A highly predictive $A_4$ flavour 3-3-1 model with radiative inverse seesaw mechanism

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We build a highly predictive 3-3-1 model, where the field content is extended by including several $SU(3)_L$ scalar singlets and six right handed Majorana neutrinos. In our model the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry is supplemented by the $A_4 \times Z_6 \times Z_{16}$ discrete group, which allows to get a very good description of low energy fermion flavor data. In the model under consideration, the $A_4 \otimes Z_6 \times Z_{16} \times Z_{16}'$ discrete group is broken at very high energy scale down to the preserved $Z_2$ discrete symmetry, thus generating the observed pattern of SM fermion masses and mixing angles and allowing the implementation of the loop level inverse seesaw mechanism for the generation of the light active neutrino masses, respectively. The model only has 9 effective free parameters (4 and 5 effective free parameters in the lepton and quark sectors, respectively), which we adjust to reproduce the experimental values of the 18 physical observables in the quark and lepton sectors. The obtained values for the physical observables in the quark sector agree with the experimental data, whereas those ones for the lepton sector also do, only for the case of inverted neutrino mass spectrum. The normal neutrino mass hierarchy scenario of the model is ruled out by the neutrino oscillation experimental data. The model predicts an effective Majorana neutrino mass parameter of neutrinoless double beta decay of $m_{ee} = 45.5$ meV, a leptonic Dirac CP violating phase of $79.11^\circ$ and a Jarlskog invariant of about $10^{-2}$ for the inverted neutrino mass hierarchy. The preserved $Z_2$ symmetry allows for a stable scalar dark matter candidate.

Keywords: Extensions of electroweak gauge sector, Extensions of electroweak Higgs sector, Electroweak radiative corrections, Neutrino mass and mixing

I. INTRODUCTION

Despite its great consistency with the experimental data, the Standard Model (SM) is unable to explain several issues such as, for example, the number of fermion generations, the large hierarchy of fermion masses, the small quark mixing angles and the sizeable leptonic mixing ones. Whereas in the quark sector, the mixing angles are small, in the lepton sector two of the mixing angles are large, and one mixing angle is small. Neutrino experiments have brought clear evidence of neutrino oscillations from the measured neutrino mass squared splittings. The three neutrino flavors mix and at least two of the neutrinos have non vanishing masses, which according to neutrino oscillation experimental data must be smaller than the SM charged fermion masses by many orders of magnitude.

Models with an extended gauge symmetry are frequently used to tackle the limitations of the SM. In particular, the models based on the gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, also called 3-3-1 models, can explain the origin of fermion generations thanks to the introduction of a family non-universal $U(1)_X$ symmetry \cite{ref1}\cite{ref10}, can provide an explanation for the origin of the family structure of the fermions. These models have the following nice interesting features: 1) The three family structure in the fermion sector naturally arises in the 3-3-1 models from the cancellation of chiral anomalies and asymptotic freedom in QCD. 2) The fact that the third family is treated under a different representation, can explain the large mass difference between the heaviest quark family and the two lighter ones. 3) The 3-3-1 models allow the quantization of electric charge \cite{ref11}\cite{ref12}. 4) These models have several sources of CP violation \cite{ref13}\cite{ref14}. 5) These models explain why the Weinberg mixing angle satisfies $\sin^2 \theta_W < \frac{1}{4}$. 6) These models contain a natural Peccei-Quinn symmetry, necessary to solve the strong-CP problem \cite{ref15}\cite{ref18}. 7) The 3-3-1 models

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with heavy sterile neutrinos include cold dark matter candidates as weakly interacting massive particles (WIMPs)\cite{19,22}. A concise review of WIMPs in 3-3-1 Electroweak Gauge Models is provided in Ref. \cite{23}.

In the 3-3-1 models, one heavy triplet field with a Vacuum Expectation Value (VEV) at high energy scale $\nu_\chi$, breaks the symmetry $SU(3)_L \otimes U(1)_X$ into the SM electroweak group $SU(2)_L \otimes U(1)_Y$, thus generating the masses of non SM fermions and non SM gauge bosons, while the another two lighter triplets with VEVs at the electroweak scale $\nu_\rho$ and $\nu_\eta$, trigger the Electroweak Symmetry Breaking \cite{24} and provide the masses for the SM particles.

On the other hand, the implementation of discrete flavor symmetries in several extensions of the SM has provided a nice description of the observed pattern of fermion masses and mixings (recent reviews on discrete flavor groups can be found in Refs. \cite{25,26}). Several discrete groups have been employed in extensions of the SM, mostly discrete groups having triplet irreducible representations, such as $A_4$ \cite{29,48}, $S_4$ \cite{19,71}, $S_4$ \cite{71,81}, $D_4$ \cite{82,91}, $Q_6$ \cite{92,95}, $T_7$ \cite{96,109}, $T_{13}$ \cite{106,109}, $T^\prime$ \cite{107}, $\Delta(27)$ \cite{118,132} and $A_5$ \cite{133,143} have been considered to explain the observed pattern of fermion masses and mixings.

Among several discrete symmetry groups, the $A_4$ group has attracted a lot of attention since it is the smallest one which admits one three-dimensional representation as well as three inequivalent one-dimensional representations. Then, the choice of the $A_4$ symmetry is natural since there are three families of fermions, i.e., the left handed leptons can be unified in triplet representation of $A_4$ while the right handed leptons can be assigned to $A_4$ singlets. This setup has been proposed for first time in Ref. \cite{29} to study the lepton masses and mixings obtaining nearly degenerate neutrino masses and allowing realistic charged leptons masses after the $A_4$ symmetry is spontaneously broken. The scalar sector of the minimal setup of Ref. \cite{29} includes one $A_4$ triplet whose components are $SU(2)_L$ doublets and one $SU(2)_L$ doublet which transforms as an $A_4$ trivial singlet. As it has been extensively discussed in the literature (for a recent reviews see Refs. \cite{25,26}) the $A_4$ group, which is the group of even permutations of four elements has been shown to generate the Tribimaximal mixing pattern which predicts solar mixing and atmospheric mixing angles consistent with the experimental data but yields a vanishing reactor mixing angle contradicting the recent experimental results from the Daya Bay \cite{144}, T2K \cite{145}, MINOS \cite{146}, Double CHOOZ \cite{147} and RENO \cite{148} experiments. In view of this the Tribimaximal mixing pattern has to be modified.

In this work we build a highly predictive $A_4$ flavor 3-3-1 model, where the $A_4$ discrete symmetry is supplemented by the $Z_6 \otimes Z_{16} \otimes Z_{16}$ discrete group, providing a framework consistent with the current low energy fermion flavor data. In the model under consideration the different discrete group factors are broken completely, excepting the $Z_6$ discrete group, which is broken down to the preserved $Z_2$ symmetry, thus allowing the implementation of the one loop level inverse seesaw mechanism for the generation of the light active neutrino masses. The SM charged fermion masses and quark mixing angles arise from the breaking of the $A_4 \times Z_6 \otimes Z_{16} \otimes Z_{16}$ discrete group. The SM Yukawa sector of our model has only 9 effective free parameters (4 and 5 effective free parameters in the lepton and quark sectors, respectively), which we adjust to reproduce the experimental values of the 18 physical observables in the quark and lepton sectors, i.e., 9 charged fermion masses, 2 neutrino mass squared splittings, 3 lepton mixing parameters, 3 quark mixing angles and 1 CP violating phase of the CKM quark mixing matrix.

The content of this paper goes as follows. In section \ref{II} we describe our model. Section \ref{III} is devoted to the implications of our model in quark masses and mixings. Section \ref{IV} deals with lepton masses and mixings. We conclude in section \ref{VI} Appendix \ref{A} provides a concise description of the $A_4$ discrete group. Appendix \ref{B} shows a discussion of the scalar potential for a $A_4$ scalar triplet and its minimization equations.

\section{The Model}\label{II}

As is well known, the $SU(3)_C \times SU(3)_L \times U(1)_X$ model (3-3-1 model) with $\beta = -\frac{1}{\sqrt{3}}$ and right-handed Majorana neutrinos in the $SU(3)_L$ lepton triplet is unsatisfactory in describing the observed SM fermion mass and mixing pattern, due to the unexplained hierarchy among its large number of Yukawa couplings. To address that problem, we propose an extension of the 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, where the scalar sector is extended to include several EW scalar singlets, the fermion sector is extended by introducing six right handed Majorana neutrinos, and the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry is supplemented by the $A_4 \times Z_6 \times Z_{16} \times Z_{16}$ discrete group, so that the
full symmetry $G$ exhibits the following three-step spontaneous breaking:

\[
G = SU(3)_C \times SU(3)_L \times U(1)_X \times A_4 \times Z_6 \times Z_{16} \times Z'_{16}
\]

\[
\downarrow \Lambda_{int}
\]

\[
SU(3)_C \times SU(3)_L \times U(1)_X \times Z_2
\]

\[
\downarrow v_\chi
\]

\[
SU(3)_C \otimes SU(2)_L \times U(1)_Y \times Z_2
\]

\[
\downarrow v_\eta, v_\rho
\]

\[
SU(3)_C \otimes U(1)_Q \otimes Z_2
\]

where the different symmetry breaking scales satisfy the following hierarchy $\Lambda_{int} \gg v_\chi \gg v_\eta, v_\rho$. Let us note that all discrete group are broken completely at the very high energy scale $\Lambda_{int} \gg v_\chi$, excepting the $Z_2$ discrete group which is broken down to the preserved $Z_2$ symmetry. That preserved $Z_2$ symmetry will allows us to implement a one loop level inverse seesaw mechanism for the generation of the light active neutrino masses.

In our model 3-$3$-1 model, the electric charge is defined in terms of the $SU(3)$ generators and the identity by:

\[
Q = T_3 + \beta T_8 + XI = T_3 - \frac{1}{\sqrt{3}} T_8 + XI,
\]

with $I = diag(1,1,1)$, $T_3 = \frac{1}{2} diag(1,-1,0)$ and $T_8 = (\frac{1}{2\sqrt{3}}) diag(1,1,-2)$ for triplet. Let us note that we have chosen $\beta = -\frac{1}{\sqrt{3}}$ because in that choice the third component of the weak lepton triplet is a neutral field $\nu_R^0$ which allows to build the Dirac matrix with the usual field $\nu_L$ of the weak doublet. The introduction of a sterile neutrino $N_R$ in the model allows the implementation of a low scale seesaw mechanism (which could be inverse or linear) for the generation of the light neutrino masses. The 3-$3$-1 models with $\beta = -\frac{1}{\sqrt{3}}$ have the advantage over other 3-$3$-1 models with different values $\beta$, of providing an alternative framework to generate neutrino masses, where the neutrino spectrum includes the light active sub-eV scale neutrinos as well as sterile neutrinos which could be dark matter candidates if they are light enough or candidates for detection at the LHC, if they have TeV scale masses. Let us note that if the TeV scale sterile neutrinos are found at the LHC, the 3-$3$-1 models with $\beta = -\frac{1}{\sqrt{3}}$ can be very strong candidates for unraveling the mechanism responsible for electroweak symmetry breaking.

The cancellation of chiral anomalies implies that quarks are unified in the following $SU(3)_C \times SU(3)_L \times U(1)_X$ left- and right-handed representations $[2, 7, 149, 150]$

\[
Q_{nL} = \begin{pmatrix} D_n \\ -U_n \\ J_n \end{pmatrix}_L \sim (3,3^*, 0), \quad Q_{3L} = \begin{pmatrix} U_3 \\ D_3 \\ T \end{pmatrix}_L \sim (3,3, \frac{1}{3}), \quad n = 1, 2,
\]

\[
D_{iR} \sim \begin{pmatrix} 3, 1, -\frac{1}{3} \end{pmatrix}, \quad U_{iR} \sim \begin{pmatrix} 3, 1, \frac{2}{3} \end{pmatrix}, \quad J_{nR} \sim \begin{pmatrix} 3, 1, -\frac{1}{3} \end{pmatrix}, \quad T_R \sim \begin{pmatrix} 3, 1, \frac{2}{3} \end{pmatrix}, \quad i = 1, 2, 3, \quad (3)
\]

where $U_{iL}$ and $D_{iL}$ ($i = 1, 2, 3$) are the left handed up and down type quarks fields in the flavor basis, respectively. The right handed SM quarks, i.e., $U_{iR}$ and $D_{iR}$ ($i = 1, 2, 3$) and right handed exotic quarks, i.e., $T_R$ and $J_{nR}$ ($n = 1, 2$) are assigned as $SU(3)_L$ singlets with $U(1)_X$ quantum numbers equal to their electric charges.

Furthermore, the requirement of chiral anomaly cancellation constrains the leptons to the following $SU(3)_C \times SU(3)_L \times U(1)_X$ left- and right-handed representations $[2, 7, 149]$

\[
L_{iL} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \sim \begin{pmatrix} 1, 3, -\frac{1}{3} \end{pmatrix}, \quad e_{iR} \sim (1, 1, -1), \quad i = 1, 2, 3, \quad (4)
\]

In the present model the fermion sector is extended by introducing six right handed Majorana neutrinos, singlet under the 3-$3$-1 group, so that they have the following $SU(3)_C \times SU(3)_L \times U(1)_X$ assignments:

\[
N_{iR} \sim (1, 1, 0), \quad \Omega_i \sim (1, 1, 0), \quad i = 1, 2, 3, \quad (5)
\]
Regarding the scalar sector of the 3-3-1 model with right handed Majorana neutrinos, we assign the scalar fields in the following $SU(3)_C \times SU(3)_L \times U(1)_X$ representations:

$$\chi = \left( \frac{\chi_1^0}{\sqrt{2}} (v_\chi + \xi_\chi \pm i\zeta_\chi) \right) \sim (1, 3, -\frac{1}{3}), \quad \rho = \left( \frac{1}{\sqrt{2}} (\rho^0_1 + \rho^+_1) \right) \sim (1, 3, \frac{2}{3}),$$

$$\eta = \left( \frac{1}{\sqrt{2}} (v_\eta + \xi_\eta \pm i\zeta_\eta) \right) \sim (1, 3, -\frac{1}{3}).$$

The scalar sector of the 3-3-1 model with right handed Majorana neutrinos includes: three 3’s irreps of $SU(3)_L$, where one triplet $\chi$ gets a TeV scale vacuum expectation value (VEV) $v_\chi$, that breaks the $SU(3)_L \otimes U(1)_X$ symmetry down to $SU(2)_L \otimes U(1)_Y$, thus generating the masses of non SM fermions and non SM gauge bosons; and two light triplets $\eta$ and $\rho$ acquiring electroweak scale VEVs $v_\eta$ and $v_\rho$, respectively, thus triggering Electroweak Symmetry Breaking and then providing masses for the fermions and gauge bosons of the SM [24].

We extend the scalar sector of the 3-3-1 model with right handed Majorana neutrinos by adding the following $SU(3)_L$ scalar singlets, with the following $SU(3)_C \times SU(3)_L \times U(1)_X$ assignments:

$$\varphi \sim (1, 0, 0), \quad \tau_n \sim (1, 0, 0), \quad \sigma \sim (1, 0, 0), \quad \phi \sim (1, 0, 0), \quad n = 1, 2,$$

$$\xi_j \sim (1, 0, 0), \quad \zeta_j \sim (1, 0, 0), \quad \Phi_j \sim (1, 0, 0), \quad \Delta_j \sim (1, 0, 0),$$

$$\Xi_j \sim (1, 0, 0), \quad \Theta_j \sim (1, 0, 0), \quad j = 1, 2, 3. \quad (6)$$

The scalar fields of our model have the following $A_4 \times Z_6 \times Z_{16} \times Z'_{16}$ assignments:

$$\chi \sim (1, 0, 0, 0), \quad \rho \sim (1, 2, 0, 0), \quad \eta \sim (1, 4, 0, 0), \quad \varphi \sim (1, -3, 0, 0),$$

$$\sigma \sim (1'\', 0, -1, 0), \quad \tau_1 \sim (1', 0, -1, -1), \quad \tau_2 \sim (1', 0, -2, -1), \quad \phi \sim (1, 0, -1, 0),$$

$$\xi \sim (3, 0, 6, -1), \quad \zeta \sim (3', 2, 0, 0), \quad \Phi \sim (3, 0, 0, -3), \quad \Delta \sim (3, 0, 0, -3),$$

$$\Xi \sim (3, 0, 0, -8), \quad \Theta \sim (3, 0, 0, 0). \quad (7)$$

Here the dimensions of the $A_4$ irreducible representations are specified by the numbers in boldface and the different $Z_6 \times Z_{16} \times Z'_{16}$ charges are written in additive notation. Let us note that all scalar fields acquire nonvanishing vacuum expectation values, excepting the $SU(3)_L$ scalar singlet $\varphi$, whose $Z_6$ charge corresponds to a nontrivial charge under the preserved $Z_2$ symmetry.

The quark assignments under the group $A_4 \times Z_6 \times Z_{16} \times Z'_{16}$ are:

$$Q_{1L} \sim (1', 0, 0, 0), \quad Q_{2L} \sim (1', 0, 2, 0), \quad Q_{3L} \sim (1, 0, 4, 0),$$

$$U_{1R} \sim (1, 4, 8, 8), \quad U_{2R} \sim (1, 4, 6, 4), \quad U_{3R} \sim (1, 4, 4, 0), \quad T_R \sim (1, 0, 4, 0),$$

$$D_R = (D_{1R}, D_{2R}, D_{3R}) \sim (3, 2, 0, 1), \quad J_{1R} \sim (1', 0, 0, 0), \quad J_{2R} \sim (1', 0, 2, 0). \quad (8)$$

Let us note that we assign the quarks fields into $A_4$ singlet representations, excepting the SM right handed down type quarks fields which are grouped in a $A_4$ triplet.

The lepton fields of our model have the following $A_4 \times Z_6 \times Z_{16} \times Z'_{16}$ assignments:

$$L_L = (L_{1L}, L_{2L}, L_{3L}) \sim (3, 3, 0, 0), \quad N_R = (N_{1R}, N_{2R}, N_{3R}) \sim (3, 3, 0, 0),$$

$$\Omega = (\Omega_1, \Omega_2, \Omega_3) \sim (3, 0, 0, 0), \quad e_{1R} \sim (1, 1, 5, 3), \quad e_{2R} \sim (1, 1, 0, 8), \quad e_{3R} \sim (1, 1, 0, 3)$$

As regards the lepton sector, we recall that the left and right-handed leptons are grouped into $A_4$ triplet and $A_4$ singlet irreducible representations, respectively, whereas the right-handed Majorana neutrinos, i.e., $N_{iR}$ and are unified $\Omega_i$ ($i = 1, 2, 3$) into the $A_4$ triplets, i.e., $N_R$ and $\Omega$.

With the above particle content, the relevant Yukawa terms for the quark and lepton sector invariant under the group
triplets. The Wolfenstein parameters) needed to naturally explain the smallness of the up quark mass, which is

\[ \mathcal{G} \text{, respectively, are:} \]

\[ -\mathcal{L}_{\text{V}}^{(q)} = y^{(U)}_3 Q^3 L \chi^T R + y^{(U)}_{33} Q^3 L \eta U^3 R + y^{(U)}_2 Q^2 L \rho^* U^3 R \frac{\sigma^2}{\Lambda^2} + y^{(U)}_1 Q^1 L \rho^* U^3 R \frac{\sigma^4}{\Lambda^4} \]

\[ + y^{(U)}_{22} Q^2 L \rho^* U^2 R \frac{\tau^1}{\Lambda^2} + y^{(U)}_{12} Q^1 L \rho^* U^2 R \frac{\tau^1 \tau_2}{\Lambda^6} + y^{(U)}_{11} Q^1 L \rho^* U^1 R \frac{\tau^1}{\Lambda^8} \]

\[ + y^{(U)}_1 Q^1 L \chi^* J^1 R + y^{(U)}_2 Q^2 L \chi^* J^2 R + y^{(U)}_1 Q^1 L \eta^* (\xi D_R)^{-1} \frac{\phi^6}{\Lambda^5} + y^{(U)}_2 Q^2 L \eta^* (\xi D_R)^{-1} \frac{\phi^4}{\Lambda^5} \]

\[ + y^{(U)}_3 Q^3 L \rho (\xi D_R) \frac{\phi^2}{\Lambda^5} + H.c. \quad (9) \]

\[ -\mathcal{L}_{\text{V}}^{(l)} = y^{(L)}_{11} (T_L \rho \Phi) \frac{e_1 R}{\Lambda^5} + y^{(L)}_{31} (T_L \rho \Delta) \frac{e_1 R}{\Lambda^5} + y^{(L)}_{22} (T_L \rho \Xi) \frac{e_2 R}{\Lambda} \]

\[ + y^{(L)}_{13} (T_L \rho \Phi) \frac{e_3 R}{\Lambda} + y^{(L)}_{33} (T_L \rho \Delta) \frac{e_3 R}{\Lambda} + y^{(L)}_{11} (N_R \Omega) \frac{\varphi}{\Lambda} + m_\Omega (\Omega) \frac{1}{\Lambda} + y^{(L)}_{22} (N_R \Omega) \frac{\xi}{\Lambda} \quad (10) \]

where the dimensionless couplings in Eq. [9] and [10] are \( O(1) \) parameters. Furthermore, as it will shown in Sect. III, the quark assignments under the different group factors of our model will give rise to SM quark mass textures where the CKM quark mixing angles only arise from the up type quark sector. As indicated by the current low energy quark flavor data encoded in the Standard parametrization of the quark mixing matrix, the complex phase responsible for CP violation in the quark sector is associated with the quark mixing angle in the 1-3 plane. Consequently, in order to reproduce the experimental values of quark mixing angles and CP violating phase, \( y^{(U)}_{13} \) is required to be complex. Besides that, as it will shown in Sect. III, the light active neutrino sector will generate the tribimaximal mixing matrix, whereas the charged lepton sector will give rise to the reactor mixing angle. In order to account for CP violation in neutrino oscillation, we will also assume that the \( y^{(L)}_{13} \) parameter is purely imaginary.

Although the flavor discrete groups in Eq. [1] look rather sophisticated, each discrete group factor is crucial for generating highly predictive SM fermion mass matrices consistent with low energy fermion flavor data. As it will shown in Sect. IV, the predictive textures for the lepton sectors will give rise to the experimentally observed deviation of the tribimaximal mixing pattern. Besides that, the resulting SM quark mass matrices will give rise to quark mixing only emerging from the up type quark sector. This is a consequence of the \( A_4 \) flavor symmetry, which needs to be supplemented by the \( A_4 \times Z_6 \times Z_{16} \times Z_{16}' \) discrete group. As we will see in the next sections, this predictive setup can successfully account for SM fermion masses and mixings. The inclusion of the \( A_4 \) discrete group reduces the number of parameters in the Yukawa and scalar sector of the \( SU(3)_C \times SU(3)_L \times U(1)_X \) model making it more predictive. We choose \( A_4 \) since it is the smallest discrete group with a three-dimensional irreducible representation and 3 distinct one-dimensional irreducible representations, which allows to naturally accommodate the three fermion families. In what follows we provide an explanation of the role of each discrete cyclic group factor introduced in our model. The \( Z_6 \) symmetry has the following roles: 1) To separate the \( A_4 \) scalar triplet \( \zeta \) participating in the Dirac neutrino Yukawa interactions from the remaining \( A_4 \) scalar triplets. 2) To forbid mixings between SM quarks and exotic quarks, thus resulting in a reduction of quark sector model parameters. 3) To allow the implementation of the one loop level inverse seesaw mechanism for the generation of the light active neutrino masses, due to the fact that the \( Z_6 \) discrete group is broken down to the preserved \( Z_3 \) symmetry. Let us note that we use the \( Z_6 \) discrete group since it is the smallest cyclic group that contains both the \( Z_3 \) and \( Z_2 \) symmetries. The \( Z_3 \) symmetry contained in \( Z_6 \) allows to decouple the exotic quarks from the SM quarks, whereas the preserved \( Z_2 \) symmetry is crucial for the implementation of the one loop level inverse seesaw mechanism for the generation of the light active neutrino masses. In what concerns, the \( Z_{16} \) symmetry, it is worth mentioning that it is crucial to generate the observed charged fermion mass and quark mixing pattern. Let us note, that the properties of the \( Z_N \) groups imply that the \( Z_{16} \) symmetry is the smallest cyclic symmetry from which the Yukawa term \( Q_L^1 \rho^* U_R^1 \frac{\tau^1}{\Lambda^8} \) of dimension twelve can be built, from a \( \frac{\tau^1}{\Lambda^8} \) insertion on the \( Q_L^1 \rho^* U_R^1 \) operator, crucial to get the required \( \lambda^5 \) suppression (where \( \lambda = 0.225 \) is one of the Wolfenstein parameters) needed to naturally explain the smallness of the up quark mass, which is \( \lambda^5 \frac{\tau^1}{\Lambda^8} \) (\( \lambda = 0.225 \) is one of the Wolfenstein parameters) times a \( O(1) \) parameter. Furthermore, the \( Z_{16} \) discrete symmetry separates the \( A_4 \) scalar triplet \( \xi \) participating in the SM down type quark Yukawa interactions from the remaining \( A_4 \) scalar triplets. The \( Z_{16}' \) symmetry has the functions: 1) To select the allowed entries of the SM quark mass matrices, thus yielding a very predictive quark sector. It is worth mentioning that the \( Z_{16}' \) is the smallest cyclic symmetry that allows us to get vanishing \((2, 1), (3, 1)\) and \((3, 2)\) entries in the SM up type quark mass matrix. 2) To separate the \( A_4 \)
scalar triplet $\xi$ participating in the SM down type quark Yukawa interactions from the remaining $A_4$ scalar triplets. 2) To distinguish the $A_4$ scalar triplet $\xi$ participating in the quark Yukawa interactions, from the ones, i.e., $\zeta$ and $\Theta$ that appear in the neutrino Yukawa terms and from the $A_4$ scalar triplets, i.e., $\Phi$, $\Delta$ and $\Xi$, contributing to the charged lepton masses, thus allowing to treat, the SM down type quark, the charged lepton and neutrino sectors independently. 3) To separate the $A_4$ scalar triplets $\Phi$ and $\Delta$ contributing to the electron and tau lepton masses as well as to the reactor mixing angle from the $A_4$ scalar triplet $\Xi$ that give rise to the muon lepton mass. This is crucial to generate the experimentally observed deviation from the tribimaximal mixing pattern, which in our model arises from the charged lepton sector.

Furthermore, since the breaking of the $A_4 \times Z_6 \times Z_16 \times Z_16'$ discrete group gives rise to the charged fermion mass and quark mixing pattern, we set the VEVs of the $SU(3)_L$ singlet scalar fields (excluding $\varphi$ which has a vanishing vacuum expectation value) with respect to the Wolfenstein parameter $\lambda = 0.225$ and the model cutoff $\Lambda$, as follows:

$$v_{\varphi} << v_{\xi} << v_{\Xi} = \lambda^2 \Lambda < v_{\Phi} = \lambda^3 \Lambda < v_{\Delta} = \lambda^4 \Lambda << v_{\zeta} \sim v_{\sigma} \sim v_{\phi} \sim v_{\tau_1} \sim v_{\tau_2} \sim \lambda$$  \hspace{1cm} (11)

Let us note that we have assumed a hierarchy between the vacuum expectation values of the $A_4$ scalar triplets, in order to simplify our analysis of the scalar potential for the $A_4$ scalar triplets. That hierarchy in their VEVs will allow us to neglect the mixings between these fields as follows from the method of recursive expansion of Ref. [151] and to treat their scalar potentials independently. Furthermore, let us note that we have assumed the relation $v_{\varphi} \sim \lambda v_{\Delta}$ for the vacuum expectation values of the $A_4$ scalar triplets $\Phi$ and $\Delta$ contributing to the electron and tau lepton masses as well as to the reactor mixing angle $\theta_{13}$. That assumption is made in order to connect the reactor mixing parameter $\sin^2 \theta_{13}$ with the Wolfenstein parameter $\lambda = 0.225$, through the relation $\sin \theta_{13} \sim \lambda$, which is suggested by the neutrino oscillation experimental data.

In the following we comment on the possible VEV patterns for the $A_4$ scalar triplets $\zeta, \xi, \Phi, \Delta, \Xi, \Theta$. Since the VEVs of the $A_4$ scalar triplets satisfy the following hierarchy: $v_{\varphi} << v_{\xi} << v_{\Xi} < v_{\Phi} < v_{\Delta} << v_{\zeta}$ the mixing angles between $\xi, \Delta, \Phi, \Xi, \zeta$ and $\Theta$ are very small since they are suppressed by the ratios of their VEVs, which is a consequence of the method of recursive expansion proposed in Ref. [151]. Thus, the scalar potentials for the $A_4$ scalar triplets $\zeta, \xi, \Phi, \Delta, \Xi, \Theta$ can be treated independently. As shown in detail in Appendix B the following VEV patterns for the $A_4$ scalar triplets are consistent with the scalar potential minimization equations for a large region of parameter space:

\[
\begin{align*}
\langle \zeta \rangle &= \frac{v_{\zeta}}{\sqrt{3}} (1, 1, 1), \\
\langle \xi \rangle &= \frac{v_{\xi}}{\sqrt{3}} (1, 0, 0), \\
\langle \Delta \rangle &= v_{\Delta} (0, 0, 1), \\
\langle \Phi \rangle &= v_{\Phi} (1, 0, 0), \\
\langle \Xi \rangle &= v_{\Xi} (0, 1, 0), \\
\langle \Theta \rangle &= -\frac{v_{\Theta}}{\sqrt{3}} (1, 2, 0).
\end{align*}
\]  \hspace{1cm} (12)

**III. QUARK MASSES AND MIXINGS.**

From the quark Yukawa interactions given by Eq. [9] we find that the SM mass matrices for quarks take the form:

\[
M_U = \frac{v}{\sqrt{2}} \begin{pmatrix}
c_1 \lambda^8 & b_1 \lambda^5 & a_1 \lambda^4 \\
0 & b_2 \lambda^4 & a_2 \lambda^2 \\
0 & 0 & a_3
\end{pmatrix}, \\
M_D = \frac{v}{\sqrt{2}} \begin{pmatrix}
g_1 \lambda^7 & 0 & 0 \\
0 & g_2 \lambda^5 & 0 \\
0 & 0 & g_3 \lambda^3
\end{pmatrix} R_D,
\]

\[
R_D = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega \\
1 & 1 & 1
\end{pmatrix}, \\
\omega = e^{2\pi i/3}.
\]  \hspace{1cm} (13)

where $c_1, b_n \ (n=1, 2)$, $a_i, g_i \ (i=1, 2, 3)$ are $O(1)$ dimensionless parameters. Here $\lambda = 0.225$ is one of the Wolfenstein parameters and $v = 246$ GeV the scale of electroweak symmetry breaking. From the SM quark mass textures given above, it follows that the quark mixing angles only arise from the up type quark sector. Besides that, the low energy quark flavor data indicates that the CP violating phase in the quark sector is associated with the quark mixing angle in the 1-3 plane, as follows from the Standard parametrization of the quark mixing matrix. Consequently, in order to get quark mixing angles and a CP violating phase consistent with the experimental data, we assume that all dimensionless parameters given in Eqs. [13] are real, except for $a_1$, taken to be complex.

Furthermore, as follows from the different $Z_6$ charge assignments for the quark fields, the exotic quarks do not mix
with the SM quarks. We find that the exotic quark masses are given by:

\[ m_T = y^{(T)} \frac{v_X}{\sqrt{2}}, \quad m_J = y^{(J)} \frac{v_X}{\sqrt{2}} = \frac{y}{y^{(T)}} m_T, \quad m_J^2 = y^{(J)} \frac{v_X}{\sqrt{2}} = \frac{y^2}{y^{(T)}} m_T. \tag{14} \]

Since the observed charged fermion mass and quark mixing pattern is generated by the breaking of the \( A_1 \times Z_6 \times Z_{16} \times Z'_{16} \) discrete group, and in order to simplify the analysis as well as motivated by naturalness arguments, we set

\[ c_1 = b_1 = b_2 = a_3 = g_2 = 1. \tag{15} \]

Consequently, there are only 5 effective free parameters in the SM quark sector of our model, i.e., \( |a_1|, a_2, g_1, g_3 \) and the phase \( \arg (a_1) \). We fit these 5 parameters to reproduce the 10 physical observables of the quark sector, i.e., the six quark masses, the three mixing angles and the CP violating phase. By varying the parameters \( |a_1|, a_2, g_1, g_3 \) and the phase \( \arg (a_1) \), we find the quark masses, the three quark mixing angles and the CP violating phase \( \delta \) reported in Table I, which correspond to the best fit values:

\[ |a_1| \simeq 3.35, \quad a_2 \simeq -0.8, \quad g_1 \simeq 0.56, \quad g_3 \simeq 1.42, \quad \arg (a_1) = -156.74^\circ. \]

| Observable          | Model value | Experimental value |
|---------------------|-------------|--------------------|
| \( m_u(\text{MeV}) \) | 1.13        | 1.45^{+0.56}_{-0.45} |
| \( m_c(\text{MeV}) \) | 460         | 635 ± 86           |
| \( m_t(GeV) \)      | 174.1       | 172.1 ± 0.6 ± 0.9  |
| \( m_d(\text{MeV}) \) | 2.9         | 2.9^{+0.5}_{-0.4}  |
| \( m_s(\text{MeV}) \) | 101         | 57.7^{+16.4}_{-15.7} |
| \( m_b(GeV) \)      | 2.82        | 2.82^{+0.09}_{-0.04} |
| \( \sin \theta_{12} \) | 0.220       | 0.2254             |
| \( \sin \theta_{23} \) | 0.0414      | 0.0413             |
| \( \sin \theta_{13} \) | 0.00354     | 0.00351            |
| \( \delta \)        | 72^\circ    | 68^\circ            |

Table I: Model and experimental values of the quark masses and CKM parameters.

In Table I we show the model and experimental values for the physical observables of the quark sector. We use the \( M_2 \)-scale experimental values of the quark masses given by Ref. [152] (which are similar to those in [153]). The experimental values of the CKM parameters are taken from Ref. [154]. As indicated by Table I, the obtained quark masses, quark mixing angles, and CP violating phase are consistent with the low energy quark flavor data.

### IV. LEPTON MASSES AND MIXINGS.

From Eqs. (10), (11), (12) and using the product rules of the \( A_4 \) group given in Appendix A we find that the charged lepton mass matrix is given by:

\[ M_l = \frac{v}{\sqrt{2}} \begin{pmatrix} x_1 \lambda^9 & 0 & z_1 \lambda^4 \\ 0 & y \lambda^3 & 0 \\ x_2 \lambda^8 & 0 & z_2 \lambda^3 \end{pmatrix} \tag{16} \]

where \( x_n, y, z_n (n = 1, 2) \) are \( \mathcal{O}(1) \) dimensionless parameters, assumed to be real, excepting \( z_1 \), taken to be complex, in order to generate a nonvanishing Dirac CP violating phase.

Since the charged lepton mass hierarchy arises from the breaking of the \( A_4 \times Z_6 \times Z_{16} \times Z'_{16} \) discrete group and in order to simplify the analysis, we consider an scenario of approximate universality in the dimensionless SM charged lepton Yukawa couplings, as follows:

\[ x_1 = x_2 = x, \quad z_1 = iz, \quad z_2 = z. \tag{17} \]
The matrix $M_l M_l^T$ is diagonalized by a rotation matrix $R_l$ according to:

$$R_l^T M_l M_l^T R_l = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}, \quad R_l = \begin{pmatrix} \cos \theta_l & 0 & -i \sin \theta_l \\ 0 & 1 & 0 \\ i \sin \theta_l & 0 & \cos \theta_l \end{pmatrix}, \quad \tan \theta_l \simeq \lambda, \quad (18)$$

where the charged lepton masses are approximately given by:

$$m_e \simeq x \lambda^0 v, \quad m_\mu = y \lambda^5 \frac{v}{\sqrt{2}}, \quad m_\tau \simeq z \lambda^4 \frac{v}{\sqrt{2}}$$

with $x, y, z$ are $O(1)$ dimensionless parameters given by:

$$x \simeq 1.37, \quad y \simeq 1.02, \quad z \simeq 0.86.$$ 

It is worth mentioning that the charged lepton masses are connected with the electroweak symmetry breaking scale $v = 246$ GeV by their scalings with powers of the Wolfenstein parameter $\lambda = 0.225$, with $O(1)$ coefficients. This is consistent with our previous assumption made in Eq. [11] regarding the size of the VEVs for the $SU(3)_L$ singlet scalars appearing in the charged fermion Yukawa terms. Furthermore, it is noteworthy that the mixing angle $\theta_l$ in the charged lepton sector is large, which gives rise to an important contribution to the leptonic mixing matrix, coming from the mixing of charged leptons.

Regarding the neutrino sector, from the Eq. [10], we find the following neutrino mass terms:

$$-\mathcal{L}^{(\nu)}_{\text{mass}} = \frac{1}{2} \begin{pmatrix} \nu^c_L & \nu^c_R \end{pmatrix} \begin{pmatrix} 0_{3 \times 3} & M_D \\ M_D^T & 0_{3 \times 3} \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_R^c \end{pmatrix} + H.c., \quad (19)$$

where the $A_3$ family symmetry constrains the neutrino mass matrix to be of the form:

$$M_\nu = \begin{pmatrix} 0_{3 \times 3} & M_{\nu D} & 0_{3 \times 3} \\ M_{\nu D}^T & 0_{3 \times 3} & M_\chi \\ 0_{3 \times 3} & M_\chi^T & M_R \end{pmatrix} \quad (20)$$

where the submatrices $M_{\nu D}$ and $M_\chi$ are generated at tree level from the nonrenormalizable $\varepsilon_{abc} \left( \bar{L}_L^a \left( L_L^b \right)^c \right) \Omega_n \left( \phi^a \right)^c \frac{v}{\Lambda}$ and renormalizable $\left( \bar{T}_L \chi N_R \right)_\Omega$ Yukawa terms, respectively, whereas the submatrix $M_R$ arises from a one loop level radiative seesaw mechanism mediated by the massive right handed Majorana neutrinos $\Omega_i (i = 1, 2, 3)$ and the real $\Re \phi$ and imaginary $\Im \phi$ parts of the $Z_8$ charged scalar field $\phi$. As previously mentioned, the facts that the $Z_8$ discrete group is broken down to the preserved $Z_2$ symmetry and the $SU(3)_L$ singlet scalar field $\phi$ (which appears in the neutrino Yukawa interaction $(\bar{N}_R \Omega) \phi$) has a $Z_8$ charge corresponding to a nontrivial $Z_2$ charge, implies that this scalar does not acquire a vacuum expectation value, thus generating the submatrix $M_R$ only at one loop level. The submatrices $M_{\nu D}, M_\chi$ and $M_R$ are given by:

$$M_{\nu D} = \frac{y_{\nu e} y_{\nu e}}{\sqrt{2} \Lambda} \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad M_\chi = y_\chi \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (21)$$

$$M_R = \begin{pmatrix} F(m_\Omega, m_{\Re \phi}, m_{\Im \phi}) m_\Omega & 0 & -2y_{\Omega l} F(-2y_{\Omega l} \frac{v_\Omega}{\sqrt{3}}, m_{\Re \phi}, m_{\Im \phi}) \frac{v_\Omega}{\sqrt{3}} \\ 0 & F(m_\Omega, m_{\Re \phi}, m_{\Im \phi}) m_\Omega & -y_{\Omega l} F(-y_{\Omega l} \frac{v_\Omega}{\sqrt{3}}, m_{\Re \phi}, m_{\Im \phi}) \frac{v_\Omega}{\sqrt{3}} \\ -2y_{\Omega l} F(-2y_{\Omega l} \frac{v_\Omega}{\sqrt{3}}, m_{\Re \phi}, m_{\Im \phi}) \frac{v_\Omega}{\sqrt{3}} & -y_{\Omega l} F(-y_{\Omega l} \frac{v_\Omega}{\sqrt{3}}, m_{\Re \phi}, m_{\Im \phi}) \frac{v_\Omega}{\sqrt{3}} & F(m_\Omega, m_{\Re \phi}, m_{\Im \phi}) m_\Omega \end{pmatrix},$$

where the following function has been introduced [155]:

$$F(m_1, m_2, m_3) = \left( \frac{y_{\phi}(\chi)}{16\pi^2} \right)^2 \left[ \frac{m_2^2}{m_2^2 - m_1^2} \ln \left( \frac{m_2^2}{m_1^2} \right) - \frac{m_3^2}{m_3^2 - m_2^2} \ln \left( \frac{m_3^2}{m_2^2} \right) \right], \quad (22)$$
In order to connect the neutrino mass squared splittings with the quark mixing parameters and motivated by the relation $\Delta m^2_{ij} \sim \lambda^4 \mathcal{O}(1) eV^2$, we set $v_0 \sim \lambda^4 m_{\Omega}$. In addition, for the sake of simplicity we assume that the $Z_3$ charged $SU(3)_L$ singlet scalar field $\varphi$ is heavier than the right handed Majorana neutrinos $\Omega_i$ ($i = 1, 2, 3$), in such a way that we can restrict to the scenario:

$$m^2_{\text{Re} \varphi}, m^2_{\text{Im} \varphi} \gg m^2_\Omega \gg y^2_0 v_\varphi^2 \sim \lambda^6 m_{\Omega}$$  \hspace{1cm} (23)$$

for which the submatrix $M_R$ takes the form:

$$M_R \simeq \left(\frac{y_\varphi^{(N)}}{\sqrt{2}}\right)^2 \frac{m^2_{\text{Re} \varphi} - m^2_{\text{Im} \varphi}}{8\pi^2} \begin{pmatrix} m_\Omega & 0 & -2y_\Omega \frac{v_0}{\sqrt{2}} \\ 0 & m_\Omega & -y_\Omega \frac{v_0}{\sqrt{2}} \\ -2y_\Omega \frac{v_0}{\sqrt{2}} & -y_\Omega \frac{v_0}{\sqrt{2}} & m_\Omega \end{pmatrix} = \begin{pmatrix} \gamma_1 & 0 & -2\gamma_2 \lambda^4 \\ 0 & \gamma_1 & -\gamma_2 \lambda^4 \\ -2\gamma_2 \lambda^4 & -\gamma_2 \lambda^4 & \gamma_1 \end{pmatrix} m_R, \hspace{1cm} (24)$$

where $\gamma_1$ and $\gamma_2$ are $\mathcal{O}(1)$ dimensionless parameters, assumed to be real for simplicity. Furthermore, $m_R$ is the mass scale for the Majorana neutrinos $\Omega_i$ ($i = 1, 2, 3$), which sets the scale of breaking of lepton number.

As shown in detail in Ref. [156], the full rotation matrix that diagonalizes the neutrino mass matrix $M_\nu$ is approximately given by:

$$U = \begin{pmatrix} V_\nu & B_3 U_\chi & B_2 U_R \\ -(B_1^T + B_1) V_\nu & (1-S) U_\chi & (1+S) U_R \\ -(B_1^T - B_1) V_\nu & -(1-S) U_\chi & (1-S) U_R \end{pmatrix}, \hspace{1cm} (25)$$

where

$$S = -\frac{1}{2\sqrt{2}y^{(L)}_\chi v_\chi} M_R, \hspace{1cm} B_2 \simeq B_3 \simeq \frac{1}{y^{(L)}_\chi v_\chi} M^*_R, \hspace{1cm} (26)$$

and the physical neutrino mass matrices are:

$$M^{(1)}_\nu = M_{\nu D} \left(M^T_\chi \right)^{-1} M_R M^{-1}_\chi M^T_D, \hspace{1cm} (27)$$

$$M^{(2)}_\nu = -\frac{1}{2} \left(M_\chi + M^T_\chi \right) + \frac{1}{2} M_R, \hspace{1cm} M^{(3)}_\nu = \frac{1}{2} \left(M_\chi + M^T_\chi \right) + \frac{1}{2} M_R, \hspace{1cm} (28)$$

where $M^{(1)}_\nu$ is the light active neutrino mass matrix whereas $M^{(2)}_\nu$ and $M^{(3)}_\nu$ are the exotic Dirac neutrino mass matrices. It is worth mentioning that physical neutrino spectrum consists of three light active neutrinos and six exotic neutrinos. The exotic neutrinos are pseudo-Dirac, with masses $\sim \pm v_\chi$ and a small splitting $\sim m_R$. Furthermore, $V_\nu$, $U_\chi$, $U_R$ and $U_\chi$ are the rotation matrices which diagonalize $M^{(1)}_\nu$, $M^{(2)}_\nu$ and $M^{(3)}_\nu$, respectively.

From Eq. [27] it follows that the light active neutrino mass matrix is given by:

$$M^{(1)}_\nu = \begin{pmatrix} 2 \left(\gamma_1 - \gamma_2 \lambda^4 \right) & 2\gamma_2 \lambda^4 & 2\gamma_2 \lambda^4 \\ 2\gamma_2 \lambda^4 & \gamma_1 & \gamma_1 \\ 2\gamma_2 \lambda^4 & \gamma_1 & \gamma_1 \end{pmatrix} m_\nu, \hspace{1cm} m_\nu = \frac{2y^2_\rho v^2_\nu v^2_\chi m_R}{(y^{(L)}_\chi)^2 v^2_\chi \Lambda^2}, \hspace{1cm} (29)$$

where $m_\nu$ is the light active neutrino mass scale, which we set as $m_\nu = 50$ meV. Let us note that the smallness of the active neutrino masses arises from their scaling with inverse powers of the high energy cutoff $\Lambda$ as well as from their linear dependence on the loop induced mass scale $m_R$ for the Majorana neutrinos $\Omega_i$ ($i = 1, 2, 3$).

The light active neutrino mass matrix $M^{(1)}_\nu$ is diagonalized by a unitary rotation matrix $R_\nu$, according to:
leptonic mixing matrix takes the form:

\[
R^T \nu M^{(1)} (1) R_\nu = \begin{pmatrix}
0 & 0 & 0 \\
0 & 2 (\gamma_1 - 2 \gamma_2 \lambda^4) m_\nu & 0 \\
0 & 0 & 2 (\gamma_1 + \gamma_2 \lambda^4) m_\nu
\end{pmatrix}, \quad R_\nu = \begin{pmatrix}
0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}
\end{pmatrix}, \quad \text{for } \text{NH}
\]

\[
R^T \nu M^{(1)} (2) R_\nu = \begin{pmatrix}
2 (\gamma_1 - 2 \gamma_2 \lambda^4) m_\nu & 0 & 0 \\
0 & 2 (\gamma_1 + \gamma_2 \lambda^4) m_\nu & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad R_\nu = \begin{pmatrix}
\frac{2}{\sqrt{6}} & 1 & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}, \quad \text{for } \text{IH}
\]

Consequently, the light active neutrino spectrum is composed of one massless neutrino and two active neutrinos, whose masses are determined from the experimental values of the neutrino mass squared splittings.

From Eqs. (18) and Eqs. (29), it follows that the normal hierarchy scenario leads to a too value for the large reactor mixing angle, which is disfavored by the neutrino oscillation experimental data. Thus, the normal neutrino mass hierarchy scenario of our model is ruled out by the current data on neutrino oscillation experiments. In what regards inverted neutrino mass hierarchy, we find from Eqs. (18) and Eqs. (29), that the corresponding PMNS leptonic mixing matrix takes the form:

\[
U = \begin{pmatrix}
-\frac{2-\lambda}{\sqrt{6}\sqrt{\lambda^2+1}} & \frac{1+i\lambda}{\sqrt{3}\sqrt{\lambda^2+1}} & \frac{i\lambda}{\sqrt{2}\sqrt{\lambda^2+1}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\
\frac{1-2\lambda}{\sqrt{6}\sqrt{\lambda^2+1}} & \frac{1+i\lambda}{\sqrt{3}\sqrt{\lambda^2+1}} & \sqrt{2}\sqrt{\lambda^2+1}
\end{pmatrix}
\]

(30)

From the standard parametrization of the leptonic mixing matrix, we predict that the lepton mixing parameters for the case of inverted neutrino mass hierarchy are given by:

\[
\sin^2 \theta_{12} = \frac{2 (1 + \lambda^2)}{3 (2 + \lambda^2)} \simeq 0.342, \quad \sin^2 \theta_{23} = \frac{1 + \lambda^2}{2 + \lambda^2} \simeq 0.512, \quad \sin^2 \theta_{13} = \frac{\lambda^2}{2 (1 + \lambda^2)} \simeq 0.0241.
\]

(31)

| Parameter | $\Delta m^2_{12} (10^{-5} \text{eV}^2)$ | $\Delta m^2_{13} (10^{-3} \text{eV}^2)$ | $(\sin^2 \theta_{12})_{\exp}$ | $(\sin^2 \theta_{23})_{\exp}$ | $(\sin^2 \theta_{13})_{\exp}$ |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Best fit  | 7.60            | 2.38            | 0.323           | 0.573           | 0.0240          |
| 1$\sigma$ range | 7.42 – 7.79    | 2.32 – 2.43  | 0.307 – 0.339  | 0.530 – 0.598  | 0.0221 – 0.0259 |
| 2$\sigma$ range | 7.26 – 7.99    | 2.26 – 2.48  | 0.292 – 0.357  | 0.432 – 0.621  | 0.0202 – 0.0278 |
| 3$\sigma$ range | 7.11 – 8.11    | 2.20 – 2.54  | 0.278 – 0.375  | 0.403 – 0.640  | 0.0183 – 0.0297 |

Table II: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [157], for the case of inverted hierarchy.

From the comparison of Eq. (31) with Table II, it follows that the leptonic mixing parameters $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ are in excellent agreement with the experimental data, whereas $\sin^2 \theta_{12}$ is deviated $2\sigma$ away from its best fit values. It is remarkable that without any free parameter, our model predict leptonic mixing parameters in very good agreement with their experimental values, for the case of inverted neutrino mass spectrum. Furthermore, the predicted Jarlskog invariant the resulting Dirac CP violating phase are given by:

\[
J = \frac{\lambda}{6 (1 + \lambda^2)} \simeq 3.57 \times 10^{-2}, \quad \delta = \arcsin \left( \frac{(2 + \lambda^2)^{\frac{1}{2}}}{\sqrt{2} (1 + \lambda^2) \sqrt{4 + \lambda^2}} \right) \simeq 79.11^\circ.
\]

(32)

Furthermore, from the experimental values of the neutrino mass squared splittings for the case of inverted neutrino mass hierarchy and setting $\gamma_2 = 1$ as suggested by naturalness arguments, we found that the $O(1)$ dimensionless parameter $\gamma_1$ is given by:

\[
\gamma_1 \simeq 0.493.
\]

(33)
Thus, our model predicts the following values for the neutrino mass squared splittings for the case of inverted neutrino mass hierarchy:

$$\Delta m^2_{21} = 12\lambda^4 \left(2\gamma_1 - \lambda^4\right) m^2_\nu \simeq 7.56 \times 10^{-5} \text{eV}^2, \quad \Delta m^2_{13} = 4 \left(\gamma_1 - 2\lambda^4\right)^2 m^2_\nu \simeq 2.38 \times 10^{-3} \text{eV}^2. \tag{34}$$

where we set the light active neutrino mass scale as $m_\nu = 50$ meV.

It is remarkable that with only one effective free parameter, i.e., $\gamma_1$, the predicted values for the neutrino mass squared splittings are inside their 1σ experimentally allowed range, thus exhibiting an excellent agreement with the experimental data on neutrino oscillations experiments, as follows from the comparison of Eq. (34) with Table II.

In the following we proceed to determine the effective Majorana neutrino mass parameter, whose value is proportional to the amplitude of neutrinoless double beta ($0\nu\beta\beta$) decay. The effective Majorana neutrino mass parameter is given by:

$$m_{ee} = \left| \sum_k U^2_{ek} m_{\nu_k} \right|, \tag{35}$$

where $U^2_{ij}$ and $m_{\nu_k}$ are the squared of the PMNS leptonic mixing matrix elements and the masses of the Majorana neutrinos, respectively.

Thus, we predict that the effective Majorana neutrino mass parameter for the inverted neutrino mass hierarchy is given by:

$$m_{ee} = \sqrt{\left[\gamma_1 \left(2 - \lambda^2\right) - 2\gamma_2 \lambda^4\right]^2 + 16\gamma_3^2} \lambda^{10} \frac{1 + \lambda^2}{1 + \lambda^2} m_\nu \simeq 45.5 \text{meV}. \tag{36}$$

Our obtained value $m_{ee} \simeq 45.5$ meV for the effective Majorana neutrino mass parameter in the case of inverted neutrino mass hierarchy, is within the declared reach of the next-generation bolometric CUORE experiment [158] or, more realistically, of the next-to-next-generation ton-scale $0\nu\beta\beta$-decay experiments. It is worth mentioning that the effective Majorana neutrino mass parameter has the upper bound of $m_{ee} \leq 160$ meV, which corresponds to $T^{0\nu\beta\beta}_{1/2}$(Xe) $\geq 1.1 \times 10^{26}$ yr at 90% C.L, which follows from the experimental data of the KamLAND-Zen experiment [159]. That limit is expected to be updated in a not too distant future. The GERDA “phase-II” experiment [160, 161] is expected to reach $T^{0\nu\beta\beta}_{1/2}$(Ge) $\geq 2 \times 10^{26}$ yr, which corresponds to $m_{ee} \leq 100$ meV. A bolometric CUORE experiment, using $^{130}\text{Te}$ [158], is currently under construction and its estimated sensitivity is about $T^{0\nu\beta\beta}_{1/2}$(Te) $\sim 10^{26}$ yr, corresponding to $m_{ee} \leq 50$ meV. In addition, there are plans for ton-scale next-to-next generation $0\nu\beta\beta$ experiments with $^{136}\text{Xe}$ [162, 163] and $^{76}\text{Ge}$ [160, 161], asserting sensitivities over $T^{0\nu\beta\beta}_{1/2} \sim 10^{27}$ yr, which corresponds to $m_{ee} \sim 12 \sim 30$ meV. Some reviews on the theory and phenomenology of neutrinoless double-beta decay are provided in Refs. [165, 166]. Our results indicate that our model predicts $T^{0\nu\beta\beta}_{1/2}$ at the level of sensitivities of the next generation or next-to-next generation $0\nu\beta\beta$ experiments.

V. DARK MATTER RELIC DENSITY.

In this section we will discuss the implications of our model in Dark matter. We will assume that the Dark matter candidate in our model is a scalar. As a result of this assumption and considering that the $SU(3)_L$ scalar singlet $\varphi$
is the only scalar field having a $Z_6$ charge corresponding to a nontrivial charge under the preserved $Z_2$ symmetry, we have that it is the only Dark matter candidate in our model.

Relic density of the dark matter in the present Universe is estimated as follows (c.f. Ref. [154])

$$\Omega h^2 = \frac{0.1 pb}{\langle \sigma v \rangle}, \quad \langle \sigma v \rangle = \frac{A}{n_{eq}},$$

where $\langle \sigma v \rangle$ is the thermally averaged annihilation cross-section, $A$ is the total annihilation rate per unit volume at temperature $T$ and $n_{eq}$ is the equilibrium value of the particle density, which are given by [167]

$$A = \frac{T}{32\pi^4} \int_0^\infty \sum_{p=W,Z,t,b,h} g_p^2 s \sqrt{s-4m_p^2} v_{rel} \sigma(\varphi \varphi \rightarrow p\bar{p}) K_1\left(\frac{\sqrt{s}}{T}\right) ds,$$

$$n_{eq} = \frac{T}{2\pi^2} \sum_{p=W,Z,t,b,h} g_p m_p^2 K_2\left(\frac{m_p}{T}\right),$$

with $K_1$ and $K_2$ being the modified Bessel functions of the second kind order 1 and 2, respectively [167]. For the relic density calculation, we take $T = m_\varphi/20$ as in Ref. [167], which corresponds to a typical freeze-out temperature. We assume that our DM candidate $\varphi$ annihilates mainly into WW, ZZ, $t\bar{t}$, $b\bar{b}$ and $hh$, with annihilation cross sections given by: [168]

$$v_{rel} \sigma(\varphi \varphi \rightarrow WW) = \frac{\lambda_{h^2\varphi^2}s}{8\pi} \left(1 + \frac{12m^4_W}{s^2} - \frac{4m_W^2}{s}\right) \sqrt{1 - \frac{4m_W^2}{s}},$$

$$v_{rel} \sigma(\varphi \varphi \rightarrow ZZ) = \frac{\lambda_{h^2\varphi^2}s}{16\pi} \left(1 + \frac{12m^4_Z}{s^2} - \frac{4m_Z^2}{s}\right) \sqrt{1 - \frac{4m_Z^2}{s}},$$

$$v_{rel} \sigma(\varphi \varphi \rightarrow q\bar{q}) = \frac{N_c \lambda_{h^2\varphi^2}m_h^2}{4\pi} \sqrt{\left(\frac{1 - 4m_h^2}{s}\right)^3 - \frac{3m_h^4}{s}},$$

$$v_{rel} \sigma(\varphi \varphi \rightarrow hh) = \frac{\lambda_{h^2\varphi^2}}{16\pi s} \left(1 + \frac{3m_h^2}{s - m_h^2} - \frac{4\lambda_{h^2\varphi^2}v^2}{s - 2m_h^2}\right) \sqrt{1 - \frac{4m_h^2}{s}}.$$

where $\sqrt{s}$ is the centre-of-mass energy, $N_c = 3$ is the color factor, $m_h = 125.7$ GeV and $\Gamma_h = 4.1$ MeV are the SM Higgs boson $h$ mass and its total decay width, respectively.

Fig. 1 displays the Relic density $\Omega h^2$ as a function of the mass $m_\varphi$ of the scalar field $\varphi$, for several values of the quartic scalar coupling $\lambda_{h^2\varphi^2}$. The curves from top to bottom correspond to $\lambda_{h^2\varphi^2} = 1, 1.2$ and 1.5, respectively. The horizontal line corresponds to the experimental value $\Omega h^2 = 0.1198$ for the relic density. The Figure 1 shows that the Relic density is an increasing function of the mass $m_\varphi$ and a decreasing function of the quartic scalar coupling $\lambda_{h^2\varphi^2}$. Consequently, an increase in the mass $m_\varphi$ of the scalar field $\varphi$ will require a larger quartic scalar coupling $\lambda_{h^2\varphi^2}$, in order to account for the measured value of the Dark matter relic density, as indicated by Fig. 2. It is worth mentioning that the Dark matter relic density constraint yields a linear correlation between the quartic scalar coupling $\lambda_{h^2\varphi^2}$ and the mass $m_\varphi$ of the scalar Dark matter candidate $\varphi$, as shown in Fig. 2. We have numerically checked that in order to reproduce the experimental value $\Omega h^2 = 0.1198 \pm 0.0026$ [169] of the relic density, the mass $m_\varphi$ of the scalar field $\varphi$ has to be in the range $0.55 \text{ TeV} \lesssim m_\varphi \lesssim 7 \text{ TeV}$, for a quartic scalar coupling $\lambda_{h^2\varphi^2}$ in the range $1 \lesssim \lambda_{h^2\varphi^2} \lesssim 4\pi$. Here we take $4\pi$ as the upper bound on the quartic coupling $\lambda_{h^2\varphi^2}$, arising from perturbativity.

VI. CONCLUSIONS

We constructed a highly predictive 3-3-1 model with right-handed neutrinos, where the symmetry is extended by $A_4 \times Z_6 \times Z_{16} \times Z'_{16}$ and the field content is enlarged by extra $SU(3)_L$ singlet scalar fields and six right handed Majorana neutrinos. Our model is consistent with the low energy fermion flavor data. The $A_4$, $Z_6$ and $Z'_{16}$ symmetries are crucial for reducing the number of fermion sector model parameters, whereas the $Z_{16}$ symmetry causes the charged
fermion mass and quark mixing pattern. In the model under consideration, the light active neutrino masses are generated from a one loop level inverse seesaw mechanism and the observed pattern of charged fermion masses and quark mixing angles is caused by the breaking of the $A_4 \times Z_6 \times Z_{16} \times Z'_{16}$ discrete group at very high energy. In our model the different discrete group factors are broken completely, excepting the $Z_6$ discrete group, which is broken down to the preserved $Z_2$ symmetry, thus allowing the implementation of the one loop level inverse seesaw mechanism for the generation of the light active neutrino masses. The resulting neutrino spectrum of our model is composed of light active neutrinos and TeV scale exotic pseudo-Dirac neutrinos. The smallness of the active neutrino masses is a natural consequence of their scaling with inverse powers of the large model cutoff $\Lambda$ and of their linear dependence on the loop induced mass scale $m_R$ for the Majorana neutrinos $N_i$ ($i = 1, 2, 3$). The SM Yukawa sector of our highly predictive $A_4$ flavor 3-3-1 model has in total only 9 effective free parameters (4 and 5 effective free parameters in the lepton and quark sectors, respectively), which we adjust to reproduce the experimental values of the 18 physical observables in the quark and lepton sectors, i.e., 9 charged fermion masses, 2 neutrino mass squared splittings, 3 lepton mixing parameters, 3 quark mixing angles and 1 CP violating phase of the CKM quark mixing matrix. The obtained values of the physical observables for the quark sector are consistent with the experimental data, whereas the ones for the lepton sector also do but only for the inverted neutrino mass spectrum. The normal neutrino mass hierarchy scenario of our model is disfavored by the neutrino oscillation experimental data, since the resulting reactor mixing parameter is much larger than its experimental upper limit. Our model predicts an effective Majorana neutrino mass parameter of neutrinoless double beta decay of $m_{ee} = 45.5$ meV, a leptonic Dirac CP violating phase of $79.11^\circ$ and a Jarlskog invariant of about $10^{-2}$ for the inverted neutrino mass spectrum. Our obtained value of meV for the effective Majorana neutrino mass is within the declared reach of the next generation bolometric CUORE experiment [168] or, more realistically, of the next-to-next generation ton-scale $0\nu\beta\beta$-decay experiments. Due to the fact that the $Z_6$ discrete group, which is broken down to the preserved $Z_2$ symmetry our model possesses a scalar DM particle candidate. The constraints arising from the DM relic density, set its mass in the range $0.55$ TeV $\lesssim m_\varphi \lesssim 7$ TeV, for a quartic scalar coupling $\lambda_{h^2\varphi^2}$ in the window $1 \lesssim \lambda_{h^2\varphi^2} \lesssim 4\pi$.
Figure 2: Correlation between the quartic scalar coupling and the mass $m_\phi$ of the scalar Dark matter candidate $\phi$, consistent with the experimental value $\Omega h^2 = 0.1198$ for the Relic density.

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Appendix A: The product rules for $A_4$

The $A_4$ group has one three-dimensional $3$ and three distinct one-dimensional $1$, $1'$ and $1''$ irreducible representations, satisfying the following product rules:

$$3 \otimes 3 = 3_s \oplus 3_a \oplus 1 \oplus 1' \oplus 1'', \quad (A1)$$

$$1 \otimes 1 = 1, \quad 1' \otimes 1'' = 1, \quad 1' \otimes 1' = 1'', \quad 1'' \otimes 1'' = 1', \quad (A2)$$

Considering $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ as the basis vectors for two $A_4$-triplets $3$, the following relations are fulfilled:

$$\begin{align*}
(3 \otimes 3)_1 &= x_1 y_1 + x_2 y_2 + x_3 y_3, \\
(3 \otimes 3)_{3_s} &= (x_2 y_3 + x_3 y_2, x_3 y_1 + x_1 y_3, x_1 y_2 + x_2 y_1), \\
(3 \otimes 3)_{3_a} &= (x_2 y_3 - x_3 y_2, x_3 y_1 - x_1 y_3, x_1 y_2 - x_2 y_1), \\
(3 \otimes 3)_{1'} &= x_1 y_1 + \omega x_2 y_2 + \omega^2 x_3 y_3, \\
(3 \otimes 3)_{1''} &= x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3,
\end{align*}$$

where $\omega = e^{i \frac{2\pi}{3}}$. The representation $1$ is trivial, while the non-trivial $1'$ and $1''$ are complex conjugate to each other. Some reviews of discrete symmetries in particle physics are found in Refs. [25–28].
Appendix B: Scalar potential for one $A_4$ scalar triplet

The scalar potential for any $A_4$ scalar triplet takes the form:

$$V(\Sigma) = -\mu_\Sigma^2 \left(\Sigma \Sigma^*\right)_1 + \kappa_{\Sigma,1} (\Sigma \Sigma^*)_1 + \kappa_{\Sigma,2} (\Sigma \Sigma^*)_1 + \kappa_{\Sigma,3} (\Sigma \Sigma^*)_1 + \kappa_{\Sigma,4} (\Sigma \Sigma^*)_1 + h.c + \kappa_{\Sigma,5} (\Sigma \Sigma^*)_1 + h.c$$

where $\Sigma = \xi, \zeta, \Phi, \Delta, \Xi, \Theta$.

That scalar potential given above has 8 free parameters: 1 bilinear and 7 quartic couplings. The scalar potential minimization conditions read:

$$\frac{\partial \langle V(\Sigma) \rangle}{\partial \Sigma_1} = 0,$$

$$\frac{\partial \langle V(\Sigma) \rangle}{\partial \Sigma_2} = 0,$$

$$\frac{\partial \langle V(\Sigma) \rangle}{\partial \Sigma_3} = 0,$$

where $\langle \Sigma \rangle = (v_{\Sigma_1} e^{i\theta_{\Sigma_1}}, v_{\Sigma_2} e^{i\theta_{\Sigma_2}}, v_{\Sigma_3} e^{i\theta_{\Sigma_3}})$. Here for the sake of simplicity we consider vanishing phases in the VEV patterns of the $A_4$ triplet scalars, i.e., $\theta_{\Sigma_1} = \theta_{\Sigma_2} = \theta_{\Sigma_3} = 0$. Then, the scalar potential minimization equations given by Eq. (B2) yields the following relations:

$$3\kappa_{\Sigma,3} - 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) + 6(\kappa_{\Sigma,4} + \kappa_{\Sigma,5}) \left(v_{\Sigma_1}^2 - v_{\Sigma_3}^2\right) = 0,$$

$$3\kappa_{\Sigma,3} - 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) + 6(\kappa_{\Sigma,4} + \kappa_{\Sigma,5}) \left(v_{\Sigma_2}^2 - v_{\Sigma_3}^2\right) = 0,$$

$$3\kappa_{\Sigma,3} - 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) + 6(\kappa_{\Sigma,4} + \kappa_{\Sigma,5}) \left(v_{\Sigma_1}^2 - v_{\Sigma_2}^2\right) = 0.$$  

From the relations given by Eq. (B3) and setting $\kappa_{\Sigma,3} = \frac{1}{3} (\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) - 2(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})$, with $S = \zeta, \Phi, \Delta, \Xi, \Theta$, we obtain that the following VEV pattern:

$$\langle \xi \rangle = \frac{\nu_{\xi}}{\sqrt{3}} (1, 1, 1), \quad \langle \Phi \rangle = v_\Phi (1, 0, 0), \quad \langle \Delta \rangle = v_\Delta (0, 0, 1),$$

$$\langle \Xi \rangle = v_\Xi (0, 1, 0), \quad \langle \zeta \rangle = \frac{\nu_{\zeta}}{\sqrt{2}} (0, -1, 1), \quad \langle \Theta \rangle = -\frac{\nu_{\Theta}}{\sqrt{3}} (1, 2, 0).$$

is a solution of the scalar potential minimization equations for a large region of parameter space.

From the expressions given above, and using the vacuum configuration for the $A_4$ scalar triplets given in Eq. (77), we find the following relation:

$$\mu_\Sigma^2 = \frac{2}{3} \left(3(\kappa_{\Sigma,1} + \kappa_{\Sigma,2}) + 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7})\right) v_\Sigma^2, \quad \Sigma = \xi, \Phi, \Delta, \Xi, \Theta.$$  

(B5)
These results indicate that the VEV pattern of the $A_4$ triplets, i.e., $\xi, \zeta, \Phi, \Delta, \Xi$ and $\Theta$ in Eq. (12), are consistent with a global minimum of the scalar potential $B_1$ of our model for a large region of parameter space.
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