A review of recent theoretical investigations on acoustic cavitation bubbles and their implications on detection of cavitation in pumps

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Abstract. Detection of cavitation in pumps is one of the essential topics in hydraulic machinery research and has been intensively investigated for several decades. In the literature, a technique based on analysis of acoustic signals generated by cavitation bubbles in the pumps has been proposed to detect cavitation activities especially incipient cavitation. In present paper, recent theoretical investigations by the author and his collaborators on acoustic cavitation bubbles (e.g. damping mechanisms, heat and mass transfer) together with their associated acoustical signals have been briefly reviewed to advance above technique.

1. Introduction
When the static pressure at some specific points (e.g. impeller) of pumps drops below the saturated vapor pressure of the liquids (e.g. water), the liquids will be vaporized forming bubbles, termed as “cavitation”. Cavitation has many negative effects on pumps and their performances, e.g. a loss of pump efficiency, damage to the impeller through micro-jets and erosion, serious vibration and prominent noise. Hence, detection of cavitation in pumps is an important topic of hydraulic machinery. Traditional techniques for detecting cavitation include observations of the drop in head, analysis of vibration in pump, fault diagnosis through hydrophone, to name a few. In the literature, a non-invasive technique based on analysis of acoustic signal of cavitation bubbles in pumps has attracted much attention [1-4]. Comparing with other techniques, this technique is able to detect cavitation activity during its infancy (i.e. incipient cavitation). In present paper, theoretical studies by the author and his collaborators [5-12] relating with fundamentals of oscillations of cavitation bubbles and their associated acoustic signals will be briefly reviewed in order to facilitate development of above technique.

2. Natural frequency and damping mechanisms
Recently, Zhang and Li [8] obtained a new expression of natural frequency ($\omega_0$) of oscillating cavitation bubbles with including effects of liquid compressibility,

$$\omega_0^2 = \frac{1}{M \rho_i R_0^2} \left[ 3K \left( \frac{P_0}{R_0} + \frac{2\sigma}{R_0} \right) - \frac{2\sigma}{R_0^2} \right],$$  \hspace{1cm} (1)

with
Here, \( \rho_l \) is the density of the liquid; \( R_0 \) is the equilibrium bubble radius; \( \kappa \) is the polytropic exponent; \( P_0 \) is the ambient pressure; \( \sigma \) is the surface tension coefficient; \( c_i \) is the undisturbed speed of sound in the liquid; \( \mu_l \) is the viscosity of the liquid; \( \mu_{th} \) is the effective thermal viscosity. Comparing with previous investigations [13, 14], we found that \( \mu_l \), \( \mu_{th} \) and liquid compressibility contributes to the natural frequency of cavitation bubbles through \( M \) in equation (2). For the values of \( \kappa \) in equation (1) and \( \mu_{th} \) in equation (2), equations of heat transfer should be solved. For details, readers are referring to section 3 of present paper.

For energy dissipation (e.g. damping) mechanisms, the total damping constant (\( \beta_{tot} \)) can be expressed as

\[
\beta_{tot} = \beta_{vis} + \beta_{th} + \beta_{ac},
\]

where

\[
\beta_{vis} = 2\mu_l / \rho_l R_0^2 \omega_0^2;
\]

\[
\beta_{th} = 2\mu_{th} / \rho_l R_0^2 \omega_0^2;
\]

\[
\beta_{ac} = \frac{R_0}{2c_i} \omega_0^2;
\]

representing the viscous, thermal and acoustic damping constants respectively. Comparing with previous works [13, 14], the predictions of acoustic damping constant (\( \beta_{ac} \)) in regions with large values of \( \omega R_0 / c_i \) (Here, \( \omega \) is the frequency of acoustic excitation) can be significantly improved by our formula (equation (5)). For a quantitative comparison between formulas in the literature [13, 14] and ours, readers are referred to Zhang and Li [8].

Figure 1 shows a demonstrating example of the values of viscous, thermal, acoustic and total damping constants during oscillations of cavitation bubbles. If frequencies are fixed, viscosity and acoustic damping are dominant ones for small and large bubbles respectively while thermal damping is dominant one for the bubbles with intermediate sizes. We further found that the contribution of thermal damping to the total damping deceases with the increase of frequency.
3. Effects of heat transfer

To close the model, the values of $\kappa$ in equation (1) and $\mu_{th}$ in equation (2) should be determined by solving the heat transfer equation of gas bubbles and surrounding liquids. Various formulas have been proposed to determine values of $\kappa$ and $\mu_{th}$ [13, 14]. For a quantitative comparison between different groups of formulas, readers are referred to Zhang [6]. Recently, Zhang and Li [9] extended formulas proposed by Prosperetti [14] into high-frequency regions. Zhang [6] further proposed reduced formulas based on correction of formulas in [14] and also validated their accuracy through comparing with exact solution. Zhang and Li [9] defined the valid regions of those formulas by employing two non-dimensional parameters (i.e. the ratios between bubble radii and wavelengths in the gases and liquids) to avoid misusing of those formulas identified in the literature. For a review of those formulas in the literature together with their valid regions, readers are referred to Zhang [6].

Figures 2 and 3 show the predictions of $\kappa$ and $\mu_{th}$ for a wide range of frequencies (represented by a non-dimensional parameter $G_1$) and bubble radii (represented by a non-dimensional parameter $G_2$). Because the solutions of $\kappa$ and $\mu_{th}$ are quite lengthy, their expressions will not be shown here. As shown in figure 2, for low-frequency acoustic excitation (e.g. $G_1=10^{-9}$), if bubble radius gradually increases, value of $\kappa$ is 1.0 for small bubbles, then gradually increases to specific heat ratio $\gamma$ ($\gamma=1.4$ for our case) for large bubbles and finally sharply drops to a value below 1.0. If $\kappa<1.0$, the effects of non-uniform pressure inside cavitation bubbles will play an important role. For high-frequency acoustic excitation (e.g. $G_1=10^{-3}$), the basic trend of variations of $\kappa$ against bubble radii remains the same as those of $G_1=10^{-9}$ with the maximum values of $\kappa$ gradually decreasing with the increase of the frequency.
4. Effects of mass transfer

During cavitation bubble oscillations, mass transfer between bubbles and surround liquids will happen due to the change of interface area of the bubble for mass transportation, internal pressure and gas concentration in the bubble together with the velocity field generated by the oscillating bubbles. Recently, Zhang and Li [12] derived generalized formulas for this problem and improved the predictions of this phenomenon near and above resonance. Zhang and Li [11] further considered the mass transfer across bubble interface during its oscillations in viscoelastic mediums (e.g. human soft tissue). Zhang [7] studied the mass transfer of cavitation bubbles under dual-frequency acoustic excitation.
5. Acoustical signals emitted by cavitation bubbles
During cavitation bubble oscillations, a scattered wave will be generated. Through analysis of above
scattered wave and related acoustic signals, information about cavitation bubbles can be obtained.
Zhang [5] derived a generalized equation for predicting the acoustic waves generated by the cavitation
bubble,

\[
\sigma_s = \frac{4\pi R_0^2}{\left[ \frac{\sigma_0^2}{4\pi R_0^2} - 1 \right]^2 M^2 + \left[ \frac{4(\mu_i + \mu_b)}{\omega \rho_l R_0^2} + \frac{R_0}{\omega \sigma_i} M \omega_0^2 \right]^2} C_1,
\]

with

\[
C_1 = \frac{\left[ \sin(2\pi R_0 / \lambda_i) / (2\pi R_0 / \lambda_i) \right]^2}{1 + (2\pi R_0 / \lambda_i)^2}.
\]

Here, \( \sigma_s \) is the acoustical scattering cross section; \( C_1 \) is the correction coefficient; \( \lambda_i \) is the
wavelength of the liquid. Comparing with previous formulas in the literature (e.g. [15]), this
generalized equation proposed by the author [5] can improve the predictions of acoustical scattering
waves in the near-resonance region with high ambient pressure and above-resonance region
(corresponding to large values of \( R_0 / \lambda_i \)).

![Figure 4. Predictions of acoustical scattering cross sections of
cavitation bubbles versus ratio between bubble radius and
wavelength in the liquid (\( R_0 / \lambda_i \)).](image)

6. Conclusions
In present paper, recent advances relating with theoretical investigations of cavitation bubbles
delivered by the author and his collaborators have been briefly reviewed. Both fundamental issues (e.g.
damping mechanisms, heat and mass transfer) and applicable issues (e.g. acoustical signal generated
by cavitation bubbles) are covered in present paper. To detect incipient cavitation in pumps using
acoustic signals generated by cavitation bubbles, the resolution of this technique (i.e. detecting
cavitation phenomenon at a very early stage) could be possibly improved through employing our
recently developed formulas presented in this paper.
7. References

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