Buckling Response Analysis of Laminated Plates Subjected to Localised Bi-axial In-plane Compressive Loading

Achchhe Lal 1, Anant Parghi 2, Anil Kumar Mahto 3, Rahul Kumar 4

1 Assistant Professor, Department of Mechanical Engineering SVNIT, Surat, India.
2 Associate Professor, Department of Mechanical Engineering SVNIT, Surat, India.
3 Assistant Professor, Department of Mechanical Engineering SVNIT, Surat, India.
4 Research Scholars, Department of Mechanical Engineering SVNIT, Surat., India.
*E-Mail: lalachchhe@yahoo.co.in

Abstract: Laminated composite plates are well known structural member since last four decades because of its high stiffness and strength to weight ratio. Usually this type of structural members is subjected to partial biaxial loading during their functioning. For better safety and stability of structure, analysis of these structural member in that condition of loading is necessary. In this present analysis buckling response of laminated composite plates under biaxial partial in-plane loading is examined and effect of various parameters like aspect ratio, stacking sequence, fiber orientation and plate thickness ratio of nondimensionalised critical buckling load (NCBL) are revealed. A MATLAB program is developed higher order shear deformation theory based on finite element method for buckling analysis of laminated composite plate. Efficiency of the program is checked by comparing the result of the present work with that of already published which shows good favour of later one.

Keywords: Laminated Composite Plate, biaxial loading

1. Introduction

Application of laminated composite plates as a structural member is very common in weight sensitive area like aerospace vehicles, submarines, bridges etc. due to its outstanding stiffness and strength to weight ratio. These plates are usually subjected partial in-plane loading by their adjacent structural member. Therefor for better safety and reliability proper analysis of these structural member are required.

Many researches are available for buckling behaviour analysis of plates under in-plane edge loadings. Osama et al. [1-2] done buckling analysis of laminated composite plate under biaxial in-plane loading. [1] used classical laminated plate theory (CLPT) and find the effect of stacking sequence, aspect ratio and modulus ratio on buckling load. [2] done survey of different plate theories and method to examine buckling behaviour of laminated plate, beside this the author achieved various other objectives like developed theoretical model to predict buckling load, studied about finite element based technique for behaviour analysis of laminated plates under buckling, accuracy of the developed technique, influence of coupling parameter of bending and twisting on buckling response and generated results based on CLPT.

Libove [3] studied buckling of orthotropic under biaxial loading when edges are simply supported and found that plates buckle with half waves in the two-principal direction. Shukla et al. [4] used first order shear deformation theory with von-Karman-type nonlinearity to estimated buckling load of laminated composite plates subjected to uniaxial and biaxial loadings and revealed the effect of support conditions, aspect ratio, stacking sequence, number of layers and properties of material on buckling loads. Tung and Surdenas [5] analysed orthotropic plate for buckling response when plate subjected to biaxial loading. Wnag and Wang [6] obtained buckling load for plate under uniaxial or biaxial compressive concentrated load using differential quadrature method and compared the results with that of existing solution.
Sometimes structural members are not subjected to load to whole of the edges besides that subjected to partial edge loading by their adjacent structural member. Analysis of these structural members in that condition of locally applied load introduces more complexity.

Spencer and Surjanhata [7] simplified buckling criteria for partial uniaxial loading and compared the results with numerical solutions. Srivastava et al. [8] studied stability of stiffened plates under localised edge loading using finite element technique and find the effect of aspect ratio, boundary conditions and different non-uniform loadings on NCBL and frequencies. Baker and Pavlovic [9] used Mathieu’s exact solution for evaluation of buckling coefficients for subjected to locally distributed load and concentrated load.

From the literature survey it is observed that stability of laminated composite plate under biaxial partial in-plane compressive load needs to be analysed for efficiently used as structural member. In the present work buckling response analysis of laminates under biaxial localised in-plane edge loading is done. Two cases for different position of load application is considered here and their effect on NCBL is investigated. The effect of increment of extent of loading on same is also studied.

2. Problem formulation

Buckling response analysis of laminated composite plate having dimension 1 x 1 x 0.1 subjected to biaxial partial in-plane loading is done with finite element based MATLAB program using nine noded isoperimetric quadrilateral element.

### Table 1: Mechanical properties of material

| $E_1$ (GPa) | $E_2$ (GPa) | $v_{12} = v_{31}$ | $v_{23}$ | $G_{12}$ (GPa) |
|-------------|-------------|-------------------|---------|----------------|
| 82.64       | 9.8034      | 0.3               | 0.0356  | 3.8545         |

2.1. Displacement field

In the present study the displacement field can be given as,

\[
\begin{align*}
    u_1 &= u_0 + z\psi_1 + z^2\phi_1, \\
    u_2 &= v_0 + z\psi_2 + z^2\phi_2, \\
    u_3 &= w_0.
\end{align*}
\]

(1)

2.2 Strain-displacement expression

\[
\begin{align*}
    \epsilon_{xx} &= \frac{\partial u_0}{\partial x}, & \epsilon_{yy} &= \frac{\partial v_0}{\partial y}, & \epsilon_{xy} &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \\
    \epsilon_{yz} &= \frac{\partial w_0}{\partial y}, & \epsilon_{zx} &= \frac{\partial u_0}{\partial z} + \frac{\partial w_0}{\partial x}, & \epsilon_{xy} &= \frac{\partial u_0}{\partial y} + \frac{\partial w_0}{\partial x}.
\end{align*}
\]

(2)

2.3. Stress-strain relations

Constitutive relation for the present work can be given as,

\[
\begin{pmatrix}
    \sigma_{xx} \\
    \sigma_{yy} \\
    \tau_{xy}
\end{pmatrix} = \begin{bmatrix}
    \epsilon_{xx} \\
    \epsilon_{yy} \\
    \gamma_{xy}
\end{bmatrix} Q
\]

(3)

Where $Q$ is reduced constitutive matrix.

Boundary condition applied in the present study is given as,

- $v = 0, w = 0, \psi_1 = 0, \phi_1 = 0; \quad \text{at} \quad x = 0, a$
- $u = 0, w = 0, \psi_1 = 0, \phi_1 = 0; \quad \text{at} \quad y = 0, b$
2.4. Finite element approach

Displacement vector can be given as,

\[ U^e = \begin{bmatrix} u_1 & v_1 & u_2 & v_2 & \ldots & u_n & v_n \end{bmatrix}^T \]

(4)

\[ U_i = \sum_{n=1}^{n_e} N'_i u_n \quad \text{and} \quad U_i = \sum_{n=1}^{n_e} N'_i v_n \]

(5)

Where, \( N'_i \) is elemental shape function.

Strain vector can be given as

\[ \{ \varepsilon \} = [L] [U^e] \]

(6)

The stiffness matrix of laminated plate can be given as,

\[ [K_i] = \int_{-1}^{1} \int_{-1}^{1} [L_i]^T [Q][L_i] |J| d\xi d\eta, \]

(7)

Where, \([L_i]\) is kinematic matrix, \([J]\) is determinant of Jacobian matrix and \([Q]\) is transformed constitutive matrix.

The geometric stiffness matrix of plate may be written as,

\[ [K_{ge}] = \int_{-1}^{1} \int_{-1}^{1} [L_{ge}]^T [N_i] [L_{ge}] |J| d\xi d\eta, \]

(8)

where, \([L_{ge}]\) is geometric kinematic matrix and \([N_i]\) is load vector.

\[ [N_i] = [N_{\alpha \alpha} \ N_{\alpha \gamma} \ N_{\gamma \alpha} \ N_{\gamma \gamma}]. \]

Where, \([N_{\alpha \alpha}] = 1/\text{thickness}, \]

\([N_{\alpha \gamma}] = 0 \) and \([N_{\gamma \gamma}] = 1/\text{thickness}\)

For partial edge loading \([N_i] = C_i\), where, \( C_i \) is x-coordinates of nodes of edge on which load is applied.

\[ [L_{ge}] = \begin{bmatrix} 0 & 0 & \frac{\partial N_\alpha}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial N_\gamma}{\partial x} & 0 & 0 & 0 \end{bmatrix}, \]

(10)

2.5. Governing equation for buckling

The governing equation for buckling response of plate acted up on by in-plane partial loading may be given as,

\[ ([K_i] - \lambda c, [K_{ge}] \} \{ \nu \} = 0, \]

(11)

NCBL for partial in-plane loading may be given as,

\[ \lambda_{cr} = \frac{E c h^2}{h}, \]

(12)

Where ‘c’ is percentage of width of plate on which partial load is acted upon.

3. Result and Discussion

NCBL of laminated composite plate subjected to biaxial partial in-plane loading for different position of loading along the edge are evaluated. Table 1 Shows validation study for NCBL for a simply supported laminated plate (0/90/0/90/0) under a/h=20 and a/h=100. It is revealed from the Table 2, that outcomes are in good agreement with the published result.
Figure 1: Biaxial partial in-plane compressive loading on 20% of edge for two different position (a) Case-I and (b) Case-II.

Table 2: Validation of non-dimensional critical buckling load for a simply supported laminated composite plate with stacking sequence (0/90/0/90/0) subjected to in-plane partial edge loadings.

| c/a  | a/h = 20 Present | Ref. [7] | a/h =100 Present | Ref. [7] |
|------|------------------|----------|------------------|----------|
| 0.2  | 15.2413          | 16.287   | 18.4578          | 19.563   |
| 0.4  | 17.3145          | 18.417   | 21.6318          | 21.590   |
| 0.6  | 20.6718          | 21.810   | 25.2101          | 25.110   |
| 0.8  | 24.8345          | 26.487   | 31.7152          | 30.205   |
| 1.0  | 30.6587          | 31.958   | 32.5478          | 35.962   |

Table 3: Variation of NCBL with aspect ratios and extent of edge loadings of simply supported laminated composite (0/90/0/90/0) with a/h=100 subjected to partial bi-axial compressive in-plane edge loading at different position.

| c/a  | NCBL a/h=1 | a/h=1.5 | a/h=2 | a/h=2.5 | a/h=3 |
|------|------------|---------|-------|---------|-------|
| Case-I |            |         |       |         |       |
| 0.2  | 10.974     | 7.2099  | 5.869 | 4.3097  | 3.414 |
| 0.4  | 8.3357     | 5.5700  | 4.604 | 3.3970  | 2.705 |
| 0.6  | 10.974     | 7.2099  | 5.869 | 4.3097  | 3.414 |
| 0.8  | 9.6664     | 6.4290  | 5.291 | 3.8988  | 3.100 |
| 1.0  | 13.629     | 8.7099  | 6.980 | 5.0807  | 4.009 |
| Case-II |            |         |       |         |       |
| 0.2  | 5.751      | 3.8004  | 3.110 | 2.2872  | 1.815 |
| 0.4  | 11.21      | 7.0919  | 5.676 | 4.1249  | 3.257 |
| 0.6  | 8.723      | 5.6510  | 4.582 | 3.3511  | 2.654 |
| 0.8  | 10.26      | 6.6931  | 5.412 | 3.9654  | 3.134 |
| 1.0  | 13.62      | 8.7099  | 6.980 | 5.0807  | 4.009 |

Table 3 shows that buckling load is decreases with increase in the length of the plate. Plate with aspect ratio 3 shows 68% less buckling load than that in case of aspect ratio is one. Plate with loading at whole edge i.e. uniform loading requires more load to buckle than the plate when subjected to nonsymmetric loading. When loading is applied at centre as in case-I plate shows 47.6% more buckling load than that in case-II loading condition.

Table 4: Effect of span to thickness ratio on NCBL of simply supported square laminated composite plate (0/90/0/90/0) under bi-axial in-plane partial (20% of edge) edge loading at different positions.
Table 4 Reveals the effect of plate span to thickness ratio of NCBL of laminated composite plate (0/90/0/90/0) under biaxial in-plane loading when loading extent is 20% of the edge of plate. With increasing thickness ratio, NCBL increases and shows 37.18% increment in buckling load when thickness ratio changes from 10 to 50 in loading case-I while only 63.6% as in case-II.

Table 5 shows variation of NCBL with fiber orientation, as fiber orientation increases from 00 to 450 NCBL increases after that from 450 to 900 it starts decreasing because fibers have highest strength along longitudinal direction.

Table 6. Variation of NCBL with stacking sequence and biaxial partial (20 % of edge) in-plane loading position for a simply supported square laminated composite plate with a/h=100.

4. Conclusion

Behaviour of Laminated composite plate when subjected to biaxial partial in-plane compressive loading for two different loading positions are examined here in the present work. The effect of span to thickness ratio, fiber orientation, stacking sequence and aspect ratio on NCBL are explored. Some important conclusions made from the present work can be given as,

- With increasing dimension of the plate NCBL decreases and plate gets easy to buckle when subjected to biaxial in-plane loading.
- Plate when subjected to biaxial in-plane loading at centre of edge (case-I) is more stable than the plate when subjected to loading at corners of edges.
- With the increment of thickness of the plate gets difficult to buckle.
- Plate with 450 fiber orientation are more stable when subjected to biaxial partial in-plane loading.
- Plate having symmetric stacking sequence is most stable when subjected to biaxial partial in-plane loading.

References

[1] Khayal O and Hussan T 2018 Biaxial buckling of laminated composite plates *International Journal of Bridge Engineering (IJBE)*, 19-44.

[2] Khayal O M E S Biaxial buckling of thin laminated composite plates Lap Lambert Academic Publishing, ISBN 978-613-8-23615-3.

[3] Libove C 1983 Buckling pattern of biaxially compressed simply supported orthotropic rectangular plates *Journal of composite material*, 17(1), 45-48.

[4] Shukla K K, Nath Y, Kreuzer E and Kumar K V S 2005 Buckling of laminated composite rectangular plates *Journal of Aerospace Engineering*, 18, 215-223.

[5] Tung K and Surdenas J 1987 Buckling of rectangular orthotropic plates under biaxial loading *Journal of Composite Materials*, 21, 124-128.

[6] Wang X, Wang Y 2015 Buckling analysis of thin rectangular plates under uniaxial or biaxial compressive point loads by the differential quadrature method *International Journal of Mechanical Sciences*, 10, 07-021.

[7] Spencer H H, Surjanhata H 1986 The simplified buckling criterion applied to plates with partial edge loading *Applied Scientific Research*, 43, 79-90.

[8] Srivastava A K L, Datta P K, Sheikh A H 2003 Buckling and vibration of stiffened plates subjected to partial edge loading *International Journal of Mechanical Sciences*, 45, 73-93.

[9] Baker G, Pavlovic M N 1982 Elastic stability of simply supported rectangular plates under locally distributed edge forces *Journal of Applied Mechanics*, 49/177.