Constraints on dark energy models from the Legacy and Gold SnIa datasets

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Abstract. A comparative analysis of three recent and reliable SnIa datasets available in the literature was made: the Full Gold (FG) dataset (157 data points $0 < z < 1.75$), a Truncated Gold (TG) dataset (140 data points $0 < z < 1$) and the most recent Supernova Legacy Survey (SNLS) dataset (115 data points $0 < z < 1$). It was found that the best fit dynamical $w(z)$ obtained from the SNLS dataset does not cross the PDL ($w = -1$) and remains above and close to the $w = -1$ line for the whole redshift range $0 < z < 1$. In contrast, the best fit dynamical $w(z)$ obtained from the Gold datasets (FG and TG) clearly crosses the PDL and departs significantly from the $w = -1$ line, while the LCDM parameter values are about $2 \sigma$ away from the best fit $w(z)$, however not excluding it. Also for the Gold dataset a scalar-tensor theory was tested, for which the best fit form of $w(z)$ was also found to cross the phantom divide line.

1. Introduction
Current cosmological observations show strong evidence that we live in a spatially flat universe [1] with low matter density [2] that is currently undergoing accelerated cosmic expansion [3, 4, 5]. The most direct indication for the current accelerating expansion comes from the accumulating type Ia supernovae (SnIa) data [4, 5] which provide a detailed form of the recent expansion history of the universe. This accelerating expansion has been attributed to a dark energy component with negative pressure which can induce repulsive gravity and thus cause accelerated expansion.

The simplest and most obvious candidate for this dark energy is the cosmological constant $\Lambda$ with equation of state $w = p/\rho = -1$. This model however raises theoretical problems related to the fine tuned value required for the cosmological constant [6]. These difficulties have lead to a large variety of proposed models where the dark energy component evolves with time usually due to an evolving scalar field (quintessence) which may be minimally [7] or non-minimally [8] coupled to gravity.

The structure of this paper is the following: In the section 2 we briefly discuss some theoretical issues about the dark energy evolution and the analysis of supernovae data. In
section 3 we present a comparative analysis of the two SNeIa datasets and we explore some aspects of an extension of general relativity, and specifically a scalar-tensor theory. Finally, some concluding remarks are included in the discussion.

2. Theory

The main prediction of the dynamical models, mentioned in the previous section, is the evolution of the dark energy density parameter $\Omega_X(z)$. Combining this prediction with the prior assumption for the matter density parameter $\Omega_{0m}$, the predicted expansion history $H(z)$ is obtained as

$$H(z)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + \Omega_X(z)]$$

The dark energy density parameter is easily obtained from the energy momentum conservation and is

$$\Omega_X(z) = \Omega_{0X} e^{3\int_0^z \frac{1+w(z')}{1+z'} dz'}$$

If the dark energy can be described as an ideal fluid with conserved energy momentum tensor $T^{\mu\nu} = \text{diag}(\rho, p, p, p)$ then the above parameter $w(z)$ is identical with the equation of state parameter of dark energy

$$w(z) = \frac{p(z)}{\rho(z)}$$

Independently of its physical origin, the parameter $w(z)$ is an observable derived from $H(z)$ (with prior knowledge of $\Omega_{0m}$) and is usually used to compare theoretical model predictions with observations.

Most evolution behaviors of $w(z)$ can be reproduced by assuming appropriate scalar field quintessence potentials. If however $w(z)$ were observationally found to cross the phantom divide line (PDL) $w = -1$ then all minimally coupled single scalar field models would be ruled out as dark energy candidates[9, 10] (this includes phantom[11] and $k$-essence models[12]). This would leave only models based on extended gravity theories[13, 14] and combinations of multiple fields [15, 16](quintessence + phantom) as dark energy candidates.

An alternative approach towards understanding the nature of dark energy is to attribute the acceleration to extensions of general relativity[8] on cosmological scales. Such extensions can be expressed for example through scalar-tensor theories[17]. In these theories the Einstein Lagrangian of general relativity is replaced by a generalized Lagrangian of the form

$$\mathcal{L} = \frac{F(\Phi)}{2} - R - \frac{Z(\Phi)}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) + \mathcal{L}_m[\psi_m; g_{\mu\nu}]$$

where we have set $8\pi G = 1$ ($F_0 = 1$) and $\mathcal{L}_m$ represents the matter fields and does not depend on $\Phi$ so that the weak equivalence principle is satisfied. A common choice in the Jordan frame is to set $Z \to 1$ by rescaling the field $\Phi$ [17].

The two most reliable and robust SNeIa datasets existing at present are the Gold dataset [4] and the Supernova Legacy Survey (SNLS) [5] dataset. The above observations provide the apparent magnitude $m(z)$ of the supernovae at peak brightness after implementing correction for galactic extinction, K-correction and light curve width-luminosity correction. The resulting apparent magnitude $m(z)$ is related to the luminosity distance $D_L(z)$ through

$$m_{\text{th}}(z) = M(M, H_0) + 5\log_{10}(D_L(z))$$
where in a flat cosmological model
\[ D_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z'; a_1, \ldots, a_n)} \] (6)
is the Hubble free luminosity distance \((H_0 d_L/c, a_1, \ldots, a_n)\) are theoretical model parameters and \(\bar{M}\) is the magnitude zero point offset and depends on the absolute magnitude \(M\) and on the present Hubble parameter \(H_0\) as
\[ \bar{M} = M + 5 \log_{10} \left( \frac{c H_0^{-1}}{M_{pc}} \right) + 25 = M - 5 \log_{10} h + 42.38 \] (7)
The parameter \(M\) is the absolute magnitude which is assumed to be constant after the above mentioned corrections have been implemented in \(m(z)\).

The evolution of Newton’s constant predicted in the context of extended gravity theories requires special care when comparing the predictions of these theories with observations [18]. This evolution induces special effects to the physics of SnIa [19, 20]. The SnIa peak luminosity varies like \(L \sim G^{-3/2}\) and the corresponding SnIa absolute magnitude evolves like
\[ M - M_0 = \frac{15}{4} \log \frac{G}{G_0} \] (8)
where the subscript 0 denotes the local values of \(M\) and \(G\). Thus, the magnitude-redshift relation of SnIa in the context of extended gravity theories is connected with the luminosity distance \(D_L(z)\) as
\[ m_{th}(z) = \bar{M} + 5 \log D_L(z) + \frac{15}{4} \log \frac{G(z)}{G_0} \] (9)
In the limit of constant \(G\) this reduces to the familiar result. On the other hand, in scalar tensor theories[17] we have
\[ \frac{G(z)}{G_0} = \frac{1}{F} \left[ 2 F + 4 \left( \frac{dF}{d\Phi} \right)^2 \right] \approx 1 \] (10)
and solar system experiments [21] indicate that \( \frac{dF(\Phi)}{d\Phi} \sim \frac{dF(z)}{dz} \approx 0 \). Assuming flatness, the expansion history \(H(z)\) is obtained from [22]:
\[ D_L(z) = (1 + z) \int_0^z \frac{dz'}{\sqrt{\frac{G_0}{G(z')} H_0 / H(z')}} \] (11)
Therefore, by fitting \(m_{th}\) of (9) to the observed Gold SnIa [4] dataset expressed as \(m_{obs}(z_i)\) and using (10) and (11) we may obtain the best fit forms of both \(H(z)\) and \(G(z)\) assuming appropriate parameterizations.

The theoretical model parameters are determined by minimizing the quantity
\[ \chi^2(a_1, \ldots, a_n) = \sum_{i=1}^N \frac{(m_{obs}(z_i) - m_{th}(z_i))^2}{\sigma_{\mu i}^2 + \sigma_{\text{int}}^2 + \sigma_{v i}^2} \] (12)
where the theoretical distance modulus is defined as \(m_{th}(z_i) \equiv m_{th}(z_i) - M\) and \(\sigma_{\mu i}^2, \sigma_{\text{int}}^2\) and \(\sigma_{v i}^2\) are the errors due to flux uncertainties, intrinsic dispersion of SnIa absolute magnitude and peculiar velocity dispersion respectively. These errors are assumed to be gaussian and uncorrelated. The steps we followed for the minimization of (12) for the Gold and SNLS datasets are described in detail in Ref.[23]. The validity of our analysis has been verified by comparing the part of our results that overlaps with the results of the original Refs [4, 5] of the Legacy and Gold datasets.
3. The Analysis

3.1. The datasets

We will consider four representative $H(z)$ parameterizations and minimize the $\chi^2$ of eq. (12) with respect to model parameters. We compare the best fit parameterizations obtained with three datasets:

- The full SNLS dataset with 115 datapoints (excluding two outliers) and $z < 1$.
- The Full Gold dataset (FG) with 157 datapoints and $0 < z < 1.7$.
- A Truncated version of the Gold dataset (TG) with 140 datapoints and $z < 1$ which can be compared in a more direct way with SNLS.

The four fitted parameterizations include the general LCDM without a flat prior:

$$w(z) = -1$$

$$H(z)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + \Omega_\Lambda + (1 - \Omega_{0m} - \Omega_\Lambda)(1 + z)^2]$$

and three dynamical dark energy parameterizations with two free parameters which allow for crossing of the PDL (assuming flatness):

- Parameterization A:
  $$w(z) = w_0 + w_1 z$$
  $$H(z)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + (1 - \Omega_{0m})(1 + z)^3(1+w_0-w_1)e^{3w_1 z}]$$

- Parameterization B:
  $$w(z) = w_0 + w_1 \frac{z}{1 + z}$$
  $$H(z)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + (1 - \Omega_{0m})(1 + z)^3(1+w_0+w_1)e^{3w_1[1/(1+z)-1]}]$$

- Parameterization C:
  $$w(z) = \frac{a_1 + 3(\Omega_{0m} - 1) - 2a_1 z - a_2(-2 + 2z + z^2)}{3(1 - \Omega_{0m} + a_1 z + 2a_2 z + a_2 z^2)}$$
  $$H(z)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + a_1(1 + z) + a_2(1 + z)^2 + (1 - \Omega_{0m} - a_1 - a_2)]$$

The motivation behind parameterization C is to mimic a two-component DE model. Alternatively, it could be viewed as a power law expansion in the scale factor dependence of the DE energy density.

In Fig.1 we show the confidence region ellipses (see Ref.[23] for details) in the $\Omega_{0m} - \Omega_\Lambda$ plane based on parameterization (14). The three plots correspond to the three datasets discussed above (SNLS, TG and FG). The following comments can be made on these plots:

- The two versions of the Gold dataset favor a closed universe instead of a flat universe ($\Omega_{tot}^{TG} = 2.16 \pm 0.59$, $\Omega_{tot}^{FG} = 1.44 \pm 0.44$). This trend is not realized by the SNLS dataset which gives $\Omega_{tot}^{SNLS} = 1.07 \pm 0.52$ and is consistent with a flat LCDM.
- The point corresponding to SCDM ($\Omega_{0m}, \Omega_\Lambda) = (1, 0)$ is ruled out by all datasets at a confidence level more than $10\sigma$. 


Figure 1. The 68% and 95% confidence region ellipses in the $\Omega_{0m} - \Omega_{\Lambda}$ plane based on parameterization (14). The three plots correspond to the three datasets discussed in the text (SNLS, TG and FG).

- If we use a prior constraint of flatness $\Omega_{0m} + \Omega_{\Lambda} = 1$ thus restricting on the corresponding dotted line of Fig. 1 and using the parameterization

$$H(z)^2 = H_0^2[\Omega_{0m}(1 + z)^3 + (1 - \Omega_{0m})]$$

we find minimizing $\chi^2(\Omega_{m})$ of eq (12)

$$\Omega_{0m}^{SNLS} = 0.26 \pm 0.04$$

$$\Omega_{0m}^{TG} = 0.30 \pm 0.05$$

$$\Omega_{0m}^{FG} = 0.31 \pm 0.04$$

The values of $\Omega_{0m}^{SNLS}$ and $\Omega_{0m}^{FG}$ are practically identical with the corresponding in the original Refs [4, 5] where the data were first published. This, along other similar tests, confirms the validity of our analysis.

Even though the cosmological constant with equation of state parameter $w = -1$ is the simplest form of dark energy consistent with the data, the possibility of evolving dark energy models with non-constant $w(z)$ remains a viable alternative which may even provide better fits to the data than LCDM. To address this issue we considered the three parameterizations (A, B, C) of eqs (15)-(20) and assuming flatness we constrained their parameters using the three datasets. Parameterizations A and B reduce to $w(z) = w_0$ in the special case when $w$ does not evolve with time ($w_1 = 0$).

Extending the analysis to the full parameter space we construct $\chi^2$ confidence contours in Fig.2 assuming a flat prior [23]. The following comments can be made regarding Fig.2:

- The minimal LCDM model ($w_0 = -1, w_1 = 0$) appears to be close to the 95% confidence contour in the analyses based on the TG and FG datasets. For the analysis based on the SNLS however, the flat LCDM model is well within the 68% contour and in fact for $\Omega_{0m} = 0.24$ it is almost identical with the best fit parameterization in both the A and B parameterization cases! Thus LCDM appears to have significantly gained in likelihood compared to dynamical dark energy models in the context of the new SNLS dataset.

- The best fit $w(z)$ parameterizations in the context of the FG and the TG datasets, not only are 1 and 2$\sigma$'s far from the LCDM point ($w_0 = -1, w_1 = 0$) (see Fig.2), however not excluding it, but they also clearly cross the PDL line. This is demonstrated in Fig.3.
Figure 2. The 68% and 95% $\chi^2$ confidence contours of parameterizations A and B assuming a flat prior using the datasets SNSL, TG and FG. A prior of $\Omega_{0m} = 0.24$ has been used. The dashed lines intersect at the parameter values of flat LCDM.

where the best fit $w(z)$ are plotted for each dataset and each parameterization [23] with a prior of $\Omega_{0m} = 0.24$.

This crossing of the PDL is not realized for the best fit A, B and C parameterizations in the context of the SNLS dataset. Several authors[15, 13, 24] have been recently motivated by the high likelihood of the PDL crossing indicated by the Gold datasets[25, 26] to explore theoretical models that predict such crossing. It has been shown that this task is not trivial and can not be achieved by a single minimally coupled field[9].

In Ref. [23] it was shown that such a PDL crossing is not favored by the new SNLS dataset and therefore the motivation for the above papers is weakened. Indeed, phantom[11] dynamical dark energy models with $w_0 < -1$ are not favored by the SNLS dataset in contrast with the Gold dataset that favored such models and their crossing of the PDL (see Figs 2 and 3 and Ref. [26, 27]).

3.2. An extension of general relativity
At this point it is important to address the following question: ‘Within the context of the Gold dataset what other options do we have and how should we proceed?’

As already discussed in section 2 an interesting option is to use a scalar-tensor theory. We considered simple polynomial parameterizations [22] for the functions $H(z)$ and $G(z)$ of the form

$$H^2(z)/H_0^2 = \Omega_{0m}(1 + z)^3 + a_1(1 + z) + a_2(1 + z)^2 + (1 - \Omega_{0m} - a_1 - a_2)$$

$$G(z)/G_0 = 1 + a z^2$$
Figure 3. The best fit $w(z)$ are plotted for each dataset (SNLS, TG and FG) and each parameterization (A, B and C) with a prior of $\Omega_{0m} = 0.24$.

Figure 4. The best fit form of $w(z)$ in the scalar-tensor (continuous line) and minimally coupled (dashed line) cases.

where the linear term in $G(z)$ has been ignored due to experimental constraints on scalar tensor theories [21]. Using the parameterizations (25) and (26) and the minimization procedure of Ref.[22] the minimum was obtained at $\chi^2 = 173.045$ for $a_1 = -12.35 \pm 8.55$, $a_2 = 5.41 \pm 3.82$ and $a = 0.05 \pm 0.04$. We compared our results with the corresponding minimally coupled case obtained by fixing $a = 0$ before minimization (setting $G(z) = G_0$ at all times). The corresponding minimum was obtained at $\chi^2 = 174.168$ for $a_1 = -4.54 \pm 2.52$, $a_2 = 1.96 \pm 1.09$.

The best fit functions for $w(z)$ for both the scalar-tensor and minimally coupled cases are shown in Fig.4 along with the $1\sigma$ (shaded) region of the scalar-tensor best fit. It is clear that the best fit equation of state parameter $w(z)$ crosses the phantom divide in both the scalar-tensor and the minimally coupled case at about $z \approx 0.2$. This type of crossing which seems to be favored by the Gold SnIa dataset [27] (but not by the more recent first year SNLS dataset [23]) has been the subject of extensive studies in the literature as its reproduction is highly non-trivial in the context of most theoretical models[9].

4. Discussion

We have performed a comparative analysis of the three most recent and reliable SnIa datasets available in the literature: the Full Gold (FG) dataset, the Truncated Gold (TG) dataset and the most recent SNLS dataset. This analysis is an extension of our earlier analyses which had focused [27] on the FG and earlier [28] datasets. We have used representative dark energy parameterizations to examine the consistency among the three datasets in constraining the corresponding parameter values. We have found that even though the constraints obtained using the three datasets are consistent with each other at the 95% confidence level, the latest (SNLS) dataset shows distinct trends which are not shared by the other (earlier) datasets.

In the case of scalar-tensor theories we assumed simple redshift parameterizations for $H(z)$ and $G(z)$ and found their best fit forms and the corresponding error regions. The best fit form of $w(z)$ was found to cross the phantom divide $w = -1$ for both a constant and a redshift dependent $G$. However, in the later case the best fit $w(z)$ was found to vary more rapidly with
redshift.

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