A simple SO(10) GUT in five dimensions

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Abstract

A simple supersymmetric SO(10) GUT in five dimensions is considered. The fifth dimension is compactified on the $S^1/(Z_2 \times Z'_2)$ orbifold possessing two inequivalent fixed points. In our setup, all matter and Higgs multiplets reside on one brane (PS brane) where the original SO(10) gauge group is broken down to the Pati-Salam (PS) gauge group, $SU(4)_c \times SU(2)_L \times SU(2)_R$, by the orbifold boundary condition, while only the SO(10) gauge multiplet resides in the bulk. The further breaking of the PS symmetry to the Standard Model gauge group is realized by Higgs multiplets on the PS brane as usual in four dimensional models. Proton decay is fully suppressed. In our simple setup, the gauge coupling unification is realized after incorporating threshold corrections of Kaluza-Klein modes. When supersymmetry is assumed to be broken on the other brane, supersymmetry breaking is transmitted to the PS brane through the gaugino mediation with the bulk gauge multiplet.

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1 Introduction

Current experimental data for the Standard Model (SM) gauge coupling constants suggest the successful gauge coupling unification in the minimal supersymmetric (SUSY) Standard Model (MSSM) and thus strongly support the emergence of a SUSY GUT around $M_{GUT} \simeq 2 \times 10^{16}$ GeV.

Among several GUTs, a model based on the gauge group $SO(10)$ is particularly attractive. In fact, $SO(10)$ is the smallest simple gauge group under which the entire SM matter content of each generation is unified into a single anomaly-free irreducible representation, $16$ representation. This $16$ representation automatically includes the right-handed neutrino and the $SO(10)$ GUT incorporates the see-saw mechanism [1] that can naturally explain the lightness of the light neutrino masses.

Among several models based on the gauge group $SO(10)$, the so-called renormalizable minimal $SO(10)$ model has been paid a particular attention, where two kinds of Higgs multiplets $\{10 \oplus 126\}$ are utilized for the Yukawa couplings with matters $16_i$ ($i =$ generation) [2,3]. A remarkable feature of the model is its high predictivity for the neutrino oscillation data as well as charged fermion masses and mixing angles. After KamLAND data [5] was released, it entered to the precise calculation phase, and many authors performed new data fitting to match up these new data [6].

High predictivity of renormalizable SUSY $SO(10)$ model was shown in constructing a concrete Higgs sector of the minimal $SO(10)$ model. A simplest and renormalizable Higgs superpotential was constructed explicitly and the patterns of the $SO(10)$ gauge symmetry breaking to the Standard Model one was shown [7,8]. This construction gives some constraints among the vacuum expectation values (VEVs) of several Higgs multiplets, which give rise to a trouble in the gauge coupling unification. The trouble comes from the fact that the observed neutrino oscillation data suggests the right-handed neutrino mass around $10^{12-14}$ GeV, which is far below the GUT scale. This intermediate scale is provided by Higgs field VEV, and several Higgs multiplets are expected to have their masses around the intermediate scale and contribute to the running of the gauge couplings. Therefore, the gauge coupling unification at the GUT scale may be spoiled. This fact has been explicitly shown in Ref. [9], where the gauge couplings are not unified any more and even the $SU(2)$ gauge coupling blows up below the GUT scale. In order to avoid this trouble and keep the successful gauge coupling unification as usual, we have several choices. One conservative approach is to add $120$ Higgs and we may adjust newly introduced parameters so as to unify gauge couplings.

In addition to the issue of the gauge coupling unification, the minimal $SO(10)$ model potentially suffers from the problem that the gauge coupling blows up around the GUT scale. This is because the model includes many Higgs multiplets of higher dimensional representations. In field theoretical point of view, this fact implies that the GUT scale is a cutoff scale of the model, and more fundamental description of the minimal $SO(10)$ model would exist above the GUT scale. As a simple realization of such a scenario, we have considered the minimal $SO(10)$ model in a warped extra dimension [10]. In this scenario, the

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1 On the other hand, there is another flow of non-renormalizable minimal $SO(10)$ GUT [4].
AdS curvature and the fifth dimensional radius were chosen so as to realize the GUT scale as an effective cutoff scale in 4D effective theory via the warped metric [11]. Furthermore, we have shown that in this context, the right-handed neutrino mass scale can be geometrically suppressed by a few order of magnitudes from the GUT scale, leaving Higgs field VEVs at the GUT scale. Thus, the gauge coupling unification remains as usual in the MSSM. This idea has been utilized in an extended model proposed in Ref. [12], where the so-called type II seesaw mechanism dominates to realize the tiny neutrino masses through the warped geometry.

In these extra-dimensional SO(10) models, it is assumed that the SO(10) gauge symmetry breaking is correctly achieved by Higgs multiplets on a brane as usual in 4D model. In addition, to realize non-trivial wave functions of matters in the bulk, non-zero VEV of the chiral adjoint multiplet $45$ in the bulk N=2 SUSY gauge multiplet is assumed, which breaks SO(10) into SU(5)$\times$U(1)$_X$. Since the $Z_2$ orbifold parity for this adjoint multiplet is assigned as odd, its VEV induces Fayet-Iliopoulos D-terms localized on the branes at the orbifold fixed points [13], which should be canceled out by some Higgs multiplets on the branes, in order to preserve SUSY. In this point, it may be not impossible but more complicated to construct a model including a complete Higgs sector in this class of extra-dimensional GUT models.

In this paper we consider another possibility for constructing extra-dimensional GUT models, the orbifold GUT [14, 15, 16, 17, 18]. In this context, the GUT gauge symmetry is broken by the orbifold boundary condition without Higgs multiplets. This boundary conditions can also realize the triplet-doublet Higgs boson mass splitting by projecting out the zero-mode of triplet Higgs while leaving the doublet Higgs one. In addition, dangerous dimension five operators causing rapid proton decay can also be projected out, the mechanism of which can be interpreted in terms of R-symmetry in 4D theoretical point of view. There are so many papers in the context of the orbifold GUT.

We propose, in this paper, a simple and clearcut scenario based on a SUSY SO(10) model in five dimensions. Usually, orbifold SO(10) models were considered in six dimensions [19], because we need at least two projections for SO(10) down to SM gauge group [20] if we break the symmetry only through boundary conditions. In our scenario, the fifth dimension is compactified on the $S^1/(Z_2\times Z'_2)$ orbifold [14, 15, 17], which has two inequivalent fixed points. By the orbifold boundary conditions, a bulk N=2 SUSY (in the sense of 4D) and SO(10) gauge symmetry are broken down to N=1 SUSY PS model with the gauge group SU(4)$_c\times$SU(2)$_L\times$SU(2)$_R$. Further gauge symmetry breaking to the SM gauge group is achieved in the usual 4D manner by VEVs of suitable Higgs multiplets on a brane. This class of SO(10) models have been proposed by several authors [21] and some improvements compared to the 6D models have been pointed out. Except for a common feature that the SO(10) gauge multiplet resides in the 5D bulk, there are many possibilities on which matter and Higgs multiplets are placed in the bulk or on one of the branes at the orbifold fixed points [21]. In our model, all matter and Higgs multiplets reside only on one brane where the PS gauge symmetry is manifest (PS brane), and thus the basic structure on the PS brane is the same as the 4D PS model. The PS gauge symmetry is broken to the SM one by VEVs of some Higgs multiplets on the brane. As in 4D PS models, there is no proton
decay problem induced by the dimension 5 operators [22]. With simple particle contents, we show that the gauge coupling unification is realized at $M_{\text{GUT}} = 4.6 \times 10^{17} \text{ GeV}$, where a more fundamental theory is assumed to take place, with the compactification scale at $M_c = 1.2 \times 10^{16} \text{ GeV}$. When we assume SUSY breaking on the other brane, the bulk gauge multiplet directly communicates with the SUSY breaking sector and transmits the SUSY breaking to the PS brane, namely the gaugino mediation [23], so that the resultant soft SUSY breaking mass spectrum is automatically flavor blind.

2 Setup

We begin with a pure SUSY SO(10) gauge theory in 5D bulk. The fifth dimension is compactified on the orbifold $S^1/\left(Z_2 \times Z'_2\right)$. A circle $S^1$ with radius $R$ is divided by a $Z_2$ orbifold transformation $y \rightarrow -y$ ($y$ is the fifth dimensional coordinate $0 \leq y < 2\pi R$) and this segment is further divided by a $Z'_2$ transformation $y' \rightarrow -y'$ with $y' = y + \pi R/2$. There are two inequivalent orbifold fixed points at $y = 0$ and $y = \pi R/2$. Under this orbifold compactification, a general bulk wave function is classified with respect to its parities, $P = \pm$ and $P' = \pm$, under $Z_2$ and $Z'_2$, respectively.

Assigning the parity $(P, P')$ as listed in Table I, only the PS gauge multiplet has zero-mode and the bulk 5D N=1 SUSY SO(10) gauge symmetry is broken to 4D N=1 SUSY PS gauge symmetry. Since all vector multiplets has wave functions on the brane at $y = 0$, SO(10) gauge symmetry is respected there, while only the PS symmetry is on the brane at $y = \pi R/2$ (PS brane).

| $(P, P')$ | bulk field | mass |
|-----------|------------|------|
| $(+, +)$  | $V(15,1,1), V(1,3,1), V(1,1,3)$ | $\frac{2n}{R}$ |
| $(+, -)$  | $V(6,2,2)$       | $\frac{(2n+1)}{R}$ |
| $(-, +)$  | $\Phi(6,2,2)$   | $\frac{(2n+1)}{R}$ |
| $(-, -)$  | $\Phi(15,1,1), \Phi(1,3,1), \Phi(1,1,3)$ | $\frac{(2n+2)}{R}$ |

Table 1: $(P, P')$ assignment and masses $(n \geq 0)$ of fields in the bulk SO(10) gauge multiplet $(V, \Phi)$ under the PS gauge group. $V$ and $\Phi$ are the vector multiplet and adjoint chiral multiplet in terms of 4D N=1 SUSY theory.

In our setup, all matter and Higgs multiplets are on the PS brane, where only the PS
symmetry is manifest so that the particle contents are in the representation under the PS gauge symmetry, not necessary to be in SO(10) representation. Thus, the particle contents do not need to include harmful Higgs fields like \( (6, 1, 1) \), which is included in 10 Higgs multiplets in a SO(10) model and mediates the dimension five operator relevant for proton decay, and there is no proton decay problem \[22\]. Even if such fields are introduced into a model in some reason, they do not need to have couplings with matter multiplets. In fact, it is easy to impose some symmetry (parity) to forbid such couplings or even if such couplings are simply neglected, they are not introduce by virtue of non-renormalization theorem. For a different setup, see \[21\].

With respect to SU(4)_c \times SU(2)_L \times SU(2)_R, we introduce matter multiplets, the left- and right-handed quarks and leptons of a given i-th generation assigned as

\[
\left( \begin{array}{cccc}
 u_r & u_y & u_b & \nu_e \\
 d_r & d_y & d_b & e
\end{array} \right)_{L(R)} \equiv F_{L(R)i},
\]

\( F_{L(R)2} \) and \( F_{L(R)3} \) are likewise defined for the 2nd and 3rd generations. Their transformation properties are \( F_{Li} = (4, 2, 1) \) and \( F_{Ri} = (4, 1, 2) \), so that \( (F_{Li} + F_{Ri}) \) yields the 16 of SO(10):

\[
\mathbf{16} = (4, 2, 1) + (\bar{4}, 1, 2).
\]

Since \((4, 2, 1) \times (\bar{4}, 1, 2) = (1, 2, 2) + (15, 2, 2)\), the Dirac masses for quarks and leptons can be generated by \((1, 2, 2)_H\) and/or \((15, 2, 2)_H\). We introduce the Higgs multiplets, which can works as \((1, 2, 2)_H + (15, 2, 2)_H\). Through the same structure as in the minimal SO(10) with \(10 + \overline{126}\), \((1, 2, 2)_H \subset 10\) is responsible for the \(b - \tau\) unification at GUT scale, while \((15, 2, 2)_H \subset \overline{126}\) can ameliorate the bad relations, \(m_e = m_u\) and \(m_\mu = m_s\), for the first two generations.

Finally, in order to break the PS symmetry to the SM one and also to generate the right-handed neutrino masses, we introduce Higgs multiplets of the fundamental and anti-fundamental representations under SU(4)_c. Here, we impose the left-right symmetry in our model, namely, the model is invariant under the exchange between \(L \leftrightarrow R\). Particle contents for matter and Higgs multiplets are summarized in Table 2. Here we have included \((6, 1, 1)_H\) but it can be decoupled to matter multiplets by imposing some symmetry (parity) as we discussed in the following.

| Matter Multiplets | \[ \psi_i = F_{Li} \oplus F_{Ri} \quad (i = 1, 2, 3) \] |
|-------------------|---------------------------------------------------|
| Higgs Multiplets  | \((1, 2, 2)_H, (1, 2, 2\)\_H, (15, 1, 1)_H, (6, 1, 1)_H, (4, 1, 2)_H, (\overline{4}, 1, 2)_H, (4, 2, 1)_H, (\overline{4}, 2, 1)_H \) |

Table 2: Particle contents on the PS brane. Here, we impose the left-right symmetry.
In the following conveniences, let us introduce the following notations:

\[
H_1 = (1, 2, 2)_H, \quad H'_1 = (1, 2, 2)'_H, \\
H_6 = (6, 1, 1)_H, \quad H_{15} = (15, 1, 1)_H, \\
H_L = (4, 2, 1)_H, \quad H_L = (4, 2, 1)_H, \\
H_R = (4, 1, 2)_H, \quad H_R = (4, 1, 2)_H.
\] (2)

Superpotential relevant for fermion masses is given by\[2\]

\[
W_Y = Y^{ij}_{L} F_{Li} F_{Rj} H_1 + \frac{Y^{ij}_{L}}{M_5} F_{Li} F_{Rj} (H'_1 H_{15}) \\
+ \frac{Y^{ij}_{R}}{M_5} F_{Ri} F_{Rj} (H_R H_R) + \frac{Y^{ij}_{L}}{M_5} F_{Li} F_{Lj} (H_L H_L),
\] (3)

where \(M_5\) is the 5D Planck scale. The product, \(H'_1 H_{15}\), effectively works as \((15, 2, 2)_H\), while \(H_R H_R\) and \(H_L H_L\) effectively work as \((10, 1, 3)\) and \((\overline{10}, 3, 1)\), respectively, and are responsible for the left- and the right-handed Majorana neutrino masses. Note that \(Y_R\) and \(Y_L\) are independent of the Dirac Yukawa couplings and there are a sufficient number of free parameters to fit the neutrino oscillation data. Assuming appropriate VEVs for Higgs multiplets, we can parameterize fermion mass matrix as the following form \([24]\):

\[
M_u = c_{10} M_{1,2,2} + c_{15} M_{15,2,2}, \\
M_d = M_{1,2,2} + M_{15,2,2}, \\
M_D = c_{10} M_{1,2,2} - 3c_{15} M_{15,2,2}, \\
M_e = M_{1,2,2} - 3M_{15,2,2}, \\
M_L = c_L M_{10,3,1}, \\
M_R = c_R M_{10,1,3}.
\] (4)

Here, the so-called Georgi-Jarlskog factor, \(-3\), appears in the lepton mass matrix as the Clebsch-Gordan coefficient associated with the basis \(\text{diag}(1, 1, 1, -3)\) for the \(SU(4)_c\) adjoint Higgs \((15, 2, 2)_H\).

We introduce Higgs superpotential invariant under the PS symmetry such as

\[
W = \frac{m_1}{2} H_1^2 + \frac{m_1'}{2} H'_1^2 + m_{15} \text{tr} [H_{15}^2] + m_4 (H_L H_L + H_R H_R) \\
+ (H_L H_R + H_R H_L) (\lambda_1 H_1 + \lambda_1' H'_1) + \lambda_{15} (H_R H_R + H_L H_L) H_{15} \\
+ \lambda \text{tr} [H_{15}^3] + \lambda_6 \left( H_L^2 + H_R^2 + H_L^2 + H_R^2 \right) H_6.
\] (5)

Parameterizing \(\langle H_{15} \rangle = \frac{v_{15}}{2\sqrt{6}} \text{diag}(1, 1, 1, -3)\), SUSY vacuum conditions from Eq. (5) and the D-terms are satisfied by solutions,

\[
v_{15} = \frac{2\sqrt{6}}{3\lambda_1} m_4, \quad \langle H_R \rangle = \langle H_R \rangle = \sqrt{\frac{8m_4}{3\lambda_1^2}} \left( m_{15} - \frac{\lambda}{\lambda_1} m_4 \right) \equiv v_{PS}
\] (6)

\[2\] For simplicity, we have introduced only minimal terms necessary for reproducing observed fermion mass matrices.
and others are zero, by which the PS gauge symmetry is broken down to the SM gauge symmetry. We choose the parameters so as to be $v_{15} \simeq \langle H_R \rangle = \langle \bar{H}_R \rangle$. Note that the last term in Eq. (5) is necessary to make all color triplets in $H_R$ and $\bar{H}_R$ heavy.

Weak Higgs doublet mass matrix is given by

$$
\begin{pmatrix}
H_1 & H'_1 & H_L
\end{pmatrix}
\begin{pmatrix}
0 & \lambda_1 \langle H_R \rangle & \lambda_1' \langle H_R \rangle & m_4
m_1 & 0 & 0 & \lambda_1 \langle H_R \rangle & \lambda_1' \langle H_R \rangle
\lambda_1 \langle H_R \rangle & \lambda_1' \langle H_R \rangle & \lambda_1 \langle H_R \rangle & \lambda_1' \langle H_R \rangle & m_4
\end{pmatrix}
\begin{pmatrix}
H_1 \\
H'_1 \\
H_L
\end{pmatrix}.
$$

(7)

In order to realize the MSSM at low energy, only one pair of Higgs doublets out of the above tree pairs should be light, while others have mass of the PS symmetry breaking scale. This doublet-doublet Higgs mass splitting requires the fine tuning of parameters to satisfy

$$
\text{det } M = m_1 m'_1 m_4 - (m_1 \lambda_1'^2 + m'_1 \lambda_1^2) v_{PS}^2 = 0.
$$

(8)

3 Gauge coupling unification

In the orbifold GUT model, we assume that the GUT model takes place at some high energy beyond the compactification scale. For the theoretical consistency of the model, the gauge coupling unification should be realized at some scale after taking into account the contributions of Kaluza-Klein modes to the gauge coupling running [17] [25].

In our setup, the evolution of gauge coupling has three stages, $G_{321}$ (SM+MSSM), $G_{422}$ (the PS) and $M_c = 1/R$. For simplicity, we assume $v_{PS} = M_c$ in our analysis. Furthermore, since we have imposed the left-right symmetry, $\text{SU}(2)_L$ and $\text{SU}(2)_R$ gauge couplings must coincide with each other at the scale $\mu = v_{PS}$. As a consequence, the PS scale is fixed from the gauge coupling running in the MSSM stage.

In the $G_{321}$ stage, we have

$$
\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{1}{2\pi} b_i \ln \left( \frac{M}{\mu} \right); \quad (i = 3, 21),
$$

(9)

were $b_i$s are

$$
b_3 = -7, \quad b_2 = -19/6, \quad b_1 = 41/10
$$

(10)

for $M_Z < \mu < M_{\text{SUSY}}$ and

$$
b_3 = -3, \quad b_2 = 1, \quad b_1 = 33/5
$$

(11)

for $M_{\text{SUSY}} < \mu < M_c = v_{PS}$. At the PS scale, the matching condition holds

$$
\begin{align*}
\alpha_3^{-1}(M_c) &= \alpha_4^{-1}(M_c) \\
\alpha_2^{-1}(M_c) &= \alpha_{2L}^{-1}(M_c) \\
\alpha_1^{-1}(M_c) &= \frac{2\alpha_4^{-1}(M_c) + 3\alpha_{2R}^{-1}}{5}
\end{align*}
$$

(12)
Figure 1: Gauge coupling unification in the left-right symmetric case. Each line from top to bottom corresponds to $g_3$, $g_2$ and $g_1$ for $\mu < M_c$, while $g_3 = g_4$ and $g_2 = g_{2R}$ for $\mu > M_c$.

In the PS stage $\mu > M_c$, the threshold corrections $\Delta_i$ due to KK mode in the bulk are added,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_c)} + \frac{1}{2\pi} b_i \ln \left( \frac{M_c}{\mu} \right) + \Delta_i. \quad (i = 4, 2_L, 2_R) \quad (13)$$

The beta functions from the matter and Higgs multiplets on the PS brane are

$$b_4 = 3, \quad b_{2L} = b_{2R} = 6. \quad (14)$$

KK mode contributions are given by

$$\Delta_i = \frac{1}{2\pi} b_i^{even} \sum_{n=0}^{N_i} \theta(\mu - (2n + 2)M_c) \ln \left( \frac{(2n + 2)M_c}{\mu} \right)$$

$$+ \frac{1}{2\pi} b_i^{odd} \sum_{n=0}^{N_i} \theta(\mu - (2n + 1)M_c) \ln \left( \frac{(2n + 1)M_c}{\mu} \right) \quad (15)$$

with

$$b_i^{even} = (-8, -4, -4),$$

$$b_i^{odd} = (-8, -12, -12) \quad (16)$$

under $G_{422}$.

Fig. 1 shows the gauge coupling unification for the left-right symmetric case. The PS (compactification) scale, $M_c$, is determined from the gauge coupling running in the MSSM stage by imposing the matching condition,

$$\alpha_2^{-1}(M_c) = \alpha_{2R}^{-1}(M_c) = (5\alpha_1^{-1}(M_c) - 2\alpha_3^{-1}(M_c))/3,$$

and we find

$$v_{PS} = M_c = 1.2 \times 10^{16}\text{GeV}. \quad (17)$$
for the inputs, $(\alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z)) = (0.01695, 0.03382, 0.1176)$ and $M_{SUSY} = 1 \, \text{TeV}$. For the scale $\mu > M_c$, there are only two independent gauge couplings $\alpha_4$ and $\alpha_2 = \alpha_{2R}$, and so the gauge coupling unification is easily realized. We find the unification scale as

$$M_{\text{GUT}} = 4.6 \times 10^{17} \, \text{GeV}. \quad (18)$$

As mentioned before, we assume that a more fundamental SO(10) GUT theory takes place at $M_{\text{GUT}}$, and it would be natural to assume $M_{\text{GUT}} \sim M_5$. In fact, the relation between 4D and 5D Planck scales, $M_5^3/M_c \simeq M_P^2$ ($M_P = 2.4 \times 10^{18} \, \text{GeV}$ is the reduced Planck scale), supports this assumption with $M_c = 1.2 \times 10^{16} \, \text{GeV}$. When we abandon the left-right symmetry, there is more freedom for the gauge coupling unification with two independent parameters $v_{PS}$ and $M_c$.

## 4 Supersymmetry breaking mediation

The origin of SUSY breaking and its mediation to the MSSM sector is still an open question of SUSY models and there have been many scenarios proposed. A mechanism which naturally transmits SUSY breaking in a flavor-blind way is the most favorable one. Here we consider such a scenario.

In higher dimensional models, the sequestering \[26\] is the easiest way to suppress flavor dependent SUSY breaking effects to the matter sector. Since all matters reside on the PS brane in our model, the sequestering scenario is automatically realized when we simply assume a SUSY breaking sector on the brane at $y = 0$. The SO(10) gauge multiplet is in the bulk and can directly communicate with the SUSY breaking sector. Here, we first consider the higher dimensional operator of the form,

$$\mathcal{L} = \delta(y) \int d^2 \theta \lambda \frac{X}{M_5^2} \text{tr} [W^a W_a], \quad (19)$$

where $\lambda$ is a dimension-less constant, and $X$ is a singlet chiral superfield which breaks SUSY by its F-component VEV, $X = \theta^2 F_X$. Therefore, the bulk gaugino first obtains SUSY breaking masses,

$$M_\lambda = \frac{\lambda F_X M_c}{M_5^2} \simeq \frac{\lambda F_X M_5}{M_P^2}, \quad (20)$$

where $M_c$ comes from the wave function normalization of the bulk gaugino, and we have used the relation $M_5^3/M_c \simeq M_P^2$ in the last equality. As usual, we take $M_\lambda = 100 \, \text{GeV}-1 \, \text{TeV}$. With this non-zero gaugino mass at high scale, SUSY breaking mass terms of sfermions are automatically generated through the renormalization group equation (RGE) from the compactification scale to the electroweak scale. Importantly, the sfermion masses generated in this way are flavor blind, because the interaction transmitting the gaugino mass to sfermion masses is the gauge interaction. This scenario is nothing but the gaugino mediation \[23\]. Comparing the gaugino mass to gravitino mass $m_{3/2} \simeq F_X/M_P$, a typical gaugino mass is smaller than the gravitino mass by a factor $\lambda M_5/M_P \sim 0.1 \lambda$. 

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If there are some extra bulk multiplets coupling with both the SUSY breaking and the 
MSSM sectors, flavor-dependent sfermion masses can be, in general, induced. Thus, it is 
important to check whether such flavor-dependent terms are small enough compared to 
the gaugino mediation contribution. Introducing an extra bulk hypermultiplet \( (H_0) \), let us 
consider effective Kahler potentials both on the PS brane and the other brane,

\[
\mathcal{L} = \delta(y) \int d^4 \theta \frac{H_0^1 H_0 X^+X}{M_5^3} + \delta(y - \pi R/2) \int d^4 \theta c_{ij} \frac{H_0^1 H_0 Q_i^1 Q_j^j}{M_5^3}, \tag{21}
\]

where \( Q_i \) stands for a MSSM matter multiplet with the generation index \( i \), and \( c_{ij} \) is a 
flavor-dependent dimensionless coefficient. Thus, one-loop corrections through \( H_0 \) lead to 
flavor-dependent contributions to sfermion masses, which are roughly estimated as

\[
\Delta \tilde{m}_{ij}^2 \sim \frac{c_{ij} F_X^2 M_c^4}{16 \pi^2 M_5^6} \left( \frac{M_c}{M_{GUT}} \right)^2 \approx 4.3 \times 10^{-6} c_{ij}, \tag{22}
\]

Comparing this to the flavor-blind sfermion mass squareds induced by the gaugino mass, 
\( \tilde{m}^2 \sim M_{\lambda}^2 \), we find

\[
\frac{\Delta \tilde{m}_{ij}^2}{\tilde{m}^2} \sim \frac{c_{ij}}{16 \pi^2} \left( \frac{M_c}{M_5} \right)^2 \approx \frac{c_{ij}}{16 \pi^2} \left( \frac{M_c}{M_{GUT}} \right)^2 \approx 4.3 \times 10^{-6} c_{ij}, \tag{23}
\]

which is negligibly small.

In the simple setup, it turns out that stau is the lightest superpartner (LSP), which is 
problematic in cosmology. It has been found that when \( M_c > M_{GUT} \), the RGE running in a 
unified theory pushes up stau mass and leads neutralino to be the LSP [27]. However, in our 
model, we cannot take such an arrangement, because \( M_c \) and \( M_{GUT} \) are fixed as \( M_c < M_{GUT} \) 
to realize the gauge coupling unification. In order to avoid this problem, we need to extend 
the SUSY breaking sector. It is possible to introduce the gauge mediation [28] on the PS 
brane, in which gravitino is normally the LSP. In general, we can introduce the messenger 
sector on the brane at \( y = 0 \). This setup is basically the same as in Ref. [29], where the 
gauge mediation was calculated in 5D with the messenger sector on one brane, sfermions on 
the other brane and gauge multiplets in the bulk. When the messenger scale is larger than 
the compactification scale \( (M_{mess} > M_c) \), the gaugino mass is given by the same formula as 
in 4D,

\[
M_{\lambda} \simeq \frac{\alpha_{GUT}}{4\pi} \frac{F_X}{M_{mess}}, \tag{24}
\]

while sfermion masses are roughly given by

\[
\tilde{m}^2 \simeq M_{\lambda}^2 \left( \frac{M_c}{M_{mess}} \right)^2. \tag{25}
\]

The sfermion mass squared is suppressed relative to the gaugino mass by a geometric factor 
\( M_c/M_{mess} \), at the messenger scale. At low energy, sfermion masses comparable to the gaugino 
mass are generated through the RGE running. In this setup, we find

\[
\frac{m_{3/2}}{M_{\lambda}} \sim \frac{M_{mess}}{\frac{\alpha_{GUT}}{4\pi} M_P} \gtrsim 10. \tag{26}
\]
for $M_{\text{mess}} \geq M_c$. Thus, in order to have gravitino the LSP, the messenger scale should be smaller than the compactification scale, namely, $M_{\text{mess}} \lesssim 10^{15}$ GeV. In this case, soft mass formulas are reduced into the usual four dimensional ones in the gauge mediation scenario.

5 Conclusion

We have proposed a simple SO(10) model in five dimensions with the 5th dimension compactified on the orbifold $S^1/(Z_2 \times Z_2')$. Due to the orbifold boundary conditions, a bulk N=2 SUSY and SO(10) gauge symmetry are broken down to 4D N=1 SUSY PS model with the gauge group, $\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$. All matter and Higgs multiplets reside only on the PS brane, while the gauge multiplet is in 5D bulk. The PS symmetry is broken to the SM one by the usual Higgs mechanism on the PS brane. Imposing the left-right symmetry, the gauge coupling unification is realized at $M_{\text{GUT}} \simeq 4.6 \times 10^{17}$ GeV with the compactification $M_c \simeq 1.2 \times 10^{16}$ GeV. There are various possibilities for SUSY breaking. When we assume SUSY breaking on the brane at $y = 0$, SUSY breaking is transmitted into the PS brane through the gaugino mediation.

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