High Performance Information Reconciliation for QKD with CASCADE

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(Dated: May 11, 2014)

It is widely accepted in the quantum cryptography community that interactive information reconciliation protocols, such as CASCADE, are inefficient due to the communication overhead. Instead, non-interactive information reconciliation protocols based on e.g. LDPC codes or, more recently, polar codes have been proposed. In this work, we argue that interactive protocols should be taken into consideration in modern quantum key distribution systems. In particular, we demonstrate how to improve the performance of CASCADE by proper implementation and use. Our implementation of CASCADE reaches a throughput above 80 Mbps under realistic conditions. This is more than four times the throughput previously demonstrated in any information reconciliation protocol.

I. INTRODUCTION

Information reconciliation (IR) in quantum key distribution (QKD) is a protocol where Alice and Bob, by public discussion over an authenticated classical channel, correct the discrepancies between the bit strings which they obtained through usage of the noisy quantum channel.

IR has been the bottleneck in many QKD systems, both discrete\cite{1,2} and continuous\cite{4,5} variable. In this paper, we address the IR problem for the binary symmetric channel, which is the model used for discrete variable QKD.

There are two main measures of performance of an IR protocol: Efficiency (e.g. the ability to correct the discrepancies without revealing more information than necessary to an eavesdropper) and throughput (e.g. how many input bits per second can be processed).

The CASCADE IR protocol\cite{6} is simple and probably the most widely used in QKD implementations. In CASCADE, Alice and Bob first permute and partition a frame of bits. They then compare the parities of each partition. When they disagree on the parity of a partition, the partition is split into two and the parities of the two halves are compared. The half where the parities disagree is then recursively split and checked until the error is found and corrected. This procedure continues for a few rounds with different permutations and partition sizes. As is easily seen, CASCADE is highly interactive which makes it very sensitive to network latencies. It is commonly believed that the interaction in CASCADE causes low throughput\cite{3,6,7,8}. In contrast to CASCADE, modern IR protocols based on forward error correction methods, such as LDPC or polar codes, are non-interactive. They do, however, require more computation than CASCADE.

Efficiency is the performance measure most commonly addressed in works on IR (See i.e. \cite{6,9,14}). However, as argued in \cite{3,6}, the trade-off between communication and computation costs must be carefully evaluated when choosing IR protocols. As long as the IR has a higher throughput than the provided raw-key rate, efficiency is the dominating performance criteria. However, if the IR throughput is low, a combination of the throughput and efficiency determines the performance of the whole QKD system.

In low latency networks, it is not a priori clear whether computationally simple but interactive IR protocols will perform worse than computationally complex but non-interactive protocols. The aim of this work is to challenge the assumption that interactive IR protocols, such as \cite{3,9,14}, have worse performance than non-interactive ones. To support our claim, we demonstrate that CASCADE, by proper implementation and usage, can outperform state of the art IR protocols\cite{2,3,5,8,13} for many settings which are relevant to QKD, such as QKD over fiber channels.

In most realistic deployments of QKD, the authenticated classical channel will have a latency of at most a couple of milliseconds. In a QKD system over fiber, it is fair to assume that the classical channel is either multiplexed with the quantum channel\cite{2,16} or sent over another fiber in the same fiber bundle. For the distances typical for a QKD system, a direct fiber connection will give latencies close to 1 ms. Even for free-space QKD with low earth orbit satellites\cite{17}, the latency will not exceed more than a couple of milliseconds. We demonstrate how CASCADE can obtain a throughput above 80 Mbps in the very common low latency scenarios. Only when the network latency exceeds 10 milliseconds, does the throughput of our implementation of CASCADE become too low for state-of-the-art QKD systems. A potential setting where the latency is high enough to rule out CASCADE is QKD with a geostationary satellite where the latency will be a few hundred milliseconds.

This paper is organized as follows: We give the theoretical background of IR performance in Section II. To justify the parameters which we use in our tests, Section III lists a few state-of-the-art QKD systems and their requirements for the IR protocol. We give a brief outline of some of the best performing IR protocols in Section IV and a description of our implementation of the CASCADE IR protocol in Section V. The results of our experiments are listed in Section VI followed by a few concluding
II. SECRET KEY RATE

After performing sifting and error estimation, Alice and Bob each have a frame of a predetermined number of bits. Alice’s and Bob’s frames are represented by random variables $A$ and $B$, respectively. During information reconciliation, Alice and Bob exchange information which will allow Bob to compute the value of $A$. The process should leak the smallest possible amount of information about $A$ to the eavesdropper. It follows from the noiseless coding theorem that the minimum amount of information which Alice and Bob need to exchange is $H(A|B)$, leaving at most

$$H(A) - H(A|B) = I(A:B)$$

(1)

bits of information which is unknown to an eavesdropper listening to the communication on the classical channel.

In a practical implementation of IR, however, Alice and Bob will exchange more than $H(A|B)$ bits of information. The efficiency of an IR protocol is a number $\alpha \in [0,1]$ such that Alice and Bob can extract $\alpha I(A:B)$ bits when IR succeeds. A further limitation of practical implementations of IR protocols is the probability that the protocol fails for a given frame. This probability is called the frame error rate (FER).

Besides the information leaked during IR, Alice and Bob also need to take the information which the eavesdropper obtained during their use of the quantum channel into account. Putting together all these factors, the maximum number of secret bits which Alice and Bob can extract from a frame is

$$(1 - FER)(\alpha I(A:B) - I_E),$$

(2)

where $I_E$ is a measure of the information which the eavesdropper has obtained during the quantum part of the protocol. The value of $I_E$ depends on both the specific QKD protocol and the security proof used.

If raw key is provided at a rate of $R_s$ bits per second, and the IR is capable of correcting at that rate, the secret key rate is at most

$$R_s(1 - FER)(\alpha I(A:B) - I_E)$$

(3)

bits per second, in which case the IR must reduce the FER and improve the efficiency in order to get the best possible utility out of the quantum channel.

If, however, the IR protocol is only capable of correcting at a rate of $R_{IR} < R_s$ bits per second, then the IR protocol becomes the bottleneck in the system. Several strategies can be used to improve on this situation. Some IR protocols, such as the ones based on LDPC codes, can improve the throughput by allowing a higher frame error rate. Other IR protocols may have a throughput-efficiency trade-off which allow them to obtain a higher throughput by sacrificing efficiency. If we let the variable $x$ describe the parameters which influence the trade-offs of a given IR protocol, the maximal secret key rate, the performance, becomes

$$R_{IR}(x)(1 - FER(x))(\alpha(x)I(A:B) - I_E)$$

(4)

bits per second, where $R_{IR}$, $FER$, and $\alpha$ are all functions of the parameters, $x$. In CASCADE, for instance, changing partition sizes will change both throughput and efficiency. In the belief propagation decoders commonly used in LDPC, the number of iterations gives a trade-off between throughput and FER. In the scenario of Eq. (4), where IR throughput is lower than raw-key rate, the trade-off must be carefully evaluated to find the maximum secret key rate.

III. STATE OF THE ART QKD

The choice of information reconciliation protocol and trade-offs between efficiency, throughput, and frame error rate depends on the raw-key rate, the latency and bandwidth of the classical channel, and the quantum bit error rate (QBER). The aim of this paper is to demonstrate that interactive IR protocols (in particular CASCADE) should be taken into consideration in current state-of-the-art QKD systems. To support our thesis, we list the properties of some of the state-of-the-art QKD systems.

Several recent QKD experimental setups have reached secret key rates of up to 1 Mbps \cite{1, 16, 18, 19}. In a series of papers \cite{1, 16, 20}, the group at Toshiba Research documents progress with their 1 Mbps QKD system based on the BB84 protocol \cite{21} with decoy states \cite{22}. While the secret key rate has been stable at 1 Mbps, the distance has increased from 20 km \cite{1} to 50 km \cite{21}, and the duration of stable operation has increased from a few seconds \cite{1} to virtually unlimited \cite{20}. They use optical fiber in their work, and experience a QBER of 2.5%–4% depending on the setting. The raw-key rate is between 1.5 Mbps \cite{16} and 3.44 Mbps \cite{1}. An important new achievement is the multiplexing of the quantum and classical channels \cite{18}. This allows cheap and very low latency classical communication. In their older work, they used the CASCADE IR protocol \cite{22}. In their high-speed QKD systems \cite{1}, they identify the need for high-speed IR protocols. In their recent work \cite{16}, the secret key rate is simulated under the assumption of an information reconciliation protocol with the same efficiency as CASCADE, but with a throughput which is fast enough to keep up with the raw-key rate.

The group at Université de Genève has a series of QKD implementations based on the COW \cite{24} protocol. Details of the work can be found in the ph.d. dissertation of Nino Walenta \cite{18}. At Qcrypt 2012, the group reported on a QKD system capable of maintaining a secret key rate of 0.88 Mbps over 1 km fiber \cite{2}. In principle, the system should be capable of keeping a secret key rate of 1 Mbps for fibers of up to 20 km. Continuous operation
of the high secret key rate was possible due to a high-speed hardware post processing system implemented in FPGA. The operational QBER is from 1.5% to 2.25%. Experiments over 25 km fiber showed a raw-key rate of 6.29 Mbps over a period of 8 hours.

Meanwhile, the group at NIST achieved 4 Mbps raw-key rate in their BB84 implementation over 1 km fiber in 2006. The classical channel used a separate 1 km fiber. The QBER was 3.42%.

The longest QKD link reported is 260 km over standard telecom fiber. The secret key rate was 1.85 bit/s and the raw key rate less than 100 bit/s. Nonetheless, the QBER was still only 3.46%. A classical link sent over a fiber of the same length will have a latency of approximately 1.5–2 ms (including media converters and authentication).

As demonstrated above, state-of-the-art QKD systems require IR protocols capable of correcting a QBER of 1.5%–4% with a throughput of at least 1–6.29 Mbps. The distance of fiber based QKD systems is still limited to a few hundred kilometers at best. With the ability to either multiplex quantum and classical channels, or just use different fibers in the same fiber bundle, latencies will be close to 1 ms. The focus of our work is this range of parameters.

IV. HIGH PERFORMANCE INFORMATION RECONCILIATION

To the best of our knowledge, the highest throughput reported for an implementation of cascade is 5.5 Mbps with a QBER of 3.8% and a frame size of 1 Mbit. The implementation uses multiple threads on quad-core computers, and the classical channel is sent over a 45 km optical fiber in the same fiber bundle as the quantum channel.

In [8], the authors present IR protocols based on both LDPC and polar codes. Their implementations are tested on a channel with 2% QBER. They obtain the highest throughput of 10.9 Mbps with polar codes on a 3.47GHz Intel Core i5 processor using a single core. The block size is 2^{16} bits, the frame error rate is 9%, and the efficiency is 93.5%. By increasing the frame size to 2^{24} bits, the authors improved the efficiency to 98%, but by sacrificing the throughput which dropped to 8.3 Mbps. They present two implementations of IR based on LDPC codes: One implemented on GPU (AMD Tahiti Graphics Processor) and one on CPU (same processor as for the polar code implementation). The GPU implementation has a throughput of 7.3 Mbps, a FER of 1%, and an efficiency of 92.9%, while the CPU implementation has a throughput of 0.83 Mbps, a FER of 3%, and an efficiency of 93.1%. In both cases the block size is 131072 bits.

Another implementation of LDPC on GPU (NVIDIA GeForce GTX 670) is presented in [9]. The authors achieve a throughput of up to 10.3 Mbps with a rate-adaptive LDPC code. The authors point out that LDPC codes have a trade-off between FER, efficiency and throughput. When the code operates close to the Shannon limit, either a non-negligible FER or a limited throughput must be accepted.

The fastest IR protocol known to us is the LDPC code implemented on FPGA (Xilinx Virtex 5) presented at Qcrypt 2012. They use standard IEEE 802.11n quasi-cyclic LDPC codes with a frame size of 1944 bits. Their IR protocol has been demonstrated to process 20 Mbps raw-key. They claim that it will be capable of operating at up to 40.8 Mbps in the future. The IR protocol has a FER of up to 10%.

For their QKD system, NIST has implemented both cascade and LDPC on FPGA, achieving 5 and 4 Mbps, respectively. In line with our thesis, the authors of [8] indicate that cascade has a higher throughput than LDPC for distances up to at least 100 km. In [10], Alan Mink of NIST reaches 12.2 Mbps for 1% QBER with an unspecified IR protocol by running 4 threads in parallel on an FPGA board.

V. CASCADE

Interaction is the main performance concern in cascade. The first natural step towards an efficient implementation of cascade is therefore the reduction of interaction. As pointed out by Louis Salvail at a SECOQC project meeting, the parity checks of partitions and sub-blocks can be done in parallel. First, the parities of all partitions of a frame are exchanged in a single message. Then, instead of correcting each partition with errors one by one, all partitions with errors are split, and the parities of their sub-blocks are sent in a single message. The binary search for errors continues in the same fashion by splitting sub-blocks with errors and exchanging their parities in single messages before splitting again. For a partition size of k bits, the total number of messages exchanged during the binary search is then \( \log_2(k) \) regardless of the number of partitions and the size of the frame. This approach is also used by the cascade implementation in the AIT QKD software project.

When an error is detected in round two or later, it implies that the error was located in partitions with even numbers of errors after each previous round. The original description of cascade has a look-back step after each round to take advantage of this fact. For instance, consider a partition which contains two undetected errors after the first round (Alice and Bob agreed on the parity of that partition after round one). If one of the errors is corrected in round two, the other error can be corrected by applying the binary search for errors on the partition from round one (which contains exactly one error after the first error was corrected). In look-back, for each error corrected in a round, all partitions from previous rounds containing that error are added to a look-back list. The binary search for errors is then applied to each of the partitions in the look-back list, starting with the
smaller partition. For each new error corrected during look-back, the partitions from all rounds containing that error are also added to the look-back list. This procedure continues until the look-back list is empty.

In the original formulation of the look-back step, only one partition can be corrected at one time. This imposes a high level of interaction. To overcome this problem, we propose a slight modification to the look-back step: Instead of only applying the binary search to the smallest partition, it is applied to all partitions of the earliest previous round which still has partitions which may contain errors. Since the corrections are done to non-overlapping partitions from the same round, we can apply the binary search in parallel as described above. During the second round, this will exactly be the same partitions as visited during the first look-back in the original protocol. As demonstrated in Section VI this modification does not reduce the efficiency noticeably for a QBER below 10%.

With our approach, the number of messages which have to be sent between Alice and Bob is

\[ r + \sum_{i=1}^{r} \lceil \log_2(k_i) \rceil + l(\text{QBER}, n, k_1, \ldots, k_r), \]  

(5)

where \( r \) is the number of rounds, \( k_i \) is the partition size used in the \( i \)th round, and \( l \) is a function describing the number of messages exchanged during look-back. For the partition sizes used in [6], \( l \) only increases logarithmic in the frame size, \( n \) (details in Section VI).

An important feature of equation 5 is that, again for the standard partition sizes, it increases sub-linearly in the frame size. To get the biggest possible advantage of this implementation, the frame size should be as large as possible. The larger the frame size, the more weight is moved away from the communication and onto the computation of the protocol. After a certain frame size, most of the time will be spend on computation (calculation of parities).

The major factor in limiting the maximum possible frame size is memory. The memory required increases linearly with the frame size. It is therefore important to use as little memory as possible. A first, trivial step to save memory is to make sure that each byte of memory is used to store 8 data bits. A more challenging situation arises because of the look-back step. Before each round in CASCADE, the data frame is permuted. In look-back, we need to find the partition of a previous round which contained the bit which has been corrected in the current round. In particular, we need to invert the permutation of the current round, and apply the permutation of the previous round. In the original description of CASCADE, permutation is done with a hash function, \( h \), from a family of universal hash functions: The bit at index \( i \) is assigned to partition number \( h(i) \). However, knowing that a bit is in partition \( h(i) \) does not allow us to compute the original index of the bit, let alone the index in a previous round, which uses a different hash function. A simple solution is to store the mapping from original indices to permuted indices. Storing this mapping, however, requires at least \( n \log_2(n) \) bits (at least 232 Mbit to store the permutation of a 10 Mbit frame).

For large block sizes, which is our aim, the overhead is prohibitively large. Instead, we permute the data bits with a random, invertible function — essentially a random number generator, where the original index of a bit is used as seed.

Once large block sizes are used, and computation has become the bottleneck, throughput is improved by speeding up the computation in CASCADE. CASCADE does not have much computation: Mostly the computation of parities. During the binary search for an error, after splitting up a block of bits, the parity of one of the sub-blocks has to be calculated: This involves revisiting half the bits which were already used to compute the parity of the original block. To limit the number of repeat look-ups, we compute a prefix parity list during the calculation of the parities of the partitions in the beginning of a round. The \( i \)th element of the prefix parity list of the permuted data frame \([d_0, \ldots, d_{n-1}]\) is

\[ pp_i = pp_{i-1} \oplus d_{i-1}, \]

(6)

for \( i \in \{1, \ldots, n\} \), and \( pp_0 = 0 \). Computing the prefix parity list takes the same time as computing the parities of the partitions, but once the prefix parity list is computed, the parity of any interval of data bits \([d_i, \ldots, d_{i+l}]\) can be computed by looking up only two values in the prefix parity list:

\[ \text{parity}(\{d_i, \ldots, d_{i+l}\}) = pp_i \oplus pp_{i+l+1}. \]

(7)

In the original description of the look-back step, only the smallest partitions of previous rounds are searched for a recently corrected bit. In [12], the authors propose an improvement where all blocks seen during CASCADE are used during look-back. This decreases the expected size of the smallest block containing the newly corrected bit, thus making the following binary search less interactive. In our implementation of CASCADE, we apply this improvement.

The source code of our CASCADE implementation is available upon request to the principal author.

VI. EXPERIMENTAL RESULTS

Our implementation of CASCADE was tested on two Intel Core i7 3.4 Ghz CPUs (comparable to the tests performed in [2]) connected by a gigabit Ethernet connection. Except for tests where we explicitly mention the number of parallel processes used, Alice and Bob use only a single CASCADE process on each their computer. All test results are the average of correcting at least 100 frames.

Our first experiment addresses the “feared” amount of interaction of CASCADE. Fig. 1 shows the number of messages exchanged between Alice and Bob as a function
FIG. 1. The number of messages exchanged to correct a single frame. The dependency on frame size comes from the lookback procedure. The logarithmic dependency on frame size encourages the use of large frames.

of frame size. As mentioned in the previous section, the number of messages increases logarithmic in the frame size. This sub-linearity results in a relative drop in the cost of communication when the frame size is increased. The figure also shows that the number of messages exchanged decreases as QBER increases. This, in part, is due to the use of smaller partition sizes for larger QBER, which reduces the number of messages exchanged in each call to the binary search for errors.

To test the effect of the interaction under different network latencies, we used facilities of the Linux kernel[31] to set the latency. All tests refer to the end-to-end latency (not round-trip latency). The time needed to authenticate the communication is ignored (included in the network latency) in all tests except for a single real-world test over optical fiber described below.

It is expected that larger frame sizes will take better advantage of the parallel binary search for errors, thus reducing the time spent on communication. However, as can be seen in Fig. 2, after a certain frame size, the throughput drops. This drop is caused by increased computation time (computation time is super-linear in the frame size). Fig. 3 shows a breakdown of the time spent on a frame into time spent communicating and time spent computing. The time spent on communication has an exponential drop-off which stabilizes after a frame size of 40–50 million bits. The computation time, on the other hand, increases steadily. From Fig. 2, we can see that the optimal throughput is reached for a frame size of 30–40 million bits for the latency of interest to us (1–3 ms latency).

The maximum throughput for a single cascade process is 18.97 Mbps and is reached with a block size of 30 Mbit, a QBER of 1%, and a network latency of 1 ms.

FIG. 2. Larger block sizes take better advantage of the parallel execution of the binary search for errors. After a certain point, the computation time overtakes the reduction of communication time.
When the QBER is increased to 5%, the throughput is still 12.35 Mbps. Even for 15% QBER (after which no secure key can be generated), the throughput is still sufficiently high for all current high speed QKD systems.

The main obstacle in using large frame sizes is memory usage. The amount of memory used increases linearly in the frame size and sub-linearly in QBER. Our implementation of *cascade* uses 303 Mb in the optimal scenario of 1% QBER and 30 million bit size. However, for a QBER of 15%, the memory usage increases to 2 Gb for a frame size of 30 million bits.

The highest reported throughput achieved with *cascade* was presented in [27]. The implementation uses 1 million bit frames and multiple threads on a quad-core computer (the exact number of threads used is not reported). It achieves a throughput of 5.5 Mbps when correcting for 3.8% QBER using a 45 km fiber link (at most 1 ms latency) for communication. In comparison, our implementation, when using 1 million bit frames, achieves 2.6 Mbps using a single thread/process when correcting 5% QBER with 1 ms latency. With 4 *cascade* processes running in parallel on eight-core computers, we reach 13.28 Mbps. As already demonstrated, the optimal frame size is approximately 30 million bits. A single process with a 30 million bit frame achieves 12.35 Mbps, while 4 parallel processes using 30 million bit frames reach 27 Mbps. With 4 processes, however, a contention for the network occurs between the 4 processes. A frame size of 15 million bits reduces this contention and results in a throughput of 36.73 Mbps.

For comparison with the IR protocol presented in [15], which has a throughput of 12.2 Mbps for a QBER of 1%, we ran 4 processes with a frame size of 15 million bits. The throughput when correcting 1% QBER with 1 ms latency was 65.99 Mbps.

By running 8 processes in parallel with a frame size of 10 million bits, we have achieved a throughput of 82.31 Mbps when correcting 1% QBER with a latency of 1 ms. This is more than four times the throughput of the previous fastest IR protocol known to us [2].

To demonstrate the performance of our implementation in a realistic, real-world scenario, we also ran tests on two computers connected by 11 km dedicated fiber, using low cost TP-LINK MC112CS/MC111CS media converters, converting 100 Mbps Ethernet connections to a 100Base-FX fiber connection. The end-to-end latency between Alice and Bob was approximately 0.4 ms. Note that most of the latency is in the media converters, as the signal only takes 0.06 ms to propagate through the fiber. To implement an authenticated channel, we created an ssh tunnel [32] between Alice and Bob. This channel is both encrypted and authenticated with HMAC SHA-2 256. SHA-2 256 is not an information theoretically secure authentication as required for QKD. Computationally, however, SHA-2 256 is slower than e.g. polynomial hashing, which is recommended for QKD. The throughput, when correcting a QBER of 1% using a frame size of 30 million bits, was 20.74 Mbps. We repeated the above experiment with 8 parallel *cascade* processes, each using a 10 million bit block size. The throughput of the parallel experiment was 83.49 Mbps. We note that by using faster media converters and authentication, the same throughput could be achieve over much larger distances.

Since we have modified the look-back step of *cascade*, we confirm that the efficiency does not deteriorate too much compared to the standard implementation of *cascade*. Table I lists the efficiency of both our implementation and the original *cascade* (as reported in [13]). The table clearly shows that our modification only has a significant influence on efficiency for high QBER (above 10%).

![Time spent on correcting a million bits of data for different frame sizes. For each frame size, we list the time for 1 ms (first column), 5 ms (second column), and 10 ms (third column) latency. The contribution of communication decreases as the block size increases. For 1 ms latency, we see that the communication becomes an insignificant contribution to the overall time spent.](image)

Table I. Efficiency of our implementation of *cascade* (column titled "this"), the original *cascade* (column titled "original"), and polar code (column titled "polar").

| QBER | This     | Original | Polar |
|------|----------|----------|-------|
| 1%   | 0.989    | 0.9889   | 0.9875 |
| 3%   | 0.96     | 0.9602   | 0.975  |
| 5%   | 0.9231   | 0.9261   | 0.9688 |
| 10%  | 0.7839   | 0.7972   | 0.95   |
| 15%  | 0.5597   | 0.5907   | —      |
more important than throughput. Does this mean that LDPC or polar code based IR protocols perform better than cascade when their throughput is high enough? The answer lies in the remaining term in Eq. (4): FER. For cascade, the FER is negligible. Our implementation of cascade successfully corrected all 701919 frames we applied it to in our tests. For the polar code based IR protocol listed in Table IV the FER is 8%. For the best LDPC based IR protocol presented in [7], the FER is 1%. When \( I_E \) is close to \( I(A:B) \), efficiency plays a much more important role than FER. However, when \( I_E \) is small compared to \( I(A:B) \), FER is the dominating term. This, then, becomes the scenario where a careful choice between the different IR protocols must be made.

In our final experiment, we consider the following question: If the throughput of cascade is significantly higher than what is needed, can we improve the overall performance by sacrificing surplus throughput to gain efficiency? One of the contributions in [12] is to use a different set of partition sizes to improve the efficiency of cascade. The partition sizes they use are \( 0.8/QBER \) for the first round, \( 4/QBER \) for the second round, and \( n/2 \) for another 8 rounds, where \( n \) is the frame size. Applying these partition sizes improves the efficiency, but significantly decreases throughput and increases FER. To avoid the increased FER and to increase throughput, we changed the partition size of the third round to \( 10^3 \), and of the last 7 rounds to \( 10^6 \). Since we use large frame sizes, the loss in efficiency caused by the extra parity bits is insignificant. However, the FER becomes negligible. On a channel with 1 ms latency and 1% QBER, we achieved 9.47 Mbps throughput and an efficiency of 0.9907 with a frame size of 30 million bits. At 5% QBER, the throughput is 6.44 Mbps and the efficiency is 0.9465 using a frame size of 30 million bits. This version of cascade has an efficiency close to that of the LDPC and polar code based IR protocols we have compared to. For all state-of-the-art QKD systems cited in this paper, the throughput of this modified cascade is sufficient. If needed, however, by running two or more processes in parallel, the throughput of this version of cascade can easily be increased. These last tests demonstrate that also when throughput is not the limiting factor, variations of cascade may still be the best performing IR protocol for many realistic scenarios.

VII. CONCLUSION

We have demonstrated that, with careful implementation and use, interactive IR protocols (in particular cascade) can reach throughput and efficiency sufficiently high for current state-of-the-art QKD systems. Considering the popularity and simplicity of cascade, we argue that cascade is a good choice for QKD implementations in most real-world scenarios. It is, however, also clear that in settings with extraordinarily high latency on the classical channel (such as geostationary satellite links), less interactive IR protocols may be preferable.

Our implementation of cascade has achieved a throughput of 83.49 Mbps over a dedicated, authenticated fiber link — more than four times faster than has previously been demonstrated by any IR protocol that we know of. The throughput is an order of magnitude higher than that needed in state-of-the-art QKD systems. In this case, the relevant performance metric is Eq. (4), which tells us that the overall performance of the system may be improved by sacrificing surplus throughput to gain efficiency. Even though LDPC and polar codes have higher efficiency than standard cascade for large QBER [18, 33], we have demonstrated that modified versions of cascade, such as the ones proposed in [12, 13], can reach comparable efficiency while still keeping throughput high enough.

The main contribution of this paper is to point out that both communication and computation cost, as well as IR protocol efficiency, must be carefully considered when choosing an IR protocol for a specific QKD realization.

Our entire focus has been on the performance of the IR protocol, while ignoring other design criteria such as the cost of the system. It is clear that the cost of dedicating a full 8 core CPU, as the one used in our experiments, to IR is more expensive than an FPGA. We do not know the performance of cascade when implemented in FPGA. One challenge with an FPGA implementation of cascade is the large amount of memory needed. For low-cost systems, a more detailed study of hardware implementations of cascade is required.

ACKNOWLEDGMENTS

This work was partially sponsored by the Turkish Republic Ministry of Development State Planning Organization — D.P.T. project no. 2009K120200.

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