Self-sustaining Star Formation Fronts in Filaments during the Cosmic Dawn

Xiawei Wang and Abraham Loeb
Department of Astronomy, Harvard University, 60 Garden Street, Cambridge, MA 02138, USA; xiawei.wang@cfa.harvard.edu
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Abstract
We propose a new model for the ignition of star formation in low-mass halos by a self-sustaining shock front in cosmic filaments at high redshifts. The gaseous fuel for star formation resides in low-mass halos that cannot cool on their own due to their primordial composition and low virial temperatures. We show that star formation can be triggered in these filaments by a passing shock wave. The shells swept up by the shock cool and fragment into cold clumps that form massive stars via thermal instability on a timescale shorter than the front’s dynamical timescale. The shock, in turn, is self-sustained by energy injection from supernova explosions. The star formation front is analogous to a detonation wave, which drives exothermic reactions powering the shock. We find that sustained star formation would typically propel the front to a speed of $\sim$300–700 km s$^{-1}$ during the epoch of reionization. Future observations by the James Webb Space Telescope could reveal the illuminated regions of cosmic filaments, and constrain the initial mass function of stars in them.

Key words: early universe – galaxies: high-redshift – galaxies: star formation – shock waves

1. Introduction
The gas reservoir of low-mass halos at high redshifts exhibits inefficient star formation due to the lack of metals, which are essential for the transition from intermediate temperature atomic gas to cold molecular gas (Krumholz & Dekel 2012; Loeb & Furlanetto 2013). Nevertheless, a significant population of star-forming galaxies beyond $z \gtrsim 10$ is required to explain the Thomson optical depth of the cosmic microwave background (Finkelstein et al. 2015; Robertson et al. 2015). A likely compensating factor for the shortage of ionizing photons is a population of faint low-mass halos (Bouwens et al. 2012; Anderson et al. 2017), observationally suggested by the steep faint-end slope of the UV luminosity function (Finkelstein et al. 2015; Anderson et al. 2017). The process by which efficient star formation is initiated in low-mass halos at high redshifts is still unknown, given the inefficient star formation rate (SFR) observed in low-mass halos at low redshifts (Behroozi et al. 2013). Therefore, it is important to probe the SFR in low-mass halos during the epoch of reionization through future observations with the James Webb Space Telescope (JWST).

Galactic outflows play an important role in the formation and evolution of low-mass galaxies (e.g., Dekel & Silk 1986; Peebles & Shankar 2011), as well as in regulating star formation (Silk 1997; Hopkins et al. 2011) and the enrichment of circumgalactic medium (CGM) and intergalactic medium (IGM; Furlanetto & Loeb 2003). Cold molecular clouds are identified in observations of such outflows (Rupke et al. 2005; Sturm et al. 2011). Numerical simulations have shown that outflowing shells tend to fragment through a thermal instability (Ferrara & Scannapieco 2016; Thompson et al. 2016; Scannapieco 2017; Schneider et al. 2018), which may lead to subsequent star formation within the outflows (Silk 2013; Zubovas & King 2014; Maiolino et al. 2017; Wang & Loeb 2018). However, previous studies of galactic outflows were limited to the scale of the host galaxy and the surrounding CGM and IGM. How these outflows may affect their neighboring halos remained unclear.

In this Letter, we propose a new model for the ignition of star formation in low-mass halos that otherwise do not form stars. Such halos are often distributed in filaments. A passing shock could trigger star formation and generate a self-sustaining starburst front. We make an analogy between this process and the propagation of a detonation wave, in that the gas reservoir of low-mass halos is analogous to gunpowder, and the burning front triggers new star formation while being dynamically maintained by the energy release from supernovae (SNe). The Letter is organized as follows. In Section 2, we describe our model in analogy to detonation wave theory. In Section 3, we calculate the propagation of the star formation front and present numerical results. Finally, Section 4 summarizes our main results and observational implications.

2. Star Formation Front
The gas reservoir of low-mass halos cannot initiate star formation on its own and remains quiescent if the virial temperature of the halo, $T_{\text{vir}}$, is below the cooling threshold temperature, $T_c$. For primordial gas composition, the cooling threshold can be at minimum $T_c = 200$ K for molecular hydrogen, $\text{H}_2$ (below which molecular transitions are not excited) or $T_c = 10^4$ K for atomic hydrogen, $\text{H}_1$, if $\text{H}_2$ is dissociated by a UV background (see review in Chapter 6 of Loeb & Furlanetto 2013). Prior to star formation and feedback, sufficient baryons have been assembled into these halos as their virial masses exceed the cosmological Jeans (filtering) mass at $z \gtrsim 20$ (Haiman et al. 1996). The halos experience a gas-poor phase when feedback partially removes the gas, but recover a gas-rich phase when the gas accretes back from the IGM. The recycling of baryons results in an average baryon fraction that is $\sim 50\%$ of the cosmic average in halos of masses $\sim 10^7 M_\odot$, with a lower fraction in lower-mass halos (Chen et al. 2014; Wise et al. 2014). However, during gas-rich phases, halos maintain a baryon fraction that is approximately the cosmic average (Chen et al. 2014). The average separation of halos with $T_{\text{vir}}$ in the range of $\sim 0.5$–1.0 $T_c$ is $\bar{I} \approx (4\pi n/3)^{-1/3}/(1 + z)$, where $n$ is the comoving number density of dark matter halos as derived from the halo mass function.
In cosmic filaments, halos are found to be closer together (Bond et al. 1996; Mo et al. 2010), with $I$ smaller by up to a factor of $\sim 10$. Figure 1 shows that $I$ is a few times $R_{\text{vir}}$, and should shrink to $\sim R_{\text{vir}}$ inside filaments. Thus, we assume that halos are contiguous with their neighbors, tightly packed along the filament.

A galactic outflow driven by active galactic nuclei or SN would propagate supersonically and sweeps up the ambient medium with a speed of hundreds of km $s^{-1}$, as based on observations and theoretical calculations (e.g., King & Pounds 2015; Wang & Loeb 2015). The shells swept up by the outflow tend to cool rapidly and fragment into cold clumps that subsequently form stars (Zubovas & King 2014; Scannapieco 2017; Wang & Loeb 2018). The outflow shock is rejuvenated as it gains energy from new SN explosions that sustain its propagation. Hence, star formation can be ignited by the front as it passes through the filament. The configuration of such a burning front of star formation is shown in Figure 2. The propagation of this self-sustaining shock is analogous to a detonation wave, which involves an igniting shock self-sustained by an exothermal chemical process (Fickett & Davis 1979).

2.1. Detonation Model

The reactive Euler equations of high-speed flows coupled to energy release can be used to describe the propagation of star formation fronts, in analogy to detonation waves. These equations are

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0 \quad (1a)$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p \quad (1b)$$

$$\frac{De}{Dt} - \frac{p}{\rho^2} \frac{D\rho}{Dt} = 0 \quad (1c)$$

$$\frac{D\lambda}{Dt} = W \quad (1d)$$

Figure 1. Comparison of the average separation of halos with $T_{\text{vir}} = 0.5-1.0 T_c$, and their $R_{\text{vir}}$, in cosmic filaments. Two panels show the cases of H$_2$ and H I cooling thresholds, respectively. The red solid lines represent $R_{\text{vir}}$, while the dashed and dotted blue lines correspond to the average separation estimated from the Press-Schechter (Press & Schechter 1974) and Sheth-Tormen (Sheth & Tormen 1999) halo mass functions, respectively ($\sim 10$ times more compact in filaments). The black lines provide the halo mass, $M_{\text{halo}}$, whose $T_{\text{vir}}$ is just below $T_c$, with the scale labeled on the right-hand vertical axes. This implies that halos just below $T_c$ are tightly packed in filaments at high redshifts.

Figure 2. Configuration of a star formation front. Panel (a) sketches the burning front sweeping through halos packed in a filament that cannot form stars before the passage of the shock, analogous to a self-propagating detonation wave in gunpowder. The width of the filament is $\sim R_{\text{vir}}$. Panel (b) shows the schematic diagram of a two-dimensional cylindrical detonation wave. The blue solid line represents the sonic locus, where the Chapman–Jouguet (CJ) condition is satisfied. The green dashed line shows the boundary of the star formation zone, behind which star formation had completed. The gray dotted lines correspond to the streamlines of post-shock flows, which are assumed to be straight but diverging (see Appendix for details). Ahead of the detonation shock front lies the unburnt fuel of low-mass halos, in which stars cannot form until the shock’s passage.
where $D/Dt = \partial / \partial t + v \cdot \nabla$ is the full time derivatives of the flow. In a steady-state, $\partial / \partial t = 0$ in the rest frames of the detonation wave. Throughout our discussion, $v$, $\rho$ and $p$ are the velocity, density, and pressure of the flow, respectively; $\epsilon$ is the internal energy per unit mass; $\lambda = \rho_* / (\rho_* + \rho_g)$ is the stellar mass fraction; and $\rho_*$ and $\rho_g$ are the stellar and gas density, respectively. The location where $\lambda = 0$ corresponds to the detonation shock front, whereas $\lambda = 1$ corresponds to the completion of star formation. We approximate the fuel distribution as uniform, i.e., smooth over the scale of individual halos when describing the global propagation of the front. This approach is similar to the description of detonation waves in gunpowder, which involves smoothing over the scale of individual grains in the fuel. We adopt the polytropic equation of state

$$e = \frac{p}{(\gamma - 1) \rho} - Q\lambda,$$

(2)

where $\gamma = 5/3$ is the polytropic index. Here $Q$ is the energy release from SN explosions per unit mass of gas:

$$Q = \frac{q_{\text{IMF}} f_{\text{SN}} E_{\text{SN}}}{\omega_{\text{SN}}},$$

(3)

where $E_{\text{SN}}$ is the energy released by each SN, and $\omega_{\text{SN}}$ is the total amount of stellar mass that must be formed in order to produce one SN. For a very massive initial mass function (IMF), $E_{\text{SN}} = 10^{52}$ ergs and $\omega_{\text{SN}} = 462 M_\odot$ (Furlanetto & Loeb 2003). The coefficient $f_{\text{SN}} \sim 0.25$ is the fraction of the energy produced by SN to power the wind, while the rest is lost mainly to radiative cooling (e.g., Mori et al. 2002).

The parameter $q_{\text{IMF}}$ quantifies a deviation of the IMF from Pop III stars. In Equation 1(d), $\Delta \lambda / D$ denotes the SFR, derived from the Kennicutt–Schmidt (KS) law (Kennicutt 1998) and converted to the volume density of SFR (Schaye & Dalla Vecchia 2008):

$$\dot{\rho}_* = A'(1 M_\odot \text{ pc}^{-2})^{-n'} \left[ \frac{\gamma}{G} (1 + \lambda) \rho \right]^{n' - 1/2} \rho_g,$$

(4)

where $G$ is Newton’s constant, and $A' = (2.5 \pm 0.7) \times 10^{-4}$ and $n' = (1.4 \pm 0.15)$ are the normalization constant and power-law index in KS law for surface density, respectively. Thus,

$$W = \epsilon \frac{\dot{\rho}_*}{\rho_* + \rho_g} = \epsilon_o Ap^n (1 - \lambda) m,$$

(5)

where $A$ is a normalization constant. We adopt $n' = 1.5$ and derive the power-law indices $n = 0.25$ and $m = 1.25$. Here, $\epsilon_o$ is a correction factor for the formation rate of Pop III stars, which could be different from KS law due to their low metallicity (Trenti & Stiavelli 2009). The fiducial values of free parameters are $f_{\text{SN}} = 0.25$, $q_{\text{IMF}} = 1.0$, $c_1 = 1.0$, $E_{\text{SN}} = 10^{52}$ ergs, $\omega_{\text{SN}} = 462 M_\odot$, $m = 1.25$, and $n = 0.25$.

To solve Equations 1(a)–(d), we follow the semi-analytical approach from Watt et al. (2012; see Appendix for details). In particular, we find that the average steady-state detonation speed, $D_0$, decreases as $R_{\text{vir}}$ decreases, or equivalently, as $z$ increases, as shown in Figure 3. The star formation front travels with a speed of $D_0 \sim 200–400 \text{ km s}^{-1}$ at the beginning of reionization ($z \sim 30$), and $D_0 \sim 300–600 \text{ km s}^{-1}$ at the end of reionization ($z \sim 6$). We show that $D_0$ is a fraction, $\sim 0.2–0.7$, of the idealized one-dimensional detonation speed $D_{\text{cl}} = [2(\gamma^2 - 1)Q/\rho_g]^{1/2} \approx 1000 \text{ km s}^{-1}$, for the free parameters set at their fiducial values. The significant deviation from a one-dimensional solution indicates that lateral expansion and energy losses along radial direction are non-negligible, particularly for high-redshift halos with a smaller $R_{\text{vir}}$.

### 2.2. Star Formation

Next, we follow the prescription of our previously derived model for star formation within shells (Wang & Loeb 2018). The swept-up gas cools and fragments into cold clumps embedded in a hot tenuous gas via a thermal instability, which occurs if the heating rate at a constant pressure rises faster than the cooling rate as a function of temperature, consistent with observations (Maiolino et al. 2017) and numerical simulations (Ferrara & Scannapieco 2016; Schneider et al. 2018). The cooler gas continues to condense at a constant pressure, leading to the formation of a two-phase medium (Field 1965; Silk 2013; Zubovas & King 2014; Inoue & Omukai 2015). The cooling timescale of the swept-up gas can be estimated as $t_{\text{cool}} \approx 3.3 \times 10^5 n_1^{-1} T_1 \lambda_{-23}^{-1} (T, Z)$ years, where $n_1 = (n_1 / 1 \text{ cm}^{-3})$ is the number density of post-shock gas. $T_1 = (T_1 / 10^4 \text{ K})$ is the post-shock gas temperature, $\lambda_{-23} = (\lambda / 10^{-23} \text{ erg cm}^{-3} \text{ s}^{-1})$ is the cooling function, and $Z$ is the metallicity (e.g., Maio et al. 2007; Arata et al. 2018). For halos of mass $\sim 10^8 M_\odot$, and size $\sim 0.5 \text{ kpc}$, the characteristic shocked gas density at redshift $z \sim 10$ is $n_1 \sim 10 \text{ cm}^{-3}$. For $Z \lesssim 10^{-2} Z_\odot$, $Z_\odot$ denotes solar metallicity, $\lambda_{-23}$ is in the range $10^{-3}–0.1$, approximately scaling as $\sim (Z/Z_\odot)$ (Sutherland & Dopita 1993; Maio et al. 2007; Inoue & Omukai 2015). Thus, $t_{\text{cool}}$ is much shorter than the dynamical timescale of the flow, $t_{\text{dyn}} \sim R_{\text{vir}} / D_0 \sim 10^7$ years. The characteristic mass and size of the clouds induced by the thermal instability can be estimated as $M_{\text{cl}} \sim 110 T_{92}^{1/2} n_{12}^{-2/3} M_\odot$ and $R_{\text{cl}} \sim 0.22 T_{92}^{1/2} n_{12}^{-7/6} h_{-1}^{-1} \text{ pc}$ (Field 1965; Wang & Loeb 2018), where $T_{92} = (T_1 / 10^4 \text{ K})$ and $n_{12} = (n_1 / 1 \text{ cm}^{-3})$ are the temperature and number density of the hot medium embedding the clouds, and $T_1 = (T_1 / 10 \text{ K})$ is the temperature of the clouds. The gas clouds induced by thermal instability have a particle number density of $n_4 \sim 10^4 \text{ cm}^{-3}$, and will therefore collapse to form stars on a free-fall timescale $t_{\text{ff}} \sim (G \rho)^{-1/2} \sim 10^6 n_{14}^{-1/2} t_{\text{dyn}} \ll t_{\text{dyn}}$, where $n_{14} = (n_1 / 10^4 \text{ cm}^{-3})$.

### 3. Numerical Results

We note that $D_0$ must exceed the maximum of the local sound speed, $c_s$, and the escape speed of the halo, $v_{\text{esc}}$, in order to remain supersonic and capable of entering neighboring halos. Figure 3 shows the critical redshift, $z_c$, beyond which the detonation mode of the star formation front fails to satisfy this requirement. Overall, we find that the star formation front is self-sustainable for a broad range of $q_{\text{IMF}}, f_{\text{SN}}$, and $\epsilon_o$. We numerically solve Equation (1) for the density, pressure, axial, and radial velocities of the flow behind the star formation shock front, as shown in Figures 4 and 5, for model parameters taken at their fiducial values. We find that two-dimensional effects are more significant in halos with smaller radii, which suffer from energy losses due to lateral expansion. In these halos, the star formation front propagates with a moderate speed of $\sim 300 \text{ km s}^{-1}$, while in halos with a larger $R_{\text{vir}}$, $D_0$ reaches...
700 km s$^{-1}$. The star formation front is curved due to lateral expansion and the streamlines in the flow diverge. The sonic locus, shown as the lower boundary in Figures 4 and 5, is the place where the flow speed is equal to the local sound speed. Star formation and energy release are incomplete in the subsonic zone between the shock front and sonic locus, behind which the flow is supersonic in the detonation front rest frame. Therefore, only the energy injection from this region, also known as the detonation driving zone (Watt et al. 2012), is available to drive the propagation of the star formation front. $D_0$ is less than the ideal one-dimensional value, $D_{CJ}$, and depends on the shock curvature and $R_{vir}$, consistently with the results shown in Figure 3. Our plots indicate that the star formation front is self-perpetuating for $q_{SN}$ and $f_{SN}$ in dwarf galaxies. This indicates that for a massive IMF, the required energy to sustain the propagation of the shock, can be as low as $\sim 10^{-4}$ of the energy produced by SN, consistent with numerical simulation (Whalen et al. 2008). The front’s characteristic speed is $\sim 300–700 \text{ km s}^{-1}$ during the epoch of reionization. This indicates that the star formation front can initiate starbursts in dwarf galaxies and supply the needed ionizing photons in the early universe (Finkelstein et al. 2015).

4. Summary and Discussion

We have explored the ignition of star formation in low-mass halos by a self-sustaining star formation front along cosmic filaments in the early universe. The gaseous fuel in these most abundant low-mass halos cannot turn to stars due to their low metallicity and low $T_{vir}$. During the front’s passage through each halo, the swept-up shell is capable of cooling rapidly and fragmenting into cold clumps that form stars on a timescale shorter than the front’s dynamical timescale. The propagation of the star formation front is maintained by energy injection from SN explosions, in analogy with the propagation of a detonation shock in gunpowder. Assuming two-dimensional cylindrical symmetry, we find that the front traverses a filament with an average speed of $\sim 300–700 \text{ km s}^{-1}$.
Figure 4. Flow density and pressure behind the detonation shock. We show the ratio of flow density (panel (a)–(d)) and pressure (panel (e)–(h)) behind the shock to the ambient medium density $\rho_0$, for $z = 5$, 20 and $T_c = 200$ K and $10^4$ K. In each panel, the solid black and dashed gray curves represent the shock front and the flow streamlines, respectively. The end of the streamlines marks the sonic locus. The values of $R_{\text{vir}}$ are shown at the bottom-right corner.
Figure 5. Axial flow speed (panel (a)–(d)) and radial flow speed (panel (e)–(h)), behind the detonation shock. The arrangement of the plot is the same as in Figure 4. The detonation wave experiences energy loss due to lateral expansion, and thus $D_0$ departs from $D_{CJ}$. 

The Astrophysical Journal Letters, 862:L14 (8pp), 2018 August 1 Wang & Loeb
As the star formation front propagates, the active region would appear to have a length of $\sim 0.3' \sim 1.5$ kpc (corresponding to $\sim 0.3'$ at $z \sim 10$, resolvable by JWST), where $t_s \sim 3 \times 10^4$ years is the lifetime of massive stars ($\gtrsim 10 M_\odot$) that should dominate the UV emission in the early universe (Bromm & Yoshida 2011; Loeb & Furlanetto 2013). We find that the length of the illuminated starburst region is up to 10 times longer than the width of the filament at $z \gtrsim 20$ for H$_2$ halos. Future observations with the JWST may reveal these elongated structures and constrain the speed of the star formation fronts.

Detection of the rest-frame UV flux from the illuminated fragment of filaments will constrain the free parameters of the detonation model because the UV fragment of the illuminated starburst region is up to 10 times longer than the length of the active region. Additionally, radio emission from the relativistic electrons produced in SN remnants can be measured to infer the detonation model of star formation fronts is viable for a SFR up to $\gtrsim 10^2$ $M_\odot$ yr$^{-1}$, where $L_{\text{UV}} = (L_{\odot}/10^{28}$ erg s$^{-1}$ Hz$^{-1}$) is the UV luminosity at a rest-frame wavelength of $\sim 1250–1500$ Å (see Loeb & Furlanetto 2013, p. 352). Additionally, radio emission from the relativistic electrons produced in SN remnants can be measured to infer the SFR, as the SN rate tracks to the production rate of massive stars. Our model assumes that SFR is proportional to the locally observed KS law with a correction factor $\epsilon_s$. We find that the detonation mode of star formation fronts is viable for a SFR up to $\gtrsim 10^4$ times less efficient than associated with the KS law, indicating that even at the beginning of reionization, low-mass halos may experience starburst activity during the passage of a shock from triggered star formation in neighboring halos. Radiative pre-processing by H II regions may be an additional source of energy injection to sustain the star formation fronts. However, H II regions produce shocks of speed $\sim 30$ km s$^{-1}$ in primordial halos (Wise et al. 2012) that are much smaller than that produced by SN, which are the dominant energy source. This self-sustaining mode of star formation fronts may account for the ionizing photons in low-mass halos at $z \gtrsim 10$, as required by current observations (Robertson et al. 2015; Anderson et al. 2017). Future probes of the faint-end slope of the UV luminosity function of the star-forming galaxies with JWST will be able to test our predictions for star-forming fronts in cosmic filaments. Even if the luminosity of an individual low-mass galaxy is below the detection threshold of JWST, filaments could be detectable as they contain many such galaxies.

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Appendix

Detonation Model

In a one-dimensional laminar flow detonation model, there is a unique solution of $D_0$ corresponding to the minimum detonation speed that satisfies the conservation laws, known as the CJ velocity, $D_{\text{CJ}}$ (Fickett & Davis 1979; Watt et al. 2012). For a two-dimensional cylindrical geometry (see Figure 2), we introduce a compressible streamline function, $\psi$, such that

$$\frac{\partial \psi}{\partial r} = -r \rho v, \quad \left(\frac{\partial \psi}{\partial z}\right)_r = r \rho u.$$  

Curves of constant $\psi$ are streamlines. We can transform $(r, z)$ to a streamline-based coordinate $(\psi, z)$, where $r = r(\psi, z)$.

Equation (1) becomes

$$\frac{\partial v}{\partial z} \left[ v^2 \left( 1 + \left( \frac{\partial r}{\partial z} \right)^2 \right) - c_s^2 \right] = c_s^2 \nu \left[ \frac{\partial r}{\partial z} + \frac{\partial^2 r}{\partial z^2} \left( \frac{\partial r}{\partial \psi} \right)^{-1} \right]$$

$$-\nu \frac{\partial r}{\partial \psi} \frac{\partial^2 r}{\partial z^2} - (\gamma - 1) QW,$$  

$$\frac{\partial \lambda}{\partial z} = \frac{W}{v}.$$  

(7a)

(7b)

If the shape of the streamlines, $r(\psi, z)$, were known a priori, then Equation 7(a) reduces to a pair of ordinary differential equations for $\nu$ and $\lambda$ along each streamline where $\psi = \text{constant}$. We assume that streamlines are straight but diverging, with their shape expressed as $r = r_1 + F(\psi)(z - z_1)$. Here $(r_1, z_1)$ denotes the shock front locus, and

$$F(\psi) = \left( \frac{\partial r}{\partial z} \right)_{r_1} = u_{\psi} / v_{\psi},$$  

(8)

where $u_{\psi}$ and $v_{\psi}$ are the post-shock flow velocities subject to shock jump conditions

$$u_{\psi} = -2 D_0 \frac{z_1'}{\gamma + 1} \left[ 1 + (z_1')^2 \right]^{-1},$$  

(9a)

$$v_{\psi} = -D_0 \left[ (z_1')^2 + \frac{\gamma - 1}{\gamma + 1} \right] \left[ 1 + (z_1')^2 \right]^{-1},$$  

(9b)

where $z_1' = dz_1/dr_1$. Streamlines ahead of the shock are parallel, and thus $\psi = r_1^2 \rho_0 D_0 / 2$, where $\rho_0$ is the density of the ambient medium. Therefore, the solutions of Equation 7(a) depend on the shock locus and shape via $(r_1, z_1, z_1', z_0')$. This results in an eigenvalue problem of $z_1''$, in that if $z_1'$ was known a priori, there would be a unique $z_0'$ that satisfies Equation 9(a) and the CJ conditions (Fickett & Davis 1979) for a given $D_0$

$$v_0 \left[ 1 + \left( \frac{\partial \lambda}{\partial y} \right)^2 \right] - c_s^2 = 0,$$

$$\left( \frac{\partial \lambda}{\partial \psi} \right)^{-1} c_s^2 \nu \frac{\partial^2 \lambda}{\partial y \partial \psi} - (\gamma - 1) QW = 0.$$  

(10)

We find that $D_0 \propto 1/R_w$ due to a two-dimensional effect known as the diameter effect of detonation waves (Watt et al. 2012).

ORCID iDs
Abraham Loeb @ https://orcid.org/0000-0003-4330-287X

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