Empirical modeling of high-intensity electron beam interaction with materials

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Abstract. The paper proposes an empirical modeling approach to the prediction followed by optimization of the exact shape of the cross-section of a welded seam, as obtained by electron beam welding. The approach takes into account the electron beam welding process parameters, namely, electron beam power, welding speed, and distances from the magnetic lens of the electron gun to the focus position of the beam and to the surface of the samples treated. The results are verified by comparison with experimental results for type 1H18NT stainless steel samples. The ranges considered of the beam power and the welding speed are 4.2 – 8.4 kW and 3.33 – 13.33 mm/s, respectively.

1. Introduction
Electron beam welding (EBW) is a highly efficient and precise welding method increasingly used in manufacturing and of growing importance in different industrial sectors, such as the automobile and aerospace industries. The advantages of the method have to do with the possibility of obtaining welds with good physical-mechanical properties with minimal structural changes and thermal deformations of the individual welded parts, especially in cases of welding expensive structures and materials with different physical properties, reactive and refractory metals, or in precision welding of thin parts and micro-welding. In order to control the dimensions and shape of the welded seams cross-sections, as well as the heat-affected areas, adequate statistical and heat models of electron beam welding are required.

The EBW process depends on a large number of parameters in what concerns the material properties and the processing parameters of the electron beam installation. This is why one uses heat models to calculate the quasi-steady-state temperature distributions and the shapes of the melted zones as an approximation when simulating EB welding or melting and other electron beam processes [1-9]. They are based on different assumptions for the heat source – point, linear, exponential, combined etc. [6]. The solutions of the quasi-steady-state thermal physical models yield approximations for the shape of the temperature distributions taking into account some process and material parameters [6-9]. Due to the assumptions made in the physical models, the exact evaluation of the experimental results is difficult. It has been demonstrated that based on statistical analysis of the experimental and/or

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calculated data one can predict the technological results, as well as choose appropriate processing parameters in view of obtaining desirable results when electron beam technologies are applied to the manufacturing of pure metals and alloys and joining of metal parts [10-12]. Additionally, regression models can be used to estimate the influence of the electron beam focus position, together with the relative position of the beam focus with respect to the surface of the processed samples [10].

This paper proposes an empirical modeling approach to predicting and optimizing the exact shape of the cross-sections of welded seams obtained by electron beam welding. The influence of the electron beam welding process parameters is taken into account, as follows: electron beam power, welding speed, the distances from the magnetic lens of the electron gun to the focus position of the beam and to the surface of the samples treated. The results are verified via comparison with the results of experiments with samples made of stainless steel type 1H18NT. The beam powers considered are in the range 4.2 – 8.4 kW, with the welding speed varying from 3.333 to 13.333 mm/s.

2. Experimental conditions and regression models
The experiment considered in this paper was electron beam welding of samples of austenitic stainless steel type 1H18NT. The following operating parameters were varied: electron beam power (P) – 4.2 kW, 6.3 kW and 8.4 kW; welding speed (v) – 80 cm/min, 20 cm/min and 40 cm/min, distance between the main surface of the magnetic lens of the electron gun and the beam focusing plane (z_0) – 176 mm, 226 mm and 276 mm; and different distances between the main surface of the magnetic lens of the electron gun and the sample surface (z_p) within 126 mm – 326 mm. The accelerating voltage was 70 kV. After polishing and etching, 81 experimental weld cross-sections were examined. The following quality characteristics were considered: weld depth H, mean weld width B and surface area of the weld cross-sections S. The geometry of the EBW experiment is presented in figure 1. The basic levels of the process parameters’ intervals of variation during the experiments and their dimensions are presented in table 1.

![Figure 1. EBW experiment geometry: 1-magnetic lens; 2-electron beam; 3- welded sample.](image)

| Parameter | Dimension | Coded | Lower level (Z_{min}) | Upper level (Z_{max}) |
|-----------|-----------|-------|-----------------------|-----------------------|
| P - z_1   | kW        | x_1   | 4.2                   | 8.4                   |
| v - z_2   | cm/min    | x_2   | 20                    | 80                    |
| z_o - z_3 | mm        | x_3   | 176                   | 276                   |
| z_p - z_4 | mm        | x_4   | 126                   | 326                   |

Table 2. Regression models for the geometry and the quality characteristics for stainless steel.

| Regression model | R² (%) | R²(adj) (%) |
|------------------|--------|------------|
| \( S_t \) =      | 96.63  | 96.03      |
| \( H_r \) =      | 89.66  | 88.01      |
| \( B_r \) =      | 81.94  | 78.44      |
Using the response surface methodology [13], polynomial regression models were estimated (table 2) for the dependence of the weld depth, weld half-width and molten area of the weld cross-section for the process parameters investigated: electron beam power, welding speed, distances from the main surface of the magnetic lens of the electron gun to the beam focusing plane, as well as to the sample surface. The data processing was carried out in a coded scale, in order to avoid problems related to a possible multi-collinearity or other numerical problems in the evaluation of the coefficients of the regression models and bad prediction values of the quality indicator.

The process parameters, shown in table 1 and used in the regression equations in table 2, were coded in the range from -1 to 1, depending on their experimental ranges. Conversion from natural (z) to coded (x) values and vice versa was performed according to the formula:

\[ x_i = \frac{2z_i - (z_{\text{max},i} + z_{\text{min},i})}{z_{\text{max},i} - z_{\text{min},i}}. \]

For example, in order to evaluate the weld depth, weld half-width and molten area of the weld cross-section for the following process parameter values: \( P = 8.4 \) kW, \( v = 20 \) cm/min, \( z_0 = 226 \) mm and \( z_p = 223 \) mm, the dimensionless coded values that should be substituted in the corresponding equations from table 2 are: \( x_1 = [2*(8.4 - (8.4+4.2))/(8.4-4.2)] = 1 \), \( x_2 = [2*(20 - (80+20))/(80-20)] = -1 \), \( x_3 = 1 \), \( x_4 = -0.1 \). Table 2 presents the estimated regression models and the corresponding determination coefficients \( R^2 \) (the square of the multiple correlation coefficient) and the adjusted \( R^2_{\text{adj}} \), which are measures for the model accuracy (the model is better if the coefficient is closer to 100%).

3. Empirical models for the welds shape

The Gaussian distribution is a good approximation of the energy distribution within the processed material in its interaction with a high-intensity electron beam. We proposed an approach to estimating empirical Gaussian models for the shape of the welds:

\[ f_1(x) = \frac{S k_1}{B \sqrt{2\pi}} e^{-\frac{x^2}{2 \left( \frac{B}{k_1} \right)^2}} \quad \text{or} \quad f_2(x) = H e^{-\frac{x^2}{2 \left( \frac{B}{k_2} \right)^2}} \quad \text{or} \quad f_3(x) = He^{-\frac{x^2}{2 \left( \frac{S}{H \sqrt{2\pi}} \right)^2}}, \]

where \( H \), \( B \) and \( S \) are corresponding to the weld depth, the weld half-width and the molten area of the weld cross-section. We estimated the coefficients \( k \), \( k_1 \) and \( k_2 \) by minimizing the loss function \( Q_{\text{res}} \), or the root-mean-square error (RMSE) criterion, namely:

\[ Q_{\text{res}} = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 \rightarrow \min ; \quad \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{n}} \rightarrow \min ; \]

where \( \hat{y}_i \) and \( y_i \) are the predicted and the experimental values of the parameters missing in the function – \( H \), \( B \) or \( S \), \( n \) being the number of data available. Table 3 presents the results obtained.

**Table 3. Empirical models for the shape of the welds.**

| Function | \( f_1(x) \) | \( f_2(x) \) | \( f_3(x) \) |
|----------|---------------|---------------|---------------|
| \( \hat{y}_i \) | \( \hat{H} = f_1(x)_{\text{max}} \) | \( \hat{S} = \frac{HB \sqrt{2\pi}}{k_2} \) | \( \hat{B} = \frac{S k_3}{H \sqrt{2\pi}} \) |
| \( y_i \) | \( H \) | \( S \) | \( B \) |
| Coefficient \( k_i \) | 2.51 | 2.51 | 2.51 |
| RSME | 0.3323 | 0.7189 | 0.0296 |
One can see that for all models the value estimated for the coefficient $k_i$ is 2.51. The RSME criterion has small values for all models, but it is the smallest for model $f_3(x)$.

Figure 2 presents the shape of the experimental and calculated by $f_3(x)$ weld cross-sections, obtained for different process parameter values for three sets of experimental condition, as follows: Experiment 1 – $P = 8.4$ kW, $v = 20$ cm/min, $z_0 = 226$ mm, $z_p = 223$ mm; Experiment 2 – $P = 6.3$ kW, $v = 40$ cm/min, $z_0 = 226$ mm, $z_p = 216$ mm; Experiment 3 – $P = 8.4$ kW, $v = 40$ cm/min, $z_0 = 276$ mm, $z_p = 308$ mm. The values of $H$ and $S$ estimated by model $f_3(x)$ for each process parameter set are calculated by the regression models in table 2 – $H_i$ and $S_i$. The results obtained show a comparatively good accuracy of prediction of the welds shape.

![Figure 2](image)

**Figure 2.** Experimental (black) and calculated by $f_3(x)$ (blue) shapes of the weld cross-sections, obtained for the process parameter values: a) Experiment 1 – $P = 8.4$ kW, $v = 20$ cm/min, $z_0 = 226$ mm, $z_p = 223$ mm; b) Experiment 2 – $P = 6.3$ kW, $v = 40$ cm/min, $z_0 = 226$ mm, $z_p = 216$ mm; c) Experiment 3 – $P = 8.4$ kW, $v = 40$ cm/min, $z_0 = 276$ mm, $z_p = 308$ mm.

**Table 4.** Weld geometry parameters – comparison between experimental results and results estimated by empirical models $f_1(x)$, $f_2(x)$, $f_3(x)$ and by regression models (table 2).

| Model Exp. | $f_1(x)$ | Abs. err. | $f_2(x)$ | Abs. err. | $f_3(x)$ | Abs. err. | Regr. | Abs. err. |
|------------|----------|-----------|----------|-----------|----------|-----------|-------|-----------|
| Param.     | $H$, mm  | $\tilde{H}_1$, mm | $|\epsilon_i|$, mm  | $\tilde{H}_2$, mm | $|\epsilon_i|$, mm  | $\tilde{H}_3$, mm | $|\epsilon_i|$, mm  | $H_0$, mm | $|\epsilon_i|$, mm  |
| Exp. 1     | 39.9     | 40.119    | 0.2190   | 40.027    | 0.1270   | 40.027    | 0.1270 | 40.03     | 0.1300    |
| Exp. 2     | 25.5     | 24.463    | 1.0370   | 25.675    | 0.1750   | 25.675    | 0.1750 | 25.68     | 0.1800    |
| Exp. 3     | 23.4     | 27.33     | 3.9300   | 21.386    | 2.0140   | 21.387    | 2.0130 | 21.39     | 2.0100    |
| Param.     | $B$, mm  | $\tilde{B}_1$, mm | $|\epsilon_i|$, mm  | $\tilde{B}_2$, mm | $|\epsilon_i|$, mm  | $\tilde{B}_3$, mm | $|\epsilon_i|$, mm  | $B_0$, mm | $|\epsilon_i|$, mm  |
| Exp. 1     | 3.5      | 3.414     | 0.0860   | 3.414     | 0.0860   | 3.422     | 0.0780 | 3.27      | 0.2300    |
| Exp. 2     | 1.9      | 2.008     | 0.1080   | 2.016     | 0.1160   | 1.924     | 0.0240 | 2.02      | 0.1200    |
| Exp. 3     | 2.8      | 2.811     | 0.0110   | 2.736     | 0.0640   | 3.48      | 0.6800 | 2.80      | 0         |
| Param.     | $S$, mm$^2$ | $\tilde{S}_1$, mm$^2$ | $|\epsilon_i|$, mm$^2$  | $\tilde{S}_2$, mm$^2$ | $|\epsilon_i|$, mm$^2$  | $\tilde{S}_3$, mm$^2$ | $|\epsilon_i|$, mm$^2$  | $S_0$, mm$^2$ | $|\epsilon_i|$, mm$^2$  |
| Exp. 1     | 140.8    | 130.94    | 9.8600   | 130.64    | 10.1600  | 130.94    | 9.8600 | 130.94    | 9.8600    |
| Exp. 2     | 48.2     | 49.45     | 1.2500   | 51.90     | 3.7000   | 49.45     | 1.2500 | 49.45     | 1.2500    |
| Exp. 3     | 65.4     | 76.50     | 11.1000  | 59.86     | 5.5400   | 76.50     | 11.1000 | 76.50     | 11.1000   |
Table 4 summarizes the experimental values and those calculated by \( f_1(x) \), \( f_2(x) \), \( f_3(x) \) and by regression models (table 2) for the weld depth \( H \), mean weld half-width \( B \) and surface area \( S \) of the welds cross-sections. The absolute error is smaller for the depth and the half-width of the welds; larger differences are obtained between the values of the surface areas of the weld cross-sections due to the accuracy of the measurements, as well as to the precision of the estimated regression models.

| Model | Error | \( f_1(x) \) | \( f_2(x) \) | \( f_3(x) \) | Regression |
|-------|-------|-------------|-------------|-------------|------------|
| \( H \) | \( \epsilon_H \) | 3.2559 | 2.4107 | 2.4107 | 2.4107 |
| \( RMSE_H \) | | 4.1705 | 3.0280 | 3.0280 | 3.0280 |
| \( B \) | \( \epsilon_B \) | 0.3458 | 0.3463 | 0.4513 | 0.3139 |
| \( RMSE_B \) | | 0.4584 | 0.4541 | 0.6293 | 0.4174 |
| \( S \) | \( \epsilon_S \) | 5.2677 | 5.2342 | 5.2631 | 5.2658 |
| \( RMSE_S \) | | 6.4933 | 6.7293 | 6.4847 | 6.4839 |

To compare the performance of the models (table 5), we used the mean absolute value of the error \( \epsilon \), calculated as the mean absolute difference between the predicted and the measured values of the weld geometry characteristics, as well as the root-mean-square error (RMSE). As seen, the three empirical models proposed \( f_1(x) \), \( f_2(x) \) and \( f_3(x) \) give approximately the same results.

### 4. Conclusions

We proposed an empirical modeling approach to predicting the shape of the welded seam cross-sections, as obtained by electron beam welding. The methodology involves two steps: a) calculation of the weld depth, weld half-width and the molten area of the weld cross-section at given exact values of the process parameters: electron beam power, welding speed, distances from the main surface of the magnetic lens of the electron gun to the beam focusing plane, as well as to the sample surface, using estimated regression models, b) calculation of the shape of the EB welds by applying empirical models \( f_1(x) \), \( f_2(x) \) or \( f_3(x) \). The calculated results were compared with the experimental data for samples from stainless steel type 1H18NT; a good prediction accuracy was found. The practical choice of the functions proposed will depend on the accuracy of the experimental measurements; some physical and metallurgical aspects of the specific process should also be considered, such as the appearance of spikes at the welds’ roots and of various defects along the crater formed during the electron beam processing.

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