The CKM Matrix and Its Origin

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Abstract

In the context of the supersymmetric unification model in which the massless sector contains extra particles beyond those in the minimal supersymmetric standard model, we obtain mixings between quarks (leptons) and the extra particles which are closely intertwined with Yukawa hierarchies. With the assumption that the unification gauge group $G$ includes $SU(2)_R$, it is shown that the non-trivial texture of the CKM matrix originates from the extra-particle mixings. The CKM matrix of quarks emerges as a consequence of the mixings between the down-type quarks and colored Higgses, both of which are $SU(2)_L$-singlets. On the other hand, the CKM matrix of leptons is due to the mixings stemming from the seesaw mechanism with the hierarchical Majorana mass matrix of right-handed neutrinos.
1 Introduction

The characteristic structure of the fermion spectra and flavor mixings likely has an important connection with the gauge symmetry and matter content above the gauge unification scale $M_U$. Unless the unification gauge group $G$ contains $SU(2)_R$, the textures of interaction terms for up-sector and down-sector of quarks (leptons) must be distinct from each other. Then, the diagonalization matrices of fermion mass matrices necessarily distinguish the up-sector from the down-sector. In this case the non-trivial structure of the Cabbibo-Kobayashi-Maskawa (CKM) matrix can essentially be traced back to the underlying theory above the scale $M_U$ such as the string theory. Alternatively, if the gauge group $G$ is large enough to contain $SU(2)_R$ as well as the standard model gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, the up-sector of quarks (leptons) shares interaction terms with the down-sector. Namely, above the scale $M_U$ the disparity between the diagonalization matrices for the up-sector and the down-sector does not emerge. This disparity occurs only below the scale $M_U$ purely within the effective field theory. In a wide class of supersymmetric unification models, the massless sector contains extra particles beyond those in the minimal supersymmetric extension of the standard model (MSSM), and then there may occur extra-particle mixings such as between quarks (leptons) and colored Higgs fields (doublet Higgs fields). As pointed out in a previous paper, it is possible that the extra-particle mixings significantly affect fermion masses and flavor mixings.

Therefore, in order to study fermion masses and flavor mixings, it is important to specify what kinds of extra particles appear in the effective field theory and to study what kinds of extra-particle mixings take place. In this paper we consider an $E_6$-type model, which is inspired by the level-one string theory.

The hierarchical structure of fermion masses strongly suggests the existence of some kinds of the flavor symmetry in the effective field theory above the scale $M_U$. If this is the case, Yukawa couplings arise from non-renormalizable interactions which respect the flavor symmetry. In other words, Yukawa couplings are generically functions of dynamical variables which carry the flavor charges and acquire non-zero VEVs around the scale $M_U$. Eventually, at low energies there appear effective Yukawa couplings as constants up to renormalization group (RG) effects. Thus the Froggatt-Nielsen mechanism is at work for the interactions. Specifically, the effective Yukawa interactions for up-type quarks are of the form

$$M_{ij}^{(q)} Q_i U_j^c H_u$$

with

$$M_{ij}^{(q)} = c_{ij} \left( \frac{X}{M_S} \right)^{m_{ij}} = c_{ij} x^{m_{ij}},$$

where the subscripts $i$ and $j$ are the generation indices, and all of the constants $c_{ij}$ are of $O(1)$ with rank $c_{ij} = 3$. The superfield $X$, which is singlet under the unifica-
tion gauge group $G$ but carries a certain flavor charge, is an appropriate composite superfield with canonical normalization. The scale $M_S$ is the intrinsic scale of the underlying string theory and is supposed to lie around the reduced Planck scale. The ratio $x = \langle X \rangle / M_S$ is assumed to be slightly less than 1. The large hierarchy results from raising $x$ to large powers. Due to the flavor symmetry, the exponents $m_{ij}$ are determined as the flavor-charges of $Q_i$, $U^c_j$, and $H_u$. Consequently, it is possible for us to obtain Yukawa hierarchies.

In this paper we solve for the extra-particle mixings which are closely intertwined with Yukawa hierarchies. With the assumption $G \supset SU(2)_R$, we show that the non-trivial texture of the CKM matrix originates from the extra-particle mixings purely within the effective field theory. The observed structure of the CKM matrix of quarks is attributable to a large mixing between the down-type quarks and colored Higgses, both of which are $SU(2)_L$-singlets. On the other hand, the CKM matrix of leptons emerges as a consequence of the seesaw mechanism [3] with the hierarchical Majorana mass matrix for right-handed neutrinos. In previous papers [1][4] the present author and his collaborators explored the fermion spectra and the CKM matrix based on a $SU(6) \times SU(2)_R$ string-inspired model. In this paper we show that the main results in the previous papers can be derived within a more general framework. Furthermore, the distinct origins of the CKM matrices of quarks and leptons are ascertained. Since we ignore the phase factors of VEVs for matter fields, we do not discuss the $CP$-violation.

This paper is organized as follows. Assuming that the unification gauge group $G$ contains $SU(2)_R$ as well as $G_{SM}$, in §2 we present the mass matrices of quarks and leptons which arise from the Froggatt-Nielsen mechanism. These mass matrices, except for up-type quarks, imply the extra-particle mixings. After diagonalizing the mass matrices by bi-unitary transformations, we trace the origin of the CKM matrices for quarks and leptons. In §3 we introduce the flavor symmetry $U(1) \times Z_2$. It is shown that the textures of the CKM matrices are controlled by the flavor charges, and so by the magnitude of the extra-particle mixings. In §4 we study the dependence of the relations among the fermion spectra, $V^Q_{CKM}$ and $V^L_{CKM}$, on the unification gauge group $G$. Section 5 is devoted to a summary.

2 Mass matrices and their diagonalization

Based on the $E_6$-type model, we assume that the unification gauge group $G$ is rank 6, and that $G$ contains $SU(2)_R$ as well as $G_{SM}$. The matter chiral superfields, if we represent them in terms of $E_6$, consist of

$$N_f \, 27 + \delta (27 + 27^*). \quad (3)$$

Here $N_f$ denotes the family number at low energies, and $\delta$ is the set number of vector-like multiplets. Note that adjoint or higher representation matter fields are
not allowed in the level-one string theory. Let us now consider the case \( N_f = 3 \) and \( \delta = 1 \). Matter fields in the 27 representation of \( E_6 \) fall in two categories,

\[
\begin{align*}
\Phi(SU(2)_R\text{-}\text{singlet}) & : \quad Q, \ L, \ g, \ g^c, \ S, \\
\Phi(SU(2)_R\text{-}\text{doublet}) & : \quad (U^c, D^c), \ (N^c, E^c), \ (H_u, H_d),
\end{align*}
\]

where \( g, g^c \) and \( H_u, H_d \) represent colored Higgs and doublet Higgs fields, respectively. \( N^c \) represents the right-handed neutrino superfield and \( S \) is an \( SO(10) \)-singlet. When the gauge group \( G \) becomes smaller than \( E_6 \) via flux breaking within the string theory, the vector-like multiplets potentially contain only some part of the 27 representation but not all of it. The concrete matter contents of the vector-like multiplets are closely linked to the topological structure of the compactified space. Although \( L \) and \( H_d \) (\( D^c \) and \( g^c \)) have the same quantum numbers under the standard model gauge group \( G_{SM} \), they reside in different irreducible representations of \( G \). Gauge invariant trilinear couplings are of the forms

\[
\Phi^3 = QQg + Qg^cL + g^cgS + QH_dD^c + QH_uU^c + LH_dE^c \\
+ LH_uN^c + SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c.
\]

We now introduce the \( Z_2 \) symmetry (R-parity) and attach odd R-parity to chiral multiplets \( \Phi_i \) \((i = 1, 2, 3)\) and even R-parity to one set of vector-like multiplets \( \Phi_0 \) and \( \overline{\Phi} \). Ordinary quarks and leptons are included in chiral multiplets \( \Phi_i \) \((i = 1, 2, 3)\). Since light Higgs scalars are even under R-parity, light Higgs doublets reside in \( \Phi_0 \) and/or \( \overline{\Phi} \). The present scheme allows the R-parity to remain unbroken all the way down to the electroweak scale. [5] Under an appropriate condition, the gauge symmetry \( G \) is spontaneously broken in two steps at the scales \( \langle S_0 \rangle = \langle S \rangle \) and \( \langle N_0^c \rangle = \langle N \rangle \) as

\[
G \longrightarrow \langle S_0 \rangle \quad G' \overset{\langle N_0^c \rangle}{\longrightarrow} \quad G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y,
\]

where \( \langle S_0 \rangle = M_U \) and \( G' \supset SU(2)_R \). [6] In this model \( SU(2)_R \) gauge symmetry is spontaneously broken at the scale \( \langle N_0^c \rangle \). In view of the phenomenological result that three gauge coupling constants of \( G_{SM} \) meet around \( 2 \times 10^{16} \text{GeV} \) in the MSSM, the scale \( \langle N_0^c \rangle \) is likely to be \( O(10^{16} \text{GeV}) \). [7] Some particles could be lying somewhere between the scale \( M_U \) and the TeV scale.

Let us discuss the mass matrices of quarks and leptons. To begin with the superpotential for up-type quarks is given by

\[
W_U \sim X^{m_{ij}} Q_i U^c_j H_{u0},
\]

where \( i, j = 1, 2, 3 \) and all terms are characterized by couplings of \( O(1) \) in units of \( M_S \). The \( G \)-singlet superfield \( X \) is assumed to carry a certain flavor charge. In the present framework the superfield \( X \) is a holomorphic function of \( S_0 \overline{S} \) and moduli
fields. The exponents \( m_{ij} \) are determined by the flavor symmetry. The \( 3 \times 3 \) mass matrix \( M^{(q)}_{ij} v_u \) of the up-type quarks can be diagonalized by a bi-unitary transformation as

\[
V_u^{-1} M^{(q)} U_u,
\]

where \( v_u = \langle H_{u0} \rangle \). Hereafter all mass matrices are assumed to be non-singular. Although \( x \) by itself is not a very small number, the physical parameters may be very small if they depend on high powers \( m_{ij} \). We are not interested in obtaining the precise values of the parameters in the mass matrix but focus on orders of magnitude.

We next study the mass matrix for down-type quarks, in which there appear mixings between \( g^c \) and \( D^c \) at energies below the scale \( \langle N^c_0 \rangle \). The superpotential of down-type colored fields is of the form

\[
W_D \sim X^{z_{ij}} g_i g_j^c S_0 + X^{g_{ij}} g_i D_j^c N_0^c + X^{m_{ij}} Q_i D_j^c H_{d0},
\]

where the exponents \( z_{ij}, g_{ij} \) and \( m_{ij} \) are determined by flavor symmetry, and all terms are also multiplied by \( O(1) \) couplings in \( M_S \) units. Below \( \langle N^c_0 \rangle \) the mass matrix of down-type colored fields is expressed as the \( 6 \times 6 \) matrix

\[
\hat{M}_d = \frac{g^c}{D} \begin{pmatrix} g^c & D^c \\ Z^{(q)} & G^{(q)} \\ 0 & \rho_d M^{(q)} \end{pmatrix}
\]

in \( M_S \) units, where we define

\[
Z^{(q)}_{ij} = O(x^{z_{ij}}) \langle S_0 \rangle / M_S, \quad G^{(q)}_{ij} = O(x^{g_{ij}}) \langle N^c_0 \rangle / M_S
\]

and \( \rho_d = \langle H_{d0} \rangle / M_S = v_d / M_S \). \( \hat{M}_d \) yields mixings between \( g^c \) and \( D^c \) explicitly. An early attempt of explaining the CKM matrix via \( g^c-D^c \) mixings appears in Ref.\[8\], in which a SUSY \( SO(10) \) model is considered. Since \( \rho_d \) is a very small number\((\sim 10^{-16})\), it is plausible that the elements of the \( 3 \times 3 \) submatrices \( Z^{(q)} \) and \( G^{(q)} \) are sufficiently large compared with those of \( \rho_d M^{(q)} \). Therefore, the left-handed light quarks consist almost completely of \( D \)-components of the quark doublet \( Q \). On the other hand, the mixing between \( SU(2)_L \)-singlet states \( g^c \) and \( D^c \) can be sizable, depending on the ratio of \( G^{(q)} \) to \( Z^{(q)} \). This type of mixing does not occur for up-type quarks. If all elements of \( G^{(q)} \) are sufficiently small relative to those of \( Z^{(q)} \), \( \hat{M}_d \) is approximately separated into two \( 3 \times 3 \) submatrices. We have three heavy modes and three light modes which are mainly determined by \( Z^{(q)} \) and \( \rho_d M^{(q)} \), respectively. The three light modes correspond to \( d \)-, \( s \)- and \( b \)-quarks. In this case the mass spectra of the light down-type quarks is the same as those of the up-type quarks up to the ratio \( v_u / v_d = \tan \beta \), and also \( V_{\text{CKM}}^Q \) becomes almost a unit matrix. This result is not
preferable phenomenologically. Alternatively, if the elements of $G^{(q)}$ are comparable to or larger than those of $Z^{(q)}$, the mass spectra of down-type quarks are generally different from those of up-type quarks. In addition, we have a non-trivial CKM matrix. However, when the elements of $G^{(q)}$ are significantly larger than those of $Z^{(q)}$, the $d$-quark Yukawa coupling becomes too small relative to the $u$-quark coupling. Then we do not consider such a case. Thus, to obtain a phenomenologically viable solution, hereafter we consider a large mixing between $g^c$ and $D^c$, that is, $G_{ij}^{(q)} \sim Z_{ij}^{(q)}$.

$\hat{M}_d$ can be diagonalized by a bi-unitary transformation as

$$\hat{V}_d^{-1} \hat{M}_d \hat{U}_d. \quad (11)$$

Using a perturbative expansion with respect to $\rho_d$, we can solve the eigenvalue problem. After some algebra the $6 \times 6$ diagonalization unitary matrices $\hat{V}_d$ and $\hat{U}_d$ turn out to be

$$\hat{V}_d \simeq \left( \begin{array}{cc} W_d & -\rho_d(A_d + B_d)^{-1}G^{(q)}M^{(q)\dagger}V_d \\ \rho_dM^{(q)}G^{(q)\dagger}(A_d + B_d)^{-1}W_d & \rho_d(A_d + B_d)^{-1}G^{(q)}M^{(q)\dagger}V_d \end{array} \right), \quad (12)$$

$$\hat{U}_d \simeq \left( \begin{array}{cc} Z^{(q)\dagger}W_d(A_d^{(0)})^{-1} & -(Z^{(q)})^{-1}G^{(q)}(M^{(q)})^{-1}V_d \Lambda_d^{(2)} \\ G^{(q)\dagger}W_d(A_d^{(0)})^{-1} & (M^{(q)})^{-1}V_d \Lambda_d^{(2)} \end{array} \right) \quad (13)$$

in the $\rho_d$ expansion, where

$$A_d = Z^{(q)}Z^{(q)\dagger}, \quad B_d = G^{(q)}G^{(q)\dagger}. \quad (14)$$

The $3 \times 3$ unitary matrices $W_d$ and $V_d$ are determined such that the matrices

$$W_d^{-1}(A_d + B_d)W_d = (\Lambda_d^{(0)})^2, \quad (15)$$

$$V_d^{-1} \left[ M^{(q)}(G^{(q)})^{-1}(A_d^{-1} + B_d^{-1})^{-1}(G^{(q)\dagger})^{-1}M^{(q)\dagger} \right] V_d = (\Lambda_d^{(2)})^2 \quad (16)$$

become diagonal. The diagonal elements of $M_S \Lambda_d^{(0)}$ represent masses of three heavy modes which are expected to be roughly of $O(M_U)$. On the other hand, three diagonal elements of $\nu_d \Lambda_d^{(2)}$ represent masses of light modes corresponding to $d$-, $s$- and $b$-quarks. From Eq.(13) the mass eigenstates of light $SU(2)_L$-singlet down-type quarks are given by

$$\hat{D}^c \simeq \Lambda_d^{(2)}V_d^{-1}(M^{(q)\dagger})^{-1}G^{(q)\dagger} \left[ -(Z^{(q)})^{-1}g^c + (G^{(q)\dagger})^{-1}D^c \right]. \quad (17)$$

As seen in Eq.(10), when the mixings between $g^c$ and $D^c$ are large, $V_d$ evidently differs from $V_u$. Therefore, we obtain a nontrivial CKM matrix, $V_{\text{CKM}}$

$$V_{\text{CKM}} = V_u^{-1}V_d. \quad (18)$$

We next proceed to study the mass matrices for the lepton sector. Under $G_{SM}$, the colorless $SU(2)_L$-doublet fields $L$ and $H_d$ are not distinguished from each other, and

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then $L$-$H_d$ mixing occurs at energies below the scale $\langle N_0^c \rangle$. Since both $L$ and $H_d$ are $SU(2)_L$-doublets, $L$-$H_d$ mixing does not trigger the disparity between diagonalization matrices for the up-sector and the down-sector of leptons. However, the disparity is possibly triggered by another mixing, namely, by the seesaw mechanism for neutrinos. For charged leptons the superpotential is

$$ W_E \sim X^{h_{ij}} H_d^i H_d^j S_0 + X^{k_{ij}} L_i H_{w_j} N_0^c + X^{l_{ij}} L_i E_d^c H_{d0} $$

(19)

in $M_S$ units, where the exponents $h_{ij}$, $k_{ij}$ and $l_{ij}$ are determined by the flavor symmetry. Below the scale $\langle N_0^c \rangle$, the mass matrix for charged leptons has the form

$$ \hat{M}_l = \frac{H_d}{L^-} \begin{pmatrix} H_u^+ & E^c+ \\ Z^{(l)} & 0 \\ G^{(l)} & \rho_d M^{(l)} \end{pmatrix} $$

(20)

in $M_S$ units, where

$$ Z^{(l)}_{ij} = O(x^{h_{ij}}) \frac{\langle S_0 \rangle}{M_S}, \quad G^{(l)}_{ij} = O(x^{k_{ij}}) \frac{\langle N_0^c \rangle}{M_S}, \quad M^{(l)}_{ij} = O(x^{l_{ij}}). $$

(21)

Note that the matrix $Z^{(l)}$ is symmetric. This $6 \times 6$ matrix $\hat{M}_l$ exhibits a texture quite similar to that of $\hat{M}_d^t$. Then, by observing the one-to-one correspondence $Z^{(l)} \leftrightarrow Z^{(q)^t}$, $G^{(l)} \leftrightarrow G^{(q)^t}$ and $M^{(l)} \leftrightarrow M^{(q)^t}$, the diagonalization of $\hat{M}_l$ is closely analogous to that in the case of down-type quarks. In fact, $\hat{M}_l$ is diagonalized by the bi-unitary transformation

$$ \hat{V}_l^{-1} \hat{M}_l \hat{U}_l $$

(22)

with

$$ \hat{V}_l \approx \begin{pmatrix} Z^{(l)} W_l (A_l^{(0)})^{-1} - (Z^{(l)^t})^{-1} G^{(l)^t} (M^{(l)^t})^{-1} V_l A_l^{(2)} \\ G^{(l)^t} W_l (A_l^{(0)})^{-1} (M^{(l)^t})^{-1} V_l A_l^{(2)} \end{pmatrix}, $$

(23)

$$ \hat{U}_l \approx \begin{pmatrix} W_l \\ \rho_d M^{(l)^t} G^{(l)} (A_l + B_l)^{-1} W_l \\ - \rho_d (A_l + B_l)^{-1} G^{(l)^t} M^{(l)} V_l \end{pmatrix} $$

(24)

in the $\rho_d$ expansion, where

$$ A_l = Z^{(l)^t} Z^{(l)}$, \quad B_l = G^{(l)^t} G^{(l)}. $$

(25)

$W_l$ and $V_l$ are unitary matrices such that

$$ W_l^{-1} (A_l + B_l) W_l = (A_l^{(0)})^2, $$

(26)

$$ V_l^{-1} [M^{(l)^t} (G^{(l)^t})^{-1} (A_l^{-1} + B_l^{-1})^{-1} (G^{(l)})^{-1} M^{(l)}] V_l = (A_l^{(2)})^2 $$

(27)
are diagonal. The three diagonal elements of $v_d \Lambda^{(2)}_l$ represent masses of light charged leptons corresponding to $e$, $\mu$ and $\tau$. Equation (24) implies that the light $SU(2)_L$-singlet charged leptons are mainly $E'$-components. From Eq.(23), the mass eigenstates of light $SU(2)_L$-doublet charged leptons are given by

$$\tilde{L}^- \simeq \Lambda_l^{(2)} \mathcal{V}^{-1}_{l} (M_l^{(l)})^{-1} G^{(l)} \left[ -(Z_l^{(l)})^{-1} H_d^- + (G_l^{(l)})^{-1} L^- \right]. \quad (28)$$

Consequently, when the elements of $(Z_l^{(l)})^{-1}$ and $(G_l^{(l)})^{-1}$ are comparable to each other, there occurs a large mixing between $H_d^-$ and $L^-$. In the neutral lepton sector we have the superpotential

$$W_N \sim X^{h_{0j}} H_{di} H_{uj} S_0 + X^{k_{0j}} L_i H_{uj} N^c_0 + X^{l_{0j}} L_i N^c_j H_{u0} + X^{n_{0j}} (N^c_i N^c_j) \left( N^c_i N^c_j \right) \quad (29)$$

in $M_S$ units. Below $\langle N^c_0 \rangle$ the mass matrix for neutral leptons becomes the $12 \times 12$ matrix

$$\tilde{M}_N = \begin{pmatrix}
H^0_u & H^0_d & L^0 & N^c \\
H^0_u & 0 & Z^{(l)} & G^{(l)} & 0 \\
H^0_d & Z^{(l)} & 0 & 0 & 0 \\
L^0 & G^{(l)} & 0 & 0 & \rho_u M^{(l)} \\
N^c & 0 & 0 & \rho_u M^{(l)T} & N
\end{pmatrix} \quad (30)$$

in $M_S$ units, where

$$N_{ij} = O(x^{n_{ij}}) \left( \frac{\langle N^c_0 \rangle}{M_S} \right)^2 \quad (31)$$

and $\rho_u = v_u/M_S$. More precisely, we have to take the contribution of the field $S$ into account. However, to illustrate the general features, the above $12 \times 12$ mass matrix suffices for us, because the main result remains unchanged. By recalling the above study in the charged lepton sector, we first carry out the unitary transformation

$$\hat{U}_{P}^{-1} \tilde{M}_N \hat{U}_P \quad (32)$$

with

$$\hat{U}_P = \begin{pmatrix}
W_l & 0 & 0 \\
0 & \hat{V}_l & 0 \\
0 & 0 & \mathcal{U}_N
\end{pmatrix} \quad (33)$$

Note that $\hat{V}_l$ is the $6 \times 6$ matrix given by Eq.(23), and $W_l$ is defined in Eq.(26). The $3 \times 3$ unitary matrix $\mathcal{U}_N$ diagonalizes the Majorana mass matrix $N$ as

$$\mathcal{U}_N^{-1} N \mathcal{U}_N = \Lambda_N, \quad (34)$$
where $\Lambda_N$ is diagonal. The scales of $M_S \Lambda_N$ are assumed to lie in the range $10^{10} \sim 10^{12}$GeV. As a result, we obtain an approximately separated matrix which is composed of the $6 \times 6$ submatrix for heavy modes and the seesaw-type $6 \times 6$ submatrix. The submatrix for heavy modes leads to the same masses as the heavy modes of charged leptons. The seesaw-type submatrix is of the form

$$
\begin{pmatrix}
0 & \rho_u \Lambda_i^{(2)} \nu_i^{-1} \mathcal{U}_N \\
\rho_u \mathcal{U}_N^{-1} \nu_i \Lambda_i^{(2)} & \mathcal{U}_N^{-1} N \mathcal{U}_N
\end{pmatrix}.
$$

Thus $\tilde{M}_N$ can be diagonalized as

$$
\tilde{U}_N^{-1} \tilde{M}_N \tilde{U}_N
$$

with

$$
\tilde{U}_N = \tilde{U}_P \times \tilde{U}_Q
$$

and

$$
\tilde{U}_Q \simeq \begin{pmatrix}
1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\
1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\
0 & 0 & \nu & \rho_u \Lambda_i^{(2)} \nu_i^{-1} N^{-1} \nu_i \\
0 & 0 & -\rho_u \mathcal{U}_N^{-1} N^{-1} \nu_i \Lambda_i^{(2)} \nu & 1
\end{pmatrix}
$$

in the $\rho_u$ expansion. The unitary matrix $\nu$ in Eq.(38) is defined such that

$$
\nu^{-1} N \nu
$$

is diagonal. From Eq.(38) the mass matrix of light neutrinos has the form

$$
\rho_u^2 \mathcal{N} = \rho_u^2 \Lambda_i^{(2)} \nu_i^{-1} N^{-1} \nu_i \Lambda_i^{(2)}.
$$

Masses of light neutrinos become

$$
m_\nu \simeq \frac{\nu_u^2}{M_S} (\Lambda_i^{(2)})^2 \Lambda_N^{-1}.
$$

It turns out that the light neutrino mass eigenstates are

$$
\tilde{L}_0 \simeq \nu^{-1} \Lambda_i^{(2)} \nu_i^{-1} (M^{(l)})^{-1} G^{(l)} \left[ -(Z^{(l)})^{-1} H_d^{(l)} + (G^{(l)})^{-1} L^{(l)} \right].
$$

Comparing these eigenstates $\tilde{L}_0$ with those of the light charged leptons $\tilde{L}_-$ given by Eq.(28), we find that $\nu$ is nothing but the CKM matrix for leptons, that is,

$$
V_{\text{CKM}}^L = \nu.
$$

As seen in Eq.(40), if the Majorana mass matrix $N$ is proportional to the unit matrix, we find that $V_{\text{CKM}}^L = 1$. Contrastively, when $N$ contains hierarchical structure, we have a non-trivial CKM matrix for leptons. In the next section we explore such a case.
3 The flavor symmetry and mass hierarchies

The superpotential above the scale $M_T$ respects flavor symmetry as well as the gauge symmetry $G$. We now introduce the global $U(1) \times \mathbb{Z}_2$ as the flavor symmetry. As mentioned above, the $\mathbb{Z}_2$-charge is identified with $R$-parity in the MSSM. The charge of the $G$-singlet superfield $X$ is assumed to be $(-1, +)$ under $U(1) \times \mathbb{Z}_2$. Due to the $U(1)$ symmetry, the exponents $m_{ij}$, $z_{ij}$, etc., are determined as the $U(1)$-charges of matter superfields. Denote the $U(1)$-charges of the $i$-th generation matter fields $\Phi_{Ai}$ as $a_{Ai}$ $(i = 1, 2, 3)$, where the $\Phi_A$ in the 27 representation of $E_6$ are classified as

$$
\Phi_A = \begin{cases} 
Q & \text{for } A = 1, \\
(U^c, D^c) & \text{for } A = 2, \\
g & \text{for } A = 3, \\
g^c & \text{for } A = 4, \\
L & \text{for } A = 5, \\
(N^c, E^c) & \text{for } A = 6, \\
(H_u, H_d) & \text{for } A = 7, \\
S & \text{for } A = 8.
\end{cases} \tag{44}
$$

Hereafter we define $\alpha_A$ and $\gamma_A$ by

$$
a_{A2} - a_{A1} = \alpha_A, \quad a_{A3} - a_{A2} = \gamma_A. \tag{45}
$$

All of the $\alpha_A$ and $\gamma_A$ are assumed to be positive. Thus the exponents which appear in the superpotential, for instance the $m_{ij}$, are of the form

$$
m_{ij} - m_{33} = \begin{pmatrix}
\alpha_1 + \gamma_1 + \alpha_2 + \gamma_2 & \alpha_1 + \gamma_1 + \gamma_2 & \alpha_1 + \gamma_1 \\
\gamma_1 + \alpha_2 + \gamma_2 & \gamma_1 + \gamma_2 & \gamma_1 \\
\alpha_2 + \gamma_2 & \gamma_1 & 0
\end{pmatrix}_{ij}, \tag{46}
$$

provided that $m_{33}$ is non-negative. If we define the diagonal matrices according to

$$
\Gamma_A = \begin{pmatrix}
x^{\alpha_A + \gamma_A} & 0 & 0 \\
0 & x^{\gamma_A} & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (A = 1, 2, \cdots, 8) \tag{47}
$$

then the $3 \times 3$ mass matrix $M^{(q)}$ for up-type quarks can be rewritten in the factorized form

$$
M^{(q)} = \Gamma_1 \left[ y_M^{(q)} M^{(q)}_0 \right] \Gamma_2, \tag{48}
$$

where $M^{(q)}_{0ij} = O(1)$ and $y_M^{(q)} = x^{m_{33}}$. Similarly, the other $3 \times 3$ submatrices can be expressed in the factorized forms

$$
\begin{align*}
Z^{(q)} &= \Gamma_3 \left[ y_Z^{(q)} Z^{(q)}_0 \right] \Gamma_4, \\
G^{(q)} &= \Gamma_3 \left[ y_G^{(q)} G^{(q)}_0 \right] \Gamma_2, \\
Z^{(l)} &= \Gamma_7 \left[ y_Z^{(l)} Z^{(l)}_0 \right] \Gamma_7, \\
G^{(l)} &= \Gamma_5 \left[ y_G^{(l)} G^{(l)}_0 \right] \Gamma_7, \\
M^{(l)} &= \Gamma_5 \left[ y_M^{(l)} M^{(l)}_0 \right] \Gamma_6, \\
N &= \Gamma_6 \left[ y_N N_0 \right] \Gamma_6,
\end{align*} \tag{49}
$$

10
where all elements of $Z_0^{(q)}$, $G_0^{(q)}$, $Z_0^{(l)}$, $G_0^{(l)}$, $M_0^{(l)}$ and $N_0$ are of $O(1)$. The constant factors are

\[
y_Z^{(q)} = x^{z_{33}} \frac{\langle S_0 \rangle}{M_S}, \quad y_G^{(q)} = x^{g_{33}} \frac{\langle N_0^c \rangle}{M_S},
\]

\[
y_Z^{(l)} = x^{h_{33}} \frac{\langle S_0 \rangle}{M_S}, \quad y_G^{(l)} = x^{k_{33}} \frac{\langle N_0^c \rangle}{M_S},
\]

\[
y_M^{(l)} = x^{l_{33}}, \quad y_N = x^{n_{33}} \left( \frac{\langle N_0^c \rangle}{M_S} \right)^2,
\]

provided that $z_{33}$, $g_{33}$, $h_{33}$, $k_{33}$, $l_{33}$, $n_{33} \geq 0$. It should be noted that due to the flavor symmetry, each mass matrix is given by a product of two $\Gamma_A$ and a matrix between them, whose elements are of the same order of magnitude. Therefore, the texture of the diagonalization matrices is essentially determined by those of these two $\Gamma_A$.

In the case of up-type quarks, from Eqs. (7) and (48), the texture of the diagonalization matrices $\mathcal{V}_u$ and $\mathcal{U}_u$ of $M^{(q)}$ is governed by $\Gamma_1$ and $\Gamma_2$, respectively. Explicitly, $\mathcal{V}_u$ and $\mathcal{U}_u$ become

\[
\mathcal{V}_u = \begin{pmatrix}
1 - O(x^{2\alpha_1}) & O(x^{\alpha_1}) & O(x^{\alpha_1 + \gamma_1}) \\
O(x^{\alpha_1}) & 1 - O(x^{2\alpha_1}) & O(x^{\gamma_1}) \\
O(x^{\alpha_1 + \gamma_1}) & O(x^{\gamma_1}) & 1 - O(x^{2\gamma_1})
\end{pmatrix},
\]

\[
\mathcal{U}_u = \begin{pmatrix}
1 - O(x^{2\alpha_2}) & O(x^{\alpha_2}) & O(x^{\alpha_2 + \gamma_2}) \\
O(x^{\alpha_2}) & 1 - O(x^{2\alpha_2}) & O(x^{\gamma_2}) \\
O(x^{\alpha_2 + \gamma_2}) & O(x^{\gamma_2}) & 1 - O(x^{2\gamma_2})
\end{pmatrix}.
\]

The diagonalization of $M^{(q)}$ yields the eigenvalues

\[
x^{m_{33}} \times \left( O(x^{\alpha_1 + \gamma_1 + \alpha_2 + \gamma_2}), \quad O(x^{\gamma_1 + \gamma_2}), \quad O(1) \right).
\]

We now assume that the global $U(1)$ charges are assigned in such a way that trilinear couplings of up-type quarks are allowed only for the third generation, i.e.

\[
m_{33} = 0.
\]

It follows that the mass eigenvalues are

\[
v_u \times \left( O(x^{\alpha_1 + \gamma_1 + \alpha_2 + \gamma_2}), \quad O(x^{\gamma_1 + \gamma_2}), \quad O(1) \right),
\]

which correspond to $u$-, $c$- and $t$-quarks, respectively.

For down-type quarks, the diagonalization matrices $\hat{\mathcal{V}}_d$ and $\hat{\mathcal{U}}_d$ of $\hat{M}_d$, which are given by Eqs. (12) and (13), are expressed in terms of $\mathcal{W}_d$ and $\mathcal{V}_d$. Here $\mathcal{W}_d$ and $\mathcal{V}_d$ are the diagonalization matrices for

\[
A_d + B_d = \Gamma_3 \left[ A_d + B_d \right] \Gamma_3, \quad M^{(q)} \left( G^{(q)} \right)^{-1} \left( A_d^{-1} + B_d^{-1} \right)^{-1} \left( G^{(q)} \right)^{-1} M^{(q)\dagger} = \Gamma_1 \left[ C_d + D_d \right]^{-1} \Gamma_1,
\]

where $A_d$, $B_d$, $C_d$, $D_d$, $\Gamma_1$, and $\Gamma_3$ are constants.
where

\[ A_d = (y_Z^{(q)})^2 Z_0^{(q)} \Gamma_4^2 Z_0^{(q)\dagger}, \]

\[ B_d = (y_G^{(q)})^2 G_0^{(q)} \Gamma_2 G_0^{(q)\dagger}, \]

\[ C_d = \left( \frac{y_G^{(q)}}{y_Z^{(q)}} \right)^2 ((Z_0^{(q)})^{-1} G_0^{(q)} (M_0^{(q)})^{-1}) \Gamma_4^{-2} ((Z_0^{(q)})^{-1} G_0^{(q)} (M_0^{(q)})^{-1}), \]

\[ D_d = (M_0^{(q)\dagger})^{-1} \Gamma_2^{-2} (M_0^{(q)})^{-1}. \]

By observing that elements of the matrices \( A_d, B_d, C_d \) and \( D_d \) are

\[ A_{dij} = (y_Z^{(q)})^2 \times O(1), \]

\[ B_{dij} = (y_G^{(q)})^2 \times O(1), \]

\[ C_{dij} = \left( \frac{y_G^{(q)}}{y_Z^{(q)}} \right)^2 \times O(x^{-2\alpha_4-2\gamma_4}), \]

\[ D_{dij} = O(x^{-2\alpha_4-2\gamma_4}), \]

it is easy to obtain the diagonalization matrices

\[ W_d = \begin{pmatrix} 1 - O(x^{2\alpha_3}) & O(x^{\alpha_3}) & O(x^{\alpha_3+\gamma_3}) \\ O(x^{\alpha_3}) & 1 - O(x^{2\alpha_3}) & O(x^{\gamma_3}) \\ O(x^{\alpha_3+\gamma_3}) & O(x^{\gamma_3}) & 1 - O(x^{2\gamma_3}) \end{pmatrix}, \]

\[ V_d = \begin{pmatrix} 1 - O(x^{2\alpha_1}) & O(x^{\alpha_1}) & O(x^{\alpha_1+\gamma_1}) \\ O(x^{\alpha_1}) & 1 - O(x^{2\alpha_1}) & O(x^{\gamma_1}) \\ O(x^{\alpha_1+\gamma_1}) & O(x^{\gamma_1}) & 1 - O(x^{2\gamma_1}) \end{pmatrix}. \]

Note that corresponding elements of the matrices \( V_u \) and \( V_d \) are of the same order of magnitude. To get a phenomenologically viable solution, we assume that a large mixing occurs between \( g^c \) and \( D^c \). This situation is realized by assuming \((Z^{(q)})_{11} \sim (G^{(q)})_{11}\); namely,

\[ y_Z^{(q)} x^{\alpha_4+\gamma_4} \approx y_G^{(q)} x^{\alpha_2+\gamma_2}. \]

In this case the two terms \( C_d \) and \( D_d \) in the brackets of Eq.(57) become comparable, and so coefficients of the leading terms in off-diagonal elements of \( V_u \) and \( V_d \) are different. Consequently, the CKM matrix is given by

\[ V_{\text{CKM}}^O = V_u^{-1} V_d = \begin{pmatrix} 1 - O(x^{2\alpha_1}) & O(x^{\alpha_1}) & O(x^{\alpha_1+\gamma_1}) \\ O(x^{\alpha_1}) & 1 - O(x^{2\alpha_1}) & O(x^{\gamma_1}) \\ O(x^{\alpha_1+\gamma_1}) & O(x^{\gamma_1}) & 1 - O(x^{2\gamma_1}) \end{pmatrix}. \]

As pointed out in the previous section, when \((Z^{(q)})_{11} \gg (G^{(q)})_{11}\), the CKM matrix becomes almost a unit matrix. It is worth noting that large \( q^c-D^c \) mixings play an
essential role in generating a non-trivial texture of the CKM matrix. Furthermore, the mass eigenvalues of light down-type quarks, which are expressed as \( \Lambda_d^{(2)} \) in Eq. (73), turn out to be

\[
v_d \times \left( O(x^{\alpha_1 + \gamma_1 + \alpha_2 + \gamma_2}), \quad O(x^{\gamma_1 + \alpha_2 + \gamma_2}), \quad O(x^{\alpha_2 + \gamma_2 - \delta_d}) \right),
\]

where

\[
\delta_d = \min(\alpha_2, \alpha_4).
\]

Note that \( \alpha_2 \) and \( \alpha_4 \) represent the differences between \( U(1) \) charges, \( a_{A_2} - a_{A_1} \), for \( A = D^c \) and \( A = g^c \), respectively. By taking \( \gamma_1 = 2\alpha_1 \) and \( x^{\alpha_1} = \lambda \simeq 0.22 \) in Eq. (73), we obtain the well-known form of the CKM matrix,

\[
V_{\text{CKM}}^Q \sim \left( \begin{array}{ccc}
1 - O(\lambda^2) & O(\lambda) & O(\lambda^2) \\
O(\lambda) & 1 - O(\lambda^2) & O(\lambda^2) \\
O(\lambda^2) & O(\lambda^2) & 1 - O(\lambda^2)
\end{array} \right),
\]

up to RG effects. In addition, if we take \( \alpha_2 \sim 2.5\alpha_1 \) and \( \gamma_2 \sim 1.5\alpha_1 \), the quark spectra are also reproduced approximately.

For the lepton sector the diagonalization matrices \( \hat{V}_l \) and \( \hat{U}_l \) of \( \hat{M}_l \) given by Eqs. (23) and (24) are described in terms of \( \mathcal{W}_l \) and \( \mathcal{V}_l \), which are the diagonalization matrices for

\[
A_l + B_l = \Gamma_7 [A_l + B_l] \Gamma_7,
\]

\[
M^{(l)\dagger} \left( G^{(l)\dagger} \right)^{-1} \left( A_l^{-1} + B_l^{-1} \right)^{-1} \left( G^{(l)} \right)^{-1} M^{(l)} = \Gamma_6 [\mathcal{C}_l + \mathcal{D}_l]^{-1} \Gamma_6,
\]

where

\[
A_l = \left( y_Z^{(l)} \right)^2 Z_0^{(l)\dagger} \Gamma_7 Z_0^{(l)},
\]

\[
B_l = \left( y_G^{(l)} \right)^2 G_0^{(l)\dagger} \Gamma_5 G_0^{(l)},
\]

\[
C_l = \left( \frac{y_G^{(l)}}{y_M^{(l)} y_Z^{(l)}} \right)^2 ((M_0^{(l)})^{-1} G_0^{(l)} (Z_0^{(l)})^{-1}) \Gamma_7^{-2} ((M_0^{(l)})^{-1} G_0^{(l)} (Z_0^{(l)})^{-1}) \Gamma_7^{-2},
\]

\[
D_l = (y_M^{(l)})^{-2} (M_0^{(l)})^{-1} \Gamma_5^{-2} (M_0^{(l)})^{-1} \Gamma_5^{-1}.
\]

Equations (73) and (74) lead to

\[
\mathcal{W}_l = \left( \begin{array}{ccc}
1 - O(x^{2\alpha_2}) & O(x^{\alpha_2}) & O(x^{\alpha_2 + \gamma_2}) \\
O(x^{\alpha_2}) & 1 - O(x^{2\alpha_2}) & O(x^{\gamma_2}) \\
O(x^{\alpha_2 + \gamma_2}) & O(x^{\gamma_2}) & 1 - O(x^{2\gamma_2})
\end{array} \right),
\]

\[
\mathcal{V}_l = \left( \begin{array}{ccc}
1 - O(x^{2\alpha_6}) & O(x^{\alpha_6}) & O(x^{\alpha_6 + \gamma_6}) \\
O(x^{\alpha_6}) & 1 - O(x^{2\alpha_6}) & O(x^{\gamma_6}) \\
O(x^{\alpha_6 + \gamma_6}) & O(x^{\gamma_6}) & 1 - O(x^{2\gamma_6})
\end{array} \right).
\]
To obtain phenomenologically viable lepton spectra, we now assume \( \alpha_7 > \xi > 0 \), where \( \xi \) is defined by
\[
x^\xi = \frac{y_Z^{(l)}}{y_G^{(l)}} x^{\alpha_7 + \gamma_7 - \alpha_5 - \gamma_5}.
\] (81)
This implies that a large mixing occurs between \( L \) and \( H_d \), and that \( C_l \) and \( D_l \) in the brackets of Eq. (74) become comparable. The mass eigenvalues of light charged leptons, which are given by \( \Lambda_l^{(2)} \) in Eq. (27), are
\[
v_d y_M^{(l)} \times \left( O(x^{\alpha_6 + \gamma_6 + \alpha_5 + \gamma_5 + \xi}), \quad O(x^{\gamma_6 + \alpha_5 + \gamma_5}), \quad O(x^{\alpha_5 + \gamma_5 - \delta_l}) \right),
\] (82)
where
\[
\delta_l = \min(\alpha_5, \alpha_7 - \xi).
\] (83)
Note that \( \alpha_5 \) and \( \alpha_7 \) represent the differences between \( U(1) \) charges, \( a_{A2} - a_{A1} \), for \( A = L \) and \( A = H_d \), respectively. Since both \( L \) and \( H_d \) are \( SU(2)_L \)-doublets, a non-trivial CKM matrix for leptons is not derived as a result of the \( L-H_d \) mixing but arises from the seesaw mechanism. As given in Eq. (39), the CKM matrix for leptons amounts to the diagonalization matrix \( V \) for the light neutrino mass matrix \( \mathcal{N} \). From Eqs. (40) and (49) we have
\[
\mathcal{N} = \frac{1}{y_N} \Lambda_l^{(2)} \Gamma_6^{-1} N_0 \Gamma_6^{-1} \Lambda_l^{(2)},
\] (84)
with
\[
N_0 = \Gamma_6 V_l^{-1} \Gamma_6^{-1} N_0^{-1} \Gamma_6^{-1} V_l \Gamma_6.
\] (85)
By observing that \( N_{0ij} = O(1) \), we find that the diagonalization matrix \( V \) is essentially determined by the hierarchical pattern of
\[
\Lambda_l^{(2)} \Gamma_6^{-1} \simeq y_M^{(l)} x^{\alpha_5 + \gamma_5} \times \text{diag} \left( x^\xi, \ 1, \ x^{-\delta_l} \right).
\] (86)
Concretely, we have
\[
V_{\text{CKM}}^L = V = \begin{pmatrix}
1 - O(x^{2\xi}) & O(x^\xi) & O(x^{\xi + \delta_l}) \\
O(x^\xi) & 1 - O(x^{2\delta_l}) & O(x^{\delta_l}) \\
O(x^{\xi + \delta_l}) & O(x^{\delta_l}) & 1 - O(x^{2\delta_l})
\end{pmatrix}.
\] (87)
It follows that the neutrino flavor mixing between \( \nu_\mu \) and \( \nu_\tau \) is expressed as
\[
\sin \theta_{23}^L \simeq x^{\delta_l}
\] (88)
up to RG effects. The neutrino masses are given by
\[
m_\nu = \frac{(v_u y_M^{(l)})^2}{M_S y_N} x^{2\alpha_5 + 2\gamma_5} \times \left( O(x^{2\xi}), \ O(1), \ O(x^{-2\delta_l}) \right),
\] (89)
which correspond to \( \nu_e, \nu_\mu \) and \( \nu_\tau \), respectively.
4 The unification gauge group

Regarding the unification gauge group $G$, up to now it has only been assumed that $G$ includes both $SU(2)_R$ and the standard model gauge group $G_{SM}$. Let us consider a larger unification group $G$ such that some of the matter fields in Eq.(44) reside in the same irreducible representations of $G$. In such a case we have several equalities among the $\Gamma_A$ with $A = 1, 2, \cdots, 8$. Specifically, when $G$ contains $SU(4)_PS \times SU(2)_L \times SU(2)_R$, quarks and leptons are unified. Matter fields $\Phi_A$ in Eq.(44) fall into five categories,

\[
\begin{align*}
(4, 2, 1) & : \quad Q, L, \\
(6, 1, 1) & : \quad g, g^c, \\
(4^*, 1, 2) & : \quad U^c, D^c, N^c, E^c, \\
(1, 2, 2) & : \quad H_u, H_d, \\
(1, 1, 1) & : \quad S,
\end{align*}
\]

under $SU(4)_PS \times SU(2)_L \times SU(2)_R$. Consequently, the relations

\[ \Gamma_1 = \Gamma_5, \quad \Gamma_2 = \Gamma_6, \quad \Gamma_3 = \Gamma_4 \]  

hold. Furthermore, from Eq.(54) we have $y_M^{(i)} = y_M^{(q)} = 1$. From Eq.(53) it follows that

\[ \delta_i = \min(\alpha_1, \alpha_7 - \xi) \leq \alpha_1. \]  

This leads us to the important relation

\[ \sin \theta_{23}^L \gtrsim \sin \theta_{12}^Q = \lambda, \]  

up to RG effects. As pointed out in Ref.[4], since matter fields of the third generation have large Yukawa couplings, it is important to investigate renormalization effects of the flavor mixings $\sin \theta_{23}^Q$ and $\sin \theta_{23}^L$ from the scale $M_S$ to the electroweak scale. Concretely, the RG evolution from the scale $M_U(= \langle S_0 \rangle)$ to the scale $\langle N_0^c \rangle$ introduces a sizable effect only to the $(3, 3)$ element of $M^{(u)}$ and $M^{(l)}$. As a consequence, we have the RG-enhanced $\sin \theta_{23}^Q$ and $\sin \theta_{23}^L$. Since effective Yukawa couplings of the first and the second generations are very small, renormalization effects of $\sin \theta_{12}$ are rather small compared with those of $\sin \theta_{23}$. Numerically, it can be shown that $\sin \theta_{23}$ is enhanced by about a factor of 2 when approaching the scale $\langle N_0^c \rangle$ from $M_U$. Then we obtain

\[ \sin \theta_{23}^L(M_Z) \gtrsim 2\lambda \simeq 0.44. \]  

This is consistent with atmospheric neutrino data. Furthermore, in the case of $G = SU(4)_P \times SU(2)_L \times SU(2)_R$, the mass eigenvalues turn out to be

\[ v_u \times \left( O(x^{\alpha_1 + \gamma_1 + \alpha_2 + \gamma_2}), \quad O(x^{\gamma_1 + \gamma_2}), \quad O(1) \right) \]  

\[ (94) \]
for up-type quarks,
\[ v_d x^{\alpha_2 + \gamma_2} \times \left( O(x^{\alpha_1 + \gamma_1}), \quad O(x^{\gamma_1}), \quad O(x^{-\delta_d}) \right) \]  
(95)

for down-type quarks,
\[ v_d x^{\alpha_1 + \gamma_1} \times \left( O(x^{\alpha_2 + \gamma_2 + \xi}), \quad O(x^{\gamma_2}), \quad O(x^{-\delta_l}) \right) \]  
(96)

for charged leptons and
\[ m_\nu \simeq \frac{v_u^2}{M_S y_N} x^{2\alpha_1 + 2\gamma_1} \times \left( O(x^{2\xi}), \quad O(1), \quad O(x^{-2\delta_l}) \right) \]  
(97)

for neutrinos, where \( \delta_d = \min(\alpha_2, \alpha_3) \) and \( \delta_l = \min(\alpha_1, \alpha_7 - \xi) \).

If \( G \) contains the traditional \( SO(10) \), matter fields \( \Phi_A \) are classified as

\[(16) : \quad Q, L, U^c, D^c, N^c, E^c, \]
\[(10) : \quad g, g^c, H_u, H_d, \]
\[(1) : \quad S, \]

under \( SO(10) \). It follows that
\[ \Gamma_1 = \Gamma_2 = \Gamma_5 = \Gamma_6, \quad \Gamma_3 = \Gamma_4 = \Gamma_7. \]  
(98)

In this case, however, it is impossible to break down the gauge group \( SO(10) \) into \( G_{SM} \) without introducing a higher representation field of \( SO(10) \), which is not allowed in the level-one string theory. \( SU(6) \times SU(2)_R \) is the largest unification gauge group from which we can implement the breakdown to \( G_{SM} \) without introducing an additional field. When \( G = SU(6) \times SU(2)_R \), matter fields \( \Phi_A \) are classified as

\( \Phi(15, 1) : \quad Q, L, g, g^c, S, \)
\( \Psi(6^*, 2) : \quad U^c, D^c, N^c, E^c, H_u, H_d. \)

Thus we obtain
\[ \Gamma_1 = \Gamma_3 = \Gamma_4 = \Gamma_5 = \Gamma_8, \quad \Gamma_2 = \Gamma_6 = \Gamma_7. \]  
(99)

Then, in the above mass eigenvalues Eqs.(94) to (97) we have \( \delta_d = \min(\alpha_1, \alpha_2) \) and \( \delta_l = \min(\alpha_1, \alpha_2 - \xi) \). Since the traditional \( SU(5) \) gauge group does not include \( SU(2)_R \), the \( SU(5) \) GUT is not in accord with the present framework.

5 Summary

The CKM matrix exhibits the disparity between the diagonalization matrices for up-type quarks (leptons) and down-type quarks (leptons) in \( SU(2)_L \)-doublets. With the
assumption that the unification gauge group $G$ includes $SU(2)_R$, we explored the origin of the disparity purely within the effective field theory below the unification scale $M_U$. In the supersymmetric $E_6$-type models, quarks and leptons mix with extra particles beyond the MSSM. It should be noted that these extra particles contain triplet and doublet Higgs superfields with odd $R$-parity in each generation. We showed that the disparity occurs when these extra-particle mixings are large.

For down-type quarks there appear mixings between $g^c$ and $D^c$, which are $SU(2)_L$-singlets. This type of the mixing does not occur for up-type quarks. When the mixings between $g^c$ and $D^c$ are large, the disparity arises in the diagonalization matrices from the mixings. Namely, a non-trivial CKM matrix is derived. By introducing the flavor symmetry $U(1) \times \mathbb{Z}_2$ and by assigning appropriate flavor charges to matter fields, we obtain a realistic solution which reproduces both the quark spectra and the CKM matrix. On the other hand, for leptons the mixings occur between $L$ and $H_d$, both of which are $SU(2)_L$-doublets. Therefore, as a result of this mixing there appears no disparity between the diagonalization matrices for charged leptons and neutrinos. However, the non-trivial texture of the CKM matrix of leptons emerges as a consequence of the seesaw mechanism with the hierarchical Majorana mass matrix for right-handed neutrinos. Appropriate assignments of flavor charges lead to a phenomenologically viable solution which explains both the lepton spectra and the large $\sin \theta_{23}$.

With the assumption $G \supset SU(4)_P \times SU(2)_L \times SU(2)_R$, we obtain phenomenologically viable relations among the fermion spectra, $V^Q_{\text{CKM}}$ and $V^L_{\text{CKM}}$. This suggests that the unification gauge group $G$ contains at least $SU(4)_P \times SU(2)_L \times SU(2)_R$. The study of these relations provides an important clue to the gauge symmetry, the matter content, and the flavor symmetry at the unification scale.

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