An intense search is underway to identify experimental realizations of topological superconductors, exotic quantum states of matter that carry robust energy currents along their boundaries. Topological superconductors can host Majorana fermions, fractional particles whose discovery could enable fault-tolerant quantum computation.

A system of identical fermionic atoms confined to two dimensions and interacting attractively through a $p$-wave Feshbach resonance was predicted to form a "$p_x + ip_y$" superfluid [1]. Here, $p_x + ip_y$ refers to a particular symmetry of the superfluid order parameter; phases of other symmetries are also possible, but are not as energetically favorable [2–4]. The excitation spectrum [5] of such a $p_x + ip_y$ state is fully-gapped, as long as the chemical potential $\mu$ is not zero. However, if the sample possesses a boundary and if $\mu > 0$, then gapless excitations appear at the superfluid edge [5, 6], propagating in a particular direction. The edge excitations can be thought of as a one-dimensional band of Majorana fermions. When the $p_x + ip_y$ superfluid is deformed by a perturbation that does not close the bulk energy gap, the gapless boundary excitations remain and retain their properties. These are said to be "topologically protected," and the phase of the superfluid with $\mu > 0$ is a two-dimensional topological superconductor. This picture applies to the weak coupling BCS regime; the strong pairing BEC phase [7, 8] has $\mu < 0$ and is topologically trivial. These are separated by a quantum critical point at $\mu = 0$ [3, 5].

Several attempts were made to create the $p$-wave superfluid experimentally in a gas of fermionic $^{40}\text{K}$ or $^6\text{Li}$ atoms. Unfortunately these gases were found to be unstable due to losses involving three-body processes [9–11], with the lifetime $t_3$ ranging from a few milliseconds in $^{40}\text{K}$ [12] to about 20ms in $^6\text{Li}$ [13, 14] at a particle density corresponding to a Fermi energy of about 10KHz. This instability prevents the gas from reaching its ground state; instead it decays with atoms simply leaving the trap where the gas is held. Interestingly, a very weakly interacting $p$-wave gas is predicted to be significantly more stable [10], yet such a gas would also have a very small gap, potentially preventing direct observation of its topological properties.

In this Letter, we show that one can induce a topological Floquet superfluid [15, 16] if weakly interacting atoms are brought suddenly close ("quenched") to such a resonance, in the time before the instability kicks in. We build off of our recent work [17], in which we determined the exact asymptotic dynamics of a BCS $p$-wave superfluid following a quantum quench. Specifically, we propose to start with a weakly interacting gas of $^{40}\text{K}$ or $^6\text{Li}$ and then suddenly tune the interaction strength to the desired value by means of a Feshbach resonance. The gas would evolve in an out-of-equilibrium fashion from its initial state. Our results determine the evolution over the time scale before the instability destroys the gas. Note that the ratio of the lifetime $t_3$ to the inverse Fermi energy $t_F = 2m/p_F^2$, can be as high as 200, which gives plenty of room for the gas to evolve and reach a quasi-stationary state before decaying, as will be elaborated below. The types of superfluid states that we describe have topologically-trivial $s$-wave analogs [18–25]. An exciting recent development is the observation of non-equilibrium order parameter dynamics in superconducting thin films [26].

Depending on the initial state and the strength of the quench, the resulting out-of-equilibrium superfluid may find itself in one of three regimes [17]: a steady state with a vanishing order parameter $\Delta(t) \to 0$ as $t \to \infty$ (Region I), a state with $\Delta(t) \to \Delta_\infty$, a non-zero constant (Region II), and a quasi-steady state with an oscillating $\Delta(t)$ (Region III). The phase diagram of all possible quenches of the superfluid is shown in Fig. 1.

To realize a topological superfluid in an ultracold gas, the most relevant quenches are those in Region III. An initial state with weak pairing is prepared far from the Feshbach resonance (i.e. $\Delta_0$ in Fig. 1 is close to zero), where three-body losses can be neglected [10]. Then the coupling in the Hamiltonian is quenched close to the resonance, and the system evolves coherently. Here we show that Region III is topologically non-trivial, and the oscillating order parameter induces Majorana edge modes.
Region III therefore realizes a Floquet topological superfluid, yet this differs from a conventional Floquet system [15, 16, 27, 28] as it is not driven externally. Instead, the periodic modulation is self-generated by the dynamics.

We briefly recount the setup of the problem from Ref. [17]. Neglecting the terms responsible for the losses, the gas can be described by the Hamiltonian [1]

\[
\hat{H} = \sum_p \frac{p^2}{2m} \hat{a}_p^\dagger \hat{a}_p - \frac{\lambda}{V} \sum_{p,q,k} q \cdot k \hat{a}_p^\dagger \hat{a}_q^\dagger \hat{a}_{-q-k} \hat{a}_{-p+k}.
\] (1)

Here \(\hat{a}_p^\dagger\) and \(\hat{a}_p\) create and annihilate fermions of mass \(m\) with momentum \(p\), \(\lambda > 0\) denotes their interaction strength, and \(V\) is the volume of the system. In the following, we imagine fixing the coupling strength to some initial value \(\lambda = \lambda_1\), and preparing the system of atoms in the corresponding ground state. Then we suddenly change (quench) the coupling to a different value \(\lambda_f\). We then evaluate how the state of the fermions evolves in time after this quench.

We compute the dynamics of Eq. (1) within self-consistent BCS mean field theory, as governed by the Hamiltonian

\[
\hat{H}_{\text{eff}} = \sum_p \frac{p^2}{2m} \hat{a}_p^\dagger \hat{a}_p + \left[\Delta(t) \sum_p' p \hat{a}_p^\dagger \hat{a}_{-p} + \text{h.c.}\right].
\] (2)

Here the symbol \(\sum'\) signifies that the summation is restricted to \(p\) that satisfy \(p_x > 0\), and \(\Delta(t)\) is the amplitude of the gap function, defined as

\[
\Delta(t) = -\frac{2\lambda}{V} \sum_p' p \langle \hat{a}_{-p} \hat{a}_p \rangle.
\] (3)

The time-dependent state of the fermions is of the BCS form [18]

\[
|\Omega(t)\rangle = \prod_p' \left[u_p(t) + v_p(t) \hat{a}_p^\dagger \hat{a}_{-p}^\dagger\right] |0\rangle,
\] (4)

where \(|0\rangle\) is the vacuum, and Eqs. (2) and (3) reduce to nonlinear differential equations satisfied by \(u_p(t)\) and \(v_p(t)\). We solve these equations exactly, exploiting the integrability of the equations of motion [21–24, 29]. The solution employs a Lax spectral method. The analysis closely parallels work done for the corresponding s-wave problem in three dimensional space [18–25]. This was carried out in Ref. [17] and the results are shown in Fig. 1.

The mean field approach differs from Eq. (1) in three ways. First, the interaction terms in Eq. (1) with \(p \neq 0\) have been removed. This is a standard approximation in the theory of superconductivity: the only terms retained in the interaction are those responsible for the pairing of the fermions into Cooper pairs or strongly-bound molecules which then Bose condense. Since our goal is to predict dynamics from a given initial state, our results will hold over a time interval in which the effects of neglected terms remain small. This is the minimum of \(t_f\) and \(t_{ph}\), where \(t_{ph}\) is the pair-breaking lifetime induced by \(p \neq 0\) terms [18, 30].

Second, the mean field approach neglects quantum fluctuations in \(\Delta(t)\). Without pair-breaking terms, this is exact in the thermodynamic limit. It is well known [31, 32] that fluctuations induce only finite-size corrections. The reason is that \(\Delta(t)\) is a global, not merely a local mean field. It becomes macroscopic and classical if the number of fermions is sufficiently large and the system exhibits superconducting order (irrespective of equilibrium). Finally, we assume an initial state with \(p_x + ip_y\) symmetry. In mean field theory this “projects out” the \(p_x - ip_y\) channel, so that it does not participate in the dynamics. The change of variables \(\hat{a}_{-k}^\dagger \hat{a}_k \to e^{i\phi_k} \hat{a}_{-k}^\dagger \hat{a}_k\) leads to Eqs. (2)–(4) [17]: \(\phi_k\) is the polar angle.

\[\text{FIG. 1: Phase diagram showing the three regimes (I-III) of non-equilibrium superfluidity reached after a quantum quench in a p-wave gas [17]. Each point in this phase diagram represents a particular quench, wherein one takes an initial state with order parameter amplitude \(\Delta_i\), and subsequently ramps the strength of attractive atom-atom interactions to weaker or stronger pairing. The initial state is specified via the vertical axis. The horizontal axis measures \(\Delta_f\), which is the amplitude one would find in the ground state of the post-quench Hamiltonian. The diagonal line \(\Delta_i = \Delta_f\) is the case of no quench; \(\Lambda_{\text{QCP}}\) locates the BCS-BEC ground state transition [33]. Each off-diagonal point to the left (right) of this line denotes a particular quench from stronger-to-weak (weaker-to-stronger) pairing. The Regions labeled I, II, III denote three different regimes of non-equilibrium superfluid dynamics. For a strong-to-weak quench in I, the order parameter \(\Delta(t)\) decays to zero. A quench in II leads to a non-zero steady-state order parameter amplitude. A weak-to-strong quench in III induces persistent oscillations in \(\Delta(t)\).} \]
Let us now examine Region III, of particular interest here. In this case, the order parameter asymptotes to
\[
\Delta(t) = \Delta_\infty(t) e^{-2i\mu_\infty t},
\]
where \(\Delta_\infty(t)\) is a complex-valued periodic function of time with some period \(T\), and \(\mu_\infty\) is a real constant. These are completely determined by the particular quench specified by \(\{\Delta_i, \Delta_f\}\) [17, 33]. In general, \(T\) and \(\pi/\mu_\infty\) are incommensurate periods. By absorbing \(\mu_\infty\) into the phase of the operators \(\hat{a}_p\) and \(\hat{a}_p^\dagger\), we can map our effective Hamiltonian in Eq. (2) to a superconductor with an oscillatory complex-valued order parameter \(\Delta_\infty(t)\) and chemical potential \(\mu_\infty\). A useful quantity to characterize such a superconductor is its retarded Green’s function \(\mathcal{G}(t,t')\), defined as the solution of the matrix Bogoliubov-de Gennes equation [33]
\[
\frac{\partial \mathcal{G}}{\partial t} - \mathcal{H} = \delta(t-t'), \quad \mathcal{H} = \left( \frac{p^2}{2m} - \mu_\infty - \Delta_\infty e^{ip\hat{p}} - \frac{\Delta^*_{\infty} p e^{-ip\hat{p}}}{2m} + \mu_\infty \right),
\]
where \(\hat{p}\) is the angle \(p\) forms with the positive \(x\)-direction. This equation is identical to that for a driven superconductor with a gap function imposed to be a given function of time. We must still keep in mind that we are describing a strongly out-of-equilibrium state, with \(\Delta_\infty\) determined by the contributions of many fluctuating Cooper pair amplitudes such as \(\langle \hat{a} - \hat{p} \hat{a}_p^\dagger \rangle(t)\) in Eq. (3).

Interestingly, in Region II where \(\Delta_\infty\) is a constant, the corresponding BdG equations formally match those of an equilibrium superconductor. The equilibrium superconductor is characterized by a topological number \(W\), which depends solely on the sign of the chemical potential [5]. If the chemical potential is positive, \(W = 1\) and the system, while gapful in the bulk, is known to have gapless edge states. Ref. [34] argued that any retarded Green’s function with a topological number \(W = 1\) when computed in a geometry with a boundary will have poles corresponding to gapless excitations in the boundary. Therefore, even the far from equilibrium superconductor discussed here will have topologically protected edge states as long as \(\mu_\infty\) is positive [33]. The range of positive \(\mu_\infty\) is shown in Fig. 1 as a subregion of Region II with \(W = 1\).

Unitary time evolution is a smooth rotation of the initial state. It may therefore appear surprising that a quench can induce a change in a winding number within a finite time interval. In fact, one must distinguish two different notions of topology here. The topology of the state (pseudospin winding) does not change, but that of the effective single particle Hamiltonian can \((W,\) as defined via the retarded Green’s function in [33]). The Green’s function determines the frequency spectrum that appears when transitions are driven by an external probe, while the state encodes the occupation of the modes [17].

In Region III, \(\Delta_\infty(t)\) is a complex-valued periodic function of time that can be determined analytically [18, 21, 22]. The parameters of this function including its turning points and the period \(T\) are computed for a particular quench by solving a certain transcendental equation [17, 33]. A periodically driven system can be topological in the Floquet sense, as was recently discussed in the literature [15, 16, 27]. What this implies is that one needs to construct its Green’s function \(U(T) = \mathcal{G}(t+T,t)\) with \(t\) being arbitrary [but sufficiently large so that the large time asymptotic for \(\Delta(t)\) applies]. The edge states of this system are then the eigenstates of \(U(T)\), with their energy related to the eigenvalues of \(U(T)\). More precisely, the eigenvalues of \(U(T)\), a unitary operator, assume the form of \(\exp(-i\epsilon T)\), where \(\epsilon\) is such an energy level, taken to reside in the compact interval \([-\pi/T,\pi/T]\). These “quasi-energies” are similar to the crystalline quasi-momentum in systems periodic in space (while here the Hamiltonian is periodic in time).

It is possible to extract whether this system is topological by analyzing [28] the winding of \(\mathcal{G}(t,t')\). In practice, this may not be easy to do. Instead, given the analytic expression for the time-dependent \(\Delta_\infty(t)\) associated to a particular quench [17], we solved the Bogoliubov-de-Gennes equation numerically in the cylinder geometry (periodic in one direction, with a hard wall boundary in the other) [33]. After computing \(U(T)\) in this geometry, we extracted its eigenvalues and checked whether the edge states appear. By doing this at various points in Region III, we can map out the topological character of this dynamical phase. At the boundary of Regions III and II where \(\Delta_\infty\) becomes a constant, we know from Fig. 1 that the system is topological. Thus we expect that within the Region III close to the boundary with Region II, the topological aspects of the phase (the boundary states) remain, even though the winding number \(W\) may change, as was recently pointed out [28], while deep within Region III there might in principle be non-topological domains or domains with a different topology from that in II.

We find that edge states are present no matter where within Region III we look. Fig. 2 exhibits a typical spectrum for \(U(T)\) at a point deep within Region III, indicated in Fig. 1 as point A. This quench is located at \(\Delta_i/\Delta_{QCP} = 0.0065, \Delta_f/\Delta_{QCP} = 0.83\). To generate the plot shown in Fig 2, we placed the superfluid on the lattice, with 50 lattice constants within the width of the cylinder [33]. The hopping amplitude on this lattice was chosen to be \(J = 1/2\), so that the system would be below half filling, yet the total bandwidth was as small as possible to prevent the spectrum from folding too many times onto itself and obscuring the graph. Crossing edge states in the center of the figure prove that the time-dependent superfluid for this particular quench is topological in the Floquet sense. We conjecture that the entire Region III is topological, but proving this requires further work.

A natural quench from the point of view of experiment would start from the non-interacting Fermi gas at \(\Delta_i = 0\). Such a quench is much harder to describe than those
FIG. 2: Majorana edge modes for a quench-induced time-dependent state of p-wave superfluidity. The Floquet spectrum (top) of ln\(U(T)\) for a system on a finite cylinder is plotted in the large time asymptotic regime for a quench in Region III, point “A” in Fig. 1. The horizontal axis represents the momentum along the boundary, the vertical axis the momentum parities multiplied by the period of oscillations, both ranging from \(-\pi\) to \(\pi\). The edge states can be clearly seen crossing in the center of the figure. The bottom left shows the orbit swept by \(\Delta_\infty(t)\) in the complex \(\Delta\) plane at this point in Region III. The bottom right plots the orbital maximum \(\Delta_{\text{max}} \equiv \max(\Delta(t))\) as a function of \(\Delta_f\) for quenches from very weak initial pairing (\(\Delta_i \to 0\)) in III. The dashed line is \(\Delta_{\text{max}} = \Delta_f\).

studies here so far. Technically, the zero-temperature Fermi-Dirac distribution is a point of unstable equilibrium for the classical equations of motion studied above, so naively it does not evolve in time. In reality, quantum or thermal fluctuations will generate an initial condition with nonzero \(p_x+i p_y\) and \(p_x-ip_y\) order parameter amplitudes, and these will compete in the subsequent dynamics. The precise outcome is difficult to predict. Instead, we assume that it is possible to first switch on very weak attraction which results in some initial very small yet nonzero \(\Delta_i\) of pure \(p_x+i p_y\) type. Then we quench this state into a far stronger interacting regime; as long as the quench resides within Region III we expect the resulting state to be in the topological Floquet phase. (Point A where \(\Delta_i \ll \Delta_f\) is a good example of such a quench.) At the same time, if the interactions after the quench are stronger than the threshold depicted Fig. 1, we will end up in the non-topological \((W = 0)\) domain of Region II.

We conclude with a discussion of relevant time scales. The main effect of the \(p \neq 0\) terms in Eq. (1) is to mediate pair-breaking collisions [18, 30], associated to a rate \(1/t_{pb}\). For quenches confined to the BCS regime, the lifetime can be estimated using Fermi liquid theory [24, 30, 35], leading to \(t_{pb}/t_F \sim \left[\frac{\epsilon_F}{E_{\text{min}}(\Delta_f)}\right]^2\), where \(t_F = 1/\epsilon_F\) is the inverse Fermi energy and \(E_{\text{min}}(\Delta) = \Delta \sqrt{2\mu - \Delta^2}\) is the ground state quasiparticle energy gap. Quenches in Region III that produce topological Floquet states reside entirely within the BCS regime; for these, the ratio \(t_{pb}/t_F\) can easily be an order of magnitude, and grows rapidly larger as \(\Delta_f\) is reduced. The inverse three-body loss rate can be estimated to be \(t_3/t_F \sim (\ell/b)^\alpha\), where \(\ell \sim 1000\) nm \((b \sim 5\) nm\) is the interparticle separation (Van der Waals length) [10, 11]. Near resonance, the exponent \(\alpha = 1\), with \(t_3 \approx 20\) ms in experiments [13, 14]. However, towards the weak BCS regime \(t_3\) becomes orders of magnitude larger with \(\alpha = 4\) [10], making three-body losses essentially irrelevant for the creation of a weakly paired initial state.

In numerical simulations of our model [17], we find that the asymptotic behavior is reached very quickly in Region III over a time \(t \lesssim t_F\). For quenches from weak initial pairing with \(\Delta_i \ll \Delta_f\), the period \(T\) of oscillations in the order parameter magnitude can be estimated as [17, 33]

\[
T \sim \frac{2}{E_{\text{min}}(\Delta_f)} \ln \left[\frac{4\epsilon_F E_{\text{min}}(\Delta_f)}{\Lambda E_{\text{min}}(\Delta_i)}\right] \sim \sqrt{t_F t_{pb}},
\]

where \(\Lambda\) is an ultraviolet energy cutoff. In the BCS regime we always have \(\epsilon_F > 2E_{\text{min}}\), so that

\[
t_F < T < \min(t_{pb}, t_3),
\]

where \(t_3\) is associated to the (larger) post-quench coupling strength. This is the window in which the topological non-equilibrium steady state can be realized. Decreasing the final pairing strength \([\text{increasing } \epsilon_F / E_{\text{min}}(\Delta_f)]\) may increase the relative size of the window, at the cost of decreasing the detectable maximum of \(|\Delta(t)|\) (see Fig. 2).

Quench-induced topological edge states could be detected using RF spectroscopy type experiments. An open question is whether these types of topological steady states support the kind of quantized thermal conductance expected in an equilibrium topological p-wave superconductor. Calculating energy transport and exploring possible quantized out-of-equilibrium transport phenomena remains the subject for future work.

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winding number \( W \) in terms of \( G \), and the details of the Floquet calculation.

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SUPPLEMENTAL MATERIAL

SCALES AND UNITS

We introduce the interaction parameter \( g \equiv \lambda m/2 \), which has units of length-squared. After dropping pair-breaking (\( p \neq 0 \)) terms, the Hamiltonian in Eq. (1) becomes

\[
\hat{H} = \frac{1}{m} \sum_{p} \frac{p^2}{2} \hat{a}_p^\dagger \hat{a}_p - \frac{g}{V} \sum_{q,k} \left[ (q^x + iq^y)(k^x - ik^y) + c.c. \right] \hat{a}_{q+k}^\dagger \hat{a}_{-q-k} \hat{a}_k \hat{a}_{q+k},
\]  

(9)

where c.c. denotes the complex conjugate. We set \( m = 1 \), since it merely specifies the units for the Hamiltonian. For a \( p_x + i p_y \) superfluid with order parameter amplitude \( \Delta \), the energy of a bulk quasiparticle excitation with momentum \( p \) is

\[
E_p = \sqrt{(p^2/2 - \mu)^2 + \Delta^2 p^2},
\]  

(10)

where \( \mu \) is the chemical potential. \( \Delta \) carries inverse length units.

The BCS mean field equations relating the ground state \( \Delta \) and \( \mu \) to the coupling strength \( g \) and fixed particle density \( n \) are given by [1]

\[
\mu = \frac{1}{g} \ln \left( \frac{2 \Lambda e}{\Delta^2} \right), \quad \mu = 2 \pi n - \frac{\Delta^2}{2} \ln \left( \frac{2 \Lambda}{e \Delta^2} \right),
\]  

(11)

where \( g_{\text{QCP}} \) denotes the coupling strength at the BCS-BEC transition \( \mu = 0 \).

\[
\frac{1}{g_{\text{QCP}}} = \frac{\Lambda}{\pi} - 4 n.
\]

Eq. (11) applies to the BCS side with \( \mu \geq 0 \); \( \Lambda \) is an ultraviolet energy cutoff. These equations are valid up to corrections of order \( 1/\Lambda \).

The natural scale for \( \Delta \) is the value at the BCS-BEC quantum phase transition [1],

\[
\Delta_{\text{QCP}} \approx \sqrt{\ln \left( \frac{4 \pi n}{2 \pi n e} \right) + \ln \left( \frac{\Delta}{2 \pi n e} \right)}.
\]  

(12)
RETARDED GREEN’S FUNCTION

The matrix retarded Green’s function $G(t, t')$ in Eq. (6) is defined as follows:

$$G(t, t') = -i \left[ \{\hat{a}_p(t), \hat{a}_p^\dagger(t')\} \{\hat{a}_p(t), \hat{a}_p^\dagger(t')\} \right] \times \theta(t - t'),$$

(13)

where $\{A, B\} \equiv AB + BA$ is the anticommutator.

Following a quench at time $t = 0$, when $\Delta(t) \to \Delta_\infty$ (constant, Region II in Fig. 1 of the main text), one has $G(t, t') = G(t - t')$ for sufficiently large times $t$ and $t'$. Then Eq. (6) is identical to its equilibrium counterpart. This is true despite the fact that the post-quench coupling constant $\lambda_f$ is by no means equal to that needed to produce $\Delta_\infty$ and $\mu_\infty$ in equilibrium [1]. To determine the presence or absence of Majorana edge modes in the strip geometry, we compute the bulk winding number [2–4]

$$W \equiv \frac{\epsilon_0 \beta}{3!} \int_{-\infty}^{\infty} d\omega \int \frac{d^2p}{(2\pi)^2} \text{tr} \left[ G^{-1}(\partial_\omega G)^{-1}(\partial_p G) G^{-1}(\partial_\omega G) \right],$$

(14)

where $G \equiv G_p(i\omega)$ is the Fourier transform of $G_p(t - t')$, continued to imaginary frequency. Edge modes are signaled by $W = 1$, while quenches that produce trivial states have $W = 0$. These are shown in Fig. 1.

REGION III DYNAMICS

The exact solution in [1] gives the asymptotic $\Delta(t)$ in Region III in terms of amplitude and phase variables,

$$\Delta(t) \equiv \sqrt{R} \exp(-i\phi).$$

These satisfy

$$\dot{\phi} = \frac{3}{2} \frac{R + m}{R}, \quad \dot{R} = (R_+ - R)(R - R_-)(R + R_+)(R + R_-),$$

(15)

The parameters in these equations can be expressed as

$$R_\pm = \frac{1}{2} \left[ \sqrt{|u_{1,2}|^2} \pm \sqrt{|u_{2,1}|^2} \sqrt{u_{1,2}^2 - u_{2,1}^2} \right],$$

(16)

$$\bar{R}_\pm = \frac{1}{2} \left[ \sqrt{|u_{1,2}|^2} \pm \sqrt{|u_{2,1}|^2} \sqrt{u_{1,2}^2 - u_{2,1}^2} \right],$$

(17)

$$m = \frac{1}{4} \left( u_{1,2} + u_{2,1} \right),$$

(18)

$$\psi = \frac{1}{8} \left[ -(u_{1,2}^2 + u_{2,1}^2) - 2u_{1,2}u_{2,1} + 2|u_{1,2}|^2 \right].$$

(19)

where $|u_{1,2}| = \sqrt{u_{1,2}^2 + u_{2,1}^2}$. The four parameters $u_{1,2}$ and $u_{2,1}$ locate the two isolated pairs of roots of the spectral polynomial for a quench in Region III [1]. These are uniquely determined by solving a certain transcendental equation for a particular quench specified by $(\Delta_1, \Delta_f)$ in III.

The period of $\Delta(t)$ is given by [1]

$$T = \frac{2}{\alpha} K \left( \frac{u_{1,2}u_{2,1}}{\alpha^2} \right),$$

$$\alpha = \frac{1}{2} \sqrt{(u_{1,2} - u_{2,1})^2 + (u_{1,2} + u_{2,1})^2},$$

(20)

where $K(M)$ is the complete elliptic integral of the first kind (and $M = k^2$).

LATTICE REGULARIZATION FOR FLOQUET

To obtain the Floquet spectrum in Fig. 2 for the quench labeled “A” in Fig. 1, we employed the following procedure. First, we computed the parameters in Eq. (16) from the isolated roots for the quench [1]. This determines the function $\Delta_\infty(t)$ and the real constant $\mu_\infty$ in Eq. (5). The orbit of the resulting $\Delta_\infty(t)$ is plotted in the lower left of Fig. 2. Here we have used the high-energy cutoff $\Lambda = 100\pi$ to fix $\Delta_{\text{QCP}}$.

We then diagonalized the Floquet operator $U(T)$ using the following lattice regularization of the $p_x + i p_y$ Bogoliubov-de Gennes Hamiltonian:

$$H = \sum_{m=1}^{M} \sum_{n=-\infty}^{\infty} \left[ -J c_{m,n}^\dagger (c_{m+1,n} + c_{m-1,n} + c_{m,n+1} + c_{m,n-1}) + (4J - \mu_\infty) c_{m,n} c_{m,n} \right]$$

$$+ \frac{i\Delta_\infty(t)}{4} \sum_{m=1}^{M} \sum_{n=-\infty}^{\infty} c_{m,n} \left( c_{m+1,n} - c_{m+1,n-1} + i c_{m,n+1} + i c_{m,n-1} \right) + \text{h.c.}$$

(21)

Here h.c. denotes the Hermitian conjugate.

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