Band structure of one-dimensional photonic crystal containing two negative index materials

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Abstract. A layered system composed of two types of metamaterials is examined. We assume the metamaterials are isotropic, homogeneous, dispersive and non-absorptive medias. We confine our attention by the case of identical permittivity and permeability of the metamaterial with the dispersion described by a single Lorentz contribution. For equal layers’ widths and several sets of metamaterials parameters we obtain the photonic crystal band structures of the system. We consider the situation, when permittivities (and, correspondingly, permeabilities) of the metamaterials coincide by absolute values and are opposite in sign. In this situation we observe absence of reflection phenomenon, which holds true with changing the metamaterials parameters.

1. Introduction

Materials, which have a negative refractive index for certain frequencies, are known as negative index materials (NIMs) or metamaterials [1]. New covered surfaces and cloaking materials can be created by using NIMs [2]. The superlens with the resolving power, exceeding the diffraction limit [3], also needs the NIMs. Among systems with NIMs (NIM systems), layered NIM systems are widely known [3-10]. The layered systems are considered also as one-dimensional photonic crystals (1DPCs) [11, 12]. Numerous investigations of 1DPC were carried out in recent years [13, 14]. But most of these ones consider systems without the frequency dispersion. The goal of our work is to describe the photonic band gap (PBG) structure for the periodic layered NIM system (photonic crystal) filled with two types of metamaterials. We assume the metamaterials are the isotropic, homogeneous, dispersive and non-absorptive medias, the permittivity and permeability stand equal and are described by using a single Lorentz contribution [4, 15]. Earlier, we have studied the similar NIM system composed of alternated metamaterial and vacuum layers [16]. We obtained absence of reflection in the case, which we called NIM situation, when the permittivity and permeability of the metamaterial are equal to the opposite their vacuum value, i.e. –1 instead of +1. Now we consider the system with two types of metamaterials, and we need to widen the NIM situation concept. The NIM situation is a case, when permittivities (and permeabilities) of the two types of the metamaterials coincide by absolute values but are opposite in sign. We are interesting in the electric field for this case and it’s dependence on metamaterials parameters.
2. Periodic NIM system

We study the periodic (infinite) NIM system composed of parallel layers (figure 1). Unit vectors \( \mathbf{e}_1 \), \( \mathbf{e}_2 \) of the Cartesian basis \( \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} \) set a plane of layer’s surfaces. Unit vector \( \mathbf{e}_3 \) sets the \( x_3 \) axis. We assume the translation invariance along the plane of layer’s surfaces. There are two types of layers. The first one is \( \Delta_1 \) in width and filled with a metamaterial (M1). The second one is \( \Delta_2 \) in width and filled with another metamaterial (M2). Metamaterials of neighbor layers have different optical properties, which are described below. The layers alternate each other. Value \( \Delta_1 + \Delta_2 \) is a period of the system. Then, the system is a 1DPC.

![Figure 1. The periodic NIM system composed of parallel alternated layers filled with the metamaterials M1 and M2. There is the translation invariance along the plane of layer’s surfaces.](image)

We consider the Maxwell’s equations in a differential form

\[
\frac{d\mathbf{D}}{dt}(x,t) = \nabla \times \mathbf{H}(x,t), \quad \frac{d\mathbf{B}}{dt}(x,t) = -\nabla \times \mathbf{E}(x,t), \quad \nabla \cdot \mathbf{D}(x,t) = 0, \quad \nabla \cdot \mathbf{B}(x,t) = 0,
\]

where \( x = \{x_1, x_2, x_3\} \) is a vector located in the Cartesian basis \( \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\} \), \( \nabla \) is the Hamilton operator, \( \times \) is a cross product symbol as well as a numerical product symbol, \( \cdot \) is an inner product symbol as well as a matrix product symbol. Also, we consider the auxiliary field equations

\[
\mathbf{D}(x,t) = \varepsilon_0 \mathbf{E}(x,t) + \mathbf{P}(x,t), \quad \mathbf{B}(x,t) = \mu_0 \left[ \mathbf{H}(x,t)^\prime + \mathbf{M}(x,t) \right],
\]

\[
\mathbf{P}(x,t) = \varepsilon_0 \int_{s=0}^{t} \chi_e(x,t-s) \cdot \mathbf{E}(x,s) \, ds, \quad \mathbf{M}(x,t) = \int_{s=0}^{t} \chi_m(x,t-s) \cdot \mathbf{H}(x,s) \, ds,
\]

where \( \varepsilon_0 \) and \( \mu_0 \) are the electric and magnetic constants, and \( \varepsilon_0 \mu_0 = 1/c^2 \), where \( c \) is the speed of light in vacuum. \( \chi_e(x,t) \) and \( \chi_m(x,t) \) are the electric and magnetic susceptibility tensors. We use the causality condition \( \chi_e(x,t) = \chi_m(x,t) = 0 \) for \( t < t_0 \) (we assume \( t_0 = 0 \)) and the passivity condition [4].

The considered system composed of layers divided by planes. The boundary conditions for each one are presented in the general form as follows:

\[
(E_j - E_2) \times \mathbf{e}_3 = 0, \quad (H_j - H_2) \times \mathbf{e}_3 = 0, \quad (D_j - D_2) \cdot \mathbf{e}_3 = 0, \quad (B_j - B_2) \cdot \mathbf{e}_3 = 0. \tag{1}
\]

\( E_j = E_j(\bar{x},t), \quad H_j = H_j(\bar{x},t), \quad D_j = D_j(\bar{x},t), \) and \( B_j = B_j(\bar{x},t) \) stand for the limits with \( x \rightarrow \bar{x} \), where \( x = x^+ + x^3 \mathbf{e}_3 \), \( \bar{x} = x^+ + \bar{x}_3 \mathbf{e}_3 \), \( x^+ = \{x_1, x_2, 0\} \) and \( x_3 \rightarrow \bar{x}_3 \), \( j \) is the index distinguishing limits calculated on different layer’s surface sides (\( j = 1, 2 \)). We use the Fourier transform with \( t \) time

\[
\hat{f}(\omega) = \int_{-\infty}^{\infty} \exp[-i\omega t]f(t) \, dt, \quad f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp[i\omega t]\hat{f}(\omega) \, d\omega,
\]

and the Fourier transform with \( x_1 \) and \( x_2 \) coordinates

\[
f_{k_1}(x_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[i(k_1 x_1 + k_2 x_2)]f(x) \, dx_1 \, dx_2 = \int_{\mathbb{R}^2} \exp[i\mathbf{k} \cdot \mathbf{x}]f(x) \, d\mathbf{x},
\]

\[
f_{k_2}(x_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[i(k_1 x_1 + k_2 x_2)]f(x) \, dx_1 \, dx_2 = \int_{\mathbb{R}^2} \exp[i\mathbf{k} \cdot \mathbf{x}]f(x) \, d\mathbf{x}.
\]
and are calculated for the M. We denote $\mathbf{k}_i$, vector, i.e. $\mathbf{k}_i = \kappa \mathbf{e}_i$, where $\kappa$ is the full wave length $\kappa = \sqrt{k_1^2 + k_2^2}$ and the unit vector $\mathbf{e}_i$, i.e. $\mathbf{k}_i = \kappa \mathbf{e}_i$, $\mathbf{k} = \{k_1, k_2, k_3\}$ is the full wave vector.

We consider the isotropic homogeneous metamaterials in layers. Therefore, the electric and magnetic permeabilities do not depend on coordinates of the x vector, i.e. $\varepsilon(x, \omega) = \varepsilon(\omega) \mathbf{U}$, $\mu(x, \omega) = \mu(\omega) \mathbf{U}$, where $\mathbf{U}$ is the 3×3 unit matrix. Also, we assume that the metamaterials are the dispersive non-absorptive medias. Then, the susceptibilities consist of a sum of Lorentz contributions [14]. We deal with a single dispersive Lorentz contribution [4]. We assume that the permittivity and permeability of each metamaterial stand equal and

$$\varepsilon(\omega) = \mu(\omega) = 1 - \frac{\Omega^2}{\omega^2 - \omega_0^2},$$

where $\Omega$ and $\omega_0$ are the plasma and resonance frequencies, respectively. Note that each metamaterial behaves like a NIM if the frequency $\omega$ is inside the interval $(\omega_{0,1}, \omega_{0,2})$ (NIM interval), where

$$\omega_{0,1} = \sqrt{\varepsilon_0^2 + \Omega^2}, \quad \varepsilon(\omega_{0,1}) = \mu(\omega_{0,1}) = 0,$$

and

$$\varepsilon(\omega_{0,2}) = \mu(\omega_{0,2}) = 0.$$ We denote $\omega_{\text{NIM}}$ as the NIM frequency, which leads to the NIM situation, i.e. $\varepsilon(\omega_{\text{NIM}}) = -\mu(\omega_{\text{NIM}})$ and $\mu(\omega_{\text{NIM}}) = -\mu(\omega_{\text{NIM}})$. It is given by the following formula:

$$\omega_{\text{NIM}} = \frac{1}{\sqrt{2}} \left[ \omega_{0,1}^2 + \omega_{0,2}^2 + \Omega^2 \right]^{1/2},$$

where parameters with the 1 and 2 subscripts correspond to the M1 and M2 metamaterials, respectively.

The Fourier transform (2) of the Maxwell’s equations leads to the following Helmholtz equation for the electric field function $\mathbf{E}(x, t)$:

$$\nabla \times \nabla \times \mathbf{E}(x, \omega) = (\omega / c)^2 \varepsilon(\omega) \mu(\omega) \mathbf{E}(x, \omega).$$ (5)

By using the Fourier transforms (2) and (3) for the Helmholtz equation (5), the boundary conditions (1) and the Floquet-Bloch theorem [17, 18] we obtain the following dispersion equation, which is similar to one obtained earlier for the NIM system composed of alternated metamaterial and vacuum layers [16]:

$$\left( e^{-i\theta(\lambda_1, \lambda_2)} \right)^2 - \left[ \frac{\sigma^2_{1,2} \sigma^2_{2,1}}{4} \left( 1 + e^{i2\beta_{1,2}} e^{-i2\beta_{2,1}} \right) + \frac{\sigma^2_{1,2} \sigma^2_{2,1}}{4} \left( e^{i2\beta_{1,2}} + e^{-i2\beta_{2,1}} \right) e^{-i\theta(\lambda_1, \lambda_2)} e^{-i\theta(\lambda_2, \lambda_1)} + 1 = 0, $$ (6)

where $\theta$ is a yet undefined wave vector, called the Bloch wave vector (the analogous vector in quantum mechanics is named quasimomentum), $\sigma^2_{j, p} = \langle \varepsilon_j k_{j}^{(p)} \pm \varepsilon_j k_{j}^{(p)} \rangle / \langle \varepsilon_j k_{j}^{(p)} \rangle$ with indices $j = 1$ and $p = 2$, or $j = 2$ and $p = 1$. Here $\varepsilon_j$ and $k_{j}^{(p)}$ are calculated for the Mj metamaterial. The dispersion equation (6) is the same for TE and TM modes. It follows from the equality of the permittivity and permeability (4). Therefore, we have the identical PBG structure for TE and TM modes.

3. Band structure
To study the PBG structure of the considered 1DPC we use a numerical approach. We consider equality (6) and seek for such values of $\omega$ and $\kappa$ that equality (6) holds true with any value of $\theta$ (it is
sufficient to consider values of $\theta$ belonging to interval $(0, 2\pi/\Delta)$. There are two cases for each metamaterial: if $(k_{\parallel})^2 > 0$, where $(k_{\parallel})^2 = |k|^2 - |k^\perp|^2 = (\omega/c)^2 \varepsilon(\omega)\mu(\omega) - \kappa^2$, then there is a radiative regime; else if $(k_{\parallel})^2 < 0$, then there is an evanescent regime. We fix the constants $\Delta_1 = \Delta_2 = 10$ nm.

The following four cases for the $\omega_{0,1}$, $\Omega_1$, $\omega_{0,2}$, and $\Omega_2$ parameters are considered:

(a) $\omega_{0,1} = 30$ THz, $\omega_{0,2} = 60$ THz, $\Omega_1 = 90$ THz, $\Omega_2 = 90$ THz (see (a) in figure 2);
(b) $\omega_{0,1} = 30$ THz, $\omega_{0,2} = 30$ THz, $\Omega_1 = 90$ THz, $\Omega_2 = 180$ THz (see (b) in figure 2);
(c) $\omega_{0,1} = 30$ THz, $\omega_{0,2} = 60$ THz, $\Omega_1 = 90$ THz, $\Omega_2 = 180$ THz (see (c) in figure 2);
(d) $\omega_{0,1} = 30$ THz, $\omega_{0,2} = 60$ THz, $\Omega_1 = 180$ THz, $\Omega_2 = 90$ THz (see (d) in figure 2).

Figure 2. Dependences of PBG structure on the frequency $\omega$ and values of $\kappa$ for TE and TM modes simultaneously. Permitted bands are grey, forbidden bands are white. The dotted curves divide the space into four parts. The bottom and top parts correspond to the radiative and evanescent regime, respectively, in the both metamaterials. The right part corresponds to the radiative regime in the $M_1$ metamaterial and the evanescent regime in the $M_2$ metamaterial for the (a), (b), and (c) cases, and vice versa for the (d) case. The left part corresponds to the evanescent regime in the $M_1$ metamaterial and the radiative regime in the $M_2$ metamaterial for the (a), (b), and (c) cases, and vice versa for the (d) case. The vertical straight dotted lines show the $\omega_\text{ZERO}^{(1)}$ and $\omega_\text{ZERO}^{(2)}$ frequencies in $M_1$ and $M_2$ metamaterials, respectively, and the $\omega_\text{NIM}$ frequency.
In all cases, we observe permitted bands near the NIM frequency for any value of $\kappa$, which become narrower and converge to the $\omega_{\text{NIM}}$ frequency. The NIM frequency is located between $\omega_{\text{ZERO}}^{(1)}$ and $\omega_{\text{ZERO}}^{(2)}$ frequencies. Then, one of two metamaterials behave like a NIM and the second one behave like a PIM (positive index material). These permitted bands keep on existing with changing the metamaterials parameters $\omega_b$, $\Omega$, and depend only on $\omega_{\text{NIM}}$ frequency. The same effect was observed for the NIM situation in the similar NIM system composed of alternated metamaterial and vacuum layers [16].

4. Electric field

Now we are interesting in solutions of the Helmholtz equation in the NIM situation. We obtained them earlier for the general case of the frequency, when we examined a periodic NIM system composed of two alternated layers filled with metamaterial and vacuum, respectively [16]. In the NIM situation for TE and TM modes they are expressed for the radiative regime as follows:

$$E_i(x_1, \omega_{\text{NIM}}, \kappa) = -4C(\omega_{\text{NIM}}, \kappa) \left( e^{i\omega_{\text{NIM}} \kappa x} - e^{i\omega_{\text{NIM}} \kappa x} e^{i\Phi_{\text{NIM}} \beta_1 x} \right) e^{-i\omega_{\text{NIM}} \kappa x},$$

(7)

$$E_i(x_2, \omega_{\text{NIM}}, \kappa) = -4C(\omega_{\text{NIM}}, \kappa) \left( e^{i\omega_{\text{NIM}} \kappa x} - e^{i\omega_{\text{NIM}} \kappa x} e^{i\Phi_{\text{NIM}} \beta_1 x} \right) e^{-i\omega_{\text{NIM}} \kappa x},$$

(8)

and for the evanescent regime as follows:

$$E_i(x_3, \omega_{\text{NIM}}) = 4C(\omega_{\text{NIM}}, \kappa) \left( e^{i\omega_{\text{NIM}} \kappa x} - e^{i\omega_{\text{NIM}} \kappa x} e^{i\Phi_{\text{NIM}} \beta_1 x} \right) e^{-i\omega_{\text{NIM}} \kappa x},$$

(9)

$$E_i(x_4, \omega_{\text{NIM}}) = 4C(\omega_{\text{NIM}}, \kappa) \left( e^{i\omega_{\text{NIM}} \kappa x} - e^{i\omega_{\text{NIM}} \kappa x} e^{i\Phi_{\text{NIM}} \beta_1 x} \right) e^{-i\omega_{\text{NIM}} \kappa x},$$

(10)

where $C(\omega, \kappa)$ is an unknown function, which can be obtained from initial conditions, and $\rho = \rho(\omega_{\text{NIM}})$ is a value of the following function:

$$\rho(\omega) = \sqrt{|k_3|^2} = \sqrt{|k|^2 - |k_1|^2} = |\omega^2 \left( 1 - \frac{\Omega^2}{\omega^2 - \omega_0^2} \right)^2 - \kappa^2 |^{1/2}.$$ 

The solutions (7) and (8) are correct only if $\omega_{\text{ZERO}}^{(1)} > \omega_{\text{ZERO}}^{(2)}$ ((d) case), i.e. if $\text{M}_1$ and $\text{M}_2$ metamaterials behave like a NIM and PIM, respectively. Else, if $\omega_{\text{ZERO}}^{(1)} < \omega_{\text{ZERO}}^{(2)}$ ((a), (b), and (c) cases), i.e. if $\text{M}_1$ and $\text{M}_2$ metamaterials behave like PIM and NIM, respectively, and, hence, solutions (7) and (8) should be changed by adding the “−” sign to the $\rho$ value. Solutions (9) and (10) are correct for the both conditions.

Note that solution (7), as well as solutions (8)-(10), has only one term responsible for propagation and no terms responsible for reflection. This effect is observed only when the dispersion equation (6) holds true, i.e. only for permitted bands. As we observed in the third section, the frequency $\omega_{\text{NIM}}$ belongs to a permitted band for any value of $\kappa$. Therefore, although the system is layered, there is no reflection at the NIM frequency. Indeed, the refractive indices in neighbor layers coincide with absolute values and are opposite in sign. Then, as we observe, the system is totally translucent at the NIM frequency, and the waves differ only by a sign of the wave phase in the radiative regime (7)-(8), and by the increasing or decreasing of its values in the evanescent regime (9)-(10).

5. Conclusion

In this paper we solved the problem of describing the photonic band gap structure of the periodic layered NIM system (1DPC) with equal layer widths, near the NIM frequency. We assumed that the permittivity and permeability of the metamaterial stand equal and are defined with the single Lorentz contribution. By the numerical approach, we observed that there is a permitted band with the “center” at the NIM frequency, i.e. at the NIM frequency the permitted band is not restricted by “angle of the wave direction”. The effect holds true with changing the metamaterials parameters. Also, we obtained
expressions for the scalar field function in the permitted bands. In the NIM situation they contain only 
one term responsible for propagation and no terms responsible for reflection, i.e. there is no reflection 
at the NIM frequency for any “wave direction”. This property can be useful for metadevice 
construction (see, e.g., [19]).

The photonic band gap structure is similar to obtained earlier in [16], where we examined the 
periodic system, composed of alternated metamaterial and vacuum layers. In [16] the NIM frequency, 
for which we observe a total translucence of the system, is defined by parameters of only the 
metamaterial (plasma and resonance frequencies). In this paper the NIM frequency is defined by 
parameters of the both metamaterials. Thus, from a practical point of view there are more ways to 
produce a NIM system with a certain NIM frequency. Also, the system containing only metamaterial 
layers seems to be more practical than the similar one containing furthermore vacuum layers.

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