A mixed-integer linear programming formulation for assembly line balancing problem with human-robot shared tasks

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Abstract. Human-robot collaboration (HRC) has arisen as a promising technology to improve the productivity of assembly processes. This paper discusses an assembly line balancing problem (ALBP) where manual, robotic, or HRC operations may be considered decision alternatives. Each assembly process task may be operated either by a human operator, a robot operator, or an HRC. This possibility of shared functions between humans and robots may result in a hybrid manual-robotic assembly line. This problem’s mathematical model is developed based on the simple ALBP and modifying the idea of two-sided ALBP, with additional aspects related to resource alternatives of human, robot, or HRC, and robot’s tool-type for the operations. The problem is formulated analytically in a mixed-integer linear programming model with a cost-oriented objective function. The exact method can be applied to obtain an optimal solution.

1. Introduction
Human-robot collaboration (HRC) is one of the emerging technologies that characterize the fourth industrial revolution [1, 2]. HRC used in assembly production systems promises high system performance in productivity, quality, occupational health and safety, and flexibility [3,4]. Assembly systems that utilize HRC are often referred to as hybrid assembly systems [5].

Researches on HRC in assembly processes have received significant attention from researchers worldwide. Comprehensive technological discussions are provided in [3–6]. Nevertheless, research on HRC integration in the assembly line balancing problem (ALBP) is still in the early phase. Papers [7–9], and [10] are those of early researches discussing HRC integration within ALBP.

This paper differs from [7, 8], and [9] in the aspects of the cost-oriented objective function formulation that considers the costs and benefits of robots and their tools. This paper mainly extends the work in [10] by contributing tool-type consideration. Different tasks may need different tool-types if performed by robot or HRC, so the assignment of functions to workstations and resources will be affected. These additional aspects will provide the model closer to the realistic condition. This paper preferred analytical modeling and exact method as a prescriptive approach rather than a heuristic or metaheuristic approach as developed in [7].
2. Mathematical modeling

An analytical model for ALBP with human-robot shared tasks is developed here, using mixed-integer linear programming formulation. The binary integer decision variables are for assigning functions to workstations and resource-types and some sequencing decisions. In contrast, the real-number decision variables are for sequencing decisions in the form of finishing tasks.

2.1. Problem description

A set of tasks \( I = \{1, 2, \ldots, n_T\} \) are to be distributed among a group of workstations \( J = \{1, 2, \ldots, n_W\} \), which are arranged in a logically serial, linear, and consecutive workflow. This task distribution is subject to precedence constraints \( P \) among the tasks and a line takt-time or cycle-time. Each task has its characteristic that defines the possibility of the task to be performed by resource type \( s \), either by a human only \((s = 1)\), by a robot only \((s = 2)\), or by HRC \((s = 3)\).

Some tasks may be feasible to be performed either by robot or HRC than manual operations. Each alternative has its task time \( t_{is} \). If a task’s robotic or HRC procedure is applicable, it has a specific end-effector requirement with tool-type \( g \) in the set \( G = \{1, \ldots, n_E\} \). Each human, robot, or HRC decision has its cost implication, i.e., recruitment and training cost for human \( c_{I1} \), rental cost for robot \( c_{I2} \), tool type-g \( c_{ITg} \), and operating cost human \( c_{O1} \), robot \( c_{O2} \), and tool type \( g \) \( c_{OTg} \).

Considering ALBP as a medium-term decision problem, or even short term, this paper assumes the provision of robots and tools by rent is possible. This is a trend arising in the 4th industrial revolution. If the robot is provided by purchase, it should be considered a long-term decision and evaluated using strategic analysis, not as ALBP, due to much higher investment cost for the robot than human recruitment and training. The utilization of a robot or HRC for each task may also benefit \( (b_{2i} \text{ or } b_{3i}) \) that can be monetarized, either from the savings of ergonomic risk or quality aspects, e.g., the reduction of reject-parts. Thus the sum of all costs minus the benefits are required to be minimized in this problem.

2.2. Mixed-integer linear programming model formulation

The mathematical model is developed mainly based on [11] and [12]. Even though the problem here is single-sided, the possibility of providing human and robot resources at the same station will borrow some ideas from the two-sided ALBP in [12]. The main decision variables are \( x_{ij}s \). Each of them has a value of 1 if task \( i \) is assigned to workstation \( j \) and performed by resource-type \( s \), or 0 otherwise. The model formulation is as follows.

\[
\min \zeta \tag{1}
\]

The objective function in Equation (1) is to minimize the total cost \( \zeta \), which is further developed in Equation (2) below. The total cost in question is the total cost of human recruitment, training costs, robot rental costs, human operation costs, robot usage costs, equipment rental, and operating costs, reduced by the total cost of robot operation and HRC operations.

\[
\zeta = c_{I1}\alpha + c_{I2}\rho + c_{O1}\alpha + c_{O2}\rho + \sum_{g=1}^{n_E} (c_{ITg}\beta_{g1} + c_{OTg}\beta_{g}) - \sum_{i=1}^{n_T} \sum_{j=1}^{n_W} b_{2i}x_{ij2} - \sum_{i=1}^{n_T} \sum_{j=1}^{n_W} b_{3i}x_{ij3} \tag{2}
\]

In Equation (2), the total number of humans \( \alpha \), the total number of robots \( \rho \), the total number of tools of type \( g \) \( \beta_{g} \) and an additional number of humans, robots, and tools \( \alpha_{I1}, \rho_{I1}, \beta_{gI} \) come from initial number \( \alpha_0, \rho_0, \beta_{g0} \) are then defined in the following Equation (3) - (8).

\[
\alpha = \sum_{j=1}^{n_W} \left[ \min \left\{ 1, \left( \sum_{i=1}^{n_T} (x_{ij1} + x_{ij3}) \right) \right\} \right] \tag{3}
\]
\[
\rho = \sum_{j=1}^{n_W} \left[ \min \left\{ 1, \left( \sum_{i=1}^{n_T} (x_{ij1} + x_{ij3}) \right) \right\} \right] \\
\beta_g = \sum_{j=1}^{n_W} \left[ \min \left\{ 1, \left( \sum_{i=1}^{n_T} (\gamma_{gi2} + \gamma_{gi3}x_{ij3}) \right) \right\} \right] \forall g \in G \\
\alpha_1 = \max \{0, (\alpha - \alpha_0)\} \\
\rho_1 = \max \{0, (\rho - \rho_0)\} \\
\beta_{gl} = \max \{0, (\beta_g - \beta_{g0})\} \forall g \in G 
\]

In Equation (5), the \(\gamma_{gi2}\) is the parameter of the requirement of tool type \(g\) for task \(i\) if it is performed by robot only, i.e., it is one if tool-type \(g\) is required, 0 otherwise. The parameter \(\gamma_{gi3}\) has a similar definition if task \(i\) is performed by HRC. For the correct parameters, it is assumed that there is only one type of unique tool that is done by the robot (HRC) with the following criteria:

(i) \(\sum_{g=1}^{n_E} \gamma_{gi2} \leq 1\), \(\sum_{g=1}^{n_E} \gamma_{gi3} \leq 1\): there is only one, or none at all, specific tool type for each task to be performed by a robot (or HRC),

(ii) if \((\sum_{g=1}^{n_E} \gamma_{gi2} = 0)\) then \((t_{i2} = \infty)\), if \((\sum_{g=1}^{n_E} \gamma_{gi3} = 0)\) then \((t_{i3} = \infty)\): if no robotic (or HRC) tool type is feasible for a task, then its robotic (or HRC) processing time must be infinite.

Besides the objective function and its related definitions for its indicators, constraints for this optimization problem are in the following Equation (9) - (22). Equation (9) is the assignment identity constraint so that each task is assigned only once to a workstation and a specific resource-type.

\[
\sum_{j=1}^{n_W} \left( \sum_{s \in \{1,2,3\}} x_{ij} \right) = 1 \quad \forall i \in I 
\]

Precendence relationships for \(P\) tasks pair are in the following Equation (10) and (11). Equation (10) is the precedence constraint for the assignment of functions at the assembly line level, while Equation (11) is the precedence constraint in each workstation. For selecting tasks in a workstation, a decision variable \(f_i\) defines the finish time of job \(i\) and relates to the sequencing of tasks in a workstation. This finish time is subject to cycle time constraint in Equation (12) and processing time constraint in Equation (13).

\[
\sum_{s \in \{1,2,3\}} x_{ij} \leq \sum_{q=1}^{i} \left( \sum_{s \in \{1,2,3\}} x_{hqs} \right) \forall (h,i) \in P, j \in J \\
f_i - f_h + M \left( 1 - \sum_{s \in \{1,2,3\}} x_{ij} \right) + M \left( 1 - \sum_{s \in \{1,2,3\}} x_{hjs} \right) \geq \sum_{s \in \{1,2,3\}} t_{is}x_{ij} \forall (h,i) \in P, j \in J \\
f_i \leq \tau \quad \forall i \in I \\
f_i \geq \sum_{j=1}^{n_W} \left( \sum_{s \in \{1,2,3\}} t_{is}x_{ij} \right) \forall i \in I
\]

Besides \(P\) as the set of pairs of tasks with precedence relationships, another set \(Q\) is defined for all pairs of tasks with no precedence relationships. For every pair of functions \((i, p)\) in \(Q\)
assigned in the same workstation, sequencing is defined by decision variable $z_{ip}$, which is set to value one if $i$ is performed before task $p$, and 0 otherwise. The following Equation (14) - (17) are constraints for sequencing tasks with no precedence relationships. Constant $M$ is a considerable number; this will ensure that if a task is performed by a human only (or robot only), then either one of Equation (14) or Equation (15) is active while the other is trivially inactive. Equations (16) and (17) have similar logic for tasks that are performed by HRC.

$$f_i - f_p + M(1 - x_{ijk}) + M\left(1 - \sum_{s \in \{k,3\}} x_{pjs}\right) + Mz_{ip} \geq t_{jk} \forall (i, p) \in Q, j \in J, k \in \{1, 2\}$$ (14)

$$f_p - f_i + M\left(1 - \sum_{s \in \{k,3\}} x_{ijs}\right) + M(1 - x_{pjk}) + M(1 - z_{ip}) \geq t_{pk} \forall (i, p) \in Q, j \in J, k \in \{1, 2\}$$ (15)

$$f_i - f_p + M(1 - x_{ijk}) + M\left(1 - \sum_{s \in \{k,3\}} x_{pjs}\right) + Mz_{ip} \geq t_{j3} \forall (i, p) \in Q, j \in J$$ (16)

$$f_p - f_i + M\left(1 - \sum_{s \in \{k,3\}} x_{pjs}\right) + M(1 - x_{pjk}) + M(1 - z_{ip}) \geq t_{p3} \forall (i, p) \in Q, j \in J$$ (17)

Equation (18) is needed to utilize the available (or planned to be known) workstations consecutively, i.e., workstation $j$ ($j > 1$) will only be used if-and-only-if workstation ($j - 1$) has already been utilized.

$$\min \left\{1, \left(\sum_{i=1}^{n_T} \sum_{s \in \{1,2,3\}} x_{ijs}\right) \right\} \geq \min \left\{1, \left(\sum_{i=1}^{n_T} \sum_{s \in \{1,2,3\}} x_{i(j+1)s}\right) \right\} \forall j \in \{1, 2, \ldots, (n_W - 1)\}$$ (18)

Equation (19) is the constraint for the maximum number of tools in a single robot. This paper assumes each robot can be loaded by multiple devices and has an automatic tool-changing capability. Still, it is limited to a certain number, regardless of the type of tool.

$$\sum_{y=1}^{n_E} \min \left\{1, \left(\sum_{g=1}^{n_G}\gamma_{gi2}x_{ij2} + \gamma_{gi3}x_{ij3}\right) \right\} \leq K \quad \forall j \in J$$ (19)

The last constraints to include are the range of decision variables. Equation (20) - (22) define those decision variables, whether binary integer (zero - one) type or positive real number type.

$$x_{ijs} = 0 \text{ or } x_{ijs} = 1 \quad \forall i \in I, j \in J, s \in \{1, 2, 3\}$$ (20)

$$z_{ip} = 0 \text{ or } z_{ip} = 1 \quad \forall (i, p) \in Q$$ (21)

$$f_i = 0 \quad \forall i \in I$$ (22)

Some equations in the model use $\min\{}$ and $\max\{}$ functions, but they include only linear formulations. Thus, they can be transformed into purely mixed-integer linear functions. This transformation can be done automatically in some optimization software, e.g., the CPLEX optimization package used in this paper.
Table 1. Model parameterization

| Model parameter             | Reference for value                                      |
|----------------------------|----------------------------------------------------------|
| Human task-times            | Base-problems [13]                                       |
| Robot & HRC task-times      | Random, based-on [3], [8], [14]                         |
| Robot’s tool-type           | Random with four tool-types                              |
| Costs of robots and tools   | Random, based on [15] and industrial samples            |
| Costs of human              | Industrial samples                                       |
| Benefits                    | Random, based-on ergonomic risk costs [16], [17]        |

3. Numerical experiments

The model has been implemented in the CPLEX optimization package to obtain results using the exact method. Initial ALBP precedence networks are taken from [13] that are also available at the www.assembly-line-balancing.de website. These initial data are further enriched by new parameters of the newly developed model. This parameterization is shown in Table 1. For a verification purpose, extreme tests are performed by setting the parameters so that only manual operations are feasible, i.e., all the robotic and HRC process times are set to infinity (immense value). If the model is correct, the solution must be the same as simple-ALBP type 1 (SALBP1). In these tests, using some precedence networks taken from [13], the answers exactly match those of SALBP1 counterparts. These conclude that the mathematical model is verified.

Experiments have been conducted using the number of tasks $n_T = 21, 25, 35, \text{ and } 45$, developed from base problems [13]. By randomization and reference values shown in Table 1, 18 different issues have been generated from those base problems. Questions were run in CPLEX using Intel i5 with a 4 MB RAM system. A computation-time limit of 7200 seconds was set. For $n_T = 21$ problems, exact (optimal) solutions were obtained within minutes. For $n_T = 25, 35, \text{ and } 45$, all the runs were terminated within the computation time limit and resulted in only feasible solutions rather than precisely optimal ones. This approach is termed the exact bounded method [18]. Some samples of the experiments are shown in Table 2. with notation * H = Human workstation, R = Robot workstation, HR = Hybrid human-robot workstation and **Time limit for computation was set at 7200 seconds

| Number of tasks | Obtained solution* | Optimal/Feasible | Computation time (sec) |
|-----------------|--------------------|------------------|------------------------|
| 21              | 5 H, 2 HR          | Optimal          | 1380                   |
| 21              | 7 H, 1 R           | Optimal          | 480                    |
| 21              | 6 H, 1 HR          | Optimal          | 1020                   |
| 25              | 7 H, 1 R, 1 HR     | Feasible         | 7200**                 |
| 35              | 6 H, 2 HR          | Feasible         | 7200**                 |
| 45              | 2 H, 4 HR          | Feasible         | 7200**                 |

4. Conclusion

An analytical model for ALBP with human-robot shared tasks has been developed using mixed-integer linear programming formulation. This model can assist the design of an assembly line, where cost orientation is considered. Experiments have shown that for a small problem size (21 tasks), the optimal solution can be obtained within acceptable computation time. For larger problem sizes, a feasible solution may still be obtained using the exact bounded method. Further research will be directed toward ALBP with human-robot shared tasks using multiple objective
optimizations, more realistic constraints, and a heuristic procedure to handle real case problem size.

References
[1] L Wang, M Torngren, and M Onori 2015 “Current status and advancement of cyber-physical systems in manufacturing” J. Manuf. Syst. 37 pp. 517-527.
[2] K-D Thoben, S Wiesner, and T Wuest 2017 “Industrie 4.0” and Smart Manufacturing - A review of research issues and application examples Int. J. Autom. Technol. 11(1) pp. 4-16.
[3] J Kruger, T K Lien, and A Verl 2009 Cooperation of human and machines in assembly lines CIRP Ann. Technol. 58(2) pp. 628–646.
[4] G Michalos, S Makris, N Papakostas, D Mourtzis, and G Chryssolouris 2010 Automotive assembly technologies review: challenges and outlook for a flexible and adaptive approach CIRP J. Manuf. Sci. Technol. 2(2) pp. 81–91.
[5] S Takata and T Hirano 2011 Human and robot allocation method for hybrid assembly systems CIRP Ann. 60(1) pp. 9–12.
[6] L Wang, R Gao, J Vancza, J Kruger, X V Wang, S Makris, and G Chryssolouris 2019 Symbiotic human-robot collaborative assembly CIRP Ann. 68(2) pp. 701–726.
[7] M D Mura and G Dini 2019 Designing assembly lines with humans and collaborative robots: A genetic approach CIRP Ann. 68(1) pp. 1–4.
[8] C Weckenborg and T S Spengler 2019 Assembly Line Balancing with Collaborative Robots under consideration of Ergonomics: a cost-oriented approach IFAC-PapersOnLine 52(13) pp. 1860–1865.
[9] C Weckenborg, K Kieckhafer, C Müller, M Grunewald, and T S Spengler 2020 Balancing of assembly lines with collaborative robots Bus. Res. 13(1) pp. 93–132.
[10] S Yaphiar, C Nugraha, and A Ma’ruf 2020 Mixed Model Assembly Line Balancing for Human-Robot Shared Tasks in iMEC-APCOMS 2019 2020 pp. 245–252.
[11] A Scholl 1999 Balancing and sequencing of assembly lines. Physica-Verlag Heidelberg, 1999.
[12] Y K Kim, W S Song, and J H Kim 2009 A mathematical model and a genetic algorithm for two-sided assembly line balancing Comput. Oper. Res. 36(3) pp. 853–865.
[13] A Scholl 1993 Data of Assembly Line Balancing Problem. assembly-line-balancing.de.
[14] J Teiwes, T Banziger, A Kunz, and K Wegener 2016 Identifying the potential of human-robot collaboration in automotive assembly lines using a standardised work description in 2016 22nd International Conference on Automation and Computing (ICAC) Sep. 2016 pp. 78–83.
[15] S Landscheidt and M Kans 2016 Method for Assessing the Total Cost of Ownership of Industrial Robots Procedia CIRP 57 pp. 746–751.
[16] L Punnett 2000 The Costs of Work-Related Musculoskeletal Disorders in Automotive Manufacturing NEW Solut. A J. Environ. Occup. Heal. Policy 9(4) pp. 403–426.
[17] D Lerner, W H Rogers, H Chang, A M Rodday, A Greenhill, V G Villagra, J R Antetomaso, A A Patel, and L Vo 2015 The Health Care and Productivity Costs of Back and Neck Pain in A Multi-Employer Sample of Utility Industry Employees J. Occup. Environ. Med. 57(1) pp. 32–43.
[18] O Battaia and A Dolgui 2013 A taxonomy of line balancing problems and their solution approaches Int. J. Prod. Econ. 142(2) pp. 259–277.