Magnetoexcitons in quantum-ring structures: a novel magnetic interference effect

S. E. Ulloa\textsuperscript{a}, A. O. Govorov\textsuperscript{a,b}, A. V. Kalameitsev\textsuperscript{b}, R. Warburton\textsuperscript{c}, and K. Karrai\textsuperscript{d}

\textsuperscript{a} Department of Physics and Astronomy, and CMSS Program, Ohio University, Athens, OH 45701, USA; ulloa@helios.phy.ohiou.edu
\textsuperscript{b} Institute of Semiconductor Physics, RAS, Siberian Branch, 630090 Novosibirsk, Russia
\textsuperscript{c} Department of Physics, Heriot-Watt University, Edinburgh, UK
\textsuperscript{d} Sektion Physik der Ludwig-Maximilians-Universität and Center for Nano-Science, Geschwister-Scholl-Platz 1, 80539 München, Germany

Abstract

A novel magnetic interference effect is proposed for a neutral, but polarizable exciton in a quantum ring with a finite width. The magnetic interference effect originates from the nonzero dipole moment in the exciton. The ground state of exciton acquires a nonzero angular momentum with increasing normal magnetic field. This leads to the suppression of the photoluminescence in defined windows of the magnetic field.

When a quantum particle moves along a closed trajectory in external electric and magnetic fields, the Aharonov-Bohm and Aharonov-Casher effects can occur, caused by quantum interference between paths with different phases. As is well known, the Aharonov-Bohm (AB) effect is related to a charged particle trajectory enclosing a magnetic flux. Here we present a novel magnetic interference effect for a neutral, but polarizable quasi-particle. In particular, we show here theoretically that the wave function of a neutral polarizable exciton acquires a nonzero phase when it moves in a quantum ring pierced by a magnetic flux. The neutral exciton wave function becomes sensitive to the magnetic flux because of a net radial electric-dipole moment induced by asymmetries in the nanostructure potential. The transition to a
finite phase and corresponding orbital momentum in the polarized exciton state strongly changes the photoluminescence (PL) spectrum of the system due to the optical selection rules for interband transitions.

We demonstrate this novel magneto-interference effect using a model of InAs self-organized quantum rings (QR’s) [1]. The calculated single-particle wave functions of electrons and holes are peaked at different radii due to the potential asymmetries and the difference in effective mass of the particles; meanwhile, their mutual interaction correlates their motion around the ring. The dipole polarization of the exciton can be strongly enhanced due to a point/impurity charge in the ring center, or by a voltage applied to a metal nano-gate in the middle of a ring. In that case, the potential minimum for the electron is shifted from that for the hole, and the ground state of the exciton acquires a finite polarization. To qualitatively demonstrate the effect, we use a model of two nested one-dimensional (1D) rings with different radii, one for the electron and one for the hole (insert of Fig. 1). For rings with relatively large radii, the motion of particles is strongly correlated, whereas, in a small system, the state is nearly single-particle-like, as the relative weight of the Coulomb interaction decreases with respect to the confinement energies.

Since the vertical size of a QR is typically much smaller than the lateral one, we will discuss only the in-plane motion. The electron and hole in-plane potentials are approximated by \( U_{e(h)}(\rho) = m_{e(h)} \Omega_{e(h)}^2 (\rho - R_{e(h)})^2 / 2 \), where \( \rho \) is the in-plane distance to the ring center, and \( m_{e(h)}, \Omega_{e(h)}, \) and \( R_{e(h)} \) are effective masses, characteristic frequencies, and ring radius, respectively; the indices \( e \) and \( h \) indicate the electron and hole quantities. In the vertical, \( z \)-direction, the motion is strongly quantized.

In a magnetic field, the Hamiltonian of an exciton confined in a quantum ring reads \( \hat{H} = \hat{T}_e + \hat{T}_h + U_e + U_h + U_C(|\mathbf{r}_e - \mathbf{r}_h|) \), where \( \mathbf{r}_{e(h)} \) are the in-plane coordinates, \( \hat{T}_{e(h)} \) are the kinetic energies in the presence of a normal magnetic field, and \( U_C \) is the Coulomb potential. Now we assume that the quantization in the radial direction is stronger than that in the azimuthal direction. It allows us to separate variables in the wave function, \( \Psi(\mathbf{r}_e, \mathbf{r}_h) = f_e(\rho_e) f_h(\rho_h) \psi(\phi_e, \phi_h) \). Here \( \mathbf{r} = (\rho, \phi) \). The radial wave functions \( f_{e(h)} \) are strongly localized near the radii, \( R_{e(h)} \). The Hamiltonian describing the wave function \( \psi(\phi_e, \phi_h) \) is (up to a \( B \)-independent constant term),
\[ \hat{h} = -\frac{\hbar^2}{2m_eR_e^2} \frac{\partial^2}{\partial \phi_e^2} + \frac{i\hbar \omega_e}{2} \frac{\partial}{\partial \phi_e} - \frac{\hbar^2}{2m_hR_h^2} \frac{\partial^2}{\partial \phi_h^2} + \frac{i\hbar \omega_h}{2} \frac{\partial}{\partial \phi_h} + \frac{m_e\omega_e^2R_e^2_e + m_h\omega_h^2R_h^2_h}{8} + u_C(|\phi_e - \phi_h|), \]

where \( \omega_{e(h)} = \frac{|e|B}{|m_{e(h)}c|} \) are the cyclotron frequencies of the particles, \( B \) is the normal magnetic field, and \( u_c \) is the Coulomb potential averaged over the coordinate \( \rho \) involving the radial wave functions. By introducing new variables, we can rewrite Eq. (1) as \( \hat{h} = \hat{h}_0(\phi_0) + \hat{h}_1(\Delta \phi) \), where \( \Delta \phi = \phi_e - \phi_h \), \( \phi_0 = (a\phi_e + b\phi_h)/(a + b) \), and \( a = m_eR_e^2 \) and \( b = m_hR_h^2 \). Then, the eigenfunctions and eigenvalues can be found in the form of \( \psi(\phi_e, \phi_h) = \psi_0(\phi_0)\psi_1(\Delta \phi) \) and \( E = E_0 + E_1 \), respectively. Here, the \( \hat{h}_0 \) operator is given by \( \hat{h}_0(\phi_0) = \varepsilon_0 \left[ -\frac{i}{\hbar} \frac{\partial}{\partial \phi_0} + \frac{\Phi_{\Delta R}}{\hbar}\phi_0 \right]^2 \), where \( M = (m_eR_e^2 + m_hR_h^2)/R_0 \), \( R_0 = (R_e + R_h)/2 \), \( \varepsilon_0 = \hbar^2/(2R_0^2M) \), \( \Phi_0 = hc/e \), and \( \Phi_{\Delta R} = \pi(R_e^2 - R_h^2)B = 2\pi\Delta RR_0B \) is the magnetic flux penetrating the area between the electron and hole trajectories (insert of Fig. 1); and \( \Delta R = R_e - R_h \). The eigenvalues of \( \hat{h}_0 \) are \( E_0(l) = \varepsilon_0[l + \Phi_{\Delta R}/\Phi_0]^2 \), where \( l \) is an integer which represents the total angular momentum of the exciton.

The relative motion in the exciton is described by the operator \( \hat{h}_1(\Delta \phi) \), which involves the Coulomb potential. The limit of strong Coulomb interaction implies the condition \( R_0 \gg a^*_0 \), where \( a^*_0 \) is the exciton Bohr radius. In this limit, the wave function \( \psi_1(\Delta \phi) \) is strongly localized near the point \( \Delta \phi = 0 \) and the ground-state energy for \( l = 0 \) can be written as \( E_1(n = 0) = E_b - 2V\cos[2\pi\Phi_{eff}/\Phi_0] \), where \( V \) is the amplitude of tunneling from \( \Delta \phi = 0 \) to \( \Delta \phi = \pi \), \( \Phi_{eff} \approx \pi R_0^2B \), and \( E_b \) is the energy of a state localized near the angle \( \Delta \phi = 0 \); \( n \) is the index of a quantum state [2]. In the strong-interaction limit, the tunneling amplitude \( V \) becomes exponentially small, and the magnetic field dispersion of the exciton energy comes mostly from the motion of a dipole. The lowest energy branches are \( E(l, n = 0) = E_1(0) + E_0(l) \sim E_b + E_0(l) \).

In the opposite case of \( R_0 \ll a^*_0 \), we can neglect the Coulomb interaction and solve Eq. (1) using the original variables. The energy spectrum reads

\[ E(l_e, l_h) = \frac{\hbar^2}{2m_eR_e^2} \left[ l_e + \frac{\Phi_e}{\Phi_0} \right]^2 + \frac{\hbar^2}{2m_hR_h^2} \left[ l_h - \frac{\Phi_h}{\Phi_0} \right]^2, \]

where \( \Phi_{e(h)} = \pi R_e^2(h)B \), and \( l_e(h) \) are electron (hole) angular momenta.

In Figs. 1 and 2 we show the energy spectra and the PL intensity of an exciton in a QR (notice that \( E_{exc} = E_{gap} + E \), where \( E_{gap} \) is the semiconductor...
gap energy). For the strong-Coulomb-interaction limit (Fig. 1), we show the lowest states with \( n = 0 \) and \( l = 0, \pm 1, \pm 2, \ldots \), where \( l \) is the total angular momentum of the exciton. We see that the ground-state momentum changes with increasing magnetic field from \( l = 0 \) to \( l = -1, -2, \ldots \), according to the equation for \( E_0(l) \). This arises as the electron and hole acquire different magnetic phases when they move along different closed trajectories (insert of Fig. 1). In the case of weak Coulomb interaction, the character of ground-state transitions in a magnetic field is a bit more complicated. The ground state \( (l_e, l_h) = (0,0) \) changes to the states \( (-1, 0), (-1, +1), (-2, +1), \text{ etc.} \), as the field increases. The total momentum of the ground state, \( l = l_e + l_h \), changes correspondingly.

According to the selection rules for optical transitions between the conduction and valence bands, only the zero-momentum excitons can emit a photon. At low temperatures, a photo-generated exciton relaxes within a short time to its ground state. Thus, in many cases the PL spectrum demonstrates mostly the line related to the ground state of an exciton. With increasing magnetic field, the exciton in its ground state acquires nonzero momentum and can not longer radiate. The darkness of the exciton in the ground state is seen as a suppression of the calculated PL in the magnetic-field intervals with the ground-state total momentum \( l = l_e + l_h \neq 0 \). The PL intensity shown in Fig. 2 was calculated from the relation \( I_{PL}(B) \propto P(l = 0, T) \), where \( P(l = 0, T) \) is the probability to find an exciton in the states with \( l = 0 \) at finite temperature \( T \). For the spacing \( \Delta R \) we have chosen \( \sim 20-30 \, \text{Å} \). As an example, an impurity charge \( +|e| \) in the middle of a ring having \( R_0 = 85 \, \text{Å}, \hbar \Omega_e = 35 \, \text{meV}, \) and \( \hbar \Omega_h = 25 \, \text{meV}, \) induces a shift \( \Delta R \sim 20 \, \text{Å} \). A similar \( \Delta R \) is obtained for the ring parameters of the system studied in Ref. [1].

A similar AB effect for a neutral exciton can occur in (spatially-indirect) type-II quantum dot embedded in a 2D quantum well. In such a system, the electron can move in a quantum-ring potential due to the joint action of the Coulomb force and the quantum-dot potential. This type-II geometry would correspond to that of GaSb/GaAs quantum dots, for example. Similarly, confinement asymmetries in a ring arising from mass differences or effective potential profiles could also give rise to a finite polarization of the exciton ground state. The details of the system would determine the strength of the magnetic field sensitivity such as shown in the figures.

It is important to emphasize that the predicted effect depends on the magnetic flux \( \Phi_{\Delta R} \) through the area between the electron and hole trajectories (insert of Fig. 1) and does not include an exponentially-small factor due to
electron-to-hole tunneling along the ring. This is in contrast to the AB effect for excitons in a 1D ring described recently in the literature [2]. This difference would make the experimental detection of the effect discussed here much more likely. As single-dot spectroscopy [6] permits one to observe the PL energies with very high accuracy, it would be a suitable method to study the predicted magnetic-field interference effects.

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**Figure captions**

Fig.1  a) The energy spectrum and PL intensity of excitons confined in a quantum ring as a function of the normal magnetic field for the strong-Coulomb-interaction limit; here, $m_e = 0.07m_0$ and $m_h = 0.2m_0$. Insert: a sketch of the quantum ring system.

Fig.2  b) The energy spectrum and PL intensity of excitons confined in a quantum ring as a function of the normal magnetic field in the limit of weak Coulomb interaction.
$E_0$ (meV)

$I_{PL}$ (arb. units)

Fig. 1 a)
(l_e, l_h) = (0,0)

(1,0)

(-1,1)

(-2,1)

Fig. 2 b)