Dissipation in Quantum Mechanics, Scalar and Vector Field Theory

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July 31, 2018

Abstract

A new minimal coupling method is introduced. A general dissipative quantum system is investigated consistently and systematically. Some coupling functions describing the interaction between the system and the environment are introduced. Based on coupling functions, some susceptibility functions are attributed to the environment explicitly. Transition probabilities relating the way energy flows from the system to the environment are calculated and the energy conservation is explicitly examined. This new formalism is generalized to the dissipative scalar and vector field theories along the ideas developed for the quantum dissipative systems.

Keywords: Dissipative Quantum Systems, Field Quantization, Environment, Scalar and Vector Fields, Coupling Functions, Transition Probabilities

1 Introduction

There are basically two approaches to study dissipative quantum systems. One is found in the interactions between two systems via an irreversible energy flow. The second approach is a phenomenological treatment under

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the assumption of nonconservative forces [3, 4]. In studying nonconservative
systems, it is essential to introduce a time dependent Hamiltonian which
describes the damped motion. Such a phenomenological approach for the study
of dissipative quantum systems, especially a damped harmonic oscillator, has
a rather long history. Caldirola and Kanai [5, 6] adopted the Hamiltonian

\[ H(t) = e^{-\beta t} \frac{p^2}{2m} + e^{\beta t} \frac{1}{2} m \omega_0^2 q^2, \]  

which leads exactly to the classical equation of motion of a damped harmonic
oscillator

\[ \ddot{q} + \beta \dot{q} + \omega_0^2 q = 0. \]

The quantum aspect of this model has been studied in a great amount of
literature. In those studies some peculiarities of this model and some fea-
tures of it have appeared to be ambiguous [7]-[15]. There are significant
difficulties in obtaining the quantum mechanical solutions for the Caldirola-
Kanai Hamiltonian. Quantization with this Hamiltonian violates the uncer-
tainty relations, i.e., the uncertainty relations vanish as time goes to infinity
[16]-[19].

Based on Caldirola-kanai Hamiltonian, it has been constructed equivalent
theories by performing a quantum canonical transformation and has been
used the path integral techniques to calculate the exact propagators of such
theories, also the time evolution of given initial wave functions, have been
studied using the obtained propagators [20].

In the framework of the phenomenological approach Lopez and Gonza-
lez [21] have taken the external non conservative forces that has linear and
quadratic dependence with respect to velocity. They have deduced classical
constants of motion and Hamiltonian for these systems and eigenvalues of
these constants have been quantized through perturbation theory.

A simple pseudo-Hamiltonian formulation has been proposed by Kupriyanov
and et al [22]. Starting from this pseudo- Hamiltonian formulation a con-
sistent deformation quantization has been developed that involve a non-
stationary star product and an extended operator of time derivative dif-
gerentiating the star product. A complete consistent quantum-mechanical
description for any linear dynamical system with or without dissipation has
been constructed in this scheme.

In another approach to quantum dissipative systems, one tries to bring about
the dissipation as a results of an averaging over all the coordinates of the bath
system, where one considers the whole system as composed of two parts, our main system and the bath system which interacts with the main system and causes the dissipation of energy on it [23]-[30].

The macroscopic description of a quantum particle with passive dissipation and moving in an external potential $v(x)$ is formulated in terms of Langevin-Schrödinger equation [31, 32, 33]

$$m\ddot{x} + \int_0^t dt' \mu(t - t')\dot{x}(t') = -\nabla v(x) + F_N(t).$$  \hspace{1cm} (3)

The coupling with the heat-bath in microscopic levels corresponds to two terms in macroscopic description. A mean force characterized by a memory function $\mu(t)$ and an operator valued random force $F_N(t)$. These two terms have a fluctuation-dissipation relation and both are required for a consistent quantum mechanical description of the particle. In [33], there are some models for interaction of the main system with the heat-bath which lead to the macroscopic Langevin-Schrödinger equation.

The layout of the paper is as follows: In section 2, we consider a particle moving in a one-dimensional external potential and take the absorptive environment of the particle as a Klein-Gordon field which interact with the momentum of the particle through a minimal coupling term. In this approach, the Langevin-Schrodinger equation [31] is obtained as the macroscopic equation of motion of the particle and the noise force $F_N(t)$, is derived in terms of the coupling function and the ladder operators of the environment. By choosing a special form for the coupling function, a friction force proportional to the velocity of the particle is obtained. For an initially exited one dimensional damped harmonic oscillator, it is shown that the entire energy of the oscillator will be absorbed by its environment. In section 3, we are concerned with a particle moving in three-dimensional absorptive environment. In this section we model the environment by two independent Klein-Gordon fields $B$ and $\tilde{B}$. The field $B$ interacts with the momentum of the particle through a minimal coupling term and the field $\tilde{B}$, interacts with the position operator of the particle similar to a dipole interaction term. A generalized Langevin-Schrodinger equation is obtained which contains two memory functions describing the absorption of energy of the particle by its environment. As an simple example, the spontaneous decay constant and the shift frequency of a two-level system, embedded in an absorptive environment, is calculated.
In sections 4 and 5, the ideas set up in sections 2 and 3, are applied to quantum dissipative systems described by scalar or vector fields.

2 One-dimensional quantum dissipative systems

2.1 Quantum dynamics

Quantum mechanics of a one-dimensional damped system can be investigated in a systematic and consistent way by modeling the environment of the system with a quantum field $B$ which interacts with the system through a minimal coupling term. For this purpose, let the damped system be a particle with mass $m$, under the influence of an external potential $v(q)$. We take the total Hamiltonian of the system and its environment as

$$H = \frac{(p - R)^2}{2m} + v(q) + H_B,$$

(4)

where $q$ and $p$ are position and canonical conjugate momentum operators of the particle respectively and satisfy the canonical commutation rule

$$[q, p] = i\hbar.$$

(5)

In Hamiltonian (4), $H_B$ is the Hamiltonian of the environment. Now we model the environment by a massless Klein-Gordon field, such that we can write the environment Hamiltonian as

$$H_B(t) = \int_{-\infty}^{+\infty} d^3k \omega_k b_k^\dagger(t)b_k(t),$$

(6)

where $\omega_k$, is the dispersion relation of the environment. The annihilation and creation operators $b_k$, $b_k^\dagger$, in any instant of time, satisfy the following commutation relations

$$[b_k(t), b_k^\dagger(t')] = \delta(\vec{k} - \vec{k}').$$

(7)

Operator $R$ in relation (4), has the basic role in interaction between the system and its environment and is defined by

$$R(t) = \int_{-\infty}^{+\infty} d^3k [f(\omega_k)b_k(t) + f^*(\omega_k)b_k^\dagger(t)],$$

(8)
where the function $f(\omega_\vec{k})$ is called the coupling function between the particle and its environment. In definition of operator $R$, the following physical assumptions are considered:

1. Since the total Hamiltonian (4) is Hermitian, so the operator $R$ should be a Hermitian operator.

2. We are interested in linear dissipative systems and for such systems, the operator $R$ should be a linear combination of annihilation and creation operators $b_\vec{k}$ and $b^\dagger_\vec{k}$.

3. Operator $R$, should depend on macroscopic characters of the environment, because we are interested in macroscopic equations of motion of the particle. It is remarkable to note that the macroscopic characters of the environment are reflected in its macroscopic susceptibility against the motion of the particle. In the following, we see that the susceptibility, is related to both the dispersion relation $\omega_\vec{k}$ and the coupling function $f(\omega_\vec{k})$ and these considerations are contained in definition of the operator $R$.

The Heisenberg equations for the position and momentum of the particle are

\[
\dot{q} = \frac{i}{\hbar} [H, q] = \frac{p - R}{m},
\]
\[
\dot{p} = \frac{i}{\hbar} [H, p] = -\frac{\partial v}{\partial q},
\]

(9)

where $H$ is the total Hamiltonian (4). By eliminating $p$ from relations (9), we obtain

\[
m \ddot{q} = -\frac{\partial v}{\partial q} - \dot{R}.\]

(10)

Also using (11), the Heisenberg equation for $b_\vec{k}$, is

\[
\dot{b}_\vec{k} = \frac{i}{\hbar} [H, b_\vec{k}] = -i\omega_\vec{k} b_\vec{k} + \frac{i}{\hbar} \dot{q} f^*(\omega_\vec{k}),
\]

(11)

which formally can be solved as

\[
b_\vec{k}(t) = b_\vec{k}(0) e^{-i\omega_\vec{k} t} + \frac{i}{\hbar} f^*(\omega_\vec{k}) \int_0^t dt' e^{-i\omega_\vec{k} (t-t')} \dot{q}(t').
\]

(12)
Now substituting $b_\vec{k}(t)$ from this equation into \(8\), we obtain

$$R(t) = R_N(t) + \int_0^{|t|} dt' \chi(|t| - t') \hat{q}(\pm t'),$$

(13)

where the upper(lower) sign corresponds to $t > 0(t < 0)$, respectively. Inspired by electrodynamics in a media, let us call the memory function

$$\chi(t) = \frac{8\pi}{\hbar} \int_0^\infty d|\vec{k}| |\vec{k}|^2 |f(\omega_{\vec{k}})|^2 \sin \omega_{\vec{k}} t, \quad t > 0$$

$$\chi(t) = 0, \quad t \leq 0$$

(14)

as the susceptibility of the environment. The operator $R_N$

$$R_N(t) = \int_{-\infty}^{+\infty} d^3k [f(\omega_{\vec{k}})b_\vec{k}(0)e^{-i\omega_{\vec{k}} t} + f^*(\omega_{\vec{k}})b_\vec{k}^\dagger(0)e^{i\omega_{\vec{k}} t}],$$

(15)

is a noise operator.

The noise forces are necessary for dealing with dissipative quantum systems in a consistent way, without these noises, some fundamental postulates of quantum mechanics like canonical commutation relations or Heisenberg uncertainty relations usually are violated.

The form of the equation (13) suggests that take this equation as a constitutive equation of the environment and interpret the operator $R$ as some kind of the polarization of the environment, induced by the particle. A feature of this approach is its flexibility to choosing an appropriate dispersion relation $\omega_{\vec{k}}$ and finding the corresponding coupling function for a given susceptibility. In fact, in definition (14) of the susceptibility, there are two main functions, namely the coupling function $f(\omega_{\vec{k}})$ and the dispersion relation $\omega_{\vec{k}}$, if one takes the simplest assumption $\omega_{\vec{k}} = c|\vec{k}|$ for the dispersion relation and write (14) as

$$\chi(t) = \frac{8\pi}{\hbar c^3} \int_0^\infty d\omega \omega^2 |f(\omega)|^2 \sin \omega t, \quad t > 0,$$

$$\chi(t) = 0, \quad t \leq 0,$$

(16)

then by inverting (16), the corresponding coupling function for a given susceptibility, can be obtained as

$$|f(\omega)|^2 = \frac{\hbar c^3}{4\pi^2 \omega^2} \int_0^\infty dt \chi(t) \sin \omega t, \quad \omega > 0,$$

$$|f(\omega)|^2 = 0, \quad \omega = 0.$$

(17)
Other choices for the dispersion relation, just lead to a redefinition of the coupling function and also more difficult mathematical expressions which basically may not allow to take an inverse of (14) and find the corresponding coupling function in terms of the susceptibility \( \chi(t) \). It is remarkable to note that one can take the real dispersion relation of the medium and enrich the model in this way but in this approach this is only a choice. From now on, we take a linear dispersion relation as \( \omega_k = c|\vec{k}| \).

Substituting \( R \) from (13) into (10), one can show easily that the equation of motion for \( q \) is

\[
m\ddot{q} \pm \frac{d}{dt} \int_0^{t_1} dt' \chi(|t| - t') \dot{q}(\pm t') = -\frac{\partial v}{\partial q} - \dot{\hat{R}}_N,
\]

this is the macroscopic Langevin-Schrödinger equation for a dissipative system [33]. If we take the Langevin-Schrödinger equation as the basic macroscopic equation of quantum dissipative systems, then the microscopic interaction of the particle with its environment can be macroscopically formulated by modeling the environment with a quantum field \( B \) and using the Hamiltonian (4), which finally leads to the correct macroscopic Langevin-Schrödinger equation.

In fact, the present approach can be considered as a generalization of the Caldeira-Legget model [25, 26], where for dissipative quantum systems, they model the environment of the system by a collection of harmonic oscillators such that the position operator of the main system, is coupled with all of the position operators of the environment oscillators. While the properties of the environment may in some cases be chosen on the basis of a microscopic model, this does not have to be the Caldeira-Legget model. As an example we mention an Ohmic resistor which as a linear electric element should be well described by the Caldeira-Leggett model. On the other hand the underlying mechanism leading to dissipation in a resistor may be more complicated than that implied by the model of a collection of harmonic oscillators. Modeling the environment with a collection of Harmonic oscillators, with a suitable interaction, means that an averaging over complicated microscopic interactions of the particle with its environment is done and the environment is effectively equivalent to this modeling.
2.2 Quantum damped harmonic oscillator

For a one dimensional harmonic oscillator with mass $m$ and frequency $\omega_0$, we have $v(q) = \frac{1}{2}m\omega_0^2q^2$ and therefore we can write (18) as

$$m\ddot{q} + m\omega_0^2q \pm \frac{d}{dt} \int_0^{[t]} dt'\chi(|t| - t')\dot{q}(\pm t') = -\dot{R}_N.$$  \hspace{1cm} (19)

This equation can be solved for negative and positive times using the Laplace transformation technique. For any time dependent operator $g(t)$, the forward and backward Laplace transformations are by definition

$$g^f(s) = \int_0^\infty dt g(t)e^{-st},$$ \hspace{1cm} (20)

and

$$g^b(s) = \int_0^\infty dt g(-t)e^{-st},$$ \hspace{1cm} (21)

respectively. Let $\chi(s)$ be the Laplace transformation of $\chi(t)$, then taking the Laplace transform of the equation (19), we obtain the forward and backward Laplace transformation of $q(t)$, i.e., $q^{f,b}(s)$, as

$$q^{f,b}(s) = \frac{ms + s\chi(s)}{m\omega_0^2 + ms^2 + s^2\chi(s)}q(0) \pm \frac{m}{m\omega_0^2 + ms^2 + s^2\chi(s)}\dot{q}(0) + \frac{\mp sR^{f,b}_N(s) \pm R_N(0)}{m\omega_0^2 + ms^2 + s^2\chi(s)}.$$

where the upper (lower) sign corresponds to forward (backward) Laplace transformation respectively.

2.2.1 A special case

Let us as a special case, take the coupling function $f(\omega)$ as

$$|f(\omega)|^2 = \frac{\beta hc^3}{4\pi^2\omega^3},$$ \hspace{1cm} (23)

where $\beta$, is a positive constant. According to (16), this choice corresponds to a step function for the susceptibility

$$\chi(t) = \begin{cases} \beta & t > 0, \\ 0 & t \leq 0, \end{cases}$$ \hspace{1cm} (24)
and accordingly, the equation (19) can be written as

\[
m\ddot{q} + m\omega_0^2 q \pm \beta \dot{q} = \xi(t),
\]

\[
\xi(t) = \frac{\beta \hbar c^3}{4\pi^2} \int \frac{d^3k}{\sqrt{\omega_k^2}} [b_k(0)e^{-i\omega_k t} - b_k^+(0)e^{i\omega_k t}] = \frac{i\sqrt{\beta\hbar c^3 \pi^2}}{4}\int d^3k \sqrt{\omega_k} \frac{b_k(0)e^{-i\omega_k t}}{\sqrt{\omega_k^2 - \frac{i\beta m \omega_k}{m}}} + C.C.
\]  

(25)

A complete solution of this equation for \(t > 0\) is as follows

\[
q(t) = e^{-\frac{\beta t}{2m}} \left\{ \frac{p(0)}{m\omega_1} \sin \omega_1 t + q(0) \cos \omega_1 t + \frac{\beta}{2m\omega_1} q(0) \sin \omega_1 t - \frac{R(0)}{m\omega_1} \sin \omega_1 t - \frac{\beta M(0)}{2m\omega_1} \sin \omega_1 t - \frac{\dot{M}(0)}{\omega_1} \sin \omega_1 t \right\} + M(t),
\]

\[
M(t) = \frac{\sqrt{\beta\hbar c^3}}{m} \int \frac{d^3k}{4\pi^2} \left\{ \frac{b_k(0)e^{-i\omega_k t}}{\sqrt{\omega_k^2 - \frac{i\beta m \omega_k}{m}}} + C.C. \right\},
\]

(26)

where \(\omega_1 = \sqrt{\omega_0^2 - \frac{\beta^2}{4m^2}}\). This solution contains two parts, the first part is an exponentially decreasing function of time which is in fact the solution of the homogeneous part of the equation (25), the second part is a non-decaying term \(M(t)\), which is the response to the noise force \(\xi(t)\). The part \(M(t)\), does not have a classical correspondence and is necessary for consistency of the quantization of the dissipative systems. From (9), (13), (24) and (26), we can obtain an asymptotic answer for the conjugate momentum \(p\) in the large-time limit as

\[
p(t) = m\dot{M}(t) + R_N(t) + \beta M(t) - \beta q(0).
\]  

(27)

Let the state of the damped harmonic oscillator at \(t = 0\) be \(|\psi(0)\rangle = |0\rangle_B \otimes |n\rangle\) where \(|0\rangle_B\) is the vacuum state of the environment and \(|n\rangle\) is an excited state of the oscillator Hamiltonian, i.e., \(H_s = \frac{p^2}{2m} + \frac{1}{2}m\omega_0^2 q^2\), then it is clear that

\[
\langle \psi(0)| \frac{p^2(0)}{2m} + \frac{1}{2}m\omega_0^2 q^2(0) |\psi(0)\rangle = (n + \frac{1}{2})\hbar\omega_0.
\]  

(28)

From (26) and (27), we find

\[
\lim_{t \to \infty} \langle \psi(0)| \left[ \frac{1}{2}m\dot{M}^2 + \frac{1}{2}m\omega_0^2 M^2 : |\psi(0)\rangle \right] = 0,
\]  

(29)
where $\langle \psi(0) | : H_B(t) : | \psi(0) \rangle$ is the normal ordering operator. Therefore, macroscopically, analogous to classical mechanics, the total energy of the particle tends to zero as time tends to infinity. The expectation value of the environment Hamiltonian $(H_B)$, in the same state $(|\psi(0)\rangle)$, and large-time limit, can be calculated by residue calculus as

$$
\lim_{t \to \infty} \left[ \langle \psi(0) | : H_B(t) : | \psi(0) \rangle \right] = \frac{\hbar \beta \omega_0}{\pi m} \left( n + \frac{1}{2} \right) \int_0^\infty \frac{\omega_0^2 + x^2}{(\omega_0^2 - x^2)^2 + \beta^2 x^2} dx = \left( n + \frac{1}{2} \right) \hbar \omega_0.
$$

(30)

So the total energy of the particle is absorbed by the environment and which is a good signal of the consistency of the approach.

### 2.3 Transition probabilities

The Hamiltonian $(H)$, for a damped harmonic oscillator can be written as

$$ H = H_0 + H', $$
$$ H_0 = (a^\dagger a + \frac{1}{2}) \hbar \omega_0 + H_B, $$
$$ H' = -\frac{p}{m} R + \frac{R^2}{2m}, $$

(31)

where $a$ and $a^\dagger$ are annihilation and creation operators of the Harmonic oscillator respectively. In interaction picture, we have

$$ a_I(t) = e^{\frac{i}{\hbar} H_0 t} a(0) e^{-\frac{i}{\hbar} H_0 t} = a(0) e^{-i \omega_0 t}, $$
$$ b_{kI}(t) = e^{\frac{i}{\hbar} H_0 t} b_k(0) e^{-\frac{i}{\hbar} H_0 t} = b_k(0) e^{-i \omega_k t}, $$

(32)

the terms $\frac{R}{m} p$ and $\frac{R^2}{2m}$, are of the first and second order of damping respectively, therefore, for a sufficiently weak damping, $\frac{R^2}{2m}$ is small in comparison with $\frac{R}{m} p$. Furthermore, in the first order perturbation, $\frac{R^2}{2m}$, has not any role in those transition probabilities where initial and final states of the harmonic oscillator are different from each other, hence we can neglect the term $\frac{R^2}{2m}$ in
$H'$. Substituting $a_I$ and $b_{kI}$ from $\mathbf{32}$ in $-\frac{\hbar}{m}p$ and using the rotating wave approximation $\mathbf{34}$, one can obtain $H'_I$ in interaction picture as

$$H'_I = -i\sqrt{\frac{\hbar \omega_0}{2m}} \int_{-\infty}^{+\infty} d^3k (f(\omega_k) a^\dagger b_k(0)e^{i(\omega_0 - \omega_k)t} - f^*(\omega_k) ab_k^\dagger(0)e^{-i(\omega_0 - \omega_k)t}).$$

(33)

In interaction picture, the density matrix $\rho(t)$, can be obtained from the initial density matrix $\rho(t_0)$ as $\mathbf{35}$

$$\rho_I(t) = U_I(t, t_0) \rho_I(t_0) U_I^\dagger(t, t_0),$$

(34)

where $U_I$, is the time evolution operator, which in the first order perturbation is

$$U_I(t, t_0 = 0) \approx 1 - \frac{i}{\hbar} \int_0^t dt_1 H_I'(t_1),$$

$$= 1 - \sqrt{\frac{\omega_0}{2m\hbar}} \int_{-\infty}^{+\infty} d^3k [f(\omega_k) a^\dagger b_k(0)e^{i(\omega_0 - \omega_k)t}]$$

$$- f^*(\omega_k) ab_k^\dagger(0)e^{-i(\omega_0 - \omega_k)t}] \sin \left(\frac{(\omega_0 - \omega_k)t}{2}\right) \frac{\sin \left(\frac{(\omega_0 - \omega_k)t}{2}\right)}{\frac{(\omega_0 - \omega_k)^2}{2}}.$$  

(35)

Using the density operator $\rho_I$, we can calculate some transition probabilities. for example let $\rho_I(0) = |n\rangle \langle n| \otimes |0\rangle_B \langle 0|_B$, where $|0\rangle_B$, is the vacuum state of the reservoir and $|n\rangle$, an exited state of the harmonic oscillator, then substituting $U_I(t, 0)$ from (35) in (34) and tracing out the environment degrees of freedom, we find

$$\rho_{st}(t) = Tr_B(\rho_I(t)),$$

$$= |n\rangle \langle n| + \frac{n\omega_0}{2m\hbar} (n - 1) \langle n - 1| \int_{-\infty}^{+\infty} d^3p |f(\omega_p)|^2 \frac{\sin^2 \left(\frac{(\omega_p - \omega_0)t}{2}\right)}{\frac{\omega_p^2 - \omega_0^2}{2}}.$$  

(36)

where we have used the formula $Tr_B[|1_{\vec{k}}\rangle_B \langle 1_{\vec{k}}|] = \delta(\vec{k} - \vec{k'})$. In large-time limit, we can write $\frac{\sin^2 \left(\frac{(\omega_p - \omega_0)t}{2}\right)}{\frac{\omega_p^2 - \omega_0^2}{2}} \approx 2\pi t \delta(\omega_p - \omega_0)$, which leads to the following relation for the density matrix

$$\rho_{st}(t) = |n\rangle \langle n| + \frac{4\pi^2 \omega_0^3 nt |f(\omega_0)|^2}{m\hbar c^3} |n - 1\rangle \langle n - 1|.$$  

(37)

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From $\rho_{st}$ we can find the probability of transition $|n\rangle \rightarrow |n-1\rangle$ as

$$
\Gamma_{n\rightarrow n-1} = Tr[(|n-1\rangle\langle n-1|)\rho(t)] = Tr_s[(|n-1\rangle\langle n-1|)\rho_{st}(t)] = \frac{4\pi^2\omega_0^3 nt |f(\omega_0)|^2}{m \hbar c^3},
$$

(38)

where $Tr_s$, denotes taking trace over harmonic oscillator eigenstates. For the special choice (23), the transition probability (38) becomes

$$
\Gamma_{n\rightarrow n-1} = \frac{n\beta t}{m}.
$$

(39)

In this case there is not any transition from the state $|n\rangle$ to state $|n+1\rangle$.

Now consider the case where the environment is an excited state in $t = 0$, for example, take the density matrix as

$$
\rho_I(0) = |n\rangle \langle n| \otimes |\tilde{p}_1, \ldots \tilde{p}_j\rangle_B \langle \tilde{p}_1, \ldots \tilde{p}_j|_B,
$$

(40)

where $|\tilde{p}_1, \ldots \tilde{p}_j\rangle_B$, denotes a state of the environment which contains $j$ quanta with the corresponding momenta $\tilde{p}_1, \ldots \tilde{p}_j$, then using the relations

$$
Tr_B[b^\dagger_k \tilde{p}_1, \ldots \tilde{p}_j]_B \langle \tilde{p}_1, \ldots \tilde{p}_j|_B b_k] = \delta(\tilde{k} - \tilde{k}'),
$$

$$
Tr_B[b_k \tilde{p}_1, \ldots \tilde{p}_j]_B \langle \tilde{p}_1, \ldots \tilde{p}_j|_B b^\dagger_{k'}] = \sum_{l=1}^{j} \delta(\tilde{k} - \tilde{p}_l)\delta(\tilde{k'} - \tilde{p}_l),
$$

(41)

we find in the large-time limit,

$$
\rho_{st}(t) = |n\rangle \langle n| + \frac{(n+1)\omega_0}{2m}\int_{-\infty}^{\infty} \frac{1}{2}\int_{-\infty}^{\infty} d^3k |f(\omega_k)|^2 \frac{\omega_k^2}{\omega_0^2} \frac{\sin^2(\omega_k t)}{\omega_k^2} + \frac{n\omega_0}{2m} |n-1\rangle \langle n-1| + \frac{n\omega_0}{2m} |n-1\rangle \langle n-1| \int_{-\infty}^{\infty} d^3k |f(\omega_k)|^2 \frac{\omega_k^2}{\omega_0^2} \frac{\sin^2(\omega_k t)}{\omega_k^2},
$$

(42)

from which the transition probabilities $|n\rangle \rightarrow |n-1\rangle$ and $|n\rangle \rightarrow |n+1\rangle$, can be obtained as

$$
\Gamma_{n\rightarrow n-1} = Tr_s[|n-1\rangle \langle n-1|\rho_{st}(t)] = \frac{4\pi^2\omega_0^3 nt |f(\omega_0)|^2}{m \hbar c^3},
$$

$$
\Gamma_{n\rightarrow n+1} = Tr_s[|n+1\rangle \langle n+1|\rho_{st}(t)] = \frac{(n+1)\pi t \omega_0}{m} |f(\omega_0)|^2 \sum_{l=1}^{j} \delta(\omega_{\tilde{p}_l} - \omega_0).
$$

(43)
Specially, for the choice (23), we have
\[ \Gamma_{n \rightarrow n-1} = \frac{n \beta t}{m}, \]
\[ \Gamma_{n \rightarrow n+1} = \frac{\beta (n+1) \delta t}{4 \pi m \omega_0^2} \sum_{l=1}^{j} \delta(\omega_{l} - \omega_0). \]  
(44)

Another important case is when the environment has a canonical thermal distribution, i.e.,
\[ \rho_I(0) = |n\rangle \langle n| \otimes \rho^T_B, \quad \rho^T_B = \frac{e^{-H_B}}{Tr_B(e^{-H_B})}, \]  
(45)
in this case, by applying the following relations
\[ Tr_B[b_{k}^\dagger \rho^T_B b_{\kappa}^\dagger] = Tr_B[b_{k}^\dagger \rho^T_B b_{\kappa}^\dagger] = 0, \]
\[ Tr_B[b_{k}^\dagger \rho^T_B b_{\kappa}] = \frac{\delta(\tilde{k} - \tilde{\kappa})}{e^{\frac{\omega}{\kappa T}} - 1}, \]
\[ Tr_B[b_{k}^\dagger \rho^T_B b_{\kappa}] = \frac{\delta(\tilde{k} - \tilde{\kappa}) e^{\frac{\omega}{\kappa T}}}{e^{\frac{\omega}{\kappa T}} - 1}, \]  
(46)
we find the density operator \( \rho_{sI}(t) \) in interaction picture as
\[ \rho_{sI}(t) := Tr_B[\rho_I(t)] \]
\[ = |n\rangle \langle n| + \frac{(n + 1) \omega_0}{2 m \hbar} |n + 1\rangle \langle n + 1| \int_{-\infty}^{+\infty} d^3 k \frac{|f(\omega_{k})|^2 \sin^2 \frac{\omega_{k} - \omega_0}{2} t}{e^{\frac{\omega_{k}}{\kappa T}} - 1} \]
\[ + \frac{n \omega_0}{2 m \hbar} |n - 1\rangle \langle n - 1| \int_{-\infty}^{+\infty} d^3 k \frac{|f(\omega_{k})|^2 e^{\frac{\omega_{k}}{\kappa T}} \sin^2 \frac{\omega_{k} - \omega_0}{2} t}{e^{\frac{\omega_{k}}{\kappa T}} - 1}, \]  
(47)
which accordingly lead to the following transition probabilities in large-time limit
\[ \Gamma_{n \rightarrow n-1} = Tr_s[|n - 1\rangle \langle n - 1| \rho_{sI}(t)] = \frac{4 \pi^2 \omega_0^3 n t |f(\omega_0)|^2 e^{\frac{n \omega_0}{\kappa T}}}{m \hbar c^3 e^{\frac{n \omega_0}{\kappa T}} - 1}, \]
\[ \Gamma_{n \rightarrow n+1} = Tr_s[|n + 1\rangle \langle n + 1| \rho_{sI}(t)] = \frac{4 \pi^2 \omega_0^3 (n + 1) t |f(\omega_0)|^2}{m \hbar c^3 e^{\frac{n + 1 \omega_0}{\kappa T}} - 1}, \]  
(48)
and for the special case (23), are reduced to

\[
\Gamma_{n \rightarrow n-1} = \frac{n \beta t e^{\frac{\pi i}{m} \omega_0}}{m(e^{\frac{\pi i}{m} \omega_0} - 1)},
\]

\[
\Gamma_{n \rightarrow n+1} = \frac{(n + 1) \beta t}{m(e^{\frac{\pi i}{m} \omega_0} - 1)}.
\] (49)

Therefore, in low temperatures regime, the energy flows from the oscillator to the environment by the rate \(\Gamma_{n \rightarrow n-1} \rightarrow \frac{n \beta}{m} \), and no energy flows from the environment to the oscillator in this case.

3 Three-dimensional quantum dissipative systems

In this section, we study a general three-dimensional quantum dissipative system. This time for generality, we model the environment of the system by two independent quantum fields, namely \(B\) and \(B'\) quantum fields. Quantum field \(B\), interacts with the momentum of the main system through a minimal coupling term and the quantum field \(B'\), interacts with the position operator of the main system similar to a dipole interaction term. For this purpose we take the Hamiltonian of the environment as

\[
H_E = H_B(t) + H_{B'}(t),
\]

\[
H_B(t) = \int_{-\infty}^{+\infty} d^3 k \hbar \omega_k b_{k}^\dagger(t)b_{k}(t),
\]

\[
H_{B'}(t) = \int_{-\infty}^{+\infty} d^3 k \hbar \omega_k d_{k}^\dagger(t)d_{k}(t),
\] (50)

where according to the explanations of the previous section, we choose \(\omega_k = c|\vec{k}|\). The annihilation and creation operators \(b_{\vec{k}}, b_{\vec{k}}^\dagger, d_{\vec{k}}, d_{\vec{k}}^\dagger\), satisfy the commutation relations

\[
[b_{\vec{k}}(t), b_{\vec{k}}^\dagger(t)] = \delta(\vec{k} - \vec{k}'),
\]

\[
[d_{\vec{k}}(t), d_{\vec{k}}^\dagger(t)] = \delta(\vec{k} - \vec{k}'),
\] (51)

and the rest of commutation relations are zero.
Let the damped system be a particle with mass \( m \) under an external potential \( v(\vec{x}) \), we take the total Hamiltonian, i.e., system plus the environment, as

\[
H = \left( \frac{\vec{p} - \vec{R}}{2m} \right)^2 + v(\vec{x}) - \vec{R} \cdot \vec{x} + H_B + H_\tilde{B},
\]

where \( \vec{x} \) and \( \vec{p} \) are position and canonical conjugate momentum operators of the particle respectively, and satisfy the canonical commutation rules

\[
[x_i, p_j] = i\hbar \delta_{ij}.
\]

Operators \( \vec{R} \) and \( \vec{\tilde{R}} \), play the basic role in interaction between the system and the environment and are defined by

\[
\vec{R}(t) = \int_{-\infty}^{+\infty} d^3k [f(\omega_k)b_k(t) + f^*(\omega_k)b_k^\dagger(t)] \hat{k};
\]
\[
\vec{\tilde{R}}(t) = \int_{-\infty}^{+\infty} d^3k [g(\omega_k)d_k(t) + g^*(\omega_k)d_k^\dagger(t)] \hat{k},
\]

where \( \hat{k} = \frac{\vec{k}}{|\vec{k}|} \). For future purposes the fields \( \vec{R} \) and \( \vec{\tilde{R}} \), are taken to have the longitudinal polarization. Let us call the functions \( f(\omega_k) \) and \( g(\omega_k) \), the coupling functions between the environment and the system. The Heisenberg equations for the position \( \vec{x}(t) \) and the momentum \( \vec{p} \) of the particle are

\[
\dot{\vec{x}} = \frac{i}{\hbar}[H, \vec{x}] = \frac{\vec{p} - \vec{R}}{m},
\]
\[
\dot{\vec{p}} = \frac{i}{\hbar}[H, \vec{p}] = -\vec{\nabla}v + \vec{\tilde{R}}.
\]

By eliminating \( \vec{p} \) in relations (55), we come to the following equation for the damped system

\[
m\ddot{\vec{x}} = -\vec{\nabla}v - \vec{R} + \vec{\tilde{R}}.
\]

Using (51), one can easily find the Heisenberg equations for the operators \( \vec{b}_k \) and \( \vec{d}_k \) as

\[
\dot{\vec{b}}_k = \frac{i}{\hbar}[H, \vec{b}_k] = -\omega_k \vec{b}_k + \frac{i}{\hbar} f^*(\omega_k) \hat{k} \cdot \vec{x},
\]
\[
\dot{\vec{d}}_k = \frac{i}{\hbar}[H, \vec{d}_k] = -\omega_k \vec{d}_k + \frac{i}{\hbar} g^*(\omega_k) \hat{k} \cdot \vec{x}.
\]
These equations can be solved formally as

\[
\begin{align*}
\vec{b}(t) &= \vec{b}(0)e^{-\omega_k t} + \frac{i}{\hbar} f^*(\omega_k) \hat{k} \cdot \int_0^t dt' e^{-\omega_k (t-t')} \vec{x}(t'), \\
\vec{d}(t) &= \vec{d}(0)e^{-\omega_k t} + \frac{i}{\hbar} g^*(\omega_k) \hat{k} \cdot \int_0^t dt' e^{-\omega_k (t-t')} \vec{x}(t').
\end{align*}
\]

(58)

Substituting these solutions into (54), one obtains what we have called the constitutive equations of the environment, i.e.,

\[
\vec{R}(t) = \vec{R}_N(t) + \int_{|t|}^0 dt' \chi(|t| - t') \vec{x}(\pm t'),
\]

\[
\vec{\tilde{R}}(t) = \vec{\tilde{R}}_N(t) + \int_{|t|}^0 dt' \tilde{\chi}(|t| - t') \vec{x}(\pm t'),
\]

(59)

where the upper(lower) sign, corresponds to \( t > 0 (t < 0) \), respectively. The functions

\[
\chi(t) = \frac{16\pi}{3\hbar c^3} \int_0^\infty d\omega \omega^2 |f(\omega)|^2 \sin \omega t \quad t > 0,
\]

\[
\chi(t) = 0 \quad t \leq 0,
\]

(60)

and

\[
\tilde{\chi}(t) = \frac{16\pi}{3\hbar c^3} \int_0^\infty d\omega \omega^2 |g(\omega)|^2 \sin \omega t \quad t > 0,
\]

\[
\tilde{\chi}(t) = 0 \quad t \leq 0,
\]

(61)

are called the susceptibilities of the environment. The operators \( \vec{R}_N \) and \( \vec{\tilde{R}}_N \), are the noise operators

\[
\vec{R}_N(t) = \int_{-\infty}^{+\infty} d^3k [f(\omega_k) b_k(0) e^{-\omega_k t} + f^*(\omega_k) b_k^\dagger(0) e^{\omega_k t}] \hat{k},
\]

(62)

\[
\vec{\tilde{R}}_N(t) = \int_{-\infty}^{+\infty} d^3k [g(\omega_k) d_k(0) e^{-\omega_k t} + g^*(\omega_k) d_k^\dagger(0) e^{\omega_k t}] \hat{k}.
\]

(63)

If we are given the macroscopic susceptibilities \( \chi(t) \) and \( \tilde{\chi}(t) \), which are zero for \( t \leq 0 \), then we can invert the equations (60) and (61) and obtain the corresponding coupling functions \( f(\omega) \) and \( g(\omega) \) as

\[
|f(\omega)|^2 = \frac{3\hbar c^3}{8\pi^2 \omega^2} \int_0^\infty dt \chi(t) \sin \omega t, \quad \omega > 0,
\]

\[
|f(\omega)|^2 = 0, \quad \omega = 0.
\]

(64)
\[
\begin{align*}
|g(\omega)|^2 &= \frac{3hc^3}{8\pi^2 \omega^2} \int_0^\infty dt \tilde{\chi}(t) \sin \omega t, \quad \omega > 0, \\
|g(\omega)|^2 &= 0, \quad \omega = 0.
\end{align*}
\]

(65)

It should be noted that the sign of the right hand side of the formulas (64) and (65) should be positive, this is because, the right hand side of these formulas is proportional to the imaginary part of the susceptibilities \(\chi(t)\) and \(\tilde{\chi}(t)\) in the frequency domain and this imaginary part is also connected to energy losses of a system in the presence of an environment, so it is necessarily positive. If these integrals would not be positive for all frequencies, the susceptibilities \(\chi(t)\) and \(\tilde{\chi}(t)\) should be discarded, as they would be unphysical. Substituting \(\vec{R}\) and \(\vec{\tilde{R}}\), from the constitutive equations (59), into (56), one can obtain the equation of motion of the operator \(\vec{x}\) as

\[
m\ddot{x} \pm \frac{d}{dt} \int_0^{\frac{|t|}{t}} dt' \chi(|t| - t') \dot{x}(\pm t') - \int_0^{\frac{|t|}{t}} dt' \tilde{\chi}(|t| - t') \ddot{x}(\pm t') = -\vec{\nabla}v(x) - \vec{R}_N + \vec{\tilde{R}}_N,
\]

(66)

where the upper (lower) sign corresponds to \(t > 0 (t < 0)\). This is the generalized macroscopic Langevin-Schrödinger equation for a dissipative quantum system. Therefore, coupling with the environment in microscopic level, is described macroscopically by two random forces \(-\vec{R}_N\) and \(\vec{\tilde{R}}_N\) and the corresponding memory functions \(\chi\) and \(\tilde{\chi}\).

As a simple example, let us consider a damping three-dimensional harmonic oscillator with resonance frequency \(\omega_0\) and mass \(m\), where the damping force is proportional to the velocity and position of the oscillator. In this case, we take the susceptibilities \(\chi(t)\) and \(\tilde{\chi}(t)\), as follows

\[
\chi(t) = \begin{cases} 
\beta & t > 0, \\
0 & t \leq 0,
\end{cases}
\]

\[
\tilde{\chi}(t) = \begin{cases} 
\frac{\alpha m \omega_0^2}{\Delta} & 0 < t < \Delta, \\
0 & \text{otherwise},
\end{cases}
\]

where \(0 < \alpha < 1\) and \(\beta\) are some positive constants. Now one can use the relations (64) and (65) to obtain the related coupling functions as

\[
|f(\omega)|^2 = \frac{3hc^3\beta}{8\pi^2 \omega^3}, \quad \omega \neq 0,
\]

(67)

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In this case, the Langevin-Schrödinger equation (66), is reduced to

\[ m \ddot{x} + m \omega_0^2 \dot{x} \pm \beta \dot{x} - \frac{\alpha m \omega_0^2}{\Delta} \int_{|t|}^{||t||} dt' \bar{x}(\pm t') = -\tilde{R}_N + \tilde{R}_N, \]

(69)

where \( \tilde{R}_N \) and \( \tilde{R} \) are the noise operators (62) and (63) with coupling functions (67) and (68) respectively. It is clear that in the limit \( \Delta \to 0 \), the coupling function (68), tends to zero and accordingly, the noise force \( \tilde{R}_N \) vanishes. Therefore, the equation (69) becomes

\[ m \ddot{x} + (1 - \alpha) m \omega_0^2 \dot{x} \pm \beta \dot{x} = \tilde{\xi}(t), \]

\[ \tilde{\xi}(t) = i \sqrt{\frac{3 \beta \hbar c^3}{8 \pi^2}} \int d^3k \left[ \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{\omega_k^2}} \frac{f(\omega_k) b_k(0)e^{-i\omega_k t} - f^*(\omega_k) b_k^*(0)e^{i\omega_k t}}{\sqrt{\omega_k^2}} \right]. \]

(70)

For an absorptive environment, \( 0 < \alpha < 1 \), otherwise the environment would behave as an amplifier, i.e., energy transfers to the system from the environment, this may be an indication for making other related models, but in our discussion, this case is not physical. As is seen from this equation, the environment affect the motion of the particle by two damping forces, one is a friction force proportional to the velocity and the other is a force proportional to the position and in opposite direction of the external force \(-\nabla v(x) = -m \omega_0^2 \dot{x}\). In fact, the later damping force, influence the frequency of the oscillator.

### 3.1 Dissipative quantum two-level systems

The rate of spontaneous decay of a two-level quantum system imbedded in an absorptive environment, can be calculated in this approach as follows. Here we assume that the interaction between the particle and its environment is so weak that we can use the Weiskopf-Wigner approximation [34] for calculating the decay constant. Let us write the total Hamiltonian (52), as the following form

\[ H = H_0 + H', \]
\[ H_0 = \frac{\vec{p}^2}{2m} + v(\vec{x}) + H_B + H_{\tilde{B}}, \]
\[ H' = -\frac{\vec{p}}{m} \cdot \vec{R} - \vec{R} \cdot \vec{x}, \]
\( (71) \)

where we have ignored from the term \( \frac{\vec{R}^2}{2m} \), because, this term does not contribute in the following calculations of the decaying rate of the system. Now for a considerable simplification, we restrict ourselves to a two-state model of a system. In fact, the Hilbert space of the main system is artificially truncated to the two states \( |1\rangle \) and \( |2\rangle \) with unperturbed energy eigenvalues \( E_1, E_2 \), respectively. Therefore, for the Hamiltonian of the system we can write
\[ H_s = \frac{\vec{p}^2}{2m} + v(\vec{x}) \equiv \frac{1}{2}\hbar \Omega_0 \sigma_z, \quad \sigma_z = |2\rangle \langle 2| - |1\rangle \langle 1|, \quad \Omega_0 = \frac{E_2 - E_1}{\hbar}, \]
\( (72) \)

and the interaction Hamiltonian \( H' \) becomes
\[ H' = -i\Omega_0 \int d^3k [\sigma b_k^f(\omega_k)\vec{x}_{12} \cdot \vec{k} + \sigma b_k^f(\omega_k)\vec{x}_{12} \cdot \vec{k} + C.C] \]
\[ - \int d^3k [\sigma d_k^g(\omega_k)\vec{x}_{12} \cdot \vec{k} + \sigma d_k^g(\omega_k)\vec{x}_{12} \cdot \vec{k} + C.C], \]
\( (73) \)

where
\[ \sigma = |1\rangle \langle 2|, \quad \vec{x}_{12} = \langle 1|\vec{x}|2\rangle. \]
\( (74) \)

and we have used this fact that the energy eigenstates of the Hamiltonian have a well defined parity, so that the diagonal elements of the operators \( \vec{x} \) and \( \vec{p} \), are zero.

To study the spontaneous decay of an initially excited two-level system, imbedded in an absorptive environment, we may look for the system wave function at time \( t \) in interaction picture
\[ |\psi(t)\rangle_I = c(t)|2\rangle_B|0\rangle_{\tilde{B}} + \int d^3k D_k(t)|1\rangle_B|0\rangle_{\tilde{B}} + \int d^3k G_k(t)|1\rangle_B|0\rangle_{\tilde{B}}, \]
\( (75) \)

where \( |0\rangle_B \) and \( |0\rangle_{\tilde{B}} \), are vacuum states of quantum fields \( B \) and \( \tilde{B} \), respectively, and the coefficient \( c(t), D_k(t) \) and \( G_k(t) \), are to be specified by the Schrödinger equation
\[ i\hbar \frac{\partial |\psi(t)\rangle_I}{\partial t} = H'_I(t)|\psi(t)\rangle_I, \quad H'_I(t) = e^{\frac{ih'_I}{\hbar}} H' e^{\frac{-ih'_I}{\hbar}}, \]
\( (76) \)
with initial conditions \( c(0) = 1, D_k(0) = G_k(0) = 0 \). Where \( H'_0 \) and \( H' \) are Hamiltonians \( H_0 \) and \( H' \) in the Schrödinger picture respectively. Substituting \( |\psi(t)\rangle \) from (75), in (76), and using the rotating wave approximation, we find the differential equations

\[
\tag{77}
\hbar \dot{c}(t) = -\int d^3k [\pm \Omega_0 \vec{x}_{12} \cdot \hat{k} f(\omega_k) e^{i(\Omega_0 - \omega_k)t} D_k(t)] + \vec{x}_{12} \cdot \hat{k} g(\omega_k) e^{i(\Omega_0 - \omega_k)t} G_k(t),
\]

\[
\tag{78}
\hbar \dot{D}_k(t) = -i \Omega_0 \vec{x}_{12} \cdot \hat{k} f^*(\omega_k) e^{-i(\Omega_0 - \omega_k)t} c(t),
\]

\[
\tag{79}
\hbar \dot{G}_k(t) = -\vec{x}_{12} \cdot \hat{k} g^*(\omega_k) e^{-i(\Omega_0 - \omega_k)t} c(t),
\]

One can solve this differential equations using the Laplace transformation technique and deduce

\[
\dot{c}(t) = \int_0^t dt' \gamma(t - t')c(t'),
\]

\[
\gamma(t - t') = -\int d^3k |\Omega_0^2| \vec{x}_{12} \cdot \hat{k} f(\omega_k)|^2 + |\vec{x}_{12} \cdot \hat{k} g(\omega_k)|^2 e^{i(\Omega_0 - \omega_k)(t - t')}.
\]

Here we restrict our attention to the weak coupling regime, where the Markov approximation applies. That is to say, we may replace \( c(t') \) in the integrand of (80) with \( c(t) \) and approximate the time integral \( \int_0^t dt' \gamma(t - t') \), in large-time limit, as

\[
\int_0^t dt' e^{i(\Omega_0 - \omega_k)(t - t')} \approx [iP \frac{1}{\Omega_0 - \omega_k} + \pi \delta(\Omega_0 - \omega_k)],
\]

where \( P \) denotes the principal Cauchy value. After some algebraic calculations, we find

\[
\dot{c}(t) = -(\beta + i\Delta)c(t),
\]

where \( \beta \) is the decay constant

\[
\beta = \frac{4\pi^2 |f(\omega_0)|^2 + 4\pi^2 |g(\omega_0)|^2}{3\hbar^2 c^3},
\]

and \( \Delta \) is the shift frequency

\[
\Delta = \frac{4\pi |x_{12}^2|^2}{3\hbar^2 c^3} P \int_0^\infty d\omega \frac{[f(\omega)]^2 + [g(\omega)]^2}{\Omega_0 - \omega}. 
\]
4 Dissipative scalar field theory

In this section, we discuss the dissipative scalar field theory. We take for convenience a 1+1-dimensional scalar field but the generalization to a general scalar field theory is straightforward. For this purpose, assume \( \psi(x, t) \) be a scalar field operator defined on a closed compact interval \([0, L]\), satisfying the boundary conditions \( \psi(0) = \psi(L) = 0 \). The field \( \psi(x, t) \), can be expanded in terms of the orthogonal wave function \( \sin \frac{n\pi x}{L} \), as

\[
\psi(x, t) = \sum_{n=1}^{\infty} \sqrt{\frac{\hbar}{L\lambda \omega_n}} [a_n(t) + a_n^\dagger(t)] \sin \frac{n\pi x}{L}, \tag{85}
\]

where \( \omega_n = \sqrt{\frac{\mu \lambda n^2}{L}} \), and \( \lambda, \mu \), are some constants. The operators \( a_n \) and \( a_n^\dagger \), are annihilation and creation operators of the scalar field \( \psi(x, t) \), respectively and satisfy the following equal-time commutation relations

\[
[a_n(t), a_m^\dagger(t)] = \delta_{nm}. \tag{86}
\]

The conjugate canonical momentum density of the field \( \psi(x, t) \), can be also expanded in the same basis as

\[
\pi_\psi(x, t) = i \sum_{n=1}^{\infty} \sqrt{\frac{\hbar \lambda \omega_n}{L}} [a_n^\dagger(t) - a_n(t)] \sin \frac{n\pi x}{L}. \tag{87}
\]

From (86), we deduce that \( \psi, \pi_\psi \), satisfy the equal time commutation relation

\[
[\psi(x, t), \pi_\psi(x', t)] = i\hbar \delta(x - x'). \tag{88}
\]

The Hamiltonian of the scalar field \( \psi(x, t) \), in normal ordered form is

\[
H_s = \int_0^L dx : \left[ \frac{\pi_\psi^2}{2\lambda} + \frac{1}{2} \mu \psi_x^2 \right] := \sum_{n=1}^{\infty} \hbar \omega_n a_n^\dagger(t)a_n(t), \tag{89}
\]

where \( \psi_x \), denotes the derivative with respect to \( x \). Now similar to what we did in previous sections, here we model the environment by a quantum field \( B \) containing an infinite number, but numerable, of Klein-Gordon fields. Therefore, the environment Hamiltonian, can be written as

\[
H_B(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} d^3 k \ h \omega_k b_n^\dagger_k(t)b_n(t), \quad \omega_k = c|k|, \tag{90}
\]
where $b_{nk}$ and $b_{nk}^\dagger$, are annihilation and creation operators of the environment respectively and satisfy the following bosonic commutation relations (bosonic bath)

$$[b_{nk}(t), b_{nk'}^\dagger(t)] = \delta_{nn'}\delta(\vec{k} - \vec{k}'),$$  \hspace{1cm} (91)

In a systematic way, and also according to what we did in the previous sections, we write the total Hamiltonian, i.e., the scalar field $\psi(x,t)$ plus the environment, as

$$H(t) = \int_0^L dx \left[ \frac{\pi \psi(x,t) - R(x,t)}{2\lambda} \right]^2 + \frac{1}{2} \mu \psi_x^2 + H_B, \hspace{1cm} (92)$$

wherein the operator $R(x,t)$, has the basic role in interaction between the scalar field and its environment and is defined by

$$R(x,t) = \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} d^3k \left[ f(\omega_k,x) b_{nk}(t) + \bar{f}(\omega_k,x) b_{nk}^\dagger(t) \right] \sin \frac{n\pi x} {L}. \hspace{1cm} (93)$$

Let us call the function $f(\omega_k,x)$, the coupling function between the scalar field and its environment. The coupling function is independent(dependent) on $x$ for a homogeneous(inhomogeneous) environment respectively.

One can easily show that the Heisenberg equations for $\psi(x,t)$ and $\pi_\psi(x,t)$ are

$$\dot{\psi}(x,t) = \frac{i}{\hbar} [H,\psi] = \frac{\psi(x,t) - R(x,t)}{\lambda},$$

$$\dot{\pi}_\psi(x,t) = \frac{i}{\hbar} [H,\pi_\psi] = -\mu \psi_{xx}, \hspace{1cm} (94)$$

we can eliminate $\pi_\psi$ between the relations (94) and finally find the following equation for the field $\psi$

$$\chi \ddot{\psi} - \mu \psi_{xx} = -\dot{R}. \hspace{1cm} (95)$$

Now similar to the previous sections, if we write the Heisenberg equation for the annihilation operator $b_{nk}$ and solve it formally then substitute this solution into (93), we can obtain a constitutive equation for the environment as

$$R(x,t) = R_N(x,t) + \int_0^{[t]} dt' \chi(x,|t| - t') \dot{\psi}(x,\pm t'), \hspace{1cm} (96)$$

where the upper(lower) sign, corresponds to $t > 0(t < 0)$ respectively and

$$\chi(x,t) = \frac{8\pi}{\hbar c^2} \int_0^{\infty} d\omega \omega^2 |f(\omega,x)|^2 \sin \omega t, \hspace{1cm} t > 0,$$

$$\chi(x,t) = 0, \hspace{1cm} t \leq 0, \hspace{1cm} (97)$$
is the susceptibility of the environment. By inverting the relation (97), we can obtain the coupling function \( f(\omega, x) \), in terms of the susceptibility, as

\[
|f(\omega, x)|^2 = \frac{\hbar c^3}{4\pi^2\omega^2} \int_0^\infty dt \chi(x, t) \sin \omega t, \quad \omega > 0,
\]

\[
|f(\omega, x)|^2 = 0, \quad \omega = 0. \quad (98)
\]

The operator

\[
R_N(x, t) = \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} d^3 k [f(\omega_k, x) b_n(0) e^{-i\omega_k t} + f^*(\omega_k, x) b_n^*(0) e^{i\omega_k t}] \sin \frac{n\pi x}{L},
\]

(99)
is a noise operator and necessary for a consistent quantization of a dissipative theory. Finally, substituting (96) in (95), leads to the following wave equation for the field operator \( \psi \)

\[
\ddot{\psi} - \mu \psi_{xx} \pm \frac{d}{dt} \int_0^{|t|} dt' \chi(x, |t| - t') \dot{\psi}(x, \pm t') = -\dot{R}_N(x, t). \quad (100)
\]

Taking the Laplace transformation of this equation, we find that the forward and backward Laplace transforms of \( \psi(x, t) \), satisfy the following equation

\[
[\lambda s^2 + s^2 \chi(x, s)] \psi^{f,b}(x, s) - \mu \psi_x^{f,b}(x, s) = J^{f,b}(x, s)
\]

(101)

where

\[
J^{f,b}(x, s) = [\lambda s + s \chi(x, s)] \psi(x, 0) \mp \lambda \dot{\psi}(x, 0) \mp R_N^{f,b}(x, s) \pm \dot{R}_N(x, 0),
\]

the upper(lower) sign, corresponds to the forward(backward) Laplace transformations respectively and \( \chi(x, s) \), is the Laplace transform of the susceptibility. One can solve equation (101) in terms of the Green function

\[
\psi^{f,b}(x, s) = \int_0^L dx' G(x, x', s) J^{f,b}(x', s), \quad (102)
\]

where \( G(x, x', s) \) satisfies

\[
(\lambda s^2 + s^2 \chi(s, x)) G(x, x', s) - \mu G_{xx}(x, x', s) = \delta(x - x'),
\]

(103)

with the boundary conditions \( G(0, x', s) = G(L, x', s) = 0 \). For a homogeneous environment, the coupling function and the susceptibility \( \chi \), are position independent and in this case, the Green function is

\[
G(x, x', s) = \frac{2}{L} \sum_{n=1}^{\infty} \frac{1}{\lambda s^2 + s^2 \chi(s) + \lambda \omega_n^2} \sin \frac{n\pi x}{L} \sin \frac{n\pi x'}{L}.
\]

(104)
4.1 A special case

For the special choice of coupling function (23), equation (100), becomes

\[ \lambda \ddot{\psi} - \mu \dot{\psi} \pm \beta \psi_{xx} = \tilde{\xi}(x, t), \]

\[ \tilde{\xi}(x, t) = i \sqrt{\frac{\beta \hbar c^3}{2\pi^2 L}} \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{\omega_k}} [b_{nk}(0)e^{-i\omega_k t} - b_{nk}^+(0)e^{i\omega_k t}] \sin \frac{n\pi x}{L}. \]

(105)

By defining

\[ x_n = \sqrt{\frac{\hbar}{2\lambda \omega_n}} (a_n + a_n^\dagger), \quad p_n = i \sqrt{\frac{\hbar \lambda \omega_n}{2}} (a_n^\dagger - a_n), \]

(106)

and using (105), we obtain

\[ \ddot{x}_n + \omega_n^2 x_n + \frac{\beta}{\lambda} \dot{x}_n = \zeta_n(t), \]

\[ \zeta_n(t) = i \sqrt{\frac{\beta}{4\pi^2 \lambda^2}} \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{\omega_k}} [b_{nk}(0)e^{-i\omega_k t} - b_{nk}^+(0)e^{i\omega_k t}], \]

(107)

a complete solution of this equation for \( t \geq 0, \) is

\[ x_n(t) = e^{-\frac{\beta t}{\lambda}} \left( x_n(0) \cos \Omega_n t + \frac{\beta}{2\lambda \Omega_n} x_n(0) \sin \Omega_n t - \frac{\beta M_n(0)}{2\lambda \Omega_n} \sin \Omega_n t \right) - M_n(0) \cos \Omega_n t + \frac{\dot{x}_n(0) - \dot{M}_n(0)}{\Omega_n} \sin \Omega_n t \} + M_n(t), \]

\[ M_n(t) = i \int_{-\infty}^{+\infty} d^3k \sqrt{\frac{\beta \hbar c^3}{4\pi^2 \lambda^2 \omega_k^3}} \left[ \frac{b_{nk}(0)}{\omega_n^2 - \omega_k^2 - \frac{i\beta}{\lambda} \omega_k} e^{-i\omega_k t} - \frac{b_{nk}^+(0)}{\omega_n^2 - \omega_k^2 - \frac{i\beta}{\lambda} \omega_k} e^{i\omega_k t} \right], \]

(108)

where \( \Omega_n = \sqrt{\omega_n^2 - \frac{\beta^2}{\lambda^2}}. \) This solution consists of two parts, the first part is an exponentially decreasing function in time which is the solution of the homogeneous part of the equation (107), i.e., when \( \zeta_n(t) = 0, \) the second part is \( M_n(t), \) which is an oscillatory exponential function and is the response to the noise force \( \zeta_n(t). \) The answer \( M_n(t), \) does not have a classical counterpart and is necessary for a consistent treatment of dissipative quantum systems. In fact, without \( M_n(t), \) the commutation relations \([x_n, p_n],\) tend to zero in
large-time limit and accordingly, the uncertainty relations would be violated. From (94), (96) and (108), we obtain the asymptotic answer

\[ p_n(t) = \lambda \dot{M}_n(t) + r_N(t) + \beta M_n(t) - \beta x_n(0), \]

\[ r_N(t) = \sqrt{\frac{\beta \hbar c^3}{4\pi^2}} \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{\omega_k^3}} [b_{nk}(0)e^{-\omega_k t} + b_{nk}^+(0)e^{\omega_k t}], \]

(109)

for \( p_n(t) \), when \( t \to +\infty \). If the state of the system in \( t = 0 \), is taken to be \( |\psi(0)\rangle = |0\rangle_B \otimes |1_{m_1}, ..., 1_{m_r}\rangle_s \), where \( |1_{m_1}, ..., 1_{m_r}\rangle_s \) is an excited state of the Hamiltonian \( H_s \), then it is clear that

\[ \langle \psi(0) | \sum_{n=1}^{\infty} \hbar \omega_n a_n^d(0)a_n(0) |\psi(0)\rangle = \sum_{i=1}^{r} \hbar \omega_{m_i}, \]

(110)
on the other hand, from (108), we find

\[ \lim_{t \to \infty} [\langle \psi(0) | : \int_0^L dx [\frac{1}{2} \lambda \dot{x}^2 + \frac{1}{2} \mu x^2] : |\psi(0)\rangle] = \langle \psi(0) | : \sum_{n=1}^{\infty} (\frac{1}{2} \lambda \dot{M}_n^2 + \frac{1}{2} \lambda \omega_n^2 M_n^2) : |\psi(0)\rangle = 0. \]

(111)

So in the large-time limit, the expectation value of total energy of the system in the initial state \( |\psi(0)\rangle = |0\rangle_B \otimes |1_{m_1}, ..., 1_{m_r}\rangle_s \), tends to zero, as expected.

Also, using (109), it is clear that

\[ \lim_{t \to \infty} [\langle \psi(0) | : \sum_{n=1}^{\infty} \frac{p_n^2}{2\lambda} + \frac{1}{2} \lambda \omega_n^2 x_n^2 : |\psi(0)\rangle] = \frac{\beta^2}{2\lambda} \langle x_n^2(0) \rangle = \frac{\beta^2 \hbar}{4\lambda^2} \sum_{i=1}^{r} \frac{1}{\omega_{m_i}}, \]

(112)

Now, if one obtains the Heisenberg equation for \( b_{nk}(t) \) and solve it formally in terms of \( \dot{x}_n(t) \), then using (108), he(she) will find

\[
\lim_{t \to \infty} [\langle \psi(0) | : \int d^3k \sum_{n=1}^{\infty} \hbar \omega_k b_{nk}^+(t)b_{nk}(t) : |\psi(0)\rangle] = \frac{\hbar \beta}{\pi \lambda} \sum_{i=1}^{r} \omega_{m_i} \int_0^\infty \frac{\omega_{m_i}^2 + x^2}{(\omega_{m_i}^2 - x^2)^2 + \frac{\beta^2}{\lambda^2} x^2} dx,
\]

\[ = \sum_{i=1}^{r} \hbar \omega_{m_i}. \]

(113)
Therefore, the energy of the system is completely transferred to the environment, as expected macroscopically.

4.2 Transition probabilities

As in the second section, we can calculate some of transition probabilities for the scalar field $\psi$ by tracing out the environment degrees of freedom. For example, Let the total density operator in the interaction picture at time $t = 0$, be

$$\rho_I(0) = |r_m\rangle_s \langle r_m| \otimes |0\rangle_B B\langle 0|,$$

where $|0\rangle_B$, is the vacuum state of the environment and $|r_m\rangle_s = (a_m^\dagger)\sqrt{r_m}|0\rangle_s$ is an excited state (Fock state) of the field $\psi$, in large-time limit, we obtain the following probability transitions

$$\Gamma_{|r_m\rangle_s \rightarrow |r_m-1\rangle_s} = Tr[|r_m - 1\rangle_s \langle r_m - 1|\rho(t)],$$

$$= Tr_s[|r_m - 1\rangle_s \langle r_m - 1|\rho (t)],$$

$$= \frac{4\pi^2 \omega_m^3 r t |f(\omega_m)|^2}{\lambda \hbar c^3},$$

(115)

where $Tr_s$, means taking trace over the scalar field eigenstates.

Another important case is when the environment has a canonical thermal distribution

$$\rho_I(0) = |r_m\rangle_s \langle r_m| \otimes \rho^T_B, \quad \rho^T_B = \frac{e^{-\frac{H_B}{kT}}}{TR_B(e^{-\frac{H_B}{kT}})},$$

(116)

then in large-time limit, we obtain the following transition probabilities

$$\Gamma_{|r_m\rangle_s \rightarrow |r_m-1\rangle_s} = \frac{4\pi^2 \omega_m^3 r t |f(\omega_m)|^2}{\lambda \hbar c^3} \frac{e^{\frac{\omega_m}{kT}}}{e^{\frac{\omega_m}{kT}} - 1},$$

$$\Gamma_{|r_m\rangle_s \rightarrow |n, r_m\rangle_s} = \frac{4\pi^2 \omega_n^3 t |f(\omega_n)|^2}{\lambda \hbar c^3} \frac{e^{\frac{\omega_n}{kT}}}{e^{\frac{\omega_n}{kT}} - 1}, \quad n \neq m,$$

$$\Gamma_{|r_m\rangle_s \rightarrow |r_m+1\rangle_s} = \frac{(r + 1)4\pi^2 \omega_m^3 t |f(\omega_m)|^2}{\lambda \hbar c^3} \frac{e^{\frac{\omega_m}{kT}}}{e^{\frac{\omega_m}{kT}} - 1}.\quad (117)$$

So in low temperatures, the energy flows from the oscillator to it’s environment by the rate $\frac{\omega_m}{kT}$ and no energy flows from the environment to the scalar field.
5 Dissipative vector field theory

Quantization of a quantum vector field $\vec{Y}$ in a three-dimensional inhomogeneous absorptive environment can be investigated by modeling the environment of $\vec{Y}$ by two independent quantum fields, namely $B$ and $\tilde{B}$ quantum fields. The susceptibility of the environment and the quantum noise fields are identified in terms of ladder operators of the environment and parameters of this model as in previous sections. We assume that the vector field $\vec{Y}$ can be propagated in infinite space with a suitable boundary condition at infinity, that is, the field $\vec{Y}$ tends to zero at infinity. In this case both the fields $B$ and $\tilde{B}$ contain a continuum of Klein-Gordon fields. It is also remarkable to note that if the volume in which the field $\vec{Y}$ can be propagated is a finite volume, for example a cubic cavity, then the fields $B$ and $\tilde{B}$ will contain a numerable set of Klein-Gordon fields, as in the previous section for the case of a scalar field.

From the interaction point of view, the field $B$ interacts with the conjugate canonical momentum density of the main vector field $\vec{Y}$ through a minimal coupling term and quantum field $\tilde{B}$ interacts with the field $\vec{Y}$ similar to a dipole interaction term. In this scheme, we take the environment Hamiltonian as

\[ H_E = H_B + H_{\tilde{B}}, \]

\[ H_B = \sum_{\nu=1}^{3} \int d^3\vec{k} \int d^3\vec{q} \omega_{\nu}(\vec{k}, \vec{q}, t) b_{\nu}(\vec{k}, \vec{q}, t), \]

\[ H_{\tilde{B}} = \sum_{\nu=1}^{3} \int d^3\vec{k} \int d^3\vec{q} \omega_{\nu}(\vec{k}, \vec{q}, t) d_{\nu}(\vec{k}, \vec{q}, t). \] (118)

where the annihilation and creation operators $b_{\nu}(\vec{k}, \vec{q}, t)$, $b_{\nu}^{\dagger}(\vec{k}, \vec{q}, t)$, $d_{\nu}(\vec{k}, \vec{q}, t)$ and $d_{\nu}^{\dagger}(\vec{k}, \vec{q}, t)$ of the environment, satisfy the commutation relations

\[ [b_{\nu}(\vec{k}, \vec{q}, t), b_{\nu}^{\dagger}(\vec{k}', \vec{q}', t)] = \delta_{\nu\nu'} \delta(\vec{k} - \vec{k}')\delta(\vec{q} - \vec{q}'), \]

\[ [d_{\nu}(\vec{k}, \vec{q}, t), d_{\nu}^{\dagger}(\vec{k}', \vec{q}', t)] = \delta_{\nu\nu'} \delta(\vec{k} - \vec{k}')\delta(\vec{q} - \vec{q}'). \] (119)

Let us assume that the Hamiltonian of the main system is

\[ H_Y = \int d^3\vec{r} \left[ \frac{\pi Y^2}{2\rho} + \frac{1}{2} \rho(\vec{r})\omega_0^2(\vec{r})Y^2 \right], \] (120)
where $\vec{\pi}_Y$, is the conjugate canonical momentum density of $\vec{Y}$. An example of this kind of Hamiltonian, the vector field $\vec{Y}$, can be an electric polarization density in an absorptive dielectric medium with related eigenfrequency $\omega_0(\vec{r})$ and density $\rho(\vec{r})$.[37]

The conjugate fields $\vec{Y}$ and $\vec{\pi}_Y$, satisfy the commutation relation

$$[\vec{Y}_i(\vec{r}, t), \vec{\pi}_{Yj}(\vec{r'}, t)] = i\hbar \delta_{ij} \delta(\vec{r} - \vec{r'}).$$ (121)

If the damping forces together with the restoring force $-\rho(\vec{r})\omega^2_0(\vec{r})\vec{Y}$, are exerted on the elements of the medium, then the equation of motion of $\vec{Y}$ should be a Langevin-Schrödinger equation (66), wherein, the external force $-\vec{\nabla}v$, is replaced with $-\rho(\vec{r})\omega^2_0(\vec{r})\vec{Y}$. For this purpose, we take the total Hamiltonian, i.e., the main system (vector field $\vec{Y}$), plus the environment as

$$H = \int d^3r \left[ \frac{(\vec{\pi}_Y - \vec{R})^2}{2\rho} + \frac{1}{2} \rho(\vec{r})\omega^2_0(\vec{r})\vec{Y}^2 - \vec{R} \cdot \vec{Y} + H_B + H_B' \right].$$ (122)

The operators $\vec{R}(\vec{r}, t)$ and $\vec{\tilde{R}}(\vec{r}, t)$, play the basic roles in the interaction between the environment and the system and are defined by

$$\vec{R}(\vec{r}, t) = \sum_{\nu=1}^{3} \int d^3\vec{k} \int \frac{d^3\vec{q}}{(2\pi)^3} \left[ f(\omega_{\vec{k}}, \vec{r})b^\dagger_{\nu}(\vec{k}, \vec{q}, t)e^{i\vec{q} \cdot \vec{r}} + f^*(\omega_{\vec{k}}, \vec{r})b_{\nu}(\vec{k}, \vec{q}, t)e^{-i\vec{q} \cdot \vec{r}} \right] \vec{u}_\nu(\vec{q}),$$ (123)

$$\vec{\tilde{R}}(\vec{r}, t) = \sum_{\nu=1}^{3} \int d^3\vec{k} \int \frac{d^3\vec{q}}{(2\pi)^3} \left[ g(\omega_{\vec{k}}, \vec{r})d_{\nu}(\vec{k}, \vec{q}, t)e^{i\vec{q} \cdot \vec{r}} + g^*(\omega_{\vec{k}}, \vec{r})d^\dagger_{\nu}(\vec{k}, \vec{q}, t)e^{-i\vec{q} \cdot \vec{r}} \right] \vec{\bar{u}}_\nu(\vec{q}),$$ (124)

where

$$\vec{u}_\nu(\vec{q}) = \vec{e}_{\nu\vec{q}}, \quad \nu = 1, 2,$$

$$\vec{\bar{u}}_3(\vec{q}) = \vec{q} = \frac{\vec{q}}{|\vec{q}|},$$

$$\vec{e}_{\nu\vec{q}} \cdot \vec{e}_{\nu'\vec{q'}} = \delta_{\nu \nu'},$$

$$\vec{q} \cdot \vec{e}_{\nu\vec{q}} = 0, \quad \nu = 1, 2,$$ (125)
are three orthogonal unit vectors for any $\vec{q}$.

The function $f(\omega_{k}, \vec{r})$ and $f(\omega_{k}, \vec{r})$, are the coupling functions, which are position dependent(independent) for an inhomogeneous(homogeneous) environment respectively. The equations of motion for the fields $\vec{Y}$ and $\vec{\pi}_{Y}$, can be obtained from the Heisenberg equations

$$\frac{\partial \vec{Y}}{\partial t} = i \frac{\hbar}{\rho} [H, \vec{Y}] = \frac{\vec{\pi}_{Y} - \vec{R}}{\rho},$$

$$\frac{\partial \vec{\pi}_{Y}}{\partial t} = i \frac{\hbar}{\rho} [H, \vec{\pi}_{Y}] = -\rho \omega_{0}^{2}(\vec{r})\vec{Y} + \vec{\tilde{R}}. \quad (126)$$

By eliminating $\vec{\pi}_{Y}$ between these equations we obtain the equation of motion of the vector field $\vec{Y}$ as,

$$\frac{\rho}{\partial t^{2}} + \rho \omega_{0}^{2}(\vec{r})\vec{Y} = -\frac{\partial \vec{R}}{\partial t} + \vec{\tilde{R}}. \quad (127)$$

Similar to what we did in the previous sections one can obtain, the constitutive equation of the environment as

$$\vec{R}(\vec{r}, t) = \vec{R}_{N}(\vec{r}, t) + \int_{0}^{\mid t \mid} dt' \chi(\vec{r}, \mid t \mid - t')\vec{Y}(\vec{r}, \pm t'),$$

$$\vec{\tilde{R}}(\vec{r}, t) = \vec{\tilde{R}}_{N}(\vec{r}, t) + \int_{0}^{\mid t \mid} dt' \tilde{\chi}(\vec{r}, \mid t \mid - t')\vec{Y}(\vec{r}, \pm t'), \quad (128)$$

where the upper(lower) sign corresponds to $t > 0 (t < 0)$, respectively. The following relations

$$\chi(\vec{r}, t) = \frac{8\pi}{\hbar c^{3}} \int_{0}^{\infty} d\omega \omega^{2} \mid f(\omega, \vec{r}) \mid^{2} \sin \omega t \quad t > 0$$

$$\chi(\vec{r}, t) = 0 \quad t \leq 0 \quad (129)$$

$$\tilde{\chi}(\vec{r}, t) = \frac{8\pi}{\hbar c^{3}} \int_{0}^{\infty} d\omega \omega^{2} \mid g(\omega, \vec{r}) \mid^{2} \sin \omega t, \quad t > 0,$$

$$\tilde{\chi}(\vec{r}, t) = 0, \quad t \leq 0, \quad (130)$$

give the susceptibilities of the environment in terms of the coupling functions. Operators $\vec{R}_{N}$ and $\vec{\tilde{R}}_{N}$, are noise fields

$$\vec{R}_{N}(\vec{r}, t) = \sum_{\nu=1}^{3} d^{3}\vec{k} \int \frac{d^{3}\vec{q}}{\sqrt{(2\pi)^{3}}} \mid f(\omega_{k}, \vec{r}) \mid b_{\nu}(\vec{k}, \vec{q}, 0) e^{-i\omega_{k} t + i\vec{q} \cdot \vec{r}}$$

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\( + f^* (\omega_{\vec{k}}, \vec{r}) b^\dagger_\nu (\vec{k}, \vec{q}, 0) e^{i\omega_{\vec{k}} t - i\vec{q} \cdot \vec{r}} \tilde{u}_\nu (\vec{q}), \)  \( \)  \( \)  \( (131) \)

\[ \tilde{R}_N (\vec{r}, t) = 3 \sum_{\nu = 1}^3 \int d^3 \vec{k} \int d^3 \vec{q} \left[ \frac{g (\omega_{\vec{k}}, \vec{r}) d_{\nu} (\vec{k}, \vec{q}, 0) e^{-i\omega_{\vec{k}} t + i\vec{q} \cdot \vec{r}}}{\sqrt{(2\pi)^3}} \right. \]

\[ + g^* (\omega_{\vec{k}}, \vec{r}) d_{\nu} (\vec{k}, \vec{q}, 0) e^{i\omega_{\vec{k}} t - i\vec{q} \cdot \vec{r}} \tilde{u}_\nu (\vec{q}). \]  \( \)

\( (132) \)

Finally, substituting (128) in (127), we find a generalized Langevin-Schrödinger equation for the damped vector field \( \vec{Y} \)

\[ \rho \ddot{\vec{Y}} + \rho \omega_0^2 (\vec{r}) \vec{Y} \pm \frac{d}{dt} \int_0^{|t|} dt' \chi (\vec{r}, |t| - t') \dot{\vec{Y}} (\vec{r}, \pm t') - \]

\[ \int_0^{|t|} dt' \chi (\vec{r}, |t| - t') \dot{\vec{Y}} (\vec{r}, \pm t') + \hat{R}_N - \tilde{R}_N \xi (\vec{r}, t) = 0, \]  \( \)

\( (133) \)

This equation can be solved using the Laplace transformation technique for negative and positive times similar to the previous sections.

6 Concluding remarks

By considering a dissipative quantum system as an open system which interacts with it’s environment, a modeling of the environment by massless Klein-Gordon fields, is achieved. Using the above idea, i.e., the field version of an environment, an arbitrary dissipative quantum system is quantized systematically and consistently. Some coupling functions are introduced which physically describe the environment under consideration and also have a basic role in interaction between the system and it’s environment. Inspired by the ideas of electrodynamics in a media, some susceptibility functions are attributed to the environment and formulas connecting these susceptibilities to the coupling function, are found. The quantum Langevin-Schrödinger equation is obtained directly from the Heisenberg equations. The explicit form of the noise terms are obtained. Some transition probabilities indicating the way energy flows from the system to it’s environment, are calculated and the energy conservation, as a signal of consistency, is explicitly worked out. The whole formalism is generalized to the case of a dissipative scalar and vector field theory straightforwardly.
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