Seismic Signal Filtering based on Pseudo Wigner-Ville Distribution and Catte Model

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Abstract—In practice, there are various noises included in the collected seismic data. As a common background noise, random noise may interfere even annihilate valid signals. In view of this, it needs to adopt effective methods to eliminate random noise as much as possible. This paper proposes a filtering method that is based on pseudo Wigner-Ville distribution (PWVD) and Catte model. PWVD is a windowed form of Wigner-Ville distribution (WVD) which can make the nonlinear signal to be locally linearized in addition to weakening the influence of cross item. And the Catte model is a kind of anisotropic diffusion algorithm which can eliminate the noise while protecting the edge of image. The implementation of the proposed method is to do PWVD for the frequency modulation form of noise-containing signal and then to filter the PWVD via Catte model, at last to adopt peak search to obtain the estimation of the valid signal.

1. Introduction

In practice, the collected seismic data are contaminated by various noises and the valid signal is interfered even annihilated [1]. As the most common background noise, the influence on valid signals caused by random noise cannot be ignored. It needs to adopt effective filtering methods to eliminate random noise and extract valid signals, which lays a good foundation for the subsequent work of data interpretation.

So far, there have been a lot of effective methods for seismic random noise reduction, such as singular value decomposition (SVD) [2], morphological filtering [3], empirical mode decomposition (EMD) [4], F-X predictive filtering [5], et al. In fact, the methods which are based on multiscale analysis [6] or time-frequency analysis [7] are widely used. One of the time-frequency methods called time-frequency peak filtering (TFPF) [8-9] is a very effective filtering method in seismic random noise reduction and many remarkable achievements [10-14] have been presented.

In TFPF, either Wigner-Ville distribution (WVD) or pseudo Wigner-Ville distribution (PWVD) are adopted. The former is known to be optimal for monocomponent signals with the quadratic phase law (linear frequency modulation, LFM) since it achieves the best energy concentration around the signal
instantaneous frequency (IF) law [9]. The latter is a windowed version of WVD that is to be used to prevent aliasing of TFPF, and the selection of the window length achieves a reduced bias.

Catte model [15,16] is a kind of anisotropic diffusion filter [17] which has solved the ill-posed problems of Perona and Malik (PM) model. It applies Gaussian filtering to implement smoothing operation and substitutes the gradient of original image with the gradient after smoothing. By virtue of this, it can overcome the sensitivity of PM model to the noise and eliminate noise effectively while protecting image edge well that is quite suitable for the reduction of strong noise.

2. The Principle of TFPF and Catte Model

2.1. The Principle of TFPF

As a time-frequency analysis method of quite good performance, TFPF adopts WVD or PWVD to obtain the time-frequency distribution (TFD) of the analytical form of a signal, and then estimates the instantaneous frequency (IF) by searching the peak of TFD. In fact, the IF estimated is the recovered waveform of the valid signal.

First, setting a noisy signal as:

\[ s(t) = x(t) + n(t) \]  

In which, \( x(t) \) is the valid signal and \( n(t) \) is the additive random noise.

Second, the analytical form of the noisy signal via frequency modulation (FM) can be obtained by the following equation:

\[ z_s(t) = e^{j2\pi\mu t} n(t) \]  

In which, \( \mu \) is the frequency modulated parameter and \( j \) is imaginary unit.

Third, doing the PWVD of \( z_s(t) \) as follows:

\[ \text{PWVD}_{z_s}(t,f) = \int_{-\infty}^{\infty} h(t) z_s(t) z_s^* \left( t - \frac{\tau}{2} \right) e^{-j2\pi \tau f} d\tau \]  

Finally, estimating the IF of the \( \text{PWVD}_{z_s} \) by searching the peak of TFD to obtain the recovered signal.

\[ \hat{f}_s(t) = \arg \max_f \left[ \text{PWVD}_{z_s}(t,f) \right] / \mu \]  

2.2. The Principle of Catte Model

Catte model is a kind of anisotropic diffusion algorithm which belongs to partial differential equation. It has demonstrated that this model satisfies the well-posedness of the solution, which ensures the stability of diffusion process. The filtering based on partial differential equation can protect the edge and detail well so that it has been widely used in noise elimination. The gradient modulus after Gaussian smoothing is adopted in the model that controls the diffusion process and overcomes the sensitivity to the noise presented by PM model. Therefore, Catte model can eliminate noise effectively while protecting image edge well. The Catte model is:

\[ \frac{\partial U(x,y,t)}{\partial t} = \nabla \cdot \left[ C \left( |\nabla G_\sigma * U(x,y,t)| \right) \nabla U(x,y,t) \right] \]

\[ U(x,y,0) = U_0(x,y) \]

In which, \( U(x,y,t) \) is the image to be processed and \( U_0(x,y) \) is the initial image. \( G_\sigma \) is the Gaussian kernel function of the scale factor \( \sigma \); \( |\nabla G_\sigma * U(x,y,t)| \) is the gradient modulus of the constant coefficient thermal diffusion equation when the scale factor is \( \sigma \). \( \nabla \cdot \) is the divergence operator; \( \nabla \) is the gradient operator; \( t \) represents time.
3. Experiments on Synthetic Seismic Signals

3.1. Experiment on a single Ricker wavelet

We take a synthetic seismic signal simulated by Ricker wavelet of which the dominant frequency is 25Hz as the experiment object, the sampling frequency is 2000Hz and the number of sampling points is 512. Adding Gaussian white noise (GWN) to the signal to make the SNR (signal-to-noise ratio) to be about -2dB. The noise-free signal and the noisy signal are shown in Fig. 1 as follows:

![Figure 1](image1)

(a) The noise-free Ricker wavelet. (b) The noisy Ricker wavelet with the SNR of -2dB.

![Figure 2](image2)

(a) The PWVD of frequency modulated noisy Ricker. (b) The PWVD of frequency modulated noisy Ricker after Catte model filtering.
The PWVD of the noisy Ricker is shown in Fig.2 (a) and the PWVD after Catte model filtering is shown in Fig.2 (b). We can see that the vibrations taken on the TFD of Fig.2 (b) are less obvious than the TFD of Fig.2 (a). That is to say, Catte model filtering plays a smoothing role for the distribution of random noise, and in fact the energy of valid signal is enhanced.

From Fig.3, it can be found that there is a large deviation between the waveform obtained by TFPF and the noise-free signal by presenting that the random noise is not suppressed well, and the wave peaks and troughs are still distorted seriously. Whereas, the waveform obtained by Catte model TFPF is more closely to the standard signal of which the fluctuation degree is smaller meanwhile the distortions at peaks and troughs are lighter. Computing the SNRs of the two signals filtered by TFPF and Catte model TFPF, we get 6.3786dB and 10.4148dB respectively. In view of above, we know that the capability in random noise reduction of the latter method is superior to the former method through both qualitative analysis and quantitative analysis.

3.2. Experiment on a synthetic seismic record

We take a synthetic seismic record including two reflection events as the experiment object. There are 30 traces in this record and the interval velocities are 2000m/s and 2200m/s respectively. The events are formed by valid reflection signals of several traces, of which two Ricker wavelets in each trace with the dominant frequencies of 25Hz and 20Hz respectively. Adding WGN to this record and make the SNR to be about -5dB. Applying TFPF and Catte model TFPF to filter the noisy record respectively and then compare and analyze the filtering results. The noise-free record, noisy record, TFPF record and Catte model TFPF record are shown in Fig.4. In different noise intensity, the SNRs and MSEs (mean square errors) of the records before and after filtering have been computed and shown in Table I.

From Fig.4, it can be found that both Catte model TFPF and TFPF can effectively suppress the background noise performed in eliminating the random noise much more, but the former method performs better since the background seems brighter. And from Table I, we can see that the SNRs of Catte model TFPF record are higher than TFPF record while the MSEs of the former are smaller than the latter for a same noisy record.

| Table 1 The SNRs and MSEs of records |
|--------------------------------------|
| SNR1       | MSE1       | SNR2       | MSE2       |
| Noisy record | -5.0304 | 0.1067 | -8.9467 | 0.2322 |
| TFPF record  | 2.6506 | 0.0182 | -1.2523 | 0.0395 |
| Catte model TFPF record | 5.1013 | 0.0103 | 1.3740 | 0.0216 |
3.3. Field seismic data processing
Through the experiments on synthetic seismic signal and records, it can be found that the results of peak estimation for the PWVD after Catte model filtering are better than the result of peak estimation for PWVD, which are presented in the smoothing degree of waveform. That is to say, the former method is superior in the random noise reduction than the latter.

Next, we take a field seismic record which is a common-shot-point (CSP) record acquired from an oil field of Northeast China as the processing object. We intercept a part of this record to process which contains 30 traces and each trace has 2000 sampling points. The original record, TFPF record and Catte model TFPF record are shown in Fig.5. In order to observe more clearly, we extract signals from a certain single trace to compare the waveforms filtered by the two methods, which are shown in Fig.6.

From Fig.5, it can be found that the strong noise of the record that appears large amplitude has not been eliminated by TFPF while being completely suppressed by Catte model TFPF. And from Fig.6, we can see that the waveform filtered by Catte model TFPF is much smoother than the waveform filtered by TFPF.

4. Conclusion
The improved method utilizes Catte model filtering conducted on the PWVD plane and adopts peak estimation to recover the valid seismic signals. Through experiments on synthetic seismic data and field seismic data, it demonstrates that the proposed method is superior to TFPF in random noise reduction and valid signal recovery by which the estimated signal is more similar to the ideal signal (derived from the experiment on synthetic data), or the waveform is smoother presented in the oscillations of waveform is slighter (derived from the experiments on both the synthetic data and the field data).

Figure 4 The experiment on a synthetic seismic record with the SNR of about -5dB. (a) The noise-free record. (b) The noisy record. (c) The TFPF record. (d) The Catte model TFPF record.
Figure 5 The processing on a part of a field CSP record by TFPF and Catte model TFPF. (a) The original record. (b) The TFPF record. (c) The Catte model FPF record.

Figure 6 The waveform comparison of single trace derived from TFPF record and Catte model TFPF record respectively. (a) The waveform comparison of 16th trace. (b) The local amplification of waveform comparison of this single trace.

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