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ABSTRACT
The interaction of an electromagnetic wave with a relativistic ionization front with frequency up-conversion has been demonstrated by the particle-in-cell (PIC) method. In the PIC simulation, the plasma ionization front is formed by using an electron beam ionizing the background gas. The PIC results are in good agreement with the basic analytic theory. In addition, the charged particles are modulated in the interaction area observed in the PIC simulation, which is hardly obtained by other methods. Based on the verified PIC methods, a relativistic hollow ionization front for frequency up-conversion of microwave to terahertz radiation is proposed for increasing the reflection cross section. Finally, the reflected energy can be increased by at least 3 orders of magnitude compared to the traditional methods.

I. INTRODUCTION
The reflection of the electromagnetic (em) wave from a moving boundary gives rise to the well-known relativistic Doppler shift, and both its frequency and its duration are altered. This mechanism may be exploited to transform the frequency spectrum of an incident pulse and provide tunable radiation from millimeter to ultraviolet wavelengths as well as subfemtosecond pulse lengths. The moving boundary can be realized with an electron beam, an optically excited silicon substrate, or electron density modulations in a wakefield. To achieve an appreciable frequency upshift factor, the moving boundary must propagate close to the light speed, c. Therefore, in most cases, it is more reliable and sufficient that the interface between the plasma and the surrounding medium is moving, producing an ionization front that can be generated by a gas ionization by an intense optical pulse. Developments in laser technology have enabled the generation of relativistic ionization fronts by intense short-pulse lasers via photoionization and tunnel ionization. The ionization front has no kinetic energy; therefore, the number of photons is not conserved upon reflection from the front and the energy in the reflected pulse may be much less than the incident pulse energy. In addition, other methods for frequency upshift about the interaction between an em wave and plasma have been studied by using photon acceleration and time-varying medium properties. The reflective Doppler upshift scheme with an overdense plasma front possesses a favorable compromise between the upshift factor and energy loss, with a straightforward experimental geometry.

In the past exploration of the ionization front, different finite-difference time-domain (FDTD) methods and only proof-of-principle experiments focusing on underdense ionization fronts have been mainly reported. In order to achieve an appreciable reflectivity, the velocity of the ionization front must be close to the speed of light and the area of the ionization front (or the reflection cross section) must be large enough. The former determines the frequency conversion, and the latter determines the energy of the reflected pulse. However, the laser spot is very small, and it causes a small reflection cross section. The corresponding reflected energy will be very low. This mainly limits the development of this technology as a source of tunable radiation. In this paper, we propose a hollow plasma front for the purpose of improving the reflection cross section to achieve the higher...
reflected pulse energy, which is verified by the particle-in-cell (PIC) simulation.

Compared with FDTD methods where the time-varying plasma is simplified as a dielectric medium (DM), the PIC method considering the interaction and motion of charged particles would be much closer to the experimental values. It also has advantages of simulating complex structures with PIC methods. At present, it seldom studies the gas ionization by an intense optical pulse from the first principle, while an equivalent PIC model for the impact ionization of the background gas to generate a plasma front with a relativistic electron beam is proposed. This model is sufficient because the core idea of this paper is to simulate a hollow plasma front. Therefore, the complex ionization process is not considered in this manuscript.

The organization of this manuscript is as follows: First, we present a brief account of the basic analytic theory describing the interaction of an em wave with a counterpropagating ionization front. After giving some details of the PIC simulations, we proceed to study and confirm the numerical results for the simplified case described by analytic theory. Then, a hollow plasma front is simulated and how to improve the reflection cross section is explained. Finally, frequency up-conversion of microwave to terahertz radiation by a relativistic hollow ionization front is understood that only increasing the area of the plasma boundary can increase the reflected pulse energy, which is verified by the particle-in-cell (PIC) simulation.

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II. FORMULATION OF THE PROBLEM

A rigorous analytic theory on the Doppler reflection of plane em waves by a propagating ionization front was given in Refs. 1 and 11. The schematic diagram is shown in Fig. 1. A uniform plane em wave with frequency \( \omega_i \) propagates in the positive z-direction with the velocity of light, \( c = c \hat{z} \). Meanwhile, an infinite plane plasma interface formed by an ionization source moves at constant velocity \( U = -U \hat{z} \). It is assumed that the plasma is lossless (collisionless) and the plasma density does not decay as the front propagates. In the reference frame of the moving plasma boundary, the spatial plasma profile is stationary and constant. According to the Doppler effect, the frequency of the input wave transforms as \( \omega' = \sqrt{\Gamma} \omega_i \), where \( \Gamma = \frac{1 + \beta^2}{1 - \beta^2} \), in terms of the velocity ratio \( \beta = U/c \). The input wave will reflect from a stationary plasma, and the reflected frequency meets \( \omega' = \omega_i' \). Transformation back to the laboratory frame yields \( \omega_r = \sqrt{\Gamma} \omega_i = \Gamma \omega_i \). Therefore, the reflected wave is upshifted relative to the input wave by the factor \( \Gamma \). The Maxwell curl equations and the equation of motion for the plasma electrons in the moving reference frame predict a special mode, which exists in the incident wave and the moving plasma boundary interaction area. By solving the boundary value problem of em wave, the special mode is a static magnetic field, which does not change with time but with space.

The coefficient for the reflected field amplitude, after transformation back to the laboratory frame, is given by

\[
R_F = \frac{E_r}{E_i} = \frac{\Gamma E'_r}{\Gamma E_i} = \frac{1 - n_i^2}{1 + n_i^2},
\]

where \( n_i = n \sqrt{1 - \omega_i^2/\omega_{cr}^2} \) corresponds to the refractive index of the plasma at the frequency \( \omega_i' \). \( \omega_{cr} = \sqrt{\omega_e^2/(\varepsilon_0 m_e)} \) is the plasma frequency, \( N_p \) is the plasma density, and \( m_e \) is the mass of the electron. Hence, \( R_F \) is mathematically equal to the Fresnel reflection coefficient of a stationary plasma at the frequency \( \omega_i' \). We define an effective critical plasma frequency as

\[
\omega_{cr} = \frac{\omega_{cr}}{\sqrt{\Gamma}},
\]

and it is obtained by

\[
\frac{n_i^2(\omega_i)}{n} = \frac{\Gamma}{\Gamma^2} \left( \frac{\omega_i}{\omega_{cr}} \right)^2 - 1, \quad \omega_i < \omega_{cr},
\]

\[
\frac{\Gamma}{\Gamma^2} \left( \frac{\omega_i}{\omega_{cr}} \right)^2, \quad \omega_i > \omega_{cr}.
\]

Here, the plasma is overdense (\( \omega_i < \omega_{cr} \)) or underdense (\( \omega_i > \omega_{cr} \)), respectively.

For an input pulse with energy \( J_i \), the coefficient for the reflected pulse energy \( J_r \) in the laboratory frame is given by

\[
\frac{J_r}{J_i} = \frac{1}{\Gamma^2} |R_F|^2,
\]

as can be understood due to the temporal compression of the pulse by a factor \( \Gamma \) (the number of oscillation periods in the pulse is preserved in the reflected pulse). Hence, the maximal energy reflection factor is \( R_F = 1/\Gamma \) for the overdense case, and an inherent compromise exists between the upshift factor and energy efficiency. The only way to increase the reflected pulse energy \( J_r \) is to increase the incident energy \( J_i \). The incident energy is mainly determined by the reflection cross section and the energy density of the incident wave. It is well understood that only increasing the area of the plasma boundary can reflect the more incident energy.

III. PIC SIMULATION AND RESULTS DISCUSSION

In this paper, the PIC simulation with the code of UNIPIC in the rectangular coordinate system is carried out to model the Doppler-reflection process. A 2D PIC model is shown in Fig. 1. The wave port is in the left and a uniform plane Gaussian pulse with the frequency of 2.45 GHz propagates in the parallel plate waveguide, which is a readily available microwave source and easily obtained.
FIG. 2. (a) The waveform of the Ey-field (black line) and the corresponding spectrogram (red line). (b) The spatial distribution of electrons on the YOZ plane and the Bx-field on the black dotted line. Parameters are \( N_p = 1 \times 10^{18} \text{ m}^{-3} \), \( U \approx v_e = 2.41 \times 10^8 \text{ m/s} \), and \( \omega_i/2\pi = 2.45 \text{ GHz} \).

with ultrahigh power. An electrode is placed in the right and a short current pulse of electrons at the voltage of \( V_F \) is emitted from its surface into the chamber filled with oxygen. We enable the impact ionization model to create the plasma in the room temperature gas having a pressure of 10 mTorr. The normal impact cross section for oxygen is superseded by a flat constant response. Without considering the energy loss of particle collision, the plasma density \( N_p \) and the plasma interface moving velocity \( U \) are only determined by the electron emission current \( I_F \) and voltage \( V_F \), respectively. The electron current determines the number of electrons emitted per unit time and per unit area, so it meets \( I_F \propto N_p \). While the electron emission voltage determines the electron emission speed \( v_e \) and it is obtained by the relativistic energy relationship,

\[
e V_F = mc^2 \left( \gamma_e - 1 \right)
\]

\( \gamma_e \) is the Lorentz factor, given by

\[
\gamma_e = \left( 1 - \left( v_e/c \right)^2 \right)^{-1/2}.
\]

Since only the elastic collisions are considered, the electron emission speed \( v_e \) is approximately equal to the plasma interface moving velocity \( U \), and they remain basically the same throughout the process. By establishing the PIC model, the theoretical results of the ionization front are verified and the PIC results are shown in Fig. 2. Figure 2(a) shows the waveform of the Ey-field (black line) and the corresponding spectrogram (red line) detected in the wave port. The Gaussian pulse with the frequency of 2.45 GHz corresponds to the incident wave, and the compressed Gaussian pulse with the center frequency of 22.73 GHz is the reflected wave by the ionization front. Figure 2(b) shows the spatial distribution of electrons on the YOZ plane and the Bx-field on the dotted line. This purely magnetic wave is the special mode, and it is of zero frequency but varies in space. In the presence of such a static magnetic field in the z-direction, electrons are modulated in this region and it produces a stationary current density that is established in the plasma. As for the spatial wavelength of the special mode, it is equal to \( 2\pi c/\omega_c \). The factor \( \Gamma \) for the up-conversion frequency and the normalized reflected field amplitude \( R_F \) are also verified with the PIC method, as shown in Fig. 3.

As mentioned above, a significant upshift \( \Gamma \) is obtained for \( U \approx c, \beta \approx 1 \). Therefore, the ionization front is often created by an ultrashort laser pulse. As is well known, the reflection cross section of the ionization front created by the laser is almost on the order of a few square millimeters. It causes the energy of the reflected wave with the up-conversion frequency to be almost small.

IV. HOLLOW IONIZATION FRONT

To obtain a higher reflection cross section, a hollow ionization front is proposed. The 3D schematic diagram with the PIC model
is shown in Fig. 4. At the frequency band of $o_i < o_{tr}$, the refractive index of the plasma for the input pulse is pure, and the input pulse will be totally reflected. The overdense plasma behaves like a good conductor (GC), and the hollow plasma can be regarded as a metal circular waveguide. In a circular waveguide, the input pulse will be totally reflected when the input pulse frequency is less than the cutoff frequency of the metal circular waveguide. If the hollow plasma also totally reflected when the input pulse frequency is less than the cutoff frequency of the metal circular waveguide. In a circular waveguide, the input pulse will be totally reflected. The overdense plasma behaves like a good conductor with the conductivity $\sigma$ good conductor with the conductivity $\sigma$ lower than that of the plasma. For example, it applies to the relation between the plasma with $o_p = 1.01o_i$ and the GC with $\sigma = 0.1$ S/m and also for $o_p = 0.579o_i$ and $\sigma = 10$ S/m. This is because at this plasma frequency range, the propagation constant of the incident wave in the plasma is a pure imaginary. However, the propagation constant in the good conductor exists in the real and imaginary parts. There will be more transmission in the good conductor, and the reflection coefficient of GD is lower than that of the plasma.

In addition, at the frequency range of $o_p < o_i$, the plasma is a dielectric medium and the relative permittivity is given by $\varepsilon_r = 1 - \frac{o_i^2}{o_p^2}$. Therefore, the relative permittivity of the plasma with $o_p = 0.12o_i$ ($N_p = 1 \times 10^{15}$ m$^{-3}$) and $o_p = 0.58o_i$ ($N_p = 2.5 \times 10^{16}$ m$^{-3}$) are $\varepsilon_{p1} = 0.99$ and $\varepsilon_{p2} = 0.66$, respectively. The results of the dielectric medium (DM) with $\epsilon_r = 1.51$ are basically the same as those of the plasma with $\varepsilon_{p2} = 0.66$. This is because $\varepsilon_{p2} \approx 1$, and the difference between the Fresnel reflection coefficient of the dielectric medium and the plasma is a negative sign, given by $\frac{1}{\sqrt{\varepsilon_r}}$, which only represents that the corresponding phase shift for the two is $180^\circ$, $r_0$ and the values are equal. Therefore, this conclusion also applies to a DM with $\varepsilon_{r1} = 1.01$ and the plasma with $\varepsilon_{p1} = 0.99$.

V. THz RADIATION BY HOLLOW IONIZATION FRONT

Based on the above theoretical analysis, we establish a 3D PIC mode to simulate a hollow ionization front to achieve a terahertz radiation. In order to meet the experimental conditions, we reasonably assume some preconditions: (1) The incident wave frequency is 2.45 GHz, and the reflected pulse frequency is 0.1 THz. So the plasma interface moving velocity $U = 2.86 \times 10^8$ m/s is modulated in the PIC simulation. (2) A typical plasma density in a laser induced filament in air is between $10^{15}$ and $10^{16}$ cm$^{-3}$, resulting in a plasma frequency $o_p > 300$ GHz. The filament thickness is almost in the order of 1 mm. On this basis, the plasma frequency is set to 300 GHz and the hollow plasma is overdense for the incident wave. The hollow
plasma thickness $R_0 - r_0$ is set to a constant amount, equal to 1 mm. It is meant to characterize that the laser or electron beam energy remains essentially the same.

The reflected waveforms of the Ey-field (a) and the corresponding spectrograms (b) with different inner radii $r_0$ are detected in the wave port, as shown in Fig. 6. The change in the inner radius causes a change in the reflection cross section, which affects the reflected field amplitude and energy. The center frequency of the reflected pulse varies from approximately 0.09 to 0.11 THz, which is basically in line with the theoretical relationship. Figure 7 shows the normalized peak field of the reflected pulse with different inner radii $r_0$ with the PIC simulation. In the reference frame of the moving plasma boundary, the frequency of the input wave transforms as $\omega'_i = \sqrt{\Gamma} \omega_i = 15.65$ GHz, and the corresponding cutoff radius is about $a'_F \approx 5.6$ mm. Therefore, the maximum peak field of the reflected pulse appears in the case of $r_0 = 5$ mm. At the situation of $r_0 < 5$ mm, the incident pulse will be totally reflected by the hollow ionization front and the reflection cross section increases as the radius increases. In addition, at the situation of $r_0 > 5$ mm, the incident pulse will be partially reflected and partially transmitted. Although the cross section of the ionization front increases with the increase in $r_0$, some waves will be transmitted into the hollow ionization front and the effective reflection cross section is actually reduced. The increase in the radius $r_0$ will cause the reduction of the laser power density (LPD) in the experiment, as shown in Fig. 7. In other words, the actual area of ionization $S = \pi (R_0^2 - r_0^2)$ increases. At $r_0 = 5$ mm, the laser power density drops by an order of magnitude compared to that of a solid laser beam ($r_0 = 0$ and $R_0 = 1$ mm). With the development of femtosecond lasers, even if the laser power density is reduced by an order of magnitude, it is also easy to generate a hollow plasma with the plasma frequency greater than 15.65 GHz. Even if the laser power density is reduced, we can still maintain the overdense plasma.

Compared to the traditional ionization front produced by a solid laser beam, the reflected pulse amplitude is increased by approximately 50 times and the energy is increased by approximately 2500 times by the hollow ionization front.

VI. CONCLUSION

In this paper, the frequency up-conversion of microwave to terahertz radiation by a relativistic hollow ionization front is demonstrated with the PIC simulation. It is a new method for a terahertz radiation source. The PIC model using the impact ionization method to generate the ionization front with an electron beam is proven to be feasible, and it can be used for the deeper mechanism exploration about the interaction between EM waves with the ionization front. In addition, a hollow ionization front is recommended in the experiments for improving the reflection cross section. According to the assumptions of related parameters in this paper, the reflected pulse energy can be increased by about 2500 times. The next work is to find a way to generate a hollow ionization front in the experiment. According to our preliminary investigation, the axicon lens may be a good choice.

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