Tracking ultrafast hot-electron diffusion in space and time by ultrafast thermomodulation microscopy

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The ultrafast response of metals to light is governed by intriguing nonequilibrium dynamics involving the interplay of excited electrons and phonons. The coupling between them leads to nonlinear diffusion behavior on ultrashort time scales. Here, we use scanning ultrafast thermomodulation microscopy to image the spatiotemporal hot-electron diffusion in thin gold films. By tracking local transient reflectivity with 20-nm spatial precision and 0.25-ps temporal resolution, we reveal two distinct diffusion regimes: an initial rapid diffusion during the first few picoseconds, followed by about 100-fold slower diffusion at longer times. We find a slower initial diffusion than previously predicted for purely electronic diffusion. We develop a comprehensive three-dimensional model based on a two-temperature model and evaluation of the thermo-optical response, taking into account the delaying effect of electron-phonon coupling. Our simulations describe well the observed diffusion dynamics and let us identify the two diffusion regimes as hot-electron and phonon-limited thermal diffusion, respectively.

INTRODUCTION
Understanding light-induced charge-carrier transport is of vital importance in modern technological applications, such as solar cells, artificial photosynthesis, optical detectors, and heat management in optoelectronic devices (1–4). For example, the pathway of energy transfer in solar cells directly influences their energy conversion efficiency, as charge carriers need to diffuse toward extraction regions before being lost to other decay channels (5). Much active research on emerging sensitized or thin-film photovoltaics aims at engineering effective carrier transport while preventing hot-carrier loss (6, 7). In modern optoelectronic devices, hot electrons are first accelerated by the incident field before they scatter and dissipate their energy on the nanometer to micrometer scale, eventually transferring excess energy either to the lattice or through Schottky junction harvesting (8–10). In the past, pump-probe techniques were widely used to study the ultrafast response of carrier and heat transport in different materials under optical excitation (11). While these time-resolved studies have uncovered numerous aspects of ultrafast carrier dynamics, the nanoscale spatial transport has remained largely unexplored. Direct real-space mapping to watch the carriers diffuse both in space and time is challenging. Recently, the combination of ultrafast spectroscopy and nanoimaging techniques, such as electron microscopy, x-ray diffraction, or near-field microscopy, has opened up new avenues to visualize microscopic transport (12–14). Unfortunately, these methods are invasive, require vacuum, or involve slow probe scanning. All-optical far-field pump-probe microscopy is an interesting alternative as it probes the system of interest in a perturbation-free manner while achieving both high temporal and spatial resolution (15, 16). Using concepts of super-resolution and localization microscopy (17), the spatiotemporal dynamics can be resolved beyond the diffraction limit by precisely measuring the nanometer spatial changes of an initially excited, diffusion-limited region. Because of the relatively facile implementation, ultrafast microscopy is now emerging as an important tool for studying exciton diffusion in semiconductors, molecular solids, and two-dimensional (2D) materials (18–21). In this context, the ultrafast carrier diffusion dynamics in noble metals, such as gold, is of particular importance for heat management in nanoscale devices, as well as femtosecond laser ablation, but has thus far only been studied in the time domain (22–30).

In this work, we use novel ultrafast microscopy to gain insight into the complex nonequilibrium dynamics of hot electrons in gold. When a metal interacts with light, its electrons are excited above the Fermi level (Fig. 1A). These initially excited electrons trigger a cascade of cooling mechanisms that involve energy transfer between different metal subsystems, of which conduction electrons and lattice vibrations (phonons) are the dominant ones. The ultrafast thermal response is commonly described by considering the conduction band electrons and the ionic lattice as separate thermodynamic subsystems with distinct thermal properties, such as heat capacity and thermal conductivity (31). Since the heat capacity of the electrons is substantially smaller than that of the lattice, the electron subsystem quickly reaches a high temperature Fermi-Dirac distribution, which we refer to as “hot electrons,” while the lattice stays close to ambient temperature under the excitation intensities considered here. The electrons subsequently cool and thermalize with the lattice within a few picoseconds. The electron relaxation and thermalization of the two subsystems are a direct result of an interplay between electron-phonon coupling and hot-electron diffusion (27). Purely time-resolved studies typically cannot separate these two contributions and have consistently neglected lateral heat flow (25, 27), which becomes particularly important when considering nanoscale systems. Directly resolving hot-electron diffusion, a crucial step for understanding the ultrafast heat dynamics, has thus far not been possible because of a lack of spatiotemporal resolution (27–29).

Here, we address this shortcoming by interrogating thin gold films with our recently developed scanning ultrafast thermomodulation microscopy (SUTM) technique. We directly measure the spatiotemporal evolution of a locally induced hot-electron distribution on nanometer length scales with femtosecond resolution. Specifically, in our experiment, an optical pump pulse illuminates a thin gold film, thus creating hot carriers in the metal (Fig. 1B). We measure the subsequent...
negligible. Then, at a sample before photoexcitation by the pump, the lattice temperature \( T_e = T_l \), while the FWHM of 0.6 and 0.9 ns components (see fig. S1). While these data contain information about the temporal carrier dynamics, the spatial diffusion information is provided by the \( \Delta R/R \) maps of Fig. 2A. Therefore, we fit the central cross sections of the images with Gaussian functions (Fig. 2C). The FWHM increases by about 100 nm from 1.05 \( \mu \)m at \( \Delta t = 0 \) to 1.15 \( \mu \)m at \( \Delta t = 10 \) ps, a result that we attribute to spatial heat diffusion. The accuracy of this method is ultimately limited by the signal-to-noise ratio of the response function, which dictates how well the profiles can be fit. We observe an FWHM accuracy of about 20 nm.

We further investigate this evident diffusion behavior, as we want to track the rate at which it takes place during the thermalization process. Therefore, we proceed to record transient reflectivity line profiles over many pump-probe delays. Figure 3A shows a typical spatio-temporal dataset for \( F = 1.0 \) mJ/cm\(^2\). We obtain an estimate of the time-dependent diffusion coefficient \( D(t) \) in a first, semiquantitative manner by assuming the following relationship between the width (FWHM) and \( D \) for an initial Gaussian profile, which we adopt from a general treatment of diffusion problems (see note S1 for details) (33)

\[
\frac{\partial \text{FWHM}^2(t)}{\partial t} = 16 (\ln 2) D(t)
\]

We extract the FWHM at each time delay by fitting Gaussian profiles to the \( \Delta R/R \) maps, as previously explained, and plot the resulting temporal evolution of the squared width, FWHM\(^2\), in Fig. 3B. We observe an initial fast spreading, revealed by an increase in FWHM\(^2\), followed by a much slower broadening at longer time delays. The two diffusion regimes appear to correlate with the fast and slow temporal decay regimes of \( \Delta R/R \). We estimate the initial and final diffusion coefficients by fitting lines to the curve in Fig. 3B (Eq. 1). Excluding the transition region, we restrict our fit to the \( \Delta t = 0 \) to 1 and 5 to 30 ps intervals, yielding \( D_{\text{fast}} = 95 \) cm\(^2\)/s and \( D_{\text{slow}} = 1.1 \) cm\(^2\)/s, respectively. We repeat the same procedure for multiple pump fluences and summarize the extracted coefficients, along with their standard errors, in the inset to Fig. 3B.

The above analysis provides a simple first measure to quantify diffusion. However, it relies on the assumption of a single diffusing profile and its proportionality to transient reflection while ignoring the underlying electron and phonon subsystems, as well as their individual thermal contributions to the reflection signal. Considering these (nonlinear) contributions to the dynamics, the asymptotic linear fitting is too simplistic. To fully understand the nature of the time-dependent diffusion mechanisms at work, particularly the transition between the two regimes of the observed spot broadening dynamics, it is necessary to model the response of the system more rigorously. Briefly, we model the spatiotemporal evolution of the pump-induced changes to the reflectivity of the sample in three basic steps (Fig. 4A). First, we calculate the electron and lattice temperature distributions in the gold film, \( T_e(r, \Delta t) \) and \( T_l(r, \Delta t) \), resolved in space and time, by means of a 3D two-temperature model (30, 31). We then calculate the spatiotemporal dynamics of the gold film permittivity

![Fig. 1. Schematic description of scanning ultrafast thermomodulation microscopy. (A) The energy distribution of the conduction band electrons at ambient temperature is perturbed by optical excitation. It quickly evolves to a quasi-thermalized “hot-electron” Fermi-Dirac distribution with high electron temperature (T\( e \)), while the lattice temperature (T\( l \)) stays close to the ambient level. Subsequent cooling due to electron-phonon coupling and hot-carrier diffusion leads to thermal equilibrium between the electron and lattice subsystems. (B) An optical pump pulse illuminates a 50-nm thin gold film, thus inducing a local hot-electron distribution. The probe pulse measures the temperature-dependent transient reflectivity (\( \Delta R/R \)) as a function of both pump-probe time delay and pump-probe spatial offset. (C) We monitor the spatiotemporal evolution of the photoinduced \( \Delta R/R \) spot size with 20-nm accuracy and 250-fs resolution to visualize and distinguish hot-electron diffusion and thermal (phonon-limited) diffusion.](http://advances.sciencemag.org/)

RESULTS

We image the light-induced thermal dynamics of a 50-nm thin gold film using SUTM. The sample is optically excited with a 450-nm (2.76 eV) pump pulse and interrogated with a 900-nm (1.38 eV) probe pulse, as outlined above (Fig. 1B). We focus the beams at the sample plane to spots of full width at half maximum (FWHM) of 0.6 and 0.9 \( \mu \)m, respectively, and record \( \Delta R/R \) maps at different times after photoexcitation by varying the pump-probe delay between -5 and 30 ps.

Figure 2A shows transient reflection images, recorded at three different pump-probe time delays \( \Delta t \) for a fixed pump fluence \( F \) of 1.0 mJ/cm\(^2\), which is well below the damage and ablation threshold of the metal (32). At \( \Delta t = -2 \) ps (i.e., when the probe interrogates the sample before photoexcitation by the pump), the \( \Delta R/R \) response is negligible. Then, at \( \Delta t = 0 \) ps, a negative \( \Delta R/R \) spot emerges around the pump beam position \( x = y = 0 \), with up to \(-10^3\) contrast. Subsequently, at \( \Delta t = 10 \) ps, we observe a reduced response, as the signal has decayed.
We define the source term $S(r,t)$ for the energy absorbed from the pump pulse via its Gaussian spatial and temporal profiles (see details in note S2). We are careful to include the lateral $(x,y)$ temperature gradient, as we are interested in the prediction of the model for the lateral heat flow out of the focal volume. A finite element method calculation yields the 3D spatiotemporal electron and lattice temperatures $T_e(r,\Delta t)$ and $T_l(r,\Delta t)$ for the sample geometry. The gold-air interface is taken to be at the $z=0$ plane. The calculated evolution of the electron and lattice temperatures $T_{el}$ at $x=y=z=0$ is shown in Fig. 4B. We observe a rapid increase in electron temperature to more than 1000 K and a subsequent cooling and thermalization with the lattice temperature at few tens of degrees above the ambient level of 293 K. Next, we calculate the spatiotemporal dependence of the gold film permittivity $\varepsilon(T_e(r,\Delta t),T_l(r,\Delta t))$ using a Drude model for the near-infrared probe wavelength (see Materials and Methods). The model includes electron-electron Umklapp scattering, electron-phonon scattering, and thermal lattice expansion in the calculation of the gold permittivity, which therefore depends on both $T_e$ and $T_l$ (see note S3). Then, the pump-induced complex permittivity is converted into transient reflectivity using the Fresnel coefficients for thin films as a function of lateral position $(x,y)$ and time $(\Delta t)$ to yield $\Delta R/R(\varepsilon(r,\Delta t))$, assuming a uniform permittivity across the film thickness (see note S4). The calculation actually predicts a higher $\Delta R/R$ contrast compared with experiment, possibly due to an overestimate of the absorbed power; however, this does not affect the width distributions. We account for the effect of the finite spatial extension of the probe pulse by convolving the resulting $\Delta R/R$ map with its measured, diffraction-limited width. Last, we analyze the predicted spatiotemporal evolution of $\Delta R/R$ by Gaussian fitting to the spatial cross sections, as described above for the experimental data. Figure 4C shows the calculated evolution of the squared width together with the experimental results for the three pump fluences under consideration. The comparison shows that the calculated evolution describes the experimental data well for both thermalization regimes, including the dependence on fluence. The relative difference $\Delta$ of the squared width between the full model and the experimental data lies below $\pm4\%$ over the entire range of studied time delays.

**DISCUSSION**

Using scanning ultrafast thermomodulation microscopy, we have imaged the thermo-optical dynamics of gold films and identified two regimes of thermal diffusion dominated by hot electrons and lattice modes (phonons), respectively. Intuitively, the two observed regimes of heat diffusion may be understood from limiting cases of the two-temperature model. At early times, when $T_e \gg T_l$, the electron-lattice thermalization time can be estimated as $\tau_{el-ph} \approx C_e/C_l$, and the electron-dominated diffusion can be estimated as $D_{fast} = k_e/C_e$, as previously predicted (22). In the long term, after thermalization of the electrons and the lattice ($T_e \approx T_l$), the two-temperature model simplifies to a diffusion equation with $D_{slow} = (k_e + k_l)/(C_e + C_l) \approx k_e/C_e$. This is the well-known thermal diffusivity of gold (35). Inserting the values of electron-phonon coupling constant, heat capacity, and thermal conductivity leads to $\tau_{el-ph} = 1$ to 3 ps for $T_e = 300$ to 1000 K, $D_{fast} = 152$ cm$^2$/s, and $D_{slow} = 1.36$ cm$^2$/s. However, $D_{fast} = k_e/C_e$, also shown as a gray dashed line in Fig. 4C, only serves as a preliminary estimation, which overestimates the values observed by our spatiotemporal imaging (see also inset to Fig. 3B). Obviously, the delaying effect of electron-phonon coupling on the diffusion dynamics needs to be taken into account. Modeling the full dynamics from absorption through...
electron-lattice thermalization to spatiotemporal permittivity dynamics allows us to predict the evolution of the spatial width of $D_R/R$, resulting in good quantitative agreement with the experimentally determined time-dependent diffusion. In particular, it illustrates how the elevated temperature of the conduction band electrons correlates with a fast carrier diffusion and how both electron-phonon coupling and hot-electron diffusion contribute to the transition between the two mentioned regimes. We note that our model relies purely on reported material constants and has no free adjustable parameters (see Materials and Methods). In addition, the dependence of the calculated width evolution on the laser fluence agrees well with the experimental data.

In the transition regime, around 5-ps time delay, we predict a subtle decrease of FWHM, which lies within the uncertainty of the experimental data. In fact, the two-temperature model predicts a substantial decrease of FWHM for the electron temperature distribution (i.e., apparent negative diffusion) in this transition regime. The effect is less visible after conversion to our observable, $\Delta R/R$, both experimentally and in simulation, presumably because of the influence of the lattice temperature in the crossover regime.

Recent experiments have studied the effects of nonthermal electron distributions, as well as a transition from ballistic to diffusive electron transport (23, 36). It will be interesting to study these effects in real space with SUTM at higher temporal resolution.

In summary, we have tracked thermally induced diffusion in a thin gold film from absorption to thermalization in time and space with femtosecond and nanometer resolution. We resolve a transition from hot-electron to phonon-limited diffusion. We interpret the electronic and phononic cooling regimes with the help of full two-temperature and thermo-optical modeling. The predicted dynamics agree well with the experimental observations. The insight gained on hot-carrier dynamics from direct spatiotemporal imaging is crucial to understand the interplay of electrons and phonons in ultrafast nanoscale photonics and thus to design nanoscale thermal management in nano-optoelectronic devices, such as phase-change memory devices and heat-assisted magnetic recording heads. In particular, the control of heat exchange between the electron and lattice systems is important in device functionality. More generally, the excess energy of hot electrons finds applications in an increasingly wide range of systems, such as thermoelectric devices, broadband photodetectors, efficient solar cells, and even plasmon-enhanced photochemistry. Here, we have applied our method to gold, the "gold" standard for many such applications of electron heat. Certainly, ultrafast thermomodulation...
microscopy is equally suitable to study a vast range of other materials and systems in the future.

MATERIALS AND METHODS

Experimental details

The sample was fabricated by thermal evaporation of 50 nm of gold onto a cleaned glass coverslip. The laser source was a Ti:Sapphire oscillator (Mira 900, Coherent) emitting at 900-nm wavelength with a repetition rate of 76 MHz and a 150-fs pulse duration. We frequency-doubled the laser using a BBO crystal to create the 450-nm pump beam. We compressed the pump and probe pulses individually with two prism compressors made of fused silica and SF10 glass, respectively. We modeled the complex permittivity at the infrared probe wavelength using the thin-film Fresnel equations and extract its spatial dynamics using the same Gaussian fitting as in the experimental data analysis. (B) Predicted temporal evolution of the electron (top) and lattice (bottom) temperatures at the beam center for the three pump fluences used in the experiment. (C) Theoretical (curves) and experimental (symbols) evolution of the squared width of $\Delta R/R$ for different pump fluences $F$ (top). In accordance with Fig. 3B, we identify a fast diffusion regime at high electron temperatures, within the first few picoseconds, followed by a thermalized regime (>5 ps) dominated by phonon-limited transport, with orders-of-magnitude lower diffusivity. The initial slope for a diffusivity of $D = k_c/C_e$ is shown for comparison. Bottom: Difference $\Delta$ between the data and the full model calculation.

Two-temperature model, finite element method implementation

For the two-temperature model calculations (Eq. 2), we used the following parameters and scaling laws: the electronic volumetric heat capacity of gold (27) $C_e(T_e) = \gamma T_e$, with $\gamma = 71 \text{ J m}^{-3} \text{ K}^{-2}$; the electronic thermal conductivity of gold (27) $k_e(T_e,T_l) = k_0 T_e/T_l$, with $k_0 = 317 \text{ W m}^{-1} \text{ K}^{-1}$; the lattice heat capacity and thermal conductivity of gold (37, 38) $C_l = 2.45 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$ and $k_l = 2.6 \text{ W m}^{-1} \text{ K}^{-1}$, respectively; the electron-phonon coupling constant (39) $G = 2.2 \times 10^{10} \text{ W m}^{-3} \text{ K}^{-1}$; and the heat capacity and thermal conductivity of glass (40) $C_g = 1.848 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$ and $k_g = 0.8 \text{ W m}^{-1} \text{ K}^{-1}$, respectively (see note S2 for further details).

Thermo-optical response calculation

We modeled the complex permittivity at the infrared probe wavelength with a Drude model permittivity (41)

$$\varepsilon = \varepsilon_\infty - \frac{\omega_p^2(T_e)}{\omega^2 + \gamma_0^2(T_e, T_l)}$$

Here, $\varepsilon_\infty = 9.5$ is the high-frequency permittivity (41). The plasma frequency $\omega_p$ depends on the lattice temperature $T_l$ due to volume expansion affecting the free conduction band electron density $n_e$. Namely, $\omega_p(T_l) = \sqrt{\varepsilon_\infty/e_0 m_e} \times n_e(T_0)/(1 + |\beta T_l|)$, where $\beta = 4.23 \times 10^{-5} \text{ K}^{-1}$ is the...
thermal expansion coefficient \( \beta_0 \), 
\[ n_c(T_0) = 5.9 \times 10^{22} \text{ cm}^{-3} \]
is the unperturbed density \( \rho_0 \), 
\[ n_{\text{eff}} \approx n_e \]
is the effective electron mass, 
\[ \epsilon_v \]
is the vacuum permittivity, and 
\[ m_e \]
is the electron charge and mass, respectively. Further, \( \gamma_{\text{ep}} \) represents the total rate of relaxation collisions that conserve momentum and energy of the electron subsystem, given by 
\[ \gamma_{\text{ep}}(T_e) = \gamma_{\text{ep-ph}}(T_e) + \gamma_{\text{ep-\text{Um}}}(T_e), \]
where \( \gamma_{\text{ep-ph}} \) is the electron-phonon collision rate, depending on the lattice temperature as 
\[ \gamma_{\text{ep-ph}}(T_e) = B T_e, \]
and \( \gamma_{\text{ep-\text{Um}}} \) is the Umklapp electron-electron collision rate, depending on \( T_e \) as 
\[ \gamma_{\text{ep-\text{Um}}}(T_e) = \Delta_{\text{Um}} A T_e^2, \]
with 
\[ A = 1.7 \times 10^{-2} \text{ K}^{-2} \text{ cm}^{-1}, B = 1.45 \times 10^{11} \text{ K}^{-1} \text{ s}^{-1}, \]
and \( \Delta_{\text{Um}} = 0.77 \) (see note S3 and fig. S2) \((43 - 44)\).

**SUPPLEMENTARY MATERIALS**

Supplementary material for this article is available at http://advances.sciencemag.org/cgi-content/full/5/5/eaae9865/DC1

Note S1. General diffusion model

Note S2. Two-temperature model

Note S3. Thermo-optical response

Note S4. Calculation of the transient reflectivity

Fig. S1. Long-time dynamics of \( \Delta R/R \).

Fig. S2. Scheme of thermo-optical calculations.

Fig. S3. Schematic of the SUTM setup.

Fig. S4. Different contributions to the thermo-optical response.

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Supplementary Materials for

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Note S1. General diffusion model

As stated in the main text, we estimate diffusivity, or diffusion coefficient, in a first approximation from the slope of the FWHM vs. time plot, before the more rigorous modeling described thereafter. Thus, to quantify a general diffusion process with a time-dependent diffusivity $D(t)$, we assume that the absorbed pump power induces a Gaussian energy distribution at the sample

$$u(x, t = 0) \propto \exp\left(-4\ln2 \frac{|x|^2}{\text{FWHM}^2_{\text{pump}}} \right),$$

where $x \in \mathbb{R}^n$ ($n = 1, 2, \text{or } 3$, spatial dimension) and that $u$ evolves according to a heat equation (33)

$$\frac{\partial u(x, t)}{\partial t} = D(t) \nabla^2 u(x, t) \tag{S1}$$

or, alternatively, a heat equation including a decay with a rate $\Gamma = 1/\tau$

$$\frac{\partial u(x, t)}{\partial t} = D(t) \nabla^2 u(x, t) - \Gamma u(x, t) \tag{S2}$$

Interestingly, both solutions of $u(x, t)$ to equations S1 and S2 are temporally decaying spatial Gaussian profiles with a time-dependent width. The full-width at half-maximum, measured in any one dimension, FWHM$(t)$, is related to $D(t)$ via

$$\frac{\partial \text{FWHM}^2(t)}{\partial t} = 16\ln2 D(t) \tag{S3}$$

This result is the generalization of the more common result known for a time-independent diffusivity $D$, $\sigma^2(t) = \sigma^2(0) + 2D \cdot t$, and converting the Gaussian width parameter $\sigma$ to FWHM using FWHM $= 2\sqrt{2\ln2} \sigma$.

Under the assumption that our observed profiles $\Delta R/R(x, t)$ scale linearly with $u(x, t)$, we can determine $D(t)$ from our space-time-resolved data. We note that this derivation is often shown for the $n$-dimensional mean square displacement $\langle x^2 \rangle$, which is dependent on the dimensionality $n$, yet the projection onto one axis, or a 1D “cutline” of the profile, is independent of $n$.

Note S2. Two-temperature model

In this model, the electron and lattice temperature $T_e$ and $T_l$ are assumed to be time- and position-dependent, with their spatio-temporal behavior being described by the following coupled equations (31)

$$C_e \left( \frac{\partial T_e}{\partial t} \right) = \nabla (k_e(T_e) \nabla T_e) - G(T_e - T_l) + S - S_{es} \tag{S4a}$$

$$C_l \left( \frac{\partial T_l}{\partial t} \right) = \nabla (k_i \nabla T_l) + G(T_e - T_l) - S_{ls} \tag{S4b}$$

$$C_s \left( \frac{\partial T_s}{\partial t} \right) = \nabla (k_s \nabla T_s) + S_{ls} + S_{es} \tag{S4c}$$
with $C_e(T_e) = \gamma T_e$, $\gamma = 71 \text{ Jm}^{-3}\text{K}^{-2}$, for the electron heat capacity (27); $C_l = 2.45 \cdot 10^6 \text{ Jm}^{-3}\text{K}^{-1}$ for the lattice heat capacity (37); $k_e(T_e, T_l) = k_0 T_e / T_l$, with $k_0 = 317 \text{ Wm}^{-1}\text{K}^{-1}$, for the electron thermal conductivity (27); $k_l = 2.6 \text{ Wm}^{-1}\text{K}^{-1}$, for the lattice thermal conductivity (38); $C_e = 1.848 \cdot 10^6 \text{ Jm}^{-3}\text{K}^{-1}$ for the glass substrate heat capacity (40); $k_e = 0.8 \text{ Wm}^{-1}\text{K}^{-1}$ for the substrate thermal conductivity (40); $G = 2.2 \cdot 10^{16} \text{ Wm}^{-3}\text{K}^{-1}$ for gold electron-phonon coupling coefficient (39); $S_{es/ls} = G_{es/ls}(T_e/T_l - T_s)$ represents the boundary interface heat exchange between the gold electron/lattice and the glass substrate, where $G_{es} = (96.12 + 0.189 T_e) \text{ MWm}^{-2}\text{K}^{-1}$ and $G_{ls} = 141.5 \text{ MWm}^{-2}\text{K}^{-1}$ are the corresponding thermal boundary conductance, respectively (34). In Eq. $S_{4a}$, the source term is defined for a Gaussian beam excitation as

$$S(r, t) = \frac{S_a}{\delta} F \cdot \exp\left[-\frac{z}{\delta} - (4 \ln 2) \frac{x^2 + y^2}{(\text{FWHM}_p^2)}\right] \cdot \frac{1}{t_{\text{pump}}} \cdot \frac{4 \ln 2}{\pi} \exp\left[-(4 \ln 2) \frac{(t - 2 t_{\text{pump}})^2}{t_{\text{pump}}^2}\right]$$

with a full-width at half-maximum FWHM$_{pump} = 0.6 \mu\text{m}$, and a pulse duration $t_{\text{pump}} = 0.2$ ps. The absorbance $S_a = 1 - R - T = 0.5$ and the penetration depth $\delta = 18.7$ nm are estimates based on tabulated optical data for gold at the pump wavelength of 450 nm (41), with $R$ and $T$ being the thin film reflectivity and transmissivity, respectively. $F$ is the pump laser fluence. Note that the $z=0$ plane indicates the air-gold interface.

**Note S3. Thermo-optical response**

Here, we describe the model used to describe the electron and lattice temperature dependence of the permittivity (or dielectric function) $\varepsilon$.

The dielectric function of the gold film as a function of electron and lattice temperature at the probe wavelength ($\lambda_{\text{probe}} = 900$ nm) is well described by the Drude model (41), in which electrons are considered as free charges moving in response to an optical field oscillating at angular frequency $\omega$. Using a classical equation of motion, the dielectric response of the plasma was shown to be

$$\varepsilon_{\text{intra}} = \varepsilon_\infty - \frac{\omega_p^2(T_l)}{\omega(\omega + i \gamma_p(T_e, T_l))}$$

with the plasma frequency $\omega_p(T_l)$ being

$$\omega_p(T_l) = \frac{e^2}{\sqrt{\varepsilon_0 m_{\text{eff}}}} \cdot n_e(T_l)$$

Here, $e$ is the electron charge; $n_e$ is the free electron density in the conduction band ($n_e = 5.9 \cdot 10^{22} \text{ cm}^{-3}$ for Au (42)); $\varepsilon_0$ is the dielectric permittivity of the vacuum; $m_{\text{eff}}$ is the effective mass (estimated as equal to the electron mass, $m_e$); and $\gamma_p$ represents the total rate of collisions that conserve the total momentum and/or energy of the electron subsystem, i.e., collisions that cause relaxation of the electron system (in contrast to like regular e-e collisions, which affect the permittivity only indirectly, by causing a thermalization, i.e., a temperature increase). It is given by
\[ \gamma_{re}(T_e, T_i) = \gamma_{e-ph}(T_i) + \gamma_{e-e}^{\text{Um}}(T_e) \]  
\( \text{(S8)} \)

where the subscript \( re \) stands for relaxation, \( \gamma_{e-ph} \) is the electron-phonon (e-ph) collision rate and \( \gamma_{e-e}^{\text{Um}} \) is the Umklapp electron-electron (e-e) collision rate.

\( \varepsilon_\infty \) represents the off-resonant (i.e., high frequency) contribution of the interband transitions (\( \varepsilon_\infty = 9.5 \) for Au (41)). Here, we neglect its dependence on the temperatures which, according to recent results (46, 47), is weak at the 900 nm probe wavelength. In addition, note that for \( 1200 K > T_e > T_{\text{Debye}}, T_i \approx 300 K \), one can neglect the temperature dependence of \( m_{\text{eff}} \) (48). Thus, heating of a metal affects the intraband contribution to the dielectric constant only via two parameters, the plasma frequency \( \omega_p \) and the constituents of the electron relaxation rate \( \gamma_{re} \). We now specify in detail the temperature dependence of these parameters. The e-ph scattering rate, \( \gamma_{e-ph} \), is given by \( \gamma_{e-ph}(T_i) = B T_i \), where for Au, \( B \approx 1.45 \cdot 10^{12} \text{K}^{-1} \text{s}^{-1} \) (43).

\( \gamma_{e-e}^{\text{Um}} \) represents the intraband transition rate due to Umklapp e-e scattering events. In normal e-e scattering, the momentum is conserved so that it does not contribute directly to the intraband collision rate, \( \gamma_{re} \). However, these collisions do cause the electron system to thermalize, hence, the electron temperature to increase, so that they contribute to the permittivity indirectly through the temperature dependence of other quantities in Eq. S6. Umklapp e-e scattering processes, however, impart momentum to the lattice as a whole, thus, the total electron momentum is not conserved, whereas the quantity that is conserved is the quasi-momentum. Violation of momentum conservation leads to contribution to the collision rate (49). As shown by Kaveh and Wiser (50), within the framework of Fermi Liquid Theory (51), it equals the fraction \( \Delta^{(\text{Um})} \) of the total e-e scattering rate, \( \gamma_{e-e} \), i.e.

\[ \gamma_{e-e}^{\text{Um}}(T_e) = \Delta^{(\text{Um})}\gamma_{e-e} \]  
\( \text{(S9)} \)

where \( \gamma_{e-e} = A T_e^2 \), and \( A = K(\pi k_B)^2 / 2 \). \( K = m_e^3 W_{e-e}/(8 \pi \hbar^6) \) is the characteristic e-e scattering constant that contains the angular-averaged scattering probability \( W_{e-e} \) (51); \( k_B \) is the Boltzmann constant. For Au, \( A \approx 1.7 \cdot 10^7 \text{K}^2 \text{s}^{-1} \) (43) and \( \Delta^{(\text{Um})} \approx 0.77 \) (44). Thus, \( \gamma_{e-e}^{\text{Um}} \) depends quadratically on \( T_e \) (52).

The plasma frequency is affected by the temperature rise via a change of \( n_e \) due to volume expansion of the metallic system (53). For the near-infrared probe wavelength, one can safely neglect the possibility of the density of the electrons in the conduction band to increase due to interband transitions. The volume reads

\[ V(T_i) = V_{eq}(1 + \beta \Delta T_i) \]  
\( \text{(S10)} \)

where \( V_{eq} \) is the equilibrium volume of the metallic system at ambient temperature \( T_0 \) and \( \beta \) is the volume thermal expansion coefficient. The thermal expansion coefficient is directly related to the Gruneisen constant \( \gamma_G \) according to the following equation (54): \( \beta \approx \gamma_G C_i / B_b \), where \( B_b \) is the bulk modulus. Inserting the values for \( \gamma_G \) and \( B_b \) for Au (55), the thermal expansion coefficient is estimated...
as $\beta = 4.23 \cdot 10^{-5} \text{ K}^{-1}$. We assume that the total number of electrons in the conduction band, $N_e$, is independent of temperature

$$N_e = n_e(T_0) V_{eq} \approx n_e(T_i) V(T_i)$$  \tag{S11}$$

so that the density is given by

$$n_e(T_i) = \frac{n_e(T_0)}{1 + \beta \Delta T_l}$$  \tag{S12}$$

Substitution of Eq. S12 in Eq. S7 leads to (S3, S6)

$$\omega_p(T_i) = \left( \begin{array}{c} \frac{\omega_p(T_0)}{1 + \beta \Delta T_l} \\ \sqrt{\epsilon_0 \mu_{eff}} \cdot \frac{n_e(T_0)}{1 + \beta \Delta T_l} \end{array} \right)$$  \tag{S13}$$

or

$$\omega_p(T_i) = \left( \begin{array}{c} e^2 \\ \sqrt{\epsilon_0 \mu_{eff}} \cdot \frac{n_e(T_0)}{1 + \beta \Delta T_l} \end{array} \right)$$  \tag{S14}$$

The dynamics of the change to the metal permittivity ensuing from the temperature dependence obtained from Eqs. S4 and Eqs. S6 - S11 are seen in Suppl. Fig. S4. One can see that the change of the imaginary part of the metal permittivity is much bigger than the change of the real part. Indeed, it can be shown that the observed increase of $\text{Im}(\Delta \varepsilon_{\text{intra}})$ causes the reflectivity to decrease; this explains the decrease-increase pattern observed in Fig. 2B and Suppl. Fig 2.

Another interesting observation is that the contribution to the change of the permittivity from the lattice heating becomes dominant over the contribution from the electron heating much earlier than the stage when the electron temperature cools down to the lattice temperature (i.e., for $T_e \gg T_l$). This happens because the derivative of the metal permittivity with respect to $T_l$ is much higher than with respect to $T_e$.

**Note S4. Calculation of the transient reflectivity**

From the spatiotemporal maps of $T_e(x, t)$ and $T_l(x, t)$, we first calculate the reflectivity $R(T_e, T_l)$, and then the transient reflectivity $\Delta R/R = [R(T_e, T_l) - R(T_0, T_0)] / R(T_0, T_0)$. We use the thin-film Fresnel equations (S58) for the reflection by a thin film (thickness $h$, refractive index $n_2 = \sqrt{\varepsilon}$ for gold (41)) under perpendicular incidence from air ($n_1 = 1$), with the film supported on a glass substrate ($n_3 = 1.5$)

$$R = |r|^2$$

with

$$r = \frac{r_{12} + r_{12} \cdot \exp(2i\beta)}{1 + r_{12} \cdot r_{23} \cdot \exp(2i\beta)}$$  \tag{S15}$$
where $\beta = 2\pi n_2 h/\lambda_0$; $r_{12} = (n_1 - n_2)/(n_1 + n_2)$; $r_{23} = (n_2 - n_3)/(n_2 + n_3)$.

We note that as an alternative approach, we have also calculated $R$ from finite $z$-slices of the 3D permittivity $\varepsilon(x,y,z)$, extracted directly from the 3D finite element method calculation mesh $T_e(x,y,z)$ and $T_l(x,y,z)$, using the transfer matrix method. As this calculation yielded nearly the same results as the simpler version (Eq. S15, assuming homogeneous permittivity along the film thickness $z$), we trust the validity of this approach.

**Fig. S1. Long-time dynamics of $\Delta R/R$.** This is an extension of Fig. 2B from the main text. (A) The red line shows the biexponential fit to extract the two decay parameters, where errors are 68% confidence intervals. (B) Same as A, on linear-log scale. Dashed lines show the exponential decay parameters taken from the above fit.
We calculate the temporal evolution of the spatial temperature profiles for the electron and lattice subsystems after optical excitation. Next, we calculate the spatio-temporal permittivity from the two-temperature maps using a temperature dependent Drude model described in Note S3. Finally, we convert the resulting permittivity to a transient reflectivity using the Fresnel coefficients for thin films (see Note S4). The evolution of the spatial width is calculated by fitting the $\Delta R/R$ maps to Gaussians.
**Fig. S3.** Schematic of the SUTM setup. BBO: \(\beta\)-Barium borate crystal. The polarization of both pump and probe beams are linear and in the same plane. Both beams enter the objective with perpendicular angle of incidence with respect to the sample plane. The sample consists of a glass substrate (facing up) and air below the evaporated gold surface. Further description of the experiment is found in the main text.

**Fig. S4.** Different contributions to the thermo-optical response. Both graphs show the temporal dynamics of the electronic and lattice temperatures (dashed red and blue lines, respectively). We show the corresponding dynamics of the (A) imaginary and (B) real parts of the metal permittivity change \(\Delta\varepsilon_{\text{intra}}\) for \(\lambda_{\text{probe}} = 900\text{nm}\). Umklapp e-e scattering, e-ph scattering, volume thermal expansion and the sum of the effects are represented respectively by green, cyan, magenta and black solid curves.