STUDY OF A TLP MOTIONS AND FORCES USING
3D SOURCE TECHNIQUE

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Abstract:

The production and consumption of oil and other petroleum products have been increasing rapidly over the years, which led to the scarcity of easily retrieved oil due to urbanization. As a result, oil producers are motivated to go to deeper ocean to extract oil and other resources. Offshore platforms in deep water like TLPs are used for exploration of oil and gas from under seabed. But it is challenging to design precisely such type of giant structure in deep sea. It experiences huge forces, motion and other environmental loads which are non-linear, need sophisticated solution techniques and expensive to apply. In the present study, wave exciting forces and motions of free floating TLP are carried out in frequency domain analysis using three-dimensional source distribution techniques within the scope of linear wave theory where six degrees of freedom have been considered. The same geometrical data are used as an input to HydroStar, which is based on linear wave theory. Results obtained from the developed program are compared with the results obtained from HydroStar. The comparison shows a very good agreement. The results obtained from the developed program and HydroStar are also validated with the published results. Forces and motions prediction of TLP is emphasized which has been done precisely in the present work. In future, it will help us to design the TLPs as well as the tendon system in deep sea. Finally, a number of recommendations have been made for further research based on the present study.

Keywords: TLP, 3D source technique, frequency domain analysis, linear wave theory

1. Introduction

Tension Leg Platform (TLP) is an offshore structure particularly well suited for deep-water operation, which is tethered to the seabed in a manner that eliminates most vertical movement of the structure. TLP and other floating structures are subjected to wave, wind and current at sea. Every floating object has six degrees of freedom. Three of them are linear and three are angular. Surge, Sway, heave are the linear motions and Roll, Pitch, Yaw are the angular motions. This oscillatory motion affects the loading, unloading system and operations.

Sujatha and Soni (1997) analyzed the forces and motion responses of a moored semi-submersible based on diffraction theory and used the potential flow theory to formulate the boundary value problem. Chandrasekaran et al. (2007) used Stokes’ fifth order wave theory for dynamic analysis of two triangular TLP models at a water depth of 1200 m and 527.8 m and it is seen that impulse loading acting on corner column of TLP significantly affect its response while that acting on pontoons does not affect TLPs behavior. Xiaobo (2011) presented a short survey of recent research and developments in the hydrodynamic domain and their applications in offshore engineering limited to the first-order and second-order potential theory of wave radiation and diffraction theory and the developed current developed present method is then applied to the computation for LNG carriers, FPSOs, barges, Semi-submersibles and TLPs. Chandrasekaran (2013) used passive control device like Tuned Mass Damper (TMD) and Multiple Tuned Mass Dampers (MTMD) to control the large amplitude of motion of TLPs and found that MTMD systems showed better response control in comparison to the single TMD. Joseph et al. (2009) presented a new geometric configuration, which could be a better alternative to an existing configuration. It is shown that 3-column mini TLP are close to the 4-column mini TLP with relatively higher surge and tether tension.

The forces and motions prediction of TLP may be carried out using frequency domain or time domain or by model tests. To compute the motions and forces of a TLP usually Frequency domain analysis is used because it is simplified and linearized form of equations of motion. Here, the wave exciting forces & motions of a free
floating (with and without tendon) TLP are carried out using 3D source distribution method within the scope of linear wave theory. Hydrodynamic coefficients of TLP are predictable by velocity potentials on the panels. After getting velocity potential, using Bernoulli’s equation leads to calculation of pressure distribution and forces over the floating structure.

The flow velocity both on and off the body surface is calculated by Hess and Smith (1964) by utilizing a source density distribution on the surface of the body and then it is compared with experimental data. A time domain model to predict dynamic response of semi-submersibles is developed program and time domain simulations are carried out to find the total extreme motions and mooring forces by Yilmaz and Incecik (1996). Islam (2001) uses 3D source density distribution technique to get the potential over the floating structure. The rapid analysis of mooring lines subject to horizontal motions carried out to assess the performance of an equivalent force model by Pascoal et al. (2006). To illustrate the analysis of hydro- elasticity Newman (2005) used a simple Euler beam modes considering the first-order and second-order mean drift forces.

In this research work, computations have been carried out using the program by Islam (2001) and are also compared with the results obtained from Hydrostar, and also with published results from which a complete behavior of TLP can be obtained and that will contribute to better design of TLP.

2. Theoretical Background

2.1 Co-ordinate System

To describe the hydrodynamic coefficient, wave exciting forces, moments and motions of a floating body, right-handed Cartesian co-ordinate systems shown in Fig. 1 are used. The reference point is taken as the Center of Gravity (CG). The positive Z-axis is vertically upward. The positive Z-axis passes through the center of gravity of the TLP in still water condition. The origin ‘O’ in the plane of the undisturbed free surface, i.e., in the calm water surface and OXY represents the calm water plane. Most relevant values of Z will be negative. The still water level is the average water level or the level of the water if no waves were present. The X-axis is positive in the direction of wave propagation. The water depth, h, (a positive value) is measured between the sea-bed (z = -h) and the still water level (z =0).

![Fig. 1: Co-ordinate systems of TLP](image)

2.2. Mathematical equations

In this paper, calculations of forces and motions of TLP have been carried out using 3D source distribution method. Hydrodynamic coefficients of TLP are predictable by velocity potentials on the panels. Using Bernoulli’s equation, the pressure distribution is obtained using the velocity potential, which leads to the pressure distribution and forces over the floating structure.
Equation of velocity potential

\[ F = R e^{i \left( x, y, z \right)} e^{-i wt} \]  

\[ \psi = \frac{-iw}{a} \left( f_0 + f_7 \right) \cos h k_z (z + h) \left( e^{ik \left( x \cos + y \sin \right)} \right) \]  

where,

- \( f_0 \) = Incident wave potential
- \( f_7 \) = Diffraction wave potential on body
- \( f_j \) = Potential due to motion of the body in \( j^{th} \) mode
- \( a \) = Circular frequency of incident wave
- \( a \) = Incident wave amplitude
- \( \alpha \) = Wave heading angle from x-axis

However, there is no analytical solution for \( f_7 \) and \( f_j \), so the problem should be solved numerically. According to the 3-D source-sink method, the potentials \( f_7 \) and \( f_j \) can be expressed in terms of well-known Green functions.

Once the velocity potential is obtained, the hydrodynamic pressure at any point on the body can be obtained from Bernoulli’s equation and can be written as.

\[ \frac{\partial}{\partial t} + \frac{1}{2} (\nabla \cdot V)^2 + P + gz = 0 \]  

Now putting the value of \( F \) in Equation (4), the following expression is obtained

\[ P = -iw + \frac{1}{2} (\nabla \cdot V)^2 + gz \]  

By neglecting the higher order terms, the linearized Bernoulli’s equation is as follows.

\[ P = gz + i \]  

As first part of Equation (6) is associated with the hydrostatic and steady forces, so neglecting this part, the first order wave exciting forces or moments and oscillatory forces and moments caused by the dynamic fluid pressure acting on the body can be obtained from the following integrals.

\[ F_k e^{i \cdot t} = i \left\{ f_0 + g \right\} n_k ds \]  

\[ F_{kj} = 2 e^{i \cdot t} \sum_{j=1}^{6} n_k ds \]
where, $F_k$ denotes the $k$-th component of wave exciting forces or moments, $F_{kj}$ denotes the $k$-th component of force arising from the $j$-th component of motion of the body. Moreover, it is customary to decompose the hydrodynamic forces resulting from motion of the bodies into components in phase with the acceleration and velocity of the rigid body motions. These yield the added mass and damping coefficients respectively. These coefficients can be expressed as.

$$a_{kj} = \Re \left[ \int_s n_k \, ds \right]$$

$$b_{kj} = \Im \left[ \int_s n_k \, ds \right]$$

where,

$a_{kj}$ = added mass co-efficient matrix of $kj$

$b_{kj}$ = damping co-efficient matrix of $kj$

The suffixes, $k, j = 1, 2, 3, 4, 5, 6$ represent surge, sway, heave, roll, pitch and yaw modes, respectively.

2.3 Non-dimensionalization

The non-dimensionalization of wave exciting forces and moments are as follows:

$$F_1' = \frac{F_1}{2 \rho g a B L} ; \quad F_3' = \frac{F_3}{2 \rho g a B L} ; \quad M_5' = \frac{M_5}{4 \rho g a B L^2}$$

where,

$F_1', F_3' =$ Non-dimensional Surge and Heave wave exciting force

$F_1', F_3' =$ Surge and Heave wave exciting force (N)

$M_5' =$ Non-dimensional Pitch wave exciting force

$M_5 =$ Pitch wave exciting force (N)

$\rho =$ Density of water (kg/m$^3$)

$a =$ Wave amplitude (m)

$B =$ Breadth of the body (m)

$L =$ Length of the body (m)

2.4 Equation of motion in frequency domain

The equation can be considered by using the following matrix relationship.

$$\sum_{j=1}^{6} (M_{kj} + a_{kj}) \ddot{X}_j + b_{kj} \dot{X}_j + CX_j = F_k$$

$k = 1, 2, 3,..., 6, j = 1, 2, 3,..., 6$

where,

$M_{kj}$ = Inertia matrix in $k$-mode due to the motion in $j$-mode

$a_{kj}$ = Added mass co-efficient matrix of $kj$
\[ b_{kj} = \text{Damping co-efficient matrix of } k \]
\[ C = \text{Hydrostatic restoring force co-efficient matrix of } k \]
\[ F_k = \text{Wave exciting force and moments vector in } k \text{-mode} \]
\[ X_j = \text{Vector containing the three-dimensional translational and three-dimensional oscillations about the co-ordinates in } j \text{-mode} \]

2.5 Validation

For the motion analysis, WAMIT 2000 (JRME, 2000) is the most widely used and well-proven computer program in frequency domain analysis considering 3-D diffraction and radiation. For the combined work such as a motion analysis, installation simulation and fatigue analysis, MOSES 2000 (JRME, 2000) may be the most popular computer program in this area. Since MOSES has the same 3-D diffraction and radiation computer module, one may use it as the tool for the global motion analysis. In this study, the hydrodynamic analysis using MOSES, WAMIT, the program developed by Islam (2001) and HydroStar are carried out. A box of 200 m \( \times \) 40 m \( \times \) 28 m size, which can represent most of the floating platforms, has been selected and the results are also compared for a wide range of wave periods. From Fig. 2, it is seen that the results for surge exciting forces from the developed program is comparable to WAMIT, MOSES and HydroStar for surge wave exciting force. From Figs. 3 to 4, the sway, heave wave exciting force are in good agreement with the results obtained from other programs. From Fig. 5, it is seen that pitch moment for WAMIT, MOSES and present program are in good agreement at higher frequencies. But the results obtained from HydroStar over predicts at lower frequency. It is also seen from Fig. 6 that yaw moment has very good agreement with the results obtained from other programs. In general, HydroStar over predicts especially when the circular frequency is lower. Basically, the input files are different in the software. So, it is difficult to find out the same value as output. For example, some software generates panel numbers automatically, some of them are not. So it is sometimes difficult to find out the same results through all points. Overall the results show a good agreement, which inspires us for further research.

3. Results and Discussions

The computations have been carried out to achieve the objective, which is to compute and analyze the wave exciting forces and motions. The principal particulars of TLP have been given in Table 1. The program developed by Islam (2001) is used to carry out the computation of the TLP. The program is validated and HydroStar is also used to compare the results. In the current study, it is seen that at 45 degree heading angle surge and sway wave exciting forces in Figs. 7 ~ 8 are found maximum at 0.6 rad/sec and 1.3 rad/sec and of similar nature for symmetry of TLP while the heave wave exciting forces in Fig. 9 are also highest at same
circular frequencies which is quite natural. Similarly, when the surge and sway wave exciting forces are minimum approximately at 0.85 rad/sec, the heave wave exciting forces are minimum. So it can be said that while surge and sway forces increasing, the heave force also increases.

Fig. 4: Comparison of heave wave exciting force $F_3'$ (Non-dimensional) at 45 degree heading angle

Fig. 5: Comparison of pitch wave exciting moment $M_5'$ (Non-dimensional) at 45 degree heading angle

Fig. 6: Comparison of yaw wave exciting moment $M_6'$ (Non-dimensional) at 45 degree heading angle

If we look at the roll and pitch wave exciting moments in Figs. 10 ~ 11, it can be seen that the maximum value of wave exciting moments is found at 0.75 rad/sec, just like the nearest result of surge and sway. But at higher frequencies, roll and pitch moments are decreasing. In Fig. 12, it is seen that yaw moments are almost zero at all frequencies. As it is a square shaped TLP, the pressure difference of both sides is almost same when the wave is coming. For that reason, yaw moments are seen almost zero. In Figs. 13 and 14, the surge and sway motions are higher at lower frequencies and it decreases slowly at higher frequencies. Heave motions are seen higher at lower frequencies in Fig. 15 and the highest value found at 0.395 rad/sec, because the natural frequency of the TLP at heave motion is 0.395 rad/sec. Heave motion drastically decreases at higher frequencies. In Fig. 16 roll motions are seen highest at lower frequencies. The natural frequency of the TLP at roll motion is 0.1382 rad/sec (wave period 45 sec). In practical experiences 45s wave period are very rare at any sea state. So, roll RAO is not calculated 0.1382 rad/sec and some deviations are seen while comparing with the developed program and HydroStar in that frequency range. In Fig.17, pitch RAO is also highest at lower frequency. The pitch natural frequency of this TLP is 0.2047 rad/sec or around 31s wave period. So, it seems the pitch motions are very high at that period. Yaw motions are almost zero at all frequencies in Fig. 18. These results of the present works meet a good agreement while comparing with other research work. So, it can be used for further research. This will help to design the structural system as well as the tendon. From the present study, the following observations can be made.

1) Results obtained from developed program are in very good agreement with the Hydrostar and WAMIT 2000.
2) The magnitude near the resonance period, there are some differences. Near the resonance period the RAO calculation is very sensitive so that the small numerical difference can cause relatively large differences.

3) Both programs have effective computing time when a moderate number of panels are used. So, further study is suggested to use larger number of panels for both programs to get more accurate results.

Table 1: Principal Particulars of TLP

| Particulars               | Symbol | Dimension       |
|---------------------------|--------|-----------------|
| Length                    | L      | 57.26 [m]       |
| Breadth                   | B      | 57.26 [m]       |
| Draught                   | T      | 25.70 [m]       |
| Displacement              | --     | 14,595 [M. Ton] |
| Wetted Surface Area       | --     | 0.915E+04 [m²]  |
| Center of Buoyancy        | X, Y, Z| -0.0008, 0.0000, -15.6336 |
| Water Plane Area          | --     | 529.00 [m²]     |
| Water Depth               | --     | 175.00 [m]      |
| Center of Gravity from Origin | -- | 0.00, 0.00, -3.43 |
| Number of Columns         | --     | 4               |
| Number of Panels          | --     | 642             |
| Number of Co-ordinates    | --     | 674             |

Fig. 7: Surge wave exciting force $F'_1$ (Non-dimensional) at 45 degree heading angle

Fig. 8: Sway wave exciting force $F'_2$ (Non-dimensional) at 45 degree heading angle

Fig. 9: Heave Wave Exciting Force $F'_3$ (Non-dimensional) at 45 degree Heading Angle

Fig. 10: Roll Wave Exciting Moment $M'_4$ (Non-dimensional) at 45 degree Heading Angle
3.1 Heave motion of TLP with tendon

TLP tendons are pre-tensioned slender members, their tops are connected to the hull and bottoms are moored to the seabed. In deep water, the TLP hull tends to interact more pronouncedly to its tendons and risers in terms of mass, stiffness and damping coupling. TLPs are compliant structures consisting of a pontoon, columns and a deck, and are vertically moored at each corner by tendons. Each tendon is pre-tensioned so that it does not go slack due to variations in the extreme ocean environment. While a buoyant hull supports the platform's topsides, an intricate mooring system keeps the TLP in place. The buoyancy of the facility's hull offsets the weight of the
platform, requiring clusters of tight tendons, or tension legs, to secure the structure to the foundation on the seabed. The foundation is then kept stationary by piles driven into the seabed. The tension leg mooring system allows for horizontal movement with wave disturbances, but does not permit vertical, or bobbing, movement, which makes TLPs a popular choice for stability. Vertical motions of the TLP are nearly non-existent due to the tendon’s high axial stiffness (low elasticity). Roll and pitch motions are also negligible. In addition, the heave response of TLP is smaller than that of the surge and sways. In Fig. 19 (representation of Fig. 15), it is seen that the highest heave RAOs occurred at 0.395 rad/sec, which is the natural period of the TLP. With some modification of the program developed by Islam (2001), four tethers at four columns are added to the TLP. Variation in the tether stiffness values for the tethering will generate different motion responses, which will help to select the proper stiffness for the TLP tendon. Such as if we add a tether stiffness of $53 \times 10^6$ N/m then the heave motion or vertical motion seen almost zero and the comparison of the heave RAOs are shown in the Fig. 19 with and without tendon. It means that it is possible to make the heave motion zero which cannot affect the loading and unloading system.

![Fig. 17: Pitch Wave Exciting Motion at 45 degree Heading Angle](image1)

![Fig. 18: Yaw Wave Exciting Motion at 45 degree Heading Angle](image2)

![Fig. 19: Heave RAO at 45 degree heading angle (With and without Tendon)](image3)
4. Conclusions

The study was set out to explore the motion characteristics of a TLP including wave forces at different wave headings. Results have been compared with the current computer program and HydroStar. The comparison is in good agreement. The forces are presented as a non-dimensional form for easy interpretation. Validity has been studied comparing with WAMIT and MOSES, the other two effective computer programs found in the published papers. Since the tether is one of the most important features of TLP, it is also discussed here. The primary thing of the tethers is stiffness, which lessens the heave motion. Choosing right stiffness of tendons, it is possible to diminish the heave motions. So it can be concluded that if forces and motions of TLP are correctly predicted, it can help to design the feasible giant structures as well as tendon system.

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