Gravitational waves from first order phase transitions during inflation

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Abstract

We study the production, spectrum and detectability of gravitational waves in models of the early Universe where first order phase transitions occur during inflation.

We consider all relevant sources. The self-consistency of the scenario strongly affects the features of the waves. The spectrum appears to be mainly sourced by collisions of bubble of the new phases, while plasma dynamics (turbulence) and the primordial gauge fields connected to the physics of the transitions are generally subdominant.

The amplitude and frequency dependence of the spectrum for modes that exit the horizon during inflation are different from those of the waves produced by quantum vacuum oscillations of the metric or by first order phase transitions not occurring during inflation.

A not too large number of slow (but still successful) phase transitions can leave detectable marks in the CMBR, but the signal weakens rapidly for faster transitions. When the number of phase transitions is instead large, the primordial gravitational waves can be observed both in the CMBR or with LISA (but in this case only marginally, for the slowest transitions) and especially with DECIGO.

We also discuss the nucleosynthesis bound and the constraints it places on the parameters of the models.

PACS number: 98.80.Cq  Keywords: inflation, gravity waves, first order phase transitions.
1 Introduction

Gravitational waves carry valuable information about the physics that produced them, as they decouple quite soon from their surrounding. In particular, waves generated during inflation could open up important opportunities to study the early Universe.

Beside the ever present generation in vacuum via quantum fluctuations, gravitational waves can also be sourced by the anisotropic stress tensor of fields and fluids. More specifically, important sources are expected to be present when first order phase transitions occur. In the literature, there has been great interest in the gravitational waves generated by this kind of transitions (see [1–7]). The analysis, however, has been mostly concerned with transitions such as the electroweak one \(^1\) or during preheating \(^2\).

In this work we instead investigate the production, features and detectability of gravitational waves from first order phase transitions during inflation.

Inflationary models exhibiting this kind of transitions have existed since the early times of inflationary theory: for example Guth’s Old Inflation was indeed driven by a first order phase transition. However, the motivation for this analysis is even stronger today, because of the appearance of many metastable vacua in effective theories of gravity and high energy physics, where tunnelings and transitions among vacua are expected to occur, possibly during inflation. For example, it has been shown that long series of connected minima can exist in the string theory landscape \(^3\).

Furthermore, recent investigations have fully analyzed \(^4\) a cosmological model alternative to slow-roll/chaotic inflation, where the inflationary dynamics is actually driven by several first order phase transitions: chain inflation.

Although expected to be important, the signatures in gravitational waves from these scenarios and models have not been studied: this work intends to fill this gap\(^5\).

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1Which however does not appear to be a first order transition for the present lower bound on the Higgs mass.

2A partial analysis of waves emission in the specific setup of Fourth Order Gravity (FOG) was done in \([10]\). Those results and approach are different from ours. In \([11]\) an analysis of gravitational waves in a specific realization of chain inflation was attempted. Our results are very different from those, as in \([11]\) the analysis appears not to be consistent. The work is not published on a scientific journal.
This paper is organized as follows: in section 2 we present the setup and the approach we follow. The detailed investigation is then divided in sections 3, 4, 5. We start in section 3 by studying our setup in terms of the relevant physical parameters and the bounds on them coming from the request of self-consistency of the scenario and its description. These bounds will affect also the emission and characteristics of the gravitational waves.

In section 4 we analyze the production of the gravitational waves and their features. We study various possible sources related to first order phase transitions during inflation. More specifically, in section 4.4 we compute the spectrum of waves emitted by the collisions of bubbles, which appear to be the strongest source. In section 4.5 we study the remaining ones.

Finally, in section 5 we discuss the detectability of the waves in the CMBR and at interferometers.

We conclude in section 6. The appendices contain useful accessory material.

2 Setup and approach

Consider a period of inflation in the early Universe where some first order phase transitions occur. The theory describing this scenario could be very complicated, with a potential exhibiting many metastable minima at different energies, a large number of fields and a complex dynamics, with rolling, tunneling and jumping phases as the fields pass through the minima.

The inflationary dynamics could occur in various ways depending on the behavior of the fields: for example via the mechanism of chain inflation, or when one scalar field undergoes slow-rolling while others tunnel through the minima.

The first order phase transitions take place via nucleation of bubbles of the new phases within the old ones. With the expansion of the bubbles and their collisions, the latent heat of the transitions is released and converted in a radiation-dominated fluid. Many sources of gravitational waves become active due to these dynamics.

We will simplify the description of this setup. In fact, knowledge of the details of the field theory is not necessary for the kind of analysis we are going to perform. Inflation and the phase transitions can be described by a series of physical parameters (for example, the time-scales of the transitions, the Hubble parameter, the nucleation size of bubbles and a few others to be introduced in section 3). We will study the production and features of the gravitational waves using these parameters, without resorting to the complicated field theory description.

The analysis is nevertheless quite complex: instead of computing the physical parameters from first principle via the field theory, we will constrain them on the basis of the requirements of consistency of the scenario. Indeed, the phase transitions must not backreact too strongly on the background, if we want inflation not to be stopped and to be efficient. At the same time they must be successful, reaching percolation and large scale thermalization, in spite of the fast expansion of the Universe.

All of this also affects the physical parameters of the sources of gravitational waves, constraining the amplitude and features of the latter. The constraints are so binding, that we will be able, using our analysis, to fully determine if a series of first order phase transitions compatible with inflation can produce a detectable spectrum of gravitational waves.

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In the following, with the term transition we will always intend a first order one, even if we do not explicitly write it to avoid repetitions.
An advantage of the approach we use is that the results are of a general nature and can be adapted to many different field theory models. Indeed, once a particular model is chosen, one can compute from first principles the physical parameters, which will now depend on the couplings of the fields and the dynamics of the model, and specialize our results on gravitational wave emission to the case of interest, being able to test it. We will do this, in the end, using the example of chain inflation.

3 Analysis of the setup: parameters and bounds

We need to consider a limited number of physical parameters to analyze the emission and features of the gravitational waves. We also need to investigate which of their values allow successful transitions together with efficient inflation, homogeneity and isotropy.

In the following we will use units for which $c = \hbar = 1$. We also use $t$ for the cosmic time, $\eta$ for the conformal and define $M_{\text{Planck}}^2 = (8\pi G)^{-1}$. The $\dot{}$ (′) indicates derivative by $t$ ($\eta$).

3.1 Evolution of the background

We need only two parameters:

- The Hubble scale $H = \frac{\dot{a}}{a}$, here written in cosmic time ($a$ is the scale factor).
- The parameter $\varepsilon = -\frac{\ddot{H}}{H}$ indicating the time evolution of the Hubble scale. We consider quasi-de Sitter scenarios, where $\varepsilon < 1$ during inflation.

These parameters come from using a Friedman-Robertson-Walker metric for the background (see appendix A.1). The latter is appropriate if there are homogeneity and isotropy at least above certain scales, that is if the bubble of the phase transitions do reach percolation and large scale thermalization, and if the radiation-fluid generated by the collisions also thermalizes. We are soon going to discuss these points.

3.2 Phase transitions

We take into account the possibility that more than one phase transition occur, by indicating each of them with a progressive integer $1 \leq n \leq N$; $N$ is kept generic.

We list here the relevant physical parameters, postponing the discussion of their bounds to section 3.2.1:

- the decay rate $\tilde{\Gamma}_n$ per unit time between phases $n$ and $n - 1$ (also called nucleation, or tunneling rate).

  $\tilde{\Gamma}_n$ is related to the decay rate per unit time and volume $\Gamma_n$, which is the quantity usually obtained in a field theory model via tunneling action (or free energy if the temperature is important) [12]: $\tilde{\Gamma}_n = \int dV_{\text{phys}} \Gamma_n$, where $V_{\text{phys}}$ is the physical volume.

- The time-scale $\beta_n^{-1}$ of the phase transition $n \rightarrow n - 1$.

  $\beta_n^{-1}$ is the lapse of cosmic time during which most of the bubble nucleate, collide and thermalize. In appendix A.2 we show its relation with the decay rate and the tunneling action. The time-scale in conformal time is indicated with $\tilde{\beta}_n^{-1}$ and defined in A.2.
the energy density $\Delta \epsilon_n$ released by the transition among phases $n, n-1$.

In each transition some energy is liberated. The energy density at disposal is $\Delta \epsilon_n \equiv \epsilon_n - \epsilon_{n-1}$, where $\epsilon_m$ indicates the energy density in the phase $m$. It is carried by the bubble walls and transferred by the transition and the collisions to the fluid velocity and heating, and ultimately to the perturbations such as the gravitational waves.

- The nucleation size $r_n$ of the bubbles of the transition between phases $n, n-1$.

We want to express $r_n$ in terms of other physical parameters. To proceed, we need to know a bit more about the process of bubble nucleation.

The growth or not of a bubble can be seen as a competition between the expansion due to the release of energy from the transition and the surface tension of the bubble wall. Only bubbles of an initial size larger or equal than a certain critical value can effectively be nucleated and grow.

A precise description is possible in terms of a tunneling action, or free energy in case the temperature is not zero, for the order parameter of the transition [12]. The tunneling action/free energy is indeed the sum of a term relative to the bubble’s wall tension minus a term for the energy of the bubble interior. The critical radius is computed by extremizing it.

In the thin wall approximation for the tunneling, we find

$$r_n = \frac{3S_n}{\Delta \epsilon_n} = \left( \frac{2}{27 \pi^2} \right)^{\frac{1}{4}} \left( \frac{S_E^{(n)}}{\Delta \epsilon_n} \right)^{\frac{1}{4}} \text{ for } T < r_n^{-1}$$  \hspace{1cm} (1)

$$r_n = \frac{2S_n(T)}{\Delta \epsilon_n(T)} = \left( \frac{2}{16 \pi^3} \right)^{\frac{1}{4}} \left( F_E^{(n)} \frac{T}{\Delta \epsilon_n} \right)^{\frac{1}{3}} \text{ for } T > r_n^{-1},$$  \hspace{1cm} (2)

where $S_n$ is the surface tension of the bubble wall and $\Delta \epsilon_n$ has been defined above. We have also expressed the critical radius in terms of the extremized tunneling action $S_E^{(n)}$ or free energy $F_E^{(n)}$, where $S_E^{(n)} = \frac{27 \pi^2 S_n^4}{\Delta \epsilon_n^4}$, $F_E^{(n)} = \frac{16 \pi S_n(T)^3}{T \Delta \epsilon_n^3}$ [12].

The two formulas above are respectively valid for vacuum or thermal nucleation. The tunneling action is used if $T < r_n^{-1}$ (vacuum description), otherwise the free energy must be employed [12]. We will show in section 3.3.1 which of the two description is more appropriate.

### 3.2.1 Conditions on the decay rates and the time-scales of the transitions

The phase transitions must be successful, reaching the stages of percolation, bubble collisions and large scale thermalization, preserving the homogeneity and isotropy of the Universe. In order to do so, the transition must cope with the inflationary expansion of the Universe.

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4We neglect gravity as it will be easy to check later that $r_n \ll R_S$, where $R_S \sim (G \Delta \epsilon_n)^{-\frac{1}{2}}$ is the gravitational radius signaling the need to consider gravity ($G$ is Newton’s constant) [13].

5Recall that the latter could be possible, as after the first transition completes there is also a radiation component of the Universe coming from the bubble walls decay.
This puts a series of conditions on the decay rate $\tilde{\Gamma}_n$ and the time scale $\beta_n^{-1}$ of the transition \[9, 14\]. In particular, the necessary conditions for successful completion of a transition during inflation is \[9\]

$$\tilde{\Gamma}_n > 3H.$$  \hfill (3)

$\tilde{\Gamma}_n$ depends in general on time\[6\]: we can have fast tunneling models (as chain inflation), where the decay rate is large enough at the onset for the transition to complete very rapidly, and evolving tunneling models, where the transition occur initially with a smaller rate (the Universe is trapped in the old vacuum for a certain time) and finishes when the tunneling rate has increased enough to satisfy (3).

However, to preserve homogeneity and isotropy, most of the bubbles must nucleate and collide in a short time, compared to the Hubble time (big bubbles are dangerous, see \[14\]). This implies

$$\beta_n^{-1} < H^{-1}.$$  \hfill (4)

We are now going to find more stringent bounds on $\beta_n$ considering the evolution of the bubbles.

Bubbles are nucleated with radius $r \sim r_n$. By the time of collision, a point on the surface of the wall of a bubble has moved by a physical distance

$$r_{f,n} - r_n = \frac{v}{H} \left( e^{\beta_n^{-1}} - 1 \right),$$

where $v$ is the wall velocity. We have assumed $H$ and $v$ to be constant during the time of the transition, as the time-scale of the latter is necessarily short, as we said. Initially the bubbles are spherical. We assume, again because of the short evolution time-scale, that they retain that shape.

In all realistic cases of successful transitions two more conditions are verified\[3–7, 14\]

$$r_{f,n} - r_n \sim \frac{v}{\beta_n}$$  \hfill (6)

$$r_n < \frac{v}{\beta_n}.$$  \hfill (7)

For (5) and (6) to be consistent at least to an acceptable value (say 5%), it must be

$$\frac{\beta_n}{H} \gtrsim 10.$$  \hfill (8)

Condition (7), instead, gives an upper bound on $\beta_n$. We start by writing $\Delta \epsilon_n$ as

$$\Delta \epsilon_n \sim -\frac{d\rho}{dt} \beta_n^{-1} \approx 6H^3 M_{\text{Planck}}^2 \beta_n^{-1} \epsilon,$$  \hfill (9)

where $\rho$ is the total energy density dominated by the vacuum component, and we have used the Friedman equation.

\[^6\text{For example through the dependence of the tunneling action on different fields.}\]

\[^7\text{In principle, we could partially relax these conditions, but those transitions would not be typical and, more importantly, the existing numerical studies \[3–6\], which we extend in section 4.4, would not be applicable.}\]
Inserting (9) in (11), we find from (7)

\[
\frac{\beta_n}{H} < \left( \frac{\pi^2}{S_E^{(n)}} \right)^{\frac{1}{5}} 10^{\frac{8}{5}}. \tag{10}
\]

We have considered only the case \( T < r^{-1} \) as we will soon show that it is the relevant one.

Summarizing, the bounds on the ratio between the scale of the transitions and the Hubble rate are

\[
10 < \frac{\beta_n}{H} < \left( \frac{\pi^2}{S_E^{(n)}} \right)^{\frac{1}{5}} 10^{\frac{8}{5}}. \tag{11}
\]

### 3.3 Radiation fluid

The collision of walls and the release of the latent heat generically produce a radiation-dominated fluid\(^8\). In order to have an effective Friedman-Robertson-Walker description of the metric, the fluid must thermalize and the time-scales for the decay of the collided walls and the fluid thermalization must be short\(^9\).

The process of bubble collision and the transfer of the energy to radiation are complex phenomena that can be studied numerically. We are not going to do so and we will assume that the thermalization of the fluid occurs rapidly.

Note that the production of radiation cannot be neglected: although during inflation it is a sub-dominant fraction of the total energy density in comparison with the vacuum, it can be important for what concerns the perturbations. This was shown, for example, in the case of chain inflation with a single scalar field, where it was actually essential to provide a spectrum of adiabatic density (scalar) perturbations in accordance with observations\(^9\).

The parameters necessary to describe the fluid are the following ones:

- The temperature \( T \) of the radiation fluid.

  Patches of the Universe in different phases have in general different temperatures, and we should distinguish them with a label \( n \). As we will see (section 4.4.1), the differences between the temperatures of the phases are (and must be) negligible for inflation to be efficient, therefore we will often omit the suffix \( n \).

- Plasma scales.

  The constituents of the fluid will in general be charge carriers. Such fluid goes under the name of plasma. More refined definition of plasma are possible, but we will not consider them.

  When it is relativistic, its dynamics is regulated only by the gauge coupling(s) \( g \) and the temperature \( T \). The relevant scales, such as the plasma frequency \( \omega_p \sim gT \) or the

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\(^8\)We will show that it is relativistic (radiation) in section 3.3.1.

\(^9\)If it were necessary to remind it: it is well known that in all FRW cosmological models matter and radiation are really never in thermal equilibrium, as those space-times do not possess a time-like Killing vector. However, in general the Universe can be, and often is, very near thermal equilibrium and this is the meaning of thermalization in this context\(^15\). See also\(^9\).
screening distance $\lambda_D \sim (gT)^{-1}$, are determined by these quantities\textsuperscript{10} \textsuperscript{16}.

Other important plasma scales and parameters will be introduced in section \textsuperscript{4.5} when studying the gravitational emission.

### 3.3.1 Conditions on the fluid temperature

A quasi-de Sitter inflationary evolution imposes an upper bound on the fluid energy density $\rho_f$ and therefore its temperature $T$.

By using the Chaudhuri equation (71) and the parametrization of the Hubble constant

$$H = \sqrt{8\pi^2 c_s \eta \varepsilon} M_{\text{Planck}} \sim 10^{-4} \sqrt{\varepsilon} M_{\text{Planck}}, \quad (12)$$

obtained normalizing the spectrum of scalar perturbations $\sim \frac{H^2}{8\pi^2 c_s \varepsilon M_{\text{Planck}}}$ to the result of the 5-years WMAP survey $\Delta^2_R = \eta \sim 2.5 \times 10^{-9}$ \textsuperscript{17}, we find

$$T = \left(\frac{\rho_f}{\kappa}\right)^\frac{1}{4} < 10^{-2} \kappa^{-1} \frac{1}{2} \varepsilon \frac{1}{2} M_{\text{Planck}}, \quad (13)$$

where $\kappa = \kappa(T)$ counts the number of relativistic degrees of freedom at temperature $T$. The inequality follows from the fact that in the specific models there can be other components entering the Chaudhuri equation beside radiation, for example the kinetic energy of scalar field(s).

If $\varepsilon$ is not pathologically small, $T \gg \text{GeV}$. At such temperature particles\textsuperscript{11} are effectively massless.

Although the temperature is constrained as shown in (13), we still have to check the modification to the tunneling dynamics due to thermal corrections. In fact, the transitions (tunneling) and the gravitational emission can change depending on the temperature. The vacuum tunneling description is well-suited to capture the dynamics only provided that the temperature $T$ is smaller than the inverse of the scale length of the field solution $r\sim R^{-1}$ \textsuperscript{12}. From (1), (11), (13), we find (for $\kappa \sim 100$)

$$\frac{T}{r\sim R^{-1}} < \left(\frac{6S_E^{(n)}}{\pi^2}\right)^\frac{1}{4} \left(\frac{\beta_n}{4\kappa H}\right)^\frac{1}{4} < 1 \quad \text{for } S_E^{(n)} < 20. \quad (14)$$

Noting that the critical tunneling action cannot be too large as it will suppress the decay rate too much, also in view of (3), we conclude that the pure vacuum description of the tunneling is appropriate.

### 3.4 Final comments

The bounds on the typical scale of the transitions and the temperature (especially (11), (13)) will strongly affect the gravitational emission and the sources.

\textsuperscript{10}One generally distinguishes between different kind of collective oscillations \textsuperscript{16}, but their characteristic frequencies are of the same order. Also, non-abelian and abelian gauge theories are generally distinguished by different constant of proportionality involved in the formulas for the typical scales. We treat both cases by indicating the plasma scales up to the proportionality constants.

\textsuperscript{11}We do not know the theory at those energies, it could be supersymmetric or also a GUT.
Let us anticipate that, in particular, the physics of the plasma will prove to be a subdominant source because of the constraint (13) on the temperature. The bounds (11) will instead be very important for discussing the suppression of the waves emitted by the collisions of bubbles.

Note also that the energy density $\Delta \epsilon_n$ released by a transition is much smaller than the total energy density, by a factor $2H/\beta_n \varepsilon$, see (9). This will have important consequences for the gravitational emission.

4 Gravitational waves

We organize the study of the gravitational waves in the following sections.

4.1 Useful definitions and notation

A gravitational wave can be seen as a ripple on the top of the background metric, as (in conformal time $\eta$)
\[
 ds^2 = a(\eta)^2 [-d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j] .
\]
(15)

In particular, gravity waves are true tensor modes and are transverse, symmetric and traceless. The linearized Einstein equation reads
\[
 h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} = 16\pi G a^2 \pi_{ij}^T .
\]
(16)

where $\mathcal{H} = aH$ and $\pi_{ij}^T$ is the traceless symmetric transverse tensor part of the anisotropic stress tensor, obtained by suitable projection from the total energy-momentum tensor.

Not indicating polarization indexes to avoid cluttering of formulas, the spectrum $P_h(k, t)$ of gravitational waves is defined as
\[
 \langle h^*(\vec{k}, t) h(\vec{k}', t) \rangle = \delta^{(3)}(\vec{k} - \vec{k}') \frac{2\pi^2}{k^3} P_h(k, t)
\]
(17)

and the energy density per octave
\[
 k \frac{dp_h(\vec{k}, t)}{dk} = k^3 \int d\Omega \frac{\langle h^*(\vec{k}, t) h(\vec{k}', t) \rangle}{(2\pi)^3 8\pi G} = k^3 \int d\Omega \frac{\langle h^s*(\vec{k}, \eta) h^s(\vec{k}', \eta) \rangle}{(2\pi)^3 8\pi G a^2} .
\]
(18)

Here, $h(\vec{k}, \eta)$ is the mode function of the graviton field $h(\vec{x}, \eta)$ expanded in eigenfunctions of the Laplace-Beltrami differential operator with eigenvalues $-k^2$. The wavenumber $k$ is related to the physical momentum by $k = a(t) p$.

The brackets in (17), (18) refer to a quantum treatment of perturbations. We will evaluate the two-point functions on the Bunch-Davies vacuum, neglecting the so-called transplanckian issue.

In cosmological perturbation theory the sources we will consider arise at second order. At that level tensor perturbations are gauge dependent.
4.2 Formal solution of the wave equation

In order to discuss the observables we will be interested in, the field equation (16) must be solved both during inflation and later on, during matter or radiation domination.

For a quasi-de Sitter background during inflation \( a = \frac{1}{\eta H(1-\varepsilon)} \), we find the general solution for the mode functions

\[
h = \left( c_1 + \frac{i\pi}{4} \int_{\eta}^{\eta'} d\eta' (\eta')^{1-\nu_T} H^{(2)}_{\nu_T}(-\eta') S(\vec{k}, \eta') \right) (-\eta)^{\nu_T} H^{(1)}_{\nu_T}(-k\eta) \\
+ \left( c_2 - \frac{i\pi}{4} \int_{\eta}^{\eta'} d\eta' (\eta')^{1-\nu_T} H^{(1)}_{\nu_T}(-\eta') S(\vec{k}, \eta') \right) (-\eta)^{\nu_T} H^{(2)}_{\nu_T}(-k\eta),
\]

where \( H^{(1,2)}_{\nu_T} \) are Hankel functions and

\[
\nu_T = \frac{3}{2} + \varepsilon \sim \frac{3}{2}, \quad S(\vec{k}, \eta) = 16\pi G a^2 \pi T(\vec{k}, \eta).
\]

The terms in the solution proportional to \( c_1, c_2 \) are related to the vacuum fluctuations of the field (homogeneous equation). We choose the Bunch-Davies vacuum setting \( c_1 = H_{\frac{\nu_T}{2}} e^{i(\nu_T + \frac{3}{2})}, \quad c_2 = 0 \).

During matter or radiation domination, when the source is no longer active and the scale has reentered the horizon \( (k \gg H) \), the solution is

\[
h = \frac{A_+(k)}{a\sqrt{k}} e^{-ik\eta} + \frac{A_-(k)}{a\sqrt{k}} e^{ik\eta}.
\]

The constants \( A_{\pm}(k) \) are determined by properly matching (19), (21) so that both \( h \) and \( h' \) are continuous. The time of matching depends on the duration of inflation and on the value of \( k\eta \), which signals when and if the scale \( k^{-1} \) is inside or outside the horizon.

4.3 Sources of gravitational waves in the presence of first order transitions during inflation

First order phase transitions provide many possible sources for gravitational waves. In particular we will consider

- the anisotropy stress tensor from bubble collision
- the velocity spectrum of the fluid
- hydrodynamical turbulence
- non-zero gauge fields and (hyper)magneto- hydrodynamical turbulence
- anisotropy tensor of the radiation fluid (viscosity)

\[\text{We will indicate with the term “hyper” the fields associated with a } U(1) \text{ gauge symmetry, although in general that will not be necessarily related to the hypercharge of the Standard Model.}\]
In the formal expansion of cosmological perturbation theory, these sources arise at second order. At that order, tensor modes are also sourced by first order metric perturbations, coming from the expansion both of the energy momentum and of the Einstein tensors [18]. We will not discuss this in the present work.

We turn now to the detailed analysis of the sources listed above and their wave emission during inflation.

4.4 From phase transitions and bubble collisions

The collisions of bubbles source gravitational waves. Instead, the evolution of bubbles prior to collision does not generate gravitational waves because of the spherical symmetry of the bubbles [7].

4.4.1 Features of the source

We have found at the end of section 3.3.1 that the appropriate description for transitions that are compatible with inflation is the vacuum one, in terms of the tunneling of an order parameter (scalar field) [12,13]. There is nevertheless an amount of radiation, due to the collisions of bubble walls of previous transitions.

The energy released by a transition goes partly in the acceleration of the bubble walls, and partly in the velocity and heating of the fluid. The features of the gravitational waves emission are different depending on how the energy is divided in these two channels.

We can understand how much of the energy released by the transition goes into the fluid by looking at the hydrodynamical equations for the fluid at the wall. In fact, the fluid can reach a steady-state configuration, at some time after the nucleation and at some distance from the wall. In that case, the transition front (related to the bubble wall) acts as a discontinuity surface where the energy and momentum fluxes must be conserved, leading to the equations of conservation in the rest frame of the wall

\[ \frac{4}{3} \kappa T^4 \nu_{(f),n} = \frac{4}{3} \kappa T^4 \nu_{(f),n-1} \]  
\[ \frac{4}{3} \kappa T^4 \nu_{(f),n} + \frac{1}{3} \kappa T^4 - \epsilon_n = \frac{4}{3} \kappa T^4 \nu_{(f),n-1} + \frac{1}{3} \kappa T^4 - \epsilon_{n-1}. \]

\[ \text{Here } \nu_{(f),n(n-1)} \text{ is the component of the four-velocity of the fluid in phase } n(n-1) \text{ locally orthogonal to the wall.} \]

We try to find what can be the values and solutions for the velocity that comply with inflation. Considering that inflation allows only small velocity and temperature perturbations, (22), (23) lead to

\[ \frac{\Delta \epsilon_n}{\kappa T^4} \lesssim \frac{1}{3} \left| \frac{\delta (\kappa T^4_{n-1})}{\kappa T^4_n} \right|, \]

where \( \delta (\kappa T^4_{n-1}) \equiv \kappa T^4_{n-1} - \kappa T^4_n \).

But from (9), (71), we find

\[ \frac{\Delta \epsilon_n}{\kappa T^4_n} > \frac{4H}{\beta_n}, \]

\[ \text{Here, for the moment, we have assumed that the interactions between the fluid and the wall are negligible during the bubble expansion prior to collisions, but it is possible to extend this to the more general case (see for example [19]).} \]
while

\[ \frac{\delta (\kappa T_{n-1}^4)}{\kappa T_n^4} = \frac{\delta \rho_f}{\rho_f} \approx \frac{\delta \rho_{\text{tot}}}{\rho_{\text{tot}}} \sim \frac{H}{M_{\text{Planck}} \sqrt{c_s \varepsilon}}. \]

By looking at (11), (12), we see that (25), (26) are in contradiction with (24), which must be satisfied if there exist steady state hydrodynamical solutions at the wall complying with inflation. Therefore there is no such solution.

The meaning of this result is that, for successful transitions compatible with inflation, the vacuum energy released in the transition goes mostly into the acceleration of the walls, which expand at a velocity rapidly approaching the one of light until collision. We can therefore assume \( v \approx 1 \) for the wall velocity. The bubble dynamics is well described by the approximation of bubbles in vacuum.

The gravitational waves are therefore mainly sourced by the stress tensor of the scalar field describing the bubbles’ configuration in vacuum, with a sub-dominant contribution from the fluid, suppressed by powers of its small velocity.

### 4.4.2 Calculation of the wave spectrum

We turn now to the detailed computation of the spectrum of gravitational waves. They are sourced because of the breaking of the spherical symmetry of bubbles at collisions. The latter are complicated events that occur out of equilibrium. It is therefore quite difficult to give an analytic description of their gravitational emission: in principle we could expect a whole range of different scales to appear in complicated inter-correlation.

The only way to deal with these problems is via numerical simulations [3–6]. On top of those, useful analytical formulas can be derived [6, 7]. We will obtain the spectrum of gravitational waves by extending the results of the numerical simulations to the case of transitions occurring during inflation.

The best simulations available today [3–6] consider short-lasting sources and static background (neglected expansion of the Universe, no inflation). For nearly vacuum collisions and wall velocity \( v \sim 1 \) in their formulas, they show that only two quantities are important in determining the spectrum of the gravitational waves: the overall energy density \( \rho_W \) released by the transition and carried by the walls that collide, and the time-scale \( \beta_n^{-1} \) of the phase transition, which sets the peak frequency of emission.

References [5, 6] also show that the energy density per octave radiated by colliding bubbles goes like \( k^3 \) for small frequencies, while for large ones it decays as \( k^{-1} \). In particular, there is a single peak when \( k \approx \beta_n \) and the maximum (normalized to the total energy density) is

\[ \sim \frac{0.013}{\pi^3 M_{\text{Planck}}^2} \frac{\rho_W}{\beta_n^2}. \]

15 We estimate the enthalpy density variation compatible with inflation using the value on superhorizon scales as it is larger than that at sub-horizon scales. The result for density perturbations that we use in the formula can be rigorously obtained in the flat gauge. We consider models of inflation, where isocurvature perturbations are negligible.

16 Here, we specifically use the results in the more recent reference [5]. In adapting from their conventions to ours, one has to compare the formula we obtain using (19) in (18), for large \( k \), with Weinberg’s formula from [20], used in [5]. Note that we define the Fourier transform of the energy-momentum tensor as \( T_{\mu \nu}(\hat{k}, k) = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} T_{\mu \nu}(x, t) \), whereas [5] uses Weinberg’s normalization for the Fourier transform. The physical equivalence of Weinberg’s formula and the one we obtain from (18), (19) for large \( k \) has been discussed in [20].
We stress that the transitions considered by [3–6] occur at the end or after inflation and the energy carried by the walls in their cases is nearly all of the total energy, so that $8\pi G \rho_W = 3H^2$.

We will now derive the radiated energy and spectrum of waves in our scenario. There are three evident differences with the setup of [3–6]:

- the Universe in our case undergoes inflation. However, the static approximation at the time of emission of the waves can be acceptable as long as their modes are $k \geq aH$. This is compatible with our scenario as the typical scales/frequencies $\beta_n$ of the phase transitions are larger than the horizon (see section 3.2.1).

- in our case, the total energy released by the walls is $\Delta \epsilon_n$ given by (9), which is much smaller than the total vacuum energy driving inflation.

- the evolution of the waves during inflation is peculiar as modes can exit the horizon.

Using this information, we can now adapt the results of the simulations and derive the energy density per octave radiated by colliding bubbles of the $n$-th transition during inflation. We find that initially, at the time $t_n$ when the sources stops being active shortly after the collisions (time of emission),

$$
\frac{1}{\rho_{tot}} \left. \frac{dp_{h}^{(n)}(p)}{dp} \right|_{t=t_n} \sim \frac{0.16}{\pi^3} \left( \frac{H}{\beta_n} \right)^4 \varepsilon^2 \left\{ \begin{array}{ll}
\left( \frac{p_n}{\beta_n} \right)^3 & H_n < p_n < \beta_n \\
\left( \frac{a}{p_n} \right) & p_n > \beta_n
\end{array} \right.,
$$

(27)

where $\rho_{tot}$ is the total energy density. We have written the result in terms of the physical momentum $p_n = k a_n^{-1}$, $a_n = a(t_n)$.

When $k \gg H$, there is a simple relation between the spectrum (17) and the energy density (18):  

$$
k \frac{dp_h(k, t)}{dk} = k^2 \frac{1}{8\pi a^2 G} P_h(k, t).
$$

(28)

Using it, we obtain the spectrum of gravitational waves at the time of emission $t_n$

$$
\left. P_h^{(n)} \right|_{t=t_n} = \frac{0.15}{\pi^2} \left( \frac{H}{\beta_n} \right)^6 \varepsilon^2 \left\{ \begin{array}{ll}
\left( \frac{p_n}{\beta_n} \right)^3 & H_n < p_n < \beta_n \\
\left( \frac{a}{p_n} \right) & p_n > \beta_n
\end{array} \right.,
$$

(29)

We need now to evolve the spectrum after the moment of emission of the waves. As this evolution is particular during inflation, further differences with respect to the results in [3–5] will arise.

At the time of emission (roughly at the completion of the transition and collisions), the modes are within the horizon. From (19), we can see that then the wave evolves as $\sim a^{-1}$ until the mode exits from the horizon at time $t_{ex}$. After that, the wave remains constant until reentering horizon.

We need to compute the proper redshift of the modes at $t > t_n$. For modes still within the horizon at time $t$, we find $\frac{a_n}{a(t)} = \frac{b(t)}{p_n}$.

For modes that have already exit the horizon at some time $t_{ex} < t$, instead, the redshift is only $\frac{a_n}{a_{ex}} = \frac{H a_n}{p_n}$. Overall, the redshift factor at time $t$ for a momentum $p$ can be written as

$$
\frac{a_n}{a(t)} \equiv \chi_n \frac{\beta_n}{p_n}, \quad \chi_n = \left\{ \begin{array}{ll}
\frac{H}{\beta_n} & \text{superhorizon modes at time } t \ (p(t) < H) \\
\frac{p}{\beta_n} & \text{subhorizon modes at time } t \ (p(t) > H)
\end{array} \right.,
$$

(30)

12
where we have used $H_\infty \sim H \sim H_n$, since the Hubble parameter does not evolve much during inflation.

Therefore, at time $t > t_n$ (still during inflation), the spectrum generated by the collisions of bubble of the $n$-th transition has evolved to

$$P_h^{(n)} = \frac{0.15}{\pi^2} \left( \frac{H}{\beta_n} \right)^6 \varepsilon^2 A_n \left\{ \begin{array}{ll}
\frac{f_{H_n}}{p} & f_{H_n} < p < f_{\beta_n} \\
\frac{f_{\beta_n}}{p} & p > f_{\beta_n}
\end{array} \right.,$$

where

$$f_{\beta_n} = \frac{a(t_n)}{a(t)} \beta_n, \quad f_{H_n} = \frac{a(t_n)}{a(t)} H_n.$$  \hspace{1cm} (32)

If we want to have a chance to detect these primordial gravitational waves, we must consider superhorizon modes, as subhorizon ones are more suppressed by the redshift. Their total spectrum at time $t$ is obtained by summing over all the phase transitions and is given by, using (30), (31) for superhorizon modes,

$$P_h^{sup} = \sum_{n=1}^{N} P_h^{sup,(n)} = \sum_{n=1}^{N} \frac{2}{\pi^2} A_n \left( \frac{H}{M_{Planck}} \right)^2 \left\{ \frac{\tilde{\beta}_n}{k} \right\}^5 a(t_n) H < k < \tilde{\beta}_n \quad \tilde{\beta}_n > \beta_n$$

$$\quad k > \tilde{\beta}_n,$$ \hspace{1cm} (33)

where

$$\tilde{\beta}_n = a_n \beta_n, \quad k = a_n p_n > a(t) H, \quad A_n \equiv 0.07 \left( \frac{M_{Planck}}{H} \right)^2 \left( \frac{H}{\beta_n} \right)^8 \varepsilon^2.$$ \hspace{1cm} (34)

This represents the main novel result of this section.

We will deal with the later evolution of the modes, after reentering the horizon, in section 5.2.

### 4.4.3 Comments on the final result

Comparing the spectrum (33) with the one generated in vacuum during inflation (see formula (82) in appendix A.3), we see that the main differences are as follows:

- the spectrum (33) is not scale invariant
- due to (11), (12), $A_n$ is an additional suppression factor, with respect to the waves generated by vacuum oscillations
- the accumulation due to a sufficiently large number of phase transitions $N$ could cope with the suppression and make the waves detectable.

In discussing the backreaction of the waves (and possibly the breaking down of the perturbative approach), we need to consider the possible number $N$ of transitions.

As we see, for a few phase transitions, the waves sourced by the collisions of bubbles are even more suppressed than those produced by vacuum quantum oscillations, and therefore their backreaction is negligible (perturbation theory is accurate).
Of a large number of transitions, we have in principle a stronger backreaction. It is easily verified, though, that the number of transitions needed to affect the background evolution is unlikely to occur (for the range of parameters given in section 3 and \( \varepsilon \sim 10^{-2} \), it would have to be at least of the order of \( N \sim 10^{12} \)).

We might also wonder if the depletion of the energy density due to gravitational radiation could affect the scalar density perturbations. However, from (53) and the above comments, it is evident that this depletion of energy for a single transition is very small (smaller than the linear order in perturbation theory). Only a large number of transitions could have an effect (in that case, invalidating the perturbation expansion). However, even in that case this would happen after many transitions have occurred, that is quite late during inflation. It would therefore affect only the late density perturbations that are not interesting for detection at the CMBR (see also section 5.1).

The spectrum (53) is different also from the one generated by the collisions of bubbles of first order phase transitions that do not occur during inflation. Indeed, beside the fact that in this second case the fluid velocity spectrum is generally an important source and that the maximum amplitude of the waves from a single transition is less suppressed, the most notable difference is that in our case the presence of modes that were superhorizon at inflation gives the spectrum a different dependence on the frequency (compare with [3–7]). If detected, this would help in distinguishing the two cases.

Another comment concerns the sum we have performed in (53). In fact, there is a non-zero probability of nucleation of bubbles of phase, say, \( n-2 \) when the bubbles of phase \( n-1 \) have not yet collided. Would this make the simple sum rule we use inadequate? The answer to this question is that it would not, as long as the moments of collisions of consecutive phase transitions are spaced out. Indeed, the evolution of the transition before collision (expansion of the bubbles) does not generate gravitational waves, if the bubbles are spherically symmetric [7].

4.5 From turbulence, (hyper)magnetic fields, viscosity

The presence of a radiation dominated fluid is a consequence of the processes of bubble collisions and walls decay after each phase transition. Although sub-dominant, this component cannot be neglected when discussing perturbations, as it is can be even essential for having the right spectrum of adiabatic density perturbations, for example in chain inflation [9].

The physics related to the plasma or hydro- dynamics could generate gravitational waves during inflation, depending on the gauge coupling and the temperature of the fluid[17]. This is what we want to investigate now.

We will make the assumption of weakly coupled plasmas, because their stress anisotropy tensor is larger (large viscosities) than that of strongly coupled plasmas, and therefore a potentially stronger source of gravitational waves. Nevertheless, the anisotropy tensor cannot be too large, otherwise it would spoil the homogeneous and isotropic description of the Universe at large scales. It will appear that this indeed does not happen when the parameters satisfy the bounds in section 3.

Note also that the validity of the hydrodynamical or plasma description depends on the relative magnitude of the Hubble scale and the microscopical ones, such as the mean-free-path or the screening distance. This tells us that only for some ranges of values of the couplings

\[ \text{For an incomplete bibliography on the processes themselves and the associated gravitational production (not during inflation), see [21].} \]
the description is self-consistent. In particular, the couplings cannot be too small, although a precise bound depends on the model-dependent numerical factors entering the formulas of the plasma scales.

We will make use of the transport coefficients characterizing the transport of energy, momentum and charge across the fluid [16]. They have been generally calculated within the most well-known theories (especially QED, QCD), but usually the results can be extended with only minor modifications to different higher energy theories, thanks indeed to the plasma or hydrodynamical approximation. We assume that this is our case.

We are now going to analyze the various sources related to hydro- and plasma dynamics. It will turn out that they are very much suppressed, contrarily to what generally happens when first order transitions occur not during inflation. The ratio \( T \beta_n \) will be particularly important in the following considerations. It is strongly constrained by the requirements of efficient inflation and self-consistency of the theory (see section 3) and this will ultimately be the reason why these sources do not produce a sizable spectrum of gravitational waves in our scenario.

4.5.1 Hydrodynamical turbulence

Turbulence is a strong source of gravitational waves. Let us study its occurrence in our scenario. In our case, the generating mechanism would be the collisions of bubbles, stirring the fluid. The typical length scale of injection is approximately the bubble size at collisions, \( \sim \beta_n^{-1} \), and this, together with the kinematic viscosity [16]

\[
\nu = \nu_0 g^{-4} \log(g^{-1})^{-1} T^{-1},
\]

yields the Reynold number

\[
Re = \frac{v_f^{(\beta_n)} \beta_n^{-1}}{\nu} \sim v_f^{(\beta_n)} \frac{T}{\beta_n} g^4 \log(g^{-1}),
\]

where \( v_f^{(\beta_n)} \) is the characteristic velocity of the fluid flow at the injection scale.

Large Reynold numbers signal the onset of turbulence. We therefore see from (36) that the relevant condition is

\[
\frac{T}{\beta_n} = \begin{cases} 
\gg 1 & \text{turbulence} \\
\lesssim 1 & \text{no turbulence}.
\end{cases}
\]

Looking at (12, 13, 11), we find that in our scenario

\[
\frac{T}{\beta_n} < 4.
\]

Therefore, turbulence does not occur and a sizable emission of gravitational waves is not possible. The reasons for this are the smallness of the scales of injection of energy and the rapidity of the bubble evolution, which are consequences of the requirements of small backreaction on inflation and self-consistency of the theory (section 3).

\[18\] Our results will therefore apply to theories and models for which this is possible. In the following, our formulas for the parameters will be written up to proportionality constant (indicated with a index 0), which depend on the peculiar details of the plasma under consideration (such as the number of light versus heavy species or the rank of the gauge symmetry group). These will not be influential for the results.
4.5.2 Plasma physics and gauge fields

The radiation fluid is likely charged (therefore it is generically a plasma) and local charge asymmetry and currents can be generated\(^{19}\). We investigate now whether non-zero long-range gauge fields generated by these currents could represent a sizable source of gravitational waves during inflation.

The only long-range fields that can survive in a plasma for enough time are magnetic fields associated with a \(U(1)\) gauge symmetry (hypermagnetic fields), see section 5.6 in \([23]\). The precise form of the generated fields depends on the details of the bubble growth, the couplings, the tunneling events, the non-equilibrium dynamics. Nevertheless, it is possible to draw sufficient conclusions from general considerations.

The fields are generated by the currents at the scale \(L_p \sim \beta_n^{-1}\) of the bubbles\(^{20}\) and then possibly enhanced by magnetic turbulence, which is signaled by a large magnetic Reynolds number. This is defined as

\[
Re_\mu = \frac{\nu_f^{(\beta_n)} L_p \sigma_c}{4\pi},
\]

where \(L_p\) is the typical scale at which magnetic fields are generated and \(\sigma_c = \sigma_0 \frac{T}{g^2 \log(g^{-1})}\) is the (hyper)electrical conductivity at high temperature.

In our case,

\[
Re_\mu \sim \nu_f^{(\beta_n)} \frac{T}{\beta_n g^2 \log(g^{-1})},
\]

which, even for reasonably small couplings \((g \sim 0.1)\) is much less than 100, given the bounds in section \(\text{3}\). Therefore there is no hypermagnetic turbulence and it is unlikely that the small scale hypermagnetic fields appreciably source gravitational waves\(^{21}\).

4.5.3 Stress tensor from fluid viscosity

The form of the anisotropic stress tensor in general relativity for a fluid with velocity \(\vec{u}\) is \([20]\)

\[
\pi^{(\text{visc})}_{ij} = -\zeta \left( \partial_j u_i^{(f)} + \partial_i u_j^{(f)} - \frac{2}{3} \nabla \cdot \vec{u}^{(f)} \delta_{ij} \right) - \zeta \nabla \cdot \vec{u}^{(f)} \delta_{ij},
\]

\(\zeta\) is the bulk viscosity. For a relativistic fluid, \(\zeta\) is vanishing, as it is given by

\[
\zeta = \zeta_0 T^3 g^{-4} \left( \frac{1}{3} - \frac{\partial \rho_f}{\partial P_f} \right),
\]

where \(\rho_f, P_f, n_f\) are respectively the energy, pressure and number density.

---

\(^{19}\)For example, it is enough to have different mean free paths (different couplings) for the various species to generate a local charge asymmetry, in presence of particle-antiparticle number asymmetry. Then, the collisions among bubbles impart a vorticity to the charged fluid, with the creation of microscopic currents. The number asymmetry can occur because bubble collisions and first order transitions are likely to provide the right conditions \([22]\). Nevertheless, a precise statement can be made only with a detailed model \([21]\).

\(^{20}\)Due to the different orientation of field lines at the bubble scale, it is also possible to assume that the field spectrum is stochastic, in this way preserving the global isotropy of spacetime.

\(^{21}\)In principle, there is still a residual but unlikely possibility that in specific models even the small scale hypermagnetic fields can be strong enough to generate a sufficiently sizable amount of gravitational emission. We will not discuss this case.
The part of $\pi_{ij}^{(\text{visc})}$ proportional to the shear viscosity $\varsigma$ could instead be important, as

$$\varsigma = \varsigma_0 \frac{T^3}{g^4 \log(g^{-1})}$$

for a weakly coupled fluid. Nevertheless, it turns out that this source is negligible as well.

In fact, the energy per octave radiated by an anisotropic stress tensor $\Pi_{ij}$, normalized by the total energy density, at emission time $\eta = \eta_*$ on a scale $k > H$ is

$$\Omega_* \equiv \frac{1}{\rho_{\text{tot}}} \frac{d\rho_*(k)}{dk} \bigg|_{\eta = \eta_*} = \frac{16}{\rho_{\text{tot}}} Gk^3 a_*^2 \int d\Omega \langle \Pi_{ij}(k, k) \Pi^*_{ij}(k, k) \rangle,$$

where $a_* = a(\eta_*)$, while $\Pi_{ij}(k, k) = \int \frac{d\eta d^3 x}{(2\pi)^2} e^{ik\eta - i\vec{k} \cdot \vec{x}} \Pi_{ij}(\vec{x}, \eta)$ and $\int d\Omega$ is the angular integral.

In the case of (41), considering only the shear viscosity part, we have

$$\Omega^{(\text{visc})} \simeq \frac{16G\varsigma^2}{\rho_{\text{tot}}} \mathcal{V}^2$$

where $\mathcal{V}^2 \equiv a_*^2 k^5 \lambda^5 v_{\beta, \lambda}^2$ and $v_{\beta, \lambda}^2(\lambda k, k)$ is the dimensionless square mean-field gradient of the velocity (integrated over the angles and Fourier transformed in the time dependence) obtained by smoothing over the comoving length scale $\lambda$ and in which we have also absorbed the dependence on $k\lambda$ from the velocity spectrum. The scales suitable for hydrodynamical treatment must be larger than the coherence scale of the velocity, therefore in our scenario $\lambda \sim \beta_n^{-1}$.

On the other hand, at the moment of emission, the energy per octave radiated by the collision of bubbles of one phase transition is (27), here written for comoving wavenumbers,

$$\Omega^{(\text{coll})} = \frac{0.16}{\pi^3} \left( \frac{H}{\beta_n} \right)^4 \varepsilon^2 \left( \frac{\beta_n}{k} \right)^{-3}.$$

We have considered wavenumbers appropriate for the comparison with the scales of the hydrodynamical description, which means $k < \beta_n$.

For scales $k \lesssim \lambda_f^{-1}$, where $\lambda_f = \lambda_0 g^{-4} \log(g^{-1})^{-1} T^{-1}$ is the mean-free-path, we therefore find

$$\frac{\Omega^{(\text{visc})}}{\Omega^{(\text{coll})}} \simeq \frac{v_{\beta, \lambda}^2}{\rho_{\text{tot}}} \bigg|_{k = \lambda_f^{-1}} \ll 1,$$

where we have used (11, 12, 13) and the final estimate takes into account that $v_{\beta, \lambda}^2$ is a small perturbation during inflation.

The energy radiated in gravitational waves from viscosity is therefore smaller than the one coming from bubble collisions.

4.5.4 Thermal fluctuations

There are also other possibilities for generating (hyper)magnetic fields in a plasma, beside local currents from bubble dynamics: for example quantum and thermal fluctuations. In both cases the generated fields can source gravitational waves.

A complete analysis of these sources would go beyond the scope of this paper, as they are not strictly present only when first order transitions occur and the results would depend on specific details of the models.
In fact, quantum fluctuations can generate gauge fields extended at appreciable scales only when their couplings to the metric are not conformal symmetric (see for example [24] and reference therein). On the other hand, the excitation of gauge fields via thermal fluctuations has been studied in flat Minkowski space via thermal field theory or the Boltzmann equation [25], and the extension of those results to a rapidly expanding Universe is not straightforward.

5 Detection

Our analysis indicates that the dominant contribution to the spectrum of gravitational waves from first order phase transitions during inflation, is sourced by collisions of bubbles and has the spectrum [33].

We will now discuss the possibility of detection of this spectrum both in the CMBR anisotropies and by direct measurement at interferometers. Although detailed, our analysis will not be precise down to numerical factors of order one, which depend on the particular high-energy models.

5.1 CMBR

Primordial gravitational waves affect both the temperature anisotropies and the polarization of the CMBR. Experiments such as CMBPol and Planck will investigate these observables in the near future. Here, we will concentrate on the temperature anisotropies, considering the so-called tensor (T)-to-scalar (S) ratio

\[ r = \frac{C_T^T}{C_T^S}, \]  

where \( C_T^T, C_T^S \) come from the decomposition of the spectrum of temperature anisotropies in two directions \( \mathbf{l}_1, \mathbf{l}_2 \) with \( \mathbf{l}_1 \cdot \mathbf{l}_2 = \cos(\theta) \),

\[ \langle \frac{\delta T}{T}(\mathbf{l}_1) \frac{\delta T}{T}(\mathbf{l}_2) \rangle = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell + 1)(C_{T}^{T} + C_{T}^{S})P_\ell(\cos \theta). \]  

The precise computation of \( r \) would require a numerical evaluation of (49) using the spectrum [33]. However, for our purposes it suffices an approximate analytical computation. We follow [26], assuming \( \ell \gg 1^{22} \).

Using \( k = \frac{H_0}{2} \), \( a_0 = 1 \), we obtain

\[ \ell (\ell + 1) C_{T}^{T} \sim \frac{5\pi^2}{64} P_{h}^{\text{sup}} \left( \frac{\ell H_0}{2} \right), \]  

and therefore, for the spectrum [33] and \( \ell (\ell + 1) C_{T}^{S} = \frac{4}{254 \pi^2 c_s c M_{\text{Plank}}^2} H_0^2 \),

\[ r \sim 12.3 c_s \varepsilon \sum_{n}^{N} A_n \frac{\ell \beta_n}{\ell}, \quad \ell \beta_n = \frac{2\beta_n}{H_0}, \]  

\(^{22}\)Recall that the multipoles we can consider are such that \( \ell < 200 \).
where we have considered only wavenumbers in the range $a_n H < k < \tilde{\beta}_n$ since the contribution from larger wavenumbers is very suppressed (see (33)).

Equation (51) is very different from the result for tensor modes generated by vacuum fluctuations in single-field slow-roll/chaotic models, which would be $r = 8\epsilon$: indeed, (51) shows i) a dependence on the multipole $\ell$ due to the lack of scale invariance in (33), ii) an additional suppression factor $A_n$ (31), and iii) a possible accumulation due to many phase transitions.

We would like to understand how many transitions are necessary to have a detectable amount of tensor modes (we require $r \sim 0.07$). To proceed, we will make a simplifying assumption: a negligible dependence on $n$ for the ratio $\frac{\tilde{\beta}_n}{H_n}$. We will also take $\epsilon \sim 10^{-2}$ as a typical value, for illustrative purposes.

We also limit our estimate to the multipoles corresponding to the frequencies $p = \tilde{\beta}_n$, which is the typical one of the collisions, and $p \simeq H_n$, which is the lower limit of validity for the numerical simulations we have extended. We indicate these multipoles in the formulas as $\ell_* = \{ \ell_{\tilde{\beta}_n}, \ell_{H_n} \}$. For these specific frequencies, also because of the simplifying assumption we have made above, the ratio $\frac{\ell_{\tilde{\beta}_n}}{\ell_*}$ is independent of $n$.

We define

$$
\sum_{n=1}^{N} A_n \frac{\ell_{\tilde{\beta}_n}}{\ell_*} \equiv A \frac{\ell_{\tilde{\beta}_n}}{\ell_*} \simeq 0.07 N 10^8 \varepsilon^{-1} \left( \frac{H}{\tilde{\beta}_n} \right)^8 \varepsilon^2 \frac{\ell_{\tilde{\beta}_n}}{\ell_*},
$$

(52)

where we have used (12), (34) and $\ell_{\tilde{\beta}} = \frac{2\tilde{\beta}}{H_0}$.

Taking into account (11), $A$ lies in the range

$$1.1 \times 10^{-6} N \varepsilon \left( \frac{S_E}{\pi^2} \right)^{\frac{\chi}{2}} \lesssim A \lesssim 0.07 N \varepsilon
$$

(53)

and therefore the scalar to tensor ratio (51) falls within the interval

$$1.5c_s 10^{-5} N \varepsilon^2 \left( \frac{S_E}{\pi^2} \right)^{\frac{\chi}{2}} \frac{\ell_{\tilde{\beta}_n}}{\ell_*} \leq r \leq 0.9c_s N \varepsilon^2 \frac{\ell_{\tilde{\beta}_n}}{\ell_*}.
$$

(54)

If $k = \tilde{\beta}$, we see that $N \sim 7.7 \times 10^2$ transitions with the minimal allowed transition rate $\left( \frac{\beta}{H} \sim 10 \right)$ would be sufficient to have $r \sim 0.07$. But the number of transitions necessary for detection rapidly increases for faster rates, up to $N \sim 3 \times 10^8$ for the fastest rate in (11).

If, instead, it is $k = a_n H_n$, we find that $N \gtrsim 77$ transitions with slow rate $\left( \frac{\beta}{H} \sim 10 \right)$ would be detectable via the CMBR, but for the fastest transitions we need at least $N \sim 6 \times 10^6$ of them.

It appears therefore that a not too large number of transitions occurring at a slower rate could be detectable, especially for lower multipoles. However, as the rate of the transitions increases, the spectrum rapidly decreases and becomes undetectable.

Note that in many of the studied models of transitions outside inflation it is found that the ratio $\frac{\beta}{H}$ is larger than $\frac{\beta}{H} \sim 10$. However, the situation for first order transitions in models and theories valid at the scales of inflation could be different. Because of our limited knowledge of the relevant theories at very high energy, such as the string landscape, we cannot discard those values of $\frac{\beta}{H}$. It is therefore important to analyze this point in future research within concrete models of interest.
The suppression of the emission from fast transitions also occur for transitions not during inflation, but in the inflationary scenario the phenomenon is much more pronounced as the suppression factor goes like \((\frac{H}{\beta})^8 \varepsilon^2\) for each transition, see (33).

5.2 Direct detection

We will investigate the direct detectability of the gravitational signal by interferometers considering the cases of LIGO, LISA and DECIGO\(^{23}\). To do so, we need to evolve the spectrum after inflation ends, from the moment when the mode reenters the horizon until now.

Since, differently from the previous sections, we are now evolving the waves also after inflation, we will indicate as \(H_{\text{infl}}\) the Hubble rate during inflation, to avoid any possible confusion.

From (21), we see that after reentering the horizon, \(h\) evolves as \(\sim a^{-1}\). We define the transfer function \(T(p) = \frac{a_p}{a_0}\), where \(a_p\) is the scale factor at the time of reentering for the physical momentum \(p\) measured today. As discussed in \(^{27}\), the sensitivities of LIGO, LISA, and DECIGO peak around frequencies which had to be within the horizon well before matter-radiation equality and nucleosynthesis. Therefore, assuming adiabatic expansion after the end of inflation, \(^{27}\)

\[
T(p) = 2.1 \times 10^{-20} \left(\frac{0.63\, \text{Hz}}{p}\right) \left(\frac{100}{\kappa_p}\right)^{\frac{1}{2}}
\]

(55)

where \(\kappa_p\) is the number of effective relativistic degrees of freedom contributing to the energy density at the moment of the reentry of the scale \(p^{-1}\).

We discuss the possible detection of the primordial waves in terms of the strain amplitude\(^{24}\)

\[
\tilde{h}_p = T(p) \sqrt{\frac{\pi P_{\text{super}}(p)}{2p}}.
\]

(56)

By using (55) and (33), we find

\[
\tilde{h}_p = 2.1 \times 10^{-20} \left(\frac{0.63\, \text{Hz}}{p}\right) \left(\frac{100}{\kappa_p}\right)^{\frac{1}{2}} \sum_n \frac{H_{\text{infl}}}{\sqrt{\pi} M_{\text{Planck}}} \sqrt{\frac{A_n}{p}} \sqrt{\frac{f_{\beta_n,0}}{p}}
\]

(57)

where we have considered only wave-numbers \(a_n H_{\text{infl}} < k < \tilde{\beta}_n\), since the contribution from larger ones is very suppressed, and we sum over the different transitions.

Here,

\[
f_{\beta_n,0} = \frac{a_n \beta_n}{a_0} = \beta_n e^{-N_n} 8.0 \times 10^{-14} \left(\frac{100}{\kappa_{\text{end}}}\right)^{\frac{1}{2}} \frac{1\, \text{GeV}}{T_{\text{end}}},
\]

(58)

\(N_n\) is the number of e-foldings from the moment of collision of bubbles of the \(n \to n-1\) transition until the end of inflation, and \(T_{\text{end}}, \kappa_{\text{end}}\) are evaluated at the end of inflation. In the following we will assume \(\kappa_n, \kappa_p, \kappa_{\text{end}} \approx 10^2\) for illustrative reasons. These are also the typical values in GUTs and minimal supersymmetric models.

\(^{23}\)We will actually consider Ultimate DECIGO.

\(^{24}\)Obtained re-writing formula (3.2) in \(^{28}\) with our conventions in \(^{17}, \, \, 18\). See also section 2.2 in \(^{28}\).
To evaluate (57, 58), we need to compute the temperature $T_{\text{end}}$, which is given by

$$T_{\text{end}} = \sum_{n=1}^{N} T_n e^{-N_n} + T_f.$$  \hfill (59)

Here $T_n \sim T$ is the temperature of a single phase transition, see (13), and $T_f$ comes from a possible final decay of the inflaton (reheating), which could be also one last phase transition with large backreaction.

We can perform the sum in (59) by considering that the last transition occurred $N_{\text{last}}$ e-foldings before the end of inflation and that the earlier transitions and collisions times were spaced out by intervals of $\Delta N_n \approx \Delta N$ e-foldings. Then we define

$$\sum_{n=1}^{N} e^{-N_n} = e^{-N_{\text{last}}} \frac{1 - e^{-N\Delta N}}{1 - e^{-\Delta N}} \equiv F(N, \Delta N, N_{\text{last}}).$$  \hfill (60)

Assuming finally, for simplicity, that $T_f < \sum_n T_n e^{-N_n}$, we obtain

$$T_{\text{end}} \approx T F(N, \Delta N, N_{\text{last}}).$$  \hfill (61)

Let us recall now that (33) is valid only for $p_n \geq H_n$. These high frequencies could fall within the range of sensitivity of the interferometers only after sufficient redshift. The sum over transitions in (57) will therefore start from the transition $n_{\text{min}}$ for which at least the smallest frequency that we can consider at the time of production $t_{n_{\text{min}}}$ has had the necessary redshift until now. That frequency is $p^{H_{n_{\text{min}}}}(t_{n_{\text{min}}}) = H_{n_{\text{min}}} \sim H_{\text{infl}}$, which is redshifted today to

$$p^{H_{n_{\text{min}}}}(t_0) = \frac{a_{n_{\text{min}}}}{a_0} p^{H_{n_{\text{min}}}}(t_{n_{\text{min}}}) = 1.2 \times 10^{19} e^{-N_{n_{\text{min}}}} F(N, \Delta N, N_{\text{last}}) \text{ Hz}$$  \hfill (62)

Comparison with experimental setups

We are now ready to compare our results with the sensitivities of LIGO, LISA and Ultimate DECIGO. The latter peak around certain frequencies as follows [28, 29]:

LIGO : $\bar{h}_f \sim 10^{-23} \text{ Hz}^{-\frac{1}{2}}$ at $f \sim 100 \text{ Hz}$ \hfill (63)

LISA : $\bar{h}_f \sim 4 \times 10^{-21} \text{ Hz}^{-\frac{1}{2}}$ at $f \sim 10^{-3} \text{ Hz}$ \hfill (64)

UDECIGO : $\bar{h}_f \sim 10^{-27} \text{ Hz}^{-\frac{1}{2}}$ at $f \sim 0.1 \text{ Hz}$. \hfill (65)

The results we list in table 1 have been obtained by asking for the strain amplitude (57) to be within the sensitivity of the detectors when evaluated at the respective peak frequencies. The strain is computed using (57, 58, 61, 62). We have chosen $S_E \approx O(1), \varepsilon \sim 10^{-2}$ as indicative values and assumed for $\beta_n$ a negligible dependence on $n$. The range of values considered for $A = \sum_n A_n$ is given by [28].

The detectability of the emitted waves depends on the number of transitions, their timescales and the number of e-foldings that must have been occurred to sufficiently redshift their frequencies after the waves were emitted. These quantities enter the strain through the function
Table 1: Bounds on $N, n_{\text{min}}, \Delta N$ via the function $F$ for the direct detection of primordial gravitational waves from phase transitions during inflation.

| $N - n_{\text{min}}$, $\Delta N/2$, 0 | Detectability | $N - n_{\text{min}}$, $\Delta N/2$, 0 |
|----------------------------------------|---------------|----------------------------------------|
| No if                                  | Yes for all possible $\frac{\beta}{H}$ if |
| LIGO 100                               | $F(N - n_{\text{min}}, \Delta N/2, 0) < 1.6 \times 10^6$ | $F(N - n_{\text{min}}, \Delta N/2, 0) \geq 4.4 \times 10^8$ |
| LISA $10^{-3}$                         | $F(N - n_{\text{min}}, \Delta N/2, 0) < 19.7$ | $F(N - n_{\text{min}}, \Delta N/2, 0) \geq 5.5 \times 10^3$ |
| UDECIGO 1                              | $F(N - n_{\text{min}}, \Delta N/2, 0) < 4.9 \times 10^{-3}$ | $F(N - n_{\text{min}}, \Delta N/2, 0) \geq 1.4$ |

To understand better what these results mean for the physical parameters, we specialize to the case of chain inflation \[9\] as an illustrative example. This one is a fast tunneling model, where inflation stops shortly after one last phase transition (so that $N_{\text{last}} \sim 0$) and where $N \gg 1$ and $\Delta N \sim \frac{H_{\text{infl}}}{\beta}$. From (60), in this case

\[
F(N - n_{\text{min}}, \Delta N/2, 0) \sim \begin{cases} 
2 \frac{\beta}{H_{\text{infl}}} & \text{for } N - n_{\text{min}} \gg \frac{\beta}{H_{\text{infl}}} \text{ and } \frac{\beta}{H_{\text{infl}}} \gg 1 \\
N - n_{\text{min}} & \text{for } \frac{\beta}{H_{\text{infl}}} \gg N - n_{\text{min}}, \frac{\beta}{H_{\text{infl}}} \gg 1
\end{cases}.
\] (66)

Using (66), (11) and table 1 we see that, for instance, in the case $N - n_{\text{min}} \gg \frac{\beta}{H_{\text{infl}}}$ (many transitions), the gravitational waves produced by bubble collisions in chain inflation will be

- detectable at Ultimate DECIGO for all allowed values of $\frac{\beta}{H_{\text{infl}}}$
- detectable at LISA for $\frac{\beta}{H_{\text{infl}}} < 11$.

In particular, if we take the most favorable case\[26\], $\frac{\beta}{H_{\text{infl}}} \approx 10$, the detected waves would have been emitted by phase transitions occurring at least 20 e-foldings before the end of inflation for Ultimate DECIGO, or 27 for LISA. Unfortunately, the gravitational waves would be undetectable at LIGO.

If detected, these waves could be distinguished from those of transitions occurring outside inflation, thanks to the different frequency dependence of the spectrum of modes that were superhorizon at inflation.\[27\]

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\[25\] Here, we have made the simplifying assumption that the intervals of e-foldings are approximately the same for all transition. The crudeness of such an approximation can be determined only by building detailed models, but this goes beyond the scope of this paper.

\[26\] Recall the comments at the end of section 5.1.

\[27\] Note however that the strain amplitude for these waves decays more rapidly with the frequency: as $p^{-2}$ for frequencies smaller than the redshifted scale of the transition and as $p^{-4}$ for larger frequencies. The signal from transitions not during inflation goes instead like $p^{-1}$ and $p^{-3}$ respectively in the two ranges of frequencies.
5.3 Nucleosynthesis bound

There is an important constraint on gravitational emission: the waves that reentered the horizon before nucleosynthesis must not interfere with it, and therefore satisfy

$$\tilde{h}_p < 1.99 \times 10^{-21} \frac{1 \text{Hz}}{p^2}.$$ (67)

We study this constraint just in the case of chain inflation, as an example. From formulas (57), (66), we see that in that case (67) implies

$$\frac{\beta}{H_{\text{infl}}} \geq 0.21 \quad \forall p,$$ (68)

which is certainly satisfied by the range (11) of allowed values for $\frac{\beta}{H_{\text{infl}}}$. We conclude that the bound from nucleosynthesis does not rule out chain inflation.

6 Conclusions

In this work we have studied the production, features and detectability of gravitational waves in models of the early Universe where first order phase transitions occur during inflation. We have described these scenarios via some physical parameters, whose values have been constrained and bounded by an analysis of the self-consistency of the theory (in particular efficient inflation taking place, homogeneity and isotropy at large scales).

The emission and features of gravitational waves are strongly affected by these bounds and by the specific dynamics during inflation (such as the exit from horizon of the modes). The resulting spectrum is different from the one due to vacuum oscillations or first order phase transitions occurring not during inflation.

The first important feature is that the waves from a single transition during inflation are very much suppressed but the accumulation due to many transitions could make them sizable. Second, turbulence and (hyper)magnetic fields are a negligible source of waves, contrary to what generally happens when the transitions occur not during inflation. The collisions of the bubbles at the end of the transitions represent the prominent source of waves, yielding a non-scale-invariant spectrum with different frequency dependence for modes that exit horizon during inflation, compared to the spectrum sourced by transitions not occurring during inflation.

We have also studied the experimental detectability of the waves. The main points emerging from this part of the analysis are that:

- a not too large number of slow (but still successful) transitions occurring during the inflationary era could leave observable marks in the CMBR anisotropies (large tensor-to-scalar ratio for reasonable values of the parameters). As the fastness of the transitions increases, the signal rapidly weakens, requiring accumulation from a large number of them to be measurable,

- direct detection via interferometers could be possible at LISA and Ultimate DECIGO for modes that were superhorizon at inflation and for a large number of transitions. However, at LISA the detection could occur only for the most optimistic scenario (slow transitions). We could distinguish transitions during inflation from those outside inflation thanks to the different frequency dependence of the spectra,
• the nucleosynthesis bound is easily satisfied by the models, in particular chain inflation is not ruled out.

A Appendices

A.1 Friedman and Chaudhuri equations

With a Friedman-Robertson-Walker ansatz
\[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \]
for the background metric, the Friedman and Chaudhuri equations read
\[ H^2 = \frac{8\pi G}{3} \sum_\ell \rho_\ell = \frac{8\pi G}{3} \rho_{\text{tot}} \]
\[ \dot{H} = -4\pi G \sum_\ell (\rho_\ell + P_\ell) , \]
where \( \rho_\ell, P_\ell \) are respectively the energy and pressure density of the component \( \ell \) of the Universe (in our case scalar fields, radiation). The sum is over all components. Note that during inflation \( \rho_{\text{tot}} \sim \rho_{\text{vacuum}} \).

The conformal time is related to the cosmic one by \( d\eta = \frac{dt}{a(t)} \).

A.2 Timescale of transitions

We call \( p_n(t) \) the vacuum persistence probability, that is the probability for a point in the Universe to remain in the \( n \)-th vacuum at time \( t \). By neglecting the possibility to tunnel directly to distant phases, it obeys the most general equation
\[ \dot{p}_n = -\tilde{\Gamma}_n p_n + \tilde{\Gamma}_{n+1} p_{n+1} \]

The conclusion of the phase transition is indicated by several markers \[9, 14\]:

\( i \) the time \( t_{c,n} \), when the patch of the Universe occupied by the old phase starts to contract,
\( ii \) the time \( t_{p,n} \), when percolation occurs,
\( iii \) the time \( t_{s,n} \), when the probability \( p_n(t) \) for a point to remain in phase \( n \) at time \( t \) has dropped below a suitable small number.

This last requirement is actually not a faithful signal of the completion of the phase transition (recall for example the issues in Old Inflation), but it yields an indication that is generally in accordance with the other two more significant conditions, when they occur, beside being physically reasonable. For a rapid transition it can be shown that \( t_{s,n} \gtrsim t_{c,n}, t_{p,n} \).

The time-scale of the transition can be defined as \[14\]
\[ \beta_n^{-1} \sim t_{s,n} - t_{i,n} \]
where \( t_{s,n}, t_{i,n} \) are such that\[28\]
\[ p_n(t_{s,n}) = e^{-M} \ll 1 \quad p_{n+1}(t_{s,n}) = e^{-M-q} \ll 1 \]
\[ p_n(t_{i,n}) = e^{-m} \sim 1 \quad p_{n+1}(t_{i,n}) = e^{-M+q'} \ll 1 \]

\[28\]Note that also the earlier and successive phases are sub-dominant at \( t_{i,n} \), but in order to estimate \( \beta_n^{-1} \) we need to consider only the phases \( n \) and \( n+1 \).
for suitable \(M, M - q, m + q' \gg 1, \ m < 1\).

If we have \(\bar{\Gamma}_n > H\) from the onset, and therefore the phase transitions are occurring very rapidly, we can expand
\[
\rho_n(t_{s,n}) \sim \rho_n(t_{i,n}) + \dot{\rho}_n(t_{i,n})(t_{s,n} - t_{i,n})
\]
and from (72), we obtain
\[
\rho_n(t_{s,n}) - \rho_n(t_{i,n}) = e^{-M} - e^{-m} \tag{77}
\]
\[
\left(-\bar{\Gamma}_n(t_{s,n} - t_{i,n}) + 1\right) e^{-m} = \left(-\bar{\Gamma}_{n+1} e^{q}(t_{s,n} - t_{i,n}) + 1\right) e^{-M}. \tag{78}
\]
Since \(e^{-M} \ll 1\),
\[
t_{s,n} - t_{i,n} \approx \bar{\Gamma}_n^{-1} \equiv \beta_n^{-1}. \tag{79}
\]

For tunneling rates depending on time, expanding the tunneling action/free energy around \(t_{s,n}\) as \(S^{(n)}_E(t) \simeq S^{(n)}_E(t_{s,n}) - \beta_n (t - t_{s,n})\), one can also find for the decay rate per unit time and volume [14]
\[
\Gamma_n = C e^{-S^{(n)}_E(t)} \quad \beta_n = -\frac{dS^{(n)}_E}{dt}\bigg|_{t_{s,n}}. \tag{80}
\]
As we see from (79, 80), \(\beta_n\) is therefore directly related to the fundamental physics governed by the tunneling action.

Finally, the time-scale of the transition in terms of conformal time is
\[
\bar{\beta}_n^{-1} = \eta_{s,n} - \eta_{i,n} = a(t_n)^{-1} \beta_n^{-1}. \tag{81}
\]

A.3 Quantum vacuum fluctuations

As a useful reference and comparison, we report here the spectrum of the gravitational waves generated via fluctuations in vacuum of the gravity field during inflation. The wave equation is the homogeneous version of (16), with \(\pi_{ij}^T = 0\). Its solution leads to the spectrum for superhorizon modes
\[
P_{h}^{supQ} = \frac{2}{\pi^2} \left( \frac{H}{M_{\text{Planck}}} \right)^2 \tag{82}
\]
using the Bunch-Davies vacuum. \(H\) is the Hubble parameter during inflation.

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