On kinetics of transfer of interphase borders during contact melting with an electric transfer

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Abstract. The mathematical laws of kinetics of the liquid zone growth under the contact melting of binary metal systems in the presence of constant electric current were described; the analytical and graphic solutions to this problem explain the experimental data.

1. Introduction

Constant electric current can affect the growth rate of the liquid layer formed during contact melting (CM). Depending on the direction of current, the speed of the process can either increase or decrease. In the second case, the growth of the liquid zone ceases, and the process goes into equilibrium [1–3].

Attempts were made [4] to find out a mathematical law expressing the kinetics of movement of interphase boundaries under the CM with an electric transfer. The solution is fraught with certain mathematical difficulties.

The present article aims to find the mathematical law that describes the movement of the melt–solid phase boundaries during the electric transfer in the melt growing during the CM.

2. Results

In the laboratory reference system, the flow of substance of one of the components (B) in the AB melt in the presence of electric current is as follows (taking into account the possible macroscopic flow [4] similar to the Kirkendall displacement) [5, 6]:

\[ J = -D \nabla c + DcFEez^*_B \]  \hspace{1cm} (1)

where \( c \) is the concentration (the number of atoms (ions)) B per unit volume, \( D \) is the coefficient of mutual diffusion, \( E \) is the electric field strength, \( z^*_B \) is the effective charge in relative units of elementary charge (e), \( F = V_A/VkT(\partial \ln a_B/\partial \ln n) \), \( V \) – volume of the solution mole, \( V_A \) – partial molar volume A, \( a_B \) – thermodynamic activity, \( n \) – atomic fractions of component B.

The kinetics of displacement of the boundaries of the melt–solid phase is determined by the difference in the flows of substance on both sides of the boundary:

\[ J_{L1} - J_{S1} = -(c_1 - c_{S1}) \frac{dl_1}{dt}, \]  \hspace{1cm} (2)

\[ J_{L2} - J_{S2} = -(c_{S2} - c_2) \frac{dl_2}{dt}, \]  \hspace{1cm} (3)

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where $J_{Li}$ and $J_{Si}$ – boundary flows from the melt and solid phase i, ($i=1,2$) respectively, $l_i$ – absolute values of coordinates of interphase boundaries, $c_i$ and $c_{Si}$ – boundary concentrations from the melt and solid phase i, respectively.

Denoting the ratio $\frac{J_{Li}}{J_{Si}}$ as $\alpha_i$ and substituting the expression for $J_{Li}$, i.e. (1), into (2) – (3), we have:

$$\frac{c_1-c_{Si}}{1-\alpha_1} \frac{dl_1}{dt} = D (\nabla c)_1 - Dc_1 FEe_z B^*_1,$$

$$\frac{c_2-c_{Si}}{1-\alpha_2} \frac{dl_2}{dt} = D (\nabla c)_2 - Dc_2 FEe_z B^*_2,$$

where indices 1 and 2 are the values taken for boundary concentrations in the melt ($c_1$ and $c_2$, respectively).

Cross-border concentration gradients can be written as:

$$\nabla c = -|\frac{c_1 - c_2}{c_1 - c_2}|,$$

where $c_1$ – concentration per initial contact plane, $c_2$ – dimensionless factor taking into account the degree of deviation of the curve $c(x)$ from the linear course (at the melt–solid phase $i$ boundary).

Equations (4) – (5) are reduced to

$$\frac{dl_1}{dt} = \frac{h_1}{l_1} = q,$$

$$\frac{dl_2}{dt} = \frac{h_2}{l_2} = q_i,$$

where $h_i = \frac{\gamma_i (1-\alpha_i) |c_0 - c_i|}{|c_1 - c_{Si}|}$, $q_i = \frac{(1-\alpha_i) Dc_i FE_e z B^*_i}{|c_i - c_{Si}|}$.

In the absence of current $q_i = 0$. The coordinates of the interphase boundaries, as can be seen from (6), will change according to the well-known parabolic law:

$$l_i^2 = 2h_{i0} t = k_i^2 t,$$

where $k_i$ – the constant of the displacement velocity of the interphase boundary under the CM in the absence of electric transfer, $h_{i0} = \frac{h_i}{E=0}$.

If the current vector coincides with the coordinate (which is measured from the plane of the initial contact in the direction of boundary 2), $E > 0$. At $zz_0 > 0$, we have: $q_i < 0$. As can be seen from (6), the electric transfer increases the speed of movement of the interphase boundaries. A similar effect will be observed at $E < 0$ and $z_0 > 0$. But if $\|Ez_0 > 0$ ($q_i > 0$), the process slows down. In this case, the value (6) decreases and the movement of the boundaries stops. Moreover, having equalized (6) to zero, we can obtain limiting values of the absolute values of the coordinates of the interphase boundaries:

$$l_i^M = \frac{h_i}{q_i} = \frac{h_i}{q_i},$$

Using the auxiliary variable $y = h_i - q_i l_i$, we transform equation (6) to the form:

$$(h_i - y) dy = -y q_i^2 dt.$$}

Its solution, taking into account the initial condition $(l_i)_{t=0} = 0$ is as follows:

$$l_i(t) + \frac{h_i}{q_i} \ln \left[ \frac{h_i - q_i l_i(t)}{h_i} \right] = q_i t.$$

Let us analyze features of solution (9) for each mode.

1. Slowing down mode ($q_i > 0$). Given (8), let us write (9) as

$$r_i + ln |1 - r_i| = -q_i t h_i,$$

where $r_i = \frac{l_i(t)}{l_i^M}$. Since $0 \leq l_i(t) \leq l_i^M$, $0 \leq r_i \leq 1$. Therefore, the sign of the module in (10) can be omitted. Since $l_i(t) = r_i l_i^M$, given (8) and (10), we have:

$$l_i^2 = \frac{r_i^2 h_i^2}{q_i} = \frac{r_i^2 h_i^2}{q_i} = \frac{r_i^2 h_i^2}{f(r_i)},$$

where $f(r_i) = -(r_i + ln(1 - r_i))$. 


Setting values of parameter $l_i^M$ and varying $b_i(t)$ from 0 to $l_i^M$, we have corresponding values of parameter $r_i = \frac{l_i(t)}{l_i^M}$ and function $f(r_i)$. (11) can be used to calculate the graph of the dependence $l_i^2(h_it)$. Figure 1 shows the curves for $l_i^M$ equal to 2, 3 and 4 mm.

(8) taking into account (6) for determining $h_i$ and $q_i$, shows that $l_i^M \sim \frac{1}{E}$. Therefore, in case of absent electric transfer ($E=0$) we have: $l_i^M \to \infty, l_i^2 \to t$, see (7).

We can determine the time for $l_i(t) = r_{ip}l_i^M$, where $r_{ip}$ – the number of target values of $r_i$. From (11), given (7) and considering $h_i \approx h_{i0} = \frac{k_i}{2}$, we have:

$$\tau = \frac{f(r_{ip})l_i^2(t)}{h_{i0}r_{ip}^2} = \frac{f(r_{ip})(l_i^M)^2}{h_{i0}r_{ip}^2} = 2f(r_{ip})(l_i^M)^2 r_{ip}^{-1}.$$  (12)

Thus, it is possible to determine the time during which the system reaches the state of stationary equilibrium. If $r_{ip} = 0.95$, a $l_i^M = 0.5$ cm, at $k_i^2 = 2 \cdot 10^{-5}$ cm$^2$/c (12) gives: $\tau = 0.51 \cdot 10^2$ c. = 14.2 h.

Given (8) and considering $h_i \equiv \frac{k_i^2}{2}$, equation (6) for the slowing down mode can be written as

$$\frac{dl_i}{dt} = \frac{k_i^2}{2} \left( \frac{1}{l_i} - \frac{1}{l_i^*} \right).$$  (13)

In case of absent electric transfer ($l_i^M \to \infty$) instantaneous velocities of movement of the interphase boundaries under the CM are:

$$\frac{k_i^2}{2l_i} = \frac{k_i}{2\sqrt{t}}.$$

Figure 1. Graph of the growth of the square coordinate of the boundary of the melt – solid phase $i$ ($i = 1,2$) under the CM with an electric transfer. The numbers of the curves correspond to the following parameters: 1–3: accelerating mode at $l_i^M = 2; 3$ and 4 mm m. 4–6: deceleration mode with $l_i^M = 4; 3$ and 2 mm.; 0 – current mode

2. The accelerating mode ($q_i < 0$). Let us introduce the notation:

$$\frac{h_i}{q_i} = -l_i^*$$  (14)

where $l_i^*$ – a value of $l_i$. Having divided both parts by $l_i^*$, we have ($r_i = \frac{l_i(t)}{l_i^*}$):
\[ r_i - \ln(1 + r_i) = \frac{q_i^2(t)}{h_i} \quad (15) \]

As far as \( l_i(t) = \eta l_i^* \), given (14) and (15), we have:

\[ l_i^2 = r_i^2 \frac{h_i^2}{h_i^*} = \frac{r_i^2 h_i t}{q_i^2} = \frac{r_i^2 h_i t}{g(r_i)} \quad (16) \]

where \( g(r_i) = r_i - \ln(1 + r_i) \). Setting the values of \( l_i^* \) and varying \( l_i(t) \), we have corresponding values of parameter \( r_i = \frac{l_i(t)}{l_i^*} \) and function \( g(r_i) \). (16) can be used to build the dependence graph \( l_i^2(h_i t) \).

Figure 1 (left part) shows the curves for \( l_i^* = 2; 3 \) and 4 mm.

Given the values of \( h_i \) and \( q_i \), ratio (14) shows that \( l_i^* \sim \frac{1}{h_i} \), \( l_i^* \to \infty \) corresponds to the special case (\( E=0 \)). The parabolic dependence (7) is valid.

Given (14) and considering \( h_i \approx h_{i0} = \frac{k_i^2}{2} \), equation (6) for the accelerating mode can be written as

\[ \frac{dl_i}{dt} = \frac{k_i^2}{2} \left( \frac{1}{l_i^*} + \frac{1}{l_i^*} \right) \quad (17) \]

Given that in the absence of electric transfer \( q_i = 0 \) \( \Rightarrow l_i^* \to \infty \), the following conclusion can be drawn from (17). Value \( l_i^* \) has a certain physical meaning: at \( l_i = l_i^* \) the instantaneous boundary velocity (melt – solid vase \( i \)) will be more than the value of this parameter in the absence of current (in the same size \( l_i \)) two times. At \( l_i = l_i^*, r_i = 1 \). \( g(r_i) = 1 - \ln 2 \approx 0,307 \). From (16), we obtain \( h_i \approx \frac{k_i^2}{2} \cdot \tau = \frac{k_i^2}{2g(1)} = 1,63k_i^2 \), while at \( E = 0 \), \( l_i^* = k_i^2 \) (see (7)).

3. Conclusion

The curves in Figure 1 are similar to the corresponding experimental graphs published in [1, 3]. Moreover, the curves reflect the general process and are valid for any binary system (where the KP process with an electric transfer is possible) and for any experimental temperature, since possible values of the parameters \( h_i \), and \( l_i^M \) (or \( l_i^* \)) must correspond to the specific experimental conditions.

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References

[1] Akkhubekov A A, Savintsev S P and Bagov A M 2005 Kinetics of the formation of a melt as a result of contact melting, under the conditions of the slowing down action of electric transfer Melts 1 54–8

[2] Bagov A M 2016 The effect of electrotransfer on mutual diffusion and macroscopic flow of a melted formed during contact melting Thesis (Nalchik: Kabardino-Balkarian State Univer.)

[3] Akkhubekov A A, Savintsev S P and Bagov A M 2004 The effect of electrotransfer on crystallization and macroscopic flow during contact melting of metals Metals 4 33–6

[4] Bagov A M, Tamaev T Kh, Khasanov A I et al 2018 Phase formation in contacting zinc melting with indium and stannic in the presence of strontium impurity and microhardness of derived alloys Advances in Engineering Research vol 177 Int. Symp. Engineer. and Earth Sci.: Appl. and Fund. Res. (ISEES 2018) pp 143–5

[5] Akkhubekov A A and Bagov A M 2009 On the shift of inert labels during contact melting of binary metal systems in the presence of electrotransfer Proc. of the Russ. Acad. of Sci. Ser. is Phys. 73(7) 961–3

[6] Akkhubekov A A 2001 Diffusion and electric transport in low-melting metal systems during contact melting Thesis (Nalchik) 312 p