A novel multi-objective-based approach to analyze trade-offs in Fair Principal Component Analysis

Guilherme Dean Pelegrina\textsuperscript{a,}\textasteriskcentered, Renan Del Buono Brotto\textsuperscript{b}, Leonardo Tomazeli Duarte\textsuperscript{a}, Romis Attux\textsuperscript{b}, João Marcos Travassos Romano\textsuperscript{b}

\textsuperscript{a}School of Applied Sciences, University of Campinas, 1300 Pedro Zaccaria Street, 13484-350 Limeira, Brazil
\textsuperscript{b}School of Electrical and Computer Engineering, University of Campinas, 400 Albert Einstein Avenue, 13083-852 Campinas, Brazil

Abstract

In dimension reduction problems, the adopted technique may produce disparities between the representation errors of two or more different groups. For instance, in the projected space, a specific class can be better represented in comparison with the other ones. Depending on the situation, this unfair result may introduce ethical concerns. Aiming at overcoming this inconvenience, a fairness measure can be considered when performing dimension reduction through Principal Component Analysis. However, a solution that increases fairness tends to increase the reconstruction error. In other words, there is a trade-off between equity and performance. In this context, this paper proposes to address this trade-off in Fair Principal Component Analysis problems by means of a multi-objective-based approach. For this purpose, we adopt a fairness measure associated with the disparity between the representation errors of different groups. Moreover, we investigate if the solution of a classical Principal Component Analysis can be used to find a fair projection. Numerical experiments attest that a fairer result can be achieved with a very small loss in the reconstruction error.

Keywords: machine learning, principal component analysis, fairness, multi-objective optimization

1. Introduction

In Machine Learning (ML), a typical concern is to project a statistical model able to perform a specific task, without an explicit knowledge of the relation between the inputs and outputs involved in the addressed problem \cite{Duda et al. 2000}. ML techniques have been highlighted in the last
years for its broad scope of application in many different areas, such as pattern recognition (Bishop, 2006), signal processing (Little, 2019), audio and image processing (Camastra & Vinciarelli, 2015) and others (Goodfellow et al., 2016).

Besides its ubiquitous use in technical problems, ML models have also been applied in tasks with social and economical impacts, e.g., credit concession (Hardt et al., 2016), recidivism prediction (Chouldechova, 2017) and gerrymandering (Kearns et al., 2018). Although these models can achieve a good performance, generally, they are not conceived in order to incorporate ethical concerns. As a consequence, ML techniques may lead to unfair decisions. Moreover, since several techniques work as black-boxes, it is difficult to understand how such unfair decisions were achieved. In this context, the Artificial Intelligence (AI) research community has been working in subjects related to ethical AI, such as the model explainability (Gilpin et al., 2018; Confalonieri et al., 2021; Kenny et al., 2021; Langer et al., 2021) and fairness (Barocas et al., 2019; Brandão et al., 2020; Saxena et al., 2020). The latter concept is the focus of this work. Since the ML approach is strongly dependent on the available data, the models can incorporate some biases present in the datasets (Barocas & Selbst, 2016). Once an unfair decision is made, the previous bias is reinforced, accentuating further the social disparities. Therefore, apart from the performance measure, such as the Mean Squared Error (MSE) or accuracy, the ML models must also incorporate additional information in order to deal with social biases. An example is to consider a fairness measure in the ML model construction (Samadi et al., 2018).

The fairness concern can be addressed in the pre-processing step (Dwork et al., 2012; Wang et al., 2018; Samadi et al., 2018), during the model training (Zafar et al., 2019) or even in a post-processing step (Lohia et al., 2019). For instance, in the pre-processing framework, which is the focus of this paper, fairness is generally associated with how one manages the acquired dataset. Suppose a dataset composed by $n$ samples and $d$ attributes; an induced bias may arise when one performs a dimension reduction in scenarios with a large number of attributes (e.g., from $d$ to $r$-dimensional space, where $d >> r$). As presented by Samadi et al. (2018), the dimension reduction provided by the classical Principal Components Analysis (PCA) (Jolliffe, 2002) may be biased towards one group of the population, leading to different reconstruction errors. For example, in a situation where sensitive attribute is the gender (male and female), the classical PCA may lead to a systematically lower reconstruction error for the male population. Since the projected data better represent males than females, the application of PCA may lead to disparate performances when these data are used in a ML model.
Motivated by the aforementioned ethical concerns, the first goal of this work is to introduce a fairness measure when dealing with dimension reduction using PCA. More specifically, alongside the “total” reconstruction error (the reconstruction error when using all samples, without a division into groups), we also consider a fairness measure as a cost function when seeking for the projection matrix. However, differently from existing works, which incorporate both objectives into a single one (Samadi et al., 2018), we address this problem in a multi-objective optimization fashion (Miettinen, 1999). Therefore, in our formulation, called Multi-Objective Fair Principal Component Analysis (MOFPCA), both criteria are optimized simultaneously. Moreover, instead of searching for a new projection matrix, we use the solution of the classical PCA to extract the projection vectors that will be used to optimize both objectives. As a result, we obtain a set of solutions that can be used to visualize the trade-offs between the reconstruction error and the fairness measure. This may be helpful in practical applications, since it provides a transparent way to support the decision under these conflicting objectives.

The fairness measure considered here is simply the disparity between the obtained reconstruction errors of each class. Since the projection matrix that minimizes the total reconstruction error does not lead, necessarily, to the minimum disparity (i.e., the objectives are conflicting), our proposal tends to lead to a set of optimal solutions (in the sense of Pareto optimality), frequently referred to as non-dominated set (Deb, 2001). Within this set, one may find the solutions that minimize each objective individually as well as some compromising solutions between them. With this framework, one can verify if it is possible to obtain a non-dominated solution associated with a small value of the reconstruction error and, at the same time, a fair representation of both classes.

The paper is organized as follows. In Section 2, we describe the existing works related to our proposal. We discuss the addressed problem in Section 3. In Section 4, we present the multi-objective optimization formulation and, in Section 5, the proposed MOFPCAm as well as the adopted evolutionary algorithm. Section 6 presents the numerical experiments and, finally, the concluding remarks and future perspectives are described in Section 7.

2. Related works

Olfat & Aswani (2019) presents a fair dimensionality-reduction method, based on a Semi Definite Program for PCA and Kernel PCA. With the projected dataset, the authors tackle a classification problem where the disparate impact was used as a fairness measure. Another work that addresses fair PCA was conducted by Samadi et al. (2018), who proposed a fairness criteria based on the
loss suffered from each protected group in the projected space with respect to their individual optimal projection. As a first step, they independently perform PCA for each protected group. For each execution of PCA, the reconstruction error for each group is calculated and the loss function indicates how far from these two benchmarks the total reconstruction error (obtained with PCA on all samples) lies. In the optimal scenario, one obtains a projection that deviates equally each population from their ideal reconstruction error. The authors have proposed an algorithm that performs this task (called FairPCA), obtaining as a solution a set of \( r+1 \) dimensions (or \( r+k \), if the sensitive attribute can assume \( k+1 \) values). This solution satisfies the loss-based fairness criterion and a reconstruction error equal to a \( r \)-dimensional PCA. The work conducted by Tantipongpipat et al. (2019) also presents some applications in the context of Fair ML, particularly in the problem of low-rank decomposition. The authors propose some algorithms able to consider the fairness requirement and that tighten the number of required extra dimensions proportional to \( \sqrt{k} \).

In the context of multi-objective optimization, fairness in PCA has also been addressed in (Kamani et al., 2019). The authors proposed a gradient descent algorithm for the Fair PCA projection matrix, where no extra dimension is required to perform the task. Moreover, the concept of non-dominance is based on the gradient direction and, therefore, only considers first order information. This may lead to dominated solutions in more general scenarios (where the cost function may not be convex) and it is necessary to use more robust algorithms to explore the feature space.

A common point on the approaches proposed by Samadi et al. (2018) and Kamani et al. (2019) is that they search for a projection matrix \( \tilde{U} \) that is different from the one obtained by the classical PCA (represented here by \( U \)). If \( \tilde{U} \) is the same as the one given by PCA, no fairness improvement is achieved. On the other hand, our proposal aims at using the principal components \( u_1, u_2, \ldots, u_d \) already obtained from the application of the classical PCA and then sorting them to attain the fairness concern. This difference is depicted in Figure 1.

In other words, with the purpose of reducing the number of attributes to a \( r \)-dimensional space, we combine the \( r \) directions of the classical PCA differently from the solution that minimizes only the total reconstruction error (which is given by the first \( r \) principal components). Therefore, the multi-objective proposed framework acts in the selection of the components that optimizes both reconstruction error and fairness measure simultaneously. This characteristic of the proposed method is interesting in already running data systems, since it introduces a re-sorting block that can be easily plugged into the data pipeline. Moreover, it takes advantage from the efficient algorithms to perform PCA even with high dimensional data (Fan et al., 2018).
3. Problem formulation

In this section, we briefly review the PCA methodology and the problem of Fair PCA.

3.1. Principal Component Analysis

Consider a data matrix \( X \in \mathbb{R}^{n \times d} \), with \( n \) as the number of \( d \)-dimensional samples. Roughly speaking, the aim of Principal Components Analysis (Jolliffe, 2002) is to seek for an orthogonal projection matrix \( U \in \mathbb{R}^{d \times r} \) that reduces the data dimension while minimizing the reconstruction error \( \mathcal{R}(U) \), given by

\[
\mathcal{R}(U) = \| X - XU^T U \|^2_F ,
\]

where \( \| \cdot \|_F \) is the Frobenius norm (Golub & Van Loan, 2013). Mathematically, the optimization problem tackled by PCA can be expressed by:

\[
\begin{align*}
\min_U & \quad \mathcal{R}(U) \\
\text{s.t.} & \quad U^T U = I,
\end{align*}
\]

where \( I \) is the identity matrix. Therefore, matrix \( U \) maps a point from \( \mathbb{R}^d \) to \( \mathbb{R}^r \) (\( U^T \) is the inverse mapping, taking a point from \( \mathbb{R}^r \) to \( \mathbb{R}^d \)). Without loss of generality, let us consider that the first \( l \) columns of \( U \) represent the first \( l \) principal components.

3.2. Fairness in PCA

Usually, the columns of \( X \) are referred to as attributes. Moreover, in social and economical problems, some of them can carry historic biases (the so-called sensitive attributes). Examples are gender, race and educational levels. As mentioned in Section 1, Samadi et al. (2018) discussed
the disparate results provided by PCA with respect to the reconstruction errors for groups with
different sensitive attribute values. This can raise ethical problems, since a specific group can be
harmed by automatic decision systems not aware of data biases.

Aiming at overcoming disparate results in dimension reduction, it is fundamental to consider an
adequate fairness measure. In this paper, we directly associate this measure with the reconstruction
errors for individual classes. Thus, in order to formulate this criterion, let us assume that the samples
can be divided as

$$X = \begin{bmatrix} X_A \\ X_B \end{bmatrix}$$

(3)

where $X_A \in \mathbb{R}^{n_A \times d}$ and $X_B \in \mathbb{R}^{n_B \times d}$ represent different sensitive groups (e.g., male and female),
respectively. The proposed fairness measure is defined as follows:

$$F(\hat{U}) = \left( \frac{1}{n_A} \| X_A - X_A \hat{U} \hat{U}^T \|_F^2 - \frac{1}{n_B} \| X_B - X_B \hat{U} \hat{U}^T \|_F^2 \right)^2.$$  

(4)

In short, the idea in (4) is that the fairness measure evaluates the disparity between the reconstruc-
tion errors for classes $X_A$ and $X_B$. Ideally, in the fairest projection, matrix $\hat{U}$ is the one leading to
$F(\hat{U}) = 0$.

In this paper, we consider that both total reconstruction error and fairness measure are equally
important in the dimension reduction problem. Therefore, we would like to optimize both cost func-
tions simultaneously. For this purpose, we consider the application of multi-objective optimization
to deal with fairness in PCA. We address this technique in the next section.

4. Multi-objective optimization

In mono-objective optimization, the notion of optimality is quite simple. One only needs to
compare the cost function values obtained by the feasible solutions and then select the one that
minimizes (or maximizes) it. However, in a multi-objective optimization, this notion must be
extended to a vector-valued cost function. Therefore, in our case, the optimal solution should be
the one that leads to minimum values of $J = [\mathcal{R}(U^*), F(U^*)]$.

Very often multi-objective optimization problems involve conflicting cost functions. As a con-
sequence, one rarely finds a single solution that optimizes all objectives simultaneously. In the ad-
dressed dimension reduction problem, it is expected the presence of conflicts between the adopted
cost functions, i.e., a solution that minimizes the reconstruction error may not be the fairest one.
On the other hand, a solution that optimizes the fairness criterion may lead to a higher reconstruc-
tion error. Although this compromise exists, we may have a set of solutions that achieve suitable
performances on both objectives. Therefore, in order to determine this set of solutions, called non-dominated set, the characterization of an optimal solution is based on the following concept of dominance (Deb, 2001; Miettinen, 1999):

**Definition 1. (Dominance):** Consider two feasible solutions \( \tilde{U} \) and \( \tilde{U}' \). We say that \( \tilde{U} \) dominates \( \tilde{U}' \) if \( \tilde{U} \) is as good as \( \tilde{U}' \) in all objectives and \( \tilde{U} \) is strictly better than \( \tilde{U}' \) in at least one objective.

Based on this concept, we say that a solution is non-dominated if none of the objectives (reconstruction error and fairness measure, in our case) can be improved without degrading some of the others (i.e., if a solution leads to a better reconstruction error, it must lead, at the same time, to a worse fairness measure and vice-versa). If we assume that both reconstruction error and fairness measure should be minimized, the solutions of the multi-objective optimization problem are the following:

**Definition 2. (Non-dominated solution):** A solution \( \tilde{U} \) is a non-dominated solution if there is no other solution \( \tilde{U}' \) such that (i) both \( R(\tilde{U}') \leq R(\tilde{U}) \) and \( F(\tilde{U}') \leq F(\tilde{U}) \) and (ii) either \( R(\tilde{U}') < R(\tilde{U}) \) or \( F(\tilde{U}') < F(\tilde{U}) \).

It is worth mentioning that the set of all non-dominated solutions composes the Pareto front (Deb, 2001; Miettinen, 1999).

5. Proposed approach

As mentioned in the previous sections, this paper deals with a Fair PCA framework by means of a multi-objective optimization problem. Moreover, for a dimension reduction problem from \( d \) to \( r \)-dimensional space, we use the solution of the classical PCA and select \( r \) principal components that optimize both Equations (1) and (4) simultaneously. Assume that \( q^* = [q^*_1, \ldots, q^*_r] \), where \( q^*_i \in \{1, 2, \ldots, d\}, i = 1, \ldots, r \), represents the selected principal components. The multi-objective optimization problem is expressed by

\[
\min_{[q^*_1, \ldots, q^*_r]} [R(U^*), F(U^*)],
\]

where \( U^* = [u_{q^*_1}, u_{q^*_2}, \ldots, u_{q^*_r}] \) is the adjusted projection matrix whose columns are composed by the principal components extracted from \( U \). Note that \( u_{q^*_1}, u_{q^*_2}, \ldots, u_{q^*_r} \) are not, necessarily, the first \( r \) columns of \( U \). For example, if \( q^* = [1, 3, 6] \) (i.e, the columns 1, 3 and 6 of \( U \) are a solution of (5)), then \( [u_{q^*_1}, u_{q^*_2}, u_{q^*_3}] = [u_1, u_3, u_6] \). Moreover, one does not need to include an orthogonality
constraint on $U^*$, since the selected columns already ensure this property. Therefore, one only needs to select a combination of the principal components provided by the classical PCA.

In the next section, we will discuss the multi-objective algorithm used in this paper to obtain the non-dominated solutions.

5.1. SPEA2 algorithm

There are several existing methods that deal with multi-objective optimization problems (Miettinen, 1999). In this paper, we tackle the MOFPCA formulation expressed in (5) by means of evolutionary computation (Deb, 2001; Zhou et al., 2011), which has been used in several data analysis problems (Mukhopadhyay et al., 2014a,b). With respect to the adopted algorithm, we considered an improved version of the Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler & Thiele, 1998), called SPEA2 (Zitzler et al., 2001). We adopted the SPEA2 since we deal with a multi-objective combinatorial problem and, as mentioned in (Ehrgott & Gandibleux, 2003), this algorithm can be used to deal with such situations. Although the implementation of SPEA2 is quite simple, one generally needs to set some specific concepts, as described in the sequel. We also highlight the particularities of each one (for more details, see (Zitzler et al., 2001)).

- **Individual**: An individual $q$ leads to a possible solution for the multi-objective problem. In our proposal, since we search for a projection matrix $U^*$ whose columns are composed by $r$ principal components extracted from the classical PCA (from a total of $d$), an individual comprises a combination of $r$ coefficient indices. For example, for a projection from $d = 10$ to $r = 4$-dimensional space, an individual could be $q = [1, 4, 8, 9]$, which leads to a projection matrix $\tilde{U} = [\tilde{u}_q, \tilde{u}_{q_1}, \tilde{u}_{q_2}, \tilde{u}_{q_3}, \tilde{u}_{q_4}] = [u_1, u_4, u_8, u_9]$.

- **Population**: The population $P$ is composed by a set of individuals. The size of population, represented by $\bar{P}$ is predefined before the algorithm starts. Moreover, in order to define the initial population, we randomly generated the individuals. In this procedure, aiming at covering a large region of the feasible space, we also ensured that they are different.

- **External set**: Another element used in SPEA2 is the external set, represented by $E$. In the beginning of the algorithm, the external set is empty. However, after each generation (or iteration) $g$, it is updated with the non-dominated solutions found so far. Similarly as in the population, we also redefine the external set size $\bar{E}$.
• **Fitness**: Based on both $R(\mathbf{U})$ and $F(\mathbf{U})$, the fitness value indicates how good (or bad) is the performance of an individual $\mathbf{q}$ in terms of dominance. Therefore, based on this measure, we can rank the individuals and select the best ones after each generation. For more details about how one calculates the fitness measure in SPEA2, see (Zitzler et al., 2001).

• **Crossover and mutation**: Both crossover and mutation are evolutionary operators used to generate new individuals. The crossover consists in generating a new individual $\mathbf{q}$, which will be used in generation $g+1$, by randomly taking parts of two different individuals of generation $g$. In mutation, we create a new individual used in generation $g+1$ by randomly modifying a part (some indices, in our case) of an individual of generation $g$. An import aspect in the addressed problem associated with both operators is that we must ensure the feasibility of the new created individuals. In our proposal, when applying both operators, we must verify that there is no repetition in the indices that compose each individual.

With the aforementioned concepts been clarified, the steps of SPEA2 are presented in Algorithm 1. As inputs, we redefine the initial population $P$, the (initially) empty external set $E$, the population size $\bar{P}$, the external set size $\bar{E}$, the maximum number of iterations $G$ and the crossover rate $\alpha$. In the first step, we calculate the fitness values for all individuals in both population and external set. These values are used in Step 2 in order to select the best individuals found so far and update the external set. At this point, if we achieve the maximum number of iterations, we set $E$ as the non-dominated solutions and stop the algorithm. Otherwise, we move to the mating selection step (Step 3). In this step, we perform a binary tournament selection on the external set in order to define the set of individuals that will be submitted to the evolutionary operators in the variation step (Step 5). It is worth mentioning that, in Step 5, $\alpha\%$ of the new population $P$ is obtained through crossover and the other $(1-\alpha)\%$ through mutation. The variation step ends by setting $g = g + 1$ and the algorithm return to Step 1.

When SPEA2 finishes, we achieve a set of non-dominated solutions. In terms of Pareto optimality, they are equally optimal. Therefore, one cannot say that a specific non-dominated solution is better than other one by taking into account the considered reconstruction error and fairness measure. In other words, we are not able to select a single optimal solution that solves our MOFPCA problem. However, an interesting aspect of the multi-objective approach is that we can visualize the compromise between the non-dominated solutions. This enhances the transparency the the ML model, since the users can see how much they are willing to lose in the reconstruction error in order
Algorithm 1 SPEA2

Input: $P$, $E$, $\bar{P}$, $\bar{E}$, $G$ and $\alpha$.

Output: External set $E$.

Set $g = 1$.

while $g \leq G$ do

Step 1: Fitness assignment. For each individual in $P$ and $E$, calculate the fitness measure.

Step 2: Selection. Based on the fitness values, select the best $\bar{E}$ individuals and update $E$.

if $g = G$ then

Step 3: Termination. Define the external set $E$ as the non-dominated set and stop the algorithm.

end if

Step 4: Mating selection. Select $\bar{P}$ individuals for the next step through a binary tournament selection in $E$.

Step 5: Variation. Apply crossover and mutation on the individuals selected on the previous step and generate the new population $P$. Set $g = g + 1$.

end while

to improve fairness.

5.2. On the selection of a non-dominated solution

Although all solutions in the non-dominated set are equally optimal, in this paper, we also propose a technique to select a single one. This will be interesting to compare our proposal with the classical PCA. In that respect, we select the $q^*$ (and, therefore, the $U^*$) that minimizes the sum of the achieved reconstruction error and fairness weighted by factors associated with the scales of each cost function. Mathematically, $U^*$ is obtained by

$$\min_{q^*} \lambda R(U^*) + (1 - \lambda) F(U^*)$$

s.t. $q^* \in \Omega$, \hspace{1cm} (6)

where $\lambda$ is the weighting factor and $\Omega$ is the set of projection matrices indices associated with the non-dominated solutions in $E$. The idea here is to assume that both cost-functions are equally important and, therefore, $\lambda$ will only compensate the difference between the scales of the reconstruction error and the fairness measure. For this purpose, we define $\lambda$ as

$$\lambda = \frac{M_{FM}}{M_{RE} + M_{FM}},$$

10
where \( M_{RE} \) is the reconstruction error achieved by projecting the dataset in the first principal component and \( M_{FM} \) is the minimum fairness level obtained by projecting the data into a 1-dimensional space. One may remark that the weight associated with \( F(U^*) \) is \( 1 - \lambda = 1 - M_{FM} / (M_{RE} + M_{FM}) = M_{RE} / (M_{RE} + M_{FM}) \).

It is important to recall that the selection of a single solution within the non-dominated set is not mandatory to analyze the trade-off between the objectives. We can use this solution in order to compare the MOFPCA approach with the classical PCA in terms of a fair dimension reduction. Figure 2 illustrates the MOFPCA scheme.

6. Numerical experiments

In order to verify the application of the proposed MOFPCA in real dimension reduction scenarios, we consider two datasets\(^1\): Default Credit [Yeh & Lien, 2009] and Labeled Faces in the Wild.

\(^1\)We considered these datasets, as well as the adopted sensitive attributes, for the purpose of comparison, since some related works [Samadi et al., 2018] [Kamani et al., 2019], mentioned in Section 2, also used them in their experiments.
The Default Credit dataset comprises $m = 23$ attributes and $n = 30,000$ samples. Among the attributes, we adopt the education level as the sensitive one. Therefore, we have $n_1 = 5,385$ and $n_2 = 24,615$ samples associated with a lower (high school and others) and a higher (graduate school and university) education levels, respectively. All the attributes were centered and normalized, in order to have zero mean and unitary variance.

Figure 3 presents the obtained reconstruction error and fairness level for different numbers of features. We compared the results provided by PCA, i.e., by taking the coefficients that minimizes the total reconstruction error of the projected data, MOFPCA (a single solution selected as described in Section 5.2) and FairPCA (Samadi et al., 2018). One may note in Figure 3a that our proposal led to reconstruction errors very close to the ones obtained by both FairPCA and PCA, the latter being the benchmark for this cost function. However, in terms of the fairness measure, Figure 3b indicates a relevant difference between the considered approaches: the MOFPCA led to a better fairness condition with fewer features than the PCA and FairPCA. Although FairPCA considers fairness in its formulation, it led to the worst adopted fairness measure for $r < 10$. It is worth recalling that the fairness measure considered in FairPCA (see (Samadi et al., 2018) for further details) is different from (4).

The difference between the reconstruction errors for each class can be visualized in Figure 4. The results in Figure 4b attested that this difference, when applying our proposal, tends to zero for all dimension reduction with $r \geq 5$. On the other hand, in both PCA and FairPCA (Figures 4a and 4c, respectively), this disparity is mitigated only for $r \geq 13$.

The aforementioned results can be explained by the presence of non-dominated solutions that have a good performance on both objectives. For instance, consider the non-dominated solutions presented in Figure 5. This is the case when we project the data into $r = 10$ dimensions. Note that, among the compromising solutions, we have the solution of a classical PCA (the one that

---

2In all experiments conducted in this paper, the SPEA2 parameters were experimentally defined by $\alpha = 50$, $P = \min \left(100, \text{round} \left(\frac{d}{r(d-r)}\right)\right)$ and $E = \text{round} \left(P/2\right)$, where $\text{round}()$ returns the closest integer. With respect to the number of iterations, we adopted $G = 30$ and $G = 50$ for the Default Credit and the LFW datasets, respectively. We consider that this parameters setting led to a good algorithm convergence.
minimizes the reconstruction error), the fairest projection (the one that minimizes the adopted fairness measure) and the selected one for this dimension reduction. Although the selected solution does not minimize both objectives simultaneously, it achieved small values for both of them. An small loss in this cost function is compensated by the gain in the fairness measure.

Table 1 compares the classical PCA and the proposed MOFPCA for some projected dimensions. We can note that a simple change in the order of the principal components significantly increased fairness (or decreased the fairness measure described in Equation (4)). For instance, with respect to the matrix associated with the selected solution presented in Figure 5, it is composed by the columns $u_1, \ldots, u_7, u_9, \ldots, u_{11}$ of $U$. Therefore, by changing the 8-th by the 11-th column of $U$, we could improve the fairness in this dimension reduction problem, and no other changes would be applied to the data processing pipeline.
6.2. Experiments with the LFW dataset

The LFW dataset consists of $m = 1764$ attributes (pixels) and $n = 13,232$ samples, divided into two groups: female ($n_1 = 2,962$) and male ($n_2 = 10,270$). As mentioned in [Samadi et al., 2018], the gender information was manually verified by [Afifi & Abdelhamed, 2017]. All pixels were normalized by $1/255$.

Figure 6 presents the obtained reconstruction error and fairness level for different numbers of features. Similarly as in the previous experiment, our proposal led to reconstruction errors close to the ones obtained by both FairPCA and PCA, specially for $r \geq 12$. However, although MOFPCA achieved better values of fairness measure in comparison with PCA, in this experiment, the FairPCA
led to the better results for $r > 4$ (for $r \leq 4$ MOFPCA performed better). Figure 7 illustrates the difference between the reconstruction errors for each class and each approach. One may note that the reconstruction errors provided by the FairPCA (Figure 7c) are closer in comparison with PCA and MOFPCA. A hypotheses for this result is that, since the LFW dataset comprises 1764 attributes (in contrast with the 23 in the Default Credit dataset) and the FairPCA search for projection vectors different from the classical PCA, this method has more flexibility to adjust them in order to enhance fairness.

If we consider the projection into $r = 10$ dimensions, the achieved non-dominated solutions...
are presented in Figure 8. Similarly as in the Default Credit dataset, we may also visualize here the trade-offs between the reconstruction error and the adopted fairness measure. In contrast with Figure 5, the non-dominated solutions in Figure 8 present a trade-off close to linear. Therefore, we could not select a single one with very good results in both cost functions.

With respect to the principal components used in the projected dimension, some instances are presented in Table 2. For example, the selected solution presented in Figure 8 comprises the columns $u_1, \ldots, u_7, u_9, \ldots, u_{11}$ of $U$. Therefore, by changing the 8-th by the 11-th column of $U$, we could slightly improve fairness in this problem.
7. Conclusions

Ethical concerns in AI have become an important subject in the last years. Automatic decision systems should take into account fairness in order to avoid disparate treatment of different sensitive groups. For instance, in dimension reduction problems, one should adopt a procedure that provides an equal (or, at least, as similar as possible) representation of different groups. In this context, this paper proposed a multi-objective framework for the Fair Principal Component Analysis. Our approach consists in a different ordering of the components given by the classical PCA, which gives rise to an approach with low computational costs and easy to deploy in already running systems. Furthermore, the set of non-dominated solutions provides a suitable portfolio of choices to the stakeholders and decision makers, presenting, clearly, the trade-off between the objectives. This characteristic is paramount in scenarios of social and economic impacts, since the system user can measure the gain in terms of fairness with a (possible) small loss in the reconstruction error.

We verified the applicability of our proposal in experiments based on two datasets frequently used in the literature. In the Default Credit dataset, we could attest that there is a non-dominated
solution that lead to very good values in both reconstruction error and fairness measure. The disparity between the representation of the two groups was considerably reduced with a small loss in the reconstruction error. Moreover, the proposed MOFPCA approach performed much better in comparison with both classical PCA and FairPCA. However, the experiments with the LFW dataset indicated that our proposal may lead to less expressive results when the number of attributes are very high. In such a scenario, we see that future works could be developed in order to generalize our proposal to the search of any projection matrix (not necessarily based on the classical PCA) while using the multi-objective framework. Therefore, one could either achieve better solutions, in terms of minimizing both objectives, and allow the trade-off analysis among the non-dominated set.

The scope of this paper lies in the fairness analysis in the pre-processing step. Note that we do not address, necessarily, a classification problem. Therefore, as another future perspective, we would like to verify the MOFPCA impact on the classification task. More specifically, we intend to investigate if looking at fairness in the pre-processing step lead to fairness in the classification task. Moreover, new researches in classification problems may consider a multi-objective formulation in which the cost functions are associated with model accuracy and fairness. Although our proposal tackled fairness in pre-processing step, it could be generalized to analyze the trade-offs in any step of a machine learning problem.

Acknowledgments

The authors would like to thank the grants #2020/10572-5, #2019/20899-4 and #2020/01089-9, São Paulo Research Foundation (FAPESP), and the grants #312228/2020-1 and #308811/2019-4, National Council for Scientific and Technological Development (CNPq), for the financial support.

References

Afifi, M., & Abdelhamed, A. (2017). AFIF4: Deep gender classification based on adaboost-based fusion of isolated facial features and foggy faces. *Journal of Visual Communication and Image Representation, 62*, 77–86.

Barocas, S., Hardt, M., & Narayanan, A. (2019). *Fairness in machine learning*. fairmlbook.org. URL: [http://www.fairmlbook.org](http://www.fairmlbook.org)
Barocas, S., & Selbst, A. D. (2016). Big data’s disparate impact. California Law Review, 104, 671–732.

Bishop, C. M. (2006). Pattern recognition and machine learning. New York: Springer.

Brandão, M., Jirotka, M., Webb, H., & Luff, P. (2020). Fair navigation planning: A resource for characterizing and designing fairness in mobile robots. Artificial Intelligence, 282, 103259.

Camastra, F., & Vinciarelli, A. (2015). Machine learning for audio, image and video analysis: Theory and applications. (2nd ed.). London: Springer.

Chouldechova, A. (2017). Fair prediction with disparate impact: A study of bias in recidivism prediction instruments. Big Data, 5, 153–163.

Confalonieri, R., Weyde, T., Besold, T. R., & Moscoso del Prado Martín, F. (2021). Using ontologies to enhance human understandability of global post-hoc explanations of black-box models. Artificial Intelligence, 296, 103471.

Deb, K. (2001). Multi-objective optimization using evolutionary algorithms. Chichester, UK: John Wiley & Sons.

Duda, R. O., Hart, P. E., & Stork, D. G. (2000). Pattern classification. (2nd ed.). Wiley.

Dwork, C., Hardt, M., Pitassi, T., Reingold, O., & Zemel, R. (2012). Fairness through awareness. In 3rd Innovations in Theoretical Computer Science Conference (ITCS 2012) (pp. 214–226). Cambridge, USA.

Ehrgott, M., & Gandibleux, X. (2003). Multiple objective combinatorial optimization - A tutorial. In Multi-Objective Programming and Goal Programming. Advances in Soft Computing (pp. 3–18). Springer, Berlin, Heidelberg volume 21.

Fan, J., Sun, Q., Zhou, W. X., & Zhu, Z. (2018). Principal component analysis for big data. arXiv preprint:1801.01602, (pp. 1–20). URL: https://arxiv.org/pdf/1801.01602.pdf

Gilpin, L. H., Bau, D., Yuan, B. Z., Bajwa, A., Specter, M., & Kagal, L. (2018). Explaining explanations: An overview of interpretability of machine learning. In 2018 IEEE 5th International Conference on Data Science and Advanced Analytics (DSAA 2018) (pp. 80–89). IEEE.

Golub, G. H., & Van Loan, C. F. (2013). Matrix computations. (4th ed.). Baltimore, Maryland: Johns Hopkins University Press.
Goodfellow, I., Bengio, Y., & Courville, A. (2016). *Deep learning*. MIT Press.

Hardt, M., Price, E., & Srebro, N. (2016). Equality of opportunity in supervised learning. In *Advances in Neural Information Processing Systems 29 (NIPS 2016)* (pp. 3315–3323). Barcelona, Spain.

Huang, G. B., Mattar, M., Berg, T., & Learned-Miller, E. (2008). Labeled faces in the wild: A database for studying face recognition in unconstrained environments. In *Workshop on Faces in 'Real-Life' Images: Detection, Alignment, and Recognition*. Marseille, France.

Jolliffe, I. T. (2002). *Principal component analysis*. (2nd ed.). New York: Springer-Verlag.

Kamani, M. M., Haddadpour, F., Forsati, R., & Mahdavi, M. (2019). Efficient fair principal component analysis. *ArXiv ID: 1911.04931v2*, . URL: [http://arxiv.org/abs/1911.04931v2](http://arxiv.org/abs/1911.04931v2)

Kearns, M., Neel, S., Roth, A., & Wu, Z. S. (2018). Preventing fairness gerrymandering: Auditing and learning for subgroup fairness. In *35th International Conference on Machine Learning (ICML 2018)* (pp. 4008–4016). Stockholm, Sweden volume 6.

Kenny, E. M., Ford, C., Quinn, M., & Keane, M. T. (2021). Explaining black-box classifiers using post-hoc explanations-by-example: The effect of explanations and error-rates in XAI user studies. *Artificial Intelligence*, 294, 103459.

Langer, M., Oster, D., Speith, T., Hermanns, H., Kästner, L., Schmidt, E., Sesing, A., & Baum, K. (2021). What do we want from Explainable Artificial Intelligence (XAI)? A stakeholder perspective on XAI and a conceptual model guiding interdisciplinary XAI research. *Artificial Intelligence*, 296, 103473.

Little, M. A. (2019). *Machine learning for signal processing - Data science, algorithms, and computational statistics*. New York: Oxford University Press.

Lohia, P. K., Ramamurthy, K. N., Bhide, M., Saha, D., Varshney, K. R., Puri, R., Road, O. R., B, E. M., & Villages, N. (2019). Bias mitigation post-processing for individual and group fairness. In IEEE (Ed.), *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP 2019)* (pp. 2847–2851).

Miettinen, K. M. (1999). *Nonlinear multiobjective optimization*. Norwell, MA, USA: International Series in Operations Research and Management Science 12, Kluwer Academic Publishers.
Mukhopadhyay, A., Maulik, U., Bandyopadhyay, S., & Coello, C. A. C. (2014a). A survey of multiobjective evolutionary algorithms for data mining: Part I. *IEEE Transactions on Evolutionary Computation, 18*, 4–19.

Mukhopadhyay, A., Maulik, U., Bandyopadhyay, S., & Coello, C. A. C. (2014b). A survey of multiobjective evolutionary algorithms for data mining: Part II. *IEEE Transactions on Evolutionary Computation, 18*, 20–35.

Olfat, M., & Aswani, A. (2019). Convex formulations for fair principal component analysis. In *Proceedings of the AAAI Conference on Artificial Intelligence* (pp. 663–670). volume 33.

Samadi, S., Tantipongpipat, U., Morgenstern, J., Singh, M., & Vempala, S. (2018). The price of fair PCA: One extra dimension. *Advances in Neural Information Processing Systems*, (pp. 10976–10987).

Saxena, N. A., Huang, K., DeFilippis, E., Radanovic, G., Parkes, D. C., & Liu, Y. (2020). How do fairness definitions fare? Testing public attitudes towards three algorithmic definitions of fairness in loan allocations. *Artificial Intelligence, 283*, 103238.

Tantipongpipat, U., Samadi, S., Singh, M., Morgenstern, J., & Vempala, S. (2019). Multi-criteria dimensionality reduction with applications to fairness. In *Advances in Neural Information Processing Systems 32 (NIPS 2019)* (pp. 15161–15171). Vancouver, Canada.

Wang, H., Ustun, B., & Calmon, F. P. (2018). Avoiding disparate impact with counterfactual distributions. In *NeurIPS Workshop on Ethical, Social and Governance Issues in AI*. Montréal, Canada.

Yeh, I.-C., & Lien, C.-H. (2009). The comparisons of data mining techniques for the predictive accuracy of probability of default of credit card clients. *Expert Systems with Applications, 36*, 2473–2480.

Zafar, B. M., Valera, I., Gomez-Rodriguez, M., & Gummadi, K. P. (2019). Fairness constraints: A flexible approach for fair classification. *Journal of Machine Learning Research, 20*, 1–42.

Zhou, A., Qu, B.-Y., Li, H., Zhao, S.-Z., Suganthan, P. N., & Zhangd, Q. (2011). Multiobjective evolutionary algorithms: A survey of the state of the art. *Swarm and Evolutionary Computation, 1*, 32–49.
Zitzler, E., Laumanns, M., & Thiele, L. (2001). *SPEA2: Improving the strength Pareto evolutionary algorithm*. Technical Report Computer Engineering and Communication Networks Lab (TIK), Swiss Federal Institute of Technology (ETH) Zurich.

Zitzler, E., & Thiele, L. (1998). *An evolutionary algorithm for multiobjective optimization: the strength Pareto approach*. Technical Report 43 Computer Engineering and Communication Networks Lab (TIK), Swiss Federal Institute of Technology (ETH) Zurich.