Two-dimensional periodic and quasiperiodic spatial structures in microchip laser resonator.

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The spatially periodic 2D patterns at output mirror of solid state microchip laser with high Fresnel number (100-1000) are discussed in view of numerical modeling with split-step FFT code comprising nonlinear gain, relaxation of inversion and paraxial diffraction.

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I. INTRODUCTION

Spatially periodic structures of electromagnetic field in optical cavities could arise not only due to boundary conditions, as for example in the case of rectangular waveguide [1]. In Talbot cavity [2] the spatially periodic layout of cavity parameters forces the lightwaves to follow the profile of index and gain. The more interesting situation occurs when nonlinear wave interaction itself arranges the sophisticated electromagnetic structures [3-5]. Regardless to the physical nature of nonlinearity the common feature of these structures is their translational symmetry: in passive systems the hexagonal spatial structures are dominant [3, 5], in active systems having optical gain the rectangular structures are more likely to survive [3, 4, 6]. The map of parameters space for each given system contains regions with hexagonal, rectangular arrays, spatial localized structures (spatial solitons or diffractive autosolitons [7, 10]) and spatial chaos [7, 11]. The location of these regions and their boundaries sensitively depends upon geometry of optical cavity, i.e. on positions and curvature of mirrors, lenses, nonlinear elements and apertures. Of course, in real experimental practice [12, 13] such separation is often ambiguous, because almost any optical element, for example gain element, could have properties of lens (linear or nonlinear), partially reflecting mirror, aperture, birefringency et al. Nevertheless, recent experimental results show the stable electromagnetic field patterns [12, 13] described by relatively simple and robust theoretical models.

This models are reduced from conventional Maxwell-Bloch equations for two-level gain medium [14], nonlinear wave equation for $\chi^2$ (parametric) [13], $\chi^3$ (Kerr) [16], or photorefractive media [17]. We will restrict here ourselves by two-level resonant nonlinearity which is conventional basis for description of the laser dynamics [18]. The most interesting feature of dynamics observed in both numerical modeling and experiments is the possibility of reducing the entire set of Maxwell-Bloch equations [14, 18] to equivalent single evolution equation: Ginzburg-Landau equation [6, 19] or to the more complicated Swift-Hohenberg equation [20, 21]. In the Swift-Hohenberg approximation the additional diffusion-like terms arise in the equation for optical envelope, because finite bandwidth of atomic gain line is taken into account and consequently, the dissipative filtering of higher harmonics. Such reducing works especially well in class - A laser, when relaxation $T_1$, $T_2$ of atomic variables $N$, $P$ (atomic inversion and polarization) is considered as “frozen”, while radiation lifetime in cavity is defined usually as $\tau = 2L/c(1 - R)$. Thus it is possible to consider atomic variables $N$, $P$ as "frozen", while radiation $E$ passes through gain medium. Then this variables are forced by constant external field $E$ (optical field), while the latter bounces between mirrors [11]. Such separation leads to discrete mapping of intracavity field from one bounce to another, the number of bounce $n$ serves here as discrete time $t$. This approach proved to be fruitful for both the

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class - B laser (ring Nd-YAG long cavity) with low Fresnel number, when only several transversal modes excited\(^\text{[11]}\) and high Fresnel-number microchip laser with short cavity\(^\text{[4]}\). In this case the spatiotemporal evolution of optical envelope is governed by iterative mapping of convolution type\(^\text{[4, 7, 11]}\):

\[
E_{n+1}(r^2) = \int K(r_1 - r^2) f(E_n(r_1)) dr_1, \\
N_{n+1} = \frac{N_n - N_0}{T_1} - \sigma N_n |E_n|^2, \tag{1}
\]

where \(\sigma\) is stimulated emission cross-section\(^\text{[11]}\).

The map\(^\text{(1)}\) is \textit{nonlocal}, i.e. it introduces the severe spatial as well as temporal dispersion at each iterate (radiation bounce). In the above mentioned mapping set the nonlinearity action \(f\), which usually acts as sharpen the field distribution, is described by local nonlinear mapping \(f\), while convolution with kernel \(K\) introduces the spatial dispersion. The most interesting feature of such approach proved to be even more general in some sense than starting point: Maxwell - Bloch equation for class B - laser, received by naive adiabatic elimination of polarisation field \(P\)\(^\text{[9, 10]}\). The case is that boundary conditions are included directly into the convolution integral and, consequently, spatial filtering provides the diffusive terms\(^\text{[7]}\) which in general are not restricted to second order. This fact leads to the following results:

1. Nonlocal iterative map\(^\text{(1)}\) contains both Ginzburg - Landau and Swift - Hohenberg equations. The latter arise even in the case of adiabatic elimination of polarisation field \(P\).
2. The existence of spatially periodic structures which are "fixed points" of the map\(^\text{(1)}\) or stable solutions of the Ginzburg - Landau or Swift - Hohenberg equations is manifestation of Talbot phenomenon, when spatial period of translationally symmetric structures is self-chosen in such a way to fulfill the condition of self-imaging. 
3. Experimentally obtained spontaneously arising vortex array structures in microchip laser resonator have period such that Talbot condition on cavity length \(L = 2P^2/\lambda\) fulfilled and Fresnel number \(F_r\) of cavity proved to be in the vicinimum of the value \(F_r = N^2_v\), predicted in\(^\text{[4]}\).

II. DIFFERENT FORMS OF THE EVOLUTION EQUATIONS

A. Partial differential equation form

The starting point of current analysis are Maxwell - Bloch equations\(^\text{[1, 14, 18]}\), written for slowly varying amplitudes and in paraxial approximation:

\[
\frac{\partial E}{\partial z} + \frac{1}{c} \frac{\partial E}{\partial t} + \frac{i}{2k} \Delta \perp E = -\frac{i2\pi\omega}{c} P - \gamma E \\\n\frac{\partial N}{\partial t} = \frac{N_0 - N}{T_1} - \frac{2i}{\hbar} (EP^* - P^*E), \\\n\frac{\partial P}{\partial t} = \frac{P}{T_2} - i(\omega - \omega_a)P + \frac{i\mu^2}{\hbar} EN, \tag{2}
\]

where \(E, N, P\) are optical field, inversion and polarization of the medium (density of the dipole moment of the resonance impurity in solid-state dielectric) respectively, \(\mu\) is electric dipole moment, \(k = 2\pi/\lambda\), \(c\) - speed of light, \(\gamma\) are nonresonant losses in medium, \(\omega_a\) is atomic resonance frequency, \(\omega\) is the carrier frequency of the optical field \(E\).

The spontaneous emission sources are neglected and polarization of electromagnetic field (the mutual orientation of electric and magnetic fields with respect to wavevector \(k\)) is assumed here to be homogeneous over the entire aperture. The further simplification of MBEs\(^\text{(2)}\) could be fulfilled traditionally\(^\text{[16, 18]}\) by virtue of longitudinal averaging of \(E\) along optical axis \(z\) over cavity length \(L_r\):

\[
\frac{\partial E}{\partial t} + \frac{i}{2k} \Delta \perp E = -i2\pi\omega P - \frac{E}{\tau} - i(\omega_c - \omega) E \\\n\frac{\partial N}{\partial t} = \frac{N_0 - N}{T_1} - \frac{i}{2\hbar} (EP^* - P^*E); \\\n\frac{\partial P}{\partial t} = \frac{P}{T_2} - i(\omega - \omega_a)P + \frac{i\mu^2}{\hbar} EN, \tag{3}
\]

where \(\tau\) is relaxation time.
This is so-called C - class laser MB equations. The next reduction could be fulfilled by two ways. The first one is in trivial elimination of the polarisation variable $P$ under "obvious" condition:

$$\frac{\partial P}{\partial t} \ll \frac{P}{T_2}$$

which leads to the quasistatic elimination of $P$ in the form:

$$P = \frac{i\mu^2 EN}{h(i(\omega - \omega_a)T_2 + 1)}.$$  

After substitution of $P$ into (3) the MBE system becomes:

$$\frac{\partial E}{\partial t} + \frac{ic}{2k}\Delta_\perp E = \frac{\sigma EN - E}{\tau_c} - i(\omega_c - \omega)E,$$

$$\frac{\partial N}{\partial t} = \frac{N_0 - N}{T_1} - \sigma|E|^2 N; \sigma = \frac{2\pi\mu^2 \omega T_2}{hc(1 + i(\omega - \omega_a))}$$

the so-called B - class laser MB equations. The system (6) has intermediate time scale, the relaxation oscillations time:

$$\tau_{rel} = \sqrt{T_1/\tau_c}/\sqrt{\sigma N_0 \tau_c - 1},$$

whose value depends upon relative values of times $T_1, \tau_c$, determined by the physical nature of gain medium. In solid - state medium, such as rare - earth doped dielectrics, the $\tau_{rel}$ lies in between $T_1 \ll \tau_{el} \ll T_2$. In the limit $\tau_{el} \gg T_1, \tau_c$, i.e. just near lasing threshold, when radical in (7) is very small, the system (7) is turned into system for class - A laser:

$$\frac{\partial E}{\partial t} + \frac{ic}{2k}\Delta_\perp E = \frac{\sigma EN - E}{2(1 + \sigma T_1|E|^2)} - i(\omega_c - \omega)E,$$

which has the form of the Ginzburg - Landau equation [7, 8, 16, 19, 21].

The more rigorous approach had been developed in [23, 24], where it was found that polarisation of medium provides natural spatial filtering, owing to finite linewidth:

$$\frac{\partial E}{\partial t} + \frac{ic}{2k}\Delta_\perp E = \sigma EN - \frac{E}{\tau_c} - i(\omega_c - \omega)E + T_2^2[(\omega_a - \omega) + \frac{ic}{2k}\Delta_\perp]^2,$$

$$\frac{\partial N}{\partial t} = \frac{N_0 - N}{T_1} - \sigma|E|^2 N;$$

This complex Swift - Hohenberg equation [6, 23, 24], where dissipative terms, containing second and fourth order spatial derivatives provide effective smoothing of abrupt changes of spatial structure of electromagnetic field and suppression of collapsing instabilities, typical to both Ginzburg - Landau equation and nonlinear Shrodinger equation [25]. These terms are disappearing in the limit $T_2 \to 0$, i.e. when atomic linewidth tends to infinity.

### B. Integral equation form

The main result of the current paper is that such spatial filtering had been already included in familiar model of laser dynamics, elaborated in [5], which had been applied afterwards to numerical simulations of optical field structure of the class - A laser with spatially periodic gain distribution [26 28]. The idea of inclusion the spatial filtering directly into the evolution equation of electromagnetic field is not new. This idea belongs to Fox and Li [29] whose model consists of the infinite sequence of periodically located nonlinear amplifying screens, which imitates the radiation bounces from one mirror of the laser resonator to another. Boundary conditions are taken into account by multiplying the kernel $K$ by both amplitude-phase masks which imitate reflections from mirrors, limitations by edges of cavity elements etc. In fact, the Fox - Lee method is no more than discrete mapping the two-dimensional complex field from one mirror to another by means of a product of two consecutive mappings:
one is local and nonlinear: it acts on each point of spatial structure independently of its neighbours, the other is nonlocal and linear: it mixes the mutual actions of adjacent points with each other. This procedure had been used each time, when robust and efficient computational scheme had been required [30]. The spatial filtering had been introduced in split-step FFT method as "windowing" and "sampling" and the authors had not lose time in vain attempts to mask the analogies with Fox-Lee method [11, 29, 30].

It was shown in [7] that infinite sequence of alternating nonlinear amplifying slices and spatial filters could be modelled by nonlocal map with real kernel when confocal cavity of the so-called 8F-type considered:

\[ E_{n+1}(\vec{r}_2) = \int K(\vec{r}_1 - \vec{r}_2) f(E_n(\vec{r}_1)) d\vec{r}_1. \]  

(10)

In same way the evolution map for radiation in optical cavity with arbitrary curvature of mirrors (fig.1) obtained:

\[ E_{n+1} = \tilde{F}_r f[E_n] = \tilde{F}_r f[E_n(\vec{r}_\perp)] = \frac{i k}{2\pi L} \int_{-\infty}^{\infty} E_n(\vec{r}_\perp) \exp\left[\frac{-ik(\vec{r}_\perp - \vec{r}_\perp')^2}{2L}\right] D(\vec{r}_\perp') d(\vec{r}_\perp'), \]  

(11)

where \( \tilde{F}_r \) is the solution of parabolic wave equation (linear Shrodinger equation) over cavity length \( L \) in standard convolution form (the kernel is Green function), \( D \) is complex aperture function [27] whose modulus corresponds to finite width of gain medium, mirrors, lenses etc., while argument imitates the phase modulation produced by curved surfaces with effective focal length \( F \) and random phase modulation produced by random field \( \psi \) of the roughness:

\[ D(\vec{r}_\perp) = D_o(\vec{r}_\perp) \exp\left(i \frac{kr^2}{2F} + i \psi(\vec{r}_\perp)\right) \]  

The equations (11-13) are the rigorous [1, 7, 11] for intracavity field dynamics during the single radiation round-trip for class-A laser, provided the gain medium and phase inhomogeneities are concentrated nearby the given plane of the cavity (fig.1). Otherwise the more complicated version of the equations (11-26) is used [30], when gain medium is represented as a sequence of thin slices, each with its own gain and phase ripple. The iterates of the equations (11-26) are equivalent, as shown in [7] by asymptotic evaluation, to solution of partial differential MBE for class-A laser, i.e. GLE [8]. The GLE had been obtained in [7] through expanding of \( E \) in Taylor series up to the second order. This is justified when \( L \) is sufficiently short and fast oscillations of the kernel of \( \tilde{F}_r \):

\[ \exp\left(\frac{ik(\vec{r}_\perp - \vec{r}_\perp')^2}{2L}\right) \]  

(13)

quench the integral for all points except for those, who are located in the vicinity of \( \vec{r}_\perp' \).

Let us consider here the more general situation, when cavity length \( L \) is not so short and fourth order terms are to be taken into account. In order to get from (11) the evolution equation with small changes from one iterate to another, let us separate local and nonlocal part of the map (11) by the following substitution [7]:

\[ E_{n+1} = E_n + \tilde{F}_r[E_n] = E_n + [f[E_n] - E_n] + O^2( \alpha ) , \]

\[ \alpha \cong \tilde{F}_r[E_n] - E_n, [f[E_n] - E_n], \]  

(14)

The physical meaning of this condition is quite natural: the first, i.e. zeroth-order term \( E_n \) is only slightly affected by the second and third terms of (14) which are responsible for diffraction and nonlinearity correspondingly and they are of the same order \( \alpha \), i.e. the changes due to nonlocal and local part of map (11) are of equal weight and additive. The fourth term is of the order \( \alpha^2 \) and it is insignificant. The second term of (14), i.e. nonlocal part of the map (11) could be evaluated by stationary phase method up to the forth order by means of decomposition of \( E \) in Taylor series in the vicinity of \( \vec{r}_\perp \):

\[ E_n(\vec{r}_\perp - \vec{r}_\perp') \cong E_n(\vec{r}_\perp) + \nabla^2[E_n(\vec{r}_\perp)] \frac{|\vec{r}_\perp - \vec{r}_\perp'|^2}{2!} + \nabla^4[E_n(\vec{r}_\perp)] \frac{|\vec{r}_\perp - \vec{r}_\perp'|^4}{4!} \]  

(15)

The substitution of (15) into (11) and integration over \( \vec{r}_\perp \) lead us to evolution equation provided time step \( n+1 \), \( n \) is considered as infinitesimally small:

\[ \frac{\partial E}{\partial t} = \gamma E + \delta E^2 + \beta E^3 + \eta E^5 + (a + ib) \nabla^2(E_n r\perp) + (c + id) \nabla^4(E_n r\perp), \]  

(16)

i.e. complex Swift-Hohenberg equation, which takes into account the higher-order spatial dispersion terms. The above procedure of derivation presents the exact values of all coefficients in (16) and their connection with geometrical parameters of the cavity. The nonzero values \( c \) and \( d \), arises here as a result of spatial filtering of high transversal harmonics on diaphragm, without the inclusion of finite gain linewidth \( T_2^{-1} \).
C. Renormalization group equation as a universal limit for nonlocal map

The conceptual progress in physical understanding of the nonlocal map properties had been made in [31], where it was shown that infinite sequence of alternating nonlinear local maps and linear nonlocal maps of convolution type behave in universal manner. It was shown by renormalization group technique, that nonlocal map with real kernel:

$$E_{n+1}(r_2) = \int K(r_1 - r_2)f(E_n(r_1))dr_1.$$  \hspace{1cm} (17)

tends to universal form, regardless to peculiarities of a given spatial filters which form the nonlinear dispersive medium:

$$E_{n+1} = \hat{G}E_n; \hat{G} = \exp\left[\frac{\Delta^2}{2} \frac{\partial^2}{\partial x^2}\right]g,$$  \hspace{1cm} (18)

where $\Delta$ is the second moment (deviation) of the kernel $K$, $\Delta^2 = \int K(\vec{r})|\vec{r}|^2$, $g$ is fixed point of the iterates of the nonlinear local map $f$ [31]. The (18) is the fixed point of renormalization group equation.

In the same way we are able to construct renormalization group equation for nonlocal complex map (11), using its simplified version (14). Really, for small longitudinal steps $\Delta L = L/m$ we may use the following asymptotic form of the nonlocal diffraction operator:

$$E_{n+1} = [1 + i\frac{\Delta L}{2k}\Delta_{\perp}]f[\hat{E}_n],$$  \hspace{1cm} (19)

Considering only the weak changes of the field at each longitudinal step we get in first order:

$$E_{n+1} = E_n + [f[\hat{E}_n] - E_n] + [1 + i\frac{\Delta L}{2k}\Delta_{\perp}][f\hat{E}_n],$$  \hspace{1cm} (20)

Because of the additive form of these map (20), we may consider the second and third terms of it as commutative operators. Thus, after $m$ infinitesimal steps $\Delta L = L/m$ we get resulting nonlocal operator in universal form:

$$\hat{K}_m = [1 + i\frac{\Delta L}{2k}\Delta_{\perp}]^m = [1 + iL/2km\Delta_{\perp}]^m,$$  \hspace{1cm} (21)

Taking into account the formal identity:

$$[1 + i\frac{L}{2km}\Delta_{\perp}]^m = [(1 + i\frac{L}{2km}\Delta_{\perp})^{2km/iL\Delta_{\perp}}]^{iL\Delta_{\perp}/2k},$$  \hspace{1cm} (22)

We get the limit of the sequence of nonlocal operators:

$$\lim_{m \to \infty} [1 + i\frac{L}{2km}\Delta_{\perp}]^m = \exp[i\frac{L}{2k}\Delta_{\perp}],$$  \hspace{1cm} (23)

having in mind the Euler's limit for $e = 2.71828...$:

$$\lim_{m \to \infty} [1 + \frac{1}{m}]^m = e,$$  \hspace{1cm} (24)

Thus the complex renormalization group equation has the fixed point in the form of the operator exponent:

$$E_{n+1} = \exp[i\frac{L}{2k}\Delta_{\perp}][f\hat{E}_n],$$  \hspace{1cm} (25)

III. SPATIALLY QUASI-PERIODIC EXACT SOLUTIONS

The model of thin nonlinear slice with any translationally symmetric $N_0(\vec{r}) = N_0(\vec{r} + \vec{p})$ gain distribution (i.e. homogeneous or spatially periodic) in Fabry-Perot cavity [4] provides the ample example of exact solutions of the eigenfunction problem of the map (11) if the aperture function has the gaussian form:
\[ D(r_{\perp}) = D_0 \exp(-\frac{r_{\perp}^2}{2D_a} + \frac{kr_{\perp}^2}{2F}) \]  

(26)

The first and most general form of solution is in Fourier series. The solution is spatially periodic with period \( p \) which is selected by field to match the Talbot condition on cavity length \( L = mp^2/\lambda \) [2, 4] :

\[ E(r_{\perp}) = \exp\left[-\frac{r_{\perp}^2}{2D_a^2(1+iN_f^{-1}+z/F)} + ikz\right] [1 + iN_f^{-1} + z/F]^{-1} \]

\[ \sum_{s,l} a_{s,l} \exp\left[i\pi(sx + ly) - i\pi(m^2 + l^2) \right] 1 + iN_f^{-1} + z/F]^{-1} \],

(27)

where \( N_f \) is Fresnel number of Talbot cavity [4] :

\[ N_f = \frac{kD_a^2}{z} = \frac{kD_a^2}{L} = \frac{2\pi D_a^2}{mp^2} \]  

(28)

where \( m \) is integer, showing how many times the half-Talbot length is contained within cavity length \( L \). From (28) it follows formally, that \( N_f \) is proportional to \( N_p^2 \) [26], i.e. number of the periods of the structure, contained within aperture, because \( D_a = N_pp \). The same connection between these quantities could be obtained from purely geometrical considerations, taking into account the number of Fresnel zones, placed within \( D_{alpha} = N_p p \) (fig. 2).

The another form of exact solution could be obtained, when spatially periodic field represented as periodic sequence of Gaussian beams which are diffracted on common Gaussian diaphragm. 

\[ E(r_{\perp}) = \exp\left[-\frac{r_{\perp}^2}{2D_a^2(1+iN_f^{-1}+z/F)} + ikz\right] [1 + iN_f^{-1} + z/F]^{-1} \]

\[ \sum_{m,n} \exp\left[-\frac{(r_{\perp} - \vec{p}_{m,n})^2}{p^2(1+iN_f^{-1}+z/F)} \right], \]

(29)

A somewhat different exact solution is obtained in the form of periodic arrays of Gauss-Laquerre beams. For the first-order Gauss-Laquerre beams each is considered as elementary optical vortex we are able to construct the rectangular grid of mutually coherent vortices with opposite topological charges:

\[ E(r_{\perp}) = \exp\left[-\frac{r_{\perp}^2}{2D_a^2(1+iN_f^{-1}+z/F)} + ikz\right] [1 + iN_f^{-1} + z/F]^{-1} \]

\[ [1 - \sum_{m,n} \exp\left[-\frac{(r_{\perp} - \vec{p}_{m,n})^2}{p^2} \right] + i\phi(-1)^{m+n} \] \[ \left(r_{\perp} - \vec{p}_{m,n}\right)^{1/2} \], \]

(30)

In the fig.2 the distribution of intensity and the phase of the in-phase vortex array shown. Note the different orientation of ”dark lattice”, which represent zeros of amplitude (i.e. positions of vortices), and ”bright lattice”, which is rotated over angle 45 degrees with respect to ”dark” one. The fig. 3 represents the out-of-phase vortex array with antiparallel topological charges.

IV. COMPARISON WITH EXPERIMENT

Recent experiments with \( L = 2mm \) long cavity diode-pumped solid-state laser [12] showed the formation of the quasi-periodic spatial lattices of vortices in the near field. The longitudinal mode spacing is \( c/(2Ln) = 50Ghz \) (experimentally measured value - 60 Ghz [12]). Such cavity length exceeds two times those predicted previously [22], although single-frequency lasing and phase-locking of the 2x2 arrays by Hermite-Gaussian TEM11-mode had been obtained readily even for the cavity length \( L = 30mm \) [13]. The transverse size of the gain region \( D \) was in between \( D = 0.5 - 1.5mm \), radius of curvature of mirror \( R = 50mm \). Thus the present microcavity had transverse size \( D \) being only somewhat smaller (by the factor 3/4) than cavity length \( L \). In general situation cavity would exhibit nonparaxial dynamics, because the partial waves (or rays in geometrical optics approximation) emitted by
edges of gain region (fig. 1) have angle of tens of degrees with the optical axis of the cavity. Nevertheless, as it was shown in [26, 32] such waves with very large tilt to optical axis do not survive in the present cavity. The case is that they greatly increase the threshold of lasing and instead of nonparaxial eigenmode the set of nearly single mode channels of “amplifiers” is formed, each having the ”local” Fresnel number smaller than unity $N_f \sim d^2/ (\lambda z_T) < 1$ [26, 32]. The ”global” Fresnel number $N$ of the cavity for $\lambda = 1.064 \mu m$ moved from 100 to 1000. The most interesting observation to my opinion is that the number of vortices in array in any direction (fig. 2) in [12] proved to be in qualitative agreement with our earlier prediction $N_f = N^2_2$ [4, 26, 32]. The observed period of arrays is equal roughly to $p = \sqrt{L \lambda} \approx 50 - 60 \mu m$. The close period value had been predicted in [32], where transverse mode-locking by periodic gain had been considered.

V. MULTISCALE STRUCTURE OF INTRACAVITY VORTEX FIELD

The interpretation of the experiment [12] as Talbot transverse mode-locking could get additional confirmations if one could prove experimentally the $p/2$ shift between intensity patterns on opposite sides of the cavity [2]. When cavity length is a half of Talbot one $L = p^2/\lambda$ the vortices are in-phase (fig. 2, see also fig. 3 from [32]). The more remarkable experimental evidence could be obtained for out-of-phase Talbot synchronisation. In this case the adjacent vortices have opposite $\pi$-shifted phases and one mirror carries $p/2$ intensity pattern [2, 32]. The cavity length in this case is the quarter of the Talbot one $L = p^2/2 \lambda$ [2, 32]. This feature of Talbot antiphase-locking is due to destructive interference of adjacent vortices within cavity, where vortex channels are not parallel, but form the bundle of intercepting threads, each with variable width. At the distances from the mirrors $L_m = /2 \lambda m$ the sequence of tiny arrays having the period $p_m = p/m$ is formed. In the fig. 4 the spatial layout of intensity between mirrors is shown. The Fourier images of the field in the planes $L_m = p^2/2 \lambda m$ have the period $p_m = p/m$.

VI. CONCLUSION

The nonlocal map approach provides simple description of microchip laser dynamics. We started from conventional Maxwell-Bloch equations and under approximation of ultrathin laser cavity we constructed integral equation (10), which takes into account the boundary conditions on cavity mirrors. The iterates of such nonlocal map are equivalent to time evolution of partial differential equations - Maxwell-Bloch equations. We got stationary nonlinear solutions of nonlocal map, i.e. nonlinear eigenfunctions of high Fresnel number cavity of microchip laser. The two-dimensional spatially periodic solutions obtained in the form of Fourier series, array of zeroth-order Gaussian beams and array of first-order Gauss-Laquerre vortices. The essential counterpart of such periodic structures is Talbot self-imaging, when cavity length is in stiff connection with period of the transverse structure. We showed the multiscale structure of intracavity vortex field when interference pattern in different planes of the cavity have fractional periods compared to mirrors patterns. Our exact solutions contain definite connection between Fresnel number and number of vortices within laser aperture predicted earlier and observed experimentally. Inspite of very large Fresnel number which offer possibility of generating the highly divergent nonparaxial waves, we found that spontaneously arising vortex structure acts as highly effective spatial filter selecting paraxial waves.
FIG. 1: Geometry of short length high Fresnel number laser cavity.

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FIG. 2: Distribution of intensity (left) and phase (right) for in-phase vortices array

FIG. 3: Distribution of intensity (left) and phase (right) for out-of-phase vortices array

FIG. 4: Distribution of intensity along microchip laser cavity. The 8 in-phase synchronized vortices are shown. The angular frequency of relaxation of relaxation oscillations $\omega_{\text{relax}} \sim 1/\sqrt{T_1}$ versus density of excited Nd ions shown.