Do Observations Exclude $\gamma$-ray bursts Originating in the Oort Cloud of Comets?

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ABSTRACT

The currently favored explanation for the origin of $\gamma$-ray bursts puts them at cosmological distances; but as long as there is no distance indicator to these events all possible sources which are isotropically distributed should remain under consideration. This is why the Oort cloud of comets is kept on the list, although there is no known mechanism for generating $\gamma$-ray bursts from cometary nuclei. Unlikely as it may seem, the possibility that $\gamma$-ray bursts originate in the solar cometary cloud cannot be excluded until it is disproved.

We use the available data on the distribution of $\gamma$-ray bursts (the BATSE catalogue up to March, 1992), and the Catalogue of Cometary Orbits by Marsden and Williams (1992) to investigate whether there is any observational indication for correlations between the angular distributions of $\gamma$-ray bursts and comets’ aphelia, assuming that the distribution of aphelia direction reflect, at least to some extent, true variations in the column density of the Oort cloud. We also apply the $\langle V/V_{\text{max}} \rangle$ test to both distributions.

We have performed a variety of statistical tests (a Kolmogorov-Smirnov test for the distributions in galactic latitude, a $\chi^2$ test for the spherical multiple moments, and a 2-D cross-correlation analysis), including testing sub-samples for isolating the effect of possible observational biases. These tests imply that it is unlikely that the two distributions agree, but the statistical significance is not sufficient for ruling out any connection with complete confidence. We performed Monte-Carlo simulations which show that only when the number of bursts exceeds $\sim 800$ it is possible to rule out a correlation between the angular
distributions. Currently, it is only the combination of these tests with the large disagreement found for the $\langle V/V_{\text{max}} \rangle$ parameter which makes the Oort cloud of comets unlikely to be related to $\gamma$-ray bursts.

*Subject headings:* Gamma Ray: bursts – comets: general
1. INTRODUCTION

Observations indicate that we are in the center of a spherically symmetric distribution of $\gamma$-ray bursts. Currently, the favored explanation for the origin of these bursts involves neutron stars at cosmological distances (e.g., Paczyński 1992; Narayan, Paczyński, and Piran 1992, and references therein) mainly due to the isotropy of the observable universe on very large scales. However, as long as there is no distance indicator for these events, all possible astrophysical objects that are isotropically distributed around us should remain under consideration. This is the reason why the Oort cloud of comets (Oort 1950; Weissman 1990) is kept on the list, although there has never been a specific suggestion how to generate $\gamma$-ray bursts from cometary nuclei in the Oort cloud. In this paper we shall use the available observational data on both $\gamma$-ray bursts and comets, and perform various statistical tests in attempt to find whether a relation between $\gamma$-ray bursts and the Oort cloud can be excluded, or alternatively, check whether there is a statistically significant correlation between these two populations.

If $\gamma$-ray bursts originate in the Oort cloud their number density in a certain direction should be roughly proportional either to the column number density of cometary nuclei in the cloud in that direction, or to the projected square of three-dimensional density (if collisions between comets is the underlying mechanism). Unfortunately, we are unable to directly observe the Oort cloud itself but only that tiny fraction of its comet population which happen to have a small perihelia distance at their present passage near the sun. We shall assume that variations in the flux of the observed comets as a function of aphelia direction reflect, at least to some extent, true variations in the column density of the Oort cloud.

Such an assumption is crucial for being able to discuss deviations of the cloud’s structure from a perfect spherical symmetry. Since this central assumption, although reasonable, cannot be verified observationally (but see §5) let us discuss some of its aspects. First, the picture of a spherical comet cloud surrounding the planetary system and stretching halfway to the nearest stars is only a rough approximation. The aphelia directions of long-period comets, calculated from their original orbits (before entering the planetary system) do not seem to be distributed at random on the celestial sphere. Variations in the flux of incoming comets from different directions are evident. For example, Oja (1975) pointed to an overdensity of comet perihelia in a direction close to the solar apex, and a concentration of comet perihelia towards a plane which is inclined at 20 degrees to the galactic plane, and Delsemme (1987) drew the attention to a deficiency of comets near the galactic poles and equator. Thus, the flux of the observed comets is already known to vary with direction (see also Fig.1), and the coincidences of the detected anisotropies
with directions which are of special interest imply that these may well reflect real features of anisotropy rather than statistical fluctuations.

Our main concern is the possibility that the currently observed variations in the distribution of comets' aphelia are attributed to some recent perturbation(s) on the cloud. It is indeed possible that the major features were caused by recent encounters with stars or molecular clouds, but Delsemme (1986, 1987) has shown that in order to reproduce the observed variations, an extremely slow perturbing body is needed, moving typically at a relative velocity of 200 $\text{ms}^{-1}$. Encounters with typical stars or clouds are very unlikely to account for such effect, but a recent slow passage of a brown dwarf, such as one of the many which may constitute the dark matter in the galactic disk, is not improbable (Delsemme 1986). An encounter with a giant molecular cloud may in principle affect the comet orbits dramatically, but the absence of a strong anisotropy feature in the observed comet distribution, and the fact that such encounters occur only about once every 70-500 myr (depends on the impact parameter of the encounter) make it very unlikely that we are currently experiencing a comet shower. In fact, the smooth variation of the galactic tidal field, rather than individual perturbations, is the dominant mechanism which drives the evolution of the Oort cloud (e.g., Heisler, Tremaine, and Alcock 1987), and the imprints of the galactic tides are even claimed to be detected in the data (Delsemme 1987). The tidal field sets limits on the outer dimensions of the cloud and shapes it as a prolate spheroid with the long axis oriented along the radius vector to the galactic center, which implies that the column density of comets in the cloud need not necessarily be identical in all directions.

Thus, it is reasonable to expect that anisotropies in the flux of the observed comets are correlated, at least to some extent, to anisotropies in the column density in the cloud. Still, this assumption is not secure (see discussion in §5). In §2 we describe the data, and in §3 the statistical tests and their results. In §4 we compare the $\langle V/V_{\text{max}} \rangle$ parameter of both distributions, and in §5 discuss some caveats and summarize the main points.

2. THE DATA

The data on cometary orbits was kindly provided by Brian Marsden from the Catalogue of Cometary Orbits (Marsden and Williams 1992), with an addition of the few most recently discovered comets. Short-period comets cannot have anything to do with $\gamma$-ray bursts since their distribution is highly anisotropic. Therefore, our main sample (hereafter denoted by $C_0$) includes only those comets which satisfy the two following conditions: a) an orbital period greater than 200 years, which is the traditional definition of a long-period
comet. b) classified in the catalogue as class 1 or 2 (the classification scheme is described by Marsden et al. 1978), namely, being widely observed and having a well determined orbit. This last condition is required in order to exclude comets with poorly determined orbits since those may be erroneously classified as long-period ones, and to avoid including poorly observed comets, such as very faint ones, which are likely to introduce selection biases (see discussion below). Finally, the trio 1992II, 1963V and 1965VIII, and the pair 1987XXX and 1988III, were each regarded as a single comet since their identical orbital parameters indicate that they separated from each other at some earlier perihelion passage. Thus, the sample \( C_0 \) contains 272 long-period comets with good orbit determinations and with no obvious trouble with non-gravitational forces.

Clearly, there are comets whose discovery has been missed, but this need not concern us since we are interested in the relative fluxes of comets incoming from different directions. However, the discovery probability of a comet depends on the number of observers that have a chance to detect it, so there might be an observational bias due to the different geographic distribution of observers in the northern and southern hemispheres. Although comets which are visible in the south are likely to be also visible sooner or later in the north, the fact that comets become brightest near perihelion implies that there may still be a smaller number of cometary perihelia discovered in the southern hemisphere which is an artifact of the observers’ depletion. Indeed, during the years 1840-1919 the number of observers in the southern hemisphere was quite low, but the situation has much improved after 1920 (Everhart 1967; Marsden 1992, private communication), so since then this selection effect is likely to be very small (the discovery probability is not linear with the number of observers, but converges rapidly when the observers’ number approaches a certain value). Therefore, in order to isolate the effect of such possible observational bias we shall examine also a subset of \( C_0 \), hereafter denoted by \( C_1 \), which contains only those 192 comets that have been discovered during or after 1920. Indeed, 102 comets (53%) in \( C_1 \) have perihelia directions in the Northern hemisphere, which implies that a strong bias is very unlikely. A comet with a large perihelion distance will always be faint and thus have a smaller discovery probability (Evhart 1967), but there is no obvious reason how this can introduce a bias in the distribution of aphelia directions.

It is also possible that some of the observed long-period comets have already completed several passages through the planetary system and thus may have changed their original aphelia direction due to planetary perturbations. Although our samples may be contaminated by such comets, this is unlikely to be a severe problem. Simulations show that originally long-period comet that enter the inner planetary system do not survive for many periods in this state but are either captured more strongly or ejected from the solar system (e.g., Fernandez and Jockers, and references therein). This is also evident from
the substantial difference between the “original” orbit parameters and the “future” ones calculated for the observed long-period comets (Marsden and Sekanina 1973). Therefore, we do not expect a substantial mixing of aphelia directions for the observed long-period comets. Most of them are probably making their first passage through the inner part of the solar system (Marsden, Sekanina, and Everhart 1978), and most of the others are very unlikely to have considerably changed their aphelia direction, the more so for comets with larger aphelia distances. Still, we shall consider also a third sample of comets, hereafter denoted by \( C_2 \), which is a subset of \( C_1 \) and contains only the very long-period comets, i.e., only those 96 comets with \((1/a)\) smaller than the median value in the sample \( C_1 \).

The data on \( \gamma \)-ray bursts is based on the BATSE catalogue and includes the 260 triggered bursts observed from April, 1991 until March, 1992 (this data set is hereafter denoted by \( B_0 \)). In order to examine possible correlations between weak bursts and very long-period comets we have constructed also a subset of \( B_0 \) (hereafter denoted by \( B_1 \)) which includes only those 130 bursts with peak flux rate (on the 64ms time scale) below the median value.

3. STATISTICAL TESTS

In this section we study the possibility that the frequency of \( \gamma \)-ray bursts is proportional to the number density of cometary nuclei. This is an adequate assumption for cases in which bursts are produced by an (inconceivable) mechanism which does not involve cometary collisions (e.g., collisions with other objects or internal processes). We shall discuss a possible relation to the square of the comet number density in §4.

We thus wish to obtain an answer to the following question: is there any observational indication that the directions of \( \gamma \)-ray bursts and comets’ aphelia are drawn from the same underlying distribution? Alternatively, can we disprove, to a certain significance level, the null hypothesis that the two data sets are consistent with a single distribution function? We shall perform three types of tests: \( K-S \) ones, a \( \chi^2 \) test for the spherical multiple moments of both distributions, and a 2-D cross-correlation test.

3.1. Kolmogorov-Smirnov tests

The \( K-S \) test is based on measuring the difference between two cumulative distributions of one-dimensional data sets, but a cumulative distribution is not well-defined for a 2-D set.
Although the $K$-S test can be generalized to distributions in a plane (Press, et al. 1992) it becomes completely meaningless for distributions on a sphere where the two coordinates are cyclical. We shall apply the familiar $K$-S test to the 1-D distributions of the data points in galactic latitude. This has the advantage of not involving any binning in galactic latitudes, but some information is still 'thrown away' due to ignoring the distribution in $l$. In general, such a test would not be of much interest due to the complete freedom in choosing the orientation of the spherical coordinate system. But in our case, galactic tides are expected to affect the shape of the Oort cloud in a way which is likely to be symmetric with respect to rotations around the galactic $\hat{z}$ direction. Thus, a $K$-S test for the distributions in galactic latitude is of special interest.

We performed such $K$-S test for each of the six possible combinations (the $B_0$ and $B_1$ sets versus $C_0$, $C_1$, and $C_2$ (see definitions in §2)). The "$K$-S" columns in Table 1 show the significance level for the null hypothesis that the two data sets were drawn from the same distribution. We see that weak bursts turn to have higher probability for correlation with the comet distribution, but these results do not give a firm answer to whether the differences between the distributions are due to statistical fluctuations or reflect real inconsistency (the values are not small enough to imply a statistically significant difference). The loss of information which is associated with reducing the spherical data to 1-D distributions is apparently too large. Let us see now how these results change in 2-D tests.

### 3.2. A $\chi^2$ Test for Spherical Multiple Moments

If the two data sets are drawn from the same underlying distribution we expect their dipole and quadrupole moments to be similar up to statistical errors. Instead of comparing each of the three dipoles and five quadrupoles separately, we shall combine all comparisons into a single $\chi^2$ test with eight degrees of freedom, one for each spherical multiple moment. To do that we need to have the error estimates (or variance) for these quantities, but how can this be done without knowing the true distribution?

A powerful technique for estimating the statistical errors is the bootstrap (or resampling) method (Efron and Tibshirani 1986; Press et al.1992; Press, Rybicki, and Schneider 1992). It uses the actual dataset with its $n$ data points to generate a large number of synthetic data sets, each also with $n$ data points. The procedure is simply to draw randomly $n$ data points at a time with replacement from the actual set. Obviously, one gets sets in which a random fraction of the original points, typically $\sim 1/e$ of them, are replaced by duplicated original points. For each of the resampled data sets we calculate
the statistical quantities, in our case the spherical multipole moments, exactly as we did for the actual data set. The simulated measured quantities can be shown to be distributed around the corresponding quantities from the actual set in the same way, both variance and covariance, that the last are distributed with respect to the true values.

We have calculated the first eight spherical moments, \( c_i \) and \( b_i \) (\( i = 1, \ldots, 8 \)) of the comets and \( \gamma \)-ray bursts distributions, respectively, and generated a 1000 repeated resampled sets for both distributions. Denoting the \( k \)-th determination of the \( i \)-th quantity by \( c_i^k \) (and \( b_i^k \)), \( k = 1, \ldots, N \), the standard 1-\( \sigma \) errors, \( \sigma(c_i) \), are estimated by

\[
\sigma^2(c_i) \approx \frac{1}{N} \sum_{k=1}^{N} (c_i^k - \bar{c}_i)^2
\]

where \( \bar{c}_i \equiv N^{-1} \sum_{k=1}^{N} c_i^k \), and \( N = 1000 \). If the errors of the various quantities had not been correlated the \( \chi^2 \) would have been

\[
\chi^2 = \sum_{i=1}^{8} \frac{(c_i - b_i)^2}{\sigma^2(c_i) + \sigma^2(b_i)}
\]

However, although the multiples are orthogonal to each other, the statistical errors of the various multipoles are not statistically independent because the same data points are used for estimating the variance of each moment. The correlation matrix among the \( c_i \) is estimated by

\[
\gamma_{ij} \approx \frac{1}{N} \sum_{k=1}^{N} (c_i^k - \bar{c}_i)(c_j^k - \bar{c}_j)
\]

and a similar matrix \( \beta_{ij} \) for the correlation between the \( b_i \) quantities. The \( \chi^2 \) test with 8 degrees of freedom (e.g., Barlow 1989) reads

\[
\chi^2 = \sum_{i,j=1}^{8} (c_i - b_i)(\gamma_{ij} + \beta_{ij})^{-1}(c_j - b_j)
\]

where the summation of the two correlation matrices is due to the fact that the variance of the difference between quantities equals to the sum of the variance of each quantity. This test was again performed for the six combinations of data sets, and the results are displayed in the "\( \chi^2 \)" columns of Table 1. Again, the weak bursts show a slightly higher probability for agreement but these significance levels are far from being conclusive.

3.3. A Cross-Correlation Test
A rapid visual assessment of the equal-area projections (Figures 1,2) seems to suggest that both distributions are roughly isotropic, but contain some slightly overdense and underdense regions. We wish to find whether over-densities (and under-densities) in the two distributions coincide, namely, whether there is a statistically significant correlation of small-scale anisotropies. In order to do that we shall employ a cross-correlation technique combined with a $\chi^2$ test.

Investigating correlations between qualitative features in the data sets requires having an estimate for the true density giving rise to the data. The key point concerning density estimation from a discrete set of points is that it involves some degree of smoothing. Essentially, the larger the data set, the less smoothing is required (a smaller smoothing scale) so in the limit of an enormous data set we would do no smoothing at all. There is much freedom in determining the best angular smoothing scale, but it should satisfy the following conditions: a) be larger than the average nearest-neighbor angular separation between data points ($\sim 7^\circ$ in our case) since otherwise we try to associate specific bursts with specific comets. b) It should be substantially smaller than $2\pi$ radians since otherwise most possible features will be erased, and all distributions will look alike. The test of the spherical multiple moments ($\S$3.2) essentially investigated correlations between large scale anisotropies. Therefore, since we are now interested in the small scale variations we shall choose a value which is closer to the lower limit, e.g, a smoothing angular scale of $20^\circ$. Enlarging the smoothing scale even up to $60^\circ$ changed all the results (the c-c columns in Table 1) by less than a factor of 3/2.

We shall use the smoothing procedure of a spherical data described by Fisher et al. (1987), and Watson (1983). If a sample contains $n$ points on a surface of a sphere, each with polar coordinates $(\theta_i, \phi_i)$, then the estimated density of points at a given direction $(\theta, \phi)$ is given by

$$f(\theta, \phi) = \frac{1}{4\pi n \Theta_e^2 \sinh(\Theta_e^2)} \sum_{i=1}^{n} \exp^{\cos \Psi_i/\Theta_e^2}$$

where $\Theta_e$ is the effective smoothing angle (in radians), $\Psi_i$ is the angular separation between the considered direction and that of the i-th data point, and $\cos \Psi_i$ is given by

$$\cos \Psi_i \equiv (\sin \theta_i \cos \phi_i \sin \theta \cos \phi) + (\sin \theta_i \sin \phi_i \sin \theta \sin \phi) + (\cos \theta_i \cos \theta)$$

The weight given to each data point depends only on its angular distance from $(\theta, \phi)$ and not on its direction, and the normalization coefficient is such that $\int f(\theta, \phi) d\Omega = 1$ for any spherical distribution of $n$ points. Denoting the dimensionless deviation of the density from the average density by $\delta f(\theta, \phi) \equiv (f - \bar{f})/\bar{f}$, where $\bar{f} = (4\pi)^{-1}$, the amount of correlation between variations in the bursts distribution, $f_b$ (Fig.4), and variations in the comet aphelia
distribution, \( f_c \) (Fig.3), is given by

\[
\xi_{bc} = \int \delta f_b(\theta, \phi) \delta f_c(\theta, \phi) \sin \theta d\theta d\phi
\]  

(7)

If variations in the two distributions correlate then \( \xi_{bc} \) should be positive, and if the two data sets were drawn from a single distribution then we should obtain \( \xi_{bc} = \xi_{bb} \) up to statistical errors, where \( \xi_{bb} \) is the correlation of \( \delta f_b \) with itself.

In order to find whether \( \xi_{bc} \) is statistically consistent with \( \xi_{bb} \) we need an estimate for the associated statistical errors, \( \sigma_{bc} \) and \( \sigma_{bb} \), respectively. These are calculated using the re-sampling method which was described in detail in §3.2, and are given by

\[
\sigma_{bb}^2 \approx N^{-1} \sum_{k=1}^{N} (\overline{\xi}_{bb} - \xi_{bb}^k)^2,
\]

and a similar expression for \( \sigma_{bc}^2 \), where the index \( k \) stands for the \( k \)-th re-sampling, and \( \overline{\xi}_{bb} \) is the average of \( \xi_{bb}^k, k = 1, \ldots, N \) \( (N = 100) \). The \( \chi^2 \) test for one degree of freedom thus reads

\[
\chi^2 = \frac{(\overline{\xi}_{bb} - \xi_{bc})^2}{\sigma_{bb}^2 + \sigma_{bc}^2}
\]  

(8)

The results (the "c-c" columns in Table 1) show confidence levels which range from 4% to 18% for agreement. Enlarging the smoothing scale up to values of 60\(^\circ\) did not yield confidence levels higher than 25%.

4. THE \( \langle V/V_{max} \rangle \) PARAMETER

The BATSE observations (Fishman et al. 1991; Meegan et al. 1992) show that \( \langle V/V_{max} \rangle \approx 0.35 \) which indicates that the sources of \( \gamma \)-ray bursts have a density distribution which falls with the distance. However, the spatial distribution of the strong bursts is found to be roughly constant \( \langle V/V_{max} \rangle \approx 0.5 \). This provides a challenge for the Oort cloud hypothesis since it is unclear why should comet dynamics, which is dominated by the potential of a single point mass, lead the comet population to a density distribution which has a roughly flat core. Indeed, numerical simulations of the origin and evolution of the solar system cometary cloud (Duncan, Quinn, and Tremaine 1987) show that the density profile of comets in the range 3000-50000 AU goes roughly as \( r^{-3.5} \), which does not agree with the roughly uniform distribution of the strong \( \gamma \)-ray bursts. This problem is even more severe if cometary collisions are assumed to produce the bursts, since the implied logarithmic slope of the square of the comets number density is \(-7\).

Let us check whether the spatial distribution of comets, as implied by the observed ones, does indeed deviate significantly from a constant density. This need not necessarily
reflect the exact density profile of the entire cloud since we are strongly biased to observe comets with a small perihelia distance, which is most likely to be correlated with the aphelia distance. Denoting the number of observed comets with a semi-major axis in the range \([a, a + da]\) by \(N(a)\), the implied total number of comets with such \(a\) is proportional to \(N(a)T(a)\), where \(T(a)\) is the orbital period (for example, if we observe the same flux of short-period and long-period comets it means that there are much more unobserved long-period comets than short-period ones). The number of comets, with a given \(a\), that can be found in any given time at a distance between \([r, r + dr]\) from the sun, is proportional to the relative period of time that those comets spend in that distance range, namely, \(\propto dr v_a^{-1}(r)T^{-1}(a)\), where \(v_a(r)\) is the radial velocity of a comet with a given \(a\) at a distance \(r\) from the sun. Thus, it is easy to verify that the spatial distribution of comets, as implied by observations, is given by

\[
n(r) \propto \frac{1}{r^2} \int_r^{a_{max}} \frac{N(a) da}{\sqrt{r^{-1} - a^{-1}}} \tag{9}
\]

In order to estimate \(N(a)\) we have extracted from the \(C_0\) data set all those comets with \(3000 \leq a \leq 50000\), and used a Kolmogorov-Smirnov test to find the best power law which models the data. The significance level for an \(a^{-0.5}\) fit was the highest (71%) and dropped rapidly to few percents (and below) for changes in the logarithmic slope of \(\pm 0.2\). Substituting the above expression for \(N(a)\) in equation (4.1) and integrating numerically we obtain a logarithmic slope for \(n(r)\) which changes from \(-1.5\) to \(-3\) between 3000 to 50000AU.

We conclude that both numerical simulations and observations imply a spatial structure for the Oort cloud which is highly non-homogeneous. An origin of \(\gamma\)-ray bursts in the Oort cloud would require a mechanism that produces a very fine-tuned luminosity as a function of distance from the sun. This, and especially a relevance of bursts’ generation to collisions between comets, are thus extremely unlikely.

5. CONCLUSION

This study does not discuss if and how \(\gamma\)-ray bursts can be produced in the Oort cloud of comets, but inquires how probable is this possibility, based on observational data. We have performed a variety of statistical tests for correlations between the angular distribution of both populations, and examined their \(\langle V/V_{max} \rangle\) parameters. The tests of the angular
distributions imply that it is unlikely that the two distributions agree, but the statistical significance is not sufficient for ruling out any connection with complete confidence. We performed Monte-Carlo simulations in which we gradually increased the $\gamma$-ray bursts population by adding randomly distributed data points and found that, assuming that $\gamma$-ray bursts are distributed isotropically, only when their number gets around 800-900 the significance levels of the angular distribution tests will be around 1% or below. Currently, it is the substantial disagreement of the $\langle V/V_{\text{max}} \rangle$ parameters, especially if cometary collisions are assumed to be relevant, which makes the Oort cloud of comets extremely unlikely to be the origin of $\gamma$-ray bursts.

It should be noticed that, although reasonable, it is not really clear how well angular variations in the observed distribution of aphelia directions represent true variations in the column density of comets in the Oort cloud. A way to check this would be to perform numerical N-body simulations and check the correlation between true anisotropies and the "observed" ones, but this is left to a future study. Also, the current number of observed comets is not large enough that statistical fluctuations in the comet flux will be negligible. Even in the absence of a recent encounter with a perturbing star or a molecular cloud, some of the detected anisotropies may result from statistical fluctuations. In any case, our main conclusion remains due to the large discrepancy in $\langle V/V_{\text{max}} \rangle$.

Finally, although it is likely to be a coincidence, we draw the attention to the peculiar resemblance of qualitative features in the angular distributions of $\gamma$-ray bursts and comets' aphelia (figures 3,4) in the celestial hemisphere defined by $180 \leq l \leq 360$, where $l$ is the galactic longitude. This coincidence does not show up in the statistical analyses due to the stronger lack of correlation in the other hemisphere.

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TABLE 1

The significance level for agreement between the angular distributions of comets’ aphelia and γ-ray bursts

|        | \( B_0 \) \((n = 260)\) | \( B_1 \) \((n = 130)\) |
|--------|------------------------|------------------------|
|        | \(K-S\) | \(\chi^2\) | \(c - c\) | \(K-S\) | \(\chi^2\) | \(c - c\) |
| \(C_0\) \((n = 272)\) | 11% | 4% | 8% | 56% | 30% | 18% |
| \(C_1\) \((n = 192)\) | 8% | 28% | 10% | 35% | 64% | 10% |
| \(C_2\) \((n = 96)\) | 14% | 53% | 9% | 38% | 53% | 4% |

Table 1 - The significance level for correlations between the distributions of γ-ray bursts and comets’ aphelia directions. \(B_0\) - all bursts, \(B_1\) - only the weak bursts, \(C_0\) - all comets, \(C_1\) - only those discovered after 1920, \(C_2\) - only the very long-period comets from \(C_1\). \(n\) is the number of data points in each sample (see details in §2).

FIGURE CAPTIONS

Fig. 1: An equal-area (Aitoff) projection of aphelia directions of long-period comets on the entire celestial sphere in galactic coordinates. (Squares – discovered before 1920, Circles – discovered after 1920, Triangles – very long-period comets discovered after 1920). The celestial equator is also drawn.

Fig. 2: An equal-area projection of the directions of the 260 γ-ray bursts in galactic coordinates. Triangles stand for weak bursts.

Fig. 3: An equal-area projection of the smoothed distribution of comets aphelia directions, based on the 192 comets (the \(C_1\) set) discovered during or after 1920, with an effective smoothing scale of 20° (see §4). Darker areas reflect higher angular densities, and the grey levels are linearly enhanced for an easy visual assessment of qualitative features.

Fig. 4: The same smoothing procedure as in figure 3, but for the γ-ray bursts distribution (the \(B_0\) set). These smoothed projections enable an easy visual comparison between angular variations in the two distributions. A quantitative analysis is described in §3.3.
REFERENCES

Barlow, R.J. 1989, In *Statistics*, John Wiley & sons, New York, p.150
Heisler, J., Tremaine, S., and Alcock, C. 1987, Icarus, 70, 269
Delsemme, A.H. 1986, in *The Galaxy and The Solar System*, R.Smoluchovski, J.N. Bahcall, M.S. Matthews, eds., U. Arizona Press, Tucson, pg.173
Delsemme, A.H. 1987, A&A, 187, 913
Duncan, M., Quinn, T., and Tremaine, S. 1987, Bull.Am.Astron.Soc, vol.19, No.2, pg.742
Efron, B., and Tibshirani, R. 1986, In *Statistical Science* vol.1, pp.54-77
Everhart, E. 1967, AJ, 72, 716
Fisher, N.I., Lewis, T., and Embleton, B.J.J. 1987, In *Statistical Analysis of Spherical Data*, Cambridge University Press.
Fishman, G. et al. 1991, presentation for BATSE at GRO Workshop in Annapolis
Marsden, B.G., Sekanina, Z. 1973, AJ, 78, 1118
Marsden, B.G., Sekanina, Z., and Everhart, E. 1987, AJ, 83, 64
Marsden, B.G., and Williams, G.V. 1992, The *Catalogue of Cometary Orbits*, 7th edition, IAU Central Bureau for Astronomical Telegrams.
Meegan, C.A., et al. 1992, Nature, 355, 143
Narayan, R., Paczyński, B., and Piran, T. 1992, ApJ Lett, 395, L83
Oja, H. 1975, A&A, 43, 317
Oort, J.H. 1950, Bull.Astr.Inst.Neth, 11, 91
Paczyński, B. 1991, ACTA Astronomica, 41, 257
Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.P. 1992, In *Numerical Recipes in Fortran*, Second edition, Cambridge University Press.
Press, W.H., Rybicki, G.B., and Schneider, D.P. 1992, CFA preprint.
Watson, G.S. 1983, In *Statistics On Spheres*, University of Arkansas Lecture Notes in Mathematical Sciences, Volume 6. New York: John Wily, pg 14.
Weissman, P.R. 1990, Nature, 344, 825

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