New interaction solutions and nonlocal symmetries for the (2 + 1)-dimensional coupled Burgers equation

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ABSTRACT
The nonlocal symmetries for the coupled (2 + 1)-dimensional Burgers system are obtained with the truncated Painlevé expansion method. The nonlocal symmetries can be localized to the Lie point symmetries by introducing auxiliary dependent variables. The finite symmetry transformations related with the nonlocal symmetries are computed. The multi-solitary wave solution of the (2 + 1)-dimensional coupled Burgers system is presented. By using the consistent tanh expansion method, many interaction solutions among solitons and other types of nonlinear excitations including the solitons, cnoidal waves, Airy waves and Bessel waves for a number of integrable systems such as the Kadomtsev-Petviashvili (KP) equation, the Burgers equation, the AKNS system, the modified Kadomtsev-Petviashvili (mKP) equation and the coupled integrable dispersionless equation. From the results of nonlocal symmetry reduction, Lou found that the symmetry related to the Painlevé truncated expansion is just the residue with respect to the singular manifold in the Painlevé analysis procedure and called residual symmetry (Gao, Lou, and Chen, 2012; Lou, Hu, and Chen, 2012). On the other hand, the consistent tanh expansion method (CTE) is proposed to identify CTE solvable systems, which is a special simplified form of the consistent Riccati expansion (CRE) method defined in Ref. (Lou, 2015). Some interaction solutions between solitons and other nonlinear excitations can be found with the help of the CTE method for many integrable systems including the Broer-Kaup (BK) system, the Boussinesq-Burgers equations and the (2 + 1)-dimensional Boiti-Leon-Pempinelli (BLP) equation (Chen and Tang, 2014; Hu, Tan, and Hu, 2016; Lou, Cheng, and Tang, 2014). There are many methods to construct the exact solutions for different integrable systems, such as the Boussinesq equation, the unsteady KdV equation, the Benney-Luke equation, etc. (Akbar and Aliz, 2011; Akbar, Ali, and Mohyud-Din, 2013; Akter and Akbar, 2015; Alam and Akbar, 2013, 2015; Alam, Hafez, Akbar, and Roshid, 2015; Alam, Hafez, Belgacem, and Akbar, 2015; Islam, Khan, Akbar, and Mastroberardino, 2014; Khan and Akbar, 2013, 2016).

In this paper, we focus on the following (2 + 1)-dimensional coupled Burgers system

\[ u_t - 2u u_x - v_{xx} = 0, \quad v_{yt} - u_{xy} - 2u v_{xy} - 2u_x v_y = 0, \]

which deserves the name as a coupled Burgers system because it is reduced to the standard Burgers equation by setting \( u = v \) and \( x = y \) (El-Wakil, Abulwafa, El-hanbaly, El-Shewy, Abd-El-Hamid, 2016; Kumar and Kumar, 2014; Su, 2017; Vaneeva, Posta, Sophocleous, 2017). The Painlevé analysis and infinite many symmetries are studied by Wang (Wang, Liang, Tang, 2014) and finite symmetry group and localized structures are given in (Lei and Yang, 2013).

The paper is organized as follows. In Section 2, the nonlocal symmetries for the coupled (2 + 1)-dimensional Burgers system are obtained with the truncated Painlevé expansion, then we localize it by introducing some dependent variables. The multi-solitary wave solution of the coupled (2 + 1)-dimensional Burgers...
the coupled Burgers system (1) such that the nonlocal symmetries become the local symmetries for the prolonged system by introducing another three new dependent variables as the following
\[ \phi_x = f, \quad \phi_t = g, \quad f_t = h. \] (9)

Then the nonlocal symmetry (6) for the coupled Burgers system (1) becomes a Lie point symmetry of the prolonged system (1), (3) and (9) and it is verified that the Lie point symmetries of the prolonged system have the form
\[ \sigma^u = \delta f, \quad \sigma^v = f, \quad \sigma^\phi = -\phi^2, \quad \sigma^f = -2\phi f, \quad \sigma^g = -2\phi g, \quad \sigma^h = -2f^2 - 2\phi h. \] (10)

Correspondingly, the initial value problem of (10) becomes
\[
\begin{align*}
\frac{du}{dt} & = \frac{\delta f}{\phi_x}, \quad \frac{dv}{dt} = f, \quad \frac{d\phi}{dt} = -\phi^2, \quad \frac{df}{dt} = -2\phi f, \\
\frac{dg}{dt} & = -2\phi g, \quad \frac{dh}{dt} = -2f^2 - 2\phi h, \\
\epsilon u|_{t=0} & = u, \quad \epsilon v|_{t=0} = v, \quad \epsilon \phi|_{t=0} = \phi, \quad \epsilon f|_{t=0} = f, \\
\epsilon g|_{t=0} & = g, \quad \epsilon h|_{t=0} = h.
\end{align*}
\] (11)

The solution of the initial value problem (11) and (12) for the enlarged system (1), (3) and (9) can be written as
\[
\begin{align*}
\epsilon u & = u + \frac{\epsilon f}{1 + \epsilon \phi}, \quad \epsilon v = v + \frac{\epsilon f}{1 + \epsilon \phi}, \\
\epsilon \phi & = \frac{\phi}{1 + \epsilon \phi}, \quad \epsilon f = \frac{f}{(1 + \epsilon \phi)^2}, \\
\epsilon g & = \frac{g}{(1 + \epsilon \phi)^2}, \quad \epsilon h = \frac{h}{(1 + \epsilon \phi)^2} - \frac{2\epsilon f^2}{(1 + \epsilon \phi)^3}.
\end{align*}
\] (13)

Using the finite symmetry transformation (13) and (14), we can obtain a new solution from the initial solution. For example, we take the trivial solution \( u = v = 0 \) of the coupled Burgers system (1) and the multi-solitary wave solution for (4) is supposed as
\[ \phi = 1 + \sum_{n=1}^{N} \exp(k_n x + w_n t). \] (15)

where \( k_n, w_n \) are arbitrary constants. The multi-solitary wave solution (15) is the solution of (4) only with the relation
\[ w_n = \delta k_n^2. \] (16)

A solution of the coupled Burgers system (1) presents in the following form by using (9), (13) and (14)
\[
\begin{align*}
u & = \frac{g}{(1 + \epsilon \phi)^2}, \quad h = \frac{h}{(1 + \epsilon \phi)^2} - \frac{2\epsilon f^2}{(1 + \epsilon \phi)^3}, \\
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\phi & = \frac{\phi}{1 + \epsilon \phi}, \quad f = \frac{f}{(1 + \epsilon \phi)^2}, \\
g & = \frac{g}{(1 + \epsilon \phi)^2}, \quad h = \frac{h}{(1 + \epsilon \phi)^2} - \frac{2\epsilon f^2}{(1 + \epsilon \phi)^3}.
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\end{align*}
\] (17)
3. Consistent tanh expansion solvability and interaction solutions for the Equation (1)

The author introduced the consistent tanh expansion method in (Lou, 2015), that is to say, for a Painlevé integrable system, we want to find the solution in the form

\[ u = \sum_{j=0}^{J} u_j \tanh^j(\omega), \]

where \( J \) can be determined by the leading order analysis of the integrable system. We can take the following truncated tanh function expansion for the Equation (1)

\[ u = u_0 + u_1 \tanh(\omega), \quad v = v_0 + v_1 \tanh(\omega). \]  

Substituting the expression (18) into the coupled Burgers system (1), we obtain two complicated expression for the function \( \tan(\omega) \)

\[
(2u_0^2 - 2v_0^2 \tanh^2(\omega) + 2u_1 \omega_1 + 2v_1 \omega_x - 2u_1 \omega_x + v_1 \omega_{xx}) \tanh^2(\omega) + (2v_1 \omega_x - 2v_0 \omega_x - 2u_0 \omega_x + u_1 \omega_x - 2u_0 \omega_{xx}) \tanh(\omega) + u_0 + v_0 - 2u_1 \omega_x + u_1 \omega_x - 2u_0 \omega_x - v_{0xx} = 0, 
\]

where

\[ C = \frac{\omega_1}{\omega_x}, \quad S = \frac{2\omega_x \omega_{xx} - 3\omega_y^2}{2\omega_x}. \]

In summary, we have the following theorem:

**Theorem 3.1.** If \( \omega \) is a solution of the consistent condition (22), then

\[ u = \delta \omega_x \tanh(\omega) + \frac{\omega_1 - \delta \omega_x}{2\omega_x}, \]

\[ v = \omega_x \tanh(\omega) + \frac{\delta \omega_x - \omega_{xx}}{2\omega_x}, \]

constitute a CTE solution of the coupled (2 + 1)-dimensional Burgers system (1).

According to Theorem 3.1, we can find various interaction solutions among different types of nonlinear excitations of the Equation (1) by solving the \( \omega \) Equation (22). It is interesting to see that Equation (22) can be written as

\[ \omega_1 = \delta \omega_x \pm 2\omega_y^2 + f_{\omega_x}, \]

\[ f_1 = \delta f_{\omega_x} + f_{\omega_x}. \]

It is not difficult to find that the four equations in (24) have similarity solutions, therefore, it is sufficient to investigate the solutions of one case of (24). Namely, \( \delta = -1 \) and the Equation (24) becomes

\[ \omega_1 = -\omega_x \pm 2\omega_y^2 + f_{\omega_x}, \]

\[ f_1 = -f_{\omega_x} + f_{\omega_x}. \]

It is clear that \( \omega \) can be obtained by solving the variable coefficient potential Burgers equation (25a) via a fixed solution of the Burgers equation (25b). Therefore, the corresponding solution of the coupled Burgers system (1) can be obtained from the CTE expression (23).

### 3.1. Single soliton solutions

In (25a), we take a trivial solution

\[ \omega = k_0 x + l_0 y + w_0 t + d_0, \quad f = \frac{w_0 - 2k_0^2}{k_0}, \]  

with \( k_0, l_0, w_0, d_0 \) being arbitrary constants. Then substituting (26) into (23) with \( \delta = -1 \) yields the single soliton solution of the coupled Burgers system (1)

\[ u = -k_0 \tanh(k_0 x + l_0 y + w_0 t + d_0) + \frac{w_0}{2k_0}, \]

\[ v = k_0 \tanh(k_0 x + l_0 y + w_0 t + d_0) - \frac{w_0}{2k_0}. \]

### 3.2. Soliton-cnoidal waves

In order to obtain the interaction solutions between soliton and cnoidal wave of the coupled Burgers system (1), letting

\[ \omega = k_0 x + l_0 y + w_0 t + W(k_1 x + l_1 y + w_1 t), \]

where

\[ C = \frac{\omega_1}{\omega_x}, \quad S = \frac{2\omega_x \omega_{xx} - 3\omega_y^2}{2\omega_x}. \]
where
\[ W(k_1x + l_1y + w_1t) = W(X) = W \]
satisfies
\[ W_{1x}^2 = C_0 + C_1W_1 + C_2W_1^2 + C_3W_1^3 + C_4W_1^4, \quad W_1 = W_x, \]
and \( C_2, C_3 \) are arbitrary constants and substituting (30) and (28) into the Equation (23) with \( \delta = -1 \), we obtain
\[
\begin{align*}
\mu_1 &= \frac{2g_x}{k_0}, \\
\nu &= (k_0 + k_1W_1) \tanh \left( k_0x + l_0y + \frac{w_1k_0}{k_1}t + W \right) - \frac{w_1}{2k_1} - \frac{k_1^2W_1}{2(k_0 + k_1W_1)}.
\end{align*}
\]

It is obvious that the Equation (29) is an equation for the definition of the elliptic functions, which can be expressed in terms of Jacobi elliptic functions. Here, we write down two types of soliton-cnoidal wave solutions of (22) with \( \delta = -1 \). The first one is a simple solution as
\[
W_1 = \mu_0 + \mu_1 \text{sn}(mX, n),
\]
where \( \text{sn}(mX, n) \) is the usual Jacobi elliptic sine function. Substituting (32) and (30) into (29) yields
\[
C_2 = \frac{2(2k_1^2\mu_0^2 - 2k_1\mu_0k_0 - k_0^2)}{k_1^2},
\]
\[
C_3 = -16\mu_0,
\]
\[
n = 1,
\]
\[
\mu_1 = \frac{k_0 + k_1\mu_0}{k_1},
\]
\[
m = \frac{-2(k_0 + k_1\mu_0)}{k_1}.
\]

Hence, one kind of soliton-cnoidal wave solutions is obtained by taking (32) and
\[
W = \mu_0X + \mu_1 \int_{y_0}^Y \text{sn}(mY, n) dY,
\]
with the parameter requirement (33) into the general solution (31). Then we can discuss the soliton-cnoidal waves for the \((2 + 1)\)-dimensional coupled Burgers

Equation (1) by selecting the proper arbitrary constants.

The second type of the soliton-cnoidal wave interaction solutions is to select
\[
\omega = k_0x + l_0y + w_0t + AE_l \left( \text{sn}(k_1x + l_1y + w_1t, m), n \right),
\]
where \( E_l \) is the first incomplete elliptic integral and \( \text{sn}(z, m) \) is the usual Jacobi elliptic sine function. Substituting (35) into (22) with \( \delta = -1 \) and setting the coefficients of different powers of Jacobi elliptic functions into zero, we can find eight arbitrary constant solutions except \( m = n \). Then substituting (35) with \( m = n \) into (23) with \( \delta = -1 \), we can obtain the soliton-cnoidal wave interaction solutions of the coupled Burgers Equation (1) with the proper arbitrary constants in the same way.

3.3. Solitons and potential Burgers wave solutions

To find out the interaction solutions of the coupled Burgers system (1), we consider \( \omega \) in the form
\[
\omega = k_0x + l_0y + w_0t + g,
\]
where \( g \) is a function of \( x, y \) and \( t \).

Applying the transformation (36) to (25a), we have
\[
g_t = -g_{xx} + 2g_x^2 + (f + 4k_0)g_x + 2k_0^2 - w_0 + f k_0.
\]

For further simplicity, we take \( f \) as the simplest constant solution
\[
f = \frac{w_0 - 2k_0^2}{k_0},
\]
on account of which, (37) becomes a constant coefficient potential Burgers equation
\[
g_t = -g_{xx} + 2g_x^2 + \left( \frac{w_0}{k_0} + 2k_0 \right) g_x.
\]

After substituting (36) and (39) into (23) with \( \delta = -1 \), we get the interaction solution between a soliton and a potential Burgers wave of the coupled Burgers system (1)
\[
\begin{align*}
\mu_1 &= \frac{2g_x}{k_0}, \\
\nu &= (k_0 + g_x) \tanh(k_0x + l_0y + w_0t + g) + g_x + \frac{w_0}{2k_0},
\end{align*}
\]

It is well known that the potential Burgers equation has many types of known exact solutions, such as resonant soliton solutions, and error function solution. In the following, we use the known solutions of (39) to construct the interaction solutions between a soliton and potential Burgers waves.
Example 1: Multiple resonant soliton solutions

Equation (25a) possesses the following multiple wave solution
\[ g = -\frac{1}{2} \ln \left[ 1 + \sum_{i=1}^{n} \exp (k_i x + l_i y + w_i t) \right], \tag{41} \]
where \( k_i, l_i \) are arbitrary constants while \( w_i \) are determined by the dispersion relations
\[ w_i = \frac{k_i}{k_0} (2 k_0^2 + w_0 - k_0 k_i). \]

Substituting (41) into (40), the \((n+1)\) resonant soliton solutions of the coupled Burgers system (1) can be directly obtained.

Example 2: Interaction solution between a soliton and an error function wave

It can be proved that the potential Burgers Equation (39) possesses an error function solution
\[ g = -\frac{1}{2} \ln \left[ \text{erf} \left( \frac{i(x + y + w_1 t)}{2\sqrt{t}} \right) \right], \quad i^2 = 1, \quad w_1 = 2 k_0 + \frac{w_0}{k_0}, \tag{42} \]
where the error function \( \text{erf}(x) \) is defined by
\[ \text{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp(-z^2)dz, \quad \tag{43} \]
which means that solution (40) with (42) present the soliton-error function interaction solution of the coupled Burgers system (1).

Example 3: Soliton interactions with periodic waves

It is known that the potential Burgers system (39) has a general solution
\[ g = -\frac{1}{2} \ln \left\{ \int_{-\infty}^{\infty} F(x + y + 2 k_0 t + \frac{w_0}{k_0} t + i^2 \sqrt{t} \xi) \exp(-\xi^2) d\xi \right\}. \tag{44} \]
where
\[ F \equiv F(z) \equiv F(x + y + 2 k_0 t + \frac{w_0}{k_0} t + i^2 \sqrt{t} \xi) < \exp(\xi^2), \quad \xi \to \infty \tag{45} \]
is an arbitrary function of the indicated variable.

If we take \( F \) as a polynomial solution of \( z \),
\[ F = \sum_{n=1}^{N} c_n z^n \tag{46} \]
with arbitrary constants \( c_n \), the \( g \) wave (44) becomes
\[ g = -\frac{1}{2} \ln \left[ \sum_{n=1}^{N} h_n \sum_{j=0}^{n} \left[ 1 + (-1)^j \right] j! \frac{t^{j/2}}{\Gamma(1+j/2) \Gamma(n-j+1)} (x + y + w_1 t)^{n-j} \right], \tag{47} \]
where \( \Gamma(x) \) is the usual Gamma function. The CTE solution (40) along with (47) becomes an interaction solution between a soliton and a rational wave.

Example 4: Soliton interactions with periodic waves

We consider the arbitrary function \( F \) in (44) as the following special simple form
\[ F = \sum_{j=1}^{N} c_j \cos \left[ a_j(x + d_j + N - 2 b_j t) \right] \exp \left[ b_j x + b_j y + (w_i b_i - b_j^2 + a_j^2) t \right]. \tag{48} \]
where \( a_j, b_j, c_j, d_j \) are arbitrary constants. In this case, we have
\[ g = -\frac{1}{2} \ln \left\{ \sum_{j=1}^{N} c_j \cos \left[ a_j(x + y + d_j + (w_1 - 2 b_j) t) \right] \exp \left[ b_j x + b_j y + (w_i b_i - b_j^2 + a_j^2) t \right] \right\}, \tag{49} \]
which means solution (40) with (49) is an interaction solution among solitons and periodic waves.

4. Conclusion and discussion

In summary, the nonlocal symmetries for the coupled \((2+1)\)-dimensional Burgers system are obtained by using the truncated Painlevé expansion. To solve the initial value problem related to the nonlocal symmetries, we prolong the coupled \((2+1)\)-dimensional Burgers equation so that nonlocal symmetries become the local Lie point symmetries for the prolonged system. The finite symmetry transformations of the prolonged coupled \((2+1)\)-dimensional Burgers equation is derived by solving the Lie’s first principle. In the meanwhile, the coupled \((2+1)\)-dimensional Burgers equation is studied by means of the CTE method and proved to be CTE solvable. Many exact interaction excitations such as the soliton-cnoidal waves, the multiple resonant soliton solutions, soliton-error function waves, soliton-rational waves and soliton-periodic waves of the coupled \((2+1)\)-dimensional Burgers equation are explicitly constructed with the help of the CTE method by selecting different solutions to the \( \omega \) equation. These new interaction wave solutions are presented analytically with the proper constant selections. The more interaction excitations from CTE method about other coupled integrable systems will be worth of further study.

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References
Akbar, M. A., Ali, N. H. M., & Mohyud-Din, S. T. (2013). Further exact traveling wave solutions to the (2 + 1)-dimensional Boussinesq and Kadomtsev-Petviashvili equation. Journal of Computational Analysis and Application, 15, 557–571.
Akbar, M. A., & Aliz, N. H. M. (2011). The modified alternative (G'/G)-expansion method for finding the exact solutions of nonlinear PDEs in mathematical physics. Physics and Astronomy, 1, 6, 7910–7920.
Akber, J., & Akbar, M. A. (2015). Exact solutions to the Benney-Luke equation and the Φ – 4 equations by using the modified simple equation method. Results in Physics, 5, 125–130.
Alam, M. N., & Akbar, M. A. (2013). Exact traveling wave solutions of the KP-BBM equation by using the new approach of generalized (G'/G)-expansion method. SpringerPlus, 2, 617.
Alam, M. N., & Akbar, M. A. (2015). Some new exact traveling wave solutions to the simplified MCH and the (1 + 1)-dimensional combined KdV-mKdV equations. Journal of Association of Arab Universities for Basic and Applied Sciences, 17, 6–13.
Alam, M. N., Hafez, M. G., Akbar, M. A., & Roshid, H. (2015). Exact solutions to the (2 + 1)-dimensional Boussinesq equation via the exp(φ(η))-expansion method. Journal of Scientific Research, 7, 1–10.
Alam, M. N., Hafez, M. G., Belgacem, F. B. M., & Akbar, M. A. (2015). Application of the novel (G'/G)-expansion method to find new exact traveling wave solutions of the nonlinear coupled Higgs field equation. Nonlinear Studies, 22, 613–633.
Chen, C. L., & Lou, S. Y. (2013). CTE solvability and exact solution to the Broer-Kaup system. Chinese Physics Letters, 30, 110202.
Chen, M. X., Hu, H. C., & Zhu, H. D. (2015). Consistent Riccati expansion and exact solutions of the Kuramoto-Sivashinsky equation. Applied Mathematics Letters, 49, 147–151.
Cheng, X. P., Lou, S. Y., Chen, C. L., & Tang, X. Y. (2014). Interactions between solitons and other nonlinear Schrödinger waves. Physical Review E, 89, 043202.
El-Wakil, S. A., Abulwafa, E. M., El-hanbaly, A. M., El-Shewy, E. K., & Abd-El-Hamid, H. M. (2016). Self-similar solutions for some nonlinear evolution equations: KdV, mKdV and Burgers equations. Journal of the Association of Arab Universities for Basic and Applied Sciences, 19, 44–51.
Gao, X. N., Lou, S. Y., & Tang, X. Y. (2013). Bosonization, singularity analysis, nonlocal symmetry reductions and exact solutions of supersymmetric KdV equation. Journal of High Energy Physics, 5, 29.
Hu, H. C., Tan, M. Y., & Hu, X. (2016). New interaction solutions to the combined KdV-mKdV equation from CTE method. Journal of the Association of Arab Universities for Basic and Applied Sciences, 21, 64–67.
Hu, X. R., Lou, S. Y., & Chen, Y. (2012). Explicit solutions from eigenfunction symmetry of the Korteweg-de Vries equation. Physical Review E, 85, 056607.
Islam, M. S., Khan, K., Akbar, M. A., & Mastroberardino, A. (2014). A note on improved F-expansion method combined with Riccati equation applied to nonlinear evolution equations. Royal Society Open Science, 1, 140038.
Khan, K., & Akbar, M. A. (2013). Exact and solitary wave solutions for the Tzitzeica-Dodd-Bullough and the modified KdV-Zakharov-Kuznetsov equations using the modified simple equation method. Ain Shams Engineering Journal, 4, 903–909.
Khan, K., & Akbar, M. A. (2016). Solving unsteady Kortweg-de-Vries equation and its two alternatives. Mathematical Methods in Applied Sciences, 39, 2752–2760.
Kumar, S., & Kumar, D. (2014). Fractional modelling for BBM-Burgers equation by using new homotopy analysis transformation method. Journal of the Association of Arab Universities for Basic and Applied Sciences, 16, 18–20.
Lei, Y., & Yang, D. (2013). Finite symmetry transformation group and localized structures of the (2 + 1)-dimensional coupled Burgers equation. Chinese Physics B, 22, 46.
Lou, S. Y. (2013). Residual symmetries and Backlund transformations. arXiv:1308.1140v1.
Lou, S. Y. (2015). Consistent Riccati expansion for integrable systems. Studies in Applied Mathematics, 134, 372–402.
Lou, S. Y., Cheng, X. P., & Tang, X. Y. (2014). Dressed dark solitons of the defocusing nonlinear Schrödinger equation. Chinese Physics Letters, 31, 070201.
Lou, S. Y., Hu, X. R., & Chen, Y. (2012). Nonlocal symmetries related to Backlund transformation and their applications. Journal of Physics A: Mathematical and Theoretical, 45, 155209.
Su, T. (2017). Explicit solutions for a modified 2 + 1-dimensional coupled Burgers equation by using Darboux transformation. Applied Mathematics Letters, 69, 15–21.
Vaneeva, O., Posta, S., & Sophocleous, C. (2017). Enhanced group classification of Benjamin-Bona-Mahony-Burgers equations. Applied Mathematics Letters, 65, 19–26.
Wang, J. Y., Liang, Z. F. & Tang, X. Y. (2014). Infinitely many generalized symmetries and Painlevé analysis of a (2 + 1)-dimensional Burgers system. Physica Scripta, 89, 025201.