Neutrino-Lasing in The Early Universe

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ABSTRACT

Recently, Madsen has argued that relativistic decays of massive neutrinos into lighter fermions and bosons may lead, via thermalization, to the formation of a Bose condensate. If correct, this could generate mixed hot and cold dark matter, with important consequences for structure formation. From a detailed study of such decays, we arrive at substantially different conclusions; for a wide range of masses and decay times, we find that stimulated emission of bosons dominates the decay. This phenomenon can best be described as a neutrino laser, pumped by the QCD phase transition. We discuss the implications for structure formation and the dark-matter problem.

Recent studies [1,2] suggest that a mixed dark-matter (MDM) universe could more readily account for the large-scale power observed in galactic surveys [3] and recent COBE measurements of the microwave background fluctuations [4]. These developments have stimulated attempts to provide a natural physical mechanism to generate MDM. We focus here on the novel and interesting idea of Madsen [5] that if a relativistic “heavy” neutrino, $\nu_H$, decays into a light fermion, F, and boson, B, then a significant fraction of the bosons may form a condensate.

Consider the processes:

$$\nu_H \leftrightarrow F + B ; \bar{\nu}_H \leftrightarrow \bar{F} + \bar{B},$$

where $m_F$ and $m_B$ are small enough ($\lesssim 100$ eV) so as not to overclose the universe and $m_F, m_B \ll m_H$. Madsen argued that if the decay processes become effective at $T = T_d$ (when the time-dilated free decay time for a $\nu_H$ of typical momentum is equal to the age of the universe) and if the $\nu_H$ are still relativistic, then the $\nu_H$, F and B populations would come into thermal (but not chemical) equilibrium. Solving for the parameters of the thermal distributions and assuming that the abundances of B and F were zero at $T_d$, he finds that, for a wide range of initial conditions, there is a Bose condensate.

While highly suggestive, this heuristic argument is somewhat unsatisfactory. First, the process is not complete. In coming to the supposed equilibrium state only about half of the $\nu_H$’s decay. What happens to the remaining $\nu_H$’s is unclear. Second, a sudden equilibration at $T_d$ may be an inadequate description. To
address these issues we have solved the Boltzmann equations for the reactions (1).

We find that if the decay rate is sufficiently high ($T_d > \sqrt{m_H m_B}$) then the equilibration will be preceded by a burst of stimulated decays into very cold ($p \ll T$) bosons. This is an exponentially unstable process \textit{i.e.} a \textit{neutrino-laser}. As a result of this non-thermal process, close to half of the $\nu_H$’s decay. The resulting $\nu_H$ and F distributions have a ‘grey-body’ form and the cold B’s are also non-thermal, with an abundance well in excess of the thermal-equilibrium Bose-condensate.

Allowing only the reactions (1), the evolution of the occupation number distributions $f_i$, $i = H,F,B$, are described by the following Boltzmann equations:

$$\dot{f}_i = (h_i m_H^2 \Gamma_0 / m_0 E_i p_i) \int dE_j \delta[f_H(1 - f_F)(1 + f_B) - f_B f_F(1 - f_H)]$$

(2)

where $\dot{f}_i$ is the time derivative of $f_i$ at a fixed comoving momentum, $h_T = -1$, $h_F = 1$ and $h_B = 2(1)$ if $B = (\neq) \bar{B}$, $p_i$ is the physical three-momentum, and $m_0/2$ is the three-momentum of the decay products in the $\nu_H$ rest-frame with $m_0^2 = m_H^2 - 2(m_B^2 + m_F^2) + (m_H^2 - m_F^2)^2/m_H^2$. $\Gamma_0$ is the free decay rate for a $\nu_H$ at rest. The integration is over the energy-conserving plane $E_H = E_F + E_B$, with limits on $E_B, E_F$

$$E_k^\pm = (m_0/2m_H)(E_H \sqrt{1 + 4m_k^2/m_0^2 \pm p_H})$$

(3)

($k = B, F$) as shown in figure 1. These limits follow from purely kinematic considerations. For high $E_H$, F and B both come off in the forward direction with similar energies. For

$$E_H = E_\ast = (m_H^2 + m_B^2 - m_F^2)/2m_B \simeq m_H^2/2m_B$$

zero-momentum bosons are accessible. If the final states were empty the $\nu_H$’s would decay into products with energies distributed uniformly on the range (3). Equation (2) then follows straightforwardly from the inclusion of the quantum mechanical statistical weights for the forward and inverse decay processes.

To set the initial conditions we assume that at some early time $\nu_H$, F and B were all in chemical and thermal equilibrium, but that the F and B decoupled prior to the QCD phase transition at $T \approx 100$ MeV while the $\nu_H$ did not; this specifically requires $F \neq \nu_e, \nu_\mu$. Following the phase transition the temperature of the $\nu_H$’s was increased by a factor $\eta^{1/3} \simeq 2$ relative to the F’s and B’s, where $\eta$ is the ratio of statistical weights before and after the phase transition. Equations (2) can be integrated numerically, but we can gain insight into the general properties of the solution by considering some limiting regimes.

Consider the equation for $\dot{f}_H$. Since $1 - f_F \leq 1$ and the range of the integral is $E_B^+ - E_B^- = m_0 p_H / m_H$, the rate for terms independent of $f_B$ is $\sim m_H \Gamma_0 / E_H$. This is just the time dilated free decay rate for a
typical energy $\nu_H$ and therefore will be negligible for $T \gg T_d \equiv \min(T_0, (T_0^2 m_H)^{1/3})$, where $T_0$ is such that $H(T_0) = \Gamma_0$. The terms involving $f_B$, however, can be important long before $T_d$. Neglecting the other terms we have\

$$f_B = (h_B \Gamma_0 m_H^2 f_B / m_0 E_B p_B) \int dE (f_H - f_F)$$

so $f_B$ will grow exponentially, driven by any initial imbalance between the $\nu_H$'s and F's. The limits on the integral in (4) are $E_H^2(E_B)$, which are given implicitly by Eq (3). The prefactor $1/E_B p_B$ suggests the growth rate will be greatest for the lowest momentum bosons which are accessible. Consider first the case $T \gg E_*$ which must hold at sufficiently early times. The minimum boson energy for a typical energy $\nu_H$ ($E_H \sim T$) is $E_B \sim (m_B / m_H)^2 E_H$. Setting $E_B, p_B \sim E_B$ and since the integral in equation (4) will be on the order $E_H \sim T$ we obtain\

$$f_B \simeq (\Gamma_0 m_H^2 / m_B T) f_B$$

so the growth rate is $\Gamma_{lase} \simeq \Gamma_0 (m_H / m_B)^3 (E_* / T)$. This increases with time while the expansion rate decreases, so these will be equal at\

$$T_{lase} \simeq (m_H^5 T_0^2 / m_B^4)^{1/3}$$

We have assumed here that $T \gg E_*$. For $T \ll E_*$ we find that the growth rate decreases faster than the Hubble rate so if the process is not effective at $T \simeq E_*$ then it never will be. The condition $T_{lase} \geq E_*$ sets a lower limit on the decay rate, or equivalently on the decay temperature: $T_d > \sqrt{m_H m_B}$.

What we have here is a neutrino laser; the low momentum boson occupation numbers will grow exponentially via stimulated decays, feeding off any initial imbalance between the $\nu_H$'s and F's, and terminating when this is driven to zero. The momentum of the bosons produced is $p_B \sim (m_B^2 / m_H^2) T$, so for a large mass ratio these will be very cold compared to the typical thermal energy.

The lasing will result in $f_H = f_F = (f_H^0 + f_F^0) / 2$ (superscript indicates initial values) which is a fermion analogue of a 'grey body' spectrum, and the total number of cold bosons produced is just $(n_H^0 - n_F^0) / 2 = (1 - \eta^{-1}) n_H^0 / 2$. The number of hot bosons is just the initial number $n_B^0 = 4 n_H^0 / (3 \eta h_B)$. This then gives the fraction of cold bosons after lasing to be $\simeq 80(65)\%$ for $B = (\neq) \bar{B}$. The result of lasing is therefore qualitatively similar to the thermalization calculation of Madsen, but with important differences: The lasing process occurs much earlier than thermalization; the initial hot bosons are unaffected and the decays occur exclusively into the cold component. In the equilibration calculation, only about half of the decays go into the cold phase, and the cold fraction is $\simeq 42(26)\%$, about half the yield from lasing (these numbers are slightly different from those calculated by Madsen as we have allowed for the finite initial boson abundance).
After lasing the temperature will eventually fall to $T_d$. What happens then depends $T_d$. For $T_d < E_*$ the low momentum bosons produced by lasing will be effectively decoupled (they can only be reached from $E_H \sim E_* \gg T$ and so this coupling will be exponentially suppressed). The hot bosons and fermions will equilibrate, but they do this without generating any further bosons. For $T_d > E_*$ on the other hand, the laser generated bosons are still accessible from typical energy decays, so we expect that some of the cold bosons will be reabsorbed and that the post-$T_d$ distributions will adopt the form predicted by Madsen.

In either case the decay process is not yet finished since there is still a finite $\nu_H$ abundance. The remaining $\nu_H$’s (58% of the original abundance) decay into hot B’s and F’s once $T \lesssim \min(m_H, T_d)$. For lasing, the final fraction of cold bosons is $(1 - \eta^{-1})/(2 + 8/3\eta h_B) \simeq 37(34)\%$. For equilibration ($T_d > E_*$) the numbers are 19(13)%.

When and how the final decays occur depends on $T_d$. For $T_d > E_*$, the populations equilibrate at $T_d$ and are then stable until $T \sim m_H$ when the heavy neutrinos go non-relativistic at which time they decay. For $m_H < T_d < E_*$ we expect the hot particles to come to thermal equilibrium at $T \sim T_d$ without generating any further cold bosons. The remaining $\nu_H$’s decay at $T \sim m_H$, with no subsequent equilibration. For $\sqrt{m_H m_B} < T_d < m_H$ no equilibration occurs and the particles decay at $T_d$ when they are non-relativistic. The last possibility is $T_d < \sqrt{m_H m_B}$ in which case no cold bosons are produced and the $\nu_H$’s decay into very hot products at $T_d$.

These predictions are confirmed by our numerical solution of (2). In Figure 2 the evolution of the occupation numbers are plotted for masses and decay temperature chosen to illustrate the lasing phenomenon.

In all of this one must be wary that the new fermionic degree of freedom F does not take up its full statistical weight prior to the weak interaction freeze-out, thereby violating the bounds from standard big bang nucleosynthesis [6]. (The B’s are not of concern if the stimulated decay is dominant since they are very low energy.) We therefore require $T_{\text{lase}} < 2.3$ MeV since the number changing reactions for the neutrinos are decoupled below this temperature. From equation (6) and the definition of $T_d$ this requires $T_d < (2.3\text{MeV})^{3/2}m_B^2m_H^{-5/2}$ if $T_d < m_H$ and $T_d < 2.3\text{MeV}(m_B/m_H)^{4/3}$ otherwise. The former, together with the lower bound $T_d > \sqrt{m_H m_B}$ sets an upper limit $m_H \lesssim \sqrt{2\text{MeV}m_B} \sim 10\text{keV}$ for a boson mass which would close the universe. These, together with the requirement that $m_H > m_B, m_F$, are shown in Fig. 3, which displays the region of $T_d - m_H$ space where cold bosons are produced.

The fraction of cold particles is important for structure formation, and provides a firm (modulo factors such as $h_B$, $\eta$) prediction of the lasing model. Also important, however, is $m_B$ and the momentum distribution of the hot bosons, as they will determine the maximum Jeans mass[7]. The attraction of the 70/30 cold/hot ratio MDM model [1] is mostly that the low hot-fraction reduces the mass of the neutrino by a factor $\simeq 3$, therefore increasing the maximum Jeans mass by a factor of $\simeq 10$. Here the final abundance of bosons is just $(1 + 4/3\eta h_B)m_H^2$. If the bosons alone constitute the dark matter then their mass will be only slightly less than
the standard value. However, the boson momentum distribution will also differ from the standard value. For 
\(T_d > E_*\) the mean momentum of the hot bosons is almost identical to the standard value so the maximum 
Jeans mass will be very similar to that in the standard HDM model. This may be problematic. In the case 
of lasing with \(T_d > m_H\) the hot bosons are \(\sim 40\%\) cooler than in standard HDM making matters even worse. 
However, for the lower triangle \(\sqrt{m_H m_B} < T_d < m_H\), the temperature of the hot bosons can be larger than 
the ambient temperature by a factor \(\sim m_H / T_d\). This can be as large as \(\sim \sqrt{m_H / m_B} \lesssim (2\text{MeV} / m_B)^{1/4} \approx 20\). 
It is also possible that the fermion and boson masses are similar, or that other particles, \(e.g.\) the \(\nu_\mu\) decay 
into the same products; in either case this would increase the maximum Jeans mass.

We note that a number of variations on the preceding discussion exist, which may have different quan-
titative predictions. For example, if \(\nu_H = \nu_\mu\), and \(\nu_\mu\) is heavier than \(\nu_\tau\), then the QCD phase transition 
would pump the \(\nu_\mu - \nu_H\) system, because charged-current production of \(\nu_\tau\) is energetically disfavored; or 
the \(\nu_H\), F and B could all be new particles outside the standard model. Investigation of such possibilities is 
beyond the scope of the present work.

We have thus far only considered the scenario where \(F \neq \nu_e, \nu_\mu\). Consider briefly the scenario where 
the light fermion is one of the known neutrinos, and is therefore coupled at \(T_{\text{QCD}}\). Madsen’s calculations 
[5] indicate that in this scenario too, Bose condensation may occur. With regard to our calculations note 
that there is no initial fermion imbalance to drive the lasing. In fact, we conclude that direct solution of the 
Boltzmann equations (Eq. 2) results in no condensation.

In conclusion: we have shown that for decay temperatures \(T_d > \sqrt{m_H m_B}\) that thermalization is in-
evitably preceded by a runaway stimulated decay into cold \((p \sim (m_B / m_H)^2 T)\) bosons. This ‘neutrino lasing’ 
provides a relatively natural way to generate MDM. For decay temperatures \(T_d \gg m_H^2 / m_B\) the populations 
will thermally equilibrate, some of the cold bosons will be reabsorbed, and the final cold boson fraction is 
\(\simeq 15\%\). For \(T_d \ll m_H^2 / m_B\) the laser generated bosons survive to the present and the cold fraction is \(\simeq 35\%\). 
If the boson alone constitutes the dark matter, then thermalization results in a maximum Jeans mass very 
similar to that in the standard HDM model; in contrast, the lasing scenario allows the interesting possibility 
of a higher Jeans mass.

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**FIGURE CAPTIONS**

1. Allowed $E_B$ for decay of $\nu_H$ with energy $E_H$ (logarithmic scales). Of particular importance for our analysis is the form of $E_B^{-}$ since the lasing occurs into the lowest momentum states which are accessible. Low energy $\nu_H$’s decay into high energy products. Particles with $E_H = E_\ast \simeq m_H^2/m_B$ can decay into zero momentum bosons and for $E_H > E_\ast$, $E_B^{-} \simeq (m_B/m_H)^2 E_H$.

2. Upper panels show $p^3 f$, the number of particles per log($p$) for three output times: the initial time, after lasing and after all $\nu_H$’s have decayed. The masses were $m_H; m_F; m_B = 1; 0.03; 0.03$ and $T_d = 1$. The lower panel shows the total number of bosons as a function of time. The lasing epoch, which injects cold bosons exclusively, followed by a trans-relativistic decay epoch, injecting hot bosons, are evident. Most striking is the bimodality of the boson distribution which is clearly divided into hot and cold components.

3. Allowed regions for generating mixed dark matter (logarithmic scales). In the upper triangle lasing occurs, but is then followed by equilibration resulting in a reduction in the number of cold bosons. In the lower triangles lasing occurs. Above the dashed line, the sequence of events is lasing; equilibration of the hot components; and finally decays at $T \sim m_H$. Below the dashed line, the particles become non-relativistic before the decay process terminates.