A linear atomic quantum coupler

Faisal A A El-Orany1,2 and M R B Wahiddin2,3

1 Department of Mathematics and computer Science, Faculty of Science, Suez Canal University 41522, Ismailia, Egypt
2 Cyberspace Security Laboratory, MIMOS Berhad, Technology Park Malaysia, 57000 Kuala Lumpur, Malaysia
3 Faculty of Science, International Islamic University Malaysia (IIUM), Jalan Istana, Bandar Indera Mahkota, 25200 Kuantan, Pahang, Malaysia

E-mail: el_orany@hotmail.com, faisal.orany@mimos.my and mridza@mimos.my

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Abstract
In this paper we develop the notion of the linear atomic quantum coupler. This device consists of two modes propagating into two waveguides, each of which includes a localized atom. These waveguides are placed close enough to allow exchange of energy between them via evanescent waves. Each mode interacts with the atom in the same waveguide in the standard way as the Jaynes–Cummings model (JCM) and with the atom-mode system in the second waveguide via the evanescent wave. We present the Hamiltonian for this system and deduce its wavefunction. We investigate the atomic inversions and the second-order correlation function. In contrast to the conventional coupler the atomic quantum coupler is able to generate nonclassical effects. The atomic inversions can exhibit a long revival–collapse phenomenon as well as subsidiary revivals based on the competition among the switching mechanisms in the system. Finally, under certain conditions the system can yield the results of the two-mode JCM.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The quantum directional coupler is a device composed of two (or more) waveguides placed close enough to allow exchange of energy between them via evanescent waves [1]. The rate of flow of the exchanged energy can be controlled by the device design and the intensity of the input flux. The outgoing fields from the coupler can be measured in standard ways to quantify the nonclassical effects. Quite recently, this device has attracted much attention in optics communication and quantum computing networks [2], which require data transmission and ultra-high-speed data processing [3]. Furthermore, the directional coupler has been experimentally implemented, e.g., in planar structures [4], dual optical fibres [5] and certain organic polymers [6]. For more details related to the quantum properties of the fields in the directional couplers the reader can consult [7] and the references therein.

The interaction between the radiation field and the matter (i.e. atom), namely the Jaynes–Cummings model (JCM) [8], is an important topic in quantum optics and quantum information theories [9]. The simplest form of the JCM is a two-level atom interacting with a single-mode radiation field. The JCM is a rich source of the nonclassical effects, e.g., the revival–collapse phenomenon (RCP) [10], sub-Poissonian statistics and quadrature squeezing [11]. Furthermore, the JCM has been experimentally implemented by various means such as the one-atom mazer [12], NMR refocusing [13], the Rydberg atom in a superconducting cavity [14], the trapped ion [15] and the micromaser [16]. Various extensions to the JCM have been reported including two two-level atoms interacting with radiation field(s) [17, 18].

Trapped atoms or molecules are promising systems for quantum information processing and communications [19]. They can serve as convenient and robust quantum memories for photons, providing thereby an interface between static and flying qubits [20]. Coupling cold atoms to a radiation field sustained by an optical waveguide has already been addressed in various contexts. For example, hollow optical glass fibres have been used to guide atoms over long distances [21], specifically when employing a red detuned light field filling out the hollow core [22, 23]. A substrate-based atom waveguide can also be realized by using guided two-colour...
evanescent light fields [24]. Moreover, the coupling of atomic dipoles to the evanescent field of tapered optical fibres has been demonstrated in [25, 26]. In this respect, optical nanofibres can be used to manipulate and probe single-atom fluorescence. Moreover, it has been suggested that using a two-colour evanescent light field around a subwavelength diameter fibre can trap and guide atoms. The optical fibre can carry a red-detuned light and a blue-detuned light far from resonance. When both input light fields are circularly polarized, a set of trapping minima of the total potential in the transverse plane appears as a ring around the fibre. This design confines the atoms in a cylindrical shell around the fibre [27]. Additionally, it has been shown that sub-wavelength diameter optical fibres can be used to detect, spectroscopically investigate and mechanically manipulate extremely small samples of cold atoms. In particular, on resonance, as little as two atoms on average, coupled to the evanescent field surrounding the fibre, already absorb 20% of the total power transmitted through the fibre [25]. By optically trapping one or more atoms around the fibres [28], it becomes possible to deterministically couple the atoms to the guided fibre mode and to establish coupling between two simultaneously trapped atoms [29]. This leads to a number of applications in quantum information processing, high precision measurements, single-photon generation in optical fibres or EIT-based parametric four-wave mixing [30] using a few atoms around optical nanofibres. Inspired by these facts, here we develop the notion of the atomic quantum coupler (AQC), in which the interaction mechanisms inside the waveguides as well as between them depend on both atomic and bosonic systems. These mechanisms are more complicated than those of the JCM, as we shall demonstrate. For this system, we obtain many interesting results. For instance, the atomic inversions can exhibit a long revival–collapse phenomenon as well as subsidiary-revival patterns based on the switching mechanisms in the system. Furthermore, under certain conditions the system can give the results of the two-mode JCM. Also, the system is able to generate nonclassical effects. We conclude this part by mentioning that the inclusion of one atom in one of the ports of the nonlinear coupler has been already considered in [31]. Nevertheless, the solution of the equations of motion there was obtained by the rotation of axes, which cannot give complete information on the system.

We restrict the study in this paper to the development of the Hamiltonian model, how it works and its dynamical wavefunction. These issues are discussed in sections 2 and 3. In section 4, we investigate the behaviour of two quantities, namely the atomic inversions and the second-order correlation functions.

2. Model formalism

In this section, we describe the linear directional atomic quantum coupler (AQC) and explain how it works. Before doing so, we shed some light on the linear directional coupler, which is described by the following Hamiltonian [7]:

\[ \frac{\hat{H}}{\hbar} = \sum_{j=1}^{2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \lambda (\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_1^\dagger \hat{a}_2), \]

where \( \hat{a}_j (\hat{a}_j^\dagger) \) and \( \hat{a}_2 (\hat{a}_2^\dagger) \) are the annihilation (creation) operators of the first and second modes in the first and second waveguides with frequencies \( \omega_1 \) and \( \omega_2 \); \( \lambda \) is the coupling constant between the waveguides. Basically this device operates as a quantum switcher since it can switch the nonclassical effects as well as the intensity of the mode propagating inside one of the waveguides to the other [32]. Nevertheless, it cannot generate nonclassical effects by itself. For reasons that will be clear shortly, we write down the mean-photon numbers of the modes controlled by Hamiltonian (1) when they are initially in the states \( |\alpha, 0\rangle \) as

\[ \langle \hat{a}_1^\dagger (T) \hat{a}_1 (T) \rangle = |\alpha|^2 \cos^2 (T), \]

\[ \langle \hat{a}_2^\dagger (T) \hat{a}_2 (T) \rangle = |\alpha|^2 \sin^2 (T), \]

where \( T = \lambda t \). These equations show a strong switching process, where the intensity \( |\alpha|^2 \) in the first waveguide can be completely transferred to the second one after a certain interaction time. It is evident that the mean-photon numbers cannot exhibit the well-known RCP.

Now we are in a position to develop the AQC, which is the main objective of the paper. The atomic coupler consists of two waveguides, each of which includes a localized or a trapped atom. The waveguides are placed close enough to each other to allow energy exchange between them. The two atoms, in the different waveguides, are located adjacent to each other. In each waveguide, one mode propagates along and interacts with the atom inside in a standard way as the JCM. The atom-mode system, in each waveguide, interacts with the other one via the evanescent wave. The outgoing fields from the coupler can be examined as single or compound modes by means of the homodyne detector to observe squeezing of the vacuum fluctuations. It can also be examined by a set of photodetectors to measure photon antibunching and sub-Poissonian photon statistics in standard ways. The scheme for the AQC is depicted in figure 1. From this figure, and in the framework of the rotating wave approximation (RWA), the Hamiltonian describing the RCP. 

\[ \frac{\hat{H}}{\hbar} = \hat{H}_0 + \hat{H}_1, \]

\[ \hat{H}_0 = \sum_{j=0}^{2} \omega_j \hat{a}_j^\dagger \hat{a}_j + \frac{\omega_0}{2} (\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}). \]

\[ \hat{H}_1 = \hat{H}_1 + \hat{H}_2 + \lambda_3 (\hat{a}_1^\dagger \hat{a}_3^\dagger \hat{a}_2 \hat{\sigma}_y^{(1)} \hat{\sigma}_y^{(2)} + \hat{a}_1 \hat{a}_3 \hat{\sigma}_y^{(1)} \hat{\sigma}_y^{(2)}), \]

\[ \hat{H}_1 = \lambda_i (\hat{a}_j \hat{\sigma}_x^{(i)} + \hat{a}_j^\dagger \hat{\sigma}_x^{(i)}), \]

\[ j = 1, 2, \]

where \( \hat{H}_0 \) and \( \hat{H}_1 \) are the free and the interaction parts of the Hamiltonian; \( \hat{\sigma}_x^{(i)} \) and \( \hat{\sigma}_x^{(i)} \) are the Pauli spin operators of the \( j \)th atom (\( j = 1, 2 \); \( \hat{a}_j \)) is the annihilation (creation) operator of the \( j \)th mode with the frequency \( \omega_j \), and \( \omega_0 \) is the atomic transition frequency (we consider that the frequencies of the two atoms are equal) and \( \lambda_3 \) is the atom–field coupling constant in the first (second) waveguide in the framework of the JCM. The derivation of the JCM Hamiltonian is well known in the literature, e.g. [33]. The interaction between the systems in the two waveguides occurs through the evanescent wave with the coupling constant \( \lambda_3 \). This term is the only one which is conservative and can execute switching of the energy between the waveguides. Thus, it plays an essential role in the behaviour of the AQC. As a
result of this we give some details related to the construction of this term. In the conventional coupler (apart from the atoms) the energy exchange between the waveguides occurs through the up-conversion process, i.e. the evanescent wave $\hat{a}_1^{\dagger}\hat{a}_2^\lambda + \hat{a}_2^{\dagger}\hat{a}_1^\lambda$ (cf (1)). In the AQC the action of the atoms in this process has to be taken into account. With this in mind, we can conclude that the switching mechanism can occur via terms of type $(\hat{a}_1^{\dagger}\hat{a}_2^{(1)}\hat{a}_1^{(2)} + \hat{a}_2^{\dagger}\hat{a}_1^{(1)}\hat{a}_2^{(2)})$ and $(\hat{a}_1^{\dagger}\hat{a}_2^{(1)}\hat{a}_2^{(2)} + \hat{a}_2^{\dagger}\hat{a}_1^{(1)}\hat{a}_1^{(2)})$. As the energy exchange occurs between two JCMs, the term which does not fulfil the RWA in its waveguide, i.e. the last term, has to be cancelled, or we can look at the problem from the reverse side. For instance, the switching mechanism is proportional to an interaction of type $\hat{H}_{3}\propto (\hat{a}_1^{\dagger}\hat{a}_2\hat{\sigma}_-^{(1)}\hat{\sigma}_+^{(2)} + \hat{a}_2^{\dagger}\hat{a}_1\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(2)}) + (\hat{a}_1^{\dagger}\hat{a}_2\hat{\sigma}_+^{(1)}\hat{\sigma}_-^{(2)} + \hat{a}_2^{\dagger}\hat{a}_1\hat{\sigma}_-^{(1)}\hat{\sigma}_+^{(2)})$, where $\hat{H}_j$ is the Hamiltonian of the $j$th JCM. The first term has to be cancelled since it does not fit with the notion of the evanescent wave. This is just a visualization for the interaction between the waveguides. We should stress that in (3) the treatment is considered only at the moment when the two fields interact with the localized atoms in the waveguides. As a final remark: when we treat the atoms (fields) classically Hamiltonian (3) tends to that of the linear directional coupler (two-atom interaction).

As we have mentioned above, one of the resources for the JCM is the trapped ion technique. This is just one justification; however, the JCM can be obtained by other means, e.g. the Rydberg atom in high-$Q$ microwave cavities [34]. Nevertheless, for the trapped ion technique various questions may be raised related to the translational degree of freedom of the trapped atoms, the quantization axes, the polarizations of the trapping fields as well as the coupling fields, Zeeman sublevels, the effect of spontaneous emission, etc. These issues have been treated in the literature, e.g. [35, 36]. Here we give a brief answer to these questions. In each waveguide, we have a JCM which can be constructed via trapped ion in the standard way as follows. In first waveguide, say, assume that we have a spin qubit with a ground state labelled $|g\rangle$ and a higher metastable state labelled $|e\rangle$ that are separated in energy by $\omega_{se}$. In single-photon transitions, these levels can be excited through a focused laser beam. Thus, the interaction between an ion and the electric field of the laser beam, i.e. the electric dipole transitions, in the interaction frame can be reduced to that of the JCM. In this case, conditions such as the rotating wave approximation, Lamb–Dicke limit and the red-sideband coupling have to be fulfilled. The scheme describing the transition between the vibrational levels of ions inside the waveguide is just figure 3(b) in [36]. As a result of the energy exchange between the waveguides, through the evanescent wave, the two atoms are entangled. There is no need to consider Zeeman sublevels, which are related to the static magnetic field. Additionally, in the trapped ion, for most applications the motion of the ions has to be considered along the axis of a linear Paul trap. Principal interest is always paid to the situation where, at any given time, there are only two internal levels of an ion. This can be established by ensuring that the internal states are nondegenerate and by using resonant excitations to couple only two levels at a time. Moreover, the spontaneous emission decoherence could, in principle, be nearly eliminated by driving single-photon transitions between ground-state-hyperfine or Zeeman levels with rf or microwave radiation since spontaneous emission from these levels is negligible. We should stress that it is difficult to solve the master equation related to the spontaneous emission in the AQC. Nevertheless, it is known that decoherence generally leads to degradation of the nonclassical effects in quantum systems.

The interaction of two two-level atoms with the two modes in the optical cavity has been considered earlier [18, 37, 38], although in a form different from that presented here. For instance, a sum of two separate Jaynes–Cummings Hamiltonians has been used to investigate the entanglement [37] as well as the entanglement transfer from a bipartite continuous-variable system to a pair of localized qubits [38]. Furthermore, the quantum properties of a system of two two-level atoms interacting with two nondegenerate cavity modes against the pair-coherent state have been investigated in [18].

3. Derivation of the wavefunction

In this section, we derive the wavefunction associated with Hamiltonian (3) using Schrödinger’s equation. We comment on some special cases of this Hamiltonian. Moreover, we
discuss some basic differences between this device and the conventional directional coupler [7].

We assume that the two modes and atoms are initially prepared in the coherent states \([a, b]\) and in the excited atomic states \([e_1, e_2]\) respectively. For the resonance case \(2\omega_0 = \omega_1 + \omega_2\) one can easily prove that \(\hat{H}_0, \hat{H}_1 = 0\). Under these conditions, the dynamical wavefunction describing the system can be expressed as

\[
|\Psi(t)\rangle = \sum_{n,m=0}^{\infty} C_{n,m}[X_1(t,n,m)|e_1, e_2, n, m\rangle + X_2(t,n,m)|e_1, g_2, n, m+1\rangle + X_3(t,n,m)|g_1, e_2, n+1, m\rangle + X_4(t,n,m)|g_1, g_2, n+1, m+1\rangle],
\]

\[C_{n,m} = \exp\left(\frac{1}{2}[a^2 - \frac{1}{2}|b|^2]g^m\right)\sqrt{a^m b^n},\]

where \(|g_j\rangle\) stands for the ground state of the \(j\)th atom. Now we give some details about the construction of wavefunction (4). In doing so we assume that the modes and the atoms are initially in the Fock states and the excited atomic states as \([e_1, e_2, n, m]\). Thus, the evolution of the system, based on Hamiltonian (3), generates entanglement among all the components of the system. A result of this, the wavefunction becomes a linear combination of the bases \(S = \{|e_1, e_2, l_1, l_2\}, |e_1, g_2, l_1, l_2\}, |g_1, e_2, l_1, l_2\}, |g_1, g_2, l_1, l_2\}\). The explicit values of \(l_i\) can be obtained in terms of \(n, m\) by showing that \(S\) is closed under the action of \(\hat{H}_1\). This is a direct consequence of the energy conservation of the system. For instance, the form \(\hat{H}_1|e_1, e_2, n, m\rangle\) leads to that \(l_1 = n, l_2 = m, l_1' = n, l_2' = m + 1, l_1 = n + 1, l_2 = m\). Also, \(\hat{H}_1|e_1, g_2, n, m+1\rangle\) gives \(l_1' = n + 1, l_2' = m + 1\). Thus, the wavefunction of the system with the initial coherent states is represented as summation over the linear combination of the elements of the set \(S\) weighted by the field probability distributions given by (4). Next, the explicit forms of the coefficients \(X_j(t)\) can be obtained by solving Schrödinger’s equation

\[
\hat{H} \frac{\partial |\Psi(t)\rangle}{\partial t} = \hat{H}_1 |\Psi(t)\rangle.
\]

Invoking Hamiltonian (3) and wavefunction (4) into (5), using simple algebra and comparing the coefficients of the bases in both sides, we arrive at

\[
\begin{align*}
\dot{X}_1(t,n,m) &= \lambda_2 \sqrt{m+1} X_2(t,n,m) + \lambda_1 \sqrt{n+1} X_3(t,n,m), \\
\dot{X}_2(t,n,m) &= \lambda_2 \sqrt{m+1} X_1(t,n,m) + \lambda_3 \sqrt{(n+1)(m+1)} \times X_3(t,n,m) + \lambda_1 \sqrt{n+1} X_4(t,n,m), \\
\dot{X}_3(t,n,m) &= \lambda_1 \sqrt{n+1} X_1(t,n,m) + \lambda_2 \sqrt{(n+1)(m+1)} \times X_2(t,n,m) + \lambda_3 \sqrt{m+1} X_4(t,n,m), \\
\dot{X}_4(t,n,m) &= \lambda_1 \sqrt{n+1} X_2(t,n,m) + \lambda_2 \sqrt{m+1} X_3(t,n,m),
\end{align*}
\]

where the dot stands for differentiation with respect to time. In the following, we give only the details related to the solution of the coefficient \(X_1(t,n,m)\), where the rest can be similarly treated. Differentiating the first and last equations in (6) and then re-substituting by \(X_2(t,n,m)\), \(X_3(t,n,m)\) we obtain

\[
\begin{align*}
\dot{(X_1^2 + A_{n,m})X_1(t,n,m)} &= -(ic_2 D + c_1)X_4(t,n,m), \\
\dot{X_2(t,n,m)} &= -(ic_2 D + c_1)X_1(t,n,m),
\end{align*}
\]

(7)

\[
\hat{D} = \frac{d}{dt} A_{n,m} = \lambda_1^2 (n+1) + \lambda_2^2 (m+1),
\]

\[c_1 = 2\lambda_1 \lambda_2 \sqrt{(n+1)(m+1)}, \quad c_2 = \lambda_3 \sqrt{(n+1)(m+1)}.\]

From (7) one can easily obtain

\[
\dot{(X_1^2 + A_{n,m})^2X_1(t,n,m)} = (ic_2 D + c_1)^2X_1(t,n,m).
\]

(8)

This equation can be easily solved using the characteristic-equation technique. By means of the initial conditions, stated above, the exact forms of the coefficients \(X_j\) can be expressed as

\[
\begin{align*}
X_1(t,n,m) &= \frac{1}{2} \exp\left(\frac{t}{2}\right) \left[\cos(t\Omega_+) - \frac{c_2}{2\Omega_+} \sin(t\Omega_-)\right] \\
&+ \frac{1}{2} \exp\left(-\frac{t}{2}\right) \left[\cos(t\Omega_-) + \frac{c_2}{2\Omega_-} \sin(t\Omega_+)\right],
\end{align*}
\]

\[
\begin{align*}
X_2(t,n,m) &= \frac{1}{2\Omega^2} \left[\lambda_2 c_1 - \frac{\lambda_1 \lambda_2}{2} \left(\frac{n}{m+1}\right)\right] \sin(t\Omega_-) \\
&+ \exp\left(-\frac{t}{2}\right) \left[\left(c_2^2 - 2 c_1\right) \frac{\lambda_2^2}{2} \left(\frac{n}{m+1}\right)\right] \\
&- \frac{1}{2\Omega^2} \left[\lambda_2 c_1 - \frac{\lambda_1 \lambda_2}{2} \left(\frac{n}{m+1}\right)\right] \sin(t\Omega_-)
\end{align*}
\]

(9)

\[
\begin{align*}
\dot{X_3(t,n,m)} &= \frac{i}{2\Omega^2} \left[\lambda_1 c_1 - \frac{\lambda_2^2}{2} \left(\frac{n}{m+1}\right)\right] \sin(t\Omega_-) \\
&- \frac{1}{2\Omega^2} \left[\lambda_1 c_1 - \frac{\lambda_2^2}{2} \left(\frac{n}{m+1}\right)\right] \sin(t\Omega_-)
\end{align*}
\]

(10)

It is obvious that the Rabi oscillations \(\Omega_\pm\) of the AQC are more complicated than those of the JCM.

The AQC includes four components, namely the first mode, the second mode, the first atom and the second atom
denoted by $F_1, F_2, A_1, A_2$, respectively. From wavefunction (4) the total density matrix of the system can be written as

$$\hat{\rho}_{F_1,F_2,A_1,A_2}(t) = |\Psi(t)\rangle \langle \Psi(t)|.$$  

(11)

By means of (11), we can study the behaviour of any subsystem by tracing out the others. In this case one has to use the completeness relations of the atoms and of the radiation fields as well. For instance, to study the behaviour of the first mode $F_1$ we have to use the relation

$$\hat{\rho}_{F_1}(t) = \text{Tr}_{F_2,A_1,A_2}[\hat{\rho}_{F_1,F_2,A_1,A_2}(t)].$$  

(12)

where $\text{Tr}$ stands for the matrix trace and the subscript $F$.

Next, from solution (9) different limits can be checked. For instance, when $(\lambda_2, \lambda_3) \to (0, 0)$ the coefficients (9) reduce to those of the standard JCM (two decoupled JCM [37]). Moreover, when $(\lambda_1, \lambda_2) \to (0, 0)$ the system reduces to a simple form, which is in good correspondence with the conventional coupler (1). Nevertheless, the device in this case is a rich source of nonclassical effects. These effects are sensitive to the type of initial atomic states, which can be explained as follows. (i) When the atoms are initially prepared in $|e_1, e_2\rangle$ (or $|g_1, g_2\rangle$) the system is reduced to the dark state, where $\hat{H}_1|e_1, e_2\rangle = 0$. These states do not evolve with time. This property has been exploited in quantum clock synchronization [39]. (ii) When the atoms are initially prepared in $|e_1, g_2\rangle$ the dynamical state of the system takes the form

$$|\Psi(T)\rangle = \sum_{n,m=0}^{\infty} C_{n,m}[\cos[T\sqrt{(n+1)(m+1)}]|e_1, g_2, n, m+1\rangle + i\sin[T\sqrt{(n+1)(m+1)}]|g_1, e_2, n+1, m\rangle],$$  

(13)

where $T = \lambda_3 t$. Expression (13) reveals that the behaviour of the radiation fields is typically that of the two-mode single-atom JCM [40]. It is worth mentioning that the quantum gates have been implemented by the two-atom system as shown in [41]. Roughly speaking, this can also be realized for the AQC. For instance, if we set $n = m = 0$ and $T = \pi/2$ in (13), the crossover as well as the phase gates can be obtained through the relation $|e_1, g_2\rangle \leftrightarrow \exp(-i\pi/2)|g_1, e_2\rangle$. By the crossover gate we mean the interchange between the excited and the ground atomic states. This is just a simple example for one of the applications of the AQC in the quantum information theory. (iii) When the two atoms are initially in the Bell state $|e_1, g_2\rangle + |g_1, e_2\rangle/\sqrt{2}$ the wavefunction takes the form

$$|\Psi(T)\rangle = \frac{1}{\sqrt{2}} \sum_{n,m=0}^{\infty} C_{n,m} \exp[-iT\sqrt{(n+1)(m+1)}]$$  

$$\times |e_1, g_2, n, m+1\rangle + |g_1, e_2, n+1, m\rangle.$$  

(14)

It is evident that the system exhibits atomic trapping, i.e. $\langle \hat{a}_1^{(1)}(T)\rangle = \langle \hat{a}_2^{(1)}(T)\rangle = 0$. Furthermore, the system is able to generate nonclassical effects, in particular, in the quantities which depend on the off-diagonal elements of the density matrix such as quadrature squeezing.

Now, we comment on the switching mechanism in the AQ. For the sake of comparison, we set $\beta = 0$ in relations (4)–(9), (13) and calculate the mean photon numbers as

$$\langle \hat{a}_1^{(1)}(T)\hat{a}_1^{(1)}(T)\rangle = |\alpha|^2 + \sum_{n=0}^{\infty} |C_{n,0}|^2[|X_2(T, n, 0)|^2 + |X_4(T, n, 0)|^2].$$  

(15)

Expression (15) reveals that the behaviour of the atomic inversions and the second-order correlation functions.

$$\langle \hat{a}_2^{(1)}(T)\hat{a}_2^{(1)}(T)\rangle = \sum_{n=0}^{\infty} |C_{n,0}|^2[|X_2(T, n, 0)|^2 + |X_4(T, n, 0)|^2].$$  

where $T = \lambda_1$. For wavefunction (13) we have $X_2(\cdot) = \cos(\cdot), X_4(\cdot) = -i\sin(\cdot), X_4(\cdot) = 0$. From these equations it is obvious that the intensity of the mode in the first waveguide cannot be switched to the other one. This is in clear contrast with the linear direction coupler (compare (2) to (15)). This behaviour is related to the nature of the atom–field interaction mechanism, which is close to the classic Lee model of quantum field theory. Moreover, this behaviour is still valid even if the interaction between the modes and the atoms in the same waveguide is neglected, i.e. $\lambda_1 = \lambda_2 = 0$, and the atoms are initially in $|e_1, g_2\rangle$ (cf (13)). In this case, expressions (15) exhibit the well-known RCP of the standard JCM [10]. This indicates that the AQC is able to generate nonclassical effects (i.e. RCP) in the second waveguide even when the second mode is initially in the vacuum state. We proceed by assuming that $\lambda_j \neq 0$ and the mode in the first waveguide is initially prepared in the even coherent state, while the second mode is still in the vacuum state. Therefore, the density matrix of the second mode takes the form

$$\hat{\rho}_2 = \sum_{n=0}^{\infty} |C_{2n}|^2[|X_1(T, 2n, 0)|^2 + |X_3(T, 2n, 0)|^2]|0\rangle\langle 0| + |X_2(T, 2n, 0)|^2 + |X_4(T, 2n, 0)|^2]|1\rangle\langle 1|].$$  

(16)

where $|C_{2n}|^2$ is the photon-number distribution of the even coherent state. It is worth recalling that even coherent states can exhibit squeezing. From (16), squeezing cannot be switched to the second mode. Nevertheless, if the second mode is prepared in the coherent state, it may exhibit squeezing. In this case, the source of the nonclassical effects may be the switching mechanism between the waveguides or the nature of the atom–field interaction, or both.

In the following section we use the above relations to investigate the atomic inversions and the second-order correlation functions.

4. Atomic inversions and second-order correlation functions

Atomic inversion of the standard JCM is well known in quantum optics by the occurrence of the RCP. The RCP has a nonclassical origin and reflects the nature of the statistics of the radiation field. The evolution of atomic inversion has been realized by the one-atom mazer [12] and using a technique similar to that of the NMR refocusing [13]. In this section, we investigate the behaviour of the atomic inversions and the second-order correlation functions of the AQC. The system includes two atoms and hence we have two forms of atomic inversion, namely single atomic inversion and total atomic...
inversion as given by $\langle \hat{\sigma}_z(T) \rangle = \frac{1}{2}[\langle \hat{\sigma}_z^{(1)}(T) \rangle + \langle \hat{\sigma}_z^{(2)}(T) \rangle]$. From wavefunction (4) one obtains the following expressions:

$$\langle \hat{\sigma}_z^{(1)}(T) \rangle = \sum_{n,m=0}^{\infty} |C_{n,m}|^2 [|X_1(T, n, m)|^2 + |X_2(T, n, m)|^2 - |X_3(T, n, m)|^2 - |X_4(T, n, m)|^2],$$

$$\langle \hat{\sigma}_z^{(2)}(T) \rangle = \sum_{n,m=0}^{\infty} |C_{n,m}|^2 [|X_1(T, n, m)|^2 - |X_2(T, n, m)|^2 + |X_3(T, n, m)|^2 - |X_4(T, n, m)|^2],$$

$$\langle \hat{\sigma}_z(T) \rangle = \sum_{n,m=0}^{\infty} |C_{n,m}|^2[|X_1(T, n, m)|^2 - |X_4(T, n, m)|^2].$$

(17)

As we have mentioned in section 2, the conventional directional coupler cannot exhibit the RCP in the evolution of the mean photon numbers. Nevertheless, the standard JCM can exhibit the RCP provided that the photon-number distribution of the initial field has a smooth envelope. A similar conclusion has been reported to the two-atom single-mode JCM [17]. For the AQC we have found that when $\alpha = \beta$ and $\lambda_j \neq 0$ the different types of atomic inversions (17) provide quite similar behaviours. It seems that the contributions of the coherence coefficients $X_2, X_3$ are comparable. Furthermore, when $\lambda_3 = 0$ and $\lambda_1 = \lambda_2$, the atomic inversions of the AQC reduce to those of the standard JCM (see figure 2(a)).

It is worth recalling that the revival patterns in the atomic inversion of the JCM occur over a certain period of the interaction time. Afterwards, they interfere with each other, providing a chaotic behaviour. Additionally, the revival time is connected with the amplitude $\alpha$ through the relation $T_r = \frac{2\pi}{\sqrt{\bar{n}}} \simeq 2\pi|\alpha|$ [10]. We proceed with the fact that for the AQC with $\lambda_1 \neq \lambda_2$ the atomic inversions provide different forms of the revival patterns. Here we restrict attention to the atomic inversion of the first atom (see figures 2(b)–(d) for the given values of the interaction parameters). We study three cases based on the relationship between the strength of the switching mechanisms in and between the waveguides, namely $\lambda_3 < \lambda_j, \lambda_3 = \lambda_j, \lambda_3 > \lambda_j$. Comparisons
between figures 2(b)–(d) and figure 2(a) are instructive. From figure 2(b) one can observe that the atomic inversion, after the zero and the first revival patterns, exhibits a long series of the subsidiary-revival patterns (see the inset in figure 2(b)). This behaviour is completely different from that of the JCM. This indicates that the nonclassical effects generated by this device can sustain for an interaction time longer than that of the JCM. It is worth mentioning that the subsidiary-revival patterns have been reported earlier for the JCM against the squeezed coherent state [42]. This was explained in conjunction with the photon-number distribution of the initial states. More illustratively, the photon-number distributions of the squeezed states exhibit many peak structures, each of which gives its own revival patterns in the evolution of the atomic inversion. These patterns interfere with each other to produce these subsidiary-revival patterns [42]. Nevertheless, for the system under consideration the occurrence of these patterns is related to the switching mechanism between the waveguides (compare figure 2(a) with figure 2(b)). This mechanism manifests itself in very complicated Rabi oscillations $\Omega_\omega$ as well as in the double summations in the atomic inversion formulae (17). Figure 2(c) presents the case when the coupling constants are different. It is noteworthy that the RCP is still remarkable and the subsidiary revivals are smoothly washed out compared to those in figure 2(b). Generally, we have found that when $\lambda_3 \geq \lambda_1 = \lambda_2$ the atomic inversion exhibits a long RCP as shown in figure 2(d). The above discussion indicates that the switching mechanism between the waveguides plays an important role in the behaviour of the AQC. Actually, we have faced a lot of difficulties in giving a mathematical treatment for the RCP of the AQC since the Rabi oscillations are rather complicated.

Now we draw the attention to the second-order correlation functions for the single-mode case, which is defined as

$$g^{(2)}_j(t) = \frac{\langle \hat{a}_j^\dagger(t)\hat{a}_j^\dagger(t)\hat{a}_j(t)\hat{a}_j(t) \rangle}{\langle \hat{a}_j^\dagger(t)\hat{a}_j(t) \rangle^2} - 1, \quad j = 1, 2,$$

(18)

where $g^{(2)}_j(t) = 0$ for Poissonian statistics (standard case), $g^{(2)}_j(t) < 0$ for sub-Poissonian statistics (nonclassical effects) and $g^{(2)}_j(t) > 0$ for super-Poissonian statistics (classical effects). The second-order correlation function can be measured by a set of two detectors such as the standard
References

[1] Jensen S M 1982 IEEE J. Quantum Electron. 18 1580
[2] Nikolopoulos G M 2008 Phys. Rev. Lett. 101 200502
[3] Ekert A and Jozsa R 1996 Rev. Mod. Phys. 68 733
Lo H-K, Popescu S and Spiller T 1998 Introduction to Quantum Computation and Information (Singapore: World Scientific)
Begie A, Braun D, Tregnna B and Knight P L 2000 Phys. Rev. Lett. 85 1762
[4] Kamwa L P, Stitch J E, Mason N J and Roberts P N 1985 Electron. Lett. 21 26

Jin R, Chuang C L, Gibbs H H, Koch S W, Polky J N and Pubanz G A 1988 Appl. Phys. Lett. 53 1791
[5] Gusovkii D D, Dianov E M, Mainer A A, Neustreuev V B, Shklovskii E I and Scherbakov I A 1985 Sov. J. Quantum Electron. 15 1523
[6] Townsend P D, Baker G L, Shelburne J L III and Etemad S 1989 Proc. SPIE 1147 256
[7] Pehfia J Jr and Pehfina J 2000 Progress in Optics vol 41, ed E Wolf (Amsterdam: Elsevier) p 361
[8] Jaynes E T and Cummings F W 1963 Proc. IEEE 51 89
[9] Bose S, Fuentes-Guridi I, Knight P L and Vedral V 2001 Phys. Rev. Lett. 87 050401
[10] Eberly J H, Narozhny N B and Sanchez-Mondragon J J 1980 Phys. Rev. Lett. 44 1323
Narozhny N B, Sanchez-Mondragon J J and Eberly J H 1981 Phys. Rev. A 23 236
Yoo H I, Sanchez-Mondragon J J and Eberly J H 1981 J. Phys. A: Math. Gen. 14 1383
Yoo H I and Eberly J H 1985 Phys. Rep. 118 239
[11] El-Orany F A A and Obada A-S 2003 J. Opt. B: Quantum Semiclass. Opt. 5 60
[12] Rempe G, Walther H and Klein N 1987 Phys. Rev. Lett. 57 353
[13] Meunier T, Gleyzes S, Maioli P, Auffeves A, Nogues G, Brune M, Raimond J M and Haroche S 2005 Phys. Rev. Lett. 94 010401
[14] Yeazell J A, Mallalieu M and Stroud C R Jr 1990 Phys. Rev. Lett. 64 1997
Brune M, Schmidt-Kaler F, Maali A, Dreyer J, Hagley E, Raimond J M and Haroche S 1996 Phys. Rev. Lett. 76 1800
[15] Vogel W and De Matos Filho R L 1995 Phys. Rev. A 52 4214
[16] Meschede D, Walther H and Muller G 1985 Phys. Rev. Lett. 54 551
Walther H 1992 Phys. Rep. 219 263
[17] Tessier T E, Deutsch H L and Delgado A 2003 Phys. Rev. A 68 062316
El-Orany F A A 2006 J. Phys. A: Math. Gen. 39 3397
[18] El-Orany F A A 2006 Phys. Scr. 74 563
[19] El-Orany F A A, Obada A-S F, Abdelslama M A and Wahiddin M R B 2008 J. Mod. Opt. 55 1649
[20] Nielsen M and Chuang I 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)
Lambropoulos P and Petrosyan D 2006 Fundamentals of Quantum Optics and Quantum Information (Berlin: Springer)
Lukin M D 2003 Rev. Mod. Phys. 75 457
Petrosyan D 2005 J. Opt. B: Quantum Semiclass. Opt. 7 S141
[21] Noh H-R and Jhe W 2002 Phys. Rep. 372 269
[22] Olshani M A, Ovchinnikov Y B and Letokhov V S 1996 Phys. Rep. 256 9
[23] Renn M J, Montgomery D, Vdovin O, Anderson D Z, Wieman C E and Cornell E A 1995 Phys. Rev. Lett. 75 3253
[24] Barnett A H, Smith S P, Olshani M, Johnson K S, Adams A W and Prentiss M 2000 Phys. Rev. A 61 023608
Sague G, Vetsch E, Alt W, Meschede D and Rauschenbeutel A 2007 Phys. Rev. Lett. 99 163602
Sague G, Baade A and Rauschenbeutel A 2008 New J. Phys. 10 113008
[25] Nayak K P, Melentiev P N, Morinaga M, Kien F L, Balykin V I and Hakuta K 2006 arXiv:quant-ph/0610136v1
[26] Kien F L, Balykin V I and Hakuta K 2004 Phys. Rev. A 70 023608
[27] Kien F L, Balykin V I and Hakuta K 2005 Phys. Rev. A 72 063815
Sague G, Vetsch E, Alt W, Meschede D and Rauschenbeutel A 2007 Phys. Rev. Lett. 99 163602
Sague G, Baade A and Rauschenbeutel A 2008 New J. Phys. 10 113008
[28] Dowling J P and Gea-Banacloche J 1996 Adv. At. Mol. Opt. Phys. 37 1
Kien F L, Dutta Gupta S, Nayak K P and Hakuta K 2005 Phys. Rev. A 72 063815
Sague G, Vetsch E, Alt W, Meschede D and Rauschenbeutel A 2007 Phys. Rev. Lett. 99 163602
Sague G, Baade A and Rauschenbeutel A 2008 New J. Phys. 10 113008

[31] Abdel-Aty M, Abdalla M S and Sanders B C 2009 Phys. Lett. A 373 315
[32] Janszky J, Sibilia C, Bertolotti M and Yushin Y 1988 J. Mod. Opt. 35 1757
[33] Louisell W H 1973 Quantum Statistical Properties of Radiation (New York: Wiley)
[34] Rempe G, Walther H and Klein N 1987 Phys. Rev. Lett. 57 353
[35] Wineland D J, Monroe C, Itano W M, Leibfried D, King B E and Meekhof D M 1998 J. Res. Natl Inst. Stand. Technol. 103 259
[36] Lee P J, Brickman K-A, Deslauriers L, CHaljan P, Duan L-M and Monroe C 2005 J. Opt. B: Quantum Semiclass. Opt. 7 S371
[37] Yøna M, Ting Y and Eberly J H 2007 J. Phys. B: At. Mol. Opt. Phys. 40 S45–59
[38] Casagrande F, Lulli A and Paris M G A 2007 Phys. Rev. A 75 032336
[39] Jozsa R, Abrams D S, Dowling J P and Williams C P 2000 Phys. Rev. Lett. 85 2010
[40] Berry C C and Eberly J H 1990 Phys. Rev. A 42 6805
Cardimona D A, Kovanis V, Sharma M P and Gavrielides A 1991 Phys. Rev. A 43 3710
El-Orany F A A, Wahiddin M R B and Obada A-S 2008 Opt. Commun. 281 2854
[41] Cirac J I and Zoller P 1995 Phys. Rev. Lett. 74 4091
[42] Satyanarayana M V, Rice P, Vyas R and Carmichael H J 1989 J. Opt. Soc. Am. B 6 228