B-Hadron Lifetimes, Width Differences and Semileptonic CP-Asymmetries

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B-hadron physics plays a fundamental role to test and improve our understanding of flavor dynamics within and beyond the Standard Model. Of particular phenomenological interest are beauty hadron lifetime ratios and, width differences and semileptonic CP-asymmetries in $B_d$ and $B_s$ systems. We discuss their theoretical predictions which, in the last years, have been improved thanks to accurate lattice calculations of operator matrix elements and perturbative computations of Wilson coefficients.

I. INTRODUCTION

Neutral $B_d$ and $B_s$ mesons mix with their antiparticles leading to oscillations between the mass eigenstates. The time evolution of the neutral meson doublet is described by a Schrödinger equation with an effective $2 \times 2$ Hamiltonian

$$i \frac{d}{dt} \langle B_q \rangle = \left[ \left( \frac{M_{12}^q}{M_{11}^q} M_{12}^q, M_{11}^q \right) - \frac{i}{2} \left( \frac{\Gamma_{11}^q}{\Gamma_{12}^q}, \frac{\Gamma_{12}^q}{\Gamma_{11}^q} \right) \right] \langle B_q \rangle.$$  \hspace{1cm} (1)

Mass and width differences are defined as $\Delta m_q = m_{11}^q - m_{12}^q$ and $\Delta \Gamma_q = \Gamma_{12}^q - \Gamma_{11}^q$, where $H$ and $L$ denote the Hamiltonian eigenstates with the heaviest and lightest mass eigenvalue. These states can be written as

$$|B_q^{L,H} \rangle = \frac{1}{\sqrt{1 + |(q/p)_q|^2}} \left( |B_q \rangle \pm \frac{(q/p)_q}{|q/p)_q|} |\overline{B}_q \rangle \right).$$  \hspace{1cm} (2)

The theoretical expressions of hadron lifetimes are related to $\Gamma_{11}$ ($\tau(B_q) = 1/\Gamma_{11}$), while width differences $\Delta \Gamma_q$ and semileptonic CP-asymmetries $A_{SL}^q$ are related to $M_{12}^q$ and $\Gamma_{12}^q$. In $B_{d,s}$ systems, the ratio $\Gamma_{12}^q/M_{12}^q$ is of $O(m_b^2/m_t^2) \approx 10^{-3}$. Therefore, by neglecting terms of $O(m_b^2/m_t^2)$, one can write

$$\Delta \Gamma_q = -2 |M_{12}^q| \text{Re} \left( \frac{\Gamma_{12}^q}{M_{12}^q} \right), \quad A_{SL}^q = \text{Im} \left( \frac{\Gamma_{12}^q}{M_{12}^q} \right).$$ \hspace{1cm} (3)

The experimental averages of the lifetime ratios of beauty hadrons are $[5]

$$\frac{\tau(B^+) \tau(B_s)}{\tau(B_{d,s})} = 1.076 \pm 0.008, \quad \frac{\tau(B_{d,s})}{\tau(B_{d,s})} = 0.957 \pm 0.027,$$

$$\frac{\tau(B_{d,s})}{\tau(B_{d,s})} = 0.84 \pm 0.05. \hspace{1cm} (4)$$

By applying the HQE, the inclusive decay width of a hadron $H_b$ can be expressed as a sum of contributions of local $\Delta B = 0$ operators with increasing dimension, as

$$\Gamma(H_b) = \sum_k \frac{\alpha_k(\mu)}{m_k^2} \langle H_b | \mathcal{O}_{k}^{\Delta B=0} (\mu) | H_b \rangle.$$

The HQE yields the separation of short-distance effects, confined in Wilson coefficients ($\alpha_k$), from long distance physics, represented by matrix elements of the local operators ($\mathcal{O}_{k}^{\Delta B=0}$).

Spectator contributions, which distinguish different beauty hadrons, appear at $O(1/m_t^2)$ in the HQE. These effects, although suppressed by an additional power of $1/m_t$, are enhanced with respect to the leading contributions by a phase-space factor of $16 \pi^2$, being $2 \to 2$ processes instead of $1 \to 3$ decays $[10, 11]$. In order to...
evaluate the spectator effects one has to calculate the matrix elements of dimension-six current-current and penguin operators, non-perturbatively, and their Wilson coefficients, in perturbation theory.

Concerning the perturbative part, NLO QCD corrections to the coefficient functions of the current-current operators have been computed \[12-14\].

Concerning the non-perturbative part, the non-valence contributions, corresponding to contractions of two light quarks in the same point, have not been computed. Their non-perturbative lattice calculation would be possible, in principle, however it requires to deal with the difficult problem of power-divergence subtractions. On the other hand, the valence contributions, which exist when the light quark of the operator enters as a valence quark in the external hadronic state, have been evaluated. For $B-$mesons, several (quenched) lattice studies \[15-17\] in QCD, HQET and NRQCD exist and yield compatible results, while for the $\Lambda_b$ baryon, only one (quenched) lattice calculation in HQET has been performed \[18\] so far.

More recently, the sub-leading spectator effects which appear at $O(1/m_b^4)$ in the HQE, have been included in the analysis of lifetime ratios. The relevant operator matrix elements have been estimated in the vacuum saturation approximation (VSA) for $B-$mesons and in the quark-diquark model for the $\Lambda_b$ baryon, while the corresponding Wilson coefficients have been calculated at leading order (LO) in QCD \[19\].

The theoretical predictions for the lifetime ratios read \[20\]

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.06 \pm 0.02, \quad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.88 \pm 0.05.$$

They turn out to be in good agreement with the experimental measurements of Eq. (3).

It is worth noting that the agreement at $1.2\sigma$ between the theoretical prediction for $\tau(\Lambda_b)/\tau(B_d)$ and its experimental value is achieved thanks to the inclusion of NLO (see Fig. 1) and $1/m_b$ corrections to spectator effects. They both decrease the central value of $\tau(\Lambda_b)/\tau(B_d)$ by 8% and 2% respectively.

Few months ago, however, the experimental picture was modified by new Tevatron measurements \[21, 22\]

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 1.041(57) \ [CDF], \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.870(102)(41) \ [D0].$$

(7)

The uncertainty of the preliminary D0 result is still too large for a significant comparison, while the CDF value is surprisingly higher ($\sim 2.5\sigma$) than the world average. Although the CDF result represents the single best measurement from fully reconstructed $\Lambda_b \to \Lambda^0 J\psi$ decay, one has to wait for experimental updates before interpreting the difference of the CDF result from theory as a $\Lambda_b$-puzzle (reversed w.r.t to the old one, as now the measurement is larger than the theoretical prediction).

On the theoretical side, further improvement of the $\tau(\Lambda_b)/\tau(B_d)$ theoretical prediction would require the calculation of the current-current operator non-valence $B-$parameters and of perturbative and non-perturbative contributions of the penguin operator, which appears at NLO and whose matrix elements present the same problem of power-divergence subtraction. These contributions are missing also in the theoretical predictions of $\tau(B^+)/\tau(B_d)$ and $\tau(B_s)/\tau(B_d)$, but in these cases they represent an effect of $SU(2)$ and $SU(3)$ breaking respectively, and are expected to be small.

III. WIDTH DIFFERENCES

The width differences between “light” and “heavy” neutral $B_q$-mesons ($q = d, s$) are defined in terms of the off-diagonal matrix elements $\Gamma_{i2}$ (see Eq. (3)).

In the HQE of $\Gamma_{i2}$, the leading contributions come at $O(1/m_b^2)$ and are given by dimension-six $\Delta B = 2$ operators. Up to and including $O(1/m_b^4)$ contribution, one can write

$$\Gamma_{i2} = \frac{G_F^2 m_b^2}{24\pi M_{B_q}} \left[ c^i_q(\mu_2)|B_q|O^2_q(\mu_2)|\bar{B}_q| + \frac{c^i_q(\mu_2)|B_q|O^2_q(\mu_2)|\bar{B}_q| + \delta_{i/m_b}^q \right],$$

(8)

where $|\bar{B}_q/O^2_q(\mu_2)|B_q|$ are the matrix elements of the two independent dimension-six operators

$$O^1_1 = \bar{b}_i \gamma^\mu (1 - \gamma_5) q_i \bar{b}_j \gamma^\mu (1 - \gamma_5) q_j, \quad O^2_2 = \bar{b}_i (1 - \gamma_5) q_i \bar{b}_j (1 - \gamma_5) q_j,$$

(9)

$c^i_q(\mu_2)$ their Wilson coefficients, known at the NLO in QCD \[23-25\], while $\delta_{i/m_b}$ represents the contribution of dimension-seven operators \[26\].

![FIG. 1: Theoretical (histogram) vs experimental (solid line) distributions of lifetime ratios. The theoretical predictions are shown at the LO (left) and NLO (right).](image-url)
Lattice results of the dimension-six operator matrix elements [27–31] have been confirmed and improved, by combining QCD and HQET results in the heavy quark extrapolation [32]. Moreover, the effect of including dynamical quarks has been examined, within the NRQCD approach, finding that these matrix elements are essentially insensitive to switching from \( n_f = 0 \) to \( n_f = 2 \) [33, 34] and to \( n_f = 2 + 1 \) [35].

Concerning the dimension-seven operators, their matrix elements have never been estimated out of the VSA. Two of these four matrix elements, however, can be related through Fierz identities and equations of motion to the complete set of operators studied in [32]. For the other two, a QCD-sum rule calculation is in progress [36].

The theoretical predictions given in [24] read

\[
\Delta \Gamma_{d}/\Gamma_{d} = (2.3 \pm 0.8) \cdot 10^{-3}, \quad \Delta \Gamma_{s}/\Gamma_{s} = (7 \pm 3) \cdot 10^{-2},
\]

compatible with the experimental averages [9, 37]

\[
\Delta \Gamma_{d}/\Gamma_{d} = (9 \pm 37) \cdot 10^{-3}, \quad \Delta \Gamma_{s}/\Gamma_{s} = (14 \pm 6) \cdot 10^{-2},
\]

within quite large uncertainties.

We note that, after the recent Tevatron measurements of \( \Delta M_{s} \) [38], both \( \Delta \Gamma_{d}/\Gamma_{d} \) and \( \Delta \Gamma_{s}/\Gamma_{s} \) can be theoretically obtained within the SM as

\[
\Delta \Gamma_{q}/\Gamma_{q} = -\text{Re} \left( \frac{\Gamma_{q}^{q}}{M_{12}^{q}} \right) (\Delta M_{q})_{\text{exp}} \cdot \tau(B_{q}),
\]

avoiding the quadratic dependence on the decay constants \( f_{B_{q}} \), whose lattice determinations have still an accuracy of about 15% [39]. In spite of that, the theoretical predictions in Eq. (10) present an uncertainty of \( \sim 40\% \), mainly due to strong cancellations coming from NLO and \( \mathcal{O}(1/m_{b}^{4}) \) contributions. The NLO corrections, indeed, decrease the values of both \( \Delta \Gamma_{d}/\Gamma_{d} \) and \( \Delta \Gamma_{s}/\Gamma_{s} \) by a factor two with respect to the LO predictions, as shown in Fig. 2. In addition, the \( \mathcal{O}(1/m_{b}^{4}) \) corrections result of comparable size relative to the leading \( \mathcal{O}(1/m_{b}^{2}) \) contributions and further decrease the width differences. The theoretical uncertainties in Eq. (10) turn out to be dominated by the VSA used for two \( \mathcal{O}(1/m_{b}^{4}) \) matrix elements, followed by the residual NNLO dependence on the renormalization scale.

It was recently observed [40] that the cancellations occurring at NLO and \( \mathcal{O}(1/m_{b}^{4}) \) are significantly reduced by expressing the width differences in the operator basis

\[
\mathcal{O}^{q}_{4} = \bar{b}_{i} \gamma^\mu (1 - \gamma_5) q_{j} \bar{b}_{j} \gamma^\mu (1 - \gamma_5) q_{i}, \\
\mathcal{O}^{q}_{3} = \bar{b}_{i} (1 - \gamma_5) q_{j} \bar{b}_{j} (1 - \gamma_5) q_{i},
\]

instead of the basis \( \{ \mathcal{O}^{q}_{1}, \mathcal{O}^{q}_{2} \} \) in Eq. (9). In the new basis, both NLO and \( \mathcal{O}(1/m_{b}^{2}) \) turn out to be smaller and have a reduced impact on the final uncertainties. Moreover, the Wilson coefficient of the operator \( \mathcal{O}_{1}^{q} \) in the new basis is about ten times larger than in the old basis. This is a very welcome feature as the \( \mathcal{O}^{q}_{1} \) contribution is the cleanest one, due to the cancellation of the corresponding \( B \)-parameter in the ratio \( \Gamma_{12}^{d}/M_{12}^{d} \). As a consequence, the theoretical predictions for the width differences that in the new basis read

\[
\Delta \Gamma_{d}/\Gamma_{d} = (4.1 \pm 0.5) \cdot 10^{-3}, \quad \Delta \Gamma_{s}/\Gamma_{s} = (13 \pm 2) \cdot 10^{-2},
\]

present relative uncertainties reduced by more than a factor two with respect to the old basis (see Eq. (10)). One has to note, however, that the significant shifts of central values due to the change of basis signal important \( \mathcal{O}(\alpha_{s}/m_{b}^{2}) \) corrections. Therefore, we quote as updated theoretical predictions the weighted averages of the results obtained in the two bases, and include in the uncertainty the effects signaled by the shifts

\[
\Delta \Gamma_{d}/\Gamma_{d} = (3.6 \pm 1.0) \cdot 10^{-3}, \quad \Delta \Gamma_{s}/\Gamma_{s} = (11 \pm 4) \cdot 10^{-2}.
\]

They result to be in good agreement with experimental data, as shown for \( \Delta \Gamma_{s}/\Gamma_{s} \) in Fig. 3.

![FIG. 2: Theoretical distributions for \( B_{d} \) and \( B_{s} \) width differences, at the LO (light/red) and NLO (dark/blue). The distributions are those obtained in the (old) basis \( \{\mathcal{O}^{q}_{1}, \mathcal{O}^{q}_{2}\} \).](image)

![FIG. 3: Theoretical (histogram) vs experimental (solid line) distribution for \( \Delta \Gamma_{s}/\Gamma_{s} \). The theoretical distribution represents the weighted average between the predictions obtained in the old and new bases.](image)

**IV. SEMILEPTONIC CP-ASYMMETRIES**

The semileptonic CP-asymmetries \( A_{S}^{q} \) (\( q = d, s \)) that describe CP-violation due to mixing, are related to \( M_{12}^{q} \) and \( \Gamma_{12}^{q} \) through Eq. (8). The theoretical predictions of \( A_{S}^{q} \) are based on the same perturbative and non-perturbative calculations discussed in Sec. III while the \( V_{CKM} \) contributions are different from those in \( \Delta \Gamma_{q}/\Gamma_{q} \).

The theoretical predictions read [24]

\[
A_{S}^{d} = -(6.4 \pm 1.6) \cdot 10^{-4}, \quad A_{S}^{s} = (2.7 \pm 0.6) \cdot 10^{-5}.
\]

The corresponding theoretical distributions are shown in Fig. 4 with an evident effect of NLO corrections.
First, still very uncertain, measurements are now available \[3, 41\]

\[A_{SL}^0 = -(30 \pm 78) \times 10^{-4},\]
\[A_{SL}^5 = (2450 \pm 1930 \pm 350) \times 10^{-5}.\]

FIG. 4: Theoretical distributions for $B_d$ and $B_s$ semileptonic CP-asymmetries, at the LO (light/red) and NLO (dark/blue).

Improved measurements are certainly needed for a significant comparison. On the theoretical side, $O(\alpha_s^2)$ and $O(\alpha_s/m_t^4)$ contributions are expected to be small in this case, since the change of basis that had an important impact for width differences has practically no effect here.

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