Modeling and optimization of dough recipe for breadsticks

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Annotation. During the work, the authors studied the combined effect of non-traditional raw materials on indicators of quality breadsticks, mathematical methods of experiment planning were applied. The main factors chosen were the dosages of flaxseed flour and grape seed oil. The output parameters were the swelling factor of the products and their strength. Optimization of the formulation composition of the dough for bread sticks was carried out by experimental-statistical methods. As a result of the experiment, mathematical models were constructed in the form of regression equations, adequately describing the process of studies. The statistical processing of the experimental data was carried out by the criteria of Student, Cochran and Fisher (with a confidence probability of 0.95). A mathematical interpretation of the regression equations was given. Optimization of the formulation of the dough for bread sticks was carried out by the method of uncertain Lagrange multipliers. The rational values of the factors were determined: the dosage of flaxseed flour - 14.22% and grape seed oil - 7.8%, ensuring the production of products with the best combination of swelling ratio and strength. On the basis of the data obtained, a recipe and a method for the production of breadsticks “Idea” were proposed (TU (Russian Technical Specifications) 9117-443-02068106-2017).

1. Introduction
In recent years, in the bakery industry, as part of the implementation of the Order of the Government of the Russian Federation of 25.10.2010, "Fundamentals of the State Policy of the Russian Federation in the Field of Healthy Nutrition of the Population for the Period to 2020", unconventional types of raw materials have been widely used in the production of baked goods of functional purpose. However, the use of such raw materials not only increases the nutritional value of the product, but also changes the properties of the dough and products for the worse. Therefore, to level the influence of non-traditional raw materials on product quality, it is necessary to purposefully combine the formulation of the product by applying modeling and optimization methods [1].

The aim of the work was to determine the rational dosages of components of the bread sticks formulation, which ensure high physicochemical indices of the product.

2. Materials and methods
To carry out the process of modeling and optimization of the formulation composition of bread sticks from first grade wheat flour, as non-traditional raw materials and input factors, the dosages of flax
flour (TU 9290-434-02068108-2016) \((x_1)\) and grape seed oil (TU 9141-0015811041-03) \((x_2)\) were used. The output parameters were the swelling factor of the products \((y_1, \text{ catch units})\) and strength \((y_2, \text{ N})\). The choice of this raw material is due to the rich chemical composition and high nutritional value.

Planning and implementation of the active full-factorial experiment by the given criteria were carried out by compiling the matrix of the central composite rotatable uniform-planning experiment. The results of the studies were processed using the statistical tests of Cochran, Student and Fisher with a confidence probability of 0.95 [2].

The dough was prepared in a free-form manner; fermentation was carried out in a thermostat for 20 minutes at a temperature of 30 °C. At the end of fermentation, the semi-finished product was molded in the form of rods 150 mm long, 10-15 mm wide. The test templates were sent to the final proofing for 30 minutes in a proofing cabinet, in which the temperature was maintained at 40 ± 1 °C and the relative air humidity was 80-85%. The products were then baked in a laboratory electric furnace at a temperature of 220-230 °C for 10-12 minutes. In the bread sticks, the authors determined the strength on the Stroganov instrument - according to GOST (Russian National Standard) 8494-96, the swelling coefficient was calculated based on the increase in mass of each sample after its swelling.

3. Research
Optimization of the formulation composition of bread sticks was carried out by experimental statistical methods [3] in three stages.

The first stage consisted in constructing mathematical models adequately describing the dependence of received response functions \(y_1\) and \(y_2\) on the factors studied. In accordance with the research task, let us complete factor experiment (CFE) type \(2^3\) in accordance with the planning matrix (table 1). To exclude the effect of uncontrolled parameters on the experimental results, the order of the experiments was randomized through a table of random numbers. In table 1, the average arithmetic values of response functions \(y_1\) and \(y_2\) are presented in two parallel experiments.

| Number of experiment | Coded factor values | The natural values of factors | Response functions |
|----------------------|---------------------|------------------------------|-------------------|
|                      | \(X_1\) \(X_2\)     | \(x_1\), % \(x_2\), %         | \(y_1\), catch units \(y_2\), N |
| 1                    | -1 \(-1\)           | 10.0 \(3.0\)                 | 1.15 \(747\)      |
| 2                    | +1 \(-1\)           | 20.0 \(3.0\)                 | 1.60 \(690\)      |
| 3                    | -1 \(+1\)           | 10.0 \(7.0\)                 | 1.48 \(825\)      |
| 4                    | +1 \(+1\)           | 20.0 \(7.0\)                 | 1.65 \(718\)      |
| 5                    | 0 \(-1.414\)        | 15.0 \(2.2\)                 | 1.40 \(795\)      |
| 6                    | -1.414 \(0\)        | 7.9 \(5.0\)                  | 1.30 \(715\)      |
| 7                    | 0 \(+1.414\)        | 15.0 \(7.8\)                 | 1.62 \(878\)      |
| 8                    | +1.414 \(0\)        | 22.1 \(5.0\)                 | 1.65 \(600\)      |
| 9                    | 0 \(0\)             | 15.0 \(5.0\)                 | 1.55 \(734\)      |
| 10                   | 0 \(0\)             | 15.0 \(5.0\)                 | 1.55 \(735\)      |
| 11                   | 0 \(0\)             | 15.0 \(5.0\)                 | 1.55 \(733\)      |
| 12                   | 0 \(0\)             | 15.0 \(5.0\)                 | 1.55 \(734\)      |
| 13                   | 0 \(0\)             | 15.0 \(5.0\)                 | 1.55 \(734\)      |

As a result of statistical processing of the experimental planning results, regression equations (in
coded form), adequately describing the dependence of swelling ratio $y_1$ and the strength of $y_2$ bread sticks on the dosage of flax flour and grape seed oil:

$$y_1 = 1.557 + 0.139X_1 + 0.086X_2 - 0.07X_1X_2 - 0.049X_1^2 - 0.031X_2^2,$$

$$y_2 = 737.55 - 40.768X_1 + 27.878X_2 - 12.5X_1X_2 - 41.372X_1^2 + 47.594X_2^2,$$

where $X_i$ – coded factor values.

The second stage of the work consisted in interpreting regression equations (1) and (2). For the study of surfaces of the second order, a method based on the calculation and analysis of the values of the orthogonal in variants of surfaces of the second order was used.

For this, let us represent regression equations (1) and (2) in the form of a general equation of the second-order surface:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_1x + 2a_2y + 2a_3z + a_0 = 0,$$

where $a_{ii}, a_{ij}$ – coefficients of the second-order surface equation; $x, y$ and $z$ – variables that correspond to factors, and response functions $y_1$ or $y_2$.

Comparing each regression equation (1) and (2) with the general equation of the surface (3), let us determine coefficients $a_0$ and $a_{ii}$ (table 2).

| Coefficient | Response function $y_1$ | Response function $y_2$ |
|-------------|-------------------------|-------------------------|
| $a_{11}$    | -0.049                  | 41.372                  |
| $a_{22}$    | -0.031                  | 47.594                  |
| $a_{33}$    | 0.000                   | 0.000                   |
| $a_{12}$    | 0.035                   | 6.250                   |
| $a_{13}$    | 0.000                   | 0.000                   |
| $a_{23}$    | 0.000                   | 0.000                   |
| $a_1$       | 0.069                   | -20.384                 |
| $a_2$       | 0.043                   | 13.939                  |
| $a_3$       | -0.500                  | -0.500                  |
| $a_0$       | 1.557                   | 737.550                 |

Information on the configuration of surfaces of the second order, which are described by equations (1) and (2), gives four basic invariants and two semi-invariants of the quadratic function (3), composed of the coefficients of the general equation (table 4).

In this case, exhaustive information about the configuration of the response surfaces can be obtained on the basis of only two invariants $I_3$ and $I_4$ and there is no need to calculate and analyze all the remaining invariants and semi-invariants.

Invariants $I_3$ and $I_4$, are composed of the coefficients of the general equation (3), and have the form:

$$I_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix};$$

$$I_4 = \begin{bmatrix} a_{11}a_{22}a_{33} - a_{12}a_{23}a_{31} - a_{13}a_{21}a_{32} \end{bmatrix};$$
Calculations of invariants show that the response surface described by equation (1) is noncentral \( I_3 = 0 \) and has the form of an elliptic paraboloid (a paraboloid of rotation) \( I_4 = -0.00007 < 0 \). The response surface described by equation (2) is also noncentral \( I_3 = 0 \), but has the form of a hyperbolic paraboloid \( I_4 = 502.03 > 0 \).

The third stage of the work consisted in optimizing the formulation of bread sticks according to two optimization criteria: swelling factor \( y_1 \) and strength of products \( y_2 \).

The optimization problem under consideration is multicriteria with the conflict between the optimal values of variables \( X_1 \) and \( X_2 \) and for each particular optimization parameter \( y_1 \) and \( y_2 \). To solve it, let us use the method of convolution of partial criteria in generalized additive criterion

\[
F = \sum_{i=1}^{n} \alpha_i \tilde{y}_i = \alpha_1 \tilde{y}_1 + \alpha_2 \tilde{y}_2 ,
\]

where \( \alpha_1, \alpha_2 \) – weight coefficients of relative importance of particular criteria \( y_1 \) and \( y_2 \); \( \tilde{y}_1, \tilde{y}_2 \) – normalized partial criteria.

Since it is difficult to prioritize the importance for each particular optimization criterion, we will assign the weights based on the values of the coefficients of the relative spread of the particular criteria. The calculated values of the weight coefficients of the partial optimization criteria are presented in table 3.

**Table 3. The results of calculating the weighting coefficients.**

| A particular optimization criterion | The greatest value of the particular criterion \( y_i^{\text{max}} \) | The smallest value of a particular criterion \( y_i^{\text{min}} \) | Coefficient of relative spread \( \delta_i \) | Weight coefficient \( \alpha_i \) |
|-----------------------------------|----------------------------------|-----------------------------|-----------------|--------------------|
| \( y_1 \)                         | 1.657                            | 1.174                       | 0.291           | 0.475              |
| \( y_2 \)                         | 880.648                          | 596.932                     | 0.322           | 0.525              |

As a result of the normalization, equations (1) and (2) for particular optimization criteria take the form:

\[
\tilde{y}_1 = 0.447 + 0.04X_1 + 0.024X_2 - 0.02X_1X_2 - 0.014X_1^2 - 0.009X_2^2 ;
\]

\[
\tilde{y}_2 = 0.442 - 0.024X_1 + 0.017X_2 - 0.008X_1X_2 - 0.025X_1^2 + 0.028X_2^2 ,
\]

In this way, generalized optimization criterion \( F \) in form (6) is defined as the weighted average sum of normalized partial criteria (7) and (8):

\[
F = 0.889 + 0.016X_1 + 0.041X_2 - 0.028X_1X_2 - 0.039X_1^2 + 0.019X_2^2 .
\]

The task of optimizing the formulation of bread sticks according to the generalized optimization criterion is formulated as follows. It is necessary to determine such optimal values of variables \( X_1^* \) and \( X_2^* \), at which the maximum value of the generalized optimization criterion is reached (9). In this case, the optimal values of variables \( X_1^* \) and \( X_2^* \) should be in the experimental area, the boundaries of which are determined by the values of the factors at the "star" points:
\begin{equation}
\begin{cases}
F = 0.889 + 0.016X_1 + 0.041X_2 - 0.028X_1X_2 - 0.039X_1^2 + 0.019X_2^2 \
\varphi(x_1, x_2) = x_1^2 + x_2^2 = R^2
\end{cases} \quad \rightarrow \quad \max
\end{equation}

The problem of conditional optimization was solved by the method of uncertain Lagrange multipliers [2], which allows the problem of conditional extremum to be reduced to the unconditioned problem of finding the extremum of the function. It is established that the conditional extremum (max) of generalized optimization criterion $F^* = 0.987$ was achieved at the point of the quotient space with coordinates $x_1^* = -0.157$ and $x_2^* = 1.405$ (table 4).

\begin{table}[h]
\centering
\caption{The results of optimization.}
\begin{tabular}{cccccc}
\hline
Number of the point & $X_1^*$ & $X_2^*$ & The values of the Lagrange multiplier $\lambda^*$ & $_{y_1}$ & $y_2$, N \\
\hline
1 & 1.413 & -0.013 & 0.033 & 0.833 & 1.656 & 597.218 \\
2 & -1.301 & -0.552 & 0.051 & 0.765 & 1.186 & 710.703 \\
3 & 0.546 & -1.305 & -0.009 & 0.884 & 1.503 & 756.518 \\
4 & -0.157 & 1.405 & -0.35 & 0.987 & 1.609 & 878.860 \\
\hline
\end{tabular}
\end{table}

4. Conclusion

In this way, by the presented practical process of modeling and optimization of the formulation composition of bread sticks, optimum values of the dosages of flex seed flour $x_1 = 14.22\%$ and grape seed oil $x_2 = 7.8\%$ provide products with the best combination of values of swelling ratio and strength. On the basis of the data obtained, a recipe and a method for the production of breadsticks "Idea" is proposed (TU 9117-443-02068106-2017).

References

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