Improved Higgs mass stability bound in the Standard Model and implications for Supersymmetry

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Abstract

We re-examine the lower bound on the mass of the Higgs boson, $M_H$, from Standard Model vacuum stability including next-to-leading-log radiative corrections. This amounts to work with the full one-loop effective potential, $V(\phi)$, improved by two-loop RGE, and allows to keep control of the scale invariance of $V$ in a wide range of the $\phi$-field. Our results show that the bound is $O(10\,\text{GeV})$ less stringent than in previous estimates. In addition we perform a detailed comparison between the SM lower bounds on $M_H$ and the supersymmetric upper bounds on it. It turns out that depending on the actual value of the top mass, $M_t$, the eventually measured Higgs mass can discard the pure SM, the Minimal Supersymmetric Standard Model or both.

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1 Introduction

The vacuum stability requirement in the Standard Model (SM) imposes a severe lower bound on the mass of the Higgs boson, $M_H$ [1-4], which depends on the mass of the top quark $M_t$ and on the cut-off $\Lambda$ beyond which the SM is no longer valid. Roughly speaking, this is due to the fact that the top Yukawa coupling, $h_t$, drives the quartic coupling of the Higgs potential, $\lambda$, to negative values at large scales, thus destabilizing the standard electroweak vacuum.

This bound is very relevant for us since it lies in a region that could be accessible for present (LEP, Tevatron) and future (LEP-200, LHC) accelerators. It is therefore worthwhile to re-examine this important issue taking into account recent improvements on the evaluation of the effective potential and the corresponding Higgs mass [5, 6]. This is the aim of this article 1.

In previous works, the stability bound was obtained from the tree level potential, improved by one-loop or two-loop renormalization group equations (RGE) for the $\beta$- and $\gamma$- functions of the running couplings, masses and the $\phi$-field (see e.g. refs. [3, 4]). However, it has been shown that the one-loop corrections to the Higgs potential are important in order to fix the boundary conditions for the electroweak breaking and calculate the Higgs mass in a consistent and scale-independent way. As we will see, they are also significant to properly understand the whole structure of the potential. In section 2 we write the one-loop corrected Higgs potential, giving the general conditions for extremals. In section 3 we apply the previous results to the determination of the realistic SM vacuum (which implies a relationship between the boundary conditions of the SM parameters) and the physical Higgs mass up to next-to-leading-log order. For a given top mass, there is a one-to-one correspondence between $M_H$ and the boundary condition for $\lambda(t)$. We discuss also in this section the differences between our approach and previous calculations. In section 4 we study the form of the Higgs potential for large $\phi$ and determine under which circumstances it develops a maximum and an additional minimum. This allows us to define the conditions for vacuum stability in the SM, from which we extract the lower bound on $M_H$ as a function of $M_t$ and $\Lambda$. In section 5 we present our complete numerical results. Typically we find that the lower bound on $M_H$ is $O(10 \text{ GeV})$ lower than in the last previous estimate [4]. In addition, we perform a detailed comparison between the SM lower bounds on $M_H$ and the supersymmetric upper bounds on it. It turns out that depending on the actual value of $M_t$, the eventually measured Higgs mass can discard the pure SM, the Minimal Supersymmetric Standard Model or both.

1The results contained in this article have been presented by one of us in Physics from Planck scale to electroweak scale, Warsaw, Poland, 21–24 September 1994 [7].
2 The effective potential of the Standard Model

The renormalization group improved effective potential of the SM, $V$, can be written in the 't Hooft-Landau gauge and the $\overline{\text{MS}}$ scheme as

$$ V[\mu(t), \lambda_i(t); \phi(t)] \equiv V_0 + V_1 + \cdots, \quad (1) $$

where $\lambda_i \equiv (g, g', \lambda, h_t, m^2)$ runs over all dimensionless and dimensionful couplings and $V_0, V_1$ are respectively the tree level potential and the one-loop correction, namely

$$ V_0 = -\frac{1}{2} m^2(t) \phi^2(t) + \frac{1}{8} \lambda(t) \phi^4(t), \quad (2a) $$

$$ V_1 = \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(\phi) \left[ \log \frac{M_i^2(\phi)}{\mu^2(t)} - c_i \right] + \Omega(t), \quad (2b) $$

with

$$ M_i^2(\phi) = \kappa_i \phi^2(t) - \kappa_i', \quad (3) $$

and

$$ n_1 = 6, \quad \kappa_1 = \frac{1}{4} g^2(t), \quad \kappa_1' = 0, \quad c_1 = \frac{5}{6}; $$

$$ n_2 = 3, \quad \kappa_2 = \frac{1}{4} [g^2(t) + g'^2(t)], \quad \kappa_2' = 0, \quad c_2 = \frac{5}{6}; $$

$$ n_3 = -12, \quad \kappa_3 = \frac{1}{2} h_t^2(t), \quad \kappa_3' = 0, \quad c_3 = \frac{3}{2}; $$

$$ n_4 = 1, \quad \kappa_4 = \frac{3}{2} \lambda(t), \quad \kappa_4' = m^2(t), \quad c_4 = \frac{3}{2}; $$

$$ n_5 = 3, \quad \kappa_5 = \frac{1}{2} \lambda(t), \quad \kappa_5' = m^2(t), \quad c_5 = \frac{3}{2}. $$

$M_i^2(\phi, t)$ are the tree-level expressions for the masses of the particles that enter in the one-loop radiative corrections, namely $M_1 \equiv m_W$, $M_2 \equiv m_Z$, $M_3 \equiv m_t$, $M_4 \equiv m_{\text{Higgs}}$, $M_5 \equiv m_{\text{Goldstone}}$. $\Omega(t) \equiv \Omega[\lambda_i(t), \mu(t)]$ is the one-loop contribution to the cosmological constant [5], which will turn to be irrelevant in our calculation.

In the previous expressions the parameters $\lambda(t)$ and $m(t)$ are the SM quartic coupling and mass, whereas $g(t), g'(t), h_t(t)$ are the SU(2), U(1) and top Yukawa couplings respectively. All of them are running with the RGE. The running of the Higgs field is

$$ \phi(t) = \xi(t) \phi_c, \quad (5) $$

$\phi_c$ being the classical field and $\xi(t) = \exp\{-\int_t^\mu \gamma(t')dt'\}$, where $\gamma(t)$ is the Higgs field anomalous dimension. Finally the scale $\mu(t)$ is related to the running parameter $t$ by

$$ \mu(t) = \mu e^{t'}, \quad (6) $$

2
where $\mu$ is a fixed scale, that we will take equal to the physical $Z$ mass, $M_Z$.

It has been shown that the L-loop effective potential improved by (L+1)-loop RGE resums all Lth-to-leading logarithm contributions. Consequently, we will consider all the $\beta$- and $\gamma$-functions of the previous parameters to two-loop order, so that our calculation will be valid up to next-to-leading logarithm approximation.

As has been pointed out in ref. [5], working with $\partial V/\partial \phi$ (and higher derivatives) rather than with $V$ itself allows to ignore the cosmological constant term $\Omega$. In fact, the structure of the potential can be well established once we have determined the $\beta$RGE resums all Lth-to-leading logarithm contributions. Consequently, we will consider all the $\beta$- and $\gamma$-functions of the previous parameters to two-loop order, so that our calculation will be valid up to next-to-leading logarithm approximation.

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up to two-loop corrections. Substituting (11a) and (11b) in (8) we obtain eq. (9).

To conclude this section, we would like to stress the fact that even though the whole effective potential is scale-invariant, the one-loop approximation is not. Therefore, one needs a criterion to choose the appropriate renormalization scale in the previous equations. As was shown in [6], a sensible criterion is to choose the scale, say \(\mu^* = \mu(t^*)\), where the effective potential is more scale-independent. \(\mu^*\) turns out to be a certain average of the \(M_i(\phi)\) masses. Actually, it was shown in [6], and we would like to emphasize it here, that any choice of \(\mu^*\) around the optimal value produces the same results for physical quantities up to tiny differences.

### 3 The realistic minimum and the Higgs mass

We follow in this section the approach in ref. [6], to which we refer the reader for further details. The boundary conditions for the parameters of the Higgs potential, \(m^2(t), \lambda(t)\), are constrained by the fact that \(V\) must develop an electroweak breaking minimum consistent with the experimental observations. In the framework of the Standard Model this “realistic” minimum corresponds to our present vacuum. Accordingly, we must impose

\[
\langle \phi(t_Z) \rangle = v = (\sqrt{2} G_{\mu})^{-1/2} = 246.22 \text{ GeV},
\]

where \(t_Z\) is defined as \(\mu(t_Z) = M_Z\) and \(v\) is the “measured” VEV for the Higgs field [10]. On the other hand \(\langle \phi(t) \rangle\) must be obtained from the minimum condition (7). If we evaluate eq. (7) at the convenient scale \(\mu^* = \mu(t^*)\), what we get is \(\langle \phi(t^*) \rangle\) rather than \(\langle \phi(t_Z) \rangle\). Then, \(\langle \phi(t_Z) \rangle\) is obtained through its RGE

\[
\langle \phi(t_Z) \rangle = \langle \phi(t^*) \rangle \frac{\xi(t_Z)}{\xi(t^*)}.
\]

The criterion to choose \(\mu^*\) has been stated in the previous section.

We can trade \(\langle \phi(t^*) \rangle\) by \(m^2(t^*)\) from the condition of minimum (7), which translates,

\[
\frac{1}{2} \beta m^2 - \gamma m^2 = \sum_i \frac{n_i \kappa_i \kappa_i'}{16 \pi^2},
\]

\[
\sum_i \frac{n_i \kappa_i'^2}{32 \pi^2} = \frac{\partial \Omega}{\partial t^*},
\]
using (13) and (12), into the boundary condition for $m^2(t)$:

$$
\frac{m^2(t^*)}{v^2} = \frac{1}{2} \lambda(t^*) \xi^2(t^*) + \frac{3}{64\pi^2} \xi^2(t^*) \left\{ \frac{1}{2} g^4(t^*) \left[ \log \frac{g^2(t^*) \xi^2(t^*) v^2}{4\mu^2(t^*)} - \frac{1}{3} \right] + \frac{1}{4} \left[ g^2(t^*) + g'^2(t^*) \right] \left[ \log \frac{g^2(t^*) + g'^2(t^*) \xi^2(t^*) v^2}{4\mu^2(t^*)} - \frac{1}{3} \right] 
- 4h^4(t^*) \left[ \log \frac{h^2(t^*) \xi^2(t^*) v^2}{2\mu^2(t^*)} - 1 \right] \right\},
$$

(14)

where we have neglected the Higgs and Goldstone boson contributions.

The running Higgs mass, $m_H^2(t)$, defined as the curvature of the scalar potential at the minimum, can be readily obtained from (8) evaluated at the scale $t^*$, i.e.

$$
m_H^2(t^*) = \frac{\partial^2 V}{\partial \phi^2(t^*)} \bigg|_{\phi(t^*) = \langle \phi(t^*) \rangle}.
$$

(15)

The scale invariance of the second derivative of the potential, $\frac{d}{dt} \left[ \xi^2(t) \frac{\partial^2 V}{\partial \phi^2(t)} \right] = 0$, allows us to write $m_H^2(t)$ at any arbitrary scale

$$
m_H^2(t) = m_H^2(t^*) \frac{\xi^2(t^*)}{\xi^2(t)}.
$$

(16)

The physical (pole) Higgs mass, $M_H^2$, is then given by

$$
M_H^2 = m_H^2(t) + \text{Re}[\Pi(p^2 = M_H^2) - \Pi(p^2 = 0)],
$$

(17)

where $\Pi(p^2)$ is the renormalized self-energy of the Higgs boson (the $t$-dependence of (17) drops out). Explicit expressions for $m_H^2(t^*)$ and $\Pi(M_H^2) - \Pi(0)$ can be found in ref. [6].

As has been stated above, the choice of $\mu^*$, i.e. the scale at which we evaluate the minimum conditions, is not important for physical quantities, provided it is within a (quite wide) region around the optimal value. This is illustrated for the Higgs mass with a representative example in Fig. 1 (solid line). The lack of flatness of $M_H$ reflects the effect of all non-considered (higher order) contributions in the calculation and, therefore, it is a measure of the total error in our estimate of $M_H$. From Fig. 1 we can see that the error is typically $\lesssim 3$ GeV, which is the uncertainty we should assign to our results. The dashed line is the corresponding result performing the previous calculations just with the (RGE improved) tree-level part of $V$ in eq. (1), as was done in refs. [3, 4]. Then, the Higgs mass has a strong dependence on $\mu^*$. Choosing $\mu^* = M_Z$, as it was done in refs. [3, 4], results in an error in the estimate of $M_H$, whose precise value depends on the top mass and is typically of $\mathcal{O}(10$ GeV), showing the need of a more careful treatment of the problem, as the one exposed above.
Finally, let us note that in the previous equations the top Yukawa coupling, $h_t(t)$, enters in several places. Therefore, the Higgs mass depends on the boundary condition chosen for $h_t(t)$, and thus on the top mass $M_t$. However the running top mass, defined as $m_t(t) = v h_t(t)$, does not coincide with the physical (pole) mass, $M_t$. In the Landau gauge the relationship between the running $m_t$ and the physical (pole) mass $M_t$ is given by [11]

$$M_t = \left\{ 1 + \frac{4 \alpha_S(M_t)}{3 \pi} + \left[ 16.11 - 1.04 \sum_{i=1}^{5} \left( 1 - \frac{M_i}{M_t} \right) \left( \frac{\alpha_S(M_t)}{\pi} \right)^2 \right] m_t(M_t) \right\}.$$  

(18)

where $M_i$, $i = 1, \ldots, 5$ represent the masses of the five lighter quarks.

Summarizing the results of this section, for a given top mass and a boundary condition for $\lambda(t)$, the boundary condition of $m^2(t)$ is obtained from the electroweak breaking constraint, and is given by eq.(14). Then, the Higgs mass, $M_H$, can be calculated from eq.(17).

### 4 The structure of the potential for large $\phi$ and the lower bound on $M_H$

The structure of maxima and minima of $V$ for large $\phi$ can be studied with the equations (7) and (9), evaluated at a scale $\mu(t)$ within the region where $V$ is scale-invariant. As was discussed in [8] and in the previous section, $\mu(t) = \phi(t)$ is always a correct choice (other choices, such that $\mu(t) = \phi(t)/2$, are equally valid and lead to the same results). Then the extremal condition (7), neglecting the Higgs and Goldstone contributions, reads

$$\phi_{\text{ext}}^2 = \frac{2 m^2}{\tilde{\lambda}},$$  

(19)

with

$$\tilde{\lambda} = \lambda - \frac{1}{16 \pi^2} \left\{ 6 h_t^4 \left[ \log \frac{h_t^2}{2} - 1 \right] - \frac{3}{4} g^4 \left[ \log \frac{g^2}{4} - \frac{1}{3} \right] \right\}.$$  

(20)

[all quantities in (19,20) are evaluated at $\mu(t) = \phi_{\text{ext}}(t)$]. From (19) we see that, if $V$ develops an extremal for large values of $\phi$, this must occur for a value of $\phi$ such that

$$0 < \tilde{\lambda}[\mu(t) = \phi(t)] \ll 1.$$  

(21)

On the other hand, for large values of $\phi$ eq. (9) can be very accurately expressed as

$$\frac{\partial^2 V}{\partial \phi^2(t)} \bigg|_{\phi(t)=\phi_{\text{ext}}(t)} = \frac{1}{2} (\beta_\lambda - 4 \gamma \lambda) \phi^2(t).$$  

(22)
Since near the extremum $\tilde{\lambda}$, and thus $\lambda$, is very small, we see from (22) that depending on the sign of $\beta_\lambda$ we will have a maximum or a minimum.

We have illustrated these features in Fig. 2 with a typical example. Fig. 2a represents the evolution of $\lambda$ (dashed line) and $\tilde{\lambda}$ (solid line) with $\mu(t)$. It is worth noticing that they do not cross the horizontal axis at the same value of $\mu(t)$, but they differ by more than one order of magnitude. This is important since the point where the maximum of the potential is located, say $\phi_{MAX}$, does correspond to $\tilde{\lambda} \sim 0$ rather than $\lambda \sim 0$ (see eq. (21)). This is apparent in Fig. 2b, where the scalar potential, $V(\phi)$, has been represented\(^4\). The open diamond and square in Fig. 2a correspond to the position of the maximum and the minimum, respectively, of $V(\phi)$. Notice also that for values of $\phi$ very slightly higher than $\phi_{MAX}$, the potential is negative and much deeper than the realistic minimum. This is simply because for values of $\mu(t)$ just beyond $\mu_{MAX} = \phi_{MAX}$, the value of $\tilde{\lambda}$ becomes negative and the potential is dominated by the contribution $\frac{1}{8} \tilde{\lambda} \phi^4$ (compare eqs. (1) and (20)). Consequently, a sensible criterion for a model to be safe is to require one of the two following conditions:

\textbf{a)} The potential has no maximum.

\textbf{b)} The maximum occurs for $\phi_M > \Lambda$,

where we recall that $\Lambda$ is the cut-off beyond which the Standard Model is no longer valid. In the following we will assume $\Lambda \leq 10^{19}$ GeV. With this criterion [in particular condition (b)] we see that the model represented in Fig. 2 is acceptable for $\Lambda \leq 2.7 \times 10^{11}$ GeV. Beyond this scale the stability of the vacuum requires the appearance of new physics. Note from this discussion that conditions (a), (b) are not equivalent to require $\lambda(\mu) > 0$ for $\mu(t) < \Lambda$, as is usually done. Instead, the significant parameter is $\tilde{\lambda}$ rather than $\lambda$.

We have represented in Fig. 3 the evolution of $\tilde{\lambda}$ for a set of models with the same value of $M_t$ as the previous one ($M_t = 160$ GeV), but with different boundary conditions for $\lambda(t)$, and thus different values of $M_H$ (obtained as explained in sect. 3). The thickest line ($M_H = 101$ GeV), which is tangent to the horizontal axis at $\Lambda_0 = 4 \times 10^{11}$ GeV, represents a limiting case: for $M_H > 101$ GeV the potential has no maximum and therefore is completely safe for any choice of $\Lambda$ [see condition (a) above]; for $M_H < 101$ GeV the models are safe depending on the value of $\Lambda < \Lambda_0$. Hence, for $\Lambda > \Lambda_0$ the lower bound on $M_H$ is insensitive to the value of $\Lambda$. The situation depicted in this example, i.e. for $M_t = 160$ GeV, typically occurs when the top mass is rather small, since then the top Yukawa coupling is too small at large scales to maintain $\beta_\lambda$ negative [see eq. (10)]. When the top mass is higher the situation is different and is illustrated with the case $M_t = 174$ GeV in Fig. 4. There we see that for each value of $M_H$, $\tilde{\lambda}(t)$ crosses the horizontal axis at most once, and there is a value of $M_H$ for which the cross occurs at $\Lambda = 10^{19}$ GeV. In consequence, for $\Lambda \leq 10^{19}$ GeV there is a one-to-one correspondence between the choice of $\Lambda$ and the value of the lower bound on $M_H$.

\(^4\)The function of $V$ that has been plotted in Fig. 2b has been chosen in order to give a continuous and faithful representation of $V$ in logarithmic units.
Finally, we would like to point out that generically the potential is *not* unbounded from below since whenever there is a maximum, there is an additional minimum for a larger value of \( \phi \), as illustrated in Fig. 2b. This is because \( \tilde{\lambda} \) gets back to the positive range for large enough scales. Hence, the potential becomes eventually positive and monotonically increasing, although in some cases, e.g. for \( M_t = 174 \) GeV, this occurs for values of \( \phi \) beyond \( 10^{19} \) GeV.

### 5 Numerical results and comparison with SUSY bounds

As stated in the Introduction and has become clear in previous sections, the lower bound on \( M_H \) is a function of \( M_t \) and \( \Lambda \). However, apart from the previously estimated error \( \lesssim 3 \) GeV in our calculation, there is an additional source of uncertainty coming from the value of \( \alpha_S \), which enters in several places in the previous calculation. The most recent estimate of \( \alpha_S \) gives \[ \alpha_S = 0.124 \pm 0.006. \]

Using the central value of (23), we have represented in Fig. 5 the lower bound on \( M_H \) as a function of \( M_t \) for different values of \( \Lambda \). The form of the curves is easily understandable from the discussion of the previous section. In Fig. 6, which corresponds to the pure SM case, we have fixed \( \Lambda \) at its maximum value, \( \Lambda = 10^{19} \) GeV, and represented the lower bound on \( M_H \) for the central value of \( \alpha_S \) in (23) (diagonal solid line) and the two extreme values (diagonal dashed lines).

If we use the recent evidence for the top quark production at CDF with a mass \( M_t = 174 \pm 17 \) GeV [13], we obtain the following lower bound on \( M_H \):

\[ M_H > 128 \pm 33 \text{ GeV}, \]

i.e. \( M_H > 95 \) GeV (1\( \sigma \)). If the Higgs is observed in the present or forthcoming accelerators with a mass below the bound of eq. (24), this would be a clear signal of new physics beyond the Standard Model.

Comparing these bounds with the last evaluation performed in ref. [1], we see that our values of \( M_H \) are lower than in [1] by an amount which increases with \( M_t \), going from \( \sim 5 \) GeV for \( M_t \sim 130 \) GeV, to \( \sim 15 \) GeV for \( M_t \sim 200 \) GeV. As has been discussed in sect. 3, the main reason of this difference is the way in which the Higgs mass was computed in ref. [1]. Accordingly, our results give more room to the Higgs mass in the framework of the Standard Model.

It is very interesting to perform a comparison between the SM *lower* bounds on \( M_H \) previously obtained and the supersymmetric *upper* bounds on \( M_H \) [14, 6]. We briefly recall that in the Minimal Supersymmetric Standard Model (MSSM) the Higgs
quartic coupling is not a free parameter, but is given by a certain combination of the $g^2$, $g'^2$ gauge couplings. Thus, experimental data constrain the boundary condition for $\lambda$, which cannot be as large as we want, leading to strong upper bounds on $M_H$.

These bounds depend on three parameters (besides $M_t$): $\Lambda_S$, i.e. the scale below which supersymmetry (SUSY) decouples (from naturality reasons $\Lambda_S \lesssim 1$ TeV); $\tan \beta$, i.e. the ratio $\langle H_2 \rangle / \langle H_1 \rangle$ of the two supersymmetric Higgs doublets; and $X_t = A_t + \mu / \tan \beta$, i.e. the mixing between stops, which is responsible for the threshold correction to the Higgs quartic coupling. The larger threshold correction and $\tan \beta$, the less stringent the SUSY bounds. Therefore, the most conservative situation takes place considering maximum threshold correction (which is achieved for $X_t^2 = 6\Lambda_S^2$) and $\tan \beta = \infty$. Likewise, the larger $\Lambda_S$, the less stringent the bounds; but, as mentioned above, it is not sensible to consider $\Lambda_S$ much larger than 1 TeV. Consequently, to be in the safe side, we have represented in Fig. 6 the MSSM upper bounds (transverse solid and dashed lines), as recently obtained up to next-to-leading-log order in ref. [6], in the most conservative situation with $\Lambda_S = 1$ TeV.

Of course, since in the SUSY case $\Lambda_S$ indicates the appearance of new physics, the supersymmetric upper bounds must be consistent with the SM lower bounds for $\Lambda = \Lambda_S$, as in fact they are. However, setting $\Lambda = 10^{19}$ GeV, i.e. the pure SM case, as in Fig. 6, we find that this is not so for all the top masses, leading to an interesting situation. We can distinguish three zones in Fig. 6:

$i)$ For $M_t = 173 \pm 4$ GeV, i.e. the crossing area of the SM and MSSM curves, the eventually measured Higgs mass will be compatible either with the pure SM or with the MSSM, but not with both at the same time (unless $M_H = 124$ with high accuracy). Accordingly, the experimental Higgs mass either will discard the MSSM or will be a clear signal of new physics beyond the SM compatible with the MSSM.

$ii)$ For $M_t < 169$ GeV, the situation is analogous, but there is a wider range of Higgs masses (area within the two curves) compatible with both SM and MSSM.

$iii)$ For $M_t > 177$ GeV, there is no region of Higgs masses compatible with the SM and MSSM simultaneously. On the contrary there is a range of $M_H$ (within the two curves) which would discard both.

Perhaps the most interesting situation is $(i)$, which occurs precisely for a top mass in the central value of the CDF experimental estimate.

In order to facilitate the comparison between the pure SM and the MSSM, we give below fits of the corresponding bounds on $M_H$, valid for $150$ GeV $< M_t < 200$ GeV and within $\pm 1$ GeV of error.

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6It is interesting to note that for the same reasons, the value of $\lambda$ at large scales remains always positive and hence there are no corresponding lower bounds on $M_H$ from vacuum stability in the MSSM.

7For more details see e.g. ref. [3].
\[ M_H > 127.9 + 1.92(M_t - 174) - 4.25 \left( \frac{\alpha_S - 0.124}{0.006} \right), \Lambda = 10^{19} \text{ GeV} \]  
(25)

\[ M_H < 126.1 + 0.75(M_t - 174) - 0.85 \left( \frac{\alpha_S - 0.124}{0.006} \right), \Lambda_S = 10^3 \text{ GeV} \]  
(26)

where \( M_H \) and \( M_t \) are expressed in GeV.

Note added: After this work was finished we received a preprint by G. Altarelli and G. Isidori [17], where the SM lower bound on \( M_H \) is refined using a different approach. Our results are typically a few GeV below theirs, though both are consistent within the estimated errors.

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Figure Captions

Fig. 1 Plot of the physical Higgs mass, $M_H$, as a function of $\mu(t^*)$ in a model with $M_t = 175$ GeV, $\Lambda = 10^{19}$ GeV and $\alpha_S = 0.124$. The solid line corresponds to our approach using the full one-loop effective potential, $V$, improved by two-loop RGE. The dashed line is the result using only the tree–level part of $V$, improved with two-loop RGE, as was done in refs. [3, 4].

Fig. 2a Plot of $\lambda$ (dashed line) and $\tilde{\lambda}$ (solid line) as a function of the scale $\mu(t)$ for $\Lambda$ and $\alpha_S$ as in Fig. 1, $M_t = 160$ GeV and $M_H = 100$ GeV.

Fig. 2b Plot of the scalar potential, $V(\phi)$, corresponding to Fig. 2a, represented in a convenient choice of units as described in the text.

Fig. 3 Plot of $\tilde{\lambda}$ as a function of the scale $\mu(t)$ for $M_t = 160$ GeV, $\Lambda = 10^{19}$ GeV and $\alpha_S = 0.124$. The curves are shown for intervals of 5 GeV in $M_H$ in the range $85$ GeV $\leq M_H \leq 115$ GeV. The thicker (tangent) line corresponds to the case $M_H = 101$ GeV.

Fig. 4 The same as Fig. 3 for $M_t = 174$ GeV and $110$ GeV $\leq M_H \leq 140$ GeV. The thicker line, with $\tilde{\lambda}(\mu = 10^{19} \text{ GeV}) = 0$, corresponds to the case $M_H = 128$ GeV.

Fig. 5 SM lower bound on $M_H$ as a function of $M_t$ for $\alpha_S(M_Z) = 0.124$ and different values of $\Lambda$ in the range $10^3$ GeV $\leq \Lambda \leq 10^{19}$ GeV. The values of $\Lambda$ for consecutive curves differ in two orders of magnitude.

Fig. 6 Diagonal (thick) lines: SM lower bound on $M_H$ as a function of $M_t$ for $\Lambda = 10^{19}$ GeV and $\alpha_S = 0.124$ (solid line), $\alpha_S = 0.118$ (upper dashed line), $\alpha_S = 0.130$ (lower dashed line). Transverse (thin) lines: MSSM upper bounds on $M_H$ for $\Lambda_S = 1$ TeV and $\alpha_S$ as in the diagonal lines.
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