Internal transitions in the confined biexciton.

Ricardo P´erez and Augusto Gonzalez

1 Instituto de Cibernetica, Matematica y Fisica, Calle E 309, Vedado, Ciudad Habana, Cuba.
2 Departamento de Fisica, Universidad de Antioquia, A. A. 1226, Medellin, Colombia.

(October 21, 2001)

Optical internal transitions relevant to ODR experiments are computed for the biexciton in a quantum dot in the strong confinement regime. Valence sub-band mixing effects are taken into account in second order perturbation theory. The transition probability from the ground state is concentrated in a few states with relatively high excitation energies. Level collisions with oscillator strength transfer are observed as the magnetic field is raised.

PACS numbers: 78.30.-j, 78.55.-m, 78.67.Hc

The Optical Detection of far-infrared Resonances (ODR) have proven to be an efficient method in the study of a sector of the exciton spectrum not accessible to inter-band transitions. A far-infrared radiation, in resonance with an internal transition, induces changes in the population of the exciton ground-state, changing in this way the amplitude of the luminescence peak. Usually, in order to tune into resonance a particular transition, a magnetic field is used instead of varying the frequency of the infrared source. By this means, many of the exciton transitions as well as electron and hole cyclotron resonance transitions have been clearly observed. In the present paper, we focus on the main biexciton line and address the question about what are the main transitions to be observed in an ODR experiment. As argued below, this question has a non-trivial answer.

Following the dictates of conclusion (a) above, we study the biexciton in a quantum dot under the strong-confinement regime. Very distinct biexcitonic and multiexcitonic lines have been observed in this regime by means of confocal microscopy techniques.

Our model quantum dot is disk-shaped with a height, w = 8.5 nm. The in-plane confinement is parabolic with hω_0^e = 7.83 meV for electrons, leading to a characteristic (oscillator) length of around 12 nm. Oscillator lengths are equal for both electrons and holes. It means that m_{xy}^eω_0^e = m_{xy}^hhω_0^hh, where m_{xy} are the in-plane masses. The Kohn-Luttinger (KL) parameters for GaAs of Ref. 2 are used: γ_1 = 6.790, γ_2 = 1.924, γ_3 = 2.681, κ = 1.2 and q = 0.04. The relative dielectric constant is ε = 12.5.

Hole levels in the dot are computed from the KL Hamiltonian. Instead of a numerical diagonalization, we use second order perturbation theory to account for the non-diagonal terms in the KL Hamiltonian. This kind of approximation has proven to capture many of the actual properties of the hole levels. Keeping the more relevant contributions for thin disk-shaped dots under low magnetic fields, we get for the heavy-hole energies

\[ E_{n\ell kjz}^{hh} = E_{n\ell kjz}^0 + \left( \frac{8\hbar\gamma_3}{\sqrt{3m_0\ell_0w}} \right)^2 \Delta E^\pm, \]

where the unperturbed energies are given by

\[ E_{n\ell kjz}^0 = \frac{\hbar^2\pi^2}{2m_{xy}^0w^2}k^2 + \hbar\Omega^{hh} (2n + |\ell| + 1) - \frac{\hbar\omega_c}{2} \ell \pm \mu_B B g_{hh}^\ell, \]

n and \ell are radial and angular momentum projection quantum numbers of the evolvent in-plane wave function. k labels the functions along the symmetry axis (z-direction). They are taken as infinite-well functions (assuming, for example, a dot formed from a AlGaAs-GaAs symmetric well with high enough Al concentration). j_z is the total (band) angular momentum projection along z. The effective frequency \( \Omega^{hh} \) is given by \( \Omega^{hh} = \sqrt{(\omega_c^h/2)^2 + (\omega_c^{hh})^2} \), where \( \omega_c \) is the cyclotron frequency. \( m_0 \) is the electron mass in vacuum, \( g^{hh} = 3\kappa + 27q/4 = 3.87 \) and \( \ell_0 = \sqrt{\hbar/(m_{xy}^e\Omega_c^e)} \).
Expressions similar to Eq. (2), in which masses and frequencies are correspondingly replaced and \( g^{th} = \kappa + q/4 = 1.21 \), are written for the light hole. The energy corrections, \( \Delta E^{\pm} \), where the + refers to \( j_z = 3/2 \), take the following form

\[
\Delta E^{\pm}_{\ell \geq 0} = \frac{n(1 + \omega(\ell_0)^2)}{E_{n,\ell,1-3/2}^0 - E_{n,\ell+1,2-,1-1/2}^0} + \frac{(n + \ell + 1)(1 - \omega(\ell_0)^2)^2}{E_{n,\ell,1-3/2}^0 - E_{n,\ell,2-,1-1/2}^0} \\
\Delta E^{\pm}_{\ell \leq 0} = \frac{(n + |\ell|)(1 + \omega(\ell_0)^2)^2}{E_{n,\ell,1-3/2}^0 - E_{n,\ell+1,2-,1-1/2}^0} + \frac{(n + 1)(1 - \omega(\ell_0)^2)^2}{E_{n,\ell,1-3/2}^0 - E_{n,\ell+1,2-,1-1/2}^0} \\
\Delta E^{\pm}_{\ell > 0} = \frac{(n + 1)(1 + \omega(\ell_0)^2)^2}{E_{n,\ell,1+3/2}^0 - E_{n,1-\ell+2,1+1/2}^0} + \frac{(n + |\ell|)(1 - \omega(\ell_0)^2)^2}{E_{n,\ell,1+3/2}^0 - E_{n,\ell-1,2,1+1/2}^0} \\
\Delta E^{\pm}_{\ell < 0} = \frac{(n + |\ell|)(1 + \omega(\ell_0)^2)^2}{E_{n,\ell,1+3/2}^0 - E_{n,1-\ell-2,1+1/2}^0} + \frac{n(1 - \omega(\ell_0)^2)^2}{E_{n,\ell,1+3/2}^0 - E_{n,1-\ell-2,1+1/2}^0}
\]

(3)

where \( \omega = eB/(2\hbar) \).

The first \( j_z = \pm 3/2 \) hole energies obtained from an exact diagonalization of the KL hamiltonian are drawn in Fig. 1 as a function of \( B \). For comparison, the unperturbed values (3) and our approximate energies (4) are also drawn, showing that band-mixing effects may lead to corrections to the single-particle energies of around 1 meV at \( B = 5 \) T.

Heavy-hole energies from (4) are included in the biexciton hamiltonian. For the wave function, however, we use the unperturbed functions, neglecting hole mixing (4). For example, for the state with total electron spin \( S_e = 1 \), and total (band) hole angular momentum \( J_z = 3 \), which is created by \( \sigma^- \) polarized light, we write the biexciton spatial wave function in the form

\[
\Psi_{1,3} = \frac{1}{2} \sum C_{n_1,1, n_2,2, n_3,3, n_4,4} \times \left\{ \phi_{n_1,1}(1)\phi_{n_2,2}(2) - \phi_{n_1,1}(2)\phi_{n_2,2}(1) \right\} \times \left\{ \phi_{n_3,3}(3)\phi_{n_4,4}(4) - \phi_{n_3,3}(4)\phi_{n_4,4}(3) \right\},
\]

(4)

where indexes 1 and 2 refer to electrons, 3 and 4 to holes, and the sum runs over states preserving the total angular momentum projection, \( M = l_1 + l_2 + l_3 + l_4 \). Up to 16 harmonic oscillator shells, i.e. 136 single-particle states \( \phi \) for electrons, and 136 for holes, are included in (4). The resulting large matrix for the biexciton hamiltonian are diagonalized by means of a Lanczos algorithm.

The results for the first 10 transitions from the lowest \( S_e = 1 \), \( J_z = 3 \) state (a \( M = 0 \) state), induced by \( \sigma^\pm \) polarized far-infrared radiation (\( \Delta M = \pm 1 \)) are presented in Fig. 2. Levels with more than 10 % of transition probability are represented by solid lines, and the probability is indicated. Levels with probability less than 10 % are represented by dashed lines. We define the normalized probabilities from the expansion coefficient of \( \Psi_{\text{final}} \) in \( D^\pm \Psi_{\text{initial}} \)

\[
\text{prob}^\pm = \frac{|\langle \Psi_{\text{final}} | D^\pm | \Psi_{\text{initial}} \rangle|^2}{\langle \Psi_{\text{initial}} | D^- D^+ | \Psi_{\text{initial}} \rangle},
\]

(5)

where \( D^\pm = r_1 e^{\pm i\theta_1} + r_2 e^{\pm i\theta_2} - r_3 e^{\pm i\theta_3} - r_4 e^{\pm i\theta_4} \) is the dipole operator for internal \( \sigma^\pm \) transitions. \( r, \theta \) are polar coordinates in the plane.

The lowest transitions, with excitation energies below 10 meV are interpreted as “single-particle” excitations. Because of their low probabilities, they are unlikely to be detected in an ODR experiment.

Collective motions, in which the electron cloud oscillates in counterphase with respect to the hole cloud, concentrate most of the transition probability. Their excitation energies take values between 15 and 20 meV (the solid lines). In the biexciton, they are a small-scale version of the giant-dipole resonances expected for larger multiexcitonic systems.
Because of the difference between electron and hole masses and the magnetic field, the dipole resonance splits into a few states. The two indicated transitions at $B = 0$, for example, account for 83% of the probability. At $B = 5$ T, there are two states concentrating 60% of the $\sigma^+$ transition probability, and a single state accounting for 87% of the $\sigma^-$ probability.

Basically, these dipole excitations are the ones to be registered by means of ODR. Following only the solid lines, one may notice in $\sigma^-$: (i) a “state” with constant excitation energy around 16 meV which ODR signal shall become very strong as B is increased, and (ii) a state starting at around 20 meV which excitation energy rises abruptly for $B > 1$ T. Whereas for $\sigma^+$ we may distinguish: (iii) a state clearly seen for $B \geq 2$ T starting at $\Delta E \approx 15$ meV and which ODR signal increases, (iv) a state starting at $\Delta E \approx 17$ meV with a very strong ODR signal at low $B$ which decreases at higher $B$ and, finally, (v) a state starting at $\Delta E \approx 20$ meV which ODR signal diffuses as $B$ is raised.

The energy of all of the $M = 1$ states increases with $B$.

A careful examination of the transitions shown in Fig. 2 reveals a complex pattern of probability transfer as the magnetic field is raised. For example, let us look at the three “colliding” levels at excitation energies around 18 meV for $\sigma^+$ polarization and $B = 2$ T. The transition probabilities are 42.7, 0.3 and 1.0% from bottom to top. After the collision, i.e. at $B = 3$ T, the probabilities become 0.0, 16.2 and 22.5% respectively. It means that ODR signals may experience significant changes even with relatively small variations of $B$.

Similar results are presented in Fig. 3 for the singlet (electron), $J_z = 0$ biexciton. The lowest state in this sector is also a $M = 0$ state. Low lying $M = \pm 1$ dark levels, a $\sim 15$ meV gap to the dipole resonances, and a complex pattern of probability transfer with increasing magnetic field are also evident in this figure.

In conclusion, we have computed the lowest lying energy levels and transition probabilities relevant for the ODR study of the biexciton in a quantum dot. The states that will induce the most pronounced ODR signals are collective dipole excitations, with excitation energies higher than $\hbar(\omega^e_0 + \omega_{hh}^0)$. A complex dependence of the transition probabilities on $B$, leading to significant variations of the position and depth of ODR signals is predicted.

The authors acknowledge support by the Committee for Research (CIDI) of the University of Antioquia and by the Caribbean Network for Theoretical Physics. Useful discussions with B. Rodriguez are gratefully acknowledged.

References:

1. J. Cerne, J. Kono, M. S. Sherwin et al., Phys. Rev. Lett. 77, 1131 (1996); M. S. Salib, H. A. Nickel, G. S. Herold et al., Phys. Rev. Lett. 77, 1135 (1996); H. A. Nickel, G. Kioseoglou, T. Yeo et al., Phys. Rev. B 62, 2773 (2000).
2. G. E. W. Bauer and T. Ando, Phys. Rev. B 38, 6015 (1988).
3. E. Dekel, D. Gershoni, E. Ehrenfreund et al., Phys. Rev. Lett. 80, 4991 (1998); E. Dekel, D. V. Regelman, D. Gershoni et al., Solid State Commun. 117, 395 (2001). Biexcitonic effects in the opposite, weak confinement, regime were studied by H. Kamada, H. Ando et al., Phys. Rev. B 58, 16243 (1998).
4. This is a common model for self-assembled or strained dots, see for example L. Jacak, P. Hawrylak and A. Wojs, Quantum Dots, Springer-Verlag, Berlin (1998). Effective dots in quantum wells arising from width fluctuations have been also modeled with a parabolic confinement, see for example, C. Riva, K. Varga et al., Phys. Stat. Sol. (b) 210, 689 (1998).
5. J. M. Luttinger and W. Kohn, Phys. Rev. 97, 869 (1955); J. M. Luttinger, Phys. Rev. 102, 1030 (1956).
6. M. Brasken, M. Lindberg and J. Tulkki, Phys. Rev. B 55, 9275 (1997).
7. For the lowest heavy hole states, the wave function almost coincides with the unperturbed function, and orbital angular momentum can be taken as a good quantum number. Wave function mixing becomes important at higher excitation energies, where other effects like anharmonicity of the lateral potential, finiteness of the barrier along $z$, etc may give corrections of the same order to the biexciton energy.
8. A. Delgado, L. Lavin, R. Capote and A. Gonzalez, Physica E 8, 342 (2000).

FIG. 1. The lowest $j_z = \pm 3/2$ states. Dashed line: Eq. (3), dotted line: Eq. (1), and solid line: exact diagonalization of the KL hamiltonian.

FIG. 2. Internal transitions in the $S^e_z = 1$, $J_z = 3$ biexciton. See explanations in the main text.

FIG. 3. Same as Fig. 2 for the singlet (electron), $J_z = 0$ biexciton.