Time-optimal control of heating process subject to phase constraints

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Abstract. The article presents the results of modeling and optimization of temperature fields by the volume of the workpiece during induction heating using a numerical two-dimensional electromagnetic thermal model of the process. Time-optimal control problem of induction heating subject to constraints on maximum permissible temperature is formulated and solved. The obtained optimal control algorithms lead to an increase in the efficiency of induction heaters by reducing total heating time and absence of defects due to technological limitations.

1. Introduction

Induction heating installations (IHI) of periodic action are widely used in practice for induction heating of metals before the subsequent processing by pressure, because it possess a number of technical and economic advantages in comparison with competitive technologies [1,2].

In a whole number of real metal heating processes it is necessary to take into account additional requirements, which, first of all, include constraints on the maximum temperature by the volume of the workpiece during heating. Taking into account such constraints significantly complicates the optimal control algorithms and their calculation [1,3].

The article formulated and solved the task of optimal control (TOC) on the criterion of speed, subject to constraints on maximum admissible temperature. Introducing of this phase constraint allowed to provide the required quality of heating at the maximum performance of the technological complex.

2. Numerical two-dimensional model of induction heating process

The article uses the finite element software product FLUX on the basis of which was developed a two-dimensional electrothermal model of the process of induction heating of metal (PIHM), focused on the use in optimization procedures.

The considered model takes into account temperature dependences of electromagnetic and thermophysical characteristics, possible nonlinearities of boundary conditions associated with heat transfer, radiation into the environment, which allows to obtain a satisfactory description of PIHM. The algorithm for solving the simulation task is a sequential iterative procedure including harmonic electromagnetic analysis and transient thermal analysis [4,5].
3. Formulation and solution of the time-optimal control problem with use of two-dimensional models

Periodic induction heating process is determined by the spatiotemporal distribution of the temperature of the heated workpiece \( T(l, y, \tau) \), where \( \tau \) is the time, and \( l, y \) are the spatial coordinates that take into account the uneven temperature distribution both along the radius \( l \in [0; R] \) and along the axis of the cylindrical workpiece of finite length \( y \in [0; L] \). Herewith, the temperature field of the workpiece is modeled with use of two-dimensional nonlinear numerical model described above [1,6,7,8,9].

Voltage of a power supply \( u(\tau) \) is considered as a control action. The constraint on the control action \( u(\tau) \) is introduced on the basis of some pre-known maximum value \( U_{\text{max}} \), determined by the energy capabilities of the IHI, and has a form:

\[
0 \leq u(\tau) \leq U_{\text{max}} \tag{1}
\]

The minimum heating time \( \tau_{\text{min}}^0 \) is considered as the optimality criterion for maximum installation performance.

With regard to the class of optimization tasks are investigating in this article, the induction installation at the final time \( \tau_k \) should provide heating of the metal workpiece to a predetermined temperature \( T^* = \text{const} \) with an acceptable temperature deviation along the cross section of the workpiece \( \varepsilon_0 \):

\[
\max_{l \in [0; R]} \max_{y \in [0; L]} \left| T(l, y, \tau_k) - T^* \right| \leq \varepsilon_0 \tag{2}
\]

The task of optimal control by the criterion of \( \text{IHI} \) speed can be formulated as follows: It is necessary to find such a variable in time control action \( u(\tau) = u_{\text{opt}}(\tau) \) constrained by the constraint (1), which provides the transfer of the heated workpiece with the initial temperature distribution \( T_0(l, y) = T_0 = \text{const} \) in a given target set (2), for the minimum possible time \( \tau_k = \tau_{\text{min}}^0 \).

For general nonlinear TOC by the processes of non-stationary thermal conductivity with internal heat radiation [1,7,10], the standard procedures of the maximum principle determine \( \Delta^{(N)} \) - parameterized representation of the control action \( u_{\text{opt}}(\tau) \) in the form of a piecewise constant function of time:

\[
u_{\text{opt}}(\tau) = \frac{u_{\text{max}}}{2} \left[ 1 + (-1)^{j+1} \right], \sum_{i=1}^{j-1} \Delta_i < \tau < \sum_{i=1}^{j} \Delta_i, j = 1, N 
\]

unambiguously giving with accuracy to the number \( N \) and durations \( \Delta_i, i = 1, N \), intervals of their constancy, acting as the desired parameters and depending only on the required heating accuracy \( \varepsilon_0 \) in (2), where \( N \) can be found by a given value \( \varepsilon_0 \) according to the general methodology of the alternance method [1,2,3]. As a result, the reduction procedure of the initial task to the task of semi-infinite optimization is carried out:

\[
I(\Delta) = \sum_{i=1}^{s} \Delta_i \rightarrow min; \Delta = (\Delta_1, \Delta_2, \ldots, \Delta_S), \Phi(\Delta) = \max_{l \in [0; R]} \max_{y \in [0; L]} \left| T(l, y, \Delta) - T^* \right| \leq \varepsilon_0,
\]

where dependences \( T(l, y, \Delta) \) determinate by the numerical FLUX-model of the object during control of view (3).

When using a numerical model of the temperature field, the computational systems of the equations of the alternance method can be solved with reference to the model at each step of the iterative procedure [5].
Based on the proposed methodology solved the tasks of time-optimal control program of the process of periodic induction heating of steel cylindrical workpieces with a diameter of 105 mm, a length of 900 mm at IHI with a length of 1010 mm.

The result of the TOC solution according to the criterion of a high speed when $u_{\text{max}} = 470V$ heating accuracy $\varepsilon_0 = \varepsilon^{(1)}_{\text{min}}$ made up 90,15°C, when the duration of the heating process 541,6 s. The non-uniformity of heating is not in line with the technological requirements of PIHM that leads to the necessity of solving this task in the class of two interval control [7].

The two interval control action includes the heating stage with the maximum voltage of the power supply and the subsequent stage of temperature equalization in the absence of voltage on the inductor. At $\varepsilon_0 = \varepsilon^{(2)}_{\text{min}} = 66,4°C$, the heating process time is 563,85 s, the temperature equalization time is 13,3 s [7].

Overheating of the workpiece during the two interval induction heating exceeded 100°C, which is unacceptable by a technological requirements. Let's consider further TOC on criterion of the high speed PIHM subject to main phase constraints on the maximum temperature by volume of the workpiece during the heating process [1,3,11,12].

3.1. Statement of TOC according to the criterion of the high speed subject to constraints on maximum admissible temperature

In real conditions of induction heating, the maximum value $T_{\text{max}}(\tau)$ of temperature over the whole volume of heated workpieces is limited throughout the heating process by a given maximum admissible value of $T_{\text{adm}}$, exceeding which leads to irreversible undesirable changes in the structural properties of the material of the treated products and (or) to their melting [1,3,13]. Therefore, it is necessary to take into account the additional constraint, which is written in the form of inequality:

\[
T_{\text{max}}(\tau) = \max_{l \in [0,K]} T(l,y,\tau) \leq T_{\text{adm}}, \quad 0 < \tau \leq \tau_{\text{e}}
\]  

(5)

Consider the above induction heating task of the workpiece to the desired temperature with a given accuracy in the shortest possible time, adding to it with condition (5).

In most practical tasks, the correction of optimal modes in connection with the considered technological limitations is necessary only within the first interval of the heating process with a maximum power $u = u_{\text{max}}$. It boils down to the selection of control actions on relevant sections within this interval, supporting $T_{\text{max}}$ at the maximum admissible levels $T_{\text{adm}}$, i.e., to implement in these areas the conditions (5) in the form of the relevant equations. Only in this case, the minimum possible total time of the workpiece heating process is kept, subject to new constraints, since it is in this case that the correction under consideration becomes minimal.

Figure 1. Control action change during heating process subject to phase constraints.

Figure 2. Piecewise constant approximation of the control $u^T(\tau)$. 

3
In accordance with this, the time-optimal algorithm control, subject to the technological limitations (Figure 1) takes the following form:

\[
    u(\tau) = \begin{cases} 
    u_{max}, & \tau \in (0; \tau_T); \\
    u^T(\tau), & \tau \in (\tau_T, \Delta \tau^0); \\
    0, & \tau \in (\Delta \tau^0, \Delta \tau^0). 
    \end{cases} 
\]  

(6)

In the first interval of duration \(\Delta \tau^0\) is forced heating with the maximum voltage of the power supply until time \(\tau = \tau_T\). Time \(\tau_T\) is finding from equality \(T_{max}(\tau_T) = T_{adm}\). Maintaining temperature at the level \(T_{max}(\tau) = T_{adm}\) in the interval \([\tau_T, \Delta \tau_T]\) provided by the stabilizing control action \(u^T(\tau)\).

The limit value of \(T_{adm}\) is reached at \(\tau = \tau_T\) at the internal point of the temperature maximum, which is the point of the extreme \((l_{max}, y_{max})\) of the temperature curve \(T(l, y, \tau)\). The values \(\tau_T\) and \((l_{max}, y_{max})\) are found as the roots of the system of equations (7), which is solved by numerical methods with a given dependence \(T(l, y, \tau)\) on its arguments [1,5,7,14].

\[
    \begin{align*}
    T_{max}(\tau_T) &= T(l_{max}, y_{max}, \tau_T) = T_{adm}; \\
    \frac{dT(l_{max}, y_{max}, \tau_T)}{dl} &= \frac{dT(l_{max}, y_{max}, \tau_T)}{dy} = 0.
    \end{align*} 
\]  

(7)

3.2. TOC solution subject to the maximum possible temperature limit

In the induction heating process of steel workpieces, the temperature stabilization period has a significant length and in order to find a control action that provides with satisfactory accuracy the stabilization of the temperature maximum at the level of \(T_{adm}\), it is necessary to approximate the desired program of changing the heating power in time by the piecewise constant function \(u^{(v)}\) (Figure 2). Here, the choice of a sufficiently large number of \(S\) stages of power change with a small step \(\tau^*\) allows to provide the necessary approximation accuracy [1,15].

Requiring each \(\tau^{(v)} = \tau_T + v \tau^*, v = 1, 2, ..., S\), the equalities implementation of the form (7), it is possible to find the required control algorithm, sequentially solving this system of equations for all \(v = 1, 2, ..., S\) relative to \(u^{(v)}\) and the point with coordinates \((l_{max}^{(v)}; y_{max}^{(v)})\). In the end, all \(u^{(v)}, v = 1, 2, ..., S\) are chosen so that they stabilize at the level \(T_{max}\) on the level \(T_{adm}\) in points \((l_{max}^{(1)}; y_{max}^{(1)}), (l_{max}^{(2)}; y_{max}^{(2)}) , ..., (l_{max}^{(S)}; y_{max}^{(S)})\) at time \(\tau^{(1)}, \tau^{(2)}, ..., \tau^{(S)}\), i.e., \(T_{max}(\tau) = T_{adm}\) for all \(\tau \in (\tau_T, \Delta \tau_t)\) with the accuracy depending on the magnitude of \(S\). The values of \(u^{(v)}\) are refined during the subsequent full calculation of the optimal heating process by the iteration method at a pre-known value \(\Delta \tau^0\).

Figures 3-6 show the results of calculation the optimal speed of periodic induction heating of steel cylindrical workpiece. The maximum allowed temperature on the volume of the workpiece during the heating process is \(T_{adm} = 1270^0\)C.

The type of time-optimal control is shown in Figure 3. Time \(\tau_T = 517\) s was found on the basis of the optimal control tasks solution without considering phase constraints from the conditions for achieving the maximum threshold value of temperature, i.e. on the basis of equality \(T_{max}(\tau) = T_{adm}\). Stabilizing control was found as a piecewise constant function of time (Figure 4). The length of the step is \(8\) s, the number of steps \(- S = 11\). In the end, the total time of the optimum process has comprised \(613.5\) s, of which the heating interval \(\Delta \tau_1 = 611\) s, stabilization \(- \Delta \tau_2 = 2.5\) s.
Figure 3. Time-optimal control subject to phase restricts

Figure 4. Stabilizing control

Figure 5 shows the resulting temperature distribution over the workpiece volume. The maximum temperature at the end of the optimum process is reached at the point with coordinates (31; 0) in the central section of the workpiece. In turn, the points with the minimum temperature are in the end section on the axis and the surface of the workpiece, respectively.

Figure 5. Temperature distribution over the workpiece volume at the end of heating process in cross sections: 1 – \( y_2 = 0 \);
2 – \( y_1^0 = y_0^0 = 450 \)

Figure 6. Maximum by volume of the workpiece temperature during heating process

The coordinates of the points with the maximum temperature during the heating process were changing significantly. Figure 6 shows a change graph of the maximum by volume of the workpiece temperature in time during the time-optimal control subject to the restrict on the maximum admissible temperature.

As can be seen from the above data, at the end of the heating process, the uneven temperature distribution over the volume of the cylindrical workpiece is \( \varepsilon^{(2)} \leq 67.8^0 \text{C} \). The temperature throughout the process does not exceed the admissible level \( T_{adm} = 1270^0 \text{C} \).

Obtained results corresponds all the basic technological requirements for the process of periodic induction heating of steel workpieces.
Obtained results show that the previously proposed method of solving TOC of the induction heating process under the conditions of achieving the phase constraint on the maximum temperature $T_{\text{max}}$, i.e. with the control algorithm of the form (6), is effective and applicable to the real INI.

4. Conclusions
The author has formulated and solved the optimization tasks of a process for periodic heating of the metal, different from the known, using sophisticated multidimensional digital models of interconnected nonlinear processes in electromagnetic and thermal fields, as well as methods for the parameterization of control actions taking into account of a various complicating factors that significantly affect on the quality of control process in a real production environment.

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