Velocity Dependence of Charmonium Dissociation Temperature in High-Energy Nuclear Collisions

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Abstract

In high-energy nuclear collisions, heavy quark potential at finite temperature controls the quarkonium suppression. Including the relaxation of the medium induced by the relative velocity between quarkonia and the deconfined expanding matter, the Debye screening is reduced and the quarkonium dissociation takes place at a higher temperature. As a consequence of the velocity dependent dissociation temperature, the quarkonium suppression at high transverse momentum is significantly weakened in high energy nuclear collisions at RHIC and LHC.

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Heavy quarkonia $J/\psi$ and $\Upsilon$ are tightly bound hadronic states. Their dissociation temperature $T_d$ is, in general, higher than the critical temperature $T_c$ for the deconfinement phase transition \cite{1} in high-energy nuclear collisions \cite{2,3,4}. Therefore, the measured cross sections of quarkonia carry the information of the early stage hot and dense medium. They have long been considered as a signature of the formation of the new state of matter, the so-called quark-gluon plasma \cite{5,6}.

The quarkonium dissociation in a static deconfined quark matter is generally described in terms of the screening effect. The heavy quark potential, which is normally taken as the Cornell form \cite{7} and can be calculated through a non-relativistic quantum chromodynamic potential \cite{8} and lattice simulations \cite{9}, is reduced to a Yukawa-like potential due to the Debye screening.
When the screening radius becomes smaller than the quarkonium size, the bound state dissociates. Substituting the screened potential, extracted from lattice simulations \cite{10,11}, into the Schrödinger equation for the wave function of the quarkonium state, one obtains the dissociation temperature that corresponds to the zero binding energy and infinite size of the di-quark system \cite{12,13}. For charmonia, while the excited states $\chi_c$ and $\psi'$ start to dissociate already around $T_c$, the calculated dissociation temperature for the ground state $J/\psi$ is much higher than the critical temperature \cite{12}.

The quarkonia produced in relativistic heavy ion collisions are, however, not at rest in the medium. There exists a relative velocity between the quarkonia and the expanding medium. The question is what is the velocity dependence of the heavy quark potential at finite temperature \cite{14,15,16}. The screening effect is due to the rearrangement of the charged particles when a pair of heavy quarks (source) is present in the medium. For a moving source, it will take a longer time for the source to interact with the medium, comparing with that of a stationary source. This ‘delay’ of the response reduces the screening charges around the source and thus weakens the screening effect. In relativistic heavy ion collisions, the average transverse momentum of the initially produced $J/\psi$s is about 2 GeV at RHIC energy \cite{17} and 3 GeV at LHC energy. \cite{18}, corresponding to an averaged relative velocity above 0.5$c$. A significant modification of the Debye screening is expected for such fast moving $J/\psi$s, especially for those produced in higher transverse momentum region \cite{19,20}. In this Letter, we study the velocity dependence of the heavy quark potential and the quarkonium dissociation temperature in a transport approach. The velocity induced effects on charmonium suppression at both Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) will be discussed. In the following calculations we take the speed of light $c = 1$.

For a static source located at $r = 0$, the ambient charge density $\rho_0(r)$ is modified by the screening potential $V_0(r)$ at finite temperature $T$ \cite{21},

$$\rho_0(r) = \sum_i q_i f_i e^{-q_i V_0(r)/T} \approx -\sum_i (q_i^2 f_i/T)V_0(r),$$

(1)

where $q_i$ is the charge of the particles of species $i$, and $f_i$ is the initial particle density without the source. The neutrality condition for the total charge has been considered here. The solution of (1) with the assumption of small $V$ gives the Debye screening of the potential. At large distance, the potential is weak, so that the approximation in (1) is appropriate, while at small distance,
the solution of (1) means a small correction to the original potential, as the lattice simulations indicated \[10, 11\].

For a source moving with velocity \( \mathbf{v} \) with respect to the medium, the non-equilibrium charge density \( \rho(\mathbf{r}, t) \) in the source-rest frame satisfies the transport equation in the relaxation time approximation,

\[
\partial_t \rho - \mathbf{v} \cdot \nabla \rho = - (\rho - \rho_0) / \tau,
\]

where \( \tau \) is the relaxation time of the medium. Taking the limit \( t \to \infty \), the final distribution \( \rho_f(\mathbf{r}, L) \equiv \lim_{t \to \infty} \rho(\mathbf{r}, t) \) becomes stable and is characterized by the equation

\[
L \cdot \nabla \rho_f = \rho_f - \rho_0,
\]

where we have introduced the relaxation length defined as \( L \equiv \mathbf{v} \tau \) which controls the velocity dependence of the Debye screening. Since the screening charge distribution is proportional to the screening potential, see equation (1), the potentials \( V_0 \) and \( V \) corresponding to a stationary and moving source, respectively, satisfy the same equation (3) which can be solved analytically,

\[
V(\mathbf{r}, L) = \int_0^\infty V_0(\mathbf{r} + \lambda L)e^{-\lambda d}\lambda.
\]

It is obvious that for a static source with \( L = 0 \) we have \( V(\mathbf{r}, 0) = V_0(\mathbf{r}) \).

With the potentials \( V_0 \) and \( V \), the screening radius \( r_d \) can be expressed as

\[
r_d(L) = \frac{1}{2} \frac{\int d^3\mathbf{r} \rho_f(\mathbf{r}, L)}{\int d^3\mathbf{r} \rho_f(\mathbf{r}, L)} = \frac{1}{2} \frac{\int d^3\mathbf{r} rV(\mathbf{r}, L)}{\int d^3\mathbf{r} V_0(\mathbf{r})}.
\]

For the second equality, we have used the total charge conservation \( \int d^3\mathbf{r} \rho_f(\mathbf{r}, L) = \int d^3\mathbf{r} \rho_0(\mathbf{r}) \) for any \( L \) which is guaranteed by integrating Eq.(3) over the whole coordinate space. The screening radius \( r_d(L) \) is in general an angle dependent function. However, for a spherically symmetric potential \( V_0(\mathbf{r}) \), the integration over the angles in the numerator of Eq.(5) can be analytically done, and the averaged screening radius can be effectively expressed as

\[
r_d(L) = \int dr^3V(r, L)/(2 \int dr^2V_0(r)) \quad \text{with the factorized averaged potential}
\]

\[
V(r, L) = V_0(r)W(r/L),
\]

where the modification factor \( W \) is defined as

\[
W(y) = 1 + \frac{2 + y^2Z(y) - (y^2 - y + 2)e^{-y}}{3y^2},
\]

\[
Z(y) = \int_y^\infty \frac{dt}{t} e^{-t}.
\]
We emphasize that the general potential $V(r, L)$ can be simplified as $V(r, L)$ only in the sense of the screening radius (5). For a more detailed calculation about a general potential, one may refer to the Refs.[20, 22] based on the linear response theory.

Now we apply the above transport solutions to the quarkonium dissociation in hot and dense matter created in high-energy nuclear collisions. The interaction between two quarks in vacuum can be well characterized by the Cornell potential [12] $V_0(r) = -\alpha/r + \sigma r$ with coupling constant $\alpha = \pi/12$ and string tension $\sigma = 0.2 \text{ GeV}^2$. At finite temperature, the screening potential for a stationary pair of heavy quarks can be written as [23, 12]

$$V_0(r) = -\frac{\alpha}{r} e^{-\mu r} - \frac{\sigma}{2 \pi (\frac{3}{4})} \left( \frac{r}{\mu} \right)^{1/2} K_{\frac{3}{4}}((\mu r)^2).$$

(8)

where $\Gamma$ and $K$ are the Gamma and modified Bessel functions. The temperature of the medium is hidden in the screening mass $\mu(T)$ which can be extracted [12] from lattice QCD calculated free energy [10, 11].

To establish a unique mapping between the relaxation length and the velocity, we estimate the relaxation time of the hot and dense matter by considering its electric analogue. When an electric charge is put into a conducting medium, the medium is neutralized in a time scale of $\tau = 1/(4\pi\sigma_e\alpha_e)$, where $\sigma_e$ is the electric conductivity of the medium, and $\alpha_e$ is the fine-structure constant. We replace $\sigma_e$ by the conductivity $\sigma_s \approx 0.4T$ for a strong field, estimated from hot quenched lattice QCD [24], and $\alpha_e$ by $\alpha$, the relaxation length becomes $L = 15v/(2\pi^2 T)$.

In Fig.1 one sees the velocity induced change in the heavy quark potential at a fixed temperature $T = 1.5T_c$. The stationary potential is taken from the lattice simulation Eq. (8). Since the screening length is proportional to the velocity and inversely proportional to the temperature of the medium, the potential well becomes deeper and screening becomes less effective, when the quarkonium velocity relative to the medium increases.

With the known potentials $V_0(T, r)$ and $V(T, r, L)$, the screen radius $r_d(T, L)$ at finite temperature $T$ can be calculated through (5), where the temperature $T$ dependent inherited from potential $V$ is written explicitly. In the rest frame of the di-quark system, the condition for dissociating a quarkonium should not depend on its relative velocity, namely the critical screening radius is a constant,

$$r_d(T, L) = C.$$

(9)
where the constant $C$ can be calculated directly at $v = 0$,

$$C = r_d(T_d(v = 0), L = 0) = \frac{1}{\mu} \left( 1 + \frac{\pi}{16\mu^2(3/4)} \frac{\sigma}{\alpha \mu^2} \right)$$

(10)

with $\mu$ the screening mass at $T_d(v = 0)$. Thus when the dissociation temperature of a $J/\psi$ at rest is given, the constant $C$ can be calculated and Eq. (9) determines the dissociation temperature $T_d(v)$ for a moving quarkonium with velocity $v$. When $T_d(0)$ runs from $T_c$ to $2.5T_c$, the screening radius $r_d(T_d(0), 0)$ runs from 0.44 fm to 0.21 fm. The velocity-dependent temperature $T_d(v)$ is shown in Fig. 2. Since the lattice calculation of the stationary dissociation temperature $T_d(0)$ is still with some uncertainty, we take $T_d(0)$ as an adjustable parameter in Fig. 2. Once we fix $T_d(0)$, its velocity dependence can be obtained from the corresponding curve. As expected, when a quarkonium moves at a large velocity relative to the medium, the screening effect becomes weaker and the dissociation temperature becomes higher. The velocity induced shift of the dissociation temperature can be as large as $\Delta T_d(v) \sim T_c$ for fast quarkonia. For charmonium, the dissociation temperature is $T_d \sim 1-2T_c$ at $v = 0$ [12, 13] but goes up to 1.2-2.7$T_c$ at $v = 0.8c$.
see Fig. 2. Considering the fact that the fireball temperature formed in heavy ion collisions at RHIC energy is in the region $T \sim 1-2T_c$, the $J/\psi$ transverse momentum spectrum should be sensitive to the velocity dependence of the dissociation temperature. The much higher dissociation temperature for fast moving $J/\psi$s will lead to a weaker suppression in the high transverse momentum region.

![Figure 2: The scaled dissociation temperature $T_d(\nu)$, starting at different stationary values $T_d(0)$. $T_c=165$ MeV is the critical temperature of the quark matter, and the upper and lower thick lines are respectively for $J/\psi$ and the excited states $\psi'$ and $\chi_c$.](image)

In order to quantitatively see the effect of the velocity-dependent temperature $T_d(\nu)$ on quarkonium suppression in high-energy nuclear collisions, we take a detailed transport approach \[25\] to describe the dynamical evolution of the hot and dense medium. The model contains transport equations for the quarkonium motion in the medium and hydrodynamic equations for the space-time evolution of the medium. The initial distribution of energy density and entropy density is based on Glauber Model. Both local temperature $T(x, t)$ and local velocity $u(x, t)$ that used in the transport equation for quarkonia are solved from the hydrodynamic equations as in our previous work \[26\]. In heavy ion collisions there are two sources for quarkonium production: the primordial production at the initial state and the regeneration in the hot medium. During the evolution, all of the produced quarkonia suffer from the medium induced dissociation, dominantly by the gluon interactions.
The model used here describes well both $J/\psi$ [27, 28] and $\Upsilon$ [29] suppression. In order to demonstrate the velocity effect on the quarkonia suppression, it is necessary to study the transverse momentum distributions. We consider the differential nuclear modification factor $R_{AA}(p_t) = N_{AA}(p_t)/(N_{coll}N_{pp}(p_t))$ as a function of quarkonium transverse momentum $p_t$. $N_{AA}(p_t)$ and $N_{pp}(p_t)$ are differential quarkonium yields in heavy ion and elementary p+p collisions, and $N_{coll}$ is the number of nucleon+nucleon collisions in heavy ion collisions. We will focus on the high $p_t$ behavior of $R_{AA}(p_t)$.

Fig. 3 shows the $J/\psi$ $R_{AA}(p_t)$ for a constant and a velocity-dependent dissociation temperature in central Au+Au collisions at top RHIC energy $\sqrt{s_{NN}} = 200$ GeV. As one can see, at the low $p_t$ region ($\leq 3.5$ GeV), the experimental results of $J/\psi$ $R_{AA}(p_t)$ is less than 0.4. However, in the higher $p_t$ region, the value of the nuclear modification factor becomes higher $R_{AA}(p_t) \approx 0.6$ indicating weaker suppression in $J/\psi$ yield. Note that this $p_t$ dependence can not be reproduced by a constant dissociation temperature unless a strong Cronin effect is assumed even at extremely high $p_t$ [26]. As discussed in [30], the strong Cronin effect at high $p_t$ region is not favored by the latest $J/\psi$ data from d+Au collisions. In our calculation, the Cronin effect has been characterized by a Gaussian smearing scheme [31], which contribute little to the high $p_t$ region $p_t > 6$ GeV. For the stationary charmonia, the dissociation temperature calculated from the Schrödinger equation is in between $(1.1-2.1)T_c$ for the ground state $J/\psi$ and $T_c$ for the excited states $\psi'$ and $\chi_c$, depending on the used heavy quark potential [32, 33, 12].

Since the fireball temperature in a central collision is much higher than $T_c$, almost all the excited states are dissociated in the medium, we will consider mainly the ground state. Considering the fact that the contribution from the decay of the excited states to the final $J/\psi$s is about 40%, there is an upper limit of 0.6 for $J/\psi$ $R_{AA}$. In Fig. 3 the three dashed lines represent the results with a constant dissociation temperature $T_d = 1.3, 1.6,$ and $1.9T_c$ from bottom to top, respectively. As one can see in the figure, the numerical results with a constant $T_d$ are all much less than 0.6 and overestimate the $J/\psi$ suppression.

We now analyze the results with the velocity-dependent temperature, see the solid line in Fig. 3. Fitting the experimental data of the nuclear modification factor $R_{AA}(N_{part})$ as a function of the number of participant nucleons $N_{part}$, we obtain $T_d(0) = 1.6T_c$. Note that for an expanding fireball, $p_t = 0$ in the laboratory frame corresponds generally to a nonzero
The $J/\psi$ nuclear modification factor $R_{AA}(p_t)$ at RHIC. The data are from the PHENIX \cite{3} at rapidity $|y| < 0.35$ and STAR \cite{34} at rapidity $|y| < 0.9$, the solid line is the calculation with a velocity-dependent temperature starting at $T_d(0)/T_c = 1.6$, and the dashed lines are the calculations with a constant dissociation temperature $T_d = 1.3T_c, 1.6T_c, 1.9T_c$ from bottom to top.

velocity in the rest frame of the fireball, therefore the velocity-dependent temperature even at $p_t = 0$ is already affected by the velocity $v$. That is why the $R_{AA}$ at $p_t = 0$ does not coincide with the calculation with a constant $T_d = 1.6T_c$. From Fig.2 one sees that the increase of the dissociation temperature is approximately linear at high velocity. On the other hand, the maximum temperature of the fireball in a central Au+Au collision is $T_{\text{max}} \approx 2T_c$ \cite{35,36}. Therefore, at sufficiently high transverse momentum, the dissociation temperature may stay above the maximum temperature. For example, the velocity of those $J/\psi$s at $p_t \sim 5$ GeV is above $0.8c$, and the dissociation temperature $T_d$ is about $2.3T_c$. As a result, those high $p_t$ $J/\psi$s will survive in the quark gluon plasma. For the same reason, those high $p_t$ excited states $\psi'$s and $\chi_c$s produced in the more peripheral region where the temperature is lower than their dissociation temperature will also survive. The competition between the velocity ($p_t$) dependent dissociation temperature and the fireball temperature leads to $R_{AA} > 0.6$ at high $p_t$, as shown in Fig.3. It is clear in the figure that our calculation with the velocity-dependent temperature is consistent with the experimental observation. High
statistics data are needed in order to confirm this ansatz for quarkonium suppression in high-energy nuclear collisions.

\[ \mu_T = 2.76 \text{ A TeV} \]

| $p_t$ (GeV) | $R_{AA}$ |
|------------|----------|
| 0          | 1.0      |
| 5          | 0.5      |
| 10         | 0.0      |

Figure 4: The $J/\psi$ nuclear modification factor $R_{AA}(p_t)$ at LHC. The data are from the CMS Collaboration at rapidity $|y| < 2.4$ [37], the solid line is the calculation with a velocity-dependent temperature starting at $T_d(0)/T_c = 1.6$, and the dashed lines are the calculations with a constant dissociation temperature $T_d = 1.3T_c, 1.6T_c$ and $1.9T_c$ from bottom to top.

In order to further test the model, the $p_t$ dependence of the $R_{AA}(p_t)$ for prompt $J/\psi$s from minimum bias Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV is also calculated in our model. The results are compared with the experimental data [37] in Fig.4. For this calculation, the charm quark production cross section is taken as $d\sigma_{NN}^c/dy = 0.62$ mb at midrapidity [38, 39]. Significant regeneration has been reported with the large charm cross section [38, 39]. Since the charm quarks interact strongly with the medium, losing its initial energy, the regenerated charmonia are soft, leading to a large $R_{AA}$ in the low $p_t$ region. Similar to the case at RHIC, the CMS experimental value of $R_{AA} \approx 0.3$ at high $p_t$ can be reproduced only when the velocity-dependent temperature is considered, while all other results with a constant $T_d$ underpredict the values of $R_{AA}$. At the same $p_t \sim 6$ GeV region, the value of $R_{AA}$ from LHC is much lower than that from RHIC, implying that a much hotter medium has been formed in heavy ion collisions at the higher energy. At $p_t \sim 10$ GeV, the velocity is above $0.9c$, so that $T_d$ is about
2.5Tc, which is still smaller than the highest temperature of the fireball at LHC.

In summary, we studied the heavy quark potential and dissociation temperature for moving quarkonia in quark gluon plasma in high-energy nuclear collisions. For a moving heavy quark pair in the hot medium, the screening potential is reduced and the dissociation temperature is enhanced. As a consequence of the velocity-dependent dissociation temperature, the J/ψ suppression becomes significantly weaker at high transverse momentum.

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