We present the leading order differential and total rates for $J/\psi$ production at LEP. By leading order we mean all terms of the form $\alpha_s[\alpha_s \log(M_Z^2/M_\psi^2)]^n$ and $\alpha_s^{n+1}\log^l(z^2)\log^m(M_Z^2/M_\psi^2)$, $(l + m = 2n - 1)$, in the regions $z = 2E_\psi/M_Z \sim O(1)$ and $z \ll 1$, respectively. In the intermediate region we interpolate using the available data. This resummation eliminates the $O[\alpha_s(M_\psi)/\alpha_s(M_Z)] \sim 2$ theoretical uncertainties in previous calculations. The $\log(z)$ resummation results in a suppression of the small $z$ region due to coherent gluon emission. Comparing the zeroth moment with the LEP data we find the value for the effective octet matrix element to be $\langle \hat{O}_8^{\psi}(3S_1) \rangle = 0.019$ GeV$^3$. The theoretical uncertainties are substantially smaller than those from Tevatron extractions. Using this value of the octet matrix element we make a prediction for the first moment of the differential rate and find that the resummed differential decay rate is in much better agreement with preliminary data than the color singlet result or the unresummed color octet prediction.
I. INTRODUCTION

The subject of quarkonium production has gained renewed interest due to the fact that it can now be understood from first principles. The production rates are calculated in a systematic expansion in $\alpha_s$ and $v$, the relative velocity of the heavy quarks in the rest frame of the quarkonium bound state. This is accomplished by working in non-relativistic QCD (NRQCD) where the expansion in $v$ is implemented by utilizing the scaling properties of non-perturbative matrix elements [1]. The use of this effective theory has clarified formal issues and allowed for better fits to data. In general, NRQCD leads to larger cross sections than its historical predecessor, the color singlet model, largely because NRQCD predicts substantial contributions to various cross sections from the color octet channel. This happens even though the octet matrix elements are suppressed by powers of $v$, because the color singlet channels can themselves be suppressed either kinematically or by powers of $\alpha_s$. For instance, $\psi'$ production at the Tevatron can now be well fit if one allows for color octet production [2].

The field of quarkonium production itself has matured to the point where we would like to go beyond order of magnitude accuracy. Indeed, once the formalism is verified quantitatively, it can be used as a tool in other areas in strong interaction physics, such as heavy ion collisions and the measurement of the spin dependent gluon distribution functions.

Presently, the values for the octet matrix element $\langle O_8^{\psi}(3S_1) \rangle$ and a certain linear combination of the $\langle O_8^{\psi}(3P_0) \rangle$, and $\langle O_8^{\psi}(1S_0) \rangle$ matrix elements have been extracted at the Tevatron. These matrix elements should be universal in nature, in that we should be able to use them as input in other processes to make definite predictions. Unfortunately, hadronic uncertainties in the Tevatron extraction make this difficult. The extraction depends sensitively on the choice of gluon distribution function and factorization scale, as well as how one treats initial gluon radiation [3]. Extractions of $\langle O_8^{\psi}(3S_1) \rangle$ are as disparate as $2.1 \times 10^{-3}$ [4], $2.7 \times 10^{-3}$ [5], $6.6 \times 10^{-3}$ [6], and $14.0 \times 10^{-3}$ [7], in GeV$^3$. A process involving smaller theoretical uncertainties is needed. Perhaps the cleanest setting to extract a value of the octet matrix
element is in prompt $J/\psi$ production at LEP, because in lepton initiated processes the theoretical errors are bounded by our computational strength rather than higher twist effects. Furthermore, at LEP we can measure the energy distribution as well, thus once we’ve extracted the value of the octet matrix element, we can then make predictions for the moments of the rate. This provides a strong test for the color octet mechanism.

Formally there are two leading order contributions in the $\alpha_s$ and $v$ expansion, both in the singlet channel, which are of order $O(\alpha_s^2 v^3)$. There is a contribution from gluon radiation in the singlet channel $Z \rightarrow \psi gg$ that is suppressed by powers of $M_Z^2/E_{\psi}^2 \ ([7]$. There is also the color singlet charm quark fragmentation process $Z \rightarrow \psi c\bar{c} \ ([8,9]$, which has no power suppression and thus dominates over non-fragmentation processes, for large $E_{\psi}$. Light quark octet fragmentation (in which the mother parton does not combine to form part of the bound state) is naively of order $\alpha_s^2 v^7$, down by $v^4 \sim 1/10$ compared to charm fragmentation. However, as it turns out, this channel is enhanced due to the presence of large logs as well as a numerical factor of five due to the number of possible quarks that initiate the process. Indeed, previous calculations of the $J/\psi$ production rate at LEP are dominated by these light quark fragmentation contributions \([10,11]\). They give cross sections that are of the correct order of magnitude when values of the non-perturbative matrix elements are taken from the Tevatron fits \([5]\). However, the same logs that enhance the octet channel also put the convergence of the perturbative expansion into question.

The tree-level calculation of the differential cross section in the color octet production channel \([10,12]\) is enhanced by a large logarithm, $\frac{d^3 \Gamma(z)}{dz}(Z \rightarrow \psi + X) \sim \alpha_s^2 \log(M_Z^2/M_\psi^2)/z$, leading to large double logs in the total rate. Since $\alpha_s \log(M_Z^2/M_\psi^2) \approx 1.5$, we should treat $\alpha_s \log(M_Z^2/M_\psi^2)$ as order one and resum all powers of the large logarithm. With this counting, the octet channel is $O(\alpha_s^0 v^7)$, on par with the singlet fragmentation contribution. More practically, the tree-level calculation has a factor of two uncertainty associated with the scale at which $\alpha_s$ is evaluated, since $\alpha_s(M_\psi)/\alpha_s(M_Z) \approx 2$ (this is just a restatement that there is a large logarithm). The resummation of the leading logarithms eliminates this uncertainty, so the resummation procedure is essential from both a practical and a
formal standpoint. We therefore calculate the quarkonium differential production rate at LEP taking all terms of the form $\alpha_s^{m+1}\log^m(M_Z^2/M_\psi^2)$ as leading order. We will see that this resummation dramatically changes the differential cross section. However, summing the above mentioned logs will only yield the correct leading order differential rate if $z$ is sufficiently large. When $z$ is parametrically small, terms of the form $\alpha_s^2 \log(z)/z$ become just as important. Furthermore, these logs will also contribute double logs to the total rate given that the lower limit on $z$ is $2M_\psi/M_Z$. This second type of log, due to soft gluon emission, is resummed using a formalism familiar from discussions of jet multiplicities $^{13,14}$. Thus, we split the calculation into two regimes, $z \sim 1$ and $z \ll 1$. We then interpolate between these regimes using the data.

II. FRAGMENTATION FORMALISM

The tree-level differential rate for color octet $J/\psi$ production is $^{11}$

$$\frac{d\Gamma}{dz}(Z \rightarrow \psi(z)qq) = \frac{4\alpha_s^2}{9}\Gamma(Z \rightarrow qq)\frac{\langle O_8^{(3S_1)} \rangle}{M_\psi^3} \times \left\{ \frac{(z - 1)^2}{z} + 2\frac{M_\psi^2}{M_Z^2}\frac{2 - z}{z} + \frac{M_\psi^4}{M_Z^2} \left[ \log \left( \frac{z + z_L}{z - z_L} \right) - 2z_L \right] \right\},$$

where the rescaled $\psi$ energy in the $Z$ rest frame $z = 2E_\psi/M_Z$ has a physical range of $2M_\psi/M_Z < z < 1 + M_\psi^2/M_Z^2$, $z_L = (z^2 - 4M_\psi^2/M_Z^2)^{1/2}$, and to the order we work, $M_\psi$ is twice the charm mass. Performing the integration over $z$ leads to the aforementioned double logs. In the fragmentation limit, Eq. (1) can be simplified to

$$\frac{d\Gamma}{dz}(Z \rightarrow \psi(z)qq) \approx \frac{4\alpha_s^2}{9}\Gamma(Z \rightarrow qq)\frac{\langle O_8^{(3S_1)} \rangle}{M_\psi^3} \left\{ \frac{(z - 1)^2}{z} \left[ \log \left( \frac{M_\psi^2}{M_Z^2} \right) + \log(z^2) \right] - 2z \right\}. $$

In this limit, the differential rate can be recast as the sum of quark and gluon fragmentation processes,

$$\frac{d\Gamma}{dz}(Z \rightarrow \psi(z)qq) = 2C_q(\mu^2,z) \ast D_q(\mu^2,z) + C_g(\mu^2,z) \ast D_g(\mu^2,z).$$

(3)
Here, $D_{q \to \psi}(z)$ and $D_{g \to \psi}(z)$ are the light quark (or anti-quark) fragmentation and gluon fragmentation functions respectively. The asterisk denotes convolution with the partonic production rates, $C \ast D \equiv \int_z^1 C(y) D(z/y) dy / y$. The $\mu$ dependence of the fragmentation functions is canceled by that of the coefficient functions, $C_q$ and $C_g$. All dependence on $M_\psi$ is contained in the fragmentation functions, while all dependence on $M_Z$ is contained in the coefficient functions. It is this factorized form that will later allow us to resum the large logarithms.

We choose to define the color octet fragmentation functions according to Collins and Soper [15],

$$D_{g \to \psi}(\mu^2, z) = \frac{-z^{d-3}}{16(d-2) \pi k^+} \int dx e^{-iP^+ x^- / z} \langle 0 | G_{b+\nu}^+(0) a^\dagger_\psi(P^+,0) a_\psi(P^+,0) G_{b\nu}^+(0,x^-,0) | 0 \rangle,$$

$$D_{q \to \psi}(\mu^2, z) = \frac{z^{d-3}}{4\pi} \int dx e^{-iP^+ x^- / z} \frac{1}{3} \text{Tr}_{\text{color}} \frac{1}{2} \text{Tr}_{\text{Dirac}} \left[ \gamma^+ (0) Q(0) a^\dagger_\psi(P^+,0) a_\psi(P^+,0) \bar{Q}(0,x^-,0) | 0 \rangle \right].$$

Here, $G_{b+\nu}^+$ is a gluon field strength tensor with color index $b$ and Lorentz indices $+$ and $\nu$, $Q$ is a quark field, $a^\dagger_\psi$ is a creation operator for a $\psi$ meson, and $d$ is the number of spacetime dimensions. The fragmentation functions are interpreted as the probabilities for a parton with momentum $k^+$ to decay into a $\psi$ with light cone momentum $P^+ = zk^+$. We have chosen to work in the light cone gauge, where eikonal factors usually written to make gauge invariance manifest reduce to the identity. An advantage of this definition over the alternative [16] is its consistency with factorization at any subtraction scale $\mu$.

For the case of quarkonium fragmentation $D_{q \to \psi}$ and $D_{g \to \psi}$ can be calculated in a systematic expansion in $\alpha_s$ and $v$ by matching onto NRQCD. Any soft divergences which may arise due to the semi-inclusive nature of the process cancel in the matching. Since we are working to leading order no such divergences occur, and the matching is trivial in the sense that there are no corrections to be calculated in the effective theory. The calculation of these fragmentation functions are well documented in the literature [17,18], so here we just present our results which are in agreement with these previous calculations.

In the $\overline{\text{MS}}$ scheme we find
\[ D_{q \rightarrow \psi}(\mu^2, z) = \frac{2\alpha_s^2}{9M_\psi^3} \langle O_8^\psi(3S_1) \rangle \left\{ \frac{(z - 1)^2 + 1}{z} \log \left[ \frac{\mu^2}{M_\psi^2(1 - z)} \right] - z \right\}, \quad (5a) \]

\[ D_{g \rightarrow \psi}(\mu^2, z) = \frac{\pi\alpha_s}{3M_\psi^3} \langle O_\delta^\psi(3S_1) \rangle \delta(1 - z), \quad (5b) \]

where

\[ O_8^\psi(3S_1) = \chi^\dagger \sigma_i T^a \psi(\bar{a}_\psi a_\psi) \psi^\dagger \sigma_i T^a \chi. \quad (6) \]

\( T^a \) is a color generator, while \( \chi \) and \( \psi \) are two component NRQCD spinors. Inserting these fragmentation functions into Eq. (3) and matching onto the QCD calculation, Eq. (2), we obtain the coefficient functions \( C_q \) and \( C_g \)

\[ C_q(\mu^2, z) = \Gamma(Z \rightarrow q\bar{q}) \delta(1 - z), \quad (7a) \]

\[ C_g(\mu^2, z) = \frac{4\alpha_s}{3\pi} \Gamma(Z \rightarrow q\bar{q}) \left\{ \frac{(z - 1)^2 + 1}{z} \log \left[ \frac{(1 - z)z^2M_Z^2}{\mu^2} \right] - z \right\}. \quad (7b) \]

Note that there is no physical distinction between what we call gluon fragmentation and what we call quark fragmentation, as it is always possible to shift some finite piece from one to the other. However, factorization dictates that we match in such a way as to make the Wilson coefficients independent of the long distance physics, i.e. \( M_\psi \).

Once we choose the scale \( \mu \) to be \( O(M_Z) \), there are no large logs in the Wilson coefficients. They have been shuffled into the fragmentation functions and can be resummed via the DGLAP equations. All the leading logs, of order \( O[\alpha_s^2 \log(M_Z^2/M_\psi^2)] \), will reside in the quark fragmentation function. The contribution from gluon fragmentation is then subleading, contributing only at \( O(\alpha_s^2) \) to the rate. We choose to keep this contribution in order to reduce the \( \mu \) dependence of our result, although we do not claim accuracy to the level \( \alpha_s^2 \). There are other terms of this order, arising from both two-loop running and the \( \alpha_s^2 \) corrections to the initial gluon fragmentation function, that have not been included.

In addition to the color octet contribution we include the contribution from the color singlet, which is formally \( O(\alpha_s^2 v^3) \), but as mentioned above is numerically smaller than the octet contribution. Again using the Collins-Soper definition for the singlet fragmentation function we find
\[ D_{c\to \psi}^{(1)}(\mu^2, z) = \frac{128\alpha_s^2}{243M_\psi^3} \langle O_1^{\psi}(3S_1) \rangle \frac{z(1-z)^2}{(2-z)^6} \left(16 - 32z + 72z^2 - 32z^3 + 5z^4\right), \] (8)

which agrees with [13].

### III. \( z \sim 1 \) Resummation

The resummation of the \( \log(M_\psi^2/M_\pi^2) \) is accomplished via the usual renormalization group analysis of the fragmentation functions. The evolution equations are given by

\[ \mu \frac{dD_q(\mu^2, z)}{d\mu} = \frac{\alpha_s(\mu^2)}{\pi} \left\{ P_{q\to qg} \ast D_q(\mu^2) + P_{q\to qg} \ast D_g(\mu^2) \right\}, \] (9a)

\[ \mu \frac{dD_g(\mu^2, z)}{d\mu} = \frac{\alpha_s(\mu^2)}{\pi} \left\{ \sum_{j=1}^{2n_f} P_{g\to q\bar{q}} \ast D_q(\mu^2) + P_{g\to gg} \ast D_g(\mu^2) \right\}, \] (9b)

where the functions \( P \) are the standard splitting functions. We solved these equations numerically using \( m_c = 1.48 \text{ GeV} \), \( \mu = M_Z \), \( \alpha_s(M_Z) = 0.118 \), and chose \( \alpha_s(M_\psi) \) to be consistent with one-loop running from \( M_Z \).

The contributions to the differential rate from both the evolved octet (solid line) and evolved singlet (dotted line) fragmentation functions are displayed in Fig. 1, in units of MeV. We used the Tevatron extraction [5] to obtain the normalization of the octet fragmentation functions. For comparison, the octet contribution without evolution is shown in dashed lines, and the singlet contribution without evolution in dot-dashed lines. Resummation of the \( \log(M_\psi^2/M_\pi^2) \) terms greatly enhances both the octet and singlet rates at small \( z \). However, as mentioned above we should not trust this result in the small \( z \) regime.

At this point we should point out that there are some values of \( z \sim 1 \) where the differential rate is not trustworthy. When \( z \) approaches within \( v^2 \) of one, we begin to probe the hadronic structure of the quarkonium state, and the expansion in \( v \) breaks down [20,21]. To correctly describe quarkonium production at the edge of phase space we must introduce a structure function [22,23] which resums all the large non-perturbative corrections. Indeed, LEP would be an ideal place to study these structure functions if there were more data available. In the total cross section however, or any sufficiently smeared version of the differential rate, such as the first ten or so moments, the expansion in \( v \) is well behaved.
FIG. 1. Differential rate $\frac{d\Gamma}{dz}$ for the octet channel with (solid line) and without (dashed line) evolution, and for the singlet channel with (dotted line) and without (dot-dashed line) evolution, as a function of $z = 2E_\psi/M_Z$. The octet matrix element has been extracted from the Tevatron (see text).

IV. RESUMMATION FOR SMALL $z$

The lower limit on $z$ is $z_{\text{min}} = 2M_\psi/M_Z$, so that when $z \sim z_{\text{min}}$, we need to include a resummation of the $\log(z)$ terms. Indeed, the generic term in the decay rate has pieces of the form $\alpha_s^{n+1}\log^l(z^2)\log^m(M^2_Z/M^2_\psi)$, ($l + m = 2n - 1$). We thus treat $\log(z^2)$ to be of the same order as $\log(M^2_\psi/M^2_Z)$, and resum all terms of the above form. This problem was first encountered in the calculation of jet multiplicities, where it was noticed that the splitting functions are highly singular at small $z$ and need to be resummed. The results of these calculations led to predictions for the shape of the hadron multiplicities (under some assumptions of quark-hadron duality) which fit the data extremely well [24]. Indeed, the results make the striking prediction that there should be a suppression at small $z$ which sets in at higher $z$ than would be expected from just phase space suppression. This suppression
is due to angular ordering of soft gluon emission and is a consequence of gluon coherence which is naively missed in canonical ladder resummations [23].

Our calculation differs from those previous calculations of hadron multiplicities in that we can actually calculate the hadronic fragmentation function in a systematic fashion, in terms of some (in our case effectively one) unknown matrix element. Thus, we are concerned with the normalization as well as the shape of the differential decay rate. There are several different formalisms for handling the coherent gluon emission problem [13,14,26]. We choose to follow the formalism developed by Mueller [26,27], and we refer the reader to these papers for details.

We begin by noticing that it is the gluon splitting function which is most singular at small $z$, $P_{g \to gg}(z) \approx 2C_A/z$. Thus, at leading order as defined above, the branching will all come from gluon splitting once the initial quarks mix into a gluon. For the moment let us consider pure gluon splitting. All the leading logs come from ladder diagrams of the form shown in Fig. 2 (without the initial quark box). The coherence issues can be skirted by imposing angular ordering on the gluons such that the angle between two branching partons is smaller than the angle of the previous pair [28]. This allows us to rewrite the series via the integral equation

$$zD_g(\hat{t}, z) = \delta(1 - z) + \frac{\alpha_s C_A}{\pi} \int_z^1 \frac{dz'}{z'} \int_{\hat{t}}^1 \frac{d\hat{t}'}{\hat{t}'} z' D_g(\hat{t}', z'),$$

(10)

where $C_A = 3$ and at the end of the calculation $\hat{t}$ is taken to be $\hat{t} = M_{\psi}^2/M_z^2 z^2$ in order
to enforce the angular ordering. For now, \( D_g(t, z) \) has been normalized to 1. Iterating this equation leads to

\[
D_g(M_Z^2/M_\psi^2, z) = \frac{1}{z} \sum_{m=1}^{\infty} \left( \frac{\alpha_s C_A}{\pi} \right)^m \frac{1}{m!} \log^m \left( \frac{M_Z^2 z^2}{M_\psi^2} \right) \frac{1}{(m-1)! \log^{m-1} \left( \frac{1}{z} \right)}.
\]  

(11)

Taking moments of Eq. (11) gives

\[
D_{gn}(M_Z^2/M_\psi^2) = \int_0^1 dz z^{n-1} D_g(M_Z^2/M_\psi^2, z) = 1 + \sum_{m=1}^{\infty} \left( \frac{\alpha_s C_A}{\pi} \right)^m \sum_{k=0}^{m} \frac{(-2)^k (m+k-1)! \log^{m-k}(M_Z^2/M_\psi^2)}{(m-1)! k! (m-k)! (n-1)^{k+m}}.
\]  

(12)

The above sums have a simple closed form. After reinserting the normalization

\[
D_{gn}(M_Z^2/M_\psi^2) = \frac{\pi \alpha_s}{3M_\psi^3} \langle O_8(\ell S_1) \rangle C_n e^{\gamma_n \log(M_Z^2/M_\psi^2)},
\]  

(13)

where

\[
\gamma_n = \frac{1}{4} \left[ \sqrt{(n-1)^2 + 8\alpha_s C_A/\pi} - (n-1) \right],
\]  

(14)

\[
C_n = 1 - \frac{2\gamma_n}{\sqrt{(n-1)^2 + 8\alpha_s C_A/\pi}}.
\]  

(15)

\( \gamma_n \) is the resummed diagonal anomalous dimension of the fragmentation function. If we expand in \( \alpha_s \), then it correctly reproduces the most singular pieces of the previously calculated two-loop splitting function [29].

At small \( z \), the leading contribution to the quark fragmentation function comes from the gluon fragmentation function above, convoluted with \( P_{q \to gg}(z) \approx 2C_F/z \). Summing up all ladder diagrams of the form shown in Fig. 2 leads to

\[
D_q(M_Z^2/M_\psi^2, z) = \frac{\pi \alpha_s}{3M_\psi^3} \langle O_8(\ell S_1) \rangle \frac{C_F}{C_A} \times \frac{1}{z} \sum_{m=1}^{\infty} \left( \frac{\alpha_s C_A}{\pi} \right)^m \frac{1}{m!} \log^m \left( \frac{M_Z^2 z^2}{M_\psi^2} \right) \frac{1}{(m-1)! \log^{m-1} \left( \frac{1}{z} \right)},
\]  

(16)

with moments

\[
D_{qn}(M_Z^2/M_\psi^2) = \frac{\pi \alpha_s}{3M_\psi^3} \langle O_8(\ell S_1) \rangle \frac{C_F}{C_A} C_n e^{\gamma_n \log(M_Z^2/M_\psi^2)}. \]  

(17)
Note that expanding Eq. (16) to leading order and convoluting with twice the coefficient function, Eq. (7a) (for quark and antiquark fragmentation), recovers the tree-level differential rate, Eq. (2), in the small $z$ limit.

We can now use the renormalization group to improve Eq. (17). After running we are left with,

$$D_{qn}(M_Z^2/M_\psi^2) = \frac{\pi \alpha_s}{3M_\psi^3} \langle O_8^\psi (3S_1) \rangle \frac{C_F}{C_A} C_n(\alpha_s(M_Z^2)) \exp \left( \int_{M_\psi^2}^{M_Z^2} \frac{d\mu^2}{\mu^2} \gamma_n(\alpha_s(\mu^2)) \right).$$

(18)

This equation can now be inverted back to $z$ space leading to the result

$$D_q(M_Z^2, z) \approx \frac{\pi \alpha_s}{3M_\psi^3} \langle O_8^\psi (3S_1) \rangle \frac{C_F}{C_A} C_{n_0}(\alpha_s(M_Z^2)) \frac{1}{2z\sqrt{\pi a}} \exp \left[ c - \frac{1}{4a} \left( \log \frac{1}{z} - b \right)^2 \right],$$

(19)

where

$$n_0 = 1 - \frac{1}{2a} \left( \log \frac{1}{z} - b \right),$$

(20)

$$a = \frac{1}{48b_0} \left( \sqrt{\frac{2\pi}{C_A\alpha_s(M_Z^2)^2}} - \sqrt{\frac{2\pi}{C_A\alpha_s(M_\psi^2)^2}} \right),$$

(21)

$$b = \frac{1}{4b_0\alpha_s(M_Z^2)} - \frac{1}{4b_0\alpha_s(M_\psi^2)},$$

(22)

$$c = \frac{1}{b_0} \left( \sqrt{\frac{2C_A}{\pi\alpha_s(M_Z^2)}} - \sqrt{\frac{2C_A}{\pi\alpha_s(M_\psi^2)}} \right),$$

(23)

and $b_0$ is the coefficient of the one-loop beta function. This result was reached in the saddle point approximation where $1 - n_0$ is small and thus should not be trusted for $z$ values larger than $z \sim 0.2$. Subleading corrections to this result can be systematically included by properly adapting the formalism discussed in [27,30], where they were found to be of order $\sqrt{\alpha_s}$ at the peak\footnote{We have checked using the results of [27], that the numerical value of the corrections to the coefficient function are actually quite a bit smaller than this.}. We should note that we expect the relative size of the subleading corrections to the total rate to be smaller than those for the differential rate, given that in the differential rate the subleading terms are down by $\log(z^2)$, whereas in the total rate they are down by $\log(M_Z^2/M_\psi^2)$. 

}\footnote{We have checked using the results of [27], that the numerical value of the corrections to the coefficient function are actually quite a bit smaller than this.}
V. EXTRACTION OF THE MATRIX ELEMENT

We now have leading log expressions for the total and differential rates in the small and large $z$ regions. Before we interpolate between the two we need to determine their respective regions of validity. Let us investigate the size of the contributions we are neglecting in the large $z$ region. The generic term in the decay rate has the form $\alpha_s^{n+1} \log^l(z^2) \log^m(M_Z^2/M_{\psi}^2)$, where $l + m = 2n - 1$. The first term that is not included in the large $z$ resummation is suppressed by $\log(z)/\log(M_{\psi}/M_Z)$. Thus a conservative value for the lower value of $z$ is $\sim 0.5$.

The small $z$ resummation is not valid at large $z$ for two reasons. First, we have used the saddle point approximation to compute the inverse Mellon transform, which is only valid for $n_0 - 1$ small. Second, we have neglected less singular terms in computing the small $z$ fragmentation result. Therefore, we will trust the small $z$ result only up to the peak, where $n_0 - 1 \approx 0$, but not for larger $z$. We believe this to be a conservative upper bound on $z$ for this approximation.

Thus, to obtain the full differential cross section, we interpolate between the small $z \lesssim 0.2$ and large $z \gtrsim 0.5$ resummations using the data in this region as our guide. We checked that varying the interpolation while still staying within the error bars, changes the result by at most 25%. Furthermore, we tested the sensitivity of our predictions to increasing and decreasing the lower bound on $z$ in the large $z$ regions as well as increasing the upper bound on the small $z$ region. These variations do not appreciably change the total rate or the first moment. Note that decreasing upper bound on small $z$ regions does drastically change the results. However, taking the maximum value of $z$ to be below $z_{\text{peak}}$ is not a reasonable thing to do however, given that the peak is the position where the saddle point approximation is trustworthy. In addition, the data for hadron multiplicites fits the resummed predictions extremely well near the peak \cite{24}.

In our final result we also included the non-fragmentation corrections given by the difference between Eqs. (1) and (3). These are significant at very small $z$, and contribute to
the total rate even when $M_\psi/M_Z \to 0$.

Since the data includes feed-down from excited charmonium states, the rate should be written in terms of the effective matrix elements [5]

\[
\langle \hat{O}_8^{\psi(n)}(3S_1) \rangle \equiv \sum_{m \geq n} \langle O_8^{\psi(m)}(3S_1) \rangle \text{BR}(\psi(m) \to \psi(n) + X),
\]

\[
\langle \hat{O}_1^{\psi(n)}(3S_1) \rangle \equiv \sum_{m \geq n} \langle O_1^{\psi(m)}(3S_1) \rangle \text{BR}(\psi(m) \to \psi(n) + X).
\] (24)

Saturating the excited states with the $\psi'$ and $\chi_{cJ}$, and using values for $\langle O_8^{\psi}(3S_1) \rangle$, $\langle O_8^{\psi'}(3S_1) \rangle$, and $\langle O_8^{\chi_{cJ}}(3S_1) \rangle$ from [5] gives $\langle \hat{O}_8^{\psi}(3S_1) \rangle = 0.014 + 0.002$ GeV$^3$. This number has at least a factor of two theoretical uncertainty. Extracting the analogous color singlet matrix elements from the $\psi$ and $\psi'$ electronic widths [1] gives $\langle \hat{O}_1^{\psi}(3S_1) \rangle = 1.45 \pm 0.10$ GeV$^3$.

Combining the singlet and octet fragmentation contributions gives the total differential rate. Integrating over $\psi$ energies yields a total branching ratio of

\[
\text{BR}(Z \to \text{prompt } J/\psi + X) = \left( 1.47 \frac{\langle \hat{O}_8^{\psi}(3S_1) \rangle}{0.014 \text{ GeV}^3} + 0.47 \frac{\langle \hat{O}_1^{\psi}(3S_1) \rangle}{1.45 \text{ GeV}^3} \right) \times 10^{-4},
\] (25)

compared to a branching ratio of $(1.93 \langle \hat{O}_8^{\psi}(3S_1) \rangle/0.014 \text{ GeV}^3 + 0.68 \langle \hat{O}_1^{\psi}(3S_1) \rangle/1.45 \text{ GeV}^3) \times 10^{-4}$ from [10].

The total branching ratio has been measured to be [31,32]

\[
\text{BR}(Z \to \text{prompt } J/\psi + X) = (1.9 \pm 0.7 \pm 0.5 \pm 0.5) \times 10^{-4} \text{ OPAL}
\]
\[
\text{BR}(Z \to \text{prompt } J/\psi + X) = (3.0 \pm 0.8 \pm 0.3 \pm 0.15) \times 10^{-4} \text{ ALEPH}
\]
\[
\text{BR}(Z \to \text{prompt } J/\psi + X) = (2.7 \pm 1.2) \times 10^{-4} \text{ L3}
\]
\[
\text{BR}(Z \to \text{prompt } J/\psi + X) = (4.4_{-3.0}^{+3.6}) \times 10^{-4} \text{ DELPHI}.
\] (26)

For OPAL and ALEPH, the uncertainties from left to right are statistical, systematic, and model-dependent, while for L3 and DELPHI, they are purely statistical. We use the LEP average to extract a value for the effective octet matrix element of

\[
\langle \hat{O}_8^{\psi}(3S_1) \rangle = (0.019 \pm 0.005_{\text{stat}} \pm 0.010_{\text{theo}}) \text{ GeV}^3,
\] (27)
Fig. 3. Differential rate $\frac{d\Gamma}{dz}$ as a function of $z = 2E_{\psi}/M_Z$ vs data. The dashed line is the sum of the tree-level octet and singlet results and the solid line is the interpolation between the large and small $z$ region octet resummation plus the singlet resummation.

where the first uncertainty is purely statistical, and the second is theoretical. The theoretical uncertainty comes from adding in quadrature roughly 30% contributions from perturbative corrections suppressed by $\alpha_s(M_\psi)$, higher order matrix elements suppressed by $v^2$, and subleading logs.

Since the LEP experiments include feed-down from $\psi'$ and $\chi_J$, we cannot directly compare our extraction of $\langle \hat{O}_8^{\psi}(^3S_1) \rangle$ with those of $\langle O_8^\psi(^3S_1) \rangle$ in [3,4,5], but our value of $\langle \hat{O}_8^{\psi}(^3S_1) \rangle$ is comparable with the central value from [5]. While our statistical uncertainties are larger, our theoretical uncertainties are under good control. Since this cannot be said of Tevatron extractions, where theoretical uncertainties dominate, we believe Eq. (27) represents the most reliable extraction currently available.

Fig. 3 shows the complete differential rate using our extracted value for $\langle \hat{O}_8^{\psi}(^3S_1) \rangle$. The solid line is the sum of the resummed singlet fragmentation and the interpolation between
the large and small $z$ regions for octet fragmentation. The dashed line is the tree-level result and the data is from the ALEPH collaboration [32] with efficiency correction from [33]. Given the large errors, it is difficult to make any definite statements, but it seems the resummed rate fits the data better at this time. The effect of evolution is to enhance the small-$z$ peak, a promising experimental signature of the octet component. A manifestation of this signature is a relatively small first moment, which we find to be

$$\frac{1}{\Gamma(Z \to \text{prompt } J/\psi + X)} \int \frac{d\Gamma(Z \to \text{prompt } J/\psi + X)}{dz} z \, dz = 0.30,$$

while the tree-level differential rate, Eq. (1), gives $\sim 0.5$. A very rough estimate of this quantity obtained from the data [32] suggests a value of $0.26 \pm 0.10$. This is in sharp contrast to the color singlet prediction. The tree-level color singlet decay rate predicts the ratio of the first moment over the zeroth moment to be 0.62. Resummation softens the color singlet decay rate, but the ratio is still too large, 0.47. The ratio is independent of the color singlet matrix element. Therefore, even if the color singlet rate were arbitrarily increased so that the color singlet model fit the experimental branching ratio, the ratio of moments would not fit the data. A rigorous extraction of the first moment by the experimental groups could provide an extremely clean, quantitative test of the NRQCD approach.

In conclusion, we have resummed large logarithms in the rate for prompt $J/\psi$ production at LEP to obtain the leading-order prediction. We predict a branching ratio that is slightly smaller than the tree-level prediction [10,11]. Moreover, we have eliminated a factor of 2 uncertainty in the tree-level result. Matching our branching ratio to LEP data yields an octet matrix element with substantially smaller theoretical uncertainties than from hadronic processes [3–6]. The differential decay rate is dramatically softer than previous calculations. The small-$z$ peak in the differential distribution represents a clear signature of the octet mechanism that we regard as strong motivation for continued analyses by the LEP experimental groups. A measurement of the first moment would be a particularly interesting.
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