Magnetic Interactions of D-branes and Wess-Zumino Terms in Super Yang-Mills Effective Actions

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Abstract

There is a close relation between classical supergravity and quantum SYM descriptions of interactions between separated branes. In the case of D3 branes the equivalence of leading-order potentials is due to non-renormalization of the $F^4$ term in $\mathcal{N} = 4$ SYM theory. Here we point out the existence of another special non-renormalized term in quantum SYM effective action. This term reproduces the interaction potential between electric charge of a D3-brane probe and magnetic charge of a D3-brane source, represented by the Chern-Simons part of the D-brane action. This unique Wess-Zumino term depends on all six scalar fields and originates from a phase of the euclidean fermion determinant in SYM theory. It is manifestly scale invariant (i.e. is the same for large and small separations between branes) and can not receive higher loop corrections in gauge theory. Maximally supersymmetric SYM theories in $D = p + 1 > 4$ contain mixed WZ terms which depend on both scalar and gauge field backgrounds, and which reproduce the corresponding CS terms in the supergravity interaction potentials between separated Dp-branes for $p > 3$. Purely scalar WZ terms appear in other cases, e.g., in half less supersymmetric gauge theories in various dimensions describing magnetic interactions between Dp and D(6 − p) branes.

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1 Introduction

The duality between the supergravity and the world-volume descriptions of D-branes has led to important advances in understanding of dynamics of supersymmetric gauge theories. Many aspects of interactions of D-branes which are transparent in the supergravity description, translate into quite non-trivial properties of the world-volume field theory. The electric and gravitational interactions between branes have been widely studied in this context (see, e.g., [1, 2, 3, 4, 5, 6] and references there). The agreement between the supergravity and the SYM descriptions of leading-order interactions between branes can be traced to the universal non-renormalization properties of the $F^4$ terms in the effective action of maximally supersymmetric gauge theories in various dimensions [7, 8].

We shall discuss magnetic interactions. The prime case of interest in connection with 4-d gauge theories is a D3 brane moving in the background of other D3 branes. The self-duality of the RR five-form field strength implies that a D3 brane carries both electric and magnetic charges. As a consequence, the probe brane will experience the Lorentz force, similar to the one an electric charge experiences in the magnetic field of a monopole. For example, the action of a D3-brane probe moving in a supergravity background produced by a D3-brane source contains [9, 10, 11] the Chern-Simons term

$$\int C_4 = \int F_5$$

which describes the interaction of an electric charge of the probe with the electric and the magnetic fields produced by the source. In the static gauge, the electric interaction produces the effective "-1" contribution to the P-even Born-Infeld part of the D3-brane action

$$S_{BI} + S^{(el)}_{SC} + S^{(mag)}_{SC}$$

This term ensures the vanishing of the interaction potential between static parallel branes. The magnetic interaction term (which is real for Minkowski signature) $S_{CS} = NS_{WZ} \sim iN \int_X \epsilon_{1\ldots6} \frac{1}{|X|^3} X^1 dX^2 \wedge \ldots \wedge dX^6$ is non-vanishing only when all 6 scalars have non-trivial gradients.

Since many features of the magnetic D3 brane interaction are similar to those of the Lorentz interaction between an electric charge and a magnetic monopole, let us briefly review some facts about the latter case. The Lorentz force acting on the charge in the field of magnetic monopole is

$$F^i = q_e \varepsilon^{ijk} \frac{q_m}{4\pi |X|^3} X_j \dot{X}_k.$$ 

1Here $T_3 = \frac{1}{2\pi g_s}, \quad Q = \frac{1}{\pi} Ng_s, \quad 2\pi \alpha' = 1, \quad |X|^2 \equiv X^i X^i$. 

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The Dirac string singularity of the gauge potential for the monopole field does not allow the Lorentz force to be a variation of a well-defined local action functional. The variational principle for a charged particle interacting with a monopole can be formulated only by adding the non-local Wess-Zumino term to the action

\[
S_{\text{WZ}} = - \frac{q_e q_m}{8\pi} \int d^2 \tau \varepsilon^{ab} \varepsilon^{ijk} \frac{1}{|X|^3} X_i \partial_a X_j \partial_b X_k
\]

\[
= - \frac{q_e q_m}{8\pi} \int \varepsilon^{ijk} n_i dn_j \wedge dn_k, \quad n_i \equiv \frac{X_i}{|X|}. \tag{3}
\]

Here the integration is over a domain whose boundary is the time axis. Then the variation of the WZ action reproduces the Lorentz force: \(\delta S_{\text{WZ}}/\delta X_i = F^i\).

According to the standard argument, there is an ambiguity in the definition of the WZ action, because \(X_i\) is defined at the boundary of the integration domain and can be continued into the interior in an arbitrary way. Since the integrand is locally a total derivative, this ambiguity is discrete, in the sense that the values of the action for different continuations differ by an integer multiple of \(q_e q_m\). Despite the multi-valuedness of the action, the path integral remains single-valued, provided \(q_e q_m\) is an integer multiple of \(2\pi\). The consistency of the quantum mechanics of an electrically charged particle thus leads to the Dirac quantization condition for the magnetic charge:

\[q_m = 2\pi n/q_e.\]

The Lorentz force experienced by a D3 brane in the background of another D3 brane (with both branes having unit charges) is

\[
F^I = \frac{1}{12\pi^2} \varepsilon^{\mu\nu\lambda\rho} \varepsilon^{IJJKLMN} \frac{1}{|X|^6} X_J \partial_\mu X_K \partial_\nu X_L \partial_\lambda X_M \partial_\rho X_N, \tag{4}
\]

\[\mu, \nu = 0, 1, 2, 3, \quad I, J, \ldots = 1, 2, 3, 4, 5, 6.\]

Here \(X_I(x)\) parametrize the position of the probe brane in the 6-d space transverse to the source brane as a function of the 4 longitudinal coordinates \(x^\mu\). The Lorentz force can be represented as a variation of the five-dimensional integral (\(m = 0, 1, \ldots, 5\))

\[
F^I = \frac{\delta S_{\text{WZ}}}{\delta X_I}, \tag{5}
\]

\[
S_{\text{WZ}} = - \frac{1}{60\pi^2} \int d^6 x \varepsilon^{mnklr} \varepsilon^{IJJKLMN} \frac{1}{|X|^6} X_I \partial_m X_J \partial_n X_K \partial_k X_L \partial_l X_M \partial_r X_N
\]

\[= - \frac{1}{60\pi^2} \int \varepsilon^{IJJKLMN} n_I dn_J \wedge dn_K \wedge dn_L \wedge dn_M \wedge dn_N, \tag{6}
\]

\(^2\)The WZ term can be written also as a 4-d integral of a local functional by using spherical \(S^5\) coordinates instead of Cartesian \(X^I\) (see [3]).
where \( n_I = \frac{X_I}{|X|} \) parametrize \( S^5 = SO(6)/SO(5) \). This action is defined up to \( 2\pi \), which reflects the fact that a D3 brane carries one unit of the quantized magnetic charge.

This “topological” term is scale-invariant and does not depend on gauge coupling. Its coefficient cannot be renormalized since any non-trivial dependence on the dilaton would break gauge invariance. Also, as for many other WZ terms, the renormalization of its coefficient would contradict the topological magnetic charge quantization condition. Like the one-loop \( F^4 \) term, this WZ term should thus be a rare example of an exact non-renormalized SYM interaction.

The scale \( (X \rightarrow aX) \) invariance and the topological nature of the above WZ term suggests that it should follow from the string-theory description of D-brane interaction at both large (supergravity) and small (gauge theory) distance regions. The non-renormalization of the coefficient of this term implies that it should be present in the effective action of quantum \( \mathcal{N} = 4 \) \( SU(N) \) SYM theory on the Coulomb branch of the moduli space in both weak and strong coupling regions.

Therefore, it should be expected, both from the weakly coupled string theory “long-distance–short-distance” duality \(^3\) and the AdS/CFT duality \(^4\), that, like the P-even “\( F^4/X^4+\)superpartners” term in \( S_{\text{BI}} \), this P-odd term should be exactly reproduced by the 1-loop computation on the \( \mathcal{N} = 4 \) SYM theory side.

Our aim below is to confirm this expectation by the explicit computation of the imaginary part of the fermion determinant in the SYM theory. As far as we are aware, the derivation of this \( D = 4 \) WZ term from \( \mathcal{N} = 4 \) SYM theory was missing in the literature (the calculations presented in \([20, 21]\), though similar, led to a different class of CS terms, see below). That this term should have, by analogy with the case discussed in \([17]\), a Berry phase interpretation was suggested to one of us (A.T.) by M. Douglas (for some related work in the context of matrix models see also \([18]\)).

Our direct perturbative derivation of the WZ term \(^5\) in the effective action of \( \mathcal{N} = 4 \) SYM theory described below in Section 2 will not be referring to the Berry phase. We will show that the WZ term originate from the same hexagon diagram which is responsible for the chiral anomaly in ten-dimensional SYM theory \([19]\).

\(^3\) Assuming that there are no phase transitions in the coupling constant, the coefficient should be a smooth function of the coupling, but an integer valued smooth function must be a constant.

\(^4\) The two terms may, in fact, be related by a non-linear part of maximal \( D = 10 \) supersymmetry as is suggested by the existence of the action for a D3-brane in \( AdS_5 \times S^5 \) space constructed in \([14]\), where the BI and CS terms in the action are related by the \( \kappa \)-symmetry. Also, there seems to be a close analogy with the \( F^4 \) and \( \epsilon_{10} F^4 C_2 \) terms in the type I superstring 1-loop effective action which are related by \( D = 10 \) supersymmetry and obey a non-renormalization theorem \([13]\).
In Section 3 we will describe a similar computation of counterparts of the ten-dimensional anomaly graph in other supersymmetric gauge theories describing systems of separated Dp branes with \( p > 3 \). In contrast to the \( D = 4 \) SYM case (and some of its analogs discussed below) where the WZ term is purely scalar and does not have a local representation, the hexagon graphs in \( D > 4 \) lead to a different class of WZ terms which involve both scalars and vectors and admit a local CS-type representation. This class of WZ terms was previously derived from SYM theories in \([20, 21]\).

Section 4 will contain some concluding remarks.

2 Wess-Zumino term in the \( D = 4 \) \( \mathcal{N} = 4 \) SYM theory

The WZ term is odd in time derivatives, so it has a factor of \( i \) in the Euclidean action (below we choose the Euclidean signature of the metric). The effective action induced by the bosonic SYM degrees of freedom is real, so the only potential source of the imaginary WZ term is the (gauge-invariant, \( O(6) \) invariant and conformal-invariant) phase of the fermion determinant. The appearance of a WZ term in the phase of a fermion determinant is not unusual, and examples of chiral WZW terms induced by fermionic loop are known \([22, 23]\). The present case of scalar fields interacting with fermions in \( \mathcal{N} = 4 \ D = 4 \) SYM theory was not explicitly discussed before.

The part of the 1-loop effective action in \( \mathcal{N} = 4 \) SYM theory which is induced by the 4 Weyl fermions has the following form (after continuation to Euclidean space)

\[
S_{\text{term}} = -\frac{1}{2} \text{Tr} \ln \left( \left( \Gamma^0 \Gamma^\mu \partial_\mu + i \Gamma^0 \Gamma^I [\Phi_I, \cdot] \right) \frac{1 + \Gamma^{11}}{2} \right) .
\]  

(7)

Here \( \Gamma^M \) are ten-dimensional Dirac matrices:

\[
\{ \Gamma^M, \Gamma^N \} = 2 \delta^{MN} , \quad (\Gamma^M)^\dagger = \Gamma^M , \quad \Gamma^{11} \equiv i\Gamma^0 \cdots \Gamma^9 .
\]  

(8)

We assume that the 10-d indices are split in the 4+6 way, \( M = (\mu, I) \), and that the \( D = 4 \) vector field has trivial background. Generalization to the case of non-trivial \( A_\mu \) background in \( D > 4 \) is straightforward (see \([20, 21]\) and Section 3 below), but in the case of \( \mathcal{N} = 4 \) SYM theory the WZ term happens to depend only on the six scalar fields.
The system of \( N \) “slowly moving” separated D3 branes is represented by slowly varying diagonal scalar fields:

\[
\Phi_I = \begin{pmatrix} X_I^1 \\ \vdots \\ X_I^N \end{pmatrix}.
\] (9)

The commutator term in the Dirac operator in this background vanishes for diagonal components of fermions, and for non-diagonal it becomes \([\Phi_I, \Psi]^{ab} = (X_I^a - X_I^b)\Psi^{ab}\). The effective action thus decomposes into a sum of pairwise interactions:

\[
S_{\text{ferm}} = \sum_{a < b} S(X^a - X^b),
\]

where

\[
S(X) = -\text{Tr} \ln \left( \Gamma^0 \Gamma^\mu \partial_\mu + i \Gamma^0 \Gamma^I X_I \left( 1 + \Gamma^{11} \right) \right).
\] (10)

Taking the variation, we find (Sp is the trace in spinor indices):

\[
\frac{\delta S}{\delta X_I(x)} = \text{Sp} \left[ \left\langle x \left| \frac{1}{i \Gamma^\mu \partial_\mu - \Gamma^J X_J} \right| x \right\rangle \Gamma^I \left( 1 + \Gamma^{11} \right) \right].
\] (11)

We are interested in the imaginary part of the effective action. Since the operator \(i \Gamma^\mu \partial_\mu - \Gamma^J X_J\) is Hermitian, and \(\Gamma^I (1 + \Gamma^{11}) = (1 - \Gamma^{11})\Gamma^I\), taking the difference of the above expression and its complex conjugate it is easy to see that

\[
\frac{\delta \text{Im} S}{\delta X_I(x)} = \frac{1}{2i} \text{Sp} \left[ \left\langle x \left| \frac{1}{i \Gamma^\mu \partial_\mu - \Gamma^J X_J} \right| x \right\rangle \Gamma^I \Gamma^{11} \right].
\] (12)

For slowly varying fields, this expression can be expanded in derivatives of \(X_I\). The term with \(n\) derivatives will be proportional to the trace of \(2n + 3\) Dirac matrices. Since this trace contains \(\Gamma^{11}\), and

\[
\text{Sp} \left( \Gamma^{M_1} \ldots \Gamma^{M_k} \Gamma^{11} \right) = 0 \quad \text{for} \quad k < 10,
\] (13)

the expansion will start with the four derivative term coming from the diagram (dashed lines are external scalar fields):
Analytically,
\[
\frac{\delta \text{Im} S}{\delta X_I} = -\frac{1}{2i} \int \frac{d^4k}{(2\pi)^4} \frac{X_J}{(k^2 + X^2)^5} \partial_{\mu} X_K \partial_{\nu} X_L \partial_\lambda X_M \partial_\rho X_N \times \text{Sp} \left( \Gamma^{\mu} \Gamma^{\nu} \Gamma^{J} \Gamma^{K} \Gamma^{M} \Gamma^{\rho} \Gamma^{N} \Gamma^{I} \Gamma^{\lambda} \Gamma^{I_1} \right) + O(\delta^5).
\] (14)

Using the identity
\[
\text{Sp} \left( \Gamma^{M_1} \ldots \Gamma^{M_{10}} \Gamma^{11} \right) = 32i\varepsilon^{M_1 \ldots M_{10}},
\] (15)
and doing the momentum integral, we find that the variation of the imaginary part of the effective action reproduces exactly the Lorentz force between a pair of D3 branes (4):
\[
\frac{\delta \text{Im} S}{\delta X_I} = \frac{1}{12\pi^2} \varepsilon^{\mu\nu\rho} \varepsilon^{IJJKLMN} \frac{1}{|X|^6} X_J \partial_{\mu} X_K \partial_{\nu} X_L \partial_{\rho} X_N + O(\delta^5).
\] (16)

For the case of a single D3 brane interacting with a cluster of \( N \) coinciding D3 branes we get \( S_{\text{ferm}} = NS(X) \), and thus rederive the magnetic Chern-Simons term in the D3 brane probe action directly from the gauge theory.

The imaginary part of the SYM effective action contains higher-derivative corrections which are not seen on the supergravity side. The conformal symmetry of \( \mathcal{N} = 4 \) SYM implies that the extra derivatives are necessarily accompanied by extra powers of \( 1/|X| \), and so the higher-derivative terms are no longer invariant under the rescaling \( X \to X/\alpha' \). From the supergravity point of view one may interpret these terms as being effectively of higher order in \( \alpha' \). Indeed, the long-distance or closed string channel representation of the full stringy expression for the D3-brane interaction amplitude should contain, in particular, all massless mode (SYM) 1-loop corrections of the open string channel.

3 WZ terms in \( D > 4 \) SYM theories

The \( D = 4 \) case discussed in the previous section is special in that the WZ term there depends only on the scalar fields. An analogous but actually different class of ‘magnetic’ WZ terms appears in the effective actions of maximally supersymmetric SYM theories in higher dimensions \( 4 < D = p + 1 < 9 \) [20, 21]. For \( D > 4 \) one needs to switch also a non-trivial gauge field background in order to get a non-zero result for the imaginary part of the fermionic determinant. This has a natural interpretation on the supergravity side: while D3 branes carry both electric and magnetic charges and thus their interaction potential contains ‘magnetic’ contribution, separated magnetic
(p > 3) Dp-branes interact only ‘electrically’, unless one switches on a gauge field background which induces effective electric charges on the Dp brane probe.

For example, the action of a D5 brane probe moving in the D5 brane background contains the CS term ($I = 1, 2, 3, 4$)

$$S_{CS} \propto i \int \eps_{IJKL} \frac{1}{|X|^4} X^I dX^J \wedge dX^K \wedge dX^L \wedge F \wedge F ,$$  \hspace{1cm} (17)

where $dC_2$ is the (magnetic) gauge field strength of the D5 brane source. The corresponding WZ term indeed arises in the effective action of $D = 6$ SYM theory describing multiple D5 branes.

Let us consider the general case of the maximal SYM theory obtained by dimensional reduction of $D = 10$ SYM theory to $D = p + 1 < 10$, and couple the fermions to both the diagonal scalar background ($9$) and the abelian gauge field background ($\mu = 0, 1, 2, \ldots, p$)

$$A_\mu = \begin{pmatrix} A_{\mu 1}^1 \\ \vdots \\ A_{\mu p}^N \end{pmatrix}.$$  \hspace{1cm} (18)

As in the $p = 3$ case ($10$), the fermionic contribution to the 1-loop effective action factorizes

$$S_{\text{ferm}} = \sum_{a < b} S(X^a - X^b, A^a - A^b) ,$$

where now

$$S(X, A) = - \text{Tr} \left[ \ln \left( \Gamma^0 \Gamma^\mu \partial_\mu + i \Gamma^0 \Gamma^\mu A_\mu + i \Gamma^0 \Gamma^I X_I \right) \frac{1 + \Gamma^{11}}{2} \right].$$  \hspace{1cm} (19)

The first term in the derivative expansion of the imaginary part of this action comes from the hexagon diagram and has the form similar to eq. ($14$) with $(p+1)$-dimensional momentum integral instead of 4-dimensional one and with some of the scalar fields replaced by the gauge potentials.

The contribution of the hexagon diagram in various dimensions $5 \leq D \leq 9$ was shown to give rise to local Chern-Simons terms in the effective action ($20$). Below we rederive these Chern-Simons terms and clarify their relation to the WZ actions.

If $p \leq 7$, we can take a variation with respect to the scalar field to get:

$$\frac{\delta \text{Im} S}{\delta X_I} = - (8 - p)d_\rho \varepsilon^{\rho_1 \ldots \rho_{p+1}} \varepsilon^{IJK_1 \ldots K_{7-p}}$$

$$\times \frac{1}{|X|^{9-p}} X_J \partial_{\mu_1} X_{K_1} \ldots \partial_{\mu_{7-p}} X_{K_{7-p}} F_{\mu_8 \ldots \mu_{9-p}} \ldots F_{\mu_p \mu_{p+1}} ,$$  \hspace{1cm} (20)
\[ d_p \equiv \frac{(-1)^{p(p-1)}}{4(p-3)!(4\pi)^{\frac{p}{2}} \Gamma\left(\frac{10-p}{2}\right)} . \]  

As a result, the effective action contains the term:

\[ \text{Im} S = d_p \int d^{p+2}x \ \varepsilon^{I_0 \ldots I_{p+1}} \varepsilon^{JK_0 \ldots K_{7-p}} \]
\[ \times \frac{1}{|X|^{9-p}} X_J \partial_{I_0} X_{K_0} \ldots \partial_{I_{7-p}} X_{K_{7-p}} F_{\mu_8 \ldots \mu_9-p} \ldots F_{\mu_p \ldots \mu_{p+1}} , \]  

or, equivalently \((n_I = X_I / |X|)\)

\[ \text{Im} S = d_p \ 2^{p-3} \int \varepsilon^{I_1 \ldots I_{9-p}} n_{I_1} dn_{I_2} \wedge \ldots \wedge dn_{I_{9-p}} (\wedge F)^{p-3} . \]  

This expression reduces to our previous \(D = 4\) SYM result \((1)\) in the case of \(p = 3\). For \(p = 5\) this WZ term reproduces the CS interaction \((17)\) in the supergravity description.

Note that the nonlocal nature of the action \((23)\) is fake. Since \(F = dA\), we can integrate by parts and that leads to the local Chern-Simons form given in \((21)\). For example, the local CS form of \((17)\) is \(\int dC_2(X) \wedge F \wedge A\). This ‘integration by parts’ is not possible in the case of purely scalar WZ terms.

Like the CS terms in the Dp brane actions, the WZ terms with different \(p\) in \((23)\) are related by dimensional reduction (‘smearing’ and T-duality, \(A_i \rightarrow X_i\)).

Instead of computing the variation of the effective action over \(X_I\), another way to obtain these ‘mixed’ WZ terms is to calculate the induced current \((24)\)

\[ J^\mu = \frac{\delta \text{Im} S}{\delta A_\mu} . \]  

For \(p = 9\) the divergence of this current produces the chiral anomaly \((19)\), which makes the ten-dimensional non-abelian SYM theory inconsistent. For \(p = 8\), the induced current is\(^5\)

\[ J^\mu = \frac{1}{12288 \pi^4} \text{sgn}(X) \ \varepsilon^{\mu_1 \ldots \mu_8} F_{\mu_1 \mu_2} \ldots F_{\mu_7 \mu_8} , \quad \text{sgn}(X) = \frac{X_9}{|X_9|} . \]  

\(^5\)If \(X = 0\) somewhere, this current is not conserved:

\[ \partial_{\mu} J^\mu = \frac{1}{12288 \pi^4} \delta(X) \varepsilon^{\mu_0 \mu_1 \ldots \mu_8} \partial_{\mu_0} X F_{\mu_1 \mu_2} \ldots F_{\mu_7 \mu_8} . \]

The equation \(X = 0\) determines a domain wall in the nine-dimensional theory, which appears in any field configuration satisfying boundary conditions \(X \rightarrow \pm X_0\) at \(x^1 \rightarrow \pm \infty\), though these domain walls do not exist as classical solutions, because there is no potential for \(X\). It is known that domain walls support fermion zero modes independently of the wall profile. As a consequence, the effective 8D field theory on the wall contains chiral fermions, which make the effective theory anomalous. The chiral anomaly of the fermion zero modes on the wall exactly compensates the divergence of the current \(J^\mu\) \((24)\). The hypersurface \(X = 0\) corresponds to the intersection of the D8-branes. The
In [21] it was suggested that the theory of multiple M5 branes should contain a similar ‘mixed’ \( \int \epsilon_{IJKLM} \frac{1}{|X|^2} X^I dX^J \wedge dX^K \wedge dX^L \wedge dX^M \), CS term related by compactification on \( S^1 \) to the \( \int A \wedge H_4(X) \) CS term in \( D = 5 \) SYM (multiple D4 brane) theory, and the one-loop microscopic derivation (equivalent to the one in [20]) of the latter term was given.

Analogous WZ terms are found in other D-brane interaction systems described by gauge theories with less than maximal supersymmetry. For example, pure-scalar WZ terms appear in the case of Dp—D(6-p) (electric-magnetic) brane interaction. It is possible to see that in the case of the D5–D1 system described by a particular \( \mathcal{N} = 4 \), \( D = 2 \) supersymmetric gauge theory [28], the relevant fermion determinant contains the WZ term \( \int \epsilon_{ijkl} \frac{1}{|X|^4} X_i dX_j \wedge dX_k \wedge dX_l \) which reproduces the CS term \( \int C_2 \) in the action of a D-string probe moving in the magnetic background produced by a D5-brane source. The same term can be obtained by starting with the D5–D5 system with the WZ term [17], and compactifying 4 parallel directions on a torus and assuming that the gauge field background has a non-zero magnetic flux \( \int F \wedge F \) representing the D1-brane charge. This is a particular example of (T-duality) relations between different magnetic WZ terms in [23].

The abelian WZ terms discussed above have natural non-abelian generalizations. In particular, they should reproduce the non-abelian CS terms in multiple D-brane action given in [21].

Let us note also that SYM theories defined on curved \( D \)-dimensional spaces should contain curvature-dependent WZ terms similar to (23) with \( F \) replaced by \( R \) (and other mixed terms). They should reproduce the corresponding \( R \)-dependent CS terms [27, 21, 31] in the Dp-brane actions (for example, \( \int R \wedge R \wedge C_2 \) in D5 brane case).

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\( \text{total inflow of the electric charge into the intersection is equal to the Chern class } c_4(F) \sim \int (\wedge F)^4, \)

where \( F \) is a (relative) magnetic field on the D8 branes. This is essentially equivalent to the D8–D0 case discussed in [23, 24], since the magnetic flux \( \int (\wedge F)^4 \) on one of the D8-branes represents a D0-brane charge. Thus the above phenomenon is a reflection of string creation phenomenon when D0 passes through D8 [24].

\( \text{Related case of M5–M2 magnetic interaction was considered in [17]. D6-D0 system was discussed in [27].} \)

\( \text{Additional } [X,X] \text{ factors originate from internal components of } F_{ij}, \text{ i.e. these CS term may be} \)

\( \text{related by dimensional reduction and T-duality to } \int \text{tr}(F \wedge ... \wedge F) \wedge C_2 + ... \text{ CS terms in D9 brane action.} \)
4 Discussion

We have shown that the 1-loop effective action of $D = 4 \, \mathcal{N} = 4$ SYM theory contains the unique WZ term (3) coming from the phase of the Euclidean fermion determinant. The presence of this term (and of its $D > 4$ analogs (22)) is related to the existence of chiral anomaly in $D = 10$ SYM theory. This term depends only on 6 scalar fields, is manifestly $SO(6)$ and conformal invariant, and its coefficient should not be renormalized by higher loop corrections.

One interesting open question is how to construct a supersymmetric generalization of this term. The answer does not seem obvious since the use of either $\mathcal{N} = 1$ or $\mathcal{N} = 2$ superfield formulation of $\mathcal{N} = 4$ SYM theory breaks the $SO(6)$ symmetry. The WZ term (3) is actually the integral part of the space-time supersymmetric and $\kappa$-symmetric action [14] for a D3 brane propagating in $AdS_5 \times S^5$ vacuum of type IIB supergravity. As in the similar superstring action case [32], this term must be added to the Born-Infeld part (1) of the action to ensure its $\kappa$-symmetry. Fixing the static gauge and a $\kappa$-symmetry gauge in the action of [14] in a suitable way one should be able to read off the 4-d supersymmetric form of this WZ term. After the gauge fixing, half of the original 32 superconformal symmetry generators become non-linearly realized, and they should be relating the WZ term to the terms in the BI part of the action.

It should be possible to rederive this WZ term directly from string theory, by taking an appropriate $\alpha' \to 0$ limit in the 1-loop expression for the interaction potential between two separated D3-branes. The topological nature of this term suggests that it should be originating from certain fermionic zero mode contribution.

Similar purely scalar magnetic WZ term $f_3 C_3$ appears in the classical action of M2 brane moving in the background of an M5 brane source. Though lack of detailed understanding of the theory of multiple M-branes prohibits us from deriving this term directly from a microscopic theory (as was possible in the D-brane case), it is natural to expect (by analogy with a related proposal about a CS term in the M5 brane theory action [21]) that this WZ term is again universal, i.e. is not renormalized.

The magnetic WZ terms are present also in some orbifold theories [33]. For example, they appear in the field theory on a stack of an equal number of electric and magnetic D3-branes in type 0 string theory [34], which is a $Z_2$ orbifold of $\mathcal{N} = 4$ SYM [35]. This theory has two sets of scalar fields, $X_{(el)}$ and $X_{(mg)}$, which correspond to the transverse coordinates of the electric and the magnetic branes. The Yukawa couplings in the diagonal background of these scalar fields are $\bar{\Psi}^{ab} \Gamma^I (X^{a}_{(el)} - X^{b}_{(mg)}) \Psi^{ab}$ [36].
Yukawa interaction induces a WZ term depending on the difference $X^a_{(el)} - X^b_{(mg)}$. The same prediction (CS term in the interaction potential) should follow from the gravity description, since there should be a Lorentz force between the separated electric and the magnetic D-branes.

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