QCD and $e^+e^- \rightarrow \text{Baryon} + \text{anti-Baryon}$

Marek Karliner
and
Shmuel Nussinov

School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv, Israel

Abstract

We discuss the QCD constraints on $e^+e^- \rightarrow \text{baryon–anti-baryon}$ close to threshold, in light of the puzzling experimental data which indicate that close to threshold $\sigma(e^+e^- \rightarrow n\bar{n}) > \sigma(e^+e^- \rightarrow p\bar{p})$. We focus on the process $e^+e^- \rightarrow \Delta\bar{\Delta}$, which is particularly simple from the theoretical point of view. In this case it is possible to make exact QCD predictions for the relative yields of the four members of the $\Delta$ multiplet, modulo one crucial dynamical assumption.

*marek@proton.tau.ac.il
**nussinov@post.tau.ac.il
It has been pointed out recently [1],[2] that the experimental results [3]-[6] indicating that
\[ \sigma(e^+e^- \rightarrow n\bar{n}) > \sigma(e^+e^- \rightarrow p\bar{p}) \] (1)
over the threshold region, \( W = 1.88 \div 2.45 \text{ GeV} \), are rather surprising.

Indeed, if the fragmentation amplitude \( F(u \rightarrow \text{proton}) = F(d \rightarrow \text{neutron}) \) is not smaller in magnitude than \( F(d \rightarrow \text{proton}) = F(u \rightarrow \text{neutron}) \), then the \( \bar{p}p \) cross section is larger than that for \( \bar{n}n \). This is so since the amplitude for \( \bar{u}u \) pair production by the virtual photon is enhanced by the ratio of quark charges \( |Q_u/Q_d| = 2 \), relative to the corresponding \( \bar{d}d \) amplitude.

It has therefore been argued [1],[2] that such reasoning – inspired by perturbative QCD, does not apply at \( N\bar{N} \) threshold. The \( q \rightarrow B \) and \( \bar{q} \rightarrow \bar{B} \) fragmentation functions are not independent, annihilation channels into multipion states dominate, and other, say Skyrme-like descriptions for the nucleons are more appropriate than the “naïve” three quark picture.

Let us briefly restate the main argument of [2]. The \( I = 1 \) (say \( \rho, \rho', \rho'' \), etc.) intermediate states leading to an even number of pions dominate the annihilation into multi-pion states. This is expected from the direct quark EM couplings of the \( \bar{u}u \) and \( \bar{d}d \) quarks to the photon in an ideal “nonet” (or “quartet” when \( s\bar{s} \) production is neglected) symmetry of the vector multiplets (we would like to emphasize that such symmetries arise when purely gluonic \( 1^- \) \( I = 0 \) channels are neglected).

If further the \( I = 1 \) amplitudes dominate the \( \bar{N}N \ 1^- \) annihilation channel – as the ratio of \( \bar{p}p \rightarrow K^+K^-/K^0\bar{K}^0 \) suggests, then \( I = 1 \) intermediaries dominate \( \bar{N}N \rightarrow e^+e^- \) at threshold.

If \( |A(I = 1)| \gg |A(I = 0)| \) then \( \sigma(e^+e^- \rightarrow \bar{p}p) = \sigma(e^+e^- \rightarrow \bar{n}n) \). Further admixture of some \( I = 0 \) amplitude can yield the desired experimentally measured cross section ratio for a range of the parameters \( \epsilon \equiv |A(I=0)/A(I=1)| \) and Arg\([A(I=0)/A(I=1)]\).

In the following we focus on another baryon-antibaryon threshold process which was briefly alluded to in [1], namely the (pair) production of \( I = 3/2, J = 3/2 \) \( \Delta \) states: \( \Delta^{++}, \Delta^+, \Delta^0 \) and \( \Delta^- \).

Our discussion does not contradict the above phenomenological considerations of the \( \bar{N}N \) threshold process for the \( I = 1/2 \) nucleons. Yet it suggests a complementary point of view and possible “exact” QCD predictions even
in this non-perturbative regime, modulo one crucial assumption, as discussed in detail below.

We start by comparing the two processes,

\[ e^+e^- \rightarrow \Delta^{++}\Delta^{++} : \quad e^+e^- \rightarrow \gamma^* \rightarrow u\bar{u} \rightarrow u\bar{u}u + \bar{u}\bar{u}\bar{u} \quad (2) \]

and

\[ e^+e^- \rightarrow \Delta^-\Delta^- : \quad e^+e^- \rightarrow \gamma^* \rightarrow d\bar{d} \rightarrow d\bar{d}d + \bar{d}\bar{d}\bar{d} \quad (3) \]

Unlike the \( e^+e^- \rightarrow \bar{N}N \) case, where one has a mixed-symmetry nucleon with two different fragmentation functions, only one quark-baryon fragmentation function is involved here, namely \( F(u \rightarrow \Delta^{++}) = F(d \rightarrow \Delta^-) \). Thus we have a clear prediction of the “naïve” quark model,

\[ \sigma(e^+e^- \rightarrow \Delta^{++}\Delta^{++}) = 4\sigma(e^+e^- \rightarrow \Delta^-\Delta^-) \quad (4) \]

The key point that we wish to emphasize is that this prediction is far from naïve. Eq. (4) above and other predicted ratios of reactions involving \( \Delta(1238) \) baryons become exact when the following assumptions are made: (a) we take the \( m_u = m_d \) limit; (b) we work to lowest order in \( \alpha_{QED} \); (c) we consider only “flavor connected” processes in which the initial \( q_i\bar{q}_i \) produced via the initial (virtual) photon do not annihilate into gluons and appear in the final hadrons.

While (a) and (b) are standard, assumption (c) is not as clear-cut. It entails a \( 1/N_c \) approximation extended to baryons, and in the present context it is equivalent to the famous Zweig (OZI) rule.\(^\dagger\)

\(^\dagger\)The extension to baryons involves some subtleties which will be discussed in some detail a bit later.
sector there are large OZI violations, at least in one case, that of $\bar{s}s$ meson production in $B\bar{B}$ annihilation at rest \cite{11,12}. It is important to see whether this is compatible with assumption (c), to be discussed in more detail later. Consequently, we now proceed on the basis of all three assumptions. Clearly, confronting the ensuing predictions with experiment will be a sensitive test of assumption (c).

First, we note that relation (4) is valid to all orders in perturbation theory. For any QCD Feynman diagram contributing to $e^+e^- \to \Delta^{++}\Delta^{++}$, there is a corresponding diagram, with all $u$-quark propagators replaced by (identical!) $d$-quark propagators, which contributes to $e^+e^- \to \Delta^-\Delta^-$. Because of the flavor independence of gluonic couplings these two diagrams will therefore be identical as far as QCD is concerned. The only difference is that $Q_d = -1/3$ replaces $Q_u = 2/3$ in the coupling to the external photon. Thus we expect that the amplitudes for the two different processes will satisfy

$$A(e^+e^- \to \Delta^{++}\Delta^{++}) = -2A(e^+e^- \to \Delta^-\Delta^-)$$

We omitted all the kinematical variables, namely the external momenta and helicities, which should be the same for the two processes.

Upon adding all the squares of the helicity amplitudes we obtain the differential cross section and (4) above follows.

Needless to say, we do not use here any approximate ($S$-wave) symmetric, naïve, quark model or any other model for the wave- (or fragmentation) functions, and allow for any admixture of orbital/radial excitations – and contribution of states with any number of gluons and/or extra $q\bar{q}$ pairs. The only issue at stake here is the $u \leftrightarrow d$ exchange symmetry. This is a vectorial flavor symmetry. The Vafa-Witten theorem \cite{13}, which relies on rigorous QCD inequalities (see \cite{14} and \cite{15} for reviews) states that such symmetries do not break down spontaneously, and hence the all-order perturbative relations should be true in the full theory.

Our argument is actually independent of any perturbative expansion. All hadronic amplitudes can be obtained by Fourier transformation and analytic continuations of Euclidean $n$-point correlators of (color singlet) local observables.

The relevant correlators for the processes of interest are the three-point
functions of the form:

\[ K_{\Delta^{++}}(x, y, z) = \langle 0 | J_{EM}(x) \Delta^{++}(y) \Delta^{++}(z) | 0 \rangle \]  

(6)

with \( J_{\mu, EM}(x) = \frac{2}{3} \bar{u}(x) \gamma_\mu u(x) - \frac{1}{3} \bar{d}(x) \gamma_\mu d(x) + \ldots \) the electromagnetic current and \( \Delta^{++}(x) \) a local operator trilinear in quark fields with the quantum numbers of the \( \Delta^{++} \) particle,

\[ \Delta^{++}(x) = u_{a,i}(x) u_{b,j}(x) u_{c,k}(x) \epsilon^{abc} \Gamma^{ijk} \]  

(7)

with \( a, b, c \) (\( i, j, k \)) color (spinor) indices. \( \epsilon \) and \( \Gamma \) guarantee coupling to an overall color-singlet baryon with total spin \( J = 3/2 \). We have an analogous expression for \( K_{\Delta^{-}}(x, y, z) \).

There are two contraction patterns of the fermionic operators in the above correlators – the “flavor connected” and the “flavor disconnected”.

In the first case the initial quark(anti-quark) created from the vacuum via \( J_{EM} \) at \( x \) propagates to the final \( y \) (or \( z \)) vertex. In the second case, these quarks annihilate, forming a closed loop which starts and terminates at the same point \( x \). The path integrals for evaluating these contributions are respectively

\[ K_{\Delta^{++}}^{\text{connected}}(x, y, z) = Q_u \int d\mu(A) \text{Tr} \left\{ \gamma_\mu S^A_u(x, y) \epsilon \Gamma S^A_u(z, y) S^A_u(z, y) \epsilon \Gamma^\dagger S^A_u(z, x) \right\} \]  

(8)

and

\[ K_{\Delta^{++}}^{\text{disconnected}}(x, y, z) = \sum_{i=u,d,s} Q_i \int d\mu(A) \text{Tr} \left\{ \gamma_\mu S^A_i(x, x) \right\} \text{Tr} \left\{ \epsilon \Gamma S^A_u(z, y) S^A_u(z, y) S^A_u(z, y) \epsilon \Gamma^\dagger \right\} \]  

(9)

where the above traces are in color/spinor space, \( S^A_i(x, y) \) is the propagator from \( x \) to \( y \) of the \( i \)-th flavor quark in a given background gauge field \( A_\mu(x) \), and

\[ d\mu(A) = DA_\mu(x) e^{-\int dt(x) (E^2 + B^2)} \prod_i \text{Det}(\partial A_i + m_i) \]  

(10)

is the measure in the path integral.

The two contraction patterns are illustrated schematically in Fig. 1(a) and 1(b), respectively.
Figure 1: The two contraction patterns for the fermionic operators determining the three-point point functions $\langle J_{EM} \Delta \Delta \rangle$ in eq. (6): (a) – the flavor-connected contraction, eq. (8); (b) – the flavor-disconnected contraction, eq. (9). The quark loop in (b) includes a sum of contributions from all the light flavors.

Baryon anti-baryon production is expected to be strongly suppressed in the large-$N_c$ limit. Still, it is meaningful to compare the $N_c$ hierarchy of the two classes of processes – 1(a) and 1(b).

Apart from an external color-singlet insertion, 1(a) is essentially the same (in $N_c$ terms) as the baryonic propagator. The contribution 1(b), on the other hand, has a quark loop in addition to the baryonic propagator. As is well known from the double(single) line counting rules for gluonic(quark) propagators [7], such a quark loop introduces an extra $g^2$ factor, yet it does not increase the number of color traces.
Hence the 1(b) flavor-disconnected contraction is suppressed by $1/N_c$ relative to the connected contraction 1(a). Analogous expressions can be written for the correlator $K_{\Delta^-} (x, y, z)$ and for the flavor-connected and flavor disconnected-parts thereof.

This argument is quite simple for mesons, but the extension to baryons, although yielding the same qualitative conclusion [9]-[10], involves some subtleties.

Although the result is not new, for the reader’s convenience we review the argument in the following. Consider the large-$N_c$ baryon propagator with the leading quark-loop contribution. It is the diagram with a quark-loop correction to a gluon propagating between two valence quarks. There are two vertices where the gluon couples to the valence quarks and two vertices where the quark loop couples to the gluon. Each vertex is associated with $1/\sqrt{N_c}$, and thus each individual diagram of this kind is suppressed by $1/N^2_c$ with respect to the diagram with only valence quarks propagating.

The subtlety arises because there are $N_c(N_c - 1)/2$ such pairs of valence quarks in the baryon, and so the total contribution of the diagrams with a quark loop is of order $O(1)$ and thus appears to be unsuppressed.

However, as stressed by Witten [8], diagrammatic expansion is not a very convenient way to study the large-$N_c$ limit of baryons, because in the perturbation series each successive order grows as a higher power of $N_c$. Thus the diagram with one gluon exchanged between the two quarks in the baryon scales like $O(N_c)$, (two vertices $\sim 1/\sqrt{N_c}$ each, multiplied by combinatorial factor of $N_c(N_c - 1)/2$ of possible quark pairs), a diagram with two gluons exchanged between two different pairs of quarks is of order $N^2_c$, etc. This divergence is simply an an artifact of the baryon mass being $\sim O(N_c)$.

The crucial point for our discussion is that the quark loop is $1/N_c$ suppressed with respect to the contribution from the same diagram without the quark loop: thus the diagram with one gluon exchanged between two valence quarks scales like $O(N_c)$, while the same diagram with an additional quark loop correction to that gluon scales like $O(1)$, etc.

The general arguments given by Witten make it clear that the total contribution of gluons to the baryon propagator is of the same order as that of the valence quarks, i.e. linear in $N_c$. Since any diagram with a quark loop is $1/N_c$ suppressed with respect to the same diagram with only valence quarks
and gluons, it follows that quark loops are suppressed by $1/N_c$ with respect to the valence quarks $[9]-[10]$.

In the real world, this suppression manifests itself, in particular via the absence of "exotic" resonances (involving extra $\bar{q}q$ pairs) and the narrowness of the ordinary, non-exotic resonances. These hold equally well both for mesons and for baryons.

Apart from the overall quark charge factors of $\frac{2}{3}$ and $-\frac{1}{3}$, respectively, the connected $K_{\Delta^+}(x, y, z)$ and $K_{\Delta^-}(x, y, z)$ are identical. This identity holds in a very strict sense, namely pointwise in the path integrals (8) and (9), i.e. for each gauge field configuration $A_\mu(x)$ separately. Hence it is guaranteed to hold also for the integrated quantities. This is true in any scheme, such as lattice gauge theory, used in order to define and regularize the path integral in a rigorous way.

Thus we conclude that

$$K_{\Delta^+}^{\text{connected}}(x, y, z) = -2K_{\Delta^-}^{\text{connected}}(x, y, z)$$

(11)

holds for any $x, y$ and $z$. It will therefore hold for the physical amplitudes for $e^+e^-$ annihilations into the corresponding $\Delta\bar{\Delta}$ baryons which are obtained by common manipulations of the two Euclidean correlators. Thus, to the extent that we neglect the flavor-disconnected part, relations (4) and (5) ensue.

It is important to note that for the case at hand the $K^{\text{disconn.}}$ contribution is suppressed not only by the $1/N_c$. There is also an independent effect which works in the same direction, namely a partial cancellation of the contributions of the three light quark flavors $u, s$ and $d$. The cancellation, due to $Q_u + Q_d + Q_s = 0$, is exact in the $SU(3)$ flavor symmetry limit. The physical world is quite well approximated by this limit, since the $m_s - m_u/d$ mass difference is negligible in comparison with the mass scale relevant for the problem $\sim m_N = 1$ GeV. In this case the three terms, due to $u$, $d$ and $s$ loops appearing in $K^{\text{disconn.}}$ above are identical as far as QCD is concerned.

$\dagger$These predictions are shared by another frequently used nonperturbative approach. This is the model utilizing $q\bar{q}$ pair production via tunneling in a chromoelectric flux tube $[16]$, incorporated in the Lund model $[17]$. Flavor-disconnected parts are neglected in such models.

§It should be emphasized that unlike the “universal” $1/N_c$ suppression, this charge-cancellation mechanism is relevant only in the specific energy range considered here. At much lower energies only $\bar{u}u$ and $\bar{d}d$ contribute, and at much higher energies $\bar{c}c$ contribute as well.
The above arguments apply also for the case of annihilations into $\bar{N}N$ states. However, in the latter case we have, even when only the flavor-connected parts are retained, two distinct contributions. One contribution occurs when the primary quark produced by the photon is one of the two same-flavour quarks in the nucleon (namely the $u$ in the proton or the $d$ in the of the neutron). The other contribution corresponds to the case when the other quark ($d$ for proton or $u$ for the neutron) is the primary one. There is no a priori model-independent relation between these two contributions, nor do we know of any systematic expansion like $1/N_c$ which could help relate them, and hence no analogous firm prediction can be made here.

In passing we note that also

$$K^{\text{disconn.}}_{\Delta^+}(x, y, z) = K^{\text{disconn.}}_{\Delta^-}(x, y, z)$$

(12)

and the resulting flavor-disconnected contributions to the physical amplitudes are also identical. This is indeed expected, as the disconnected amplitudes correspond to purely gluonic and hence $I=0$ intermediate states, which couple equally to all members of the $\Delta(1238)$ multiplet.

Considering next the $I=1$ and $I=0$ isospin channels we find that eq. (11) above implies that for the $\gamma^* \rightarrow \Delta \bar{\Delta}$,

$$A(I=1) = 2A(I=0).$$

(13)

Interestingly, this is consistent with the dominance of the $I=1$ channel in $e^+e^- \rightarrow \bar{N}N$, as inferred from phenomenological analysis of the data in Ref. [2]. It is reassuring that here it follows directly from QCD, augmented by assumption $c$.

Isospin symmetry and the above amplitude ratio imply

$$A(\Delta^{++}) : A(\Delta^+) : A(\Delta^0) : A(\Delta^-) = 2 : 1 : 0 : -1$$

(14)

Thus the $e^+e^- \rightarrow \Delta \bar{\Delta}$ cross section ratios 4:1:0:1 discussed in [1], are in fact exact QCD results, modulo assumption (c) above, i.e. neglect of the “flavor disconnected” parts.$^{\star}$

$^{\star}$For the processes $e^+e^- \rightarrow N\Delta$, involving nucleon and delta production, only the $I=1$ amplitude contributes and the ratios are fixed by isospin alone.
It is interesting to consider the effect of possible resonances in the $I = 1$ ($\rho, \rho', \rho''$) and the $I = 0$ channel ($\omega, \omega', \omega''$). Clearly, if at any particular energy an $I = 0$ or an $I = 1$ resonance dominates, the above ratio of $A(I = 1)/A(I = 0)$ cannot be maintained.

We note however that precisely in the large $N_c$ limit, when the quark annihilation diagrams are neglected, the $I = 1$ and $I = 0$ states are degenerate: $m_\omega = m_\rho$, $m_{\omega'} = m_{\rho'}$, etc. In reality, it is much easier to maintain the $A(I = 1)/A(I = 0)$ ratio, since the resonances in the $W > 2$ GeV region are broad and overlapping.

Independently of the ordinary $\rho^{(n)}$ and $\omega^{(n)}$ resonances and/or annihilation channels, dynamical enhancement of the gluonic $I = 0$ intermediate state could result if a relatively narrow glueball state with a vector $1^{--}$ quantum numbers occurs somewhat above the $\Delta(1238)\bar{\Delta}(1238)$ threshold. It would manifest itself via a transient deviation from (5) when the resonant energy is traversed.

Qualitative arguments [14] suggest that the lightest $1^{--}$ state in the “three-gluon” sector is at least 50% heavier than the lightest scalar glueball in the “two-gluon” sector, with $m_{0^{++}} \simeq 1.6 \div 1.7$ GeV. In other words, $m_{1^{--}} \gtrsim 2m_\Delta$. Lattice calculations [18] indicate a much larger value, $m_{1^{--}} \sim 3.8$ GeV. If this is indeed the case, the $1^{--}$ glueball is too far above the $\Delta\bar{\Delta}$ threshold. It would be very interesting to settle this issue.

Following the suggestion of Refs. [1] and [2], we next consider the $\gamma\gamma \to B\bar{B}$ processes. We again compare $\Delta^{++}$ and $\Delta^-$ pair productions. If we neglect the disconnected part, then the same arguments imply a ratio of 4 of the flavor-connected amplitudes and 16(!) for the corresponding cross sections, as discussed briefly in [1].

It should be emphasized, however, that the neglect of the flavor-disconnected part relies here, unlike for the previous single-photon case, on the $1/N_c$ suppression only and not on cancellation of charges as well. There is no such cancellation here, as only the squares of the charges enter.

In addition, in the two photon case we have also the $0^- \eta'$ anomalous channel, which implies that the flavor-disconnected parts may be enhanced. However, as we move even slightly above the $B\bar{B}$ threshold, the number of partial waves contributing increases rapidly and the relative importance of the pseudoscalar channel decreases.
The present work is indirectly triggered by present data on $e^+e^- \rightarrow \bar{n}n/\bar{p}p$ at threshold, which seem to confound the naïve QCD expectations. Our main point is to note that there is a similar process, in which the theoretical analysis is much more straightforward, namely $e^+e^- \rightarrow \Delta\bar{\Delta}$. In this case QCD, augmented by a well-motivated dynamical assumption, makes clearcut and largely model-independent predictions. Future experimental tests of these predictions will be a sensitive probe of this dynamical assumption.

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