On Stable Exponential Cosmological Solutions in the EGB Model with a Cosmological Constant in Dimensions $D = 5, 6, 7, 8$

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Abstract—A $D$-dimensional Einstein–Gauss–Bonnet (EGB) flat cosmological model with a cosmological constant $\Lambda$ is considered. We focus on solutions with an exponential time dependence of the scale factor. Using the previously developed general stability analysis of such solutions by V.D. Ivashchuk (2016), we apply the criterion from that paper to all known exponential solutions up to the dimension $7 + 1$. We show that this criterion, which guarantees the stability of solutions under consideration, is fulfilled for all combinations of the coupling constant of the theory except for some discrete set.

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1. INTRODUCTION

Lovelock gravity [1] can be considered as the most conservative modification of general relativity (GR) in the sense that the equations of motion of this theory are second-order differential equations (as in GR), in contrast to other metric theories, usually leading to fourth-order equations (though there are some other approaches with the same property, for example, the Palatini version of $f(R)$ theory or $f(T)$ theory). Usually, an increase in the order of equations leads to a variety of new solutions, some of them without a GR limit (for example, famous “false radiation” vacuum isotropic solution in $f(R)$), Lovelock gravity being a second-order theory is free from this feature. However, due to rather complicated equations of motion, the Lovelock theory can also contain some solutions without a GR analog. One of such examples are exponential solutions in anisotropic cosmology.

In GR there is a unique vacuum solution for a flat anisotropic Universe, the Kasner solution (which, strictly speaking, is a one-dimensional set of solutions). The scale factors of this solution have a power-law behavior in time. A version of a power-law solution is known in general Lovelock gravity. However, when higher Lovelock terms (starting from the Gauss–Bonnet term) are taken into account, there emerges a new type of solutions with exponential time behavior of the scale factors (i.e., with constant Hubble parameters). Such solutions also exist for a non-vacuum Universe and belong to two different cases. If the matter content is different from the cosmological constant, the solution exists only in a very special case of the Universe with a constant volume. For matter in the form of a cosmological constant, there is no restriction on the volume. In this latter case, all exponential solutions appear to be a subject of rather a strict condition: space is divided into a restricted number (for Gauss–Bonnet theory, maximum three) of isotropic subspaces. The fact that this division is not introduced “by hand” and appears naturally from the equations of motion makes the exponential solutions interesting for model building in multidimensional cosmology. Any application should be preceded by stability studies. The stability of exponential solutions has been recently considered in several papers, in particular, it was shown that in the Einstein–Gauss–Bonnet (EGB) theory, a necessary condition for stability is volume increasing. As for a sufficient condition, a special algebraic relation should be satisfied.

In this paper we consider a $D$-dimensional gravitational model with a Gauss–Bonnet term and a cosmological constant $\Lambda$. Our goal is to check explicitly this relation for all known exponential solutions up to seven spatial dimensions. We note that so-called Gauss–Bonnet term has appeared in string theory as a correction to the (Fradkin–Tseytlin) effective action [2–6].

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We note that at present the EGB gravitational model and its modifications, see [9–30] and references therein, are intensively studied in cosmology, in particular, for a possible explanation of the accelerating expansion of the Universe which follows from supernovae (type Ia) observational data [33–35]. Other applications are related to Gauss–Bonnet–AdS black holes and a holographic description of certain quantum systems (e.g., superfluids), see [7, 8] and references therein.

Here we consider examples of solutions in the dimensions $D = 1 + d = 5, 6, 7, 8$. We study the stability of these solutions in a class of cosmologies with diagonal metrics by using the results of [28, 29], see also the approach of [26].

Several sets of special stable exponential solutions with zero variation of the effective gravitational constant for two and three factor spaces were recently found in [30, 31] and [32], respectively. It should be noted that two special solutions from [31] for $D = 22, 28$ and $\Lambda = 0$ were found earlier in [25]. In [28] it was proved that these solutions are stable.

2. THE SETUP

The action of the model reads

$$S = \int_M d^Dz \sqrt{|g|}\{\alpha_1 (R[g] - 2\Lambda) + \alpha_2 L_2[g]\},$$

(1)

where $g = g_{MND}dz^M \otimes dz^N$ is the metric defined on the manifold $M$, $\dim M = D$, $|g| = \det(g_{MND})$, $\Lambda$ is the cosmological constant,

$$L_2 = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2$$

is the standard Gauss–Bonnet invariant, and $\alpha_1, \alpha_2$ are nonzero constants.

We consider the manifold

$$M = \mathbb{R} \times M_1 \times \ldots \times M_n$$

(2)

with the metric

$$g = -dt \otimes dt + \sum_{i=1}^n B_i e^{2\psi_i} dt^i \otimes dt^i,$$

(3)

where $B_i > 0$ are arbitrary constants, $i = 1, \ldots, n$, and $M_1, \ldots, M_n$ are one-dimensional manifolds (either $\mathbb{R}$ or $S^1$) and $n > 3$.

The equations of motion for the action (1) give us the set of polynomial equations [28]

$$G_{ij}v^i v^j + 2\Lambda - \alpha G_{ijkl}v^i v^j v^k v^l = 0,$$

(4)

$$2G_{ij}v^j - \frac{4}{3} \alpha G_{ijkl}v^i v^j v^k v^l = \sum_{i=1}^n v^i,$$

$$-\frac{2}{3} G_{ij}v^i v^j + \frac{8}{3} \Lambda = 0,$$

(5)

where $i = 1, \ldots, n$, and $\alpha = \alpha_2/\alpha_1$. Here

$$G_{ij} = \delta_{ij} - 1, \quad G_{ijkl} = G_{ijG_{ikG_{il}}} G_{jG_{kl}}$$

(6)

are the components of two metrics on $\mathbb{R}^n$ [20, 21]. The first one is a 2-metric, and the second one is a Finslerian 4-metric. For $n > 3$ we get a set of forth-order polynomial equations.

We note that for $\Lambda = 0$ and $n > 3$ the set of equations (4) and (5) has an isotropic solution $v^1 = \ldots = v^n = H$ only if $\alpha < 0$ [20, 21]. This solution was generalized in [23] to the case $\Lambda \neq 0$.

It was shown in [20, 21] that there are no more than three different numbers among $v^1, \ldots, v^n$ when $\Lambda = 0$. This is also valid for $\Lambda \neq 0$ if $\sum_{i=1}^n v^i \neq 0$ [29].

3. STABILITY CONDITIONS

Here, as in [28, 29], we deal with exponential solutions (3) with a nonstatic volume factor, which is proportional to $\exp \left( \sum_{i=1}^n v^i t \right)$, i.e., we put

$$K = K(v) = \sum_{i=1}^n v^i \neq 0.$$  

(7)

We use the restriction

$$\det(L_{ij}(v)) \neq 0$$

(8)

on the matrix

$$L = (L_{ij}(v)) = (2G_{ij} - 4\alpha G_{ijkl}v^i v^j v^k v^l).$$

(9)

For a general cosmological setup with the metric

$$g = -dt \otimes dt + \sum_{i=1}^n e^{2\beta_i(t)} dy^i \otimes dy^i,$$

(10)

we have the (mixed) set of algebraic and differential equations [20, 21]

$$E = G_{ij}h^i h^j + 2\Lambda - \alpha G_{ijkl}h^i h^j h^k h^l = 0,$$

(11)

$$Y_i = \frac{dL_i}{dt} + \sum_{j=1}^n h^j L_i$$

$$-\frac{2}{3} (G_{sj} h^s h^j - 4\Lambda) = 0,$$

(12)

where $h^i = \dot{\beta}_i$,

$$L_i = L_i(h) = 2G_{ij} h^j - \frac{4}{3} \alpha G_{ijkl} h^i h^j h^k h^l,$$

(13)

$$i = 1, \ldots, n.$$

It has been proved in [29] that a fixed-point solution $(h^i(t)) = (v^i)$ $(i = 1, \ldots, n; n > 3)$ to Eqs. (11), (12), obeying the restrictions (7) and (8), is stable under perturbations of the form

$$h^i(t) = v^i + \delta h^i(t), \quad i = 1, \ldots, n$$

(14)
as \( t \to +\infty \) if
\[
K(v) \equiv \sum_{k=1}^{n} v^k > 0, \quad (15)
\]
and it is unstable (as \( t \to +\infty \)) if \( K(v) < 0 \).

We remind the reader that the perturbations \( \delta h^i \) obey (in the linear approximation) the following set of equations [28, 29]:
\[
C_i(v)\delta h_i = 0, \quad (16)
\]
\[
L_{ij}(v)\delta h^j = B_{ij}(v)\delta h^j, \quad (17)
\]
where
\[
C_i(v) = 2v_i - 4aG_{ijk}\alpha v^j v^k, \quad (18)
\]
\[
B_{ij}(v) = -\left(\sum_{k=1}^{n} v^k\right) L_{ij}(v) - L_{ii}(v) + \frac{4}{3} v_j, \quad (19)
\]
\[
v_i = G_{ij}v^j, \quad \text{and} \quad i, j, k, s = 1, \ldots, n.
\]

It has been proved in [29] that the set of linear equations for perturbations (16), (17) has the following solution:
\[
\delta h^i = A^i \exp(-K(v)t), \quad i = 1, \ldots, n, \quad (20)
\]
\[
\sum_{i=1}^{n} C_i(v)A^i = 0, \quad (21)
\]
if the restrictions (7), (8) are imposed.

It has also been shown in [29] that in the case where we have two different Hubble-like parameters \( H \) and \( h \), i.e., if the vector \( v \) is
\[
v = (H, H, H, \ldots, H, h, \ldots, h),
\]
and \( K = mH + lhi \neq 0 \), the matrix \( L \) has a block-diagonal form: \( L = \text{diag}(L_{\mu\nu}, L_{ab}, L_{\alpha\beta}) \), where
\[
L_{\mu\nu} = (1 - \delta_{\mu\nu})(2 + 4aS_{HH}),
\]
\[
L_{ab} = (1 - \delta_{ab})(2 + 4aS_{hh}),
\]
\[
L_{\alpha\beta} = (1 - \delta_{\alpha\beta})(2 + 4aS_{hh}), \quad (22)
\]
and \( S_{HH} \), \( S_{hh} \) are functions of \( H, h, m, l \). Hence we immediately find that if \( h \) corresponds to a one-dimensional subspace, i.e., \( l = 1 \), then \( L_{\alpha\beta} \) is a 1 \times 1 block which equals zero since \( \delta_{\alpha\beta} = \delta_{m+1m+1} = 1 \) and \( \det(L) = 0 \) in this case.

Analogously, it has been shown in [32] that in the case where we have three different Hubble-like parameters \( H, \mathcal{H}, h \), i.e., if the vector \( v \) is
\[
v = (H, H, H, \ldots, H, \mathcal{H}, \ldots, H, h, \ldots, h),
\]
and \( K = mH + k_1\mathcal{H} + k_2h \neq 0 \), the matrix \( L \) has a block-diagonal form again: \( L = \text{diag}(L_{\mu\nu}, L_{ab}, L_{\alpha\beta}) \) with
\[
L_{\mu\nu} = (1 - \delta_{\mu\nu})(2 + 4aS_{HH}),
\]
\[
L_{ab} = (1 - \delta_{ab})(2 + 4aS_{hh}),
\]
\[
L_{\alpha\beta} = (1 - \delta_{\alpha\beta})(2 + 4aS_{hh}), \quad (23)
\]
where \( \mu, \nu = 1 \ldots m, \quad a, b = m + 1, \ldots, m + k_1, \quad \alpha, \beta = m + k_1 + 1, \ldots, n \), and \( S_{HH} \), \( S_{\mathcal{H}\mathcal{H}} \), \( S_{hh} \) are functions of \( H, \mathcal{H}, h, k_1, k_2 \). If \( h \) corresponds to a 1-dimensional subspace, then \( \det(L) = 0 \) for precisely the same reason as in the previous case.

We will see particular examples of this situation in the next section. Moreover, the solutions in question will leave this particular Hubble parameter \( h \) unconstrained. From the continuity of \( \det(L) \) as a function of \( h \) we can conclude that in this case \( \det(L) = 0 \) for all \( h \), i.e., also when \( h \) either coincides with one of the other Hubble-like parameters (\( H \) or \( \mathcal{H}, H \)) or when the sum of all Hubble-like parameters \( K \) is zero.

4. STABILITY OF FIXED-POINT SOLUTIONS

IN \( d = 4, 5, 6, 7 \)

Now we apply the stability criterion (15) to (4 + 1)-, (5 + 1)-, (6 + 1)- and (7 + 1)-dimensional exponential solutions with a nonstatic volume factor, obtained in [23, 24], and present the data on the stability of these solutions in Tables 1–4 below.

4.1. \( d = 4 \) and \( d = 5 \)

Since the criterion works under the restriction (7) only, first of all we evaluate the determinant of the matrix (9) for each solution and check if it equals to zero. In the case of a singular matrix \( L \) we cannot say anything about the stability of the corresponding solutions; such solutions are marked in tables 1 and 2 by the symbol \( \bullet \). One can see that \( \det(L) = 0 \) for all those solutions which exist for a single value of \( \Lambda \) (given fixed \( \alpha \)); in all these cases \( \alpha < 0 \). This is the solution with 3D isotropic subspace and one extra dimension (we already know from the previous section that the determinant is zero for this solution) and two particular cases of isotropic and 2D+2D solution. Note that the former solution is in fact a one-dimensional set of solutions (because \( h \) is a free parameter there), and two special cases of other solutions with zero determinant appear to be particular points in this set. In this sense, only 3D+1D solution (existing only for a particular combination of the coupling constants) has a vanishing determinant. It is interesting to note that both the \( \Lambda \)-term and vacuum solutions with 3D isotropic subspace are stable, and the stability condition requires expansion of this 3D...
subspace. As well as in the case \(d = 4\), we see that \(\det(L) = 0\) for all those solutions which exist for a single value of \(\Lambda\) (at fixed \(\alpha\)). There are two one-dimensional sets of solutions and a special case of an isotropic solution when it coincides with a point in the 4D+1D set.

The case \((H, H, h, h)\) with a 3D isotropic subspace is more complex we consider it separately:

**\(\Lambda\)-term solution with \((H, H, h, h)\)**

\[
192 H^6 \alpha^3 - 112 H^4 \alpha^2 + (64\Lambda\alpha + 4)H^2\alpha - 1 = 0, \quad (*)
\]

\[
h = -\frac{4 H^2\alpha + 1}{8 H\alpha}. \quad (**)
\]

It is easy to check that if \(\alpha > 0\), Eqs. (*) and (**) have at least one solution for any \(\Lambda\); if \(\alpha < 0\), Eqs. (*) and (**) have at least one solution if \(\Lambda \geq -5/(12\alpha)\). In this case

\[
\det(L) = -\frac{248832(H^2\alpha - 1/4)^3(H^2\alpha + 1/12)^5}{H^6\alpha^5}.
\]

One can easily check that \(\det(L) = 0\) if \(H^2 = 1/(4\alpha)\) or \(H^2 = -1/(12\alpha)\); \(H^2 = 1/(4\alpha)\) is a solution of Eqs. (*) and (**) in a special case of the family \((H, H, -H, -H, h); H^2 = -1/(12\alpha)\) is their solution.
In this case, \[ \det(L) = 0 \] lead to the solution of Eqs. (\(*\)) and (\(**\)) is stable if

\[ 3H + 2h > 0 \iff 16H^2\alpha - 1 < 0 \]

or

\[ \alpha < 0, \quad H > \frac{1}{\sqrt{16\alpha}}, \quad \alpha > 0. \]

### Vacuum solution with \((H, H, H, h, h)\)

\( H_1 = H_2 = H_3 = H, \quad H_4 = H_5 = \xi H, \)

\[ H^2 = -\frac{\xi^2 + 6\xi + 3}{12\alpha(3\xi + 2)} \]

\[ \xi = \frac{3}{4}, \quad \alpha = 0.722, \quad \alpha < 0. \]

In this case,

\[ \det(L) = -1769472 \cdot \left[H^4\alpha^2(\xi^2 + \frac{2}{3}\xi + \frac{1}{3}) - \frac{H^2\alpha^2(\xi^2 - 10\xi - 3) + 1}{36} \right] \]

\[ \times \left[\xi(\xi + 2)H^2\alpha + \frac{1}{4} \right] \left[ H^2\alpha + \frac{1}{12} \right]. \quad (\ast) \]

It is easy to check that the expression (\ast) does not lead to \( \det(L) = 0 \), so this solution is stable if \( H > 0 \).

Note also that all vacuum solutions in both cases \( d = 4 \) and \( d = 5 \) exist only for \( \alpha < 0 \) and are stable for \( H > 0 \); there are no vacuum solutions with singular matrix \( L \).

### 4.2. \( d = 6 \)

In this subsection we extend the above results to the case \( d = 6 \). There are two special solutions which exist for a single value of \( \Lambda \) (at a fixed \( \alpha \)) such that \( \det(L) = 0 \):

- \((H, H, H, h, h, \tilde{h})\):
  \[ H^2 = 1/(12\alpha), \quad h = -2H, \quad \tilde{h} \in \mathbb{R}, \]
  \[ \Lambda = 1/(4\alpha), \quad \alpha > 0; \]

- \((H, H, H, H, h)\):
  \[ H^2 = -1/(24\alpha), \quad h \in \mathbb{R}, \quad \Lambda = -5/(16\alpha), \]
  \[ \alpha < 0. \]

Here we face a situation qualitatively different from what we see in the \((4 + 1)\)- and \((5 + 1)\)-dimensional models: the matrix \( L \) is singular not only for one-dimensional sets of solutions which exist at a single value of \( \Lambda \) and has the free parameter \( h \), but now this matrix turns out to be singular for other solutions (however, still for some special values of \( \Lambda \)); in the Table 3, for each family, we write \((H, h, \Lambda)\) for solutions with two different Hubble parameters and \((H, \Lambda)\) for an isotropic solution such that \( \det(L) = 0 \). We do not write down these solutions themselves due to their awkwardness and describe each family by its splitting into isotropic subspaces; the reader can find the formulas in \([23, 24]\). Some solutions listed in Table 3 are special cases of the solutions (24), (25); we point out overlapping families in the last column of Table 3. There are two solutions (both have 3D isotropic
4.3. $d = 7$

In this subsection we extend the above results to the case $d = 7$. There are three special solutions which exist for a single value of $\Lambda$ (at fixed $\alpha$), such that $\det(L) = 0$:

\[
(H, H, H, H, h, \tilde{h}) : \\
H^2 = 1/(24\alpha) , \ h = -3H , \ \tilde{h} \in \mathbb{R}, \\
\Lambda = 13/(48\alpha) , \ \alpha > 0 
\]

\[
(H, H, H, -H, -H, -H, h) : \\
H^2 = 1/(8\alpha) , \ h \in \mathbb{R}, \\
\Lambda = 3/(16\alpha) , \ \alpha > 0; 
\]

\[
(H, H, H, H, H, h) : \\
H^2 = -1/(40\alpha), \ h \in \mathbb{R}, \\
\Lambda = -21/(80\alpha), \ \alpha < 0. 
\]

The other families of solutions have only special solutions with $\det(L) = 0$; in Table 4, for each family we write $(H, h, \tilde{h}, \Lambda)$ for solutions with three different Hubble parameters, $(H, h, \Lambda)$ for solutions with two different Hubble parameters, and $(H, \Lambda)$ for an isotropic solution. As in the previous case, we do not write down the solutions themselves due to their awkwardness, all formulas can be found in [23, 24]. As in the case $d = 6$, we list overlapping families in the last column of Table 4. We see again that there are several solutions with 3D isotropic subspaces that do not overlap with the solutions (26)–(28); we mark these solutions with a bullet. There are no $(7 + 1)$-dimensional vacuum solutions with $\det(L) = 0$ again.
5. CONCLUSIONS

We have considered a $D$-dimensional EGB model with a $\Lambda$ term and two constants $\alpha_1$ and $\alpha_2$. The full list of solutions with exponential time dependence of the scale factors have been found in [23, 24], and here we consider the stability of these solutions. As has been described in [29], a necessary condition for stability is the condition that the total spatial volume has been described in [29], a necessary condition for the scale factors have been found in [23, 24], and a full list of solutions with exponential time dependence for all known exponential solutions up to dimension $7+1$.

To summarize the results obtained, it is worth to remember that exponential solutions can be divided into two groups. Solutions of the first group (we can call them special solutions) exist only for the coupling satisfying an additional linear relation. On the other hand, one of the Hubble parameters of the solution remained unconstrained. Our results shown that all such solution considered in the present paper do not satisfy the sufficient condition for stability, and so the dynamics in the vicinity of these solutions requires a further investigation.

One the contrary, the second group of solutions (existing for a nonzero measure of possible couplings and with all Hubble parameters fully determined) satisfy the sufficient condition for stability except for very few special sets of couplings. One of the reasons for this situation may be the fact that branches of this second group of solutions can intersect with branches of the special solutions. On the other hand, we have identified several particular couplings which do not satisfy the sufficient stability condition and do not originate from an intersection with any other branches of solutions. Curiously, such a situation occurs only for solutions with a 3D isotropic subspace.

To conclude, in brief, our results shows that (at least up to the dimension $7+1$) the stability of exponential solutions with growing spatial volume in Gauss–Bonnet cosmology cannot be proved in the linear perturbation analysis only for a discrete set of couplings. What happens in this case (when the necessary stability condition is fulfilled but the sufficient condition is not) needs a further analysis.

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