Modeling of UTSG in the Pressurized Water Reactor Using Accurate Formulae of Thermodynamic Properties

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Abstract

In nuclear power plants, steam generators are important parts. This paper introduces a U-tube steam generator (UTSG) model implemented completely using MATLAB environment. The UTSG consists of four regions: upward and downward primary regions, upward and downward metal tube regions, and secondary regions, which contain heat transfer region, steam separation region, and subcooled water region. Governing equations are derived by applying energy and mass conservation equations in all regions. Accurate functions that describe the relationships between thermodynamic properties of the saturated steam are introduced instead of interpolation method that is widely used. Steady state and one transient case are presented as well.

Keywords

UTSG Modeling, Steam Tables, Thermodynamic Properties

1. Introduction

Steam generators (SGs) of the vertical recirculating type (UTSG) are the dominant types that are used in the pressurized water reactors (PWRs). SGs are heat exchangers containing thousands of tubes to transfer heat from the primary coolant which represents the hot side to the secondary coolant which represents the cold side to produce steam, that produce electricity by powering turbine generators. A large number of nuclear power plants (NPPs) have from 2 to 6 SGs; however some have up to 12 SGs [1]. In nuclear power plants overall the world, there are around 1300 SGs being in service.
Water/Steam thermodynamic properties are required in equation derivation of SG models. These properties are available in the form of plots or tables. Extracting data directly from these plots or tables is not suitable for dynamic modeling. Therefore, most of the researchers expressed them in linear mathematical forms, which can be effective for simulation purposes. In UTSG mathematical models, the relations of the specific volumes/densities of the vapor and liquid, and their specific enthalpy of the vapor and liquid and saturated temperature that depend on the saturation pressure ($P_{\text{sat}}$) are required. Typically, such relationships can be found for pressure intervals in thermodynamic properties tables [2]. Many of these models develop approximating formulae using an interpolation method. In addition, the relationships of the derivatives of these parameters are also needed [3]. That is the reason that most of the models use linear approximating formulae.

Recently, many researchers reported the modeling of the UTSG using different platforms. For example, the derivation of the UTSG model was done using MATLAB as a part of a complete power plant simulator [4]. They used steam tables from Ref. [5] to get water/steam properties using interpolation functions programmed in C++ and FORTRAN languages. Reference [6] used an external software package to calculate the physical parameters of water and vapor. Then, a dynamic model for steady-state and transient responses was programmed on a personal computer using MATLAB/SIMULINK. While in Ref. [7], a dynamic model of AP1000 steam generator introduced using the following assumption: linear approximate profiles for volume weighted specific volume and specific enthalpy are used in secondary side tube bundle region. Other researchers reported the using of MATLAB to model Steam Generators as in [8] [9] [10] [11] [12] to do the same for calculating water/steam properties.

In the present work, a mathematical model for the UTSG is introduced using a home-made computer model written fully using MATLAB programming environment. It is well known that if there are many measurement data, MATRIXx environment provides a quick solution to the least square’s problems, so matrix calculations can be performed easily using commercial software packages such as MATHCAD and MATLAB [1]. MATLAB is widely used in modeling of steam generators because of its interactive programmable matrix calculator with graphics capabilities [13].

The Steam Generator is divided into four regions according to Ali’s second intermediate model (model C) for the UTSG, which is utilized in this work using a MATLAB Program [14]. The regions are: upward and downward primary regions which represent primary water pumped from the reactor cooling pump going up then down inside thousands of inverted U tubes, and during this movement heat transferred through upward and downward metal tube regions. Finally, the heat is transferred to the secondary regions where the cold feed water uses this heat to generate steam; then steam is separated during its way to turbine. Governing equations are derived by applying energy, mass, and momentum conservation equations on all regions.
The most popular design for steam generators is UTSG [1]. Figure 1 represents a schematic diagram of an inverted U-shaped vertical steam generator, in which all parts of the steam generator are contained within a cylindrical vessel and the pre-cooling water coming from the reactor core enters the inlet at a pressure of about 15 MPa facing upward at the inverted U. This flowing water transfers the heat to the secondary water in the steam generator through the pipe walls without mixing. The primary cooling water enters the steam generator at approximately 583 - 600 K and exits at 530 - 560 K. Also, the steam mass ratio varies from 17% - 33%. The higher this percentage, the higher the efficiency of the steam generator and the better for the turbine because this wet steam cannot be sent to the high-pressure turbine, which increases the need for high techniques to remove the liquid from the steam as much as possible [15].
2. The UTSG Model Equations

2.1. Assumptions

The first step in the UTSG modeling is to make appropriate assumptions. These assumptions are introduced to smoothing calculations and minimizing the complexity. The basic assumptions on which our model is constructed are:

1) One dimensional flow for primary and secondary cooling water.
2) Primary flow \( (W_{\text{p}}) \) is constant.
3) Steam exit flow \( (W_{\text{s}}) \) is proportion to saturated pressure \( (P_{\text{sat}}) \).
4) Thermal conductivity of the inverted metal tube is constant.
5) Heat transfer coefficients are assumed to be constant during transients.
6) Heat transfer between the tube bundle area and the downcomer is neglected.
7) Flow of exit Steam \( (W_{\text{s}}) \) = flow of feedwater \( (W_{\text{f}}) \).

2.2. UTSG Parameters and Design State Values

The current mathematical model of UTSG is based on the geometrical parameters and the numerical constants for the four regions listed in Table 1 and Table 2, respectively [4]. In addition, Table 3 shows symbols and units of the variables used in the thermodynamic equations in the current model.

Table 1. Parameters of UTSG.

| Symbol | Description | Value and Units |
|--------|-------------|-----------------|
| \( A_d \) | Area of the coolant in the accumulation tank | 3 m² |
| \( A_{\text{dr}} \) | Area of coolant in recirculation region | 10.3 m² |
| \( C_m \) | Specific heat capacity of metal tubes | 460 J/(kg°C) |
| \( C_s \) | Conversion factor between \( W_{\text{s}} \) and \( P_{\text{sat}} \) | 80 kg/(s MPa) |
| \( K \) | Thermal conductivity of metal tube | 55.0012 J/(s m°C) |
| \( L \) | Height of U-tube | 10.83 m |
| \( L_d \) | Height of accumulation tank | 10.83 m |
| \( L_{\text{lb}} \) | Height of liquid | 1.057 m |
| \( M_3 \) | Mass of half of total metal tube upward | 25,600 kg |
| \( M_4 \) | Mass of half of total metal tube downward | 25,600 kg |
| \( N \) | Total number of U-tubes | 3388 |
| \( R_{\text{avg}} \) | \( (R_{\text{in}} + R_{\text{out}})/2 \) | 10.357 cm |
| \( R_{\text{in}} \) | Inner radius of U-Tube | 9.75 cm |
| \( R_{\text{out}} \) | Outer radius of U-Tube | 11 cm |
| \( T_{\text{fi}} \) | Temperature of inlet feed water | 223°C |
| \( T_p \) | Temperature of primary water | 312°C |
| \( U_{\text{ms}} \) | Heat transfer coefficient between the metal tubes and the secondary water | J/(s m²°C) |
| \( V_{\text{dr}} \) | Volume of drum | 124.5 m³ |
| \( V_{\text{v}} \) | Internal volume of Liquid/Vapor separator | 13.3 m³ |
| \( V_{\text{c}} \) | Volume of the secondary coolant in the effective heat exchange region | 94.4 m³ |
| \( W_{\text{p}} \) | Flow rate of primary water | 4950 kg/s |
| \( \rho_{\text{d}} \) | Metal tube density | 8490 |
| \( \rho_{\text{m}} \) | Density of metal tube | 8500 kg/m³ |
Table 2. Design steady state values of UTSG.

| Symbol | Description                                      | Value and Units |
|--------|--------------------------------------------------|-----------------|
| $T_1$  | Temperature of primary coolant flowing upward     | 576 K           |
| $T_2$  | Temperature of primary coolant flowing downward   | 569 K           |
| $T_3$  | Temperature of metal tube of coolant flowing upward | 571 K         |
| $T_4$  | Temperature of metal tube of coolant flowing downward | 565 K         |
| $T_5$  | Temperature of recirculated water                | 535 K           |
| $T_6$  | Temperature of water in the mixing region        | 535 K           |
| $L_s$  | Level of water of mixing region of secondary coolant | 3.2 m       |
| $x_s$  | Quality of coolant at the end of the boiling region | 0.233        |
| $P_{sat}$ | Saturation Pressure of secondary water in boiling region | 5.9 MPa |

Table 3. Symbols and units of the variables used in the current model.

| Symbol | Description                                      | Units             |
|--------|--------------------------------------------------|-------------------|
| $A_{ms}$ | Surface area in contact between secondary water and metal tubes | m$^2$           |
| $A_{pm}$ | Surface area in contact between metal tubes and primary coolant | m$^2$           |
| $C_p$  | Specific heat of subcooled water                 | J/(kg°C)         |
| $h_f$  | Saturated water specific enthalpy                | kJ/kg            |
| $h_{fg}$ | $H_f - h_g$                                       | kJ/kg            |
| $h_v$  | Saturated vapor specific enthalpy                | kJ/kg            |
| $h_s$  | Average enthalpy of secondary water              | kJ/kg            |
| $L_b$  | Height of boiling column                         | m                |
| $M_d$  | Mass of water in annular tank                    | kg               |
| $M_{sw}$ | Mass of drum water                              | kg               |
| $M_p$  | Mass of half of the primary water                | kg               |
| $M_s$  | Mass of secondary coolant                        | kg               |
| $\dot{Q}_{ms1}$ | Upward metal tube to secondary coolant heat transfer rate | J/s            |
| $\dot{Q}_{ms2}$ | Downward metal tube to secondary coolant heat transfer rate | J/s          |
| $T_s$  | Average bulk mean temperature in the secondary lump | °C             |
| $T_{sat}$ | Saturation temperature                           | °C               |
| $U'_{ms}$ | Coefficient of heat transfer between secondary water and metal tubes | J/(s m$^2$°C)  |
| $U_{pm}$ | Coefficient of heat transfer between primary coolant and metal tubes | J/(s m$^2$°C)  |
| $W_1$  | Mass flow rate of fluid from the downcomer to the riser | Kg/s            |
| $W_2$  | Mass flow rate of outlet secondary water          | Kg/s            |
| $W_3$  | Mass flow rate of secondary coolant leaving separator | kg/s         |
| $W_6$  | Flow rate of feed water                          | kg/s            |
| $W_{sat}$ | Flow rate of saturated steam                     | kg/s            |
| $\rho_f$ | Saturated liquid density                         | kg/m$^3$         |
| $\rho_v$ | Saturated vapor density                         | kg/m$^3$         |
| $\nu_f$ | Saturated liquid specific volume                 | m$^3$/kg         |
| $\nu_g$ | $\nu_f - \nu_v$                                 | m$^3$/kg         |
| $\nu_v$ | Saturated vapor specific volume                  | m$^3$/kg         |
2.3. State Equations for UTSG Model

Nodalization scheme of the UTSG model is shown in Figure 2. The model governing the equations is derived by applying energy, mass, and momentum conservation equations.

Primary coolant equations derivation is done by applying the energy conservation equation to both PRIMARY_UP and PRIMARY_DOWN, which represent the moving of primary coolant up and down inside the inverted U-tube respectively (Figure 2).

\[
\frac{d}{dt} \left( M_p \cdot C_p \cdot T_i \right) = W_{pi} \cdot C_p \left( T_p - T_i \right) - U_{pm} \cdot A_{pm} \cdot (T_i - T_3) \tag{1}
\]

\[
M_p \cdot C_p \cdot \frac{dT_i}{dt} = W_{pi} \cdot C_p \cdot T_p - W_{pi} \cdot C_p \cdot T_1 - U_{pm} \cdot A_{pm} \cdot T_1 + U_{pm} \cdot A_{pm} \cdot T_3 \tag{2}
\]

\[
U_{pm} = \frac{1}{k} + \frac{R_{in}}{R_{avg}} \cdot \log \left( \frac{R_{avg}}{R_{in}} \right) \tag{3}
\]

so

\[
\frac{dT_i}{dt} = \left( \frac{-W_{pi} \cdot U_{pm} \cdot A_{pm}}{M_p \cdot C_p} \right) T_1 + \left( \frac{U_{pm} \cdot A_{pm}}{M_p \cdot C_p} \right) T_3 + \left( \frac{W_{pi}}{M_p} \right) T_{pi} \tag{4}
\]

In the same way, we have

\[
\frac{dT_i}{dt} = \left( \frac{-W_{pi} \cdot U_{pm} \cdot A_{pm}}{M_p \cdot C_p} \right) T_2 + \left( \frac{U_{pm} \cdot A_{pm}}{M_p \cdot C_p} \right) T_3 + \left( \frac{W_{pi}}{M_p} \right) T_{pi} \tag{5}
\]

**Figure 2.** Schematic diagram of the UTSG.
Also, U-Tube metal region equations derivation is done by applying the energy conservation equation on both METAL_UP and METAL_DOWN, which represent the inverted U-metal tube up flow and downflow respectively as shown in Figure 2.

\[
\frac{d}{dt} M_s \cdot C_m \cdot T_s = U_{pm} \cdot A_{pm} \cdot (T_i - T_s) - U_{ms} \cdot A_{ms} \cdot (T_s - T_i)
\]

(6)

\[
U_{ms} = \frac{1}{1 + \frac{R_{ms} \cdot \log \left( \frac{R_{ms}}{R_{avg}} \right)}{k}}
\]

(7)

\[
\frac{dT_s}{dt} = \left( \frac{-U_{pm} \cdot A_{pm} - U_{ms} \cdot A_{ms}}{M_s \cdot C_m} \right) T_s + \left( \frac{U_{pm} \cdot A_{pm}}{M_s \cdot C_m} \right) T_i + \left( \frac{U_{ms} \cdot A_{ms}}{M_s \cdot C_m} \right) T_i
\]

(8)

In the same way, we have

\[
\frac{dT_i}{dt} = \left( \frac{-U_{pm} \cdot A_{pm} - U_{ms} \cdot A_{ms}}{M_s \cdot C_m} \right) T_i + \left( \frac{U_{pm} \cdot A_{pm}}{M_s \cdot C_m} \right) T_i + \left( \frac{U_{ms} \cdot A_{ms}}{M_s \cdot C_m} \right) T_i
\]

(9)

Secondary coolant region equations derivation is done applying energy and mass conservation equations on the three secondary coolant regions: Effective heat transfer region (EHT), upper tank region (ACCU_T), and Inlet annular tank region (IA_T) as shown in Figure 2.

EHT region of secondary cooling water is the area of heat accumulation in the metal tubes. Because of the heat build-up, the secondary cooling water is pushed up, and the energy conservation equation can be written as follows:

\[
\frac{d}{dt} (h_s \cdot M_s) = \dot{Q}_{ms1} + \dot{Q}_{ms2} + W_1 \cdot C_{ps} \cdot T_s - W_2 \cdot h_e
\]

(10)

\[
V_s \frac{d}{dt} (\rho_s \cdot h_s) = U_{ms} \cdot A_{ms} \cdot (T_i - T_s) + U_{ms} \cdot A_{ms} \cdot (T_s - T_i)
\]

\[
+ W_1 \cdot C_{ps} \cdot T_s - W_2 \cdot h_e
\]

(11)

Now the following equations are deduced for the average thermodynamic properties of the secondary coolant in the effective heat exchange mass.

\[
L = L_{sh} + L_b
\]

(12)

\[
T_s = \frac{T_i + T_{ms} \cdot L_{sh}}{2} + \frac{L_{sh}}{T_s}
\]

(13)

\[
\rho_s = \frac{\rho_d + \rho_f \cdot L_{sh}}{2} \cdot \frac{L_{sh}}{L} \cdot \left( \frac{U_f + \frac{x}{2} \cdot U_{sh}}{L} \right)
\]

(14)

\[
h_e = C_{ps} \cdot \frac{T_i + T_{ms} \cdot L_{sh}}{2} + \left( h_f + \frac{x}{2} \cdot h_{fe} \right) \cdot \frac{L_b}{L}
\]

(15)

\[
h_e = h_f + x_e \cdot h_{fe}
\]

(16)

The heat transfer in the ACCU_T zone is not carried out immediately when the secondary coolant meets the metal tubes. It is in this region specifically that the secondary cooling water is in the state of saturation, as a mixture of water
and steam. And dryers are used to separate the steam from the water to allow the steam to flow into the turbine while the rest of the water is returned to the inlet ring tank to be mixed with the feed water. Now by applying the continuity equations only to the mixture of water vapor and mass of vapor respectively, and by applying both the continuity equations and the energy balance to the mass of water only:

\[
\frac{d(V_\ast \rho_\ast)}{dt} = W_2 - W_3
\] (17)

\[
\frac{d(L_{dw} \ast A_{dw} \ast \rho_f)}{dt} = W_{fi} + (1-x_e) \ast W_3 - W_1
\] (18)

\[
\frac{d(M_{dw} \ast C_{ps} \ast T_b)}{dt} = W_{fi} \ast M_{dw} \ast C_{ps} \ast T_{bi} + (1-x_e) \ast W_3 \ast C_{ps} \ast T_{s\text{sat}} - W_1 \ast C_{ps} \ast T_b
\] (19)

\[
\frac{d((V_{dw} - A_{dw} \ast L_{dw}) \ast \rho_e)}{dt} = x_e \ast W_3 - W_{so}
\] (20)

Subcooled water in IA_T region of UTSG is a mixture of both recycled water from the previous zone and the main feed water. By applying the energy balance:

\[
\frac{d(M_d \ast C_{ps} \ast T_5)}{dt} = W_1 \ast C_{ps} \ast T_0 - W_1 \ast C_{ps} \ast T_5
\] (21)

We can rewrite Equation (21) as:

\[
\frac{dT_5}{dt} = \frac{W_1}{M_d} \ast T_0 - \frac{W_1}{M_d} \ast T_5
\] (22)

From Equations (18) and (19):

\[
\frac{dT_b}{dt} = \left(\frac{W_{fi} \ast (x_{e1} - 1) \ast W_1 \ast T_{bi}}{L_{dw} \ast A_{dw} \ast \rho_f}\right) T_b + \frac{W_{fi} \ast (1-x_e) \ast W_3 \ast T_{s\text{sat}}}{L_{dw} \ast A_{dw} \ast \rho_f} \ast T_{s\text{sat}}
\] (23)

From Equation (20):

\[
W_5 = \frac{1}{x_e} \left[\left(V_{dw} - A_{dw} \ast L_{dw}\right) \frac{d\rho_e}{dP} + \frac{\rho_e}{\rho_f} \ast A_{dw} \ast L_{dw} \ast \frac{d\rho_f}{dP} \right] \left[1 + \frac{(1-x_e) \ast \rho_e}{\rho_f} \right]
\] (24)

From Equations (17) and (24):

\[
W_2 = \left\{ \frac{V_r \ast \left(\frac{dV_{fr}}{dP} + x_e \ast \frac{dV_{fr}}{dP}\right)}{(v_{fr} + x_e \ast v_{fr})^2} \right\} \frac{dP_{sat}}{dt} - \left\{ \frac{V_r \ast v_{fr}}{(v_{fr} + x_e \ast v_{fr})^2} \right\} \frac{dx_e}{dt}
\] (25)
From Equation (18):

\[
\frac{dL_{dw}}{dt} = \left( -\frac{1}{\rho_f} \frac{d\rho_f}{dP} + \frac{dP_{sat}}{dP} \right) L_{dw} + \frac{1}{\rho_f + A_{dw}} \left( W_f + (1 - x_f) * W_g - W_i \right) \quad (26)
\]

From Equations (10), (12), (24), (25), and (26), the state equations of both \( P_{sat} \) and \( x_c \) can be written as:

\[
\frac{dx_c}{dt} = \frac{V_s * G_5 + \left( 1 / x_c * G_6 - \frac{V_s * \left( \frac{dV_f}{dP} + x_c * \frac{dV_{fg}}{dP} \right)}{\left( v_f + x_c * v_{fg} \right)} \right) * U_{sat} * A_{ms}}{G_1} * (T_3 + T_4)
\]

\[
\frac{dP_{sat}}{dt} = \left( \frac{V_s * L_{sh} * \frac{v_{fg}}{2}}{L * \left( v_f + x_c * v_{fg} \right)^2} \right) * U_{sat} * A_{ms} * (T_3 + T_4)
\]

\[
\frac{dP_{sat}}{dt} = \left( \frac{V_s * L_{sh} * \frac{v_{fg}}{2}}{2 * L * M_d * G_1} + G_2 \right) * V_s * \rho_s * \left( C_{ps} * L_{sh} \right) * W_i
\]

\[
+ \left( 2 * L * M_d * G_1 \right) * T_5 + T_6
\]

where

\[
G_1 = V_s * \left( \frac{L_{sh} * \frac{d\rho_f}{dP}}{2 * L} - \frac{\frac{dV_f}{dP} + x_c * \frac{dV_{fg}}{dP}}{\left( v_f + x_c * v_{fg} \right)^2} \right)
\]

\[
+ \left( \frac{1}{x_c} * G_6 - \frac{V_s * \left( \frac{dV_f}{dP} + x_c * \frac{dV_{fg}}{dP} \right)}{\left( v_f + x_c * v_{fg} \right)^2} \right) * V_s
\]

\[
* \rho_s * \left( \frac{L_{sh} * h_{fg}}{2 * L} \right) - h_s * \left( \frac{L_{sh} * \frac{v_{fg}}{2}}{v_f + x_c * v_{fg}^2} \right)
\]
\[-h_e \left( \frac{V_r \cdot v_{fg}}{(v_f + x_e \cdot v_{fg})} \right) + \left( \frac{V_r \cdot \frac{L_{sb}}{L} \cdot \frac{v_{fg}}{2}}{(v_f + x_e \cdot v_{fg})^2} \right) + \left( \frac{V_r \cdot v_{fg}}{(v_f + x_e \cdot v_{fg})^2} \right) \]

\[
V_e \cdot (\rho_e \cdot G_3 + h_e \cdot G_4) + h_e \cdot \left( \frac{1}{x_e} \cdot G_6 - \frac{V_r \cdot v_{fg}}{(v_f + x_e \cdot v_{fg})^2} \right)
\]

\[
G_2 = \left[ \frac{h_e \cdot \rho_e \cdot \frac{W_{fi} - W_i}{\rho_f} - 2 \cdot U_{mi} \cdot A_{mi} \cdot \left( \frac{L_{sb}}{2 \cdot L} + \frac{L_{sb}}{L} \right) \cdot T_{sat}}{1 - \frac{x_e}{\rho_f}} \right]
\]

\[
G_3 = -G_1
\]

\[
G_4 = \left( \frac{W_i}{C_{pi}} + V_i \cdot \rho_s \cdot \frac{C_{pi} \cdot L_{sb}}{2 \cdot L} \cdot \frac{W_i}{M_f} - U_{mi} \cdot A_{mi} \cdot \frac{L_{sb}}{L} \right)
\]

\[
G_5 = \frac{C_{pi} \cdot L_{sb}}{2 \cdot L} \cdot \frac{dT_{sat}}{dP} + \frac{L_{sb}}{L} \cdot \left( \frac{dL_f}{dP} + \frac{x_e}{2} \cdot \frac{dL_{fg}}{dP} \right)
\]

\[
G_6 = \left( \frac{(V_{if} - A_{dif} \cdot L_{dif}) \cdot \frac{d\rho_s}{dP} + \rho_s \cdot A_{dif} \cdot \frac{d\rho_f}{dP}}{1 - \frac{x_e}{\rho_f}} \right)
\]

\[
G_7 = V_c \cdot G_5 + \frac{1}{x_e} \cdot G_6 - \frac{V_r \cdot v_{fg}}{(v_f + x_e \cdot v_{fg})^2}
\]

\[
G_8 = -G_1
\]
2.4. Accurate Formulae of Steam/Water Properties

From Equations (15) to (36), we need to calculate the terms: \( \rho_v, \rho_L, h_v, h_L \) and \( T_{sat} \), which are functions of \( P_{sat} \). Also we need their derivatives: \( \frac{d\rho_v}{dP_{sat}}, \frac{d\rho_L}{dP_{sat}}, \frac{dh_v}{dP_{sat}} \), \( \frac{dh_L}{dP_{sat}} \) and \( \frac{dT_{sat}}{dP_{sat}} \).

We used MATLAB Curve Fitting Toolbox to provide an accurate functions or equations of steam/water properties taken from [16]. Also, the derivatives of functions are calculated easily. The MATLAB Curve Fitting Toolbox uses Linear and Nonlinear Regression to fit the data. It introduces extensive fitting capabilities. These functions are constant and can be used easily by itself or by their derivatives or integrals. These accurate functions introduced more accuracy for calculating of the thermodynamic properties of steam/water. Also, these functions give very fast computation, because steam properties are often called millions of times within the run of the model.

The formulae which are fitted are:

1) Vapor Saturated Density in terms of \( P_{sat}, \rho_v(P_{sat}) \) as shown in Figure 3 and Table 4.

2) Liquid Saturated Density in terms of \( P_{sat}, \rho_L(P_{sat}) \) as shown in Figure 4 and Table 5.

3) Vapor Saturated Enthalpy in terms of \( P_{sat}, h_v(P_{sat}) \) as shown in Figure 5 and Table 6.

\[
V \cdot (\rho_v * G_i + h_v * G_i) + h_v \cdot \left( \frac{1}{x_v} \cdot G_i - \frac{V}{x_v} * \left( G_i * \left( \frac{V}{P} + x_v * \frac{dV}{dP} \right) \right) \right) - h_v * G_v
\]

\[
G_i * x_v * \left( 1 + \frac{1-x_v}{x_v} * \frac{P_{sat}}{P_v} \right)
\]

\[ W_{sat} (36) \]
Table 4. (a) General form of fitted curve of saturated vapor density in terms of $P_{sat}$ (b) Coefficients of the equation with confidence Interval. (c) Goodness of fit.

| Goodness of fit | Coefficients (with 95% confidence bounds) |
|-----------------|------------------------------------------|
| SSE: 1.81e−07   | $a = 14.41 (14.4, 14.42)$                |
| R-square: 1     | $b = -0.9291 (-0.9293, -0.9289)$        |
| Adjusted R-square: 1 | $c = -0.005371 (-0.005414, -0.005329)$ |
| RMSE: 6.206e−05 |                                          |

$$\rho_v = 14.41 \times (P_{sat})^{-0.9291} - 0.005371$$ (37)

Figure 4. Fitting of saturated liquid density in terms of $P_{sat}$

Table 5. (a) General form of fitted curve of Saturated Liquid Density in terms of $P_{sat}$ (b) Coefficients of the equation with confidence Interval. (c) Goodness of fit.

| Goodness of fit | Coefficients (with 95% confidence bounds) |
|-----------------|------------------------------------------|
| SSE: 1.621e−10  | $p_1 = -5.342e−16 (-6.484e−16, -4.2e−16)$ |
| R-square: 0.9997| $p_2 = 1.383e−12 (1.148e−12, 1.617e−12)$ |
| Adjusted R-square: 0.9997 | $p_3 = -1.276e−09 (-1.437e−09, -1.116e−09)$ |
| RMSE: 1.898e−06 | $p_4 = 8.258e−07 (7.85e−07, 8.665e−07)$ |
|                 | $p_5 = 0.001053 (0.00105, 0.001056)$      |

$$\rho_L = -5.342e−16 \times (P_{sat})^{-1.383e−12} + 1.383e−12 \times (P_{sat})^{-1.276e−09} - 8.258e−07 \times (P_{sat})^2 - 8.258e−07 \times P_{sat} + 0.001053$$ (38)

4) Liquid Saturated Enthalpy in terms of $P_{sat} h_v(P_{sat})$ as shown in Figure 6 and Table 7.

5) Saturated Temperature in terms of $P_{sat} T_{sat}(P_{sat})$ as shown in Figure 7 and Table 8.

It is clear that the results of the statistical data in Tables 4-8 show good compatibility according to four important statistical indicators; the first one is the (SSE), which represents the sum of squares due to the error and is considered a measure of the total deviation, the second one is R-Square, which is a measure of the variance of the data, the third is the Adjusted R-Square, which is closer to
Figure 5. Fitting of saturated vapor enthalpy in terms of $P_{sat}$.

Table 6. (a) General form of fitted curve of saturated vapor enthalpy in terms of $P_{sat}$ (b) Coefficients of the equation with confidence interval. (c) Goodness of fit.

| Goodness of fit | Coefficients (with 95% confidence bounds) |
|-----------------|-------------------------------------------|
| SSE: 9.775      | $p_1 = 1.141e^{-22} (8.235e^{-23}, 1.458e^{-22})$ |
| R-square: 0.9997| $p_2 = -5.663e^{-19} (-7.121e^{-19}, -4.205e^{-19})$ |
| Adjusted R-square: 0.9996| $p_3 = 1.199e^{-15} (9.165e^{-16}, 1.481e^{-15})$ |
| RMSE: 0.4943   | $p_4 = -1.413e^{-12} (-1.712e^{-12}, -1.114e^{-12})$ |
|                | $p_5 = 1.016e^{-09} (8.273e^{-10}, 1.205e^{-09})$ |
|                | $p_6 = -4.604e^{-07} (-5.327e^{-07}, -3.881e^{-07})$ |
|                | $p_7 = 0.0001316 (0.0001152, 0.0001481)$ |
|                | $p_8 = -0.02346 (-0.02553, -0.02139)$ |
|                | $p_9 = 2.531 (2.407, 2.655)$ |
|                | $p_{10} = 2665 (2663, 2668)$ |
| $h_v = 1.141e^{-22} \cdot (P_{sat})^2 - 5.663e^{-19} \cdot (P_{sat})^3 + 1.199e^{-15} \cdot (P_{sat})^4 - 1.413e^{-12} \cdot (P_{sat})^5$ |
| $+ 1.016e^{-09} \cdot (P_{sat})^6 - 4.604e^{-07} \cdot (P_{sat})^7 + 0.0001316 \cdot (P_{sat})^8 - 0.02346 \cdot (P_{sat})^9$ |
| $+ 2.531 \cdot P_{sat}^2 + 2665$ |

Figure 6. Fitting of saturated liquid enthalpy in terms of $P_{sat}$.
Table 7. (a) General form of fitted curve of Saturated Liquid Enthalpy in terms of \( P_{sat} \) (b) Coefficients of the equation with confidence Interval. (c) Goodness of fit.

| Goodness of fit | Coefficients (with 95% confidence bounds) |
|-----------------|------------------------------------------|
| SSE: 515.8      | \( a = 146.5 \) (135.2, 157.7)           |
| R-square: 0.9998| \( b = 0.3114 \) (0.3027, 0.32)          |
| Adjusted R-square: 0.9998| \( c = 143.5 \) (120.3, 166.7)          |
| RMSE: 3.313     |                                          |

\[ h_s = 146.5 \times \left( P_{sat} \right)^{0.3114} + 143.5 \] (40)

Figure 7. Fitting of saturated temperature in terms of \( P_{sat} \).

Table 8. (a) General form of fitted curve of Saturated Temperature in terms of \( P_{sat} \) (b) Coefficients of the equation with confidence Interval. (c) Goodness of fit.

| Goodness of fit | Coefficients (with 95% confidence bounds) |
|-----------------|------------------------------------------|
| SSE: 11.73      | \( a = 84.17 \) (81.84, 86.51)           |
| R-square: 0.9999| \( b = 0.2052 \) (0.2025, 0.208)         |
| Adjusted R-square: 0.9999| \( c = -37.1 \) (−40.27, −33.93)       |
| RMSE: 0.4997    |                                          |

\[ T_{sat} = 84.17 \times \left( P_{sat} \right)^{0.2052} - 37.1 \] (41)

one, which indicates better fit to the data, and finally, the RMSE, which expresses the standard deviation.

3. State Variables of UTSG

The above Equations ((4), (5), (8), (9), (22), (23), (26)-(28)) define the following main UTSG variables:

- \( T_1 \): temperature of the upward direction primary coolant in inverted U-Tube.
- \( T_2 \): temperature of the downward direction primary coolant in inverted U-Tube.
- \( T_3 \): temperature of the metal tube containing upward direction primary w-
ter.

- $T_i$: temperature of the metal tube containing downward direction primary water.
- $T_r$: recirculated water temperature before mixing.
- $T_a$: annular tank temperature.
- $L_{dw}$: water level of recirculated water plus feedwater (mixing region).
- $x$: steam quality at the exit.
- $P_{sat}$: secondary coolant saturation pressure.

Now we can arrange these equations in matrix form to be suitable for MATLAB as the following:

$$\frac{dX}{dt} = A \times X + B$$  \hspace{1cm} (42)

and state vector is:

$$X = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ L_{dw} \\ x \\ P_{sat} \end{bmatrix}$$  \hspace{1cm} (43)

and state matrix is:

$$A = \begin{bmatrix} A_{1,1} & 0 & A_{1,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{2,1} & A_{2,2} & 0 & A_{2,4} & 0 & 0 & 0 & 0 & 0 \\ A_{3,1} & 0 & A_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{4,2} & 0 & A_{4,4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{5,5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{6,6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{7,7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{8,8} & A_{8,4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{9,9} \end{bmatrix}$$  \hspace{1cm} (44)

and input matrix is:

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \end{bmatrix}$$  \hspace{1cm} (45)
To make the model works independently, the model must be maintained in a closed-loop situation. This is done by adding the following two control conditions:

1) The flow rate of the exit steam is equal to the flow rate of the feedwater ($W_{so} = W_{fi}$).

2) The flow rate of the exit steam is proportional to the saturated pressure of the steam ($W_{so}/P_{sat} = \text{constant}$).

Now, by solving Equation (42), and taking into consideration the two control conditions, the UTSG model is presented. It was totally implemented in MATLAB environment. In the next section, some results of running the model in both steady state and transient state will be presented.

4. Results

Simulations are carried out by running the MATLAB model for 3 hours (1800 sec). Simulations are programmed on a personal computer (Intel Core™ i5-11600T Processor (12M Cache, up to 4.10 GHz)). The first step is using the initial design data from Table 2 to simulate the steady-state condition. Figures 8-11 show the steady state simulation of the thermodynamic properties $T_1$, $T_2$, $T_3$, $T_4$, $P_{sat}$, and $x$, respectively.

Simulation of the transient performance of the UTSG is also done. As shown in Figure 12, the transient case increases the steam flow by 5% starting from initial time $t_i = 500$ seconds till final time $t_f = 1500$ seconds. Figure 13 through Figure 16 show the transient state simulation of the response of thermodynamic properties $T_1$, $T_2$, $T_3$, $T_4$, $P_{sat}$, and $x$ respectively.

5. Conclusion

An accurate mathematical modeling of the U-Tube steam generator, which is introduced in this work, offers a good start to assist a complete simulation for
Figure 9. The Tm-up (Tm-dn) inside the U-tube in the steady state.

Figure 10. The Psat, in steady state.

Figure 11. The Xe, in steady state.
Figure 12. The steam flow rate.

Figure 13. The Tp-up (Tp-dn) inside the U-tube in Steam Flow Increase 5%.

Figure 14. The Tm-up (Tm-dn) inside the U-tube in Steam Flow Increase 5%.
the pressurized water reactor. The main advantage of using such highly precision expressions for deducing the thermodynamic relationships is the reduction in the simulation time as these functions provide a very fast calculation and control the safety usage of the PWR. This is because the properties of the vapor are called millions of times within the framework of the model while running and used the interpolation technique with the steam tables. The evidence of the high precision is shown through the statistical quantities computed in the fitting process with R-square being less than 1 in all cases, and can be easily used on their own or through their derivatives or integrals. These computed functions increase the precision for the calculation of the thermodynamic properties of steam/water. Moreover, this model was tested with both static persistence and 5% transient changes and gave typical and reasonable results that reflect the dynamic characteristics of the steam generator in real time. It shows a good adaptability in a wide range of industrial control.

**Figure 15.** The Psat, in steam flow increase 5%.

**Figure 16.** The Xe, in steam flow increase 5%.
Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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