Reply to Comment on “Proposal for the Measurement of Bell-Type Correlations from Continuous Variables”

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Abstract

Recently K. Banaszek, I. A. Walmsley, K. Wodkiewicz [quant-ph/0012097] commented on our Proposal for the Measurement of Bell-Type Correlations from Continuous Variables [T. C. Ralph, W. J. Munro, R. E. S. Polkinghorne, Phys. Rev. Lett. 85, 2035 (2000)]. Their comment is based on a blatant misreading and misunderstanding of our letter and as such is simply wrong.

The comment by K. Banaszek, I. A. Walmsley, K. Wodkiewicz [1] on our letter [2] purports to present a hidden variable theory which describes our results. The hidden variable scheme described is based on the positive Wigner distribution which describes quadrature measurements on the parametric amplifier and vacuum background. It is well known, and was pointed out on three separate occasions in our paper, that such a description is possible if only quadrature measurements are performed. This is the reason why (as was also pointed out on three separate occasions) auxiliary intensity measurements must also be made. The comment incorrectly identifies these auxiliary measurements as being quadrature measurements of the vacuum. Instead, as is stated in the paper, they are quantum limited intensity measurements on the vacuum.

As is also clearly stated in the paper, this auxiliary intensity measurement guarantees the positivity of the underlying (but inaccessible) individual measurements which make up our ensemble average. Without such a measurement the positivity of the individuals is not guaranteed and the Bell test cannot be applied. The authors say nothing that was not already stated in our letter when they make this point in their comment.

It seems a cursory reading of our letter can lead to misconceptions. Given the brevity of the letter format this is perhaps understandable. A more detailed account is presently in preparation. Here we present a brief discussion aimed particularly at the point which the comment missed.

The crux of the issue is whether the positivity of our local realities, the count rates \( R_A^i(\theta_A) \) and \( R_B^j(\theta_B) \), given by

\[
R_A^i(\theta_A) = (\hat{X}_{A;1}^i)^2 - (\hat{X}_{va;1}^i)^2 + (\hat{X}_{A;2}^i)^2 - (\hat{X}_{va;1}^i)^2
\]
\[
R_B^j(\theta_B) = (\hat{X}_{B;1}^j)^2 - (\hat{X}_{vb;1}^j)^2 + (\hat{X}_{B;2}^j)^2 - (\hat{X}_{vb;1}^j)^2
\]

(1)
can be guaranteed. Here as usual the quadrature operators are related to their corresponding annihilation ($\hat{C}$) and creation ($\hat{C}^\dagger$) operators via

$$
\begin{align*}
\hat{X}_{C;1} & = \hat{C}^\dagger + \hat{C} \\
\hat{X}_{C;2} & = i(\hat{C}^\dagger - \hat{C}) \\
\hat{C} & = \hat{A}, \hat{B}, \hat{V}_A, \hat{V}_B
\end{align*}
$$

where $\hat{A}$ and $\hat{B}$ represent the polarization entangled beams at stations $A$ and $B$, whilst $\hat{V}_A$ and $\hat{V}_B$ represent vacuum modes which enter the detectors when the entangled beams are blocked. The problem is that the different quadrature components cannot be measured simultaneously (just as results for different polarization angles cannot be measured simultaneously). Thus, although the ensemble averages are positive, we cannot guarantee the positivity of individual realizations $R_{A}^i(\theta_A)$ or $R_{B}^j(\theta_B)$ from separate quadrature measurements. However, by substituting Eq.2 into Eq.1, it is straightforward to prove the equalities

$$
\begin{align*}
R_{A}^i(\theta_A) & = \hat{A}^\dagger i \hat{A}^i - \hat{V}_A^\dagger i \hat{V}_A^i \\
R_{B}^j(\theta_B) & = \hat{B}^\dagger i \hat{B}^j - \hat{V}_B^\dagger i \hat{V}_B^j
\end{align*}
$$

It is very important to note that these equalities hold for both a quantum and a classical treatment of the observables. That is Eq.3 is equally true whether the variables are treated as operators or c-numbers. It is then trivial to see from Eq.3 that provided $\hat{V}_{A,B}$ represents an unoccupied mode (ie is in a vacuum state) then the positivity of our local realities is guaranteed. In other words to guarantee the positivity of $R_{A}^i$ we must show explicitly through measurement that $V^\dagger V = 0$. This is what our auxiliary measurement does. Thus the measurement of the intensity of the vacuum is a necessary but sufficient condition for the positivity of the local realities and hence the validity of the Bell inequality for the quadrature ensemble averages.

The auxiliary measurement could be included in the measurement protocol by randomly swapping between it and the various quadrature measurements, thus avoiding conspiracy loopholes. For sufficiently large data sets no information would need to be discarded. Indeed in principle the auxiliary measurements could be made with the same detectors as the quadrature measurements by simply blocking both the local oscillator and the signal at various times. In practice, though, detectors with sufficient dynamic range to make this possible do not presently exist.

In summary the comment is based on an erroneous assumption: the incorrect identification of the auxiliary measurement in our letter as a quadrature measurement. As a result it fails to raise any issues which were not in fact discussed in our letter.
REFERENCES

[1] K. Banaszek, I. A. Walmsley, K. Wodkiewicz, quant-ph/0012097.
[2] T. C. Ralph, W. J. Munro, R. E. S. Polkinghorne, Phys. Rev. Lett. 85, 2035 (2000)