New Experimental Results on Steady Streaming in Oscillating Deformable Spherical Container

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Abstract. The paper presents an experimental study of steady streaming generated by harmonic oscillations of the boundaries of a deformable spherical container. The container is filled with a viscous fluid and its oscillations are produced by means of two linear servo motors, installed symmetrically. The shape of the boundary during deformation is studied by processing images recorded with a high speed camera. The flows are investigated using the particle image velocimetry. In the analysis of results, the focus is made on the relation between the shape of the oscillating boundary, the dimensionless frequency and the streaming velocity. The possibility of controlling the steady streaming pattern in order to affect the convective mass transfer is discussed.

1. Introduction
In modern industry, the problem of mass transfer intensification in chemical systems remains actual and very important [1–3]. For instance, in the process of liquid–liquid extraction the disturbances of the interface between dispersed phase (drops) and surrounding liquid lead to the excitation of drops’ oscillations, accompanied by the increase in the reaction rate [4]. Drop oscillations are an efficient method of action, because they lead to the mixing of fluid inside and outside by means of the forced vibrational convection [5, 6]. In order to excite the drop oscillations various techniques are applied: extraction columns with special geometry, alternating electric fields, microchannels with special shape, ultrasound, etc. [3, 7, 8]. In a number of theoretical and experimental works, steady streaming generated by the oscillations of boundaries in closed containers of different type (drops, cavities with elastic walls) was studied [5, 9–12], as well as its impact on mass transfer [6, 13].

The mechanism of vibrational convection consists in the generation of average vorticity near a solid boundary due to the viscous interaction, when a fluid oscillates about this boundary [14, 15]. This leads to the establishment of steady streaming. If the angular frequency of oscillations is \( \Omega \) and the kinematic viscosity of the fluid is \( \nu \), the average vorticity is localized near the boundary within a region of thickness about the Stokes boundary layer thickness \( \delta = \sqrt{2\nu \Omega} \). Such flows are referred to as inner with respect to the boundary layer, or primary. At a distance much larger than \( \delta \), the secondary vortical flows are formed, characterized by the opposite sense of rotation. Depending on the ratio between the characteristic inner size of the container and the boundary layer thickness, \( R / \delta \), they distinguish ranges of low and high dimensionless frequencies \( \omega \sim (R / \delta)^2 \). The magnitude of the dimensionless frequency determines the structure of steady streaming. In the limit of low dimensionless frequency, the primary flows are dominating, while the secondary ones may be merely absent. In the limit of high frequency, the secondary flows prevail, while the primary vortices are localized in thin, visually vanishing boundary layers.

The steady streaming velocity \( v \) in a closed container oscillating with amplitude \( b \) is determined by the pulsation Reynolds number \( \text{Re}_p = b^2 \Omega / \nu \) and the dimensionless frequency \( \omega = \Omega R^2 / \nu \). The
former is the governing parameter responsible for the intensity of pulsations, while the latter determines the streaming structure. After rescaling the dimensionless streaming parameter \( \frac{v}{R} \) with \( \text{Re}_p \), to describe the average dynamics, it is sufficient to use only two parameters: the dimensionless frequency \( \omega = \Omega R^2 / \nu \) and the dimensionless velocity \( \tilde{v} = vR/(b^2 \Omega) \) [12]. At the variation of all experimental parameters, all experimental data coincide on the plane \((\omega, \tilde{v})\), and this is typical for the steady streaming in general, considered in containers of different configuration [11, 16, 17]. In the range of low dimensionless frequency, \( \omega \sim (1 - 10) \), the primary flows obey the scaling law \( \tilde{v} \sim \omega \). With the increase of the dimensionless frequency until the range of intermediate values, \( \omega \sim (10^2 - 10^3) \), a mixed flow structure is realized in the container. In this situation the velocity of the primary flows reaches a plateau and then, as the secondary flows grow stronger, it decreases. In the high-frequency limit, where the secondary flows dominate \( (\omega \sim 10^4) \), \( \omega \) does not affect their dimensionless velocity anymore, and \( \tilde{v} \) approaches asymptotically some constant value, \( \tilde{v} = \text{const} \). Such behavior agrees with the asymptotic high-frequency theory [18].

In the proposed work, the steady streaming is studied in a spherical container whose elastic boundary oscillates symmetrically according to a harmonic law. Experiments are conducted in the range of low and moderate dimensionless frequency, the focus is made on the primary flows. This domain of parameters corresponds to the eigenfrequencies of water drops of the characteristic size about 1 mm that are encountered in liquid–liquid extraction devices.

2. Experimental setup and technique

The experimental model was a spherical container \( I \) made of silicone and having radius \( R = 24.0 \text{ mm} \) and the walls of thickness \( 2.0 \text{ mm} \) (figure 1). On the sphere’s wall, joints 2 were fixed; each of them consisted of two disks of diameter \( d = 15.0 \text{ mm} \) that squeezed the wall on the two sides. The joints were then connected to thrusters 3 that transmitted the force from linear servo motors of the type Dunkermotoren STA1112. The container was placed horizontally (with respect to the thrusters’ axes) in a cubic cooling jacket \( 4 \) that was filled with water and assured simultaneously the thermal stabilization and the optical correction. The cube side was \( L = 100.0 \text{ mm} \). The motors and the thrusters were installed symmetrically on the two sides of the container in such a way that the latter was situated in the center of the cube (figure 2).

![Figure 1. Diagram of the experimental setup: front view (a) and top view (b)](image)
The container was filled with a working liquid – the aqueous solution of glycerin with the mass fraction 90%. At the temperature \( T = 20^\circ\text{C} \), its kinematic viscosity and thermal diffusivity are, respectively, \( \nu = 1.90 \text{cm}^2/\text{s} \) and \( \chi = 9.8 \times 10^{-4} \text{cm}^2/\text{s} \). The temperature in different experimental realizations varied in the range \( T = (18 \pm 22) \text{C} \), but it was maintained constant in each experiment with the deviation not higher than \( \pm 0.3 \text{C} \). The temperature in the water-cooling jacket was controlled during experiments by means of a thermocouple of type K (figure 1a) whose measurements were acquired using a device TermoDat 22K5. Channels 6 and 7 were used as the input and output for the recirculation of the cooling liquid.

![Figure 2. Photograph of the experimental setup: in the central part one may see the cubic water-cooling jacket with the spherical container inside it, illuminated with the laser](image)

The thrusters were driven to oscillate in a synchronous manner by a harmonic law with the frequency and amplitude varying in the ranges, respectively, \( f = (1-10) \text{Hz} \) and \( b = (1-7) \text{mm} \). During this process the container’s wall underwent a periodic deformation as is shown schematically in figure 1 and by dashed lines traced after the photographs in figure 3. The container was squeezed and extended with the period of oscillations, \( f^{-1} \). The amplitude of oscillations was measured from oscillograms that were plotted after the data from the Hall sensors implemented in the motors. That data was acquired by means of controllers Copley Accelnet ACJ-055-18-S that were used to command the motors, as well. The discrepancy in amplitudes between the left and the right thrusters remained within 0.04 mm.

To study the shape of container-boundary oscillations, a high-speed recording was done with the frame rate \( 100/f \). The superposition of the photographs that corresponded to the extreme positions of the thrusters allowed to locate the nodes of boundary oscillations: these were the points that remained immobile during the oscillation period (\( A \) and \( B \) in figure 3, \( C \) and \( D \) were estimated by symmetry).

In order to study the structure and velocity of the steady streaming, light-scattering tracer particles of diameter 40 μm were added to the liquid, and laser 8 (figure 1a) was used to create a laser light sheet that cut the container along the diameter. The frame rate of image recording was synchronized with the container’s oscillations. This allowed to automatically subtract the oscillatory part of the velocity and
measure directly the steady time-average part. The velocity vector fields were obtained by particle image velocimetry (PIV). The images within each pair were taken with a time step $n/f$ (here, $n$ is a natural number), and processed by means of PIVlab software [19] with four passes, while decreasing the interrogation window on each pass as follows: $64 \times 64$, $32 \times 32$, $24 \times 24$, $20 \times 20$ (pixel $\times$ pixel).

Figure 3. Structure of the steady streaming at $f = 3.0$ Hz, $b = 3.4$ mm ($\omega = 67.3$). In the background, the superposition of two photographs, taken at the maximum contraction and extension of the container, is shown. In the foreground, the vector plot of the steady time-average velocity is presented.

3. Experimental results

On the background of fluid oscillations, in the volume the steady steaming in the form of tori is generated. In figures 3 and 4b, a system of two symmetric tori, which in the cross-section have the form of four coordinated vortices, is shown. Their centers, marked with symbols +, are displaced from the cavity center towards the boundary and located near the nodes of boundary oscillations, A, B, C, D. Along the axis of thrusters’ vibration, two opposite streams coming from the poles are observed. The maximum horizontal velocity in these streams, $v_j$, is going to be used as a measure of the steady streaming intensity. With a decrease in the streaming intensity due to the decrease in either $\omega$, or $Re_p$, the flow structure becomes less symmetric (for instance, see figure 4a). In the studied range of dimensionless frequencies, two types of flow structure are observed. At lower values of $\omega$, two tori are observed: one on the left and one on the right; the right torus is denoted as 1 in figure 4b. At higher $\omega$, the second pair of tori emerges near the poles of the sphere (thrusters), denoted as 2 in figure 4d.

The elastic sphere possesses some slight inner asymmetry due to the manufacturing process. Besides, at rest the container reposes only on the thrusters fixed to it on the sides, and the center of the sphere is not fixed explicitly. So, during the oscillations the forcing provided by each thruster affects the entire container, and it turns out to be extremely difficult to make the container oscillations perfectly symmetrical. Some asymmetry between the left and the right tori is present in all experiments and it results in the deviation of $v_j$ values. For this reason, during the quantitative evaluation the velocity values are averaged.
Figure 4. Structure of steady streaming at $(\omega, \text{Re}_p) = (22.4, 1.6)$ (a), $(67.3, 1.4)$ (b), $(84.6, 2.5)$ (c), $(169, 4.8)$ (d). The vector plots show the streaming velocity, while the color maps depict the time-average vorticity whose color bars are scaled in s$^{-1}$.

The steady-streaming velocity increases as the square of the amplitude of oscillations at a fixed frequency (figure 5). This is a common feature of steady streaming generated in the Stokes boundary layers due to viscous effects [14, 15], and this result indicates that the Schlichting’s mechanism is the governing one in the present experiments.

4. Discussion of results
On the plane of the dimensionless frequency and velocity all experimental points follow the same dependence that is in qualitative agreement with the trend common to various oscillating containers (figure 6). This means that at lower $\omega$, $\bar{V}$ demonstrates the linear growth, then at $\omega \approx 50$ there is a maximum. In the studied frequency range, no secondary flows were observed. The experimental results in the case of the spheroidal container [12] reveal a less intensive vibrational convection, while at $\omega > 100$ a clearly pronounced plateau is observed: $\bar{V} = \text{const}$ (dashed line in figure 6). The optimal values of $\omega$, at which the maxima of $\bar{V}$ are observed, are rather close in the two cases.
Figure 5. Dependence of the steady velocity $v_j$ on the amplitude of vibrations $b$ at different frequencies. Points 1: $f = 1.0$ Hz, $v = 1.61$ cm$^2$/s; points 2: $f = 10.0$ Hz, $v = 2.01$ cm$^2$/s

Figure 6. Dependence of the dimensionless velocity $\tilde{v}$ on the dimensionless frequency $\omega$. Points 1 and 4 correspond to the experiments at fixed frequencies that are shown in figure 5. Points 2 and 3 were obtained at fixed amplitudes. The data for the dashed line was interpolated from the experimental points from figure 9 in [12].

The comparison of velocities in the present experiments and the earlier research [12] is not strictly correct. Indeed, the selection of a quantifying parameter is the question that arises at every different container configuration. Let us refer to the boundary shape. In both cases it is reasonable to define “poles” – the centers of flat areas that move along the axis of vibration. In figure 7, the “poles” are in the center of actuators. The polar regions of the boundary do not undergo any deformation, in contrast to the equatorial region – a circumference, equidistant from both poles – that is extended and contracted periodically, in the opposite oscillation phases. The third important location on the boundary is the oscillation nodes that are circumferences contained in the planes parallel to that of the equator. It is reasonable to consider that the nodes do not deform in experiments, as well. Taking into account theoretical works [5, 6], as well as experiments with a cylindrical container [20], one may notice that virtually radial streams exist at the oscillation nodes under certain conditions. However, in the present experiments they are not observed. In our opinion, this is determined by the dimensionless frequency and the boundary shape.

In theory [5, 6] the oscillations of a spherical drop are defined by the law $R = R_0 + b e^{-i\omega t} P_2(\cos \theta)$, where $P_2(\cos \theta)$ is a Legendre polynomial of the second kind. In the high-frequency limit, considered in these works, the steady streaming has the form of four tori (eight vortices in the cross-section) that are delimited approximately by the coordinates of oscillation nodes and the equator. The angular coordinate $\theta_n$ of a nodal circumference, counted from a pole, makes 55 degrees in [5, 6], while in the present experiments it is approximately 38 deg., in [12] – about 70 deg. To compare the position of nodes in different experiments, please, refer to figure 7. At relatively low $\omega$, as the thickness of the Stokes layers grows, a competition between the neighboring tori leads to the situation when one of the two prevails. In this case, the node coordinate determines the flow structure. In the present experiments, the nodes are close to the poles ($\theta_n$ is relatively small), and the equatorial tori dominate, while the polar ones are suppressed (figure 7a). In the case of spheroid, on the contrary, $\theta_n$ is large, and the polar tori dominate (figure 7b). As a consequence, the inward streaming (denoted by red arrow) is located at the poles in the case of the sphere, and at the equator in the case of the spheroid. As the reader may see from
experiments on the cylinder [20], at low \( \omega \) the inward streams are located at the contact between the neighboring vortices, at the nodes. This comparison allows us to distinguish the polar and equatorial tori by the direction of their rotation. Returning at this point to the question on the selection of a measure, one may see that in the present case \( v_j \) is the velocity of two axial jets going through the tori (figure 7a), while in [12] it is the velocity of the flow converging towards the center in the equatorial plane (figure 7b).

![Figure 7](image_url)

**Figure 7.** Comparison of the steady-streaming fields in the cases of sphere (a) and spheroid (b). The results of the present study are for \( \omega = 67.3 \) and \( \text{Re}_p = 1.4 \) (a). The data for spheroid for \( \omega = 56 \) and \( \text{Re}_p = 4.9 \) (b) were kindly provided by S. Subbotin and correspond to the results published in [12].

It is worth mentioning that with the increase in the dimensionless frequency the polar tori become bigger. For instance, at \( \omega \approx 100 \) the polar tori are already clearly distinguished in the present experiments (figure 4d, 2).

5. **Conclusion**

In the present experiments, the flows in a drop-like container have been successfully modeled by the steady streaming generated in the elastic sphere filled with viscous liquid. In the considered range of dimensionless frequency (\( \omega < 200 \)) only the primary flows have been observed. The dimensionless streaming velocity has a maximum about \( \omega \approx 50 \), when the flows consist of two symmetric equatorial tori. The flow structure varies with the dimensionless frequency, and at \( \omega > 80 \), when the velocity \( \bar{v} \) decreases with \( \omega \), two additional, polar tori emerge on the axis of thrusters vibration. As the comparison with the works by other researchers shows, the dimensionless frequency \( \omega \) and the angular position \( \theta_b \) of the oscillation nodes of the boundary determine the flow structure. For instance, these parameters influence the choice in favor of the polar or equatorial tori that are in competition at low \( \omega \). Understanding the impact of these parameters is a key to the control of mass transfer intensity in liquid drops or similar systems.

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