We examine lepton number violation (LNV) in theories with a saturated black hole bound on a large number of species. Such theories have been advocated recently as a possible solution to the hierarchy problem and an explanation of the smallness of neutrino masses. The violation of lepton number can be a potential phenomenological problem of this N-copy extension of the Standard Model as due to the low quantum gravity scale black holes may induce TeV scale LNV operators generating unacceptably large rates of LNV processes. We show, however, that this does not happen in this scenario due to a specific compensation mechanism between contributions of different Majorana neutrino states to these processes. As a result rates of LNV processes are extremely small and far beyond experimental reach, at least for the left-handed neutrino states.

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\section{Introduction}

Very recently the existence of a large number of copies of Standard Model (SM) particles has been proposed as a possibility to lower the Planck scale and solve the electroweak hierarchy problem \cite{1,2}. It was shown that in this scenario the fundamental quantum gravity scale $\Lambda$ is related to the effective Planck scale $M_P$ as

$$\Lambda \simeq \frac{M_P}{\sqrt{N}}. \quad (1)$$

This implies $\Lambda \sim \mathcal{O}(\text{TeV})$ for $N \simeq 10^{32}$ and thus solves the hierarchy problem \cite{1,2}. The above bound is imposed by consistency of large distance black hole dynamics \cite{1,2} in the presence of $N$ copies of the SM fields.

Moreover, in \cite{4} this scenario has been advocated also as a mechanism for generating small neutrino masses, providing an attractive alternative for seesaw, extra dimensional and other known mechanisms. It is assumed that there exists one SM singlet right-handed neutrino $\nu_{Rj}$ per SM$_j$ copy, so that $j = 1, \ldots, N$. The mechanism relies on the fact that the right-handed neutrinos, being SM singlets, couple to all the SM copies “democratically”. This SM singlet democracy, combined with the requirement of unitarity of the theory, leads to a $1/\sqrt{N}$ suppression of the Yukawa couplings to the left-handed neutrinos $\nu_{Lj}$ and thus a suppression of the corresponding Dirac neutrino mass terms. Thus the minimalistic approach to the problem of small neutrino masses advocated in \cite{4} suggests that $B - L$ violating Majorana masses of $\nu_{Rj}$ are unnecessary in this scenario and lepton number could assumed to be conserved.

The assumption of lepton number conservation is however rather ad hoc, as there is no fundamental reason to forbid Majorana masses for the right-handed neutrinos and lepton number conservation appears as an accidental symmetry. Moreover, quantum gravity breaks global symmetries, and then conserved lepton number requires a gauged $B - L$ symmetry $U_{1(B-L)}$. The latter should be spontaneously broken to avoid the existence of the corresponding massless gauge boson, stringently constrained by phenomenology. On the other hand, lepton number violation might be helpful for successful baryogenesis.

In the following we analyze the issue of lepton number violation within the $N$-copies SM.

\section{Model Framework}

We assume

$$\prod_i ((SU_{3c} \times SU_{2W} \times U_y)_i \times U_{1(B-L)} \times Z_N) \quad (2)$$

gauge symmetry of the $N$-copies SM including a common anomaly free gauge factor $U_{1(B-L)}$. This gauge factor prevents the appearance of phenomenologically dangerous Lepton Number Violating (LNV) operators induced by TeV black holes. An additional permutation symmetry $Z_N$ acting in the space of the SM, species ($i = 1, 2, \ldots, N$) is also imposed \cite{4} which, to be unaffected by black holes, should be considered as a gauged symmetry in the sense of being a discrete subgroup $Z_N \subset G$ of some continuous gauge group $G$ spontaneously broken down to $Z_N$.
The Lagrangian terms relevant for our discussion are the following:

$$\mathcal{L}_{\nu HS} = \lambda_{ij} \overline{\nu_{Ri}} (LH)_{ij} + \beta_{ij} \overline{\nu_{Ri}} \nu_{Rj} S + \kappa_{ij} (H^\dagger H)_{ij} S \dagger S,$$

The model involves $N$ right-handed SM singlet neutrinos $\nu_{Ri}$ and one SM singlet complex scalar field $S$ having the $B-L$ charge equal to $+2$. Then the trilinear $HHS$ couplings are forbidden in $\mathcal{L}_{\nu HS}$, which is suggested by the observation that these two quantities are of the same nature and there exists no permutation symmetry acting on the space of species. This permutation symmetry constrains the Yukawa coupling matrix to be of the form

$$\lambda_{ij} = \begin{pmatrix} a & b & b & \ldots \\ b & a & b & \ldots \\ b & b & a & \ldots \end{pmatrix}. \quad (4)$$

This matrix, combined in the first term in $\mathcal{L}_{\nu HS}$ with the SM Higgs expectation value $\langle H \rangle = \langle H_i \rangle$, results in the Dirac neutrino mass matrix

$$m_D^{ij} = \lambda_{ij} \langle H \rangle. \quad (5)$$

Here following Ref. [4] we assume that the electroweak symmetry breaking leaves the permutation symmetry unbroken. This implies that the VEVs of all the Higgs species are equal to the same value $\langle H \rangle$. A key point ensuring in the scenario of Ref. [4] the smallness of neutrino mass matrix entries is the smallness of the Yukawa coupling matrix $\lambda_{ij}$, which follows from the requirement of unitarity of the theory. This can be shown by considering right-handed neutrino inclusive production in the scattering of the SM particles, as displayed in Fig. 1(a).

At high energies the rate of this process grows like

$$\Gamma \simeq N b^2 E, \quad (6)$$

as follows from dimensional analysis. Here we assumed $a \sim b$, which is suggested by the observation that these two quantities are of the same nature and there exists no fundamental reason for them to be very different in magnitude $[4]$. Unitarity below the gravity cutoff is preserved only for

$$b \lessapprox \frac{1}{\sqrt{N}}. \quad (7)$$

Thus the neutrino mass matrix $m_D^{ij}$ results in $N-1$ Dirac neutrinos with tiny masses $m_D \simeq \langle H \rangle / \sqrt{N} \lessapprox \mathcal{O}(\text{eV})$ $[4]$, which fulfill the experimental bounds constraining them to the sub-eV scale. One neutrino state in this framework is very heavy, with mass of the order $M^D \simeq \sqrt{N} \langle H \rangle$, which is comparable with the effective Planck scale.

Now let us turn to the second term of Eq. (3) which upon breaking the $B-L$ symmetry leads to the Majorana mass matrix of the right-handed neutrinos

$$m_M^{ij} = \beta_{ij} \langle S \rangle. \quad (8)$$

The permutation symmetry constrains the Majorana type Yukawa coupling $N \times N$ matrix in the same way as for the case of Dirac type Yukawa couplings to be of the form

$$\beta_{ij} = \begin{pmatrix} c & d & d & \ldots \\ d & c & d & \ldots \\ d & d & c & \ldots \end{pmatrix}. \quad (9)$$
The scalar quartic couplings \( \kappa \), following form,\( \kappa \)ing. The mass matrix written in the basis of the 2

\[
\text{neutrino Majorana mass matrix entries (8) are even more strongly suppressed as the Dirac masses discussed above.}
\]

\[
\text{This limit remains unaffected if additional S-branches are inserted in the diagram in Fig. 2. Consequently the neutrino Majorana mass matrix entries (5) are even more strongly suppressed as the Dirac masses discussed above.}
\]

\[
\text{III. NEUTRINO SPECTRUM AND LEPTON NUMBER VIOLATION}
\]

We now turn to the neutrino mass spectrum and mixing. The mass matrix written in the basis of the 2N fields \( \nu_{\alpha} = \{ \nu_{L\beta}, \nu_{R\beta+N} \} \ (\alpha = 1, \ldots, 2N, \beta = 1, \ldots, N \) has the following form,

\[
M^\nu = \begin{pmatrix} 0 & m^D \\ m^D & m^M \end{pmatrix},
\]

where \( m^D \) and \( m^M \) are \( N \times N \) submatrices given by Eqs. 3-5. The set of 2N mass eigenstates \( \nu_i = U_{i\alpha} \nu_{\alpha} \) of this symmetric matrix splits into two groups of \( (N - 1) \) degenerate states \( \nu^+ \) and \( \nu^- \) and another two states \( N^\pm \). These groups correspond to two \( N - 1 \) dimensional and two singlet representations of the permutation group \( Z_N \) \[4\]. The resulting eigenstates are

\[
(N - 1) - \text{plet : } \nu^+_{k}, \quad k = 1, \ldots, N - 1
\]

\[
m_+ = \frac{1}{2} (\sigma_0 + \Delta_0),
\]

\[
U_{1\nu^+_{k}} = \frac{(-1)^{k-1}}{\sqrt{k(k+1)}} \frac{m_-}{\sqrt{m^2_+ + g_0^2}}
\]

\[
(N - 1) - \text{plet : } \nu^-_{k}, \quad k = 1, \ldots, N - 1
\]

\[
m_- = \frac{1}{2} (\sigma_0 - \Delta_0),
\]

\[
U_{1\nu^-_{k}} = \frac{(-1)^{k-1}}{\sqrt{k(k+1)}} \frac{m_+}{\sqrt{m^2_+ + g_0^2}}
\]

singlet : \( N^+, \ M_+ = \frac{1}{2} (\sigma_N + \Delta_N) \),

\[
U_{1N^+} = \frac{M_+}{\sqrt{N(M^2_+ + g_0^2)}},
\]

singlet : \( N^-, \ M_- = \frac{1}{2} (\sigma_N - \Delta_N) \),

\[
U_{1N^-} = \frac{M_-}{\sqrt{N(M^2_- + g_0^2)}},
\]

with

\[
\sigma_n = [c + (n - 1)d] \langle S \rangle, \quad g_n = [a + (n - 1)b] \langle H \rangle,
\]

\[
\Delta_n = \sqrt{(4g_n^2 + \sigma_n^2)}, \quad n = 0, N.
\]

Here we denoted the mixing matrix element of each state with the neutrino of the original SM copy by \( U_{1\alpha} \). All phenomenological manifestations of the two sets of \( (N - 1) \) degenerate states specified above are identical to two effective fields \( \nu^+ \) and \( \nu^- \) with masses \( m_+ \) and \( m_- \), respectively. They are defined as

\[
\nu^\pm = \frac{\sum_{k=1}^{N-1} \nu^+_{k} U_{1\nu^+_{k}}}{\sum_{k=1}^{N-1} U^2_{1\nu^+_{k}}},
\]

Thus, the left-handed neutrino interaction eigenstate of the original SM copy (\( \alpha = 1 \)) can be written as

\[
\nu_{L1} = U_{1\nu^+} \nu^+ + U_{1\nu^-} \nu^- + U_{1N^+} N^+ + U_{1N^-} N^-, \quad (17)
\]

where the effective mixing matrix element of \( \nu^\pm \) with \( \nu_{L1} \) is defined as

\[
U_{1\nu^\pm} = \frac{\sum_{k=1}^{N-1} U^2_{1\nu^+_{k}}}{\sqrt{m^2_+ + g_0^2}} \frac{\sqrt{N - 1}}{N}, \quad (18)
\]
Assuming \( a \sim b \) and \( c \sim d \) as discussed above we find

\[
m_{\pm} \approx \pm m_\nu (1 \pm \delta_m) \sim \frac{1}{\sqrt{N}},
\]

\[
M_\pm \approx \pm M_N (1 \pm \delta_M) \sim \sqrt{N},
\]

whith

\[
m_\nu = (a - b) \langle H \rangle \sim \frac{1}{\sqrt{N}}, \quad \delta_m = \frac{1}{2} \frac{c - d}{a - b} \langle S \rangle \sim \frac{1}{\sqrt{N}},
\]

\[
M_N = N b \langle S \rangle \sim \sqrt{N}, \quad \delta_M = \frac{1}{2} \frac{d \langle S \rangle}{b \langle H \rangle} \sim \frac{1}{\sqrt{N}}.
\]

Thus there are two light states \( \nu^\pm \) and two very heavy states \( N^\pm \) with a mass ratio \( M_\pm / m_\pm \) of \( \sim N \). For consistency with the neutrino phenomenology one needs one neutrino at sub-eV scale, say, \( m_\nu \sim 10^{-2} \text{eV} \). Then, as seen from Eqs. (20), the states \( N^\pm \) are pushed in mass towards the effective Planck scale and, therefore, their phenomenological impact is negligible. The light Majorana states \( \nu^\pm \) have a very small mass splitting \( \delta_m \sim 1/\sqrt{N} \) and form a quasi Dirac state with mass \( m_\nu \). Thus these light states are expected to induce lepton number violating processes at rates \( \sim 1/N \). However, due to the structure of the mass matrix (14) with zero submatrix in the upper-left corner, LNV processes are even more strongly suppressed. The contribution of light \( \nu_k \) and heavy \( N_k \) Majorana neutrinos with the masses \( m_{\nu_k} \) and \( M_{N_k} \), respectively, to the amplitude \( A_{LNV} \) of a generic LNV processes can be schematically written as

\[
A_{LNV} \sim \sum_k \frac{U_{1k}^2 m_{\nu_k}}{p_0 + m_k} \approx \frac{1}{p_0} \langle m_\nu \rangle + \frac{1}{p_0} \langle m_\nu^3 \rangle (21)
\]

\[
+ \left\langle \frac{1}{M_N} \right\rangle.
\]

Here \( m_k \) denotes all mass eigenstates, while \( m_{\nu_k} \) and \( M_{N_k} \) denote the light and heavy sets \( m_{\nu_k} \ll p_0 \) and \( M_{N_k} \gg p_0 \), respectively, and \( p_0 \) is the characteristic momentum of the LNV process under consideration. Here we defined

\[
\langle m_\nu \rangle = \sum_k m_{\nu_k} U_{1k}^2, \quad \langle m_\nu^3 \rangle = \sum_k m_{\nu_k}^3 U_{1k}^2, (22)
\]

\[
\left\langle \frac{1}{M_N} \right\rangle = \sum_k \frac{U_{1k}^2}{M_{N_k}}.
\]

For neutrinoless double beta decay \( (0\nu\beta\beta) \), which is the most sensitive probe of LNV, the characteristic momentum is \( p_0 \approx 105 \text{ MeV} \). For other LNV processes such as meson decays and equal sign dileptons in pp-collisions, this characteristic momentum \( p_0 \) is even larger.

From the definition (22) and Eqs. (15-18) it follows that the leading term of the expansion (21) vanishes, \( \langle m_\nu \rangle = 0 \). This result can be understood by taking into account the following two facts. First, the following relation holds,

\[
\sum_k m_k U_{1k}^2 = M_1^{\nu} = 0, (23)
\]

where now the summation runs over all masses of both heavy and light neutrinos. Here the mass matrix \( M^\nu \) with zero \( N \times N \) upper-left corner is given in (14).

Second, as we mentioned before in Eqs. (15), the light neutrino states belong to two \( (N - 1) \) dimensional representations of the permutation group \( Z_N \) while the two heavy states are \( Z_N \)-singlets. Thus, due to symmetry reasons the cancellation in the sum (23) can only happen within the same representation, in other words, within \( (N - 1) + (N - 1) \) group of light neutrino states and \( 1 + 1 \) heavy neutrino states. This can be directly confirmed by substituting Eqs. (15-18) into Eq. (22).

Thus the \( N = 10^{32} \text{ SM} \) exhibits the curious property that the expression (23) vanishes individually also if the sum runs only over the eigenstates being much lighter than \( p_0 \), which corresponds exactly to the usually dominating contribution to neutrinoless double beta decay originating from the first term in Eq. (21). Consequently the first non-vanishing contribution to the LNV amplitude starts from the second term in Eq. (21). This fact can also be illustrated diagrammatically as in Fig. 5. From Eqs. (15-20) and (22) it follows that

\[
\langle m_\nu^3 \rangle = (a - b)^2 (c - d) \langle S \rangle^2 \sim N^{-2}, (24)
\]

\[
\left\langle \frac{1}{M_N} \right\rangle \approx \frac{\langle H \rangle^2}{d \langle S \rangle^2 b N^2} \sim N^{-2}.
\]

Therefore, the amplitudes (21) of LNV processes in the studied framework are extremely small.

However, this conclusion is based on the assumption \( a \sim b \) and \( c \sim d \) which does not have a firm physical motivation since the parameters \( a, c \) are not directly subject to the unitarity constraints like in Eq. (7) and (13). We thus examine what are the typical rates of LNV processes in the scenario in which the latter assumption is relaxed. Now the parameters \( a, c \) should not be expected very small because they are not suppressed by any symmetry. Assuming \( \langle S \rangle > \langle H \rangle \) to avoid phenomenological problems with a new light gauge boson in Eqs. (15) we have

\[
m_+ \approx c \langle S \rangle, \quad m_- \approx \frac{a^2 \langle H \rangle^2}{c \langle S \rangle}, (25)
\]

\[
M_+ \approx \frac{1}{2} (c + dN) \langle S \rangle \pm bN \langle H \rangle \sim \sqrt{N}. (26)
\]

Thus, there are still two very heavy states with masses \( M_+ \sim \sqrt{N} \), which contribute to the amplitude of LNV processes via the last term in Eq. (21) and this contribution is negligible. The other two Majorana states with masses \( m_+ \) are not necessarily light, since the suppression due to multiple SM copies is absent now. Their masses are related as

\[
m_+ = \left( \frac{\langle S \rangle}{\langle H \rangle} \right)^2 \left( \frac{c}{a} \right)^2 m_- . (27)
\]

To ensure the phenomenological consistency of this scenario one has to require that there exists a light neutrino. Thus, we assume that \( m_- \approx 10^{-2} \text{ eV} \) and study
the corresponding mass $m_+$. The condition $\langle S \rangle \gg \langle H \rangle$ is required in order to push the masses of the $B - L$ gauge boson and the singlet Higgs $S$ towards sufficiently large values to elude current experimental limits. However in the $N = 10^{32}$ SM \footnote{G. Dvali, arXiv:0706.2050 [hep-th].} the values of $\langle S \rangle$ larger than the gravity cutoff $\Lambda \sim \mathcal{O}(\text{TeV})$ make little sense. Reasonable values are around $\langle S \rangle \sim 1 \text{ TeV}$. In the present scenario there are no symmetry or other reasons supporting any significant difference between the diagonal Yukawa couplings of $H$ and $S$ Higgs fields in Eq. (3). Therefore, for estimations it is reasonable to assume $a \sim c$. Then one obtains $m_+ \sim 1 \text{ eV}$. Thus for this rather typical case the natural values of both masses $m_-$ and $m_+$ are much smaller than the typical momentum scales of LNV processes $p_0 \sim 100 \text{ MeV}$. This means that again $\langle m_\nu \rangle = 0$ and the leading contribution to the LNV amplitude \footnote{G. Dvali and M. Redi, Phys. Rev. D 77, 045027 (2008) arXiv:0710.4344 [hep-th].} is

$$A_{\text{LNV}} p_0^2 \sim \frac{1}{p_0^2} \langle m_\nu^3 \rangle = m_- \left( \frac{m_+}{p_0} \right)^2 \sim m_- \times 10^{-16} \sim 10^{-18} \text{eV}. \quad (28)$$

This is far beyond the sensitivity of possible experimental observations. The most sensitive $0\nu\beta\beta$ experiments have reached the limit on the double beta decay observable $A_{\text{LNV}} p_0^2 \sim \langle m_\nu \rangle^{\text{exp}} \leq 0.38 \text{ eV} \footnote{G. Dvali and C. Gomez, Phys. Lett. B 674, 303 (2009) arXiv:0812.1940 [hep-th].}$. This conclusion is valid unless a strong hierarchy $c \sim 10^9 a$ between the lepton number violating and lepton flavor conserving Yukawa couplings $c$ and $a$, respectively, is considered. Then $m_+ \sim 100 \text{ MeV}$ and $\langle m_\nu \rangle \neq 0$ and the LNV amplitude \footnote{G. Dvali and M. Redi, Phys. Rev. D 80, 055001 (2009), arXiv:0905.1709 [hep-ph].} may become large. However, as we commented above, this situation is rather unnatural since in the present scenario there is no symmetry or other mechanism supporting this hierarchy.

\section*{IV. CONCLUSIONS}

In this paper we have addressed a problem arising in any scenario with a low quantum gravity scale: do LNV operators induced by TeV scale black holes invalidate the model? For the case of the $N = 10^{32}$-copies SM we have shown that this consequence is avoided due to a non-trivial cancelation mechanism. This property should be considered as an important benefit of the model.

Nevertheless, the presence of a large number of right-handed Majorana states may have interesting phenomenological consequences. For example, a very naive estimate of the right-handed neutrino decay diagrams on tree and one-loop level, which give rise to the baryon asymmetry in leptogenesis, scale as $(\sqrt{N})^2$ from the Yukawa coupling with $N$ copies of $\nu_{R_i}$, contributing potentially to the decay, and the $\nu_{R_i}$ propagator in the loop diagram. So the process may be relevant despite the fact that LNV signals are strongly suppressed for the left-handed neutrino states. A similar line of reasoning may apply e.g. to single $\nu_{R_1}$ production at the LHC. Finally we did not address the effects of the new neutrino degrees of freedom on big-bang nucleosynthesis \footnote{G. Dvali, arXiv:0706.2050 [hep-th].} and the phenomenology of baryon number violation and proton decay \footnote{The latest limits are quoted in \cite{Knochel}.} in the present scenario. These and other phenomenological consequences will be discussed elsewhere.

We conclude that the $N = 10^{32}$-copies SM is safe from LNV in the SM sector, and leave the potentially interesting phenomenology of $\nu_{R_i}$ production and decay for further study.

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