Boundary layer Darcy-Forchheimer couple stress hybrid nanofluid flow over a quadratic stretching surface due to nonlinear thermal radiation

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1. Introduction

The exploration of boundary layer flows caused via stretching/shrinking surface has been experimentally utilized by different researchers due to its comprehensive implementation. Stretching/shrinking surfaces produce heat transport and flow properties which are extensively used in engineering applications mechanisms including coating and molten metal, polyethylene manufacturing sector, spray coating, fibrous rolling, hydrodynamic extruding of plastic covers, materials management sorters, and condensation processes with a water phase. Sakiadis [1] is the first one to find out that the boundary layer on a limited surface differs from the boundary layer on a hard object. Miklavcic and Wang [2] investigated the fluid motion generated by a shrinking surface and discovered precise solutions, either numerically or in closed form. These results clearly state that adequate suction should be supplied on the surface to maintain the flow characteristics. Although Sakiadis and Crane pioneered research on the stretching/shrinking surfaces, several scholars have investigated this topic in the context of many factors and components, see in articles [3–9].

Maxwell [10] highlighted in the early 1900s that increasing fluid thermal conductivity may be accomplished simply by holding numerous objects having higher conductivity, such as solid materials. Choi and Eastman [11] later announced the clue of nanoliquids as just a novel type of energy transport fluid developed by hanging nanomaterials in a common working liquid to enhance the fluid’s conductivity. Nanofluid would be a heterogeneous solution of nanomaterials of dimensions from 1 nm up to 100 nm and a base fluid that is used as a heat transport medium in advanced nanotechnology applications. By adding frequently utilized nanomaterials to the base fluids, like metal, graphite substance, and iron oxide, the thermal conduction of the resulting fluids is enhanced, resolving the poor thermal conductivity of common heat transfer fluids, that restricts cooling efficiency. Nanofluids have attracted scientists’ effort and attention because of the numerous applications in the engineering and technological sectors, such as cooling applications, converter circuits, energy engines, and cooling of vehicle engines. It’s important to keep in mind that several researchers have performed comprehensive evaluations on nanofluids as illustrated by [12,13]. However, hybrid nanofluid has recently been developed as amplifying the concept of nanofluid, which further comprises multiple nanosize particles distributed in the working fluids which is estimated to deliver additional significant rheological and physicochemical features and also improved heat transport characteristics. Because of its capabilities in improving thermal characteristics, such innovative heat transfer fluid has concerned the concentration of various scientists and researchers who want to explore the actual problem of energy exchange. Devi and Devi [14] determined the Cu-Al2O3/water in three-dimensional movement through a stretched surface under the influence of MHD (magnetohydrodynamic). Although since, various scholars and scientists have taken into consideration several scientific hypotheses over the extended...
and shrinking surface dealing with hybrid nanoliquids including Shehzad et al. [15], Bhatti et al. [16,17], Waini et al. [18], Ali et al. [19], Swain et al. [20] and Khan et al. [21].

Generally, thermal radiation is an important component of the heat transport mechanism. In various industrial and technological applications, the thermal radiation consequence is potentially utilized, like energy production, nuclear reactors, astronomical processes, communications satellites, solar systems, natural gas, and so on. In the proposed investigation, we design numerous simulations that explain the impact of the radiation of the heat in different circumstances on the MHD fluid flow. Mbeledogu and Ogulu [22] designed an impressive theoretical framework for heat and mass transfer, using a radiative heat transfer and natural convection, of a steady MHD flow of a circulating liquid flowing over a vertical porous flat plate. Enhancing the Prandtl number and radiating factor was predicted to reduce the fluid temperature inside the boundary layer. Ansari et al. [23] explored the stream via a straight stretched surface of non-Newtonian viscous nanofluid under the influence of constant magnetic field and nonlinear transmission of radiative heat. In the study of Kumar et al. [24], Ali and Sandeep [25], Souayeh et al. [26], and Ibrahim et al. [27] highlight the outstanding and creative research of the radiative phenomenon.

It is noticeable that the flow characteristics across porous media having consistent porosity and permeability have been examined by most scholars and experts. Furthermore, such quantities are not uniform in the implementations of technology, industry, and daily situations. The usage includes chemical fixed bed reactors, steam generators, and drying systems packaged in bed. In general, the strength of porosity increases as you get close to a surface/wall and diminishes as you get further away. Darcy’s law has been used to study many aspects of porous media; however, assembling a working framework scenario needs an inhomogeneous permeability circulation. As a result, the Non-Darcian phenomena are significant for the motion on permeable medium. Inertia impacts in porous material are explored by Forchheimer [28], who uses a quadratic component in the equation of motion. Hong et al. [29] analyzed the Darcy–Forchheimer movement past a vertical surface. The Darcy Brinkman Forchheimer model for iso-thermally mixed convection flow heated stream in the permeable medium was established by Hadim and Chen [30]. The publications Saif et al. [31] and Fares et al. [32] summarize some of the significant and noteworthy investigations on this topic.

Furthermore, scientists are looking into the significances of a steady motion of hybrid nanoliquids across a stretching and shrinking surface, which has a high demand in engineering and industrial usages. Nobody has analyzed the steady motion of a couple stress hybrid nanoliquids across a spongy stretching and shrinking medium where Darcy-Forchheimer, heat source, radiative heat transfer, and quadratic velocity are involved. The authors’ key purpose in this work is to depict a mathematical model comprising a steady incompressible flow of a hybrid nanofluid in a horizontal direction through a porous stretching/shrinking surface under the influence of Darcy-Forchheimer, heat source, radiative heat transfer, and quadratic velocity. Three nanoparticles such as MgO (Magnesium Oxide), CuO (Copper Oxide), and MWCNTs (Multi-walled carbon nanotubes) with H2O as a base fluid are used. On an industrial scale, these nanoparticles have better structural integrity, are more environmentally sound, and can be well-dispersed in the base fluids. The resulting non-linear PDEs are transformed into non-linear ODEs via resemblance alterations. HAM method is utilized to solve the resulting nonlinear coupled ODEs analytically. The effects of altering relevant factors on the distribution of velocity and temperature are examined in depth graphically and in tabular form.

2. Mathematical modelling

Considering a 2D, the time-independent motion of couple stress hybrid nanoliquids approaching a porous stretching and shrinking surface. The proposed methodology is used in mathematical modelling in the current problem.

(a) We assume that x and y are normalized and specified towards the stretching/shrinking sheet, with y = 0, so the liquid movement is restricted to y ≥ 0.

(b) Also, the sheet is stretched/shrunk quadratically, that executed at the x-axis is estimated to generate the flow, where sheet velocity is u_w(x) = ax + bx^2.

(c) The velocity u(x) represents the liquid away from the sheet where the mass velocity is v_w(x) forced to the surface of the sheet. So, v_w(x) < 0 denote suction, v_w(x) > 0 denote injection where the state of the impermeable sheet is v_w(x) = 0.

(d) The permeable region of Darcy-Forchheimer is considered.

(e) The heat generation/absorption and radiation reveal heat transport properties.

(f) T_w signify a temperature of hybrid nanofluid at wall and T∞ symbolize temperature away from the stretch/shrink sheet, so T_w(x) = T∞ + T_0(x^2), T_0 > 0 temperature characteristics of fluid, whereas the typical measurement of the sheet.

(g) Water is utilized as the basic solvent, and nanomaterials are comprised of (CuO, MgO, MWCNTs).

(h) The working liquid of the hybrid nanofluid is kept at a constant temperature.

We may formulate the governing equations as follows by using the conventional boundary layer
The essential non-dimensional factors are identified as \[7,8,9,13,14\]

\[
\begin{align*}
    u &= ax f'(\eta) + bx^2 g'(\eta), \\
    \nu &= -\sqrt{a \nu_f} f(\eta) - 2bx \sqrt{\frac{\nu}{\nu_f}} g(\eta), \\
    \Theta(\eta) &= T_{\infty} (1 + (\Theta_w - 1) \Theta), \\
    \eta &= y \sqrt{\frac{b}{\nu_f}},
\end{align*}
\]

In the light of Equation (10), the Equations (1)--(4) becomes

\[
\begin{align*}
    f''' + \frac{\rho \nu}{\rho f} \frac{\mu f}{\mu_{nf}} [f f'' - (1 + Fr)(f')^2] - \lambda^* f' - k^* f'' &= 0, \\
    g''' + \frac{\rho \nu}{\rho f} \frac{\mu f}{\mu_{nf}} [f g'' - 3(f'g') + 2(f''g) - (Fr)(g'^2)] - \lambda^* g' - k^* g'' &= 0, \\
    \Theta(\eta) &= T_{\infty} (1 + (\Theta_w - 1) \Theta), \\
    \eta &= y \sqrt{\frac{b}{\nu_f}}.
\end{align*}
\]

With inter-related boundary constraints are as:

\[
\begin{align*}
    f(0) &= S_1, \\
    g(0) &= S_2, \\
    f'(0) &= g'(0) = \lambda_1, \\
    \Theta(0) &= 1, \\
    f'(\infty) &= 0, \\
    g'(\infty) &= 0, \\
    \Theta(\infty) &= 0.
\end{align*}
\]

Solving Equation (14) with the help of the physical conditions (15) to obtain.

\[
g(\eta) = S_2 \Theta(\eta),
\]

With the help of Equation (16), the equations (12,13 and 15) are summarized as:

\[
\begin{align*}
    f''' + \frac{\rho \nu}{\rho f} \frac{\mu f}{\mu_{nf}} [f f'' - (1 + Fr)(f')^2] - \lambda^* f' - k^* f'' &= 0, \\
    \Theta(\eta) &= T_{\infty} (1 + (\Theta_w - 1) \Theta), \\
    \eta &= y \sqrt{\frac{b}{\nu_f}}.
\end{align*}
\]

In above expressions, \( Fr = \frac{C_p}{\kappa f} \), denoted the Inertia coefficient, \( Rd = 4 \frac{g^* T_f^2}{k^* \nu_f} \), designate Radiation factor, \( \lambda^* = \frac{\lambda}{\nu_f} \), signify Porosity factor, \( g_w = \frac{T_w}{\nu_f} \), Temperature ratio parameter, \( Pr = \frac{\nu}{\nu_f} \), represent Prandtl number, \( Q = \frac{Q_o}{\nu_f} \), symbolize the Heat source/sink factor.
Significant physical quantities:
The important engineering quantities are $C_{fx}$ and $Nu_x$:

$$C_{fx} = \frac{\tau_w}{2 \rho_\text{inf}(u_w)^2}, \quad Nu_x = \frac{xq_w}{k_{\text{inf}}(T_w - T_\infty)}.$$  \hfill (20)

Where $\tau_w$ denote the shear stress and $q_w$ signifies the surface heat flux. Employing Equations (10,16), so the Equation (20) becomes

$$C_{fx}Re_x^{-0.5} = \frac{\mu_{\text{inf}}}{\mu_f}(f''(0) + \beta xg''(0)) - \frac{NuxRe_x^{-0.5}}{4}e_2^2 e_3^2 \frac{1}{\epsilon^2} \phi''(0).$$ \hfill (21)

The local Reynolds number and dimensionless parameter $\beta$ are defined as, $Re_x = \frac{\text{ax}x}{\nu_f}, \beta = \frac{2}{3}$.

2.1. HAM solution

For the solution procedure, the optimal approach is employed. The best analytical method HAM is used to solve Equations (17–18) with boundary constraints (19). Mathematica software is utilized for this purpose. The following description provides a fundamental illustration of the model equation employing HAM method.

$$f(\eta) = S_1 + \lambda_1 (1 - e^{-\eta}), \quad g(\eta) = S_2 e^{-\eta},$$ \hfill (22)

Where the linear operators denoted by $L_f$ and $L_g$

$$L_f(f) = f''', \quad L_g(g) = g'. \hfill (23)$$

Where

$$L_f(e_1 + e_2 \eta + e_3 \eta^2) = 0, \quad L_g(e_4 + e_5 \eta) = 0.$$

Where $e_1, e_2$ and $e_3$ are constants.

3. Results and discussion

Under some boundary conditions, the influences of the couple stress parameter, porosity, Quadratic Stretching Surface, Darcy-Forchheimer, heat generation and absorption, thermal radiations are considered in the analytical analysis of the current mathematical model.

A set of time independent nonlinear partial differential equations comprising equations of motion and heat characterize the current model of the physical problem. The influence of different values of specified factors on the flow field, such as the couple stress parameter, porosity, Quadratic Stretching Surface, Darcy-Forchheimer, heat source, and thermal radiation, are studied and represented visually from a physical perspective. The default values of important parameters are used in the present section for numerical computation as $\lambda^* = 0.2, S_1 = S_2 = 1, \lambda = 0.1, \phi_1 = \phi_2 = 0.02, Fr = 0.1, Q = Rd = 0.2, Pr = 6.2$. Figure 1 depicts the geometry of the model problem. Figure 2 is used to show the total square residual error for the obtained results through HAM technique. The obtained results authenticate the proposed problem solution and the increasing number of iteration improve the accuracy of the problem as shown in the problem.

Figures 3–6 highlight how different values of relevant factors affect $\text{MWCNT}$ nanofluid and $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids $f'(\eta)$ (velocity profiles). In Figure 3 the velocity $f'(\eta)$ variations for $\lambda^*$ (porous media factor) for $\text{MWCNT}$ nanofluid and $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids are highlighted. The velocity $f'(\eta)$ distributions shrink as the range of $\lambda^*$ rises, as shown in the plot. Physically, growing the magnitude of $\lambda^*$ leads to decline the spongy zone’s transparency. As a result, there is a tiny hole on the sheet surface that inhibits the movement of $\text{MWCNT}$ nanofluid, $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids, and the velocity $f'(\eta)$ filed of liquids slow. Figure 4 depicts the steady-state $f'(\eta)$ velocity distributions of $\text{MWCNT}$ nanofluid and $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids at multiple values of $\phi_1, \phi_2$. The figure illustrates also that stronger the magnitudes of $\phi_1, \phi_2$, lesser the $f'(\eta)$ velocity distributions of liquids. Physically, when the value of $\phi_1, \phi_2$ strengthens, the colliding effect between the nanoparticles ($\text{CuO, MgO, MWCNT}$) accelerates, and as a result, the $f'(\eta)$ profile of $\text{MWCNT}$ nanofluid, $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids drop. The influence of the $k^*$ (Couple Stress factor) on the $f'(\eta)$ velocity profile of $\text{MWCNT}$ nanofluid and $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids inside the boundary layer is shown in Figure 5. As predicted form the plot, the raising values of $k^*$ causes a decrease the hybrid nanofluid movement owing to an increase in drag force, that corresponds to an apparent reduction in fluid viscosity. Physically, it is evident that the flow is delayed as a result of the addition of viscous effects caused by the $k^*$, resulting a reduction in velocity profiles of $\text{MWCNT}$ nanofluid, $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids. Figure 6 depicts the fluctuations $Fr$ relative $f'(\eta)$ velocity distributions for $\text{MWCNT}$ nanofluid and $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids. The plots emphasize that for nanofluid and hybrid nanofluid, $Fr$ is the decreasing function of $f'(\eta)$. In short, an increase in $Fr$ helps fluids to become more robust, resulting in decreased the velocity $f'(\eta)$. Physically, expanding the intensity of $Fr$ diminishes the inward nanofluid speed and seems to have no influence on liquid thicknesses. As a response, an elevation in $Fr$ produces effectively stream restriction, reducing the $\text{MWCNT}$ nanofluid, $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids’ profile.

Figures 7–10 show how slight changes in significant factors affect $\text{MWCNT}$ nanofluid, $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids $g(\eta)$ thermal performance. Figure 7 illustrates the steady-state $g(\eta)$ of $\text{MWCNT}$ nanofluid, $\text{MgO + MWCNT}$, $\text{CuO + MWCNT}$ hybrid nanofluids for different value of $\phi_1, \phi_2$. According to the
Figure 1. The physical illustration of the present flow model.

Figure 2. The total square residual error for the obtained results through HAM technique.

Figure 3. Variation of $\lambda^*$ via $f'(\eta)$.

Figure 4. Variation of $\phi_1, \phi_2$ via $f'(\eta)$.

Figure 5. Variation of $k^*$ via $f'(\eta)$.

Figure 6. Variation of $Fr$ via $f'(\eta)$.

Figure 7. Variation of $\phi_1, \phi_2$ via $g(\eta)$.

Figure 8 demonstrates the influence of $Pr$ (Temperature ratio) on the $g(\eta)$ for MWCNT nanofluid and MgO + MWCNT, CuO + MWCNT hybrid nanofluids. The $g(\eta)$ improves as the $g_w$ rises as shown in the plot. So, the $g_w$ is...
the temperature ratio of the wall temperature to ambient temperature and $T_w > T_\infty$ or $T_w - T_\infty > 0$. Physically, this parameter describes the thermal state of the hybrid nanofluid and in case of the larger values of this improves the temperature profile. Figure 9 indicates the $Q$ (heat generation factor) influence on the $g(\eta)$ temperature profile for $\text{MWCNT}$ nanofluid and $\text{MgO} + \text{MWCNT}$, $\text{CuO} + \text{MWCNT}$ hybrid nanofluids. As the values of the $Q$ are elevated, the temperature of the fluid goes up rapidly. The reason for this is that the exterior source of heat delivers excessive energy into the flow area, which causes the fluid temperature to rise. According to the impact of the $Rd$ (thermal radiation parameter) as seen in Figure 10, the $g(\eta)$ temperature profile of $\text{MWCNT}$ nanofluid and $\text{MgO} + \text{MWCNT}$, $\text{CuO} + \text{MWCNT}$ hybrid nanofluids is enhanced. Enhancing heat flow via radiation stimulates particle movement inside the frame, which results in a frequent collision of molecules into heat energy. With the maximum value of, a broader $g(\eta)$ has been recorded. The thermophysical values for the solid nanomaterials and base solvent are revealed in Table 1. The % enhancement in the heat transfer rate for the $\text{MgO} + \text{MWCNT}$, $\text{CuO} + \text{MWCNT}$ and $\text{MWCNT}$, are displayed in Table 2. It is concluded from the obtained results that (3%Wt) $\text{MgO} + \text{MWCNT}$ is more effective from the (4%wt) of the $\text{CuO} + \text{MWCNT}$. Similarly, the thermal efficiency of the hybrid nanofluids is comparatively larger than the nanofluids containing same kind of nanoparticles.

4. Conclusion

The primary goal of this study is to examine the characteristics of a steady two-dimensional flow of a couple stress hybrid nanofluid over a porous stretching/shrinking surface under the influence of Darcy-Forchheimer, heat production, thermal radiation and other factors. Nobody has analyzed the steady motion of a couple stress hybrid nanoliquids across a spongy stretching and shrinking medium where Darcy-Forchheimer, heat source, radiative heat transfer, and quadratic velocity are involved. The fluid motion is produced by, a quadratic stretching/shrinking surface that stretched/shrunk at a general quadratic rate. Numerous graphs and tables describe the physical influence of different flow factors. The following are the significant outcomes of the current investigation:

- Porosity factor $\lambda^*$, Couple stresses $k^*$ and Darcy-Forchheimer factor $Fr$ decline the speed $f'(\eta)$ of the $\text{MWCNT}$ nanofluid and $\text{CuO} + \text{MWCNT}$ $\text{MgO} + \text{MWCNT}$ hybrid nanofluids while $\phi_1$, $\phi_2$ enhancing $f'(\eta)$.
- According to the observations, higher $g_w$ improve the fluid temperature values.
- Temperature profile increasing for large values of $Q$ and $\phi_1$, $\phi_2$.
- The expansion of the $g(\eta)$ temperature profile is mostly due to an increase in the intensity of $Rd$ thermal radiation.
- The % analysis shows that the hybrid nanofluids are more effective to improve the heat transfer rate.
- $\text{MgO} + \text{MWCNT}$ are more efficient to improve the heat transfer rate as compared to the other fluids.

| Table 1. Base fluid and solid nanomaterials thermo-physical properties [19]. |
| --- |
| Property | MWCNT | H$_2$O | MgO | CuO |
| $\sigma$(Sm$^{-1}$) | $10^{-7}$ | $5.5 \times 10^{-6}$ | $1.42 \times 10^{-3}$ | $6.9 \times 10^{-2}$ |
| $\rho$(kgm$^{-3}$) | 2100 | 997.1 | 3580 | 6320 |
| $C_p$(Jkg$^{-1}$K$^{-1}$) | 711 | 4179 | 960 | 531.8 |
| $k$(WmK$^{-1}$) | 3000 | 0.613 | 48.4 | 76.5 |

Figure 8. Variation of $g_w$ via $g(\eta)$.

Figure 9. Variation of $Q$ via $g(\eta)$.

Figure 10. Variation of $Rd$ via $g(\eta)$.

Table 1. Base fluid and solid nanomaterials thermo-physical properties [19].
Table 2. % analysis versus $\text{Nu} \times Re_x^{-0.5}$.

| $\phi_1, \phi_2$ | $\text{Nu} \times Re_x^{-0.5}$ (MgO & MWCNT) | % | $\text{Nu} \times Re_x^{-0.5}$ CuO & MWCNT | % | $\text{Nu} \times Re_x^{-0.5}$ MWCNT | % |
|----------------|---------------------------------|---|---------------------------------|---|---------------------------------|---|
| 0.0            | 1.34556                         | ...| 1.34556                         | ...| 1.34556                         | ...|
| 0.01           | 1.371567                        | 1.93280| 1.370348                        | 1.84220| 1.35982                         | 1.059781|
| 0.02           | 1.38686                         | 3.20089| 1.38552                         | 2.9697| 1.376512                        | 2.30030|
| 0.03           | 1.399892                        | 3.96565| 1.38695                         | 3.07604| 1.38241                         | 2.7386|
| 0.04           | 1.41214                         | 4.94812| 1.39789                         | 3.8890| 1.38962                         | 3.27447|

Disclosure statement
No potential conflict of interest was reported by the author(s).

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