DETERMINATION OF IMPACT FORCE ON A NON-SPINNING SPHERE DURING WATER ENTRY

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Manuscript History
Number: IJIRAE/RS/Vol.04/Issue08/AUAE10083
DOI: 10.26562/IJIRAE.2017.AUAE10083
Received: 21, July 2017
Final Correction: 31, July 2017
Final Accepted: 08, August 2017
Published: August 2017

Citation: Shivkumar, A. K. & A.K, D. (2017), 'DETERMINATION OF IMPACT FORCE ON A NON-SPINNING SPHERE DURING WATER ENTRY', Master’s thesis, Department of Aerospace Engineering.

Editor: Dr.A.Arun L.S, Chief Editor, IJIRAE, AM Publications, India
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Abstract: This paper extends the study to determine the Impact forces for generalized Oblique water entry. The forces of impact for various cases along with their trajectories are exhibited in the results. All the cases considered are of super cavitating flow conditions. The detailed formulation of how the results are obtained is also discussed with appropriate assumptions and justifications.

Keywords: supercavitating speeds, axisymmetric, Surface wetting

INTRODUCTION

A high-speed water entry is accompanied by a huge amount of impact force [3] and can cause structural distortion, internal component damage or mission failure. Hence, it is important to determine the impact force accurately. In majority of realistic cases, the impact scenarios usually include conical or ogival projectile noses interacting with the water surface. Thus, the impact forces achieved for those cases are smaller as compared to that of a sphere. At the moment of impact [2] there is a sudden change in the pressure gradient at the water surface and leads to acceleration of water under and near the object in the downward and radially outward direction respectively. Cavity formation is initiated as it starts to enter through the water and the splash at the water surface has a transverse velocity component which gives a radially outward motion. The splash moves upward and then radially inward before it finally domes over and seals the cavity at the surface or at a certain depth depending on entry and flow conditions. The closed cavity continues to grow in length until the hydrostatic and dynamic pressures of the surrounding fluid cause the cavity closure at an intermediate depth. In this paper, we will focus on determining impact on projectile during water entry especially for sphere because during water entry at supercavitating speeds, the initial instances of impact which are very small the part of the cavity formed and the impact surface can be considered to be a small part of a large sphere. And as sphere is the most simple and ideal axisymmetric geometry it can help in easy understanding of the approach. Hence, we will study impact estimation on spheres in detail along with their trajectory.
The data obtained from the physics of impact of spheres can be used as a base for experiments involving supercavitation, ship slamming etc. Although experimental modelling is very expensive for this field of study.

**NOMENCLATURE**

- $F$ = Resultant force (N).
- $F_I$ = Impact force (N).
- $C_I$ = Coefficient of Impact.
- $B$ = Buoyancy force (N).
- $B_0$ = Normal Buoyancy force (N).
- $\eta$ = Coefficient of Viscosity (N.sec/m$^2$).
- $U$ = Velocity (m/sec).
- $U_0$ = Impact velocity (m/sec).
- $U_d$ = Desired/Terminal velocity (m/sec).
- $m$ = Mass of sphere (Kg).
- $m_a$ = Added mass of sphere (Kg).
- $\dot{m}_a$ = Time rate of change of Added mass of sphere (Kg/sec).
- $\rho$ = Density of surrounding fluid/water (Kg/m$^3$).
- $\rho_b$ = Density of sphere (Kg/m$^3$).
- $r$ = Radius of sphere (m).
- $A$ = Cross-section area of sphere (m$^2$).
- $t$ = Time (sec).
- $dt$ = Small time interval (sec).
- $g$ = Acceleration due to gravity (m/sec$^2$).
- $a$ = Acceleration (m/sec$^2$).
- $\alpha$ = Angle of impact (rad).
- $\theta_0$ = Initial Wetting angle (rad).
- $\Theta$ = Wetting angle (rad).
- $b$ = Part of sphere diameter underwater (m).
- $D$ = Depth reached by the sphere (m).
- $\phi$ = Velocity potential.
- $T$ = Fluid kinetic energy (J).

**IMPACT FORMULATION**

To determine the impact on the sphere it is assumed to have a density higher than water. The study on determination of impact on sphere was referred from [1] which specialized on Vertical water entries. The study is further extended in this paper for generalized Oblique water entry where Vertical water entry is just a special case at $\alpha = 90^0$ and the instantaneous variation in impact along with trajectory plotting [3] are also obtained. The free body diagram of the sphere is shown in Figure 1.

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**Figure 1:** Free-body diagram for Oblique entry of the Sphere.

As per the Newton’s second law of motion,
\[
\Sigma F = ma
\]
\[
mgsina - B(t)sina - F_1(t) = \frac{d}{dt}\left[mU(t)\right]
\] (1)

Using the conservation momentum along the line of motion, the velocity can be expressed as,
\[
mU_0 = (m + m_a(t))U(t)
\] (3)
\[
U(t) = \frac{mU_0}{m + m_a(t)}
\] (4)

Substituting the Equation (4) into (2) we get,
\[
F_i(t) = mgsina - B(t)sina - mU_0 \frac{d}{dt} \frac{m}{m + m_a(t)}
\] (5)
\[
F_I(t) = mgsina - B(t)sina + m^2U_0 \frac{m}{(m + m_a(t))^2}
\] (6)
\[
F_I(t) = mgsina - B(t)sina + \left(\frac{m}{m + m_a(t)}\right)^2 \left[m_a(t)U_0\right]
\] (7)

During the first moments of impact very small quantity of liquid is in contact with water, hence it can be assumed that \(m_a<< m\). Hence, neglecting it in Equation (7) we get,
\[
F_I(t) = mgsina - B(t)sina + (m_a(t)U_0)
\] (8)

Gives the final simplified expression for impact.

**To obtain the variation in Velocity, \(U(t)\) with time:**

The force equation for spherical projectile moving in a fluid is,
\[
\Sigma F = mg sina - B_0 sina - \frac{6\pi\eta U(t)}{2}
\] (9)
\[
m\frac{dU(t)}{dt} = mg sina - B_0 sina - \frac{6\pi\eta U(t)}{2}
\] (10)

Where, 
‘\(mg\)’ is the Weight of the sphere in downward direction,
‘\(6\pi\eta U(t)\)’ is the Viscous force on the sphere along the Line of motion but only a half of the total value is considered because during water entry and supercavitation, approximately only the frontal half portion of the sphere is in contact with the fluid(water) and the remaining sphere is inside the cavity which doesn’t account for viscous force due to water and ‘\(B_0\)’ is the Buoyancy force on the sphere in normal case given by,
\[
B_0 = \rho \left(\frac{4}{3}\pi r^3\right) g
\] (11)

Thus, a 1<sup>st</sup> order differential equation for velocity is obtained. The solution depends on the impact velocity (\(U_0\)) and the terminal/desired velocity (\(U_d\)). The terminal velocity, \(U_d\) is obtained by:-
\[
\Sigma F = 0
\] (12)

Which gives,
\[
U_d = \frac{(mg - B_0) sina}{3\pi\eta}
\] (13)

Thus, after solving the Equation (10) by using \(U_0\) and \(U_d\) we obtain the time variation of velocity with time.

**To obtain the variation in Depth, \(D(t)\) with time:**

From the velocity, the Instantaneous Acceleration along the line of motion is obtained by,
\[
a(t) = \frac{U_{t+dt} - U_t}{dt}
\] (14)

Where ‘\(dt\)’ is the small-time interval which decides the number of iterations.

Then the Instantaneous Depth along the vertically downward direction is,
\[
D(t) = \frac{U_t^2}{2a(t)} sina
\] (15)

which is obtained from the simple kinematical equations.
The instantaneous values of depth for instantaneous change in velocity are obtained from Equation (15).

**To obtain the variation in Wetting angle, \(\theta(t)\) with time:**
To obtain the part of sphere underwater at every instant we have two conditions,

(i) If we start the analysis from the touchdown point of the sphere then the initial wetting angle $\Theta_0$ will be zero and we get,

$$ b(t) = U(t) \, \sin \alpha \cdot t \quad \text{(16)} $$

And for $b(t) \leq 2r$,

$$ \Theta(t) = 2 \cos^{-1} \left( \frac{r - b(t)}{r} \right) \quad \text{(17)} $$

(ii) However, if the analysis is started when the sphere is partially immersed, then the part of the sphere already underwater must be considered and there will be a certain finite value of $\Theta_0$ which will lead to,

$$ b(t) = r \left( 1 - \cos \frac{\Theta_0}{2} \right) + U(t) \, \sin \alpha \cdot t \quad \text{(19)} $$

For $b(t) \leq 2r$,

$$ \Theta(t) = \Theta_0 + 2 \cos^{-1} \left( \frac{r - b(t)}{r} \right) \quad \text{(20)} $$

And for $b(t) > 2r$,

$$ \Theta(t) = 2\pi \quad \text{(21)} $$

Where $\Theta_0$ is the wet angle subtended at the center if the sphere is partially submerged at the start of analysis and $b(t)$ is the instantaneous part of the diameter below the free surface. The impact analysis can be initiated from any phase of immersion i.e. when it has just touched the water free surface for $\Theta_0 = 0^\circ$ when it is already half immersed for $\Theta_0 = 180^\circ$ or when it is partially immersed by mentioning $\Theta_0$ as per requirement. Thus, Equations (17) and (20) suggest that till sphere reaches a depth equal to its diameter i.e. till is fully immersed, the wetting angle will vary from zero or from $\Theta_0$ to $360^\circ$ respectively. Once it is fully immersed, then the value of $\Theta = 360^\circ$ will remain the same for entire onward analysis as shown in Equations (18) and (21). Certain conditions are assigned to ensure that value of $\Theta$ doesn't exceed $360^\circ$ in any of the cases. The Equations can be derived from Figure 2a and b.

![Figure 2: Surface wetting angle variation.](image)

(a) When $\Theta_0 = 0^\circ$ i.e. touchdown. (b) When $\Theta_0$ is finite i.e. partial immersion.

To obtain the variation in Buoyancy force, $B(t)$ with time:

The immersed volume for a sphere [1] of radius 'r' as a function of depth 'D(t)' is,

$$ V(t) = \pi r \left( D(t) \right)^2 - \frac{\pi}{3} \left( D(t) \right)^3 \quad \text{(22)} $$

Thus, the weight of the fluid displaced gives the buoyancy force as,

$$ B(t) = \rho g V(t) \quad \text{(23)} $$

Substituting the Equation (22) in (23),

$$ B(t) = \rho g \left[ \pi r \left( D(t) \right)^2 - \frac{\pi}{3} \left( D(t) \right)^3 \right] \quad \text{(24)} $$

Gives the final simplified equation of buoyancy force.

To obtain the variation in Added-mass coefficient, $m_a(t)$ and its rate of change, $m'_a(t)$ with time:

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IJIRAE: Impact Factor Value – SJIF: Innospace, Morocco (2016): 3.916 | PIF: 2.469 | Jour Info: 4.085 | ISRAJIF (2016): 3.715 | Indexcopernicus: (ICV 2015): 47.91
We will just discuss a brief overview to obtain the added mass [1] [5] [6] of a sphere by Miloh method. The boundary value problem for Velocity potential $\varphi(r, z, t)$ is governed by,

$$\nabla^2 \varphi = 0 \quad z \geq 0 \quad (25)$$

With body and free surface boundary conditions:-

$$\frac{\partial \varphi}{\partial n} = \mathbf{U} \cdot \mathbf{n} \quad \text{on } S \quad (26)$$

$$\varphi = 0 \quad z = 0 \quad (27)$$

$$\varphi \rightarrow 0 \quad r^2 + z^2 \rightarrow 0 \quad (28)$$

The origin of the coordinate system is at the free surface with positive $z$ pointing downward as shown in Figure 3. $S$ is the sphere’s boundary and $n$ is the normal vector pointing outward from the surface of the sphere.

**Figure 3:** Boundary conditions and Surface deformation after impact [1].

By Generalized-Wagner method the kinetic energy of the surrounding fluid is,

$$T(\theta_0) = \frac{2}{3} \pi \rho a^3 U^2 \int_0^\infty (4p^2 + 1) \frac{\cosh(p(\pi - \theta_0))}{\sinh(2p\pi) \cosh(p\theta_0)} \times [3 \sinh(p\theta_0) \cosh(p(\pi - \theta_0)) - \cosh(p\theta_0) \sinh(p(\pi - \theta_0))] \quad (29)$$

Where ‘p’ is a constant which depends on the flow conditions, projectile geometry and other such factors. The parameter instantaneous Wetting angle at the free surface [1], $\theta(t)$ is defined as shown in the Figure 4.

**Figure 4:** The instantaneous Wetting angle at the free surface $\theta$ [1].

By conservation of energy, the fluid kinetic energy in Equation (29) is related to the added mass, $m_a$ by the following equation:-

$$T(\theta_0) = \frac{1}{2} m_a U^2 \quad (30)$$

Then the Equation (25) was solved for a small-time interval of the fluid kinetic energy in Equation (29). Then the kinetic energy was expressed as:-

$$T(\tau) = \frac{4}{3} \rho r^3 U^2 \tau^2 (1 - 0.35\tau - 0.17\tau^2) \quad (31)$$

Where,

$$\tau = \frac{b(t)}{r} = \frac{U(t) \tau}{r} = \frac{\tau^2}{2} \quad (32)$$

$$\epsilon(\tau) = \pi - \theta_0(\tau) \quad (33)$$

Equation (29) implies that the added mass coefficient $\lambda(\tau)$ of a double spherical bowl with semi-axis ‘b’, with $b = U^*t$ is given by:-

$$\lambda(\tau) = \frac{2T}{\pi \rho r^3 U^2} = \frac{16\sqrt{2}}{3\pi} \tau^2 - 1.19\tau^2 - 0.837\tau^2 \quad (34)$$
The added mass coefficient of a half spherical bowl, such as the bottom of the sphere is half of Equation (34). The added mass of the projectile after ignoring higher order terms is then given by:

\[ m_a(t) = \frac{1}{2}A(\tau)\pi r^3 \]  
(35)

\[ m_a(t) = \frac{1}{2} \left( \frac{16\sqrt{2}}{3\pi} \tau - 1.19\tau^2 - 0.837\tau^{\frac{3}{2}} \right) \pi r^3 \]  
(36)

\[ m_a(t) = \frac{1}{2} \left( \frac{16\sqrt{2}}{3\pi} \left( \frac{Ut}{r} \right)^2 - 1.19 \left( \frac{Ut}{r} \right)^2 - 0.837 \left( \frac{Ut}{r} \right)^{\frac{3}{2}} \right) \pi \rho r^2 \]  
(37)

As seen from Equation (37) the added mass of the sphere for a small-time interval of impact is very small as per our initial assumption. Taking the time derivative of the Equation (37) gives the desired time rate of change in added mass as:

\[ \dot{m}_a(t) = \frac{1}{2} \left( \frac{8\sqrt{2}}{\pi} \tau^{\frac{1}{2}} - 2.38\tau - 2.092\tau^{\frac{3}{2}} \right) \pi \rho Ur^2 \]  
(38)

To obtain the variation in Impact force, \( F(t) \) with time:-

As we know from the studies the Impact force is given by,

\[ F(t) = mg \sin \alpha - B(t) \sin \alpha + \dot{m}_a(t)U_0 \]  
(39)

As the instantaneous values of all the parameters required in Equation (39) are determined we can get the time variation of impact force which is plotted.

To obtain the variation in Coefficient of Impact, \( C_i(t) \) with time:-

The Coefficient of Impact [1] [3] is then defined by,

\[ C_i(t) = \frac{F_i(t)}{\frac{1}{2} \rho U^2 A} = \frac{F_i(t)}{\frac{1}{2} \rho \rho U^2 r^2} \]  
(40)

\[ C_\rho(t) = \frac{mg \sin \alpha - B(t) \sin \alpha + \dot{m}_a(t)U_0}{\frac{1}{2} \pi \rho U^2 r^2} \]  
(41)

The final expression for the Coefficient of Impact.

RESULTS AND CONCLUSIONS

Effect of Angle of Impact, \( \alpha \):-

The common input parameters for the results of Impact and Trajectory shown in Figures 5, 6 and 7 are:-

- Density of sphere, \( \rho_0 = 2500 \text{ kg/m}^3 \).
- Radius of sphere, \( r = 6 \text{ cm} \).
- Density of fluid, \( \rho = 1000 \text{ kg/m}^3 \).
- Coefficient of viscosity of fluid, \( \eta = 8.234 \times 10^{-4} \text{ Ns/m}^2 \).
- Initial wetting angle subtended at center of the sphere, \( \Theta_0 = 0^\circ \).
- Impact velocity at free surface, \( U_0 = 150 \text{ m/s} \).

The sphere undergoes water entry at different angles of impact and the variation in Impact during water entry along with trajectory for the same duration are exhibited in the results below. As the formulations are valid for impact analysis during water entry, the results are exhibited only for a short duration of '5 milliseconds' which is sufficient enough for sphere to attain full immersion at high velocities. As seen from the results above, the depth reached increases with increasing angle of impact [4] because with increase in ‘\( \alpha \)’ value of ‘sin \( \alpha \)’ gets closer to 1. Thus, the sphere will take less time to attain full immersion with increasing angle of impact. As Buoyancy force is a direct function of Depth reached as seen in Equation (24), greater values will be obtained for vertical water entry as compared to oblique water entry.

With increasing \( \alpha \), the velocity and ‘sin \( \alpha \)’ increase, thus leading to an increase in the dominant term the rate of change of added mass in Equation (38). This rate of change of added mass is observed to have values in the order of \( 10^3 \sim 10^4 \) and can have positive or negative values.
Figure 5: Impact and Trajectory at $\alpha = 30^\circ$.

Figure 6: Impact and Trajectory at $\alpha = 60^\circ$.

Figure 7: Impact and Trajectory at $\alpha = 90^\circ$. 
This leads to increase in impact force and the Coefficient of impact in Equations (39) and (41) with increasing angle of impact [3] [4]. Hence, it is concluded that Vertical water entries lead to greater magnitudes of impact force than Oblique ones for the same geometric and flow conditions.

**Effect of State of Immersion / Initial Wetting Angle, θ₀:**

The state of immersion also has an influence on impact and the trajectory of the sphere. The Wetting angle, θ as shown in Figure 2 is the angle subtended by the wet surface of the sphere in contact with water at the center. Depending on the value of wetting angle given as the input parameter, the analysis can be initiated from any immersed phase of the sphere during water entry.

The various phases of immersion are shown in Figure 8.

i) Sphere ‘A’ refers to Touchdown phase where the Initial Wetting angle, θ₀ = 0° is given as input. ii) Spheres ‘B’, ‘C’ and ‘D’ represent Partial immersion where the value of θ₀, as shown in Figure 2 is in the range 0° < θ₀ < 180°. iii) Sphere ‘E’ represents Full immersion where θ₀ = 360° for entire analysis after this phase.

**Figure 8: Phases of immersion due to variation in Initial Wetting angle.**

The common input parameters for the results shown in Figures 9, 10 and 11 are:-

- Density of sphere, ρ₀ = 1300 kg/m³.
- Radius of sphere, r = 4 cm.
- Density of fluid, ρ = 1000 kg/m³.
- Coefficient of viscosity of fluid, η = 8.234 x 10⁻⁴ Ns/m².
- Impact velocity at free surface, U₀ = 100 m/s.
- Angle of Impact, α = 60°.

**Figure 9: Impact and Trajectory at θ₀ = 0°.**
As seen from the above trajectory results, with increasing values of Initial wetting angle, $\theta_o$, the depth reached by the sphere increases. The trajectory starts from origin for $\theta_o = 0^o$ and then from a finite value of depth along Y-axis as $\theta_o$ increases. This finite value on Y-axis at $X = 0$ of the trajectory plot indicates the part of the sphere initially underwater due to partial immersion for the values of $\theta_o > 0^o$ which is represented by the first term in Equation (19). Thus, increasing values of $\theta_o$ leads to downward shift of trajectory curve. The time to attain full immersion reduces with increasing values of $\theta_o$. Due to increase in depth reached for increasing values of $\theta_o$, there is an increase in Buoyancy force which is direct function of depth as shown in Equation (24). On increasing $\theta_o$, the parameter ‘$c$’ in the Equation (38) increases thereby leading to an increase in the rate of change of added mass. Hence, this dominant term leads to an overall increase in Impact force and the Coefficient of Impact in Equations (39) and (41). Thereby, it concludes that increase in initial immersion of sphere leads to greater depth reached and higher impact forces for similar flow conditions.

**Effect of Increase in Mass of the Sphere, $m$:**

As we are well aware that the mass of the sphere is defined by the Equation (42) which shows that is dependent on both radius, $r$ and the density, $\rho_b$ of the sphere. Hence, the mass of the sphere can be increased by either increasing the radius or its density and thereby we will study their effect on Impact and Trajectory.

\[
m = \rho_b \left( \frac{4}{3} \pi r^3 \right)
\]  

(42)
First, we will study the effect of increase in Sphere mass by increasing the sphere density and keeping the radius constant. The input parameters for Oblique water entry as shown in Figures 12, 13 and 14 are:

- Radius of sphere, \( r = 4 \) cm.
- Density of fluid, \( \rho = 1000 \) kg/m\(^3\).
- Coefficient of viscosity of fluid, \( \eta = 8.234 \times 10^{-4} \) Ns/m\(^2\).
- Initial wetting angle subtended at center of the sphere, \( \theta_0 = 0^\circ \).
- Impact velocity at free surface, \( U_0 = 120 \) m/s.
- Angle of Impact, \( \alpha = 45^\circ \).

As shown in the results above, there are slight increments in Impact force and Impact coefficient but due to high order values and high range of plots for these parameters, the small changes are not noticeable. But with increase in sphere density, the Buoyancy increases due to slight increase in depth achieved and Impact forces increase due to increase in mass which is shown in Equations (23) and (39). The increase in sphere density also causes a slight increase in rate of change of added mass due to increase in velocity as seen in Equation (38) which is the high value dominating term leading to resultant increase in Impact force and coefficient. However, a significant increase in depth is not achieved in case of increase in Sphere density as the displacements plotted in the trajectory plots are of the center of the sphere rather than the lowest point on the surface of the sphere. This is the criteria followed for all the trajectory plots in this paper.

**Figure 12:** Impact and Trajectory at \( \rho_b = 1500 \) kg/m\(^3\).

**Figure 13:** Impact and Trajectory at \( \rho_b = 1800 \) kg/m\(^3\).
Now the mass of the sphere will be increased by increasing the radius of the sphere. We are well aware that if the radius increases, the mass of the sphere will increase for a constant density and vice-versa. The results are obtained for $\alpha = 45^\circ$ for Oblique water entry. The input parameters for Oblique water entry as shown in Figures 15, 16, 17 and 18 are:

- Density of sphere, $\rho_b = 2000 \text{ kg/m}^3$.
- Density of fluid, $\rho = 1000 \text{ kg/m}^3$.
- Coefficient of viscosity of fluid, $\eta = 8.234 \times 10^{-4} \text{ Ns/m}^2$.
- Initial wetting angle subtended at center of the sphere, $\Theta_0 = 0^\circ$.
- Impact velocity at free surface, $U_0 = 120 \text{ m/s}$.
- Angle of Impact, $\alpha = 45^\circ$.

As seen from the above results for Oblique water entry, the depth attained by the spheres of different radii are the same for respective cases as they are the instantaneous values of the center of the sphere. Hence, although having different radii, the center of sphere for such small-time interval almost have same values of depth with minor variations. Greater the radius, closer is the sphere from the free water surface for approximately same values of depth leading to less Buoyancy force. From Equation (39) it can be well observed that increase in mass and decrease in Buoyancy due to increase in radius leads to overall increase in Impact force. Further the Equation (38) suggests that rate of change of added mass is proportional to square of radius, thereby also increasing the value of dominant term.
Figure 16: Impact and Trajectory at r = 6 cm.

Figure 17: Impact and Trajectory at r = 8 cm.

Figure 18: Impact and Trajectory at r = 10 cm.
Although Impact force increases, the Coefficient of impact decreases with increasing radius due to the inverse proportionality of the radius as seen in Equation (41). Now we will compare the results in Figures 12 – 14 with Figures 15 – 18 to distinguish between the effect due increase in Sphere radius and Sphere density. As observed the depth reached by the center of the sphere is not altered at a significant level due to increase in mass for respective cases. Although increase in radius and density both lead to increase in mass, noticeable finite changes are observed in Impact for the case of change in radius rather than change in density. The reason is well reflected in Equation (42) for the mass of a sphere. As we know, the mass is proportional to density to the power 1 and to radius to the power 3. The density is in kg/m³ (SI units) and radius in cm (centimeters). When the density is increased, the mass of the sphere increases only by a small amount. However, due to dependency on the higher power of the radius, a small increase in radius leads to considerable increase in mass which is demonstrated in the Table 1.

| Sphere Density (ρ in kg/m³) | Sphere Radius (r in cm) | Sphere Mass (m in kg) |
|-----------------------------|-------------------------|-----------------------|
| Increasing Density          |                         |                       |
| 1500                        | 4                       | 0.402                 |
| 1800                        | 4                       | 0.482                 |
| 2100                        | 4                       | 0.562                 |
| Increasing Radius           |                         |                       |
| 2000                        | 4                       | 0.536                 |
| 2000                        | 6                       | 1.808                 |
| 2000                        | 8                       | 4.288                 |
| 2000                        | 10                      | 8.376                 |

Table 1: Variation in Mass of the Sphere.

Thus, this noticeable rise in mass leads to finite increase in impact force due to increase in radius. Similar explanation follows for the decrease in the Buoyancy force with the increase in sphere radius which brings about a noticeable change in the results rather than increase in density of the sphere where the Buoyancy force is almost unchanged because buoyancy is mainly dependent on the density of the surrounding fluid rather than the density of the body immersed in the fluid. Similarly, for the Coefficient of impact a significant decrease is observed with the increase in radius due to higher power inverse proportionality as seen in Equation (41) and is exhibited in the results above. But for increasing Sphere density slight increase is observed which are not predictable in high range plots in Figures 12, 13 and 14. Thus it can be concluded that the increase in Sphere radius has more adverse effect on Impact rather than increase in Sphere density which is mainly due to increase in volume and area of impact of the sphere during water entry due to increase in radius.

**Effect of Fluid density, ρ:**

We will now analyze the effect of change of fluid (liquid) medium. Change in fluid (liquid) leads to change in Density and Coefficient of Viscosity of the surrounding fluid (liquid) in the input parameters. There are three fluids that are considered namely- water, ethanol (less dense than water) and nitrobenzene (denser than water). It should be noted that density and viscosity are the parameters that are temperature dependent. Both the density and viscosity decrease with increase in temperature for most of the common fluids although there are some exceptions. Hence, all the fluids are considered at a temperature of 25°C for justified comparison of the results at similar flow conditions and the properties of the fluids at the temperature of 25°C are in Table 2. To understand the significance of the change in surrounding fluid medium, all the input parameters except fluid density and viscosity are kept unchanged for Oblique liquid entry.

| Fluid (Liquid) | Density (ρ in kg/m³) | Coefficient of Viscosity (η in Ns/m²) |
|----------------|-----------------------|---------------------------------------|
| Temperature at 25°C |                       |                                       |
| Ethanol        | 789                   | 1.074 X 10⁻³                         |
| Water          | 1000                  | 8.94 X 10⁻⁴                          |
| Nitrobenzene   | 1200                  | 1.863 X 10⁻³                         |

Table 2: Surrounding fluid mediums and their Properties.
As different surrounding mediums are involved, Oblique liquid entry will be more suitable term here than Oblique water entry. The density of all the surrounding fluid mediums is assumed to be constant with depth throughout the analysis as the time interval and depth reached are very small. Now the Impact and Trajectory analysis is carried out on these fluids and the results are exhibited. The input parameters for Oblique liquid entry as shown in Figures 19, 20 and 21 are:

- Density of sphere, \( \rho_b = 1800 \text{ kg/m}^3 \).
- Radius of sphere, \( r = 3.5 \text{ cm} \).
- Initial wetting angle subtended at center of the sphere, \( \theta_0 = 0^\circ \).
- Impact velocity at free surface, \( U_0 = 180 \text{ m/s} \).
- Angle of Impact, \( \alpha = 60^\circ \).

![Figure 19: Impact and Trajectory for Ethanol.](image1)

![Figure 20: Impact and Trajectory for Water.](image2)

As we know, with increase in fluid density, the viscous and surface tension forces also increase, thereby offering greater resistance to the sphere during liquid entry. As seen from the results for Oblique liquid entry, the final velocity reached by the sphere will decrease with increasing fluid density as the sphere has to overcome greater hydrodynamic drag. The depth reached in Oblique liquid entry for all the fluids is marginally the same because the time interval considered is very small. Significant difference in depth can be noticed if analysis is carried out for longer time durations. The Buoyancy force is directly proportional to fluid density as seen in Equation (24). So, increase in fluid density leads to increase in Buoyancy force. The rate of change of added mass is directly proportional to fluid density as seen in Equation (38). The increase in the values of this dominating term leads to overall increase in the Impact force with increase in fluid density.
The Coefficient of impact remains the same as it is a non-dimensional representation of impact force and will not vary with surrounding fluid for Oblique liquid entry. The reason being, the proportional rise in Impact force is overcome by the inversely proportional rise in fluid density as seen in Equation (41), thereby leading to same values of the non-dimensional parameter in different mediums. Hence, it can be concluded that the increase in surrounding fluid density leads to increase in Impact force due to greater hydrodynamic resistive loads but has no effect on the non-dimensional Coefficient of impact.

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