Proportional-Derivative State-Feedback Control for Singular Systems With Input Quantization

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This work was supported in part by the National Research Foundation of Korea (NRF) through the Basic Science Research Program funded by the Ministry of Education under Grant 2020R1F1A1064330, and in part by Yeungnam University, through a research grant, in 2019.

ABSTRACT This paper deals with an admissibilization problem of singular systems with uniform input quantization. The aim is to design a controller to guarantee the admissibility of the closed-loop system. To achieve this, this paper proposes a proportional-derivative state-feedback controller which includes non-linear control part to reject the effect of uniform input quantization. Based on the proposed controller, sufficient conditions are obtained in terms of linear matrix inequalities. Two examples show the feasibility of the proposed controller.

INDEX TERMS Singular system, input quantization, admissibility, linear matrix inequalities.

I. INTRODUCTION

In the field of control theory, to effectively handle the problems such as stability analysis or controller synthesis, the dynamic systems are generally modeled as the state-space systems which consist of the first-order differential equations of the system states. In the real world, however, there are many cases where algebraic equations as well as differential equations are required to model the practical systems. For example, since the interconnection of system states can be represented as algebraic equations, both differential and algebraic equations are required when modeling the dynamic systems in which interconnection of system states exist such as large-scale power systems [1].

Singular systems, which are also referred to as differential-algebraic equation systems or descriptor systems, include both differential and algebraic equations. For this reason, the singular systems have drawn extensive consideration of researchers and have been used in many practical systems [2]–[4]. Apart from their practical importance, they are of theoretical significance and have attracted a lot of attention due to the fact that there is a fundamental difference from the state-space systems. Based on the state-space system theory, many researches have been actively extended to the singular systems, such as stability analysis [5], [6], controller synthesis [7], $H_{\infty}$ control [8], $H_{\infty}$ filtering [9], and dissipativity analysis [10]. At this time, the important thing in the study of the singular systems is to ensure admissibility: regularity, impulse-freeness, and stability.

In modern control theory, on the other hand, there are systems that require analog-to-digital and digital-to-analog converters or encoders and decoders, such as network control systems (NCSs), cyber-physical-systems (CPSs). These cause quantization errors which are closely related to the general performance of the systems. Sometimes these can also cause stable closed-loop system unstable [11]. For this reason, the stabilization problems of the systems with quantized input have been widely studied for various systems [12]–[16]. In the existing studies, there are two main types of static quantizers: logarithmic and uniform. The authors of [12] introduced the sector bound approach to state- and output-feedback control for linear systems with logarithmic quantizer. On the other hand, a simple but powerful method for eliminating the effect of uniform input quantization via state-feedback control was introduced in [17]. Also, the authors of [18] proposed dynamic output-feedback control for linear system with uniform input quantization. Obviously, a study considering input quantization for the singular systems is necessary, and such studies have been recently researched. In [19], network-based event-triggered control for singular systems with logarithmic quantization was introduced. The authors of [20] proposed state-feedback control for singular Markovian jump systems with input quantization. The proposed controller in [20] guarantees that the closed-loop
system is regular and stable. To the best of our knowledge, however, there is no related articles considering the admissibility conditions for the singular systems with uniform input quantization which is the motivation behind this study.

This paper proposes the state-feedback control for the singular systems with uniform input quantization. The proposed controller comprises the main and nonlinear control parts. The main control part is designed as a proportional-derivative (PD) state-feedback controller for guaranteeing the admissibility of the closed-loop system and the nonlinear control part leads to eliminate the effect of uniform quantization. Based on the proposed controller, the admissibilization conditions are obtained in terms of LMIs. Finally, two examples are provided to demonstrate the validity of the proposed PD state-feedback control with additional nonlinear control part.

The notations used in this paper are fairly standard. For vector or matrix $x$, the superscript $T$ denotes its transpose. For the symmetric matrices $X$ and $Y$, $X \geq (\geq)Y$ means that $X-Y \geq (\geq)Y$ is positive (semi-) definite. The notation $E \in \mathbb{R}^{n \times m}$ means the sum of itself and its transpose, i.e., $E^T = E$. The notation $\text{round}(\cdot)$ represents an ellipsis of the terms which can be induced by symmetry. The notation $e_k$ indicates a unit vector with a single non-zero entry at the $k$th position, i.e. $e_k = [0 \ldots 1_{(k\text{-th component)}} \ldots 0]^T$.

II. PROBLEM STATEMENT

Consider a singular system such that

$$E \dot{x}(t) = Ax(t) + BQ(u(t)), \quad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ denote system state, control input, respectively, and matrix $E$ is supposed to be singular, i.e., $\text{rank}(E) = r < n$. Further, $Q(\cdot)$ is the quantization operator defined by a function $\text{round}(\cdot)$ that rounds a number to the nearest integer, i.e.,

$$Q(u(t)) \triangleq e_u \text{round}(u(t)/e_u), \quad (2)$$

where $e_u(>0)$ is called a quantization level and $Q(\cdot)$ is the uniform quantizer with the fixed $e_u$. We note that the quantization error $\nabla u(t)$ is defined as $\nabla u(t) \triangleq Q(u(t)) - u(t)$. From the definition of $Q(u(t))$ and $\nabla u(t)$, it can be seen that each component of $\nabla u(t)$ at time $t$ is bounded by the half of the quantization level $e_u$, i.e.,

$$\|\nabla u(t)\|_\infty \leq e_u/2. \quad (3)$$

Also, the system (1) can be described by

$$E \dot{x}(t) = Ax(t) + B(u(t) + \nabla u(t)). \quad (4)$$

The definition below generalizes the concept of admissibility for descriptor systems.

**Definition 1 [3]:**

a. The singular system (1) is said to be regular if, for all $Ex(0-) \rightarrow U(s)$, the state $x(t)$ can be uniquely determined that means $\text{det}(sE-A)$ is not identically zero.

b. The singular system (1) is said to be impulse-free if, the unique solution $x(t)$ does not have any Dirac impulse function $\delta(\cdot)$ that is equivalent to $\text{deg}(\text{det}(sE-A)) = \text{rank}(E)$.

c. The singular system (1) is said to be stable if, for any $x_0 \in \mathbb{R}^n$, there exists a scalar $M(x_0) > 0$ such that

$$\int_0^\infty \|x(t)\|^2 dt \leq M(x_0). \quad (5)$$

d. The singular system (1) is said to be admissible if it is regular, impulse-free and stable.

The aim of this paper is to design a PD state-feedback control which has the following form:

$$u(t) = Kx(t) - K_D \dot{x}(t) + u_c(t) \quad (6)$$

where $u_c(t)$ is an additional nonlinear control part to reject the effect of quantization error. From (1) and (6), the closed-loop system can be described as

$$(E + BK_D) \dot{x}(t) = (A + BK)x(t) + B(u_c(t) + \nabla u(t)). \quad (7)$$

Remark 1: If the derivative matrix $(E + BK_D)$ is nonsingular, then the closed-loop system (7) can be rewritten as

$$\dot{x}(t) = (E + BK_D)^{-1} \times [(A + BK)x(t) + B(u_c(t) + \nabla u(t))] \quad (8)$$

Since the system (8) has unique solution for any initial condition, (8) is regular system. Besides, since all the eigenvalues of the system (8) are finite value, the system (8) is impulse-free. Therefore, if the system (8) could be stable with nonsingular matrix $E + BK_D$, then the closed-loop system (8) is admissible.

III. MAIN RESULTS

In this section, the admissibility problem of the singular system (1) with uniform input quantization is considered. First, the admissibility conditions for the closed-loop system (7) will be introduced. Then, the LMI conditions for the obtained admissibility conditions will be derived in the next subsection.

A. ADMISSIBILITY ANALYSIS FOR CLOSED-LOOP SYSTEM

To facilitate the derivation of the following theorem, we redefine the closed-loop system as

$$\tilde{E} \dot{x}(t) = \tilde{A}x(t) + B(u_c(t) + \nabla u(t)) \quad (9)$$

where $\tilde{A} = A + BK$, $\tilde{E} = E + BK_D$. Also, for slack matrices $S_1$ and $S_2$ with appropriate dimensions, it follows from (9) that

$$S(t) \triangleq \text{He} \left[(S_1 \dot{x}(t) + S_2 \dot{x}(t))^T \times (\tilde{E} \dot{x}(t) - \tilde{A}x(t) - B(u_c(t) + \nabla u(t))\right] \equiv 0. \quad (10)$$

**Theorem 1:** The singular system (1) with quantized input is admissible via the PD state-feedback controller (6), if there
exist symmetric matrix \( P \in \mathbb{R}^{n \times n} \) and matrices \( K \in \mathbb{R}^{m \times n} \), \( K_D \in \mathbb{R}^{m \times n} \), \( S_1 \in \mathbb{R}^{n \times n} \) and \( S_2 \in \mathbb{R}^{n \times n} \) such that
\[
0 < P, \quad 0 > \begin{bmatrix}
\Sigma_{11} \\
(*)
\end{bmatrix} \Sigma_{12}
\begin{bmatrix}
(*) \\
\Sigma_{22}
\end{bmatrix}
\]  
(11)

where
\[
\Sigma_{11} = \mathbf{He}(-S_1^T (A + BK)), \quad \Sigma_{12} = P - (A + BK)^T S_1 + S_2^T (E + BK_D), \quad \Sigma_{22} = \mathbf{He}(S_1^T (E + BK_D)).
\]
(12)

Further, each component of the nonlinear control part \( u_c(t) \) is defined as
\[
e_k^T u_c(t) = -\epsilon_u \text{sgn}(e_k^T \sigma(t)) \text{max}(0, \text{sgn}(\sigma^T(t) \nabla u(t))).
\]
(16)

where
\[
\sigma(t) \triangleq -B^T (S_1 \dot{x}(t) + S_2 x(t)).
\]
(17)

Proof: Let us choose a Lyapunov function candidate \( V(t) = x^T(t) P x(t) + S_1(t) \), where \( P \) is a positive definite matrix (11). Combining the condition (10) and the derivative of \( V(t) \) with respect to time gives
\[
\dot{V}(t) = 2 \dot{x}^T(t) P x(t) + S_1(t)
\]
(18)
\[
= x^T(t) \Sigma_{11} x(t) + \mathbf{He}(x^T(t) \Sigma_{12} \dot{x}(t))
\]
\[
+ x^T(t) \Sigma_{22} \dot{x}(t) + 2 \sigma^T(t) \nabla u(t) + u_c(t).
\]
(19)

Using the conditions (3) and (16), it is shown that \( u_c(t) \) ensures that the last term in (19) is negative, i.e.,
\[
\bullet \sigma^T(t) \nabla u(t) \leq 0
\]
(20)
\[
\bullet \sigma^T(t) \nabla u(t) > 0
\]
(21)

Then, the derivative of \( V(t) \) can be rewritten as
\[
\dot{V}(t) \leq \begin{bmatrix}
x(t) \\
\dot{x}(t)
\end{bmatrix}^T \begin{bmatrix}
\Sigma_{11} \\
(*)
\end{bmatrix} \begin{bmatrix}
\Sigma_{12} \\
(*)
\end{bmatrix} \begin{bmatrix}
x(t) \\
\dot{x}(t)
\end{bmatrix}.
\]
(22)

Therefore, the stability criterion is given by (12). Further, as explained in Remark 1, the stability of the system (8) guarantees the admissibility of the closed-loop system. Thus, the PD state-feedback controller (6) with the conditions (11) and (12) ensures the admissibility of the closed-loop system (7).
for the singular system with uniform input quantization (1) in terms of LMIs.

Remark 4: From (3) and (4), it can be seen that the system has a bounded matched disturbance. At this time, the proposed nonlinear control part provides an effective and robust means of controlling dynamic systems with bounded and matched disturbance [14]. That is, the nonlinear control part enables us to eliminate the energy in the sense of Lyapunov function caused by the quantization error $\nabla u(t)$.

IV. EXAMPLES

In this section, two examples will be given to verify the validity of the proposed PD state-feedback controller (6).

Example 1: Consider the singular system (1) with input quantization, whose system matrices are

$$A = \begin{bmatrix} 0.5 & 0 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$ (32)

Also, it is assumed that the quantization level $\epsilon_u$ is given as 0.01. The state trajectories of the unforced system are shown in Fig. 1. It can be known that the unforced system is unstable. On the other hand, by Theorem 2, the controller gain matrices are obtained as

$$K = \begin{bmatrix} 1.2868 & 2.3286 \end{bmatrix}, \quad K_D = \begin{bmatrix} -4.3392 & 2.1942 \end{bmatrix}.$$ (33) (34)

Fig. 2 shows the state trajectories of the closed-loop system. It can be seen that all of the states go to zeros, which guarantees that the proposed controller successfully stabilizes the singular system with input quantization.

Example 2 (Practical Example): In this example, we will consider an oil catalytic cracking process [21], [22] as follows:

$$\begin{align*}
\dot{x}_1(t) &= R_{11}x_1(t) + R_{12}x_2(t) + B_1Q(u(t)), \\
0 &= R_{21}x_1(t) + R_{22}x_2(t) + B_2Q(u(t)),
\end{align*}$$ (35)

where $x_1(t)$ denotes a state vector to be regulated, such as blower capacity, regenerate temperature, or valve position; $x_2(t)$ denotes a state reflecting business benefits, administration, etc.; $u(t)$ denotes regulation value. Let $R_{11} = 0.6, R_{12} = 0, R_{21} = 0.4, R_{22} = 0.5, B_1 = -1.1$, and $B_2 = 0.9$. Using the above parameters, the system (35) can be expressed as

$$E\ddot{x}(t) = Rx(t) + BQ(u(t)),$$ (36)

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0.6 & 0 \\ 0.4 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -1.1 \\ 0.9 \end{bmatrix}.$$ (37)

Also, it is assumed that the quantization level $\epsilon_u$ is given as 0.01. By Theorem 2, the controller gains are obtained as follows:

$$K = \begin{bmatrix} -44.3236 & -12.7626 \end{bmatrix}, \quad K_D = \begin{bmatrix} 6.1939 & 4.4000 \end{bmatrix}.$$ (38)
Fig. 3 illustrates the state trajectories of the closed-loop system and Fig. 4 presents the quantized control input for the proposed PD state-feedback controller. From the figures, it can be seen that the proposed controller effectively stabilizes the oil catalytic cracking process system with uniform input quantization.

V. CONCLUSION

This paper has considered the PD state-feedback control for singular system with uniform input quantization. In contrast to the previous research work about quantized feedback control for singular systems, the proposed controller guarantees the admissibility of the closed-loop system in spite of the error of uniform quantizer. Two examples were provided to demonstrate the validity of the proposed controller. In the future work, we will extend our research into two perspectives. The first is to change the type of the controller such as output-feedback control, and the second is to design a controller applicable to the stochastic systems such as singular Markovian jump systems.

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