Tensor susceptibility of the QCD vacuum from an effective quark-quark interaction

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Abstract

Treating the bilocal quark-quark interaction kernel as an input parameter, the self-energy functions can be determined from the “rainbow” Dyson-Schwinger equation, which is obtained in the global color symmetry model. The tensor susceptibility of QCD vacuum can be calculated directly from these self-energy functions. The values we obtained are much smaller than the estimations from QCD sum rules and from chiral constituent quark model.

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The tensor susceptibility of QCD vacuum is relevant for the determination of nucleon tensor charge ([1], [2]), which is related to the first moment of the transversity distribution $h_1(x)$ [3], where $h_1(x)$ is chiral-odd spin-dependent structure function and can be measured in the polarized Drell-Yan process [4]. The previous estimations for the value of tensor susceptibility were obtained by QCD sum rules techniques ([3]-[8]) or from chiral constituent quark model [9]. In this letter, we report the different results of tensor susceptibility from the global color symmetry model (GCM) approximation ([10]-[19]) to QCD.

GCM based upon an effective quark-quark interaction can be defined through a truncation of QCD as follows. The QCD partition function for massless quarks in Euclidean space can be written as

$$Z = \int \mathcal{D}q\mathcal{D}\bar{q}e^{-\int dx\bar{q}\gamma^\mu q W[J]}$$

(1)

with $W[J]$ given by

$$e^{W[J]} = \int \mathcal{D}Ae^{\int dx\left(-\frac{1}{4}G_{\mu\nu}^aG_{\mu\nu}^a + J_\mu^aA_\mu^a\right)}$$

(2)

where $J_\mu^a(x) = ig\bar{q}(x)\gamma_\mu \frac{\lambda^a}{2} q(x)$. The functional $W[J]$ can be formally expanded in terms of the current $J_\mu^a$:

$$W[J] = \frac{1}{2} \int dx dy J_\mu^a(x) D_{\mu\nu}^{ab}(x,y) J_\nu^b(y) + \frac{1}{3!} \int J_\mu^a J_\nu^b J_\rho^c D_{\mu\nu\rho}^{abc} + \cdots$$

(3)

The GCM is defined through the truncation of the functional $W[J]$ in which the higher order $n(\geq 3)$-point functions are neglected, and only the gluon 2-point function $D_{\mu\nu}^{ab}(x,y)$ is retained. This is an effective model based on the bilocal quark-quark interaction $D_{\mu\nu}^{ab}(x,y) = D_{\mu\nu}^{ab}(x-y)$. This model
maintains global color symmetry of QCD. The primary loss by this truncation
is local SU(3) gauge invariance.

By the functional integration approach, the partition function of this
truncation can be given by

$$Z_{GCM} = \int Dq D\bar{q} \exp \left( -\int d\bar{q} q - \frac{g^2}{2} \int dx dy j^a_\mu(x) D^{ab}_{\mu\nu}(x-y) j^b_\nu(y) \right),$$

(4)

where $j^a_\mu(x) = \bar{q}(x)\bar{\gamma}_\mu \frac{\lambda^a}{2} q(x)$ is the quark color current. In [19], the gluon 2-
point function $D^{ab}_{\mu\nu}(x-y)$ is treated as the model input parameter, which is
chosen to reproduce the pion decay constant in the chiral limit $f_\pi = 87$ MeV
and moreover reproduce values for the chiral low energy coefficients. For
simplicity we use a Feynman-like gauge $D^{ab}_{\mu\nu}(x-y) = \delta_{\mu\nu} \delta^{ab} D(x-y)$. By the
standard bosonization procedure, the resulting expression for the partition
function in terms of the bilocal field integration is $Z_{GCM} = \int DB e^{-S[B]}$, where
the action is given by

$$S[B] = -\mathrm{Tr} \ln[G^{-1}] + \int dx dy \frac{B^\theta(x,y) B^\theta(y,x)}{2g^2 D(x-y)},$$

(5)

and the quark inverse Green’s function $G^{-1}$ is defined as

$$G^{-1}(x,y) = \partial \delta(x-y) + \Lambda^\theta B^\theta(x,y).$$

(6)

Here the quantity $\Lambda^\theta$ arises from Fierz reordering of the current-current in-
teraction term in (4)

$$\Lambda^\theta_{ji} = \left( \gamma_\mu \frac{\lambda^a}{2} \right)_{jk} \left( \gamma_\mu \frac{\lambda^a}{2} \right)_{li}$$

(7)

and is the direct product of Dirac, flavor SU(3) and color matrices:

$$\Lambda^\theta = \frac{1}{2} (1_D, i\gamma_5, \frac{i}{\sqrt{2}} \gamma_\mu, \frac{i}{\sqrt{2}} \gamma_\mu \gamma_5) \otimes \left( \frac{1}{\sqrt{3}} 1_F, \frac{1}{\sqrt{2}} \lambda^a_F \right) \otimes \left( \frac{4}{3} 1_c, \frac{i}{\sqrt{3}} \lambda^a_c \right).$$

(8)
The vacuum configurations are defined by minimizing the bilocal action:
\[ \frac{\delta S[B]}{\delta B} \bigg|_{B_0} = 0, \]
which gives
\[ \mathcal{B}_0^\theta(x - y) = g^2 D(x - y) \text{tr}[\Lambda^\theta G_0(x - y)]. \] (9)

These configurations provide self-energy dressing of the quarks through the definition
\[ \Sigma(p) \equiv \Lambda^\theta \mathcal{B}_0^\theta(p) = i \not{p} [A(p^2) - 1] + B(p^2). \]

The self-energy functions \( A \) and \( B \) satisfy the so-called “rainbow” Dyson-Schwinger equation,
\[ [A(p^2) - 1]p^2 = \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} g^2 D(p - q) \frac{A(q^2) \cdot p}{q^2 A^2(q^2) + B^2(q^2)} \]
\[ B(p^2) = \frac{16}{3} \int \frac{d^4 q}{(2\pi)^4} g^2 D(p - q) \frac{B(q^2)}{q^2 A^2(q^2) + B^2(q^2)}. \] (10)

In terms of \( A \) and \( B \), the quark Green’s function at \( \mathcal{B}_0^\theta \) is given by
\[ G_0(x, y) = G_0(x - y) = \int \frac{d^4 p}{(2\pi)^4} \frac{-i \not{p} A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} e^{ip(x-y)}. \] (11)

The vacuum expectation value of any operator of the form
\[ Q_n \equiv (\bar{q}_{j_1} \Lambda^{(1)}_{j_1i_1} q_{i_1})(\bar{q}_{j_2} \Lambda^{(2)}_{j_2i_2} q_{i_2}) \cdots (\bar{q}_{j_n} \Lambda^{(n)}_{j_ni_n} q_{i_n}) \]
is
\[ \langle Q_n \rangle = (-1)^n \sum_p (-1)^p \left\{ \Lambda^{(1)}_{j_1i_1} \cdots \Lambda^{(n)}_{j_ni_n} (G_0)_{i_1j_1} \cdots (G_0)_{i_nj_n} \right\}, \] (13)
where \( \Lambda^{(i)} \) represents an operator in Dirac, flavor and color space and \( p \) stands for a permutation of \( n \) indices (19, 20).

With the above preparation, we are now able to calculate the QCD vacuum tensor susceptibility readily. Through the 2-point correlator of tensor current
\[ j_{\mu\nu}(x) = \bar{q}(x)\sigma_{\mu\nu} q(x), \]
\[ \Pi_{\mu\nu;\alpha\beta}(p) = \int d^4 x e^{ip\cdot x} \langle 0 | T[j_{\mu\nu}(x) j_{\alpha\beta}(0)] | 0 \rangle, \] (14)
the tensor susceptibility $\chi$ is defined as \[5\]

$$\chi \equiv \frac{\Pi(0)}{6\langle \bar{q}q \rangle}, \quad \Pi(q^2) \equiv \Pi_{\mu\nu;\mu\nu}(q^2).$$

Using eq. (13), we get

$$\langle 0|\bar{q}(x)\sigma_{\mu\nu}q(x)\bar{q}(0)\sigma_{\mu\nu}q(0)|0\rangle = \text{tr } \gamma^c \int \frac{d^4p}{(2\pi)^4} \frac{-i\not{p}A(p^2) + B(p^2)}{X(p^2)} \text{tr } \gamma^c \int \frac{d^4q}{(2\pi)^4} \frac{-i\not{q}A(q^2) + B(q^2)}{X(q^2)}$$

$$- \int \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} e^{i(p-q)\cdot x} \text{tr } \gamma^c \left[ \sigma_{\mu\nu} \frac{-i\not{p}A(p^2) + B(p^2)}{X(p^2)} \sigma_{\mu\nu} \frac{-i\not{q}A(q^2) + B(q^2)}{X(q^2)} \right]$$

$$= -48N_c \int \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} e^{i(p-q)\cdot x} \frac{B(p^2)B(q^2)}{X(p^2)X(q^2)},$$

(16)

$$\Pi_{\mu\nu;\mu\nu}(k) = -48N_c \int \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} e^{i(p-q)\cdot x} \frac{B(p^2)B(q^2)}{X(p^2)X(q^2)} \delta(p-q+k),$$

(17)

where $N_c = 3$ is the number of colors, $X(s) = sA^2(s) + B^2(s)$. So we have the quantity

$$\frac{1}{12}\Pi(0) = -\frac{3}{4\pi^2} \int_0^\mu \frac{dss}{X(s)} \left[ \frac{B(s)}{X(s)} \right]^2,$$

(18)

where $\mu$ is the renormalization scale which we chose to be 1 GeV$^2$.

As a typical example, we let $g^2D(s) = 4\pi^2d\frac{s^2}{s^2+\Delta}$, $d = \frac{12}{27}$ and choose three sets of different parameters for $\lambda$ and $\Delta$ by fixing the pion decay constant in the chiral limit to $f_\pi = 87$ MeV \[19\]. In Table 1 we display the values for $\frac{\Pi(0)}{12}$, and the corresponding values for quark condensate $\langle \bar{q}q \rangle$ and the mixed quark-gluon condensate $g\langle \bar{q}\sigma Gq \rangle$ are also displayed \[19\]. The values of quantity $\frac{\Pi(0)}{12}$ vary with different input parameters for a specific gluon 2-point function.

Our results

$$\frac{\Pi(0)}{12} = -(0.0013-0.0016) \text{ GeV}^2$$

(19)
Table 1: The values of $\frac{\Pi_{\chi}(0)}{12}$ at $\mu = 1$ GeV$^2$ for $g^2 D(s) = (4\pi^2 d)^{\frac{1}{2}}$, $d = \frac{12}{27}$ with three sets of different parameters. The quark condensate $\langle \bar{q}q \rangle$ and the mixed quark-gluon condensate $g\langle \bar{q}\sigma Gq \rangle$ are also presented.

| $\Delta$ [GeV$^4$] | $\lambda$ [GeV] | $-\langle \bar{q}q \rangle^{\frac{1}{4}}$ [MeV] | $-g\langle \bar{q}\sigma Gq \rangle^{\frac{1}{4}}$ [MeV] | $\Pi_{\chi}(0)/12$ [GeV$^2$] |
|---------------------|-----------------|----------------|----------------|-------------------|
| $10^{-1}$           | 1.77            | 183            | 460            | -0.0016           |
| $10^{-2}$           | 1.33            | 178            | 456            | -0.0014           |
| $10^{-4}$           | 0.95            | 175            | 458            | -0.0013           |

are much smaller than the estimations which were obtained recently \[\text{[8]}\] from QCD sum rules with nonlocal condensates

$\frac{\Pi_{\chi}(0)}{12} = -0.0055 \pm 0.0008$ GeV$^2$ \hspace{1cm} (20)

and from the standard sum rules

$\frac{\Pi_{\chi}(0)}{12} = -0.0053 \pm 0.0021$ GeV$^2$. \hspace{1cm} (21)

Their results are similar to the estimations $\frac{\Pi_{\chi}(0)}{12} = -0.008$ GeV$^2$ given by Belyaev and Oganesian from QCD sum rules \[\text{[8]}\] and $\frac{\Pi_{\chi}(0)}{12} = -(0.0083-0.0104)$ GeV$^2$ from the chiral constituent quark model \[\text{[9]}\]. The earliest estimation obtained by He and Ji \[\text{[1, 5]}\] has opposite sign. The tensor susceptibility given by Kisslinger from the QCD sum rules for three-point functions is large in magnitude and also has opposite sign.

In conclusion, the QCD vacuum tensor susceptibility can be calculated from GCM other than QCD sum rules. The value of tensor susceptibility is
uniquely determined by the self-energy functions $A$ and $B$ for a given quark-quark interaction, which is chosen to reproduce $f_{\pi}$. The values of tensor susceptibility obtained in GCM are smaller than all the previous estimations.

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