The Effect of Various Types of Constant and Time Dependent Heating on Human Tissue: A Finite Element Approach

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ABSTRACT

Cancer is one of the most leading causes of death now worldwide. Most of the cancer therapy aims to raise the temperature of the cancerous tissue above a therapeutic value and thermally kill or destroy it. Minimizing the damage of the healthy cells surrounding the infected cells is one of the major concerns of these therapies. Precise acknowledgment of the temperature profile of living tissue during therapy is of utmost necessity for this purpose. Towards that direction, this paper presents an unsteady finite element model of the bioheat equation to analyze the temperature distribution during the thermal therapy. A C language based system has been developed to solve the unsteady part of the problem employing Crank-Nicolson method and to solve the linear problem employing the Gauss elimination technique. Using this system, we investigate thermal behaviors in human tissues subjected to constant, sinusoidal spatial and surface, point, and stochastic heating. It was found that surface heating is beneficial for treating skin surface cells, while laser heating for the cells that lie below the skin surface. Moreover, for deep cell, the point heating style can bring the most desirable outcome. Results describe in this paper could be useful for researchers and doctors to optimize the treatment procedure, even protocols.

Keywords: FEM; Bioheat Transfer; Human Tissue; Pennes Equation; Spatial Heating.
equations can meet the purpose. The objective of this research is to develop a one dimensional finite element code for the solution of both steady and transient bio-heat equation. The popular Crank-Nicolson method was used in time discretization of the transient analysis. The developed finite element code was used to simulate the thermal response of tissue during cancer hyperthermia, laser surgery, tissue heating with a hot disk, and point heating. Moreover, time-dependent spatial and surface heating were incorporated. The effect of surface heating, step heating, sinusoidal heating, and point heating was thoroughly investigated. The temperature profile for all cases is found with valuable information for the physicians and researchers.

2. Bio Heat Transfer

For the study of bio-heat transfer in human tissue, the most useful one is Pennes equation which can be expressed as

$$ \frac{\partial q}{\partial x} + \omega_b \rho_b c_b (T_a - T) + Q_m + Q_r = \rho c \frac{\partial T}{\partial t} $$

(1)

For steady state case the Eq. 1(a) is reduced as

$$ \frac{\partial q}{\partial x} + \omega_b \rho_b c_b (T_a - T) + Q_m + Q_r = 0 $$

(2)

Where $\rho$, c, k are respectively the density, the specific heat, and the thermal conductivity of the tissue; $\rho_b$, $c_b$ denote density, and specific heat of blood, respectively. The $\omega_b$ is the blood perfusion, $T_a$ the known arterial temperature, and $T(x,t)$ is the unknown tissue temperature. Where $Q_m$ is the metabolic heat, and $Q_r(x,t)$ is the heat source due to spatial heating.

Let the one dimensional problem of length L, where the skin surface is defined at $x = 0$ and the body core at $x = L$. The constant body core temperature is defined as $T_c$, $h_o$ is the ambient heat convection coefficient between the skin surface and the surrounding air, and $T_a$ is the ambient temperature. At the skin surface ($x = 0$), the temperature $T(x, t)$ is the unknown tissue temperature. Where $Q_m$ is the metabolic heat, and $Q_r(x,t)$ is the heat source due to spatial heating.

In section 3.2, 3.3, 3.4, and 3.5 at $x=0$. Where the force boundary condition Eq. (5) is used at $X=L$, the temperature boundary condition (3), (5) and (6) is used. When the value of $\alpha$ is considered 0.5, the process is called the popular Crank-Nicolson method. The discretized Eq. (11) can be written as:

$$ \frac{C \frac{T^m+1-T^m}{\Delta t} + \alpha KT^n+1 + (1-\alpha)KT^n}{Q} = \frac{C \frac{1}{\Delta t} + \alpha K}{T^n + Q + q} $$

(12)

The Eq. (12) was solved using an iterative procedure. The initial temperature is known and then the temperature of the next step is calculated from the solution of Eq. (12) through the Gauss elimination technique.

2.3 Boundary Conditions and Input Parameters

Throughout the study at $X=L$, the temperature boundary condition is used.

However, depending upon the types of heating boundary condition (3), (5) and (6) is used at $X=0$.

In section 3.2, 3.3, 3.4, and 3.5 at $X=0$. Where the force boundary condition Eq. (5) is used in sections 3.6 and 3.7. The input parameters used in this study is summarized in Table 1 [12].
### Table 1 Input Parameters.

| Parameters                                | Value          |
|-------------------------------------------|----------------|
| Thermal conductivity (k)                  | 0.5 w/m²       |
| Convection Coefficient (h_o)              | 10 w/m²        |
| Forced convection coefficient (h_f)       | 100 w/m²       |
| Environmental Temperature (T_0)           | 25 °C          |
| Temperature of the Artery (T_a)           | 37 °C          |
| Body core temperature (T_c)               | 37 °C          |
| Metabolic heat generation (Q_m)           | 33800 w/m²     |
| Density of blood (ρ_b)                    | 1000 kg/m³     |
| Density of tissue (ρ)                     | 1000 kg/m³     |
| Specific heat of blood (c_b)              | 4200 J/kg.°C   |
| Specific heat of tissue (c)               | 4200 J/kg.°C   |
| Blood perfusion (ω_b)                     | 0.0005 ml/s/ml |

#### 3. Results and Discussion

For the simple thermal analysis, from the skin surface to tissue body is enough to consider. So to avoid the computational complexity, a 1D tissue of length (L) of 30 mm is considered as a computational model.

**3.1 Code Verification**

Fig. 1 shows a comparison between numerical result and the analytical result obtained from [12] for the steady state case considering the Eq. (3-6). The comparison shows a better agreement. The boundary conditions are defined as describe in Eq. (3) and Eq. (4).

![Fig. 1 Comparison with analytical solution.](image1)

In case of heating by laser and microwave, the heat absorption rate can simply be approximated by Beer's Law, which can be expressed as \( Q_r = \eta P_0(t)e^{-\eta x} \) in which heat flux decays exponentially with respect to distance from the skin surface [13]-[15]. Here \( P_0(t) \) is the time-dependent heating power on the skin surface, and \( \eta \) is the scattering coefficient. Since \( P_0(t) \) and \( \eta \) vary from one apparatus to another, so it is important to know the influence of these parameters on tissue temperature. \( P_0(t) \) can be either constant and time-dependent. In our study, we have considered both cases.

**3.2 Spatial Heating**

Laser and microwave therapy are some of the most widely used non-invasive techniques to destroy malignant cells. In this section, we aim to know the temperature distribution of human tissue during heating by laser or microwave.

![Fig. 2 Temperature distribution at different times; (η= 200 m⁻¹ , (a)P_0(t)=250 W/m², (b) P_0(t)=250+200cos(0.02t) W/m²).](image2)

Fig. 2 depicts the transient temperature at different times when tissues subject to two different spatial heating. Fig. 2 (a) shows the case of constant spatial heating and while Fig. 2 (b) is for sinusoidal spatial heating. In both cases, at the early stage, the tissue temperature increases along with the distance from the skin surface due to external heating, but later it decreases towards the body core. Moreover, there is an inter-cross...
between temperature curves at different times in Fig. 2 (b), which indicates the oscillation of the temperature inside the tissue due to sinusoidal spatial heating. These figures also reveal that the temperature within the tissue increases along with time and finally reached the steady state condition. Here the maximum temperature is about 47°C, and that lies 7 mm below the skin surface.

3.3 Effect of Scattering Co-efficient

Fig. 3 shows the effect of the scattering coefficient, where Fig. 3 (a) shows the result for constant heating, and Fig. 3 (b) depicts the result for sinusoidal heating. In both cases, the larger coefficient results in a higher temperature. Moreover, in the case of sinusoidal spatial heating, the larger coefficient returns higher amplitude. Fig. 3 (b) indicates the sinusoidal effect. Fig. 3 (a) shows that after about 3000 seconds (approximately), tissue temperature begins to stabilize.

3.4 Surface Heating

Heating with a hot plate or pad is a traditional approach to retain from pain. Depending upon the temperature and thermal properties of heating disk, this approach can be used for cell repair or to destroy affected cells. In this section, we analysed the thermal behaviour of living tissue subjected to time-dependent surface heat flux. Both constant and step heating are considered in this study. In constant heating, human tissue is heated with a heating pad at a constant rate. In step heating after heating for a certain period heat source is removed and allows it to cool. Results are calculated at different heat flux and time. Here Eq. 3(a) is used for skin surface boundary conditions.

\[ \text{Constant Heating:} \]

The calculated tissue temperature for constant surface heating is shown in Fig. 4 (a) at different heat flux. And skin temperature at different times, along with the distance from the skin surface, is shown in Fig. 4 (b). From Fig. 4 (a) higher heat flux results in a higher temperature, and temperature increases as time increases. At the early stage, temperature increases rapidly, but as time increases, increasing rate decreases and tends to be stabilized. From Fig. 4 (b) it is clear that temperature decreases towards the body core.
**Step Heating:** In this case, after heating for 1200 seconds the heat source is removed. In this section, temperature distribution at three different locations is obtained over time. It is very useful in eye surgery via a single laser pulse due to a flash fire, heating using a hot plate for a short period of time [10]. The heating power used in this particular type of investigation is expressed as

\[
Q(t) = \begin{cases} 
1000 \text{ W/m}^2, & t \leq 1200 \text{ s} \\
0 \text{ W/m}^2, & t > 1200 \text{ s} 
\end{cases}
\]

(13)

The transient temperature at three different locations of the skin is shown in **Fig. 5**. Where \( Q_r = 0 \), in **Fig. 5**. The result also carried out for two different value of blood perfusion \( \omega_b = 0.0005 \text{ ml/s/ml} \) and \( \omega_b = 0.004 \text{ ml/s/ml} \).

![Fig. 5 Transient temperature at three positions (Q_r=0); (a) \( \omega_b = 0.0005 \text{ ml/s/ml} \); (b) \( \omega_b = 0.004 \text{ ml/s/ml} \).](image)

Both figures show that as time increases, temperature also increases, but after 1200 seconds when the heat source is removed, tissue temperature decreases as time passes. Moreover, these figures show us the effect of blood perfusion in surface heating. Also the higher blood perfusion results in lower temperature and quick temperature loss (after 1200 seconds when \( Q(t) = 0 \)). This happens as a higher blood flow rate carried away excess heat. Such information is valuable in thermal comfort analysis. In practice, the temperature of the surrounding fluid temperature and duration should be in the safe range. A high temperature or long durable process may encounter pain, even burning of the skin.

### 3.5 Effect of Heating Frequency and Blood Perfusion

The calculated result for different heating frequency and blood perfusion is shown in **Fig. 6** subjected to sinusoidal surface heating. The sinusoidal heating at the skin surface can be expressed as

\[
Q(t) = q_0 + q_w \cos(\omega_1 t)
\]

(14)

Where \( q_0 \) and \( q_w \) are the constant terms, and the oscillation amplitude of sinusoidal heat flux and \( \omega_1 \) represents the heating frequency.

![Fig. 6 Effect of heating frequency and blood perfusion on sinusoidal surface heat flux.](image)

![Fig. 7 Different heating condition and its impact on skin surface temperature.](image)
In Fig. 6 curves A and B, we use blood perfusion as 0.0005, where in curve C & D it is 0.004. While in curve A and C, we use a heating frequency of value, 0.02 were in B & D, it is 0.01. From these figures, we can say that high blood perfusion results in lower temperatures where temperature response under two different heating frequencies almost negligible.

The calculated tissue temperature result subject to simultaneously surface and spatial heating is shown in Curve A of Fig. 7. While Curve B represents a single sinusoidal surface heating \((Q_r=0)\), and curve C represents only sinusoidal spatial heating \((f_1(t)=0)\). The applied surface and spatial heating are \(Q(t)=1000+500\cos(0.02t)\) W/m² and \(P_o(t)=250+200\cos(0.02t)\) respectively.

However, Fig. 8 illustrates the impact of frequencies of surface heating. Here the applied surface and spatial heating are \(Q(t)=1000+500\cos(0.02t)\) W/m² and \(P_o(t)=250+200\cos(0.01t)\) respectively. In curve A, we applied simultaneously sinusoidal surface and spatial heating. Curve B represents a single sinusoidal surface heating \((Q_r=0)\) when curve C represents only spatial heating \((f_1(t)=0)\).

![Fig. 8 Temperature distribution at different heating frequency.](image)

In curve A of Fig. 8, the frequency of surface heating and spatial heating was the same, and thus, due to the same frequency, the resultant temperature appears having the same frequency as external heating. However, in curve A of Fig. 8, different heating frequency was applied to spatial and surface heating; that’s why irregular frequency has appeared in tissue temperature.

3.6 Impact Forced Convection Boundary Condition

In this section, we concentrate on temperature profiles under different kinds of surrounding medium classified by their temperature. Fig. 9 depicts the tissue temperature distribution under different cooling medium temperature. Here force cooling significantly reduces the skin surface temperature. Moreover, lower cooling medium temperature results in lower skin surface temperature. However, the effect of forced cooling is negligible for the deep tissues, as shown in Fig. 9, the temperature of the cells over approximately \(x=12\) mm line remains changeless for different cooling temperature.

![Fig. 9 Influence of cooling medium temperature on tissue temperature.](image)

Using a cooling medium on the skin surface may be a good approach during hyperthermia treatment as it can reduce the skin temperature even below the core temperature, which may result in hypothermia. Hence, concentration should be given selecting proper cooling medium. Here the force convection coefficient of the cooling medium is considered as 100 W/(m². °C).

3.7 Point Heating

Treating deep tumors- located at kidney, lung, or rectum- it is very difficult to adopt surgical treatment. In such a case, point heating can be an alternative to surgery due to its ability to treat a tumor with a defined volume. In such a case, the total heating power is deposited at the tumor site with the help of a microwave probe, radio-frequency probe etc. In this heating type, the target region is heated more than 50°C within few minutes. This heating is very beneficial in the case of thermal ablation when a target tissue is destroyed, injecting thermal energy at the tumor site [5],[6]. There is an inverse relationship between elevated temperature and exposure duration. For the same amount of tissue necrosis, the high temperature needs low exposure duration. On the contrary, the low temperature needs high exposure duration. So for effective treatment, we need to know the required temperature and exposure duration precisely. Moreover, in some cases, to protect the skin surface cells from excess heat, the cooling medium is used on the skin surface during treatment, which is a very efficient approach to reduce the skin surface temperature. In this investigation, we calculate the tissue temperature at different times, cooling medium properties, and heating power. To deposit total heating power at the desired site, we use the expression of the external heating as [12],[13].

\[
Q_r(x,t)=P_1(t)\delta(x-x_0)
\]

\((15)\)
Where \( P_1(t) \) is the point, heating power, and \( \delta(x-x_0) \) is the Dirac delta function. It has a value 1 at our desired point \((x_0)\), and at all other points, its value is 0. That’s why it allows depositing total heating power at the tumor site. Where \( x_0 \) is the distance of tumor site from the skin surface. Here we consider the distance of the tumor site from the skin surface is 21 mm \((x_0)\). In this case, convection boundary condition is applied \((h=100 \text{ W/(m}^2 {\text{°C}}) \) and \(T_f=15 \text{ °C}) at the skin surface.

In Fig. 11 the temperature response under different temperatures of cooling fluid is analysed where in Fig. 12 influence of tissue temperature under different heating power is shown. Both results are computed for the steady state condition. In Fig. 11, point source of \( P_1(t)=2500 \text{ W/m}^2 \) is used. Fig. 11 shows that the magnitude and position of the highest steady state temperature are changeless at different cooling medium temperature. It reduces skin surface temperature considerably.

From Fig. 12 it is clear that higher power of the point heat leads to a higher temperature. Moreover, tissue temperature sensitivity due to point heating power decreases along with the distance from point heat source.

In Fig. 10 the impact of point heating on tissue temperature distribution is shown where point heating with a point heat source of \( P_1(t)=2500 \text{ W/m}^2 \) is applied. This figure demonstrates that due to point heat source, the position of the maximum temperature remains constant at the site of the point source at different times.

**Fig. 10** Impact of point heating on tissue temperature distribution.

**Fig. 11** Influence of cooling medium temperature to steady state temperature distribution.

3.8 Tissue Temperature Fluctuation under Stochastic Cooling Medium Temperature

Earlier, we consider the surrounding fluid temperature as constant. However, practically surrounding fluid temperature does not remain constant; rather, it fluctuates over time. So it is necessary to know the impact of such stochastic behavior. For this purpose, we use the following expression for flowing medium temperature.

\[
T_e = T_f + \varepsilon(t) \tag{16}
\]

Where \( \varepsilon(t) \) the stochastic variance in \( T_e \) and \( T_f \) is the equilibrium value if the environmental temperature. This variance gives the environmental temperature a stochastic value. We assume

\[
\varepsilon = \lambda t (0.05 - \sigma) \tag{17}
\]

Where \( \varepsilon \) and \( t \) is the stochastic variance in environmental temperature and the discrete-time, respectively; \( \sigma \) is the random number between 0 and 1.

**Fig. 13** and **Fig. 14** depict the influences of variance in environmental temperature with different convection coefficient between the cooling medium and skin surface. **Fig.**
13 (a) and Fig. 14 (a) shows the tissue temperature fluctuation over time where Fig. 13 (b) and Fig. 14 (b) shows the fluctuation of stochastic variance. These figures demonstrate that due to irregular cooling medium temperature, the tissue temperature fluctuates within a certain range. Moreover, the frequency of the tissue temperature is much smaller than that of the stochastic variance. It may be noticed that as convection coefficient gets larger the temperature fluctuation magnitude also increases slightly.

Fig. 13 Impact of stochastic temperature variance on tissue temperature \( (h_f=100 \text{ Wm}^{-2}) \).

Fig. 14 Impact of stochastic temperature variance on tissue temperature \( (h_f=25 \text{ Wm}^{-2}) \).

Where \( Q_{m}(t) \) is the stochastic variance in metabolic rate, and in the initial state, the metabolic rate was considered as constant \( Q_M \). Here we assumed that

\[
Q_{m}(t) = \lambda q(0.5 - \sigma)
\]  

Where \( Q_{m} \) and \( t \) is the stochastic variance in metabolic heat generation and the discrete-time respectively; \( \sigma \), the random number between 0 and 1 and \( t \) is the discrete stochastic variance in environmental temperature. And \( \lambda q \) is a constant, which was regarded as \( Q_m/10 \) in this study.

The calculated results are shown in Fig. 15 which indicates that due to stochastic heating the temperature fluctuates within a small range \( \pm 0.1 \). Fig. 15 (a) shows the tissue temperature fluctuation over time where Fig. 15 (b) shows the fluctuation of stochastic variance. Section 4.8 and Section 4.9 clearly indicates that the biological body, tends to keep its temperature balance.
Temperature is almost negligible, as shown in stochastic heating. The different heating fluctuation of environmental fluid temperature may be out of consideration as its impact on tissue temperature is almost negligible, as shown in stochastic heating. The different heating styles used for investigation in this study are generally carried out in clinical trials. Hence results described in this paper could be beneficial to predict the treatment outcome before the treatment. This will help to detect the possible risk as well as increasing the effectiveness of the treatment. Moreover, the developed Finite Element Model and computer code can be further use to solve more practical bio-heat transfer problems.

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4. Summary

In this paper, a one dimensional Finite Element Model was developed to know the temperature profile inside the human tissue subject to numerous heating pattern i.e., spatial heating, surface heating, point heating, and stochastic heating. Effect of heating frequency, blood perfusion, and scattering coefficient are also discussed briefly. Which can be used in parameter estimation. It is found that for destroying a target cell point, heating is more suitable than other heating as it increases the effectiveness of the treatment. Moreover, the developed Finite Element Model and computer code can be further use to solve more practical bio-heat transfer problems.