Digital approximation to extended depth of field in no telecentric imaging systems

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Abstract. A method used to digitally extend the depth of field of an imaging system consists to move the object of study along the optical axis of the system and different images will contain different areas that are sharp; those images are stored and processed digitally to obtain a fused image, in that image will be sharp all regions of the object. The implementation of this method, although widely used, imposes certain experimental conditions that should be evaluated for to study the degree of validity of the image final obtained. An experimental condition is related with the conservation of the geometric magnification factor when there is a relative movement between the object and the observation system; this implies that the system must be telecentric, which leads to a reduction of the observation field and the use of expensive systems if the application includes microscopic observation. This paper presents a technique that makes possible to extend depth of filed of an imaging system non telecentric; this system is used to realize applications in Optical Metrology with systems that have great observation field.

1. Introduction

The optical systems that are constructed with conventional lens have limited depth of field and only the regions that are inside of depth of field (DOF) will appear in focus in the final image. Thus the regions of the object surface that are out of DOF will appear out of focus on the acquired image. Physical optics establishes that the diffraction pattern diameter of pupil imaging system defines the DOF of the imaging system. The diameter of this central spot, in experimental situations is of the order of tenths of a micron, while that the digital cameras have micro-sensor dimensions of some microns. Thus, diffraction-related effects as predicted by physical optics are not significant, since the spatial resolution of most sensor arrays is quite below that of the diffraction limit. It implicates that the geometrical optics defines the depth of field of a digital imaging system like CCD or CMOS. According to geometrical model of thin lenses, if the sensor plane does not coincide with the focal plane, the image of a punctual object will be a circle for a circular pupil of the imaging system and known as circle of confusion. The diameter of the circle confusion is calculated from the parameters of the optical system and the defocus distance, displacement from sensor plane to focal plane. Thus, the PSF can be approximated to a cylindrical function of brightness constant, filtering high frequency information. However, due to lens aberrations, diffraction and noise, the brightness decreases gradually to brim. Frequently is suggested a two-dimensional Gaussian function as PSF model of the optical system. In this way a defocused imaging system is considered as a low pass filtering system. Thus a blurry image have attenuated information of high frequency, without image details and
degraded image quality. As a result, limited DOF poses a significant problem when automatic analysis of images is desired. Several methods have been proposed to increment DOF of a classical imaging system. Basically the increment of the depth of field can be carry out: (1) physically adjusting the PSF of the optical system without post-capture digital treatment, (2) digitally making a digital treatment on acquired images and (3) a combination of the previous methods: physically introducing external devices to the traditional systems (platforms of movement, phase masks, etc) and computationally making a restoration operation through digital treatments. Several methods have been proposed to extended-DOF [1-7]. A method used frequently and used in the most commercial systems with E-DOF consists in to acquire several images for several positions on the optical axis. Thus if $N$ images of $m \times n$ pixels has been acquired, each pixel has an intensity vector of $N$ values corresponding to $N$ different axial positions.

In order to extract the intensity value corresponding to focused axial position for each pixel, a function or mathematical criterion must be defined to evaluate the focus quality, called focus measure. Several methods have been proposed to define the focus measure (local variance, gradient, wavelet transform, Fourier transform, etc.). All of them rely on the fact that a focused image has high contents of higher frequencies with higher local variations of intensity values. Thus, for each pixel the focus measure (FM) is evaluated and the intensity that produces the maximum value of FM is extracted and used to generate a fused image with E-DOF. The procedure described above for E-DOF imposes the condition that the $N$ intensity values correspond to the same surface point and imaged in the same pixel $(i,j)$ for each axial position. Thus, if the imaging system is non-telecentric, the magnification factor changes and the surface point is imaged in different pixels according to the axial position. Additionally a lateral displacement in the perpendicular direction of the optical axis produces the same problem: Intensity vector for pixel $(i,j)$ does not correspond to the same surface point on the object.

In this work we propose a new method to Extended DOF for a non-telecentric imaging system. The focus measure of a specific pixel is evaluated using the Tenengrad method with the appropriated correction in order to reduce the non-telecentric influence. A mathematical correction is proposed to extract from each image the coordinates in pixels that correspond to the same surface point.

2. E-DOF for a non-telecentric system

Figure 1 shows the images of a surface point projected by a non-telecentric system. In the figure $P_1$ has coordinates $(x,y,z)$ while $P_2$ has coordinates $(x,y,z+\Delta z)$ the images of these points appear located in $x'_1$ and $x'_2$ respectively; $P_1$ and $P_2$ correspond to same surface point for two axial positions. The relative displacement in the axial direction between object and optical system is $\Delta z$. It is evident the problem because the system is non-telecentric: the same surface point located on the object has two different positions in the image space, these points are $x'_1$ and $x'_2$ respectively. The solution of this problem is to find the corrected pixel position in each acquired images which corresponds to the same surface point and it is used to evaluate the FM. In the non-telecentric condition the lateral displacement between $x'_1$ and $x'_2$ depends on optical system parameters, object distance and $z$ value of surface point. A mathematical structure is proposed in order to eliminate the $z$ value influence using a first order approximation in the compute of $x'_1 - x'_2$ distance.

From figure 2 can be deduced that,

$$x'_i = \frac{vx}{u - x \tan \theta}. \quad (1)$$

Now for $x'_2$, that is the position of the point $P_2$ in the image space, is obtained,
\[ x'_n = \frac{uvx'_1}{uv - (n-1)\Delta z(x'_1 \tan \theta + v)} \]  
\[ y'_n = \frac{uvy'_1}{uv - (n-1)\Delta z(x'_1 \tan \theta + v)} \]  

The equations (3) and (4) enable to calculate the coordinates in the image space of a specific surface point for each acquired image in a non-telecentric system. The problem lies in these equations is the influence of unknown values \( z \) and \( \theta \). However if is take into account the approximation \( \theta_2 \approx \theta_1 \) and that \( \theta_2^a = \theta_1^a \) (see figure 1), then,

\[ \tan \theta_2^a = \frac{x_2^a - x}{z + \Delta z} \]  
\[ x_1 - x = z \tan \theta_1 \]  
\[ \tan \theta_1 = \frac{x_1}{u} \]  

It is possible to express \( x_2^a \) in terms of optical system parameters,
Moreover it is taking into account that  $x_{z}^{a} = x_{z}^{r} \frac{v}{u}$, is obtain that,

\[ x_{z}^{a} = \frac{v x_{1}}{u} \left( 1 + \frac{\Delta z}{u} \right). \]  

(8)

The equation (9) can be generalized as follows,

\[ x_{n}^{a} = \frac{v x_{1}}{u} \left[ 1 + (n-1) \frac{\Delta z}{u} \right]. \]  

(10)

Similarly for the $Y'$ coordinate,

\[ y_{n}^{a} = \frac{v y_{1}}{u} \left[ 1 + (n-1) \frac{\Delta z}{u} \right]. \]  

(11)

However taking into account that $x_{i}^{r} = \frac{v x_{1}}{u}$ and $y_{i}^{r} = \frac{v y_{1}}{u}$, the equations (13) and (14) can be expressed as

\[ x_{n}^{a} = x_{i}^{r} \left[ 1 + (n-1) \frac{\Delta z}{u} \right], \]  

(12)

and

\[ y_{n}^{a} = y_{i}^{r} \left[ 1 + (n-1) \frac{\Delta z}{u} \right]. \]  

(13)

The equations (12) and (13) enable to calculate the position of a specific pixel for each image acquired for a non-telecentric system knowing $\Delta z$.

3. Experimental evaluation

In order to evaluate the proposed equations a non-telecentric imaging system was implemented. A FUJINON HF35HA-1B objective of focal distance 35 mm and $f/1.6$ was used in a CCD camera iVC500 (640 pixels $\times$ 480 pixels, 16.5 $\mu$m $\times$ 16.5 $\mu$m). A translation stage of 10 $\mu$m precision was used to generate the relative displacement between object and CCD camera. An integrated circuit was used as test object. A set of 29 images with $\Delta z = 0.50$ mm was acquired. Figure 2 shows the acquired image for $z = 0.00$ mm.

Figure 3 shows the fused image E-DOF procedure using the Tenengrad method with a 15 pixels $\times$ 15 pixels window to calculate the focus measure. From visual analysis it is appreciated that the fused image quality is poor because it does not consider that a specific pixel does not has the same position in each acquired image. In the central zone of the image the influence of non-telecentric condition is appreciably smaller than exterior zone: The fused image is severely degraded in the exterior zone. Figure 4 shows the fused image using the proposed method with the Tenengrad focus measure for a
window size of 15 pixels × 15 pixels. From visual analysis it is evident that the quality image is augmented because equations (12) and (13) correct the lateral displacement introduce by the non-telecentric condition.

Figure 2. Acquired image for \( z = 0.00 \) mm.

Figure 3. Extended depth of field image using the Tenengrad method.

Figure 4. Extended depth of field image using the proposed method.

From the E-DOF procedure it is possible to compute the axial position that has the maximum value of the FM. This value is related to 3D information of surface object. Figure 5 shows the tridimensional
reconstruction. The $z$ value is obtained by using the $n$th position of the maximum focus measure for each $(x, y)$ pixel of the image.

4. Conclusions
In this paper has been proposed a method (based in the first order approximation) that enables to extend the depth of field of a non-telecentric imaging system. A non-telecentric imaging system has been designed and implemented in order to evaluate the performance of proposed method. The results obtained show that this method enables to extend the depth of field of a non telecentric imaging system. It is showed that is necessary to know certain optical parameters of the system. Additionally, it is possible to obtain tridimensional information of the study object.

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