Fermion confinement induced by geometry

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We consider a five-dimensional model in which fermions are confined in a hypersurface due to an interaction with a purely geometric field. Inspired by the Rubakov-Shaposhnikov field-theoretical model, in which massless fermions can be localized in a domain wall through the interaction of a scalar field, we show that particle confinement may also take place if we endow the five-dimensional bulk with a Weyl integrable geometric structure, or if we assume the existence of a torsion field acting in the bulk. In this picture, the kind of interaction considered in the Rubakov-Shaposhnikov model is replaced by the interaction of fermions with a geometric field, namely a Weyl scalar field or a torsion field. We show that in both cases the confinement is independent of the energy and the mass of the fermionic particle. We generalize these results to the case in which the bulk is an arbitrary n-dimensional curved space.

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I. INTRODUCTION

It has been suggested in recent times that our world may be viewed as a hypersurface (brane) $\Sigma$ embedded in a higher-dimensional manifold $M$, often referred to as the bulk [1]. As far as the geometry of this hypersurface is concerned, it has been generally assumed that it has a Riemannian geometrical structure. This hypothesis has the great advantage of avoiding possible conflicts with the well-established theory of general relativity which is formulated in a Riemannian geometrical setting. Likewise there has not been much discussion on what kind of geometry the bulk possesses, which is generally supposed to be also Riemannian. Some attempts to broaden this scenario have appeared recently in the literature, where non-Riemannian frameworks, in particular, Weyl and Riemann-Cartan geometries, are taken into consideration as viable possibilities to describe the geometry of the bulk [2–5].

In some brane-world scenarios it is postulated that the particles and fields of the standard model are confined to the brane universe. Classically, test particles are confined in spacetime if the hypersurface $\Sigma$ has vanishing extrinsic curvature $\vec{e}$. This kind of confinement is purely geometrical, which means that the only force acting on the particles is the gravitational force. Classical confinement has been investigated recently in different contexts \cite{6,7}. At the quantum level, the stability of the confinement of matter fields is made possible by assuming an interaction of matter with a scalar field. An example of how this mechanism works is clearly illustrated by a field-theoretical model devised by Rubakov and Shaposhnikov, in which fermions may be trapped to a brane by interacting with a scalar field that depends only on the extra dimension $\vec{e}$. On the other hand, it has been shown that in a purely classical (non-quantum) picture, there are geometrical mechanisms which play the role of a quantum scalar field so as to constrain massive particles to move on hypersurfaces in a stable way. For instance, it has been shown that by modelling the bulk as a five-dimensional manifold endowed with some kind of non-Riemannian connection we may provide the mechanism necessary for confinement and stabilization of the motion of particles in the brane in a purely geometrical way \cite{8,9}.

The paper is organized as follows. In Sections II and III, we present the basic mathematical facts that define the geometries of Weyl and Riemann-Cartan, respectively. We proceed in Section IV to briefly describe the Rubakov-Shaposhnikov model with its mechanism of fermion confinement. In Section V, we discuss the extension of the Dirac equation to a class of non-Riemannian geometries. In Sections VI and VII, we investigate the confinement of fermions induced by geometrical fields, namely, a Weyl scalar field and a torsion field. We conclude with a few remarks in Section VIII.
II. WEYL GEOMETRY

We can say that the geometry conceived by Weyl is a kind of simple generalization of Riemannian geometry in the sense that in the former one replaces the assumption that the covariant derivative of the metric tensor $g$ is zero by the more general condition

$$\nabla_a g_{bc} = \sigma_a g_{bc}$$

where $\sigma_a$ denotes the components with respect to a local coordinate basis of a one-form field $\sigma$ defined on $M$. This, in fact, represents a generalization of the Riemannian condition of compatibility between the connection $\nabla$ and $g$, which is equivalent to requiring the length of a vector to remain unaltered by parallel transport. If $\sigma = d\phi$, where $\phi$ is a scalar field, then we have what is called an integrable Weyl geometry. A differentiable manifold $M$ endowed with a metric $g$ and a Weyl scalar field $\phi$ is usually referred to as a Weyl frame. It is interesting to note that the Weyl condition (1) remains unchanged when we go to another Weyl frame $(M, g, \phi)$ by performing the following simultaneous transformations in $g$ and $\phi$:

$$g = e^{-\phi} g$$

$$\phi = \phi - df$$

where $f$ is a scalar function defined on $M$.

Quite analogously to Riemannian geometry, the condition (1) is sufficient to determine the Weyl connection $\nabla$ in terms of the metric $g$ and the Weyl one-form field $\sigma$. Indeed, a straightforward calculation shows that one can express the components of the affine connection with respect to an arbitrary vector basis completely in terms of the components of $g$ and $\sigma$:

$$\Gamma^a_{bc} = \{^a_{bc}\} - \frac{1}{2} g^{ad}[g_{db,c} + g_{dc,b} - g_{bc,d}]$$

where $\{^a_{bc}\} = \frac{1}{2} g^{ad}[g_{db,c} + g_{dc,b} - g_{bc,d}]$ represents the Christoffel symbols, i.e., the components of the Levi-Civita connection.

A clear geometrical insight on the properties of Weyl parallel transport is given by the following proposition: Let $M$ be a differentiable manifold with an affine connection $\nabla$, a metric $g$ and a Weyl field of one-forms $\sigma$. If $\nabla$ is compatible with $g$ in the Weyl sense, i.e. if (1) holds, then for any smooth curve $\alpha = \alpha(\lambda)$ and any pair of two parallel vector fields $V$ and $U$ along $\alpha$, we have

$$\frac{d}{d\lambda} g(V, U) = \sigma(\frac{d}{d\lambda}) g(V, U)$$

where $\frac{d}{d\lambda}$ denotes the vector tangent to $\alpha$.

If we integrate the above equation along the curve $\alpha$, starting from a point $P_0 = \alpha(\lambda_0)$, then we obtain

$$g(V(\lambda), U(\lambda)) = g(V(\lambda_0), U(\lambda_0)) e^{\int_{\lambda_0}^{\lambda} \sigma(\frac{d}{d\lambda}) d\lambda}$$

Putting $U = V$ and denoting by $L(\lambda)$ the length of the vector $V(\lambda)$ at an arbitrary point $P = \alpha(\lambda)$ of the curve, then it is easy to see that in a local coordinate system $\{x^a\}$ the equation (5) reduces to

$$\frac{dL}{d\lambda} = \frac{\sigma_a}{2} \frac{dx^a}{d\lambda} L$$

Consider the set of all closed curves $\alpha : [a, b] \in R \rightarrow M$, i.e., with $\alpha(a) = \alpha(b)$. Then, we have the equation

$$g(V(b), U(b)) = g(V(a), U(a)) e^{\int_{a}^{b} \sigma(\frac{d}{d\lambda}) d\lambda}.$$
It follows from Stokes’ theorem that if \( \sigma \) is an exact form, that is, if there exists a scalar function \( \sigma \), such that \( \sigma = \frac{d\phi}{dx} \), then

\[
\oint \sigma \left( \frac{d}{dx} \right) d\lambda = 0
\]

for any loop. In other words, in this case the integral \( e^{I(a,b)} \sigma \left( \frac{d}{dx} \right) dp \) does not depend on the path.

Historically, the geometrical theory developed by Weyl was used as a framework of his gravitational theory, one of the first attempts to unify gravity and electromagnetism. As is well known, although admirably ingenious, Weyl’s gravitational theory turned out to be unacceptable as a physical theory, as was immediately realized by Einstein who raised objections to the theory [12, 13]. Einstein’s argument was that in a non-integrable Weyl geometry the existence of sharp spectral lines in the presence of an electromagnetic field would not be possible since atomic clocks would depend on their past history [12]. However, it has been shown that a variant of Weyl geometries, known as Weyl integrable geometry, does not suffer from the drawback pointed out by Einstein. Indeed, it is the integral \( I(a,b) = \int_a^b \sigma \left( \frac{d}{dx} \right) d\lambda \) that is responsible for the difference between the readings of two identical atomic clocks following different paths. Because in Weyl integrable geometry \( I(a,b) \) is not path-dependent it has attracted the attention of many cosmologists in recent years [14]. In the opinion of some authors, Weyl theory “contains a suggestive formalism and may still have the germs of a future fruitful theory” [12].

### III. RIEMANN-CARTAN GEOMETRY

Another example of non-Riemannian geometry is the so-called Riemann-Cartan geometry, which also represents one of the simplest generalizations of Riemannian geometry. It constitutes the geometrical framework of a theory formulated by E. Cartan [17] in an attempt to extend general relativity when matter with spin is present. In spite of the limited interest it has arisen among theoretical physicists since its conception (perhaps due to the fact that it differs very little from general relativity), some authors believe that the Einstein-Cartan theory can have an important role in a future quantum theory of gravitation [17]. Moreover, torsion cosmology has been investigated recently in connection with the acceleration of the Universe [18]. Finally, it should be mentioned that torsion has been also considered in the context of higher-dimensional scenarios, particularly in Kaluza-Klein theory and brane models [3].

A concept that is basic to the Riemann-Cartan geometry is that of torsion, which is given by the following definition: Let \( \nabla \) be an affine connection defined on a manifold \( M \) and \( U, V \in T(M) \), where \( T(M) \) denotes the set of all tangent vector fields of \( M \). We define the torsion \( T \) of \( M \) as the mapping \( T : T^2(M) \times T(M) \to T(M) \), such that

\[
T(U,V) = \nabla_U V - \nabla_V U - [U,V].
\]

(7)

If the torsion vanishes identically we say that the affine connection \( \nabla \) is symmetric (or, simply, torsionless). A manifold on which a nonvanishing torsion is present is called a Riemann-Cartan manifold. Now, to establish a link between the affine connection \( \nabla \) and the metric \( g \) we need a second definition: Let \( M \) be a differentiable manifold endowed with an affine connection \( \nabla \) and a metric tensor \( g \) globally defined in \( M \). We say that \( \nabla \) is compatible with \( g \) if, for any vector fields \( U, V, W \in T(M) \), the condition below is satisfied:

\[
\nabla[U,V] = g(\nabla V U, W) + g(U, \nabla V W).
\]

(8)

We now state an important result, which may be called the Levi-Civita extended theorem: In a given differentiable manifold \( M \) endowed with a metric \( g \) on \( M \), there exists only one affine connection \( \nabla \) such that \( \nabla \) is compatible with \( g \) (10). In other words, this means that the affine connection \( \nabla \) is uniquely determined from the metric \( g \) and the torsion \( T \). (In the torsionless case, \( \nabla \) is determined from \( g \) alone [19].)

Now, a tensor that is naturally associated with \( T \) is the torsion tensor \( \mathcal{T} \), defined by the mapping \( T^* : T^3(M) \times T^2(M) \to R \), such that \( \mathcal{T}(w, U, V) = \tilde{w}(T(U, V)) \), where \( T^*(M) \) denotes the set of all differentiable one-form fields on \( M \) and \( \tilde{w} \in T^*(M) \). It is easy to see that the components of \( \mathcal{T} \) in a coordinate basis associated with a local coordinate system \( \{x^a\} \) are simply given in terms of the connection coefficients, i.e. \( \mathcal{T} \equiv \Gamma^a_{bc} - \Gamma^a_{cb} \), where \( \Gamma^a_{bc} \equiv dx^a(\nabla_{\partial_b} \partial_c) \). A straightforward calculation shows that one can express the components of the affine connection as

\[
\Gamma^a_{bc} = \{\gamma_{bc}\} + K^a_{bc}.
\]

(9)
where \( \{ \gamma^i \} \) denotes the Christoffel symbols of second kind and \( K^a_{bc} = -\frac{1}{2}(\mathcal{T}^a_{cb} + \mathcal{T}^a_{bc} + \mathcal{T}^a_{bc}) \) represents the components of another tensor, called the \textit{torsion tensor} \(^2\).

Thus, we see that what basically makes the geometry discovered by Cartan distinct from Riemannian geometry is simply the fact that in the latter the affine connection \( \nabla \) is not supposed to be symmetric. As a consequence, the affine connection \( \nabla \) is no longer a Levi-Civita connection and for this reason affine geodesics do not coincide in general with metric geodesics.

### IV. FERMIONS CONFINEMENT AND THE RUBAKOV-SHAPOSHNIKOV MODEL

In the context of theories of Kaluza-Klein type the compactness of extra dimensions ensures that spacetime is effectively four-dimensional. On the other hand, in higher-dimensional scenarios that postulate the existence of non-compact dimensions the assumption that matter should somehow be confined to a four-dimensional hypersurface is certainly a prerequisite for the theory to be consistent with observation. One of the first attempts to construct a higher-dimensional model that exhibits a confinement mechanism was devised by Rubakov and Shaposhnikov \(^8\). In this section we give a brief account of this model.

Let us begin by considering a theory of one real scalar field \( \varphi \) whose action is given by

\[
S_\varphi = \int d^4x dl \left[ \frac{1}{2} \partial_a \varphi \partial^a \varphi - V(\varphi) \right],
\]

where \( a = 0, 1, \ldots, 4 \), leading to the equation

\[
\Box \varphi + \frac{dV}{d\varphi} = 0 . \tag{10}
\]

The scalar field potential is assumed to have the form \( V(\varphi) = \lambda (v^2 - \varphi^2)^2 \), and if \( \varphi \) is assumed to depend only on the extra coordinate \( l \), then (10) yields the classical solution (often referred to as a \textit{kink})

\[
\varphi(l) = v \tanh(2\lambda v).
\]

As is well known, this solution describes a domain wall separating two classical vacua of the model and has the following asymptotic behaviour

\[
\varphi(l \to \pm \infty) = \pm v.
\]

The basic idea in the Rubakov-Shaposhnikov model is to introduce a fermionic field \( \Psi \), in five-dimensional Minkowski spacetime, that interacts with the scalar field \( \varphi \) (Yukawa-type interaction) through the action

\[
S_\Psi = \int d^4x dl \left[ i \bar{\Psi} \gamma^a \partial_a \Psi - h \varphi \bar{\Psi} \Psi \right], \tag{11}
\]

where \( h \) is the coupling constant, and the matrices \( \gamma^0, \gamma^1, \gamma^2, \gamma^3 \) are the standard Dirac matrices in four dimensions, with \( \gamma^4 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \). It is worth noting that in each of the scalar field vacua, i.e. when \( \varphi = \pm v \) the fermions acquire a mass given by \( m = hv \).

The Dirac equation resulting from the action (11) is

\[
i \gamma^a \partial_a \Psi - h \varphi \Psi = 0 . \tag{12}
\]

It turns out that for \( m = 0 \) there exists a zero mode solution, which is given by

\[
\Psi_0 = \Psi_w(p) e^{-\int_0^l dy h \varphi(y)} , \tag{13}
\]

where \( \Psi_w(p) \) is the usual solution of the four-dimensional Weyl equation \(^8\). Note that for large values of \( |l| \) we have

\[
\Psi_0 \propto e^{-m|l|}.
\]

It is clear that the zero mode (13) is localized near the hypersurface \( \Sigma \) defined by \( l = 0 \), and the probability of finding the particle outside \( \Sigma \) goes down exponentially. This result is valid for massless fermions (e.g. neutrinos that satisfy the equation \( i \gamma^\mu \partial_\mu \Psi_w = 0 \), where here \( \mu = 0, \ldots, 3 \)). Our purpose now is to show that it is possible to construct field-theoretical models in which fermions are localized by means of pure geometrical fields, i.e. models in which the fermion confinement has a geometrical origin.

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\(^2\) Note that the indices appearing in the components of the torsion are raised and lowered with \( g^{ab} \) and \( g_{ab} \), respectively.
V. DIRAC EQUATION IN NON-RIEMANNIAN GEOMETRIES

In this section we will assume that the geometry of the bulk is modelled by a non-Riemannian five-dimensional manifold $M$ endowed with an affine connection, which has a Weyl or Riemann-Cartan structure. We will assume, as in Rubakov-Shaposhnikov model, the existence of five-dimensional fermions that live in a four-dimensional hypersurface $\Sigma$ embedded in $M$. However, instead of postulating an Yukawa interaction of the fermions with a scalar field $\varphi$, we will seek a geometrical mechanism capable of confining the fermions in $\Sigma$ without the intervention of any other physical field. As far as the trapping mechanism is concerned the new degrees of freedom coming from the non-Riemannian character of $M$ will play the same role as $\varphi$. We thus need to write the Dirac equation in $M$ in such a way as to take into account the fact that the geometry now has a non-Riemannian character. We will then look for solutions of the fermionic field in that geometry that describe confinement in $\Sigma$, corresponding, in a certain way, to a geometric analogue of the Rubakov-Shaposhnikov model.

As is well known, the Dirac equation is a quantum mechanical wave equation proposed by Dirac in 1928 that describes the dynamics of elementary spin $-1/2$ particles in flat spacetime and that is fully consistent with the principles of quantum mechanics and special relativity. In generalizing it to Riemannian curved spacetime, one often uses the minimal coupling procedure (MCP) and assumes that the covariant derivative of the Dirac matrices is zero \[ \nabla \Psi = 0 \], whereas the components of $\eta_{AB}$ are given by $g(e_A,e_B) = \eta_{AB} = \text{diag}(1 - 1 - 1 - 1 - 1)$. This procedure also allows the Dirac equation to be written in curvilinear coordinates and in non-inertial frames. However, if the spacetime is non-Riemannian, we are in presence of an ambiguity. Indeed, in this case we need to decide whether the MCP should be applied to the Dirac Lagrangian, or directly to the field equations \[ \Box \Psi = 0 \]. As it happens, different choices lead to different field equations. In this paper, we will consider, for simplicity, that the minimal coupling of the fermionic field with the geometry is implemented directly in the Dirac equation.

In what follows, we will make use of the moving frame formalism, in which the components of the metric $g$, when written with respect to a vector basis \( \{ e_A \} \), are given by $g(e_A,e_B) = \eta_{AB} = \text{diag}(1 - 1 - 1 - 1 - 1)$. The dual basis of \( \{ e_A \} \) will be denoted by \( \{ \theta^A \} \), whereas the components of $\epsilon_A$ and $\theta^A$ in a coordinate basis will be denoted, respectively, by $e_A^\mu$ and $a^A\mu$. Tetrad indices will be raised and lowered with Minkowski five-dimensional metric tensor $\eta_{AB} = g(e_A,e_B)$.

Now, in the case of a non-Riemannian connection, the application the MCP directly to the Dirac equation will consist of the following general procedure: we start with the Dirac equation written in curved space \[ \Box \Psi = 0 \] and then replace the Riemannian metric connection by a non-Riemannian affine connection.

\[
i\gamma^C \left( \partial_C + A_C \| + \frac{1}{8} \omega_{ACB}[\gamma^A, \gamma^B] \right) \Psi - m \Psi = 0, \tag{14}\]

where $m$ is the fermionic mass, $A_C$ denote the components of an arbitrary vector field $A$ (that can be regarded as an external electromagnetic potential minimally coupled to $\Psi$), $I$ is the unit matrix, $\gamma^A$ are the Dirac matrices, and $\omega_{ACB} = \theta^A(\nabla_C e_B)$ gives the components of the affine connection in a tetrad basis. Considering the rather general case of a non-Riemannian manifold in which the affine connection may depend on both the torsion and the Weyl field, the connection components $\omega_{ACB}$ can be written as

\[
\omega_{ACB} = e_B^\lambda c_{CA\lambda} + \binom{\lambda}{\alpha\mu} e_A^\alpha e_C^\mu + \Gamma_{ACB}^{CB} + K_{ACB}, \tag{15}\]

where

\[
\Gamma_{ACB}^{CB} = -1/2(\sigma_C \eta_{AB} + \sigma_B \eta_{AC} - \eta_{CB} \sigma_A), \tag{16}\]

and

\[
K_{ACB} = -1/2(T_{CBA} + T_{BCA} - T_{ACB}) \tag{17}\]

represents, respectively, the non-metric contributions of the Weyl and contorsion field to the affine connection. Here, the components of the torsion tensor are given by $T_A^{BC} = \theta^A(\mathcal{T}(e_B,e_C))$.

VI. FERMION CONFINEMENT INDUCED BY A WEYL SCALAR FIELD

Working along the same lines of the Rubakov-Shaposhnikov model \[ \tilde{M} \], i.e. considering a five-dimensional torsionless Minkowski space-time $M$ and the same convention for the Dirac matrices in four dimensions, as mentioned above, we will also make the additional assumption that $M$ is a Weyl integrable space. In Cartesian coordinates are $g_{\mu\nu} = \eta_{\mu\nu}$.
so we choose $\epsilon_{A}^{\mu} = \delta_{A}^{\mu}$. In addition, in these coordinates we also have $\{\lambda_{\mu}\} = 0$. Setting $K_{\mu\nu\alpha} = 0$ the general expression \[15\] for the affine connection $\omega_{ABC}$ reduces to $\omega_{ABC} = \Gamma_{ABC}$. By substituting $\omega_{ABC}$ into Eq. \[14\], it is not difficult to see that we finally get 
\[18\]
\[i\gamma^{\mu} \partial_{\mu} \Psi - i\sigma_{\mu} \gamma^{\nu} \Psi - m \Psi = 0,\]
where we have set $A = 0$ as we are not considering electromagnetic interaction, and $\sigma_{\mu}$ denotes the components of $\sigma = d\phi$ with respect to the chosen basis.

We now need to solve the Dirac equation \[18\]. If we define the new variable $\psi$ by writing $\Psi = e^{\phi(x^{\mu})}\psi$, then \[18\] can be put in the simpler form
\[19\]
\[i\gamma^{\mu} \partial_{\mu} \psi - m \psi = 0.\]
This equation is formally identical to the Dirac equation in five-dimensional Minkowski spacetime for a free particle, whose solutions are given, in standard notation, by
\[20\]
\[\psi = u_{\nu} e^{-ip_{\alpha}x^{\alpha}},\]
where the spinors $u_{\nu}$ can be normalized in a Lorentz-covariant way, similarly to the four-dimensional case \[23\]. Because the wave functions \[20\] do not diverge as $l$ goes to $\pm\infty$, it is easy to choose a particular functional form of $\sigma$ such that particle confinement takes place for any given hypersurface $\Sigma$ described by $\sigma = \text{constant}$.

Thus, in turn, gives rise to a torsion field. We then postulating the existence of a Weyl field, we will now assume that the bulk is modeled by a $n$-dimensional Riemann-Cartan manifold $M$ endowed with a non-symmetric affine connection $\nabla$, thus giving rise to a torsion field. We then turn to the general form of Dirac equation \[14\] and set $\sigma_{A} = 0$. In addition, we choose the torsion tensor as given by the special form
\[23\]
\[T^{\alpha}_{\beta\lambda} = f_{\nu}(x) \left(\delta^{\alpha}_{\nu} \delta^{\beta}_{\lambda} - \delta^{\alpha}_{\nu} \delta^{\beta}_{\lambda}\right),\]
where $f = f(x)$ is an arbitrary function to be determined later and $f_{\nu}$ stands for $\partial f/\partial x^{\nu}$. Writing \[17\] in coordinates and taking $T^{\alpha}_{\beta\lambda}$ as in \[23\], we get $K_{\alpha\mu\beta} = f_{\nu} (g_{\alpha\mu} \delta^{\nu}_{\beta} - g_{\alpha\nu} \delta^{\nu}_{\beta}$. It is not difficult to verify that Eq. \[14\] reduces to
\[24\]
\[i\gamma^{\mu} \left(\partial_{\mu} + \Gamma_{\mu} + \frac{n-1}{2} f_{\mu}\right) \Psi - m \Psi = 0,\]
which, in turn, leads again to Dirac equation in a $n$-dimensional Riemannian spacetime
\[24\]
\[i\gamma^{\mu} (\partial_{\mu} + \Gamma_{\mu}) \psi - m \psi = 0,\]
where this time we have put $\Psi = \psi(x)e^{-(n-1)f/2}$. Therefore, we recover the same result obtained in Section VI with the torsion field \[23\] replacing the Weyl scalar field. In this case, it is the torsion field that provides the geometrical mechanism capable of trapping matter to the brane.
VIII. FINAL REMARKS

In this paper, we have shown, inspired by the well-known Rubakov-Shaposhnikov five-dimensional model, how it is possible to trap fermions to a brane through purely geometrical fields. We established this result by allowing the bulk to be described either by a Weyl space or a Riemann-Cartan manifold. In both cases, by considering the Dirac equation in such non-Riemannian geometries, we have shown that with an appropriate choice of the Weyl field or the torsion it is fairly straightforward to construct models in which fermions are localized on a brane. We also have shown that in both cases the confinement is independent of the energy and the mass of the fermionic particle.

The Dirac equation in non-Riemannian geometries has been studied in different contexts by some authors. However, in the present work we have not assumed a priori any particular theory of gravity which takes into account the Weyl field or torsion. In other words, the geometry of the higher-dimensional space is not assumed to come from a particular set of field equations or theory. In a certain way, this seems to give some generality to our results, although, for the sake of completeness, it would be desirable to work out the problem in the setting of a concrete higher-dimensional gravitational theory, a task that we leave for the future.

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