DECAYING COLD DARK MATTER AND THE EVOLUTION OF THE CLUSTER ABUNDANCE

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ABSTRACT

The cluster abundance and its redshift evolution are known to be powerful tools for constraining the amplitude of mass fluctuations $\sigma_8$ and the mass density parameter $\Omega_{m0}$. We study the impact of the finite decay rate of cold dark matter particles on the cluster abundances. On the basis of a spherical model in a decaying cold dark matter universe, we calculate the mass function of clusters and compare it with observed cluster abundance. We find the decay of cold dark matter particles significantly changes the evolution of the cluster abundance. In particular, we point out that the lifetime of dark matter particles comparable to the age of the universe lowers the ratio of the local cluster abundance to the high-redshift cluster abundance and can account for the observed evolution of the cluster abundance quite well. The strong dependence of the cluster abundance on the decay rate of dark matter suggests that distant cluster surveys may offer clues to the nature of dark matter.

Subject headings: cosmological parameters — cosmology: theory — dark matter — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

The abundance of cluster of galaxies places a strong constraint on fundamental cosmological parameters such as the amplitude of mass fluctuations $\sigma_8$ and the present mass density parameter, $\Omega_{m0}$ (Henry & Arnaud 1991; Bahcall & Cen 1992; White, Efstathiou, & Frenk 1993; Eke, Cole, & Frenk 1996; Kitayama & Suto 1997). Although the abundance of clusters is a strong function of both $\Omega_{m0}$ and $\sigma_8$, the evolution of the cluster abundance with redshift breaks the degeneracy between $\Omega_{m0}$ and $\sigma_8$ (Eke et al. 1996; Henry 1997, 2000; Bahcall & Fan 1998; Bahcall & Bode 2003). For instance, Bahcall & Bode (2003) pointed out that the relatively high abundance of massive clusters at $z > 0.5$ prefers large mass fluctuations $\sigma_8 = 0.9 - 1$. On the other hand, the local cluster abundance in both optical (Bahcall et al. 2003) and X-ray (Ikebe et al. 2002; Seljak 2002) wave bands suggests low mass fluctuations $\sigma_8 = 0.7 - 0.8$ for $\Omega_{m0} \sim 0.3$. The combination of local and high-$z$ cluster abundances yields relatively low matter density in the universe, e.g., $\Omega_{m0} = 0.17 \pm 0.05$ (Bahcall & Bode 2003). However, other observations, such as the anisotropy of cosmic microwave background (Spergel et al. 2003) and Type Ia supernovae (Tonry et al. 2003), prefer somewhat larger matter density: $\Omega_{m0} = 0.27 \pm 0.04$ from Wilkinson Microwave Anisotropy Probe plus large-scale structure (Spergel et al. 2003) or $\Omega_{m0} = 0.28 \pm 0.05$ from Type Ia supernovae (Tonry et al. 2003). Although all these results are consistent with each other within 2 $\sigma$, the slight difference of $\sigma_8$ or $\Omega_{m0}$ might be interpreted to mean that underlying models are wrong (Bridle et al. 2003).

Although the above constraints are derived assuming the usual stable cold dark matter model, the evolution may be considerably different if cold dark matter particles are not perfectly stable and decay into relativistic particles with a decay rate $\Gamma$ (Cen 2001). Therefore, the evolution of the cluster abundance may become a useful test to explore the nature of dark matter. The decaying cold dark matter model has been studied so far in several contexts, e.g., to reconcile the Einstein–de Sitter universe with the current low-matter universe (Turner, Steinman, & Krauss 1984), to explain unexpected X-ray observations (Dilella & Zioutas 2003), or to reduce possible overconcentration of dark matter and overabundance of substructures in cold dark matter model (Cen 2001; Sanchez-Salcedo 2003). Moreover, cold dark matter with a similar property, disappearing dark matter, has also attracted much attention in the context of the brane world scenario (Randall & Sundrum 1999a, 1999b). In the brane world scenario, a bulk scalar field can be trapped on the brane, which is identified with our universe, and become cold dark matter. However, since the scalar field is expected to be metastable, it decays into continuum states in the higher dimension with some decay width $\Gamma$, which is determined by the mass of the scalar field and the energy scale of the extra dimension (Dubovsky, Rubakov, & Tinyakov 2000). An observer on the brane sees this as if the scalar field disappears into the extra dimension, leaving some energy on the brane. This energy behaves in the same way as energy of relativistic particles but does not correspond to real particles. Thus, this is called “dark radiation.” Although the disappearing dark matter and the decaying dark matter are not the same, they contribute to the expansion of the universe in the same way. Hereafter we call both of them decaying dark matter. Ichiki et al. (2003) studied the universe with decaying dark matter as a test of the brane world scenario. They found that a decaying cold dark matter model with a lifetime about the age of the universe improves the fit of Type Ia supernovae observations and the evolution of mass-to-light ratios of clusters.

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In this paper, we study structure formation in the decaying cold dark matter universe; we present the spherical model, make theoretical predictions for the cluster abundance, and show that the evolution of the cluster abundance is quite sensitive to the decay rate of dark matter particles. We also compare our predictions with observed low-z and high-z cluster abundances and see if the decaying cold dark matter model explains the relatively large cluster abundance at the high-z universe.

2. THEORETICAL MODEL

To study the cluster abundance, we need the mass function of dark halos in the decaying cold dark matter model. In this section, we construct the mass function using the spherical model as in Press & Schechter (1974). We also present the nonlinear overdensity \( \Delta_c \), which is important in studying the cluster abundance. Throughout this paper we assume a flat universe.

2.1. Cosmology and Linear Growth Rate

We consider the case that dark matter particles decay into relativistic particles, which are assumed to contribute to the structure formation only through the change of the expansion law of the universe by their energy behavior. Rate equations of matter and radiation components in the decaying dark matter model are

\[
\dot{\rho}_m + 3H\rho_m = -\Gamma \rho_m \quad \text{and} \quad \dot{\rho}_r + 4H\rho_r = \Gamma \rho_m.
\]

This results in the following evolution of matter and radiation:

\[
\frac{\dot{\rho}_m}{\rho_m} = \frac{\rho_m}{\rho_0} a^{-3} e^{-\Gamma t} \quad \text{and} \quad \frac{\dot{\rho}_r}{\rho_r} = \Gamma \rho_0 a^{-3} \int_0^t e^{-\Gamma(t-t_d)} dt,
\]

respectively. Here we assume no radiation component except for decay products. The Friedmann equation then becomes

\[
\frac{H^2(a)}{H_0^2} = \Omega_{m0} a^{-3} e^{-\Gamma(t-t_d)} + \Omega_{\Lambda0} a^{-4} \int_0^t e^{-\Gamma(t-t_d)} dt + \frac{\rho_0}{H^2}.
\]

This equation should be solved simultaneously with the equation of the cosmological time \( t = \int_0^a da/a[H(a)] \). By combining these two equations, we obtain the differential equation

\[
\ddot{a} - \frac{\dot{a}^2}{a} = -\frac{H_0^2}{2} \left( \frac{\Omega_{m0} e^{-\Gamma(t-t_d)}}{a^2} + 4a\lambda_0 \right) + \frac{\dot{\rho}}{\rho},
\]

where \( \dot{\rho} \equiv d\rho/da \). Note that the current density parameter \( \Omega_{m0} \) should satisfy the Friedmann equation at \( z = 0 \):

\[
1 = \Omega_{m0} + \frac{\Omega_{m0}}{H_0^2} \int_0^{t_0} a^2 e^{-\Gamma(t-t_d)} dt + \lambda_0.
\]

Therefore, the current density parameter \( \Omega_{m0} \) is uniquely determined for a given \( \Gamma \) and \( \lambda_0 \), i.e., \( \Omega_{m0} = \Omega_{m0}(\Gamma, \lambda_0) \).

Next, consider the linear perturbation. The density perturbation is denoted by \( \delta_m \equiv (\rho_m - \rho_0)/\rho_0 \), where \( \rho_0 \) is the background matter density. Then the perturbation equation becomes

\[
\ddot{\delta}_m + 2\frac{\dot{a}}{a} \dot{\delta}_m + 4\pi G \rho_0 \delta_m = 0.
\]

We solve this differential equation with the boundary condition \( \delta_m \propto a \) at \( a \ll 1 \).

2.2. Spherical Model

In this subsection, we describe spherical collapse in a decaying dark matter model and give the methodology to calculate nonlinear overdensity \( \Delta_c \). First, we consider a spherical overdensity with initial mass \( M \) and radius \( R \). The equation of motion of such a spherical shell is given by

\[
\frac{\ddot{R}}{R} = -\frac{GM}{R^2} e^{-\Gamma t} - H_0^2 \Omega_{m0} \int_0^t e^{-\Gamma(t-t_d)} dt + H_0^2 \lambda_0.
\]

We define the following quantities:

\[
x \equiv a/a_{\text{ta}}, \quad y \equiv R/R_{\text{ta}}, \quad \tau \equiv H(x = 1) [\Omega_m(x = 1)]^{1/2} t,
\]

\[
\gamma \equiv \Gamma / \left[ H(x = 1) [\Omega_m(x = 1)]^{1/2} \right], \quad \eta \equiv \lambda(x = 1) / [\Omega_m(x = 1)].
\]

Here, \( a_{\text{ta}} \) represents the scale factor at turnaround. Then the equations we should solve are the two dimensionless equations,

\[
\frac{d^2 y}{d\tau^2} + \frac{1}{2} \left[ e^{\gamma(x-x_0)}/x^2 + 4\eta x \right] \left( \frac{d\tau}{x} \right)^2 + \frac{1}{x} \frac{dy}{d\tau},
\]

and

\[
\frac{d^2 x}{d\tau^2} = \frac{d^2 y}{d\tau^2} \left( \frac{d\tau}{dx} \right)^2 \frac{dy}{d\tau} + \left( \frac{d\tau}{dx} \right)^2 \left[ \frac{1}{2} y^2 - 2\zeta e^{-\gamma(x-x_0)} - \gamma y x^4 \right.
\]

\[
\left. \times \int_0^\tau x e^{-\gamma(x-x_0)} d\tau + \eta y^2 \right],
\]

where \( \zeta \) is the density contrast of the spherical overdensity region at turnaround, \( \zeta \equiv \rho_{\text{spherical}} / \rho_m |_{x = 1} \). We solve these equations from \( \tau = 0 \) to the virialization of the overdensity region with boundary conditions \( \tau(x = 1) \approx 2(3/2)e^{-\gamma x_0/2} x^{3/2} \), \( \tau(x = 1) \approx \tau_{\text{ta}} \), \( y(x = 0) = 0 \), \( y(x = 1) = 1 \), and \( dy/dx(x = 1) = 0 \) in order to derive \( \tau_{\text{ta}} \) and \( \zeta \). Although the conventional choice of the virialization epoch is the collapse time (\( y = 0 \)), in this paper we define the virialization epoch when the virial theorem in the following holds:

\[
\left( \frac{dy}{d\tau} \right)^2 = \frac{1}{2} y^2 e^{-\gamma(x-x_0)} - \gamma y x^4 \int_0^\tau x e^{-\gamma(x-x_0)} d\tau + \eta y^2.
\]

Given the virialization epoch and virial radius, the nonlinear overdensity can be calculated from \( \Delta_c(x_{\text{vir}}) = (x_{\text{vir}}/v_{\text{vir}})^{3/2} - 1 \). To calculate the cluster abundance, we also need the extrapolation of the linear fluctuation \( b_c \) at virialization. We use \( b_c = 1.58 \), which is the value in Einstein–de Sitter universe for all cosmological models, mainly because of the computational cost. This simplification does not change our results, because we confirmed that \( b_c \) only weakly depends on the cosmological parameters.

2.3. Mass Function

We derive the mass function of clusters using the theory of Press & Schechter (1974). We start from an initial density field \( \delta(x, M_i, z_i) \) smoothed over the region containing mass \( M_i \). If the initial density field is a random Gaussian, a probability distribution function of \( \delta \) at any point is given
by

\[ P(\delta(M_t, z_i)) = \frac{1}{(2\pi)^{1/2} \sigma_M(z_i)} \exp \left[ -\frac{\delta^2(M_t, z_i)}{2\sigma_M^2(z_i)} \right], \]

where \( \sigma_M(z_i) = \sigma(R_M, z_i) \) is the mass variance. From the spherical model, it can be interpreted that the region is already virialized at \( z \) if the linearly extrapolated density contrast \( \delta_{\text{linear}}(M_t, z) \) exceeds the critical value \( \delta_c \). Therefore, the probability that the region with mass \( M \) is already virialized is given by

\[ f(M, t) = \frac{1}{2} \text{erfc} \left( \frac{\delta_c(z)}{\sqrt{2} \sigma_M} \right), \]

where \( \text{erfc}(x) \) is the complementary error function, \( \delta_c(z) \equiv \delta_c D(z = 0)/D(z) \), and \( \sigma_M \equiv \sigma_M(z = 0) \), where \( D(z) \) is linear growth rate calculated from equation (4). We assume the mass variance for the cold dark matter fluctuations with the primordial spectral index \( n = 1 \) and adopt a fitting formula presented by Kitayama & Suto (1996). From equation (10), we finally obtain the comoving number density of halos of mass \( M \) at time \( z \),

\[ \frac{dN}{dM}(M, z) = e^{2\delta_{\text{crit}}(z)} \sqrt{\frac{2}{\pi M \sigma_M^2}} \exp \left[ -\frac{\delta_c^2(z)}{2\sigma_M^2} \right] \Bigg|_{M = M_{\text{crit}}} \]

with \( \rho_0 = \rho_{\text{crit}}(z = 0) \Omega_m \theta e^{-\Gamma(t_0-t)} \).

3. EVOLUTION OF THE CLUSTER ABUNDANCE

In the model described in the previous section, the abundance and its redshift evolution of clusters are fully determined by three independent parameters: \( \lambda_0, \sigma_8 \), and \( \Gamma \). In this section, we see the dependence of the cluster abundances on these parameters.

Figure 1 plots the number density of clusters as a function of redshift. We consider the cases with the lifetime of dark matter \( \Gamma^{-1} = 10 \, h^{-1} \) Gyr, and also with the usual infinite lifetime of dark matter for reference. Following Bahcall & Bode (2003), we adopt the mass within a comoving radius of 1.5 \( h^{-1} \) Mpc, \( M_{1.5\,h^{-1}\text{Mpc}} \), to compare our theoretical predictions with observed cluster abundance. For the extrapolation to 1.5 \( h^{-1} \) Mpc, we use the observed cluster profile, \( N(<R) \sim R^{4.6} \) (Carlberg, Yee, & Ellingson 1997; Fischer & Tyson 1997). For the observed cluster abundance, we plot the data compiled by Bahcall & Bode (2003): abundance at \( z \sim 0.05 \) from Ikebe et al. (2002), at \( z \sim 0.38 \) from Henry (2000), at \( z \sim 0.5-0.65 \) and \( z \sim 0.65-0.9 \) from Bahcall & Fan (1998). Obviously, inclusion of finite lifetime greatly improves the situation: for \( \Gamma^{-1} \sim 10 \, h^{-1} \) Gyr, both local and high-\( z \) cluster abundance can be reproduced with the same \( \sigma_8 \). For \( \Gamma^{-1} = \infty \), it is difficult to explain local \( z \sim 0.05 \) and high-\( z \) \( z \sim 0.3-0.9 \) cluster abundances simultaneously, although both can be reconciled if we adopt larger \( \lambda_0, \lambda_0 \sim 0.8 \). This drastic change of the evolution of the cluster abundance implies that it can place a strong constraint on the decay rate of cold dark matter.

To see how large a decay rate is required to explain the observations, in Figure 2 we show the constraints from low-\( z \) (Bahcall et al. 2003) and high-\( z \) (Bahcall & Bode 2003) cluster abundances, assuming \( \lambda_0 = 0.7 \). We find that the lifetime of \( \Gamma^{-1} \leq 10 \, h^{-1} \) Gyr can explain both low-\( z \) and high-\( z \) cluster abundances. Interestingly, this decay rate is roughly consistent with that derived from the evolution of mass-to-light ratios \( \Gamma^{-1} \sim 10^{-1} \) Gyr (Ichiki et al. 2003).

**Fig. 1.—** Evolution of the cluster abundance for \( \lambda_0 = 0.7 \). The number of clusters with mass \( M_{1.5\,h^{-1}\text{Mpc}} > 8 \times 10^{14} \, h^{-1} \) Mpc, where \( M_{1.5\,h^{-1}\text{Mpc}} \) is the mass within a comoving radius of 1.5 \( h^{-1} \) Mpc, is plotted as a function of redshift. Filled circles with error bars: Observed abundance (see text for details). Solid line: Predicted cluster abundance with lifetime of dark matter \( \Gamma^{-1} = 10 \, h^{-1} \) Gyr, while we assume no decay for the dotted lines. We plot curves with \( \sigma_8 = 0.7, 0.8, 0.9, 1.0, 1.1, \) and 1.2 (only solid lines are labeled). Since adopted mass function in this paper is based on the theoretical model of Press & Schechter (1974), which is now known to be inaccurate, curves may be different in more accurate calculations. In particular, our mass function may underpredict the large-mass halos, and this may result in faster decline of the curves with \( z \).
large mass fluctuations $\sigma_8 \sim 1.1$ are required to account for this excess if it is interpreted as the Sunyaev-Zeldovich effect of the cluster at $z \sim 1$ (Komatsu & Seljak 2002; Bond et al. 2003).

While we have concentrated on the evolution of the cluster abundance, it is interesting to ask whether the decaying cold dark matter model is consistent with other observations, such as the anisotropy of cosmic microwave background and large-scale structure of the universe. For instance, Percival et al. (2001) constrained the cosmological parameters as $\Omega_{m0} h = 0.20 \pm 0.03$ from the power spectrum of galaxy clustering. Since the power spectrum primarily measures the epoch of the matter-radiation equality, $\Omega_{m0} h$ should be interpreted as $\Omega_{m0} e^{\gamma/2}$ in the decaying cold dark matter model. In the case of $h = 0.7$, $\Gamma^{-1} = 7 h^{-1}$ Gyr, we constrained $1 - \lambda_0$ as $0.19 < 1 - \lambda_0 < 0.23$ with a 68% confidence level. This allowed range corresponds to $0.23 < \Omega_{m0} e^{\gamma/2} h < 0.27$, and is thus marginally consistent with the observed power spectrum. Moreover, the situation becomes better if the Hubble constant is smaller than 0.7. For the anisotropy of the cosmic microwave background, it is difficult to infer the consequence of the decaying cold dark matter model because many parameters degenerate. We plan to study comprehensive constraints of these observations on decaying cold dark matter model, which will be presented elsewhere.

We note that our modeling of mass function may be inaccurate because it has been claimed that the mass function of Press & Schechter (1974), particularly in high-mass regions, tends to underestimate the abundance of the massive halos compared with $N$-body simulations (e.g., Jenkins et al. 2001). This may be a part of reason that the cluster abundance in our modeling based on the theory of Press & Schechter (1974) seems to decline slightly faster with redshift than that in Bahcall & Bode (2003), who used the result of $N$-body simulations. Thus, the constraints on the decay rate $\Gamma$ (e.g., Fig. 2) may be weaken a bit if we use more accurate mass function in decaying cold dark matter model. One way to refine the model is to resort to $N$-body simulations, as in the case of the standard cold dark matter model. Nevertheless, we believe that our qualitative result that decaying cold dark matter model tends to lower the ratio of the local cluster abundance to the high-redshift cluster abundance is still unchanged even when we adopt more accurate mass functions. The cluster abundance at higher redshift universe $z \sim 1$–2, which will be obtained by future surveys of X-ray or Sunyaev-Zeldovich effects, can tightly constrain the decay rate of dark matter.

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Fig. 3.—Constraints in the $(1 - \lambda_0)$–$\sigma_8$ plane assuming $\Gamma^{-1} = 7 h^{-1}$ Gyr. We also plot the 68% and 95% confidence limits from 172 Type Ia supernovae compiled by Tonry et al. (2003). The best-fit parameter set of the combination of these three constraints is $(1 - \lambda_0, \sigma_8) = (0.2, 1.2)$.

4. SUMMARY

We have presented structure formation model in a decaying cold dark matter universe. More specifically, we have calculated the spherical model and constructed the mass function in a similar way to Press & Schechter (1974). Using this mass function, we have shown that a decaying cold dark matter model with $\Gamma^{-1} \lesssim 10 h^{-1}$ Gyr can explain observed cluster abundances at both the low-$z$ and high-$z$ universe even better than the usual stable cold dark matter model. Our model might also be consistent with the excess of cosmic microwave background fluctuations on small scales.

**REFERENCES**

Bahcall, N. A., & Bode, P. 2003, ApJ, 588, L1

———. 2000, ApJ, 534, 565

Henry, J. P., & Arnould, K. A. 1991, ApJ, 372, 410

Ilievi, K., Garnavich, P. M., Kajino, T., Mathews, G. J., & Yahiro, M. 2005, Phys. Rev. D, in press

Ichiki, K., Narukage, T., & Suto, Y. 1996, ApJ, 469, 480

Kamada, T., Sato, Y., & Kitayama, T. 2004, Phys. Rev. D, in press

Kitayama, T., & Suto, Y. 1996, ApJ, 469, 480

———. 1997, ApJ, 490, 557

Komatsu, E., & Slosar, A. 2002, MNRAS, 336, 1256

Lahav, O., & Steinhardt, P. J. 2003, Science, 299, 1532

Liddle, L., & Zois, K. 2003, Astropart. Phys., 19, 145

Dubovsky, S. L., Rubakov, V. A., & Tinyakov, P. G. 2000, Phys. Rev. D, 62, 105011

Eke, V. R., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263

Fischer, P., & Tyson, J. A. 1997, AJ, 114, 14

Henry, J. P. 1997, ApJ, 489, L1

———. 2000, ApJ, 534, 565

Henry, J. P., & Arnould, K. A. 1991, ApJ, 372, 410

Ilievi, K., Garnavich, P. M., Kajino, T., Mathews, G. J., & Yahiro, M. 2005, Phys. Rev. D, in press

Ikebe, Y., Reiprich, T. H., Böhringer, H., Tanaka, Y., & Kitayama, T. 2002, A&A, 383, 773

Jenkins, A., Frenk, C. S., White, S. D. M., Cole, S., Evrard, A. E., Couchman, H. M. P., & Yoshida, N. 2001, MNRAS, 321, 372

Kitayama, T., & Suto, Y. 1996, ApJ, 469, 480

———. 1997, ApJ, 490, 557

Komatsu, E., & Slosar, A. 2002, MNRAS, 336, 1256

Percival, W. J., et al. 2001, MNRAS, 327, 1297

Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Randall, L., & Sundrum, R. 1999a, Phys. Rev. Lett., 83, 3370
———. 1999b, Phys. Rev. Lett., 83, 4690
Sanchez-Salcedo, F. J. 2003, ApJ, 591, L107
Seljak, U. 2002, MNRAS, 337, 769
Spergel, D. N., et al. 2003, ApJS, 148, 175

Tonry, J. L., et al. 2003, ApJ, 594, 1
Turner, M. S., Steigman, G., & Krauss, L. M. 1984, Phys. Rev. Lett., 52, 2090
White, S. D. M., Efstathiou, G., & Frenk, C. S. 1993, MNRAS, 262, 1023