One-loop renormalization factors and mixing coefficients of bilinear quark operators for improved gluon and quark actions

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We calculate one-loop renormalization factors and mixing coefficients of bilinear quark operators for a class of gluon actions with six-link loops and O(a)-improved quark action. The calculation is carried out by evaluating on-shell Green’s functions of quarks and gluons in the standard perturbation theory. We find a general trend that finite parts of one-loop coefficients are reduced approximately by a factor two for the renormalization-group improved gluon actions compared with the case of the standard plaquette gluon action.

1. Introduction

In numerical studies of field theories through computer simulations finite lattice spacing effects poses an obstacle in extracting physical predictions for observable quantities. In an effort to reduce this problem, a recent full QCD simulation by the CP-PACS Collaboration[1] employs a combination of the O(a)-improved clover quark action[2] and a renormalization-group-improved gluon action with 6-link loops[3], based on previous results of a comparative study of various action combinations[4]. With this development it has become necessary to calculate renormalization factors and mixing coefficients of quark operators for improved gluon and quark actions. We have carried out such a calculation to one-loop order of perturbation theory for bilinear quark operators[5,6]. In this article we summarize the results.

Our calculations are performed within standard perturbation theory for on-shell Green’s functions of quarks and gluons. For the plaquette gluon action, perturbative calculation of the mixing coefficients of bilinear quark operators has been previously carried out using the Schrodinger functional technique[7,8]. A recent work with the same technique[9] extends the results to the same set of improved gluon actions as examined by us. We find a good agreement between results by us and those reported in these references.

2. Choice of actions

We adopt the clover action for quark, which is given by adding the clover term

\[ S_C = -c_{SW}a^3 \sum_n \sum_{\mu,\nu} i g \frac{\tau}{4} \bar{\psi}(n) \sigma_{\mu\nu} P_{\mu\nu}(n) \psi(n) \]

to the ordinary Wilson fermion action. The gluon action we consider is defined by

\[ S_{gluon} = \frac{1}{g^2} \text{Tr} \left\{ c_0 U_{pl} + c_1 U_{rtg} + c_2 U_{chr} + c_3 U_{plg} \right\} , \]

where the first term represents the standard plaquette term, and the remaining terms are six-link loops formed by a 1 x 2 rectangle, a bent 1 x 2 rectangle (chair) and a 3-dimensional parallelogram. The coefficients \( c_0, \cdots, c_3 \) satisfy \( c_0 + 8c_1 + 16c_2 + 8c_3 = 1 \). At the one-loop level, the choice of the gluon action is specified by the pair of numbers \( c_1 \) and \( c_23 = c_2 + c_3 \). We adopt the following five cases in our calculation: (i) the standard plaquette action \( c_1 = 0 \), \( c_23 = 0 \), (ii) the tree-level improved action in the Symanzik approach \( c_1 = -1/12 \), \( c_23 = 0 \)[10], and (iii) three choices suggested by approximate renormalization-group analysis, \( c_1 = -0.331 \), \( c_23 = 0 \) and \( c_1 = -0.27 \), \( c_23 = -0.04 \) by Iwasaki[11], and \( c_1 = -0.252 \), \( c_23 = -0.17 \) by Wilson[11].
Table 1
Finite part $z_\Gamma$ of renormalization factor for bilinear operator of form $(1 + ma)\overline{\psi} \Gamma \psi$ (i.e., $z = 0$ in (1)) with $m$ the subtracted quark mass incorporating one-loop self-energy correction. Coefficients of the term $c_{SW}^n(n = 0, 1, 2)$ in an expansion $z_\Gamma = z_\Gamma^{(0)} + c_{SW}^{(1)} + c_{SW}^{(2)}$ are given in the column labeled $(n)$. Errors are at most in the last digit given.

| gauge action | c_1 | c_{23} | V | A | S | P |
|-------------|-----|--------|---|---|---|---|
| (0) (1) (2) | (0) (1) (2) | (0) (1) (2) | (0) (1) (2) | (0) (1) (2) | (0) (1) (2) |
| 0 | 0 | -20.618 4.745 0.543 | -15.797 -0.248 2.251 | -12.953 -7.738 1.380 | -22.596 2.249 -2.036 |
| -1/12 | 0 | -16.603 4.228 0.464 | -12.540 -0.198 2.021 | -9.607 -6.836 1.367 | -17.734 2.015 -1.745 |
| -0.331 | 0 | -11.099 3.326 0.336 | -8.192 -0.125 1.610 | -4.858 -5.301 1.266 | -10.673 1.601 -1.281 |
| -0.27 | -0.04 | -11.540 3.418 0.353 | -8.523 -0.131 1.657 | -5.260 -5.545 1.292 | -11.292 1.644 -1.316 |
| -0.252 | -0.17 | -10.525 3.248 0.338 | -7.707 -0.117 1.587 | -4.366 -5.166 1.287 | -10.001 1.565 -1.212 |

3. Bilinear operators and improvement

We consider bilinear quark operators of form,

$$O^\Gamma = \overline{\psi} \Gamma \psi, \quad \Gamma = 1, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu \nu}. \quad (1)$$

At the tree-level, there exists a one-parameter family of $O(a)$-improved operators given by

$$O_0^\Gamma = \left[ 1 + ar(1 - z)m_0 \right] \overline{\psi} \Gamma \psi + z\overline{\psi} \Gamma_0^\psi - z^2 \overline{\psi} \Gamma_1^\psi, \quad (2)$$

where $\Gamma_0$ and $\Gamma_1^\psi$ are $O(a)$ and $O(a^2)$ vertices, and $m_0$ is the bare quark mass.

At the one-loop level, on-shell Green’s functions of these operators do not have terms of $O(g^2a)$ when we set the clover term coefficient $c_{SW} = 1$ [22]. The $O(g^2a)$ terms still remain, however. In order to extract the renormalized operators without $O(g^2a)$ errors we need to calculate mixing coefficients $B_\Gamma$ and $C_\Gamma$ as well as the renormalization factor $Z_\Gamma$ defined by

$$O_0^\Gamma = Z_\Gamma^{-1}O^\Gamma_R - g^2C_{\Gamma a}m_R B_\Gamma O^\Gamma_R - g^2C_{\Gamma a} C_{\Gamma} \tilde{O}_R^\Gamma, \quad (3)$$

where $C_{\Gamma a}$ denotes the second-order Casimir eigenvalue for the quark field, and the last two terms are needed to remove $O(g^2a)$ errors from on-shell matrix elements, with $\tilde{O}_R^\Gamma$ a dimension 4 operator with derivative, e.g., $\tilde{O}^A = \partial_\mu O^\mu$ and $\tilde{O}^V = \partial_\mu O^\Gamma$.

4. Strategy of one-loop calculation

We apply standard perturbation theory to the Green’s functions of the operator $O_0^\Gamma$ and two on-shell external quarks to calculate renormalization factors and mixing coefficients. In order to isolate terms of $O(g^2a)$, one-loop amplitudes are expanded in powers of $a$, or equivalently in the external momenta $p$ and quark mass $m$. Infrared divergences that generally appear in on-shell amplitudes are regularized by a gluon mass $\lambda$.

For this procedure to work, infrared divergences that contribute to the mixing coefficients $B_\Gamma$ and $C_\Gamma$ have to cancel out among diagrams. We have explicitly checked that this is in fact the case; infrared divergences appear only in the ordinary renormalization factor $Z_\Gamma$, which, however, cancel against those in the renormalization factor for the continuum operator. As a result, the renormalization factor in the $\overline{MS}$ scheme takes the form,

$$Z_\Gamma^{-1} = 1 + \frac{g^2C_F}{16\pi^2} \left( \left( \frac{h_2(\Gamma)}{4} - 1 \right) \log(ma)^2 + z_\Gamma \right),$$

where $h_2(\Gamma)$ is an integer given by $h_2(\Gamma) = 4(A), 4(V), 16(P), 16(S), 0(T)$.

We note that the bare quark mass $m_0$ in [3] may be replaced by a subtracted mass $m = m_0 - g^2C_F m_0/a$ incorporating the one-loop self-energy correction by a redefinition of $z_\Gamma$, $B_\Gamma$ and $C_\Gamma$. We adopt this definition for numerical results presented below.

4.1. Results

The calculational procedure described above yields $z_\Gamma$, $B_\Gamma$ and $C_\Gamma$ as sums of one-loop integral constants. While straightforward in principle, the actual algebra to evaluate the integrands
Table 2
Mixing coefficients for axial vector, vector, pseudo scalar and scalar density at $z = 0$.

| $c_1$ | $c_{23}$ | $C_A$     | $C_V$     | $B_A$     | $B_V$     | $B_P$     | $B_S$     |
|-------|----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0     | 0        | $-0.005680(2)$ | $-0.01226(3)$ | 0.1141(1) | 0.1150(2) | 0.1148(1) | 0.1444(2) |
| $-1/12$ | 0        | $-0.00451(1)$ | $-0.01030(4)$ | 0.0881(1) | 0.0886(2) | 0.0890(1) | 0.1144(2) |
| $-0.331$ | 0        | $-0.00285(1)$ | $-0.00729(4)$ | 0.0547(1) | 0.0550(2) | 0.0561(1) | 0.0747(2) |
| $-0.27$ | $-0.04$  | $-0.00302(1)$ | $-0.00757(4)$ | 0.0572(1) | 0.0575(2) | 0.0586(1) | 0.0777(2) |
| $-0.252$ | $-0.17$  | $-0.00281(1)$ | $-0.00705(4)$ | 0.0512(1) | 0.0514(2) | 0.0527(5) | 0.0706(2) |

is extremely tedious. We employ Mathematica to carry out this task, and also to generate the FORTRAN code from the results. Infrared divergences that can appear in individual integrals are subtracted by working out the leading term of integrands for small loop momenta. Integrals are then evaluated by the Monte Carlo routine VEGAS in double precision. We generally employ 20 sets of $10^5$ points for integration, except for $C_A$ for the plaquette action for which 20 sets of $10^6$ points are used. Errors are estimated from variation of integrated values over the sets.

We present our results for $z\Gamma$, $C\Gamma$ and $B\Gamma$ for the case of $z = 0$ in Tables 1 and 2. Results for $z \neq 0$ can be found in Refs. [5,6]. The Wilson parameter is taken to be $r = 1$. For $z\Gamma$, the coefficients of the expansion $z\Gamma = z\Gamma^{(0)} + c_{SW}z\Gamma^{(1)} + c_{SW}^2z\Gamma^{(2)}$ are given (Table 1), while the constants $C\Gamma$ and $B\Gamma$ are evaluated for $c_{SW} = 1$ (Table 2).

We observe in the results that the renormalization factors and mixing coefficients are reduced by about a factor two for renormalization-group improved gluon actions compared to those for the plaquette action. More generally we find that $z\Gamma$’s monotonically decrease as $c_1$ and $c_{23}$ become large and negative.

Numerical calculations have been carried out at Center for Computational Physics, University of Tsukuba and at Research Institute for Fundamental Physics, Kyoto University. This work is supported in part by the Grants-in-Aid of the Ministry of Education (Nos. 2373, 09304029,10640246). KN and YT are JSPS Research Fellows.

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