Exploring the Design Space of Static and Incremental Graph Connectivity Algorithms on GPUs

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ABSTRACT

Connected components and spanning forest are fundamental graph algorithms due to their use in many important applications, such as graph clustering and image segmentation. GPUs are an ideal platform for graph algorithms due to their high peak performance and memory bandwidth. While there exist several GPU connectivity algorithms in the literature, many design choices have not yet been explored. In this paper, we explore various design choices in GPU connectivity algorithms, including sampling, linking, and tree compression, for both the static as well as the incremental setting. Our various design choices lead to over 300 new GPU implementations of connectivity, many of which outperform state-of-the-art. We present an experimental evaluation, and show that we achieve an average speedup of 2.47x speedup over existing static algorithms. In the incremental setting, we achieve a throughput of up to 48.23 billion edges per second. Compared to state-of-the-art GPU implementations on a 72-core machine, we achieve a speedup of 8.26–14.51x for static connectivity and 1.85–13.36x for incremental connectivity using a Tesla V100 GPU.

CCS CONCEPTS

• Theory of computation → Parallel algorithms.

KEYWORDS

Connected components, Graph algorithms, GPU algorithms, Spanning forest

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1 INTRODUCTION

Connected components (connectivity) is a fundamental graph problem that plays a critical role in many graph applications. Given an undirected graph with \( n \) vertices and \( m \) edges, the problem assigns each vertex a label such that vertices that are reachable from each other have the same label, and otherwise have different labels [25]. Connectivity algorithms are used in many applications, such as computer vision [31, 43], VLSI design [61], and social analysis [41]. Graph connectivity is also a key subroutine to solve other graph algorithms, such as biconnectivity [86] and clustering [33, 72, 90, 91], and some of these algorithms require many calls to graph connectivity. As such, there has been a large amount of work on efficient parallel connectivity algorithms [2–7, 11, 12, 14, 15, 17, 20–23, 27, 34, 36–40, 42, 46–48, 50, 55–59, 62, 63, 67, 68, 73–75, 77–79, 81–83, 85, 87].

Graphics processing units (GPUs) are attractive devices for performing graph computations because of their high computing power and memory bandwidth. However, achieving high performance using GPUs is challenging due to several factors, including uncoalesced memory access, insufficient parallelism to tolerate high memory latency, load imbalance, and thread divergence. Several GPU connectivity implementations have been proposed in the literature [13, 24, 49, 65, 71, 85, 88], but we found that there are many algorithmic choices and optimizations that have not been thoroughly explored for GPU connectivity. The goal of this paper is to explore this large design space to better understand how different choices affect performance.

In this paper, we study min-based connectivity algorithms [52, 53, 62, 73, 78, 83], which are based on vertices propagating labels to other connected vertices, and keeping the minimum label received. At convergence, vertices will have the same label if and only if they are in the same connected component. A large number of our algorithms are based on using a union-find data structure for maintaining disjoint sets. The data structure maintains a tree for each sub-component found so far, and joins trees to merge sub-components that are connected. We also study several other min-based algorithms that maintain trees, but not using a disjoint set data structure. All existing GPU connectivity implementations are min-based; however, there are various design choices that have not been explored, such as different rules for searching and compressing union-find trees and for propagating labels. Furthermore, these algorithms can be improved by using sampling as a preprocessing step to remove vertices in a large component from consideration, so that the remaining steps are more efficient. Sutton et al. [85] provide one instantiation of sampling combined with a particular union-find algorithm. Inspired by their work, we explore different sampling strategies in this paper, and combine them with each of our algorithms to sweep the search space.

To implement different connectivity algorithms, we designed the ConnectIt framework, which is an extension of the GCorr framework [29] for multicore CPUs. However, achieving high performance for connectivity algorithms on GPUs requires significant effort beyond what is provided in ConnectIt for two reasons. The first reason is that the programming models on CPUs and GPUs are
different, which required us to significantly rewrite the codebase. The second reason is that the bottlenecks on CPUs and GPUs are different (e.g., on GPUs, performance can be easily degraded from uncoalesced memory accesses, low parallelism, and heavy use of atomics), and this required us to apply GPU-specific optimizations to achieve high performance. GConn contains several GPU-specific optimizations: edge reorganization, which is specific to our connectivity algorithms, and CSR coalescing and vertex gathering, which are commonly used in other graph algorithms.

In this paper, we generate a total of 339 different connectivity implementations and evaluate their performance. With our comprehensive study, we are able to obtain the fastest GPU connectivity algorithms to date. Figure 1 shows the normalized performance of the fastest implementation in GConn compared to four state-of-the-art GPU implementations: GPU-CC [82], GSWITCH [65], ECL-CC [49], and Afforest [85]. We achieve an average speedup of 2.48x over the fastest implementation for each graph input. Although we consider many connectivity implementations, we note that based on the results of our experimental study, practitioners only need to consider a handful of these implementations to obtain high performance.

In addition to connected components, most of our algorithms solve the related problem of computing a spanning forest of a graph. Furthermore, we extend our algorithms to the incremental setting, where the connected components or spanning forest is updated upon new edge arrivals. Our incremental algorithms are able to achieve throughputs of up to 48.23 billion edges per second, which improves upon state-of-the-art for GPUs—EvoGraph [76]—by orders of magnitude based on a rough comparison of reported numbers since their code is not available. Additionally, we compare our GPU implementations to the CPU implementations in ConnectIt on a 72-core CPU and show that we achieve speedups of 8.27–17.48x for static connectivity, and 1.85–13.36x for incremental connectivity on a Tesla V100 GPU that costs about half as much as the CPU. Finally, we perform an analysis of different design choices to explain where the performance benefits of our fastest implementations are coming from. GConn is publicly available at https://github.com/hochawa/gconn.

**Summary of Contributions.** We believe that this paper presents the most comprehensive study of GPU implementations of connectivity for both static and incremental connectivity to date. By performing this extensive study, we provide an understanding of how different algorithmic choices affect performance and where the performance benefits of fast connectivity implementations come from. Our paper provides the fastest GPU implementations of connectivity, which we obtain by combining many combinations of algorithmic choices that prior work did not explore. Building high-performance implementations of these optimizations and combining them is key to achieving high performance. We believe that our proposed techniques can be integrated into graph processing compilers and other frameworks to significantly improve performance.

The remainder of the paper is organized as follows. Section 2 discusses graph terminology and prior work. Section 3 describes the design choices in GConn, and their relation to prior work. We perform a comprehensive evaluation of our implementations in Section 4, and conclude in Section 5.

## 2 NOTATION, PRELIMINARIES, AND PRIOR WORK

**Graph Notation and Formats.** In this paper, we focus on undirected, unweighted graphs, which we denote as $G = (V, E)$. $V$ and $E$ are the set of vertices and edges, and $n = |V|$ and $m = |E|$ are the number of vertices and edges, respectively. We consider two graph formats, coordinate list (COO) and compressed sparse row (CSR), with 0-based index notation (i.e., each vertex is labeled with a unique identifier in the range $[0, \ldots, n-1]$). The COO format is represented using an array of edges, where each edge contains a source and a destination vertex. The CSR format contains two arrays, $\text{Offsets}$ and $\text{Edges}$, of lengths $n + 1$ and $m$, respectively. The edges for vertex $i$ are stored in $\text{Offsets}[i]$, $\ldots$, $\text{Offsets}[i + 1] - 1$, and $\text{Offsets}[n] = m$, and we assume they are in sorted order.

**Graph Connectivity Problems.** A **connected component (CC)** in $G$ is a maximal set of vertices in which each pair of vertices in the component is connected by a path. A connected components algorithm computes a vertex label $\text{labels}(v)$ for each $v \in V$. Two vertices $u, v \in V$ are in the same component if and only if $\text{labels}(u) = \text{labels}(v)$. A **connectivity query** returns true if and only if two input vertices belong to the same component. A **spanning forest (SF)** maintains a connected tree for each connected component. A **breadth-first search (BFS)** takes a graph with a source vertex and returns an array $A$ where $A(v)$ stores the shortest path between the source and $v$. If a vertex $v$ is unreachable from the source vertex, $A(v)$ is set to infinity.

A **union-find** (disjoint set) data structure keeps track of a number of disjoint sets such that elements in each set have the same labels. Each set is represented by a tree and each element in the set has a parent pointer. The parent of a tree root points to itself. In this paper, we assume that elements either point to themselves or point to a parent with a smaller ID. The data structure provides three operations: $\text{MAKESET}$, $\text{UNION}$, and $\text{FIND}$ [25]. $\text{MAKESET}(u)$ generates a new tree that is composed of only one node $u$, which is a root. $\text{UNION}(u, v)$ joins two trees, $u$ and $v$, into a single tree. $\text{FIND}(u)$ returns the label of the root of the tree containing $u$. $\text{FIND}$ and $\text{UNION}$ operations can perform path compression during execution to speed up subsequent operations. Union-find can be used for connectivity by creating a set for each vertex, and calling $\text{UNION}$ on the endpoints of each edge. The labels of the vertices can be obtained in the end by running $\text{FIND}$ on each vertex.
In this paper, we also consider connectivity algorithms that maintain trees, where we can move vertices between trees without completely joining the trees (as needed for merging sets in union-find). We refer to an algorithm as root-based if it only modifies the parent pointers of tree roots to point to vertices in other trees (vertices are still free to update their pointer to point to other vertices in the same tree). All of the union-find algorithms are root-based.

**Compare-and-Swap.** In this paper, we use the atomic compare-and-swap (CAS) primitive, which takes as input a memory location, an old value, and a new value. If the value stored in the memory location is equal to the old value, then the CAS atomically replaces the old value with the new value, and returns true. Otherwise, the CAS does not update the value, and returns false.

**Prior work on GPUs.** Soman et al. [82] provide GPU-CC, the first high-performance implementation of the Shiloach-Vishkin algorithm [77] on GPUs. Many libraries (e.g., Gunrock [89], IrGL [71], and Groute [13]) also adopt a variant of this approach. Note that GPU-CC uses the COO format so the performance does not suffer from poor load-balance on GPUs. GSWITCH [65] is a framework for graph processing which provides different combinations of optimization strategies for different graph algorithms, including connectivity. Hence, GSWITCH allows tens of different combinations of optimizations (e.g., different load-balancing strategies can be combined with other optimization strategies). The best set of optimizations is found using a machine learning approach. ECL-CC [49] uses a concurrent union-find algorithm for connectivity in the CSR format. Afforest [85] incorporates the k-out sampling strategy (described in Section 3.3) that significantly improves performance on many real-world graphs. Afforest uses the CSR format, and uses the load-balancing technique from [13].

### 3 GConn Overview

We first describe the overview of ConnectIt in Section 3.1. Then we provide an overview of the GConn framework in Section 3.2. In Sections 3.3 and 3.4, we provide an overview of specific algorithmic choices that we explore. Due to space constraints, we defer the complete description and pseudocode for some of these methods to the full version of our paper [45].

#### 3.1 Overview of ConnectIt

ConnectIt is a framework for multicore connectivity algorithms that outperforms existing state-of-the-art multicore algorithms and generates various implementations for connectivity based on a two-phase execution model: a sampling phase and a finish phase. In the sampling phase, a subset of edges are inspected to partially form connected components. Next, the most frequently occurring label \( L_{\text{max}} \) is identified. In the finish phase, only vertices whose label is not equal to \( L_{\text{max}} \) need to process their outgoing edges. Vertices with label \( L_{\text{max}} \) skip processing their edges, since any neighbor with label \( L_{\text{max}} \) is already in the same component, and any neighbor with label other than \( L_{\text{max}} \) will process an edge to this vertex. This two-phase execution can significantly reduce the number of edges processed. ConnectIt provides correctness proofs for the two-phase execution. GConn is an extension of ConnectIt that we developed for GPUs, which enables easy exploration of different algorithmic choices.

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**Algorithm 1 GConn for Static Connectivity**

```plaintext
1: procedure StaticConn(G, V, E, sample, finish, compress)
2: labels ← InitLabel(V)
3: edges ← SamplePhase(G, labels, sample, finish, compress)
4: \( L_{\text{max}} \) ← GetMostFrequentLabel(labels)
5: labels ← FinishPhase(G, labels, finish, comp)
6: labels ← LabelFinalization(V, labels)
7: return labels
```

**Algorithm 2 GConn for Spanning Forest**

```plaintext
1: procedure SpanningForest(G, V, E, sample, finish, compress)
2: labels ← InitLabel(V)
3: \( (\text{labels}, \text{edges}) \) ← SamplePhase(G, labels, sample, finish, compress)
4: \( L_{\text{max}} \) ← GetMostFrequentLabel(labels)
5: \( (\text{labels}, \text{edges}) \) ← FinishPhase(G, labels, finish, comp)
7: return edges
```

### 3.2 GConn Framework

In this section, we give an overview of the GConn that we use to obtain our GPU implementations for static connectivity, spanning forest, and incremental connectivity. **Static Connectivity.** Algorithm 1 presents the main steps in GConn. GConn takes in a graph in either CSR or COO format, as well as parameters for the sampling algorithm (sample), the finish algorithm (finish), and the compression algorithm (compress). The label of each vertex is initialized to its own ID on Line 2, and the sampling phase is performed on Line 3. On Line 4, the most frequently occurring label \( L_{\text{max}} \) is identified from the result of the sampling phase. On Line 5, GConn performs the finish phase, processing edges of vertices with label not equal to \( L_{\text{max}} \). Finally, on Line 6, we finalize the labels of each vertex by assigning each vertex the label of the root of its tree. We use C++ templates and inline functions to achieve high-performance implementations while keeping the GConn implementations high-level. Our implementations modularize the routines for the sampling algorithm, finish algorithm, compression algorithm, the load-balancing strategy, and graph format, making it easy to test different implementations and add new variants.

**Static Spanning Forest.** As shown in Algorithm 2, implementations for spanning forest are similar to those for connectivity; GConn supports different combinations of sampling and finish methods while generating correct spanning forest algorithms. In conjunction with the label initialization (Line 2), we also maintain the edges in the spanning forest using an auxiliary array of size \( n \) (Line 3). The key idea behind our implementations of spanning forest is to assign each edge of the discovered spanning forest to a unique vertex that is one of the endpoints of the edge. This special vertex is the root vertex in the union-find structure that updates its parent pointer to point to another vertex, so that it is no longer a root. Hence, the root-based algorithms for connectivity (to be described in Section 3.4) can be converted to compute spanning forests. The sampling phase simply creates a subset of the edges for the spanning forest (Line 4), while computing partially connected components and determining \( L_{\text{max}} \) as in static connectivity (Line 5). After that, using \( L_{\text{max}} \), the finish phase computes the rest of the edges for the spanning forest (Line 6). The finalization step is not required for spanning forest since the edges of the spanning forest have already been generated in previous steps, which eliminates the overhead for the post-processing. In our experiments, we found
that spanning forest algorithms are 6% faster on average than their static connectivity counterpart.

Incremental Connectivity. As many real-world graphs are being updated frequently, many connectivity algorithms have been proposed for dynamic graphs [1, 28, 30, 64, 76, 80]. Many of our algorithms are a natural fit for the incremental setting, where edges are inserted but not deleted. We designed GConn to support incremental connectivity (and spanning forest) algorithms that receive batches of operations consisting of edge insertions and connectivity queries that can be executed in parallel. We will describe how GConn supports incremental connectivity algorithms in Section 3.6.

3.3 Sampling Algorithms

As done in ConnectIt [29], we decompose connectivity algorithms into the sampling and finish phases. The sampling phase traverses a subset of edges in the graph to update the labels of vertices. The sampling phase can reduce the number of edges inspected in the finish phase, since in practice we expect that a large fraction of the vertices will already be settled in the \( L_{\text{max}} \) component after applying the sampling phase. All connectivity algorithms in the literature today, except for Afforest [85], only support a finish phase. We implement sampling for graphs in CSR format due to the ease of skipping over all edges for particular vertices (i.e., the ones with label \( L_{\text{max}} \) after sampling). Below we introduce the different sampling methods implemented in GConn. We discuss them in the context of connectivity, although they are used similarly in spanning forest. Pseudocode for these methods can be found in the full version of our paper [45].

\( k \)-out Sampling. Given a parameter \( k \), \( k \)-out sampling computes connected components on a sampled graph constructed by uniformly sampling \( k \) edges out of each vertex [44]. Sutton et al. [85] use a type of \( k \)-out sampling strategy where the first \( k \) edges out of each vertex are used. The vertices obtain labels as a result of running a parallel connected components algorithm on the sampled graph. In practice, after applying \( k \)-out sampling, many of the vertices in the largest connected component will have the same label, since many real-world graphs have a single massive component that most vertices will be a part of in the sampled graph. In GConn, we implemented several variants of \( k \)-out sampling for different values of \( k \), and found that taking the first 2 edges, as in [85], gave the best performance overall. Taking the first 2 edges per vertex enabled us to label most of the vertices in the largest component with the final label, and minimized the overall number of edge inspections during the sampling and finish phases. Moreover, since in many real-world graphs, the edges for a vertex are sorted and if all vertices choose their first two edges (neighbors), there are likely to be many shared neighbors, which improves locality and increases the size of the large component found. For running connectivity on the subgraph, we use different variants of union-find (discussed in Section 3.4).

Hook-Based Search (HB) Sampling. Inspired by ECL(CC) [49], we designed the hook-based search (HB) sampling approach to potentially reduce the number of edge traversals compared to \( k \)-out sampling. This approach uses a union-find structure to maintain vertex labels. First, each vertex \( v \) inspects its smallest (i.e., first) neighbor \( w \), and updates its label to \( labels(v) = \min(labels(v), labels(w)) \).

This step is efficient since there is no contention. Second, for all vertices that are still roots (i.e., \( labels(v) = v \)), we inspect their first \( N \) edges and apply the union operation to these edges. The goal of these steps is to minimize the total number of roots after sampling, since fewer roots mean that the graph is more connected. This approach can potentially require fewer edge traversals than \( k \)-out sampling for \( k \geq 2 \) if there are few roots after the first step.

Breadth-First Search (BFS) Sampling. In breadth-first search (BFS) sampling, we run a BFS from a chosen source vertex, which discovers the connected component of this source. If the graph has a massive connected component (containing a large fraction of the vertices), we have a high probability of finding the largest connected component. Connected component algorithms typically incur a large overhead in a concurrent setting, whereas BFS with idempotent operations can incur a smaller overhead [66]. Furthermore, the performance of BFS can be improved using direction-optimizing to reduce unnecessary edge traversals for many real-world graphs [9]. In our BFS sampling strategy, we applied a BFS from a source vertex, which we chose by sampling a subset of vertices and using the vertex with the largest degree from the sample. This idea was used by Slota et al. [81] to speed up label propagation.

3.4 Finish Algorithms

Here we introduce the finish algorithms in GConn. Again, we discuss them in the context of connectivity, but they are applied similarly in spanning forest. Our algorithms are min-based algorithms, where vertices maintain labels, propagate labels to other connected vertices, and keep the minimum label received. When the algorithms converge, vertices will have the same label if and only if they are in the same component. Pseudocode for our algorithms can be found in the full version of our paper [45].

Union-Find. Union-find algorithms are a special case of min-based algorithms that use a disjoint set data structure to maintain and propagate labels. GConn includes a broad set of concurrent union-find algorithms that are obtained by combining different union operations with different path compression strategies, all of which are root-based algorithms. In particular, it contains concurrent GPU implementations of union used in Rem’s algorithm (Union-Rem-Lock [73]), randomized linking by index (Union-Async [52]), and randomized linking by rank (Union-JTB [53]). Note that Union-Async and Union-JTB are lock-free compare-and-swap (CAS) implementations, whereas Union-Rem-Lock is a lock-based implementation. Spin-locks are used in Union-Rem-Lock, which can significantly degrade parallelism on GPUs [32], so we also implemented a lock-free version using CAS (Union-Rem-CAS). We also implement two variations of Union-Async: Union-Early, which traverses the paths of the two inputs simultaneously and terminates once a common node is reached [52]; and Union-Hooks, which performs CAS operations on an auxiliary hooks array so that writes to the labels array are uncontented (on the other hand, Union-Async directly performs CAS operations directly on the labels array). All of our union operations link from larger to smaller ID to ensure that there are no cycles. FIND is implemented by traversing to the root of the tree of the input vertex.
In our implementations, path compression is done on-the-fly when calling either \texttt{Union} or \texttt{Find}, and GConn includes five options: no path compression, path-splitting, path-halving, full path compression, and path-splicing. We describe these operations in detail in the full version of our paper [45]. Each union operation can be combined with a subset of these path compression rules. ConnectIt [29] proves which of the combinations are valid, and GConn supports the valid combinations from [29].

All of the implementations are asynchronous, meaning that \texttt{Union} and \texttt{Find} calls can be executed concurrently without synchronization. Furthermore, all of the implementations, except for \texttt{Union-Rem-Lock}, are wait-free. As far as we know, none of the variants of union-find above have been implemented for GPUs in the literature. The only union-find variants that have been implemented on GPUs are ECL-CC [49] and Afforest [85]. ECL-CC [49] implements a union-find algorithm, which uses the Union-Async rule, but has a separate path compression step to fully compress the paths, which requires synchronization. Afforest [85] implements a variant of the classic Shiloach-Vishkin algorithm [77]. Their algorithm also has a separate compression step, and requires synchronization. Both ECL-CC and Afforest are root-based algorithms. We also implemented these algorithms using GConn.

\textbf{Other Min-based Algorithms.} Besides the union-find algorithms previously described, we also implement the following min-based algorithms in GConn: Shiloach-Vishkin (SV) algorithms [3], Liu-Tarjan (LT) algorithms [62], Stergiou’s algorithm [83], and the Label Propagation (LP) algorithm [78]. These other min-based algorithms generalize the union-find algorithms by allowing a vertex \( \nu \) in tree \( T \) to be moved to another tree \( T' \), without requiring that all vertices in \( T \) be moved to \( T' \). The algorithms still link from larger to smaller ID to prevent cycles.

The SV algorithm is a classical parallel connectivity algorithm, and many variants of it have been proposed in the literature, e.g., \cite{3, 5, 10, 13, 24, 36, 65, 71, 82, 89, 92}. Besides Afforest, all other GPU implementations \cite{13, 65, 71, 82, 89} are based on a GPU implementation by Soman et al. \cite{82}. Soman et al.’s algorithm alternates between hooking from smaller to larger ID and from larger to smaller ID (the idea was originally proposed by Greiner \cite{36}), but does not guarantee that only roots are hooked. We also implemented a root-based variant of SV, where on every round each vertex uses an atomic minimum operation to update their neighbors’ labels, followed by full path compression via pointer jumping.

The LT algorithms are generated by combining several simple rules about how to update the \texttt{labels} array by using edges to transfer connectivity information. The updates are done using atomic min operations. In their paper \cite{62}, only five algorithms are considered, but ConnectIt supports 16 variants by considering more combinations of rules (all are min-based, and 6 variants are root-based as well). Stergiou’s algorithm \cite{83} is very similar to one of the LT variants, but it maintains two \texttt{labels} arrays, one for the previous iteration and one for the current iteration.

Many implementations and frameworks for connectivity adopt the LP algorithm, including \cite{69, 78, 81}. In each round of the LP algorithm, the labels corresponding to the endpoints of each edge are compared, and if they are different, the larger label updates itself to be equal to the smaller label. The LP algorithm terminates when there is a round in which no labels are updated.

\subsection{3.5 Iterative GPU Optimizations}

This section describes three key GPU optimizations—CSR coalescing (CSR-C), edge reorganization (ER), and vertex gathering (VG)—that we apply to our implementations in GConn. For practical purposes, we only consider the optimization strategies that improve the performance of the fastest GConn implementations. Since we found Afforest \cite{85} to be the fastest existing implementation in most cases, we identify the performance bottlenecks in Afforest, and show how to iteratively apply our optimizations to achieve the performance of our fastest implementations. We first substituted their union-find implementation with Union-Async, which we found to be faster, and use this as the baseline. Figure 2 shows how performance improves over the baseline (base) with each additional optimization applied.

\textbf{CSR Coalescing (CSR-C).} We identified that the sampling phase requires a significant amount of time in Afforest (35.15–98.63\%), with a median of 88.80\%, as shown in our experimental evaluation in Section 4.2. During \( k \)-out sampling with \( k = 2 \), Afforest has two sub-phases, the first which traverses the first edge out of every vertex, and the second which traverses the second edge out of every vertex. This strategy traverses the scattered entries in the CSR array twice, which wastes memory bandwidth. Since the first two edges of a vertex are contiguous in the memory layout, the two edges of a vertex can be processed simultaneously by two adjacent threads, which makes memory accesses for the CSR array coalesced and halves the data volume needed to access the CSR array. We observe that applying this optimization on top of the baseline (base+CSR-C) in Figure 2) improves performance by an average of 20.68\%.

\textbf{Edge Reorganization (ER).} During \( k \)-out sampling, we found that CAS operations make the warp-eficiency on the GPU very low due to only a few threads in a warp being active. For these algorithms, if two edges having the same endpoint try to update a memory location at the same time, then the updates can become serialized. This is often the case in CSR format since edges incident to a single vertex are processed in the same warp. To alleviate this issue, we could have each thread read the first two edges of a...
vertex and process them sequentially; however, this would cause threads to have scattered accesses to the CSR edge array, which decreases memory efficiency. The edge reorganization optimization groups threads into pairs, where each pair reads the two edges for one vertex together, and then reads the two edges for another vertex together. Each time, the pair of threads reads contiguous locations in memory. Then, using warp shuffling [18, 84] the four edges can be reorganized so that the edges from the same vertex are contiguous in memory, enabling each thread to process the two edges of a vertex serially to avoid CAS operations. Base+CSR-C+ER+VG in Figure 2 shows the performance improvement after applying ER on top of CSR-C. We achieve an average performance improvement of 28.79% over the baseline.

**Vertex Gathering (VG).** The previous optimizations improve the performance of the sampling phase. However, we found that the performance of the finish phase can also be improved. In the kernel for the finish phase in Afforest, each thread accesses entries in the labels array in a cyclic fashion, and if the entry of the i’th label is not equal to L_{max}, then the edges of vertex i are processed in a load-balanced fashion by distributing the edges across threads. However, as we show in Section 4.2, only a small subset of vertices have labels not equal to L_{max}. Therefore, many threads will be idle during the finish phase, which degrades performance. To improve performance in the finish phase, we first aggregate the vertices whose labels are not equal to L_{max} into an active vertex set, and in the finish phase, each thread reads a vertex in the active vertex set, and distributes the edges for load-balancing. We show the performance improvement of adding the vertex gathering optimization in Base+CSR-C+ER+VG of Figure 2. With all three optimizations, the overall performance is 41.55% faster than the baseline on average.

### 3.6 Incremental Connectivity Support

**Algorithm 3 GConn for Incremental Connectivity**

1. **procedure** IncrementalG(V, E), sample, finish, compress
2. labels ← StaticConn(G, V, E), sample, finish, compress
3. for B ∈ Batches do
4. (labels, query_results) ← FinishPhaseBatch(R, labels, finish, comp)

This section describes how GConn supports incremental connectivity given batches of updates. We support incremental connectivity for the root-based algorithms, and the pseudocode is shown in Algorithm 3. We first generate the initial graph (which can be empty) by initializing the labels array for the vertices in the graph (Line 2) using the StaticConn procedure from Algorithm 1. We assume that the labels array is large enough to hold all of the vertices that will be encountered, but we only initialize the entries for vertices present in the initial graph. Batches of insertions and connectivity queries arrive in an online fashion, and are processed using one of the finish algorithms on just the batch (Lines 3–4). The batches are given in COO format, which is a natural input for streaming algorithms. However, sampling is inefficient for graphs in COO format, and hence we do not use sampling for incremental algorithms.

Each batch B is composed of a mix of inserts and queries. For an update, FinishPhaseBatch updates the labels array as FinishPhase from Algorithm 1 does. Furthermore, a thread that inserts a new vertex will initialize its entry in the labels array by acquiring a spin lock. For a query, FinishPhaseBatch calls the Find function (one of the compression algorithms GConn provides) for both query endpoints to check whether they are in the same component. The query results (query_results in Line 4) are stored in a bitvector: the i’th entry in the bitvector is true if the i’th edge of B is a query, and the two endpoints of it are in the same component.

As shown in Algorithm 3, the previous batches are never inspected because the root-based algorithms guarantee correctness without requiring inspecting edges in a previous batch [29]. The root-based algorithms incorporated into GConn are all union-find algorithms used for static connectivity, Shiloach-Vishkin, and the root-based Liu-Tarjan algorithms. However, when Union-Rem-CAS and Union-Rem-Lock with SpliceAtomic is used, the updates and queries need to be processed separately to guarantee correctness [29].

### 4 EVALUATION

In this section, we provide an experimental evaluation and analysis of GConn. All numbers reported in this section are the median of five runs on the Volta machine unless noted otherwise. We found that the trends for spanning forest are similar to the trends for connectivity, with our spanning forest implementations obtaining an average speedup of 6% over the connectivity implementations due to not requiring the label finalization step.

**Overview of Results.** The results of this section can be summarized as follows:

- We provide an experimental evaluation of GConn connectivity implementations in the no sampling setting (Section 4.1). Without sampling, Union-Async and Union-Rem-CAS are the fastest implementations.
- In the sampling setting, we provide a detailed analysis of different sampling procedures and find that k-out sampling or HB sampling can significantly improve the performance unless the average degree of vertices is low (Section 4.2).
- The fastest GConn algorithms consistently and significantly outperform state-of-the-art GPU connectivity implementations (Section 4.3).
- GConn incremental connectivity algorithms can achieve a throughput of tens of billions of edges per second. We also evaluate the throughput for different batch sizes, and ratios of insertions to queries (Section 4.4). In the incremental setting, Union-Async is usually the fastest implementation.
- Compared to ConnectIt, a framework for CPU connectivity algorithms, GConn achieves 8.26–14.51x speedup for static connectivity and 1.85–13.36x speedup for incremental connectivity. Our analyses also show GPUs are an attractive platform for connectivity algorithms in terms of both speed and cost (Section 4.6).

**Experimental Setup.** Our GPU evaluation is performed on two machines. The first is an NVIDIA Tesla V100, which is a Volta generation GPU with 32GB that offers a 900 GB/sec memory bandwidth, 6MB of L2 cache, and 128KB of L1 cache per Streaming Multiprocessor (SM) with a total of 80 SMs. The second is an NVIDIA TITAN Xp, which is a Pascal generation GPU with 12GB that offers a 547.6 GB/sec memory bandwidth, 3MB of L2 cache, and 48KB of L1 cache per SM with a total of 30 SMs. All implementations are compiled with NVCC v10.0 using the --03 and --use_fast_math flags.
Table 1: Graph inputs, including number of vertices \((n)\), edges \((m)\), diameter, number of connected components, and the largest connected component. The graphs are symmetric, and edges are counted once in each direction. For graphs on which we were unable to compute the exact diameter, we compute the effective diameter (marked with ‘*’), which is a lower bound on the actual diameter.

| Dataset              | \(n\)  | \(m\)  | Diam.  | Num. Comps. | Largest Comp. |
|----------------------|--------|--------|--------|-------------|---------------|
| coPapersDBLP         | 540.4K | 30.49M | 15*    | 1           | 540.49K       |
| cit-Patents          | 3.77M  | 33.04M | 20*    | 3,627       | 3.76M         |
| road_usa             | 23.95M | 57.71M | 6,809  | 1           | 23.95M        |
| soc-LiveJournal1     | 4.85M  | 85.70M | 16     | 1,876       | 4.84M         |
| ljournalt           | 5.36M  | 99.03M | 31*    | 75          | 5.36M         |
| delanuay_n24        | 16.78M | 100.66M| 1,720* | 1           | 16.78M        |
| europe_cem          | 50.91M | 108.12M| 19,314 | 50.91M      |
| hollywood-2009      | 1.14M  | 112.75M| 11     | 44,508      | 1.07M         |
| kron_g500-logn21    | 2.10M  | 182.08M| 6      | 553,159     | 1.54M         |
| com-Orkut           | 3.07M  | 234.37M| 9      | 1           | 3.07M         |
| indochina-2004      | 7.41M  | 301.97M| 28     | 299         | 7.32M         |
| uk-2002             | 18.52M | 523.57M| 29*    | 38,359      | 18.46M        |
| twitter7             | 41.65M | 2.41B  | 23*    | 1           | 41.65M        |
| com-Friendster      | 65.61M | 3.61B  | 32     | 1           | 65.61M        |

Table 2 shows the results of the fastest implementations of each graph in the no-sampling setting. The graphs are sorted in order of increasing diameter (presented with the red curve).

From Figure 4, we observe that the fastest implementation is either Union-Async or Union-Rem-CAS in the No Sampling setting. Union-Rem-CAS is 1.02x slower on average than Union-Async across all graphs, and Union-JTB is 1.26x slower on average than Union-Async due to the usage of 64-bit CAS operations. Union-Hooks is designed to reduce the overhead for atomic operations, but it is 1.44x slower on average than Union-Async due to a costly memory barrier needed to avoid race conditions. Union-Rem-Lock is 3.81x slower on average than Union-Rem-CAS due to the poor performance of spin locks on GPUs.

Evaluation of Other Min-based Algorithms. Figure 4 shows the slowdown of the other min-based algorithms compared to the fastest variant across all algorithms in the no-sampling setting. The graphs that are sorted in order of increasing diameter (presented with the red curve).

As shown for the three graphs located on the far right in Figure 4, the performance of LabelPropagation degrades significantly as the diameter increases. This is because LabelPropagation needs a large number of rounds to propagate the minimum label to all vertices, and during that time most vertices are active. Even on low-diameter graphs (diameter at most 32 in our experiments), LabelPropagation is still 7.76x slower than the union-find algorithms since LabelPropagation still requires multiple traversals per edge, whereas the union-find algorithms do not.

As shown in Table 2, we also incorporate the finish algorithms used in two fastest state-of-the-art works, ECL-CC [49] (denoted as G_ECL-CC) and Afforest [85] (denoted as G_Afforest), into GConn, and evaluate them in Section 4.3.
4.2 Static Parallel Connectivity with Sampling

This section studies how our three sampling strategies affect performance, in terms of their execution time and the quality of the resulting sub-problem that they generate for the finish step in performance. To see the performance improvement with respect to the analysis for Union-Async, or we defer the number of rows in Table 2, presents the results of the fastest implementations of each algorithm using k-out sampling. The fastest implementation is Union-Async, Union-Rem-CAS, or G_ECL-CC, Union-Rem-CAS is 1.03x slower on average than Union-Async, and we defer the analysis for G_ECL-CC to Section 4.3. Figure 5 shows how each sampling algorithm improves performance. To see the performance improvement with respect to the average degree of the vertices (i.e., m/n), we sort the graphs in ascending order of the average degree.

For the union-find variants, except on the two road-network graphs (road_use and europe_omsm), k-out sampling improves performance over the unsampled versions by 6.16x on average due to the significant reduction of edges that need to be inspected, as we will show shortly. For the other min-based algorithms, other

| Group Algorithm | coPapers | cit-Papers | road_use | soc-LiveJournal | journal | delaunay | n2n | europe | hollywood | kron_g500 | com-linkon | indochina | com-Friendster |
|-----------------|----------|-----------|---------|----------------|---------|---------|-----|--------|-----------|-----------|------------|-----------|----------------|
| Union-Early     | 1.49     | 4.79      | 5.68    | 8.18           | 8.83    | 17.65   | 6.67 | 8.14   | 16.23     | 14.11     | 26.39      | 32.89     | 375.50        |
| Union-Hooks     | 0.68     | 3.44      | 5.56    | 3.94           | 5.26    | 5.98    | 11.67| 1.95   | 4.65      | 4.02      | 7.36       | 13.97     | 155.98        |
| Union-Async     | 0.66     | 2.02      | 3.39    | 3.07           | 3.76    | 6.76    | 16.24| 3.62   | 3.85      | 5.32      | 9.09       | 127.78    | 365.92        |
| Union-Rem-CAS   | 0.45     | 1.88      | 3.83    | 2.98           | 3.20    | 4.09    | 5.90 | 1.53   | 3.60      | 3.70      | 5.39       | 9.26      | 133.46        |
| Union-Rem-Lock  | 0.90     | 4.66      | 9.57    | 5.73           | 7.59    | 10.31   | 11.45| 2.73   | 6.52      | 4.19      | 65.96      | 93.69     | 484.23        |
| Union-JTB       | 0.53     | 2.84      | 5.53    | 5.25           | 5.10    | 10.85   | 5.62 | 5.78   | 6.40      | 11.07     | 30.01      | 117.03    | 140.01        |
| Liu-Tarjan      | 1.20     | 6.73      | 12.13   | 3.72           | 11.27   | 5.20    | 6.50 | 4.02   | 6.67      | 12.39     | 21.53      | 31.44     | 644.76        |
| Shiloach-Vishkin | 1.47     | 13.05     | 15.65   | 12.23          | 17.17   | 18.63   | 34.49| 7.57   | 15.93     | 34.09     | 35.01      | 47.70     | 1053.16       |
| LabelPropagation| 2.57     | 8.05      | 38.89   | 59.01          | 50.88   | 1032.09 | 2.144 | 6.18   | 12.50     | 18.58     | 79.25      | 112.74    | 1361.37       |
| G_ECL-CC        | 0.51     | 2.26      | 3.76    | 3.18           | 3.80    | 4.19    | 10.09| 1.92   | 3.84      | 5.01      | 6.73       | 11.58     | 161.13        |
| G_Afforest      | 1.96     | 23.07     | 40.70   | 25.46          | 42.12   | 6.70    | 15.92| 8.52   | 91.66     | 97.24     | 43.12      | 73.80     | 143.75        |

Table 2: Running times of implementations in GConn and state-of-the-art static connectivity algorithms in milliseconds on a V100 GPU. We report running times for five groups: implementations with No sampling, k-out sampling, HB sampling, BFS sampling, and existing algorithms. Within each of the first four groups, we display the fastest variant for each graph in green. For each graph, we also display the fastest variant across all groups in bold font. x means that we were unable to obtain results due to the graph not fitting in the GPU memory for the given implementation.
than on the two road-network graphs, $k$-out sampling improves performance by 4.61x on average over the unsampled versions.

As shown in Figure 5, when the average degree is small (e.g., road_usa and europe_osm), $k$-out sampling as well as other sampling algorithms degrades performance. This is because most of the edges in the graph get inspected in the sampling phase, and there is additional overhead to split the computation across two phases. In general, $k$-out sampling decreases the number of edge inspections by up to $m - kn$, and so as the average degree increases, we can expect a higher potential performance improvement. As shown in Figure 5, $k$-out sampling usually shows better performance when the average degree is large.

**Evaluation of HB sampling.** The third group of rows in Table 2 presents the fastest variants of each algorithm using HB sampling with a default value of $N = 4$, which we found to work the best on average across our input graphs. Unlike $k$-out sampling, HB sampling does not always improve performance; in many cases, the performance is degraded. The overall trend of HB sampling is similar to that of $k$-out sampling. As shown in Figure 5, when the average degree of vertices is small, HB sampling significantly degrades performance, and otherwise HB sampling can greatly improve performance. We will discuss how HB sampling compares with our other sampling schemes shortly.

**Evaluation of BFS sampling.** The fastest variants using BFS sampling are presented in the fourth group in Table 2. The effectiveness of BFS sampling is dependent on the diameter of an input graph and the parameters for the direction-optimizing BFS. As shown in Figure 5, for the high-diameter graphs (the three graphs on the far left), BFS sampling degrades performance by 24.59x on average over the unsampled versions due to the very small active vertex set for each BFS iteration and since a GPU kernel must be launched for each iteration. For the other graphs, BFS sampling achieves a 0.07–174.87x speedup over the unsampled versions. Note that as shown in the last row of Table 2, we also implemented the BFS-CC algorithm introduced in Ligra [78], in which a BFS is repeated on each new component until all components are found. BFS-CC performs poorly due to the GPU kernel launch overhead for high-diameter graphs and for graphs with many connected components.

**Comparing Different Sampling Strategies.** Table 3 shows how many vertices and edges are covered by each sampling algorithm. As shown in Table 3, since BFS traverses all vertices connected to a source vertex likely to be in the largest connected component, the fraction of vertices covered (Cov) and the fraction of remaining edges covered (Over) are both high. However, the fraction of remaining edges is not a good indicator of the overall performance gain. The IC columns show the percentage of vertices that are in the largest connected component after the sampling phase. Hence, the edges incident to these vertices are not inspected in the finish phase. The Cov columns show the percentage of vertices that are in the largest connected component after the sampling phase. Hence, the edges incident to these vertices are not inspected in the finish phase.
inter-component edges (IC) are maximized and minimized, respectively. Unfortunately, as shown in Table 2, BFS sampling takes significantly longer than other sampling algorithms, especially for the high-diameter graphs (delaunay_n24, road_usa, and europe_osm) due to the kernel launch overhead and low parallelism on these graphs. It also inspects significantly more edges than the other two schemes, as we will show shortly. Although BFS sampling usually takes longer than k-out sampling, we observe that Cov and IC for BFS sampling and k-out sampling are very similar on our inputs. The sampling time for HB sampling is much lower than for the others due to the reduced number of edge inspections, but for some graphs, Cov and IC are significantly lower than for the other two schemes (especially for high-diameter graphs). The Time columns in Table 3 show the ratio of the sampling time to the total execution time. We see that the sampling phase takes most of the time, and hence, the design of an efficient, high-quality sampling algorithm is of paramount importance.

Figure 6 shows the normalized total number of edges inspected during the execution of each sampling algorithm. For each graph, Figure 6 shows three bars. The first bar (green only) corresponds to the number of edges inspected by k-out sampling using union-find. The second bar (red and green) shows the number of edges inspected during the label initialization step, and the union-find step of HB sampling, respectively. Finally, the third bar (blue) shows the number of edges inspected by BFS sampling. From Table 2 and Figure 6, we observe that BFS sampling is never the fastest, and HB sampling is the fastest when the number of edge inspections for union-find is small during the sampling phase. In this case, the total number of edge inspections is also minimized. For com-Friendster, as shown in Table 2, BFS sampling is faster than k-out sampling; the number of edge inspections with BFS sampling is usually the highest, but for com-Friendster, the total number of edge inspections for both BFS sampling and k-out sampling is similar, and thus BFS sampling is faster because the edge inspection step performed by the BFS (a single CAS on one of the endpoints) is much faster than the edge inspection by a union-find algorithm (a loop that must potentially run multiple times due to contention). As seen in Figures 5 and 6, for some graphs (e.g., indochina-2004, which has a speedup of 0.07x), the number of edge inspections is much higher for BFS, leading to significant performance degradation.

4.3 Comparison with State-of-the-art

This section compares our implementations with current state-of-the-art connectivity implementations on GPUs, which are shown in the last group of rows in Table 2. Figure 7 presents the normalized execution time over the fastest current state-of-the-art implementation for each graph with our fastest variant without and with sampling. Note that we did not report results for Groute [13], Gunrock [88], and IrGL [71] as they are outperformed by ECL-CC [49], which we compare against. We also tried the strategy by Cong and Muzio [24], but it never gave the best performance.

GPU-CC [82] and GSWITCH [65]. GPU-CC adopts a classic SV algorithm, which turns out to be slow as shown in Figure 7. Our fastest implementations without sampling and with sampling are 7.06x and 26.68x faster on average than GPU-CC, respectively. GSWITCH also applies a classic SV algorithm, but outperforms GPU-CC due to several additional optimizations. Our fastest implementations without sampling and with sampling are 3.24x and 8.88x faster on average than GSWITCH.

ECL-CC [49]. ECL-CC is faster than both GPU-CC and GSWITCH due to using a more efficient algorithm based on union-find. As shown in Figure 7, our fastest implementations without sampling and with sampling are 2.78x and 10.15x faster on average than ECL-CC, respectively, mainly because we use a more efficient load-balancing strategy, and more efficient find and compress rules.

We also implemented the finish algorithm G_ECL-CC that is used in ECL-CC. As shown in Table 2, G_ECL-CC without sampling is 2.30x faster on average than ECL-CC. The speedup comes from using a different load-balancing strategy: in ECL-CC, one thread handles all edges of a vertex with degree at most 16, which can lead to significant load-imbalance. In contrast, GConn enables those edges to be processed in a load-balanced fashion using a variation of a strategy by Merrill et al. [66] that we designed. Without sampling and with sampling, G_ECL-CC is 1.19x and 1.04x slower on average than our fastest variants, respectively.

Afforest [85]. Afforest also implements a root-based algorithm with k-out sampling. As shown in Figure 7, our fastest implementations without sampling are 1% slower on average than Afforest with sampling due to the fact that many edges are not inspected in Afforest. Our fastest implementations with sampling are 2.51x faster on average because variants using HB sampling sometimes outperform those using k-out sampling, and the finish algorithm used in Afforest is slower than those used in our fastest variants. We also implemented G_Afforest, the finish algorithm used in Afforest, in GConn. We found that compared to other finish algorithms in the no sampling setting, G_Afforest is 12.6x slower on average than our fastest finish algorithm. The main reason seems to be due to some union-find algorithms, such as Union-Async and Union-Rem-CAS, handling path compression more efficiently than the method used in G_Afforest.

4.4 Incremental Parallel Graph Connectivity

In this section, we evaluate incremental connectivity algorithms on GPUs. We achieve a raw speedup of 2,482x over EvoGraph [76] which is the fastest current state-of-the-art streaming connectivity implementation. Furthermore, the memory bandwidth-normalized
speedup is 794x over EvoGraph. Unfortunately, we are not able to directly compare our streaming connectivity implementations with existing implementations on the same machine, although we provide indirect comparisons based on the numbers provided by the authors in their paper.7 cuSTINGER [35] and Hornet [16] are GPU frameworks for streaming graphs, but their code does not contain streaming connectivity algorithms.

We conduct two types of streaming experiments. In the first type of experiment, we generate a stream of edge updates from the input graphs in Table 1. The second type of experiment models real-world graph streams using synthetic graph generators. Specifically, we use the RMAT generator [19] with parameters \((a, b, c) = (0.5, 0.1, 0.1)\) and the Barabasi-Albert (BA) generator [8]. For both generators, we use \(n = 2^{27}\) and \(m = 10n\). The edges in a batch are given in COO format, and are unsorted.

**Throughput.** We evaluate the throughput of our incremental connectivity implementations on all of the input graphs in Table 2, and the two large synthetic graphs generated from the RMAT and BA generators. Table 4 reports the streaming throughput achieved by the fastest variant of each algorithm for each input when all of the edges are treated as a single batch of updates.

As shown in Table 4, the Union-Async algorithm usually achieves the highest throughput. Recall that for static parallel connectivity, Union-Rem-CAS is 1.02x slower on average than Union-Async, but here Union-Rem-CAS is 3.41x slower on average than Union-Async. In the incremental setting, when accessing an element of the labels array, the algorithm must check whether the element has been initialized, which incurs an additional overhead: for instance, Union-Async when used in the incremental setting is 1.73x slower on average than when it is used in the static setting. Compared to Union-Async, in Union-Rem-CAS, the labels array is accessed much more often, which decreases the throughput.

The other min-based algorithms are much slower than the union-find algorithms because the other min-based algorithms inspect more edges. In particular, Liu-Tarjan is 2.86x slower on average than Union-Async, and Shiloach-Vishkin is 4.87x slower on average than Union-Async.

**Throughput vs. Batch Size.** Figure 8 shows the throughput of the fastest variant for each algorithm with respect to different batch sizes. When the batch size is small, the performance significantly degrades mainly due to the GPU kernel launch overhead: the time to launch the GPU kernel takes tens of times longer than the time to run it for a small batch size. For small batch sizes, the Liu-Tarjan and Shiloach-Vishkin algorithms are much slower because for each

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7We contacted the authors for their code, but were unable to obtain it.
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Table 4: Throughput achieved by incremental connectivity algorithms in GConn on a V100 GPU machine when all of the edges in the graph are treated as a single batch of updates. For each graph, the highest throughput is shown in green.

| Algorithm          | Average slowdown against V100 | Average speedup over state-of-the-art |
|--------------------|-------------------------------|---------------------------------------|
| Static conn.       | 1.62                          | 1.57                                  |
| Span. forest       | 1.59                          | 1.60                                  |
| Streaming conn.    | 1.79                          | 2.25                                  |

Table 5: Summary of results on the Titan Xp machine.

4.6 Performance analysis on CPUs vs. GPUs

In this section, we compare the performance of GConn [29] on the V100 with the performance of ConnectIt. ConnectIt’s experiments were performed on a Dell PowerEdge R930 with 4 × 2.4GHz Intel 18-core E7-8864 x4 Xeon processors, a 45MB L3 Cache and a memory bandwidth of 85 GB/sec.

We compare CPU and GPU performance with the five graphs used in ConnectIt that can fit in the GPU memory [29]. To understand the relation between the execution time and the total data movement from/to memory, Figure 11 shows the speedup in execution time achieved by GConn over ConnectIt (blue bars), and the speedup in time for the data movement from/to memory at peak memory bandwidth using the GPU compared to the CPU (red bars). Our GPU provides 900/85 = 10.59 times higher memory bandwidth compared to the CPU, but the CPU L2 cache is much larger than the GPU L2 cache. Hence, there can be more memory transactions on the GPU, depending on the graph. As shown in Figure 11, there is a strong correlation between the execution time and the time for data movement (a Pearson correlation coefficient of 0.854). Note that the first five pairs of bars are without sampling, and the rest of them are with k-out sampling. For a fair comparison, we did not apply HB sampling because the best variants in ConnectIt for these graphs always adopt k-out sampling. The best variant of GConn without HB sampling achieves 8.26–14.51x speedup over the best variant with ConnectIt. For incremental algorithms, when we treat all of the edges as one large batch, GConn only achieves 1.85–13.36x speedup over ConnectIt due to the spin-locks on the labels array being more expensive on GPUs than CPUs.

To obtain a rough estimate of the monetary cost savings for running the connectivity algorithms on the GPU vs. the CPU, we compare two machines on Amazon EC2: the p3.2xlarge configuration, which is very similar to our V100 setup, and the x1.6xlarge configuration, which provides a multicore similar to the Dell PowerEdge R930. On-demand pricing of the p3.2xlarge and x1.6xlarge instances is $3.06 and $6.669 per hour, respectively. The fastest variant of GConn is $3.02x faster on average than that of ConnectIt. Hence, for our input graphs, we can expect to save roughly a factor of (6.669/3.06) × 12.02 = 26.2 in costs by using a similar GPU compared to a similar CPU.

4.7 Takeaways and Guidelines

Based on our experimental study, we found that variants of union-find that have not been studied in prior work on GPU connectivity performed the best. We discovered that the sampling phase mostly dominates the execution time, which indicates that reducing the overhead for the sampling phase is crucial for high performance. We found that many of our static connectivity algorithms can be extended to support spanning forest and incremental connectivity, and can achieve high performance as well. Finally, we found GPUs to be a cost-efficient option for connectivity algorithms compared to CPUs, as long as the graph can fit in the GPU memory.

The fastest implementation is dependent on a given input graph, and as GConn supports several hundred variants, trying all variants to find the fastest one can be overwhelming. We provide some guidelines below on how to choose an implementation with high performance.
Figure 12: A decision tree for the sophisticated dynamic mechanism.

First, a practitioner can simply use the best variant overall based on our experimental evaluation, which is to use Union-Async or Union-Rem-CAS combined with k-out sampling. For our input graphs, using this approach achieves performance that is within 20.8% on average of the performance of the fastest implementation for each graph.

A more sophisticated dynamic mechanism for selecting sampling methods for a fixed finish method is as follows. First, we observed that if the average degree in the graph is small enough, most edges are inspected in the sampling phase, and an additional overhead to access the edge array multiple times is incurred. Therefore, we found that when the average degree is small, sampling is not worthwhile optimization. Second, we observed that one does not need to consider BFS sampling, as it is always outperformed by either k-out sampling or hook-based search (HB) sampling. From Table 3, we see that when HB Cov (the percentage of vertices in the largest connected component after HB sampling) is high enough, HB sampling is always the best strategy. As shown in Figure 6, in this case, HB sampling samples a minimal number of edges. A rough lower bound of HB Cov can be obtained by applying the first step of HB for a few edges, which is inexpensive, and then using this value to choose between HB or k-out sampling.

Our study also provides insights on choosing finish methods that complement a particular sampling method. First, we found that the fastest finish method is always one of [Union-Async, Union-Rem-CAS, or G_ECL-CC]. In the no-sampling setting, either Union-Async or Union-Rem-CAS is the fastest. G_ECL-CC performs fewer on-the-fly compressions, which normally leads to more path traversal (and thus more cache misses). On the other hand, when HB sampling is used, which implies the input graph is very well connected, G_ECL-CC is recommended as only a very few edges are inspected by the union-find algorithms. In this case, G_ECL-CC has the lowest overhead as it does not perform much on-the-fly compression, which requires performing additional writes. On our input graphs, using this strategy for choosing the sampling and finish methods gives a slowdown of only 9.4% on average compared to using the fastest implementation for each graph.

Figure 12 presents a decision tree for the strategy above. HB Cov is the lower bound mentioned above, and Union-Async can be substituted with Union-Rem-CAS as their performance is similar.

5 CONCLUSION

We have designed the GConn framework, which supports several hundred efficient implementations on GPUs for static connectivity, spanning forest, and incremental connectivity. To the best of our knowledge, this paper provides the most comprehensive study of different variants of connectivity algorithms on GPUs. Extensive evaluations show that the best connectivity implementations in GConn significantly outperform other state-of-the-art libraries and implementations. For future work, we are interested in extending our framework to the multi-GPU or distributed memory settings.

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REFERENCES

[1] Umut A. Acar, Daniel Anderson, Guy E. Blelloch, and Laxman Dhulipala. 2019. Parallel Batch-Dynamic Graph Connectivity. In ACM Symposium on Parallelism in Algorithms and Architectures (SPAA). 381–392.
[2] Alexandre Andoni, Zhao Song, Clifford Stein, Zhengyu Wang, and Petrin Zhong. 2018. Parallel graph connectivity in log diameter rounds. In IEEE Symposium on Foundations of Computer Science (FOCS). 674–685.
[3] B. Awerbuch and Y. Shiloach. 1987. New Connectivity and MSF Algorithms for Shuffle-Exchange Network and PRAM. IEEE Trans. Comput. C-36, 10 (1987). 1258–1263.
[4] David A. Bader and Guojing Cong. 2005. A fast, parallel spanning tree algorithm for symmetric multiprocessors (SMPs). Journal of Parallel and Distrib. Comput. 65, 9 (2005), 994–1008.
[5] David A. Bader, Guojing Cong, and John Feo. 2005. On the Architectural Requirements for Efficient Execution of Graph Algorithms. In International Conference on Parallel Processing (ICPP). 547–556.
[6] David A. Bader and Joseph Jaja. 1996. Parallel Algorithms for Image Histogramming and Connected Components with an Experimental Study. J. Parallel Distrib. Comput. 35, 2 (1996), 173–190.
[7] Dip Sankar Banerjee and Kishore Kothapalli. 2011. Hybrid Algorithms for List Ranking and Connected Components. In International Conference on High Performance Computing (HiPC). 1–10.
[8] Albert-Laszlo Barabasi and Eric Bonabeau. 2003. Scale-Free Networks. Scientific American (2003).
[9] Scott Beamer, Ksite Asanovic, and David Patterson. 2012. Direction-optimizing breadth-first search. In ACM/IEEE International Conference for High Performance Computing, Networking, Storage and Analysis (SC). Article 12, 12:1–12:10 pages.
[10] Scott Beamer, Ksite Asanovic, and David A. Patterson. 2013. The GAP Benchmark Suite. CoRR abs/1508.03619 (2015). http://arxiv.org/abs/1508.03619
[11] Soheil Behnezhad, Laxman Dhulipala, Hossein Esfandiari, Jakub Lacki, and Vahab Mirrokni. 2019. Near-optimal massively parallel graph connectivity. In IEEE Symposium on Foundations of Computer Science (FOCS). 1615–1636.
[12] Soheil Behnezhad, Laxman Dhulipala, Hossein Esfandiari, Jakub Lacki, Vahab Mirrokni, and Warren Schudy. 2019. Massively Parallel Computation via Remote Memory Access. In ACM Symposium on Parallelism in Algorithms and Architectures (SPAA). 59–68.
[13] Tal Ben-Nun, Michael Sutton, Srerathai Pati, and Keshav Pingali. 2017. Groute: An Asynchronous Multi-GPU Programming Model for Irregular Computations. In ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP). 235–248.
[14] Guy E. Blelloch, Jeremy T. Fineman, Phillip B. Gibbons, and Julian Shun. 2012. Internally Deterministic Paral lel Algorithms Can Be Fast. In ACM SIGPLAN Symposium on Proceedings of Principles and Practice of Parallel Programming (PPoPP). 181–192.
[15] Libor Bus and Pavel Tvrdirk. 2001. A Parallel Algorithm for Connected Components on Distributed Memory Machines. In Recent Advances in Parallel Virtual Machine and Message Passing Interface. 280–287.
[16] Federico Busato, Oded Green, Nicola Bompieri, and David A. Bader. 2018. HorNet: An efficient data structure for dynamic sparse graphs and matrices on GPUs. In IEEE High Performance Extreme Computing Conference (HPEC). 1–7.
[17] E.N. Caceres, F. Dehne, H. Mongelli, SW. Song, and J.L. Szwarcfiter. 2004. A Coarse-Grained Parallel Algorithm for Spanning Tree and Connected Components. In Euro-Par.
[18] Bryan Catanzaro, Alexander Keller, and Michael Garland. 2014. A Decomposition for In-Place Matrix Transposition. In ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP). 193–206.
D. S. Hirschberg, A. K. Chandra, and D. V. Sarwate. 1979. Computing Connected Components. *Commun. ACM*, 22, 9 (Aug. 1979), 461–464.

Laukik Chitnis, Anish Das Sarma, Ashwin Machanavajjhala, and Vibhor Rastogi. 2013. Finding Connected Components in Map-Reduce in Logarithmic Rounds. In *IEEE International Conference on Data Engineering (ICDE)*.

K.W. Chong and T.W. Lam. 1995. Finding Connected Components in O(log in log n) Time on the EREW PRAM. *J. Algorithms* 18, 3 (1995), 374–402.

Richard Cole and Uzi Vishkin. 1986. Approximate parallel scheduling. II. Applications to logarithmic-time optimal parallel graph algorithms. *Information and Computation* 92, 1 (1991), 1–47.

Gangong Cong and Paul Muzio. 2014. Fast Parallel Connected Components Algorithms on GPUs. In *Euro-Par*. 153–164.

Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. 2009. *Introduction to Algorithms* (3rd ed.). MIT Press.

Timothy A. Davis and Yifan Hu. 2011. The University of Florida Sparse Matrix Collection. *ACM Trans. Math. Software* 38, 1 (Nov. 2011), 1:1–1:25.

Laxman Dhillapala, Guy E. Blelloch, and Julian Shun. 2018. Theoretically Efficient Parallel Graph Algorithms Can Be Fast and Scalable. In *ACM Symposium on Parallelism in Algorithms and Architectures (SPAA)*. 393–404.

Laxman Dhillapala, David Durfee, Janardhan Kulkarni, Richard Peng, Saurabh Sawlani, and Xiaorui Sun. 2020. Parallel Batch-Dynamic Graphs: Algorithms and Lower Bounds. In *ACM-SIAM Symposium on Discrete Algorithms (SODA)*. 1390–13919.

Laxman Dhillapala, Changwan Hong, and Julian Shun. 2020. connect4ltt: A framework for static and incremental parallel graph connectivity algorithms. https://arxiv.org/abs/2008.03909

D. Ediger, R. McColl, J. Riedy, and D. A. Bader. 2012. STINGER: High performance data structure for streaming graphs. In *IEEE Conference on High Performance Extreme Computing (HIPEC)*.

Ehsan Elhamifar and Rene Vidal. 2013. Sparse subspace clustering: Algorithm, theory, and applications. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 35, 11 (2013), 2765–2781.

Ahmed ElTantawy and Tor M Aamodt. 2018. Warp scheduling for fine-grained Inter-Component Edges. In *ACM Symposium on Parallelism in Algorithms and Architectures (SPAA)*. 14–25.

Martin Ester, Hans-Peter Kriegel, Jörg Sander, and Xiaowei Xu. 1996. A Density-Connected Clustering Approach for Some Graph Problems. *Inf. Process. Lett.* 61, 1 (1997), 75–82.

Donald Nguyen, Andrew Lenharth, and Keshav Pingali. 2013. A Lightweight Connected Components Implementation for GPUs. In *Graph Connectivity*. *J. Algorithms* 70, 1 (2013), 43–64.

John Greiner. 1994. A Comparison of Parallel Algorithms for Connected Components Algorithms for the CREW PRAM. *J. Comput. System Sci.* 52, 4 (1996), 227–242.

David Karger, Noam Nisan, and Michal Parnas. 1999. Fast Connected Components Algorithms for the EREW PRAM. *SIAM J. Comput.* 28, 3 (Feb. 1999), 1021–1034.

Raimondas Kiveris, Silvio Lattanzi, Vibhor Rastogi, and Sergei Vassilvitskii. 2014. Connected Components in MapReduce and Beyond. In *Proceedings of the ACM Symposium on Cloud Computing (SOCC)*. Article 18, 18:1–18:13 pages.

Vaclav Koubek and Jana Krnkova. 1985. Parallel algorithms for connected components in a graph. In *Fundamentals of Computational Theory*. 208–217.

Clyde P. Kruskal, Larry Rudolph, and Marc Snir. 1990. Efficient parallel algorithms for graph problems. *Algorithmica* 5, 1–4 (1990), 43–64.

Jure Leskovec and Andrej Krevl. 2019. SNAP Datasets: Stanford Large Network Dataset Collection. http://snap.stanford.edu/data.

Jianhua Li and Lahore Behjat. 2006. A connectivity based clustering algorithm with application to VLSI circuit partitioning. *IEEE Transactions on Circuits and Systems II: Express Briefs* 53, 5 (2006), 384–388.

Sixue Liu and Robert E. Tarjan. 2019. Simple Concurrent Labeling Algorithms for Connected Components. In *Symposium on Simplicity in Algorithms (SOSA)*. 3.1–3.20.

Kamesh Madduri and David A. Bader. 2009. Compact graph representations and parallel connectivity algorithms for massive dynamic network analysis. In *IEEE International Parallel and Distributed Processing Symposium (IPDPS)*. 1–11.

R. McColl, O. Green, and D. A. Bader. 2013. A new parallel algorithm for connected components in dynamic graphs. In *IEEE International Conference on High Performance Computing (HiPC)*.

Ke Meng, Jiujia Li, Guangming Tan, and Ninghui Sun. 2019. A Pattern Based Algorithmic Autotuner for Graph Processing on GPUs. In *ACM SGCPLN Symposium on Principles and Practice of Parallel Programming (PGPP)*.

Duanmei Merrill, Michael Garland, and Andrew Grunshaw. 2012. Scalable GPU Graph Transversal. In *ACM SGCPLN Symposium on Principles and Practice of Parallel Programming (PGPP)*. 117–128.

Dhruba Nath and S. N. Maheshwari. 1982. Parallel Algorithms for the Connected Components and Minimal Spanning Tree Problems. *Inf. Process. Lett.* 14, 1 (1982), 7–11.

Donald Nguyen, Andrew Lenharth, and Keshav Pingali. 2013. A Lightweight Infrastructure for Graph Analytics. In *ACM Symposium on Operating Systems Principles (SOSP)*. 456–474.

Donald Nguyen, Andrew Lenharth, and Keshav Pingali. 2013. A Lightweight Infrastructure for Graph Analytics. In *ACM Symposium on Operating Systems Principles (SOSP)*. 395–416.

Donald Nguyen, Andrew Lenharth, and Keshav Pingali. 2013. A Lightweight Infrastructure for Graph Analytics. In *ACM Symposium on Operating Systems Principles (SOSP)*. 395–416.

R. McColl, O. Green, and D. A. Bader. 2013. A new parallel algorithm for connected components in dynamic graphs. In *IEEE International Conference on High Performance Computing (HiPC)*.

Yue-Jin Meng, Jiujia Li, Guangming Tan, and Ninghui Sun. 2019. A Pattern Based Algorithmic Autotuner for Graph Processing on GPUs. In *ACM SGCPLN Symposium on Principles and Practice of Parallel Programming (PGPP)*.

Ke Meng, Jiujia Li, Guangming Tan, and Ninghui Sun. 2019. A Pattern Based Algorithmic Autotuner for Graph Processing on GPUs. In *ACM SGCPLN Symposium on Principles and Practice of Parallel Programming (PGPP)*.

Ke Meng, Jiujia Li, Guangming Tan, and Ninghui Sun. 2019. A Pattern Based Algorithmic Autotuner for Graph Processing on GPUs. In *ACM SGCPLN Symposium on Principles and Practice of Parallel Programming (PGPP)*.

Ke Meng, Jiujia Li, Guangming Tan, and Ninghui Sun. 2019. A Pattern Based Algorithmic Autotuner for Graph Processing on GPUs. In *ACM SGCPLN Symposium on Principles and Practice of Parallel Programming (PGPP)*.
structure. In ACM/IEEE International Conference on High Performance Computing, Networking, Storage and Analysis (SC). 62:1–62:11.

[73] Md. Mostofa Ali Patwary, Peder Refsnes, and Fredrik Manne. 2012. Multi-core Spanning Forest Algorithms using the Disjoint-set Data Structure. In IEEE International Parallel and Distributed Processing Symposium (IPDPS). 827–835.

[74] C. A. Phillips. 1989: Parallel Graph Contraction. In ACM Symposium on Parallelism in Algorithms and Architectures (SPAA). 148–157.

[75] J. H. Reif. 1985. Optimal Parallel Algorithms for Integer Sorting and Graph Connectivity. TR-08-85, Harvard University (1985).

[76] Dipanjan Sengupta and Shuaiwen Leon Song. 2017. EvoGraph: On-the-Fly Efficient Mining of Evolving Graphs on GPU. In High Performance Computing. 97–119.

[77] Yossi Shiloach and Uzi Vishkin. 1982. An $O(\log n)$ Parallel Connectivity Algorithm. J. Algorithms 3, 1 (1982), 57–67.

[78] Julian Shun and Guy E. Blelloch. 2013. Ligra: A Lightweight Graph Processing Framework for Shared Memory. In ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP). 135–146.

[79] Julian Shun, Laxman Dhulipala, and Guy E. Blelloch. 2014. A Simple and Practical Linear-Work Parallel Algorithm for Connectivity. In ACM Symposium on Parallelism in Algorithms and Architectures (SPAA). 143–153.

[80] Natcha Simsir, Kanat Tangwongsan, Srikantha Thirthapura, and Kun-Lung Wu. 2017. Work-efficient parallel union-find.Concurrency and Computation: Practice and Experience 30, 4 (2017).

[81] George M. Slota, Sivaskaran Rajamanickam, and Kamesh Madduri. 2014. BFS and Coloring-based Parallel Algorithms for Strongly Connected Components and Related Problems. In IEEE International Parallel and Distributed Processing Symposium (IPDPS). 550–559.

[82] J. Somar, K. Kishore, and P. J. Narayanan. 2010. A fast GPU algorithm for graph connectivity. In IEEE International Parallel and Distributed Processing Symposium (IPDPS). 1–8.

[83] Stergios Stergiou, Dipen Bughwani, and Kostas Tsoucoulidis. 2018. Shortcutting Label Propagation for Distributed Connected Components. In ACM International Conference on Web Search and Data Mining (WSDM). 540–546.

[84] I-Jui Sung, Juan Gómez-Luna, José María González-Linares, Nicolás Guíl, and Wen-Mei W Hwu. 2014. In-place transposition of rectangular matrices on accelerators. In ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP). 207–218.

[85] M. Sutton, T. Ben-Nun, and A. Barak. 2018. Optimizing Parallel Graph Connectivity Computation via Subgraph Sampling. In IEEE International Parallel and Distributed Processing Symposium (IPDPS). 12–21.

[86] Robert E Tarjan and Uzi Vishkin. 1985. An efficient parallel biconnectivity algorithm. SIAM J. Comput. 14, 4 (1985), 862–874.

[87] Uzi Vishkin. 1984. An optimal parallel connectivity algorithm. Discrete Applied Mathematics 9, 2 (1984), 197–207.

[88] Yangzhao Wang, Andrew Davidson, Yuechao Pan, Yuduo Wu, Andy Riffel, and John D. Owens. 2016. Gunrock: A High-performance Graph Processing Library on the GPU. In ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP).

[89] Yangzhao Wang, Yuechao Pan, Andrew Davidson, Yuduo Wu, Carl Yang, Leyuan Wang, Muhammad Osama, Chenshan Yuan, Weitang Liu, Andy T. Riffel, and John D. Owens. 2017. Gunrock: GPU Graph Analytics. ACM Trans. Parallel Comput. 4, 1, Article 3 (Aug. 2017), 3:1–3:49 pages.

[90] Dong Wen, Lu Qin, Ying Zhang, Lijun Chang, and Xue Min Lin. 2017. Efficient structural graph clustering: an index-based approach. Proceedings of the VLDB Endowment 11, 3 (2017), 245–255.

[91] Yongzhe Zhang, Ariful Azad, and Zhenjiang Hu. 2019. FastSV: A Distributed-Memory Connected Component Algorithm with Fast Convergence. CoRR abs/1910.05971 (2019). http://arxiv.org/abs/1910.05971