Supplementary Material for ‘Semiparametric Regression on Cumulative Incidence Function with Interval-Censored Competing Risks Data and Missing Event Types’

JUN PARK\(^1,2\), GIORGOS BAKOYANNIS\(^*\), YING ZHANG\(^3\), CONSTANTIN T. YIANNOUTSOS\(^1\)

\(^1\)Department of Biostatistics, Indiana University, Indianapolis, IN 46202, USA
\(^2\)Merck & Co., Inc., North Wales, PA 19454, USA
\(^3\)Department of Biostatistics, University of Nebraska Medical Center, College of Public Health, Omaha, NB 68198, USA

gbakogia@iu.edu

\(^*\)Giorgos Bakoyannis, Email: gbakogia@iu.edu
Appendix I. Proof of Theorem 1 (double-robustness)

To show the double robustness property of the proposed estimator we will use empirical process theory (Kosorok, 2008; Van der Vaar and Wellner, 1996). In this section we use the standard empirical process notations $P f = \int_{X} f(x) dP(x)$ and $P_n = n^{-1} \sum_{i=1}^{n} f(X_i)$ for a measurable function $f : \mathcal{X} \to \mathbb{R}$, where $\mathcal{X}$ is the sample space. Also, let $K$ be a generic constant, that could differ from place to place. We now define empirical process. Now, define the functions

$$\tilde{l}_{\theta, \xi, \psi}(X) = \sum_{j=1}^{k} \tilde{\Delta}_j^{(1)}(\xi, \psi) \log \{ F_j(U; Z, \theta_j) - F_j(V; Z, \theta_j) \}$$

$$+ \sum_{j=1}^{k} \tilde{\Delta}_j^{(2)}(\xi, \psi) \log \{ F_j(U; Z, \theta_j) \}$$

$$+ (1 - \Delta) \log \left\{ 1 - \sum_{j=1}^{k} F_j(V; Z, \theta_j) \right\}$$

for a generic observation $X \in \mathcal{X}$, and

$$l_{\theta}(X) = \sum_{j=1}^{k} \Delta_j^{(1)} \log \{ F_j(U; Z, \theta_j) - F_j(V; Z, \theta_j) \}$$

$$+ \sum_{j=1}^{k} \Delta_j^{(2)} \log \{ F_j(U; Z, \theta_j) \}$$

$$+ (1 - \Delta) \log \left\{ 1 - \sum_{j=1}^{k} F_j(V; Z, \theta_j) \right\}$$

Note that based on this notation, obtaining the proposed augmented inverse probability weighting sieve estimator of $\theta$ requires to maximize $\mathbb{P}_n \tilde{l}_{\theta, \xi_n, \psi_n} \equiv \tilde{M}_n(\theta; \hat{\xi}_n, \hat{\psi}_n)$. If there were no missing event types one would need to maximize $\mathbb{P}_n l_{\theta} \equiv M_n(\theta)$. Bakoyannis and others (2017) The latter objective function can be seen as an estimator of $P l_{\theta} \equiv M(\theta)$. In this work, similarly to Bakoyannis et al. Bakoyannis and others (2017), we assume the following regularity conditions:

C1. $Z$ and $A$ are bounded in the sense that there exists a $K \in (0, \infty)$ such that $\text{Pr}(\|Z\| \vee \|A\| \leq K) = 1$. Moreover, $E(ZZ^\top)$ is a non-singular.
C2. For $j = 1, 2, \cdots, k$, $\beta_{0,j} \in \mathcal{B}$, where $\mathcal{B}$ is a compact subset of $\mathbb{R}^d$.

C3. There exists $\eta > 0$ such that $P(U - V \geqslant \eta) = 1$ and the unions of the supports of $U$ and $V$ are contained in $[a, b]$ for $0 < a < b < \infty$. Also, and $0 < \min_{j \in \{1, 2, \ldots, k\}} F_j(a; Z = 0) < \sum_{j=1}^k F_j(b; Z = 0) < 1$.

C4. $\phi_{0,j} \in \Phi$, where $\Phi$ is a set of functions whose $p$th derivative is bounded in $[a, b]$ for $p \geq 1$, and the first derivative of $\phi_{0,j}$ is strictly positive and continuous on $[a, b]$, for $j = 1, \ldots, k$.

C5. The joint density of $(V, U)$ conditional on $Z$ has bounded partial derivatives with respect to $(v, u)$, whose bounds do not depend on $(v, u, z)$.

C6. There exists $\kappa$ for $0 < \kappa < 1$ such that
\[
a^\top \text{Var}(Z|V)a \geq \kappa a^\top E(ZZ^\top|V)a \quad \text{and} \quad a^\top \text{Var}(Z|U)a \geq \kappa a^\top E(ZZ^\top|U)a
\]
a.s. for all $a \in \mathbb{R}^d$.

C7. The parametric model $\pi_j(O_i; \psi)$, $j = 1, \ldots, k$, is continuously differentiable in $\psi$. Moreover, $\hat{\psi}_n \xrightarrow{p} \psi^*$ and $\sqrt{n}(\hat{\psi}_n - \psi^*) = n^{-1/2} \sum_{i=1}^n \omega_i + o_p(1)$, where $E\omega_1 = 0$ and $E\|\omega_1\|^2 < \infty$.

C8. The parametric model $\rho(O_i; \xi)$ is continuously differentiable in $\xi$ and satisfies $\rho(O_i; \xi) > 0$ a.s.. Moreover, $\hat{\xi}_n \xrightarrow{p} \xi^*$ and $\sqrt{n}(\hat{\xi}_n - \xi^*) = n^{-1/2} \sum_{i=1}^n \phi_i + o_p(1)$, where $E\phi_1 = 0$ and $E\|\phi_1\|^2 < \infty$.

Conditions C1–C6 guarantee the consistency of $\hat{\theta}_n$ and the $\sqrt{n}$-consistency and asymptotic normality of the regression coefficient estimator for the B-spline sieve maximum likelihood estimator for interval-censored competing risks data without missing event types. Bakoyannis and others (2019) Conditions C7 and C8 are required for the proposed augmented inverse probability weighted sieve maximum likelihood estimator for dealing with missing event types. These additional conditions are satisfied if the parametric models $\rho$ and $\pi_j$, $j = 1, \ldots, k$, are specified.
as regular generalized linear models and estimated through maximum likelihood. The positivity condition \( \rho(O_i; \xi) > 0 \) a.s. is expected to be satisfied in general in practice.

To show the consistency of the proposed estimator we need to prove the following conditions:

(i) \( \sup_{\theta \in \Theta_n} |\tilde{M}_n(\theta; \hat{\xi}_n, \hat{\psi}_n) - M(\theta)| \equiv |\tilde{M}_n(\theta; \hat{\xi}_n, \hat{\psi}_n) - M(\theta)||_{\Theta_n} \overset{p}{\to} 0 \)

(ii) \( \sup_{d(\theta, \theta_0) \geq \varepsilon} M(\theta) < M(\theta_0) \)

(iii) The sequence of the estimators \( \hat{\theta}_n \) satisfies \( \tilde{M}_n(\hat{\theta}_n; \hat{\xi}_n, \hat{\psi}_n) \geq \tilde{M}_n(\theta_0; \hat{\xi}_n, \hat{\psi}_n) - o_p(1) \)

For condition (i) we have

\[
\|\tilde{M}_n(\theta; \hat{\xi}_n, \hat{\psi}_n) - M(\theta)||_{\Theta_n} \lesssim \|\tilde{M}_n(\theta; \hat{\xi}_n, \hat{\psi}_n) - \tilde{M}_n(\theta; \xi^*, \psi^*)\|_{\Theta_n} \\
+ \|\tilde{M}_n(\theta; \xi^*, \psi^*) - M_n(\theta)\|_{\Theta_n} \\
+ \|M_n(\theta) - M(\theta)\|_{\Theta_n} \\
\equiv A_n + B_n + C_n. \tag{0.1}
\]

For the first term we have

\[
A_n \leq \sum_{j=1}^{k} \left\| \frac{1}{n} \sum_{i=1}^{n} \left\{ \tilde{\Delta}_{ij}^{(1)}(\hat{\xi}_n, \hat{\psi}_n) - \tilde{\Delta}_{ij}^{(1)}(\xi^*, \psi^*) \right\} \log \{ F_j(U_i; Z_i, \theta_j) - F_j(V_i; Z_i, \theta_j) \} \right\|_{\Theta_n} \\
+ \sum_{j=1}^{k} \left\| \frac{1}{n} \sum_{i=1}^{n} \left\{ \tilde{\Delta}_{ij}^{(2)}(\hat{\xi}_n, \hat{\psi}_n) - \tilde{\Delta}_{ij}^{(2)}(\xi^*, \psi^*) \right\} \log \{ F_j(U_i; Z_i, \theta_j) \} \right\|_{\Theta_n} \\
\leq K \sum_{j=1}^{k} \left[ \frac{1}{n} \sum_{i=1}^{n} \left\{ \tilde{\Delta}_{ij}^{(1)}(\hat{\xi}_n, \hat{\psi}_n) - \tilde{\Delta}_{ij}^{(1)}(\xi^*, \psi^*) \right\} \frac{\log \{ F_j(U_i; Z_i, \theta_j) - F_j(V_i; Z_i, \theta_j) \}}{\Theta_n} \right] \\
+ \frac{1}{n} \sum_{i=1}^{n} \left\{ \tilde{\Delta}_{ij}^{(2)}(\hat{\xi}_n, \hat{\psi}_n) - \tilde{\Delta}_{ij}^{(2)}(\xi^*, \psi^*) \right\} \left[ \log \{ F_j(U_i; Z_i, \theta_j) \} \right]_{\Theta_n}
\]

Under this inequality, conditions C1, C7, C8, and Taylor expansion it follows that \( A_n \overset{p}{\to} 0 \). For
If either \( C_1, C_7, \) and \( C_8 \) it follows that
\[
d \rightarrow 0
\]
and others. Bakoyannis (2017) showed that \( C_n \) a.s. or
\[
E(\Delta_{ij}^{(1)}|O_i) = \pi_j(O_i; \psi^*) \text{ a.s.},
\]
then, in light of condition C8, it follows that
\[
E \left\{ \hat{\Delta}_{ij}^{(1)}(\xi^*, \psi^*) - \Delta_{ij}^{(1)} \right\} = 0.
\]
Similarly, if either \( \rho(O_i; \xi^*) \) or \( \pi_j(O_i; \psi^*) \) is correctly specified, then
\[
E \left\{ \hat{\Delta}_{ij}^{(2)}(\xi^*, \psi^*) - \Delta_{ij}^{(2)} \right\} = 0.
\]
Therefore, in light of (0.2), \( B_n \) a.s. Finally, Bakoyannis and others (2017) showed that \( C_n \) a.s. and, thus, based on (0.1), condition (i) is satisfied.

Condition (ii) has been shown by Bakoyannis and others (2017). Finally, by Taylor expansion and conditions C1, C7, and C8 it follows that
\[
\hat{\Delta}_{ij}^{(1)}(\hat{\xi}_n, \hat{\psi}_n) = \hat{\Delta}_{ij}^{(1)}(\xi^*, \psi^*) + o_p(1)
\]
and
\[
\hat{\Delta}_{ij}^{(2)}(\hat{\xi}_n, \hat{\psi}_n) = \hat{\Delta}_{ij}^{(2)}(\xi^*, \psi^*) + o_p(1).
\]
Thus,
\[
\hat{M}_n(\theta_n; \hat{\xi}_n, \hat{\psi}_n) - \hat{M}_n(\theta_0; \hat{\xi}_n, \hat{\psi}_n) = \hat{M}_n(\theta_n; \xi^*, \psi^*) - \hat{M}_n(\theta_0; \xi^*, \psi^*) + o_p(1).
\]
Now, using the same arguments to those used in the consistency proof in Bakoyannis and others (2017) leads to the conclusion that condition (iii) is satisfied. Therefore,
\[
d(\hat{\theta}_n, \theta_0) \overset{p}{\to} 0.
\]
Table 1. The set of variables in \textit{simdata\_aipw}

| Variables | Description                  |
|-----------|------------------------------|
| id        | an unique individual identifier |
| v         | a last observation time prior to the event |
| u         | a first observation time after the event |
| c         | an event type                |
| z1        | a binary covariate           |
| z2        | a continuous covariate       |
| a         | an auxiliary variable        |

Appendix II: Illustration of the R function \textit{ciregic\_aipw}

We implemented the proposed augmented inverse probability weighted method in the existing R package \textit{intccr} (Park \textit{et al.}, 2019). The corresponding function \textit{ciregic\_aipw} for the analysis of interval-censored competing risks data and missing event types is provided in R version 3.5.2 or higher (R Core Team, 2019). Currently, the function allows for only two event types. The package installation and loading can be performed as follows:

R> install.packages("intccr")
R> library(intccr)

In this illustration we will analyze the simulated data set \textit{(simdata\_aipw)} which is available in the \textit{intccr} package. This data set consists of 200 observations with 7 variables: id, v, u, c, z1, z2, and a. The description of these variables is provided in Table 1. The first 6 observations in the data set \textit{(simdata\_aipw)} are listed below.

R> head(simdata\_aipw)

| id  | v       | u     | c   | z1        | z2     | a       |
|-----|---------|-------|-----|-----------|--------|---------|
| 1   | 1.0000000 | 0.1779317 | 2   | 1.2239254 | 0.6279651 |        |
| 2   | 1.4760692 | 1.9341271 | NA  | -1.1562233 | 1.0021440 |        |
| 3   | 0.5704245 | 1.5265510 | 2   | 1.3247553 | 0.2843777 |        |
| 4   | 1.0087580 | 1.7452873 | NA  | 1.3247553 | -1.0017791 |        |
Table 2. The argument of function `ciregic_aipw`.

This is not applicable in most applications. However, in some special cases such as our motivating study, only a subset of the observations with an event type are subject to missingness.

| Variables | Description |
|-----------|-------------|
| formula   | a model formula |
| aux       | a set of auxiliary variables (optional) |
| data      | a data frame |
| sub       | an indicator of the subset of the observation |
|           | that are subject to being missing (optional) |
| alpha     | a parameter specifying the link functions |
| k         | a parameter that controls the number of internal knots |
| nboot     | a number of bootstrap replications for standard error estimation |
| do.par    | a logical constant to utilize parallel computing |

5 5 0.1232930 0.3463802 2 0 0.1410843 -0.6172219

6 6 2.6582404 Inf 0 0 -0.5360480 1.8281942

The first observation \((id = 1)\) is left-censored and the corresponding event type is \(c = 2\). The second observation \((id = 2)\) is interval-censored. The event occurred in \((v, u) = (1.476, 1.934)\), but the corresponding event type is missing.

R> `table(simdata_aipw$c)`

|   | 0 | 1 | 2 |
|---|---|---|---|
|   | 31| 45| 39|

R> `sum(is.na(simdata_aipw$c))`

[1] 85

The `simdata_aipw` has 50.3 % missing event types among 169 observations that are not right-censored.

The function `ciregic_aipw` fits two parametric models; one is a logistic regression model for the probability of non-missingness using 169 observations and the other is for the probability of event type using 84 observations. Table presents the arguments of the function `ciregic_aipw`. The argument `formula` consists of the object function `Surv2(v, u, c)` and a linear combination
of covariates (for the simdata_aipw data set the formula is \( \text{Surv2}(v = v, u = u, \text{event} = c) \sim z1 + z2 \)). A set of auxiliary variables is allowed in the argument aux. Multiple auxiliary variables can be put into the argument. For example, users simply type aux = a for a single auxiliary variable or aux = a + b for two auxiliary variables. However, the default setting of aux is NULL, which means that the models for the probability of missingness and event type do not contain an auxiliary variable. The argument alpha is a vector of nonnegative values that govern the link function under the class of odds rate transformation models (for more details see Section 2.1 in the main text). Note that the function allows for different models for each event type. The argument k requires a value between 0.5 and 1, with a default value of 1. Based on k, the number of internal knots is defined as largest integer which is less than or equal to \( kn^{1/3} \). Using a smaller number k reduces the computation time at the expense of a crude B-spline approximation in finite samples. The remaining arguments, nboot and do.par, are options to define the number of bootstrap replications and the use of parallel computing. If the nboot = 0 then the ciregic_aipw function returns only point estimates without standard errors and p-values. If nboot > 0, one needs to set a seed number for reproducibility of the bootstrap standard errors is as follows:

```r
R> set.seed(2019)
```

Obtaining point estimates for the regression coefficients using this data set requires the code

```r
R> fit <- ciregic_aipw(formula = Surv2(v = v, u = u, event = c) ~ z1 + z2,
                       aux = a, data = simdata_aipw, alpha = c(1, 1),
                       nboot = 0, do.par = FALSE)
R> summary(fit)
```

Call:
ciregic_aipw.default(formula = Surv2(v = v, u = u, event = c) ~ 

\[ z1 + z2, \text{aux} = a, \text{data} = \text{simdata_aipw}, \alpha = c(1, 1), \text{do.par} = \text{FALSE}, \]
Event type 1
Coefficients:
  \[ z_1 \quad z_2 \]
  0.25067 0.01175

Event type 2
Coefficients:
  \[ z_1 \quad z_2 \]
  -0.19678 0.08918

Point estimates for the regression coefficients and bootstrap standard errors based on 50 replications without parallel computing are obtained as follows:

```r
R> set.seed(2019)
R> fit.npar <- ciregic_aipw(formula = Surv2(v = v, u = u, event = c) ~ z1 + z2,
+ aux = a, data = simdata_aipw, alpha = c(1, 1),
+ nboot = 50, do.par = FALSE)

Completed bootstrapping: 50 out of 50
```

> summary(fit.npar)

Call:
ciregic_aipw.default(formula = Surv2(v = v, u = u, event = c) ~
  z1 + z2, aux = a, data = simdata_aipw, alpha = c(1, 1), do.par = FALSE,
  nboot = 50)
Event type 1

| Estimate | Std. Error | z value | Pr(>|z|) |
|----------|------------|---------|----------|
| z1       | 0.2507     | 0.3450  | 0.727    | 0.468    |
| z2       | 0.0118     | 0.1888  | 0.062    | 0.950    |

Event type 2

| Estimate | Std. Error | z value | Pr(>|z|) |
|----------|------------|---------|----------|
| z1       | -0.1968    | 0.3640  | -0.541   | 0.589    |
| z2       | 0.0892     | 0.1902  | 0.469    | 0.639    |

A warning message is automatically generated if there are bootstrap replications that result in non-convergence in the numerical algorithm. The generic function `summary` provides the summary table for both event types. The output consists of the function call, estimates with its bootstrap standard error, z score, and p-value with significant stars. The significant stars appears when at least one covariate satisfies levels of significance.

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `. ' 0.1 ` ' 1

The parallel computing option `do.par = TRUE` selects the maximum number of cores minus one. For example, 3 available cores are assigned in quad core system because the user needs one core to run the operating system. The parallel computing offers faster bootstrap standard error computation, and returns the same result if the same seed number is defined. Moreover, we provide a function that returns the covariate-specific predicted cumulative incidence function (CIF). The generic function `predict` provides a corresponding predicted CIF to a sequence of time points and a combination of covariates. The following R code shows how to draw a plot for the predicted baseline CIFs. The resulting plot is depicted in Figure ??, a different value in the argument `covp` provides the predicted CIFs with for the required covariate pattern (e.g. `covp = c(1, .5)`).
R> t <- seq(from = fit$tms[1], to = fit$tms[2], by = diff(fit$tms) / 99)
R> pred <- predict(object = fit, covp = c(0, 0), times = t)
R> plot(pred$t, pred$cif1, type = "l", ylim = c(0, .6), lwd = 2,
       main = "Predicted cumulative incidence function",
       xlab = "time", ylab = "cumulative incidence function")
R> points(pred$t, pred$cif2, type = "l", col = 2, lty = 2, lwd = 2)
R> legend("topleft", legend = c("Event type 1", "Event type 2"), lty = 1:2,
          col = 1:2, lwd = c(2, 2))
Appendix III: Additional simulation results

Table 3. Simulation results regarding the regression coefficients under an average right censoring rate of 13.6% and $\xi_4 = -0.5$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|-------------|-----------|-----------|
| $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | | | | |
| % bias | -18.165 | -30.041 | -2.499 | -1.649 | -21.094 | -30.982 | -5.913 | -1.879 |
| MCSD | 0.336 | 0.174 | 0.320 | 0.164 | 0.247 | 0.123 | 0.234 | 0.113 |
| ASE | 0.357 | 0.173 | 0.337 | 0.163 | 0.246 | 0.119 | 0.232 | 0.112 |
| ECP | 0.950 | 0.899 | 0.958 | 0.949 | 0.934 | 0.864 | 0.940 | 0.943 |
| ii. MI | | | | |
| % bias | -6.846 | -7.710 | -6.099 | -6.623 | -7.978 | -7.565 | -8.581 | -7.284 |
| MCSD | 0.318 | 0.163 | 0.313 | 0.157 | 0.233 | 0.116 | 0.228 | 0.114 |
| ASE | 0.339 | 0.166 | 0.335 | 0.164 | 0.235 | 0.115 | 0.233 | 0.114 |
| ECP | 0.960 | 0.950 | 0.963 | 0.960 | 0.947 | 0.953 | 0.960 | 0.947 |
| iii. AIPW | | | | |
| % bias | 0.883 | -1.016 | 1.705 | 0.216 | -1.520 | -0.114 | -2.070 | 0.302 |
| MCSD | 0.341 | 0.181 | 0.340 | 0.174 | 0.245 | 0.124 | 0.241 | 0.124 |
| ASE | 0.362 | 0.180 | 0.354 | 0.177 | 0.242 | 0.121 | 0.240 | 0.119 |
| ECP | 0.964 | 0.945 | 0.956 | 0.945 | 0.939 | 0.935 | 0.946 | 0.947 |

| 50% of missing | $n = 200$ | $n = 400$ |
|-------------|-----------|-----------|
| $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | | | | |
| % bias | -32.616 | -46.683 | 0.387 | 10.913 | -35.139 | -48.949 | -3.645 | 9.304 |
| MCSD | 0.416 | 0.208 | 0.376 | 0.191 | 0.298 | 0.144 | 0.268 | 0.130 |
| ASE | 0.457 | 0.210 | 0.402 | 0.193 | 0.302 | 0.141 | 0.272 | 0.130 |
| ECP | 0.956 | 0.888 | 0.964 | 0.944 | 0.914 | 0.805 | 0.956 | 0.939 |
| ii. MI | | | | |
| % bias | -12.990 | -11.383 | -11.257 | -9.198 | -13.237 | -11.231 | -13.187 | -10.474 |
| MCSD | 0.373 | 0.191 | 0.361 | 0.185 | 0.271 | 0.137 | 0.263 | 0.134 |
| ASE | 0.396 | 0.192 | 0.391 | 0.190 | 0.271 | 0.133 | 0.268 | 0.131 |
| ECP | 0.955 | 0.949 | 0.960 | 0.950 | 0.947 | 0.937 | 0.946 | 0.940 |
| iii. AIPW | | | | |
| % bias | 0.766 | 1.002 | 2.289 | 4.154 | -1.677 | 1.563 | -1.460 | 2.930 |
| MCSD | 0.433 | 0.239 | 0.429 | 0.234 | 0.307 | 0.158 | 0.306 | 0.158 |
| ASE | 0.519 | 0.233 | 0.473 | 0.230 | 0.306 | 0.152 | 0.303 | 0.151 |
| ECP | 0.967 | 0.949 | 0.961 | 0.950 | 0.947 | 0.935 | 0.936 | 0.935 |
Table 4. Simulation results regarding the regression coefficients under an average right censoring rate of 13.6% and $\xi_4 = -0.1$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|----------------|----------|----------|
|                | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC          | -15.210  | -22.154  | 2.878  | 9.021  | -18.530  | -23.714  | -1.259  | 8.238  |
| % bias         | -29.659  | -40.710  | 7.552  | 21.368 | -32.481  | -43.319  | 2.410  | 20.366 |
| MCSD           | 0.334    | 0.173    | 0.323  | 0.164  | 0.247    | 0.120    | 0.237  | 0.113  |
| ASE            | 0.357    | 0.173    | 0.345  | 0.166  | 0.245    | 0.118    | 0.237  | 0.114  |
| ECP            | 0.955    | 0.925    | 0.966  | 0.943  | 0.932    | 0.898    | 0.947  | 0.943  |
| ii. MI         | -6.658   | -6.866   | -5.754 | -5.642 | -8.839   | -7.688   | -9.456 | -7.372 |
| % bias         | -11.915  | -11.380  | -9.832 | -8.757 | -13.455  | -11.622  | -13.255 | -10.797 |
| MCSD           | 0.323    | 0.166    | 0.317  | 0.160  | 0.235    | 0.115    | 0.231  | 0.113  |
| ASE            | 0.343    | 0.167    | 0.339  | 0.166  | 0.237    | 0.117    | 0.234  | 0.116  |
| ECP            | 0.962    | 0.955    | 0.967  | 0.959  | 0.948    | 0.953    | 0.950  | 0.947  |
| iii. AIPW      | 0.883    | -1.016   | 1.705  | 0.216  | -2.053   | 0.059    | -2.574 | 0.463  |
| % bias         | 2.345    | 0.521    | 4.691  | 3.885  | -2.800   | 1.058    | -2.602 | 2.344  |
| MCSD           | 0.341    | 0.181    | 0.340  | 0.174  | 0.248    | 0.122    | 0.245  | 0.122  |
| ASE            | 0.362    | 0.180    | 0.354  | 0.177  | 0.243    | 0.122    | 0.242  | 0.121  |
| ECP            | 0.964    | 0.945    | 0.956  | 0.945  | 0.937    | 0.943    | 0.941  | 0.949  |

| 50% of missing | $n = 200$ | $n = 400$ |
|----------------|----------|----------|
|                | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC          | -29.659  | -40.710  | 7.552  | 21.368 | -32.481  | -43.319  | 2.410  | 20.366 |
| % bias         | -29.659  | -40.710  | 7.552  | 21.368 | -32.481  | -43.319  | 2.410  | 20.366 |
| MCSD           | 0.404    | 0.203    | 0.389  | 0.194  | 0.293    | 0.143    | 0.284  | 0.132  |
| ASE            | 0.442    | 0.207    | 0.422  | 0.196  | 0.297    | 0.139    | 0.281  | 0.132  |
| ECP            | 0.958    | 0.904    | 0.970  | 0.937  | 0.923    | 0.833    | 0.945  | 0.920  |
| ii. MI         | -11.915  | -11.380  | -9.832 | -8.757 | -13.455  | -11.622  | -13.255 | -10.797 |
| % bias         | -11.915  | -11.380  | -9.832 | -8.757 | -13.455  | -11.622  | -13.255 | -10.797 |
| MCSD           | 0.375    | 0.192    | 0.368  | 0.186  | 0.273    | 0.137    | 0.269  | 0.134  |
| ASE            | 0.397    | 0.192    | 0.394  | 0.191  | 0.273    | 0.135    | 0.270  | 0.133  |
| ECP            | 0.952    | 0.947    | 0.962  | 0.952  | 0.949    | 0.945    | 0.949  | 0.935  |
| iii. AIPW      | 2.345    | 0.521    | 4.691  | 3.885  | -2.800   | 1.058    | -2.602 | 2.344  |
| % bias         | 2.345    | 0.521    | 4.691  | 3.885  | -2.800   | 1.058    | -2.602 | 2.344  |
| MCSD           | 0.434    | 0.236    | 0.428  | 0.229  | 0.301    | 0.157    | 0.303  | 0.156  |
| ASE            | 0.472    | 0.227    | 0.470  | 0.226  | 0.300    | 0.151    | 0.302  | 0.151  |
| ECP            | 0.966    | 0.943    | 0.953  | 0.943  | 0.954    | 0.941    | 0.946  | 0.943  |
Table 5. Simulation results regarding the regression coefficients under an average right censoring rate of 13.6% and $\xi_4 = 0.1$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| of missing | $n = 200$ | $n = 400$ |
|------------|-----------|-----------|
|            | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC      |           |           |           |           |           |           |           |           |
| % bias     | -12.628   | -16.674   | 5.670     | 13.793    | -15.684   | -18.440   | 1.203      | 12.647     |
| MCSD       | 0.335     | 0.173     | 0.327     | 0.165     | 0.244     | 0.118     | 0.238      | 0.113      |
| ASE        | 0.350     | 0.171     | 0.343     | 0.165     | 0.241     | 0.117     | 0.235      | 0.113      |
| ECP        | 0.953     | 0.924     | 0.961     | 0.939     | 0.932     | 0.921     | 0.948      | 0.935      |
| ii. MI     |           |           |           |           |           |           |           |           |
| % bias     | -6.153    | -5.730    | -5.096    | -4.411    | -8.572    | -7.238    | -8.971     | -6.994     |
| MCSD       | 0.326     | 0.166     | 0.319     | 0.160     | 0.235     | 0.116     | 0.229      | 0.114      |
| ASE        | 0.339     | 0.166     | 0.337     | 0.165     | 0.236     | 0.116     | 0.233      | 0.115      |
| ECP        | 0.959     | 0.950     | 0.965     | 0.952     | 0.945     | 0.946     | 0.958      | 0.949      |
| iii. AIPW  |           |           |           |           |           |           |           |           |
| % bias     | 0.539     | 0.056     | 1.324     | 1.283     | -1.862    | 0.143     | -2.373     | 0.531      |
| MCSD       | 0.341     | 0.180     | 0.338     | 0.173     | 0.247     | 0.123     | 0.242      | 0.122      |
| ASE        | 0.353     | 0.178     | 0.353     | 0.176     | 0.241     | 0.121     | 0.240      | 0.119      |
| ECP        | 0.956     | 0.944     | 0.956     | 0.950     | 0.945     | 0.943     | 0.947      | 0.941      |
| 50%        |           |           |           |           |           |           |           |           |
|            | $n = 200$ | $n = 400$ |           |           |           |           |           |           |
| i. CC      |           |           |           |           |           |           |           |           |
| % bias     | -26.485   | -35.389   | 10.377    | 25.324    | -29.191   | -36.689   | 4.878      | 24.550     |
| MCSD       | 0.396     | 0.202     | 0.389     | 0.196     | 0.289     | 0.140     | 0.286      | 0.132      |
| ASE        | 0.428     | 0.203     | 0.423     | 0.196     | 0.290     | 0.137     | 0.280      | 0.132      |
| ECP        | 0.963     | 0.912     | 0.971     | 0.933     | 0.931     | 0.853     | 0.941      | 0.914      |
| ii. MI     |           |           |           |           |           |           |           |           |
| % bias     | -11.110   | -11.276   | -9.079    | -8.630    | -12.792   | -11.016   | -12.923    | -10.242    |
| MCSD       | 0.373     | 0.190     | 0.365     | 0.185     | 0.270     | 0.134     | 0.267      | 0.132      |
| ASE        | 0.393     | 0.190     | 0.391     | 0.189     | 0.270     | 0.134     | 0.267      | 0.132      |
| ECP        | 0.955     | 0.945     | 0.962     | 0.958     | 0.940     | 0.943     | 0.944      | 0.938      |
| iii. AIPW  |           |           |           |           |           |           |           |           |
| % bias     | 2.526     | -0.374    | 4.668     | 2.802     | -2.595    | 1.113     | -2.628     | 2.342      |
| MCSD       | 0.427     | 0.234     | 0.426     | 0.227     | 0.301     | 0.153     | 0.302      | 0.154      |
| ASE        | 0.457     | 0.224     | 0.466     | 0.233     | 0.297     | 0.150     | 0.298      | 0.150      |
| ECP        | 0.956     | 0.943     | 0.958     | 0.937     | 0.951     | 0.934     | 0.935      | 0.933      |
Table 6. Simulation results regarding the regression coefficients under an average right censoring rate of 13.6% and $\xi_4 = 0.5$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|----------------|-----------|-----------|
| $\beta_{11}$  | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | -8.424 | -7.855 | 10.944 | 22.273 | -10.788 | -9.978 | 6.006 | 20.756 |
| % bias | 0.334 | 0.170 | 0.333 | 0.165 | 0.235 | 0.118 | 0.243 | 0.115 |
| MCSD | 0.348 | 0.168 | 0.349 | 0.166 | 0.237 | 0.116 | 0.237 | 0.114 |
| ASE | 0.954 | 0.938 | 0.967 | 0.931 | 0.933 | 0.937 | 0.946 | 0.912 |
| ECP | 0.324 | 0.165 | 0.318 | 0.160 | 0.235 | 0.115 | 0.234 | 0.114 |
| ii. MI | -5.830 | -5.538 | -4.734 | -4.006 | -7.316 | -6.868 | -7.745 | -6.489 |
| % bias | 0.324 | 0.165 | 0.318 | 0.160 | 0.235 | 0.115 | 0.234 | 0.114 |
| MCSD | 0.337 | 0.166 | 0.336 | 0.164 | 0.235 | 0.115 | 0.234 | 0.114 |
| ASE | 0.959 | 0.944 | 0.969 | 0.952 | 0.939 | 0.949 | 0.953 | 0.951 |
| ECP | 1.562 | 1.050 | 2.554 | 2.395 | -1.620 | -1.042 | -2.090 | 0.300 |
| % bias | 0.345 | 0.181 | 0.345 | 0.176 | 0.252 | 0.126 | 0.247 | 0.125 |
| MCSD | 0.359 | 0.180 | 0.371 | 0.179 | 0.244 | 0.121 | 0.243 | 0.121 |
| ASE | 0.953 | 0.940 | 0.962 | 0.947 | 0.938 | 0.947 | 0.941 | 0.948 |
| ECP | 0.373 | 0.190 | 0.362 | 0.186 | 0.273 | 0.133 | 0.267 | 0.131 |
| iii. AIPW | 0.391 | 0.201 | 0.442 | 0.199 | 0.282 | 0.136 | 0.287 | 0.134 |

| 50% of missing | $n = 200$ | $n = 400$ |
|----------------|-----------|-----------|
| $\beta_{11}$  | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | -21.830 | -26.442 | 15.977 | 33.185 | -24.989 | -27.229 | 10.162 | 33.219 |
| % bias | 0.387 | 0.197 | 0.391 | 0.197 | 0.285 | 0.137 | 0.288 | 0.134 |
| MCSD | 0.417 | 0.201 | 0.442 | 0.199 | 0.282 | 0.136 | 0.287 | 0.134 |
| ASE | 0.959 | 0.915 | 0.972 | 0.923 | 0.930 | 0.899 | 0.945 | 0.886 |
| ECP | 1.050 | 1.064 | 8.141 | 7.964 | -11.575 | -10.141 | -11.457 | -9.226 |
| % bias | 0.373 | 0.190 | 0.362 | 0.186 | 0.273 | 0.133 | 0.267 | 0.131 |
| MCSD | 0.391 | 0.190 | 0.391 | 0.189 | 0.270 | 0.133 | 0.268 | 0.132 |
| ASE | 0.953 | 0.946 | 0.965 | 0.946 | 0.940 | 0.937 | 0.944 | 0.935 |
| ECP | 3.026 | 0.118 | 5.326 | 3.423 | -2.882 | 0.722 | -2.672 | 2.091 |
| % bias | 0.438 | 0.236 | 0.441 | 0.233 | 0.311 | 0.160 | 0.311 | 0.162 |
| MCSD | 0.468 | 0.232 | 0.577 | 0.232 | 0.306 | 0.153 | 0.309 | 0.154 |
| ASE | 0.954 | 0.956 | 0.966 | 0.940 | 0.946 | 0.930 | 0.951 | 0.943 |
Table 7. Simulation results regarding the regression coefficients under an average right censoring rate of 30% and $\xi_4 = -0.5$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|----------------|-----------|-----------|
| i. CC % bias | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| % bias | -28.175 | -51.080 | 3.964 | 12.910 | -29.538 | -49.967 | 0.754 | 12.769 |
| MCSD | 0.383 | 0.191 | 0.347 | 0.176 | 0.269 | 0.135 | 0.246 | 0.121 |
| ASE | 0.402 | 0.194 | 0.366 | 0.175 | 0.272 | 0.133 | 0.250 | 0.119 |
| ECP | 0.948 | 0.858 | 0.967 | 0.947 | 0.932 | 0.789 | 0.956 | 0.935 |
| ii. MI % bias | -3.486 | -5.631 | -2.598 | -3.435 | -4.569 | -4.179 | -6.059 | -3.315 |
| MCSD | 0.347 | 0.179 | 0.339 | 0.173 | 0.251 | 0.123 | 0.241 | 0.122 |
| ASE | 0.366 | 0.183 | 0.360 | 0.177 | 0.251 | 0.126 | 0.248 | 0.122 |
| ECP | 0.958 | 0.951 | 0.959 | 0.951 | 0.960 | 0.953 | 0.960 | 0.953 |
| iii. AIPW % bias | 0.612 | -1.992 | 1.373 | 0.684 | -0.905 | -0.221 | -2.517 | 0.836 |
| MCSD | 0.370 | 0.195 | 0.360 | 0.187 | 0.258 | 0.130 | 0.250 | 0.130 |
| ASE | 0.401 | 0.198 | 0.382 | 0.192 | 0.258 | 0.132 | 0.256 | 0.127 |
| ECP | 0.952 | 0.950 | 0.960 | 0.947 | 0.949 | 0.950 | 0.959 | 0.945 |

| 50% of missing | $n = 200$ | $n = 400$ |
|----------------|-----------|-----------|
| i. CC % bias | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| % bias | -49.188 | -80.533 | 12.376 | 35.192 | -49.183 | -79.499 | 7.937 | 34.212 |
| MCSD | 0.473 | 0.227 | 0.405 | 0.203 | 0.325 | 0.158 | 0.280 | 0.138 |
| ASE | 0.550 | 0.230 | 0.449 | 0.200 | 0.330 | 0.155 | 0.287 | 0.136 |
| ECP | 0.961 | 0.799 | 0.969 | 0.919 | 0.895 | 0.649 | 0.957 | 0.889 |
| ii. MI % bias | -7.640 | -7.954 | -5.118 | -3.517 | -7.640 | -5.944 | -8.026 | -3.811 |
| MCSD | 0.410 | 0.208 | 0.396 | 0.201 | 0.293 | 0.143 | 0.279 | 0.141 |
| ASE | 0.419 | 0.207 | 0.412 | 0.200 | 0.289 | 0.143 | 0.284 | 0.140 |
| ECP | 0.954 | 0.948 | 0.958 | 0.943 | 0.950 | 0.948 | 0.948 | 0.951 |
| iii. AIPW % bias | -0.654 | -1.178 | 1.512 | 5.385 | -1.167 | 0.683 | -1.643 | 3.320 |
| MCSD | 0.455 | 0.248 | 0.439 | 0.239 | 0.312 | 0.160 | 0.307 | 0.160 |
| ASE | 0.597 | 0.252 | 0.526 | 0.243 | 0.325 | 0.160 | 0.313 | 0.156 |
| ECP | 0.975 | 0.962 | 0.963 | 0.940 | 0.953 | 0.946 | 0.948 | 0.942 |
Table 8. Simulation results regarding the regression coefficients under an average right censoring rate of 30% and $\xi_4 = -0.1$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|----------------|----------|----------|
|                | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC          |           |           |           |           |           |           |           |           |
| % bias         | -24.568  | -42.747  | 9.637    | 22.488    | -25.978  | -42.111  | 5.396     | 21.564     |
| MCSD           | 0.377    | 0.186    | 0.352    | 0.175     | 0.261    | 0.130    | 0.247     | 0.122      |
| ASE            | 0.389    | 0.190    | 0.371    | 0.176     | 0.264    | 0.129    | 0.252     | 0.121      |
| ECP            | 0.953    | 0.888    | 0.960    | 0.937     | 0.918    | 0.826    | 0.960     | 0.916      |
| ii. MI         |           |           |           |           |           |           |           |           |
| % bias         | -3.070   | -4.738   | -2.004   | -2.366    | -4.631   | -4.076   | -5.990    | -3.116     |
| MCSD           | 0.352    | 0.177    | 0.340    | 0.171     | 0.249    | 0.122    | 0.240     | 0.121      |
| ASE            | 0.363    | 0.182    | 0.359    | 0.176     | 0.250    | 0.126    | 0.248     | 0.122      |
| ECP            | 0.959    | 0.952    | 0.960    | 0.946     | 0.958    | 0.959    | 0.956     | 0.954      |
| iii. AIPW      |           |           |           |           |           |           |           |           |
| % bias         | -0.077   | -0.788   | 0.780    | 1.866     | -1.214   | -0.227   | -2.737    | 0.690      |
| MCSD           | 0.366    | 0.190    | 0.356    | 0.184     | 0.255    | 0.129    | 0.248     | 0.128      |
| ASE            | 0.380    | 0.192    | 0.378    | 0.189     | 0.255    | 0.130    | 0.254     | 0.126      |
| ECP            | 0.953    | 0.947    | 0.956    | 0.955     | 0.945    | 0.953    | 0.957     | 0.949      |

| 50% of missing | $n = 200$ | $n = 400$ |
|----------------|----------|----------|
|                | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC          |           |           |           |           |           |           |           |           |
| % bias         | -46.856  | -75.525  | 20.008   | 45.882    | -46.331  | -74.528  | 14.307    | 45.695     |
| MCSD           | 0.450    | 0.216    | 0.422    | 0.205     | 0.314    | 0.154    | 0.289     | 0.142      |
| ASE            | 0.502    | 0.221    | 0.479    | 0.205     | 0.317    | 0.149    | 0.296     | 0.139      |
| ECP            | 0.948    | 0.810    | 0.978    | 0.898     | 0.889    | 0.653    | 0.949     | 0.833      |
| ii. MI         |           |           |           |           |           |           |           |           |
| % bias         | -6.590   | -7.664   | -3.273   | -2.596    | -6.936   | -5.710   | -7.042    | -3.174     |
| MCSD           | 0.409    | 0.206    | 0.402    | 0.202     | 0.290    | 0.143    | 0.278     | 0.142      |
| ASE            | 0.415    | 0.206    | 0.415    | 0.201     | 0.287    | 0.143    | 0.284     | 0.141      |
| ECP            | 0.951    | 0.946    | 0.966    | 0.949     | 0.955    | 0.948    | 0.962     | 0.946      |
| iii. AIPW      |           |           |           |           |           |           |           |           |
| % bias         | 0.380    | -1.540   | 3.886    | 4.614     | -1.854   | 0.283    | -2.337    | 3.028      |
| MCSD           | 0.446    | 0.239    | 0.443    | 0.232     | 0.303    | 0.158    | 0.297     | 0.159      |
| ASE            | 0.529    | 0.244    | 0.533    | 0.240     | 0.306    | 0.157    | 0.307     | 0.155      |
| ECP            | 0.967    | 0.968    | 0.963    | 0.952     | 0.955    | 0.951    | 0.956     | 0.946      |
Table 9. Simulation results regarding the regression coefficients under an average right censoring rate of 30% and $\xi_4 = 0$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| of missing | $n = 200$ | $n = 400$ |
|------------|-----------|-----------|
|            | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC      |           |           |            |           |           |           |            |            |
| % bias     | -23.189   | -40.630   | 11.393     | 24.763    | -25.370   | -39.849   | 6.327       | 24.124     |
| MCSD       | 0.379     | 0.186     | 0.356      | 0.177     | 0.261     | 0.128     | 0.249       | 0.122      |
| ASE        | 0.385     | 0.188     | 0.375      | 0.177     | 0.262     | 0.129     | 0.253       | 0.121      |
| ECP        | 0.944     | 0.892     | 0.964      | 0.934     | 0.914     | 0.834     | 0.956       | 0.915      |
| ii. MI     |           |           |            |           |           |           |            |            |
| % bias     | -2.559    | -5.139    | -1.554     | -2.617    | -4.827    | -3.956    | -6.202      | -2.988     |
| MCSD       | 0.352     | 0.179     | 0.341      | 0.174     | 0.250     | 0.122     | 0.239       | 0.121      |
| ASE        | 0.363     | 0.182     | 0.360      | 0.176     | 0.250     | 0.126     | 0.247       | 0.122      |
| ECP        | 0.958     | 0.953     | 0.962      | 0.948     | 0.956     | 0.959     | 0.956       | 0.956      |
| iii. AIPW  |           |           |            |           |           |           |            |            |
| % bias     | 0.338     | -0.815    | 1.037      | 1.877     | -1.387    | -0.022    | -2.939      | 0.979      |
| MCSD       | 0.367     | 0.190     | 0.360      | 0.184     | 0.256     | 0.129     | 0.247       | 0.128      |
| ASE        | 0.379     | 0.194     | 0.376      | 0.189     | 0.254     | 0.129     | 0.253       | 0.126      |
| ECP        | 0.947     | 0.953     | 0.957      | 0.955     | 0.935     | 0.945     | 0.960       | 0.942      |

| of missing | $n = 200$ | $n = 400$ |
|------------|-----------|-----------|
|            | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC      |           |           |            |           |           |           |            |            |
| % bias     | -44.638   | -74.617   | 22.175     | 47.723    | -45.682   | -72.581   | 15.532      | 47.959     |
| MCSD       | 0.442     | 0.217     | 0.426      | 0.206     | 0.316     | 0.152     | 0.296       | 0.142      |
| ASE        | 0.486     | 0.220     | 0.495      | 0.206     | 0.315     | 0.148     | 0.300       | 0.139      |
| ECP        | 0.951     | 0.810     | 0.973      | 0.894     | 0.882     | 0.671     | 0.951       | 0.824      |
| ii. MI     |           |           |            |           |           |           |            |            |
| % bias     | -5.874    | -8.353    | -2.370     | -3.009    | -7.336    | -5.457    | -7.416      | -3.111     |
| MCSD       | 0.407     | 0.207     | 0.400      | 0.202     | 0.296     | 0.142     | 0.284       | 0.140      |
| ASE        | 0.417     | 0.206     | 0.420      | 0.201     | 0.287     | 0.143     | 0.285       | 0.141      |
| ECP        | 0.955     | 0.948     | 0.964      | 0.945     | 0.946     | 0.948     | 0.947       | 0.947      |
| iii. AIPW  |           |           |            |           |           |           |            |            |
| % bias     | 0.935     | -3.187    | 4.531      | 3.101     | -2.443    | 0.122     | -3.174      | 2.480      |
| MCSD       | 0.445     | 0.241     | 0.441      | 0.233     | 0.308     | 0.156     | 0.302       | 0.157      |
| ASE        | 0.533     | 0.242     | 0.543      | 0.238     | 0.306     | 0.156     | 0.308       | 0.154      |
| ECP        | 0.962     | 0.953     | 0.967      | 0.952     | 0.950     | 0.947     | 0.958       | 0.942      |
Table 10. Simulation results regarding the regression coefficients under an average right censoring rate of 30% and $\xi_4 = 0.1$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|-----------------|----------|----------|
|                 | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC           |          |          |          |          |          |          |          |          |
| % bias          | -22.969  | -38.340  | 12.710   | 27.540   | -24.854  | -37.823  | 7.320    | 26.466   |
| MCSD            | 0.378    | 0.186    | 0.360    | 0.179    | 0.263    | 0.128    | 0.251    | 0.123    |
| ASE             | 0.383    | 0.187    | 0.375    | 0.177    | 0.261    | 0.128    | 0.255    | 0.122    |
| ECP             | 0.941    | 0.896    | 0.962    | 0.929    | 0.924    | 0.843    | 0.952    | 0.903    |
| ii. MI          |          |          |          |          |          |          |          |          |
| % bias          | -2.590   | -4.662   | -1.428   | -2.039   | -4.872   | -3.692   | -6.224   | -2.664   |
| MCSD            | 0.353    | 0.179    | 0.343    | 0.175    | 0.252    | 0.122    | 0.240    | 0.121    |
| ASE             | 0.362    | 0.181    | 0.360    | 0.176    | 0.250    | 0.125    | 0.247    | 0.122    |
| ECP             | 0.954    | 0.948    | 0.961    | 0.945    | 0.956    | 0.960    | 0.955    | 0.949    |
| iii. AIPW       |          |          |          |          |          |          |          |          |
| % bias          | 0.217    | -0.309   | 0.893    | 2.271    | -1.580   | -0.172   | -3.070   | 0.796    |
| MCSD            | 0.368    | 0.190    | 0.362    | 0.185    | 0.258    | 0.130    | 0.249    | 0.129    |
| ASE             | 0.380    | 0.194    | 0.387    | 0.189    | 0.255    | 0.129    | 0.253    | 0.127    |
| ECP             | 0.946    | 0.947    | 0.962    | 0.948    | 0.943    | 0.955    | 0.954    | 0.938    |

| 50% of missing | $n = 200$ | $n = 400$ |
|-----------------|----------|----------|
|                 | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC           |          |          |          |          |          |          |          |          |
| % bias          | -42.944  | -72.520  | 24.281   | 50.345   | -44.564  | -70.271  | 17.046   | 50.504   |
| MCSD            | 0.441    | 0.215    | 0.436    | 0.210    | 0.314    | 0.152    | 0.299    | 0.143    |
| ASE             | 0.475    | 0.219    | 0.503    | 0.208    | 0.312    | 0.147    | 0.302    | 0.141    |
| ECP             | 0.949    | 0.799    | 0.971    | 0.884    | 0.878    | 0.679    | 0.952    | 0.814    |
| ii. MI          |          |          |          |          |          |          |          |          |
| % bias          | -5.320   | -8.920   | -1.549   | -3.404   | -6.906   | -5.015   | -6.917   | -2.591   |
| MCSD            | 0.412    | 0.205    | 0.409    | 0.203    | 0.297    | 0.143    | 0.285    | 0.141    |
| ASE             | 0.417    | 0.205    | 0.421    | 0.201    | 0.286    | 0.143    | 0.285    | 0.141    |
| ECP             | 0.953    | 0.941    | 0.960    | 0.949    | 0.943    | 0.952    | 0.950    | 0.947    |
| iii. AIPW       |          |          |          |          |          |          |          |          |
| % bias          | 1.263    | -3.955   | 5.032    | 2.571    | -2.189   | 0.487    | -3.015   | 2.878    |
| MCSD            | 0.449    | 0.241    | 0.450    | 0.236    | 0.310    | 0.156    | 0.302    | 0.158    |
| ASE             | 0.506    | 0.241    | 0.557    | 0.238    | 0.306    | 0.156    | 0.309    | 0.155    |
| ECP             | 0.960    | 0.958    | 0.965    | 0.949    | 0.948    | 0.949    | 0.949    | 0.950    |
Table 11. Simulation results regarding the regression coefficients under an average right censoring rate of 30% and $\xi_4 = 0.5$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing | $n = 200$ | $n = 400$ |
|----------------|----------|----------|
| $\beta_{11}$  | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | | |
| % bias | -18.531 | -28.981 | 17.612 | 35.953 | -19.860 | -29.087 | 11.976 | 33.953 |
| MCSD | 0.373 | 0.182 | 0.363 | 0.180 | 0.261 | 0.126 | 0.255 | 0.126 |
| ASE | 0.375 | 0.184 | 0.387 | 0.181 | 0.256 | 0.126 | 0.260 | 0.124 |
| ECP | 0.947 | 0.921 | 0.958 | 0.909 | 0.922 | 0.882 | 0.956 | 0.878 |
| ii. MI | | |
| % bias | -2.167 | -3.902 | -0.741 | -0.927 | -4.181 | -3.084 | -5.269 | -1.871 |
| MCSD | 0.353 | 0.177 | 0.342 | 0.173 | 0.252 | 0.123 | 0.239 | 0.122 |
| ASE | 0.360 | 0.180 | 0.360 | 0.176 | 0.250 | 0.125 | 0.248 | 0.122 |
| ECP | 0.951 | 0.954 | 0.962 | 0.952 | 0.943 | 0.958 | 0.954 | 0.948 |
| iii. AIPW | | |
| % bias | 1.069 | -0.307 | 2.316 | 2.832 | -1.199 | -0.108 | -2.508 | 0.991 |
| MCSD | 0.372 | 0.191 | 0.367 | 0.188 | 0.265 | 0.133 | 0.253 | 0.134 |
| ASE | 0.393 | 0.195 | 0.425 | 0.193 | 0.257 | 0.131 | 0.258 | 0.129 |
| ECP | 0.948 | 0.942 | 0.963 | 0.948 | 0.938 | 0.947 | 0.951 | 0.941 |

| 50% of missing | $n = 200$ | $n = 400$ |
|----------------|----------|----------|
| $\beta_{11}$  | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | | |
| % bias | -38.601 | -61.912 | 29.332 | 58.422 | -40.134 | -59.790 | 21.818 | 57.892 |
| MCSD | 0.431 | 0.205 | 0.438 | 0.216 | 0.301 | 0.146 | 0.310 | 0.147 |
| ASE | 0.453 | 0.212 | 0.541 | 0.215 | 0.301 | 0.143 | 0.315 | 0.146 |
| ECP | 0.952 | 0.848 | 0.981 | 0.875 | 0.889 | 0.746 | 0.949 | 0.784 |
| ii. MI | | |
| % bias | -4.011 | -8.004 | -0.035 | -2.164 | -6.312 | -5.232 | -6.112 | -2.536 |
| MCSD | 0.402 | 0.202 | 0.401 | 0.203 | 0.295 | 0.142 | 0.286 | 0.140 |
| ASE | 0.413 | 0.204 | 0.421 | 0.201 | 0.286 | 0.142 | 0.285 | 0.141 |
| ECP | 0.953 | 0.955 | 0.964 | 0.954 | 0.943 | 0.947 | 0.943 | 0.945 |
| iii. AIPW | | |
| % bias | 0.761 | -3.191 | 4.929 | 3.871 | -2.529 | -0.899 | -2.588 | 2.210 |
| MCSD | 0.456 | 0.238 | 0.464 | 0.241 | 0.318 | 0.167 | 0.313 | 0.167 |
| ASE | 0.536 | 0.245 | 0.698 | 0.250 | 0.314 | 0.160 | 0.322 | 0.161 |
| ECP | 0.965 | 0.952 | 0.968 | 0.947 | 0.940 | 0.943 | 0.953 | 0.942 |
Table 12. Simulation results regarding the regression coefficients under an average right censoring rate of 45% and \( \xi_4 = -0.5 \). CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30% of missing |  |  |  |  |  |  |  |  |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | \( \beta_{11} \) | \( \beta_{12} \) | \( \beta_{21} \) | \( \beta_{22} \) | \( \beta_{11} \) | \( \beta_{12} \) | \( \beta_{21} \) | \( \beta_{22} \) |
| i. CC          |                 |                 |                 |                 |                 |                 |                 |                 |
| % bias         | -33.579         | -59.865         | 10.785          | 23.729          | -33.404         | -58.904         | 5.963           | 21.783          |
| MCSD           | 0.430           | 0.208           | 0.384           | 0.187           | 0.295           | 0.146           | 0.275           | 0.131           |
| ASE            | 0.455           | 0.212           | 0.414           | 0.189           | 0.300           | 0.145           | 0.272           | 0.129           |
| ECP            | 0.954           | 0.857           | 0.963           | 0.934           | 0.921           | 0.749           | 0.950           | 0.919           |
| ii. MI         |                 |                 |                 |                 |                 |                 |                 |                 |
| % bias         | -2.961          | -3.050          | 0.037           | 0.031           | -2.720          | -2.949          | -5.036          | -2.501          |
| MCSD           | 0.389           | 0.198           | 0.380           | 0.190           | 0.273           | 0.135           | 0.271           | 0.133           |
| ASE            | 0.396           | 0.198           | 0.394           | 0.189           | 0.274           | 0.137           | 0.269           | 0.131           |
| ECP            | 0.954           | 0.956           | 0.963           | 0.945           | 0.955           | 0.951           | 0.942           | 0.949           |
| iii. AIPW      |                 |                 |                 |                 |                 |                 |                 |                 |
| % bias         | -0.260          | -1.630          | 2.491           | 1.971           | -0.099          | 0.240           | -2.372          | 0.787           |
| MCSD           | 0.405           | 0.212           | 0.394           | 0.200           | 0.277           | 0.137           | 0.276           | 0.139           |
| ASE            | 0.453           | 0.218           | 0.439           | 0.205           | 0.280           | 0.143           | 0.277           | 0.135           |
| ECP            | 0.958           | 0.962           | 0.962           | 0.946           | 0.952           | 0.964           | 0.948           | 0.943           |

| 50% of missing |  |  |  |  |  |  |  |  |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | \( \beta_{11} \) | \( \beta_{12} \) | \( \beta_{21} \) | \( \beta_{22} \) | \( \beta_{11} \) | \( \beta_{12} \) | \( \beta_{21} \) | \( \beta_{22} \) |
| i. CC          |                 |                 |                 |                 |                 |                 |                 |                 |
| % bias         | -58.047         | -93.203         | 25.295          | 50.877          | -55.270         | -92.671         | 16.247          | 47.299          |
| MCSD           | 0.530           | 0.250           | 0.536           | 0.218           | 0.361           | 0.172           | 0.316           | 0.147           |
| ASE            | 0.760           | 0.257           | 0.559           | 0.217           | 0.375           | 0.173           | 0.315           | 0.146           |
| ECP            | 0.979           | 0.804           | 0.980           | 0.890           | 0.914           | 0.617           | 0.947           | 0.843           |
| ii. MI         |                 |                 |                 |                 |                 |                 |                 |                 |
| % bias         | -8.069          | -6.298          | -0.155          | -0.135          | -5.074          | -4.387          | -5.914          | -2.363          |
| MCSD           | 0.451           | 0.229           | 0.478           | 0.217           | 0.312           | 0.155           | 0.312           | 0.153           |
| ASE            | 0.466           | 0.226           | 0.466           | 0.216           | 0.313           | 0.155           | 0.307           | 0.149           |
| ECP            | 0.966           | 0.952           | 0.962           | 0.948           | 0.960           | 0.955           | 0.938           | 0.944           |
| iii. AIPW      |                 |                 |                 |                 |                 |                 |                 |                 |
| % bias         | -2.970          | -2.787          | 5.387           | 5.641           | 0.035           | 0.874           | -0.734          | 3.494           |
| MCSD           | 0.484           | 0.261           | 0.575           | 0.247           | 0.331           | 0.165           | 0.333           | 0.167           |
| ASE            | 0.860           | 0.282           | 0.692           | 0.268           | 0.365           | 0.173           | 0.345           | 0.165           |
| ECP            | 0.979           | 0.960           | 0.970           | 0.946           | 0.964           | 0.957           | 0.941           | 0.941           |
Table 13. Simulation results regarding the regression coefficients under an average right censoring rate of 45% and $\xi_4 = -0.1$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| of missing | $n = 200$ | $n = 400$ |
|------------|-----------|-----------|
|            | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC      |            |            |            |            |            |            |            |            |
| % bias     | -30.116   | -53.047    | 16.549     | 32.112     | -30.420    | -51.982    | 10.355      | 30.023      |
| MCSD       | 0.421     | 0.201      | 0.391      | 0.189      | 0.286      | 0.139      | 0.279       | 0.131       |
| ASE        | 0.436     | 0.207      | 0.423      | 0.192      | 0.290      | 0.141      | 0.276       | 0.130       |
| ECP        | 0.950     | 0.876      | 0.971      | 0.934      | 0.918      | 0.786      | 0.945       | 0.902       |
| ii. MI     |            |            |            |            |            |            |            |            |
| % bias     | -2.941    | -2.833     | 0.657      | 0.410      | -2.889     | -2.420     | -4.806      | -1.826      |
| MCSD       | 0.390     | 0.195      | 0.383      | 0.188      | 0.268      | 0.135      | 0.269       | 0.134       |
| ASE        | 0.394     | 0.197      | 0.394      | 0.190      | 0.271      | 0.136      | 0.269       | 0.131       |
| ECP        | 0.954     | 0.959      | 0.959      | 0.943      | 0.954      | 0.946      | 0.949       | 0.940       |
| iii. AIPW  |            |            |            |            |            |            |            |            |
| % bias     | 0.130     | -0.549     | 3.113      | 3.035      | -0.175     | 0.606      | -2.340      | 1.026       |
| MCSD       | 0.397     | 0.208      | 0.388      | 0.199      | 0.272      | 0.136      | 0.273       | 0.137       |
| ASE        | 0.433     | 0.213      | 0.439      | 0.204      | 0.275      | 0.140      | 0.273       | 0.134       |
| ECP        | 0.959     | 0.960      | 0.969      | 0.947      | 0.951      | 0.957      | 0.947       | 0.946       |
| 50% of missing |            |            |            |            |            |            |            |            |
| i. CC      |            |            |            |            |            |            |            |            |
| % bias     | -55.888   | -90.460    | 32.170     | 62.220     | -53.278    | -89.044    | 21.591      | 58.782      |
| MCSD       | 0.507     | 0.240      | 0.483      | 0.219      | 0.345      | 0.167      | 0.323       | 0.152       |
| ASE        | 0.673     | 0.245      | 0.647      | 0.224      | 0.355      | 0.165      | 0.329       | 0.150       |
| ECP        | 0.969     | 0.795      | 0.984      | 0.878      | 0.898      | 0.611      | 0.949       | 0.798       |
| ii. MI     |            |            |            |            |            |            |            |            |
| % bias     | -6.804    | -6.372     | 1.964      | 1.384      | -6.081     | -4.472     | -6.407      | -2.078      |
| MCSD       | 0.449     | 0.230      | 0.463      | 0.219      | 0.313      | 0.154      | 0.314       | 0.151       |
| ASE        | 0.455     | 0.224      | 0.471      | 0.217      | 0.311      | 0.155      | 0.308       | 0.149       |
| ECP        | 0.963     | 0.949      | 0.966      | 0.940      | 0.957      | 0.945      | 0.943       | 0.944       |
| iii. AIPW  |            |            |            |            |            |            |            |            |
| % bias     | -2.368    | -2.536     | 5.366      | 5.863      | -1.924     | 0.151      | -2.814      | 2.903       |
| MCSD       | 0.479     | 0.257      | 0.488      | 0.241      | 0.319      | 0.163      | 0.325       | 0.166       |
| ASE        | 0.693     | 0.268      | 0.747      | 0.262      | 0.335      | 0.168      | 0.334       | 0.163       |
| ECP        | 0.973     | 0.950      | 0.970      | 0.957      | 0.964      | 0.952      | 0.957       | 0.934       |
Table 14. Simulation results regarding the regression coefficients under an average right censoring rate of 45% and $\xi_4 = 0$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| 30\% | $n = 200$ | $n = 400$ | 50\% | $n = 200$ | $n = 400$ |
|-------|----------|----------|-------|----------|----------|
| of missing | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | $\%$ bias | -28.943 | -51.351 | 18.264 | 34.498 | -30.003 | -49.707 | 11.285 | 32.456 |
| | MCSD | 0.423 | 0.201 | 0.393 | 0.190 | 0.286 | 0.137 | 0.281 | 0.132 |
| | ASE | 0.432 | 0.205 | 0.428 | 0.193 | 0.287 | 0.140 | 0.277 | 0.131 |
| | ECP | 0.950 | 0.884 | 0.970 | 0.929 | 0.923 | 0.795 | 0.948 | 0.890 |
| ii. MI | $\%$ bias | -2.862 | -2.846 | 0.942 | 0.435 | -3.293 | -2.218 | -5.234 | -1.658 |
| | MCSD | 0.392 | 0.196 | 0.386 | 0.189 | 0.268 | 0.135 | 0.269 | 0.134 |
| | ASE | 0.393 | 0.197 | 0.394 | 0.190 | 0.271 | 0.136 | 0.268 | 0.131 |
| | ECP | 0.954 | 0.955 | 0.961 | 0.947 | 0.955 | 0.950 | 0.953 | 0.951 |
| iii. AIPW | $\%$ bias | 0.030 | -0.668 | 3.052 | 2.648 | -0.534 | 0.721 | -2.704 | 1.087 |
| | MCSD | 0.397 | 0.205 | 0.391 | 0.200 | 0.272 | 0.136 | 0.274 | 0.137 |
| | ASE | 0.424 | 0.215 | 0.438 | 0.209 | 0.275 | 0.140 | 0.274 | 0.135 |
| | ECP | 0.957 | 0.955 | 0.965 | 0.947 | 0.957 | 0.948 | 0.949 | 0.949 |
| 50\% | $n = 200$ | $n = 400$ |  | $n = 200$ | $n = 400$ |
| of missing | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | $\%$ bias | -52.405 | -87.260 | 34.807 | 62.057 | -51.157 | -85.400 | 22.118 | 59.515 |
| | MCSD | 0.494 | 0.237 | 0.650 | 0.218 | 0.337 | 0.162 | 0.322 | 0.151 |
| | ASE | 0.601 | 0.239 | 0.651 | 0.223 | 0.344 | 0.161 | 0.330 | 0.150 |
| | ECP | 0.956 | 0.781 | 0.982 | 0.889 | 0.896 | 0.631 | 0.948 | 0.788 |
| ii. MI | $\%$ bias | -7.168 | -7.021 | 0.607 | -0.374 | -5.506 | -4.019 | -5.870 | -1.669 |
| | MCSD | 0.443 | 0.223 | 0.460 | 0.214 | 0.309 | 0.152 | 0.310 | 0.150 |
| | ASE | 0.453 | 0.220 | 0.467 | 0.213 | 0.307 | 0.153 | 0.305 | 0.149 |
| | ECP | 0.960 | 0.947 | 0.966 | 0.946 | 0.957 | 0.947 | 0.943 | 0.944 |
| iii. AIPW | $\%$ bias | -2.221 | -4.253 | 6.116 | 3.532 | -1.846 | 0.311 | -2.738 | 2.697 |
| | MCSD | 0.475 | 0.254 | 0.570 | 0.240 | 0.316 | 0.160 | 0.323 | 0.164 |
| | ASE | 0.667 | 0.262 | 0.720 | 0.257 | 0.326 | 0.167 | 0.329 | 0.162 |
| | ECP | 0.972 | 0.961 | 0.975 | 0.954 | 0.958 | 0.957 | 0.950 | 0.934 |
Table 15. Simulation results regarding the regression coefficients under an average right censoring rate of 45% and $\xi_4 = 0.1$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| of missing | $n = 200$ | $n = 400$ |
|------------|-----------|-----------|
| $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | | | | | | | |
| % bias | -28.275 | -49.605 | 19.867 | 37.015 | -29.476 | -48.030 | 12.149 | 34.224 |
| MCSD | 0.420 | 0.201 | 0.397 | 0.192 | 0.284 | 0.137 | 0.281 | 0.132 |
| ASE | 0.427 | 0.203 | 0.435 | 0.193 | 0.286 | 0.139 | 0.280 | 0.131 |
| ECP | 0.949 | 0.877 | 0.969 | 0.926 | 0.919 | 0.802 | 0.948 | 0.891 |
| ii. MI | | | | | | | |
| % bias | -2.645 | -2.638 | 1.348 | 0.865 | -3.198 | -2.442 | -5.119 | -1.849 |
| MCSD | 0.392 | 0.196 | 0.387 | 0.191 | 0.269 | 0.135 | 0.269 | 0.133 |
| ASE | 0.393 | 0.197 | 0.395 | 0.190 | 0.271 | 0.136 | 0.269 | 0.131 |
| ECP | 0.952 | 0.955 | 0.956 | 0.941 | 0.962 | 0.955 | 0.950 | 0.949 |
| iii. AIPW | | | | | | | |
| % bias | 0.125 | -0.281 | 3.282 | 3.373 | -0.556 | 0.253 | -2.640 | 0.809 |
| MCSD | 0.397 | 0.206 | 0.393 | 0.202 | 0.273 | 0.136 | 0.274 | 0.137 |
| ASE | 0.426 | 0.214 | 0.463 | 0.211 | 0.275 | 0.140 | 0.274 | 0.135 |
| ECP | 0.951 | 0.949 | 0.967 | 0.946 | 0.963 | 0.955 | 0.952 | 0.948 |

| of missing | $n = 200$ | $n = 400$ |
|------------|-----------|-----------|
| $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| i. CC | | | | | | | |
| % bias | -50.782 | -85.256 | 35.604 | 64.385 | -50.346 | -82.834 | 23.424 | 61.693 |
| MCSD | 0.494 | 0.238 | 0.550 | 0.220 | 0.334 | 0.162 | 0.328 | 0.151 |
| ASE | 0.588 | 0.237 | 0.683 | 0.225 | 0.343 | 0.161 | 0.332 | 0.152 |
| ECP | 0.963 | 0.787 | 0.982 | 0.881 | 0.897 | 0.641 | 0.942 | 0.786 |
| ii. MI | | | | | | | |
| % bias | -6.428 | -8.069 | 1.876 | -0.934 | -5.537 | -4.029 | -5.942 | -1.697 |
| MCSD | 0.445 | 0.222 | 0.463 | 0.214 | 0.312 | 0.152 | 0.311 | 0.150 |
| ASE | 0.453 | 0.219 | 0.473 | 0.213 | 0.308 | 0.153 | 0.306 | 0.149 |
| ECP | 0.964 | 0.950 | 0.970 | 0.947 | 0.952 | 0.948 | 0.941 | 0.948 |
| iii. AIPW | | | | | | | |
| % bias | -1.049 | -5.031 | 6.811 | 3.129 | -2.301 | 0.339 | -3.328 | 2.694 |
| MCSD | 0.474 | 0.250 | 0.529 | 0.238 | 0.319 | 0.162 | 0.322 | 0.165 |
| ASE | 0.637 | 0.262 | 0.775 | 0.256 | 0.328 | 0.166 | 0.332 | 0.163 |
| ECP | 0.969 | 0.961 | 0.971 | 0.953 | 0.958 | 0.953 | 0.953 | 0.937 |
Table 16. Simulation results regarding the regression coefficients under an average right censoring rate of 45% and $\xi_4 = 0.5$. CC refers to complete case method. MI refers to the multiple imputation method. AIPW refers to the augmented inverse probability weighting method. MCSD refers to Monte Carlo standard deviation. ASE refers to average standard error. ECP refers to empirical coverage probability.

| % of missing | $n = 200$ | $n = 400$ |
|--------------|-----------|-----------|
| i. CC        | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| % bias       | -23.424   | -37.740   | 22.582 | 44.353 | -24.223 | -37.758 | 14.878 | 39.640 |
| MCSD         | 0.409     | 0.194     | 0.403 | 0.193 | 0.279      | 0.134     | 0.286 | 0.135 |
| ASE          | 0.413     | 0.200     | 0.440 | 0.196 | 0.278      | 0.136     | 0.284 | 0.133 |
| ECP          | 0.945     | 0.908     | 0.974 | 0.901 | 0.929      | 0.845     | 0.945 | 0.875 |
| ii. MI       | -2.441    | -0.652    | 1.943 | 3.215 | -2.976     | -1.934    | 4.640 | 1.023 |
| % bias       | 0.389     | 0.192     | 0.386 | 0.189 | 0.267      | 0.133     | 0.266 | 0.132 |
| MCSD         | 0.389     | 0.195     | 0.396 | 0.190 | 0.268      | 0.134     | 0.267 | 0.130 |
| ASE          | 0.950     | 0.951     | 0.969 | 0.948 | 0.957      | 0.953     | 0.948 | 0.948 |
| iii. AIPW    | -0.055    | 1.304     | 3.491 | 5.245 | -1.306     | 0.014     | 3.443 | 0.731 |
| % bias       | 0.398     | 0.204     | 0.399 | 0.203 | 0.276      | 0.139     | 0.276 | 0.140 |
| MCSD         | 0.436     | 0.217     | 0.513 | 0.216 | 0.275      | 0.141     | 0.280 | 0.137 |
| ASE          | 0.963     | 0.943     | 0.968 | 0.950 | 0.954      | 0.953     | 0.950 | 0.941 |
| 50% of missing | $n = 200$ | $n = 400$ |
| i. CC        | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{21}$ | $\beta_{22}$ |
| % bias       | -45.094   | -75.234   | 45.117 | 70.900 | -45.102   | -72.015 | 27.893 | 68.414 |
| MCSD         | 0.475     | 0.227     | 1.114 | 0.226 | 0.325      | 0.155     | 0.337 | 0.159 |
| ASE          | 0.518     | 0.228     | 0.764 | 0.236 | 0.327      | 0.155     | 0.347 | 0.157 |
| ECP          | 0.956     | 0.823     | 0.983 | 0.870 | 0.910      | 0.695     | 0.949 | 0.763 |
| ii. MI       | -4.835    | -7.481    | 4.239 | 0.338 | -4.590    | -3.838    | -4.525 | -1.008 |
| % bias       | 0.442     | 0.221     | 0.489 | 0.213 | 0.310      | 0.150     | 0.312 | 0.150 |
| MCSD         | 0.444     | 0.218     | 0.470 | 0.214 | 0.305      | 0.152     | 0.307 | 0.149 |
| ASE          | 0.946     | 0.944     | 0.965 | 0.948 | 0.949      | 0.948     | 0.943 | 0.943 |
| iii. AIPW    | -0.310    | -4.569    | 9.605 | 4.948 | -2.112    | -0.743    | -2.382 | 2.593 |
| % bias       | 0.490     | 0.254     | 0.646 | 0.256 | 0.327      | 0.173     | 0.335 | 0.175 |
| MCSD         | 0.651     | 0.273     | 0.980 | 0.274 | 0.344      | 0.172     | 0.376 | 0.171 |
| ASE          | 0.967     | 0.959     | 0.977 | 0.959 | 0.955      | 0.942     | 0.951 | 0.936 |
Appendix IV: Analysis of cause-specific hazards in the HIV data example

It has been argued that, in real-world analyses with competing risks data, one should analyze all the CSHs and CIFs to obtain a more complete understanding of the competing risks process under study (Latouche and others, 2013). However, to the best of our knowledge, there are no methods for semiparametric regression analysis of the CSH under both interval censoring and missing event types. Thus, in order to additionally analyze the CSHs, we used the maximum pseudo-partial-likelihood estimator (MPPLE) for the semiparametric proportional hazards model that accounts for missing event types (Bakoyannis and others, 2020), while we implemented the naïve midpoint imputation method to address the interval censoring issue in the data. The results from this analysis are provided in Table 1. There is no statistically significant evidence that male gender is associated with the CSH of disengagement. In contrast, the effect of male gender on the CIF of disengagement is statistically significant. This can be explained by the fact that males have a higher CSH of death compared to females. Thus, males appear to disengage less than females since males die more and this precludes them from experiencing disengagement. This can be seen more precisely by the relationship between the CIF and the CSH. Let \( F_1(t; Z) \) and \( F_2(t; Z) \) represent the CIFs of disengagement and death, respectively, conditional on the covariates \( Z \). The corresponding CSHs are denoted as \( \lambda_1(t; Z) \) and \( \lambda_2(t; Z) \). Then

\[
F_1(t; Z) = \int_0^t \lambda_1(u; Z) \exp \left[ - \int_0^u \lambda_1(s; Z) ds - \int_0^u \lambda_2(s; Z) ds \right] du \\
\equiv \int_0^t \lambda_1(u; Z) S(u; Z) du.
\]

Therefore, even if \( \lambda_1(s; Z) \) does not depend on gender, males will have a lower overall survival \( S(u; Z) \) as a result of their CSH hazard of death \( \lambda_2(s; Z) \) and, thus, a lower CIF of disengagement \( F_1(t; Z) \). This means that, even if male gender is not a risk factor for disengagement (defined in terms of the CSH), it is an important prognostic factor of disengagement (defined in terms of the CIF). This result can be used for risk prediction purposes. The rest of the results from the
analysis of the CSHs are qualitatively similar to those from the analysis of the CIFs with the proposed AIPW method. Similarly to the analysis of the CIFs, a naïve complete case analysis using the Cox’s partial likelihood with a midpoint imputation of the interval-censored event times provides substantially different results compared to the MPPLE method which addresses the issue of missing event types.

Table 1. Covariate effects on the cause-specific hazard of disengagement from care and death based on the naïve complete case analysis using the Cox’s partial likelihood (CC) and the maximum pseudo-partial-likelihood estimation (MPPLE) method

| Outcome      | Covariates                          | CC          | MPPLE       |
|--------------|-------------------------------------|-------------|-------------|
|              |                                     | $\hat{\beta}$ (p-value) | $\hat{\beta}$ (p-value) |
| Disengagement| Gender                              | 0.178 (< 0.001) | 0.012 (0.611) |
|              | Male versus Female                   | 0.008 (0.286) | 0.076 (< 0.001) |
|              | CD4 at ART initiation per 100 cells/µl | -0.235 (< 0.001) | -0.147 (< 0.001) |
|              | Age at ART initiation per 10 years   |             |             |
| Death        | Gender                              | 0.356 (< 0.001) | 0.242 (< 0.001) |
|              | Male versus Female                   | -0.044 (< 0.001) | -0.215 (< 0.001) |
|              | CD4 at ART initiation per 100 cells/µl |         |             |
|              | Age at ART initiation per 10 years   | 0.040 (0.848) | 0.089 (< 0.001) |

References

Bakoyannis, Giorgos, Yu, Menggang and Yiannoutsos, Constantin T. (2017). Semiparametric regression on cumulative incidence function with interval-censored competing risks data. *Statistics in Medicine* 36(23), 3683–3707.

Bakoyannis, Giorgos, Zhang, Ying and Yiannoutsos, Constantin T. (2019). Nonparametric inference for Markov processes with missing absorbing state. *Statistica Sinica* 29(4), 2083–2104.

Bakoyannis, Giorgos, Zhang, Ying and Yiannoutsos, Constantin T. (2020). Semipara-
metric regression and risk prediction with competing risks data under missing cause of failure. 

_Lifetime Data Analysis_ **26**(4), 659–684.

**Kosorok, Michael R.** (2008). _Introduction to Empirical Processes and Semiparametric Inference_. Springer, New York.

**Latouche, Aurelien, Allignol, Arthur, Beyersmann, Jan, Labopin, Myriam and Fine, Jason P.** (2013). A competing risks analysis should report results on all cause-specific hazards and cumulative incidence functions. _Journal of Clinical Epidemiology_ **66**(6), 648–653.

**Park, Jun, Bakoyannis, Giorgos and Yiannoutsos, Constantin T.** (2019). Semiparametric competing risks regression under interval censoring using the R package intccr. _Computer Methods and Programs in Biomedicine_ **173**, 167 – 176.

**R Core Team**. (2019). _R: A Language and Environment for Statistical Computing_. R Foundation for Statistical Computing, Vienna, Austria.

**Van der Vaar, Aad W. and Wellner, Jon A.** (1996). _Weak convergence and empirical processes with applications to statistics_. Springer, New York.

[Received August 1, 2010; revised October 1, 2010; accepted for publication November 1, 2010]
Fig. 1. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 13.6% and $\xi_4 = -0.5$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 2. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 13.6% and $\xi_d = -0.1$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 3. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 13.6% and $\xi_4 = 0.1$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 4. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 13.6% and \( \xi_t = 0.5 \). The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 5. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 30% and \( \xi_4 = -0.5 \). The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 6. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 30% and $\xi_4 = -0.1$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 7. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 30% and $\xi_4 = 0$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 8. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 30% and $\xi_4 = 0.1$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 9. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 30% and $\xi_4 = 0.5$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 10. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 45% and $\xi_4 = -0.5$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 11. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 45% and $\xi_4 = -0.1$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 12. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 45% and $\xi_4 = 0$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 13. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 45% and $\xi_4 = 0.1$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.
Fig. 14. Simulation results regarding the baseline cumulative incidence functions (CIF) under an average right censoring rate of 45% and $\xi_4 = 0.5$. The estimated CIFs correspond to the average of the estimated baseline CIFs from the 1000 simulated datasets.