Abstract: We discuss semileptonic and leptonic $B$ decays, $B \to D(\ast)\tau\nu$ and $B \to \tau\nu$, in the chiral $U(1)'$ models which were proposed by the present authors in the context of the top forward-backward asymmetry ($A_{\text{FB}}^t$) observed at the Tevatron. In these models, extra Higgs doublets with nonzero $U(1)'$ charges are required in order to make the realistic mass matrix for up-type quarks. Then the extra (pseudo)scalars contribute to $A_{\text{FB}}^t$ with large flavor-changing Yukawa couplings involving top quark. The contribution of the charged Higgs to $A_{\text{FB}}^t$ is negligible, but it may significantly affect $B$ decays: especially, $B \to D(\ast)\tau\nu$ and $B \to \tau\nu$. We investigate constraints on the $B$ decays, based on the recent results in BaBar and Belle experiments, and discuss the possibility that the allowed parameter region in the $B$ decays can achieve large $A_{\text{FB}}^t$. 

$B \to D(\ast)\tau\nu$ and $B \to \tau\nu$ in chiral $U(1)'$ models with flavored multi Higgs doublets

P. Ko$^a$ Yuji Omura$^b$ Chaehyun Yu$^{a,c}$

$^a$School of Physics, KIAS, Seoul 130-722, Korea
$^b$Physik Department T30, Technische Universität München,
James-Franck-Straße, 85748 Garching, Germany
$^c$SLAC National Accelerator Laboratory,
2575 Sand Hill Rd, Menlo Park, CA 94025, USA

E-mail: pko@kias.re.kr, yuji.omura@tum.de, chyu@kias.re.kr
1 Introduction

Recently, the BaBar collaboration analyzed semileptonic $B$ decays, $B \to D\tau \nu$ and $B \to D^*\tau \nu$ [1], and investigated the ratios of the branching ratios for $B \to D^{(*)}\tau \nu$ to those for $B \to D^{(*)}l \nu$ ($l = e, \mu$),

$$R(D^{(*)}) = \frac{\text{BR}(B \to D^{(*)}\tau \nu)}{\text{BR}(B \to D^{(*)}l \nu)}.$$  (1.1)

The results are $R(D) = 0.440 \pm 0.072$ and $R(D^*) = 0.332 \pm 0.030$ which deviate from the Standard Model (SM) predictions, $R(D)_{\text{SM}} = 0.297\pm0.017$ [2] and $R(D^*)_{\text{SM}} = 0.252\pm0.003$ [3] by 2.2σ and 2.7σ, respectively [4]. The combined discrepancy is about 3.4σ [1], which might be an evidence of new physics as discussed in some recent works [4–10]. One good candidate for new physics for this anomaly is a charged Higgs boson in the extended SM with extra Higgs doublets [4, 5].

On the other hand, a leptonic $B$ decay $B \to \tau \nu$ was measured at BaBar [11] and Belle [12]. The average of the branching ratios is $\text{BR}(B \to \tau \nu) = (1.67 \pm 0.3) \times 10^{-4}$ [13]. In the SM calculation, there are some uncertainties from $|V_{ub}|$ and the $B$ meson decay constant $f_B$. Still the measured number is slightly inconsistent with the SM prediction, for example, given by UTfit Collaboration, $\text{BR}(B \to \tau \nu)_{\text{SM}} = (0.84 \pm 0.11) \times 10^{-4}$ [14]. The Belle experiment recently presented a new result, $\text{BR}(B \to \tau \nu) = 0.72^{+0.27}_{-0.25} \pm 0.11$ by making use of a hadronic tagging method for $\tau$ decays with the full data sample [15], and
the combined average is consistent with the SM prediction. Since both semileptonic and leptonic $B$ decays may be affected by the same new physics (e.g., charged Higgs boson), such new physics scenario will be strongly constrained by combined analysis of the $B$ decays.

An interesting point is that it is difficult for the so-called type-II two-Higgs-doublet model (2HDM), which could be well motivated by minimal supersymmetric SM (MSSM), to explain the discrepancies in $R(D)$ and $R(D^*)$ \([1]\). Other types of 2HDMs with natural flavor conservation, where Yukawa couplings are controlled by $Z_2$ \([16]\) or $U(1)_H$ \([17]\) symmetry, also allow only the so-called minimal flavor violation (MFV) in the charged Higgs sector, and it is impossible to accommodate $R(D)$ and $R(D^*)$ at BaBar simultaneously \([4, 5, 9, 10]\). Eventually, we would have to introduce non-minimal-flavor-violating (non-MFV) terms in order to explain the $R(D)$ and $R(D^*)$ data, which would tend to generate too large flavor changing neutral currents (FCNCs). It would be highly nontrivial to introduce non-MFV interactions in the Yukawa couplings in 2HDMs without too excessive flavor violations in the $K$ and $B$ meson sectors.

In ref. \([18, 19]\), the present authors proposed flavor models with extra Higgs fields, where gauged $U(1)'$ symmetry forbids the potentially problematic FCNCs for $B$-$\bar{B}$ and $K$-$\bar{K}$ mixings but allows certain amounts of FCNCs which are still consistent with experimental data, by slightly breaking the criteria of ref. \([17]\). There, neutral CP-even scalars and a CP-odd scalar can have large $(t, q)_{\ell=u,c,t}$ elements of Yukawa couplings. Their tree-level mediations enhance the top forward-backward asymmetry ($A_{FB}$) at the Tevatron, while accommodating with the newest strong constraints from LHC thanks to the destructive interference among the scalars \([18–21]\). In previous works, phenomenology of the charged Higgs boson in our models was not considered carefully because the charge Higgs boson cannot have sizable contribution to the top quark production at hadron colliders. However, couplings of the charged Higgs boson to the bottom quark may be large so that the models could be strongly constrained by the $B$ decays. For example, the $(b, u)$ element of the charged Higgs coupling, which is constrained by $B \rightarrow \tau \nu$, may become large if the $(t, u)$ element of the Yukawa coupling of the pseudoscalar boson is large. Besides, the $(b, c)$ element, which modifies the branching ratios of $B \rightarrow D^{(*)} \tau \nu$, could be large, if the $(t, c)$ element of the pseudoscalar Yukawa coupling is large (see eq. (3.6), for example). In this paper, we first investigate if our $U(1)'$ flavor models can explain the discrepancies in the $B$ semileptonic decays while keeping consistency with $\text{BR}(B \rightarrow \tau \nu)$. Then we discuss the possibility that the allowed parameter regions can achieve the large enough $A_{FB}$ which was observed at the Tevatron.

This paper is organized as follows. In section 2, we give a brief review of our models proposed in ref. \([18, 19]\). We assign family dependent $U(1)'$ charges to the right-handed (RH) up-type quarks in order to generate flavor changing $Z' - u_R - t_R$ couplings in the mass eigenstates. Then we will immediately realize that it is mandatory to introduce extra $U(1)'$-charged Higgs doublets in order to write Yukawa couplings for the up-type quarks including top quark, which have been first realized in ref. \([18, 19]\). Thus we are led to multi-Higgs doublet models in the presence of a new spin-1 $Z'$ boson with chiral couplings to the SM fermions. Sections 3 and 4 are devoted to discussion of the phenomenology.
for $B \rightarrow D^{(*)}\tau\nu$ and $B \rightarrow \tau\nu$ in our 2HDM and three-Higgs-doublet model (3HDM), respectively. We also give comments on the constraints on the charged Higgs boson from the $B \rightarrow X_s\gamma$ process in section 5. Finally, we summarize our results in section 6. The general Higgs potential in multi Higgs doublet models is discussed in appendix A.

2 Models with extra Higgs and gauged flavor $U(1)'$

Adding extra Higgs doublets to the SM is one of the interesting extensions to the SM. However such an extension generically suffers from neutral Higgs-mediated FCNC problem at tree level, if up- and down-type quark masses get contributions from the vacuum expectation values (VEVs) of more than one Higgs doublets. For example, when the mass matrix of up-type quarks ($M^u_{ij}$) depends on two Higgs doublets, ($H_1, H_2$), as

$$(M^u)_{ij} = (y_1)_{ij}\langle H_1 \rangle + (y_2)_{ij}\langle H_2 \rangle,$$

all scalar components of ($H_1, H_2$) would have flavor-dependent couplings. This is because $(y_1)_{ij}$ and $(y_2)_{ij}$ cannot be diagonalized simultaneously without any relation between $(y_1)_{ij}$ and $(y_2)_{ij}$. A simple way to control the flavor structures of Yukawa couplings is to assign a symmetry to extra Higgs and matter fields. The most popular symmetry is an extra $Z_2$ symmetry which is softly broken [16]. In ref. [17–19], gauged $U(1)'$ symmetry is assigned to the SM fermions and newly introduced extra Higgs doublets instead of $Z_2$ symmetry. Especially, in ref. [18, 19], only right-handed (RH) up-type quarks are charged flavor-dependently under $U(1)'$, and FCNCs involving only top quark can be enhanced. The authors constructed both 2HDM and 3HDM depending on the $U(1)'$ charge assignments to the SM fermions.

When only the RH up-type quarks are charged in a flavor dependent way, the extra Higgs doublets charged under $U(1)'$ couple with the RH up-type quarks in the form

$$V_y = y_{1j}^u Q_j^H H_j U_{Rj} + y_{1j}^c Q_j^H H_2 D_{Rj} + y_{1j}^t \overline{L}_j H_2 E_{Rj} + h.c.. \quad (2.1)$$

Here $H_j (j = u, c, t)$ are charged under $U(1)'$, but $H_2$ is neutral under $U(1)'$ and has the same quantum numbers as the SM Higgs doublet. The SM leptons and the down-type quarks get masses from VEV of $H_2$. Note that the up-type quarks cannot have gauge invariant mass terms without new $U(1)'^{'}$-charged Higgs doublets $H_j$.

In general, there may be up to four Higgs doublets: $H_2$ and $H_{u,c,t}$. The actual number of Higgs doublets will depend on the $U(1)'$ charge assignment. Motivated by the $A_{FB}$ at the Tevatron, we use the charge assignment $(u_j) = (0, 0, 1)$ on the RH up-type quarks in the 2HDM, where we identify $H_u$ and $H_c$ as $H_2$ and $H_t$ as $H_1$, respectively. In the 3HDM, we use the charge assignment $(u_j) = (-1, 0, 1)$ and we identify $H_u$ as $H_1$, $H_c$ as $H_2$, and $H_t$ as $H_3$, respectively.* In addition to the extra Higgs doublets $H_j$, a SM singlet scalar with nonzero $U(1)'$ charge, $\Phi$, may exist in order to break $U(1)'$. In general, its CP-even component mixes with CP-even components of $H_j$ after electroweak and $U(1)'$ symmetry breaking.

*There could be other assignment of $(u_j)$ but we consider only these two cases for simplicity.
In the following two sections, we will concentrate on these 2HDM and 3HDM. Then we investigate the impact of the charged Higgs boson sector on the (semi)leptonic $B$ decays and discuss the results in the context of the $A_{FB}$ at the Tevatron.

3 2HDM

In this section, we consider the chiral $U(1)'$ flavor model with the charge assignment $(u_i) = (0,0,1)$ in the interaction eigenstates. There are a neutral Higgs doublet, $H_2$, and $U(1)'$-charged Higgs doublet, $H_1$, as we discussed in section 2.

3.1 Yukawa couplings

In the 2HDM case, there are three CP-even neutral scalars ($h_1, h_2, h_3$), one CP-odd pseudoscalar ($a$), and one charged Higgs pair ($h^\pm$) after electroweak and $U(1)'$ symmetry breaking. Note that there is one more CP-even scalar from the $U(1)'$-charged singlet scalar $\Phi$ compared with the usual 2HDM.

Let us define the Yukawa couplings for the neutral scalar bosons in the mass bases as follows:

$$ V_{y} = Y_{ij}^{u(k)}(k)h_L^i \bar{u}^j - iY_{ij}^{au}(k)\bar{u}^i_L u^j_R + h.c. \quad (3.1) $$

$Y_{ij}^{u(k),au}$ depend on the Higgs VEVs and the diagonalizing matrices for the quark mass matrices. The Yukawa couplings can be derived in a straightforward manner. For example, the Yukawa couplings of the lightest CP-even scalar boson $h_1$ and pseudoscalar boson $a$ are given by

$$ Y_{ij}^{u(1)} = \frac{m^u_i \cos \alpha}{v \cos \beta} \cos \alpha \Phi \delta_{ij} + \frac{2m^u_i}{v \sin \beta} (g_R^u)_{ij} \sin(\alpha - \beta) \cos \alpha \Phi, \quad (3.2) $$

$$ Y_{ij}^{au} = \frac{m^u_i \tan \beta}{v} \delta_{ij} - \frac{2m^u_i}{v \sin \beta} (g_R^u)_{ij}. \quad (3.3) $$

The parameters $v$ and $\beta$ are defined by $(\langle H_1 \rangle, \langle H_2 \rangle) = (v \cos \beta / \sqrt{2}, v \sin \beta / \sqrt{2})$, and $\alpha$ and $\alpha \Phi$ are the mixing parameters among the 3 CP-even neutral scalars.

In the above equations, the mixing matrix $g_R^u$ is defined as

$$ (g_R^u)^{ij}_{kl} = (R_u)^{ik}_{jl}(R_u)^{kj}_{il} \quad (3.4) $$

where the matrix $(R_u)_{ij}$ is defined by

$$ (M^u_u M_u)_{ij} = (R_u)^{ik}_{jl}(m^u_k)_{lj}(R_u)_{kj}. $$

For the 2HDM case with $(u_k) = (0,0,1)$, $g_R^u$ is reduced to

$$ (g_R^u)_{ij} = (g_R^u)^*_{ji} = (R_u)^{i3}(R_u)^{j3}. $$

Note that the neutral Higgs bosons have flavor-dependent couplings to the up-type quarks (see the second terms in eq. (3.2) and (3.3)). The flavor-dependent couplings of $Z'$ which

\[ ^1 \text{We use the same notations as in ref. [18, 19].} \]
is the gauge boson of $U(1)'$ are also linear in $(g_{R}^{u})_{ij}$ in our 2HDM [18, 19]. These flavor-dependent couplings make additional contributions to $A_{FB}^{t}$ at the Tevatron, $t\bar{t}$ cross section, and the same-sign top-quark pair production cross section. Recently, the CMS collaboration has announced stringent bounds on the cross section for the same-sign top-quark pair production: $\sigma^{tt} \leq 0.39\text{ pb}$ at 95% confidence level [22, 23]. This strong bound excludes simple scenarios such as the original $Z'$ model that only one mediator contribution is taken into account. However, flavor models usually have several mediators to couple with the SM particles flavor-dependently, and they interfere with each other. In fact, the pseudoscalar, CP-even scalars, and $Z'$ have destructive interference in our $U(1)'$ flavor models and we could find the points evading the stringent upper bound on the same-sign top-quark pair production while enhancing $A_{FB}^{t}$ [18–21].

The origin of this non-MFV couplings in our models is the flavor-dependent $U(1)'$ interactions of $Z'$. (Recall that $(g_{R}^{u})_{ij} \propto \delta_{ij}$ if the $U(1)'$ were flavor-independent, i.e., $(u_{k}) \propto (1, 1, 1)$.) Thus our 2HDM is a nice realization of non-MFV models where the flavor non-universality has its origin in the flavor-dependent $U(1)'$ gauge interactions. The amount of flavor non-universality is related with the local gauge symmetry and its spontaneous breaking. It is worthwhile to emphasize that the nature of flavor non-universality is neither completely arbitrary nor ad hoc. These are very unique features of the 2HDM (and 3HDM described in the next section) we have proposed in ref. [18, 19].

Similarly to the neutral Higgs Yukawa couplings, the charged Higgs Yukawa couplings are defined by

$$V_{c} = -Y_{ij}^{-u} h^{-d_{Li}} u_{Rj} + Y_{ij}^{+d} h^{+d_{Li}} d_{Rj} + Y_{ij}^{+l} h^{+l_{Li}} e_{Rj} + h.c.. \hspace{1cm} (3.5)$$

There are definite relations between Yukawa couplings of the pseudoscalar boson $(a)$ and those of the charged Higgs bosons $(h^{\pm})$:

$$Y_{ij}^{-u} = \sum_{l} V_{il}^{-u} Y_{lj}^{a} \sqrt{2},$$

$$Y_{ij}^{+d} = \sum_{l} V_{il}^{+d} Y_{lj}^{a} \sqrt{2},$$

$$Y_{ij}^{+e} = \sum_{l} (V_{PMNS})_{il}^{+e} Y_{lj}^{a} \sqrt{2}, \hspace{1cm} (3.6)$$

where $V_{ij}$ is the CKM matrix and $(V_{PMNS})_{ij}$ is the Pontecorvo-Maki-Nakagawa-Sakata matrix.

In order to accommodate the large $A_{FB}^{t}$ observed at the Tevatron, large flavor-changing Yukawa couplings of the pseudoscalar are inevitable [18, 19]. As the authors pointed out in ref. [18–21], the pseudoscalar contribution to the same-sign top-quark pair production has destructive interference with other neutral scalar and $Z'$ contributions. Once we consider a large FCNC in the pseudoscalar Yukawa couplings in the up-type quark sector in order to enhance the $A_{FB}^{t}$, it is mandatory to have large flavor-changing couplings in the charged Higgs sector, which would affect various phenomenology including the $B$ meson system.

In the following, we will discuss phenomenology of the charged Higgs boson.
3.2 \( R(D^{(*)}) \) and \( BR(B \to \tau \nu) \) in 2HDM

The charged Higgs boson in our model will contribute to the (semi)leptonic \( B \) decays and would modify the SM lepton universality which is a result of the \( W^\pm \) contributions derived from the underlying \( SU(2)_L \) gauge theory. Since the charged Higgs contributions are proportional to the final lepton mass, we consider the \( h^\pm \) contributions only to \( B \to D^{(*)}\tau \nu \) and \( B \to \tau \nu \), and use the SM predictions for other leptonic channels.

In the effective Hamiltonian approach, the semileptonic decays \( B \to q \tau \nu \) and leptonic decay \( B \to \tau \nu \) are described by the effective Hamiltonian [4],

\[
H_{\text{eff}} = C_{SM}^{q} (\overline{q} \gamma_\mu b_L)(\tau_L \gamma^\mu \nu_L) + C_{R}^{q} (\overline{q} L b_R)(\tau_R \nu_L) + C_{L}^{q} (\overline{q} R b_L)(\tau_R \nu_L),
\]

where \( q = u, c \) is the up-type quark flavor. In the above equation, \( C_{SM}^{q} \) is the Wilson coefficient for the \( W \) exchange in the SM and \( C_{R}^{q}, C_{L}^{q} \) are those for the charged Higgs exchange present in our models. \( R(D^{(*)}) \) and \( BR(B \to \tau \nu) \) are given by the following expressions depending on the above Wilson coefficients [4]:

\[
R(D) = R_{SM}^{(*)} \left( 1 + 1.5 \ Re \left( \frac{C_{R}^{q} + C_{L}^{q}}{C_{SM}^{q}} \right) + \left| \frac{C_{R}^{q} + C_{L}^{q}}{C_{SM}^{q}} \right|^2 \right),
\]

\[
R(D^{*}) = R_{SM}^{(*)} \left( 1 + 0.12 \ Re \left( \frac{C_{R}^{q} - C_{L}^{q}}{C_{SM}^{q}} \right) + 0.05 \left| \frac{C_{R}^{q} - C_{L}^{q}}{C_{SM}^{q}} \right|^2 \right),
\]

\[
BR(B \to \tau \nu) = \frac{G_F^2 |V_{ub}|^2 m_B^2 f_B^2 m_t \tan \beta \left( 1 - \frac{m_\tau^2}{m_b^2} \right)^2}{8 \pi} \left[ 1 + \frac{m_B^2}{m_b^2} \cos^2 \beta \left( \frac{C_{R}^{q} - C_{L}^{q}}{C_{SM}^{q}} \right)^2 \right] (3.10)
\]

where each Wilson coefficient is at the \( B \) meson scale [6]. Here, \( R_{SM}^{(*)} \) are given by \( BR(B \to D^{(*)}\tau \nu)/BR(B \to D^{(*)}\nu \nu) \) in the SM.

Integrating out the heavy degrees of freedoms \( (h^\pm) \) in our 2HDM, we can obtain the Wilson coefficients at the charged Higgs mass scale:

\[
C_{SM}^{q} = 2 V_{ub} (V_{PMNS})^* \frac{s_{\tau}}{v^2}, \quad C_{R}^{q} = m_q m_\tau \tan^2 \beta - \sum_l \frac{V_{ub} m_q^u m_\tau (g_{R}^u)_{lq}}{V_{qb} m^2 m_\tau \cos^2 \beta}, \quad C_{L}^{q} = - \frac{m_q m_\tau}{m^2} \tan^2 \beta.
\]

(3.11) (3.12) (3.13)

We note that \( C_{R}^{q}/C_{SM}^{q} \) is flavor-blind, but \( C_{L}^{q}/C_{SM}^{q} \) depends on the flavor, \( q = u \) or \( c \). If \( (g_{R}^u)_{ij} = \delta_{ij} \) were satisfied, \( C_{L,R}^{q} \) would be the same as the type-II 2HDM and one could not accommodate the \( R(D) \) and \( R(D^{*}) \) simultaneously. However in our flavor-dependent \( U(1)' \) models, we have \( (g_{R}^u)_{ij} \neq \delta_{ij} \) and there appears a new term in eq. (3.12) which is absent in the type-II 2HDM. This new term may give rise to a possibility to accommodate \( R(D) \) and \( R(D^{*}) \) unlike the type-II 2HDMs. As discussed in the previous subsection, we could generate this non-MFV interactions from flavor-dependent \( U(1)' \) gauge couplings, which are very interesting aspects of our multi-Higgs doublet models with flavor-dependent \( U(1)' \) gauge interactions.
The contribution of the charged Higgs boson to the semileptonic $B$ decays, $B \to D^{(*)} \tau \nu$, is controlled by $C_{L,R}^{ub}$ while that to the leptonic $B$ decay, $B \to \tau \nu$, is affected by $C_{L,R}^{ub}$. As discussed in ref. [1, 4-7], it is impossible to explain $R(D^{(*)})$ and $BR(B \to \tau \nu)$ simultaneously within the 2HDMs with MFV, where the Yukawa couplings are fixed by the angles $\alpha$ and $\beta$ up to the quark mass. However, if $C_{L,R}^{ub}$ and $C_{L,R}^{ub}$ are mutually independent as in our models, both the measured values of $R(D^{(*)})$ and $BR(B \to \tau \nu)$ might be explained by taking appropriate parameters for the Wilson coefficients [4].

In ref. [18, 19], the present authors suggested that large $Y_{tu}^{au}$ is required in order to achieve the large $A_{FB}^u$ and evade the strong bound from the same-sign top-quark pair production signal. Such large $Y_{tu}^{au}$ can be realized by large $(g_{R}^{u})_{tu}$ and small $\sin 2\beta$, but it leads to large $Y_{bu}^{-u}$ according to eq. (3.6) in our 2HDM. Therefore the charged Higgs contributions to the (semi)leptonic $B$ decays could be too large, and have to be studied carefully.

In numerical analysis for (semi)leptonic $B$ decays in our 2HDMs, we take the following parameter regions: $1 \leq \tan \beta \leq 100$, $200$ GeV $\leq m_{h^{+}} \leq 1$ TeV, and $0 \leq |(g_{R}^{u})_{tu}|, |(g_{R}^{d})_{tc}| \leq 1$, respectively. We use the input parameters in the SM which are given by the global fit [24].

First, we consider only the $B \to \tau \nu$ decay. In figure 1 (a), we show constraints on $|C_{L}^{ub}/C_{SM}^{ub}|$ and $|C_{R}^{ub}/C_{SM}^{ub}|$. The red points are allowed by the combined data by Heavy Flavor Average Group (HFAG) [13] without the recent Belle data in the 1$\sigma$ level. The blue points are consistent with the recent Belle data on $B \to \tau \nu$ in the 1$\sigma$ level [15]. The SM point of $C_{L}^{ub}/C_{SM}^{ub} = C_{R}^{ub}/C_{SM}^{ub} = 0$ is slightly deviated from the red region, but it is in good agreement with the recent Belle data. If the new Belle data is combined together with the old data, the small discrepancy in $BR(B \to \tau \nu)$ might disappear. Then, $BR(B \to \tau \nu)$ will give a strong constraint on the parameters related with charged Higgs boson. In figure 1 (b), we depict the allowed regions in our 2HDM for $m_{h^{+}}$ and $|Y_{tu}^{au}|$, with both being scaled by $\tan \beta$. The red and blue regions are consistent with the combined data and new Belle data, respectively.\footnote{By using the averaged value of the combined data by HFAG and the new Belle data for $BR(B \to \tau \nu)$, one may obtain similar results. But there is no official averaged value up to now [25] and we considered the new Belle data separately in this paper.}

In our $U(1)'$ models, there exists an additional gauge boson $Z'$. One of the Higgs fields is charged under the extra $U(1)'$, so that the parameter $\rho$ deviates from the SM prediction at tree level by

$$\Delta \rho_{\text{tree}} = \left\{ h_i (\langle H_i \rangle \sqrt{2}/v)^2 \right\} \frac{2 g^2}{g_Z^2} \frac{m_Z^2}{m_{Z'}^2} - m_{Z'}^2, \tag{3.14}$$

where $m_Z^2 = g_Z^2 v^2$ and $m_{Z'}^2 = g^2 v^2 \{ h_i^2 (\langle H_i \rangle \sqrt{2}/v)^2 \} + g^2 h_{\phi}^2 v_\phi^2$. This form can be applied to 2HDM, 3HDM and etc., fixing the charge assignment, $\{ h_i \}$ of $H_i$ and $h_{\phi}$ of $\Phi$. In the case of our 2HDM, $h_i (\langle H_i \rangle \sqrt{2}/v)^2 = \sin^2 \beta$. Therefore, the tree-level $\rho$ parameter favors small $\tan \beta$ region [17]. This in turn implies that $Y_{tu}^{au}$ of $O(1)$ can be realized for $m_{h^{+}}/\tan \beta$ of $O(100)$ GeV.
Figure 1. Bounds on Yukawa couplings and $m_{h^+}/\tan \beta$ from $B \to \tau \nu$. The red region is the allowed region for the combined data [13] without the recent Belle data and the blue one is allowed by the recent Belle experimental result [15]. We used the relation $Y_{ta}^u = \sqrt{2} V_{tb}^* Y_{ta}^{cu}$, ignoring the other elements of the Yukawa coupling.

On the other hand, the BaBar data on $R(D)$ and $R(D^*)$ [4] prefers a large $C_{L,R}^{cb}/C_{SM}^{cb}$ (see eq. (3.12) and (3.13)). In figure 2 (a) and (b), we show favored regions (a) for Yukawa couplings $|Y_{ta}^u|$ and $|Y_{ta}^{cu}|$, and (b) $\tan \beta$ and $m_{h^+}$, which are consistent with $R(D)$ and $R(D^*)$ at BaBar within 1σ, respectively. The red points are consistent with the combined data for $BR(B \to \tau \nu)$ while the blue points agree with the recent Belle data for $BR(B \to \tau \nu)$. $|Y_{ta}^u|$ is restricted to be less than 0.05 while $|Y_{ta}^{cu}|$ is allowed to be $O(1)$. For the new Belle data $|Y_{ta}^u|$ is more constrained because the data are more consistent with the SM prediction and leave little room for the charged Higgs contributions to $B \to \tau \nu$. In order to account for the discrepancies in $R(D^{(*)})$, the Yukawa coupling $|Y_{ta}^u|$ has to be sizable and its lower bound is about 0.2. As we have discussed in figure 1, $|Y_{ta}^u|$ of $O(1)$ might be consistent with $BR(B \to \tau \nu)$ experiments if $R(D^{(*)})$ are not taken into account. Basically $|Y_{ta}^u|$ is constrained by $BR(B \to \tau \nu)$ while $|Y_{ta}^{cu}|$ is by $R(D^{(*)})$. However they are related to each other through $\tan \beta$ and $(g_R^b)$ (see eq. (3.3)). Hence, parameters which generate large $Y_{ta}^u$ are excluded by $R(D^{(*)})$ data at BaBar, and the large top FB asymmetry at the Tevatron cannot be realized in our 2HDM. In figure 2 (b), $\tan \beta \gtrsim 3$ is required in both the combined and new Belle data. For large $\tan \beta$, $m_{h^+}$ tends to be large. This is natural since $C_{L,R}^{cb}$ in (3.12) and (3.13) are proportional to $\tan \beta / m_{h^+}$ except for the last term in eq. (3.12).

Since only the RH up-type quarks are charged non-universally under $U(1)'$, our models do not generate the dangerous tree-level FCNC contributions to $B_d \overline{B}_d$, $B_s \overline{B}_s$, and $K^0-\overline{K^0}$ mixings. The $(u,c)$ elements of Yukawa couplings for neutral scalars and pseudoscalar, which may generate a tree-level FCNC contributing to $D^0-\overline{D^0}$ mixing, are small due to the suppression factor of the light-quark mass. If a pseudoscalar has large $(t,u)$ and $(t,c)$ elements of Yukawa couplings, they may enhance $D^0-\overline{D^0}$ mixing at the one-loop level. The loop of the pseudoscalar would induce the operators, $C_1 (\overline{\tau R} \gamma^\mu c_R)(\overline{\tau R} \gamma^\mu c_R)$ and $C_2 (\overline{\tau R} c_L)(\overline{\tau R} c_L)$, but the contribution to $C_1$ vanishes if external momenta are set to be zero. Only $C_2$ is non-vanishing, but the contribution is suppressed by the factor, $m_c^2/m_a^2$. 


Figure 2. Bounds on (a) $|Y_{tu}^a|$ and $|Y_{tc}^a|$ and (b) $\tan \beta$ and $m_{h^+}$ in 2HDM. The points are consistent with $R(D^{(*)})$ within 1σ. The red points are consistent with the combined data for $\text{BR}(B \to \tau\nu)$ [13] while the blue points are in agreement with the new Belle data for $\text{BR}(B \to \tau\nu)$ [15].

The upper bound on $C_2$ is discussed in ref. [26, 27]: $|C_2| \lesssim 1.6 \times 10^{-7}$ TeV$^{-2}$. Based on the general Higgs potential analysis in appendix A, the mass difference between the charged Higgs and the pseudoscalar boson is at most the weak boson mass scale. Roughly speaking, $C_2$ could be estimated as $O((m_c^2/m_h^2)(Y_{ta}^a Y_{tc}^a)^2/(16\pi^2))$, which is much less than the upper bound on $C_2$. Hence we can expect that the points in figure 2 do not disturb the SM prediction in $D^0\overline{D^0}$ mixing very much.

As discussed in ref. [18, 19], $Y_{ta}^a \sim O(1)$ is required to generate the large $A_{FB}^t$ at the Tevatron and to evade the strong constraint from the same-sign top-quark pair production at the LHC. However, figure 2 tells that $Y_{ta}^a$ should be less than $5 \times O(10^{-2})$, and $\tan \beta \gtrsim 3$ for $m_{h^+} > 200$ GeV. There is also a strong constraint from $\Delta \rho$. According to eq. (3.14), the lower bound on $\tan \beta$ indicates $\Delta \rho_{\text{tree}} \gtrsim 0.81 \times (g^2/m_{Z'}^2)(m_{Z'}^2/g_Z^2)$ in the limit, $m_{Z'}^2 \ll m_{Z}^2$, in our 2HDM. This leads that the size of the $Z'$ interaction, $g^2/m_{Z'}^2$, should be by at least $O(10^{-3})$ times smaller than the size of the $Z$ interaction, $g_Z^2/m_{Z}^2$, to achieve the $\Delta \rho$ within 1σ. This is too small to enhance the $A_{FB}^t$ [18–20]. This implies that we cannot consider a sizable $Z'$ interaction while achieving the small $\Delta \rho$.

Therefore in our 2HDM, it is difficult to find a favored region which is consistent with $R(D^{(*)})$ and $\text{BR}(B \to \tau\nu)$ as well as $A_{FB}^t$ at the Tevatron. If $R(D)$ and $R(D^*)$ become consistent with the SM prediction in the future, a tiny $Y_{ta}^a$ is favored and the only constraint on $Y_{ta}^a$ will come from figure 1 (b). According to figure 2, small $m_{h^+}/\tan \beta$ is required by the large deviations of $R(D^{(*)})$ and large $Y_{tc}^a$, so that a large $m_{h^+}/\tan \beta$, where the new physics contribution is more suppressed, could be chosen to realize a large $Y_{ta}^a$, if the deviations in $R(D^*)$ become smaller.

§ The region $m_{h^+} < 200$ GeV is strongly constrained by $B \to X_s\gamma$ and search for the exotic top decay.
4 3HDM

In order to enhance the $A_{FB}^{t}$, we need to have $O(1) \ (t, u)$ elements of Yukawa couplings for both a CP-even scalar and a pseudoscalar [18, 19]. On the other hand, the $R(D^{(*)})$ data reported by the BaBar experiments required very tiny $Y_{tu}^{u}$ in our 2HDM, which cannot produce large $A_{FB}^{t}$. One simple solution to achieve both of the large $A_{FB}^{t}$ and the BaBar discrepancies is to consider 3HDM. In the 3HDM of chiral $U(1)'$ models, we have one more pair of charged Higgs and one more pseudoscalar, so that we can realize a scenario that one charged Higgs pair is constrained by $B$ decays, but the other decouples with the $B$ physics. In fact, we will find the parameter set that the Yukawa coupling of one pseudoscalar is $O(1)$.

4.1 Yukawa couplings

In this section, we consider the 3HDM with the $(u_k) = (-1, 0, 1)$ $U(1)'$ charge assignment of the RH up-type quarks, which was introduced in ref. [18, 19]. In our 3HDM, there are 4 CP-even neutral scalars ($h_1, h_2, h_3, h_4$), 2 CP-odd pseudoscalars ($a_1, a_2$), and 2 charged Higgs pair ($h_{1}^\pm, h_{2}^\pm$), after the gauge symmetry breaking. The Goldstone modes which are eaten by $W$ and $Z$ bosons are linear to the VEVs, ($\langle H_1 \rangle, \langle H_2 \rangle, \langle H_3 \rangle$), and the orthogonal directions correspond to the mass eigenstates of pseudoscalars ($\xi_{1,2}^{a,c}$) and charged Higgs ($\xi_{1,2}^{c}$). Defining ($\langle H_1 \rangle, \langle H_2 \rangle, \langle H_3 \rangle$) $= \frac{v}{\sqrt{2}}(\sin \beta \cos \gamma, \cos \beta, \sin \beta \sin \gamma)$, we find that $\xi_{1,2}^{a,c}$ are given by

$$
\xi_{1}^{a,c} = \cos \alpha_{a,c} \begin{pmatrix}
- \cos \beta \cos \gamma \\
\sin \beta \\
- \cos \beta \sin \gamma
\end{pmatrix} + \sin \alpha_{a,c} \begin{pmatrix}
\sin \gamma \\
0 \\
- \cos \gamma
\end{pmatrix}, \quad (4.1)
$$

$$
\xi_{2}^{a,c} = - \sin \alpha_{a,c} \begin{pmatrix}
- \cos \beta \cos \gamma \\
\sin \beta \\
- \cos \beta \sin \gamma
\end{pmatrix} + \cos \alpha_{a,c} \begin{pmatrix}
\sin \gamma \\
0 \\
- \cos \gamma
\end{pmatrix}. \quad (4.2)
$$

The mixing parameters, $\alpha_{a,c}$, relate to the terms, $M_{ij}$ and $\tilde{\lambda}_{ij}$, in the Higgs potential, as described in appendix A. The Yukawa couplings of pseudoscalars are given by

$$
Y_{ij}^{u(1)} = - \hat{Y}_{ij}^{u(1)} \cos \alpha_{a} + \hat{Y}_{ij}^{u(2)} \sin \alpha_{a}, \quad (4.3)
$$

$$
Y_{ij}^{u(2)} = \hat{Y}_{ij}^{u(2)} \cos \alpha_{a} + \hat{Y}_{ij}^{u(1)} \sin \alpha_{a}, \quad (4.4)
$$

where $\hat{Y}_{ij}^{u(1)}$ and $\hat{Y}_{ij}^{u(2)}$ are defined as

$$
\hat{Y}_{ij}^{u(1)} = \frac{m_{u}^{u}}{u} \left\{ \frac{1}{\tan \beta} \delta_{ij} - \frac{2}{\sin 2\beta} (R_{i2} R_{j2}^{*}) \right\}, \quad (4.5)
$$

$$
\hat{Y}_{ij}^{u(2)} = \frac{m_{u}^{u}}{u} \left\{ \frac{\tan \gamma}{\sin \beta} \delta_{ij} - \frac{\tan \gamma}{\sin \beta} (R_{i2} R_{j2}^{*}) - \frac{2}{\sin 2\gamma} (R_{i3} R_{j3}^{*}) \right\}. \quad (4.6)
$$
The down-type quark sector and lepton sector Yukawa couplings are

\[
Y_{ij}^{\text{ad}(1)} = \delta_{ij} \frac{m_i^d}{v} \tan \beta \cos \alpha_c,
\]

(4.7)

\[
Y_{ij}^{\text{ad}(2)} = -\delta_{ij} \frac{m_i^d}{v} \tan \beta \sin \alpha_c,
\]

(4.8)

\[
Y_{ij}^{\text{al}(1)} = \delta_{ij} \frac{m_i^l}{v} \tan \beta \cos \alpha_c,
\]

(4.9)

\[
Y_{ij}^{\text{al}(2)} = -\delta_{ij} \frac{m_i^l}{v} \tan \beta \sin \alpha_c.
\]

(4.10)

The magnitude of off-diagonal elements of \(\hat{Y}_{ij}^{u(1)}\) is the same as the corresponding Yukawa coupling in our 2HDM except for replacement of \((R_{i2}R_{j2}')\) by \((g_R^{ij})\). The Yukawa couplings of the charged Higgs are obtained from the relation (3.6) with replacement of \(\alpha_a\) by \(\alpha_c\) in \(Y_{ij}^{a(1,2)}, Y_{ij}^{a(1,2)}, \) and \(Y_{ij}^{a(1,2)}\).

### 4.2 \(R(D^{(*)})\) and \(\text{BR}(B \to \tau \nu)\) in 3HDM

Now let us consider \(R(D^{(*)})\) and \(\text{BR}(B \to \tau \nu)\) in our 3HDM. There are two charged Higgs pairs that contribute to \(C_L^{eb}\) and \(C_R^{eb}\), which are estimated at the charged Higgs mass scale as follows:

\[
\frac{C_L^{eb}}{C_{SM}^{eb}} = v m_{\tau} \tan \beta \left( \frac{Y_{ll}^{u(1)}}{V_{ql}} \right) \left\{ -\hat{Y}_{kq}^{u(1)} \left( \frac{\cos^2 \alpha_c}{m_{h_1^+}^2} + \frac{\sin^2 \alpha_c}{m_{h_2^+}^2} \right) + \hat{Y}_{kq}^{u(2)} \frac{2 \sin \alpha_c}{2 m_{h_1^+}^2 - 1} \left( \frac{1}{m_{h_1^+}^2} - \frac{1}{m_{h_2^+}^2} \right) \right\},
\]

(4.11)

\[
\frac{C_R^{eb}}{C_{SM}^{eb}} = -m_b m_{\tau} \tan^2 \beta \left( \frac{\cos^2 \alpha_c}{m_{h_1^+}^2} + \frac{\sin^2 \alpha_c}{m_{h_2^+}^2} \right).
\]

(4.12)

\(R(D^{(*)})\) and \(\text{BR}(B \to \tau \nu)\) depend on several parameters. In order to find the regions favored by the BaBar and Belle data, we vary each parameter in the following range:

- \(1 \leq \tan \beta \leq 100\), 200 GeV \(\leq m_{h_1^+} \leq 1\) TeV, 200 GeV \(\leq m_{h_2^+} \leq 400\) GeV, \(0 \leq \alpha_{a,c} \leq 2\pi\), and \(0 \leq |R_{i2}R_{j2}'|, |R_{i3}R_{j3}'| \leq 1\), respectively, in addition to constraints on the Yukawa couplings, \(|Y_{tu(ud)}^{u(1,2)}| \leq 1.5\). In the numerical analyses, we take \(\tan \gamma = 1\) in order to realize a small tree-level contribution to the \(\rho\)-parameter \(^*\). After imposing the experimental constraints from \(R(D^{(*)})\) at BaBar [1], BR(\(B \to \tau \nu\)) [13], and the \(D^0\overline{D^0}\) mixing, we could find parameter regions consistent with all the experimental constraints. For the bound on \(\text{BR}(B \to \tau \nu)\), we use the HFAG value, because the result does not change much even if we adopt the new data at Belle.

Let us discuss a few specific cases for simplicity. First, we consider the case where \(\cos \alpha_c = 1\). In this case, \(C_L^{eb}/C_{SM}^{eb}\) do not depend on \(m_{h_i^+}\), so that \(h_i^+\) decouples from \(B \to D^{(*)}\tau \nu\) and \(B \to \tau \nu\), but we note that the contribution of the pseudoscalar exchanges to the \(D^0\overline{D^0}\) mixing also constrains the model parameters. Then, we can apply

\(^*\)Note that eq. (3.14) and \(\{h_i((H_i)\sqrt{2}/v)^2\} \propto (1 - \tan^2 \gamma)\) in the 3HDM.
the same discussion as in the 2HDM by replacing \((R_{i3}R_{j3}^{*})\) with \((g_{ij}^R)^{R,L}/C_{SM}^{R,L}\). In this case, \(\bar{Y}_{tu}^{u(1)}(R_{i3}R_{j3}^{*}) = \bar{Y}_{tu}^{u(2)}(R_{i3}R_{j3}^{*})\) is satisfied, so that \(\bar{Y}_{tu}^{u(1)}(R_{i3}R_{j3}^{*})\) should be smaller than 0.05 according to the discussion in our 2HDM. On the other hand, \(\bar{Y}_{tu}^{u(2)}(R_{i3}R_{j3}^{*})\) depends not only on \((R_{i3}R_{j3}^{*})\) but also on \((R_{i3}R_{j3}^{*})\) (see eq. (4.6)). Since the combination \((R_{i3}R_{j3}^{*})\) certainly enhances \(\bar{Y}_{tu}^{u(2)}\), the exchange of the heavier pseudoscalar boson \(a_2\) could realize the large \(A_{FB}^t\) when \(\cos \alpha_a\) is also 1.\(^\text{10}\)

Secondly, we consider the case where \(\sin \alpha_c = 1\). In this case, \(C_{R,L}^{R,L}/C_{SM}^{R,L}\) is independent of \(h_+^1\). Then \(h_+^1\) decouples from \(B \rightarrow D(\pm)\tau\nu\) and \(B \rightarrow \tau\nu\). In figure 3, we depict \(|Y_{tc}^{au(2)}|\) vs. \(|Y_{tu}^{au(2)}|\) and \(\tan \beta\) vs. \(m_{h_+^2}\) for \(\sin \alpha_c = 1\) with the constraints from the \(D(\pm)\tau\nu\) mixing. In figure 3 (a), the red points are allowed points for 200 GeV \(\leq m_{h_+^1} \leq 400\) GeV while the blue ones are for 400 GeV \(\leq m_{h_+^1} \leq 1\) TeV, respectively. As we see in figure 3 (a), we can find the allowed points with small \(|Y_{tc}^{au(2)}|\) and \(|Y_{tu}^{au(2)}|\) of \(O(1)\), which are in agreement with \(R(D(\pm))\) and \(BR(B \rightarrow \tau\nu)\). As we have already discussed, \(|Y_{tu}^{au(2)}|\) of \(O(1)\) might realize the large \(A_{FB}^t\) at the Tevatron. In this scenario, \(h_+^1\) contributes to the \(B\) decays and \(a_2\) could enhance the \(A_{FB}^t\). As we see in figure 3 (b), \(\tan \beta\) tends to increase slightly as \(m_{h_+^2}\) becomes large. Hence, small \(\tan \beta\) is also favored to increase the enhancement, because the mass difference between \(h_+^2\) and \(a_2\) would be an order of electroweak scale.

Finally, we consider the degenerate case of the charged Higgs pairs with \(m_{h_+^1} = m_{h_+^2}\). In this case, \(C_{R,L}^{R,L}/C_{SM}^{R,L}\) and \(C_{R,L}^{R,L}/C_{SM}^{R,L}\) are independent of \(\alpha_c\) and \(\bar{Y}_{h_+}^{u(2)}\). We show the scattered plots for \(|Y_{tu}^{au(2)}|\) vs. \(|Y_{tc}^{au(2)}|\) in figure 4 (a) and for \(\tan \beta\) vs. \(m_{h_+^1}\) in figure 4 (b), respectively. We find that the tendency between \(\tan \beta\) and \(m_{h_+^1}\) looks similar to that in 2HDM or in the \(\cos \alpha_c = 1\) case in 3HDM. As discussed in ref. \[18–21\], the small mass of the pseudoscalar of around 300 GeV, is favored to enhance \(A_{FB}^t\). As we see in figure 4 (a), one can find a few red points with \(O(1)\) \(|Y_{tu}^{au(2)}|\) and light \(m_{h_+^1}\) which could realize the large \(A_{FB}^t\) at the Tevatron. If \(m_{h_+^1}\) is heavier than 400 GeV, very large \(\lambda_{ij}\) in the Higgs potential is required as discussed in appendix A.

\(^{10}\)In appendix A, the condition for \(\cos \alpha_c = \cos \alpha_a = 1\) is discussed.
In conclusion, we can find the parameter regions which could explain the deviation of $R(D^{(*)})$ in the BaBar experiment and be consistent with BR($B \to \tau\nu$) at the $B$ factories in 3HDM. We further investigate three interesting cases, (i) $\cos\alpha_c = 1$, (ii) $\sin\alpha_c = 1$, and (iii) $m_{h_1^+} = m_{h_2^+}$. In all the three cases, we could find the allowed regions with large $|Y_{tc}^{au(2)}|$, which in turn could generate large top forward-backward asymmetry at the Tevatron.

5 The Other Constraint on $m_{h^+}$

It is well known that $B \to X_s\gamma$ can give a stringent bound on the charged Higgs mass depending on the details of models. In the type-II 2HDM, the lower bound of $m_{h^+}$ is about 300 GeV at next-to-next-to-leading order [28–31]. In our multi-Higgs doublet models with flavor-dependent $U(1)'$ gauge interactions, the Yukawa couplings of the charged Higgs have extra parameters. The $B \to X_s\gamma$ decay occurs through the loop diagram involving the top quark and the charged Higgs boson, where the relevant element of Yukawa couplings is the $(b, t)$ element. According to eq. (3.6), we can expect that the $(b, t)$ element of the charged Higgs is governed by the $(t, t)$ elements of the pseudoscalar bosons. In principle, the $(t, t)$ elements of the pseudoscalar bosons have other mixing parameters, such as $(g^u_R)_{tt}$ in our 2HDM, which, however, are not directly constrained by the semileptonic and leptonic $B$ decays. There is a theoretical relation, $|Y_{tc}^{au(2)}|^2 = (g^u_R)_{qq}(g^u_R)_{tt}$, in our 2HDM, so that $O(1)$ $(g^u_R)_{tt}$ for $R(D^{(*)})$ requires $O(1)$ $(g^u_R)_{tt}$.

The bound on $B \to X_s\gamma$ at leading order (LO) up to $O(100 \text{ GeV}/m_{h^+})^2$ is given by

$$-0.20 \lesssim \left\{ -\left(46.26 + 46.83 \ln\left(\frac{100 \text{ GeV}}{m_{h^+}}\right)\right) Y_{tt}^{au}\tan\beta + 9.00(Y_{tt}^{au})^2 \right\} \left(\frac{100 \text{ GeV}}{m_{h^+}}\right)^2 \lesssim 0.79,$$

where two relations $Y_{bt}^{au} = \sqrt{2}V_{tb}Y_{tt}^{au}$ and $Y_{tb}^{au} = \sqrt{2}V_{tb}Y_{bb}^{ad} = m_b\tan\beta/v$ are used [32]. If we assume $\tan\beta = 1$ and $m_{h^+} = 300$ GeV, then we obtain a constraint $-0.077 \lesssim Y_{tt}^{au} \lesssim 0.262$. Therefore we can expect that $(g^u_R)_{tt}$ can be $O(1)$ without conflict with the $B \to X_s\gamma$ constraint.
6 Summary

In this paper, we investigated the constraints from the semileptonic and leptonic $B$ decays on our 2HDM and 3HDM, which were proposed in ref. [18–21] in order to accommodate the top forward-backward asymmetry ($A_{FB}^t$) observed at the Tevatron. In ref. [18–21], the $U(1)'$-charged extra Higgs doublets were introduced in order to generate the realistic Yukawa couplings for the up-type quarks (especially the top quark mass) [18, 19]. In the previous study, not only CP-even scalar bosons but also pseudoscalar bosons are required to have $O(1)$ $(t, u)$-element Yukawa couplings in order to achieve the large $A_{FB}^t$ and to evade the strong bound from the same-sign top-quark pair production signal at the LHC [18–21].

On the other hand, such a large $(t, u)$ element of the pseudoscalar Yukawa coupling allows a large $(b, c)$ element to appear in the charged Higgs Yukawa couplings. This implies that our model might predict large deviations from the SM predictions in $B$ physics. For example, the $(t, u)$ element of the pseudoscalar boson is constrained indirectly by the $B \rightarrow \tau \nu$ decay.

Recently, the BaBar collaboration reported the interesting results for $R(D^{(*)})$ in the semileptonic $B$ decays, $B \rightarrow D^{(*)}\tau\nu$. The combined results deviate from the SM predictions by 3.4σ and require large FCNCs beyond MFV if the signal comes from charged Higgs exchanges. Our $U(1)'$-flavored multi-Higgs doublet models naturally realize a large $(b, c)$ element of the charged Higgs. Therefore in this paper, we investigated if our models explain both the BaBar discrepancies and $A_{FB}^t$ at the Tevatron.

In our 2HDM, only one Higgs doublet is charged under $U(1)'$, so that small $\tan \beta$ is required to avoid the constraint on the tree-level $\rho$ parameter. The $R(D^{(*)})$ discrepancies at BaBar require large new physics effects, so that small $m_{h^+}/\tan \beta$ is required as we see in figure 2 (b). However, a small $m_{h^+}/\tan \beta$ requires a very small $|Y_{ta}^{au}|/\tan \beta$ according to the bound from $B \rightarrow \tau \nu$ as shown in figure 1 (b), so that we could not find the points in figure 2 (a) with large $Y_{ta}^{au}$ which is needed for large $A_{FB}^t$. If $R(D^{(*)})$ converge to the SM prediction in the future, we would not need consider such large new physics effect and large $Y_{tc}^{au}$, so that we would be able to choose the points with large $Y_{ta}^{au}$ and small $\tan \beta$ in figure 1 (b).

In our 3HDM, we have more freedom: one more charged Higgs pair and one more pseudoscalar boson. It is not difficult to find the allowed points for $R(D^{(*)})$ and $BR(B \rightarrow \tau \nu)$ in the general case. In the limit, $\cos \alpha_c \rightarrow 1$, one of the charged Higgs bosons does not contribute to the (semi)leptonic $B$ decay, so that we can describe the scenario that one of the charged Higgs bosons explains the BaBar results and the other becomes independent of the $B$ physics. The explicit relation like eq. (3.6) is not respected in this 3HDM because of the extra freedom, $\alpha_{a,c}$. We also investigated the scalar potential in appendix A and derived the condition for $\cos \alpha_{a,c} = 1$, where both of $A_{FB}^t$ at the Tevatron and $R(D^{(*)})$ at BaBar could be achieved. In figures 3 and 4, we also plotted the allowed region, setting $\tan \gamma = 1$ which corresponds to $\Delta \rho_{\text{tree}} = 0$, and discussed the two cases: $\sin \alpha_c = 1$ and $m_{h_1^+} = m_{h_2^+}$. We can find the points with $O(1)$ $(t, u)$ Yukawa coupling and 300 – 400 GeV charged Higgs mass, which are consistent with $R(D^{(*)})$ and $D_{s0}^0-D_{s0}^0$ mixing. $\tan \beta \lesssim 40$ should be satisfied in $\sin \alpha_c = 1$ case, and the pair, $(h_2^+, a_2)$, could achieve the BaBar discrepancies and $A_{FB}^t$. In the degenerate limit, $m_{h_1^+} = m_{h_2^+}$, the $\alpha_c$ dependence disappears in the $B$ decays and...
we find the tendency between \( \tan \beta \) and \( m_{h_1^+} \) as shown in figure 4. However, the new contributions from the extra scalars are also constrained by the \( D^0 - \bar{D}^0 \) mixing, so that only a few points with a large \((t, u)\) Yukawa coupling are allowed if \( m_{h_1^+} \) is lighter than 400 GeV.

We also commented on the other bound on the charged Higgs boson. One of the most important bounds on the charged Higgs boson comes from \( B \to X_s \gamma \). In our models, the \((t, t)\) element of Yukawa coupling contributes to the process at LO and we derived the bound in the limit that the charged Higgs boson is heavy. As discussed in ref. [28–31], the lower bound on the charged Higgs mass in the type-II 2HDM is around 300 GeV, but the mass below the bound is possible in our models, depending on the \((t, t)\) element.

Before closing this paper, let us emphasize once more the importance to study phenomenology in the framework of a well defined consistent renormalizable lagrangian. The original \( Z' \) model for the top forward-backward asymmetry has been excluded several times by the upper bound on the same-sign top-quark pair production cross section. However the model with only \( Z' \) is not well defined since it is not renormalizable and not realistic because the up-type quarks including top quark are massless. It is mandatory to extend the Higgs sector, by introducing new Higgs doublets with nonzero \( U(1)' \) charges. Making such an extension actually affects the top phenomenology a lot. Basically all the top-related observables are affected by extra Higgs doublets, both neutral and charged Higgs bosons. Also the \( B \) meson and \( D \) meson sectors are modified too. Maybe it is timely to remind ourselves that the SM with three quarks \( u, d, s \) produces too large FCNCs in the kaon sector, and is immediately excluded. This disaster can be cured only by introducing the 4th quark, the charm quark, which also solves the problem of gauge anomaly. The experience with the charm quark already teaches us that it is important to work in a minimal consistent (anomaly free) and renormalizable model in order to do a meaning phenomenology.

### A Potential Analysis

The general potential in multi-Higgs-doublet models with \( U(1)' \) is given by

\[
V = m^2_i H_i^\dagger H_i - \left( m^2_{ij}(\Phi) H_i^\dagger H_j + h.c. \right) \\
+ \frac{\lambda_i}{2} (H_i^\dagger H_i)^2 + \lambda_{ij} (H_i^\dagger H_i)(H_j^\dagger H_j) + \tilde{\lambda}_{ij} (H_i^\dagger H_j)(H_j^\dagger H_i),
\]

(A.1)

where \( m^2_i(\Phi) = \lambda_{ii} = \tilde{\lambda}_{ii} = 0 \), \( \lambda_{ij} = \lambda_{ji} \) and \( \tilde{\lambda}_{ij} = \tilde{\lambda}_{ji} \) are satisfied. \( m^2_{ij}(\Phi) \) is the function of \( \Phi \), which is the complex scalar to break \( U(1)' \) and the function is fixed by the charge assignment of the fields. Each \( H_i \) includes neutral, pseudoscalars and charged Higgs,

\[
H_i = \left( \frac{v_i}{\sqrt{2}} + \frac{1}{\sqrt{2}} (h_i + i \chi_i) \right).
\]

(A.2)

\( v_i \) satisfies the stationary condition,

\[
0 = m^2_i v_i - (m^2_{ij} + m^2_{ji}) v_j + \lambda_i v_i^3 + \frac{\lambda_{ij} v_i v_j^2}{2} + \frac{\tilde{\lambda}_{ij} v_i v_j^2}{2}.
\]

(A.3)
Assuming $\Phi$ gets nonzero VEV, the mass matrices of pseudoscalars and charged Higgs are

$$
(M_a^2)_{ij} = -M_{ij}^2 + \frac{v_i v_j}{v} M_{ik}^2 \delta_{ij},
$$
(A.4)

$$
(M_{h+}^2)_{ij} = (M_a^2)_{ij} + \frac{\tilde{\lambda}_{ij} v_i v_j}{2} - \frac{\tilde{\lambda}_{ik} v_k^2}{2} \delta_{ij},
$$
(A.5)

where $M_{ij}^2 = m_{ij}^2 + m_{ji}^2$ and we assume that $M_{ij}^2$ is real.

In 2HDM, we can find one simple relation between $m_{h+}^2$ and $m_a^2$,

$$
m_{h+}^2 = m_a^2 - \frac{\tilde{\lambda}_{12} v^2}{2}.
$$
(A.6)

This means that the mass difference is at most the electroweak scale.

In 3HDM, the directions of massive modes in pseudoscalar and charged Higgs sectors can not be fixed, as we discuss in section 4. The condition for $\cos \alpha_{c,a} = 1$ is obtained from the following calculation,

$$
M_a^2 \begin{pmatrix} \sin \gamma \\ 0 \\ -\cos \gamma \end{pmatrix} = \begin{pmatrix} M_{12}^2 \tan \gamma + M_{13}^2 \frac{1}{\sin \gamma} \\ -M_{12}^2 \sin \gamma + M_{23}^2 \cos \gamma \\ -M_{13}^2 \frac{1}{\sin \gamma} - M_{23}^2 \frac{1}{\tan \gamma} \end{pmatrix},
$$
(A.7)

$$
M_{h+}^2 \begin{pmatrix} \sin \gamma \\ 0 \\ -\cos \gamma \end{pmatrix} = M_a^2 \begin{pmatrix} \sin \gamma \\ 0 \\ -\cos \gamma \end{pmatrix} - \frac{v^2}{2} \begin{pmatrix} \sin \gamma (\tilde{\lambda}_{12} \cos^2 \beta + \tilde{\lambda}_{13} \sin^2 \beta) \\ -\cos \gamma (\tilde{\lambda}_{23} - \tilde{\lambda}_{12}) \cos \beta \sin \beta \cos \gamma \sin \gamma \\ -\cos \gamma (\tilde{\lambda}_{23} \cos^2 \beta + \tilde{\lambda}_{13} \sin^2 \beta) \end{pmatrix}.
$$
(A.8)

That is, $M_{12}^2 \tan \gamma = M_{23}^2 \cos \gamma$ and $\tilde{\lambda}_{23} = \tilde{\lambda}_{12}$ are required.

**Acknowledgments**

We thank Korea Institute for Advanced Study for providing computing resources (KIAS Center for Advanced Computation Abacus System) for this work. This work is supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education Science and Technology 2011-0022996 (CY), by NRF Research Grant 2012R1A2A1A01006053 (PK and CY), and by SRC program of NRF funded by MEST (20120001176) through Korea Neutrino Research Center at Seoul National University (PK). The work of YO is financially supported by the ERC Advanced Grant project FLAVOUR (267104).

**References**

[1] J. P. Lees et al. [BaBar Collaboration], Phys. Rev. Lett. 109, 101802 (2012) [arXiv:1205.5442 [hep-ex]].

[2] J. F. Kamenik and F. Mescia, Phys. Rev. D 78, 014003 (2008) [arXiv:0802.3790 [hep-ph]].

[3] S. Fajfer, J. F. Kamenik and I. Nisandzic, Phys. Rev. D 85, 094025 (2012) [arXiv:1203.2654 [hep-ph]].
[4] A. Crivellin, C. Greub and A. Kokulu, Phys. Rev. D 86, 054014 (2012) [arXiv:1206.2634 [hep-ph]].
[5] S. Fajfer, J. F. Kamenik, I. Nisandzic and J. Zupan, Phys. Rev. Lett. 109, 161801 (2012) [arXiv:1206.1872 [hep-ph]].
[6] A. Datta, M. Duraisamy and D. Ghosh, Phys. Rev. D 86, 034027 (2012) [arXiv:1206.3760 [hep-ph]].
[7] D. Becirevic, N. Kosnik and A. Tayduganov, Phys. Lett. B 716, 208 (2012) [arXiv:1206.4977 [hep-ph]].
[8] A. Celis, M. Jung, X. -Q. Li and A. Pich, arXiv:1210.8443 [hep-ph].
[9] X. -G. He and G. Valencia, arXiv:1211.0348 [hep-ph].
[10] M. Tanaka and R. Watanabe, arXiv:1212.1878 [hep-ph].
[11] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 81, 051101 (2010).
[12] K. Hara et al. [Belle Collaboration], Phys. Rev. D 82, 071101 (2010) [arXiv:1006.4201 [hep-ex]].
[13] D. Asner et al. [Heavy Flavor Averaging Group Collaboration], arXiv:1010.1589 [hep-ex].
[14] M. Bona et al. [UTfit Collaboration], Phys. Lett. B 687, 61 (2010) [arXiv:0908.3470 [hep-ph]].
[15] I. Adachi et al. [Belle Collaboration], arXiv:1208.4678 [hep-ex].
[16] S. L. Glashow and S. Weinberg, Phys. Rev. D 15 (1977) 1958.
[17] P. Ko, Y. Omura and C. Yu, Phys. Lett. B 717, 202 (2012) [arXiv:1204.4588 [hep-ph]].
[18] P. Ko, Y. Omura and C. Yu, Phys. Rev. D 85, 115010 (2012) [arXiv:1108.0350 [hep-ph]].
[19] P. Ko, Y. Omura and C. Yu, JHEP 1201 (2012) 147 [arXiv:1108.4005 [hep-ph]].
[20] P. Ko, Y. Omura and C. Yu, arXiv:1205.0407 [hep-ph].
[21] P. Ko, Y. Omura and C. Yu, Nuovo Cim. C 035N3, 245 (2012) [arXiv:1201.1352 [hep-ph]].
[22] S. Chatrchyan et al. [CMS Collaboration], JHEP 1208, 110 (2012) [arXiv:1205.3933 [hep-ex]].
[23] CMS Collaboration, CMS-PAS-SUS-12-017 (2012).
[24] J. Charles et al. [CKMfitter Group Collaboration], Eur. Phys. J. C 41, 1 (2005) [hep-ph/0406184], and updated results and plots available at http://ckmfitter.in2p3.fr.
[25] Y. Kwon (Belle collaboration), private communication.
[26] K. Blum, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. 102, 211802 (2009) [arXiv:0903.2118 [hep-ph]].
[27] O. Gedalia, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. D 80, 055024 (2009) [arXiv:0906.1879 [hep-ph]].
[28] M. Misiak and M. Steinhauser, Nucl. Phys. B 764, 62 (2007) [hep-ph/0609241].
[29] M. Misiak, H. M. Asatrian, K. Bieri, M. Czakon, A. Czarnecki, T. Ewerth, A. Ferroglia and P. Gambino et al., Phys. Rev. Lett. 98, 022002 (2007) [hep-ph/0609232].
[30] T. Becher and M. Neubert, Phys. Rev. Lett. 98, 022003 (2007) [hep-ph/0610067].
[31] T. Hermann, M. Misiak and M. Steinhauser, JHEP 1211, 036 (2012) [arXiv:1208.2788 [hep-ph]].

[32] F. Borzumati and C. Greub, Phys. Rev. D 58, 074004 (1998) [hep-ph/9802391].