Dynamic Responses of RC Girder Bridge under Heavy Truck and Seismic Loads Combined

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Abstract: Overloaded truck and earthquake have become two main factors responsible for bridge damage, consequently the combination of heavy truck and seismic loads as a typical occurrence of extreme events is likely to lead to bridge collapse or destructive damage, in which the crucial issues of coupling load model, dynamic equations and bridge responses have not been adequately addressed. In this study, a simplified vehicle-bridge model consisting of many containers is established to simulate vehicle passage, and the dynamic equations are derived for a 5-axle truck on a simply supported beam as an illustration. Then, five ground motions selected from PEER with appropriate peak ground accelerations and durations and the three truck models specified in American Association of State Highway and Transportation Officials, Caltrans and Chinese codes are applied on the finite element model of a typical reinforced concrete continuous girder bridge, in which, vehicle speed, number of trucks, ground motion and vehicle type are assumed to be random variables and their influences on dynamic responses of the bridge are analyzed. The results show seismic load is the governing factor in dynamic responses but truck load may change displacement shapes; in addition, dynamic responses present a high sensitivity with the number of trucks (set as truck platoon) and gross vehicle weight but rare with vehicle speed. Specifically, the presence of a few trucks could serve as energy dissipation facilities for the bridge under seismic motions but may amplify the response when more trucks involved; some combinations of truck platoon with seismic excitation produce very large displacements and even cracks on the bridge, therefore, such an extreme event requires higher robustness in bridge design to make it be sustainable and serviceability after earthquakes.

Keywords: random loads; structure safety under extreme event; combination of truck and seismic loads; dynamic responses

1. Introduction

The seismic analysis has been a very conventional part of structural analysis [1]. During the past years, the dynamic behaviors of many kinds of structures, such as buildings, concrete bridges, soil-steel composite bridges and tunnels etc. under seismic excitation have been widely studied by simulations or experiments [2–7]. On the other hand, vehicle loads represent a major live load to highway bridges, especially for short- and medium-span bridges, vehicle loads even become the governing live load, therefore the effects of moving vehicles are always of special concerns in bridge engineering [8]. Normally, vehicles are approximated as a series of concentrated loads moving on bridges, which in many cases involving light vehicles allows the numerical method to implement it accurately. However, it is observed that both the amount and loading level of vehicles have significantly increased during recent years [9], as increasingly larger transportation vehicles are utilized, bridges have to subject to larger and heavier loads so that the inertia effect of vehicles can no longer be neglected. To take this effect into account, many vehicle models were developed by researchers to address the issue of vehicle-bridge interaction.
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(VBI) and great leaps have been achieved [10–13]. However, the combined effect of vehicle load and seismic load has not been adequately addressed. The previous studies have shown girder displacements and internal forces induced by heavy traffic loads are likely to exceed design values [14]. If earthquake occurs at the same time, then much severer load effects will be produced, due to the vibration and inertia loads of superstructure and heavy vehicles as well as their interaction, which possibly leads to badly damage or even collapse of bridges, or remarkably reduces their post-earthquake serviceability. As pointed out by Frangopol etc., it is important to study not only structural performance of the bridge during earthquake, but also to investigate incidents that could lead to incapacity to carry traffic after the earthquake [15,16].

Several accidents about the presence of vehicles on highway bridges during earthquakes have been reported, such as the Bay bridge in Oakland California during the Loma Pretia earthquake [17], Gavin Canyon bridge during the Northridge earthquake [18], a RC girder bridge during the Wenchuan earthquake, and the Yokohama-Bay Bridge during the Great East Japan Earthquake [19,20]. Some researches assessed the combined effect of vehicle presence and seismic excitations on the seismic performance of bridges, but most of them focus on the railway bridges in presence of trains [10,21,22]. However, characteristics of trains and highway vehicles, such as mass and suspension system are quite different, these findings cannot be directly used to understand the combined effect of seismic and vehicle load. A few studies addressed the issue of seismic excitation and live load acting on bridge structures simultaneously. Kim et al. [23] found that acceleration responses of the bridge subject to moderate earthquakes were amplified when the vehicles were treated as additional mass, whereas they were reduced when the vehicles were simulated as a dynamic system. Siringoringo et al. [19] analyzed the stability problem of a vehicle against rollover and side slip when crossing a bridge during an earthquake, the results showed that the significant bridge-deck vibration due to an earthquake reduces the effective normal force on vehicle wheels. Borjigin et al. [24] reported that continuously moving vehicles might yield larger longitudinal displacement responses at pier tops and plastic deformations at pier bottoms than those of the bridge alone, implying that ignoring vehicles’ additional mass effect and dynamic effects during earthquakes might be on the non-conservative side. Cui and Xu [25] statistically investigated the seismic responses of both vehicle and bridge under real earthquakes and explained the interaction mechanism between vehicle and bridge based on energy analysis.

In addition, a few experimental studies were also conducted to investigate vehicle dynamic effects on bridge seismic response. Shaban et al. [26] conducted a large-scale experiment with a real vehicle parked on the deck of a simple-span bridge. The results showed that top slab transverse accelerations and bearing displacements were reduced in the presence of a vehicle during seismic excitation, in which, the vehicle can be treated as a tuned mass damper. Besides, an extensive testing of the influences of vehicle dynamics on bridge seismic response was conducted at the University of Nevada at Reno [27]. This study addressed that if the seismic level was rather low, then vehicles had a beneficial effect, but they had an adverse effect if the level was great.

These previous studies mentioned above mainly focused on normal vehicles, and few studies have specifically concerned about heavy trucks, which have obvious differences in characteristics of configurations. Moreover, the presence of vehicles is a random process, which has not been deeply discussed in these studies. With the increase of traffic volume and loading level, the likelihood of heavy vehicle presences on a bridge during an earthquake is expected to increase, therefore, the influence of highway vehicles on bridge seismic responses is still a subject of ongoing research [28,29] it is necessary to better understand the performance of bridges during earthquakes in presence of heavy vehicles. Therefore, this study adopts a simplified VB model to simulate vehicle passage and the dynamic equations are derived. Ground motions selected from PEER and heavy truck models specified in AASHTO, Caltrans and Chinese codes are applied on the finite element model of a typical RC continuous girder bridge, specifically, vehicle speed, number of
vehicles, ground motion and vehicle type are assumed to be random variables and their influences on dynamic responses of the bridge are analyzed.

2. Modeling Approach

2.1. Simplified Vehicle-Bridge System

Dynamic responses will be induced by vehicle passage as a moving load. It is a complex nonlinear system and a simplified model needs to be established for analyzing bridge response without involving too much uncertainties. Generally, the hypothesis that there is no relative displacement between wheels and the bridge deck is made, if the coordination of displacements and forces between vehicles and the bridge is satisfied. If vehicle mass is relatively small, then the inertial force could be calculated by a single mass model moving on bridges. Whereas, vehicle suspension system has to be taken into account for bridge dynamic response when heavy vehicles travelling on bridges, for this end, sprung mass model is the prior one due to its simplicity and efficiency but without consuming accuracy. According to M. H. Scott and Zhu [30], the sprung mass model could be further simplified as a structure with two mass nodes, one node is the mass of vehicle body \( M_v \) and the other is that of vehicle wheels \( M_w \); \( K_v \) and \( C_v \) represent the spring and the damper concerning the force between wheels and the bridge deck, as shown in Figure 1a. In this section, the analytical model for bridge dynamic response is to be established and derived, with two mass nodes moving on a simply supported beam, as illustrated in Figure 1b.

![Figure 1. The sprung mass model representing vehicles. (a) two mass nodes. (b) analytical model.](image)

To reflect the fact that vehicular properties associated with bridge dynamic response will vary with time, and also to guarantee no relative slips between vehicle wheels and then bridge deck, many containers are developed to bound with the bridge deck to carry vehicle properties, by which, vehicular properties could move in the longitudinal direction and be transferred from one container to the other, such that the important properties of axle load, damping and stiffness will be endowed to the very containers when the vehicle arrives at the position, while other containers will be set as non-occupation [30]. In this manner, the entire process of vehicle passage is simulated, and what is more, the important parameters of mass, stiffness and damping could be reasonably accounted. This approach is illustrated by Figure 2.
wheels can overlap certain containers for each step forward. Note that the appearance of
proportion of traffic configurations according to findings from WIM data research [31].
properties of containers are set as zero except those associating with vehicle loads, by
placement of vehicle wheels
sponse even though the static degree of freedom (DOF) is increased, therefore, these DOFs
occupation containers are zero so that they do not have any contribution to dynamic re-
loads, by means of a special mass element in ANSYS. Also note that the masses of non-occupation
containers are zero so that they do not have any contribution to dynamic response even
though the static degree of freedom (DOF) is increased, therefore, these DOFs will not be
included in the dynamic analysis.
As usual, there are various categories of vehicles running on bridges, but every vehicle
could be regarded as a combination of several independent sprung mass models according
to the number of axles, so the kinematic equation of vehicle-bridge system is derived
based on the single sprung mass. To illustrate the basic principle, the kinematic equation
is developed for a 5-axle vehicle moving on the simply supported beam in Figure 1,
equivalently, to simulate 5 pairs of node mass models moving on the beam bounded with
containers. Also note 5-axle vehicle is chosen as the example since they represent the highest
proportion of traffic configurations according to findings from WIM data research [31].

As discussed before, there is no displacement between vehicle wheels and the bridge
deck, so the equilibrium equation of vehicle-bridge system is derived
based on the single sprung mass. To illustrate the basic principle, the kinematic equation
was derived based on the single sprung mass. To illustrate the basic principle, the kinematic equation
is developed for a 5-axle vehicle moving on the simply supported beam in Figure 1,
equivalently, to simulate 5 pairs of node mass models moving on the beam bounded with
containers. Also, the masses of non-occupation containers are zero so that they do not have any
contribution to dynamic response even though the static degree of freedom (DOF) is increased,
therefore, these DOFs will not be included in the dynamic analysis.

Moreover, to make the simulation of vehicle passage considerably accurate but without
time-consuming, a lot of containers are designed along the bridge spans so that the vehicle
wheels can overlap certain containers for each step forward. Note that the appearance of
containers will change the original stiffness matrix and geometry of the bridge since they are
bounded together and their interface is set as no-displacement. To overcome this problem
and reduce the interference of containers with dynamic analysis as much as possible, all
properties of containers are set as zero except those associating with vehicle loads, by
means of a special mass element in ANSYS. Also note that the masses of non-occupation
containers are zero so that they do not have any contribution to dynamic response even
though the static degree of freedom (DOF) is increased, therefore, these DOFs will not be
included in the dynamic analysis.

For \( M_v \), the equilibrium equation could be written as:

\[
M_v \ddot{Z}(t) + K_v[Z(t) - y(x,t)|_{x=vt}] + C_v[\dot{Z}(t) - \frac{d y(x,t)}{dt} \bigg|_{x=vt}] = 0 \tag{1}
\]

where \( Z(t) \) is the displacement of vehicle body \( M_v \), and \( y(x,t) \) represents the identical
displacement of vehicle wheels \( M_w \) and the bridge; \( K_v \) and \( C_v \) are the spring stiffness and
damping of the vehicle, respectively.

When the node mass models moving on the bridge at a speed of \( v \), the bridge has to
bear four kinds of forces, namely self-weight of node mass, inertia force of vehicle body,
elastic force and damping force imposed by the spring and damper, respectively. As a
result, the external force \( P(x,t) \) acts on the bridge segment contacting with the node mass
can be obtained as

\[
P(x,t) = (M_v + M_w)g - M_w \frac{d^2 y(x,t)}{dt^2} + K_v[Z(t) - y(x,t)|_{x=vt}] + C_v \left[ \dot{Z}(t) - \frac{d y(x,t)}{dt} \bigg|_{x=vt} \right] \tag{2}
\]

As discussed before, there is no displacement between vehicle wheels and the bridge deck, so
that

\[
Z = y(x,t)|_{x=vt} = y(vt,t) \tag{3}
\]

by inserting Equation (3) into Equations (1) and (2), then

\[
M_v \ddot{Z}(t) + K_v[Z(t) - y(x,t)|_{x=vt}] + C_v[\dot{Z}(t) - \frac{d y(x,t)}{dt} - v \frac{\partial y(x,t)}{\partial x}] = 0 \tag{4}
\]

\[
P(x,t) = (M_v + M_w)g - M_w \left( \frac{d^2 y(x,t)}{dt^2} + 2v \frac{\partial^2 y(x,t)}{\partial vt} + v^2 \frac{\partial^2 y(x,t)}{\partial x^2} \right) + K_v[Z(t) - y(x,t)] + C_v[\dot{Z}(t) - \frac{\partial y(x,t)}{\partial t} - v \frac{\partial y(x,t)}{\partial x}] \tag{5}
\]
consequently, the kinematic equation for the simply supported beam as shown in Figure 1b can be obtained as
\[
EI \frac{\partial^4 y(x,t)}{\partial x^4} + M \frac{\partial^2 y(x,t)}{\partial t^2} + C \frac{\partial y(x,t)}{\partial t} = \delta(x - vt)P(x,t)
\]
(6)
in which, \(\delta\) is the Dirac function, when \(x = vt, \delta = 1\), otherwise, \(\delta = 0\).

Note that the displacement \(y(x,t)\) can be decomposed into \(y(x,t) = \sum_{i=1}^{\phi} \phi_i(x) \eta_i(t)\) with the modal analysis method, \(\phi_i(x)\) is the \(i\)th mode shape and expressed as \(\phi_i(x) = \sin \frac{Ni\pi x}{L}\), for \(L = \) span length and \(N = \) the modal order; \(\eta_i(t)\) is usually defined as generalized displacement. Insert the decomposed \(y(x,t)\) into the kinematic Equation (6), it becomes
\[
EI \sum_{i=1}^{\phi} \frac{d^4 \phi_i(x)}{dx^4} \eta_i(t) + M \sum_{i=1}^{\phi} \phi_i(x) \eta_i(t) + C \sum_{i=1}^{\phi} \frac{d \phi_i(x)}{dt} \eta_i(t) = \delta(x - vt)P(x,t)
\]
(7)
by multiplying \(\phi_i(x)\) and making integration along the length, then taking advantage of the Reyleigh damping and orthogonality of mode shapes, the oscillation equation for the \(n\)th mode shape can be written:
\[
\omega_n^2 \eta_n(t) + 2\xi_n \omega_n \eta_n(t) + \eta_n(t) = \frac{2}{mL} \int_0^L \delta(x - vt)P(x,t) \phi_n(x) dx
\]
(8)
where \(\omega_n = (\frac{N\pi}{L})^2 \sqrt{\frac{EI}{m}}\) and \(\xi_n\) is the damping ratio of the \(n\)th natural mode shape. Combining equations of (6)–(8), it could be obtained
\[
\eta_n(t) + \gamma_w \sin N\Omega t \sum_{i=1}^{\infty} \sin(\Omega t) \eta_i(t) + 2\xi_n \omega_n \eta_n(t) + 2\gamma_w \Omega \sum_{i=1}^{\infty} \cos(\Omega t) \eta_i(t) +
\gamma_c \sin N\Omega t \sum_{i=1}^{\infty} \sin(\Omega t) \eta_i(t) + \omega_n^2 \eta_n(t) - \gamma_w \Omega^2 \sum_{i=1}^{\infty} \sin^2(\Omega t) \eta_i(t) +
\gamma_k \sin N\Omega t \sum_{i=1}^{\infty} \cos(\Omega t) \eta_i(t) + \gamma_k \sin N\Omega t \sum_{i=1}^{\infty} \cos(\Omega t) \eta_i(t) + \gamma_k \sin N\Omega t - \gamma_k \Omega \sum_{i=1}^{\infty} \cos(\Omega t) \eta_i(t)\]
(9)
in which, \(\gamma_w = \frac{2mL}{\omega_n^2}, \gamma_c = \frac{2mL}{\omega_n^2}, \gamma_k = \frac{2mL}{\omega_n^2}\) and \(\Omega = \frac{\omega_n}{T}\), \(m\) is the mass per unit length of beam being a constant.

Meanwhile, the Equation (4) is rewritten by inserting the decomposed displacement \(y(x,t)\):
\[
M_x \ddot{Z}(t) + K_w Z(t) + C_w \dot{Z}(t) - K_w \sum_{i=1}^{\infty} \sin(\Omega t) \eta_i(t) - C_w \sum_{i=1}^{\infty} \cos(\Omega t) \eta_i(t) = 0
\]
(10)
and the matrix form is \(M \{\ddot{q}(t)\} + C \{\dot{q}(t)\} + K \{q(t)\} = F(t), \) for \(\{q(t)\} = [\eta_1(t) \ \eta_2(t) \ \ldots \ \eta_n(t) \ \ldots \ \eta_N(t) \ U(t)]^T\); Matrices of mass, stiffness and damping are displayed as
\[
M = \begin{bmatrix}
1 + \gamma_1 \Omega & \gamma_1 D_1 & \ldots & \gamma_1 D_{1N} & 0 \\
\gamma_1 D_1 & 1 + \gamma_2 \Omega & \ldots & \ldots & \gamma_1 D_{2N} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\gamma_1 D_{N1} & \ldots & 1 + \gamma_1 \Omega & \ldots & \gamma_1 D_{NN} \\
0 & 0 & \ldots & 0 & \gamma_1 D_1 \\
\end{bmatrix}
\]
(11)
\[
C = \begin{bmatrix}
2\xi_1 \Omega & 2\gamma_2 \Omega \gamma_1 \Omega E_{12} + \gamma_c D_{11} & \ldots & 2\gamma_2 \Omega \Omega E_{1N} + \gamma_c D_{1N} & -\gamma_c D_1 \\
2\gamma_2 \Omega E_{21} + \gamma_c D_{11} & 2\gamma_2 \Omega E_{22} + \gamma_c D_{12} & \ldots & 2\gamma_2 \Omega \Omega E_{2N} + \gamma_c D_{2N} & -\gamma_c D_2 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
2\gamma_2 \Omega E_{N1} + \gamma_c D_{11} & 2\gamma_2 \Omega E_{N2} + \gamma_c D_{12} & \ldots & 2\gamma_2 \Omega \Omega E_{NN} + \gamma_c D_{NN} & -\gamma_c D_N \\
-C_v D_1 & -C_v D_2 & \ldots & -C_v D_N & \gamma_c \Omega
\end{bmatrix}
\]
(12)
\[
K = \begin{bmatrix}
\omega_n^2 & -\gamma_1 \Omega & \ldots & -\gamma_1 \Omega & \gamma_c \Omega D_{11} + \gamma_c \Omega D_{12} + \gamma_c \Omega E_{12} & \ldots & -\gamma_1 \Omega \Omega G_{1N} + \gamma_c \Omega D_{1N} + \gamma_c \Omega E_{1N} & -\gamma_c D_1 \\
-\gamma_1 \Omega \Omega G_{12} + \gamma_c \Omega D_{11} + \gamma_c \Omega E_{12} & \omega_n^2 & \ldots & -\gamma_1 \Omega \Omega G_{12} + \gamma_c \Omega D_{12} + \gamma_c \Omega E_{12} & \ldots & -\gamma_1 \Omega \Omega G_{12} + \gamma_c \Omega D_{12} + \gamma_c \Omega E_{12} & -\gamma_c D_2 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
-\gamma_1 \Omega \Omega G_{1N} + \gamma_c \Omega D_{11} + \gamma_c \Omega E_{1N} & -\gamma_1 \Omega \Omega G_{1N} + \gamma_c \Omega D_{12} + \gamma_c \Omega E_{1N} & \ldots & \omega_n^2 & -\gamma_1 \Omega \Omega G_{1N} + \gamma_c \Omega D_{1N} + \gamma_c \Omega E_{1N} & -\gamma_c D_N \\
-K_c D_1 - C_v \Omega \Omega G_{1N} + \gamma_c \Omega D_{11} + \gamma_c \Omega E_{11} & -K_c D_2 - C_v \Omega \Omega G_{1N} + \gamma_c \Omega D_{12} + \gamma_c \Omega E_{12} & \ldots & -K_c D_N - C_v \Omega \Omega E_{NN} + \gamma_c \Omega D_{NN} + \gamma_c \Omega E_{NN} & -K_c D_N
\end{bmatrix}
\]
(13)
where $D_i = \sin i\Omega t$, $D_{ij} = D_i D_j = \sin i\Omega t \sin j\Omega t$, $E_i = i \cos i\Omega t$, $G_i = i^2 \sin i\Omega t$, $E_{ij} = E_i D_j = \sin i\Omega t \times j \cos j\Omega t$, $G_{ij} = G_i D_j = \sin i\Omega t \times j^2 \sin j\Omega t$.

By the usage of Newmark-β method, the displacement, velocity and acceleration at $t_i$ and $t_{i+1}$ time can be solved. Finally, the displacement and acceleration of the bridge and vehicle are obtained, respectively:

\[ y(x, t) = \sum_{n=1}^{\infty} \frac{N\pi x}{L} \eta_n(t) \sin \frac{n\pi x}{L} \]
\[ \dot{y}(x, t) = \sum_{n=1}^{\infty} \frac{N\pi x}{L} \dot{\eta}_n(t) \sin \frac{n\pi x}{L} \]
\[ \ddot{z}(t) = \sum_{n=1}^{N} n\Omega \dot{\eta}_n(t) + 2 \sum_{n=1}^{N} n\Omega \cos n\Omega t \eta_n(t) - \sum_{n=1}^{N} (n\Omega)^2 \sin n\Omega t \eta_n(t) \]

2.2. Bridge Model

To make structural dynamic analysis under the combined seismic and truck loads, a prestressed continuous box-girder bridge with a span arrangement of 36 + 56 + 36 m is modelled. The details of the superstructure and the column can be seen in Figure 3. The superstructure is supported by pot bearings. The prototype bridge was designed for a site in class II with a PGA of 0.1 g and the live load of highway class-I. Based on the design parameters, the finite element model (FEM) of this bridge is built. In which, the girder and columns are modelled with BEAM 188 element provided in ANSYS software; this element was developed based on the Timoshenko beam theory for any of elastic, plastic and other nonlinear material analyses. By which, shear deformation could be considered besides flexural deformation and suitable for box-girder used in the present study. There are at least 6 DOFs in the element, including three translational DOFs and three rotational DOFs at X, Y and Z directions, respectively. The FEMs of superstructure and substructure are displayed in Figure 4. All DOFs are fixed at the bottom of piers, and the abutments are fixed in both lateral and vertical directions, while bearings are modelled by coupled nodes.

Figure 3. Cont.
Figure 3. The design details of the superstructure and columns. (a) Layout of the RC continuous girder bridge. (b) I-I section profile. (c) II-II section profile.
2.3. Vehicle Model

Three extensively used types of vehicle models are used to apply live loads to the bridge deck, including the truck model specified in AASHTO bridge design code, Caltrans DOT heavy truck model and the one in Chinese provision, they are separately combined with seismic load to compare and analysis if travelling speed, truck weight and axle load etc. would significantly affect the dynamic response. These truck models are concisely introduced in this section.

2.3.1. Truck Model in AASHTO Code

The type 362 truck specified in AASHTO code is shown in Figure 5a, it is a 5-axle truck with the gross weight of 320.28 kN. The detailed axle loads and spaces are marked in the figure.
2.3.2. Heavy Truck Model in Caltrans

California is well known as an industrial state in the U.S. so that many kinds of huge (on width, height, trailer length, and number of trailers) and heavy (on GVW, axle weight, and wheel weight) trucks appear on highway bridges to carry some high-density commodities with divisible loads, they have been indispensable components of the traffic configuration, indicating the vehicular overload standard issued by the Federal government was not compatible with the real social requirement, therefore, the Caltrans regulated their permission truck level by themselves, embodied by a series of very heavy trucks. To be consistent, the 5-axle truck of this series is used herein. Each tandem has a weight of 213.51 kN and the first axle load is 115.65 kN, as shown in Figure 5b.

2.3.3. Truck Model in Chinese Provision

A simplified 5-axle truck model was stipulated in the Chinese bridge design specification as an alternative of the lane load, accounting for the live loads applying to bridges. The layout of axle loads and spaces is plotted in Figure 5c.

As discussed before, the vehicle is modelled by two nodes of vehicle body and wheels denoting the weight of $M_v$ and $M_w$, and its moving process is simulated by a cluster of containers associated with the bridge, transferring the properties of the vehicle step by step. The MASS21 element with 6 DOFs is used to simulate vehicle body and wheels, and COMBIN14 element for the suspension system as a one-dimension tensioned or compressed spring-damper. The interface element of CONTA 175 is utilized to deliver vehicle weight to the girder and make sure no slips between vehicle wheels and the containers involved.

3. Dynamic Characteristic of the Bridge Model

As a MDOF structure system, bridges have many inherent vibration frequencies and mode shapes, but the dynamic characteristics of the 1st–6th orders are adequate for dynamic response analysis of normal RC girder bridges, since the higher orders are rarely excited and have negligible effects on the whole responses. Therefore, the first 6th frequencies and mode shapes are calculated and demonstrated in Table 1 and Figure 6.
Table 1. Natural frequencies and mode shapes of the bridge.

| Vibration Mode | Frequency (Hz) | Mode Shape                        |
|----------------|---------------|-----------------------------------|
| 1              | 1.94          | 1st order longitudinal bending    |
| 2              | 3.69          | antisymmetric vertical bending    |
| 3              | 4.81          | 2nd order longitudinal bending    |
| 4              | 5.73          | 1st order torsion                 |
| 5              | 7.50          | dissymmetric vertical bending     |
| 6              | 7.72          | dissymmetric transverse bending   |

Figure 6. The first 6th mode shapes of the bridge. (a) The 1st order. (b) The 2nd order. (c) The 3rd order. (d) The 4th order. (e) The 5th order. (f) The 6th order.

4. Vehicle-Bridge System Equation under Combined Seismic and Vehicle Loads

So far, the concerned question of bridges under combined seismic and vehicle loads has been divided into two parts, one is the simplified vehicle-bridge (VB) system established in the previous section, the remaining part concerning seismic load can be treated as an external excitation to apply to
the VB system. If we let \( U_b \) denote the displacement vector of the container nodes, then the enlarged bridge equation could be expressed as:

\[
\begin{bmatrix}
M_v & 0 \\
0 & M_b
\end{bmatrix}
\begin{bmatrix}
\bar{U}_v \\
\bar{U}_b
\end{bmatrix}
+ 
\begin{bmatrix}
C_v & C_{vb} \\
C_{bv} & C_b
\end{bmatrix}
\begin{bmatrix}
\bar{U}_v \\
\bar{U}_b
\end{bmatrix}
+ 
\begin{bmatrix}
K_v & K_{vb} \\
K_{bv} & K_b
\end{bmatrix}
\begin{bmatrix}
\bar{U}_v \\
\bar{U}_b
\end{bmatrix}
= 
\begin{bmatrix}
F_v + F_{\text{eff}} \\
F_b + F_{\text{eff}}
\end{bmatrix}
\tag{15}
\]

in which, \( F_{\text{eff}} \) and \( F_{\text{eff}} \) represent dynamic responses of the vehicle and the bridge associated with containers under seismic motions, respectively; subscripts v and b denote vehicle and bridge, respectively. \( F_v \) and \( F_b \) are force vectors. Other parameters have been discussed before. Moreover, the vector \( U_b \) can be divided into pseudo-static displacement and dynamic displacement, written as

\[
\{ U_b \} = \{ U_{bs} \} + \{ U_{bd} \},
\]

where superscripts s and d denote pseudo-static and dynamic displacement, respectively; subscripts s denotes bridge nodes except bearings, while b denotes bearings. Since the seismic effects are treated as uniform excitation, then the pseudo-static displacements at supports are identical and no relative slips between bearings.

It is supposed that there are \( N \) vehicles moving on the bridge and \( N_1 \) mode shapes of the bridge have contributions to the dynamic response when earthquake occurs, then, \( \{ P_{\text{eff}} \} = \{ P_{\text{eff}1} \ P_{\text{eff}2} \ldots \ P_{\text{eff}N} \}^T \) and \( \{ P_{\text{eff}} \} = \{ P_{\text{eff}1} \ P_{\text{eff}2} \ldots \ P_{\text{eff}N} \}^T \). To be consistent with vehicle passage, ground motions are input in longitudinal direction, then the generalized seismic load on the bridge can be expressed by the normalized \( \text{ith} \) mode as \( P_{\text{eff}i} = a_x^i(t) \{ \phi \}^T \{ M \} \{ \phi \} a_x^i(t) \) is the longitudinal ground motion component; \( \{ D \} \) denotes directional vector, and \( \{ \phi \} \) is the \( \text{ith} \) generalized mode vector. Based on these derived expressions, the Equation (15) can be solved by the Newmark-\( \beta \) method further.

5. Dynamic Response Analysis of the Bridge under Combined Seismic and Vehicle Loads

To address the influences of vehicle speed, number of vehicles, intensity and duration of ground motion and vehicle type on bridge dynamic responses, these factors are analyzed separately in this section, incorporating with site class, damping ratio, design spectral acceleration etc. of the prototypic bridge, to understand the performance of the bridge under different seismic and/or vehicle loading conditions.

5.1. Ground Motions Selection

Un-scaled ground motions (GMs) are selected from the PEER strong ground motion database. Out of three ground motion components in each record set, two horizontal and one vertical, the horizontal motion is selected. Besides, the design spectral acceleration in Chinese Specifications for Seismic Design of Highway Bridges is taken as reference for selecting ground motion records, based on the design parameters, such as site class of I, damping ratio of 0.05 and characteristic period of 0.3 s etc., the calculated spectral peak acceleration \( S_{\text{max}} = 0.195 \) g is for the basic seismic design level, for severe earthquake with a return period of 1600 years it should be enlarged 1.6~2.3 times according to the specification, let it be 2 times and finally a 0.4 g of \( S_{\text{max}} \) is set for scaling ground motions. Note that the duration of the selected ground motions needs to be determined additionally rather than using the recorded time, to make sure the time period with dominant acceleration/energy of ground motions complies with the estimated time for vehicles passing through the bridge with a certain speed. Therefore, the 70% of energy duration is taken as seismic duration because of its widespread use. As a consequence, 5 potential ground motions are firstly selected and their processed parameters are listed in Table 2.

| No. | Earthquake Event     | Energy Duration(s) | Normalized PGA(g) |
|-----|----------------------|--------------------|-------------------|
| T1  | Sagueneay, 25 November 1988 | 3.93               | 0.95              |
| T2  | AuSableForks, 20 April 2002  | 15.22              | 1.0               |
| T3  | RiviereDuLoup, 6 March 2005 | 14.73              | 1.0               |
| T4  | MtCarmel, 18 April 2008    | 7.10               | 0.72              |
| T5  | Mineral, 23 August 2011   | 5.86               | 0.95              |

As can be seen, T2 and T3 have longer durations to present the combination of seismic load and vehicle load of interest in this study. In addition, the standard deviations in energy durations of T2 equals to 0.24 and less than that of 0.26 of T3, thus, T2 is selected as the seismic excitation
plus live-load for the bridge. The normalized T2 ground motion and its energy duration are shown in Figure 7.

![Figure 7](image-url)

**Figure 7.** The ground motion selected. (a) The acceleration time history of GM T2. (b) The duration of the GM T2.

5.2. Dynamic Response Analysis of the Bridge

Based on the VB system and ground motion time history demonstrated before, some important factors associating with vehicle load and seismic load are to be investigated specifically, to understand the influence of each parameter on bridge dynamic responses. For this end and also making the results more comparable, the Caltrans truck model and ground motion T2 are set as benchmark for vehicle load and seismic load, respectively. If not reported particularly, then they are used in default. In addition, vehicles are supposed to move at uniform speeds and the headways are constant if more than one vehicle involved.

5.2.1. Vehicle Speed

Vehicle speed determines the travelling time and therefore the loading time of the vehicle on the bridge, and it may have effect on displacements of the bridge. To be comparable, five common passing speeds from 36 km/h to 108 km/h with an interval of 18 km/h are considered. Figure 8a shows the displacement time history under the combined seismic load and heavy truck load, and Figure 8b shows their counterparts under traffic loading only.

![Figure 8](image-url)

**Figure 8.** Displacements of the bridge taking different vehicle speeds. (a) combined loads. (b) vehicle loads only.
It can be seen in Figure 8b, when the vehicle passing through the bridge at different speeds, the peak displacement very slightly increases with increasing travelling speed, indicating larger impact and dynamic response of the bridge are produced. Whereas, the displacements are very close and the maximum difference is only 0.15 mm, so vehicle speed has very limited influence on bridge displacement. On the other hand, when vehicle loads combined with seismic loads, the displacements are amplified dramatically and the displacement induced by seismic excitation is overwhelming component of the total response. However, the displacement slightly decreases with increasing travelling speed, and the maximum value is 0.01 mm, denoting vehicle speed has negligible effect on the dynamic response. As shown, the dynamic response induced by the combined seismic load plus live-load is approximately 10 times that under live load only.

It is notable the largest displacements in (a) always occur when the vehicle just appears at the side-span, also the ground acceleration has relatively small value; the second largest values appear at the time when the vehicle arrives at the mid-span, while in the case of (b), it has the largest displacements. Also note the GM excitation time in (a) depends on the travelling time, namely the vehicle speed, so that the fast speed of 108 km/h and the slowest speed of 36 km/h have the shortest and longest excitation time, respectively. Even though the latter gets the peak acceleration value of 1.0 g while the former only gets half of that, but their peak displacements are very close, implying this bridge structure may be sensitive to certain frequency components but not necessarily the ground acceleration value. This is also seen for other excitation time (or vehicle speed).

5.2.2. Multiple Presence of Vehicles

Vehicles usually appear on a bridge span in one or more lanes simultaneously, which is known as multiple truck presence. The previous studies reported the multiple loading configuration would induce higher static load effects to bridge components than those of single one [2]. To address if the dynamic responses will still be governed by multiple truck presence when the live loads are combined with seismic loads, five platoons consisting of one to five trucks are applied on the bridge separately, denoting case 1 to case 5 herein, for each platoon, the travelling speed is set as 72 km/h and the distance between platoons is 10 m. The platoons are assumed to move on the centerline of lane 2. The presence of the platoon consisting of two trucks on the bridge can be viewed in Figure 9 as an example.

![Figure 9. Application of truck platoon to the bridge.](image)

Figure 10a displays displacement time histories under combined seismic motion and truck platoon, and Figure 10b displays displacements under each truck platoon. As observed, displacements in Figure 10b have nonlinear relationship with the number of trucks, when the number increases from 1 to 2 or 3, the displacements increase noticeably and the maximum increment is about 3.5 mm, but when the number continually increases to 4 or 5, the displacements do not necessarily increase and even decrease by 2 mm at most. It can be concluded that multiple truck presence would induce higher load effects to bridges than single truck, and the severest spatial loading configuration would govern the maximum effects rather than the number of trucks.

When the ground motion excitation is taken into account, displacements in Figure 10a generally increase with the truck number, and a significant difference of 30 mm exits between one truck and four trucks, indicating the number of trucks has remarkable influence on the total response, but the influence is to be limited when the number increase to a certain degree. However, the responses slightly decrease when comparing with the case of GM exciting only, indicating vehicles may make a positive effect on energy dissipation, due to their body mass and suspension system, to reduce the dynamic responses to some extent. In this study, case 3, 4 and 5 generally have observable dissipation effect, but one should keep in mind that the number of trucks, their presence on lanes, headway distances as well as truck system will affect this result, so it would be a case-specified analysis rather a general conclusion.
5.2.3. Ground Motions

Ground motion has very strong stochastic behavior, to better understand the random variables associated with seismic loads in the previous analyses, another three ground motions in widespread use are selected as the inputs and combined with live loads to apply to the bridge as before. The three ground motions RiviereDuLoup, Kobe and Chi-Chi are adopted because they have similar PGAs and other features with the scaled ground motion T2. They are also processed accordingly, and the normalized time histories are displayed in Figure 11 as below.

For consistency, similar analyses are implemented in this section. Table 3 and Figure 12a,b show the displacements under the four ground motions plus vehicle-load taking different speeds and platoons, respectively. Here vehicle speeds and number of trucks are identically set with the previous ones. For analysis of vehicle speed, one single Caltrans truck model is applied, while for that of vehicle number, a speed of 72 km/h is used. It is shown no matter which ground motion is adopted, the displacements have almost the same variation tendency with the changes of vehicle parameters. Displacements slightly decrease with faster speeds while increase when more trucks running on the bridge. As the vehicle speed or vehicle number isthe same, the displacements for the four GMs are very close, and the differences are within 0.5 mm and 15 mm, respectively. As discussed above, the number of trucks has remarkable influence on the total response, whereas the randomness of ground motion barely affects the dynamic response of interest herein.

Figure 11. Cont.
Figure 11. Three supplemented ground motions. (a) Chi-Chi ground motion. (b) Kobe ground motion. (c) RiviereDuLoup ground motion.

Table 3. Maximum midspan displacements considering different ground motions.

| GM               | Vehicle Speed (km/h, One Caltrans) Number of Vehicles (at 72 km/h) |
|------------------|---------------------------------------------------------------------|
|                  | 36 | 54 | 72 | 90 | 108 | 1  | 2  | 3  | 4  | 5  |
| AuSableForks     | 63.75 | 63.75 | 63.75 | 63.74 | 63.75 | 64.15 | 79.16 | 91.70 | 87.46 |
| RiviereDuLoup    | 63.71 | 63.71 | 63.70 | 63.70 | 63.70 | 63.71 | 61.48 | 87.35 | 106.94 | 99.22 |
| Chi-Chi          | 63.67 | 63.67 | 63.67 | 63.66 | 63.68 | 61.45 | 87.31 | 106.88 | 99.17 |
| Kobe             | 63.58 | 63.58 | 63.58 | 63.57 | 63.59 | 61.36 | 87.20 | 106.73 | 99.06 |
5.2.4. Vehicle Type

Three truck models in different specifications have been introduced above, among them the Caltrans truck model induces the largest load effects so it has been used in the previous analysis. As known, vehicles represent a major live load to bridges and uncertainties associated with it are very high, therefore, this study also has a consideration on the randomness of vehicle loads as one of critical variables. For the focused objective, the randomness of vehicle loads herein includes speed, model type and presence number, which are to be analyzed by grouping.

Firstly, each truck model running on the bridge at different speeds is considered with the scaled ground motion T2. To be consistent, all speeds set in Section 5.2.1 are used, and the midspan displacements under combined loads and truck loads only are listed in Table 4 below. The results are further plotted in Figure 13a,b to compare.

![Figure 12. Displacements of the bridge under different ground motions. (a) at different vehicle speeds. (b) with different vehicle numbers.](image)

Table 4. Maximum midspan displacements considering different vehicle models.

| Truck Model | Load Type     | Vehicle Speed (km/h) |
|-------------|---------------|----------------------|
|             |               | 36       | 54       | 72       | 90       | 108      | 108      |
| Caltrans    | Combined load | 63.75   | 63.75   | 63.75   | 63.75   | 63.74    |           |
|             | Truck load only | 6.34   | 6.36   | 6.40   | 6.44   | 6.49    |           |
| AASHTO      | Combined load | 63.67   | 63.67   | 63.67   | 63.67   | 63.66    |           |
|             | Truck load only | 3.48   | 3.49   | 3.504  | 3.52  | 3.56    |           |
| China       | Combined load | 63.73   | 63.73   | 63.728  | 63.726  | 63.72    |           |
|             | Truck load only | 6.14   | 6.16   | 6.194  | 6.230  | 6.29    |           |
In Figure 13a, the displacements induced by light vehicle and heavy vehicle are quite different, for example, the displacement under the Caltrans model having the largest GVW is nearly two times that of the lightest AASHTO model, indicating vehicle weight has significant influence on bridge responses under live loads only. In addition, the GVWs of Caltrans and China models are close but their axle loading configurations are different, causing the displacements produced are also different, especially when the moving speed is high, thus, vehicle speed is also a notable factor enlarging the dynamic responses when vehicle loads act alone. On the contrary, in Figure 13b, the influence is remarkably weakened when seismic loads and live loads work together, since the former becomes the governing one, leading to a difference less than 3 mm between each pair of truck models.

Secondly, truck platoons are assembled by the same truck model with different numbers of one to five moving on the bridge at a speed of 72 km/h, as a result, 15 platoons are combined with the scaled ground motion T2, respectively, to calculate the displacements shown in Table 5. The results are also plotted in Figure 13c,d for comparison.

Figure 13. Displacements of the bridge taking different truck models in codes. (a) vehicle loads only. (b) combined loads. (c) vehicle loads only. (d) combined loads.
Table 5. Maximum midspan displacements considering different vehicle platoons.

| Truck Platoon | Load Type          | Number of Vehicles in the Platoon |
|---------------|--------------------|-----------------------------------|
|               | Combined load      | 1  | 2   | 3  | 4   | 5   |
| Caltrans      |                    | 63.75 | 64.15 | 79.16 | 91.70 | 87.46 |
|               | Truck platoon only | 6.40 | 9.55 | 9.49 | 7.53 | 7.69 |
| AASHTO        |                    | 66.45 | 67.90 | 84.70 | 86.96 | 82.35 |
|               | Truck platoon only | 3.50 | 4.99 | 4.32 | 3.78 | 3.90 |
| China         |                    | 63.73 | 65.94 | 94.42 | 99.62 | 91.21 |
|               | Truck platoon only | 6.19 | 8.60 | 7.55 | 6.57 | 6.29 |

It can be observed from Table 5 and Figure 13c,d, no matter the seismic load is combined or not, the number of vehicles apparently affects the displacements. For the situation without seismic loads, the largest response is produced when two vehicles moving on the bridge, whichever vehicle model is used, and the maximum difference also exists between Caltrans and AASHTO models, which is about twice as high. For another situation with seismic loads, when platoons consisting of a few trucks, variables of truck model and number hardly generate disparity, trucks have somehow tuned-mass-damper behavior and make positive contribution for reducing bridge dynamic response, but when more trucks come into platoons, the displacements go up, implying these variables play important roles in the total responses. Note the AASHTO platoon of three trucks induces larger displacement than the Caltrans platoon, even though the former’s GVW is much lesser, and the maximum responses occur for platoons of four trucks rather than five trucks. Among the three specified models, the platoon designed by the Chinese provision induces the largest displacement, which is about 15 mm higher than that of the other two. As a consequence, the variables associated with the number of trucks, especially the spatial loading configurations would remarkably affect the bridge responses, sometimes it may govern the load effects rather than the number.

6. Discussion

In this study, dynamic responses of a RC bridge subjected to combined truck load and seismic load are analyzed, as a representative occurrence of extreme events for bridge structures. Numerical simulations are performed on this continuous girder bridge, presenting a simplified vehicle-bridge system associated with containers, for which, the kinematic equations are established and derived accordingly. Then, vehicle speed, multiple presence of vehicles, ground motions and vehicle type are assumed to be random variables, and the midspan displacements are compared under combined loads and live loads alone, taking these randomness and uncertainties into account. The main findings could be summarized as below.

1. The results show that the simplified VB model has sufficient accuracy in simulating the inertia force on vehicle body induced by GMs, and it also has many conveniences in presenting vehicle passage and seismic excitation. Seismic load is always the governing one in all combined loading cases, no matter what kind of vehicle loads are considered, but heavy truck loads may change displacement shapes of girders.

2. For these key random variables concerned in this study, vehicle speed almost has no influence on dynamic response, since the maximum difference of midspan displacements is less than 7%, but multiple presence of vehicles has significant effects on dynamic response, showing the maximum difference of about 40%, for the severest case, the largest vertical displacement is close to 10cm, which would cause destructive damage for the bridge.

3. When presence number is rather small, like one or two, the dynamic responses slightly decrease, comparing with their counterparts excited by ground motions only, indicating vehicles somehow make a positive effect on energy dissipation, provided by their body mass and suspension system, to reduce the dynamic responses to some extent.

4. Vehicle type presented by configurations of axle load and spacing also has some contributions to the comprehensive responses, where the gross vehicle weight would not be the determining factor rather than the spatial arrangement of total weight. In addition, displacements under the four GMs are very close, the differences are within 0.5 mm~15 mm, therefore, the randomness of ground motion barely affects the dynamic response of interesting herein.
The analysis concentrates on the dynamic responses of a RC girder bridge subjected to combined seismic and live loads. Also, the analysis considers some randomness and uncertainties related. To report more comprehensive findings, the effect of bridge bearings, bridge type as well as span lengths and number of lanes require further investigation.

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References
1. Chopra, A.K. Dynamics of Structures: Theory and Applications to Earthquake Engineering, 2nd ed.; Prentice Hall: Hoboken, NJ, USA, 2001; pp. 1–3.
2. Malecka, T.; Beben, D. Behaviour of Soil-Steel Composite Bridge with Various Cover Depths under Seismic Excitation. Steel Compos. Struct. 2022, 42, 747–764. [CrossRef]
3. Flener, E.B.; Karoumi, R. Dynamic Testing of A Soil–Steel Composite Railway Bridge. Eng. Struct. 2009, 31, 2803–2811. [CrossRef]
4. Malecka, T.; Beben, D.; Nowacka, J. Vulnerability of A Soil-Steel Composite Tunnel–Norway Tolpinrud Railway Tunnel Case Study. Tunn. Undergr. Space Technol. 2021, 110, 103808. [CrossRef]
5. Miao, Y.; He, H.; Liu, H.; Wang, S. Reproducing ground response using in-situ soil dynamic parameters. Earthq. Eng. Struct. Dyn. 2022, 51, 2449–2465. [CrossRef]
6. Wang, S.Y.; Zhuang, H.Y.; Zhang, H.; He, H.-J.; Jiang, W.-P.; Yao, E.-L.; Ruan, B.; Wu, Y.-X.; Miao, Y. Near-surface softening and healing in eastern Honshu associated with the 2011 magnitude-9 Tohoku-Oki Earthquake. Nat. Commun. 2021, 12, 1215. [CrossRef] [PubMed]
7. Long, H.; Zhuang, K.; Deng, B.; Jiao, J.; Zuo, J.; You, E. Dynamic Characteristics of Coral Sand in the Condition of Particle Breakage. Geofluids 2022, 2022, 5304179. [CrossRef]
8. Fu, G.K.; Liu, L.; Bowman, M.D. Multiple Presence Factor for Truck Load on Highway Bridges. J. Bridge Eng. 2013, 18, 240–249. [CrossRef]
9. Wang, F.Y.; Xu, Y.L. Traffic Load Simulation for Long-Span Suspension Bridges. J. Bridge Eng. 2019, 24, 05019005. [CrossRef]
10. Yang, J.P. Theoretical Formulation of Amplifier-Vehicle-Bridge System Based on Sophisticated Vehicle Model. J. Vib. Eng. Technol. 2018, 31, 1–12. [CrossRef]
11. Deng, L.; Cai, C.S. Development of Dynamic Impact Factor for Performance Evaluation of Existing Multi-Girder Concrete Bridges. Eng. Struct. 2010, 32, 21–31. [CrossRef]
12. Han, W.S.; Yuan, Y.G.; Huang, P.M.; Wu, J.; Wang, T.; Liu, H. Dynamic Impact of Heavy Traffic Load on Typical T-beam Bridges Based on WIM Data. J. Perfor. Constr. Facil. 2017, 31, 1–14. [CrossRef]
13. Yang, J.P. Theoretical Formulation of Amplifier-Vehicle-Bridge System Based on Sophisticated Vehicle Model. J. Vib. Eng. Technol. 2012, 10, 789–794. [CrossRef]
14. Han, W.; Liu, X.; Gao, G.; Xie, Q.; Yuan, Y. Site-Specific Extra-Heavy Truck Load Characteristics and Bridge Safety Assessment. J. Aerosp. Eng. 2018, 31, 1–12. [CrossRef]
15. Dong, Y.; Frangopol, D.M.; Saydam, D. Sustainability of Highway Bridge Networks Under Seismic Hazard. J. Earthq. Eng. 2014, 18, 41–66. [CrossRef]
16. Dong, Y.; Frangopol, D.M. Risk and Resilience Assessment of Bridges under Mainshock and Aftershocks Incorporating Uncertainties. Eng. Struct. 2015, 83, 198–208. [CrossRef]
17. Yashinsky, M. The Loma Prieta, California, Earthquake of October 17, 1989-Highway Systems; US Government Printing Office: Washington, DC, USA, 1998.
18. Mitchell, D.; Bruneau, M.; Saatcioglu, M.; Williams, M.; Anderson, D.; Sexsmith, R. Performance of bridges in the 1994 Northridge earthquake. Can. J. Civ. Eng. 1995, 22, 415–427. [CrossRef]
19. Siringoringo, D.M.; Fujino, Y. Lateral Stability of Vehicles Crossing a Bridge during an Earthquake. J. Bridge. Eng. 2018, 23, 1–22. [CrossRef]
20. Siringoringo, D.M.; Fujino, Y.; Yabe, M. Investigation on vehicle lateral instability when crossing a curved highway bridge during an earthquake. Struct. Infrastruct. Eng. 2020, 17, 1–21. [CrossRef]
21. Xia, H.; Han, Y.; Zhang, N.; Guo, W. Dynamic analysis of train–bridge system subjected to non-uniform seismic excitations. *Earthq. Eng. Struct. Dyn.* 2006, 35, 1563–1579. [CrossRef]

22. Yang, Y.B.; Wu, Y.S. Dynamic stability of trains moving over bridges shaken by earthquakes. *J. Sound. Vib.* 2002, 258, 65–94. [CrossRef]

23. Kim, C.W.; Kawatani, M.; Konaka, S.; Kitaura, R. Seismic responses of a highway viaduct considering vehicles of design live load as dynamic system during moderate earthquakes. *Struct. Infrastruct. Eng.* 2011, 7, 523–534. [CrossRef]

24. Borjigin, S.; Kim, C.W.; Chang, K.C.; Sugiura, K. Nonlinear Dynamic Response Analysis of Vehicle-Bridge Interaction System under Strong Earthquakes. *Eng. Struct.* 2018, 176, 500–521. [CrossRef]

25. Cui, C.; Xu, Y. Mechanism study of vehicle-bridge dynamic interaction under earthquake ground motion. *Earthq. Eng. Struct. Dyn.* 2021, 50, 1931–1947. [CrossRef]

26. Shaban, N.; Caner, A.; Yakut, A.; Askan, A.; Karimzadeh Naghshineh, A.; Domanic, A.; Can, G. Vehicle Effects on Seismic Response of A Simple-Span Bridge During Shake Tests. *Earthq. Eng. Struct. Dyn.* 2015, 44, 889–905. [CrossRef]

27. Zhou, Y.; Chen, S. Full-response prediction of coupled long-span bridges and traffic systems under spatially varying seismic excitations. *J. Bridge Eng.* 2018, 23, 04018031. [CrossRef]

28. Li, R.W.; Wu, H.; Yang, Q.T.; Wang, D.F. Vehicular impact resistance of seismic designed RC bridge piers. *Eng. Struct.* 2020, 220, 111015. [CrossRef]

29. Wibowo, H.; Sanford, D.M.; Buckle, I.G.; Sanders, D.H. The Effect of Live Load on the Seismic Response of Bridges; Reno Research Report No. CCEER 13–10; University of Nevada: Reno, NV, USA, 2013.

30. Scott, M.H.; Zhu, M.J. Combined Seismic Plus Live-Load Analysis of Highway Bridge; Final Report OTREC-RR-11-20; Transportation Research and Education Center (TREC): Portland, OR, USA, 2011.

31. Lang, L.; Chen, D.J.; Ren, Q.Y. Overloaded Truck Models and Their Load Effects on Multiple-One Lane for Highway Bridges. *J. Southwest Jiaotong Univ.* 2019, 54, 1169–1176. (In Chinese)