Existence of Traveling Waves for Ratio-dependent Predator-prey System with Nonlocal Diffusion

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Abstract: In this paper, we study the traveling waves for the ratio-dependent predator-prey model with nonlocal diffusion, which is devoted to the existence and nonexistence of traveling wave solution. This model incorporates the ratio-dependent functional response into the Lotka-Volterra type system, and both species obey the logistic growth. Firstly, we construct a nice pair of upper and lower solutions when the wave speed is greater than the minimal wave speed. Then by applying Schauder’s fixed point theorem with the help of suitable upper and lower solutions, we can obtain the existence of traveling waves when the wave speed is greater than the minimal wave speed. Moreover, in order to prove the limit behavior of the traveling waves at infinity, we construct a sequence that converges to the coexistence state. Finally, by using the comparison principle, we obtain the nonexistence of the traveling waves when the wave speed is greater than 0 and less than the minimal wave speed. The difficulty of this paper is to construct a suitable upper and lower solution, which is also the novelty of this paper. Under certain restricted condition, this paper concludes the existence and the nonexistence of the traveling waves for the ratio-dependent predator-prey model with nonlocal diffusion.

Keywords: Traveling Wave Solution, Predator-prey Model, Nonlocal Diffusion, Ratio-dependent Functional Response, Schauder’s Fixed Point Theorem, Comparison Principle

1. Introduction

The interaction between the predator and the prey constitutes a dynamic relationship that has been one of the main topics in ecological research, which is important for studying the distribution of organisms and the balance of the environment.

This relationship can be described by the dynamic behavior of some mathematical models. To describe the predator-prey model, Tanner in [28] considered the following ordinary differential system

\[
\begin{align*}
\frac{du}{dt} &= u(t)[1 - u(t)] - H(u(t))v(t), \\
\frac{dv}{dt} &= r v(t) \left(1 - \frac{v(t)}{u(t)} \right),
\end{align*}
\]

(1)

where \(H(u) = \frac{mu}{A+u} \quad (m > 0, A > 0)\) denotes the functional response to predation suggested by Holling in [15] and \(r\) denotes the growth rate of predator. The second equation of (1) means that the intrinsic population growth rate \(r\) affects not only the potential increase of the population but also its decrease. The classical Lotka-Volterra model and its modified models have been studied for the stability of equilibria and the existence of traveling waves, see [8, 14, 17, 19, 29, 30].

By taking into account the effect of the diffusion, reaction-diffusion predator-prey models have been established to describe the invasion of a predator species [11, 12, 22, 26]. Some previous work for dynamics of diffusive Holling-Tanner predator-prey systems on a bounded region can be found in [3, 25]. Zuo and Shi in [31] has studied the reaction-diffusion Holling-Tanner type predator-prey system with ratio-dependent functional response

\[
\begin{align*}
u_t(x, t) &= du_x(x, t) + u(x, t)(1 - u(x, t)) - \frac{Au(x, t)v(x, t)}{u(x, t) + av(x, t)}, \\
v_t(x, t) &= v_x(x, t) + Bv(x, t)\left(1 - \frac{v(x, y)}{u(x, t)} \right),
\end{align*}
\]

for the ratio-dependent functional response.
the two species move randomly along a one-dimensional region $R$, and the parameter $d > 0$ is a rescaled diffusion coefficient of the prey species while the diffusion coefficient for the predator is rescaled to be $1$. They established the existence of traveling wave solution by using the upper and lower solutions method and proved the existence of periodic traveling wave train by using the Hopf bifurcation theorem. Upper-lower solution and Fixed point theory have also be used to prove the existence of traveling wave solutions in a quasimonotone system, see [13, 16, 20, 21].

About the discrete diffusive ratio-dependent predator-prey model, Zhang and Su in [27] considered the following system

$$\begin{align}
\frac{du(t)}{dt} &= D[u(t)] + u(t)\left(1 - u(t)\right) - \frac{a_u(t)v(t)}{u(t) + a_v(t)}, \\
\frac{dv(t)}{dt} &= dD[v(t)] + \beta v(t)\left(1 - \frac{v(t)}{u(t)}\right),
\end{align}$$

where $j \in \mathbb{Z}$, $t > 0$, $d > 0$, $\mathcal{D}[z_j] = z_{j+1} + z_{j-1} - 2z_j$ and $A, a, \beta$ are positive constants. Zhang and Su obtained the existence of invasion traveling wave solution of (2) by applying Schauder’s fixed point theorem with the help of suitable upper and lower solutions.

Considering of special diffusion, the standard Laplacian operator is corresponding to expected values for individuals moving under a Brownian process. But the movement of individuals is free and random which can not be limited in a small area. So various integral operators have been widely applied to describe the nonlocal diffusion. The nonlocal reaction-diffusion system takes the form

$$\begin{align}
\frac{\partial u(x, t)}{\partial t} &= d_1(j * u(x, t) - u(x, t)) + f(u(x, t), v(x, t)), \\
\frac{\partial v(x, t)}{\partial t} &= d_2(j * v(x, t) - v(x, t)) + g(u(x, t), v(x, t)),
\end{align}$$

where $f(x)$ is the diffusion kernel given by

$$J * u - u(x, t) \text{ and } (J * v - v)(x, t) \text{ represent nonlocal diffusion processes [1, 10, 23, 24].}$$

Meanwhile, many researchers study the properties of the traveling wave solution for the reaction-diffusion systems with nonlocal diffusion term, see [2, 4, 5, 6, 7, 9].

Inspired by these results, we consider the ratio-dependent predator-prey model with nonlocal diffusion, that is

$$\begin{align}
\frac{\partial u(x, t)}{\partial t} &= d_1(j * u(x, t) - u(x, t)) + u(x, t)(1 - u(x, t)) - \frac{\beta u(x, t)v(x, t)}{u(x, t) + a_v(x, t)}, \\
\frac{\partial v(x, t)}{\partial t} &= d_2(j * v(x, t) - v(x, t)) + rv(x, t)\left(1 - \frac{v(x, t)}{u(x, t)}\right),
\end{align}$$

where $u(x, t)$ and $v(x, t)$ are the population densities of the prey and predator species at the location $x$ and time $t$ respectively, $j * u(x, t)$ and $j * v(x, t)$ are the same as the previous (3); the parameters $\beta$, $a$, $d_i(i = 1, 2)$ and $r$ are positive constants. The parameters $d_i(i = 1, 2)$ are diffusion rates for the prey and predator individuals, respectively, $r$ is the intrinsic growth rate of predator, $\beta$ is the capturing rate, and $a$ is the half-capturing saturation constant. If $\beta = 0$, then the first equation of system (4) is simplified to fisher's KPP equation. If the preys are only as food for the predator, that is, $u \equiv 1$, then the second equation of system (4) is simplified to fisher's KPP equation. Some authors obtained the nonexistence of traveling wave solutions of predator systems by considering the related Cauchy problem of fisher's equation, see [31]. Throughout this paper, we need the below assumptions of the kernel function $J$.

Assumption 1.1 (K1) The function $J$ is a smooth function in $\mathbb{R}$ and satisfies $J \in C^1(\mathbb{R})$, $J(y) = J(-y) \geq 0$, $\int_{\mathbb{R}} J(y) dy = 1$.

(K2) There exists $\lambda_0 \in (0, +\infty]$ such that

$$\int_{\mathbb{R}} J(y)e^{-\lambda y} dy < +\infty \text{ for any } \lambda \in [0, \lambda_0),$$

and

$$\int_{\mathbb{R}} J(y)e^{-\lambda y} dy \to +\infty \text{ as } \lambda \to \lambda_0 - 0.$$
Then we can get characteristic equations
\[
\Delta_{d,b}(\lambda, c) := d_1 \int_B f(y) e^{-\lambda y} dy - d_1 c + b,
\]
(6)
Where \(d_1\) and \(b\) are non-negative constants. By a direct calculation, for \(c > 0\) and \(\lambda > 0\), we can obtain
\[
\Delta_{d,b}(0, c) = b, \\
\frac{\partial \Delta_{d,b}(\lambda, c)}{\partial c} = -\lambda < 0, \\
\frac{\partial \Delta_{d,b}(0, c)}{\partial \lambda} = -c < 0 \text{ and } \frac{\partial \Delta_{d,b}(\lambda, c)}{\partial \lambda^2} > 0.
\]
In view of the above properties of the function \(\Delta_{d,b}(\lambda, c)\), we can get the following lemma.

Lemma 2.1. For any given \(0 < U < U^*\), then \(Y, Z, Q, P, U > 0\) for any \(L > 0\). Moreover, for any \(U > U^*\), there exist positive constants \(L^* < L < L_0\) such that
\[
\Delta_{d,b}(L, U) = 0, L = L^*, L = L_0, L = L^* + \infty, \text{ and } L = L_0.
\]

In the sequel, we always assume that \(U > U^*\) and simply denote \(L = U^*\) for \(G = 1, 2, 3\), respectively.

Definition 2.1. If the functions \(u^+, u^-, v^+, v^-, \), \(\xi_1, \xi_2, \cdots, \xi_n\) satisfy the following inequalities
\[
E(u^+, v^+)(\xi) := c(u^+)(\xi) - d_1(f * u^+)(\xi) - u^+\xi(1 - u^\xi) + \frac{\beta u^+\xi}{u^+ + v^+\xi} \geq 0,
\]
(7)
\[
E(u^-, v^-)(\xi) := c(u^-)(\xi) - d_1(f * u^-)(\xi) - u^-\xi(1 - u^-\xi) + \frac{\beta u^-\xi}{u^- + v^-\xi} \leq 0,
\]
(8)
\[
E(v^+, u^+)(\xi) := c(v^+)(\xi) - d_2(f * v^+)(\xi) - v^+\xi - r v^+\xi + r\frac{(v^+\xi)^2}{u^+\xi} \geq 0,
\]
(9)
\[
E(v^-, u^-)(\xi) := c(v^-)(\xi) - d_2(f * v^-)(\xi) - v^-\xi - r v^-\xi + r\frac{(v^-\xi)^2}{u^-\xi} \leq 0,
\]
(10)
for \(\xi \in \mathbb{R} \setminus D\) with some finite set \(D = \{\xi_1, \xi_2, \cdots, \xi_n\}\) and have no derivatives at \(\xi_i (i = 1, 2, \cdots, n)\), then the functions \((u^+, v^+)\) and \((u^-, v^-)\) are called a pair of upper and lower solutions of the system (5).

3. The Existence of Traveling Waves

3.1. Upper and Lower Solutions of The System (5)

Define
\[
u^+ = \min\left\{\frac{a}{\beta} e^{\lambda_2\xi}, \xi \leq \frac{1}{\beta} \ln \frac{\beta}{\xi}\right\},
\]
\[
u^- = \max\left\{0, \frac{a}{\beta} e^{\lambda_2\xi}(1 - Me^{\eta\xi})\right\},
\]
(12)
\[
u^+ = \min\left\{\frac{a}{\beta} e^{\lambda_2\xi}, \xi \leq \frac{1}{\beta} \ln \frac{\beta}{\xi}\right\},
\]
\[
u^- = \max\left\{0, \frac{a}{\beta} e^{\lambda_2\xi}(1 - Me^{\eta\xi})\right\},
\]
where
\[
0 < \beta < a, \quad 0 < a < \lambda_2 \text{ and } d_1 \int_B f(y) e^{-\alpha y} dy - \alpha c - d_1 < 0,
\]
(13)
\[
\delta \in (0, \nu^+\xi), \eta \in (0, \min(\lambda_2, \lambda_3 - \lambda_2)),
\]
(14)
and \(M > 1\) satisfies
\[ M = \frac{ra^2}{-\beta^2(a-\rho)da(d^2+\rho)} + \rho^2 + 1. \]  

(15)

Lemma 3.1. Let \( c > c^* \), the functions \((u^+, v^+)\) and \((u^-, v^-)\) are a pair of upper and lower solutions of the system (5) which are defined as in (11)-(12).

Proof. Firstly, we show that (7) holds. If \( \xi \leq \frac{2}{\alpha} \ln \frac{\beta}{a}, u^+(\xi) = 1 \), we can obtain

\[ E(u^+, v^+)(\xi) = \frac{\beta v^+(\xi)}{1 + \alpha v^+(\xi)} > 0, \text{ where } v^+(\xi) > 0. \]

If \( \xi > \frac{2}{\alpha} \ln \frac{\beta}{a} \), then

\[ E(u^+, v^+)(\xi) = u^+(\xi) \left[ u^+(\xi) - 1 + \frac{\beta v^+(\xi)}{u^+(\xi) + \alpha v^+(\xi)} \right] \geq u^+ \left( \frac{\beta v^+(\xi)}{1 + \alpha v^+(\xi)} - \frac{\beta}{1 + a\delta} \right) > 0, \]

where we use \( \delta < v^+(\xi) \) and the function \( \frac{\beta}{1 + ax} \) is increasing in \( x \), when \( \beta > 0 \).

Next, if \( \xi > \frac{2}{\alpha} \ln \frac{\beta}{a} \), then

\[ E(u^-, v^+)(\xi) = -d_1 \left( 1 - \frac{\beta}{a} \right) \int_{\mathbb{R}} J(y)dy + d_1 \left( 1 - \frac{\beta}{a} \right) - \left( 1 - \frac{\beta}{a} \right) \frac{\beta}{a} + \frac{\beta(1 - \frac{\beta}{a} v^+(\xi))}{1 - a + \alpha v^+(\xi)} \]

\[ = \left( 1 - \frac{\beta}{a} \right) \frac{\beta v^+(\xi)}{1 - a + \alpha v^+(\xi)} - \frac{\beta}{a} \leq \left( 1 - \frac{\beta}{a} \right) \frac{\beta}{a} - \frac{\beta}{a} = 0. \]

For \( \xi \leq \frac{2}{\alpha} \ln \frac{\beta}{a} < 0 \), \( u^-(\xi) = 1 - \frac{a}{\beta} e^{\alpha \xi} \) and \( v^+(\xi) = \min \left( \frac{a}{\beta^2} e^{\lambda_2 \xi}, 1 \right) \leq \frac{a}{\beta^2} e^{\lambda_2 \xi} \leq \frac{a}{\beta^2} e^{\alpha \xi} \) since \( 0 < \alpha < \lambda_2 \). By easy calculation and the similar above argument, we can get that

\[ 1 - u^-(\xi) - \frac{\beta v^+(\xi)}{u^+(\xi) + \alpha v^+(\xi)} = \frac{a}{\beta} e^{\alpha \xi} - \frac{\beta v^+(\xi)}{1 - a + \alpha v^+(\xi)} \]

\[ \geq \frac{a}{\beta} e^{\alpha \xi} - \frac{a}{\beta} e^{\alpha \xi} = 0. \]

Hence, with (13), we obtain

\[ E(u^-, v^+)(\xi) = -c \alpha \frac{a}{\beta} e^{\alpha \xi} - d_1 \int_{\mathbb{R}} J(y) \left( 1 - \frac{a}{\beta} e^{\alpha \xi} \right) dy + d_1 \left( 1 - \frac{a}{\beta} e^{\alpha \xi} \right)\]

\[ - u^-(\xi) \left[ 1 - u^-(\xi) - \frac{\beta v^+(\xi)}{u^+(\xi) + \alpha v^+(\xi)} \right] \leq - u^-(\xi) \left[ 1 - u^-(\xi) - \frac{\beta v^+(\xi)}{u^+(\xi) + \alpha v^+(\xi)} \right] \leq 0. \]

So the (8) holds.

Then, we show that (9) holds. It is clear that

\[ v^+(\xi) = \begin{cases} 1, & \xi \geq \frac{1}{\lambda_2} \ln \frac{\beta}{a}, \\ \frac{a}{\beta^2} e^{\lambda_2 \xi}, & \xi < \frac{1}{\lambda_2} \ln \frac{\beta}{a}. \end{cases} \]

For \( \xi \geq \frac{1}{\lambda_2} \ln \frac{\beta}{a} \), \( v^+(\xi) = 1 \). That is

\[ E(v^+, u^+)(\xi) = -r + \frac{r}{u^+(\xi)} \geq 0. \]

For \( \xi < \frac{1}{\lambda_2} \ln \frac{\beta}{a} \), \( v^+(\xi) = \frac{a}{\beta^2} e^{\lambda_2 \xi} \). We can deduce

\[ E(v^+, u^+)(\xi) \geq - \frac{a}{\beta^2} e^{\lambda_2 \xi} (d_2 \int_{\mathbb{R}} J(y) e^{-\lambda_2 y} dy - d_2 - c\lambda_2 + r) + \frac{a^2}{\beta^2} e^{\lambda_2 \xi}. \]
By the definition of $\Delta_d, (\lambda, c)$ and Lemma 2.1, we can get

$$E(u^+, u^+)(\xi) \geq r \frac{\alpha}{\beta^2} e^{2\lambda_2^\xi} > 0.$$  

Lastly, we prove that (10) holds. For $\xi \geq \frac{1}{n} \ln \frac{1}{M}$, $v^-(\xi) = 0$ and with (14), we get

$$E(u^-, u^-)(\xi) = 0.$$  

For $\xi < \frac{1}{n} \ln \frac{1}{M} < 0$, $v^-(\xi) = \frac{a}{\beta^2} e^{2\lambda_2^\xi} (1 - Me^{\eta})$, we have

$$E(v^-, u^-)(\xi) \leq c \left[ \lambda_2 \frac{a}{\beta^2} e^{2\lambda_2^\xi} - (\lambda_2 + \eta) \frac{a}{\beta^2} Me^{(\lambda_2 + \eta)^\xi} \right] - d_2 \int_R f(y) \left( \frac{a}{\beta^2} e^{2\lambda_2^\xi} - \frac{a}{\beta^2} Me^{(\lambda_2 + \eta)^\xi} \right) dy - \frac{\alpha}{\beta^2} e^{2\lambda_2^\xi} + \frac{a}{\beta^2} Me^{(\lambda_2 + \eta)^\xi} \right] - r \left( \frac{a}{\beta^2} e^{2\lambda_2^\xi} - \frac{a}{\beta^2} Me^{(\lambda_2 + \eta)^\xi} \right) + \frac{\alpha}{\beta^2} e^{2\lambda_2^\xi} \right] \leq - \frac{\alpha}{\beta^2} e^{2\lambda_2^\xi} - \frac{a}{\beta^2} Me^{(\lambda_2 + \eta)^\xi} \right] \leq - \frac{\alpha}{\beta^2} e^{2\lambda_2^\xi} - \frac{a}{\beta^2} Me^{(\lambda_2 + \eta)^\xi} \right].$$

where use the facts that $v^-(\xi)^2 \leq v^-(\xi) \frac{a}{\beta^2} e^{2\lambda_2^\xi}$, (6) and Lemma 2.1, (14)-(15), this completes the proof.

Now we define $A > \max \left( \frac{1}{2} \ln M, \frac{\alpha}{\beta^2} \ln \frac{a}{\beta^2} \right)$ and a function set

$$\Gamma_A = \left\{ (\phi(\cdot), \varphi(\cdot)) \in C([-A, A], \mathbb{R}^2) : \begin{cases} \phi(-A) = u^-(A), \varphi(-A) = v^-(A), \\ u^-(\xi) \leq \phi(\xi) \leq u^+(\xi), v^-(\xi) \leq \varphi(\xi) \leq v^+(\xi) \end{cases} \right\},$$

For any $(\phi(\cdot), \varphi(\cdot)) \in C([-A, A], \mathbb{R}^2)$, we define

$$\hat{\phi}(\xi) = \begin{cases} \phi(A), & \xi > A, \\ \phi(\xi), & |\xi| \leq A, \end{cases} \quad \hat{\varphi}(\xi) = \begin{cases} \varphi(A), & \xi > A, \\ \varphi(\xi), & |\xi| \leq A, \end{cases}$$

and consider the following initial value problems

$$cu'(\xi) = d_1 \int_R f(y) \left( \hat{\phi}(\xi - y) - u(\xi) \right) dy + \phi(\xi) \left( 1 - u(\xi) \right) - \frac{\beta u(\xi) \varphi(\xi)}{\phi(\xi) \varphi(\xi)} = \xi > A, \quad \hat{\phi}(\xi), \quad |\xi| \leq A, \quad \hat{\varphi}(\xi), \quad \hat{\varphi}(\xi), \quad |\xi| \leq A, \quad \hat{\phi}(\xi), \quad \hat{\varphi}(\xi), \quad |\xi| \leq A,$$

with

$$u(-A) = u^-(A), \quad v(-A) = v^-(A).$$

Obviously, the problems (16)-(18) admit a unique solution $(u_A(\cdot), v_A(\cdot))$ satisfying $u_A(\cdot) \in C^1([-A, A])$ and $v_A(\cdot) \in C^1([-A, A])$. Then, we define an operator $F = (F_1, F_2) : \Gamma_A \rightarrow C([-A, A])$ by $F_1[\phi, \varphi](\xi) = u_A(\xi)$ and $F_2[\phi, \varphi](\xi) = v_A(\xi)$ for $\xi \in [-A, A]$.

Lemma 3.2. The operator $F$ maps $\Gamma_A$ into $\Gamma_A$.

Proof. For any $(\phi(\cdot), \varphi(\cdot)) \in \Gamma_A$, we should prove that

$$F_1[\phi, \varphi](-A) = u^-(A), \quad F_2[\phi, \varphi](-A) = v^-(A),$$

By the definition of $\Delta_d, (\lambda, c)$ and Lemma 2.1, we can get

$$E(u^+, u^+)(\xi) \geq r \frac{\alpha}{\beta^2} e^{2\lambda_2^\xi} > 0.$$
and
\[ u^- (\xi) \leq \mathcal{F}_1[\phi, \varphi](\xi) \leq u^+ (\xi), \quad v^- (\xi) \leq \mathcal{F}_2[\phi, \varphi](\xi) \leq v^+ (\xi) \quad \text{for any} \quad \xi \in [-A, A]. \]

By the definition of the operator \( \mathcal{F} \), it is obvious to see that
\[ \mathcal{F}_1[\phi, \varphi](-A) = u_A(-A) = u^-(A), \quad \mathcal{F}_2[\phi, \varphi](-A) = v_A(-A) = v^- (A). \]

Then according to Lemma 3.1 and direct calculation, we can directly obtain
\[ u^- (\xi) \leq \mathcal{F}_1[\phi, \varphi](\xi) \leq u^+ (\xi), \quad v^- (\xi) \leq \mathcal{F}_2[\phi, \varphi](\xi) \leq v^+ (\xi) \quad \text{for any} \quad \xi \in [-A, A]. \]

This ends the proof.

Lemma 3.3. The operator \( \mathcal{F} : \Gamma_A \to \Gamma_A \) is completely continuous.

Proof. We first show that \( \mathcal{F} \) is continuous. By a direct calculation, we get that
\[ \mathcal{F}_1[\phi, \varphi] = u_A(\xi) = u^-(A) \exp \left\{ -\frac{1}{c} \int_{-A}^{A} \left[ d_1 + \phi(s) + \frac{\beta \varphi(s)}{\phi(s) + \alpha \varphi(s)} \right] ds \right\} \]
and
\[ \mathcal{F}_2[\phi, \varphi] = v_A(\xi) = v^-(A) \exp \left\{ -\frac{1}{c} \int_{-A}^{A} \left[ d_2 + r \varphi(s) \right] ds \right\} \]

where
\[ f_\varphi (\eta) = \int_{-\infty}^{A} f (\eta - y) u^-(y)dy + \int_{-A}^{A} f (\eta - y) \varphi(y)dy + \int_{A}^{\infty} f (\eta - y) \varphi(\phi(y))dy, \]
and
\[ g_\varphi (\eta) = \int_{-\infty}^{A} f (\eta - y) v^-(y)dy + \int_{-A}^{A} f (\eta - y) \varphi(y)dy + \int_{A}^{\infty} f (\eta - y) \varphi(\phi(y))dy. \]

For \( \forall (\phi_1(\cdot), \varphi_1(\cdot)), (\phi_2(\cdot), \varphi_2(\cdot)) \in \Gamma_A \), we have that
\[ \left| f_{\phi_1}(\eta) - f_{\phi_2}(\eta) \right| \leq \left| \int_{-A}^{A} f (\eta - y) \left[ \phi_1(y) - \phi_2(y) \right] dy \right| + \left| \int_{A}^{\infty} f (\eta - y) \left[ \phi_1(A) - \phi_2(A) \right] dy \right| \leq 2 \max_{y \in [-A, A]} \left| \phi_1(y) - \phi_2(y) \right|. \]

Combining with the continuity of the compound function, \( \mathcal{F} \) is continuous.

Next we confirm that \( \mathcal{F} \) is compacted, thus we should prove that for any bounded subset \( \Omega \subset \Gamma_A \), \( \mathcal{F}(\Omega) \) is precompact. By the definition of \( \mathcal{F} \), we have that for all \( (u_A, v_A) \in \mathcal{F}(\Omega) \), there exists \( (\phi, \varphi) \in \Omega \) such that
\[ \mathcal{F}[\phi, \varphi](\xi) = (u_A, v_A)(\xi), \quad \forall \xi \in [-A, A]. \]

Since \( (\phi, \varphi) \in \Omega \), in (19) and (20), we obtain that
\[ \left| u_A(\xi) \right| \leq M_1 \quad \text{and} \quad \left| v_A(\xi) \right| \leq M_1, \quad \forall \xi \in [-A, A], \]
where \( M_1 > 0 \) is a constant. That is, \( \mathcal{F}(\Omega) \) is uniformly bounded. Further, according to the equations (16), (17) and the above inequality, then there exists some constant \( M_2 > 0 \) such that
\[ \left| u'_A(\xi) \right| \leq M_2 \quad \text{and} \quad \left| v'_A(\xi) \right| \leq M_2, \quad \forall \xi \in [-A, A]. \]

So we can get that \( \mathcal{F}(\Omega) \) is equicontinuous. By Arzela-Ascoli Theorem, then we have that \( \mathcal{F}(\Omega) \) is precompact. Thus we establish that \( \mathcal{F} : \Gamma_A \to \Gamma_A \) is completely continuous with respect to the maximum norm.

Theorem 3.1. The operator \( \mathcal{F} \) has a fixed point in \( \Gamma_A \).

Proof. By the definition of \( \Gamma_A \), it is easy to see that \( \Gamma_A \) is closed and convex. Thus, according to Lemma 3.3 and using Schauder’s fixed point theorem, there exists \((u_A^*(\cdot), v_A^*(\cdot)) \in \Gamma_A \) such that
To obtain the existence of solutions for (5), we need some estimates about \( u_A^*(\cdot), v_A^*(\cdot) \). For the sake of convenience, we use \( (u_A^*(\cdot), v_A^*(\cdot)) \) instead of \( (u_A^*(\cdot), v^*(\cdot)) \).

Assumption 3.1. The kernel function \( f \) is compactly supported.

Lemma 3.4. Assume that \((K_2)\) hold, then there exists some constant \( C > 0 \) such that

\[
\| u_A^* \|_{C^1([0,1])} \leq C \quad \text{and} \quad \| v_A^* \|_{C^1([0,1])} \leq C
\]

for any \( 0 < B < A \), where \( A > \max\{\frac{\ln M}{n}, \frac{1}{\alpha} \ln \frac{\beta}{\alpha} \} \).

Proof. By Theorem 3.1, we have that \( u_A^*(\cdot), v_A^*(\cdot) \) satisfies

\[
u_A^*(\xi) = d_A \left( \int_0^\xi (y) \hat{u}_A(y) \, dy - u_A^*(\xi) \right) + u_A^*(\xi) \left( 1 - u_A^*(\xi) \right) - \frac{\beta u_A^*(\xi) v_A^*(\xi)}{u_A^*(\xi) + v_A^*(\xi)}
\]

(21)

and

\[
u_A^*(\xi) = d_A \left( \int_0^\xi (y) \hat{v}_A(y) \, dy - v_A^*(\xi) \right) + v_A^*(\xi) \left( 1 - v_A^*(\xi) \right),
\]

(22)

where

\[
\hat{u}_A(\xi) = \begin{cases} 
\begin{array}{ll}
  u_A(A), & \xi > A, \\
  u_A^*(\xi), & |\xi| \leq A, \\
  u^-(A), & \xi < -A,
\end{array}
\end{cases} \quad \hat{v}_A(\xi) = \begin{cases} 
\begin{array}{ll}
  v_A(A), & \xi > A, \\
  v_A^*(\xi), & |\xi| \leq A, \\
  v^-(A), & \xi < -A.
\end{array}
\end{cases}
\]

Following that \( 1 - \frac{\beta}{\alpha} \leq u_A(\xi) \leq 1, \ 0 < v_A(\xi) \leq 1 \) for \( \xi \in [-B, B] \), we have

\[
|u_A^*(\xi)| \leq \frac{d_1}{c} \left| \int_0^\xi (y) \hat{u}_A(y) \, dy \right| + \frac{d_1}{c} |u_A(\xi)| + \frac{1}{c} |u_A(\xi)(1 - u_A(\xi))| + \frac{\beta |u_A(\xi)v_A(\xi)|}{c |u_A(\xi) + v_A(\xi)|}
\]

\[
\leq \frac{2d_1 + \frac{\beta}{\alpha} + \frac{\beta}{\alpha} + E_0}{c},
\]

and

\[
|v_A^*(\xi)| \leq \frac{d_2}{c} \left| \int_0^\xi (y) \hat{v}_A(y) \, dy \right| + \frac{d_2}{c} |v_A(\xi)| + \frac{1}{c} |v_A(\xi)(1 - v_A(\xi))| + \frac{\beta |u_A(\xi)v_A(\xi)|}{c |u_A(\xi) + v_A(\xi)|}
\]

\[
\leq \frac{2d_2 + \frac{\beta}{\alpha} + \frac{\beta}{\alpha} + E_0}{c}.
\]

So there exists some constant \( C_1 > 0 \) such that \( \| u_A^* \|_{C^1([0,1])} \leq C_1 \) and \( \| v_A^* \|_{C^1([0,1])} \leq C_1 \). It is obvious to obtain that

\[
|u_A(\xi) - u_A(\eta)| < C_1 |\xi - \eta| \quad \text{and} \quad |v_A(\xi) - v_A(\eta)| < C_1 |\xi - \eta|
\]

(23)

for any \( \xi, \eta \in [-B, B] \). In view of (3.11), we have

\[
c |u_A^*(\xi) - u_A^*(\eta)| \leq d_1 \left| \int_0^\xi (y) \hat{u}_A(y) \, dy - \int_0^\eta (y) \hat{u}_A(y) \, dy \right|
\]

\[
+ |u_A(\xi)(1 - u_A(\xi))| - u_A(\eta)(1 - u_A(\eta))| + \frac{\beta |u_A(\xi)v_A(\xi)|}{(\alpha - \beta)|u_A(\xi) - u_A(\eta)|}
\]

\[
+ \frac{\beta |u_A(\xi)v_A(\xi)|}{(\alpha - \beta)|v_A(\xi) - v_A(\eta)|} =: d_1 U_1 + d_1 U_2 + U_3 + \frac{\beta |u_A(\xi)v_A(\xi)|}{(\alpha - \beta)} U_4,
\]

(24)

By the conditions \( (K_1) \), we can assume that \( L \) is its Lipschitz constant and \( R \) is the radius of \( \text{supp} f \). Then, we have

\[
U_1 = \left| \int_0^\xi (y) \hat{u}_A(y) \, dy - \int_0^\eta (y) \hat{u}_A(y) \, dy \right|
\]

\[
= \left| \int_0^{\xi + R} (y) \hat{u}_A(y) \, dy - \int_0^{\eta + R} (y) \hat{u}_A(y) \, dy \right|
\]

\[
\leq \left| \int_0^{\xi + R} (y) \hat{u}_A(y) \, dy - \int_0^{\eta + R} (y) \hat{u}_A(y) \, dy \right|
\]

\[
+ \left| \int_0^{\eta + R} (y) \hat{u}_A(y) \, dy - \int_0^{\eta + R} (y) \hat{u}_A(y) \, dy \right|
\]

\[
\leq 2 \| f \|_{L^\infty + RL}[\xi - \eta],
\]
\[ U_3 = |u_\xi(\xi) - u_\eta(\xi) - u_\eta(\eta)| \]
\[ \leq |u_\xi(\xi) - u_\eta(\eta)| + |u_\eta(\xi) - u_\eta(\eta)| \]
\[ \leq 3|u_A(\xi) - u_A(\eta)|. \]

and

\[ U_4 = |u_\xi(\xi) v_\xi(\xi) - u_\eta(\eta) v_\eta(\eta)| \]
\[ \leq |u_\xi(\xi) - u_\eta(\eta)||v_\xi(\xi) - v_\eta(\eta)| + |u_\eta(\xi) - u_\eta(\eta)||v_\eta(\xi) - v_\eta(\eta)|. \]

Combining (23) and (24), we obtain that there exists some constant \( \beta > 0 \) such that
\[ |u'_A(\xi) - u'_A(\eta)| \leq \beta |v_A(\xi) - v_A(\eta)|. \]

Then applying with (19), we also have
\[ \text{c} |u'_A(\xi) - u'_A(\eta)| \leq d_2 \left| \int \mathbb{R} f(y)[\theta_A(\xi - y) - \theta_A(\eta - y)]dy \right| + d|v_A(\xi) - v_A(\eta)| \]
\[ + r \left| v_A(\xi) (1 - \frac{v_A(\xi)}{u_A(\xi)}) - v_A(\eta) (1 - \frac{v_A(\eta)}{u_A(\eta)}) \right| \]
\[ := d_2 V_1 + d_2 V_2 + r V_3. \]

(25)

Similar to the same argument of \( \text{c} \), we have that
\[ \text{c} |u'_A(\xi) - u'_A(\eta)| \leq 2\| f \|_{L^\infty} \| RL \|_{L^1} |\xi - \eta|. \]

Then, by (23), (25) with
\[ V_3 \leq |v_A(\xi) - v_A(\eta)| + \frac{\| v_A(\xi) \|_{L^1} - \| v_A(\xi) \|_{L^1}}{u_A(\xi) u_A(\eta)} |u_A(\xi) - u_A(\eta)| \]
\[ \leq |v_A(\xi) - v_A(\eta)| + \| v_A(\xi) \|_{L^1} - \| v_A(\xi) \|_{L^1} |u_A(\xi) - u_A(\eta)| \]
\[ \leq \frac{3a - \beta}{a - \beta} |v_A(\xi) - v_A(\eta)| + \frac{a^2}{(a - \beta)^2} |u_A(\xi) - u_A(\eta)| \]
we can get that
\[ |u'_A(\xi) - u'_A(\eta)| \leq L_2 |\xi - \eta| \]
for any \( \xi, \eta \in [-B, B] \). So we have obtained that there exists a constant \( C > 0 \) for any \( B < A \) independent of \( \xi \) such that
\[ \| u_A \|_{C^{1,1}([-B,B])} < C \text{ and } \| v_A \|_{C^{1,1}([-B,B])} < C. \]

3.2. Existence of Traveling Waves

Theorem 3.2. Assume that \((K_1) - (K_3)\) hold. For any \( c > c^* \), there exists a pair function \((\bar{u}(\xi), \bar{v}(\xi))\) satisfying (5), \((\bar{u}(-\infty), \bar{v}(-\infty)) = (1,0)\) and \((\bar{u}(+\infty), \bar{v}(+\infty)) = (k,k)\), where \( k = 1 - \frac{\beta}{\alpha + 1} \).

Proof. Choosing an increasing sequence \( \{A_n\}_{n=1}^{\infty} \) such that \( \lim_{n \to +\infty} A_n = +\infty \) and \( A_n > \max \left( \frac{\ln n}{\eta}, \frac{2a}{a - \beta} \right) \) for each \( n \). For every \( c > c^* \), there exists \( (u_{A_n}, v_{A_n}) \in \mathcal{L} \) which satisfies Lemma 3.4 and equations (21), (22) in each \( \xi \in [-A_n, A_n] \). According to the estimates for the sequence \( \{(u_{A_n}, v_{A_n})\} \) in Lemma 3.4, we can extract a subsequence by a standard diagonal extract argument, denoted by \( \{(u_{A_{n_k}}, v_{A_{n_k}})\}_{k \in \mathbb{N}} \), tending towards \((\bar{u}, \bar{v}) \in C^1(\mathbb{R})\) in the following topologies
\[ u_{A_{n_k}} \to \bar{u} \text{ and } v_{A_{n_k}} \to \bar{v} \text{ in } C^{1,1}_{loc}(\mathbb{R}) \text{ as } k \to +\infty. \]

By the assumption of the kernel function \( f(y) \) and applying the dominated convergence theorem, we can obtain that
\[ \lim_{k \to +\infty} \int \mathbb{R} f(y) \bar{u}_{A_{n_k}}(\xi - y) dy = \int \mathbb{R} f(y) \bar{u}(\xi - y) dy, \]
and
\[
\lim_{\xi \to \infty} \int_{\mathbb{R}} f(y) \delta_{A_n}(\xi - y) dy = \int_{\mathbb{R}} f(y) \delta(\xi - y) dy
\]
for any \( \xi \in \mathbb{R} \). Then, it is easy to show that \((\bar{u}, \bar{v})\) satisfies system (5) and
\[
u^{-}(\xi) \leq \bar{u}(\xi) \leq \nu^{+}(\xi), \quad v^{-}(\xi) \leq \bar{v}(\xi) \leq v^{+}(\xi).
\]
By the above inequality, we can further get that \((\bar{u}, \bar{v})\) satisfies \(\bar{u}(-\infty), \bar{v}(-\infty) = (1,0)\), and
\[
1 - \frac{\beta}{a} \leq \liminf_{\xi \to +\infty} \bar{u}(\xi) \leq \limsup_{\xi \to +\infty} \bar{u}(\xi) \leq 1 - \frac{\beta\delta_0}{1+a\delta_0}, 0 \leq \liminf_{\xi \to +\infty} \bar{v}(\xi) \leq \limsup_{\xi \to +\infty} \bar{v}(\xi) \leq 1
\]
(26)
Since \(\limsup_{\xi \to +\infty} \bar{u}(\xi) \leq 1 - \frac{\beta\delta_0}{1+a\delta_0}\) and \(\liminf_{\xi \to +\infty} \bar{v}(\xi) > 0\), there exist \(\xi > \xi_0\) and \(\delta_0 \in (0, 1 - \frac{\beta}{a})\) such that \(\bar{u}(\xi) \leq 1 - \frac{\beta\delta_0}{1+a\delta_0}\) and \(\delta_0 \leq \bar{v}(\xi)\) for any \(\xi > \xi_0\), then by (26), we get that
\[
\delta_0 < \bar{u}(\xi) \leq 1 - \frac{\beta\delta_0}{1+a\delta_0} \quad \text{and} \quad \delta_0 \leq \bar{v}(\xi) \leq 1
\]
(27)
for any \(\xi > \xi_0\). Now, we introduce a sequence \(\{y_n\}_{n \geq 0}\) defined by
\[
\begin{align*}
y_0 &= 1, y_1 = \delta_0, \\
1 - y_{n+1} &= \frac{\beta y_n}{y_0 + y_n}, n \geq 1.
\end{align*}
\]
By direct calculation, we can get that the sequences \(\{y_{2n}\}_{n \geq 0}\) and \(\{y_{2n+1}\}_{n \geq 0}\) are adjacent. They converge to \(K\) and satisfy for each \(s \geq 0\),
\[
\begin{align*}
u^{-} &< \nu^{+} < \cdots < \nu_{2n+1} < \cdots < 1 - \frac{\beta}{1+a} < \cdots < \nu_{2n} < \cdots < \nu_2 < \nu_0.
\end{align*}
\]
Next, we prove that
\[
\begin{align*}
y_{2n+1} \leq \bar{u}(\xi) \leq y_{2n+2} \quad \text{and} \quad y_{2n+1} \leq \bar{v}(\xi) \leq y_{2n}
\end{align*}
\]
(28)
for all \(n \geq 0\) and \(\xi > \xi_0\). According to the inequality (27), this inequality (28) holds true for \(n = 0\). Let us now argue by induction on \(n\). Assume that (28) hold true for all \(n \geq 1\) and let us prove that (28) holds true for \(n+1\). Since \(\bar{u}(\xi) \leq y_{2n+2}\), then \(\bar{v}(\xi)\) satisfies
\[
c\nu' \left( \xi - d_2 \int_{\mathbb{R}} f(y) \left[ \bar{v}(\xi - y) - \bar{v}(\xi) \right] dy + r \bar{v}(\xi) \left( 1 - \frac{\bar{v}(\xi)}{y_{2n+2}} \right) \right) \leq 0 \quad \text{for} \quad \xi \geq \xi_0.
\]
which means that \(\bar{v}(\xi)\) is the subsolution of the equation
\[
c\nu' \left( \xi - d_2 \int_{\mathbb{R}} f(y) \left[ \bar{v}(\xi - y) - \bar{v}(\xi) \right] dy + r \bar{v}(\xi) \left( 1 - \frac{\bar{v}(\xi)}{y_{2n+2}} \right) \right) = 0 \quad \text{for} \quad \xi \geq \xi_0.
\]
(29)
Since \(y_{2n+2}\) is a solution of the equation (29), we can get that \(\bar{v}(\xi) \leq y_{2n+2}\) for all \(\xi \geq \xi_0\). Then we can get that \(\bar{u}(\xi)\) satisfies
\[
c\bar{u}' \left( \xi - d_1 \int_{\mathbb{R}} f(y) \left[ \bar{u}(\xi - y) - \bar{u}(\xi) \right] dy - \bar{u}(\xi) \left( 1 - \bar{u}(\xi) \right) + \frac{\beta y_{2n+2} \bar{u}(\xi)}{y_{2n+1} + \alpha y_{2n+2}} \right) \geq 0 \quad \text{for} \quad \xi \geq \xi_0.
\]
which means, \(\bar{u}(\xi)\) is the supersolution of the equation
\[
c\bar{u}' \left( \xi - d_1 \int_{\mathbb{R}} f(y) \left[ \bar{u}(\xi - y) - \bar{u}(\xi) \right] dy - \bar{u}(\xi) \left( 1 - \bar{u}(\xi) \right) + \frac{\beta y_{2n+2} \bar{u}(\xi)}{y_{2n+1} + \alpha y_{2n+2}} \right) = 0 \quad \text{for} \quad \xi \geq \xi_0.
\]
(30)
Using the fact that \(1 - y_{2n+3} = \frac{\beta y_{2n+2}}{y_{2n+1} + \alpha y_{2n+2}}\) for \(n \geq 0\), we can get that \(\bar{u}(\xi) \geq y_{2n+3}\) for \(\xi \geq \xi_0\). By the same arguments as before and \(\bar{u}(\xi) \geq y_{2n+3}\), one can easy to conclude that \(\bar{u}(\xi) \geq y_{2n+3}\) for \(\xi \geq \xi_0\). Then we can use the result to get that \(\bar{u}(\xi) \leq y_{2n+4}\) for \(\xi \geq \xi_0\). Thus (28) holds true for \(n+1\). Letting \(n \to +\infty\) of (28), we can obtain that
\[
\bar{u}(\xi) \equiv 1 - \frac{\beta}{a+1}, \quad \bar{v}(\xi) \equiv 1 - \frac{\beta}{a+1}
\]
for \(\xi \geq \xi_0\). So we can get \(\bar{u}(+\infty) = \bar{v}(+\infty) = 1 - \frac{\beta}{a+1}\). This completes the proof of the theorem.

4. Nonexistence of Traveling Waves

In this section, we will establish the nonexistence of traveling waves for (5) when \(0 < c < c^*\). When the initial value is given, the traveling wave system becomes a single fisher's equation. So we can use the comparison principle to prove the non-existence of traveling wave solutions. Firstly, we consider an associated Cauchy problem for Fisher's KPP equation with the nonlocal diffusion.
where $f$ satisfies condition $(K_1)-(K_2)$, $d$ is a constant, and the initial function $\chi(x)$ is uniformly continuous and bounded. In view of [18], we have the following Lemmas of system (30).

Lemma 4.1. Assume that $0 \leq \chi(x) \leq 1$. Then system (30) admits a solution for all $x \in \mathbb{R}$ and $t > 0$. If $W(x,0)$ is uniformly continuous and bounded, and $W(x,t)$ satisfies

$$
\begin{align*}
\frac{\partial W(x,t)}{\partial t} & \geq \langle \xi \rangle d(f(W(x,t) - W(x,t)) + W(x,t)(1 - W(x,t)) + W(x,t) \rangle(1 - W(x,t)), \\
W(x,0) & \leq \langle \xi \rangle \chi(x), x \in \mathbb{R},
\end{align*}
$$

then there holds

$$
W(x,t) \geq \langle \xi \rangle U(x,t), x \in \mathbb{R}, t > 0.
$$

Lemma 4.2. Assume that $\chi(x) > 0$. Then for any $0 < c < c^*$, we have

$$
\liminf_{t \to +\infty} \inf_{|x|<ct} U(x,t) = \limsup_{t \to +\infty} \sup_{|x|<ct} U(x,t) = 1.
$$

Lemmas 4.1-4.2 can be directly derived from the theorem 2.3 and 3.5 of [18].

Theorem 4.1. For any speed $0 < c < c^*$, there exists no nontrivial positive solution $(u(\xi), v(\xi))$ of (5) satisfying

$$
\lim_{\xi \to +\infty} (u(\xi), v(\xi)) = (1,0) \quad \text{and} \quad \lim_{\xi \to -\infty} (u(\xi), v(\xi)) = (k,k).
$$

Proof. By contradiction, we suppose that there exists some $c_1 < c^*$ such that system (5) has a positive solution $(u(\xi), v(\xi))$ satisfying (31). Then $u(\xi)$ is bounded on $\mathbb{R}$. We can find a positive constant $R_2$ such that $v(\xi) = v(x + c_1 t)$ satisfies

$$
\frac{\partial v(x,t)}{\partial t} \geq \langle \xi \rangle d(f(v(x,t) - v(x,t)) + rv(x,t)(1 - R_2 v(x,t)), \\
v(x,0) = v(x) > 0.
$$

Let $x(t) = -\frac{(c_1 + c^*)t}{2}$. Then $|x(t)| = \frac{(c_1 + c^*)t}{2} < c^* t$.

From Lemmas 4.1-4.2, we obtain

$$
\liminf_{t \to +\infty} \inf_{|x|=\frac{(c_1 + c^*)t}{2}} v(x,t) \geq \frac{1}{R_2} > 0.
$$

On the other hand, let $x(t) + c_1 t = \frac{(c_1 - c^*)t}{2}$, then

$$
\xi = x + c_1 t \to -\infty \quad \text{as} \quad t \to +\infty,
$$

and

$$
\limsup_{t \to +\infty} v(x(t),t) = \lim_{\xi \to -\infty} v(\xi) = 0,
$$

which is a contradiction. Hence, the proof is completed.

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