Field Theoretic Approaches
To Early Universe

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by
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I am dedicating my thesis to my beloved parents and well wishers
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ABSTRACT

This thesis compiles the results of six works which deal with - inflationary model building and estimation of cosmological parameters from various field theoretic setup, quantification of reheating temperature, studies of leptogenesis in braneworld and estimation of primordial non-Gaussianity from $\mathcal{N}=1$ supergravity using $\delta N$ formalism.

We start our discussion with exploring the possibility of MSSM inflation in the light of recent observed data from various D-flat directions using the saddle and inflection point techniques. The effective inflaton potential around saddle point and inflection point have been utilized in estimating the observable parameters and confronting them with WMAP7 and Planck dataset.

Next we explore the possibility of inflation from the five dimensional $\mathcal{N}=2$ supergravity setup by deriving the effective potential in the context of Randall-Sundrum like braneworld model and Dirac Bonn Infeld Galileon. After deriving an four dimensional effective potential, we obtain the inflationary observables from both the scenarios and confront them with the WMAP7 data. Further we fit the CMB angular power spectra from TT anisotropy and other polarization data obtained from WMAP7.

Further, we discuss the non-trivial features of reheating from supergravity inspired braneworld model, where the results are to some extent different from that of the usual low energy General Relativistic counterpart, because of the modified Friedmann equations in this setup. We explicitly derive the analytical expressions for the reheating temperature and further solve the evolution equation of the number density of thermal gravitino which results in the gravitino abundance.

Finally, we study the primordial non-Gaussian features using $\delta N$ formalism of unavoidable higher dimensional non-renormalizable Kähler operators for $\mathcal{N}=1$ supergravity framework. In particular we study the nonlinear evolution of cosmological perturbations on large scales which enable us to compute the curvature perturbation, without solving the exact perturbed field equations. Hence we compute the various non-Gaussian parameters for local type of non-Gaussianities, for a generic class of sub-Planckian models dominated by the Hubble-induced corrections.
PUBLICATIONS
(Thesis is based on the papers marked by the symbol *)

1. Brane inflation in background supergravity
   *Sayantan Choudhury* and Supratik Pal
   Physical Review D 85, 043529 (2012).

2. Reheating and leptogenesis in a SUGRA inspired brane inflation
   *Sayantan Choudhury* and Supratik Pal
   Nuclear Physics B 857 (2012) pp. 85-100.

3. Fourth level MSSM inflation from new flat directions
   *Sayantan Choudhury* and Supratik Pal
   Journal of Cosmology and Astroparticle Physics 04 (2012) 018.

4. DBI Galileon inflation in background SUGRA
   *Sayantan Choudhury* and Supratik Pal
   Nuclear Physics B 874 (2013) pp. 85-114.

5. Primordial non-Gaussian features from DBI Galileon inflation
   *Sayantan Choudhury* and Supratik Pal
   arXiv:1210.4478.

6. Features of warped geometry in presence of Gauss-Bonnet coupling
   *Sayantan Choudhury* and Soumitra SenGupta
   Journal of High Energy Physics 02 (2013) 136.

7. Higgs inflation from new Kähler potential
   *Sayantan Choudhury*, Trina Chakraborty and Supratik Pal
   Nuclear Physics B 880 (2014) pp. 155-174.

8. Low & High scale MSSM inflation, gravitational waves and constraints from Planck
   *Sayantan Choudhury*, Anupam Mazumdar and Supratik Pal
   Journal of Cosmology and Astroparticle Physics 07 (2013) 041.

9. Thermodynamics of Charged Kalb Ramond AdS black hole in presence of Gauss-Bonnet coupling
   *Sayantan Choudhury* and Soumitra SenGupta
   arXiv:1306.0492.

10. An accurate bound on tensor-to-scalar ratio and the scale of inflation
    *Sayantan Choudhury* and Anupam Mazumdar
    Nuclear Physics B 882 (2014) pp. 386-396.

11. Primordial blackholes and gravitational waves for an inflection-point model of inflation
    *Sayantan Choudhury* and Anupam Mazumdar
    Physics Letters B 733 (2014) 270-275.
12. Collider constraints on Gauss-Bonnet coupling in warped geometry model
   Sayantan Choudhury, Soumya Sadhukhan and Soumitra SenGupta
   arXiv:1308.1477.

13. Galileogenesis: A new cosmophenomenological zip code for reheating through
    R-parity violating coupling
   Sayantan Choudhury and Arnab Dasgupta
   Nuclear Physics B 882 (2014) pp. 195-204.

14. A step towards exploring the features of Gravidilaton sector in Randall-
    Sundrum scenario via lightest Kaluza-Klein graviton mass
   Sayantan Choudhury and Soumitra SenGupta
   European Physical Journal C 74 (2014) 3159.

15. Constraining $\mathcal{N} = 1$ supergravity inflationary framework with non-minimal
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   Sayantan Choudhury, Anupam Mazumdar and Ernestas Pukartas
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16. Constraining $\mathcal{N} = 1$ supergravity inflation with non-minimal Kähler oper-
    ators using $\delta N$ formalisms
   Sayantan Choudhury
   Journal of High Energy Physics 04 (2014) 105.

17. Inflamagnetogenesis redux: Unzipping inflection-point inflation via various
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   Sayantan Choudhury
   Physics Letters B 735 (2014) 138-145.

18. Reconstructing inflationary potential from BICEP2 and running of tensor
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   arXiv:1403.5549.

19. Sub-Planckian inflation & large tensor to scalar ratio with $r \geq 0.1$
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   arXiv:1404.3398.

20. Modulus stabilization in higher curvature gravity
   Sayantan Choudhury, Joydip Mitra and Soumitra SenGupta
   Journal of High Energy Physics 08 (2014) 004.

21. Can Effective Field Theory of inflation generate large tensor-to-scalar ratio
    within Randall Sundrum single braneworld?
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   arXiv:1406.7618.

22. Measuring CP violation within EFT of inflation from CMB
   Sayantan Choudhury, Barun Kumar Pal, Banasri Basu and Pratul Bandyopadhyay
   arXiv:1409.6036.
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1

Introduction

1.1 Prologue

Mayhap the most important picture of the modern day science is the evolution of the field “particle cosmology” in theoretical physics. Within this subject area particle physics is one of the prime components which specifically tests the nature on the very very smallest length scales (or equivalently in a very high energy scale), on the other hand the subject cosmology deals with the description of the universe on the very very largest length scales. Apart from this fact that these two fields of theoretical physics are distinct in the length scales of the specific features they analyse, it is totally impracticable to comprehend the exact theoretical origin as well as the growth of large-scale structure formation in the universe without knowing the exact “initial conditions” that finally led to the structures that we observe today through various observational probes. The preferred “initial conditions” was set in the very early epoch of universe when all the four fundamental forces are active so that all of them can able to produce the cosmological perturbations in the smoothed density field \[1\]. A detailed knowledge of the presently observed large-scale structure of the universe will also be impractical without considering the specific effects of the dark component in the cosmological density field in perturbation theory. Though things are not fully settled within this subject area, possibly this omnipresent dark component of the universe can be treated as an elementary particle constituent from the early universe. Additionally it is important to note that the detailed knowledge of the structure of the presently observed universe may divulge intuition into various facts which happened in the very early universe. This also implies that the essence of the four fundamental forces and various particle contents at an energy scale afar the extend of terrestrial accelerators. Mayhap the early universe was the ideal particle accelerator which will provide the first glance of theoretical physics at the scale of Grand Unified Theories (GUTs) \[2, 3\], or comparable to the Planck energy scale. The detailed study of the early universe deals with the physics of Standard Model and its beyond. This involves exploring ideas with Supersymmetry (SUSY) and Supergravity \[4\] (SUGRA), String Theory \[5, 6, 7, 8\], Braneworld gravity \[9\] and analyzing fundamental aspects like- reheating \[10, 11, 12\], leptogenesis \[13, 14\], baryogenesis \[15, 16\] etc. In fact, the exciting opportunities presented by links to the early Universe and fundamental physics provided essential motivation for collecting the new data in the first place. Since the information about the early universe is encoded in the spectrum of cosmological fluctuations about the expanding background, it is these fluctuations which provide a link between the physics of the very early universe and cosmological observations till date. At the present stage observational cosmology is one of the strongest probes by which one can, at least attempt to check the fundamental components of the early universe. A wealth of new observational results are being uncovered. The cosmic microwave background (CMB) has been measured to high precision, the distribution of visible and dark matter is being mapped out to greater and greater depths. New observational windows to probe the structure of the universe are opening up. To explain these observational results it is necessary to consider processes which happened in the very early universe.
The most natural proposal to explore the physics of early universe is Inflationary Cosmology. Originally *inflation* was introduced by Guth [17] in order to explain the initial conditions forced on the standard big bang model, but later, it has been found to have more important role: a favored candidate for the origin of structure in our universe. Thirty years later *inflation* is still alive in a much stronger position than ever based on highly precise observational data available of late. Inflation is said to complement the older standard big cosmology (SBBC) in a much sophisticated way. Although the idea of Standard Big Bang Cosmology (SSBC) very successfully explains expanding universe, nucleosynthesis, formation of Cosmic Microwave Background Radiation (CMBR)[18], the temperature anisotropies [19] in the CMB challenged SBBC as it can not satisfactorily explain those observations. The CMB which gives us a snapshot of the very early universe shows that at last scattering surface the universe was almost perfectly homogeneous and isotropic on all scales. Then what initial conditions could lead to such homogeneity and isotropy? Inflation not only provides natural answers to those but also solves an additional puzzle of the SBBC, i.e., of the generation of cosmological perturbations.

*Cosmic inflation* was proposed to solve those pathological problems of SBBC and to get rid of unwanted relics generally predicted by high energy theories. The simplest definition of inflation is that it corresponds to a phase of super-fast acceleration in the very early (time scale $t \sim 10^{-34}$ sec) universe. During inflation, the potential energy of slowly rolling scalar field(s), called inflaton, is supposed to dominate the energy density of our universe by a false vacuum. Also at that time the quantum fluctuations imprinted on space-time are stretched outside the Hubble patch. These primordial fluctuations eventually re-enter the Hubble patch, whence their form can be extracted by observing the perturbations in the CMBR. In the past three decades numerous inflationary models have been proposed, some of them [20] are more or less consistent with the observations but the nature of the field driving inflation is still unknown. The prime input of Inflationary Cosmology is inflaton potential which is originated from the various background field theoretic prescriptions [12, 21]. The end of inflation can be considered as a paradigm for the origin of matter, since all matter arises from the vacuum energy stored in the inflaton field. Slow-roll inflationary framework generically predicts almost Gaussian adiabatic perturbations with a nearly flat spectrum, which has met with an unprecedented success with the observational results obtained from CMB experiments like WMAP [23], PLANCK [24, 25] etc. Apart from the cosmological issues, one of the theoretical challenges is to understand the proper origin of the scale of inflation which is in principle below the UV cut-off scale of the gravity in the effective theory prescription. But if the inflation is embedded on the SM framework then in such scenario the masses are not protected from the quantum (loop) corrections and in theoretical physics this which is known as the *hierarchy* or *fine-tuning* problem [26, 27]. The most popular and successful remedy is the extension of SM by including new symmetry, commonly this known as supersymmetry (SUSY) [4, 28, 29], which is presumably broken first at high scales in some hidden sector and then transmitted to the global SUSY extension of the SM, by gravitational or gauge interactions [12]. When SUSY is broken locally just similarly like a gauge symmetry, an intimate connection with gravity emerges, commonly known as the supergravity (SUGRA) [4, 28, 29], which is valid in the sub-Planckian scale of the effective low energy field theory. In a much broader sense, the unification of gravity along with the other gauge interactions, treated all the fundamental particles as the excitations of extended objects within the framework of string theory [5, 6, 7, 8]. Therefore, it is a significant question to ask whether beyond the SM physics prescription can provide all the right ingredients for inflation or not? As there is still no unique model of inflation consistent with all the theoretical and observational requirements, there
1. Introduction

are lots of open spaces to work on in this field.

Additionally it is important to note that, observational cosmology has indeed made very rapid progress in recent years. The ability to quantify the universe has largely improved due to various available observational constraints coming from large scale structure formation of the universe. The transition to precision cosmology has been spear-headed by measurements of the anisotropy in the cosmic microwave background (CMB) over the past decade. Observations of the large scale structure formation of the universe in the distribution of galaxies, high red-shift supernova, have provided the required complementary information in this context. But there are also other open as well as challenging issues are appearing, which strongly motivates us to do the work on inflationary cosmology and on the particle physics and cosmology overlap. These are-

- breaking the degeneracy among the cosmological parameters using the CMB polarization data,
- dealing with sophisticated and precise data obtained from various probes with cosmic variances etc.

In a nutshell, the aim of our work presented in the thesis is two fold:-

- To derive inflationary models from beyond the SM prescription, specifically from SUSY and SUGRA, DBI Galileon framework and explore their pros and cons in the light of recent observational data,
- To propose new observational tools for inflationary cosmology i.e. non-Gaussianity, precise constraint on scale of inflation, primordial gravitational waves (PGW) via tensor-to-scalar ratio and primordial black hole (PBH) formation etc. We have addressed some of these issues in this thesis.

1.2 Field theoretic tools for early universe

1.2.1 Supersymmetry

Supersymmetry (SUSY) is a generalization of the space time symmetries in quantum field theory that transforms fermions into bosons and vice versa \(^4, 28, 29\). SUSY is appealing for various reasons both for phenomenological and cosmological points of view:

- It provides a framework for the unification of the particle physics and gravity, which is governed by the Planck scale where the gravitational interactions of elementary particles is supposed to be comparable to the gauge interactions.
- It is possible to explain the large hierarchy of the energy scales from W and Z boson masses to the Planck scale using SUSY.
- Stability of the hierarchy in presence of radiative corrections is not possible in SM, but it is possible for supersymmetric theories. SUSY eliminates the quadratic divergence in the mass of fundamental light scalar fields,

\[
\delta m^2 \sim \Lambda_{UV}^2,
\]  \hspace{1cm} (1.2.1)

where \(\Lambda_{UV}\) is the scale where low energy theories no longer apply.
• SUSY is the only framework in which we seem to be able to understand the light fundamental scalars in the context of model building in particle physics and cosmology interface.

• Although initially investigated for other reasons, SUSY turns out to have a significant impact on the cosmological constant problem, and may even be said to solve it halfway.

Only to mention a few drawbacks, the theory has too many parameters, it does not estimate the fermion masses and why the number of generations is three. However these problems can be addressed in the modern framework of SUSY. If SUSY is an exact symmetry of nature, then particles and their superpartners would have degenerated in mass. Since this is usually not observed in experimental data from particle accelerators, SUSY is not an exact symmetry and must be broken. The stability of hierarchy of scales mentioned above can still be maintained if the symmetry breaking is soft (corresponding symmetry breaking mass terms are no longer more than a few TeV). The most interesting theories of this type are theories of low energy or weak-scale SUSY where the effective scale of SUSY breaking is tied to the scale of electroweak symmetry breaking. At present, there are no experimental support for the existence of low energy SUSY. However the unification of the three gauge couplings at an energy scale close to the Planck scale may serve as the indirect evidence. The unification of gauge coupling is not possible in Standard model but it is achievable in minimal supersymmetric extension of the Standard model (MSSM) [4, 12, 28, 30, 31] and provides an additional motivation for low energy SUSY. If Large Hadron Collider (LHC) [32] uncovers the evidence of SUSY, this would have a profound effect on the study of TeV scale physics and development of more fundamental theory of mass and symmetry breaking in particle physics. During the early stages of the evolution of the universe, more precisely during inflation SUSY plays an important role.

The generator (Q) of the supersymmetric transformation is characterized by the spin-1/2 degrees of freedom. The SUSY generator $Q_\alpha (\alpha = 1, 2)$ can be chosen in such a way that it belongs to the family of a left handed Weyl spinor which transforms as $(1/2, 0)$ under Lorentz transformations. Its Hermitian conjugate is designated by $\overline{Q}_\beta$, which belongs to the family of right handed Weyl spinor. Since the anti-commutator of any operator with its Hermitian adjoint is nonzero, therefore, it does not vanish in this context. It transform as $(1/2, 1/2)$ under Lorentz transformations. Similarly it also transforms as $(1/2, 1/2)$ under Lorentz transformation like the anti-commutator stated above.

The superalgebra is defined by the following expression [4, 21, 28, 29]:

$$[Q_\alpha, P_\mu] = 0$$  \hspace{1cm} (1.2.2)
$$\{Q_\alpha, \overline{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu$$  \hspace{1cm} (1.2.3)

where $\sigma^\mu = (-1, \vec{\sigma})$ and $\vec{\sigma}$ denotes Pauli spin matrices and $P_\mu$ are conserved by Coleman Mandula theorem [33]. Fermionic behavior of the SUSY generator indicates that the probability of appearing bosonic (spin-0) and fermionic (spin-1/2) states are equal in number. Here we introduce the anti-commutating parameters as $\theta_\alpha (\alpha = 1, 2)$ and $\overline{\theta}^\beta (\beta = 1, 2)$, which are the elements of Grassmann super-algebra

$$\{\theta^\alpha, \theta^\beta\} = \{\theta^\alpha, \overline{\theta}^\beta\} = 0.$$  \hspace{1cm} (1.2.4)

Including the anti-commutating parameters the Grassmann super-algebra can be modified to [4, 21, 28, 29]:

$$[\theta Q, \overline{Q} \overline{Q}] = 2\theta \sigma_{\mu}, \overline{\theta} P_\mu,$$  \hspace{1cm} (1.2.5)
$$[\theta Q, \theta Q] = [\overline{Q} \overline{Q} \overline{Q} \overline{Q}] = 0.$$  \hspace{1cm} (1.2.6)
\[ \theta Q = \theta^\alpha Q_\alpha = \theta^\alpha Q^\beta \epsilon_{\alpha\beta} \]  

and \( \epsilon_{\alpha\beta} \) is an antisymmetric tensor with \( \epsilon_{12} = 1 \). Using this input further one can decompose the superfield into left and right handed representation by the following relation

\[ \phi(x_\mu, \theta, \bar{\theta}) = \phi_L(x_\mu + i \theta \sigma_\mu \bar{\theta}, \theta, \bar{\theta}) = \phi_R(x_\mu - i \theta \sigma_\mu \bar{\theta}, \theta, \bar{\theta}). \]  

\( L \) and \( R \) superfield transform in the same way as the superfield \( \phi(x_\mu, \theta, \bar{\theta}) \) transform under SUSY transformation. Let us now expand \( \phi(x, \theta) \) in power series keeping in mind that \( \theta \) is anti-commuting parameter. As a consequence the power series terminates after the third term of the left chiral superfield. So finally we may write

\[ \phi(x, \theta) = \varphi(x) + \theta^\alpha \psi_\alpha(x) + \theta^\alpha \theta^\beta \epsilon_{\alpha\beta} F(x), \]  

where \( \varphi \) and \( F \) (auxiliary field) are complex scalar fields and \( \psi \) is a left/right-handed Weyl spinor. Taking the infinitesimal transformation of equation (1.2.9) we obtain [4, 21, 28, 29]:

\[ \delta \phi(x, \theta) = \delta \varphi + \theta \delta \psi + \theta \theta \delta F \]  

Under a SUSY transformation the component fields can be shown to transform as [4, 21, 28, 29]:

\[ \delta \varphi = \sqrt{2} \eta \psi, \]  

\[ \delta \psi = \sqrt{2} \eta F + \sqrt{2} i \sigma_\mu \eta \partial_\mu \varphi, \]  

\[ \delta F = - \sqrt{2} i \partial_\mu \psi \sigma_\mu \eta. \]

A scalar superfield contains spin 0 bosons and spin \( \frac{1}{2} \) fermions. For a gauge theory one has to introduce spin 1 vector superfields, which can be expressed in the reducible representation as: [4, 34]:

\[ V(x, \theta, \bar{\theta}) = \left( 1 + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \right) C + \left( i \theta + \frac{1}{2} \theta \theta \sigma_\mu \bar{\theta} \partial_\mu \right) \chi + \left( - \bar{\theta} + \frac{1}{2} \bar{\theta} \bar{\theta} \theta \sigma_\mu \partial_\mu \right) \bar{\chi} \]

\[ + \frac{1}{2} \theta \theta (M + i N) - \frac{i}{2} \bar{\theta} \bar{\theta} (M - i N) - \theta \sigma_\mu \bar{\theta} \partial_\mu \chi + i \theta \theta \bar{\theta} \lambda - i \bar{\theta} \bar{\theta} \theta \lambda + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D \]

where \( \Box = \partial_\mu \partial^\mu \); \( C, M, N, D \) are all real scalar fields; \( \chi \) and \( \lambda \) are Weyl spinors and \( V^\mu \) is a vector field. In an Abelian gauge theory (e. g. QED) the gauge field \( V^\mu \) transforms under gauge transformation as,

\[ V_\mu \to V_\mu + \partial_\mu \varphi, \]

where \( \varphi \) is a real scalar field. Here the gauge field \( V^\mu \) is a partner of the supermultiplet \( V(x, \theta, \bar{\theta}) \). In the context of Abelian gauge theory we need the supersymmetric extension of the gauge theory. So \( \varphi \) is also the part of the supermultiplet. For supersymmetric generalization Wess and Zumino [35] showed that the vector superfield transforms as,

\[ V \to V + i (\Lambda - \Lambda^\dagger), \]

where \( \Lambda \) represents a chiral superfield. Further, using Eq (1.2.14) one can show that the fields \( C, \chi, M \) and \( N \) are gauge artifacts which can be gauge away using the Wess-Zumino gauge while \( \lambda \) and \( D \) are gauge invariant quantities. Consequently, the vector superfield reduces to [4, 35]:

\[ V_{WZ}(x, \theta, \bar{\theta}) = - \theta \sigma^\mu \bar{\theta} V_\mu + i \theta \theta \bar{\theta} \lambda - i \bar{\theta} \theta \bar{\theta} \lambda + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D \]
where $\lambda_\alpha$ is the gaugino, $D$ is the auxiliary field and $V_\mu$ is the gauge field. However, in strict sense the Wess-Zumino gauge is incomplete in the context of gauge fixation. Here our goal is to obtain the supersymmetric generalization of the electromagnetic field strength. To fulfill our need we will start with a spinor chiral superfield $W^\alpha = 4i\lambda^\alpha - 4\theta_\alpha D + 4i\theta^\beta \sigma_{\alpha\dot{\beta}} \sigma^\dot{\beta} \partial^\mu V^\nu - 4(\theta\theta) \sigma_{\alpha\beta} \partial^\mu \lambda^\beta$ (1.2.18)

where the field strength tensor is defined as

$$F_{\mu\nu} = V_{\mu\nu} = \partial_{[\mu} V_{\nu]}.$$  (1.2.19)

Under the infinitesimal SUSY transformation the chiral field transform as $\delta \lambda = i\eta D + \eta \sigma^\mu \sigma^\nu F_{\mu\nu}$.  (1.2.20)

The generic Lagrangian for global supersymmetry (GSUSY) can be decomposed into two parts $L = L_F + L_D$, (1.2.21)

where $L_F$ is the sum of scalar superfields and $L_D$ is a sum of vector superfields. These are called F and D terms respectively. F and D terms are defined through the following integral over $\theta$ and $\bar{\theta}$ as:

$$\int d\theta = 0, \int d\theta^\alpha \theta_\alpha = 1.$$  (1.2.22)

Using equation (4.3.7) one can have $L = \frac{1}{32}(W^\alpha W_\alpha) = -\frac{1}{4} V_{\mu\nu} V^\mu V^\nu - \frac{i}{2} \lambda^\alpha \sigma_{\mu\gamma} \partial^\mu \lambda^\gamma - \frac{i}{2} \sigma^\alpha_{\mu\beta} \partial^\mu \lambda^\beta \lambda^\alpha + \frac{D^2}{2}$. (1.2.23)

which is the Lagrangian density of pure SUSY extension of Abelian Yang-Mills theory. To make the theory more general we introduce here non-Abelian gauge theory [36]. Here we redefine the vector and chiral field as

$$V_\mu = V^a_\mu T^a$$  (1.2.24)

and

$$\lambda^a = \Lambda^a T^a$$  (1.2.25)

where $T^a$ are the generators of the non-Abelian gauge group and satisfy the basic propositions of Lie algebra. Thus in the context of pure SUSY Yang-Mills theory we get [36]:

$$L = \frac{1}{32g^2} W^\alpha W_\alpha = -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} + \frac{D^2}{2} - \frac{i}{2} \lambda^\alpha \sigma_{\mu\beta} \partial^\mu \lambda^\beta \lambda^\alpha$$  (1.2.26)

where

$$G^{\mu\nu}_{\mu\nu} = \partial_{[\mu} V^a_{\nu]} + ig[V_{\mu}, V_{\nu}]^a = \partial_{[\mu} V^a_{\nu]} - gf_{abc} V_{b\mu} V_{c\nu}$$  (1.2.27)

and

$$\lambda = \lambda^a T^a$$  (1.2.28)

in the adjoint representation of the gauge group. Gauge group generator satisfies the commutation relation

$$[T^a, T^b] = if^{abc} T^c,$$  (1.2.29)
where $T^a$’s are the generator of the gauge group and $f^{abc}$ are real constants, called the structure constants of the group which satisfy the famous Jacobi identity. So the most general renormalizable Lagrangian in superspace can be written as \[ L = \sum_n \int d^2 \theta \, d^2 \bar{\phi}_n e^{\phi^\dagger} \phi_n + \frac{1}{64g^2} \int d^2 \theta W_\alpha W^\alpha + \int d^2 \theta W(\phi_n) + h.c. \] (1.2.30)

where $h.c.$ signifies the hermitian conjugate and in the adjoint representation

$$Tr(T^a T_b) = \delta^a_b.$$ (1.2.31)

Here $W(\phi_n)$ is the holomorphic superpotential which will play the most crucial role in deriving the F and D term potentials, as revealed subsequently. The single $U(1)$ gauge group with coupling $g$ is simple. But in the case of several $U(1)$’s, there are no cross-terms in the potential from the D-terms, i.e.

$$V_D = \sum_n (V_D)_n.$$ (1.2.32)

In terms of component fields the Lagrangian for Abelian theory can be written as \[ L = \sum_n [(D_\mu \phi^*_n)(D^\mu \phi_n) + i \bar{\psi}_n D_\mu \sigma^\mu \psi_n + |F_n|^2] - \left[ \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + i \lambda \sigma^\mu \partial_\mu \bar{X} - \frac{1}{2} D^2 - \frac{g D}{2} \sum q_n |\phi_n|^2 \right] \]

$$- \left[ i \sum \frac{g}{\sqrt{2}} \bar{\psi}_n \bar{\psi}_{n,m} \frac{1}{2} \frac{\partial^2 W}{\partial \phi_n \partial \phi_m}, \psi_{n,m} + \sum_n F_{\alpha} \left( \frac{\partial W}{\partial \phi_n} \right) \right] + h.c.$$ (1.2.33)

where the covariant derivative is defined as $D_\mu = \partial_\mu + ig V_\mu$ and its non-Abelian generalization can be written as [36]:

$$L = \sum_n [(D_\mu \phi^*_n)(D^\mu \phi_n) + i \bar{\psi}_n D_\mu \sigma^\mu \psi_n + |F_n|^2] - \frac{1}{4} G_{\mu \nu} G^{\mu \nu} + \frac{1}{2} D^2 - i [\sigma^\alpha \sigma_{\mu \beta}(\partial^\mu \bar{X}^\beta + ig[V^\mu, \bar{X}^\beta])]$$

$$- (\partial^\mu \bar{X}^\beta + ig[V^\mu, \bar{X}^\beta]) \sigma_{\mu \beta} \lambda^\alpha] + \frac{g D}{2} \sum q_n |\phi_n|^2 - \left[ i \sum \frac{g}{\sqrt{2}} \bar{\psi}_n \bar{\psi}_{n,m} \psi_{n,m} + \sum_n \left( \frac{\partial W}{\partial \phi_n} \right) + \sum_n F_{\alpha} \left( \frac{\partial W}{\partial \phi_n} \right) \right] + h.c.$$ (1.2.34)

where the covariant derivative is defined as:

$$D_\mu = \partial_\mu + ig T^\alpha_\mu V^\alpha_\mu$$ (1.2.35)

and $q_n$ are the $U(1)$ charges (the response of interaction) of the fields $\phi_n$. Here in this expression for Lagrangian both for Abelian and non-Abelian gauge singlet kinetic term appears with overall family of fields and it reflects the gauge interaction with coupling strength $g$. The constraints of $F_n$ and D are given by [4, 21, 28]:

$$F_n = - \left( \frac{\partial W}{\partial \phi_n} \right)^\ast$$ (1.2.36)

$$D = \frac{g}{2} \sum_n q_n |\phi_n|^2.$$ (1.2.37)
These equations basically give the equation of motion of auxiliary fields. Using equation (2.3.31) and (1.2.37) the scalar field potential can be split into two parts [4, 21, 28]:

\[ V = V_F + V_D \]  

(1.2.38)

where

\[ V_F \equiv \sum_n |F_n|^2 \]  

(1.2.39)

\[ V_D \equiv \frac{1}{2} D^2. \]  

(1.2.40)

This splitting of potential into F and D term is very much significant in the context for model building for inflation.

In order to get a renormalizable theory, one can investigate that the superpotential \( W \) is at most cubic in the fields, which corresponds to the potential is at most quartic. From (6.0.38) we observe that the overall phase factor appearing in \( W \) is not physically significant at all. Due to R-symmetry a sign flip occurs in \( W \). Internal symmetries respect holomorphicity of \( W \) and thus restrict its form much more with respect to the case of potential \( V \). For completeness In the context of \( U(1) \) gauge symmetry it is instructive to add an extra contribution to the Lagrangian, which is called Fayet-Iliopoulos term and the new form of the generalized Lagrangian is [21, 37],

\[
\mathcal{L} = \sum_n \int d^2 \theta \; d^2 \theta \phi_n^d e^{g V} \phi_n + \frac{1}{64 g^2} \int d^2 \theta W_\alpha W^\alpha + \int d^2 \theta W(\phi_n) - 2 \xi \int d^2 \theta \; d^2 \theta V + h.c. \]  

(1.2.41)

where the underbrace term is the Fayet-Iliopoulos term and the corresponding modified D field can be expressed as [21, 37]:

\[ D = -\frac{g_c}{2} \sum_n q_n |\phi_n|^2 - \Lambda. \]  

(1.2.42)

where \( \Lambda \) appears as an effective coupling strength in the Fayet-Iliopoulos term which plays similar role as in the first term in \( D \). Here one can identify \( \Lambda \) as effective \( U(1) \) Fayet-Iliopoulos charge. The modified D term of the potential can then be written as [21, 37]:

\[ V_D = \frac{1}{2} \left( \frac{g_c}{2} \sum_n q_n |\phi_n|^2 + \Lambda \right)^2. \]  

(1.2.43)

For convenience again we can redefine the D term in such a way that it can identify the effective Fayet-Iliopoulos \( U(1) \) charge as \( g_c \Lambda \) and the corresponding modified potential by redefining the charges and \( \Lambda \) as [21, 37]:

\[ D = -g_c \left( \sum_n q_n |\phi_n|^2 + \Lambda \right) \]  

(1.2.44)

and the D-term potential can be written as:

\[ V_D = \frac{g^2_c}{2} \left( \sum_n q_n |\phi_n|^2 + \Lambda \right). \]  

(1.2.45)

The Fayet-Iliopoulos term appears as a heavy degrees of freedom. However, at the end of the calculation, the heavy degrees of freedom have been integrated out.
1.2.2 Minimal Supersymmetric Standard Model (MSSM)

The Minimal Supersymmetric Standard Model (MSSM) [4, 12, 28, 30, 31, 38] is a minimal extended version to the Standard Model which realizes basic principles of $\mathcal{N} = 1$ SUSY as introduced in the earlier subsection. The prime incentive for introducing MSSM was to stabilize the physics of weak scale via solving the well known hierarchy or naturalness problem. This is a very well known fact that the Higgs boson mass of the Standard Model (SM) is very much unstable in presence of quantum corrections and the corresponding theory also predicts that weak energy scale should be much weaker than collider experiment bound. In the context of MSSM, the Higgs boson has a fermionic superpartner within $\mathcal{N} = 1$ SUSY. This is known as the Higgsino which has the exactly same mass as it would if also $\mathcal{N} = 1$ SUSY were an exact symmetry of the present setup. In this physical scenario all the fermion masses and Higgs mass are stable under radiative correction in the matter sector of the theory. However, within the framework of $\mathcal{N} = 1$ SUSY based MSSM there is a need for more than one Higgs field and this is explicitly described later in detail. There are three principal incentives for the $\mathcal{N} = 1$ SUSY based MSSM over the other theoretical extensions of the Standard Model, namely [4, 12, 28, 40, 41]:

1. To solve the naturalness or hierarchy problem in the context of $\mathcal{N} = 1$ SUSY,
2. To unify all gauge couplings and
3. To identify the proper dark matter candidate in the context of $\mathcal{N} = 1$ SUSY.

These motivations are the primary reasons for considering the MSSM as the leading candidate for a new theory to be discovered at collider experiments such as the LHC [32], ILC [39] etc in future.

In addition to the usual quark and lepton superfields as appearing in the context of $\mathcal{N} = 1$ SUSY, MSSM has two Higgs fields, $H_u$ and $H_d$. In this context two Higgses are are the building block to construct the mathematical structure of the superpotential and for such construction $H^\dagger$ and any other combination of this term is completely redundant to maintain the perfect holomorphicity. The superpotential in the context of MSSM for $\mathcal{N} = 1$ SUSY is described by the following expression [4, 28, 40, 41]:

$$W_{\text{MSSM}} = \lambda_u Q H_u u + \lambda_d Q H_d d + \lambda_e L H_d e + \mu H_u H_d,$$

(1.2.46)

where $H_u, H_d, Q, L, u, d, e$ in Eq. (1.2.46) characterizing the chiral superfields, and the dimensionless Yukawa couplings $\lambda_u, \lambda_d, \lambda_e$ are $3 \times 3$ matrices in the SUSY family space. We have suppressed the gauge and family indices for clarity in the present context. Here $H_u, H_d, Q, L$ fields are actually $SU(2)$ doublets and $u, d, e$ fields are the $SU(2)$ singlets. Additionally, the last term is called the $\mu$ term in MSSM, which is a $\mathcal{N} = 1$ SUSY version of the SM Higgs boson mass in the context of MSSM. The prime reason for the requirement of two higgs field $H_u$ and $H_d$ are two fold in the present context of discussion-

1. to give masses to all the quarks and leptons via spontaneous SUSY breaking (SSB) in the context of MSSM and
2. for the cancellation of gauge anomalies in the context of MSSM.

It is important to note that as the top quark, bottom quark and tau lepton are the heaviest fermionic contents in the SM family, only the third family element of the matrices $\lambda_u, \lambda_d, \lambda_e$ plays significant role in the context of MSSM.
In this context the \( \mu \) term provides masses to the fermionic superpartner of Higgs field, which is known as the Higgsino and can be represented by the following Lagrangian density:

\[
\mathcal{L} \supset -\mu (\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + c.c.,
\]

(1.2.47)

and actively contributes to the Higgs mass terms in the scalar potential via the following expression:

\[-\mathcal{L} \supset V \supset |\mu|^2 (|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2) > 0.\]

(1.2.48)

Consequently, here \(|\mu|^2\) should approximately cancel the negative soft mass term in order to accommodate for a Higgs VEV of order \( \sim 246\) GeV. Additionally it is important to note that, a general gauge invariant as well as renormalizable superpotential would also include baryon number \( B \) or lepton number \( L \) violating terms in the present context. There exists a discrete \( Z_2 \) symmetry in the present context, which can easily forbid baryon and lepton number violating terms. This is commonly known as \( R \)-parity \([28, 42]\). For each particle one can write the expression for \( R \)-parity as:

\[P_R = (-1)^{3(B-L)+2s},\]

(1.2.49)

with

\[P_R = +1\] for the SM particles and the Higgs bosons, while

\[P_R = -1\] for all the sleptons, squarks, gauginos, and Higgsinos. Additionally, \( s \) signifies the spin of the particle.

Also it is important to note that, the general soft \( \mathcal{N} = 1 \) SUSY breaking terms in the total MSSM Lagrangian can be expressed as \([4, 12, 28, 40, 41]\):

\[
\mathcal{L}_{soft} = -\frac{1}{2} (M_\lambda \lambda^\dagger \lambda^\dagger + c.c.) - (m_i^2) \phi_i^+ \phi_i - \left( \frac{1}{2} b_{ij} \phi_i \phi_j + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + c.c. \right),
\]

(1.2.52)

where \( M_\lambda \) signifies the common gaugino mass as given by:

\[(m_i^2) \sim m_0^2 \sim O(100)\text{GeV}^2\]

(1.2.53)

which are \( 3 \times 3 \) matrices determining the masses for squarks and sleptons given by:

\[m_Q^2, m_u^2, m_d^2, m_L^2, m_e^2, m_{H_u}^2, m_{H_d}^2, b \sim m_0^2 \sim O(100))^2 \text{GeV}^2.\]

(1.2.54)

Additionally, \( b_{ij} \) is the mass term for the specific combination \( H_u H_d \) and also \( a^{ijk} \) are the complex \( 3 \times 3 \) matrices in the family space which yield the trilinear \( A \)-terms as given by:

\[a_u, a_d, a_e \sim m_0 \sim O(100) \text{ GeV}.\]

(1.2.55)

It is important to note that, there are a total of 105 new entries in the MSSM Lagrangian which have not any counterpart in the context of SM. The only mechanism of SUSY breaking where the breaking scale is not introduced either at the level of superpotential or in the gauge sector is through dynamical SUSY breaking and this appears to be useful in MSSM.
Also it is important to mention here that the $N = 1$ SUSY field configurations satisfying simultaneously the following sets of constraint equation of motions in the context of MSSM [43, 44, 45]:

$$D^a \equiv X^\dagger T^a X = 0,$$

$$F_{X_i} \equiv \frac{\partial W}{\partial X_i} = 0.$$

where these are called the $D$-flat and $F$-flat respectively for $N$ chiral superfields $X_i$ defined in MSSM supermultiplet. Also here $D$-flat directions are parameterized by gauge invariant monomials of the chiral superfields in the context of MSSM.

In many cosmological applications of flat directions, it is important to consider the running of mass term below the GUT scale,

$$M_{\text{GUT}} \sim \mathcal{O}(10^{16} \text{ GeV})$$

to study the effective field theory at low and high energy scale. For simplicity we can also assume that it is the scale where SUSY breaking is transmitted to the visible sector.

A most generalized structural form of one loop Renormalization Group (RG) equations can be written in terms of $\beta$ function as [4, 12, 28]:

$$\beta_{m^2_i} := \frac{\partial m^2_i}{\partial t} = \sum_g a_{ig} m_g^2 + \sum_a h_a^2 \left( \sum_j b_{ij} m_j^2 + A^2 \right),$$

where $a_{ig}$ and $b_{ij}$ are two constants, $m_g$ characterizes the gaugino mass, $h_a$ is the Yukawa coupling, $A$ is the supersymmetry trilinear $A$-term, and

$$t = \ln \frac{M_X}{q}$$

characterizes the background scale of RG.

In this context the one-loop RG equations for the three gaugino mass parameters are determined by the following expression [4, 12, 28]:

$$\frac{d}{dt} m_i = \frac{1}{8\pi^2 g_i^2} b_i g_i^2 m_i, \quad (b_i = 33/5, 1, -3)$$

where $i = 1, 2, 3$ correspond to $U(1)$, $SU(2)$, $SU(3)$. An interesting property is that the three ratios $m_i/g_i^2$ are RG scale independent. Therefore at the GUT scale, it is assumed that gauginos masses also unify with a value $m_{1/2}$. Then at any scale:

$$m_i/g_i^2 = m_{1/2}/g_{\text{GUT}}^2,$$

where $g_{\text{GUT}}$ is the unified gauge coupling at the GUT scale. The RG evolution due to Yukawa interactions are small except for top.

### 1.2.3 Next to Minimal Supersymmetric Standard Model (NMSSM)

NMSSM is an acronym for Next-to-Minimal Supersymmetric Standard Model. It is a $N = 1$ supersymmetric extended version to the Standard Model that incorporates the effect of an additional singlet chiral superfield to field content of the MSSM as already introduced in the previous section. This simple extended version of MSSM can be obtained just by including a new gauge-singlet chiral
supermultiplet with even matter parity in $\mathcal{N} = 1$ SUSY sector. In this context the superpotential can be expressed as [46, 47, 48]:

\[
W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda S H_u H_d + \frac{1}{3}\kappa S^3 + \frac{1}{2}\mu S^2, \tag{1.2.63}
\]

where $S$ is the new gauge-singlet chiral supermultiplet added in the field content of MSSM. Additionally it is important to note that, NMSSM also introduces extra coefficients, which play crucial role for the successful realization of the electroweak symmetry breaking and can be easily done by choosing appropriately all such coefficients. One of the advantage of the NMSSM framework is that it can provide an answer to solve the $\mu$ problem \(^1\). Additionally, it is important to mention here that an effective $\mu$-term:

\[
\mu_{\text{eff}} = \lambda S \tag{1.2.64}
\]

for the contribution $H_uH_d$ can be easily able to generate from Eq. (1.2.63) in the present context. For the determination of such term dimensionless coupling parameters as well as the soft SUSY breaking mass term $m_{\text{soft}}$ are required. In the most generalized physical prescription, it is important to note that the natural framework of NMSSM also provides an extra sources for the large amount of CP violation. Apart from this fact NMSSM also provides the conditions for electroweak baryogenesis which is a very important fact for the study of physics of early universe. In eq. (1.2.63) the symbol $\cdots$ characterizes the scale invariant cubic superpotential for the $\mathbb{Z}_3$-invariant NMSSM, where accidental $\mathbb{Z}_3$ symmetry is maintained due to a overall multiplication of a phase factor $e^{2\pi i/3}$ to all the components of all chiral superfields as appearing in $\mathcal{N} = 1$ extended supermultiplet. Here it is important to mention that any of the dimensionful contributions in the general $\mathcal{N} = 1$ superpotential (which is not cubic in general) breaks the $\mathbb{Z}_3$ symmetry in a explicit fashion within the framework of NMSSM. As a consequence the extended version corresponding to the general full modified superpotential will be designated as the generalized NMSSM.

### 1.2.4 Supergravity

Till now in the introduction of the thesis we have discussed the various field theoretic features of global supersymmetry (GSUSY). Now it is important to note that, in the context of the physics of early universe, especially during the epoch of inflation, one needs to consider effect of local version of supersymmetry, which is commonly known as supergravity (SUGRA). Within the framework of SUGRA one can describe the most general non-renormalizable structure of GSUSY [4, 12, 21, 28]. Sometimes this can also be treated as the low energy version of string theory. The mathematical structure of all such SUGRA operators mimics the role of effective field theory operators which we will discuss in the context of inflation. Within the effective description such a non-renormalizable field theory is valid upto the ultra-violet cutoff scale $\Lambda_{\text{UV}} \sim M_p$, beyond which it is invalid.

\(^1\)In the context of SUSY a very important question one can ask regarding the issue that how SUSY mass parameter $\mu$ can assume a value of the order of the SUSY breaking scale $M_{\text{SUSY}}$. In theoretical particle physics this commonly known as the “$\mu$-problem” of the MSSM [49] theory. Here we mention a possible graceful way to resolve this problem. Within this study one needs to generate an SUSY effective mass term $\mu$ by exactly following the generation mechanism of quark and lepton masses in the context of SM. In this case the mass term $\mu$ is exactly replaced by a Yukawa coupling of $H_u$ and $H_d$ to a scalar field and specifically the soft SUSY breaking induced VEV of the scalar field is consistent with the requirement.
Prime components of SUGRA

In the context of GSUSY we start our discussion with the definition of various crucial components—chiral and gauge supermultiplets and also the characteristic superalgebra for GSUSY. In the context of SUGRA \(^2\) one need the following components to construct the background field theoretic setup:

- The superpotential for \(\mathcal{N} = 1\) SUGRA is characterized by holomorphic function \(W\) which is in principle function of complex scalar fields.

- The Kähler potential is characterized by the real function \(K\) which is not at all holomorphic in nature.

- The holomorphic gauge kinetic function \(f\) \(^3\).

Within the framework of SUGRA one can start discussion with two crucial components— the superpotential \(W\) and Kähler potential \(K\). In terms of these crucial components one can further define a generating functional using which one can easily able to generate the effective potential required for the study of the physics of early universe, specifically the physics of inflation. Such generating functional for \(\mathcal{N} = 1\) SUGRA can be expressed as \([4, 21]\):

\[
G \equiv \frac{K}{M_p^2} + \ln \frac{|W|^2}{M_p^6}
\]

where the Kähler potential \(K\) and superpotential \(W\) transform under the Kähler transformation defined within the framework of \(\mathcal{N} = 1\) SUGRA as \([4, 21]\):

\[
\frac{K}{M_p^2} \rightarrow \frac{K}{M_p^2} - X - \bar{X}, \quad W \rightarrow e^X W
\]

where \(X\) and \(\bar{X}\) characterize any arbitrary holomorphic and anti-holomorphic function of the superfield contents of \(\mathcal{N} = 1\) SUGRA multiplet. Throughout the thesis henceforth we will follow the following list of crucial conventions within the framework of \(\mathcal{N} = 1\) SUGRA:

- All the scalar components \(\phi^n\) and also the auxiliary components \(F^n\) in the SUGRA multiplet are identified by a superscript.

- In this context a subscript \(n\) characterizes \(\partial/\partial \phi^n\), and a subscript \(n^\dagger\) characterizes \(\partial/\partial \phi^{n^\dagger}\).

- Lowering operation of the indices can be performed within the present framework as,

\[
\phi_n \equiv K_{nm^\dagger} \phi^{m^\dagger}
\]

and

\[
F_n \equiv K_{nm^\dagger} F^{m^\dagger}
\]

\(^2\)In the rest of the thesis we will concentrate on the \(\mathcal{N} = 1\) and \(\mathcal{N} = 2\) SUGRA sector which is relevant for both cosmological and particle physics model building purpose for early universe \([12, 21]\).

\(^3\)In the more generalized physical prescription one can think of other class of holomorphic functions which incorporates the effect of higher order simple derivatives, higher order non-minimal derivatives and various other complicated functions in principle. But the general framework of effective field theory indicates that those type of non-trivial complicated contributions are highly suppressed by the UV cut-off scale (i.e. \(\Lambda_{UV} \sim M_p\)) of the theory and consequently all such contributions are negligibly small in the present context and at the leading order of SUGRA theories one can easily neglect such complicated non-trivial functions from the computation. See \([50, 51, 52, 53]\) for the details on these aspects.
Let us briefly discuss about the technical details of superpotential $W$, Kähler potential $K$ and gauge kinetic function $f$ within the framework of $\mathcal{N}=1$ SUGRA. To serve this purpose first of all in this context we consider a most generalized expansion about a preferred choice of origin in $\mathcal{N}=1$ superfield space within the framework of $\mathcal{N}=1$ SUGRA [4, 12, 21, 28].

### The superpotential: In the most general physical prescription the holomorphic superpotential can be expanded as a power series expansion [4, 12, 21] within the framework of $\mathcal{N}=1$ SUGRA as:

\[
W = \sum_{d=0}^{\infty} W_d(\phi^n) \frac{M_p^d}{M_p^3}
\]  

(1.2.70)

where $W_d$ is the contribution in the $\mathcal{N}=1$ SUGRA superpotential from the superfield sector in $d$ mass dimension. Additionally it is important to note that, for $d \geq 4$ the above mentioned $\mathcal{N}=1$ SUGRA operators are non-renormalizable in nature and also suppressed by the UV cut-off scale i.e Planck scale, $M_p$ in the present context. Now here one can also think of a physical situation where the computation starts with an expression in which the $\mathcal{N}=1$ SUGRA field contents only appears at lower order. In that case one can easily forbid additional terms appearing in the above mentioned generic series expansion and truncate the series up to a finite order by imposing an additional discrete $\mathbb{Z}_N$ symmetry in the present context. Also one can truncate the series up to a finite order by imposing a continuous symmetry in the context of $\mathcal{N}=1$ SUGRA.

### The Kähler potential: Within the framework of $\mathcal{N}=1$ SUGRA the Kähler potential determines the structure of the kinetic terms of the superfields as given by [4, 12, 21]:

\[
\mathcal{L}_{\text{kin}} = (\partial^\mu \phi^n) K_{n^\dagger m} (\partial_\mu \phi^m).
\]  

(1.2.71)

In the $\mathcal{N}=1$ superfield space one can use the following series expansion ansatz of Kähler potential as given by [4, 21]:

\[
K = K_{nm^\dagger} \phi^n \phi^{m^\dagger} + \sum_{d=3}^{\infty} K_d(\phi^n, \phi^{n^\dagger}) \frac{M_p^{d-2}}{M_p^2},
\]  

(1.2.72)

where $K_{nm^\dagger}$ characterizes the $\mathcal{N}=1$ Kähler metric evaluated at the specified preferred choice of origin $^4$. In the most simplest physical situation the scalar fields are chosen to be canonically normalized at the preferred choice of origin, which directly implies that in such a case $\mathcal{N}=1$ Kähler metric is diagonal and can be expressed as:

\[
K_{nm^\dagger} = \delta_{nm^\dagger}.
\]  

(1.2.73)

In Eq (1.2.72) the higher mass dimensional $\mathcal{N}=1$ SUGRA Kähler operators as denoted by $\cdots$ symbol and most importantly are highly suppressed by the Planck scale. Additionally, it is important to note that $\mathcal{N}=1$ Kähler metric $K$ is not a holomorphic function and consequently using the physical symmetries of the setup it is not possible to constrain the mathematical structural form of the function in a very strong fashion.

$^4$It is important to note that, in the present context any constant term or linear contribution has been absorbed into the $\mathcal{N}=1$ SUGRA superpotential by a Kähler transformation in this context.
1. Introduction

The gauge kinetic function: Finally, within the framework of $N = 1$ SUGRA the gauge kinetic function determines the kinetic terms of the gauge and gaugino field contents. Here one can easily choose all of them to be canonically normalized when the scalar fields are specified at a preferred choice of origin and in that case one can use the following series expansion ansatz of gauge kinetic function as given by [4, 21]:

$$ f = 1 + \sum_{d=1}^{\infty} \frac{f_d(\phi^n)}{M_p^d}, $$

Planck scale suppressed operators

where it is important to note that the mathematical structural form of $f$ is constrained via various physical symmetries of the $N = 1$ SUGRA setup due to its holomorphicity.

Scalar potential for SUGRA

Here we will explicitly write down the total structure of $N = 1$ SUGRA effective potential which is very relevant for the discussion of the physics of early universe. It is very well know fact that $N = 1$ SUGRA can be broken only using the mechanism of spontaneously SUSY breaking but not explicitly. But in the case of GSUSY it is possible to break SUSY explicitly. Within the framework of $N = 1$ SUGRA the auxiliary fields which are determined from the equations of motion are are given by [4, 12, 21]:

$$ D = -g(q_n K_n \phi^n + \xi), $$

$$ F^n = -e^{K/2}K^{nm} (W_m + M_p^{-2} W K^m)\dagger. $$

Using Eq (1.2.75) the D-term and F-term potentials are given by [4, 12, 21]:

$$ V_D = \frac{1}{2} (\text{Re}, f)^{-1} g^2 (q_n K_n \phi^n + \xi)^2, $$

$$ V_F = F^2 - 3e^{K/M_p^2} |W|^2 \frac{M_p^2}{M_p^2} $$

$$ = e^{K/M_p^2} \left[ \left( W_n + \frac{W}{M_p^2} K_n \right) K^{m\dagger} (W_m + \frac{W}{M_p^2} K_m)\dagger - 3|W|^2 \frac{M_p^2}{M_p^2} \right]. $$

Finally adding the contribution from Eq (1.2.77) and Eq (1.2.78) the total tree level potential within $N = 1$ SUGRA is given by the following expression [4, 12, 21]:

$$ V = V_D + V_F. $$

1.2.5 String theory and its low energy realizations

In this subsection we will highlight specific aspects of the ultra-violet (UV) completion of quantum field theoretic prescription and gravity sector. Here the field content and the corresponding interactions are described in the vicinity of Planck scale, which can be treated as the UV cut-off scale in the present context. This type of crucial issues are most naturally studied in the vicinity of Planck-scale and for this purpose string theory is the conceivably the best developed field theoretic

\[5\text{Within the framework of } N = 1 \text{ SUGRA, if we choose canonical SUGRA, } K_{nm} = \delta_{nm} \text{ then one can explicitly show that } V_D \text{ is proportional to } D^2 \text{ while } F^2 \text{ is equal to } \sum |F_n|^2.\]
prescription [54, 55, 56, 57, 58, 59, 60]. This motivates one to understand the various aspects of the physics of early universe within the framework of string theory. String theory is a very very successful theoretical framework within which all the point like particles as appearing in the context of particle physics are replaced by one dimensional objects called “strings”. Within this prescribed field theoretic setup different types of observed elementary particles in various experiments usually appear from the different quantum mechanical states of these one dimensional strings. Additionally it is importnat to note that, apart from hypothesizing various types of particles within SM of particle physics, string theory normally includes the effect of gravity. Consequently this can be treated as a most sucessful candidate for a self-contained and well established mathematical framework that unifies all the four fundamental forces and various forms of matter. Additionally, string field theoretic framework is now extensively used as a theoretical tool in various areas in theoretical physics and it has shed light on many unknown aspects of quantum field theory as well as quantum gravity. It is very well known fact that initially string theory is described by the bosonic sector only which incorporated the effet of bosonic degrees of freedom. Later a more sophisticated and upgraded version of the string theory was proposed, known as “superstring theory” which incorporates SUSY within this present field theoretic framework. Also it is important to mention here that, String theory requires the existence of extra spatial dimensions for its mathematical consistency. But all such extra dimensions are compactified to extremely small scales in realistic physical models constructed from the basic priciples from string theory. In our discussion we will mainly focus on the four-dimensional effective actions which can be used in the context of inflation.

D-branes

In this subsection let us briefly discuss about the very crucial component of the string theory which is made up of various solitonic degrees of freedom. It is important to note that, in this context of discussion $D$ branes are the most successful candidate. In case of $D_p$-brane one can describe this as an object which has $p$ spatial dimensions and also this is electrically charged under $C_{p+1}$. Additionally this is described by the Chern Simons (CS) action [61, 62, 63, 64] as given by:

$$S_{CS} = \mu_p \int_{\Sigma_{p+1}} C_{p+1}$$  \hspace{1cm} (1.2.80)

where $\Sigma_{p+1}$ physically represents the $D_p$-brane worldvolume and also $\mu_p$ characterizes the brane charge.

It is very well know fact that D-branes can be treated as surfaces on which open strings can end. Usually D-brane stands for Dirichlet brane which deals with the open strings and satisfies Dirichlet boundary conditions in the transverse directions D-brane. Also it is important to note that the open strings satisfies the Neumann boundary conditions as well in the directions along the spatial extension of a $D_p$-brane with the additional restriction $p > 0$. For the sake of simplicity let us concentrate on the bosonic sector for the discussion in the present context. In the context of our discussion a charge less $p$-dimensional membrane moving in a curved space-time with the curved background metric $G_{MN}$ can be directly explained by the well known Dirac action and within the framework of string theory this can be described by a extra dimensional generalization of the Polyakov action [61, 62, 63, 64] given by:

$$S_D = -T_p \int d^{p+1}\sigma \sqrt{-\det(G_{ab})} \quad \text{where} \quad G_{ab} \equiv \partial_{\sigma^n} X^M \partial_{\sigma^p} X^N G_{MN}.$$  \hspace{1cm} (1.2.81)
Here in Eq (1.2.81), $G_{ab}$ characterizes the usual pullback of the metric of the target space-time and also $T_p$ describes the membrane tension in the present context. To construct a most generalized version of D-brane within the framework of string theory one can firstly consider the effect of non-linear electromagnetism and this can be successfully described by Born-Infeld theory in the present context and described by the following action in $p + 1$ flat space-time dimensions [65, 66, 67]:

$$S_{BI} = -Q_p \int d^{p+1}\sigma \sqrt{-\det(\eta_{ab} + 2\pi \alpha' F_{ab})} = -Q_p \int d^{p+1}\sigma \left[ 1 + \frac{(2\pi \alpha')^2}{4} F_{ab} F^{ab} \right], \quad (1.2.82)$$

where $F_{ab}$ characterizes the field strength for Abelian gauge field $A_a$ and also $Q_p$ serves the role of a constant in the present discussion with the dimensions of brane tension. By computing string field theoretic amplitudes one can explicitly show that the effective action for a D$p$-brane in a most generalized string theoretic prescription can be written as a combination of the Dirac actions and Born-Infeld actions. This combined version is commonly known as the Dirac-Born-Infeld action [64, 68, 69] given by:

$$S_{DBI} = -g_s T_p \int d^{p+1}\sigma e^{-\Phi} \sqrt{-\det(G_{ab} + {\cal F}_{ab})}. \quad (1.2.83)$$

where

$${\cal F}_{ab} \equiv B_{ab} + 2\pi \alpha' F_{ab} \quad (1.2.84)$$

is a gauge invariant object. Here $\alpha'$ is the Regge slope parameter. Also the D$p$-brane tension is given by

$$T_p \equiv (2\pi)^{-p} g_s^{-1} (\alpha')^{-(p+1)/2}. \quad (1.2.85)$$

It is important to note that when $N$ number of D$p$-branes coincide in the present context, then the world volume gauge theory transformed to a non-Abelian gauge theory and in that case the representative action takes the following form [64, 69]:

$$S_{DBI} = -g_s T_p \int d^{p+1}\sigma \text{Tr} \left[ e^{-\Phi} \sqrt{-\det(G_{ab} + {\cal F}_{ab})} \right] \quad (1.2.86)$$

where the mathematical operation for the trace is performed over the gauge field indices. In that case the generalized version of the Chern-Simons action for $N$ number of D$p$-branes in the presence of various background fields can be written as [61, 62, 63, 64]:

$$S_{CS} = i\mu_p \int_{\Sigma_{p+1}} \text{Tr} \left[ \sum_n C_n \wedge e_2^F \right] \quad (1.2.87)$$

Finally, it is important to mention here that the complete bosonic action for D-branes in a low energy supergravity background can be expressed as the sum of the Dirac-Born-Infeld action and the Chern-Simons action as

$$S_{Dp} = S_{DBI} + S_{CS}. \quad (1.2.88)$$

**Dimensional Reduction**

Here our prime objective is to compute the effective action in $D = 4$ effective action using the basic principles of string compactification. To serve this purpose in this context one starts with the $D = 10$ string theory action and then use Kaluza-Klein (KK) reduction technique to derive an
1.2 Field theoretic tools for early universe

To give clear picture about this technique here we will discuss a very specific example in the present context.

to implement this technique let us start with a $D = 10$ geometry described by the following background metric:

$$G_{MN}dX^MdX^N = e^{-6U(x)}g_{\mu\nu}dx^\mu dx^\nu + e^{2U(x)}\hat{g}_{mn}dy^m dy^n \quad (1.2.89)$$

where $e^{U(x)}$ characterizes a “breathing mode” and $\hat{g}_{mn}$ signifies a reference metric with fixed six volume as given by

$$\int_{X_6} d^6 y \sqrt{\hat{g}} \equiv V. \quad (1.2.90)$$

Now it is important to mention here that, using the background metric mentioned in Eq (1.2.89) one can explicitly study the dimensional reduction technique of $D = 10$ Einstein Hilbert action to its $D = 4$ counterpart. To show this explicitly here we start with $D = 10$ Einstein Hilbert action which can be written as:

$$S^{(10)} = \frac{1}{2k_{10}^2} \int d^{10}X \sqrt{-G}e^{-\Phi}R_{10} \quad (1.2.91)$$

where $k_{10}$ be the ten dimensional gravitational coupling strength and $R_{10}$ be the Ricci scalar at $D = 10$. After performing dimensional reduction Eq (1.2.91) can be recast as:

$$S^{(10)} = \frac{1}{2k_{10}^2} \int d^4x \sqrt{-g} \int_{X_6} d^6 y \sqrt{\hat{g}}e^{-2\Phi} \left[ R_4 + e^{-8U(x)}\hat{R}_6 + \frac{12}{U^2(x)}\partial_\mu U(x)\partial^\mu U(x) \right] \quad (1.2.92)$$

Here we define $R_4$ to be the $D = 4$ counterpart of the Ricci scalar constructed from the metric $g_{\mu\nu}$ and also $\hat{R}_6$ characterizes $D = 6$ counterpart of the Ricci scalar constructed from the metric $\hat{g}_{mn}$. Here $U(x)$ are the scalar moduli field appearing from string theory. Most importantly the second term in the above mentioned dimensionally reduced version of the effective action is non-canonical in nature and interacted with the dilaton modes represented by $e^{-2\Phi}$. Also it is important to note that $D = 6$ counterpart of the Ricci scalar $\hat{R}_6$ interacted in a non-minimal fashion with both scalar field $U(x)$ and the dilaton modes. For the sake of simplicity here one can also consider a physical situation where the string theoretic coupling strength is given by

$$g_s \equiv e^\Phi \quad (1.2.93)$$

and in that case the effective action corresponding to the 4D counterpart of the Einstein- Hilbert action term can be written in the following form as:

$$S^{(4)} = \frac{M_p^2}{2} \int d^4x \sqrt{-g}e^{-\Phi}R_4 \quad (1.2.94)$$

where it is important to note that the four-dimensional effective (reduced) Planck mass scale appearing through dimensional reduction is defined as:

$$M_p^2 \equiv \frac{V}{g_s^2 k_{10}^2}. \quad (1.2.95)$$

1.2.6 Braneworld gravity

A very well known low energy version of String Theoretic prescription is Braneworld Gravity within which the observable universe is usually described by 1+3-surface. In technical language
this is known as “brane” or “membrane”. According to construction of braneworld gravity such membranes are embedded in a 1+3+d-dimensional space-time within the “bulk”, with Standard Model degrees of freedom and fields trapped on the brane. On the other hand in this context graviton and other field contents (example: dilaton, scalar field, antisymmetric higher rank tensor fields etc.) are freely propagate within the bulk [9]. In this discussion \( d \) characterizes the extra spatial dimensions. Brane-world gravity models includes a purely phenomenological way to study the features of modifications to general theory of relativity via warp geometry and extra dimensions. The Randall Sundrum braneworld model (RS) [70, 71], one of the pioneering warped geometry model, was proposed to resolve the long standing problem in connection with the fine tuning of the mass of Higgs (also known as gauge hierarchy or naturalness problem) in an otherwise successful Standard Model of elementary particles. In particle phenomenology, one of the important experimental signatures of such extra dimensional models is the search of the Kaluza-Klien (KK) gravitons in pp collision leading to dilepton decays in the Large Hadron Collider (LHC) [32]. The couplings of the zero mode as well as the higher KK modes are determined by assuming the standard model fields to be confined on a 3-brane located at an orbifold fixed point. Such a picture is rooted in a string-inspired model where the standard model fields being open string-excitations are localized on a 3-brane. This led to the braneworld description of extra dimensional models with gravity only propagating in the bulk as a closed string excitation. But apart from graviton, string theory admits of various higher rank antisymmetric tensor excitations as closed string modes which can also propagate as a bulk field. It was found that remarkably such fields are heavily suppressed on the brane in such warped geometry model and thus offers a possible explanation of invisibility of these fields in current experiments [72, 73, 74, 75]. Subsequently going beyond the stringy description, the implications of the presence of standard model fields in the bulk were also studied in different variants of warped braneworld models. All these models in general assumed the 3-brane hypersurface to be flat. These models were subsequently generalized to include non-flat branes [76, 77, 78] and also braneworld with larger number of extra dimensions [79, 80, 81].

From a theoretical standpoint, warped geometry model has its underlying motivation in the backdrop of string theory where the Klevanov-Strassler (KS) throat geometry solution exhibits warping character. While the Randall-Sundrum model starts with Einstein’s gravity in AdS\(_5\) manifold in five dimensional space-time, there have been efforts to include the higher curvature effects in the nature of the warped geometry. Such corrections originate naturally in string theory where power expansion in terms of inverse string tension yields the higher order corrections to pure Einstein’s gravity. Supergravity, as the low energy limit of heterotic string theory yields the Gauss-Bonnet (GB) term as the leading order correction and therefore became an active area of interest as a modified theory of gravity. In addition to this string theory admits of higher loop corrections which is a further modification on Einstein-Gauss-Bonnet correction on Einstein’s gravity [82, 83, 84].

**Higher dimensional gravity**

For an Einstein–Hilbert gravitational action in (4+d) dimension we have [9]:

\[
S_{\text{EH}}^{(4+d)} = \frac{1}{2\kappa_{4+d}^2} \int d^4 x \, d^d y \sqrt{-(4+d)g} \left[ (4+d)R - 2\Lambda_{4+d} \right]. \tag{1.2.96}
\]
After varying this action with respect to the (4+d) dimensional metric \((4+d)g_{AB}\) one can derive the equation of motion as [9]:

\[
(4+d)G_{AB} = (4+d)R_{AB} - \frac{1}{2}(4+d)R(4+d)g_{AB} - \Lambda_{4+d}(4+d)g_{AB} + \kappa_{4+d}^2(4+d)T_{AB},
\]

(1.2.97)

where the 4 + d coordinates can be identified as \(X^A = (x^\mu, y^1, \ldots, y^d)\), and in the present context \(\kappa_{4+d}^2\) signifies the (4+d) dimensional gravitational coupling constant and defined as:

\[
\kappa_{4+d}^2 = 8\pi G_{4+d} = \frac{1}{M_{4+d}^2} \sim \frac{L^d}{M_p^2}.
\]

(1.2.98)

where the length scale of the extra dimensions is \(L\) and we have used the fact that the fundamental scale via the volume of the extra dimensions can be written as:

\[
M_p^2 \sim M_{4+d}^2 L^d.
\]

(1.2.99)

If we incorporate the further modification on Einstein-Gauss-Bonnet dilaton correction on Einstein’s gravity then the (4+d) dimensional higher dimensional gravity is described by the following action [82, 84]:

\[
S_{EHGB}^{(4+d)} = \frac{1}{2\kappa_{4+d}^2} \int d^4x d^d y \sqrt{-(4+d)g} \left[(4+d)R - 2\Lambda_{4+d}\right.
\]

\[
+ \alpha_{4+d}(1 - A_{4+d}^{loop} \exp[\Theta_{4+d} \chi(y)]) \left\{(4+d)R^{ABCD}(4+d)R_{ABCD}
\right.\]

\[
- 4(4+d)R^{AB}(4+d)R_{AB} + \left(4+d\right)R^2\right\},
\]

(1.2.100)

where \(\alpha_{4+d}, A_{4+d}^{loop\text{,} ab\text{,} cd}\) and \(\Theta_{4+d}\) represent the Gauss-Bonnet, string two-loop and dilaton coupling respectively coming from the interaction with dilatonic degrees of freedom coming from the computation of Conformal Field Theory (CFT) disk amplitude in the bulk geometry. After varying Eq (1.2.100) with respect to the (4+d) dimensional metric \((4+d)g_{AB}\) one can derive the equation of motion as [82, 84]:

\[
(4+d)G_{AB} + \alpha_{4+d}(1 - A_{4+d}^{loop} \exp[\Theta_{4+d} \chi(y)]) (4+d)H_{AB} = -\Lambda_{4+d}(4+d)g_{AB} + \kappa_{4+d}^2(4+d)T_{AB},
\]

(1.2.101)

where the Gauss-Bonnet tensor \((4+d)H_{AB}\) in (4+d) dimension is defined as [82, 84]:

\[
(4+d)H_{AB} = 2(4+d)R_{ACDE}(4+d)R^{CDE} - 4(4+d)R_{ACBD}(4+d)R^{CD} - 4(4+d)R_{AC}(4+d)R^C_B + 2(4+d)R(4+d)R_{AB} - \frac{1}{2}(4+d)g_{AB} \left((4+d)R^{ABCD}(4+d)R_{ABCD} - 4(4+d)R^{AB}(4+d)R_{AB} + (4+d)R^2\right).
\]

(1.2.102)

Let us now discuss about three well known braneworld gravity models elaborately:

### A. Randall–Sundrum braneworlds

Let us start our discussion with Randall–Sundrum (RS) braneworld gravity model. Before going to details of RS brane-worlds it is importnat to mention here that such type of frameworks do not
rely on the compactification technique to localize the gravity at the orbifold fixed points where the brane is fixed. But within this framework the curvature of the bulk plays a significant role as in warped geometry background one can easily implement the method of compactification very easily in the present context. In the slice of $AdS_5$ to make the curvature of the space-time real and positive one puts necessary restriction on the signature of the five dimensional bulk cosmological constant and in the present context it is given by:

$$\Lambda_5 = \frac{6}{f^2} = -6q^2 < 0,$$

(1.2.103)

where $f$ signifies the radius of curvature $AdS_5$ and $q$ characterizes the corresponding energy scale associated with this setup. Now introducing Gaussian normal coordinates $X^A = (x^\mu, y)$ localized on the brane at the orbifold point $y = 0$ one can re-express the $AdS_5$ metric in the following form as $[70, 71]$:

$$(5)^2ds^2 = e^{-\frac{2\text{y}}{f}} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

(1.2.104)

with $\eta_{\mu\nu}$ being the usual flat Minkowski metric. It is important to mention here that here $Z_2$-symmetry is imposed at the orbifold fixed point $y = 0$. In the bulk the 5D Einstein equations $[70, 71]$ can be written as:

$$(5)^2G_{AB} = -\Lambda_5 (5)^2g_{AB}.$$  

(1.2.105)

Here it is additionally important to note that to avoid back reaction effect we set

$$ (5)^2T_{AB} = 0$$  

(1.2.106)

in the 5D bulk Einstein equations.

Similarly neglecting the contribution from back reaction effect within the bulk the Einstein Gauss Bonnet in presence of dilaton coupling turns out to be $[82, 84]$:

$$(5)^2G_{AB} + \alpha_5 (1 - A_5^{loop} \exp(\Theta_5 \chi(y))) (5)^2H_{AB} = -\Lambda_5 (5)^2g_{AB},$$

(1.2.107)

where the Gauss-Bonnet tensor $(5)^2H_{AB}$ is defined in (4+1) dimension and the dilaton field $\chi(y)$ is given by $[82, 84]$:

$$\chi(y) = c_1 |y| + c_2$$

(1.2.108)

where $c_1$ and $c_2$ are dimensionful arbitrary integration constants which can be precisely determined by imposing the boundary condition at the orbifold fixed points. Now using these solutions in the higher curvature gravity setup, the warped metric in $AdS_5$ slice takes the form $[82, 84]$:

$$(5)^2ds^2 = e^{-\frac{3}{4\pi} \frac{1}{\alpha_5 (1 - A_5^{loop} \exp(\Theta_5 \chi(y)))} \left[ 1 \pm \sqrt{1 - \frac{8\pi^2}{\alpha_5^2 (1 - A_5^{loop} \exp(\Theta_5 \chi(y)))}} \right]} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

(1.2.109)

where we get two branch of warping solutions. In the limit $\alpha_5 \to 0$, $A_5^{loop} \to 0$ and $\Theta_5 \to 0$ the -ve branch of solution exactly reduces to RS metric as stated in Eq (1.2.104). On the other hand, the +ve branch diverges in such weak coupling limit and incorporates ghost fields.

The two RS models are distinguished as follows:

1. **RS 2-brane:** In this prescription there are two branes are localized at two orbifold fixed points $y = 0$ and $y = L \sim \pi$. Most importantly in this context the branes separated by a
distance $r_c$ have equal and opposite membrane tensions $\pm \lambda_b$ and it is given by the following expression:

$$\lambda_b = \frac{3M_5^2}{4\pi f^2}. \quad (1.2.110)$$

The positive-tension brane has fundamental scale $M_5$ called hidden and negative-tension brane is called visible brane where all the SM fields are embedded. In this context the effective 4D mass scale can be expressed in terms of 5D quantum gravity cut-off scale as:

$$M^2_5 = M^3_5 \left[ e^{2\pi f} - 1 \right] f. \quad (1.2.111)$$

Additionally it is important to note that using this set of braneworld construction it is possible to address the well known hierarchy problem [82, 84].

2. RS 1-brane: In this prescription there is a single brane which is localized at the orbifold fixed point $y = 0$ with positive membrane tension. Here one can think that the single braneworld prescription is arises from the two braneworld prescription, provided the negative brane tension brane is infinitely far from the other brane which is fixed at $y = 0$. In this context the effective 4D mass scale can be expressed in terms of 5D quantum gravity cut-off scale as:

$$M^3_5 = \frac{M^2_5}{f}. \quad (1.2.112)$$

Surprisingly within the single brane framework we get a finite contribution to the 5D world volume because of the warp factor $f$ and:

$$V_5 = \int d^5 X \sqrt{-\tilde{g}} = \frac{f}{2} \int d^4 x. \quad (1.2.113)$$

This directly implies that the effective size of the extra dimension realized by the 5D graviton degrees of freedom is of the order of $f$.

B. Other contemporary models

In this section we will give a short introduction about other well known contemporary techniques applicable in the context of braneworld (modified) gravity.

1. DGP model:

The Dvali-Gabadadze-Porrati model is a model of modified gravity theory which consists of a 4D Minkowski brane embedded in a 5D Minkowski bulk like Randall Sundrum (RS) 2 model as discussed earlier. In the present context the 5th extra dimension is flat as well as infinitely large. Most importantly in this context one can easily recover the Newton’s law by just adding a 4D Einstein-Hilbert term generated by the braneworld curvature contribution to the 5D action. Additionally it is important to note that, in the present context at very small length scales the standard 4D gravity theory can be retrieved. On the other hand the effect from the 5D gravity plays significant role in large length scales.

The DGP model is described by the following 5D action [85] as given by:

$$S = \frac{1}{2\kappa^2(5)} \int d^5 X \sqrt{-\tilde{g}} \tilde{R} + \frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} R + \int d^4 x \sqrt{-g} L_{brane}^M, \quad (1.2.114)$$
where the Lagrangian $L_{\text{brane}}$ describes matter localized on the 3-brane. Additionally $\tilde{g}_{AB}$ characterizes the metric in the 5D bulk and the corresponding 4D counterpart of the metric can be written in terms of the 5D metric as:

$$g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \tilde{g}_{AB}. \quad (1.2.115)$$

This can be physically interpreted as the induced metric on the brane with bulk coordinates $X^A(x^c)$ and the brane coordinates are labeled by $x^c$. Also it is important to note that, the first and second terms as appearing in Eq. (1.2.114) correspond to Einstein-Hilbert actions in the 5D bulk and on the brane respectively. In the present context $\kappa^2_{(5)}$ and $\kappa^2$ are the 5D and 4D gravitational coupling constants, respectively, which are related with 5D and 4D Planck masses, $M_5$ and $M_4$, via the following relationship:

$$\kappa^2_{(5)} = \frac{1}{M_5^3}, \quad \kappa^2 = \frac{1}{M_4^2} = \frac{1}{M_p^2} = 8\pi G. \quad (1.2.116)$$

The bulk equations of motion in 5D can be expressed as:

$$G^{(5)}_{AB} = 0, \quad (1.2.118)$$

where $G^{(5)}_{AB}$ represents the 5D counterpart of the Einstein tensor. Further applying the well known Israel junction conditions for non compact extra dimension on the brane the effective equation of motion can be expressed as:

$$G_{\mu\nu} - \frac{1}{r_c} (K_{\mu\nu} - g_{\mu\nu} K) = \kappa^2_{(4)} T_{\mu\nu}, \quad (1.2.119)$$

where $K_{\mu\nu}$ characterizes the extrinsic curvature computed on the brane, $T_{\mu\nu}$ signifies the energy-momentum tensor for localized matter on the brane. Additionally, here $r_c$ characterizes the crossover or critical length scale defined by:

$$r_c = \frac{M_4^2}{2M_5^3}. \quad (1.2.120)$$

Such length scale sets an upper limit above which the effect of extra dimension becomes important in the present discussion and below such scale it is also possible to retrieve the GR limiting results.

The modified Friedmann equations in this DGP model are given by [85]:

$$H^2 = \left( \sqrt{\frac{\kappa^2 \rho}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right)^2, \quad (1.2.121)$$

$$\dot{H} + H^2 = -\frac{\kappa^2}{6} (\rho + p) \left[ 1 + \left( \frac{\kappa^2 \rho}{3} + \frac{1}{4r_c^2} \right)^{-1/2} \right] + \left[ \sqrt{\frac{\kappa^2 \rho}{3} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right]^2. \quad (1.2.122)$$

In the present analysis, we have set the extra dimension as time-like in DGP model. But in generalized prescription in principle one can consider both space and time like extra dimensions in the present context.

---

*In the next section we have introduced the Friedmann equation in the context of General Relativity (GR) for flat FLRW spacetime.*
However apart from the various success of the DGP model in the context of modification in the gravity sector it has crucial problems as well. These are:

- The model deals with ghost degrees of freedom, which cannot be possible to get read off from the present setup. Additionally, applying a quasi-static approximation to linear cosmological perturbations on the length scales required for the large scale structure formation directly shows that the DGP model contains a ghost mode.
- In the quantum regime of gravity the model has strong coupling problem for the length scale smaller than $\mathcal{O}(1000)$ km.
- Additionally this model contains super-luminal modes.

2. **ADD model:**

This model proposes that the scale of gravity is $M_{\text{ew}}$ in the bulk space-time. To make this proposal consistent with observations we need to change the nature of gravity so that this scale is related to 4D Planck mass in a natural way. This is achieved by assuming ordinary matter and gauge fields are localized on a 3-brane which are embedded in a higher-dimensional bulk. Only gravity propagates in higher dimensions and becomes strongly coupled at the TeV scale. This proposal ignores brane tension (mass per unit volume) and consequently back-reaction of it on the geometry transverse to the brane. In the simplest form it considers compact extra dimensions of radius $R$. Only gravity becomes higher dimensional at length scales smaller than $R$. Since Newton’s law in 4D is experimentally tested up to 0.2 mm, to be consistent with it, extra dimensions should be smaller than 0.2 mm but can be as large as, say, 0.1 mm. In other words, fundamental scale of higher-dimensional gravity is $M \sim 10^3$ GeV. One starts with a $(4+d)$ dimensional Einstein action,

$$S^{(4+d)} = \frac{1}{2\kappa_4^{2+d}} \int d^4x \, d^dy \sqrt{-g^{(4+d)}} R.$$  \hspace{1cm} (1.2.123)

4D gravity is obtained by simple dimension reduction and by ignoring massive KK modes. Since $d$-dimensional space is compactified on a flat torus, wave function of the zero mode is homogeneous in extra dimensions. 4D effective action for gravity is as:

$$S_{\text{eff}} = \frac{V_d}{2\kappa_4^{2+d}} \int d^4x \sqrt{-g^{(4)}} R$$  \hspace{1cm} (1.2.124)

where $V_d \sim R^d$ is the volume of extra dimensions. We can now read off the 4D Newton’s constant

$$G_N = G^{4+d}/V_d = M^{-(2+d)} R^{-d}$$  \hspace{1cm} (1.2.125)

or equivalently,

$$M_p = M(MR)^{d/2}.$$  \hspace{1cm} (1.2.126)

Planck mass $M_p$ is a derived quantity and can be made larger by having large extra dimensions. While this proposal removes $M_{\text{ew}}/M_p$ hierarchy, it resurfaces as a hierarchy of scales between the compactification scale $\sim 10^{-2}$ cm and the electroweak scale $\sim 10^{-16}$ cm. Alternatively, we have very light KK mass spectrum $m \sim \text{MeV}$. In ADD model [26, 27], KK modes are uniformly distributed in the internal dimensions. We can get weak gravity coupling if we make them spend most of their time away from the visible brane thereby reducing their overlap with the physical brane.
1. Introduction

1.3 Applications of Field Theory to Early Universe

1.3.1 Inflation

Inflation is a well accepted proposal to explain the physics of early universe based on the prescription of quantum field theory minimally or non-minimally coupled to the gravity sector. It is a very well known fact that for a given a field theoretic prescription, it is generally simple to compute the time evolution of the homogeneous cosmological background, and many many quantum field theoretic prescriptions that support the existence of cosmological inflationary phases have been proposed. However, deriving the effective action in four dimension for the inflationary paradigm from a more fundamental physical principle, or specifically in the context of a well-motivated original background theory, notwithstanding an important problem.

There are two parallel and well known approaches which can be implemented to finally obtain an ultimate quantum field theoretic prescription well acceptable in the context of inflation [54, 86, 87, 88, 89, 90]. Specifically in the ‘top-down’ approach, the general notion is to start the computation with a UV-complete field theoretic prescription, and using this approach one finally attempts to derive the theory of inflation as one of the low energy effective theory. On the other hand in the ‘bottom-up’ approach one generally start the computation from the low-energy (IR) degrees of freedom and try to predict the background UV complete field theory. Most importantly both the parallel approaches appear at an effective field theory prescription which is justifiable at inflationary energy scales.

The original incentive for implementing the idea of Inflation was to explain the initial conditions for the Big-Bang theory or more precisely the initial condition for large scale structure formation observed at present epoch, by hypothesizing a much earlier epoch during which our Universe expanded and the scale factor varies quasi exponentially. But, at present inflation is considered as the most favored theoretical prescription in physics to describe the cosmological evolution of Early Universe. Throughout the thesis we will focus only on the single field models of inflation which are embedded within the framework of effective field theory prescription [54, 86, 87, 88, 89, 90].

Conditions for inflation

The conditions for inflation are described by the following conditions [12, 21, 54]:

- **I. Decreasing comoving horizon:**
  During the epoch of inflation the comoving Hubble sphere shrinks and after inflation it expands. So one can explain inflation as a physical mechanism to “zoom-in” on a smooth sub-horizon patch which finally generates required amount of cosmological fluctuations consistent with various observational probes. Using the Friedmann Equations a shrinking comoving Hubble radius can be expressed in terms of the acceleration and the pressure of the universe:

\[
\frac{d}{dt}(aH)^{-1} < 0. \tag{1.3.127}
\]

- **II. Accelerated expansion:**
  Using the relation

\[
\frac{d}{dt}(aH)^{-1} = -\frac{\ddot{a}}{(\dot{a})^2} \tag{1.3.128}
\]
it can be easily shown that shrinking comoving Hubble radius directly implies accelerated expansion of universe given by:
\[
\frac{d^2a}{dt^2} > 0. \tag{1.3.129}
\]

• III. Slowly-varying Hubble parameter:
Similarly using the relation
\[
\frac{d}{dt}(aH)^{-1} = -\frac{1}{a}(1 - \epsilon), \tag{1.3.130}
\]
where
\[
\epsilon = -\frac{\dot{H}}{H^2} > 0 \tag{1.3.131}
\]
it can be easily shown that shrinking comoving Hubble radius implies the following constraint:
\[
\epsilon = -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN} < 1. \tag{1.3.132}
\]
where we have defined \(dN = Hdt = d\ln a\), which measures the number of e-folds \(N\) of inflationary expansion. Also to achieve sufficient number of e-foldings during inflationary epoch the necessary condition is the slow roll parameter \(\epsilon\) must be small compared to unity for a sufficient large number of Hubble times. This condition is explicitly taken care of by the second slow roll parameter defined as:
\[
\eta = \frac{\dot{\epsilon}}{H\epsilon} = \frac{d\ln \epsilon}{dN}. \tag{1.3.133}
\]
For \(|\eta| < 1\) the fractional change of \(\epsilon\) per Hubble time is small compared to unity and inflation continues.

• IV. Negative pressure:
Although this condition essentially follows from the above discussion, assuming a perfect fluid with pressure \(p\) and density \(\rho\), we can write the Friedmann Equations in the following form as:
\[
\dot{H} + H^2 = -\frac{1}{6M_p^2}(\rho + 3p) = -\frac{H^2}{2} \left(1 + \frac{3p}{\rho}\right). \tag{1.3.134}
\]
Now here it is important to note that:
\[
\epsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1 + w) \quad \Leftrightarrow \quad w = \frac{p}{\rho} < -\frac{1}{3}. \tag{1.3.135}
\]
which directly implies that inflation requires negative pressure or more precisely a violation of the strong energy condition (SEC).

Effective field theory of inflation
It is very well known fact that for single scalar field models, cosmological dynamics for inflation is directly explained by the scalar inflaton field \(\phi\) which is minimally coupled to Einstein gravity sector
\[
S_{\text{EFT}} = \int d^4x\sqrt{-g} \left[ \frac{M_p^2}{2} \left(\frac{4}{4}\right) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \tag{1.3.136}
\]

\(^7\)In principle, a non-minimal coupling between the inflaton and the graviton sector can be considered in the present context. However, in most of the realistic physical situations, such non-minimally coupled theories can be transformed to minimally coupled form by a field redefinition or via special type of conformal transformation in the
where $^{(4)}R$ is the four-dimensional Ricci scalar derived from the metric $g_{\mu\nu}$ in FLRW background and $V(\phi)$ be the inflationary potential. In principle $V(\phi)$ can be expressed in terms of any arbitrary mathematical function of the inflation field $\phi$ in the present context of discussion. However, the background field theoretic prescriptions always restrict the 4D field theory to take such an arbitrary mathematical form. In a generic physical prescription the inflaton potential $V(\phi)$ can be expressed as:

$$V(\phi) = V_{\text{ren}}(\phi) + \sum_{\alpha=5}^{\infty} c_\alpha \frac{\phi^\alpha}{\Lambda_{\text{UV}}^{\alpha-4}}$$  \hfill (1.3.137)

where $\Lambda_{\text{UV}}$ is the UV cut-off of gravity. In our discussion we take $\Lambda_{\text{UV}} \sim M_p$ and $c_\alpha$ be the expansion co-efficient of the non-renormalizable contribution in the potential. In the effective field theory prescription $c_\alpha$ can be treated as the Wilson coefficients as appearing in the context of RG flow. Also the contribution from the renormalizable part of the inflationary potential can be expressed as:

$$V_{\text{ren}}(\phi) = \sum_{\delta=0}^{4} a_\delta \frac{\phi^\delta}{\Lambda_{\text{UV}}^{\delta-4}}$$  \hfill (1.3.138)

where $a_\delta$ is the expansion co-efficient in the renormalizable sector in EFT and can also be treated as Wilson coefficients as appearing in the context of RG flow. However in this most generalized prescription various types of interactions between the scalar inflaton field and other matter degree of freedom are allowed via higher mass dimensional UV scale suppressed non-renormalizable operators in the kinetic as well the potential sector of the field theory arising from effective field theory (EFT) \cite{54, 86, 87, 88, 89, 90} prescription. Let us briefly discuss two parallel theoretical approaches of EFT through which one can be able to study the details of inflationary paradigm. These approaches are-

**Top down approach:**

To construct an effective field theory setup following the top down approach one needs to first identify the physical degrees of freedom that are significant to explain the observational results. More precisely in this context we introduce a cutoff scale $\Lambda_{\text{UV}}$ of gravity sector to specify the regime of validity of the EFT prescription in the context of inflation. In the present discussion light particles $\phi$, with very small masses $m < \Lambda_{\text{UV}}$, are included within the framework of effective field theory of inflation. On the other hand usually the heavy particles $\Psi$, with large masses $M > \Lambda_{\text{UV}}$, are integrated out from the framework of effective field theory. Within this physical prescription the most generalized action for EFT of inflation can be recast from Eq (1.3.136) in the following form as:

$$S_{\text{EFT2}} = \int d^4x \sqrt{-g} \left[ \frac{\Lambda_{\text{UV}}^{4}\Lambda_{\text{UV}}^{4}}{2}R + L_{\text{inf}}[\phi] + L_{\text{heavy}}[\Psi] + L_{\text{int}}[\phi, \Psi] \right].$$  \hfill (1.3.139)

where $L_{\text{inf}}[\phi]$ and $L_{\text{heavy}}[\Psi]$ represent the part of total Lagrangian density $\mathcal{L}$ involving only the light and heavy fields, and $L_{\text{int}}[\phi, \Psi]$ includes all types of possible interactions involving the light and heavy fields within EFT prescription for inflation. Now to integrate out the effects of all such heavy field from EFT of inflation picture here one need to preform the present context. Example: Higgs inflation in presence of non-minimal coupling \cite{91}. Similarly, one can think of the possibility that the leading order the UV incomplete Einstein-Hilbert part of the action is modified at high energies in presence of perturbative UV corrections in the gravity sector. The simplest examples are: $f(R)$ gravity \cite{92, 93, 94}, Gauss-Bonnet gravity \cite{92, 93, 94, 95}, which can be transformed into a minimally coupled scalar field.
path integral over the heavy degrees of freedom and over the all possible high frequency contributions of the light degrees of freedom within the framework of EFT of inflation and it can be written in terms of the following effective action for inflation as:

$$e^{iS_{EFT2}[\phi]} = \int [D\Psi] e^{iS_{EFT2}[\phi, \Psi]}.$$  (1.3.140)

In EFT prescription this serves the purpose for Wilsonian effective action from which one can easily study RG flow. The effective action for inflation in 4D admits a systematic series expansion in terms of the light degrees of freedom for which the total action within EFT action for inflation as stated in Eq (1.3.139) using Eq (1.3.140) can be re-expressed as:

$$S_{EFT2} = \int d^4x \sqrt{(-g)} \left[ \frac{\Lambda_{UV}^2}{2} R + \mathcal{L}_{inf}[\phi] + \sum_{\alpha} J_\alpha(g) \frac{\mathcal{O}_\alpha[\phi]}{M_{\Delta_\alpha-4}} \right].$$  (1.3.141)

where $J_\alpha(g) \forall \alpha$ are the dimensionless coupling constants that depend on the coupling parameter $g$ of the background UV complete higher dimensional field theoretic prescription. Also $O_\alpha[\phi]$ characterizes the local EFT renormalizable or non-renormalizable higher mass dimensional EFT operators for inflation of mass dimension $\Delta_\alpha$. Following this prescription one can easily able to generate all such relevant local EFT renormalizable or non-renormalizable higher mass dimensional EFT operators for inflation $O_\alpha[\phi]$ physically consistent with the symmetries of the background UV complete higher dimensional field theory. If we switch off all such relevant contribution from the EFT of inflation which are physically allowed by the symmetries of the the background UV complete higher dimensional field theoretic prescription is generally described as a “fine-tuning”[54, 86, 87, 88, 89, 90] mechanism or “naturalness” in EFT picture.

**Bottom up approach:**

In the bottom up approach the situation is completely opposite as discussed above. Here we do not know the structural form of the complete UV complete higher dimensional field theoretic prescription and due to this reason one cannot at all construct the EFT explicitly by integrating out the heavy degrees of freedom always from the background field theoretic picture. In the present context we usually parameterize the various choices of UV field theoretic setup and then we explicitly use various assumptions about the symmetries of the UV field theoretic setup and finally we write down the most general EFT action for inflation which is consistent with all of these symmetries appearing in the context of UV theory. In this case the 4D effective action can be expressed as:

$$S_{EFT3} = \int d^4x \sqrt{(-g)} \left[ \frac{\Lambda_{UV}^2}{2} R + \mathcal{L}_{inf}[\phi] + \sum_{\alpha} z_\alpha(g) \frac{\mathcal{O}_\alpha[\phi]}{M_{\Delta_\alpha-4}} \right].$$  (1.3.142)

where it is important to note that the sum runs over all local EFT renormalizable or non-renormalizable higher mass dimensional EFT operators for inflation $O_\alpha[\phi]$ of mass dimension $\Delta_\alpha$, favored by the symmetries appearing in the context of UV field theoretic setup. The size of the local EFT renormalizable or non-renormalizable higher mass dimensional EFT operators for inflation is estimated in terms of the EFT UV cutoff scale $\Lambda_{UV}$, while the pre-factors appearing in Eq (1.3.142) $z_\alpha$ serves the purpose of dimensionless Wilson coefficients in the context of EFT of inflation.
1. Introduction

Slow-roll technique

To illustrate the features of slow-roll, we start with the Klein Gordon equation and the Friedmann equations computed from the simplest action (1.3.136). The equations governing the cosmological dynamics are:

\[
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \\
H^2 = \frac{\rho}{3M_p^2} = \frac{1}{6M_p^2} \left[ \dot{\phi}^2 + 2V(\phi) \right], \\
\dot{H} + H^2 = -\frac{1}{6M_p^2} (\rho + 3p) = -\frac{1}{3M_p^2} \left[ \dot{\phi}^2 - V(\phi) \right].
\]

The slow-roll approximation in EFT of inflation states that-

- The contribution from the kinetic term is negligibly small in the effective pressure \( p \) and density \( \rho \). Consequently the Friedmann equations can be recast as:

\[
H^2 \approx \frac{V(\phi)}{3M_p^2}, \quad \dot{H} + H^2 \approx \frac{V(\phi)}{3M_p^2}.
\]

- The contribution from the acceleration term, \( \ddot{\phi} \), is negligibly small compared to the damping term \( 3H\dot{\phi} \) and \( V'(\phi) \). Consequently the Klein-Gordon equation is modified to:

\[
3H\dot{\phi} + V'(\phi) \approx 0.
\]

Furthermore, the slow-roll technique is characterized by a set of flatness parameters which can be expressed in terms of the inflationary potential and its higher powers of the derivatives as:

\[
\epsilon_V = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_V = M_p^2 \left( \frac{V''}{V} \right), \\
\xi_V^2 = M_p^4 \left( \frac{V'V''}{V^2} \right), \quad \sigma_V^3 = M_p^6 \left( \frac{V'V''' - V''^2}{V^3} \right),
\]

where within the slow-roll regime,

\[
\epsilon_V, |\eta_V|, |\xi_V^2|, |\sigma_V^3| \ll 1.
\]

Inflation ends when the slow-roll approximation is violated and the corresponding field value of the inflaton is computed from the following equation:

\[
\max_{\phi} \{ \epsilon_V, |\eta_V|, |\xi_V^2|, |\sigma_V^3| \} = 1
\]

The amount of inflaton is generally expressed as the logarithmic difference between the final and initial values of the scale factor, and is called number of e-foldings, \( N \):

\[
N \equiv \ln \left( \frac{a_f}{a_i} \right) = \int_{t_i}^{t_f} H dt \approx -\frac{1}{M_p} \int_{\phi_i}^{\phi_f} \frac{d\phi}{\sqrt{2\epsilon_V}}.
\]

---

8By assuming the isotropy and homogeneity in the background geometry here we use the Friedmann-Robertson-Walker (FRW) metric with spatially flat hypersurface.

9The Hubble slow roll parameter \( \epsilon, \eta \) and the potential dependent slow-roll parameters \( \epsilon_V, \eta_V \) are connected via the relations, \( \epsilon \approx \epsilon_V \), and \( \eta \approx \eta_V - \epsilon_V \).
1.3 Applications of Field Theory to Early Universe

where \(a_i\) and \(a_f\) and \(\phi_i\) and \(\phi_f\) are the values of the scale factor and inflaton field at the start and at the end of inflation respectively. The largest scales observed in the CMB are produced some 50 to 70 e-folds before the end of inflation

\[
N_{\text{cmb}} = \frac{1}{M_p} \int_{\phi_{\text{cmb}}}^{\phi_f} \frac{d\phi}{\sqrt{2\epsilon V}} \approx 50 - 70. \quad (1.3.153)
\]

A successful solution to the horizon problem requires at least \(N_{\text{cmb}}\) e-folds of inflation.

The dynamics of the inflaton field, from the time when CMB fluctuations were created at \(\phi_{\text{cmb}}\) to the end of inflation at \(\phi_f\), is determined by the shape of the inflationary potential \(V(\phi)\). The different possibilities for \(V(\phi)\) can be classified in a useful way by determining whether they allow the inflaton field to move over a large or small distance via field excursion,

\[
\Delta \phi \equiv \phi_{\text{cmb}} - \phi_f, \quad (1.3.154)
\]

as measured in Planck units. Depending on the numerical values of the field excursion the single field inflationary models are classified into two categories \(^{10}\):

- **A. Small field model:** Within this category the field excursion becomes sub-Planckian, \(\Delta \phi \leq M_p\) and the VEV of the inflaton field is also sub-Planckian \(^{11}\),

\[
\langle \phi \rangle \equiv \phi_0 \leq M_p. \quad (1.3.155)
\]

Small field models are characterized by the concave potentials i.e.,

\[
V''(\phi) \leq 0. \quad (1.3.156)
\]

Example: MSSM inflation from saddle point \(V'(\phi_0) = 0 = V'''(\phi_0)\) \([99, 100]\) and inflection point \(V''(\phi_0) = 0\) \([101, 102]\), Assisted inflation \([103, 104]\) satisfy this condition.

- **B. Large field model:** Within this category the field excursion becomes super-Planckian, \(\Delta \phi > M_p\) and the VEV of the inflaton field is super-Planckian \(^{12}\),

\[
\langle \phi \rangle \equiv \phi_0 > M_p. \quad (1.3.157)
\]

Small field models are characterized by the concave potentials i.e.,

\[
V''(\phi) > 0. \quad (1.3.158)
\]

Example: Chaotic inflation \([106, 107]\), Natural inflation \([108]\) satisfy this condition.

---

\(^{10}\)In figure(1.1) we have shown the behaviour of various types of single field inflationary models obtained from sub-Planckian and super-Planckian field excursion.

\(^{11}\)This is consistent with the EFT prescription for inflation where the particle theory is embedded in EFT setup which is trustable below the UV cut-off of the scale of gravity \([96, 97, 98]\).

\(^{12}\)This is not consistent with the EFT prescription for inflation. However string theory models can be embedded within this prescription within which it is possible to go beyond the UV cut-off of the scale of gravity \([105]\).
Possible extensions of the paradigm

In this subsection we mention the various physical possibilities for getting inflationary paradigm in extended picture. It is very well known fact that inflation is a theoretical framework for the early universe, but it is not at all a unique prescription to describe the physics of early universe. From phenomenological point of view various models of inflation have been proposed with distinctive theoretical incentives as well to describe various observational predictions. In this context the most straightforward inflationary paradigm described by single field effective actions in 4D can be modified the following ways:

- **A. Non-minimal coupling to gravity:**
  In principle, we could imagine a non-minimal coupling between the inflaton and the graviton, however, in practical purpose, non-minimally coupled gravity theories can be easily expressed as minimally coupled gravity theories by just a field re-definition via conformal transformations on the background FLRW metric. Higgs inflation \([91, 109]\) is one of the examples of the inflation in presence of non-minimal coupling with the Einstein-Hilbert term in the representative action of EFT.

- **B. Modified gravity:**
  In this specific case the Einstein-Hilbert part of the effective action is modified at high energy scales in presence of higher curvature perturbative corrections motivated from quantum gravity sector. Nevertheless, the simplest possibilities for this UV extension of gravity sector, so-called \(f(R)\) theory of gravity and Gauss-Bonnet gravity non-minimally coupled with a scalar field in 4D. Here both of the extended theories of gravity can again easily be transformed

---

Figure 1.1: Examples of different classes of slow-roll potentials: (a) hilltop inflation, (b) infection point inflation, (c) chaotic inflation and (d) natural inflation. The light gray regions indicate the parts of the potential where slow-roll inflation occurs. The dark gray regions denote regions of eternal inflation. Here (a) and (b) correspond to small-field models \((\Delta \phi < M_p)\), while (c) and (d) are large-field models \((\Delta \phi > M_p)\) \([59]\).
into a gravity theory minimally coupled scalar field in 4D. Starobinsky inflation \cite{110,111} is one of the famous examples of the inflationary model driven by modified gravity prescription, where the usual Einstein-Hilbert term, $R$, is modified by $R + \alpha R^2$.

\begin{itemize}
  \item **C. Non-canonical kinetic term:**
    Within this prescription the inflaton action in presence of non-canonical term is modified as \cite{54}:
    \[
    S_{\phi} = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left[ P(\phi, X) - V(\phi) \right]
    \]
    where $P(\phi, X)$ is arbitrary function of the inflaton field and its derivative,
    \[
    X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.
    \]
    In this specific situation one can also consider a possibility where it is easily possible that inflation is driven by the kinetic term as appearing in the effective action in 4D and occurs even in the presence of a steep inflationary potential, which is different from usual slow-roll inflationary models explained through canonical kinetic terms in the matter sector. Single field DBI inflation \cite{112,113} and Galileon inflation \cite{114,115} are the famous examples of the inflation driven by $P(X, \phi)$ theory.
  \\
  \item **D. Multifield model:**
    In this context we incorporate more than one scalar field in the background field theoretic picture and all of them are dynamically relevant during the epoch of inflation. Consequently the possibilities to explain the inflationary paradigm and the dynamical mechanisms for the production of quantum fluctuations expand in very dramatic fashion and finally the effective theory loses various important predictions in the present context. Multifield DBI \cite{116} and DBI Galileon \cite{117} inflation are the famous examples in this area.
\end{itemize}

**Quantum fluctuations within EFT**

The quantum fluctuations of inflaton produce a spectrum that matches the CMB observations to quite a good extent. The inflaton evolution $\phi(t)$ governs the energy density of the early universe $\rho(t)$ and hence controls the end of inflation. Essentially, $\phi$ plays the role of a local clock reading off the amount of inflationary expansion remaining. Because microscopic clocks are quantum mechanical objects with necessarily some variance by the uncertainty principle, the inflaton will have spatially varying fluctuations
\[
\delta \phi(t, \mathbf{x}) \equiv \phi(t, \mathbf{x}) - \bar{\phi}(t).
\]
These fluctuations imply that different regions of space inflate by different amounts. In other words, there will be local differences in the time when inflation end $\delta t(\mathbf{x})$. Moreover, these differences in the local expansion lead to differences in the local densities after inflation.

Herein we will briefly study both of the scalar and tensor fluctuations in the present context. For the scalar modes we have to be careful to identify the true physical degrees of freedom. A priori, we have 5 scalar modes which come from 4 metric perturbations given by $\delta g_{00}$, $\delta g_{ii}$, $\delta g_{0i}$ and $\delta g_{ij}$ and 1 scalar field perturbation $\delta \phi$. Gauge invariance associated with the invariance of the effective action of the EFT as stated in Eq (1.3.136) under scalar coordinate transformations:
\[
t \to t + \epsilon_0
\]
1. Introduction

and

\[ x_i \rightarrow x_i + \partial_i \theta \]  \hspace{1cm} (1.3.163)

will further remove two modes. The Einstein equations remove two more modes from this picture, so that we are left with only 1 physical scalar mode.

Here we will work in comoving gauge, defined by the vanishing of the momentum density,

\[ \delta T_{0i} \equiv 0. \]  \hspace{1cm} (1.3.164)

For slow-roll inflation this condition can be translated into

\[ \delta \phi = 0. \]  \hspace{1cm} (1.3.165)

It is important to note that the additional non-dynamical metric perturbations \( \delta g_{00} \) and \( \delta g_{0i} \) in terms of the comoving curvature perturbation \( \zeta \) is replaced by the Einstein equations in the present context. In this gauge, perturbations are characterized purely by fluctuations in the metric \( \delta g_{ij} \),

\[ \delta g_{ij} = a^2 \left[ e^{-2\zeta} \delta g_{ij} + h_{ij} \right] \approx a^2 \left[ (1 - 2\zeta) \delta g_{ij} + h_{ij} \right] \]  \hspace{1cm} (1.3.166)

Here, \( h_{ij} \) is a spin-2 graviton degrees of freedom, which is transverse \( (\nabla_i h_{ij} = 0) \), traceless \( (h_i^i = 0) \) tensor and \( \zeta \) is a scalar. It is important to note that in the comoving spatial slices \( \phi = const \) have extrinsic three-curvature

\[ R_{(3)} = \frac{4}{a^2} \nabla^2 \zeta. \]  \hspace{1cm} (1.3.167)

In the present context the comoving curvature perturbation \( \zeta \) has the crucial property for adiabatic matter fluctuations which is time-independent on superhorizon scales after Fourier decomposition i.e.

\[ \lim_{k \ll aH} \zeta_k = 0. \]  \hspace{1cm} (1.3.168)

The constancy of \( \zeta \) on superhorizon scales allows us to relate CMB observations directly to the inflationary dynamics at the time when a given fluctuation crosses the horizon.

Further substituting the metric fluctuations \( \delta g_{00} \) and \( \delta g_{0i} \) into the effective action of EFT as stated in Eq (1.3.136) and expanding in the powers of \( \zeta \) and \( h_{ij} \) up to the second order, one can write the free field action for scalar and tensor modes respectively as:

\[ S^{(2)}_\zeta = \frac{M_p^2}{2} \int dt \, d^3 x \, a^3 \frac{\dot{\zeta}^2}{H^2} \left[ \zeta^2 - \frac{1}{a^2} (\partial_i \zeta)^2 \right], \]  \hspace{1cm} (1.3.169)

\[ S^{(2)}_h = \frac{M_p^2}{8} \int dt \, d^3 x \, a^3 \left[ h_{ij}^2 - (\partial_i h_{ij})^2 \right] \]  \hspace{1cm} (1.3.170)

Now we define the canonically- normalized Mukhanov variable as,

\[ v = z\zeta M_p \]  \hspace{1cm} (1.3.171)

13In some literature the notation \( R \) is used for the comoving curvature perturbation, to distinguish it from the curvature perturbation on uniform density hypersurfaces, which sometimes is also denoted by \( \zeta \).

14The constraint equations are solved most conveniently in the ADM formalism, where the metric fluctuations become non-dynamical Lagrange multipliers. See [54] for more details on technical aspects.
and consequently one can write\textsuperscript{15}:

\[ u_\gamma = \frac{a}{\sqrt{2}} h_\gamma M_p, \]  

(1.3.172)

where

\[ z = \frac{a \phi}{H} = \sqrt{2} \epsilon a \]  

(1.3.173)

and transitioning to conformal time \( \eta \) leads to the action for a canonically normalized scalar:

\[
S^{(2)}_\zeta = \frac{1}{2} \int d\eta d^3x \left[ (v')^2 - (\partial_i v)^2 - m_{\text{eff};\zeta}^2(\eta)v^2 \right], 
\]

(1.3.174)

\[
S^{(2)}_h = \frac{1}{2} \sum_{\gamma = +, \times} \int d\eta d^3x \left[ (u_\gamma')^2 - (\partial_i u_\gamma)^2 - m_{\text{eff};h}^2(\eta)u_\gamma^2 \right].
\]

(1.3.175)

where \( ' \) represents the differentiation with respect to conformal time \( \eta \). We recognize this as the action of a harmonic oscillator with a time-dependent effective mass for scalar and tensor modes as:

\[
m_{\text{eff};\zeta}^2(\eta) \equiv -\frac{z''}{z} = \frac{H}{a \dot{\phi}} \frac{a \ddot{\phi}}{H}, \]

(1.3.176)

\[
m_{\text{eff};h}^2(\eta) \equiv -\frac{a''}{a^2}.
\]

(1.3.177)

The time-dependence of the effective mass accounts for the interaction of the scalar field \( \zeta \) and graviton field \( h_{ij} \) with the gravitational background respectively.

After decomposing \( v \) and \( u_\gamma \) in terms of the Fourier modes and varying the action (1.3.174) and (1.3.175), we get the following equation of motions for the scalar and tensor modes respectively:

\[
v_k'' + w_k^2(\eta)v_k = 0,
\]

(1.3.178)

\[
u_k'' + w_k^2(\eta)u_k = 0.
\]

(1.3.179)

which is commonly known as the Mukhanov-Sasaki equation\textsuperscript{16,17}. Here also the effective frequency of the harmonic oscillator \( w_k(\eta) \) can be expressed as:

\[
w_{k;\zeta}^2 = \left( k^2 - \frac{z''}{z} \right),
\]

(1.3.180)

\[
w_{k;h}^2 = \left( k^2 - \frac{a''}{a} \right),
\]

(1.3.181)

which depends only on the magnitude of the momentum vector, \( k \equiv |k| \). The general solution of Eq (1.3.178) and Eq (1.3.179) after quantization can be written as:

\[
v_k \equiv a^{-k}_k v_k(\eta) + a^+_k v^*_k(\eta),
\]

(1.3.182)

\[
u_k \equiv a^{-k}_k u_k(\eta) + a^+_k u^*_k(\eta).
\]

(1.3.183)

\textsuperscript{15}Here \( \gamma \) stands for the helicity index for the transverse and traceless spin-2 graviton degrees of freedom. In general tensor modes can be written in terms of the two orthogonal polarization basis vectors.

\textsuperscript{16}In general the Mukhanov-Sasaki equation is hard to solve analytically since in general \( z(\eta) \) depends on the background dynamics. For a given inflationary background, one may, of course, solve this equation numerically. However, for the simplicity, we will discuss approximate analytical solutions: in the pure de Sitter limit, as well as in the slow-roll expansion of quasi-de Sitter space.

\textsuperscript{17}In Eq (1.3.179) the helicity index \( \gamma \) is summed over in the Fourier modes for the tensor contribution \( u_k \).
Here, \(v_k(\eta)\) and its complex conjugate \(v_k^*(\eta)\) are two linearly independent solutions of Eq (1.3.178). Also the creation and annihilation operators \(a_k^{+}\) and \(a_k\) satisfy the canonical commutation relations. Now the Wronskian of the mode functions for scalar and tensor modes are given by:

\[
W_\xi[v_k, v_k^*] = v_k v_k^* - v_k v_k^* = 2i \text{Im}(v_k v_k^*),
\]
\[
W_h[u_k, u_k^*] = u_k u_k^* - u_k u_k^* = 2i \text{Im}(u_k u_k^*),
\]

which is time independent and by rescaling the mode functions via the scale transformation \(v_k \rightarrow \lambda v_k, u_k \rightarrow \lambda u_k\) (giving \(W_\xi[v_k, v_k^*] \rightarrow |\lambda|^2 W_\xi[v_k, v_k^*]\) and \(W_h[u_k, u_k^*] \rightarrow |\lambda|^2 W_h[u_k, u_k^*]\)) one can always normalize \(v_k\) and \(u_k\) such that:

\[
W_\xi[v_k, v_k^*] = -i,
\]
\[
W_h[u_k, u_k^*] = -i.
\]

We must choose a vacuum state for the fluctuations,

\[
a_k|0\rangle = 0
\]

where \(|0\rangle\) is the time dependent vacuum, commonly known as Bunch-Davies vacuum and quite different from the usual Minkowski vacuum. This corresponds to specifying an additional boundary conditions for \(v_k\). Now when all comoving scales were far inside the Hubble horizon at the far past, \(\eta \rightarrow -\infty\) or \(|k\eta| >> 1\) or \(k >> aH\) fixes the solution as:

\[
\lim_{\eta \rightarrow -\infty} (v_k, u_k) = \frac{e^{-ik\eta}}{\sqrt{2k}}.
\]

Further we will concentrate on two limiting situation to get the closed form of the analytical solution of \(v_k(\eta)\) and \(u_k(\eta)\):

- **A. Pure de Sitter limit:** In this case using

\[
a = (H\eta)^{-1},
\]

the effective frequency reduces to

\[
w_{k,\xi}(\eta) = w_{k,h}(\eta) = \left(\frac{k^2}{\eta^2} - \frac{2}{\eta^2}\right).
\]

Consequently the exact solution of the Eq (1.3.178) and Eq (1.3.179) can be written as:

\[
v_k(\eta) = \alpha \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) + \beta \frac{e^{ik\eta}}{\sqrt{2k}} \left(1 + \frac{i}{k\eta}\right),
\]
\[
u_k(\eta) = \mu \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) + \sigma \frac{e^{ik\eta}}{\sqrt{2k}} \left(1 + \frac{i}{k\eta}\right).
\]

The above mentioned boundary condition stated in Eq (1.3.189) fixes the co-efficients \(\alpha = 1, \beta = 0, \mu = 1, \sigma = 0\) and leads to the unique solution of the Bunch-Davies mode functions at the subhorizon scale as:

\[
v_k(\eta) = u_k(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right).
\]

This determines the future evolution of the mode including its superhorizon dynamics at \(k << aH\) or \(|k\eta| << 1\) or \(\eta \rightarrow 0\) as:

\[
v_k(\eta) = u_k(\eta) = \frac{1}{i\sqrt{2} k^{3/2} \eta}.
\]
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- **B. Quasi de Sitter limit:** In this case the effective frequency reduces to

\[ w^2_{k,\zeta}(\eta) = w^2_{k,\lambda}(\eta) = \left( k^2 - \frac{z''}{z} \right), \]  

where

\[ \frac{z''}{z} = \frac{a''}{a} = \left( \frac{\nu^2 - \frac{1}{4}}{\eta^2} \right) \]

(1.3.197)

with

\[ \nu = \frac{3}{2} + \epsilon + \frac{\eta}{2}, \]

(1.3.198)

in the first order of slow-roll approximation. Consequently the exact solution of the Eq (1.3.178) and Eq (1.3.179) can be written as:

\[ v_k(\eta) = \sqrt{-k\eta} \left[ \alpha H^{(1)}_\nu(-k\eta) + \beta H^{(2)}_\nu(-k\eta) \right], \]

(1.3.199)

\[ u_k(\eta) = \sqrt{-k\eta} \left[ \mu H^{(1)}_\nu(-k\eta) + \sigma H^{(2)}_\nu(-k\eta) \right], \]

(1.3.200)

To impose the Bunch-Davies boundary condition at early times we have used:

\[ \lim_{k\eta \to -\infty} H_{\nu}^{(1,2)}(-k\eta) = \sqrt{\frac{2}{\pi}} \frac{e^{\pm ik\eta}}{\sqrt{-k\eta}} e^{\pm \frac{i\pi}{2} (\nu + \frac{1}{2})}. \]

(1.3.201)

As a result we get

\[ \alpha = \mu = \sqrt{\frac{\pi}{2}} \]

(1.3.202)

and

\[ \beta = \sigma = 0 \]

(1.3.203)

and this leads to the unique solution of the Bunch-Davies mode functions at the subhorizon scale as:

\[ v_k(\eta) = u_k(\eta) = \sqrt{\frac{\pi}{2}} e^{\pm \frac{i\pi}{2}(\nu + \frac{1}{2})} \sqrt{-k\eta} H^{(1)}_\nu(-k\eta). \]

(1.3.204)

This determines the future evolution of the mode including its superhorizon dynamics at \( k << aH \) or \( |k\eta| << 1 \) or \( \eta \to 0 \) as:

\[ v_k(\eta) = u_k(\eta) = \frac{i}{\sqrt{\pi}} e^{\pm \frac{i\pi}{2}(\nu + \frac{1}{2})} \left( -\frac{k\eta}{2} \right)^{\frac{1}{2} - \nu} \Gamma(\nu). \]

(1.3.205)

**Inflationary observables and EFT**

Here we will start with the two-point correlation functions computed from scalar and tensor contribution of inflationary EFT using *Bunch-Davies* vacuum. For both the cases the final results are proportional to the power spectrum at any arbitrary momentum scale \( k \). In the in-in picture
(discussed in details in section 1.3.3) the two-point correlator can be written as:

\[ \langle \zeta_k \zeta_k' \rangle = \left( \frac{H}{\phi} \right)^2 \langle \delta \phi_k \delta \phi_{k'} \rangle = \frac{1}{z^2 M_p^2} \langle v_k v_{k'} \rangle = (2\pi)^3 \delta^3(k + k') \frac{1}{z^2 M_p^2} P_v(k) \]

\[ = (2\pi)^3 \delta^3(k + k') \frac{2\pi^2}{k^3} \left( \frac{H}{\phi} \right)^2 \left( \frac{H}{2\pi} \right)^2, \]

\[ = (2\pi)^3 \delta^3(k + k') \frac{2\pi^2}{k^3} P_s(k), \quad (1.3.208) \]

\[ \langle h_k h_{k'} \rangle = \frac{4}{M_p^2} \langle \delta \psi_k \delta \psi_{k'} \rangle = \frac{2}{a^2 M_p^2} \langle u_k u_{k'} \rangle = (2\pi)^3 \delta^3(k + k') \frac{2}{a^2 M_p^2} P_u(k) \]

\[ = (2\pi)^3 \delta^3(k + k') \frac{2\pi^2}{k^3} \frac{4}{M_p^2} \left( \frac{H}{2\pi} \right)^2, \]

\[ = (2\pi)^3 \delta^3(k + k') \frac{2\pi^2}{k^3} P_t(k) \quad (1.3.209) \]

where the primordial power spectrum for scalar and tensor modes and tensor-to-scalar ratio at any arbitrary momentum scale \( k \) can be written within the slow-roll regime as:

\[
P_s(k) \equiv \frac{k^3 P_s(k)}{2\pi^2} = \frac{k^3 |v_k|^2}{2\pi^2 z^2 M_p^2} = \left( \frac{H}{\phi} \right)^2 \left( \frac{H}{2\pi} \right)^2 \sim \frac{V}{24\pi^2 \epsilon \phi^2(k) M_p^4}, \]

\[ = P_s(k_s) \left( \frac{k}{k_s} \right)^{n_s(k_s) - 1 + \frac{\alpha_s(k_s)}{2} \ln \left( \frac{k}{k_s} \right) + \frac{\kappa_s(k_s)}{3} \ln^2 \left( \frac{k}{k_s} \right) + \cdots}, \quad (1.3.210) \]

\[
P_t(k) \equiv 2 \times \frac{k^3 P_h(k)}{2\pi^2} = \frac{2k^3 |u_k|^2}{\pi^2 a^2 M_p^2} = \frac{8}{M_p^2} \left( \frac{H}{2\pi} \right)^2 \sim \frac{2V}{3\pi^2 M_p^4}, \]

\[ = P_t(k_s) \left( \frac{k}{k_s} \right)^{n_t(k_s) + \frac{\alpha_t(k_s)}{2} \ln \left( \frac{k}{k_s} \right) + \frac{\kappa_t(k_s)}{3} \ln^2 \left( \frac{k}{k_s} \right) + \cdots}, \quad (1.3.211) \]

\[
r(k) \equiv \frac{P_t(k)}{P_s(k)} = 16\epsilon(k) \sim 16\epsilon \phi^2(k) \]

\[ = r(k_s) \left( \frac{k}{k_s} \right)^{n_t(k_s) - n_s(k_s) + 1 + \frac{\alpha_t(k_s)}{2} \ln \left( \frac{k}{k_s} \right) + \frac{\kappa_t(k_s)}{3} \ln^2 \left( \frac{k}{k_s} \right) + \cdots}, \quad (1.3.212) \]

---

\[ ^{18} \text{Here to compute the two-point correlator for slow-roll limit we use expression for the comoving curvature perturbation and tensor perturbation in terms of time delay during inflation as,} \]

\[ \zeta = H \frac{\delta \phi}{\phi} \equiv -H \delta t \quad (1.3.206) \]

and

\[ h = \frac{2}{M_p} \psi \quad (1.3.207) \]

in a spatially flat gauge [54].

\[ ^{19} \text{In Eq (1.3.211) an extra } 2 \text{ factor appears due to the appearance of two orthogonal polarization basis for the spin-2 helicity degrees of freedom [54].} \]
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where in CMB experiments, \( k_s \) is chosen at the scale at which the modes cross the horizon, \( k_s = aH \). The power spectrum for scalar and tensor modes and tensor-to-scalar ratio are given by [118, 119]:

\[
P_s(k_s) = \frac{V_s}{24\pi^2 eV(k_s)M_p^2}, \quad P_t(k_s) = \frac{2V_s}{3\pi^2 M_p^2}, \quad r(k_s) = \frac{P_t(k_s)}{P_s(k_s)} \sim 16\epsilon_V(k_s)
\]

and \( k_s \) is the momentum pivot or scale of normalization of the power spectrum around which the logarithm of the power spectrum as well as the slow-roll parameters are expanded in Taylor series. In both of the Eq (1.3.210) and Eq (1.3.211) \( n_s, n_t \) stand for spectral tilt, \( \alpha_s, \alpha_t \) represent running of the tilt and \( \kappa_s, \kappa_t \) signifies the running of the running of the spectral tilt for the scalar and tensor modes respectively.

Further the spectral tilt, running of the tilt and running of the running of spectral tilt can be expressed at any arbitrary momentum scale within the slow-roll regime as [120]:

\[
n_s(k) - 1 = \frac{d\ln P_s(k)}{d\ln k} = n_s(k) - 1 + \alpha_s(k_s) \ln \left( \frac{k}{k_s} \right) + \frac{\kappa_s(k_s)}{2} \ln^2 \left( \frac{k}{k_s} \right) + \cdots
\]

\[
n_t(k) = \frac{d\ln P_t(k)}{d\ln k} = n_t(k) + \alpha_t(k_s) \ln \left( \frac{k}{k_s} \right) + \frac{\kappa_t(k_s)}{2} \ln^2 \left( \frac{k}{k_s} \right) + \cdots
\]

\[
\alpha_{s,t}(k) = \frac{d^n_{s,t}(k)}{d\ln k} = \alpha_{s,t}(k_s) + \kappa_{s,t}(k_s) \ln \left( \frac{k}{k_s} \right) + \cdots
\]

\[
\kappa_{s,t}(k) = \frac{d\alpha_{s,t}(k)}{d\ln k} \approx \kappa_{s,t}(k_s) + \cdots
\]

where at the scale \( k_s \) the spectral tilt, running of the tilt, running of the running of spectral tilt for scalar and tensor modes and consistency relation for tensor-to-scalar ratio are given by [118, 119]:

\[
n_s(k_s) - 1 = \left[ \frac{d\ln P_s(k)}{d\ln k} \right]_{k_s} = 2\eta_V(k_s) - 6\epsilon_V(k_s),
\]

\[
n_t(k) = \left[ \frac{d\ln P_t(k)}{d\ln k} \right]_{k_s} = -2\epsilon_V(k_s),
\]

\[
\alpha_s(k_s) = \left[ \frac{d^n_s(k)}{d\ln k} \right]_{k_s} = 16\eta_V(k_s)\epsilon_V(k_s) - 24\epsilon_V^2(k_s) - 2\xi_V^2(k_s),
\]

\[
\alpha_t(k_s) = \left[ \frac{d^n_t(k)}{d\ln k} \right]_{k_s} = 4\eta_V(k_s)\epsilon_V(k_s) - 8\epsilon_V^2(k_s),
\]

\[
\kappa_s(k_s) = \left[ \frac{d^2n_s(k)}{d\ln k^2} \right]_{k_s} = 192\epsilon_V^2(k_s)\eta_V(k_s) - 192\epsilon_V^3(k_s) + 2\sigma_V^2(k_s) - 24\epsilon_V(k_s)\xi_V^2(k_s) + 2\eta_V(k_s)\xi_V^2(k_s) - 32\epsilon_V^2(k_s)\xi_V^2(k_s) + 8\eta_V^2(k_s)\epsilon_V(k_s) - 4\epsilon_V(k_s)\xi_V^2(k_s),
\]

\[
\kappa_t(k_s) = \left[ \frac{d^2n_t(k)}{d\ln k^2} \right]_{k_s} = 56\eta_V(k_s)\epsilon_V(k_s) - 64\epsilon_V^2(k_s) - 8\eta_V^2(k_s)\epsilon_V(k_s) - 4\epsilon_V(k_s)\xi_V^2(k_s),
\]

\[
r(k_s) = -8n_t(k_s).
\]
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The present observational status for the inflationary paradigm is shown in Table (1.1) where the constraints are quoted from WMAP9 (WP) [23], Planck+ WP [24, 25] and Planck+WP+BICEP2 [121] dataset.

1.3.2 Reheating after inflation

Reheating at the end of the period of accelerated expansion is a significant component of inflationary paradigm. Without reheating phenomena, inflation cannot able to generate matter in universe. Reheating appears through coupling of the inflaton degrees of freedom, $\phi$, the scalar field generating the accelerated expansion, to Standard Model (SM) matter. Such couplings must be present at least in gravitational interactions. However, in many inflationary models, instead of such gravitational interactions direct couplings through the matter sector play crucial role. Reheating mechanism was initially proposed using first order perturbation theory within EFT prescription and further analyzed in terms of the decay of an inflaton degrees of freedom into SM particle constituents [10, 11, 12].

Initial condition for reheating

As discussed, inflationary paradigm generically requires a scalar field. A scalar field is postulated to exist in the SM too, where the Higgs field used to give elementary fermions their masses via spontaneous symmetry breaking mechanism. To serve the scalar degrees of freedom as a Higgs field, its potential energy must have a minimum at a non-trivial field value, known as vacuum expectation value (VEV). To demonstrate this let us start with standard Higgs potential:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$  \hspace{1cm} (1.3.227)

where $v$ represents the VEV of $\phi$. To fix the initial condition for reheating it is assumed that at high temperatures the symmetry is restored by twofold components-

1. finite temperature effects,

2. the Higgs field value at $\phi = 0$.

\hspace{1cm} See figure (1.2) for the details.
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Figure 1.2: Schematic diagram of reheating phenomena which starts just after the end of inflation [54].

During reheating one can note down the following characteristic features:

- When the temperature falls below a critical value $T_c$, the scalar field $\phi$ ceases to be trapped and starts to roll towards one of the lowest energy states $\phi = \pm v$.

- The SM Higgs must have a self coupling constant $\lambda$ and its numerical value cannot be sufficiently small to achieve slow rolling of $\phi$, which is required to obtain enough inflation.

- Inflation takes place during the period when $\phi$ is undergoing the symmetry-breaking phase transition and slowly rolling towards $\phi = \pm v$.

- Here the model of reheating was based on scalar field dynamics obtained by replacing the potential (1.3.227) by a symmetry breaking potential of Coleman-Weinberg form, where the mass term at the field origin is set to zero and symmetry breaking is obtained through quantum corrections to the effective potential.

Large-field inflation is an alternative prescription where inflation is triggered by a period of slow-rolling of $\phi$. This situation can be described using the monomial potential:

$$V(\phi) = \frac{1}{2} m^2 \phi^2,$$  \hspace{1cm} (1.3.228)

where $m$ is the mass of $\phi$. One can avoid this situation by adding a second scalar field $\chi$ to the potential sector of the theory responsible for inflation and to invoke a hybrid potential of the form

$$V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \frac{g^2}{2} \chi^2 \phi^2 + \frac{\lambda}{4} (\chi^2 - \theta^2)^2,$$  \hspace{1cm} (1.3.229)

where $g$ and $\lambda$ are dimensionless coupling constants and $\theta$ is the VEV of $\chi$. For large values of $|\phi|$, the potential in $\chi$ direction has a minimum at $\chi = 0$, whereas for small values of $|\phi| < M_p$, $\chi = 0$ becomes an unstable point.

**Perturbative decay of inflaton**

Reheating takes place at the end of inflation when the energy density is stored overwhelmingly in the oscillations of $\phi$. We assume that the inflaton $\phi$ is coupled to another scalar field $\chi$ or fermionic
field $\psi$. Taking the interaction Lagrangian to be
\[
L_{\text{int}}^{(1)} = -g\sigma \phi \chi^2, \quad (1.3.230)
\]
\[
L_{\text{int}}^{(2)} = -h\phi \bar{\psi} \psi, \quad (1.3.231)
\]
where $g$ and $h$ are dimensionless coupling constants and $\sigma$ is a mass scale. When the mass of the inflaton is much larger than those of $\chi$ and $\psi$ ($m >> m_{\chi}, m_{\psi}$), the decay rate are given by:
\[
\Gamma(\phi \rightarrow \chi\chi) = \frac{g^2 \sigma^2}{8\pi m}, \quad (1.3.232)
\]
\[
\Gamma(\phi \rightarrow \psi\bar{\psi}) = \frac{h^2 m}{8\pi}. \quad (1.3.233)
\]
The energy loss of the inflaton due to the production of $\chi$ and $\psi$ particles can be taken into account by adding a damping term to the inflaton equation of motion which in the case of a homogeneous inflaton field is [54]:
\[
\ddot{\phi} + 3H \dot{\phi} + \Gamma_{\text{total}} \dot{\phi} = -V'(\phi). \quad (1.3.234)
\]
For small coupling constant, the total interaction rate
\[
\Gamma_{\text{total}} = \Gamma(\phi \rightarrow \chi\chi) + \Gamma(\phi \rightarrow \psi\bar{\psi}) \quad (1.3.235)
\]
is typically much smaller than the Hubble parameter at the end of inflation. Thus, at the beginning of the oscillations, the energy loss into particles is initially negligible compared to the energy loss due to the expansion of space. Once the Hubble expansion rate decreases to a value comparable to $\Gamma_{\text{total}}$, $\chi$ and $\psi$ particle production becomes effective. In the context of Einstein GR it is the energy density at the time when
\[
H = \Gamma_{\text{total}} = \Gamma(\phi \rightarrow \chi\chi) + \Gamma(\phi \rightarrow \psi\bar{\psi}) = \sqrt{\frac{\rho}{3M_{PL}^2}}. \quad (1.3.236)
\]
Assuming all the energy density $\rho$ of the universe is in the form of relativistic matter with
\[
\rho = N^* \pi^2 T^4 / 30, \quad (1.3.237)
\]
where $N^*$ is the effective number of mass-less degrees of freedom ($N^* = 10^2 - 10^3$), we obtain the reheat temperature:
\[
T_R \sim 0.2 \left(\frac{100}{N^*}\right)^{1/4} (\Gamma_{\text{total}} M_{PL})^{1/2}. \quad (1.3.238)
\]
Since
\[
\Gamma_{\text{total}} \propto \sqrt{g}, \quad (1.3.239)
\]
which is generally very small, perturbative reheating is slow and produces a reheating temperature,
\[
T_R < < \sqrt[4]{V_*}. \quad (1.3.240)
\]
There are two prime problems with the perturbative approach as described above. First of all, even if the inflaton decay were perturbative, it is not justified to use the heuristic equation (1.3.234) since it violates the fluctuation-dissipation theorem: in systems with dissipation, there are always fluctuations, and these are missing in (1.3.234). Also the prescribed analysis does not take into account the coherent nature of the inflaton field. However, the matter fields can be assumed to start off in their vacuum state. Thus, matter fields $\chi$ and $\psi$ must be treated quantum mechanically.
1.3 Applications of Field Theory to Early Universe

Preheating

In this case the energy is rapidly transferred from the inflaton degrees of freedom to the other bosonic fields interacting with it in the present context. In principle this process occurs very far away from thermal equilibrium and commonly known as preheating. In the context of physics of early universe this plays a very significant role. Preheating is in general follows the non-perturbative nature of the particle creation and consequently the theory of preheating is complicated in nature and different from the reheating phenomena as introduced earlier. We will present the preheating mechanism [122, 123] for the simple toy model with interaction Lagrangian:

\[ L_{\text{int}} = -\frac{1}{2}g^2 \chi^2 \phi^2, \]

where, the parameter \( g \) plays the role of a dimensionless coupling in the present context. Here the time period of preheating is small compared to the Hubble expansion time \( H^{-1} \). The quantum theory of \( \chi \) particle production in the external classical inflaton background begins by expanding the quantum field \( \hat{\chi}(t, x) \) as:

\[ \hat{\chi}(t, x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{(2\pi)^3/2} \left( \chi_k^*(t) \hat{a}_k e^{ikx} + \chi_k(t) \hat{a}_k^\dagger e^{-ikx} \right), \]

In the present context the mode functions \( \chi_k \forall k \) satisfy the Mathieu equation in Fourier space as:

\[ \chi_k'' + \frac{\omega_k^2}{m^2} \chi_k = 0, \]

where

\[ \omega_k = \sqrt{k^2 + m^2 + g^2 \Phi(t)^2 \sin^2 z}, \]

where we introduce a dimensionless variable \( z = mt \). Here \( \Phi \) is the amplitude of oscillation of \( \phi \). The growth of the mode function corresponds to particle production, analogous to the situation in case of an external gravitational field. Eq (1.3.243) leads to exponential growth,

\[ \chi_k \propto \exp(\mu_k z), \]

where \( \mu_k \) is called the Floquet exponent. For

\[ \frac{g^2 \Phi^2}{4m^2} \ll 1, \]

resonance occurs in a narrow instability band around \( k = m \). The resonance is much more efficient if

\[ \frac{g^2 \Phi^2}{4m^2} \gg 1 \]

where it occurs in broad bands. In particular, the bands include all long wavelength modes \( k \to 0 \). A condition for particle production is that the WKB approximation for the evolution of \( \chi \) is violated. Here we write,

\[ \chi_k \propto e^{\pm i \int \omega_k dt}, \]

which is valid as long as the adiabaticity condition

\[ \frac{d\omega_k^2}{dt} \leq 2\omega_k^3 \]

is satisfied. The adiabaticity condition is violated for momenta satisfying,

\[ k^2 \leq \frac{2}{3} \mu^2 m \Phi - m^2 \chi. \]

For modes with these values of \( k \), the adiabaticity condition breaks down in each oscillation period when \( \phi \) is close to zero.
Phenomenological consequences

Reheating or preheating lead to non-thermal particle production, as we have mentioned earlier. In cosmology it is usually assumed that all particles start out in thermal equilibrium at the beginning of the Standard Cosmology phase. However, reheating begins with out-of-equilibrium decay of the inflaton oscillations and decay products may not reach full thermal equilibrium immediately [124, 125, 126]. During the transition from inflation to the Standard Cosmology various non-thermal processes take place and the assumption of thermal equilibrium of all particles clearly breaks down. In the following, we briefly mention a few applications of non-thermal particle production.

1. Baryogenesis and Leptogenesis:

The first application is leptogenesis [13, 14] and baryogenesis [15, 16]. One of the several possible mechanisms to explain the observed asymmetry between baryons and antibaryons is to make use of out-of-equilibrium decay of superheavy Higgs and gauge particles. If reheating were purely perturbative, particles as heavy as the inflaton could have been created either in inflaton decay or from scatterings of inflaton decay products. Preheating, however, provides a mechanism to produce a large population of superheavy scalar particles much heavier than the inflaton. Another way to generate observed baryon to entropy ratio is via leptogenesis, a scenario in which initially an asymmetry in the lepton number is produced which is then partially converted into baryon asymmetry via SM sphalerons. Preheating after inflation is a way to generate the initial lepton asymmetry. For example, preheating can produce a large number density of super-massive right handed neutrinos in a model in which the inflaton couples to these neutrinos $\psi$ via the standard fermionic preheating interaction term:

$$L_{\text{int}} = -h\phi\bar{\psi}\psi.$$  (1.3.251)

If hybrid inflation occurs at a scale close to the electroweak scale, then the non-thermal production of particles may provide the out-of-equilibrium condition that is necessary in order to achieve electroweak baryogenesis [16, 127].

2. Dark matter:

Another application of non-thermal particle creation during reheating is to excite dark matter [128, 129]. It is usually assumed that the dark matter particles are thermally distributed. This assumption is implicit in most current analyses of the prospects for dark matter detection in direct and indirect experiments. However, if the dark matter particles couple to the inflaton, then non-thermal production of dark matter during reheating is to be expected. If the dark matter particles have sufficiently strong interactions which allows them to thermalize during reheating, then the signatures of the initial non-thermal distribution will be washed out. However, if the interactions do not permit thermalization after inflation, then the predictions concerning the dark matter distribution will be quite different. The dark matter abundance which can be obtained by the preheating channel is very model-dependent, whereas, direct gravitational particle production produces dark matter of the required abundance for particle masses of

$$M_X \sim g^{1/2} 10^{15} \text{ GeV}.$$  (1.3.252)
3. Moduli and Gravitino Production

Preheating could also produce unwanted particles. An example are particles with gravitationally suppressed couplings and weak scale masses that arise in many theories beyond the SM. Overproduction of these particles could overclose the universe, if they are stable, or ruin the success of Big Bang nucleosynthesis (BBN) in the case of unstable relics. Here we consider moduli and gravitino production during preheating. Moduli (bosonic modulus, \( \chi \), and fermionic modulus, \( \psi \)) are typically coupled to the inflaton via non-renormalizable UV scale suppressed such as:

\[
\mathcal{L}_{\text{int}} \sim \frac{\phi^4}{\Lambda_{UV}} \chi \quad \text{(bosonic)}, \quad \mathcal{L}_{\text{int}} \sim \frac{\phi^2}{\Lambda_{UV}} \bar{\psi} \psi \quad \text{(fermionic)}. \tag{1.3.253}
\]

where \( \Lambda_{UV} \sim M_{PL} \) is the UV cut-off scale of the effective theory. It was shown that moduli field can be parametrically amplified. Another important example is the gravitino, the spin 3/2 partner of the graviton. Gravitinos are produced thermally from scatterings of light particles in the thermal bath. The number density of gravitinos thus produced can be obtained by solving the Boltzmann equation:

\[
\dot{n}_X + 3Hn_X \simeq \langle \sigma v \rangle n_l^2, \tag{1.3.254}
\]

where \( n_X \) is the number density of the gravitinos, \( \sigma \) is the production cross section which scales as \( M_{PL}^{-2} \), and \( v \sim c \) is the relative velocity of scatterers \( l \) whose number density is \( n_l \). The resulting abundance is found to be:

\[
\Omega_X = \frac{n_X}{s} \sim 10^{-2} \frac{T_R}{M_{PL}} \sim 2 \times 10^{-3} \left( \frac{100}{N^*} \right)^{1/4} \left( \frac{\Gamma_{\text{total}}}{M_{PL}} \right)^{1/2}, \tag{1.3.255}
\]

where \( s \) is the entropy density and \( T_R \) is the reheat temperature of the universe. BBN gives rise to an absolute upper bound

\[
(n_X/s) < 10^{-12}, \tag{1.3.256}
\]

which in turn leads to an upper bound

\[
T_R < 10^9 \text{ GeV}. \tag{1.3.257}
\]

The presence of the oscillating inflaton field leads to a periodically varying correction to the effective gravitino mass that results in an instability in the same way that there is an instability for spin 0 and 1/2 particle modes. The exact strength of the instability depends sensitively on the precise SUSY inflationary model one is considering. Gravitino with helicity \( \pm 1/2 \) component mainly contain the Goldstino component- the inflatino (superpartner of the inflaton), whose interactions are not suppressed by \( \Lambda_{UV} \). One would naturally expect them to be created in large abundance. However, in realistic scenarios, where the scale of inflation is much higher than the scale of SUSY breaking, e.g.

\[
H_{inf} \gg \mathcal{O}(100 \text{ GeV}) \tag{1.3.258}
\]

and the helicity \( \pm 1/2 \) states that are produced during preheating mainly decay in the form of inflatinos along with the inflaton.
1.3.3 Primordial non-Gaussianity

The CMB power spectrum analysis reduces the WMAP data from about $10^6$ pixels to $10^3$ multipole moments. In principle, there can be a wealth of information that is contained in deviations from the perfectly Gaussian distribution. So far CMB experiments haven’t had the sensitivity to extract this information from the data. However, searches are going on with improved precision which will presumably provide accurate measurements of higher-order CMB correlations. So, it is instructive to study non-Gaussian features which will also constrain various classes of inflationary models. The precise measurements of primordial non-Gaussianity are also a powerful way to exploring the various hidden aspects of particle physics, which is to determine the action (i.e. the fields, symmetries and couplings) as a function of energy scale. The prime sources for the non-Gaussianity in CMB are:

- **1. Primordial non-Gaussianity**: Non-Gaussianity in the primordial curvature perturbation $\zeta$ produced in the very early universe by inflation or in an alternative prescription.

- **2. Second-order non-Gaussianity**: Non-Gaussianity arising from non-linearities in the transfer function relating $\zeta$ to the CMB temperature anisotropy $\Delta T$ at recombination.

- **3. Secondary non-Gaussianity**: Non-Gaussianity generated by late time effects after recombination e.g. gravitational lensing.

- **4. Foreground non-Gaussianity**: Non-Gaussianity created by Galactic and extra-Galactic sources.

We briefly discuss here about the two essential tools - the in-in formalism and the $\delta N$ formalism - for computing non-Gaussianities in the models of the early universe.

A. In-In formalism

The specific computation of $n$-point correlation functions in the field of inflationary cosmology is significantly different in various manner from the analysis used in the context of flat space quantum field theory applicable to particle physics theory. In the framework of flat space quantum field theory or in particle physics the prime motivation is to study various effective interactions via the S-matrix which physically describes the transition probability for a state vector $|\text{in}\rangle$ in the very far past to some state vector $|\text{out}\rangle$ realized in the very far future. This can be technically expressed by the following expression:

$$\langle \text{out}|S|\text{in}\rangle = \langle \text{out} (+\infty)|\text{in} (-\infty)\rangle$$

(1.3.259)

Here one needs to impose the asymptotic boundary conditions at very early and very late time scales. This is because of the fact that in Minkowski space all the quantum states are assumed to be non-interacting in the far past and the far future when the scattering particles are far from the interaction region.

In the present context the quantum fluctuations by the field content is collectively denoted by the following way:

$$\psi = \{\zeta, h^+, h^x\}$$

(1.3.260)

---

21Here we mostly concerned about primordial non-Gaussianity and second order non-Gaussianity.
and our here prime objective is to compute the exact \( n \)-point correlator or more precisely the expectation value of the filed theory operator defined as:

\[
A = \prod_{i=1}^{n} \psi_{k_i},
\]

and in the present context this can be expressed as,

\[
\langle A \rangle = \langle in|A|in \rangle.
\]

Here the state vector \(|in\rangle\) characterizes the structure of the quantum field theoretic vacuum of the interacting theory at the moment in the far past in FLRW curved space-time. Here in this computation we introduce the interaction picture in which the leading order time-dependence of the fields is exactly determined by the quadratic Hamiltonian or equivalently by the linear equations of motion. Consequently corrections arising from the various effective interactions in the quantum field theory can be treated as a power series in the interaction Hamiltonian, \( H_{int} \) in interaction picture. Consequently the expectation value of the quantum field theoretic operator can be recast in the following form \(^{22}\):

\[
\langle A \rangle = \langle 0|\tilde{T} \exp \left[ i \int_{-\infty}^{t} H_{int}^{I}(t')dt' \right] A^{I}(t) T \exp \left[ -i \int_{-\infty}^{t} H_{int}^{I}(t'')dt'' \right] |0\rangle
\]

where \( T \) and \( \tilde{T} \) signify the time ordering and anti-time ordering symbolic operations respectively. The superscript \( I \) is specifically used for interaction picture in quantum field theory. The standard \( i\epsilon \) prescription in quantum field theory has been used in the present context to effectively turn off the interaction in the very far past time scale and finally project the interacting quantum \(|in\rangle\) state vector onto the free vacuum state vector represented by \(|0\rangle\). Here we also introduce time evolution operator \( U \) \(^{23}\) may be used to relate the interacting vacuum state at arbitrary any time \(|\Omega(\tau)\rangle\) to the free Bunch-Davies vacuum \(|0\rangle\). We first expand \(|\Omega(\tau)\rangle\) in its eigenstates of the free Hamiltonian in the following fashion:

\[
|\Omega\rangle = \sum_{n} |n\rangle\langle n|\Omega(\tau)\rangle
\]

and after time evolution this can be expressed as:

\[
|\Omega(\tau_2)\rangle = U(\tau_2, \tau_1)|\Omega(\tau_1)\rangle = |0\rangle\langle 0|\Omega\rangle + \sum_{n \geq 1} e^{iE_n(\tau_2 - \tau_1)} |n\rangle\langle n|\Omega(\tau_1)\rangle
\]

\(^{22}\)It is important to note that, in Eq (1.3.263) the integration open contour exactly goes from \(-\infty(1 - \epsilon)\) to \(t\) in time scale where all the correlation function is evaluated for quantum field theory in curved FLRW background and back to \(-\infty(1 + \epsilon)\). This will directly imply that the Wick contraction of quantum field theoretic operators finally lead to real-valued Wightman Green’s functions, rather than complex-valued Feynman Green’s functions in the present context of discussion.

\(^{23}\)The interaction Hamiltonian in interaction picture of quantum field theory defines the evolution of quantum states via the well known unitary time evolution operator:

\[
U(\tau_2, \tau_1) = T \exp \left[ -i \int_{\tau_1}^{\tau_2} H_{int}^{I}(t'')dt'' \right].
\]

By directly solving the mode function from the Mukhanov-Sasaki equation one can directly choose the Bunch-Davies initial condition in the interaction Hamiltonian. In principle one can also study the consequences from arbitrary vacuum as well, but Bunch-Davies vacuum is more consistent with the various observed data sets.
Further in the present context we choose \( \tau_2 = -\infty(1 - i\epsilon) \) which finally projects out all excited quantum states in curved FLRW space-time. Consequently we get the following relation between the interacting vacuum at the time scale \( \tau = \tau_2 = -\infty(1 - i\epsilon) \) and the free Bunch-Davies vacuum state \( |0\rangle \) as:

\[
|\Omega(-\infty(1 - i\epsilon))\rangle = |0\rangle\langle 0|\Omega\rangle. \tag{1.3.267}
\]

Using this finally the interacting vacuum at an arbitrary time scale \( t \) can be expressed as:

\[
|in\rangle \equiv |\Omega(t)\rangle = U(t, -\infty(1 - i\epsilon))|\Omega(-\infty(1 - i\epsilon))\rangle = T \exp \left[-i \int_{-\infty(1 - i\epsilon)}^{t} H^I_{\text{int}}(t'') dt''\right]|0\rangle\langle 0|\Omega\rangle. \tag{1.3.268}
\]

Using this useful computational tool it can be easily shown that the three and four point correlator of curvature perturbation in momentum space can be written as \(^{24}\):

\[
\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} \rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3)B_\zeta(k_1, k_2, k_3), \tag{1.3.269}
\]

\[
\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\zeta_{k_4} \rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3 + k_4)T_\zeta(k_1, k_2, k_3, k_4). \tag{1.3.270}
\]

where \( B_\zeta(k_1, k_2, k_3) \) and \( T_\zeta(k_1, k_2, k_3, k_4) \) represent the bispectrum and trispectrum which only depend on the magnitude of the momentum vectors due to isotropy or rotational invariance. In case of scale invariant quantum fluctuations, the bispectrum and trispectrum are homogeneous functions of degree \(-6\) and \(-8\) respectively. Further, the bispectrum and trispectrum can be expressed in terms of the non-Gaussian observable parameter \( f_{NL}, \tau_{NL}, g_{NL} \) and the power spectrum \( P_\zeta \) as \(^{25}\):

\[
B_\zeta(k_1, k_2, k_3) \propto f_{NL} \sum_{i<j=1}^{3} P_\zeta(k_i)P_\zeta(k_j), \tag{1.3.271}
\]

\[
T_\zeta(k_1, k_2, k_3, k_4) \propto g_{NL} \sum_{i<j<p=1}^{3} P_\zeta(k_i)P_\zeta(k_j)P_\zeta(k_p) + \tau_{NL} \sum_{j<p,i\neq j,p=1}^{11} P_\zeta(k_{ij})P_\zeta(k_j)P_\zeta(k_p). \tag{1.3.272}
\]

Here the proportionality constants for both the cases are determined by various configurations and types of the non-Gaussianity to be discussed later. It will be convenient to define the shape function in terms of the bispectrum to determine the configurations of non-Gaussianity by the following expression:

\[
S(k_1, k_2, k_3) \equiv N_{\text{norm}}(k_1k_2k_3)^2 B_\zeta(k_1, k_2, k_3) \tag{1.3.273}
\]

where \( N_{\text{norm}} \) is the normalization factor which are different for various non-Gaussian configurations.

\(^{24}\)In this definition of bispectrum and trispectrum all the contributions of diagrams-connected and disconnected are taken care of.

\(^{25}\)In Eq (1.3.286) the symbol \( k_{ij} \) is defined as, \( k_{ij} := |\mathbf{k}_i - \mathbf{k}_j| = \sqrt{k_i^2 + k_j^2 - 2k_i k_j \cos \theta_{ij}} \), where \( \theta_{ij} \) be the angle between the momentum vectors \( \mathbf{k}_i \) and \( \mathbf{k}_j \).
B. $\delta N$ formalism

In the previous section we computed the non-Gaussianity generated at horizon crossing. In this section we discuss a second source of non-Gaussianity arising from non-linearities after horizon crossing when all modes have become classical. A convenient way to describe these non-Gaussianities is the $\delta N$ formalism ($N$ being the number of e-foldings) [130, 131, 132, 133, 134, 135] on large scales ($k \ll aH$). It provides a fruitful technique to compute the expression for the curvature perturbation $\zeta$ without explicitly solving the perturbed field equations. This formalism has twofold advantages-(1) it perfectly holds good at the super-horizon scales and (2) is also independent of any kind of intrinsic non-Gaussianities generated at the scale of horizon crossing. Let us mention the algorithm for $\delta N$ formalism point wise:

1. One can define $\delta N(x, t)$ as the number of e-folds from a fixed flat slice $26(\psi = 0)$ to a uniform density slice ($\psi = \zeta$) at time $t$.

2. Then, one can write, $\zeta(x, t) = \delta N(x, t)$. This leads to a simple algorithm to compute the superhorizon evolution of the primordial curvature perturbation $\zeta$ to illustrate the procedure consider a set of scalars $\phi_i$. A linear combination of these fields can be identified to be the inflaton degrees of freedom. The remaining background fields are known as ‘isocurvatons’. We also assume that all fields have become superhorizon at some initial time at which we choose a spatially flat time-slice, on which there are no scalar metric fluctuations, but only fluctuations in the matter fields, $\bar{\phi}_i + \delta \phi_i(x)$.

3. Next we choose the final time-slice to have uniform density, i.e. the inflaton field is unperturbed and all fluctuations are in the metric and the isocurvatons.

4. Further we evolve the unperturbed fields $\bar{\phi}_i$ in the initial slice ‘classically’ to the unperturbed final slice, and denote the corresponding number of e-folds as $\bar{N}(\bar{\phi}_i)$.

5. Next, we evolve the perturbed initial field configuration $\bar{\phi}_i + \delta \phi_i(x)$ classically to the perturbed final slice. In this prescription the $\delta N$ is defined as, $\delta N = N(\bar{\phi}_i + \delta \phi_i(x)) - \bar{N}(\bar{\phi}_i)$. Further, Taylor expanding $\delta N$, we obtain an expression for $\zeta$ in terms of the scalar field fluctuations $\delta \phi_i$ and derivatives of $N$ is defined on the initial slice as,

$$\zeta = N_i \delta \phi_i + \frac{1}{2!} N_{ij} \delta \phi_i \delta \phi_j + \frac{1}{3!} N_{ijk} \delta \phi_i \delta \phi_j \delta \phi_k + \cdots$$ (1.3.274)

where $N_i \equiv \partial_i N$, $N_{ij} \equiv \partial_i \partial_j N$ and $N_{ijk} \equiv \partial_i \partial_j \partial_k N$ are derivatives evaluated on the initial slice.

Consequently the three and four point correlation function can be recast as the analytical expressions written in Eq (1.3.269) and Eq (1.3.270). In this context the non-Gaussian parameter

26In the earlier discussions we have introduced $\zeta$ as the curvature perturbation in ‘comoving’ gauge. Now in the context of $\delta N$ formalism on the superhorizon scales this is exactly equal to the curvature perturbation in ‘uniform density’ gauge.
$f_{NL}$, $\tau_{NL}$ and $g_{NL}$ can be expressed in terms of the derivatives of $N$ as:

\begin{align}
    f_{NL} & \propto \frac{N_i N_j N_{ij}}{(N_1^2)^2}, \\
    \tau_{NL} & \propto \frac{N_i N_j N_{ik} N_{jk}}{(N_1^2)^3}, \\
    g_{NL} & \propto \frac{N_i N_j N_k N_{ijk}}{(N_1^2)^3}.
\end{align}

(1.3.275, 1.3.276, 1.3.277)

where all the repeated indices are summed over and the proportionality constants are determined by the shapes and configurations of the non-Gaussianities.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(a) Visual representations of triangles forming the primordial bispectrum. (b) Shapes of the primordial bispectra.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Figure 1.3: (a) Visual representations of triangles forming the primordial bispectrum, with various combinations of wave numbers satisfying $k_3 \leq k_2 \leq k_1$ [136] and (b) shapes of the primordial bispectra in which we show the normalized amplitude of $S(k_1, k_2, k_3)(k_2/k_1)^2(k_3/k_1)^2$ as a function of $k_2/k_1$ and $k_3/k_1$ for a given $k_1$, with a condition that $k_3 \leq k_2 \leq k_1$ is satisfied [136].}
\end{figure}

C. Shapes of non-Gaussianity

- **Local non-Gaussianity**: One of the first ways to parameterize non-Gaussianity phenomenologically was via a non-linear correction to a Gaussian perturbation $\zeta_g$,

\[
    \zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL}^{loc} \left[ \zeta_g^2(x) - \langle \zeta_g^2(x) \rangle \right] + \frac{9}{25} g_{NL}^{loc} \zeta_g^3(x) + \cdots
\]

(1.3.278)

where $\cdots$ are higher order non-Gaussian contributions. This definition is local in real space and therefore called local non-Gaussianity. The bispectrum, trispectrum and the shape func-
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The bispectrum for local non-Gaussianity is then largest when the smallest \(k\) (i.e. \(k_1\)) is very small, \(k_1 \ll k_2 \sim k_3\). By momentum conservation, the other two momenta are then nearly equal. This is known as squeezed limit. The bispectrum, shape function and the non-Gaussian parameter \(f_{\text{loc}}^{NL}\) for local non-Gaussianity become:

\[
B_{\zeta}^{\text{loc}}(k_1, k_2, k_3) = \frac{6}{5} f_{\text{loc}}^{NL} \sum_{i<j=1}^{3} P_{\zeta}(k_i)P_{\zeta}(k_j),
\]

\[
T_{\zeta}^{\text{loc}}(k_1, k_2, k_3, k_4) = \frac{54}{25} g_{\text{loc}}^{NL} \sum_{i<j<p=1}^{3} P_{\zeta}(k_i)P_{\zeta}(k_j)P_{\zeta}(k_p)
+ \tau_{\text{loc}}^{NL} \sum_{j<p, i\neq j, p=1}^{11} P_{\zeta}(k_{ij})P_{\zeta}(k_j)P_{\zeta}(k_p),
\]

\[
S^{\text{loc}}(k_1, k_2, k_3) = \frac{K_3}{3K_{111}},
\]

\[
\lim_{k_1 \ll k_2 \sim k_3} B_{\zeta}^{\text{loc}}(k_1, k_2, k_3) = \frac{12}{5} f_{\text{loc, sq}}^{NL} P_{\zeta}(k_1)P_{\zeta}(k_2),
\]

\[
\lim_{k_1 \ll k_2 \sim k_3} S^{\text{loc}}(k_1, k_2, k_3) = \frac{2k_2}{3k_1},
\]

\[
j_{\text{loc, sq}}^{NL} = \lim_{k_1 \ll k_2 \sim k_3} f_{\text{loc, sq}}^{NL} = \frac{5}{12}(1 - n_s).
\]

**Equilateral non-Gaussianity:** Higher-derivative corrections during inflation can lead to large non-Gaussianities in presence of sound speed \(c_s < 1\). A key characteristic of derivative

\[\text{Equilaterial non-Gaussianity:}\]

Higher-derivative corrections during inflation can lead to large non-Gaussianities in presence of sound speed \(c_s < 1\). A key characteristic of derivative

\[\text{Equilaterial non-Gaussianity:}\]
interactions is that they are suppressed when any individual mode is far outside the horizon. This suggests that the bispectrum is maximal when all three moves have wavelengths equal to the horizon size. The bispectrum therefore has a shape that peaks in the equilateral configuration, \( k_1 = k_2 = k_3 = k \). Consequently the bispectrum and the shape function can be expressed as:

\[
B^{\text{equil}}(k, k, k) = \frac{18}{5} f^{\text{equil}}_{NL} P^2(k),
\]

\[
S^{\text{equil}}(k, k, k) = \lim_{k_1=k_2=k_3=k} \frac{k_1 k_2 k_3}{K_{111}} = 1.
\]

**Orthogonal non-Gaussianity:** Just like in the previous case each higher-derivative interaction of the inflaton field generically gives rise to a bispectrum with a shape which is similar but not identical to the equilateral form. Therefore it has been shown, using an effective field theory approach to inflationary perturbations, that it is possible to build a combination of the corresponding similar equilateral shapes to generate a bispectrum that is orthogonal to the equilateral one, the so-called orthogonal shape. This can be approximated by the template:

\[
B^{\text{orth}}(k_1, k_2, k_3) = 6 f^{\text{orth}}_{NL} P_s(k_*) \left( \frac{3}{k_1^2 k_2^2} - \frac{3}{k_2^2 k_3^2} - \frac{3}{k_3^2 k_1^2} \right. \\
\left. - \frac{8}{k_1^2 k_2^2 k_3^2} + \frac{3}{k_1 k_2 k_3} + (5 \text{ perm.}) \right)
\]

\[
S^{\text{orth}}(k_1, k_2, k_3) = \frac{(k_1 k_2 k_3)^2}{P_s(k_*)} B^{\text{orth}}(k_1, k_2, k_3)
\]

where \( k_* \) is the pivot momentum scale.

### 1.4 Plan of the thesis

The plan of the thesis is as follows: In chapter 2 we have studied the features of MSSM inflation from various supersymmetric D-flat directions using the saddle and inflection point techniques. In the case of saddle point inflation we derive the one loop effective potential by introducing non-renormalizable higher mass dimensional operators in the context of low scale MSSM. We have explored the possibility of PBH formation from this proposed model. Also we have solved the one-loop Renormalization Group (RG) flow equation to extract the behaviour of one-loop couplings and then the inflationary observable parameters as estimated from the model are then confronted with observational data from WMAP. Next introducing an inflection point feature we have derived the expression for the effective potential in the context of high scale MSSM inflation within \( \mathcal{N} = 1 \) SUGRA which can able to generate large tensor-to-scalar ratio. Hence the inflationary observable parameters as estimated from the model are then confronted with observational data from Planck data and fit the observed CMB TT spectra within low and high multipole region.

Further, in chapter 3 we have explored the possibility of inflation from the five dimensional background supergravity setup, in the context of Randall-Sundrum (RS) like two braneworld model

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30 Additionally the other limits-folded \((k_1 = 2k_2 = 2k_3)\), elongated \((k_1 = k_2 + k_3)\) and isosceles \((k_1 > k_2 = k_3)\) configurations can be studied from the non-Gaussian template.

31 An example is provided by the two interaction terms for an inflaton with a non-standard kinetic term.
and Dirac Bonn Infeld (DBI) Galileon embedded in Klebanov-Strassler (KS) throat geometry. In both the cases we have derived the model from the background five dimensional $\mathcal{N} = 2$ supergravity setup by implementing dimensional reduction technique. Hence we have derived the inflationary observables from both of the scenarios and confront them with the WMAP data. Further using a cosmological code CAMB we fit the CMB angular power spectra from TT anisotropy and other polarization data obtained from WMAP and also estimate various cosmological parameters from these models.

Hence in chapter 4 we have discussed the various features of reheating phenomenology in modified gravity framework embedded in the SUGRA inspired RS like two braneworld model where the results are subsequently different from that of the usual low energy General Relativity (GR) prescribed phenomenological counterpart as the Friedmann equations are modified in RS braneworld. In this chapter we have explicitly derived the analytical expressions for the reheating temperature and using this we have further solved the evolution equation of the number density of thermal gravitino in perturbative regime which results in the gravitino abundance in RS two braneworld model. Next we have compared the results with that of its low energy GR limiting results.

Further, we have studied the primordial non-Gaussian features using $\delta N$ formalism of unavoidable higher dimensional non-renormalizable Kähler operators for $\mathcal{N} = 1$ SUGRA framework in chapter 5. In particular we have studied the nonlinear evolution of cosmological perturbations on large scales which enables us to compute the curvature perturbation, without solving the exact perturbed field equations. Hence we compute the various non-Gaussian parameters for local type of non-Gaussianities and CMB dipolar asymmetry parameter, for a generic class of sub-Planckian models induced by the Hubble-induced corrections for a MSSM D-flat direction where inflation occurs at the point of inflection within the visible sector of effective theory. Next using the constraint on sound speed from Planck data we determine the stringent bound on the non-Gaussian parameters and CMB dipolar asymmetry parameter.

Finally we summarize our works in chapter 6. In this we also mention the future prospects of our works and overall comments made from our study.
MSSM inflation from various flat directions

2.1 Introduction

The observational success of primordial inflation arising from the Cosmic Microwave Background (CMB) radiation \([25, 23]\) has led to an outstanding question how to embed the inflationary paradigm within a particle theory \([143]\). Since inflation dilutes all matter except for the quantum vacuum fluctuations of the inflaton, it is pertinent that the end of inflation creates all the relevant Standard Model degrees of freedom for the success of Big Bang Nucleosynthesis \([144]\), without any extra relativistic degrees of freedom, i.e. dark radiation \([24]\). This immediately suggests that the inflationary vacuum cannot be arbitrary and the inflaton must decay solely into the Standard Model degrees of freedom. Furthermore, the recent WMAP \([23]\) and Planck data \([24]\) separately indicate that the perturbations in the baryons and the cold dark matter are adiabatic in nature, it is, therefore, evident that there must be a single source of perturbations, which is responsible for seeding the fluctuations in all forms of matter \(^1\). This can be realized conveniently if the inflaton itself carries the Standard Model charges as in the case of Minimal Supersymmetric Standard Model (MSSM) flat-directions \([148]\), where the lightest supersymmetric particle could be the dark matter candidate and can be created from thermal annihilation of the MSSM degrees of freedom \([99, 100, 84, 150]\). The inflaton candidates are made up of gauge invariant combinations of squarks (supersymmetric partners of quarks) and sleptons (supersymmetric partners of leptons).

One of the key ingredients for embedding inflation within MSSM is that the inflaton VEV must be below the Planck scale. This justifies the application of an effective field theory treatment at low energies. It is well-known that the potential for the MSSM flat-direction inflaton has high degree of flexibility – the potential can accommodate both the saddle-point and inflection-point below the Planck VEV, which allows a rich class of flat potentials, which has been studied analytically and numerically \([100, 151]\). The application of saddle point and inflection point inflation is not just limited to particle theory, but such potentials have also found their applications in string theory \([152]\). It is conceivable that, at high energies the universe is dominated by a large cosmological constant arising from a string theory landscape. Our own patch of the universe could be locked in a false vacuum within an MSSM landscape, or there could be hidden sector contributions, or there could

\(^1\)Embedding the last 50 -70 e-foldings of inflation within string theory has a major disadvantage. Due to large number of hidden sectors arising from any string compactifications, it is likely that the inflaton energy density will get dumped into the hidden sectors instead of the visible sector \([145, 146]\). The branching ratio for the inflaton decay into the visible sector is very tiny, therefore, reheating the Standard Model degrees of freedom is one of the biggest challenges for any string motivated models of inflation. Furthermore, many of the compactifications generically lead to extra dark radiation (massless axions) which are already at the verge of being ruled out by the present data \([147]\).

\(^2\)In principle more than one fields can still participate during inflation, but they must do so in such a way that here exists an attractor solution which would yield solely adiabatic perturbations and no isocurvature perturbations, such as, in the case of assisted inflation \([149]\).
also be a combination of these effects. For the purpose of illustration, we will model earlier phases of inflation driven by the superpotential of type [50, 51]:

\[ W = W(\Phi) + W(S) = \frac{\lambda \phi^n}{nM_p^{n-3}} + \frac{M_s}{2} S^2, \]  

(2.1.1)

where \( n \geq 3 \) and \( \lambda \sim \mathcal{O}(1) \). Here \( M_s \) governs the scale of heavy physics which dictates the initial vacuum energy density, and \( S(= s e^{\imath \theta}) \) is the hidden sector superfield. The total \( \text{Kähler potential} \) can be of the form [50, 51] of:

\[ K = S^\dagger S + \Phi^\dagger \Phi + \delta K, \]  

(2.1.2)

where the non-minimal term \( \delta K \) can be any one of these functional forms:

\[ \delta K = f(\Phi^\dagger \Phi, S^\dagger S) , \ f(S^\dagger \Phi \Phi) , \ f(S^\dagger S^\dagger \Phi \Phi) , \ f(S \Phi^\dagger \Phi). \]  

(2.1.3)

We will always endeavour to treat the fields \( s, \phi \ll M_p \). The higher order Planck scale suppressed corrections to the \( \text{Kähler potential} \) are extremely hard to compute. Within the regime of effective field theory it is possible to constrain the co-efficients of all of these effective operators and one can easily envisage various cosmological consequences from such contributions. See the ref. [50, 51] for details. The scalar potential in \( \mathcal{N} = 1 \) supergravity can be written in terms of superpotential, \( W \), and \( \text{Kähler potential}, \ K, \) as

\[ V = e^{K(\Phi, \Phi^\dagger)/M_p^2} \left[ (D_\Phi W(\Phi))K^{\Phi, \Phi^\dagger} (D_{\Phi^\dagger} W^*(\Phi^\dagger)) - \frac{3}{M_p^2} |W(\Phi)|^2 \right] + (D - \text{terms}), \]  

(2.1.4)

where \( D_\Phi, W = W_{\Phi_i} + K_{\Phi_i} W/M_p^2 \), and \( K^{\Phi_i, \Phi_j} \) is the inverse matrix of \( K_{\Phi_i \Phi_j} \), and the subscript \( (\Phi_i) \) denotes derivative with respect to the field. Hereafter, we neglect the contribution from the \( D \)-term, since the MSSM inflatons are D-flat directions. After minimizing the potential along the angular direction, \( \theta \ (\Phi = \phi e^{\imath \theta}) \), we get [102]:

\[ V(\phi, \theta) = V_0 + \left( m_\phi^2 + c_H H^2 \right) \frac{1}{2} |\phi|^2 + (a_H H + a_\lambda m_\phi) \frac{\lambda \phi^n}{nM_p^{n-3}} \cos(n\theta + \theta_{a_H} + \theta_{a_\lambda}) + \frac{\lambda^2 |\phi|^{2(n-1)}}{M_p^{2(n-3)}} \]  

(2.1.5)

where the cosmological constant will be determined by the overall inflationary potential,

\[ V_0 = \langle V(s) \rangle = M_s^4 \approx 3H^2M_p^2. \]  

(2.1.6)

Usually this bare cosmological term can be set to zero from the beginning by tuning the graviton mass. We will consider scenarios, where we will have \( V_0 \neq 0 \) (for \( n = 6 \)) and \( V_0 = 0 \) (for \( n = 4 \)). Note that \( m_\phi \sim \mathcal{O}(1) \) TeV and \( a_H \) are soft-breaking mass and the non-renormalizable \( A \)-term respectively (\( A \) is a positive quantity since its phase is absorbed by a redefinition of \( \theta \) during the process) \(^3\). The potential also obtains Hubble-induced corrections, with coefficients \( c_H, a_H \sim \mathcal{O}(1) \). Their exact numerical values will depend on the nature of \( \text{Kähler corrections} \) and compactification.

\(^3\)The masses of the various flat directions are given by [100]:

\[ m_\phi^2 = \frac{\sum_i m_i^2}{n} \]

where \( 3 \leq n \leq 9 \) and the symbol \( \tilde{i} \) is used for flat direction contents. In this chapter we will concentrate only on \( n = 4 \) and \( n = 6 \) level flat directions. Typically these masses are set by the scale of SUSY, in the low scale case the masses will be typically of order \( \mathcal{O}(1) \) TeV.
which are hard to compute for a generic scenario [153], but the corrections typically yield $\sim \mathcal{O}(1)$ coefficients. The non-renormalizable terms have a periodicity of $2\pi$ in $(\phi, \theta)$ 2D plane, $\theta_{a_H}, \theta_{a_A}$ are the extra phase factors.

Further using the potential stated in Eq (2.1.5) we have studied the inflationary paradigm from various supersymmetric D-flat direction in section (2.3) and section (2.4) from the low and high scale limiting situations by implementing saddle and inflection point techniques respectively. Hence we have estimated various cosmological parameters and studied the features of CMB TT angular power spectra from these two limiting situations. We have also analyzed other crucial astro-particle features- Primordial Black Hole (PBH) formation, one loop Renormalization Group (RG) flow and reheating temperature from the proposed models.

### 2.2 Saddle point and inflection point in MSSM

Let us briefly discuss saddle point and inflection point and there importance in the context of inflationary model building within MSSM:

1. **Saddle point**: It is characterized by a point on a curve which is a stationary point but not an extremum. In terms of potential the saddle point is defined as, $V'(\phi_0) = 0$ and $V''(\phi_0) \neq 0$.

2. **Inflection point**: Point of inflection is characterized by a point on a curve at which the curve changes from being concave to convex, or vice versa. Equivalently, an inflection point is defined by a point where the tangent meets the curve to order at least three. Points of inflection can also be categorized according to whether the first derivative of the potential with respect to inflaton field i. e. $V'(\phi)$ is zero or not zero. If $V'(\phi_0) = 0$ then the inflection point $\phi_0$ is a stationary point. On the other hand, if $V'(\phi_0) \neq 0$, then the inflection point $\phi_0$ is identified with a non-stationary point. A necessary condition for $\phi_0$ to be an inflection point is $V''(\phi_0) = 0$. A sufficient condition requires $V''(\phi_0 \pm \epsilon)$ have opposite signs in the neighborhood of $\phi_0$.

Additionally in the present context if at a point the curvature vanishes but does not change sign is sometimes called a undulation point.
It is important to note that in this chapter we discuss the cosmological consequences from non-stationary inflection point \( V'(\phi_0) \neq 0, V''(\phi_0) = 0 \) and stationary saddle point \( V'(\phi_0) = 0, V''(\phi_0) \neq 0 \) within MSSM.

### 2.3 MSSM inflation using saddle point technique

Let us start with \( n = 4 \) level, D-flat directions given by, \( QQL, QuQd, QuLe, uude \), where we have considered vacuum energy correction, \( V_0 = 0 \) and the contribution from the soft SUSY breaking mass term is larger than the Hubble induced corrections i.e. \( m_\phi \gg H \).

#### 2.3.1 Flat potential around saddle point

For \( n = 4 \) level the superpotential can be written as

\[
W_{nr}^4 = \frac{1}{M_p} \left[ \sum_{I=1}^{24} \alpha_I(QQQL)_I + \sum_{I=1}^{81} \beta_I(QuQd)_I + \sum_{I=1}^{81} \gamma_I(QuLe)_I + \sum_{I=1}^{27} \delta_I(uude)_I \right]; (2.3.7)
\]

The renormalizable flat directions of the MSSM at \( n = 4 \) level correspond to the gauge invariant monomials subject to the four additional complex constraints two each from

\[
F^u_H = \mu H^u_d + \lambda^u Q^u_u u_b = 0,
\]

\[
F^d_H = -\mu H^d_u + \lambda^d Q^d_d d_b + \lambda^d E^d_e l_e = 0,
\]

which can lift the flat directions which do not contain a Higgs field. Here \( \lambda^U, \lambda^D \) and \( \lambda^E \) are the Yukawa couplings, \( H_u, H_d \) are the Higgs superfield and the \( \mu \)-term appears in the renormalizable part of the superpotential of MSSM. Consequently the equation(2.3.7) breaks into four parts, each one of them now being flat:

\[
W_{4}^{(1)} = \frac{1}{M_p} \sum_{I=1}^{24} \alpha_I(QQQL)_I, (2.3.10)
\]

\[
W_{4}^{(2)} = \frac{1}{M_p} \sum_{I=1}^{81} \beta_I(QuQd)_I, (2.3.11)
\]

\[
W_{4}^{(3)} = \frac{1}{M_p} \sum_{I=1}^{81} \gamma_I(QuLe)_I, (2.3.12)
\]

\[
W_{4}^{(4)} = \frac{1}{M_p} \sum_{I=1}^{27} \delta_I(uude)_I, (2.3.13)
\]

resulting in \( W_{4}^{(i)} \approx \frac{\lambda_{4 i}}{4M_p} \Phi^4 \forall i(= (1, 2, 3, 4)) \). Considering any one of the above flat directions leading to the one loop corrected effective potential:

\[
V(\phi, \theta) = \frac{1}{2} m_\phi^2 |\phi|^2 + \frac{\lambda_4 A}{4M_p} \phi^4 \cos(4\theta + \theta_A) + \frac{\lambda_2}{M_p^2} |\phi|^6, (2.3.14)
\]
for all \(i\). Here we define:

\[
\lambda_i = \lambda_{i,0} \left[ 1 + D_3 \log \left( \frac{\phi^2}{\mu_0^2} \right) \right],
\]

(2.3.15)

\[
A = A_0 \left[ 1 + D_2 \log \left( \frac{\phi^2}{\mu_0^2} \right) \right] \frac{1}{1 + D_3 \log \left( \frac{\phi^2}{\mu_0^2} \right)},
\]

(2.3.16)

\[
m_{\phi}^2 = m_0^2 \left[ 1 + D_1 \log \left( \frac{\phi^2}{\mu_0^2} \right) \right],
\]

(2.3.17)

and in \(G_{\text{MSSM}} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) the representative flat direction field content is given by

\[
Q_{Ia}^I = \frac{1}{\sqrt{2}} (\Phi, 0)^T, \quad Q_{Ib}^I = \frac{1}{\sqrt{2}} (\Phi, 0)^T, \quad Q_{Ic}^I = \frac{1}{\sqrt{2}} (\Phi, 0)^T, \quad L_{I3}^I = \frac{1}{\sqrt{2}} P_d(0, \Phi)^T, \quad d_a^{B_1} = \frac{\Phi}{\sqrt{2}}, \quad d_b^{B_2} = \frac{\Phi}{\sqrt{2}}, \quad u_c^{B_3} = \frac{\Phi}{\sqrt{2}}, \quad e_3 = \frac{\Phi}{\sqrt{2}}.
\]

(2.3.18)

Here \(m_0, A_0\) and \(\lambda_{i,0}\) are the values of the respective parameters at the scale \(\mu_0\) and \(D_1, D_2\) and \(D_3\) \((|D_i| \ll 1\text{v}i)\) are the fine tuning parameters. Additionally in the field contents \(1 \leq B_1, B_2, B_3 \leq 3\) are color indices, \(1 \leq a, b, c \leq 3\) denote the indices for quark and lepton families and \(1 \leq I_1, I_2, I_3, I_4 \leq 2\) are the weak isospin indices. The flatness constraints require that \(B_1 \neq B_2 \neq B_3\) for quarks, \(I_1 \neq I_2 \neq I_3 \neq I_4\), \(\sum_{d=1}^3 P_d^2 = 1 \forall P_d \in \mathbb{R}\) for leptons and \(a \neq b \neq c\) for both. In Eq. (2.3.14) \(m_\phi\) represents the soft SUSY breaking mass term, \(\phi\) the radial coordinate of the complex scalar field \(\Phi = \phi \exp(i \theta) \in \mathbb{C}\) and the second term is the so called A-term which has a periodicity of \(2\pi\) in \(2\text{D}\) along with an extra phase \(\theta_\Lambda\). The radiative correction slightly affects the soft term and the value of the saddle point.

For \(n = 4\) we get an extremum for the principal values of \(\theta\) at \(\theta = \frac{(m_\pi - \theta_\Lambda)}{4}\) (where \(m \in \mathbb{Z}\))

\[
\phi_0 = \sqrt{\frac{M_p}{4\lambda_4(3 + D_3)} \left[ A \left(1 + \frac{D_2}{2}\right) \pm \sqrt{A^2 \left(1 + \frac{D_2}{2}\right)^2 - 8m_{\phi}^2(1 + D_1)(3 + D_3)} \right]}^{\frac{1}{2}},
\]

(2.3.19)

which appears from the constraint \(V'(\phi_0) = 0\) as a necessary condition for saddle point. However, this condition alone will not lead to saddle point. Rather, we have to make the potential sufficiently flat which can be achieved by vanishing higher derivatives of the potential. In this article, we consider non-vanishing fourth derivative of the potential resulting in saddle point. This will imply more fine-tuning but increased precision level in the information obtained from RG flow.

As discussed, \(V''''(\phi_0) < 0\) will give us secondary local minimum. This leads to constraint relations:

\[
A = \sqrt{2(3 + D_3)G_1G_2G_3 m_{\phi}(\phi_0)},
\]

(2.3.20)

\[
D_3 = \frac{M_p A_0}{4\lambda_{4,0} \phi_0^2 \left(37 + 60 \log \left( \frac{\phi_0}{\mu_0} \right) \right)} \left\{ D_2 \left(13 + 12 \log \left( \frac{\phi_0}{\mu_0} \right) \right) - \frac{2m_{\phi}^2(\phi_0) D_1 M_p}{\lambda_{4,0} A_0 \phi_0^2} + 6 \left(1 - \frac{20 \lambda_{4,0} \phi_0^2}{M_p A_0} \right) \right\},
\]

(2.3.21)
2.3 MSSM inflation using saddle point technique

one each for \( V''(\phi_0) = 0 \) and \( V'''(\phi_0) = 0 \). In this context:

\[
G_1 = \left[ \frac{(1 + D_1)}{(3 + D_3)} (15 + 11D_3) - (1 + 3D_1) \right]^2, \tag{2.3.22}
\]

\[
G_2 = \left[ (1 + D_1) \left( 3 + \frac{7}{2}D_3 \right) - (1 + 3D_1) \left( 1 + \frac{D_3}{2} \right) \right]^{-1}, \tag{2.3.23}
\]

\[
G_3 = \left[ (1 + \frac{D_3}{2}) (15 + 11D_3) - \left( 3 + \frac{7}{2}D_3 \right) \right]^{-1}. \tag{2.3.24}
\]

For the limit \(|D_1| \ll 1, |D_2| \ll 1 \) and \(|D_3| \ll 1 \) which gives:

\[
\phi_0 = \phi_{\text{tree}}^{\frac{1}{2}} \left[ 1 + D_1 \left( \frac{1}{2} - \frac{D_3}{6} \right) \right], \tag{2.3.25}
\]

\[
A \approx A_{\text{tree}} \left[ 1 + D_1 \left( \frac{1}{2} - \frac{D_3}{6} \right) \right]. \tag{2.3.26}
\]

where

\[
\phi_{\text{tree}} = \sqrt{\frac{m_\phi(\phi_0) M_p}{\lambda_4 \sqrt{6}}}, \tag{2.3.27}
\]

\[
A_{\text{tree}} = 2 \sqrt{6} m_\phi(\phi_0), \tag{2.3.28}
\]

represents tree level expressions. This means, during RG flow mentioning two parameters only \((D_1 \text{ and } D_2)\) will suffice instead of the usual three parameters in earlier MSSM models. This results in more precise information in RG flow. One may get tempted to vanish further higher derivatives of the potential in order to evaluate other unknown parameters \((D_1 \text{ and } D_2)\) without going into RG flow but this will make the effective inflaton potential in the vicinity of saddle point non-renormalizable. So, this is the highest level of precision constraint one can impose on RG flow parameters.

Consequently, around the saddle point \(\phi_0\), the inflaton potential can be expanded in a Taylor series as,

\[
V(\phi) = \tilde{C}_0 + \tilde{C}_4 (\phi - \phi_0)^4, \tag{2.3.29}
\]

where

\[
\tilde{C}_0 = V(\phi_0) = \frac{m_\phi^3(\phi_0) M_p}{6 \sqrt{6} \lambda_4} \left\{ 3 \left( 1 + \frac{D_1}{2} - \frac{D_3}{6} \right) \left[ 1 + D_1 \log \left( \frac{\phi_0^2}{\mu_0^2} \right) \right] - 3 \left( 1 + \frac{D_1}{2} - \frac{D_3}{6} \right)^2 \left[ 1 + D_2 \log \left( \frac{\phi_0^2}{\mu_0^2} \right) \right] \right. \left. + \left( 1 + \frac{D_1}{2} - \frac{D_3}{6} \right)^2 \left[ 1 + D_2 \log \left( \frac{\phi_0^2}{\mu_0^2} \right) \right] \right\}, \tag{2.3.30}
\]

and

\[
\tilde{C}_4 = \frac{1}{4!} V''''(\phi_0) = \frac{m_\phi^5(\phi_0)}{24 \sqrt{6} \phi_0^2} \left\{ \left( 1 + \frac{D_1}{2} - \frac{D_3}{6} \right) \left\{ \left( \frac{360}{\sqrt{6}} - 12 \sqrt{6} \right) + (684D_3 - 50 \sqrt{6}D_2) \left( 1 + \frac{D_1}{2} - \frac{D_3}{6} \right) - \frac{2 \sqrt{6}D_3}{(1 + \frac{D_1}{2} - \frac{D_3}{6})} \right\} + \left( 1 + \frac{D_1}{2} - \frac{D_3}{6} \right) \left( \frac{360D_3}{\sqrt{6}} - 12 \sqrt{6}D_2 \right) \log \left( \frac{\phi_0^2}{\mu_0^2} \right) \right\}. \tag{2.3.31}
\]
In what follows we shall model MSSM inflation with the above potential.

### 2.3.2 Modeling inflation & parameter estimation

For the best fit value of the model parameters:

\[
\tilde{C}_0 = 2.867 \times 10^{-36} M_p^4, \quad \tilde{C}_4 = -1.685 \times 10^{-13}
\]  \hspace{1cm} (2.3.32)

for the no. of e-foldings \( N = 70 \) the cosmological parameters obtained from our model is:

\[
P_s = \Delta_s^2 = 2.498 \times 10^{-9}, \quad n_s = 0.957, \\
r = 1.240 \times 10^{-29}, \quad \alpha_s = -0.612 \times 10^{-3} \quad \kappa_s = 1.749 \times 10^{-5}.
\]

Further, we use the publicly available code CAMB [155] to verify our results directly with observation. To operate CAMB at the pivot scale \( k_0 = 0.002 \) Mpc\(^{-1}\) the values of the initial parameter space are taken for \( \tilde{C}_0 = 2.867 \times 10^{-36} M_p^4 \) and \( N = 70 \). Additionally WMAP dataset [156, 22] for \( \Lambda \)CDM background has been used in CAMB to obtain CMB angular power spectrum. In Table 2.1 we have given all the input parameters for CAMB. Table 2.2 shows the CAMB output, which is in good agreement with WMAP seven years data. In Fig. 2.2 we have plotted CAMB output of CMB TT angular power spectrum \( C_{l}^{TT} \) for best fit with WMAP seven years data for scalar mode, which explicitly show the agreement of our model with WMAP dataset.

| \( H_0 \) \((\text{km/sec/Mpc})\) | \( \tau_{\text{Reion}} \) | \( \Omega_b h^2 \) | \( \Omega_c h^2 \) | \( T_{\text{CMB}} \) \((\text{K})\) |
|------------------|-----------------|-----------------|-----------------|------------------|
| 71.0             | 0.09            | 0.0226          | 0.1120          | 2.725            |

**Table 2.1: Input parameters [84].**

| \( t_0 \) \((\text{Gyr})\) | \( z_{\text{Reion}} \) | \( \Omega_m \) | \( \Omega_\Lambda \) | \( \Omega_k \) | \( \eta_{\text{Rec}} \) \((\text{Mpc})\) | \( \eta_0 \) \((\text{Mpc})\) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 13.707          | 10.704          | 0.2670          | 0.7330          | 0.0             | 285.10          | 14345.1         |

**Table 2.2: Output obtained from CAMB [84].**

### 2.3.3 Analyzing primordial Black hole formation

Now in the context of any running mass model one can expand the spectral index with the following parameterization [157]:

\[
n(\mathcal{R}) = n_z(k_0) - \frac{\alpha_z(k_0)}{2!} \ln(k_0\mathcal{R}) + \frac{\kappa_z(k_0)}{3!} \ln^2(k_0\mathcal{R}) + \ldots.
\]  \hspace{1cm} (2.3.33)

with \( \mathcal{R} \ll 1/k_0 \), i.e. \( \ln(k_0\mathcal{R}) < 0 \). This is identified to be the significant contribution to the Primordial Black Hole (PBH) formation [158]. Here the parameterization index \( z : [s(\text{scalar}), t(\text{tensor})] \) and the explicit form of the first term in the above expansion is given by

\[
n_z(k_0) = \begin{cases} 
  n_s(k_0) - 1 & \text{if } z = s \\
  n_t(k_0) & \text{if } z = t.
\end{cases}
\]  \hspace{1cm} (2.3.34)
Existence of the running and running of the running is the key feature in the formation of PBH in the radiation dominated era just after inflation [159] which could form CDM in the Universe. The initial PBHs mass $M_{\text{PBH}}$ is related to the particle horizon mass $\mathcal{M}$ by:

$$M_{\text{PBH}} = M_\gamma = \frac{4\pi}{3} \gamma \rho H^{-3} \quad (2.3.35)$$

at horizon entry, $R = (aH)^{-1}$. This is formed when the density fluctuation exceeds the threshold for PBH formation given as in Press–Schechter theory by

$$f(\geq \mathcal{M}) = \gamma \text{erf}\left[\frac{5\pi (1 + \frac{2}{3}w) \Theta_{\text{th}}}{8(1 + w)\sqrt{2\pi} \Gamma\left[\frac{3}{2}\right]}\right]. \quad (2.3.36)$$

Here $\Theta_{\text{th}}$ be the threshold value of the linearized density field $\Theta$ smoothed on a comoving scale $\mathcal{R}$ with $n_s(R) > 3$.

In general the mass of PBHs is expected to depend on the amplitude, size and shape of the perturbations. As a consequence the PBH mass is given by:

$$M_{\text{PBH}} = \gamma M_{\text{eq}}(k_{\text{eq}} R)^2 \left(\frac{g_{\star,\text{eq}}}{g_{\star}}\right)^{\frac{1}{2}}, \quad (2.3.37)$$

where the subscript “eq” refers to the matter–radiation equality. Here we use $g_{\star} = 228.75$ (all degrees of freedom in MSSM), while $g_{\star,\text{eq}} = 3.36$ and $k_{\text{eq}} = 0.07\Omega_m h^2 \text{ Mpc}^{-1}$ [160]. Consequently the relation between comoving scale and the PBH mass in the context of MSSM is given by:

$$\frac{R}{1 \text{ Mpc}} = 1.250 \times 10^{-23} \left(\frac{M_{\text{PBH}}}{1 \text{ g}}\right)^{\frac{1}{2}}. \quad (2.3.38)$$

Fig(2.3) shows the behavior of Press–Schechter mass function with respect to PBH mass. With the values of the parameters as obtained earlier, we have $\mathcal{M}_{\text{PBH}} \approx 10^{13} \text{gm}$ and the corresponding fractional energy density $f = 0.170$.\footnote{In this chapter we fix $\gamma \approx 0.2$ during the radiation dominated era [160] for numerical estimations.} \footnote{Here we use $\Omega_m h^2 = 0.2670$ from the CAMB output.}
2.3.4 One loop Renormalization Group flow

For the flat direction \( QQQL, QuQd, QuLe, uude \) the soft SUSY breaking masses can be expressed as:

\[
\begin{align*}
(m^2_\phi)_{QQQL} &= \frac{1}{4}(m^2_{Q_a} + m^2_{Q_b} + m^2_{Q_c} + m^2_{L_3}), \\
(m^2_\phi)_{QuQd} &= \frac{1}{4}(m^2_{Q_a} + m^2_{Q_b} + m^2_{\bar{u}} + m^2_{\bar{d}}), \\
(m^2_\phi)_{QuLe} &= \frac{1}{4}(m^2_{Q_a} + m^2_{\bar{u}} + m^2_{e} + m^2_{e_3}), \\
(m^2_\phi)_{uude} &= \frac{1}{4}(m^2_{u_a} + m^2_{\bar{u}} + m^2_{d} + m^2_{e_3}),
\end{align*}
\]

(2.3.39)

where \( 1 \leq a, b, c \leq 3 \) and \( a \neq b \neq c \). After neglecting the contribution from the all Yukawa couplings except from the top we can express the one-loop beta function as: [4]

\[
\beta_{m^2_a} = \dot{\mu} m^2_a = \frac{1}{8\pi^2} (m^2_a + |\lambda^3|^2) (\lambda^3)^2 - \frac{1}{2\pi^2} \sum_{\alpha=1}^{3} g^2_\alpha |\tilde{m}_\alpha|^2 X_{aa}
\]

(2.3.40)

where \( X_{aa} \) are the quadratic Casimir Group Invariants for the superfield \( \Phi \), defined in terms of Lie Algebra generators \( T^a \) by

\[
(T^a T^a)^\phi = X_{aa} \delta^a_b
\]

(2.3.41)

and \( \dot{\mu} = \frac{\partial}{\partial \mu} \). Here \( \mu \) is the Renormalization Group (RG) scale of the effective theory.

In the context of MSSM:

\[
X_{1a} = \frac{3 Y_a^2}{5} \quad \text{(for each } \Phi_a \text{ with weak hyper charge } Y_a),
\]

(2.3.42)

\[
X_{2a} = \frac{3}{4} \quad \text{(for } \Phi_a = Q, L, H_u, H_d),
\]

(2.3.43)

\[
= 0 \quad \text{(for } \Phi_a = \bar{u}, \bar{d}, \bar{e}),
\]

(2.3.44)

\[
X_{3a} = \frac{4}{3} \quad \text{(for } \Phi_a = Q, \bar{u}, \bar{d}),
\]

(2.3.45)

\[
= 0 \quad \text{(for } \Phi_a = L, \bar{e}, H_u, H_d),
\]

(2.3.46)
For Soft mass

\[ \mu \dot{m}_\phi^{QQQL} = \frac{1}{8\pi^2} \left( 3m_{Q_3}^2 + m_{U_3}^2 + |A^3_{U}|^2 \right) \left( \lambda_{U}^{33} \right)^2 - \frac{1}{8\pi^2} \left( 3g_2^2|m_2|^2 + 4g_3^2|m_3|^2 \right), \]

\[ \mu \dot{m}_\phi^{QuQd} = \frac{1}{2\pi^2} \left( m_{Q_3}^2 + m_{U_3}^2 \right) \left( \lambda_{U}^{33} \right)^2 - \frac{1}{8\pi^2} \left( \frac{3}{2}g_2^2|m_2|^2 + \frac{16}{3}g_3^2|m_3|^2 \right), \]

\[ \mu \dot{m}_\phi^{QuLe} = \frac{3}{8\pi^2} \left( m_{Q_3}^2 + m_{U_3}^2 \right) \left( \lambda_{U}^{33} \right)^2 - \frac{1}{8\pi^2} \left( \frac{3}{2}g_2^2|m_2|^2 + \frac{8}{3}g_3^2|m_3|^2 \right), \]

\[ \mu \dot{m}_\phi^{uude} = \frac{1}{2\pi^2} \left( m_{Q_3}^2 + m_{U_3}^2 \right) \left( \lambda_{U}^{33} \right)^2 - \frac{1}{2\pi^2}g_3^2|m_3|^2, \]

(2.3.47)

For Trilinear A-term

\[ \dot{\mu}A_D^{ab} = \delta_{b3} \left( \lambda_{U}^{33} \right)^2 \frac{A_D^{3b}}{8\pi^2} - \frac{1}{4\pi^2} \left( \frac{7}{18}g_1^2|m_1|^2 + \frac{3}{2}g_2^2|m_2|^2 + \frac{8}{3}g_3^2|m_3|^2 \right), \]

\[ \dot{\mu}A_U^{ab} = \frac{3(1 + \delta_{a3})A_D^{3b}}{8\pi^2} \left( \lambda_{U}^{33} \right)^2 - \frac{1}{4\pi^2} \left( \frac{13}{18}g_1^2|m_1|^2 + \frac{3}{2}g_2^2|m_2|^2 + \frac{8}{3}g_3^2|m_3|^2 \right), \]

\[ \dot{\mu}A_E^{ab} = -\frac{1}{4\pi^2} \left( \frac{3}{2}g_2^2|m_2|^2 + \frac{3}{2}g_3^2|m_3|^2 \right), \]

(2.3.48)

For Fourth level Yukawa coupling

\[ \dot{\mu} \lambda_U^{aa} = \frac{3(1 + \delta_{a3})}{8\pi^2} \left( \lambda_{U}^{33} \right)^3 - \frac{\lambda_U^{aa}}{4\pi^2} \left( \frac{13}{18}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 \right), \]

\[ \dot{\mu} \lambda_D^{ab} = \delta_{b3} \left( \lambda_{U}^{33} \right)^2 \frac{\lambda_D^{3b}}{8\pi^2} - \frac{\lambda_D^{ab}}{4\pi^2} \left( \frac{7}{18}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 \right), \]

\[ \dot{\mu} \lambda_E^{aa} = -\frac{\lambda_E^{aa}}{4\pi^2} \left( \frac{3}{2}g_2^2 + \frac{3}{2}g_3^2 \right). \]
Figure 2.5: Running of soft mass squared ratio \( \left( \frac{m_{\Phi}^2(\mu)}{m_{\Phi}^2(\mu_0)} \right) \) in one loop RGE for MSSM with the logarithmic scale \( \log_{10}(\mu) \) where \( \mu_0 = 2.6 \times 10^7 \) GeV for \( \zeta = 0.5, 1, 2 \) for \( n = 4 \) level QuQd \( \forall i \). Similar plots can be obtained for QuLe, QQQL and uuude flat directions also \( \forall i \) [84].

where all the superscript \( a \) and \( b \) represent generation or family indices run from 1 to 3 physically representing the first, second and third generation respectively. For the one-loop renormalization of gauge couplings and gaugino masses, one has in general

\[
\beta_{g_\alpha} = \mu g_\alpha = \frac{g_\alpha^3}{16\pi^2} \left[ \Sigma_a I_{aa} - 3X_{\alpha G} \right], \tag{2.3.49}
\]

\[
\beta_{m_\alpha} = \mu m_\alpha = \frac{g_\alpha^2 m_\alpha}{8\pi^2} \left[ \Sigma_a I_{aa} - 3X_{\alpha G} \right] \tag{2.3.50}
\]

where \( X_{\alpha G} \) quadratic Casimir invariant of the group \([ 0 \text{ for } U(1) \text{ and } N \text{ for } SU(N) \]), \( I_{aa} \) is the Dynkin index of the chiral supermultiplet \( \Phi_a \) [ normalized to \( \frac{1}{2} \) for each fundamental representation of \( SU(N) \) and to \( 3Y^2/5 \) for \( U(1)_Y \)]. For the above mentioned flat direction the running of gauge couplings \( (g_\iota(\mu)) \) and gaugino masses \( (m_\iota(\mu)) \) obey,

\[
\mu g_i = -\frac{d_i}{2} g_i^3, \tag{2.3.51}
\]

\[
\mu \left( \frac{m_i}{g_i^2} \right) = 0 \forall i \tag{2.3.52}
\]

where for \( i = 1(U(1)_Y), 2(SU(2)_L), 3(SU(3)_C) \) here \( d_1 = \frac{11}{8\pi^2}, d_2 = \frac{1}{8\pi^2}, d_3 = -\frac{3}{8\pi^2} \) which is the simpler version of the equation(2.3.49). Now to show explicitly that the contributions from the top Yukawa coupling \( (\lambda_{23}^{u3}) \) are very small for an induced electroweak group \( G_{EW} = SU(2)_L \otimes U(1)_Y \) breakdown, let us start with the Higgs potential [4]

\[
V_{Higgs}(H, \bar{H}) = m_1^2 |H|^2 + m_2^2 |\bar{H}|^2 + m_3^2 \left( H^\dagger H + H^\dagger \bar{H} \right) + \frac{1}{8} \left( g_1^2 + g_2^2 \right) \left[ |H|^2 - |\bar{H}|^2 \right]^2, \tag{2.3.53}
\]

where \( H = H_u \) and \( \bar{H} = H_d \) represent the Higgs superfields and the relative vev of the two Higgses are given by

\[
v = \sqrt{\langle H \rangle^2 + \langle \bar{H} \rangle^2} = \sqrt{\frac{2 \left[ m_1^2 - m_2^2 - (m_1^2 + m_2^2) \cos(2\theta) \right]}{g_1^2 + g_2^2} \cos(2\theta)} \tag{2.3.54}
\]
2.3 MSSM inflation using saddle point technique

\[ A_1 \left( \frac{\mu}{M} \right) A_1 \left( \frac{\mu_0}{M} \right) U_1 \ (a) \]

\[ A_2 \left( \frac{\mu}{M} \right) A_2 \left( \frac{\mu_0}{M} \right) D_2 \ (b) \]

\[ A_3 \left( \frac{\mu}{M} \right) A_3 \left( \frac{\mu_0}{M} \right) E_3 \ (c) \]

Figure 2.6: Running of trilinear A-term ratio \( \left( \frac{A_\beta(\mu)}{A_\beta(\mu_0)} \right) \) in one loop RGE for MSSM with the logarithmic scale \( \log_{10}(\mu) \) where \( \mu_0 = 2.6 \times 10^7 \text{GeV} \), \( \zeta = 0.5(dotteddashed), 1(solid), 2(dashed) \) and \( \beta = 1(U), 2(D), 3(E) \) for \( n = 4 \) level \( \forall i \ [84] \).

\[
\tan(\theta) = \frac{\langle \tilde{H} \rangle}{\langle H \rangle}.
\]

(2.3.55)

Here \( \theta \) represents an angular parameter which parameterizes MSSM. For the sake of convenience let us now write \( \cos(2\theta) \) appearing in equation\( (2.3.54) \) introducing new parameterization as [4]:

\[
\cos(2\theta) = \frac{w^2 - 1}{w^2 + 1}
\]

(2.3.56)

where

\[
w = \frac{\langle H \rangle}{\langle \tilde{H} \rangle} \frac{1}{\sqrt{1 - \left( \frac{\langle H \rangle}{\langle \tilde{H} \rangle} \right)^2}}.
\]

(2.3.57)

Consequently the top Yukawa coupling can be expressed as:

\[
\lambda_{33}^{33} U = \frac{m_U}{\sin(\theta)}
\]

(2.3.58)

where \( 0 \leq \theta < \frac{\pi}{2} \) and the top mass: \( 43 \text{GeV} \leq m_U \leq 170 \text{GeV} \ll \mu_{\text{GUT}} \) comes from the RG flow [4]. It is evident from the above parameterization that as \( w \to 1, \theta \to \frac{\pi}{4} \) which implies \( \langle H \rangle \) and \( \langle \tilde{H} \rangle \) is very large and have the same order of magnitude. As a result the relative vev \( v \) is also large and the top Yukawa coupling is very small for which one can easily neglect it from the RG flow at the energy scale of MSSM inflation as mentioned earlier. The consequence of the large vev of Higgs field can be taken care of by introducing strongly interacting gauge group \( G_{\text{NEW}} = G_S \otimes \text{SU}(3)_C \) and its superconformal version \( G_{\text{SCONF}} = \text{SU}(3)_{\text{SC}} \otimes \text{SU}(3)_C \).

In table\( (2.6) \) we have tabulated the numerical values of vev of \( H \) and \( \tilde{H} \), the angular parameter \( \theta \), \( \tan(\theta) \), \( w \), the top mass \( m_U \) and the top Yukawa coupling \( \lambda_{33}^{33} \) contributing to the parameter space of MSSM for the \( n = 4 \) level flat directions \( \text{QQQL, QuQd, QuLe} \) and \( \text{uude} \). It should be noted that appearance of large VEV of Higgses as mentioned in table\( (2.6) \) can easily be interpreted when Einstein Hilbert term appears in the total action of the theory at lowest order approximation which is our present consideration. Consequently the contributions from the hard cutoff is sub-leading.
due to the soft conformal symmetry breaking. This leads to small top Yukawa coupling in the restricted parametric space of MSSM characterized by the phenomenological bound:

\[ 43 \text{GeV} \leq m_U \leq 170 \text{GeV}, \quad 1.006 \leq \tan(\beta) \leq 1.025 \]  

(2.3.59)

for the n=4 flat directions.

Neglecting all the sub-leading contributions arising from the top Yukawa coupling in the restricted parameter space of the MSSM, the solutions of these RGE for n=4 level flat directions can be written as:

\[
\begin{align*}
g_i(\mu) &= \frac{g_i(\mu_0)}{\sqrt{1 - d_i g_i^2(\mu_0) \ln \left( \frac{\mu}{\mu_0} \right)}}, \quad (2.3.60) \\
m_i(\mu) &= m_i(\mu_0) \left( \frac{g_i(\mu)}{g_i(\mu_0)} \right)^2, \quad (2.3.61) \\
\Delta m_i^2 &= \sum_{i=1}^{3} f^i_i \Delta m_i^2, \quad (2.3.62) \\
\Delta A^a_b(\beta) &= \frac{1}{2} \sum_{i=1}^{3} (C^i_\beta)^{ab} \Delta m_i, \quad (2.3.63) \\
\lambda^a_b(\mu) &= \lambda^a_b(\mu_0) \prod_{i=1}^{3} \left( \frac{g_i(\mu)}{g_i(\mu_0)} \right)^{(C^i_\beta)^{ab}}, \quad (2.3.64)
\end{align*}
\]

Here \( g_i(\mu_0), m_i(\mu_0), A_\beta(\mu_0), m_\phi(\mu_0) \) and \( \lambda_\beta(\mu_0) \) represent the value of the gauge couplings, gaugino masses, trilinear couplings, soft SUSY braking masses and Yukawa couplings at the characteristic scale \( \mu_0 \). In equation (2.3.60-2.3.64) we have used the following shorthand notation: \( \Delta W = W(\mu) - W(\mu_0) \), where \( W = \{ A_\beta, m_\phi^2, m_i^2, m_i \} \) and the \( \beta \) indices 1,2,3 represent U, D, E respectively. In equation (2.3.60-2.3.64) \( f^i_F \) and \( (C^i_\beta)^{ab} \) are \((4 \times 3)\) and \((3 \times 3)\) matrices whose entries are tabulated.
2.3 MSSM inflation using saddle point technique

| \( f_i^F \) | \( i = 1(\text{U}(1)Y) \) | \( i = 2(\text{SU}(2)\text{L}) \) | \( i = 3(\text{SU}(3)\text{C}) \) |
|----------|----------------|----------------|----------------|
| \( F=1(\text{QQQL}) \) | 0 | \( \gamma \) | \( -\gamma \) |
| \( F=2(\text{QuQd}) \) | 0 | \( \gamma \) | \( -\gamma \) |
| \( F=3(\text{QuLe}) \) | 0 | \( \gamma \) | \( -\gamma \) |
| \( F=4(\text{ude}) \) | 0 | 0 | \( \gamma \) |

Table 2.3: Entries of \( f_i^F \) matrix obtained from the solution of RGE [84].

| \( \beta^a_{\beta^b} \) | \( i = 1(\text{U}(1)Y) \) | \( i = 2(\text{SU}(2)\text{L}) \) | \( i = 3(\text{SU}(3)\text{C}) \) |
|-----------------|----------------|----------------|----------------|
| \( \beta=1(\text{U}), a=b \) | \( \frac{26}{7} \) | 6 | \( \frac{14}{3} \) |
| \( \beta=2(\text{D}), a=b \) | \( \frac{26}{7} \) | 6 | \( \frac{14}{3} \) |
| \( \beta=3(\text{E}), a \neq b \) | \( \frac{6}{7} \) | 6 | 0 |

Table 2.4: Entries of \( \beta^a_{\beta^b} \) matrix obtained from the solution of RGE [84].

in Table(2.3) and Table(2.4) respectively. It is obvious from the RGE that \( \beta = 1, 2 \) implies \( a = b \) and \( \beta = 3 \) implies \( a \neq b \).

Using the solutions of RGE along with the approximation that the running of the gaugino masses and gauge couplings is very very small we get:

\[
D_1 = -\frac{1}{8\pi^2} \sum_{i=1}^{3} J_i \left( \frac{m_i}{m_{\phi_0}} \right)^2 g_i^2(\mu_0), \tag{2.3.65}
\]

\[
D_2^\beta = -\frac{1}{4\pi^2} \sum_{i=1}^{3} K_i^\beta \left( \frac{m_i}{A_0} \right) g_i^2(\mu_0), \tag{2.3.66}
\]

where we have \( J_1 = 0, J_2 = 3 \) and \( J_3 = 4 \) for \( i = 1, 2, 3 \) and all the entries of \( K^{\beta i} (3 \times 3) \) matrix are tabulated in Table(2.5).

| \( K^{\beta i} \) | \( i = 1(\text{U}(1)Y) \) | \( i = 2(\text{SU}(2)\text{L}) \) | \( i = 3(\text{SU}(3)\text{C}) \) |
|-------------|----------------|----------------|----------------|
| \( \beta=1(\text{U}) \) | \( \frac{26}{7} \) | \( \frac{2}{7} \) | \( \frac{2}{7} \) |
| \( \beta=2(\text{D}) \) | \( \frac{26}{7} \) | \( \frac{2}{7} \) | \( \frac{2}{7} \) |
| \( \beta=3(\text{E}) \) | \( \frac{6}{7} \) | \( \frac{2}{7} \) | 0 |

Table 2.5: Entries of \( K^{\beta i} \) matrix [84].

In this context the subscript ‘0’ represents the values of parameters at the high scale \( \mu_0 \). As discussed in section III, constraining only \( D_1 \) and \( D_2^\beta \) is sufficient here. Eqn(2.3.21) provides an extra constraint relation which restricts the parameters further leading to more precise information in RG flow. For universal boundary conditions, the high scale is identified to be the GUT scale:

\[
\mu_{\text{GUT}} \approx 3 \times 10^{16} \text{ GeV}, \tag{2.3.67}
\]

\[
\tilde{m}_1(\mu_{\text{GUT}}) = \tilde{m}_2(\mu_{\text{GUT}}) = \tilde{m}_3(\mu_{\text{GUT}}) = \tilde{m}, \tag{2.3.68}
\]

\[
A_E(\mu_{\text{GUT}}) = A_U(\mu_{\text{GUT}}) = A_D(\mu_{\text{GUT}}) = A_0 \tag{2.3.69}
\]

with \( g_1 \approx 0.56, \ g_2 \approx 0.72, \ g_3 \approx 0.85 \). Now depending upon the different phenomenological situations the \( n = 4 \) level flat directions are divided into two classes. The first class deals with
2. MSSM inflation from various flat directions

| Flat direction | $\mu_0 = \phi_0$ GeV | $A_{0,\text{tree}}$ GeV | $m_{\phi_0}$ GeV | $(H)$ GeV | $(\bar{H})$ GeV | $\tan \theta$ | $v$ GeV | $m_{\tilde{U}}$ GeV | $\lambda_{U}^{33} = \lambda_{0}$ |
|----------------|---------------------|----------------------|----------------|-------------|-------------|-------------|------|----------------|------------------|
| QuLe          | $2.6 \times 10^2$   | 36.967               | 7.546          | $0.200 \times 10^{16}$ | $0.458 \times 10^{16}$ | $1.006 \times 10^{16}$ | $1.000 \times 10^{16}$ | 43   | $1.212 \times 10^{-14}$ |
| QuQd          | $2.6 \times 10^2$   | 36.967               | 7.546          | $0.450 \times 10^{16}$ | $0.423 \times 10^{16}$ | $1.013 \times 10^{16}$ | $1.010 \times 10^{16}$ | 170  | $7.106 \times 10^{-14}$ |
| QQQL          | $1.344 \times 10^{18}$ | $892 \times 10^4$ | $182 \times 10^4$ | $0.188 \times 10^8$ | $0.124 \times 10^8$ | $1.025 \times 10^8$ | $0.226 \times 10^8$ | 80   | $4.945 \times 10^{-14}$ |
| uude          | $2.896 \times 10^2$ | $4.142 \times 10^6$ | $845 \times 10^6$ | $0.174 \times 10^8$ | $0.157 \times 10^8$ | $1.019 \times 10^8$ | $0.235 \times 10^8$ | 135  | $8.047 \times 10^{-14}$ |

Table 2.6: MSSM parameter values obtained from RG flow for $n=4$ level flat directions [84].

QuQd, QuLe which is lifted completely at $n = 4$ level. The other class which is lifted by higher dimensional operators deals with uude, QQQL. Most importantly uude, QQQL take part in the proton decay ($p \rightarrow \pi^0 e^+$, $p \rightarrow \pi^+ \nu_e$ etc.) [84] which introduces a stringent constraint on the Yukawa coupling $\lambda_0$ at $n = 4$ level. Additionally the neutrino-antineutrino oscillation data restricts $\lambda_0$ again. Then we just use RG equations along with these restrictions to run the coupling constants and masses to the scales as mentioned in table 2.6.

Considering all these values we obtain effectively:

$$D_1 \approx -0.056\zeta^2,$$
$$D_2 \approx -0.074\zeta, \quad D_2^2 \approx -0.071\zeta, \quad D_3^2 \approx -0.031\zeta,$$
$$D_3^1 = D_3^2 = D_3^3 \approx -0.048 - 0.168\zeta^2,$$

where $\zeta = m/m_\phi$ is calculated at the GUT scale. Typically the running based on gaugino loops alone results in negative values of $D_1 \nu i$. Positive values can be obtained when one includes the Yukawa couplings, practically the top Yukawa, but the order of magnitude remains the same. The choice of fine tuned initial conditions directly shows more fine tuning is required compared to other models. It is a straightforward exercise to verify that even if one considers all the flat directions at $n = 4$ level one will arrive at the potential eqn. (2.3.29) with same $\bar{C}_0$ and $\bar{C}_4$. This is precisely what we have done in this paper.

The results of RG flow have been demonstrated in figs (2.4)-(2.7). In fig (2.4) and fig (2.5) ‘dashed’, ‘solid’ and ‘dotdashed’ line represents $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge group content respectively. Fig (2.4)-fig (2.7) explicitly showing the behavior of the RGE flow of gaugino masses, soft SUSY breaking mass, trilinear couplings and Yukawa couplings respectively. Additionally fig (2.4)-fig (2.6) give consistent GUT scale unification.

2.4 MSSM inflation using inflection point technique

In the last section we have explored the possibility of low scale MSSM inflation using saddle point technique originating from D-flat directions. But effectively one can uplift the scale of inflation by adding an extra vacuum energy dominated SUGRA correction term to the low scale inflationary potential. In this section instead of saddle point technique we use inflection point technique to construct the effective potential within MSSM and we explore both the cases-low scale as well as high scale inflationary paradigm. We will keep $V_0$ in Eq (2.1.5) for this case, and do the computation for $n = 6$ flat directions, uudd and LLe.
2.4 MSSM inflation using inflection point technique

2.4.1 Flat potential around the inflection point

Applying inflection-point technique as discussed in section 2.2, inflationary potential \( V(\phi) \) can be expanded in Taylor series as [101]:

\[
V(\phi) = \alpha + \beta(\phi - \phi_0) + \gamma(\phi - \phi_0)^3 + \kappa(\phi - \phi_0)^4 + \cdots,
\]

where any generic potential, \( V(\phi) \), has been expanded around the inflection-point, \( \phi_0 \), where \( \alpha \) denotes the cosmological constant which will further determine the scale of inflation, and coefficients \( \beta, \gamma, \kappa \) determine the shape of the potential in terms of the model parameters. Typically, \( \alpha \) can be set to zero by fine tuning, but here we wish to keep this term for generality as we are interested here for \( V_0 \neq 0 \). Note that not all of the coefficients are independent once we prescribe inflaton within MSSM.

In the present context let us consider two \( D \)-flat directions- \( \tilde{u}\tilde{d} \) and \( \tilde{L}\tilde{L}\tilde{e} \) as the inflaton candidates [99, 100]. Both \( \tilde{u}\tilde{d} \), where \( \tilde{u}, \tilde{d} \) correspond to the right handed squarks, and \( \tilde{L}\tilde{L}\tilde{e} \), where \( \tilde{L} \) is the left handed slepton, and \( \tilde{e} \) is the right handed (charged) leptons, flat directions are lifted by higher order superpotential terms of the following simple form:

\[
W(\Phi) = \frac{\lambda}{6} \Phi^6 M_\rho^3,
\]

where \( \lambda \sim O(1) \) coefficient. The scalar component of \( \Phi \) superfield, denoted by \( \phi \), is given by

\[
\phi = \frac{\tilde{u} + \tilde{d} + \tilde{d}}{\sqrt{3}}, \quad \phi = \frac{\tilde{L} + \tilde{L} + \tilde{e}}{\sqrt{3}}
\]

and the masses are given by:

\[
m_\phi^2 = \frac{m_L^2 + m_{\tilde{L}}^2 + m_{\tilde{e}}^2}{3}, \quad m_\phi^2 = \frac{m_\tilde{u}^2 + m_{\tilde{d}}^2 + m_{\tilde{d}}^2}{3}
\]

for the \( \tilde{u}\tilde{d}\tilde{d} \) and \( \tilde{L}\tilde{L}\tilde{e} \) flat directions, respectively.

2.4.2 Low scale inflation

Since \( c_H, a_H \sim O(1) \) the Hubble-induced terms do not play any crucial role in this case, and the scale of inflation remains very low. As a result the tensor-to scalar ratio, \( r \), become too small to be ever detectable. This was the scenario studied in Refs. [99, 100]. In the low scale scenario, the value of \( V_0 \leq m_\phi^2 \phi_0^2 \), is negligible and does not contribute to the dynamics. In this case we can set its value to \( V_0 = 0 \) from the beginning by tuning the gravitino mass [4]. The potential can be minimized along the \( \theta \) direction, which reduces to [99, 100]:

\[
V(\phi) = \frac{m_\phi^2}{2} |\phi|^2 - a_\lambda m_\phi \frac{\lambda \phi^6}{6 M_\rho^3} + \frac{\lambda^2 |\phi|^{10}}{M_\rho^6}
\]

For,

\[
a_\lambda^2 \equiv 1 - 4\delta^2,
\]

and \( \delta^2 \ll 1 \), there exists a point of inflection \( (\phi_0) \) in \( V(\phi) \), where

\[
\phi_0 = \left( \frac{m_\phi M_\rho^3}{\lambda \sqrt{10}} \right)^{1/4} + O(\delta^2),
\]

and

\[
V''(\phi_0) = 0,
\]
For large scale inflation, $H \gg m_\phi$, we have shown the variation of $P_S$ vs $n_S$ [102]. The red curve shows the model parameters, $\delta \sim 10^{-4}$, $\lambda = 1$, $a_H = 2.108$, $\phi_0 = 1.129 \times 10^{16}$ GeV, for the pivot scale $k_*=0.002$ Mpc$^{-1}$. The green shaded region shows the $2\sigma$ CL. range in $n_s$ allowed by the Planck data [25]. Instead of getting a single solid red curve we get a black shaded region if we consider the full parameter space for high scale ($H \gg m_\phi$) inflation given by Eq. (5.5.64).

The potential is specified completely by $m_\phi$ and $\lambda$. However $m_\phi$ is determined by the soft-SUSY breaking mass parameter, which is well constrained by the current ATLAS [161] and CMS [162] data, and we shall take $m_\phi = 1$ TeV. For $m_\phi \sim 1$ TeV, $H_* \sim 0.1$ GeV, and our assumption of neglecting $H$ in such a case is well justified. We will always consider $\lambda = 1$ in our analysis.

### 2.4.3 High scale inflation

The SUGRA corrections become important, the Hubble-induced terms dominate the potential. This can happen quite naturally if there exists a previous source of effective cosmological constant term described in Ref. [163]. In this case one can safely ignore the soft SUSY breaking mass term, and since $a_\lambda \sim O(1)$, one can safely consider only the Hubble-induced non-renormalizable term.

One advantage of considering such a potential is to obtain large tensor-to-scalar ratio, $r$, which can be within the range of Planck and other future CMB B-mode polarization experiments. We will keep $V_0$ in this case, and the potential simplifies to [163]:

$$V(\phi) = V_0 + \frac{c_H H^2}{2} |\phi|^2 - \frac{a_H H \phi^6}{6 M_p^3} + \frac{|\phi|^{10}}{M_p^6}.$$  

---

7Within the setup of effective field theory without losing the generality one can consider the non-renormalizable operators. In such a case the co-efficients of the non-renormalizable operators are suppressed by UV cut-scale $\Lambda_{UV} \sim M_p$ of the effective theory.
where we have taken $\lambda = 1$. The potential admits inflection point for $a_H^2 \approx 40e_H^2$. We characterize the required fine-tuning by the quantity, $\delta$, defined as [100]

$$\frac{a_H^2}{40e_H^2} = 1 - 4\delta^2. \quad (2.4.84)$$

When $|\delta|$ is small, a point of inflection $\phi_0$ exists such that $V''(\phi_0) = 0$, with

$$\phi_0 = \left(\sqrt{\frac{c_H}{10} H M_p^2}\right)^{1/4}. \quad (2.4.85)$$

For $\delta < 1$, we can Taylor-expand the inflaton potential around the inflection point $\phi = \phi_0$ similar to Eq. (2.4.73), where the coefficients are now given by:

$$\alpha = V(\phi_0) = V_0 + \left(\frac{4}{15} + \frac{4}{3} \delta^2\right) c_H H^2 \phi_0^2 + \mathcal{O}(\delta^4), \quad (2.4.86)$$

$$\beta = V'(\phi_0) = 4\delta^2 c_H H^2 \phi_0 + \mathcal{O}(\delta^4), \quad (2.4.87)$$

$$\gamma = \frac{V'''(\phi_0)}{3!} = \frac{c_H H^2}{\phi_0^3} \left(32 - 80\delta^2\right) + \mathcal{O}(\delta^4), \quad (2.4.88)$$

$$\kappa = \frac{V''''(\phi_0)}{4!} = \frac{c_H H^2}{\phi_0^4} \left(384 - 1260\delta^2\right) + \mathcal{O}(\delta^4). \quad (2.4.89)$$

Note that once we specify $c_H$ and $H$, all the terms in the potential can be determined. In this regard the potential indeed simplifies a lot to study the cosmological observables.

One must also ensure that the vacuum energy density which generated the large cosmological constant in the first place vanishes by the end of slow-roll inflation. This typically happens in the case of hybrid inflation [164], and as discussed in [101]. In the string landscape, or in the case of MSSM, this can happen through bubble nucleation, provided the rate of nucleation is such that $\Gamma_{\text{nucl}} \gg H$. In the latter case all the bubbles will belong to the MSSM vacuum—similar to the first order phase transition in the electroweak symmetry breaking scenario. However, one has to make sure that the cosmological constant disappears in the MSSM vacuum right at the end of inflation.

### 2.4.4 Parameter estimation and CMB observables

In this section our primary focus is to study the cosmological observables to match the CMB data for an inflection-point inflation whose potential is given by Eq. (5.4.35). Here the consistency relations are modified at the second order due to the presence of running. Cosmological parameter estimation can be done more precisely once we allow the higher order radiative corrections to the slow-roll parameters [165], which we have listed in Appendix A (see Eqs. (6.0.1)-(6.0.9)). In our case we have obtained the predicted power spectrum from the higher order radiative corrections to the slow-roll parameters. In order to illustrate this, let us consider the case when $H \gg m_\phi$. In this case there is a possibility of detecting large tensor-scalar ratio, $r$.

We need to compute the pivot scale, $k_*$, when the relevant perturbations had left the Hubble patch during inflation. We can compute by expressing the number of e-foldings during inflation, which is given by [25]:

$$N_* \approx 71.21 - \ln \left(\frac{k_0}{k_*}\right) + \frac{1}{4} \ln \left(\frac{V_*}{V_{P}}\right) + \frac{1}{4} \ln \left(\frac{V_{P}}{\rho_{\text{end}}}\right) + \frac{1-3w_{\text{int}}}{2(1+w_{\text{int}})} \ln \left(\frac{\rho_{P}}{\rho_{\text{end}}}\right), \quad (2.4.90)$$

where $\rho_{\text{end}}$ is the energy density at the end of inflation, $\rho_{P}$ is an energy scale during reheating, $k_0 = a_0 H_0$ is the present Hubble scale, $V_*$ corresponds to the potential energy when the relevant
2. MSSM inflation from various flat directions

(a) \( r \) vs \( n_S \). By varying \( H_* \) we can probe a wide range of tensor-to-scalar ratio: \( 10^{-29} < r_* \leq 0.12 \). The vertical line on the left corresponds to \( N = 50 \), while the right line corresponds to \( N = 70 \).

(b) \( r \) vs \( n_S \). By varying \( H_* \) we can probe a wide range of tensor-to-scalar ratio: \( 10^{-29} < r_* \leq 0.12 \). The vertical line on the left corresponds to \( N = 50 \), while the right line corresponds to \( N = 70 \).

Figure 2.9: We show the joint 1σ and 2σ CL. contours in \( r - n_S \) plane using (a) Planck+WMAP-9 data with \( \Lambda \)CDM+r(Blue region), and \( \Lambda \)CDM+r + \( \alpha_S \)(Red region), (b) Planck+WMAP-9+BAO data with \( \Lambda \)CDM+r(Blue region) and \( \Lambda \)CDM+r + \( \alpha_S \)(Red region) [102]. The straight lines parallel to \( n_S \) axis are drawn by varying the Hubble parameter \( H_* \) within the range \( 10^{-1} \mathrm{GeV} < H_* \leq 9.241 \times 10^{13} \mathrm{GeV} \). The deep green line and the yellow line correspond to the upper and lower bound of \( H_* \) respectively. The green shaded region bounded by orange lines represent the allowed region obtained from the model. Additionally, the black thick line divides the low scale (\( m_\phi \gg H \)) and the high scale (\( H \gg m_\phi \)) regions of inflation.

where we have used \( g_* = 228.75 \) (all the degrees of freedom in MSSM). Since the temperature of the universe is so high, the lightest supersymmetric particle (LSP) relic density is then given by the standard (thermal) freeze-out mechanism [166]. In particular, if the neutralino is the LSP, then its relic density is determined by its annihilation and coannihilation rates. The advantage of realizing inflation in the visible sector is that it is possible to nail down the thermal history of the universe precisely \(^8\).

For low scale models of inflation, i.e. \( m_\phi \gg H \), the tensor modes are utterly negligible. For \( m_\phi \sim 1 \text{TeV} \), and \( \phi_0 \sim 3 \times 10^{14} \text{GeV} \), the value of \( H_* \sim 10^{-1} \text{GeV} \), see Eq.(2.4.78). The estimation of the reheat temperature is given by the equality of the above Eq. (2.4.91). The reheat temperature is typically \( 3 \times 10^8 \text{GeV} \) for the above parameters. Note that for \( m_\phi \gg H \), the tensor to scalar ratio \( r \) does not scale with the reheat temperature.

\(^8\)At temperatures below 100 GeV there will be no extra degrees of freedom in the thermal bath except that of the SM, therefore BBN can proceed without any trouble.
Note that saturating the upper-bound on $r \sim 0.12$ would yield a large reheating temperature of the universe. It is sufficiently large to create gravitino from a thermal bath. The gravitino production from the direct decay of the inflaton will be suppressed. In this case, the gravitino abundance is compatible with the BBN bounds, provided the gravitino mass, $m_{3/2} \geq O(10)$ TeV, see [167]. The bound holds only for a decaying gravitino, for which the graviton will decay before the BBN. If gravitino happens to be the LSP, then such a high scale model of inflation with large Hubble-induced corrections will be ruled out, unless there is some late entropy injection or there are some kinematical reasons for which the gravitino production is highly suppressed.

The Planck constraint implies that the tensor-to-scalar ratio, $r$, at the pivot scale $k = k_*$, corresponds to an upper bound on the energy scale of the Hubble induced inflection point inflation $^9$:

$$V_* \leq (1.96 \times 10^{16}\text{GeV})^4 \frac{r_*}{0.12} \Rightarrow H_* \leq 9.241 \times 10^{13} \sqrt{\frac{r_*}{0.12}} \text{ GeV}. \quad (2.4.92)$$

Using this input we scan the model parameters for obtaining large tensor to scalar ratio, $r$, for the following values:

$$c_H \sim O(1 - 10), \; a_H \sim O(10 - 100), \; \lambda \sim O(1), \; \phi_0 \sim O((1 - 3) \times 10^{16}\text{GeV}). \quad (2.4.93)$$

Now including the higher order corrections to the slow-roll parameters, the inflationary observables are estimated from our model as following:

$$2.092 < 10^0 P_S < 2.297, \quad 0.958 < n_S < 0.963, \quad r < 0.12, \quad -0.0098 < \alpha_S < 0.0003, \quad -0.0007 < \kappa_S < 0.006 \quad (2.4.94)$$

$^9$Here in eqn (2.4.91) and eqn (5.2.17) the equalities hold good in high scale inflationary regime ($H \gg m_\phi$). The inequalities are more significant once we enter low scale inflationary ($m_\phi \gg H$) region.
which confronts the Planck+WMAP-9+BAO data set, well within 2σ CL. Furthermore, we consider the following values of the model parameters which match the TT-spectrum of the CMB data for high scale model of inflation, i.e. $H \gg m_{\phi}$,

$$
\delta \sim 10^{-4}, \quad \lambda = 1, \quad c_H = 2, \quad a_H = 12.650, \quad \phi_0 = 1.129 \times 10^{16} \text{ GeV}. \quad (2.4.95)
$$

In principle, we can vary $H_*$ from high scales to low scales. Since in our case the advantage is that the thermal history is well established, we can trace the relevant number of e-foldings, by varying $10^{-1} \text{ GeV} < H_* < 9.241 \times 10^{13} \text{ GeV}$ and consequently we can probe tensor-to-scalar ratio for a wide range: $10^{-29} \leq r_* \leq 0.12$.

Using Eq. (2.4.95), in Fig. (2.8) we have shown the behavior of the amplitude of the the power spectrum, $P_S$ with respect to spectral tilt, $n_S$ at the pivot scale $k_* = 0.002 \text{ Mpc}^{-1}$ by a red curve. If we take care of the full parameter space, see Eq. (5.5.64), there are solutions which have been shown in a black shaded region. Furthermore, by using Planck+WMAP-9 and Planck+WMAP-9+BAO datasets with $\Lambda$CDM background along with different combined constraints, we have shown the status of inflection point inflationary model in the marginalized $1\sigma$ and $2\sigma$ CL. contours in Fig. (2.9). The allowed region from the model is explicitly shown by the green shaded region bounded by orange vertical lines parallel to $r$-axis. Along the vertical lines the number of e-foldings varies within $50 < N_* < 70$ (from left to right) for various ranges of $H_*$ as mentioned earlier. We have also shown a thick black line parallel to $n_S$ axis in Fig. (2.9) which discriminates between the low scale ($m_{\phi} \geq H$) and high scale ($H \gg m_{\phi}$) inflationary scenarios. Additionally, we have depicted various straight lines for the intermediate values of $H_*$ within the allowed region. The model also provides very mild running, $\alpha_S$, and running of running, $\kappa_S$, which is also shown in the marginalized $1\sigma$ and $2\sigma$ CL. contours in Fig. (2.10), where we have used Planck+WMAP-9 and Planck+WMAP-9+BAO datasets with $\Lambda$CDM background along with different combined observational constraints.

In this section we study the TT-angular power spectrum for the CMB anisotropy. For our present setup at low $\ell$ region ($2 < l < 49$) the contributions from the running ($\alpha_S, \alpha_T$), and running of
running $(\kappa_S, \kappa_T)$ are very small. Consequently their additional contribution to the power spectrum for scalar and tensor modes becomes unity ($\sim O(1)$) (this is consistent with the initial condition at the pivot scale $k = k_*$) and the original power spectrum becomes unchanged. As a result the proposed model will be well fitted with the Planck low-$l$ data within high cosmic variance except for a few outliers. On the other hand, when we move towards high $\ell$ regime ($50 < l < 2500$) the contribution of running and running of running become stronger and this will enhance the power spectrum to a permissible value such that it will accurately fit Planck high-$l$ data within very small cosmic variance. In this way one can easily survey over all the multipoles starting from low-$l$ to high-$l$ using the same parameterizations as mentioned in Eqs. (2.4.95). From Figs. (2.11), we see that the Sachs-Wolfe plateau obtained from our model is non flat, confirming the appearance of running, and running of the running in the spectrum observed for low $l$ region ($l < 50$). For larger value of the multipole ($50 < l < 2500$), CMB anisotropy spectrum is dominated by the Baryon Acoustic Oscillations (BAO), giving rise to several ups and downs in the spectrum, see [168]. Note that high $l$ regions of our model are well fitted within the small cosmic variance observed by Planck. In the low $l$ region due to the presence of very large cosmic variance there may be other pre-inflationary scenarios which might be able to fit the TT-power spectrum better.

2.5 Chapter summary

In this chapter by implementing the saddle point and inflection point mechanism we have proposed two different models of inflation in the framework of MSSM constructed from various D-flat directions. The major outcomes of our study are:

- We have demonstrated how we can construct the effective inflationary potential in the vicinity of the saddle point and inflection point starting from $n = 4$ and $n = 6$ level superpotential for the D-flat direction content $\text{QQQL}$, $\text{QuQd}$, $\text{QuLe}$, $\text{uude}$ and $\text{udd}$, $\text{LLe}$ respectively.

- The effective inflaton potential around saddle point and inflection point, has then been utilized in estimating the observable parameters and confronting them with WMAP7 and Planck dataset using the publicly available code CAMB [155], which reveals consistency of our model with latest observations.

- We have then explored the possibility of Primordial Black Hole formation from the running-mass model by estimating the mass of PBH from the model derived from $n = 4$ flat directions.

- Subsequently, we have engaged ourselves in finding out the effective parameter space and the coupling constants appearing in the saddle point analysis for the MSSM inflation by exactly solving the one loop RGE in this context.

- In case of inflection-point inflation we have taken potentials from two situations where the SUGRA corrections are important and as well as negligible. In the former case, we yield significantly large tensor-to-scalar ratio, $r \leq 0.12$, for $H_* \sim 9.24 \times 10^{13}$ GeV and the VEV $\phi_0 = 1.12 \times 10^{16}$ GeV. The model tends to predict a perfect match for the spectral tilt even for a small tensor-to-scalar ratio, $r$ for the situation where SUGRA corrections are negligible.

- The inflection point model fits the amplitude of the power spectrum and the spectral tilt. The model predicts mild running and running of the running of the spectral tilt well within the $2\sigma$ uncertainties.
2. MSSM inflation from various flat directions

- In particularly, the high scale inflection-point inflation model fits the high \( l \) multipoles of the Planck data quite well with the \( \Lambda \)CDM parameters. Inflection point technique plays another crucial role in fitting Planck data for low \( l \) in CMB. In CMB TT spectra the low \( l \) multipoles have high uncertainties and they are within the cosmic variance. The forthcoming polarization data from Planck will hopefully further constrain the inflection-point model of inflation.

- The perturbations created from the slow roll evolution of the inflaton are Gaussian and adiabatic. The amplitude and the spectral tilt match very well with the Planck data. One of the advantages of the proposed model is that it is embedded fully within MSSM, and therefore, it predicts the right thermal history of the universe with no extra relativistic degrees of freedom other than that of the Standard Model.
Inflation from background supergravity

3.1 Introduction

Investigations for the crucial role of Supergravity (SUGRA) in explaining cosmological inflation date back to early eighties of the last century. Inflation can be caused by the potential energy of a scalar field. Such a potential must be relatively flat in order to guarantee long duration of inflation and small deviation of scale invariance of primordial density fluctuations. However, the flatness of the scalar potential can be easily destroyed by radiative corrections. Supersymmetry (SUSY) is one of the leading theoretical proposals to protect a scalar field from radiative corrections, which also gives an attractive solution to the hierarchy problem of the standard model (SM) of particle physics as well as the unification of the strong and electro-weak gauge couplings. In particular, its local version, SUGRA, would govern the dynamics of the early Universe. However, in fact, it is a non-trivial task to incorporate inflation in SUGRA. This is primarily due to the well-known $\eta$-problem of SUGRA inflation, which appears in the F-term inflation due to the fact that the energy scale of F-term inflation is induced by all the couplings via vacuum energy density. Precisely, in the expression of F-term inflationary potential a factor $\exp(K/M_p)$ appears, leading to the second slow roll parameter $\eta \gg 1$, thereby violating an essential condition for slow roll inflation. The usual wayout is to impose additional symmetry to the framework. One such symmetry is Nambu-Goldstone shift symmetry [169, 170] under which Kähler metric becomes diagonal which serves the purpose of canonical normalization and stabilization of the volume of the compactified space. Consequently, the imaginary part of the scalar field gives a flat direction leading to a successful model of inflation. An alternative approach is to apply non-compact Heisenberg group transformations of two or more complex scalar fields where one can exploit Heisenberg symmetry [171] to solve $\eta$-problem. In addition, it prevents a scalar field from acquiring a value larger than $M_p$. This fact implies that it is almost impossible to realize large field inflation like chaotic inflation in SUGRA.

Of late the idea of braneworlds came forward [70, 71] as elaborately discussed in chapter 1. From cosmological point of view the most appealing feature of brane cosmology is that the 4 dimensional Friedmann equations are to some extent different from the standard ones due to the non-trivial embedding in the $S^1/Z_2$ orbifold [9]. This opens up new perspectives to look at the nature in general and cosmology in specific. Brane inflation in the above framework has also been studied to some extent in Refs. [172, 173]. In the same vein, we construct the brane inflaton potential of our consideration starting from 5D SUGRA. In brane inflation the modified Friedman equations lead to a modified version of the slow roll parameters [9]. So, by construction, $\eta$-problem is smoothened

There are some other approaches as well which are more appealing in dealing with fundamental aspects such as possible realization in string theory can be found in Refs. [174, 175]. For example, an apparent conflict between self-tuning mechanism and volume stabilization has been shown in [176], subsequently, this problem has been resolved in [177] where the credentials of the dilatonic field in providing a natural explanation for dark energy by an effective scalar field on the brane has been demonstrated using self-tuning mechanism in six dimensional bulk.
to some extent by modification of Friedmann equations on the brane [178]. In a sense, this is a parallel approach to the usual string inflationary framework where η-problem is resolved by fine-tuning [179]. As it will appear, there is still some fine-tuning required in brane inflation, which arises via a new avatar of five-dimensional Planck mass but it is softened to some extent due to the modified Friedmann equations.

On the other hand, in higher-dimensional setups as in the case of DGP model [85] where self-acceleration is sourced by a scalar field, Infra-Red (IR) modification of gravity [180] play a crucial role. Despite its profound success it has got some serious limitations [93], which are resolved by introducing a dynamical field, aka, Galileon [181] arising on the brane from the bulk in the DGP setup. Very recently, a natural extension to the scenario has been brought forward by tagging Galileon with the good old DBI model [68, 184], resulting in “DBI Galileon” [117], that has reflected a rich structure from four dimensional cosmological point of view. However, in most of the physical situations, this type of effective gravity theories are plagued with additional degrees of freedom which often results in unwanted debris like ghosts, Laplacian instabilities etc [185]. In the second portion of this chapter we introduce a single scalar field model described by the D3 DBI Galileon originated from D4-¯D4 brane anti-brane setup in the background of 5D SUGRA. This prevents the framework from having extra degrees of freedom as well as Ostrogradski instabilities [186]. Nevertheless, a consistent field theoretic derivation of the effective potential commonly used in the context of DBI Galileon cosmology has not been brought forth so far. On top of that, it is imperative to point out that the SUGRA origin of D3 DBI Galileon is yet to be addressed. In this chapter we plan to address both of these issues explicitly by deriving the inflaton potential from our proposed framework of DBI Galileon. Moreover, in general appearance of non-vanishing frame functions in the 4D action expedites breakdown of shift symmetry. Without shift symmetry, it may happen that the theory is unstable against large renormalization. The background action chosen in our model preserves shift symmetry of the single scalar field which gives it a firm footing from phenomenological point of view as well.

The plan of this chapter is as follows. First we propose a fairly general framework in the background of bulk $\mathcal{N}=2, D=5$ SUGRA by taking the Randall Sundrum braneworld scenario including Einstein’s Hilbert term in the gravity sector and the full DBI action in D4 brane including the quadratic modification in Einstein’s Hilbert action via Gauss-Bonnet correction in the bulk. Hence, using dimensional reduction technique, we derive the effective action for brane inflation and DBI Galileon in D3 brane induced by the quadratic correction in the geometry sector in the background of $\mathcal{N}=1, D=4$ SUGRA and study cosmological inflationary scenario therefrom. We next engage ourselves to calculate the primordial power spectrum of the scalar and tensor modes, their running and other observable parameters for both the frameworks. We further confront our proposed models with observations by using the publicly available code CAMB [155], and find them to fit well with observational data from WMAP7 [22].

### 3.2 Brane inflation

#### 3.2.1 The background model in $\mathcal{N} = 2, D = 5$ supergravity

For systematic development of the formalism, let us demonstrate briefly how one can construct the effective 4D inflationary potential of our consideration starting from $\mathcal{N} = 2, D = 5$ SUGRA in the

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$^{2}$The cosmological consequences of the Galileon models have been studied in Ref. [182, 183].
bulk which leads to an effective $\mathcal{N} = 1, \mathcal{D} = 4$ SUGRA in the brane. As mentioned, we consider the bulk to be five dimensional where the fifth dimension is compactified on the orbifold $S^1/Z_2$ of comoving radius $R$. The system is described by the following action [187]:

$$S = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_5} \left[ M_5^3 \left( R(5) - 2\Lambda_5 \right) + \mathcal{L}_{\text{bulk}} + \sum_i \delta(y - y_i) \mathcal{L}_{i4} \right]$$  \hspace{1cm} (3.2.1)$$

where $M_5$ be the 5D quantum gravity scale, $\Lambda_5$ be the 5D bulk cosmological constant and $\mathcal{L}_{\text{bulk}}$ contains bulk field contents. Also the sum includes the walls at the orbifold points $y_i = (0, \pi R)$ and 5-dimensional coordinates $x^m = (x^a, y)$, where $y$ parameterizes the extra dimension compactified on the closed interval $[-\pi R, +\pi R]$ and $Z_2$ symmetry is imposed. The metric in $\mathcal{D} = 5$ in conformal form is given by,

$$ds^2_5 = g_{mn} dx^m dx^n = e^{2A(y)} \left( ds^2_4 + R^2 \beta^2 dy^2 \right),$$  \hspace{1cm} (3.2.2)$$

where the $\mathcal{D} = 4$ metric $ds^2_4 = g^{\alpha\beta} dx_\alpha dx_\beta$ is the well known FLRW metric and $g_{5} = \text{det}(g_{mn})$. The numerical constant $\beta$ has been introduced just for convenience and physically determines the slope of the warp factor $e^{2A(y)}$. Also the product $R^2\beta$ stands for compact dimension which will stabilize the modulus fields appearing in the present context. Further solving $\mathcal{D} = 5$ Einstein Equations the warp factor can be expressed as:

$$e^{2A(y)} = \frac{b_0^2}{R^2 \left( e^{\beta y} + \frac{\Lambda_5 b_0^4}{24 R^2} e^{-\beta y} \right)},$$  \hspace{1cm} (3.2.3)$$

where $b_0$ is a constant having dimension of length.

For $\mathcal{N} = 2, \mathcal{D} = 5$ SUGRA in the bulk Eq (3.2.1) can be written as [188]:

$$S = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_5} \left[ M_5^3 \left( R(5) - 2\Lambda_5 \right) + \mathcal{L}^{(5)}_{\text{SUGRA}} + \sum_i \delta(y - y_i) \mathcal{L}_{i4} \right],$$  \hspace{1cm} (3.2.4)$$

which is a generalization of the scenario described in [187]. Written explicitly, the contribution from bulk SUGRA in the action is given by:

$$e^{\frac{1}{2}A} \mathcal{L}^{(5)}_{\text{SUGRA}} = -\frac{M_5^3 R^{(5)}}{2} + \frac{i}{2} \Psi_{\tilde{m}q} \nabla_{\tilde{q}} \Psi_{\tilde{m}} - S_{IJJK} F_{\tilde{m}nk} F^{\tilde{m}nk} - \frac{g_{\mu\nu}}{2} (D_{\tilde{m}} \phi^\mu)(D_{\tilde{m}} \phi^\nu) + \text{Fermionic + Chern – Simons},$$  \hspace{1cm} (3.2.5)$$

Including the contribution from the radion fields:

$$\chi = -\psi_5^2, \quad T = \frac{1}{\sqrt{2}} \left( \epsilon_5^2 - i \sqrt{\frac{2}{3}} A_5^0 \right)$$  \hspace{1cm} (3.2.6)$$

the effective brane SUGRA counterpart turns out to be \(^3\)

$$\delta(y) L_4 = -e^{\frac{3}{2}} \delta(y) \left[ (\partial_\alpha \phi)^\dagger (\partial^\alpha \phi) + i \bar{\sigma}^a D_\alpha \chi \right].$$  \hspace{1cm} (3.2.7)$$

\(^3\)In this context $\Delta(y) = e^{\frac{3}{2}} \delta(y)$ is the modified Dirac delta function which satisfies the normalization conditions:

$$\int_{-\pi R}^{+\pi R} dy \ e^3 \Delta(y) = 1, \quad \int_{-\pi R}^{+\pi R} dy \ e^3 = \mathcal{V}_5,$$

where $\mathcal{V}_5$ is the 5 dimensional volume.
The Chern-Simons terms can be gauged away assuming cubic constraints and $Z_2$ symmetry. It is useful to define the five dimensional generalized Kähler function ($G$) in this context as:

$$G = -3 \ln \left( \frac{T + T^\dagger}{\sqrt{2}} \right) + \delta(y) \frac{\sqrt{2}}{T + T^\dagger} K(\phi, \phi^\dagger),$$

(3.2.8)

which precisely represents interaction of the radion with gauge fields. Including the kinetic term of the five dimensional field $\phi$ the singular terms measured from the modified Dirac delta function can be rearranged into a perfect square thereby leading to the following expression for the action

$$S \supset \frac{1}{2} \int d^4 x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_5} \epsilon_5(4) e_5^5 \left[ g^{\alpha\beta} G^n_m(\partial_\alpha \phi^{m})^\dagger(\partial_\beta \phi^{n}) + \frac{1}{g_{55}} \left( \partial_5 \phi - \sqrt{H(G)} \Delta(y) \right)^2 \right],$$

(3.2.9)

where the bulk F-term potential in terms of generating function can be written as:

$$H(G) = \exp \left( \frac{G}{M_p^5} \right) \left[ \left( \frac{\partial W}{\partial \phi_m} + \frac{\partial G}{\partial \phi_m} W \right)^\dagger (G_m^n)^{-1} \left( \frac{\partial W}{\partial \phi^n} + \frac{\partial G}{\partial \phi^n} W M_p^5 \right) - \frac{3}{4} |W|^2 \right].$$

(3.2.10)

In this context we introduce the 4D Planck scale ($M_p$), which can be expressed in terms of the 5D scale ($M_5$) as:

$$M_p = \sqrt{\frac{e_4}{\lambda_0}} = \sqrt{\frac{6\epsilon(5)}{4\pi\lambda}} M_5^3$$

(3.2.11)

where $\lambda = \frac{\Lambda_5^5}{2M_5^5}$. Further, imposing $Z_2$ symmetry to $\phi$ via $\phi(0) = \phi(\pi R) = 0$ and compactifying around a circle ($S^1$) by imposing the constraint condition, $\partial_5 \phi = \sqrt{H(G)} \Delta(y) - \frac{1}{2\pi R}$ we get,

$$S = \frac{1}{2} \int d^4 x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_5} M_5^3 \left( R(5) - 2\Lambda_5 \right) + \epsilon_5(4) e_5^5 \left\{ g^{\alpha\beta} G^n_m(\partial_\alpha \phi^{m})^\dagger(\partial_\beta \phi^{n}) - \frac{g_{55}}{4\pi^2 R^2} H(G) \right\}.$$  

(3.2.12)

Now to trace out all the significant contribution from the fifth dimension using dimensional reduction technique here we use method of separation of variable $\phi^m = \phi(x^\mu, y) = \phi(x^\mu) \chi(y)$ which leads to,

$$S = \frac{M_p^2}{2} \int d^4 x \sqrt{-g_4} \left[ R(4) - P \int_{-\pi R}^{+\pi R} dy \frac{4(3\epsilon_4^{\alpha\beta} y + 3\lambda^2 e^{-2\beta y} - 2\lambda)}{R^2(e^{\beta y} + \lambda e^{-\beta y})^5} \right.$$

$$\left. + \left( \frac{\partial^2 K}{\partial \phi_\alpha \partial \phi^\alpha} \right)(\partial_\beta \phi^\alpha)^\dagger(\partial^\alpha \phi_\beta) - QV_F \right].$$

(3.2.13)

where we define:

$$P = \frac{2M_5^3 \beta_0^5}{M_p^2 R^5}, \quad Q = \frac{C(T, T^\dagger)}{4\pi^2 R^2}.$$  

(3.2.14)

Here $C(T, T^\dagger)$ represents an arbitrary function of stabilized modulus $T$ and $T^\dagger$. Eqn(6.0.32) explicitly shows that the theory is reduced to an effective $N = 1, D = 4$ SUGRA theory. For a general physical situation of $N = 1, D = 4$ SUGRA in the brane where the F-term potential on the brane defined earlier is modified as $[4]$:

$$V_F = \exp \left( \frac{K(\phi, \phi^\dagger)}{M_p^2} \right) \left[ \left( \frac{\partial W}{\partial \phi_\alpha} + \frac{\partial K}{\partial \phi_\alpha} W \right)^\dagger \left( \frac{\partial^2 K}{\partial \phi_\alpha \partial \phi^\alpha} \right)^{-1} \left( \frac{\partial W}{\partial \phi^\beta} + \frac{\partial K}{\partial \phi^\beta} W \right) - \frac{3}{4} |W|^2 \right].$$

(3.2.15)
Here $\Psi^{\alpha}$ is the chiral superfield and $\phi^{\alpha}$ be the complex scalar field. From now on the inflaton field $\phi$ appears to be 4-dimensional as demonstrated earlier. Finally, in the canonical basis Eq. (6.0.32) takes the following form:

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g_4} \left[ R_{(4)} - P \int_{-\pi R}^{+\pi R} dy 4(3e^{2\beta y} + 3\lambda e^{-2\beta y} - 2\lambda) \right] \left( \phi, \partial_4 \phi \right) - Q V_F \right) ,$$

where the F-term potential can be recast as $[189]$:

$$V = V_F = \exp \left[ \frac{1}{M_p^2} \sum _{\alpha} \phi_\alpha^d \phi^\alpha \right] \left[ \sum _{\beta} \left| \frac{\partial W}{\partial \phi_\beta} \right|^2 - 3 \frac{|W|^2}{M_p^4} \right] .$$

Next we expand the slowly varying inflaton potential derived from F-term around the value of the inflaton field where the quantum fluctuation is governed by, $\phi \to \tilde{\phi} + \phi$, ($\tilde{\phi}$ being the value of the inflaton field where structure formation occurs). Further we impose $Z_2$ to remove all odd order terms responsible for gravitational instabilities. Finally, the required renormalizable inflaton potential turns out to be:

$$V(\phi) = \Delta^4 \sum _{m=0} ^2 C_{2m} \left( \frac{\phi}{M_p} \right) ^{2m} ,$$

with an additional constraint on the tree-level constant $C_0 = 1$. The mass term decides the steepness of the potential. Absence of this term indicates that process is slow which is compensated by brane tension in the braneworld scenario. For the phenomenological purpose this specific choice is completely viable. Now translating the momentum integral within a specified UV cut-off ($\Lambda$) the effective potential turns out to be:

$$V(\phi) = \Delta^4 + \frac{9}{4!} \phi^4 + \frac{g^2 \phi^4}{(16\pi)^2} \left[ \ln \left( \frac{\phi^2}{\Lambda^2} \right) - \frac{25}{6} \right] + O(\lambda^3) ,$$

where the coupling constant $g = \frac{24\Lambda^4 C_4}{M_4^4}$ which is, in general at the scale $M$, defined as:

$$g(M) = \frac{dV(\phi)}{d\phi^4} \bigg| _{\phi=M} = g + \frac{6g^2}{(8\pi)^2} \ln \left( \frac{M^2}{\Lambda^2} \right) + O(g^3) ,$$

so that the general expression for the effective potential in terms of all finite physical parameters is given by:

$$V(\phi) = \Delta^4 + g(M) \phi^4 + \frac{g^2(M) \phi^4}{(16\pi)^2} \left[ \ln \left( \frac{\phi^2}{M^2} \right) - \frac{25}{6} \right] + O(g^3(M)) .$$

which is the Coleman Weinberg potential $[190]$. After substituting the expression for $g$ in terms of $C_4$ the one loop corrected potential can be expressed at the mass scale $M \sim M_p$ as:

$$V(\phi) = \Delta^4 \left[ 1 + \left\{ D_4 + K_4 \ln \left( \frac{\phi}{M} \right) \right\} \left( \frac{\phi}{M} \right) ^4 \right] ,$$

where the one-loop co-efficients are given by: $K_4 = \frac{9\Delta^4 C_4^2}{2\pi^2 M_4^4}$, $D_4 = C_4 - \frac{25K_4}{12}$.

$^4$In this context we assume that the Kähler potential is dominated by the leading order term in canonical basis of the series representation i.e. $K = \sum \phi_\alpha^\dagger \phi^\alpha$. The superpotential in Eq (3.2.15) is given by $W = \sum_{n=0} ^\infty D_n W_n (\phi^\alpha)$ with the constraint $D_0 = 1$. Here $W_n (\phi^\alpha)$ is a holomorphic function of $\phi^\alpha$ in the complex plane.

$^5$In Eq (3.2.16) the second term plays the role of effective cosmological constant in 4D. For further discussion we absorb this contribution to the scale of the effective potential $\Delta$.

$^6$In this analysis we assume that the $U(1)$ gauge interaction is absent, which implies $V_D = 0$. 

3.2 Brane inflation
3. Inflation from background supergravity

Figure 3.1: Variation of (a) one loop corrected potential \( V(\phi) \) vs inflaton field \( \phi \) \([188]\), (b) \(|\eta_V|\) vs inflaton field \( \phi \) for \( C_4 = -0.68 \) \([188]\) and (c) number of e-foldings \( N \) vs inflation field \( \phi \) within the range \(-0.70 < D_4 < -0.60\) \([188]\).

Fig. (3.1(a)) represents the inflaton potential for different values of \( C_4, D_4 \) and \( K_4 \). From the observational constraints the best fit model is given by the range \(-0.70 < D_4 < -0.60\) so that while doing numerical analysis we shall restrict ourselves to this range of \( D_4 \). In what follows our primary intention will be to engage ourselves in modeling brane inflation and to search for its pros and cons with the derived effective potential.

### 3.2.2 Modeling brane inflation in \( \mathcal{N} = 1, \mathcal{D} = 4 \) supergravity

The most appealing feature of brane cosmology is that the 4D Friedmann equations are to some extent different from the standard ones due to the non-trivial embedding in the \( S^1/Z_2 \) manifold \([9]\). At high energy regime one can neglect the contribution from Weyl term and consequently, the brane Friedmann equations within slow-roll regime are given by:

\[
H^2 = \frac{V(\phi)}{3M^2} \left( 1 + \frac{V(\phi)}{2\lambda} \right),
\]

\[
\dot{H} + H^2 = \frac{V(\phi)}{3M^2} \left( 1 + \frac{V(\phi)}{2\lambda} \right).
\]

In the high energy regime the contribution of energy density of the scalar field is significant compared to the brane tension i.e. \( \rho \sim V(\phi) >> \lambda \) within slow-roll. Consequently the Friedmann Eqn can be modified as, \( H = V(\phi)/\sqrt{6\lambda M} \). On the other hand, in the GR limit the energy density...
of the inflaton field is suppressed compared to the brane tension i.e. \( \rho \sim V(\phi) < < \lambda \) and we get the usual Friedmann Eqn, \( H = \sqrt{V(\phi)/\sqrt{3}M} \). In the braneworld scenario the modified Friedmann equations, along with the Klein Gordon equation, lead to new slow roll conditions and new expressions for observable parameters as well. Incorporating the potential of our consideration from Eq (3.2.22) the slow roll parameters turn out to be 7:

\[
\epsilon_V = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \frac{1 + \frac{V}{2\lambda}}{(1 + \frac{V}{2\lambda})^2} = \frac{U^2(\phi)T(\phi)}{2S^2(\phi)L^2(\phi)} \left( \frac{\phi}{M} \right)^6, \tag{3.2.26}
\]

\[
\eta_V = \frac{M_p^2}{2} \left( \frac{V''}{V} \right) \frac{1}{(1 + \frac{V}{2\lambda})} = \frac{E(\phi)}{S(\phi)L(\phi)} \left( \frac{\phi}{M} \right)^2, \tag{3.2.27}
\]

\[
\xi_V = \frac{M_p^4}{V^2} \left( \frac{V'}{V} \right)^2 \frac{1}{(1 + \frac{V}{2\lambda})^2} = \frac{U(\phi)F(\phi)}{S^2(\phi)L^2(\phi)} \left( \frac{\phi}{M} \right)^4, \tag{3.2.28}
\]

\[
\sigma_V = \frac{M_p^6}{V^3} \left( \frac{V'}{V} \right)^2 \frac{1}{(1 + \frac{V}{2\lambda})^3} = \frac{U^2(\phi)J(\phi)}{S^3(\phi)L^3(\phi)} \left( \frac{\phi}{M} \right)^6. \tag{3.2.29}
\]

Fig. (3.1(b)) depict how the slow roll parameter \( \eta_V \) vary with the inflaton field for the allowed range of \( D_4 \) and they give us a clear picture of the starting point as well as the end of the cosmic inflation. Nevertheless, it further reveals that the \( \eta \)-problem is smoothened to some extent in brane cosmology 8.

The number of e-foldings are defined in brane cosmology [9] for our model as:

\[
N \simeq \frac{1}{M_p^2} \int_{\phi_i}^{\phi_f} \left( \frac{V}{V'} \right) \left( 1 + \frac{V}{2\lambda} \right) d\phi \simeq \frac{M_p^2}{U} \left[ \frac{1}{2} \left( 1 + \frac{\alpha}{2} \right) \left( \frac{1}{\phi_f^2} - \frac{1}{\phi_i^2} \right) \right. \\
+ \left. \frac{D_4}{2M^4}(1 + \alpha)(\phi_f^2 - \phi_i^2) + \frac{\alpha D_4^2}{12M^6}(\phi_f^6 - \phi_i^6) \right]. \tag{3.2.30}
\]

Here \( \phi_i \) and \( \phi_f \) are the corresponding values of the inflaton field at the start and end of inflation. Fig. (3.1(c)) represents a graphical behavior of number of e-folding versus the inflaton field in the high energy limit for different values of \( D_4 \) and the most satisfactory point in this context is the number of e-folding lies within the observational window 56 < \( N \) < 70. The end of the inflation leads to the constraint, \( \alpha = \frac{2}{M}\sqrt{E} \beta^{3/2} \), which is required for numerical estimations.

Let us now engage ourselves in analyzing quantum fluctuation in our model and its observational imprints via primordial spectra generated from cosmological perturbation. In brane inflation the expressions for amplitude of the scalar perturbation, tensor perturbation and tensor to scalar ratio

\footnote{For convenience throughout the analysis we define the following functions:}

\[
L(\phi) = \left[ 1 + \frac{2}{3}S(\phi) \right], \quad T(\phi) = \left[ 1 + \alpha S(\phi) \right], \quad S(\phi) = \left[ 1 + \{D_4 + K_4 \ln \left( \frac{\phi}{M} \right) \} \left( \frac{\phi}{M} \right)^4 \right],
\]

\[
U(\phi) = \left[ (K_4 + 4D_4)/4K_4 \ln \left( \frac{\phi}{M} \right) \right], \quad E(\phi) = \left[ (7K_4 + 12D_4)/12K_4 \ln \left( \frac{\phi}{M} \right) \right],
\]

\[
F(\phi) = \left[ 26K_4 + 24D_4 \right] + 24K_4 \ln \left( \frac{\phi}{M} \right), \quad J(\phi) = \left[ 50K_4 + 24D_4 \right] + 24K_4 \ln \left( \frac{\phi}{M} \right),
\]

\[
P(\phi) = \sqrt{\left[ 1 + 2\alpha S(\phi)L(\phi) \right] - 2\alpha S(\phi)L(\phi) \sinh^{-1} \left[ 2\alpha S(\phi)L(\phi) \right]^{-1/2} \right] \left( \phi_f^6 - \phi_i^6 \right)}, \tag{3.2.25}
\]

with \( \alpha = \Delta^4/\lambda \).

\footnote{However, we are yet to figure out if there is any underlying dynamics that may lead to the solution of this generic feature of SUGRA.}
are given by \(^9\) \(^10\):

\[
\Delta_s^2 \simeq \frac{512\pi}{75M_p^2} \left[ \frac{V^3}{(V')^2} \left[ 1 + \frac{V}{2\lambda} \right]^3 \right]_{k=aH} = \frac{M^2\alpha\lambda S^3(\phi_*)L^3(\phi_*)}{75\pi^2 U^2(\phi_*)\phi_*^2}, \quad (3.2.31)
\]

\[
\Delta_l^2 \simeq \frac{32}{75M_p^4} \left[ \frac{V [1 + \frac{V}{2\lambda}]}{\sqrt{1 + \frac{2V}{\lambda} (1 + \frac{V}{2\lambda}) - \frac{2V}{\lambda} (1 + \frac{V}{2\lambda}) \sinh^{-1}\left[ \frac{1}{\sqrt{\frac{2V}{\lambda} (1 + \frac{V}{2\lambda})}} \right]}} \right]_{k=aH}
= \frac{\lambda\alpha}{150\pi^2 M^4} \frac{S(\phi_*)L(\phi_*)}{P(\phi_*)}, \quad (3.2.32)
\]

\[
r = \frac{16\Delta_s^2}{\Delta_l^2} \simeq \frac{8(\phi_*)^6 U^2(\phi_*)}{M^6 S^2(\phi_*) L^2(\phi_*) P(\phi_*)}. \quad (3.2.33)
\]

Finally, to estimate five dimensional Planck mass from the observational parameters we use the relation \(M \sim M_p = \frac{M_5}{\sqrt{\alpha}} \sqrt{\frac{3}{15}}\) and consequently from Eq (5.2.9) we get an analytical expression for 5D cut-off scale in terms of model parameters:

\[
M_5 = \sqrt{\frac{800\pi^4\Delta_s^2 U^2(\phi_*)}{\alpha S^3(\phi_*)L^3(\phi_*) \phi_*}}. \quad (3.2.34)
\]

\(^9\) In this context \(\phi_*\) represents the value of the inflaton field at the horizon crossing represented by \(k = aH\).

\(^10\) In Appendix B we have also mentioned a set of consistency relations applicable for brane inflation.
3.2.3 Parameter estimation

Direct numerical analysis

| $C_4 \approx D_4$ | $\alpha$ | $\lambda \times 10^{-14} M^4$ | $\phi_f / M$ | $\phi_i / M$ | $N$ | $\phi_s / M$ | $\Delta_s^2 / 10^{-9}$ | $n_s$ | $r / 10^{-5}$ | $\alpha_s / 10^{-3}$ | $M_5 / 10^{-3} M$ |
|------------------|---------|-----------------------------|--------------|--------------|-----|--------------|-------------------|--------|----------------|-------------------|-------------------|
| -0.70            | 17.389  | 2.553                       | 1.017        | 0.147        | 70  | 0.158        | 3.126             | 0.951  | 2.176          | -0.798            | 11.792            |
|                  |         |                             |              | 0.158        | 60  | 0.173        | 1.835             | 0.941  | 3.706          | -1.142            |                  |
|                  |         |                             |              | 0.164        | 56  | 0.180        | 1.440             | 0.936  | 4.723          | -1.345            |                  |
| -0.65            | 16.757  | 2.632                       | 1.036        | 0.150        | 70  | 0.161        | 2.902             | 0.951  | 2.176          | -0.798            | 11.865            |
|                  |         |                             |              | 0.161        | 60  | 0.176        | 1.704             | 0.941  | 3.706          | -1.142            |                  |
|                  |         |                             |              | 0.167        | 56  | 0.184        | 1.327             | 0.936  | 4.723          | -1.345            |                  |
| -0.60            | 16.099  | 2.758                       | 1.057        | 0.153        | 70  | 0.165        | 2.679             | 0.951  | 2.176          | -0.798            | 11.944            |
|                  |         |                             |              | 0.165        | 60  | 0.180        | 1.573             | 0.941  | 3.706          | -1.142            |                  |
|                  |         |                             |              | 0.170        | 56  | 0.187        | 1.234             | 0.936  | 4.723          | -1.345            |                  |

Table 3.1: Different observational parameters related to the cosmological perturbation for our proposed model in Eq (3.2.22) [188].

Table 3.1 represent numerical estimation for different observational parameters related to the cosmological perturbation as estimated from our model. It is worthwhile to point out to the salient features of estimated inflationary parameters obtained from our proposed model:

- The observable parameters help us have an estimation for the brane tension to be $\lambda \gg 1 \text{ MeV}^4$ provided energy scale of the inflation is in the vicinity of GUT scale. Also the 5D cut-off scale turns out to be $M_5 \sim (11.792 - 11.944) \times 10^{-3} M$.

- The amplitude of scalar power spectrum corresponding to different best fit values of $D_4$ is of the order of $5 \times 10^5$ and it perfectly matches with WMAP7 [22]. The scalar spectral index for lower values of $N \sim 55$ are pretty close to observational window $0.948 < n_s < 1$ [22] whereas for higher values of $N \sim 70$ this lies well within the window. Also for our model running of the scalar spectral index $\alpha_s \sim -10^{-3}$.

- Though the tensor to scalar ratio as estimated from our model is well within its upper bound fixed by WMAP7 [22] and Planck [24, 25], thereby facing no contradiction with observations, its value is even small to be detected.

Data analysis with CAMB

In this context we shall make use of the cosmological code CAMB [155] in order to confront our results directly with observation. To operate CAMB, the values of the initial parameters associated with inflation are taken from the Table 3.1 for $D_4 = -0.60$. Additionally WMAP7 dataset in ΛCDM background has been used in CAMB to obtain CMB angular power spectrum at the pivot scale $k_* = 0.002 \text{ Mpc}^{-1}$. Table 3.2 and table 3.3 shows input from the WMAP7 dataset and the output obtained from CAMB respectively.

The curvature perturbation is generated due to the fluctuations in the *inflaton* and at the end of inflation it makes horizon re-entry creating matter density fluctuations, which is the origin of the structure formation in Universe. In Fig. 3.2(a), Fig. 3.2(b) and Fig. 3.2(c) we confront CAMB output of CMB angular power spectrum $C^TT_l$, $C^TE_l$ and $C^{EE}_l$ for best fit with WMAP seven years...
Table 3.2: Input in CAMB [188].

| $H_0$ (km/sec/Mpc) | $\tau_{\text{Reion}}$ | $\Omega_b h^2$ | $\Omega_c h^2$ | $T_{\text{CMB}}$ (K) |
|------------------|----------------|---------------|---------------|------------------|
| 71.0             | 0.09           | 0.0226        | 0.1119        | 2.725            |

Table 3.3: Output from CAMB [188].

| $t_0$ (Gyr) | $z_{\text{Reion}}$ | $\Omega_m$ | $\Omega_\Lambda$ | $\eta_{\text{Rec}}$ (Mpc) | $\eta_0$ (Mpc) |
|-------------|----------------|-----------|----------------|-------------------|---------|
| 13.708      | 10.692         | 0.2669    | 0.7331         | 285.15            | 14347.5 |

data for the scalar mode. From Fig. 3.2(a) we see that the Sachs-Wolfe plateau obtained from our model is almost flat confirming a nearly scale invariant spectrum. For larger value of the multipole $l$, CMB anisotropy spectrum is dominated by the Baryon Acoustic Oscillations (BAO) giving rise to several ups and downs in the spectrum. Also the peak positions are sensitive on the dark energy and other forms of the matter. Also fig. 3.2(a) is in good agreement with WMAP7 data for $\Lambda$CDM background apart from the two outliers at $l \sim 21$ and $l \sim 42$.

3.3 DBI Galileon inflation

3.3.1 The background model in D4 brane

As discussed in introduction, now we will describe the features of DBI Galileon inflation in this section. Let us demonstrate briefly the construction of DBI Galileon starting from $N=2,D=5$ SUGRA along with Gauss Bonnet correction in D4 brane set up. The full five dimensional model is described by [191]:

$$S_{\text{Total}}^{(5)} = S_{EH}^{(5)} + S_{GB}^{(5)} + S_{D4\text{brane}}^{(5)} + S_{\text{BulkSugra}}^{(5)}$$

(3.3.35)

where

$$S_{EH}^{(5)} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left[ R^{(5)} - 2\Lambda_{(5)} \right],$$

(3.3.36)

$$S_{GB}^{(5)} = \frac{\alpha_{(5)}}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} \left[ R^{ABC(D(5)} R^{(5)}_{ABCD} - 4 R^{AB(5)} R^{(5)}_{AB} + R_5^{(5)} \right]$$

(3.3.37)

where $\alpha_{(5)}$ and $\kappa_{(5)}$ represent Gauss-Bonnet coupling and 5D gravitational coupling strength respectively. Additionally, $\Lambda_{(5)}$ and $g^{(5)}$ represent the 5D cosmological constant and the determinant of the 5D metric explicitly mentioned in equation(3.3.46). The D4 brane action decomposed into two parts as [191]:

$$S_{D4\text{brane}}^{(5)} = S_{DBI}^{(5)} + S_{WZ}^{(5)}$$

(3.3.38)
where the DBI action and the Wess-Zumino action are given by respectively [191]:

\[
S_{DBI}^{(5)} = -\frac{T_4}{2} \int d^5x \exp(-\Phi) \sqrt{- (\gamma^{(5)} + B^{(5)} + 2\alpha' F^{(5)})}, \tag{3.3.39}
\]

\[
S_{WZ}^{(5)} = -\frac{T_4}{2} \int \sum_{n=0,2,4} \tilde{C}_n \wedge \exp \left( \tilde{B}_2 + 2\alpha' F_2 \right) \big|_{4 \text{ form}} \tag{3.3.40}
\]

\[
= \frac{1}{2} \int d^5x \sqrt{-g^{(5)}} \left\{ \epsilon^{ABCD} \left[ \partial_A \Phi^I \partial_B \Phi^J \left( C_{IJ} B_{KL} \partial_C \Phi^K \partial_D \Phi^L + \frac{\pi \alpha'}{2} C_{IJ} F_{CD} \right) + \frac{C_0}{8 T_4} B_{IJ} B_{KL} \partial_C \Phi^K \partial_D \Phi^L + \frac{\pi \alpha' C_0}{2} B_{IJ} F_{CD} \right] + 2\pi^2 \alpha'^2 T_4 C_0 F_{AB} F_{CD} - T_4 \nu(\Phi) \right\}
\]

where \(T_4\) is the D4 brane tension, \(\alpha'\) is the Regge Slope, \(\exp(-\Phi)\) is the closed string dilaton and \(C_0\) is the Axion. Here and throughout the article that denotes a pull-back onto the D4 brane so that \(\gamma_{AB}\) is the 5D induced metric on the D4 brane explicitly defined in equation (3.3.48). Here \(\gamma^{(5)}\), \(B^{(5)}\) and \(F^{(5)}\) represent the determinant of the 5D induced metric \((\gamma_{AB})\) and the gauge fields \((B_{AB}, F_{AB})\) respectively. The gauge invariant combination of rank 2 field strength tensor, appearing in D4 brane, is \(F_{AB} = B_{AB} + 2\pi \alpha' F_{AB}\) and \(\{F_2, B_2\}\) represents 2-form U(1) gauge fields which have the only non-trivial components along compact direction. On the other hand \(C_4\) has components only along the non-compact space-time dimensions. In a general flux compactification all fluxes may be turned on as the Ramond-Ramond (RR) forms \(F_{n+1} = dC_n\) (along with their duals) with \(n = 0, 2, 4\) and the Neveu Schwarz-Neveu Schwarz (NS-NS) flux \(H_3 = dB_2\). Additionally the D4 brane frame function is defined as [191]:

\[
\nu(\Phi) = \left( \nu_0 + \frac{\nu_4}{\Phi^4} \right) \tag{3.3.41}
\]

which is originated from interaction between D4-\(\overline{D}4\) brane in string theory. Here \(\nu_0\) and \(\nu_4\) represent the constants characterizing the interaction strength between D4-\(\overline{D}4\) brane.

In Eq (3.3.35) \(N = 2, D = 5\) bulk SUGRA action is exactly similar as described in Eq (3.2.1) in 3.2.1. In this context the 5-dimensional coordinates \(X^A = (x^i, y)\), where \(y\) parameterizes the extra dimension compactified on the closed interval \([-\pi R, +\pi R]\) and \(Z_2\) symmetry is imposed. For computational purpose it is useful to define the five dimensional generating function \((G)\) of SUGRA in this setup as [191]:

\[
G = -3 \ln \left( \frac{T + T_4^1}{\sqrt{2}} \right) + K(\Phi, \Phi^\dagger), \tag{3.3.42}
\]

where the SUGRA Kähler moduli fields are given by \(T = \frac{(e^{\phi} - i \sqrt{2} \phi A_2)}{\sqrt{2}}\) which is assumed to be stabilized and \(K(\Phi, \Phi^\dagger)\) represents generalized Kähler function.

Including the kinetic term of the five dimensional field \(\Phi\) and rearranging into a perfect square, the 5D bulk SUGRA action can be expressed as

\[
S \supset \frac{1}{2} \int d^5x \int_{-\pi R}^{+\pi R} dy \sqrt{-g_{(4)}} e^\phi \left[ g^{\alpha \beta} G_M^N (\partial_5 \Phi^M)^\dagger (\partial_5 \Phi^N) + \frac{1}{g_{55}} \left( \partial_5 \Phi - \sqrt{V_{bulk}^{(5)}}(G) \right) \right]^2, \tag{3.3.43}
\]

where the 5D potential described by

\[
V_{bulk}^{(5)}(G) = \exp \left( \frac{G}{M^2} \right) \left[ (\partial W / \partial \Phi^M) + (\partial G / \partial \Phi^M) W \right]^\dagger (G_M^N)^{-1} \left( \partial W / \partial \Phi^N + (\partial G / \partial \Phi^N) W \right) - 3 \left( W^2 / M^2 \right) \right] \tag{3.3.44}
\]
where $W$ physically represents the superpotential in the context of $\mathcal{N} = 2, D = 5$ SUGRA theory and expressed in terms of the holomorphic combination of the fields $\Phi, \Phi^\dagger, T$ and $T^\dagger$. The field equations in presence of Gauss-Bonnet term can be expressed as [191]:

$$G^{(5)}_{AB} + \alpha^{(5)} H_{AB}^{(5)} = 8\pi G_{(5)}T_{AB}^{(5)} - \Lambda_{(5)} g_{AB},$$  \hspace{1cm} (3.3.45)

where $H_{AB}^{(5)}$ covariantly conserved Gauss-Bonnet tensor as defined in Eq (1.2.102). It is useful to introduce the 5D metric in conformal form:

$$ds^2_{4+1} = g_{AB}dX^AdX^B = \frac{1}{\sqrt{h(y)}} ds^2_4 + h(y)G(y)dy^2,$$  \hspace{1cm} (3.3.46)

with metric function:

$$\frac{1}{\sqrt{h(y)}} = \frac{R^2}{b_0^2\beta^2} \tilde{g}(y) = \frac{R^2}{\sqrt{\exp(\beta y) + \frac{\Lambda_{(5)} b_0^4}{24R^2} \exp(-\beta y)}}$$  \hspace{1cm} (3.3.47)

and $ds^2_4 = g_{\alpha\beta}dx^\alpha dx^\beta$ is FLRW counterpart. In order to write down explicitly the expression for D4 brane action, the induced metric can be shown as

$$\gamma_{CD} = \frac{1}{\sqrt{h(y)}} (g_{AB} + h(y)G_{AB}\partial_C\Phi^A\partial_D\Phi^B).$$  \hspace{1cm} (3.3.48)

Now using the scaling relations

$$\Phi^A = \sqrt{T_{(4)}} \tilde{\Phi}^A, \quad G_{AB} = \exp(-\Phi)g_{AB}, \quad b_{AB} = \frac{\sqrt{h(y)}}{T_{(4)}} B_{AB}$$  \hspace{1cm} (3.3.49)

the 5D action for D4 brane can be expressed in more convenient form as

$$S_{D4 \text{ brane}}^{(5)} = \int d^5x \sqrt{-g^{(5)}} \left[K(\Phi, X) - G(\Phi, X)\Box^{(5)}\right],$$  \hspace{1cm} (3.3.50)

where

$$K(\Phi, X) = -\frac{1}{2f(\Phi)} \left(\sqrt{D} - 1\right) - \frac{V_{\text{brane}}^{(5)}(\Phi)}{2}$$  \hspace{1cm} (3.3.51)

where the determinant can be expressed as

$$D \simeq 1 - 2f(\Phi)G_{AB}X^{AB} + 4f^2(\Phi)X_A^A X_B^B - 8f^3(\Phi)X_A^A X_B^B X_C^C + 16f^4(\Phi)X_A^A X_B^B X_C^C X_D^D$$  \hspace{1cm} (3.3.52)

which is expressed in terms of the kinetic term $X_B^B = \frac{1}{2} G_{DA} \partial^D \Phi^A \partial_C \Phi^B$. In this context the 5D D’Alembertian Operator is defined as, $\Box^{(5)} = \frac{1}{\sqrt{-g^{(5)}}} \partial_A \left(\sqrt{-g^{(5)}} g^{AC} \partial_C\right)$. Here we use the fact that no spatial direction along which the scalar fields are only time dependent lead to $B_\mu^\nu = 0$ and $F_{\mu\nu} = 0$ in the background. Consequently Maxwell’s field equations are unaffected in 4D after dimensional reduction. In this context the D4 brane potential is given by:

$$V_{\text{brane}}^{(5)}(\Phi) = T_{(4)} \nu(\Phi) + \frac{1}{f(\Phi)},$$  \hspace{1cm} (3.3.53)

where 5D warped geometry motivated $Z_2$ symmetric frame function

$$f(\Phi) = \frac{\exp(\Phi)h(y)}{T_{(4)}} \simeq \frac{1}{(f_0 + f_2\Phi^2 + f_4\Phi^4)}$$  \hspace{1cm} (3.3.54)

is originated from higher dimensional field theory and the implicit D4 brane function defined as:

$$G(\Phi, X) = \frac{g(\Phi)}{2(1 - 2f(\Phi)X)},$$  \hspace{1cm} (3.3.55)

with $g(\Phi) = g_0 + g_2\Phi^2$. Here $g_0$ and $g_2$ are model dependent constants characterizes the effects of possible interactions on the D4 brane.
3.3.2 Modeling DBI Galileon inflation in D3 brane

The technical details of the dimensional reduction technique are elaborately discussed in the Appendix C which can generate an effective D3 DBI Galileon theory in 4D. Summing up all the contributions from Eq (6.0.22,6.0.25,6.0.28,6.0.35), the model for D3 DBI Galileon is described by the following effective action \[ [191]:

\[
S = \int d^4x \sqrt{-g^{(4)}} \left[ \hat{K}(\phi, X) - \tilde{G}(\phi, X) \square^{(4)} \phi + \tilde{t}_4 R^{(4)} \right. \\
+ \tilde{t}_4 \left( C(1) R^{\alpha \beta \gamma \delta} R^{(4)}_{\alpha \beta \gamma \delta} - 4\pi(2) R^{\alpha \beta (4)} R^{(4)}_{\alpha \beta} + A(6) R^{2}_{(4)} \right) + \tilde{t}_3, \\
\]

where

\[
\hat{K}(\phi, X) = -\frac{\hat{D}}{\hat{f}(\phi)} \left[ \sqrt{1 - 2Q X f} - Q_1 \right] - \hat{C}_2 \hat{G}(\phi, X) - 2X \hat{M}(T, T^\dagger) - V(\phi), \\
\hat{M}(T, T^\dagger) = \frac{\hat{M}(T, T^\dagger)}{2\hat{v}(\phi)} - \frac{\hat{D}}{2\hat{v}(\phi)}, \\
\hat{G}(\phi, X) = \left( 2(1 - 2\gamma f(\phi, X^2)) \right), \\
\hat{g}(\phi) = \hat{g}_0 + \hat{g}_2 \phi^2, \\
\hat{f}(\phi) \simeq \frac{1}{(f_0 + f_2 \phi^2 + f_4 \phi^4)} \\
\tilde{t}_1 = \left\{ \frac{1}{\alpha(4)} \left[ 1 + \frac{\alpha(4)}{R_{\alpha \beta}^2} (24\hat{A}(2) - 24\hat{A}(9) - 16\hat{A}(10)) \right] - \frac{\alpha(4)}{R_{\alpha \beta}^2} \right\}, \\
\tilde{t}_3 = \frac{1}{\alpha(4)} \left[ 24\hat{C}(4) - 144\hat{C}(4) - 64\hat{A}(5) + 144\hat{A}(7) - 64\hat{A}(8) + 192\hat{A}(11) \right] - \frac{3M^2_{DBI} \phi^2}{2\kappa(4)^2 M^2_{DBI} R_{\alpha \beta}} \hat{T}(1), \\
\]

where \( \alpha(4), \tilde{t}_1, \tilde{t}_3, \tilde{t}_4 \) are effective 4D couplings and \( \kappa(4) \) is the gravitational coupling strength. Here \( X \) represents the 4D kinetic term after dimensional reduction given by \( X := -\frac{1}{2} g_{\mu \nu} \partial^\mu \phi \partial^\nu \phi \). In this context \( (T, T^\dagger) \) are the four dimensional background SUGRA moduli fields which are constant after dimensional reduction. The collective effect of Eq (6.0.30) and Eq (6.0.38) gives the total D3 DBI Galileon potential as \[ [191]:

\[
V(\phi) = Q_2 \hat{D}V_{brane}^{(4)} + \hat{Z}(T, T^\dagger) V_{bulk}^{(4)}(\phi) = \sum_{m=-2, m \neq -1}^{2} C_{2m} \phi^{2m}, \\
\]

where

\[
C_0 = \left( T_3 \hat{p}_0 + \beta R \hat{I}(2) \hat{f}_0 + \hat{Z}(T, T^\dagger) A(13) \right)^0, \\
C_{-4} = T_3 \hat{p}_4, \\
C_2 = \left( \beta R \hat{I}(2) \hat{f}_2 - g_{\mu \nu} \hat{Z}(T, T^\dagger) A(13) \right), \\
C_4 = \left( \beta R \hat{I}(2) \hat{f}_4 + \frac{1}{4} \hat{Z}(T, T^\dagger) A(13) \right)^2, \\
\]

are tree level constants. Now we want to see the effect of one-loop radiative correction to the derived potential. After doing proper analysis throughout it comes out that the one-loop correction does not affect the superpotential due to the cancellation of all tadpole terms appearing in the theory. On the other hand one-loop radiative correction in the Kähler potential results in \[ [191]:

\[
\delta K^{1-loop}(\phi, \phi^\dagger) = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left[ \frac{1}{2} Tr ln \hat{K}(\phi, \phi) \\
+ \frac{1}{2} Tr \left( \hat{K}(\phi, \phi) p^2 - \hat{W}(\phi, \phi^\dagger) \hat{K}(\phi, \phi^\dagger)^{-1} \hat{W}(\phi, \phi^\dagger) \right) \right] \\
= \frac{\Lambda^2_{UV}}{16\pi^2} \ln \left( det \left[ \hat{K}(\phi, \phi^\dagger) \right] \right) - \frac{1}{32\pi^2} Tr \left( \hat{M}_\phi^2 \left[ \frac{M^2_{DBI}}{\Lambda^2_{UV}} - 1 \right] \right), \\
\]

(3.3.60)
where $\Lambda_{UV} = M \sim M_p$ is used as a UV cut-off of the theory appearing in the context of cut-off regularization. In this connection the chiral mass matrix is given by

$$
\mathcal{M}^2_{\phi} = \mathcal{K}^{-\frac{1}{2}}(\phi, \phi^\dagger) \mathcal{W}^\dagger(\phi, \phi^\dagger) \left( \mathcal{K}(\phi, \phi^\dagger)^{-1} \right)^\dagger \mathcal{W}(\phi, \phi^\dagger) \mathcal{K}^{-\frac{1}{2}}(\phi, \phi^\dagger) .
$$

(3.3.61)

Now including the contribution from one-loop radiative correction both from brane and bulk SUGRA, the renormalizable Coleman Weinberg potential is as under\footnote{To compute the trace part here we use the supertrace identity, $STr(\mathcal{M}^n) \equiv \sum_i (-1)^{2j_i} (2j_i + 1) m_i^n$.} [191]:

$$
V(\phi) = V_{\text{tree}}(\phi) + \delta V_{1-loop}(\phi) = \sum_{m=-2, m\neq -1}^2 C_{2m} \phi_{2m} + \lim_{\epsilon \to 0} \sum_{n=-2, n \neq -1}^0 B_{2m} \left( \int_{p}^{A_{UV}=M} \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - 2C_2 + i\epsilon)^2} \right) \phi_{2n}^{2n} + \sum_{q=0}^2 \frac{\phi_{2q}^2}{64\pi^4} \left[ \Lambda_{UV}^2 STr(\mathcal{M}^0) \ln \left( \frac{\Lambda_{UV}^2}{\phi^2} \right) + 2\Lambda_{UV}^2 STr(\mathcal{M}^2) + STr \left( \mathcal{M}^4 \ln \left( \frac{\mathcal{M}^2}{\Lambda_{UV}^2} \right) \right) \right]
$$

(3.3.62)

One–loop correction in the bulk $\mathcal{N}=1$, $D=4$ SUGRA

where $D_0 = 0$, $D_{2m} = \frac{B_{2m} + 4C_{2m}}{C_{2m}}$. Here the 4D effective potential respect the Galilean symmetry: $\phi \to \phi + \bar{b}_\mu x^\mu + c$ which taskse both shift and spacetime translational symmetry.

Fig. (3.3(a)) represents the inflaton potential for different values of $C_{2m}$ and $D_{2m}$. From the observational constraints the best fit model is given by the range: $5.67 \times 10^{-11} M_p^4 < C_0 < 6 \times 10^{-11} M_p^4$, $1.01 \times 10^{-16} M_p^8 < C_{-4} < 2 \times 10^{-16} M_p^8$, $7.27 \times 10^{-10} M_p^2 < C_2 < 7.31 \times 10^{-10} M_p^2$, $2.01 \times 10^{-14} < C_4 < 2.45 \times 10^{-14}$, $0.014 < D_{-4} < 0.021$, $0.002 < D_2 < 0.012$ and $0.011 < D_4 < 0.019$ so that while doing numerical analysis, we shall restrict ourselves to this range.

Hence using Eq (3.3.56) the modified Friedmann and Klein-Gordon equations can be expressed as:

$$
H^4 = \frac{\Lambda_{(4)} + 8\pi G_{(4)} V(\phi)}{\bar{g}_1},
$$

(3.3.63)

$$
\dot{\phi}^2 \left( e_2(\phi) + 9e_3(\phi) H^2 \right) = \left\{ V'(\phi) + \tilde{C}_5 \tilde{g}'(\phi) k_1 - \frac{\bar{D} \tilde{f}(\phi)}{\tilde{f}(\phi)} (1 - Q_1) \right\},
$$

(3.3.64)

where $e_2(\phi) = \tilde{M}(T, T^\dagger) J_\phi + 2\tilde{g}(\phi) \tilde{f}(\phi) k_1 k_2 + 8 \tilde{f}(\phi) \tilde{g}(\phi) k_1 k_2^2 + 2 \tilde{g}(\phi) \tilde{f}(\phi) k_1 k_2 - g''(\phi) k_1$ and $e_3(\phi) = 2C_4 \bar{f}(\phi) \tilde{g}(\phi) k_1 k_2$ provided $|e_3(\phi)| \gg |e_2(\phi)|$ in the slow-roll regime. Here we have fixed the signature of $\dot{\phi}$ so that the scalar field rolls down the potential. Additionally ghost instabilities are avoided provided the coefficient of $\dot{\phi}^2 > 0$. Consequently the potential dependent slow-roll
parameters can be expressed as:

\[ \varepsilon_V : = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \frac{1}{\sqrt{G(\phi)V'(\phi)}} \]
(3.3.65)

\[ \eta_V : = M_p^2 \left( \frac{V''}{V} \right) \frac{1}{\sqrt{G(\phi)V(\phi)}} \]
(3.3.66)

\[ \xi_V : = M_p^4 \left( \frac{V'V'''}{V^2} \right) \frac{1}{G(\phi)V'(\phi)} \]
(3.3.67)

\[ \sigma_V : = M_p^6 \left( \frac{(V')^2V'''}{V^3} \right) \frac{1}{\left( G(\phi)V'(\phi) \right)^2} \]
(3.3.68)

where \( G(\phi) = \frac{4\epsilon_{\text{Gal}} M_p^2}{g_1 V(\phi)} \). In this connection Galileon terms effectively flatten the potential due to the presence of the flattening factor \( \frac{1}{\sqrt{G(\phi)V'(\phi)}} \ll 1 \). This implies that in the presence of Galileon like derivative interaction slow-roll inflation can take place even if the potential is rather steep.
3. Inflation from background supergravity

The number of e-foldings for $D3$ DBI Galileon can be expressed as

$$N = \frac{1}{8M_p} \int_{\phi_i}^{\phi_f} \frac{\sqrt{G(\phi)V'(\phi)}}{\sqrt{\epsilon V}} d\phi,$$

where $\phi_i$ and $\phi_f$ are the corresponding values of the inflaton field at the beginning and end of inflation.

Fig. (3.3(b)) represents a graphical behavior of number of e-folding versus the inflaton field for different values of $C_i \forall i$.

### 3.3.3 Quantum fluctuations and CMB observables

Let us now engage ourselves in analyzing quantum fluctuation in our model and its observational imprints via primordial spectra generated from cosmological perturbation. To serve this purpose we use $ADM$ formalism [192] to expand the 4D effective action up to second order as:

$$S_2^\xi = \int d^4x \, a^3 \left[ -3t_1 \zeta^2 + \frac{2w_1}{a^2} \zeta \partial^2 \psi - \frac{t_2}{a^2} \alpha \partial^2 \zeta - \frac{2t_1}{a^2} \alpha \partial^2 \zeta + 3t_2 \alpha \zeta + \frac{1}{3} t_3 a^2 + \frac{t_4}{a^2} \partial_1 \zeta \partial_1 \zeta \right],$$

where the effect of effective Gauss-Bonnet coupling and the DBI Galileon features are explicitly appearing in the coefficients of the second order perturbative action as:

$$t_1 = t_4 \approx \tilde{l}_1,$$

$$t_2 \approx \left( 2H \tilde{l}_1 - 2\dot{\phi} X \tilde{G}_X \right),$$

$$t_3 \approx -9\tilde{l}_1^2 H^2 + 3 \left( X \tilde{K}_X + 2X^2 \tilde{K}_{XX} \right) + 18H \dot{\phi} \left( 2X \tilde{G}_X + X^2 \tilde{G}_{XX} \right) - 6(XG_{\phi} + X^2 G_{\phi X}).$$

It is important to mention here that, in the action (3.3.70), both the coefficients of the terms $\alpha \zeta$ and $\zeta^2$ vanish by using the background equations of motion. Furthermore, in (3.3.70), the term quadratic in $\psi$ vanishes by making use of integrations by parts. The equations of motion for $\psi$ and

---

12 Here the end of the inflation leads to the extra constraint $V'(\phi_f) = \sqrt{V'(\phi_f)V(\phi_f)}$.

13 In $ADM$ formalism the line element can be written as:

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j),$$

where $N$ and $N^i$ ($i = 1, 2, 3$) are the lapse and shift functions, respectively. In this context we consider scalar metric perturbations about the flat FLRW background. Here we expand the lapse $N$ and the shift vector $N^i$, as $N = 1 + \alpha$ and $N_i = \partial_i \psi + \tilde{N}_i$, respectively. Here $\partial_i \psi$ is the irrotational part and $\tilde{N}_i$ be the incompressible vector part ($\tilde{N}_{i,i} = 0, \tilde{N}_i = 0$). These are actually non-dynamical Lagrange multipliers in the action, so that it is sufficient to know $N$ and $N^i$ up to first order. This implies their equation of motion is purely algebraic. To fix the time and spatial reparameterization we choose the uniform-field gauge with $\delta \phi = 0$, which fixes the temporal component of a gauge-transformation vector $\xi^\mu$. After that by fixing the spatial part of $\xi^\mu$ we gauge away a field $\epsilon$ that appears as a form $\epsilon_{ij}$ inside $h_{ij}$. Consequently the metric on three dimensional constant time slice can be expressed as $h_{ij} = a^2(t) \epsilon^\mu \delta_{ij}$. Finally at linear level of the perturbation theory one can write:

$$ds^2 = -(1 + 2\alpha) dt^2 + 2(\partial_i \psi + \tilde{N}_i) dt dx^i + a^2(t)(1 + 2\zeta) \delta_{ij} dx^i dx^j.$$

---
\(\alpha\), derived from (3.3.70), lead to the following two-fold constraint relations\(^{14}\): \[
\begin{align*}
\frac{1}{a^2} \partial^2 \psi &= \frac{2t_3}{3t_2} \alpha + 3\dot{\zeta} - \frac{2t_1}{t_2} \frac{1}{a^2} \partial^2 \zeta, \\
\mathcal{J} &= \frac{2t_1}{t_2} = -\frac{2l_1}{2Hl_1 - 2\dot{\phi}XG_X} = \frac{1}{H} \left[ 1 + \delta_{GX} + \mathcal{O}(\epsilon_V^2) \right].
\end{align*}
\] (3.3.72) (3.3.73) (3.3.74)

where
\[
\mathcal{J} = \frac{2t_1}{t_2} = -\frac{2l_1}{2Hl_1 - 2\dot{\phi}XG_X} = \frac{1}{H} \left[ 1 + \delta_{GX} + \mathcal{O}(\epsilon_V^2) \right].
\]

Then substituting Eq (3.3.72) and Eq (3.3.73) into Eq (3.3.70) and integrating the term \(\dot{\zeta} \partial^2 \zeta\) by parts the second order action stated in Eq (3.3.70) can be re-expressed as:
\[
S_2^\zeta = \int dtd^3 x a^3 Y_S \left[ \dot{\zeta}^2 - \frac{c_s^2}{a^2} (\partial \zeta)^2 \right],
\] (3.3.75)

where
\[
Y_S = \frac{t_1 (4t_1 t_3 + 9t_2^2)}{3t_2^2}, \quad c_s^2 = \frac{3 (2Ht_2 l_1^2 - t_2 l_1^2 + 2t_1 l_2)}{t_1 (4t_1 t_3 + 9t_2^2)}.
\] (3.3.76)

It is important to mention here that \textit{ghosts} and \textit{Laplacian} instabilities can be avoided iff \(c_s^2 > 0\), \(Y_S > 0\). Now using Eq (3.3.72) and Eq (3.3.76) in Eq (3.3.73) we get:
\[
\psi = -\mathcal{J} \zeta + \partial^{-2} \left( \frac{a^2 Y_S \dot{\zeta}}{t_1} \right).
\] (3.3.77)

For future convenience, we have introduced a new parameter defined as:
\[
\epsilon_s = \frac{Y_S c_s^2}{l_1} = \frac{(2Ht_2 l_1^2 - t_2 l_1^2 - 2t_1 l_2)}{t_2 l_1} = \epsilon_V + \delta_{GX} + \mathcal{O}(\epsilon_V^2).
\] (3.3.78)

Now varying the action stated in Eq (3.3.75) and expressing the solution at the linear level in terms of Fourier modes, we arrive at the \textit{Mukhanov Sasaki Equation} for Galileon scalar mode.
\[
v''_{\hat{k}} + \left( c_s^2 k^2 - \frac{z''}{z} \right) v_{\hat{k}} = 0,
\] (3.3.79)

where \(c_s^2\) takes into account the nontrivial modification due to Galileon. Similarly for tensor modes, Eq (3.3.75) can be recast as:
\[
S_2^h = \int dtd^3 x a^3 Y_T \left[ \hat{h}_{ij}^2 - \frac{c_s^2}{a^2} (\partial h_{ij})^2 \right],
\] (3.3.80)

\(^{14}\)Here we introduce new set of parameters:
\[
\begin{align*}
\epsilon_V &= \frac{\dot{\epsilon}_s}{H \epsilon_s} = \frac{4 \sqrt{V(\phi)} M_p^2}{\sqrt{g(\phi)}} \frac{d}{d \phi} \left( \ln \epsilon_s \right), \quad \epsilon_T = \frac{\dot{\epsilon}_T}{H \epsilon_T} = \frac{M_p^2 \sqrt{\tilde{g}_i V(\phi)}}{2 \sqrt{v_0} V(\phi) \phi} \frac{d}{d \phi} \left( \ln \left[ 1 + \mathcal{O}(\epsilon_V^2) \right] \right), \quad \delta_{GX} = \frac{\dot{\phi}XG_X}{l_1}, \\
\eta_s &= \frac{\epsilon_s}{H \epsilon_s} = \frac{4 \sqrt{V(\phi)} M_p^2}{\sqrt{g(\phi)}} \left[ \frac{\epsilon_V (1 + \mathcal{O}(\epsilon_V)) + \delta_{GX} \phi}{[\epsilon_V + \delta_{GX} + \mathcal{O}(\epsilon_V^2)]} \right], \quad \delta_V = \frac{\dot{Y}_c}{HY_s} = \frac{4 \sqrt{V(\phi)} M_p^2}{\sqrt{g(\phi)}} \frac{d}{d \phi} \left( \ln Y_s \right).
\end{align*}
\]
where

\[ Y_T = \frac{t_1}{4} = \frac{\bar{t}_1}{4}, \quad c_T^2 = \frac{t_4}{t_1} = 1 + O(\epsilon^2). \]  

(3.3.81)

For tensor modes we use the normalization condition \( e_{ij}^\lambda e_{ij}^{\lambda'} = 2 \delta^{\lambda\lambda'} \) and traceless condition \( e_{ii} = 0 \) for polarization tensor. Following the same prescription we can establish Eq (3.3.79) for tensor modes provided \( c_s \) is replaced by \( c_T \). The Bunch-Davies mode function turns out to be (Throughout the paper we have used DS for de-Sitter results and BDS for beyond de-Sitter results.)

\[ u_\zeta(\eta, k) = \begin{cases} 
\frac{iH \exp(-ikc_s\eta)}{2\sqrt{2Y_S(c_s k)^2}} (1 + ikc_s\eta) & :DS \\
\sqrt{-k\eta c_s} \mathcal{H}_{\nu_s}^{(1)}(-k\eta c_s) & :BDS.
\end{cases} \]

(3.3.82)

where \( \nu_s = \left( \frac{3 - \nu_V - 2s^2 + \delta_V}{2(1 - \nu_V - s^2)} \right) \) and in the super-Hubble limit we have:

\[ \mathcal{H}_{\nu_s}^{(1)} \rightarrow \frac{(\nu_s)^{-\nu_s} \exp(i[\nu_s - \frac{1}{2}]\frac{\pi}{2})}{\sqrt{2c_s k}} 2^{\nu_s - \frac{3}{2}} \left( \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right) \]

(3.3.83)

Now using eqn(3.3.82) the two-point correlation function for scalar modes can be expressed as:

\[ \langle 0|\zeta(\vec{k})\zeta(\vec{k}')|0\rangle = \frac{2\pi^2}{k^3} (2\pi)^3 \mathcal{P}_\zeta(k) \delta^3(\vec{k} + \vec{k}') = (2\pi)^3 |u_\zeta(\eta, k)|^2 \delta^3(\vec{k} + \vec{k}'), \]  

(3.3.84)

where the dimensionless Power spectrum for scalar modes \( \mathcal{P}_\zeta(k) \) at the horizon crossing turns out to be:

\[ \mathcal{P}_\zeta(k_*) = \frac{k^3}{2\pi^2} |u_\zeta(k_*)|^2 = \begin{cases} 
\left( \frac{\sqrt{V(\phi)}}{2\pi^2 c_s l_{1} \sqrt{y_1 M_p}} \right)^* & :DS \\
\left( \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right)^2 \left( 1 - \epsilon_V - s^2 \right)^2 \sqrt{V(\phi)} & :BDS.
\end{cases} \]

(3.3.85)

\* corresponds to the horizon crossing. Similarly using the tensor version of eqn(3.3.82) the two-point correlation function for tensor modes can be expressed as:

\[ \langle 0|h_{ij}(\vec{k})h_{ij}(\vec{k}')|0\rangle = \frac{2\pi^2}{k^3} (2\pi)^3 \mathcal{P}_T(k) \delta^3(\vec{k} + \vec{k}') = (2\pi)^3 |u_T(\eta, k)|^2 \delta^3(\vec{k} + \vec{k}'), \]

(3.3.86)

where \( \mathcal{P}_T(k) = [\mathcal{P}_T(k)]_{ij;ij} \) and the corresponding dimensionless Power spectrum for tensor modes reads:

\[ \mathcal{P}_T(k_*) = \frac{k^3}{2\pi^2} |u_T(k_*)|^2 \left( \sum_{\lambda=+,\times} e_{ij}^\lambda e_{ij}^{\lambda'} \right) = \begin{cases} 
\left( \frac{\sqrt{V(\phi)}}{2\pi^2 c_T \epsilon_T l_1 \sqrt{y_1 M_p}} \right)^* & :DS \\
\left( \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right)^2 \left( 1 - \epsilon_V - s^2 \right)^2 \sqrt{V(\phi)} & :BDS.
\end{cases} \]

(3.3.87)
Consequently the ratio of tensor to scalar power spectrum can be expressed as:

\[
    r(k_*) = \frac{\mathcal{P}_T(k_*)}{\mathcal{P}_S(k_*)} = \left\{ \begin{array}{l}
    \left(16\epsilon_s c_s \left[1 - \frac{3}{2} \mathcal{O}(\epsilon_T^2)\right]\right)^*, \quad \text{:DS} \\
    \left(16.2^{(n_T - n_S)} \frac{\Gamma(n_T)}{\Gamma(n_S)}\right)^2 \left(\frac{1 - \epsilon_V - s_V^T}{1 - \epsilon_V - s_V^S}\right)^2 c_s \left[1 - \frac{3}{2} \mathcal{O}(\epsilon_T^2)\right], \quad \text{:BDS}.
\end{array} \right.
\]

Further, the scale dependence of the perturbations, described by the scalar and tensor spectral indices, as follows:

\[
    n_\zeta - 1 = \left(\frac{d \ln \mathcal{P}_S}{d \ln k}\right)_* = \left\{ \begin{array}{l}
    \left( -2\epsilon_V - \eta_s - s_V^S \right)_* = \left( -2\epsilon_s - \eta_s - s_V^S + 2\delta_{G_X} + 2\mathcal{O}(\epsilon_T^2)\right)_*, \quad \text{:DS} \\
    \left(3 - 2n_\zeta\right)_* = -\left(2\epsilon_V + s_V^S + \delta_V\right), \quad \text{:BDS}.
\end{array} \right.
\]

\[
    n_T = \left(\frac{d \ln \mathcal{P}_T}{d \ln k}\right)_* = \left\{ \begin{array}{l}
    \left(-2\epsilon_V\right)_* = \left(-2\epsilon_s + 2\delta_{G_X} + 2\mathcal{O}(\epsilon_T^2)\right)_*, \quad \text{:DS} \\
    \left(3 - 2n_T\right)_* = -\left(\frac{s_V^T}{1 - \epsilon_V - s_V^T}\right)_*, \quad \text{:BDS}.
\end{array} \right.
\]

Consistently, the consistency relation is also modified to:

\[
    r = \left\{ \begin{array}{l}
    \left( -8c_s \left( n_T - 2\delta_{G_X} - 2\mathcal{O}\left(n_T^2\right)\right) \left[1 - \frac{3}{2} \mathcal{O}(\epsilon_T^2)\right]\right)_*, \quad \text{:DS} \\
    \left(8.2^{(n_\zeta - n_T)} \frac{\Gamma(3 - n_T)}{\Gamma(3 - n_\zeta)}\right)^2 \left(\frac{s_V^T}{n_T} \left(1 - \frac{1}{n_T} - s_V^T\right) - \frac{s_V^S}{n_\zeta}\right)^2 c_s \left[2 \left[1 + s_V^T \left(\frac{1}{n_T} - 1\right)\right]\right)_*, \quad \text{:BDS}.
\end{array} \right.
\]

The expressions for the running of the scalar and tensor spectral index in this specific model with respect to the logarithmic pivot scale at the horizon crossing are given by:

\[
    \alpha_\zeta = \left(\frac{dn_\zeta}{d \ln k}\right)_* = \left\{ \begin{array}{l}
    \left\{ \frac{4\sqrt{V'(\phi)M_p^2}}{\sqrt{G(\phi)}} \left( -2\epsilon_V - \eta_s - s_V^S \right)^\prime \right\}_*, \quad \text{:DS} \\
    \left\{ \frac{4\sqrt{V'(\phi)M_p^2}}{\sqrt{G(\phi)}} \left(1 - \epsilon_V - s_V^S\right)^2 \left[s_V^S \epsilon_V^\prime + \delta_V^\prime + s_V^S \delta_V^\prime + (2\epsilon_V + s_V^S \delta_V^\prime)\right]\right\}, \quad \text{:BDS}.
\end{array} \right.
\]

\[
    \alpha_T = \left(\frac{dn_T}{d \ln k}\right)_* = \left\{ \begin{array}{l}
    \left\{ -2 \left(\frac{4\sqrt{V'(\phi)M_p^2}}{\sqrt{G(\phi)}} \epsilon_V^\prime\right)^\prime \right\}_*, \quad \text{:DS} \\
    \left\{ -\left(\frac{4\sqrt{V'(\phi)M_p^2}}{\sqrt{G(\phi)}} \left(1 - \epsilon_V - s_V^T\right)^2 \left[ -2\epsilon_V^\prime + s_V^T \delta_V^\prime\right]\right\}_*, \quad \text{:BDS}.
\end{array} \right.
\]

Here we have used a shorthand notation \(ab = a' b - a b'\) where \(\prime = \frac{d}{d\phi}\). We also use the operator identity \(\frac{d}{d \ln k} = \frac{4\sqrt{V'(\phi)M_p^2}}{\sqrt{G(\phi)}} \frac{d}{d\phi}\) to compute all the inflationary observables.

### 3.3.4 Parameter estimation using CAMB

Using the parameter space for the model parameters \((C_i, D_i)\) we have estimated the window of the cosmological parameters from our model which confronts observational data well in \(56 < N < 70\).
Figure 3.4: Variation of CMB angular power spectrum for (a) TT, (b) TE and (c) EE correlation with respect to multipole l for the best fit model parameters. Also in (d) we show the variation of matter power spectrum with respect to the momentum scale [191].

In Table(3.4) we have tabulated the relevant observational parameters estimated from our model for both DS and BDS limit.

| Scheme | $P_\zeta \times 10^{-9}$ | r       | $n_\zeta$ | $\alpha_\zeta \times 10^{-3}$ |
|--------|------------------------|---------|-----------|-------------------------------|
| DS     | 2.401 - 2.601          | 0.215 - 0.242 | 0.964 - 0.966 | 2.240 - 2.249               |
| BDS    | 2.471 - 2.561          | 0.232 - 0.250 | 0.962 - 0.964 | 4.008 - 4.012              |

Table 3.4: Model Dependent Observational Parameters [191].

Further, we use the publicly available code CAMB [155] to verify our results directly with observation. To operate CAMB at the pivot scale $k_0 = 0.002 \text{Mpc}^{-1}$ the values of the initial parameter space are taken for lower bound of $C_i's$ and $N = 70$. Additionally WMAP7 years dataset for $\Lambda CDM$ background has been used in CAMB to obtain CMB angular power spectrum. In Table(3.5) we have given all the input parameters for CAMB. Table(3.6) shows the CAMB output, which is in good fit with WMAP7 [22] data. In fig. (3.4(a)-figure(3.4)(c) we have plotted CAMB output of CMB TT, TE and EE angular power spectrum $C^{TT}_l, C^{TE}_l, C^{EE}_l$ for the best fit with WMAP7 data for scalar mode, which explicitly show the agreement of our model with WMAP7 dataset. The small scale modes have no impact in the CMB anisotropy spectrum only the large scale modes have little contribution. Hence in fig. (3.4(d)) we have plotted the variation of matter power spectrum with respect to the momentum scale which is in concordance with observational results.
### Table 3.5: Input parameters in CAMB [191].

| Parameter | Value |
|-----------|-------|
| $H_0$     | 71.0  |
| $\tau_{\text{Reion}}$ | 0.09  |
| $\Omega_b h^2$ | 0.0226 |
| $\Omega_c h^2$ | 0.1119 |
| $T_{\text{CMB}}$ | 2.725 |

### Table 3.6: Output parameters from CAMB [191].

| Parameter | Value |
|-----------|-------|
| $t_0$     | 13.707 |
| $z_{\text{Reion}}$ | 10.704 |
| $\Omega_m$ | 0.2670 |
| $\Omega_k$ | 0.7329 |
| $\eta_{\text{Rec}}$ | 0.0 |
| $\eta_0$ | 285.10 |
|               | 14345.1 |

## 3.4 Chapter summary

In this chapter we have studied single field inflation in the context of Randall-Sundrum brane and DBI Galileon induced D3 brane respectively. The major outcomes of our study are:

- We have demonstrated the technical details of construction mechanism of an one-loop 4D inflationary potential via dimensional reduction starting from $\mathcal{N} = 2, D = 5$ SUGRA in the bulk which leads to an effective field theoretic picture within $\mathcal{N} = 1, D = 4$ SUGRA embedded in the brane for both the cases.

- Hence we have studied inflation using the one loop effective potential by estimating the observable parameters originated from primordial quantum fluctuation for scalar and tensor modes.

- We have further confronted our results with WMAP7 [22] dataset by using the cosmological code CAMB [155].

- Hence we have generated the theoretical CMB angular power spectra from TT, TE and EE correlation for scalar modes from both the proposed inflationary frameworks and fit with the observed CMB angular power spectra obtained from WMAP7 data.

- On the top of that in this chapter we have proposed new sets of inflationary consistency relations in the case of braneworld and DBI Galileon framework which is quite different from the results obtained from usual General Relativistic framework.
4 Reheating & Leptogenesis in brane inflation

4.1 Introduction

It is now well accepted fact that the post big bang universe \cite{193} passed through different phases having cosmo-phenomenological significance. One of the significant phases, namely, reheating \cite{12, 194} plays the pivotal role in explaining production of different particles from inflaton and vacuum energy. As we look back in time reheating was completed within approximately the first second after the big bang. At that time nucleosynthesis \cite{195}, or the formation of light nuclei occurred. On the other hand the mysterious force that drives the inflationary phase is conventionally described by a scalar field, named inflaton which oscillates near the minimum of its effective potential and produces elementary particles \cite{196}. These particles interact with each other and eventually they come to a state of thermal equilibrium at some arbitrary temperature. This process completes when all the energy of the classical scalar field transfer to the thermal energy of elementary particles. Since long theoretical physicists have been investigating reheating as a perturbative phase \cite{11}, or one in which single inflaton quanta decayed individually into ordinary matter and radiation \cite{1}. In short there is no existence of a complete theory which explains non-perturbative effects during reheating for the total time scale.

Besides production of gravitinos during perturbative reheating \cite{198} its decay plays a significant role in the context of leptogenesis \cite{14}. More precisely two types of gravitinos are produced in this epoch - stable \cite{199} and unstable \cite{200}. Stable ones and decay products of unstable ones directly or indirectly stimulate the light element abundances during big bang nucleosynthesis. Most importantly the unstable one has important cosmological consequences out of which the major one directly affects the expansion rate of the universe. In order to explain cosmological consequences at a time by a single physical entity, it is customary to explain everything in terms of gravitino energy density which is directly proportional to the gravitino number density or gravitino abundance. This gravitino abundance is obtained by considering gravitino production in the radiation dominated era following reheating \cite{201}. Gravitinos are originated through thermal scattering \cite{202} in the early universe and are usually related to the reheating temperature. Particle physics phenomenology usually requires that under instantaneous decay approximation \cite{1} reheating temperature is maximum during reheating.

In this chapter we have studied extensively reheating and leptogenesis in a typical brane inflation model in the framework of $N = 1, D = 4$ SUGRA in braneworld which was proposed in the chapter \ref{sec:brane} in section \ref{sec:brane_inflation}. The standard results of reheating and leptogenesis are obtained considering four dimensional Einsteinian gravity \cite{1, 203} \footnote{The recent theoretical studies have shown that in many cases the decay occurs through a non-perturbative process \cite{197}, in which the particles behave in an ordered manner. Non-perturbative processes involved at reheating are extremely more efficient than the perturbative ones \cite{125} and often more difficult to investigate in practice.}. Here we show that the results are dramatically

\footnote{For standard results of reheating mechanism and leptogenesis in Einsteinian GR see also Appendix D.}
different if one considers brane inflation starting from higher dimension resulting in effective non-
Einsteinian gravity in four dimension. This has serious implication for the production of the heavy
Majorana neutrinos needed for leptogenesis [204]. We further estimate different parameters related
to reheating and leptogenesis at the epoch of phase transition. Finally, we have given an estimate
of CP violation which is the indirect evidence of the baryon asymmetry in the present context.

4.2 Background model

From the knowledge of particle physics it is known that during the epoch of reheating inflatons
decay into different particle constituents [194] are directly related to the trilinear coupling of the
inflaton field. There might be a possibility of collision originated through quartic coupling and
driven by background scalar field. For example here the contribution from the heavy Majorana
neutrinos comes from the seesaw Lagrangian

\[ L_{Majo} = -h_j l_i \mathcal{H} \bar{\psi}_j - \frac{1}{2} \sum_i \mathcal{M}_i \bar{\psi}_i \psi_i + h.c., \]  

(4.2.1)

where where \( i, j = 1, 2, 3 \) denote the generation indices, \( h \) is the Yukawa coupling, \( l \) and \( \mathcal{H} \) are
the lepton and the Higgs doublets, respectively, and \( \mathcal{M}_i \) is the lepton-number-violating mass term
of the right-handed neutrino. In this chapter, we assume the hierarchical mass spectrum for the
heavy neutrinos, \( \mathcal{M}_1 \ll \mathcal{M}_2, \mathcal{M}_3 \), for simplicity. Now using the another assumption, inflaton mass
\( m_\phi \gg m_\sigma, m_\psi \gg m_\psi \) \(^3\) the total inflaton decay width for the positively and negatively charged
\( \phi(\phi^+, \phi^-) \) scalar fields as well as the fermionic field \( \psi \) \(^4\) is given by:

\[ \Gamma_{total} \sim \frac{C^2}{16\pi m_\phi} + \frac{h^2 m_\psi}{4\pi} \sim \frac{1}{(2\pi)^3} \left( \frac{\Delta^6}{M^9} \right) \]  

(4.2.2)

where the coupling strength \( C \sim m_\phi \left( \frac{A^2_M}{M} \right) \) and \( h \sim \left( \frac{A^2_M}{M} \right) \) and the background scalar field is \( \sigma \).

Now to construct the thermodynamical observable the effective number of particles incorporating
relativistic degrees of freedom is defined \([1]\) as \(^5\):

\[ N^* = N^*_B + \frac{7}{8} N^*_F, \]  

(4.2.3)

where \( N^*_B = \sum_i N^*_B i \) and \( N^*_F = \sum_j N^*_F j \). Here \( N^*_B \) represents the number of bosonic degrees
of freedom with mass \( m_\phi \ll T \) and \( N^*_F \) represents number of fermionic degrees of freedom with
mass \( m_\psi \ll T \). Here \( i \) and \( j \) stand for different bosonic and fermionic species respectively. For
convenience let us express reheating temperature on the brane as:

\[ \Gamma_{total} = 3H(T_{br}) = \sqrt{\frac{3\rho(t_{rech})}{M^2} \left[ 1 + \frac{\rho(t_{rech})}{2\lambda} \right]}, \]  

(4.2.4)

where \( \lambda, H(T_{br}) \) and \( \rho(t_{rech}) \) be the brane tension, Hubble parameter and energy density during
reheating respectively. In Eq (4.2.4) the correction term in the Hubble parameter is significant in
the high energy limit where the energy density is large compared to the brane tension i.e. \( \rho >> \lambda \).
On the other hand, in the limit \( \rho << \lambda \) we get back the standard result in Einsteinian gravity. It
is worth mentioning that the brane reheating temperature does not depend on the initial value of
the inflaton field and is solely determined by the elementary particle theory of the early universe.

---

\(^3\) Here \( m_\sigma \) and \( m_\phi \) be the background scalar field mass and fermion mass respectively.
\(^4\) For the heavy Majorana neutrinos the decay process \( \psi \rightarrow l_i \mathcal{H}, \psi \rightarrow l_i \mathcal{H} \) predominates.
\(^5\) For the phenomenological estimation [205] \( N^* \sim 10^2 - 10^4 \) and for realistic models \( N^* \sim 10^2 - 10^3 \).
4.3 Phase transition in braneworld inflation

Phase transition in braneworld scenario is weakly first order in nature [206]. So it is convenient to write the brane reheating temperature in terms of the critical parameters. To serve this purpose the critical density and the critical temperature or transition temperature can be written as:

$$\rho(t_c) = 2\lambda = \frac{3}{16\pi^2} \frac{M_5^6}{M^2}, \quad T_c = \sqrt{\frac{3}{\pi} \frac{5}{\pi N^*} \frac{M_5^3}{M}}$$

(4.3.5)

which makes a bridge between the phenomenology and observation. In the high energy limit 5D Planck mass ($M_5$) can be expressed in terms of our model parameters as:

$$M_5 = \sqrt{\frac{6400\pi^4 \Delta^2_s (K_4 + 4D_4)^2}{\alpha^4 M}}$$

(4.3.6)

The major thermodynamic quantities – critical density ($\rho_c$), critical pressure ($P_c$), critical entropy ($S_c$) – and the Hubble parameter at the critical temperature ($H_c$) related to the phase transition designated by the following fashion for our model:

$$\rho_c = 1200\phi^4_\star A(\phi_\star) \quad \forall \gamma \in J,$n

(4.3.7)

$$P_c = 400\phi^4_\star A(\phi_\star) = \frac{P_c}{3} \quad \forall \gamma \in J,$n

(4.3.8)

$$S_c = 1600\phi^4_\star A(\phi_\star) \quad \forall \gamma \in J,$n

(4.3.9)

$$H_c = 20\sqrt{A(\phi_\star)\phi^2_\star} \quad \forall \gamma \in J$$

(4.3.10)

where we have defined a dimensionless characteristic quantity:

$$A(\phi_\star) = \frac{\pi^2(K_4 + 4D_4)^2 \Delta^2_s \phi^2_\star}{\alpha^4 M^2}$$

(4.3.11)

at the horizon crossing in this context. The above mentioned physical quantities are function of the critical or transition temperature which is defined as:

$$T_c := \sqrt{\left\{ C_{\gamma} A(\phi_\star)\phi^4_\star \right\} \pi^2 N^*_\gamma} \quad \text{with} \quad C_{\gamma} = \left( 36000, \frac{28800}{7}, 19200 \right) \quad \forall \gamma \in J$$

(4.3.12)

with gauge group $J := SU(2)_L \otimes U(1)_Y$ and the species index $\gamma = 1(B \Rightarrow \text{Boson}), 2(F \Rightarrow \text{Fermion}), 3(M \Rightarrow \text{Mixture})$.

4.4 Reheating temperature

In the present context the reheating temperature can be written [207] as:

$$T^{br} = \frac{T_c}{\sqrt{2}} \left[ \sqrt{1 + \frac{5}{\pi^3 N^*} \left( \frac{\Gamma_{\text{total}} M_{PL}}{T_c^2} \right)^2} - 1 \right].$$

(4.4.13)

---

6Here $\Delta^2_s$ represents the amplitude of the scalar perturbation defined in Eq (5.2.9) in chapter 3. Most importantly here the subscript $\star$ represents here the epoch of horizon crossing ($k = aH$) and $\alpha$ represents a dimensionless model parameter defined as, $\alpha = \frac{\Delta^4}{\lambda}$, where $\Delta$ represents the energy scale of brane inflation.
4.5 Gravitino production via leptogenesis

In the high energy regime the reheating temperature can be cast as:

\[
T_{brh}^{*} = 4 \left\{ \sqrt{\frac{10}{N^s} \frac{2\sqrt{2} M \Gamma_{total} T_c^2}{3\pi}} \right\}.
\] (4.4.14)

But in this context we are confining ourselves into the Standard Model regime. So to construct a fruitful model of reheating in the context of Standard Model gauge group, we rewrite all general principal components in terms of physical degrees of freedom in a compact fashion. We consider a one to one high energy mapping such that:

\[
T_{br}^{*} = T_{c}^{*} \frac{\sqrt{2}}{4 \sqrt{2}} \left[ 1 + \frac{Z_{\gamma}}{\pi^3 N^*_{\gamma}} \left( \frac{\Gamma_{total} M_{PL}}{T_{c\gamma}^2} \right)^2 - 1 \right] \Rightarrow T_{brh}^{*} = \sqrt{\frac{W_{\gamma} (K_4 + 4D_4) \Delta_{\phi_0^2} \Gamma_{total}}{\pi N^*_{\gamma} a^2}} \forall \gamma \in J
\] (4.4.15)

it maps the actual brane reheating temperature \((T_{br}^{*})\) to its high energy value \((T_{brh}^{*})\) in the Standard Model gauge group \(J := SU(2)_L \otimes U(1)_Y\) with \(Z_{\gamma} = (5, \frac{40}{7}, \frac{50}{7}), W_{\gamma} = (600, \frac{4800}{7}, 320)\) and \(\gamma = 1(B), 2(F), 3(M)\). Most importantly the superscript ‘br’ and ‘brh’ stands for parameters before and after high energy mapping respectively. Here it should be mentioned that the brane reheating temperature incorporates all the effects of heavy Majorana neutrinos as well as the other fermions and bosons through the total decay width \(\Gamma_{total}\).

The reheating temperature for different species can readily be calculated from our model as proposed in chapter 3. For a typical value of model parameters, \(C_4 \simeq D_4 = -0.7\), we have:

1. For boson: \(T_{brh}^{B} \simeq 7.6 \times 10^{10}\) GeV,
2. For fermion: \(T_{brh}^{F} \simeq 7.8 \times 10^{10}\) GeV,
3. For mixture of boson and fermion: \(T_{brh}^{M} \simeq 6.5 \times 10^{10}\) GeV.

These results are significantly different from GR value \(T_{reh} \leq 10^9\) GeV and is a characteristic feature of brane inflation with non-Einstenian framework.

4.5 Gravitino production via leptogenesis

Let us now move on to studying how the self interacting term of our model is directly related to the leptogenesis through the production of thermal gravitinos which is a special ingredient for the heavy Majorana neutrinos in the leptogenesis. Let us start with a physical situation where the inflaton field starts oscillating when the inflationary epoch ends at a cosmic time \(t = t_{osc} \simeq t_f\). Throughout the analysis we have assumed that the universe is reheated through the perturbative decay of the inflaton field for which the reheating phenomenology in brane is described by the Boltzmann equation [1]:

\[
\dot{\rho}_r + 4H \rho_r = \Gamma_{\phi} \rho_\phi,
\] (4.5.16)

where in braneworld

\[
H^2 = \frac{8\pi}{3M_{PL}^2} (\rho_r + \rho_\phi) \left[ 1 + \frac{(\rho_r + \rho_\phi)}{2\lambda} \right] = H_{osc}^2 \left( \frac{a_{osc}}{a} \right)^4 \left[ 1 + \frac{\alpha}{2} \left( \frac{a_{osc}}{a} \right)^4 \right].
\] (4.5.17)
Here $\rho_r$ and $\rho_\phi$ represent the energy density of radiation and inflaton respectively and $\Gamma_\phi$ is the rate of dissipation of the inflaton field energy density. At $t = t_{osc}$ epoch the Hubble parameter is designated by [1]:

$$H_{osc} = \sqrt{\frac{8\pi}{3}} \frac{\Delta^2}{M_{PL}} = \frac{\Delta^2}{\sqrt{3}M}$$  \hspace{1cm} (4.5.18)

Assuming $\Gamma_\phi \gg H$ from we get

$$\rho_\phi = \Delta^4 \left(\frac{a_{osc}}{a}\right)^4 \exp\left[-\Gamma_\phi(t - t_{osc})\right].$$  \hspace{1cm} (4.5.19)

It is worthwhile to mention here that the inflaton field $\phi$ follows an equation of state similar to radiation rather than matter i.e. $\omega_\phi = \frac{P_\phi}{\rho_\phi}$  \hspace{1cm} (4.5.18)

Now solving Friedmann equation the dynamical character of the scale factor can be expressed as

$$a(t) = a_{osc} \sqrt{\left[\left(\sqrt{1 + \frac{\alpha}{2} + 2H_{osc}(t - t_{osc})}\right)^2 - \frac{\alpha}{2}\right]},$$  \hspace{1cm} (4.5.20)

where we use a specific notation $a(t_{osc}) = a_{osc}$.

Plugging Eq (4.5.20) and Eq (4.5.19) in Eq (4.5.16) we get:

$$\dot{\rho}_r + \frac{2H_{osc}}{\left[\left(\sqrt{1 + \frac{\alpha}{2} + 2H_{osc}(t - t_{osc})}\right)^2 - \frac{\alpha}{2}\right]}\rho_r = \frac{\Gamma_\phi \Delta^4 \exp\left[-\Gamma_\phi(t - t_{osc})\right]}{\left[\left(\sqrt{1 + \frac{\alpha}{2} + 2H_{osc}(t - t_{osc})}\right)^2 - \frac{\alpha}{2}\right]} \rho_r.$$  \hspace{1cm} (4.5.21)

As a whole phenomenological construction of gravitino abundance is governed by the above equation. But Eq (4.5.21) is not exactly analytically solvable. So we are confining our attention to the high energy limit where the Friedmann equation (4.5.17) can be approximated as:

$$H^2 = \frac{8\pi}{6\lambda M_{PL}^2} (\rho_r + \rho_\phi)^2 = \frac{\alpha}{2} H_{osc}^2 \left(\frac{a_{osc}}{a}\right)^4,$$  \hspace{1cm} (4.5.22)

whose solution is given by:

$$a(t) = a_{osc} \sqrt{\left[\left(\sqrt{1 + \frac{\alpha}{2} + 2H_{osc}(t - t_{osc})}\right)^2 - \frac{\alpha}{2}\right]},$$  \hspace{1cm} (4.5.23)

Now using an physically viable assumption $t \leq \Gamma_\phi^{-1}$ the exact solution of the Eq (4.5.21) in the high energy limit can be written as:

$$\rho_r \approx \frac{3M^2 H_{osc}^2 \Gamma_\phi(t - t_{osc})}{1 + 2\sqrt{2\alpha}H_{osc}(t - t_{osc})} = \frac{3M^2 H_{osc} \Gamma_\phi}{2\sqrt{2\alpha}} \left(\frac{a_{osc}}{a}\right)^4 \left[\left(\frac{a}{a_{osc}}\right)^4 - 1\right].$$  \hspace{1cm} (4.5.24)

Our intention is to find out the extremum temperature during reheating epoch which is one of the prime components for the determination of gravitino abundance. In the braneworld scenario this extremum temperature is given by:

$$T_{ex}^{bh} = \sqrt{\sqrt{\frac{13\sqrt{5} \Delta^2 M \Gamma_\phi}{N^* \pi^2}} \sqrt{\frac{1}{2\alpha}}} = \sqrt{\sqrt{\frac{45 \Gamma_\phi M_{Pl}^2}{8N^* \pi^3}}}$$  \hspace{1cm} (4.5.25)

---

7 Here pressure can be expressed as, $P_\phi = \rho_\phi - 2V(\phi)$, where $V(\phi)$ is the potential derived in Eq (3.2.22) of chapter 3.
and it is less than the reheating temperature in brane \( T_{brh} \). This phenomenon is different from standard GR results \([203]\) where we see that the reheating temperature shoots up to a maximum value and it gives the upper bound of the reheating temperature. But in the present context of brane inflation this situation is completely different i.e. at first temperature falls down to a minimum which fixes the lower bound of the reheating temperature and rises to a maximum at the end of reheating epoch. Using Eq (4.5.21), Eq (4.5.25) and the thermodynamic background of energy density of radiation we can express the scale factor in terms of temperature as

\[
a(T) = \begin{cases} 
\frac{a_{osc}}{\sqrt{\frac{1}{4} + \frac{32}{3} \left( \frac{T}{T_{bh}} \right)^4 \left( \frac{T}{T_{osc}} \right)}} & \text{if } t = t_{osc} \simeq t_f \\
\frac{a_{osc}}{\sqrt{\frac{1}{4} + \frac{32}{3} \left( \frac{T}{T_{osc}} \right)^4 \left( \frac{T}{T_{osc}} \right)}} & \text{if } t_{osc}(\simeq t_f) < t \leq t_{reh},
\end{cases}
\]  

(4.5.26)

It is worth mentioning that if we break the time scale into two parts \( t_{osc} < t < t_{ex} \) and \( t_{ex} < t < t_{reh} \), as done in GR the scale factor and hence the remaining results have same expressions in these two different zones. This is in sharp contrast with standard GR results except at \( t = t_f \), where they have different values in the two different regimes.

Let us now use this phenomenological background to derive the expression of the gravitino production during two thermal epochs - reheating and radiation dominated era. It is well known that gravitinos are produced by the scattering of the inflaton decay products \([208]\). The master
equation of gravitino as obtained from ‘Boltzmann equation.’ is given by [202]:

\[
\frac{dn_{\tilde{G}}}{dt} + 3Hn_{\tilde{G}} = \langle \Sigma_{\text{total}} | v | \rangle n^2 - \frac{m_{\tilde{G}} n_{\tilde{G}}}{\langle E_0 \rangle T^2},
\] (4.5.27)

For convenience let us recast Eq (4.5.27) as

\[
\dot{T} \frac{dn_{\tilde{G}}}{dT} + 3Hn_{\tilde{G}} = \langle \Sigma_{\text{total}} | v | \rangle n^2,
\] (4.5.28)

where a boundary condition \( T = T_{\text{ex}}^{bh}, \dot{T} = 0 \) is introduced. In terms of a dimensionless variable

\[
x = 32 \left( \frac{T}{T_{\text{ex}}^{bh}} \right)^4 - 1
\] (4.5.29)

Eq (4.5.28) can be expressed as

\[
\frac{dn_{\tilde{G}}}{dx} + \frac{d_1}{x} n_{\tilde{G}} = - \frac{d_3 (x + 1)^2}{x^2}
\] (4.5.30)

where

\[
d_1 = -\frac{3}{4}, \quad d_3 = \frac{\left( T_{\text{ex}}^{bh} \right)^6}{32} \frac{\tilde{\alpha}}{M^2} \left( \frac{\zeta(3)}{\pi^2} \right)^2 \frac{\sqrt{\lambda}}{4\sqrt{3} H_{\text{osc}} M}.
\] (4.5.31)

The exact solution of the Eq (4.5.30) is given by [209]:

\[
n_{\tilde{G}}(x) = \frac{2d_3}{x^d_1} \sqrt{x + 1} \left(-2_{-} F_1 \left[ \frac{1}{2}; 1 - d_1; \frac{3}{2}; x + 1 \right] + 2_{+} F_1 \left[ \frac{1}{2}; -d_1; \frac{3}{2}; x + 1 \right]
\]

\[
+ 2_{+} F_1 \left[ \frac{1}{2}; d_1; \frac{3}{2}; x + 1 \right]\right).
\] (4.5.33)

---

\[8^*\]In this context, \( n = \frac{\langle 3 \rangle}{4 \pi} \) is the number density of scatterers bosons in thermal bath with \( \zeta(3) = 1.20206 \ldots \). Here \( \Sigma_{\text{total}} \) is the total scattering cross section for thermal gravitino production, \( v \) is the relative velocity of the incoming particles with \( \langle v \rangle = 1 \) where \( \langle \cdots \rangle \) represents the thermal average. The factor \( \frac{m_{\tilde{G}}}{\langle E_0 \rangle} \) represents the averaged Lorentz factor which comes from the decay of gravitinos can be neglected due to weak interaction. For the gauge group \( E := SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) the thermal gravitino production rate is given by,

\[
\langle \Sigma_{\text{total}} | v | \rangle = \frac{\tilde{\alpha}}{M^2} = \frac{3 \pi}{16 \zeta(3) M^2} \sum_{i=1}^{3} \left[ 1 + \frac{M_i^2}{3 m_{\tilde{G}}^2} \right] C_i g_i^2 \ln \left( \frac{K_i}{g_i} \right),
\]

where \( i = 1, 2, 3 \) stands for the three gauge groups \( U(1)_Y, SU(2)_L \) and \( SU(3)_C \) respectively. Here \( M_i \) represent gaugino mass parameters and \( g_i(T) \) represents gaugino coupling constant at finite temperature from MSSM RGE

\[
g_i(T) \approx \frac{1}{\sqrt{\frac{1}{2} (\ln \frac{\tau}{M_Z})^2 - \frac{b_i}{8\pi} \ln \left( \frac{\tau}{M_Z} \right)}
\]

with \( b_1 = 11, b_2 = 1, b_3 = -3 \). Here \( C_i \) and \( K_i \) represents the constant associated with the gauge groups with \( C_1 = 11, C_2 = 27, C_3 = 72 \) and \( K_1 = 1.266, K_2 = 1.312, K_3 = 1.271 \).

\[9\]Using the properties of Gaussian hypergeometric function for \( x >> 1 \) Eq (5.5.56) reduces to the following simpler form [209]:

\[
n_{\tilde{G}}(x) \approx 2d_3 x^d_1 \sqrt{1 + x} \left[ \frac{\Gamma(\frac{1}{2}) \Gamma(\frac{3}{2})}{\Gamma(\frac{1}{2})} + \frac{1}{\Gamma \left( -\frac{1}{2} \right)} - \frac{2}{\Gamma \left( -\frac{3}{2} \right)} \right]
\] (4.5.32)

Using the boundary condition at \( T = T_{\text{ex}}^{bh} \) in Eq (4.5.32) the numerical value of gravitino abundance turns out to be \( n_{\tilde{G}}(x_{ex}) = 6.023d_3 \).
where \( _2F_1[a, b, c, d] \) be the Hypergeometric function defined for \( |d| < 1 \) by the power series as:

\[
_2F_1[a, b, c, d] = \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n} d^n n!
\]  

(4.5.34)

It is undefined (or infinite) if \( c \) equals a non-positive integer. Here \((a)_n\) is the (rising) Pochhammer symbol, which is defined by:

\[
(a)_n = \begin{cases} 
1 & \text{if } n = 0 \\
(a)(a+1)\ldots(a+n-1) & \text{if } n > 0.
\end{cases}
\]  

(4.5.35)

Let us now find out the exact analytical expression for the gravitino abundance at reheating temperature \( T^{brh} \) in the high energy limit. To serve this purpose substituting \( T = T^{brh} \) in Eq (5.5.56) we get [209]:

\[
n_{\tilde{G}}(T^{brh}) = 8\sqrt{2}d_3 \left( 32 \left( \frac{T^{brh}}{T_{bhex}} \right)^4 - 1 \right)^{\frac{3}{4}} \left( \frac{T^{brh}}{T_{bhex}} \right)^2 G \left( \frac{T^{brh}}{T_{bhex}} \right)
\]  

(4.5.36)

where

\[
G \left( \frac{T^{brh}}{T_{bhex}} \right) = \left( -2_2F_1 \left[ \begin{array}{c} 1 \quad 3 \\ \frac{1}{4} \quad \frac{3}{2} \end{array} \right] ; 32 \left( \frac{T^{brh}}{T_{bhex}} \right)^2 \right)^2 + _2F_1 \left[ \begin{array}{c} 1 \quad 3 \\ \frac{1}{4} \quad \frac{3}{2} \end{array} \right] ; 32 \left( \frac{T^{brh}}{T_{bhex}} \right)^2
\]  

(4.5.37)

along with an extra constraint [209]:

\[
G \left( \frac{T^{brh}}{T_{bhex}} \gg \frac{1}{\sqrt{32}} \right) = \frac{\pi}{2} \left( 32 \left( \frac{T^{brh}}{T_{bhex}} \right)^4 - 1 \right)^{-\frac{1}{2}} \left\{ \frac{1}{\Gamma\left(\frac{3}{4}\right)} + \frac{1}{\Gamma\left(\frac{5}{4}\right)} - \frac{2}{\Gamma\left(-\frac{1}{4}\right)} \right\}.
\]  

(4.5.38)
It is convenient to express the abundance of any species 's' as \( \sigma \), where \( n_b \) is the number density of the species 'b' in a physical volume and 's' is the entropy density given by \( s = \frac{25}{43} N^* T^3 \).

Here the master equation for gravitino can be expressed as [209]:

\[
T \frac{dY_G^{br}}{dT} = \langle \Sigma_{\text{total}}(v) \rangle Y_G^{br} n
\]  

Using Eq (4.5.20) and Eq (4.5.26) the time-temperature relation can be found as [209]:

\[
T = \frac{T^{br}}{\sqrt{\left( \left[ \sqrt{1 + \frac{\pi}{2}} + 2H_{reh}(t - t_{reh}) \right]^2 - \frac{\alpha}{2} \right)}}
\]  

Eliminating \( T \) we get the solution of the master Eq (4.5.39) in the radiation dominated era as [209]:

\[
Y_G^{br}(T_f) = Y_G^{br}(T^{br}) + Y_G^{br-rad}(T_f)
\]  

where

\[
Y_G^{br-rad}(T_f) = \sqrt{\frac{90}{\pi^2 N^*}} \left( \frac{45\sqrt{2}}{2\pi^2 N^* \sqrt{\alpha}} \right) \left( \frac{\bar{\alpha}}{M} \right) \left( \frac{\zeta(3)}{\pi^2} \right)^2 \times \frac{T^{br}}{T_f \sqrt{1 + \frac{\pi^2}{60N^*} (T^{br})^4}} \left( T^{br} \right)^4_2 \left[ \frac{1}{4}; \frac{1}{2}; \frac{5}{4}; -\frac{2(T^{br})^4}{\alpha T_f^4} \right] - T_f \left( \frac{1}{4}; \frac{1}{2}; \frac{5}{4}; -\frac{2}{\alpha} \right)
\]  

But in Eq (4.5.41) the first term on the right-hand side is not exactly computable. As mentioned earlier to find out exact expression we have used here the high energy mapping.

In the radiation dominated era the dynamical behavior of temperature can be mapped as

\[
T = \left( \frac{T^{br}}{\sqrt{\left( \left[ \sqrt{1 + \frac{\pi}{2}} + 2H_{reh}(t - t_{reh}) \right]^2 - \frac{\alpha}{2} \right)}} \right)^{\frac{1}{2}} \Rightarrow \left( \frac{T^{br}}{\sqrt{\left( \left[ \sqrt{1 + \frac{\pi}{2}} + 2H_{reh}(t - t_{reh}) \right]^2 - \frac{\alpha}{2} \right)}} \right)^{\frac{1}{2}}
\]  

Using this map we finally have [209]:

\[
Y_G^{br}(T_f) = \left( \frac{\bar{\alpha}}{M} \right) \left( \frac{\zeta(3)}{\pi^2} \right)^2 \left( \frac{45\sqrt{3}\lambda}{2\pi^2 \Delta^2 N^*} \right) \left( \frac{60\sqrt{\lambda}}{\pi N^* T_f} \right) \left( 1 - \frac{T_f}{T^{brh}} \right)
\]  

\[
+ \left( \frac{(T_{ex}^{brh})^4}{16\Delta^2 T^{brh}} \right) \left( 32 \left( \frac{T^{brh}}{T_{ex}^{brh}} \right)^4 - 1 \right)^{\frac{1}{2}} G \left( \frac{T^{brh}}{T_{ex}^{brh}} \right)
\]  

where

\[
Y_G^{br-rad}(T_f) = \left( \frac{6\bar{\alpha}}{M} \right) \left( \frac{\zeta(3)}{\pi^2} \right)^2 \left( \frac{3\lambda}{\pi^2 N^*} \right)^2 \left( \frac{1}{T_f} - \frac{1}{T^{brh}} \right)
\]  

and

\[
Y_G^{brh} \simeq Y_G^{br} = \frac{n_G}{s} = \left( \frac{360 \sqrt{2} d_3}{2\pi^2 N^* (T^{brh})^3} \right) \left( 32 \left( \frac{T^{brh}}{T_{ex}^{brh}} \right)^4 - 1 \right)^{\frac{1}{2}} \left( \frac{T^{brh}}{T_{ex}^{brh}} \right)^2 G \left( \frac{T^{brh}}{T_{ex}^{brh}} \right)
\]
The gravitino dark matter abundance and the baryon asymmetry is connected through \[209\]:

\[
\frac{Y_{\text{brh}}^{\tilde{G}}}{G} \simeq \frac{\Theta_{\text{CP}} D}{N^*} \tag{4.5.47}
\]

where \( D(\leq 1) \) is the dilution factor and the leading contribution is given by the interference between the tree level and the one-loop level decay amplitudes. In the case of the hierarchical mass spectrum for the heavy neutrinos, the lepton asymmetry in the universe is generated dominantly by CP-violating out-of-equilibrium decay of the lightest heavy neutrino. In the present context the CP-violating parameter is described as \[210\]:

\[
\Theta_{\text{CP}} = \frac{\Gamma(\psi \rightarrow \bar{l}_L H) - \Gamma(\psi \rightarrow l_L H^*)}{\Gamma(\psi \rightarrow \bar{l}_L H) + \Gamma(\psi \rightarrow l_L H^*)} = \frac{3M_1m_\nu}{16\pi v^2} \sin \delta_{\text{CP}} \tag{4.5.48}
\]

where \( m_\nu \) is the heaviest light neutrino mass, \( v = 174 \text{ GeV} \) is the VEV of Higgs and \( \delta_{\text{CP}} \) is an effective CP phase which parameterize each entries of the CKM matrix. Particularly \( \delta_{\text{CP}} \) acts as a probe of flavor structure in SUGRA theories. The complete wash out situation corresponds to \( D = 1 \).

**Figure 4.3:** Here we have plotted the variation of total gravitino abundance vs temperature in the domain \(-0.70 < D_4 < -0.60\), which clearly shows that gravitino abundance at zero temperature shoots up initially to maximum and then becomes constant with respect to temperature during radiation dominated era in braneworld scenario \[209\]. As mentioned earlier we have used the fundamental scale \( d_3 \equiv 4.596 \times 10^{-44} \tilde{\alpha} \), where \( \tilde{\alpha} \) is a dimensionless number which depends on the species of the MSSM and \( M = 2.43 \times 10^{18} \text{ GeV} \).

### 4.6 Numerical analysis for MSSM flat direction

Embedding MSSM from String theory via braneworld is a long standing unresolved problem in the context of theoretical physics. If such embedding is really possible then one can study various particle cosmological aspects from MSSM flat direction in braneworld scenario. The procedure of embedding needs to be checked throughly, but there are some positive efforts towards this field of research is already been discussed in refs. \[211\]. In this chapter we assume that such embedding is true and just as an example we have further studied the numerical estimations for MSSM D-flat directions proposed in chapter 2 section 2.3. Through out all the numerical estimation we have taken decay width \( \Gamma_\phi \simeq 2.9 \times 10^{-3} \text{ GeV} \), mass of the inflaton \( m_\phi \simeq 10^{13} \text{ GeV} \), final temperature and time at the end of reheating \( T_f \simeq 10^9 \text{ GeV} \) and \( t_f \simeq 1.4 \times 10^{31} \text{ GeV} \) respectively. Let us discuss our results in following:
For a typical value of $C_4 \simeq D_4 = -0.7$ extremum (minimum) temperature during reheating can be estimated as $T_{e3}^{bh} \simeq 7.0 \times 10^{10}$ GeV. This clearly shows deviation from standard GR phenomenology [203] where the extremum (maximum) temperature during reheating $T_{\text{max}} \simeq 1.3 \times 10^{12}$ GeV.

Similarly for $C_4 \simeq D_4 = -0.7$ the critical temperature for different particle species and gravitino abundance at different temperatures obtained from our model are: for boson $T_{c,B} \simeq 3.2 \times 10^{14}$ GeV, for fermion $T_{c,F} \simeq 3.3 \times 10^{14}$ GeV, for mixture of species $T_{c,M} \simeq 2.8 \times 10^{14}$ GeV, at reheating temperature $Y_G^{1/3}(T_{brh}) \simeq 8.1 \times 10^{-34}$ GeV$^{-3}d_3$ and at the end of reheating $Y_G^{b-rad}(T_f) \simeq 2.1 \times 10^{-13}$ GeV$^{-3}$.

Most significantly for different flat direction contents the phenomenological parameter is different and can be calculated from MSSM RGE flow at the one-loop level for that flat direction. To obtain a conservative estimate of gravitino abundance we have taken here gaugino masses $M_i \to 0$ for all gauge subgroups within MSSM. For example the fourth level flat directions $QQQL$, $QuQd$, $QuLe$, $uude$ give $\tilde{a} = 15.694$ for a specific choice of the $U_Y(1)$, $SU_L(2)$ and $SU_C(3)$ gauge couplings $g_1 = 0.56$, $g_2 = 0.72$ and $g_3 = 0.85$ respectively obtained from the universal mSUGRA boundary condition and consistent with electroweak extrapolation of the solution of MSSM RGE flow from the energy scale of brane inflation $\Delta = 0.2 \times 10^{16}$ GeV for our model.

The linear dependence on $T_{brh}$ makes simple to revise the constraints on $T_{brh}$ based on the lower limit on the gravitino abundance - the lower bound on $T_{brh}$ is increased by a factor of 1.074. Since $T_{e3}^{bh} \propto T_{brh}$, $T_{e3}^{bh}$ is not affected much. Therefore models of leptogenesis that invoke a small $T_{e3}^{bh}$ to create heavy Majorana neutrinos are not significantly affected.

Within $55 < N < 70$ and $T_{brh} \simeq 6.5 \times 10^{10}$ GeV the entropy density changes. As a consequence the total gravitino abundance changes according to fig (4.3). It is easily seen that $P = \frac{\dot{\tilde{a}}}{\tilde{a}}$, $S = \frac{\dot{\tilde{a}}P}{\tilde{a}}$ consistency relations are valid in this context.

It is worthwhile to mention here that in brane pressure and entropy density of the universe falls down to a minimum due to the minimum temperature during reheating epoch. However during radiation dominated era total entropy density is almost constant for both the cases. This clearly shows the deviation from standard GR phenomenology.

Throughout the analysis we have not included the effect of $\exp[-\Gamma_{\phi}(t - t_{\text{osc}})]$ in the energy density of inflaton $\rho_{\phi}$. One might be concerned that this will lead to inaccuracies close to $t_{brh}$ when most of the gravitinos are produced. However if one writes $\rho_{\phi} \simeq a^{-4} \exp(-\Gamma_{\phi}t) \simeq t^{-2} \exp(-\Gamma_{\phi}t)$ for $t >> t_{brh}^{bh}$ then $\dot{\rho}_{\phi}/\rho_{\phi} = -2/t - \Gamma_{\phi}$. Therefore even till close to $t_{brh} = \Gamma_{\phi}^{-1}\rho_{\phi}$ decreases primarily due to the expansion of the universe. Furthermore, near $t_{brh}$ it increases as $T^{-1/2} \simeq \sqrt{a} \simeq t^{1/4}$ in brane which is again different from GR phenomenology where $T^{-1/2} \simeq \sqrt{a} \simeq t^{1/4}$.

The thermal leptogenesis in the braneworld can take place if the lightest heavy neutrino mass lying in the range $T_{brh} < M_1 < T_c$. This confirms that the upper bound of 5D Planck mass

\footnote{We have calculated all the abundances in the fundamental unit of $d_3$ i.e. $d_3 = 6.5945 \times 10^{11}$ GeV$^3$, where $\tilde{a}$ is a dimensionless characteristic constant originated through the thermal gravitino production rate in the context of MSSM.}
$M_5 < 10^{16}$ GeV (for our model $M_5 \approx 7.8 \times 10^{15}$ GeV for $C_4 \approx D_4 = -0.7$), which coincides with the leptogenesis bound implied by the observed baryon asymmetry.

- It is important to mention here that in the standard cosmology, the thermal leptogenesis in SUGRA models is hard to be successful, since the reheating temperature after inflation is severely constrained to be $T_{reh} \leq 10^9$ GeV due to the gravitino problem. However, as pointed out in [207], the constraint on the reheating temperature is replaced by the transition temperature in the brane world cosmology. As a result the gravitino problem can be solved even if the reheating temperature is much higher. In fact, such inflation models are possible but limited and our model is also in that category.

- Here we are using a preferable value of the heaviest light neutrino mass from atmospheric neutrino oscillation data $m_{\nu} \approx 0.05$ eV and for sufficient baryon asymmetry the lightest neutrino mass $M_1 \approx 10^{10}$ GeV.

- For complete washout situation ($D = 1$) in our model the effective CP phase lying within the window $2.704 \times 10^{-9} < \delta_{CP} < 2.784 \times 10^{-9}$, where $\delta_{CP}$ is measured in degree. Most significantly it indicates that the amount of CP violation in braneworld scenario is very small and identified with the soft CP phase. Consequently it has negligibly small contribution to $K$ and $B$ physics phenomenology.

### 4.7 Chapter summary

In this chapter we have explored the phenomenological features of reheating in brane cosmology on the background of SUGRA. The major outcomes of our study are:

- We have exhibited the process of construction of a fruitful theory of reheating for an effective 4D inflationary potential in $N = 1, D = 4$ SUGRA in the brane derived from $N = 2, D = 5$ SUGRA in the bulk.

- We have employed the proposed setup in reheating model building by analyzing the reheating temperature in the context of brane inflation, followed by analytical and numerical estimation of different phenomenological parameters.

- It is worthwhile to mention that we get a lot of new results in the context of braneworld compared to standard GR case. Most importantly we get a different numerical value of reheating temperature as well the extremum temperature compared to the standard GR results. Next using the extremization principle we justify that the extremum temperature is the minimum temperature during reheating which again shows deviation from standard GR inspired phenomenology. All these facts are reflected in the numerical results of the gravitino abundance in reheating and radiation dominated era. In the context of phase transition we also get different numerical results for different parameters for standard model particle constituents.

- We have further engaged ourselves in investigating for the effect of perturbative reheating. To this end we propose a theory which reflects the effect of particle production through collision and decay thereby showing a direct connection with the thermalization phenomena. To show this internal link more explicitly we put forward both analytical and numerical expressions
for the gravitino abundance in a physical volume in the reheating epoch. Next we have found out the gravitino abundance in the radiation dominated era. Last but not the least we have expressed the total gravitino abundance in terms of final temperature during reheating. Most significantly the precision level of all estimated numerical results is the outcome of the 4D effective field theory which is analyzes with the arrival of lots of sophisticated techniques.

- Apart from the aforesaid success in estimating phenomenological parameters there are some added advantages of our model with reheating in brane which are worth mentioning. One of the most significant features in the context of braneworld phenomenology is the validity of leptogenesis for our model which consequently shows the production of heavy Majorana neutrinos in the brane.
5 Primordial non-Gaussianity from $\mathcal{N} = 1$ supergravity using $\delta N$ formalism

5.1 Introduction

The inflationary paradigm is a very rich idea to explain various aspects of the early universe, which creates the perturbations and the matter. The simplest prescription to explain inflation is made via a single scalar field slowly rolling down a flat potential, which has unique predictions for the Cosmic Microwave Background (CMB) observables. In this scenario, the induced cosmological perturbations are generically random Gaussian in nature with a small tilt and running in the primordial spectrum, which can be conveniently described in terms of two-point correlation function. But a big issue may crop up in model discrimination and also in the removal of the degeneracy of cosmological parameters obtained from CMB observations [24, 25, 156, 212]. Non-Gaussianity has emerged as a powerful observational tool to discriminate between different classes of inflationary models. The Planck data show no significant evidence in favour of primordial non-Gaussianity, the current limits [212] are yet to achieve the high statistical accuracy expected from the single-field inflationary models and for this opportunities are galore for the detection of large non-Gaussianity from various types of inflationary models. To achieve this goal, apart from the huge success of cosmological linear perturbation theory, the general focus of the theoretical physicists has now shifted towards the study of nonlinear evolution of cosmological perturbations. Typically any types of nonlinearities are expected to be small; but, that can be estimated via non-Gaussian n-point correlations of cosmological perturbations.

An overview of calculating non-Gaussianity from different methods have been discussed in chapter 1 section 1.3.3. Here we will employ the “$\delta N$ formalism” (where $N$ being the number of e-foldings) [130, 131, 132, 133, 134, 135], which is a well accepted tool for computing non-linear evolution of cosmological perturbations on large scales ($k << aH$), which is derived using the “separate universe” approach [130, 131]. Particularly, it provides a fruitful technique to compute the expression for the curvature perturbation $\zeta$ without explicitly solving the perturbed field equations from which the various local non-Gaussian parameters, $f_{NL}^{\text{local}}, \tau_{NL}^{\text{local}}, g_{NL}^{\text{local}}$ and CMB dipolar asymmetry parameter [213, 214], $A_{\text{CMB}}$ are easily computable.

We will be using the following constraints on the amplitude of the power spectrum, $P_s$, spectral tilt, $n_s$, tensor-to-scalar ratio, $r$, sound speed, $c_s$, local type of non-Gaussianity, $f_{NL}^{\text{local}}$ and $\tau_{NL}^{\text{local}}$ and CMB dipolar asymmetry from Planck data throughout the paper [24, 25, 212, 215]:

\[P_s, n_s, r, c_s, f_{NL}^{\text{local}}, \tau_{NL}^{\text{local}}, A_{\text{CMB}}\]

One can also compute all the non-Gaussian parameters, $f_{NL}^{\text{local}}, \tau_{NL}^{\text{local}}, g_{NL}^{\text{local}}$ and CMB dipolar asymmetry parameter, $A_{\text{CMB}}$ using In-In formalism in the quantum regime. But the inflationary dynamics responsible for the interactions between the modes occurs at the super-horizon scales within the effective theory setup proposed in this chapter. Here we use $\delta N$ formalism as-(1) it perfectly holds good at the super-horizon scales and (2) are also independent of any kind of intrinsic non-Gaussianities generated at the scale of horizon crossing.
5. Primordial non-Gaussianity from $\mathcal{N} = 1$ supergravity using $\delta N$ formalism

\[
\ln(10^{10} P_s) = 3.089^{+0.024}_{-0.027} \quad (\text{at } 2\sigma \text{ CL}),
\]
\[
n_s = 0.9603 \pm 0.0073 \quad (\text{at } 2\sigma \text{ CL}),
\]
\[
r \leq 0.12 \quad (\text{at } 2\sigma \text{ CL}),
\]
\[
0.02 \leq c_s \leq 1 \quad (\text{at } 2\sigma \text{ CL}),
\]
\[
f_{NL}^{\text{local}} = 2.7 \pm 5.8 \quad (\text{at } 1\sigma \text{ CL}),
\]
\[
\tau_{NL}^{\text{local}} \leq 2800 \quad (\text{at } 2\sigma \text{ CL}),
\]
\[
A_{\text{CMB}} = 0.07 \pm 0.02 \quad (\text{at } 2\sigma \text{ CL}).
\]

In this chapter we will concentrate our study for Hubble induced inflection point MSSM inflation derived from various higher dimensional Planck scale suppressed non-minimal Kähler operators in $\mathcal{N} = 1$ supergravity (SUGRA) which satisfies the observable universe, and it is well motivated for providing an example of visible sector inflation.

In section 5.2 we start our discussion with cosmological perturbation scenario for sound speed $c_s \neq 1$. Hence in section 5.3, we will describe the setup with one heavy and one light super field which are coupled via non-minimal interactions through Kähler potential. In section 5.4 we discuss very briefly the role of various types of Planck suppressed non-minimal Kähler corrections to model a Hubble induced MSSM inflation for any D-flat directions. Hence in section 5.5 we present a quantitative analysis to compute the expression for the local types of non-Gaussian parameters and CMB dipolar asymmetry parameter which characterize the bispectrum and trispectrum using the $\delta N$ formalism. For the numerical estimations we analyze the results in the context of two D-flat direction, $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$ within the framework of MSSM inflation [99].

### 5.2 Cosmological perturbations for $c_s \neq 1$

In this section we briefly recall some of the important formulæ when $c_s \neq 1$, the scalar and tensor perturbations are given by [25]:

\[
P_S(k) = \mathcal{P}_S \left( k c_s k_\star \right)^{n_s - 1},
\]
\[
P_T(k) = \mathcal{P}_T \left( k c_s k_\star \right)^{n_T},
\]

where the speed of sound at the Hubble patch is given by, $c_s k_\star = aH$ (where $k_\star \sim 0.002$ $\text{Mpc}^{-1}$).

The amplitude of the scalar and tensor perturbations can be recast in terms of the potential, as [25]:

\[
\mathcal{P}_S = \frac{V_\star}{24\pi^2 M_p^4 c_s \epsilon_V},
\]
\[
\mathcal{P}_T = \frac{2V_\star}{3\pi^2 M_p^4 \epsilon_s^2 \epsilon_V},
\]

where running of the spectral tilt for the scalar and tensor modes can be expressed at $c_s k_\star = aH$, as:

\[
n_s - 1 = 2\eta_V - 6\epsilon_V - s,
\]
\[
n_T = -2\epsilon_V.
\]
where running of the sound speed is defined by an additional slow-roll parameter, \( s \), as:

\[
s = \frac{\dot{c}_s}{Hc_s} = \sqrt{\frac{3}{V}} \frac{\dot{c}_s}{c_s} M_p. \tag{5.2.13}
\]

In all the above expressions, the standard slow-roll parameters are defined by:

\[
\epsilon_V = \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_V = M_p^2 \left( \frac{V''}{V} \right). \tag{5.2.14}
\]

Finally, the single field consistency relation between tensor-to-scalar ratio and tensor spectral tilt is modified by [25]:

\[
r_* = 16\epsilon_V c_s \frac{1 + \epsilon_V}{1 - \epsilon_V} = -8n_T c_s \frac{1 - \eta_V}{\epsilon_V}. \tag{5.2.15}
\]

Using the results for \( c_s \neq 1 \) stated in Eqs. (5.2.9-5.2.15), the upper bound on the numerical value of the Hubble parameter during inflation is given by:

\[
H \leq 9.241 \times 10^{13} \sqrt{\frac{r_*}{0.12} \frac{\epsilon_V}{c_s^{\frac{2}{3}}}} \text{ GeV} \tag{5.2.16}
\]

where \( r_* \) is the tensor-to-scalar ratio at the pivot scale of momentum \( k_* \sim 0.002 Mpc^{-1} \). An equivalent statement can be made in terms of the upper bound on the energy scale of inflation for \( c_s \neq 1 \) as:

\[
V_* \leq (1.96 \times 10^{16} \text{GeV})^4 \frac{r_*}{0.12} \frac{\epsilon_V}{c_s^{\frac{2}{3}}}. \tag{5.2.17}
\]

Here in Eqs. (5.2.16) and (5.2.17), the equalities will hold good for a high scale model of inflation.

Furthermore, for a sub-Planckian slow-roll models of inflation, one can express the tensor-to-scalar ratio, \( r_* \), at the pivot scale, \( k_* \sim 0.002 Mpc^{-1} \), in terms of the field displacement, \( \Delta \phi \), during the observed \( \Delta N \approx 17 \) e-foldings of inflation, for \( c_s \neq 1 \) [96, 120]:

\[
\frac{3}{25\sqrt{c_s}} \sqrt{\frac{r_*}{0.12}} \left\{ \frac{3}{400} \left( \frac{r_*}{0.12} \right) - \frac{n_T(k_*)}{2} \left( \frac{1}{2} \right) \right\} \approx \left| \frac{\Delta \phi}{M_p} \right|. \tag{5.2.18}
\]

where \( \Delta \phi = \phi_{\text{cmb}} - \phi_e < M_p \), where \( \phi_{\text{cmb}} \) and \( \phi_e \) are the values of the inflaton field at the horizon crossing and at the end of inflation.

### 5.3 Planck suppressed non-minimal Kähler operators within \( \mathcal{N} = 1 \) SUGRA

#### 5.3.1 The Superpotential

In this section we concentrate on two sectors; heavy hidden sector denoted by the superfield \( S \), and the light visible sector denoted by \( \Phi \) where they interact only via gravitation. Specifically the inflaton superfield \( \Phi \) is made up of \( D \)-flat direction within MSSM and they are usually lifted by the \( F \)-term [148] of the non-renormalizable operators as appearing in the superpotential. In the present setup for the simplest situation we start with the following simplified expression for the superpotential made up of the superfields \( S \) and \( \Phi \) as given by:

\[
W(\Phi, S) = W(\Phi) + W(S) = \frac{\lambda \Phi^n}{n M_p^{n-4}} + \frac{M_s}{2} S^2, \tag{5.3.19}
\]
where for MSSM D-flat directions, $n \geq 3$ (in the present context $n$ characterizes the dimension of the non-renormalizable operator) and the coupling, $\lambda \sim \mathcal{O}(1)$. The scale $M_c$ characterizes the scale of heavy physics which belongs to the hidden sector of the effective theory. Furthermore, I will assume that the VEV, $\langle s \rangle = M_s \leq M_p$ and $\langle \phi \rangle = \phi_0 \leq M_p$, where both $s$ and $\phi$ are fields corresponding to the superfield $S$ and $\Phi$. We also concentrate on two MSSM flat directions, $\bar{\tilde{L}} \bar{\tilde{L}} \bar{\tilde{e}}$ and $\bar{\tilde{u}} \bar{\tilde{d}}$, which can drive inflation with $n = 6$ via $R$-parity invariant $(\bar{\tilde{L}} \bar{\tilde{L}})(\bar{\tilde{L}} \bar{\tilde{L}})/M_p^3$ and $(\bar{\tilde{u}} \bar{\tilde{d}})(\bar{\tilde{u}} \bar{\tilde{d}})/M_p^3$ operators in the visible sector, which are lifted by themselves [44], where $\tilde{u}, \tilde{d}$ denote the right handed squarks, and $\tilde{L}$ denotes that left handed sleptons and $\bar{c}$ denotes the right handed slepton.

5.3.2 The Kähler potential

In this paper I consider the following simplest choice of the holomorphic Kähler potential which produces minimal kinetic term, and the Kähler correction of the form:

$$K(s, \phi, s^\dagger, \phi^\dagger) = \delta K_{\text{minimal part}} + \delta K_{\text{non-minimal part}},$$

(5.3.20)

where $\delta K$ represent the higher order non-minimal Kähler corrections which are extremely hard to compute from the original string theory background. In a more generalized prescription such corrections allow the mixing between the hidden sector heavy fields and the soft SUSY breaking visible sector MSSM fields. Using Eq (5.3.20) the most general $\mathcal{N} = 1$ SUGRA kinetic term for $(s, \phi)$ field can be written in presence of the non-minimal Kähler corrections through the Kähler metric as:

$$L_{Kin} = \left(1 + \frac{\partial \delta K}{\partial \phi \partial s^\dagger}\right)(\partial^\mu \phi)(\partial^\mu s^\dagger) + \left(1 + \frac{\partial \delta K}{\partial s \partial s^\dagger}\right)(\partial^\mu s)(\partial^\mu s^\dagger) + \left(\frac{\partial \delta K}{\partial \phi \partial s}\right)(\partial^\mu s^\dagger)(\partial^\mu \phi) + \left(\frac{\partial \delta K}{\partial s \partial \phi^\dagger}\right)(\partial^\mu \phi^\dagger)(\partial^\mu s).$$

(5.3.21)

In this paper I consider the following gauge invariant non-minimal Planck scale suppressed Kähler operators within $\mathcal{N} = 1$ SUGRA [44]:

$$\delta K^{(1)} = \frac{a}{M_p^2} \phi^\dagger \phi s^\dagger s + \text{h.c.} + \cdots,$$

(5.3.22)

$$\delta K^{(2)} = \frac{b}{2M_p} s^\dagger s \phi^\dagger \phi + \text{h.c.} + \cdots,$$

(5.3.23)

$$\delta K^{(3)} = \frac{c}{4M_p^2} s^\dagger s^\dagger \phi^\dagger \phi + \text{h.c.} + \cdots,$$

(5.3.24)

$$\delta K^{(4)} = \frac{d}{M_p} s \phi^\dagger \phi + \text{h.c.} + \cdots,$$

(5.3.25)

where $a, b, c, d$ are dimensionless non-minimal coupling parameters. The $\cdots$ contain higher order non-minimal terms which has been ignored in this paper.

5.4 Effective field theory potential construction from $\mathcal{N} = 1$ SUGRA

In this section we will consider two interesting possibilities, one which is the simplest and provides an excellent model for inflation with a complete decoupling of the heavy field. Inflation occurs via the slow roll of $\phi$ field within an MSSM vacuum, where inflation would end in a vacuum with an enhanced gauge symmetry, where the entire electroweak symmetry will be completely restored.
5.4.1 Heavy field is dynamically frozen

Let us first assume that the dynamics of the heavy field $s$ is completely frozen during the onset and the rest of the course of slow roll inflation driven by $\phi$. The full potential can be found in Table 5.1. Note that the potential for $s$ field, $V(s)$ contains soft term and the corresponding $A$-term:

$$V(s) \sim M_s^2 |s|^2 + A'M_s s^2,$$

where $A'$ is a dimensional quantity, and it is roughly proportional to $A' \sim M_s \gg \text{TeV}$. In this case there are two possibilities which we briefly mention below:

- We can imagine that the heavy field, $s$, to have a global minimum at:

$$\langle s \rangle = 0, \quad \dot{\langle s \rangle} = 0, \quad V(s) = 0. \quad (5.4.27)$$

In this particular setup, the kinetic terms for each cases, i.e. $1, 2, 3, 4$, become canonical for the $\phi$ field, therefore the heavy field is completely decoupled from the dynamics. One can check them from Table-5.1. This is most ideal situation for a single field dominated model of inflation, where the overall potential for along $\phi$ direction simplifies to:

$$V(\phi) = m_{\phi}^2 |\phi|^2 + \left( A \frac{\lambda \phi^n}{n M_p^{n-3}} + \text{h.c.} \right) + \lambda^2 |\phi|^{2(n-1)} M_p^{2(n-3)} \quad (5.4.28)$$

The overall potential is solely dominated by the $\phi$ field, therefore Hubble expansion rate becomes, $H_{in,f} \propto V(\phi)/M_p^2$.

In this setup inflation can occur near a saddle point or an inflection point, where $\phi_0 \ll M_p$, and $m_{\phi} \gg H_{in,f}$, first discussed in Refs. \cite{99, 100}. During inflation the Hubble expansion rate is smaller than the soft SUSY breaking mass term and the $A$-term, i.e. $A \sim m_{\phi} \gg H(t)$ for $a_H \sim c_H \sim \mathcal{O}(1)$ in Eq. (2.1.5), such that the SUGRA corrections are unimportant. This scenario has been discussed extensively, and has been extremely successful with the Planck data explaining the spectral tilt right on the observed central value, with Gaussian perturbations with the right amplitude \cite{102}.

- On the other hand, if

$$\langle s \rangle \sim M_s \ll M_p, \quad \langle \dot{s} \rangle = 0, \quad V(s) = M_s^4, \quad (5.4.29)$$

then the kinetic term for $\phi$ field will be canonical for cases $K^2$ and $K^3$ by virtue $\dot{s} = 0$, see Table-5.1. However for cases $K^1$ and $K^4$, the departure from canonical for the $\phi$ field will depend on $M_s$. If $\langle M_s \rangle \ll M_p$, and $a, d \sim \mathcal{O}(1)$, see Table-5.1, then the kinetic term for $\phi$ will be virtually canonical, and as a consequence $c_s \approx 1$, while the potential will see a modification:

$$V_{total} = M_s^4 + c_H H^2 |\phi|^2 + \left( a_H H \frac{\lambda \phi^n}{n M_p^{n-3}} + \text{h.c.} \right) + \lambda^2 |\phi|^{2(n-1)} M_p^{2(n-3)} \quad (5.4.30)$$

This large vacuum energy density, i.e. $M_s^4 \gg (\text{TeV})^4$, would yield a large Hubble expansion rate, i.e. $H_{in,f}^2 / M_s^4 / M_p^2 \gg m_{\phi}^2 \sim \mathcal{O}(\text{TeV})^2$. Therefore, the Hubble induced mass and the $A$-term would dominate the potential over the soft terms. Inspite of large mass, $c_H$, and $a_H$-term, there is no SUGRA-$\eta$ problem, provided inflation occurs near the saddle point or the inflection point \cite{102, 163}. We will not discuss this case any further, we will now focus on a slightly non-trivial scenario, where high scale physics can alter some of the key cosmological predictions.
5.4.2 Heavy field is oscillating during the onset of inflation

One dramatic way the heavy field can influence the dynamics of primordial perturbations is via coherent oscillations around its minimum, while $\phi$ still plays the role of a slow roll inflaton. Furthermore, the heavy field would only influence the first few e-foldings of inflation, once the heavy field is settled down its effect would be felt only via the vacuum energy density. Inspite of this short-lived phase, the heavy field can influence the dynamics and the perturbations for the light field as we shall discuss below.

Let us imagine the heavy field is coherently oscillating around a VEV, $\langle s \rangle \sim M_s$, during the initial phase of inflation, such that

$$V(s) \neq 0, \quad \langle s \rangle \neq 0, \quad \langle \dot{s} \rangle \neq 0.$$  \hspace{1cm} (5.4.31)

The origin of coherent oscillations of $s$ field need not be completely ad-hoc, such a scenario might arise quite naturally from the hidden sector moduli field which is coherently oscillating before being damped away by the initial phase of inflation, see for instance [217]. This is particularly plausible for high string scale moduli, where the moduli mass can be heavy and can be stabilised early on in the history of the universe. There could also be a possibility of a smooth second order phase transition from one vacuum to another during the intermittent phases of inflation [218]. Such a possibility can arise within MSSM where there are multiple false vacua at high energies [219].

Irrespective of the origin of this heavy field, during this transient period, the heavy field with an effective mass, $M_s \gg H_{inf}$, can coherently oscillate around its vacuum. We can set its initial amplitude of the oscillations to be of the order $M_s$.

$$s(t) \sim M_s - M_s \sin(M_s t).$$ \hspace{1cm} (5.4.32)

This also implies that at the lowest order approximation, $\langle s \rangle \sim M_s$ and $\langle \dot{s} \rangle \sim M_s^2$. The contribution to the potential due to the time dependent oscillating heavy field, see Eq (5.4.32), is averaged over a full cycle $(0 < t_{osc} < H_{inf}^{-1})$ is given by:

$$\langle V(s) \rangle \approx M_s^2 \langle s^2(t) \rangle \sim H_{inf}^2 M_p^2$$ \hspace{1cm} (5.4.33)

The $s$ field provides at the lowest order corrections to the kinetic term for the $\phi$ field, and to the overall potential, see Table (5.1), for both kinetic and potential terms.

At this point one might worry, the coherent oscillations of the $s$ field might trigger particle creation from the time dependent vacuum, see Refs. [197, 220], for a review see [11]. First of all, if we assume that the heavy field is coupled to other fields gravitationally, then the particle creation may not be sufficient to back react into the inflationary potential. Furthermore, inflation would also dilute the quanta created during this transient phase.

Since the kinetic terms for the 4 cases tabulated in Table (5.1) are now no longer canonical, they

\footnote{There could be other scenarios where the influence of heavy field is felt throughout the inflationary dynamics, see for instance in Refs. [52, 216]. Here we will discuss a slightly simpler scenario where both heavy and light sectors are coupled gravitationally via the Kähler correction.}

\footnote{At this point one might say why we had taken the amplitude of oscillations for the heavy field to be $M_s$. In some scenarios, it is possible to envisage the amplitude of the oscillations to be $M_p$. This would not alter much of our discussion, therefore for the sake of simplicity we will consider the initial amplitude for the $s$ field to be displaced by $M_s$, the same as that of the VEV.}
would contribute to the speed of sound, \( c_s \neq 1 \), which we can summarize case by case below:

\[
c_s = \sqrt{\frac{\hat{p}}{\hat{p}}}, \quad \hat{p} = \frac{1}{2} \left( X_1(t) - X_2(t) - \dot{V} \right) \frac{X_1(t) + X_3(t) + \dot{V}}{X_1(t) + X_3(t) + \dot{V}}
\]

for Case I

\[
\frac{\hat{Y}_1(t) - \hat{Y}_2(t) - \dot{V}}{\hat{Y}_1(t) + \hat{Y}_3(t) + \dot{V}}
\]

for Case II

\[
\frac{\hat{Z}_1(t) - \hat{Z}_2(t) - \dot{V}}{\hat{Z}_1(t) + \hat{Z}_3(t) + \dot{V}}
\]

for Case III

\[
\frac{\hat{W}_1(t) - \hat{W}_2(t) - \dot{V}}{\hat{W}_1(t) + \hat{W}_3(t) + \dot{V}}
\]

for Case IV.

where \( \hat{p} \) is the effective pressure and \( \hat{p} \) is the energy density. The dot denotes derivative w.r.t. physical time, \( t \). All the symbols, i.e. \( X_1, X_2, Y_1, Y_2, Z_1, Z_2, W_1, W_2 \), appearing in Eq (5.5.60) are explicitly mentioned in the Appendix F. Additionally, here we have defined, \( \hat{V} = V(\phi) - V(s) \).

### 5.4.3 Constraining non-renormalizable operators, i.e. \( a, b, c, d, \) and \( M_s \)

For the potential under consideration, we have \( V(s) = 3H^2M_p^2 \sim M_s^2s^2 \gg m_\phi^2|\phi|^2 \), where \( m_\phi \sim \mathcal{O}(\text{TeV}) \) is the soft mass. In this case the contributions from the Hubble-induced terms are important compared to the soft SUSY breaking mass, \( m_\phi \), and the A term for all the four cases tabulated in Table-(5.1). After stabilizing the angular direction of the complex scalar field \( \phi = |\phi|\exp[i\theta] \) [99, 100, 163], reduces to a simple form along the real direction, which is dominated by a single scale, i.e. \( H \sim H_{inf} \):

\[
V(\phi) = V(s) + c_HH^2|\phi|^2 - \frac{a_HH\phi^n}{nM_p^{n-3}} + \frac{\lambda^2|\phi|^{2(n-1)}}{M_p^{2(n-3)}},
\]

where we take \( \lambda = 1 \), and, the Hubble-induced mass parameter \( c_H \), for \( s << M_p \), is defined as:

\[
c_H = \begin{cases} 
3(1 - a), & \text{for Case I} \\
3(1 + b^2), & \text{for Case II} \\
3, & \text{for Case III} \\
3(1 + d^2), & \text{for Case IV} 
\end{cases}
\]

Note that for only third case, i.e. \( K^2 \), the Hubble induced mass term does not contain any Kähler correction, i.e. \( \delta K \). Similarly, we can express \( a_H \), see Appendix H for full expressions. Note that for all 4 cases, the kinetic terms are all non-minimal, and we have already listed in Table-(5.1). Fortunately for this class of potential given by Eq (5.4.35), inflection point inflation can be

\footnote{As a side remark, our analysis will be very useful for the Affleck-Dine (AD) baryogenesis [44], especially when the minimum of the AD field is rotating in presence of the inflaton oscillations. Effectively, the AD field will have non-canonical kinetic terms, this has never been taken into account in the literature and one should take the non-canonical kinetic terms for the AD field in presence of the inflaton oscillations in order to correctly estimate the baryon asymmetry. The role of \( s \) field will be that of an inflaton and \( \phi \) field will be that of an AD field.}

\footnote{See Appendix-G and Appendix-H for details.}
accommodated, when $a_H^2 \approx 8(n - 1)c_H$. This can be characterized by a fine-tuning parameter, $\delta$, which is defined as \[ 100 \]:

$$
a_H^2 = 8(n - 1)c_H = 1 - \left( \frac{n - 2}{2} \right)^2 \delta^2. \tag{5.4.37}
$$

When $|\delta|$ is small $^6$, a point of inflection $\phi_0$ exists, such that $V''(\phi_0) = 0$, with

$$
\phi_0 = \left( \frac{c_H}{(n - 1)}HM_p^{n-3} \right)^{1/n-2} + \mathcal{O}(\delta^2). \tag{5.4.38}
$$

For $\delta < 1$, one can Taylor-expand the inflaton potential around an inflection point, $\phi = \phi_0$, as \[ 101 \], \[ 102 \], \[ 163 \], \[ 219 \]:

$$
V(\phi) = \alpha + \beta(\phi - \phi_0) + \gamma(\phi - \phi_0)^3 + \kappa(\phi - \phi_0)^4 + \cdots, \tag{5.4.39}
$$

where $\alpha$ denotes the height of the potential, and the coefficients $\beta$, $\gamma$, $\kappa$ determine the shape of the potential in terms of the model parameters $^7$. Note that once the numerical values of $c_H$ and $H$ are specified, all the terms in the potential are determined.

For $c_s \neq 1$, the upper bound on the numerical value of the scale of the heavy string moduli field ($M_s$) are expressed as:

$$
M_s \leq 1.77 \times 10^{16} \times \left( \frac{r_\ast}{0.12} \right) c_s^{\frac{2.56}{5/8 \sqrt{c_s} - 1}} \text{GeV}. \tag{5.4.40}
$$

where $r_\ast$ is the tensor-to-scalar ratio at the pivot scale of momentum $k_\ast \sim 0.002 \text{Mpc}^{-1}$.

5.5 Calculation of non-Gaussianity in $\delta N$ for $c_s \neq 1$

In this section we have used the $\delta N$ formalism \[ 130 \], \[ 131 \], \[ 132 \], \[ 133 \], \[ 134 \], \[ 135 \] to compute the local type of non-Gaussianity, $f_{NL}^{\text{local}}$ from the prescribed setup for $c_s \neq 1$. In the non-attractor regime, the $\delta N$ formalism shows various non trivial features which has to be taken into account during explicit calculations.

It is important to mention that during the computation the solution attains the attractor behaviour in the present context and consequently the significant contribution comes from only on the cosmological perturbations of the scalar field trajectories with respect to the inflaton field value defined at the initial hypersurface, $\phi$. This is because the velocity of the inflaton field, $\dot{\phi}$, is solely characterized by the inflaton field $\phi$. Nonetheless, during the computation, specifically in the non-attractor regime of physical solution, both the information from the inflaton field value $\phi$ and also the velocity of the inflaton field $\dot{\phi}$ are necessarily required to determine the cosmological trajectory \[ 221 \] relevant for present discussion.

Additionally it is important to note that, to compute the scalar-field cosmological trajectories explicitly, here we first of all we need to solve the classical equation of motion of the scalar field and this is in general a second-order differential equation in a prescribed theoretical background. This can be solved by providing two initial conditions on the inflaton field $\phi$ and he velocity of the inflaton field, $\dot{\phi}$ on the initial hypersurface.

$^6$We will consider a moderate tuning of order $\delta \sim 10^{-4}$ between $c_H$ and $a_H$.

$^7$The analytical expressions for the co-efficients appearing in the inflection point potential, $\alpha, \beta, \gamma$ and $\kappa$, can be expressed in terms of the mass parameter $c_H$, Hubble scale $H$ and, the VEV of the inflaton $\phi_0$ and tuning parameter $\delta$ are explicitly mentioned in the appendix E.
| Non-minimal Kähler potential | Non-canonical kinetic term | Potential $V(\phi)$ for $|s| \ll M_p$ |
|-----------------------------|---------------------------|-------------------------------------|
| $K^{(1)} = \phi^1 \phi + s^1 s$ | $\mathcal{L}_{\text{kin}} = \left(1 + \frac{|s|}{M_p^2}\right)(\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}\phi)$ | $V(s) + \left(m_\phi^2 + 3(1 - a)H^2\right)|\phi|^2 - \frac{A\phi^3}{nM_p^{n-3}}$ |
| $+ \frac{b}{2M_p} s^1 \phi + h.c.$ | $+ \frac{b s^1}{2M_p}(\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ | $- \left(1 - \frac{3}{n}\right)\phi + \frac{b s^1}{nM_p}$ $\phi^3 M_p^{-n+1}$ |
| $K^{(2)} = \phi^1 \phi + s^1 s$ | $+ \frac{b s^1}{2M_p}(\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ | $- \left(1 - \frac{3}{n}\right)\phi + \frac{b s^1}{nM_p}$ $\phi^3 M_p^{-n+1}$ |
| $+ \frac{b}{2M_p} s^1 \phi + h.c.$ | $+ \frac{b s^1}{2M_p}(\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ | $- \left(1 - \frac{3}{n}\right)\phi + \frac{b s^1}{nM_p}$ $\phi^3 M_p^{-n+1}$ |
| $K^{(3)} = \phi^1 + s^1$ | $+ \frac{c s^1}{4M_p} (\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ | $+ \frac{c s^1}{4M_p} (\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ |
| $+ \frac{c s^1}{4M_p} (\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ | $+ \frac{c s^1}{4M_p} (\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ | $+ \frac{c s^1}{4M_p} (\partial_{\phi}\phi)(\partial_{\phi}^{\dagger}s^1)$ |
| $K^{(4)} = \phi^1 + s^1$ | $+ \frac{d}{4M_p} s^1 \phi + h.c.$ | $+ \frac{d}{4M_p} s^1 \phi + h.c.$ |
| $+ \frac{d}{4M_p} s^1 \phi + h.c.$ | $+ \frac{d}{4M_p} s^1 \phi + h.c.$ | $+ \frac{d}{4M_p} s^1 \phi + h.c.$ |

Table 5.1: Various supergravity effective potentials and non-canonical kinetic terms for $|s| \ll M_p$ in presence non-minimal Kähler potential [54]. Here both $\phi$ and $s$ are complex fields, and so are the $A$-terms.
5.5.1 General conventions

In the present context, further we have neglected the canonical kinetic term during the non-attractor phase for simplicity. The background equation of motion for the four physical situations are given by

\[
0 = \begin{cases} 
(\ddot{\phi} + 3H\dot{\phi}) \left[ 1 + \frac{2M^2}{M_p^2} (1 + \sin(M_st))^2 \right] + \frac{2aM^2}{M_p^2} \left[ \left( 2\dot{\phi} + 3H\phi \right) \cos(M_st) - \phi M_s \sin(M_st) \right] (1 + \sin(M_st)) + V'(\phi) & \text{for Case I} \\
\ddot{\phi} + 3H\dot{\phi} + \frac{bM^2}{M_p^2} \left[ \left( \dot{\phi} + 3H\phi \right) \cos(M_st) (1 + \sin(M_st)) + M_s\phi \cos^2(M_st) - \sin(M_st) (1 + \sin(M_st))) \right] + V'(\phi) & \text{for Case II} \\
\ddot{\phi} + 3H\dot{\phi} \left[ 1 + \frac{2dM^2}{M_p^2} (1 + \sin(M_st)) \right] + \frac{2dM^2}{M_p^2} \left[ \left( 2\dot{\phi} + 3H\phi \right) \cos(M_st) - \phi M_s \sin(M_st) \right] + V'(\phi) & \text{for Case III} \\
\ddot{\phi} + 3H\dot{\phi} \left[ 1 + \frac{2eM^2}{M_p^2} (1 + \sin(M_st)) \right] + \frac{2eM^2}{M_p^2} \left[ \left( 2\dot{\phi} + 3H\phi \right) \cos(M_st) - \phi M_s \sin(M_st) \right] + V'(\phi) & \text{for Case IV} 
\end{cases}
\]  

(5.5.41)

From the Eq (5.5.41), it is obvious that the determination of a general analytical solution is too much complicated. To simplify the task here we consider a particular solution,

\[
\phi = \phi_L \propto e^{\theta H t} \quad (\text{i.e.} \quad \phi = \phi_L(N) = \phi_s e^{-\theta N}),
\]

(5.5.42)

and further my prime objective is to obtain a more generalized solution for the background up to the second order in perturbations around this particular solution. Here we also assume that the non-attractor phase ends when the inflaton field value is achieved at \( \phi = \phi_s \). Let me define a perturbative parameter,

\[
\Delta \equiv \phi - \phi_0 - \phi_L = \Delta_1 + \Delta_2 + \cdots,
\]

which represents the difference between the true background solution and the reference solution to solve the background Eq (5.5.41) perturbatively. Here \( \Delta_1 \) and \( \Delta_2 \) are the general linearized and second order perturbative solution of the background field equations. The \( \cdots \) contribution comes from the higher order perturbation scenario which we will neglect for further computation.

5.5.2 Linearized perturbation

Let us consider the contribution from the linear perturbation, \( \Delta_1 \). Consequently in the leading order the background linearized perturbative equation of motion takes the following form:

\[
0 \approx \begin{cases} 
\left( \ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L \right) \left[ 1 + \frac{aM^2}{4M_p^2} \right] - \frac{aM^3}{M_p^2} (\Delta_1 + \phi_L) M_s + \beta & \text{for Case I} \\
\ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L + \beta & \text{for Case II} \\
\ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L + \beta & \text{for Case III} \\
\left( \ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L \right) \left[ 1 + \frac{2dM^2}{M_p^2} \right] + \beta & \text{for Case IV} 
\end{cases}
\]  

(5.5.43)
where we have neglected the higher powers of $\Delta_1$ in the linearized approximation. The general solution is given by

$$\Delta_1 \approx \begin{cases} 
\frac{1}{2} \left( -3H - \sqrt{\frac{4aM_p^2}{M_P^2} + 9H^2} \right) t + C_1 e^{\frac{1}{2} \left( -3H + \sqrt{\frac{4aM_p^2}{M_P^2} + 9H^2} \right) t} & \text{for Case I} \\
C_3 - \frac{C_4}{3H} e^{-3Ht} - \frac{\beta t}{3H} - \phi_s e^{\phi_H t} & \text{for Case II} \\
C_5 - \frac{C_6}{3H} e^{-3Ht} - \frac{\beta t}{3H} - \phi_s e^{\phi_H t} & \text{for Case III} \\
C_7 - \frac{C_8}{3H} e^{-3Ht} - \frac{\beta t}{3H(1 + \frac{24M_p}{aM^2})} - \phi_s e^{\phi_H t} & \text{for Case IV.}
\end{cases}$$

(5.5.44)

where $C_i \forall i(= 1, 2, ..., 8)$ are dimensionful arbitrary integration constants which can be fixed by imposing the boundary conditions.

### 5.5.3 Second-order perturbation

Next we have considered the contribution from the second-order perturbation, $\Delta_2$. Consequently in the leading order the background Second-order perturbative equation of motion takes the following form:

$$\Pi_s \approx \begin{cases} 
\left( \Delta_2 + 3H \Delta_2 + \vartheta H^2 (3 + \vartheta) \phi_L \right) \left[ 1 + \frac{aM^2}{M_P^2} \right] - \frac{aM^2}{M_P^2} (\Delta_2 + \phi_L) M_s + \beta & \text{for Case I} \\
\Delta_2 + 3H \Delta_2 + \vartheta H^2 (3 + \vartheta) \phi_L + \beta & \text{for Case II} \\
(\Delta_2 + 3H \Delta_2 + \vartheta H^2 (3 + \vartheta) \phi_L) \left[ 1 + \frac{24M_p}{aM^2} \right] + \beta & \text{for Case III} \\
(\Delta_2 + 3H \Delta_2 + \vartheta H^2 (3 + \vartheta) \phi_L) \left[ 1 + \frac{24M_p}{aM^2} \right] + \beta & \text{for Case IV.}
\end{cases}$$

(5.5.45)

where the source term, $\Pi_s$, for the sub-Planckian Hubble induced inflation point inflation within $N = 1$ SUGRA is given by

$$\Pi_s = 3\gamma (\Delta_1 + \phi_L)^2.$$  

(5.5.46)

Now to solve Eq (5.5.45) in presence of non-linear source term, let me assume that the contribution from $\phi_L$ is sub-dominant. Consequently the general solution in presence of second-order perturbation is given by:

$$\Delta_2 \approx \begin{cases} 
\frac{1}{2} \left( -3H - \sqrt{\frac{4aM_P^2}{M_P^2} + 9H^2} \right) t + \frac{1}{2} \left( -3H + \sqrt{\frac{4aM_P^2}{M_P^2} + 9H^2} \right) t + G_1 e^{\frac{1}{2} \left( -3H + \sqrt{\frac{4aM_P^2}{M_P^2} + 9H^2} \right) t} + G_2 e^{\frac{1}{2} \left( -3H - \sqrt{\frac{4aM_P^2}{M_P^2} + 9H^2} \right) t} + \Sigma_s(t) & \text{for Case I} \\
G_3 - \frac{G_4}{3H} e^{-3Ht} + \Xi_s(t) & \text{for Case II} \\
G_5 - \frac{G_6}{3H} e^{-3Ht} + \Psi_s(t) & \text{for Case III} \\
G_7 - \frac{G_8}{3H} e^{-3Ht} + \Theta_s(t) & \text{for Case IV.}
\end{cases}$$

(5.5.47)

where the time dependent functions $\Sigma_s(t), \Xi_s(t), \Psi_s(t)$ and $\Theta_s(t)$ are explicitly mentioned in the Appendix J. Here $G_i \forall i(= 1, 2, ..., 8)$ are dimensionful arbitrary integration constants which can be fixed by imposing the boundary conditions.
5.5.4 $\delta N$ at the final hypersurface

Here our prime objective is to compute the perturbations of the number of e-folds, $\delta N$. The truncated background solution of $\phi$ up to the second-order perturbations around the reference trajectory, $\phi_L \propto e^{-\theta N}$ in terms of $N$ is given by,

$$
\phi = \phi_0 + \left\{ \begin{array}{l}
\frac{\phi_*}{1+C_1+C_2 - \frac{3M^2}{\alpha_*M_*^2} + G_1 + G_2 + \Sigma(t)} \left( e^{-\theta N} + \hat{\Delta}_1(N) + \hat{\Delta}_2(N) \right) \quad \text{for Case I} \\
\frac{\phi_*}{1+C_3 - \frac{C_3}{G_1} + G_3 + \frac{12G_3}{\alpha_*M_*^2} + \Psi_s(0)} \left( e^{-\theta N} + \hat{\Delta}_1(N) + \hat{\Delta}_2(N) \right) \quad \text{for Case II} \\
\frac{\phi_*}{1+C_5 - \frac{C_5}{G_5} + G_5 + \frac{12G_5}{\alpha_*M_*^2} + \Theta_s(0)} \left( e^{-\theta N} + \hat{\Delta}_1(N) + \hat{\Delta}_2(N) \right) \quad \text{for Case III} \\
\frac{\phi_*}{1+C_7 - \frac{C_7}{G_7} + G_7 + \frac{12G_7}{\alpha_*M_*^2} + \Theta_s(0)} \left( e^{-\theta N} + \hat{\Delta}_1(N) + \hat{\Delta}_2(N) \right) \quad \text{for Case IV}.
\end{array} \right.
$$

(5.5.48)

where the symbol $\hat{\cdot}$ is introduced to rescale the integration constants as well as the perturbative solutions by $\phi_*$. Here we have neglected the contribution from $e^{-\theta N}$ in $\hat{\Delta}_2(N)$ to avoid over counting in the Eq (5.5.48). It is important to note that in the present context of all these sets of scaled integration constants parameterizes different trajectories, and we have set $\phi(0, \tilde{W}_k) = \phi_*$ for any value of $\tilde{W}_k \forall k = [1, 2, [3, 4, [5, 6, [7, 8]])$ in accordance with the assumption that the end of the non-attractor phase is determined only by the value of the scalar field, $\phi = \phi_*$. Here $\tilde{W}_k = \tilde{C}_k, \tilde{G}_k$ represent collection of all integration constants.

Further inverting Eq (5.5.48) for a fixed set of $\tilde{W}_k$, we have obtained $N$ as an implicit function of $\phi$ and $\tilde{W}_k$. Then the $\delta N$ formula can be expressed in the present context as:

$$
\delta N = N(\phi + \delta \phi, \tilde{W}_k) - N(\phi, 0) = \sum_k \sum_{n,m} \frac{1}{n!m!} \frac{\partial^n \phi}{\partial \phi^n} \frac{\partial^m \tilde{W}_k}{\partial \tilde{W}_k^m} N(\phi, 0) \delta \phi^n \tilde{W}_k^m.
$$

(5.5.49)

Here we have introduced the shift in the inflaton field $\phi \to \phi + \delta \phi$ and the number of e-folds $N \to N + \delta N$ on both sides of Eq (5.5.48) to compute $\delta N$ iteratively. In the present context we have obtained, perturbative solutions of the scalar-field trajectories around the particular reference solution, $\phi_L = \phi_* e^{\theta H t}$, which are valid only when the perturbed trajectories are not far away from the reference solution. Additionally, since we have neglected the sub-dominant solution, $\Delta_1 \propto e^{\theta H t}$, my approximation holds good only at sufficiently late times. These imply that here we should choose the initial time as close as possible to the final time for which $N \lesssim 1$. Then the simplest choice is to take the initial time to be infinitesimally close to $t = t_*$.

Now perturbing the number of e-folds $N$ up to the second order at the epoch $t = t_*$, we get [222]:

$$
\zeta = \delta N = N_\phi \delta \phi + \frac{1}{2} N_{\phi \phi} \delta \phi^2 + \frac{1}{6} N_{\phi \phi \phi} \delta \phi^3 + \cdots.
$$

(5.5.50)

where we have used the $\tilde{W}$-independence of $N$ at $N = 0$ for which, $N_{\tilde{W}_k} = 0 = N_{\tilde{W}_q} \tilde{W}_q$. Here $\cdots$ corresponds to the higher order contributions, which are negligibly small compared to the leading order contributions. By taking the derivatives of both sides of Eq. (5.5.48) and setting $N = 0 = \tilde{W}_k (= \tilde{C}_k, \tilde{G}_k) \forall k$ at the end, my next task is to identify $\delta \phi_*$ and $\tilde{W}_k (= \tilde{C}_k, \tilde{G}_k)$ which are actually generated from quantum fluctuations on flat slicing, $\delta \phi$. To serve this purpose let me consider the evolution of $\delta \phi$ on super-horizon scales. The shift in the inflaton field can be expressed...
5.5 Calculation of non-Gaussianity in $\delta N$ for $c_s \neq 1$

Here as:

$$\delta \phi(N) = \delta \phi_1(N) + \delta \phi_2(N) = \phi_s \left( \hat{\Delta}_1(N) + \hat{\Delta}_2(N) \right)$$  \hspace{1cm} (5.5.51)

where the subscript “1” and “2” represent the solution at the linear and the second order respectively. It is important to note that both the solutions include the features of growing and decaying mode. Now imposing the boundary condition from the end of the non-attractor phase, where $N = 0$, we get:

$$\delta \phi_1(0) = \delta \phi_{1*} = \phi_s \hat{\Delta}_1(0) = \begin{cases} \phi_s \left( \hat{C}_1 + \hat{C}_2 - \frac{\beta M^2}{a \phi_s M^2} \right) & \text{for Case I} \\ \phi_s \left( \hat{C}_3 - \frac{\beta}{3 H} \right) & \text{for Case II} \\ \phi_s \left( \hat{C}_5 - \frac{\beta}{3 H} \right) & \text{for Case III} \\ \phi_s \left( \hat{C}_7 - \frac{\beta}{3 H} \right) & \text{for Case IV} \end{cases}$$  \hspace{1cm} (5.5.52)

$$\delta \phi_2(0) = \delta \phi_{2*} = \phi_s \hat{\Delta}_2(0) = \begin{cases} \phi_s \left( \hat{G}_1 + \hat{G}_2 + \hat{\xi}_s(0) \right) & \text{for Case I} \\ \phi_s \left( \hat{G}_3 - \frac{12 H^2}{25} + \hat{\xi}_s(0) \right) & \text{for Case II} \\ \phi_s \left( \hat{G}_5 - \frac{12 H^2}{25} + \hat{\zeta}_s(0) \right) & \text{for Case III} \\ \phi_s \left( \hat{G}_7 - \frac{12 H^2}{25} + \hat{\theta}_s(0) \right) & \text{for Case IV} \end{cases}$$  \hspace{1cm} (5.5.53)

from which we have obtained:

$$\delta \phi_s = \delta \phi(0) = \delta \phi_{1*} + \delta \phi_{2*} = \phi_s \left( \hat{\Delta}_1(0) + \hat{\Delta}_2(0) \right).$$  \hspace{1cm} (5.5.54)

Further neglecting the mixing between the solutions corresponding to the linearized and second order perturbation, the analytical expression for $\delta N$ can be expressed as:

$$\zeta = \delta N = -\frac{(\delta \phi_{1*} + \delta \phi_{2*})}{\delta \phi_s} + \frac{(\delta \phi_{1*} + \delta \phi_{2*})}{2 \delta \phi_s^2} + \cdots$$  \hspace{1cm} (5.5.55)

### 5.5.5 Computation of local type of non-Gaussianity and CMB dipolar asymmetry

Further using the results obtained for $\delta N$ as mentioned in the in chapter 1 section 1.3.3, the non-Gaussian parameter corresponding to the local type of non-Gaussianity $f_{NL}^{local}$, $g_{NL}^{local}$ and $\tau_{NL}^{local}$ can be computed as:

$$f_{NL}^{local} = \frac{5}{6} N_{,\phi \phi} N_{,\phi}^2 + \cdots = \frac{5 \theta}{6} + \cdots$$  \hspace{1cm} (5.5.56)

$$\tau_{NL}^{local} = \frac{N_{,\phi \phi}^2}{N_{,\phi}^4} + \cdots = \theta^2 + \cdots$$  \hspace{1cm} (5.5.57)

$$g_{NL}^{local} = \frac{25}{54} N_{,\phi \phi \phi} N_{,\phi}^3 + \cdots = \frac{25 \theta^2}{108} + \cdots$$  \hspace{1cm} (5.5.58)
where the parameter $\vartheta$, appearing in all the physical situations, can be expressed in terms of the sound speed ($c_s$), potential dependent slow roll parameter ($\epsilon_V,\eta_V$) and the model parameters ($\alpha,\beta,\gamma$) as:

$$\vartheta \approx \left[ \eta_V \left( 1 + \frac{1}{c_s^2} \right)^2 + \epsilon_V \left( 1 - \frac{1}{c_s^2} \right) \right] \approx \left[ \frac{6\gamma \phi_* M_p^2}{\alpha} \left( 1 + \frac{1}{c_s^2} \right)^2 + \frac{\beta^2 M_p^2}{2\alpha^2} \left( 1 - \frac{1}{c_s^2} \right) \right]. \quad (5.5.59)$$

where the sound speed $c_s$ can be expressed in terms of non-canonical Kähler corrections, $a,b,c,d$ and the scale of heavy field, $M_s$, as:

$$c_s \approx \left[ \frac{\Sigma_1(t) - \Sigma_2(t) - \hat{V}}{\Sigma_1(t) + \Sigma_3(t) + \hat{V}} \right]_{t=t_*} \quad (5.5.60)$$

The dot denotes derivative w.r.t. physical time, $t$. Here $\hat{V} = V(\phi) - V(S)$ and the symbol $\Sigma = X,Y,Z,W$, appearing for the four cases in Eq (5.5.60) are mentioned in the Appendix F.

Additionally, it is important to note that the well-known Suyama-Yamaguchi consistency relation [223, 224] between the three and four point non-Gaussian parameters, $f_{NL}^{\text{local}},f_{NL}^{\text{local}}$ and $g_{NL}^{\text{local}}$ violates [142, 225] in the present context due to the appearance of $\cdots$ terms in Eq (5.5.59). As the contributions form $\cdots$ terms are positive, the consistency relation is modified as:

$$g_{NL}^{\text{local}} = \frac{25}{108} f_{NL}^{\text{local}} = \frac{9}{25} \left( f_{NL}^{\text{local}} \right)^2 + \cdots. \quad (5.5.61)$$

However, it is important to note that since $\cdots$ terms are small, the amount of violation is also small.

Further using $\delta N$ formalism, the CMB dipolar asymmetry parameter for single field inflationary framework can be expressed as [226]:

$$A_{\text{CMB}} = \frac{1}{4} \frac{\Delta P_s(k)}{P_s(k)} \approx \frac{1}{2} \frac{\Delta(\delta N)}{\delta N} = \frac{3}{5} f_{NL}^{\text{local}} |N_\phi \Delta \phi| + \frac{27}{50} f_{NL}^{\text{local}} |N_\phi \Delta \phi|^2 + \cdots \quad (5.5.62)$$

where $|N_\phi \Delta \phi| < 1$ for which the perturbative expansion is valid here. In it the field excursion $\Delta \phi = \phi_{\text{cmb}} - \phi_* \approx \phi_* - \phi_e$. Hence substituting Eq (5.5.61) in Eq (5.5.62) we derive the following simplified expression in terms of the tensor-to-scalar ratio and sound speed as:

$$A_{\text{CMB}} = \frac{1}{2} \frac{|\Delta \phi|}{\phi_*} + \frac{1}{8} \left( \frac{|\Delta \phi|}{\phi_*} \right)^2 + \cdots \approx \frac{3 M_p}{50 \phi_* \sqrt{c_s}} \left[ \frac{3}{400} \left( \frac{r_*}{0.12} \right) - \frac{3\gamma \phi_* M_p^2}{\alpha} - \frac{1}{2} \right] + \cdots \quad (5.5.63)$$

where $|\Delta \phi|/\phi_* < 1$ in the present sub-Planckian setup.

### 5.5.6 Constraining local type of non-Gaussianity and CMB dipolar asymmetry via multi parameter scanning

Our present job is to scan the parameter space for $c_H$, $a_H$, by fixing $\lambda = \mathcal{O}(1)$ and $\delta \sim 10^{-4}$. In order to satisfy the inflationary paradigm, the Planck observational constraints, as stated in the introduction of this chapter, we obtain the following constraints on our parameters for $H_{\text{inf}} \geq$
\[ m_\phi \sim \mathcal{O}(\text{TeV}) : \]
\[ c_H \sim \mathcal{O}(10 - 10^{-6}) , \quad (5.5.64) \]
\[ a_H \sim \mathcal{O}(30 - 10^{-3}) , \quad (5.5.65) \]
\[ M_s \sim \mathcal{O}(9.50 \times 10^{10} - 1.77 \times 10^{16}) \text{ GeV} , \quad (5.5.66) \]

Inflation would not occur outside the scanning region since, at least, one of the constraints would be violated. Note that for the above ranges, the VEV of the inflaton, \( \langle \phi \rangle = \phi_0 \), gets automatically fixed by Eq. (5.4.38), in the sub-Planckian scale as:
\[ \phi_0 \sim \mathcal{O}(10^{14} - 10^{17}) \text{ GeV} \quad (5.5.67) \]

which bounds the tensor-to-scalar ratio within, \( 10^{-22} \leq r_s \leq 0.12 \) for the present setup. This analysis will further constrain the non-minimal Kähler coupling parameters \( a, b, c, d \) appearing in the higher dimensional Planck scale suppressed operators within the following range:
\[ a \sim \mathcal{O}(1 - 0.99) , \quad (5.5.68) \]
\[ b \sim \mathcal{O}(1 - 0.92) , \quad (5.5.69) \]
\[ c \sim \mathcal{O}(0.3 - 1) , \quad (5.5.70) \]
\[ d \sim \mathcal{O}(1 - 0.5) . \quad (5.5.71) \]

In Fig (5.2) and Fig (5.3) we have shown the behaviour of the local type of non-Gaussian parameter \( f_{NL}^{local} \) and \( \tau_{NL}^{local} \) with respect to the sound speed \( c_s \) in the Hubble induced inflationary regime (\( H >> m_\phi \)). In Fig (5.2), the shaded yellow region represent the allowed parameter space for Hubble induced inflation which satisfies the combined Planck constraints on the \( f_{NL}^{local} \) and \( c_s \). For all the four cases, the region above the \( f_{NL}^{local} = 8.5 \) line is observationally excluded by the Planck data. The four distinctive features are obtained by varying the model parameters of the effective theory of \( \mathcal{N} = 1 \) SUGRA, \( c_H, a_H \) and \( M_s \) subject to the constraint as stated in Eq (5.5.64-5.5.67). As Planck puts an upper bound, \( \tau_{NL}^{local} \leq 2800 \), the rest of the region above the \( \tau_{NL}^{local} = 2800 \) line in Fig (5.3) is excluded. In the present setup we have obtained the following stringent bound on the \( f_{NL}^{local} \), \( \tau_{NL}^{local} \) and \( g_{NL}^{local} \) within the following range:
\[ 5 \leq f_{NL}^{local} \leq 8.5 , \quad 100 \leq \tau_{NL}^{local} \leq 2800 , \quad 23.2 \leq g_{NL}^{local} \leq 648.2 \text{ GeV} \quad \text{for Case I} \]
\[ 1 \leq f_{NL}^{local} \leq 8.5 , \quad 150 \leq \tau_{NL}^{local} \leq 2800 , \quad 34.7 \leq g_{NL}^{local} \leq 648.2 \text{ GeV} \quad \text{for Case II} \]
\[ 5 \leq f_{NL}^{local} \leq 8.5 , \quad 75 \leq \tau_{NL}^{local} \leq 2800 , \quad 17.4 \leq g_{NL}^{local} \leq 648.2 \text{ GeV} \quad \text{for Case III} \]
\[ 2 \leq f_{NL}^{local} \leq 8.5 , \quad 110 \leq \tau_{NL}^{local} \leq 2800 , \quad 25.5 \leq g_{NL}^{local} \leq 648.2 \text{ GeV} \quad \text{for Case IV} \quad (5.5.72) \]

Here the theoretical upper and lower bound on \( f_{NL}^{local} \) satisfy both the constraints on the \( f_{NL}^{local} \) and \( c_s \) observed by Planck data. Also it is important to note that, within this prescribed framework, \( \tau_{NL}^{local} \) is bounded by below for all the four cases and consequently it is possible to put a stringent lower bound on \( f_{NL}^{local} \) which satisfies the constraints on \( \tau_{NL}^{local} \) and \( c_s \) both. Till date the observational results obtained from Planck do not give any significant constraint on \( g_{NL}^{local} \). However in this paper

\footnote{The analytical expressions for the non-minimal coupling parameters, \( a, b, c, d \) can be expressed in terms of the scale (VEV) of the heavy field \( M_s \) are explicitly mentioned in the Appendix I.}

\footnote{In the prescribed setup the consistency relation between the non-Gaussian parameter \( f_{NL}^{local} \) and the spectral tilt \( n_s [227] \), \( f_{NL}^{local} \sim \frac{1}{12}(1 - n_s) \), does not hold as in the present setup sound speed, \( c_s \neq 1 \) and for such non-minimal \( \mathcal{N} = 1 \) SUGRA setup, Planck data favours lower values of the sound speed (within 0.02 < \( c_s < 1 \)).}
we have provided a theoretical lower and upper bound of $g_{NL}^{local}$ using the consistency relation between $\tau_{NL}^{local}$ and $\alpha_{NL}^{local}$ as stated in Eq (5.5.61).

Finally, in Fig (5.4) we have shown the behaviour of the CMB dipolar asymmetry parameter $A_{CMB}$ with respect to the tensor-to-scalar ratio $r_\ast$, within, $10^{-22} \leq r_\ast \leq 0.12$, at the pivot scale, $k_\ast \sim 0.002$ Mpc$^{-1}$ for the Hubble induced inflation. Here the red and blue coloured boundaries are obtained by fixing the sound speed at $c_S = 0.02$ and $c_S = 1$. The orange dark coloured region satisfied the Planck constraint on the $A_{CMB}$ i.e. $0.05 \leq A_{CMB} \leq 0.09$ for $10^{-22} \leq r_\ast \leq 0.12$ within our proposed framework. In Fig (5.4) only the region bounded by the red, blue and brown colour is the allowed one and the rest of the region ($A_{CMB} < 0.02$ and $A_{CMB} > 0.09$) is excluded by the Planck data.

5.6 Conclusion

In this chapter we have studied primordial non-Gaussian features from $\mathcal{N} = 1$ SUGRA inflationary framework in presence of Planck scale suppressed non-minimal Kähler operators using $\delta N$ formalism. The major outcomes of our study in this chapter are:

- Here we have shown that in any general class of $\mathcal{N} = 1$ SUGRA inflationary framework, the behaviour of Kähler potential in presence of non-minimal Kähler corrections in effective theory setup are constrained via the non-minimal couplings of the non-renormalizable gauge invariant Kähler higher dimensional Planck scale suppressed operators from the observational constraint on non-Gaussianity, sound speed and CMB dipolar asymmetry as obtained from the Planck data.

- In the present setup the hidden sector based heavy field is settled down in its potential via its Hubble induced vacuum energy density. In particular, for the numerical estimations in this paper we have used a very particular kind of inflection point inflationary model, which is fully embedded within MSSM, where the inflaton is made up of $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$ gauge invariant D-flat directions. However the prescribed methodology holds good for other kinds of inflationary models too.

- Further, we have scanned the multi-parameter region characterized by the Hubble induced mass parameter, $c_H$, A-term, $a_H$ and the scale of the heavy field $M_s$, where we have satisfied the current Planck observational constraints on the, inflationary parameters: $P_S$, $n_S$, $c_\tau$, $r_\ast$ (within $2\sigma$ CL), non-Gaussian parameters: $f_{NL}^{local}$, $\alpha_{NL}^{local}$ (within $1\sigma - 1.5\sigma$ CL) and CMB dipolar asymmetry parameter $A_{CMB}$ (within $2\sigma$ CL).

- In our analysis the non-minimal Kähler couplings, $a, b, c, d$ are fixed within $\sim \mathcal{O}(1)$ in the proposed effective theory setup.

- Finally, using this methodology, we have obtained the theoretical upper and lower bound on the non-Gaussian parameters within the range, $\mathcal{O}(1 - 5) \leq f_{NL} \leq 8.5$, $\mathcal{O}(75 - 150) \leq \alpha_{NL} < 2800$ and $\mathcal{O}(17.4 - 34.7) \leq g_{NL} \leq 648.2$, and the CMB dipolar asymmetry parameter within, $0.05 \leq A_{CMB} \leq 0.09$, which satisfy the observational constraints stated in Eq (5.1.1-5.1.7), as obtained from Planck data.

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\textsuperscript{10}The upper bound of the CMB dipolar asymmetry parameter ($A_{CMB}$) can be expressed in terms of the non-Gaussian parameter $f_{NL}^{local}$ through a consistency relation as [228], $A_{CMB} \lesssim 10^{-1} f_{NL}^{local}$, which perfectly holds good in the present effective theory setup.
Figure 5.1: We show the constraints on the non-renormalizable Kähler operators, “a”, “b”, “c” and “d” with respect to the tensor-to-scalar ratio $r_*$ at the pivot scale $k_\star = 0.002 \, \text{Mpc}^{-1}$ when the heavy field $s$ is oscillating during the initial phase of inflation, especially at the time when the interesting perturbations are leaving the Hubble patch for $H_{inf} \gg m_\phi \sim \mathcal{O}$(TeV) [51]. All the shaded regions represent the allowed parameter space for the Hubble induced inflation satisfying the Planck $2\sigma$ constraints on the amplitude of power spectrum $2.092 \times 10^{-9} < P_S < 2.297 \times 10^{-9}$ and spectral tilt $0.958 < n_S < 0.963$. The dark coloured boundaries are obtained from the allowed range of the speed of sound $c_s$, within the window $0.02 \leq c_s \leq 1$. 

(a) Case I  
(b) Case II  
(c) Case III  
(d) Case IV
130 5. Primordial non-Gaussianity from $\mathcal{N} = 1$ supergravity using $\delta\mathcal{N}$ formalism

Figure 5.2: Behaviour of the local type of non-Gaussian parameter $f_{\text{NL}}^\text{local}$ computed from the effective theory of $\mathcal{N} = 1$ supergravity with respect to the sound speed $c_s$ in the Hubble induced inflection point inflationary regime, represented by $H \gg m_\phi$ [51]. The shaded yellow region represents the allowed parameter space for Hubble induced inflation which satisfies the combined Planck constraints on the $f_{\text{NL}}^\text{local}$ (within 1 $\sigma$ CL) and sound speed $c_s$ (within 2 $\sigma$ CL). The red, blue coloured boundaries and the bounded dark coloured regions are obtained from the scanning range of the scale of the heavy scalar degrees freedom $M_s$ corresponds to the hidden sector, within the window $9.50 \times 10^{10}$ GeV $\leq M_s \leq 1.77 \times 10^{16}$ GeV. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, $c_H$, $\alpha_H$, $M_s$ and $\phi_0$, subject to the constraint as stated in Eq (5.5.64-5.5.67). The overlapping region between the dark coloured and yellow region satisfied the combined constraints on the $f_{\text{NL}}^\text{local}$ and $c_s$ within our proposed framework and the rest of the regions are excluded.
Figure 5.3: Behaviour of the local type of non-Gaussian parameter $\tau_{NL,local}$ computed from the effective theory of $\mathcal{N} = 1$ supergravity with respect to the sound speed $c_s$ in the Hubble induced inflationary regime is represented by $H >> m_\phi$ [51]. The red and blue coloured boundaries are obtained from the scanning range of the scale of the of heavy scalar degrees freedom $M_s$ corresponds to the hidden sector, within the window $9.50 \times 10^{10}$ GeV $\leq M_s \leq 1.77 \times 10^{16}$ GeV. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, $c_H$, $a_H$, $M_s$ and $\phi_0$, subject to the constraint as stated in Eq (5.5.64-5.5.67). The dark coloured region satisfied the combined constraints on the $f_{NL}^{local}$ and $c_s$ within the proposed framework. As Planck puts an upper bound, $\tau_{NL,local}^{local} \leq 2800$, the rest of the region above the $\tau_{NL,local}^{local} = 2800$ line is excluded.
5. Primordial non-Gaussianity from $\mathcal{N} = 1$ supergravity using $\delta \mathcal{N}$ formalism

Figure 5.4: Behaviour of the CMB dipolar asymmetry parameter $A_{\text{CMB}}$ computed from the effective theory of $\mathcal{N} = 1$ supergravity with respect to the tensor-to-scalar ratio $r_*$ at the pivot scale, $k_* \sim 0.002$ Mpc$^{-1}$ for the Hubble induced inflation [51]. The red and blue coloured boundaries are obtained by fixing the sound speed at $c_S = 0.02$ and $c_S = 1$. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, $c_H$, $a_H$, $M_s$ and $\phi_0$ subject to the constraint as stated in Eq (5.5.64-5.5.67). The orange dark coloured region satisfied the Planck constraint on the $A_{\text{CMB}}$ within the proposed framework. Here only the region bounded by the red, blue and brown colour is the allowed one and the rest of the region is excluded by the Planck data.
The early universe provides an arena where various ideas about quantum field theory can be tested and the initial singularity of the Big Bang model is a prime example where a quantum field theoretic prescription is compulsory. Fluctuations of the metric during inflation, imprinted in primordial B-mode perturbations of the CMB, are the most vivid evidences conceivable for the reality of field theory. Inflation defers the singularity problem, allowing us to make predictions for the initial conditions that emerge from the aftermath of the Big Bang. However, as we have discussed in the thesis, the inflationary paradigm retains a subtle sensitivity to (sub) Planckian-scale interactions. This is a challenge for microscopic theories of inflation, as well as, an opportunity for using the early universe as a natural laboratory to test (sub) Planckian-scale physics. To fulfill this expectation, inflationary scenarios must be developed to an unprecedented level of completeness and sophistication. The last decade of research on inflation has witnessed a number of significant advances. The development of various field theoretic tools has led to vastly improved understanding of the background models, and in turn, to a sharply improved understanding of the associated inflationary models. In special cases it has been possible to characterize the Planck-scale suppressed non-renormalizable operators to the inflatonary action. On the other hand, many critical challenges still remain in existence, as for examples reheating phenomena, leptogenesis, the connection among the inflationary sector and the Standard Model and Supersymmetric degrees of freedom are not clearly known. These continue to be zeroth-order challenge for deriving inflation using various field theoretic tools and have stymied many attempts to construct inflationary models. Furthermore, in most of the cases, the non-renormalizable Planck-suppressed corrections and other renormalizable loop corrections to the inflationary action are only partially characterized. Finally, and most importantly, there is not a single observation that gives direct evidence for a field-theoretic origin for inflation and reheating, although an unambiguous detection of primordial gravitational waves produced by quantum fluctuations of the metric during inflation would directly prove the quantization of the gravitational field. A striking feature of present day observations is the extraordinary simplicity of the primordial curvature fluctuations, which are, approximately Gaussian, adiabatic, and, nearly scale-invariant. In contrast, the ultraviolet completions, involving many interacting fields and a landscape of quantized parameters, are made. The simplicity of the data motivates various theories of inflation but it does not constrain the ultraviolet completions in the same way. It is important to understand whether the simplicity of the data can emerge from the apparent complexity of the ultraviolet completion. To serve this purpose one should determine which details of the short-distance physics decouple and which should leave subtle traces in the data. On the other hand, it is an important open problem to determine the relative probabilities of different inflationary models in a broader setting. More generally, deriving specific predictions from the field theoretic tools as a whole, rather than from individual models, are distant goal that could require a new approach to measure problem.

Keeping the above points in mind, in this thesis we have studied the following aspects:-

1. Inflationary model building from various field theoretic setup covering both low-scale and
high-scale models.

2. Estimation of cosmological parameters from the proposed models of inflation and confronting them with observational data.

3. Estimation of scale of inflation from the proposed setup and generation of primordial gravitational waves.

4. Quantification of reheating temperature and study of non-trivial features of leptogenesis scenario in braneworld.

5. Estimation of primordial non-Gaussianity and CMB asymmetry parameter using $\delta N$ formalism.

We briefly summarize our works discussed in the various chapters of the thesis as follows:-

- In the chapter 1 we review the various field theoretic approaches applicable to modeling early universe- especially in the context of cosmological inflation. Here we first start our discussion with -Supersymmetry, Supergravity and braneworld scenario and then we have discussed the particle physics and cosmology connection in the context of inflation, reheating, PBH formation and primordial non-Gaussianity in which all of these field theoretic tools are applied.

- In the chapter 2 we have proposed two different models of inflation in the framework of MSSM with various flat directions using saddle point and inflection point mechanism. We have demonstrated how we can construct the effective inflationary potential in the vicinity of the saddle point starting from $n = 4$ and $n = 6$ level superpotential for the D-flat direction content $QQL$, $QuQd$, $QuLe$, $uude$ and $udd$, $LLe$ respectively. The effective inflaton potential around saddle point and inflection point have been utilized in estimating the observable parameters and confronting them with WMAP7 and Planck dataset using the publicly available code CAMB, which reveals consistency of our model with latest observations. One of the advantages of the proposed model is that it is embedded fully within MSSM, and therefore, it predicts the right thermal history of the universe with no extra relativistic degrees of freedom other than that of the Standard Model.

- In the chapter 3 we have studied single field inflation in the context of Randall-Sundrum braneworld and DBI Galileon induced D3 brane respectively. We have demonstrated the technical details of construction mechanism of an one-loop 4D inflationary potential via dimensional reduction starting from $\mathcal{N} = 2, \mathcal{D} = 5$ supergravity in the bulk which leads to an effective field theoretic picture within $\mathcal{N} = 1, \mathcal{D} = 4$ supergravity embedded in the brane for both the cases. Hence we have studied inflation using the one loop effective potential by estimating the observable parameters originated from primordial quantum fluctuation for scalar and tensor modes. We have further confronted our results with WMAP7 [22] dataset by using the cosmological code CAMB [155]. Additionally we have proposed new sets of inflationary consistency relations in the case of Randall-Sundrum braneworld and DBI Galileon framework which is different from the results obtained from usual GR framework.

- In the chapter 4 we have explored the features of reheating in brane cosmology on the background of supergravity. We have exhibited the process of construction of a fruitful theory of
reheating for an effective 4D inflationary potential in $\mathcal{N} = 1, D = 4$ supergravity in the brane derived from $\mathcal{N} = 2, D = 5$ supergravity in the bulk. We have employed this setup in reheating model building by analyzing the reheating temperature in the context of brane inflation, followed by analytical and numerical estimation of different phenomenological parameters. It is worthwhile to mention that we get a lot of new results in the context of braneworld compared to standard GR case. We further we propose a theory which reflects the effect of particle production through collision and decay thereby showing a direct connection with the thermalization phenomena. To show this internal link more explicitly we put forward both analytical and numerical expressions for the gravitino abundance in a physical volume in the reheating epoch. Also the validity of leptogenesis for our model shows the production of heavy Majorana neutrinos in the brane.

- In the chapter 5 we have shown that in any general class of $\mathcal{N} = 1$ SUGRA inflationary framework, the behaviour of Kähler potential in presence of non-minimal Kähler corrections in effective theory setup are constrained via the non-minimal couplings of the non-renormalizable gauge invariant Kähler higher dimensional Planck scale suppressed operators from the observational constraint on non-Gaussianity, sound speed and CMB dipolar asymmetry as obtained from the Planck data. In the present setup the hidden sector based heavy field is settled down in its potential via its Hubble induced vacuum energy density. In particular, for the numerical estimations in this paper we have used a very particular kind of inflection point inflationary model, which is fully embedded within MSSM, where the inflaton is made up of $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$ gauge invariant D-flat directions. However the prescribed methodology holds good for other kinds of inflationary models too.

The future prospects of the work studied in the thesis are:-

- **Cosmology from Effective Field Theory:**

  One of the prime goals is to study the various cosmo-phenomenological aspects of Effective Field Theory (EFT) of (eternal) inflation in a model independent fashion by imposing the constraints on various cosmological parameters obtained from the recent observational probe which we have not addressed in the thesis elaborately. The beauty of the EFT technique is not to bother about the background field theoretical origin and also to tightly constrain (and rule out) various existing models of inflation available in literature. But this has a generic power to modify the background dynamics of inflation and other cosmological features by incorporating the effects of interactions in the effective action via higher order radiative corrections. Specifically in the context of eternal inflation it resolves the Infra Red (IR) problem by taking into account all possible higher order loop contributions. The earlier works in this area did not address this issue properly. Our future aim is to address this serious issue in a very comprehensive manner. Additionally we want to extend the EFT approach in the context of dark matter, dark energy and large scale structure formation.

- **Primordial non-Gaussianity:**

  Single field models of inflation are expected to produce small local type of non-Gaussianity in the squeezed limit according to consistency relation proposed in [227]. Nevertheless it is not necessary that it will always hold good in every physical prescription. One of my future interests is to compute the various types of non-Gaussianity-local, orthogonal and equilateral from bispectrum and trispectrum and to study the features in squeezed limit
configuration from the proposed inflationary models discussed in the thesis. Another focus of mine is to study the violations/modifications of various inflationary consistency relations by proposing model independent techniques from which one can constrain various proposed model of inflation available in literature. We are also keenly interested in studying the features of various CMB cross correlations from scalar and tensor cosmological perturbations from the proposed models. Further using using EFT approach we want to constrain the features of local type of primordial non-Gaussianity computed from the bispectrum and trispectrum using -In-In formalism and $\delta N$ formalism.

- **CMB Polarization:**
  At present, one of the most challenging area of research in the field of theoretical physics is to explain the unexplored features of Polarization data which is already released by Planck. Another future goal of mine is to constrain/ rule out various models of inflation using the results of CMB Polarization (via TT, TE, BB, EB correlations) to be observed by Planck. Besides, our aim is to study reconstruction technique of the primordial power spectrum and also the inflationary potential using the polarization data in a model independent fashion by which it is possible to rule out various models available in literature and also to break the degeneracy among the cosmological parameters as obtained from various models.

- **Gravitational Waves:**
  One of the important focusing issues of the present day research, is to search for the Primordial Gravitational Waves (PGW) through the large scale B-mode signal. It is a common notion amongst theoretical physicists that the gravitational waves are generated just after the Big-Bang and during inflationary era their amplitude will be amplified to a permissible value. But the signatures of such gravity waves are yet to be detected. This, in turn, will provide the best information along with CMB about the era of cosmic inflation. Even if the gravity waves are detected, we will not be able to draw positive conclusion about its origin- primordial or stochastic. As the CMB Polarized B-mode signals are generated from lensing of purely E-mode signals, the separation of pure primordial gravity waves from observed data will be an essential subject matter for the present day research. In future I would like to carry on my research in the extraction of pure (unlensed) B-mode signal from the lensed data by introducing a completely new theoretical tool. As the South Pole Telescope (SPT) has claimed about the detection of CMB (lensed) B- mode polarization [229], a lot of avenues would remain open for future work. This methodology can also be applied to the inflationary scenario to constrain inflationary observables further as inflationary tensor perturbations are believed to be the main source of primordial gravity waves. After extraction of the unlensed B-mode signal, our aim is to propose a theoretical/semi-analytical tool by which it is possible to subtract the effect of primordial non-Gaussianity. This will clearly quantify the effect of inflationary tensor perturbations via primordial gravity waves (or tensor-to-scalar ratio) present in the unlensed B-mode signal.

- **Particle Cosmology:**
  Apart from that one of the most promising branches of research in present day theoretical physics is “particle cosmology”. Particle physics examines nature on the smallest scales, while cosmology studies the universe on the largest scales. One of the focus of our future research is to elaborately study various cosmo-particle phenomenological aspects like- re/preheating,
leptogenesis, baryogenesis and dark matter etc from Beyond the Standard Model (BSM) physics (Example: Supersymmetry and Supergravity, MSSM, NMSSM, CMSSM etc.) where the structure of the effective four dimensional Field equations are subsequently modified in presence of modifications in the background geometry sector in light of recent collider and observational constraints.
Appendix

A. Higher order slow-roll corrections within GR

\[ P_S(k_\ast) = \left[ 1 - (2C_E + 1)\epsilon_V + C_E\eta_V \right]^2 \frac{V}{24\pi^2 M_p^4 \epsilon_V} \], \quad (6.0.1) \\
\[ P_T(k_\ast) = \left[ 1 - (C_E + 1)\epsilon_V \right]^2 \frac{2V}{3\pi^2 M_p^4}, \quad (6.0.2) \]

\[ \kappa S(k_\ast) - 1 \approx (2\eta_V - 6\epsilon_V) - 2C_E\xi_V^2 + \frac{2}{3}\eta_V^2 + 2(8C_E + 3)\epsilon_V^2 + 2\epsilon_V \eta_V \left( 6C_E + \frac{3}{2} \right) \]

\[ - 4C_E(C_E + 1)\xi_V^2 \epsilon_V + 2C_E^2 \eta_V \xi_V^2, \quad (6.0.3) \]

\[ \eta T(k_\ast) \approx -2\epsilon_V + 2 \left( \frac{2C_E + 5}{3} \right) \epsilon_V \eta_V - 2 \left( \frac{4C_E + 13}{3} \right) \epsilon_V^2, \quad (6.0.4) \]

\[ r(k_\ast) = 16\epsilon_V \frac{[1 - (C_E + 1)\epsilon_V]^2}{[1 - (2C_E + 1)\epsilon_V + C_E\eta_V]^2} \approx 16\epsilon_V \left[ 1 + 2C_E(\epsilon_V - \eta_V) \right], \quad (6.0.5) \]

\[ \alpha S(k_\ast) \approx (16\epsilon_V \epsilon_V - 24\epsilon_V^2 - 2\xi_V^2) - 2C_E(4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) + \frac{4}{3} \eta_V (2\eta_V \epsilon_V - \xi_V^2) \]

\[ + 4(8C_E + 3)\epsilon_V (4\epsilon_V^2 - 2\eta_V \epsilon_V) \]

\[ - 4C_E(C_E + 1) \left[ \epsilon_V (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) + \xi_V^2 (4\epsilon_V^2 - 2\eta_V \epsilon_V) \right] + 2C_E^2 \xi_V^2 (2\eta_V \epsilon_V - \xi_V^2) \]

\[ + 2C_E^2 \eta_V (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2), \quad (6.0.6) \]

\[ \alpha T(k_\ast) \approx (4\eta_V \epsilon_V - 8\epsilon_V^2) + 2 \left( \frac{2C_E + 5}{3} \right) \left[ \epsilon_V (2\eta_V \epsilon_V - \xi_V^2) + \eta_V (4\epsilon_V^2 - 2\eta_V \epsilon_V) \right] \]

\[ - 4 \left( \frac{4C_E + 13}{3} \right) \epsilon_V (4\epsilon_V^2 - 2\eta_V \epsilon_V), \quad (6.0.7) \]

\[ \kappa S(k_\ast) \approx 192\epsilon_V^2 \eta_V - 192\epsilon_V^2 + 2\sigma_V^2 - 24\epsilon_V \xi_V^2 + 2\eta_V \xi_V^2 - 32\eta_V^2 \epsilon_V \]

\[ - 8C_E \left[ \epsilon_V (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) + \xi_V^2 (4\epsilon_V^2 - 2\eta_V \epsilon_V) \right] \]

\[ + 2C_E \left[ \eta_V (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) + \xi_V^2 (2\eta_V \epsilon_V - \xi_V^2) \right] + 4C_E \sigma_V^2 (3\epsilon_V - \eta_V) + \frac{4}{3} (2\eta_V \epsilon_V - \xi_V^2)^2 \]

\[ + \frac{4}{3} \left[ 2\eta_V (4\epsilon_V^2 - 2\eta_V \epsilon_V) + 2\epsilon_V (2\eta_V \epsilon_V - \xi_V^2) - (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) \right] \]

\[ + 4 (8C_E + 3) (4\epsilon_V^2 - 2\eta_V \epsilon_V) + 16 (8C_E + 3) \epsilon_V \left[ (2\epsilon_V - \eta_V) (4\epsilon_V^2 - 2\eta_V \epsilon_V) - \epsilon_V (2\eta_V \epsilon_V - \xi_V^2) \right] \]

\[ + 4 \left( \frac{6C_E + \frac{7}{3}}{3} \right) (2\eta_V \epsilon_V - \xi_V^2) (4\epsilon_V^2 - 2\eta_V \epsilon_V) \]

\[ + 2 \left( \frac{6C_E + \frac{7}{3}}{3} \right) \epsilon_V \left[ 2(2\eta_V \epsilon_V - \xi_V^2) \epsilon_V + 2\eta_V (4\epsilon_V^2 - 2\eta_V \epsilon_V) \right] \]

\[ - (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) - 4C_E (C_E + 1) (4\epsilon_V^2 - 2\eta_V \epsilon_V) (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) \]

\[ - 4C_E (C_E + 1) \epsilon_V \left[ (4\epsilon_V - \eta_V) (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) + (16\epsilon_V^2 + \xi_V^2 - 10\eta_V \epsilon_V) \xi_V^2 \right] \]

\[ - 2\sigma_V^2 (3\epsilon_V - \eta_V) - 4C_E (C_E + 1) \left[ (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) (4\epsilon_V^2 - 2\eta_V \epsilon_V) \right] \]

\[ + 2\epsilon_V ((4\epsilon_V - \eta_V) [4\epsilon_V^2 - 2\eta_V \epsilon_V] - \epsilon_V (2\eta_V \epsilon_V - \xi_V^2)) \]

\[ + 2C_E^2 [(4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2) (2\eta_V \epsilon_V - \xi_V^2) + \xi_V^2 (2\eta_V (4\epsilon_V^2 - 2\eta_V \epsilon_V) + 2\epsilon_V (2\eta_V \epsilon_V - \xi_V^2) \]

\[ - [4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2]) + 2C_E^2 [(2\eta_V \epsilon_V - \xi_V^2) (4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2)] \]

\[ + \eta_V [(4\epsilon_V - \eta_V) [4\epsilon_V \xi_V^2 - \eta_V \xi_V^2 - \sigma_V^2] + \xi_V^2 [16\epsilon_V^2 + \xi_V^2 - 10\eta_V \epsilon_V] - 2\sigma_V^2 [3\epsilon_V - \eta_V]], \quad (6.0.8) \]
\[ \kappa_T(k_s) \approx 56\eta_V \epsilon_v^2 - 64\epsilon_v^3 - 8\eta_V^2 \epsilon_v - 4\epsilon_v \xi_v^2 + 2(2C_E + \frac{2}{3}) \left[ (2\eta_V \epsilon_v - \xi_v^2) (4\epsilon_v^2 - 2\eta_V \epsilon_v) \right. \\
+ \epsilon_v (2\eta_V [4\epsilon_v^2 - 2\eta_V \epsilon_v] + 2\epsilon_v [2\eta_V \epsilon_v - \xi_v^2] - 4\epsilon_v \xi_v^2 - \eta_V \xi_v^2 - \sigma_v^2) \\
+ \eta_V (8\epsilon_v [4\epsilon_v^2 - 2\eta_V \epsilon_v] - 2\eta_V [4\epsilon_v^2 - 2\eta_V \epsilon_v] - 2\epsilon_v [2\eta_V \epsilon_v - \xi_v^2]) \\
- 4 \left( 4C_E + \frac{13}{3} \right) [(4\epsilon_v^2 - 2\eta_V \epsilon_v)^2 + \epsilon_v ((8\epsilon_v - 2\eta_V) [4\epsilon_v^2 - \epsilon_v] - 2\epsilon_v [2\eta_V \epsilon_v - \xi_v^2])] \right]. \]

(6.0.9)

B. Consistency relations in brane inflation

In the context of RS single braneworld the spectral indices \((n_S, n_T)\), running \((\alpha_S, \alpha_T)\) and running of the running \((\kappa_T, \kappa_S)\) at the momentum pivot scale \(k_s \approx k = aH\) can be expressed as [188]:

\[
\begin{align*}
n_S(k_s) - 1 &= 2\eta_V (\phi_s) - 6\epsilon_V (k_s), \\
n_T(k_s) &= -3\epsilon_V (k_s) - \frac{r_V (k_s)}{8}, \\
\alpha_S(k_s) &= 16\eta_V (k_s) \epsilon_V (k_s) - 18\epsilon_v^2 (k_s) - 2\xi_v^2 (k_s), \\
\alpha_T(k_s) &= 6\eta_V (k_s) \epsilon_V (k_s) - 9\epsilon_v^2 (k_s), \\
\kappa_S(k_s) &= 152\eta_V (k_s) \epsilon_v^2 (k_s) - 32\epsilon_V (k_s) \eta_v^2 (k_s) - 108\epsilon_v^2 (k_s) - 24\xi_v^2 (k_s) \epsilon_V (k_s) \\
&\quad + 2\eta_V (k_s) \xi_v^2 (k_s) + 2\sigma_v^2 (k_s), \\
\kappa_T(k_s) &= 66\eta_V (k_s) \epsilon_v^2 (k_s) - 12\epsilon_V (k_s) \eta_v^2 (k_s) - 54\epsilon_v^2 (k_s) - 6\epsilon_V (k_s) \xi_v^2 (k_s). \end{align*}
\]

from which we get the following set of consistency relations:

\[
\begin{align*}
n_T(k_s) - n_S(k_s) + 1 &= \left( \frac{d \ln r(k)}{d \ln k} \right)_s = \frac{r(k_s)}{8} - 2\eta_V (k_s), \\
\alpha_T(k_s) - \alpha_S(k_s) &= \left( \frac{d^2 \ln r(k)}{d \ln k} \right)_s = \left( \frac{r(k_s)}{8} \right)^2 - \frac{20}{3} \left( \frac{r(k_s)}{8} \right) + 2\xi_v^2 (k_s), \\
\kappa_T(k_s) - \kappa_S(k_s) &= \left( \frac{d^2 \ln r(k)}{d \ln k} \right)_s \\
&= 2 \left( \frac{r(k_s)}{8} \right)^3 - \frac{86}{9} \left( \frac{r(k_s)}{8} \right)^2 \\
&\quad + \frac{4}{3} \left( 6\xi_v^2 (k_s) + 5\eta_v^2 (k_s) \right) \left( \frac{r(k_s)}{8} \right) \\
&\quad + 2\eta_V (k_s) \xi_v^2 (k_s) + 2\sigma_v^2 (k_s). \end{align*}
\]

Here Eq (6.0.16-6.0.18) represent the running, running of the running and running of the double running of tensor-to-scalar ratio.

C. Dimensional reduction technique for DBI Galileon

In this section we employ dimensional reduction technique to derive a \(\mathcal{N}=1, D=4\) SUGRA and the inflaton potential therefrom that results in DBI Galileon on the D3 brane. In this framework the 5D and 4D \(\text{Riemann tensor}, \text{Ricci tensor} \) and \(\text{Ricci scalar} \) are related through the following expressions:

\[
R_{\alpha\beta\gamma\delta}^{(5)} = R_{\alpha\beta\gamma\delta}^{(4)} + \frac{\exp(2A(y))}{R^2 \beta^2} \left( \frac{dA(y)}{dy} \right)^2 \left[ g_{\gamma\beta}^{(4)} g_{\alpha\delta}^{(4)} - g_{\alpha\gamma}^{(4)} g_{\beta\delta}^{(4)} \right].
\]

(6.0.19)
The scaling relationship between 4D and 5D coupling constant is given by:

\[ R_{\alpha\beta}^{(5)} = R_{\alpha\beta}^{(4)} - \frac{3g_{\alpha\beta}^{(4)}}{R^2\beta^2} \left( \frac{dA(y)}{dy} \right)^2 \]  \hspace{1cm} (6.0.20)

\[ R_{(5)} = \exp(2A(y)) \left[ R_{(4)} - \frac{12}{\beta^2R^2} \left( \frac{dA(y)}{dy} \right)^2 - \frac{8}{\beta^2R^2} \left( \frac{d^2A(y)}{dy^2} \right) - 2\Lambda_5 \exp(2A(y)) \right] \]  \hspace{1cm} (6.0.21)

which is necessarily required for dimensional reduction. For convenience we deal with different contributions to the action \((3.3.35)\) separately.

I. The Einstein-Hilbert action:

After integrating out the contribution from the five dimension, the Einstein-Hilbert action in four dimension can be written as:

\[ S_{EH}^{(4)} = \frac{1}{2\kappa_4^2} \int d^4 x \sqrt{-g^{(4)}} \left[ R_{(4)} - \frac{3M_5^2b_0^2}{M_\beta^2R^5} I(1) \right], \]  \hspace{1cm} (6.0.22)

where the explicit expression for \(I(1)\) is mentioned in Appendix C. In this context \(R_{(4)}\) is the 4D Ricci scalar. It is important to mention here that the 5D Planck mass \((M_5)\) and 4D Planck mass \((M_P)\) are related through the following relation:

\[ M_P^2 = M_5^3 \beta R \int_{-\pi R}^{+\pi R} dy \exp(3A(y)) = \frac{M_5^3b_0^2}{3R^2\beta^{3/2}} \left[ \exp(\beta\pi R)P_1 - \exp(-\beta\pi R)P_2 \right] \]  \hspace{1cm} (6.0.23)

where \(P_1\) and \(P_2\) is defined as:

\[ P_1 = \left\{ \frac{3\sqrt{I(4)}}{\sqrt{\exp(\beta\pi R) + I(4)}} - \frac{\exp(2\pi R) + I(4)}{\exp(\beta\pi R) + I(4)} \right\}; \]

\[ P_2 = \left\{ \frac{3\sqrt{I(4)}}{\sqrt{\exp(-\beta\pi R) + I(4)}} - \frac{\exp(-2\pi R) + I(4)}{\exp(-\beta\pi R) + I(4)} \right\}. \]  \hspace{1cm} (6.0.24)

II. The higher curvature gravity action:

Using Eq \((6.0.19)\)-Eq \((6.0.21)\) in Eq \((3.3.37)\) we get

\[ \kappa_{GB}^{(4)} = \frac{\alpha_{(4)}}{2\kappa_4^{(4)}} \int d^4 x \sqrt{-g^{(4)}} \left[ (C(1)R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}^{(4)} - 4\mathcal{I}(2)R_{\alpha\beta}^{(4)}R_{\alpha\beta}^{(4)} + \mathcal{A}(6)R_{(4)}^2 \right]

+ \frac{2C(2)}{R^2\beta^2} R_{\alpha\beta\gamma\delta}^{(4)} \left( g^{\gamma\beta}(4)g^{\delta\alpha}(4) - g^{\gamma\alpha}(4)g^{\delta\beta}(4) \right) + \frac{G(1)}{R^4\beta^4} + \frac{G(2)}{R^2\beta^2} \right] \]  \hspace{1cm} (6.0.25)

where the co-efficients after dimensional reduction are given by:

\[ G(1) = 24C(4) - 144\mathcal{I}(4) - 64A(5) + 144A(7) + 64A(8) + 192A(11), \]

\[ G(2) = 24\mathcal{I}(2) - 24A(9) - 16A(10). \]  \hspace{1cm} (6.0.26)

The scaling relationship between 4D and 5D coupling constant is given by:

\[ \alpha_{(4)} = \frac{\kappa_4^{(4)} \beta R}{\kappa_5^{(5)} \alpha_{(5)}} \]  \hspace{1cm} (6.0.27)

where \(\kappa_{(4)}\) and \(\kappa_{(5)}\) are gravitational couplings in 4D and 5D respectively. Explicit form of each of the constants appearing in Eq \((6.0.25)\) are mentioned in \([191]\).
III. The D3 Brane Action:

To reduce the D4 brane action we employ the method of separation of variable \( \Phi(X^A) = \Phi(x^\mu, y) = \phi(x^\mu) \exp(\frac{2\pi i y}{R}) \). Consequently the D3 brane action turns out to be

\[
S_{D3}^{(4)} = \int d^4x \sqrt{-g^{(4)}} \left[ \tilde{K}(\phi, \tilde{X}) - \tilde{G}(\phi, \tilde{X}) \Box^{(4)}\phi \right],
\]

where

\[
\tilde{K}(\phi, X) = \left\{ -\frac{\partial}{\partial \phi} \left[ \sqrt{1 - 2Q\tilde{X}} \tilde{f} - Q_1 \right] - \tilde{C}_5 \tilde{G}(\phi, \tilde{X}) - \tilde{Q}_2 \tilde{D}V^{(4)}_{brane} \right\},
\]

\[
\tilde{G}(\phi, \tilde{X}) = \left( \frac{\tilde{g}_2(\phi) \tilde{C}_4}{2(1 - 2f(\phi)X_{35})} \right), \quad \tilde{g}(\phi) = \tilde{g}_0 + \tilde{g}_2\phi^2,
\]

\[
\tilde{D} = \frac{\partial}{2\kappa_4}, \quad \tilde{C}_4 = \frac{C_4}{2\kappa_4}, \quad \tilde{C}_5 = \frac{C_5}{\kappa_2}, \quad \tilde{Q}_2 = \tilde{Q}_2 \tilde{D} = \beta R.
\]

The effective Klebanov Strassler and Coulomb frame function on the D3 brane are hereby expressed as \( \tilde{f}(\phi) \approx \frac{1}{(f_0 + f_2\phi^2 + f_4\phi^4)} \) and \( \nu^{(4)}(\phi) = \tilde{V}_0 + \frac{\tilde{V}_4}{\phi} \) with \( \tilde{V}_0 = \nu_0A(1) \), \( \tilde{V}_4 = \nu_4A(12) \). The scaled D3 brane potential turns out to be:

\[
\tilde{V}_{brane}^{(4)} = \tilde{Q}_2 \tilde{D}V^{(4)}_{brane} = T_{(3)}\nu^{(4)}(\phi) + \frac{\beta RT(2)}{f(\phi)}
\]

where the D3 brane tension \( T_{(3)} \) can be expressed in terms of the D4 brane tension \( T_{(4)} \), compactification radius \( R \) and the slope parameter \( \beta \) as, \( T_{(3)} = \beta RT_{(4)} \).

IV. The \( \mathcal{N}=1, D=4 \) Supergravity Action:

Further, imposing \( Z_2 \) symmetry to \( \phi \) via \( \Phi(0) = \Phi(\pi R) = 0 \) and compactifying around a circle \( S^1 \) \( \partial_5 \Phi = \sqrt{V^{(5)}_{bulk}(G)} (1 - \frac{1}{2\pi R}) \) we get,

\[
S_{Bulk \ Supra}^{(5)} = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{-g^{(5)}} \left\{ e^{(4)}e^5 \left\{ g^{\alpha\beta} G^\alpha_m (\partial_\mu \phi^m) (\partial_\nu \phi_n) - g^{\nu\alpha} V^{(5)}_{bulk}(G) \right\} \right\}.
\]

Now using the above mentioned ansatz for method of separation of variable we get

\[
S_{Supra}^{(4)} = \frac{1}{2 \kappa_4^2} \int d^4x \sqrt{-g^{(4)}} \left\{ M(T, T^\dagger)J_\nu(\phi, \phi^\dagger)g^{\alpha\beta}(\partial_\alpha \phi^\mu) (\partial_\beta \phi^\nu) - Z(T, T^\dagger)V_F^{(4)}(\phi) \right\}.
\]

where we define

\[
J_\nu(\phi, \phi^\dagger) = \int_{-\pi R}^{+\pi R} dy \exp(-A(y)) \left( \frac{\partial^2 K(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R}))}{\partial \phi^\dagger_\nu \partial \phi^\nu} \right),
\]

\[
M(T, T^\dagger) = \frac{\sqrt{2} \beta R^2}{(T + T^\dagger)} Z(T, T^\dagger) = \frac{1}{8\pi^2 R^2 \beta |T + T^\dagger|^2}.
\]

Here we used the ansatz \( \mathcal{W}(\phi, \phi^\dagger, T, T^\dagger) = \frac{1}{2} \mathcal{W}_1(\phi, \phi^\dagger)|T + T^\dagger|^2 \) for superpotential and the factorization ansatz \( \mathcal{K}(\phi \exp(\frac{2\pi i y}{R}), \phi^\dagger \exp(-\frac{2\pi i y}{R})) = \mathcal{K}_1(\phi, \phi^\dagger)\mathcal{K}_2(\exp(\frac{2\pi i y}{R}), \exp(-\frac{2\pi i y}{R})) \) with \( \mathcal{K}_1(\phi, \phi^\dagger) = \mathcal{K}_1^{\alpha\beta} \phi^\alpha \phi^\dagger_\beta \) and \( \mathcal{K}_2(\exp(\frac{2\pi i y}{R}), \exp(-\frac{2\pi i y}{R})) = 1 \) for the Kähler using which the effective
Appendix

4D F-term potential can be expressed as:

$$V_F^{(4)} = A(13) \exp \left( \frac{K_{1\alpha}^\alpha \phi_\alpha \phi_\beta^\beta}{M^2} \right) \left[ \left( \frac{\partial W_1}{\partial \phi_\alpha} + K_{1\alpha}^\alpha \phi_\beta^\beta W_1 \right) \frac{\partial W_1}{\partial \phi_\beta} + K_{1\eta}^\alpha \phi_\eta W_1 - 3 \frac{|W_1|^2}{M^2} \right]$$

with the general Kähler metric $K_{1\alpha}^\alpha = \frac{\partial^2 K_{1}}{\partial \phi_\alpha \partial \phi_\beta}$. In most of the simple situations, we are interested in the Canonical metric structure defined by $K_{1\alpha}^\alpha = \delta_{\alpha \beta}$. Consequently the $N = 1, D = 4$ SUGRA action turns out to be

$$S_{Can \ SUGRA}^{(4)} = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g^{(4)}} \left[ M(T, T^\dagger) g^{\alpha\beta}(\partial_\alpha \phi_\mu) (\partial_\beta \phi_\mu) - Z(T, T^\dagger) V_{Can}^{(4)}(\phi) \right].$$

where the canonical F-term potential can be recast as

$$V^{(4)} = V_F^{(4)} = A(13) \exp \left( \frac{\phi_\alpha \phi_\beta}{M^2} \right) \left[ \left( \frac{\partial W_1}{\partial \phi_\beta} \right)^2 - 3 \frac{|W_1|^2}{M^2} \right].$$

To derive the expression for the specific form of the inflaton potential we start with a specific superpotential $W_1 = v\phi - \frac{g}{n+1} \phi^{n+1}$ with $n \geq 2$. Here $g$ and $v$ is the coupling constant and the VEV of $\phi$ respectively. This leads to the following form of the bulk contribution to the potential:

$$V_{bulk}^{(4)}(\phi) = A(13) \exp \left( |\phi|^2 \right) \left[ \left( 1 + |\phi|^2 \right) v^2 - \left( 1 + \frac{|\phi|^2}{n+1} \right) g \phi^n \right]^2 - 3 |\phi|^2 \left. v^2 - \frac{g}{n+1} \phi^n \right|^2.$$  

Identifying $\phi \to \sqrt{2} \\text{Re}(\phi)$ and imposing renormalization condition, here we restrict ourselves to $n = 2$ leading to effective $N = 1, D = 4$ SUGRA potential:

$$V_{bulk}^{(4)}(\phi) = A(13) \left( v^4 - g v^2 \phi^2 + \frac{g^2}{4} \phi^4 \right).$$

D. Standard results of reheating mechanism and lepto-genesis in Einsteinian gravity:

- **I. Reheating temperature:**

  $$T_R \sim 0.2 \left( \frac{100}{N^*} \right) \left( \frac{\Gamma_{total} M_{PL}}{1/4} \right)^{1/2}.$$  

- **II. Extremum (maximum) temperature:**

  $$T_{max} \sim 0.8 \left( \frac{N+1/4}{V_0^{1/8}} \right) \left( \frac{\Gamma_{total} M_{PL}}{1/2} \right)^{1/2}$$

  where $V_0$ is the vacuum energy.

- **III. Temperature-time relationship:**

  $$T = \frac{T_R}{\sqrt{2 H_R(t-t_R)} + 1}$$

  where $H_R$ is the Hubble parameter at reheating time scale $t_R$. 

• IV. Number density of Gravitino during reheating:

\[ Y_\zeta(T_R) = \frac{2\alpha}{M_{PL}^2} \left( \frac{\zeta(3)}{\pi^2} \right)^2 \left( \frac{45}{2\pi^2 N^*} \right) \frac{T_{\text{max}}^4}{H_0 T_R} \]  

where \( H_0 = \frac{\sqrt{3} s}{\sqrt{2M_{PL}}} \).

E. The model parameters \( \alpha, \beta, \gamma, \kappa \):

The model parameters characterizing the potential stated in Eq (5.4.39) can be expressed as:

\[ \alpha = M_s^4 + \left( \frac{(n-2)^2}{n(n-1)} + \frac{(n-2)^2}{n} \right) c_H H^2 \phi_0^2 + \cdots, \]  

\[ \beta = 2 \left( \frac{n-2}{2} \right)^2 \delta^2 c_H H^2 \phi_0 + \cdots, \]  

\[ \gamma = \frac{c_H H^2}{\phi_0^2} \left( 4(n-2)^2 - \frac{(n-1)(n-2)^2 \delta^2}{2} \right) + \cdots, \]  

\[ \kappa = \frac{c_H H^2}{\phi_0^2} \left( 12(n-2)^3 - \frac{(n-1)(n-2)(n-3)(7n^2 - 27n + 26) \delta^2}{2} \right) + \cdots \]  

where the higher order \( \cdots \) terms are neglected due to \( \delta^2 << 1 \). During numerical estimations I fix \( n = 6 \) for \( \tilde{L}\tilde{L}\tilde{e} \) and \( \tilde{u}\tilde{d}\tilde{d} \) D-flat directions respectively.

F. The symbol \( \Sigma = X, Y, Z, W \):

The symbols appearing in the Eq (5.5.60), in the definition of the sound speed \( c_s \) for \( s << M_p \), after imposing the slow-roll approximation are given by:

\[ X_1(t) = \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \frac{aM_s^3}{M_p^2} \left[2\sin(2M_s t) + 4\cos(M_s t)\right] \right. \]

\[ - \left. \frac{aM_s^4}{M_p^2} |\phi| \cos \Theta \left[ \cos(2M_s t) - \sin(M_s t) \right] \right\}, \]

\[ Y_1(t) = \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \frac{bM_s^2}{M_p} \cos(M_s t) + \frac{bM_s^4}{M_p} |\phi| \cos \Theta \sin(M_s t) \right\}, \]

\[ Z_1(t) = \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \frac{cM_s^3}{4M_p^2} \left[2\sin(2M_s t) + 4\cos(M_s t)\right] \right. \]

\[ - \left. \frac{cM_s^4}{4M_p^2} |\phi| \cos \Theta \left[ \cos(2M_s t) - \sin(M_s t) \right] \right\}, \]

\[ W_1(t) = \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2c_V(\phi)V(\phi)}{3}} \frac{dM_s^2}{M_p} \cos(M_s t) + \frac{dM_s^3}{M_p} |\phi| \cos \Theta \sin(M_s t) \right\}, \]

\[ X_2(t) = \left( Y_2(t) + \frac{a|\phi|^2 M_s^6}{M_p^2} \sin(2M_s t) \right), \]

\[ Y_2(t) = Z_2(t) = W_2(t) = 5M_s^5 \sin(2M_s t) + 8M_s^3 \cos(M_s t), \]

\[ X_3(t) = \left( Y_3(t) - \frac{a|\phi|^2 M_s^6}{M_p^2} \sin(2M_s t) \right), \]

\[ Y_3(t) = Z_3(t) = W_3(t) = 3M_s^5 \sin(2M_s t) - 8M_s^3 \cos(M_s t). \]
Here the complex inflaton field $\phi$ is parameterized by, $\phi = |\phi| \exp(i\Theta)$. Here the new parameter $\Theta$ characterizes the phase factor associated with the inflaton and it has a two dimensional rotational symmetry.

G. Case -1, 2, 3, 4

- Case 1

$$ K = \phi^\dagger \phi + s^\dagger s + \frac{a}{\lambda M_p^2} \phi^\dagger \phi s^\dagger s $$

For the above non-minimal Kähler interaction with $'a'$ being a dimensionless number. We have also computed the correction to the Hubble-induced mass term, for $c_H$ for $|s| << M_p$:

$$ c_H = \left\{ 3 \left[ (1 - a) + (1 + a) a \frac{|s|^2}{M_p^2} \right] + \left[ (1 + 3a) + (1 - 3a) a \frac{|s|^2}{M_p^2} \right] \left( \frac{e^K |F_s|^2}{V(s)} - 1 \right) \right\} \approx 3(1 - a), $$

(6.0.52)

where we used the fact that: $V(s) = |W_s|^2 = 3H^2M_p^2 = 4M_p^2|s|^2$. Next we compute the correction to the Hubble-induced A term, $a_H H \frac{\phi^n}{n M_p^2}$, in presence of the non-minimal Kähler correction:

$$ a_H H \frac{\phi^n}{n M_p^2} = \left\{ \left[ 1 + a \frac{|s|^2}{M_p^2} \right] W_\phi \phi - 3W(\phi) \right\} \frac{e^K W^*(I) I}{M_p^2} + \left[ W(\phi) \frac{I}{M_p} - aW(\phi) \frac{|s|^2}{M_p^2} \right] \frac{e^K \phi^*}{M_p} + h.c. $$

(6.0.53)

$$ \approx \left\{ \left[ 1 + a \frac{|s|^2}{M_p^2} \right] \left( 1 - a \frac{|s|^2}{M_p^2} \right) \frac{s^2}{M_p^2} + \left( 1 - a \frac{|s|^2}{M_p^2} \right) \left( a - a \frac{e^2}{M_p^2} \right) \frac{\lambda M_p s^n}{M_p^2} + h.c. \right\}, $$

which explicitly shows the Planck suppression for $|s| << M_p$ in the Hubble-induced A term.

- Case 2

$$ K = \phi^\dagger \phi + s^\dagger s + \frac{b}{2M_p} s^\dagger s^\dagger \phi + h.c. $$

Similarly, for the above non-minimal kähler correction where $'b'$ is a dimensionless number we can compute the correction to the Hubble-induced mass term, $c_H H^2 |\phi|^2$, for $|s| << M_p$:

$$ c_H = \left[ 3 + b^2 \frac{e^K |W_s|^2}{H^2 M_p^2} + \left( \frac{e^K |F_s|^2}{V(s)} - 1 \right) \right] \approx 3(1 + b^2), $$

(6.0.54)

where $V(s) = |W_s|^2 = 3H^2M_p^2 = 4M_p^2|s|^2$. And similarly the Hubble-induced A term, $a_H H \frac{\phi^n}{n M_p^2}$, in presence of non-minimal kähler correction read as:

$$ a_H H \frac{\phi^n}{n M_p^2} = \left\{ W_\phi \phi - 3W(\phi) + bW_\phi \phi^\dagger \frac{\phi^*}{M_p} \right\} \frac{e^K W^*(I) I}{M_p^2} - b \frac{e^K W^*(I) I}{M_p^2} + \frac{W(\phi) \frac{I}{M_p} - bW_\phi \phi^\dagger}{M_p} \frac{e^K \phi^*}{M_p} + h.c. $$

(6.0.55)

$$ \approx \left\{ \left[ 1 - \frac{3}{n} \right] \phi + \frac{b\phi^*}{M_p} \right\} \frac{\lambda M_p s^{n-1} s^2}{M_p^2} + \left( \frac{s\phi^*}{M_p} - b n \phi^\dagger \right) \frac{2M_p \lambda M_p^{n-1} s^2}{M_p^2} + \frac{4M_p |s|^2 s^2 |s|^2}{M_p^2} \phi \phi + h.c. $$
Case 3. \( K = \phi\phi^\dagger + ss^\dagger + \frac{c}{4M_p^2}ss^\dagger\phi\phi + h.c. \)

In a similar way we can analyze the above non-minimal Kähler interaction, where \( c\) is the dimensionless number. We have computed the correction to the Hubble-induced mass term, \( c_H H^2|\phi|^2 \) for \( |s| \ll M_p \) as:

\[
c_H = \left[ 3 + \frac{3c}{2} \frac{|s|^2}{M_p^2} \left( 1 + \frac{3c}{2} \frac{|s|^2}{M_p^2} - \frac{c^2}{4} \frac{|s|^4}{M_p^4} \right) \right] \left( \frac{e^K|F_s|^2}{V(s)} - 1 \right) \approx 3
\]

where we have used \( V(s) = |W_s|^2 = 3H^2M_p^2 = 4M_s^2|s|^2 \). Next we compute the Hubble-induced A term, \( a_H H\frac{\phi^n}{nM_p^{-n}} \):

\[
a_H H\frac{\phi^n}{nM_p^{-n}} = \left( W_\phi \phi - 3W(\phi) + \frac{c}{2} W_\phi \phi \frac{s\phi^\dagger}{M_p} - \frac{c}{2} \frac{W_s}{M_p} \phi \right) \frac{e^K|F_s|^2}{M_p^2} \left( W(\phi) \frac{s^\dagger}{M_p} - cW_\phi \phi \frac{s}{M_p} \right) \frac{e^K|F_s|^2}{M_p^2}
\]

\[
= \left\{ (1 - \frac{3}{n}) \phi + \frac{c\phi^\dagger ss}{2M_p^2} \right\} \frac{\lambda nM_p^{n-1}s^2}{M_p} + \frac{cM^2\phi^\dagger ss}{M_p^2} \left( \frac{ss^\dagger}{M_p} \right) \frac{2M_\phi^2 \lambda nM_p^{n-1}s^2}{nM_p^{-n}} + \frac{M_p^2|s|^2s\phi^\dagger}{M_p^2} \phi + h.c.
\]

Case 4. \( K = \phi\phi^\dagger + ss^\dagger + \frac{d}{M_p} s\phi^\dagger \phi + h.c. \)

For the above non-minimal Kähler potential, where \( d\) is the dimensionfull number, we can compute the Hubble-induced mass term, \( c_H H^2|\phi|^2 \), for \( |s| \ll M_p \):

\[
c_H = \left[ 3 + \frac{d + \lambda ss}{M_p^2} + \frac{d^2}{M_p^2} \left( 1 + \frac{d + \lambda ss}{M_p^2} \right)^{-1} \right] \left[ 1 + \frac{d + \lambda ss}{M_p^2} \right] \left( \frac{e^K|F_s|^2}{V(s)} - 1 \right) \approx 3(1 + d^2),
\]

where we used \( V(s) = |W_s|^2 = 3H^2M_p^2 = 4M_s^2|s|^2 \). Next we compute the correction to the Hubble-induced A term, \( a_H H\frac{\phi^n}{nM_p^{-n}} \):

\[
a_H H\frac{\phi^n}{nM_p^{-n}} = \left( W_\phi \phi - 3W(\phi) + \frac{c}{2} W_\phi \phi \frac{s\phi^\dagger}{M_p} - \frac{c}{2} \frac{W_s}{M_p} \phi \right) \frac{e^K|F_s|^2}{M_p^2} + \left( W(\phi) \frac{s^\dagger}{M_p} - dW_\phi \phi \frac{s}{M_p} \right) \frac{e^K|F_s|^2}{M_p^2} + h.c.
\]

\[
= (1 - \frac{3}{n}) \frac{\lambda nM_p^{n-1}s^2}{M_p} + \left( \frac{ss^\dagger}{M_p} - d \right) \frac{2M_\phi^2 \lambda nM_p^{n-1}s^2}{nM_p^{-n}} + h.c.
\]
H. Expression for $a_H$

Using these results in Hubble induced A-term, $a_H$ can be computed from Eqs. (6.0.53), Eq (6.0.55), Eq (6.0.57) and Eq (6.0.59) for the four physical situations, the simplified expressions turn out be:

$$a_H \sim \begin{cases} 
\frac{\sqrt{2}}{2} \left( \frac{\phi}{\sqrt{\phi}} \right)^4 \sqrt{\frac{H_{\inf}}{M_p}} \left[ 1 + a - \frac{4}{n} + \frac{35a}{8} \sqrt{\frac{2}{3}} \left( 2 - \frac{3}{n} \right) \frac{H_{\inf}}{M_p} - \frac{35a^2}{4} \sqrt{\frac{2}{3}} \frac{H_{\inf}}{M_p} \right] & \text{for Case I} \\
\frac{1}{2} \left[ 3 \left( 1 - \frac{1}{n} \right) + \frac{5a}{n} \sqrt{\frac{2}{3}} \frac{H_{\inf}}{M_p} \right] \left( \frac{3}{2} \right)^{\frac{n-2}{2}} - \frac{b}{2} \left( \frac{3}{2} \right)^{\frac{n-2}{2}} \frac{H_{\inf}}{M_p} - bn \right] + 10b \left( \frac{3}{2} \right)^{\frac{n-2}{2}} \left( \frac{M_p}{\sqrt{2}} \right)^{\frac{n-2}{2}} \frac{H_{\inf}}{M_p} & \text{for Case II} \\
\sqrt{\frac{2}{3}} \left( \frac{3}{2} \right)^{\frac{n-2}{2}} \sqrt{\frac{2}{3}} \frac{H_{\inf}}{M_p} + (1 - cn)^{\frac{3}{2}} \frac{H_{\inf}}{M_p} & \text{for Case III} \\
\sqrt{\frac{2}{3}} (n-3) \sqrt{\frac{H_{\inf}}{M_p}} + 2 \frac{2}{3} (\sqrt{\frac{2}{3}} \frac{H_{\inf}}{M_p} - d) & \text{for Case IV} 
\end{cases}$$

I. Expression for the non-minimal couplings $a, b, c, d$:

The expressions for the non-minimal supergravity coupling parameter $a, b, c, d$ for all the four physical cases within $\mathcal{N} = 1$ SUGRA with $H_{\inf} \gg m\phi$ can be expressed in terms of the VEV of the heavy field, $\langle s \rangle = M_s$, as:

$$a \sim O \left( 1 - 1.06 \times 10^{-5} \frac{n^2}{(n-1)} \frac{M_s^2}{M_p^2} \right)$$

$$b \sim O \left( \sqrt{ \frac{(3 - \frac{1}{n})^2 M_s^2}{100(n-1) M_p^2} - 1} \right)$$

$$c \sim O \left( \frac{1}{500} \left| \pm 8.16 \frac{M_s}{M_p} \sqrt{n-1} - \sqrt{\frac{2}{3}} \left( \sqrt{\frac{2}{3}} (n-3) + 1 \right) \right| \right)$$

$$d \sim O \left( \frac{2.54 \times 10^{-4} (n-1 + \sqrt{6})^2 M_s^2}{(n-1)} \frac{M_s^2}{M_p^2} - 1 \right)$$

for Case I, Case II, Case III, Case IV.
J. Expression for $\Sigma_s(t), \Xi_s(t), \Psi_s(t), \Theta_s(t)$:

$$
\Sigma_s(t) = \frac{e^{-3Ht}}{\sqrt{\frac{4aM^4}{M_p^2(1 \pm \frac{aM^2}{4M_p^2})} + 9H^2}} \left\{ \frac{3\gamma}{(1 + \frac{aM^2}{4M_p^2})} \left[ \left( \frac{aM^4}{M_p^2(1 + \frac{aM^2}{4M_p^2})} + 3H^2 \right) - H \right] \sqrt{\frac{4aM^4}{M_p^2(1 + \frac{aM^2}{4M_p^2})} + 9H^2} \right\},
$$

$$
\Xi_s(t) = \gamma \left[ \frac{C_2^2}{54H^2} e^{-6Ht} + \frac{3}{84H^2} \left\{ \beta^2 (2 - 3Ht) - 3H^2 \beta (9Ht C_3) - \left[ \beta t^2 + 6C_3 \right] - 81H^4 \left( \frac{\beta}{\gamma} - C_3^2 \right) \right\} - \frac{C_2}{243H^7} e^{-3Ht} \right. 
\times \left\{ \beta (2 + 6Ht + 9H^2 t^2) - 18H^2 (1 + 3Ht) C_3 \right\}],
$$

$$
\Psi_s(t) = \gamma \left[ \frac{C_2^2}{54H^2} e^{-6Ht} + \frac{3}{84H^2} \left\{ \beta^2 (2 - 3Ht) - 3H^2 \beta (9Ht C_5) - \left[ \beta t^2 + 6C_5 \right] - 81H^4 \left( \frac{\beta}{\gamma} - C_5^2 \right) \right\} - \frac{C_2}{243H^7} e^{-3Ht} \right. 
\times \left\{ \beta (2 + 6Ht + 9H^2 t^2) - 18H^2 (1 + 3Ht) C_5 \right\}],
$$

$$
\Theta_s(t) = \frac{\gamma}{(1 + \frac{24M^4}{M_p^2})} \left[ \frac{C_2^2}{54H^2} e^{-6Ht} + \frac{3}{84H^2} \left\{ \beta^2 (2 - 3Ht) - \frac{3H^2 \beta}{(1 + \frac{24M^4}{M_p^2})} (9Ht C_7) - \left[ \frac{\beta t^2}{(1 + \frac{24M^4}{M_p^2})} + 6C_7 \right] - 81H^4 \left( \frac{\beta}{\gamma} - C_7^2 \right) \right\} - \frac{C_2}{243H^7} e^{-3Ht} \right. 
\times \left\{ \frac{\beta}{(1 + \frac{24M^4}{M_p^2})} (2 + 6Ht + 9H^2 t^2) - 18H^2 (1 + 3Ht) C_7 \right\} \right].
$$
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