Supersymmetric contributions to $B \to DK$ and the
determination of angle $\gamma$

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Abstract

We analyze supersymmetric contributions to the branching ratios and CP asymmetries of $B^- \to D^0K^-$ and $B^- \to \bar{D}^0K^-$ processes. We investigate the possibility that supersymmetric CP violating phases can affect our determination for the angle $\gamma$ in the unitary triangle of Cabibbo-Kobayashi-Maskaw mixing matrix. We calculate the gluino and chargino contributions to $b \to u(\bar{c}s)$ and $b \to c(\bar{u}s)$ transitions in a model independent way by using the mass insertion approximation method. We also revise the $D^0 - \bar{D}^0$ mixing constraints on the mass insertions between the first and second generations of the up sector. We emphasize that in case of negligible $D^0 - \bar{D}^0$ mixing, one should consider simultaneous contributions from more than one mass insertion in order to be able to obtain the CP asymmetries of these processes within their $1\sigma$ experimental range. However, with a large $D^0 - \bar{D}^0$ mixing, one finds a significant deviation between the two asymmetries and it becomes natural to have them of order the central values of their experimental measurements.
1 Introduction

Recently, the BaBar collaborations have measured the charge CP asymmetries $A_{CP}$ and the branching ratios $R_{CP}$ of the $B^{-} \rightarrow D^{0}K^{-}$ and $B^{-} \rightarrow \bar{D}^{0}K^{-}$ decays [1]. The following results have been reported:

$$A_{CP^{+}} = 0.35 \pm 0.13({\text{stat}}) \pm 0.04({\text{syst}}), \quad A_{CP^{-}} = -0.06 \pm 0.13({\text{stat}}) \pm 0.04({\text{syst}}),$$

(1)

$$R_{CP^{+}} = 0.90 \pm 0.12({\text{stat}}) \pm 0.04({\text{syst}}), \quad R_{CP^{-}} = 0.86 \pm 0.10({\text{stat}}) \pm 0.05({\text{syst}}).$$

(2)

These results, with all other $B$-factories measurements, provide a stringent test of the Standard Model (SM) picture of flavor structure and CP violation and open the possibility of probing virtual effect from new physics at low energy.

In the SM, CP violation arises from complex Yukawa couplings which lead to the angles $\alpha, \beta$ and $\gamma$ in the unitary triangle of Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. The angle $\beta = \arg \left( \frac{V_{cd}V_{cb}^{*}}{V_{ud}V_{ub}^{*}} \right)$ has been determined by the CP asymmetry in $B^{0} \rightarrow J/\psi K_{S}$ process which is dominated by tree level contribution. Concerning the angle $\gamma = \arg \left( \frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}} \right)$, it is believed that a theoretically clean measurement of this angle can be obtained from exploiting the interference between $B^{-} \rightarrow D^{0}K^{-}$ and $B^{-} \rightarrow \bar{D}^{0}K^{-}$ when $D^{0}$ and $\bar{D}^{0}$ mesons decay to the same CP eigenstate [2].

At the quark level, the $B^{-} \rightarrow D^{0}K^{-}$ and $B^{-} \rightarrow \bar{D}^{0}K^{-}$ decays are based on $b \rightarrow u(\bar{c} \bar{s})$ and $b \rightarrow c(\bar{u} \bar{s})$ transitions respectively. Therefore, their SM contributions at tree level are suppressed by the CKM factors $V_{cs}V_{ub}^{*}$ and $V_{us}V_{cb}^{*}$ which are of order $10^{-3}$. This gives the hope that it may be possible for a new physics beyond the SM, like supersymmetry, which contributes to these decays at one loop level to manifest itself and compete the SM. In this paper we aim to investigate this possibility and check, in a model independent way, whether supersymmetry can significantly modify the CP asymmetries in $B^{-} \rightarrow D^{0}K^{-}$ processes and hence affects the determination of the angle $\gamma$. Therefore, we perform a systematic analysis of the SUSY contributions to $B \rightarrow DK$ processes. We compute SUSY contributions to $b \rightarrow u(\bar{c} \bar{s})$ and $b \rightarrow c(\bar{u} \bar{s})$ transitions through the gluino and chargino exchange, using the mass insertion approximation method. This approximation is quite useful tool for studying the SUSY contributions to the flavor processes in a model independent way. We show that the gluino box diagrams give the dominant SUSY contribution while the chargino exchanges lead to subdominant contributions.

It turns out that the $D^{0} - \bar{D}^{0}$ mixing may limit the gluino contribution to $B^{-} \rightarrow DK^{-}$ due to the stringent constraints on the mass insertions between the first and second generations in the up sector, $(\delta_{AB}^{a})_{12}$. Thus in our analysis, we revise the $D^{0} - \bar{D}^{0}$ mixing constraints [3] and take them into account. We find that with a single mass insertion, the SUSY contribution to $B^{-} \rightarrow DK^{-}$ decay will be much smaller than the SM result. Nevertheless, with simultaneous contributions from more than one mass insertion, the
SUSY effect can be enhanced and the results of the CP asymmetries become within 1σ experimental range, while the $D^0 - \bar{D}^0$ mixing constraints are satisfied.

The paper is organized as follows. In section 2 we study the CP asymmetries and the branching ratios of $B^- \rightarrow D K^-$ in the SM. We show that in the SM the branching ratios $R_{CP \pm}$ are within the experimental range. While the CP asymmetry $A_{CP \pm}$ is below its 1σ experimental lower bound and the value of $A_{CP}$ is typically negative. In section 3 we compute the gluino and chargino contributions to $b \rightarrow u(\bar{c}s)$ and $b \rightarrow c(\bar{u}s)$ transitions in terms of the mass insertions. Section 4 is devoted for analyzing the SUSY contribution to $D^0 - \bar{D}^0$ mixing and revise the possible constrain on the mass insertions ($\delta_{AB}^{u}$).

The analysis of SUSY contribution to CP asymmetries $A_{CP \pm}$ and branching ratios $R_{CP \pm}$ is given in section 5. We show that in case of negligible $D^0 - \bar{D}^0$ mixing, one should consider simultaneous contributions from more than one mass insertion in order to obtain $A_{CP \pm}$ within their 1σ experimental range. Nevertheless, the usual relation: $A_+ \simeq - A_-$ which is valid in the SM remains hold. With a large $D^0 - \bar{D}^0$ mixing, one finds a significant deviation between $A_+$ and $A_-$ and it becomes natural to obtain $A_{CP \pm}$ of order the central values of their experimental measurements. Finally, we give our conclusions in section 6.

2 $B^- \rightarrow D K^-$ in the Standard Model

In this section we analyze the CP violation in $B^- \rightarrow D K^-$ decays within the SM. The possible quark level topologies of $B^- \rightarrow D K^-$ that contribute to the amplitude $A(B^- \rightarrow D^0 K^-)$ and $A(B^- \rightarrow \bar{D}^0 K^-)$ in the SM can be classified to the following three categories, as shown in Fig.(1): color-favored tree (T), color-suppressed tree (C) and annihilation (A). These processes are given in terms of the CKM factors $\lambda_c = V_{cb} V_{us}^*$, $\lambda_u = V_{ub} V_{cs}^*$. The decay $B^- \rightarrow D^0 K^-$ receives contributions from $T$ and $C$ with factor $\lambda_c$, while $B^- \rightarrow \bar{D}^0 K^-$ get contributions from $C$ and $A$ in terms of $\lambda_u$. Since the contributions from the annihilation process to the matrix elements are quite suppressed at the leading order correction [4], it is quite reasonable to assume that $A = 0$. In our analysis we will adopt this approximation and therefore the general parametrization of the SM amplitudes of $B^- \rightarrow D K^-$ decays can be given by

$$A^{SM}(B^- \rightarrow D^0 K^-) = |A_1|e^{i\delta_1} \equiv \bar{T} + \bar{C},$$

$$A^{SM}(B^- \rightarrow \bar{D}^0 K^-) = |A_2|e^{i\delta_2}e^{i\gamma} \equiv C,$$

where $\delta_i, i = 1,2$ are the strong (CP-conserving) phases. $\bar{T}$ and $\bar{C}$ refer to the color allowed and color suppressed tree amplitudes involving $b \rightarrow c(\bar{u}s)$ while $C$ is related to the process $b \rightarrow u(\bar{c}s)$. In terms of the two CP-eigenstates of the neutral $D$ meson system, $D^0_{CP \pm} = (D^0 \pm \bar{D}^0)/\sqrt{2}$, one considers the ratios $R_{CP \pm}$ of charged averaged partial rates.
Figure 1: SM contributions to $B^− → DK^−$: color-favored tree (left up), color-suppressed tree (right-up and left-down) and annihilation (right-down)

and the charge asymmetries $A_{CP±}$:

$$R_{CP±} = \frac{2 [Γ(B^− → D_{CP±}^0 K^−) + Γ(B^+ → D_{CP±}^0 K^+)]}{Γ(B^− → D^0 K^−) + Γ(B^+ → D^0 K^+)}$$

(5)

$$A_{CP±} = \frac{Γ(B^− → D_{CP±}^0 K^−) - Γ(B^+ → D_{CP±}^0 K^+)}{Γ(B^− → D_{CP±}^0 K^−) + Γ(B^+ → D_{CP±}^0 K^+)}.$$  

(6)

We define the ratio of the SM amplitudes of $B^− → \bar{D}^0 K^−$ and $B^− → D^0 K^−$ as

$$r_B e^{iδ_B} = \frac{A_{SM}^{\bar{D}^0 K^−}}{A_{SM}^{D^0 K^−}}.$$  

(7)

According to Eqs.(3,4), $r_B = |A_2/A_1|$ and $δ_B = δ_2 − δ_1$. Using this parametrization, one finds that $R_± ≡ R_{CP±}$ is given by

$$R_± = 1 + r_B^2 ± 2r_B \cos δ_B \cos γ,$$

(8)

and $A_± ≡ A_{CP±}$ takes the form

$$A_± = \frac{±2r_B \sin δ_B \sin γ}{1 + r_B^2 ± 2r_B \cos δ_B \cos γ} = \frac{±2r_B \sin δ_B \sin γ}{R_{CP±}}.$$  

(9)

From Eq.(8) one gets

$$\cos γ = \frac{R_+ - R_-}{4r_B \cos δ_B}.$$  

(10)
Thus, by using the expressions for the CP asymmetries $A_{\pm}$ in Eq.(9), one can factorize the dependence on the strong phase and gets the following expression for the angle $\gamma$ in terms of $R_{\pm}, A_{\pm}$ and $r_B$ only:

$$\sin \gamma = \frac{2 \cos \gamma (A_+ - A_-)}{\sqrt{16 r_B^2 \cos^2 \gamma - (R_+ - R_-)^2 R_+ + R_-}}. \quad (11)$$

From this expression, one can easily see that the central experimental values of $R_{\pm}$ and $A_{\pm}$ with $r_B \simeq 0.1$ implies that the angle $\gamma$ is of order $\gamma \simeq 71^\circ$. It is worth mentioning that within the SM, the effect of the $D^0 - \bar{D}^0$ mixing is very small on extracting the angle $\gamma$ using the $B^- \rightarrow DK^-$ decays. As emphasized in Ref.[5], neglecting this mixing implies an error in determining $\gamma$ of order $0.1 \sim 1^\circ$.

In order to analyze the SM predictions for the $A_{\pm}$ and $R_{\pm}$ and compare them with the experimental results reported in Eqs.(1,2), let us consider the SM contributions to the $b \rightarrow u(\bar{c}s)$ and $b \rightarrow c(\bar{u}s)$ transitions. As shown in Fig. 1, within the SM the $B^- \rightarrow DK^-$ are pure ‘tree’ decays. The effective Hamiltonian of this transition is given by

$$H_{eff}^{b \rightarrow u(\bar{c}s)} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* \left[ C_1(\mu) Q_1^u + C_2(\mu) Q_2^u \right], \quad (12)$$

where $C_i$ and $Q_i^u$ are the Wilson coefficients and operators of this transition renormalized at the scale $\mu$ with

$$Q_1^u = (\bar{u}^\alpha \gamma_\mu L b^\alpha) \left( \bar{s}^\beta \gamma_\mu L c^\beta \right), \quad Q_2^u = (\bar{u}^\alpha \gamma_\mu L b^\alpha) \left( \bar{s}^\beta \gamma_\mu L c^\beta \right), \quad (13)$$

where $L = (1 - \gamma_5)$. The effective Hamiltonian for the $b \rightarrow c(\bar{u}s)$ transition can be obtained from the effective Hamiltonian in Eq.(12) by exchanging $u \leftrightarrow c$. The SM results for the corresponding Wilson coefficients are:

$$C_1(m_W) = 1 - \frac{11}{6} \frac{\alpha_s}{4\pi}, \quad C_2(m_W) = \frac{14\alpha_s}{16\pi}. \quad (14)$$

However, due to the QCD renormalization to the scale $\mu \simeq m_b$, $C_1$ and $C_2$ get mixed, as will be discussed in more details in the next section, and one finds

$$C_1(\mu) = 1.07, \quad C_2(\mu) = -0.17. \quad (15)$$

To evaluate the SM results to the decay amplitude of $B^- \rightarrow DK^-$, we have to determine the matrix elements for the operators $Q_{1,2}^u$. A detailed analysis for the matrix elements will be given in the next section. Here, we just give the matrix elements for these four operators in naive factorization:

$$\langle \bar{D}^0 K^- | Q_1^u | B^- \rangle = -\frac{X}{3}, \quad \langle \bar{D}^0 K^- | Q_2^u | B^- \rangle = -X, \quad (16)$$

$$\langle D^0 K^- | Q_1^c | B^- \rangle = -\frac{1}{3}X - Y, \quad \langle D^0 K^- | Q_2^c | B^- \rangle = -X - \frac{1}{3}Y, \quad (17)$$

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where
\[ X = iF_0^{B \rightarrow K}(m_D^2)f_D(m_B^2 - m_K^2), \quad Y = iF_0^{B \rightarrow D}(m_K^2)f_K(m_B^2 - m_D^2). \] (18)

There are two comments in order: i) The naive factorization can not determine the strong phases, therefore, in our analysis we consider these phases as free parameters. ii) As mentioned above, the factorized matrix element \( \langle \bar{D}^0 K^-|((s\gamma^\mu L)c)|0\rangle \langle 0|\bar{u}\gamma_\mu Lb|B^-\rangle \) corresponding to an annihilation process is suppressed as showed in Ref.[4], and can be neglected. Therefore,
\[ A^{SM}(B^- \rightarrow \bar{D}^0 K^-) = -\frac{G_F}{\sqrt{2}}V_{ub}V^*_{cs}Y \left( C_1 + C_2 \right), \] (19)
and
\[ A^{SM}(B^- \rightarrow D^0 K^-) = -\frac{G_F}{\sqrt{2}}V_{ub}V^*_{as} \left[ X \left( \frac{C_1}{3} + C_2 \right) + Y \left( C_1 + \frac{C_2}{3} \right) \right]. \] (20)

Fixing the hadronic parameters as follows: \( f_D = 0.2, f_K = 0.16, F_0^{B \rightarrow D} = 0.34, F_0^{B \rightarrow K} = 0.62, \) and the meson masses as: \( m_K = 0.49, m_D = 1.86, \) and \( m_B = 5.278 \) GeV. One finds
\[ r_B \simeq 0.05 \] (21)

Note that it is customary assumed that with a large uncertainty, the SM prediction for \( r_B \) may be much larger than the above value (can be \( \mathcal{O}(0.1) \), see Ref.[6]). Here we will use the value that we obtained in Eq.(21) as a typical value for the SM contribution. In order to have a general picture of the SM predictions for the CP asymmetries \( A_\pm \) and the branching ratios \( R_\pm \), we plot in Fig.2 \( R_\pm \) versus \( A_\pm \). Here, we vary the parameter \( \delta_B \) in the range \([0, \pi]\) and the angle \( \gamma \) is also considered between 0 and \( \pi \).

![Figure 2](image_url)

**Figure 2:** \( R_+ \) versus \( A_+ \) and \( R_- \) versus \( A_- \) within the Standard Model. The horizontal line in the left figure represents the lower bound of \( A_+ \) at 1\( \sigma \) experimental range.

As can be seen from the results in Fig.2, the SM predictions for the branching ratios \( R_\pm \) are within the 1\( \sigma \) experimental range. However, the results for the CP asymmetry \( A_+ \)
are below its experimental lower bound. Also the SM leads to a negative CP asymmetry \( A_\pm \) which is still consistent with its experimental results in Eq(1), due to the large uncertainties in these measurement. Therefore, more precise measurements would be very important in analyzing the SM predictions for \( R_\pm \) and \( A_\pm \) and, hence, in determining the value of the angle \( \gamma \).

3 SUSY contributions to \( b \to u(\bar{c}s) \) and \( b \to c(\bar{u}s) \)

The crucial point to note from the previous section, is that the SM contributions to the amplitudes of \( b \to u(\bar{c}s) \) and \( b \to c(\bar{u}s) \) transitions are suppressed by the CKM factors \( V_{ub} \simeq \mathcal{O}(10^{-3}) \) and \( V_{us}V_{ub} \simeq \mathcal{O}(10^{-3}) \) respectively. Therefore, it may be possible to have a comparable effect from new physics at one loop level which can compete with the SM tree level contribution. In this section we study the supersymmetric contributions to the \( b \to u(\bar{c}s) \) and \( b \to c(\bar{u}s) \) transitions. In this case, the effective Hamiltonian \( H_{\text{eff}}^{\Delta C=1} \) for the \( b \to u(\bar{c}s) \) can be expressed as

\[
H_{\text{eff}}^{\Delta C=1} = \sum_{i=1}^{10} \left( C_i^u(\mu) \, Q^u_i(\mu) + \tilde{C}_i^u(\mu) \, \tilde{Q}_i^u(\mu) \right),
\]

(22)

where \( C_i^u \) are the Wilson coefficients and \( Q_i^u \) are the relevant local operators at low energy scale \( \mu \simeq m_b \). The operators \( Q_i^u \) are given by

\[
\begin{align*}
Q_1^u &= (\bar{u}^a\gamma_\mu Lb^a) \left( \bar{s}^\beta \gamma^\mu Lc^\beta \right), \\
Q_2^u &= (\bar{u}^a\gamma_\mu Lb^a) \left( \bar{s}^\beta \gamma^\mu Lc^\beta \right), \\
Q_3^u &= (\bar{u}^a\gamma_\mu Lb^a) \left( \bar{s}^\beta \gamma^\mu Rc^\beta \right), \\
Q_4^u &= (\bar{u}^a\gamma_\mu Lb^a) \left( \bar{s}^\beta \gamma^\mu Rc^\beta \right), \\
Q_5^u &= (\bar{u}^a Lb^a) \left( \bar{s}^\beta Lc^\beta \right), \\
Q_6^u &= (\bar{u}^a Lb^a) \left( \bar{s}^\beta Lc^\beta \right), \\
Q_7^u &= (\bar{u}^a Lb^a) \left( \bar{s}^\beta Rc^\beta \right), \\
Q_8^u &= (\bar{u}^a Lb^a) \left( \bar{s}^\beta Rc^\beta \right), \\
Q_9^u &= (\bar{u}^a \sigma_{\mu\nu} Lb^a) \left( \bar{s}^\beta \sigma^{\mu\nu} Lc^\alpha \right), \\
Q_{10}^u &= (\bar{u}^a \sigma_{\mu\nu} Lb^a) \left( \bar{s}^\beta \sigma^{\mu\nu} Lc^\alpha \right),
\end{align*}
\]

(23)

where \( \alpha \) and \( \beta \) refer to the color indices. \( L, R \) are given by \( (1 \mp \gamma_5) \) respectively and \( \sigma_{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \). The operators \( \tilde{Q}_i^u \) are obtained from \( Q_i^u \) by the chirality exchange \( L \leftrightarrow R \). In the SM, the coefficients \( \tilde{C}_i^u \) are identically vanish, while in SUSY models, they receive contributions from both gluino and chargino exchanges. The corresponding operators for \( b \to c(\bar{u}s) \) can be obtained from the above expression by exchanging \( u \leftrightarrow c \).

The dominant SUSY contribution to the \( b \to u(\bar{c}s) \) transition can be generated through the box-diagrams with gluino exchange, as in Fig.3, and chargino exchange, as in Fig.4. From these figures, one can see that the \( b \to u(\bar{c}s) \) transition is based on two topologically distinct box diagrams only for gluino or chargino exchange. This is unlike the \( b \to d \) and \( b \to s \) transitions that contribute to \( B - \bar{B} \) mixing, where four topologically distinct box diagrams are included [7]. Therefore, it is expected that the Wilson coefficients for this
Figure 3: Box diagrams for $B^- \rightarrow K^- \bar{D}^0$ ($b \rightarrow u \bar{c}$ transition) with gluino exchanges, where $h, k, m, n = \{L, R\}$.

Figure 4: Box diagrams for $B^- \rightarrow K^- \bar{D}^0$ ($b \rightarrow u \bar{c}$ transition) with chargino exchanges, where $U = \{u, c, t\}, D = \{d, s, b\}$ and $h, k, m, n = \{L, R\}$.

process are different from those obtained in the literature for $b \rightarrow s$ transition. It is also worth mentioning that contributions through penguin diagrams to these transitions are always hybrid (i.e., contain internal SUSY and SM particles). Therefore, they are suppressed by $V_{ub}$ in addition to the usual loop suppression factor, hence they are much smaller than the pure SM or pure SUSY contributions. Thus, the Wilson coefficients at $m_W$ scale can be expressed as follows

$$C_i^u = (C_i^u)^{SM} + (C_i^u)\tilde{g} + (C_i^u)\tilde{\chi},$$

(24)

We evaluate the SUSY contributions to the Wilson coefficients by using the mass insertion approximation. The Mass insertion approximation is quite useful method in order to perform model independent analysis of flavor changing processes in general SUSY models. In our analysis we set to zero the contributions that are proportional to to the Yukawa coupling of light quarks. Also, we use the approximation of retaining only terms proportional to order $\lambda$. In the case of the gluino exchange all the above operators give significant contributions and the corresponding Wilson coefficients are given by

$$C_1^g(m_W) = \frac{\alpha_s^2}{48m_W^2}(\delta_{LL}^d)_{23}(\delta_{LL}^u)_{12}\left[7\hat{f}_6(x) - 4xf_6(x)\right],$$

(25)

$$C_2^g(m_W) = \frac{\alpha_s^2}{144m_W^2}(\delta_{LL}^d)_{23}(\delta_{LL}^u)_{12}\left[\hat{f}_6(x) + 20xf_6(x)\right],$$

(26)
\[ C^\tilde{g}_3(m_W) = \frac{5\alpha_s^2}{48\tilde{m}^2}(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12}\tilde{f}_6(x), \]  
\[ C^\tilde{g}_4(m_W) = \frac{11\alpha_s^2}{144\tilde{m}^2}(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12}\tilde{f}_6(x), \]  
\[ C^\tilde{g}_5(m_W) = \frac{2\alpha_s^2}{3\tilde{m}^2}(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12}xf_6(x), \]  
\[ C^\tilde{g}_6(m_W) = \frac{-\alpha_s^2}{9\tilde{m}^2}(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12}xf_6(x), \]  
\[ C^\tilde{g}_7(m_W) = \frac{\alpha_s^2}{12\tilde{m}^2}(\delta_{RR}^d)_{23}(\delta_{LL}^u)_{12}[-\tilde{f}_6(x) + 7xf_6(x)], \]  
\[ C^\tilde{g}_8(m_W) = \frac{\alpha_s^2}{36\tilde{m}^2}(\delta_{RR}^d)_{23}(\delta_{LL}^u)_{12}[5\tilde{f}_6(x) + xf_6(x)], \]  
\[ C^\tilde{g}_9(m_W) = -\frac{\alpha_s^2}{48\tilde{m}^2}(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12}xf_6(x), \]  
\[ C^\tilde{g}_{10}(m_W) = \frac{5\alpha_s^2}{144\tilde{m}^2}(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12}xf_6(x). \]

where \( x = m_{\tilde{g}}^2/\tilde{m}^2 \). The \( m_{\tilde{g}} \) is the gluino mass and the \( \tilde{m}^2 \) is an average squark mass. The functions \( f_6(x) \) and \( \tilde{f}_6(x) \) are the same as the loop function obtained in case of \( b \to d(\bar{q}q) \) and are given by

\[ f_6(x) = \frac{6(1 + 3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}, \]  
\[ \tilde{f}_6(x) = \frac{6x(1 + x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x - 1)^5}. \]

The Wilson coefficients \( \tilde{C}_i^\tilde{g} \) are simply obtained by interchanging \( L \leftrightarrow R \) in the mass insertions appearing in \( C_i^\tilde{g} \). The above Wilson coefficients are due to the gluino exchange of \( b \to u \) transition, the corresponding coefficients for \( b \to c \) transition can be obtained by changing the mass insertions \( (\delta_{AB}^u)_{12} \) to \( (\delta_{AB}^u)_{21} \) where \( \{A, B\} = \{L, R\} \).

Note that the discrepancy between the above SUSY Wilson coefficients of \( b \to u \) transition and those of \( b \to d \) or \( b \to s \) \( \Delta B = 2 \) transition is due to the following reasons: 1) the \( b \to u \) transition is based, as mentioned above, on two distinct box diagrams only in contrast of the \( \Delta B = 2 \) transition where four distinct box diagrams are involved. 2) All the external quarks in the box diagrams of \( b \to u(\bar{c}s) \) are different, therefore, one can not use Fierz transformation to relate any operator with the other unlike the case in \( \Delta B = 2 \). For instance, in \( B_d - \bar{B}_d \) mixing the operate \( Q_2 = (\bar{d}\gamma_\mu Lb^3) (\bar{d}\gamma_\mu Lb^3) \) is equivalent to the operator \( Q_2 = (\bar{d}\gamma_\mu Lb^3) (\bar{d}\gamma_\mu Lb^3) \). In this case, the Wilson coefficients \( C_1 \) and \( C_2 \) in Eqs.(25,26) are combined together and leads to to the usual \( \Delta B = 2 \) Wilson coefficient: \( C_1 \propto \alpha_s/(108\tilde{m}^2) \left[ 24xf_6(x) + 66\tilde{f}_6(x) \right] \). In this respect, it is clear that the expression used in Eq.(9) in Ref.[3] for \( H_{\text{eff}}^{b\to u(\bar{c}s)} \) is incorrect.
Now let us turn to the chargino contributions to the effective Hamiltonian in Eq. (22) in the mass insertion approximation. The leading diagrams are illustrated in Fig. 4, where the cross in the middle of the squark propagator represents a single mass insertion. Within the above mentioned approximation where we neglect contributions proportional to the light quark masses, one finds that the relevant chargino exchange affects only the operator $Q_1$, as in the SM and the corresponding Wilson coefficient is given by

$$C_1^\tilde{\chi}(m_W) = \frac{\alpha_s}{16\pi^2} \left[ \sqrt{\alpha} V_{\chi^1 L}^i U_{\chi^0 L}^i (\delta_{LL})_{12} ((\delta_{LL})_{23} + \lambda (\delta_{LR})_{13}) + \frac{y_t}{\sqrt{4\pi}} U_{\chi^1 L}^i U_{\chi^0 L}^j V_{\chi^0 L}^j (\delta_{LL})_{12} ((\delta_{LR})_{23} + \lambda (\delta_{LR})_{13}) \right] (L_2(x_i, x_j) - 2L_0(x_i, x_j)), \quad (37)$$

where $\alpha = g^2/4\pi$ and $g$ is the $SU(2)$ gauge coupling constant. The $\lambda$ parameter stands for the Cabibbo mixing, i.e., $\lambda = 0.22$. The $U_{ij}$ and $V_{ij}$ are the unitary matrices that diagonalise the chargino mass matrix and $y_t$ is the top yukawa coupling. The $x_i = m_{\chi_i}^2/m^2$, and the functions $L_0(x, y)$ and $L_2(x, y)$ are given by [7]

$$L_0(x, y) = \sqrt{xy} \frac{xh_0(x) - yh_0(y)}{x - y}, \quad h_0(x) = -11 + 7x - 2x^2 - \frac{6\ln x}{(1 - x)^4},$$

$$L_2(x, y) = \frac{xh_2(x) - yh_2(y)}{x - y}, \quad h_2(x) = \frac{2 + 3x - x^2}{(1 - x)^3} + \frac{6\ln x}{(1 - x)^4}. \quad (38)$$

Finally, we have also neglected the small contributions from the box diagrams where both gluino and chargino are exchanged as in Ref. [8].

To obtain the Wilson-coefficients at the scale $m_b$ one has to solve the corresponding renormalisation group equations. The solution is generally expressed as

$$C_i(m_b) = \sum_j U_{ij}(m_b, m_W) C_j(m_W), \quad (39)$$

where $U_{ij}(m_b, m_W)$ is the evolution matrix given by the $8 \times 8$ anomalous dimension matrix of leading order (LO) corrections in QCD [9]. Note that we have not included the operators $Q_{9,10}$ since they have zero matrix elements at LO and also they do not mix with the other operators in the evolution from $m_W$ scale down to $m_b$ scale.

$$U(m_b, m_W) = \hat{V} \left( \begin{bmatrix} \alpha_s(m_W) \\ \alpha_s(m_b) \end{bmatrix} \left( \frac{\alpha_s(m_B)}{\alpha_s(m_C)} \right) \right) \hat{V}^{-1}, \quad (40)$$

where $\hat{V}$ diagonalizes the $\hat{\gamma}^{(0)T}$

$$\hat{\gamma}^{(0)}_{D} = \hat{V}^{-1} \hat{\gamma}^{(0)T} \hat{V} \quad (41)$$

and $\hat{\gamma}^{(0)}$ is the diagonal elements of $\hat{\gamma}^{(0)}_{D}$. The value of $\beta_0$ is given by $\beta_0 = \frac{4}{3} N_c - \frac{2}{3} f$ where $N_c$ is the number of colors and $f$ is number of active flavors. Finally, the anomalous
dimension matrix \( \hat{\gamma}^0 \) at the leading order is given by

\[
\hat{\gamma}^0 = \begin{pmatrix}
-2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -16 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2 & 6 & 0 & 0 \\
0 & 0 & 0 & 0 & 6 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -16
\end{pmatrix}.
\] (42)

As can be seen from the above matrix that the mixing between different operators is divided into blocks. Each block contains two operators \( Q_i, Q_{i+1} \), \( i = 1, 3, 4, 7 \) and with no mixing between different blocks [10].

Let us now consider the evaluation of the hadronic matrix elements of the above operators which represents the most uncertain part in this calculation. In the limit of neglecting QCD corrections and \( m_b \gg \Lambda_{QCD} \), the hadronic matrix elements of \( B^- \rightarrow DK^- \) decay can be factorized. The hadronic matrix elements for the operators \( Q_i^u \) are given by

\[
\langle \bar{D}^0 K^- | Q_i^u | B^- \rangle = 0,
\] (43)

and

\[
\langle \bar{D}^0 K^- | Q_1^u | B^- \rangle = \frac{X}{3}, \\
\langle \bar{D}^0 K^- | Q_2^u | B^- \rangle = -X, \\
\langle \bar{D}^0 K^- | Q_3^u | B^- \rangle = \frac{2m_D^2}{3(m_b - m_s)(m_u + m_c)}X, \\
\langle \bar{D}^0 K^- | Q_4^u | B^- \rangle = \frac{2m_D^2}{(m_b - m_s)(m_u + m_c)}X, \\
\langle \bar{D}^0 K^- | Q_5^u | B^- \rangle = \langle \bar{D}^0 K^- | Q_6^u | B^- \rangle = 0, \\
\langle \bar{D}^0 K^- | Q_7^u | B^- \rangle = \frac{X}{6} \\
\langle \bar{D}^0 K^- | Q_8^u | B^- \rangle = \frac{X}{2}, \\
\langle \bar{D}^0 K^- | Q_9^u | B^- \rangle = \langle \bar{D}^0 K^- | Q_{10}^u | B^- \rangle = 0.
\] (44)

While the hadronic matrix elements for the operators \( Q_i^c \) are given as follows:

\[
\langle \bar{D}^0 K^- | Q_i^c | B^- \rangle = 0,
\] (45)

and

\[
\langle \bar{D}^0 K^- | Q_1^c | B^- \rangle = -Y - \frac{1}{3}X,
\]
\[ \langle D^0 K^- | Q_2^c | B^- \rangle = -\frac{1}{3} Y - X, \]
\[ \langle D^0 K^- | Q_3^c | B^- \rangle = Y + \frac{2m_D^2}{3(m_b - m_s)(m_u + m_c)} X, \]
\[ \langle D^0 K^- | Q_4^c | B^- \rangle = \frac{1}{3} Y + \frac{2m_D^2}{(m_b - m_s)(m_u + m_c)} X, \]
\[ \langle D^0 K^- | Q_5^c | B^- \rangle = \langle D^0 K^- | Q_6^c | B^- \rangle = 0, \]
\[ \langle D^0 K^- | Q_7^c | B^- \rangle = \frac{m_p^2}{(m_b - m_s)(m_u + m_c)} Y + \frac{1}{6} X, \]
\[ \langle D^0 K^- | Q_8^c | B^- \rangle = -\frac{1}{3} \frac{m_p^2}{(m_b - m_c)(m_u + m_c)} Y + \frac{1}{2} X, \]
\[ \langle D^0 K^- | Q_9^c | B^- \rangle = \langle D^0 K^- | Q_{10}^c | B^- \rangle = 0, \] (46)

where \( X \) and \( Y \) are given in Eq.(18).

Having evaluated the SUSY contributions to the Wilson coefficients and determined the hadronic matrix elements of the relevant operators, one can analyze the decay amplitude of \( B^- \rightarrow DK^- \) and study the SUSY effect on the CP asymmetries \( A_{\pm} \) and branching ratios \( R_{\pm} \). As can be observed, the Wilson coefficients depend on several mass insertions which are in general complex and provide new sources for the CP violation beyond the SM phase in the CKM mixing matrix. These new CP violating phases may contribute significantly to the \( b \rightarrow u \) transition and affect the determination of the angle \( \gamma \). Nevertheless, one should be very careful with the constraints imposed on these parameters. In fact, the dominant gluino contributions depend on the mass insertions: \( \langle \delta_{AB}^u \rangle_{12} \) and \( \langle \delta_{AB}^d \rangle_{23} \). The mass insertions \( \langle \delta_{AB}^d \rangle_{23} \) are constrained by the experimental results for the branching ratio of \( B \rightarrow X_s \gamma \) [11–13]. These constraints are very weak on the LL or RR mass insertion and more stronger for LR or RL mass insertions. Concerning the mass insertion \( \langle \delta_{AB}^u \rangle_{12} \), the chargino contributions to the \( K^0 \rightarrow \bar{K}^0 \) impose constraint on the LL mass insertion only [14]. However, the \( D^0 \rightarrow \bar{D}^0 \) mixing may induce more strangest constraints on both \( LL(RR) \) and \( LR(RL) \) mass insertions. Therefore, before we proceed in analyzing the decay amplitude of \( B^- \rightarrow DK^- \), we will take a short detour and give a detail analysis for the SUSY contributions to \( D^0 \rightarrow \bar{D}^0 \) mixing and revise the corresponding constraints on the \( \langle \delta_{AB}^u \rangle_{12} \) mass insertions.

4 Constraints from \( D^0 - \bar{D}^0 \) mixing

We start this section by summarizing the SM results for the \( D^0 - \bar{D}^0 \) mixing, then we consider the supersymmetric contribution to the effective Hamiltonian for \( \Delta C = 2 \) transitions given by gluino and chargino box exchanges.

In the \( D^0 \) and \( \bar{D}^0 \) systems, the flavor eigenstates are given by \( D^0 = (\bar{u}c) \) and \( \bar{D}^0 = (u\bar{c}) \).
The formulism for $D^0 - \bar{D}^0$ mixing is the same as the one used for $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing. The mass eigenstates are given in terms of the weak eigenstates as:

$$D_{1,2} = p\ D^0 \pm q\ \bar{D}^0,$$

(47)

where the ratio $q/p$ can be written in terms of the off-diagonal element of the mass matrix: $q/p = \sqrt{M_{12}^* / M_{12}}$ and $q/p \neq 1$ is an indication for the CP violation through mixing. The strength of $D^0 - \bar{D}^0$ mixing is described by the mass difference

$$\Delta M_D = M_{D_1} - M_{D_2}.$$  

The present experimental results for $\Delta M_D$ is given by [15]

$$(\Delta M_D)_{\text{exp}} < 1.7 \times 10^{-13} \text{ GeV}.$$  

(48)

The CP asymmetry of the $D^0$ and $\bar{D}^0$ meson decay to CP eigenstate $f$ is given by

$$a_f(t) = \frac{\Gamma(D^0 \to f) - \Gamma(\bar{D}^0 \to f)}{\Gamma(D^0 \to f) + \Gamma(\bar{D}^0 \to f)} = S_f \sin(\Delta M_D t) + C_f \cos(\Delta M_D t),$$

(49)

where $S_f$ and $C_f$ represent the mixing and direct CP asymmetry respectively and they are given by

$$S_f = \frac{2\text{Im}[\frac{\bar{\rho}(f)}{\rho(f)}]}{\mid\rho(f)\mid^2 + 1}, \quad C_f = \frac{\mid\bar{\rho}(f)\mid^2 - 1}{\mid\rho(f)\mid^2 + 1}.$$  

(50)

The parameter $\bar{\rho}(f)$ is defined by $\bar{\rho}(f) = \frac{A(D^0 \to f)}{A(\bar{D}^0 \to f)}$. Generically, the $\Delta M_D$ and $S_f$ can be calculated by

$$\Delta M_D = 2\mid\langle D^0 | H^{\Delta C=2}_{\text{eff}} | \bar{D}^0 \rangle \mid, \quad S_f = \sin \left( \arg \left[ \langle D^0 | H^{\Delta C=2}_{\text{eff}} | \bar{D}^0 \rangle \right] \right).$$  

(51)

Here $H^{\Delta C=2}_{\text{eff}}$ is the effective Hamiltonian responsible for $\Delta C = 2$ transition. In the framework of the SM, this transition occurs via box diagram in which two virtual down quarks and two virtual W bosons are exchanged. The $\Delta M_D^{SM} = 2\langle D^0 | (H^{\Delta C=2}_{\text{eff}})^{SM} | \bar{D}^0 \rangle$ is given by [16]

$$\Delta M_D^{SM} \simeq \frac{G_F^2 M_{D_f}^2}{3\pi^2} |V_{cs}V_{cd}|^2 \frac{(m_s^2 - m_d^2)^2}{m_c^2} \simeq 1.4 \times 10^{-18} \text{ GeV}.$$  

(52)

As can be seen from this expression, the SM predicts a very small $D^0 - \bar{D}^0$ mixing. Note that, in the above estimation for $\Delta M_D^{SM}$, the $b$-quark contribution has been neglected since it is much smaller due to the CKM suppression. Also, the CP violation is absent in the mixing and in the dominant tree level decay due to the involving of the first two generations only.
In supersymmetric theories, the dominant contributions to the off diagonal entry in the $D^0$-meson mass matrix, $\mathcal{M}_{12} = \langle D^0 | H_{\text{eff}}^{C=2} | \bar{D}^0 \rangle$, is given by

$$
\mathcal{M}_{12} = \mathcal{M}^{\text{SM}}_{12} + \mathcal{M}_{12}^\vartheta + \mathcal{M}_{12}^{\tilde{c}^+},
$$

(53)

where $\mathcal{M}_{12}^\vartheta$ and $\mathcal{M}_{12}^{\tilde{c}^+}$ correspond to the gluino and chargino contributions respectively. The effect of supersymmetry can be parameterized as follows

$$
\frac{r^2_c e^{2i\theta_c}}{\mathcal{M}^{\text{SM}}_{12}} = \mathcal{M}_{12},
$$

(54)

where $\Delta M_D = 2|\mathcal{M}^{\text{SM}}_{12}| r^2_c$ and $2\theta_c = \arg\left(1 + \frac{M_{\text{SUSY}}}{\mathcal{M}^{\text{SM}}_{12}}\right)$. As in the case of $K^0$ and $B^0$ systems, The most general effective Hamiltonian for $\Delta C = 2$ processes, induced by gluino and chargino exchanges through box diagrams, can be expressed as

$$
H_{\text{eff}}^{\Delta C=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu) \tilde{Q}_i(\mu) + \text{h.c.},
$$

(55)

where $C_i(\mu)$, $\tilde{C}_i(\mu)$ and $Q_i(\mu)$, $\tilde{Q}_i(\mu)$ are the Wilson coefficients and operators respectively renormalized at the scale $\mu$, with

$$
Q_1 = \bar{u}^\alpha_c \gamma^\mu c_L \bar{u}^\beta_c \gamma^\mu c_L, \quad Q_2 = \bar{u}^\alpha_R c_L \bar{u}^\beta_R c_L, \quad Q_3 = \bar{u}^\alpha_R c_R \bar{u}^\beta_R c_R, \\
Q_4 = \bar{u}^\alpha_R c_L \bar{u}^\beta_R c_R, \quad Q_5 = \bar{u}^\alpha_R c_L \bar{u}^\beta_R c_L.
$$

(56)

In addition, the operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging $L \leftrightarrow R$.

In the case of the gluino exchange all the above operators give significant contributions and the corresponding Wilson coefficients are given by [17]

$$
C^{\tilde{g}}_1(m_W) = -\frac{\alpha_s^2}{216m_\tilde{g}^2} (24x f_6(x) + 66 \bar{f}_6(x)) (\delta_{12}^u)^2_{LL}, \\
C^{\tilde{g}}_2(m_W) = -\frac{\alpha_s^2}{216m_\tilde{g}^2} 204x f_6(x) (\delta_{12}^u)^2_{RL}, \\
C^{\tilde{g}}_3(m_W) = \frac{\alpha_s^2}{216m_\tilde{g}^2} 36x f_6(x) (\delta_{12}^u)^2_{RL}, \\
C^{\tilde{g}}_4(m_W) = -\frac{\alpha_s^2}{216m_\tilde{g}^2} \left(504x f_6(x) - 72 \bar{f}_6(x)\right) (\delta_{12}^u)^2_{LR} - 132 \bar{f}_6(x) (\delta_{12}^u)^2_{LR} (\delta_{12}^u)^2_{RL}, \\
C^{\tilde{g}}_5(m_W) = -\frac{\alpha_s^2}{216m_\tilde{g}^2} \left(24x f_6(x) + 120 \bar{f}_6(x)\right) (\delta_{12}^u)^2_{LR} - 180 \bar{f}_6(x) (\delta_{12}^u)^2_{LR} (\delta_{12}^u)^2_{RL},
$$

(57)

where $x = m_\tilde{g}^2/\bar{m}^2$ and $\bar{m}^2$ is an average squark mass. The functions $f_6(x)$ and $\bar{f}_6(x)$ are given in Eqs.(35,36). The Wilson coefficients $\tilde{C}_{1-3}$ are simply obtained by interchanging $L \leftrightarrow R$ in the mass insertions appearing in $C_{1-3}$. 

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In the case of the chargino exchange the operator $Q_1$ only gives a significant contribution [14]. At the first order in the mass insertion approximation, the Wilson coefficient $C_1^\chi(m_W)$ is given by

$$
C_1^\chi(m_W) = \frac{g^4}{768\pi^2\bar{m}^2} \sum_{i,j} |V_{i1}|^2 |V_{j1}|^2 (\delta^d_{21})^2_{LL} L_2(x_i, x_j).
$$

(58)

where $x_i = m_{\chi_i}^2/\bar{m}^2$, and the function $L_2(x, y)$ is as given in Eq.(38).

As usual, the Wilson coefficients $C_i(\mu)$ are related to $C_i(m_W)$ by [18]

$$
C_r(\mu) = \sum_i \sum_s \left( b_i^{(r,s)} + \eta c_i^{(r,s)} \right) \eta^a_i C_s(m_W),
$$

(59)

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$ and the coefficients $b_i^{(r,s)}$, $c_i^{(r,s)}$, and $a_i$ appearing in (59) can be found in Ref.[18]. Also the matrix elements of the operators $Q_i$ in the vacuum insertion approximation are given by [17]

$$
\langle D^0|Q_1|\bar{D}^0\rangle = \frac{1}{3} M_D f_D^2,
$$

$$
\langle D^0|Q_2|\bar{D}^0\rangle = -\frac{5}{24} \left( \frac{M_D}{m_c + m_u} \right)^2 M_D f_D^2,
$$

$$
\langle D^0|Q_3|\bar{D}^0\rangle = \frac{1}{24} \left( \frac{m M_D}{m_c + m_u} \right)^2 M_D f_D^2,
$$

$$
\langle D^0|Q_4|D^0\rangle = \left[ \frac{1}{24} + \frac{1}{4} \left( \frac{M_D}{m_c + m_u} \right)^2 \right] M_D f_D^2,
$$

$$
\langle D^0|Q_5|\bar{D}^0\rangle = \left[ \frac{1}{8} + \frac{1}{12} \left( \frac{M_D}{m_c + m_u} \right)^2 \right] M_D f_D^2.
$$

The same results are also valid for the corresponding operators $\tilde{Q}_i$ since strong interactions preserve parity.

We now discuss the results of SUSY contribution to $D^0 - \bar{D}^0$ mixing. It is worth mentioning that the mass insertions $(\delta^d_{AB})_{12}$ are strongly constrained by the experimental limits of $K^0 - \bar{K}^0$ mixing. In particular, the $\Delta M_K$ upper bound implies that $|\langle \delta^d_{LL} \rangle_{12}| \leq 10^{-4}$ [17]. Therefore, the chargino contribution to $\Delta M_D$ becomes very suppressed and can be neglected with respect to the gluino contributions which depend on less constrained mass insertions $(\delta^u_{AB})_{12}$. As an example, we present the gluino contribution to $\Delta M_D$, with $m_\tilde{g} \simeq m_\tilde{q} \simeq 500$ GeV:

$$
\frac{\Delta M_D^{\text{SUSY}}}{1.7 \times 10^{-13}} \simeq \begin{vmatrix}
33.4(\delta^u_{LL})_{12}^2 + 1733.6(\delta^u_{LR})_{12}^2 - 3178.5(\delta^u_{LR})_{12}(\delta^u_{RL})_{12} + 1733.6(\delta^u_{RL})_{12}^2 \\
- 12946.9(\delta^u_{LL})_{12}(\delta^u_{RR})_{12} + 33.4(\delta^u_{RR})_{12}^2
\end{vmatrix} < 1.
$$

(61)
From this expression, we can see that the strongest constraint will be imposed on the product $(\delta_{u LR})_{12}(\delta_{u RL})_{12}$ while the constraint obtained on the individual mass insertion $(\delta_{u LL})_{12}$ or $(\delta_{u RR})_{12}$ is less stringent. As usual in this kind of analysis, the most conservative constraints on the mass insertions can be obtained by considering the contribution due to a single mass insertion per time and set all other ones to zero. In table I, we present the results for the upper bounds on the relevant mass insertions from the experimental constraint on $\Delta M_D$ for $x = 1/4, 1, 4$. We find that these bounds on $(\delta_{u AB})_{12}$ are more stringent than those obtained from the chargino contribution to the $K^0 - \bar{K}^0$ in Ref.[14]. In fact, the $(\delta_{u LR})_{12}$ and $(\delta_{u RL})_{12}$ are completely unconstrained by the chargino contribution to $K^0 - \bar{K}^0$ mixing. Therefore, their bounds in the above table are the only known constraints. However, we should mention that these constraints may be relaxed if one consider simultaneous contributions from more than one mass insertion. In this case, a possible cancellation may occur which reduce the SUSY contribution significantly and leave a room for a larger mass insertion.

Finally, we comment on the CP violation in this process. As emphasized above, the SM contribution to $D^0 - \bar{D}^0$ is real since it is proportional to $V_{cs}^* V_{cd}$. Furthermore, it is much smaller than the dominant gluino contribution. Therefore CP violating phase $\theta_c$ in Eq.(54) can be written as

$$\theta_c = \frac{1}{2} \arg \left( \frac{M_{12}}{M_{SM}^{12}} \right) \simeq \frac{1}{2} \arg \left( \tilde{M}_{12}^{\tilde{q}} \right).$$

(62)

In case $(\delta_{u LR})_{12}$ gives a dominant contribution to $\tilde{M}_{12}^{\tilde{q}}$, $\theta_c$ will be given by

$$\theta_c = \frac{1}{2} \arg \left( (\delta_{u LR})_{12} \right) \simeq \mathcal{O}(1),$$

(63)

which means that the mixing CP asymmetry of $D^0 - \bar{D}^0$, $S_f$, could be quite large. Therefore, one can conclude that the new physics in general and supersymmetry in particular could enhance the $D^0 - \bar{D}^0$ mixing significantly.

| $x$ | $\sqrt{|(\delta_{u LL}^{u RR})_{12}|^2}$ | $\sqrt{|(\delta_{u LR}^{u RL})_{12}|^2}$ | $\sqrt{|(\delta_{u LL}^{u RR})_{12}(\delta_{u RR}^{u RL})_{12}|}$ | $\sqrt{|(\delta_{u LR}^{u RL})_{12}(\delta_{u RL}^{u LR})_{12}|}$ |
|-----|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1/4 | $7.5 \times 10^{-2}$          | $7.5 \times 10^{-2}$          | $1 \times 10^{-2}$             | $1 \times 10^{-2}$             |
| 1   | $1.7 \times 10^{-2}$          | $2.4 \times 10^{-2}$          | $8.7 \times 10^{-3}$           | $1.7 \times 10^{-2}$           |
| 4   | $0.4$                          | $3.3 \times 10^{-2}$          | $1.2 \times 10^{-2}$           | $4 \times 10^{-2}$             |

Table 1: Upper bounds on $\sqrt{|(\delta_{AB}^{u AB})_{12}|^2}$ from $\Delta M_D < 1.7 \times 10^{-13}$ GeV, for $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2 = 1/4, 1, 4$. 
5 Supersymmetric contribution to $R_{\pm}$ and $A_{\pm}$

In this section we study the supersymmetric contributions to the CP asymmetries and the branching ratios of $B^- \rightarrow DK^-$ decay in the following cases: 1) negligible $D^0 - \bar{D}^0$ mixing. 2) Large $D^0 - \bar{D}^0$ mixing due to a possible significant SUSY contribution as advocated in the previous section.

In general, applying the naive factorization approximation implies that the amplitudes $A(B^- \rightarrow DK^-)$ are given by

$$A(B^- \rightarrow D^0 K^-) = \sum_{i=1}^{8} \left(C_i^c - \tilde{C}_i^c\right) |D_i^0 K^-| Q_i^c |B^-|,$$

and

$$A(B^- \rightarrow \bar{D}^0 K^-) = \sum_{i=1}^{8} \left(C_i^u - \tilde{C}_i^u\right) |\bar{D}_i^0 K^-| Q_i^u |B^-|. \quad (64)$$

The sign difference between the Wilson coefficients $C_i$ and $\tilde{C}_i$ in the above equations is due to the fact that the initial and final states of $B^- \rightarrow DK^-$ decays have opposite parity and therefore $\langle DK^- | Q_i | B^- \rangle = -\langle DK^- | \tilde{Q}_i | B^- \rangle$ [19].

5.1 $R_{\pm}$ and $A_{\pm}$ with negligible $D^0 - \bar{D}^0$ mixing

In case of neglecting the effect of $D^0 - \bar{D}^0$ mixing, it is useful to parameterize the SUSY contribution by introducing the ratio of the SM and SUSY amplitudes as follows:

$$\frac{A^{SUSY}(B^- \rightarrow D^0 K^-)}{A^{SM}(B^- \rightarrow D^0 K^-)} = R_1 e^{i(\phi_1 - \gamma)} e^{i\delta_1}, \quad (66)$$

and

$$\frac{A^{SUSY}(B^- \rightarrow \bar{D}^0 K^-)}{A^{SM}(B^- \rightarrow D^0 K^-)} = R_2 e^{i\phi_2} e^{i\delta_2}, \quad (67)$$

where $R_i$ stands for the corresponding absolute value of $|A^{SUSY}/A^{SM}|$, the angles $\phi_i$ are the corresponding SUSY CP violating phase, and $\delta_i = \delta_i^{SM} - \delta_i^{SUSY}$ are the strong phases. In this respect, our previous definition for the SM ratio of the amplitudes of $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow D^0 K^-$ in Eq.(7) will be generalized as follows

$$\frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} = \frac{A^{SM}(B^- \rightarrow \bar{D}^0 K^-) + A^{SUSY}(B^- \rightarrow \bar{D}^0 K^-)}{A^{SM}(B^- \rightarrow D^0 K^-) + A^{SUSY}(B^- \rightarrow D^0 K^-)}$$

$$= r_B e^{i\delta_B} \left[ e^{i\gamma} + R_1 e^{i\phi_1} \right] \equiv R_B e^{i\delta_B} e^{i\phi_B}, \quad (68)$$

where

$$R_B = r_B \left| \frac{e^{i\gamma} + R_1 e^{i\phi_1}}{1 + R_2 e^{i\phi_2}} \right|, \quad \text{and} \quad \phi_B = \text{arg} \left[ \frac{e^{i\gamma} + R_1 e^{i\phi_1}}{1 + R_2 e^{i\phi_2}} \right], \quad (69)$$

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Note that, for simplicity, we have assumed that the SM and SUSY strong phases are equal. In this case, the ratios $R_\pm$ and the CP asymmetries $A_\pm$ take the form

$$R_\pm = 1 + R_B^2 \pm 2R_B \cos \delta_B \cos \phi_B,$$

and

$$A_\pm = \frac{\pm 2R_B \sin \delta_B \sin \phi_B}{1 + R_B^2 \pm 2R_B \cos \delta_B \cos \phi_B}.$$

As shown in Eq.(69), the deviation of $R_B$ from the standard model value $r_B$ is governed by the size of $R_1$ and $R_2$. Therefore, we start our analysis by discussing the dominant gluino contributions to $R_1$ and $R_2$. We choose the input parameters as $\tilde{m} = 250$ GeV, $x = 1$ we obtain

$$R_1 = 0.15(\delta_{LL}^d)_{23}(\delta_{LL}^u)_{12} - 0.17(\delta_{RR}^d)_{23}(\delta_{LL}^u)_{12} + 0.18(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{12} - \{L \leftrightarrow R\},$$

and

$$R_2 = -0.01(\delta_{LL}^d)_{23}(\delta_{LL}^u)_{21} - 0.015(\delta_{RR}^d)_{23}(\delta_{LL}^u)_{21} + 0.03(\delta_{RL}^d)_{23}(\delta_{LR}^u)_{21} - \{L \leftrightarrow R\}.$$  

Using the fact that the mass insertion is less than or equal one, we find that $R_2 \ll 1$, i.e. $A^{\text{SUSY}}(B^- \rightarrow D^0 \bar{K}^-) \ll A^{\text{SM}}(B^- \rightarrow D^0 \bar{K})$. It is worth mentioning that $(\delta_{AB}^d)_{23}$ is constrained by the experimental results for $B \rightarrow X_s \gamma$ decay. These constraints are very weak on the $LL$ and $RR$ mass insertions and they can be of order one. However, they impose stringent upper bounds on the $LR$ and $RL$ mass insertions, namely $|(\delta_{LR(RL)}^d)_{23}| \lesssim 1.6 \times 10^{-2}$ [11]. Concerning the $(\delta_{AB}^u)_{12}$, the important constraints on these mass insertions are due to the $D^0 - \bar{D}^0$ mixing. Applying these constraints one finds that $R_1$ is also quite small and the SM gives the dominant contribution. Therefore, there will be no chance to modify the results obtained in Fig. 2. However, as advocated in the previous section, these constraints can be relaxed if one allows for simultaneous contributions from more than one mass insertion, which is the case in any realistic model. In this case, there may be cancellation between different contributions which reduces the SUSY contribution to $D^0 - \bar{D}^0$ mixing without severely constraining the mass insertion. If we adopt this scenario and assume, for instance, that $(\delta_{LL}^d)_{23} \simeq -(\delta_{RR}^d)_{23}$ and $(\delta_{LL}^u)_{12} \simeq -(\delta_{RR}^u)_{12}$, then one can easily see that $R_1 \simeq \mathcal{O}(0.6)$ and the phase $\phi_1$ is given by $\arg \left[ (\delta_{LL}^d)_{23} + (\delta_{LL}^u)_{12} \right]$.

In this case, one can easily observe that different combinations of $(\gamma, \phi_1)$ can lead to values for the $A_\pm$ within the experimental range. Therefore, the supersymmetric CP violating phases may affect the extraction of the angle $\gamma$. As an example, let us consider the case where $R_B$ is enhanced from 0.05 (SM value) to 0.1 and the phase $\phi_B$ is given by $70^\circ$, which can be obtained by $\gamma \sim \pi/3$ and $\phi_1 \sim \pi/2$ or $\gamma \sim \pi/2$ and $\phi_1 \sim \pi/3$. In this case, one finds that

$$R_+ \simeq 1.1, \quad R_- \simeq 0.94, \quad A_+ \simeq -A_- \simeq 0.2.$$  

$$\text{(74)}$$
Therefore, we can conclude that the SUSY contributions to $B^- \to DK^-$ imply that $A_+$ and $A_-$ are within their $1\sigma$ experimental range simultaneously, unlike the SM results.

Finally, it is important to mention that in this scenario it is a challenge to find a realistic SUSY model that accommodates these results and satisfies all other constraints. Also the observation of $A_+$ indicates that the ratio of the amplitudes for the processes $B^- \to \bar{D}^0K^-$ and $B^- \to D^0K^-$ is larger than 0.1 which is rather difficult to obtain in the SM, so it may be a hint for a new physics effect.

### 5.2 $R_\pm$ and $A_\pm$ with large $D^0 - \bar{D}^0$ mixing

In the previous analysis, we have ignored the effect of the $D - \bar{D}^0$ mixing. Now we consider this effect and define the time dependent meson state $|D_1\rangle = |D^0(t)\rangle$ and $|D_2\rangle = |\bar{D}^0(t)\rangle$ as

$$
|D_1\rangle = g_+(t)|D^0\rangle + \frac{q}{p} g_-(t)|\bar{D}^0\rangle, \quad (75)
$$

$$
|D_2\rangle = g_+(t)|\bar{D}^0\rangle + \frac{p}{q} g_-(t)|D^0\rangle, \quad (76)
$$

where $q/p$ is defined, as in the previous section, by

$$
\frac{q}{p} = \sqrt{\frac{M_{12}}{M_{12}}} = e^{-2i\theta_c}. \quad (77)
$$

As shown in Eq.(63), the phase $\theta_c$ is of order one. The functions $g_\pm(t)$ is given by [20]

$$
g_\pm = \frac{1}{2} \left( e^{-\mu_i t} \pm e^{-i\mu_2 t} \right), \quad (78)
$$

with $\mu_i = M_{Di} - i\Gamma_{Di}/2$. In terms of $x_D = \frac{\Delta M_{D^0}}{\Gamma}$ and $y_D = \frac{\Delta \Gamma}{2\Gamma}$, where $\Gamma = \Gamma_{D_1} + \Gamma_2$, one finds

$$
g_+(t) = e^{(-iM_{D^0}t-\tau/2)} \left[ 1 + (x_D - i y_D)^2 \tau^2/4 + \ldots \right], \quad (79)
$$

$$
g_-(t) = e^{(-iM_{D^0}t-\tau/2)} \left[ (-ix_D - y_D)^2 \tau/2 + \ldots \right]. \quad (80)
$$

Here $\tau = \Gamma t$. In this case, the decay amplitudes of $B^- \to DK^-$ are given by

$$
A(B^- \to D_1K^-) = A(B^- \to D^0K^-)g_+(t) + A(B^- \to \bar{D}^0K^-) \frac{q}{p} g_-(t), \quad (81)
$$

and

$$
A(B^- \to D_2K^-) = A(B^- \to \bar{D}^0K^-)g_+(t) + A(B^- \to D^0K^-) \frac{p}{q} g_-(t). \quad (82)
$$

Also the decay rates are defined as [20]

$$
\Gamma(B^- \to DK^-) = \int dt \ |A(B^- \to DK^-)|^2. \quad (83)
$$
Therefore, one finds that
\[
\Gamma(B^- \to D_1 K^-) = |A(B \to D^0 K^-)|^2 \left( G_+ + R_B^2 G_- + 2 R_B \text{Re} \left[ G_{+ \to} e^{-i(\delta_B + \phi_B - 2 \theta_c)} \right] \right),
\]
where \( G_i \) are given by
\[
\begin{align*}
G_+ &= \int_0^\infty |g_+(t)|^2 dt \approx \frac{1}{\Gamma} \left( 1 + \frac{y_D^2 + x_D^2}{2} \right), \\
G_- &= \int_0^\infty |g_-(t)|^2 dt \approx \frac{1}{\Gamma} \left( \frac{y_D^2 + x_D^2}{2} \right), \\
G_{+ \to} &= \int_0^\infty g_+(t) g_+(t) dt \approx \frac{1}{\Gamma} \left( \frac{-y_D - i x_D}{2} \right).
\end{align*}
\]

The CP asymmetries \( A_{CP_{1,2}} \) are defined by
\[
A_{CP_{1,2}} = \frac{\Gamma(B^- \to D_{1,2} K^-) - \Gamma(B^+ \to D_{1,2} K^+)}{\Gamma(B^- \to D_{1,2} K^-) + \Gamma(B^+ \to D_{1,2} K^+)},
\]
Thus one can easily prove that
\[
A_{CP_1} = \frac{R_B \left[ y_D \sin \delta_B \sin(\phi_B - 2 \theta_c) - x_D \sin \delta_D \cos(\phi_B - 2 \theta_c) \right]}{G'_+ + R_B^2 G'_- - R_B \left[ y_D \cos \delta_B \cos(\phi_B - 2 \theta_c) + x_D \cos \delta_B \sin(\phi_B - 2 \theta_c) \right]},
\]
while
\[
A_{CP_2} = \frac{R_B \left[ y_D \sin \delta_B \sin(\phi_B - 2 \theta_c) + x_D \sin \delta_D \cos(\phi_B - 2 \theta_c) \right]}{R_B^2 G'_+ + G'_- - R_B \left[ y_D \cos \delta_B \cos(\phi_B - 2 \theta_c) - x_D \cos \delta_B \sin(\phi_B - 2 \theta_c) \right]}.
\]

Here \( G'_+ = \Gamma G_{+ \to} = (1 + \frac{y_D^2 - x_D^2}{2}, \frac{y_D^2 + x_D^2}{2}) \) respectively. The parameters \( x_D \) and \( y_D \) are subjected to stringent experimental bounds in case of \( \theta_c = 0 \): \( x_D^2 + y_D^2 \leq (6.7 \times 10^{-2})^2 \). For non-vanishing \( \theta_c \), this bound is no longer valid. However it is believed that in general \( x_D \sim y_D \sim 10^{-2} \). In this case, it is clear that \( G'_+ \simeq 1 \) and \( G'_- \simeq 10^{-4} \) which imply that
\[
A_{CP_1} \simeq 10^{-2} \times R_B \simeq O(10^{-3}),
\]
and
\[
A_{CP_2} \simeq \frac{10^{-2}}{R_B} \simeq O(0.1).
\]

From these results, it is remarkable that the effect of \( D^0 - \bar{D}^0 \) mixing breaks the usual relation between the CP asymmetries \( A_{CP_1} \equiv A_- \) and \( A_{CP_2} \equiv A_+ \): \( A_+ \simeq - A_- \) which is satisfied in the SM and SUSY models with negligible \( D^0 - \bar{D}^0 \) mixing, as we have emphasized in the previous sections. As an example to show how natural to obtain in this case CP asymmetries of order their central values of the experimental results in Eq.(1), let
us consider \( R_B \simeq 0.15, x_D \simeq 3 \times 10^{-2}, y \simeq 5 \times 10^{-2}, \delta_B \sim \pi \) and \( \phi_B \sim \theta_c \simeq \pi/4 \). One case easily find that

\[
A_{CP_1} \simeq 0.002, \quad A_{CP_2} \simeq 0.3, \quad (93)
\]

It is interesting to note that these values of the CP asymmetries depend on the CP violating SM phase \( \gamma \) and the SUSY phase in the \( b \to u \) transition \( \phi_1 \), which contribute together to \( \phi_B \) as in Eq.(69), in addition to the \( D^0 - \bar{D}^0 \) mixing phase \( \theta_c \). Therefore, the determination of the angle \( \gamma \) relies on the new SUSY phases \( \phi_1 \) and \( \theta_c \). This confirms the fact the our determination of the SM angle might be influenced by a new physics effect.

6 Conclusions

In this paper we have studied supersymmetric contributions to \( B^- \to D^0K^- \) and \( B^- \to \bar{D}^0K^- \) processes. We have shown that in the SM, the branching ratios \( R_{CP\pm} \) of these processes are within the experimental range. However the CP asymmetry \( A_{CP\pm} \) is below its \( 1\sigma \) experimental lower bound and the value of \( A_{CP-} \) is always negative. We have performed a model independent analysis of the gluino and chargino contributions to \( b \to u \) and \( b \to c \) transitions. We have used the mass insertion approximation method to provide analytical expressions for all the relevant Wilson coefficients.

The \( D^0 - \bar{D}^0 \) mixing experimental limits imply strong constraints on the mass insertions \( (\delta_{AB})_{12} \) which affect the dominant gluino contribution to \( B^- \to DK^- \). We have revised these constraints and took them into account. We showed that in case of negligible \( D^0 - \bar{D}^0 \) mixing, it is possible to overcome these constraint and enhance the SUSY results for the CP asymmetries in \( B^- \to DK^- \) if one consider simultaneous contributions from more than one mass insertion. In this case, the \( A_{CP+} \) becomes within \( 1\sigma \) experimental range. However, with a large \( D^0 - \bar{D}^0 \) mixing, one finds a significant deviation between the two asymmetries and it becomes natural to have them of order the central values of their experimental measurements.

In general, We have emphasized that SUSY CP violating phases may contribute significantly to the CP asymmetries in \( B^- \to DK^- \) and therefore, they may affect our determination for the angle \( \gamma \) in the unitary triangle of the CKM mixing matrix.

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References

[1] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0512067.

[2] M. Gronau and D. London., Phys. Lett. B 253, 483 (1991).
   M. Gronau and D. Wyler, Phys. Lett. B 265, 172 (1991),
   D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3257 (1997).

[3] D. Chang, W. F. Chang, W. Y. Keung, N. Sinha and R. Sinha, Phys. Rev. D 65, 055010 (2002).

[4] Z. z. Xing, Phys. Rev. D 53, 2847 (1996).
   B. Blok, M. Gronau and J. L. Rosner, Phys. Rev. Lett. 78, 3999 (1997).

[5] Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D 72, 031501 (2005).

[6] M. Gronau, Phys. Rev. D 58, 037301 (1998); M. Gronau, Phys. Lett. B 557, 198 (2003).

[7] E. Gabrielli and S. Khalil, Phys. Rev. D 67, 015008 (2003).

[8] D. Chakraverty, E. Gabrielli, K. Huitu and S. Khalil, Phys. Rev. D 68, 095004 (2003).
   E. Gabrielli, K. Huitu and S. Khalil, Nucl. Phys. B 710, 139 (2005).

[9] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).

[10] A. J. Buras, M. Jamin, M. E. Lautenbacher and P. H. Weisz, Nucl. Phys. B 400, 37 (1993).

[11] S. Khalil, Phys. Rev. D 72, 035007 (2005).

[12] T. Besmer, C. Greub and T. Hurth, Nucl. Phys. B 609, 359 (2001).

[13] M. Ciuchini, E. Franco, A. Masiero and L. Silvestrini, Phys. Rev. D 67, 075016 (2003)
    [Erratum-ibid. D 68, 079901 (2003)].

[14] S. Khalil and O. Lebedev, Phys. Lett. B 515, 387 (2001).

[15] C. Cawlfield et al. [CLEO Collaboration], Phys. Rev. D 71, 077101 (2005).

[16] A. Datta and D. Kumbhakar, Z. Phys. C 27, 515 (1985).

[17] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. B 477, 321, 1996.

[18] M. Ciuchini et al., JHEP 9810, 008 (1998).
[19] S. Khalil and E. Kou, Phys. Rev. Lett. 91, 241602 (2003),
    A. L. Kagan, talk at SLAC Summer Institute, August 2002.

[20] J. P. Silva and A. Soffer, Phys. Rev. D 61, 112001 (2000).

[21] H. N. Nelson, arXiv:hep-ex/9909028.