \left( \frac{t - \tau}{\tau_{ch}} \right) + \tanh \left( \frac{t + \tau}{\tau_{ch}} \right) \right],
\]

\[
\delta_{a}(t) = -\left(\kappa_{s} - 1\right)h_{3} \frac{\Omega_{0}}{2} \left[ \tanh \left( \frac{t - \tau}{\tau_{ch}} \right) + \tanh \left( \frac{t + \tau}{\tau_{ch}} \right) \right],
\]

so that

\[
\delta_{m}(t) - \delta_{a}(t) = \delta_{s}(t) = -\frac{\chi_{s}h_{3}}{2} \left[ \tanh \left( \frac{t - \tau}{\tau_{ch}} \right) + \tanh \left( \frac{t + \tau}{\tau_{ch}} \right) \right].
\]

The shapes of these coupling and detuning pulses are shown in figure 1(b). The choice of these pulse shapes for the magnomechanical coupling and the detunings can be understood by looking at the instantaneous eigenvalues of the system. In the rotating wave approximation the Hamiltonian (5) is given by

\[
H = \begin{bmatrix}
0 & \Omega_{p}(t)/2 & 0 \\
\Omega_{p}(t)/2 & \delta_{m}(t) & \Omega_{l}/2 \\
0 & \Omega_{l}/2 & \delta_{a}(t)
\end{bmatrix}
\]

(11)

which is similar to the Hamiltonian used in STIRAP in a three-level atom with states \( |0, 1, 2\rangle \), where the transfer is sought from \( |0\rangle \) to \( |2\rangle \) without populating \( |1\rangle \). In our system this corresponds to the transfer from \( |a\rangle \) to \( |b\rangle \) and vice versa in the retrieval stage of the scheme, with no occupation of \( |m\rangle \) during the storage stage. The instantaneous eigenvalues \( \lambda_{0}, \lambda_{1}, \lambda_{2} \) of the Hamiltonian in equation (11), when the time modulated pulses are applied, are shown in figure 2(a) (solid lines). If we consider the magnomechanical coupling \( G_{mb}(t) \) to vanish (i.e. \( \Omega_{p}(t) = 0 \)), the Hamiltonian is given by

\[
H_{i} = \begin{bmatrix}
0 & 0 & 0 \\
0 & \delta_{m}(t) & \Omega_{l}/2 \\
0 & \Omega_{l}/2 & \delta_{a}(t)
\end{bmatrix}
\]

(12)

and it acts only on the cavity–magnon subspace, i.e. it does not involve the mechanical mode. This yields the asymptotic eigenstates \( |s_{0}(t = \pm \infty)\rangle \), and \( |s_{a}(t = \pm \infty)\rangle \), where

\[
|s_{+}(\pm \infty)\rangle \simeq |\Psi_{b}\rangle \rightarrow |s_{+}(\pm \infty)\rangle \simeq |\Psi_{a}\rangle,
\]

\[
|s_{-}(\pm \infty)\rangle \simeq |\Psi_{b}\rangle \rightarrow |s_{-}(\pm \infty)\rangle \simeq |\Psi_{a}\rangle.
\]

(13)

(14)

Here \( |\Psi_{a}\rangle \) (\( |\Psi_{b}\rangle \)) are the states of the microwave cavity (mechanical mode) and the corresponding eigenvalues are

\[
S_{0} = 0, \quad S_{a} = \frac{\delta_{a} + \delta_{m}}{2} \pm \sqrt{\left(\frac{\delta_{a} - \delta_{m}}{2}\right)^{2} + \Omega_{l}^{2}}.
\]

(15)

The time evolution of the eigenvalues of this Stokes Hamiltonian results in the eigenvalues \( S_{a}(t) \) crossing the eigenvalue \( S_{0} \) twice at \( t \sim t_{c1} \) and \( t \sim t_{c2} \) as shown by the dashed lines in figure 2(a). However, the application of the magnomechanical coupling \( \Omega_{p} \) lifts the degeneracy between the eigenstates \( S_{-} \) and \( S_{0} \) in the first avoided crossing, and \( S_{0} \) and \( S_{+} \), in the second avoided crossing (shown by the solid lines in
state, $\rho$ with nearly 100% fidelity from the cavity mode these two points for an initial Fock state in the cavity are depicted in figure 2(b), where we show the time fidelity, $F$, from the microwave cavity mode to the mechanical resonator mode and then back to the cavity mode, (c) unitary transfer dynamics of a coherent state with $\alpha = 0.5$ in terms of the fidelity, $F$, from the microwave cavity mode to the mechanical resonator mode and then back to the cavity mode. The pulse parameters considered are same as in figure 1.

Figure 2. (a) Evolution of the 'Stokes' eigenvalues, $\{S_+, S_-, S_0\}$ (dashed lines), and the instantaneous eigenvalues of the system under the STIRAP-like pulses explained in the text (solid lines), (b) unitary population transfer dynamics (without any damping) of the Fock state $|1, 0, 0\rangle$ showing the transfer from the microwave cavity mode to the mechanical resonator mode and then back to the cavity mode, (c) unitary transfer dynamics of a coherent state with $\alpha = 0.5$ in terms of the fidelity, $F$, from the microwave cavity mode to the mechanical resonator mode and then back to the cavity mode. The pulse parameters considered are same as in figure 1.

Next, we consider the situation where the cavity mode is initially in a coherent state $|\Psi\rangle = D(\alpha)|0\rangle$, where $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$. The performance of our quantum memory scheme is evaluated in terms of the fidelity, $F = \sqrt{\langle \Psi | \rho(t) | \Psi \rangle}$, which measures the overlap of the density matrix of the instantaneous cavity state, $\rho(t) = Tr_{b,m}[\rho(t)]$, with the initial cavity state $|\Psi\rangle$. For an initial coherent state with $\alpha = 1$ the transfer and retrieval processes are shown in figure 2(c) and values of $F \sim 1$ indicate that the transferred state closely resembles the original cavity state intended to be transferred, stored and retrieved [43]. In figure 2(c) the rapid oscillations in the fidelity at the starting and ending portions of the pulse are due to the rapid rotation of the cavity state in phase space with large detunings. By halting the pulse at the opportune time we obtain near-unit fidelity, which means near-perfect transfer and retrieval.

It is to be noted that until now in our analysis, we have not taken into account any damping existing in the system. We will therefore, in the following, study the state transfer dynamics for various input states in a realistic open system by coupling all modes to a thermal bath. We consider the open quantum system dynamics using the quantum master equation,

$$\dot{\rho} = i [\rho, H] + \left\{ \kappa_a (\hat{n}_a + 1) \mathcal{L}[a_1] + \kappa_a \hat{n}_a \mathcal{L}[a_1^\dagger] + \kappa_m (\hat{n}_m + 1) \mathcal{L}[m_1] + \kappa_m \hat{n}_m \mathcal{L}[m_1^\dagger] \right\} \rho,$$

(16)

with $\mathcal{L}[A] \rho \equiv A \rho A^\dagger - 1/2 \{A^\dagger A, \rho\}$ representing the dissipation and noises in the system. We consider the phonon, magnon and cavity mode frequencies as $\omega_b/2\pi = 10$ MHz and $\omega_m/2\pi = \omega_m/2\pi = 10$ GHz, and the bath temperature is chosen to be $T_{ab} = 1$ mK. Since the cavity and magnon modes oscillate at high frequencies, coupling these to a thermal bath at mK temperatures yields almost zero thermal occupancy, however the phonon mode is occupied. The damping rates are considered as $\kappa_b = 100$ Hz and $\kappa_m = 10$ kHz. In the following we also consider $\kappa_a = 0$, as we are only interested in the loss of fidelity caused by the transfer and storage processes. We start the transfer from an initial state given by $|\alpha, 0, 0\rangle$,
where the cavity mode contains a coherent state, the magnon mode occupation is zero and the mechanical mode has been precooled to the ground state [44–46].

In figures 3(a)–(c) we show the instantaneous fidelities for the transfer and retrieval processes for the coherent state from the cavity to the mechanical mode and back to the cavity mode for various values of \( \alpha \). For all examples the retrieval fidelity is nearly 100\%. To further quantify the effectiveness of the photon–phonon–photon transfer, we calculate the Wigner function for the initial cavity state and for the cavity state after the transfer. One can see from figures 3(d) and (e) that both these Wigner functions are virtually identical with only a small shift after storage and retrieval towards the vacuum state.

Next, we consider two slightly more complicated states. In figure 4, we show the storage and retrieval fidelity of an initial cat state given by

\[
|\Psi_i\rangle = N (|\alpha\rangle + |-\alpha\rangle),
\]

where \( N \) is a normalization parameter. Panels (a)–(c) show the results for the transfer fidelity for \( \alpha = 0.5, 0.75, 1 \) and panels (d) and (e) show the Wigner functions for the cavity state before and after the transfer for the state with \( \alpha = 1 \). We also consider in figure 5 the squeezed vacuum state as the initial state in the cavity given by

\[
|\Psi_i\rangle = e^{(r^2 - r^2 \phi^2)/2} |0\rangle,
\]

where \( r \) is the squeezing parameter. The storage fidelities for three different squeezing strengths \( r = 0.5, 0.75, 1 \) are shown in figures 5(a)–(c) and panels (d) and (e) depict the Wigner functions for the cavity state before and after the transfer with \( r = 1 \). One can see in both cases that, even though the retrieval fidelity is rather high, it does no longer reach the value of \( F = 1 \). For the cat state it decreases with increasing values of \( \alpha \), which suggests that the overall fluctuations of the two terms, \(|\alpha\rangle\) and \(|-\alpha\rangle\) become more prominent and are the
Figure 4. Transfer fidelities for an input cat state in the cavity mode with (a) $\alpha = 0.5$, (b) $\alpha = 0.75$, and (c) $\alpha = 1$. Figures (d) and (e) show the input and output Wigner functions for the cavity mode with input cat state with $\alpha = 1$. Parameters are same as in figure 3.

reason for the loss of fidelity. Similarly for squeezed states the decoherence distorts the squeezed feature in phase space, resulting in the lower fidelity. As $r$ increases the distortion is more pronounced, which is visible in the shape of the Wigner function of the retrieved photon in figure 5(e). However, for all the input states described here, the storage fidelity is still very high which shows that the transfer is highly efficient.

The above analysis shows that the mechanical mode can be used as a storage mode for the photonic state. However, as the fidelity inevitably decays for longer storage times, it is interesting to study how this decay happens. For this we introduce a time-delay of the retrieval pulse by $\Delta t$, so that

$$\Omega_{p_2}(t) = \Omega_0 e^{\left(\frac{t - (\tau + \Delta t)}{\tau_{ch}}\right)^2},$$

and similar for the cavity and magnon detunings

$$\delta_m(t) = -\kappa_d h_0 \frac{\Omega_0}{2} \left[ \tanh\left(\frac{t - (\tau + \Delta t)}{\tau_{ch}}\right) \right] + \tanh\left(\frac{t + \tau}{\tau_{ch}}\right),$$

$$\delta_a(t) = -(\kappa_d - 1) h_0 \frac{\Omega_0}{2} \left[ \tanh\left(\frac{t - (\tau + \Delta t)}{\tau_{ch}}\right) \right] + \tanh\left(\frac{t + \tau}{\tau_{ch}}\right).$$

For a cavity mode that is initially occupied by a coherent state with $\alpha = 1$ we show the dynamics of the fidelity for delay time $\Delta t = 14 \times t_{ch}$ in figure 6(a). Before the transfer the fidelity is equal to one, but after the first set of pulses is applied, the cavity empties and the fidelity drops to the value for the vacuum, $F \sim |\langle \Psi | 0 \rangle|^2$ (indicated by the orange-colored line). This means that the quantum state initially present in
the cavity mode has been almost fully transferred to the mechanical mode, from which it will be retrieved when the second set of pulses is applied. At this point the cavity state fidelity increases again and in figure 6(b) the corresponding mode occupations are shown. One can see that the cavity mode occupation goes down to zero after the initial transfer, while the mechanical mode occupation rises. The latter also slowly increases during the storage period due to its coupling to the thermal environment at $T = 1$ mK, the steady-state mechanical occupation tending towards $\bar{n}_b \sim 1.6$. During application of the transfer pulses the magnon mode occupation rises briefly, but also quickly returns to zero. In figure 6(c), we show the cavity fidelity after retrieval, $F_r$ (red dots) as a function of the storage time, $t_s$, with the blue line corresponding to an exponential fitting of these results. The storage time can be calculated as the time difference between the application of the transfer and retrieval pulses, which can be given approximately as $t_s = (t_c^2 + \Delta t - t_c)$. The half-life of the fitted exponential decay rate is given by 6 ms, where the primary loss arises from the mechanical mode damping, which itself has a lifetime of 10 ms. The additional small losses can therefore be attributed to magnon mode decay that occurs during the transfer.

For all the analysis above, we have assumed rather low values for the bath temperature and the magnon damping rate, which are currently not experimentally accessible. Let us therefore in the following explore the effects of higher bath temperature and higher magnon damping. Since the numerical treatment becomes very resource intensive for higher bath occupations [47, 48], we first solve for the average values of the second-order moments of the system analytically as described in the following. Applying the master equation given in equation (16), a linear set of differential equations for the second-order moments can be...
obtained as

\[ \hat{\partial}_t \langle \hat{n} \rangle = \text{Tr} \left( \hat{\rho} \hat{\partial}_t \hat{n} \right) = \sum_{m,n} \mu_{m,n} \langle \hat{n}_m \hat{n}_n \rangle , \]

where the \( \hat{n}_1, \hat{n}_2, \hat{n}_m, \hat{n}_n \) are one of the operators: \( \hat{a}_1^\dagger, \hat{m}_1^\dagger, \hat{b}_1^\dagger, \hat{a}_1, \hat{m}_1 \) and \( \hat{b}_1 \); and \( \mu_{m,n} \) are the corresponding coefficients. We use this approach to time evolve the mean occupations in the photon (\( \langle \hat{a}_1^\dagger \hat{a}_1 \rangle \)), phonon (\( \langle \hat{b}_1^\dagger \hat{b}_1 \rangle \)) and magnon (\( \langle \hat{m}_1^\dagger \hat{m}_1 \rangle \)) modes, considering that initially only the cavity mode is occupied with \( \langle \hat{N}_0 \rangle = \langle \hat{a}_1^\dagger \hat{a}_1 \rangle (t = 0) = 1 \), and all the other second-order moments are zero, with state \( |N_0, N_m, N_n \rangle (t = 0) = |1, 0, 0 \rangle \). Considering \( T_{th} = 10 \text{ mK} \), we show in figure 7 the retrieved photon number in the cavity at the end of the scheme as a function of a decreasing magnon damping rate. Here the storage time is chosen to be \( \omega_0 t_s = 1224.4 \) which for \( \omega_0 / 2\pi = 10 \text{ MHz} \) corresponds to \( t_s = 0.19 \text{ ms} \). One can see that, as expected, lower magnon damping rates result in improved photon retrieval.

4. Conclusions

In conclusion, we have presented a scheme to transfer quantum states from a microwave cavity mode to a mechanical mode in a photon–magnon–phonon hybrid system where a YIG sphere is placed in a
microwave cavity with the magnon mode coupled both to the photonic and phononic modes. We have shown that using time-modulated shapes for the detunings for the magnon and cavity modes and also the magnetostrictive coupling, transfer with high fidelity between non-directly coupled modes can be obtained for a number of different states including coherent states, cat states and squeezed vacuum states. Given the flexibilities in controlling the magnomechanical coupling, it is interesting to consider this work as a first step for implementing mechanical bosonic quantum error correction codes on the stored phononic quantum information to further increase the mechanical storage times [49].

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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