Mesoscopic fluctuations of the Coulomb drag at $\nu=1/2$

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We consider mesoscopic fluctuations of Coulomb drag transresistivity between two layers at a Landau level filling factor $\nu = 1/2$ each. We find that at low temperature sample to sample fluctuations exceed both the ensemble average and the corresponding fluctuations at $B = 0$. At the experimentally relevant temperatures, the variance of the transresistivity is proportional to $T^{-1/2}$. We find the dependence of this variance on density and magnetic field to reflect the attachment of two flux quanta to each electron.

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Measurements of Coulomb drag between two parallel electronic systems at close proximity are a useful tool for studies of electron-electron interactions. In these measurements a current $I_1$ is driven in one ("active") of the systems. As a consequence of inter-layer electron-electron scattering, a momentum $q$ is transferred from the active system to the other ("passive") one with a voltage $V_2$ developing in the latter. The ratio $\rho_D = -V_2/I_1$ is known as the transresistivity or the drag coefficient. When the Landau level filling factor in each layer is $\nu = 0$, in the presence of disorder, would fluctuate with the same correlation functions as the longitudinal one, $\rho_D$. We note in passing that the Hall drag resistivity, which vanishes when disorder-averaged, would fluctuate with the same correlation functions as the longitudinal one, $\rho_D$.

It is convenient to calculate mesoscopic fluctuations for the drag conductivity, $\sigma_D$, rather than for $\rho_D$. In fact, at $\nu = 1/2$, to the lowest non-vanishing order in the screened interlayer interaction $t$, $\rho_D \approx \left( \frac{2\hbar}{e^2} \right)^2 \sigma_D$. In the same approximation, $\sigma_D$ can be expressed in terms of the non-linear susceptibility of the system $\Gamma$ (to be defined below) and the propagator $\mathcal{D}$ of the screened interlayer Coulomb interaction:

$$\sigma_D = \frac{1}{4S} \int \frac{d\omega}{2\pi\hbar} \left( \frac{\partial}{\partial \omega} \coth \frac{\hbar \omega}{2T} \right) D^{R \ast}_{12} \Gamma_{13} D_{34} \Gamma_{41}^\ast. \quad (2)$$

Here, $S$ is the area of the sample, numerical subscripts indicate spatial coordinates, and are implied to be in-
interlayer interaction. Ref. [15]). In momentum representation, 
\[ \rho_{\alpha\beta}(\omega) = \frac{1}{e\omega + Dq^2} \]
with \( \rho_{\alpha\beta} \) being the induced dc current. From gauge invariance 
\[ \int d\mathbf{r}_1 \int d\mathbf{r}_2 \Gamma_{\alpha\beta}(\omega) = \int d\mathbf{r}_2 \Gamma_{\alpha\beta}(\omega) = 0. \]

The retarded (advanced) propagators of the screened interlayer interaction \( \mathcal{D}^{R(A)} \) are calculated by means of a standard random phase approximation (see, e.g. Ref. [15]). In momentum representation,
\[ \mathcal{D}^{R(A)} = \frac{\Pi^{R(A)}}{1 + 2\pi e^2 d^2 \Pi^{R(A)}}, \quad \Pi^{R(A)} = N(0)Dq^2 + e\omega + Dq^2. \]

In the diffusive regime the non-linear susceptibility \( \Gamma \) can be obtained from Ohm law, \( \tilde{j} = \sigma \tilde{E} - eD\nabla n \), where \( \tilde{E} \) is the electric field. Combined with the continuity equation, Ohm law yields for the linear response of the density to an applied field,
\[ \langle n(q,\omega) \rangle = \frac{1}{e} q^2 \frac{\omega^2 \Gamma(\omega)}{\omega^2 + Dq^2}. \]
where \( \langle \ldots \rangle \) indicate disorder-averaged quantities.

A non-linear response to the electric field from the density dependence of the conductivity \( J_{dc} = \text{Re} (\alpha_{\alpha\beta}) n(q,\omega)E(-q,-\omega) \), and yields,
\[ \langle \Gamma \rangle = \frac{1}{e} \frac{\partial \langle \sigma \delta \rangle}{\partial n} q^2 \frac{\omega \sigma_{xx} q^2}{\omega^2 + Dq^2} = e \frac{\partial (\sigma \delta)}{\partial n} q^2 \text{Im} \Pi^{R}. \]

The density dependence of the conductivity is a measure of electron-hole asymmetry, which is essential for Coulomb drag [14]. The fluctuations in \( \rho_D \) result from mesoscopic fluctuations in \( \sigma_{\alpha\beta} \).

In the absence of a magnetic field \( \Gamma \) is parallel to \( \tilde{q} \). In contrast, at \( \nu = 1/2 \) the large Hall component of the conductivity leads to \( \Gamma \) which is approximately perpendicular to \( \tilde{q} \). In both cases the disorder-averaged conductivity is linear in the density, and \( \frac{\partial \sigma_{\alpha\beta}}{\partial n} \approx \frac{\omega}{\epsilon} \). Substituting this approximation in Eq. (5), and using Eqs. (3) and (4), we arrive at the familiar result
\[ \langle \rho_D \rangle = \frac{2\pi e h}{3e^2 (kp_d)^4} \left( \frac{T}{T_0} \right)^2 \ln \left( \frac{T}{T_0} \right) \]
with \( T_0 = 4\pi \sigma_{xx}/e\epsilon \). Note that Eq. (6) holds both at zero magnetic field and at \( \nu = 1/2 \). Quantitatively, it yields very different results in the two cases, since the electronic longitudinal conductivity \( \sigma_{xx} \) at \( \nu = 1/2 \) is about three orders of magnitude smaller than at \( B = 0 \).

We now turn to the discussion of mesoscopic fluctuations of \( \rho_D \). The first step is the estimate of fluctuations in \( \Gamma(q,\omega) \). At \( \nu = 1/2 \) the main source of these fluctuations is the derivative \( \frac{\partial \sigma}{\partial n} \). The rest of the parameters, i.e., the compressibility and diffusion constant, can be approximated by their average values (as in Eq. (3)), since their fluctuations are much smaller than their average. To estimate the fluctuations of \( \frac{\partial \sigma}{\partial n} \) we express this derivative in terms of response functions for composite fermions, which are the Fermi liquid quasi-particles of the \( \nu = 1/2 \) state. The conductivity matrices of electrons and composite fermions are related by
\[ \langle \sigma_{\alpha\beta} \rangle = \frac{1}{e} \hat{\Gamma} \partial \sigma_{\alpha\beta} \partial n \hat{\delta} \langle \sigma_{\gamma\delta} \rangle \]
with \( \hat{\delta} \) being the two dimensional anti-symmetric tensor.

On average \( \sigma_{\alpha\beta} \), the composite fermion conductivity matrix at \( \nu = 1/2 \), is diagonal with both diagonal elements being \( \frac{e^2}{\hbar} g_{cf} \) (where \( g_{cf} \) is the dimensionless conductance of composite fermions). In the limit of \( g_{cf} \gg 1 \), the electronic longitudinal conductivity is inversely proportional to \( g_{cf} \), \( \sigma_{xx} \approx \frac{e^2}{\hbar} \frac{1}{g_{cf}} \), and so is also the electronic diffusion constant.

For a particular disorder realization the system is not isotropic, and \( \sigma_{\alpha\beta} \) is not diagonal. In the limit of \( g_{cf} \gg 1 \), Eq. (6) leads to
\[ \delta \sigma \approx \frac{1}{4g_{cf}} \hat{\delta} \delta \sigma_{\alpha\beta} \hat{\delta} \]
where \( \delta \sigma, \delta \sigma_{\alpha\beta} \) are the deviations of \( \sigma, \sigma_{\alpha\beta} \) of a particular sample from their average value. The fluctuations of \( \hat{\delta} \sigma_{\alpha\beta} \) then stem from fluctuations of \( \partial \sigma_{\alpha\beta} /\partial n \). This derivative is taken with the external magnetic field \( B \) kept constant. At the same time, the magnetic field experienced by the composite fermions, \( \Delta B = B - 2\Phi_0 n \), is not constant: when the density is varied, \( \Delta B \) varies as well. The derivative must then be taken with respect to both \( n \) and \( \Delta B \),
\[ \frac{\partial \sigma_{\alpha\beta}}{\partial n} \bigg|_{\Delta B} = \frac{\partial \sigma_{\alpha\beta}}{\partial n} \bigg|_{\Delta B} - 2\Phi_0 \frac{\partial \sigma_{\alpha\beta}}{\partial \Delta B} \bigg|_{n} \]

We now estimate the fluctuations of \( \Gamma \) for a phase coherent sample of size \( L \). The scale for the variance of the conductivity \( \sigma_{\alpha\beta} \) at \( T = 0 \) is \( \frac{e^2}{\hbar} \). The typical magnetic field scale is \( \Delta B \sim \Phi_0 / L^2 \), therefore the estimate for the fluctuations of \( 2\Phi_0 \frac{\partial \sigma_{\alpha\beta}}{\partial \Delta B} \bigg|_{n} \) is \( \frac{e^2}{\hbar} \). The fluctuations of \( \frac{\partial \sigma_{\alpha\beta}}{\partial n} \bigg|_{\Delta B} \) are smaller by a factor of \( g_{cf} \), and can therefore be neglected. Consequently, the variance of both components of \( \Gamma \) is
\[ \delta \Gamma \sim \frac{e^2}{\hbar} \left( \frac{L}{g_{cf}} \right)^2 \text{Im} \Pi^{R}, \]

Since on average \( \frac{\partial \sigma_{\alpha\beta}}{\partial n} \bigg|_{B} = \frac{e^2}{2\hbar n} \), the fluctuations of \( \Gamma \) can be re-written as \( \langle \delta \Gamma \rangle \sim \left( \frac{k_{B} e}{g_{cf}} \right)^4 \), which in the diffusive regime is much larger than unity. This by itself is a
measurable conclusion: In the diffusive regime in a fully coherent sample the fluctuations of the acousto-electric current are much larger than its average. Equation (11) holds as long as the thermal length $L^f_T \equiv \sqrt{hD^f_T/T}$ and the phase breaking length $L^\varphi_f$ are much larger than $L$.

Having now an estimate for the fluctuations of $\Gamma$, we estimate the fluctuations of $\rho_D$ as:

$$\delta \rho_D \sim T \int_{-T}^{T} \frac{d\omega}{h^2 \omega^2} \int_{|q| \geq 1/\min(L,L^f_T)} dq \, \delta \Gamma^2 |D(q,\omega)|^2 \quad (12)$$

As commonly happens in mesoscopic fluctuations, the integral over $q$ is dominated by its lower limit. We start with the case where the phase breaking length is the largest scale in the problem. We describe the temperature dependence of the transresistance, and denote the r.m.s. of the fluctuations by $\delta \rho_D^0$.

At extremely low temperatures $T \ll (kd)hD/L^2$, all the excitations with energies smaller than $T$ contribute and we obtain with the help of Eqs. (12) and (4)

$$\delta \rho_D^0 \sim \frac{h}{e^2 (kd)^2} \left( \frac{T}{E^f_T} \right)^2, \quad T \ll (kd)E^f_T/g^f_{cf}. \quad (13a)$$

where $E^f_T = hD^f_T/L^2$ is the Thouless energy for the composite fermions. At higher temperature $(kd)hD/L^2 \ll T \ll E^f_T$, the processes with the energy transfer $\omega > (kd)D/L^2$ are suppressed, leading to

$$\delta \rho_D^0 \sim \frac{h}{e^2 (kd)g^f_{cf}} \frac{T}{E^f_T}, \quad (kd)E^f_T/g^f_{cf} \ll T \ll E^f_T. \quad (13b)$$

At yet higher temperature, $T > E^f_T$, $\delta \Gamma$ itself is temperature dependent, and it is suppressed by a factor of $\sqrt{E^f_T/T}$ similar to conductance fluctuations, see e.g. [10]. This yields a temperature independent result

$$\delta \rho_D^0 \sim \frac{h}{e^2 (kd)g^f_{cf}}, \quad T \gg E^f_T. \quad (13c)$$

Let us now discuss the effect of a finite phase breaking length $L^\varphi_f$. As is well known for mesoscopic fluctuations, averaging in a large sample, $L \gg L^\varphi_f$, is carried out by summing over the statistically independent contributions of $(L/L^\varphi_f)^2$ patches of a size $L^\varphi_f$ each. The contribution of each patch is given by Eqs. (13), where the Thouless energy $E^\varphi_T$ is set to $h/\tau^\varphi_f \equiv hD^\varphi_f/(L^\varphi_f)^2$. As a result of the averaging $\langle \delta \rho^2_D \rangle = \langle \delta \rho^0_D \rangle^2 (L^\varphi_f/L)^2$. Using Eqs. (13) we obtain

$$\langle \delta \rho^2_D \rangle = \frac{h^2}{e^2 g^f_{cf}(kd)^2} \left( \frac{L^\varphi_f}{L} \right)^2 \min \left[ 1, \frac{1}{\alpha_1 \left( \frac{g^f_{cf} T \tau^\varphi_f}{kd\hbar} \right)^2} \right] \times \min \left[ \alpha_3, \alpha_2 \left( T \tau^\varphi_f / \hbar \right)^2 \right], \quad (14)$$

where the coefficients $\alpha_{1,2}$ are of order unity and we were able to calculate $\alpha_3 \approx 0.2 \frac{23}{9\pi} = 0.23$.

While the actual magnitude of the mesoscopic fluctuations depends on the precise source of phase breaking, their temperature dependence is robust. All generic models of phase breaking in two dimensions lead to $1/\tau^\varphi \propto T$, so that the temperature dependence following from Eq. (13) is $\langle \delta \rho^2_D \rangle \propto 1/T$.

The dependence of the mesoscopic fluctuations on $g_{cf}$ requires further specification of the model. Our preliminary study suggests that the main mechanism for the phase breaking is the quasi-elastic scattering of composite fermions from the thermal quasi-static fluctuations of the Chern-Simons magnetic field, $(2\Phi_0)^{-2} \langle B(r)B(r') \rangle = \langle \delta n(r)\delta n(r') \rangle$ $\approx \delta(r-r')\frac{T N(0)}{\kappa d} = \delta(r-r')\frac{T}{2\pi e^2 d}$ (with $\epsilon$ being the bulk dielectric constant). Due to the accumulation of the Aharonov - Bohm phase from such fluctuations, $(L^\varphi_f)^2$ is the area at which the thermal fluctuations of the electron number are of the order of one $\int_{|r|<L^\varphi_f} d^2 r \langle n(r)\delta n(r') \rangle \sim 1$, which result in

$$T \frac{\epsilon(L^\varphi_f)^2}{2\pi e^2 d} \simeq 1, \quad \frac{h}{\tau^\varphi} \sim \frac{g_{cf} T}{kd}. \quad (15)$$

This estimate can be understood very simply: a thermal fluctuation of a charge $e$ in one layer and $-e$ in the other layer over a scale $L^\varphi_f$ involves a charging energy cost of $2\pi e^2 d (L^\varphi_f)^2$. The energy available for that fluctuation is $T$. Balancing the two energies determines $L^\varphi_f$.

Substituting estimate (15) into Eq. (14) and using the condition $g_{cf} \gg kd$, we find up to a numerical constant

$$\langle \rho^2_D \rangle \simeq \frac{h^2}{e^2} \left( \frac{1}{g^f_{cf}} \right) \left[ \frac{2\pi e^2 d}{\epsilon L^2 T} \right] \quad (16)$$

Our estimate (13) for the phase breaking rate implies $T \tau^\varphi \ll h$. This by no means indicates a collapse of the Fermi liquid picture of composite fermions, since most of the phase breaking results from scattering off the Chern-Simons field fluctuations whose dynamics [with characteristic frequency $T/(\hbar g_{cf})$] is very slow compared to $\tau^\varphi$, but fast compared to the time of the experiment. Field fluctuations which are static on the scale of the experiment time affect the mesoscopic fluctuations only by affecting $g_{cf}$. Field fluctuations that are faster than that scale make the potential landscape seen by the composite fermions time dependent, and lead to a suppression of the mesoscopic fluctuations by partial ensemble averaging.

We now use Eq. (16) to estimate the value of the fluctuations of the transresistance of a realistic sample. At $T = 0.6K$ the average drag [11] $\langle \rho_D \rangle = 150\Omega/\square$ with the interlayer spacing $d = 300\AA$. For that sample, the single layer resistance at $\nu = 1/2$ is $3k\Omega/\square$, which gives
the value \( g_{ef} \approx 8 \). Given the sample’s size \( L \approx 100 \mu m \), we estimate the magnitude of the fluctuations Eq. (10) as \( \delta \rho_D \approx 0.3 \Omega \). For lower temperatures and smaller samples we expect the fluctuations to be stronger, possibly exceeding the average. Increasing current suppresses the mesoscopic fluctuations mostly by heating of the electrons.

FIG. 1. Electronic non-linear response within the composite fermion RPA. Wavy lines are gauge field propagators \([3]\). Bubbles are composite fermion response functions.

We now turn to sketch the calculation, whose details will be published separately. Within the composite fermion random phase approximation (RPA), \( \Gamma \) is depicted diagrammatically on Fig. 1. The center triangle is the non-linear response of the composite fermions \( \Gamma^{cf} \) to a driving field. Since composite fermions interact with scalar and vector potentials, \( \Gamma^{cf} \) is a tensor with component \((\Gamma^{cf})_{\alpha\beta}^\mu \), where \( \alpha\beta \) are the directions of the driving fields, and \( \mu \) is the direction of the induced current.

In the limit of \( g_{ef} \gg 1 \) the analytic expression corresponding to the diagram on Fig. 1 is approximated by

\[
\Gamma_{12}^{\alpha} \approx \frac{2\hbar^2 e^4}{e^2 g_{ef}} \left[ \Gamma^{cf} \right]^{\gamma'}_{\beta4\mu\nu} \frac{\epsilon^{\mu\nu}}{\epsilon^2} \Pi_{31}^R(\omega) \frac{\epsilon^{\nu\sigma}}{\epsilon^2} \Pi_{32}^A(\omega),
\]

where numerical subscripts indicate spatial coordinates, we imply integration over \( r_{3,4} \), and the polarization operators are defined in Eq. (9). The mesoscopic fluctuations in \( \Gamma \) result from fluctuations in \( \Gamma^{cf} \). Within the RPA, the latter is approximated by the corresponding response function for non-interacting particles:

\[
[\Gamma^{cf}]_{12;\mu\nu}^{\alpha} = \int d\epsilon \frac{e}{2\pi} \left[ J^{\alpha} \left( J^{\alpha}_{12;\mu\nu}(\omega, \epsilon) + J^{\alpha}_{21;\mu\nu}(-\omega, \epsilon) \right) \right],
\]

\[
J^{\alpha}_{12;\mu\nu}(\omega, \epsilon) = \left( \tanh \frac{\epsilon - \omega}{2T} - \tanh \frac{\epsilon}{2T} \right) \times \left[ G^R_{12}(\epsilon - \omega) - G^R_{12}(\epsilon + \omega) \right] \frac{\epsilon^{\mu\nu}}{\epsilon^2} \Pi_{31}^R(\omega) \frac{\epsilon^{\nu\sigma}}{\epsilon^2} \Pi_{32}^A(\omega),
\]

The further analysis of the statistics of the vertex Eq. (17) follows the lines of Ref. [12], and results in Eq. (14).

Closing the paper, we point out that two analyses of mesoscopic fluctuations of the Coulomb drag near \( \nu = 1/2 \) are of interest. First, one varies the magnetic field and measures the correlation function \( \langle \rho_D(B)\rho_D(B + \delta B) \rangle - \langle \rho_D(B) \rangle \langle \rho_D(B + \delta B) \rangle \), with \( B \) close to the \( \nu = 1/2 \) value. An experimental study of the decay of this correlation function is a way to measure \( L_{cf}^2 \): the characteristic magnetic field of the decay is \( \delta B^* \sim \Phi_0/[L_{cf}^2 \mu^*] \). Second, one varies the electron density in one of the layers and measures the correlation function \( \langle \rho_D(n)\rho_D(n + \delta n) \rangle - \langle \rho_D(n) \rangle \langle \rho_D(n + \delta n) \rangle \). Again, the decay of this function is governed by \( L_{cf}^2 \): the characteristic density change \( \delta n^* \) at which it decays is expected to correspond to half of an electron in a phase coherent region, i.e., \( \delta n^* = 1/2[L_{cf}^2] \). This statement should hold as long as the composite fermion cyclotron radius is much larger than its mean free path, i.e., \( |\nu - 1/2| < (2g_{ef})^{-1} \).

It is noteworthy that near zero magnetic field \( \delta B^* \) is \( \Phi_0/L_{cf}^2 \) but \( \delta n^* = k_F l/L_{cf}^2 \). Since at \( B = 0 \) the electrons do not carry flux tubes, the density change \( \delta n^* \) has to be such that the chemical potential changes by \( h/\tau_\varphi \).

Thus an experimental observation of \( \delta B^*/\delta n^* = 2\Phi_0 \) would in some sense be a verification of the attachment of flux to the Fermi liquid quasi-particles at and close to \( \nu = 1/2 \).

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