A method for determining the internal DC magnetic field inside a superconducting cavity is presented. The method implemented uses the magnetic field dependence of the frequency of the Kittel mode of a ferrimagetic sphere, hybridised in the dispersive regime of the superconducting cavity. Results were used to experimentally determine the level of screening a superconducting Nb cavity provides as it transitions from perfect diamagnetism to no screening. Two cavity geometries were tested, a cylinder and single post re-entrant cavity. Both demonstrated a consistent value of field that enters the cavity, expected to be the superheating critical field. Hysteresis in the screened field during ramp up and ramp down of the external magnetic field due to trapped vortices was also observed. Some abnormal behaviour was observed in the cylindrical cavity in the form of plateaus in the internal field above the first critical field, and we discuss the potential origin of this behaviour.

The measurement approach would be a useful diagnostic for axion dark matter searches, which plan on using superconducting materials but need to know precisely the internal magnetic field.

Superconducting cavities are of great benefit for many scientific and engineering disciplines, which require low loss and high quality-factor resonators. For example, frequency metrology, particle accelerators and tests of fundamental physics. Axion haloscope dark matter experiments, which rely on low loss microwave cavities in high DC magnetic fields, may be enhanced by the use of type II superconductors operated between the first and second critical fields, thus allowing flux into the cavity but retaining some superconductivity - interest in this has recently grown. Such experiments require that the DC magnetic field inside the cavity be both large and precisely known for a sensitive experiment. Thus, a system to measure the internal field is necessary for the calibration and development of such experiments, and it must be verified that large DC magnetic fields can penetrate superconducting cavities whilst retaining high quality factors. Any sensor to measure the internal field would need to operate in vacuum, and at cryogenic temperatures. Additionally, operating between the first and second critical field would open up the use of type II superconductors for improving loss in hybrid quantum systems based on cavities and magnetic materials, including ferrimagetic-axion haloscopes. This work presents a novel method for making measurements of the internal DC magnetic field inside superconducting microwave cavities immersed in an external magnetic field.

Superconductors interact with nearby magnetic fields. Type I and II superconductors below their critical field (or first critical field for type II), behave as perfect diamagnets, generating supercurrents by the Meissner effect to perfectly screen external magnetic fields such that none is contained internally. Type I superconductors, fully explained by Bardeen-Cooper-Schrieffer (BCS) theory, will undergo a phase transition returning to normal conductivity above their critical field, allowing external DC fields to pass through with negligible screening. Type II superconductors on the other hand, above their first critical field will enter a mixed state, allowing some field to pass through. This occurs via formation of superconducting vortices surrounding a small region of normal conductor, with each vortex allowing a flux quantum to penetrate. As the field approaches the second critical field, all external flux can penetrate the superconductor as it an Abrikosov flux lattice, determined by Ginzburg-Landau (GL) theory. Above this second critical field, the system returns to its normal state.

Whether a superconductor is type I or II is determined by the Ginzburg-Landau parameter, , where for type I , and type II . A further distinction can be made in type II superconductors, where if , it is denoted Type II/1. Such materials, including very pure Niobium, can form an intermediate mixed state (IMS) above the first critical field, in which the superconductor forms some domains in the Meissner state, and other domains containing vortices. Vortex matter can appear in type II superconductors in the mixed phase, where vortices become pinned in place forming solid like states. Vortex motion can produce an additionally channel for losses. This includes vibration in all cases and flow in vortex liquids. Flux pinning, where vortices can become trapped by defects or impurities in the crystal, can provide a barrier for vortex motion however, even in vortex glass states, flux creep allows loss to occur. Vortex matter, typically arising due to thermal fluctuations compared to the mean field GL theory, are primarily found in high temperature superconductors, however, they are also expected and observed in low temperature superconductors like Niobium. Heat treatment of the material also significantly affects how materials like Niobium will trap or expel magnetic flux as they cool.
Ferrimagnetic materials contain a collection of spins, in which the collective excitations are called magnons. Spherical geometries of ferrimagnetic materials such as lithium ferrite (LiFe), have a uniform precession mode, otherwise known as a Kittel mode which has a linear relation between external magnetic field, $B$, and its mode frequency, $\omega_m [33]$.

\[
\omega_m = (2\pi) \frac{g_{\text{eff}}\mu_B}{\hbar} (B_{DC} + B_{\text{off}}),
\]

where $\mu_B$ is the Bohr magneton, $B_{\text{off}}$ is an offset field typically due to magneto-crystalline anisotropies and $g_{\text{eff}}$ is the effective Landé $g$-factor. Thus the procedure for its use as a magnetic field sensor was simple. The sensor was placed inside a normal conducting cavity, as shown in fig[1], and the Kittel mode frequency was measured over the desired magnetic field range. A fit for calibration was implemented, which relates DC magnetic field incident on the LiFe, to the magnon Kittel mode frequency the field incident on the sphere was taken to be the same as the external applied field, owing to the normal conducting nature of the cavity. After this calibration, the sensor could then be placed in a series of superconducting cavities to measure the penetrating DC magnetic field, by measuring and mapping the magnon frequency as the applied external DC magnetic field was varied. A detailed description of the setup for these kinds of measurements has been covered in past work[2,3]. In particular for this experiment, the probes were coupled primarily to the cavity. The magnon mode, measured in the dispersive regime, was thus primarily seen indirectly through its coupling to the fundamental cavity mode. These measurements may thus be undertaken with minimal alterations to a cavity setup used for other purposes, with only the addition of a small LiFe sphere.

Methods of detecting DC magnetic fields at cryogenic temperatures already exist. Flux gate magnetometers, for example, are a popular choice and have been used to detect DC field external to radio frequency (RF) superconducting cavities[22,23,32]. One could easily conceive of such methods being applied to internal fields, however, other than measuring field close to the walls, this requires the insertion of conductor into the cavity which would prevent simultaneous measurements of the cavity mode. This would be the case for many commercial magnetometers, especially those that rely on conductive pickup loops. The main benefit of our method, therefore, was that it was noninvasive. Due to the high spin density and low loss of LiFe, the sphere can be extremely small and still couple to the cavity mode enough in the dispersive regime to be visible. With the dielectric permittivity and loss of PTFE being small, the presence of the sensor minimally alters the mode shape of the cavity, and thus the sensor can be positioned deep inside the cavity. This allows accurate simultaneous measurements of the cavity mode and DC field anywhere in the cavity.

The material of choice for many cavity magnon experiments is usually Yttrium Iron Garnet (YIG), due to its high spin density and low loss. In this experiment, LiFe was chosen instead. This was due to its higher spin density and relatively low loss, as well as the relatively low number of spurious modes visible at low temperatures in transmission data[9,10] relative to YIG[11], where a large number of spurious modes would make comparison with the calibration data more difficult. The orientation of the crystal axis of the LiFe sphere relative to the external magnetic field was maintained between calibration and experiments, as it affects $B_{\text{off}}$ in equation [1]. Calibration measurements of the LiFe sensor were undertaken in a Copper cavity of identical dimensions to the subsequent cylindrical Niobium cavity. The measurements revealed two similar magnon modes, which needed to be taken into account when calibrating measurements. The results of the fitting the two modes gave $g_{\text{eff}} = 1.90$ and $B_{\text{off}} = 0.034T$ for the first mode and $g_{\text{eff}} = 1.97$ and $B_{\text{off}} = 0.019T$ for the second. To help with observing the modes and fitting, static features were removed from transmission plots by subtracting the transmission value by the average of all magnetic field values for each frequency. The LiFe sphere used in this experiment was a 0.58mm diameter sphere, mounted on a 0.72mm diameter, 4mm tall, PTFE post, where the (111) axis of the crystal was oriented along the direction of the mount. More detailed analysis of the spectra of the LiFe sphere used, can be found in past work[9].

After calibration, the first superconducting cavity measured was a cylinder of internal diameter 10mm and height 8mm, corresponding to a fundamental $TM_{0,1,0}$ mode frequency at 22.45GHz. The base and walls were cut out of a single piece of Niobium with 3mm walls and a 4mm base, including probe holes and a mount for the LiFe sphere. A 3mm thick lid was made from a second piece of Niobium using six evenly spaced 6mm M2 bolts. See fig. 1 for a sketch.

Both the cavity and magnon modes were measured to determine how the penetrating field and superconducting material losses changed with varying DC magnetic field at 4.5K in temperature. From the calibration data, the frequency of the magnon mode determined the internal magnetic field for several experimental runs. This included ramping the field up and down with the probes weakly coupled to the cavity mode (and thus even more
The measured transition at which field enters the cavity was therefore interpreted as the superheating field. This corresponds to \( H_{sh}/\mu_0 = 0.2712 \pm 0.0005 \) T for the cavity under coupled case, which was identical to the value that can be obtained from the first drop in cavity transmission and quality factor from fig. 4, and \( H_{sh}/\mu_0 = 0.2793 \pm 0.0001 \) T for the case of directly coupling to the sphere. The discrepancy between the measured transition field could come from the fact that different runs have likely small differences in temperature and different magnetic field ramp rate. The measured field was initially screened by the walls and eventually returns to the un-screened case at large field. The presence of vortex motion explains why changes in field correspond to changes in cavity loss in the mixed phase. Hysteretic behaviour was observed as the ramp down measurements have more field internally than provided by the external magnet. This is likely due to the vortices generated to allow field to pass through, being trapped during the ramp down, thus generating additional internal field even when the external magnet was ramped down. Future work could investigate the effect of ramp rate or observe the decay of trapped vortices. Additionally, in fig. 4 the transmission and Q factors were observed to return to a lower value on the ramp down, due to trapped vortices. A dip in the quality factor and transmission of the cavity mode was observed around 0.8 T, this was simply due to a cavity mode interaction with the magnon mode, which was tuning through at this field value.

Significant deviations from expectations were observed in the form of plateaus visible in fig. 5. In this case,
rather than a smooth transition, a constant internal field was measured as the external field was ramped, corresponding to a constant quality factor and peak transmission of the resonant cavity. On further ramping up, the system sees another transition as the measured field jumps at \( H = 0.3613 \pm 0.0005 \text{T} \), corresponding to the end of a plateau in loss. There are several expected transition fields between the first and second critical fields which could account for the observed behaviour. If the Nb cavity was pure enough to exhibit an IMS, the plateau could correspond to the field experienced by a local cluster of vortices with field lines in the vicinity of the sensor. Alternatively, the transition field could correspond to the flux depinning field \( H_{dp} \), in which the vortex matter transitions from solid to liquid. This explanation was consistent with the loss data as vortex pinning prevents a significant loss mechanism from vortex motion. The plateau in field could therefore be explained by a fixed number of vortices with magnetic field lines pinned in the vicinity of the sensor. Further work, including spatial characterization of the field, would need to be done to explore this hypothesis. Characterization of the material properties of the sample, including \( \kappa \), would be necessary to determine if the sample could exhibit an IMS. A SANS experiment would also be a benefit to determine vortex matter structure\(^{25,37} \).

For this particular cavity, the probes were inserted from the bottom. This means the probe holes were aligned with the external magnetic field, and could thus play a role in behaviour seen. Intuitively, the system should favour threading flux through the empty hole rather than creating vortices, and would localize flux near the centre of the cavity. This is something that can be quantified, however. The energy associated with flux through a loop of superconductor is minimized when an integer number of flux quanta, \( \Phi_0 \), is threaded through. Assuming this energy was minimized (the system was in equilibrium), the energy cost, \( E_B \), of \( n \) flux quanta is equal to the energy associated with the magnetic field\(^22 \).

\[
E_B = \frac{l}{2\mu_0 A} (n\Phi_0)^2, \tag{2}
\]

where \( \mu_0 \) is the permeability of free space, \( l \) is the length of the probe holes and \( A \) is their cross-sectional area. Similarly, the energy associated with a single vortex, \( E_v \), is approximately given by\(^223 \).

\[
E_v \approx \frac{l\Phi_0^2}{4\pi\mu_0 \lambda^2} \ln\left(\frac{\lambda}{\zeta}\right), \tag{3}
\]

where \( \lambda \) is the London penetration depth, and \( \zeta \) is the coherence length, both material and temperature dependent parameters. Using values for these constants from the literature\(^{23} \) and the probe hole geometry, it is possible to estimate the point at which creating a vortex is energetically favourable to threading an additional flux quantum through the loop. This occurs at \( n \approx 10^7 \), which corresponds to an average field in the probe holes of \( \sim 10^{-11} \text{T} \). Clearly, the flux through the probes holes does not play a large part in the observed behaviour.

For comparison and confirmation of these results, it was decided to measure another kind of superconducting cavity, a re-entrant cavity consisting of a cylinder of internal diameter 13mm and height 4mm, with a post of diameter 1mm and height 3.9mm. This cavity supports a fundamental reentrant mode at 5.62 GHz. A mount was machined for the PTFE post of the LiFe sensor, such that the sphere sat 2mm into the cavity. From a previous experiment a 1x6mm slot had been machined on the opposite side of the post. See fig. 5 for a drawing. Loop probes were inserted into the cavity along the axis perpendicular to this cross-section. In this case, the post mode was close in frequency to where the magnon mode should favour threading flux through the loop. This occurs at \( n \approx 10^7 \), which corresponds to an average field in the probe holes of \( \sim 10^{-11} \text{T} \). Clearly, the flux through the probes holes does not play a large part in the observed behaviour.

The results of measured internal against external field are shown in fig. 6, the first appearance of the magnon mode provides a consistent measurement of \( H_{sh}/\mu_0 = 0.272 \pm 0.001 \text{T} \). These results also show that the field measured inside the cavity becomes higher than the field outside, even on the ramp up. This can potentially be explained by the presence of the post inside the cavity. It too will begin acting as a diamagnet, due to some of the field being expelled from the post by the Meissner effect. With the sphere nearby to the post, this would enhance the local field measured by the sphere. Thus if the experiment were repeated, ramping up to larger magnetic fields, we would expect the increase in internal field to eventually return to the no screening case. It can also be noted that no plateaus were observed, although the usual hysteric behaviour due to trapped vortices was observed on the ramp down.

FIG. 5: Cross-section of the Nb re-entrant cavity.

FIG. 6: Re-entrant cavity internal magnetic field against external magnetic field for both the ramp up and down.
In summary, a new method of intracavity DC magnetic field sensing was tested on two Niobium cavities. The screening effects of these cavities were measured, demonstrating the transition from perfect diamagnetism, by the Meissner effect, to negligible screening effects. Both cavities measured a consistent transition field for DC magnetic field to enter the material, interpreted as the superconducting Meissner effect, to negligible screening effects. Both cavities demonstrated some observed plateaus just above the first critical field, the origin of which are unknown, however could be due to a transition in the structure of vortices in the material. The re-entrant cavity did not demonstrate the same plateaus, however did see an enhancement of field on the LiFe sphere over some range, most likely due to the post screening field. Possible future investigations include observing the decay of persistent fields by this method, measurement of the penetrating field as it varies spatially within the cavity, and the effect of cavity geometry including the use of thin films compared to the bulk case here.

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1 M. E. Tobar and D. G. Blair, IEEE Transactions on Microwave Theory and Techniques 42, 344 (1994).
2 D. Broemmelsiek, B. Chase, D. Edstrom, E. Harms, J. Leibfritz, S. Nagaitsev, Y. Pischalnikov, A. Romanov, J. Ruan, W. Schappert, V. Shiltsev, R. Thurman-Keup, and A. Vaishev, New Journal of Physics 20, 113018 (2018).
3 T. Mason, D. Abernathy, I. Anderson, J. Anknr, T. Egami, G. Ehlers, A. Elkebus, G. Granroth, M. Hagen, K. Herwig, J. Hodges, C. Hoffmann, C. Horak, L. Horton, F. Klose, J. Larese, A. Mesecar, D. Myles, J. Neufeind, M. Ohl, C. Tulk, X.-L. Wang, and J. Zhao, Physica B: Condensed Matter 385-386, 955 (2006).
4 F. Behnke, J. E. Brau, B. Foster, J. Fuster, M. Harrison, J. M. Paterson, M. Peskin, M. Stanitzki, N. Walker, and H. Yamamoto, (2013) see also http://www.linearcollider.org/ILC/TDR . The full list of contributing institutes is inside the Report, arXiv:1306.6327 %CITATION = arXiv : 1306.6327 ;%
5 O. S. Brning, P. Collier, P. Lebrun, S. Myers, R. Ostojc, J. Poole, and P. Proudlock, LHC Design Report CERN Yellow Reports: Monographs (CERN, Geneva, 2004).
6 R. Janish, V. Narayan, S. Rajendran, and P. Riggins, Phys. Rev. D 100, 015036 (2019).
7 F. Stern, A. A. Chisholk, J. Hoskins, P. Skivik, N. S. Sullivan, D. B. Tanner, G. Carosi, and K. van Bibber, Review of Scientific Instruments 86, 123305 (2015) http://dx.doi.org/10.1063/1.4938164.
8 D. Ahn, O. Kwon, W. Chung, W. Jang, D. Lee, J. Lee, S. W. Youn, D. Youn, and Y. K. Semertzidis, (2019), arXiv:1904.05111 [physics.ins-det].
9 M. Goryachev, S. Watt, J. Bourhill, M. Kostylev, and M. E. Tobar, Phys. Rev. B 97, 155129 (2018).
10 D. Lachance-Quirion, Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, and Y. Nakamura, Science Advances 6, 973107 (2020).
11 Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Science 349, 405 (2015).
12 Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Comptes Rendus Physique 17, 729 (2016) quantum microwaves / Micro-ondes quantiques.
13 R. Hisatomi, A. Osada, Y. Tabuchi, T. Ishikawa, A. Noguchi, R. Yamazaki, K. Usami, and Y. Nakamura, Phys. Rev. B 93, 174427 (2016).
14 J. Bourhill, N. Kostylev, M. Goryachev, D. L. Creedon, and M. E. Tobar, Phys. Rev. B 93, 144420 (2016).
15 M. Goryachev, W. G. Farr, D. L. Creedon, Y. Fan, M. Kostylev, and M. E. Tobar, Phys. Rev. Applied 2, 054002 (2014).
16 Y. Tabuchi, S. Ishino, K. Usami, and Y. Nakamura, Phys. Rev. Lett. 113, 083603 (2014).
17 G. Flower, J. Bourhill, M. Goryachev, and M. E. Tobar, Physics of the Dark Universe 25, 100306 (2019).
18 N. Crescini, D. Alesini, C. Braggio, G. Carugno, D. Di Gioacchino, C. S. Gallo, U. Gambardella, C. Gatti, G. Lannone, G. Lamanu, C. Ligi, A. Lombardi, A. Ortolan, S. Pagano, R. Perno, G. Ruso, C. C. Speake, and L. Taffarello, The European Physical Journal C 78, 704 (2018).
19 W. Meissner and R. Ochsenfeld, Naturwissenschaften 21, 78788 (1933).
20 J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 100, 014501 (1955).
21 A. Abrnikosov, Journal of Physics and Chemistry of Solids 2, 199 (1957).
22 X. B. Xu, H. Fenghui, S. Y. Ding, F. Zhou, X. N. Xu, Z. H. Wang, M. Gu, D. Q. Shi, and S. X. Dou, Phys. Rev. B 83, 014501 (2011).
23 X. B. Xu, H. Fenghui, S. Y. Ding, F. Zhou, X. N. Xu, Z. H. Wang, M. Gu, D. Q. Shi, and S. X. Dou, Phys. Rev. B 83, 014501 (2011).
24 X. B. Xu, H. Fenghui, S. Y. Ding, F. Zhou, X. N. Xu, Z. H. Wang, M. Gu, D. Q. Shi, and S. X. Dou, Phys. Rev. B 83, 014501 (2011).
25 X. B. Xu, H. Fenghui, S. Y. Ding, F. Zhou, X. N. Xu, Z. H. Wang, M. Gu, D. Q. Shi, and S. X. Dou, Phys. Rev. B 83, 014501 (2011).
26 A. A. Chisholk, J. Hoskins, P. Sikivie, N. S. Sullivan, D. B. Tanner, G. Carosi, and K. van Bibber, Review of Scientific Instruments 86, 123305 (2015) http://dx.doi.org/10.1063/1.4938164.
27 A. Romanenko, A. Grassellino, A. C. Crawford, D. Sergatovskiy, A. Martiello, O. S. Melnychuk, A. Romanenko, D. A. Sergatovskiy, and Y. Trenikhina, Journal of Applied Physics 119, 213903 (2016) https://doi.org/10.1063/1.4950807.
28 A. Romanenko, A. Grassellino, A. C. Crawford, D. A. Sergatovskiy, and O. Melnychuk, Applied Physics Letters 105, 234103 (2014) https://doi.org/10.1063/1.4909008.
29 S. Posen, N. Valles, and M. Liepe, Phys. Rev. Lett. 115, 047001 (2010).
30 S. Posen, N. Valles, and M. Liepe, Phys. Rev. Lett. 115, 047001 (2010).
31 M. Harder, L. Bai, C. Match, J. Sirker, and C. Hu, Science China Physics, Mechanics & Astronomy 59, 117511 (2016).
32 W. G. Farr, D. L. Creedon, M. Goryachev, K. Benmessal, and M. E. Tobar, Phys. Rev. B 88, 224426 (2013).
33 M. Goryachev, W. G. Farr, D. L. Creedon, and M. E. Tobar, Phys. Rev. B 89, 224407 (2014).
34 P. Dhakal, G. Cavoti, and A. Gurevich, Phys. Rev. Accel. Beams 23, 023102 (2020).
35 D. B. Liarte, S. Posen, M. K. Transtrum, G. Catelani, M. Liepe, and J. P. Sethna, Superconductor Science and Technology 30, 033002 (2017).
36 A. R. Pack, J. Carlson, S. Wadsworth, and M. K. Transtrum, Phys. Rev. B 101, 144504 (2020).
37 D. K. Christen, F. Tasset, S. Spooner, and H. A. Mook, Phys. Rev. B 15, 4506 (1977).
38 C. P. Poole, Handbook of Superconductivity (Academic Press, 2000).