Suppression of Unwanted \(ZZ\) Interactions in a Hybrid Two-Qubit System

Jaseung Ku, Xuexin Xu, Markus Brink, David C. McKay

1Department of Physics, Syracuse University, Syracuse, New York 13244, USA
2Peter Grünberg Institute, Forschungszentrum Jülich, Jülich 52428, Germany
3Jülich-Aachen Research Alliance (JARA), Fundamentals of Future Information Technologies, Jülich 52428, Germany
4IBM, T. J. Watson Research Center, Yorktown Heights, New York 10598, USA

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Mitigating crosstalk errors, whether classical or quantum mechanical, is critically important for achieving high-fidelity entangling gates in multi-qubit circuits. For weakly anharmonic superconducting qubits, unwanted \(ZZ\) interactions can be suppressed by combining qubits with opposite anharmonicity. We present experimental measurements and theoretical modeling of two-qubit gate error for gates based on the cross resonance interaction between a capacitively shunted flux qubit and a transmon and demonstrate the elimination of the \(ZZ\) interaction.

Superconducting qubits are a promising candidate for building fault-tolerant quantum computers [1–4]. However, the gate errors in current devices are not definitively below the threshold required for fault-tolerance. Despite tremendous improvements in qubit coherence, circuit design, and control, two-qubit gate errors remain in the range of \(4 − 9 \times 10^{-3}\) [5, 6]. This is worse than what would be naively expected based on current device coherences [7]. One limiting factor to these errors is crosstalk in the device corresponding to unwanted terms in the Hamiltonian. This is a particular concern for one of the more common superconducting qubit architectures fixed-frequency transmons [8] coupled to nearest neighbors via a static exchange term \(J\). In this architecture, the two-qubit gate is enabled by activating the cross-resonance (CR) effect [9–11], where a \(ZX\) interaction term is generated by driving one qubit (the control) at the frequency of the neighboring qubit (the target).

CR has several advantages: it allows for all-microwave control of a fixed-frequency device, and is thus simple from a control perspective; also, the use of non-tunable qubits removes a source of decoherence. The strength of the CR effect is proportional to \(J\) [12]. However, for transmons, which have a negative value of the anharmonicity – the difference between the primary qubit transition out of the qubit subspace and the qubit transition – this \(J\) also produces an always-on \(ZZ\) coupling term. Such a \(ZZ\) interaction, whether static or driven during the CR gate [12], is an ever-present source of error. Unlike classical crosstalk, which can be cancelled by the application of compensation tones [6, 12], the \(ZZ\) term leads to unwanted entanglement between pairs and so is not easily mitigated unless, for example, additional circuitry, such as a tunable coupler, is added [13].

As an alternative approach, if the transmon qubit can be combined with a qubit design where the anharmonicity is positive, the \(ZZ\) term can be cancelled at specific qubit-qubit detunings, and the CR effect between the two qubits utilized to form a high-fidelity gate. Fortunately, such a qubit exists: the capacitively shunted flux qubit (CSFQ) [14]. Recently, the CSFQ has regained attention, in part, due to its greatly improved coherence time [15]. Although the CSFQ is a flux-tunable device, it can be operated at a flux sweet spot (flux bias \(\Phi/\Phi_0 = 0.5\)), where it is first-order insensitive to flux noise. The anharmonicity at the sweet spot can be positive and large (> +500 MHz), which provides a parameter regime that is otherwise inaccessible in all-transmon devices [16, 17].

In this manuscript we present measurements of the first such hybrid CSFQ-transmon device and theoretical modeling to investigate its performance. First, we experimentally demonstrate and theoretically model the suppression of the static \(ZZ\) interaction for a particular detuning of the CSFQ and transmon. Second, we investigate the characteristic behavior of the CR effect as a function of CSFQ-transmon detuning. Third, we explore the dependence of two-qubit gate error on both flux and gate length. Finally, we use our model to describe the requirements for a future device capable of achieving a two-qubit gate error of \(1 \times 10^{-3}\).

The device consists of a fixed-frequency transmon and CSFQ coupled via a bus cavity resonator [Fig. 1(a)]. Each qubit has its own readout resonator with a microwave input/output port. Details on sample fabrication, measurement setup, and device parameters can be found in the Supplemental Material [18]. This coupled two-qubit system can be described by the Hamiltonian in the bare basis:

\[
H = \sum_{q=1,2} \sum_{n_q} \omega_q(n_q) |n_q\rangle \langle n_q| + \sqrt{(n_1 + 1)(n_2 + 1)} J_{n_1,n_2} \langle n_1 + 1, n_2 | n_1, n_2 + 1 | + h.c., \]

(1)

where \(\omega_q(n_q)\) is the transition frequency between energy levels \(n_q\) and \(n_q + 1\) for qubit \(q\). The primary qubit transition is thus \(\omega_q(0)\). The coupling strength \(J_{n_1,n_2}\) provides an indirect two-photon interaction via a bus resonator between energy levels \(n_1\) and \(n_1 + 1\) in qubit 1 and levels \(n_2\) and \(n_2 + 1\) in qubit 2 (see Supplementary Material [18] for details). We take \(\hbar = 1\) throughout.
The qubits were measured using conventional circuit-QED techniques in the dispersive regime [19]. The measured qubit frequency, anharmonicity, and qubit-qubit detuning for the CSFQ and transmon at the sweet spot are shown in Fig. 1(b). We fit the anticrossing between the CSFQ and transmon [Fig. 1(c) inset] to obtain the zeroth-order exchange coupling strength $J_{00}/2\pi = 6.3$ MHz. The average single-qubit gate fidelity was measured with the standard randomized benchmarking (RB) protocol (details in Supplement [18]), giving the average gate error lower than $1 \times 10^{-3}$. For a CR drive, we take the CSFQ (transmon) as the control (target) qubit. The tunability of the CSFQ spectrum as a function of flux [Fig. 1(c)] allows us to explore a range of qubit-qubit detuning in the following experiments.

Before applying the CR drive, we investigate how the static $ZZ$ interaction of the system varies with the flux bias of the CSFQ. The effective Hamiltonian that is diagonal in the dressed frame is,

$$H_{\text{eff}} = -\tilde{\omega}_1 \frac{Z I}{Z} - \tilde{\omega}_2 \frac{I Z}{Z} + \zeta \frac{ZZ}{4},$$

(2)

where $\tilde{\omega}_1(\equiv \tilde{\omega}_1(0))$ and $\tilde{\omega}_2(\equiv \tilde{\omega}_2(0))$ are the dressed qubit frequencies. $\zeta$ is the frequency shift of one qubit when the other qubit is excited from the ground state: $\zeta = (E_{11} - E_{10}) - (E_{01} - E_{00})$, where $E_{ij}$ is an energy eigenvalue of the Hamiltonian [Eq. (1)] for qubit 1 (2) at state $|i\rangle$ ($|j\rangle$). Such a static $ZZ$ term arises when higher energy levels are involved in the two-qubit Hamiltonian [Eq. (1)]. A large $ZZ$ results in an additional phase rotation depending on the state of either qubit, thus contributing to two-qubit gate error. For our device, the static $ZZ$ strength has a maximum value of 140 kHz at the flux sweet spot, but away from this point it decreases and eventually crosses zero near $\Phi/\Phi_0 = 0.496$ and 0.504 (Fig. 2), where the CSFQ-transmon detuning is 191 MHz. ZZ-free qubit pairs can be obtained if $\zeta$ vanishes in Eq. (2). A detailed analysis involving block-diagonalization of the multilevel Hamiltonian [Eq. (1)] into the qubit subspace shows that $\zeta$ can be expressed as (see Ref. 18 for details):

$$\zeta = -\frac{2J_{01}^2}{\Delta + \delta_2} + \frac{2J_{10}^2}{\Delta - \delta_1},$$

(3)

where $\Delta = \omega_2(0) - \omega_1(0)$ is the qubit-qubit detuning, and $\delta_i = \omega_i(1) - \omega_i(0)$ is the anharmonicity of qubit $i$. Within the limit $|\Delta| < |\delta|$, where the CR effect is strongest [21], for a transmon-transmon device, both terms of Eq. (3) are positive, and thus $ZZ$ interactions will always be present in all-transmon circuits with fixed couplings. However, in a CSFQ-transmon circuit the second term in Eq. (3) can be negative, due to the large and positive anharmonicity of the CSFQ. This allows the hybrid CSFQ-transmon combination to be ZZ-free.

Eq. (3) was used to compute the flux dependence of the static $ZZ$ strength using separately extracted device parameters, including the flux-dependent anharmonicity and transition frequencies of the CSFQ (red solid line in Fig. 2). The agreement between theory and experiment is quite good except near the zero-crossing points, where the experimental $ZZ$ data exhibits a kink. We speculate that this could be due to the breakdown of our pertur-
bative treatment of the effective Hamiltonian, and thus Eq. (3). Away from the flux sweet spot, the qubit-qubit detunings decreases, while \( J_{10} \) increases, thus pushing the ratio \( J/\Delta \) beyond the dispersive limit. A framework for treating such situations is discussed in Ref. 22.

For the CR effect, a drive tone applied to the control qubit at the frequency of the target qubit induces a rotation of the target qubit with the direction of rotation dependent on the state of the control qubit, thus corresponding to a \( ZX \) term in the effective Hamiltonian [12]. Due to other terms in the full CR Hamiltonian, an echoed CR protocol is commonly used, which removes \( ZX \) and \( IJ \) contributions [10]. We performed echoed CR to measure the rotation rate, \( f_{CR} \), as a function of CR amplitude at different flux points (Fig. 3). The echoed CR pulse consists of two Gaussian flat-top CR pulses with \( \pi \) phase difference, and a \( \pi \)-pulse on the control qubit after each CR pulse. With variable CR length \( \tau \), the oscillation frequency of the transmon was measured for a range of CR amplitude (Fig. 3 inset). The CR amplitude was calibrated in terms of the Rabi frequency of the CSFQ at the flux sweet spot. The echoed CR rate increases linearly at low CR amplitude, while for the stronger CR drive it slows down as the CSFQ is driven off-resonance [9]. Eventually, the rate levels off to a maximum as energy levels \( E_{11} \) and \( E_{02} \) get closer and finally anticross each other at the CR amplitude corresponding to the maximum. We solved the full Hamiltonian, including the effects of CR driving, using separately measured device parameters with a non-perturbative diagonalization scheme (details in Supplement [18]). The resulting theoretical curves for \( f_{CR} \) vs. CR amplitude agree well with the experimental points (Fig. 3).

The average two-qubit error per gate was measured via standard randomized benchmarking (RB) [23] at various flux points and gate lengths \( t_g \) of the \( ZX_{90} \), which serves as the pulse primitive for the two-qubit entangling gate [10] (Fig. 4). For each flux point, the primitive single-qubit gate \( (X_{90}) \) and two-qubit gate \( (ZX_{90}) \) were re-calibrated. The RB data was fit to the standard fidelity decay curve \( A\alpha^m + B \), where \( m \) is the number of Clifford gates and \( \alpha \) the depolarization parameter [23]. The average two-qubit error per gate \( \epsilon \) was then calculated using the expression, \( \epsilon = (3/4) \cdot (1 - \alpha^{1/N}) \), where \( N \) is the average number of \( ZX_{90} \) gates per two-qubit Clifford gate [24, 25].

The smallest gate error, \( 1.6 \times 10^{-2} \), occurs for \( t_g = 200 \) ns and \( (\Phi/\Phi_0) - 0.5 = \pm 0.004 \) (Fig. 4). By increasing the gate time, a characteristic “W”-shaped pattern develops with respect to flux, corresponding to larger errors at the sweet spot with minima to either side, followed by increasing error for further flux biasing away from 0.5. This behavior can be described by the interplay between fidelity loss from the \( ZZ \) interaction and classical crosstalk on the one hand with fidelity gain from longer coherence times near the sweet spot on the other hand. Away from the sweet spot, the \( ZZ \) interaction and classical crosstalk decrease and the gate fidelity approaches the coherence limit. Including the \( ZZ \) interaction and classical crosstalk in our simulation was sufficient to reproduce the flux-dependence of the experimental gate errors.

The dashed lines in Fig. 4 correspond to the coherence-limited gate error, which is mainly dominated by the

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**Fig. 3.** Echoed CR rate vs. CR amplitude at four representative flux points. The corresponding qubit-qubit detunings are \((234, 217, 199, 166)\) in MHz. Solid lines correspond to theoretical model. (Inset) Color density plot of the oscillation of target qubit driven with various CR amplitudes at the flux sweet spot. Colorbar represents the first excited state probability of the target qubit. Echoed CR pulse sequence is illustrated above the density plot.

**Fig. 4.** Average error per two-qubit gate plotted versus flux for four representative two-qubit gate lengths. Dashed lines indicate theoretical coherence-limited two-qubit gate errors with no \( ZZ \) interactions; full theory simulations are shown by solid lines.
CSFQ’s $T_2$. Due to flux noise, the CSFQ has a maximum $T_2$ at this point, which quickly decreases away from the sweet spot (see $T_2$ vs. flux in Supplement [18]). As is clear from Fig. 4, the coherence-limit curves alone are not sufficient to reproduce the measured flux-dependence of the gate error. The static $ZZ$ strength plotted in Fig. 2 has a significant impact on the gate error. In addition, we model classical crosstalk in a similar manner to Ref. 12, by including in the driving Hamiltonian a modified amplitude $R(f, t_g)\Omega$ and shifted phase, where $R(f, t_g)$ is a scaling factor of the crosstalk-term, and $\Omega$ is the CR amplitude. For our theoretical modeling, $\Omega$ was obtained from the experimental $ZX_{90}$ pulse calibrations for each flux and gate length. $R$ was modeled using a CR tomography measurement [6] (more details in Supplemental Material [18]). Theoretical simulations agree well with experimental data (solid lines in Fig. 4).

Based on the success of our theoretical model in describing the measured flux dependence of the two-qubit gate error, we consider target parameters for a future device to achieve further reductions in gate error. In Fig. 5, we illustrate the two-qubit gate error vs. $t_g$ for various conditions. In simulating the gate error, we considered three sets of coherence times: $(T_1^{(1)}, T_2^{(1)}, T_1^{(2)}, T_2^{(2)})$, where the superscripts indicate the qubit, are (18, 15, 40, 45), corresponding to the present device, (40, 54, 43, 67), corresponding to the two-transmon device in Ref. 6, and (200, 200, 200, 200) for a hypothetical, but not out of reach, device (all times in $\mu$s). From the discussion above, we know that one of the most prominent advantages of a CSFQ-transmon device over a transmon-transmon device is that the static $ZZ$ interaction can be cancelled inherently by carefully choosing qubit frequency and anharmonicity. To model an idealized device, we consider a CSFQ-transmon with static $ZZ = 0$, which could be made by potentially keeping the CSFQ at the sweet spot, while making the transmon slightly tunable [26]. Such a device results in a comparable fidelity (1b) for the relatively short coherence times of the present experimental device as compared to the transmon-transmon (2). For the projected longer coherence times (200 $\mu$s) [27, 28], the gate error (3b) of such a device subject to elimination of classical crosstalk can reach $1 \times 10^{-3}$. This level is inaccessible for a transmon-transmon device, even with the projected longer coherence times (3).

Coherence-limited gate errors (dashed lines in Fig. 5) decrease monotonically as $t_g$ decreases. The $ZZ$ interaction as well as classical crosstalk add error on top of the coherence limit. This error depends on the gate length, so that the total error reaches a minimum at an optimum gate time. This is a universal behavior, even for a device with no static $ZZ$ or classical crosstalk, see line (3b) in Fig. 5. The $ZZ$ interaction has two parts: a static (undriven) term $\zeta$, and a dynamical term depending quadratically on driving amplitude, $\eta \Omega^2$, with $\eta$ being a device-dependent quantity (see Supplementary Material [18] for details). Fig. 3 shows the rate of target qubit rotation in the weak-driving regime is $f_{CR} \sim 0.1\Omega$ at the sweet spot, therefore $ZX_{90}$ limits the CR gate time to satisfy $(2\pi f_{CR})\tau = \pi/2$. Consequently, the dynamical $ZZ$ interaction scales with gate length as $6.25\eta/\tau^2$. This shows that even in the absence of static $ZZ$, the dynamical part can still produce large errors at short gate time.

In conclusion, we have characterized the CR gate on a CSFQ-transmon device. This hybrid system with opposite anharmonicity between the qubits allows for the complete suppression of the static $ZZ$ interaction, which becomes essential for achieving a high-fidelity CR gate. Our theoretical analysis shows that suppressing the $ZZ$ interaction is just as important as enhancing coherence times. By eliminating the spurious $ZZ$ interaction, a CSFQ-transmon gate can achieve comparable fidelities to a transmon-transmon gate despite having shorter coherence times. With longer coherence times that are not too far beyond current experimental capabilities (200 $\mu$s), two-qubit gate errors of $1 \times 10^{-3}$ should be feasible.

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Supplementary Material: Suppression of Unwanted $ZZ$ Interactions in a Hybrid Two-Qubit System

Jaseung Ku, Xuexin Xu, Markus Brink, David C. McKay, Jared B. Hertzberg, Mohammad H. Ansari, and B.L.T. Plourde

1 Department of Physics, Syracuse University, Syracuse, New York 13244, USA
2 Peter Grünberg Institute, Forschungszentrum Jülich, Jülich 52428, Germany
3 Jülich-Aachen Research Alliance (JARA), Fundamentals of Future Information Technologies, Jülich 52428, Germany
4 IBM, T. J. Watson Research Center, Yorktown Heights, New York 10598, USA

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FIG. S1. SEM micrographs of CSFQ similar to the one used in this work. (a) Shunt capacitors, SQUID loop and coupling capacitors in gap within opening in chip ground plane. (b) Close-up of SQUID loop. Image of full chip of this type may be found in Ref. 1. $J_1$ and $J_2$ indicate two large Josephson junctions, and $J_3$ is a smaller Josephson junction.

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DEVICE AND MEASUREMENT SETUP

The device was fabricated at IBM. The $4 \times 8$ mm chip contains one fixed-frequency transmon, one bus resonator, one CSFQ, and readout resonators for each qubit. A photo of a similar chip appears in reference [1]. We fabricated the device in a manner described in Ref. 1 and 2. We sputter-deposited a $\sim 200$ nm niobium film on a $730 \mu$m-thick silicon substrate, followed by photolithography and plasma-etch to define the microwave structures. Bus and readout resonators comprise half-wave sections of coplanar waveguide terminated by metal pads that define coupling capacitors. We formed Al/AlOx/Al tunnel junctions and the CSFQ loop using e-beam lithography, Manhattan-style double-angle shadow-evaporation [3], and lift-off. The CSFQ contains three junctions in a $30 \times 20 \mu$m$^2$ loop. We formed the aluminum elements of the transmon and CSFQ simultaneously into identical shunting capacitors. We diced the chip, installed it into a package comprising a circuit board, a copper backing-plate, coaxial connectors and a superconducting bobbin coil. Similar packaging is described in Ref. 4, with the exception that the package is not potted into epoxy but is mounted inside a light-tight magnetically shielded sample can. The device was measured on a dry dilution refrigerator with a base temperature below 10 mK and heavily filtered cryogenic microwave lines. We show our room-temperature microwave electronics setup in Fig. S2(a) and cryogenic wiring in Fig. S2(b). For the
FIG. S2. (a) Room-temperature microwave control electronics. (b) Cryogenic wiring for one of the two qubits. Wiring for other qubit is identical.

flux-bias, we used a battery-operated voltage source (SRS SIM928) and a 10 kΩ room-temperature standard resistor in series for a current-bias. The flux line is filtered through a π-filter at the 3 K stage and an Eccosorb filter at the mixing chamber stage before it reaches the superconducting bobbin coil inside the Cryoperm magnetic shield.

DEVICE PARAMETERS

In Table S.I, S.II, S.III, and S.IV, we list device parameters. These parameters were either directly measured in experiments or calculated based on the measured parameters.
TABLE S.I. Frequency scales on device with CSFQ at the sweet spot.

| Description                        | Symbol   | Frequency   |
|------------------------------------|----------|-------------|
| Transmon bare frequency            | $\omega_b^2(0)/2\pi$ | 5.2920 GHz  |
| Transmon dressed frequency         | $\tilde{\omega}_2(0)/2\pi$ | 5.2855 GHz  |
| Transmon anharmonicity             | $\delta_2/2\pi$ | -326.6 MHz  |
| Transmon bare readout frequency    | $\omega_a/2\pi$ | 6.8050 GHz  |
| Transmon dressed readout frequency | $\tilde{\omega}_a/2\pi$ | 6.8059 GHz  |
| Transmon-readout coupling          | $g_{aT}/2\pi$ | 37 MHz      |
| Transmon-readout dispersive shift  | $\chi_{aT}/2\pi$ | 200 kHz     |
| CSFQ bare frequency                | $\omega_1^2(0)/2\pi$ | 5.0616 GHz  |
| CSFQ dressed frequency             | $\tilde{\omega}_1(0)/2\pi$ | 5.0511 GHz  |
| CSFQ anharmonicity                 | $\delta_1/2\pi$ | +592.7 MHz  |
| CSFQ bare readout frequency        | $\omega_h/2\pi$ | 6.9065 GHz  |
| CSFQ dressed readout frequency     | $\tilde{\omega}_h/2\pi$ | 6.9074 GHz  |
| CSFQ-readout coupling              | $g_{hm}/2\pi$ | 41 MHz      |
| CSFQ-readout dispersive shift      | $\chi_{hm}/2\pi$ | 550 kHz     |
| Bus bare frequency                 | $\omega_r/2\pi$ | 6.3062 GHz  |
| Bus dressed frequency              | $\omega_r/2\pi$ | 6.3226 GHz  |
| Bus-Transmon dispersive shift      | $\chi_{rT}/2\pi$ | -2.2 MHz    |
| Bus-CSFQ dispersive shift          | $\chi_{rm}/2\pi$ | 5.9 MHz     |
| Bus-Transmon coupling              | $g_{rT}/2\pi$ | 76.9 MHz    |
| Bus-CSFQ coupling                  | $g_{rm}/2\pi$ | 111.7 MHz   |
| Transmon-CSFQ exchange coupling    | $J_{01}/2\pi$ | 6.3 MHz     |

TABLE S.II. Junction parameters of the transmon and CSFQ. CSFQ Josephson energy is for the larger junctions. The two transmon parameters were calculated using the measured dressed qubit frequency. Meanwhile, the three CSFQ parameters were obtained by fitting the spectroscopy data of the dressed qubit frequencies, $\tilde{\omega}_1(0)/\pi$ and $\tilde{\omega}_1(1)/2\pi$ vs. flux with 1D potential approximation [5].

| Description          | Symbol | Value      |
|----------------------|--------|------------|
| Transmon Josephson energy | $E_{JT}$ | 13.7 GHz   |
| Transmon charging energy   | $E_{CT}$ | 0.286 GHz  |
| CSFQ Josephson energy    | $E_{Jm}$ | 123.1 GHz  |
| CSFQ charging energy     | $E_{Cm}$ | 0.268 GHz  |
| CSFQ critical current ratio of the small to large junction | $\alpha$ | 0.43       |

TABLE S.III. Coherence times for the transmon and CSFQ at the sweet spot

| T1 (µs) | T2 (µs) | T1 (µs) | T2 (µs) | T2 (µs) |
|---------|---------|---------|---------|---------|
| 40      | 25      | 45      | 18      | 12      | 18      |

TABLE S.IV. Two-photon virtual exchange coupling strength ($J_{01}$ and $J_{10}$), qubit-qubit detuning ($\Delta$), and anharmonicities ($\delta_i$) at $\Phi/\Phi_0 = 0.504$, where $ZZ = 0$

| $J_{01}$ (MHz) | $J_{10}$ (MHz) | $\Delta$ (MHz) | $\delta_1$ (MHz) | $\delta_2$ (MHz) |
|---------------|---------------|----------------|------------------|------------------|
| 4.9           | 8.1           | 192            | 560              | -327             |

THEORY

Circuit Hamiltonian

The circuit schematic and parameters are shown in Fig. S3 and Table S.V, respectively. The Lagrangian $\mathcal{L} = T - U$ with $T$ being electrostatic energy and $U$ potential energy of the Josephson junctions. We define $\varphi_m \equiv (\varphi_e - \varphi_g)/2$
and \( \varphi_p \equiv (\varphi_e + \varphi_g)/2 - \varphi_f \) in order to simplify the Lagrangian into the following form:

\[
\mathcal{L} = \frac{1}{2} \left( \frac{\Phi_0}{2\pi} \right)^2 \left[ C_{rT} \dot{\varphi}_a + C_{ab} (\varphi_a - \varphi_b)^2 + C_{ba} \varphi_b^2 + C_{cd} (\varphi_c - \varphi_T - \varphi_d)^2 \\
+ (C_{shT} + C_T) \varphi_T^2 + C_{da} (\varphi_b - \varphi_T)^2 + C_R \varphi_d^2 + C_{de} (\varphi_e - \varphi_d)^2 + C_{g0} \varphi_c^2 \\
+ C_{gh} (\varphi_e - 2\varphi_m - \varphi_h)^2 + 2C (\varphi_m^2 + \varphi_p^2) + 4(C_3 + C_{shCSFQ}) \varphi_m^2 \\
+ C_{g0} (\varphi_e - 2\varphi_m)^2 + C_{rCSFQ} \varphi_h^2 \right] + E_{JT} \cos \varphi_T + 2E_J \cos \varphi_p \cos \varphi_m \\
+ \alpha E_J \cos (2\pi f - 2\varphi_m) - \left( \frac{\Phi_0}{2\pi} \right)^2 \left( \frac{\varphi_a^2}{2L_{rT}} + \frac{\varphi_b^2}{2L_T} + \frac{\varphi_h^2}{2L_{rCSFQ}} \right),
\]

where \( f = \Phi/\Phi_0 \) is the magnetic flux, \( \Phi_0 = h/2e \) the flux quantum, \( h \) is the Planck's constant, \( e \) the electron charge, \( C_1 = C_2 \equiv C \), and \( E_{J1} = E_{J2} \equiv E_J \). The Hamiltonian is calculated using the usual definition of \( H \) as the Legendre transformation of the Lagrangian \( \mathcal{L} \),

\[
H = \sum_i \dot{\varphi}_i \frac{\partial \mathcal{L}}{\partial \varphi_i} - \mathcal{L} = T + U,
\]

\[
T = \frac{1}{2} \left( \frac{\Phi_0}{2\pi} \right)^2 \ddot{\varphi}^T \mathbf{C} \ddot{\varphi},
\]

\[
U = E_{Ld} \varphi_a^2 + E_{Ld} \varphi_b^2 + E_{Lh} \varphi_h^2 + E_{JT} \cos \varphi_T \\
- 2E_J \cos \varphi_p \cos \varphi_m - \alpha E_J \cos (2\pi f - 2\varphi_m),
\]

where the phase vector in the circuit is defined as \( \ddot{\varphi}^T = (\varphi_b, \varphi_e, \varphi_a, \varphi_T, \varphi_d, \varphi_m, \varphi_p, \varphi_h) \), and the energies stored in the readout resonators for the transmon and CSFQ and the bus resonator are \( E_{La} = \Phi_0^2/8\pi^2L_{rT} \), \( E_{LCSFQ} = \Phi_0^2/8\pi^2L_{rCSFQ} \), and \( E_{LR} = \Phi_0^2/8\pi^2L_R \), respectively. Fig. S4 indicates the potential energies associated with the
FIG. S4. Potential profiles. (a) Readout of transmon. (b) Bus cavity. (c) Readout of CSFQ. (d) Transmon. (e) CSFQ along the $\varphi_m$ direction at $f = 0.5$. (f) CSFQ along the $\varphi_m$ direction at $f = 0$.

In this experiment, the ratio is designed to be less than 0.5 to be in the CSFQ regime. Since the potential does not depend on $\varphi_b$ and $\varphi_e$, and also because the kinetic energy of $\varphi_p$ in the CSFQ is superior to its contribution in the qubit potential, we use standard methods to safely remove these three phases, and reduce the circuit Hamiltonian matrix size from $8 \times 8$ to the following $5 \times 5$ matrix:

$$H = 4 \vec{n}^T \frac{e^2}{2C} \vec{n} + U,$$

where $\vec{n} = (n_a, n_T, n_d, n_m, n_h)$ is the canonical term of $\vec{\varphi}$ and

$$C' = \begin{pmatrix}
C_a' & -C_{ab}C_{dT}/C_{T0} & -C_{ab}C_{cd}/C_{T0} & 0 & 0 \\
-C_{ab}C_{dT}/C_{T0} & C_T' & C_{a0}C_{cd}/C_{T0} & 0 & 0 \\
-C_{ab}C_{cd}/C_{T0} & C_{a0}C_{cd}/C_{T0} & C_r' & -2C_{de}C_{h0}/C_{m0} & -C_{de}C_{gh}/C_{m0} \\
0 & 0 & -2C_{de}C_{h0}/C_{T0} & C_m' & 2C_{dm}C_{gh}/C_{m0} \\
0 & 0 & -C_{de}C_{gh}/C_{m0} & 2C_{dm}C_{gh}/C_{m0} & C_h'
\end{pmatrix}.$$
FIG. S5. Bare CSFQ frequency and anharmonicity versus flux. (a) Bare transition frequency $\omega_{b}^{(n)}(n)$ between levels $n$ and $n+1$. (b) Bare CSFQ anharmonicity $\delta_{b}^{(n)}(n) = \omega_{b}^{(n+1)}(n) - \omega_{b}^{(n)}(n)$.

Capacitances are listed as the following:

$$C_{T0} = C_{ab} + C_{b0} + C_{c0} + C_{cd}, \quad C_{h0} = C_{g0} + C_{gh}$$
$$C_{m0} = C_{dc} + C_{e0} + C_{gh}, \quad C_{dm} = C_{de} + C_{e0}$$
$$C_{dT} = C_{cd} + C_{c0} + C_{a0} = C_{ab} + C_{b0}$$
$$C_{T} = C_{dT} + C_{shT} + C_{T} - C_{dT}^{2}/C_{T0}$$
$$C_{m} = 2C + 4(C_{3} + C_{shCSFQ}) - 4C_{h0}^{2}/C_{m0} + 4C_{h0}$$
$$C_{r} = -C_{cd}^{2}/C_{T0} + C_{cd} + C_{de} + C_{m} - C_{dT}^{2}/C_{m0}$$
$$C_{a} = -C_{ab}/C_{T0} + C_{ab} + C_{rT}$$
$$C_{h} = -C_{gh}/C_{m0} + C_{gh} + C_{rCSFQ}.$$

Analytical expressions for transmon frequency and anharmonicity can be obtained using the systematic perturbation theory to large orders [6]. Similarly, the quantization of the CSFQ requires that we define the following operators in the Fock space [7]:

$$\varphi_{m} = \xi (m + m^{\dagger}), \quad n_{m} = \frac{i}{2\xi} (m^{\dagger} - m),$$

where $\xi$ is a device-dependent parameter. Fig. S5 shows the theoretical flux dependence of the bare frequency and anharmonicity in our experimental device. After quantizing the circuit, we simplify its Hamiltonian by taking it to a rotating frame and applying the Rotating Wave Approximation (RWA), which results in:

$$H_{\text{circuit}} = \omega_{a} a^{\dagger} a + g_{aT} (a^{\dagger} T + a T^{\dagger}) + \omega_{h} h^{\dagger} h + g_{hm} (h^{\dagger} m + h m^{\dagger})$$

$$+ \omega_{r} r^{\dagger} r + \sum_{j} \omega_{T}(j) | j \rangle \langle j | + \sum_{k} \omega_{m}(k) | k \rangle \langle k | + g_{rm} (r^{\dagger} m + r m^{\dagger})$$

$$+ g_{rT} (r^{\dagger} T + r T^{\dagger}) + g_{mT} (m^{\dagger} T + m T^{\dagger}),$$

where $a$ and $h$ represent readout resonators, $r$ is the bus resonator and we use $m = \sum_{k} \sqrt{k+1} | k \rangle \langle k + 1 |$ and $T = \sum_{j} \sqrt{j+1} | j \rangle \langle j + 1 |$ for the CSFQ and transmon, respectively. We use the notation $\omega_{2}^{(j)}(\omega_{n}^{(k)})$ to denote the
transition energy between the energy levels, \( j+1(k+1) \) and \( j(k) \) in the transmon (CSFQ). The relationships between the various coupling strengths \( g_{ij} \) and the relevant capacitances are given by the following expressions:

\[
\begin{align*}
g_{hm} & \sim -\frac{2C_{gh}C_{dm}}{(C_{gh} + C_{rCSFQ})(C_{gs}C_{m0} - 4C_{h0}^2)} \\
g_{rm} & \sim \frac{2C_{de}C_{h0}}{C_{cder}(4C_{h0}^2 - C_{gs}C_{m0})} \\
g_{nT} & \sim \frac{(C_{ab} + C_{rT})(C_{gT}C_{T0} - C_{ab}^2)}{C_{cd}C_{a0}} \\
g_{rT} & \sim -\frac{C_{cder}(C_{dT}^2 - C_{gT}C_{T0})}{C_{cd}C_{a0}} \\
g_{mT} & \sim -\frac{2C_{cd}C_{de}C_{a0}C_{h0}}{C_{cder}(C_{gT}C_{T0} - C_{ab}^2)(C_{gs}C_{m0} - 4C_{h0}^2)},
\end{align*}
\]

where \( C_{gs} = 2C_0 + 4C_g + 4(C_3 + C_{shCSFQ}), C_{gT} = C_{cd} + C_{de} + C_{shT} + C_T, \) and \( C_{cder} = C_{cd} + C_{de} + C_R. \) In the limit that the qubit-resonator detuning is much larger than the coupling between the qubits and resonators, we can use the Schrieffer-Wolff transformation to simplify the Hamiltonian. Here we first eliminate the readout resonators and then the bus, and obtain the multilevel version of the qubit-qubit effective Hamiltonian:

\[
H_{qr} = H_r + H_q = \omega_r r^1 r + \sum_{q=1,2} \sum_{n_q} \omega_q(n_q) |n_q\rangle \langle n_q| + \sqrt{(n_1 + 1)(n_2 + 1)}J_{n_1 n_2} (|n_1 + 1, n_2 \rangle \langle n_1, n_2 + 1| + |n_1 + 1, n_2 + 1\rangle \langle n_1 + 1, n_2|),
\]

where the dressed bus frequency is \( \tilde{\omega}_r = \omega_r + \chi_q \omega_q(n_q) |n_q\rangle \rangle \) and \( \chi \) is the dispersive shift of the resonator frequency and can be solved using Eq. (9) in Ref. [8], \( J \) is the two-photon virtual coupling rate defined as \( J_{j,k} = J_{dir} + J_{indir} \) with the direct coupling being \( J_{dir} = g_{mT} \), and the indirect coupling \( J_{indir} \):

\[
\begin{align*}
J_{indir}^{j,k} & = -\frac{g_{rT}g_{rT+1}}{2} \left( \frac{1}{\Delta_m(k)} + \frac{1}{\Delta_T(j)} + \frac{1}{\Sigma_m(k)} + \frac{1}{\Sigma_T(j)} \right) \\
\Delta_m(k) & = \omega_r - \omega_m(k) \\
\Delta_T(j) & = \omega_r - \omega_T(j) \\
\Sigma_m(k) & = \omega_r + \omega_m(k) \\
\Sigma_T(j) & = \omega_r + \omega_T(j).
\end{align*}
\]

In the limit of \(|\Delta| \gg J\), the Hamiltonian (9) can be diagonalized into the Hamiltonian in the dressed frame, using a unitary operator \( U \):

\[
\hat{H}_q = U \hat{H}_q U = \sum_{q=1,2} \sum_{n_q} \tilde{\omega}_q(n_q) |n_q\rangle \langle n_q|.
\]

Dressed qubit frequencies, anharmonicity, bare bus frequency, coupling strength, and two-photon exchange rate are presented in Table S.VI (|\tilde{\omega}_q| = |\tilde{\omega}_q(0)|, \( g_{\alpha \beta} = g_{\alpha \beta}(0) \)).

| CSFQ freq. \( \tilde{\omega}_1 \) | 5.051 GHz | \( g_{\alpha T} \) | 36.2 MHz | \( J_{00} \) | 5.7 MHz |
|---------------------|------------------|------------------|------------------|------------------|------------------|
| transmon freq. \( \tilde{\omega}_2 \) | 5.286 GHz | \( g_{\alpha T} \) | -76.4 MHz | \( J_{01} \) | 4.8 MHz |
| bus freq. \( \tilde{\omega}_r \) | 6.306 GHz | \( g_{\alpha m} \) | 111.7 MHz | \( J_{10} \) | 8.0 MHz |
| CSFQ anharmonic. \( \delta_1 \) | +593 MHz | \( g_{\alpha m} \) | -34 MHz |  |  |
| transmon anharmonic. \( \delta_2 \) | -327 MHz | \( g_{\alpha m} \) | -2.7 MHz |  |  |

**TABLE S.VI.** Qubit and bus resonator frequencies, anharmonicities, and coupling strengths.
Cross-Resonance Gate

A cross-resonance gate is enabled by driving the control qubit at the frequency of the target qubit and this allows for entanglement between the two qubits, where additional single-qubit rotations can implement a CNOT operation. In the dressed frame, the CR driving Hamiltonian, 

$$\hat{H}_d = U^\dagger H_d U = \Omega \cos(\omega_d t) \sum_{n_1} U^\dagger (|n_1\rangle \langle n_1 + 1| + |n_1 + 1\rangle \langle n_1|) U. \quad (17)$$

Moving into the rotating frame by RWA, 

$$H_r = R^\dagger(\hat{H} + \hat{H}_d)R - iR^\dagger R,$$  \quad (18)

where $$R = \sum_n \exp(-i\omega_d t\hat{n}) |n\rangle \langle n|.$$ For our device, we consider the total number of excitations to be limited to 4, therefore we consider the states \{00, 01, 10, 11, 02, 20, 03, 30, 12, 21, 31, 22, 32, 40\}. Next we block diagonalize it into two individual qubit blocks and all higher excited levels: \(2 \times 2, 2 \times 2,\) and \(11 \times 11\) to decouple higher levels from the computational subspace under the principle of least action \[9\]. This method aims to find a unitary operator \(T\), which is closest to the identity operation. The least action unitary operator \(T\) that satisfies \(H_{BD} = T^\dagger H_r T\) is given by \[10, 11\]

$$T = XX_B^\dagger X_P^{-\frac{1}{2}},$$  \quad (19)

where \(X\) is the nonsingular eigenvector matrix of \(H_r\), \(X_{BD}\) is the block-diagonal matrix of \(X\), and \(X_P = X_{BD}X_{BD}^\dagger\). Finally, the driven Hamiltonian in the computational subspace can be written as

$$H_{CR} = \alpha_{ZI} \frac{ZI}{2} + \alpha_{IX} \frac{IX}{2} + \alpha_{ZX} \frac{ZX}{2} + \zeta(\Omega) \frac{ZZ}{4}. \quad (20)$$

The CR gate is accompanied with some unwanted interactions such as \(ZZ, IX,\) and \(ZI\). The latter two can be cancelled by echoed CR sequences \[12\], while the \(ZX\) term remains and results in the oscillation behavior in the target qubit. On top of the static \(ZZ\) interaction \(\zeta\), which solely comes from the contribution of higher excitations in the qubit-qubit interaction, the CR drive with the amplitude \(\Omega\) introduces an additional \(ZZ\) interaction that depends quadratically on \(\Omega\). The two together produce the total \(ZZ\) interaction, \(\zeta(\Omega) = \zeta(0) + \eta \Omega^2\), where \(\zeta(0)\) is the static \(ZZ\) interaction, and \(\zeta(\Omega)\) is what we refer to as the dynamic \(ZZ\) interaction. In this manuscript, we look into schemes for eliminating the static \(ZZ\) interaction, while in Ref. \[7\], we develop a scheme for eliminating the total \(ZZ\) interaction.

Classical Crosstalk

In the presence of classical crosstalk, the normal driving Hamiltonian is modified as,

$$H_d^{cl} = \Omega \cos(\omega_d t + \phi_0) \sum_{n_1} (|n_1\rangle \langle n_1 + 1| + |n_1 + 1\rangle \langle n_1|)$$

$$+ R\Omega \cos(\omega_d t + \phi_1) \sum_{n_2} (|n_2\rangle \langle n_2 + 1| + |n_2 + 1\rangle \langle n_2|). \quad (21)$$

When the Hamiltonian is taken to the dressed frame and block diagonalized, one can find the terms \(IY\) and \(ZY\) in the effective driving Hamiltonian below:

$$H^{cl}_{CR} = \beta_{ZI}\frac{ZI}{2} + \beta_{ZX}\frac{ZX}{2} + \beta_{ZY}\frac{ZY}{2} + \beta_{IX}\frac{IX}{2} + \beta_{IY}\frac{IY}{2} + \beta_{ZZ}\frac{ZZ}{4},$$  \quad (22)

where the Pauli coefficients are

$$\beta_{ZX} \approx (B_f \Omega + C_f \Omega^3) \cos \phi_0$$

$$\beta_{ZY} \approx (B_f \Omega + C_f \Omega^3) \sin \phi_0$$

$$\beta_{IX} \approx (D_f \Omega + E_f \Omega^3) \cos \phi_0 + RK_f \Omega \cos \phi_1$$

$$\beta_{IY} \approx (D_f \Omega + E_f \Omega^3) \sin \phi_0 + RK_f \Omega \sin \phi_1$$

$$\beta_{ZI} \approx \alpha_{ZI}$$

$$\beta_{ZZ} \approx \zeta(\Omega), \quad (23)$$
FIG. S6. Pauli coefficients versus driving amplitude at the sweet spot. The parameters used for simulation are $R = 0.0125$, $\phi_0 = \pi$ and $\phi_1 = \pi + 0.4$.

where $B_f$, $C_f$, $D_f$, $E_f$ and $K_f$ are flux-dependent quantities that can be evaluated numerically. In this experiment, the driving phase has been calibrated as $\phi_0 = \pi$ and $\phi_1 = \pi + 0.4$. All Pauli coefficients at the sweet spot are plotted in Fig. S6. One can see from Fig. S6 that the unwanted $ZY$ vanishes in the device, and the $IY$ component can be classically removed by applying the same term with a negative phase to the target qubit (see Ref. [1] for more details).

Echoed CR Oscillation Frequency

After eliminating all unwanted components, the CR gate will effectively behave like a two-qubit gate corresponding to $ZX_{\theta}$ [13]:

$$ZX_{\theta} = \exp \left[ -i \theta \left( ZX/2 \right) \right] = \begin{pmatrix}
\cos(\theta/2) & -i \sin(\theta/2) & 0 & 0 \\
-i \sin(\theta/2) & \cos(\theta/2) & 0 & 0 \\
0 & 0 & \cos(\theta/2) & -i \sin(\theta/2) \\
0 & 0 & -i \sin(\theta/2) & \cos(\theta/2)
\end{pmatrix}, \quad (24)
$$

where $\theta = -2\pi f_{CR} \tau$, and $\tau$ is the single CR pulse length. For our particular entangling gate, we choose $\theta = -\pi/2$. Following the Echo CR gate shown in Fig. 3 inset of the main text, one can find that in the presence of a ZZ interaction as well as all other unwanted terms, the frequency of the echoed CR oscillation can be determined from the following relation (see Ref. [7] for more details):

$$f_{CR} = \sqrt{(\beta_{ZX} + \beta_{IX})^2 + (\beta_{ZY} + \beta_{IV})^2 + (\beta_{ZZ}/2)^2} + \sqrt{(\beta_{ZX} - \beta_{IX})^2 + (\beta_{ZY} - \beta_{IV})^2 + (\beta_{ZZ}/2)^2}. \quad (25)
$$

If both the classical crosstalk and ZZ interaction are eliminated, the equation above reduces to $2\beta_{ZX}$. As shown in Fig. 3 of the main text, there are upper limits at the oscillation frequency, and this is because two energy levels, 11 and 02, get closer together as the driving amplitude becomes stronger, such that leakage occurs at the specific amplitude. The energy eigenvalues of the 11 and 02 states in the rotating frame are shown in Fig. S7. The maximum oscillation frequency ($f_{CR}$) occurs where the anti-crossing between the 11 and 02 levels takes place.

For $ZX_{90}$ rotation, the amplitude and frequency of the echoed CR gate satisfy $(2\pi f_{CR}) \tau = \pi/2$. Fig. 3 in the main text shows that for a weak driving regime, $f_{CR} \approx \gamma(f) \Omega$ with a flux-dependent coefficient $\gamma(f)$, e.g., $\gamma(0.5) \approx 0.1$. The exact flux-dependent $\gamma(f)$ can be found from Eq. (25). Putting those two together indicates that $ZX_{90}$ gate length inversely scales with its amplitude: $\tau = 1/(4\gamma(f) \Omega)$.

Two-qubit Gate Error

We simulate an echoed CR pulse sequence, i.e., $ZX_{90}$, to compute the two-qubit error per gate by a density matrix starting in the ground state in the Pauli basis. Here, the ZZ interaction is a global error, and for each time step we
apply corresponding operators and decoherence terms. The total map is,

\[ \rho_t = \Lambda_{T1,T2,Q1} \circ \Lambda_{T1,T2,Q2} \circ \Lambda_{ZZ} \circ \Lambda_{XI} \circ \Lambda_{CR-} \circ \Lambda_{XI} \circ \Lambda_{CR+}[\rho_i], \]  

(26)
where each map is defined by,

\[
\Lambda_{ZZ}[\rho] = U_{ZZ} \cdot \rho \cdot U_{ZZ}^\dagger \\
\Lambda_{XI}[\rho] = XI \cdot \rho \cdot XI \\
\Lambda_{CR\pm}[\rho] = U_{CR\pm} \cdot \rho \cdot U_{CR\pm}^\dagger \\
\Lambda_{t_1, t_2}[\rho] = \frac{1 - e^{-t_2/T_2}}{2} \rho \cdot Z + \frac{1 + e^{-t_1/T_1}}{2} \rho + \frac{1 - e^{-t_1/T_1}}{2} |0\rangle \langle 1| \cdot \rho \cdot |1\rangle \langle 0| - \frac{1 - e^{-t_2/T_2}}{2} |1\rangle \langle 1| \cdot \rho \cdot |1\rangle \langle 1|.
\]

Operators are defined as \( U_{ZZ} = e^{-i2\pi\zeta(t)f}Z/Z^2 \) and \( U_{CR\pm} = e^{-i2\pi\tau H_{CR}^\pm(\pm\Omega)} \), where \( t_g \) is the total gate length including two Gaussian flat-top pulses with \( \pi \) phase shift and a \( \pi \)-pulse on the control qubit after each CR pulse.

To explore the influence of unwanted interactions, we plot the gate error of echoed CR pulses at different CSFQ flux points for \( t_g = 560 \text{ ns} \) in Fig. S8. In this plot, we show the coherence limit without unwanted interactions by a thick dashed line, without ZZ but with classical crosstalk in a thin dashed line, and with both ZZ and classical crosstalk in a solid line. For this plot, we take all parameters from the present device; for the static ZZ interaction, we use experimental data shown in Fig. 2 in the main text; for the flux dependence of \( T_2 \) in the CSFQ, we used the results in Fig. S9. For the gate error simulations for different flux bias points, we must model the appropriate flux-dependent dephasing of the CSFQ. Simply taking \( T_2 \) to be the value obtained by a Ramsey measurement with a single echo refocusing pulse is insufficient and overestimates the gate errors away from the sweet spot. In order to capture the coherence limit for our measurements, we find that we must use the measured flux dependence of a Ramsey sequence with 20 echo pulses, plus an additional flux-independent dephasing term that could be caused, for example, by photon fluctuations in the readout cavity [16].

For Eq. (23), the phase difference \( \phi_1 \) is a constant, and \( R(f, \tau) = (0.2 - 50|f - 0.5|^{1.2})\tau \), which is a phenomenological fitting function for the experimental data. Fig. S8 shows that both classical crosstalk and ZZ interaction suppress the fidelity the most at the sweet spot where the ZZ interaction and also \( R \) are maximal (see Fig. 2 in the main text). Away from the sweet spot, all unwanted interactions become suppressed, and therefore the gate fidelity approaches its coherence limit.

To make a comparison of the gate error between a CSFQ-transmon hybrid device and an all-transmon device, we considered a state-of-the-art transmon-transmon device [10] and also an ideal CSFQ-transmon device. For the ideal CSFQ-transmon device, we set the static ZZ = 0 at the sweet spot for which we use the current circuit parameters and only change the Josephson energy \( E_J \). By changing the gate length, we determine the gate fidelity \( F \) and plot the gate error, defined as \( 1 - F \), in Fig. S10. It shows that the error rate of \( 1 \times 10^{-3} \) can be achieved in a CSFQ-transmon device.
device with no static ZZ term or no classical crosstalk and enhanced coherence ($T_1, T_2 = 200 \mu s$). The corresponding coherence times are listed in Table S.VII.

FIG. S10. Two-qubit gate error for three sets of coherence times ($T_1^{(1)}, T_2^{(1)}, T_1^{(2)}, T_2^{(2)}$), where the superscripts indicate the qubit, are (18, 15, 40, 45), corresponding to the present device, (40, 54, 43, 67), corresponding to the two-transmon device in Ref. 1, and (200, 200, 200, 200) for an ideal CSFQ-transmon device (all times in $\mu s$). (a) Present CSFQ-transmon device. (b) Transmon-transmon device. (c) Ideal CSFQ-transmon device. Note that all three figures share the same legend as in (a). CSFQ is assumed to be at the sweet spot in the simulation.

| Device                  | $T_1^{(1)}$ $\mu s$ | $T_2^{(1)}$ $\mu s$ | $T_1^{(2)}$ $\mu s$ | $T_2^{(2)}$ $\mu s$ | $\tilde{\omega}_1$ GHz | $\tilde{\omega}_2$ GHz | $\delta_1$ MHz | $\delta_2$ MHz | $\eta$ $1/\text{MHz}$ |
|------------------------|---------------------|---------------------|---------------------|---------------------|------------------------|---------------------|-----------------|-----------------|---------------------|
| Present CSFQ-Transmon  | 18                  | 15                  | 40                  | 45                  | 5.051                  | 5.286               | 593             | −327            | 6.0 $\times$ 10$^{-6}$ |
| Transmon-Transmon      | 40                  | 54                  | 43                  | 67                  | 5.114                  | 4.914               | −330            | −330            | 1.6 $\times$ 10$^{-6}$ |
| Ideal CSFQ-Transmon    | 200                 | 200                 | 200                 | 200                 | 5.094                  | 5.286               | 593             | −327            | 8.0 $\times$ 10$^{-6}$ |

TABLE S.VII. Coherence time, transition frequency, anharmonicity and nonlinear ZZ interaction rate for the current device, transmon-transmon and ideal CSFQ-transmon device, respectively.
The static ZZ strength was measured at different flux points by the JAZZ (Joint Amplification of ZZ) protocol [17, 18]. This measurement protocol involves a Ramsey measurement on one qubit with an echo \( \pi \)-pulse inserted to both qubits. The pulse sequence is executed twice – once with the qubit that is not manipulated by the Ramsey measurement in the ground state, then again in the excited state. The frequency difference between the two resultant Ramsey fringes then corresponds to the static ZZ strength. It is necessary to vary the phase of the second \( \pi/2 \)-pulse to observe fringes, since the \( \pi/2 \)-pulses are on resonance for each qubit. The oscillation frequency of the fringes, and hence the extracted ZZ strength, is independent of the choice of qubit for the Ramsey sequence. Because the transmon has better coherence, we chose it for the Ramsey measurement.

**FIG. S11.** CSFQ coherence measured versus flux. (a) \( T_1 \). (b) \( T_2^* \) measured with standard Ramsey sequence. (c) \( T_2 \) measured with Hahn-echo sequence with single echo pulse.

In Fig. S11, we show the \( T_1, T_2^* \), and \( T_2 \) times for the CSFQ measured as a function of flux. During the measurements, the \( \pi/2 \)-pulse was recalibrated at each flux point. The \( \pi \)-pulse was composed of two \( \pi/2 \) pulses.

**Single-qubit RB**

All the gate pulses were generated with single-side-band (SSB) modulation. For single-qubit pulses (\( X_{90} \) and \( Y_{90} \)), we used 20 ns (= 4\( \sigma \)) Gaussian pulses including the derivative removal via adiabatic gate (DRAG) corrections [19], where \( \sigma \) is a standard deviation. We use two \( X_{90} \) (\( Y_{90} \)) pulses back-to-back for a \( X \) (\( Y \)) pulse, i.e., \( \pi \)-pulse. For a pulse calibration of \( X_{90} \) gate, we first performed a Ramsey measurement to find the qubit transition frequency. We then calibrated the amplitude for the \( X_{90} \) pulse via phase estimation [20], followed by the DRAG calibration. The pulse amplitude and DRAG calibration were executed twice to make sure both converged. The \( Y_{90} \) pulse was not separately calibrated, but was assumed to have the same amplitude as \( X_{90} \).

For single-qubit RB [21, 22], we used the pulse primitives \{\( \pm X_{90}, \pm Y_{90}, \pm X, \pm Y \}\) to create a group of 24 single-qubit Clifford gates. In RB measurements, we created 30 randomly chosen Clifford sequences for each number of Clifford gates and averaged 2000 times for each pulse sequence to obtain reasonable error bars. By fitting the data to a fidelity decay function of the form \( A \alpha^m + B \), we calculated the single-qubit error per gate \( \epsilon = 1/2 \cdot (1 - \alpha^{1/N}) \), where \( N \) is the average number of pulse primitives in the 24 single-qubit Clifford gates [23]: \( N = 1.875 \) [24].

In Fig. S12(a) and S12(b), we show two representative RB measurements for the transmon and CSFQ at the sweet spot, and their gate errors. The typical error per gate was lower than \( 1 \times 10^{-3} \) for the transmon and CSFQ over the entire flux range of our experiments.
Two-qubit RB

The pulse primitive for two-qubit gate is the $ZX_{90}$, which is an echoed CR pulse. CR pulses consist of a flat-top waveform with 20 ns Gaussian rise and fall times ($≈ 2\sigma$). The pulse calibration for the $ZX_{90}$ was performed in two steps: first, we calibrated the phase of the CR pulse so that the rotation axis of the target in the Bloch sphere matches the x-axis; next, we calibrated the amplitude of the $ZX_{90}$ gate via phase estimation [20].

To create the set of two-qubit Clifford gates, we followed Ref. [12]. Each two-qubit Clifford gate was generated from single-qubit primitive gates $\{\pm X_{90}, \pm Y_{90}, \pm X, \pm Y\}$ for the transmon and CSFQ, and the two-qubit primitive gate $ZX_{90}$. The ground state probability of the target qubit was measured as a function of the number of randomly chosen two-qubit Clifford gates for each Clifford sequence. To sample more Clifford gates, the total 20 different Clifford sequences for each length of the Clifford sequence were used. For every Clifford sequence, each measurement was averaged 2000 times for obtaining reasonable statistics. As explained in the main text, the gate error $\epsilon$ was calculated by $\epsilon = 3/4 \cdot (1 - \alpha^{1/N})$, where $\alpha$ is the depolarization parameter from the same fidelity decay function as in the single-qubit RB, and $N$ is the average number of two-qubit primitive gates ($ZX_{90}$); $N = 1.5$ [12, 23]. In Fig. S12(c) we show a representative two-qubit RB for a 200 ns gate length.

Simultaneous RB

FIG. S13. Simultaneous single-qubit RB and addressability for (a) transmon and (b) CSFQ as a function of CSFQ flux bias.
In this section, we show simultaneous single-qubit randomized benchmarking data for the transmon and CSFQ as a function of CSFQ flux bias. The simultaneous RB was performed by applying two different sets of single-qubit RB sequences to the CSFQ and transmon simultaneously. Next, we measured each single-qubit RB, which, combined with the previous simultaneous RB, allows us to measure the addressability for each qubit. For the transmon, which, again is a fixed-frequency non-tunable qubit, the gate error decreases for CSFQ flux bias points $f \sim 0.496$ and $0.504$, where $ZZ = 0$. This is attributed to the static $ZZ$ interaction, which has a maximum at the CSFQ flux sweet spot. The addressability [25] – a measure of how much the average error per Clifford gate changes – is defined by $\delta r_T|C = r_T - r_T|C$, where $r_T$ is the error per gate of transmon without simultaneous RB, and $r_T|C$ is the error per gate with simultaneous RB. Clearly, the addressability shows the same dependence on flux as the gate error for the transmon. These results are consistent with the static $ZZ$ measurement and show that the $ZZ$ interaction is a source of error when the two qubits are driven simultaneously, even without performing a two-qubit entangling gate.

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