THEORY OF SUB-10 FS GENERATION IN KERR-LENS MODE-LOCKED SOLID-STATE LASERS WITH A COHERENT SEMICONDUCTOR ABSORBER

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Abstract

The results of the study of ultra-short pulse generation in continuous-wave Kerr-lens mode-locked (KLM) solid-state lasers with semiconductor saturable absorbers are presented. The issues of extremely short pulse generation are addressed in the frames of the theory that accounts for the coherent nature of the absorber-pulse interaction. We developed an analytical model that bases on the coupled generalized Landau-Ginzburg laser equation and Bloch equations for a coherent absorber. We showed, that in the absence of KLM semiconductor absorber produces $2\pi$ - non-$\text{sech}$-pulses of self-induced transparency, while the KLM provides an extremely short $\text{sech}$-shaped pulse generation. $2\pi$- and $\pi$ - $\text{sech}$-shaped solutions and variable-area chirped pulses have been found. It was shown, that the presence of KLM removes the limitation on the minimal modulation depth in absorber. An automudational stability and self-starting ability were analyzed, too.

Key words: ultrashort laser pulse, solid-state laser, Kerr-lens mode locking, coherent semiconductor saturable absorber

PACS: 42.55.A,P,R, 42.65, 42.65.T

$^1$ The author is a gratefull to Dr. I. G. Poloyko and D. O. Krimer for the helpful discussions and suggestions.
1 Introduction

A recent progress in ultra-fast lasers has resulted in the generation of sub-10 fs pulses, which is close to the fundamental limit defined by the light wave period in visible and near-IR [1]. Today a basic technique for fs-generation is the Kerr-lens mode locking (KLM) [2] in the combination with a slow saturation of interband or excitonic transitions in semiconductor structure [3]. It was shown [4], that in both cases a quasi-soliton formation plays a key role in the stabilization of ultra-short pulse generation down to shortest possible pulse durations. Moreover, the quasi-soliton generation due to its highest stability and simple pulse form is the subject for the use of analytical and semi-analytical approaches, which are of highest theoretical and practical interest that stimulates the investigations in this field.

As was shown in [5,6], the strong nonlinear effects in semiconductor absorbers, such as absorption linewidth enhancement and Stark effect, transform the laser dynamics essentially, that causes an additional pulse stabilization and compression. Since the pulse durations in modern ultrafast lasers are comparable or shorter than the absorber dephasing time $t_{coh}$, the coherent nature of the pulse-semiconductor interaction has to be taken into consideration. A coherent absorber mode locking has been analyzed in refs. [7–10]. In particular, it was shown in [7–9] that the dynamical gain saturation is essential for the coherent quasi-soliton generation in the lasers. However, the dynamical gain saturation is negligible in femtosecond solid-state lasers, where the dominating nonlinear factors are the self-phase modulation (SPM) and the self-focusing. Numerical simulations [10] have demonstrated the generation of fs-pulses of self-induced transparency in solid-state laser with semiconductor quantum-well absorber in the absence of KLM. However, as was shown in [11], the self-focusing is essential in femtosecond time domain and should be taken into account in the analysis. Furthermore, an analytical approach to this problem can provide a new insight into the physics of fs-lasers and semiconductor optical devices.

Here we present a study of fs-pulse generation in cw solid-state KLM - laser with semiconductor absorber. We developed an analytical model that accounts for a coherent pulse-semiconductor interaction, SPM, KLM, group-velocity dispersion (GVD) and gain saturation by the full pulse energy. The condition of the $sech$-shaped pulse formation was found and the contribution of Kerr-lensing to pulse characteristics was considered. We showed that the generation of chirp-free $2\pi$- and $\pi$- pulses as well as chirped pulses with the variable area is possible. The obtained solutions are stable against laser noise and automodulational instability.
2 Model

Based on the slowly varying envelope approximation for the field amplitude \( a(t) \), let consider a distributed system including saturable gain, linear loss and SPM with coefficients \( \alpha \), \( \gamma \) and \( \beta \), respectively, Kerr-lens-induced fast saturable absorption with saturation intensity \( 1/\sigma \), GVD with coefficient \( D \) and bandwidth limiting element with transmission bandwidth \( 1/t_f \), where \( t_f \) defines the minimal pulse duration and is equal to 2.5 fs for Ti: sapphire laser. For the sake of staying in framework of analytical approach we use a two-level model for quantum-well semiconductor absorber. This assumption is valid for quantum-confined semiconductor structures utilized in mode-locked fs-lasers (see, for example, [4,10]; last work contains some generalizations of model in framework of numerical analysis).

When the pulse duration \( t_p \) is much shorter than the dephasing time in absorber \( t_{coh} \) and the field intensity \( |a(t)|^2 \) is not enough for the Stark effect manifestation, the pulse-semiconductor interaction obeys the Bloch equations [12]:

\[
\frac{du}{dt} = (\Delta - \frac{d\phi}{dt})v + qaw, \quad \frac{dv}{dt} = -(\Delta - \frac{d\phi}{dt})u, \quad \frac{dw}{dt} = -qau, \tag{1}
\]

where \( u(t), v(t) \) and \( w(t) \) are the slowly varying envelopes of the polarization quadrature components and the population difference, respectively, \( q = d/h, \quad d = 0.28 \cdot e \ [\text{Coulomb} \times \text{nm}] \) is the dipole momentum, \( e \) is the elementary charge (in our calculations we used the parameters of GaAs/AlAs absorber, the saturation energy \( E_a = 50 \mu J/cm^2 \) and \( t_{coh} = 50 \text{ fs} \)), \( \Delta \) is the mismatch between optical resonance and pulse carrier frequency, \( \phi \) is the instant field phase. An initial saturable absorption \( \gamma_a = 2\pi N d z a t_{coh} / (c \hbar) = 0.01 \) (\( \omega \) is the field frequency) corresponds to the carriers density \( N = \gamma_a E_a / (h \cdot \omega \cdot za) = 2 \cdot 10^{18} \text{ cm}^{-3} \) and the thickness of semiconductor absorber \( za = 10 \text{ nm} \).

The laser part of the master equation is the generalized Landau-Ginzburg equation [2]. Then the master equation can be written as:

\[
\frac{\partial a(z, t)}{\partial z} = \left[ \alpha - \gamma + i\theta + \delta \frac{\partial}{\partial t} + (n^2 + iD) \frac{\partial^2}{\partial t^2} + \frac{\sigma - i\beta}{n^2} |a|^2 \right] a + \left[ \frac{2\pi N za \omega d}{c} u - \frac{2\pi N za d \omega v}{c t} \right], \tag{2}
\]

where \( z \) is the longitudinal coordinate normalized to the cavity length, i.e.
the number of the cavity round-trip, \(c\) is the light velocity, \(\theta\) and \(\delta\) are the phase and time field delays of the field after the cavity round-trip, respectively. We neglected the spatial effects in the absorber. Later we will consider only steady-state pulse-like solutions, which allows to eliminate the dependence on \(z\). Let normalize the times to \(t_f\) and the field to \(q\) (note, however, that we will use a dimensional pulse duration in Figs. 2 and 4). Then \(\beta\) and \(\sigma\) are normalized to \(2(\sigma) = 5 \cdot 10^{-12} \text{cm}^2/W\), where \(n\) is the index of refractivity, \(\varepsilon_0\) is the permittivity and \(t_f = 2.5 \text{ fs} (\text{Ti: sapphire laser})\). With these normalizations, \(\sigma = 0.14\) corresponds to the saturation parameter of Kerr-lens induced fast saturable absorber of \(10^7 \text{ W}\) and 30 \(\mu\text{m}\) spot size in active medium. Dimensionless SPM parameter \(\beta\) is equal to 0.26 for 1 \(\text{mm}\) Ti: sapphire crystal. Additionally, we introduce an important control parameter \(\eta\), which is governed by 1) the ratio between the size of generation mode in active medium and in semiconductor absorber or by 2) the reflectivity of the upper surface of the semiconductor saturable device. Formally, the variation of \(\eta\) means the variation of relative contribution of SPM, Kerr-lens-induced saturable absorption and saturable gain with respect to the saturable absorption in semiconductor.

As we will see later, the gain saturation by the full pulse energy is an important factor for the pulse stability and thus should be taken into account. The simplest way to do this is to use a quasi-two level model for active medium. After some calculations for the gain saturated by the full pulse energy \(E\) in the steady-state condition we have \(\alpha = \frac{P_{\alpha_{max}}}{P_0 + E/\eta^2 + 1/\tau_r}\), where \(P_{\alpha_{max}}\) is the gain for the full population inversion, \(\tau_r\) is the gain relaxation time normalized to the cavity period \(T_{\text{cav}}\), \(P_0\) is the gain the dimensionless pump intensity, \(\nu_p\) is the pump frequency, \(\sigma_{14}\) is the absorption cross section of active medium, \(I_p\) is the pump intensity, \(\tau = 6.25 \cdot 10^{-4}\) is the normalized inverse energy of the gain saturation.

Later we will consider the different realizations of our model aimed to investigation of the soliton-like solution of the system (1, 2).

### 3 Pulse of the self-induced transparency in the absence of KLM

In the beginning we consider the case of chirp-free pulse-like solutions. After integration of the equations (1), the master equation (2) reads as:

\[
\frac{\partial a(z,t)}{\partial z} = \left[ \alpha - \gamma + i\theta + \delta \frac{\partial}{\partial t} + (1 + iD) \frac{\partial^2}{\partial t^2} + \frac{\sigma - i\beta}{\eta^2} |a|^2 \right] a - \frac{\gamma}{t_{\text{coh}}} \sin(\psi(z,t)),
\]

(3)
where $\psi(z, t) = \int_{-\infty}^{t} a(z, t') dt'$ is the pulse area (note, that the field and time are the dimensionless quantities here). Under steady-state condition (the pulse envelope is independent on $z$), an integro-differential Eq. (3) results in differential equation

$$
\left[ (\alpha - \gamma + i\theta) \frac{d}{dt} + \delta \frac{d^2}{dt^2} + (1 + iD) \frac{d^3}{dt^3} + \frac{\sigma - i\beta}{\eta^2} \left( \frac{d\psi(t)}{dt} \right)^2 \frac{d}{dt} \right] \psi(t) - \frac{\gamma a}{t_{coh}} \sin(\psi(t)) = 0. \tag{4}
$$

In the absence of the lasing factors such as linear loss, frequency filtering, SPM and GVD we have a well-known nonlinear equation with $2\pi$-soliton solution in the form $a(t) = a_0 \text{sech}(t/t_p)$, where $a_0$ is the amplitude, $t_p$ is the duration [12]. But this solution does not satisfy the full Eq. (4) in the absence of KLM ($\sigma = 0$).

Now let consider the Eq. (4) without KLM, SPM and GVD ($\sigma = \beta = D = \theta = 0$). The substitution $\psi(t) = x$, $d\psi(t)/dt = y(x)$ (“area-amplitude” representation) reduces the third-order Eq. (4) to the second-order one:

$$
\left[ \left( \frac{d^2 y}{dx^2} \right) y + \left( \frac{dy}{dx} \right)^2 + \delta \frac{dy}{dx} + (\alpha - \gamma) \right] y - \frac{\gamma a}{t_{coh}} \sin(x) = 0. \tag{5}
$$

We solved this equation numerically and found a non-sech-shaped $2\pi$-solutions for it (see Fig. 1, where numerical solution is shown in comparison with the sech-shaped pulse). This clearly indicates on some additional nonlinear factors which are necessary for the sech-pulse formation and which are lacking in Eq. (5).

However, $2\pi$- non-sech solution needs a more detailed investigation because of it may be relevant mechanism for extremely short pulse generation in real lasers. As is known [4], the main mechanism of destabilization of fs-pulses is the noise generation as result of loss saturation in absorber. In the case of $2\pi$-pulse formation, a Rabi flopping of the absorber population suppresses the noise behind the pulse tail, that stabilizes fs-generation [10].

To investigate the pulse-like solutions of Eq. (5) analytically, we used a harmonic approximation: $y(x) = a_1 \sin(x/2) + \sum_{m>2} a_m \sin\left(\frac{2m\pi x}{2}\right)$, which is usual for analysis of oscillating processes (see, for example, [13]). In our case this approximation is most appropriate for “area-amplitude” representation because
of that allows to describe the full pulse area but not only behavior of the field amplitude in the vicinity of pulse maximum as in the case of polynomial expansion. We select the subharmonic corresponding to $2\pi$-pulse generation (first term) and “high frequency” corrections to it. Retaining only the first term, in the “area-amplitude” representation, we arrive to the solution $a_1 = 2\sqrt{2(\alpha - \gamma)}$, $\delta = \gamma_a/(2(\alpha - \gamma)t_{coh})$, $t_p = 2/a_1$, which corresponds to sech-shaped solution in the “time-amplitude” representation. The relations between pulse parameters are analogues to that ones for $2\pi$-sech-shaped solution, but an additional relation between pulse amplitude and dissipative coefficients $\alpha$ and $\gamma$ appears.

Fig. 2 (curves 1 and 2) presents the pulse durations for two physical approximated solutions of Eq. (5). One can see, that the coherent absorber provides sub-10 fs pulse generation starting from some minimal pump. An important feature of the obtained solution is the positive difference between saturated gain and linear loss coefficients $\alpha - \gamma > 0$, which imposes an important requirement on the possible minimal saturable loss coefficient $\gamma_a$, which is necessary for stable pulse generation. So, there is the minimal modulation depth of absorber that confines the region of the pulse stability against laser noise. As it was shown in [2], the pulse is stable if the net-gain outside pulse is negative that results in the condition $\alpha - \gamma - \gamma_a < 0$, i.e. $\gamma_a > \alpha - \gamma$. The dependence of the minimal modulation depth on the pump is shown in Fig. 3 for two depicted in Fig. 2 solutions of Eq. (5) (lower curve corresponds to the solution with larger duration, the dashed curve depicts the generation threshold, hatching shows a corresponding stability zone). As one can see, the pulse stabilization against laser noise is possible only for the solution with longer duration. The stability range widens in $\gamma_a$, however at the cost of pump growth. It should be noted, that in the absence of a coherent mechanism the pulse stabilization can be caused only by essential contribution of dynamical gain saturation [14,15], that is not possible in fs-time domain, or by “soliton mode locking” [4] due to SPM and GVD balance. Self-induced transparency in semiconductor absorber produces a quite different mechanism of pulse stabilization, which does not involve any additional nonlinear processes.

Now let introduce into equations the SPM and GVD terms, that corresponds to the real-world femtosecond lasers. Assuming a chirp-free (i.e. pure real) nature of possible solution we can reduce Eq. (4) to the first-order equation:

$$
\left[ \delta \frac{dy(x)}{dx} + \frac{\beta}{\eta^2 D} y(x)^2 + (\alpha - \gamma - \frac{\theta}{D}) \right] y(x) - \frac{\gamma_a}{t_{coh}} \sin x = 0. \quad (6)
$$

With the above described harmonic approximation we have the following solutions: $\theta = 3\beta \cdot a_1^2/(4\eta^2) + D(\alpha - \gamma)$, $\delta = 4\gamma_a/(t_{coh} \cdot a_1^2)$, $a_1 = 2\sqrt{3(\alpha - \gamma)}$. 

6
\( t_p = 2/a_1 \) for \( 2\pi \)-pulse. The pulse durations are presented in Fig. 2 by curves 1’ and 2’. The stable solution has a slightly longer duration than the unstable one.

Formally, the obtained solution is the solution of the laser part of master equation that in the same time satisfies the Bloch equations. The stability of the solution results from the self-induced transparency in absorber, when the pulse propagates in the condition of the positive net-gain but noise is suppressed due to Rabi flossing of the absorber population. We do not analyze the automodulational stability [16] of the solution since it has been demonstrated directly by the numerical simulation in [10].

Thus we can conclude, that there is not the generation of coherent sech-shaped pulse in the absence of Kerr-lens-induced fast saturable absorption. But the generation of non-sech \( 2\pi \)-pulse takes place, which imposes a limitation on minimal modulation depth of the absorber with subsequent growth of generation threshold. Nevertheless, as result of the coherent \( 2\pi \)-pulse formation, the pulse stabilization in this case is possible without contribution of dynamical gain saturation, or SPM and GVD. In the next section we take into account the contribution of KLM.

4 Coherent \( 2\pi \)-sech-pulse in the presence of KLM

The presence of KLM is described by the term \( \sigma |a|^2 \) in Eq. (3). In this case there is an exact \( 2\pi \)-sech-pulse solution of Eq. (3), however for a strict relation between \( \sigma \) and \( \eta \), so that \( \sigma = \frac{\gamma^2}{\tau} \). The sech-shaped solution has the following parameters:

\[
\begin{align*}
  a_0 &= \frac{2}{t_p}, \\
  t_p &= \frac{1}{\sqrt{\gamma - \alpha}}, \\
  \delta &= \frac{\gamma a_0}{t_{coh}(\gamma - \alpha)}. 
\end{align*}
\]

There are two distinct features of \( 2\pi \)-sech-pulse generation: 1) the condition \( \alpha - \gamma < 0 \) is satisfied automatically and, consequently, there is no limitation on the minimal modulation depth of the absorber; 2) the expression for the pulse duration is precisely same as for the case of pure fast saturable absorber mode locking [2], that suggests that the KLM is the main mechanism determining the pulse duration. This conclusion corroborates with the results of ref. [11]. The action of the coherent absorber determines the pulse delay \( \delta \) and imposes a restriction on the pulse area, i. e. the relation between pulse duration and amplitude. Pulse duration in the presence of KLM is shown in Fig. 2 by dotted curve 3. As is seen, the pulse duration is much shorter than for the case with
no KLM, especially for the small pump. As the contribution of semiconductor absorber is increased ($\eta$ approaches 1, curve 1 in Fig. 4) the pulse duration is reduced down to the limit of the validity of the slowly varying envelope approximation.

An explanation of the additional relation between the parameters of Kerr-lens-induced saturable absorber and semiconductor absorber is as follows: a sech-shaped solution satisfies both pure laser equation and the Bloch equations, but the coherent interaction discriminates the special cases of $n\pi$-area pulses, in particular $2\pi$-pulses, which are provided by $\sigma = \eta^2/2$ relation.

Let us study an automodulational stability of the coherent laser pulse, which we showed before to be very important factor in fs-lasers [16]. We used an aberrationless approximation, which assumes an approximately unchanged form of solution and $z$-dependence for the pulse parameters. The substitution of the pulse envelope in Eq. (3) with following expansion into the time series yields:

$$
\frac{da_0}{dz} = 2 \frac{(\alpha - \gamma)\eta^2 t_p^2 - \eta^2 + 4\sigma}{\eta^2 t_p^3}, \quad \frac{dt_p}{dz} = 4 \frac{\eta^2 - 2\sigma}{a_0 \eta^2 t_p^2},
$$

$$
\theta = \frac{2(D + 4\frac{\beta}{\eta^2})}{a_0 t_p^3}, \quad \delta = \frac{2\gamma a_0 t_p}{t_{coh} a_0}.
$$

Eqs. (8) were derived for a chirp-free solution with the dispersion $D = -2\beta/\eta^2$ exactly compensating SPM. To be self-consistent the system (8) should be completed by the additional relation between pulse duration and amplitude arising from the Bloch equations. After some calculations we have the explicit expressions, which determine the pulse stability. The pulse is stable if the Jacobian of the right-hand sides of first two Eqs. (8) has only non-positive real parts of eigenvalues. The condition for amplitude perturbation decay $-4(\gamma - \alpha)^2 < 0$ is satisfied automatically. For $\sigma = \eta^2/2$ the pulse possesses a marginal stability with respect to the evolution of pulse duration. However for any $\sigma < \eta^2/2$ the pulse stability condition with respect to duration evolution is satisfied.

Thus, we have analyzed the characteristics of $2\pi$-sech-pulses generated in KLM-laser with semiconductor coherent absorber. The main features here are: the “automatic” stabilization against laser noise and against pulse automodulations, and, also, the pulse duration decrease down to sub-10 fs. However, equations describing this physical situation allow yet another type of analytical solutions.
5 Coherent $\pi$-sech-pulse and chirped pulses in the presence of KLM

As one can see from the previous part of our work, the ultrashort pulse is the soliton-like solution for the both laser part of the master equation and the Bloch equations. Now we will consider the complex anzatz $a(t) = a_0 \text{sech}(t/\tau_p)^{1-i\zeta}$ describing a pulse with chirp $\zeta$. It is known [12], that the Eqs. (1) have a sech-shaped solutions in form of chirp-free $\pi$-pulse or sech-shaped chirped solution, when the following relations hold:

$$u(t) = u_0 \text{sech}(t/\tau_p), \quad v(t) = v_0 \text{sech}(t/\tau_p), \quad w(t) = \tanh(t/\tau_p), \quad \frac{d\phi(t)}{dt} = \frac{\zeta}{\tau_p} \tanh(t/\tau_p),$$

where

$$a_0 = \sqrt{1+\zeta^2}, \quad u_0 = -\frac{1}{\sqrt{1+\zeta^2}}, \quad v_0 = \frac{\zeta}{\sqrt{1+\zeta^2}}.$$

A chirp-free solution is a $\pi$-pulse, which is obviously unstable in the absorber since the full population inversion behind the pulse tail amplifies the noise. However, another nonlinear factors in KLM-laser can stabilize the pulse and this requires a corresponding consideration.

Parameters of the chirp-free $\pi$-pulse in KLM-laser are:

$$a_0 = \frac{1}{\tau_p}, \quad \tau_p = \frac{1}{\sqrt{\gamma - \alpha}}, \quad D = -\frac{2\beta}{2\eta^2}, \quad \theta = \frac{\beta(\gamma - \alpha)}{2\eta^2}, \quad \sigma = 2\eta^2. \quad (9)$$

This result is very similar to $2\pi$-solution (7), however there is some difference: as the amplitude of the $\pi$-pulse is two times smaller than the amplitude of $2\pi$-pulse, the KLM-parameter should be increased four times and the dispersion should be decreased accordingly in order to produce the same effect and support coherent pulse generation.

Curve 4 in Fig. 2 depicts the duration of $\pi$-pulse. As is seen, the pulse duration slightly differs from the duration of $2\pi$-pulse (curve 2) and is shorter in the region of small $\eta$ (curve 2 in Fig. 4).

As was said before, the $\pi$-pulse inverts the population difference in the absorber that causes the noise amplification behind the pulse tail. Hence, the laser pulse stabilization is possible if the condition $\alpha + \gamma_a - \gamma < 0$ is satisfied. This defines the maximal initial loss in the absorber (dotted curve in Fig. 3). It is seen, that the maximal modulation depth exceeds the threshold (dashed curve) and, consequently, the generation of the stable $\pi$-pulse is possible.
When the condition \( \eta^2 < \frac{3\sqrt{\alpha^2 + \beta^2 - \sigma}}{4} \) holds there are the chirped pulses with \textit{sech}-shape. In this case, the expressions for \( D \) and pulse parameters are bulky and we do not write them here \cite{17}. The pulse duration is presented in Fig. 2 by curve 5. As is seen, the duration of chirped pulse can be very short even for the moderate value of \( P \). There is a minimum in the dependence of the pulse duration on \( \eta \) (an optimal reflectivity of semiconductor absorber device, curve 3 in Fig. 4) and it does not coincide with the point of precise chirp compensation (curve 1 in Fig. 5, a). Additionally, the pulse area is variable for this type of solution (curve 1 in Fig. 5, b).

There is a sharp minimum in the dependence of the pulse duration on \( \sigma \) (curve 4 in Fig. 4). In our case a corresponding KLM-parameter is \( 7 \cdot 10^7 \text{ W} \). As it was in the previous case, the minimum of the pulse duration does not coincide with the point of chirp compensation (curve 2 in Fig. 5, a). Unlike the case of variation of \( \eta \), the change of \( \sigma \) causes only a slight variation of the pulse area (curve 2 in Fig. 5, b).

Summarizing, the generation of \textit{sech}-shaped \( \pi \)-pulses and chirped pulses with variable area is possible in KLM-laser with coherent semiconductor absorber as a result of definite relation between KLM’s and saturable absorber’s contribution. A larger KLM contribution (for a fixed \( \eta \)) is needed to produce the pulse in the comparison with the case of \( 2\pi \)-pulse generation.

6 Self-starting ability

Our results suggest that the pulse duration is determined rather by KLM, whereas a saturable absorber puts a limitation on the pulse area and stabilizes the pulse against automodulations. But another important feature of KLM in the presence of semiconductor absorber is the self-starting ability. To estimate it in our model we analyzed an evolution of the initial noise spike, which is much longer than the relaxation time of the excitation in absorber \( T_a = 1 \text{ ps} \). For such noise spike an absorber is fast and the action of SPM and self-focusing is negligible. Using the normalization of the time, gain saturation energy and field intensity to \( T_{\text{cav}} \), \( E_a \) and \( E_a/T_{\text{cav}} \), respectively, an evolution equation for the pulse seed is:

\[
\frac{\partial a(z, t)}{\partial z} = \left( \frac{P_{\alpha_{\text{max}}} T_r}{1 + \frac{2\tau_r a_0^2(z) \eta(z)}{\eta^2} + PT_r} - \frac{\gamma a}{1 + 2a_0(z) t_p T_a - \gamma + t_f^2 \frac{\partial^2}{\partial t^2}} \right) a(z, t),
\]

(10)
where all notations have the meaning as before, and the field parameters refer to the noise spike. To solve Eq. (10) we used, as before, an aberrationless approximation. The decay of the field (growth of the pulse duration and decrease of its intensity) means in our model that the system will not self-start. An opposite situation with an asymptotic growth of the pulse seed testifies about ability of the system to self-start.

Fig. 6 demonstrates the regions of the initial pulse parameters corresponding to the self-starting. The dark zone corresponds to the pump $P = 8.5 \times 10^{-4}$, which is close to the threshold of mode locking self-starting. Lower pump can not provide the self-starting while a higher pump ($P = 8.8 \times 10^{-4}$) causes an expansion of self-starting region.

7 Conclusion

In conclusion, we investigated analytically the conditions of the coherent pulse formation in cw solid-state laser with the semiconductor absorber. It was found, that the mode locking in the absence of Kerr-lens-induced fast saturable absorption does not produce $sech$-shaped pulse. However, there exists $2\pi$-pulse (pulse of self-induced transparency), which has fs-duration and is stabilized by the defined minimal modulation depth of absorber. The stabilization results from the coherent interaction with absorber in the condition of the positive difference between saturated gain and linear loss coefficients. But positive value of this difference increases the threshold of sub-10 fs pulse generation due to growth of the modulation depth of absorber, that is required by stability condition. A combined action of KLM and coherent absorption produces the $sech$-shaped pulse. In this case there is no a requirement to the minimal modulation depth of semiconductor absorber. The pulse duration, which is close to the fundamental limit, is defined by KLM and the coherent absorber defines the pulse area, stabilizes the pulse against the automodulations and self-starts the mode locking operation. As result, there are the generation of $2\pi$, $\pi$-pulses and chirped pulses with variable area.

Our results can be useful for the development of high-efficient self-starting generators of extremely short pulses for fs-spectroscopy, X-ray and THz generation.

All calculations in this paper were carried out in computer algebra system Maple V (r5.0) and Maple 6, the corresponding commented programs are presented on http://www.geocities.com/optomaplev and summarized in [15].
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[17] corresponding expressions can be found on http://www.geocities.com/optomaplev/programs/laser_pi.html
8 Figure captions

Fig. 1. 2π-pulse envelope in the coordinates “pulse area – pulse amplitude” as resulted from numerical solution of Eq. (5) (solid curve) and the sech-shaped pulse envelope (dashed curve). \(\gamma = 0.04, \gamma_a = 0.01, \delta = 0.042, t_f = 2.5 fs\).

Fig. 2. Pulse duration \(t_p\) versus pump \(P\). 2π-pulses (1, 2) in the absence and (1′, 2′) in the presence of SPM. (2, 2′) – unstable against the noise solutions. (3) 2π-pulse; (4) π-pulse; and (5) chirped pulse (\(\sigma = 0.14, \beta = 0.26\)) in the presence of KLM. \(\alpha_{\text{max}} = 0.1, T_r = 3 \mu s, T_{\text{cav}} = 10 ns, \tau = 6.25 \times 10^{-4}, \gamma = 0.01, \eta = 1 (1, 1', 2, 2'), 0.5 (3), 0.2 (4), 0.3 (5)\).

Fig. 3. Minimal modulation depth of absorber \(\gamma_a\) for (1) stable and (2) unstable 2π-pulse, (dashed curve) generation threshold and (dotted curve) maximal modulation depth for π-pulse. Dashed region is a stability zone. \(\eta = 1 (1, 2), 0.2 \text{ (dotted curve)}\).

Fig. 4. Pulse duration \(t_p\) versus (solid and dashed curves) \(\eta\) and (dotted curve) \(\sigma\) in the presence of KLM for (1) 2π-pulse; (2) π-pulse; (3) chirped pulse for \(\sigma = 0.14, \beta = 0.26\); (4) chirped pulse for \(\eta = 0.2, \beta = 0.26\). \(P = 0.001\) for all curves.

Fig. 5. a) chirp \(\varsigma\) and b) pulse area \(\psi/\pi\) versus \(\eta\) and \(\sigma\) in the presence of KLM: (1) \(\sigma = 0.14\); (2) \(\eta = 0.2, \beta = 0.26, P = 0.001\).

Fig. 6. Self-starting ranges on the plane “peak intensity of initial noise spike – duration of initial noise spike“. Dark and hatched (together with dark) regions correspond to the self-starting for \(P = 8.5 \times 10^{-4}\) and \(8.8 \times 10^{-4}\), respectively. \(T_a = 1 ps, \gamma = 0.01, \eta = 1, \tau = 6.25 \times 10^{-5}\).
Figure 1.
Figure 2.
Figure 3.
Figure 4.
Figure 6.