The Odd-Parity Galaxy Bispectrum

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(Dated: June 13, 2019)

The galaxy bispectrum contains a wealth of information about the early universe, gravity, as well as astrophysics such as galaxy bias. In this paper, we study the parity-odd part of the galaxy bispectrum which is hitherto unexplored. In the standard cosmological model, the odd-parity bispectrum is generated by galaxy velocities through redshift-space distortions. While small in the case of General Relativity coupled with smooth dark energy, the signal could be larger in modified gravity scenarios. Thus, apart from being a very useful consistency test of measurements of galaxy clustering, the odd bispectrum offers a novel avenue for searching for new physics.

INTRODUCTION — In the standard cosmological model, the \(n\)-point correlation functions of the matter distribution in the Universe are invariant under the parity transformation, or have even parity. This is because the formation and evolution of large-scale structure is mainly driven by scalar perturbations. We can, therefore, pursue the signatures of physics beyond the standard cosmological model by searching for odd-parity correlation functions, as suggested by \cite{1} in the context of the cosmic microwave background (CMB) bispectrum.

In this paper, we explore a novel method of measuring the parity-violating (odd-parity) part of the correlation functions in galaxy clustering. Unlike the case for the CMB where, apart from instrumental systematics, only parity-violating new physics can generate odd-parity correlation \cite{1}, the correlation functions of galaxies possess odd-parity parts even in the absence of parity-violating new physics. It is therefore important to quantify their amplitude in order to use the odd-parity correlation functions as probes of new physics. Moreover, we will see that this signal contains valuable cosmological information even without parity-violating new physics.

Ref. \cite{1} argued that there is no odd-parity bispectrum in three dimensions. However, this only holds in the galaxy rest frame, i.e. if there are no preferred directions; for observed galaxy statistics, the line of sight provides a preferred direction, as the observed redshift is given by \(z \equiv \bar{z}(\chi) + v_{\parallel}\), where \(v_{\parallel}\) is the galaxy’s radial velocity.\textsuperscript{1} Thus, there is a non-vanishing parity-odd galaxy bispectrum.

IMAGINARY PART OF GALAXY CORRELATION FUNCTIONS — The reality condition for the galaxy density field is translated into its Fourier representation as \(\delta(-\bm{k}) = \delta^*(\bm{k})\). For the three-dimensional power spectrum, which is defined as

\[
P(\bm{k}_1) \equiv \langle \delta(\bm{k}_1) \delta(\bm{k}_2) \rangle' = \langle \delta(\bm{k}_1) \delta(-\bm{k}_1) \rangle' = \langle |\delta(\bm{k}_1)|^2 \rangle',
\]

the reality condition implies that the auto power spectrum is a positive-definite real quantity and parity-even:

\[
P(-\bm{k}) = P(\bm{k}).
\]

For the cross power spectrum of two tracers 1 and 2, on the other hand,

\[
P_{12}(\bm{k}_1) \equiv \langle \delta_1(\bm{k}_1) \delta_2(\bm{k}_2) \rangle' = \langle \delta_1(\bm{k}_1) \delta_2(-\bm{k}_1) \rangle',
\]

the reality condition only implies \(P_{12}(-\bm{k}) = P_{12}^*(\bm{k})\), so that one can decompose the cross power spectrum into parity-odd and -even parts as

\[
P_{12}^+(\bm{k}) = \frac{1}{2} [P_{12}(\bm{k}) + P_{12}(-\bm{k})] = \text{Re}[P_{12}(\bm{k})]
\]

\[
P_{12}^-(\bm{k}) = \frac{1}{2i} [P_{12}(\bm{k}) - P_{12}(-\bm{k})] = \text{Im}[P_{12}(\bm{k})].
\]

This means that the imaginary part of the cross power spectrum probes the parity-odd part of the clustering \cite{3,4}.

Similarly, the reality condition for the bispectrum leads to \(B_g(-\bm{k}_1,-\bm{k}_2,-\bm{k}_3) = B_g^*(\bm{k}_1,\bm{k}_2,\bm{k}_3)\), which allows us to decompose the bispectrum into even-parity and odd-parity pieces:

\[
B_g(\bm{k}_1,\bm{k}_2,\bm{k}_3) = B_g^+(\bm{k}_1,\bm{k}_2,\bm{k}_3) + iB_g^-(\bm{k}_1,\bm{k}_2,\bm{k}_3)
\]

where the parity-even part is real, while the parity-odd part \(iB_g^-\) is imaginary.

We see that, among the auto-correlations of any tracer, the bispectrum is the lowest-order statistic that is sensitive to parity. Including the imaginary, parity-odd part of the bispectrum, which has hitherto been unexplored, in observational analyses means that we double the number of observables. Thus, this is an observable in search of a signal.

COSMOLOGICAL SIGNAL IN THE ODD-PARITY BISPECTRUM — Due to the peculiar velocity contribution to the observed redshift, the observed galaxy density contrast is distorted from the one in the galaxy rest-frame, an effect called redshift-space distortion \cite{5}. The galaxy density contrast in redshift space (defined with the radial distance from the observed redshift \(z\) up to second order in perturbations may be written as

\[
\delta_g = b_1 \delta + b_{v_{\parallel}} \left[v_{\parallel} (1 + b_1 \delta) + \frac{1}{2} b_2 [\delta^2] + b_K [K^2] \right] - \frac{1}{H} \partial_{\parallel} \left[v_{\parallel} (1 + b_1 \delta) + \frac{1}{2H^2} \partial_{\parallel}^2 v_{\parallel}^2 \right],
\]

\textsuperscript{1} We will adopt the notation of [2] throughout.
where \( b_1, b_2, b_{K^2} \) are the well-known LIMD (local in matter density) and tidal bias parameters. Note that in addition to the usual redshift-space density contrast expression, for example in \([6]\), we include the term proportional to \( v_{||} \), the peculiar velocity along the line-of-sight direction. This term is frequently dropped, as it is suppressed on small scales compared to terms involving derivatives of the velocity. However, as we will see, the term proportional to \( v_{||} \) in Eq. \((3)\) gives rise to the odd-parity bispectrum.

The coefficient \( b_{v_{||}} \) can be derived from the linear-order general relativistic treatment of galaxy clustering (e.g., \([7\,\text{and}\,12]\)), and is given by

\[
b_{v_{||}} = b_e - 1 - 2Q + \frac{1}{4\mathcal{H}} \frac{\partial}{\partial z} + 2(Q - 1) \frac{1}{\mathcal{H} \chi}
= \frac{d \ln (a^3 \bar{n}_g \chi^2 / \mathcal{H})}{d \ln a} - Q \frac{d \ln (a \chi)^2}{d \ln a}.
\]

Here, \( \bar{n}_g \) is the comoving number density of galaxies and \( b_e \equiv d \ln (a^3 \bar{n}_g) / d \ln a \), \( \chi \) is the comoving radial distance to the galaxies, and \( Q \equiv d \ln \bar{n}_g / d \ln M \) parameterizes the change of the observed density contrast due to gravitational lensing (\( M \) stands for the magnification). For a magnitude-limited sample, with the cumulative luminosity function \( \bar{n}_g(\geq L_{\text{min}}) \), \( Q = -d \ln \bar{n}_g(\geq L_{\text{min}}) / d \ln L_{\text{min}} \) is given by the slope of the cumulative luminosity function at the limiting luminosity. All quantities in Eqs. \((3)\)–\((4)\) are defined at the observed redshift. Note that the two terms in Eq. \((4)\) have a straightforward interpretation: the first quantifies the evolution of the mean physical number density of galaxies, while the second does the same for the angular diameter distance squared. Added together, these two contributions in \( b_{v_{||}} \) quantify the fractional change of the galaxy number density due to the change in redshift \( \delta z = v_{||} \) (Fig. 1).

It is worth noting that this simple and physically clear expression for \( b_{v_{||}} \) is only obtained once a proper relativistic calculation is done, which in particular includes terms \( \propto \partial_z \Phi \). This latter term is proportional to \( v_{||} \), and hence equally relevant at this order, but has been dropped in quasi-Newtonian calculations of redshift-space distortions such as that in \([\text{6}]\) (and many followup papers), leading to a coefficient that differs from, and is not as simple as, Eq. \((4)\).

The leading odd-parity bispectrum comes from the three-point correlations involving one power of \( b_{v_{||}} v_{||} \). It is in fact equivalent to the bispectrum dipole recently derived in \([13]\) using a full second-order relativistic formalism for galaxy clustering. Here, we include the terms that dominate the signal-to-noise and correspond to those with the highest number of spatial derivatives. Correspondingly, in Eq. \((3)\) we have neglected other relativistic contributions proportional to the gravitational potential, Sachs-Wolfe effect, integrated Sachs-Wolfe effect and Shapiro time delay, because their contributions are much smaller than those considered here.

In perturbation theory, the line-of-sight velocity field and the density contrast are, to second order, given by

\[
\frac{v_{||}(k)}{Hf} = \frac{ik}{k^2} \left[ \delta_L(k) + \int q G_2(q, k - q) \delta_L(q) \delta_L(k - q) \right],
\]

\[
\delta(k) = \delta_L(k) + \int q F_2(q, k - q) \delta_L(q) \delta_L(k - q),
\]

with the second-order velocity and density kernels

\[
F_2(k_1, k_2) = \frac{5}{7} + \frac{2}{7} \mu_1^2 + \frac{1}{2} \mu_2 \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right),
\]

\[
G_2(k_1, k_2) = \frac{3}{7} + \frac{4}{7} \mu_1 + \frac{1}{2} \mu_2 \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right),
\]

and the linear density contrast field \( \delta_L(k) \). Here, we define \( \mu_{ij} \) as the cosine of the angle between two Fourier vectors \( k_i \) and \( k_j \): \( \mu_{ij} = k_i \cdot k_j / (k_i k_j) \). In Fourier space, the tidal field is related to the density contrast by \( K_{ij}(k) = (\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij}) \delta(k) \).

The leading-order expression for the odd-parity bispectrum is then,
Fourier-space vector or flattened) triangles. Gaussianities, it roughly peaks for degenerate (elongated, isosceles) configurations. Although the angle average complicates the details of the shape-dependence, the basic scale-dependence is very similar to what is generated in case of the folded-type primordial non-Gaussianity.

\[
\frac{B_g^- (k_1, k_2, k_3)}{b_{||} f_{H}} = 2 \left\{ b_1 F_2(k_1, k_2) + f \mu_3^2 G_2(k_1, k_2) + \frac{b_2}{2} + b_{K^2} \left( \mu_1^2 - \frac{1}{3} \right) - b_1 \frac{f k_3 \parallel}{2} \left( \mu_1^2 \frac{k_1}{k_2} + \frac{f^2 k_3 \parallel}{2} \frac{\mu_1 \mu_2}{k_1 k_2} \right) \right. \\
\times \left. \left[ (b_1 + f \mu_3^2) \frac{\mu_2}{k_2} + (b_1 + f \mu_3^2) \frac{\mu_1}{k_1} \right] \right. \\
+ \left. \left( b_1 + f \mu_3^2 \right) \left( b_1 + f \mu_3^2 \right) \left[ \mu_3^2 G_2(k_1, k_2) - \frac{b_1}{2} \left( \frac{\mu_1}{k_1} + \frac{\mu_2}{k_2} \right) \right] \right\} P_l(k_1) P_l(k_2) + (2 \text{ cyclic perm.}) ,
\] (6)

with \( \mu_i \equiv \hat{n} \cdot k_i / k_i \) the cosine of the angle between the Fourier-space vector \( k_i \) and the line-of-sight direction \( \hat{n} \). Note that for an order-unity parameter \( b_{||} \), all contributions to the odd-parity bispectrum are suppressed compared to the leading parity-even part of the bispectrum by a factor of \((H/k_i)\). The scale-dependence of the odd-parity bispectrum is, therefore, very similar to the case for the “folded” primordial bispectrum sourced by initial-state modifications \([14–16]\). Fig. 2 shows the configuration dependence of the dipole of the odd-parity bispectrum at tree level. Like the case for the folded-type non-Gaussianities, it roughly peaks for degenerate (elongated, or flattened) triangles.

As Eq. (6) is proportional to \( \mu_i \), the odd-parity bispectrum vanishes when \( k_i \perp \hat{n} \) for all three Fourier-space vectors. This happens when \( k_i \) are on the plane perpendicular to the line-of-sight direction. Notice further that the odd-parity bispectrum is free of shot noise, and the shot noise only contributes to the covariance matrix.

**Covariance matrix** — At leading order in perturbation theory, the parity-odd part of the bispectrum is Gaussian distributed with a diagonal covariance matrix given by

\[
\sigma^2 [B_g^- (k_1, k_2, k_3)] = s_B \frac{V_{\text{survey}}}{2 N_t} P_g(k_1) P_g(k_2) P_g(k_3),
\] (7)

where \( s_B \) is the symmetry factor (\( s_B = 6 \) for equilateral configuration, \( s_B = 2 \) for isosceles configurations, and \( s_B = 1 \) for all others), and \( N_t^- \) is the number of triangles contributing to the estimation of \( B_g^- \) within the angular bin \( d\mu d\phi \):

\[
N_t^-(k_1, k_2, k_3, \mu, \phi) d\mu d\phi \simeq d\mu d\phi \left( \frac{3}{\pi} \frac{k_1 \Delta k_i}{k_{F_i}} \right) \times \left\{ \begin{array}{ll} \pi/2, & k_i = k_j + k_k \\ \pi, & \text{otherwise} \end{array} \right.
\] (8)

Here, \( \Delta k_i \) and \( k_{F_i} = (2\pi)/L_i \) are, respectively, the width of the wavenumber bin and the fundamental wavenumber in the \( i \)-th direction. We parameterize the angles between the line-of-sight direction and the plane of the three wavevectors (the Fourier-space plane embedding \( k_1, k_2, k_3 \)) by two parameters: \( \mu \), the cosine of the angle between the line-of-sight direction and the plane, and \( \phi \), the angle between the \( k_3 \) vector and the line-of-sight direction projected onto the plane. In this parametrization,
the parallel components of all three vectors are given as
\( k_{i\parallel} = k_i \mu \cos \phi \), \( k_{i\parallel} = -k_\mu \cos (\alpha + \phi) \), \( k_{i\parallel} = -k_\mu \), with \( \cos \alpha \equiv -\mu_2 \) being the cosine of the inner angle of the Fourier-space triangle.

Note that we choose the full angular range \( (\mu \in [-1, 1] \text{ and } \phi \in [0, 2\pi]) \) and take half of the available triangular configurations in each angular configuration. When integrating over the angular configurations \( (\mu \text{ and } \phi) \), \( \Delta b_{v||} \) is expected to yield \( \Delta b_{v||} \) variance-limited galaxy survey in the range \( 1 \lesssim |\kappa| \lesssim 40 \) Mpc

When using the range of \( k_{i\parallel} \) in \( \left| k_{i\parallel} \right| \lesssim 40 \) h Mpc\(^{-1} \), we have studied the corresponding signal-to-noise ratio of detecting the odd-parity bispectrum for a concordance \( \Lambda \)CDM cosmology will be at most of order unity in any realistic case, unless the galaxy number density or luminosity function depend extremely sharply on redshift.

This conclusion, however, relies on the fiducial \( \Lambda \)CDM model. In case gravity is modified, one expects an increase in velocities due to enhanced gravitational forces. Moreover, the effects of modified gravity can in general be scale-dependent if the additional degree of freedom has a finite mass (as is the case, for example, in \( f(R) \) gravity and symmetron scenarios). As a toy model, let us consider the case where velocities are enhanced by

\[
\mathbf{v}(\mathbf{k}) = (1 + R^2 \mathbf{k}^2) \mathbf{v}_{\Lambda \text{CDM}}(\mathbf{k}).
\]

Here, \( R_v \) corresponds to the length scale (inverse mass, or Compton length) associated with the additional degree of freedom. We find that using the odd-parity bispectrum, Euclid and DESI will be able to constrain \( b_v R_v^2 \lesssim 150 \, h^{-2}\text{Mpc}^2 \), while the above-mentioned full-sky cosmic-variance-limited survey could achieve \( b_v R_v^2 \lesssim 40 \, h^{-2}\text{Mpc}^2 \). The galaxy power spectrum is in general expected to yield a tighter constraint on such a scale-dependent modification of gravity; however, the odd-parity bispectrum is a much cleaner probe, as it can only be sourced by velocities which are unbiased on large scales (unlike the galaxy density, and even RSD terms such as \( \partial_{\mu} v|| \), which can be biased through selection effects). Thus, the odd-parity bispectrum could be used as a rigorous cross-check of any signs of new physics found in the galaxy power spectrum.

CONCLUSION — In this paper, we have studied the imaginary part of the galaxy bispectrum that is the lowest-order correlation function sensitive to the odd-parity component in the galaxy density field. In \( \Lambda \)CDM cosmological models without parity-violating primordial perturbations, only galaxy velocities generate the odd-parity bispectrum through redshift-space distortions.

A related complementary approach is to correlate galaxies with an observable that is itself parity-odd like the velocity \( v_\parallel \). One such observable is the kinetic Sunyaev-Zel’dovich (kSZ) effect, which is proportional to the line-of-sight momentum in ionized gas. The resulting odd-parity galaxy-galaxy-kSZ bispectrum was recently explored by [17]. Here, we have studied the observational prospects for this guaranteed signal in the odd galaxy bispectrum and showed that the corresponding signal-to-noise ratio is at most of order unity even for idealistic (cosmic-variance limited, full-sky) galaxy surveys, which is smaller than that reported for the kSZ signal in [17]. This pessimistic outlook changes however if modifications of gravity affect galaxy velocities significantly. While the detection prospects for such effects are generally higher in lower-order statistics such as the galaxy power spectrum, the odd galaxy bispectrum is an exceptionally clean probe since it can only be sourced by parity-odd terms such as velocities. Thus, the odd-parity galaxy bispectrum clearly deserves further attention.

ACKNOWLEDGMENTS — DJ acknowledges support from National Science Foundation grant (AST-1517363) and NASA ATP program (80NSSC18K1103). FS acknowledges support from the Starting Grant (ERC-2015-STG 678652) “GrInflaGal” of the European Research
Council.

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