$B^0 - \bar{B}^0$ mixing in supersymmetry with gauged baryon and lepton numbers

Fei Sun$^{a,b,}$*, Tai-Fu Feng$^{a,b,c,d}$, Shu-Min Zhao$^{a,c,d}$, Hai-Bin Zhang$^{a,b}$, Tie-Jun Gao$^c$, Jian-Bin Chen$^e$

$^a$Department of Physics, Hebei University, Baoding, 071002, China
$^b$Department of Physics, Dalian University of Technology, Dalian, 116024, China
$^c$Institute of theoretical Physics, Chinese Academy of Sciences, Beijing, 100190, China
$^d$The Key Laboratory of Mathematics-Mechanization (KLMM), Beijing, 100190, China
$^e$Department of Physics, Taiyuan University of Technology, Taiyuan, 030024, China

Abstract

We perform an analysis on $B^0 - \bar{B}^0$ mixing in the extension of the minimal supersymmetric standard model where baryon and lepton numbers are local gauge symmetries (BLMSSM) by using the effective Hamiltonian method. And the constraint of a 125 GeV Higgs to the parameter space has also been considered. The numerical results indicate that the contributions of the extra particles can be sizeable in $B^0 - \bar{B}^0$ mixing. For certain parameter sets, the theoretical prediction of mass differences $\Delta m_B$ agrees with the current experimental result. Furthermore, $B^0 - \bar{B}^0$ mixing in the BLMSSM can preliminarily constrain the parameter space. With the development of more precise theoretical analysis and experimental determinations, the $B^0 - \bar{B}^0$ mixing in the BLMSSM will have a clearer picture and the parameter space in this model will also be further constrained.

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* Corresponding author. email:sunfei@mail.dlut.edu.cn
I. INTRODUCTION

The Minimal Supersymmetric Standard Model (MSSM) \cite{1-5}, as one of the most appealing options for the physics beyond the Standard Model (SM), has drawn the physicists' attention for a long time. As the simplest soft broken supersymmetry (SUSY) theory, the MSSM can solve hierarchy problem, ensure that the gauge couplings unify at high energies and provide a good dark matter candidate. To search for new particles predicted by SUSY, the Large Hadron Collider (LHC) has collected huge amounts of data, the CMS \cite{7} and ATLAS \cite{8} experiments now set strong limits on these parameter space \cite{9-12}. However, the present searches are largely based on the assumption of conserved R-parity \cite{6}. Some studies in the low-energy SUSY have been motivated by the results of the LHC \cite{13-23}, and R-parity violating scenarios of general MSSM have been proposed \cite{24-48}.

A model based on the gauge symmetry group $SU(3) \otimes SU(2) \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ has been investigated at the TeV scale recently \cite{49-52}, where $B$ stands for baryon number and $L$ stands for lepton number. In this theory, the baryon and lepton numbers are local gauge symmetries spontaneously broken at the TeV scale. Breaking baryon number can explain the origin of the matter-antimatter asymmetry in the Universe. And breaking lepton number can explain the smallness of neutrino masses \cite{53-57}. Two extensions of the SM where $B$ and $L$ are spontaneously broken gauge symmetries near the weak scale are constructed \cite{58}: model I is a non-supersymmetric extension \cite{59,60}; model II (BLMSSM) is a supersymmetric extension and is more favoured by the experiments \cite{61}. The BLMSSM has been studied in great detail and could avoid the current LHC bounds on the SUSY mass spectrum \cite{62,63,65}. Some further phenomenology analysis based on the BLMSSM coincide with the current experimental data well, the mass and decays of the lightest CP-even Higgs have been investigated in Refs. \cite{65,66}, and the neutron electric dipole moment in CP violating BLMSSM has also been studied \cite{67}.

The flavor changing neutral current (FCNC) processes are highly suppressed in the SM, therefore it is a fertile ground to search for physics beyond SM (BSM). FCNC processes such as $b \to s \gamma$, $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing have played an important role in particle physics over the last four decades. It is well known that CP violation was first observed
in the decays of $K_L^0$ meson in 1964 [68], and CP violation of the neutral $B$ meson system was observed in 2001 [69]. The first indication of a large top quark mass was also given by $B^0 - \bar{B}^0$ mixing [70, 71]. $B$-system decays have an advantage over the $K$-system to provide a direct test of the CP violating of SM and is free of corrections from strong interactions [72–74]. The experiment results of $B^0 - \bar{B}^0$ mixing have been published by the ALEPH [75], DELPHI [76, 77], L3 [78], OPAL [79, 80], BaBar [81], Belle [82], CDF [83], DØ [84], and LHCb [85] collaborations. Current experimental result of mass difference is $\Delta m_{B}^{Exp} = 0.507 \pm 0.004 \text{ ps}^{-1} = (3.337 \pm 0.033) \times 10^{-13} \text{ GeV}$ [86]. Calculations for $B^0 - \bar{B}^0$ mixing have been done in the SM, the two-Higgs doublet model (2HDM), the MSSM and other models [87–96]. The SM prediction for mass difference is $\Delta m_B^{SM} = 0.543 \pm 0.091 \text{ ps}^{-1}$ [97], which has a good agreement with the experiment. However, the theoretical error is around 17%, which is considerably larger than the experimental error. The running of LHC will resume in 2015 with higher energy and luminosity. Proposals for next-generation B-factories including SuperKEKB in Japan whose target luminosity is $8 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ will start collecting data in the near future [98]. This may also give some hints on physics beyond the SM. So it is important for experimental and theoretical physicist to search for new physics. As a candidate of new physics, the BLMSSM provides new FCNC at loop level in the $B^0 - \bar{B}^0$ mixing. We will carry out our calculations for $B^0 - \bar{B}^0$ mixing in this model.

Our presentation is organized as follows. In Section II, we briefly summarize the main features of the BLMSSM and introduce the superpotential as well as soft breaking terms, then we obtain the mass matrices and couplings needed for $B^0 - \bar{B}^0$ mixing. In Section III, we give the analytical formulae of the $B^0 - \bar{B}^0$ mixing in BLMSSM. The numerical analysis are shown in Section IV. Section V presents our conclusions. Finally, some related formulae are given in Appendix A–B.

II. BLMSSM

In this section, we briefly review some main features of the BLMSSM. In the BLMSSM with gauged baryon ($B$) and lepton ($L$), by adding the new quarks with baryon number $B_4 = \frac{3}{2}$ and the new leptons with lepton number $L_4 = \frac{3}{2}$, one can cancel the baryonic and
leptonic anomalies respectively \cite{58}. Compared with the MSSM, the BLMSSM includes many new fields. Tables II-IV list the superfields including the new quarks, new leptons, new Higgs, the exotic superfields $\tilde{X}$ and $\tilde{X}'$, respectively. As one can see, the left-handed superfields have the same absolute value of $U(1)_B$ as that of the right-handed superfields but with a contrary sign to cancel baryonic anomalies in the quark sector, similarly for the $U(1)_L$ in the leptonic sector to cancel leptonic anomalies.

**TABLE I: Superfields including the new quarks in the BLMSSM.**

| Superfields | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_B$ | $U(1)_L$ |
|-------------|-----------|-----------|----------|----------|----------|
| $\tilde{Q}_4$ | 3         | 2         | 1/6      | $B_4$    | 0        |
| $\tilde{U}^c_4$ | 3         | 1         | -2/3     | $-B_4$   | 0        |
| $\tilde{D}^c_4$ | 3         | 1         | 1/3      | $-B_4$   | 0        |
| $\tilde{Q}^c_5$ | 3         | 2         | -1/6     | $(1 + B_4)$ | 0 |
| $\tilde{U}_5$ | 3         | 1         | 2/3      | $1 + B_4$ | 0        |
| $\tilde{D}_5$ | 3         | 1         | -1/3     | $1 + B_4$ | 0        |

**TABLE II: Superfields including the new leptons in the BLMSSM.**

| Superfields | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_B$ | $U(1)_L$ |
|-------------|-----------|-----------|----------|----------|----------|
| $\tilde{L}_4$ | 1         | 2         | -1/2     | 0        | $L_4$    |
| $\tilde{E}^c_4$ | 1         | 1         | 1        | 0        | $-L_4$   |
| $\tilde{N}^c_4$ | 1         | 1         | 0        | 0        | $-L_4$   |
| $\tilde{L}^c_5$ | 1         | 2         | 1/2      | 0        | $(3 + L_4)$ |
| $\tilde{E}_5$ | 1         | 1         | -1       | 0        | $3 + L_4$ |
| $\tilde{N}_5$ | 1         | 1         | 0        | 0        | $3 + L_4$ |

In order to break baryon number spontaneously, we need to introduce the superfields $\hat{\Phi}_B$ and $\hat{\varphi}_B$ to acquire nonzero vacuum expectation values (VEVs), which also generate large
TABLE III: Superfields including the new Higgs in the BLMSSM.

| Superfields | SU(3) | SU(2) | U(1)Y | U(1)B | U(1)L |
|-------------|-------|-------|-------|-------|-------|
| \(\hat{\Phi}_B\) | 1     | 1     | 0     | 1     | 0     |
| \(\hat{\varphi}_B\) | 1     | 1     | 0     | -1    | 0     |
| \(\hat{\Phi}_L\) | 1     | 1     | 0     | 0     | -2    |
| \(\hat{\varphi}_L\) | 1     | 1     | 0     | 0     | 2     |

TABLE IV: Superfields avoiding stability for the exotic quarks in the BLMSSM.

| Superfields | SU(3) | SU(2) | U(1)Y | U(1)B | U(1)L |
|-------------|-------|-------|-------|-------|-------|
| \(\hat{X}\) | 1     | 1     | 0     | 2/3 + B_4 | 0     |
| \(\hat{X}'\) | 1     | 1     | 0     | -(2/3 + B_4) | 0     |

mass for the new quarks. Similarly, we introduce the superfields \(\hat{\Phi}_L\) and \(\hat{\varphi}_L\) to acquire VEVs spontaneously breaking lepton number. Finally, the exotic quarks should be unstable, so the model also includes the superfields \(\hat{X}\) and \(\hat{X}'\) to avoid the stability for the exotic quarks. Here \(\hat{\Phi}_B\) and \(\hat{\varphi}_B\) have \(U(1)_B\) charge 1 and -1, respectively, \(\hat{\Phi}_L\) and \(\hat{\varphi}_L\) have \(U(1)_L\) charge -2 and 2, respectively. For superfields \(\hat{X}\) and \(\hat{X}'\), \(U(1)_B\) charge is \(2/3 + B_4\) and \(-(2/3 + B_4)\), respectively. Here the lightest \(X\) could be a dark matter candidate.

The superpotential in BLMSSM is written as

\[ W_{BLMSSM} = W_{MSSM} + W_B + W_L + W_X, \tag{1} \]

where \(W_{MSSM}\) is the superpotential of MSSM, and

\[ W_B = \lambda_Q \hat{Q}_4 \hat{Q}_5 \hat{\Phi}_B + \lambda_4 \hat{U}_4^c \hat{U}_5 \hat{\varphi}_B + \lambda_D \hat{D}_4^c \hat{D}_5 \hat{\varphi}_B + \mu_B \hat{\Phi}_B \hat{\varphi}_B + Y_{u4} \hat{Q}_4 \hat{H}_u \hat{E} + Y_{u5} \hat{Q}_5 \hat{H}_u \hat{E}_5 + Y_{d4} \hat{Q}_4 \hat{H}_d \hat{D} + Y_{d5} \hat{Q}_5 \hat{H}_d \hat{D}_5, \]

\[ W_L = Y_{e4} \hat{L}_4 \hat{H}_u \hat{E}_4 + Y_{e5} \hat{L}_5 \hat{H}_u \hat{E}_5 + \mu_L \hat{\Phi}_L \hat{\varphi}_L, \]

\[ W_X = \lambda_1 \hat{Q}_5 \hat{Q}_5 \hat{X} + \lambda_2 \hat{U}_5^c \hat{X} + \lambda_3 \hat{D}_5^c \hat{D}_5 \hat{X}' + \mu_X \hat{X} \hat{X}'. \tag{2} \]
In the superpotential above, the exotic quarks obtain TeV scale masses after $\Phi$, and the nonzero VEV of $\varphi_L$ implements the seesaw mechanism for the tiny neutrino masses. Correspondingly, the soft breaking terms are generally given as

$$
\mathcal{L}_{soft} = \mathcal{L}_{soft}^{MSSM} - (m_{\tilde{N}_c}^2)_{ij} \tilde{N}_i^c \tilde{N}_j^c - m_{\tilde{Q}_4}^2 \tilde{Q}_4^c \tilde{Q}_4^c - m_{\tilde{U}_4}^2 \tilde{U}_4^c \tilde{U}_4^c - m_{\tilde{D}_4}^2 \tilde{D}_4^c \tilde{D}_4^c
$$

respectively. After the $L$-soft breaking terms are generally given as

$$
\mathcal{L}_{soft} = (m_{\tilde{N}_c}^2)_{ij} \tilde{N}_i^c \tilde{N}_j^c - m_{\tilde{Q}_4}^2 \tilde{Q}_4^c \tilde{Q}_4^c - m_{\tilde{U}_4}^2 \tilde{U}_4^c \tilde{U}_4^c - m_{\tilde{D}_4}^2 \tilde{D}_4^c \tilde{D}_4^c
$$

where $\mathcal{L}_{soft}^{MSSM}$ is the soft breaking terms of MSSM, $\lambda_B$, $\lambda_L$ are gauginos of $U(1)_B$ and $U(1)_L$, respectively. After the $SU(2)_L$ doublets $H_u$, $H_d$ and $SU(2)_L$ singlets $\Phi_B$, $\varphi_B$, $\Phi_L$, $\varphi_L$ acquiring the nonzero VEVs $v_u$, $v_d$, $v_B$, $v_L$, $\nu_L$.

$$
H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}(v_u + H_u^0 + iP_u^0) \end{pmatrix}, \\
H_d = \begin{pmatrix} H_d^- \\ \frac{1}{\sqrt{2}}(v_d + H_d^0 + iP_d^0) \end{pmatrix},
$$

$$
\Phi_B = \frac{1}{\sqrt{2}}(v_B + \Phi_B^0 + iP_B^0), \\
\varphi_B = \frac{1}{\sqrt{2}}(v_B + \varphi_B^0 + iP_B^0), \\
\Phi_L = \frac{1}{\sqrt{2}}(v_L + \Phi_L^0 + iP_L^0), \\
\varphi_L = \frac{1}{\sqrt{2}}(v_L + \varphi_L^0 + iP_L^0),
$$

the local gauge symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ is broken down to the electromagnetic symmetry $U(1)_e$.  

6
The exotic bottom quark mass matrix is given by

\[ M = \text{diag}(m_{\tilde{b}_1}, m_{\tilde{b}_2}) \]  

(5)

The exotic bottom quark mass matrix is given by

\[ M_b' = \begin{pmatrix} -\frac{1}{\sqrt{2}} Y_{Q} v_b & -\frac{1}{\sqrt{2}} Y_{Q} v_u \\ -\frac{1}{\sqrt{2}} Y_{d} v_d & \frac{1}{\sqrt{2}} \lambda_d \eta \end{pmatrix}, \]

(6)

and this mass matrix is diagonalized by two rotation matrices \( W_b' \) and \( U_b' \)

\[ W_b' \cdot M_b' \cdot U_b' = \text{diag}(m_{\tilde{b}_1}', m_{\tilde{b}_2}'). \]

(7)

The mass matrix of the first three families up-type scalar quark is given as follow

\[ M_{U_i}^2 = \begin{pmatrix} m_{U_i}^2 + m_{\tilde{b}_i}^2 & D_{L_i}^{\tilde{b}_i} + \frac{2}{3} m_{\tilde{b}_i}^2 \cos 2\beta_B & m_{\tilde{u}_i} (A_{U_i} - \mu^* \cot \beta) \\ m_{\tilde{u}_i} (A_{U_i} - \mu^* \cot \beta) & m_{U_i}^2 + m_{\tilde{b}_i}^2 + D_{R_i}^{\tilde{b}_i} - \frac{2}{3} m_{\tilde{b}_i}^2 \cos 2\beta_B \end{pmatrix}, \]

(8)

which has some differences from that of MSSM, here \( m_{\tilde{b}_i}^2 = g_b^2 (v_b^2 + \bar{\tau}_b^2) \) is the mass squared of \( U(1)_B \) gauge boson \( Z_B \), and the \( D \)-terms are

\[ D_{L}^{\tilde{b}_i} = \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) m_B^2 \cos 2\beta, \]

\[ D_{R}^{\tilde{b}_i} = -\frac{2}{3} \sin^2 \theta_W m_B^2 \cos 2\beta. \]

(9)

In the basis \((\tilde{Q}_4^2, \tilde{D}_4^c, \tilde{Q}_5^{1c}, \tilde{D}_5)\), the mass term for the exotic bottom scalar quarks in the Lagrangian reads as

\[ -\mathcal{L}_{\text{mass}} = \left( \tilde{Q}_4^2, \tilde{D}_4^c, \tilde{Q}_5^{1c}, \tilde{D}_5 \right) \cdot \mathcal{M}_{\tilde{b}_i}^2 \cdot \left( \tilde{Q}_4^2, \tilde{D}_4^c, \tilde{Q}_5^{1c}, \tilde{D}_5 \right)^\dagger, \]

(10)

where \( \mathcal{M}_{\tilde{b}_i}^2 \) is a \( 4 \times 4 \) matrix, and the matrix elements are listed as follows

\[ (\mathcal{M}_{\tilde{b}_i}^2)_{11} = m_{U_4}^2 + \frac{1}{2} Y_{Q} v_u^2 + \frac{1}{2} Y_{d} v_d^2 + \frac{1}{2} \lambda_Q v_B^2 - \left( \frac{1}{2} - \frac{2}{3} s_w^2 \right) m_B^2 \cos 2\beta + \frac{B}{2} m_{\tilde{b}_i}^2 \cos 2\beta_B, \]

\[ (\mathcal{M}_{\tilde{b}_i}^2)_{12} = (\mathcal{M}_{\tilde{b}_i}^2)_{21} = -\frac{1}{\sqrt{2}} Y_{Q} v_d A_{d_4} + \frac{1}{\sqrt{2}} Y_{d} \mu v_d, \]

\[ (\mathcal{M}_{\tilde{b}_i}^2)_{13} = (\mathcal{M}_{\tilde{b}_i}^2)_{31} = -\frac{1}{\sqrt{2}} \lambda_Q v_B A_{B_4} + \sqrt{2} \lambda_B \mu v_B, \]
(M^2_{\tilde{b}^\prime})_{14} = (M^2_{\tilde{b}^\prime})^{*}_{41} = -\frac{1}{\sqrt{2}} Y_{d_4} \lambda_d v_d \bar{\sigma}_B + \frac{1}{\sqrt{2}} Y_{d_5} \lambda_Q v_u v_B ,

(M^2_{\tilde{b}^\prime})_{22} = m_{\tilde{b}_4}^2 + \frac{1}{2} Y_{d_4} v_d^2 + \frac{1}{2} \lambda_d^2 \bar{\sigma}_B - \frac{1}{3} s_w m_Z^2 \cos 2\beta - B_i \frac{1}{2} m_{Z_B}^2 \cos 2\beta ,

(M^2_{\tilde{b}^\prime})_{23} = (M^2_{\tilde{b}^\prime})^{*}_{32} = \frac{1}{2} \lambda_Q Y_{d_4} v_d v_B + \frac{1}{2} \lambda_d Y_{d_5} v_u \bar{\sigma}_B ,

(M^2_{\tilde{b}^\prime})_{24} = (M^2_{\tilde{b}^\prime})^{*}_{42} = -\frac{1}{\sqrt{2}} \lambda_d A_{BD} \bar{\sigma}_B + \frac{1}{\sqrt{2}} \lambda_d \mu_B v_B ,

(M^2_{\tilde{b}^\prime})_{33} = m_{\tilde{b}_5}^2 + \frac{1}{2} Y_{d_5} v_u^2 + \frac{1}{2} \lambda_d^2 \bar{\sigma}_B + \frac{1}{3} s_w m_Z^2 \cos 2\beta - \frac{1}{2} B_i m_{Z_B}^2 \cos 2\beta ,

(M^2_{\tilde{b}^\prime})_{44} = m_{\tilde{b}_4}^2 + \frac{1}{2} Y_{d_4} v_d^2 + \frac{1}{2} \lambda_d^2 \bar{\sigma}_B + \frac{1}{3} s_w m_Z^2 \cos 2\beta + \frac{1}{2} B_i m_{Z_B}^2 \cos 2\beta ,

(M^2_{\tilde{b}^\prime})_{34} = (M^2_{\tilde{b}^\prime})^{*}_{43} = Y_{d_5} A_{d_5} v_u + \frac{1}{\sqrt{2}} Y_{d_5} \mu v_d .

The mass-squared matrix \( M^2_{\tilde{b}^\prime} \) is diagonalized by the unitary matrix \( Z_{\tilde{b}^\prime} \)

\[
Z_{\tilde{b}^\prime}^\dagger \cdot M^2_{\tilde{b}^\prime} \cdot Z_{\tilde{b}^\prime} = \text{diag}\left( m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{b}_3}^2, m_{\tilde{b}_4}^2 \right),
\]

and the physical states are related to the gauge states by

\[
\begin{pmatrix}
\tilde{b}_1' \\
\tilde{b}_2' \\
\tilde{b}_3' \\
\tilde{b}_4'
\end{pmatrix} = Z^\dagger_{\tilde{b}^\prime} \cdot
\begin{pmatrix}
\tilde{Q}_{4_1}^2 \\
\tilde{D}_{4_5}^{1c*} \\
\tilde{Q}_{5_4}^{1c*} \\
\tilde{D}_{5_4}^c
\end{pmatrix} .
\]

The mass squared matrix in the basis \((X^*, X^\prime)\) is

\[
M^2_{X^*} = \begin{pmatrix}
\mu_X^2 + \frac{1}{2} (\frac{2}{3} + B_4) m_{Z_B}^2 \cos 2\beta & -\mu_X^* B_X^* \\
\mu_X B_X & \mu_X^2 - \frac{1}{2} (\frac{2}{3} + B_4) m_{Z_B}^2 \cos 2\beta
\end{pmatrix} .
\]

Adopting the unitary transformation, the mass eigenstates are

\[
\begin{pmatrix}
X_1 \\
X_2
\end{pmatrix} = Z^\dagger_X \cdot
\begin{pmatrix}
X \\
X^\prime
\end{pmatrix} ,
\]

and the mass squared matrix \( M^2_X \) is diagonalized by

\[
Z^\dagger_X \cdot M^2_X \cdot Z_X = \text{diag}\left( m_{X_1}^2, m_{X_2}^2 \right).
\]

In four-component Dirac spinors, the mass term for superfields \( \tilde{X} \) is given by

\[
-\mathcal{L}^{\text{mass}}_X = \mu_X \bar{\tilde{X}} \tilde{X} ,
\]
here, we have defined
\[ \tilde{X} = \begin{pmatrix} \psi_X \\ \bar{\psi}_X' \end{pmatrix}. \] (18)

So the parameter \( \mu_X \) is the mass of the particle \( \tilde{X} \).

In mass basis, we obtain the couplings of quark-exotic quark and the superfields \( X \)
\[ \mathcal{L}_{X'u'd} = \sum_{\delta, \epsilon = 1}^2 \left( -\lambda_1 (W_{\nu}^\dagger)_{\delta 1} (Z_X)_{1, \delta} X_i \bar{b}_\delta P_L d^I - \lambda_3^* (U_{\nu}^\dagger)_{\delta 2} (Z_X)_{2, \delta} X_i \bar{b}_\delta P_R d^I \right) + \text{h.c.}, \] (19)

We also obtain the couplings of quark-exotic scalar quark and the field \( \tilde{X} \)
\[ \mathcal{L}_{\tilde{X}'d} = \sum_{\rho = 1}^4 \left( -\lambda_1 (Z_{\nu}^\dagger)_{3\rho} \bar{b}_\rho \tilde{X} P_L d^I - \lambda_3^* (Z_{\nu}^\dagger)_{4\rho} \bar{b}_\rho \tilde{X} P_R d^I \right) + \text{h.c.}, \] (20)

where \( \lambda_1, \lambda_3 \) are the coupling coefficients, and \( \delta, \epsilon, \rho \) are the indices of the flavor.

Considering the radiative corrections, the mass squared matrix for the neutral CP-even Higgs in the basis \( (H_d^0, H_u^0) \) is written as \[99\text{–}110\]
\[ \mathcal{M}_{\text{even}}^2 = \begin{pmatrix} M_{11}^2 + \Delta_{11} & M_{12} + \Delta_{12} \\ M_{12} & M_{22} + \Delta_{22} \end{pmatrix}, \] (21)

where
\[ M_{11}^2 = m_Z^2 \cos^2 \beta + m_{A^0}^2 \sin^2 \beta, \]
\[ M_{12}^2 = -(m_Z^2 + m_{A^0}^2) \sin \beta \cos \beta, \]
\[ M_{22}^2 = m_Z^2 \sin^2 \beta + m_{A^0}^2 \cos^2 \beta, \]
\[ \Delta_{11} = \Delta_{11}^{\text{MSSM}} + \Delta_{11}^B + \Delta_{11}^L, \]
\[ \Delta_{12} = \Delta_{12}^{\text{MSSM}} + \Delta_{12}^B + \Delta_{12}^L, \]
\[ \Delta_{22} = \Delta_{22}^{\text{MSSM}} + \Delta_{22}^B + \Delta_{22}^L, \] (22)

and the expressions of \( \Delta_{11}^B, \Delta_{12}^B, \Delta_{22}^B \) can be found in Refs. \[65, 66\]. A Higgs around 125 GeV has been observed at the LHC by ATLAS \[111\] and CMS \[112\] with the combined significances of 5.9 and 5.0 standard deviations, respectively. So after diagonalizing the mass
squared matrix, the lightest neutral CP even Higgs \( m_{h_0} \) should satisfy this constraint. To obtain the Higgs \( h_0 \) with mass of 125 GeV gives a strong limit on the parameter space. Considering this constraint, we can also obtain \( m_{A^0}^2 \) from the inverse solution of Eq. (21). We have

\[
m_{A^0}^2 = \frac{m_{h_0}^2 (m_{h_0}^2 - m_{h_0}^2 + \Delta_{11} + \Delta_{22}) - m_{h_0}^2 \Delta_A + \Delta_{12}^2 - \Delta_{11} \Delta_{22}}{-m_{h_0}^2 + m_e^2 \cos^2 2\beta + \Delta_B},
\]

where

\[
\Delta_A = \sin^2 \beta \Delta_{11} + \cos^2 \beta \Delta_{22} + \sin 2\beta \Delta_{12},
\]

\[
\Delta_B = \cos^2 \beta \Delta_{11} + \sin^2 \beta \Delta_{22} + \sin 2\beta \Delta_{12}.
\]

For the charged Higgs scalars, \( H_{1,2}^\pm \) are related to the initial Higgs by the matrix \( Z_H \), and the charged Higgs mass \( m_{H^\pm_1} \) satisfy a relation with the pseudo-scalar Higgs mass \( m_{A^0} \) at tree-level:

\[
m_{H^\pm_1} = \sqrt{m_{A^0}^2 + m_W^2},
\]

Using the Feynman–t’Hooft gauge, another charged Higgs boson \( H_2^\pm \) has the same mass as the gauge boson \( W \).

III. \( B^0 - \bar{B}^0 \) MIXING

When external masses and momenta are neglected, the general form of the effective Hamiltonian for \( B^0 - \bar{B}^0 \) mixing at the weak scale can be expressed as

\[
H_{\text{eff}} = \frac{1}{4} \frac{G_F^2}{\pi^2} m_\pi^2 \sum_{\alpha=1}^8 C_\alpha O_\alpha,
\]

where \( G_F \) denotes the Fermi constant, \( C_\alpha \) are the corresponding Wilson coefficients, \( O_\alpha \) are the effective operators, which read as

\[
O_1 = \bar{d} \gamma_\mu P_L b d \gamma^\mu P_L b,
\]

\[
O_2 = \bar{d} \gamma_\mu P_L b d \gamma^\mu P_R b,
\]

\[
O_3 = \bar{d} P_L b d P_R b,
\]
\[ O_4 = \bar{d}P_L b \bar{d}P_L b, \]
\[ O_5 = \bar{d}\sigma_{\mu\nu} P_L b \bar{d}\sigma^{\mu\nu} P_L b, \]
\[ O_6 = \bar{d}\gamma_{\mu} P_R b \bar{d}\gamma^{\mu} P_R b, \]
\[ O_7 = \bar{d}P_R b \bar{d}P_R b, \]
\[ O_8 = \bar{d}\sigma_{\mu\nu} P_R b \bar{d}\sigma^{\mu\nu} P_R b, \]
(27)

where \( P_{R,L} = (1 \pm \gamma_5)/2 \) denote the chiral projectors, \( \sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2 \), the \( SU(3) \) color indices here have omitted for simplicity.

FIG. 1: The box diagram contributions to \( B^0 - \bar{B}^0 \) mixing in the SM.
FIG. 2: The box diagrams contributing to $B^0 - \bar{B}^0$ mixing in the BLMSSM.

The box diagram contributions to $B^0 - \bar{B}^0$ mixing from the SM are displayed in Fig. 1 and the box diagrams contributing to $B^0 - \bar{B}^0$ mixing in the BLMSSM are shown in Fig. 2. Note that the diagrams including the particles $\tilde{\chi}$ and $\tilde{X}$ should make a Fierz rearrangement to ensure that the operators are color singlet states as follows

$$O'_1 = O_1,$$
$$O'_2 = O_2,$$
$$O'_3 = -\frac{1}{2}O_2,$$
$$O'_4 = -\frac{1}{2}O_4 - \frac{1}{8}O_5,$$
After this, the Wilson coefficients are given as follows

\[
O' = -6O_4 + \frac{1}{2}O_5, \\
O' = O_6, \\
O' = -\frac{1}{2}O_7 - \frac{1}{8}O_8, \\
O' = -6O_7 + \frac{1}{2}O_8.
\]

The operators with a prime stand for the product of two color non-singlet quark current.

After this, the Wilson coefficients are given as follows

\[
C_1 = V_{ib} V_{id} V_{jd} V_{j}^* \left( f_p^2(x_{u_i}, x_{W}, x_{u_j}, x_{W}) - 2\frac{x_{u_i} x_{u_j}}{\sin^2 \beta} (Z_H^{2k})^2 f_1(x_{u_i}, x_{H^-}, x_{u_j}, x_{W}) \\
+ \frac{x_{u_i} x_{u_j}}{4 \sin^2 \beta} (Z_H^{2k})^2 f_p^2(x_{u_i}, x_{H^-}, x_{u_j}, x_{H^-}) + Z_{i\alpha}^\lambda Z_{j\beta}^\nu Z_{i\alpha}^\nu f_p^2(x_{u_i}, x_{\chi^-}, x_{u_j}, x_{\chi^-}) \right) \\
+ \frac{1}{32\alpha^2 m_W^2} |\lambda_1|^4 \left( |(W_{\nu})_\delta,1|^2 |(W_{\nu})_\delta,1|^2 |(Z_X)_{1,l}|^2 |(Z_X)_{1,l}|^2 f_p^2(x_{u_i}, x_{u_j}, x_{W}) \right) \\
+ |Z_{\nu}^\nu|_3 f_p^2(x_{X}, x_{u_i}, x_{u_j})
\]

\[
C_2 = V_{ib} V_{id} V_{jd} V_{j}^* \left( \frac{1}{4 \sin^2 \beta} (x_{u_i} + x_{u_j}) Z_{i\alpha}^\lambda Z_{j\beta}^\nu Z_{i\alpha}^\nu f_p^2(x_{u_i}, x_{H^-}, x_{u_j}, x_{H^-}) \right) \\
+ (Z_{i\alpha}^\lambda Z_{j\beta}^\nu Z_{i\alpha}^\nu + Z_{i\alpha}^\lambda Z_{j\beta}^\nu Z_{i\alpha}^\nu f_p^2(x_{u_i}, x_{\chi^-}, x_{u_j}, x_{\chi^-}) \right) \\
- 2 \sqrt{\frac{x_{u_i} x_{u_j}}{4 \sin^2 \beta}} (Z_{i\alpha}^\lambda Z_{j\beta}^\nu Z_{i\alpha}^\nu + Z_{i\alpha}^\lambda Z_{j\beta}^\nu Z_{i\alpha}^\nu f_p^2(x_{u_i}, x_{\chi^-}, x_{u_j}, x_{\chi^-}) \right) \\
+ \frac{1}{64\alpha^2 m_W^2} |\lambda_1|^2 |\lambda|^2 \left( |(W_{\nu})_\delta,1|^2 |(W_{\nu})_\delta,1|^2 |(Z_X)_{1,l}|^2 |(Z_X)_{1,l}|^2 f_p^2(x_{u_i}, x_{u_j}, x_{W}) \right) \\
+ |(W_{\nu})_\delta,1|^2 |(W_{\nu})_\delta,1|^2 |(Z_X)_{1,l}|^2 |(Z_X)_{1,l}|^2 f_p^2(x_{u_i}, x_{u_j}, x_{W}) \\
+ 2 ((Z_{\nu}^\nu)_{3\alpha} (Z_{\nu}^\nu)_{4\alpha} (Z_{\nu}^\nu)_{4\alpha} + (Z_{\nu}^\nu)_{3\alpha} (Z_{\nu}^\nu)_{4\alpha} (Z_{\nu}^\nu)_{4\alpha} f_p^2(x_{X}, x_{u_i}, x_{u_j}) \right) \\
+ 2 |(Z_{\nu}^\nu)|_3 |(Z_{\nu}^\nu)|_3 f_p^2(x_{X}, x_{u_i}, x_{u_j}) \right)
\]

(28)
\[ C_3 = V_{ib} V^{*}_{id} V_{jb} V^{*}_{jd} \left( - \frac{2 \sqrt{x_{u_1} x_{u_2} x_{d_1}}}{\sin^{2} \beta \cos^{2} \beta} (Z_H^1)^2 f_p^2 (x_{u_1}, x_{H^+_i}, x_{u_j}, x_W) \
+ \frac{x_{u_1} x_{u_2} \sqrt{x_{u_2} x_{d_1}}}{\sin^{2} \beta \cos^{2} \beta} ((Z_H^1)^2 (Z_H^2)^2 + (Z_H^2)^2 (Z_H^1)^2) f_1 (x_{u_1}, x_{H^+_i}, x_{u_j}, x_{H^-_i}) \right) \
+ 4 \sqrt{x_{\kappa} x_{\eta} x_{\kappa} x_{\eta}} (Z^{\lambda}_{\kappa \lambda} Z^{\kappa \
\delta}_{\lambda \gamma} Z^{\eta \kappa \
\nu}_{\gamma \xi} Z^{\xi x_{\lambda} x_{\eta}} \
\delta_{\gamma \
\beta} Z^{\eta \gamma}_{\beta \xi} Z^{\lambda \kappa}_{\xi \eta} f_1 (x_{u_1}, x_{H^-_j}, x_{u_1}, x_{H^+_i}) \right) \)
\]

\[ C_4 = V_{ib} V^{*}_{id} V_{jb} V^{*}_{jd} \left( \frac{x_{u_1} x_{u_2} x_{d_1}}{\sin^{2} \beta \cos^{2} \beta} (Z_H^1)^2 (Z_H^2)^2 f_1 (x_{u_1}, x_{H^+_i}, x_{u_j}, x_{H^-_i}) \right) \
+ \sqrt{x_{\kappa} x_{\eta} x_{\kappa} x_{\eta}} (Z^{\lambda}_{\kappa \lambda} Z^{\kappa \
\delta}_{\lambda \gamma} Z^{\eta \kappa \
\nu}_{\gamma \xi} \delta_{\gamma \
\beta} Z^{\eta \gamma}_{\beta \xi} f_1 (x_{u_1}, x_{H^-_j}, x_{u_1}, x_{H^+_i}) \right) \)
\]

\[ C_5 = - \frac{1}{4} V_{ib} V^{*}_{id} V_{jb} V^{*}_{jd} \sqrt{x_{\kappa} x_{\eta} x_{\kappa} x_{\eta}} (Z^{\lambda}_{\kappa \lambda} Z^{\kappa \
\delta}_{\lambda \gamma} Z^{\eta \kappa \
\nu}_{\gamma \xi} \delta_{\gamma \
\beta} Z^{\eta \gamma}_{\beta \xi} f_1 (x_{u_1}, x_{H^-_j}, x_{u_1}, x_{H^+_i}) \right) \]

\[ - \frac{1}{128 \sqrt{\beta} \cos \beta} \lambda_1^{2} \lambda_2^{2} (Z^{\beta}_{\alpha x_{\lambda}} Z^{\alpha \
\gamma}_{x_{\beta}} Z^{\gamma \alpha}_{x_{\lambda}} f_1 (x_{\beta}, x_{\lambda}, x_{\alpha}, x_{\eta}) \right) \]
\[ C_6 = V_{ib} V_{id}^* V_{jd} V_{jb}^* \left( \frac{x_{ib} x_{id}}{4 \sin^2 \beta \cos^2 \beta} (Z^1_{i \alpha})^2 (Z^1_H)^2 f_{p^2} (x_{ui}, x_{H^-}, x_{uj}, x_{H^+_l}) \right. \\
\left. + Z^b_{i \alpha} Z^{b \alpha} Z_{j \beta}^b Z^{b \eta} f_{p^2} (x_{\bar{u}^i}, x_{\bar{\chi}^-}, x_{\bar{u}^j}, x_{\bar{\chi}^-}) \right) \\
\left. + \frac{1}{32 G_F m_W^2} |\lambda^3|^4 \left( |(U^b_L)^\dagger \sigma_2|^2 |(U^b_L)^\dagger \sigma_2|^2 |(Z_X)_{2, k}|^2 |(Z_X)_{2, l}|^2 f_{p^2} (x_{b^i}, x_{X_k}, x_{b^j}, x_{X_l}) \right. \\
\left. + |(Z_{b^i})_{4 \rho}|^2 |(Z_{b^j})_{4 \sigma}|^2 f_{p^2} (x_{\bar{X}}, x_{\bar{b^i}}, x_{\bar{X}}^*, x_{\bar{b^j}}) \right) \right) \]

\[ C_7 = V_{ib} V_{id}^* V_{jd} V_{jb}^* \left( \frac{x_{ib} x_{id} x_{tb}}{4 \sin^2 \beta \cos^2 \beta} (Z^k_{i \alpha} Z^H_{2k} Z^1_H Z^H_{2 l} f_1(x_{ui}, x_{H^-}, x_{uj}, x_{H^+_l}) \right. \\
\left. + \sqrt{x_{\kappa \lambda} x_{\kappa \eta} Z^\lambda_{i \alpha} Z^{b \eta} f_1(x_{\bar{u}^i}, x_{\bar{\chi}^-}, x_{\bar{u}^j}, x_{\bar{\chi}^-}) \right) \\
\left. + \frac{1}{32 G_F m_W^2} |\lambda^3|^2 \left( 4 (W^b_L)^\dagger (W^b_L)^\dagger (U^b_L)^\dagger \sigma_2 (U^b_L)^\dagger (Z_X)_{1, k} (Z_X)_{2, l} \right. \\
\left. \times \sqrt{x_{b^i} x_{b^j} f_1(x_{b^i}, x_{X_k}, x_{b^j}, x_{X_l}) + (Z_{b^i})_{3 \rho} (Z_{b^j})_{3 \sigma} f_1(x_{\bar{X}}, x_{\bar{b^i}}, x_{\bar{X}}^*, x_{\bar{b^j}}) \right) \right) \right) \]

\[ C_8 = -\frac{1}{4} V_{ib} V_{id}^* V_{jd} V_{jb}^* \sqrt{x_{\kappa \lambda} x_{\kappa \eta} Z^\lambda_{i \alpha} Z^{b \alpha} Z^{b \eta} f_1(x_{\bar{u}^i}, x_{\bar{\chi}^-}, x_{\bar{u}^j}, x_{\bar{\chi}^-}) \\
- \frac{1}{128 G_F m_W^2} |\lambda^3|^2 (Z_{b^i})_{3 \alpha} (Z_{b^j})_{3 \beta} (Z_{b^j})_{4 \beta} f_1(x_{\bar{X}}, x_{\bar{b^i}}, x_{\bar{X}}^*, x_{\bar{b^j}}) \right) \]

(29)

For convenience, we have defined the ratio of mass square as: \( x_i = m_i^2 / m_W^2 \), and here \( Z^\lambda_{i \alpha} \), \( Z^{b \lambda}_{i \alpha} \) ... have been defined as

\[
Z^\lambda_{i \alpha} = \frac{-Z^1_{i \alpha} Z^1_{i \alpha}}{Z^1_{i \alpha} + \frac{\sqrt{2 x_u}}{2 \sin \beta} Z^{2 \alpha}_{i \alpha} Z^{2 \alpha}_{i \alpha}} \\
Z^{b \lambda}_{i \alpha} = \frac{\sqrt{2 x_d}}{2 \cos \beta} Z^{1 \alpha}_{i \alpha} Z^{2 \alpha}_{i \alpha} \\
\ldots
\]

(30)
Here $f_1$ and $f_{p^2}$ are the functions related to the one-loop integral functions.

$$
\mu^{2\epsilon}\int \frac{d^Dp}{(2\pi)^D} \frac{1}{p^2 - m_1^2} \frac{1}{p^2 - m_2^2} \frac{1}{p^2 - m_3^2} \frac{1}{p^2 - m_4^2} = \frac{1}{16\pi^2 m_W^4} f_1(x_1, x_2, x_3, x_4) \quad (31)
$$

$$
\mu^{2\epsilon}\int \frac{d^Dp}{(2\pi)^D} \frac{1}{p^2 - m_1^2} \frac{1}{p^2 - m_2^2} \frac{1}{p^2 - m_3^2} \frac{1}{p^2 - m_4^2}^2 = \frac{1}{16\pi^2 m_W^4} f_{p^2}(x_1, x_2, x_3, x_4) \quad (32)
$$

The analytical expressions for the functions $f_{p^2}(x_1, x_2, x_3, x_4)$ and $f_1(x_1, x_2, x_3, x_4)$ are listed in Appendix A. It should be noted that we need perform summation over the repeated indices in the calculations.

The matching scale is chosen as $\mu_0 = \mu_W$ in our calculations. Now we should evolve the coefficients from the scale $\mu_W$ down to the $B$-meson scale $\mu_b$.

$$
\left[ \mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} - \frac{\hat{\gamma}_T}{2} \right] \bar{C}(\mu, \alpha_s) = 0. \quad (33)
$$

By solving the remormalization group equation [114], we have

$$
\bar{C}(\mu_b) = W(\mu_b, \mu_W) \bar{C}(\mu_W) \quad (34)
$$

with

$$
W(\mu_b, \mu_W) = \left[ \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right]^{\frac{\gamma^{(0)}}{2\beta_0}}, \quad (35)
$$

where $\gamma^{(0)}$ is the anomalous dimensions matrix (ADM) [114, 115], and $\beta_0 = \frac{11N_c - 2n_f}{3}$ with $N_c$ denoting the number of colors and $n_f$ denoting the number of active quark flavors.

The mass difference of $B^0 - \bar{B}^0$ mixing can be expressed as

$$
\Delta m_B = \frac{\left| \left\langle \bar{B}^0 | H_{\text{eff}} (\Delta B = 2) | B^0 \right\rangle \right|}{m_B}. \quad (36)
$$

After substituting Eq. (26) into the above equation, at $B$-meson scale, the mass difference $\Delta m_B$ can be written by

$$
\Delta m_B = \frac{1}{4} \frac{G_F^2}{\pi^2 m_W^2} \sum_{\alpha=1}^{8} \left| C_\alpha(\mu_b) \left\langle B^0 | O_\alpha(\mu_b) | B^0 \right\rangle \right| \frac{1}{m_B}, \quad (37)
$$
where, the matrix elements $\langle B^0 | O_\alpha | B^0 \rangle$ require non-perturbative QCD calculations by the lattice Monte Carlo estimates. The matrix element is parameterized as $\langle B^0 | O_1 | B^0 \rangle = \frac{2}{3} B_B(\mu) f_B^2 m_B^2$, and the other hadronic matrix elements parameterized are listed in Appendix B.

**IV. THE NUMERICAL ANALYSIS**

In our calculations for the CKM matrix, we apply the Wolfenstein parametrization and set $A = 0.81$, $\lambda = 0.22$, $\rho = 0.135$, $\eta = 0.349$. For the hadronic matrix element, the recent average of the lattice results is $f_{B_d} \sqrt{B_{B_d}} = 216 \pm 15$ (MeV) [116], and we adopt the central value of the $f_{B_d} \sqrt{B_{B_d}}$ in our calculations. The other SM parameters are chosen as $m_W = 80.385$ GeV, $m_u = 2.3 \times 10^{-3}$ GeV, $m_c = 1.275$ GeV, $m_t = 173.5$ GeV, $m_b = 4.18$ GeV, $m_d = 4.8 \times 10^{-3}$ GeV, $m_B = 5.279$ GeV, $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$, $\alpha_s(m_W) = 0.12$, $\alpha_s(m_b) = 0.22$ [86].

Now we investigate the numerically behavior of these parameters to the $B^0 - \bar{B}^0$ mixing in BLMSSM. This model contains many parameters. In our following discussions, the parameters needed to study contain $\lambda_{1,3}$, $\mu_B$, $m_{Z_B}$, $m_{D_5}$, $\mu_X$. The other parameters are adopted as Refs. [66, 67] which have been analyzed in the signals of decay channels $h \to \gamma \gamma$ and $h \to VV^*(V = Z, W)$ with the Higgs mass around 125 GeV.

\begin{align*}
&m_{\tilde{Q}_{1,2,3}} = m_{\tilde{U}_{1,2,3}} = m_{\tilde{D}_{1,2,3}} = 1 \text{ TeV}, \\
&A_{d,s,b} = A_{u,c,t} = -1 \text{ TeV}, \\
&M_2 = 750 \text{ GeV}, \\
&B_4 = L_4 = \frac{3}{2}, \\
&\tan \beta = \tan \beta_B = \tan \beta_L = 2, \\
&\mu = -800 \text{ GeV}, \\
&m_{\tilde{B}_4} = m_{\tilde{D}_4} = m_{\tilde{E}_5} = 1 \text{ TeV}, \\
&m_{\tilde{U}_4} = m_{\tilde{E}_4} = m_{\tilde{E}_5} = m_{\tilde{L}_5} = m_{\tilde{N}_5} = 1 \text{ TeV}, \\
&A_{s_4} = A_{s_5} = A_{s_5} = A_{u_4} = A_{u_5} = A_{d_4} = A_{d_5} = 550 \text{ GeV},
\end{align*}
\(v_B = \sqrt{v_B^2 + \tau_B^2} = 3 \text{ TeV},\)
\(A_{BQ} = A_{BU} = A_{BD} = 1 \text{ TeV},\)
\(Y_{u_4} = 0.76Y_t,\)
\(Y_{d_4} = Y_{u_5} = 0.7Y_b,\)
\(Y_{d_5} = 0.13Y_t,\)
\(\lambda_Q = \lambda_u = \lambda_d = 0.5,\)
\(m_{\nu_4} = m_{\nu_5} = 90 \text{ GeV},\)
\(m_{\nu_4} = m_{\nu_5} = B_X = 100 \text{ GeV},\)
\(m_{\tilde{Q}_4} = 790 \text{ GeV}.\) (38)

In order to see the dependence of the mass difference \(\Delta m_B\) on the parameters space in the BLMSSM, we fix \(m_{\tilde{Q}_5} = 1 \text{ TeV}, m_{\tilde{D}_5} = 1 \text{ TeV}, \mu_B = 500 \text{ GeV}, \mu_X = 2.4 \text{ TeV}, m_{Z_B} = 1 \text{ TeV.}\) From the Wilson coefficients listed in Section III one can see that the mass difference \(\Delta m_B\) is the continuous function of the parameters \(\lambda_1\) and \(\lambda_3,\) and because of the fourth power of \(\lambda_{1,3}\) the \(\Delta m_B\) should remarkably increase with the increasing of \(|\lambda_1|\) and \(|\lambda_3|\). So \(\lambda_1\) and \(\lambda_3\) play an important role to the theoretical prediction on \(\Delta m_B.\) Next, the influence of the parameters \(\lambda_{1,3}\) to \(\Delta m_B\) will be discussed in detail. We plot the contours corresponding to the mass difference \(\Delta m_B\) in the parameter space of \(\lambda_1\) and \(\lambda_3\) in Fig. 3. We can see that \(\Delta m_B\) increases as \(|\lambda_{1,3}|\) increases, and sensitively depends on \(|\lambda_{1,3}|\) when \(|\lambda_1|\) and \(|\lambda_3|\) are both larger than 0.2. As one can see, the values of \(|\lambda_1|\) and \(|\lambda_3|\) that all is larger than 0.25 are disfavored by experiment results under this given assumption.

Next, we investigate the dependence of \(\Delta m_B\) on the parameter \(m_{Z_B}.\) In Fig. 4 we plot \(\Delta m_B\) varying with the mass of neutral \(U(1)_B\) gauge boson \(Z_B,\) when \(\lambda_1 = 0.25\) and \(\lambda_3 = 0.2.\) The figure shows that \(\Delta m_B\) decreases as the \(m_{Z_B}\) increases. However, it should be noted that the value of the \(m_{Z_B}\) should not be too large, in order to avoid some tachyons appearing, as well as to coincide with the current experimental result on the mass of squarks. Actually, the corrections of some other parameters to \(\Delta m_B\) are small, such as \(m_{\tilde{D}_4}, m_{\tilde{Q}_4}\) and \(B_X,\) which we would not discuss in this paper.

In the following discussions, we choose \(\lambda_1 = 0.2\) for simplicity. Now, we investigate the
FIG. 3: (Color online) Contour plots of $\Delta m_B$ in the parameter space of $\lambda_1$ and $\lambda_3$.

FIG. 4: The mass difference $\Delta m_B$ versus the new gauge boson mass $m_{Z_B}$.

dependence of $\Delta m_B$ on the parameter $\mu_B$. Considering that $\mu_B$ is the mass parameter of the "brand new" Higgs superfields $\Phi_B$ and $\phi_B$, the behavior of the $\Delta m_B$ versus $\mu_B$ when $\lambda_3 = 0.25$ is shown in Fig. [3] The numerical result shows that the contribution of the
FIG. 5: The mass difference $\Delta m_B$ as a function of $\mu_B$.

...parameter $\mu_B$ to $\Delta m_B$ is quite small, when $\mu_B$ is lighter than 500 GeV. When $\mu_B$ is heavier than 500 GeV, $\Delta m_B$ decreases sharply with the increasing of $\mu_B$.

We plot $\Delta m_B$ as a function of the exotic right-handed soft-SUSY-breaking squark mass $m_{\tilde{D}_5}$ for three values of $\lambda_3$ in Fig. 6, the dotted line corresponds to the result of $\lambda_3 = 0.2$, the dashed line corresponds to the result of $\lambda_3 = 0.25$, the dot-dashed line corresponds to the result of $\lambda_3 = 0.3$. The light gray area denotes the $\Delta m_B^{SM}$ at 1$\sigma$, and the gray area denotes the $\Delta m_B^{Exp}$ at 1$\sigma$. As one can see, $\Delta m_B$ decreases along with the increasing of $m_{\tilde{D}_5}$ for a given value of $\lambda_3$. Fig. 6 also exhibits that $\Delta m_B$ has a strong dependence on $m_{\tilde{D}_5}$ for large values of $\lambda_3$. However, this figure indicates that the $\Delta m_B$ declines slowly with the increasing of $m_{\tilde{D}_5}$, when the value of $\lambda_3$ is small. Generally speaking, the influence of the $m_{\tilde{D}_5}$ to $\Delta m_B$ can be neglected as $\lambda_3$ is enough small. Considering the constraint from the $\Delta m_B^{SM}$ at 1$\sigma$, one can see that small values of $m_{\tilde{D}_5}$ can be excluded for large value of $\lambda_3$ as well as large values of $m_{\tilde{D}_5}$ can be excluded for small value of $\lambda_3$ under the given assumption.

In Fig. 7, we study the dependence of $\Delta m_B$ on the particle $\tilde{X}$ mass $\mu_X$. The dotted line corresponds to the result when $\lambda_3 = 0.2$, the solid line corresponds to the result when $\lambda_3 = 0.25$, the dashed line corresponds to the result when $\lambda_3 = 0.3$. The light gray area...
FIG. 6: The mass difference $\Delta m_B$ varies with the parameter $m_{\tilde{D}_8}$ for three values of $\lambda_3$. The light gray area denotes the $\Delta m_B^{SM}$ at 1$\sigma$, and the gray area denotes the $\Delta m_B^{Exp}$ at 1$\sigma$.

FIG. 7: The mass difference $\Delta m_B$ as a function of $\tilde{X}$ mass $\mu_X$ for three values of $\lambda_3$. The light gray area denotes the $\Delta m_B^{SM}$ at 1$\sigma$, and the gray area denotes the $\Delta m_B^{Exp}$ at 1$\sigma$. 

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denotes the $\Delta m_{B}^{SM}$ at 1$\sigma$, and the gray area denotes the $\Delta m_{B}^{Exp}$ at 1$\sigma$. It clearly shows a large influence of the new particle $\tilde{X}$ on the mixing of $B^0 - \bar{B}^0$. The mass difference $\Delta m_{B}$ decreases with increasing of the $\mu_X$ in a very similar manner as that in Fig. 6. We find the mass of the exotic particle $\tilde{X}$ should not be too light for large values of $\lambda_3$, however, the heavy mass of the exotic particle $\mu_X$ is also constrained for small values of $\lambda_3$.

V. CONCLUSIONS

With the constraint of a 125 GeV Higgs, we analyze the correction of the extra fermions and scalars to $B^0 - \bar{B}^0$ mixing in the extension of the MSSM where baryon number and lepton number are local gauge symmetries. In this framework, the new particles’ LO correction to $B^0 - \bar{B}^0$ mixing is significant in some parameter space. The numerical evaluations indicate that the parameters $\lambda_{1,3}$, $m_{\tilde{D}_5}$ and $\mu_X$ are sensitive to the process of $B^0 - \bar{B}^0$ mixing. It is well known that the space that is left for hiding some new physics effects in the $B^0 - \bar{B}^0$ mixing is mainly given by the theoretical error. With the development of more precise theoretical analysis (especially the lattice calculations) and accurate experimental measurements, the $B^0 - \bar{B}^0$ mixing in the BLMSSM will have a clearer picture and the parameters space will also be further constrained.

Many experiments have been performed to search for baryon number violation (BNV). Belle and BaBar have obtained the upper limits on the branching fraction of BNV $\tau$ decays $\tau^- \to \Lambda\pi^-$ and $\tau^- \to \Lambda k^-$ \cite{117, 118}. Some $B$ meson decays $B^0 \to \Lambda^+_c l^-$, $B^- \to \Lambda l^-$ and $B^- \to \bar{\Lambda} l^-$ have been investigated by BaBar \cite{119}. Charged lepton flavour violation (CLFV) and BNV decays $\tau^- \to \bar{\nu}\mu^+\mu^-$ and $\tau^- \to p\mu^+\mu^-$ have been carried out by LHCb \cite{120}. Searching for baryon number violation in top-quark decays has been done by CMS \cite{121}. However, these experimental searches for BNV have yield only upper limits. On the other hand, the branching fractions of CLFV process ($\mu \to e\gamma$, $\mu \to eee$, $\tau \to l\gamma$ and $\tau \to lll$ (with $l = e, \mu$), et al.) are predicted very small in the SM. For instance, the SM prediction for branching fractions in muon decays is smaller than $10^{-50}$. In the BLMSSM, there are some new contributions to these BNV and CLFV processes. And the contributions of BLMSSM may significantly enhance these branching fractions. One can have BNV signals from the
decays of squarks and gauginos without conflict with the current experiments. For instance, if the gluino is the lightest supersymmetric particle one could have signals with multitops and multibottoms such as $pp \rightarrow \tilde{g}\tilde{g} \rightarrow ttbbjj$ (j stands for a light jet), which may be observed at the LHC [63, 64]. The projected sensitivity for future experiments that searching for the CLFV processes will be largely improved [122, 128]. And the running of LHC will resume in 2015 with higher energy and luminosity. So, it would be interesting to investigate this model. Any observation of BNV or CLFV whose branching fractions is large than that of SM prediction would be a clear sign for BSM physics. Investigating these BNV and CLFV processes can test the BLMSSM and provide constraints on the parameter space.

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Appendix A: Integral function

The functions related to the one-loop integral functions are given as

$$f_1(x_1, x_2, x_3, x_4) =$$

$$\begin{align*}
&\left\{-\frac{\log(x_1)x_1}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + \frac{\log(x_2)x_2}{(x_1-x_2)(x_2-x_3)(x_2-x_4)} + \frac{\log(x_3)x_3}{(x_1-x_3)(x_3-x_2)(x_3-x_4)} + \frac{\log(x_4)x_4}{(x_1-x_4)(x_4-x_2)(x_4-x_3)}, (x_1 \neq x_3 \text{ and } x_2 \neq x_4)\right\} \\
&\left\{-\frac{\log(x_1)x_1}{(x_1-x_2)^2(x_1-x_3)} + \frac{\log(x_2)x_2}{(x_1-x_2)^2(x_2-x_3)} + \frac{\log(x_3)x_3}{(x_1-x_3)^2(x_3-x_2)} + \frac{1}{(x_1-x_3)^2(x_4-x_3)}, (x_1 \neq x_3 \text{ and } x_2 = x_4)\right\} \\
&\left\{-\frac{\log(x_2)x_2}{(x_1-x_2)^2(x_2-x_3)} + \frac{\log(x_3)x_3}{(x_1-x_2)^2(x_3-x_2)} + \frac{\log(x_4)x_4}{(x_1-x_3)^2(x_3-x_2)} - \frac{1}{(x_1-x_3)(x_4-x_2)(x_4-x_3)}, (x_1 = x_3 \text{ and } x_2 \neq x_4)\right\} \\
&\left\{\frac{2\log(x_1)x_1x_2}{(x_1-x_2)^3} - \frac{2\log(x_2)x_1x_2}{(x_1-x_2)^3} - \frac{x_1+x_2}{(x_1-x_2)^3}, (x_1 = x_3 \text{ and } x_2 = x_4)\right\}
\end{align*}$$

$$f_{\mu^2}(x_1, x_2, x_3, x_4) =$$

$$\begin{align*}
&\left\{-\frac{\log(x_1)x_1}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + \frac{\log(x_2)x_2}{(x_1-x_2)(x_2-x_3)(x_2-x_4)} + \frac{\log(x_3)x_3}{(x_1-x_3)(x_3-x_2)(x_3-x_4)} + \frac{\log(x_4)x_4}{(x_1-x_4)(x_4-x_2)(x_4-x_3)}, (x_1 \neq x_3 \text{ and } x_2 \neq x_4)\right\} \\
&\left\{-\frac{\log(x_1)x_1}{(x_1-x_2)^2(x_1-x_3)} + \frac{\log(x_2)x_2}{(x_1-x_2)^2(x_2-x_3)} + \frac{\log(x_3)x_3}{(x_1-x_3)^2(x_3-x_2)} + \frac{1}{(x_1-x_3)^2(x_4-x_3)}, (x_1 \neq x_3 \text{ and } x_2 = x_4)\right\} \\
&\left\{-\frac{\log(x_2)x_2}{(x_1-x_2)^2(x_2-x_3)} + \frac{\log(x_3)x_3}{(x_1-x_2)^2(x_3-x_2)} + \frac{\log(x_4)x_4}{(x_1-x_3)^2(x_3-x_2)} - \frac{1}{(x_1-x_3)(x_4-x_2)(x_4-x_3)}, (x_1 = x_3 \text{ and } x_2 \neq x_4)\right\} \\
&\left\{\frac{2\log(x_1)x_1x_2}{(x_1-x_2)^3} - \frac{2\log(x_2)x_1x_2}{(x_1-x_2)^3} - \frac{x_1+x_2}{(x_1-x_2)^3}, (x_1 = x_3 \text{ and } x_2 = x_4)\right\}
\end{align*}$$
Appendix B: Hadronic matrix elements

The hadronic matrix elements can be written as
\[
\langle \bar{B}^0 | O_1 | B^0 \rangle = \frac{2}{3} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_2 | B^0 \rangle = -\frac{1}{6} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_3 | B^0 \rangle = -\frac{5}{12} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_4 | B^0 \rangle = \frac{5}{12} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_5 | B^0 \rangle = \frac{1}{3} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_6 | B^0 \rangle = \frac{2}{3} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_7 | B^0 \rangle = \frac{5}{12} B_B(\mu) f_B^2 m_B^2 \\
\langle \bar{B}^0 | O_8 | B^0 \rangle = \frac{1}{3} B_B(\mu) f_B^2 m_B^2.
\]

Here $f_B$ is the $B$-meson decay constant constant, $B_B$ is the bag parameter.

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