Computation of the Heavy-Light Decay Constant with NRQCD

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Non-relativistic QCD is applied for a lattice computation of the heavy-light meson decay constant in quenched approximation at $\beta = 6.0$. Clear signals are obtained for the ground state at large times in the correlators, allowing a reliable extraction of the decay constant. Estimating the current renormalization factor by the tadpole improvement procedure, we find $f_B = 164(17)$ MeV for the $B$ meson with $a^{-1} = 2.3$ GeV, while an extrapolation to the static limit yields $f_B^{\text{static}} = 247(26)$ MeV.

1. Introduction

In recent years large effort has been directed toward a lattice QCD calculation of the decay constant of heavy-light mesons. Uncertainties in the results, however, are still quite large\cite{1}. In the static approximation for heavy quarks, the problem manifests in the presence of large noise in the correlators, which makes it difficult to extract ground state signals. Smearing techniques have not resolved the problem, failing to yield results independent of the smearing size\cite{2}.

In this work we report on a calculation of heavy-light decay constants using non-relativistic QCD (NRQCD)\cite{3}. Our motivation stems from the expectation that noise in correlators should be reduced for a finite heavy quark mass\cite{4} and hence cleaner results could be obtained in NRQCD. In addition NRQCD allows incorporation of $1/m_Q$ corrections in a systematic way. This is an important point since available results indicate that $1/m_Q$ corrections to the static limit is fairly large for the $B$ meson\cite{5}.

2. Simulation

For the heavy quark we employ the standard NRQCD action given by\cite{3}

$$S_Q^{(n)} = a^3 \sum_{xt} Q_{xt}^I \left[ \Delta_4 - H^{(n)} - c_1 \frac{\vec{\sigma} \cdot \vec{B}}{2m_Q a} \right] Q_{xt} \tag{1}$$

where $H^{(1)} = H = \sum_j \Delta_j \Delta_j/2m_Q a$ and $H^{(2)} = H - H^2/4$, the latter being a modified choice in order to stabilize high frequency modes for $m_Q a < 3$. The spin-magnetic interaction term is included for keeping consistency of the $1/m_Q$ expansion. We use the tree-level value $c_1 = 1$ for the coefficient.

Our simulation is carried out with 40 quenched configurations on an $16^3 \times 48$ lattice at $\beta = 6.0$. The heavy quark masses used are $m_Q a = 1000, 10.0, 7.0, 5.0, 4.0$ with the $n = 1$ action and $5.0, 4.0, 3.0, 2.5$ with $n = 2$. The value $m_Q = 1000$ is taken to compare with the results of the static approximation. For the light quark we used the Wilson action with $K = 0.1530, 0.1540$ and 0.1550. The critical hopping parameter is $K_c = 0.15708(2)$.

The heavy-light meson decay constant is extracted from the correlator of the axial-vector current and the heavy-light meson in the standard manner. For the meson operator we use the local form and also employ the cube smearing of sizes $3^3, 5^3, 7^3$ and $9^3$, fixing gauge configurations to the Coulomb gauge to avoid the necessity of inserting gauge link factors.

3. Results

3.1. Signal to noise ratio

In Fig.\textsuperscript{1} we show the effective mass of the local-local correlation function

$$C(t) = \langle \bar{Q}(t) \gamma_4 \gamma_5 q(t) \bar{q}(0) \gamma_5 Q(0) \rangle \tag{2}$$

for $m_Q a = 1000$ and 5.0. Clearly the signal is far better for $m_Q a = 5.0$ for which we observe a plateau beyond $t \approx 10 - 12$. 


The improvement of ground state signals for smaller values of $m_Qa$ can be qualitatively understood from the estimate of the relative error

$$\frac{\delta C(t)}{C(t)} \propto \exp \left[ \left( E(Q\bar{q}) - \frac{E(Q\bar{Q}) + m_\pi}{2} \right) t \right]$$

(3)

where $E(Q\bar{q})$ and $E(Q\bar{Q})$ are binding energies of heavy-light and heavy-heavy mesons. For finite $m_Qa$ the negative contribution of $E(Q\bar{Q})$ reduces the value of the exponential slope from that in the static limit where $E(Q\bar{Q})$ vanishes. We found that our data for $\delta C(t)/C(t)$ are consistent with this estimate. Typical examples are shown in Fig. 2 where solid lines indicate the slope expected from the measured values of the binding energies and $m_\pi$. 

Figure 1. Effective mass of the local-local correlation function for $m_Qa = 1000$ (open circles) and $m_Qa = 5$ (filled circles), both with $K = 0.1530$ for light quark.

Figure 2. Relative error of local-local correlation function for $m_Qa = 1000$ (open circles) and $m_Qa = 5$ (filled circles), both with $K = 0.1530$ for light quark.
3.2. Dependence on smearing size

In Fig. 3 we plot the raw value of the combination \( f_P \sqrt{m_P} \) of the decay constant \( f_P \) and the heavy-light meson mass \( m_P \) extracted from fits of the correlators over the interval \( t_{\text{min}} \leq t \leq t_{\text{min}} + 4 \) for various smearing sizes. For each group of data points \( t_{\text{min}} \) increases as \( t_{\text{min}} = 6, 8, 10, 12 \) from left to right. We observe that the estimates converge to the same value after \( t \approx 10 \) for all the smearing sizes including the case of no smearing. This gives us confidence that the asymptotic region is really reached at \( t \approx 10 \sim 12 \). We also note that the magnitude of errors are similar for various smearing sizes. In particular reliable results can be obtained without smearing in NRQCD. In the following analysis we use the cube smearing of size \( 5^3 \) and extract \( f_P \sqrt{m_P} \) from a global fit over the interval \( 10 \leq t \leq 20 \). Other choices give similar results.

3.3. Heavy-light decay constant

In order to obtain \( f_P \sqrt{m_P} \) as a function of the heavy quark mass \( m_Qa \) we extrapolate the results at three values of \( K \) for the light quark linearly in \( 1/K \) to \( K_c \) for each \( m_Q \). For the axial-vector current renormalization factor \( Z_A \) we take the value \( Z_A = 0.65 \) obtained by applying the improvement procedure with the coupling \( g_2^2 (1/a) \) to the one-loop result, disregarding a small \( m_Q \) dependence of a few % over our range of \( m_Qa \). The results for \( f_P \sqrt{m_P} \) are plotted in Fig. 4. Circles and triangles are for the results obtained with the \( n = 1 \) and 2 action in (1). We see that they yield consistent values for \( f_P \sqrt{m_P} \) in the region of \( m_Qa \) where both actions can be employed.

![Figure 3. Dependence of \( f_P \sqrt{m_P} \) on the fitting range \( t_{\text{min}} \leq t \leq t_{\text{min}} + 4 \) for various smearing sizes at \( m_Qa = 5.0 \) and \( K = 0.1530 \).](image)

![Figure 4. \( 1/m_Q \) dependence of \( f_P \sqrt{m_P} \) after extrapolation the limit \( K = K_c \) for light quark.](image)
with the form,

\[ f_P \sqrt{m_P} = (f_P \sqrt{m_P})^{\text{static}} (1 - \frac{c}{m_Qa}), \]  

(4)

and obtain

\[ (f_P \sqrt{m_P})^{\text{static}} = 0.163(17), \quad c = 0.60(20). \]  

(5)

With \( a^{-1} = 2.3(1) \) GeV obtained from the \( \rho \) meson mass on the same set of configurations, this gives

\[ f_B^{\text{static}} = 247(26) \left( \frac{a^{-1}}{2.3 \text{ GeV}} \right)^{3/2} \text{ MeV} \]  

(6)

for the \( B \) meson decay constant in the static approximation.

Substituting \( m_Qa = 1.8 \) for the \( b \) quark mass which is estimated from the \( \Upsilon \) mass including the heavy quark mass renormalization\,[7], we find a substantially smaller value for the \( B \) meson decay constant in NRQCD:

\[ f_B = 164(17) \left( \frac{a^{-1}}{2.3 \text{ GeV}} \right)^{3/2} \text{ MeV}. \]  

(7)

The large coefficient \( c=0.60(20) \) in (4) leads to a significant \( 1/m_Q \) correction (\( \sim 30 \% \)) to the static approximation, which is consistent with the results obtained with propagating heavy quarks\,[5].

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