Newtonian Quantum Gravity

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Abstract

The quantum treatment of gravity has been researched for more than fifty years. The focus of the efforts was the investigation of the “gravitational quantum field” concept. This article paves the way to solving this puzzle. Gravity is a phenomenon that emerges from the undulating nature of the universe. To support this claim, I deduce a more generalized expression than that of Newton’s law of gravity from simple considerations related to wave function, probability density function, and a hypothesis of how all particles in the universe are related. This expression uncovers the true nature of the gravitational phenomenon, dark matter, and dark energy. This true nature of the gravitational field allows its renormalization and the definition of its elementary charge. The result obtained considers the Mach’s principle and deduces the origin of the inertial force. Finally, it is concluded that a new model of the universe is necessary.

Keywords

Extended Theories of Gravity, Dark Energy, Dark Matter, Black Holes, Accelerated Cosmic Expansion, Renormalization

1. Introduction

The quantum treatment of gravity has been researched for more than fifty years. The focus of the research efforts was the investigation into the concept of “gravitational quantum field”. This article presents a new method that can solve this puzzle. Gravity is a phenomenon generated by the undulating nature of the universe. To support this claim, I deduce a more generalized expression than that of Newton’s law of gravity from simple considerations related to wave function, probability density function, and a hypothesis of how all particles in the universe are related. I also consider space to be a three-dimensional surface of a sphere with radius equal to the age of the universe, so that the universe can be represented as a sphere of radius equal to the distance from the observer to
the current particle horizon, which can be approximately expressed as $R_h = \pi R$, where $R$ is the Hubble radius. I have used the basic concept of the probability function of a particle and considered that the energy of a particle is distributed in space according to this function of probability density. The objective of this theory of “quantum gravity” is the identification of the quantum substrate from which macroscopic gravity emerges [1] [2] and to pave the way for the complete quantum analysis of the gravitational phenomenon. Likewise, both the problem of dark matter and dark energy may, in principle, be solved by extended theories of gravity [3]; an example of this can be the results obtained.

The hypothesis of this work is that the wave function of each particle deforms the wave functions of all particles of the universe, thereby generating the gravitational phenomenon.

2. Methods

We define the probability $\Psi(r, t)^2 = \sum_{i=1}^{N} \frac{\phi_i(r, t)^2}{N}$, where $N$ is the number of particles in the entire universe and $\phi_i$ is the wave function of each particle. Let us now consider a closed spherical surface $S$ such that the value of $\Psi(r, t)^2$ at all points on that surface is the same, $\phi_s$; $S$ is an equiprobable surface. The interior volume of that closed surface is $V_s$. If we define a reference point 0, the value of $V_s$ will depend on $\phi_s$ and given that the particles in the universe are in perpetual motion, there will be a probability density flow entering and exiting through the surface $S$ because of which $V_s$ and $S$ will be time-dependent. Consider the situation prior to the detection of the particle, when the parameter $t$ has the value $t_0$. The particles do not always have a definite energy, but irrespective of their energy and for the specific case that I discuss, I consider that the energy is distributed throughout the universe according to its probability density function. The hypothesis of this work is that the probability density functions of the remaining particles affect the probability distribution of each of them so that the probability density function of a particle includes the probability distribution $\Psi(r, t_0)^2$ defined above. We can call $\phi(r, t_0)$ as the wave function of the particle already deformed by $\Psi(r, t_0)^2$ and $\phi_0(r, t_0)$ to the intrinsic part of the particle. Therefore, the probability density function of a particle can be given as:

$$\phi(r, t_0)^2 = \frac{1}{2} \phi_0(r, t_0)^2 + \frac{1}{2} \Psi(r, t_0)^2.$$  \hspace{1cm} (1)

It is not easy to calculate this probability density function, $\Psi(r, t_0)^2$, but by doing some simplification, it is feasible to calculate the total probability of finding a particle inside $V_s$; this probability is denoted by $P_s$. Because no particle in the universe is excluded, it can be calculated as the quotient obtained by dividing the energy contained within the volume $V_s$ by the total energy of the universe; $V_u$ is the volume of the universe. This probability would also be the result of applying the volume integral of $\Psi(r, t_0)^2$ throughout the volume $V_s$. With $\rho(r)$ as the average energy density in $V_s$, we obtain:
\[ \int_{S} \Psi(r, t_0)^2 \, dV = \frac{\rho(r) V_s(r)}{\rho(R_h) V_s(R_h)} \]  

Equation (2) is an expression of the probability of finding some particle inside \( V_s \). For a spherical surface of radius \( r \) and with centre at the centre of mass, which is the point \( O \), we have \( V_s = \frac{4}{3} \pi r^3 \). Moreover, because the universe is considered a sphere of radius \( R_h \), (2) can be expressed as

\[ \int_{r_s} \Psi(r, t_0)^2 \, dV = \frac{\rho(r) r^3}{\rho(R_h) R_h^3}. \]  

Applying the same reasoning to the area from the outer system to \( V_s \), we can arrive at a similar expression

\[ \int_{r_0} \Psi(r, t_0)^2 \, dV = \frac{\rho(R_h - r) (R_h^3 - r^3)}{\rho(R_h) R_h^3}. \]  

where \( \rho(R_h - r) \) is the average density outside of \( S \). This would be the probability of finding a particle outside \( S \). If we assume that \( r \ll R_h \), then we can approximate \( \rho(R_h - r) \) to \( \rho(R_h) \) and the expression for the whole universe can be given as

\[ \int \Psi(r, t_0)^2 \, dV = \frac{\rho(r) r^3}{\rho(R_h) R_h^3} + \frac{(R_h^3 - r^3)}{R_h^3} = 1. \]  

This function, as defined above, is a sum of probabilities. Now, consider a particle that is free and at rest, i.e. the indeterminacy of \( r \) is of the order of \( R_h \); its wave function is evenly distributed throughout the universe. As mentioned previously, the hypothesis of this work is that the probability density functions of the particles affect the probability distribution of each of them so that the probability density function of a particle includes the previously defined density function. All particles in the universe are related. Substituting in (1) the values of \( \Psi(r, t_0)^2 \) from above, the expression for the probability density of the particle is obtained as

\[ \varphi(r, t_0) = \frac{1}{2} \left( \varphi_0(r, t_0) + \frac{\rho(r) r^3}{\rho(R_h) R_h^3} + 1 - \frac{r^3}{R_h^3} \right). \]  

This would be valid for a free particle before being detected on surface \( S \). The normalizing constant of the complete wave function is \( \frac{1}{2} \). In the expression \( \frac{\rho(r) r^3}{\rho(R_h) R_h^3} \), we can see a product of four factors corresponding to four degrees of freedom of the system. The expression \( \frac{r^3}{R_h^3} \) corresponds to the three degrees of
freedom of a three-dimensional space. The factor \( \frac{\rho(r)}{\rho(R_h)} \) is independent of the previous degrees of freedom and depends on the history of the system; the density can vary over time. To analyze this phenomenon, we can study the collapse of the wave function only in that spatial degree of freedom. The particle is detected at the distance \( r \), e.g. on the surface of the sphere indicated above, with an indeterminacy of \( \Delta r \); the corresponding statistical factor, \( \frac{r}{R_h} \), is reduced to the factor \( \frac{\Delta r}{\Delta R_h} \). This new factor must meet two conditions to be able to have a defined measure. First, its value should be 1; given that there is certainty, this condition is fulfilled if \( \Delta r = \Delta R_h \), that is, the indeterminacy does not depend on the observer. Second, both \( \Delta r \) and \( \Delta R_h \) should be greater than 0. Both conditions are assured by the principle of uncertainty. In this case,

\[
\Delta r p_g \geq \frac{\hbar}{2}.
\] (7)

The momentum acquired by the particle is \( p_g \). After the collapse of the wave function, the probability distribution of the particle, with \( \phi' \) and \( \phi'_0 \) as the wave functions of the particle after detection, is as follows:

\[
\phi'(r, t) = \frac{1}{2} \left[ \phi'_0(r, t) + \frac{\rho(r)}{\rho(R_h) R_h^2} r^2 + 1 - \frac{r^2}{R_h^2} \right].
\] (8)

This expression remains normalized. The \( \frac{\rho(r)}{\rho(R_h) R_h^2} \) factor of the probability density function maintains the degree of freedom due to the density factor and two additional spatial degrees of freedom. That is, neither the system knows the location of the particle (i.e. it ignores the \( \phi \) and \( \theta \) coordinates) nor the particle knows how energy is distributed inside \( V \). From this and by applying the formalism of quantum mechanics to the left side of the previous equality, we can obtain the total average energy of the particle, \( E_i \), which includes the average mechanical energy as the average kinetic energy due to the gravitational phenomenon \( K_g \), that is, \( E_i = mc^2 + K_g \). If we apply the same to the first term on the right, we would obtain \( \frac{1}{2} mc^2 \), where \( mc^2 \) is the average mechanical energy of the particle. The second, third, and fourth terms in the second part of the equality are probability distributions. If, as we indicated at the beginning, the energy of the particle can be considered distributed according to its probability density function, we would obtain:

\[
E_i = \frac{1}{2} mc^2 + \frac{1}{2} mc^2 \frac{\rho(r) r^2}{\rho(R_h) R_h^2} + \frac{1}{2} mc^2 - \frac{mc^2 r^2}{R_h^2}.
\] (9)

If the average mechanical energy \( \left( \frac{1}{2} mc^2 + \frac{1}{2} mc^2 \right) \) is denoted by \( E_o \), we
obtain the following result:

$$E_i = E_0 + \frac{1}{2} mc^2 \frac{\rho(r) r^2}{\rho(R_h) R_h^2} - \frac{1}{2} mc^2 \frac{r^2}{R_h^2}, \quad (10)$$

$$mc^2 = mc^2 + K_g - \frac{1}{2} mc^2 \frac{\rho(r) r^2}{\rho(R_h) R_h^2} + \frac{1}{2} mc^2 \frac{r^2}{R_h^2}. \quad (11)$$

Finally, we obtain the energy balance for a particle linked to a gravitational system as

$$0 = K_g - \frac{1}{2} mc^2 \frac{\rho(r) r^2}{\rho(R_h) R_h^2} + \frac{1}{2} mc^2 \frac{r^2}{R_h^2},$$

$$K_g = \frac{1}{2} mc^2 \left( \frac{r^2}{R_h^2} \left( \frac{\rho(r)}{\rho(R_h)} - 1 \right) \right). \quad (12)$$

If we calculate $K_g$ for a sphere of radius $r$ and $M + m_0$ is its total internal energy, where $m_0$ is its component that contributes to $\rho(R_h)$, i.e.,

$$m_0 = \rho(R_h) \frac{4}{3} \pi r^3,$$

and assuming that $\rho(R_h)$ is equal to the critical density ($\frac{3c^2}{8\pi G R_h^2}$), then we find that

$$K_g = \frac{1}{2} mc^2 \frac{r^2}{R_h^2} \left[ \frac{M + m_0}{\frac{4}{3} \pi r^3} \frac{m_0}{\frac{4}{3} \pi r^3} - 1 \right],$$

$$K_g = \frac{1}{2} mc^2 \frac{r^2}{R_h^2} \frac{M}{m_0},$$

$$K_g = \frac{GMm}{r}. \quad (13)$$

Expression (12) is the kinetic energy of a free-falling particle in a gravitational system when it falls with initial velocity 0 from an infinite distance or $R_h$. When it is at rest on the surface of Earth, i.e., $K_g = 0$, the energy indicated in (13) would be as follows:

$$E_p = -\frac{GMm}{r}. \quad (14)$$

If the particle has a kinetic energy not due to gravity, its energy can be expressed as

$$E = K - \frac{GMm}{r}. \quad (15)$$

As we shall see below, it is important to note that the contribution of the energy $m_0$ is not included in (13). In systems with high densities, this contribution is irrelevant; however, in systems with small densities, the expression of
gravity given in (12) does not coincide with Newton’s law. The force of gravity calculated with Newton’s law will always be greater than that obtained with (13).

For example, observations of the large-scale structure of the universe have given rise to the need to postulate the concept of dark energy because they are very low-density systems; the application of formulation (13) instead of Newton’s law eliminates the need to postulate this concept of dark energy.

The expression \( -\frac{1}{2}mc^2\rho(r)r^2\rho(R_s)R_s^2 \) can be called the “gravitational term”; the expression \( \frac{1}{2}mc^2\rho(r)r^2\rho(R_s)R_s^2 \) can be called the “expansive term”. It must be considered that the temporal evolution of the wave function indicated in (1) depends on the whole universe; therefore, the observed energy of the particle depends on how the whole universe evolves, both inside \( S \) and outside of \( S \). This study considers the Mach’s principle from this point of view. The acceleration of a particle “moves” the entire universe by modifying the probability density function of all particles in the universe (see Section 3.1).

3. Results and Discussion

3.1. Force of Inertia and Mach’s Principle

The expansive term introduced above is equivalent to the energy of a simple harmonic oscillator with an elastic constant of \( \frac{mc^2}{R_s^2} \). It does not depend on the mass of the system, but only on the distance to the origin of coordinates and the mass of the particle. It is valid for any other frame of reference. This leads us to think that the particle is attached to the rest of the universe by multitude of springs with the same elastic constant and different elongations. The particle is confined at the centre of a scalar field of the form \( V = \frac{1}{2}mc^2\rho(r)r^2\rho(R_s)R_s^2 \). If the particle is at rest or in uniform rectilinear motion, the force resulting from these springs is 0. Thus, just as the force represented by the weight is the reaction to the force due to the gravitational term, the force of inertia can be identified as the reaction from the scalar field that binds the particle to the universe through any force that is applied on the particle. The centrifugal force must be considered to be of the same nature as the gravitational force.

If a particle is accelerated from rest with an acceleration \( a \), it will have travelled a distance \( dr \) in time \( dt \), where \( dr = \frac{1}{2}adt^2 \). The force exerted by the oscillators in the direction of the movement of the particle will be \( F = K(r - dr) \), the force in the opposite direction is \( F = -K(r + dr) \), and the resultant of both will be \( F = -2Kdr \). If we calculate the sum of all the oscillators of both hemispheres, we obtain the following result:

\[
F = -m4\pi\frac{c^2}{R_s}.
\]
If the expression \( 4\pi \frac{c^2}{R_h} \) is called \( a_0 \), then for another acceleration, the displacement in the same time \( dt \) will be \( \Delta r \) so that \( \frac{\Delta r}{dr} = \frac{a}{a_0} \); consequently, we can deduce that for any acceleration,

\[
F = -m 4\pi \frac{c^2}{R_h} \frac{a}{a_0}.
\] 

This expression is equivalent to Newton’s 2nd law. The force of inertia has the same nature as the gravitational force.

### 3.2. Gravitational Scalar Field Model

Consider the scalar field of the previous point \( V(r) = \frac{1}{2} m \frac{c^2}{R_h^2} r^2 \). Now consider the fundamental oscillator with energy \( \frac{1}{2} \frac{c^2}{R_h^2} r^2 \), \( m = 1 \). The natural frequency of this elementary oscillator will be \( \omega_0 = \frac{c}{R_h} \), its energy in the ground state will be \( E_0 = \frac{1}{2} \hbar \omega_0 \), and the mass corresponding to that energy will be \( m_0 = \frac{1}{2} \frac{\hbar}{c R_h} \); we could say that based on the known values of the constants, the value of the mass is approximately \( 4.3 \times 10^{-70} \) kg. Its wavelength according to the expression, \( E_0 = \frac{1}{2} \hbar \omega_0 \), would be \( 2\pi R_h \), its spatial indeterminacy is of the order of the particle horizon, that is to say, is at rest with respect to the universe. We can imagine that the universe is filled with these particles (henceforth called as gravitons) and I consider that they are characterized by an elementary charge of mass at rest of value 1, which we can call the elementary charge of mass at rest. Their wavelength indicates that they would be standing waves whose main nodes could be identified with the gravitons indicated. The interactions between them are given by the previous scalar field. Each graviton is connected to the rest by springs with the same constant but with different elongations. The uniformity of the universe implies that the force of the springs located in one hemisphere exerting force at one point will be compensated for by the force exerted by the springs of the other hemisphere so that, in principle, these springs would tend to be at rest. We have seen in the previous discussion that an external force exerted on some of them would trigger a reaction from the rest, which we have called inertia.

These springs would tend to completely fill the universe due to the initial inequalities between the hemispheres, eliminating these inequalities. Let \( r_0 \) be called the average distance between them at equilibrium. The force exerted between two of them separated by \( r_0 \) will be \( F_0 = \frac{c^2}{R_h^2} r_0 \). However, this force is what a frame of reference anchored in one of them would observe; if we chose
the mass centre as the origin of the reference frame (an inertial frame of reference), the force applied to each of them will be \( F_0 = \frac{1}{2} \frac{c^2}{R_0} r_0 \). From the above, it can be said that the force of attraction between the gravitons, with respect to an inertial frame of reference, is of the form \( F = \frac{1}{2} \frac{c^2}{R} r \). The average separation distance between the adjacent gravitons will be \( r_0 \) and will have an average density of \( \rho_0 = 1 \); therefore, there would be a graviton in each cube of edge \( r_0 \). If we identify the universe as a 3-sphere, the gravitons will tend to cover the entire universe uniformly.

Let us now imagine a sphere with radius \( nr_0 \). Within that sphere there is an excess of \( N \) free gravitons with centres of mass at the centre of the sphere. Symmetrical to this sphere and to its right is another identical sphere but with gravitons corresponding to the average density \( \rho_0 \); in each of these spheres, there will be \( \frac{4}{3} \pi n^3 r_0^3 \) gravitons corresponding to this density. The density of the left sphere will be \( \frac{4}{3} \pi n^3 r_0^3 + N \). At the point of contact of both spheres, there is another graviton called the test graviton. By choosing the mass centre of the complete system of the free \( N \) gravitons, the test graviton and the gravitons of both spheres corresponding to the density \( \rho_0 \) as the origin of the frame of reference (an inertial frame of reference), the acceleration induced by the gravitons of both spheres on the test graviton will be as follows:

\[
g = \frac{1}{2} \frac{c^2}{R^2} n r_0 \left( \frac{4}{3} \pi n^3 r_0^3 + N \right) \left( \frac{4}{3} \pi n^3 r_0^3 \rho_0 - 1 \right) = -\frac{3c^2}{8\pi R^2 \rho_0} \frac{N}{n^2 r_0^2}.
\]

If \( \frac{3c^2}{8\pi R^2 \rho_0} \) is termed as \( G \), then the above expression is similar to that for Newton’s law of gravity except that in (18), the mass corresponding to the density-\( \rho_0 \) \( \left( \frac{4}{3} \pi n^3 r_0^3 \right) \) is not considered, which can lead to errors for very low-density systems. A particle model could be postulated such that a particle of mass at rest \( m \) could be identified with a stable set of \( N \) coordinated gravitons that move at the same time. As an example, it can be stated that an electron would be a coordinated set of \( 2 \times 10^{98} \) gravitons, which is approximately the same ratio as that of a hurricane to an air molecule.
It is important to say here that expression (12) has been applied to the system described here because, as indicated previously, neither the system of $N$ gravitons knows where the test graviton is (it ignores the $\phi$ and $\theta$ coordinates), nor does the test graviton know how the $N$ gravitons are distributed within the sphere; that is, the gravitons have not been detected, and the gravitational force is the effect of the interaction between the gravitons caused by their asymmetric distribution. The expression $\frac{M}{m_0}$ is the only information we can have of the internal distribution of gravitons. If the anterior sphere is large enough so that its inner density is $\rho_0$, the acceleration described in (18) would be null. This force between the free system of $N$ gravitons and the rest of the gravitons deforms the sphere, making it smaller until the force is balanced by that exerted by the rest of the gravitons in the universe. This deformation can be identified as the deformation considered for space by the General Theory of Relativity (GTR); in fact, it would be possible to identify this network of gravitons with the space concept of the GTR. The distance from which it seems that gravity does not cause a curvature in space is the distance from which it seems that dark energy accelerates the expansion. However, the reality is that from that distance, the curvature is always null because the gravitons are in equilibrium. The photons can be identified by the excitations transmitted throughout this network of gravitons. I propose that the true gravitational field is the scalar field indicated above,

$$V(r) = \frac{1}{2} m c^2 \frac{1}{R_i^2} r^2,$$

and the gravitational field which was considered until now is a side effect of the unequal distribution of energy.

The fact that the gravitational potential between the gravitons depends on $r^2$ and not $\frac{1}{r}$ can allow the quantification of this field model without the problems encountered in previous attempts to find the gravitational quantum field. Additionally, a quantum theory of gravity based on this model would be compatible with GTR.

3.3. Anomalous Velocities in the Halos of Spiral Galaxies and Dark Matter

The anomalous speeds of rotation in the halos of the spiral galaxies have forced researchers to postulate the existence of some type of invisible matter that can justify those speeds. The nature of this dark matter is a mystery. However, the model presented in the previous section can illuminate this enigma. I have indicated before that the material particles can be considered as coordinated sets of gravitons travelling at the same time. Therefore, there may also be large accumulations of gravitons that are not yet in equilibrium, which travel freely through the universe and have not yet coordinated to form material particles. These gravitons would have the same effect as the so-called dark matter.

Additionally, the eventual coordinated union of a sufficient number of gravi-
tons to form material particles may justify the enigmatic presence of gas and dust in the spiral galaxies. Considering the rate that stars are created in these galaxies, the proportion of gas in them must have reduced to zero millions of years ago.

3.4. Gravitational Constant

The constant $G$ was discovered by Newton. It is an empirical constant that does not focus on the nature of the gravitational phenomenon. The expression $\frac{G M m}{r^2}$ is an empirical formula deduced from experience, and not from the knowledge of the nature of the gravitational phenomenon. However, a correct formulation of the gravitational phenomenon derived from the knowledge of its nature can add a practical structure to $G$ as opposed to a constant proportionality that is adjusted according to experience. The expression (18) allows us to obtain a response to this problem. If we substitute $\rho_0$ for $\frac{3 M_h}{4 \pi R^3_h}$, where $M_h$ is the total energy of the universe and $R_h$ is the radius of the particle horizon, and then compare the results with a system with spherical symmetry, we obtain the following value for $G$:

$$G = \frac{c^2 R_h}{2 M_h}. \quad (19)$$

$G$ gives information about the relationship between the radii of particles in the universe and the total energy of the universe.

3.5. Average Density of the Universe

Heretofore, we have been postulating that $\rho_0$ is equal to the critical density. However, the gravitational term of the expression (18) implies that the average density of the universe is intrinsically equal to the critical density. Let us imagine that the average density of the universe is not $\rho_0$ but $\rho'$. In that case, Newton would have measured another gravitational constant $G'$. Considering a system with spherical symmetry, we have:

$$\frac{1}{2} mc^2 \frac{r^2}{R_h^2} \left( \frac{\rho}{\rho_0} - 1 \right) = \frac{G' M m}{r}. \quad (20)$$

By substituting $\rho$ for $\frac{3 M}{4 \pi r^3}$, we obtain

$$\rho' (R_h) = \frac{3 c^2}{8 \pi G' R_h^2}. \quad (21)$$

The same expression of critical density again. The universe would be inherently flat, and its average density would always coincide with the critical density. It is not necessary to postulate the existence of an inflationary period to justify the flatness of the universe.
3.6. Cosmological Implications

The expression (18) involves changing the hitherto accepted conception of the origin of the universe. Measurements of the brightness of distant stars indicate that Newton’s constant $G$ has been the same for all such observations. However, expression (18), which is equivalent to Newton’s law of gravity for denser systems such as stars, implies that the relationship between the mass of the universe and the radius of the universe, $\frac{R}{M}$, has been constant throughout the life of the universe, as is apparent from the result 3.4. There are two options that fulfill this relationship, a stationary universe or the black hole proposed by Schwarzschild. In a black hole, it is true that $c^2 = \frac{2GM}{r}$, where $M$ is the mass of the black hole and $r$ is its radius. If we discard the stationary universe option, then the universe would be a black hole contained in another outer universe [4] [5]. Therefore, our universe cannot be considered an isolated system. The expansion of the universe is comparable to the expansion of the 3-sphere that would form the event horizon of a black hole as it grows.

4. Conclusion

Although the results of this work allow a macroscopic view of the gravitational phenomenon in the field of quantum mechanics, it is necessary to achieve a complete formulation of gravity that includes elementary particles at small distances or with great energies. I believe that a complete gravitational theory of the quantum field that includes the range of elementary particles is possible if we consider that the graviton is not directly affected by the gravitational interaction, but that the real interaction between them is long range and directly proportional to distance. The gravitational theory of the quantum field could explain the true nature, birth, and evolution of our universe.

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