We first confirm that previous findings in cognitive psychology
favor the restricted focus viewing interface to further analyze the strategies people
used to trace through programs, and the relationship of tracing strategy to WM. Given a straight-line program, we find half of
our participants traced a program from the top-down line-by-line
(linearly), and the other half start at the bottom and trace upward based on data dependencies (on-demand). Participants with an on-demand strategy made more WM errors while tracing straight-line code than with a linear strategy, but the two strategies contained an equal number of WM errors when tracing code with functions. We conclude with the implications of these findings for the design of programming tools: first, programs should be analyzed to identify and refactor human-memory-intensive sections of code. Second, programming environments should interactively visualize variable metadata to reduce WM load in accordance with a person’s tracing strategy. Third, tools for program comprehension should enable externalizing program state while tracing.

CCS CONCEPTS
• Human-centered computing → Laboratory experiments.

KEYWORDS
Program tracing, program comprehension, working memory, restricted focus viewing

1 INTRODUCTION
Program tracing is the task of mentally simulating a program on concrete inputs to produce a concrete output. To trace a program, a person follows a procedure step-by-step as specified in the source code. Go here, add this, store that, repeat — a literal human information processor. Program tracing encompasses questions such as:

- For some function \( f \) what is the concrete value of \( f(1) \)? By contrast, program comprehension involves tasks about abstract relationships, e.g. what is the symbolic value of \( f(x) \) in terms of any \( x \in \mathbb{N} \)?

Prior work on human-centered design of programming tools has focused on measuring and improving comprehension, not tracing. After all, the entire point of a computer is to automatically trace programs, and have the human operate at a higher level of abstraction. However, cognitive psychologists have repeatedly demonstrated the close relationship between action and understanding in comprehension of language [28] and gestures [34]. By extrapolation, program tracing, the ability to execute a computational process, is likely a component skill of program comprehension, the ability to understand that same process. This claim has support from research in computing education and software engineering. Students who could correctly trace programs performed better on program comprehension questions [21,22]. Students’ comprehension errors can often be attributed to a flawed mental model (notional machine) of how a computer would trace individual instructions [29]. Expert programmers use tracing to understand code when its structure is not similar to a previously observed pattern (schema) [12,20].

Hence, to design for program comprehension, we ought to better understand what makes program tracing difficult. Consider the myriad differences between a computer tracing a program versus a person doing the same. A computer can instantaneously parse a program, effortlessly keep its runtime state in memory, and infallibly apply the language’s rules to compute the program’s output. Humans, by contrast, are much worse at these tasks. To understand why, take a moment to trace this Python program to determine what value it prints.

```python
1  def f(x, r, q):
2      return r + q + x
3      q = 1 * 4
4      e = 8 - q
5  print(f(q, e, f(3, 5, e)))
```

While tracing this program, you may have found yourself thinking — “what was the value of \( e \)? Which was the second argument in the function call? Which \( q \) is this referring to?” These questions all point to failures of memory to maintain program state during tracing. Specifically, failures of working memory (WM): the “brain system that provides temporary storage and manipulation of information” [4]. The universal limitations of WM have been studied since the early days of cognitive psychology [23]: WM has a capacity of roughly \( 4 \pm 1 \) chunks [9], and chunks are forgotten.
from WM as a function of time and interference [26]. Mental tasks which exceed these limitations (high cognitive load) become difficult to correctly and efficiently execute. Prior qualitative studies have shown that students encounter WM difficulties while tracing; they remember “at most one value for all the variables in the program” [32]. WM has also inspired the development of program comprehension tools such as the Code Bubbles IDE [5]. However, to date, no work has systematically explored the role of WM in program tracing.

As demonstrated in the example above, maintaining program state during tracing likely imposes demands on WM. Program state requires WM because it is intangible. If a person is tracing a procedure with physical state like a recipe or a furniture manual, they can quickly survey ingredients on the kitchen counter, or find the missing parts of a half-built chair. But programs have purely virtual state — variables, instruction pointers, call stacks — meaning a person cannot rely on their environment to maintain state for them. Therefore in this paper, we focus on two high-level questions:

1. How much program state can a person hold in WM?
2. How do WM limitations for program state influence tracing strategies?

WM theory suggests a few coarse answers: people can’t remember much state, and they’ll forget relevant state while tracing. But in this paper, our goal is to dive deeper into the nuances of each question: how much program state can a person remember in pure recall vs. interleaved with mental arithmetic? Do people linearly trace in the same fashion as a computer, or do they optimize their strategies for WM? To that end, we ran four controlled experiments to elucidate the influence of working memory for program state on tracing. In each experiment, we carefully restrict the participants’ view on the program, for example by seeing a single line/function at a time, or blurring lines of code until hovered. Then we use the participants’ responses and their mouse movements to derive quantitative measures of working memory influence. Our main contributions are:

- Verifying whether results from prior cognitive psychology research on similar tasks (e.g. WM capacity for paired-associates) will transfer to program tracing.
- Designing novel experimental methodologies to analyze participants’ behavior at a fine-grained level while tracing programs.
- Applying WM theory to predict participants’ tracing behavior, and to generate design principles for programming tools that can reduce WM load.

## 2 BACKGROUND AND RELATED WORK

To relate program tracing back to prior research on working memory, we can break down the general task of tracing into components that look more like tasks in prior work. Specifically, we will focus on how program state in tracing arises from a few basic programming concepts: arithmetic, variables, computation order, and functions.

### 2.1 Arithmetic

Mental arithmetic is a well-studied task within the context of working memory [11]. It is also a “program” that people trace every day — adding multi-digit numbers is a procedure many people are taught to memorize and mentally simulate from a young age. For example, computing $47 + 18$ is similar to tracing the program in Figure 1 (left).

Graham Hitch [17] first found in 1978 that a working memory analysis of mental arithmetic could explain many observed calculation errors. While calculating, a person must maintain a number of intermediates in working memory, such as all previously computed digits and a carry bit. Based on a mathematical model fit to experimental data, Hitch’s theory was that a person’s probability of forgetting an intermediate increased exponentially with the number of calculations since computing the intermediate. For example, in mentally computing $138 + 326$, if a person first computes $6 + 8 = 14$, then they will forget the ones-place digit 4 with exponentially increasing probability after computing each subsequent digit.

Within the domain of mental computation, prior cognitive psychology research has focused on memorized procedures like addition, as opposed to tracing unfamiliar procedures presented through an external medium. Zhang and Norman [35] use a representational analysis to justify why Arabic numerals are an ideal number representation for performing mental multiplication. Campbell and Charness [7] ran an experiment where participants memorized an
eight-step procedure for squaring two-digit numbers, shown in Figure 1 (right). They found that the majority of errors occurred within 5 stages of the procedure, and most errors could be explained as working memory errors (not calculation errors) where one intermediate was substituted for another.

2.2 Variables
To understand the influence of WM on variables in program tracing, we can extrapolate the concept of an intermediate in mental arithmetic. For example, consider tracing this program:

```
1. x = 12 + 9 - 2
2. print(x + 3)
```

This trace involves three kinds of intermediates. First, computing \( x = 12 + 9 - 2 \) involves the intermediates of multi-digit addition, e.g. a carry bit. Second, the expression \( x = 19 \) requires remembering the intermediate 21 while computing \( 21 - 2 \). Finally, the variable assignment \( x = 19 \) must be remembered while tracing the second line. The key idea is that each intermediate must be stored in WM by association to its role in the program. That association could be behavioral (carry bit), positional (left hand side of an expression), or symbolic (the letter \( x \)). In this view, variables in program tracing are intermediates with association to strings. A person remembers \( x = 19 \), then retrieves the value 19 when cued with the variable \( x \).

In the vocabulary of cognitive science, remembering a variable/value pair is paired-associate learning, and retrieving the value from memory given a variable is cued recall. Cued recall for paired-associate learning has been studied in a variety of domains. The simplest form of study is memory span: how many pairs can a person remember at once with no distractions? Multiple experiments on cued recall for pairs of common English nouns found that participants could remember on average 5 out of 10 pairs (when presented for 2s/pair) [25, 31].

Program tracing is not a pure memorization task, however — a person must interleave the storage of variables in WM with the calculation of expressions. This raises the question: would calculation interfere with WM for variables? While this question has not been studied directly, its inverse has prior work: WM used by mental computation can be interfered with by certain competing tasks. In one experiment, participants had to mentally add three-digit numbers while concurrently performing a dual task. Participants were significantly more likely to forget the carry bit under a dual task which involves central executive WM ( Trails task, speaking aloud an alternating sequence e.g. “A-1-B-2-…” versus a dual task which involves phonological WM (articulatory suppression, repeatedly saying the word “the” aloud) [14].

2.3 Computation order
An important distinction between program tracing and mental arithmetic is that program tracing is accompanied by the program source code, while mental arithmetic uses a memorized procedure. The source code acts an external memory of the process’s instructions, but also potentially for program state. For example, try tracing this program:

```
1. x = 8
2. q = 3 + x
3. r = 6
4. print(x - q + r)
```

A “linear” strategy, the most similar to how a computer actually executes a program, would trace from line 1 to line 4, committing each variable/value pair to memory along the way. But you might have used an “on-demand” strategy: skim to line 4, then look up the value of each variable as needed. Either strategy produces the correct answer, although on-demand strategies nominally become more challenging in the presence of side effects (e.g. writing to a file) because out-of-order tracing may not produce the same ordering of effects as linear tracing.

Vainio and Sajaniemi [32] observed that students tracing programs would adopt a strategy they called “single value tracing.” In our terminology, students would trace on-demand with respect to variables assigned to constants. In the example above, a student might ignore line 1 (a “trivial” value in their words) and skip to line 2, looking up the value of \( x \) on demand. However, we do not know generally how often people will adopt a particular strategy, or how strategy relates to WM.

For more complex programs (e.g. with control flow, indentation, methods, etc.) eye-tracking studies of programmers have shown that programmers will read programs non-linearly. Busjahn et al. [6] found that the gaze path of expert programmers was more similar to following control flow from top-to-bottom than to reading line-by-line like a book. A replication of Busjahn et al. by Peitek et al. [27] further found that more linear programs (e.g. with fewer function calls) correlated to less vertical eye movement. Unlike these studies, we use much simpler programs and combine eye tracking with dataflow to identify WM errors from the gaze path.

2.4 Control flow
When tracing a program, a person needs to know what instruction to execute next, i.e. how to follow the program’s control flow. If a person adopts a linear tracing strategy in a straight-line program, they only need to know the “instruction pointer”, or the current position in the program. A person can trivially determine whether a line has been previously executed (above the pointer), or still needs to be executed (below the pointer).

In what situations, then, does a person need to remember more about control flow than the instruction pointer? Consider the three equivalent representations of an arithmetic expression shown in Figure 2. A trace through these programs is akin to a walk around the expression tree. For example, linearly tracing the variable program in Figure 2 starts from the leaves: observing 1 and 4, then computing 1 - 4, and so on. By contrast, an on-demand tracing strategy for the variable program starts at the root: observing that the goal is \( u - s \), then computing \( u = v + j \), and so on. A standard tracing strategy for the function program should look similar to the on-demand tracing strategy for the variable program: start with \( x() \), observe that \( x \) depends on \( u() \) - \( s() \), then compute \( u() \) and so on.

Because the variable program must be a topological sort of the tree, tracing linearly ensures that an expression’s dependencies...
We ran four experiments to explore the role of WM for tracing with working memory errors:

1. How many variable/value pairs can a person keep in WM?
2. How does the kind of variable name affect its memorability?

In a realistic program, both of these questions are inevitably confounded by the semantic relationship between a variable and its value. For example, greeting = "Hello" is likely easier to remember than foo = "Hello" because "Hello" is a kind of greeting. To avoid this confounder, we will consider the case where a variable has no particular relationship to its value. We only consider numeric values, and we consider variable names that are either common English nouns or single letters.

For the first question, our hypothesis is that participants should hold about 5 variable/value pairs in WM, consistent with prior work on cued recall in paired-associate learning [25, 31]. For the second question, we hypothesize that English words should be more memorable than single letters due to having some meaning if not a relevant meaning, and therefore should correspond to more pairs remembered on average.

3.1 Methodology. In this experiment, we tested WM capacity via cued recall. Participants were presented with several expressions of the form variable = value for 2s each. They were then prompted to answer variable = ? for every observed pair. The specific methodology and parameters are similar to those used in Nobel and Shiffrin’s paired-associate experiments [25]. Within a given trial, we randomly generated 10 variable/value pairs. Each value is a digit from 0 to 9. We have two different conditions for variable names: single-letters (A to Z), and common English nouns (e.g. cave, tax, cherries). The participant was presented with one pair at a time for 2s per pair. The presentation used programming syntax, e.g. x = 4.

After the final pair, the participant was prompted to input the corresponding value of all variables, e.g. x = ?; f = ?. The order of prompts was randomized with respect to the order of initial presentation. (The randomization was used to defeat mnemonic strategies that simply memorized variables and values as two separate streams of serial data which we observed in a pilot.) We then measured the participant’s accuracy as the number of values correctly recalled.

Unlike Nobel and Shiffrin, we did not add a distractor task (e.g. mental arithmetic) inbetween the presentation and prompt phases. Their goal was to measure long-term memory capacity, while our goal is to measure WM capacity. As a result, we may expect to see a larger number of recalled pairs than in their experiment.

Each participant completed 4 trials per condition. The order of trials was counterbalanced, and this experiment had 15 participants.
In this and all other experiments, we recruited participants with the following criteria and instructions:

- **Participant pool:** Participants were recruited from Amazon Mechanical Turk within the United States and Canada. They were compensated a fixed amount, estimated by timing the average duration of a pilot study and paid a corresponding amount prorated at $15/hr.
- **Required expertise:** We required that participants have a basic competency with tracing Python programs, which we ensured with a short pre-test before each experiment. Participants also needed to correctly complete a sample trial before doing an experiment.
- **No memory aids:** Participants were instructed to not use pen/paper or any other external media as a memory aid.
- **No distractions:** Participants were asked to complete the experiment without any visual or audio distractions, and to complete the experiment in one sitting.

### 3.2 Variable recall with arithmetic

Program tracing involves the maintenance of program state while performing operations like arithmetic. So next, we consider: how does WM capacity for variable/value pairs change when interleaved with mental arithmetic? Within the multi-component model of WM [4], prior work has found that three-digit mental arithmetic interferes with simultaneous tasks that use central executive WM, but not phonological WM [14]. Single-digit mental arithmetic problems have also been repeatedly confirmed to use central executive WM [11]. The cognitive action of updating WM in a running memory task is coordinated by central executive WM [24], therefore we hypothesize that participants should have a lower effective WM capacity for variable/value pairs than in the prior experiment.

#### 3.2.1 Methodology

In this experiment, participants traced a straight-line program of variables assigned to arithmetic expressions, one line at a time and without the ability to look back. We measured WM capacity by the point at which the participant provides an incorrect response. Specifically, we randomly generated programs of the form:

```plaintext
1. x = 3 + 4
2. t = x - 1
3. b = t + x
4. z = x - b
5. # ...and so on
```

Each line assigns a new variable to an arithmetic expression that involves randomly selected variables from the previous lines (or a constant for lines 1-2). Variables names are lower-case single letters and all constants are between 0 and 9. The only arithmetic operations are addition and subtraction to keep the mental effort of any individual operation relatively small. The program is 11 lines long, such that the participant would have to remember 10 variables by the last line.

Within a given trial, we present the participant one line at a time. In the above example, the participant would start by seeing: “x = 3 + 4. What is the value of x?” After entering 7, that line disappears and the participant sees: “t = x - 1. What is the value of t?” This way, the participant must commit the variable/value pairs to working memory, instead of looking them up later. After entering a value, the software waits for two seconds before proceeding to the next line. This process repeats until the participant responds incorrectly.

An incorrect response cannot necessarily be attributed to a working memory failure. The participant could simply add two numbers incorrectly (which could itself be a working memory failure, but for now we ignore that possibility). Using the methodology of Campbell and Charness two-digit squaring experiment [7], we classify the incorrect response as either a substitution error or calculation error. Substitution means the participant entered an a number that could have been computed by mixing up two previously seen variables. For example, if the current state is x = 7; t = 6; b = 13 and for x - b the participant enters 1, then we classify that as
a substitution of t for b, a failure of working memory. All other incorrect numbers are considered calculation errors.

Each participant completed 10 trials, and this experiment had 15 participants (one participant was excluded for on average failing at the first line, leaving 14 participants).

3.2.2 Results. Each participant finished the experiment at a particular line, either by providing an incorrect response (final line 1-11) or completing the task without error (final line 12). The histogram of participants’ average final line is shown in Figure 4. Across all participants, the average final line was 7.9 (σ = 3.7) and median was 8.0.

To compare these results to Experiment 1, first we have to model WM capacity in terms of the line of failure. If participants failed at line 8, then they either calculated incorrectly or forgot at least one of 7 preceding variables, suggesting a WM capacity of 6 variables. Hence, we model effective WM capacity as line number – 2, indicating the average WM capacity for this experiment was 5.9. To test the hypothesis of whether mental calculations reduced WM capacity, we compared the distribution of WM capacity in Experiment 2 vs. the distribution of WM capacity measured in Experiment 1 in the letter condition. Using a Kruskal-Wallis test, the difference was not significant (H(1) = 1.51, p = 0.21).

For each trial where the participant gave an incorrect response, we classified the error as substitution or calculation. The distribution of errors by type is shown in Figure 5. Of the 101 total errors, 53 were substitution and 48 were calculation. The median calculation error happened at line 3.5, while the median substitution error happened at line 7.0. Using a Kruskal-Wallis test, the difference was significant (H(1) = 25.57, p < 0.001).

3.2.3 Discussion. These results do not support the hypothesis that performing mental arithmetic reduces WM capacity. Participants in Experiment 2 on average remembered fewer variables than Experiment 1, but the difference is not statistically significant. However, the experimental designs are somewhat different (e.g. continually prompting for information vs. asking for it all at once) and the test is between-subjects, so future work can control for these factors. Consistent with Campbell and Charness [7], we found that over half of mistakes can be attributed to substitution in WM. Specifically, errors could be explained by accidentally swapping the association between variables in WM, as opposed to forgetting them entirely.

3.3 Straight-line tracing strategy

Next, we consider how WM influences tracing for straight-line programs when the complete program is available at all times. We investigate two questions:

(1) Do participants demonstrate tracing strategies that diverge from linear execution?

(2) Where in the program are participants most likely to have working memory errors?

As described in Sections 2.3 and 2.4, we expect there to be two basic tracing strategies: linear and on-demand. Prior work has shown that people will trace somewhat out of linear order [32], but we do not know precisely to what extent or how often. Hence our first goal for this experiment was to design a code viewing interface such that we can deduce the participants’ strategies from their viewing patterns. Then once we have identified which strategy a participant is using, we can hypothesize where errors are likely to occur:

- For linear tracing, the primary source of WM load should be remembering variable/value bindings. Based on Hitch’s model of errors in mental arithmetic [17], a person is more likely to forget values computed earlier in the program than later. Then participants should forget computations in the top lines of the program (leaf nodes) more frequently than later lines (inner nodes).
- For on-demand tracing, the primary source of WM load should be tracking the current path in the expression tree. Based on the predictions of cognitive load theory for hierarchical task structures [1–3], participants should make the most WM errors at the deepest sub-goal (i.e. leaf nodes). A WM error would cause a participant to forget their path to the leaf (i.e. inner nodes), so participants should revisit inner nodes more frequently than leaf nodes.
3.3.1 Methodology. In this experiment, we track where a participant’s attention goes as they trace through a straight-line program. Given the participant’s attention over time, we classify their tracing strategy as linear or on-demand. Then we identify working memory errors in their traces based on re-visits to lines of the program.

To track attention during tracing, we created a code viewing interface where each expression is blurred by default, i.e. a restricted focus viewer [19] (we did not use an eye-tracker so the experiment could be performed in a browser on Mechanical Turk). The participant can move their mouse over an individual line to bring it into focus. This way, the mouse acts like a foveal region in an eye tracking experiment — it represents the user’s current focus. W Figure 6 shows the interface. Because the mouse may briefly hover over other lines while moving to a destination, we discard any hover actions that occurred for less than 300ms. This number was selected by inspecting the noise in the data, but in the spirit of multiverse analysis [30] we confirmed that all significant results still hold with $p < 0.05$ for a threshold of 100ms and 500ms.

To generate straight-line programs, we start by randomly generating an arithmetic tree of size 7 with constants at the leaves and binary operators at the inner nodes. Every node is given a random single-letter variable name. To map a tree to a straight-line program, we have to pick an ordering that respects the dependencies in the tree, i.e. any topological sort. The main difference between sorts is the distance from a variable definition to its usage. For this experiment, we consider two conditions: sorts with the highest average distance (“Far”), and lowest average distance (“Close”). Figure 7 shows an example translation.

Each participant completed 5 trials in each condition (Far/Close), and this experiment had 15 participants. 3 participants were removed due to exceptionally poor performance (accuracy less than 20%), leaving 12 participants.

3.3.2 Results. First, for each trial, we classified the participant’s mouse movements as indicating an on-demand or linear strategy. As a simple heuristic, we classify a trial as linear if the first line is first visited before the last line, and on-demand otherwise. Across all trials, 55% were classified as on-demand and 45% as linear. Most participants appeared to individually prefer one strategy over another — 9 out of 12 participants adopted a single strategy at least 70% of the time. Figure 8 shows a sample of actual traces classified as linear (top) and on-demand (bottom) in the Far condition. In these examples, we can already observe a few WM influences:

- The top-left and top-center participants with the linear strategy traced linearly in the set of binary operations, but looked up constant values on-demand (consistent with Vainio and Sajanlemi [32]). Others like the top-middle-left participant traced linearly in both variables and binary operations.

- Some actions are clearly attributable to strategy vs. WM error. For example, the bottom-left participant goes from $b = o + s$ to $o = 6$, indicating an intentional on-demand shift in attention, but then returns to $b$ before moving to $s = c - y$. This indicates a WM error of forgetting the second operand to $b$. By contrast, the bottom-middle-left participant goes straight from $u$ to $t$ after observing $v = u - t$, indicating no comparable WM error.

To quantitatively analyze forgetting, we counted the number of times a participant re-visited each line of the program. We then categorized the lines as either leaves of the expression tree (constant variables) or inner nodes (binary operations). Figure 9 shows the distribution of average re-visits separated by participant’s strategy and the program sorting condition. To test for differences between distributions, we fit a linear mixed model with a three-way interaction of strategy, node type, and variable distance as the fixed effects and participant as the random effect. The number of revisits was the dependent variable. A one-way ANOVA showed a significant difference between revisits in the interaction of node type and strategy ($p < 0.001$), but not variable distance. We ran six post-hoc T-tests on all contrasts between pairs of the interaction, adjusted by the Tukey method. We found:

- Participants on average revisited nodes 1.37 more times per node in the on-demand strategy than the linear strategy ($T(200) = 8.66, p < 0.001$).
- Participants revisited inner nodes more than leaf nodes on average 2.1 more times in the on-demand strategy ($T(200) = 9.95, p < 0.001$), and 0.63 times in the linear strategy ($T(200) = 2.69, p = 0.038$).
- Participants revisited inner nodes an average of 1.11 more times in the on-demand strategy than the linear strategy ($T(216) = 4.23, p < 0.001$).
3.3.3 Discussion. We have found clear evidence that participants will adopt both on-demand and linear strategies for tracing straight-line code. The results also support our hypothesis that WM theory predicts where a participant will make errors given a strategy: at leaves for linear, and at inner nodes for on-demand. Moreover, we also found evidence that the linear strategy causes fewer WM errors than on-demand. That suggests that the cognitive load caused by tracking paths through the tree is greater than storing variable/value pairs in WM.

3.4 Function tracing strategy

Finally, we extend the methodology of the prior experiment to programs with functions. Similarly, we want to understand: what strategies will people use to trace programs with functions, and where are they most likely to make errors? Recall from Section 2.4 that tracing a function program starting from the top-level function call is equivalent to an on-demand tracing strategy in a straight-line program. When projected onto the abstract expression tree, both traces start at the root and proceed to the leaves.

However, now a linear strategy should be harder than before, because a person has to mentally reconstruct the dependency graph and topological sort before tracing linearly. Hence we hypothesize that more people should adopt an on-demand strategy than in the prior experiment. For a given strategy, though, we hypothesize that WM errors should occur most frequently as predicted in the prior experiment: on-demand traces forget the inner nodes, and linear traces forget the leaves.

3.4.1 Methodology. In this experiment, participants trace through a program with functions while we track which function has their attention. Like the prior experiment, we randomly generate an expression tree, and then to track the participant’s attention, we created a code viewing environment where one function is displayed at a
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Figure 11: Experiment interface for showing participants one function at a time.

time, shown in Figure 11. The participant is given a bank of buttons with the name of each function, and clicking on the button goes to the function’s source, similar to an IDE. A participant’s trace through the expression tree corresponds to their sequence of button clicks. The order of buttons is randomized, so it bears no relationship with respect to the functions’ order of usage in the program.

Each participant completed 10 trials, and this experiment has 15 participants (1 was removed for poor performance, leaving 14 participants).

3.4.2 Results. First, we classified each trace as an on-demand or linear strategy. We used a simple heuristic similar to before: if the participant visited the root function node first, they used an on-demand strategy, and they used a linear strategy for visiting any other node first. Figure 13 shows several examples of traces around the expression tree within each strategy. Out of 140 trials, 54% used a linear strategy and 46% used an on-demand strategy. Like the prior experiment, 11/14 participants consistently picked one strategy at least 70% of the time.

Next, we computed the average number of revisits to inner and leaf nodes in each trace. The distribution of revisits by strategy and node type is shown in Figure 12. As in the previous experiment, we fit a linear mixed model with strategy and node type as fixed effects and the participant as a random effect, and the number of revisits as the dependent variable. Using a one-way ANOVA, the only statistically significant difference in revisits is the node type ($F(2,63) = 41.06, p < 0.001$).

3.4.3 Discussion. These results provide mixed support for our first hypothesis. It was not true that most participants preferred an on-demand strategy. In fact, we found the preponderance of linear traces somewhat bewildering — participants appeared to just click through each function, remembering what they could and then synthesizing the information into an answer at the end. However, the results do suggest that the linear strategy had increased WM load for function programs compared to straight-line programs. In Experiment 4, there was not a statistically significant difference between WM errors in the on-demand and linear strategies, whereas on-demand traces contained more errors in Experiment 3.

The results do not support our second hypothesis, that participants revisit nodes as predicted by WM theory. The relative revisits to inner vs. leaf nodes was not different between strategies. One possible explanation is that the notion of a linear strategy is different for the straight-line interface than the function interface. The function interface does not expose the call graph, so a participant cannot easily walk from the leaves to the root as they can when scanning top-to-bottom in the straight-line code.

4 GENERAL DISCUSSION

In summary, we made six findings from our experiments:

- A person can hold about 7 variable/value pairs in working memory, and can remember variables named as random nouns equally well as random letters.
- Single-digit mental arithmetic does not reduce WM capacity for variable/value pairs.
- When recalling values for mental arithmetic, a majority of errors could be explained by recalling a value bound to a different variable held in WM.
- People will trace a straight-line program both linearly and on-demand, and a given person is likely to stick with one strategy.
- People will make WM errors in different parts of the program based on their tracing strategy, and on-demand strategy causes more WM errors than the linear strategy for a straight-line program.
- People will not necessarily trace a function in order of execution.

These findings occurred within a controlled experimental setting and within a specific application domain. As with all laboratory/cognitivist research, this naturally raises the question — do these results generalize to realistic programs? And how can we practicably apply these results to high-level design?

4.1 Limitations

Our findings are not universal statements about e.g. all variable/value pairs. For example, if a person tries to remember $x = [1, 5, 7]$, the list is a more complex object to be stored in WM than a single digit. A person likely cannot remember as many variable/list pairs as variable/number pairs. More generally, several factors limit the
Figure 13: A sample of traces from the participants, grouped by strategy. Each tree represents a dependency graph of function calls e.g. \( j \to e \) means the definition of \( j \) calls \( e() \).

The generality of our current results and point to interesting future work.

- **Data types:** we focus on integers in this work. But practical programs have booleans, strings, structs, classes, and so on. Variables assigned to values of different data types might be less likely to be swapped in WM.

- **Mental operations:** we focus on single-digit mental arithmetic with addition and subtraction. Harder mental operations may shift the balance e.g. in Experiment 2 and interfere with WM for program state.

- **Mutability:** we only test programs that assign to variables once, i.e. immutable state. Imperative programs often rely on in-place updates to databases, files, or so on. Mutating variables means updating an existing association between a variable/value in WM rather than adding a new one, which may be subject to different kinds of interference. Mutation also potentially increases the difficulty of on-demand tracing.

- **Control flow constructs:** we used functions due to their close relationship to on-demand tracing. Standard structured programming constructs like if-statements and for-loops should also be investigated for their WM influence.

- **Schematic structure:** our programs do not solve any common problem, e.g. sorting a list or computing the quadratic formula. A person’s tracing strategies and WM encodings of program state would likely differ for programs with a recognizable structure, i.e. pieces that fit into a known schema.

### 4.2 Implications for theory

One goal of this work is to contribute towards a broader theory of program comprehension: how do people understand programs, and how do aspects of cognition influence that process? We focused on program tracing because it is a likely component skill of comprehension. We have shown that WM both limits how much state a person can remember, and causes different kinds of errors...
based on tracing strategies. However, most popular models of program comprehension focus largely on schemata/plans as a mechanism of comprehension, e.g. the seven models in the survey of von Mayrhauser and Vans [33]. We think that tracing deserves a closer look to understand its role in comprehension.

One major finding of our work is that different people will adopt substantially different tracing strategies for the same program. We demonstrated that both the linear and on-demand strategies are common, and that individuals seem to prefer one strategy. But we do not yet know: what factors made a person pick one strategy over the other? And in what situations is a particular strategy better than the other? We found that linear tracing caused fewer WM errors than on-demand tracing for straight-line programs, but not for function programs. Future work should identify other factors that influence this trade-off.

Additionally, tracing as a task likely lies on a continuum of abstraction. We focused on tracing with concrete inputs and outputs, but one can imagine tracing over symbolic values, akin to the method of abstract interpretation in static analysis [8]. In this setting, variables are represented not by values but by properties. For example, if $x > 0$ and $y = x + 1$, then we know $y > 1$. Abstract tracing likely introduces even more WM load than concrete tracing. For example, while tracing "if ($x \neq \text{NULL}$) \{ ... \}", the property "$x \neq \text{NULL}$" must be kept in working memory while reasoning about the body of the if-statement. A WM account of tracing should consider tracing at all levels of abstraction.

### 4.3 Implications for design

Another goal of our research is to generate design principles for reducing WM load that can be applied to IDEs, refactoring tools, style guides, and so on. We suggest a few guidelines that extend from our findings. As a general framing, it’s important to observe that every experiment had significant between-subjects variability in memory capacity, time to completion, accuracy, strategy, and so on. Accessible programming tools need to account for working memory differences, e.g. tool designers should not assume that their users will be able to remember as much as themselves.

#### 4.3.1 Reduce variable scope

If a program has many overlapping variables, it will be hard for a person to keep the variables’ values in working memory as shown in Experiments 1 and 2. Therefore, programs should avoid having many variables within a given scope where possible. One implementation of this suggestion is to put a variable’s definition as close as possible to its usage. This advice is consistent with most modern style guides, for example the Google C++ Style Guide [15]. Our work confirms that this style has genuine cognitive benefits, as opposed to being a matter of preference.

This advice could also be mechanized as a complexity metric. The compiler technique of liveness analysis can identify when the live ranges (point from definition of variable to its last use) of variables will overlap. To test the technique, we applied it to every function in the Python standard library. We identified that some functions have up to 18 overlapping variables! We also found that functions with many parameters had a high complexity score, since they effectively are declaring dozens of variables at the top of a block rather than close to their usage, a de facto violation of the style rule.

#### 4.3.2 Visualize variable context

As shown in Experiment 3, people with different tracing strategies will make different kinds of WM errors, and so will need different kinds of tools to augment WM. A person tracing linearly needs to keep track of previously seen variables, and so a programming environment can visualize information about all the variables in scope up until a particular line. For example, the Lean interactive theorem prover [10] in Emacs (Figure 14, left) will show all the variables in scope at the current line based on the editor’s mouse position. Lean only shows the
name and type of the variable, but a visualization could also show the last line of code modifying that variable, or other semantic information.

By contrast, a person tracing on-demand needs to keep track of their path through the variable dependency graph so they can follow it back. If a person forgets a part of their path, a programming environment can visualize information about where a variable is used to help refresh their memory. For example, the DrRacket IDE [13] (Figure 14, right) will show all references to a variable on mouse hover. The point is that these ideas already exist in some form within (admittedly niche) programming tools, but every IDE should support both of these visualizations to reduce WM load during program tracing.

4.3.3 Externalize program state. Many of our experiments would be significantly different if the participants had been tracing on paper with a pen, able to write down program state such as variable values or flow markers. Providing programmers the ability to externalize their tracing process could significantly reduce working memory load.

However, nearly all interfaces for displaying code outside of an IDE are read-only. For example, GitHub has a specialized interface for pull requests that propose a change to a code base. The entire point of this interface is for a reviewer to understand a change, but the interface provides no “margin notes” or other features for throwaway annotations. Consistent with the theory of distributed cognition [18], code comprehension tools should consider how to enable programmers to externalize their thought process to the environment without needing to print out code.

5 CONCLUSION

In this work, we have shown that WM has a major role in maintaining program state during tracing. Beyond the basic predictions of WM theory (it is hard to remember a lot of state), we contribute a nuanced understanding of the type of WM errors a person can make. For example, programs involve associations between objects like variables and values, and WM errors involve not just forgetting but swapping these associations. A person will need to remember different kinds of information about a program based on their tracing strategy, and so they will make WM errors in different ways. With greater awareness of these issues, we can better design our programming tools to provide cognitive support for WM.

More broadly, we believe that our work demonstrates the enduring value of bridging the cognitive and computer sciences. Within HCI for programmers, constructivist theory-building has largely fallen by the wayside in favor of contextual inquiry and system-building. But programming is a wicked problem, and cognitive science offers a unique lens for understanding challenges that involve (mostly) universal human capabilities. The last major cogsci contribution of HCI to programming, the Cognitive Dimensions of Notation [16], happened 30 years ago. It is high time to revitalize the goal of building a theory of how people program, one that spans from low-level cognitive resources to high-level sociological phenomena.

REFERENCES

[1] John R Anderson and Robin Jefferies. 1985. Novice LISP errors: Undetected losses of information from working memory. Human–Computer Interaction 1, 2 (1985), 107–131.

[2] Paul Ayres and John Sweller. 1990. Locus of difficulty in multistage mathematics problems. The American Journal of Psychology 103, 2 (1990), 167–193.

[3] Paul L Ayres. 2001. Systematic mathematical errors and cognitive load. Contemporary Educational Psychology 26, 2 (2001), 227–248.

[4] Alan Baddeley. 2010. Working memory. Current biology 20, 4 (2010), R136–R140.

[5] Andrew Bragdon, Robert Zeleznik, Steven P Reiss, Suman Karumuri, William Chiang, Joshua Kaplan, Christopher Coleman, Ferdi Addepurut, and Joseph J LaVo- ola Jr. 2010. Code bubbles: a working set-based interface for code understanding and maintenance. In Proceedings of the SIGCHI Conference on Human Factors in Computing Systems. ACM, Atlanta, GA, 2503–2512.

[6] Teresa Bujalski, Roman Bednarik, Andrew Begel, Martha Crosby, James H Paterson, Carsten Schultz, Bonita Sharif, and Sascha Tamm. 2015. Eye movements in code reading: Relaxing the linear order. In 2015 IEEE 23rd International Conference on Program Comprehension. IEEE, IEEE, Florence, Italy, 255–265.

[7] Jamie JD Campbell and Neil Charness. 1990. Age-related declines in working-memory skills. Evidence from a complex calculation task. Developmental Psychology 26, 6 (1990), 879.

[8] Patrick Cousot and Radhia Cousot. 1977. Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints. In Proceedings of the 4th ACM SIGACT-SIGPLAN symposium on Principles of programming languages. ACM, Los Angeles, CA, 238–252.

[9] Nelson Cowan. 2001. The magical number 4 in short-term memory: A reconsideration of mental storage capacity. Behavioral and brain sciences 24, 1 (2001), 87–114.

[10] Leonardo de Moura, Soonho Kong, Jeremy Avigad, Floris Van Doorn, and Jakob von Raumer. 2015. The Lean theorem prover (system description). In International Conference on Automated Deduction. Springer, CADE, Berlin, Germany, 378–388.

[11] Diana DeStefano and Jo-Anne LeFevre. 2004. The role of working memory in mental arithmetic. European Journal of Cognitive Psychology 16, 3 (2004), 353–386.

[12] François Détienne and Elliot Soloway. 1990. An empirically-controlled contrast structure for the process of explaining program understanding. International Journal of Man-Machine Studies 33, 3 (1990), 323–342.

[13] Robert Bruce Findler. 2020. DrRacket: The Racket Programming Environment. https://docs.racket-lang.org/dracket.

[14] Assar J Füst and Graham J Hitch. 2000. Separate roles for executive and phonological components of working memory in mental arithmetic. Memory & Cognition 28, 5 (2000), 774–782.

[15] Google. 2020. Google C++ Style Guide. https://google.github.io/styleguide/cppguide.html.

[16] Thomas RG Green. 1989. Cognitive dimensions of notations. People and computers V, 5 (1989), 443–460.

[17] Graham J Hitch. 1978. The role of short-term working memory in mental arithmetic. Cognitive Psychology 10, 3 (1978), 302–323.

[18] Edwin Hutchins. 1995. Cognition in the Wild. MIT press, Cambridge, MA.

[19] Anthony R Jansen, Alan F Blackwell, and Kim Marriott. 2003. A tool for tracking visual attention: The restricted focus viewer. Behavior research methods, instruments, & computers 35, 1 (2003), 57–69.

[20] Stanley Letovsky. 1987. Cognitive processes in program comprehension. Journal of Systems and software 7, 4 (1987), 325–339.

[21] Raymond Lister, Colin Fidge, and Donna Teague. 2009. Further evidence of the role of the central executive. Behavioral and brain sciences 32, 10 (2009), 161–165.

[22] Mike Lopez, Jacqueline Whalley, Phil Robbins, and Raymond Lister. 2008. Relationships between reading, tracing and writing skills in introductory programming. In Proceedings of the fourth international workshop on computing education research. ACM, Sydney, Australia, 101–112.

[23] George A Miller. 1956. The magical number seven, plus or minus two: Some limits on our capacity for processing information. Psychological review 63, 2 (1956), 81.

[24] Neil Morris and Dylan M Jones. 1990. Memory updating in working memory: The role of the central executive. British journal of psychology 81, 2 (1990), 111–121.

[25] Peter A Nobel and Richard M Shiffrin. 2001. Retrieval processes in recognition and cued recall. Journal of Experimental Psychology: Learning, Memory, and Cognition 27, 2 (2001), 384.

[26] Klaus Oberauer, Stephan Lewandowsky, Edward Awh, Gordon DA Brown, An- drew Conway, Nelson Cowan, Christophon Donkin, Simon Farrell, Graham J Hitch, Mark J Hurlstone, et al. 2018. Benchmarks for models of short-term and working memory. Psychological Bulletin 144, 9 (2018), 885.

[27] Norman Petek, Janet Siegmund, and Sven Apel. 2020. What Drives the Reading Order of Programmers? An Eye Tracking Study. In Proceedings of the 28th International Conference on Program Comprehension. IEEE/ACM, Seoul, South Korea, 342–353.

[28] Martin J Pickering and Simon Garrod. 2007. Do people use language production to make predictions during comprehension? Trends in cognitive sciences 11, 3 (2007), 105–110.

[29] Juha Sorva. 2007. Notional machines and introductory programming education. Trans. Comput. Educ. 13, 2 (2007), 1–31.
[30] Sara Steegen, Francis Tuerlinckx, Andrew Gelman, and Wolf Vanpaemel. 2016. Increasing transparency through a multiverse analysis. *Perspectives on Psychological Science* 11, 5 (2016), 702–712.

[31] Nash Unsworth. 2009. Examining variation in working memory capacity and retrieval in cued recall. *Memory* 17, 4 (2009), 386–396.

[32] Vesa Vainio and Jorma Sajaniemi. 2007. Factors in novice programmers’ poor tracing skills. *ACM SIGCSE Bulletin* 39, 3 (2007), 236–240.

[33] Anneliese Von Mayrhauser and A Marie Vans. 1995. Program comprehension during software maintenance and evolution. *Computer* 28, 8 (1995), 44–55.

[34] Margaret Wilson and Gunther Knoblich. 2005. The case for motor involvement in perceiving conspecifics. *Psychological bulletin* 131, 3 (2005), 460.

[35] Jiajie Zhang and Donald A Norman. 1995. A representational analysis of numeration systems. *Cognition* 57, 3 (1995), 271–295.