Abstract. We show that a very light scalar field experiencing quantum fluctuations during primordial inflation can explain the current cosmic acceleration. Provided its mass does not exceed the Hubble parameter today, this field has been frozen during the cosmological ages to start dominating the universe only recently. Assuming this scenario to be correct, and using supernovae data, the model predicts the energy scale of primordial inflation to be around a few TeV and suggests that it has lasted for an extremely long period. Dark energy could therefore be a natural consequence of cosmic inflation close to the electroweak energy scale.

1. Introduction

The success of Cosmic Inflation certainly relies in its ability of providing an unified paradigm for some unaddressed questions of the standard Big-Bang model of Friedmann and Lemaître [1, 2, 3, 4]. Among those questions are the apparent fine-tuning of some cosmological parameters. For instance, the density parameter of spatial curvature is currently bounded to $|\Omega_k| < 2\%$ [5] whereas the spectral index $n_S$ of a primordial power law power spectrum is close to unity, but not equal to unity [5] ($0.015 < n_S - 1 < 0.071$ at two-sigma). Inflation provides a simple answer to those two apparently unrelated problems while addressing many others such as the adiabaticity and Gaussianity of the initial conditions. In its simpler form, the inflationary paradigm can be implemented with a self-gravitating scalar field in a potential dominated regime. At the background level, such a system triggers an accelerated expansion of the universe solving the flatness problem whereas its quantum fluctuations predicts an almost scale invariant power spectrum of the cosmological perturbations [6].

Another unaddressed question of the $\Lambda$CDM model concerns the present value of the cosmological constant $\Omega_\Lambda \simeq 0.723 \pm 0.016$, which is required to explain the current acceleration of the universe [7, 8, 9]. According to the interpretation given to the cosmological constant, such a measurement can be either viewed as a severe fine-tuning problem or a failure in our understanding of the gravitational properties of the vacuum state [10].

In this context, one may wonder whether Cosmic Inflation could also provide some insight to the current value of the cosmological constant, or more straightforwardly, to the current acceleration of the universe.
Most alternative models of Dark Energy assume that, from still unknown symmetry reasons, the bare cosmological constant in the Einstein equation vanishes and that the current cosmic acceleration is sourced by a new dynamical degree of freedom. As a result, these models genuinely introduce extra-parameters that may need to be fine-tuned to ensure that their energy density dominates the universe only recently. This is the so-called coincidence problem. Quintessence models belong to this class as they introduce a new scalar field. However, if the potential is of the Ratra–Peebles kind, they solve the coincidence problem by exhibiting a tracking behaviour which washes out memory of the initial conditions [11, 12, 13, 14]. But still, the normalization of the potential has to be tuned to the observed cosmological constant. According to the classification of Ref. [15], these quintessence models belong to the “freezing” class as opposed to the “thawing” class. The later models are exactly as Cosmic Inflation, a field slowly rolls down a potential thereby triggering acceleration. Since the initial conditions are not erased by a tracking mechanism in the thawing models, they appear as additional and unwanted model parameters. But this is without considering the effect of Cosmic Inflation.

Inflationary effects in the freezing quintessence models have been first explored in Ref. [16] to determine how inflation affects the initial conditions of the field. As shown in Ref. [17], they have an unfortunate tendency to push the field away from the region where the tracker behavior can efficiently wash out the initial conditions. As a result, freezing quintessence on the tracker today requires that inflation lasted a low number of e-folds.

As we have shown in Ref. [18], the situation is different for the thawing models. Cosmic Inflation actually gives natural initial conditions for a thawing quintessence field which address simultaneously the coincidence problem and the apparent small value of the cosmological constant today. For this reason, thawing models within inflation have less fine-tuning than any other Dark Energy scenario and are actually predictive: the energy scale of Cosmic Inflation should be at a few TeV.

2. Storing quantum fluctuations from inflation

As an illustrative example, let us consider a thawing model which is simply a massive scalar field during inflation

$$\varphi(\mathbf{x}, t) = \phi(t) + \delta \phi(\mathbf{x}, t), \quad (1)$$

where $\phi$ is the homogeneous mode and $\delta \phi$ are its perturbations. Furthermore, assuming that inflation is almost de-Sitter with an Hubble parameter given by $H_{\text{inf}}$, it is straightforward to solve the Klein–Gordon equation [2, 3, 19]. The homogeneous part evolves as (in Planck units)

$$\phi(N) = \phi_{\text{ini}} \exp\left(-\frac{m^2}{3H_{\text{inf}}^2} N\right), \quad (2)$$

where $N \equiv \ln(a/a_{\text{ini}})$ is the number of e-folds and $a$ the scale factor. As a result, if inflation lasts long enough, the homogeneous mode reaches the minimum of its potential, as one may expect. Starting from the Bunch–Davis vacuum on sub-Hubble scales, the linear fluctuations of the field in Fourier space $\mu \equiv a \delta \phi(k, t)$ have also an exact solution as

$$\mu(\eta, k) = e^{i(\nu+1/2)\pi/2} \sqrt{\frac{\pi}{4k}} \sqrt{k \eta} H^{(1)}(k \eta), \quad \nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H_{\text{inf}}^2}}, \quad (3)$$

where $\eta$ is the conformal time. From this expression, the primordial power spectrum of the field fluctuations after Hubble exist reads

$$P_{\delta \phi} = \lim_{k \eta \ll 1} \frac{k^2}{8\pi^2} \left| \frac{\mu}{a} \right|^2 \simeq \frac{H_{\text{inf}}^2}{4\pi^2} \left( \frac{k}{aH_{\text{inf}}} \right)^{2m^2/(3H_{\text{inf}}^2)}. \quad (4)$$
This expression shows that if the field is very light during inflation, \( m \ll H_{\text{inf}} \), its Fourier modes around the Hubble scale develop a scale invariant power spectrum of amplitude \([20]\)

\[
\mathcal{P}_{\delta \phi}(k) = \frac{H_{\text{inf}}^2}{4\pi^2} + \cdots .
\] (5)

Up to a numerical factor accounting for the polarization states, this spectrum is of comparable amplitude with the one associated with the tensor modes during inflation. As a result, if inflation occurs at low energy scales, the field perturbations that re-enters the Hubble radius during the radiation and matter era are of extremely low amplitude thereby preventing the field to grow sub-Hubble inhomogeneities \([21, 22, 23]\). As the result, only the homogeneous part can have observable effects. As we have just discussed, its classical value vanishes but not its quantum generated variance. Indeed, going back to real space and summing only on all super-Hubble modes gives, after \( N \) e-folds of inflation

\[
\langle \delta \phi^2 \rangle = \int_{a_{\text{ini}}H_{\text{inf}}}^{aH_{\text{inf}}} \frac{d^3k}{(2\pi)^3} |\delta \phi_k|^2 = \frac{3H_{\text{inf}}^4}{8\pi^2 m^2 \left[ 1 - \exp\left( -\frac{2m^2}{3H_{\text{inf}}^2}N \right) \right]} \rightarrow \frac{3H_{\text{inf}}^4}{8\pi^2 m^2} .
\] (6)

The last limit is applicable if inflation lasts long enough, by precisely the same amount necessary to cancel the classical homogeneous mode. Let us notice that this field variance can only be realized as an homogeneous mode for an observer since we have restricted the summation on super-Hubble scales. Strictly speaking, the brackets refer to a quantum average in the de-Sitter vacuum state and, as such, the above equation is of quantum probabilistic nature. This result can be recovered and interpreted from the stochastic inflation formalism \([24, 25, 26]\). It shows that the field reaches an equilibrium distribution on super-Hubble scales resulting from the competition between quantum kicks that tend to push it out of the potential minimum and the classical force which drives the field towards the minimum.

For our purpose, we notice that the potential energy of such a field develops an universal value, i.e. it depends only on the Hubble scale during inflation

\[
\langle V(\phi) \rangle = \frac{1}{2} m^2 \langle \delta \phi^2 \rangle = \frac{3H_{\text{inf}}^4}{16\pi^2} .
\] (7)

In order to explain the current cosmic acceleration, it is necessary that the field remains frozen after inflation. This is satisfied as long as \( m < H_0 \), the Hubble parameter today. In other words, the field should be extremely light and as such, inflation should be extremely long. In Ref. \([18]\), a lower limit for the total number of e-folds is estimated to be \( N > 10^9 \) whereas a natural number would be \( N \simeq 10^{60} \). Another observable prediction of this model is therefore that the spatial curvature today should be infinitely small.

Finally, we want the field potential today to be of the same order than the energy density necessary to trigger the current acceleration. This is trivially satisfied provided

\[
H_{\text{inf}} \simeq (\Omega_\Lambda)^{1/4} \sqrt{4\pi H_0 M_P} ,
\] (8)

where we have restored the reduced Planck mass \( M_P \) for clarity. This expression makes clear that, in our model, the apparent fine-tuning of the cosmological constant value is recast into the question of knowing the energy scale of inflation, i.e. a prediction. Plugging numbers into this expression gives

\[
H_{\text{inf}} \simeq 6 \times 10^{-3} \text{eV} \quad \Rightarrow \quad E_{\text{inf}} \equiv \rho_{\text{inf}}^{1/4} = (3M_P^2 H_{\text{inf}}^2)^{1/4} \simeq 5 \text{TeV} .
\] (9)

In the context of inflation, the small value of \( \rho_{\Lambda}^{1/4} \) has a natural origin. It is typical of the energy density associated with de-Sitter quantum fluctuations at TeV scale.
3. Conclusion
In Ref. [18], we have consistently taken into account the probabilistic nature associated with the theoretical predictions of our model by using Bayes statistics, and compared them with the current supernovae, BAO and CMB data [5, 9, 27]. The result is consistent with the order of magnitude considerations presented above. More quantitatively, the energy scale of inflation lies in the 95% confidence interval

$$3.8 \text{ TeV} < E_{\text{inf}} < 12.1 \text{ TeV},$$

(10)

whereas $m < 75 \text{ km/s/Mpc}$ (also at 95%).

The mechanism described above actually works with any spectator field, not necessarily scalar, experiencing quantum fluctuations during inflation provided its potential is bounded from below. Indeed, in some extension of electromagnetism, gauge fields have also been shown to be able to store inflationary quantum fluctuations [28, 29]. The fact that the TeV scale seems to be universal directly stems from Eq. (8), and thus from Eq. (7). As already mentioned, this is the energy scale at which de-Sitter quantum fluctuations match the cosmological constant energy scale.

For instance, if instead of having considered a massive field, we would have assumed the potential to be $V(\phi)$, the stochastic inflation formalism gives the stationary probability distribution [26]

$$P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H_{\text{inf}}^4} V(\phi)\right].$$

(11)

As opposed to the massive case, the field distribution is no longer Gaussian, but the potential value remains universal $\langle V(\phi) \rangle \simeq 3H_{\text{inf}}^4/(8\pi^2)$ and our results still hold.

Finally, let us notice that because this model is a late-time inflationary era, triggered by the primordial one, the current acceleration of the universe will come to an end. Would our universe be filled with various uncoupled sectors having completely different mass hierarchies, one could even imagine a succession of inflationary eras, temporarily interrupted with decelerating eras, and re-triggered by quantum fluctuations stored in the lightest fields. In a more pragmatic way, if the mass of the field is of comparable value with $H_0$ (as opposed as being much smaller), one could expect a disambiguation possible with respect to a pure cosmological constant through the redshift evolution of equation of state parameter [30].

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