C-Axis Tunneling Spectra in High-$T_c$ Superconductors in the Presence of a d Charge-Density Wave

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The optimally doped and underdoped region of the $t-J$ model at large $N$ ($N$ is the number of spin components) is governed by the competition of d-wave superconductivity (SC) and a d Charge-Density Wave (d-CDW). The partial destruction of the Fermi surface by the d-CDW and the resulting density of states are discussed. Furthermore, c-axis conductances for incoherent and coherent tunneling are calculated, considering both an isotropic and an anisotropic in-plane momentum dependence of the hopping matrix element between the planes. The influence of self-energy effects on the conductances is also considered using a model where the electrons interact with a dispersionless, low-lying branch of bosons.

We show that available tunneling spectra from break-junctions are best explained by assuming that they result from incoherent tunneling with a strongly anisotropic hopping matrix element of the form suggested by band structure calculations. The conductance spectra are then characterized by one single peak which evolves continuously from the superconducting to the d-CDW state with decreasing doping. The intrinsic c-axis tunneling spectra are, on the other hand, best explained by coherent tunneling. Calculated spectra show at low temperatures two peaks due to SC and d-CDW. With increasing temperature the BCS-like peak moves to zero voltage and vanishes at $T_c$, exactly as in experiment. Our results thus can explain why break junction and intrinsic tunneling spectra are different from each other. Moreover, they support a scenario of two competing order parameters in the underdoped region of high-$T_c$ superconductors.

I. INTRODUCTION

One important topic in high-$T_c$ superconductors is the question of how many order parameters are needed for a proper description of the optimally doped and underdoped cases. One scenario assumes that only the superconducting order parameter is relevant. The decrease in the transition temperature $T_c$ is then caused by fluctuations of its phase and the pseudogap is locally just the superconducting gap. A second scenario assumes that the physics in the underdoped and optimally doped region is mainly determined by the competition of the superconducting order parameter with a second one in the particle-hole channel. Examples could be the antiferromagnetic, s- and d- charge density wave or stripe order parameters.

Many experiments such as angle-resolved photoemission [1] or tunneling in break-junctions [2–5] suggests that there is only one energy scale related with the gap and that this scale increases monotonically with decreasing doping. Recent intrinsic c-axis tunneling spectra in several cuprates [6–13] seem to modify this picture. Optimally doped or underdoped samples show at low temperatures two peaks for positive or negative voltages. The peak at larger voltages stays essentially at the same position, but becomes broader with increasing temperature. With decreasing doping it moves towards larger voltages. Though this peak behaves similar to the one seen in tunneling in break-junctions, it has recently been argued that heating effects could seriously affect this peak [14,15]. The peak at smaller voltages moves towards zero voltage with increasing temperature, loses hereby intensity and vanishes at $T_c$. Heating effects should be unimportant for the behavior of this peak. Intrinsic tunnel spectra of this kind have been found both for double layer and as well as single layer materials [12]. On the other hand, strongly overdoped samples show only one sharp peak with properties as expected from BCS theory [6].

It is tempting to associate the two peaks observed in the optimally doped and the underdoped region with the SC and the pseudogap, as has been done in some of the above references. In the following we will investigate whether the widely accepted $t-J$ model supports such a picture. To this end we will present calculations for the conductance based on a $t$-$J$ model where the two spin components have been generalized to $N$ components and the leading diagrams at large $N$ are taken into account. As discussed in detail in Ref. [16] the phase diagram in this limit has a quantum critical point (QCP) separating at $T=0$ the normal phase at large dopings from a d-CDW state at lower dopings if superconductivity is omitted.
Allowing also for superconductivity the QCP separates a pure superconducting state from a ground state containing both superconductivity and a d-CDW. This model thus represents an example for the above second scenario.

II. DENSITY OF STATES AND FERMISURFACE IN THE PRESENCE OF SC AND D-CDW

The d-CDW order parameter, appropriate for the t-J model at large N, is given by \( \Phi(\mathbf{k}) = -i/2N_c \sum_{\mathbf{q}} J(\mathbf{k} - \mathbf{q})(\bar{\sigma}^\dagger_{\mathbf{q}}\sigma_{\mathbf{q+Q}^\sigma}) \). \( J \) is the Heisenberg coupling, \( \bar{\sigma}^\dagger \) are creation and annihilation operators for electrons under the constraint that double occupancies of lattice sites are excluded, \( N_c \) is the number of primitive cells, \( \langle ... \rangle \) denotes an expectation value, and \( \mathbf{Q} \) is the wave vector of the d-CDW. Keeping only the instanton term in the effective interaction, the superconducting order parameter is \( \Delta(\mathbf{k}) = 1/2N_c \sum_{\mathbf{q}} (J(\mathbf{k} - \mathbf{q}) - V_C(\mathbf{k} - \mathbf{q}))(\bar{\sigma}^\dagger_{\mathbf{q^+}}\sigma_{\mathbf{q}}). \) As shown in Ref. [16] it is in general necessary to include the Coulomb potential \( V_C \) in order to stabilize the d-CDW with respect to phase separation. In the presence of the two order parameters the operators \( \bar{\sigma}^\dagger_{\mathbf{k}^\dagger,\mathbf{k}}\bar{\sigma}_{\mathbf{k}^\dagger,\mathbf{k}+\mathbf{Q}^\sigma,\mathbf{k}^\dagger,\mathbf{k}+\mathbf{Q}^\sigma} \) are coupled leading to the following Green’s function matrix [16]

\[
G_0^{-1}(z, \mathbf{k}) = \begin{pmatrix}
  z - \epsilon(\mathbf{k}) & -\Delta(\mathbf{k}) & -i\Phi(\mathbf{k}) & 0 \\
  -\Delta(\mathbf{k}) & z + \epsilon(\mathbf{k}) & 0 & i\Phi(\mathbf{k}) \\
  i\Phi(\mathbf{k}) & 0 & z - \epsilon(\mathbf{k}) & -\Delta(\mathbf{k}) \\
  0 & i\Phi(\mathbf{k}) & -\Delta(\mathbf{k}) & z + \epsilon(\mathbf{k})
\end{pmatrix}
\]  

(1)

\( \epsilon(\mathbf{k}) \) is the one-particle energy, \( \Delta(\mathbf{k}) = -\delta t + \alpha J(\cos(k_x) + \cos(k_y)) - 2t'\cos(k_x)\cos(k_y) - \mu \), with \( \alpha = 1/N_c \sum \cos(q_x) f(\epsilon(q)) \). \( f \) is the Fermi function, \( \delta \) the doping away from half-filling, \( \mu \) a renormalized chemical potential, \( t \) and \( t' \) are nearest and second-nearest neighbor hopping amplitudes, \( z \) a complex frequency, and \( \bar{k} = k - Q \).

Expressing the expectation values in the order parameters by \( G_0 \) and using Eq.(1) one obtains two coupled equations for the order parameters. In the interesting doping region the order parameters have d-wave symmetry, \( \Phi(\mathbf{k}) = \Phi(\gamma(\mathbf{k}), \Delta(\mathbf{k}) = -\Delta(\mathbf{k}), \) with \( \gamma(\mathbf{k}) = (\cos(k_x) - \cos(k_y))/2 \). The equations for \( \Delta \) and \( \Phi \) read then

\[
1 = \frac{2J - V_C}{N_c} \sum_{\mathbf{k}} \sum_{\alpha=1}^4 \frac{f(E_{\alpha}(\mathbf{k}))\gamma^2(\mathbf{k})}{\Pi_{\beta\neq\alpha}(E_{\alpha}(\mathbf{k}) - E_{\beta}(\mathbf{k}))} \left( \gamma^2(\bar{\mathbf{k}})(\Delta^2 + \Phi^2) - (E_{\alpha}(\mathbf{k}) - \epsilon(\bar{\mathbf{k}}))(E_{\alpha}(\mathbf{k}) + \epsilon(\bar{\mathbf{k}})) \right).
\]  

(2)

\( V_C \) is the expansion coefficient of the Coulomb potential in the d-wave channel with the basis function \( \gamma \), and \( E_{\alpha}(\mathbf{k}) \) are the four poles of \( G_0(z, \mathbf{k}) \) in the \( z \)-plane. At zero temperature \( \Phi \) decreases monotonically with increasing \( \delta \), whereas \( \Delta \) first increases, passes then through a maximum at \( \delta_0 \) and finally decreases again, as shown in Fig.1 of Ref. [17]. Fig.1 shows the temperature dependence of \( \Phi \) and \( \Delta \), calculated with \( t'/t = -0.35 \) and \( J/t = 0.3 \). The energy unit is \( t \). A repulsive nearest-neighbor Coulomb interaction was also included with \( V_C/t = 0.06 \). According to Fig.1 the temperature dependence of the order parameters is mean-field like sufficiently away from \( \delta_0 \). Near \( \delta_0 \) the two order parameters strongly interact with each other. For instance, for the slightly underdoped case of \( \delta_0 = 0.114 \), the increase of \( \Delta \) at low temperatures is accompanied by a decrease of \( \Phi \) so that the “total gap” \( \sqrt{\Delta^2 + \Phi^2} \) is rather constant at low temperatures.

Fig.2 contains quasi-particle densities for \( \delta = 0.114 \) and \( T = 0 \). The thin dotted line denotes the density for vanishing order parameters. It shows a logarithmic divergence due to the van Hove singularity. The latter lies for the chosen parameters just below the Fermi energy which corresponds to zero energy. The long-dashed curve describes the case where the correct finite value for \( \Phi \) has been used, but \( \Delta \) has been put to zero. This density shows a strongly asymmetric gap structure with a strong peak on the low and a weaker, splitted peak at the high-energy side. A closer analysis shows that the lower peak of this dublett comes from \( \mathbf{k} \)-points near the antinodal points \( X \) and \( Y \). States near these points are...
FIG. 2. Density of states for $\Phi = \Delta = 0$ (thin dotted line), $\Phi \neq 0, \Delta = 0$ (long-dashed line), $\Phi = 0, \Delta \neq 0$ (dash-dotted line), $\Phi \neq 0, \Delta \neq 0$ (solid line) for $T = 0$ and $\delta = 0.114$.

coupled by the d-CDW and their energies are shifted by the formation of the d-wave gap. This explains why this peak moves upwards (see the dashed and solid lines in Fig. 2) in the presence of an additional BCS gap. In contrast to that, the upper peak of the doublet originates from $k$-states on the boundaries of the reduced Brillouin zone near the points $(\pi/2 - \delta, \pi/2 - \delta)$ and equivalent points with $\delta << \pi/2$. Their energies are mainly determined by the one-particle energies near this point relative to the Fermi energy and thus less sensitive to the formation of the gap. This explains why the position of this higher peak of the doublet is unchanged by the BCS gap, see the dashed and solid lines in Fig. 2. The density is everywhere nonzero in the d-CDW state, in particular, at the Fermi energy. The asymmetry of the density with respect to zero energy is caused by the asymmetric bare density due to the proximity of the van Hove singularity. The finite density of states at the Fermi energy allows to lower further the ground state energy by introducing a superconducting gap. The density of states becomes then strictly zero at the Fermi level and the additional d-wave gap is rather symmetric with respect to the Fermi energy. The resulting density of states for this case $\Phi \neq 0, \Delta \neq 0$ is given by the solid line in Fig.2. Comparing this line with the dot-dashed line, which corresponds to a pure SC state $\Phi = 0, \Delta \neq 0$, one recognizes that the inner part of the gap structure looks like a reduced SC gap with weakly developed edges, at least on the low-energy side. Fig.3 shows the density of states for $\delta = 0.114$ and three different temperatures. These temperatures are low enough so that the main gap edges do not change much because $\sqrt{\Phi^2 + \Delta^2}$ is nearly constant. However, the opening of the SC gap in the inner part of the gap structure can clearly be seen as a function of temperature.

FIG. 3. Density of states for $T = 0$. (solid line), $T = 0.007$ (dashed line), $T = 0.011$ (dotted line) and $\delta = 0.114$.

FIG. 4. Density of states for three different dopings at $T = 0$.

Figs.2 and 3 illustrate the fact that the total gap is not just one single d-wave gap with an amplitude given by the square root of the sum of the square of the two gaps. Instead, the SC and CDW gaps interact with each other, however in such a way that their individual structures can still be seen in the density of states. This is also apparent in the density plots for three different dopings in Fig.4. The upper and lower panels illustrate the difference between the density of states for a SC and a d
CDW gap, respectively. Some features of the individual gaps are still present in the middle panel of Fig.4 which describes the case of coexisting SC and d CDW gaps. We find in contrast to Ref. [18] that our self-consistently calculated order parameters yield for all considered dopings and hopping parameters densities where the SC gap lies inside the d-CDW gap.

One important feature of the coexisting SC and d-CDW state is that the two gaps have different locations in k-space: The CDW gap mainly resides near the X and Y points, and the SC gap near the diagonal. This becomes clear by looking at the Fermi lines as a function of Φ in the absence of superconductivity. Since a finite Φ implies a doubling of the elementary cell we have plotted Fermi lines in Fig.5 in the reduced zone scheme, e.g., the new Brillouin zone (BZ) is bounded by a straight line between the points $X = (\pi, 0)$ and $Y = (0, \pi)$. The black line corresponds to $\Phi = 0$ and describes the usual normal state Fermi line in the reduced Brillouin zone for the parameters $t'/t = 0.35$ and $J/t = 0.3$. For $\Phi = 0.02$ the Fermi line consists of a long arc around the diagonal ending at the new boundary of the BZ and two pieces near the X and Y points. This means that the region around the hot spots becomes first gapped. Increasing Φ only the arc around the diagonal survives and becomes shorter. The finite density of states at the Fermi energy in the CDW state is due to this arc. Allowing also for SC the arc becomes gapped and the Fermi line shrinks to one point on the diagonal. The coexistence of SC and CDW thus becomes possible because the CDW state can lower the free energy by introducing a SC gap along the arc.

### III. CONDUCTANCE

Using lowest-order perturbation theory in the interlayer hopping the quasi-particle c-axis current $J$ between superconducting layers (SIS junction) is given by [19]

$$J(V) = \frac{e}{\pi N_c} \sum_{k,q,k',q'} \langle T_{kq} T_{k'q'}^{*} \rangle \int_{-\infty}^{\infty} d\omega (f(\omega) - f(\omega + eV)) A_{11}(k',\omega + eV) A_{11}(q',\omega). \quad (4)$$

In Eq.(4) $T_{kq}$ denotes the hopping matrix element between states with momenta $k$ and $q$ in adjacent layers and $\langle ... \rangle$ an average over quenched disorder. $f$ is the Fermi function, $V$ the applied voltage, and, using the Nambu representation, $A_{11}$ the spectral function of the element 11 of the 2x2 Green’s function matrix. The momenta in the above formula refer to the original (large) BZ. The differential conductance $G(V)$, which is the main quantity of interest in the following, is defined as the first derivative of $J$ with respect to $V$.

We make the following Ansatz for the averaged squared tunneling matrix element

$$\langle T_{kq} T_{k'q'}^{*} \rangle = t^2 \gamma(k) \gamma(q) \gamma(k') \gamma(q') N_c \delta_{k'-q-k-q'} (u \delta_{k,q} + g(k-k')). \quad (5)$$

The form factors $\gamma(k)$ determine which electrons in the BZ are mainly involved in the tunneling process. Results from band structure calculations [20] suggest the Ansatz [21]

$$\gamma(k) = 1 - u + u/2 |\cos k_x - \cos k_y|, \quad (6)$$

with $0 \leq u \leq 1$. The parameter $u$ interpolates between the isotropic case $u = 0$ and the strongly anisotropic case $u = 1$. The latter is typical for tunneling within a double layer of $CuO_2$ planes, whereas tunneling between layers in different elementary cells may include also an isotropic component. The first term in the brackets in Eq.(5) accounts for coherent scattering with strength $u$. The second term in the brackets describes incoherent scattering where $g$ is a smooth function of the momentum. One may distinguish two cases for the momentum dependence of $g$. In the case of strong, localized scatterer $g$ may assumed to be completely independent on momentum. If long-ranged random fields are present $g$ is large (small) mainly at small (large) momentum transfers. The momentum dependence of $g$ thus can be modelled by

$$g(k) = g \cdot \exp(-|k|^2/\Lambda^2), \quad (7)$$

where the momentum $\Lambda$ interpolates from isotropic to forward scattering, described by large and small values for $\Lambda$, respectively.

Since the case of coherent scattering can be obtained from that of incoherent scattering by replacing $g$ by $4\pi/\Lambda^2$ and taking the limit $\Lambda \rightarrow 0$ we will first consider
incoherent scattering. Using Eq.

\[ J(V) = \frac{e^2}{N^2} \int_{-\infty}^{\infty} d\omega (f(\omega) - f(\omega + eV)) \]

\[ \rho(\omega) = \frac{1}{N_e} \sum_{\mathbf{k}} \gamma^2(\mathbf{k}) \int_{-\infty}^{\infty} d\omega (f(\omega) - f(\omega + eV)) \]

with the weighted density

\[ \rho(\omega) = \frac{1}{N_e} \sum_{\mathbf{k}} \gamma^2(\mathbf{k}) (A_{11}c(\mathbf{k},\omega) + A_{33}(\mathbf{k},\omega)) \]

For a SIN junction one usually assumes that \( g \) is independent of momentum. Its current is then obtained from Eq.

\[ G_{SIN}(V) = \frac{e^2 \rho_M}{N^2} \rho(eV) \]

Since we will mainly consider SIS junctions in the following conductance will always refer to SIS junctions unless it is stated otherwise.
broad compared to that in the intrinsic c-axis tunneling spectra of Ref. [6], the lower peak in the underdoped cases is caused by the relaxation of electronic states around the CDW gap to states near the arcs or the nearby BCS gap and thus does not approach zero at $T_c$ as in intrinsic tunneling spectra. Similar conclusions are reached, following Eq.(12), by comparing the experimental conductance curves of SIN junctions [3,4] with the densities of Fig.4. The monotonic increase of the distance between the two main peaks with decreasing doping occurs in both cases but the experiment does not show the asymmetry of the theoretical conductance curve in the underdoped case as well as the additional structures obtained in the region of coexisting SC and d CDW.

Things change substantially if the form factor $\gamma(k)$ with a non-zero value for $u$ is taken into account. Assuming still a momentum-independent function $g$ the tunnel current is now to be calculated from the weighted density $\tilde{\rho}$ as given by Eqs.(10) and (11). Fig.7 shows $\tilde{\rho}(\omega)$ for the extreme anisotropic case $u = 1$ for three different dopings $\delta$.

FIG. 7. Weighted density $\tilde{\rho}$ as a function of frequency for $T=0$, $J=0.3$, $\eta = 0.004$, $t'=-0.35$, and three different dopings $\delta$.

and the low-energy structures for $\delta = 0.114$ become invisible. The dip on the high-energy sides of the main peaks in the experimental spectra is, however, missing in $\tilde{\rho}$ indicating the presence of self-energy effects beyond a constant damping.

Performing the frequency integral in Eq.(10) with the weighted densities $\tilde{\rho}$ one obtains the curves of Fig.8. The main effect of the inclusion of the anisotropic form factors $\gamma$ with $u = 1$ is the suppression of the small quasi-
particle excitations near the nodal regions. This means in the overdoped case $\delta = 0.178$ that the slowly decaying tail of the main peak towards lower voltages seen in the upper panel of Fig.6 is substantially suppressed making the peak much sharper. The dip above $2\Delta$ is also more pronounced than in Fig.6 and the conductance assumes (small) negative values over a wide region towards higher voltages. The reason for this negative resistance becomes clear from a comparison of Figs.4 and 7. Most of the rather constant background density in Fig.4 has been removed by the anisotropic form factor. However, just this constant background density is responsible for a positive and structureless conductance outside of the gap region. Similar considerations apply to the underdoped case $\delta = 0.077$. The anisotropic form factor sharpens up somewhat the high voltage peak and suppresses the lower peak at around $\Phi$ because the states near the arcs can no longer contribute much. At the same time the conductance shows a well-pronounced dip above $2\Phi$ with large negative values due to the eliminated background density of states. Similar statements hold for the slightly underdoped case. Here the lower peak at around $\Delta + \Phi$, which was in Fig. 6 still the strongest one, is suppressed but still visible.

The above calculations show that an incoherent tunneling model with a momentum independent function $g$ is not able to produce a peak in the coexistence region which moves towards zero voltage if $T$ approaches $T_c$.

$YBa_2Cu_3O_{x+y}$ [22]. Using the isotropic form factor $u = 0$ and $\Lambda = \pi/8$ Fig.9 shows the conductance for three different dopings at $T=0$. The spectra are dominated by a peak at approximately the frequencies $2\sqrt{\Phi^2 + \Delta^2}$. This peak reflects the doping dependence of the “total” gap which increases monotonically with decreasing doping. In the upper panel the gap describes a superconducting gap, in the lower panel a d-CDW gap and in the middle panel a combination of both. The doping dependence of the main peak in Fig.9 agrees well with experimental SIS spectra, see, for instance, Figs. 1 and 2 in [5], though the dips above the main line are more pronounced than in the experiment. Also negative conductances are only very rarely observed experimentally. In the slightly underdoped regime (middle panel in Fig.9) the two order parameters coexist. The conductance shows in this case besides of the dominating high-frequency peaks a peak near the superconducting part of the gap. This peak moves towards smaller frequencies with increasing temperature and vanishes at $T_c$. The exact energy position of this peak is somewhat below the superconducting part of the gap, $2\Delta$. This can easily be understood from Figs.2 and 3: Due to the interaction between the two gaps part of the BCS shoulder has been removed by the d-CDW so that the superconducting part of the gap appears smaller than the canonical value for $2\Delta$. This reduced gap exhibits the expected temperature dependence as can be seen from Fig.3. The upper and lower panels in Fig.9 also show weak structures at low voltages. A closer inspection, however, reveals that these structures are caused by the underlying band structure and are unrelated to the BCS gap. The spectra in Fig.9 exhibit well-pronounced

![FIG. 9. Incoherent c-axis conductance calculated for isotropic $u = 0$ form factors, a momentum cutoff $\Lambda = \pi/8$, $T = 0$, $J = 0.3$, $t' = -0.35$ and three different dopings $\delta$.](image)

![FIG. 10. Incoherent c-axis conductance for five temperatures using a Gaussian distributed hopping matrix element with width $\Lambda = \pi/8$. Increasing temperatures correspond to curves with energetically decreasing low-energy peaks.](image)
dips at energies somewhat above the main peaks. These
dips, which are also seen in tunneling spectra from break
junctions, have been associated with self-energy effects
due to the coupling to some boson [2,5]. We would like
to stress that no self-energy effects have been taken into
account in calculating Fig.9. Simple model calculations
indicate that one finds easily a dip in SIS spectra if the
size of the region above 2Δ where spectral weight piles
up because of the formation of the gap is comparable or
smaller than the gap.

Fig.10 shows the temperature dependence of the inco-
herent conductance for δ = 0.114 and Λ = π/8. The
dominating higher peak is practically temperature inde-
dendent for the temperatures shown in the figure. On
the other hand, Fig.1 indicates that both order param-
eters vary in the considered temperature interval. One
concludes from this that the higher peak reflects the to-
tal gap which is rather independent of temperature. In
contrast to that, the lower peak depends strongly on tem-
perature. It moves towards zero frequency with increasing
temperature, loose spectral weight and vanishes with
vanishing Δ. Intrinsically a-axis tunneling spectra in various
cuprates show essentially the same features as in Fig. 10.
In particular, the observed low-frequency peak, which
seems not to be affected much by heating effects, also
moves towards zero frequency with increasing tempera-
ture and vanishes near Tc. We would like to point out
that this BCS-like peak can be seen in incoherent scat-
tering only for a rather isotropic form factor γ. Other-
wise, the tunneling of electrons near the nodal direction,
where the SC gap is located, is too much suppressed. An-
other prerequisite is that the averaged tunneling matrix
element must be strongly momentum-dependent causing
strong forward scattering.

Besides of incoherent tunneling the quasiparticle cur-
rent always contains a contribution Jcoh due to coherent
tunneling, originating from the first term in the paran-
theses in Eq.(5). Jcoh is given by Eq.(4) with g replaced
by aδk,q. The explicit expression for Jcoh thus becomes
\[ J_{coh}(V) = \frac{e\alpha \tau^2}{\pi} \sum_{\delta} \int_{-\infty}^{\infty} d\omega (f(\omega) - f(\omega + eV)) \]
\[ \sum_{\omega' = 0,1} A_{1+2\delta,1+2\delta'} (\vec{k}, \omega + eV) A_{1+2\delta',1+2\delta(\vec{k}, \omega)}. \] (13)

If the superconducting order parameter is zero, Eq.(1)
reduces to a 2x2 matrix. Calculating explicitly the spectral
functions from this matrix and performing the frequency
integration in the above integral one finds then that the
sum over i, i' yields zero without any further approxima-
tion. One thus obtains the important result that coherent
tunneling is zero in a pure d-CDW.

Fig.11 shows the coherent conductance for u = 1,
T = 0, and three different dopings. The spectral func-
tions were obtained from the Green’s functions using the
frequency ω + iη with η = 0.01. In the overdoped and
slightly underdoped case the curves are similar to those
in Fig.9. In particular, corresponding curves have neg-
ative conductances above the main peak caused by the
restriction to small momentum transfers in the tunneling
process. For δ = 0.114 both curves show besides of the
main peak associated with the total gap a second peak
at smaller energies with BCS properties. However, one
should note that Fig.9 was calculated with u = 0 whereas
Fig.11 with u = 1. It is somewhat surprising that coher-
ent tunneling shows still the BCS peak though most elec-
trons near the nodal direction are prevented from tunnel-
ing due to the employed strongly anisotropic form factor.
The dominance of small energy features in coherent tun-
neling also is present in the underdoped case δ = 0.077
where the tiny BCS gap causes a sharp structure at very
low energies. One important feature in Fig.11 is related
to the absolute values for the conductance depicted along
the y-axis. The coherent conductance drops dramatically
with decreasing doping. Going to smaller doping values
one finds that Jcoh becomes zero if the superconducting
order parameter vanishes in agreement with the above
analytic result. Coherent tunneling is non-zero in case of
a pure superconductor as shown by the upper panel in
Fig. 11. However, it is zero for a pure d-CDW state and
the d-CDW gap can only be probed in the presence of
superconductivity. If the identification of the pseudogap
phase with a d-CDW is correct coherent tunneling should
vanish in the pseudogap phase.

Fig.12 illustrates the dependence of the coherent con-
ductance on temperature for δ = 0.114. With increasing
temperature the position of the low-energy peak and its intensity decrease and approach zero at around
$T = 0.0081$ where the superconducting order parameter vanishes. The position of the high-energy peak as well

as the dip are rather independent of temperature but the intensity of the whole high-energy part drops dramatically with temperature and vanishes at $T = 0.0081$. This again demonstrates that contributions from the d-CDW can only be seen in the conductance if the superconducting order parameter is finite, i.e., in the coexistence regime.

**IV. SELF-ENERGY EFFECTS**

According to angle-resolved photoemission experiments the generic spectral function in the superconducting state consists of a well pronounced peak followed by a dip and a hump towards larger energies [1]. In accordance with that the electron dispersion shows a kink very smoothly through the coexistence region of SC and/or d-CDW state. The panels in the figure illustrate the very smooth transition from a SC to a d-CDW gap with decreasing doping, passing also very smoothly through the coexistence region of SC and d-CDW. We used in this and in all the following figures the value 1 for the dimensionless coupling constant $\lambda$. This value corresponds to a change of the slope of the electron dispersion at the kink by a factor two in rough agreement with the photoemission data.

![FIG. 12. Coherent c-axis conductance for anisotropic \( u=1 \) form factors, \( J=0.3, t'=-0.35, \eta = 0.01, \) and various temperatures labeled according to decreasing maxima.](image)

The position of the high-energy peak as well and it is $\Sigma_{11} = \Sigma_{22} = \Sigma_{33} = \Sigma_{44}$, with

$$\Sigma_{11}(z) = -\frac{g^2}{N_c} \sum_k \left( \sum_{\alpha=1}^{4} \frac{2\omega_0 s(E_\alpha) f(E_\alpha)}{(z - E_\alpha)^2 - \omega_0^2} \prod_{\beta \neq \alpha} (E_\alpha - E_\beta) + \frac{b(-\omega_0)s(z - \omega_0) - b(\omega_0)s(z + \omega_0)}{\prod_{\alpha} (z - \omega_0 - E_\alpha)} \right).$$

(15)

$g^2$ is related to $\lambda$ by $g^2 = \lambda \omega_0/(2N(0))$, where $N(0)$ is the density of states for one spin direction and $\omega_0$ the frequency of the boson. $E_\alpha$ denote the four poles of $G(0)$, $\delta$ the Bose function, and $N_c$ is two times the number of allowed momenta $k$. $s(z)$ is given by

$$s(z) = 2z^2 - (\epsilon^2(k) + \epsilon^2(k - Q))/2 - \delta^2(k) - \Phi^2(k).$$

(16)

![FIG. 13. Real (dotted line) and imaginary (solid line) parts of the self-energy for $T=0$, $\omega_0 = 0.065$, $\eta = 0.004$, and $\lambda = 1$.](image)
FIG. 14. SIN conductances with self-energy corrections for $T=0, \upsilon=1, \omega_0=0.065, \eta=0.004, \lambda=1,$ and three dopings.

analogous Fig.7 where self-energy effects have been omitted. The BCS-structure seen in Fig.7 for $\delta=0.114$ has practically vanished in Fig.14. The peaks in Fig.14 are slightly broader, but the main effect of the self-energy is to move spectral weight from the main peaks to the sidebands. In the pure superconducting state at $\delta=0.178$ the sidebands consists of a clear dip and hump whereas in the two other cases the dip-hump feature is less pronounced. Both dip and hump move monotonically towards larger voltages with decreasing doping. The position of the dip in a pure superconductor is approximately half of the gap plus the boson energy. This rule also holds in the d-CDW and the mixed states. The exact differences between the main peaks and the dip, however, fluctuate between 0.065 and 0.087 in Fig.14. Though the dip in the SIN spectra is solely caused by the interaction with the bosons it may thus be difficult to determine precisely the boson energy from it.

The solid and dashed lines in Fig.15 are conductance curves for incoherent tunneling with and without self-energy effects, respectively. The bosons do not contribute to the non-diagonal self-energy because of the assumed momentum-independent coupling to the electrons. As a result, the bosons diminish both the SC and the d-CDW gaps via their diagonal self-energies. Consequently, the main peak moves towards lower frequencies but, considered as a function of doping, this peak increases monotonically with decreasing doping as in the case without self-energy. Fig.15 also illustrates that the distance between the dip and the main peak is similar in the curves with and without self-energy effects and thus is rather unrelated with the boson energy. For instance, at $\delta=0.178$ the distance between the dip and the main peak is 0.020 and 0.018 in the case with and without self-energy, respectively, and thus much smaller than the boson energy of 0.065.

If the experimental SIS spectra correspond to incoherent tunneling the boson energy cannot be obtained from the distance between the main peak and dip. Since the incoherent SIS spectrum is the folding of the SIN spectrum in energy the incoherent SIS spectrum could have, in principle, a dip when the lower main peak is multiplied by the dip at positive voltages and this dip position would be equal to the full gap plus the boson energy. The upper panel of Fig.15, however, shows that the folding in energy does not lead to a dip just below the maximum of the sideband. The main effect of the self-energy in Fig.15 is to shift spectral weight from the main peak to the sideband consisting of a broad hump which monotonically moves towards larger voltages with decreasing doping. This hump is due to the folding of the (occupied) lower main peak with the (unoccupied) upper sidebands in Fig.14. The solid and dashed lines in Fig.16 represent coherent conductance curves with and without self-energy effects, respectively. We have omitted curves for the strongly underdoped case $\delta=0.077$ because they are smaller by two order of magnitudes due to the smallness of the supercondcuting order parameter in this case. Self-energy effects shift the main peaks to smaller energies, diminish somewhat the regions of negative conductance, and create weak sidebands. Figs.7 and 14-16 suggest that self-energy effects and thus the nature of the boson spectrum appear more clear-cut in the SIN than in the SIS spectra. For instance, the dip in the SIN spectrum is solely caused by self-energy effects whereas that in the SIS spectrum is present even in the absence of any self-energy. The curves for the purely su-
perconducting case in Figs.14 and 15 are similar to those published in Refs. [24,27].

![Graphical representation of coherent conductances with and without self-energy corrections.](image)

**FIG. 16.** Coherent conductances with (solid line) and without (dashed line) self-energy corrections for T=0, u=1, ω₀ = 0.065, η = 0.004, λ = 1, and two dopings.

### V. CONCLUSIONS

The $t-J$ model exhibits in the employed large-N limit a d-CDW phase at lower dopings besides of the superconducting phase which is a natural candidate for the pseudogap phase observed in the cuprates. The density of states in the pure d-CDW state is strongly reduced near the Fermi level but still everywhere finite. This means that only part of the Fermi lines of the normal state are destroyed by the d-CDW and that the remaining Fermi lines form arcs around the nodal direction ending at the boundaries of the reduced Brillouin zone. With decreasing doping the length of the arcs become shorter. The ground state energy of the d-CDW can be lowered by introducing a d-wave superconducting gap near the arcs which explains the occurrence of a coexistence region of SC and d-CDW. The appearance of a low-energy peak in the calculated coherent tunneling spectrum, which moves to 0 if $T_c$ is approached from below, is unique for intrinsic tunneling spectra. From this we conclude that tunneling in stacked, intrinsic junctions is dominated by coherent tunneling and that the appearance of the low-energy peak related to superconductivity supports models with two competing order parameters in the underdoped region. Including self-energy effects due to the coupling of electrons to a dispersionless boson branch as suggested by ARPES removes part of the regions of negative resistances and also creates sidebands which, at least in the case of the intrinsic tunneling spectra, resemble those which have been measured.

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