Analysis of $^6\text{He}+^{12}\text{C}$ elastic scattering and breakup reactions using a microscopic optical potential model

V K Lukyanov$^1$, D N Kadrev$^2$, E V Zemlyanaya$^1$, A N Antonov$^2$, K V Lukyanov$^1$ and M K Gaidarov$^2$

$^1$ Joint Institute for Nuclear Researches, Dubna, 141980, Russia
$^2$ Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
E-mail: vlukyanov@jinr.ru

Abstract. Theoretical analysis is made for the $^6\text{He}+^{12}\text{C}$ elastic scattering data at three different beam energies. The breakup effect of the $^6\text{He}$ at higher energies is also studied. Calculations were performed using microscopic optical potentials (OP's) obtained by a double-folding procedure and also those inherent in the high-energy Glauber-Sitenko approximation. The problem of ambiguity of the adjusted depths of these potentials is resolved. The role of breakup processes in formation of the imaginary potential in elastic channel is also discussed.

$^6\text{He}$ is a typical nucleus having the weak binding energy and extended neutron halo in its periphery. The latter is the reason why in collisions with the proton and nuclear targets the projectile nucleus $^6\text{He}$ is breaking up with a comparably large probability that causes the flux loss in the elastic channel. Therefore, the study of elastic scattering of $^6\text{He}$ on protons or light targets is a powerful tool to get information on peculiarities of the mechanism of such processes.

We perform an analysis of the $^6\text{He}+^{12}\text{C}$ elastic scattering data at three beam energies $E = 3$ [1], 38.3 [2] and 41.6 MeV/N [3] using the microscopically calculated OP. The main expressions for the real and imaginary parts are

$$U(r) = V^{DF}(r) + iW(r), \quad V^{DF}(r) = V^D(r) + V^{EX}(r). \quad (1)$$

The real part $V^{DF}$ consists of the direct and exchange double-folding (DF) integrals that include an effective $NN$ potential and density distribution functions of colliding nuclei. The formalism of the DF potentials is described in Refs. [4, 5]. In the considered case $V^D$ and $V^{EX}$ include only the isoscalar part of the $NN$-potential while its isovector part is omitted because $Z = N$ in the target nucleus $^{12}\text{C}$. All details of the mathematical treatments and calculations are given in Ref. [6].

Concerning the imaginary part of our OP, we take it in two forms. The first one has the same form as the real part of OP but with different strength. Also we test another shape of the imaginary part that corresponds to the full microscopic OP derived in Refs. [7, 8] within the HEA [9, 10] whose imaginary part is as follows

$$W^H(r) = -\frac{1}{2\pi^2} \frac{E}{k} \sigma_N \int_0^\infty j_0(qr) \rho_p(q) \rho_t(q) f_N(q) q^2 dq. \quad (2)$$
Here $\bar{\sigma}$ is the isotopically averaged total NN cross section, and $f_N$ is the form factor of the NN amplitude. The final microscopic volume OP has the following form:

$$U(r) = N_R V^{DF}(r) + i N_I W(r),$$

where $W(r)$ is taken to be equal either to $V^{DF}(r)$ or to $W^H(r)$. The parameters $N_R$ and $N_I$ entering Eq. (3) renormalize the strength of OP and are fitted by comparison with the experimental cross sections. In the work [11] we attempt to simulate the surface effects caused by the polarization potential by adding to [Eq. (3)] the different surface terms:

$$W^{sf}(r) \to -i N_I^{sf} \frac{dW(r)}{dr}; -i N_I^{sf} r \frac{dW(r)}{dr}; -i N_I^{sf} r^2 \frac{dW(r)}{dr}; -i N_I^{sf} \frac{dW(r - \delta)}{dr}$$

where $N_I^{sf}$ is a fitting parameter, and the shift $\delta$ is fixed to be $\delta = 1$ fm.

We calculated the $^6$He$+^{12}$C elastic scattering differential cross sections using the program DWUCK4 [12] and the microscopically calculated OP [Eq. (3)]. The inclusion of different forms of the surface potential leads to almost similar curves of cross sections displayed in the gray area at Fig. 1. The ambiguity of the values of $N$’s arises. We tried to choose the most physical values of $N$’s by adding the physical criterion that the obtained potentials should obey a determined behavior of the volume integrals dependent on the energy [4]

$$J_V(E) = -\frac{4\pi}{A_p A_t} \int N_R V^{DF}(r) r^2 dr, \quad J_W(E) = -\frac{4\pi}{A_p A_t} \int N_I W(r) r^2 dr,$$

where the OP’s were fitted at different energies. It was shown in Refs. [13, 14, 15]) that the volume integrals $J_V$ decrease with the energy increase at $E < 100$ MeV/N, while $J_W$ increases up to 10-20 MeV/N and then saturates. In Fig. 2 we select those curves whose parameters $N$’s lead to the mentioned behavior of $J_V$ and $J_W$. In Fig. 3 the deviations of the OP’s from their volume parts are presented. In our theoretical scheme they are related to the surface parts of the ImOP in the form of $N_I^{sf} r dW^H^{(DF)}/dr$ and $N_I^{sf} r^2 dW^H^{(H)DF}/dr$. One can suppose that these dynamical polarization potentials arrive due to effects of the breakup of $^6$He.

We consider the simplest breakup $ha$-model of $^6$He, where it is suggested consisting of two clusters $^4$He and $h$, the correlated pair of two neutrons $h = 2n$ [3]. The s-wave function $\varphi_0(s)$ of relative motion of clusters was obtained by fitting to the known binding energy and rms-radius of $^6$He and defines the density distribution of the latter

$$\varrho_0(s) = |\varphi_0(s)|^2 = (1/4\pi)|\varphi_{l=0}(s)|^2$$

It then used for calculations of the ground state matrix elements of breakup processes. One can see in Fig. 4 that $\varrho_0(s)$ coincides fairly well with $\varrho_L(s)$. Thus, we may apply the 2-cluster $ha$-model for the further calculations.
Figure 2. The $^{6}$He+$^{12}$C elastic scattering calculations with the volume (solid for $W = W^{H}$, dashed for $W = V^{DF}$) and surface part $-iN I_{r}^{2} dW(r)/dr$ in OP. For details see Ref. [11].

Figure 3. The surface ImOP’s used in the calculations of the cross sections. Solid and dot-dashed curves are for $W = W^{H}$ and others for $W = V^{DF}$, respectively. For details see Ref. [11].

In the framework of the $h\alpha$-model of $^{6}$He one can estimate the $^{6}$He+$^{12}$C OP as folding of two OP’s of interaction of clusters $\alpha$ and $h$ with $^{12}$C:

$$U^{(b)}_{HeC}(r) = V^{DF}(b) + iW^{(b)} = \int d^{3}s \varrho_{h}(s) \left\{ U_{\alpha}(r + (1/3)s) + U_{h}(r - (2/3)s) \right\} =$$

$$= 2\pi \int_{0}^{\infty} \varrho_{h}(s) s^{2} ds \int_{-1}^{1} dx \left\{ U_{\alpha} \left( \sqrt{r^{2} + (1/9)s^{2}} + r(2/3) sx \right) + 
+ U_{h} \left( \sqrt{r^{2} + (4/9)s^{2}} - r(4/3) sx \right) \right\}.$$

(7)

Here the $h$-$^{12}$C potential is taken as the twice neutron-$^{12}$C OP $U_{h} = 2U_{n}$. In turn, potentials $U_{\alpha}$ and $U_{n}$ are calculated within the microscopic hybrid model of OP [7]. Doing so, one can see from Fig.5 that angular distributions for different kinds of ImOP in the $h\alpha$-model based on the $W^{H}$ and $V^{DF}$ forms (solid and dashed curves), as well as in the DF-model (dash-dotted and dotted curves) are closely displayed, and the corresponding total reaction cross sections are almost equal in value. As a result the $h\alpha$-model of $^{6}$He seems to be reliable for the further evaluations of total breakup cross sections with a help of the HEA theory. The earlier HEA theory for the breakup processes was developed in Refs.[17],[18]. In recent papers [19],[20] this method was generalized to study breakup reactions of lightest nuclei.

In the Ref.[21] we showed that this method is applicable for the $^{6}$He+$^{12}$C at energies about 40 MeV/N. Thus, we can obtain the total absorption cross section for the $h\alpha$-model of $^{6}$He withen the HEA method as follows:

$$\sigma^{tot}_{abs} = 2\pi \int_{0}^{\infty} b_{h} db_{h} \left( 1 - |S_{h}(b_{h})|^{2} \right) \left( 1 - I(b_{h}) \right)$$

(8)
where

$$I(b_h) = \int d^3s g_b(s)|S_\alpha(b_\alpha)|^2, \quad b_\alpha = \sqrt{s^2 \sin^2 \vartheta + b_h^2 - 2sb_h \sin \vartheta \cos \phi}.$$  \hspace{1cm} (9)

Here the relation is used of impact parameters $b_\alpha = b_h - b$ with $b = s \sin \vartheta$ being the projection of the $h - \alpha$ vector $s$ on the plane normal to the $0z$-axis along the straight line trajectory of an incident nucleus.

In the case of the stripping reaction with removing $h$-particle from $^6\text{He}$ to the target nucleus, one should use the probability of $h$ to leave the elastic channel $(1 - |S_h(b_h)|^2)$, and for $\alpha$ to continue its elastic scattering with probability $|S_\alpha(b_\alpha)|^2$. Then the probability of the whole process is $|S_\alpha(b_\alpha)|^2 \cdot (1 - |S_h(b_h)|^2)$, and to get the total stripping cross section one must average over $g_b(s)$. In a similar manner the transfer of the $\alpha$ particle can be constructed, and the net contribution of both removal reactions yields the total breakup cross section

$$\sigma_{bu}^{tot} = 2\pi \int_0^\infty b_h db_h \left\{ |S_h(b_h)|^2 + \left[ 1 - 2|S_h(b_h)|^2 \right] \cdot I(b_h) \right\}. \hspace{1cm} (10)$$

The sum of the absorption (8) and breakup (10) cross sections results in the total reaction cross section

$$\sigma_R^{tot} = 2\pi \int_0^\infty b_h db_h \left( 1 - |S_h(b_h)|^2 \cdot I(b_h) \right).$$  \hspace{1cm} (11)

### Table 1.
The HEA estimations within the $h\alpha$-model of total cross sections of $^6\text{He}+^{12}\text{C}$ at $E=38.3\text{ MeV/N}$.

| Potential | $\sigma_{tot}^{abs}$, mb | $\sigma_{tot}^{bu}$, mb | $\sigma_{tot}^{R}$, mb |
|-----------|--------------------------|----------------------|----------------------|
| ImOP=$N_f W^H(b)$ | 392                      | 412                  | 804                  |
| ImOP=$N_f V^{DF}(b)$ | 447                      | 383                  | 830                  |

### Results and conclusion

The results of the present work can be summarized as follows:

(i) The microscopic OP and cross sections of $^6\text{He}+^{12}\text{C}$ elastic scattering were calculated at 3 different energies. In contrast to the phenomenological and semi-microscopic models we deal
with a fully microscopic approach as a physical ground to account for the single-particle structure of the colliding nuclei.

(ii) At higher energies additional surface terms in OP having a form of a derivative of the imaginary part of the OP became necessary.

(iii) The depths of the real and imaginary parts of the microscopic OP’s are considered as fitting parameters. To overcome the ambiguity of these parameters, additional physical constraints were imposed.

(iv) Estimations of the total cross sections were made by means of the preliminary calculated imaginary parts of OP’s $U_h$ and $U_\alpha$ for scattering of $h$- and $\alpha$-clusters on $^{12}$C. The obtained results are shown in Table 1. The breakup cross sections constitute approximately a half of the total reaction cross section. Therefore the significant amount of the imaginary potential of the $^6$He+$^{12}$C in elastic scattering channel is formed due to transmittions of the incident flux of $^6$He into breakup channels.

One can summarize that the elastic scattering data can be explained in a reasonably good agreement by using two folding models. These both models explain fairly well the separation energy and $rms$ radius of the nucleus. The $ha$-model can also estimate the other characteristics of the reaction mechanism such as the $^6$He breakup cross sections. This gives a possibility to understand the significant role of breakup processes in formation of the imaginary part of the $^6$He+$^{12}$C elastic scattering OP.

**Acknowledgments**

The work is partly supported by the Project from the Agreement for co-operation between the INRNE-BAS (Sofia) and JINR (Dubna). Three of the authors (D.N.K., A.N.A. and M.K.G.) are grateful for the support of the Bulgarian Science Fund under Contract No. 02–285 and one of them (D.N.K.) under Contract No. DID–02/16–17.12.2009. The authors E.V.Z. and K.V.L. thank the Russian Foundation for Basic Research (Grant No. 09-01-00770) for the partial support.

**References**

[1] Milin M, Cherubini S, Davinson T et al 2004 *Nucl. Phys.* **A730**, 285
[2] Lapoux V, Alamanos N, Auger F et al 2002 *Phys. Rev.* **C 66**, 034608
[3] Al-Khalili J S, Cortina-Gil M D, Roussel-Chomaz P et al 1996 *Phys. Lett.* **B 378**, 45
[4] Satchler G R and Love W G 1979 *Phys. Rep.* **55**, 183
[5] Khoa D T and Satchler G R 2000 *Nucl. Phys. A* **668**, 3
[6] Lukyanov K V 2007 *Comm. JINR* P11-2007-38, Dubna
[7] Lukyanov K V, Zemlyanaya E V, and Lukyanov V K 2004 *JINR Preprint* P4-2004-115, Dubna; 2006 *Phys. At. Nucl.* **69**, 240
[8] Shukla P 2003 *Phys. Rev.* **C 67**, 054607
[9] Glauber R J 1959 *Lectures in Theoretical Physics* (New York, Interscience), p.315
[10] Sitenko A G 1959 *Ukr. Fiz. J.* **4**, 152
[11] Lukyanov V K, Kadrev D N, Zemlyanaya E V et al 2010 *Phys. Rev.* **C 82**, 024604
[12] Kunz P D and Rost E 1993 *Computational Nuclear Physics*, edited by K. Langanke et al. (Springer-Verlag, New York), Vol.2, p.88
[13] Romanovsky E A 1998 et al., *Bull. Rus. Acad. Sci.*, Physics, **62**, 150
[14] Mahaux C and Ngo H 1982 *Nucl. Phys.* **A378**, 205
[15] Brandan M E and Satchler G R 1997 *Phys. Rep.* **285**, 143
[16] Karataglidis S, Dortmans P J, Amos K and Bennhold C 2000 *Phys. Rev.* **C 61**, 024319
[17] Glauber R J 1955 *Phys. Rev.* **99**, 1515 (1955); 1955 *ibid.* **100**, 242
[18] Akhiezer A I and Sitenko A G 1955 *Sci. Notes of Kharkov Univ.* **64**, 9 (1955); 1957 *Phys. Rev.* **106**, 1236
[19] Hencken K, Bertsch G, Esbensen H 1996 *Phys. Rev.* **C 54** 3043
[20] Bertulani C A and Hansen P G 2004 *Phys. Rev.* **C 70** 034609
[21] Lukyanov V K, Zemlyanaya E V, Lukyanov K V 2011 *Int. J. Mod. Phys.* **E 20**, 9