Parameter Constrain of Nonminimal Derivative Coupling of Scalar Field Using Strong Energy Condition

M. R. Taufani$^{1,a}$, A. Suroso$^{1,2,b}$, and F. P. Zen$^{1,2,c}$

$^1$Theoretical Physics Laboratory, THEPI Division, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jl. Ganesha 10, Bandung 40132, Indonesia
$^2$Indonesia Centre for Theoretical and Mathematical Physics (ICTMP), Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jl. Ganesha 10, Bandung 40132, Indonesia
E-mail: $^a$muhammad.rizka@itb.ac.id, $^b$agussuroso@fi.itb.ac.id, $^c$fpzen@fi.itb.ac.id

Abstract. The study aims to determine the constrain of the coupling parameter /xi of nonminimal derivative coupling theory (NMDC) of scalar field. The calculation of the parameter was build from the strong energy conditions combined to modified Einstein theory (i.e. the scalar filed act as the external source of gravity beside the ideal fluid). The violation of strong energy condition (SEC) indicate the attracting gravity phenomenon. Then one can expecting two equations that consist of of energy density $\rho$ and the momentum $p$, then one defined the deacceleration $q$ to these equation. The present value of cosmological constant such as Hubble constant $H$, $q$, and scale factor $a$ need to be put into the equations to determine the boundaries of the strong energ. The results show the coupling parameter must be $\xi \geq 1.2 \times 10^{34} s^2$ for the SEC to be valid in the theory.

1. Introduction

General relativity is a theory of gravity that sees the correlation between the geometry of spacetime and its filling matter, and it deals with a field equation (Einstein’s field equation) that describes the connection between them. Specifically, the equation can be interpreted that matter tells how the geometry to curve and the curvature of spacetime explain the motion of the matter [1]. That is, that all kinds of geometric dynamics or curvature of the universe are influenced by the matter inside. It is interesting to examine further that there have been many astronomical observations which state that the universe is now expanding at an accelerated rate [2]. This phenomenon can not be separated from the success of Einstein’s theory in explaining the dynamics of the universe on the cosmological scale. It should be noted that the causes of the accelerating universe are some complex physical phenomena, by which various types of models such as extra dimensions and involvement of external fields in the universe in the early ages as being part of the Einstein theory modification. Both are interesting studies for researchers because it is known that Riemann’s geometry is not only limited to $3+1$ dimensions and how the dynamics that occur if there is an addition of external fields to Einstein’s theory [3]. This study focus on the modification of Einstein’s theory with an external scalar field. The Horndeski’s
theory is a theory of gravity with the most common scalar-tensor action with the term of a second order differential equation [4]. The non minimal derivative coupling (NMDC) is a coupling theory which is part of Horndeski theory has been widely used to explain the phenomenon of the universe which is first introduced by [5]. Some studies related to NMDC applications include the study by [6] concerning the NMDC theory for analyzing the inflation and the study by [7] has shown the results of NMDC analysis for the study of accelerating expansion of the universe. Whereas, NMDC studies on extra dimensions have been carried out by [8]. In some examples of studies that have already been mentioned, there has been no boundary involvement of the exact value of the coupling parameter $\xi$ in the theory. Therefore, in this study we will look for the limits of coupling constants that are in accordance with the cosmological material conditions that are generally used, namely through energy conditions. Energy conditions are some mathematical conditions that restrain the arbitrariness of the matter in the theory of gravity. There are four conditions that commonly used; strong energy condition (SEC), weak energy condition (WEC), null energy condition (NEC), and dominant energy condition (DEC) [1]. They used as a tool to determining the constrains of the coupling constant, then present coupling constants with other cosmological parameters such as the Hubble constant $H$ and scale factor $a$. SEC violation are related to the occurrence of the repulsive gravity phenomenon and violations of this condition occur earlier than the other energy conditions [9]. In addition, the study by [10] shows that in the early era of the universe strong energy condition were violated first. The follow-up study of this paper is to obtain a list of coupling constant restrictions that are valid for each other energy conditions; WEC, NEC, and DEC. That way, for other models we can use the same method to produce different physical results.

2. Perfect Fluids Equations of NMDC

Let the scalar field $\phi(t)$ as an additional field in the NMDC scalar field theory besides the ordinary matter so that it can be written the action as

$$S = S_H + S_\phi + S_M,$$  \hspace{1cm} (1)

with first two terms state as follows

$$S_H = \int \sqrt{-g} R d^4x,$$  \hspace{1cm} (2)

and

$$S_\phi = \int d^4x \sqrt{-g} \left( \frac{1}{2} g_{\mu\nu} D^\mu \phi D^\nu \phi + \frac{\xi}{2} R g_{\mu\nu} D^\mu \phi D^\nu \phi + \frac{\eta}{2} R_{\mu\nu} D^\mu \phi D^\nu \phi \right),$$  \hspace{1cm} (3)

using $2\xi + \eta = 0$ to reducing the high order of differentiation, so from the action above can be obtained the expression of modified Einstein equation $G_{\mu\nu} + \kappa^2 H_{\mu\nu} = \kappa^2 T_{\mu\nu}$, where $G_{\mu\nu}$ is the Einstein tensor from the variation of Hilbert action $S_H$ respect to $g^{\mu\nu}$, $\kappa^2$ is Einstein constant, $H_{\mu\nu}$ is a tensor that produced from the variation of the NMDC action $S_\phi$ by [8] respect to $g^{\mu\nu}$, and the energy momentum tensor $T_{\mu\nu} = \frac{\sqrt{-g}}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}$. For the case to be reviewed, flat FRW metrics are used, which are applied to four energy conditions in cosmology by [1], namely

Weak Energy Condition (WEC)

$$T_{\mu\nu} t^\mu t^\nu \geq 0, \quad t^\mu t_{\mu} < 0.$$  \hspace{1cm} (4)

Null Energy Condition (NEC)

$$T_{\mu\nu} l^\mu l^\nu \geq 0, \quad l^\mu l_{\mu} = 0.$$  \hspace{1cm} (5)
Strong Energy Condition (SEC)

\[ R_{\mu\nu} t^\mu t^\nu \geq 0, \quad t^\mu t_\mu < 0. \]  

(6)

Dominant Energy Condition (DEC)

\[ T_{\mu\nu} t^\mu t^\nu \geq 0, \quad t^\mu t_\mu < 0. \]  

(7)

This study concerns only the SEC, and the vector \( T_{\mu\nu} t^\mu \) must obey nonspacelike vectors. From the definition above one can have the two equations of states that describe the relation between the density \( \rho \) and the pressure \( p \) of the perfect fluids also the scalar field \( \phi \). Indeed, the equation is complicated and we need to reduce its complexity by removing the term of the first and the second differentiation by time of the scalar field (i.e. \( \dot{\phi} \) and \( \ddot{\phi} \)) with the equation of states from the components of the Einstein equation above. To determine the value of the coupling parameter \( \xi \) we can use the current data on a cosmological variable such as \( H, a \), and the deacceleration parameter \( q \equiv -\frac{1}{H^2}(\dot{H} + H) \) which introduced by \([11]\). After putting the simplifications before, the equation of states from the strong energy conditions reads

\[
\frac{7}{2} + 3\xi H_0^2(4q_0 - 1) \leq \frac{\rho_0 + \frac{\frac{1}{2} H_0^2(2q_0 - 1)}{\rho_0 - \frac{3H_0^2}{\kappa^2}}}{\frac{1}{2} + 9\xi H_0^2} \]  

(8)

and

\[
\frac{\rho_0 + \frac{\frac{1}{2} H_0^2(2q_0 - 1)}{\rho_0 - \frac{3H_0^2}{\kappa^2}}}{\frac{1}{2} + 9\xi H_0^2} \geq \frac{\frac{7}{2} + 3\xi H_0^2(7 + 4q_0)}{\frac{1}{2} + 9\xi H_0^2}. \]  

(9)

The subscript indicate that each variable was measured in current values. Finally, to determining the restriction of the coupling constant \( \xi \) one can use the current values of \( \rho_0 \approx 9.73 \times 10^{-27} \text{ kg/m}^3 \), \( H_0^2 \approx 5.3 \times 10^{-36} \text{ /s}^2 \), and \( \kappa^2 \approx 1.86 \times 10^{-26} \text{ m/kg} \) [1]. The parameter \( q \) can be found in [11] which the value is \( q_0 = -0.81 \pm 0.14 \). After putting the cosmological values into the equations (8) and (9) we have \( \xi \geq 1.2 \times 10^{34} \text{ s}^2 \) for the SEC is not violated. If we compare the value to the Planck mass which is has the order of \( 10^{42} \), the coupling \( \xi \) then acts like the perturbation of the the gravity. Just the common sense that the perturbation must be smaller than the background. After taking this calculation, there are three energy conditions remaining that need to be concerned. If we have all the constrains, one can determine the violated condition of \( \xi \) for the reasons by [9] that in our current universe all the energy condition must be violated.

3. Conclusion

The strong energy conditions in cosmology tell us how the changes in conditions that occur are observed by timelike observers if they fulfill Einstein’s modified theory with the NMDC of scalar field as the external field. The number of equations that result from the calculation of the SEC with modified Einstein’s theory is the equation of state between the energy density of \( \rho \) and the momentum of \( p \). The \( q \) deacceleration parameter was introduced so that the boundary of the coupling constant value in the NMDC theory of the scalar field can be known. The value of parameters \( H, \rho, \) and \( q \) in the equation are now taken to find the limit of \( \xi \), the same method by [12]. The result is that the strong energy condition of the modified Einstein theory for NMDC cosmology the scalar field must be \( \xi \geq 1.2 \times 10^{34} \text{ s}^2 \). This result is far from the order of the Planck’s mass. It needs to be calculated the restrictions of the \( \xi \) by other energy conditions so that the one can be determined the limit according to the current state of the universe.
References
[1] Sean M C 2004 Spacetime and geometry, an introduction to general relativity *Addison-Wesley*
[2] Gerson G 2009 *AIP Conf. Proc.* **1166** 53
[3] Clifton T, Ferreira P G, Padila A and Skordis C 2012 *Phys. Rep.* **513** 1-189
[4] Gergely L A and Tsujikawa S 2014 *Phys. Rev. D* **89** 064059
[5] Lucia A 1993 *Phys. Lett. B* **301** 175-182
[6] Capozziello S, Lambiase G and Schmidt H J 2000 *Ann. Phys.* **9** 39-48
[7] Granda L N 2010 *JCAP* **1007** 006
[8] Agus S and Freddy P Z 2013 *Gen. Relativ. Gravit.* **45** 799-809
[9] Visser M and Barcelo C 1999 *Plenary talk delivered at Cosmo99, Trieste*
[10] Santos J, Alcaniz J S, Pires N and Robucas M J 2007 *Phys. Rev. D* **75** 083523
[11] David R, Steven W A, Mustafa A A and Roger D B 2007 *Mon. Not. R. Astron. Soc.* **375** 1510-1520
[12] Santos J, Alcaniz J S, Rebouças M J and Carvalho F C 2007 *Phys. Rev. D* **76** 083513