Elastic $pp$-scattering at $\sqrt{s}=7$ TeV with the genuine Orear regime and the dip

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The unitarity condition unambiguously requires the Orear region to appear in between the diffraction cone at low transferred momenta and hard parton scattering regime at high transferred momenta in hadron elastic scattering. It originates from rescattering of the diffraction cone processes. It is shown that such region has been observed in the differential cross section of the elastic $pp$-scattering at $\sqrt{s}=7$ TeV. The Orear region is described by exponential decrease with the scattering angle and imposed on it damped oscillations. They explain the steepening at the end of the diffraction cone as well as the dip and the subsequent maximum observed in TOTEM data. The failure of several models to describe the data in this region can be understood as improper account of the unitarity condition. It is shown that the real part of the amplitude can be as large as the imaginary part in this region. The overlap function is calculated and shown to be small outside the diffraction peak. Its negative sign there indicates the important role of phases in the amplitudes of inelastic processes.

I. INTRODUCTION

The TOTEM collaboration has published

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experimental results on the differential cross section of the elastic $pp$-scattering at the total cms energy $\sqrt{s}=7$ TeV. Among the most interesting features they observe the steepening of the diffraction cone near the squared transferred momentum 0.3 GeV$^2$, the dip at 0.53 GeV$^2$ and the maximum at 0.7 GeV$^2$. We explain them as resulting from the rigorous requirements of the unitarity condition. It prescribes the Orear regime characterized by exponential decrease with the scattering angle to start at transferred momenta just above the diffraction cone. The damped oscillations imposed on it lead to the dip in the differential cross section. No particular model has been used.

At the same time there exist several models mostly based on reggeon approach. Their predictions are extensively cited in

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Being rather successful in the diffraction regime were obtained even earlier

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but the results did not fit new experimental findings.

At the same time the simple theoretical explanation based on rigorous model-independent consequences of the unitarity condition was proposed

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and a careful fit to experimental data showed good quantitative agreement with experiment.

We follow these ideas to demonstrate that they are also applicable to the recent data of the TOTEM collaboration at the LHC at energies as high as 7 TeV.

II. THEORETICAL DESCRIPTION

The elastic scattering proceeds mostly at small angles. The diffraction peak has a Gaussian shape in the scattering angle in the center of mass system and

\[ \theta \leq \theta_d \ll 1 \]

The special session was devoted to these findings at the 1968 Rochester conference in Wien.

The theoretical indications on the possibility of such region were obtained even earlier

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\[ \frac{d\sigma}{dt} \left( \frac{d\sigma}{dt} \right)_{t=0} = e^{Bt} = e^{-B\theta^2}, \]

where the four-momentum transfer squared is

\[ t = -2p^2(1 - \cos \theta) \approx -p^2\theta^2 \quad (\theta \leq \theta_d \ll 1) \]

with $p$ and $\theta$ denoting the momentum and the scattering angle in the center of mass system and $B$ known as the diffraction slope.

At large energies the forward scattering amplitude has a small real part as known from the dispersion relations

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Therefore, in the first approximation, it is reasonable to assume that its real part is negligible within the diffraction peak $\theta \leq \theta_d$. Then the elastic scattering in this region can be described by the amplitude

\[ A(p, \theta) \approx 4ip^2\sigma_1 e^{-Bp^2\theta^2/2} \]
with a proper optical theorem normalization to the total cross section $\sigma_t$ in the forward direction. We stress that

$$\text{Im} A(p, \theta) = \text{Im} A_2(p, \theta) + F(p, \theta) = \frac{1}{32\pi^2} \int \int d\theta_1 d\theta_2 \sin \theta_1 \sin \theta_2 A(p, \theta_1) A^*(p, \theta_2) \frac{\sqrt{\cos \theta - \cos(\theta_1 + \theta_2)} \sqrt{(\cos \theta_2 - \cos(\theta_1 + \theta_2))}}{\cos(\theta_1 + \theta_2) - \cos \theta} + F(p, \theta).$$

(4)

The region of integration in (4) is given by the conditions

$$|\theta_1 - \theta_2| \leq \theta, \quad \theta \leq \theta_1 + \theta_2 \leq 2\pi - \theta.$$  

(5)

The integral term represents the two-particle intermediate states of the incoming particles. The function $F(p, \theta)$ represents the shadowing contribution of the inelastic processes to the elastic scattering amplitude. Following Van Hove [7] it is called the overlap function. It determines the shape of the diffraction peak and is completely non-perturbative. Only some phenomenological models pretend to describe it (see also [13,15] where its shape is obtained using the unitarity relation in combination with experimental data).

Now, let us consider the integral term $I_2$ outside the diffraction peak. Because of the sharp fall-off of the amplitude with angle, the principal contribution to the integral arises from a narrow region near the line $\theta_1 + \theta_2 \approx \theta$. Therefore one of the amplitudes should be inserted at small angles within the cone while another one is kept at angles outside it. At the beginning, let us neglect the real parts of the amplitude both in the diffraction region and at large angles. We insert Eq. (3) for one of the amplitudes in $I_2$ and integrate over one of the angles. Then the linear integral equation is obtained:

$$\text{Im} A(p, \theta) = \frac{\rho_1}{4\pi \sqrt{2\pi B}} \int_{-\infty}^{\infty} d\theta_1 e^{-B\rho^2(\theta - \theta_1)^2/2} \text{Im} A(p, \theta_1) + F(p, \theta).$$

(6)

It can be solved analytically (for more details see [11,14]) with the assumption that the role of the overlap function $F(p, \theta)$ is negligible outside the diffraction cone.

To account for the real part of the amplitude, one replaces $\sigma_1$ by $\sigma_1 f_\rho$ where $f_\rho = 1 + \rho_4 \rho_t$ with average values of ratios of real to imaginary parts of the amplitude in and outside the diffraction cone denoted as $\rho_4$ and $\rho_t$ correspondingly. It follows from Eq. (4) that $A_1 A_2^* \rightarrow \text{Im} A_1 \text{Im} A_2 (1 + \rho_4 \rho_t)$.

Using the Fourier transformation one gets the solution

$$\text{Im} A(p, \theta) = C_0 e^{-\sqrt{2B \ln \frac{2\pi}{\sigma_1 \rho_t}}} + \sum_{n=1}^{\infty} C_n e^{-|\text{Re} b_n| p_\theta} \cos(|\text{Im} b_n| p_\theta - \phi_n),$$

(7)

$$b_n \approx \sqrt{2\pi B n}(1 + isign n) \quad n = \pm 1, \pm 2, \ldots$$

(8)

Eq. (3) follows directly from experimental results and does not appeal to any particular model.

Let us have a look at the unitarity condition which is

This shape has been obtained from contributions due to the pole on the real axis and a set of the pairs of complex conjugated poles. Correspondingly, it contains the exponentially decreasing with $\theta$ (or $\sqrt{t}$) term (Orear regime) with imposed on it damped oscillations. Let us mention the papers [17] where non-damped oscillations were predicted in the reggeon exchange model but they are not observed in experiment.

The elastic scattering differential cross section outside the diffraction cone (in the Orear regime region) is

$$\frac{d\sigma}{p_1 dt} = \left( e^{-\sqrt{2B|t| \ln \frac{2\pi}{\sigma_1 \rho_t}}} + p_2 e^{-\sqrt{2\pi B|t| \cos(\sqrt{2\pi B|t| - \phi)}} \right)^2.$$  

(9)

The first (Orear) term is exponentially decreasing with $\theta$ (or $\sqrt{t}$) and the second term demonstrates the damped $(n = 1)$ oscillations which are in charge of the dip-maximum structure near the diffraction cone. The omitted terms with larger $n$ in Eq. (7) are damped stronger because they contain $\sqrt{n}$ in exponents. Let us note that the exponents of the damped terms are much larger numerically than that of the Orear term if the experimentally measured values of the diffraction cone slope $B$ and the total cross section $\sigma_t$ are inserted. Namely $B$ and $\sigma_t$ determine mostly the shape of the elastic differential cross section in the Orear region between the diffraction peak and the large angle parton scattering. The value of $4\pi B/\sigma_t$ is so close to 1 that the first term is very sensitive to $\rho_t$. Thus it becomes possible for the first time to estimate the ratio $\rho_t$ from fits of experimental data.

Beside the overall normalization constant $p_1$ this formula contains the constants $p_2$ and $\phi$ which determine the strength and the phase of the oscillation [12]. They can be found from fits of experimental data. The constant $p_1$ is determined by the transition point from the diffraction cone to the Orear regime. The constants $p_2$ and $\phi$ define the depth of the dip and its position.

What concerns the ratios $p_\theta$s, one can choose $\rho_t \approx 0.14$ as prescribed by the dispersion relations for its value at $t = 0$ [13,14] and use $p_1$ as another fitted parameter which influences the exponents in Eq. (9).

Let us note that all parameters can depend on energy as well as the values of the diffraction cone slope $B$ and the total cross section $\sigma_t$. Surely, this is unimportant if the fit is done at a fixed energy as in the present paper.

The unitarity condition is not a complete theory. It
imposes some restrictions on its consequences however. Its solution predicts the dependence on $p^\theta \approx \sqrt{|t|}$ but not the dependence on the collision energy! Nevertheless, main exponents in Eq. (9) depend on energy. We are able to predict them at different energies if the dependence of the diffraction slope $B$ and the total cross section $\sigma_t$ is known from experiment. In this way different reactions (including $\bar{p}p$, in particular) may be analysed.

Apart from comparison of theoretical predictions with experimental data one can get some knowledge about the overlap function $F(p, \theta)$ (see [15]). It is important, in particular, to confirm the assumption about its smallness outside the diffraction peak. Then the equation (4) is used as an expression for $F(p, \theta)$:

\[
F(p, \theta) = 16\rho^2 \left( \pi \frac{d\sigma}{dt} / (1 + \rho^2) \right)^{1/2} - \frac{8\rho^2(1 + \rho_4\rho)}{\pi \sqrt{(1 + \rho_5^2)(1 + \rho^2)}} \int_{-1}^{1} dz_2 \int_{z_1}^{z_2+} \frac{d\sigma}{d\tau_1} \cdot \frac{d\sigma}{d\tau_2} K^{-1/2}(z, z_1, z_2),
\]

where $z_i = \cos \theta_i$; $K(z, z_1, z_2) = 1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2$, and the integration limits are $z_1^\pm = z_2 \pm [(1 - z^2)(1 - z_2^2)]^{1/2}$.

At $\sqrt{s} = 7$ TeV the angles are extremely small so that the kernel becomes very singular. $K$ is close to 0 but integrable. The divergence is of the type $\int dz/\sqrt{z}$ and can be computed. Computing $F$ in the diffraction cone one uses $\rho = \rho_4$. Outside it $\rho = \rho_l$.

Let us mention that the inhomogeneous equation (11) has been solved [11] by iterations with the overlap function approximated by $F/s\sigma_{inel} = \exp[-B_{in}p^2\theta^2/2]$. Its more precise approximation is required to get accurate results but it is important that the conclusion about the phase $\phi$ remains valid.

Below we show and discuss the obtained results.

**III. A FIT OF THE EXPERIMENTAL DATA**

Having at our disposal Eq. (11) we try to fit experimental distribution of elastic $pp$-scattering at $\sqrt{s} = 7$ TeV. The experimental values of $B = 20.1$ GeV$^{-2}$ and $\sigma_t = 98.3$ mb were used in [9]. We ask that Eq. (9) must be applicable from the end of the diffraction cone at $|t| = 0.3$ GeV$^2$ to the beginning of hard parton processes at $|t| > 1$ GeV$^2$. The result is shown in Fig. 1.

It is seen that the fit is quite successful in the expected applicability region. First of all, we notice the steeper decrease in the region $0.3 < |t| < 0.36$ GeV$^2$ compared to the slope of the diffraction cone at $|t| < 0.3$ GeV$^2$ as observed in experiment. It is explained here as the negative contribution of the oscillating term in Eq. (9). That determines the phase $\phi$. The dip develops at $|t| = 0.53$ GeV$^2$ where the cosine in the second term is close to -1. Then this term increases, becomes positive and leads to the maximum at $|t| \approx 0.7$ GeV$^2$. The positions of the dip and of the subsequent maximum are uniquely determined by the period of oscillations $\Delta t = 2\pi/B$ which is predicted by the unitarity condition and depends only on the well measured slope of the diffraction peak $B$. The damping exponent in front of the cos-term becomes so strong at larger $|t|$ that the simple Orear regime with the first term in Eq. (9) prevails. Let us note that the exponent in this term is rather small because the ratio $4\pi B/\sigma_t$ is very close to 1 [15]. Therefore it is extremely sensitive to the parameter $\rho_l$. That helps determine this parameter.

Hardly any oscillations will be observed at large $|t|$. The exponent in the oscillation term is very large and strongly damps it. One could pretend to observe the next weak oscillation at $|t| \approx 0.9 - 1.0$ GeV$^2$. However it would require very high precision. It is interesting to note that the damping increases with energy due to increase of the slope $B$. At the same time the shrinkage of the cone leads to the shift of the Orear regime (and the dip) to smaller angles at higher energies so that the oscillations are still noticeable there.

Let us list and discuss the parameters in Eq. (9) which we found by the fitting procedure: $p_1 = 18.71$; $p_2 = 115.6$; $\phi = -0.845$; $\rho_l \approx -2$. The large value of $\rho_l$ demonstrates that the dip is well pronounced in the data. Up to now the only possible model-independent estimate of the ratio of real to imaginary parts of the elastic scattering amplitude was available from the dispersion relations at $t = 0$. It is for the first time that it is done at large $|t|$ in a model-independent way and shows that this ratio is of the order of 1 there. Surely,
are many models where this ratio is calculated in a wide range of t-values. There is no common consensus about their validity however. The parameter $\phi$ is so close to its theoretical estimate that it was not even necessary to use it as a free one.

Now we discuss the role of the parameters.

1. The parameter $p_1$ is in charge of the overall normalization and, consequently, of the smooth transition from the diffraction cone to the Orear region. 

2. The parameter $p_2$ defines the amplitude of the oscillations and, consequently, the depth of the dip. In combination with $\phi$ it leads to the steepened slope at $0.3 < |t| < 0.36 \text{ GeV}^2$.

3. The phase $\phi$ determines the position of the dip and the beginning of the steepened slope. Actually, it was shown in [11] that it can be obtained from the iterative solution of the non-linear equation [4]. It is almost independent of the form of $F(p, \theta)$ so that $|\phi| \approx \pi/4$. Nevertheless this problem asks for further studies.

4. The parameter $\rho_1$ in $f_p$ is in charge of the exponential slope at $|t|$ above the maximum (together with $B$ and $\sigma_1$ in the first term of (9)). It is negative and rather large (in the absolute value).

5. The relative position of the dip and the maximum (the period of oscillations) is determined only by the diffraction cone slope $B$ (the second term in (9)).

IV. THE OVERLAP FUNCTION

As follows from experiment, the inelastic cross section is much larger than the cross section of elastic scattering at high energies. Therefore the overlap function is much larger than the integral term in the unitarity relation at small $t$. To see what is the contribution of inelastic processes to the unitarity relation at any values of $t$, it is instructive to calculate the overlap function according to Eq. (10). In [15] that has been done at the assumption of ratios of real to imaginary parts $\rho$ equal to zero both at small and large $t$. Now with the above estimate of $\rho_1$ we can take it into account. Nevertheless, the calculations were done with and without account of $\rho$ to compare with previous results and estimate the role of $\rho$. The results are shown in Fig. 2.

There are several distinctive features observed. First of all, as expected, the overlap function drops down very fast with increase of the transferred momentum $|t|$ and determines the shape of the diffraction cone. Second, it crosses the abscissa axis at $|t| \approx 0.3$ and becomes negative. Namely there the Orear regime starts working. If compared to low energies [15], the overlap function becomes narrower at higher energies. Third, it is small and changes very slowly outside the diffraction cone similarly to the low energy behavior. Intuitively, this smallness may be understood as a consequence of strong destructive interference between amplitudes of inelastic processes with very different kinematics. In one of these amplitudes the final state must be turned to the large angle $\theta$ relative to the direction of initial particles. Thus the overlap of these two processes is small. Fourth, the account of $\rho$ does not change qualitatively this conclusion in general even though somewhat changes the numerical estimates diminishing $|F|$ further. This follows from better fit of experimental data with $\rho_1$ different from zero. Fifth, the negative sign of $F$ imposes a severe problem to theorists because it shows the important role of the phases of matrix elements of inelastic processes and their strong interference when trying to reconstruct elastic scattering from two inelastic processes turned by $t$ one to another.

V. CONCLUSIONS

Thus we conclude:

- At intermediate angles between the diffraction cone and hard parton scattering region the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations. Earlier, this solution was helpful in explaining this regime at lab. energies $8 - 20 \text{ GeV}$.

- The experimental data on elastic $pp$ differential cross section at $\sqrt{s}=7 \text{ TeV}$ in this region are fitted by it with well described position of the dip at $|t| \approx 0.53 \text{ GeV}^2$, its depth and subsequent damped oscillations with the predicted period about $0.3 \text{ GeV}^2$. The large amplitude of the oscillations and their negative sign explain the steepened slope at $0.3 < |t| < 0.36 \text{ GeV}^2$. The positive sign of the oscillating term at $|t| \approx 0.7 \text{ GeV}^2$ leads to the maximum. Strong damping of the oscillations at higher
values of $|t|$ results in clear signature of the simple exponential (in $\sqrt{|t|}$) behavior observed first by Orear which extends up to $|t| \approx 1.5$ GeV$^2$.

- A good fit allows without using any definite model for the first time to estimate the ratio of real to imaginary parts of the elastic scattering amplitude in this region ($\rho_l \approx -2$) far from forward direction $t=0$.

- The overlap function at 7 TeV has been calculated using only the experimental differential cross section and the above estimate of the ratio of real to imaginary parts. As at low energies, it is small and negative in the Orear region. That confirms the assumption used when solving the unitarity equation and shows that the phases of inelastic amplitudes become crucial in any model of inelastic processes.

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[1] G. Antchev et al. (TOTEM Collaboration), e-print arXiv:1110.1385 (2011); e-print arXiv:1110.1395 (2011).
[2] V. Uzhinsky and A. Galoyan, e-print arXiv:1111.4984 (2011).
[3] O. Selyugin, e-print arXiv:1201.4458 (2012).
[4] G. Cocconi et al., *Phys. Rev.* **138**, B165 (1965).
[5] J. Orear et al., *Phys. Rev.* **152**, 1162 (1966).
[6] J. Allaby et al., *Phys. Lett.* **B 27**, 49 (1968); **28**, 67 (1968).
[7] L. Van Hove, *Nuovo Cim.* **28**, 798 (1963).
[8] D. Amati, M. Cini, and A. Stanghellini, *Nuovo Cim.* **30**, 193 (1963).
[9] W. Cottingham and R. Peierls, *Phys. Rev.* **137**, B147 (1965).
[10] I. Andreev and I. Dremin, *JETP Lett.* **6**, 262 (1967).
[11] I. Andreev and I. Dremin, *Sov. J. Nucl. Phys.* **8**, 473 (1968).
[12] I. Andreev, I. Dremin, and I. Gramenitskii, *Nucl. Phys.* **10**, 137 (1969).
[13] I. Dremin and M. Nazirov, *JETP Lett.* **37**, 198 (1983).
[14] M. Block and F. Halzen, e-print arXiv:1102.3163 (2011).
[15] I. Andreev, I. Dremin, and D. Steinberg, *Sov. J. Nucl. Phys.* **11**, 261 (1970).
[16] The results of the paper [15] (see the Figure in there) as well as our results (see Eq. (10) and Fig. 2 below with the subsequent discussion) where the elastic rescattering $I_2$ was subtracted from experimental data give strong support to this assumption.
[17] A. Anselm and I. Dyatlov, *Phys. Lett.* **B 24**, 479 (1967); *Sov. J. Nucl. Phys.* **6**, 430 (1968).
[18] The phase was determined in [10, 11] from the iterative solution of the unitarity equation to be equal $\phi \approx \pm \pi / 4$ (actually with somewhat larger absolute value) but we use it here as a free parameter. The first (weaker damped) oscillating term in the exact solution of the equation (6) has only been taken into account in Eq. (9). Let us note the same values of the exponential damping and the period of the oscillations.
[19] This happens at all energies!