The quantum approach to human reasoning does explain the belief-bias effect

E. D. Vol

B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine 47, Lenin Ave., Kharkov 61103, Ukraine.

(Dated: May 22, 2014)

Based on the ideas of quantum physics and dual-process theory of human reasoning that takes into account two primary mechanisms of reasoning: 1) deductive rational thinking and 2) intuitive heuristic judgment, we proposed the "quantum" approach to practical human logic that allows one to specify the most distinctive peculiarities in activity of two reasoning systems mentioned above and in addition to describe phenomenologically well-established experimentally belief-bias effect.

PACS numbers: 05.40.-a

I. INTRODUCTION

The idea that some essential human values and concepts may be incompatible with each other had originated long before the beginning of scientific psychology. By distinct ways this idea was justified by such outstanding philosophers and thinkers as G.W. Leibniz, N.Machiavelli and I. Kant. The interested reader can find detail account of the history of this idea with relevant references in [1]. However only in the XX century with the rise of quantum theory this idea has received adequate scientific expression in the language of the Bohr’s Complementarity Principle. We shall give here only two distinctive quotations of founding fathers of quantum mechanics that are clearly demonstrating their profound understanding of the inconsistency of some basic concepts relating to human psychology. So, in the paper of 1948 "On the Notions of Causality and Complementarity" N. Bohr wrote:"Recognition of complementary relationship is not least required in psychology, where the conditions for analysis and synthesis of experience exhibit striking analogy with the situation in atomic physics. In fact, the use of words like thoughts, and sentiments, equally indispensable to illustrate the diversity of psychological experience, pertain to mutually exclusive situations characterized by a different drawing of the line of separation between subject and object. In particular, the place left for the feeling of volition is afforded by the very circumstance that situations where we experience freedom of will are incompatible with psychological situations where causal analysis is reasonably attempted. In other words, when we use the phrase "I will" we renounce explanatory argumentation. In fact, the use which we make of words like "thought" and "feeling," or "instinct" and "reason" to describe psychic experiences of different types, shows the existence of characteristic relationships of complementarity conditioned by the peculiarity of introspection" [2]. On the other hand W. Pauli drew particular attention to the problem of relation between complementarity of mental and physical aspects of the same reality. In his

*Electronic address: vol@ilt.kharkov.ua
The remainder of the paper is organized as follows. In chapter 2 we briefly remind basic facts relating to discrete-continuous logic (DCL) that are necessary for the understanding of the present paper. The main contribution of this chapter is the interpretation of the general propositions in DCL as the integral mental structures that consist both of logical and heuristic constituents. Under such interpretation these two constituents of the proposition can be considered as complementary to each other exactly like two noncommuting observables in quantum mechanics. In chapter 3 we state the uncertainty relation that just reflects the complementary nature of such concepts as logical rigour and the heuristic grasp. And finally in chapter 4 using only logical tools we give the phenomenological explanation of the belief-bias effect. Now let us go to the details.

II. PRELIMINIRIES

In this part we briefly remind for the reader convenience the necessary facts relating to the discrete-continuous logic that were outlined more detail in author preprint [2] So, we will consider as the primary objects of our study the set of general propositions (GP) \{-A_1\} that may be represented by \(2 \times 2\) positive definite matrices with unit trace of the following form:

\[
A_j = \begin{pmatrix} p_j & i\alpha_j \\ -i\alpha_j & 1 - p_j \end{pmatrix},
\]

(1)

(where \(i\) is imaginary unit). In this case the negation of such proposition - (not \(A_j\)) may be defined as (not \(A\)) = \(\begin{pmatrix} 1-p_j & -i\alpha_j \\ i\alpha_j & p_j \end{pmatrix}\). It turns out that in addition to negation another but already two place operation - \(\triangle\) (which is the analogue of strong disjunction in ordinary Boolean logic) can be introduced in DCL according to the next definition:

\[
(A \triangle B) = \begin{pmatrix} p & i\alpha \\ -i\alpha & 1 - p \end{pmatrix} + \begin{pmatrix} q & i\beta \\ -i\beta & 1 - q \end{pmatrix} = \begin{pmatrix} p + q & i\alpha + i\beta \\ -i\alpha - i\beta & 1 - (p + q) \end{pmatrix}.
\]

(2)

where \(R = p + q - 2pq + 2\alpha\beta, \gamma = \alpha(1 - 2q) + \beta(1 - 2p)\). Comparing representation Eq. (1) with standard form of density matrix of the mixed state of two-level quantum system that looks as \(\rho = \frac{1}{2} \begin{pmatrix} 1 + P_x & iP_y \\ ip_y & 1 - P_x \end{pmatrix}\) (where \(P = (P_x, P_y, P_z)\) is the Bloch vector of the state) we see that GP may be represented by the similar way but in this case \(x\)-component of the Bloch vector is equal to zero. In the rest of the paper we will use such reduced Bloch representation for the arbitrary proposition \(A\), that is:

\[
A = \begin{pmatrix} \frac{1 + P_x}{2} & \frac{-iP_y}{2} \\ \frac{ip_y}{2} & \frac{1 - P_x}{2} \end{pmatrix}
\]

with \(P = (P_y, P_z)\). In this case it is convenient to introduce the complex vector \(P = P_x - iP_y\) which we call further as representing vector (RV) of proposition \(A\). It is easy to verify directly that the RV of proposition (not \(A\)) is equal to (-\(P\)) and RV of proposition (\(A \triangle B\)) is equal to (-\(PQ\)) (where \(Q\) is RV of \(B\)). Note also the useful relation connecting negation with operation \(\triangle\):

\[
\text{not}(A \triangle B) = \text{not}(A) \triangle B = A \triangle \text{not}(B).
\]

(3)

It should be noted that unlike of ordinary Boolean logic in DCL it is possible to define the whole one-parameter group of continuous logical operations (logical rotations of propositions in the plane \(P_y - P_z\)) according to the following rule: if proposition \(A\) has the RV - \(P\) then rotated at an angle \(\Phi\) proposition \(A^\Phi\) has RV - \(P^\Phi\) with components:

\[
P^\Phi_y = P_y \cos \phi + P_z \sin \phi
\]

\[
P^\Phi_z = P_z \cos \phi - P_y \sin \phi
\]

(4)

It is easy to see that the negation of any proposition coincides with logical rotation of it at an angle \(\pi\) and in addition that if one rotates the GP \(A\) at an angle \(\Phi_1\) and the other proposition \(B\) at an angle \(\Phi_2\) then the proposition \((A \triangle B)\) will be rotated at an angle \(\Phi_1 + \Phi_2\). Thus all logical operations in DCL obtain quite clear geometric meaning. Now after describing the syntax of DCL we can pass to the more difficult task: clarification of its semantics that is the interpretation both the meaning of general propositions and logical operations with them. It should be noted that interpretation that we are going to propose here is not the only possible but it is appropriate for our ultimate goal namely to explain the belief-bias effect in human reasoning from pure logical point of view. So, as before we will assume that diagonal elements of representing matrix for arbitrary GP coincides with logical validity (from DRS point of view) while its non-diagonal elements we will interpret as the believability of the same proposition inspired by the heuristic reasoning system (HRS). This interpretation can be expressed more precisely as follows. Let us introduce two projection operators: \(P_1\) and \(P_2\) \((P_1^2 = P_1, P_2^2 = P_2)\) according to the definition: \(P_1 = \frac{1 + \gamma}{2}\) and \(P_2 = \frac{1 + \sigma}{2}\). It is easy to see that average values of these operators in the state whose density matrix coincides with representing matrix of proposition \(A\) of the form \(\frac{p}{-\alpha} 1 - p\) give us the probabilities of its logical plausibility \(p_l\) and its believability \(p_b\) respectively. Thus we obtain

\[
p_l = \langle P_1 \rangle = \text{Sp}(P_1A) = p
\]

and

\[
p_b = \langle P_2 \rangle = \text{Sp}(P_2A) = \frac{1 - 2\alpha}{2}
\]

(5)

In connection with above interpretation we want to point out two important marginal GP: 1) \(T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\)
true proposition, and 2) $B = \left( \frac{i}{2}, -\frac{i}{2}, \frac{1}{2} \right)$ - highest possible believable proposition and their negations: $F = (\text{not}T)$—false proposition and $U = (\text{not}B)$—unbelievable proposition. Note in addition that noncommutativity of operators $P_1$ and $P_2$ implies that main predicates of arbitrary GP (plausibility and belief) may be considered as complementary (in the sense of quantum theory) aspects of the same proposition. This important fact implies specific uncertainty relation for the observables $P_1$ and $P_2$ connected with any GP. The simple derivation of this relation is the subject of the next section of the present paper.

III. THE UNCERTAINTY RELATION BETWEEN PREDICATES PLAUSIBILITY AND BELIEVABILITY IN DCL.

To derive the required uncertainty relation it is convenient to represent any GP $A$ in the Bloch form:

$A = \left( \frac{1+P_1}{2}, \frac{-iP_2}{2}, \frac{1-P_2}{2} \right)$. According to definition the uncertainty of logical truth for the proposition $A$ can be written with the help of operator $P_1 = \frac{1+P_1}{2}$ as:

$\Delta p_1^2 = \left( \frac{1+P_1}{2} - \frac{1-P_1}{2} \right)^2 = \frac{1}{4} (1-\sigma_z^2) = \frac{1-P_z^2}{4}$. In the similar manner the uncertainty of believability of the same proposition is equal to: $\Delta p_c = \frac{1}{4} (1-\sigma_z^2) = \frac{1}{4} (1-P_z^2)$. By adding these two expressions we obtain:

$\Delta p_A^2 \equiv \Delta p_1^2 + \Delta p_c^2 = \frac{1}{4} (2-P_y^2-P_z^2)$. Finally taking into account that $P_y^2 + P_z^2 \leq 1$ we get the desired relations:

$$\frac{1}{4} \leq \Delta p_A^2 \leq \frac{1}{2}. \quad (6)$$

The notable fact should be mentioned here: if one takes two propositions $A$ and $B$ with RV $P$ and $Q$ respectively then according above calculation one can write two equations 1) $\Delta p_A^2 = \frac{(2-P_y^2)}{4}$ and 2) $\Delta p_B^2 = \frac{(2-Q_z^2)}{4}$.

On the other hand as we marked earlier the proposition $(A \triangledown B)$ has RV $(-PQ)$ and hence its uncertainty is equal to $\Delta p_{(A\triangledown B)}^2 = \frac{(2-P_y^2Q_z^2)}{4}$. As long as $P_y^2Q_z^2 \leq P^2, Q^2$ one can conclude that $\Delta p_{(A\triangledown B)}^2 \geq \Delta p_A^2, \Delta p_B^2$ and hence as a final result of logical operation $\triangledown$ the ending uncertainty of proposition can only increases. We would like to hope that properly organized experiments with specially selected reasoning tasks will be able to confirm (or may be disprove) the proposed uncertainty relations (6). Now we come back to the main goal of present paper: the explanation of the belief-bias effect in human reasoning.

IV. MANY VALUED PROBABILISTIC LOGIC AND THE BELIEF-BIAS EFFECT.

In this part we will try to describe (phenomenologically) the belief-bias effect in human reasoning by purely logical tools. For this purpose it is convenient to use some version of probabilistic many-valued logic that in some sense can be considered as simplified version of original DCL. Really if in original version of DCL we restrict ourselves only by discrete set of logical rotations with angles: $0, \frac{2\pi}{N}, ... \frac{2\pi}{N}(N-1)$ we obtain the closed logic with N marginal propositions which possess representing matrices: $A_0, A_1, ..., A_{N-1}$ (where $A_0 = \left( \begin{array}{c} 1 \\ 0 \end{array} \right)$, $A_1 = \left( \begin{array}{c} 0 \\ 1 \end{array} \right)$, ..., $A_{N-1} = \left( \begin{array}{c} 0 \\ 0 \end{array} \right)$). Here we mean that the space of these propositions is a tensor product of two spaces with $2 \times 2$ diagonal matrices, that is:

$$A = \sum_1 a_i T_i \otimes B_i, \quad (7)$$

where $T_i = \left( \begin{array}{cc} p_i & 0 \\ 0 & 1-p_i \end{array} \right)$, $B_i = \left( \begin{array}{cc} q_i & 0 \\ 0 & 1-q_i \end{array} \right)$ and $\sum a_i = 1$. In addition we assume that matrices $T_i$ in the decomposition Eq. (7) are associated with the activity of deductive cognitive subsystem (DRS), while matrices $B_i$ are connected with its heuristic subsystem (HRS). Thus the basis of this logic consists of four propositions: 1) truth-believable $TB = diag(1, 0, 0, 0, 2)$, 2) truth-unbelievable $TU = diag(0, 1, 0, 0, 3)$, 3) false-believable $FB = diag(0, 0, 1, 0)$ and 4) false-unbelievable $FU = diag(0, 0, 0, 1)$. Our next step is to define basic logical operations that can be implemented with such propositions. The interpretation that we have adopted above implies that the negation of proposition $A$ must be defined as $\text{not} A = diag(P_1, P_2, P_3, P_4)$. The certain dilemma arises however when we want to define the conjunction of two propositions $A = diag(P_1, P_2, P_3, P_4)$ and $B = diag(Q_1, Q_2, Q_3, Q_4)$. We have proposed here the following definition:

$$C \equiv (A \text{and} B) = diag(C_1, C_2, C_3, C_4), \quad (8)$$

where $C_1 = P_1 (Q_1 + Q_2) + P_2 Q_1$, $C_2 = P_2 Q_2$, $C_3 = P_1 (Q_3 + Q_4) + P_2 Q_3 + P_3 (Q_1 + Q_3)$, $C_4 = P_2 Q_4 + P_4 (Q_2 + Q_4)$. This definition of conjunction namely Eq.
First of all we note that definition Eq. \( \text{o} \) satisfies to the necessary symmetry condition: \( (A \land B) = (B \land A) \) as it should be. In addition if one takes the projection of conjunction Eq. \( \text{o} \) in DRS (first reasoning subsystem) the result is: \( (A \land B)_1 = \begin{pmatrix} pq & 0 \\ 0 & 1 - pq \end{pmatrix} \equiv (A_1 \land B_1) \) where \( p = P_1 + P_2 \) and \( q = Q_1 + Q_2 \). This result obviously consistent with the definition of conjunction in ordinary probabilistic Boolean logic. On the other hand if one takes the projection of Eq. \( \text{o} \) in HRS (second reasoning subsystem) the obtained result reads as:

\[
(A \land B)_2 = \begin{pmatrix} 1 - (P_2 + P_4)(Q_2 + Q_4) \\ (P_2 + P_4)(Q_2 + Q_4) \end{pmatrix}.
\]

We see that conjunction in heuristic system differs from standard logical conjunction. In our view this distinction explicitly reflects (from phenomenological point of view) the essential difference existing between two reasoning systems when they operate jointly. In particular the definition Eq. \( \text{o} \) implies for two basic marginal propositions in second reasoning subsystem: \( B = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \) responding to the statement of unconditional belief and \( D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) -which is the most doubtful statement, the next conjunction relations: \( (B \land D) = (D \land B) = B \) and \( (D \land D) = D \). Thus we obtain that the unconditional belief when it conflicts with certain doubtful one always overcomes it. Now if one takes the expression Eq. \( \text{o} \) for granted then he (she) can define another logical operations (in particular implication that we especially interested in) without any obstacles. To this end one should be guided by two relations of ordinary logic which as we assume continue to be valid in our case as well: 1) \( (A \lor B) = \neg([\neg(A) \land \neg(B)] \) and 2) \( (A \Rightarrow B) = (\neg(A) \lor B) \). Acting in this manner we obtain for the implication \( (A \Rightarrow B) \) the required relation:

\[
I \equiv (A \Rightarrow B) = \text{diag}(I_1, I_2, I_3, I_4),
\]

where \( I_1 = p_4(q_1 + q_3) + p_2q_4, I_2 = p_3 + q_2(1 - p_3) + p_1q_1 + p_4q_4, I_3 = p_2q_3, I_4 = p_1(q_3 + q_4) + p_2q_4 \). The expression Eq. \( \text{o} \) for the implication of two probabilistic propositions in four-valued logic is the foundation for our following explanation of bias-belief effect. Note that here we are going to demonstrate only the simplest case of the application of the approach proposed. The detail quantitative analysis of numerous possible situations connected with the interaction between DRS and HRS will be realized by us at length in separate publication. So, let us take the proposition \( B \) - (consequent of the implication) in the form: \( B = \text{diag}(1, 0, 0, 0) \), that means that consequent is both true and believable proposition. Then the expression Eq. \( \text{o} \) implies that matrix \( (A \Rightarrow B) \) has the form:

\[
(A \Rightarrow B) = \text{diag}(p_2 + p_4, p_1 + p_3, 0, 0),
\]

and hence its projections in DRS (1) and HRS (2) systems are respectively:

\[
(A \Rightarrow B)_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

\[
(A \Rightarrow B)_2 = \begin{pmatrix} p_2 + p_4 \\ p_1 + p_3 \end{pmatrix}.
\]

On the other hand if one choose the consequent \( B \) in the form \( B = \text{diag}(0, 1, 0, 0) \) that means that consequent \( B \) is true but unbelievable proposition then according to expression (10) one obtain for the implication \( (A \Rightarrow B) \) the relation:

\[
(A \Rightarrow B) = \text{diag}(0, 1, 0, 0),
\]

and hence the projections of this proposition in two cognitive systems are:

\[
(A \Rightarrow B)_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},
\]

\[
(A \Rightarrow B)_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]

Now if we make the natural assumption that after the first (unconscious) stage of reasoning, when two cognitive systems operate jointly, at the second stage the conscious evaluation of the validity of a conclusion \( V \) occurs in accordance with the simple rule:

\[
V = aP_1 + (1 - a)P_2,
\]

where \( a \) (\( 0 \leq a \leq 1 \)) is certain number coefficient depending on age, intellect, training of the subject and possibly some other factors). Note that this assumption in fact coincides with similar rule which was used in the paper [4]. Now returning to the above example of interest we result in that the magnitude of the bias-belief effect \( V \) can be evaluate quantitatively as \( V = V_1 - V_2 = (1 - a)(p_2 + p_4) \). We believe that although the value of coefficient \( a \) is unknown in advance nevertheless the validity of the Eq. \( \text{13} \) can be explicitly verified in seria of properly organized psychological experiments with various subjects using the identical cognitive tasks.

In conclusion of our study let us formulate once more the central results of the present paper:

1) We introduced the novel version of DCL with both discrete and continuous logical operations between generalized propositions.

2) We proposed the concrete interpretation of propositions in DCL as integral mental structures that include both logical and heuristic constituents.

3) We stated the specific uncertainty relation between logic rigour and heuristic grasp that reflect complementary aspects of human reasoning process.

4) We proposed phenomenological model of human reasoning based on simplified version of DCL and demonstrated that it is able to explain belief-bias effect qualitatively and possibly quantitatively as well.

All these conclusions we hope to discuss more detail in our further publications.
[1] Stent GS. Proc Am Philos Soc. 2004 Jun;148(2): 205-12.
[2] N. Bohr, Science, New Series, Vol. 111, No. 2873 (Jan. 20, 1950), pp. 51-54
[3] Enz, P.; von Meyenn, Karl (editors); Schlapp, Robert (translator) Wolfgang Pauli, Writings on physics and philosophy, pp218 - 279, Berlin: Springer Verlag,(1994)
[4] Jonathan St. B. T. Evans, Thinking & Reasoning, V13, Issue 4, pp321-339, (2007)
[5] E. D. Vol, physics.soc-ph 1306.2433, (2013)