The role of hyperfine mixing in $b \rightarrow c$ semileptonic and electromagnetic decays of doubly-heavy baryons

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Abstract. We analyze the effects of hyperfine mixing in $b \rightarrow c$ semileptonic and electromagnetic decays of doubly heavy baryons.

Keywords: doubly-heavy baryons, semileptonic decay, electromagnetic decay, hyperfine mixing

PACS: 12.39.Jh, 13.30.Ce, 13.40.Hq

INTRODUCTION

In the infinite heavy quark mass limit, and according to heavy quark spin symmetry, the total spin of the heavy quark subsystem in a doubly heavy baryon can be selected to be $S_h = 0, 1$. Indeed, this has been the criterion for the most common classification scheme of these baryons. Table 1 summarizes the quark content and quantum numbers of the baryons considered in this study, classified so that $S_h$ is well defined, and to which we shall refer as the $S_h$-basis. Being ground state baryons, a total orbital angular momentum $L = 0$ is assumed.

**TABLE 1.** Quantum numbers and quark content of ground-state doubly heavy baryons.

| Baryon | Quark content | $S_h$ | $J^z$ | Baryon | Quark content | $S_h$ | $J^z$ |
|--------|--------------|-------|-------|--------|--------------|-------|-------|
| $\Xi_{cc}$ | [c c] | 1 | 1/2$^+$ | $\Omega_{cc}$ | [c c] | s | 1 | 1/2$^+$ |
| $\tilde{\Xi}_{cc}$ | [c c] | 1 | 3/2$^+$ | $\tilde{\Omega}_{cc}$ | [c c] | s | 1 | 3/2$^+$ |
| $\Omega_{bb}$ | [b b] | 1 | 1/2$^+$ | $\Omega_{bb}$ | [b b] | s | 1 | 1/2$^+$ |
| $\Omega'_{bc}$ | [b c] | 1 | 3/2$^+$ | $\Omega'_{bc}$ | [b c] | s | 1 | 3/2$^+$ |
| $\Xi_{bc}$ | [b c] | 1 | 1/2$^+$ | $\Xi_{bc}$ | [b c] | s | 0 | 1/2$^+$ |
| $\Omega'_{bc}$ | [b c] | 1 | 0 | $\Omega'_{bc}$ | [b c] | s | 0 | 1/2$^+$ |

Due to the finite value of the heavy quark masses, the hyperfine interaction between

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1 Supported by DGI and FEDER funds, under contracts FIS2008-01143/FIS, FIS2006-03438, FPA2007-65748, CSD2007-00042, by Junta de Castilla y León under contracts SA016A07 and GR12, and by the EU HadronPhysics2 project
TABLE 2. Masses (in MeV) for unmixed states.

| State | This work | [5] | [1] | This work | [5] | [1] |
|-------|-----------|-----|-----|-----------|-----|-----|
| $M_{\Xi_{cc}}$ | 3613 | 3620 | 3676 | $M_{\Omega_{cc}}$ | 3712 | 3778 | 3815 |
| $M_{\Xi_{cc}^*}$ | 3707 | 3727 | 3753 | $M_{\Omega_{cc}^*}$ | 3795 | 3872 | 3876 |
| $M_{\Xi_{bb}}$ | 10198 | 10202 | 10340 | $M_{\Omega_{bb}}$ | 10269 | 10359 | 10454 |
| $M_{\Xi_{bb}^*}$ | 10237 | 10237 | 10367 | $M_{\Omega_{bb}^*}$ | 10307 | 10389 | 10486 |
| $M_{\Xi_{bc}}$ | 6928 | 6933 | 7020 | $M_{\Omega_{bc}}$ | 7013 | 7088 | 7147 |
| $M_{\Xi_{bc}^*}$ | 6958 | 6963 | 7044 | $M_{\Omega_{bc}^*}$ | 7038 | 7116 | 7166 |
| $M_{\Xi_{bc}^{**}}$ | 6996 | 6980 | 7078 | $M_{\Omega_{bc}^{**}}$ | 7075 | 7130 | 7191 |

the light quark and any of the heavy quarks can admix both $S_h = 0$ and $S_h = 1$ spin components into the wave function. This mixing is negligible for $cc$ and $bb$ baryons, as the antisymmetry of the wave function would require higher orbital angular momenta or radial excitations. On the other hand, in the $bc$ sector, one would expect the role of the mixing to be noticeable and actual physical states to be admixtures of the $B_{bc}$ and $B'_{bc}$ ($B = \Xi, \Omega$) states listed in Table 1.

Masses are rather insensitive to the mixing, and most calculations simply ignore it and use the $S_h$-basis. Roberts and Pervin [1] took the issue of the hyperfine mixing and its role in semileptonic decays. They noticed that, working in the $S_h$-basis, the decay width of $\Xi_{bc}$ ($\Omega_{bc}$) greatly differs from that of $\Xi'_{bc}$ ($\Omega'_{bc}$) so that mixing could be of relevance for these processes. In Ref. [2] they found that indeed mixing had an enormous impact in semileptonic decays of doubly-heavy baryons.

We have studied the effect of this mixing in both semileptonic and electromagnetic decays in Refs. [3] and [4]. Our study on the semileptonic decay widths of $bc$ baryons qualitatively corroborate the findings of Roberts and Pervin. In the electromagnetic case the decay widths are proportional to $(M_I - M_F)^3$, with $M_I, M_F$ the initial and final baryon masses, showing thus a strong dependence on the actual baryon masses.

RESULTS AND DISCUSSION

Table 2 shows our results for the masses of the unmixed states. We compare them to the results from Refs. [5] and [1]. Details on the model used can be found in Ref. [6]. Mixed $bc$ states are obtained by diagonalizing the corresponding mass matrices. In our calculation, the mixed states and masses are given by [3]

$$
\begin{align*}
\Xi_{bc}^{(1)} &= 0.902 \Xi'_{bc} + 0.431 \Xi_{bc}, \\
M_{\Xi_{bc}^{(1)}} &= 6967 \text{ MeV}, \\
\Xi_{bc}^{(2)} &= -0.431 \Xi'_{bc} + 0.902 \Xi_{bc}, \\
M_{\Xi_{bc}^{(2)}} &= 6919 \text{ MeV}, \\
\Omega_{bc}^{(1)} &= 0.899 \Omega'_{bc} + 0.437 \Omega_{bc}, \\
M_{\Omega_{bc}^{(1)}} &= 7046 \text{ MeV}, \\
\Omega_{bc}^{(2)} &= -0.437 \Omega'_{bc} + 0.899 \Omega_{bc}, \\
M_{\Omega_{bc}^{(2)}} &= 7005 \text{ MeV}.
\end{align*}
$$

(1)

Comparing with Table 2, we see small changes in the masses when mixing is taken into account. However, as shown in Eq. (1), the admixture is important and it can affect the
TABLE 3. Semileptonic decay widths (in units of 10^{-14} GeV) for unmixed states. We use $|V_{cb}| = 0.0413$, $l = e, \mu$.

| Decay                      | This work [7] | [8] | [2] | This work [7] | [8] | [2] |
|----------------------------|---------------|-----|-----|---------------|-----|-----|
| $\Gamma(\zeta_{bb} \rightarrow \zeta_{bc}^{*}/\nu_{l})$ | 1.08          | 0.82 | 0.36 | --            | 1.14 | 0.85 | 0.42 |
| $\Gamma(\zeta_{bb} \rightarrow \zeta_{bc}^{*}/\nu_{l})$ | 0.36          | 0.28 | 0.14 | --            | 0.38 | 0.29 | 0.15 |
| $\Gamma(\zeta_{bb} \rightarrow \zeta_{bc}^{*}/\nu_{l})$ | 1.09          | 0.82 | 0.43 | 0.41 | 1.16 | 0.83 | 0.48 | 0.51 |
| $\Gamma(\zeta_{bb} \rightarrow \zeta_{bc}^{*}/\nu_{l})$ | 2.00          | 1.63 | 0.80 | 0.69 | 2.15 | 1.70 | 0.86 | 0.92 |
| $\Gamma(\zeta_{bc} \rightarrow \zeta_{cc}^{*}/\nu_{l})$ | 1.36          | 0.88 | 1.10 | --            | 1.36 | 0.95 | 0.98 | --       |
| $\Gamma(\zeta_{bc} \rightarrow \zeta_{cc}^{*}/\nu_{l})$ | 2.57          | 2.30 | 2.10 | 1.38 | 2.58 | 2.48 | 1.88 | 1.54 |
| $\Gamma(\zeta_{bc} \rightarrow \zeta_{cc}^{*}/\nu_{l})$ | 2.35          | 1.70 | 2.01 | --            | 2.35 | 1.83 | 1.93 | --       |
| $\Gamma(\zeta_{bc} \rightarrow \zeta_{cc}^{*}/\nu_{l})$ | 0.75          | 0.72 | 0.64 | 0.52 | 0.76 | 0.74 | 0.62 | 0.56 |

TABLE 4. Electromagnetic decay widths (in units of 10^{-8} GeV) for unmixed states.

| Decay                      | This work [9] | [9] | [9] | This work [9] | [9] | [9] | [9] |
|----------------------------|---------------|-----|-----|---------------|-----|-----|-----|
| $\Xi_{bcs}^{*} \rightarrow \Xi_{bcs}^{*}/\gamma$ | 4.04          | 0.28 | ±0.01 | 3.69          | 0.16 | ±0.01 | -- |
| $\Xi_{bcd}^{*} \rightarrow \Xi_{bcd}^{*}/\gamma$ | 4.04          | 0.28 | ±0.01 | --            | --   | --   | -- |
| $\Xi_{bcu}^{*} \rightarrow \Xi_{bcu}^{*}/\gamma$ | 105           | 49  | ±9   | 20.9          | 0.12 | ±0.02 | -- |
| $\Xi_{bcd}^{*} \rightarrow \Xi_{bcd}^{*}/\gamma$ | 50.5          | 24  | ±4   | --            | --   | --   | -- |
| $\Xi_{bcu}^{*} \rightarrow \Xi_{bcu}^{*}/\gamma$ | 0.992         | 1.56 | ±0.08 | 0.568         | 1.26 | ±5    | -- |
| $\Xi_{bcd}^{*} \rightarrow \Xi_{bcd}^{*}/\gamma$ | 0.992         | 1.56 | ±0.08 | --            | --   | --   | -- |

decay widths.

The results for $b \rightarrow c$ semileptonic decays in the unmixed case are shown in Table 3, where for comparison we also show the results obtained in Refs.[7, 8], within different relativistic approaches, and in the nonrelativistic calculation of Ref.[2]. Our results are in a global fair agreement with the ones in Ref.[7]. As for the other relativistic calculation in Ref.[8], the agreement is fair for transitions with a $bc$ baryon in the initial state but there is an approximate factor of 2 discrepancy for transitions with a $bc$ baryon in the final state. The nonrelativistic calculation in Ref.[2] also gives results that are roughly a factor of 2 smaller than ours for all decays. A very interesting feature of the decay widths shown in Table 3 is that they are very different for transitions involving $\Xi_{bc}$ or $\Xi_{bc}^{*}$ ($\Omega_{bc}$ or $\Omega_{bc}^{*}$). This means, as suggested in Ref.[1], that mixing in those states, provided the admixture coefficients are large, can have a great impact on the decay widths.

In Table 4 we show our results for electromagnetic decays in the unmixed case. Branz et al. [9] have also studied electromagnetic decays of doubly heavy baryons within a relativistic constituent quark model. The agreement with our results is very poor in this case in part due to the different masses used in both calculations. The agreement improves in most cases if we divide out the $(M_{l} - M_{F})^{3}$ mass factor discussed above. Still the differences are in the range 50-80%. This can be seen in Table 5.

$b \rightarrow c$ semileptonic decay widths involving the mixed states $\Xi_{bc}^{(1)}, \Xi_{bc}^{(2)}$, and $\Omega_{bc}^{(1)}, \Omega_{bc}^{(2)}$ are now given in Table 6. We see rather big changes from the values in Table 3 where unmixed states were used. Special attention deserves the $B_{bc}^{(2)} \rightarrow B_{cc}^{*}$ transitions where the width reduces by a large factor of 44 (54) for the $\Xi_{bc}^{(2)} \rightarrow \Xi_{cc}^{*}$ ($\Omega_{bc}^{(2)} \rightarrow \Omega_{cc}^{*}$) decay compared to the unmixed case. This can be easily understood by taking into account
that $B_{bc}^{(2)} \approx \langle |qc; 0 \rangle \otimes |b; \frac{1}{2} \rangle \rangle^{1/2}$. In the latter state the light and $c$ quarks are coupled to spin 0, whereas in the $B_{bc}^{*}$ the light and any of the $c$ quarks are in a relative spin 1 state. In any spectator calculation, as the ones here and in Ref.[2], the amplitude for the $(|qc; 0 \rangle \otimes |b; \frac{1}{2} \rangle \rangle^{1/2} \rightarrow B_{bc}^{*}$ transition cancels due to the orthogonality of the different spin states of the spectator quarks in the initial and final baryons. The fact that $B_{bc}^{(2)}$ slightly deviates from $(|qc; 0 \rangle \otimes |b; \frac{1}{2} \rangle \rangle^{1/2}$ produces a non zero, but small, decay width.

Our results for the electromagnetic decay width for mixed states are enclosed in

### Table 5. Electromagnetic decay widths, divided by $(M_j - M_F)^3$, (in units of $(10^{-5}$ $GeV^{-2}$) for un mixed states.

| This work | Branz et al. | This work | Branz et al. |
|-----------|-------------|-----------|-------------|
| $\Xi_{bcu}^* \rightarrow \Xi_{bcu}^* \gamma$ | 73.6 | 57.0 | $\Omega_{bcs}^* \rightarrow \Omega_{bcs}^* \gamma$ | 72.8 | 58.3 |
| $\Xi_{bcd}^* \rightarrow \Xi_{bcd}^* \gamma$ | 73.6 | 57.0 | |
| $\Xi_{bcu}^* \rightarrow \Xi_{bcu}^* \gamma$ | 333.9 | 471 | $\Omega_{bcs}^* \rightarrow \Omega_{bcs}^* \gamma$ | 87.7 | 160 |
| $\Xi_{bcd}^* \rightarrow \Xi_{bcd}^* \gamma$ | 160.6 | 231 | |
| $\Xi_{bcs}^* \rightarrow \Xi_{bcs}^* \gamma$ | 36.7 | 57.8 | $\Omega_{bcs}^* \rightarrow \Omega_{bcs}^* \gamma$ | 36.3 | 57.4 |
| $\Xi_{bcd}^* \rightarrow \Xi_{bcd}^* \gamma$ | 36.7 | 57.8 | |

### Table 6. Semileptonic decay widths (in units of $10^{-14}$ $GeV$) for mixed states.

| This work | [2] | This work | [2] |
|-----------|-----|-----------|-----|
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{cc}^* (\gamma))$ | 0.47 | – | $\Gamma(\Omega_{bcs}^* \rightarrow \Omega_{cc}^* (\gamma))$ | 0.48 | – |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{cc}^* (\gamma))$ | 0.99 | – | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{cc}^* (\gamma))$ | 1.06 | – |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{bc}^* (\gamma))$ | 2.21 | 0.95 | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{bc}^* (\gamma))$ | 2.36 | 0.99 |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{bc}^* (\gamma))$ | 0.85 | 0.33 | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{bc}^* (\gamma))$ | 0.91 | 0.30 |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{bc}^* (\gamma))$ | 0.38 | – | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{bc}^* (\gamma))$ | 0.37 | – |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{bc}^* (\gamma))$ | 3.50 | 1.92 | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{bc}^* (\gamma))$ | 3.52 | 1.99 |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{bc}^* (\gamma))$ | 3.14 | – | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{bc}^* (\gamma))$ | 3.14 | – |
| $\Gamma(\Xi_{bc}^* \rightarrow \Xi_{bc}^* (\gamma))$ | 0.017 | 0.026 | $\Gamma(\Omega_{bc}^* \rightarrow \Omega_{bc}^* (\gamma))$ | 0.014 | 0.013 |

### Table 7. Electromagnetic decay widths (in units of $10^{-8}$ $GeV$) for mixed states.

| This work | [9] | This work | [9] |
|-----------|-----|-----------|-----|
| $\Xi_{bcs}^* \rightarrow \Xi_{bcs}^* (\gamma)$ | 6.05 | 0.15 ± 0.02 | $\Omega_{bcs}^* \rightarrow \Omega_{bcs}^* (\gamma)$ | 0.31 | (1 ± 1) • 10^{-4} |
| $\Xi_{bcd}^* \rightarrow \Xi_{bcd}^* (\gamma)$ | 0.12 | (2 ± 2) • 10^{-4} | |
| $\Xi_{bcs}^* \rightarrow \Xi_{bcs}^* (\gamma)$ | 73.9 | 46 ± 10 | $\Omega_{bcs}^* \rightarrow \Omega_{bcs}^* (\gamma)$ | 50.2 | 29 ± 3 |
| $\Xi_{bcd}^* \rightarrow \Xi_{bcd}^* (\gamma)$ | 103 | 51 ± 6 | |
| $\Xi_{bcs}^* \rightarrow \Xi_{bcs}^* (\gamma)$ | 12.4 | 14 ± 3 | $\Omega_{bcs}^* \rightarrow \Omega_{bcs}^* (\gamma)$ | 8.52 | 21 ± 2 |
| $\Xi_{bcd}^* \rightarrow \Xi_{bcd}^* (\gamma)$ | 20.9 | 31 ± 4 | |
Table 7. To the best of our knowledge ours was the first calculation which took into account the effect of the mixing in the electromagnetic decay width of $bc$-baryons. As for the unmixed case, we find a better qualitative agreement with the results of Ref. [9] when we show the decay widths divided by the $(M_I - M_F)^3$ mass factor, see Table 8.

**CONCLUSIONS**

We qualitatively confirm the findings in Refs. [1, 2] as to the relevance of hyperfine mixing in $b \to c$ semileptonic decays of doubly heavy baryons. Actual results differ by a factor of two. We find mixing is also very important for electromagnetic decays. In this latter case decay widths are very sensitive to the actual baryon masses. We have compared our results for the electromagnetic decay widths with those from Ref. [9]. Predictions are rather different, although a better agreement is found if the dependence of the width on $(M_I - M_F)^3$ is divided out. Electromagnetic decay studies demand an accurate determination of the masses.

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