Research on Hedging Ratio of Power Futures Based on 0LS Model

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Abstract. With the further development of power market reform, the demand for energy trading entities to avoid the risk of electricity price is becoming more and more prominent. This paper uses the method of minimizing the variance of the portfolio return rate combined with the electricity futures price data from the European Energy Exchange to estimate the optimal hedging ratio and perform the residual test. Finally, the decision of power producers to use the power futures hedging strategy to avoid the risk of spot price fluctuations will be discussed.

1. Introduction
Although the establishment of China's futures market is relatively late and its system is not sound enough, the pace of opening up the power sales market will promote the formation of power futures trading. At present, power futures market has yet been established in China. By using the ordinary least squares regression (OLS) model combined with the spot and futures price data from European power futures commodities, this paper estimates the hedging rate, tests its stability and residuals, and uses power futures to help generators avoid the risk of spot price. The conclusion can be taken as the theoretical basis for the realization of power futures trading in China.

2. Feasibility analysis of hedging function of power futures

2.1 Verifying the cointegration of electricity spot and futures prices
Based on the real historical data from the European energy trading market, the daily system clearing price of the market trading data from August 31, 2016 to May 3, 2018 is selected as the spot price data, while the closing price of each trading day is taken as the futures electricity price data. From the power spot and futures prices we can get the spot price $P_s$, the futures price log $P_f$, the spot price rate $\Delta P_s = P_t - P_{t-1}$ and the futures price rate $\Delta P_f = P_t - P_{t-1}$ at the moment of $t_0$.

By the Analysis of the power futures spot and futures price series, it is shown in Figure 1 that the trend curves of power spot and futures prices are highly consistent, which indicates that they are correlated and therefore suitable for hedging.
2.2 Analysis of electricity market yield

Generally, we assume that the rate of return on electricity futures is normally distributed, but in reality this assumption will lead to the underestimation of risk and pricing bias, and as a result the risk cannot be managed rationally [1].

The statistics in Table 1 show that both the power spot and futures price returns are relatively small, the sequence standard deviation is small, the sequence dispersion is relatively low. Moreover, the spot yield is a reverse distribution whose sequence belongs to the sharp peak, while the futures yield is positively distributed with a sharp peak sequence. The J-B statistic of the two is much larger than the p value. In other words, the statistics of the power spot and futures returns do not meet the null hypothesis, and therefore their yield sequences are not normally distributed.

Table 1. Statistics of the yield series

| Statistic      | $R_s$ | $R_f$ |
|----------------|-------|-------|
| Mean Value     | 0.001108 | 0.001074 |
| Std.Error      | 0.193885 | 0.199969 |
| Skewness       | 0.514087 | 1.071082 |
| Kurtosis       | 8.466529 | 13.89138 |
| J-B Statistic  | 562.0782 | 2238.333 |

Based on the principle of minimizing the variance of investment yield, this paper uses OLS traditional regression model to calculate the optimal hedging ratio of electricity futures[2].

Assume that the spot price of electricity in the past is $P_{st-1}$, and the futures price is $P_{ft-1}$. And assume the current spot price is $P_s$ and the futures price is $P_f$. Through the price fluctuation between the two moments and the amount of energy that is hedged, we obtain the yield of this portfolio as well as determine that its minimum variance of N such moments is the covariance and variance ratio $h^*$. The slope $\beta$ of the basic least squares regression equation obtained at N such moments is analyzed by least squares regression, that is the hedge ratio $h^*$.

\[
h^* = \frac{C_t (P_u - P_f)}{C_s (P_{st} - P_f)}
\]  

(1)

On the basis of the optimal hedging rate obtained from the historical electricity price data and knowing the current spot and futures price and the forecast on the changes of electricity price in the future, the current trading decision situation can be selected. In other words, the number of trading and whether the generator should sell, buy or hold futures can be decided, so as to avoid the risk of fluctuations in electricity prices and minimize the loss of traders.
3. Hedging model based on traditional portfolio theory

Electric hedging is a kind of behavior that temporarily replaces physical electricity trading by futures trading in order to avoid or reduce the loss of unfavorable changes in electricity prices [3].

Summarizing the existing research results [4], the calculation of the hedging ratio is mainly based on the minimum variance method, using spot and futures as a portfolio to calculate the hedging ratio of the portfolio with the minimum risk.

Hypothetical combination: there is a need to buy in the spot market which has \( C_s \) units when there are \( C_t \) units available for trading in the futures market. Assume \( P_s \) and \( P_t \) are the spot price and futures price at \( t_0 \), then the return of this combination \( R_h \) can be expressed as follows:

\[
R_h = \frac{C_s \Delta P_s - C_t \Delta P_t}{C_s P_s} = R_s - hR_f
\]

Among them, \( R_h = \Delta P_s (P_s)^{-1} \), \( R_t = \Delta P_t (P_t)^{-1} \), \( \Delta P_s = P_s - P_{t-1} \), \( \Delta P_t = P_t - P_{t-1} \), and \( h = C_t (C_s)^{-1} \) is the hedging ratio.

The variance of the rate of return is:

\[
Var(R_h) = Var(R_s) + h^2 Var(R_f) - 2h Cov(R_s, R_f)
\]

On this basis, the first-order partial derivative of \( h \) can be found and made equal to 0 to get the minimum variance hedge ratio:

\[
h^* = \frac{Cov(R_s, R_f)}{Var(R_f)} = \frac{\rho \sigma_s}{\sigma_f}
\]

\( \rho \) is the correlation coefficient of \( R_s \) and \( R_f \), \( \sigma_s \) is the standard deviation of \( R_s \), \( \sigma_f \) is the standard deviation of \( R_f \).

The following part introduces the traditional regression model OLS based on the principle of variance minimization to calculate the hedging ratio [5-6]. The traditional regression model of OLS least squares is as follows:

\[
\Delta P_s = \alpha + \beta \Delta P_t + \epsilon_i
\]

Specifically, \( P_s \) and \( P_t \) are the natural logarithm of spot and futures electricity price respectively, \( \Delta P_s \) is the first-order difference of \( P_s \), and \( \Delta P_t \) is the first-order difference of \( P_t \), namely, the yield of spot and futures. Equation (5) is the regression on them. The connotations of each symbol in the equation are: \( \alpha \) is a constant term, \( \epsilon_i \) is the residual term of the model, the slope \( \beta \) of the equation is actually the optimal hedge ratio. Assume there are N samples, according to the least squares regression rule, the estimated value of is:

\[
\beta = \frac{\sum_{i=1}^{N} (\Delta P_s - \bar{\Delta P}) (\Delta P_t - \bar{\Delta P})}{\sum_{i=1}^{N} (\Delta P_t - \bar{\Delta P})^2} = \frac{Cov(\hat{\Delta P}_s, \hat{\Delta P}_t)}{Var(\hat{\Delta P}_t)} = \frac{\sigma_{\epsilon s}}{\sigma_{\epsilon f}} = \hat{h}
\]

4. Empirical Analysis of Hedging of Power Futures

4.1 Collection of data

The power spot and futures price data of this paper are all from the European Energy Exchange, and the spot and futures prices of electricity futures commodities from August 31, 2016 to May 3, 2018 are collected manually. The analysis software Eviews8.0 is used to do data processing.
4.2 Analysis of least squares regression model

Applying the ADF test method, the AIC and SC criteria are used to test the station's stationarity, and the unit root test is performed on the logarithm of the spot price, the logarithm of the futures price, the spot price of the spot price, and the rate of return of the futures price. The results in Table 2 show if the ADF test statistic of the spot and futures price returns is greater than the critical value of the significance level, which means the null hypothesis cannot be rejected, then there is no unit root and the sequence is stable. Similarly, if the statistic is less than the critical value of the significance level, then there is no unit root the sequence is unstable.

Table 2. $\Delta P_s$, $\Delta P_t$ unit root check results

| Null Hypothesis: $\Delta P_s$ has a unit root | Null Hypothesis: $\Delta P_t$ has a unit root |
|---------------------------------------------|---------------------------------------------|
| Lag Length: 5 (Automatic-based on AIC, maxlag=17) | Lag Length: 8 (Automatic-based on AIC, maxlag=10) |
| Augmented Dickey-Fuller test statistic | Augmented Dickey-Fuller test statistic |
| 1% level | 1% level |
| -11.525 63 | -9.969 366 |
| Test critical values | Test critical values |
| 5% level | 5% level |
| -3.979 543 | -3.979 695 |
| 10% level | 10% level |
| -3.420 308 | -3.420 382 |
| -3.132 826 | -3.132 869 |

*MacKinnon(1996) one-sided p-values

OLS regression is performed on the stationary sequence to estimate the optimal hedge ratio, and the least squares regression model is used to construct the regression equation. The regression results are shown in Table 3:

Table 3. OLS regression results with constant terms

| Dependent Variable: $P_t$ | Method: Least Squares |
|---------------------------|-----------------------|
| Date: 09/04/18 Time: 14:34 | Sample(adjusted): 8/31/2015 5/03/2018 |
| Include observations: 436 after adjustments |

| Variable | Coefficient | Std.Error | t-Statistic | Prob* |
|----------|-------------|-----------|-------------|-------|
| C        | 0.0000 149  | 0.003 625 | 0.041 120  | 0.967 2 |
| $P_t$    | 0.892 818   | 0.018 149 | 49.192 80  | 0.000 0 |

Get the regression equation:

$$\Delta P_s = 0.892818 \Delta P_t + \varepsilon_t$$  \hspace{1cm} (7)$$

Thus, the hedging ratio is obtained: $h^* = \beta = 0.892818$. The unit root test is carried out on each of the $P_s$, $P_t$, $\Delta P_s$ and $\Delta P_t$ sequences by the ADF test. It is found that the price series of the power spot and the futures are first-order stable, which is consistent with the basic financial data features [6].

5. Hedging ratio and research on the influence of power producers

Influenced by the fluctuation of spot electricity prices, power producers will make different trading decisions based on the fluctuations of spot and futures electricity prices. The power futures hedging strategy of power producers is discussed below.

Set $t_0$ which is the initial spot price $P_s$, futures price $P_t$, $t$ moment as the futures delivery date, the power producer’s predicted electricity price $P_t$ for the forward spot is equal to the futures price. According to the initial fluctuation forecast on forward spot price and futures price, combined with the hedging rate, the power producer adopts a suitable hedging strategy. The generator’s power futures hedging strategy is selected as shown in Table 4.
The purpose of power futures trading is to reduce the risk of spot price fluctuation caused by different factors such as peak and valley electricity price. Starting from the power futures hedging function that can avoid the risk of electricity price fluctuation, this paper analyzes the feasibility of hedging. Based on the traditional investment portfolio theory model combined with historical data, it calculates the hedging ratio of power futures, and uses the hedging ratio and the power producer's forecast of forward spot price to analyze the hedging strategy of generators in various situations. China's futures market starts relatively late and its development is still not mature enough. Therefore, it is of particular importance to find a proper method to establish electricity futures trading. The futures hedging function is used to improve the market effect of power futures. Power producers use power futures to avoid electricity price risks, thereby realizing the scientific optimization of resources.

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References
[1] Chao J.X., Liu D.G.(2007) Simulation of power generation strategy for power generators under hedging of power futures. J. Journal of Hunan University (Natural Science), 24(1):34-40.
[2] Liu H.(2007) Combined hedging strategy model. J. Knowledge Jungle, 229(1):130-131.
[3] Shen Y.L.(2008) International Trade Theory and Practice. Qinghua University Press, Beijing.
[4] Zhang Y.D.(2013) Research on Hedging Model of Aluminum Futures Based on Minimum Variance. D. Central South University, 2013.
[5] Daniel, S.Kirschen, Goran, Stsbac.(2004) Fundamentals of Power System Economics.
[6] Zhu T.(2008) Power generators use power futures to avoid spot trading risk models. J. Power Grid Technology, 32(6);76-80.
[7] Tang M.(2015) The formation and development of Australian power futures market. J. Financial Science, 325(4);23.