The study of $^{12}\text{C}(\alpha,\gamma)$ astrophysical reaction using $^{12}\text{C} (^6\text{Li},d)$ and $^{12}\text{C} (^7\text{Li},t)$ reaction at 20 MeV and in the framework of the potential model

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Abstract. The triton angular distribution in the $^{12}\text{C}(^7\text{Li},t)^{16}\text{O}$ reaction has been measured at 20 MeV incident energy. Comparison of the data with Finite Range DWBA and CDCC-CRC calculations show that breakup coupling effects are prominent in the transfer to the ground state. This observation is similar to that in the $^{12}\text{C}(^6\text{Li},d)$ reaction at the same incident energy. The alpha spectroscopic factor of the $^{16}\text{O}$ ground state is determined ($S_{\alpha,\gamma}$=0.25) from a comparison of the measured angular distribution with respect to the CDCC-CRC calculations. The E2 S-factor of the $^{12}\text{C}(\alpha,\gamma)$ reaction at 300 keV in the framework of a potential model is determined to be about 118 keV-barn.

1 Introduction

The $^{12}\text{C}(\alpha,\gamma)$ fusion (capture) cross-section at Ecm.$\sim$300 keV determines the ratio of the abundance of $^{16}\text{O}$ to $^{12}\text{C}$ in helium burning stars [1]. A direct measurement of the cross-section at such low energy ($\sigma \sim 10^{-19}$ barn) is severely hampered by the presently available techniques. The lowest energy at which the reaction has been measured in the laboratory is at Ecm.<0.9 MeV. So generally an extrapolation of the astrophysical S-factor instead of the cross-sections ($S(E)=\sigma(E)\times E\exp(2\eta(E))$, where $\eta$ is the Sommerfield parameter) is carried out using theoretical models to evaluate $S(E)$ at 300 keV. The total S-factor is considered to have contributions from primarily E1 and E2 capture processes and their interferences. The energy dependence of both these processes are different. The E1 contribution to the S-factor (SE1) is reasonably well understood in comparison to the E2 contribution. The E2 contribution to the S-factor (SE2) at 300 keV is due to a E2 transition from the 6.92 MeV ($2^+$) state to the ground state following a subthreshold $\alpha+^{12}\text{C}$ capture process. The subthreshold capture process is possible due to the extended tail of the alpha+$^{12}\text{C}$ bound state wave-function that goes above the threshold (7.16 MeV) to enable the capture process. The magnitude of this tail is determined by the Asymptotic Normalization Constant (ANC) of the state and is related to the reduced alpha width of the state. It is one of the main inputs in the theoretical model for extrapolation and is generally obtained by fitting the available experimental S-factor data [2]. The reduced alpha width of $^{16}\text{O}$ can be additionally constrained by $^{12}\text{C}(\alpha,\alpha)$ elastic scattering data and alpha transfer reaction on $^{12}\text{C}$. The alpha transfer reactions such as $^{12}\text{C}(^6\text{Li},d)$ [3,4] and $^{12}\text{C}(^7\text{Li},t)$ [5,6] at various incident energies have been used to determine the reduced alpha width of $^{16}\text{O}$ states. The Trojan Horse method has been also used to study the $^{12}\text{C}(\alpha,\gamma)$ reaction [7]. The transfer reaction method in principle is applicable at higher energy where cross-sections are larger. A comparison of the measured angular distributions with respect to an appropriate calculation yields the alpha spectroscopic factor ($S_{\alpha}$). The alpha spectroscopic factor can be used to obtain the ANC if the reaction is peripheral. Moreover if one uses the potential model the E2 S factor to the ground state depends on the ANC of the 6.92 MeV state and the alpha spectroscopic factor of the ground state. The determination of $S_{\alpha}$ however depends critically on the model used to analyse the data. In earlier calculations, the Finite Range Distorted Wave Born Approximation (FRDWBA) has been used assuming a direct alpha transfer process. However, $^6\text{Li}$ being loosely bound nuclei are likely to breakup and can affect the transfer process and thereby the $S_{\alpha}$. This can be investigated in the framework of the Continuum Discretized Coupled Channel (CDCC-CRC) theory. In two separate experiments angular distributions were measured for the $^{12}\text{C}(^6\text{Li},d/t)$ reactions at 20 MeV populating the discrete states of $^{16}\text{O}$. The analysis of the $^{12}\text{C}(^6\text{Li},d)$ reaction indicated the effect of very strong breakup coupling for the population of the $^{16}\text{O}$ ground state [4]. In this respect the analysis of the $^{12}\text{C}(^7\text{Li},t)$ reaction is interesting because $^7\text{Li}$ has a higher breakup threshold (2.47 MeV) in comparison to $^6\text{Li}$. As such whether breakup influences the angular distributions as for $^6\text{Li}$ is one of the motivation of this paper.
2 Experiment

The experiment was carried out at the General Purpose Scattering Chamber (GPSC) facility of the Inter University Accelerator Centre, New Delhi. A 7Li3+ beam at 20 MeV was bombarded onto a 200μg/cm2 12C self supporting target in the 1.5 m scattering chamber. Three DE-E silicon telescopes each 200-2000micron were setup to perform the angular distribution measurements at 18-124 degrees in the laboratory. The average beam current was about 15-20 nA.

In fig.1 we show the measured energy spectra for the 12C(7Li,t)16O* reactions at 40 degrees. The energies of the discrete states of 16O populated in the reaction are marked in the figure.

3 Results and discussions

The alpha spectroscopic factor of a nuclear state [8] is obtained as structure factor taken out of the many body nuclear state in a single particle approximation of the state. Thus in the single particle approximation the 16O nuclear state in a single particle approximation of the many body wave function can be written as \( \psi = S_{\alpha} u \)

where \( S_{\alpha} \) is the alpha spectroscopic factor and \( u \) is the \( \alpha^+12C \) bound state wavefunction generated from some single particle potential. The tail part of both \( y \) and \( u \) tend to be Whittaker functions with constants \( C \) and \( b \) respectively. \( C \) and \( b \) are the ANC and single particle ANC of the corresponding state [8]. Thus the relation \( C^2 = S_{\alpha}^2 \)

arrived at can be used to determine the ANC of the 6.92 MeV state if the alpha spectroscopic factor is determined from comparison of the measured transfer angular distributions with respect to an appropriate theoretical calculations [9]. The 1C(7Li,d) or 1C(7Li,t) reactions at 20 MeV populating discrete states of 16O can have a complex reaction mechanism. These states can be populated from either a compound nuclear reaction or a direct alpha transfer process (BIT). The compound nuclear emissions in the reaction can be calculated by using Hauser Feshbach theory. The Finite range Distorted Wave Born Approximation (FRDWBA) theory can explain emissions resulting from direct alpha transfer. The Continuum Discretized Coupled Channel Reaction (CDCC-CRC) theory is the most appropriate formalism to understand the BIT process.

3.1 CDCC-CRC theory

In the CDCC-CRC theory [5], the cross-section is calculated by solving the coupled differential equation

\[
(T - E - \langle \alpha | V | \alpha \rangle) \psi_{\alpha} = \sum_{\beta} (\alpha | U | \beta) \psi_{\beta} r^{-1} \tag{1}
\]

Where \( \langle \alpha | U | \beta \rangle \) is the coupling matrix and \( \alpha, \beta \) are intrinsic states involved in the inelastic or transfer process. The breakup continuum and resonant states of \( ^7Li \) couple inelastically with the ground state as well as with each other. The transfer coupling is between the projectile states (ground and the breakup states) and the discrete states of \( ^{16}O \) populated in the reaction. The intrinsic states are in general many body nuclear states. These are approximated in terms of single particle states in a two body cluster model with an appropriate binding potential. Thus \( ^7Li \) and \( ^{16}O \) nuclei are modelled in terms of \( \alpha^+d \) and \( \alpha^+12C \) two body structure. The alpha particle is considered as the valence or the transferred particle and \( d \) and \( ^12C \) are considered the core nuclei. In the DWBA approximation there is only one coupled state other than the ground state and the cross-section is linearly dependent on the spectroscopic factor. Unlike in DWBA the summation on the RHS of the coupled equation can involve many states and their corresponding spectroscopic amplitudes and the cross-section is no longer a linear function of the amplitudes.

3.1.1 FRESCO calculations

The angular distributions in terms of the FRDWBA and CDCC-CRC formalism can be conveniently carried out in terms of the Coupled Channel program FRESCO [8,10]. In this program the coupled equations are solved by Numerov method using appropriate interaction potentials.

The potentials required for the FRESCO calculations of the 1C(7Li,d) angular distributions are the optical potentials in the entrance and exit channel, the real binding potentials of \( ^6Li \) and \( ^{16}O \) and the \( d/t-12C \) core optical potentials. In the CDCC-CRC formalism the entrance channel potential is obtained by a folding of the cluster target potentials. The continuum of \( ^7Li \) in the present work extends from \( \varepsilon=2.47 \) to 8.00 MeV (\( \varepsilon \) being the relative energy) and contains two resonances at 4.63 MeV (7/2-) and 6.68 MeV (5/2-). The calculations were performed for \( l=0,1,2,3 \). The two resonances of \( ^7Li \) were generated from an appropriate \( \alpha+d \) binding potential. The phase shift variation with respect to the relative energy of the \( \alpha+d \) system was studied in terms of the depth of the binding potential. The resonance can be observed as a rapid increase of the phase shift near the resonance energy (\( \varepsilon=90 \text{ deg} \)). The width of the resonance is
obtained by 2 times the inverse of the slope of the $\delta \, v/s \, \epsilon$ curve at the resonance energy. The continuum bin size for $^7$Li should be chosen wide enough so that the entire resonance width calculated from the model falls within the bin.

Figure 2. Comparison of the FRDWBA and CDCC-CRC calculations in $^{12}$C($^7$Li,t)$^{16}$O$_{gs}$ reaction at 20 MeV incident energy.

Figure 3. Comparison of the FRDWBA and CDCC-CRC calculations for the $^{12}$C($^6$Li,d)$^{16}$O$_{gs}$ reaction at 20 MeV incident energy, data are from [3].

3.2 Results of the calculations

The triton angular distributions measured in $^{12}$C($^7$Li,t)$^{16}$O$_{gs}$ reaction at 20 MeV populating the ground state of $^{16}$O is shown in fig.2. The direct alpha transfer process is calculated using the FRDWBA formalism in FRESCO. This calculation is shown by black line in the figure. The alpha spectroscopic factor in this calculation is assumed to be 1. It is clear that the FRDWBA calculations are unable to explain the observed angular distributions. The CDCC-CRC calculations are then performed and shown by red line. It is now seen that the inclusion of breakup coupling in the calculation improves the agreement significantly. This is a similar observation as in the $^{12}$C($^6$Li,d) reaction at 20 MeV. The BIT mechanism is thus found to be significant for the $^{12}$C($^7$Li,t)$^{16}$O$_{gs}$ reaction. The spectroscopic factor using CDCC-CRC theory for the $^{12}$C($^7$Li,t) is 0.25 and that from $^{12}$C($^6$Li,d) is $0.1^{+0.09}_{-0.06}$.

4 Potential Model calculation of SE2

In this work we present a potential model calculation of the E2 astrophysical S-factor in the framework of a potential model [8]. In the potential model we consider a potential that generates a $\alpha^{12}$C scattering state through which the capture occurs and finally decays to the ground state by E2 gamma transition.

Figure 4. Potential model calculation of the E2 S-factor using the ANC of the 6.92 MeV states from Ref [5,11,12] and the ground state $\alpha$ from the present work. For details please see the text. Experimental data are taken from [13] and [14].

It is well known that this scattering state should have the property of the 6.92 MeV subthreshold state responsible for the capture. With an appropriate choice of a Wood Saxon potential we reproduced an average ANC of $5.11 \pm 1.52 \times 10^{10}$ fm$^{-1}$ of the 6.92 MeV state from earlier works [5,11,12]. The SE2 also depends in the potential model on the alpha ground state spectroscopic factor. The red line shows the calculations using the spectroscopic factor extracted from the $^7$Li+$^{12}$C transfer reaction and the green dashed line shows the calculation using the spectroscopic factor extracted from the $^6$Li+$^{12}$C transfer reaction. The blue dashed line represents the calculation where the scattering state reproduces the features of 9.84 MeV resonance state by a choice of different Wood Saxon potential parameters. The black solid line is the incoherent sum of the red line and the blue line. This simple potential model calculation yield SE2(300)=118 keV-barn using the $S_\alpha$ extracted from $^{12}$C($^7$Li,t) reaction in the present work.

S. Adhikari acknowledges DST grant SR/WOS-A/PS-50/2011(G) for this work.

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