Hybrid algorithm for two-terminal reliability evaluation in communication networks

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ABSTRACT
Network reliability is valuable in establishing a survivable communication network. Reliability evaluation algorithms are used in the design stage and during the network deployment. This work presents a new multistage hybrid technique for two-terminal reliability evaluation problem. It is based on a combination of graph reduction techniques and tie-set method. A new approach has been introduced for deducing tie-sets in a network containing both unidirectional and bi-directional edges. The proposed algorithm can be applied for both simple and complex networks without restrictions. The results confirm that new algorithm evaluates network's reliability with decreasing computing time compared to classical algorithms. The results for a case study of a 20-node network have demonstrated that the required time for reliability evaluation is decreased from (t>1 hour) in the case of using a classical algorithm, to (t<1 second) for the new algorithm.

Keywords: Graph reduction, Hybrid algorithm, Tie-set, Two-terminal reliability

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1. INTRODUCTION
Nowadays, data communication systems and especially computer networks are the major characteristic of the modern world technology. The network reliability is one of the important Quality of Service (QoS) factors beside security, availability, and delay. Network reliability can be defined as the probability of performing the mentioned functionality of a network in a successful way [1]. A further definition of the network reliability states that it is the probability of achieving successful communication operation between the transmitter and the receiver in a network [2]. Thus, high reliability has become an inevitable consequence for various controlled applications such as military, aircraft systems, and banking systems, where faults may cause financial and human life damages [3]. The network reliability requirement depends on the application and type of network such as wired network [4], wireless sensor network [5], mobile system [6], electrical distribution networks [7], and the electrical grid reliability assessment [8, 9]. The network reliability is classified into three types according to the number of involved source-destination nodes. The first, is the two-terminal reliability, concerning the measurement of the reliability between one source and one destination node. This problem has been treated widely because it is the basic of other reliability types [10-12]. The second form is the all-terminal reliability where all nodes are considered as possible source-destination pair as presented in [13]. Finally, the most general term is the k-terminal reliability where the problem is defined according to the value of k [14, 15]. Various techniques and
algorithms are available for computing the network reliability, including state space decomposition [16],
Graph Reduction Technique (GRT) [17], cut-sets, tie-sets [18], Monte Carlo method [19], and conditional
probability and connection matrix techniques [20]. Some of the mentioned methods may fail depending on
the complexity of the network, the required accuracy, and the algorithm convergence speed. Different
attempts have been continuously exerted to the development of new exact algorithms as in [21], or
approximated methods algorithms as presented in [22, 23]. Exact methods are usually preferred when
the purpose is an accurate calculation of static network's reliability, whereas approximation becomes the best
solution in the case of dynamic network topology, as in military or mobile networks in general. The
combination of more than one method can improve the reliability evaluation process by taking the
advantages of each basic method used as components [24]. The major factors affecting the network
availability from different layers are classified in order to indicate their impact. A review has recently been
published concerning the probability methods in [25], where the most important trends and achievements in
network reliability algorithms are presented.

The present work proposes a new algorithm for two-terminal reliability calculation; the Multistage
Hybrid Technique (MHT), which is based on the successive application of two known techniques: the GRT,
and the Tie-Set Method (TSM). The application of the algorithm is simulated by MATLAB with a complex
network of 20-node and the results are compared with those obtained from the well-known classical tie-set
algorithm. The new algorithm has proved its capability for real time reliability evaluation of random
networks. The remaining parts of this paper are organized as follows. Section 2 introduces the basic
theoretical backgrounds. Section 3 presents the research method and the MHT algorithm steps. The results
from algorithm simulation are discussed in Section 4. Finally, Section 5 concludes this work.

2. PRELIMINARIES
2.1. Network modeling

The communication network (CN) is one of the many physical problems that can be modeled
graphically in order to be treated easily during design and enhancement phases. Any CN consists basically of
computer devices, communication links, routers, switches, and other components connected together. It can be
represented by an equivalent graph \( G = (N, E) \), where \( N (n_1, n_2, ..., n_k) \) is the set of \( k \)-nodes and \( E \)
\((e_1, e_2, ..., e_m)\) is a set of \( m \)-links. Each link \( e_j \) has two states i.e. operational state with probability \( p_j \),
and failing state with probability \( q_j = 1 - p_j \). The network topology can be presented as a matrix where element
\( L_{jk} \), represents the probability of a link between nodes \( n_j \), and \( n_k \). The diagonal elements give nodes
probability. Assuming that the network satisfies the following assumptions:

a) Perfectly reliable nodes by the use of redundant materials \((L_{ii} = 1)\),
b) Network components fail independently,
c) Each edge is either in working state or in failing state with known constant discrete probability.

The assumption of perfect nodes will not be negatively reflected to the generality of the proposed
algorithm because a non-perfect node affects only the composition of connectivity matrix. A non-perfect node
is replaced by two perfect nodes with a link between then with a probability equal to the probability of the
original non-perfect node. This will increase the dimension of the connectivity matrix by one.

2.2. TSM and GRT theoretical principles

A tie-set is a group of network components with the property that if all components are in an
operating state, then there is a path between the source node \( n_s \) to the target node \( n_d \). If none of its
components can be removed without the loss of the above property, then the tie-set is minimal [26]. TSM
consists of listing all minimal tie-set, and followed by the application of the inclusion equation called Poincare
equation. \((T_n)\), is defined as the group of successive links forming the minimal path between \( n_s \), and \( n_d \). If
there are \((i)\) tie-sets, then the two-terminal reliability is given by:

\[
R_{sd} = P(T_1 \cup T_2 \cup T_3 \ldots \cup T_i)
\]  

Where \( P \) is the event probability If the tie sets are, all disjoints or mutually exclusive, then (1) can be
written as

\[
R_{sd} = P(T_1) + P(T_2) + P(T_3) + \cdots + P(T_i)
\]  

Since tie sets are not disjoint events in general, then (1) can be written as [27]:

\[
R_{sd} = P(T_1 + T_2 + T_3 + \cdots + T_i)
\]
Hybrid algorithm for two-terminal reliability evaluation in ... (Musaria Karim Mahmood)
then there are no parallel links and series reduction can be directly applied. If at least one element of \( M_{ij2} \) matrix is not null, then a parallel reduction is required. For all elements of \( M \) matrix, if \( p_{ij2} \neq 0 \), then:

\[
\begin{align*}
p_{ij1} &= p_{ij1} \cup p_{ij2} = p_{ij1} + p_{ij2} = 1 - (q_{ij1} \times q_{ij2}) \\
p_{ij2} &= 0
\end{align*}
\]

(7)

(8)

In case \( (t > 2) \) parallel edges in the last layer matrix \( M_{i,j,t>2} \) are treated firstly by considering layer \( M_{i,j,t>2} \) as the base layer. After applying parallel reduction procedure on the two previous matrices, the first one is deleted (all its elements become zeros), while the other one is updated to contain the new calculated values. The flowchart of the parallel reduction procedure is illustrated in the Figure 2(a).

3.3. Series reduction procedure

The series reduction technique is applied after the parallel procedure as shown in Figure 2(b). It starts by checking the initial condition of node validation that is, the node \( n_i \) is neither a source node \( n_s \) nor a destination node \( n_d \). The connectivity of \( n_i \) is inspected from \( M \). If \( n_i \) is connected to only two terminal nodes \( n_j \) and \( n_m \), then it is removed from the graph by series simplification and the dimension of \( M \) is decreased by eliminating row \((i)\), and column \((i)\). The application of the following equations declares the addition of a new link between \( n_j \) and \( n_m \) and the removal of node \( n_i \).

\[
\begin{align*}
p_{j,m,1} &= P_{j,1,1} \times P_{l,m,1} \\
p_{j,1,1} &= P_{l,m,1} = P_{i,j,m} = 0
\end{align*}
\]

(9)

(10)

There is a possibility of new addition of parallel links between \( n_j \) and \( n_m \) to be checked where the parallel reduction must be repeated to resolve this problem before continuing. This is done via the inspection of the parallel_index number.

![Flowchart of the parallel and series reduction algorithm](image-url)

Figure 2. Parallel and series reduction algorithm
3.4. Tie-sets generation

On completing the reduction stage the matrix $M_{N,N}$ is converted into two-dimensional matrix $R_{N_r,N_r}$ which can be used to generate arrangement matrix, $T(v,N_r)$, where $(N_r)$ is the number of nodes after the application of GRT, and $v$ is the number of different arrangements for $(N_r - 1)$ nodes (excluding $n_y$), that is:

$$v = (N_r - 1)!$$

(11)

The tie-sets generation step starts by denoting the source node $n_s$, and the destination node $n_d$, then seeking for all the possible minimum paths connecting the pair $(n_s,n_d)$. A single route may be composed of one or a group of edges and nodes. Since the remaining nodes $N_r$ are less than the original number of nodes before reduction, the arrangement operation does not demand a very large memory or consuming a long processing time. Matrix $T$ is formed by enumerating all possible combinations of the remaining $(N_r - 1)$ nodes. The elements of $T$ are node numbers giving the location of links between nodes in the matrix $R$. The first column is filled with all elements that are equal to the number of the source node $n_s$. For example: if $T_{3,2} = 5$, and $T_{3,3} = 3$, it means that node number (5) is located in rows (3) and column number (2) in the $T$ matrix, and the element after (in the same row) is node number (3). In order to find the corresponding link probability between node (5), and node (3), element $R[5,3] = P_{S,3}$, is copied from matrix $R$. If $P_{S,k} = 0$, then there is no direct connection between node $n_i$, and $n_k$. An action is made by replacing node $n_k$ by $n_i$ in matrix $T$. This measure ensures the correctness of the connection check operation since no possible connecting case will be skipped. In order to get the correct sequence for each tie-set, the following three simplification steps have to be followed:

a) Eliminate all the repeated nodes in one row,

b) Remove each node after the destination $n_d$, and

c) Eliminate redundant tie-sets.

Finally, we will get a matrix $TS(t_{ts},t_r)$, which contains the entire tie-sets. The number of rows $t_{ts}$ is equal to the number of the tie-sets, while the number $t_r$ is the node number in a specific tie-set with $(t_r \leq N_r)$. Figure 3 depicts tie-set generation procedure.

![Diagram of Tie-sets generation algorithm](image-url)
Table 1. Simulation results

| S  | D  | $M_{PH}$ | $T_{PH}(s)$ | $M_{PT}$ | $T_{PT}(s)$ | R  |
|----|----|----------|-------------|----------|-------------|----|
| 1  | 2  | 0.042    | 30          |          |             | R  |
| 1  | 3  | 0.0141   | 20          |          |             | R  |
| 1  | 4  | 0.0408   | 28          |          |             | R  |
| 1  | 5  | 0.0176   | 21          |          |             | R  |
| 1  | 6  | 0.0079   | 25          |          |             | R  |
| 1  | 7  | 0.0028   | 9           |          |             | R  |
| 1  | 8  | 0.0027   | 12          |          |             | R  |
| 1  | 9  | 0.0029   | 12          |          |             | R  |
| 1  | 10 | 0.0019   | 12          |          |             | R  |
| 1  | 11 | 0.0017   | 21          |          |             | R  |
| 1  | 12 | 0.0099   | 3           | 1.21665  | 0.9622     | R  |
| 1  | 13 | 0.0145   | 4           | 1.06430  | 0.9468     | R  |
| 1  | 14 | 0.0129   | 4           | 1.01374  | 0.9404     | R  |
| 1  | 15 | 0.0142   | 4           | 0.95241  | 0.9362     | R  |
| 1  | 16 | 0.0027   | 3           | 0.95085  | 0.9497     | R  |
| 1  | 17 | 0.0027   | 3           | 1.04519  | 0.9622     | R  |
| 1  | 18 | 0.0026   | 3           | 0.97193  | 0.9414     | R  |
| 1  | 19 | 0.0024   | 3           | 0.94365  | 0.9372     | R  |
| 1  | 20 | 0.0023   | 3           | 0.94788  | 0.8435     | R  |
| 2  | 3  | 0.3155   | 12          | 13.5680  | 0.9888     | R  |
| 2  | 4  | 0.4314   | 19          | 894.000  | 0.9778     | R  |
| 2  | 5  | 0.0293   | 15          | 19.2411  | 0.9865     | R  |
| 2  | 6  | 0.0136   | 11          | 14.8627  | 0.9888     | R  |
| 2  | 7  | 0.0234   | 22          |          |             | R  |

Figure 4. Simulated topology

4. RESULT AND DISCUSSION

The MHT performance is evaluated by a 20-node random topology implementation as shown in Figure 4. The topology is simulated using $P_k = 0.9$ as link probability, and perfect nodes. For clarity, only a sample of the results is listed in Table 1, where nodes ($n_1, n_2, n_3, n_4$) are considered successively as source nodes for all possible destinations. The results of the MHT Algorithm are compared against existing classical tie-sets algorithms. $S$ is the source node, while $D$ is the destination node. $R$ is the reliability, which must be the same for both simulated algorithms. $M_{PH}$ and $T_{PH}(s)$ are respectively the number of tie-sets and time required for reliability evaluation for the new MHT algorithm. $M_{PT}$ and $T_{PT}(s)$ are the same variables for the classical tie-sets algorithms, the Path Tracing Algorithm (PTA). The results show a clear improvement by a clear reduction in the number of tie-sets using MHT compared to PTA. This decrease will have a direct impact on decreasing the computing time required for reliability evaluation. For example, for the commodity $(S = n_1; D = n_2)$, the number of tie-set has decreased from (30) to (8) as shown in the first row of Table 1. This decrease is positively reflected in the computing time required for reliability evaluation. This time is reduced from $(t > 1 \text{ Hour})$ in PTA to (0.042 sec) when MHT is applied. For complicated networks with high node number, the improvement is expected to be much more significant.
An efficient algorithm named MHT is proposed. The new algorithm has its efficiency due to the fact that it combines the advantages of both TSM and GRT algorithms, and can be applied to any network regardless of whether the network topology is simple, medium or complex. MHT efficiency is demonstrated by the application of GRT first, which yields to simplifying the network topology resulting a simple application of the TSM. The obtained simulation results show a significant improvement in the required time for network reliability evaluation. In this context, MHT can be considered as a real-time tool for network reliability calculation.

5. CONCLUSION

Several methods have been proposed to compute network reliability evaluation. TSM, and GRT are very often used, especially in probabilistic context. Tie-set can treat small to medium complexity networks. GRT is used to evaluate all network types. In this paper, an efficient algorithm named MHT is proposed. The new algorithm has its efficiency due to the fact that it combines the advantages of both TSM and GRT algorithms, and can be applied to any network regardless of whether the network topology is simple, medium or complex. MHT efficiency is demonstrated by the application of GRT first, which yields to simplifying the network topology resulting a simple application of the TSM. The obtained simulation results show a significant improvement in the required time for network reliability evaluation. In this context, MHT can be considered as a real-time tool for network reliability calculation.

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