Chapter 11

Top Quark Mass

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Ever since the discovery of the top quark at the Tevatron collider in 1995 the measurement of its mass has been a high priority. As one of the fundamental parameters of the Standard Theory of particle physics, the precise value of the top quark mass together with other inputs provides a test for the self-consistency of the theory, and has consequences for the stability of the Higgs field that permeates the Universe. In this review I will briefly summarize the experimental techniques used at the Tevatron and the LHC experiments throughout the years to measure the top quark mass with ever improving accuracy, and highlight the recent progress in combining all measurements in a single world average combination. As experimental measurements became more precise, the question of their theoretical interpretation has become important. The difficulty of relating the measured quantity to the fundamental top mass parameter has inspired alternative measurement methods that extract the top mass in complementary ways. I will discuss the status of those techniques and their results, and present a brief outlook of further improvements in the experimental determination of the top quark mass to be expected at the LHC and beyond.

1. A Brief History of the Top Quark

When the existence of the top quark was first postulated in 1973 to explain the observation of CP symmetry in the kaon system, few could have imagined that its mass would turn out to be comparable to a gold atom, more than a hundred times heavier than the charm quark, the heaviest quark known at the time. Even today, after the recent discovery of a Higgs boson with a mass of 125 GeV, the top quark remains the heaviest known elementary particle, a striking empirical fact for which the Standard Theory offers no explanation.

Searches at $e^+e^-$ colliders during the 1970s and 80s looking for a narrow $t\bar{t}$ resonance failed to find the top quark, and a mass below 23.3 GeV or 30.2 GeV was ruled out at the PETRA and TRISTAN colliders respectively.⁵ Even at the SLC

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and LEP colliders operating at the Z resonance energy no evidence was found for any $Z \rightarrow t\bar{t}$ decays, raising the bar to 45.8 GeV by Spring 1990.

In the meantime hadron collider experiments UA1 and UA2 at the ISR searching for the process $p\bar{p} \rightarrow W \rightarrow t\bar{b}$ had not fared any better, and by 1990 had set the limit at 69 GeV. By that time the Tevatron collider at Fermilab had started operations, and first results by CDF including the now dominant $p\bar{p} \rightarrow t\bar{t}$ process extended the exclusion up to 91 GeV, until finally in 1995 the CDF and D0 collaborations announced the discovery of the top quark with a mass of $175 \pm 8$ GeV.

While the Standard Theory does not prescribe what should be the value of the top quark mass $m_t$, its value in turn has big consequences for the phenomenology of the Standard Theory as we know it, and its consistency with experimental observations.

The special value of $m_t$ has a direct impact on the physics of on-shell top quarks, as will be discussed in Section 2. However, the top quark indirectly plays an important role in many other areas of the Standard Theory as well. In particular, the top quark is present in radiative quantum-loop corrections that appear everywhere in theoretical calculations, and often has a dominant effect due to the large value of $m_t$. For example, corrections to the W boson mass are proportional to $m_t^2$, a fact which was used in the early 1990s to constrain the mass of the top quark through its effect in virtual one-loop corrections before it was discovered. A comparison of the indirect prediction of $m_t$ from a global electroweak fit with the direct measurements of $m_t$ as function of time is shown in Fig. 1. A step-wise improvement in the indirect prediction of $m_t$ is visible at the time of the Higgs boson discovery in 2012.

Precision measurements of the Higgs boson mass, W mass $m_W$, and $m_t$ together with other electroweak parameters allow a strong test of the self-consistency of the

![Fig. 1](image-url)
The indirect predictions of $m_W$ and $m_t$ are compared to the direct measurements in Fig. 2. Radiative corrections and the global electroweak fit are discussed in more detail elsewhere in this book.

While electroweak precision tests are currently limited more by the uncertainty in $m_W$ than in $m_t$, an improvement in our knowledge of $m_t$ would directly benefit calculations of the Higgs potential at high energy, allowing tighter constraints on the stability of the electroweak vacuum and on some cosmological models.

Through its strong Yukawa coupling $Y_t \approx 1$ the top quark also plays a special role in Higgs physics. While at the time of writing this review the associate production of $t \bar{t}$ with a Higgs boson has not yet been observed, the top quark has contributed already to the Higgs discovery and mass measurements by enabling the production of Higgs bosons in processes containing virtual top quark loops, such as the gluon fusion channel at the LHC and the subsequent decay to two-photon final states.

And finally, top quarks play an important role in flavour physics, as they contribute to many rare processes through virtual top quark loops. Thus the observation of $B^0 - \overline{B^0}$ oscillations in 1988 allowed the ARGUS collaboration to derive a lower limit $m_t > 50$ GeV. Similarly, the predicted rate of a rare decay such as $B_s \rightarrow \mu \mu$, recently observed at the LHC, depends strongly on the value of $m_t$.

Thus many motivations exist to measure $m_t$ with the best possible experimental precision. And as always, the quest for precise measurements doubles as a search for possible minute deviations from the predictions that could reveal signs of new physics processes beyond the Standard Theory.
2. The Short Life of a Top Quark

The large mass of the top quark allows it to decay to an on-shell $W$ boson and a $b$ quark, resulting in a very short lifetime ($\approx 5 \times 10^{-25}$s) that prevents the formation of $t\bar{t}$ bound states like the characteristic $J/\psi$ and $\Upsilon$ resonances for the charm and bottom quarks respectively, and it also means that top quarks decay before hadronizing into jets. However, the lifetime is still long enough that the corresponding decay width $\Gamma_t$ is narrow compared to the top quark mass $m_t$. The prediction at NLO precision in quantum chromodynamics (QCD) is given by:\[21\]

$$\Gamma_t = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + 2\frac{m_W^2}{m_t^2}\right) \left[1 - \frac{2\alpha_s}{3\pi} \left(\frac{2\pi^2}{3} - \frac{5}{2}\right)\right]. \quad (1)$$

The top-quark decay width depends on the top-quark mass ($m_t$), the $W$ boson mass ($m_W$), the Fermi coupling constant ($G_F$), the strong coupling constant ($\alpha_s$) and the magnitude of the top-to-bottom-quark coupling in the quark-mixing matrix ($V_{tb}$).\[22–24\] The most recent calculations at next-to-next-to leading order (NNLO) including QCD and electroweak corrections predict $\Gamma_t = 1.32$ GeV,\[25\] for $m_t = 172.5$ GeV and other parameters fixed to their best-known values.\[26\]

Indeed, a relatively narrow mass peak can be observed experimentally by reconstructing the invariant mass of the top quark decay products, as shown in Fig. 3 both for doubly-resonant $t\bar{t}$ events and for singly produced top quarks. The width of the observed mass peak is dominated by the experimental resolution, and has been shown to be consistent with an underlying mass resonance with the narrow width predicted by the Standard Theory,\[27\] also confirmed by indirect estimates based on top quark production and decay rates.\[28–30\]

The position of the mass peak is strongly related to the value of $m_t$, which makes this a suitable observable for experimental determinations of the top quark mass.

![Fig. 3. (a) Invariant mass of top-quark decay products in $t\bar{t}$ events reconstructed in CDF Template analysis using the full Tevatron Run II data set.\[31\] (b) Reconstructed top quark invariant mass in CMS single top $t$-channel cross-section analysis using the 2012 data set.\[32\]](image-url)
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mass. Generally Monte Carlo (MC) programs are used to model the full complexity of the candidate top quark events, and the top mass value that is measured is the MC top mass parameter $m_{t}^{MC}$ for which the observable of interest (in this case the shape of the invariant mass peak) agrees best between the MC simulation and the experimental data.

However, to make a theoretically well-defined connection between a bare parameter like $m_t$ and a physical observable, a mass renormalization scheme must be used in which higher-order quantum corrections are accounted for in the definition. Many possible choices for mass renormalization schemes exist. Two commonly used mass definitions are the pole mass scheme ($m_{t}^{pole}$) and the modified minimal subtraction scheme ($\overline{MS}$), also known as a “running mass” scheme.

The $\overline{MS}$ scheme is a so-called “short distance” mass, suitable for calculations involving high scales, such as the global electroweak fit or the calculation of the electroweak vacuum stability. The pole mass remains attractive, however, because $m_{t}^{pole}$ has a numerical value close to the position of the top invariant mass peak, while $\overline{MS}$ has a value that is about 10 GeV lower.

The relation between $m_{t}^{pole}$ and other schemes can generally be expressed as

$$m_{t}^{pole} = m_{t}(R, \mu) + \delta m_{t}(R, \mu);$$

$$\delta m_{t}(R, \mu) = R \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} [\alpha_s(\mu)]^n \ln \left( \frac{\mu^2}{R^2} \right)$$

(2)

where $R$ is the subtraction scale associated with the scheme; for the $\overline{MS}$ scheme $R \sim m_t$. The translation between the pole and $\overline{MS}$ masses was recently calculated at four-loop precision in QCD $^{33}$ with a convergence to about 200 MeV, the size of the four-loop term. Relations between proper short-distance mass schemes are known with even better precision. However, in the case of $m_{t}^{pole}$ an additional non-perturbative term remains, called the renormalon ambiguity.

While transformations between field-theoretical mass schemes are well described, no formal connection exists between $m_{t}^{MC}$ and any quantum-field theoretical mass definition. Based on analogies between the models in MC programs and field-theoretical QCD calculations it is often assumed that $m_{t}^{pole}$ and $m_{t}^{MC}$ are close to within about 1 GeV. It has been suggested $^{34}$ that $m_{t}^{MC}$ may be similar to a mass scheme with a renormalization scale $R$ close to the cut-off scale of the parton shower (typically 1–3 GeV) used in the MC simulation. Eventually a more quantitative numerical correspondence between $m_{t}^{MC}$ and a suitably chosen theoretical short-distance low-scale mass may possibly be established by comparing the predictions of physical observables between the two approaches. Now that the most precise invariant-mass based measurements of $m_{t}^{MC}$ have reached sub-GeV precision this is an important question, and a topic of active discussion $^{34-36}$.

Measurements of $m_t$ based on observables that can be calculated directly in a QCD calculation with a well-defined theoretical mass definition avoid this issue. An example of such an observable is the inclusive $t\bar{t}$ cross-section, or the $t\bar{t}$ production
threshold at a potential future $e^+e^-$ collider. A brief overview of the various approaches and their potential performance is given in the following sections.

3. Conventional Top Quark Mass Measurements at Hadron Colliders

Throughout the twenty years since the discovery of the top quark, the measurement of its mass has been an active field of development for novel analysis methods and techniques. In the quest for experimental precision, the initial focus was above all the improvement of the statistical precision of the top mass determination, by far the dominant source of uncertainty during the early years.

The experimentally most precise determinations of the top quark mass have traditionally been obtained with full kinematic reconstruction of $t\bar{t}$ events, and reconstruction of the invariant mass of the decay products of the top (and anti-top) quarks: $t \to Wb$. Depending on the decay modes of the two $W$ bosons in the event, three final state topologies are possible, listed with their relative abundance assuming lepton universality:

(a) dilepton: $t\bar{t} \to W^+b\bar{W}^-\bar{b} \to \ell\nu_b\ell\bar{\nu}\bar{b}$ (10.5%)
(b) lepton+jets: $t\bar{t} \to W^+b\bar{W}^-\bar{b} \to q\bar{q}b\ell\bar{\nu}\ell\bar{b}$ or charge conjugate (43.8%)
(c) all-jets: $t\bar{t} \to W^+b\bar{W}^-\bar{b} \to q\bar{q}bq\bar{q}b$ (45.7%)

The $b$ quarks and light-flavour quarks ($q$) hadronize and are observed as jets of particles. Additional jets may be produced through the radiation of energetic gluons from the hard scattering production process and in the top quark decay. The lepton $\ell$ can be an electron, muon or tau lepton but in top quark mass measurements tau leptons have hardly ever been used explicitly (with one exception), due to more challenging detection and reduced kinematic information due to the presence of neutrinos in their decay. To reconstruct the invariant mass of the top decay products with the best possible resolution advanced statistical methods can be applied, often including one or more of the following commonly used techniques:

**Kinematic constraint fits** In the lepton+jets and all-jets channel a kinematic constrained fit can be used to improve the reconstruction of the event final state kinematics beyond the detector resolution. The five constraints that are typically used are equality of the top and anti-top mass, transverse momentum balance of the full event, and knowledge of the mass of the two $W$ bosons in the event. In the case of the dilepton channel the final state is underconstrained, with 6 missing momentum components of the two energetic neutrinos, and dedicated techniques were developed to estimate as well as possible the top quark invariant mass in these events.

**Monte Carlo Template method** In this approach the distribution of a variable that is sensitive to the top quark mass is compared between data and the Monte Carlo (MC) prediction. The MC prediction (also called “Template”) is
produced for different values of the MC top mass parameter, and the top mass parameter is varied until the best agreement is obtained between data and the MC Template, using a maximum-likelihood fit. This method is conceptually simple and elegant, but the information that can be extracted event-by-event is limited. In its simplest form only a single variable per event is considered, typically the reconstructed top mass after a kinematic fit, but also 2-dimensional and 3-dimensional Template fits have been used. The Template method was the method of choice in the first generation of top mass measurements in the lepton+jets channel by CDF and D0, and has been used in many analyses at the Tevatron and LHC since.

**Matrix Element and Ideogram techniques** In these methods an event-by-event likelihood is calculated as function of the top quark mass, allowing to extract more statistical information from the events by including a more complete picture of the complexity of the $t\bar{t}$ events. The event likelihoods can take into account the probability that a given event is a background event, all relevant ambiguities such as multiple possible jet assignments, and an estimated mass resolution for every mass solution depending on the event topology and compatibility with the $t\bar{t}$ hypothesis. In the case of the Matrix Element (ME) method the $t\bar{t}$ hypothesis is defined by a (typically) LO theoretical matrix element, and the signal likelihood is calculated by integration over a multi-dimensional phase space taking into account detector resolutions. In the Ideogram approach the $t\bar{t}$ event hypothesis is defined by the constraints of a kinematic fit, and the signal likelihood is based on the post-fit variables including the goodness-of-fit. Both methods effectively allow events that are most signal-like and have less ambiguity or better resolution to have a bigger impact on the top mass determination.

By employing a more detailed signal model contained in the matrix element, the ME method is statistically the more powerful technique, but the calculations involved make this method very CPU intensive. The Ideogram method is faster to execute and less model dependent. The ME technique is used in the most precise measurement at the Tevatron to date, while the most precise determinations by CMS are based on the Ideogram approach.

Regardless of the exact method used to reconstruct the top invariant mass, all measurements in this class of analyses have to rely on the same (or very similar) MC simulation programs to apply corrections for experimental and theoretical effects that play a role in the mass determination, including detector resolution, trigger effects, ambiguities and approximations in the event reconstruction and interpretation, and effects of perturbative and non-perturbative QCD as modeled by the MC simulation. For all mass determinations based on direct reconstruction of the invariant mass of the top decay products and calibrated this way one can therefore assume that within the uncertainties assigned, effectively the same “MC mass definition” ($m_t^{MC}$) applies.
3.1. World average anno 2014

In March 2014 the four collaborations ATLAS, CDF, CMS and D0 joined forces and prepared a combination of the most precise top quark mass measurements available at the time. Possible correlations between each source of systematic uncertainty of the different input analyses were evaluated and taken into account, yielding the following world-average value of the top quark mass\(^{44}\) (using the \(m_{t}^{\text{MC}}\) definition):

\[
m_{t} = 173.34 \pm 0.76 \text{ GeV}.
\]  

As shown in Fig. 4 the combination was also provided per experiment, per collider and per decay channel. It is interesting to see that at this point in time the combined measurements of the LHC had reached a precision equal to the Tevatron measurements, and that the central values are in good agreement in spite of the big change in collision energy from 1.96 TeV to 7 TeV. Measured top mass values also agree well between the various decay channels with very different event topologies and background conditions. While the lepton+jets channel still yields slightly better results than the dilepton and all-jets final states, they all have reached a relative experimental precision of well below 1%. The consistency of the results across different event topologies, collider energies, luminosity conditions and detectors is impressive, and provides a confirmation of the ability to understand the relevant physics effects and experimental conditions with great accuracy.

![Fig. 4. World average combination\(^{44}\) of top mass results from the LHC and Tevatron collaborations available in March 2014, and for subsets in different channels, experiments and colliders.](image-url)
3.2. **New results in $m_t^{MC}$ measurements since 2014**

The latest ATLAS measurements using the full 7 TeV dataset were not yet included in the world average, but are in good agreement. The combination of the results in the dilepton and single lepton channel yields $m_t = 172.99 \pm 0.91$ GeV.

The CDF collaboration published their final measurements with the full Tevatron Run II data set in the all-jets and di-lepton channel, and D0 in the lepton+jets channel. The D0 measurement uses the Matrix Element technique and reaches a precision equal to the 2014 world average and central value confirming earlier D0 measurements: $m_t = 174.98 \pm 0.76$ GeV.

The most precise measurements all use the hadronic decay of the $W(\rightarrow jj)$ and the known value of $m_W$ to determine an overall jet-energy scale factor (JSF) in situ in the $t\bar{t}$ events. By correcting all measured jet energies with this factor, the main experimental systematic uncertainty due to jet energy calibration is reduced. However, the ATLAS analysis in the lepton+jets channel goes one step further by performing a 3-dimensional fit of top quark mass, the overall JSF, and an additional scale factor $b$-JSF describing any possible deviation between the jet response for jets from light-flavour quarks and $b$ quarks. To constrain $b$-JSF, a new variable is introduced probing the transverse momentum balance of $b$-tagged jets versus non-$b$-tagged jets. A simulated distribution of this variable, $R_{\text{reco}}$, and its dependence on $b$-JSF is shown in Fig. 5(a), and the reconstructed top mass in Fig. 5(b). This novel analysis approach reduces flavour-dependent uncertainties in jet energy calibration, at the cost of an increased statistical uncertainty. With larger data sets at 8 TeV and in the upcoming LHC Run 2 prospects are excellent for further reduction of the overall measurement uncertainties.

![Fig. 5. Mass analysis in the lepton+jets channel using the full 7 TeV dataset in ATLAS using transverse momentum balance of $b$-tagged jets versus other jets (a) to improve the jet energy scale calibration and reduce systematic uncertainty on the reconstructed top mass (b).](image-url)
3.3. Prospects for $m^\text{MC}_t$

The prospects for further improvements in measurements of $m_t$ in the $m^\text{MC}_t$ definition are good. Experimental uncertainties are well understood and expected to become better with increasing statistics of data samples for calibration. While the modelling of QCD effects at perturbative and non-perturbative level is far from trivial, the MC simulation programs currently in use by experimental collaboration generally describe the data very well, and theoretical uncertainties on the determination of $m^\text{MC}_t$ are of the order of 0.2 GeV or less per effect studied. The latest tools with full NLO matching between matrix elements and parton shower and calculations taking into account off-shell NLO effects, or calculations achieving NNLO+NNLL accuracy promise further improvements.

Increased top quark data samples will also allow more detailed studies of the stability of the top mass observable as a function of kinematic variables, which may help to pinpoint issues with QCD modeling and could possibly even help in the future to shed more light on the question of a correspondence between $m^\text{MC}_t$ and well-defined theoretical mass definitions.\textsuperscript{36}

3.4. Extraction of $m^\text{MC}_t$ with different observables

The top mass determinations discussed so far were all based on full reconstruction of the invariant mass of top decay products. Various other observables have been proposed to minimize the effects of certain modeling uncertainties or experimental effects, or more generally to have different and therefore less correlated uncertainties.

One example is the use of only leptonic variables,\textsuperscript{47} which has been proposed to minimize potentially poorly understood and modeled effects of non-perturbative QCD. Another approach advocates the use of the position of the peak in the $b$ jet energy spectrum,\textsuperscript{48} a method which is claimed to reduce sensitivity to the production mechanism, but does not mitigate uncertainties related to $b$ jet reconstruction.

To avoid reliance on jet reconstruction and energy calibration, methods have been proposed that are purely based on charged particles: the $L_{xy}$ method determines the boosts of $b$ jets by measuring the $b$-hadron decay length, a method proposed initially in Ref. 49 and applied by CDF.\textsuperscript{27,50} Another method is based on the invariant mass of a $J/\psi$ from the $b$-jet and the isolated lepton from the $W$ decay of the same top,\textsuperscript{51} a Lorentz-invariant quantity and potentially less sensitive to top quark production modeling. However, statistical uncertainties are large and the modeling of the $b$ jet fragmentation becomes an important source of uncertainty.

Finally the use of the single top channel is being considered. As suggested by the invariant mass peak clearly visible in Fig. 3 it should be possible to perform a mass measurement using single top events at the LHC. The production mechanism is different so some of the theory modeling systematics would be different from measurements based on $t\bar{t}$ events.
4. Top Mass Extraction Using Other Top Mass Definitions

The idea to extract a well-defined (pole or $\overline{\text{MS}}$) top quark mass by comparing the $t\bar{t}$ cross section to a theoretical prediction as function of $m_t$ was pioneered by D0. Recalling more precise measurements by CMS\textsuperscript{53} and ATLAS\textsuperscript{54} were obtained using the $t\bar{t}$ cross section calculated at NNLO+NNLL precision.\textsuperscript{55} The combination of the ATLAS 7 and 8 TeV results yields $m_t = 172.9^{+2.5}_{-2.6}$ GeV, and the method is illustrated in Fig. 6(a). While this is a theoretically very clean method to extract the top quark mass, it is hard to imagine significant further improvements as this analysis already has excellent experimental precision on the cross-section with minimal dependence on the assumed $m_t$, and theoretical predictions beyond the current NNLO+NNLL accuracy are not expected in the near future.

Rather than using the inclusive production cross-section, differential distributions of observables that can be calculated using first-principles QCD may allow to obtain a theoretically well-defined top quark mass with improved precision. One example is the normalized inverse of the $t\bar{t}+\text{one jet}$ invariant mass which has been calculated to NLO precision in perturbative QCD\textsuperscript{56} and was recently employed by the ATLAS collaboration in the lepton+jets channel to extract a top quark pole mass\textsuperscript{57} $m_t^{\text{pole}} = 173.7 \pm 1.5(\text{stat}) \pm 1.4(\text{syst})^{+1.0}_{-0.5}$ GeV.

Another promising observable that may provide for theoretically well defined measurements is the $m_{\ell b}$ distribution which can be calculated with NLO precision in QCD,\textsuperscript{58} also including off-shell non-factorizable corrections.\textsuperscript{59–61}

The CMS collaboration has performed a radically different measurement using the kinematic endpoints of various distributions in the dilepton channel.\textsuperscript{62} The endpoints of various lepton-related distributions are used, as well as the $m_{\ell b}$ distribution, shown in Fig. 6(b). No MC simulation is used to calibrate the measurement, nor does the analysis include any QCD corrections or calculations. The endpoints

![Fig. 6. Extraction of the top pole mass from the $t\bar{t}$ cross-section by ATLAS (a) and the $m_{\ell b}$ mass distribution used in the CMS endpoint analysis (b).](image-url)
are predicted based on the picture of a narrow mass resonance, extracting a value close to the position of the underlying mass peak. By definition the mass extracted is not $m_t^{\text{MC}}$ nor is it $m_t^{\text{pole}}$. Yet the result is numerically in good agreement with the top mass measurements obtained with those definitions: $m_t = 172.9^{+1.9}_{-2.3}$ GeV.

5. Top Mass Prospects at Lepton Colliders

Prospects for top mass measurements at a possible future $e^+e^-$ collider at or above the $t\bar{t}$ production threshold have been investigated in detail and several useful overviews and reports exist.\textsuperscript{63–67} Theoretical calculations of the $e^+e^-\to t\bar{t}$ threshold shape are advanced and studies show that a precise experimental scan of the cross-section shape around the threshold would allow a determination of $m_t$ with a precision below 100 MeV in a well-defined short-distance scheme optimized for the threshold scan.

At energies above the threshold, it would also be possible to perform invariant mass measurements of the top decay very similar to methods used at hadron colliders before. This would have the advantage of a much cleaner $e^+e^-$ environment without the additional underlying hadronic event activity from proton remains. The question of interpretation in a well-defined mass scheme, while not fundamentally different from hadron collider measurements, will be easier to address theoretically and perhaps better controlled for the $e^+e^-$ case.\textsuperscript{67–70}

6. Summary and Outlook

In the 20 years since the discovery of the top quark, measurements of its mass at hadron colliders have made huge strides, evolving towards a precision well below 1% and even approaching 0.5 GeV. A further step forwards is critically dependent on a detailed and accurate treatment of the effects of perturbative and non-perturbative QCD that link the $m_t$ parameter to the physical particles observed in experiment. The more than 100-fold increase in top quark data sets anticipated at the LHC, innovative analysis approaches, and state-of-the-art theoretical calculations and MC tools will offer further opportunities to make a fundamental step forwards in measuring this fundamental constant of Nature. Reaching a precision of the order of $\Lambda_{\text{QCD}}$ or better is not excluded at the LHC, and a future lepton collider would open a window to even greater levels of scrutiny and precision.

The question why the top mass is so much heavier than other known fermions remains an unexplained mystery of particle physics. Knowing its precise value allows to better calculate the experimental implications of the Standard Theory and test its consistency, but also to constrain possible effects of new physics beyond it.
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