Electromagnetic coupling to centimeter-scale mechanical membrane resonators via RF cylindrical cavities

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Abstract
We present experimental and theoretical results for the excitation of a mechanical oscillator via radiation pressure with a room-temperature system employing a relatively low-Q centimeter-size mechanical oscillator coupled to a relatively low-Q standard three-dimensional radio-frequency (RF) cavity resonator. We describe the forces giving rise to optomechanical coupling using the Maxwell stress tensor and show that nanometer-scale displacements are possible and experimentally observable. The experimental system is composed of a 35 mm diameter silicon nitride membrane sputtered with a 300 nm gold conducting film and attached to the end of a RF copper cylindrical cavity. The RF cavity is operated in its TE011 mode and amplitude modulated on resonance with the fundamental drum modes of the membrane. Membrane motion is monitored using an unbalanced, non-zero optical path difference, optically filtered Michelson interferometer capable of measuring sub-nanometer displacements.

1. Introduction
We are entering a new realm of experimental quantum mechanics where it is possible to observe quantum-mechanical behavior in relatively large mesoscale objects. In recent years high-Q, high-frequency mechanical oscillators have emerged as platforms for quantum optomechanics, electromechanics, parametric amplifiers, phononic and nanomechanical systems [1–8]. The majority of optomechanical systems use high-finesse optical cavities, in which the underlying optomechanical coupling force is due to the radiation pressure arising from transverse-electromagnetic (TEM) modes in the optical cavity [9, 10]. More recently, microwave cavity optomechanics has shown great promise for the study of the coupling of electromagnetic energy and mechanical motion at the quantum level [11–13]. We emphasize that the term cavity is generally used in microwave cavity optomechanics to describe two-dimensional superconducting microstrip/co-planar waveguides, or one-dimensional transmission line resonator type architectures. Microwave cavity optomechanical systems operate at lower electromagnetic frequencies which reduces the magnitude of the optomechanical forces. However, significant couplings to micro-mechanical resonators can be achieved due to the small electromagnetic mode volumes achievable with such schemes [14].

In contrast, optomechanics with standard, macroscale (3D) RF cavities has not been an attractive avenue primarily because of the large electromagnetic mode volumes which may lead to weak coupling strengths [10, 14], and the lack of centimeter-size high-Q mechanical oscillators. For example, a theoretical estimate based on the frequency pull parameter \( G \equiv \frac{d\omega}{dx} = \frac{\pi \omega_0^2}{\omega_0^2 L^3} \) of a 10 GHz cylindrical RF cavity operating in the TE011 mode yields an optomechanical coupling strength of \( g_{\text{rf}} = \frac{G}{2\pi} = G_{\text{ZPF}} \approx 1 \times 10^{-6} \text{ Hz} \) for the room temperature system presented here [9]. Yet, among the first experimentally optomechanical couplings explored were with three-dimensional RF cavities by Braginsky [15, 16], whose work on coupling a rectangular RF cavity to a pendulum ultimately lead to the optical spring effect and the field of optomechanics. The ongoing development of high-Q superconducting radio frequency (SRF) cavity resonators and macro-scale mechanical membrane oscillators [2] now motivate a natural platform for the study of macro-scale optomechanical systems.
Chemical surface treatments have now become rather standard in SRF cavities, and quality factors on the order of 100 million or above can be relatively easily achieved [17–21]. Thus, it is natural to envision macro-scale optomechanical systems consisting of SRF cavity resonators and macro-scale, high-Q, high-frequency mechanical oscillators. For example, an optomechanical system consisting of superfluid helium coupled to a SRF niobium cavity has been demonstrated [22]. Or more recently, microkelvin cooling of a millimeter-sized silicon nitride mechanical resonator coupled to a 3D aluminum superconducting microwave cavity was achieved [23].

In an effort to move towards SRF centimeter-scale cavity optomechanical systems, we present theoretical and experimental results which demonstrate that optomechanical couplings can be achieved with low-Q centimeter-size mechanical resonators coupled to the electromagnetic fields of RF cavities exhibiting relatively low Q-factors. The source of the optomechanical coupling is the radiation pressure which arises from transverse-magnetic (TM) or transverse-electric (TE) electromagnetic modes found in RF cavities. A centimeter-scale room temperature system with a relatively low-quality-factor mechanical oscillator coupled to a RF copper cylindrical cavity is used to experimentally confirm that the radiation pressure, due to the magnetic component of the Maxwell stress tensor in a cylindrical cavity, is sufficiently strong enough to excite the drum modes of a 35 mm diameter, 500 nm thick, silicon nitride membrane attached to one end of the cavity.

These results suggest that optomechanics with SRF microwave frequency cavities might hold promise for future systems which can exploit their extremely high Q-factors. We add that future configurations using membrane-SRF-cavities schemes might pave the way for macroscopic parametric amplifier/oscillator systems [24, 25], which could then be used to perform experiments probing the quantum/classical boundary. Finally, hybrid systems that coherently couple microwave fields to optical fields [26] and, therefore, efficiently transfer squeezed states from the microwave to the optical regime, might also be a feasible future application for such SRF cavity systems.

2. Calculations

2.1. Membrane coupling via magnetic component of Maxwell stress tensor in a cylindrical cavity

For a cylindrical cavity operating in a $TE_{011}$ mode the relevant fields at the end-walls are the azimuthal component of the electric field $E_\phi \propto \sin(\pi z/L)$, the radial component of the magnetic field $H_r \propto \cos(\pi z/L)$, and the longitudinal component of the magnetic field $H_z \propto \sin(\pi z/L)$; where $L$ is the length of the cavity, and $z$ is along the axial direction. For perfect conductor boundary conditions the electric field component of the Maxwell stress tensor in the cylindrical cavity vanishes at the end-walls. However, the magnetic field components are at a maximum at the end-walls and dominate the force exerted on the membrane.

Now consider a large mechanical membrane attached at one end-wall of a cylindrical cavity (figure 2). We analyze the electromagnetic coupling with a thin circular silicon nitride membrane attached to one end of a cylindrical RF cavity of radius $R$. For this calculation, the cavity is assumed to be excited in its resonant $TE_{011}$ mode which sets up an azimuthally symmetric electromagnetic pressure on the silicon nitride membrane. The boundary conditions of the conductor, cavity geometry, and membrane (clamped boundary) are assumed to be ideal. This procedure can be generalized to other cavity modes [27].

The displacement ($u$) for the damped-driven elastic membrane is given by the following equation of motion [28]

$$\rho \ddot{u} + \gamma \dot{u} - TV^2u = e^{i\Omega t}F(r), \quad (1)$$

where $\rho$ is the mass per unit area of the membrane (assumed to be uniform), $\gamma \equiv 2\rho\dot{u}$ is the damping coefficient, $T$ is the tension per unit length, $\Omega$ is the modulation driving frequency, and $F(r)$ is an azimuthally symmetric driving pressure. The particular solution to equation (1) is obtained via a Bessel-series solution method with the following anzatz $^2$

$$u(r, t) = e^{i\Omega t} \sum_{n=1}^{\infty} A_n f_0\left(x_{0n}r/R \right), \quad (2)$$

where $f_0$ is the zero order Bessel function, $x_{0n}$ is the $n$th zero of the zeroth order Bessel function, and $R$ is the radius of the membrane which is assumed to coincide with the radius of the RF cylindrical cavity.

The force per unit area along the normal direction on the membrane is computed from the Maxwell stress tensor $T_{\phi\theta}$ [29]

$^2$ Clamped boundary conditions, $u(r = R, \theta) = 0$. 

\[ T_{ij} = \varepsilon_0 \left( E_i E_j - \frac{1}{2} \mu_0 B_i B_j \right) + \frac{1}{\mu_0} \left( B_i E_j - \frac{1}{2} \varepsilon_0 E_i B_j \right), \]  

(3)

where the \( E \) and \( B \) fields are those of a cylindrical cavity \( \text{TE}_{011} \) mode which are readily obtainable \([19, 30]\). For the \( \text{TE}_{011} \) mode, the force per unit area along the normal direction on the membrane is

\[ T_{zz} = \frac{\mu_0 \mathcal{H}_0^2}{2} \left[ J_0^2 \left( k_{0n} r \right) \right], \]

(4)

where \( k_{0n} = \frac{x_{0n}}{R} \) is the \( n \)th zero of the derivative of the zeroth order Bessel function \( J_0'(x) \), and \( \mathcal{H}_0 \) is the peak magnetic field strength inside the cavity. Since the driving force along the normal direction \( F(r) \) is proportional to \( T_{zz} \), we write the driving pressure on the membrane as

\[ F(r) \equiv -\frac{1}{2} \frac{\mu_0 \mathcal{H}_0^2}{2} \left[ J_0^2 \left( x_{0n} r \right) \right], \]

(5)

where we have included an extra factor of 1/2 for the time averaging of the fast oscillating RF frequency \( (\omega_{011}) \). This is because the RF power is amplitude modulated at the low acoustical frequency \( \Omega \), which drives the membrane at resonance.

We compute \( F(r) \) as a Bessel series \([31]\) and calculate the coefficients numerically. The particular solution for the membrane’s displacement is

\[ u(r, t) = e^{i\omega t + i\phi} \sum_{n=1}^{\infty} A_n J_0 \left( \frac{x_{0n} r}{R} \right), \]

(6)

\[ A_n = \frac{\mu_0 \mathcal{H}_0^2}{\rho} \frac{c_n}{\sqrt{\left( \omega_{0n}^2 - \Omega^2 \right)^2 + 4\beta^2 \Omega^2}}, \]

(7)

\[ c_n = \int_{0}^{\infty} J_1^2 \left( \frac{x_{0n} \zeta}{R} \right) \frac{d\zeta}{J_1^2 \left( x_{0n} \right)}, \]

(8)

where \( \rho \) is the mass per unit area, \( \phi \) is a phase factor, \( \beta \) is the HWHM decay rate of the membrane’s resonance in angular units, \( \mathcal{H}_0 \equiv H_0(\pi R/L x_{11}) \) with \( H_0 \) the peak magnetic field inside the cavity \([27]\), and \( \zeta \equiv r/R \) is a dimensionless integration variable. Because we have assumed an azimuthally symmetric driving force, the solution is restricted to azimuthally symmetric mechanical drum modes with eigenfrequencies of the form \( \omega_{0n} \).

However, as with any mechanical resonator all natural modes can be excited and in general all natural drum modes should be observable near their natural resonances

\[ \omega_{nm} = \frac{x_{mn} v}{R}, \]

(9)

where \( v \) is the speed of sound for transverse vibrations of the silicon nitride membrane, and \( x_{mn} \) are the \( n \)th zeros of the \( m \)th order Bessel function.

### 2.2. Displacement amplitude estimate

This calculation serves as an order of magnitude estimate of the displacement because the silicon nitride membrane is a composite structure whose mechanical properties are difficult to fully describe analytically, hence, several simplifications have been made: (1) the effective-mass is assumed equivalent to the sum of the 500 nm silicon nitride window mass and the 300 nm gold-film mass. (2) The effects of the elastic properties of the 300 nm gold film on the 500 nm silicon nitride membrane are not considered. (3) Ideal clamped-boundary conditions are assumed for the membrane which are difficult to achieve in practice. And (4) it does not consider the effects of the 500 \( \mu \)m annular silicon frame which also has its own set of mechanical resonances and boundary conditions. Lastly, we emphasize that our linear harmonic oscillator model does not incorporate any nonlinear effects that might arise, for example, from a quadratic drag term. With this in mind, an estimate for the maximum displacement at the center of the membrane \( (r = 0) \) follows from equations (6)–(8)

\[ u_{\text{max}} \approx \frac{15 P_f Q_{\text{RF}}}{m f_{011}} \sum_{n} \frac{c_n}{\sqrt{\left( \omega_{0n}^2 - \Omega^2 \right)^2 + 4\beta^2 \Omega^2}}, \]

(10)

where we have assumed perfect power coupling to the RF cavity, \( Q_{\text{RF}} \) is the cavity’s internal Q factor, \( P_f \) is the RF forward traveling power, and \( m \) is the assumed effective-mass of the membrane. When the membrane is driven at its fundamental resonance with \( m = 10 \text{ mg}, P_f = 25 \text{ mW}, Q_{\text{RF}} = 20,000, \omega_{011} = 2\pi \times 10.332 \text{ GHz}, \)

In the simplification of this solution the following Bessel relations were used: \( J_0'(x) = 2J_1(x)/x - J_0(x), \) and \( \int_{0}^{\infty} z J_0^2 (x) dz = J_1^2 (x_{0n})/2, \) where \( x_{0n} \) is the \( n \)th zero of the zeroth order Bessel function.
\[ \beta = 2\pi \times 10 \text{ Hz}, \quad \Omega = 2\pi \times 5, \quad 878 \text{ Hz}, \] 
we find \( u_{\text{max}} \sim 6.0 \text{ nm} \). A plot of the solution for the first 20 terms in the series of the membrane driven at the first resonance with the given parameters is plotted in figure 1.

3. Experiment

The experiment used a cylindrical cavity made from high purity oxygen free copper excited in its \( \text{TE}_{011} \) mode and a 35 mm diameter, 500 nm thick silicon nitride membrane acquired from Norcada. The membrane was sputtered with a 300 nm gold film and attached to one end of the cavity (figure 2). Excitation of the membrane’s mechanical modes was achieved by amplitude modulation of the cavity’s input power at a frequency \( \Omega \).

Modulation of the RF input power directly modulates the radiation pressure on the silicon nitride membrane, thus exciting the fundamental drum mode resonances of the membrane.

3.1. The RF cylindrical cavity

Measurement of the RF resonant frequencies and Q-factor were made with an HP 8720C Network Analyzer. Figure 3 shows a typical S11 reflection measurement on our copper cylindrical cavity with \( D = 3.81 \text{ cm} \) and overall length of 4.229 cm, however, the relevant internal length was \( L = 3.81 \text{ cm} \). The RF cavity was designed with a \( D/L = 1 \) ratio and a resonance at 10.332 GHz for the \( \text{TE}_{011} \) mode. A small gap in the solid end-cap shifts the degenerate \( \text{TM}_{111} \) mode. Measurement of the cavity’s loaded (external) Q-factor was determined experimentally from \( Q_L = f/2\Delta f \), where \( \Delta f \) is the HWHM value at the resonant frequency. The loaded quality factor of the copper cavity with the gold silicon nitride membrane attached at one end was 10 400 ± 5% and slightly undercoupled with a coupling coefficient of 0.9.

3.2. Low-frequency membrane displacement measurement

Sub-nanometer displacement measurements using Michelson interferometers have been well studied and play an important role in the study of piezoelectric devices [32–35]. Here we employ an unbalanced, non-zero optical path difference Michelson interferometer for the membrane’s displacement measurement. We implement the use of an optical filter via a single mode fiber optic to avoid interference pattern distortions caused by poor reflection quality from the coated membrane, and to compensate for the non-zero optical path difference. Noise
are measured relative to the DC offset voltage of the detector when no light is incident. The optical phase added by the DC optical path difference is tuned via a piezoelectric actuator prior to any modulation so that the displacement of the membrane is extracted from a measurement of the DC and rms voltages from each arm of the interferometer; $\Delta V = \Delta V_1 + \Delta V_2$. After proper tuning, the displacement of the membrane is extracted from a measurement of the DC and rms voltages from equation (11) with $\sin(2kd) = 1$.

Measurements were performed with a 10 mW helium–neon 633 nm laser as the light source. The DC voltages ($\Delta V_1$ and $\Delta V_2$) were measured with an oscilloscope. Solenoid-actuated blockers were used to block the beams as necessary to measure each arm’s intensity. A piezoelectric actuator was used to tune the interference pattern to its most sensitive setting as described above. The rms voltage was measured with an SRS 830 DSP lock-in amplifier, and the lock-in reference signal was used to amplitude modulate the Agilent N5183A RF signal generator at the reference frequency. A Labview VI program automatically swept the reference frequency and recorded the rms voltage measurement from the lock-in. Since the measurements can be done relatively quickly, no feedback loop was implemented (figure 4).

4. Results and discussion

The resonant frequencies of the mechanical drum modes are theoretically calculated from equation (9). We use the experimentally determined value of the first resonance as a reference point for calculating the expected resonant frequencies of the higher order modes. That is, $\omega_{n1} = \omega_{01}/x_{01}$, where $\omega_{01}$ is experimentally determined, etc. The gold-coated silicon nitride membrane’s fundamental resonance was experimentally observed at 5878 Hz. Based on this value the second ($x_{11}$), third ($x_{21}$), and fourth ($x_{02}$) modes are expected at 9366, 12554, and 13493 Hz, respectively. The second drum mode was experimentally measured at 9380 Hz (figure 5). The third mode was observed at approximately 12700 Hz which is also in good agreement with the expected theoretical resonance value. Finally, the fourth mode was not discernible due to the numerous signals detected in the range from 13500–14200 Hz (figure 5).

Figure 6 shows the experimental data for the first mode at 5878 Hz for different RF forward traveling powers. The solid curves are fits to equations (6)–(8) from which we extract the resonance frequency and quality factor. Equation (11) is used to determine the membrane’s vibration amplitude. A maximum vibration amplitude of approximately 0.9 nm was observed which is consistent with the order of magnitude estimate of 6 nm. To confirm excitation of the main resonance, we varied the RF forward traveling power. From equation (10) we see...
Figure 4. Experimental configuration for membrane displacement measurement.

Figure 5. Log-linear spectrum plot from 5000–15 000 Hz shows the detected modes. The first two resonances at \( \sim 5900 \) and \( \sim 9400 \) Hz correspond to the first two fundamental drum modes; the third mode is observed near 12 700 Hz. The fourth mode is hard to discern since there are several other peaks detected in the relevant region.

Figure 6. Log-linear plot of experimental data and fits to equation (6) (solid curves) for different RF powers of the TE\(_{011}\) mode. For 25 mW of forward traveling power the membrane’s displacement amplitude is approximately 0.9 nm at its resonance frequency of 5878 Hz.
that the power scales linearly with the amplitude of oscillation of the membrane. This linear relationship is displayed in figure 7. The mechanical $Q$ of the membrane is calculated from the experimental results and is $Q_M \approx 300$.

It was observed that the resonance frequency of the membrane slightly decreased as the RF power was increased. The observed shift in frequency is on the order of 4–10 Hz as seen in figure 6. Note that during actual measurements the vacuum pump was shut off to reduce vibrational noise which lead to a gradual pressure increase during the experimental runs. We speculate that the frequency-shift may be due to viscous damping associated with air resistance (drag) and/or the DC force created by the electromagnetic fields in the RF cavity. Based on the damped driven linear oscillator model used, the observed frequency-shift as a function of power is not expected in the solution since it does not include the DC component of electromagnetic force or account for any nonlinear effects. Thus, nonlinear effects should not be ruled out, and may be investigated further. However, to test the effects of viscous damping due to increased air pressure, the air pressure of the chamber was increased and the relevant excited drum modes were observed to decrease in frequency. This behavior is consistent with the solution of a damped-driven linear oscillator. Finally, we add that the excitation of the membrane resonance was confirmed using an acoustical measurement with a speaker tuned to the main fundamental resonant frequency.

5. Conclusions and future work

We demonstrated that a copper cylindrical microwave cavity with a loaded $Q$ of 10 400 can drive into motion a 35 mm diameter, 500 nm thick silicon nitride membrane coated with a 300 nm gold film placed at one end. The driving mechanism is dominated by the magnetic field pressure of the radio frequency electromagnetic fields exerted on the membrane as described by Maxwell’s stress tensor. This work demonstrates that radio frequency electromagnetic fields of traditional RF cavities can couple to macroscopic mechanical oscillators even in room temperature situations with low powers and relatively low $Q$'s.

Indeed, couplings with three-dimensional RF cavities can be achieved when the size of the mechanical resonator becomes comparable with the size of the cavity resonator. Future direction of this work includes using a high-$Q$ SRF cavity coupled to a centimeter-sized mechanical resonator consisting of a flexible superconducting niobium coated membrane. Stronger couplings are expected since the forces scale with resonator quality factors. Such three-dimensional macroscopic configurations seem to be a natural direction away from the currently used two-dimensional micro-sized architectures in optomechanics. Three-dimensional architectures lead naturally to much higher quality-factor cavities, since the surface current densities are much lower due to the much smaller surface-to-volume ratios of such 3D microwave cavities as that illustrated in figure 2. To test such a scheme we would like to excite higher frequency dilatational modes as observed in [5], but with high-$Q$ SRF cavities instead of high repetition lasers.
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