Reconstruction of the Extended Gauge Structure from $Z'$ Observables at Future Colliders

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Abstract

The discovery of a new neutral gauge boson $Z'$ with a mass in the TeV region would allow for determination of gauge couplings of the $Z'$ to ordinary quarks and leptons in a model independent way. We show that these couplings in turn would allow us to determine the nature of the extended gauge structure. As a prime example we study the $E_6$ group. In this case two discrete constraints on experimentally determined couplings have to be satisfied. If so, the couplings would then uniquely determine the two parameters, $\tan \beta$ and $\delta$, which fully specify the nature of the $Z'$ within $E_6$. If the $Z'$ is part of the $E_6$ gauge structure, then for $M_{Z'} = 1$ TeV $\tan \beta$ and $\delta$ could be determined to around 10% at the future colliders. The NLC provides a unique determination of the two constraints as well as of $\tan \beta$ and $\delta$, though with slightly larger error bars than at the LHC. On the other hand, since the LHC primarily determines three out of four normalized couplings, it provides weaker constraints for the underlying gauge structure.
I. INTRODUCTION

New neutral ($Z'$) and charged ($W'$) gauge bosons are a generic prediction of theories beyond the standard model. If their mass turns out to be in the TeV region future hadron colliders (the Large Hadron Collider (LHC) at CERN) as well as $e^+e^-$ colliders (the New Large Collider (NLC)) would provide an ideal environment for their discovery as well as for further testing of their properties.

In recent years it has been demonstrated that for $M_{Z',W'} \sim 1$ TeV a diagnostic study of heavy gauge bosons is possible at the LHC (integrated luminosity $L_{int} = 100$ fb$^{-1}$, center of mass energy $\sqrt{s} = 14$ TeV) \[1,2\] as well as at the NLC ($L_{int} = 20$ fb$^{-1}$, $\sqrt{s} = 500$ GeV) \[4\]. Both machines turn out to have complementary diagnostic power for $Z'$ physics \[5\].

For $M_{Z'} \leq 1 - 2$ TeV the future colliders allow for model independent determination of gauge couplings to quarks and leptons. The analysis assumes family universality; $[Q', T_i] = 0$, where $Q'$ is the $U(1)'$ charge and $T_i$ are the generators of $SU(2)_L$; and that $Z - Z'$ mixing is negligible. The LHC can probe the absolute magnitude of the overall strength of the new interaction $g_2$, as well as the magnitude of (primarily) three out of four normalized gauge couplings. On the other hand, the NLC with longitudinal electron beam polarization and heavy flavor tagging available is sensitive to the ratio of $g_2^2/M_{Z'}^2$ and four normalized gauge couplings. The latter can also be determined in terms of normalized couplings probed directly at the LHC; the error bars are typically smaller by a factor of $\sim 2$ than those at the NLC \[3\], but there is a few-fold ambiguity. Thus, the LHC and the NLC are complementary and together have the potential to uniquely determine the couplings with small error-bars. For $M_{Z'} \sim 1$ TeV the error bars are in the 10%-20% region.

In this note we point out that a model independent determination of gauge couplings in turn provides a tool to learn more about the nature of the extended gauge structure. In particular, one would be able to gain information about the underlying gauge group and subsequently probe the type of symmetry breaking associated with the new heavy gauge boson.

To illustrate the method we study as a prime example the Cartan subalgebra of the $E_6$ gauge group, which provides a general enough framework for grand unified symmetry with canonical embedding of electric charge and simple cancellation of anomalies. In addition, it is motivated by string theory. $E_6$ imposes two discrete constraints on the $Z'$ gauge couplings. Provided these are satisfied by the measured couplings, one can further determine the two parameters $\tan \beta$ and $\delta$. These two parameters, together with the overall strength of the new gauge couplings $g_2$, contain the information brought by the new interaction concerning the pattern of symmetry breaking, possible intermediate mass scales, and renormalization effects above the $Z'$. This window to higher scales, as in the case of the weak angle, is not sufficient to fix the full theory at the Planck scale, but it should allow for constraining possible unification schemes, excluding many possibilities. See, for instance, Refs. \[6,7\] for a discussion of the predictions for the new parameters of extended models from the heterotic string.

\[1\] Production rates have recently been recalculated in Ref. \[3\].
In Section II we spell out the formalism and parameterization of the $Z'$ interaction with the corresponding quark and lepton neutral currents. In Section III we recapitulate the results for the model independent determination of gauge couplings. In Section IV we illustrate how such couplings allow for a determination of the underlying gauge symmetry and subsequently for determination of the symmetry breaking pattern with a particular gauge structure. Conclusions are given in Section V.

II. FORMALISM

The coupling of an additional neutral gauge boson $Z'$ and the neutral standard model bosons $W_3, B$ to ordinary fermions are parameterized in the neutral current Lagrangian as [7,8]:

$$\mathcal{L}_{NC} = \bar{\psi}_k \gamma^\mu [T_3 k g W_3 + Y_k g_1 B_\mu + (Q'_k g_2 + Y_k g_{12}) Z'_\mu] \psi_k,$$

(1)

where $T_3$ and $Y$ are the standard model isospin and hypercharge charges, respectively, and $Q'$ is the charge of the extra $U(1)'$. A sum over all the quarks and leptons $\psi_k$ is implied. This Lagrangian describes the general coupling of a new gauge boson $Z'$ with charge $Q'$ which commutes with the $SU(2)_L$ generators $T_i$, i.e., $[Q', T_i] = 0$. This is the case for all of the extended gauge structures for which the generator of the $U(1)'$ associated with $Z'$ lies in its Cartan subalgebra. In the following we also assume family universality and neglect $Z - Z'$ mixing, as indicated by present bounds [9] and the expected accuracy at future colliders.

The $Z'$ couplings are specified by the (normalized) charges $Q'_k$ and the overall strength $g_2$, as well as by $g_{12}$, which is the coupling of $Z'$ to the weak hypercharge. Thus, the neutral current Lagrangian of the ordinary fermions, which includes an additional gauge boson, is specified by five charges $g_{12} Q'_k$ and the relative strength of the $Z'$ gauge coupling to the hypercharge $Y$ and the $Q'$ currents, $\delta \equiv g_{12}/g_2$. The charges $Q'_k$ are characteristic of the underlying extended gauge structure. $Q'_k$ charges for typical models based on $SO(10)$ and $E_6$ grand unified symmetries are given in Table I. The $SO(10)$ symmetry includes the general left-right symmetric models with [4]

$$g_2 = \frac{e}{c_W} \sqrt{\frac{2}{5}} \frac{\rho^2 + 1}{\rho}, \quad \delta = \frac{g_{12}}{g_2} = \sqrt{\frac{1}{6}} \frac{3 \rho^2 - 2}{\rho^2 + 1}, \quad \rho = \sqrt{\frac{\kappa^2 \cot^2 \theta_W - 1}{\kappa^2 \cot^2 \theta_W - 1}},$$

(2)

where $\kappa = \frac{g_R}{g_L}$ is the ratio of the gauge couplings $g_{L,R}$ for $SU(2)_{L,R}$, respectively. In general, $\kappa > \frac{s_W}{c_W}$ [10]. Here $c_W \equiv \cos \theta_W$ and $s_W \equiv \sin \theta_W$ where $\theta_W$ is the weak angle.

For a particular extended gauge symmetry the charges $Q'_k$ are fixed numbers which usually satisfy specific constraints. For $SO(10)$ the $Q'_k$ satisfy the four conditions:

$$Q'_{e_L} = Q'_{\nu_L} = Q'_{q_L}, \quad Q'_{\ell_L} = Q'_{d_L}, \quad Q'_{d_L} = 3 Q'_{q_L},$$

(3)

whereas for $E_6$ only the first three are satisfied in general.

The gauge couplings, $g, g_{1,2,12}$, specify the overall strength of the interactions. They are fixed at the large scale, where the full underlying gauge structure is present, e.g., in string theory the gauge couplings of all the group factors are related at the string compactification.
TABLE I. Isospin $T_3$, hypercharge $Y$, and $Q'$ charges for ordinary quarks and leptons. $Q_X$ specifies the additional $SO(10)$ charge, whereas $Q_\psi$ is the additional charge in the breaking of $E_6$ to $SO(10)$. Thus, $Q_\beta = Q_X \cos \beta + Q_\psi \sin \beta$ is the general $E_6$ charge. The charges $Q_{X,\psi,\eta}$ of the particular models correspond to $Q_\beta$ with $\beta = 0, \frac{\pi}{2}, -\tan^{-1} \sqrt{\frac{3}{5}}$, respectively.

| Fermions | $T_3$ | $\sqrt{\frac{2}{3}} Y$ | $Q'$ | $2\sqrt{10} Q_X$ | $Q_\beta$ |
|----------|-------|-----------------|------|-----------------|--------|
| $u_L$, $d_L$ | $\frac{1}{2}$ | $\frac{1}{6}$ | $Q'_{uL}$ | $-1$ | $-\frac{1}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta$ |
| $u_L^c$, $d_L^c$ | 0 | $-\frac{2}{3}$ | $Q'_{uL^c}$ | $-1$ | $-\frac{1}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta$ |
| $\nu_L$, $e_L^c$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $Q'_{\nu_L}$, $Q'_{e_L^c}$ | 3 | $\frac{3}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta$ |

scale. However, these couplings are renormalized at low energies, where their values crucially depend on the matter content, the symmetry breaking pattern, and the threshold effects. In particular, $g_{12}$ may vanish at the large scale, but it is in general non-zero at low energy. It allows for description of the left-right models (see Eq. (2)) within the $SO(10)$ symmetry. Introduction of $g_{12}$ is also necessary for a complete parameterization of low energy models when only the extended gauge structure or its Cartan subalgebra, within which $Q'$ lies, is known.

III. MODEL INDEPENDENT DETERMINATION OF Z' COUPLINGS AT THE LHC AND THE NLC

The NLC would be able to probe in a model independent way the following suitable combinations of normalized gauge couplings $\hat{g}$:

$$P_V^\ell = \frac{\hat{g}_L^\ell + \hat{g}_R^\ell}{\hat{g}_L^\ell - \hat{g}_R^\ell}, \quad P_L^\ell = \frac{\hat{g}_L^\ell}{\hat{g}_L^\ell - \hat{g}_R^\ell}, \quad P_R^{u,d} = \frac{\hat{g}_R^{u,d}}{\hat{g}_L^\ell},$$

as well as the ratio

$$\epsilon_A = (\hat{g}_L^\ell - \hat{g}_R^\ell)^2 \frac{g_2^2}{4\pi \alpha} \frac{s}{M_{Z'}^2 - s}. \quad (5)$$

2Without loss of generality the gauge coupling $g_{21}$, which parameterizes the strength of the $B$ coupling to the neutral currents specified by $Q'$, can be set to zero. Gauge couplings of $B$ and $Z'$ to the neutral currents specified by $T_3$ vanish by gauge invariance.
Here $\hat{g}_k$ are the normalized $Z'$ couplings to ordinary fermions in Eq. (4), $\hat{g}_k = Q'_k + \delta Y_k$ (see Ref. [11] for conventions and notation). The values of these couplings within the general underlying $E_6$ gauge structure are given in Table II. In Table III we define typical models ($\chi, \psi, \eta, LR$, and general $LR$ models) in terms of the parameters $\beta$ and $\delta$ and the overall $Z'$ gauge coupling, expressed in terms of the electric charge $e$ and the weak angle.

On the other hand, in addition to the mass $M_{Z'}$, the LHC will probe [1] an overall strength $g_2$ of the $Z'$ couplings as well as three normalized couplings $\gamma_{LL}$, $\tilde{U}$, and $\tilde{D}$, where:

$$\gamma_{LL} = \frac{(\hat{g}_{LL})^2}{(\hat{g}_{LL})^2 + (\hat{g}_{LR})^2}, \quad \tilde{U} = \frac{(\hat{g}_{uR})^2}{(\hat{g}_{LL})^2}, \quad \tilde{D} = \frac{(\hat{g}_{dR})^2}{(\hat{g}_{LL})^2}.$$

A fourth coupling,

$$\gamma_{L} = \frac{(\hat{g}_{L})^2}{(\hat{g}_{LL})^2 + (\hat{g}_{LR})^2}$$

(7)
could be determined, provided the $Z'$ cross section into quark pairs could be measured with sufficient precision. Recent studies indicate [12,13] that this might be feasible. It turns out [12], however, that for $M_{Z'} \geq 1$ TeV and the typical models specified in Table III the $Z'$ gauge couplings are too small to allow for determination of $\gamma_{L}$ with sufficient precision at the LHC. Thus, in the analysis the determination of $\gamma_{L}$ is not included.

The couplings (4), probed directly at the NLC, are determined with a few-fold ambiguity in terms of the couplings (6) and (7) probed directly at the LHC. Namely, couplings probed at the NLC (Eq. [4]) are related to the couplings probed at the LHC (Eq.[6]) in the following way:

$$P_{LL}^\ell = \frac{1 + 2\epsilon_\ell \gamma_{LL} ^{1/2}(1 - \gamma_{LL}^{1/2})^{1/2}}{2\gamma_{LL}^{1/2} - 1},$$

$$P_{LL}^q = \frac{1}{2\epsilon_q \gamma_{L}^{1/2}} \left[ \frac{1 + 2\epsilon_q \gamma_{L} ^{1/2}(1 - \gamma_{L}^{1/2})^{1/2}}{2\gamma_{L}^{1/2} - 1} \right]^{1/2},$$

$$P_R^u = \epsilon_u \tilde{U}^{1/2},$$

$$P_R^d = \epsilon_d \tilde{D}^{1/2},$$

(8)

where $\epsilon_\ell, \epsilon_q, \epsilon_u, \epsilon_d$ can take $\pm$ values and give rise to a sixteen-fold sign ambiguity. The inability to probe $\gamma_{L}$ directly implies that $P_{LL}^q$ is not probed either.

For typical models (within the $E_6$ gauge structure), and $M_{Z'} = 1$ TeV the values and expected statistical error bars for the three couplings [4] and [4] and the four couplings [4] [4] are given in Table IV. The error bars at the LHC update the analysis of Ref. [1]; the updated numbers correspond to the lower c.m. energy (14 TeV) and the (more optimistic) assumption that the branching ratios include $Z'$ decays into ordinary three family fermions only.

In the analysis only statistical errors for the observables are included and error correlations for the input parameters are neglected. Experimental cuts and detector acceptances are not included either. The results should thus be interpreted as a limit on how precisely the couplings can be determined for each model for the given c.m. energy and the integrated
TABLE II. $Z'$ couplings to ordinary fermions for $E_6$ models ($\beta = 0$ corresponds to $SO(10)$ models). The corresponding right-handed couplings are $\tilde{g}_R = -\tilde{g}_L^c$.

| $\tilde{g}_k$ | $Q'_k + \delta Y_k$ |
|---------------|----------------------|
| $\tilde{g}_q^q$ | $-\frac{1}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta + \frac{\delta}{3} \sqrt{\frac{3}{5}}$ |
| $\tilde{g}_L$ | $-\frac{1}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta - \frac{\delta}{3} \sqrt{\frac{3}{5}}$ |
| $\tilde{g}_L^c$ | $\frac{3}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta + \frac{\delta}{3} \sqrt{\frac{3}{5}}$ |
| $\tilde{g}_L^f$ | $\frac{3}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta - \frac{\delta}{3} \sqrt{\frac{3}{5}}$ |
| $\tilde{g}_L^c$ | $-\frac{1}{2\sqrt{10}} \cos \beta + \frac{1}{2\sqrt{6}} \sin \beta + \delta \sqrt{\frac{3}{5}}$ |

TABLE III. Parameterization of typical $Z'$ models within $E_6$. The value of $\rho$ specifies a general $LR$ model.

| Model | $\beta$ | $\delta$ | $g_2$ |
|-------|--------|--------|----------|
| $\chi$ | 0 | 0 | $\sqrt{\frac{5}{3}} e^{c_W}$ |
| $\psi$ | $\frac{5}{2}$ | 0 | $\sqrt{\frac{5}{3}} e^{c_W}$ |
| $\eta$ | $-\tan^{-1} \sqrt{\frac{5}{3}}$ | 0 | $\sqrt{\frac{5}{3}} e^{c_W}$ |
| $LR$ | 0 | 0.615 | 1.382 $\frac{5}{3} e^{c_W}$ |
| general $LR$ | 0 | $\sqrt{\frac{3\rho^2 - 2}{\rho^2 + 1}}$ | $\sqrt{\frac{3\rho^2 - 1}{\rho^2 + 1}} e^{c_W}$ |
| $\beta$ | $\beta$ | 0 | $\sqrt{\frac{5}{3}} e^{c_W}$ |
TABLE IV. Values and statistical error bars for $\gamma_\ell^L$, $\tilde{U}$, $\tilde{D}$ at the LHC and for $P_V^q$, $P_L^q$, $P_R^q$, $P_R^d$ at the NLC for the $\chi$, $\psi$, $\eta$, LR models, with $M_{Z'} = 1$ TeV. The error bars in parentheses are for the probes without polarization $^5$. We do not include a possible determination of $\gamma_L^q$.

|       | $\chi$     | $\psi$    | $\eta$    | $LR$          |
|-------|-------------|------------|------------|---------------|
| $P_V^q$ | 2 ± 0.08(0.26) | 0 ± 0.04(1.5) | -3 ± 0.5(1.1) | -0.15 ± 0.018(0.072) |
| $P_L^q$ | -0.5 ± 0.04(0.10) | 0.5 ± 0.10(0.2) | 2 ± 0.3(1.1) | -0.14 ± 0.037(0.07) |
| $P_R^q$ | -1 ± 0.15(0.19) | -1 ± 0.11(1.2) | -1 ± 0.15(0.24) | -6.0 ± 1.4(3.3) |
| $P_R^d$ | 3 ± 0.24(0.51) | -1 ± 0.21(2.8) | 0.5 ± 0.09(0.48) | 8.0 ± 1.9(4.1) |
| $\gamma_L^q$ | 0.9 ± 0.016 | 0.5 ± 0.02 | 0.2 ± 0.012 | 0.36 ± 0.007 |
| $\tilde{U}$ | 1 ± 0.16 | 1 ± 0.14 | 1 ± 0.08 | 37 ± 6.6 |
| $\tilde{D}$ | 9 ± 0.57 | 1 ± 0.22 | 0.25 ± 0.16 | 65 ± 11 |

luminosity of the NLC and the LHC. Realistic fits are expected to give larger uncertainties.

IV. RECONSTRUCTION OF THE EXTENDED GAUGE STRUCTURE

We now demonstrate how the model independent determination of the normalized couplings in Eq. (4) or Eq. (6) allows for the testing of a particular underlying symmetry structure and, subsequently, how such couplings further constrain the symmetry breaking pattern. We demonstrate the procedure for models based on $E_6$ gauge symmetry.

As the first step in establishing the underlying $E_6$ symmetry structure, the normalized couplings should be compatible with the discrete constraints on the couplings of the type (3), which, in terms of the $\hat{g}$ couplings, are:

$$2\hat{g}_L^q + \hat{g}_R^u + \hat{g}_R^\ell = 0,$$
$$\hat{g}_L^q - \hat{g}_R^d - \hat{g}_L^\ell + \hat{g}_R^\ell = 0,$$
$$3\hat{g}_L^q + \hat{g}_L^\ell = 0.$$  \hspace{1cm} (9)

Only the first two equalities hold for general $E_6$ models. The third holds in the special case of $SO(10)$. At the NLC these constraints, expressed in terms of (4), are of the form:

$$C_1 \equiv 2P_L^q(2 + P_R^q) + P_V^\ell - 1 = 0,$$
$$C_2 \equiv P_L^q(1 - P_R^d) - 1 = 0,$$
$$C_3 \equiv 6P_L^q + P_V^\ell + 1 = 0.$$  \hspace{1cm} (10)

The underlying gauge structure is therefore determined by the above constraints; the symmetry group corresponds to $SO(10)$ if the measured normalized couplings are compatible with all three constraints (10), while the symmetry group is $E_6$ if the measured couplings are compatible with the first two only.
The next step is to address the nature of the symmetry breaking pattern within the underlying symmetry structure.

The normalized couplings probed at the NLC (see Eq. (4)) in turn determine $\tan \beta$ and $\delta$, parameterizing the most general symmetry breaking pattern within the $E_6$ group (recall $\beta = 0$ corresponds to the $SO(10)$ group). $\beta$ and $\delta$ can be expressed in terms of the normalized couplings by:

$$\tan \beta = \sqrt{15} \frac{P^q_L(1 - P^u_R) + 1}{P^q_L(1 + 3P^u_R - 4P^d_R) - 1 + 2P^\ell_V},$$

$$\delta = 2\sqrt{6} \frac{P^q_L(1 + 3P^u_R - 4P^d_R) - 1 + 2P^\ell_V}{P^q_L(1 + 3P^u_R - 4P^d_R) - 1 + 2P^\ell_V}. \tag{11}$$

Determination of the symmetry breaking pattern within the underlying gauge structure can be thought as replacing the phenomenological variables in (4) by the four independent variables $C_1, C_2, \beta, \delta$ in (10) and (11). ($C_3$ is a function of $C_1$ and $\beta$.) $\beta$ and $\delta$ are interpreted as determining the pattern of symmetry breaking within $E_6$ for $C_1 = C_2 = 0$ only.

- One can determine the parameters in (4) and their correlation matrix from the NLC probes, and from these determine $C_1, C_2, \beta, \delta$ (step one). If $C_1$ and $C_2$ are compatible with zero, then one assumes $C_1 = C_2 = 0$ (step two), i.e., one assumes $E_6$ to be the underlying symmetry. In this case only two normalized couplings in (4) are independent, thus in turn implying that $\beta$ and $\delta$ can be determined with better precision.

- Equivalently, one can rewrite the NLC probes as functions of $C_1, C_2, \beta, \delta$ and fit them directly to experimental data (step one). If $C_1$ and $C_2$ are compatible with zero, then one can fix them to be zero (step two), i.e., one is assuming $E_6$ as the underlying symmetry, and then fit $\beta$ and $\delta$ with higher precision.

We will use the second approach in our numerical examples below.

Before proceeding with the numerical analysis, we will introduce new convenient combinations of couplings, which in the case of $E_6$ symmetry are set to zero. The choice of variables $C_1, C_2, \beta, \delta$ is suggested by the $E_6$ parameterization of charges. There is an equivalent choice of parameters:

$$S_1 = C_2, \quad S_2 = 2(2 + P^u_R) + (1 - P^d_R)(P^\ell_V - 1), \quad \beta, \quad \delta. \tag{12}$$

$S_2$ is a linear combination of $C_1$ and $C_2$, which does not depend on $P^q_L$. Thus, $C_1 = C_2 = 0$ if and only if $S_1 = S_2 = 0$, i.e., the latter set of constraints uniquely fix $E_6$ as the underlying symmetry as well. In the following we shall use variables (12) $S_1, S_2, \beta, \delta$ because they are better adapted to the analogous analysis at the LHC, and thus allow for an easy comparison of the NLC and the LHC potentials.

The analogous analysis at LHC requires one to rewrite (10), (11), (12) in terms of (9) and (9). This can be done using the relation (8). The sign ambiguity due to different sign assignments for $\epsilon_\ell, \epsilon_q, \epsilon_u$, and $\epsilon_d$ give rise to a sixteen-fold sign ambiguity due to the fact that at the LHC only the magnitude of the corresponding couplings is determined. In general,
TABLE V. Values and 1 σ statistical error bars for $S_1, S_2, \beta, \delta$, and $\epsilon_A$ at the NLC for the typical models. $M_{Z'} = 1$ TeV. The error bars in parentheses are determined by setting $S_1 = S_2 = 0$.

|     | $\chi$     | $\psi$     | $\eta$     | $LR$     |
|-----|-------------|-------------|-------------|----------|
| $S_1$ | 0 ± 0.074   | 0 ± 0.18    | 0 ± 0.22    | 0 ± 0.074 |
| $S_2$ | 0 ± 0.45    | 0 ± 0.37    | 0 ± 0.51    | 0 ± 0.76  |
| $\beta$ | 0 ± 0.028(0.28) | 1.57 ± 0.043(0.027) | −0.912 ± 0.038(0.028) | 0 ± 0.058(0.047) |
| $\delta$ | 0 ± 0.038(0.015) | 0 ± 0.035(0.014) | 0 ± 0.059(0.019) | 0.615 ± 0.032(0.008) |
| $\epsilon_A$ | 0.071 ± 0.005(0.005) | 0.121 ± 0.017(0.010) | 0.012 ± 0.003(0.003) | 0.255 ± 0.016(0.009) |

$C_1 = C_2 = 0$ ($S_1 = S_2 = 0$) can be satisfied only for a specific choice of the corresponding signs.

A. Determination of the symmetry breaking structure at the NLC

As stated above we fit the expected NLC observables for $S_1, S_2, \beta, \delta$, and $\epsilon_A$. In Table V the values and 1 σ uncertainties for these quantities are given for the models specified in Table III and $M_{Z'} = 1$ TeV. The parameters $\beta$ and $\delta$ can be determined with uncertainties between 0.02 and 0.06 for a large class of typical models. The large uncertainties for the $\eta$ model are primarily due to the small value of $\epsilon_A$. In Figure 1 we plot the 90% confidence level ($\Delta \chi^2 = 4.6$) contours (dashed lines) for two constraints ($S_1$ and $S_2$ as defined in (12)) for the models specified in Table III and $M_{Z'} = 1$ TeV. For the $\eta$ model the uncertainties are slightly too large to allow for the corresponding 90% confidence level plot. In Figure 2 (dashed lines) we plot the 90% confidence level ($\Delta \chi^2 = 4.6$) contours in the $\beta$ vs. $\delta$ plane for the typical models and $M_{Z'} = 1$ TeV.

As a second step we fix $S_1, S_2$ to zero, fitting the values of $\beta, \delta$ and $\epsilon_A$. Clearly, the uncertainties are reduced. In Table V the corresponding 1 σ uncertainties are given in parentheses. In this case even the parameters of the $\eta$ model can be determined with a sufficient accuracy. The uncertainties are now reduced by a factor of 1 to 4 compared to the step one analysis. In Figure 2 the 90% confidence level ($\Delta \chi^2 = 4.6$) contours in the $\beta$ vs. $\delta$ are plotted with solid lines.

Provided the mass $M_{Z'}$ is known by the time the NLC is turned on, the determination of the $\epsilon_A$ parameter would yield additional useful information on the overall strength $g_2$ and thus on the symmetry breaking patterns [14].

3Note, however, that for special values of $\delta$ and/or $\beta$ parameters a few-fold sign ambiguity is not removed.
FIG. 1. 90% confidence level ($\Delta \chi^2 = 4.6$) contours for $S_1$ vs. $S_2$ (constraints defined in Eq. (12) for the typical models at the NLC. $M_{Z'} = 1$ TeV. Only statistical error bars for the probes are used.

FIG. 2. 90% confidence level ($\Delta \chi^2 = 4.6$) contours (dashed lines) for $\beta$ vs. $\delta$ for the typical models at the NLC. $M_{Z'} = 1$ TeV. The solid lines correspond to the assumption that the constraints $S_1 = S_2 = 0$ are satisfied. Only statistical error bars for the probes are used.
TABLE VI. Values and 1 σ statistical error bars for $S_2, \beta, \delta$ for the typical models at the LHC. $S_1 = 0$ and $M_{Z'} = 1$ TeV. The error bars in parentheses are determined by setting $S_2 = 0$.

|       | $\chi$     | $\psi$     | $\eta$     | $LR$      |
|-------|-------------|-------------|-------------|-----------|
| $S_2$ | 0 ± 0.40    | 0 ± 0.16    | 0 ± 0.71    | 0 ± 0.54  |
| $\beta$ | 0 ± 0.024(0.024) | 1.57 ± 0.028(0.028) | −0.912 ± 0.010(0.010) | 0 ± 0.034(0.029) |
| $\delta$ | 0 ± 0.019(0.014) | 0 ± 0.022(0.016) | 0 ± 0.016(0.011) | 0.615 ± 0.021(0.004) |

B. Determination of the symmetry breaking structure at the LHC

At the LHC primarily three couplings 4 would be probed for a large class of models. Namely, for $M_{Z'} = 1$ TeV and typical models specified in Table III $\gamma^{\ell}(P^L_L)$ cannot be determined with sufficient precision. Consequently, one can only determine the three combinations of $S_2, \beta, \delta$. We therefore set $S_1 = 0$, thus expressing the undetermined coupling $P^L_L$ in terms of the three left-over couplings $P^\ell_V, P^u_R$, and $P^d_R$. This constraint also removes one sign ambiguity ($\epsilon_q$). Then, we fit for $S_2, \beta, \delta$. Thus, at the LHC one has to assume that one ($S_1 = 0$) of the two $E_6$ discrete constraints on the gauge couplings is already satisfied in order to further test whether or not the second discrete constraint $S_2 = 0$ is satisfied and then ultimately determine the values of $\beta$ and $\delta$.

In Table VI the values and expected 1 σ uncertainties for these quantities are given for the models specified in Table III and $M_{Z'} = 1$ TeV. In general, there are eight disjoint regions corresponding to the different sign assignments for $\epsilon_u, \epsilon_d, \epsilon_\ell$. We only quote the results for the region in the vicinity of $S_2 = 0$. The parameters $\beta$ and $\delta$ can be determined with uncertainties between 0.02 and 0.04 for a large class of typical models.

In Figure 3 we plot the 90% confidence level ($\Delta \chi^2 = 4.6$) contours (dashed lines) in the $\beta$ vs. $\delta$ plane for the typical models specified in Table III with $M_{Z'} = 1$ TeV, and for the region in the vicinity of $S_2 = 0$.

4The LHC would also determine the $Z'$ mass. It would furthermore establish whether there is a corresponding $W'$, as expected in LR models, and the ratio $M_{W'}/M_{Z'}$, which probes the LR-breaking mechanism 5.

5In general, the region in the vicinity of $S_2 = 0$ removes the few-fold sign ambiguity, thus uniquely fixing the value of $\beta$ and $\delta$. However, for special models, such a region can be fitted with more than one value of $\beta$ and $\delta$. E.g., the couplings corresponding to the $\chi$, and $\eta$ models, can also be fitted to the models with $\delta = 0, \beta = \tan^{-1}(3\sqrt{3}/5)$, and $\delta = 0, \beta = -\tan^{-1}(7\sqrt{3}/5)$, respectively. The model with $\delta = 0, \beta = \tan^{-1}(\sqrt{3}/5)$ is an alternative of the $\psi$ model. In this case constraints $C_1 = C_2 = 0$ (or $S_1 = S_2 = 0$) are formally undetermined because they are obtained by dividing the first two constraints in 4 by $\hat{g}_L^\ell - \hat{g}_R^\ell = 0$. However, the first two constraints in 4 are satisfied. In Figure 3 we do not display any of the alternative regions.
FIG. 3. 90% confidence level ($\Delta \chi^2 = 4.6$) contours for $\beta$ vs. $\delta$ for the typical models at the LHC. $M_{Z'} = 1$ TeV. Only statistical error bars for the probes are used. Dashed lines are determined by fixing $S_1 = 0$, while the solid ones correspond to setting $S_2 = 0$ as well.

As a second step, we set $S_2$ to zero as well. In this case the fitted values for $\beta$ and $\delta$ have smaller uncertainties. In Table VI the corresponding 1 $\sigma$ uncertainties are given in parentheses. The uncertainties are now reduced by a factor of 1 to 5 compared to the step one analysis. In Figure 3 the 90% confidence level ($\Delta \chi^2 = 4.6$) contours in $\beta$ vs. $\delta$ are plotted with solid lines. The parameters can be determined with a precision of about a few percent. For the typical models the uncertainties are smaller than those at the NLC (see Figure 2).

Although the overall strength of the $Z'$ gauge couplings does not enter in the above analysis (except for the estimate of typical statistical error), the overall strength of the interactions $g^2_2((\hat{g}^L)^2 + (\hat{g}^R)^2)$ can also be measured at the LHC, by measuring the cross-section in the main production channel and the total width. Determination of this coupling can be used to constrain further the symmetry breaking pattern [14].

V. CONCLUSIONS

We have demonstrated how the model independent determination of gauge couplings of a possible $Z'$ at future colliders would allow one to gain information about the nature of the extended gauge structure associated with the $Z'$.

As a prime case we study $E_6$ [$SO(10)$] as the underlying gauge symmetry. In the case of $E_6$ [$SO(10)$] $Z'$ gauge couplings have to satisfy two [three] discrete constraints. Provided such constraints are satisfied, the parameters $\tan \beta$ [$\beta = 0$ for $SO(10)$] and $\delta$, which charac-
terize the effects of the symmetry pattern within $E_6 \ [SO(10)]$ on the $Z'$ couplings, can be determined.

For $M_{Z'} \sim 1–2$ TeV and typical models the statistical uncertainties for parameters $\beta$ and $\delta$ are in the 0.01–0.04 range, once the corresponding discrete constraints are fixed. We included only statistical uncertainties for the probes. Realistic fits, which include experimental cuts and detector acceptances, are expected to give larger uncertainties.

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