Research Article

Simulated Annealing Method-Based Flight Schedule Optimization in Multiairport Systems

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Received 3 November 2019; Revised 3 May 2020; Accepted 14 May 2020; Published 2 June 2020

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Integrated development and operational collaboration of regional airport groups have the potential to improve capacity and safety and also reduce environmental impacts and operational costs. However, research in multiairport systems (MASs), especially in China, is still in its infancy, with the consequences of unbalanced development, inadequate coordination, unclear function partitioning, difficulty in air traffic management, and poor service quality of regional airports. Considering these characteristics influencing effective interaction and collaboration of regional airports, this paper formulates a model to optimize the flight schedules in the MAS with multiple objectives of minimizing the maximum displacement of all flights, the weighted sum of total flight adjustment of each airport, and flight delays. An improved simulated annealing algorithm (SAA) is designed to solve the proposed multiobjective optimization problem. The model is applied to a case of the Beijing-Tianjin-Hebei Airport Group. The computational results demonstrate that the model generates significant reductions in maximum displacement, average displacement, and average delay, compared to the First-Come-First-Served (FCFS) principle. The model proposed in this paper can be used by civil aviation authorities, air navigation service providers, and airlines to facilitate the integrated management of flight schedules in the MAS.

1. Introduction

In order to improve the integrated development and operation of a group of airports in a multiairport system, the allocation of achievable flight resources should be fully optimized to significantly reduce congestion and delays. In a multiairport system, the relevant airports work together in a collaborative way towards their common goals. To date, research has focused on optimizing flight time resources at the tactical level. However, delivering real change in resolving congestion and delay requires the consideration of strategic level of operations, through optimizing the flight schedules.

Recently, the problem of flight scheduling and demand management has attracted considerable attention. Several studies focus on slot allocation based on market-based mechanisms such as congestion pricing and slot auctions [1–4]. Other studies seek an integrated approach to improve flight schedule coordination, through a joint optimization of flight schedules at the strategic level and airport operation efficiency at the tactical level [5, 6]. Simiaikis et al. optimize the allocation of airport capacity to airlines at slot-controlled airports in Europe by minimizing the difference between the requested and allocated scheduled times [7]. Based on the evaluation of the impact of flight continuity and passenger demands on airport capacity, Yang et al. [8] formulate a model to reduce flight delays in airport systems. The squeaky-wheel optimization (SWO) algorithm is improved and used to search for the minimal flight adjustment allocation scheme at the Beijing Capital airport.

The existing studies above are limited as they only consider adjusting the demands through flight schedules, while ignoring the interaction of several airports in a region that compete for limited airspace resources. The former is
addressed by [9] through a simulation-based optimization of flight schedule in the Pearl River Delta (PRD) region. Postorino and Pratichò [10] illustrate the application of the multiple-criteria decision-making (MCDM) method to validate the role and position of each airport within an MAS. To solve the multiairport ground-holding problem, some researchers have proposed multiobjective optimization approaches considering the objectives of minimizing the total delay time, total delay cost, total number of adjusted flights, and the total number of delayed flights, subject to the constraints such as airspace capacity and aircraft turnaround process [11]. Sidiropoulos et al. [12] propose a framework for the design and optimization of dynamic arrival and departure routing in airport terminal maneuvering areas (TMAs), which significantly improves the operational efficiency in MAS airspaces compared to the conventional segregated approach. Yin et al. [13] propose a methodological and assessment framework for runway configuration with a focus on the exploitation of multiple active runways in metroplex airports.

The optimization of flight allocation has been widely studied through various operation research approaches. Unfortunately, most of the corresponding models are formulated as a single-objective optimization which mainly focuses on minimizing flight delay [10, 11]. Ma et al. [14] consider a multiairport system with its surrounding airspace to attempt to solve the problem of optimal aircraft routing and scheduling. These research efforts have achieved some progress in improving slot management. However, they ignore the imbalance between air traffic demand and airport capacity in the MAS. Therefore, to significantly improve airport operations and alleviate congestion in terminal maneuvering areas, this paper focuses on flight schedule optimization in regional multiairport systems through the dual dimensions of space and time. In this paper, an optimization model of flight schedule in multiairport systems is formulated to minimize the maximum displacement of all flights, considering a weighted flight adjustment for each airport in the multiairport systems. To date, most of the algorithms used in flight management in individual airports and multiairport systems are heuristics and metaheuristics [15–18]. Additionally, branch and bound, branch and price, and queuing theory have also been adopted to solve specific problem formulations [19, 20]. The simulated annealing algorithm (SAA) is a metaheuristic algorithm which can be used to approximate the global solution in a large search space for mathematical optimization problems. Although the SAA has been used in other research fields, e.g., job-shop problem, sequencing problem, and resource allocation [21–23], it is not widely used in civil aviation research and in particular flight scheduling in multiairport systems.

When optimizing flight schedule, the first thing is to comply with the safety requirements and relevant operational regulations. All aircraft’s takeoff and landing times must be assigned as soon as possible to achieve optimal allocation of airspace and runway resources and minimal total delay time of TMA departures and arrivals. Besides, a scheduling scheme must be set without extra workload on air traffic controllers. In addition, the system should facilitate a flight to land at the most economic speed, minimizing the total fuel cost resulting from deviation of aircraft start-times from respective ready-times. The scheduling should be generated to facilitate the required arrival within a TMA as a flight can be delayed by waiting at holding points, speed control, and maneuver operation. In a word, the current optimization of a single airport flight schedule without the consideration of the other proximate airports in a multi-airport system is suboptimal. To solve this problem, this paper develops a framework for the optimization of the operation of multiairport systems. The contributions of this paper are summarized as follows: we formulate a model to optimize the flight schedules in multiairport systems with the objectives of minimizing the maximum displacement of all flights, the weighted sum of total flight adjustment of each airport, and flight delays. An improved simulated annealing algorithm (SAA) is designed to solve the proposed multiobjective optimization problem. The model is applied to the case of the Beijing-Tianjin-Hebei Airport Group. The results demonstrate that the model generates significant reductions in maximum displacement, average displacement, and average delay, compared to the First-Come-First-Served (FCFS) principle.

The paper is structured as follows. In Section 2, we describe the problem of flight schedule and formulate an optimized model for flight scheduling in multiairport systems. Then, an improved simulated annealing algorithm is proposed in Section 3 to solve the formulated model. In Section 4, we conduct a case study for the Chinese Beijing-Tianjin-Hebei Airport Group based on real data. Section 5 presents the comparison between the FCFS and SAA and discusses the computational results. Finally, the paper is concluded in Section 6.

2. Mathematical Formulation

2.1. Notations. Consider a set of flights $F = \{1, 2, \ldots, f\}$ in a multiairport system $J = \{1, 2, \ldots, e\}$, and $i \in f$ and $j \in J$. The planning horizon is divided into a set of time intervals $V = \{1, 2, \ldots, n\}$, and $v \in V$. In this paper, the length of a single time interval is set to 15 min. The notations of our mathematical formulation are shown in Table 1.

2.2. Flight Scheduling Model in MAS. The flight scheduling problem in a multiairport system involves different objectives from different perspectives that consider, for example, displacement of flight adjustment and flight delay. In this paper, we seek to balance a number of optimization objectives including the maximum displacement of all flights, weighted sum of total flight adjustment of each airport in the multiairport system, and the average flight delay in the multiairport system. The optimization model of flight scheduling in the MAS is presented as follows:
\[
\min z = \omega_1 \max_{i \in F, j \in J} |\mu_{ij}| + \omega_2 \sum_{j \in J} \left( \epsilon_j \sum_{i \in F_j} |\mu_{ij}| \right) + \frac{\omega_3}{s} \sum_{i \in F, j \in J} d_{ij},
\]

s.t. \[
\sum_{k=1}^{3} w_k = 1, \tag{2}
\]
\[
\sum_{v \in V} x_{ij}^{uv} = 1, \tag{3}
\]
\[
\sum_{v \in V} y_{ij}^{uv} = 1, \tag{4}
\]
\[
\sum_{u \in U} x_{ij}^{uv} = 1, \tag{5}
\]
\[
\sum_{u \in U} y_{ij}^{uv} = 1, \tag{6}
\]
\[
\sum_{v \in V} t_{x_{ij}^{uv}} = \sum_{v \in V} t_{a_{ij}^{uv}} + \mu_{ij}, \tag{7}
\]
\[
\sum_{v \in V} t_{y_{ij}^{uv}} = \sum_{v \in V} t_{d_{ij}^{uv}} + \mu_{ij}, \tag{8}
\]

\[
x_{ij}^{uv} = \begin{cases} 1, & \text{if flight } i \text{ is assigned a period } v \text{ to land/take off from airport } j \text{ and go through waypoint } u, \\ 0, & \text{otherwise,} \end{cases} \tag{9}
\]
\[
y_{ij}^{uv} = \begin{cases} 1, & \text{if flight } i \text{ originally scheduled to land/take off from airport } j \text{ and go through waypoint } u \text{ during period } v, \\ 0, & \text{otherwise,} \end{cases} \tag{10}
\]
\[
\begin{pmatrix} \sum_{v \in V} y_{ij}^{uv} - \sum_{v \in V} x_{ij}^{uv} \end{pmatrix} \geq Z_{ij} t_{ij}^{min}, \tag{11}
\]
Mathematical Problems in Engineering

The first objective function (i.e., the first part in equation (1)) minimizes the maximum displacement of all flights in the multiairport system. The second objective function (i.e., the first part in equation (2)) minimizes the weighted sum of the total flight adjustment of each airport in a multiairport system. The third objective function (i.e., the first part in equation (3)) minimizes the average flight delays in the multiairport system.

The constraints in equations (2)–(10) ensure that each flight ∀i ∈ F is related to an arrival/departure period, airport, and waypoint, which make the unique characteristics of variable setting. The scheduled time interval between each pair of connecting flights should meet particular values considering the regulated turn-around time. For ∀i, k ∈ F, aircraft connectivity constraints (11)–(13) ensure that the ground turn-around time is larger than the minimum turn-around time and smaller than the maximum turn-around time. The following three capacity constraints are related to the slot limits which we want to apply to the airport and route. Constraint (14) reflects the limit of flight schedule capacity curve at the airport j. Constraints (15) and (16) ensure that the number of arrival and departure flights is less than the route and the waypoint capacity in all periods. Moreover, these capacity constraints represent capacity limit of multiairport systems in regional airspace. Constraints (17)–(18) present the restriction of the maximum deviation and total deviation of flights.

3. Algorithm Design

To address the limitations of existing approaches discussed in Section 1, an improved simulated annealing algorithm (SAA) framework is developed below for the optimization of the flight operations in multiairport systems. In this section, we proposed an improved SAA to solve the formulated flight schedule optimization model in Section 2. In order to investigate the optimization performance of the SAA, we take the classical First-Come-First-Served (FCFS) as a baseline. In the real practice of Chinese airport operations, the FCFS principle is the most frequently adopted strategy to manage the landings and takeoffs.

3.1. Parameter Setting. The SAA is a random optimization algorithm based on the Monte Carlo iterative solution strategy and is an extension of the local search algorithm. Specifically, it is a metaheuristic algorithm to approximate global optimization in a large search space for an optimization problem. The starting point is the similarity between the annealing process of solid substances and general combinatorial optimization problems. By simulating the solid annealing process, starting from a certain initial temperature, as the temperature decreases, the optimal solution is searched in the solution space in combination with the probability jump characteristic tending to a global optimal. It uses the Metropolis-Hastings acceptance criteria and controls the algorithmic process with a set of parameters called a cooling schedule.

The parameters of this model are set as follows:

(i) Initial optimization sequence A is opt_A.
(ii) Optimal objective function value at each temperature: total delay = optimization (1).
(iii) Initial global optimal objective function value total delay = optimization_value.
(iv) Initial optimized scheduling time: X = X_opt.
(v) Initial temperature: temperature (1) = 50 + round (((0.98)^P) * 100). P is the Metropolis acceptance criteria, a certain probability at which the degraded solution is accepted so as to escape local extremes and avoid premature convergence.
(vi) Optimization times at the same temperature: iter = 10 + round ((1 – ((0.95)^P)) * P/4) * 10.
(vii) Initial value of repeat number: $l = 1$.
(viii) Initialize weight $w_1$, weight $w_2$, and weight $w_3$.
(ix) Execute expression (1).
(x) Start time = now.

3.2. SAA Framework. Based on the description above, the SAA implementation framework is presented in Figure 1, followed by the corresponding pseudocode in Algorithm 1. The new flight sequence temp_A is obtained by Monte Carlo simulation. A decision is first made on the feasibility of the new solution. If it is infeasible, a new solution will be generated iteratively. The maximum number of iterations is set to 30, after which the next cycle is initiated to prevent going into a dead cycle.

3.3. Baseline. FCFS is a type of service rules used in the Chinese air traffic management center, and it provides an efficient, simple, and error-free process scheduling algorithm that saves valuable computational resources [13]. Actually, it represents the actual operational data in the multiairport system. Under the rule of FCFS, the flight shall land in accordance with their estimated departure/arrival time. The first flight which abides by its estimated runway time will be given priority for takeoff/landing.

4. Data Source

In this paper, we conduct a case study of flight scheduling in the multiairport system of the Beijing-Tianjin-Hebei Airport Group, which includes three major airports: Beijing Capital International Airport (PEK), Tianjin Binhai International Airport (TSN), and Shijiazhuang Zhengding International Airport (SJW).

The airspace resource in the region when used individually by airports does not meet the traffic demand, causing severe congestion. One-day flight plan data in this airport group are selected for analysis in the case study. In the above regional airspace, capacity is limited with a few way points. These waypoints result in severe levels of airspace congestion in this multiairport system. Table 2 shows the basic information of the Beijing-Tianjin-Hebei region multiairport group.

5. Results and Discussion

This section presents and discusses the computational results from the FCFS and SAA methods. First, we illustrate the
result of FCFS, and then the result of the SAA is produced with the same weight settings, $w_1 = w_2 = w_3$. Finally, we interpret the results for the corresponding results with different weights.

5.1. Comparisons between FCFS and SAA with Equal Weights. The computational results of max. displacement, average displacement, and average delay with the FCFS and SAA methods are illustrated in Tables 3 and 4.

From the results of the FCFS and SAA methods with equal weights in Tables 3 and 4, the values of the max. displacement, average displacement, and average delay are significantly smaller for the simulated annealing algorithm than the FCFS method.

5.2. Results of SAA with Different Weights. When simulated annealing is applied to solve the multiobjective optimization of takeoff and landing problem, the weights of each objective will have a strong influence on the optimization results, which are shown in Table 5.

We can see from Table 5 that, if we only focus on minimizing the maximum displacement of all flights, the weight setting $w_1 = 1, w_2 = w_3 = 0$ is the best policy, and the minimal max. displacement is 16.4 min. If we only focus on minimizing the weighted sum of total flight adjustment of each airport in the multiairport system, the weight setting $w_2 = 1, w_1 = w_3 = 0$ is the best policy, and the minimal max. displacement is 6.55 min. If we only focus on minimizing the average flight delays in the multiairport system, the weight setting $w_3 = 1, w_1 = w_2 = 0$ is the

WHILE temperature ($l$) > 0.0001
FOR $i=1$ : iter
    Calculate $F(X')$ in accordance with expression (1).
    $\Delta F = \text{optimization_value} - F(X)$
    IF $\Delta F < 0$
        $A = \text{temp}_A$; (Accept $\text{temp}_A$ as the new taking off/landing sequence and enter the next cycle.)
        $X = X''$; (Accept $X''$ as a new current solution)
        Optimization ($l$) = $F(X'')$; (Update the optimal value of objective function at $l$th cycle)
        $\Delta d = F(X'')$-optimization_value;
        IF $\Delta d < 0$
            opt_ $A = \text{temp}_A$,
            $X_{\text{opt}} = ''$,
            optimization_value = $F(X'')$,
            optimization_value ($l$) = $F(X'')$,
        END
    ELSEIF exp($-\Delta F/\text{temperature} (l)$) > rand()
        $A = \text{temp}_A$,
        $X = ''$,
        optimization_value ($l$) = $F(X'')$,
    END
END FOR
endtime1 = now;
epstime1 = (endtime1 - starttime) * 24 * 3600;
IF epstime1 > 160
    break;
END
$\text{I} = \text{I} + 1$,
$\text{temperature} (l) = \text{temperature} (l - 1) * 0.99$;
IF $\text{temperature} (l) < \text{temperature} (l) * 0.618$
    $\text{temperature} (l) = \text{temperature} (l) * 0.6$;
END
END WHILE

Algorithm 1: Iteration procedure of the SAA.

| Airport | Runway quantity | Hour capacity | Terminal area (kilo square meters) | Number of aircraft movements in 2018 | 2018 handing capacity |
|--------|----------------|--------------|----------------------------------|-------------------------------------|-----------------------|
|        |                |              |                                  |                                     |                       |
| PEK    | 3              | 88           | 1414                             | 614022                              | 100,983               | 2074                  |
| TSN    | 2              | 34           | 364                              | 179414                              | 23,591                | 258                   |
| SJW    | 1              | 20           | 5.5                              | 89717                               | 11,333                | 46                    |

Table 2: Basic information of the Beijing-Tianjin-Hebei Airport Group.
best policy, and the minimal max. displacement is 10.12 min. Obviously, if we focus on the trade-offs among the three parts of objective with weight setting $w_1 = w_2 = w_3 = 1/3$, the values will be located in the interval from instance nos. 2–4 in Table 5.

Due to the characteristics of the heuristic algorithm, the simulated annealing algorithm has a challenging task to get the optimal solution by merely running once, even with appropriate parameter settings. Therefore, it is recommended to conduct multiple experiments and get the results with different weights. The evaluation method proposed in this paper can be employed to select the result which best satisfies the interests of multiple stakeholders in air transportation industry.

The limitation of our method is the sensitivity to the weights in making trade-offs among different parts of the objective function, while ignoring the flexible trade-off among different optimization objectives. We will address this limitation in future research.

6. Conclusions

This paper proposes a framework for flight schedule optimization in multiairport systems at the strategic level, while enforcing a set of constraints and an objective function with three different parts. We design an improved SAA to seek for the Pareto-optimal solutions of this computationally challenging problem, which can make trade-offs amongst three parts of the optimization objectives of the model, including the maximum displacement of all flights, weighted sum of total flight adjustment of each airport in the multiairport system, and the average flight delay in the multiairport system.

The method is applied to the Beijing-Tianjin-Hebei multiairport system, with the results showing that, compared with the classical FCFS as the baseline strategy, our optimized SAA strategy can improve the operation performance in the MAS. Improvements have been achieved in the efficiency of flight schedule, which leads to the realization of the full functionality of the international airport hub of PEK. In summary, the multiairport system schedule optimization model and algorithm can be applied to optimize flight schedule, which can significantly alleviate the congestion and delay problems in the multiple airport systems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (grant nos. 71731001 and 61671237).

References

[1] J. Brueckner, “Price vs. Quantity-based approaches to airport congestion management,” *Journal of Public Economics*, vol. 93, no. 5-6, pp. 681–690, 2009.
[2] L. Basso and A. Zhang, “Pricing vs. slot policies when airport proms matter,” *Transportation Research Part B*, vol. 44, no. 3, pp. 381–391, 2010.
[3] A. Czerny, “Airport congestion management under uncertainty,” *Transportation Research Part B*, vol. 44, no. 3, pp. 371–380, 2010.
[4] F. Liu and M. Hu, “Auction method for airspace congesting resource allocation and game equilibrium analysis,” *Transactions of Nanjing University of Aeronautics & Astronautics*, vol. 28, no. 3, pp. 282–293, 2011.
[5] A. Jacquillat and A. Odoni, *Congestion Mitigation through Schedule Coordination at JFK: An Integrated Approach*, Massachusetts Institute of Technology, Cambridge, MA, USA, ESD Working Paper Series, 2013.
[6] A. Jacquillat, *A Queuing Model of Airport Congestion and Policy Implications at JFK and EWR*, Massachusetts Institute of Technology, Cambridge, UK, 2012.
[7] I. Simaiaikis, M. Sandberg, and H. Balakrishnan, “Dynamic control of airport departures: algorithm development and field evaluation,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 1, pp. 285–295, 2014.

[8] X. Yang, L. Qi, and W. Zhai, “Optimal allocation and delay assessment of flight time resource,” *Journal of Henan University of Science and Technology (Natural Science)*, vol. 3, pp. 19–23, 2016.

[9] J. Hu, H. Y. Zhao, and L. Diao, “Simulation optimization of flight schedule of multi-airport system in PRD region,” *Computer Engineering and Applications*, vol. 49, no. 4, pp. 236–239, 2013.

[10] M. N. Postorino and F. G. Praticò, “An application of the multi-criteria decision-making analysis to a regional multi-airport system,” *Research in Transportation Business & Management*, vol. 4, pp. 44–52, 2012.

[11] B. Ye, M. Hu, and Y. Tian, “Multi-airport ground holding problem based on airline schedule optimization: models and algorithm,” *Journal of Southwest Jiaotong University*, vol. 45, no. 3, pp. 464–469, 2010.

[12] S. Sidiropoulos, A. Majumdar, and K. Han, “A framework for the optimization of terminal airspace operations in multi-airport systems,” *Transportation Research Part B: Methodological*, vol. 110, pp. 160–187, 2018.

[13] J. Yin, Y. Ma, W. Tian, D. Chen, Y. Hu, and W. Ochieng, “Impact analysis of demand management on runway configuration in metroplex airports,” *IEEE Access*, vol. 8, pp. 66189–66212, 2020.

[14] Y. Ma, M. Hu, J. Yin, and W. Tian, “Collaborative sequencing and scheduling method for arrival and departure traffic flow in metroplex terminal area,” *Acta Aeronautica et Astronautica Sinica*, vol. 38, no. 2, p. 320222, 2017.

[15] K. N. Androutsopoulos, E. G. Manousakis, and M. A. Madas, “Modeling and solving a bi-objective airport slot scheduling problem,” *European Journal of Operational Research*, vol. 284, no. 1, pp. 135–151, 2020.

[16] M. Sama, A. D’Ariano, F. Corman, and D. Pacciarelli, “Coordination of scheduling decisions in the management of airport airspace and taxiway operations,” *Transportation Research Part A: Policy and Practice*, vol. 114, pp. 398–411, 2018.

[17] J. Yin, M. Hu, H. Zhang, Y. Ma, and F. Wu, “Optimized method for multi-runway spatio-temporal resource scheduling in the mode of independent departures,” *Acta Aeronautica et Astronautica Sinica*, vol. 36, no. 5, pp. 1574–1584, 2015.

[18] J. Yin, M. Hu, H. Zhang, and Y. Ma, “Optimization approach for collaborative operating modes of multi-runway systems,” *Acta Aeronautica et Astronautica Sinica*, vol. 35, no. 3, pp. 795–806, 2014.

[19] K. N. Androutsopoulos and M. A. Madas, “Being fair or efficient? A fairness-driven modeling extension to the strategic airport slot scheduling problem,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 130, pp. 37–60, 2019.

[20] K. K. H. Ng, C. K. M. Lee, F. T. S. Chan, and Y. Qin, “Robust aircraft sequencing and scheduling problem with arrival/departure delay using the min-max regret approach,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 106, pp. 115–136, 2017.

[21] A. Elmi, M. Solimanpur, S. Topaloglu, and A. Elmi, “A simulated annealing algorithm for the job shop cell scheduling problem with intercellular moves and reentrant parts,” *Computers & Industrial Engineering*, vol. 61, no. 1, pp. 171–178, 2011.

[22] G. B. Murali, B. B. V. L. Deepak, M. R. Bahubalendruni, and B. B. Biswal, “Optimal assembly sequence planning towards design for assembly using simulated annealing technique,” in *Proceedings of the International Conference on Research into Design*, pp. 397–407, Singapore, July 2017.

[23] F. Ahmed and O. Tirkkonen, “Simulated annealing variants for self-organized resource allocation in small cell networks,” *Applied Soft Computing*, vol. 38, pp. 762–770, 2016.