A High-Precision Moving Target Tracking Algorithm Based on UKF

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Abstract. Since Kalman Filter is not applicable to non-linear systems, a moving target tracking algorithm based on Untracked Kalman Filter (UKF) is proposed. The algorithm performance is analyzed by algorithm principle analysis, building mathematical model of the system, and simulation GUI design. Simulation results show that the algorithm has an expected tracking precision and high reliability in 6-dimensional CA mode, and has a good application prospect.

1. Introduction
In recent years, moving robot technology gains continuous development and increasingly wide applications. In the process of target tracking, there are such random factors as odometer error accumulation, sensor noise, and ambient noise which reduce the precision of target tracking. Therefore, Kalman Filter (KF) receives wide applications for its optimal estimation and recursive calculation.

Standard KF is applicable to systems where the process and measurement are linear and the errors submit Gaussian distribution. In fact, many systems have some nonlinearity, for example, the state equation or measurement equation is nonlinear. Standard KF no longer applies to these systems. The solution is to transform the nonlinear system into an approximate linear system. The two most widely used methods are Expand Kalman Filter (EKF) and Untracked Kalman Filter (UKF). For UKF, the posterior probability is approximated by a set of determined sampling points by Unscented Transform (UT). UKF has the advantages of less calculation and high convergence rate with no higher derivative ignored. The paper focuses on the moving target tracking algorithm based on UKF and 6-dimensional parameters. The simulation model is established to verify the effectiveness of the target tracking algorithm in 6-dimensional constant accelerated (CA) mode.

2. Modeling Moving Target Tracking System
Aiming at establishing the trajectory of the target, the target tracking system consists of three components: pre-processing, position estimation and tracking filter. Pre-processing de-noises the initial data to minimize the initial error of the system. Position estimation is to determine the initial position of the target by the signals obtained from sensors. The key component, tracking filter reduces the influence of random factors on the precision and implements continuous prediction and estimation of the target’s motion states.

The fundamental principles of target tracking are shown in Figure 1, where X represents state vector, including space parameters of position, velocity and acceleration, V is the observed noise, and Z stands for observed values, the expression of which is shown in equation (1).

\[ Z = HX + V \] (1)
Target tracking mainly include the following three steps.

The prediction value of target state at the time of $k + 1$, $HX_{k+1/k}$, is acquired by the target state equation at the time of $k$.

The moving state of the target is determined by the residual vector $D$, the difference between the observed value $Z$ and the predicted value $HX_{k+1/k}$ at the time of $k + 1$.

$$D = Z_{k+1} - HX_{k+1/k} \tag{2}$$

Filtering algorithm is applied to seek $X_{k+1}$, the estimation value of target state at the time of $k + 1$.

It can be seen from the above steps that prediction and filtering are the two key steps in target tracking system, and the key to prediction is to model moving target tracking system.

**3. Constant Acceleration (CA) Model**

To keep the real-time tracking, modeling moving target tracking system should consider both characteristics of moving target and mathematical calculation quantity. The performance of target tracking system directly depends on the construction of moving target model. Since the target has randomness in the process of motion, and there are also uncertain external noises, the motion process cannot be accurately described by mathematical models. Approximate estimation methods are generally adopted. Several classical target motion models are CV model, CA model, CT model, Singer model, etc. CA model is applied in the paper.

In CA model, target moves with fixed acceleration. For a moving target in 2-dimensional space, its position, velocity and acceleration at time of $k$ can be represented by state vector $x_k$ shown in equation (3).

$$x(k) = \begin{bmatrix} x_k \ y_k \ x_k' \ y_k' \ x_k'' \ y_k'' \end{bmatrix} \tag{3}$$

Assuming that a moving target makes approximate linear motion with constant acceleration in the horizontal direction and approximate linear motion with constant acceleration in the vertical direction, its state equation of discrete time in the Cartesian coordinate system can be expressed as equation (4),
where $F_k = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $T$ is the sampling time, and $w(k)$ represents horizontal and vertical additive system noises with the mean of 0 and the covariance of Q.

$$x(k + 1) = F_k x(k) + w(K)$$  \hspace{1cm} (4)

4. **Untracked Kalman Filter (UKF)**

The major function of tracking filtering algorithm is to estimate and predict the target's current motion state. UKF is a production of the fusion of UT and standard KF. UT makes non-linear system equations apply to the linear standard KF system. UKF uses fewer sample points to represent the distribution of states. These sample points can accurately capture the mean and covariance matrices of Gaussian random variables. Applied to nonlinear systems, UKF receives a more than third-order precision.

UKF algorithm mainly consists of the following four steps.

**Initialization.**

$$\hat{x}_0 = \mathbb{E}[x_0]$$  \hspace{1cm} (5)

**Calculation of Sigma**

$$x_k = [x_k, x_k + \gamma \sqrt{P_k}, x_k - \gamma \sqrt{P_k}]$$  \hspace{1cm} (6)

**Update Time**

$$\dot{X}_{k+1/k} = \sum_{i=0}^{2^n} W_i^m X_{i,k+1/k}$$  \hspace{1cm} (7)

$$z_k = H[x_{k+1/k}]$$  \hspace{1cm} (8)

$$\hat{Z}_{k+1/k} = \sum_{i=0}^{2^n} W_i^m Z_{i,k+1/k}$$  \hspace{1cm} (9)

**Update Observation Values.**

$$\hat{X} = X_{k+1/k} + K_{k+1}(Z - \hat{Z}_{k+1/k})$$  \hspace{1cm} (10)

The implementation steps show the advantages of UKF. Calculation of the Jacobian matrix is omitted. It is not necessary to linearize the state equation and the measurement equation in the nonlinear system. System functions can be discontinuous.

5. **Simulation & Analysis**

It is designed a moving target tracking algorithm combining UKF and 6-dimensional parameters. As is shown in Figure2, the GUI is designed to facilitate the comparison of algorithm performances.
Figure 2. GUI for System Simulation

It is analyzed that the performance of UKF in 6-dimensional CA motion model. Suppose the system noise \( w(k) \) has covariance matrix \( Q_k \), and \( v(k) \) has the covariance matrix \( R_k \). \( w(k) \) and \( v(k) \) are not correlated. The number of observations \( N \) is 2000, sampling time \( t \) is 0.5s, initial state \( x(0) = [1000,0,0,0,0,0] \), the variance of input noise is 1-20, and the variance of output noise is 200. Simulation results are shown in Figure 3 and Figure 4.

Figure 3. Moving trajectory of UKF in 6D CA Model
Figure 4. Error Analysis of UKF in 6D CA Model

It can be seen from the simulation results that the actual trajectory of the target is basically consistent with the average trajectory predicted by filtering, which reveals that the mobile target tracking system based on UKF has the high precision, and has no target tracking loss in 6-dimensional CA model. Despite that the covariance error of UKF is still large, the mean error is very small. Simulation results clearly show that UKF has better performance than standard KF in the case of high non-linearity.

Consider the effects of different motion parameters on moving target tracking system. Figure 5 respectively shows the trajectory diagrams in six dimensions and two dimensions generated by the moving target tracking system based on the UKF. The comparison results show that the higher the spatial parameter dimension, the better the precision of the system.

Figure 5. Comparison between the moving trajectory of 6D parameters and that of 2D parameters
6. Conclusion
It is presented in the paper a moving target tracking algorithm based on UKF according to UKF principles and moving target tracking mathematical models. Finally, the target tracking in 6-dimensional CA mode is achieved. Simulation results reveal that the algorithm has high tracking precision, good real-time performance and high reliability. The algorithm is especially applicable to nonlinear moving target tracking systems and has a good application prospect.

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