Numerical study of electro-vortex flow in long cylinder with localized current supply

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Abstract. Numerical simulations of the electrovortex flow (EVF) of the GaSnZn alloy in a closed cylindrical volume are performed. The electric current is applied with two localized opposed electrodes on the cylinder side wall. It is shown that in the entire considered range of electric current values from 50 to 1000 A the EVF arises in a cylinder, in a form of the two-toroidal flow. The flow intensity increases with increasing current values and its maximum intensity is 12 cm/s (at 1000 A). Even at the lowest electric current value of 50 A, a non-stationary behavior of EVF is observed. However, the two-toroidal flow pattern is present in all considered regimes. It is shown that with increasing current the velocity pulsations in the near-electrode region have a frequency up to 0.26 Hz.

1. Introduction

Electrovortex flows (EVF) arise due to the interaction between the electric current in conducting medium and the magnetic field, induced by the current itself, in case of inhomogeneous electric current distribution in the medium [1]. EVFs are found in many industrial applications. For example, in titanium arc furnaces [2, 3], which are used in modern industrial metallurgy.

In dependence on the problem configuration, EVF can be stationary, or not. A number of academic tasks are known, where the possibility of obtaining stationary EVF is show. EVFs are well-studied in canonical configurations, such as hemispheres [4] and flat layers [5, 6]. In most industrial configurations EVFs appear in non-stationary form. The instability of such flows is shown to be either positive (for example, better mixing of a liquid metal) or negative factor. Liquid metal batteries [7, 8] — a promising power storage unit, in which the arising non-stationary EVF may result in device malfunction, are good example of undesired EVFs. At the same time, as mentioned before, EVFs can help in better liquid metal stirring, improving crystalline structure of obtained ingot. Thus, it is of interest to study EVF in close to industrial ones configurations, with the aim of investigating the possibilities of controlling their intensity.

An additional parameter of the task is the method of electric current supply to the conducting medium. Of particular interest is the localized current application, since it is the method used in described above industrial applications, and also in aluminum reduction cells [9] (including the multi-electrode current supply [5]).

In this work, the EVF in a closed cylindrical volume, in the case of inconsistency of the cavity geometry and topology of the arising vortices, is studied numerically. A localized electric current application with two opposed electrodes on the cylinder side wall is considered.
2. Methods

The electrovortex flow of a gallium alloy GaSnZn arising in a cylindrical volume is considered. The problem is solved numerically. The solution of the electromagnetic part is carried out in the ANSYS Emag software, and the hydrodynamic one — in ANSYS Fluent. The computational domain is a cylinder 1 with a diameter of \( D = 0.068 \) m and a height of \( L = 0.2 \) m (see Fig. 1). The Cartesian coordinate system is used in calculations and in the presentation of results. Its origin is located at the bottom of the cylinder, the \( y \) axis is parallel to the line connecting the electrodes 2, and the \( z \) axis coincides with its vertical symmetry axis. The electrodes are located oppositely on the side wall at \( z = 0.1 \) m and have a diameter of 0.02 m. The current values were varied from 50 to 1000 A. The cylinder is filled with GaSnZn gallium alloy with the following material parameters: kinematic viscosity \( \nu = 3.1 \times 10^{-7} \) m\(^2\)/s, density \( \rho = 6265 \) kg/m\(^3\), electric conductivity \( \sigma = 3.56 \times 10^6 \) Sm. An unstructured mesh with tetrahedral elements is used whose linead size does not exceed 0.002 m. To resolve the boundary layer, computational nodes distribution density is increased as it approaches the side walls of the cylinder. The problem is considered to be isothermal. No-slip conditions are set on all boundaries for the velocity. All simulations began from a state of zero velocity.

For additional data processing, wavelet analysis [10, 11] was used. This type of analysis was selected due to the appearance of non-harmonic oscillations. The wavelet transform translates the one-dimensional time series \( f(t) \) into a two-dimensional frequency-time plane \((\nu, t)\), according to

\[
W(\nu, t) = \sqrt{\nu} \int f(\tau) \psi(\nu, (\tau - t)) d\tau.
\]

The Morlet wavelet was selected as an analyzing wavelet — one of the most popular wavelets:

\[
\psi(t) = \exp\left[-t^2/(2\sigma^2)\right] \exp[i2\pi t],
\]

which better suits for signals of complex nature. Here \( \sigma \) is an adjustable parameter. By changing its value one can obtain optimal resolutions of time \( t \) and frequency \( \nu \). Low \( \sigma \) values provide better time resolution, while large ones can help achieving better frequency resolution of wavelet. However, an uncontrolled increase in \( \sigma \) is undesirable, since it leads to attenuation of the analyzing function. The commonly adopted value of \( \sigma \) is 1 [12], while the limit \( \sigma \to \infty \) corresponds to the Fourier transform.

The map \( W(\nu, t) \) has an easily interpreted structure only in case of the simple data analysis. But in the case of the real data, wavelet map interpretation can be much more difficult. The
wavelet transform integral spectrum is one of the simplest its characteristic:

\[ M(\nu) = \int |W(\nu, t)|^2 dt , \]

which is calculated as a convolution of \( W(\nu, t) \) along the time axis. The integral spectrum is a smoothed Fourier transform. Increasing the \( \sigma \) parameter reduces the degree of spectrum smoothing.

3. Results

The three-dimensional velocity fields of gallium alloy in the cylinder, as well as statistical characteristics of the process, are obtained. The current values were varied in the range from 50 to 1000 A, which corresponds to the MHD interaction parameter \( S = \mu_0 I^2/(\rho \nu^2) \) values in range \( S = (0.52 \div 208) \cdot 10^7 \). In an experimental study [13, 14] the lower current value was determined by the resolution of the Doppler ultrasound velocimeter [15], and the upper one — by the technological properties of the setup. In numerical simulations such limitations are not imposed, which made it possible to expand the range of considered regimes. The intensity of emerging EVF is moderate — the average flow rate does not exceed 12 cm/s for the highest current value of 1000 A, which corresponds to the Reynolds number determined through the diameter \( (\text{Re} = \langle V \rangle D/\nu) \) in the range of 438 \( \div \) 26761.

![Figure 2](image1.png)

Figure 2. The fields of instant velocity \( z \)-component in sections \( zOx \) (a, b, c) and \( zOy \) (d, e, f) for the current values of 50 A (a, d), 200 A (b, e), 1000 A (c, f)

![Figure 3](image2.png)

Figure 3. The fields of instant velocity \( z \)-component in section \( yOx \) at \( z = 0.1 \) m for the current values 50 A (a), 200 A (b), 1000 A (c)

The visualization of the instant velocity field in sections \( zOx \), \( zOy \) and \( yOx \) is pictured in Figs. 2, 3 at \( t = 400 \) s. Here, the color indicates the amplitude of the velocity \( z \)-component and
the velocity vector field interpretation in current section is shown with lines. The figure shows that the flow intensity is the highest in areas close to the electrodes and that it is noticeably lower in other parts of the cylinder. Also, one can clearly see the distorted two-toroidal flow pattern. The observed asymmetry stands for non-stationarity of the EVF, even for the lowest electric current value of 50 A.

**Figure 4.** Liquid metal velocity profiles along the line ($-0.025, 0, z$) m for the current values a — 50 A, b — 200 A, c — 1000 A

**Figure 5.** Liquid metal velocity profiles along the line ($0.025, 0, z$) m for the current values a — 50 A, b — 200 A, c — 1000 A

Figs. 4, 5 presents the liquid metal velocity time-averaged profiles in a cylinder, obtained along the lines parallel to the cylinder vertical symmetry axis. Namely, ($0.025, 0, z$) m and ($-0.025, 0, z$) m. Here, a negative velocity value means the flow towards the bottom cylinder face, and a positive value — towards the top one. Thus, the presented profiles indicate the presence of a large-scale two-toroidal vortex structure in volume. The velocity pulsations depicted by vertical lines have an amplitude of up to 6 cm/s, which indicates a strong oscillatory behavior of EVF. It can be seen that the velocity profiles are slightly asymmetric with respect to the center of the vertical axis of the cylinder, and this asymmetry differs for profiles along the lines ($0.025, 0, z$) m and ($-0.025, 0, z$) m, which indicates the distortion of the two-toroidal flow pattern.

To analyze the statistical characteristics of the process, the spatiotemporal maps of the vertical velocity component were produced (see Fig. 6). As in the Fig. 10, one can see here that the highest flow amplitude is observed in areas near the electrodes. The change in the velocity amplitude in time indicated that the process is non-stationary. Also, an increase in electric current value leads to an increase in both oscillations intensity and frequency.

Next, points with $z = 0.10$ m were selected on the spatiotemporal maps, that correspond to the areas with the highest velocity values, for obtaining oscillation characteristics. For them, the wavelet spectrograms were obtained (see Figs. 7, 8). Here $a = f_s/\nu$, where $f_s = 0.1$. The Fig. 7a
Figure 6. Spatiotemporal maps of the liquid metal flow along the line \((-0.025, 0, z)\) m for the current values a — 50 A, b — 200 A, c — 1000 A

shows that even at the electric current value of 50 A it is possible to resolve oscillations with frequency approx 0.069 Hz, which can be also seen from Fig. 6. With subsequent increase in the electric current oscillation frequency increases to 0.075 Hz (see Fig. 7b). This observed peak remains at higher current values, while at 490 A additional peaks are present, corresponding to 0.147 and 0.222 Hz. Further increase in the electric current results in moderate oscillation frequency growth, while the peaks positions on wavelet spectrograms change. This can indicate a traversing of large-scale vortices with time.

Figure 7. Wavelet spectrograms for the flow velocity at \((0.025, 0, 0.10)\) m and current values a — 50 A, b — 100 A, c — 490 A

Figure 8. Wavelet spectrograms for the flow velocity at \((0.025, 0, 0.10)\) m and current values a — 700 A, b — 900 A, c — 1000 A

The dependencies of obtained in the wavelet analysis frequencies on the electric current value are pictured in Fig. 9. The figure shows that low-frequency oscillations are present at all electric current values. From 200 A and above, additional high-frequency oscillations appear. While the
number of dominant frequencies increases with increase in electric current, the overall change in those frequencies values is rather small.

![Graph showing oscillation frequency in point (0.025, 0, 0.10) m versus electric current value](image1)

**Figure 9.** Oscillation frequency in point (0.025, 0, 0.10) m versus electric current value

An estimation of the maximum mean flow velocity shows an increase in the EVF intensity with an increase in the electric current magnitude (see Fig. 10). In the considered range of current values, this dependence is best fitted with quadratic trend. The figure shows that at the highest considered electric current value of 1000 A the maximum mean flow velocity reaches 12 cm/s.

![Graph showing maximum mean flow velocity versus the electric current value](image2)

**Figure 10.** Maximum mean flow velocity versus the electric current value

4. **Conclusions**

The EVF in a closed cylindrical volume, generated by a direct electric current supply with two localized opposite electrodes on the side wall, is studied numerically. For all electric current values a large-scale two-toroidal vortex flow is formed in the volume, supplemented with several small-scale vortices. The flow is non-stationary even at the lowest current of 50 A. With increasing current, the velocity pulsations amplitude and the flow asymmetry increase, as well as the number of secondary small-scale vortices. Maximum EVF intensity is 12 cm/s, at 1000 A (which is the highest electric current value in this work). The distorted two-toroidal flow pattern remains at all studied regimes. At all electric current values the presence of a dominant velocity pulsations frequency, up to 0.26 Hz, is observed.
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