Comments on some recent papers on the hyperspherical approach in few–body systems

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Abstract

Recent papers by Kievsky, Viviani, and Rosati are commented. It is pointed out that the techniques for treatment few–body reactions and bound states in the framework of the hyperspherical approach presented in those papers as original ones were in fact known and have been applied previously.

The purpose of these notes is to demonstrate the fact of misleading presentation in the recent papers by Kievsky, Viviani, and Rosati (Refs. [1–4]). Namely, some aspects of the hyperspherical harmonic (HH) method in few–body systems are presented in those papers as original ones, though, in reality, they have been developed in the literature previously. And the authors were aware of the latter (which can be proved). Just this, and not a mere point of priority, deserves a reaction. Therefore I feel obliged to discuss this issue in public.

Below the technique for treatment reactions in the framework of the (HH) approach is discussed. In Appendix the procedure for reduction of the number of HH entering calculations is considered.

It is not my aim to discuss here the usefulness of the large–scale calculations performed by the above mentioned group.

TREATMENT OF REACTIONS IN THE FRAMEWORK OF THE HH APPROACH

The paper [1] by Kievsky, Viviani, and Rosati starts with:

"In a recent paper [5] (by the same authors, V.D.E.) a variational technique to calculate scattering states below the deuteron breakup threshold has been developed for the three–nucleon $N – d$ system. In this approach, the wave function of the system is written as a sum of two terms

$$\Psi = \Psi_C + \Psi_A.$$  \hspace{1cm} (1)

The first term $\Psi_C$ is responsible for the description of the system when the three nucleons are close to each other. It is decomposed in channels labeled by the angular–spin–isospin quantum numbers and the corresponding two–dimensional spatial amplitudes are expanded in terms of the pair correlated hyperspherical harmonic (PHH) basis [6]. The second term $\Psi_A$ is a solution of the Schrödinger equation in the asymptotic region where the incident nucleon and the deuton are well apart.”
This statement, repeated in subsequent papers by Kievsky et al. [2,3], is misleading since in reality the quite similar technique was used before in the literature. The above extract is to be compared to that from the introductory section of Ref. [7]:

"...In the present work the K–harmonics method is generalized to the scattering problem for the three–nucleon system. However, in that case in no approximation in the expansion of the wave function can one ignore harmonics with large K. This is due to the fact that that part of the wave function which corresponds to the asymptotics of two–fragment channels (free motion of the nucleon with respect to the deuteron) requires for its description a large number of harmonics.

It is natural to write Ψ in the form

\[ Ψ = Φ + X, \]

(2)

where Φ describes the two–fragment of Ψ, and to search for an expansion in K harmonics of the part X only. It then turns out that it is possible to restrict oneself to a small number of terms in such an expansion.

The function Φ is known accurate to within the sought for amplitudes \( f_L \)...

Taking into account that "K-harmonics" mean the HH basis, the main difference between the two extracts lies merely in the notation in Eqs. (1) and (2). A minor difference is that in the latter case the pure HH basis is discussed while in the former one the correlated HH basis is mentioned. Obviously, the correlation factors are included into the wave function exactly in the same way as for bound states, and this has nothing to do with the specific character of the continuum spectrum problem. (In fact, the correlated HH basis was considered first in 1972 in Ref. [8] in the bound state problem. It has been shown there that the inclusion of the correlations leads to a great increase in the convergence rate. In the commented papers the correlations are applied in a modified form.)

The convenience of the HH expansion for the description of the three–body breakup reaction has been recognized for the first time in Ref. [7]. However, the presence of the two–fragment reaction channels led to difficulties. To solve the problem, in the subsequent paper [10] the author attempted to combine the HH approach and the resonating group approach. This, however, led to integro-differential equations being very involved even when only the lowest HH term is retained. In the next paper [11] the author renounced of the treatment of the two–body channels at all simply disregarding them. But that led to unrealistic results for the three–body photodisintegration cross section. (See Ref. [12] on possible approximate interpretation of such type calculations.) One more attempt to generalize the HH approach to the scattering problem was undertaken in Ref. [13] independently of Ref. [7] but it was also not successful enough. The difficulty has been removed in the above mentioned paper where the procedure leading to a simple set of simultaneous differential equations for the coefficients of the HH expansion and algebraic equations for the scattering or reaction amplitudes has been given. It corresponds to the first order Hulthén–Kohn type calculation. The second order variational Hulthén–Kohn corrections were calculated in Refs. [14–16]. Among the other papers on the subject, we mention Ref. [17] where the photodisintegration of \(^3\)H was studied using the modified version of the procedure, and the calculations above the breakup threshold were performed with a noncentral realistic NN potential of those days.

Claiming the developement of a variational technique to calculate scattering states,
Kievsky, Viviani, and Rosati have never mentioned the papers \cite{7,14–17}. While the studies performed in those papers required less intensive calculations than those performed by Kievsky et al., the whole procedure itself for treatment continuum states used by the latter authors in Refs. \cite{7,14} was essentially the same as in Refs. \cite{7,14}. I did not find any new point concerning that procedure in \cite{5,1}, and hence there is no possibility to understand what the variational technique to calculate scattering states "has been developed" there.

I have recently got to know from my Russian colleagues\footnote{I could have listed the names.} that they in fact discussed the considered approach with Viviani at the Uzhgorod Few–Body Conference in 1990 and provided him with the corresponding references, in particular. In addition, in 1991 they visited Pisa and delivered a University seminar on HH three–body calculations of continuum and bound states. Kievsky, Viviani and Rosati were present at the seminar, and had further discussions with them on the procedure in question during that visit. Therefore, concerning the presentation in Refs. \cite{1–3}, there is no question whether the "Pisa group" was aware of the preceding literature on the subject while performing their calculations.

APPENDIX. AN ALGORITHM FOR REDUCTION OF THE NUMBER OF HH BASIS STATES

A quite similar comment applies to the paper \cite{4} by Kievsky et al. In that paper, the authors have managed to solve accurately the bound–state three nucleon problem for the Argonne \nn interaction due to selecting in the calculation certain subsystems from complete sets of HH at given $K$ values. The authors discuss their selection comparing it with other "various selections of the HH functions" in the literature but do not mention that the selection prescription they are just using was known before. This prescription for selecting (or adding consecutively) the subsystems has in fact been proposed in Ref. \cite{18}.

Earlier than in Ref. \cite{4} by Kievsky et al. this prescription has been used in Refs. \cite{19–21} and \cite{22} in the three– and four–nucleon calculations, respectively. In Ref. \cite{4} the first of these papers is mentioned. It is stated there that Ref. \cite{19}, among others, "...revealed a slow rate of convergence and provided satisfactory results only for soft NN interaction models. It turned out that the main difficulty to be overcome arises from the large degeneracy of the HH basis." This presentation is misleading since the authors do not say that in Ref. \cite{19} the prescription for the selection of important contributions has already been used and the difficulty with the slow rate of convergence has been overcome with its help. Moreover, the selection used by the authors of the commented paper is in fact identical to that used in Refs. \cite{18–22}.

This can be clarified by mentioning the following point. Despite the terminological differences with the previous papers, such as use of the term "Faddeev decomposition" by Kievsky et al., the ansatz for the wave function they use is in fact identical to that used previously\footnote{A slight and only apparent difference is that in \cite{18–22} a "symmetrization" of the spatial channel...}. The dynamical equations are also the same. Apart from the inserted correlation factors, the same applies to other papers by Kievsky et al.
(In reality, in the above mentioned earlier papers by the Russian group physically accurate bound state \(A=3\) and 4 results had been obtained with the HH approach for softer core realistic \(NN\) interactions. For \(A=3\) they compare well with more accurate Faddeev results obtained by that time [23]. While in Ref. [4] by Kievsky et al. the authors solved the problem with the Argonne potential (with the accuracy that far exceeds a physically meaningful one) due solely to substantial increase in the number of the retained HH within the same known selection prescription.)

basis functions is performed, and then the functions with proper symmetries obtained are coupled to the spin–isospin basis functions with conjugate symmetries to obtain the totally antisymmetric basis functions. While in Ref. [4] products of the spatial and spin–isospin channel basis functions are symmetrized directly. The resulting basis functions may be grouped in pairs spanning two–dimensional subspaces that are obviously just the same for the both ways of the antisymmetrization. In particular, this applies to ”important” basis functions retained in [4] which are thus completely equivalent to those defined in Ref. [18]. The choice of different \(K_{\text{max}}\) values for HH components of different types was done e.g. in Refs [20,22].
REFERENCES

[1] Kievsky A., Viviani M., Rosati S.: Phys. Rev. C 52, R15 (1995)
[2] Kievsky A., Viviani M., Rosati S.: Phys. Rev. C 56, 2987 (1997)
[3] Kievsky A.: Nucl. Phys. A624, 125 (1997)
[4] Kievsky A., Marcucci L.E., Rosati S., Viviani M.: Few–Body Systems, 22, 1 (1997)
[5] Kievsky A., Viviani M., Rosati S.: Nucl. Phys. A577, 511 (1994)
[6] Kievsky A., Viviani M., Rosati S.: Nucl. Phys. A551, 241 (1993)
[7] Zakhar’ev B.N., Pustovalov V.V., Éfros V.D.: Yad. Fiz. 8, 406 (1968) [Sov. J. Nucl. Phys. 8, 234 (1969)]
[8] Fenin Yu.E., Éfros V.D.: Yad. Fiz. 15, 887 (1972) [Sov. J. Nucl. Phys. 15, 497 (1972)]
[9] Delves L.M.: Nucl. Phys. 9, 391 (1959); 20, 275 (1960)
[10] Delves L.M.: Nucl. Phys. 26, 136 (1961)
[11] Delves L.M.: Nucl. Phys. 29, 268 (1962)
[12] Levinger J.S.: Phys. Rev. C 19, 2083 (1979)
[13] Zickendraht W.: Phys. Rev. 159, 1448 (1967)
[14] Pernyakov V.P., Pustovalov V.V., Fenin Yu.E., Éfros V.D.: Yad. Fiz. 14, 567 (1971) [Sov. J. Nucl. Phys. 14, 317 (1972)]
[15] Zhukov M.V., Éfros V.D.: Yad. Fiz. 14, 577 (1971) [Sov. J. Nucl. Phys. 14, 322 (1972)]; Éfros V.D., Zhukov M.V.: Phys. Lett. B37, 18 (1971)
[16] Raynal J.: in: The Nuclear Many–Body Problem, F. Calogero, C. Ciofi degli Atti eds., vol. 1, p. 589 (Editrice Compositori–Bologna 1973)
[17] Vostrikov A.N., Zhukov M.V.: Yad. Fiz. 34, 344 (1981) [Sov. J. Nucl. Phys. 34, 196 (1981)]
[18] Éfros V.D.: Sov. J. Nucl. Phys. 15, 128 (1972)
[19] Demin, V.F., Pokrovsky, Yu.E., Efros, V.D.: Phys. Lett. B44, 227 (1973)
[20] Demin V.F., Pokrovsky, Yu.E.: Phys. Lett. B47, 394 (1973)
[21] Mukhtarova, M.I.: Yad. Fiz. 49, 338 (1989) [Sov. J. Nucl. Phys. 49, 208 (1989)]
[22] Fomin, B.A., Éfros, V.D.: Yad. Fiz. 34, 587 (1981) [Sov. J. Nucl. Phys. 34, 327 (1981)]; Phys. Lett. B98, 389 (1981)
[23] J.L. Friar: Few–Body Sys. Suppl. 6, 538 (1992)