The transverse mass spectra of $J/\psi$ and $\psi'$ mesons and $\Omega$ hyperons produced in central Au + Au collisions at RHIC energies are discussed within a statistical model used successfully for the interpretation of the SPS results. The comparison of the presented model with the future RHIC data should serve as a further crucial test of the hypothesis of statistical production of charmonia at hadronization. Finally, in case of validity, the approach should allow to estimate the mean transverse flow velocity at the quark–gluon plasma hadronization.

The concepts of chemical (hadron multiplicities) and kinetic (hadron momentum spectra) freeze-outs were introduced to interpret data on hadron production in relativistic nucleus–nucleus ($A + A$) collisions. The equilibrium hadron gas (HG) model describes remarkably well the light hadron multiplicities measured in $A + A$ collisions at the SPS [1] and RHIC [2] energies, where the creation of a quark–gluon plasma (QGP) is expected. Recently it was found [3] that also charmonium yield systematics in nuclear collisions at the SPS [4] follow the pattern predicted by the hadron gas model. The hadronization temperature parameter extracted from the fits to the hadron multiplicities is similar for both energies: $T_H = 170 \pm 10$ MeV. It is close to an estimate of the temperature $T_C$ for the QGP-HG transition at zero baryonic density.

Experimental results on inclusive hadron spectra and correlations show evidence for a hydrodynamic expansion of the matter created in heavy ion collisions. Strong transverse flow effects in Pb + Pb collisions at the SPS (average collective transverse velocity is approximately 0.5–0.6) are firmly established from the combined analysis [5,6] of the pion transverse mass spectra and correlations. The kinetic (‘thermal’) freeze-out of pions and nucleons seems to occur at a rather late stage of an $A + A$ reaction. The thermal freeze-out temperature parameter of pions measured by the NA49 Collaboration [7] for central Pb + Pb collisions at the SPS is $T_T \approx 120$ MeV.

Further exploration of idea of the statistical $J/\psi$ production [3] led to the formulation of the hypothesis that the kinetic freeze-out of $J/\psi$ and $\psi'$ mesons takes place directly at hadronization [8]. This means that chemical and thermal freeze-outs occur simultaneously for those mesons and they, therefore,
carry information on the flow velocity of strongly interacting matter just after the transition to the HG phase. A possible influence of the effect of rescattering in the hadronic phase on the transverse momentum \((p_T)\) spectra was recently studied within a “hydro + cascade” approach [9,10]. \(A + A\) collisions are considered there to split into three stages: hydrodynamic QGP expansion (“hydro”), transition from QGP to HG (“switching”) and the stage of hadronic rescattering and resonance decays (“cascade”). The switching from hydro to cascade takes place at \(T = T_C\), where the spectrum of hadrons leaving the surface of the QGP-HG transition is taken as an input for the subsequent cascade calculations. The results [9,10] suggest that the \(p_T\) spectrum of \(\Omega\)s is only weakly affected during the cascade stage. The corresponding calculation for charmonia are not yet performed within this model, but a similar result may be expected due to their very high masses and low interaction cross sections.

In previous work [11] devoted to the analysis of \(\Omega\) + \(\rho\) + \(\omega\) and \(J/\psi\) + \(\psi'\) mesons [13] produced in \(Pb + Pb\) at 158 \(\text{A GeV}\) collisions can be reproduced within a hydrodynamical approach using the same freeze-out parameters: hadronization temperature \(T \approx 170\ \text{MeV}\) and the mean transverse flow velocity \(\bar{v}_T \approx 0.2\). Within our approach the value of the \(\bar{v}_T\) parameter extracted in this way should be interpreted as the mean flow velocity of the hadronizing QGP.

In the present Letter we discuss the transverse mass spectra of \(J/\psi\) and \(\psi'\) mesons and \(\Omega\) hyperons in central \(Au + Au\) collisions at RHIC energies within the statistical approach to charmonium production successfully used for the interpretation of the SPS data [3,11].

Assuming kinetic freeze-out of matter at constant temperature \(T\), the transverse mass spectrum in clyndrically symmetric and longitudinally boost invariant fluid expansion can be approximated as [14]:

\[
\frac{dN_i}{m_T dm_T} \propto m_T \int_0^R \frac{y_T}{\bar{y}_T} I_0 \left( \frac{p_T \sinh y_T}{T} \right) e^{-\frac{m_T}{T^*}}.
\]

where \(y_T = \tanh^{-1} \bar{v}_T\) is the transverse fluid rapidity, \(R\) is the transverse system size, \(K_1\) and \(I_0\) are the modified Bessel functions. The spectrum (1) is obtained under the assumption that the freeze-out occurs at constant longitudinal proper time \(\tau = \sqrt{t^2 - z^2}\) (\(\tau\) is the time and \(z\) is the longitudinal coordinate), i.e., the freeze-out time and \(\bar{v}_T\) is independent of the transverse coordinate \(r\). The analysis of the numerical calculations done within a hydrodynamical model [10] shows that the latter is approximately fulfilled. The quality of the approximation made gets better for heavy particles considered here because a possible deviation from Eq. (1) decreases with increasing particle mass at constant \(p_T\). In order to calculate (1) the function \(y_T(r)\) has to be given. A linear flow profile, \(y_T(r) = y_T^{\text{max}} r / R\), is often assumed in phenomenological fits [14] and we used it as well. Recent numerical hydrodynamical calculations [10] show that deviations from this simplified profile are smaller than 10% for both SPS and RHIC energies. Their importance for our results is discussed later in the Letter.

In Figs. 1–3 the measured \(m_T\)-spectra of \(\Omega^+\) [12], \(J/\psi\) and \(\psi'\) [13] produced in \(Pb + Pb\) collisions at 158 \(\text{A GeV}\) are shown together with the fit performed using Eq. (1). The model parameters are: \(T = 170\ \text{MeV}\), \(y_T^{\text{max}} = 0.28\) [11]. The value of \(y_T^{\text{max}}\) found from fitting the SPS data appears to be close to the number estimate done within a hydrodynamical approach \(y_T^{\text{fit}} \approx 0.30\) [10]. This encourages us to use a corresponding estimate of \(y_T^{\text{fit}} \approx 0.55\) made for \(Au + Au\) collisions at \(\sqrt{s_{NN}} = 130\ \text{GeV}\) [10] for the first guess of the \(m_T\)-spectra of charmonia at RHIC energies within our approach. As follows from the analysis [2] of hadron yields within the HG model, in our calculations we use a hadronization temperature \(T = 170\ \text{MeV}\), the same as in the case of the SPS data. The resulting \(m_T\)-spectra for \(\Omega\), \(J/\psi\) and \(\psi'\) are shown in Figs. 1, 2 and 3, respectively. The normalization of the SPS and RHIC spectra is arbitrary. Due to large transverse flow velocity of hadronizing QGP at RHIC energies, the particle spectra are expected to significantly deviate from the simple exponential form of the \(m_T\)-distribution:

\[
\frac{dN}{m_T dm_T} = C m_T^{1/2} e^{-\frac{m_T}{T^*}}.
\]

where \(C\) and \(T^*\) are normalization and inverse slope parameters, respectively.
Fig. 1. The transverse mass spectra (in arbitrary units) of $\Omega^-$ hyperons produced in Pb + Pb (Au + Au) collisions. The points indicate experimental results measured for Pb + Pb collisions at 158 A GeV. The dashed line shows the fit of the hydrodynamical model to these data ($T = 170$ MeV and $y_{\text{max}}^T = 0.28$). The prediction of our approach for central Au + Au collisions at $\sqrt{s}_{NN} = 130$ GeV is indicated by the solid line ($T = 170$ MeV and $y_{\text{max}}^T = 0.55$). The long dashed and dashed-dotted lines correspond to the exponential spectrum fitted in the intervals $(m_T - m) \in [0, 0.6]$ GeV and $(m_T - m) \in [0.6, 1.6]$ GeV, respectively.

In order to quantify the deviations of the RHIC spectra from Eq. (2) we plotted in Fig. 4 the local inverse slope defined as:

$$T^* \equiv -\left[\frac{d}{dm_T} \ln \left(m_T^{-1/2} \frac{dN}{dm_T} \right)\right]^{-1},$$

versus $m_T - m$ for $J/\psi$, $\psi'$ and $\Omega$. The $T^*$ parameter was calculated using Eq. (1) with the values of parameters used previously for the SPS and RHIC energies. No significant dependence of $T^*$ is observed in the SPS case $y_{\text{max}}^T = 0.28$, whereas a very strong decrease of $T^*$ with increasing $m_T$ in the region $m_T - m < 1$ GeV is seen for $J/\psi$ and $\psi'$ at RHIC ($y_{\text{max}}^T = 0.55$) and the moderate one is expected for $\Omega$ hyperons. Thus the shape of the $m_T$-spectrum is
Fig. 4. The dependence of the inverse slope parameter $T^\ast$ of $J/\psi$ (solid line) and $\psi'$ (long dashed line) mesons and $\Omega$ hyperons (short dashed line) on $m_T - m$ expected within the model of statistical production at QGP hadronization. The left panel indicates the behaviour given by the fit of the hydrodynamical ansatz to the data on Pb + Pb collisions at 158 A GeV, whereas the right panel shows the predictions for central Au + Au collisions at $\sqrt{s_{NN}} = 130$ GeV.

strongly dependent on the magnitude of the transverse flow and the mass of the particle. These characteristic “hydrodynamical” features of the spectrum should allow for a clear test of our approach in the near future.

To be more specific, we follow the suggestion of Ref. [10] and consider two domains $0 < m_T - m < 0.6$ GeV and $0.6 < m_T - m < 1.6$ GeV of $m_T$-spectra. Within these $m_T$ domains the $\Omega$, $J/\psi$ and $\psi'$ spectra at RHIC calculated according to Eq. (1) were approximated by Eq. (2) and the values of $C$ and $T^\ast$ were found by the maximum likelihood method. The inverse slopes in low $m_T$ domain (long dashed lines in Figs. 1–3) are larger than the high $m_T$ ones (dashed-dotted lines in Figs. 1–3). However, the difference between low and high $m_T$ inverse slopes strongly depends on the particle mass—for $\Omega$ the inverse slopes differ by 13 per cent only, whereas for $\psi'$ by about 40 per cent.

It is important to check the sensitivity of the above results to the assumed flow profiles (density and velocity) of hadronic matter at freeze-out. We first note that because of the basic assumption of our model ($\Omega$, $J/\psi$ and $\psi'$ freeze-out at QGP hadronization) the freeze-out density at the boundary between the mixed and hadronic phases is determined by the hadronization temperature $T = 170$ MeV and hence the density profiles of these particles are constant as the function of $r$. Thus the uncertainty is due to the velocity profile only. The numerical hydrodynamical calculations [10] indicate that the transverse rapidity profile may deviate from the linear dependence assumed above by less than 10%. In order to estimate a possible influence of these deviations we calculated the spectra of $\Omega$, $J/\psi$ and $\psi'$ using a profile of the form $y_T(r) = y_T^{\text{max}}(r/R)^\alpha$ for $\alpha = 0.9$ and 1.1. In the calculations we keep the average transverse velocity as for $y_T^{\text{max}} = 0.55$ and $\alpha = 1$ case. It turns out that the shape of the spectra is almost insensitive to the assumed value of $\alpha$ parameter in the whole range of transverse mass for $\Omega$ and for $m_T - m > 0.6$ GeV for $J/\psi$ and $\psi'$ mesons. In the low $m_T$ domain ($m_T - m < 0.6$ GeV) the fitted inverse slope parameters for $J/\psi$ and $\psi'$ mesons vary by 3 per cent and 5 per cent, respectively, under considered variations of the $\alpha$ parameter. Therefore we conclude—the principal results of our analyt-
Theoretical model are weakly affected by the approximations made.

The transverse flow velocity of hadronizing matter depends on the energy and centrality of the collision. Thus, in general, we consider the parameter $y_T^{\text{max}}$ as a free parameter, who’s value should be extracted from the fit of Eq. (1) to the measured spectrum. Providing the fit is successful, the fitted $y_T^{\text{max}}$ may be treated as an estimate of the transverse flow velocity of hadronizing matter.

The $m_T$-spectra in Au + Au collisions at RHIC were also calculated within a hydrodynamical ansatz in Ref. [15]. There is however a principal difference between our approach [11] and the one used in Ref. [15]. We consider the early kinetic freeze-out for $\Omega$, $J/\psi$ and $\psi'$ only, whereas in [15] the simultaneous (chemical and thermal) freeze-out was postulated for all hadrons. This generates a substantial difference in the model predictions for the RHIC freeze-out parameters.

The results on charmonium production in central Au + Au collisions at RHIC energies are expected to be available very soon. They will, obviously, allow to test various approaches [16–20] developed in the course of the analysis of the corresponding SPS data. In particular, a statistical approach to charmonium production in nuclear collisions yields well defined and characteristic predictions for charmion multiplicities [3] and the transverse mass spectra discussed in this Letter. The analysis of the $m_T$-spectra of $J/\psi$ and $\psi'$ mesons and $\Omega$ hyperons within our approach should yield a unique information on the transverse flow velocity of hadronizing QGP in $A + A$ collisions at RHIC energies.

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