Exploring the Hubble Tension and Spatial Curvature from the Ages of Old Astrophysical Objects

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Abstract

We use the age measurements of 114 old astrophysical objects (OAO) in the redshift range $0 \lesssim z \lesssim 8$ to explore the Hubble tension. The age of the universe at any $z$ is inversely proportional to the Hubble constant, $H_0$, so requiring the universe to be older than the OAO it contains at any $z$ will lead to an upper limit on $H_0$. Assuming flat $\Lambda$CDM and setting a Gaussian prior on the matter density parameter $\Omega_m = 0.315 \pm 0.007$ informed by Planck, we obtain a 95% confidence level upper limit of $H_0 < 70.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, representing a 2$\sigma$ tension with the measurement using the local distance ladder. We find, however, that the inferred upper limit on $H_0$ depends quite sensitively on the prior for $\Omega_m$, and the Hubble tension between early-time and local measurements of $H_0$ may be due in part to the inference of both $\Omega_m$ and $H_0$ in Planck, while the local measurement uses only $H_0$. The age-redshift data may also be used for cosmological model comparisons. We find that the $K_0 = cR_0$ universe accounts well for the data, with a reasonable upper limit on $H_0$, while Einstein–de Sitter fails to pass the cosmic-age test. Finally, we present a model-independent estimate of the spatial curvature using the ages of 61 galaxies and the luminosity distances of 1048 Pantheon Type Ia supernovae. This analysis suggests that the geometry of the universe is marginally consistent with spatial flatness at a confidence level of 1.6$\sigma$, characterized as $\Omega_k = 0.43^{+0.27}_{-0.37}$.

Unified Astronomy Thesaurus concepts: Cosmological models (337); Cosmological parameters (339); Hubble constant (758); Galaxy ages (576); Quasars (576); Type Ia supernovae (728)

1. Introduction

Since the first discovery of the expansion of the universe more than 90 yr ago (Lemaître 1927; Hubble 1929), the Hubble constant $H_0$ characterizing its current expansion rate has been of great interest to astronomers. In the last decade, however, a significant mismatch has emerged between several early-time and local measurements of $H_0$ (see Verde et al. 2019; Di Valentino et al. 2021 for recent reviews). The latest value of $H_0 (= 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$; Riess et al. 2021) measured from local Type Ia supernovae (SNe Ia), calibrated by the Cepheid distance ladder, is in 4.2$\sigma$ tension with that inferred from Planck cosmic microwave background (CMB) observations interpreted in the context of the standard $\Lambda$CDM model ($H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$; Planck Collaboration et al. 2020). If the unknown systematics cannot be responsible for the discrepancy, the Hubble tension may imply new physics beyond $\Lambda$CDM (Melia 2020; Vagnozzi 2020).

In order to resolve the Hubble tension, more independent methods of measuring $H_0$ are required. For example, the age of the oldest stellar populations in our galaxy can provide an independent local determination of $H_0$ (Trenti et al. 2015; Jimenez et al. 2019; Valcin et al. 2020; Bernal et al. 2021; Boylan-Kolchin & Weisz 2021). But most age measurements use objects at higher redshifts, which can also constrain other cosmological parameters (e.g., Bolte & Hogan 1995; Krauss & Turner 1995; Dunlop et al. 1996; Alcaniz & Lima 1999; Lima & Alcaniz 2000; Jimenez & Loeb 2002; Jimenez et al. 2003; Capozziello et al. 2004; Friaça et al. 2005; Simon et al. 2005; Jain & Dev 2006; Pires et al. 2006; Dantas et al. 2007, 2009, 2011; Samushia et al. 2010; Bengaly et al. 2014; Wei et al. 2015; Rana et al. 2017; Nunes & Pacucci 2020; Vagnozzi et al. 2021a). Very recently, Vagnozzi et al. (2021b) used the age estimates of high-redshift (up to $z \sim 8$) old astrophysical objects (OAO) to derive an upper limit on $H_0$ by requiring that all OAO at any $z$ must be younger than the age of the universe at that redshift. Their study shed some light on the ingredients needed to resolve the Hubble tension, but to constrain $H_0$ in this manner, one has to assume a background cosmology. Assuming the validity of $\Lambda$CDM at late times, Vagnozzi et al. (2021b) found a 95% confidence level upper limit of $H_0 < 73.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, marginally consistent with that measured using the local distance ladder.

Of direct relevance to the principal aim of this paper is the fact that, unlike the cosmic distance ladder methods that rely on the distances of primary or secondary indicators, the age measurements of distant objects are independent of each other. The age-redshift relationship of high-$z$ OAO may therefore provide a new perspective on one of the most frontier issues in modern cosmology, i.e., the spatial curvature of the universe. Knowing whether the universe is open, closed, or flat is crucial for a complete understanding of its evolution and the nature of dark energy (Ichikawa et al. 2006; Clarkson et al. 2007; Gong & Wang 2007; Virey et al. 2008). A significant deviation from zero spatial curvature would have far-reaching consequences for the inflationary paradigm and its underlying physics (Eisenstein et al. 2005; Tegmark et al. 2006; Wright 2007; Zhao et al. 2007; Melia 2020).

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Although a spatially flat universe ($\Omega_k = 0$) is strongly favored by most of the current cosmic probes, especially by the Planck 2018 CMB observations (Planck Collaboration et al. 2020), these curvature determinations are based on the pre-assumption of a particular cosmological model (e.g., $\Lambda$CDM). But there is a strong degeneracy between the curvature parameter and the dark-energy equation of state, so it would be better to measure the purely geometric quantity $\Omega_k$ from the data using a model-independent method. A nonexhaustive set of references attempting to constrain the value of $\Omega_k$ in a model-independent way includes Bernstein (2006), Clarkson et al. (2007), Shafieloo & Clarkson (2010), Li et al. (2014, 2016, 2018a, 2018b, 2019a, 2019b, 2020), Sapone et al. (2014), Räsänen et al. (2015), Cai et al. (2016), Yu & Wang (2016), L’Huillier & Shafieloo (2017), Liao et al. (2017), Rana et al. (2017), Wang et al. (2017, 2020, 2021b), Wei & Wu (2017), Xia et al. (2017), Denissenya et al. (2018), Wei (2018), Witzemann et al. (2018), Yu et al. (2018), Collett et al. (2019), Cao et al. (2019a, 2019b, 2022), Liao (2019, 2020), Ruan et al. (2019), Qi et al. (2019a, 2019b), Liu et al. (2020), Wei & Melia (2020a, 2020b), Zhou & Li (2020), Dhawan et al. (2021), Jesus et al. (2021), Vagnozzi et al. (2021a), Yang & Gong (2021), Zheng et al. (2021) and Wei et al. (2022).

In this paper, we broaden the base of support for the age measurements of high-$z$ OAO by demonstrating their usefulness in testing the late-time expansion history and arbitrating the Hubble tension in different cosmological models. Further, we propose a new model-independent method of determining the spatial curvature by combining the OAO age-$z$ data with SNe Ia luminosity distances. Using a polynomial fitting technique, we reconstruct a continuous age-$z$ function representing the discrete age measurements of OAO without the pre-assumption of any specific cosmological model. The time-redshift derivative $dt/dz$ can then be approximately obtained by differentiating the age-$z$ function. Then, $dt/dz$ can be transformed into the curvature-dependent luminosity distance $D_L(\Omega_k; z)$ according to the geometric relation derived from the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. Finally, by carrying out the joint maximum likelihood analysis on the polynomial fitting and the observed differences between $D_L(\Omega_k; z)$ and the curvature-independent luminosity distances inferred from SNe Ia, one can simultaneously constrain the curvature parameter $\Omega_k$, the polynomial coefficients, and the SN nuisance parameters in a model-independent way.

The paper is arranged as follows. In Section 2, we briefly describe the age-redshift test, and then constrain $H_0$ in different cosmological models. In Section 3, we introduce the methodology of measuring $\Omega_k$ using OAO age-$z$ and SN Ia data, and then present the results of our analysis. We summarize our main conclusions in Section 4.

2. Exploration on the Hubble Tension

2.1. The Age-redshift Test

The theoretical age of the universe at redshift $z$ is given as

$$t(z) = \int_z^\infty \frac{dz'}{(1 + z')H(z', \theta)},$$

where $H(z, \theta)$ is the Hubble parameter and $\theta$ stands for the parameters of the specific cosmological model. All of our analysis in this paper is based on this expression, which is derived from the FLRW metric. In so doing, we restrict our attention to the spacetime predicted in the context of general relativity only, though we shall consider possible model variations consistent with this constraint as prescribed via the choice of stress-energy tensor in Einstein’s equations, which are characterized by the specific model parameters $\theta$.

The age $t_{\text{obj}, i}$ of an object (e.g., a passive galaxy or quasar) at redshift $z_i$ is defined as the difference between the age of the universe at $z_i$ and that when the object was formed at redshift $z_f$. Given that no object was born at the Big Bang ($z_f \to \infty$), the age of the universe at any redshift should always be greater than or equal to the age of the oldest astrophysical object (OAO) at the same redshift, i.e., $t(z_i) \geq t_{\text{obj}, i}$. The difference between $t(z_i)$ and $t_{\text{obj}, i}$, which we denote by $\tau_{\text{inc}}$, represents the “incubation” time, or delay factor, and accounts for the amount of time elapsed since the Big Bang to the formation of the object.

Equation (1) shows that the age of the universe at any given redshift is inversely proportional to the Hubble constant $H_0 \equiv H(z = 0)$. An upper limit on $H_0$ can therefore be obtained by requiring that the universe be at least as old as the oldest objects at the corresponding redshifts (Vagnozzi et al. 2021b). If the value of $H_0$ is too high, then we are in an awkward position that the universe is younger than the oldest objects it contains at a given redshift. In Equation (1), $t(z)$ receives most of its contribution at late times ($z \lesssim 10$), and is scarcely sensitive to pre-recombination physics. Therefore, consistency between the high-$z$ upper limits on $H_0$ and the local $H_0$ measurements offers a stringent test of late-time and/or local new physics, potentially suggesting the necessity for the latter to operate together with early-time new physics to completely address the Hubble tension (Krishnan et al. 2020, 2021, 2022; Dainotti et al. 2021, 2022; Jedamzik et al. 2021; Lin et al. 2021; Vagnozzi 2021; Vagnozzi et al. 2021b).

Using a combination of galaxies and high-$z$ quasars, Vagnozzi et al. (2021b) constructed an age-redshift diagram of OAO up to $z \sim 8$. Most of their galaxy data come from the Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (CANDELS) observing program (Grogin et al. 2011), and the remaining galaxy data are from the observations of 32 old passive galaxies in the redshift range $0.117 \leq z \leq 1.845$ (Simon et al. 2005). For high-$z$ quasars, they considered the following observations: 7446 quasars from SDSS DR7 in the range $3 \leq z \leq 5$ (Shen et al. 2011), 50 quasars detected by the GNIRS spectrograph in the range $5.5 \leq z \leq 6.5$ (Shen et al. 2019), 15 quasars detected by Pan-STARRS1 in the range $6.5 \leq z \leq 7.0$ (Mazzucchelli et al. 2017), and 9 of the most distant quasars ever discovered in the range $7.0 \leq z \leq 7.642$ (Mortlock et al. 2011; Bañados et al. 2018; Wang et al. 2018, 2021a; Matsuoka et al. 2019a, 2019b; Yang et al. 2019, 2020). Basically, the ages of the CANDELS galaxies were estimated by fitting the photometric spectral energy distribution, whereas for the quasars, a specific growth model of black hole seeds developed by Pacucci et al. (2017) was adopted. Applying severe quality cuts to these observations, and selecting only those objects that are among the oldest ones within each redshift bin, Vagnozzi et al. (2021b) compiled a final catalog of 114 OAO with reliable redshift and age measurements, in which 61 OAO are galaxies and the other...
53 are quasars. We adopt this high-z OAO catalog covering the redshift range $0 < z < 8$ for our assessment of the $H_0$ limits. Figure 1 shows the age measurements as a function of redshift for these 114 OAO. In this plot, we also illustrate the dependence of the universe’s age $t(z)$ (estimated using flat $\Lambda$CDM with a fixed matter density $\Omega_m = 0.3$) on the value of the Hubble constant $H_0$.

2.2. Upper Limits on $H_0$

We are now in position to use the selected 114 age measurements of OAO as a function of redshift to derive upper limits on $H_0$. Given the observed data $D$ (with the OAO ages at redshifts $z_i$ being $t_{\text{obj},i}$ $\pm$ $\sigma_{t_{\text{obj}},i}$; see solid points in Figure 1) and some prior knowledge about the hypothetical models (for which the parameters are denoted by the vector $\theta$), the posterior probability distributions of the free parameters can be modeled through the half-Gaussian (log-)likelihood (Vagnozzi et al. 2021b):

$$\ln L(\theta | D) = -\frac{1}{2} \sum_{i} \frac{\Delta_i^2(\theta)}{\sigma_{i}^2}$$

if $\Delta_i(\theta) < 0$

$$\ln L(\theta | D) = 0$$

if $\Delta_i(\theta) \geq 0$, (2)

where $\Delta_i = t(\theta, z_i) - t_{\text{obj},i}$ is defined as the age of the universe minus the age of the $i$th OAO at redshift $z_i$. The expression in Equation (2) is based on the fact that (a) since the universe must not be younger than its oldest inhabitants, parameters for which the universe is younger than the OAO (i.e., $\Delta_i(\theta) < 0$) are exponentially unlikely, and this means the more the universe is younger than the OAO, the worse the fit; (b) parameters for which the universe is older than the OAO (i.e., $\Delta_i(\theta) \geq 0$) are equally likely, and cannot be distinguished solely on the basis of the OAO age.

To calculate model predictions for the age $t(\theta, z_i)$ in Equation (1), we need an expression for $H(z, \theta)$. As the cosmic expansion rate within the context of specifically selected models is significantly different, it is interesting to examine the upper limits on $H_0$ derived from the OAO ages using different background cosmologies. Here we discuss how these limits are obtained for $\Lambda$CDM, the Einstein–de Sitter universe, and the $R_0 = ct$ universe.

2.2.1 $\Lambda$CDM

In flat $\Lambda$CDM, the Hubble parameter is well approximated by

$$H^{\Lambda\text{CDM}}(z, \theta) = H_0 [\Omega_m (1 + z)^3 + \Omega_\Lambda]^{1/2},$$

where $\Omega_\Lambda = 1 - \Omega_m$ is the cosmological constant energy density. Note that we ignore the contribution from radiation, which is negligible compared to that of matter and dark energy in the late-time expansion history. The analysis of the OAO ages provides a valuable consistency test: if we trust the data, a disagreement between our upper limit on $H_0$ and the value measured from the local distance ladder may indicate new physics beyond $\Lambda$CDM, at least in the late-time expansion history.

For the basic $\Lambda$CDM model, the free parameters to be constrained are $\theta = \{H_0, \Omega_m\}$. We adopt the Python Markov chain Monte Carlo (MCMC) module, EMCEE (Foreman-Mackey et al. 2013), to explore the posteriors probability distributions of these parameters. Note that Vagnozzi et al. (2021b) expressed $\Delta_i$ in Equation (2) as $\Delta_i \equiv t(\theta, z_i) - t_{\text{obj},i} - \tau_{\text{inc}}$, and modeled the incubation time $\tau_{\text{inc}}$ as a prior distribution derived by Jimenez et al. (2019), based on the assumption that the formation redshift $z_f$ for the oldest observed galaxies is $z_f > 11$. After marginalizing over $H_0$, $\Omega_m$, and $\tau_{\text{inc}}$ this approach yields a prior peaked at $\tau_{\text{inc}} \sim 0.1 - 0.15$ Gyr, which Vagnozzi et al. (2021b) labeled as J19 and adopted its fitting function provided in Appendix G of Valcin et al. (2020).

For the quasars, Vagnozzi et al. (2021b) fixed $\tau_{\text{inc}} = 0.7$, under the assumption that they were all seeded at redshift $z_f \sim 20$. In their baseline analysis, Vagnozzi et al. (2021b) set flat priors on $H_0 \in [40, 100]$ $\text{km s}^{-1} \text{Mpc}^{-1}$ and $\Omega_m \in [0.2, 0.4]$, the J19 prior on $\tau_{\text{inc}}$ for the galaxies, and fixed $\tau_{\text{inc}} = 0.7$ for the quasars. To verify the reliability of our calculations, we have carried out a parallel analysis with the same priors on $H_0$, $\Omega_m$, and $\tau_{\text{inc}}$ to ensure that our results are consistent with each other. Figure 2 shows the joint $H_0 - \Omega_m - \tau_{\text{inc}}$ posterior distributions obtained from the baseline analysis suggested by Vagnozzi et al. (2021b). Our 95% confidence level upper limit on the reduced Hubble constant $h_0 \equiv H_0/(100 \text{ km s}^{-1} \text{Mpc}^{-1}) < 0.732$ (all quoted upper limits will hereafter be at the 95% confidence level) is the same as that obtained by Vagnozzi et al. (2021b). Our methodology can thus reliably incorporate the constraints of Vagnozzi et al. (2021b), producing results consistent with their analysis.

As one can see from Equations (1) and (2), however, the inclusion of $\tau_{\text{inc}}$ clearly results in a more stringent, less conservative limit on $H_0$ depending on one’s choice of the initial conditions. In addition, the derived $\tau_{\text{inc}}$ distribution from Jimenez et al. (2019) depends (though only weakly) on the assumed $\Lambda$CDM cosmology. In order to be as conservative as possible, and to provide the most reliable upper limits, we suggest to avoid introducing $\tau_{\text{inc}}$ in Equation (2). For the rest of this section, we shall therefore begin by conservatively constraining $H_0$ without the inclusion of this incubation time.

In our analysis, we choose wide flat priors for $H_0 \in [0, 150]$ $\text{km s}^{-1} \text{Mpc}^{-1}$ and $\Omega_m \in [0.2, 0.4]$. The 1D marginalized posterior distributions and 2D plots of the $1-\sigma$ confidence regions for these two parameters, constrained by the 114 age-redshift data, are displayed in Figure 3 (red contours). These contours show that, whereas $\Omega_m$ is not as well constrained, we can set an upper limit on $H_0$ whose 95% confidence level value

![Figure 1](image.png)
is $h_0 < 0.755$. This is roughly consistent with its latest local measurement ($h_0 = 0.732 \pm 0.013$; Riess et al. 2021). To explore the impact of a $\tau_{\text{inc}}$ prior, Vagnozzi et al. (2021b) also analyzed the data without its inclusion, i.e., by setting $\tau_{\text{inc}} = 0$ Gyr, finding in this case that $h_0 < 0.791$, which is somewhat incompatible with our result ($h_0 < 0.755$). The difference appears to be due to the fact that

Vagnozzi et al. (2021b) set a narrower prior on $h_0 \in [0.4, 1]$, while we put $h_0 \in [0, 1.5]$. The relatively low values of $H_0$ are equally favored by the half-Gaussian likelihood (Equation (2)).

The results of Next, we explore the impact of an $\Omega_m$ prior by fixing its value to be 0.1, 0.3, 0.5, 0.7, and 0.9, respectively. The outcome of each case is presented in Table 1. One can see that the inferred upper limit on $H_0$ at low and high redshifts in this model is that $H_0$ is constrained on its own for the former, but only in concert with other parameters, particularly $\Omega_m$, for the latter. The 95% Confidence Level Upper Limits on $H_0$ with Different $\Omega_m$ Priors

| $\Omega_m$ (fixed) | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  |
|-------------------|-----|-----|-----|-----|-----|
| $H_0/(\text{km s}^{-1}\text{Mpc}^{-1})$ | <112.4 | <72.2 | <56.6 | <47.9 | <42.4 |

The expansion rate in the $R_h = c t$ universe (Melia 2003, 2007, 2013; Melia & Shevchuk 2012; Wei et al. 2015; Melia 2020), is given as

$$H^{R_h=ct}(z, \theta) = H_0(1 + z).$$

The $R_h = c t$ cosmology has only one free parameter, i.e., $\theta = \{H_0\}$. Here, we also set a flat prior on $H_0 \in [0, 150]$ km s$^{-1}$ Mpc$^{-1}$. The results of fitting the 114 age-redshift data with this cosmology are shown in the left panel of Figure 4. We find an upper limit of $h_0 < 0.861$ at the 95% confidence level, in good agreement with the locally measured $H_0$.

2.2.3 Einstein–de Sitter

The Einstein–de Sitter universe is characterized by a cosmic fluid containing only matter. In this model, $H_0$ is the sole free parameter, i.e., $\theta = \{H_0\}$, and the Hubble rate is expressed as

$$H^{EdS}(z, \theta) = H_0(1 + z)^{3/2}.$$
In principle, we may use the age-redshift data of all 114 OAO to estimate the spatial curvature constant by comparing it with the empirically derived distance modulus of SNe Ia. Thus, if one can access the quantity $\frac{dt}{dz}$ required redshift, without pre-assuming a particular cosmological model, one may reconstruct the line-of-sight comoving distance $D_L(z) = c \int z_0^z (1 + z') \frac{dt}{dz'} dz'$ along with the curvature-dependent luminosity distance, $D_L(\Omega_k; z)$. The idea that $\frac{dt}{dz}$ may be obtained from the age-redshift measurements of old objects has been suggested on various occasions (Jimenez & Loeb 2002; Jesus et al. 2017; Rana et al. 2017).

Since we are primarily interested in the derivative $\frac{dt}{dz}$ and the present age of the universe from other observations, and much less so in the incubation time $\tau_{inc}$, we choose to directly fit the original estimated ages $t_{obj}(z)$ of the OAO. Taking $\tau_{inc}$ to be constant, one may see that $t(z)$ differs from $t_{obj}(z)$ by just a constant. That is,

$$t(z) = t_{obj}(z) + \tau_{inc}. \quad (7)$$

In principle, we may use the age-redshift data of all 114 OAO up to $z \sim 8$ compiled by Vagnozzi et al. (2021b) to estimate $\frac{dt}{dz}$. However, this catalog includes two different kinds of source, viz., 61 galaxies and 53 quasars, each of which has its own distinct incubation time. For this analysis, we therefore only employ the age of 61 galaxies distributed over the redshift interval $0.001 \leq z \leq 6.689$ to estimate $\frac{dt}{dz}$. The advantage of solely using galaxies is the relative uniformity of the sample. The originally estimated ages, $t_{obj}$, of the 61 galaxies are indicated as a function of redshift by the black points in Figure 1.

In our analysis, we construct the age function $t_{obj}(z)$ in a cosmology-independent way by fitting a third-order polynomial, with the initial condition $t_{obj}(z = \infty) = 0$, to the age-redshift data. To mitigate the convergence problem that the polynomial fit encounters at high redshifts, we recast the $t_{obj}(z)$ function in the form of the $y$ redshift, defined by the relation $y = z/(1 + z)$. In this way, the age in $z \in [0, \infty)$ is mapped into $y \in [0, 1]$, so that the polynomial fit is well behaved throughout the redshift range from our local universe to the Big Bang. This polynomial is then expressed as

$$t_{obj}(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3, \quad (8)$$

where $a_1$, $a_2$, and $a_3$ are three free parameters (all in units of gigayears). With the initial condition $t_{obj}(z = \infty) = t_{obj}(y = 1) = 0$, it is easy to identify $a_0 = -a_1 - a_2 - a_3$. For $z = 0$, Equation (7) simplifies to $t_0 = a_0 + \tau_{inc}$. Once we have the inferred value of $a_0$ and know the present age of the universe $t_0$, we can also estimate $\tau_{inc}$.

As we assume $\tau_{inc}$ to be constant, we have $\frac{dt}{dz} = \frac{dt}{dy}$. Thus, by differentiating the polynomial (Equation (8)), we obtain

$$\frac{dt}{dz} = \frac{a_1}{(1 + z)^2} + \frac{2a_2 z}{(1 + z)^3} + \frac{3a_3 z^2}{(1 + z)^4}. \quad (9)$$

Then, the curvature-dependent luminosity distance can be derived by substituting Equation (9) into (6), i.e.,

$$D_L(z) = \frac{c (1 + z)}{H_0 \sqrt{\Omega_k}} \sinh \left( H_0 \sqrt{\Omega_k} \int z^0 (1 + z') \frac{dt}{dy} \ dz' \right). \quad (10)$$

We can further obtain the reconstructed distance modulus $\mu_{age}(\Omega_k, a_1, a_2, a_3; z)$ using the age-redshift data:

$$\mu_{age}(\Omega_k, a_1, a_2, a_3; z) = 5 \log_{10} \left[ \frac{D_L(z)}{\text{Mpc}} \right] + 25. \quad (11)$$

### 3.2. Distance from Observations of SNe Ia

By comparing the curvature-dependent luminosity distance $D_L(\Omega_k, z)$ derived from the age-redshift data with the empirically derived luminosity distance (at similar redshifts), we can obtain a model-independent measurement of $\Omega_k$. For the latter, we use the largest Pantheon SN Ia sample, consisting of 1048 SNe Ia in the redshift range $0.01 < z < 2.3$ (Scolnic et al. 2018).
The observed distance modulus of each SN is given as
\[ \mu_{SN} = m_B + \alpha_1 - \beta C - M_B^* , \] (12)
where \( m_B \) is the observed B-band apparent magnitude, \( \alpha_1 \) is the light-curve stretch factor, and \( C \) is the SN color at maximum brightness. The absolute B-band magnitude \( M_B^* \) is correlated with the host galaxy mass \( M_{stellar} \) via a simple step function (Betoule et al. 2014; Scolnic et al. 2018):
\[ M_B^* = \begin{cases} M_B + \Delta M & \text{for } M_{stellar} > 10^{10} M_\odot \\ M_B & \text{otherwise,} \end{cases} \] (13)
where \( \Delta M \) corresponds to a distance correction based on \( M_{stellar} \). Note that \( \alpha, \beta, M_B, \) and \( \Delta M \) are nuisance parameters that need to be constrained simultaneously with the cosmological parameters. As such, the derived SN distance is typically dependent on the chosen cosmology. To avoid this, Kessler & Scolnic (2017) introduced an approximate method called BEAMS with Bias Corrections (BBC) to correct those expected biases and simultaneously fit for the SN nuisance parameters.

The BBC fit produces a bin-averaged Hubble diagram of SNe Ia, and then the nuisance parameters \( \alpha \) and \( \beta \) are constrained by fitting to a reference cosmological model with fixed values of the matter density \( \Omega_m \) and equation of state of dark energy \( w \). Within each redshift bin, the local shape of the Hubble diagram is assumed to be well described by the reference cosmological model. If there are sufficient redshift bins, the fitted parameters \( \alpha \) and \( \beta \) will converge to consistent values (Marriner et al. 2011; Kessler & Scolnic 2017).

With the BBC method, Scolnic et al. (2018) report the corrected apparent magnitudes \( m_{corr} = m_B + \alpha x_1 - \beta C - \Delta M + \Delta B \) for all the SNe, where \( \Delta B \) is the added distance correction. Given these corrected apparent magnitudes, we just need to subtract the absolute magnitude \( M_B \) from \( m_{corr} \) to derive the observed distance moduli:
\[ \mu_{SN} = m_{corr} - M_B. \] (14)

The caveat with this approach, however, is that the format assumes all cosmological models are nested, which is not true in general. This formulation may be used approximately for various versions of \( \Lambda \)CDM, but not for other models, such as \( R_0 = c t \), whose luminosity distance does not depend on parameters such as \( \Omega_k \). The caveat here is that the results we report below pertain specifically to \( \Lambda \)CDM, not necessarily to other FLRW models, or models based on alternative theories of gravity.

Even within the context of \( \Lambda \)CDM, however, there may still be some residual model dependence, so to test how serious this limitation might be, we take the following approach. The inferred values of \( \alpha \) and \( \beta \) in the BBC method are valid only for the reference model. We therefore consider two different cases: first, the determination of \( \alpha \) and \( \beta \) is assumed to be independent of the model, and we directly use those corrected apparent magnitudes reported by Scolnic et al. (2018) for our purpose; second, we carry out a parallel analysis of the uncorrected SN magnitudes by reconstraining \( \alpha \) and \( \beta \) as nuisance parameters, and we compare the results.

### 3.3. Analysis and Results

We constrain all of the free parameters via a joint analysis involving the galaxy age and SN Ia data. The final log-likelihood sampled by the Python MCMC module EMCEE is a sum of the separate likelihoods of the galaxy ages and SNe Ia:
\[ \ln(L_{tot}) = \ln(L_{tot}) + \ln(L_{SN}), \] (15)
where
\[ \ln(L_{tot}) = -\frac{1}{2} \sum_i \left( \frac{\Delta \mu_{obs} - \Delta \mu_{fit}(\alpha_i, \beta, \alpha_1, \alpha_2, \alpha_3; z_i)}{\sigma_{abc,i}^2} \right)^2 \] (16)
and
\[ -2 \ln(L_{SN}) = \Delta \mu T \cdot \text{Cov}^{-1} \cdot \Delta \mu. \] (17)

In Equation (16), \( \sigma_{abc,i} \) is the uncertainty of the \( i \)th age measurement \( \mu_{obs} \) and \( \mu_{fit} \) is obtained from Equation (8). In Equation (17), \( \Delta \mu = \mu_{SN}(M_B; z) - \mu_{age}(\Omega_k, a_1, a_2, a_3; z) \) is the data vector, defined by the difference between the distance modulus \( \mu_{SN} \) of SNe Ia (Equation (14)) and the constructed distance modulus \( \mu_{age} \) from the galaxy age-redshift data (Equation (11)), and \( \text{Cov} \) is a full covariance matrix that contains both statistical and systematic uncertainties of SNe. Note that, in the SN likelihood estimation, there is a degeneracy between \( H_0 \) and \( M_B \). We therefore adopt a fiducial \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for the sake of constraining \( M_B \). In this case, the free parameters are the spatial curvature parameter \( \Omega_k \), the three polynomial coefficients \( (a_1, a_2, a_3) \), and the SN absolute magnitude \( M_B \).

The 1D marginalized posterior distributions and 2D regions with 1–2\( \sigma \) contours corresponding to these five free parameters, constrained by the galaxy ages and corrected SN magnitudes, are presented in Figure 5. These contours show that, at the 1\( \sigma \) confidence level, the inferred parameter values are \( \Omega_k = 0.43^{+0.27}_{-0.27} \), \( a_1 = -14.21^{+1.14}_{-1.44} \), \( a_2 = -5.37^{+0.91}_{-0.95} \), \( a_3 = 6.76^{+1.15}_{-1.13} \) and \( M_B = -19.40^{+0.23}_{-0.21} \). The corresponding results for the galaxy + corrected SN data are summarized in Table 2. With this approach, we find that the spatial geometry of the universe is marginally consistent with spatial flatness at a 1.6\( \sigma \) level of confidence.

As noted earlier, our procedure allows us to determine the inferred value of \( a_0 \) along with the best-fit polynomial coefficients, i.e., \( a_0 \equiv -a_1 - a_2 - a_3 = 12.82 \pm 2.06 \text{ Gyr} \). Considering the present age of the universe as inferred from Planck in the context flat \( \Lambda \)CDM \( (t_0 = 13.80 \pm 0.02 \text{ Gyr}; \) Planck Collaboration et al. 2020), we can further estimate the incubation time as \( t_{inc} = t_0 - a_0 = 0.98 \pm 0.06 \text{ Gyr} \).

Next, to investigate how sensitive our results of \( \Omega_k \) are to the choice of corrected SN magnitudes provided by the Pantheon team, we also perform a (parallel) comparative analysis of the galaxy + uncorrected SN data by simultaneously constraining the nuisance parameters along with \( \Omega_k \). The likelihood function of SNe now becomes
\[ L_{SN} = \prod_{i=1}^{1048} \frac{1}{\sqrt{2\pi \sigma_{stat,i}}} \exp \left( \frac{-(\Delta \mu_i^2)}{2\sigma_{stat,i}^2} \right), \] (18)
where \( \Delta \mu_i = \mu_{SN}(\alpha, \beta, M_B, \Delta M; z_i) - \mu_{age}(\Omega_k, a_1, a_2, a_3; z_i) \) is the difference between the distance modulus \( \mu_{SN} \) of SN Ia (Equation (12)) and the distance modulus \( \mu_{age} \) constructed from the galaxy age-redshift data (Equation (11)), and \( \sigma_{stat,i} \) is the statistical uncertainty of each SN, given by the
Here, \( \sigma_{mb,i}, \sigma_{xi,i} \), and \( \sigma_{z,i} \) stand for the uncertainties of the peak magnitude and light-curve parameters of the \( i \)th SN, the terms \( C_{mb,xi}, C_{mx,xi}, C_{xi,zi} \) represent the covariances among \( m_B, x_1, C \) for the \( i \)th SN, \( \sigma_{lens,i} \) is the uncertainty from stochastic gravitational lensing, and \( \sigma_{int} \) is the unknown intrinsic uncertainty. The dispersion \( \sigma_{z-i} = 5 \sqrt{\sigma_{pec}^2 + \sigma_{z}^2} / (z_i \ln 10) \) accounts for the uncertainty from the peculiar velocity uncertainty \( \sigma_{pec} \) and redshift measurement uncertainty \( \sigma_z \), in quadrature. We follow Scolnic et al. (2018) in using \( c \sigma_{pec} = 240 \text{ km s}^{-1} \), as well as \( \sigma_{lens,i} = 0.055 z_i \).

Only the statistical uncertainties are considered since the six-parameter systematic covariance matrices \( (m_B, x_1, C, m_B C, x_1 m_B, x_1 C) \) are not available in Scolnic et al. (2018). In this case, the free parameters are the curvature parameter \( \Omega_k \), the three polynomial coefficients \( (a_1, a_2, a_3) \), and the SN nuisance parameters \( (\alpha, \beta, M_B, \Delta M, \sigma_{int}) \). These nine parameters are constrained to be \( \Omega_k = 0.59^{+0.18}_{-0.17}, a_1 = -15.28^{+1.53}_{-1.59}, a_2 = -3.78^{+0.71}_{-0.74}, a_3 = 6.02^{+1.06}_{-1.05}, \alpha = 0.132^{+0.005}_{-0.005}, \beta = 2.595^{+0.057}_{-0.059}, M_B = -19.48^{+0.23}_{-0.21}, \Delta M = 0.052^{+0.009}_{-0.009}, \) and \( \sigma_{int} = 0.079^{+0.006}_{-0.006} \), which are displayed in Figure 6 and summarized in Table 2.

**Figure 5.** 1D and 2D marginalized posterior distributions with the \( \pm 2\sigma \) contours for the cosmic curvature \( \Omega_k \), the polynomial coefficients \( (a_1, a_2, a_3) \), and the SN absolute magnitude \( M_B \), based on the joint analysis of the galaxy age and corrected SN magnitude data. The vertical solid lines represent the medium parameter values, whereas the vertical dashed lines indicate \( \pm 1\sigma \) deviations from their respective means.
The comparison between lines 1 and 2 of Table 2 suggests that simply using the corrected SN magnitudes introduces a nonnegligible disparity in the results. The value of $\Omega_k$ inferred from the galaxy + corrected SN data represents a 0.5$\sigma$ tension with that measured from the galaxy + uncorrected SN data.
4. Summary and Discussion

In this work, we have used the age measurements of 114 OAO (including 61 galaxies and 53 quasars) in the redshift range $0 \lesssim z \lesssim 8$ to constrain the late-time cosmic expansion history and explore the Hubble tension in several cosmological models. Owing to the age of the universe at any redshift being inversely proportional to the Hubble constant $H_0$, the requirement that the universe be older than the OAO it contains at any redshift provides an upper limit to $H_0$.

Assuming the validity of flat ΛCDM at late times, and setting wide flat priors on $H_0$ and $\Omega_m$, we have obtained $H_0 < 75.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ at the 95% confidence level, roughly consistent with local $H_0$ measurements. However, if a Gaussian prior of $\Omega_m = 0.315 \pm 0.007$ informed by Planck is used, then the 95% confidence level upper limit on $H_0$ turned out to be $H_0 < 70.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, representing a 2σ tension with the locally measured value. It is compatible with the Planck inference, however. This is interesting because, in this scenario, both $H_0$ and $\Omega_m$ are mutually consistent for both Planck and the OAO age-redshift data. We found that the inferred upper value to $H_0$ does depend quite significantly on $\Omega_m$. Since the local measurement of $H_0$ does not require $\Omega_m$, while Planck uses both, we conclude that the Hubble tension between the two measurements may be due in part to the use of $\Omega_m$ in one case and not the other.

Besides ΛCDM, we also discussed how the $H_0$ limits may be obtained for $R_0 = ct$ and Einstein–de Sitter. The $R_0 = ct$ universe fits the age-redshift data with an upper limit of $H_0 < 86.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$. By comparison, the Einstein–de Sitter universe fits the same data with an upper limit of $H_0 < 40.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Obviously, Einstein–de Sitter fails to pass the cosmic-age test, because the inferred upper limit to $H_0$ in this model represents a 25.5σ tension with the locally measured $H_0$. Our overall results affirm the idea that cosmic ages are an extremely valuable probe in the quest toward uncovering the nature of the Hubble tension.

We have also proposed a novel method of estimating the spatial curvature, avoiding possible biases introduced by the pre-assumption of a specific cosmological model. To perform our analysis, we have considered the following cosmological data: 61 age measurements of galaxies and 1048 SNe Ia from Pantheon compilation. Based on the geometric relation in the FLRW metric, we have shown the possibility of obtaining the curvature-dependent luminosity distance from a best-fit polynomial to the age-redshift data of old objects. By comparing this curvature-dependent luminosity distance with the empirical luminosity distance inferred from SNe Ia, we obtained a somewhat model-independent estimate of the curvature parameter $\Omega_k$ based on the parameterization in ΛCDM.

Scolnic et al. (2018) applied the BBC method to determine the SN nuisance parameters and reported the corrected apparent magnitudes for all the Pantheon SNe. Combining the age-redshift measurements of galaxies with these corrected SN magnitudes, we have placed limits simultaneously on the cosmic curvature $\Omega_k$, the polynomial coefficients $(a_1, a_2, a_3)$, and the SN absolute magnitude $M_B$. This analysis suggests that the curvature parameter is constrained to be $\Omega_k = 0.43^{+0.27}_{-0.37}$, which is marginally compatible with zero. That is, the spatial geometry of the universe is marginally consistent with spatial flatness at the 1.6σ level.

As the inferred values of the SN nuisance parameters in the BBC method may depend on the reference cosmological model, even within the context of ΛCDM, we also carried out this type of analysis using the combined galaxy + uncorrected SN data sets by simultaneously constraining the curvature parameter $\Omega_k$, the polynomial coefficients $(a_1, a_2, a_3)$, and the SN nuisance parameters $(\alpha, \beta, M_B, \Delta_{M_B}, \sigma_{\Delta M_B})$. In this case, we found that the constraint is $\Omega_k = 0.59^{+0.18}_{-0.17}$. The value of $\Omega_k$ changes slightly, by about 0.5σ, when the SN nuisance parameters are reconstrained along with the cosmology, implying that simply using the corrected SN magnitudes would introduce a nonnegligible disparity in the results.

Such deviations from zero are not yet compelling enough to initiate a detailed investigation of their implications. We point out, however, that there are several rather essential consequences, should such an outcome be realized. First, spatial flatness is assumed to be an indicator of inflation (Guth 1981). If the universe is not spatially flat after all, this would cast serious doubt on the possibility that inflation could have happened. At the very least, it would require major modifications to most of the inflation potentials proposed thus far. Note, however, that a de Sitter expansion need not necessarily proceed solely with spatial flatness. As such, several attempts have been made to create an inflationary scenario with negative spatial curvature, leading to an open universe. This may happen, e.g., in the context of quantum tunneling-induced false vacuum decay (Ratra 1994; Ratra & Peebles 1994, 1995; Bucher et al. 1995; Kleban & Schillo 2012). Second, if it turns out that $\Omega_k$ is definitely positive, the universe must also have net positive energy density (Melia 2020). This would be very alarming in the context of a quantum-fluctuation origin for the Big Bang, since it would rule out a “creation from nothing” scenario, in which all the laws of physics, initial conditions, and all the structure appeared as a quantum fluctuation at $t=0$ without any prehistory. It would at the very least imply a preexisting vacuum prior to the expansionary event. Even so, one would then need to contend with the very serious problem of how a universe with the known value of Planck’s constant and such an enormous amount of energy could have lived long enough to classicize and evolve into the large-scale structure we see today (Melia 2020).

There are good philosphical, if not empirical, reasons for believing that $\Omega_k$ must be zero. But we cannot yet make that claim without at least some doubt, certainly not based on the analysis of the oldest astronomical objects in the universe that we have carried out in this paper.

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