The purpose of this Letter is to present a simple and transparent explanation for why the IC-SDW order is observed only in FFLO phase and the IC wave vector is robust against the magnetic field. 

The mechanism of incommensurate (IC) spin-density-wave (SDW) order in the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase of CeCoIn$_5$ is discussed on the basis of new mode-coupling scheme among IC-SDW order, two superconducting orders of FFLO with B$_{1g}$ $(d_{x^2-y^2})$ symmetry and $\pi$-pairing of odd-parity. Unlike the mode-coupling schemes proposed by Kenzelmann et al, Scienceexpress, 21 August (2008), that proposed in the present Letter can offer a simple explanation for why the IC-SDW order is observed only in FFLO phase and the IC wave vector is rather robust against the magnetic field.

KEYWORDS: FFLO phase in CeCoIn$_5$, incommensurate spin-density-wave order,

Recent discovery of incommensurate (IC) spin-density-wave (SDW) order in the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase of CeCoIn$_5$ has attracted intense attention. This result is consistent with that of NMR Knight shift measurements suggesting the existence of field-induced magnetism. Kenzelmann et al reported that an IC-SDW order in the $c$-direction with the wave vector $Q = (\pi/a, \pi/a, 0.5\pi/c)$, $h = 0.56$ and $a$ being the lattice constant in the $ab$-plane, exists in the so-called FFLO phase under the magnetic field $H$ along the (1,1,0) direction. They also claimed that the wave vector $Q$ is essentially constant against the magnetic field strength $H$ within an experimental resolution. In order to explain their results, they proposed a scenario on the basis of mode-couplings among the IC-SDW order $M_q$, $d$-wave pairing $\Delta_d$, additional superconducting orders with finite center-of-mass momentum $\Delta_n = -q_n$ with different combination of $(i, n)$ on the symmetry requirements. While the scenario explains the appearance of the IC-SDW order, it fails to explain the reason why the IC-SDW is observed only in the FFLO phase and the ordering wave vector $Q$ is robust against the magnetic field.

The purpose of this Letter is to present a simple and transparent explanation for these questions. It is crucial to examine an explicit form of the mode-coupling term on the semi-microscopic level beyond conventional group theoretical arguments which include ambiguities concerning combination of the wave vectors of order parameters. The mode-coupling term relevant to the present situation, in the GL region, is given by the Feynman diagram shown in Fig. 1, and the analytic expression is given as follows:

$$V_\infty = C\Delta^{(c)}_q M_{Q_0 + q} \Delta^{(o)}_{-Q_0},$$

where $M_{Q_0 + q}$, $\Delta^{(c)}_q$, and $\Delta^{(o)}_{-Q_0}$ denote the IC-SDW magnetization in the $c$-direction, the $d$-wave superconducting (SC) gap in the FFLO state, and an additional odd-parity "equal-spin" SC gap with finite center-of-mass momentum $-Q_0$, $Q_0 \equiv (0.5\pi/a, 0.5\pi/a, 0.5\pi/c)$, (the so-called $\pi$-pairing), respectively. The coefficient $C$ in eq.(1) is given in terms of the Green function $G_\sigma (k, i\epsilon_n)$ of the quasiparticles in the normal state as follows:

$$C = g q g Q_0 + q/2 T \sum_{q} \sum_{p} \phi_{o}(p) \chi_{\sigma\bar{\sigma}}(p - Q_0/2) \chi_{\bar{\sigma}\sigma}^{(o)} \times$$

$$G_\sigma (p + q/2, i\epsilon_n) G_\sigma (p - Q_0 - q/2, i\epsilon_n) \times$$

$$G_\sigma (-p + q/2, -i\epsilon_n),$$

where $g$'s denote the pairing interactions for different center-of-mass momentum, and $\phi_{o}(p)$ is the wave function of $d$-wave pairing; e.g., $\phi_{o}(p) = (\cos p_x a - \cos p_y a)$, and $\phi_{o}(p)$ is the wave function of odd-parity state. $\chi^{(c)}_{\sigma\bar{\sigma}}$ and $\chi^{(o)}_{\bar{\sigma}\sigma}$ denote the spin part of gap functions for even- and odd-parity state, respectively. Here, the quantization axis of "spin" (label of crystalline-electric-field ground doublet) is taken as parallel to the magnetic field, i.e., in the (1,1,0) direction, and the direction of IC-SDW magnetization is along the (0,0,1) direction. Since $\chi^{(c)}_{\sigma\bar{\sigma}}$ is antisymmetric with respect to interchange of $\sigma$ and $\bar{\sigma}$, i.e., $\chi^{(c)}_{\sigma\bar{\sigma}} = -\chi^{(c)}_{\bar{\sigma}\sigma}$, and $\chi^{(o)}_{\bar{\sigma}\sigma}$ is not antisymmetric, the coefficient $C$ vanishes without the magnetic field: In other word, $C$ is proportional to the magnetic field $H$ or odd-function of $H$. As a result, the mode-coupling term eq.(1) becomes the time-reversal invariant quantity.

In order to explore the condition for the IC-SDW to be realized in practice, let us perform the analysis based on the GL free energy. For concise presentation, we denote $M_{Q_0 + q}$ as $\bar{M}$ and $\Delta^{(o)}_{-Q_0}$ as $\bar{\Psi}$. Then, the GL free energy $F$ is expressed as

$$F = F_0 + \frac{a_M}{2} \bar{M}^2 + \frac{b_M}{4} \bar{M}^4 + \frac{a_{\Psi}}{2} \bar{\Psi}^2 + \frac{b_{\Psi}}{4} \bar{\Psi}^4 +$$

$$+ C\Delta_\bar{M} \bar{\Psi},$$

where the coefficients $a_M$, $a_{\Psi}$, $b_M$, and $b_{\Psi}$ are assumed to be positive because both of $\bar{M}$ and $\bar{\Psi}$ are assumed to be vanishing without the FFLO gap $\Delta_\bar{M}$. The extremum conditions for $\bar{M}$ and $\bar{\Psi}$ are

$$0 = \frac{\partial F}{\partial \bar{M}} = a_M \bar{M} + b_M \bar{M}^3 + C\Delta_\bar{M} \bar{\Psi},$$

$$0 = \frac{\partial F}{\partial \bar{\Psi}} = a_{\Psi} \bar{\Psi} + b_{\Psi} \bar{\Psi}^3,$$

The mechanism of incommensurate (IC) spin-density-wave (SDW) order in the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase of CeCoIn$_5$ is discussed on the basis of new mode-coupling scheme among IC-SDW order, two superconducting orders of FFLO with B$_{1g}$ $(d_{x^2-y^2})$ symmetry and $\pi$-pairing of odd-parity. Unlike the mode-coupling schemes proposed by Kenzelmann et al, Scienceexpress, 21 August (2008), that proposed in the present Letter can offer a simple explanation for why the IC-SDW order is observed only in FFLO phase and the IC wave vector is rather robust against the magnetic field.

KEYWORDS: FFLO phase in CeCoIn$_5$, incommensurate spin-density-wave order,
is not the same as that of FFLO

SDW wave vector is along \((0.5+q,0.5+q,0.5)\) direction in

In the experiment of Ref. 1, the FFLO wave vector is con-

H

the first order transition at

q

where the SC gap \(\Delta\)

C

H

latter condition is met in the FFLO state near the

This has \(\tilde{\Psi}\):

Substituting this into eq.(4), we obtain the equation for \(\tilde{M}\):

This has \(\tilde{M} \neq 0\) solution if the condition

is satisfied, giving the IC-SDW magnetization as

The condition (8) will be satisfied if \(a_M\) is small enough and \(C\Delta_q\) is large enough. The former condition may be guaranteed by some circumstantial evidence that CeCoIn\(_5\) is located not far from the AF-QCP,\(^4,5\) and the latter condition is met in the FFLO state near the \(H_{c2}\) where the SC gap \(\Delta_q\) remains to be large enough due to the first order transition at \(H_{c2}\). The rather large value of \(H_{c2}\) (comparable to the effective Fermi energy) supports the large value of the coefficient \(C\).

In the present model, the wave-vector of IC-SDW \(Q\) is not the same as that of FFLO \(q\), but is given by

In the experiment of Ref. 1, the FFLO wave vector is con-

FFLO vector was parallel to \((1,0,0)\) or \((0,1,0)\), the expected IC-SDW wave vector would be in \((0.5+q,0.5,0.5)\) or \((0.5,0.5+q,0.5)\) direction, respectively. This is a prediction of the present theory.

If the magnitude of FFLO wave-vector \(q\) is given by

\[ q = \frac{2\mu_B H}{\hbar v_F} \]

where we have used \(H \approx 10\) Tesla and \(v_F \approx 7.5 \times 10^3\) m/s. Therefore, the ratio of \(q\) and \(Q_0\) (= \(a/\pi\)) is

This value is about 3 times smaller than the observed value \(0.56-0.5)/0.5 = 0.12\). However, the Fermi velocity \(v_F\) used for the estimation in eq.(11) is that near the SC transition temperature \(T_{c0}\) at \(H = 0\), and the mass enhancement at temperatures in FFLO phase is about 5-6 times larger than that at \(T \sim T_{c0}\) as estimated by the entropy balance argument. Therefore, it is reasonable that \(q\), the deviation from the AF wave vector \(Q_0\), is given by the wave-vector of FFLO state.

The robustness of IC-SDW wave vector \(Q\) against the magnetic field \(H\) is understood as follows: The range of variation of \(H\) is \(10.6 < H/(\text{Tesla}) < 11.4\) so that the relative variation from the central value \(H = 11\) is about \(3.6 \times 10^{-2}\). This means the absolute value of variation of \(Q\) is \(0.06 \times 3.6 \times 10^{-2} = 2.1 \times 10^{-3}\) which seems to be comparable to the error bar of IC-SDW wave vector although there is no error bar shown in the figure of Ref. 1. Another factor, which reduces the effect of the magnetic field \(H\), is the \(H\)-dependence of the Fermi velocity which is an increasing function of \(H\) in general.

The existence of the \(\pi\)-pairing of odd-parity can be detected, in principle, by the NQR measurement of the dynamical susceptibility along the magnetic field (in \((1,1,0)\) direction), which can give rise to an anomalous relaxation of NQR spectrum as observed in Sr\(_2\)RuO\(_4\)\(^1\) with the same tetragonal symmetry as in CeCoIn\(_5\). Indeed, since the wave function of \(\phi_0\) of the Cooper pair can be doubly degenerate in the odd-parity manifold, the spin-orbit coupling associated with relative motion of Cooper pairs can give rise to anomalous relaxation of NQR corresponding to the oscillations of magnetization along the \((1,1,0)\) direction.\(^8\) However, it may be left for future studies to analyze in more detail how to detect the \(\pi\)-pairing.

In conclusion, the mechanism of IC-SDW order in the FFLO phase of CeCoIn\(_5\) observed by neutron scattering has been clarified on the basis of a new mode-coupling scheme among IC-SDW order, two SC orders of FFLO with \(B_{1g}\) \((d_{x^2-y^2})\) symmetry and \(\pi\)-pairing with odd-parity. The mode-coupling term proposed in the present Letter gives a simple explanation for why the IC-SDW order is observed only in FFLO phase and the IC wave vector is rather robust against the magnetic field. The
difference between the case of CeRhIn$_5$, which is a sister compound of CeCoIn$_5$ and IC-SDW state at $H = 0$, should be clarified by further investigations.

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