Wave functions for arbitrarily Polarized Quantum Hall States

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Abstract

We determine the wave functions for arbitrarily polarized quantum Hall states by employing the doublet model which has been proposed recently to describe arbitrarily polarized quantum Hall states. Our findings recover the well known fully polarized Laughlin wave functions and unpolarized Halperin wave function for the filling fraction $\nu = 2/5$. We have also confirmed by an explicit One-loop computation that the Hall conductivity does indeed get quantized at those filling fractions that follow from the model. Finally, we have given a physical picture for the non-analytic nature of the wave functions, and shown that quantum fluctuations restore the Kohn mode.

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I. INTRODUCTION

Recently Fradkin [1] has proposed a method for determining the absolute square of the many particle wave function for the ground state of a system from a knowledge of correlation functions which are generally computed in field theory. This beautiful method which employs the generating functions for equal-time correlation functions is applicable to any field theoretic problem. If one further knows before hand, or assumes, that the many particle system is non-degenerate, the many particle wave function is also determined thereby, apart from the gauge dependent phase which is essential in expectation values of observables such as velocity. Using this formalism, Lopez and Fradkin [2] have extracted the wave functions for fully polarized quantum Hall (QH) states within the composite fermion model (CFM) [3,4]. Remarkably, they recover the Jastrow part of the Laughlin wave functions [5] unambiguously for filling fractions $\nu = 1/(2k + 1)$ $(k$ an integer) which have been shown to be exact by Kivelson and Trugman [6] and Haldane [7]. Note that this result vindicates the mean field (MF) ansatz which one normally employs for the Chern-Simons (CS) field. More generally, Lopez and Fradkin [2] show that the Jastrow form is indeed generic to all states with filling fractions $\nu = p/(2sp\pm1)$ $(p, s$ are integer) which occur in CFM. Further, the long distance properties of the many body wave function are universal, being independent of the microscopic charge-charge interaction. Yet another interesting aspect is that the wave functions are in general non-analytic. That is, the exponents that occur in the Jastrow form are arbitrary rational numbers. (We discuss the origin of the non-analyticity in section–III-C).

The above analysis of Lopez and Fradkin [2] is based on the strict assumption that the spin degree of freedom of the electrons is frozen in the direction of the magnetic field, i.e., they are assumed to be spinless. Such an assumption which is valid for large magnetic fields ($B \sim 10T$) breaks down for small values of $B$: the Zeeman splitting is now not that large (also partly due to small $g$ value, $g \sim 0.4$) and, as Halperin [8] has observed, these systems are not fully polarized. Indeed, experiments reveal that at relatively small values of $B$, the QH states at filling factors $\nu = \frac{1}{3}, \frac{5}{8}, \frac{10}{7}$ (Ref. [10]) and $\frac{2}{3}$ (Ref. [11]) are
unpolarized while the states at $\nu = \frac{3}{5}$ (Ref. [12]) and $\frac{7}{5}$ (Ref. [9]) are partially polarized. Further, it is also known experimentally that the states which are at maximum polarization to start with pass over to partially polarized or unpolarized states as the Zeeman energy is lowered sufficiently — either by reducing the tilting angle of the magnetic field [9, 12] or by decreasing the electron density [11]. In the vanishing Zeeman splitting (VZS) limit, it has been found from numerical computations [13] that the states with $\nu = 2/(2n+1)$ are unpolarized and those of the Laughlin sequence [5] with $\nu = 1/(2n+1)$ are fully polarized, in the thermodynamic limit. Also the state at $\nu = \frac{3}{5}$ has been found to be partially polarized by an exact diagonalization study [14], in agreement with experiments.

Wu, Dev and Jain [15] have studied this problem and constructed trial wave functions by employing the CFM. These trial wave functions are confirmed to be exact by numerical computation. They report that, in the VZS limit, all even numerator QH states are unpolarized and all those states with both the numerator and denominator (of $\nu$) odd are partially/fully polarized. Further, Belkhir and Jain [16] have proposed that the CFM accommodates the sequence $\nu = 2n/(3n+2)$ all of which are spin unpolarized. From the wave functions that they construct, they also interpret that these states possess a new feature where each composite fermion carries two different types of vortices — one of which seen by all electrons while the other is visible only to an electron of like spin.

Recently we have proposed [17] a global model which employs a doublet of CS gauge fields, with the strength of the CS term given by a symmetric coupling matrix. The model is within the composite fermion framework, i.e., each fermion has an even number of vortices attached to it. The model accounts for all the observed as well as proposed [15] QH states with the correct spin polarization. Further it predicts new possible QH states characterized by two gaps (corresponding to two spin species) of excitations.

In this context we mention that a very similar model has been proposed by Lopez and Fradkin [23] to describe QH effects in double layered systems. In comparing the two models, we note that while the same kind of the CS action is employed in both the models, the problems addressed are otherwise of entirely different nature. The problem at hand is the
description of QH systems with the spin degree of freedom, restricted to a single layer; Lopez and Fradkin [23] study spinless fermions in double layered systems. This leads to different consequences. For instance, in the model of Lopez and Fradkin, “the spin singlet state \((3,3,2)\), which has filling fraction \(\nu = 2/5\), cannot be described within the Abelian Chern-Simons approach” [23]. On the other hand, the 2/5 state which is the most important for us here emerges naturally in our model. More importantly, the inclusion of the spin degree of freedom in the paper of Lopez and Fradkin [23] involves the more complicated non-abelian SU(2) CS model, which was first introduced by Frohlich et al [24] in CS action to study QH systems. The inclusion of spin degree is only to the extent of describing the singlet states, unlike the present paper where arbitrary polarization are discussed. In that sense, we claim that the present model provides a more comprehensive and a simpler picture than the non-abelian models employed in Ref. [23]. A detailed comparative study between our approach and results with those of Lopez and Fradkin [23] will be provided in the Appendix.

It is the purpose of this paper to extract wave functions for arbitrary polarized states by employing the doublet model [17]. For simplicity’s sake, we shall restrict to only those sequences which have been seen significantly. They correspond to states with a single gap of excitation. We show that the asymptotic properties of the wave function, in the VZS as well as thermodynamic limits, are completely determined by the long-distance behaviour of the equal-time density-density correlation functions of different spin species of electrons. We explicitly determine the square of the absolute value of the wave function for (i) spin-unpolarized, (ii) partially polarized and (iii) fully polarized QH states. Gratifyingly, we recover the well-known wave function for spin-unpolarized state \(\nu = 2/5\) which was predicted by Halperin [8], in our analysis. All the unpolarized states, having numerator 2, emerge as analytic functions (which is not the case for spinless electrons). However, the wave functions for integer QH states with \(\nu > 2\) and all the fractional QH states having numerator greater than 2 continue to remain non-analytic. Interestingly we also find that, for the integer QH states, the particles with dissimilar spins are completely uncorrelated. Finally, we predict that the wave function for even denominator unpolarized QH states are analytic in nature.
In the next section we briefly discuss the doublet model and determine the equal-time generating functional for mixed spin density-density correlations. Section 3 is devoted to a determination of the wave functions and the origin of non-analyticity is briefly discussed. Finally in section 4, we perform the linear response analysis to demonstrate explicitly the quantization of Hall conductivity $\sigma_H$ at appropriate filling fractions and show that the Kohn-mode is restored by the fluctuations. We conclude the paper in section 5.

II. THE GENERATING FUNCTIONAL

A. The Doublet Model

We first briefly discuss the model [17] employed here.

Consider a two-dimensional system of non-relativistic spin-1/2 interacting fermions in the presence of magnetic field perpendicular to the plane. In their study of spinless fermions, Lopez and Fradkin [19] have shown that such a system is equivalent to the one interacting with a CS gauge field provided the CS parameter is such that the statistics of the particles remains fermionic. Using this generic argument, we propose a generalized Lagrangian density

$$
\mathcal{L} = \bar{\psi}_\uparrow^\dagger D(\mathbf{A}_\uparrow + \mathbf{a}_\uparrow) \psi_\uparrow + \bar{\psi}_\downarrow^\dagger D(\mathbf{A}_\downarrow + \mathbf{a}_\downarrow) \psi_\downarrow + \frac{1}{2} \bar{\mathbf{a}} \epsilon^{\mu\nu\lambda} \partial_\nu \mathbf{a}_\lambda - e A_0^\mu \rho + \frac{1}{2} \int d^3 x' A_0^\mu(x) V^{-1}(x - x') A_0^\mu(x') .
$$

(2.1)

Here $\psi$ is the fermionic field and $\uparrow$ (down) represents spin-up (down),

$$
\mathcal{D}(\mathbf{A}_\mu + \mathbf{a}_\mu) = iD_0^\mu + (1/2m^*) D_k^2 + \mu + (g/2) \mu_B (B + B^r + b^r) \sigma ,
$$

(2.2)

with $D_\mu^r = \partial_\mu - ie(A_\mu + A^r_\mu + a^r_\mu)$ where $A_\mu$ is the external electro-magnetic field which interacts with all the electrons while $A^r_\mu$ and $a^r_\mu$ are the external probe [20] and the CS gauge field respectively, interacting with only the particles having spin indices $r = \uparrow, \downarrow$. The field $A_0^\mu$ is identified as an internal scalar potential. Particles with an effective mass $m^*$ and charge $e$ have mean density $\rho$ which is fixed by the introduction of chemical potential $\mu$ as a Lagrange multiplier. (We have chosen the units $\hbar = c = 1$). Note that the Zeeman term
includes all the three kinds of magnetic fields, $\mu_B$ is the Bohr-magneton, and $\sigma = +1(-1)$ for spin-up (down) electrons. We have introduced a doublet of CS gauge fields in (2.1) as

$$a_\mu = \begin{pmatrix} a_\mu^+ \\ a_\mu^- \end{pmatrix},$$

(2.3)

and the strength of the CS parameter is taken to be

$$\Theta = \begin{pmatrix} \theta_1 & \theta_2 \\ \theta_2 & \theta_1 \end{pmatrix}.$$  

(2.4)

$\tilde{a}_\mu$ is the transpose of the doublet field $a_\mu$. The fourth term in Eq. (2.1) describes the charge neutrality of the system. Finally, $V^{-1}(x - x')$ is the inverse of the electron interaction potential (in the operator sense). The usual fermion interaction term in quartic form would be achieved by an integration over $A_{0}^{\text{in}}$ field. The action given by Eq. (2.1) is invariant under the gauge transformations $a^\pm_\mu \rightarrow a^\pm_\mu + \partial_\mu \lambda^\pm(x), \psi^\pm(x) \rightarrow \exp\left[ie\lambda^\pm(x)\right]\psi^\pm(x)$. In other words, the doublet model is abelian.

We diagonalize the matrix $\Theta$, with the eigen values $\theta_{\pm} = \theta_1 \pm \theta_2$, and denote $a_\mu$ in the eigen basis by

$$a_\mu = \begin{pmatrix} a_\mu^+ \\ a_\mu^- \end{pmatrix}.$$  

(2.5)

By simple rescalings, Eq. (2.1) may be written as

$$\mathcal{L} = \psi_\uparrow D(A_{\mu}^\uparrow + a_\mu^+ + a_\mu^-)\psi_\uparrow + \psi_\downarrow D(A_{\mu}^\downarrow + a_\mu^+ - a_\mu^-)\psi_\downarrow + \frac{e}{2} \epsilon^{\mu\nu\lambda} a_\mu^+ \partial_\nu a_\lambda^+$$

$$+ \frac{\theta^-}{2} \epsilon^{\mu\nu\lambda} a_\mu^- \partial_\nu a_\lambda^- - eA_{0}^{\text{in}}(x) V^{-1}(x - x') A_{0}^{\text{in}}(x').$$  

(2.6)

This incorporates the idea that each electron, in general, has two kinds of vortices associated with it — while the contributions of vortices are added for spin up particles, the spin down particles get their subtracted contribution.
B. Case–1: No Polarization

We first study the case $\theta_1 = \theta_2 = \theta$ (say). Here $\theta_+ = 2\theta$ and $\theta_- = 0$. Hence the gauge field $a^-_\mu$ decouples dynamically and merely plays the role of a Lagrange multiplier: $(\partial L/\partial a^-_\mu) = \rho_\uparrow - \rho_\downarrow \equiv 0$, where $\rho_\uparrow (\rho_\downarrow)$ is the density for spin-up (down) particles. Thus the unpolarized case is accomplished by the choice $\theta_1 = \theta_2$. Rescaling $\theta$ by $\theta/2$, we parametrize $\theta = (e^2/2\pi)(1/2s)$ ($s$ is an integer) in order to impose the composite fermion picture.

The generating functional of the system (in uniform magnetic field $B$) can be written as

$$Z\left[A^\uparrow_\mu, A^\downarrow_\mu\right] = \int[d\psi^\dagger_\uparrow][d\psi_\downarrow][d\psi^\dagger_\downarrow][d\psi_\downarrow][da^\dagger_\mu][dA^\text{in}_0]e^{i \int d^3x L}. \quad (2.7)$$

The terms (quadratic) in $\psi_\uparrow$ and $\psi_\downarrow$ fields can be integrated out to produce the fermion determinants for spin-up and down respectively. The other terms remain as they are in the Lagrangian density (2.6). Now the fermion determinants are to be expanded about a saddle point of gauge fields which we fix as follows.

It is clear that the fermions are associated with an even number ($2s$) of flux quanta. In the mean field (MF) ansatz, these fluxes produce an average CS magnetic field $\langle b^+ \rangle = -e\rho/\theta$ which is seen by all the electrons. Demanding that the effective Landau levels (LL) formed by the effective magnetic field $\bar{B}^+ = B + \langle b^+ \rangle$ accommodate all the particles at an even integer filling factor $2p$, ($p$ for up spin and $p$ for down), the actual filling fraction $\nu$ is obtained as

$$\nu = \frac{2p}{4sp + 1}. \quad (2.8)$$

The energy corresponding to each level is obtained as $\varepsilon_{n\sigma} = (n + 1/2)\bar{\omega}_c - \frac{2\mu_B}{2m^*}\bar{B}^+ \sigma$ ($n = 0, 1, \ldots$), where the effective cyclotron frequency $\bar{\omega}_c = \frac{e}{m^*} \bar{B}_c$. The actual cyclotron frequency $\omega_c$ of the system is related to $\bar{\omega}_c$ via $\omega_c = \bar{\omega}_c(1 + 4sp)$. Recall that $p$ can be a negative integer (meaning $\bar{B}^+$ is antiparallel to $B$). All the states obeying Eq. (2.8) are spin unpolarized. It is known [19] that a liquid-like solution exists for the vanishing average of $A^\text{in}_0$ field. Therefore the saddle point is fixed at $\langle b^+ \rangle = -e\rho/\theta$ and $\langle A^\text{in}_0 \rangle = 0$. We remark that this model does not accommodate the sequence $\nu = 2n/(3n + 2)$ proposed by Belkhir and Jain [16]. Of course, our model does include all the unpolarized states that are seen experimentally.
Expanding the fermion determinants about the above mentioned saddle point, up to terms quadratic in the gauge field fluctuations and the external probes, we obtain

\[ Z \left[ A_\mu^+, A_\mu^- \right] = \int [da_\mu^+][dA_0^\text{in}] \exp \left[ iS_{\text{eff}} \left( A_\mu^+, A_\mu^-, a_\mu^+, A_0^{\text{in}} \right) \right], \quad (2.9) \]

where \( S_{\text{eff}} \) is identified as one-loop effective action and has the form

\[
S_{\text{eff}} = -\frac{1}{2} \int d^3x \int d^3x' (A_\mu^+ + a_\mu^+ + A_0^{\text{in}} \delta_{\mu0})(x) \Pi^{\mu\nu}(x, x')(A_\nu^+ + a_\nu^+ + A_0^{\text{in}} \delta_{\nu0})(x') \\
- \frac{1}{2} \int d^3x \int d^3x' (A_\mu^+ + a_\mu^+ + A_0^{\text{in}} \delta_{\mu0})(x) \Pi^{\mu\nu}(x, x')(A_\nu^+ + a_\nu^+ + A_0^{\text{in}} \delta_{\nu0})(x') \\
+ \int d^3x \frac{\theta}{2} \varepsilon^{\mu\nu\lambda} a_\mu^+ \partial_\nu a_\lambda^+ + \frac{1}{2} \int d^3x \int d^3x' A_0^{\text{in}}(x)V^{-1}(x - x') A_0^{\text{in}}(x'). \quad (2.10) \]

\( a_\mu^+ \) and \( A_0^{\text{in}} \) now represent fluctuations about their corresponding mean values. The polarization tensors \( \Pi^{\mu\nu}_{\uparrow, \downarrow} \) will be evaluated below.

The procedure for the evaluation of polarization tensor is well known in literature (see e.g., [19]) for spinless particles. The additional feature is that, here we evaluate \( \Pi^{\mu\nu}_{\uparrow, \downarrow} \) for two different spin species respectively. In brief, it follows from translational and gauge invariance that \( \Pi^{\mu\nu}_{\uparrow, \downarrow} \) have the form (in momentum space)

\[
\Pi^{\mu\nu}_{\uparrow, \downarrow} = \Pi_0^{\uparrow, \downarrow}(\omega, \mathbf{q}^2)(q^2 g^{\mu\nu} - q^\mu q^\nu) + \left( \Pi_2^{\uparrow, \downarrow} - \Pi_0^{\uparrow, \downarrow} \right) (\omega, \mathbf{q}^2) \\
\times \left( q^2 \delta^{ij} - q^i q^j \right) \delta^{\mu i} \delta^{\nu j} + i\Pi_1^{\uparrow, \downarrow}(\omega, \mathbf{q}^2) \varepsilon^{\mu\nu\lambda} q_\lambda. \quad (2.11) \]

The form factors are then evaluated in the lowest order in \( \mathbf{q}^2 \) and we find

\[
\Pi_0^{\uparrow, \downarrow} = -\frac{e^2}{2\pi} \frac{\tilde{\omega}_c}{\omega^2 - \tilde{\omega}_c^2} \equiv \Pi_0; \quad \Pi_1^{\uparrow, \downarrow} = \Pi_0^{\uparrow, \downarrow} \tilde{\omega}_c \equiv \Pi_1; \quad (2.12a) \]

\[
\Pi_2 = -\frac{e^2}{4\pi m^*} \tilde{\omega}_c^2 \left[ \frac{3}{\omega^2 - \tilde{\omega}_c^2} - \frac{4}{\omega^2 - 4\tilde{\omega}_c^2} \right] p(p - 1); \quad (2.12b) \]

\[
\Pi_2^\dagger = -\frac{e^2}{4\pi m^*} \tilde{\omega}_c^2 \left[ \frac{3}{\omega^2 - \tilde{\omega}_c^2} - \frac{4}{\omega^2 - 4\tilde{\omega}_c^2} \right] p(p + 1). \quad (2.12c) \]

To evaluate the effective action for the external probes \( A_\mu^{\uparrow, \downarrow} \), we integrate over the internal fields \( a_\mu^+ \) and \( A_0^{\text{in}} \) in (2.3). Thus we obtain (in momentum space),
\[
S_{\text{eff}} [A^\mu_\uparrow, A^\mu_\downarrow] = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} A^\mu_q(q) K^\mu_{rr'}(\omega, q^2) A^\mu_{r'}(-q),
\]

(2.13)

where the indices \( r, r' = \uparrow, \downarrow \). Since we need only the density-density correlations between different spin species, it is sufficient that we evaluate \( K^0_{rr'} \). We find (for small \( q^2 \))

\[
K^0_{\uparrow\uparrow} = K^0_{\uparrow\downarrow} = \frac{1}{2} \left[ \Pi_0 - \frac{\Pi_0 \theta^2}{4\Pi_0^2 \omega^2 - (2\Pi_1 + \theta)^2} \right] q^2 + O((q^2)^2); \tag{2.14a}
\]

\[
K^0_{\uparrow\downarrow} = K^0_{\downarrow\uparrow} = -\frac{1}{2} \left[ \Pi_0 + \frac{\Pi_0 \theta^2}{4\Pi_0^2 \omega^2 - (2\Pi_1 + \theta)^2} \right] q^2 + O((q^2)^2), \tag{2.14b}
\]

subject to the condition \( \lim_{q^2 \to 0} V(q^2) q^2 = 0 \). In other words, Eq. (2.14) is valid for any potential which is short ranged compared to \( \ln r \). Here \( K^0_{\uparrow\uparrow} \), \( K^0_{\uparrow\downarrow} \), \( K^0_{\downarrow\uparrow} \) and \( K^0_{\downarrow\downarrow} \) represent the density-density correlations among spin up-up, up-down, down-up and down-down species of the particles respectively. We next write the generating functional for equal-time density-density correlations among mixed spins in the form,

\[
Z [A^\mu_\uparrow_0, A^\mu_\downarrow_0] = \exp \left[ -\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} A^\mu_0(q) K_{rr'} A^\mu_{r'}(-q) \right],
\]

(2.15)

with

\[
K_{rr'} = \int \frac{d\omega}{2\pi i} K^0_{rr'}(\omega, q^2)
\]

(2.16)

which is given by

\[
K = \frac{q^2}{2\pi} \left( \frac{e^2}{2} \right) p \begin{bmatrix} 2sp + 1 & -2sp \\ -2sp & 2sp + 1 \end{bmatrix}.
\]

(2.17)

C. Case 2: Partial Polarization

Now consider the case \( \theta_1 \neq \theta_2 \), i.e., \( \theta_{\pm} \neq 0 \). The generating functional, in this case, reads

\[
Z [A^\mu_\uparrow, A^\mu_\downarrow] = \int [d\psi^\dagger_\uparrow][d\psi_\downarrow][d\psi^\dagger_\downarrow][d\psi_\uparrow][dA^\mu_0][dA^\mu_0][dA^\mu_0^\dagger][dA^\mu_0^\dagger] e^i \int d^3x L, \tag{2.18}
\]

where there is an additional functional integral over the field \( a^-_\mu \). The fermionic fields are then integrated out to produce the fermion determinants which are to be expanded about
the saddle point of the gauge fields. We parametrize \( \theta_\pm = (e^2/2\pi)(1/s_\pm) \). As discussed in Ref. [17], we choose \( s_- = 0 \), i.e., mean CS magnetic field \( \langle b^- \rangle = 0 \). In this case, the field \( a^-_\mu \) provides a vanishing mean field \( \langle b^- \rangle \), and does not contribute at tree level (in contrast to the unpolarized case where \( a^-_\mu \) is completely non-dynamical). We impose the composite fermion requirement, which fixes \( s_+ = 2s \) (an even integer) [17]. Since \( \langle b^- \rangle = 0 \), the effective magnetic field for both spin up and down particles is same and is given by \( \vec{B}^+ = B + \langle b^+ \rangle \) where \( \langle b^+ \rangle = -e\rho/\theta_+ \) in the MF ansatz. Let \( p_t(p_d) \) be the number of effective LL formed by \( \vec{B}^+ \) filled by up (down) particles. This leads to the actual filling fraction and the spin density to be

\[
\nu = \frac{p_t + p_d}{2s(p_t + p_d) + 1} ; \quad \Delta \rho = \rho \left( \frac{p_t - p_d}{p_t + p_d} \right).
\]

Note that \( p_t \) and \( p_d \) can be negative integers as well (when \( \vec{B}^+ \) is antiparallel to \( B \)). Further, it is easy to check that the choice \( \Delta \rho = 0 \), i.e., \( p_t = p_d \) simply collapses to case-1. We thus require that \( p_t \neq p_d \), which naturally leads to partial polarization. The effective cyclotron frequency \( \bar{\omega}_c \) is related to \( \omega_c = eB/m^* \) by \( \omega_c = \bar{\omega}_c[2s(p_t + p_d) + 1] \). For small Zeeman energies, we may take \( p_t = p_d + 1 = p \) (say). Then

\[
\Delta \rho \rho = \frac{1}{2p - 1} ; \quad \nu = \frac{2p - 1}{2s(2p - 1) + 1}.
\]

The sequence (2.20) is indeed partially polarized becoming fully polarized for \( p = 1 \). Then \( \nu_{p=1} = 1/(2s + 1) \) which is simply the Laughlin sequence [3] known theoretically to be fully polarized [13]. In short, for obtaining partially/fully polarized QH states we fix the saddle point at \( \langle b^+ \rangle = -(e\rho/\theta_+) \), \( \langle b^- \rangle = 0 \) and \( \langle A_0^{\text{in}} \rangle = 0 \).

An expansion of the fermion determinants about the above mentioned saddle point yields the on-loop effective action for the gauge fields to be

\[
S_{\text{eff}} = -\frac{1}{2} \int d^3x \int d^3x' (A^+_\mu + a^+_\mu + a^-_\mu + A_0^{\text{in}} \delta_{\mu 0})(x)\Pi^\mu_{\nu} (x, x') (A^+_\nu + a^+_\nu + a^-_\nu + A_0^{\text{in}} \delta_{\nu 0})(x')
\]

\[
- \frac{1}{2} \int d^3x \int d^3x' (A^+_\mu + a^+_\mu - a^-_\mu + A_0^{\text{in}} \delta_{\mu 0})(x)\Pi^\mu_{\nu} (x, x') (A^+_\nu + a^+_\nu - a^-_\nu + A_0^{\text{in}} \delta_{\nu 0})(x')
\]

\[
+ \int d^3x \left[ \frac{\theta_+}{2} e^{\mu \nu \lambda} a^+_{\mu} \partial_{\nu} a^+_{\lambda} + \frac{\theta_-}{2} e^{\mu \nu \lambda} a^-_{\mu} \partial_{\nu} a^-_{\lambda} \right] + \frac{1}{2} \int d^3x \int d^3x' A_0^{\text{in}}(x)V^{-1}(x - x') A_0^{\text{in}}(x').
\]

(2.21)
The polarization tensors $\Pi_{\text{irr}}^{\mu\nu}$ have the same form (in momentum space) as Eq. (2.11) with the form factors in the lowest order in $q^2$ to be
\[
\Pi_0^{\uparrow\downarrow} = -\left(\frac{e^2}{2\pi}\right)\frac{\tilde{\omega}_c}{\omega^2 - \omega_c^2} p_{\uparrow\downarrow} \quad ; \quad \Pi_{\text{irr}}^{\uparrow\downarrow} = \Pi_{00}^{\uparrow\downarrow} \tilde{\omega}_c ;
\] (222a)
\[
\Pi_2^{\uparrow} = -\frac{e^2}{4\pi m^* \tilde{\omega}_c^2} \left[ \frac{3}{\omega^2 - \omega_c^2} - \frac{4}{\omega^2 - 4\omega_c^2} \right] p_{\uparrow} (p_{\uparrow} - 1) ;
\] (222b)
\[
\Pi_2^{\downarrow} = -\frac{e^2}{4\pi m^* \tilde{\omega}_c^2} \left[ \frac{3}{\omega^2 - \omega_c^2} - \frac{4}{\omega^2 - 4\omega_c^2} \right] p_{\downarrow} (p_{\downarrow} + 1) .
\] (222c)
Note that $\Pi_0^{\uparrow} \neq \Pi_0^{\downarrow}$ any more.

The effective action for external probes $A^{\mu\nu}_{\text{irr}}$, which is obtained by the integration over internal fields $a_{\mu}^{\uparrow}, a_{\mu}^{\downarrow}$ and $A_0^{\text{in}}$ has the same form as Eq. (2.13). Now the density-density correlations among different spin species $K_{\mu\nu}^{00}$, are obtained (for small $q^2$) as below:
\[
K_{\uparrow\uparrow}^{00} = \frac{q^2}{\Pi_0^{\uparrow} + \Pi_0^{\downarrow}} \left[ \Pi_0^{\uparrow} \Pi_0^{\downarrow} - \frac{\Pi_0^{\uparrow} \theta_+^2}{\left(\Pi_0^{\uparrow} + \Pi_0^{\downarrow}\right)^2} \left(\Pi_0^{\uparrow} + \Pi_0^{\downarrow} + \theta_+^2\right)^2 \right] + \mathcal{O}(q^2) ;
\] (232a)
\[
K_{\downarrow\downarrow}^{00} = \frac{q^2}{\Pi_0^{\uparrow} + \Pi_0^{\downarrow}} \left[ \Pi_0^{\uparrow} \Pi_0^{\downarrow} - \frac{\Pi_0^{\downarrow} \theta_+^2}{\left(\Pi_0^{\uparrow} + \Pi_0^{\downarrow}\right)^2} \left(\Pi_0^{\uparrow} + \Pi_0^{\downarrow} + \theta_+^2\right)^2 \right] + \mathcal{O}(q^2) ;
\] (232b)
\[
K_{\uparrow\downarrow}^{00} = K_{\downarrow\uparrow}^{00} = -\frac{q^2}{\Pi_0^{\uparrow} + \Pi_0^{\downarrow}} \left[ \Pi_0^{\uparrow} \Pi_0^{\downarrow} + \frac{\Pi_0^{\uparrow} \theta_+^2}{\left(\Pi_0^{\uparrow} + \Pi_0^{\downarrow}\right)^2} \left(\Pi_0^{\uparrow} + \Pi_0^{\downarrow} + \theta_+^2\right)^2 \right] + \mathcal{O}(q^2) ,
\] (232c)
in the limit $\theta_+ \to \infty$ ($s_+ = 0$) and $\lim_{q^2 \to 0} V(q^2)q^2 = 0$.

The generating functional, for equal-time density-density correlations among mixed spins, is given by Eq. (2.13) with the following $\mathcal{K}$ matrix:
\[
\mathcal{K} = \frac{q^2}{2\pi} \left(\frac{e^2}{2}\right) \frac{1}{(p_{\uparrow} + p_{\downarrow})} \begin{bmatrix}
 p_{\uparrow} p_{\downarrow} + \frac{p_{\uparrow}^2}{2(n(p_{\uparrow} + p_{\downarrow})+1)} & -p_{\uparrow} p_{\downarrow} + \frac{p_{\uparrow} p_{\downarrow}}{2(n(p_{\uparrow} + p_{\downarrow})+1)} \\
 -p_{\downarrow} p_{\downarrow} + \frac{p_{\uparrow} p_{\downarrow}}{2(n(p_{\uparrow} + p_{\downarrow})+1)} & p_{\downarrow} p_{\downarrow} + \frac{p_{\uparrow}^2}{2(n(p_{\uparrow} + p_{\downarrow})+1)}
\end{bmatrix}
\] (224)

III. WAVE FUNCTIONS

Using the same procedure as employed by Lopez and Fradkin [2], we write the square of the modulus of the ground state (non-degenerate) wave function for QH states of mixed spin as
\[ |\Psi[\rho_\uparrow, \rho_\downarrow]|^2 = \int [dA_0^\uparrow][dA_0^\downarrow][A_0^\uparrow, A_0^\downarrow] \exp \left[ -ie \int \frac{d^2q}{(2\pi)^2} A_0^\uparrow(q) \delta\rho_r(-q) \right], \tag{3.1} \]

where \( \delta\rho_r(q) \) is the Fourier transform of the density fluctuation

\[
\delta\rho_r(X) = \sum_{i=1}^{N_r} \delta(X - X_i^r) - \rho_r,
\tag{3.2}
\]

where \( N_r \) is the number of electrons with spin index \( r (= \uparrow, \downarrow) \) and \( \rho_r \) is the corresponding mean density. Now the integrations over \( A_0^\uparrow \) and \( A_0^\downarrow \) in Eq. (3.1) yield

\[
|\Psi[\rho_\uparrow, \rho_\downarrow]|^2 = \exp \left[ \frac{e^2}{2} \int \frac{d^2q}{(2\pi)^2} \delta\rho_r(q) K_{rr'} \delta\rho_{r'}(-q) \right], \tag{3.3}
\]

where \( K_{rr'} \) are given by Eqs. (2.17 and 2.24) for two different cases.

### A. Unpolarized States

For the unpolarized states, both up and down spins are equally populated. Therefore, \( \rho_\uparrow = \rho_\downarrow = \rho/2 \) and \( N_\uparrow = N_\downarrow = N/2 \) where \( N \) is the total number of particles. Transforming Eq. (3.3) back into the real space and using Eq. (2.17) for this case, we find

\[
|\Psi[\rho_\uparrow, \rho_\downarrow]|^2 \equiv \exp \left[ \frac{e^2}{2} \int \frac{d^2q}{(2\pi)^2} \delta\rho_r(q) K_{rr'} \delta\rho_{r'}(-q) \right], \tag{3.4}
\]

where \( K_{rr'} \) are given by Eqs. (2.17 and 2.24) for two different cases.

\[
|\Psi[\rho_\uparrow, \rho_\downarrow]|^2 \equiv \exp \left[ \frac{e^2}{2} \int \frac{d^2q}{(2\pi)^2} \delta\rho_r(q) K_{rr'} \delta\rho_{r'}(-q) \right], \tag{3.5}
\]

where \( R \) is the radius of the system which serves as a long-distance cut-off such that the magnetic length and inter-electron distance are much less than \( R \). Thus, from Eqs. (3.3 and 3.4), we obtain the modulus square of the wave function for the ground state of filling fraction (2.8),

\[
|\Psi \left( X_1^\uparrow, \cdots, X_{N/2}^\uparrow, X_1^\downarrow, \cdots, X_{N/2}^\downarrow \right)|^2 = \prod_{i<j}^{N/2} \left| X_i^\uparrow - X_j^\downarrow \right|^{2(2sp+1)/p} \times \prod_{k<l}^{N/2} \left| X_k^\downarrow - X_l^\uparrow \right|^{2(2sp+1)/p} \prod_{i,k}^{N/2} \left| X_i^\uparrow - X_k^\downarrow \right|^{2(2s)} \times \exp \left[ -\frac{1}{2\ell_0^2} \left( \sum_{i=1}^{N/2} \left| X_i^\uparrow \right|^2 + \sum_{k=1}^{N/2} \left| X_k^\downarrow \right|^2 \right) \right], \tag{3.5}
\]
where $X_i^\uparrow(X_i^\downarrow)$ represents the co-ordinate of i-th spin-up (down) particle and $l_0 = (eB)^{-1/2}$ is the magnetic length.

Eq. (3.5) reproduces the wave functions for unpolarized QH states at filling fractions (i) $\nu = 2(s = 0, p = 1)$ and, (ii) $\nu = \frac{2}{3}(s = 1, p = 1)$ as proposed by Halperin [8]. It agrees with the trial wave functions proposed by Jain et al. [4,15]. However, the wave functions differ (except the state $\nu = \frac{2}{3}$) from the trial wave functions for the sequence $\nu = 2n/(3n + 2)$, proposed by Belkhir and Jain [16]. Note that while the infra red cut-off is required to capture the exponential part, the Jastrow part of the wave function is obtained unambiguously. Clearly, the wave functions are analytic for all states with $\nu = 2p/((4sp + 1)$ for $p = \pm 1$. On the other hand, all the states with finite $p$ are non-analytic. However, the even denominator states ($p = \infty$) are again analytic in nature. The reason behind analyticity/non-analyticity will be discussed below. Interestingly, for all the integer states ($s = 0$), the exponent of the Jastrow form $|X_i^\uparrow - X_k^\downarrow|$ vanishes. Therefore the model predicts that the particles with unlike spins are uncorrelated for the integer states.

B. Partially/Fully Polarized States

Partially polarized states have unequal population in spin species. As discussed earlier, for small Zeeman energy, let $p_\uparrow = p_\downarrow + 1 = p$ (say). Hence, $\rho_\uparrow = (p/(2p - 1))\rho$, $\rho_\downarrow = ((p - 1)/(2p - 1))\rho$, $N_\uparrow = (p/(2p - 1))N$ and $N_\downarrow = ((p - 1)/(2p - 1))N$. Using the same procedure as discussed for unpolarized states in section (III-A), we obtain the wave function for partially/fully polarized states from Eqs. (2.24, 3.2 and 3.3),

$$|\Psi(X_1^\uparrow, \cdots, X_{N_\uparrow}^\uparrow, X_1^\downarrow, \cdots, X_{N_\downarrow}^\downarrow)|^2 = \prod_{i<j}^{N_\uparrow} |X_i^\uparrow - X_j^\downarrow|^{2(2sp+1)/p} \times \prod_{k<l}^{N_\downarrow} |X_k^\downarrow - X_l^\uparrow|^{2(1+2s(p-1))/(p-1)} \prod_{i,k}^{N_\uparrow,N_\downarrow} |X_i^\uparrow - X_k^\downarrow|^{2(2s)} \times \exp \left[ -\frac{1}{2l_0^2} \left( \sum_{i=1}^{N_\uparrow} |X_i^\uparrow|^2 + \sum_{k=1}^{N_\downarrow} |X_k^\downarrow|^2 \right) \right]. \quad (3.6)$$

It is clear that the wave functions are non-analytic for $p > 1$, which is the case for partial
polarization. However, for \( p = 1 \) which essentially gives fully polarized Laughlin sequence [4], (3.6) is in agreement with the result of Lopez and Fradkin [3] for spinless system.

C. Source of Analyticity/non-analyticity

Let us write the Jastrow part of the wave function for unpolarized states (3.5) in the form,
\[
|\Psi|^2 \approx \prod_{i<j}^{N/2} |X_i^\uparrow - X_j^\uparrow|^2((1/\theta)+(1/\Pi_1^\uparrow))e^2/(2\pi) \prod_{k<l}^{N/2} |X_k^\downarrow - X_l^\downarrow|^2((1/\theta)+(1/\Pi_1^\downarrow))e^2/(2\pi)
\times \prod_{i,k}^{N/2} |X_i^\uparrow - X_k^\downarrow|^2(1/\theta)e^2/(2\pi),
\] (3.7)

where \( \Pi_1^{\uparrow,\downarrow} \) are evaluated at \( \omega = 0, q^2 = 0 \). Note that the exponents of the Jastrow forms are determined by the parity and time reversal violating factors of the effective action given by Eqs. (2.10, 2.11). \( 1/\theta \) is always an even integral multiple of \((2\pi/e^2)\), and depending on the effective filling factor \( (p) \), \( 1/\Pi_1^{\uparrow,\downarrow} \) should either be an integral or fractional multiple of \((2\pi/e^2)\). Therefore the wave function becomes analytic when \( 1/\Pi_1^{\uparrow,\downarrow} \) is an integral multiple of \((2\pi/e^2)\), i.e., only for \( p = \pm 1 \), while for other values of \( p \), \( 1/\Pi_1^{\uparrow,\downarrow} \) is fractional multiple of \((2\pi/e^2)\) and hence the wave function becomes non-analytic.

Indeed, the exponents describe the number of effective vortices associated with a particle which is seen by others and therefore the exponents of the Jastrow form between like spin particles differ from the same between unlike spins. It is natural that \( \Psi \) should reflect the nature of vortices associated with the fermion. \( \Psi \) is determined from the density-density correlations which represent, in fact, the change in local density of the system and hence the change in CS magnetic field. This causes a change in the local current which is represented by the vortices.

A similar argument for analyticity/non-analyticity also holds for partially/fully polarized QH states.
We have introduced the external probes $A^\uparrow_\mu$, mainly for the computational purposes, viz., for determining the mixed spin density-density correlations which are used to determine the ground state wave functions of QH states. All the electro-magnetic responses of the system are determined by the physical electro-magnetic probe $A_\mu$ which couples to both the spins. It is not necessary to compute the response function de novo, since, the correlations found from the probes $A^\uparrow_\mu$ are related to that from the probe $A_\mu$. If we write

$$S_{\text{eff}}[A_\mu] = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} A_\mu(q) K^{\mu\nu} A_\nu(-q) ,$$

(4.1)

the electromagnetic response tensor $K^{\mu\nu}$ is related to $K^{\mu\nu}_{rr'}$ (which we have evaluated with the probes $A^\uparrow_\mu$), through

$$K^{\mu\nu} = \sum_{r,r'} K^{\mu\nu}_{rr'} .$$

(4.2)

Having thus determined $K^{\mu\nu}$, considering translational and gauge invariance, we write

$$K^{\mu\nu} = K_0(q^2 g^{\mu\nu} - q^\mu q^\nu) + (K_2 - K_0)(q^2 \delta^{ij} - q^i q^j)\delta^{\mu i} \delta^{\nu j} + iK_1 \epsilon^{\mu\nu\lambda} q^\lambda ,$$

(4.3)

where $K_0$, $K_1$ and $K_2$ are functions of $\omega$ and $q^2$.

The density-density correlation function can then be evaluated in the limit $q^2 \rightarrow 0$ as

$$K^{00}(\omega, q^2) \equiv -K_0 q^2 = -\left(\frac{e^2 \rho}{m^*}\right) \frac{q^2}{\omega^2 - \omega_c^2} + O((q^2)^2)$$

(4.4)

which is the same for both the cases, i.e., unpolarized and polarized states. Note that the pole occurring in the density-density correlation function is at $\omega = \omega_c$, which is the cyclotron frequency due to the applied field only. This is the well known as Kohn’s mode [21] which has got restored by the fluctuation of CS gauge fields over their mean values. Further, to the leading order in $q^2$, $K^{00}$ also saturates the $f$-sum rule as Lopez and Fradkin [22] have observed for the spinless case.

We may now obtain the Hall conductivity of the system to be
\[ \sigma_H \equiv K_1(0,0) = \begin{cases} \frac{2\Pi_1(0,0)\theta}{2\Pi_1(0,0) + \theta}, & \text{for unpolarized states} \\ \frac{(\Pi^+_1(0,0) + \Pi^-_1(0,0))\theta_1}{(\Pi^+_1(0,0) + \Pi^-_1(0,0)) + \theta}, & \text{for partially polarized states} \end{cases} \]  \tag{4.5}

Therefore \( \sigma_H = \nu(e^2/2\pi) \), which is quantized for the corresponding filling fractions \( \nu \) (2.8, 2.20).

V. CONCLUSION

In conclusion, we have determined the wave functions for arbitrarily polarized QH states within the doublet model [17] proposed recently. Our findings reduce to that of Lopez and Fradkin [2] for fully polarized states. We are able to recover the wave function proposed by Halperin [8] for \( \nu = \frac{2}{5} \). Our wave functions do not agree with those of Belkhir and Jain [10], except for \( \nu = \frac{2}{5} \). This disagreement is not surprising since the sequence for unpolarized states obtained here is different from the sequence \( \nu = 2n/(3n + 2) \) employed by Belkhir and Jain [10] to write their wave functions. We have also confirmed by an explicit One-loop computation that the Hall conductivity does indeed get quantized at those filling fractions that follow from the model. Finally, we have given a physical picture for the non-analytic nature of the wave functions, and shown that quantum fluctuations restore the Kohn mode. It would be interesting if spin correlations are measured and compared with the findings here. It would be equally interesting if the other states corresponding to two gaps of excitation are also discovered experimentally. One simple way of identifying them would be by measuring the activation in diagonal resistivity. Finally, there remains the possibility of multilayered arbitrarily polarized QH states. A study of such systems would require a fusion of the model used here with that of Lopez and Fradkin [23] which discusses double layered spinless fermion (fully polarized) systems. We believe that this fusion might well be possible since the two models are so very similar to each other.
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We thank the referee for bringing the paper of Lopez and Fradkin [23] to our notice.

APPENDIX: COMPARISON WITH BILAYERED SYSTEMS

Recently Lopez and Fradkin (LF) [23] have studied QH effects in bilayered systems with complete spin polarization. There is a close resemblance between their approach with the one we have taken here: The Lagrangians are formally the same, with a fermion doublet (while there is layered index in LF, we have the spin index here) and a matrix valued CS strength. In bilayered system the interlayer and intralayer interaction potentials are different, unlike in the case of spin-1/2 fermions for which the interaction potential does not depend on spin. However, this aspect is not important as long as we consider short-range interactions.

The crucial difference, in fact, is in the (physical) choice of Θ. In their work, LF [23] have chosen the Θ matrix as

\[
\Theta_{\text{LF}} = \frac{e^2}{2\pi} \frac{1}{4s_1s_2 - n^2} \begin{pmatrix} 2s_2 - n \\ -n & 2s_1 \end{pmatrix},
\]

where \(s_1, s_2\) and \(n\) are integers. Clearly \(\Theta_{\text{LF}}\) has more parameters compared to our choice of Θ in Eq. (2.4) since LF have assumed more generically that the particles in two different layers feel unequal number of flux quanta which is, in fact, not the case for spin-1/2 fermions in a single layer. In contrast, our case corresponds to \(s_1 = s_2\). Note that this choice is a strict requirement that the parity operation transforms the up-spin to down-spin electron and vice versa. There is no such requirement in the LF case. More importantly, note that \(\Theta_{\text{LF}}\) is ill-defined for \(s_1 = s_2 = n/2\). As a consequence, there are fundamental differences in results and interpretation in the two approaches which we discuss below.

Consider the spin unpolarized (singlet) state first. The corresponding sequence of states obtained by LF [23] for equal population in the two layers is identical to Eq. (2.8) here for the choice of \(s_1 = s_2 = n/2\) in Eq. (A1). A closer look however shows that it is precisely for...
these states, (characterized by the filling fractions $\nu_1 = \nu_2$ in the two layers and the number of particles $N_1 = N_2$ in two layers in Ref. [23]) that the CS strength $\Theta_{LF}$ becomes ill-defined. The ensuing dynamics is also ill-defined. Indeed as we have quoted earlier, LF [23] point out in their paper “the spin singlet state (3,3,2), which has filling fraction $\nu = 2/5$, cannot be described within the Abelian Chern-Simons approach”. In fact, no spin unpolarized states given by filling fraction in (2.8), can be described in the approach of LF [23]. In contrast, our $\Theta$ is given by

$$\Theta = \frac{e^2}{2\pi} \frac{1}{4s} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

(A2)

corresponding to $\theta_1 = \theta_2$, with eigen values $\theta_+ = (e^2/2\pi)(1/2s)$ and $\theta_- = 0$, which keeps the composite fermion picture intact. This well defined matrix naturally leads to unpolarized states (See Ref. [17]). The dynamics is also well defined, allowing us to obtain quantization of Hall conductivity (1.5) and many-body wave functions (3.5) for these states. Again, as we have stated earlier, we recover Halperin wave function [8] for spin unpolarized $\nu = 2/5$ state. Note that since all the elements in $\Theta$ in Eq. (A2) are same, the flux seen by all the particles are same irrespective of their spins, unlike $\Theta_{LF}$ for which flux seen by particles in the same layer are different from particles in different layers.

In our approach, $\nu = 1/m$ ($m$ odd) states (which are the Laughlin sequence) are always fully polarized (see Eq. 2.20). This, in fact, is true as it is seen both in the experiment [12] and numerical calculations [13] in a single layer. Contrarily in the work of LF [23], these states can also be obtained for equal population of particles in the two layers corresponding to filling fractions $\nu_1 = \nu_2 = 1/2m$ (where 1 and 2 refer to two different layers). The wave functions for fully polarized $\nu = 1/m$ states are well known Laughlin wave functions which is also derived here (See Eq. (3.6) for fully polarized limit). On the other hand, LF have obtained the wave functions for $\nu = 1/m$ states, in their above construction, as $(m,m,m)$ Halperin wave functions with an extra non-analytic piece (which we do not encounter) due to the presence of a gapless mode in the spectrum of collective excitations for these states.
The $\nu = 1/2$ state in bilayered systems corresponds to the assignment of filling fractions $\nu_1 = \nu_2 = 1/4$ in two layers which yield the gaps $\bar{\omega}_c^{1,2} = \omega_c/4$. With their choice, LF obtain the wave function for which coincides with the one obtained by numerical computation in double layers. On the other hand, the present model yields $\bar{\omega}_c^{\uparrow,\downarrow} = 0$. This is again closer to several experiments \cite{25–27} which have unambiguously verified that $\bar{\omega}_c = 0$ for $\nu = 1/2$ in a single layer.

It is of course not that the two models are entirely different in all aspects. Indeed, the double layered systems with dissimilar gaps in two layers are exact analogues of the spin systems with dissimilar gaps for spin up and down states because $\Theta_{\text{LF}}$ becomes identical to $\Theta$ here, provided one puts $s_1 = s_2$ in Eq. (A1). Note, however, that spin unpolarized states (which are seen experimentally \cite{9–12}) can never arise if $\bar{\omega}_c^{\uparrow} \neq \bar{\omega}_c^{\downarrow}$. Experimentally observed partially polarized or fully polarized states which are given by Eq. (2.19) also correspond to $\bar{\omega}_c^{\uparrow} = \bar{\omega}_c^{\downarrow}$. It will be interesting to observe experimentally whether there is any such states for which $\bar{\omega}_c^{\uparrow} \neq \bar{\omega}_c^{\downarrow}$ in which case there would be a one-to-one correspondence between the two models.

Finally, we remark that LF \cite{23} have also studied spin unpolarized states in a single layer by employing a non-abelian CS interaction. In their picture, electrons are composite of holons and spinons. Charged spinless holons interact with $U(1)$ CS gauge field where as neutral spin-1/2 spinons interact with $SU(2)$ CS gauge field. They both obey semionic statistics – which indicates a departure from composite fermion model (where spin and charge are not separated) to which we completely adhere.
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