Collective Modes in a Symmetry-Broken Phase: Antiferromagnetically Correlated Quantum Wells

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We investigate the intersubband spin-density-excitation spectrum of a double quantum well in a low-density symmetry-broken phase with interwell antiferromagnetic correlations. This spectrum is related to the intensity measured in depolarized inelastic light scattering (ILS) experiments and therefore provides a means of empirically identifying the antiferromagnetic phase. Our computations reveal the existence of two collective modes, a damped Nambu-Goldstone (NG) mode arising from the broken spin symmetry and an undamped optical mode. Since the NG mode contains most of the spectral weight, ILS experiments will need to examine the low-frequency response for signatures of the antiferromagnetic phase.

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Electronic correlations become increasingly important as either the dimensionality or the effective electron density is lowered. These enhanced correlations may give rise to new states of matter, of which the best-known example is the fractional quantum Hall liquid. This phase occurs in 2D electron gases in high magnetic fields, but correlation-induced quantum phase transitions should also occur in sufficiently pure low-dimensional, low-density quantum systems in the absence of magnetic fields. At very low densities, energetic considerations suggest that the electron liquid in these systems will crystallize to form a Wigner solid and this transition has been observed in a classical 2D electron gas on the surface of liquid helium.

The possibility of other novel quantum phases in the density regime between the Wigner solid and the electron liquid has been explored both experimentally and theoretically in several recent publications. These publications consider effectively multilayer systems such as double quantum wells (DQWs) or wide single quantum wells, in which the layer or subband index acts as an additional degree of freedom. Of particular current interest are correlation-driven transitions resulting in a spontaneous charge transfer between the layers, which is expected theoretically whether a magnetic field is present or not and which may have been observed experimentally.

Another correlation-induced phase transition, which awaits experimental verification, involves the development of nontrivial, subband-coupled antiferromagnetic order between the layers in certain two-layer quantum well geometries in zero field. The entrance into this antiferromagnetic phase is marked by the collapse of the usual intersubband spin-density excitations (SDEs) as the density is lowered. These excitations can be measured by depolarized inelastic light scattering (ILS) and so the approach to the antiferromagnetic phase boundary as a function of density should be detectable. Nonetheless, unambiguous identification of this phase would require the observation of the modified elementary excitations of the symmetry-broken phase, as occurred in the Wigner solid with the detection of its phonons.

In this paper, we assume uncritically the existence of the antiferromagnetic phase and calculate its elementary excitations in the intersubband spin-density channel as a guide for future experiments. Since the antiferromagnetic ground state is not reproducible within the dynamical spin-polarized local-density-approximation approach underlying the original calculations, we start from a simple mean-field theory and compute the collective modes in a conserving approximation. These calculations yield an intersubband response which is dominated at low frequencies by the Nambu-Goldstone (NG) mode associated with the broken spin-rotation invariance of the antiferromagnetic phase. Thus, ILS experiments will need to examine the low-frequency portion of the response for signatures of this phase. We believe that the results we present transcend the specific details of our model and are, to the best of our knowledge, the first determination of the collective modes of a low-density, double-quantum-well-type structure in a symmetry-broken phase.

We consider an interacting electron gas subject to a confining potential along one direction which results in a bimodal distribution of the self-consistent charge density. The antiferromagnetic phase may arise when the two layers are sufficiently close that the lowest two subbands are well separated in energy from the higher subbands. At the temperatures and densities relevant for this study, we may therefore neglect the higher subbands. Furthermore, since the antiferromagnetic phase is driven by the Coulomb interaction $V(r)$, we take a simple parameterization of this interaction in order to obtain the generic features of this phase: $V(r) = V\delta(r)$. Even at this very simple level of approximation, ferromagnetic phases may arise at low densities due to the intrasubband matrix elements of $V(r)$. Since the aim of this paper is to obtain the collective modes of the antiferromagnetic phase and not to establish its existence, we may set the intrasubband matrix elements to zero without loss of generality.

The self-energy for this simplified model is computed
within self-consistent Hartree-Fock theory, which is illustrated by the diagrams in Fig. 1(a). In the subband representation, the antiferromagnetic phase corresponds to a mixing of the non-interacting states with opposite spin and different subband index. Thus, we look for solutions to the self-energy equations with non-zero off-diagonal elements. Such solutions exist and are stable in the range of parameters denoted by “AF” in Fig. 1(a). For a more detailed discussion of this phase diagram and the self-energy and model underlying it, see Ref. 10.

We compute the collective modes in the usual way within a conserving approximation for the electronic polarizability [cf. Fig. 1(b)]. One complication that arises in this calculation is that the mixing of the wave functions from different subbands in the antiferromagnetic phase causes the “bare” polarizability to become off-diagonal. Hence, the resulting polarizability matrices have 16 × 16 elements, making analytical work nearly impossible. However, the specific form of the interaction we use allows a straightforward numerical solution for the interacting polarizability through a matrix inversion of the equations. The collective modes can then be obtained from the study of the resulting interacting polarizability. Since the intersubband spin-density excitations were the first sign of the antiferromagnetic phase,9 we restrict attention to these excitations in what follows and perform all our calculations at zero temperature.

Before proceeding to our results, a few remarks on our calculational scheme are in order. At the level of the self-energy, we know that the Hartree-Fock approximation is a poor one for the interacting electron gas because it neglects screening effects. Including these effects realistically is a difficult problem which has not yet been solved. In computing the collective modes, this problem is amplified by the distinction one should draw between the interaction between the bubbles, which is unscreened, and the interaction within the bubbles, which should be screened. Summing a particular set of screening diagrams may reduce the error introduced into the self-energy, but would render the collective mode calculation completely intractable. Thus, we adopt a strong approximation to the actual Coulomb interaction, a point-contact interaction, which should reproduce the approximate qualitative features of the screening effects while leaving a solvable set of equations.

To get a clear picture of the effect on the collective modes of the transition from the normal paramagnetic
phase to the antiferromagnetic phase, we present calculations for four representative points in the parameter space of our model. These points are labeled in Fig. 3(a) and sample both the normal phase with one subband occupied and the antiferromagnetic phase. The two phases are separated by a second-order phase transition at zero temperature.

In the normal phase [points $A_1$ and $A_2$ in Fig. 3(a)], a single, collective spin-density excitation exists below the energy of the renormalized subband splitting, as shown by the solid lines in Fig. 3(b). This mode disperses into the continuum of intersubband particle-hole excitations denoted by the shaded areas in Fig. 3(b), where it becomes damped. As one approaches the phase transition from the normal side ($A_1 \rightarrow A_2$), the $q = 0$ frequency of the spin-density excitation $\omega_0$ decreases according to the relation $\omega_0/\Delta_{\text{SAS}} = \sqrt{1 - V_{01} n/\Delta_{\text{SAS}}}$, where $\Delta_{\text{SAS}} = \Delta_{\text{SAS}}^0 + V_{01} n/2$ is the renormalized subband splitting, $n$ is the electronic density, $V_{01}$ the intersubband Coulomb repulsion, and $\Delta_{\text{SAS}}^0$ is the $V_{01} = 0$ subband splitting. We see that the $q = 0$ spin density excitation softens completely ($\omega_0 \rightarrow 0$) when $V_{01} n \rightarrow 2\Delta_{\text{SAS}}^0$. This signals the onset of the antiferromagnetic phase.

In the antiferromagnetic phase, the pattern of the collective modes changes, as plotted in Fig. 3. The most obvious difference is the presence of an additional region of particle-hole excitations at low frequencies. This continuum arises because the wave functions in the antiferromagnetic phase are linear combinations of wave functions from different subbands, which results in a mixing of intra- and intersubband excitations. A second feature in these spectra is the reappearance of an optical spin-density excitation whose $q = 0$ frequency increases as one goes deeper into the antiferromagnetic phase ($B_1 \rightarrow B_2$). The dependence of the frequency of this mode, along with that of the charge-density mode and the renormalized subband splitting, is displayed in Fig. 3(a). Finally, one observes a strong, linearly dispersing feature within the lower particle-hole continuum. This mode is the Nambu-Goldstone mode arising from the broken spin-rotation invariance in the antiferromagnetic phase. A unique feature of this mode is that it is damped by particle-hole excitations.

This damping leads naturally to the question of how much spectral weight is associated with the Nambu-Goldstone mode. This question has important implications for ILS experiments because the imaginary part of the polarizability is related to the light scattering cross section. In particular, the spin-density excitations are directly observable in depolarized ILS, in which the polarization of the scattered light is rotated by $90^\circ$ with respect to the incident light. We display in Fig. 3(b), therefore, the imaginary part of the intersubband polarizability at point $B_2$ of Fig. 3(a) at a representative wave vector. We see that the Nambu-Goldstone mode, the optical mode, and the intersubband particle-hole continuum are all present, but that the Nambu-Goldstone mode contains most of the spectral weight. Since most ILS experiments are done at small wave vector and moderate frequencies on this scale, this mode may be difficult to observe. In fact, the empirical signature of the new phase may simply be the apparent disappearance of all ILS intensity.

In summary, we have computed the collective modes in antiferromagnetically correlated quantum wells for a simple model within a conserving approximation. We find that two collective excitations in the intersubband spin-density channel are associated with the antiferromagnetic phase, one optical and one acoustic. The acoustic excitation is the Nambu-Goldstone mode produced by the broken spin symmetry in the antiferromagnetic state. This mode is damped by particle-hole excitations but nonetheless retains the majority of the spectral weight, indicating that most of the inelastic light scattering in-
FIG. 4. (a) Frequency at \( q = 0 \), \( \omega_0 \), of the intersubband spin- (SDE, solid line) and charge-density excitations (CDE, dotted line) as a function of electronic density \( n \) for \( N_0V_{01} = 1 \). Also shown is the renormalized subband splitting \( \Delta_{SAS} \) (SPE, dot-dashed line). The top axis marks the points from the phase diagram in Fig. 2(a). Note the re-emergence of the SDEs in the antiferromagnetic phase (\( n/2N_0\Delta_{SAS} \geq 1 \)).

(b) Negative of the imaginary part of the intersubband polarizability \(-\text{Im} \Pi_{\text{inter}}\) in arbitrary units as a function of frequency \( \omega \) in the antiferromagnetic phase (point \( B_2 \) in the phase diagram of Fig. 3(a)). This quantity is related to the intensity of the intersubband response in inelastic light scattering experiments. As emphasized by the inset, the acoustic (Nambu-Goldstone) mode dominates the low-\( \omega \) intensity profile. The spectrum is computed at \( q/q_\Delta = 0.4 \) and incorporates a finite scattering rate \( \gamma = 0.01\Delta_{SAS} \) to simulate the effect of impurities; the notation is the same as in Fig. 3.

tensity will be at low frequencies in the antiferromagnetic phase. The qualitative features of the collective excitations in this symmetry-broken phase follow from very general principles, such as the conservation of particle number, momentum, and energy\(^1\) and Goldstone’s theorem for phase transitions involving a spontaneously broken continuous symmetry\(^2\). Thus, while the quantitative details of our calculated collective mode dispersion are undoubtedly model-specific, the qualitative features should remain valid independent of our rather crude treatment of the interaction term.

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