Subhalo abundance matching using progenitor mass

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ABSTRACT

We propose a novel subhalo abundance matching (SHAM) model that uses the virial mass of the main progenitor of each (sub)halo $M_{\text{prog}}$ as a proxy of the galaxy stellar mass $M_*$ at the time of observation. This $M_{\text{prog}}$ model predicts the two-point correlation functions depending on the choice of the epoch $z_{\text{prog}}$ at which $M_{\text{prog}}$ is measured. With $z_{\text{prog}}$ as a fitting parameter, we apply the $M_{\text{prog}}$ model to the latest observed angular correlation functions of galaxy samples at $z \approx 0.4$ with varying stellar mass thresholds from $M_{\ast,\text{lim}}/(h^{-2}M_\odot) = 10^{11}$ to $10^{8.6}$. The $M_{\text{prog}}$ model can reproduce the observations very well over the entire scale range of $0.1$–$10$ $h^{-1}$ Mpc. We find that, for the samples of $10^9 \leq M_{\ast,\text{lim}}/(h^{-2}M_\odot) \leq 10^{10.2}$, the correlation functions predicted by the widely-used $V_{\text{peak}}$ model lacks amplitudes at the scale of $\leq 1$ $h^{-1}$ Mpc, demonstrating the high capability of our $M_{\text{prog}}$ model to explain observed clustering measurements. The best-fit $z_{\text{prog}}$ parameter is highest ($z_{\text{prog}} \approx 5$) for intermediate mass galaxies at $M_* \approx 10^{9.7} h^{-2}M_\odot$, and becomes smaller towards $z_{\text{prog}} \approx 1$ for both lower- and higher-mass galaxies. We interpret these trends as reflecting the downsizing in the in-situ star formation in lower-mass galaxies and the larger contribution of ex-situ stellar mass growth in higher-mass galaxies.

Key words:

1 INTRODUCTION

The observed spatial distribution of galaxies contains a huge amount of physical information on cosmology, and galaxy formation and evolution. To extract such information accurately from modern wide-field galaxy surveys, it is necessary to correctly model the relations between the observed galaxies and their host dark matter structures, i.e., halos and their substructures, subhalos\textsuperscript{1}.

One of the empirical methods widely used for modeling the galaxy-subhalo connection is the subhalo abundance matching (SHAM) method (Kravtsov et al. 2004; see Wechsler & Tinker 2018 for a recent review). The SHAM method assumes a monotonic relation with some scatter between a galaxy observable and a simulated subhalo property. In the simplest form, known as the rank-ordering SHAM, for a sample of observed galaxies selected by a threshold in some physical property (e.g., stellar mass or luminosity), the corresponding subhalo sample is constructed by taking the threshold of the selected property so that the subhalo number density matches that of the galaxy sample. Unlike the halo occupation distribution (HOD) method (see Cooray & Sheth 2002, for a review), which is also a frequently used method, SHAM can incorporate subhalo distributions on small scales realized in cosmological N-body simulations.

In performing SHAM, one should use a subhalo property that is expected to strongly correlate with the target galaxy property. The choice of such a property would have a profound impact on the predicted galaxy statistics, including clustering measurements (e.g., Conroy et al. 2006; Behroozi et al. 2013c; Reddick et al. 2013; Moster et al. 2013; Chaves-Montero et al. 2016; Lehmann et al. 2017; Moster et al. 2018; Behroozi et al. 2019). Thus, comparing the SHAM predictions with observational results can validate the chosen subhalo property. Reddick et al. (2013) used various subhalo properties to study how the predictions for galaxy statistics at $z \approx 0$ are affected. They showed that using the peak maximum circular velocity of each subhalo in its lifetime, $V_{\text{peak}}$, can reproduce the observed projected correlation functions (PCFs) of the galaxy samples constructed with a threshold of stellar mass and luminosity. Hereafter, we refer to this rank-ordering approach as the $V_{\text{peak}}$ model.

However, the validity of using $V_{\text{peak}}$ as a proxy for stellar mass or luminosity is still unclear, despite its successful reproduction of observed two-point correlation functions (2PCFs) of some types of galaxies (Nuza et al. 2013; Rodríguez-Torres et al. 2016; Saito et al. 2016; Alam et al. 2017; Dong-Páez et al. 2022). Since satellite subhalos reach $V_{\text{peak}}$ before getting accreted onto host halos as shown by Behroozi et al. 2014, the physical relation between $V_{\text{peak}}$ and star formation activity in satellite galaxies is uncertain. Campbell et al. (2018) argued that stellar mass growth implied by the $V_{\text{peak}}$ model is too early. The implied early growth does not agree with the mass-based SHAM models tuned to match the observed evolution of stellar mass functions (Yang et al. 2012; Behroozi et al. 2013c; Moster et al. 2013). This inconsistency is due to the earlier formation of the gravitational potential well of subhalos than mass (van den Bosch et al. 2014). Interestingly, the rank-ordering model using the

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\textsuperscript{1} For simplicity, unless explicitly noted, hereafter we shall refer to all halos and subhalos simply as subhalos.

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peak mass $M_{\text{peak}}$ predicts the growth history similar to the stellar mass function-tuned models. Furthermore, Leauthaud et al. (2017) reported that SHAM and HOD models, including the $V_{\text{peak}}$ model (Reid et al. 2014; Rodríguez-Torres et al. 2016; Saito et al. 2016; Alam et al. 2017), can reproduce the observed PCF of the CMASS galaxies (Ahn et al. 2014) but overpredict the lensing profile. There are attempts to reconcile the inconsistency between clustering and lensing by incorporating assembly bias (Wechsler et al. 2006; Gao & White 2007) in the HOD modeling (Yuan et al. 2020) or re-estimating the lensing measurements (Lange et al. 2021). However, the situation has not been settled yet. Hearin et al. (2013) also showed that the SHAM models including the $V_{\text{peak}}$ model exhibit tensions with the observed luminosity functions of field galaxies and group member galaxies.

Campbell et al. (2018) also studied galaxy clustering predicted by the mass-based SHAM models including the rank-ordering model using the peak mass $M_{\text{peak}}$, and more detailed models for the stellar mass-halo mass relation (Yang et al. 2012; Behroozi et al. 2013c; Moster et al. 2013). They showed that these models do not reproduce the observed galaxy clustering without introducing assembly bias as the secondary subhalo property (Masaki et al. 2013b; Hearin & Watson 2013) or invoking the usage of substantial ‘orphans’, i.e., galaxies hosted by subhalos that fall below the numerical resolution limit.

In this paper, we propose a novel rank-ordering mass-based SHAM model. Our model uses the virial mass of the progenitor at a redshift $z_{\text{prog}}$ of each subhalo, $M_{\text{prog}}$, as a proxy of the galaxy stellar mass at the time of observation. We refer to this model as the $M_{\text{prog}}$ model. We show that our $M_{\text{prog}}$ model with a certain choice of $z_{\text{prog}}$ can reproduce the observed 2PCFs with the same or higher amplitudes than the $V_{\text{peak}}$ model, without the need to invoke orphan galaxies or any secondary subhalo properties. We apply the $M_{\text{prog}}$ and $V_{\text{peak}}$ models to the latest observed angular correlation functions (ACFs) of the galaxy samples with several stellar mass thresholds at $z \approx 0.4$ obtained from the Subaru Hyper-Suprime Cam (HSC) survey (Aihara et al. 2018; Ishikawa et al. 2020). For the samples with the thresholds of $9 \leq \log_{10}[M_{\ast, \text{lim}}/(h^{-2} M_\odot)] \leq 10.2$, we find that the predictions of the $M_{\text{prog}}$ model agree better with the observation than the $V_{\text{peak}}$ model at the scales of $< 1 h^{-1}$ Mpc. We also find that the best-fit $z_{\text{prog}}$, the primary parameter of the $M_{\text{prog}}$ model and the characteristic epoch for stellar mass growth, is qualitatively consistent with the two-phase scenario of stellar mass growth in galaxies, i.e., in-situ star formation and ex-situ star accretion (Oser et al. 2010).

This paper is structured as follows. In Sec. 2, we describe the simulation used in this paper. Then we introduce the $M_{\text{prog}}$ model, study how this model predicts the 2PCFs, and discuss how to fit the predictions to the observed ACFs with the stellar mass thresholds. Sec. 3 presents the fitting results on the observed ACFs by both $M_{\text{prog}}$ and $V_{\text{peak}}$ models, the constrained parameters, the inferred satellite fraction and halo occupation numbers. We summarize our results and conclude in Sec. 4.

## 2 METHODS

We first present the details of the simulations used in this work, then describe the SHAM model that uses the virial mass of the progenitor at redshift $z_{\text{prog}}$ of each subhalo, $M_{\text{prog}}$. We study the 2PCFs predicted by the $M_{\text{prog}}$ model and interpret their $z_{\text{prog}}$-dependence. We conclude this section by discussing how we fit the model predictions to observed clustering.

### 2.1 the mini-Uchuu simulations

We use the publicly available halo/subhalo catalogs produced from the high-resolution mini-Uchuu simulations (Ishiyama et al. 2021) carried out with the GReM code (Ishiyama et al. 2009). The simulation adopts the Planck 2018 Λ-cold dark matter (LCDM) cosmological parameters as $\Omega_m = 0.3089$, $\Omega_{\Lambda} = 0.6911$, $h = 0.6774$, $\sigma_8 = 0.8159$, $\Omega_b = 0.0486$ and $n_s = 0.9667$, where $h$ is the dimensionless Hubble constant defined by $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$. Other aspects of the simulation are summarized as follows: the number of simulation particles: $N_{\text{part}} = 2560^3$, the simulation box length: $L_{\text{box}} = 400$ $h^{-1}$ Mpc, the softening length: $\epsilon = 4.27$ $h^{-1}$ kpc, and the mass of a simulation particle: $m_{\text{part}} = 3.27 \times 10^8 h^{-1}$ M$_\odot$. The halos and subhalos are identified by the ROCKSTAR finder (Behroozi et al. 2013a). We utilize the NBODYkit package (Hand et al. 2018) to handle the halo/subhalo catalogs.

### 2.2 Our SHAM model

#### 2.2.1 Motivation

It is known that the downsizing scenario that higher-mass galaxies tend to assemble their stellar mass and cease star formation at earlier epochs, while lower-mass ones tend to continue star formation to later times (Cowie et al. 1996; Guzmán et al. 1997; Brinchmann & Ellis 2000; Kodama et al. 2004; Bell et al. 2005; Jimenez et al. 2005; Juneau et al. 2005; Bundy et al. 2006; Neistein et al. 2006). In this picture, it is naturally expected that the stellar mass of massive galaxies is better correlated with their progenitors’ host subhalo mass at earlier epochs and vice versa. In other words, we expect that higher-mass galaxies reside in subhalos that were sufficiently massive at higher-$z$.

The above downsizing-based expectation would be the case for galaxies whose stellar mass growth is dominated by the “in-situ” star formation. For most massive galaxies, we also need to account for stellar mass growth via galaxy mergers, i.e., the “ex-situ” star accretion (Oser et al. 2010; Lackner et al. 2012; Pillepich et al. 2015; Rodríguez-Gomez et al. 2016; Pillepich et al. 2018b; Davison et al. 2020; Cannarozzo et al. 2022). The process of galaxy mergers increases both the host subhalo and stellar masses. Cosmological simulations of galaxy formation suggest that the fraction of the ex-situ stars in more massive galaxies increases rapidly and can be dominant over or comparable to those formed in-situ at later epochs (Rodríguez-Gomez et al. 2016). We thus expect that the observed stellar mass of most massive galaxies is represented better by the subhalo mass at epochs nearer the time of observation, instead of some earlier epochs as expected from the downsizing scenarios.

Motivated by these expected correlations between the observed stellar mass and the host subhalo mass, we propose a novel SHAM model. Our model uses the progenitor subhalo mass $M_{\text{prog}}$ at an epoch $z = z_{\text{prog}}$ as a proxy of the observed stellar mass, where $z_{\text{prog}}$ can vary as a function of the stellar mass.

As we shall see later in Sec. 2.3, the predicted 2PCFs depend on the choice of $z_{\text{prog}}$ non-trivially in amplitude and shape. Treating $z_{\text{prog}}$ as a free parameter in fitting to observed clustering measurements, the obtained best-fit $z_{\text{prog}}$ values should reflect the characteristic epoch of stellar mass growth as a function of the galaxy stellar mass.

As we discussed above, the best-fit $z_{\text{prog}}$ values are expected to be lower toward the higher and lower stellar mass ends and have a peak at the intermediate mass range. The lower-mass and higher-mass sides of the peak reflect downsizing in-situ star formation and ex-situ star accretion, respectively.
2.2.2 The implementation

We now describe the implementation of the \( M_{\text{prog}} \) model. Among the various definitions of the subhalo mass, we use the virial mass given by the Rockstar halo finder as the progenitor mass \( M_{\text{prog}} \). We fit the model predictions to observed galaxy clustering with two free parameters. The primary parameter is \( z_{\text{prog}} \), the redshift at which we evaluate the virial mass of the progenitor \( M_{\text{prog}} \).

The second parameter is to control the scatter between \( M_{\ast} \) and \( M_{\text{prog}}, \sigma_{M} \). Although we assume a tight correlation between the two, there could be a non-negligible scatter in the relation. To incorporate such a scatter, we perturb \( M_{\text{prog}} \) by multiplying the logarithm of \( M_{\text{prog}} \) with a random number drawn from a Gaussian distribution \( N \) with the mean of 0 and the standard deviation of \( \sigma_{M} \) (Rodríguez-Torres et al. 2016; Yu et al. 2022) as

\[
\log_{10} M_{\text{pert}} = [1 + N(0, \sigma_{M})] \log_{10} M_{\text{prog}}.
\]

Such a perturbation leads to more low mass subhalos in the resultant subhalo samples for larger \( \sigma_{M} \) because they are more abundant than high mass ones. Hence, a larger \( \sigma_{M} \) suppresses the overall amplitude of 2PCFs.

We construct the mass accretion histories (MAHs; see e.g., Wechsler et al. 2002; McBride et al. 2009) of the most massive progenitors (MMPs) to evaluate \( M_{\text{prog}} \) using the halo merger trees obtained with the ConsistentTrees code (Behroozi et al. 2013b). The MAHs of MMPs can be seen as the main trunk of each merger tree. The available number of the model parameter \( z_{\text{prog}} \) is limited by the number of outputs of the mini-Uchuu simulations, i.e., 50 outputs from \( z = 13.93 \) to \( z = 0 \).

The \( M_{\text{prog}} \) model is similar to the SHAM model for luminous red galaxies (LRGs) developed by Masaki et al. (2013a). Following observational suggestions on the growth of LRGs, they assumed that the most massive distinct halos at \( z = 2 \) are the progenitors of LRGs, and identified their descendants at \( z \approx 0.3 \) as LRGs. They found that their model reproduces observed clustering and lensing profiles qualitatively well. Our \( M_{\text{prog}} \) model differs in the inclusion of satellite subhalos at \( z_{\text{prog}} \) and has higher flexibility as \( z_{\text{prog}} \) is a model parameter.

2.3 The impact of \( z_{\text{prog}} \) on 2PCFs

We study the impacts of \( z_{\text{prog}} \) on predicting galaxy clustering. We use the CorrFunc package (Sinha & Garrison 2019; Sinha & Garrison 2020) to measure the real-space 2PCF \( \xi \) as a function of the comoving distance \( r \). For this, we construct subhalo samples at \( z = 0 \) by taking the threshold \( M_{\text{prog}} \) value with several \( z_{\text{prog}} \) so that the number density equals to \( n_{\text{gal}} = 10^{-2} \) \( h^{-1} \) Mpc\(^{-3}\). We estimate the error bars for 2PCFs by the ‘omit-one’ jackknife resampling using the 27 subvolumes. We compare the results with those from the \( V_{\text{peak}} \) model, which are taken as the fiducial. Below we denote the 2PCFs from the \( M_{\text{prog}} \) and \( V_{\text{peak}} \) models as \( \xi_{M} \) and \( \xi_{V} \), respectively. For simplicity, we do not perturb \( M_{\text{prog}} \) and \( V_{\text{peak}} \).

Fig. 1 shows the impact of \( z_{\text{prog}} \) on \( \xi_{M} \) at \( z = 0 \). For ease of comparison, we show \( \xi_{V} \) scaled by \( \xi_{M} \). The horizontal thin solid line is unity, i.e., \( \xi_{V} \) with the scaled error bars. For understanding the impact of \( z_{\text{prog}} \), we show the halo occupation numbers, the average number of galaxies in a halo, \( \langle N_{\text{gal}} \rangle \) at \( z = 0 \) with varying \( z_{\text{prog}} \) as a function of the virial mass of host central subhalos \( M_{\text{vir}} \) in Fig. 2. We found a very similar \( z_{\text{prog}} \)-dependence for two other samples constructed with lower number densities of \( n_{\text{gal}} = 10^{-3} \) and \( 10^{-4} \) \( h^{3} \) Mpc\(^{-3}\) at \( z = 0 \), as well as for the samples with the same three \( n_{\text{gal}} \) values at \( z = 0.5 \) and 1.

It is naively expected that taking a higher \( z_{\text{prog}} \) amplifies \( \xi_{M} \) because most massive subhalos at higher-\( z \) formed in more biased regions. However, we observe an ‘up-and-down’ trend in the amplitudes of \( \xi_{M} \) for decreasing redshift. That is, it rises from \( z_{\text{prog}} \approx 8 \) to \( z_{\text{prog}} \approx 3-4 \) and then turns downward to \( z_{\text{prog}} \approx 0 \). The up trend from \( z = 7.77 \) to \( z \approx 3-4 \) is mainly due to the larger variations in the future MAHs that higher-redshift subhalos will undergo: the descendants of the most massive subhalos at very high-\( z \) can be not only well-grown high-mass subhalos but also less-grown low-mass ones at low redshifts. This is clearly seen in Fig. 2 with \( \langle N_{\text{gal}} \rangle \) of \( z_{\text{prog}} \approx 7.77 \). As a consequence of more low-mass subhalos in the sample, the amplitude of \( \xi_{M} \) is suppressed. This particularly decreases the one-halo term at \( r \lesssim 1 \) \( h^{-1} \) Mpc of \( \xi_{M} \) as Fig. 1 clearly shows the up trend is more prominent at smaller-\( r \). This is because the number of central-satellite pairs in a halo is decreased.

We next discuss the down trend. Fig. 1 shows that the overall amplitude of \( \xi_{M} \) peaks at \( z_{\text{prog}} \approx 3-4 \). \( \xi_{M} \) with a lower-\( z_{\text{prog}} \) is...
Table 1. Summary of the incompleteness-corrected number density of each stellar mass threshold sample at $0.30 \leq z < 0.55$ in Ishikawa et al. (2020). The threshold mass is in units of $\log_{10} \left[ M_{\star, \lim} / (h^{-2} M_\odot) \right]$.

| threshold mass | $10^3 n_{\text{gal}}$ [h$^{-3}$ Mpc$^{-3}$] |
|---------------|----------------------------------|
| 11.0          | 0.175                            |
| 10.8          | 0.596                            |
| 10.6          | 1.458                            |
| 10.4          | 2.689                            |
| 10.2          | 4.838                            |
| 10.0          | 7.799                            |
| 9.8           | 10.42                            |
| 9.6           | 12.68                            |
| 9.4           | 16.14                            |
| 9.2           | 19.50                            |
| 9.0           | 23.36                            |
| 8.8           | 27.03                            |
| 8.6           | 30.74                            |

more suppressed, especially in the one-halo term range. This is due to the mass stripping of satellite subhalos during accretion on their host halos (e.g., Reddick et al. 2013). Rank ordering using subhalo mass near the observation time loses satellite subhalos in the resultant sample. This is reflected in Fig. 2 which clearly shows that $\langle \dot{N}_{\text{gal}} \rangle$ with lower $z_{\text{prog}}$ are more suppressed at the high-mass range due to the loss of satellites. We found that the $z_{\text{prog}}$ values at the transition of up and down trend of $\xi_{\text{M}}$ for the sample with $n_{\text{gal}} = 10^{-3}$ h$^{-3}$ Mpc$^{-3}$ is $0.6$ and lower than the sample with $n_{\text{gal}} = 10^{-2}$ h$^{-3}$ Mpc$^{-3}$. This is because the satellite fraction of the low-$n_{\text{gal}}$ threshold samples is intrinsically low, and then the impact coming from satellite subhalos becomes relatively small. Hence the overall amplitude of $\xi_{\text{M}}$ can keep high for low $z_{\text{prog}}$.

It is known that the $V_{\text{peak}}$ model reproduces observed galaxy clustering well. As shown in Fig. 1, the 2PCFs predicted by the $M_{\text{prog}}$ model are similar to or even more amplified than those by the $V_{\text{peak}}$ model. It is also known that introducing a scatter between the subhalo and the galaxy properties decreases clustering amplitudes. Therefore, by varying $z_{\text{prog}}$ and $\sigma_{M}$, the $M_{\text{prog}}$ model is expected to reproduce the observed clustering. In Appendix A, we discuss the best matching $\xi_{M}$ and $\xi_{V}$ for the subhalo samples constructed with different number density threshold ($n_{\text{gal}} = 10^{-2}$, $10^{-3}$ and $10^{-4}$ h$^{-3}$ Mpc$^{-3}$) at $z = 0$, 0.5 and 1.

2.4 Fitting to observed clustering

In this paper, we fit the predictions of both the $M_{\text{prog}}$ and $V_{\text{peak}}$ models to the observed ACFs of the photo-$z$ galaxies at $0.30 \leq z < 0.55$ from the Subaru HSC survey measured by Ishikawa et al. (2020). They reported the ACFs for the 13 stellar mass threshold samples with the lower limits from $\log_{10} \left[ M_{\star, \lim} / (h^{-2} M_\odot) \right] = 11$ to 8.6 with a bin size of 0.2 dex. Their observations are attractive because they can measure the ACFs in a stellar mass bin as fine as 0.2 dex and down to a small scale of $\approx 4 \times 10^{-4}$ deg which corresponds to $\approx 8$ h$^{-1}$ kpc at $z = 0.43$, using data over a large area of 178 deg$^2$ of the HSC survey.

Table 1 summarizes the number density of each stellar mass threshold sample for which Ishikawa et al. (2020) measured the ACFs. Note that these values are corrected for incompleteness. For the implementation of the two SHAM models, we use the halo/subhalo catalog from the mini-Uchuu simulations at $z = 0.43$, which is close to the peak of the redshift distributions of the observed galaxies.

In fitting the predictions of the $M_{\text{prog}}$ model to observed clustering, as we stated, the two parameters $z_{\text{prog}}$ and $\sigma_{M}$ are the free parameters. Specifically, we take 40 values for $z_{\text{prog}}$ from 0.49 to 13.93, and 20 values for $\sigma_{M}$ from 0 to 0.19 with a linear spacing of 0.01. The virial mass at high-$z$ often becomes zero due to the limitation of particle resolution. Large $\sigma_{M}$ values include subhalos with $M_{\text{prog}} = 0$ in resultant samples meaning that subhalos are selected randomly in part. Hence, we perturb $M_{\text{prog}}$ within only 0.2 dex to avoid this random selection. As discussed in Sec. 2.2.1, we allow for a stellar mass dependence of $z_{\text{prog}}$. As well as $z_{\text{prog}}$, we treat $\sigma_{M}$ as a function of the galaxy stellar mass.

We also use the $V_{\text{peak}}$ model for comparison. We perturb $V_{\text{peak}}$ by the same method as in the $M_{\text{prog}}$ model to account for the scatter between $V_{\text{peak}}$ and $M_{\star}$ as

$$\log_{10} V_{\text{peak}} = \left[ 1 + N(0, \sigma_{V}) \right] \log_{10} V_{\text{peak}}.$$  

where $\sigma_{V}$ is the standard deviation of the Gaussian distribution $N$ with the zero-mean. $\sigma_{V}$ is the only free parameter and taken to be from 0 to 0.49 with the linear spacing of 0.01. We also treat this parameter as a function of the galaxy stellar mass.

Allowing all free parameters in both models to depend on stellar mass, we construct the subhalo samples corresponding to the stellar mass threshold galaxy samples in a self-consistent manner as follows. First, we assume that the free parameters are constant for the most massive sample, i.e., the sample of $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \geq 11$ in this paper. For this sample, we simply perform the rank-ordering SHAM using the perturbed $M_{\text{prog}}$ or $V_{\text{peak}}$. We measure the ACFs for each parameter set and find the best-fit set. Then we temporally exclude the subhalos which are assigned with the sample galaxies by the best-fit parameter set from the whole subhalo catalog. Next, we abundance-match using the rest of the subhalos for the galaxy sample of $10.8 \leq \log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] < 11$ assuming that the free parameters are constant in this narrow range of the stellar mass. The number density of subhalos in this bin given by the difference between the two samples of $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \geq 11$ and $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \leq 10.8$. Combining the subhalos assigned with galaxies of $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \geq 11$ by the best-fit parameter set and the subhalos assigned with galaxies of $10.8 \leq \log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] < 11$ by each parameter set, we obtain the subhalo sample for the galaxies with $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \geq 10.8$. By comparing the predicted ACFs with the observation for the galaxies with $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \geq 10.8$, we obtain the best-fit parameter set for the galaxies of $10.8 \leq \log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] < 11$. We repeat this procedure every 0.2 dex bin until reaching the galaxy sample with $\log_{10} \left[ M_{\star} / (h^{-2} M_\odot) \right] \geq 8.6$.

To compute the model prediction of ACFs of $\omega(\theta)$, we first measure the real-space 2PCFs $\xi$ as a function of the comoving distance $r$ for the subhalo samples constructed with the $M_{\text{prog}}$ and $V_{\text{peak}}$ models. Then we project $\xi$ along the line of sight to obtain $\omega$ using the Limber approximation (Limber 1953; Simon 2007) as

$$\omega(\theta) = 2 \int_0^\infty dr \left[ \frac{p^2(z)}{\frac{d}{dz}} \right] \int_0^\infty d\xi \left[ r = \sqrt{\xi(z)^2 + \chi^2(z)^2} \right]$$ \hspace{1cm} (3)

$$\omega(\theta) = 2 \int_0^\infty dr \left[ \frac{p^2(z)}{\frac{d}{dz}} \right] \int_0^\infty d\xi \left[ r = \sqrt{\xi(z)^2 + \chi^2(z)^2} \right].$$ \hspace{1cm} (4)

where $p(z)$ is the normalized redshift distribution of the observed galaxies, $\chi(z)$ is the comoving distance to the redshift $z$ and $u$ is the comoving distance along the line-of-sight. In doing so, the evolution of $\xi$ over the redshift range $0.30 \leq z < 0.55$ is ignored.

2 The observed $p(z)$ in Ishikawa et al. (2020) is kindly provided by S. Ishikawa.
To find the best-fit parameters, we calculate the chi-square values as a function of the parameter set for each threshold sample defined as

$$\chi^2 = \sum_i \frac{(\omega_{\text{obs}}, i - \omega_{\text{model}}, i)^2}{\sigma_{\text{obs}}, i + \sigma_{\text{model}}, i}^2,$$

where \(i\) denotes the \(i\)-th bin of \(\theta\), \(\omega_{\text{obs}}\) is the observed ACFs, \(\omega_{\text{model}}\) is the ACFs predicted by the \(M_{\text{prog}}\) or \(V_{\text{peak}}\) models, and \(\sigma_{\text{obs}}\) and \(\sigma_{\text{model}}\) are the \(1\sigma\) error for the \(i\)-th bin of the observation and the SHAM models, respectively. The \(1\sigma\) range of the best-fit parameters is given by \(\Delta \chi^2 = \chi^2 - \chi_{\text{min}}^2\) where \(\chi_{\text{min}}^2\) is the least chi-square value, \(\Delta \chi^2 = 1\) for the \(V_{\text{peak}}\) model and \(\Delta \chi^2 = 2.3\) for the \(M_{\text{prog}}\) model (Press et al. 1986). We found that the slope of \(\omega_{\text{obs}}\) at \(\theta \leq 1 \times 10^{-3}\) deg can be approximated to \(\omega_{\text{obs}} \approx \theta^{-1}\) for almost all samples. This slope is steeper than that at larger angles and thus implies that baryonic effects induce stronger clustering. Hence, for the calculation of the chi-square values, we only use the bins with \(\theta > 1 \times 10^{-3}\) deg. Taking \(z = 0.43\) as a representative redshift of the observed galaxy sample, the angle corresponds to the comoving distance of \(\approx 20\ h^{-1}\) kpc.

3 RESULTS

Here we present the results of fitting to the observed ACFs and show that the \(M_{\text{prog}}\) model matches with the observations better than the \(V_{\text{peak}}\) model. We discuss the constrained free parameters in the two models. Finally, we study the inferred satellite fractions and halo occupation numbers.

3.1 The ACFs \(\omega(\theta)\) at \(z = 0.4\): the observation versus the SHAM models

Fig. 3 compares the observed ACFs and the best-fit predictions from the \(M_{\text{prog}}\) and \(V_{\text{peak}}\) models. For clarity, we show \(\theta \times \omega(\theta)\) as the vertical axis rather than \(\omega(\theta)\). The vertical gray thin lines are the angles corresponding to the comoving distances of 0.1, 1 and 10 \(h^{-1}\) Mpc at \(z = 0.43\). The blue data points with error bars are the observed ACFs measured by Ishikawa et al. (2020). This figure omits the ACFs for the sample with \(\log_{10}[M_{\star}/(h^{-2}M_{\odot})] \geq 11\) due to the large error bars. The black solid and the red dashed lines with the error bars show the ACFs with the least chi-square values of the \(M_{\text{prog}}\) and \(V_{\text{peak}}\) models, respectively. The ACFs given by the parameter sets within the \(1\sigma\) range are plotted by the light gray and the light pink lines for the \(M_{\text{prog}}\) and \(V_{\text{peak}}\) models, respectively.

For the most massive samples, i.e., the samples with the threshold mass of \(\log_{10}[M_{\star}/(h^{-2}M_{\odot})] = 10.8, 10.6\) and 10.4, we see that the best-fit ACFs of both models agree with the observations within the error bars at all scales. This is consistent with the fact that the \(V_{\text{peak}}\) model works well for clustering of the CMASS galaxies at \(z \approx 0.5\) (Nuza et al. 2013; Rodriguez-Torres et al. 2016; Saito et al. 2016). The situation is different for the samples with \(9 \leq \log_{10}[M_{\star}/(h^{-2}M_{\odot})] \leq 10.2\). The \(M_{\text{prog}}\) model predicts ACFs with higher amplitudes than the \(V_{\text{peak}}\) model over the entire angular range we consider. This is due to the higher satellite fractions of the \(M_{\text{prog}}\) model (see Sec. 3.3). The best-fit ACFs of both models are consistent with the observations within the error bars at \(> 1\ h^{-1}\) Mpc. We found that the \(M_{\text{prog}}\) model agrees with the observations within the error bars at 0.1–1 \(h^{-1}\) Mpc. On the other hand, the \(V_{\text{peak}}\) model typically underpredicts the observed amplitudes by 20% at the scales of \(< 1\ h^{-1}\) Mpc. At smaller scales, \(< 0.1\ h^{-1}\) Mpc, the \(M_{\text{prog}}\) model still works better than the \(V_{\text{peak}}\) model but underpredicts the observation typically by 15%. This implies that the \(M_{\text{prog}}\) model needs fine-tuning for more accurate modeling of galaxy-subhalo connections. It is unlikely due to the lack of resolution of the mini-Uchuu simulations because the ACFs of the two lowest mass samples, i.e., the samples with the lower limits of \(\log_{10}[M_{\star}/(h^{-2}M_{\odot})] = 8.8\) and 8.6, are reproduced well by both models.

3.2 The constrained parameters

The best-fit parameters \(\sigma_{\text{M}, \text{prog}}\) in the \(M_{\text{prog}}\) model and \(\sigma_{\text{V}}\) in the \(V_{\text{peak}}\) model for the stellar mass bin samples are summarized in Table 2 and plotted in Figs. 4 and 5 as functions of the stellar mass range.

The parameters of the \(M_{\text{prog}}\) model, \(\sigma_{\text{prog}}\) and \(\sigma_{\text{M}}\), are poorly constrained in our considered range for the three highest mass ranges, i.e., \(> 11, 10.8–11\) and 10.6–10.8 in units of \(\log_{10}[M_{\star}/(h^{-2}M_{\odot})]\). The large error bars of \(\omega(\theta)\) prevent us from breaking the degeneracy between the two parameters.

The scatter parameters, \(\sigma_{\text{M}}\) and \(\sigma_{\text{V}}\), are constrained weakly for the lowest mass range, \(8.6 \leq \log_{10}[M_{\star}/(h^{-2}M_{\odot})] < 8.8\), despite the high precision of the observed ACF. This is because the galaxies in the mass range are assigned in the lowest mass and the most abundant subhalos, and perturbing the stellar mass proxy is less effective in predicting galaxy clustering. For the other intermediate-mass ranges, the scatter parameters are close to zero. This is simply because the ACFs predicted by the \(M_{\text{prog}}\) and \(V_{\text{peak}}\) models lack amplitude compared to the observations at small scales. The only exception is the mass range of \(9.6 \leq \log_{10}[M_{\star}/(h^{-2}M_{\odot})] < 9.8\). The large error bar of \(\sigma_{\text{M}}\) for this range is caused by the ‘up-and-down’ dependence of clustering on \(\sigma_{\text{prog}}\) (see Sec. 2.3). The best fit \(\sigma_{\text{prog}}\) for this mass range is at the peak of the up-and-down trend of clustering. Hence the ACFs with similar amplitude and shape can be given by the parameter sets of \(\sigma_{\text{prog}}\) around the best-fit values and the varying \(\sigma_{\text{M}}\) values.

The constrained characteristic redshift of the \(M_{\text{prog}}\) model, \(z_{\text{prog}}\), displays an interesting feature. As shown in the lower panel of Fig. 4, \(z_{\text{prog}}\) as a function of the stellar mass range appears to peak at \(9.6 \leq \log_{10}[M_{\star}/(h^{-2}M_{\odot})] < 9.8\). As discussed in Sec. 2.2.1, this feature is expected and related to the two phases of stellar mass growth in galaxies. In the two-phase formation scenario, the best-fit \(z_{\text{prog}}\) would increase toward the high mass range of the in-situ star-
forming galaxies and decrease toward massive end galaxies in which the ex-situ star accretion is effective. In other words, the best-fit \( z_{\text{prog}} \) as a function of the galaxy stellar mass has a peak at the intermediate mass. This is indeed seen in the lower panel of Fig. 4. Also, it is suggested that the ex-situ star accretion is efficient for the galaxies with \( \log_{10}(M_*/(h^{-2}M_\odot)) \geq 9.8 \) at \( z = 0.4 \), and the in-situ star formation is dominant in the lower mass galaxies.

### 3.3 The inferred satellite fraction

Fig. 6 shows the satellite fraction \( f_{\text{sat}} \) for the threshold samples as a function of the threshold mass \( \log_{10}(M_{*,\lim}/(h^{-2}M_\odot)) \) inferred by the \( M_{\text{prog}} \) and \( V_{\text{peak}} \) models. For each threshold sample with the best-fit parameter set, \( f_{\text{sat}} \) is simply measured as

\[
 f_{\text{sat}} = \frac{\text{number of galaxies hosted by satellite subhalos}}{\text{total number of galaxies}}. 
\]

The error bars are the range given by the parameter sets within the 1\( \sigma \) error.

The overall shape of the satellite fraction differs between the \( M_{\text{prog}} \) and \( V_{\text{peak}} \) models. The \( V_{\text{peak}} \) model yields a monotonically decreasing form with increasing stellar mass. One would naively expect that the satellite fraction to be higher for lower mass galaxies, as the \( V_{\text{peak}} \) model predicts. However, in the \( M_{\text{prog}} \) model, \( f_{\text{sat}} \) as a function of the lower stellar mass limit has a single peak at \( \log_{10}(M_{*,\lim}/(h^{-2}M_\odot)) \approx 9.3 \).

The amplitude of galaxy clustering is strongly related to the satellite fraction. For the threshold samples with \( \log_{10}(M_{*,\lim}/(h^{-2}M_\odot)) \geq 10.4 \) and \( \log_{10}(M_{*,\lim}/(h^{-2}M_\odot)) = 8.6 \), the ACFs of the \( M_{\text{prog}} \) and \( V_{\text{peak}} \) models are consistent with each other (and the observations). The two models predict the satellite fractions very close to each other for these samples. For the other threshold samples with \( 8.8 \leq \log_{10}(M_{*,\lim}/(h^{-2}M_\odot)) \leq 10.2 \), the \( M_{\text{prog}} \) model gives the higher \( f_{\text{sat}} \) than the \( V_{\text{peak}} \) model as well as the amplitudes of the ACFs. This implies the higher amplitude of the correlation functions predicted by the \( M_{\text{prog}} \) model is due to the higher satellite fraction.

Fig. 7 is very similar to Fig. 6 but shows the inferred satellite fraction \( f_{\text{sat}} \) for the stellar mass bin samples as a function of the mass range. Compared to \( f_{\text{sat}} \) for the threshold samples, the peak of the \( M_{\text{prog}} \) model is shifted to a higher stellar mass, \( \log_{10}(M_*/(h^{-2}M_\odot)) \approx 10.7 \), because \( f_{\text{sat}} \) for the threshold samples is the cumulation of \( f_{\text{sat}} \) for the bin samples.

Knobel et al. (2013) used the spectroscopic galaxy sample at \( 0.1 < z < 0.8 \) from the zCOSMOS survey to evaluate the satellite fractions (see van den Bosch et al. 2008, for the measurements at \( z = 0 \)). They showed that the satellite fractions at \( 0.4 < z < 0.6 \) and \( 0.6 < z < 0.8 \) are not a monotonic function of the stellar mass, and have a peak at \( \log_{10}(M_*/(h^{-2}M_\odot)) \approx 10.1 \) and 10.3, respectively. This is similar to the prediction of the \( M_{\text{prog}} \) model.

The stellar mass dependence of the satellite fraction of the \( M_{\text{prog}} \) model comes from the best-fit \( z_{\text{prog}} \). The peak positions and shapes of the satellite fraction and the best-fit \( z_{\text{prog}} \) (the lower panel of Fig. 4) are very similar to each other. This is because a larger \( z_{\text{prog}} \) generally leads to a higher satellite fraction as discussed in Sec. 2.3.

### 3.4 The inferred halo occupation numbers

Fig. 8 shows the inferred halo occupation numbers of the subhalo samples given by the best-fit parameter sets for each threshold sample.
**Figure 4.** The best-fit parameters of the $M_{\text{prog}}$ model, $\sigma_M$ (upper panel) and $z_{\text{prog}}$ (lower panel) for the stellar mass bin samples. The error bars represent 1σ uncertainty. Note that only the highest mass range represents the threshold sample of $\log_{10}[M_*/(h^{-2}M_\odot)] \geq 11$, not a binned one.

**Figure 5.** Similar to Fig. 4 but for the best-fit parameter in the $V_{\text{peak}}$ model, $\sigma_V$.

**Figure 6.** The satellite fraction $f_{\text{sat}}$ of the threshold sample as a function of the threshold mass $\log_{10}[M_{\text{lim}}/(h^{-2}M_\odot)]$ inferred by the $M_{\text{prog}}$ model (the black square points) and the $V_{\text{peak}}$ model (the red small dot points).

**Figure 7.** Very similar to Fig. 6 but the inferred satellite fractions for the stellar mass bin samples.

As the solid lines. The dashed and dotted lines represent the halo occupation numbers of central and satellite galaxies, respectively. The thick black and thin red lines are the results with the best-fit parameter sets of the $M_{\text{prog}}$ and $V_{\text{peak}}$ models, respectively. The halo occupation numbers given by the parameter sets within the 1σ range are represented by the light gray (the $M_{\text{prog}}$ model) and pink (the $V_{\text{peak}}$ model) lines.

For the three highest and the two lowest mass samples for which both models give the ACFs consistent with the observation, the halo occupation numbers of the $M_{\text{prog}}$ and $V_{\text{peak}}$ models are consistent with each other within the 1σ error. For the other mass thresholds with $9 \leq \log_{10}[M_{\text{lim}}/(h^{-2}M_\odot)] \leq 10.2$, the $M_{\text{prog}}$ model preferentially assigns both central and satellite galaxies in lower mass subhalos than the $V_{\text{peak}}$ model. It should be noted that the $M_{\text{prog}}$ model predicts ACFs with higher amplitudes than the $V_{\text{peak}}$ model even in the two-halo term regime, i.e., $> a$ few $h^{-1}$ Mpc, as shown in Fig. 3. This implies that the $M_{\text{prog}}$ model preferentially selects highly biased (or early-forming) low-mass subhalos.

We also study the halo occupation numbers for the stellar mass bin samples in Fig. 9. The biggest difference from the halo occupation
Figure 8. The solid lines show the inferred halo occupation numbers $\langle N_{\text{halo}} \rangle$ as a function of the virial mass of the host central subhalo $M_{\text{vir}}$ for each threshold sample. The mass ranges are noted in each panel, where the stellar mass $M_*$ is in units of $h^{-2}M_\odot$. The dashed and dotted lines represent the halo occupation numbers of central and satellite galaxies, respectively. The thick black and thin red lines are the results with the best-fit parameter sets of the $M_{\text{prog}}$ and $V_{\text{peak}}$ models, respectively. The halo occupation numbers given by the parameter sets within the 1$\sigma$ range are represented by the light gray (the $M_{\text{prog}}$ model) and pink (the $V_{\text{peak}}$ model) lines.

Figure 9. Same as Fig. 8 but the halo occupation numbers inferred for the stellar mass bin samples.
numbers for the threshold samples is that the $M_{\text{prog}}$ model assigns the central galaxies in higher mass halos for all mass bins. This means that there are more central-satellite galaxy pairs in the $M_{\text{prog}}$ model than in the $V_{\text{peak}}$ model. Hence, together with the higher satellite fractions, the $M_{\text{prog}}$ model predicts an enhanced galaxy clustering signal at the smallest scales than the $V_{\text{peak}}$ model. It also can be said that using observed clustering of stellar mass bin samples is more useful to examine SHAM models because the differences between the two models are larger than the halo occupation numbers for the threshold samples.

4 SUMMARY AND CONCLUSION

We have proposed a novel rank-ordering SHAM model using the progenitor virial mass of each subhalo at redshift $z_{\text{prog}}$, $M_{\text{prog}}$, as a proxy of galaxy stellar mass at the time of observation. In this model, the characteristic redshift $z_{\text{prog}}$ at which we evaluate $M_{\text{prog}}$, and the scatter parameter $\sigma_{z_{\text{prog}}}$ (see Eq. 1) are the free fitting parameters. The motivation of this model is related to the two-phase scenario of stellar mass growth in galaxies, i.e., in-situ star formation and ex-situ star accretion (see Sec. 2.2.1).

We have studied the $z_{\text{prog}}$-dependence of the 2PCFs $\xi$ for the subhalo samples with the number density of $n_{\text{gal}} = 10^{-2}$ $h^3$ Mpc$^{-3}$ at $z = 0$ (Fig. 1). The $z_{\text{prog}}$-dependence can be understood by the variation of subhalo mass accretion histories and the subhalo mass stripping during accretion as shown in the halo occupation number (Fig. 2). We have shown that the $M_{\text{prog}}$ model with certain $z_{\text{prog}}$ value gives the 2PCFs similar to or even more amplified than those by the $V_{\text{peak}}$ model. We expect the $M_{\text{prog}}$ model to be able to well reproduce the observed galaxy clustering signal.

We have applied the $M_{\text{prog}}$ and $V_{\text{peak}}$ models to the observed ACFs $\omega$ of the photo-$z$ selected galaxies at $z = 0.4$ from the Subaru HSC survey (Ishikawa et al. 2020). Both models can reproduce the observed ACFs for the stellar mass threshold samples with $\log_{10}[M_{\text{gal}}/(h^{-2}M_\odot)] \geq 10.4$ and $\log_{10}[M_{\text{gal}}/(h^{-2}M_\odot)] \leq 8.8$ (Fig. 3). We have also shown that the $V_{\text{peak}}$ model underpredicts the amplitude of ACFs at the scale of $< 1$ $h^{-1}$ Mpc and fails to match the observed ACFs for the samples of $9 \leq \log_{10}[M_{\text{gal}}/(h^{-2}M_\odot)] \leq 10.2$. On the other hand, the $M_{\text{prog}}$ model gives sufficiently higher amplitudes and agrees with the observations within the error bars at $0.1 - 1$ $h^{-1}$ Mpc for these samples. We have found that $z_{\text{prog}}$ constrained by the observed ACFs has an interesting dependence on the stellar mass (the lower panel of Fig. 4). The best-fit $z_{\text{prog}}$ is lower toward the lowest and highest stellar mass ranges and has a single peak at $\log_{10}[M_{\text{gal}}/(h^{-2}M_\odot)] = 9.7$. This trend is qualitatively consistent with the in-/ex-situ scenario of stellar mass growth. It is clearly important to quantitatively examine whether the obtained values of $z_{\text{prog}}$ are physically reasonable. Specifically, investigating what events related to stellar mass growth happened at $z = z_{\text{prog}}$ should be an interesting topic. The recent cosmological simulations of galaxy formation (e.g., EAGLE: Crain et al. 2015; Schaye et al. 2015; Illustris TNG: Weinberger et al. 2017; Pillepich et al. 2018; FIREbox; Feldmann et al. 2022) might give clues to understanding the physical origin of the constrained $z_{\text{prog}}$ values, although it is beyond the scope of this work.

We have studied the inferred satellite fractions in Figs. 6 & 7. The successful agreement of the $M_{\text{prog}}$ model with the observed ACFs for the samples with the thresholds of $9 \leq \log_{10}[M_{\text{gal}}/(h^{-2}M_\odot)] \leq 10.2$ is attributed to the higher satellite fraction than in the $V_{\text{peak}}$ model. For the other mass-threshold samples for which both models can reproduce the observed ACFs, the predicted satellite fractions from the two models agree with each other. Thus the satellite fraction is a crucial factor for determining the strength of galaxy clustering.

We have found notable differences in the inferred halo occupation numbers between the $M_{\text{prog}}$ and $V_{\text{peak}}$ models (Figs. 8 & 9). For the three highest and the two lowest mass samples for which both models match the observed ACFs, the halo occupation numbers are consistent with each other within the 1σ error of the fitted parameters. For the samples with $9 \leq \log_{10}[M_{\text{gal}}/(h^{-2}M_\odot)] \leq 10.2$, the $M_{\text{prog}}$ model predicts more amplified ACFs and is more successful in reproducing the observations by populating galaxies with lower mass subhalos.

In future work, we plan to examine the $M_{\text{prog}}$ SHAM model by comparing it with observed galaxy statistics as a function of the stellar mass including clustering measurements at various redshifts (e.g., Yang et al. 2012; Ishikawa et al. 2020; Shuntov et al. 2022), the satellite fractions as a function of the stellar mass (van den Bosch et al. 2008; Knobel et al. 2013), the mass profiles around galaxies (Mandelbaum et al. 2006; Leauthaud et al. 2012) and the group statistics (Hearin et al. 2013). Especially, it is interesting to apply to the “lensing is low” problem on the CMASS galaxies. Leauthaud et al. (2017) showed that SHAM and HOD models, including the $V_{\text{peak}}$ model, can reproduce the observed PCF of the CMASS galaxies but overpredict the lensing profile. Since, as we stated, galaxies tend to live in lower mass halos in the $M_{\text{prog}}$ model than the $V_{\text{peak}}$ model while keeping the clustering amplitudes, the $M_{\text{prog}}$ model may be able to match better the observed CMASS clustering and lensing simultaneously.

With only one more free parameter than the $V_{\text{peak}}$ model, the $M_{\text{prog}}$ model is shown to be highly flexible and can more faithfully reproduce the observed ACFs, providing a more physical way to interpret the observed clustering measurements. The findings in this paper would be an important step toward accurate modeling of the galaxy-halo connection.

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DATA AVAILABILITY

The Uchuu simulations are available at http://skiesanduniverses.org/Simulations/Uchuu/. Other data presented in this paper can be provided by the authors upon request.

REFERENCES

Ahn C. P., et al., 2014, ApJS, 211, 17
Aihara H., et al., 2018, PASJ, 70, 54
Alam S., Miyatake H., More S., Ho S., Mandelbaum R., 2017, MNRAS, 465, 4853
We here compare the 2PCFs given by the $M_{\text{prog}}$ and $V_{\text{peak}}$ models denoted as $\xi_M$ and $\xi_V$, respectively. As in Sec. 2.3, we measure $\xi_M$ and $\xi_V$ for the subhalo samples with the number densities of $n_{\text{gal}} = 10^{-2}$, $10^{-3}$, and $10^{-4} \ h^3 \text{Mpc}^{-3}$ at $z = 0$, 0.5, and 1. We seek $z_{\text{prog}}$ that gives the best matched-$\xi_M$ to $\xi_V$. For simplicity, we do not perturb either $M_{\text{prog}}$ or $V_{\text{peak}}$.

We find that $\xi_M$ with a certain $z_{\text{prog}}$ value matches with $\xi_V$ very well for all three samples at all three redshifts. Table A1 summarizes the best match $z_{\text{prog}}$ values for each sample at $z = 0$, 0.5 and 1. As in Sec. 2.3, we measure $\xi_M$ and $\xi_V$ for the subhalo samples with the number densities of $n_{\text{gal}} = 10^{-2}$, $10^{-3}$, and $10^{-4} \ h^3 \text{Mpc}^{-3}$ at $z = 0$, 0.5, and 1.

### Table A1: Summary of the $z_{\text{prog}}$ values in the $M_{\text{prog}}$ model which gives the best matched-2PCFs to the $V_{\text{peak}}$ model, and $\langle z_{\text{peak}} \rangle$ for the $V_{\text{peak}}$ model at $z = 0$, 0.5 and 1 for the three samples.

| $n_{\text{gal}}$ | $z = 0$ | $z = 0.5$ | $z = 1$ |
|------------------|--------|----------|--------|
| $10^{-2}$        | 1.12   | 1.43     | 1.77   |
| $10^{-3}$        | 0.63   | 1.03     | 1.54   |
| $10^{-4}$        | 0.56   | 0.86     | 1.43   |

We seek $z_{\text{prog}}$ that gives the best matched-$\xi_M$ to $\xi_V$. For simplicity, we do not perturb either $M_{\text{prog}}$ or $V_{\text{peak}}$.

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Table A2. Summary of the matching rates between the subhalo samples from the $M_{\text{prog}}$ model with the best match $z_{\text{prog}}$ for each sample and from the $V_{\text{peak}}$ model at $z = 0$, 0.5 and 1.

| $n_{\text{gal}}$ [$h^3 \text{ Mpc}^{-3}$] | $z = 0$ | $z = 0.5$ | $z = 1$ |
|----------------------------------|--------|--------|--------|
| $10^{-2}$                        | 94.6%  | 94.3%  | 94.1%  |
| $10^{-3}$                        | 94.3%  | 93.8%  | 93.2%  |
| $10^{-4}$                        | 93.4%  | 92.5%  | 91.7%  |

matching rates are summarized in Table A2. The $M_{\text{prog}}$ model can select more than 90% of subhalos which are selected by the $V_{\text{peak}}$ model by taking a certain $z_{\text{prog}}$ value. Thus the $M_{\text{prog}}$ model can mimic the $V_{\text{peak}}$ model in predicting galaxy clustering.

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