A Two-Step Damage Identification Based on Cross-Model Modal Strain Energy and Simultaneous Optimization

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Abstract. Aiming at the problem of the effect on damage identification caused by noise and modal incompleteness, a two-stage damage identification method based on cross-model modal strain energy and simultaneous optimization was proposed. This paper proposed an improved damage index (Modified Index of Cross-model Modal Strain Energy, MICMSE) based on cross-model modal strain energy. This method first locates the damaged elements by MICMSE, and introduces simultaneous optimization to quantify the damage ratios of damaged elements. The two-step method avoids the calculation of the sensitivity of the modal parameters and the reanalysis of the characteristic equation, speeds up the convergence speed, and guarantees the recognition accuracy. The results of the numerical example show that the method proposed in this paper can effectively locate the damage elements and identify the damage ratios under different damage cases and noise levels.

1. Introduction

During the service period of civil engineering structures, due to long-term load, fatigue effects, environmental corrosion, material aging, and mutation effects, etc., the internal structure will inevitably cause damage to the structure. When the damage accumulates to a certain extent, it may cause serious accidents such as collapse [1-2]. Therefore, structural health monitoring has become one of the key and hot issues in the field of civil engineering [3].

As the core part of structural health monitoring, structural damage identification is particularly important. Structural damage identification method based on dynamic characteristics is currently the most commonly used method [4]. It can be divided into dynamic fingerprint method without inversion and model correction method with inversion according to whether inversion is required. The dynamic fingerprint method directly uses the modal parameters and their deformation for damage identification. The expression is clear and does not require inversion, such as frequency [4-5], mode shape [6-7], modal strain energy [8-11]. The modal strain energy method is widely used because of its stability and effectiveness. Wu et al [10] proposed a location index based on cross-model modal strain energy and found its robustness was better than traditional modal strain energy. However, the mode shape is only a series of proportional coefficients and has no actual size significance, so it is necessary to study the difference between the cross-model modal strain energy of different modes.

Although the dynamic fingerprint method is simple in form and clear in the physical meaning, it can only achieve damage location but not damage quantification. The commonly used sensitivity analysis method firstly calculates the sensitivity of modal parameters to structural parameters and then obtains the changes of structural parameters by model updating [12]. This method needs to calculate the first and second derivatives of modal parameters (frequency and mode), which leads to low solution efficiency and complicated solving process. Simultaneous optimization can solve this problem well and also avoid...
the errors produced by modal expansion. Zhang et al [13] took a time-domain square distance between the measured and the computed responses as an objective function and used the virtual distortion method to solve it. Jayalakshmi [14] formulated the inverse problem associated with system identification as a simultaneous optimization problem and used a newly developed dynamic hybrid adaptive firefly algorithm to solve this problem. However, when there are too many parameters to be modified or the noise level is too high, the result of simultaneous optimization will produce a large error.

Therefore, combining the advantages of cross-model modal strain energy and synchronization optimization, this paper proposed a two-stage damage identification method. First, the damage index was constructed based on the cross-model modal strain energy. Then, aiming at the problem of a large amount of calculation in traditional sensitivity analysis, simultaneous optimization was used to identify structural damage. Finally, a numerical simulation example was used to verify the effectiveness of the method proposed in this paper.

2. Theory

2.1. Definition of damage

Structural damage usually causes the degradation of structural stiffness without affecting the structural quality. Therefore, this paper defines structural damage as the degradation coefficient of the element stiffness matrix of each element.

For a structure with n DOFs and ne elements, the element stiffness matrix of the jth element after structural damage is

$$
K^d = \sum_{j=1}^{ne} K_j (1 - \alpha_j)
$$

where $j = 1, 2, \ldots, ne$ denotes the number of elements. Additionally, $d$ represents the damage structure and $K$ denotes the element global stiffness matrix. And $\alpha_j$ denotes the damage coefficient of $j$th element.

2.2. Cross-model modal strain energy

In 1988, Chen et al [8] proposed the concept of modal strain energy (MSE) and used it for damage location. Define the modal strain energy of $j$th element corresponding to $i$th mode before and after structural damage as [8]

$$
MSEC_{ij} = MSE^a_{ij} - MSE^d_{ij} = \phi_i^T K_j \phi_i - \phi_i^T K_j \phi_i
$$

where $i = 1, 2, \ldots, nm$ denotes the number of measured modes. And $\phi_i$ is $i$th eigenvector of the structure. Additionally, $MSEC_{ij}$ represents the change of modal strain energy.

From equation (2), the larger $MSEC_{ij}$, the more likely $j$th element is damaged. Due to the incomplete modal and noise interference, the location result of modal strain energy will produce errors. In order to solve this problem, an indicator based on cross-model modal strain energy was proposed as follows [11]:

$$
F_{ij} = E^a_{ij} - E^d_{ij} = \phi_i^T K_j \phi_i - \phi_i^T K_j \phi_i
$$

where $E^a_{ij}$ denotes cross-model modal strain energy. And $F_{ij}$ represents the change of cross-model modal strain energy.

Considering the difference between different modes, this paper proposed an improved damage index (Modified Index of Cross-model Modal Strain Energy) based on the change of cross-model modal strain energy, it is given by
where mean represents the average value and std denotes the standard deviation.

2.3. Simultaneous optimization

Simultaneous optimization mode is to unify the state variables and design variables in the traditional structural optimization problem as the design variables of the new problem, thereby transforming the complex structural optimization problem into a simple nonlinear numerical optimization problem [15]. This method can avoid structural reanalysis and complicated sensitivity calculations to improve the computational efficiency and identification result.

2.3.1. Design variables.

When simultaneous optimization is used for damage identification, the design variables of the optimization problem need to be determined first. Considering that the frequency usually has high measurement accuracy in actual engineering, in order to further improve the calculation efficiency, this paper only used the damage coefficients and dynamic feature vectors of the structure as design variables.

The design variables are given by

$$\{d_1, d_2, \ldots, d_{n_m}, d_{11}, d_{21}, \ldots, d_{n_m 11}, d_{12}, d_{22}, \ldots, d_{n_m 22}, \ldots, d_{1n_m}, d_{2n_m}, \ldots, d_{nn_m}\}$$  \hspace{1cm} (5)

The boundary constraints that the design variables need to satisfy are as follows:

$$0 \leq \alpha_j \leq 1$$
$$d_{lb} \leq \phi_i \leq d_{ub}$$  \hspace{1cm} (6)

where \(lb\) and \(ub\) are the lower bound and upper bound.

For the mode shape, it can be divided into two cases whether the test can be obtained. For the mode value that cannot be obtained by the test, this paper assigns a larger boundary as the limit constraints:

$$d_{lb} = -\infty, d_{ub} = +\infty$$  \hspace{1cm} (7)

For the mode value that can be obtained by the test, considering the error in the actual measurement, if the noise level can be determined in advance, then its approximate range can be given by

$$\begin{cases} 
\phi_i^{d,lb} = \phi_i^d / (1 + \sigma \gamma), & \phi_i^{d,ub} = \phi_i^d / (1 - \sigma \gamma), \\
\phi_{i1}^d \geq 0, & \phi_{i2}^d < 0 
\end{cases}$$  \hspace{1cm} (8)

where \(\sigma\) and \(\gamma\) are the guarantee factor and estimated noise level.

2.3.2. Objective function.

In the real damage process of the structure, the damage is generally sparse damage which means only a few components in the structure are damaged. Therefore, this paper takes the L2 norm of the structural damage coefficient vector as the objective function of the optimization problem:

$$\min f = ||\alpha||_2$$  \hspace{1cm} (9)

2.3.3. Constrained function.

Without considering the damping, the characteristic equation of the damaged structure is

$$K^d \phi^d - \lambda^d \phi^d = 0$$  \hspace{1cm} (10)
where $K$ and $M$ are the global stiffness and mass matrix of the structure. And $\lambda_i^d$ denotes the $i$th modal eigenvalue of the damaged model.

Substitute equation (1) into equation (10), the first constrained function can be expressed as follow:

$$ \sum_{r=1}^{n} \left[ \sum_{j=1}^{n_e} k_{ij} (1 - \alpha_j) \right] \phi_i^d - \lambda_i^d \sum_{r=1}^{n} m_r \phi_i^d = 0 \tag{11} $$

where $k_{ij}$ denotes the contribution of the $j$th element to the global stiffness matrix in a specific degrees of freedom. And $m_r$ is the component of the global mass matrix in the corresponding degrees of freedom.

Additionally, for the mode shape, the conditions of mass normalization need to be satisfied. Then, the second constrained function is given by

$$ \sum_{j=1}^{n_e} \sum_{r=1}^{n} m_r \phi_i^d \phi_j^d = 1 \tag{12} $$

2.3.4. Optimization model. Finally, the optimization model determined in this paper is as follows

$$ \min f = \| \alpha \|_2 $$

s.t. 

$$ \sum_{r=1}^{n} \left[ \sum_{j=1}^{n_e} k_{ij} (1 - \alpha_j) \right] \phi_i^d - \lambda_i^d \sum_{r=1}^{n} m_r \phi_i^d = 0 $$

$$ \sum_{j=1}^{n_e} \sum_{r=1}^{n} m_r \phi_i^d \phi_j^d = 1 $$

$$ 0 \leq \alpha_j \leq 1 $$

$$ \phi_{i_{\text{lb}}} \leq \phi_i^d \leq \phi_{i_{\text{ub}}} $$

where $l = 1,2,3,...,n$, $r = 1,2,3,...,n_e$, $j = 1,2,3,...,n_e$, $i = 1,2,3,...,nm$

This paper used the trust region-based SQP algorithm to solve the above optimization problem.

2.4. Noise
In actual measurement, the data will inevitably be interfered by noise, so it is necessary to consider the influence of noise on the modal test data. But the frequency can be approximated as high accuracy in most cases. Therefore, this paper only used numerical simulation to impose noise on the mode shape as follow:

$$ \phi_i^d = \phi_i^d (1 + \eta R_i \phi_{i_{\text{max}}}) \tag{14} $$

where $\phi_i^d$ and $\phi_i^d$ are the $i$th mode value before and after imposing noise. And $\eta$ denotes the disturbance level, $R_i$ is the matrix of Gaussian random numbers with mean value 0 and variance of 1. Additionally, $\phi_{i_{\text{max}}}$ represents the maximum value of the $i$th mode shape.

3. Numerical example
In order to verify the effectiveness of the method proposed in this paper, a numerical simulation of damage identification was carried out on a 45-element two-span steel truss shown in figure 1. The area of the truss members is $A = 0.02 m^2$, the modulus of elasticity is $E = 206 GPa$, and the density is $\rho = 7850 kg \cdot m^3$. 
It is assumed that the first 3 modal information before and after structural damage can be obtained. Table 1 indicates the sensor placement of the structure, and the sensor placement method adopts the sensor placement method proposed by Zeng [16].

Table 1. The sensor placement of the structure.

| Sensor number | Node | DOF direction | Sensor number | Node | DOF direction |
|---------------|------|---------------|---------------|------|---------------|
| 1             | 2    | y             | 9             | 13   | y             |
| 2             | 3    | y             | 10            | 14   | y             |
| 3             | 4    | y             | 11            | 16   | y             |
| 4             | 7    | y             | 12            | 17   | y             |
| 5             | 8    | y             | 13            | 18   | y             |
| 6             | 9    | y             | 14            | 19   | x             |
| 7             | 10   | y             | 15            | 19   | y             |
| 8             | 12   | y             | 16            | 20   | y             |

Three damage conditions are set up (see Table 2), and the effects of three noise levels of 0%, 3% and 5% on the results of structural damage identification are considered.

Table 2. Damage cases.

| Damage cases | Damage elements and ratios |
|--------------|---------------------------|
| 1            | The damage element is 8, the damage ratio is 25% |
| 2            | The damage elements are 16 and 18, and the damage ratios are 20% and 20% |
| 3            | The damage elements are 9, 33 and 45, and the damage ratios are 25%, 20% and 30% |

Figure 2 ~ figure 4 indicate different values of MICMSE for different cases with considering different noise levels. It can be seen from figure 2 that, for single damage, the MICMSE can accurately locate the damage element even considering 5% noise level and modal incompleteness. And from figure 3 and figure 4, for multiple damages, although there exist false locations, the damaged elements can still be identified under 5% noise level. It can be also seen that the elements that are mistakenly located are all adjacent elements of the damaged element, which is also one of the common problems of modal strain energy methods mentioned in chapter 2. Additionally, the figures show that the location results remains basically unchanged under the influence of noise. Hence, the MICMSE has strong locating ability and robustness in different damage cases.

Figure 2. Values of MICMSE for case 1.  
Figure 3. Values of MICMSE for case 2.
To study the effect of location for simultaneous optimization, this paper used simultaneous optimization with pre-location by MICMSE and direct simultaneous optimization for structural damage identification respectively. Figure 5 ~ figure 7 indicate the damage identification results of simultaneous optimization with pre-location by MICMSE for different cases with considering different noise levels. And figure 8 ~ figure 10 indicate the damage identification results of direct simultaneous optimization for different cases with considering different noise levels. From figure 5 ~ figure 7, it can be seen that simultaneous optimization with pre-location can identify the damage accurately even under 5% noise level. And for different damage cases, the identification result of this method can still maintain high accuracy. On the other hand, the simultaneous optimization can make up for the mistakenly location of MICMSE if the location result of MICMSE contains damaged elements. As a comparison, figure 8 and figure 9 show that simultaneous optimization can identify damage elements accurately under low noise level, but excessively high noise level could make the results of simultaneous optimization occur large errors. Additionally, from figure 10, the direct simultaneous optimization method may occur mistakes no matter how large the noise level is. In short, the number of...
parameters to be modified and noise level have a strong impact on the results of simultaneous optimization. Therefore, pre-location by MICMSE has a significant influence on the identification result’s accuracy of structural simultaneous optimization.

Table 3 indicates the number of iterations for the 2 methods in different damage cases. It can be seen that, in most cases, the two-step method has fewer iterations than direct simultaneous optimization, which means the two-step method’s problem-solving efficiency is stronger. As for case 1 with 1% noise, case 1 with 5% noise, and case 3 with 5% noise, although the number of iterations of direct simultaneous optimization is less, it can be seen from figure 8 and figure 10 that it sacrifices solution accuracy. The results of the numerical example show that the two-step method proposed in this paper can identify single and multiple damage cases accurately and has strong robustness and solve efficiency.

Table 3. Number of iterations of different methods.

| Methods                        | Case 1 no noise | Case 1 1% noise | Case 1 5% noise | Case 2 no noise | Case 2 1% noise | Case 2 5% noise | Case 3 no noise | Case 3 1% noise | Case 3 5% noise |
|--------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| MICMSE-Simultaneous optimization | 6               | 6               | 6               | 6               | 7               | 7               | 5               | 5               | 6               |
| Simultaneous optimization      | 7               | 3               | 4               | 8               | 10              | 10              | 6               | 8               | 4               |

4. Conclusions
In the practice of structural damage identification, the limited sensors and noise can make the test data incomplete and exist errors. To solve this problem, this paper proposed a two-step method based on cross-model modal strain energy and simultaneous optimization. Therefore, a steel truss is utilized to assess the performance of the two-step method and direct simultaneous optimization. Different noise levels are applied to the mode shapes of damaged structures to assess the robustness of the two methods. The results showed that the damage locating index (MICMSE) proposed in this paper could locate damage elements under different noise levels. But there also existed extra locations because the adjacent elements were affected by damaged elements so that the index may be large too. This problem could be solved by simultaneous optimization in the two-step method. Additionally, the two-step method had better performance on the robustness, accuracy, and efficiency than the direct simultaneous optimization method. Moreover, simultaneous optimization avoided the structural reanalysis and calculation of sensitivity. Finally, the combination of MICMSE and simultaneous optimization took the location ability of MICMSE to improve the accuracy and efficiency of simultaneous optimization, and it also used the latter one’s identification to solve the extra location problem of the former one.

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