Linear period dependencies from free parameters in RR Lyrae models

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Abstract. Detailed analysis of free parameters of the Florida-Budapest turbulent convective stellar model were carried out. A recent explanation of the Blazhko effect ties the observed modulation to convective structure changes inside the star. Before modelling this process, we test relations of growth rates and linear periods to the free parameters. This information is required to model the varying physical characteristics in the code appropriately. Our analysis shows that parameters related to the properties of the turbulent convective zone have the potential to create variations in pulsation periods and amplitudes, however the magnitudes may not be sufficient.

1. Introduction
A new explanation of the still-unresolved Blazhko-phenomenon was recently suggested by Stothers (2006). The model focuses on the cyclic weakening and strengthening of stellar convection caused by turbulent magnetic dynamo, and varying the pulsation properties of the star. Despite the novel approach, no detailed calculations were carried out, whether the mechanism is able to support Blazhko-type amplitude and timescale variations. In his review, Kovács (2009) also points out the lack of elaboration and some inconsistencies of Stothers’ idea.

As the proposed mechanism strongly depends on the magnetic field and the turbulent convection inside the star, exact modelling would require a 3 dimensional approach. An overall effect can be introduced to a 1D model as well, by varying the parameters connected to turbulent convection.

The calculations can be further simplified using amplitude equation (AE) formalism (Buchler & Goupil 1984). With the AE method large number of models can be created, but one has to determine the required coefficients. To do this, we started with calculating a few hydrodinamical models to determine growth rates (\(\kappa\)) and amplitudes without introducing any modulation.

2. The model parameters
The Florida-Budapest code involves seven independent dimensionless \(\alpha\) parameters connected to turbulent convection. The details of the model can be found in Kolláth & Buchler (2001). Two parameters out of seven, the parameters controlling convective flux (\(\alpha_c\)) and eddy viscosity (\(\alpha_\nu\)) are the dominant ones governing the amplitude of the pulsation. By modulating these two parameters within physically reasonable values the predictions of the Stothers-model can be inspected. As a further control, we determined all seven \(\kappa(\alpha)\) and \(P(\alpha)\) relations. Yecko et al. (1998) already published tests regarding the dimensionless parameters. Using the same
hydrodinamical code to Cepheids, the effects of the convective flux $\alpha_c$, mixing length $\alpha_\lambda$ and eddy viscosity $\alpha_\nu$ parameters on the shape and location of the linear instability strip and the internal structure of stellar models were investigated. We treat the mixing length differently, with a fixed value ($\alpha_\lambda = 1.5$) and we fine-tune the model with the other parameters because of their internal dependencies.

Properties of our model star are listed in Table 1.

| Mass     | $T_{\text{eff}}$ | Luminosit y | Z    |
|----------|-----------------|-------------|------|
| 0.77 $M_\odot$ | 6100 K | 50 $L_\odot$ | $10^{-4}$ |

3. Linear periods

The dependence of the linear fundamental period is shown on Figure 1. The most dominant parameter is clearly the parameter governing the convective flux ($\alpha_c$). The turbulent source function controlled by $\alpha_s$ follows the same trend at low values but it saturates around $\alpha_s \sim 0.4$. Higher values make the model numerically unstable, which clearly shows up in the $\kappa$ values. Changing the eddy viscosity with $\alpha_\nu$ does not affect the stellar structure hence the periods are independent from it. The remaining three parameters have also little if any effect on periods, so for clarity they were omitted from the plot.

Besides the amplitude modulation, Blazhko-variables show phase modulation as well, which can be explained by the variation of pulsation period similar to the amplitude variations. Measuring the magnitude of period changes is not an easy task as it requires good coverage for an entire Blazhko period. A current example is V783 Cyg where the one month long, continuous observations of the Kepler satellite allowed the direct calculation of an instantaneous period that shows period changes with $\sim 0.55\%$ half-amplitude (see Fig. 2 in Kolenberg et al. 2010). The same method applied to four CoRoT RR Lyrae stars revealed variations between $0.1 - 0.7\%$ (Szabó et al. 2009). We estimated period changes from published phase modulation plots and pulsation-modulation frequency values as well. Four stars, SS Cnc (Jurcsik et al. 2006), RR Lyr (Kolenberg et al. 2006), AR Her (Smith et al. 1999) and MW Lyr (Jurcsik et al. 2008) have quite similar values to each other and to V783 Cyg, between $0.4 - 0.6\%$ while for RR Gem (Jurcsik et al. 2006) and DM Cyg (Jurcsik et al. 2009) it is somewhat lower, $\sim 0.15\%$. O-C curves with good coverage over Blazhko cycles (e.g. RR Lyr) confirm these values. Blazhko-periods seem to have little effect on the variation of the pulsational periods: for example the two stars with the longest (RR Lyr, 39 days) and shortest (SS Cnc, 5.3 days) modulations have essentially the same amplitude, $\sim 0.44\%$.

To see how these values compare to our calculations, we indicated variations from our $P = 0.6567d$ reference period with $\pm 0.1\%$ (dark grey area) and $\pm 0.5\%$ (light grey area) on Figure 1. The results indicate that large amplitude variations need large variations in the parameters, and the only suitable candidate is the convective flux $\alpha_c$. We note again that these are linear results where only the eigenvalues of the equilibrium model have been calculated, not the full hydrodinamical code. However, no significant corrections are expected between linear and nonlinear (limit cycle) periods, our first calculations show $0.1 - 0.2\%$ shift only.

Data from the GEOS maxima database: http://dbrr.ast.obs-mip.fr/
Figure 1. Linear fundamental period values versus normalized $\alpha$ parameters. The black square denotes the reference values of all four parameters. Dark and light grey areas show the $\pm 0.1\%$ and $\pm 0.5\%$ changes to the reference periods respectively.

4. Growth rates and amplitudes

Similarly to the periods, the largest effect on growth rates ($\kappa$) come from $\alpha_c$ and $\alpha_s$, but the eddy viscosity ($\alpha_\nu$) now follows them closely. The other four $\alpha$ parameters also have some, albeit minor effect on growth rates.

We estimated amplitude variations with a simple relation derived from AEs. The simplest form of an AE with a single mode present is $\dot{A} = \kappa A - qA^3$. Considering a limit cycle solution ($\dot{A} = 0$) the amplitude is given by $A = \sqrt{\kappa/q}$. Although the $q$ coefficient is not independent from the other variables, we assume a constant value for our estimate. Variations from the reference value are now simple to compute, $\Delta A/A \sim \sqrt{\kappa}$ as it is shown on Figure 2.

Such very basic estimate suggests a magnitude of $\pm 10 - 20\%$ variations in amplitude-like quantities. Deriving real variations in physical characteristics require running the hydrodinamical code and compute considerably large number of pulsation cycles to sort out all transient features. In the case of $\alpha_c$, our preliminary results with stellar radius variations show $q$ increasing towards higher $\alpha$ values, making the amplitude-dependence steeper thus allowing slightly higher modulation amplitudes.

5. Conclusions

We investigated RR Lyrae model sequences with the Florida-Budapest code to derive relations between free parameters of the model and linear periods and growth rates of the star. Although our results are only preliminary estimates, these exercises can give us early results regarding the mechanism for the Blazhko-effect proposed by Stothers. The most potent parameters, the convection strength, eddy viscosity and turbulent source function are all connected to the convective structures in the star, in accordance with Stothers’ idea. However the actual values seem to reach magnitudes of low-amplitude Blazhko-modulation only. To explain all Blazhko-
variables, huge modulation of $\alpha$ parameters has to be introduced, which causes significant changes in the convective structure of the stellar interior. The reality of such large variations over a few weeks in the stellar structure is doubtful.

Our analysis will follow with the more detailed nonlinear model calculations. Modulation properties will be investigated with amplitude equations and a few modulated hydrodinamical model runs to verify the AE results. The parameter space is of course larger than the values we covered here, but these early results already indicate that the mechanism proposed by Stothers faces difficulties.

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