Computational simulations of the influence of noise in optical CT reconstruction

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Abstract. In 3D radiation dosimetry with optical CT scanning readout, projections of transmitted light are recorded in either lines or planar. The projections are then transferred to optical density maps by use of filtered back-projection. Absolute dose maps can be derived from the optical density maps by calibration of the optical absorption coefficient to dose values. The transmission profiles will be subject to a certain level of detector noise and noise arriving from fluctuations in the light source. Different reconstruction filters in the frequency domain can be applied in the image reconstruction procedure. The noise level in the final reconstructed images is determined by the noise in the projections, the spatial resolution and the reconstruction algorithm. The reconstruction filters may also have an effect on the geometrical precision as a spatial frequency filter may also affect the spatial resolution. We here propose an easy method to assess both the noise sensitivity and the performance in terms of mapping dose distributions with various theoretical dose gradients.

1. Introduction
Three-dimensional radiation dosimetry has received growing interest with the implementation of highly conformal radiotherapy treatments [1]. Over the last two decades, much research has been conducted towards both the development of different radiation sensitive three-dimensional dosimeters and adequate scanning techniques. Although an acceptable precision and accuracy has been achieved with polymer gel dosimeters and MRI scanning [2-4], the procedure is relatively labor intensive and some expertise is needed to acquire reliable quantitative MR images. As an alternative to MR scanning, optical CT scanning has been proposed as an alternative [5]. Different staining dosimeters have been proposed as a better alternative to polymer gel dosimeters including polyurethane based dosimeters, PRESAGE™ [6] and micelle gel dosimeters [7, 8] and another system will be proposed at this conference. For a comprehensive discussion on optical CT scanning and different methods we refer to another communication and references herein [9]. An analysis of the propagation of noise in reconstructed images for different frequency filters has already been performed by Krstajic and Doran [10] for a uniform circular phantom.

In this study, we extend this approach to a circular phantom containing a theoretical square field. This allows not only to observe the effect of the filter on the noise propagation in the reconstructed images but also the effect of the frequency filter on the spatial integrity of resolving dose gradients that occur in the dose distribution. In addition to the study by Krstajic and Doran, we also note that the noise in the reconstructed image is not uniformly distributed but displays a radial dependence.
2. Methods and Materials

2.1. SNR of reconstructed images

All image reconstructions were performed using Matlab R2008 using the built-in function “iradon” with the following spectral filters: Hamming, Hann, Ram-Lak and Cosine. A circular phantom with diameter 5 cm was considered. The number of projections from which the image was calculated was 180 over a demi-arc, corresponding to a radial increment of one degree. The attenuation through the phantom was considered to occur according to an inverse exponential power law. Additive stochastic white noise was added to the projection profiles prior to reconstruction that corresponds with the combined effect of read noise, thermal noise and shot noise of the detector.

The signal-to-noise in the projection is a function of the light intensity falling on the detector as described by equation 1.

\[
SNR = \frac{S}{N} = k \Phi h \nu t \eta A n CCD + n readout = k \cdot SS + n CCD + n readout
\]

where \( \Phi \) is the light power falling on the detector (W/m²), \( h\nu \) is the photon energy (J), \( t \) is the exposure time, \( A \) is the surface of the detector element, \( \eta \) is the quantum efficiency of the detector, \( n CCD \) is the CCD noise (electrons) and \( n readout \) is the noise (electrons) due to read out of the CCD-image sensor (i.e. the electronics). The scaling constant \( k \) accounts for the binning and averaging during readout.

Several of these parameters are detector specific. In order to come up with some realistic values, we performed some noise measurements on the CCD camera that we used in house. To derive the relation of SNR versus signal intensity \((S)\), 512 projection images where recorded with a resolution of 0.2 mm x 0.2 mm. The standard deviation and mean signal values over the 512 projection images were derived on a pixel-by-pixel basis and plotted in a scatter plot (SNR(S)). The relation was fitted to the function:

\[
SNR_{proj} = k \cdot SS + n cam 2
\]

where \( n cam \) combines both \( n CCD \) and \( n readout \) (i.e. \( n cam 2 = n CCD + n readout \)).

Figure 1 shows the scatter plot of SNR_{proj} as a function of the detected normalized signal intensity \( SS_0 \) in a realistic projection image obtained with a commercial CCD camera (DragonFly2 DR2-HIBW).

![Figure 1. SNR_{proj} as a function of the detected normalized signal intensity S/S_0 in a realistic projection image obtained with a commercial CCD camera (DragonFly2 DR2-HIBW)](image)

2.2. Spatial integrity study

To study the effect of frequency filters in the reconstruction in the presence of noise on the measured penumbra, a synthetic circular phantom with square field was constructed (figure 2). The steepness of the gradient in the square field can be varied by only a single parameter. The theoretical equation for the optical density offset created by the square field is given by equation:
\[ ODFx,y,a = ODR \cdot \sinh(\xi a) \cosh(\xi x - x0) + \cosh(\xi a) \cdot \sinh(\xi y - y0) + \cosh(\xi a) \]

where \( ODR \) is the optical density in the flat (irradiated) region of the field, \( a \) is the field size, \( \xi \) is a parameter inversely correlated with the penumbra width and \( x0, y0 \) are the spatial coordinates of the offset of the field with respect to the geometrical centre. The theoretical equation allows to perform the performance study without the need of spatial discretization of the synthetic phantom which avoids any bias in the results caused by regridding along the projection lines.

Figure 2. A synthetic circular phantom with square field in the centre (a). Note that in the simulations the field was chosen acentric to generate a more general case (b).

3. Results and Discussion

3.1. SNR characteristics for different filters
The SNR as a function of optical absorbance coefficient is plotted for two different frequency filters in figure 3. All other parameters were kept the same, thus illustrating the strong influence of the frequency filter on the noise performance. A more than two fold increase in SNR is observed by applying a Hann filter instead of a Ram-Lak filter.

3.2. Spatial integrity of dose gradients
The spatial performance in terms of the ability to acquire the penumbra width is demonstrated in figure 4.

4. Conclusion
An easy way of simulating the performance of an optical scanner and the image reconstruction algorithm for 3D radiation dosimetry is described. These simulations can be used to predict and optimize the precision in terms of dose resolution, for any detector, number of projections, spatial binning and reconstruction algorithm. It is also helpful in describing the effect of the reconstruction software on the spatial integrity. It is shown that for a phantom of 10 cm diameter, the optimal average optical density of the dosimeter should be around 0.2 cm\(^{-1}\). Note that this value is dependent on the size of the phantom. It is also found that the noise is not uniformly distributed in the reconstructed dose maps. From the four studied frequency filters, the Hann filter gives the best results, both in terms of precision and spatial integrity at steep dose gradients but it is likely that other filters can still improve the noise performance.
Figure 3. SNR in OD maps reconstructed using a Ram-Lak filter (a) and a Hann filter (b). The SNR is plotted for different circular strips showing that the noise is non-uniform in the entire phantom. The different graphs are for different circular shells with inner and outer radius as indicated in the figure inset. A reconstructed image for OD = 0.2 cm\(^{-1}\) is also shown.

Figure 4. Performance of filtered back projection (Hann filter) on the measured penumbra in the presence of noise. OD maps of simulated square fields with different dose gradients (a) and derived penumbra width as compared to the theoretical penumbra (b).

5. References

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