Notes on Bit-reversal Broadcast Scheduling

Marcin Kik*
Faculty of Fundamental Problems of Technology
Wrocław University of Technology
ul. Wybrzeże Wyspiańskiego 27
PL-50-370 Wrocław
Poland

December 21, 2013

Abstract

This report contains revision and extension of some results about RBO from [14]. RBO is a simple and efficient broadcast scheduling of $n = 2^k$ uniform frames for battery powered radio receivers. Each frame contains a key from some arbitrary linearly ordered universe. The broadcast cycle – a sequence of frames sorted by the keys and permuted by $k$-bit reversal – is transmitted in a round robin fashion by the broadcaster. At arbitrary time during the transmission, the receiver may start a simple protocol that reports to him all the frames with the keys that are contained in a specified interval of the key values $[\kappa', \kappa'']$. RBO receives at most $2k + 1$ other frames' keys before receiving the first key from $[\kappa', \kappa'']$ or noticing that there are no such keys in the broadcast cycle. As a simple corollary, $4k + 2$ is upper bound the number of keys outside $[\kappa', \kappa'']$ that will ever be received. In unreliable network the expected number of efforts to receive such frames is bounded by $(8k + 4)/p + 2(1 - p)/p^2$, where $p$ is probability of successful reception, and the reception rate of the frames with the keys in $[\kappa', \kappa'']$ is $p$ – the highest possible.

The receiver’s protocol state consists of the values $k$, $\kappa'$ and $\kappa''$, one wake-up timer and two other $k$-bit variables. Its only nontrivial computation – the computation of the next wake-up time slot – can be performed in $O(k)$ simple operations, such as arithmetic/bit-wise operations on $k$-bit numbers, using only constant number of $k$-bit variables.

1 Introduction

RBO [14] is a simple and efficient method of periodic broadcasting of a large sequence of uniform radio messages for radio receivers with a limited source of energy. Examples of such receivers are battery powered sensors or portable devices. In modern devices, the receiver can save the energy by keeping it’s radio device switched off for long periods of time.

The broadcaster transmits in a round robin fashion a large sequence of frames. Such sequence is called a broadcast cycle. Each frame is of the same length (we call it a time slot) and contains in its header a key from an arbitrary linearly ordered universe of key values.

The receiver may decide at arbitrary time (usually somewhere in the middle of the broadcast cycle) to locate and receive all the frames in the stream that contain the keys from some specified range $[\kappa', \kappa'']$. The receiver may wake-up (switch on its radio) at arbitrary time slot to receive the transmitted frame. However, the radio consumes energy while it is switched on. We want to minimize the energy dissipated by the receiver, i.e. to minimize the number of the wake-ups. In RBO, the receiver is able to receive all the requested frames transmitted since that moment. Roughly speaking: the receiver listens to some keys of the broadcast cycle and learns the interval

* Web: http://www.im.pwr.wroc.pl/~kik/
of positions in the sorted sequence with the keys in \([\kappa', \kappa'']\). After that, it only listens in the time slots that contain the keys from these positions.

RBO requires that the length of the broadcast cycle is an integer power of two. This can be achieved by duplicating some of the frames. If \(n'\) denotes the number of frames that must be transmitted, then the length of the broadcast cycle is \(n = 2^k\), where for integer \(k\), \(k \geq \lceil \log_2 n' \rceil\).

We assume that the length of each frame is the same, i.e. a single time slot. However, the same key may be repeated many times in the broadcast cycle. Thus, as single long information attributed with some key can be split among many frames with the same key. We can also repeat many times, the frames that should be delivered more frequently to the receivers. (The frames with the same key are scattered uniformly over the transmission cycle).

The keys may be arbitrary values from arbitrary linearly ordered domain. The receiver does not have any knowledge of the distribution of the keys in the cycle. RBO is energetically efficient for the receiver (Section 3), robust to the radio interferences (Section 3.1), and its implementation is very simple and efficient and requires little memory (Section 4), thus it is suitable even for very weak sensor devices (see e.g. [15]).

This report updates [14] as follows:

- New, simpler proof of the main theorem (Theorem 1) is based on a simpler decomposition of the time-slots sequence.
- We focus on the application of the RBO to filtering the frames with the keys from specified interval \([\kappa', \kappa'']\). In Corollary 1 we show that the receiver has to listen to no more than \(4k + 2\) frames with keys outside \([\kappa', \kappa'']\), to learn which are the time-slots of the frames with keys in \([\kappa', \kappa'']\).
- The expected energetic costs for the receiver in unreliable network has been estimated in Section 3.1.
- A simpler and more efficient algorithm for computing the next wake-up time slot has been proposed in Section 4.

1.1 Example Applications

The protocol can be applied to the dissemination of information or to centralized controlling or synchronizing of large populations of energy constrained devices. Some examples are following:

- The keys may be identifiers of records from a huge database transmitted in the stream.
- The keys may be identifiers of the receiver. The broadcaster may send commands or messages to individual receivers.
- The keys may be identifiers of groups of mutually non-interfering sensors. Each frame with such key would contain only the header, while the rest of the time slot can be used for transmission by the sensors from this group.
- The keys may be coordinates of the objects on the plane encoded by Morton z-ordering [16]. In such ordering the receiver may limit an approximately square region containing the objects that are interesting to him.

Diverse applications could be mixed within a single stream of frames by assigning to them disjoint intervals of key values. The sorted sequence of keys is permuted by bit-reversal permutation, which scatters the keys from each interval uniformly over the whole stream.
1.2 Related Work

Broadcast scheduling for radio receivers with low access time (i.e. the delay to the reception of the required record) and low average tuning time (i.e. the energetic cost) was considered by Imielinski, Viswanathan, and Badrinath (see e.g. [8], [9], [10]). In [9], hashing and flexible indexing for finding single records in broadcast cycle have been proposed and compared. In [10], a distributed index based on an ordered balanced tree has been proposed. The broadcast sequence consists of two kinds of buckets. Groups of index buckets, containing parts of the index tree, are interleaved with the groups of data buckets containing proper data and a pointer (i.e. time offset) to the next index bucket. Each group of index buckets consists of the copy of upper part of the index tree together with the relevant fragment of the lower part of the tree. This mechanism has found useful application even in more complex scenarios of delivering data to mobile users [5].

Khanna and Zhou [11] proposed a sophisticated version of the index tree aimed at minimizing mean access and tuning time, for given probability of each data record being requested. The broadcast cycle contains multiple copies of data items, so that spacing between copies of each item is related to the optimal spacing, minimizing mean access time derived in [19]. However the keys are not arbitrary. The key of the item is determined by its probability of being requested.

Indexing of broadcast stream for XML documents [1] or for full text search [2] have also been considered.

If the broadcast cycle contains indexing tree structure, then the reception of data in current broadcast cycle depends on the successful reception of the path to this data. Instead of separate index buckets RBO uses short headers of the frames. Each such header contains the key assigned to the frame. As a consequence, in unreliable network the receiver has much more chances of efficient navigation towards the desired frames.

In practical applications, due to imperfect synchronization between the broadcaster and the receiver, the header should also contain either the time-slot number or its bit reversal – the index of the frame. To enable changing the contents and the length of the sequence of the transmitted keys by the broadcaster, the header may also include the parameter $k$, such that $2^k$ is the length of the broadcast cycle, and some bits used to notify the receiver that the that the sequence of keys has been changed. For RBO, these issues have been discussed in [14].

Recall that each step of the classic binary search algorithm actually clips the interval of the possible locations of the searched key in the sorted sequence of keys. The customary presentation is that the keys of the sequence are organized in a balanced binary search tree, and the searched key is compared with a sequence of keys from subsequent levels of this tree. Bit-reversal permutes the sorted sequence of keys so that the broadcast cycle is a sequence of the subsequent levels of a balanced binary search tree for the keys. Moreover, each level is recursively so permuted. We show that it enables efficient search in the periodic transmission of the broadcast cycle even if the search is started at arbitrary time slot. We also exploit this property in the computation of the next time slot that should be listened by the receiver. Bit-reversal permutation has been found useful in many contexts. Some examples of its applications are in FFT algorithm [3] [4], lock-free extensible hash arrays [17], distributed arrays in P2P [6], address mapping in SDRAM [18], scattering of video bursts in transmission scheduling in mobile TV [7]. In RBO, bit-reversal emerged from updating the recursive definition of the rbo permutation used in the underlying ranking procedure in [13] in such a way that zero became a fixed point. The simplicity of bit-reversal computation is a great advantage for practical implementations.

2 Notation and preliminaries

Let $\mathbb{Z}$ denote the set of integers. Let $\mathbb{R}$ denote the set of real numbers. For simplicity and generality, we assume that the keys are from $\mathbb{R}$. By $[a, b]$ we denote the interval of real numbers $\{x \in \mathbb{R} | a \leq x \leq b\}$. If $a > b$ then $[a, b] = \emptyset$. By $[a, b]$ we denote we denote $[a, b] \cap \mathbb{Z}$ (i.e. interval of integers between $a$ and $b$). For a set $S$, we denote the number of its elements by $|S|$.

For $x \in \mathbb{Z}$, $x \geq 0$, for $i \geq 0$, let bit$_i(x)$ be the $i$th least significant bit of the binary representation
of $x$, i.e. \( \text{bit}_t(x) = \lfloor (x \mod 2^{l+1}) / 2^l \rfloor \). For \( l \geq 0 \), a number with binary representation \( x_1 \ldots x_0 \) is denoted by \( (x_1, \ldots, x_0)_2 \), i.e. \( (x_1, \ldots, x_0)_2 = \sum_{i=0}^{l} 2^i \cdot x_i \).

For \( x \in [0, 2^k - 1] \) let \( \text{rev}_k(x) \) denote the bit-reversal of \( x \), i.e. if \( x_i = \text{bit}_i(x) \) then \( x = (x_{k-1}, x_{k-2}, \ldots, x_0)_2 \) and \( \text{rev}_k(x) = (x_0, x_1, \ldots, x_{k-1})_2 \).

For a set \( S \subseteq [0, 2^k - 1] \), \( \text{rev}_k S \) denotes the image of \( S \) under \( \text{rev}_k \), i.e \( \text{rev}_k S = \{ \text{rev}_k(x) \mid x \in S \} \).

Let \( n \) denote the length of the broadcast cycle, \( n = 2^k \), for integer \( k \geq 0 \). Let \( \kappa_{-1}, \kappa_0, \ldots, \kappa_{n-1}, \kappa_n \) be a sequence defined as follows:

- \( \kappa_{-1} = -\infty \)
- \( \kappa_n = +\infty \)
- \( \kappa_0, \ldots, \kappa_{n-1} \) is a sorted sequence of \( n \) finite real values of the keys (i.e. \( \kappa_i \leq \kappa_{i+1} \), for \( -1 \leq i \leq n-1 \)).

Let \( \text{KEYS} = \{ \kappa_0, \ldots, \kappa_{n-1} \} \) (the set of the values of the keys in the sequence).

Let \( \kappa' \) and \( \kappa'' \) be finite real key values such that \( \kappa' \leq \kappa'' \). \([\kappa', \kappa'']\) is the interval of the searched keys.

\( E[X] \) denotes expected value of random variable \( X \).

### 2.1 The description of the protocol

The broadcaster at time-slot \( t \) broadcasts the frame with the key \( \kappa_{\text{rev}_k(t \mod n)} \). The receiver searching for the \([\kappa', \kappa'']\) has two variables \( \text{lb} \) and \( \text{ub} \) initialized to 0 and \( n-1 \), respectively. The receiver may start at arbitrary time slot \( s \), and executes the following algorithm:

- While \( \text{lb} \leq \text{ub} \):
  - In time-slot \( t \) if \( \text{lb} \leq \text{rev}_k(t \mod n) \leq \text{ub} \), then the receiver receives the message with the key \( \kappa = \kappa_{\text{rev}_k(t \mod n)} \) and
    - if \( \kappa < \kappa' \) then it sets \( \text{lb} \) to \( \text{rev}_k(t \mod n) + 1 \), else
    - if \( \kappa'' < \kappa \) then it sets \( \text{ub} \) to \( \text{rev}_k(t \mod n) - 1 \), else
    - if \( \kappa' \leq \kappa \leq \kappa'' \) then it reports reception of the key \( \kappa \) from \([\kappa', \kappa'']\)
  - if \( \text{lb} > \text{ub} \) then the receiver reports that \([\kappa', \kappa''] \cap \text{KEYS} = \emptyset \)

In the above description we used broadcaster time slot numbers. By receiver time we mean the number of time slots that elapsed since the start of the receiver’s protocol. Thus, just before the time slot \( s \) the receiver time is zero, just after time slot \( s \) the receiver time is one, and so on. However, the receiver knows the broadcaster time modulo \( n \) (this information may be included in the frame header) and uses it to compute the timer waking-up the radio for next reception of the frame.

### 2.2 Subsets \( Y_{k,s,i} \) and \( X_{k,s,i} \)

In the analysis of the receiver’s protocol (Section 3), we split the sequence of the time slots following the starting slot \( s \) into segments \( Y_{k,s,i} \). The set \( X_{k,s,i} \) is the set of indexes of the elements transmitted during time slots \( Y_{k,s,i} \). We show that the “density” of initially transmitted indexes bounds the length of \([\text{lb}, \text{ub}]\) and the “sparsity” of the set of indexes of the next segment bounds the number of needed receptions. Finally we sum up the bounds on receptions in all segments. In Section 3 we use this decomposition and also the binary search tree on the elements of \( X_{k,s,i} \) embedded on the graph of the permutation \( \text{rev}_k \), for efficient computation of the wake-up timer.

For the starting time slot \( s \in [0, 2^k - 1] \), for \( i \geq 0 \), let \( t_{k,s,i} \) and \( l_{k,s,i} \) be defined as follows:

- \( t_{k,s,0} = t \) and \( l_{k,s,0} = \max\{l \leq k | t_{k,s,0} \mod 2^l = 0 \} \).
• For $i > 0$, $t_{k,s,i} = (t_{k,s,i-1} + 2^{k,s,i-1}) \bmod n$ and $l_{k,s,i} = \max\{l \leq k | t_{k,s,i} \bmod 2^l = 0\}$. $l_{k,s,i}$ is the maximal length of of the suffix of the zero bits in binary representation of $t_{k,s,i}$. $t_{k,s,i+1}$ is the next time slot after $t_{k,s,i}$ (modulo $n$), that has longer such suffix. Note that $t_{k,s,0}, t_{k,s,1}, t_{k,s,2}, \ldots$ is a (possibly empty) increasing sequence of some integers from $[1, 2^k - 1]$ followed by infinite sequence of zeroes.

Let $last_{k,s} = \min\{i \geq 0 | t_{k,s,i} = 0\}$. Note that $l_{k,s,0}, \ldots, l_{k,s, last_{k,s}}$ is an increasing sequence of integers from $[0, k]$. For $0 \leq i < last_{k,s}$, let $Y_{k,s,i} = \{(t_{k,s,i}, t_{k,s,i+1} - 1)\}$ and let $Y_{k,s,last} = \{(0, 2^k - 1)\}$. For $0 \leq i < last_{k,s}$, let $X_{k,s,i} = \text{rev}_k(Y_{k,s,i})$.

**Lemma 1** $X_{k,s,i} = \{\text{rev}_k(t_{k,s,i}) + 2^{k-l_{k,s,i}} \cdot x'|x' \in ([0, 2^{k-l_{k,s,i}} - 1])\}$ and $\text{rev}_k(t_{k,s,i}) < 2^{k-l_{k,s,i}}$.

**Proof** Let $y_0 = \text{bit}_j(t_{k,s,i})$, let $l = l_{k,s,i}$. Then $Y_{k,s,i}$ is the set of all numbers $(y_{k-1}, \ldots, y_l, y_{l-1}, \ldots, y_0)_2$ such that $y_j \in \{0, 1\}$. Thus $X_{k,s,i} = \text{rev}_k(Y_{k,s,i})$ is the set of all numbers $(x_{l-1}, \ldots, x_0, y_l, \ldots, y_{k-1})_2$ such that $x_i \in \{0, 1\}$. Note that $\text{rev}_k(t_{k,s,i}) = (0, \ldots, 0, y_l, \ldots, y_{k-1})_2 = (0, \ldots, 0, x_{k-l-1}, \ldots, x_0)_2$, where $x_i = y_{k-i-1}$. Thus $\text{rev}_k(t_{k,s,i}) < 2^k - l$. This completes the proof of Lemma 1.

For $0 \leq i < last_{k,s}$, for $0 \leq l \leq l_{k,s,i}$, let $Y_{k,s,i,l} = \{(t_{k,s,i}, t_{k,s,i} + 2^l - 1)\}$. Note that $Y_{k,s,i,l}$ is a disjoint union of the sets $Y_{k,s,i,l}$, for $0 \leq l \leq l_{k,s,i}$. For $0 \leq i < last_{k,s}$, let $X_{k,s,i,l} = \text{rev}_k(Y_{k,s,i,l})$.

**Lemma 2** For $l \in ([0, l_{k,s,i}])$, $X_{k,s,i,l} = \{(\text{rev}_k(t_{k,s,i} + 2^{l-1}) + 2^{k-l-1} \cdot x'|x' \in ([0, 2^{l-1}] - 1))\}$ and $\text{rev}_k(t_{k,s,i} + [2^{l-1}]) < 2^{k-l} - l + 1$.

**Proof** If $l = 0$, then $Y_{k,s,i,l} = \{t_{k,s,i}\}$ and, $X_{k,s,i,l} = \{\text{rev}_k(t_{k,s,i})\}$ and $\text{rev}_k(t_{k,s,i}) < 2^{k-1}$.

Consider the case: $l > 0$. $Y_{k,s,i,l} = \{(t_{k,s,i} + 2^{l-1}), (t_{k,s,i} + 2^{l-1} + 2^{l-1} - 1)\}$. Since $t_{k,s,i} \bmod 2^{l_{k,s,i}} = 0$ and $l - 1 < l_{k,s,i}$, we have $(t_{k,s,i} + 2^{l-1}) \bmod 2^l = 0$ and $Y_{k,s,i,l}$ is the set of all numbers $(y_{k-1}, \ldots, y_l, y_{l-1}, \ldots, y_0)_2$ such that $y_j = \text{bit}_j(t_{k,s,i} + 2^{l-1})$ and $y_j \in \{0, 1\}$. Thus $\text{rev}_k(t_{k,s,i} + 2^{l-1}) < 2^{k-l-1}$ and $X_{k,s,i,l}$ is the set of all numbers $(x_{l-2}, \ldots, x_0, x_{k-l-1}, \ldots, x_0)_2$ such that $x_j = y_{k-j-1}$. This completes the proof of Lemma 2.

**Lemma 3** $\bigcup_{j=0}^3 X_{k,s,i,j} = \{(\text{rev}_k(t_{k,s,i} + 2^{k-j} \cdot x'|x' \in ([0, 2^j - 1]))\}$ and $\text{rev}_k(t_{k,s,i}) < 2^{k-j}$.

**Proof** $\bigcup_{j=0}^3 X_{k,s,i,j} = \text{rev}_k(\bigcup_{j=0}^3 Y_{k,s,i,j}) = \text{rev}_k(\{(t_{k,s,i}, t_{k,s,i} + 2^l - 1)\})$. Since $t_{k,s,i} \bmod 2^l = 0$, the proof follows as in the previous lemmas. This completes the proof of Lemma 3.

### 3 The analysis of the receiver’s process for $[\kappa', \kappa''']$

Let $r'$ and $r''$ be defined as follows:

- $r' = \min\{r \in [-1, n]|\kappa' \leq \kappa_r\}$, and $r'' = \max\{r \in [-1, n]|\kappa_r \leq \kappa''\}$.

For each $r \in [r', r'')$, $\kappa_r \in [\kappa', \kappa''']$. If $[\kappa', \kappa'''] \cap \text{KEYS} = \emptyset$ then, for some $r \in [-1, n - 1]$, $\kappa_r < \kappa'$ and $\kappa'' < \kappa_{r+1}$, and $r' = r + 1$ and $r'' = r$. If $[\kappa', \kappa'''] \cap \text{KEYS} \neq \emptyset$ then, since $-\infty < \kappa' \leq \kappa'' < +\infty$, we have $0 \leq r' \leq r'' < n - 1$.

Let $s$ be the first time slot of the receiver’s protocol. We assume w.l.o.g. that $s \in [0, n - 1]$. For $t \geq 0$: Let $l_b$ and $u_b$ be the values of the variables $l_b$ and $u_b$, respectively, at receiver time $t$. (Thus $l_b = 0$ and $u_b = n - 1$.) Let $x_t = \text{rev}_k((s + t) \bmod n)$. Let used$_t = 1$ if $l_b \leq x_t \leq u_b$ and used$_t = 0$ otherwise. (used$_t = 1$ if the receiver wakes the radio at receiver time $t$.) Let hit$_t = 1$ if $r' \leq x_t \leq r''$ and hit$_t = 0$ otherwise. (hit$_t = 1$ if the requested frame is received at receiver time $t$.) The energy used in the initial $t$ time slots is $\text{en}(t) = \sum_{j=1}^t \text{used}_j$. The extra energy is the energy used for the reception of messages with the keys outside $[\kappa', \kappa''']$. ee($t$) = $\text{en}(t) - \sum_{j=1}^t \text{hit}_j$. Let $HY_t = \{(s + y) \bmod n | y \in [0, t - 1]\}$. $HY_t$ is the set of the
broadcasters' time-slot numbers modulo $n$ of the receiver's initial $t$ slots. Let $HX_t = rev_k HY_t$.

Note that $HX_0 = HY_0 = \emptyset$. A history of the lower (respectively, upper) bounds up to time $t$ for $[\kappa', \kappa'']$ is the sequence \( HL(\kappa', \kappa'', t) = (lb_0, \ldots, lb_t) \) (respectively, \(HU(\kappa', \kappa'', t) = (ub_0, \ldots, ub_t)\)).

Let $\tau = \min\{t \geq 0 | hit_t = 1 \lor lb_{t+1} > ub_{t+1}\}$. Note that $\tau$ is the time until the first hit or noticing that $[\kappa', \kappa''] \cap KEYS = \emptyset$. By the first receiver cycle we mean the first $n$ slots of the receiver time. For $y \in [[0, n]]$, let $tt(y)$ denote the receiver time just before the transmission of the broadcast time slot $y \mod n$, in the first receiver cycle, i.e. $tt(y) = \min\{t \geq 0 | (s + t) \mod n = y\}$.

Note that, since $HX_n = [[0, n - 1]]$, we have $\tau < n$.

**Theorem 1** We have $\tau < n$ and $en(\tau) \leq 2 \cdot k + 1$.

**Proof** We prove Lemmas 4, 5, 6, 7 to show the theorem in the case $[\kappa', \kappa''] \cap KEYS = \emptyset$, and then conclude the general case.

**Lemma 4** If $[\kappa', \kappa''] \cap KEYS = \emptyset$, then for $t \geq 0$, $[[lb_t, ub_t]] \subseteq [[0, n - 1]] \setminus HX_t$.

**Proof** Note that the Lemma follows directly from the algorithm: $lb_0 = 0$, $ub_0 = n - 1$, and, since $[\kappa', \kappa''] \cap KEYS = \emptyset$, for each $x \in HX_t$, either $x < lb_t$ or $ub_t < x$. \(\Box\)

Since $k$ and $s$ are fixed, we use the following notation: last = last$_{k,s}$, $t_i = t_{k,s,i}$, $k_i = k_{k,s,i}$, $Y_i = Y_{k,s,i}$, $X_i = X_{k,s,i}$, $Y_{i,j} = Y_{k,s,i,j}$, and $X_{i,j} = X_{k,s,i,j}$.

**Lemma 5** If $[\kappa', \kappa''] \cap KEYS = \emptyset$, then $\sum_{s \leq t} \text{mod } n \in Y_0 \text{ used}_t \leq k_0 + 1$.

**Proof** Since $Y_0 = \{t_0\}$, and only the first time slot congruent modulo $n$ to $t_0$ is used, we have $\sum_{s \leq t} \text{mod } n \in Y_0 \text{ used}_t = 1$.

Since $Y_0 = \{t_0\}$, and only the first time slot congruent modulo $n$ to $t_0$ is used, we have $\sum_{s \leq t} \text{mod } n \in Y_0 \text{ used}_t = 1$.

For $0 < t \leq k_0$, we show that $\sum_{s \leq t} \text{mod } n \in Y_0 \text{ used}_t \leq 1$: By Lemma 4, $[[lb_{2^t-1}, ub_{2^t-1}]] \subseteq [[0, n - 1]] \setminus HX_{2^t-1}$, and $HX_{2^t-1} = \bigcup_{j=0}^{2^t-1} X_{0,j}$, which, by Lemma 3, contains all the integers from $[[0, n - 1]]$ congruent modulo $2^{t-1}$ to $rev(t_0)$. Hence $ub_{2^t-1} - 1 \leq lb_{2^t-1} \leq 2^{k_t-1} - 1$. Therefore, $X_{0,t}$ contains only the integers from $[[0, n - 1]]$ congruent modulo $2^{k_t-1}$ to $rev_k(t_0 + 2^{l} - 1)$. Hence, $X_{0,t} \cap [[[lb_{2^{t-1}}, ub_{2^{t-1}}]]] \subseteq 1$, and, since only the first time slot congruent modulo $n$ is used, we have $\sum_{s \leq t} \text{mod } n \in Y_0 \text{ used}_t \leq 1$. Since $Y_0 = \bigcup_{t_0} Y_{0,t}$, the Lemma follows. \(\Box\)

**Lemma 6** If $[\kappa', \kappa''] \cap KEYS = \emptyset$, then, for $1 \leq t \leq last$, $\sum_{s \leq t} \text{mod } n \in Y_{t-1} \text{ used}_t \leq k_t - k_{t-1} + 1$.

**Proof** Let $\kappa'' = tt(t_{i-1})$. By Lemma 4, $[[lb_{2^{t-1}}, ub_{2^{t-1}}]] \subseteq [[0, n - 1]] \setminus HX_{2^{t-1}}$. We have $X_{t-1} \subseteq HX_{2^{t-1}}$, and, by Lemma 1, $X_{t-1}$ contains all the integers from $[[0, n - 1]]$ congruent modulo $2^{k_t-1}$ to $rev(t_{i-1})$. Thus, $ub_{2^{t-1} + 1} \leq lb_{2^{t-1}} + 1 + 2^{k_t-1}$. By Lemma 3, $\bigcup_{j=0}^{2^{t-1}} X_{t-1,j}$ contains only the integers from $[[0, n - 1]]$ congruent modulo $2^{k_t-1}$ to $rev_k(t_{i-1} + 2^{l-1} - 1)$. Hence, we have $\bigcup_{j=0}^{2^{t-1}} X_{t-1,j} \cap [[[lb_{2^{t-1}}, ub_{2^{t-1}}]]] \subseteq 1$, and $\sum_{s \leq t} \text{mod } n \in Y_{t-1} \text{ used}_t \leq 1$.

For $k_t - 1 \leq t \leq k_t$, we show that $\sum_{s \leq t} \text{mod } n \in Y_{t-1,j} \text{ used}_t \leq 1$: We have $[[lb_{2^{t+1} - 2^{t-1}}, ub_{2^{t+1} - 2^{t-1}}]] \subseteq [[0, n - 1]] \setminus HX_{2^{t} - 2^{t-1}}$ and $HX_{2^{t} - 2^{t-1}}$ is a super-set of $\bigcup_{j=0}^{2^{t-1}} X_{t-1,j}$, which, by Lemma 3, contains all the integers from $[[0, n - 1]]$ congruent modulo $2^{k_t-1} - l$ to $rev(t_{i-1})$. By Lemma 2, $X_{t-1}$ contains only the integers from $[[0, n - 1]]$ congruent modulo $2^{k_t-1} - l$ to $rev_k(t_{i-1} + 2^{l-1} - 1)$. Hence $X_{t-1} \cap [[[lb_{2^{t+1} - 2^{t-1}}, ub_{2^{t+1} - 2^{t-1}}]]] \subseteq 1$.

Thus $\sum_{s \leq t} \text{mod } n \in Y_{t-1,j} \text{ used}_t \leq k_t - k_{t-1} + 1$ and the Lemma follows. \(\Box\)

**Lemma 7** If $[\kappa', \kappa''] \cap KEYS = \emptyset$, then $\sum_{t \geq 0} \text{used}_t \leq 2k + 1$.

**Proof** $[[0, n - 1]] = \bigcup_{t=0}^{2^k} Y_t$, and $\sum_{t \geq 0} \text{used}_t = \sum_{s \leq t} \text{mod } n \in Y_0 \text{ used}_t + \sum_{t=1}^{last} \sum_{s \leq t} \text{mod } n \in Y_t \text{ used}_t.$

Thus, by Lemma 5 and Lemma 6, $\sum_{t \geq 0} \text{used}_t \leq k_0 + 1 + \sum_{t=1}^{last} (k_t - k_{t-1} + 1) = k_{last} + last + 1$.

Since $k_0, \ldots, k_{last}$ is increasing sequence of values from $[[0, k]]$, we have $k_{last} \leq k$ and last $\leq k$. \(\Box\)

In Lemma 7 we assumed that $[\kappa', \kappa''] \cap KEYS = \emptyset$. Note that we have:
Consider a model of the network, where the probability of successful reception is \( p \).

### Unreliable network

#### Corollary 1

For arbitrary \( t > 0 \), \( ee(t) \leq 4k + 2 \).

**Proof** If \([k', k''] \cap \text{KEYS} = \emptyset\), then \( ee(t) \leq en(t) \) and, by Theorem 1, \( en(t) \leq 2k + 1 \).

Consider the case \([k', k''] \cap \text{KEYS} \neq \emptyset\). Then \(-1 < r' < r'' < n\). Let \( \gamma' \) and \( \gamma'' \) be such that \( \kappa_{r'-1} < \gamma' < \kappa_{r'} \) and \( \kappa_{r''} < \gamma'' < \kappa_{r'+1} \). Then, for arbitrary \( t \geq 0 \), \( \text{HL}(\gamma', \gamma', t) = \text{HL}(k', k'', t) \) and \( \text{HU}(\gamma'', \gamma'', t) = \text{HU}(k', k'', t) \). Any reception of the key that is outside \([k', k'']\) updates either the lower or the upper bound: For \( t > 0 \), \( u_b = 1 \) and \( h_t = 0 \) if and only if either \( \kappa_i < 1 \) or \( u_b + 1 \). Thus \( ee(t) \) is equal to the total number of changes in both \( \text{HL}(k', k'', t) \) and \( \text{HU}(k', k'', t) \). Since \([\gamma', \gamma'] \cap \text{KEYS} = \emptyset\), the number of changes in \( \text{HL}(\gamma', \gamma', t) \) is not greater than \( 2k + 1 \). Similarly, the number of changes in \( \text{HU}(\gamma'', \gamma'', t) \) is not greater than \( 2k + 1 \).

### 3.1 Unreliable network

Consider a model of the network, where the probability of successful reception is \( p \), \( 0 \leq p \leq 1 \). Thus the receiver may wake up to listen in some time slot, and still fail to receive the frame with probability \( q = 1 - p \). Thus the unit of energy used for the wake-up is lost. We state that in the case of reception failure, the receiver’s protocol leaves its variables \( lb \) and \( ub \) unchanged and waits for the next time slot from \( \text{rev}_k([lb, ub]) \).

We split the wake-ups of the receiver into hits – the wake-ups in the time slots from \( \text{rev}_k([r', r'']) \), and misses – the remaining wake-ups. The hits are unavoidable: the requested keys are transmitted during the hits. The penalty for unreliability here is that the reception rate drops from 1 to \( p \) – which is the highest possible in this model. Another penalty is the increase in the number of misses. We show the bound on the number of the misses in unreliable network. Recall that the first wake up of the protocol is in time slot \( s \). For \( t > 0 \), let \( \text{success}(t) \) be true if the transmission in the \( t \)th receiver’s time slot is successful, and false – otherwise.

**Lemma 8** The expected number of misses after the first receiver cycle (i.e. after the initial \( n \) time slots) is not greater than \( 2 \cdot q/p^2 \).

**Proof** The misses in the cycle following the first cycle are the wake-ups during the time slots in \( \text{rev}_k([lb_n, r' - 1]) \cup [r'' + 1, ub_n]) \). The values of \( lb_n - 1 \) and \( ub_n + 1 \) are the following random variables:

- \( lb_n - 1 = \max\{-1\} \cup \{i \in [0, r' - 1] \mid \text{success}(\text{rev}_k(i))\} \)
- \( ub_n + 1 = \min\{n\} \cup \{i \in [r'' + 1, n - 1] \mid \text{success}(\text{rev}_k(i))\} \)
Each of \( r' - (\text{lb}_n - 1) \) and \((\text{ub}_n + 1) - r''\) can be bound by a random variable with geometric distribution (see e.g. [4]) and expected value \(1/p\). Hence, \(\max\{E[r' - \text{lb}_n], E[\text{ub}_n - r'']\} \leq 1/p - 1 = 1/(1 - q) - 1\).

After the \( j \)th cycle, for \( j \geq 1 \), each position has been tested \( j \) times. Thus \(\max\{E[r' - \text{lb}_{jn}], E[\text{ub}_{jn} - r'']\} \leq 1/(1 - q^j) - 1\) and the expected number of misses in the \((j + 1)\)st cycle is not greater than \(2(1/(1 - q^j) - 1)\). Finally, note that \(\sum_{j=1}^{\infty}(1/(1 - q^j) - 1) = \sum_{j=1}^{\infty}(q^j/(1 - q^j)) \leq \frac{1}{1-q} \sum_{j=1}^{\infty} q^j = q/(1-q)^2\).

The more complex task is to bound the number of misses during the first cycle.

**Lemma 9** If \(\text{KEYS} \cap [\kappa', \kappa''] = \emptyset\), then the expected number of wake-ups (all of them are misses) during the first cycle is not greater than \((4k + 2)/p\).

**Proof** Since \(\text{KEYS} \cap [\kappa', \kappa''] = \emptyset\), we have \(r' = r'' + 1\). Let us use the notation from the proof of Theorem 1.

First consider the time-slots in \(Y_0\). There is one wake-up in \(Y_{0,0} = \{t_0\}\). For each \(l \geq 0\), \(\bigcup_{j=0}^{l} X_{0,j} \subseteq \text{xxt}(\min Y_{0,l+1})\). Hence, by Lemma 3

- \(\text{lb}_{tt}(\min Y_{0,l+1}) - 1 \geq \max\{-1\} \cup \{i \in [0,r' - 1] \mid (i - \text{rev}_k(t_0)) \mod 2^{k-l} = 0 \land \text{success}(\text{tt}(\text{rev}_k(i)))\},\)
- and \(\text{ub}_{tt}(\min Y_{0,l+1}) + 1 \leq \min\{n\} \cup \{i \in [r'' + 1, n-1] \mid (i - \text{rev}_k(t_0)) \mod 2^{k-l} = 0 \land \text{success}(\text{tt}(\text{rev}_k(i)))\}\).

Note that \([0, r' - 1] \cup [r'' + 1, n-1] = [0, n-1]\). Thus, \(E[(\text{ub}_{tt}(\min Y_{0,l+1}) - \text{lb}_{tt}(\min Y_{0,l+1}))/2^{k-l}] < 2/p\) — the expected number of integers congruent modulo \(2^{k-l}\) to \(\text{rev}_k(t_0)\) in \([\text{lb}_{tt}(\min Y_{0,l+1}), \text{ub}_{tt}(\min Y_{0,l+1})]\). Since, by Lemma 2, all elements of \(X_{0,l+1}\) are congruent modulo \(2^{k-l}\) to \(\text{rev}_k(t_0) + \lfloor 2^{k-l}\rfloor\), the expected number of wake-ups during time slots \(Y_{0,l+1}\) is bounded by \(2/p\). Thus the expected number of wake-ups in \(Y_0\) is not greater than \(2k_0/p + 1 \leq 2(k_0 + 1)/p\).

Now consider \(Y_i\), for \(i \in [1, \text{last}]\). Since \(X_{i-1} \subseteq \text{xxt}(\min Y_i)\) and, by Lemma 1, \(X_{i-1}\) contains all integers congruent modulo \(2^{k-i-1}\) to \(\min X_{i-1}\) and, by Lemma 3, \(\bigcup_{j=0}^{k_i} X_{i,j}\) contains only integers congruent modulo \(2^{k-k_i-1}\) to \(\min X_i\), the expected number of wake-ups in \(\bigcup_{j=0}^{k_i} Y_{i,j}\) can be bound, as above, by \(2/p\).

For each \(l \in [k_i - k_i - 1, k_i]\), we use \(\bigcup_{j=0}^{l} X_{i,j} \subseteq \text{xxt}(\min Y_{i,l})\), to bound the expected number of wake-ups in \(Y_{i,l}\) by \(2/p\). Thus the expected number of wake-ups in \(Y_i\) is not greater than \(2(k_i - k_i - 1 + 1)/p\).

Summing up, as in the proof of Lemma 7, the expected number of wake-ups during the first cycle is at most \(\frac{3}{2} (k_0 + 1 + \sum_{i=0}^{\text{last}} (k_i - k_i - 1 + 1)) \leq (4k + 2)/p\).

**Theorem 2** The expected number of misses during the infinite execution of the protocol is not greater than \((8k + 4)/p + 2(1 - p)/p^2\).

**Proof** If \(\text{KEYS} \cap [\kappa', \kappa''] = \emptyset\), then the theorem follows directly from Lemmas 8 and 9.

Consider the case \(\text{KEYS} \cap [\kappa', \kappa''] \neq \emptyset\). As in Corollary 1, let \(\gamma'\) and \(\gamma''\) be key values such that \(\kappa_{r'} < \gamma' < \kappa_{r'}\) and \(\kappa_{r''} < \gamma'' < \kappa_{r''+1}\). Let \(E_{\gamma}\) denotes the expected number of misses in the first cycle when the protocol is started for interval \([\gamma, \gamma]\). By Lemma 8, \(\max\{E_{\gamma'}, E_{\gamma''}\} \leq (4k + 2)/p\). The expected number of misses during the first cycle of the protocol for \([\kappa', \kappa'']\) is the sum of the expected number of misses on both sides of \([r', r'']\) which is not greater than \(E_{\gamma'} + E_{\gamma''}\).

**4 Implementation issues**

We present an efficient algorithm for computing the time slot of the reception of the next frame required by the protocol. The efficiency of this algorithm is based on the observation that elements of \(X_{k,s,i}\) are organized by \(\text{rev}_k\) into subsequent levels of an almost balanced binary search tree.
4.1 Binary search tree on $X_{k,s,i}$

For $d \geq 0$, for any sequence $c = (c_1, \ldots, c_d) \in \{-1, 1\}^d$, let a descendant of $x$ by path $c$ be defined as $\text{dsc}_k(x,c) = x + \sum_{i=1}^{d} 2^{k-i}c_i$. Note that $\text{dsc}_k(x,(c_1, c_2, \ldots, c_d)) = \text{dsc}_{k-1}(\text{dsc}_k(x,(c_1))),(c_2, \ldots, c_d)$. Note that $(\text{dsc}_k(x,(c_1, \ldots, c_d)) - x) \mod 2^{k-d} = 0$. Let a level at depth $d$ rooted at $x$ be defined as $L_{k,d}(x) = \{\text{dsc}_k(x,c) | c \in \{-1,1\}^d\}$. Let a sub-tree of depth $d$ rooted at $x$ be defined as $\text{ST}_{k,d}(x) = \{\text{dsc}_k(x,c) | \exists x \in [0,d] | c \in \{-1,1\}^d\}$. The following properties are easy to note without the proof:

**Lemma 10** For $k \geq 0$, for $d \in [0, k]$, we have the following properties:

a) $|L_{k,d}(x)| = 2^d$.

b) $L_{k,0}(x) = \text{ST}_{k,0}(x) = \{x\}$ and, for $d > 1$, $L_{k,d}(x) = \text{ST}_{k,d}(x) \setminus \text{ST}_{k,d-1}(x)$.

c) $|\text{ST}_{k,d}(x)| = 2^{d+1} - 1$.

d) $\text{ST}_{k,d}(x) = \{x + i \cdot 2^{k-d} | i \in [-2^d + 1, 2^d - 1]\}$.

e) If $d \geq 1$ then $\{x\} \cup \text{ST}_{k-1,d-1}(\text{dsc}_k(x,(1))) = \{x + i \cdot 2^{k-d} | i \in [0, 2^d - 1]\}$.

f) $\text{ST}_{k,d}(x) = \text{ST}_{k-1,d-1}(\text{dsc}_k(x,(1))) \cup \{x\} \cup \text{ST}_{k-1,d-1}(x, \text{dsc}_k(x,(1)))$.

g) $\max \text{ST}_{k-1,d}(\text{dsc}_k(x,(1))) + 2^{k-1-d} = x = \min \text{ST}_{k-1,d}(\text{dsc}_k(x,(1))) - 2^{k-1-d}$.

**Lemma 11** shows that each $X_{k,s,i}$ is organized by $\text{rev}_k$ in a binary search tree with the root at $\min X_{k,s,i} = \text{rev}_k(t_{k,s,i})$, without the left sub-tree and with a totally balanced right sub-tree, see Figure 1.

**Lemma 11** states that the elements of the levels closer to the root have lower values of their $k$-bit reversals than the elements of the more distant levels.

![Figure 1: The binary search trees for $X_{5,12,0}$ (a), $X_{5,12,1}$ (b) and $X_{5,12,2}$ (c), on the graph of $y = \text{rev}_5(x)$. Note that the $y$ axis of the graph is directed downwards.](image)

**Lemma 11** For $k \geq 0$, $t \in [0, 2^k - 1]$, $i \in [0, \text{last}_{k,s}]$, let $r = \text{rev}_k(t_{k,s,i})$ and $l = k_{s,i}$.

Then we have:

a) $X_{k,s,i} = \{r\}$ and, for $d \in [1,l]$, $\bigcup_{j=0}^{d} X_{k,s,i,j} = \{r\} \cup \text{ST}_{k-1,d-1}(\text{dsc}_k(r,(1)))$.

b) If $l > 0$ then $X_{k,s,i} = \{r\} \cup \text{ST}_{k-1,l-1}(\text{dsc}_k(r,(1)))$. If $l = 0$ then $X_{k,s,i} = \{r\}$.

c) $X_{k,s,i,0} = \{r\}$ and, for $d \in [1,l]$, $X_{k,s,i,d} = L_{k-1,d-1}(\text{dsc}_k(r,(1)))$.

d) If $c \in \{-1, 1\}^d$ and $c' \in \{-1, 1\}^d$, where $0 \leq d' < d'' \leq l$, and $x' = \text{dsc}_k(r, c')$ and $x'' = \text{dsc}_k(r, c'')$ and $x', x'' \in X_{k,s,i}$ and $y' = \text{rev}_k(x')$ and $y'' = \text{rev}_k(x'')$ then $y' < y''$. 


Proof

Lemma 11a. By Lemma 10d, $\{r\} \cup ST_{k-1,d-1}(\text{dsc}_k(r,(1))) = \{r + i \cdot 2^{k-d} | i \in [0,2^d]\}$ which, by Lemma 5 is equal to $\bigcup_{j=0}^{d} X_{k,s,i,j}$. Lemma 11b follows from $X_{k,s,i} = \bigcup_{j=0}^{d} X_{k,s,i,j}$ and from Lemma 11a.

Lemma 11c. If $d' = 0$ then $x' = r$ and the lemma follows, since $t_{k,s,i} = \min Y_{k,s,i}$. Otherwise, we have $0 < d < d'$, $x' \neq r$ and $x'' \neq r$ and, by Lemma 11d, $x', x'' \in ST_{k-1,d-1}(\text{dsc}_k(r,(1)))$. Thus $x' \in L_{k-1,d-1}(\text{dsc}_k(r,(1)))$ and $x'' \in L_{k-1,d''-1}(\text{dsc}_k(r,(1)))$. By Lemma 11e, $x' \in X_{k,s,i,d'}$ and $x'' \in X_{k,s,i,d''}$. To conclude, note that $\max Y_{k,s,i,d'} < \min Y_{k,s,i,d''}$.

4.2 Implementation of nsi

In realistic implementation, after each reception, the receiver has to compute the next time slot with the index of the transmitted key in the interval $[lb, ub]$, and switch off the radio for the time remaining to this event.

By nsi$_k(t, r_1, r_2)$ we denote the next slot number (modulo $2^k$) after the slot $t$ with its $k$-bit reversal in $[r_1, r_2]$: For $r_1, r_2 \in [0,2^k-1]$, $r_1 \leq r_2$, and $t \in [0,2^k-1]$, nsi$_k(t, r_1, r_2) = (t + \tau(t, r_1, r_2)) \mod 2^k$, where $\tau(t, r_1, r_2) = \min \{ d > 0 | \text{rev}_k((t + d) \mod 2^k) \in [r_1, r_2]\}$.

One could naively test subsequent values after $t$ or all values in $\text{rev}_k([r_1, r_2])$. However, both these methods are time consuming, when both $2^k/(r_2 - r_1)$ and $r_2 - r_1$ are large.

We present an efficient algorithm for the computation of nsi$_k(t, r_1, r_2)$:

1. $t'' \leftarrow (t + 1) \mod 2^k$
2. $l \leftarrow 0$
3. repeat
   a) $t' \leftarrow t''$
   b) while $l < k \land t' \mod 2^{l+1} = 0$ do $l \leftarrow l + 1$
   c) $x_1 \leftarrow \text{rev}_k(t'')$
   d) $t'' \leftarrow (t' + 2^l) \mod 2^k$
   e) $x_2 \leftarrow \text{rev}_k(t' + 2^l - 1)$
4. until $r_1 \leq x_2 \land r_2 \geq x_1 \land \lfloor (r_1 - x_1)/2^{k-l} \rfloor \leq \lfloor (r_2 - x_1)/2^{k-l} \rfloor$
5. $c \leftarrow 2^{k-1}$
6. while $x_1 < r_1 \lor x_1 > r_2$ do
   a) if $x_1 < r_1$ then $x_1 \leftarrow x_1 + c$ else $x_1 \leftarrow x_1 - c$
   b) $c \leftarrow c/2$
7. return $\text{rev}_k(x_1)$

Correctness of the algorithm:

Let $s = (t + 1) \mod 2^k$. Let the iterations of the “repeat-until” loop be numbered starting from `zero`. After the $i$th iteration, at line 3 we have $l = l_{k,s,i}$, $t' = t_{k,s,i}$, $x_1 = \min X_{k,s,i}$, $x_2 = \max X_{k,s,i}$, and $t'' = t_{k,s,i+1}$. Let $i' = \min \{ i \geq 0 | X_{k,s,i} \cap [r_1, r_2] = \emptyset \}$. Since $r_1, r_2 \in [0,2^k-1]$, $r_1 \leq r_2$, and $X_{k,s,last_{k,s}} = [0,2^k-1]$, we have $0 \leq i' \leq \text{last}_{k,s}$. Thus, by Lemma 10f, $i'$ is the number of the first iteration, after which $x_1 \leq x_2 \land r_2 \geq x_1 \land \text{min} \{ j \mid x_1 + 2^{k-l} \cdot j \leq r_2 \} \leq \text{max} \{ j \mid x_1 + 2^{k-l} \cdot j \leq r_2 \}$, which is equivalent to $r_1 \leq x_2 \land r_2 \geq x_1 \land \lfloor (r_1 - x_1)/2^{k-l} \rfloor \leq \lfloor (r_2 - x_1)/2^{k-l} \rfloor$.

After the “repeat-until” loop finishes, at line 5 we have $x_1 = \text{rev}_k(t_{k,s,i'})$ and, by Lemma 11e, $X_{k,s,i'} = \{ x_1 \} \cup S$, where either $S = ST_{k-1,l-1}(\text{dsc}_k(x_1,(1)))$, if $l > 0$, or $S = \emptyset$, if $l = 0$. Since
We do a binary search in $X_{k,s,i'}$ until we enter the interval $[r_1, r_2]$ for the first time. By the Lemma [11], the returned value is $\min \{ \text{rev}_k(x) | x \in X_{k,s,i'} \}$.

Complexity of the algorithm:

The memory complexity: Only the constant number of $k$-bit variables are used.

The time complexity: The number of iterations of the “repeat-until” loop is never greater than $k + 1$. Since the value of $l$ never decreases, the total number of iterations of the internal “while” loop (line [33b]) in all iterations of the “repeat-until” loop is never greater than $k + 1$. The total number of iterations of the binary search loop (starting at line [6]) is never greater than $k$. Thus the total complexity is $O(k)$ elementary operations on $k$-bit integers.

Multiplication, division and modulo operations by the powers of two can be replaced by shifting or bit-masking operations. The implementation of this algorithm in programming language, with optimizations of bit-wise operations can be found on [12].

Some technical aspects of the implementation, such as dealing with imperfect synchronization and proposed structure of the frame header has been discussed in technical report [14].

References

[1] Yon Dohn Chung and Ji Yeon Lee. An indexing method for wireless broadcast XML data. Inf. Sci., 177(9):1931–1953, 2007.

[2] Yon Dohn Chung, Sanghyun Yoo, and Myoung-Ho Kim. Energy- and latency-efficient processing of full-text searches on a wireless broadcast stream. IEEE Trans. Knowl. Data Eng., 22(2):207–218, 2010.

[3] James Cooley and John Tukey. An algorithm for the machine calculation of complex Fourier series. Mathematics of Computation, 19(90):297–301, 1965.

[4] Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest. Introduction to Algorithms. The MIT Press and McGraw-Hill Book Company, 1989.

[5] Anindya Datta, Debra E. VanderMeer, Aslihan Celik, and Vijay Kumar. Broadcast protocols to support efficient retrieval from databases by mobile users. ACM Trans. Database Syst., 24(1):1–79, 1999.

[6] Daisuke Fukuchi, Christian Sommer, Yuichi Sei, and Shinichi Honiden. Distributed arrays: A P2P data structure for efficient logical arrays. In INFOCOM, pages 1458–1466. IEEE, 2009.

[7] Mohamed Hefeeda and Cheng-Hsin Hsu. On burst transmission scheduling in mobile TV broadcast networks. IEEE/ACM Trans. Netw., 18(2):610–623, 2010.

[8] Tomasz Imielinski, S. Viswanathan, and B. R. Badrinath. Energy efficient indexing on air. In Richard T. Snodgrass and Marianne Winslett, editors, SIGMOD Conference, pages 25–36. ACM Press, 1994.

[9] Tomasz Imielinski, S. Viswanathan, and B. R. Badrinath. Power efficient filtering of data on air. In Matthias Jarke, Janis A. Bubenko Jr., and Keith G. Jeffery, editors, EDBT, volume 779 of Lecture Notes in Computer Science, pages 245–258. Springer, 1994.

[10] Tomasz Imielinski, S. Viswanathan, and B. R. Badrinath. Data on air: Organization and access. IEEE Trans. Knowl. Data Eng., 9(3):353–372, 1997.

[11] Sanjeev Khanna and Shiyu Zhou. On indexed data broadcast. Journal of Computer and System Sciences, 60(3):575 – 591, 2000.

[12] Marcin Kik. http://sites.google.com/site/rboprotocol/.
[13] Marcin Kik. Ranking and sorting in unreliable single hop radio network. In David Coudert, David Simplot-Ryl, and Ivan Stojmenovic, editors, ADHOC-NOW, volume 5198 of Lecture Notes in Computer Science, pages 333–344. Springer, 2008.

[14] Marcin Kik. RBO protocol: Broadcasting huge databases for tiny receivers. CoRR, abs/1108.5095, 2011.

[15] Philip Levis and David Gay. TinyOS Programming. Cambridge University Press, New York, NY, USA, 2009.

[16] G.M. Morton. A computer oriented geodetic data base and a new technique in file sequencing. IBM technical report. Ottawa, Canada, 1966.

[17] Ori Shalev and Nir Shavit. Split-ordered lists: Lock-free extensible hash tables. J. ACM, 53(3):379–405, 2006.

[18] Jun Shao and Brian T. Davis. The bit-reversal SDRAM address mapping. In Krishna M. Kavi and Ron Cytron, editors, SCOPES, volume 136 of ACM International Conference Proceeding Series, pages 62–71, 2005.

[19] Nitin H. Vaidya and Sohail Hameed. Scheduling data broadcast in asymmetric communication environments. Wireless Networks, 5(3):171–182, 1999.