Designing learning activities on conditional probability

Benidiktus Tanujaya\(^1\), Rully Charitas Indra Prahmana\(^2\) and Jeinne Mumu\(^1\)

\(^1\)University of Papua, Jl. Gunung Salju, Manokwari, West Papua 98314, Indonesia
\(^2\)Universitas Ahmad Dahlan, Jl. Pramuka Kav. 5, Yogyakarta 55161, Indonesia

E-mail: b.tanujaya@unipa.ac.id

Abstract. This study aims to design student-learning activities on the concept of conditional probability using the development of test items. The method used is design research implemented in three stages, namely preliminary design, teaching experiment, and retrospective analysis. The study conducted on 47 mathematics education students of the University of Papua from mathematics statistics lesson. The research produces a learning trajectory, which contains a series of the learning process that students can develop conditional probability test items in three versions, determine sample space, formulate, and use conditional probability formula. The research shows that the development of test items with the approach namely observe, imitate, and modify can help students understand the concept of conditional probability.

1. Introduction
Mathematics is one of the most critical subjects in the education system of Indonesia. The subject studied at all levels of education in Indonesia, from elementary school to university [1]. One of the courses reviewed in the issue is statistics. Students learn several concepts in statistics instruction, including the idea of probability. Probability, a mathematics lesson, is also taught at all education levels including elementary, junior and senior high school.

However, including a subject in the curriculum does not automatically guarantee its correct instruction. The specific characteristics of probability, such as a multidimensional view of possibility or the lack of reversibility of random experiments, are not usually found in other subjects and create specific challenges for teachers and students [2, 3]. Mathematics teachers frequently lack detailed preparations to teach the lesson comprehensively. The teachers do not have particular knowledge and skill to teaching probability [4]. Furthermore, probability in an elementary statistics class is challenging to teach because there is not much time, the concepts and procedures are complicated, and the students do not see the importance of learning it [5]. The situation is even worse for textbooks, and curriculum documents of probability prepared for elementary and high school teachers do not offer adequate support [4].

The mathematics textbooks sometimes provide a too limited understanding of probability (only the conventional approach), and applications at other times limited to games of chance, and the definitions of concepts are inappropriate in several textbooks [6]. In Indonesia, especially in West Papua, mathematics’ teachers generally use only one to three mathematics textbooks in mathematics instruction [1]. The books have a standard procedure for teaching probability, i.e., there are only definitions, example, and exercises. This situation leads to insufficient teacher learning resources,
especially in teaching about the concept of probability. As a result, mathematics teachers often start the lecture by write down definitions, followed by providing examples and counterexamples [7]. The teachers are not creative in teaching probability. Examples of questions given by those contained in textbooks are of a very limited. As a result, it is urgent to offer the mathematics teachers a better preceding knowledge as well as adequate skills to teach probability creatively.

Probability is one of the subjects in the lesson of Mathematical Statistics in Mathematics Education program at the University of Papua. The mathematics education students’ University of Papua, as a mathematics teacher candidate, need to be well prepared for an understanding concept and teaching skill of probability concept. Probability, especially conditional probability concept is an essential concept for mathematics education students. Conditional probability is the basic concept for understanding further concepts in mathematical statistics and other lessons. The idea is a basic so that students can better understand other concepts, such as Bayes theory, random variables, and parameter estimation. While in other lessons, the idea of conditional probability is foundation for the development of queuing theory, decision-making, and games theory. Consequently, it is essential to find effective strategies and methods to teach conditional probability for the mathematics education students, the teacher candidate.

The teachers should be able to predict students' learning difficulties, mistakes, obstacles and problem-solving strategies' in conditional probability lessons to provide some effective strategies of instruction. For example, teachers should know students’ difficulties and mistakes in comparing two terms in conditional probability [4]. Moreover, since the teachers want students to build their knowledge actively by solving problems, the teachers should use this same approach in teaching, especially if they want the students’ later use a constructivist and social approach in their teaching [1]. The students are required to have more responsibility in their studying and to think creatively and critically. Therefore, the teachers should create suitable atmosphere for students to reflect on their previous beliefs about teaching and discuss these ideas with other classmates [8].

By knowing the students’ problems comprehensively, teachers can design various strategies in instruction [1]. The issue of students in learning conditional probability is the inability of students to understand the problem. Examples of test given by teachers are very different from the exam questions. Moreover, Tanujaya [9] stated that there are two main problems of students in solving mathematics problems, that is they seem still face difficulty in understanding the issues, and they have not been able to use the formula that has been studied. The problem is known as the connection in mathematics. Mathematical connections are a bridge where prior knowledge is used to build an understanding of the relationship between mathematical ideas, concepts, or representations [10].

Probability is a subject of mathematics that consists of randomness. The conflict between probability theory and students' view of the world is due at least in part to students' limited contact with randomness. It causes the students to have difficulty in solving the probability problem. Students only memorize the formulas and examples of questions given by teachers without understanding the concept of probability correctly and adequately. Once another matter is given, they cannot solve it. Therefore, the mathematics teachers must prepare the way for studying probability by providing experience with random behavior in the mathematics curriculum [11].

Therefore, developing students' understanding using test item development is proposed to solve those problems. By using test item development, teachers can also find information about students understanding through various instruction activities, comments, explanations, questions, answers, and other students' activities in the classroom. Based on the instruction activities, the teachers can develop students' understanding of the concept of conditional probability [9].

The development of test items can be done with some approach strategy, one of them is through observation, imitation and then modification. These three are the learning strategies that students can do to improve their thinking skills as well as to practice what they think and develop their understanding of a particular concept. It is by the Bandura’s social learning theory posits that students learn from one another through observation, imitation, and modeling. The approach is often
considered as a bridge between behaviorist and cognitive learning theories. Learning is a change in performance because of practice.

Does it correct that the development of test item can be used to develop students’ understand? How to use the development techniques of test item to improve students' understanding of the concept of conditional probability? What procedures and strategies should be undertaken for the development of test item to develop students' thinking skills in learning? The following will describe the methods used in answering these questions.

2. Method
The research conducted in the even semester of the academic year 2017/2018. The method used in this research is design research as a series of approaches that the intent of producing new theories, artefacts, and practices that account for and potentially influence learning and teaching in naturalistic settings [12, 13]. The method has five characteristics namely interventionist nature, process-oriented, the reflective component, cyclic character, and theory-oriented [14]. Primary objective of design research is to develop theories together with instructional materials [15]. Therefore, the purpose of this research is to develop a learning theory of conditional probability using the development of instrument test. This learning theory can be used to improve students' understanding of the concept of conditional probability.

Moreover, three phases of conducting design research are that preparing for the experiment, experimenting in the classroom, and conducting retrospective analyses [15] while the research method also has a cyclic character that consists of three phases: design, teaching experiment and retrospective analysis [16]. Thus, this research used three stages, namely preparation of experiment, experiment design, and retrospective analysis.

2.1. Preparation of experiment
The stage is also called Preparation and Design [17] or Preliminary Design [18]. At this stage, a literature review of conditional probability learning materials was conducted. After that, a suspected strategy and thought of students are in the learning process. Next, continue designing learning trajectory (LT) and the hypothetical learning trajectory (HLT). The LT contains the trajectory of learning material and concept map that students will go through during the instruction. Meanwhile, HLT is a hypothesis of how students’ thinking and understanding develop in an instruction [18]. The HLT consists of learning goals, starting points, the description of mathematics activities, conjectures of students’ thinking and strategies, and suggestions for teacher regarding what and how to react students’ particular response.

2.2. Design experiment
Design experiment comprises pilot and teaching experiment [18]. In the pilot experiment stage, the researcher involves six students. Selected students have different levels of ability, two students with the high knowledge, two students with average ability and two students with low strength. In teaching experiment stage, learning activities designed in the preliminary design stage was tested. At this stage, all the students who take the mathematical statistics courses that amount to 47 people involved in this study.

2.3. Retrospective analysis
Data from the teaching experiment were analyzed in this stage to plan activities and to develop the design of activities in subsequent learning. The study aims to explain how students can generalize from learning activities such as observing and analyzing examples of given questions, developing items to understand the concept of conditional probabilities. The retrospective analysis is to create Local Instructional Theory (LIT). At this stage, HLT compares with actual student learning.
3. Result and discussion
This study designed three activities consisting of activity determining sample space and sample point, and the probability of an event. Here is a description of these activities.

3.1. Preparation of experiment
At this stage, the researcher designed LT and HLT based on the literature review. There is a critical point that the students must pass in studying the conditional probability concept, i.e., the students must be able to define two events appropriately [19]. The two events have members and are not members.

There are two statements that students can use in developing conditional probability instruments test to help students to understand the critical point as follows:
- Determine a sample space (S) of an experiment, with at least two events, e.g., event A and event B, where both events have several corresponding members.
- Develop some items of test instrument about the concept of conditional probability.

Students determine both matters based on the test items obtained, either from textbooks, modules or various other teaching materials. Students then classify the test items into several categories. Based on these categories, students develop new test items, through observations, imitations, and modifications.

3.2. Design experiment
At this stage, the researcher involves six students. The results show that the students understand and carry out the test questions but only imitate examples of problems contained in textbooks. They have not been able to develop more varied test item. It led to the researcher to redesign the learning activity with the following statement:
- Determine a sample space (S) of an experiment, with at least two events, event A, and event B. Both events have several corresponding members.
  **For example**, it is known that in a container there are five marbles, three are red, two are white, and each marble is numbered. From the container is taken a marble at random. If it is known that the marbles are taken in red, then what is the probability of marbles is numbered 1?
- Determine the two events, where the events are one because of another event, event A, and event B. Thus, there are four events, namely the event A and event, not A, event B and event not B. If an event A occurs, event B is more probable to occur than an event, not B. In other words, if an event, not A occurs, then event not B is more probable than event B.
  **For example**, based on the records at Rendani Airport of Manokwari, the flight probability of aircraft A departs on time is 0.65; while the flight probability is landing on time is 0.86. For additional information, it also known that the flight opportunity departs and landing on time is 0.57. Calculate how many chances the flight landed on time if known it departed on time.

In teaching experiment stage, learning activities designed in the preliminary design stage tested [20-22]. All of 47 students who take the mathematical statistics courses involved in this experiment. The results of the experiments show that all students are at least able to develop two different test items.

Some students can prepare an instrument test that asks about conditional probabilities, \( P(A|B) \), Probability of event A and event B simultaneously, \( P(A \cap B) \), and mutually probability of event A, \( P(A) \) and event B, \( P(B) \). As the result, the students would be able to construct formula of conditional probability.

3.3. Retrospective analysis
Based on results, obtained Local Instructional Theory (LIT) for conditional probability learning, which compared with actual student learning are:
Students should understand some concepts of probability: sample space, events, independent events, and non-mutually exclusive events.

Students should be able to determine two independent events and not be independent.

Students should be able to provide actual examples of sample space that could be countered.

Students should be able to develop an item of the test instrument, using observation, adaptation, and modification approaches.

Students should be able to develop conditional probability formulas based on developed sample test questions.

\[ P(A|B) \]

is the probability of A given B to denote a conditional probability [3, 19, 23]. \( P(A|B) \) refers to the probability of A when B has already been known. It is assumed that A and B are two events containing a and b outcomes, respectively, and let c denote the number of outcomes in the intersection of A and B. The conditional probability of A given B is the ratio of c to b. However, \( \frac{c}{b} \) can be written as the quotient of two other ratios,

\[
\frac{c}{b} = \frac{c}{b} \cdot \frac{n^{-1}}{m^{-1}}
\]

So, for the particular case,

\[
P(A \setminus B) = P(A \cap B)[P(B)]^{-1}
\]

4. Conclusion

The research reveals that the development of test items with the approach: observe, imitate and modify can help students understand the concept of conditional probability. Students conduct observation by collecting various test items of conditional probability from different teaching materials, such as textbooks, and modules. The test items collected are grouped into several categories to facilitate observation. Students then develop similar test items by referring to the type of test items that have been obtained. Finally, the student modifies the test item that has been designed to achieve the test item about the conditional probability of his development.

References

[1] Tanujaya B, Prahmana R C I and Mumu J 2017 Mathematics instruction, problems, challenges, and opportunities: A case study in Manokwari regency, Indonesia World Transactions on Engineering and Technology Education 15 287

[2] Batanero C, Chernoff E J, Engel J, Lee H S and Sanches E 2016 Research on Teaching and Learning Probability (Hamburg: Springer)

[3] Lee C, Li H C and Shahrrill M 2018 Utilising the think-pair-share technique in the learning of probability International Journal on Emerging Mathematics Education 2 51

[4] Ortiz J J, Cañizares M J, Batanero C and Serrano L 2002 An experimental study of probabilistic language in secondary school textbooks The International Conference on Teaching Statistics 6 Cape Town

[5] Keeler C and Steinhorst K 2001 A new approach to learning probability in the first statistics course Journal of Statistics Education 9 3

[6] Batanero C, Juan D, Godino and Rafael R 2004 Training teachers to teach probability, Journal of Statistics Education 12 1

[7] Mumu J, et al 2018 J. Phys.: Conf. Ser. 943 012011

[8] Jaworski B 2001 Developing mathematics teaching: teachers, teacher educators and researchers as co-learners in Making sense of mathematics teacher education eds. Lin L and Cooney T J (Dordrecht: Kluwer)

[9] Tanujaya B 2017 Application of assessment as learning in mathematics instruction Advances in Social Science, Education and Humanities Research 100 140
[10] Hiebert J and Carpenter T P 1992 Learning and teaching with understanding, *Handbook of research on mathematics teaching and learning* ed. Grouws D A 65-97 (New York: Mc Millan)
[11] Moore D S 1990 Uncertainty *On the shoulders of giants: New approaches to numeracy* (Washington: National Academy Press)
[12] Barab S and Squire K 2004 Design-based research: Putting as take in the ground *Journal of the Learning Sciences* 13 11
[13] Ploom T 2013 Educational design research in Ploom T and Nieveen N (eds.) Educational Design Research- Part A: An Introduction (Enschede: SLO)
[14] Akker J V D, Gravemeijer K, Mckenney S and Nieveen N 2006 *Educational Design Research* (London: Routledge Taylor and Francis Group)
[15] Bakker A 2004 *Design Research in Statistics Education on symbolizing and Computer Tools* (Amersfoort: Wilco Press)
[16] Gravemeijer K and Cobb P 2006 Design Research from A Learning Design Perspective In Akker J V D, Gravemeijer K, Mckenney S, Nieveen N (eds.) *Educational Design Research* (London: Routledge Taylor and Francis Group)
[17] Eerde D V 2013 Design Research: Looking into the Heart of Mathematics Education *The First SEA-DR Proceeding*
[18] Prahmana R C I 2017 *Designing Mathematics Learning Trajectory: An Introduction* (Beau Bassin: Lambert Academic Publishing)
[19] Larsen J L and Marx M L 2011 An Introduction to Mathematical Statistics and Its Applications (New York: Prentice Hall)
[20] Prahmana R C I, et al 2017 *J. Phys.: Conf. Ser.* 893 012001
[21] Setyawan F, et al 2018 *J. Phys.: Conf. Ser.* 943 012004
[22] Hendroanto A, et al 2018 *J. Phys.: Conf. Ser.* 943 012029
[23] Purnami A S, et al 2018 *J. Phys.: Conf. Ser.* 948 012020