Implications of the Quark Mass Hierarchy on Flavor Mixings

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Abstract

We stress that the observed pattern of flavor mixings can be partly interpreted by the quark mass hierarchy without the assumption of specific quark mass matrices. The quantitatively proper relations between the Kobayashi-Maskawa matrix elements and quark mass ratios, such as

$$|V_{cb}| \approx |V_{ts}| \approx \sqrt{2} \left( \frac{m_s}{m_b} - \frac{m_c}{m_t} \right) \left[ 1 + 3 \left( \frac{m_s}{m_b} + \frac{m_c}{m_t} \right) \right],$$

are obtainable from a simple Ansatz of flavor permutation symmetry breaking at the weak scale. We prescribe the same Ansatz at the supersymmetric grand unified theory scale, and find that its all low-energy consequences on flavor mixings and CP violation are in good agreement with current experimental data.

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1 Introduction

The discovery of the top quark at Fermilab fulfilled the three-family quark picture in the standard electroweak model. Up to now, some knowledge on the mass spectra of \((u, c, t)\) and \((d, s, b)\) quarks has been accumulated through both experimental and theoretical (or phenomenological) attempts \([1]\). The ratios of quark mass eigenvalues are obtainable after one renormalizes them to a common reference scale, e.g., \(\mu = 1\ \text{GeV}\) or \(M_Z\). There exists a clear mass hierarchy in each quark sector:

\[
m_u \ll m_c \ll m_t ; \quad m_d \ll m_s \ll m_b .
\]  

(1.1)

In comparison, the masses of three charged leptons manifest a similar hierarchical pattern \([2]\).

Quark mass eigenstates are related to quark weak (flavor) eigenstates by the Kobayashi-Maskawa (KM) matrix \(V\) \([3]\), which provides a quite natural description of flavor mixings and \(CP\) violation in the standard model. To date, many experimental constraints on the magnitudes of the KM matrix elements have been achieved. The unitarity of \(V\) together with current data requires a unique hierarchy among the nine matrix elements \([4]\):

\[
|V_{tb}| > |V_{ud}| > |V_{cs}| \gg |V_{us}| > |V_{cd}|
\]

\[
\gg |V_{cb}| > |V_{ts}|
\]

\[
\gg |V_{td}| > |V_{ub}| > 0 .
\]  

(1.2)

Here \(|V_{ub}| \neq 0\) is a necessary condition for the presence of \(CP\) violation in the KM matrix.

How to understand the hierarchies of quark masses and flavor mixings is an important but unsolved problem in particle physics. A natural approach to the final solution of this problem is to look for the most favorable pattern of quark mass matrices (see, e.g., Refs. \([5, 6]\)), which can account for all low-energy phenomena of quark mixings and \(CP\) violation. The relevant symmetries hidden in such phenomenological schemes are possible to provide useful hints toward the dynamical details of fermion mass generation.

It has been speculated by some authors that the realistic fermion mass matrices could arise from the flavor permutation symmetry and its spontaneous or explicit breaking \([7, 8, 9]\). Under exact \(S(3)_L \times S(3)_R\) symmetry the mass spectrum for either up or down quark sector consists of only two levels: one is of 2-fold degeneracy with vanishing mass eigenvalues, and the other is nondegenerate (massive). An appropriate breakdown of the above symmetry may lead to the observed mass hierarchy and flavor mixings. Although the way to introduce the minimum number of free parameters for permutation symmetry breaking is technically trivial, its consequences on quark mixings and \(CP\) violation may be physically instructive and may even shed some light on the proper relations between the KM matrix elements and quark mass ratios. Indeed there has not been a satisfactory symmetry breaking pattern with enough predictive power in the literature.

In this work we first stress that some observed properties of the KM matrix can be interpreted by the quark mass hierarchy without the assumption of specific mass matrices. In the quark
mass limits such as $m_u = m_d = 0$, $m_t \to \infty$ or $m_b \to \infty$, we find that simple but instructive relations between the KM matrix elements and quark mass ratios are suggestible from current experimental data. Then we present a new quark mass Ansatz through the explicit breakdown of flavor permutation symmetry at the weak scale ($M_Z = 91.187$ GeV). This Ansatz contains seven free parameters, thus it can give rise to three predictions for the phenomena of quark mixings and $CP$ violation. The typical results are $|V_{cb}| \approx |V_{ts}| \approx \sqrt{2} \left( m_s/m_b - m_c/m_t \right)$, $|V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c}$ and $|V_{td}/V_{ts}| \approx \sqrt{m_d/m_s}$ in the leading order approximation. Prescribing the same Ansatz at the supersymmetric grand unified theory (GUT) scale ($M_X = 10^{16}$ GeV), we derive the renormalized quark mass matrices at $M_Z$ for small $\tan \beta_{\text{susy}}$ (the ratio of Higgs vacuum expectation values in the minimal supersymmetric model). We also renormalize some relations between the KM matrix elements and quark mass ratios at $M_Z$ for arbitrary $\tan \beta_{\text{susy}}$, and find that the relevant results are in good agreement with experimental data. The scale-independent predictions of our Ansatz for the characteristic measurables of $CP$ asymmetries in weak $B$ decays, i.e., $0.18 \leq \sin(2\alpha) \leq 0.58$, $0.5 \leq \sin(2\beta) \leq 0.78$ and $-0.08 \leq \sin(2\gamma) \leq 0.5$, can be tested at the forthcoming KEK and SLAC $B$-meson factories.

The remaining part of this paper is organized as follows. Some qualitative implications of the quark mass hierarchy on the KM matrix elements, which are almost independent of the specific forms of quark mass matrices, are discussed in section 2. In section 3 we suggest a new quark mass Ansatz from the flavor permutation symmetry breaking at the weak scale, and study its various consequences on flavor mixings and $CP$ violation. The same Ansatz is prescribed at the supersymmetric GUT scale in section 4. By use of the one-loop renormalization group equations, we run the mass matrices from $M_X$ to $M_Z$ and then discuss the renormalized relations between the KM matrix elements and quark mass ratios. Section 5 is devoted to a brief summary of this work.

2 Flavor mixings in quark mass limits

Without loss of any generality, the up and down quark mass matrices (denoted by $M_u$ and $M_d$, respectively) can be chosen to be Hermitian. After the diagonalization of $M_u$ and $M_d$ through the unitary transformations

$$O_u^\dagger M_u O_u = \text{Diag}\{m_u, m_c, m_t\},$$
$$O_d^\dagger M_d O_d = \text{Diag}\{m_d, m_s, m_b\},$$

one obtains the KM matrix $V \equiv O_u^\dagger O_d$, which describes quark flavor mixings in the charged current. Explicitly, the KM matrix elements read

$$V_{ij} = \sum_{k=1}^3 \left( O_{ki}^u O_{kj}^d \right),$$

depending upon the quark mass ratios $m_u/m_c$, $m_c/m_t$ (from $O_u$) and $m_d/m_s$, $m_s/m_b$ (from $O_d$) as well as other parameters of $M_u$ and $M_d$ (e.g., the non-trivial phase shifts between $M_u$ and $M_d$). In view of the distinctive mass hierarchy in Eq. (1.1), we find that some interesting properties of $V$ can be interpreted without the assumption of specific forms of $M_u$ and $M_d$. 
A. $|V_{us}|$ and $|V_{cd}|$ in the limits $m_t \rightarrow \infty$ and $m_b \rightarrow \infty$

Since the mass spectra of up and down quarks are absolutely dominated by $m_t$ and $m_b$ respectively, the limits $m_t \rightarrow \infty$ and $m_b \rightarrow \infty$ are expected to be very reliable when we discuss flavor mixings between $(u, d)$ and $(c, s)$. In this case, the effective mass matrices turn out to be two $2 \times 2$ matrices and the resultant flavor mixing matrix (i.e., the Cabibbo matrix $|11|$) cannot accommodate CP violation. The magnitudes of $V_{us}$ and $V_{cd}$ can be obtained from Eq. (2.2), since $O_{i3} = O_{3i} = \delta_{i3}$ holds for both sectors in the above-mentioned mass limits. We find that $|V_{us}| = |V_{cd}|$ is a straightforward result guaranteed by the unitarity of $O_u$ and $O_d$. The current experimental data, together with unitary conditions of the $3 \times 3$ KM matrix, have implied $|V_{us}| \approx A^2 \mathcal{A} \left( \frac{1}{2} - \rho \right) < 10^{-3}$,

which is insensitive to allowed errors of the Wolfenstein parameters $A$, $\lambda$ and $\rho$ $|12|$. From the discussions above we realize that the near equality of $|V_{us}|$ and $|V_{cd}|$ is in fact a natural consequence of $m_t \gg m_c, m_u$ and $m_b \gg m_s, m_d$.

The magnitude of $V_{us}$ (or $V_{cd}$) must be a function of the mass ratios $m_u/m_c$ and $m_d/m_s$ in the limits $m_t \rightarrow \infty$ and $m_b \rightarrow \infty$, if $M_u$ and $M_d$ have parallel or quasi-parallel structures. Considering the experimentally allowed regions of $m_u/m_c (\sim 5 \times 10^{-3} \ [2])$, $m_s/m_d (= 18.9 \pm 0.8$ $|13|$) and $|V_{us}| (= 0.2205 \pm 0.0018 \ [4])$, one may guess that $|V_{us}|$ is dominated by $\sqrt{m_d/m_s}$ but receives small correction from $\sqrt{m_u/m_c}$. Indeed such an instructive result for $|V_{us}|$ or $|V_{cd}|$ can be derived from $2 \times 2$ Hermitian mass matrices of the form $[5]$

$$
\begin{pmatrix}
0 & A \\
A^* & B
\end{pmatrix}
$$

(2.4)

where $|B| \gg |A|$. Denoting the phase difference between $A_u$ and $A_d$ as $\Delta \phi$, we obtain

$$
|V_{us}| = |V_{cd}| = \left| \sqrt{\frac{m_c}{m_u + m_c}} \sqrt{\frac{m_d}{m_d + m_s}} - \exp(i \Delta \phi) \sqrt{\frac{m_u}{m_u + m_c}} \sqrt{\frac{m_s}{m_d + m_s}} \right|. \tag{2.5}
$$

Although the $2 \times 2$ flavor mixing matrix cannot accommodate CP violation, the phase shift $\Delta \phi$ is non-trivial on the point that it sensitively determines the value of $|V_{us}|$. For illustration, we calculate the allowed region of $\Delta \phi$ as a function of $m_u/m_c$ in Fig. 1. It is clear that the possibilities $\Delta \phi = 0^\circ, 90^\circ$ and $180^\circ$ have all been ruled out by current data on $V_{us}$ and $m_s/m_d$, since $m_u/m_c \geq 10^{-3}$ is expected to be true. We conclude that the presence of $\Delta \phi$ in the quark mass Ansatz above is crucial for correct reproduction of $|V_{us}|$ and $|V_{cd}|$. Such a non-trivial phase shift will definitely lead to CP violation, when the limits $m_t \rightarrow \infty$ and $m_b \rightarrow \infty$ are discarded.

B. $|V_{cb}|$ and $|V_{ts}|$ in the limit $m_u = m_d = 0$

Considering the fact that $m_u$ and $m_d$ are negligibly small in the mass spectra of up and down quarks, one can take the reasonable limit $m_u = m_d = 0$ to discuss flavor mixings between the second and third families. In this case, there is no mixing between $(u, d)$ and $(c, s)$ or between
(u, d) and (t, b). Thus \( M_{11} = M_{22} = 0 \) holds for both up and down mass matrices, and then we get \( O_{11} = O_{22} = \delta_{11} \). The relation \(|V_{cb}| = |V_{ts}|\) is straightforwardly obtainable from Eq. (2.2) by use of the unitary conditions of \( O_u \) and \( O_d \). In contrast, the present data and unitarity of the KM matrix requires

\[
|V_{cb}| - |V_{ts}| \approx A \lambda^4 \left( \frac{1}{2} - \rho \right) < 10^{-2}.
\]  

(2.6)

We see that the near equality between \(|V_{cb}|\) and \(|V_{ts}|\) can be well understood, because the quark mass limit \( m_u = m_d = 0 \) is a good approximation for \( M_u \) and \( M_d \).

We expect that \(|V_{cb}|\) and \(|V_{ts}|\) are functions of the mass ratios \( m_c/m_t \) and \( m_s/m_b \) in the limit \( m_u = m_d = 0 \). Current experimental data give \(|V_{cb}| = 0.0388 \pm 0.0032 \) \( [14] \), while \( m_c/m_t \approx 10^{-3} \) \( [2] \) and \( m_s/m_b = 34 \pm 4 \) \( [15] \) are allowed. Thus \(|V_{cb}| \) (or \(|V_{ts}|\)) should be dominated by \( m_s/m_b \), and it may get a little correction from \( m_c/m_t \). To obtain a linear relation among \( V_{cb}, m_s/m_b \) and \( m_c/m_t \) in the leading order approximation, one can investigate mass matrices of the following Hermitian form:

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & A & B \\
0 & B^* & C
\end{pmatrix},
\]

(2.7)

where \( A \neq 0 \) and \(|C| \gg |B| \sim |A|\) for both quark sectors. This generic pattern can also be regarded as a trivial generalization of the Fritzsch Ansatz, in which \( A = 0 \) is assumed \( [3] \), but they have rather different consequences on the magnitudes of \( V_{cb} \) and \( V_{ts} \). Denoting \( \Delta \varphi = \arg(B_u/B_d) \), \( R_u = |B_u/A_u| \) and \( R_d = |B_d/A_d| \), we find the approximate result

\[
|V_{cb}| = |V_{ts}| \approx \left| R_d \frac{m_s}{m_b} \exp(i\Delta \varphi) R u \frac{m_c}{m_t} \right|.
\]

(2.8)

One can see that \(|V_{cb}| \propto m_s/m_b\) holds approximately, if \( R_u \) is comparable in magnitude with \( R_d \). Here the phase shift \( \Delta \varphi \) plays an insignificant (negligible) role in confronting Eq. (2.8) with the experimental data on \(|V_{cb}|\), since the term proportional to \( m_c/m_t \) is significantly suppressed. To determine the values of \( R_u \) and \( R_d \), however, one has to rely on a more specific Ansatz of quark mass matrices.

**C.** \(|V_{ub}/V_{cb}|\) in \( m_b \to \infty \) and \(|V_{td}/V_{ts}|\) in \( m_t \to \infty \)

Now let us take a look at the two smallest matrix elements of \( V \), \(|V_{cb}|\) and \(|V_{td}|\), in the quark mass limits. Taking \( m_b \to \infty \), we have \( O_{i3}^d = O_{3i}^d = \delta_{i3} \), because \( M_d \) turns out to be an effective \( 2 \times 2 \) matrix in this limit. Then the ratio of \(|V_{ub}|\) to \(|V_{cb}|\) reads

\[
\lim_{m_c \to \infty} \frac{|V_{ub}|}{|V_{cb}|} = \frac{|O_{31}^u|}{|O_{32}^2|},
\]

(2.9)

obtained from Eq. (2.2). Contrary to common belief, \(|V_{ub}/V_{cb}|\) is absolutely independent of the mass ratio \( m_d/m_s \) in the limit \( m_b \to \infty \)! Therefore one expects that the left-handed side of Eq. (2.9) is dominated by a simple function of the mass ratio \( m_u/m_c \), while the contribution from \( m_c/m_t \) should be insignificant in most cases. The present numerical knowledge of \(|V_{ub}/V_{cb}|\)
\[ (= 0.08 \pm 0.02 \text{~[2]} ) \text{ and } m_u/m_c \sim 5 \times 10^{-3} \text{~[2]} \] implies that \[ |V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c} \] is likely to be true. Indeed such an approximate result can be reproduced from the Fritzsch Ansatz and a variety of its modified versions \[ [16] \].

In the mass limit \[ m_t \to \infty \], \[ M_u \] becomes an effective \[ 2 \times 2 \] matrix, and then \[ O_{i3}^{u} = O_{3i}^{u} = \delta_{i3} \] holds. The ratio of \[ |V_{td}|/|V_{ts}| \] is obtainable from Eq. (2.2) as follows:

\[
\lim_{m_t \to \infty} \left| \frac{V_{td}}{V_{ts}} \right| = \frac{|O_{31}|}{|O_{32}|}.
\] (2.10)

Here again we find that \[ |V_{td}/V_{ts}| \] is independent of both \[ m_u/m_c \] and \[ m_c/m_t \] in the limit \[ m_t \to \infty \], thus it may be a simple function of the mass ratios \[ m_d/m_s \] and \[ m_s/m_b \]. The current data give \[ 0.15 \leq |V_{td}/V_{ts}| \leq 0.34 \text{~[17]} \] and \[ m_s/m_d = 18.9 \pm 0.8 \text{~[13]} \] and \[ m_b/m_s = 34 \pm 4 \text{~[15]} \]. We expect that \[ |V_{td}/V_{ts}| \approx \sqrt{m_d/m_s} \] has a large chance to be true in the leading order approximation. Note that this approximate relation can also be derived from the Fritzsch Ansatz or some of its revised versions \[ [16] \].

The qualitative discussions above have shown that some properties of the KM matrix \[ V \] can be well understood just from the quark mass hierarchy. For example, \[ |V_{us}| \approx |V_{cd}| \] and \[ |V_{cb}| \approx |V_{ts}| \] are natural consequences of arbitrary (Hermitian) mass matrices with \[ m_3 \gg m_2, m_1 \] and \[ m_1 \ll m_2, m_3 \] respectively, where \[ m_i \] denote the mass eigenvalues of each quark sector. To a good degree of accuracy, \[ |V_{us}| \] and \[ |V_{cd}| \] are expected to be independent of the mass ratios \[ m_c/m_t \] and \[ m_s/m_b \], while \[ |V_{cb}| \] and \[ |V_{ts}| \] are independent of \[ m_u/m_c \] and \[ m_d/m_s \]. The ratios \[ |V_{ub}/V_{cb}| \] and \[ |V_{td}/V_{ts}| \] may be simple functions of \[ m_u/m_c \] and \[ m_d/m_s \], respectively, in the leading order approximations. These qualitative results should hold, in most cases and without fine tuning effects, for generic (Hermitian) forms of \[ M_u \] and \[ M_d \]. They can be used as an enlightening clue for the construction of specific and predictive Ansätze of quark mass matrices.

### 3 A quark mass Ansatz at the weak scale

We are now in a position to consider the realistic \[ 3 \times 3 \] mass matrices in no assumption of the quark mass limits. Such an Ansatz should be able to yield the definite values of \[ R_u \] and \[ R_d \] in Eq. (2.8), and account for current experimental data on flavor mixings and \( CP \) violation at low-energy scales.

#### A. Flavor permutation symmetry breaking

We start from the flavor permutation symmetry to construct quark mass matrices at the weak scale, so that the resultant KM matrix can be directly confronted with the experimental data. The mass matrix with the \( S(3)_L \times S(3)_R \) symmetry reads

\[
M_0 = \frac{c}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix},
\] (3.1)
where \( c = m_3 \) denotes the mass eigenvalue of the third-family quark (\( t \) or \( b \)). Note that \( M_0 \) is obtainable from another rank-one matrix

\[
M_H = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\] (3.2)

through the unitary transformation \( M_0 = U^\dagger M_H U \), where

\[
U = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix}.
\] (3.3)

To generate masses for the second- and first-family quarks, one has to break the permutation symmetry of \( M_0 \) to the \( S(2)_L \times S(2)_R \) and \( S(1)_L \times S(1)_R \) symmetries, respectively. Here we assume that the up and down mass matrices have the parallel symmetry breaking patterns, corresponding to the parallel dynamical details of quark mass generation. We further assume that each symmetry breaking chain (i.e., \( S(3)_L \times S(3)_R \rightarrow S(2)_L \times S(2)_R \) or \( S(2)_L \times S(2)_R \rightarrow S(1)_L \times S(1)_R \)) is induced by a single real parameter, and the possible phase shift between two quark sectors arises from an unknown dynamical mechanism.

With the assumptions made above, a new Ansatz for the up and down mass matrices can be given as follows:

\[
M_0' = \frac{c}{3} \left[ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \sigma \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \right],
\] (3.4)

where \( \epsilon \) and \( \sigma \) are real (dimensionless) perturbation parameters responsible for the breakdows of \( S(3)_L \times S(3)_R \) and \( S(2)_L \times S(2)_R \) symmetries of \( M_0 \), respectively. In the basis of \( M_H \), the mass matrix \( M_0' \) takes the form

\[
M_H' = c \begin{pmatrix} 0 & \frac{\sqrt{3}}{3} & 0 \\ \frac{\sqrt{3}}{3} \sigma & -\frac{2}{9} \epsilon & -\frac{2\sqrt{2}}{9} \epsilon \\ 0 & -\frac{2\sqrt{2}}{9} \epsilon & 1 + \frac{5}{9} \epsilon \end{pmatrix},
\] (3.5)

which has three free parameters and three texture zeros. Diagonalizing \( M_H' \) through the unitary transformation \( O^\dagger M_H' O = \text{Diag}\{m_1, m_2, m_3\} \), one can determine \( c, \epsilon \) and \( \sigma \) in terms of the quark mass eigenvalues. In the next-to-leading order approximations, we get

\[
c \approx m_3 \left( 1 + \frac{5 m_2}{2 m_3} \right), \\
\epsilon \approx -\frac{9 m_2}{2 m_3} \left( 1 - \frac{1}{2} \frac{m_2}{m_3} \right), \\
\sigma \approx \frac{\sqrt{3} m_1 m_2}{m_3} \left( 1 - \frac{5}{2} \frac{m_2}{m_3} \right).
\] (3.6)
Then the elements of $O'$ are expressible in terms of the mass ratios $m_1/m_2$ and $m_2/m_3$.

The flavor mixing matrix can be given as $V = O'_u P O'_d$, where $P$ is a diagonal phase matrix taking the form $P = \text{Diag}\{1, \exp(i\Delta \phi), \exp(i\Delta \phi)\}$. Here $\Delta \phi$ denotes the phase shift between up and down mass matrices, and its presence is necessary for the Ansatz to correctly reproduce both $|V_{us}|$ (or $|V_{cd}|$) and CP violation.

**B. Flavor mixings and CP violation**

Calculating the KM matrix elements $|V_{us}|$ and $|V_{cd}|$ in the next-to-leading order approximation, we obtain

$$|V_{us}| \approx |V_{cd}| \approx \sqrt{(m_u - m_d) m_s m_c \cos \Delta \phi \left(1 - \frac{m_u}{m_c} + \frac{m_d}{m_s}\right)}.$$

(3.7)

This result is clearly consistent with that in Eq. (2.5). The allowed region of $\Delta \phi$ has been shown by Fig. 1 with the inputs of $m_s/m_d$ and $|V_{us}|$. We find $73^0 \leq \Delta \phi \leq 82^0$ for reasonable values of $m_u/m_c$. In the leading order approximation of Eq. (3.7) or Eq. (2.5), it is easy to check that $|V_{cd}|$, $\sqrt{m_u/m_c}$ and $\sqrt{m_d/m_s}$ form a triangle in the complex plane

In the next-to-leading order approximation, $|V_{cb}|$ and $|V_{ts}|$ can be given as

$$|V_{cb}| \approx |V_{ts}| \approx \sqrt{2 \left(\frac{m_s}{m_b} - \frac{m_c}{m_t}\right) \left[1 + 3 \left(\frac{m_s}{m_b} + \frac{m_c}{m_t}\right)\right]}.$$

(3.8)

Comparing between Eqs. (3.8) and (2.8), we get $R_u = R_d = \sqrt{2}$, determined by the quark mass Ansatz in Eq. (3.4). By use of $m_b/m_s = 34 \pm 4$ [15], we illustrate the allowed region of $|V_{cb}|$ as a function of $m_c/m_t$ in Fig. 2, where the experimental constraint on $|V_{cb}|$ ($= 0.0388 \pm 0.0032$ [14]) has also been shown. We see that the result of $|V_{cb}|$ obtained in Eq. (3.8) is rather favored by current data. This implies that the pattern of permutation symmetry breaking (i.e., $S(3)_L \times S(3)_R \rightarrow S(2)_L \times S(2)_R$) in Eq. (3.4) may have a large chance to be true.

The ratios $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ are found to be

$$\frac{|V_{ub}|}{|V_{cb}|} \approx \sqrt{\frac{m_u}{m_c}}, \quad \frac{|V_{td}|}{|V_{ts}|} \approx \sqrt{\frac{m_d}{m_s}}$$

(3.9)

to a good degree of accuracy \footnote{More precisely, we obtain $|V_{ub}/V_{cb}| \approx \sqrt{m_u/m_c \left(1 - \delta\right)}$ with $\delta = \sqrt{(m_c m_d)/(m_u m_s)} \left(m_u/m_s\right) \cos \Delta \phi$. The magnitude of $\delta$ may be as large as 10\% to 15\% for $\Delta \phi \sim 0^0$ or $180^0$, but it is only about 2\% for $73^0 \leq \Delta \phi \leq 82^0$ allowed by Eq. (3.7).}. By use of Leutwyler’s result $m_s/m_d = 18.9 \pm 0.8$ [13], we get $0.225 \leq |V_{td}/V_{ts}| \leq 0.235$. In comparison, the current data together with unitarity of the $3 \times 3$ KM matrix yield $0.15 \leq |V_{td}/V_{ts}| \leq 0.34$ [14]. The allowed region of $|V_{ub}/V_{cb}|$ is constrained by that of $m_u/m_c$, which has not been reliably determined. We find that $0.0036 \leq m_u/m_c \leq 0.01$ is necessary for the quark mass Ansatz in Eq. (3.4) to accommodate the experimental result $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [3].
In the leading order approximations, we have $|V_{ud}| \approx |V_{cs}| \approx |V_{tb}| \approx 1$. Small corrections to these diagonal elements are obtainable with the help of the unitary conditions of $V$. If we rescale three sides of the unitarity triangle $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$ by $V_{ub}^*$, then the resultant triangle is approximately equivalent to that formed by $V_{cd}$, $\sqrt{m_u/m_c}$ and $\sqrt{m_d/m_s}$ in the complex plane [18]. This interesting result can be easily shown by use of Eqs. (3.7), (3.8) and (3.9). Three inner angles of the unitarity triangle turn out to be

$$
\alpha = \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{tb}^* V_{td}} \right) \approx \Delta \phi ,
$$

$$
\beta = \arg \left( -\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} \right) \approx \tan \left( \frac{\sin \Delta \phi}{\sqrt{m_u m_d - m_c m_s}} \right) ,
$$

$$
\gamma = \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{ub}^* V_{ud}} \right) \approx 180^0 - \alpha - \beta
$$

(3.10)
in the approximations made above. At the forthcoming $B$-meson factories, these three angles will be determined from $CP$ asymmetries in a variety of weak $B$ decays (e.g., $B_d \to J/\psi K_S$, $B_d \to \pi^+ \pi^-$ and $B_s \to \rho^0 K_S$). For illustration, we calculate $\sin(2\alpha)$, $\sin(2\beta)$ and $\sin(2\gamma)$ by use of Eq. (3.10) and plot their allowed regions in Fig. 3. Clearly the quark mass Ansatz under discussion favors $0.18 \leq \sin(2\alpha) \leq 0.58$, $0.5 \leq \sin(2\beta) \leq 0.78$ and $-0.08 \leq \sin(2\gamma) \leq 0.5$. These results do not involve large errors, and they can be confronted with the relevant experiments of $B$ decays and $CP$ violation in the near future.

Finally we point out that $CP$ violation in the KM matrix, measured by the Jarlskog parameter $J$ [19], can also be estimated in terms of quark mass ratios. It is easy to obtain

$$
J \approx 2 \sqrt{\frac{m_u}{m_c}} \sqrt{\frac{m_d}{m_s}} \left( \frac{m_s - m_c}{m_b} \right)^2 \left[ 1 + 6 \left( \frac{m_s}{m_b} + \frac{m_c}{m_t} \right) \right] \sin \Delta \phi .
$$

(3.11)

Typically taking $m_u/m_c = 0.005$, $m_s/m_d = 19$, $m_c/m_t = 0.005$, $m_b/m_s = 34$ and $\Delta \phi = 80^0$, we get $J \approx 2.3 \times 10^{-5}$. This result is of course consistent with current data on $CP$ violation in the $K^0 - \bar{K}^0$ mixing system [2]

## 4 A quark mass Ansatz at the GUT scale

It is interesting to speculate that the quark mass hierarchy and flavor mixings may arise from a certain symmetry breaking pattern in the context of supersymmetric GUTs [20, 21]. Starting from the flavor permutation symmetry, here we prescribe the same Ansatz for quark mass matrices as that proposed in Eq. (3.4) at the supersymmetric GUT scale $M_X$. For simplicity we use $\hat{M}_0$ and $\hat{M}_H$, which correspond to $M_0'$ in Eq. (3.4) and $M_H'$ in Eq. (3.5), to denote the mass matrices at $M_X$ in two different bases. They are related to each other through the unitary transformation $\hat{M}_0 = U^\dagger \hat{M}_H U$. The flavor mixing matrix derived from $\hat{M}_0$ (or $\hat{M}_H$) is denoted by $\hat{V}$. The subsequent running effects of $\hat{M}_0$ and $\hat{V}$ from $M_X$ to $M_Z$ can be calculated with the help of the renormalization group equations in the minimal supersymmetric standard model.
Table 1: The values of $c^n_i$, $b_i$ and $g^2_i(0)$ in the minimal supersymmetric standard model.

| $i$ | $c^n_i$ | $c^d_i$ | $c^e_i$ | $b_i$ | $g^2_i(0)$ |
|-----|---------|---------|---------|-------|-----------|
| 1   | 13/9    | 7/9     | 3       | 11    | 0.127     |
| 2   | 3       | 3       | 3       | 1     | 0.42      |
| 3   | 16/3    | 16/3    | 0       | −3    | 1.44      |

### A. Renormalized mass matrices at $M_Z$

The simplicity of $\hat{M}_0$ (or $\hat{M}_H$) may be spoiled after it evolves from $M_X$ to $M_Z$. To illustrate this point, here we derive the renormalized mass matrices $\hat{M}^u_0$ and $\hat{M}^d_0$ at $M_Z$ by use of the one-loop renormalization group equations for the Yukawa matrices and gauge couplings [22]. To get instructive analytical results, we constrain the ratio of Higgs vacuum expectation values $\tan \beta_{\text{susy}}$ to be small enough ($\tan \beta_{\text{susy}} < 10$), so that all non-leading terms in the Yukawa couplings different from that of the top quark can be safely neglected [23]. In this approximation, the evolution equations of $\hat{M}^u_0$ and $\hat{M}^d_0$ read

\[ 16\pi^2 \frac{d\hat{M}^u_0}{d\chi} = \left[ \frac{3}{v^2} \text{Tr} \left( \hat{M}^u_0 \hat{M}^u_0\dagger \right) + \frac{3}{v^2} \left( \hat{M}^u_0 \hat{M}^u_0\dagger \right) - G_u \right] \hat{M}^u_0, \]
\[ 16\pi^2 \frac{d\hat{M}^d_0}{d\chi} = \left[ \frac{1}{v^2} \left( \hat{M}^u_0 \hat{M}^u_0\dagger \right) - G_d \right] \hat{M}^d_0, \]

where $\chi = \ln(\mu/M_Z)$, $G_u$ and $G_d$ are functions of the gauge couplings $g_i$ ($i = 1, 2, 3$), and $v$ is the overall Higgs vacuum expectation value normalized to 175 GeV. For the charged lepton mass matrix $\hat{M}^e_0$, its evolution equation is dominated only by a linear term $G_e$ in the case of small $\tan \beta_{\text{susy}}$. Thus the Hermitian structure of $\hat{M}^e_0$ will be unchanged through the running from $M_X$ to $M_Z$ (in our discussions the neutrinos are assumed to be massless). The quantity $G_n$ ($n = u, d$ or $e$) obeys the following equation:

\[ G_n(\chi) = 8\pi^2 \sum_{i=1}^{3} \left[ \frac{c^n_i \ g^2_i(0)}{8\pi^2 - b_i \ g^2_i(0)} \chi \right], \]

where $c^n_i$ and $b_i$ are coefficients in the context of the minimal supersymmetric standard model. The values of $g^2_i(0)$, $c^n_i$ and $b_i$ are listed in Table 1. In order to solve Eq. (4.1), we diagonalize $\hat{M}^u_0$ through the unitary transformation $\hat{O}^\dagger \hat{M}^u_0 \hat{O} = \hat{M}^u_0''$. Making the same transformation for $\hat{M}^d_0$, i.e., $\hat{O}^\dagger \hat{M}^d_0 \hat{O} = \hat{M}^d_0''$, we obtain the simplified evolution equations as follows:

\[ 16\pi^2 \frac{d\hat{M}^u_0''}{d\chi} = \left[ 3f^2_t \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (3f^2_t - G_u) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \hat{M}^u_0'', \]
\[
16\pi^2 \frac{d\hat{M}_0^{i'}}{d\chi} = \left[ f_i^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} - G_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \hat{M}_0^{i'}, \tag{4.3}
\]

where \( f_i = m_t/v \) is the top quark Yukawa coupling eigenvalue. For simplicity in presenting the results, we define

\[
\Omega_n = \exp \left[ + \frac{1}{16\pi^2} \int_0^{\ln(M_X/M_Z)} G_n(\chi) \, d\chi \right],
\]

\[
\xi_i = \exp \left[ - \frac{1}{16\pi^2} \int_0^{\ln(M_X/M_Z)} f_i^2(\chi) \, d\chi \right] \tag{4.4}
\]

with \( i = t \) (or \( i = b \)). By use of Eq. (4.2) and the inputs listed in Table 1, one can explicitly calculate the magnitude of \( \Omega_n \). We find \( \Omega_u = 3.47, \Omega_d = 3.38 \) and \( \Omega_e = 1.49 \) for \( M_X = 10^{16} \) GeV and \( M_Z = 91.187 \) GeV. The size of \( \xi_t \) depends upon the value of \( \tan \beta_{\text{susy}} \) and will be estimated in the next subsection. Solving Eq. (4.3) and transforming \( \hat{M}_0^{i'} \) back to \( \hat{M}_0^n \), we get

\[
\hat{M}_0^n(M_Z) = \Omega_u \xi_t^3 \hat{O} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \xi_t \end{pmatrix} \hat{O}^\dagger \hat{M}_0^{i'}(M_X),
\]

\[
\hat{M}_0^d(M_Z) = \Omega_d \hat{O} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \xi_t \end{pmatrix} \hat{O}^\dagger \hat{M}_0^{d'}(M_X) \tag{4.5}
\]

in the leading order approximation.

Since \( \hat{O} \) can be easily determined from \( \hat{M}_0^{u'}(M_X) \) and \( \hat{M}_0^{d'}(M_X) \) in the approximation of \( \hat{M}_0^{u'}(M_X) \approx \text{Diag}\{0, 0, m_t\} \) made above, we explicitly express \( \hat{M}_0^{u}(M_Z) \) and \( \hat{M}_0^{d}(M_Z) \) as follows:

\[
\hat{M}_0^{u}(M_Z) = \frac{c_u}{3} \Omega_u \xi_t^3 \left[ \xi_t^3 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \epsilon_u \begin{pmatrix} x_u & x_u & y_u \\ y_u & x_u & y_u \end{pmatrix} + \sigma_u \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \right] \tag{4.6}
\]

with \( x_u = (\xi_t^3 - 1)/9, y_u = (7\xi_t^3 + 2)/9 \) and \( z_u = (13\xi_t^3 - 4)/9 \); and

\[
\hat{M}_0^{d}(M_Z) = \frac{c_d}{3} \Omega_d \left[ \xi_t \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \epsilon_d \begin{pmatrix} x_d & x_d & y_d \\ y_d & x_d & y_d \end{pmatrix} + D_x + \sigma_d \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} \right] \tag{4.7}
\]

with \( x_d = (\xi_t - 1)/9, y_d = (7\xi_t + 2)/9, z_d = (13\xi_t - 4)/9 \) and

\[
D_x = 2(\epsilon_d - \epsilon_u)x_d \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{4.8}
\]

If one takes \( M_Z = M_X \), which leads to \( \Omega_n = 1, \xi_i = 1 \) and in turn \( x_n = 0, y_n = 1, z_n = 1 \) and \( D_x = 0 \), then Eqs. (4.6) and (4.7) recover the form of \( \hat{M}_0(M_X) \) as assumed in Eq. (3.4). To a good degree of accuracy, \( \hat{M}_0^{u}(M_Z) \) remains Hermitian. The Hermiticity of \( \hat{M}_0^{d}(M_Z) \) is violated by \( D_x \), which would vanish if the top and bottom quark masses were identical (i.e., \( \epsilon_d = \epsilon_u \)).
The presence of nonvanishing $D_\epsilon$ reflects the fact that $m_\epsilon$ dominates the mass spectra of both quark sectors. Of course, one can transform the mass matrices obtained in Eqs. (4.6) and (4.7) into the basis of $\hat{M}_H$. In doing so, we will find the inequality between (2,3) and (3,2) elements of $\hat{M}_H^0(M_Z)$, arising from $D_\epsilon$.

**B. Renormalized flavor mixings at $M_Z$**

Calculating the magnitudes of flavor mixings from $\hat{M}_0$ or $\hat{M}_H$ at $M_X$, we can obtain the same asymptotic relations between the KM matrix elements and quark mass ratios as those given in Eqs. (3.7), (3.8) and (3.9). Now we renormalize such relations at the weak scale $M_Z$ by means of the renormalization group equations. The quantities $\xi_t$ and $\xi_b$ defined in Eq. (4.4) will be evaluated below for arbitrary $\tan\beta_{\text{susy}}$, so that one can get some quantitative feeling about the running effects of quark mass matrices and flavor mixings from $M_X$ to $M_Z$.

The one-loop renormalization group equations for quark mass ratios and elements of the KM matrix $\hat{V}$ have been explicitly presented by Babu and Shafi in Ref. [22]. In view of the hierarchy of Yukawa couplings and quark mixing angles, one can make reliable analytical approximations for the relevant evolution equations by keeping only the leading terms. It has been found that (1) the running effects of $m_u/m_c$ and $m_d/m_s$ are negligibly small; (2) the diagonal elements of the KM matrix have negligible evolutions with energy; (3) the evolutions of $|\hat{V}_{us}|$ and $|\hat{V}_{cd}|$ involve the second-family Yukawa couplings and thus they are negligible; (4) the KM matrix elements $|\hat{V}_{ub}|$, $|\hat{V}_{cb}|$, $|\hat{V}_{td}|$ and $|\hat{V}_{ts}|$ have identical running behaviors. Considering these points as well as the dominance of the third-family Yukawa couplings (i.e., $f_t$ and $f_b$), we get three key evolution equations in the minimal supersymmetric standard model:

$$
\frac{m_d}{m_b}_{M_Z} = \frac{1}{\xi_t \xi_b^3} \frac{m_s}{m_b}_{M_X},
$$

$$
\frac{m_c}{m_t}_{M_Z} = \frac{1}{\xi_t^3 \xi_b} \frac{m_c}{m_t}_{M_X},
$$

$$
|\hat{V}_{ij}|_{M_Z} = \frac{1}{\xi_t \xi_b} |\hat{V}_{ij}|_{M_X},
$$

with $(ij) = (ub), (cb), (td)$ or $(ts)$. In the same approximations, the renormalization group equations for the Yukawa coupling eigenvalues $f_t$, $f_b$ and $f_\tau$ read [22]:

$$
16\pi^2 \frac{df_t}{d\chi} = f_t \left(6f_t^2 + f_b^2 - G_u\right),
$$

$$
16\pi^2 \frac{df_b}{d\chi} = f_b \left(f_t^2 + 6f_b^2 + f_\tau^2 - G_d\right),
$$

$$
16\pi^2 \frac{df_\tau}{d\chi} = f_\tau \left(3f_b^2 + 4f_\tau^2 - G_e\right),
$$

where the quantities $G_n$ have been given in Eq. (4.2).

With the typical inputs $m_t(M_Z) \approx 180$ GeV, $m_b(M_Z) \approx 3.1$ GeV, $m_\tau(M_Z) \approx 1.78$ GeV and those listed in Table 1, we calculate $\xi_t$ and $\xi_b$ for arbitrary $\tan\beta_{\text{susy}}$ by use of the above
equations. Our result is illustrated in Fig. 4. We see that $\xi_b \approx 1$ for $\tan \beta_{\text{susy}} \leq 10$. This justifies our approximation made previously in deriving $\hat{M}_0^0(M_Z)$ and $\hat{M}_H^0(M_Z)$. Within the perturbatively allowed region of $\tan \beta_{\text{susy}}$ [21], $\xi_b$ may be comparable in magnitude with $\xi_t$ when $\tan \beta_{\text{susy}} \geq 50$. In this case, the evolution effects of quark mass matrices and flavor mixings are sensitive to both $f_t$ and $f_b$.

Clearly the analytical results of $|\hat{V}_{us}|$, $|\hat{V}_{cd}|$, $|\hat{V}_{ub}/\hat{V}_{cb}|$ and $|\hat{V}_{td}/\hat{V}_{ts}|$ as those given in Eqs. (3.7) and (3.9) are almost scale-independent, i.e., they hold at both $\mu = M_X$ and $\mu = M_Z$. Non-negligible running effects can only appear in the expression of $|\hat{V}_{cb}|$ or $|\hat{V}_{ts}|$, which is a function of the mass ratios $m_s/m_b$ and $m_c/m_t$ (see Eq. (3.8) for illustration). With the help of Eq. (4.9), we find the renormalized relation between $|\hat{V}_{cb}|$ (or $|\hat{V}_{ts}|$) and the quark mass ratios at the weak scale $M_Z$:

$$|\hat{V}_{cb}| \approx |\hat{V}_{ts}| \approx \sqrt{2} \left( \xi_b \frac{m_s}{m_b} - \xi_t \frac{m_c}{m_t} \right) \left[ 1 + 3 \xi_t \xi_b \left( \xi_b \frac{m_s}{m_b} + \xi_t \frac{m_c}{m_t} \right) \right]. \quad (4.11)$$

This result will recover that in Eq. (3.8) if one takes $M_Z = M_X$ (i.e., $\xi_t = \xi_b = 1$). Using $m_b/m_s = 34 \pm 4$ [13] and taking $m_c/m_t = 0.005$ typically, we confront Eq. (4.11) with the experimental data on $\hat{V}_{cb}$ (i.e., $|\hat{V}_{cb}| = 0.0388 \pm 0.0032$ [14]). As shown in Fig. 5, our result is in good agreement with experiments for $\tan \beta_{\text{susy}} < 50$. This implies that the quark mass pattern $\hat{M}_0$ or $\hat{M}_H$, proposed at the supersymmetric GUT scale $M_X$, may have a large chance to survive for reasonable values of $\tan \beta_{\text{susy}}$.

Note that evolution of the $CP$-violating parameter $J$ is dominated by that of $|\hat{V}_{cb}|^2$. Note also that three sides of the unitarity triangle $\hat{V}_{ub}^* \hat{V}_{ud} + \hat{V}_{cb}^* \hat{V}_{cd} + \hat{V}_{tb}^* \hat{V}_{td} = 0$ have identical running effects from $M_X$ to $M_Z$, hence its three inner angles are scale-independent and take the same values as those given in Eq. (3.10) or Fig. 3. As a result, measurements of $\alpha$, $\beta$ and $\gamma$ in the forthcoming experiments of $B$ physics may check both the quark mass Ansatz at the weak scale and that at the supersymmetric GUT scale.

5 Summary

Without the assumption of specific mass matrices, we have pointed out that part of the observed properties of flavor mixings can be well understood just from the quark mass hierarchy. In the quark mass limits such as $m_u = m_d = 0$, $m_t \to \infty$ or $m_b \to \infty$, a few instructive relations between the KM matrix elements and quark mass ratios are suggestive from current experimental data. We stress that such Ansatz-independent results may serve as a useful guide in constructing the specific quark mass matrices at either low-energy scales or superheavy scales.

Starting from the flavor permutation symmetry and assuming an explicit pattern of symmetry breaking, we have proposed a new quark mass Ansatz at the weak scale. We find that all experimental data on quark mixings and $CP$ violation can be accounted for by our Ansatz. In particular, we obtain an instructive relation among $|V_{cb}|$, $m_s/m_b$ and $m_c/m_t$ in the next-to-leading approximation (see Eq. (3.8)). The scale-independent predictions of our quark mass
pattern, such as $0.18 \leq \sin(2\alpha) \leq 0.58$, $0.5 \leq \sin(2\beta) \leq 0.78$ and $-0.08 \leq \sin(2\gamma) \leq 0.5$, can be confronted with the forthcoming experiments at KEK and SLAC $B$-meson factories.

With the same Ansatz prescribed at the supersymmetric GUT scale $M_X$, we have derived the renormalized quark mass matrices at the weak scale $M_Z$ for small $\tan \beta_{\text{susy}}$ and calculated the renormalized flavor mixing matrix elements at $M_Z$ for arbitrary $\tan \beta_{\text{susy}}$. Except $|\hat{V}_{cb}|$ and $|\hat{V}_{ts}|$, the other asymptotic relations between the KM matrix elements and quark mass ratios are almost scale-independent. We find that the renormalized result of $|\hat{V}_{cb}|$ (or $|\hat{V}_{ts}|$) is in good agreement with the relevant experimental data for reasonable values of $\tan \beta_{\text{susy}}$.

In this work we neither assumed a specific form for the charged lepton mass matrix nor supposed its relation with the down quark mass matrix within the supersymmetric GUT framework. Of course, this can be done by following the strategy proposed in Ref. [25]. Then one may obtain the relations between $m_d$, $m_s$, $m_b$ and $m_e$, $m_\mu$, $m_\tau$. Such an Ansatz, based on the specific GUT scheme and flavor permutation symmetry breaking, will be discussed somewhere else.

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Figure 1: The region of $\Delta \phi$ (as a function of $m_u/m_c$) allowed by Eq. (2.5), where $m_s/m_d = 18.9 \pm 0.8$ [13] and $|V_{us}| = 0.2205 \pm 0.0018$ [2] have been used.

Figure 2: The possible region of $|V_{cb}|$ (as a function of $m_c/m_t$) obtained from Eq. (3.8). Here we have used $m_b/m_s = 34 \pm 4$ [15], and the experimental data on $|V_{cb}|$ are cited from Ref. [14].
Figure 3: The allowed region of $C\bar{P}$-violating measurables $\sin(2\alpha)$, $\sin(2\beta)$ and $\sin(2\gamma)$. Here we have used the inputs $m_s/m_d = 18.9 \pm 0.8$ [13], $|V_{us}| = 0.2205 \pm 0.0018$ [2] and $0.0036 \leq m_u/m_c \leq 0.01$ (favored by the experimental value of $|V_{ub}/V_{cb}|$).
Figure 4: The magnitudes of $\xi_t$ and $\xi_b$ (defined in Eq. (4.4)) changing with $\tan \beta_{\text{susy}}$ in the minimal supersymmetric standard model.

Figure 5: The possible region of $|\hat{V}_{cb}|$ at the weak scale $M_Z$ (as a function of $\tan \beta_{\text{susy}}$) obtained from Eq. (4.11). Here we have used $m_b/m_t = 34 \pm 4$ [15] and taken $m_c/m_t = 0.005$ typically.