Proposed mathematical model for stress-strain behaviour of geopolymer concrete

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Abstract. In the present study, appropriate analytic stress-strain mathematical model is developed that can capture the real (observable) stress-strain behaviour of geopolymer concrete. The geopolymer concrete mixes have shown improved stress values for the same strain levels compared to that of controlled concrete mix in M20 grade. The analytical equations for the stress-strain response of conventional and geopolymer concrete mixes have been proposed in the form of $y = \frac{Ax}{1+Bx+Cx^2}$, both for ascending and descending portions of the curves with different set of values for constants. The proposed equations have shown good correlation with experimental values. The proposed empirical equations can be used as stress block in analyzing the flexural behavior of sections of controlled and geopolymer concrete. The stress-strain curves obtained in the experiment for M20 & G20 grades of controlled and geo polymer concrete exhibit a similar trend when compared to the empirical equations of modified Saenz model. So Saenz mathematical model is successfully evaluated and validated for geopolymer concrete.

1 Introduction

One of the most essential tasks in studying the stress-strain behaviour of Geopolymer concrete is to develop adequate analytic stress-strain models that represent the true (observable) behaviour. The more accurate the stress-strain model, the more trustworthy the estimation of concrete structural member strength and deformation behaviour. The development of an appropriate analytic stress-strain mathematical model that can represent the real (observable) stress-strain behaviour of geopolymer concrete is completed. This may be accomplished by combining the best features of previous models to develop a stress-strain model that accurately represents the total stress-strain behaviour of Geopolymer concrete. Empirical equations are constructed to characterise uni-axial stress-strain behaviour of conventional and Geo polymer concrete mixes of standard grade concrete after experimentally acquiring the stress-strain behaviour of conventional and Geo polymer concrete (M20). Theoretical stresses for conventional and Geo polymer concrete are computed using these empirical formulae and compared to experimental data. Numerous models for predicting concrete stress-strain behaviour have been developed by many researchers. The following are some important models to consider:

1) The models of Desai and Krishnan (1964)
2) Saenz Model with Changes (1964)
3) Model Hognestad (1955)
4) Model by Wang et al (1978)
5) Models Carriera and Chu (1985)

Simplified and modified single variable polynomial equations based on modified Saenz's model that fit with the produced normalised stress-strain curves appear to be valid for both ascending and descending sections of the curve, out of all the aforementioned stress-strain models. The equations derived for the ascending and descending sections of the analytical stress-strain curve are in the form of:

$$y = \frac{Ax}{1+Bx+Cx^2}$$

and

$$y = \frac{Dx}{1+Ex+Fx^2}$$

(1)

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where \( y \) is the stress at any level; \( x \) is the corresponding strain at that level; \( A, B, C \) are the constants for the ascending section of the analytical stress-strain curve, and \( D, E, F \) are the constants for the descending portion.

The equations for the ascending and descending sections of a non-dimensional stress-strain curve (normalised) are similar.

\[
\frac{f}{f_0} = A'\left(\frac{e}{e_0}\right) / (1 + B'\left(\frac{e}{e_0}\right) + C'\left(\frac{e}{e_0}\right)^2)
\]

(2)

and

\[
\frac{f}{f_0} = D'\left(\frac{e}{e_0}\right) + (1 + E'\left(\frac{e}{e_0}\right) + F'\left(\frac{e}{e_0}\right)^2)
\]

(3)

The constants for the climbing section of the non-dimensional stress-strain curve are \( A_1, B_1, C_1 \), while the constants for the descending portion are \( D_1, E_1, F_1 \). The normalised stress (stress ratio) is equal to \( f / f_0 \), and the normalised strain is equal to \( f / f_0 \) (strain ratio). The boundary conditions of normalised stress-strain curves for both conventional and geopolymer concrete are used to estimate constants. Boundary conditions for ascending and descending portions of stress-strain curves are,

1. At the origin the ratio of stresses and strains are zero
   \( i.e. \ 0 = \frac{f}{f_0} = 0\)

2. The strain ratio \( \left(\frac{e}{e_0}\right) \) and stress ratio at the peak of the non-dimensional stress-strain curve is unity.
   \( i.e. \ 1 = \frac{f}{f_0} = 1\)

3. The slope of non-dimensional stress-strain curve at the peak is zero
   \( i.e. \ 0 = \frac{d(f/f_0)}{d(e/e_0)}\)

4. At 85% stress ratio, the corresponding values of strain ratio is recorded
   \( i.e. \ (f/f_0) = 0.85, \ (e/e_0) = \text{strain ratio} \)

The constants \( A_1, B_1, C_1 \) in the ascending section of the normalised stress-strain curve are determined by boundary conditions (1), (2), and (3), whereas the constants \( D_1, E_1, F_1 \) in the descending portion of the curve are determined by boundary conditions (2), (3), and (4). Equations are then used to get the corresponding \( A, B, C \) constants for the ascending section and \( D, E, F \) constants for the descending portion of the analytical stress-strain curve.

\[
A = A_1 \left(\frac{f_0}{f_0}\right) = 1 \quad B = B_1 \left(\frac{1}{e_0}\right) \quad C = C_1 \left(\frac{1}{e_0}\right)^2
\]

(4)

And

\[
D = D_1 \left(\frac{f_0}{f_0}\right) = 1 \quad E = E_1 \left(\frac{1}{e_0}\right) \quad F = F_1 \left(\frac{1}{e_0}\right)^2
\]

(5)

### 2 Theoretical Stresses

Theoretical stresses were computed using provided empirical equations for conventional and geopolymer concrete obtained from a modified Saenz model in the form of

\[
y = \frac{Ax}{1 + Bx + Cx^2} \quad \text{For ascending portion}
\]

(6)

\[
y = \frac{Dx}{1 + Ex + Fx^2} \quad \text{For descending portion}
\]

(7)

Where \( y \) is the stress at any level; \( x \) is the corresponding strain at that level.

| Strain | Stress N/mm² | Normalized stress | Normalized strain |
|--------|--------------|-------------------|------------------|
| 0.0000 | 0.00         | 0.00              | 0                |
| 0.0001 | 2.26         | 0.079             | 0.018            |
| 0.0002 | 4.43         | 0.155             | 0.050            |
| 0.0004 | 6.58         | 0.230             | 0.101            |
| 0.0008 | 8.45         | 0.295             | 0.181            |
| 0.0013 | 11.82        | 0.413             | 0.295            |
| 0.0015 | 13.79        | 0.481             | 0.350            |
| 0.0018 | 15.83        | 0.553             | 0.407            |
| 0.0020 | 17.09        | 0.597             | 0.462            |
| 0.0023 | 19.23        | 0.671             | 0.522            |
| 0.0026 | 21.62        | 0.755             | 0.602            |
| 0.0032 | 24.36        | 0.851             | 0.741            |
| 0.0036 | 25.42        | 0.888             | 0.819            |
| 0.0040 | 27.68        | 0.966             | 0.920            |
| 0.0044 | 28.64        | 1.000             | 1.000            |
| 0.0048 | 28.41        | 0.992             | 1.103            |
Table 2. Experimental stress strain values of geopolymer concrete

| Strain | Stress N/mm² | Normalized stress | Normalized strain |
|--------|--------------|-------------------|------------------|
| 0.0000 | 0            | 0                 | 0                |
| 0.0001 | 2.26         | 0.074             | 0.018            |
| 0.0003 | 4.43         | 0.145             | 0.061            |
| 0.0006 | 6.58         | 0.216             | 0.104            |
| 0.0008 | 8.05         | 0.264             | 0.151            |
| 0.0014 | 11.82        | 0.387             | 0.247            |
| 0.0016 | 13.79        | 0.452             | 0.283            |
| 0.0019 | 15.83        | 0.519             | 0.337            |
| 0.0021 | 17.09        | 0.560             | 0.380            |
| 0.0023 | 19.23        | 0.630             | 0.419            |
| 0.0028 | 21.62        | 0.708             | 0.493            |
| 0.0032 | 23.85        | 0.781             | 0.566            |
| 0.0035 | 25.25        | 0.827             | 0.622            |
| 0.0044 | 28.88        | 0.946             | 0.785            |
| 0.0049 | 29.79        | 0.976             | 0.885            |
| 0.0052 | 30.34        | 0.994             | 0.935            |
| 0.0056 | 30.52        | 1.000             | 1.000            |
| 0.0057 | 29.02        | 0.951             | 1.029            |
| 0.0059 | 26.62        | 0.872             | 1.056            |
| 0.0060 | 24.89        | 0.816             | 1.066            |

Table 3. Constants for ascending and descending portions of non-dimensional stress strain curve

| Grade of Concrete | Conventional Concrete | Geopolymer concrete |
|-------------------|------------------------|---------------------|
|                   | Ascending portion      | Descending portion  |
|                   | Constants              | Constants           |
|                   | A¹                    | B¹                  | C¹                  | D¹        | E¹        | F¹        |
|                   | A¹                    | B¹                  | C¹                  | D¹        | E¹        | F¹        |
| M20               | 0.51                  | -1.49               | 1                   | 0.16      | -1.84     | 1         |
|                   | 1.07                  | -0.93               | 1                   | 0.02      | -1.98     | 1         |

Table 4. Analytical equations for non-dimensional stress-strain curve

| Grade of Concrete | Conventional Concrete | Geopolymer concrete |
|-------------------|------------------------|---------------------|
|                   | Ascending portion      | Descending portion  |
|                   | y= 0.51x / 1-1.49x+y²  | y= 0.16x / 1-1.84x+y² |
| M20               | y= 1.07x / 1-0.93x+y²  | y= 0.02x / 1-1.98x+y² |

Table 5. Constants for ascending and descending portions of dimensional analytical stress-strain curve

| Grade of Concrete | Conventional Concrete | Geopolymer concrete |
|-------------------|------------------------|---------------------|
|                   | Ascending portion      | Descending portion  |
|                   | A                      | B                    | C                    | D        | E        | F        |
|                   | A                      | B                    | C                    | D        | E        | F        |
| M20               | 3342                   | -341                 | 52365                | 1049     | -421     | 52365    |
| G20               | 5791                   | -167                 | 32117                | 109      | -355     | 32117    |
Table 6. Experimental and theoretical stress-strain values of M20 and G20

| Strain | Conventional concrete | Geopolymer concrete |
|--------|------------------------|---------------------|
|        | Experimental Stress N/mm² | Theoretical stress | Strain | Experimental Stress N/mm² | Theoretical stress |
| 0.0000 | 0.00                    | 0.00                | 0.0000 | 0.00                    | 0.00                |
| 0.0001 | 2.26                    | 0.27                | 0.0001 | 2.26                    | 2.24                |
| 0.0002 | 4.43                    | 0.79                | 0.0003 | 4.43                    | 4.39                |
| 0.0004 | 6.58                    | 1.71                | 0.0006 | 6.58                    | 6.51                |
| 0.0008 | 8.45                    | 3.46                | 0.0008 | 8.05                    | 7.97                |
| 0.0013 | 11.82                   | 6.66                | 0.0014 | 11.82                   | 11.70               |
| 0.0015 | 13.79                   | 8.51                | 0.0016 | 13.79                   | 13.65               |
| 0.0018 | 15.83                   | 10.64               | 0.0019 | 15.83                   | 15.67               |
| 0.0020 | 17.09                   | 12.86               | 0.0021 | 17.09                   | 16.92               |
| 0.0023 | 19.23                   | 15.40               | 0.0023 | 19.23                   | 19.04               |
| 0.0026 | 21.62                   | 18.89               | 0.0028 | 21.62                   | 21.40               |
| 0.0032 | 24.36                   | 24.34               | 0.0032 | 23.85                   | 23.61               |
| 0.0036 | 25.42                   | 26.57               | 0.0035 | 25.25                   | 25.00               |
| 0.0040 | 27.68                   | 28.26               | 0.0044 | 28.88                   | 28.59               |
| 0.0044 | 28.64                   | 28.65               | 0.0049 | 29.79                   | 29.49               |
| 0.0048 | 28.41                   | 26.99               | 0.0052 | 30.34                   | 30.04               |
| 0.0051 | 27.67                   | 25.06               | 0.0056 | 30.52                   | 30.21               |
| 0.0052 | 23.54                   | 24.23               | 0.0057 | 29.02                   | 28.73               |
|        |                         |                     | 0.0059 | 26.62                   | 26.35               |
|        |                         |                     | 0.0060 | 24.89                   | 24.64               |

Fig.1. Experimental and theoretical stress-strain curves of conventional concrete
3 Discussions

Theoretical non-dimensional stress-strain data is derived from experimental non-dimensional stress-strain data. The practical and theoretical results correspond well, indicating that the suggested model for studying the stress-strain behaviour of controlled and geopolymer concrete of grade M20 is viable. The stress-strain curve for mix is drawn using the values of stresses and strains, using the average values of the three cylinders' findings. The related normalised stress-strain values are derived by dividing each stress value by the peak stress and dividing each strain value by strain at peak strain from the stress-strain values of controlled and geopolymer concrete mixes. The average normalised stress-strain curves for controlled and geopolymer concrete mix are plotted separately from the normalised stress-strain values, and empirical equations in the form of $y = A x / (1 + B x + C x^2)$ are proposed for ascending and descending portions of controlled and geopolymer concrete mix for M20 grade of concrete. Theoretical stresses are assessed and compared to experimental stress levels, and it is discovered that there is very little fluctuation, indicating that the mathematical model presented is correct. The stress-strain behaviour of all the controlled and geopolymer concrete mixes is virtually same, according to the observations derived from stress-strain curves. The main difference is that when compared to controlled concrete mixes, geopolymer concrete mixes have demonstrated better stress values for the same strain levels. The form of the ascending section of the stress-strain curve for typical concrete is more linear and steeper, as can be seen from stress-strain curves.

In comparison to standard strength concrete, the strain at peak stress is somewhat greater, and the slope of the falling section is steeper. This was because the degree of internal micro cracking in greater strength concrete was reduced. The suggested equations have demonstrated satisfactory agreement with experimental results for grade M20 of controlled and geopolymer concrete. According to the literature, the modified second degree polynomial proposed by L.P. Saenz looks to be a better match with acceptable constants for the current curves.

4 Conclusions

The following conclusions may be derived from the experimental data gathered during the course of this study:

1. When compared to a controlled concrete mix in M20 grade, the geopolymer concrete mixes showed better stress values at the same strain levels.
2. The average strain at peak stress for controlled and geopolymer concrete is extremely near to the strain at peak stress for controlled concrete in axial compression, which is 0.002 according to IS 456-2000.
3. In the form of $y = A x / (1 + B x + C x^2)$, analytical equations for the stress-strain response of conventional and geopolymer concrete mixes have been developed, both for ascending and descending sections of the curves with various sets of constants. The suggested equations exhibit a high degree of agreement with experimental results.
4. The suggested empirical equations may be utilised to analyse the flexural behaviour of sections of controlled and geo polymer concrete as a stress block.
5. When compared to the empirical equations of the modified Saenz model, the stress-strain curves produced in the experiment for M20 and G20 classes of controlled and geopolymer
concrete show a similar tendency. As a result, the Saenz mathematical model for geopolymer concrete has been effectively assessed and verified.

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