Self similarity of the expanding universe as understood by quantum-phase-fields

I. Steinbach‡, J. Kundin† and F. Varnik‡
Ruhr-University Bochum, ICAMS, Universitaetsstrasse 150, DE-44801 Bochum, Germany
(Dated: March 2, 2020)

PACS numbers: 04.20.Cv, 04.50.Kd, 05.70.Fh

I. INTRODUCTION

Each categoric feature of our physical world is commonly attributed by a typical ‘scale’. In the anthropic environment we use units related to the human body, like meter and feet, seconds (= one heart beat), pounds or kilograms corresponding to our body weight. In rational physics we use Planck units. They are defined exclusively in terms of five universal physical constants: Speed of light c, Planck’s constant ħ, vacuum permittivity ε₀, Boltzmann constant k_B and the coefficient of gravitation G. In the context of this essay we will deal only with speed of light c, Planck’s constant ħ and the coefficient of gravitation G, i.e. charge and temperature are not discussed. The most clear meaning of these fundamental constants has the speed of light which relates two spacial dimensions of our daily experience: the time needed for traveling and the distance of travel. We do not travel with speed of light, of course, but c is the accepted upper limit of traveling speed and independent of the observer. We can specify any speed as a fraction of the speed of light, or measure any distance by the time light would need to travel through. Planck’s constant ħ is more difficult to interpret. It relates the energy of light to its frequency, or, if speed of light is included, to its wave length. Thereby, as speed of light relates length- and time-scale, Planck’s constant relates energy- and time-scale. We accept Planck’s constant ħ as the minimum ‘quantum of action’ according to Heisenberg’s uncertainty principle. For now let us formulate the question: If c and ħ only relate different scales of our physical world: What is the scale in itself? In the above mentioned canon of universal physical constants, the coefficient of gravitation G fixes the scale of length, time or mass: Planck length \( l_P = \sqrt{\frac{\hbar G}{c^2}} \), Planck time \( \tau_P = \sqrt{\frac{\hbar G}{c^2}} \) and Planck mass \( M_P = \sqrt{\frac{\hbar G}{c^2}} \). It is argued, that our present understanding of physics breaks down below these scales, but they are definitely inaccessible by any means of experimentation in an anthropic environment. Who, however, has ever measured the gravitational constant G outside of the solar system? Or even outside of earth? Measurements are difficult and subject to large scatter and systematic errors [1]. Therefore, assuming G as a fundamental constant must be considered as a ‘reasonable hypothesis’, rather than a fact. It is evident also that, if G is not a fundamental constant, the ‘scale’ of the universe by itself is a priori undetermined. Although such a statement seems to knock over the foundations of physics, one may argue that it would be even more natural to accept that ‘no fundamental scale’ exists, instead of that such a scale is ‘God-given’. If, on the other hand, no fundamental scale is given, how can we explain the empirical fact of typical scales of structures in the universe?

Here we can learn from condensed matter physics, from the discipline of ‘pattern formation in mesoscopic systems’ [2]. A ‘mesoscopic’ system is defined small against a body where it is embedded in, that it feels no influence from macroscopic boundary conditions. The typical pattern arises from intrinsic mechanisms of self-organization. The scale of the system itself does not have to be stationary. It can be constantly evolving or approach some oscillating or chaotic limit cycle, as described by the logistic function [3]. The only necessary condition for establishing a ‘scale’ is, that characteristic features of the system are approaching some self-similar distribution in reduced coordinates. ‘Self-similar in reduced coordinates’ denotes, that the size distribution function of objects I of a size (or another characteristic scale) \( \Omega_I \), evaluated relative to an appropriate mean size \( \Omega = \text{mean}(\{\Omega_I\}) \), i.e. \( \frac{\Omega_I}{\Omega} \) becomes stationary. Examples are coarsening of foams or grains [4] (see figure [1]). The scale emerges from internal interactions in combination with appropriate conservation constraints. It shall be remarked that self-similarity of a size distribution and expansion of the characteristic scale are not in contradiction!

Recently, these concepts of mesoscopic systems have been applied by the authors to ‘pattern formation in the universe’, where gravitational interaction is shown to be
emergent from the action of quantum fluctuations in finite space objects (quantum-phase-fields)\cite{9–7}. A short summary of the concept will be presented in section II. An important outcome of the analysis is a modified law of summary of the concept will be presented in section II.

An important outcome of the analysis is a modified law of summary of the concept will be presented in section II.\cite{13}. All manifestations of being thereby have to originate from an antisymmetric process of decomposition from the homogeneous state of ‘nothing’ \cite{22}. The stage of decomposition introduces a second category of being: ‘time’ with a given direction, related to entropy production, the second law of thermodynamics (2nd Law) \cite{13}. Form this principle we take the statement that a symmetry broken state cannot be reverted to the original symmetric state, i.e. the symmetry breaking introduces a topological asymmetry, but does not violate 1st Law.

The first stage of decomposition is the separation of positive states of energy $U_i$ and negative states of energy $E_I$, $i = 1...n$, $I = 1...N$ which have to sum to $0 = \sum_{i=1}^{n} U_i + \sum_{I=1}^{N} E_I$. According to 2nd Law we will attribute the elements $U_i$ and $E_I$ with different characteristics in a common network where the respective elements separate / connect the respective other elements. Without proof we postulate the following topology as the simplest form consistent with the above principles:

- One $E$-element $E_K \div E_{km}$ connects two $U$-elements $U_k$ and $U_m$, $k, m \in 1...n$.
- One $U$-element $U_k$ connects a number of $M > 2$, $M \leq N$ $E$-elements $E_K$, $K \in 1...M$.

Obviously this leads to a network as depicted in figure 2 where the $U$-elements are ‘nodes’ and the $E$-elements are ‘edges’, for details see \cite{7}. Without loss of generality we associate the $U$-elements with positive energy, called ‘masses’ or ‘particles’, and the $E$-elements with negative energy, called ‘space’. Additional attributes, like charge, may be associated to further symmetry relations, but are not included in the present state of development of the concept.

It is noteworthy that the connectivity of the network does not necessarily have to be complete. In other words, not all particles ($U$-elements) have to be connected to each other in general. An important property of the model also is that the energy associated with the connection of two particles always depends on a one-dimensional object, the $E$-element connecting two particles, and is thus independent of the dimensionality of the space-time in which the objects are embedded.

Thus, we start from a set of elements $U_i$ and $E_I$, which fall into different categories and are ordered by a network topology. Both elements can formally be described by one mathematical object, called ‘quantum-phase-field’ $\phi_I = \phi_I(s)$ defined on a 1-dimensional space coordinate $s$. This space coordinate is related to the E-element $E_{km}$ connecting two $U$-elements $U_k$ and $U_m$ according to the above detailed topological relations. The quantum-phase-field $\phi_I(s)$ represents both $U$- and $E$-states. $U$-states will be related to gradients of the field with positive energy within the Ginsburg-Landau (GL) formalism \cite{14}, where the Hamiltonian is expanded as function of the fields $\phi_I$. Also we will include time $t$, $\phi_I(s) \rightarrow \phi_I(s, t)$. Time $t$ here is a sequence of configurations of the set of states, and we will not discuss the quest of time related
FIG. 2: Network of 4 \(U\) and 5 \(E\)-elements (dashed lines, edges). This 3-dimensional space filling configuration may relate to a neutron-neutrino, or proton-electron pair. It is ‘closed in itself’, i.e. there is no ‘loose end’. Though it needs nor necessary to be ‘com-

protons-electron pair. It is ‘closed in itself’, i.e. there is space filling configuration may relate to a neutron-neutrino, to transportation of information. The set of fields \(\phi_I(s, t)\) forms an isomorphism to the set of energetic states \(U_i(t)\) and \(E_i(t)\). They are the basic ingredients to draw the physical world in a wave-mechanical picture, as outlined in section III. We will use these fields in two different respects. Firstly in section III we treat the fields as waves in the de Broglie wave mechanical pictures and particles as realistic objects in Bohms interpretation. Second we invert the picture to a doublon network description in section IV. In this picture the waves are confined between the particles which form edges between 1-dimensional fields. Quantization follows canonically from the finiteness of spaces related to the fields \(\phi_I\) in section III.

III. PARTICLE MOTION BY WAVE GUIDANCE

Here we relate to the isomorphism between the algebra of the set of \(U\)- and \(E\)-elements of energy and their topological relations, defining space and time, and the set of fields \(\phi_I(s, t)\). The fields allow us to derive principal concepts in physics, like the de Broglie-Bohm (dBB) double solution program. In this framework we consider the structure of the Universe from the view in which particles are related to \(U\)-elements embedded in \(E\)-elements. The latter are the quantum statistical waves which obey the Klein-Gordon or Schrödinger equations. The energy of the \(E\)-elements can be defined to be positive or negative, but energy has no absolute scale in this case.

In our previous work [17], we have defined a superwave, \(\Phi = a\Phi e^{i\psi}\), as a solution of a phase-filed equation derived from a GL Hamiltonian, where \(a\Phi\) is an amplitude and \(\psi\) is a phase. We have shown that there is the analogy to the particle motion in the water wave (the \(U\)-element embedded in the \(E\)-element). The set of fields \(\phi_I(s, t)\) with common nodes. It forms an isomorphism to the set of energetic states \(E_i\) with orthogonal unit vectors \(\vec{e}_s\) and \(\vec{e}_z\).

\[
\mathbf{v}_p = \nabla \psi_v. \tag{1}
\]

Then the continuity equation for the constant density model is

\[
\nabla \cdot \mathbf{v}_p = \nabla^2 \psi_v = 0 \text{ at } z < \zeta, \tag{2}
\]

where \(\zeta\) is the surface profile function. The components of the particle velocity are defined as

\[
v_{p,x} = \frac{\partial \psi_v}{\partial x}, \quad v_{p,z} = \frac{\partial \zeta}{\partial z} = \frac{\partial \psi_v}{\partial z}. \tag{3}
\]

According to the simplified Bernoulli’s equation, we can write

\[
\frac{\partial \psi_v}{\partial t} = -g\zeta, \tag{4}
\]

where \(g\) is the acceleration due to a virtual force in \(z\)-direction by analogy to the gravity.

By differentiating eq. (4) we obtain

\[
\frac{\partial^2 \psi_v}{\partial t^2} = -g\frac{\partial \zeta}{\partial t} = -g v_{p,z} = -g \frac{\partial \psi_v}{\partial z}. \tag{5}
\]

The full set of equation for \(\phi_v\) reads

\[
\frac{\partial^2 \psi_v}{\partial s^2} + \frac{\partial^2 \psi_v}{\partial z^2} = 0 \quad \text{at } z < 0, \tag{6}
\]

\[
\frac{\partial^2 \psi_v + g }{\partial t^2} \frac{\partial \psi_v}{\partial z} = 0 \quad \text{at } z = 0, \tag{7}
\]

\[
\frac{\partial \psi_v}{\partial z} = 0 \quad \text{at } z \to -\infty. \tag{8}
\]
A solution of equations is
\[ \psi_v(s, z, t) = Ae^{kz} \cos(ks - \omega t), \tag{9} \]
and the corresponding surface profile is
\[ \zeta(s, z, t) = \eta \sin(ks - \omega t), \tag{10} \]
where \( \eta = \frac{A\omega}{g} \) is a length to be determined, \( \omega \) is the angular frequency, \( k = \frac{2\pi}{\lambda} \) is the wave number, \( \lambda \) is the wave length. From eq. \[7\], \( \omega = \sqrt{gk} \) because
\[-\omega^2 \cos(ks - \omega t) + gk \cos(ks - \omega t) = 0, \]
and from eq. \[10\] the wave velocity and the wave length are related by
\[ v_\zeta = -\frac{\partial \zeta}{\partial \zeta} = \frac{\omega}{k}. \]
From the velocity potential the coordinates of the particle velocity are defined as follows:
\[ v_{p, s} = -\eta \omega e^{kz} \sin(ks - \omega t) \tag{11} \]
\[ v_{p, z} = \eta \omega e^{kz} \cos(ks - \omega t). \tag{12} \]
Using a Taylor series about the point \( s_0 = 0, z_0 = 0 \) up to second order, the following traveling velocity in \( s \) direction can be obtained
\[ v_{p, s} = -\eta \omega \sin(k s_0 + \omega t) + \frac{\eta^2 \omega^2}{v_\zeta}. \tag{13} \]
Here the second term on the right hand side provides the drift velocity of the particle in the direction of the wave propagation.

From the dBB theory \[16\], the traveling velocity of the particle due to the guidance by a wave \( \psi \) is defined as
\[ v_{p, \text{trav}} = \frac{c^2}{v_\psi}, \]
where \( c \) is the the speed of light and
\[ v_\psi = -\frac{\partial \psi}{\partial \psi}. \]
Since the second term on the right-hand side of eq. \[13\] is the traveling velocity of the particle due to the guidance by the wave \( \zeta \), we can deduce that \( n \omega = c \). Finally, the particle velocity is defined as
\[ v_{p, s} = -c \sin(k s_0 - \omega t) + \frac{c^2}{v_\zeta}. \tag{14} \]
where the first term is the oscillation around point \( s_0 \) and the second term is the traveling velocity \( v_{p, \text{trav}} \).

Since the \( \zeta \) wave can be considered as the guidance wave in the dBB theory, it should obey the Klein-Gordon equation in relativistic case:
\[ \frac{1}{c^2} \frac{\partial^2 \zeta}{\partial t^2} - \frac{\partial^2 \zeta}{\partial s^2} = \frac{m^2 c^2}{\hbar^2} \zeta, \tag{15} \]
where \( m \) is the mass of a particle. The Hamiltonian from which this equation is derived is similar to the Hamiltonian of a phase-field equation for a relativistic singularity \[7\], where \( \zeta \) corresponds to a phase-field variable \( \psi \), which is a guiding wave. More precisely, the phase-field equation has the form of the Klein-Gordon equation with advection. The corresponding potential for the phase-field variable is equal to
\[ f(\psi) = \frac{m^2 c^2}{2\hbar^2} \psi^2. \]

According to eq. \[15\], the angular frequency and the wave number of \( \zeta \) are related by
\[ \frac{\omega^2}{c^2} = k^2 + \frac{m^2 c^2}{\hbar^2}. \tag{16} \]
From this, we can define the acceleration by
\[ g = \frac{\omega^2}{k} = c^2 k + \frac{m^2 c^2}{\hbar^2 k}. \tag{17} \]
Using these relations we can find the limits of the traveling velocity:
- if \( v_{p, \text{trav}} \to 0 \), then \( \zeta \to \infty, k \to 0 \) (because the momentum is 0), \( \lambda \to \infty, \omega \to \frac{mc^2}{\hbar}, g \to \infty, \eta \to \frac{\hbar}{mc} \).
- if \( v_{p, \text{trav}} \to c \), then \( \zeta \to c, k \to \infty, \lambda \to 0, \omega \to \infty, g \to \infty, \eta \to 0 \).

In this paper, we consider the non-relativistic case which is similar to the Klein-Gordon equation with advection. Hence, if
\[ k = \sqrt{2m(E - U)} \tag{19} \]
Hence
\[ g = \frac{\omega^2}{k} = E^2 \frac{2}{2m(E - U)}. \tag{20} \]
In the limit \( v_{p, \text{trav}} \to 0 \) we have \( k \to 0 \) because \( v_{p, \text{trav}} \sim k \). In order to get \( \omega \to \frac{\hbar}{k} \to \infty, \) we should have \( \omega \to constant \). This can be achieved if \( E \to U \to U_0 \) in eq. \[20\]. Hence, if \( v_{p, \text{trav}} \to 0 \), then \( \zeta \to \infty, k \to 0, \lambda \to \infty, \omega \to \frac{mc^2}{\hbar}, g \to \infty, \eta \to \frac{c}{\omega} \to \frac{\hbar}{U_0} = \frac{mc}{U_0} \).

Here by the comparison to the relativistic Klein-Gordon equation, it can be seen that for the case \( v_{p, \text{trav}} \to 0 \), \( \eta \to \frac{\hbar}{mc} \). Hence our assumption \( |U_0| = mc^2 \) is valid. Note that the potential can be negative or positive according to eq. \[20\]. Finally, we can see that there exists
a ground state energy $U_0$ of a pilot wave corresponding to massive energy. The cause of this energy is given by the oscillations of the guiding waves even in the case of zero particle traveling velocity, inducing oscillations of particles around their stationary positions. To estimate the minimal amplitude of the oscillations, we introduce a minimal distance $\eta_{\text{min}} = \frac{\hbar}{m v_{\text{trav}}^0}$, at which a particle position can be defined. Note that in this relation, in comparison to the classical definition of the de Broglie wave length $\lambda_{\text{dB}} = \frac{\hbar}{m v_{\text{trav}}}$, the particle occupies a finite space even for $v_{\text{trav}}^0 \to 0$.

### IV. CLOSED DOUBLON NETWORK

In the previous section a single field $\psi$ had been analyzed in regard to its individual components as the particle component $\zeta$. As said before, this analysis in the dBB wave mechanical framework can only be solved for systems with given boundary conditions, as in a quantum mechanical experiment like the double slit experiment where the experimental set up sets the boundary conditions. For discussion see [5] [17].

The identical formalism, however, can be applied in a ‘moving boundary’ setting where the particles define the (moving) boundary conditions for the waves $\psi$ which follow a linear, Schrödinger type equation on the one hand. The particles are described by the solution of a phase-field, or soliton type, non-linear wave equation with a particle solution $\phi$. They are guided, on the other hand, by the $\psi$ waves they are connected to. The coupled system evolves according to internal mechanisms of self organization. The solutions of the wave equations are called ‘doublons’ as a combination of two antisymmetric solitons [7]. This solution is schematically depicted in figure 2 for two single fields $\phi_1$ and $\phi_2$. Each field is normalized by $0 \leq \phi_i \leq 1$. The state $\phi_i = 0$ of an individual field has no physical meaning, $\phi_i = 1$ represents the $i$-element while intermediate states $0 < \phi_i < 1$ correspond to $U$-elements, see figure 2. Different fields are connected by a conservation constraint $\sum_{i=1}^N \phi_i = 1$ as postulated in [18]. Now we can reason this constraint from the second topological principle, that each $U$-element connects a number of $E$-elements, i.e. there are no ‘loose ends’ of fields but all fields end in an $U$-element. By the sum constraint the set of fields is closed in itself, forming a ‘universe’. For details see [5] [7].

The solution is depicted in figure 3 for two particle waves $\phi_1$ and $\phi_2$, also called ‘doublon’ since it consists of two half sided solitons.

The doublon forms a container of finite size for quantum oscillations $\psi$ with a discrete spectrum. The energy of this wave spectrum defines the apparent scale of the doublon $\Omega_{ij} = \frac{U_0 \eta}{E_{ij}}$ normalized by the energy quantum $U_0$ corresponding to the rest mass and the length quantum $\hbar$. In the general case the density of vacuum fluctuations on the doublon, however, are not in equilibrium (equally distributed). Then the scales seen from different particles will not be equal $\Omega_{ij} \neq \Omega_{ji}$, where $\Omega_{ij}$ denotes the scale seen from particle $i$ and $\Omega_{ji}$ the scale seen from particle $j$. The energy $E_{ij}$ associated with the ‘space’ connecting the observer $i$ with a $U$-element $j$ is defined from the Casimir forces on a 1-dimensional edge between two nodes [5] [19]:

$$E_{ij} = -\alpha_i \frac{m_j}{\Omega_{ij}}.$$ (21)

It is noteworthy that $E_{ij} \neq E_{ji}$ and depends on the observer $i$ via the coefficient $\alpha_i$ to be determined later. As will be shown below, $\alpha_i$ (and consequently the energy $E_{ij}$) depends on the set of space-like elements connecting $i$ to other particles. Since this set is in general different among observers, $\alpha_i$ depends on the specific observer. Without loss of generality, $m_j$ can be viewed as the mass of the particle $j$.

The energy $E_i = \sum_{j=1}^{N_i} E_{ij}$ attributed to the node $i$ connected to a number of nodes $j$ via the edges $ij$ is thus given by,

$$E_i = -\alpha_i \sum_{j=1}^{N_i} \frac{m_j}{\Omega_{ij}}.$$ (22)

where $N_i$ is the number of fields, which meet at the particle or $U$-element $i$. Here it must be noted that $E_i$ is a local quantity and no ‘instantaneous action at a distance’ is necessary to evaluate the energy balance according to the principle of neutrality [5] [6] $E_i + p U_i = 0$, where $p = 1$ for periodic systems, i.e. where each two particles are connected twice, and $p = 2$ for non-periodic systems [23]. We calculate the constant $\alpha_i$ using $E_i = -p U_i$ as

$$\alpha_i = p U_i \left[ \sum_{j=1}^{N_i} \frac{m_j}{\Omega_{ij}} \right]^{-1}.$$ (23)
Further, we define a characteristic size, $\tilde{\Omega}_i$, with respect to the locus $i$ as the geometric mean

$$\tilde{\Omega}_i = N_i \left[ \sum_{j=1}^{N_i} \frac{m_j}{\Omega_{ij}} \right]^{-1} \sum_{j=1}^{N_i} \frac{m_j}{N_i}. \quad (24)$$

Importantly, the scale $\tilde{\Omega}_i$ is not a universal one but depends on the specific observer $i$ and the set of E-elements, which connect it to the 'world'. In the context of this essay we will only deal with quantities observed in the local environment of the earth at present time.

Using (24) we can rewrite (23) as $\alpha_i = 2U_i \tilde{\Omega}_i / \sum_j m_j$ which after insertion in to (21) gives

$$E_{ij} = -\frac{pU_i \tilde{\Omega}_i}{N_{ij} \tilde{\Omega}_{ij}} \Omega_{ij}, \quad (25)$$

where we define $\tilde{N}_{ij} = (\sum_{j=1}^{N_i} m_k) / m_j$. This interaction energy depends on the scale of the doublon $\Omega_{ij}$ and on the characteristic scale $\tilde{\Omega}_i$. Both scales have the dimension of length, however they must not be considered as (relative) space coordinates in classical mechanics, since they depend on the position of the local observer (see above, for discussion see [5, 6]). Only if the time $\tau_{ij}$ for equilibration of the vacuum fluctuation defining the scale $\Omega_{ij}$ is negligible compared to the inverse minimum frequency of fluctuation $\nu_{\min} = c / \Omega_{ij}$, one can use the formal relation between the scales (24) to determine the microscopic force acting on $j$ by the observer $i$, $f_{ij}^{\text{micro}}$.

$$f_{ij}^{\text{micro}} = -\left( \frac{\Omega_{ij}}{\Omega_{ij}} + \frac{\tilde{\Omega}_i}{\tilde{\Omega}_{ij}} \right) \left[ \frac{\tilde{\Omega}_i}{N_{ij} \tilde{\Omega}_{ij}} \right]. \quad (26)$$

For distances $\Omega_{ij} < \tilde{N}_{ij} \tilde{\Omega}_i$ we note repulsive action while for $\Omega_{ij} \gg \tilde{N}_{ij} \tilde{\Omega}_i$ Newton’s law of gravity is recovered. Identifying the prefactor in (20) with $Gm_jm_i$ ($G$=the coefficient of gravitation), one obtains

$$\tilde{\Omega}_i = \frac{Gm_im_jN_{ij}}{pU_i} = \frac{GM_i}{pc^2}, \quad (27)$$

where we used $U_i = m_i c^2$ and defined the mass of the universe, seen by the ‘observer’ $U_i$, to be $M_i = m_j \tilde{N}_{ij} = \sum_{k=1}^{N_i} m_k$. Note that the mass of the universe depends on the specific observer, $i$. When measured from a basis on Earth, the gravitational constant is given by $G \approx 6.67 e^{-11} \left( \frac{m^3}{kg^2s^2} \right)$. Further, for the mass of universe, measured by an observed on Earth, we set $M_{\text{Earth}} \approx 10^{52} \text{kg}$ [20]. We thus obtain an estimate of the scale of the universe viewed from the Earth $\tilde{\Omega}_i = \tilde{\Omega}_{\text{Earth}}$:

$$\tilde{\Omega}_{\text{Earth}} \approx 7.4 \times 10^{24} \text{ m} \approx \frac{1}{p} 240 \text{ Mpc}, \quad (28)$$

which is, for a non periodic system, $p = 2$ consistent with observations of the size of the large voids in the universe. Regarding the meaning of the mass, $m_i$, we set for the number of particles in the observable universe $N \approx 10^{79}$ and obtain $m_j = M/N \approx 10^{-27}$ kg. This is roughly equal to the mass of a proton.

Instantaneous ‘action at a distance’ and repulsive gravitational action at the micro-scale is thereby limited to distances $\frac{\tilde{\Omega}_{\text{Earth}}}{pc}$ below the Plack length. We leave closer consideration to future work.

On ‘cosmological’ distances, one must clearly treat $\tilde{\Omega}_i$ and $\Omega_{ij}$ as independent, since the cosmological scale $\tilde{\Omega}_i$ cannot be directly related to a length defined in a current microscopic environment, or even the solar system. Variation of the energy of space (25) with respect to $\tilde{\Omega}_i$ and $\Omega_{ij}$ independently gives:

$$f_{ij} = -\left( \frac{\partial E_{ij}}{\partial \Omega_{ij}} + \frac{\partial E_{ij}}{\partial \tilde{\Omega}_i} \right) \frac{pU_i \tilde{\Omega}_i}{N_{ij} \tilde{\Omega}_{ij}} \left[ 1 - \frac{\tilde{\Omega}_i}{N_{ij} \tilde{\Omega}_{ij}} \right]. \quad (29)$$

For illustration we treat the limit of 3 particles with $\tilde{N}_{ij} = 2$. There are three possible cases:

- if $\Omega_{ij} = \Omega_{ik}$, $\tilde{\Omega}_i = \Omega_{ij}$, then $f_{ij} = 0$,
- if $\Omega_{ij} \ll \Omega_{ik}$, $\tilde{\Omega}_i = 2 \Omega_{ij}$, then $f_{ij} = \frac{pU_i}{2 \Omega_{ij}}$ and $f_{ik} = -\frac{pU_i}{2 \Omega_{ik}}$,
- if $\Omega_{ij} \gg \Omega_{ik}$, $\tilde{\Omega}_i = 2 \Omega_{ik}$, then $f_{ij} = -\frac{pU_i}{2 \Omega_{ij}}$ and $f_{ik} = \frac{pU_i}{2 \Omega_{ik}}$.

In the last case, if the distance between the particles is much larger then the average value dominated by two close particles due to the homogeneous mean, the forces becomes repulsive.

Treating the system as periodic, $p = 1$, we can correlate this length to the size of large voids [8] which seems reasonable at least in the visible universe. Also a non-periodic system $p = 2$ lies in the span of the large uncertainties of these estimations. Therefore it is hard to draw a conclusion on periodicity from these analogies. Structures beyond the marginal distance $\tilde{\Omega}_{\text{Earth}}$ repel each other, leading to an accelerating expansion. In the limit $\Omega_{ij} \gg \tilde{\Omega}_{\text{Earth}}$ we further see from the generalized gravitational law (29) that the force scales as $f_{ij} \propto -\frac{1}{\Omega_{ij}}$ instead of $f_{ij} \propto \frac{1}{\Omega_{ij}}$ for small distances, i.e. that repulsive gravitational action on ultra-long distances decays more slowly with distance than attractive gravitational action on small distances. Consequences of this statement deserve further considerations in the future.
V. DISCUSSION AND CONCLUSION

In section [III] we considered a model of the universe in a wave mechanical picture according to the dBB double solution program with a guiding wave, oscillating in a virtual direction $z$. These waves are related to particle waves, which oscillate with the same phase as the guiding wave and move in analogy to particles on water waves. If the traveling velocity of a particle is not zero, the corresponding wave has a finite phase velocity moving in the direction of the particle on the line coordinate $s$. The oscillations of waves have the minimum energy $U_0$ corresponding to the massive energy of the particles. Assigning this energy to the pilot wave instead of the particle wave allows us to motivate a kind of uniformly distributed ‘the ground state potential’ with a total energy density in the visual universe $NU_0/V_H$, where $V_H$ is the Hubble volume and $N$ is the number of particles.

According to the standard cosmological model of expanding universe, the energy conservation equation for a particle of mass $m$ moving from a center of mass $M$ with the velocity $v$ can be written as

$$\frac{mv^2}{2} - \frac{mMG}{a} - Nmc^2\frac{4\pi a^2\eta}{V_H} = 0, \quad (30)$$

where $a$ is the distance between the particle and the center, also known as scale factor, $4\pi a^2\eta$ is a volume of a thin layer which is responsible for the ground state potential of one particle. Then by substituting $v = \dot{a}$, $\frac{4\pi}{3} p_U = \frac{M}{a^3}$, $\eta = \frac{Ω_{\text{earth}}}{N}$, and $V_H = \frac{4\pi R_H^3}{3}$, where $R_H$ is the Hubble radius, we obtain

$$\frac{\dot{a}^2}{a} - \frac{8\pi G}{3} p_U - \frac{3c^2Ω_{\text{earth}}}{R_H^3} = 0. \quad (31)$$

It can be seen that if the ground state potential is negative it should cause the acceleration of the expansion of the universe. The comparison with the cosmological constant gives $Λ \approx 9\frac{Ω_{\text{earth}}}{R_H^3} \approx 1.2 \times 10^{-52}$ m$^{-2}$. This value is close to the value defined as $Λ = 3\frac{H_0^2}{c^2}Ω_Λ = 3\frac{Ω_Λ}{R_H^3}$, where $H_0$ the measured Hubble constant and $Ω_Λ$ is the ratio between the energy density due to the cosmological constant and the critical density. Hence the last term in eq. (31) gives the acceleration behavior similar to the cosmological constant for todays values of $Ω_{\text{earth}}$ and $R_H$.

On the other hand, $U_0$ is related to the mass of a particle, hence we can estimate the total energy density related to $U_0$ of all particles in universe using the mass density of visual universe $p_U = 3 \times 10^{-28}$ kg/m$^3$.

This is a factor 20 smaller than the vacuum energy density $ρ_{\text{vacuum}} = 5.96 \times 10^{-27}$ kg/m$^3$ defined by the standard model of cosmology, using $Λ = \frac{8\pi G}{c^2}ρ_{\text{vacuum}}$ [21]. Hence, we can state that the ground state potential can be the rational of a cosmological constant $Λ$ in the case or an open universe as described by the dBB double solution program. The factor 20 between the massive energy and the estimated ‘vacuum energy’ of the universe is well in agreement with common assumptions about dark energy. The new point here is the interpretation in the wave mechanical picture of the dBB double solution program, where we have the traditional understanding of massive particles embedded in an ‘open space’. We may argue that this picture overestimates energetic contributions related to space largely compared to energy related to mass. This has to be corrected by introducing ‘dark energy’ in order to be consistent with observations.

Reverting the picture to a closed doublon network model, section [IV] gives reasonable agreement without introducing an additional ground state energy. Here the prediction of repulsive gravitational action on ultra-long distances is a consequence of neutrality and energy conservation, which leads to the repulsive term in the gravitational force between two bodies [29]. Nevertheless, we can show in this picture an analogy to the standard model or cosmological expansion, if we define from [29] the coefficient of gravitation dependent on the distances

$$G(Ω_{ij}, Ω_{\text{Earth}}) = G_0 \left[1 - \frac{Ω_{ij}}{Ω_{\text{Earth}}}ight]. \quad (32)$$

In the case of self similarity the last term in (32) should approach a constant, i.e. $Ω_{\text{Earth}} \propto a$. The model predicts an accelerating expansion $\frac{\dot{a}}{a} \propto p_U$ which should fade out proportionally to $\frac{1}{a^2}$ for constant mass of the universe.

We leave closer consideration to future work.

In conclusion, we have outlined a wave mechanical framework of the ‘universe’ along the double solution approach by de Broglie-Bohm with wave representation of particles and space. The traditional dBb picture was extended to a model of the particles ‘swimming’ on the pilot wave by analogy to the particles in the dense water waves. The particles have a traveling velocity related to quantum oscillations of the pilot wave. This model predicts the existence of a ground state energy of the quantum oscillations with a finite vacuum energy rationalizing the assumption of a cosmological constant in the standard model of cosmology. Reverting this picture we define a closed doublon network, where particles form the boundary for vacuum fluctuation along the 1-dimensional path between particles. According to Casimir [19] we define the negative vacuum energy of the quantum fluctuations. Finally, we have shown that the gravitational force [29], resulting from this model can be displayed in a form consistent with a positive cosmological constant. Finally we state that a self similarity of the distribution of masses in the universe, corresponding to a constant relation between scale factors $a \propto Ω_{\text{Earth}}$, is not in contradiction to the observation of an ever expanding universe.
[1] G. Rosi F. Sorrentino L. Cacciapuoti M. Prevedelli and G.M. Tino. Precision measurement of the newtonian gravitational constant using cold atoms. Nature, 510:518–21, 2014.

[2] H. Zapolsky. Kinetics of Pattern Formation: Mesoscopic and Atomistic Modelling, pages 153–192. 05 2015.

[3] H. Hofer and E. Zehnder. Symplectic Invariants and Hamiltonian Dynamics. 1994.

[4] R. D. Kamachali, A. Abbondandolo, K. F. Sieburg, and I. Steinbach. Geometrical grounds of mean field solutions for normal grain growth. Acta Materialia, 90, 2015.

[5] I. Steinbach. Quantum-phase-field concept of matter: Emergent gravity in the dynamic universe. Z. Naturforschung A, 72:5158, 2017.

[6] I. Steinbach. Erratum to: Quantum-phase-field concept of matter: Emergent gravity in the dynamic universe. Zeitschrift fur Naturforschung A, pages 89–91, 2020.

[7] J. Kundin and I. Steinbach. Quantum-phase-field: from de broglie–bohm double solution program to doublon networks. Z. Naturforschung A, 75(2):155–170, 2020.

[8] V. Müller, S. Arbabi-Bidgoli, J. Einasto, and D. Tucker. Voids in the las campanas redshift survey versus cold dark matter models. Mon. Not. R. Astron. Soc., 318:280–288, 2000.

[9] A.G. Riess et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astronomical J., 116:1009–38, 1998.

[10] A. Einstein. Die Grundlage der allgemeinen Relativitstheorie. Annalen der Physik, 49:769–822, 1916.

[11] A. Einstein. Kosmologische Betrachtungen zur Allgemeinen Relativitätstheorie. Sitzungsberichte der Königlische Preussischen Akademie der Wissenschaften (Berlin), page 142152, 1917.

[12] A. Hebecker and C. Wetterich. Quintessential adjustment of the cosmological constant. Phys. Rev. Lett, 85:3339–3342, 2000.

[13] E.H. Lieb and J. Yngvason. The physics and mathematics of the second law of thermodynamics. Physics Reports, 310:1–96, 1999.

[14] L.D. Landau and E.M. Lifshitz. Statistical Physics Part 1, third revised edition 1980. Pergamon, Oxford, 1959.

[15] S.O. Olsen. A natural way of quantization. Acta Physica Academiae Scientiarum Hungaricae, 37:97–103, 1974.

[16] L. de Broglie. L'interprétation de la mécanique ondulatoire par la théorie de la double solution. Proc. Znt. Sch. Phys. Enrico Fermi, 49:346–367, 1971.

[17] S. Colin M. Hatifi, R. Willox and T. Durt. Bouncing oil droplets, de broglie's quantum thermostat, and convergence to equilibrium. Entropy, 20:32, 2018.

[18] I. Steinbach and F. Pezzolla. A generalized field method for multiphase transformations using interface fields. Physica D, 134:385–393, 1999.

[19] H. Casimir. On the attraction between two perfectly conducting plates. Proc. Kon. Nederland. Akad. Wetensch., B51:793–795, 1948.

[20] M.A. Persinger. A simple estimate for the mass of the universe: Dimensionless parameter A and the construct of "pressure". J. of Physics Astrophysics and Physical Cosmology, 3:1–3, 2009.

[21] Planck collaboration (2016). Planck 2015 results. xiii. cosmological parameters. Astronomy & Astrophysics, page 594:A13, 2016.

[22] Here we anticipate that 'nothing' is the only amount of a substance which needs no creation.

[23] p in this context needs not to be strictly an integer, since not all particles have to be connected, but may be used as an adjustable parameter to account for 'dark matter'.