Towards a realistic interpretation of quantum mechanics providing a model of the physical world

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Abstract

It is argued that a realistic interpretation of quantum mechanics is possible and useful. Current interpretations, from “Copenhagen” to “many worlds” are critically revisited. The difficulties for intuitive models of quantum physics are pointed out and possible solutions proposed. In particular the existence of discrete states, the quantum jumps, the alleged lack of objective properties, measurement theory, the probabilistic character of quantum physics, the wave-particle duality and the Bell inequalities are analyzed. The sketch of a realistic picture of the quantum world is presented. It rests upon the assumption that quantum mechanics is a stochastic theory whose randomness derives from the existence of vacuum fields. They correspond to the vacuum fluctuations of quantum field theory, but taken as real rather than virtual.

1 Introduction

Quantum mechanics is extremely efficient for the prediction of experimental results. In contrast, the interpretation of the quantum formalism has been the subject of continuous debate since the very begining of the theory[1], [2], [3], [4], [5] and it lasts until today[6], [7], [8]. Is there a real problem? Feynman believed that a problem exists when he stated: “Nobody understands quantum mechanics” [9], and many people agree with that opinion.
The fact is that none of the different interpretations proposed till now offers a clear intuitive picture of the quantum world. Nevertheless most physicists do not worry for the lack of a picture and embrace a pragmatic approach, good enough in practice. In contrast with this attitude, in this paper a realistic interpretation of quantum theory is supported, the difficulties for that interpretation are analyzed and a picture of the quantum world is proposed.

The plan of the paper is as follows. In this introduction section, a few general comments are made on two opposite approaches to the interpretation of quantum mechanics, namely pragmatic and realistic. After that the sketch is presented of a picture of the quantum world. In section 2 a number of empirical facts of the quantum domain are analyzed in order to show that none of them prevents the existence of an intuitive picture of the microworld. In section 3 the most popular interpretations proposed till now, from ‘Copenhagen’ to ‘many worlds’, are revisited critically. In section 4 it is discussed the ‘ensemble interpretation’, that supports an epistemological rather than ontological treatment of the wave function. Also the closely related subjects of hidden variables models and Bell’s inequalities are commented. Finally in section 5 the proposed picture of the quantum world is discussed in some detail.

1.1 The pragmatic approach to quantum mechanics

Many physicists, not too interested in foundations, accept a minimal interpretational framework with the following key features[10]:

1. Quantum theory is viewed as a scheme for predicting the probabilistic distribution of outcomes of measurements made on suitable prepared copies of a system.

2. The probabilities are interpreted in a statistical way as referring to relative frequencies.

In general a physical theory has at least two components[11]: (1) the formalism, or mathematical apparatus, of the theory, and (2) the rules of correspondence that establish a link between the formalism and the results of measurements. For instance, the standard formalism of quantum mechanics is based on the mathematical theory of Hilbert spaces. In it there are two essential kinds of operators, density operators, \( \hat{\rho} \), that represent states, and self-adjoint operators, \( \hat{A} \), that represent observables. The link with the measurement results is given by the Born rule where the ‘expectation value’,
\( Tr(\hat{\rho}\hat{A}) \), is assumed to correspond to the statistical mean of the values obtained when one realizes several measurements on identically prepared systems (which determines \( \hat{\rho} \)) by means of an apparatus which corresponds to \( \hat{A} \). If we assume that the formalism and the correspondence rules are the only objects required to define a physical theory, in the sense that the statistical regularities need not be further explained, then we get what has been called a *minimal instrumentalistic interpretation* of the theory[12]. It might be named *pragmatic approach* or even qualified as rejection of any interpretation[13].

Most people claiming to support that approach accept the following positions which go beyond the purely pragmatic attitude:

1. The notion of an individual physical system ‘having’ or ‘possessing’ values for all its physical quantities is *inappropriate* in the context of quantum theory.

2. The concept of ‘measurement’ is fundamental in the sense that the scope of quantum theory is *intrinsically* restricted to predicting the results of measurements.

3. The spread in the results of measurements on identically prepared systems must not be interpreted as reflecting a ‘lack of knowledge’ of some objectively existing state of affairs.

Actually these propositions define an interpretation that has been also called *instrumentalistic*[10]. It is quite different from, even opposite to, the *realistic* view traditional of classical physics. Between these two extremes there are a variety of approaches.

### 1.2 Realistic interpretations

In this paper it is supported a realistic interpretation that demands physical models for the quantum phenomena. This position is not new, it has been advocated by many people including Einstein as the most distinguished author. Indeed the celebrated article by Einstein, Podolsky and Rosen[14] (EPR) begins: “Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with *the objective reality*, and by means of these concepts we picture this reality to ourselves” (my emphasis).

It is true that in the 80 years since the EPR paper the concept of “ob-
jective reality” has been questioned as not clear. Due to the difficulties with the interpretation of quantum mechanics, many people working on foundations dismiss the ‘realism’ of EPR as ‘naive’. Thus more sophisticated forms or realism have been proposed\[15\], \[16\]. In any case a discussion about the philosophical aspects of reality or realism is outside the scope of this paper.

The starting point of this article is the claim that any physical theory should contain a physical model in addition to the formalism and rules for the connection with the experiments. The latter are obviously essential because they are required for the comparison of the theory with empirical evidence, which is the test for the validity of the theory. But in my opinion physical models are also necessary in order to reach a coherent picture of the world. Many quantum physicists apparently support the useless of pictures, but it is the case that when they attempt popular explanations of quantum mechanics they propose models, and in fact rather bizarre ones. For instance it is claimed that quantum mechanics compel us to believe that there are a multiplicity of “me” in parallel universes or that an atom may be present in two distant places at the same time. For me this is an indication that the need of “picture the reality to ourselves”\[14\] cannot be easily rejected. Furthermore the existence of physical models might open the possibility for new developments and applications of quantum theory and therefore it is not a purely academic question.

It is interesting the contrast between the two great theories of the 20th century, quantum mechanics and relativity. The latter provides a beautiful physical model: There is a four-dimensional manifold with intrinsic curvature and all material objects (e. g. particles or fields) are defined in that continuum. This is fundamental even for the formulation of quantum (field) theory. But the calculational tool of general relativity (derived from the Riemann geometry) is rather involved, the fundamental (Einstein) equations being nonlinear. In quantum mechanics there is a relatively simple formalism involving vectors and operators in a Hilbert space. Indeed the fundamental (Schrödinger) equation is linear. However there is no coherent physical model behind it. I would say that general relativity has physical beauty, the quantum formalism possesses mathematical elegance.

Historically the renunciation to physical models in quantum mechanics was a consequence of frustration caused by the failure of the models proposed during the first quarter of the 20th century. This was specially the case after Bohr’s atom, consisting of point electrons moving in circular orbits around the nucleus. The model, generalized with the inclusion of elliptical
orbits, produced some progress in the decade after 1913. However it was increasingly clear that the model was untenable. In 1926 an alternative model was proposed by Schrödinger, who interpreted his wave mechanics as showing that electrons are continuous charge distributions. As is well known that model was soon abandoned after the correct criticisms by Bohr, Heisenberg and other people. Independently Heisenberg had proposed a formalisms, with the name of quantum mechanics, that explicitly rejected any model. Indeed he supported the view that the absence of a picture was a progress towards a more refined form of scientific knowledge. The success of the new quantum mechanics in the quantitative interpretation of experiments, toghether with the failure to find a good physical model of the microworld, led to the almost universal acceptance of the current view that models are unnecessary or even misleading.

I do not agree with that wisdom, and this paper is a defence of a real-istic interpretation of the quantum phenomena. I am aware that the task is extremely difficult, but convinced that many of the obstacles derive from assumptions unnecessary for the interpretation of the experiments. These assumptions have been included along the time and are now a part of the common wisdom. Pointing out the main obstacles and how they might be removed is the main purpose of this article. It does not pretend to be a coherent and complete realistic interpretation.

1.3 A note on epistemology of physics

In order to make science some previous philosophical questions should be answered. For instance, what is science?, or what are the criteria to distinguish science from nonscientific knowledge? I accept the definition of Karl Popper[17]: “A claim is scientific if it may be refuted by observations or experiments”. This definition is a consequence of a well known fact, that is the possible existence of several different theories all of them predicting correctly the results of experiments in a given domain. In other words the correctness of a theory is sufficient, but not necessary, for the appropriate prediction of the empirical facts. For this reason a single experiment may refute a theory but a theory can never be fully confirmed empirically, and this is essentially the Popper thesis. As a consequence several different theories may exist able to predict correctly the empirical results, but suggesting quite different pictures of the microworld.

Popper´s criterion is good enough as a matter of principle, but it is not so
good in practice. In fact it is the case that rarely an established theory breaks down as a consequence of a single experiment contradicting it. As Lakatos has correctly pointed out, well tested theories are protected in the sense that the empirical refutation of a single prediction may be interpreted without rejecting the theory, for instance assuming that the particular model used to analyze the experiment was too coarse. Indeed it is a historical fact that established theories are only abandoned, or better superseded, when there is a new theory in agreement with the former one in its domain of validity, but possessing a wider domain or other virtues.

Quantum mechanics is today a fully established theory and therefore it is very well protected in the sense of Lakatos. I do not only mean protection in the domain where the theory has been tested. What I want to stress is that along the years people has introduced a number of assumptions, today widely accepted, that are additions beyond any possible empirical test. These unnecessary additions are also protected and, in my opinion, they are the main cause of the strong difficulties in reaching a realistic physical model of the quantum world.

Most working quantum physicists adhere to the pragmatic approach as commented above. The support has its roots in a “positivistic” attitude. Positivism is the philosophical doctrine that, in a broad sense, states that all knowledge should be founded on empirical evidence. If the statement is applied to scientific knowledge, it is accepted by everybody. But in a more strict sense it is a tendency to give value to the empirical data in detriment of the theoretical elaborations. For instance this was the opinion of Ernst Mach, who rejected the concept of atom because at that time (around 1900) atoms had not been directly observed.

Positivism was also behind Heisenberg initial formulation of quantum mechanics resting upon the belief that only sets of numbers corresponding to the possible results of measurements should enter the theory. This led him to elaborate quantum mechanics as a calculational tool involving matrices (that was sometimes called ‘matrix mechanics’.) The combination of mathematical formalism and empirical results almost without further theoretical elaboration permeates the interpretation of quantum mechanics till now. A clear confrontation between the positivistic and realistic epistemologes appears in the conversation of Heisenberg with Einstein that took place in Berlin 1926, as remembered by Heisenberg himself. The most relevant part is reproduced in the following.

\[\text{“Einstein opened the conversation with a question that bore on the philo-}\]
sophistical background of my recent work. 'What you have told us sounds extremely strange. You assume the existence of electrons inside the atom, and you are probably quite right to do so. But you refuse to consider their orbits, even though we can observe electron tracks in a cloud chamber. I should very much like to hear more about your reasons for making such strange assumptions'. 'We cannot observe electron orbits inside the atom', I must have replied, 'but the radiation which an atom emits during discharges enables us to deduce the frequencies and corresponding amplitudes of its electrons. After all, even in the older physics wave numbers and amplitudes could be considered substitutes for electron orbits. Now, since a good theory must be based on directly observable magnitudes, I thought it more fitting to restrict myself to these, treating them, as it were, as representatives of the electron orbits.' 'But you don't seriously believe', Einstein protested, 'that none but observable magnitudes must go into a physical theory?'. 'Isn't that precisely what you have done with relativity?' I asked in some surprise. 'After all, you did stress the fact that it is impermissible to speak of absolute time, simply because absolute time cannot be observed; that only clock readings, be it in the moving reference system or the system at rest, are relevant to the determination of time'. 'Possibly I did use this kind of reasoning,' Einstein admitted, 'but it is nonsense all the same. Perhaps I could put it more diplomatically by saying that it may be heuristically useful to keep in mind what one has actually observed. But on principle, it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens, it is the theory which decides what we can observe. You must appreciate that observation is a very complicated process. The phenomenon under observation produces certain events in our measuring apparatus. As a result, further processes take place in the apparatus, which eventually and by complicated paths produce sense impressions and help us to fix the effects in our consciousness. Along this whole path - from the phenomenon to its fixation in our consciousness - we must be able to tell how nature functions, must know the natural laws at least in practical terms, before we can claim to have observed anything at all. Only theory, that is, knowledge of natural laws, enables us to deduce the underlying phenomena from our sense impressions. When we claim that we can observe something new, we ought really to be saying that, although we are about to formulate new natural laws that do not agree with the old ones, we nevertheless assume that the existing laws - covering the whole path from the phenomenon to our consciousness - function in such a way that we can rely upon them and hence speak of observations.'
The conversation continued for a while and at the end Einstein warned: “You are moving on very thin ice. For you are suddenly speaking of what we know about nature and no longer about what nature really does. In science we ought to be concerned solely with what nature does.” Einstein arguments are a clear support to a realistic epistemology, and I fully agree with his views.

1.4 Sketch of a realistic interpretation of quantum mechanics

In this paper it is supported the hypothesis that quantum theory is a peculiar stochastic theory. The stochasticity derives from the existence of random fields in the vacuum. That is I assume that the vacuum is not empty but full of fluctuating fields, Planck’s constant, \( h \), fixing the scale of the fields. More specifically every vacuum field in free space may be expanded in plane waves, whose amplitudes are a set of statistically independent random variables with zero mean. The square mean is such that the average energy of one of the plane waves is \( \frac{1}{2} h \nu \), \( \nu \) being the frequency. The stochasticity of quantum theory is peculiar because the field components of low frequency are weak but those of high frequency are strong. This contrasts with the best known stochastic theory, namely Brownian motion, where all components have equal strength (the spectrum corresponds to white noise). The vacuum fields act on particles, thus producing a random motion that departs from the classical motion. Also the presence of matter (particles) modifies the vacuum fields, as shown for example in the Casimir effect commented in the following.

The existence of virtual fluctuating fields in the vacuum is recognized in the most advanced form of quantum theory, namely quantum field theory, but virtual is not a well defined concept in the first place. Thus I assume that the fields are real. A support to the reality of the vacuum fields is provided by the Casimir effect\(^{20, 21}\), that is an attraction between two parallel metallic plates placed in vacuum, say at a distance \( d \). In fact, due to the boundary conditions of the electromagnetic field on a metallic surface, the (mean) energy density and pressure of the vacuum field near the surface is different from those quantities in free space, which gives rise to a net force per unit area, \( F/A \), between the plates. For perfectly conducting metal and large enough plates (i.e. \( A >> d^2 \)) so that border effects are negligible, \( F/A \) may depend only on the distance, \( d \), the Planck constant, \( h \), and the velocity
of light, $c$, whence dimensional considerations lead to

$$\frac{F}{A} = K \frac{hc}{d^4}.$$ 

A detailed calculation (which may use purely classical electrodynamics\cite{22}) gives $K = -\pi/480$, the negative value meaning attractive force. The measurement results agree with the prediction\cite{23}.

Elementary quantum mechanics (QM) is an approximation to field theory where the vacuum fields do not appear explicitly. Thus QM looks like a stochastic theory where the source of randomness is hidden. This is one of the main obstacles for a realistic understanding of the theory. In fact in sections 2 to 4 several examples will be presented showing that quantum field theory allows a better intuitive understanding than QM. The examples deal with the quantum theory of the electromagnetic field (i. e. quantum electrodynamics, QED) because that field is the most relevant in low energy phenomena, related to atoms, molecules or condensed matter.

2 Specific features of quantum physics

In this paper it is proposed that the difficulties for a realistic interpretation of quantum phenomena do not derive from the empirical facts, or not only. Thus in the following I shall briefly revisit the most relevant of those facts in order to analyze whether the nude empirical facts put actual difficulties for a physical model of the microworld, independently of the quantum formalism. The difficulties for the interpretation of the formalism will be treated in another section.

2.1 Discrete energy states

As is well known the assumption that material systems may possess only energies belonging to a discrete set was the first quantum hypothesis, introduced by Planck in 1900. It was reinforced by the Einstein 1905 proposal that light consists of discrete pieces of energy (photons). In 1913 Bohr incorporated this idea to his atomic model postulating that atoms can only exist in states having energies within a discrete set, $E_0, E_1, E_2,...$. The model also assumed that the absorption and emission of light takes place with transitions between these states, the frequency, $\nu_{jk}$, of the light related to the
difference of atomic energies by

\[ h\nu_{jk} = E_j - E_k. \] (1)

In practice the frequencies are observed whilst the existence of energy states is derived from eq. (1). The success of Bohr’s model gave support to the hypothesis of discrete atomic energy states and the assumption was confirmed by the experiment of Frank and Hertz in 1914. It consisted of the scattering of electrons on mercury atoms in vapour state with the result that, for high enough electron energies, inelastic scattering was observed with a decrease of the electron energy by 4.9 eV. This quantity precisely corresponds to a frequency of the mercury spectrum via the relation eq. (1). Thus the said quantity was interpreted as the energy difference between the ground state and the first excited state of the atom. As a consequence of these facts, and others, it has been fully accepted the hypothesis that the set of energy states of atoms is discrete. Furthermore the discontinuities have been incorporated to the quantum formalism, assuming that physical quantities should correspond to operators (in a Hilbert space), many of them having a discrete spectrum. Also the discreteness has been adscribed to other dynamical quantities.

The quantum discontinuities give rise to difficulties for an intuitive understanding of quantum physics. In fact it is difficult to picture how material systems may make transitions between two different energy states never possessing any intermediate energy. Hints for the solution of the difficulties is provided in the following. Firstly it is necessary to distinguish the discreteness of the energies in the electromagnetic radiation, that is the assumption that light consists of particles (photons), from the discreteness of the energies of many-body systems like atoms, molecules or nuclei. The nature of photons will not be discusses here, the discontinuity of the atomic energies will be considered in the following.

Quantum electrodynamics predicts that spectral lines are not sharp, but possess some width. Thus Bohr’s eq. (1) should be taken as an approximation, the possible atomic energies actually consisting of a continuous set. Thus eq. (1) simply recalls that the probabilities of the energy states are strongly concentrated near some discrete values. This solves one paradox which appears when the emission or absorption of light is presented at an elementary level, namely the contradiction between assuming that atomic transitions are instantaneous and assuming that the emitted light has a sharp
frequency. The fact is that the transition has a finite duration, $\Delta t$, and the emitted light has a finite linewidth, $\Delta \omega$, fulfilling the inequality

$$\Delta \omega \Delta t \gtrsim 2\pi,$$

which is well known from classical optics. The inequality holds true for any periodic motion and the quantum formalism also predicts it. Indeed sharp energies of atoms appear only in a calculation to lowest order of approximation (i.e., in the limit when the electron charge $e \to 0$). However when radiative corrections of quantum electrodynamics are taken into account the calculation leads to spectral lines with a finite width. The corrections are small and may be neglected in elementary calculations but they are essential for a realistic interpretation. This is a typical example of how the emphasis on the simplicity of the calculations, rather than the clarity of the concepts, has the consequence that quantum mechanics appears as counterintuitive. A realistic picture of the atomic emission is possible assuming that light is emitted in a continuous process lasting a time $\Delta t$ that fulfils the inequality \(2\), the total energy of atom plus light being conserved at all instants of time. Indeed this fits with the quantum electrodynamical evolution equation of the atom coupled to the electromagnetic field. Of course the realistic picture is incomplete because some explanation should be found for the relation between frequency and energy, which is quite strange from a classical perspective. But the lack of a realistic model (till now) does not imply that a model is impossible.

Similar arguments may be used in order to understand the quantization of angular momentum, as shown for instance in the Stern-Gerlach experiment. In popular expositions the experimental results are presented as if all atoms arrive at one amongst two sharp lines in a screen. Then it is difficult to reach a picture of what is taking place in the interaction between the atom and the inhomogenous magnetic field. However the truth is that what appears in the screen are two wide spots, something much less counterintuitive. It is the case that an accurate quantum mechanical treatment of the experiment precisely predicts that \(2\). A picture of the phenomenon may be reached assuming that during the interaction of the atom with the magnetic field some fluctuation and dissipation takes place which tends to alineate (approximately) the atomic magnetic moment with the field.
2.2 Heisenberg uncertainty principle

The Heisenberg uncertainty principle is the most frequently quoted evidence of the dramatic splitting between classical and quantum physics. In fact the principle appears in popular writings like a kind of mysterious property of our world. I shall not discuss here the general principle dealing with conjugate dynamical variables. I will restrict attention to the experimentally proved impossibility of determining simultaneously the position and the velocity (or the momentum) of a particle. This implies that it is not possible to prepare a particle with both position and velocity sharply defined, and also that no measurement may provide the values of both these quantities at the same time. A consequence is the impossibility of finding empirically the path of a particle. In the following it is shown that a realistic interpretation is possible by analogy with what happens in Brownian motion. A Brownian particle possesses a highly irregular path whose instantaneous velocity cannot be measured (with ordinary, macroscopic set-ups). Only the mean velocity, \( \bar{\nu} \), during some time interval may be measured, that is

\[
\bar{\nu} = \frac{\Delta r}{\Delta t},
\]

where \( |\Delta r| \) is the distance between the initial and final positions in the time interval \( \Delta t \). On the other hand there is a relation, derived by Einstein in 1905, between the expected value of the square of the distance, \( |\Delta r|^2 \), and the time interval, \( \Delta t \). That is

\[
\langle |\Delta r|^2 \rangle = D\Delta t,
\]

where \( D \) is called diffusion constant and \( \langle \rangle \) means ensemble average, that is the average over many measurements involving the same time interval. If we eliminate \( \Delta t \) amongst the two equalities we get

\[
\langle |\Delta r|^2 \rangle = \langle \nu^{-2} \rangle \Delta t^2 \Rightarrow \langle |\Delta r|^2 \rangle \langle \nu^{-2} \rangle \approx D^2,
\]

a relation having some similarity with the Heisenberg inequality. In conclusion a plausible interpretation of the Heisenberg principle is that the quantum motion possesses a random component having similarity (not identity!) with Brownian motion. This similarity has been the basis for the development of
stochastic mechanics\cite{25}, which provides an intuitive picture of some typically quantum phenomena. However this theory presents difficulties that will not be discussed here.

The Heisenberg principle becomes an obstacle for a realistic interpretation of quantum mechanics when the empirically found practical difficulty (or impossibility) of simultaneous knowledge of position and velocity is elevated to the category of an ontological statement: “Trajectories of quantum particles do not exist”.

2.3 The apparent lack of objective properties.

In classical physics it is an implicit hypothesis that any observation or measurement just reveals (‘removes a veil’) a property which exists objectively with independence of any observation. In quantum mechanics this seems to be untrue. Let us clarify the motivation for that belief with an example. We consider a physical system possessing three observable properties which I shall label $A$, $B$ and $C$.

I will assume that the observables $A$ and $C$ may be measured in the same experiment and similarly for $B$ and $C$, but for some reasons $A$ and $B$ cannot be measured with the same experimental arrangement. Then with repeated measurements in identically prepared systems it is possible to obtain the joint probability distribution for the results of the former measurement, which I will represent by the density, $\rho(a,c)$, that the observable $A$ takes the value $a$, and the observable $C$ the value $c$. Similarly we may obtain $\rho(b,c)$, but it is not possible to obtain empirically a joint probability density $\rho(a,b)$ because we cannot measure $A$ and $B$ simultaneously. Up to here no problem arises, everything agrees with the intuition.

Now if we think that the measurement just reveals preexisting values of the observable quantities we are compelled to assume that, in every state, the system possesses the values $a$, $b$ and $c$, independently of any observation or measurement. More generally the preparation procedure will lead to a state with a joint probability distribution, $\rho(a,b,c)$, for the three observables. If this is the case the joint probabilities for two observables should be the marginals of the former distribution, that is

$$\rho(a,c) = \int \rho(a,b,c) \, db, \quad \rho(b,c) = \int \rho(a,b,c) \, da. \quad (3)$$

However it has been shown in some experiments that there are particular
cases of states and observables where no (positive) joint probability density \( \rho(a,b,c) \) exists such that the marginals eqs.\((3)\) agree with the empirical results. The non-existence of a joint probability fulfilling eqs.\((3)\) in general is predicted from the quantum formalism, and it is the essential content of the Kochen-Specker theorem (see e.g. Mermin\([26]\).) As in the case of the Heisenberg principle, the practical impossibility has been raised to the rank of an ontological statement: “\textit{Physical systems do not possess properties independently of measurements.}”

Is that statement justified? It is not\([27]\). What the experiments have shown is that the observed properties depend not only on the state of the system but on the whole experimental set-up. In fact, we may assume that physical systems possess some properties, called ‘elements of reality’ by EPR\([14]\), which in specific experimental set-ups give rise to observable quantities, but the observables may not exist independently of the experiment. Actually a similar situation also happens in classical physics, as for instance when we play dice. If we get a number, say 2, we cannot claim that the value 2 was preexistent to our experiment. In fact the result 2 is actually ‘created’ by the experiment of throwing the dice. Returning to quantum physics, there is a simple explanation for the frequent inexistence of properties independent of measurements (some particular properties do exist, for instance the rest mass of particles). We may assume that the measured properties are contextual, that is they depend not only on the state of the system but on the whole experimental context. This point was correctly emphasized by Bohr and, in my opinion, solves all problems of interpretation which might follow from the Kochen-Specker theorem. (Of course the theorem provides some quantitative statements which should be explained, but here I am addressing the question whether the practical impossibility of getting joint probabilities prevents a realistic interpretation.)

The real difficulty arises when people attempts to reach conclusions which go beyond what follows from the facts. Indeed we can state that \textit{some properties} do not exist independently of measurements \textit{in some particular instances}, but we should not extrapolate telling that in nature \textit{there are no properties} independent of the observation. This absurd extrapolation was correctly criticized by Einstein with his celebrated rhetorical question “\textit{Is the moon there when nobody looks?}”\([29]\).

One might ask why in the microscopic domain it is frequent that values of the observables are created by the experiments whilst this situation is rare at the macroscopic level. An explanation may be as follows. In the macroscopic
world we may study systems with instruments more fine than the object to be studied. E. g. we may look at the interior of an orange using a knife. In the microscopic domain any macroscopic equipment used for the study of atoms will consist of atoms. This makes our knowledge less direct in the micro than in the macroscopic domain, and more dependent on the context.

The fact that the measurement cannot be understood as simply revealing the values of preexisting properties has led to the introduction of some ‘postulates of the measurement’ in the standard quantum formalism as discussed in the next section.

2.4 Statistical character

Typical experiments are affected by statistical errors even in the classical domain. That is the same experiment performed in similar conditions may give rise to (slightly) different results. For this reason it is a standard practice to report the results of measurements accompanied by an uncertainty interval. In the macroscopic domain the uncertainty is attributed to the difficulty in controlling a very large number of parameters, with the consequence that never (or rarely) an experiment may be repeated in exactly the same conditions. In any case it is usual that the uncertainty is only a very small fraction of the measured quantity. In contrast in the microscopic domain it is frequent that the uncertainties are of the same order than the measured result. This is equivalent to say that the same experiment may give rise to a number of different results, every one with some probability. For this reason it is said that any theory of the microworld should be statistical, that is giving predictions in the form of probabilities of several different outcomes. However, at a difference with macroscopic (classical) physics, in quantum physics the probabilities are usually not attributed to lack of control in the experiment.

The current wisdom is that quantum probabilities are radically different from the classical, ordinary life, probabilities. The latter derive from incomplete knowledge (‘ignorance’), maybe unavoidable, about the truth of some assertion. For instance we may attach a probability 1/2 to the appearance of head in throwing a coin, because we cannot control all relevant variables in the experiment. In contrast it is assumed that quantum probabilities are quite different, that they derive from a lack of strict causality of the natural laws, that is the fact that different effects may follow to the same cause. This is usually called the fundamental or essential probabilistic character of the physical laws. Again a practical difficulty has been raised to the rank of an
ontological statement: “Natural laws are not strictly causal”.

Einstein disliked that assumption and strongly criticized it, as shown by his celebrated sentence “God does not play dice”. I understand very well Einstein’s opinion. For him the rational understanding of nature was a kind of religion. As more loose (strict) are the natural laws smaller (greater) could be our rational understanding of nature. Accepting a weak causality is like accepting poor science. Nevertheless there are people happy with the absence of determinism implied by the nonexistence of strict causality. For instance some claims have been made that the quantum lack of determinism may explain human freedom. This question lies outside the scope of this paper and will not be further commented.

But I do not support determinism in the mechanicistic view of Laplace. In my opinion quantum mechanics is a stochastic theory. There are strictly causal laws, but there is also an universal noise (the random vacuum fields) which permeates everything and prevents any practical determinism in the evolution (see the Introduction section). Strict causality combined with stochasticity (randomness) is in practice indistinguishable from essential probability, and the former is more plausible. In order to clarify the matter let us think again in Brownian motion. Under macroscopic observations the random motion of a Brownian particle may appear as lacking causality, but we assume that, taking into account the molecules of the liquid where the particle is immersed, the whole motion is governed by Newtonian dynamics, which is causal.

2.5 Wave-particle duality

The assumption that all quantum entities have a dual nature, particle and wave, is the source of most difficulties for an intuitive understanding of quantum mechanics. But if we do not want to destroy the basic properties of space, the wave-particle duality really involves a contradiction. In fact particle means something localized, wave means something extended. More precisely, particle (wave) means smaller (much greater) than some reference length, usually a few times the size of an atom. For this reason it is bizarre to say that an atom (with radius about 10 nm) passes simultaneously through two slits (distant about 1µm). Thus it is not strange that for some people, like Feynman, the interference experiments contain all the mysteries of quantum mechanics. The problem posed by the wave-particle duality for a realistic interpretation of the quantum phenomena is certainly big. It will be
considered elsewhere, but in the following I sketch a possible solution. We may assume that in nature there are both particles and fields (waves), the particle behaviour of fields deriving from the interaction with particles and the wave behaviour of particles from the interaction with fields. The difference with the macroscopic world, where there are also particles and fields, is that interactions are more relevant and complicated in the microscopic domain.

I think that the electrons (or protons, neutrons, atoms, molecules) are particles, whilst radiation consists of waves. ‘Photons’ are not particles but mathematical constructs useful for the description of some phenomena.[30] Then, how may we interpret the interference experiments where we observe fringes typical of waves, but these fringes appear as sets of localized events which are typical of particles? In the case of radiation the interference may be easily understood in classical terms, the problem is the particle behaviour in the detection. The opposite is true for particles like atoms. Its localized detection is easy to understand but their interference puts the problem. Let us study the two cases separately.

The detection of ‘individual photons’ in a photographic plate is due to the atomic nature of the plate. In this case saying that radiation are particles because they give rise to individual blackened grains is like saying that wind is corpuscular because the number of trees falling in the forest is an integer. Of course in both cases, the photo and the forest, there is a random element. It is obvious for the wind but there is also a random element in the radiation: the quantum noise or quantum vacuum fluctuations. The detection process in a photon counter may be explained as a transfer of energy from the field to individual atoms or molecules, this producing excitation that in some cases results in one count. I believe that the vacuum fields play a relevant role in this process.

The wave behaviour of neutrons, atoms or molecules, for instance in the two-slits experiments, is more difficult to understand. We might assume that it is caused by the vacuum fields, mainly the electromagnetic radiation. For instance let us consider a metallic plate with two holes. The electromagnetic vacuum fields near the plate will be different from the fields when the plate is not present (the difference gives rise to the Casimir effect, see the Introduction section above). Thus it is plausible to assume that the waves traveling from the left (right) of the plate give an interference pattern at the right (left) side. Thus any particle with net charge (e. g. an electron) or charged parts (a neutron or an atom) crossing one of the holes will have a
motion modified by the action of those waves, which provides a qualitative understanding of the particle interference. The picture has some similarity with the old proposal by L. de Broglie (the pilot wave theory) or to the picture offered by ‘Bohmian mechanics’ \[31\], but there are important differences. Firstly in our view there is a clear physical entity causing the interference, namely the vacuum fluctuations, whilst the particle remains localized all the time. Secondly there is a random element which is not present in Bohmian mechanics. Of course any model resting upon these ideas would be rather involved and it is not easy to understand why quantum mechanics provides so simple rules for the quantitative prediction of the empirical results. In fact the interference experiments with particles put a big challenge for a realistic interpretation of quantum theory.

2.6 Conclusions

The analysis of the most characteristics quantum phenomena leads me to emphasize a point that is crucial for the attempt of reaching a picture of the quantum world. The difficulties for a realistic interpretation of quantum mechanics may derive from a number of unneeded assumptions, adhered to the minimal quantum formalism for historical reasons. In some cases the difficulties are caused by an excess of idealization in the interpretation of the experiments. An example of the former is the claim that quantum particles have no trajectories. An instance of the latter is the usage of idealizations and the adscription of any deviation from the idealized result to accidental errors. It is assumed that this contributes to the clarity but in my opinion it is the opposite, it contributes to misunderstanding. It is true that the method may simplify the teaching of how to use quantum mechanics, but it puts a strong obstacle for getting an intuitive picture of the quantum world. A typical example, commented above, is the use of first order perturbation theory in the study of emission or absorption of light, which hides the fact that the formalism (quantum electrodynamics) predicts a continuous evolution of atom plus field.
3 Critical comments on current interpretations

As said above, Heisenberg quantum mechanics was proposed as an abstract formalism without any physical picture behind. Bohr justified the absence of a model and, on this basis he elaborated the ‘Copenhagen interpretation’[32]. Later on several modifications or novel interpretations have appeared[1], [5]. Relevant papers up to 1983 are reprinted in a book by Wheeler and Zurek[2]. I shall comment briefly on the most popular interpretations in the following.

For the sake of clarity I will illustrate the comments with the celebrated ‘Schrödinger cat’ gedanken experiment[33]. It consists of a box containing a radioactive atom and a cat together with a device that kills the cat, say instantaneously, when the atom decays. I will assume that both the atom in the excited state and the live cat are put inside the box at time $t_1$. The question is what may be said about the atom and the cat at times $t > t_1$. In particular, what is the prediction of quantum mechanics for the states of both the cat and the atom when the box is open at time $t_2$. Any person with knowledge of the law of radioactive decay, but ignorant of quantum mechanics, would claim that the probability of being both the cat alive and the atom excited at time $t \in [t_1, t_2]$ is

$$P(t) = exp\left[-\lambda (t - t_1)\right],$$

(4)

$\lambda^{-1}$ being the mean lifetime of the atom. In particular the probability at the moment of opening the box will be $P(t_2)$, eq.(4). This may be named the response of a ‘naive realist’. In contrast, the answer of an educated quantum physicist will depend on the interpretation that she/he supports, as I comment in the following.

3.1 Copenhagen interpretation

According to the Copenhagen interpretation (CI) the referent of quantum mechanics is not the material world but the experiments. That is the theory deals with the relations between the world and the observers. As Bohr put it “the finite magnitude of the quantum of action prevents altogether a sharp distinction being made between a phenomenon and the agency by which it is observed”[32]. Thus CI is close to the pragmatic approach as commented
in the Introduction section. According to this approach we should not make assertions about the bodies, but about the results of possible observations or measurements. Thus a sentence like “the probability that the atom is in the excited state at time $t$” is considered meaningless. A meaningful assertion should be something like “if we perform a measurement of the state of the atom at time $t$, the probability that we get the result ‘excited’ is given by eq. (4).” The approach is instrumentalist and it might be called a ‘protocol for the use of the quantum formalism’ rather than an interpretation. Bohr elaborated a philosophical background with the introduction of the ‘complementarity principle’ and the ‘correspondence principle’, in order to solve two theoretical difficulties of the formalism. Firstly it is unsatisfactory that quantum mechanics applies only to the microscopic world whilst the macroscopic one is governed by classical theories. Bohr’s solution to this problem was to assume that there is a smooth transition, quantum laws approaching the classical ones in the limit when Planck constant becomes negligible, formally when $\hbar \to 0$. This is the essential content of the correspondence principle, that Bohr also applied to several instances deriving some relevant results. The second theoretical problem was the existence of apparent contradictions, in particular those derived from the fact that quantum entities behave sometimes like particles and other times like waves. In order to solve that problem Bohr proposed the complementarity principle, which stresses the incompatibility between causal laws and spacetime description, due to the finite (nonzero) value of the quantum of action. After that he showed that there is no contradiction in practice because the behaviour of the quantum entities does not derive from the microscopic system alone, but also depends on the full context, including macroscopic measuring devices.

The rules of CI for the use of quantum mechanics are to some extent independent on the two mentioned Bohr’s principles, and I will comment only on the rules. CI assumes (or at least it does not reject the assumption) that macroscopic bodies have objective properties (that is independent of any measurement) and its evolution is governed by the laws of classical physics. Thus it is meaningful to ask whether a cat is either alive or dead at any time. A more difficult question is whether we are allowed to assign a probability to every one of these possibilities. The application of quantum mechanics, with the CI rules, to the “cat experiment” is that for $t \in (t_1, t_2)$ the atomic
state should be represented by the state vector

\[ | \psi(t) \rangle = c_g(t) | g \rangle + c_e(t) | e \rangle, \]

where \( | g \rangle \) (\( | e \rangle \)) is the state vector of the atom in the ground (excited) state. Now there are two possibilities depending on what is supposed to be a measurement: 1) If we assume that the actual measurement takes place when the box is open, then quantum mechanics says nothing about the atom and the cat for times \( t \in (t_1, t_2) \). At time \( t_2 \) it predicts that the probability of both the cat being alive and the atom being excited is given by the modulus square of the amplitude \( c_g(t) \), eq.(5), which precisely agrees with the naive prediction \( P(t_2) \), eq.(4). 2) We might assume that the cat, being a macroscopic system, may act as measuring device. In this case, CI tells us that, for any time \( t \in (t_1, t_2) \), the probability of both the cat being alive and the atom excited is eq.(4). The latter interpretation (the cat as measuring device) is consistent with the fact that, if the cat is found dead at time \( t_2 \), a careful study of the corpse (involving macroscopic manipulations) might determine the time of death, say \( t_d \). This would allow reconstructing the whole history: The cat was alive and the atom excited until \( t_d \). We must assume that, if a similar experiment is performed many times, the distribution of times \( t_d \) would converge to an agreement with the probability eq.(4).

It is interesting the Bohr approach to the problem of the ‘state vector (or wave function) collapse’. This is the discontinuous change of the state vector when a measurement is made, e.g. a change from eq.(5) to \( | \psi \rangle = | g \rangle \), at the time of opening the box. In our example we may naively believe that the collapse is just a change of our information as a result of the observation. However Bohr strongly opposed to the belief that the wave function just represents our information about the system, with the implicit consequence that this information may be incomplete. See section 4.1 for a more detailed discussion of the completeness question.

### 3.2 John von Neumann

CI is very good from the practical point of view and avoids any bizarre assumption (which is not the case in more elaborated interpretations commented below.) The problem with the CI is that it creates what has been called an ‘infamous boundary’ [35], that is a discontinuity between micro and
macrophysics. The former should be studied within quantum mechanics, the latter using classical physics. In order to remove the boundary and get an interpretation where quantum mechanics is valid also for macroscopic systems, John von Neumann introduced a theory of measurement and even he gave a model for it. His approach has been also currently named Copenhagen interpretation, but this is misleading because von Neumann’s interpretation is different from Bohr’s as a matter of principle. However it was an elaboration of the Copenhagen interpretation rather than an alternative, which may justify the name. For short I shall label it MCI with M standing for modified or measurement. MCI has been supported in most papers and books of quantum mechanics until around 1980.

The modification introduced by von Neumann with respect to Bohr was to take seriously the assumption that quantum mechanics is the universal theory and classical theories are just approximations. Thus he proposed studying the measurement within quantum mechanics and made a model involving the coupling of the microscopic system with the measuring apparatus. However this gave rise to a number of difficulties that will be commented below, but previously we clarify the matter studying the application of von Neumann’s ideas to the cat experiment.

In MCI both the cat and the atom should be treated as quantum objects. Therefore eq.(5) is no longer appropriate and we should represent the state of the whole system, atom plus cat, by

\[ |\psi(t)\rangle = c_g(t) |g\rangle |\text{deadcat}\rangle + c_e(t) |e\rangle |\text{livecat}\rangle. \tag{6} \]

Of course one may point out that a dead cat does not correspond to a pure state to be represented by the vector $|\text{deadcat}\rangle$. Indeed there may be very many quantum states corresponding to a dead cat and similarly for a live cat. However this is not a real problem because MCI assumes that any physical system is associated to a well defined state vector. (When the appropriate state vector is not known we should use a probability distribution over those vectors, which may be formalized via a density matrix. But for simplicity we may use a single state vector as in eq.(6))

Eq.(6) represents a typical ‘entangled state’, a name introduced by Schrödinger in 1935. If CI had been modified with the assumption that state vectors actually represent statistical ensembles this would have lead to the ensemble interpretation, to be commented below. However the mainstream of the scientific community rejected it and supported the ‘completeness’ of quantum
mechanics, in the sense that the state vector represents the actual state of an individual physical system, as opposed to a statistical ensemble. With this assumption the MCI leads to bizarre consequences, which was the point that Schrödinger attempted to stress with his cat example. Indeed for many people it is impossible to understand the meaning of a state represented by a superposition of alive and dead cat. Is it something intermediate between life and death?

The problem is not only the highly counterintuitive character of the superpositions of macroscopic systems, it is the disagreement with empirical evidence. Indeed it is the case that those macroscopic superpositions cannot be manufactured in practice (there is a lot of literature about the actual preparation of ‘Schrödinger cats’, but they always involve mesoscopic rather than truly macroscopic systems.) Thus it seems that the quantum evolution (the Schrödinger equation) is violated at the macroscopic level. This has been called the problem of the objectification or individuation. That is the fact that a particular value is obtained in the measurement amongst several possible values, something not predicted by the quantum formalism except if an explicit postulate is included. This postulate forces us to change the state vector at the time of measurement, a change usually called the ‘state vector, or wave function, collapse’. That change is not predicted by the Schrödinger equation and in fact it precludes the validity of that equation during measurements. In the CI the collapse was just a change of the mathematical representation needed for the analysis of the experiment. However in MCI it becomes a real physical change because it is assumed that the state vector corresponds to an individual system (rather than a statistical ensemble). In the next subsection I discuss possible solutions that have been proposed.

A problem related to the objectification is the existence of quantum jumps, the typical example being the decay of a radioactive atom. For instance an atom of uranium 238 may remain as such during million years but, at some unexpected time, it decays with the emission of an alpha particle (a nucleus of helium 4). The sudden decay (within a small fraction of one second) apparently contradicts Schrödinger equation. People like to say that the observation of the (spontaneous) decay is a particular case of measurement and therefore the problem of the quantum jumps becomes an example of objectification after a measurement. In my opinion however there is a clear difference between objectification and quantum jump. The former is more properly the ‘disentanglement’ of an entangled state involving macroscopic bodies. For instance the fact that we see the cat alive or dead at the time
of opening the box in the Schrödinger cat example. In contrast a quantum jump refers to a discontinuous change of a microscopic system. In any case the difficulties with both objectification and jump may be solved simultaneously, for instance in either hidden variables or collapse theories, the latter discussed in the next subsection and the former later on.

3.3 The objectification and the quantum jumps problems

There are a variety of proposed solutions to the objectification problem. In the original (Bohr) Copenhagen interpretation there is no real problem: the Schrödinger equation is just a mathematical tool able to relate the preparation of a (microscopic) system to measurements made on it. The wave function (or state vector) is just a convenient form of dealing with the probabilities involved. That is a preparation gives rise, after some time, to a set of probabilities when the system is placed in an appropriate experimental context. The objectification is a change due to the measurement. But the change must be postulated because the interaction between the microscopic system and the macroscopic apparatus can be described neither by quantum nor by classical theories in CI. The objectification problem also does not exist in the ‘many worlds interpretation’ (MWI) that will be discussed in the next subsection.

From the time of von Neumann’s book[36] (1932) until around 1980’s, and for a fraction of the scientific community until today, the MCI has been the most popular interpretation. For this reason a very large number of papers and books have been devoted to propose possible solutions to the objectification problem.

John von Neumann pointed out that the measurement only finish when a (human) observer is conscious of the result of the experiment. This would solve the objectification problem if we assume that the mind is not governed by quantum mechanics. The proposition was also supported by London and Bauer[39] and commented by Wigner[40]. In the cat experiment, this seems to imply that the cat really dies when we look at the box after opening it, or even when we are informed by another person of the result of the experiment (this leads to the ‘Wigner’s friend’ paradox.) The solution dislikes many people.

In practice many authors (maybe not too fond of the subtleties of foun-
dational questions) accepted a kind of peaceful coexistence of the two (contradictory) postulates: the Schrödinger equation and the quantum theory of measurement. This attitude however has been strongly criticized by philosophers of science like Karl Popper[41] or Mario Bunge[42]. In particular the latter stresses that a physical theory should not include a general theory of measurement, but particular recipes or protocols for every specific measurement. This is most clear in chemistry. There are recipes for, say, the preparation of pure alcohol or the analysis of water of a river. However it would be absurd to search for a ‘general theory of preparation or analysis’ in chemistry. In my opinion the same is true in physics, including quantum physics. Actually the existence of a ‘theory of measurement’ is peculiar, it does not exist in any other theory in physics (or more generally in natural science). It is true that from a philosophical (epistemological) point of view any theory requires some assumptions for the connection with the results of observations or experiments. For instance in classical mechanics we use the concepts of time, space, particle, isolated system, etc., and there are rules telling us how these concepts should be related to the (mathematical) formalism. However it would be absurd to search for a ‘general theory of preparation or measurement’ in physics, including quantum physics.

A proposal that has been popular since around 1985 is to modify the Schrödinger equation in such a way that the change fulfil two consistency requirements: 1) For microscopic systems it produces an extremely weak, practically undetectable, modification in the evolution of the wave function, and 2) For macroscopic systems it gives rise to a rapid disentanglement, that is an evolution from any superposition to a single term. There have been several explicit models of this type, called ‘collapse theories’. The most satisfactory has been proposed by Ghirardi, Rimini, and Weber in 1985, and usually referred to as the GRW theory[43], [44]. At present, it involves phenomenological parameters that, if the theory is taken seriously, acquire the status of new constants of nature. There have been also attempts at deriving the parameters from fundamental arguments, like the action of gravitational forces (effects of general relativity.)

In spite of their phenomenological character, the collapse theories have relevance since they have made clear that there are new ways to overcome the difficulties of the quantum formalism. Moreover, they have allowed a clear identification of the formal features which should characterize any unified theory of micro and macro processes. Last but not least, collapse theories qualify themselves as rival theories of quantum mechanics and one can identify some
of their physical implications which, in principle, would allow crucial tests discriminating between the two. This possibility, for the moment, seems to require experiments which go beyond the present technological possibilities. I shall not review here the collapse theory, which would lead far from the main purpose of the paper. The interested reader may look at a good review by Ghirardi[45].

3.4 Many-worlds

The many worlds interpretation (MWI) offers a radical solution to the objectification problem, it assumes that objectification never takes place. That is, the evolution of an isolated system is always governed by the Schrödinger equation. Now no system involving a macroscopic body may be completely isolated, so that in the study of its evolution we should consider the wave vector of the whole universe. In particular, in the cat experiment we should include, in addition to the atom and the cat, also the box, the human observer and everything else. Thus eq.(6) should be replaced by

\[ |\psi(t)\rangle = c_g(t) |g\rangle |\text{deadcat}\rangle |\text{world} - g\rangle + c_e(t) |e\rangle |\text{livecat}\rangle |\text{world} - e\rangle, \]  

where \(|\text{world} - g\rangle\) represents the rest of the world associated to the atom in the ground state and the cat dead, and similarly for \(|\text{world} - e\rangle\). Eq.(7) seems to tell that there are two copies of the human observer and of the whole world. In the latter copy the observer sees the cat alive and the atom excited, in the former she/he sees the cat dead and the atom in the ground state. The state vector of the universe is a linear combination of these copies.

MWI is the unavoidable end of the logical path if we believe that quantum mechanics (as defined by the standard postulates excluded those of measurement) is universally valid. It was initially proposed by Everett[46] with the name of ‘relative states interpretation’ and elaborated later by de Witt[47], who introduced the name ‘many worlds’. The aims of MWI are: 1) retain the unrestricted validity of the quantum formalism, 2) remove the need of the state vector collapse, 3) remove the need of an external observer, and 4) derive the Born rule[48]. The latter is the rule for finding the probabilities of the different possible outcomes as a result of a measurement.

Apart from the difficulty of understanding the real meaning of ‘multiplicity of worlds’, the main problem of MWI is to reproduce the Born rule
without introducing any explicit probabilistic postulate. The standard approach to do that is the theory of decoherence\cite{49}, \cite{50}. Decoherence is the evolution predicted by quantum mechanics when a system possesses very many degrees of freedom, as is the case for a measuring device in contact with the environment. It involves a loss of coherence which leads from a state vector (representing a pure quantum state) to a density matrix, as a result of the interaction with the environment. That density matrix is approximately diagonal in an appropriate basis (the preferred basis), so that it looks like a probability distribution defined on a set of pure states, as is exhibited in the example eq.(8) below. In the context of MWI the density matrix may be seen as coming from taking the partial trace over those degrees of freedom which are not of interest. For instance if we take the partial trace, with respect to the world states, of the (idempotent) density matrix associated to the state vector eq.(7) we get with very good approximation

$$Tr_{\text{world}} |\psi\rangle \langle \psi| \simeq |c_g(t)|^2 |g - d\rangle \langle g - d| + |c_e(t)|^2 |e - l\rangle \langle e - l|,$$

(8)

where $|g - d\rangle$ is short for $|g\rangle |\text{deadcat}\rangle$ and $|e - l\rangle$ for $|e\rangle |\text{livecat}\rangle$ and the orthogonality of the state vectors $|\text{world} - g\rangle$ and $|\text{world} - e\rangle$ has been taken into account. Eq.(8) is mathematically identical to the representation of the quantum state of the atom plus the cat that we should use when we do not know its actual state, and consequently we attribute the probability $|c_g(t)|^2$ to the atom being in the ground state and the cat dead, and $|c_e(t)|^2$ the probability of the alternative. The question, to be discussed below, is whether eq.(8) actually corresponds to a mixture or not.

Actually decoherence theory is more involved than it may appear from our example. Firstly we should consider very many terms in the sum which represents the quantum state of the world, rather than only two as in our simplified example eq.(7). Also there is an ambiguity in the world state vector because, it being a linear combination of (tensor) products of state vectors, it could be written in many different forms depending on the choice of basis in the Hilbert space. This leads to the problem of the preferred basis, whose solution is one of the achievements of decoherence theory. I shall not discuss here in more detail the different approaches and the technical issues of the MWI and decoherence, and refer to the vast literature on the subject (see e. g.\cite{48} and references therein). Related to decoherence is the “consistent histories” approach\cite{51}, which will not be commented here.

MWI has the virtue that it makes quantum mechanics a selfconsistent theory resting upon a simple hypothesis: its universal validity. In this re-
spect it is superior to the old-fashioned CI and MCI. However as usually understood it leads to a rather bizarre picture of the world. For the sake of clarity I will consider a measurement with reference to eq.(8), although now ‘cat’ means the macroscopic measuring device able to suffer an irreversible evolution. Once MWI plus decoherence theory leads to a reduced density matrix like eq.(8), it seems plausible to interpret it as representing a mixture, \(|c_g(t)|^2\) and \(|c_e(t)|^2\) being probabilities in the usual sense of mathematical measures of information. However this interpretation is not compatible with the assumption that quantum mechanics is complete. That is, the hypothesis that eq.(8) represents a mixture is not compatible with the assumption that the state vector of the universe corresponds to an individual world (although with many branches), rather than a statistical ensemble of possible worlds. However it is irrelevant in practice whether we assume that the world state vector represents complete or incomplete information. In fact a detailed knowledge of that state vector would always lie beyond the human capabilities. Therefore the assumption that eq.(8) represents an actual mixture, and quantum mechanics is incomplete, is in my opinion most plausible.

In contrast, the conjunction of assuming universal validity (i.e. MWI) and completeness of quantum mechanics leads to the extravagant view that there are many parallel worlds\[48\]. I think that this belief is unnecessary. Actually the view rests on what has been termed a Platonist paradigm by M. Tegmark\[52\], who defines it as follows: “The outside view (the mathematical structure) is physically real, and the inside view and all the human language we use to describe it is purely a useful approximation for describing our subjective perceptions.” The mathematical structure referred to by Tegmark is the formalism of quantum mechanics. Thus the Platonist paradigm is equivalent to assuming that standard quantum mechanics is the absolute truth and everything else are shadows.

In my opinion scientific theories, quantum mechanics in particular, are something more modest. They are attempts at describing, rather imperfectly, “the objective reality, which is independent of any theory”\[14\]. In consequence I prefer to retain as much as possible of the MWI, logically superior to CI or MCI, but without adhering to the Platonist paradigm. The choice is obvious to me: we should reject the completeness of quantum mechanics, that rejection leading to the ensemble interpretation to be commented in the next subsection. (But most people assume that MWI is not compatible with an ensemble interpretation. Even if it is compatible the relation is not trivial and will not be discussed here.)
We have seen that in CI and MCI, above commented, it is necessary to introduce a probabilistic postulate, which is substituted for Schrödinger evolution equation during measurement. That postulate (Born’s rule) allows calculating the probabilities of the different possible outcomes of a measurement and the corresponding state vector after the collapse. In MWI it is controversial whether a probabilistic postulate is introduced. Many authors consider that this is not the case, that the quantum probabilities may be got from the formalism. Actually Everett introduced, in his original formulation, a measure given by the squares of the amplitudes in the sum (of normalized state vectors) which the world state vector consists of. In our example, eq.(7), that measure may allow to assume that $|c_g(t)|^2$ and $|c_e(t)|^2$ are probabilities. Therefore it is my opinion that MWI does introduce a probabilistic postulate, even if it is most natural, as Everett emphasized[46].

An interesting consequence of the MWI is that a state vector is only appropriate for the whole world. In contrast, the states of the systems which we may actually study (subsystems of the universe) should be represented by density matrices. This leads to the conjecture that only a subset of the whole set of possible density matrices represent physical states. This conjecture strongly limits the validity of the superposition principle and gives rise to the problem of determining what is the subset of the whole set of density matrices which correspond to physical (realizable) states. This problem will be discussed elsewhere.

3.5 Ensemble interpretation and hidden variables

The Copenhagen, von Neumann and many worlds interpretations have in common the assumption that the description offered by quantum mechanics is complete. They may be grouped within the class of ‘orthodox’ interpretations. An alternative to completeness is the assumption that the wave function just represents our knowledge about the actual state of a system. This hypothesis has been called the ‘ensemble interpretation’ and it was supported by Einstein[14], [29], and also by some authors in recent times [53].

The ensemble interpretation poses a question: What is the ensemble and what is the probability distribution on the ensemble?. Answering the question implies searching for an ontology behind the quantum formalism. That research has been usually known as the hidden variables programme. The ensemble interpretation and the hidden variables approach are crucial for a realistic understanding of quantum physics and consequently the whole
section 4 will be devoted to them.

3.6 Conclusions

In comparison with the Copenhagen and the von Neumann interpretations, the many worlds (MWI) has the advantage that it does not require the measurement postulates. It follows rigorously from the universal validity of quantum mechanics. However in order to avoid a Platonic paradigm (see section 3.4), strange to natural science, it should be combined with (or replaced by) an ensemble interpretation, so giving rise to an interpretation which may be realistic and not bizarre.

4 The hypothesis that quantum theory is not complete

4.1 The epistemological versus ontological treatment of the wavefunction

Since the very early days of quantum mechanics the possibility was put forward that the probabilistic character of the theory is due to the fact that the description offered by the wave function is not complete. If this is the case additional variables might be included in order to complete the description. They do not appear explicitly in the quantum formalism whence the name of ‘hidden variables’. However the mainstream of the scientific community has been positioned against the hidden variables (HV). Possibly the origin of this fact lies in the strong personality of Bohr, opposed to HV, combined with the confort produced by the belief that one possesses the final theoretical framework of physics, that is quantum mechanics. The rejection to HV theories was reinforced by the failure to find a useful one. In addition John von Neumann included in his celebrated 1932 book[36] a theorem apparently proving that any hidden variables model would contradict the predictions of quantum mechanics. The theorem was an obstacle for the research on the subject during more than three decades. In 1965 Bell[27] showed that the physical assumptions of von Neumann were too restritive and that (contextual) hidden variables are possible[26]. Indeed it is a simple matter to find a specific contextual hidden variables model for any (simple) experiment consisting of a preparation followed by a single measurement[54]. What is
difficult is to get a HV model valid for different experiments, for instance for a set of identical preparations followed by several alternative measurements, as in typical tests of Bell inequalities. Actually many of the supporters of the incompleteness of the quantum description have not proposed a search for specific HV models. This position would make the ensemble interpretation of quantum mechanics a rather philosophical belief.

The dichotomy between completeness versus incompleteness of quantum mechanics or, in modern language, epistemological versus ontological treatment of the wavefunction, has been the subject of a controversy lasting along the whole existence of quantum mechanics. As is well known, in the early period the most famous debate took place between Bohr and Einstein (see, e. g., [1]). An important contribution to that debate was the 1935 paper by Einstein, Podolsky and Rosen[14]. Although the paper is currently celebrated for having put forward the relevance of the entanglement between distant particles, its declared purpose was to provide (supposedly strong) arguments against the completeness of quantum mechanics. In the paper the authors considered a system consisting on two particles placed at a distance in a quantum state such that the particles are correlated in both position and momentum. The state is possible according to the quantum formalism and in fact the authors wrote explicitly the wave function of the composite system. According to Heisenberg uncertainty principle it is not possible to determine simultaneously the position and the momentum of one of the particles but nothing forbids measuring only one of the two observables with good accuracy. Due to the correlation, if the position of one particle, say number 1, is measured we will know the position of particle number 2 without interacting with it in any way. Thus, after the measurement, we may attribute to the second particle a wavefunction representing a state with definite position (but indefinite momentum). On the other hand a measurement of the momentum of the first particle allows attaching to the second particle a wavefunction corresponding to a definite momentum (but indefinite position). The point of the argument is that the state of the second particle should be the same in both cases, because nothing has perturbed its state, and nevertheless we may describe that state by means of two different wave functions. Hence the authors concluded that the wave function just describes our knowledge and not the real state of the particle. That is the wave function should be treated as epistemological rather than ontological.

Crucial for the EPR[14] argument is the assumption that no influence could exist on a particle due to a measurement performed on another dis-
tant particle, a hypothesis known as ‘locality’. Bohr rebutted\[34\] the EPR argument claiming that in quantum mechanics there is a kind of wholeness such that the assumed locality is not true. The current wisdom, resting upon Bell’s theorem to be discussed below, is that Bohr was right and EPR were wrong.

In addition to nonlocality, a bizarre consequence of assuming an ontological status for the wave function is exhibited in the EPR example. Indeed the wave function of the two-particle system represents a pure state but the state of each particle is not pure, according to the quantum rules. In fact the state of the two-particle system is represented by a wave function, what in standard quantum mechanics means that our knowledge of the two-particle state is complete. However the state of one of the particles cannot be represented by a wave function, but by a density operator obtained by taking the partial trace of the density operator associated with the two-particle wave function (the process is similar to the one leading to eq.\(8\)). That density operator represents a statistical mixture, meaning incomplete knowledge. (I must point out that this fact does not contradict the representation by a wave function made by EPR as commented above. In the EPR argument the quantum state attributed to particle 2 follows from a measurement performed on particle 1, but now we are considering the state when no measurement is made). The conclusion is that we have complete information about a composite system, but incomplete about every part, contrary to the usual definition of ‘complete’. It is as if a student claims to know completely the subject matter of a given book, but she/he is admittedly ignorant about every chapter. In my opinion this behaviour of entangled quantum systems is another argument for the epistemological character of the wavefunction. If that character is assumed, our knowledge will be incomplete for both the composite (entangled) system and every one of its parts, whence no paradox would arise.

In spite of the above arguments, during the whole history of quantum mechanics the ‘orthodox view’ has been that the theory is complete, as stressed by Bohr and his followers. However Bohr’s completeness may be seen as a support to the ‘instrumentalistic approach’ rather than a statement about the relation between the wavefunction and reality. In contrast for Einstein the relevant question was whether “the \(\psi\) – function corresponds to a single system or to a (statistical) ensemble of systems”\[29\] (Einstein carefully avoided the name wave function -not to be committed himself to the existence of waves associated to particles- and he used instead the name \(\psi\) – function). He clearly supported the latter assumption, which may be stated saying that
he adhered to the interpretation of the wave function as information. This interpretation has been vindicated by recent authors, for instance Chris Fuchs who has written “quantum states are states of information, knowledge, belief, pragmatic gambling commitments, not states of nature.” [57]. See also Englert and references therein [58].

At this moment it is appropriate to emphasize that an epistemological interpretation of the wave function does not always imply for it a purely subjective character. In many cases the available information is such that everybody would attribute the same wave function to the physical system, whence it acquires some objective character. A related question is whether the wave function collapse after a measurement is a physical change or just a change in our information (see section 3.3). In my opinion it is wrong to adhere exclusively to one of the possibilities. Actually both, or a combination of both, may appear in measurements. The EPR argument provides an example of a pure change of information, but an atom crossing a Stern-Gerlach apparatus may suffer an actual physical change in the direction of the spin. In any case the quantum postulate that ‘in a measurement the state of the system goes to an eigenstate of the measured observable’ may be appraised as an elegant formal statement, but in actual experiments things are more involved.

4.2 Recent approach to realistic interpretations

The advances in quantum information theory during the last three decades have had an important influence on foundations. For our purposes three aspects closely related are relevant: 1) A renewed support to the assumption that the quantum wave function (or state vector) represents information, 2) A vindication of some Einstein’s views on the interpretation of quantum mechanics, and 3) A study of the foundational problems from the point of view of realism. The recent vindication of Einstein does not refer to his opinions (it is generally assumed that he was wrong in his beliefs on locality, allegedly refuted by Bell’s theorem, but see section 5.) Rather he is vindicated as having pointed out what are the relevant questions to be answered. In fact a close scrutiny of Einstein’s letters to different authors shows that his main interest was not the question whether the wavefunction \( \psi \) represents an ensemble of possible systems - or, what is almost equivalent, if it only represents our information - but on whether a given real (ontic) state may correspond to different quantum-mechanical states \( \psi \). Einstein clarified the point in a
letter to Schrödinger[60]. An extended discussion about Einstein’s opinions, with many references, appears in a paper by Harrigan and Spekkens[59].

In recent times it has become popular to study the possibility of realistic interpretations of quantum theory resting upon the concept of ‘ontic states’, that is real physical states of systems not necessarily completely described by quantum theory, but objective and independent of the observer[59]. In the following I will use the names ‘hidden variables models’ or ‘ontological models’ as equivalent.

Thus the standard assumption in classical physics that measurements just reveal existing properties may be formalized stating that the observed results depend on both the ontic state, λ, of the system and the measuring apparatus, A, appropriate for a given observable quantity. That is the observed result, a, will be a function

\[ a = a(\lambda, A). \]

Let us analyze whether a similar analysis can be made in quantum physics. If we assume that the results of all observations on a system derive from functions like \( a(\lambda, A) \), then the correlation between several observable properties, \( \{A, B, ...C\} \), may be written

\[ \langle AB...C \rangle = \int f_\psi(\lambda) a(\lambda, A) b(\lambda, B) ...c(\lambda, C) d\lambda, \tag{9} \]

where \( f_\psi(\lambda) d\lambda \) gives the probability distribution of the ontic states in a given quantum state, either pure or mixed (but we use the subindex \( \psi \) for both cases). Without loss of generality we may consider that the properties \( \{A, B, ...C\} \) can take only the values \( \{0, 1\} \) because any property may be defined in terms of yes-no questions. Thus the knowledge of all correlations like eqs.(9) determines the joint probability distribution of all observable properties and the reciprocal is also true. We may define noncontextual hidden variables models (HVM) (or noncontextual ontological models) as those where all correlations may be got from eqs.(9). The predictions of quantum mechanics not always can be interpreted that way. In fact the Kochen-Specker theorem proves that noncontextual HVM are not always possible.

The correlations involved in eq.(9) may appear in two different scenarios: 1) Correlations between properties of a system localized in a small region of space, 2) Correlations between distant systems. Actually the difference between the two scenarios is not sharp, but there is an important case which
belongs clearly to the latter class, namely in EPR type experiments to be discussed below, when dealing with Bell’s theorem.

Recently a theorem has been proved by Pusey, Barrett and Rudolph\[61\] which apparently implies that quantum states are physical properties of a system, contrary to Einstein’s opinion. If this implication is correct, the theorem would be very important, because it would contradict the assumption that the wave function represents only information and this is the cornerstone for the realistic interpretation supported in this paper. The authors claim to have proven that, for two different quantum states represented by the wavefunctions $\psi$ and $\phi$, “the distributions $f_\psi(\lambda)$ and $f_\phi(\lambda)$ (of ontic states $\lambda$) cannot overlap. If the same can be shown for any pair of quantum states, then the quantum state can be inferred uniquely from $\lambda$. In this case, the quantum state is a physical property of the system”.

In order to see the relevance of the theorem for a realistic interpretation let us return to the example of the Schrödinger cat. In standard quantum mechanics it is assumed that any system is in a quantum state (represented by a wave function), although the most useful representation for macroscopic bodies is a density operator (equivalent to a probability distribution of state vectors or wave functions). Thus our living cat will be in some quantum state represented by one of the state vectors $|\text{livecat},j\rangle$, $j = 1,2,...$ Similarly a dead cat may be represented by $|\text{deadcat},k\rangle$, $k = 1,2,...$ But standard quantum mechanics also assumes that a linear combination like
\begin{equation}
\frac{1}{\sqrt{2}}(|\text{livecat},1\rangle + |\text{deadcat},1\rangle),
\end{equation}
also represents a possible quantum state. Now the commented theorem\[61\] implies that all ontic states associated to the quantum state eq.\((10)\) should be different from every ontic state of living cat, associated to the quantum state $|\text{livecat},j\rangle$, and also from every ontic state of dead cat, associated to $|\text{deadcat},k\rangle$. But no plausible realistic interpretation may assume the existence of ontic states associated specifically to the quantum state eq.\((10)\) (of partially living cat!) Any realistic interpretation of that quantum state should associate to it a statistical mixture of ontic states of living cat and dead cat.

Thus a careful scrutiny of the assumptions of the theorem is necessary. According to the authors the assumptions are: “a system has a real physical (ontic) state... This assumption only needs to hold for systems that are isolated, and not entangled with other systems... The other main assumption
is that systems that are prepared independently have independent physical states”[61]. However aside from the explicit hypotheses there are implicit assumptions, for instance that linear combinations like eq. (10) represent possible quantum states. A critical analysis of all these implicit assumptions lies beyond the scope of this paper.

4.3 Bell’s theorem

The original theorem of Bell[55], [28], [26] provides necessary conditions for the possibility that measurements performed in two distant regions are independent, i.e. they cannot influence each other. More formally Bell’s fundamental hypothesis may be stated as follows. Let us assume that pairs of particles (more generally subsystems) are produced in a source and the two particles of every pair move in different directions. Eventually one of the particles arrives at Alice, who measures some observable \( A \), and the other one arrives at Bob, who measures the observable \( B \). If the result obtained by Alice (Bob) in one run of the experiment is \( a_j \) (\( b_j \)), after a large number, \( n \), of similar runs the relevant quantity is the correlation

\[
\langle AB \rangle_n \equiv \frac{1}{n} \sum_{j=1}^{n} a_j b_j.
\] (11)

Here ‘similar’ means that the runs of the experiments, each consisting of the preparation of a pair and the subsequent measurements, are performed in identical conditions, as far as they may be controlled (we cannot exclude that perturbations out of control may arise in every run.) Bell assumed that the result of Alice’s measurement depends on the values of the (hidden) variables, collectively labelled \( \lambda_a \), that specify the real state (the ‘ontic’ state) of the Alice’s particle and, obviously, on the Alice’s measuring set up, which will be labelled \( A \) here. We shall write that dependence in the form of a function \( a (\lambda_a, A) \). Similarly for Bob’s particle we write the function \( b (\lambda_b, B) \). Thus the theoretical correlation, see eq. (11), may be written

\[
\langle AB \rangle = \int \rho (\lambda_a, \lambda_b) a (\lambda_a, A) b (\lambda_b, B) d\lambda_a d\lambda_b,
\] (12)

where \( \rho (\lambda_a, \lambda_b) \) is the joint probability density for the variables \( \lambda_a \) and \( \lambda_b \) (compare with eq.(9)). The essential assumption of locality is that neither \( a \) depends on \( B \) nor \( b \) depends on \( A \), nor \( \rho \) depends on either \( A \) or \( B \).
As usual, the probabilities involved in eq.(12) are tested by measuring the frequencies appearing in eq.(11) with \( n \) large. Actually we might include in the functions \( a(b) \) additional hidden variables \( \mu_a(\mu_b) \) taking into account the fact that no measuring set up may be completely controlled. After that we should perform appropriate averages over those variables, which is equivalent to assuming that \( a(\lambda_a, A) \) and \( b(\lambda_b, B) \), eq.(12), are already averages over the variables \( \mu_a \) and \( \mu_b \), respectively.

From Bell’s proposal, eq.(12), it is a trivial task to derive inequalities that are necessary conditions for the existence of local models of the correlation experiments. Bell also proved that there are (ideal) experiments where quantum mechanics predicts violations of one of the inequalities. Hence Bell’s theorem follows: “Local hidden variables models of quantum mechanics are not possible”.

Actually there is an important consequence of Bell’s work that is independent of the existence of quantum mechanics, and not always has been duly appreciated. That is, if there are correlations between distant systems which violate a Bell inequality then those correlations cannot be explained as deriving from a common cause. In fact any such explanation could be formalized by eq.(12) and it would imply a Bell inequality. The relevance of this result is that the explanation of correlations between distant bodies as deriving from a common past is a cherished hypothesis, not only in physics but in all branches of science and even in ordinary life. For instance everybody will believe that the similarity between twins is due to the common genetic code.

For many quantum physicists, not too fond of foundations, the merit of Bell’s theorem was to “show the absurdity of searching for hidden variables of quantum mechanics, an useless goal in the first place.” However the relevance of Bell’s theorem is greater than just to refute a class of hidden variables theories of quantum mechanics, as pointed out above. Furthermore the theorem implies that there is some conflict between relativity theory and quantum mechanics. Indeed Bell himself reinterpreted eq.(12) in the sense that \( \lambda_a \) and \( \lambda_b \) mean the set of all events in the past light cones of the measurements performed by Alice and Bob, respectively. Thus if the measurements are space-like separated, in the sense of relativity theory, then Bell’s theorem seems to prove the incompatibility of quantum mechanics with relativity theory. The contradiction is dramatic, but people have found an escape after the proof that experiments violating a Bell inequality do not allow sending superluminal signals from Alice to Bob (or from Bob to Alice) and this is
the only think forbidden by relativity theory. In my opinion this is not a satisfactory solution. I am convinced that we should hold strong on the validity of the principles of realism and (relativistic) locality in physics. This was also the belief of Einstein until his death\cite{29}.

Quantum mechanics has had so spectacular a success in predicting the results of experiments that for most authors any proposal to change the quantum formalism seems a blunder. But I am convinced that a solution must be found to the conflict posed by Bell’s theorem. Thus I propose the following restriction - not a modification - of the quantum formalism.

**Conjecture 1** *Experiments showing a (loophole-free) violation of a Bell inequality are not feasible.*

To many readers this conjecture may appear as a pure speculation. But it is a scientific statement in Popper’s sense because it can be tested, and eventually refuted, by experiments. On the other hand the conjecture will be increasingly confirmed as time passes without a refutation, and half a century has already elapsed from Bell’s work\cite{56}. It is true that the experiments have been refined along the time and that quantum mechanics has been vindicated in those experiments, with a few exceptions not too significant. In any case we should conclude that the question is open. In the following an attempt will be made to convince readers that the above conjecture is not crazy.

Firstly it is necessary to point out a fact usually neglected. The most spectacular agreement between the quantum predictions and the experiments may be explained from the quantum equations and a little more. E.g. the calculation of the anomalous magnetic moment of the electron, in agreement with experiments up to 10 decimal figures, derives from the solution of the quantized coupled Dirac and Maxwell equations for an electron placed in a homogeneous magnetic field. Asides from the quantized Dirac and Maxwell equations no additional quantum assumptions are needed. In particular we may dispense with the quantum postulates of measurement because in the experiment the light detection may be described as a macroscopic process. *The precision of the agreement between theory and experiment compel us to admit that the quantum equations are correct.* In contrast the measurement of probabilities rarely provides an agreement better than a few percent. For instance in performed tests of Bell inequalities the measured parameter, that is a linear combination of probabilities, is typically reported with uncertainty greater than one per thousand\cite{56}. 

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Secondly it is physically absurd, although may appear as mathematically elegant, to assume an one-to-one correspondence between density matrices and states of a system (or between ‘positive operator valued measures’, POVM, and feasible measurements). In particular only a relatively small number of different states of a system may be actually prepared in the laboratory whilst the set of quantum state vectors consists of infinitely many, and similarly for the measurements. Most textbooks are aware of the problem and present the quantum postulates without assuming one-to-one correspondence. It is stated, for instance, that “to every state of a physical system we associate a density matrix”, but without postulating the reciprocal. Nevertheless in the proof of Bell’s theorem a quantum state vector violating a Bell inequality is used without investigating whether the corresponding state may be actually produced in the laboratory. In other words Bell’s, as any theorem, is a mathematical statement, a contradiction between the quantum formalism and the assumption eq. (12). The true, very important, physical consequence of Bell’s work is to suggest experiments that may eventually refute local realism, but directly the theorem proves nothing about nature.

Thirdly quantum theory itself puts some constraints on the feasibility of experiments, difficulties usually dismissed as (small) practical deficiencies. For instance in recent tests involving entangled photons produced in a non-linear crystal an efficient photon detection cannot be achieved using too short a time window, which puts a challenge for the spacelike separated detections needed to close the locality loophole[56].

5 Proposed physical model of the quantum world

A realistic interpretation, giving rise to an ontology, that is a physical model of the world, would make quantum mechanics more palatable to lovers of theory, in the ethimological sense of contemplation. It would allow “understanding quantum mechanics”[9]. In the model here proposed I shall not consider the quantization of gravity because I think that only after having a good understanding of quantum mechanics in Minkowsi space might we try to understand quantum gravity. The picture here supported was sketched in the Introduction section and further discussed in the following, where the most relevant assumptions will be presented as ‘propositions’.
In spacetime there are fields, a form of ‘matter’ giving rise to phenomena that may be observed. In quantum theory every field at a spacetime point is represented by a (scalar, vector or tensor) operator belonging to some non-commutative algebra (whose full mathematical structure is well known and will not be specified here). I shall refrain from making any hypothesis about the state of the universe as a whole. I will consider only the study of systems of ‘human size’ (rather than cosmic size). For instance an atom or a small piece of bulk matter.

The crucial assumption is that the representation of fields by operators, rather than classical functions, should be interpreted as the fact that the fields are stochastic. That is, I assume the following

Proposition 2 The quantum formalism is a disguised form of dealing with peculiar stochastic fields. Furthermore the quantum vacuum fluctuations of all those stochastic fields are real. Commutation or anticommutation rules of the field operators characterize the stochastic properties of the fields.

The assumption puts a well known and strong problem, namely that the free fields are ultraviolet divergent. Solving the problem lies outside the scope of this paper. As possible solutions I suggest either some kind of cancelation for interacting fields or general relativistic effects. An important consequence of the proposition is

Proposition 3 No small system can be isolated from the rest of the world, even approximately.

Indeed every system should be effectively interacting with many other systems via the vacuum fields. But in order to be able to make physics we should assume that microscopic systems, even if not isolated, may be treated with a formalism that in some form takes into account the interaction. I believe that this formalism is quantum mechanics. For instance, if we represent the state of an atom by a state vector it is plausible to assume that this representation corresponds to the atom ‘dressed’ by all fields that interact with it. This is consistent with the fact that in quantum electrodynamics the physical electrons are never ‘bare’ but ‘dressed with virtual photons and electron-positron pairs’. The word ‘virtual’ is just a name for something that we know to have observable effects, but we cannot consider ‘real’ without a conflict with the cherished (but for me wrong) assumption that quantum
systems may exist as isolated. The point is that the representation of an atom by a state vector takes into account the (approximate) action of the vacuum fields as is shown by the use of the physical, rather than bare, mass and charge of the electrons. And similarly for other quantum systems.

As a consequence it is a rather presumptuous attitude to pretend that a state vector represents faithfully the actual state of an individual system. It is more plausible to assume that the state vector represents the relevant information available about the system. The conclusion is that the quantum-mechanical representation of the state of a system is incomplete. This incompleteness is the cause of the claimed ‘irreducible probabilistic character of the physical laws’. For instance the fact that an atom decays at a time that cannot be predicted derives from the fluctuations of the vacuum fields that actually stimulate the decay.

The concept of isolated system is the cornerstone of classical physics and, therefore, it is not strange that it was also introduced in quantum physics. It is true that early authors dealing with quantum theory, like Planck, Einstein and Nernst, studied the possible existence and influence of vacuum (nonthermal) fluctuations\[21\. However the success of the Bohr atomic model, where the concept of fluctuation was absent, reinforced the idea that quantum systems may be treated as isolated. Nonthermal fluctuations reappeared in modern quantum mechanics associated to the zero point energy of bounded quantum systems. However in the alternative of either rejecting the assumption of isolated system or dismissing the reality of the quantum fluctuations, the mainstream of the community choosed the latter. This compelled people to introduce the ill-defined concept of ‘virtual’. In my opinion that choice has been the source of most difficulties for a realistic interpretation of the quantum formalism.

The existence of real vacuum fluctuations gives rise to two characteristic traits of quantum physics. Firstly quantum theory should be probabilistic. Secondly it should present a kind of wholeness, quite strange to classical physics where the concept of isolated system is crucial. The fact that the vacuum fluctuations at different points may be correlated is the origin of the wholeness, which manifests specially in the phenomenon of entanglement.

**Proposition 4** In addition to the usual correlations between (two or more) physical systems, involving directly observable quantities, there are additional correlations via the quantum vacuum fluctuations interacting with the systems. These correlations give rise to the phenomenon of entanglement.
This leads to the following *picture of the quantum world*. Fundamental fermions, like leptons or quarks, are (localized) particles, but fundamental bosons, in particular photons, are actually (extended) fields. Gravity plays a special role, I support the view that general relativity determines the structure of (curved) spacetime and its relation with matter, so that gravity is not a field in the same sense than other fields. The wave behaviour of particles derives from the unavoidable interaction with fields, and the particle behaviour of fields derives from the interaction with particles, e. g. during detection. A fundamental property of the universe is the existence of fluctuations of all fields in the vacuum (i. e. at zero Kelvin.) In the case of Bose fields there are random fluctuations similar to the zeropoint fluctuations of the electromagnetic field. In the case of Fermions the fluctuations may correspond to the existence of a kind of Dirac sea of particles and antiparticles that may be created and annihilated. There should be also metric fluctuations of spacetime itself, possibly stronger than those deriving from the fluctuations of stress-energy of the particles and fields. That is a background of gravitational waves with wavelengths of atomic or subatomic size.

Now the question is to what extent the quantum formalism is compatible with, or better encodes, this picture. It is necessary to distinguish the core from the rest of the quantum formalism. The core consists of the Hilbert space (or $C^*$-algebra) mathematical structure and the fundamental equations of motion (Dirac, Maxwell, Klein-Gordon, etc.) in terms of that structure. To the mathematical theory and the equations it is necessary to add Born’s rule. In the non-relativistic approximation to particle motion, Born’s rule is the interpretation of the modulus squared of the wave function as a probability density. It should be appropriately generalized for fields.

*I do not propose any modification of that core, but claim that the rest of the quantum formalism is dispensable*, although it may be useful in practice. I think that it is flawed to introduce physical operations as a part of the postulates of quantum mechanics, for instance ‘preparation’ or ‘measurement’. Therefore it is not appropriate to establish any rigid correspondence between ‘preparation’ and ‘density matrix’ or between ‘measurement’ and ‘selfadjoint operator’ (or POVM). Preparation is a rather complex set of physical manipulations whence the density matrix appropriate for a microscopic system may be guessed, rather than derived, most times after a process of trial and error on the part of the scientists performing the particular preparation. Similarly for the measurement. Indeed an empirical result is taken as a valid discovery only after the relevant experiment has been critically analyzed and repeated.
by different groups of researchers. This view agrees with what Bell wrote: “I am now convinced that the word ‘measurement’ has now been so abused that the field would be significantly advanced by banning its use altogether, in favour for example of the word ‘experiment’.” (p. 166)

As a consequence I believe that a formalization like eq. (??) or theorems derived from it, like the one by Pusey et al. are of limited value. Furthermore, as pointed out in the subsection devoted to Bell’s theorem, it is an undue extrapolation to assume that all density matrices may correspond to physical states. Hence it follows that the boundary of the set of (physical) density operators is not the set of idempotent operators (or what is equivalent, the set of state vectors) and the very concept of ‘quantum pure state’ is not well defined.

In summary I propose to search for a realistic interpretation of quantum mechanics, or equivalently a physical model of the microworld, using only the core of the formalism as defined above. That is the Hilbert space structure, the equations and Born’s rule, without attempting to attach a meaning to the remaining postulates, which does not preclude their practical usefulness.

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