Coupled Responses in a Partially Liquid-Filled Container with the Multi-Elastic Baffles Subjected to Pitching Excitation

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Abstract. The coupled responses in the partially liquid filled container with multi-elastic annular baffles are investigated by the semi-analytical method subjected to the pitching excitations. The trial solutions of the dynamic deflections of the multi-elastic baffles, the surface wave height and the velocity potential can be obtained by introducing the time-dependent generalized coordinates. The Stokes-Joukowski potentials, which are relevant to the kinematical boundary condition of the container, can be obtained analytically. Based on the coupled vibration equations of the multi-elastic baffles and the boundary conditions on the free surface, the coupled dynamic response equations can be established. Parameter studies are carried out to investigate the effects of the parameters of the multi-elastic baffles on the coupled responses of the system.

1. Introduction
Liquid sloshing in the moving container is of great importance to the design of the liquid storage containers which are often encountered in many engineering applications [1, 2]. Anti-sloshing devices (such as baffles and internal structures) installed have been devised as effective components to reduce the liquid sloshing in the container [3].

In the literature, the interaction of the anti-sloshing devices and the liquid sloshing in the container has been investigated in many different ways. Akyildiz [4] studied the liquid sloshing in a rectangular tank with one vertical baffle at the bottom of the container using the finite difference method. Cho et al. [5] investigated the sloshing reduction effect of the horizontal porous baffle in the 2D tank. Chu et al. [6] investigated the liquid sloshing in a rectangular water tank with multiple bottom-mounted baffles by the experimental and numerical method which is based on the Large Eddy Simulation (LES) model. Sanapala et al. [7] carried out the shake table experiments to investigate the interactions between the liquid sloshing and the vibration of the internal structure. Konopka et al. [8] developed the active sloshing device to reduce the surface wave height in the cylindrical container. Kim et al. [9] conducted a series of experiments to observe the sloshing reduction effect of the moving baffles. Ye et al. [10] proposed the semi-analytical scheme to study the liquid sloshing in cylindrical container with the coaxial dual circular or arc-shaped porous structures. Yu et al. [11] carried out the model tests of suppressing sloshing fitted with two perforated floating plates. Zhang et al. [12] conducted a series of experiments to investigate the hydrodynamics of the anti-sloshing technique using floating foams in the rectangular liquid container.

In this paper, the semi-analytical scheme is developed to investigate the coupled responses for the partially liquid filled container equipped with the multiple elastic annular baffles under the pitching excitations. The effects of the inner radius and the thickness of the elastic baffles on the coupled response are discussed.
2. Governing Equations

The partially filled cylindrical container with multi elastic baffles is illustrated in Figure 1 and Figure 2. As shown in Figure 1 and Figure 2, O-XYZ is the inertial frame, both o-xyz and o-ritz denote the Cartesian and cylindrical coordinate systems fixed to the moving container, which is set as rigid body. The baffles are made of linear elastic material and the liquid is assumed as ideal liquid. The notation \( \Sigma_0 \) is introduced to denote the mean free surface of the liquid, \( \Sigma_0 \) denotes the mean wetted surface of the container and the multi baffles. The surface wave height relative to \( \Sigma_0 \) is expressed by the equation \( z = \eta(r, \theta, t) \). The container is undergoing the pitching excitation \( \Theta_2(t) \), and the pitching axis is the \( y \)-axis of the moving coordinate system o-xyz. Under these assumptions, the velocity potential \( \Phi \) satisfies the following equations:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \text{ in } \Omega, \tag{1}
\]

\[
\frac{\partial \Phi}{\partial r} - z \dot{\Theta}_2 \cos \theta \text{ on the container wall}, \tag{2}
\]

\[
\frac{\partial \Phi}{\partial z} = -r \dot{\Theta}_2 \cos \theta \text{ on the container bottom}, \tag{3}
\]

\[
\frac{\partial \Phi}{\partial z} = -r \dot{\Theta}_2 \cos \theta + \frac{\partial \eta}{\partial t} \text{ on the free surface}, \tag{4}
\]

\[
\frac{\partial \Phi}{\partial t} = g r \dot{\Theta}_2 \cos \theta - \eta g \text{ on the free surface}, \tag{5}
\]

\[
\int_{\Sigma_0} \eta d\rho d\theta = 0, \tag{6}
\]

in which, \( \Omega \) denotes the mean liquid domain in the baffled container, \( g \) denotes the gravitational acceleration.

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**Figure 1.** Cylindrical container with multi-elastic baffles under pitching excitations.

**Figure 2.** Sub-domains and interfaces.

As shown in Figure 2, the multi elastic baffles are located at \( z_1, z_2, \ldots, z_M \) (\( M \) denotes the number of the baffles). As illustrated in Figure 3, the liquid domain is partitioned into \( 2M+2 \) sub-domains with
2M+1 artificial interfaces. The notations \( \Phi_i \) are introduced as the velocity potential corresponding to the sub-domains \( \Omega_i \) \((i=1,2,\ldots,2M+2)\). The notations \( w_j \) \((j=1,2,\ldots,M)\) are introduced as the dynamic deflections of the elastic baffles. On the baffles, the dynamic deflections satisfy the coupled kinematic and dynamic equations:

\[
\frac{\partial \Phi_i}{\partial z} \bigg|_{z=z_j} = -r_\Omega \cos \theta + \frac{\partial w_j}{\partial t} \quad (j=1,2,\ldots,M),
\]

\[
D_j \nabla^4 w_j + \rho_j \tau_j \frac{\partial^2 w_j}{\partial t^2} = \rho_j \frac{\partial \Phi_{2j+1}}{\partial t} \bigg|_{z=z_j} - \rho_j \frac{\partial \Phi_{2j+1}}{\partial t} \bigg|_{z=z_j} \quad (D_j = \frac{E_j r_j^3}{12(1-\nu_j^2)}).
\]

in which, \( \rho_j, \tau_j, E_j \) and \( \nu_j \) denote the density, the thickness, the elasticity modulus and the Poisson ratio of the elastic baffle located at \( z=z_j \), respectively. \( \rho \) denotes the density of the liquid. The notations \( h \) and \( R_1 \) are introduced as the height of liquid and the inner radius of the baffles, respectively. \( R_2 \) denotes the container inner radius and the outer radius of the baffles. The dimensionless parameters:

\[
\xi = r/R_2, \quad \zeta_1 = R_1/R_2, \quad \zeta_2 = z/R_2, \quad \zeta_3 = h/R_2 \text{ are introduced.}
\]

Introducing the generalized coordinates \( q_{mn}(t), q_{mn}'(t) \) and \( p_{mn}(t) \), the free surface height \( \eta \), the dynamic deflections \( w_j \) and the velocity potential \( \Phi \) are expanded as the series of the coupled modes. One can obtain

\[
\eta = \sum q_{mn}(t)f_{mn}(\xi)\cos m\theta,
\]

\[
w_j = \sum q_{mn}'(t)W_{mn}(\xi)\cos m\theta,
\]

\[
\Phi = \tilde{\phi}_2 R_2^3 (\zeta_3 - \psi)\cos \theta + \sum p_{mn}(t)\phi_{mn}(\xi, \zeta)\cos m\theta.
\]

in which, \( W_{mn} \) denote the coupled modes of the elastic baffles, \( \phi_{mn} \) denote the coupled modes of the liquid and \( f_{mn}(\xi) = \phi_{mn}(\xi, \zeta_0) \). The coupled modes \( W_{mn} \) and \( \phi_{mn} \) can be defined by the coupled eigenvalue problem [13]. According to the reference [13], the semi-analytical method is proposed to solve the coupled eigenvalue problem in the partially filled container with multi elastic baffles. The function \( \psi \) satisfies the following equations:

\[
\frac{\partial^2 \psi}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial \psi}{\partial \xi} - \psi + \frac{\partial^2 \psi}{\partial \zeta^2} = 0 \text{ in } \Omega,
\]

\[
\frac{\partial \psi}{\partial \xi} = 0 \text{ on the wall of the container},
\]

\[
\frac{\partial \psi}{\partial \zeta} = 2\zeta \text{ on the bottom of the container, the free surface and the baffles.}
\]

where the function \( \psi \) can be obtained by the semi-analytical scheme, which was developed by the authors [3]. The notation \( \phi_{mn}^i \) is introduced as the coupled mode corresponding to the sub-domains \( \Omega_i \).

It is known from the literature [13] that \( W_{mn}^j \) and \( \phi_{mn}^i \) satisfy the following equations:

\[
\frac{\partial \phi_{mn}^{j+1}}{\partial \xi} \bigg|_{z=z_j} = R_j W_{mn}^j, \quad \frac{\partial \phi_{mn}}{\partial \xi} \bigg|_{z=z_j} = R_j W_{mn}^j,
\]

\[
D_j \nabla^4 \left(W_{mn}^j \cos m\theta\right) = \rho \Lambda_m^2 k_j^2 \sum \rho_j W_{mn}^j + \rho_j \left(\phi_{mn}^{j+1} - \phi_{mn}^j\right) \bigg|_{z=z_j} \cos m\theta.
\]
in which, $\Lambda_{n-m}^2$ denote the dimensionless coupled frequencies. Substituting equations (9)-(11) into the kinematic conditions (4) and (7) on the free surface and the elastic baffles gives the following equations

$$\sum p_{mn}(t)\Lambda_{mn}^2\phi_{mn} = \sum \dot{q}_{mn}(t)\phi_{mn} \text{ on the free surface}, \quad (17)$$

$$\sum p_{mn}(t)g\Lambda_{mn}^2R_z^2\left[\tau_j W_{mn}\rho_j \rho_j^{-1} + \left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right)\right] = \sum \dot{q}_{mn}(t)g\Lambda_{mn}^2R_z^2\left[\tau_j W_{mn}\rho_j \rho_j^{-1} + \left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right)\right] \text{ on the baffles.} \quad (18)$$

According to the orthogonality condition of the coupled modes proved in the literature [13], one can obtain the following relations among the generalized coordinates

$$\sum q_{mn}(t)\Lambda_{mn}^2 = R_z\dot{q}_{mn}(t)\Lambda_{mn}^2, \quad \sum \dot{q}_{mn}(t)\Lambda_{mn}^2 = R_zq_{mn}(t)\Lambda_{mn}^2. \quad (19)$$

3. Dynamic Response Equation

Substituting equations (10)-(11) into the dynamics conditions (8) on the multi elastic baffles gives

$$\sum \ddot{q}_{mn}(t)\Lambda_{mn}^2gR_z^2\left[\rho_j \tilde{T}_j W_{mn} + \rho_l \left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right)\right] \cos\theta = \sum p_{mn}(t)g\Lambda_{mn}^2R_z^2\left[\tau_j W_{mn}\rho_j \rho_j^{-1} + \left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right)\right] \cos\theta. \quad (20)$$

According to the relations (19) among the generalized coordinates, equation (20) can be rewritten as

$$\sum q_{mn}(t)g\left[\rho_j \tilde{T}_j W_{mn} + \rho_l \left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right)\right] \cos\theta + \sum \dot{q}_{mn}(t)g\Lambda_{mn}^2R_z^2\left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right) \cos\theta = \sum p_{mn}(t)g\Lambda_{mn}^2R_z^2\left(\phi_{mn}^{2j-1} - \phi_{mn}^{2j+1}\right) \cos\theta. \quad (21)$$

Substituting equations (9) and (11) into the dynamics condition (5) on the free surface gives

$$\sum R_z\ddot{q}_{mn}(t)\Lambda_{mn}^2f_{mn}(\xi)\cos\theta + g\sum q_{mn}(t)\Lambda_{mn}^2f_{mn}(\xi)\cos\theta - g\Theta_2\xi + \bar{\Theta}_2\left(\psi_{\theta_0} - \zeta_0\xi\right)R_z\cos\theta = 0 \text{ on the free surface}, \quad (22)$$

Because of the orthogonality of the trigonometric functions, equations (21) and (22) can be rewritten as the following expressions:

$$\sum \ddot{q}_{lm}(t)\Lambda_{lm}^2\tilde{T}_{lm} + \sum \dot{q}_{lm}(t)\Lambda_{lm}^2f_{lm}(\xi) + gR_z^2\sum q_{lm}(t)\left[\tau_{lm} W_{lm}\rho_l \rho_l^{-1} + \left(\phi_{lm}^{2l-1} - \phi_{lm}^{2l+1}\right)\right] = 0. \quad (23)$$

$$\sum \ddot{q}_{lm}(t)\Lambda_{lm}^2f_{lm}(\xi) + gR_z^2\sum q_{lm}(t)\left[\tau_{lm} W_{lm}\rho_l \rho_l^{-1} + \left(\phi_{lm}^{2l-1} - \phi_{lm}^{2l+1}\right)\right] = 0. \quad (24)$$

Because of the orthogonality of the coupled modes proved in the literature [13], the dynamics equation for the generalized coordinate $q_{lm}(t)$ can be found:

$$M_{ln}\ddot{q}_{lm}(t) + q_{lm}(t)K_{ln} = F(t), \quad (25)$$

in which, the mass coefficient $M_{ln}$ and the stiffness coefficient $K_{ln}$ are
\[ M_{in} = \Lambda_{in}^2 \int_{f_{in}} f_{in} \xi \, d\xi + \Lambda_{in}^2 \sum_{j=1}^{M} \left[ \tau W / \rho_j \rho_l^1 + \left( \phi_{in}^{j-1} - \phi_{in}^j \right) \right] W_{in} \xi \, d\xi, \quad (26) \]

\[ K_{in} = \Lambda_{in}^2 M_{in} g R_z^1. \quad (27) \]

The right-hand sides of the equation (25) is prescribe as

\[ F(t) = g\Theta_2 \int_0^1 f_{in} \xi \, d\xi + \tilde{\Theta}_2 \int_0^1 f_{in} \psi \xi \, d\xi - \tilde{\Theta}_2 \int_0^1 f_{in} \xi \, d\xi \right] R_z + \]

\[ R_z \tilde{\Theta}_2 \sum_{j=1}^{M} \left[ \psi_{j-1} - \psi_{j+1} \right] W_{in} \xi \, d\xi. \quad (28) \]

The hydrodynamic pressure due to liquid movement can be determined from the following equation:

\[ P(\xi, \theta, \zeta) = -\rho \frac{\partial \Phi}{\partial t} = -\rho R_z \cos \theta \left[ \tilde{\Theta}_2 \left( \xi \xi - \psi \right) R_z + \sum \tilde{q}_{in} (t) \phi_{in} \Lambda_{in}^2 \right]. \quad (29) \]

The resultant force acting on the container wall can be obtained by integrating the pressure over the wall. The resultant forces along the \( \theta=0 \) direction is given by:

\[ F_x = -\rho R_z^2 \pi \sum \tilde{q}_{in} (t) \Lambda_{in}^2 \Phi_{in} \int_0^\infty \xi \, d\xi + \rho R_z^2 \tilde{\Theta}_2 \left[ \int_0^\infty \psi \xi \, d\xi - \frac{\xi_0^2}{2} \right]. \quad (30) \]

The hydrodynamic moments due to liquid pressure acting on the wall about the \( \theta=\pi/2 \) axe is given by:

\[ M_{wall} = \rho R_z^2 \pi \sum \tilde{q}_{in} (t) \Lambda_{in}^2 \Phi_{in} \int_0^\infty \xi \, d\xi - \rho R_z^2 \tilde{\Theta}_2 \left[ \int_0^\infty \psi \xi \, d\xi - \frac{\xi_0^2}{3} \right]. \quad (31) \]

The hydrodynamic moments due to liquid pressure acting on the bottom about the \( \theta=\pi/2 \) axe is given by:

\[ M_{bottom} = \rho R_z^2 \pi \sum \tilde{q}_{in} (t) \Lambda_{in}^2 \Phi_{in} \int_{-\infty}^0 \xi \, d\xi - \rho R_z^2 \tilde{\Theta}_2 \left[ \int_0^\infty \psi \xi \, d\xi - \frac{\xi_0^2}{3} \right]. \quad (32) \]

The hydrodynamic moments due to liquid pressure acting on the multi-elastic baffles about the \( \theta=\pi/2 \) axe is given by:

\[ M_{baffle} = \rho R_z^2 \pi \sum \tilde{q}_{in} (t) \Lambda_{in}^2 \Phi_{in} \int_{-\infty}^0 \xi \, d\xi - R_z \tilde{\Theta}_2 \left[ \int_0^\infty \psi_{j+1} - \psi_{j-1} \right] \xi \xi \, d\xi]. \quad (33) \]

According to equations (31)-(33), the resultant moment acting on the container can written as

\[ M_y = M_{wall} + M_{bottom} + M_{baffle}. \quad (34) \]

4. Numerical Examples

The numerical examples about the coupled responses in a partially liquid-filled cylindrical tank with the four elastic baffles under pitching excitations are investigated in this section. The following parameters are used in all numerical examples such as the inner radius of the cylindrical container \( R_z=1m \), the liquid height in the container \( h=1m \) and the density of the liquid \( \rho_l=1000kg/m^3 \). These four baffles are positioned are located at \( z_1=0.2m, z_2=0.4m, z_3=0.6m \) and \( z_4=0.8m \), respectively. All the baffles have the same parameters. The density of the baffles is \( \rho_j=2780 \text{ kg/m}^3 \). The elasticity modulus and the Poisson ratio are taken as \( E_j=7.2\text{e}10 \text{ Pa} \) and \( \nu_j=0.3 \), respectively. The angular displacement of the pitching excitation follows a sinusoidal function \( \Theta_2 = \Theta_0 \sin \omega t \), where the amplitude \( \Theta_0 \) is fixed.
at 0.01 rad. The frequency of the excitation is taken as $\omega_0 = 4.14 \text{rad/s}$, which is close to the first natural frequency of the liquid sloshing in the container with no baffle.

First of all, the effect of inner radius of the multi-elastic baffles on the dynamic response was studied. Three different inner radii of the baffles ($R_1 = 0.3 \text{m}$, $0.5 \text{m}$ and $0.7 \text{m}$) were considered when the thickness of the baffles is 2 mm. Time history curves of the coupled responses corresponding to different inner radii of the baffles are presented in Figure 3. As can be seen from those figures, the magnitudes of the coupled responses are increased with the increased inner radius of the baffles. It indicates that the smaller the inner radius of the baffles is, the more significant the suppressing effect of liquid sloshing is. Moreover, when the inner radius of the baffles is 0.8 m, its inherent frequency is close to the excitation frequency. Therefore, the amplitude of the coupled response is great.
Secondly, the effect of flexibility of the multi-elastic baffles on the coupled responses was studied. The inner radius of the baffles was 0.5m. It should be noted that the flexibility of the baffles mainly depends upon the thickness of the baffles. And the smaller the thickness of the baffles is, the greater the flexibility is. Time history curves of the coupled response corresponding to three different thicknesses of the baffles (τ_j=1mm, 2mm and 3mm) are presented in Figure 4. As can be seen from those figures, the coupled responses are firstly decreased and then increased with the increase in the thickness of the baffles. Moreover, the variation of the magnitude of the coupled responses with the thickness of the baffles is given in Table 1, so as to analyse the effect of the flexibility of the baffles in details.

Table 1. The variation of the magnitude of the coupled responses versus the thickness of the baffles

| τ_j (mm) | 0.5  | 1.0  | 1.5  | 2.0  | 2.5  | 5.5  | 6.0  |
|----------|------|------|------|------|------|------|------|
| f_max (m) | 0.110| 0.059| 0.029| 0.035| 0.040| 0.042| 0.043|
| F_max (N) | 1131 | 1011 | 351.3| 243.6| 209.8| 212.2| 219.9|
| M_max (N•m) | 1294 | 631.6| 452.5| 397.8| 454.3| 496.0| 498.8|
5. Conclusion
The coupled dynamic response of partially liquid-filled cylindrical tank equipped with multi-elastic baffles subjected to the pitching excitation has been analytically investigated. The following observations are drawn:

1. The smaller the inner radius of the baffles is, the more significant the suppressing effect of liquid sloshing is.
2. The coupled responses are firstly decreased and then increased with the increase in the thickness of the baffles.

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