Magnetic-field dependence of the critical current of Abrikosov – Josephson junction in superconducting film

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Abstract. Low-angle boundaries between single-crystal domains (with misorientation angles \(\theta \sim 1^\circ\) in epitaxial films of high-temperature superconductors are crucial for achievement of high values of critical current and low microwave losses. Such boundaries with high transparency for superconducting current (as compared with usual Josephson junctions) for temperatures \(T\) rather close to the critical one \(T_c\) possess properties of Abrikosov – Josephson (A-J) junctions, as well as boundaries with \(\theta \sim 5^\circ - 10^\circ\) in films on bi-crystal substrates for the whole temperature range. Magnetic flux penetrates into such junctions by the means of A-J vortices, which differ from the Abrikosov ones by the presence of phase cores rather then normal-metallic cores, and from the Josephson ones by non-local dependence between the phase difference and magnetic field. Generalization of the well-known Fraunhofer-like magnetic-field dependence of the Josephson critical current is presented for the case of A-J junction in the field normal to the film plane.

1. Introduction.
The concept of Abrikosov – Josephson (A-J) vortex was introduced by A. Gurevich [1] for a moderately weak superconducting link with non-local electrodynamics. Such junction is characterized by its “transparency” to superconducting current, i.e. by the locally (at the plane of the link) suppressed values of the order parameter \(\Delta\) and the depairing critical current density \(j_c^{tr}\), while non-local (in the link plane direction) properties are essential if the bulk London penetration depth \(\lambda\) exceeds the value of Josephson magnetic penetration depth in the junction \(\lambda_J = (c \Phi_0/16\pi^2 \lambda_c^{tr})^{1/2}\), where \(\Phi_0\) is the magnetic flux quantum and \(c\) is the velocity of light. The last condition is equivalent to

\[
j_0 J_c^{tr}/\kappa J_c^{tr} < J_0,
\]

where \(j_0 = c \Phi_0/12\sqrt{3}\pi^2 \lambda_c^{tr} \zeta\) is the depairing critical current density in the bulk (far from the link), \(\kappa = \lambda/\xi\) is the Ginzburg – Landau parameter and \(\zeta\) is the coherence length. The criterion (1.1) of A-J vortices existence is fulfilled easily for planar defects in the wide range of transparency \(j_c^{tr}/J_0\) in cuprate high-temperature superconductors (HTS) (such as in the most suitable for technical aims material YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\) (YBCO) with \(\kappa \approx 100\), especially in epitaxial thin films and designed on their base for high-current applications “coated conductors” [2].

The examples of A-J junctions are low-angle boundaries (LABs) in thin YBCO films on bi-crystal substrates (with misorientation angles \(\theta \sim 5^\circ - 10^\circ\)) [3,4] and anti-phase boundaries (APB) in vicinal YBCO films [5,6]. In the former case theoretical predictions for the dissipation in the flux-flow regime of A-J vortices’ chain were confirmed experimentally for the \(\theta = 7^\circ\) sample [4].
It should be noted that A-J vortices may be present even in usual epitaxial YBCO films on single-crystal high-symmetry-plane substrates, at least in the near vicinity (of a few kelvins) to the critical temperature $T_c$. Out-of-plane dislocations with non-superconducting cores are inherent for epitaxial film growth due to misfit with a substrate. For sufficiently thin films ($\sim 100$ nm) the dislocations are threading films from the interface to the very surface [7]. Edge dislocations form quasi-periodical rows within intrinsic LABs between single-crystal domains (with $\theta \sim 1^\circ$). These LABs possess properties of A-J junctions if the temperature dependent coherence length $\xi(T) = \xi_0(1-T/T_c)^{1/2}$ exceeds average distance between dislocations in the row [8,9]. In the case of additional suppression of superconductivity in the inter-crystalline LABs (for example, due to enhanced oxygen depletion during film deposition process [10]) they may reveal A-J properties at much lower temperatures.

To describe peculiarities of magnetic field dependences of critical current in various YBCO films either inter-crystalline boundaries [10] or APBs [6] were treated as usual Josephson junctions distributed by their lengths $L$. In such models the well-known Fraunhofer-like magnetic-field dependence [11] of the critical current $I_c(H)$ for each individual junction was assumed:

$$I_c(H) = I_{c0} \sin \left( \frac{\pi \Phi / \Phi_0}{\pi \Phi / \Phi_0} \right).$$  \hspace{1cm} (1.2)

Here $\Phi = 2\lambda LH$ is the magnetic flux through the junction area. Nevertheless, if any kind of junctions exists in some epitaxial YBCO films, they obviously correspond to A-J range of parameters (1.1). The expression (1.2) is inapplicable in this non-local case. Despite detailed theoretical investigation of A-J electrodynamics [12] and pinning [13], there was not given explicit generalization of (1.2). Such one is presented below.

2. Macroscopic current of A-J junction in magnetic field.

Let us consider A-J junction in magnetic field $H = H_z$ normal to the plane of thin film $(x, y)$ and parallel to the junction plane $(x, z)$. It is described by the set of integral equations for the local magnetic field $H(x,y)$ and phase difference $\phi(x)$ across the junction [1] which in the simplest stationary case reads:

$$H(x, y) = \frac{\Phi_0}{4\pi^2 \lambda^2} \int_{-\infty}^{\infty} K_0 \left[ \frac{y^2 + (x-u)^2}{\lambda} \right] \frac{\partial \phi}{\partial u} du,$$ \hspace{1cm} (2.1)

$$\sin \phi = \frac{\lambda^2}{\pi \lambda} \int_{-\infty}^{\infty} K_0 \left[ \frac{|x-u|}{\lambda} \right] \frac{\partial^2 \phi}{\partial u^2} du,$$ \hspace{1cm} (2.2)

where $K_0(x)$ is a modified Bessel function.

In the non-local limit $\lambda_j \ll \lambda$ this set of equations (2.1), (2.2) has asymptotically exact phase-kink-like solution for isolated A-J vortex [1]:

$$\phi(x) = \pi + 2 \tan^{-1} \left( \frac{x}{l} \right),$$ \hspace{1cm} (2.3)

$$l = \frac{\lambda_j^2}{\lambda} = \frac{3\sqrt{5}}{4} \frac{\Phi_0}{j_c \xi}.$$ \hspace{1cm} (2.4)

Here characteristic length $l$ gives the size of phase core of A-J vortex.

Let us consider a bridge of width $W$ made from thin superconducting film (with thickness $d \ll W$). The order parameter is slightly suppressed at the plane $y = 0$. Otherwise the film is assumed to be perfect (pinning is absent along the junction as well as in its banks). In this case magnetic flux penetrates the junction with the regular chain of A-J vortices continued on the both sides (due to the demagnetization factor close to unity) by the regular Abrikosov vortex lattice (AVL) with the constant $a_0 \approx (\Phi_0/H)^{1/2}$ which is imposed through inter-vortex magnetic interaction to the inoculating A-J chain.

Macroscopic current density (averaged over the area of junction) reads
where $x_0$ is the coordinate of the A-J vortex centre playing a part of an arbitrary phase constant. By variation relative to this constant the maximal value $J_c(H)$ for given field $H$ is obtained. In the $z$ direction current density distribution is assumed to be uniform due to small film thickness. For the usual Josephson case magnetic field penetrates only into the junction plane, and for the narrow ($W << \lambda_J$) contact $\varphi(H, x) \sim H x$ \cite{11}. After substitution into (2.5) and optimization relative to the phase constant $\varphi_0$ this yields Fraunhofer-like dependence (1.2).

In the case of stationary infinite A-J chain with period $a_0$ distribution of the phase difference along the junction is given by asymptotically exact (in the non-local limit $\lambda_J << \lambda$) solution \cite{12}:

$$\varphi(H, x, x_0) = \pi + 2 \tan^{-1}\left[ M(h) \cdot \tan\left( \frac{\pi}{a_0} (x - x_0) \right) \right],$$

where $M(h) = (1 + \sqrt{1 + \tilde{h}}) \sqrt{\tilde{h}}$, and $h = (2\pi l / a_0)^2$ is normalized magnetic field.

Substitution of (2.6) into (2.5) and optimization of the absolute value of current with respect to $x_0$ yields the expression for the maximal macroscopic current density through A-J junction in the absence of pinning:

$$J_c(H) = \frac{j_c^\mu \cdot M \cdot a_0}{\pi \cdot W \cdot (M^2 - 1)} \ln \left[ \frac{1 + M^2 \tan^2 \left( \varphi_0 + \varphi_h \right)}{1 + \tan^2 \left( \varphi_0 + \varphi_h \right)} \cdot \frac{1 + \tan^2 \varphi_0}{1 + M^2 \tan^2 \varphi_0} \right],$$

where

$$\varphi_h = \pi \cdot \left[ W / a_0 - \text{Int}(W / a_0) \right]$$

and

$$\varphi_0 = \frac{\pi}{a_0} x_0^{\max} = \tan^{-1} \left[ \sqrt{1 + \left( \frac{2 \cot \varphi_h}{M + M^{-1}} \right)^2 - 1} \cdot \tan \varphi_h \cdot (1 + M^{-2}) / 2 \right].$$

Here $x_0^{\max}$ is the coordinate of centre of the first A-J vortex in the chain which maximizes the absolute value of macroscopic current density through the junction.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Magnetic field dependence (2.7) of the macroscopic critical current density through the A-J junction with $W / l = 100$ normalized by the microscopic “transparency” current density $j_c^\mu$ (a); the same with tenfold scaled axis of normalized magnetic field (b).}
\end{figure}

3. Discussion.
Let us consider some peculiarities of the magnetic-field dependence obtained. It follows easily from (2.7) – (2.9) that critical current density \( J_c(H) \) equals to zero at \( a_0 = W/n \) \((n = 1, 2, 3\ldots)\) and possesses maximums at \( a_0 = W/(n + 1/2) \) (at \( H = H_n \)), where it takes on the values \( \frac{2 j^\omega_0 \cdot M \cdot a_n(H_n)}{\pi \cdot W \cdot (M^2 - 1)} \cdot \ln M \). (3.1)

The first (rather trivial) difference with the usual case of Josephson junction is non-equidistance (see Figure 1) of adjusting zeroes (and maximums) in the linear \( H \) plot. This is just a consequence of flux penetration in the geometry considered into the whole film rather than only into the junction area. At the zero field macroscopic (averaged) critical current density takes on the value

\[
J_c(0) = \frac{1}{W} \cdot \ln \left[ 1 + \frac{W}{I} + \frac{1}{2} \frac{W}{I} \right],
\]

which tends to microscopic transparency current density \( j_c^\omega \) for \( W \ll l \), but is essentially lower for the actual opposite case \( W \gg l \) (see Fig. 1). At finite but moderately low fields (\( h \ll 1 \)), when \( M \approx a_0/\pi l \), the expression (3.1) (the envelope of magnetic-field dependence (2.7)) yields:

\[
J_c(H_n) = \frac{2 j^\omega_0}{W} \cdot a_n(H_n) = \frac{j_0}{\pi l} \cdot \ln \left[ \frac{\Phi_0}{H_n} \cdot \frac{1}{\pi \cdot \xi^2 \cdot j_c^\omega} \right].
\]

Electromagnetic longitudinal pinning of A-J vortices at junction imperfections as well as interaction with AVL pinned at the banks [13] may enhance the value of critical current, especially by smearing the minimums of the \( J_c(H) \) dependence (2.7).

In conclusion, macroscopic critical current density of A-J junction in the absence of pinning and within logarithmic accuracy is proportional to \( W^{-1} \), reduced temperature \( \tau = (1-T/T_c) \) (with \( j_0 \sim \tau^{3/2} \) and \( \xi \sim \tau^{-1/2} \)), \( \ln(1/H) \) and \( \ln j_c^\omega \), rather then \( j_c^\omega \) and \( 1/H \) for the envelope of (1.2) in the local limit \( \lambda_J \ll \lambda \).

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