MONEY DEMAND IN VENEZUELA:
MULTIPLE CYCLE EXTRACTION IN A COINTEGRATION FRAMEWORK

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ABSTRACT

Money demand in Venezuela is modeled using structural time series and error correction approaches, for the period 1993.1 to 2001.4. The preferred model features seasonal cointegration and was estimated following a structural time series approach. There are similarities in the long-run behavior of money demand associated with the structural time series and error correction approaches. Estimated short-run dynamics are more fragile, with the structural time series modeling approach providing richer insights into the adjustment dynamics of money demand. A cycle with a 3-year period has been found to be common to money demand, real GDP and opportunity cost variables. This cycle is robust to changes in model specification, including choice of opportunity cost variable. Higher frequency cycles are also found to exist but are more sensitive to model specification. Results are also presented for a combined approach that takes advantage of error correction models as well as the type of insights into short-run dynamics afforded by the structural time series modeling approach.

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INTRODUCTION AND APPROACH

Background

The demand for real money balances in Venezuela has experienced sharp, recurrent fluctuations, as the economy adjusted to external and internal shocks. Real money balances (as measured by M1) was 41 percent higher at the end of 2001 than in 1993, but it was still 7 percent below the peak reached at the end of 1997. Despite the severity of these fluctuations, real money balances expanded at an annual rate of about 4 percent since 1993 (see Figure “Venezuela: Real Money Balances”).

![Venezuela: Real Money Balances](image1)

![Venezuela: Standard Deviation of Changes in Real Money Balances](image2)

*M1 deflated by the CPI index. Quarterly frequency.

*Rolling standard deviation using a 4 quarter window.

The severity of shocks to real money balances as measured by the rolling standard deviation of its changes, also fluctuated greatly during the 1993.1-2001.4 period, multiplying eightfold from a low in 1994 to a peak in 1996, and falling thereafter by about one fifth (see Figure “Venezuela: Standard Deviation of Changes in Real Money Balances”).

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2 This section focuses on M1 because no satisfactory empirical results have been obtained using base money and M2 as measures of money demand.
Conventional economic theory generally explains fluctuations in the demand for real money balances as resulting from changes in other macroeconomic aggregates, which are broadly grouped in three types. Firstly, aggregates that measure the level of economic activity and that are positively associated with the demand for money; secondly, variables that measure the opportunity cost of holding money and that are negatively related to the demand for money; and finally, variables that measure the rate of return to holding money and that are positively associated with money demand. Usually, the level of aggregate economic activity is measured by real GDP (although GNP, aggregate consumption and measure of industrial production have also been used in some countries). In certain contexts, the opportunity cost of holding money has been measured by the rate of inflation (thus measuring, for example, the opportunity cost of holding money vis-à-vis holding real assets), or alternatively by the rate of currency depreciation (which closely tracks the opportunity cost of holding domestic monetary assets relative to foreign assets). The own rate of return to holding money has often been measured by various alternative definitions of domestic interest rates.

As pointed out in Sriram (2001), the conventional theoretical approach to money demand modeling may be summarized by a general function of the form

\[ \frac{M}{P} = f(Y, OC) \]

where \( M \) is a measure of aggregate demand for money (in nominal terms) deflated by an appropriate aggregate price index \( P \); \( OC \) measures the opportunity cost (or the rate of return) to holding money. In general, we expect \( \frac{\partial f}{\partial Y} \geq 0 \). If \( OC \) is a measure of opportunity cost, then \( \frac{\partial f}{\partial OC} \leq 0 \); by contrast, if \( OC \) measures rates of return, \( \frac{\partial f}{\partial OC} \geq 0 \).

It is generally accepted that economic theory provides guidance on long-run behavior of money demand and its association with other macroeconomic variables, but that little can be said from a theoretical standpoint regarding the structure of short-run dynamics of money demand.
Empirical approaches to money demand estimation

There have been repeated efforts at providing empirical content in a country-specific context, to the theoretical approach to money demand briefly described above. In recent years in particular, there has been a proliferation of studies using a vector error correction approach to modeling money demand (VECM). This approach has, among many others, the advantage of jointly estimating the long- and short-run components of the demand for money, thus facilitating the task of ensuring that short-run specifications are associated with long-run components consistent with established economic theory. In practice, the VECM approach does appear to perform relatively well in the estimation of models that exhibit long-run behavior consistent with economic theory. This appears to be the case despite the fact that changes to the short-run model specification often dramatically alter associated estimates of the long-run component.

In many cases, however, the short-run autoregressive modeling framework used in the VECM approach becomes parameter intensive, and more often than not, there is little guidance from economic theory on the appropriate lag structure to be selected. The autoregressive framework may sometimes obscure, or at least does not help in making transparent, the presence of potentially interesting short-run components (e.g. of one or more cycles) common to the endogenous variables in the system. Conventional unit root and cointegration tests, as well as parameter estimates using a VECM approach, may also be overly sensitive to the presence of seasonal unit roots.

As in the VECM approach, the structural time series modeling approach allows joint estimation of long- and short-run components of money demand. By imposing the restriction that the endogenous variables in the system share a common stochastic trend (i.e., that variables are cointegrated), it is possible to find specifications that are consistent with economic theory at the zero frequency.

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3 For a recent survey of empirical studies using error-correction methods in the estimation of money demand please refer to Sriram (2001). Cartaya et al (1997) used a cointegration framework in the study of money demand at the monthly frequency in Venezuela. More recently, Ramajo (2001) used time-varying parameter error correction models, applied at a quarterly frequency to Venezuela.

4 Seasonal error-correction models have been proposed and applied to the estimation of money demand. See for example a recent application to Chile, in Soto and Tapia (2001).
Unlike in the VECM approach, however, structural time series models also facilitate the explicit estimation of complex short-run behavior, in a relatively transparent manner. For example, one or more cycles, as well as non-stationary seasonal components common to the endogenous variables, can be explicitly specified and estimated without eliciting an explosion in the number of model parameters. Thus structural time series models parsimoniously provide greater insight into the short-run dynamics of money demand, without necessarily relaxing theory-inspired constraints on the long-run behavior of system variables.

For convenience, it is possible to combine both approaches into a two-step framework that takes advantage of the simplicity of VECMs in the estimation of long-run parameters, as well as the flexibility and transparency of structural time series for modeling complex short-run dynamic structures. In this framework, referred to in this paper as the “merged approach”, a statistically satisfactory VECM is estimated and a theory-consistent cointegrating vector is selected.

The fitted values of the long-run component embedded in the VECM are then computed, normalizing to unity the coefficient on money demand (following conventional practice). Subsequently, a structural time series model for the first differences of the endogenous variables is estimated, including the fitted values of the error-correction term lagged one period as an exogenous variable.

The presence of the exogenous error-correction term in the structural time series model (in differences), incorporates information about adjustment to a long-run equilibrium in the estimation of short-run dynamics in a manner that broadly resembles a standard VECM. However, in the merged approach it is possible to proceed with the specification of richer and more transparent short-run dynamic structures than is usually possible within a purely autoregressive framework. Short-run dynamics estimated following the merged approach would closely track the dynamics associated with the pure structural time series approach, at least to the extent that the long-run components associated with the structural time series and merged approaches are comparable.
The structural time series approach to the estimation of money demand

In the structural time series modeling approach, the log of the demand for real money balances ($m_t$) could be represented as,

$$m_t = \mu_t^m + \psi_{1,t}^m + \psi_{2,t}^m + \xi_t^m + \epsilon_t^m \quad t = 1,\ldots,T$$

where $\mu_t^m$ represents the stochastic trend (unit root) component of the demand for real money balances, $\psi_{i,t}^m$ ($i=1,2$) represents a stochastic (trigonometric) cycle, $\xi_t^m$ is a seasonal component, and $\epsilon_t^m$ is an innovation.

Similarly, the variable representing the level of real economic activity expressed in logs ($y_t$), is assumed to have a representation

$$y_t = \mu_t^y + \psi_{1,t}^y + \psi_{2,t}^y + \xi_t^y + \epsilon_t^y \quad t = 1,\ldots,T$$

where $\mu_t^y$ represents a stochastic trend component of real economic activity, $\psi_{i,t}^y$ ($i=1,2$) represents a stochastic cycle, $\xi_t^y$ is a seasonal component and $\epsilon_t^y$ is an innovation.

Finally, we represent the opportunity cost or rate of return variable (which may or may not be expressed in logs, depending on whether interest is in calculating elasticities or semi-elasticities of money demand, respectively) as

$$p_t = \mu_t^p + \psi_{1,t}^p + \psi_{2,t}^p + \xi_t^p + \epsilon_t^p \quad t = 1,\ldots,T$$

where $\mu_t^p$ represents the stochastic trend component of an opportunity cost or rate of return variable, $\psi_{i,t}^p$ ($i=1,2$) represents a stochastic cycle, $\xi_t^p$ is a seasonal component, and $\epsilon_t^p$ is an innovation.\(^5\)

\(^5\) For a discussion of structural (unobserved components) time series models in a multivariate context, please refer to Harvey (1989) or Harvey (1993).
In the above representations, the \( \varepsilon_t \) are assumed to be normally distributed, mutually independent, serially uncorrelated innovations with zero mean and finite variance, that are also uncorrelated with any other stochastic elements in the representations of \( m_t, y_t \) and \( p_t \).

In addition, it is worth noting that we will specify models such that the slope components of \( \mu_t^m, \mu_t^y \) and \( \mu_t^p \) are not fixed but could themselves be random walks, thus allowing for the possibility of time-varying rates of drift.

Using conventional notation, we let the generic trend components \( (\mu_t) \) be expressed as

\[
\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t,
\]
\[
\beta_t = \beta_{t-1} + \zeta_t,
\]

where \( \eta_t \) is an innovation in the level of the trend, \( \beta_t \) is a time-varying slope component, and \( \zeta_t \) is an innovation in the slope component. The innovations \( \eta_t \) and \( \zeta_t \) are assumed to be independent and serially uncorrelated, with zero mean and finite variance. Cointegration requires the restriction that \( \mu_t^m, \mu_t^y \) and \( \mu_t^p \) have a common root—this restriction is imposed on all structural time series models subsequently discussed in this paper.\(^6\)

Turning now to short-run dynamics and following conventional notation, the cyclical components \( \psi_{i,t} \) \((i=1,2)\) have a general representation of the form

\[\psi_{i,t} = \psi_{i,t-1} + \xi_{i,t} + \zeta_{i,t}\]

\(^6\) In addition, please note that if the innovation in the slope process \( \zeta_t \) has zero variance, then \( \mu_t \) is a random walk with a constant rate of drift, i.e. an I(1) process. However, if \( \zeta_t \) has non-zero variance then \( \mu_t \) is an I(2) process. Since we are not imposing the prior restriction that \( \zeta_t \) has zero variance, it will be possible to use the estimated variance (or the signal-to-noise ratio) of the \( \zeta_t \) to check the statistical properties of \( \mu_t \). This will turn out to be an important step in the proposed model selection strategy.
\[
\begin{pmatrix}
\psi_t \\
\psi_{t-1}^*
\end{pmatrix}
= \rho
\begin{pmatrix}
\cos \lambda & \sin \lambda \\
-\sin \lambda & \cos \lambda
\end{pmatrix}
\begin{pmatrix}
\psi_{t-1} \\
\psi_{t-1}^*
\end{pmatrix}
+ \begin{pmatrix}
\omega_t \\
\omega_t^*
\end{pmatrix}
\]

where \( \psi_t \) represents the cyclical component, \( \omega_t \) and \( \omega_t^* \) are uncorrelated innovations with zero mean and finite variance, and \( \psi_{t-1}^* \) is introduced by construction of the cycle. The parameters \( \lambda, \rho \) are interpreted as the cycle frequency and damping factor on the amplitude of the cycle, respectively.\(^7\)

Furthermore, we will generally impose the restriction that cyclical behavior is common (up to sign and scaling factors) to all endogenous variables, since this specification yields the most interesting results for the analysis of the short-run dynamics of money demand. Initially, fixed seasonal components were specified and no commonality restrictions at the seasonal frequency were imposed.\(^8\) The seasonal components in the above representations (\( \xi_t \)) nevertheless satisfy the restriction that

\[
\xi_t = -\sum_{j=1}^{3} \xi_{t-j}
\]

i.e. the seasonal components sum to zero over a suitable period of time.

Structural time series models are put in state space form, and the Kalman filter is used to compute the likelihood function under the assumption that all disturbances are normally distributed. The likelihood function is optimized to obtain estimates of model parameters. Once parameters have been estimated, smoothed estimates of structural components can be obtained. Estimation can be carried out using STAMP 6.0.\(^9\)

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\(^7\) Note that in general the cycle is stochastic, not deterministic. The cyclical process has an equivalent reduced form representation as an ARMA(2,1) process in which the autoregressive component has complex roots. It may be shown that when \( \lambda \) equals either zero or \( \pi \), the model collapses to a first-order autoregressive process. The cycle is stationary when the damping factor \( \rho \) is strictly less than unity.

\(^8\) We tried several alternative specifications for the seasonal component. When the log of currency depreciation is used as measure of opportunity cost, we found a preferred alternative model specification which involves non–stationary seasonal components and seasonal cointegration. In this connection, please refer to Annex V.

\(^9\) See Doornik, J., et al (2000).
Tests for normality, autocorrelation and heteroskedasticity of residuals are used as specification tests. Model specifications are also assessed with respect to conventional goodness of fit and information criteria. Moreover, the model selection strategy will also take into account (a) the consistency of models with prior information regarding the order of integration of variables; (b) the consistency of a model with the cointegration hypothesis (tested for, in advance, using conventional cointegration tests); and (c) consistency of the long-run parameters of the demand for money associated with a model, with standard macroeconomic theory.

Using VECMs in the estimation of money demand

Generally, macroeconomic variables involved in money demand studies in several countries have been found to be non-stationary. Using conventional unit root tests, we establish formally whether or not we accept the hypotheses that these variables are indeed first-order integrated processes, in light of the particular data set in use. Should this be the case, it is in principle possible to use a conventional VECM approach in the estimation of a money demand function. As has already been mentioned, a VECM explicitly distinguishes between short- and long-run behaviors. Economic theory provides guidance on the types of restrictions and general functional forms that may apply in the context of long-run money demand estimation, but little guidance is offered regarding the structure of short-run dynamics. In this paper, we specify VECMs such that by judiciously choosing the form of the variables to be included and evaluating the resulting long-run parameters, the implied steady-state behavior of variables in the model is consistent with economic theory. Short-run dynamic specification is generally left unconstrained, since we have little prior (theoretical) information on its characteristics.

Finding the precise order of integration of a variable can nevertheless be rather problematic. For example, Barkoulas et al (1998) discuss the possibility that U.S. monetary aggregates are fractionally integrated, suggesting that monetary aggregates may have long-memory properties that are not fully exploited in a conventional autoregressive modeling approach.

For an introductory discussion of stationary and non-stationary processes, please refer to “Unit Roots and Cointegration for the Applied Economist” by D. Holden and R. Perman (in “Cointegration for the Applied Economist”, edited by B. Rao, St. Martin’s Press, 1994). For a detailed discussion of the theoretical underpinnings of the VECM approach, please see Chapter 16 in “Econometric Theory” by Davidson (2000).
Following standard notation, consider a general $p$-th order VAR model

$$\tilde{y}_t = A_1 \tilde{y}_{t-1} + \ldots + A_p \tilde{y}_{t-p} + B \tilde{x}_t + \eta_t, \quad t = 1, \ldots, T$$

where $\tilde{y}_t$ is a $n \times 1$ vector of I(1) endogenous variables (in this case consisting of elements that are measures of the demand for real money balances, the level of economic activity, as well as the opportunity cost or rate of return to holding money in period $t$); $\tilde{x}_t$ is a generic $r \times 1$ vector of exogenous variables; $A$ and $B$ are parameter matrices of suitable dimensions; and $\eta_t$ is a vector of innovations.

It can be shown that the $p$-th order VAR model presented above also has a representation as a general VECM of the form

$$\Delta \tilde{y}_t = \Pi \tilde{y}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \tilde{y}_{t-i} + B \tilde{x}_t + \epsilon_t,$$

where $\Pi = \sum_{i=1}^{p} A_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$. $\epsilon_t$ is a vector of innovations of suitable dimension.\(^{12}\)

From Granger’s Representation Theorem,\(^{13}\) it can be asserted that if $r \equiv \text{rank}(\Pi) < n$ then there exist $n \times r$ matrices $P, Z$ of $\text{rank}(P) = \text{rank}(Z) = r$ such that

(a) $\Pi = PZ^T$ and (b) $Z^T \tilde{y}_t$ is I(0). In this situation, the I(1) elements in vector $\tilde{y}_t$ are said to be cointegrated.

A VECM explicitly distinguishes between long-run behavior and short-run dynamics. In the RHS of the VECM equation, $\Pi \tilde{y}_{t-1}$ summarizes the long-run (or steady state) behavior of the variables. In this framework $Z$ is a matrix of cointegrating vectors,\(^{13}\)

\(^{12}\) Please note that there is no implication that $\eta_t$ and $\epsilon_t$ in the VECM representation are necessarily the same as in the structural time series representation.

\(^{13}\) For a more detailed discussion of Granger’s Representation Theorem, see for example Proposition 19.1 in Hamilton (1994). Johansen and Schaumburg (1999) present a general version of Granger’s Representation Theorem which encompasses seasonal cointegration.
at least one of which should have an economic interpretation as a vector representing the steady-state parameters of the system. $P$ is usually referred to as an “adjustment matrix” and is interpreted as measuring the speed to which the system corrects for deviations (or “errors”) from long-run equilibrium. The rest of terms in the RHS of the VECM equation represent the short-run dynamics of the system, up to a well-behaved innovation. The preferred estimation method for VECMs is maximum likelihood, under the conventional assumption that disturbances are normally distributed with zero mean and finite variances. As is now customary, to prevent spurious equation specifications we conduct cointegration tests on the various sets of I(1) variables used in the alternative money demand specifications already discussed.\textsuperscript{14}

It is worth noting that $n \times r$ matrices $P, Z$ are not identified, since for any non-singular matrix $F$, the product of matrices $PF$ and $Z[F^T]^{-1}$ yields $\Pi$. In view of this and following conventional practice, we normalize some of the elements in $Z$ by imposing the restriction of a unit coefficient associated with one variable in each equation. To facilitate interpretation of results, in what follows we have chosen always to normalize to unity the coefficient associated with the demand for real money balances, leaving unconstrained the coefficients on the measures of the level of economic activity and opportunity cost (or rate or return).

VECMs have some important properties. By virtue of the cointegration of the elements of $\tilde{y}_t$, the steady-state parameters of the system are estimated super-consistently.\textsuperscript{15} This means that greater confidence can be placed on parameter estimates for a given sample size. Moreover, the estimated parameters (elasticities and semi–elasticities of money demand) are robust to measurement errors in variables, as well as residual heteroskedasticity and other anomalies. In practice, however, short-run dynamic specification (i.e. the autoregressive lag structure in the VECM) heavily influences estimates of long-run parameters.

\textsuperscript{14}The types of tests conducted are as in “Likelihood-based Inference in Cointegrated Vector Autoregressive Models” by S. Johansen (Oxford University Press, 1995).

\textsuperscript{15} For a discussion of super-consistency properties, see Chapter 14 in J. Davidson (ibid.)
In this modeling framework, all elements in $\tilde{y}$ are treated as endogenous variables, thus circumventing possible issues of simultaneity. However, it is worth noting that endogeneity of the elements of $\tilde{y}$ in the estimation of a VECM does not necessarily imply that a particular estimated model necessarily has a structural interpretation vis-à-vis all of the elements in $\tilde{y}$. What allows us to give a structural interpretation to a particular estimated equation and its associated cointegrating vector is the set of identifying restrictions that are imposed on the system. Normalization to unity of the coefficient associated with money demand, as well as prior (theory inspired) information on the acceptable signs of the coefficients on other variables, together with the assumption that the supply of money adjusts to meet demand, lay the foundations for a structural interpretation of an estimated model as a money demand function. However, there is no implication that the system of estimated equations also contains a structural representation of variables other than money demand in $\tilde{y}$.$^{16}$ Estimation of a VECM is carried out using maximum likelihood methods and a conventional model selection strategy has been followed.$^{17}$

**A combined approach to estimating money demand in a cointegration framework**

The first step in the proposed approach consists of estimating a VECM for money demand as described in the earlier section. In the second step of the combined modeling approach, we represent the change in the log of the demand for real money balances ($\Delta m_t$) as,

$$\Delta m_t = \pi^m E_{-1} + \psi_{1,t}^m + \psi_{2,t}^m + \xi_t^m + e_t^m \quad t = 1, \ldots, T$$

where $\pi^m$ represents an adjustment coefficient associated with the error correction term, assumed to be exogenous for estimation purposes; $E_{-1}$ is the lagged value of the error-

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$^{16}$ What cointegrating relations do imply is that there is a set of long-run economic relations among the elements in $\tilde{y}$ that behave as “attractors” i.e. they draw economic variables together in the long run. For a discussion of structural cointegrating relations, including issues of identification and parallels with I(0) system estimation, please refer to section 16.6 in J. Davidson (ibid).

$^{17}$ We used the EViews 4 software to estimate the VECMs discussed in this paper. Please refer to Quantitative Micro Software (2000) for information on EViews 4.
correction term obtained in the first step (having previously selected a cointegrating vector with a normalizing restriction on money demand and theory-consistent coefficients on the other endogenous variables); \( \psi_{i_t}^{m} \) \((i=1,2)\) represent stochastic (trigonometric) cycles, \( \xi_{i_t}^{m} \) is a seasonal component, and \( \varepsilon_{i_t}^{m} \) is an innovation. In a similar fashion, the variable representing changes in the level of log real economic activity, \( (\Delta y_{t}) \), is assumed to have a representation,

\[
\Delta y_{t} = \pi^{y} E_{-1} + \psi_{1_t}^{y} + \psi_{2_t}^{y} + \xi_{t}^{y} + \varepsilon_{t}^{y} \quad t = 1,\ldots,T
\]

where \( \pi^{y} \) represents the adjustment coefficient associated with the exogenous error correction term; \( E_{-1} \) is the lagged value of the error-correction term obtained in the first step; \( \psi_{i_t}^{y} \) \((i=1,2)\) represent stochastic cycles, \( \xi_{t}^{y} \) is a seasonal component, and \( \varepsilon_{t}^{y} \) is an innovation.

Finally, we represent changes in the opportunity cost or rate of return variable \( (\Delta p_{t}) \), expressed in levels or in logs, as

\[
\Delta p_{t} = \pi^{p} E_{-1} + \psi_{1_t}^{p} + \psi_{2_t}^{p} + \xi_{t}^{p} + \varepsilon_{t}^{p} \quad t = 1,\ldots,T
\]

where \( \pi^{p} \) represents the adjustment coefficient of the exogenous error correction term; \( E_{-1} \) is the lagged value of the error-correction term (as in the other equations); \( \psi_{i_t}^{p} \) \((i=1,2)\) represents stochastic cycles, \( \xi_{t}^{p} \) is a seasonal component, and \( \varepsilon_{t}^{p} \) is an innovation.

Please note that consistency with a cointegration framework is achieved by including the lagged error correction term in each equation. No prior restrictions are imposed on the adjustment coefficients associated with the error correction term. As in the pure structural time series modeling approach, we do impose the restriction that cyclical behavior is common (up to sign and scaling factors) to the endogenous variables \( \Delta m_{t} \), \( \Delta y_{t} \) and \( \Delta p_{t} \). Initially, fixed seasonal components are specified (with no commonality restrictions). Innovations are assumed to be normally distributed, with zero
mean and finite variance. Parameter estimation is carried out by putting the models in state-space form, using the Kalman filter and optimizing the likelihood function. Estimation can be carried out using STAMP 6.0. Tests for normality, autocorrelation and heteroskedasticity of residuals are used as specification tests. Model specification is also assessed in light of conventional goodness of fit and information criteria.

**ESTIMATION AND STATISTICAL RESULTS**

**General comments on estimation and results**

Quarterly data used range from 1993.1 to 2001.4 (please see Annex I for a description of the data set). Results are presented for real M1 balances, which was the only money monetary measure for which consistently good results were obtained using all modeling approaches. The same estimation procedures presented here for M1, were applied to data for base money and M2, with generally unsatisfactory results. The measure of real economic activity was real GDP in all models. Relatively satisfactory results were obtained for inflation, log of inflation and log of the nominal depreciation rate, as measures of the opportunity cost of holding money. In all cases, the magnitude and signs of the estimated long-run elasticities (or semi-elasticities) conformed to our expectations based on economic theory. We also attempted to use (nominal) deposit interest rates as a measure of the rate of return to holding money, but results were not very satisfactory (mainly because the deposit rate tended to be negatively associated with demand for M1, suggesting that deposit rates are if anything a weak measure of opportunity cost not of the own rate of return to holding M1-type assets).

**Estimation results following the structural time series approach**

A model taking as endogenous variables log real M1, log real GDP and the inflation rate was estimated and convergence was achieved in 100 iterations. The value of the log-likelihood was 271.43. The estimated (common) stochastic cycles have periods of 2.78

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18 See Doornik, J., et al (2000).
19 For a limited sample (1997.1 to 2001.4), moderately satisfactory results could be obtained for M2 using the VECM approach; these results, however, are based on a very short sample and could not be strengthened by using alternative modeling approaches. We have therefore chosen to disregard these results.
and 2.85 years. For the money demand equation, heteroskedasticity, autocorrelation and normality statistics are satisfactory at conventional critical levels. The estimated equations for log real GDP and inflation have satisfactory heteroskedasticity and normality statistics; but they nevertheless appear to exhibit residual (higher-order) autocorrelation. Goodness of fit measures indicate a generally good fit, including that there is an improvement with respect to alternative random walk with drift models.

| Statistic                                         | Log Real M1 | Log Real GDP | Inflation |
|---------------------------------------------------|-------------|--------------|-----------|
| Normality (Bowman-Shenton): $n \sim \chi^2$       | 2.28        | 0.71         | 3.69      |
| Skewness: $s \sim \chi^2_1$                       | 0.18        | 0.07         | 2.65      |
| Kurtosis: $k \sim \chi^2_1$                       | 2.10        | 0.64         | 1.04      |
| Heteroskedasticity: $H(11) \sim F_{11,11}$       | 0.31        | 0.13         | 0.21      |
| Autocorrelation (up to 14th order) (Box-Ljung): $Q(14,6) \sim \chi^2_6$ | 8.85        | 31.18        | 15.13     |
| Autocorrelation (first order) Durbin-Watson: $DW \sim N(2, \frac{4}{T})$ | 1.98        | 1.66         | 1.47      |
| Goodness of Fit (improvement over random walk plus drift model) $R^2_d = 1 - \frac{(T - d)\hat{\sigma}^2}{\sum_{t=1}^{T} (y_t - \hat{y})^2}$ | 0.82        | 0.74         | 0.30      |
| Goodness of Fit (ordinary $R^2$)                   | 0.85        | 0.76         | 0.75      |
| Akaike Information Criterion                      | -3.57       | -5.29        | -4.76     |
| Bayes Information Criterion                       | -1.83       | -3.55        | -3.03     |

We note, however, that the estimated variance of the slope disturbances is non-zero, suggesting that the estimated trends are I(2), instead of I(1) as we would have expected based on the results of conventional unit root tests. Moreover, the periods of the
estimated (common) cycles are very close, in contrast to other model specifications that yield distinct high and low frequency cycles.

Results for a second model, taking log real M1, log real GDP and the log of the inflation rate as endogenous variables, are presented below. In this case, strong convergence was achieved in 87 iterations. The value of the log-likelihood is 275.82. High and low frequency (common) cycles were estimated with periods 1.71 and 2.97 years, respectively.

| Statistic                                      | Log Real M1 | Log Real GDP | Log of Inflation |
|-----------------------------------------------|-------------|--------------|------------------|
| Normality (Bowman-Shenton): \( n \sim \chi^2_2 \) | 1.80        | 0.07         | 1.31             |
| Skewness: \( s \sim \chi^2_1 \)               | 0.39        | 0.00         | 1.15             |
| Kurtosis: \( k \sim \chi^2_1 \)               | 1.41        | 0.07         | 0.16             |
| Heteroskedasticity: \( H(10) \sim F_{10,10} \) | 0.33        | 0.13         | 0.19             |
| Autocorrelation (up to 14\textsuperscript{th} order) (Box-Ljung): \( Q(14,6) \sim \chi^2_6 \) | 10.41       | 20.17        | 15.92            |
| Autocorrelation (first order) Durbin-Watson: \( DW \sim N(2, \frac{4}{T}) \) | 1.80        | 1.85         | 1.63             |
| Goodness of Fit (improvement over random walk plus drift model) \( R_D^2 = 1 - \frac{(T - d)\hat{\sigma}^2}{\sum_{t=1}^T (y_t - \bar{y})^2} \) | 0.82        | 0.77         | 0.37             |
| Goodness of Fit (ordinary \( R^2 \)) | 0.84        | 0.78         | 0.77             |
| Akaike Information Criterion | -3.47       | -5.36        | -5.01            |
| Bayes Information Criterion | -1.72       | -3.60        | -3.26            |

For the money demand equation, estimation results were generally satisfactory in terms of the normality of the residuals, heteroskedasticity as well as autocorrelation. The
equations for log real GDP and log inflation have satisfactory normality and heteroskedasticity statistics, but there is evidence of (higher-order) residual autocorrelation. Goodness of fit measures are also satisfactory, indicating that this model is indeed an improvement over random walk with drift models. Nevertheless, the estimated variance of the slope disturbances is non-zero, suggesting that the estimated trends are I(2), instead of I(1), as we expected based on the results obtained from preliminary conventional unit root tests.

The results of a third model, taking logs of real M1, real GDP and the rate of nominal currency depreciation as endogenous variables, are presented below. Very strong convergence was achieved in 79 iterations.

| Venezuela: Structural Time Series Model for Logs of Real Money Balances, Real GDP and Currency Depreciation |
|---------------------------------------------------------------|
| **Key Summary Statistics (T=34)**                             |
| **Statistic**        | **Log Real M1** | **Log Real GDP** | **Log of Depr.** |
| Normality (Bowman-Shenton): \( n \sim \chi^2 \)              | 0.36            | 0.52            | 0.28            |
| Skewness: \( s \sim \chi^2 \)                                | 0.05            | 0.26            | 0.15            |
| Kurtosis: \( k \sim \chi^2 \)                                | 0.30            | 0.26            | 2.36            |
| Heteroskedasticity: \( H(10) \sim F_{10,10} \)              | 0.74            | 0.19            | 0.13            |
| Autocorrelation (up to 14th order) (Box-Ljung): \( Q(14,6) \sim \chi^2 \) | 17.10          | 15.50           | 17.27           |
| Autocorrelation (first order)                                 | 1.87            | 2.06            | 2.62            |
| Durbin-Watson: \( DW \sim N(2, \frac{4}{T}) \)              |                  |                  |                 |
| Goodness of Fit (improvement over random walk plus drift model) | 0.84            | 0.79            | 0.69            |
| \( R_D^2 = 1 - \frac{(T - d) \hat{\sigma}^2}{\sum_{t=1}^{T} (y_t - \bar{y})^2} \) |                 |                  |                 |
| Goodness of Fit (ordinary \( R^2 \))                         | 0.84            | 0.81            | 0.24            |
| Akaike Information Criterion                                  | -3.54           | -5.37           | -1.71           |
| Bayes Information Criterion                                   | -1.77           | -3.60           | 0.06            |
The value of the log-likelihood is 218.30. Common cycles were estimated with periods 1.36 and 2.99 years. Estimation results for all equations are generally satisfactory in terms of residual normality and heteroskedasticity. There is nevertheless some evidence of (higher-order) residual autocorrelation. Goodness of fit measures are also satisfactory, indicating that this model, just as the models before, represents an improvement over random walk with drift models. Finally, we note that as in the other specifications based on the structural time series approach with fixed seasonality, the estimated variance of the slope disturbances is non-zero, suggesting that the estimated trends are I(2), instead of I(1), as we would have expected based on the results of preliminary (conventional) unit root tests.

Please refer to Annex V for a discussion of the preferred model, which features non-stationary seasonal components with seasonal cointegration, using currency depreciation as the opportunity cost measure, following the structural time series approach.

**Estimation results following the VECM approach**

A VECM model taking log of real M1, log of real GDP and the inflation rate as endogenous variable was estimated. Intercepts (but no trends) are included in the cointegrating relation as well as in the short-run model. The signs of the estimated long-run parameters are consistent with economic theory.

| Statistic                             | Log Real M1 | Log Real GDP | Inflation |
|---------------------------------------|-------------|--------------|------------|
| Coefficient in cointegrating equation | 1           | 0.26         | -0.68      |
| Goodness of Fit ($R^2$)               | 0.68        | 0.63         | 0.19       |
| Akaike Information Criterion         | -1.88       | -3.80        | -3.48      |
| Schwarz Criterion                     | -1.52       | -3.43        | -3.11      |
A second model VECM was estimated taking logs of real M1, real GDP and the inflation rate as endogenous variables. As before, intercepts (but no trends) appear in the cointegrating relation as well as in the short-run model. We also find that the signs of the estimated long-run parameters are consistent with economic theory.

| Statistic                                      | Log Real M1 | Log Real GDP | Log Inflation |
|-----------------------------------------------|-------------|--------------|---------------|
| Coefficient in cointegrating equation         | 1           | 0.24         | -0.48         |
| Goodness of Fit (ordinary $R^2$)              | 0.74        | 0.59         | 0.33          |
| Akaike Information Criterion                  | -1.99       | -3.76        | -3.96         |
| Schwarz Criterion                              | -1.62       | -3.38        | -3.59         |

Finally, we estimated a VECM model taking as endogenous variables the logs of real M1, real GDP and the nominal rate of currency depreciation. Intercepts but no trends appear in both the cointegrating relations and the short-run model. As before, the signs of the estimated long-run parameters are consistent with economic theory.

| Statistic                                      | Log Real M1 | Log Real GDP | Log Depr. |
|-----------------------------------------------|-------------|--------------|-----------|
| Coefficient in cointegrating equation         | 1           | 0.26         | -0.68     |
| Goodness of Fit (ordinary $R^2$)              | 0.68        | 0.63         | 0.19      |
| Akaike Information Criterion                  | -1.88       | -3.80        | -3.48     |
| Schwarz Criterion                              | -1.52       | -3.43        | -3.11     |

**Estimation results following the combined approach**

A model taking as endogenous variables the first differences of log real M1, log real GDP and the inflation rate, was estimated. Very strong convergence was achieved in 86 iterations. The value of the log-likelihood is 276.18. Estimated high and low
frequency cycles, common to all variables in the system, have periods of 1.55 and 3.04 years respectively.

| Statistic                                      | Log Real M1 | Log Real GDP | Inflation |
|------------------------------------------------|-------------|--------------|-----------|
| Normality (Bowman-Shenton): $n \sim \chi^2_2$ | 1.81        | 0.54         | 4.41      |
| Skewness: $s \sim \chi^2_1$                   | 1.65        | 0.44         | 2.29      |
| Kurtosis: $k \sim \chi^2_1$                   | 0.15        | 0.10         | 2.12      |
| Heteroskedasticity: $H(11) \sim F_{1,1,1}$   | 0.42        | 0.15         | 0.15      |
| Autocorrelation (up to 12th order) (Box-Ljung): $Q(12,6) \sim \chi^2_6$ | 11.85 | 18.02 | 9.82 |
| Autocorrelation (first order)                  | 2.11        | 1.84         | 2.00      |
| Goodness of Fit (w.r.t. seasonal mean)         | 0.72        | 0.53         | 0.52      |
| Goodness of Fit (ordinary $R^2$)               | 0.85        | 0.73         | 0.29      |
| Akaike Information Criterion                   | -4.22       | -5.70        | -5.22     |
| Bayes Information Criterion                    | -2.87       | -4.36        | -3.87     |

Heteroskedasticity, normality and autocorrelation statistics are satisfactory for the first differences of log real M1 and the inflation rate. Nevertheless, there appears to be (higher-order) residual autocorrelation in the case of the change in log real GDP. Goodness of fit measures indicate a generally good fit, including with respect to the seasonal mean.

Results for a second model, where first differences of log real M1, log real GDP and the log of the inflation rate are taken as endogenous variables, are presented below. In this case, very strong convergence is achieved in 43 iterations. The value of the log-likelihood is 260.81. Common cycles were estimated with periods 1.70 and 2.86 years. In the case of the equation for the first difference of log real M1, results were generally
satisfactory in terms of normality of the residuals, heteroskedasticity and autocorrelation. For the first difference of log real GDP, statistics for normality and heteroskedasticity are satisfactory, but there is nevertheless some evidence of (higher-order) residual autocorrelation.

| Statistic                              | ?Log Real M1 | ?Log Real GDP | ?Log Inflation |
|----------------------------------------|--------------|---------------|---------------|
| Normality (Bowman-Shenton): $n \sim \chi^2_2$ | 0.66         | 0.07          | 9.61          |
| Skewness: $s \sim \chi^2_1$            | 0.42         | 0.02          | 6.66          |
| Kurtosis: $k \sim \chi^2_1$            | 0.24         | 0.05          | 2.94          |
| Heteroskedasticity: $H(10) \sim F_{10,10}$ | 0.39         | 0.24          | 0.14          |
| Autocorrelation (up to 12th order)     | 6.47         | 16.63         | 15.03         |
| (Box-Ljung): $Q(12,6) \sim \chi^2_6$  |              |               |               |
| Autocorrelation (first order)           | 1.92         | 1.85          | 2.02          |
| Goodness of Fit (w.r.t. seasonal mean)  | 0.70         | 0.55          | 0.55          |
| Goodness of Fit (ordinary $R^2$)       | 0.84         | 0.74          | 0.26          |
| Akaike Information Criterion           | -3.62        | -5.24         | -4.88         |
| Bayes Information Criterion            | -1.87        | -3.48         | -3.13         |

The statistics suggest (a) the non-normality of the residuals of the equation corresponding to the change in the log of the inflation rate, and (b) the presence of higher-order residual autocorrelation in the model for the change in the log of the inflation rate. Goodness of fit measures are otherwise also satisfactory, including with respect to the seasonal mean.

Finally, the results of a third model, taking as endogenous variables the first differences of the logs of real M1, real GDP and the rate of nominal currency
depreciation, are presented below. Very strong convergence was achieved in 28 iterations. The value of the log-likelihood is 198.18. High and low frequency cycles (common to all endogenous variables in the system) were estimated, with periods 1.55 and 2.92 years, respectively. For the first difference of log real M1 and GDP, estimation results are satisfactory, in terms of the normality and heteroskedasticity of the residuals.

| Statistic                              | ?Log Real M1 | ?Log Real GDP | ?Log Depr. |
|----------------------------------------|--------------|---------------|------------|
| Normality (Bowman-Shenton): \( n \sim \chi_2^2 \) | 1.25         | 0.31          | 3.87       |
| Skewness: \( s \sim \chi_1^2 \)        | 0.32         | 0.05          | 3.34       |
| Kurtosis: \( k \sim \chi_1^2 \)        | 0.93         | 0.25          | 0.52       |
| Heteroskedasticity: \( H(10) \sim F_{10,10} \) | 0.30         | 0.20          | 0.09       |
| Autocorrelation (up to 12th order)     | 16.17        | 12.57         | 23.70      |
| (Box-Ljung): \( Q(12,6) \sim \chi_6^2 \) |              |               |            |
| Autocorrelation (first order)          | 2.28         | 1.83          | 3.05       |
| Goodness of Fit (w.r.t. seasonal mean) | 0.60         | 0.54          | 0.70       |
| Goodness of Fit (ordinary \( R^2 \))   | 0.79         | 0.73          | 0.19       |
| Akaike Information Criterion          | -3.68        | -5.55         | -1.23      |
| Bayes Information Criterion           | -2.20        | -4.07         | 0.25       |

The skewness of the distribution of the residuals of the equation corresponding to the first difference of log of the rate of depreciation, is somewhat higher than has been the case with most of the other models; however, at standard 5 percent critical values it is still consistent with the null of normality of residuals. At conventional critical values, there is nevertheless evidence of (higher-order) residual autocorrelation, except for the equation associated with the change in log real GDP. Goodness of fit measures, including with respect to seasonal means, are otherwise satisfactory.
Analysis of results at the zero frequency

We now present an analysis of findings at the zero frequency, for each alternative opportunity cost variable that has been used. Taking the level of inflation as a measure of the opportunity cost of holding money, it has been found that the demand for real money balances is inelastic with respect to real GDP. Depending on the modeling approach used to estimate elasticities at the zero frequency, structural time series (STSM) or VECM, the elasticity with respect to real GDP is found to be 0.71 or 0.26 respectively. Similarly, the semi-elasticity of money demand with respect to the inflation rate is −0.37 and −0.68, using STSM and VECM approaches respectively.

The trend component of log real money demand, estimated using alternative approaches, is shown in Figure “M1-Demand at the Zero Frequency: Inflation as Measure of Opportunity Cost”. The trend estimated following the STSM approach is clearly smoother than with the VECM approach (and thus the combined approach).\footnote{The trend component in the combined approach is identical to the pure VECM approach, since the former uses the same long-run information as the latter.} The key difference is an important dip in the estimated trend under the VECM approach, in 1994 and 1996. Otherwise, the estimated trends appear to track each other closely.

| Long-Run Responsiveness of M1-Demand | Elasticity | Semi-Elasticity |
|--------------------------------------|------------|-----------------|
| $\text{Approach}$                    | $\text{Real GDP}$ | $\text{Inflation}$ |
| STSM                                 | 0.71       | −0.37           |
| VECM and Merged                       | 0.26       | −0.68           |
Similar results are obtained when the log of the inflation rate is used as the opportunity cost variable. The demand for real money balances is inelastic with respect to real GDP. Depending on the modeling approach used to estimate elasticities at the zero frequency, the elasticity with respect to real GDP is 0.86 or 0.24, with the STSM or VECM approaches, respectively. The elasticity of money demand with respect to the inflation rate is −0.31 and −0.48, using STSM and VECM approaches respectively.

The trend component of log real money demand under alternative approaches, is shown in the Figure “M1-Demand at the Zero Frequency: Log Inflation as Measure of Opportunity Cost”.

### Long-Run Responsiveness of M1-Demand

| Approach          | Elasticity Real GDP | Elasticity Inflation |
|-------------------|---------------------|----------------------|
| STSM              | 0.86                | −0.31                |
| VECM and Merged   | 0.24                | −0.48                |
The trend estimated following the STSM approach is smoother than with the VECM and combined approaches. The most noticeable differences between the estimated trends are (a) the somewhat higher level of the VECM trend in the early years; (b) an important dip in the estimated trend under the VECM approach in 1996 (which does not appear in the STSM-based trend of money demand); and (c) the higher level of the VECM trend in 1998-1999. It is worth noting that both VECM- and STMS-based trends in the case of log inflation as the opportunity cost variable, appear to be “noisier” than when inflation appears as the opportunity cost variable.

When the log of the depreciation rate is used as the opportunity cost variable, the demand for real money balances is also inelastic with respect to real GDP. Depending on the modeling approach used to estimate elasticities at the zero frequency, the elasticity with respect to real GDP is 0.68 or 0.74, using the STSM and VECM approaches respectively. The elasticity of money demand relative to the depreciation rate is −0.43 and −0.26, using STSM and VECM approaches respectively.
At this stage, there are two things to highlight. Firstly, unlike with the other opportunity cost variables, in this case STSM and VECM modeling approaches both yield very similar results for the elasticity of money demand with respect to real GDP. Secondly, the STSM approach consistently yielded an elasticity with respect to real GDP in a relatively “high” range of estimates (0.68, 0.71 and 0.86), regardless of the opportunity cost variable chosen. By contrast, the VECM approach yielded real GDP-elasticities of about 0.25 with the inflation rate (in levels or in logs), but an estimate of magnitude about three times higher when depreciation is the opportunity cost variable.

| Approach          | Elasticity Real GDP | Elasticity Depreciation |
|-------------------|---------------------|-------------------------|
| STSM              | 0.68                | -0.43                   |
| VECM and Merged   | 0.74                | -0.26                   |

*Using fitted values of cointegrating equation as VECM long-run component. Using stochastic trend of M1 in the STSM approach.
The trend component of log real money demand, under alternative approaches, is shown in the Figure “M1-Demand at the Zero Frequency: Depreciation as Measure of Opportunity Cost”.

The trend estimated following the STSM approach is much smoother than with the VECM and combined approaches. The most noticeable differences between the estimated trends are (a) the somewhat higher level of the VECM trend in the early years of the series, and (b) sharp dips in the trend estimated following the VECM approach in 1993, 1995 and 1996 (something that does not appear in the STSM-based trend). Since 1998, the VECM-based trend appears to have fluctuated around the STSM-based trend.

Please refer to Annex V for an analysis of results using a model with seasonal cointegration and exchange rate depreciation as a measure of opportunity cost.

Analysis of results at cyclical and seasonal frequencies

In a standard VECM approach there is no explicit structural decomposition at the cyclical and seasonal frequencies. Thus in order to compare the short-run dynamics implied by the various VECMs that have been estimated, with the dynamics obtained using the STSM and merged approaches, we first have to aggregate the STSM-based cycles and seasonal components. We will first compare short-run dynamics using the fitted values of the VECM model, with the “composite” dynamics based on the STSM approach. Later in the paper, decompositions of money demand at cyclical and seasonal frequencies based on structural time series models will be presented. The spectral densities of the estimated (composite) short-run dynamics of money demand using the VECM, STSM and merged approaches can be found in Annex II.

We begin by presenting results using inflation as a measure of opportunity cost. The first thing to notice is that the short-run dynamics obtained under the VECM approach are distinctively different from the dynamics implied by the STSM and merged approaches. Under the VECM modeling approach, there is but a mild cyclical downturn

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21 Seasonal cointegration can be modeled in the VECM approach, but there is still no obvious way of carrying out further decomposition of the cyclical frequencies. In addition, modeling cointegration at the zero and seasonal frequencies using an autoregressive approach can become very parameter intensive.
in 1996; by contrast, the STSM and merged approaches suggest that there is a strong cyclical downturn in 1996. This occurs because the sharp fall in money demand that took place in 1996 has been attributed in the VECM approach to the trend component; by contrast, STSM-based trends turned out to be smoother and the sharp dip in 1996 was captured as a cyclical phenomenon (see Figure “M1-demand at Cycle and Seasonal Frequencies: Inflation as Measure of Opportunity Cost”).

When log inflation is used as the measure of opportunity cost, the general features of short-run dynamics based on the STSM and merged approaches are similar. By contrast, the VECM-based short-run dynamics are somewhat different from the STSM and merged approaches. It is important to note, however, that the most salient characteristics of short-run dynamics are fairly robust to the change in the measure of opportunity cost from inflation to log inflation, but more so in the case of the STSM and merged approaches (see Figure “M1-demand at Cycle and Seasonal Frequencies: Log Inflation as Measure of Opportunity Cost”).

Finally, when the log of the nominal exchange rate depreciation is used as the opportunity cost variable, the general patterns of short-run dynamics remain unchanged.
In particular, the similarity of the short-run dynamics based on the STSM and merged approaches, together with the distinctiveness of the VECM-based short-run dynamics, is a characteristic that has been robust to changes in the definition of the opportunity cost variable. Moreover, notice that whereas VECMs attributed the sharp fall in money demand in 1996 to the zero frequency (i.e. a shock in the level of the trend component),
the alternative approaches consistently attributed it to cyclical dynamics (see Figure “M1-demand at Cycle and Seasonal Frequencies: Depreciation as Measure of Opportunity Cost”).

We now present further decompositions of the short-run dynamics of money demand, using different measures of the opportunity cost of holding money. We only present results obtained using the pure structural time series modeling approach, since results obtained using the merged approach are very similar. As may be recalled, we sought to specify structural time series models that would decompose short-run dynamics, specifically, into two cycles plus a seasonal component. The optimal frequency (and by implication the optimal period) and amplitude of the cycles is determined from the data—not imposed by assumption. However, it is worth recalling that the restriction that cycles be common to all endogenous variables was imposed in all structural time series models that have been estimated.

It is worth noting that the lower frequency cycles of money demand obtained using alternative measures of opportunity cost, are well defined and almost identical (see

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22 No further decomposition of short-run dynamics is possible using the standard VECM approach.
Figure: “M1-demand: Lower Frequency Cycles). However, higher frequency cycles were not as robust to changes in the definition of the opportunity cost variable. The higher frequency cycle appears to be most important when depreciation is used as opportunity cost measure, and least important when inflation is used as the opportunity cost measure. Nevertheless, for each of these models the higher frequency cycle is still relatively well defined (see Figure: “M1-demand: Higher Frequency Cycles).

![M1-Demand Higher Frequency Cycles](image1)

The estimated (fixed) seasonal components of money demand are virtually identical, regardless of the opportunity cost variable that is used (see Figure: “M1-demand: Seasonal Frequency, where series appear to be almost perfectly superimposed on each other). The spectral densities of the cyclical and seasonal components of money demand based on the structural time series and merged approaches, can be found in Annexes III and IV, respectively.

![M1-Demand Seasonal Frequency](image2)

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23 In the Figure, “CPI” is the cycle associated with the model using log inflation, “NER” is the cycle associated with the model that uses depreciation, “INF” is the cycle associated with the model that uses log inflation, as measures of opportunity cost.
Annex V contains an analysis of results obtained using a model with non-stationary seasonal components, seasonal cointegration and depreciation as opportunity cost measure, following a structural time series approach.

**SUMMARY AND CONCLUSIONS**

Using structural time series models, we have estimated common stochastic trend and cycle models of money demand (M1) for Venezuela in the 1993.1-2001.4 period, using the inflation rate, log of the inflation rate and log of the nominal rate of currency depreciation as measures of the opportunity cost of holding money. The estimated common trends are subsequently used to obtain long-run elasticities of money demand. Based on these estimates, the elasticity of the demand for real money balances with respect to real GDP would initially appear to be in the range 0.7–0.9. We also find that the semi-elasticity of money demand with respect to the inflation rate is about –0.4, the elasticity with respect to the inflation rate is in the vicinity of –0.4, and the elasticity with respect to the nominal depreciation rate is close to –0.4.

After carrying out conventional cointegration tests, we also estimated VECM models for money demand (using the same data set as before). The long-run behavior of money demand obtained using the VECM approach is broadly similar to the long-run behavior implied by the structural time series approach. When inflation is used as the opportunity cost variable, the elasticity of money demand to real GDP is about 0.3, while the semi-elasticity to the rate of inflation is in the vicinity of –0.7. When log inflation is used as the opportunity cost variable, the elasticity of money demand to real GDP is in the 0.2–0.3 range, and the elasticity with respect to the inflation rate is close to –0.3. Also, when the log of the rate of currency depreciation is used as the measure of opportunity cost, the elasticities of money demand with respect to real GDP and nominal depreciation are in the vicinity of 0.7 and –0.3, respectively.

In the preferred model, the log of the depreciation rate is taken as the opportunity cost variable, using a seasonal cointegration specification (following the structural time series approach). In this case, the estimated elasticity of money demand with respect to
real GDP is in the 0.5–0.6 range, while the elasticity with respect to the rate of depreciation is close to -0.5. We could not find an alternative seasonal cointegration model with satisfactory statistical properties, using inflation as measure of the opportunity cost of holding money.

The sensitivity of long-run elasticities to changes in modeling approach, model specification, and choice of opportunity cost variable, can partly be explained by the relatively short length of the series (in terms of calendar years) that is being used in model estimation. With longer time series, it could be expected that somewhat more robust estimates of long-run elasticities can be obtained.

However, another reason for unstable estimates of long-run elasticities appears to be the presence of non-stationary seasonal components. In this connection, we note that non-stationary seasonality could only be adequately captured in the case of currency depreciation as measure of opportunity cost and using the structural time series approach—i.e., in the preferred model. Notice that the elasticity of money demand with respect to real GDP estimated using a seasonal cointegration specification, is between the lower and higher range of estimates obtained using fixed seasonality.

The composite short-run dynamics of money demand have nevertheless been found to be robust to changes in the definition of the opportunity cost variable, and to the presence of non-stationary seasonal components. However, the structural time series modeling approach does provide richer insights into the short-run adjustment dynamics of money demand than a VECM approach. A combined approach takes advantage of VECM models to estimate long-run relationships, but uses a structural time series approach to obtain further insights into short-run dynamics.

Using the structural time series and combined approaches, it has been found that a lower frequency cycle (with period close to 3 years) is uniformly present in the dynamics

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24 Monetary and opportunity cost variables are available at a quarterly frequency for longer periods. Unfortunately, we could not find GDP series at a quarterly frequency for periods before 1993.

25 We note that seasonal cointegration models can also be specified using a seasonal vector error-correction model. However, the autoregressive framework of such a model does not facilitate further decomposition of short-run dynamics. We thus prefer to follow the structural time series approach.
of real GDP and the opportunity cost variables. This cycle is a key feature of the short-run dynamics of money demand. Higher frequency cycles in money demand are present but are more sensitive to the choice of opportunity cost variable, as well as to the specification of the seasonal components.

Accurately forecasting money demand can be very important for policy-making purposes. Structural time series models can be used for forecasting aggregated (composite) series as well as individual structural components. Composite forecasts using the preferred structural time series model have been generated. The Figure “Venezuela: Real Money Balances—Smoothed Trend and Forecasts” presents the historical series of demand for M1 (expressed in real terms, in levels and not in logs) from the last quarter of 1997 to the end of 2001. Composite forecasts of the demand for M1 are presented through the end of 2004.

Venezuela: Real Money Balances

Smoothed Trend and Forecasts *

*M1 deflated by the CPI index. 2002.1-2004.4 is a forecast.
The forecasts of money demand presented above are accompanied by RMSE forecast error bounds on either side. For reference, the Figure also shows the smoothed and forecast trend (in levels, not in logs) of the demand for M1 in Venezuela.

Since multiple stochastic cyclical components have been explicitly modeled in the structural time series approach, it is also possible to generate forecasts at cyclical frequencies. We present below the smoothed and forecast cycles (high and low frequency) for money demand in Venezuela (expressed in real terms, in levels and not in logs).

Notice how important the low frequency cycle is in shaping forecast deviations from trend values. For example, the fall in money demand in early 2002 is mostly the result of a forecast cyclical downturn. Downward pressure on demand for money due to cyclical factors is expected to subside later in the year and a cyclical upturn is forecast to begin in the second half of 2002. Given the information available to date, a cyclical peak in money demand is forecast to occur in 2004. Forecasts of seasonal effects can also be generated and added to forecasts of trend and cyclical components, to construct the composite forecast of money demand.

Venezuela: Real Money Balances

Smoothed Cycles and Forecasts*

*M1 deflated by the CPI index. 2002.1-2004.4 is a forecast.
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ANNEX I
DESCRIPTION OF STATISTICAL SERIES

- **Real GDP.** Quarterly real GDP series for 1993.1-2001.4 period, as published by the Central Bank of Venezuela. Series expressed in billions in constant local currency terms.

- **M1.** Quarterly series for the M1 definition of money, for 1993.1-2001.4 period, taken from the International Financial Statistics published by the IMF. Series expressed in millions in current local currency terms.

- **Exchange rate.** Quarterly Bolivar/US$ exchange rate series, for the 1993.1-2001.4 period, taken from the International Financial Statistics published by the IMF.

- **CPI.** Quarterly CPI series for 1993.1-2001.4 period taken from the International Financial Statistics published by the IMF.

| Year | Real GDP | M1   | NER  | CPI   |
|------|----------|------|------|-------|
| 1993.1 | 134.83  | 320763 | 84.4 | 33.76 |
| 1993.2 | 136.33  | 330766 | 89.3 | 36.59 |
| 1993.3 | 144.05  | 310746 | 97.6 | 40.11 |
| 1993.4 | 142.99  | 407219 | 105.6 | 45.06 |
| 1994.1 | 132.41  | 489851 | 114.7 | 49.76 |
| 1994.2 | 138.70  | 592969 | 198.3 | 56.15 |
| 1994.3 | 134.07  | 756981 | 169.8 | 67.36 |
| 1994.4 | 139.91  | 974120 | 170.0 | 76.87 |
| 1995.1 | 135.78  | 1002836 | 170.0 | 84.44 |
| 1995.2 | 139.46  | 1048031 | 170.0 | 94.20 |
| 1995.3 | 144.14  | 1028981 | 170.0 | 103.29 |
| 1995.4 | 147.25  | 1343964 | 290.0 | 118.08 |
| 1996.1 | 136.53  | 1501817 | 290.0 | 145.19 |
| 1996.2 | 138.47  | 1562422 | 469.0 | 186.60 |
| 1996.3 | 142.44  | 1805943 | 473.3 | 221.23 |
| 1996.4 | 148.07  | 2777650 | 476.5 | 246.49 |
| 1997.1 | 137.11  | 2783313 | 478.0 | 265.82 |
| 1997.2 | 150.51  | 3377482 | 485.8 | 284.50 |
| 1997.3 | 155.87  | 3861088 | 497.8 | 308.95 |
| 1997.4 | 158.04  | 4917782 | 504.3 | 340.32 |
| 1998.1 | 150.43  | 4566350 | 523.5 | 364.84 |
| 1998.2 | 153.55  | 4663692 | 547.3 | 396.70 |
| 1998.3 | 147.89  | 4314370 | 576.8 | 420.90 |
| 1998.4 | 150.69  | 5149172 | 564.5 | 446.36 |
| 1999.1 | 137.63  | 4630083 | 583.5 | 468.70 |
| 1999.2 | 142.19  | 4824230 | 606.0 | 489.20 |
| 1999.3 | 141.56  | 4758389 | 628.0 | 511.70 |
| 1999.4 | 144.51  | 6412637 | 648.3 | 533.40 |
| 2000.1 | 138.86  | 5788165 | 669.5 | 554.20 |
| 2000.2 | 146.09  | 5838165 | 682.0 | 572.90 |
| 2000.3 | 146.57  | 6133311 | 690.8 | 591.30 |
| 2000.4 | 152.68  | 8037221 | 699.8 | 609.30 |
| 2001.1 | 144.16  | 7516310 | 707.8 | 624.00 |
| 2001.2 | 149.87  | 7536509 | 718.8 | 643.80 |
| 2001.3 | 150.74  | 7523932 | 743.0 | 666.50 |
| 2001.4 | 158.70  | 9208993 | 763.0 | 685.10 |
ANNEX II

ESTIMATED SPECTRAL DENSITIES OF COMPOSITE SHORT-RUN DYNAMICS OF MONEY DEMAND UNDER ALTERNATIVE MODELING APPROACHES
Spectral density

- MERGED INF
- MERGED CPI
- MERGED NER
ANNEX III

ESTIMATED SPECTRAL DENSITIES OF MONEY DEMAND AT CYCLICAL AND SEASONAL FREQUENCIES IN THE STRUCTURAL TIME SERIES APPROACH

[Graph showing spectral densities for different cycles and variables.]
ANNEX IV

ESTIMATED SPECTRAL DENSITIES OF MONEY DEMAND AT CYCLICAL AND SEASONAL FREQUENCIES IN THE COMBINED APPROACH
A structural time series model with full seasonal cointegration (i.e. cointegration at all frequencies), taking logs of real M1, real GDP and the rate of nominal currency depreciation as endogenous variables, has been specified. Although similar specifications for alternative measures of opportunity cost and rates of return were tried, the statistical results that were obtained were not very satisfactory.\(^{26}\)

In the preferred model, seasonal components \(\xi_j\) are stochastic and a common factor restriction is imposed.\(^{27}\) The general representation of a trigonometric stochastic seasonal component (when \(s\) is even), is given by

\[
\xi_j = \sum_{j=1}^{s/2} \xi_{j,t} \quad t = 1,\ldots,T
\]

where the \(\xi_{j,t}\) have the following representation

\[
\begin{bmatrix} \xi_{j,t} \\ \xi_{j,t}^* \end{bmatrix} = \phi_j \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \begin{bmatrix} \xi_{j,t-1} \\ \xi_{j,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{j,t} \\ \omega_{j,t}^* \end{bmatrix} \quad j = 1,\ldots,s/2
\]

where the innovations \(\omega_{j,t}\) and \(\omega_{j,t}^*\) have zero mean and (common) finite variance, and are mutually uncorrelated; \(\lambda_j = \frac{2\pi j}{s}\) is a frequency expressed in radians and \(\phi_j\) is a damping factor.\(^{28}\)

\(^{26}\) The results obtained at cyclical and seasonal frequencies are virtually identical to the results presented in this Annex, when following a merged approach with seasonal cointegration.

\(^{27}\) Please refer to Harvey (1989) for a further discussion of common trigonometric (stochastic) seasonality in structural time series models. We note that it is also possible to model seasonal cointegration using seasonal error-correction models. However, the autoregressive framework of the standard approach does not facilitate the specification of models with multiple (possibly common) stochastic cycles as well as stochastic seasonality.

\(^{28}\) In this paper, we let \(\phi_j = 1\) at \(j = 1,2\). It follows that the seasonal component is non-stationary at both frequencies (fundamental and harmonic). Moreover, by imposing the restriction that the seasonal
When \( s \) is even, the component at \( s/2 \) collapses to

\[
\xi_{j,t} = \xi_{j,t-1} \cos \lambda_j + \omega_{j,t}
\]

In this case, very strong convergence was achieved in 51 iterations. The value of the log-likelihood is 232.60. Higher and lower frequency cycles have been estimated, with periods 1.54 and 2.91 years. Estimation results for all models are satisfactory, in terms of the normality and heteroskedasticity of the residuals. There is nevertheless some statistical evidence of residual (higher-order) autocorrelation in the model, except in the case of the equation representing the dynamics of log real GDP. Goodness of fit measures are also satisfactory.

An interesting feature of this specification is that the estimated variance of the slope disturbances is zero (thus the signal-to-noise ratio of the slope component of the stochastic trend is nil)—this contrasts with all specifications with fixed seasonality using the structural time series approach, where the estimated slope disturbances have non-zero variance. It is as if the seasonal cointegration specification has “absorbed” the instability of the rates of drift of the stochastic trend process common to all endogenous variables in money demand.

This characteristic may also provide an explanation, at least on a tentative basis, of why we persistently found an inconsistency between the results of the conventional unit root tests that we have performed, and what the signal-to-noise ratios of the estimated slope processes are saying when following the structural time series approach. Whereas conventional unit root tests uniformly indicated that variables are I(1), the results obtained from structural time series modeling had consistently signaled a higher order of integration of the endogenous variables in money demand. Thus, the presence of non-stationary seasonal components may initially have confounded the statistical results.

component is common (up to sign and scaling factors) to all endogenous variables in the system, we are thus specifying a model with full seasonal cointegration at all frequencies. Please note that fixed seasonality is a special case, which occurs when \( \text{Var}(\omega_{j,t}) = \text{Var}(\omega^*_{j,t}) = 0 \).
It is found that the estimated long-run elasticities of money demand are somewhat sensitive to the change in the specification of the seasonal component from fixed to stochastic seasonality. With fixed seasonality and no commonality restrictions, the elasticity with respect to real GDP was estimated to be 0.68; however, if the seasonal cointegration specification is selected, the estimated elasticity falls to 0.55. However, it is important to note that an elasticity of 0.55 is about the mid-point of the elasticities with respect to GDP estimated using alternative specifications and modeling approaches. Models with fixed seasonality following the structural time series approach tended to yield higher estimates of the elasticity with respect to GDP; by contrast, conventional VECM models yielded lower estimates of this elasticity (in two of three cases).

| Long-Run Responsiveness of M1-Demand |
|--------------------------------------|
| Depreciation as Opportunity Cost Variable |

| Approach                  | Elasticity | Elasticity |
|---------------------------|------------|------------|
| Real GDP                  | 0.68       | -0.43      |
| Depreciation              | 0.55       | -0.52      |

By contrast, the magnitude of the elasticity with respect to the rate of depreciation increases from –0.43 to –0.52, once the seasonal cointegration specification is selected. This is also higher than the elasticity estimated using the VECM approach (–0.26).

Further analysis of the estimated trends suggests that most of the impact of the switch from fixed seasonality to a seasonal cointegration specification, was on the smoothed estimates of the stochastic trend component of the log of the rate of depreciation. As can be seen in the charts below, the smoothed trends of log money demand and real GDP are essentially unchanged; by contrast, the smoothed trend component of log depreciation rate was much more visibly changed. This provides a plausible reason as to why the adoption of the seasonal cointegration specification leads to a non-trivial change in the estimates of the long-run elasticities of money demand.
Otherwise, the salient features of short-run dynamics at the cyclical frequency are not greatly affected by the change in the specification of the seasonal component. In particular, the lower frequency cycle of money demand remains virtually unchanged after the seasonal cointegration specification is adopted (see Figure “M1-Demand and Seasonal Cointegration: Lower Frequency Cycle”). This highlights the robustness of the lower frequency cycle. The higher frequency cycle of money demand is nevertheless more sensitive to the change in seasonal specification, although its most salient features do remain unchanged in broad terms (see Figure “M1-Demand and Seasonal Cointegration: Higher Frequency Cycle”).
For money demand, the composite short-run dynamics at the cyclical frequency implied by the structural time series model with seasonal cointegration, are otherwise almost identical to the structural time series model with fixed seasonality (key differences being attributed to the very slight changes in the estimated higher-frequency cycle).

The lower frequency cyclical dynamics for both real GDP and the depreciation rate are also robust to the change in specification to seasonal cointegration. A similar statement can be made about the higher frequency cyclical dynamics associated with the rate of depreciation. By contrast, the higher frequency cyclical dynamics associated with real GDP are much more sensitive to the switch to the alternative specification (please see the Figures below).

We turn now to the analysis of the seasonal frequencies. The estimated seasonal pattern of money demand is virtually identical under both the fixed and seasonal cointegration specifications (see Figure “M1-Demand and Seasonal Cointegration: Seasonal Frequency”). Similarly, the seasonal dynamics of log real GDP are not visibly affected by the change in specification (see Figure “Log Real GDP and Seasonal Cointegration: Seasonal Frequency”). By contrast, the seasonal dynamics of the log of the depreciation rate are now visibly evolving over time (please refer to Figure “Log Depreciation and Seasonal Cointegration: Seasonal Frequency”).
It is important to note that seasonal cointegration does not necessarily imply that seasonal patterns have to be identical, only that the disturbances driving changes in seasonal patterns have a common source. The estimated seasonal components for money demand, real GDP and depreciation rate are cointegrated (by construction of the preferred model), but an evolution in the composite seasonal pattern is visible to the naked eye only in the case of the depreciation rate—the cumulative impact of common disturbances on seasonal patterns is simply less important in the case of the other two variables. This is consistent with the finding that the variance of the disturbances of the seasonal component for log depreciation is several times the variance of the disturbances of the
other seasonal components (and the q-ratio of the seasonal component of log depreciation is about twice the q-ratios, or signal-to-noise ratios, of the other seasonal components).

That seasonal components are evolving over time for money demand, real GDP and currency depreciation, becomes visibly clear once seasonal effects are graphed for each series. As the graphs below show, in the seasonal cointegration specification all seasonal effects have been changing over time. With a fixed seasonal specification, seasonal effects would have been constants.
It is especially noteworthy that changes in individual seasonal effects of money demand are positively associated with changes in the corresponding seasonal effect of real GDP. By contrast, changes in individual seasonal effects of money demand are negatively associated with changes in individual seasonal effects of nominal currency depreciation.

Thus, although the composite seasonal patterns appear to be distinctively different for money demand, real GDP and depreciation rate, the evolution of the individual seasonal effects of each variable is in fact closely related to the evolution of the corresponding seasonal effects of the other two variables. Note that at any point in time, the seasonal effect of a given variable is itself the sum of two stochastically evolving elements (at fundamental and harmonic frequencies). In addition, it is important to remember that the composite seasonal effects for each variable are constructed such that their sum over four consecutive periods is equal to zero.

Notice also that the seasonal effects have experienced sharp changes after 1996, with the (positive) fourth quarter seasonal effect becoming more marked for both money demand and real GDP, whereas it first became negligible and then turned slightly negative for nominal currency depreciation. The first quarter effect for money demand appears to have fallen sharply since 1996; for real GDP, it has become increasingly
negative over the same period, although the fall has been proportionately mild; finally, the first quarter effect for currency depreciation has remained negative, but its magnitude has fallen sharply since 1996. The second quarter effect for money demand has remained negative since 1996, but its magnitude has fallen somewhat; for real GDP, it appears that the second quarter effect turned from negative to positive; finally, for currency depreciation, the second quarter effect has remained positive but with a sharp fall in magnitude since 1996. Finally, the third quarter effect has remained negative but with a relatively mild tendency towards an increase in magnitude in the case of money demand; the third quarter effect in the case of real GDP has been consistently positive but with a tendency towards a decrease in its magnitude; in the case of currency depreciation, the third quarter effect has been negative, with its magnitude falling since 1996.

The structural time series model with cointegrating seasonality comfortably satisfies CUSUM tests for the stability of the means of each equation, at a 10 percent level of significance (please see below). In the following pages, we also present (a) the estimated spectral densities of the (common) lower and higher frequency cycles of each series; and (b) the estimated probability densities of the irregular components, plotted against suitably parameterized theoretical normal distributions.
Although the model with seasonal cointegration has statistical properties that are generally satisfactory, it does not necessarily represent a uniform improvement (in purely statistical terms) over the model with fixed seasonality. In fact, in terms of goodness of fit measures, normality of the residuals, and the AIC and BIC criteria, the model performs “better” with fixed seasonality than with seasonal cointegration. By contrast, in terms of residual heteroskedasticity and autocorrelation, the model with seasonal cointegration performs “better” than a model with fixed seasonality. In view of these results alone and following a principle of parsimony in model selection, we would be somewhat reluctant to dismiss the model with fixed seasonality in favor of one with seasonal cointegration.  

| Statistic                                         | Log Real M1 | Log Real GDP | Log of Depr. |
|--------------------------------------------------|-------------|--------------|--------------|
| Normality (Bowman-Shenton): $n \sim \chi^2$      | 0.45        | 2.10         | 1.78         |
| Skewness: $s \sim \chi^2$                        | 0.43        | 1.08         | 0.81         |
| Kurtosis: $k \sim \chi^2$                        | 0.02        | 1.02         | 0.96         |
| Heteroskedasticity: $H(10) \sim F_{10,10}$      | 0.55        | 0.25         | 0.05         |
| Autocorrelation (up to 15th order) (Box-Ljung): $Q(15,6) \sim \chi^2$ | 13.70       | 12.37        | 12.68        |
| Autocorrelation (first order)                     | 2.12        | 1.69         | 1.88         |
| Durbin-Watson: $DW \sim N(2, \frac{4}{T})$      |             |              |              |
| Goodness of Fit (w.r.t. seasonal mean)            | 0.59        | 0.09         | 0.60         |
| Goodness of Fit (ordinary $R^2$)                  | 0.88        | 0.77         | 0.19         |
| Akaike Information Criterion                     | -3.62       | -5.16        | -1.67        |
| Bayes Information Criterion                      | -1.75       | -3.29        | 0.19         |

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29 It is worth stressing that with inflation as measure of opportunity cost, specifications with seasonal cointegration did not display statistical properties that were at all satisfactory. What appear to be mixed
However, besides statistical tests we also have the additional prior information that (a) the empirical models with fixed seasonality that have been estimated are associated with slope innovations that have non-zero estimated variances, i.e. with unstable rates of drift; (b) having slope innovations with non-zero variances is inconsistent with the results of the conventional unit root tests that have been performed; and (c) conventional unit root tests are not likely to capture the presence of non-stationary seasonal components.

Moreover, we also note that (a) the model with seasonal cointegration is associated with stable rates of drift; (b) the behavior of the non-stationary seasonal effects is consistent with economic theory insofar as changes in the seasonal effects of money demand are positively associated with changes in seasonal effects of real GDP, but negatively associated with changes in the seasonal effect of the rate of depreciation; and (c) the non-stationary seasonal components that have been estimated are cointegrated, thus adding an additional (economically meaningful) restriction on the behavior of the system.

This information suggests that the model with seasonal cointegration does have the capacity to explain more features of the data than a model with fixed seasonality. We do not have a particularly interesting interpretation of unstable rates of drift, whereas observed co-movement of seasonal effects does have an economically meaningful interpretation. In view of this, we are led to prefer the seasonal cointegration specification.\(^{30}\)

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\(^{30}\) Complementary evidence in favor of the seasonal cointegration specification could in principle be obtained by directly testing for non-stationary seasonality and seasonal cointegration. Such tests, as well as tests for non-stationary components in series with known structural breaks, are under development within the framework of the structural time series approach. Busetti and Harvey (2000), Harvey (2001) and Busetti (2002) present recent advances in this area.