Steady-state mechanical squeezing in a double-cavity optomechanical system

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We study the physical properties of double-cavity optomechanical system in which the mechanical resonator interacts with one of the coupled cavities and another cavity is used as an auxiliary cavity. The model can be expected to achieve the strong optomechanical coupling strength and overcome the optomechanical cavity decay, simultaneously. Through the coherent auxiliary cavity interferences, the steady-state squeezing of mechanical resonator can be generated in highly unresolved sideband regime. The validity of the scheme is assessed by numerical simulation and theoretical analysis of the steady-state variance of the mechanical displacement quadrature. The scheme provides a platform for the mechanical squeezing beyond the resolved sideband limit and solves the restricted experimental bounds at present.

The optomechanical system is a rapidly growing field in which researchers study the interaction between the optical and mechanical degrees of freedom via radiation pressure, optical gradient, or photothermal forces. In optomechanical systems, quantum fluctuations become the dominant mechanical driving force with strong radiation pressure, which leads to correlations between the mechanical motion and the quantum fluctuations of the cavity field. Originally, the goal of studying the optomechanical interaction is to detect gravitational wave. As research continues, the optomechanical system has been developed to investigate quantum coherence for quantum information processing and quantum-to-classical transition studying in macroscopic solid-state devices. Many projects of cavity optomechanics systems have been conceived and demonstrated experimentally, including red-sideband laser cooling in the resolved or unresolved sideband regime, coherent-state transiting between the cavity and mechanical resonator, normal-mode splitting, quantum network, backaction-evading measurements, entanglement between mechanical resonator and cavity field or atom, induced transparency, macroscopic quantum superposition, squeezing light, and squeezing resonator.

In the above applications, quantum squeezing is important for studying the macroscopic quantum effects and the precision metrology of weak forces. In the above schemes of squeezing, the theory of most schemes is based on the nonlinear property. The history of squeezing is linked intimately to quantum-limited displacement sensing, and many schemes have been proposed to generate squeezing states in various systems. The squeezing of light is proposed for the first time using atomic sodium as a nonlinear medium. In recent years, researchers have found that the optomechanical cavity, in which radiation pressure proportional to optical intensity changes the cavity length, could act as a low-noise Kerr nonlinear medium. So the optomechanical cavity could be a better candidate to generate squeezing of the optical and mechanical modes. The squeezing of optical field is easy to be achieved in the optomechanical systems, and has been reported experimentally. Furthermore, many theoretical schemes have been proposed to generate mechanical squeezing in the optomechanical systems by using different methods. For example, in 2010, Nunnenkamp et al. proposed a scheme to generate mechanical squeezing via the quadratically nonlinear coupling between optical cavity mode and the displacement of a mechanical resonator. In 2011, Liao et al. proposed a scheme to generate mechanical squeezing via periodically modulating the driving field amplitude at a frequency matching the frequency shift of the resonator. In 2013, Kronwald et al. proposed a scheme to generate mechanical squeezing by driving the optomechanical cavity with two controllable lasers with differing amplitudes in a dissipative mechanism. In 2015, Lü et al. proposed a scheme to generate steady-state mechanical squeezing via utilizing the mechanical intrinsic nonlinearity. With the deepening of research, the squeezing of mechanical mode has finally been observed experimentally by Wollman et al. In most theoretical schemes, the mechanical resonator squeezing must rely on the resolved sideband limit, requiring a cavity decay rate smaller than the mechanical resonator frequency, which restricted the progress of the experiment.

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Traditionally and generally, the decay rate of cavity, which is a dissipative factor in optomechanical systems, is considered to have negative effect on the performance of quantum manipulation and quantum information processing. The optomechanical coupling strength \( g = (\omega_c / L) \sqrt{\hbar / m \omega_m} \) (with the cavity frequency \( \omega_c \), the mechanical resonator mass \( m \), and the mechanical resonator frequency \( \omega_m \)) is inverse relation to the cavity length \( L \). While the cavity quality factor \( Q \) increases with increasing the cavity volume \( V \). Thus it is difficult to achieve small decay and strong optomechanical coupling strength simultaneously. Here we propose a method to generate steady-state mechanical squeezing in a double-cavity optomechanical system with the highly dissipative cavity \( (\kappa_2 / \omega_m = 100) \). The scheme does not need to satisfy the conditions of the small cavity decay rate and the strong optomechanical coupling strength simultaneously. The coherently driving on the cavity mode is a monochromatic laser source which can generate strong optomechanical coupling between the mechanical and cavity modes. We show that, based on the mechanical nonlinearity and cavity cooling process in transformed frame, the steady-state mechanical squeezing can be successfully and effectively generated in the highly unresolved sideband regime via the coherent auxiliary cavity interfering. The result indicate that the squeezing can reach 4.4 dB, beyond the so-called 3 dB limit. Different from the hybrid atom-optomechanical systems\(^8\),\(^{23}\),\(^{37}\), the scheme does not have the challenge of putting a large number of atoms in the cavity. Unlike the dissipative coupling mechanism\(^{32}\),\(^{41}\),\(^{43}\),\(^{49}\),\(^{54}\),\(^{55}\), our scheme utilizes the destructive interference coming from the coherent auxiliary cavity to resist the influence of cavity decay.

The paper is organized as follows: In Sec. II, we describe the model of a double-cavity optomechanical system and derive the linearized Hamiltonian and the effective coupling between the auxiliary cavity and the mechanical resonator. In Sec. III, we engineer the mechanical squeezing and derive the analytical variance of the displacement quadrature of the mechanical resonator in the steady-state. In Sec. IV, we study the relationship between the variance of mechanical mode and the system parameters and obtain the steady-state mechanical squeezing in the highly unresolved sideband regime by numerical simulations method. A conclusion is given in Sec. V.

**Results**

**Basic model.** We consider a double-cavity optomechanical system, which is composed of a mechanical resonator and two coupled single-mode cavities, depicted in Fig. 1. The mechanical resonator couples to the first dissipative cavity which is driven by an external laser field, forming the standard optomechanical subsystem. The second high Q optical cavity is regarded as the auxiliary part, which couples to the first dissipative cavity with the coupling strength \( J \). The total Hamiltonian \( H = H_0 + H_1 + H_{\text{pump}} \) describes the double-cavity optomechanical system, consists of three parts, which reads \( (b = 1) \), respectively,

\[
H_0 = \omega_1 a_1^+ a_1 + \omega_2 a_2^+ a_2 + \omega_m b^+ b + \frac{g}{2} (b + b^d)^4,
\]

\[
H_1 = J (a_1^+ a_2 + a_1 a_2^+) - g a_1^+ a_1 (b + b^d),
\]

\[
H_{\text{pump}} = \Omega_d (e^{-i \omega_d t} a_1^+ + e^{i \omega_d t} a_1).
\]

(1)

The part \( H_0 \) accounts for the free Hamiltonian of the two cavity modes (with frequency \( \omega_1, \omega_2 \) and decay rate \( \kappa_1, \kappa_2 \), respectively) and the mechanical resonator (with frequency \( \omega_m \) and damping rate \( \gamma_m \)). Here \( a_i (a_i^+) \) is the bosonic annihilation (creation) operator of the first optical cavity mode, \( a_2 (a_2^+) \) is the bosonic annihilation (creation) operator of the second optical cavity mode, and \( b (b^d) \) is the bosonic annihilation (creation) operator of the mechanical mode. The last term of \( H_0 \) describes the Duffing nonlinearity\(^{56},57\) of the mechanical resonator with amplitude \( \eta \). The intrinsic nonlinearity of the gigahertz mechanical resonator is usually very weak with nonlinear amplitude smaller than \( 10^{-5} \omega_m \). We can obtain a strong nonlinearity through coupling the mechanical mode to an auxiliary system\(^2\), such as the nonlinear amplitude of \( \eta = 10^{-4} \omega_m \) can be obtained when we couple the mechanical resonator to an external qubit\(^4\). The resulting model is known as the Duffing oscillator and exhibits a bifurcation phenomenon as the strength of the mechanical driving is increased\(^5\). In our scheme, the bifurcation...
the Hamiltonian \( H(\eta) \) is related to the input laser power \( \kappa \).

 modes satisfy the corresponding semiclassical equations:

\[
\text{denoting the mean values of the optical and mechanical modes. The mean values of the optical and mechanical decay rate of cavity 1 here,}
\]

\[
\text{where the corresponding noise operators}
\]

\[
\text{are written as}
\]

\[
\text{Considering the effect of the thermal environment, the quantum Heisenberg-Langevin equations for the system is given by}
\]

\[
H' = -\delta_1 a_1^\dagger a_1 + \delta_2 a_2^\dagger a_2 + \omega_m b^\dagger b + \eta (b + b^\dagger)^3 + f (a_1^\dagger a_2 + a_2 a_1^\dagger) - g a_1^\dagger (a_1 + a_1^\dagger),
\]

\[
\text{where} \delta_1 = \omega_d - \omega_1 \text{ and } \delta_2 = \omega_d - \omega_2 \text{ are the detunings of the two cavity modes from the driving field, respectively. Considering the effect of the thermal environment, the quantum Heisenberg-Langevin equations for the system are written as}
\]

\[
\begin{align*}
\dot{a}_1 &= \left(i \delta_1 - \frac{\kappa_1}{2}\right) a_1 - i f a_2 + i g a_1 (b + b^\dagger) - \Omega_d - \sqrt{\gamma_m} a_{1\text{m}}, \\
\dot{a}_2 &= \left(i \delta_2 - \frac{\kappa_2}{2}\right) a_2 - i f a_1 - \sqrt{\gamma_m} a_{2\text{m}}, \\
\dot{b} &= -i \omega_m - \frac{\gamma_m}{2} \left[b - 2 i \eta (b + b^\dagger)^3 + i g a_1 a_1^\dagger - \sqrt{\gamma_m} b_{\text{m}}\right].
\end{align*}
\]

\[
\text{where the corresponding noise operators } a_{1\text{m}}, a_{2\text{m}} \text{ and } b_{\text{m}} \text{ satisfy the following correlations:}
\]

\[
\begin{align*}
\langle a_{1\text{m}}(t) a_{1\text{m}}^\dagger(t') \rangle &= \langle a_{2\text{m}}(t) a_{2\text{m}}^\dagger(t') \rangle = \delta(t - t'), \\
\langle a_{1\text{m}}^\dagger(t) a_{2\text{m}}(t') \rangle &= \langle a_{2\text{m}}^\dagger(t) a_{1\text{m}}(t') \rangle = 0, \\
\langle b_{\text{m}}(t) b_{\text{m}}^\dagger(t') \rangle &= \langle \bar{n}_\text{m} + 1 \rangle \delta(t - t'), \\
\langle b_{\text{m}}^\dagger(t) b_{\text{m}}(t') \rangle &= \bar{n}_\text{m} \delta(t - t').
\end{align*}
\]

\[
\text{here, } \bar{n}_\text{m} = \langle \exp[\hbar \omega_m/(k_B T)] - 1 \rangle^{-1} \text{ is the mean thermal excitation number of bath of the mechanical resonator at temperature } T, k_B \text{ is the Boltzmann constant. And under the assumption of Markovian baths, the noise operators } a_{1\text{m}}, a_{2\text{m}} \text{ and } b_{\text{m}} \text{ have zero mean values.}
\]

Since the system is driven by a classical laser field, in the case of strong driving field, we can treat the field operators as the sum of their mean values and small quantum fluctuation. So we can apply a displacement transformation to linearize the equations, \( a_1 \rightarrow \alpha_1 + a_1, \ a_2 \rightarrow \alpha_2 + a_2, \ b \rightarrow \beta + b \), where \( \alpha_1, \alpha_2, \text{ and } \beta \) are complex numbers denoting the mean values of the optical and mechanical modes. The mean values of the optical and mechanical modes satisfy the corresponding semiclassical equations:
\[\alpha_1 = \left[ i(\delta_1 + g\beta + g\beta^*) - \frac{\kappa_1}{2} \right] \alpha_1 - i\gamma \alpha_2 - i\Omega_d,\]
\[\alpha_2 = \left( i\beta - \frac{\kappa_2}{2} \right) \alpha_2 - i\gamma \alpha_1,\]
\[\beta = \left( -i\omega_m - \frac{\gamma_m}{2} \right) \beta - 2i\gamma \beta^* + 3\beta\beta^* \beta + 3\beta^*\beta^* \beta + 3\beta^* + 3\beta + i\gamma \alpha^* \alpha_1.\]  

(5)

The steady-state amplitudes of the optical and mechanical modes are relative to the driving power \(P\), and the relationship can be derived by solving the above equations under the condition of steady situation. One can see that when the driving power \(P\) is in the microwatt range, the amplitudes of the cavity and mechanical modes satisfy the relationships:

\[\alpha_\beta,11,\text{as shown in Fig. 2.}\]

And the amplitudes of the cavity and mechanical modes increase with increasing the driving power. At the point of the driving power \(P = 0.53\ \text{mW}\), the result of \(\alpha_\beta,3901\) and \(\beta,40\) can be obtained, respectively.

Under the conditions of strong driving, the nonlinear terms are neglected. The quantum fluctuations satisfy the following linearized equations:

\[\kappa = \left( i\delta + \frac{\kappa_1}{2} \right) \kappa - iG \kappa + i\Lambda,\]
\[\omega = \left( i\delta + \frac{\kappa_2}{2} \right) \omega - iG \omega + i\Lambda,\]
\[\gamma = \left( -i\omega_m - \frac{\gamma_m}{2} \right) \gamma - 2i\gamma \gamma^* + 3\gamma^* \gamma^* \gamma + 3\gamma^* \gamma^* \gamma + 3\gamma^* + 3\gamma + i\gamma \kappa^* \kappa_1.\]  

(6)

The linearized Hamiltonian is given by

\[H_L = -\Delta a_1 a_1 - \delta a_2 a_2 + \frac{1}{2} \gamma_m b^* b,\]
\[-2G(a_1 a_2 + a_1^* a_2^*).\]  

(8)

When considering the system-reservoir interaction, which results in the dissipations of the system, the full dynamics of the system is described by the master equation

\[\dot{\rho} = -i[H_L, \rho] + \kappa_1 \mathcal{L}[a_1] \rho + \kappa_2 \mathcal{L}[a_2] \rho + \gamma_m \mathcal{L}[b] \rho + \gamma_m \mathcal{L}[b^*] \rho,\]  

(9)

where \(\mathcal{L}[\rho] = \rho \mathcal{O}^\dagger - (\mathcal{O}^\dagger \rho + \rho \mathcal{O})/2\) is the standard Lindblad operators. \(\kappa_1, \kappa_2, \text{ and } \gamma_m\) are the decay rate of cavity mode \(a_1, a_2\), and the damping rate of mechanical resonator, respectively. \(\mathcal{O}\) is the average phonon number in thermal equilibrium.

Effective coupling between the auxiliary cavity and the mechanical resonator. Since the decay rate of cavity 1 (\(\kappa_1\)) is much larger than the decay rate of cavity 2 (\(\kappa_2\)) and the damping rate of mechanical resonator (\(\gamma_m\)), the cavity mode \(a_1\) can be eliminated adiabatically for the time scales longer than \(\kappa_1^{-1}\). The steady solution of the first equation in Eq. (6) about cavity mode \(a_1\) can be written as

\[a_1 = \frac{-iG}{-i\Delta_1 + \frac{\kappa_1}{2} a_2 + \frac{G}{-i\Delta_1 + \frac{\kappa_1}{2} a_2} (b + b^*) - \frac{\gamma_m}{-i\Delta_1 + \frac{\kappa_1}{2} a_2} a_1.\]  

(10)

Substituting Eq. (10) into the rear two equations of Eq. (6), we can obtain the effective coupling between the cavity mode \(a_2\) and the mechanical mode \(b\), which can be described by the following equations:

\[\dot{a}_2 = \left( i\Delta_2 - \frac{\kappa_2}{2} \right) a_2 + iG a_2 (b + b^*) - A_{2m},\]
\[\dot{b} = \left( -i\omega_m' - \frac{\gamma_m}{2} \right) b + iG (a_2 + a_2^*) - 2i\Lambda b^* - B_m,\]  

(11)

where \(A_{2m}\) and \(B_m\) denote the modified noise terms, the effective parameters of the mechanical frequency, optomechanical coupling strength, detuning, decay rate, and coefficients of bilinear terms are given by...
\[\omega_m' = \omega_m + \frac{2G^2\Delta_1}{\Delta_1^2 + \left(\frac{\kappa_1}{2}\right)^2} = \omega_m + 2\Lambda',\]

\[G_{\text{eff}} = \frac{Gf}{\Delta_1 \pm \left(\frac{\kappa_1}{2}\right)^2},\]

\[\Delta_{\text{eff}} = \delta_2 - \frac{f^2\Delta_1}{\Delta_1^2 + \left(\frac{\kappa_1}{2}\right)^2},\]

\[\kappa_{\text{eff}} = \kappa_2 + \frac{f^2\kappa_1}{\Delta_1^2 + \left(\frac{\kappa_1}{2}\right)^2},\]

\[\Lambda' = \Lambda + \frac{G^2\Delta_1}{\Delta_1^2 + \left(\frac{\kappa_1}{2}\right)^2}.\]  

Thus the effective Hamiltonian, describing the effective coupling between the auxiliary cavity mode and the mechanical resonator, is written as

\[H_{\text{eff}} = -\Delta_{\text{eff}} a_2^\dagger a_2 + \omega_m' b^\dagger b - G_{\text{eff}}(a_2 + a_2^\dagger)(b + b^\dagger) + \Lambda'(b^2 + b'^2),\]  

and the master equation becomes

\[\rho = -i[H_{\text{eff}}, \rho] + \kappa_{\text{eff}}\mathcal{L}[a_2] \rho + \gamma_m(\pi_{\text{th}} + 1)\mathcal{L}[b] \rho + \gamma_m\pi_{\text{th}}\mathcal{L}[b^\dagger] \rho.\]  

The effective Hamiltonian describes the effective interaction between the cavity and the mechanical resonator. As we all know, if the Hamiltonian in the interaction picture has the form \(b^2 + b'^2\), the corresponding evolution operator is a squeezed operator.

**Engineering the mechanical squeezing.** Applying the unitary transformation \(S(\zeta) = \exp[\zeta(b^2 - b'^2)/2]\), which is the single-mode squeezing operator with the squeezing parameter

\[\zeta = \frac{1}{4} \ln\left(1 + \frac{4\Lambda'}{\omega_m}\right),\]

\[\zeta_n = \zeta + \frac{1}{4} \ln\left(1 + \frac{4\Lambda'}{\omega_m}\right),\]

\[\zeta_n = \pi_{\text{th}} + \zeta + \frac{1}{4} \ln\left(1 + \frac{4\Lambda'}{\omega_m}\right)\]

\[\pi_{\text{th}} = \pi_{\text{th}} + \zeta + \frac{1}{4} \ln\left(1 + \frac{4\Lambda'}{\omega_m}\right)\]

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\[\zeta_n = \pi_{\text{th}} + \zeta + \frac{1}{4} \ln\left(1 + \frac{4\Lambda'}{\omega_m}\right)\]

where \(\omega_m'\) is the transformed effective mechanical frequency and \(G'\) is the transformed effective optomechanical coupling. The transformed Hamiltonian is a standard cavity cooling Hamiltonian and the best cooling in the transformed frame is at the optimal detuning \(\Delta_{\text{eff}} = -\omega_{m}'\). The transformed Hamiltonian describes the effective interaction between the cavity and the mechanical resonator. As we all know, if the Hamiltonian in the interaction picture has the form \(b^2 + b'^2\), the corresponding evolution operator is a squeezed operator.

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Discussion

In this section, we solve the original master equation Eq. (9) numerically to calculate the steady-state variance of the mechanical displacement quadrature $X$. Firstly, we should provide the time evolution of variance $\langle \delta X^2 \rangle$ about the mechanical displacement quadrature, which is shown in Fig. 3. It indicates that the variance $\langle \delta X^2 \rangle$ gradually tends to be stable after a period of time. For simplicity, we have assumed that the system is initially prepared in its ground state and the system parameters are chosen to be the same as in Fig. 2.

The relationship between the steady-state variance and intercavity coupling strength is shown in Fig. 4. Before we study their relationship, we should recalculate the steady-state amplitudes of the optical and mechanical modes $|\alpha_1|$, $|\alpha_2|$, and $|\beta|$ with the different intercavity coupling strengths. We can find that the steady-state mechanical squeezing can be achieved effectively when the intercavity coupling strength is appropriate, which reaches a balance between the enough large photons number in cavity 1 and the coherent auxiliary cavity interferences. However, when we remove the coherent auxiliary cavity interferences ($J = 0$), the mechanical steady-state squeezing cannot be obtained effectively under the present condition.

The relationship between the steady-state variance and driving power is shown in Fig. 5. One can see from Fig. 5 that the steady-state squeezing of the mechanical resonator changes observably with the laser driving power. We can obtain the steady-state mechanical squeezing effectively when the driving power is in milliwatts level. At last, we consider the effect of the cavity 1 decay $\kappa_1$. When calculate the relationship between the steady-state variance and cavity 1 decay, we consider a more variable $J$ as shown in Fig. 6. The result shows that the maximum value of squeezing can be reached with an appropriate $J$ when the cavity 1 decay is certain. In Fig. 6, the minimum value of the steady-state variance is 0.36, corresponding to the 4.4 dB.

In the above, we study the steady-state squeezing of the mechanical resonator in a double-cavity optomechanical system and illustrate that the steady-state squeezing can be effectively generated in highly unresolved sideband regime with appropriate intercavity coupling strength and driving power. When the decay rate of cavity is known, the maximum value of the squeezing parameter $\zeta$ is achieved at the point of $\Delta_0 = \kappa_1/2$, which can be easily seen from Eq. (12). The experimental studies of the double-cavity optomechanical system with whispering-gallery microcavities have been reported. The tunable nonlinearity of the mechanical resonator has been greatly improved by exploring the anharmonicity in chemical bonding interactions. And our method, utilizing the coherent auxiliary cavity 2 to resist the influence of decay coming from cavity 1, is also feasible with the cubic nonlinearity of mechanical resonator, which is easy to prove as refs 37, 44. We also notice another approach beyond the resolved sideband limit and demonstrated experimentally in optomechanical system.
Furthermore, the generated steady-state mechanical squeezing in the present scheme can be detected based on the method proposed in refs 22, 44. As illustrated in refs 22, 44, for detecting the mechanical resonator, we consider another auxiliary cavity mode $a_1$ (another mode of the cavity $a$, or adding another cavity on the right) with resonant frequency $\omega_s$, which is driven by a weak pump laser field of amplitude $\Omega_p$ and frequency $\omega_p$. The presence of the cavity $a_1$ will affect the mirror dynamics, which is no more exactly described by Eqs (6). The original Hamiltonian Eq. (2) should be added the new detection parts $\delta = -\omega_p - \omega_s + \Omega_p (a_1^\dagger + a_1)$, where $\delta = \omega_p - \omega_s$ and $g_s$ is the strength of the single-photon optomechanical coupling. However, if the intracavity field is very weak under the weak driving field (the cavity mode steady-state amplitude $\alpha_s \ll |\alpha_1|$, the cavity backaction on the mechanical mode can be neglected and the relevant dynamics is still well described by Eq. (6). Through homodyning detection of the output field of another auxiliary cavity mode with an appropriate phase, we can obtain the information of the position and the momentum quadratures of the mechanical resonator. Effective detection of the mechanical state requires that $\alpha_s \ll |\alpha_1|$ while $g_s |\alpha_s| \ll \kappa_s$, where $\kappa_s$ is decay rate of the another auxiliary cavity. The experimental detection technology of the output field has also been realized.

In conclusion, we have proposed a scheme for generating the steady-state squeezing of the mechanical resonator in a double-cavity optomechanical system via the mechanical nonlinearity and cavity cooling process in transformed frame. The steady-state squeezing of the mechanical resonator can be obtained in the highly unresolve sideband regime through the coherent auxiliary cavity interferences. Since the auxiliary cavity mode is not directly coupled to the mechanical resonator, it can be a high $Q$ optical cavity with big cavity volume $V$, while another cavity coupling with the mechanical resonator can have a short cavity length $L$ to possess good mechanical properties. The effective coupling between the mechanical resonator and the auxiliary cavity can be obtained by reducing the cavity mode adiabatically. We simulate the steady-state variance of the mechanical displacement quadrature numerically at a determinate laser driving power and find that under an appropriate intercavity coupling strength $J$ by solving the master equation numerically.

![Figure 5. The variance of the mechanical displacement quadrature $X$ relates to the driving power $P$ by solving the master equation numerically. The other parameters are chosen to be the same as in Fig. 2.](image)

![Figure 6. The variance of the mechanical displacement quadrature $X$ relates to the cavity 1 decay $\kappa_1$ and the intercavity coupling strength $J$ by solving the master equation numerically. The other parameters are chosen to be the same as in Fig. 2.](image)
The effective interaction between the auxiliary cavity and the mechanical resonator. Here, we will introduce another way to derive the effective coupling between the auxiliary cavity and the mechanical resonator. The quantum Langevin equations Eq. (6) can be formally integrated as

\[ a_1(t) = a_1(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} + e^{i \Delta_1 t - \frac{\kappa_1}{2} t} \int_0^t \left[ \left[ -i a_2(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} + i G b(t) e^{-i \Delta_1 t + \frac{\kappa_1}{2} t} \right] e^{-i \Delta_1 t + \frac{\kappa_1}{2} t} \right] dt. \]

\[ a_2(t) = a_2(0) e^{i \Delta_1 t - \frac{\kappa_2}{2} t} + e^{i \Delta_1 t - \frac{\kappa_2}{2} t} \int_0^t \left[ \left[ -i a_1(0) e^{-i \Delta_1 t + \frac{\kappa_2}{2} t} + i G b(t) e^{i \Delta_1 t - \frac{\kappa_2}{2} t} \right] e^{-i \Delta_1 t + \frac{\kappa_2}{2} t} \right] dt. \]

\[ b(t) = b(0) e^{i \omega_m t - \frac{\gamma_m}{2} t} + e^{i \omega_m t - \frac{\gamma_m}{2} t} \int_0^t \left[ \left[ i G a(t) + i G b(t) - 2i \kappa_1 \right] e^{-i \omega_m t + \frac{\gamma_m}{2} t} \right] dt. \]

(19)

Since the decay rate of cavity 1 \( \kappa_1 \) is much larger than the decay rate of cavity 2 \( \kappa_2 \) and the damping rate of mechanical resonator \( \gamma_m \), the dynamics of mode \( b \) and \( a_1 \) are only slightly affected by mode \( a_2 \). We obtain the approximated expressions

\[ a_2(t) = a_2(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} + A_{2in}^{(t)}, \]

\[ b(t) = b(0) e^{-i \omega_m t + \gamma_m t} + B_{m}^{(t)}, \]

(20)

where \( A_{2in}^{(t)} \) and \( C_{m}^{(t)} \) denote the noise terms. By Plugging Eq. (20) into the first equation of Eq. (19), we obtain

\[ a_1(t) \approx a_1(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} + e^{i \Delta_1 t - \frac{\kappa_1}{2} t} \int_0^t \left[ \left[ -i a_2(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} + i G b(t) e^{-i \Delta_1 t + \frac{\kappa_1}{2} t} \right] e^{-i \Delta_1 t + \frac{\kappa_1}{2} t} \right] dt + A_{1in}^{(t)}(t). \]

\[ = a_1(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} + \frac{-i a_2(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t}}{-i (\Delta_1 - \gamma_m) + \frac{\kappa_1}{2}} + \frac{i G b(t)}{e^{-i \Delta_1 t + \frac{\kappa_1}{2} t} - i (\Delta_1 - \gamma_m) + \frac{\kappa_1}{2}} \]

\[ + \frac{i G b(0)}{e^{-i \Delta_1 t + \frac{\kappa_1}{2} t} - i (\Delta_1 - \gamma_m) + \frac{\kappa_1}{2}} + A_{1in}^{(t)}, \]

(21)

where \( A_{1in}^{(t)} \) denote the noise term. Under the conditions of \( |\Delta_1| \gg |\Delta_2|, \gamma_m \) and \( \kappa \gg (\kappa_2, \omega_m) \), we obtain

\[ a_1(t) \approx a_1(0) e^{i \Delta_1 t - \frac{\kappa_1}{2} t} - \frac{i a_2(t)}{-i \Delta_1 + \frac{\kappa_1}{2}} + \frac{i G [b(t) + b'(t)]}{-i \Delta_1 + \frac{\kappa_1}{2}} + A_{1in}^{(t)}. \]

(22)

Neglecting the fast decaying term which contains \( \exp(-\kappa_1 t/2) \), and the above Eq. (22) is same form as Eq. (10).

References
1. Purdy, T. P., Peterson, R. W. & Regal, C. A. Observation of Radiation Pressure Shot Noise on a Macroscopic Object. Science 339, 801 (2013).
2. Barish, B. C. & Weiss, R. LIGO and the Detection of Gravitational Waves. Phys. Today 52, 44 (1999).
3. Marquardt, F. & Girvin, S. M. Optomechanics. Physics 2, 40 (2009).
4. Blencowe, M. Quantum electromechanical systems. Phys. Rep. 395, 159 (2004).
5. Kippenberg, T. J. & Vahala, K. J. Cavity Optomechanics: Back-Action at the Mesoscale. Science 321, 1172 (2008).
6. Xu, X. & Nori, F. Squeezed Phonon States: Modulating Quantum Fluctuations of Atomic Displacements. Phys. Rev. Lett. 76, 2294 (1996).
7. Wilson-Rae, I., Nooshi, N., Zwerger, W. & Kippenberg, T. J. Theory of Ground State Cooling of a Mechanical Oscillator Using Dynamical Backaction. Phys. Rev. Lett. 99, 093901 (2007).
8. Liu, Y. C., Xiao, Y. F., Luan, X. S., Gong, Q. H. & Wong, C. W. Coupled cavities for motional ground-state cooling and strong optomechanical coupling. Phys. Rev. A 91, 033818 (2015).
9. Chen, X., Liu, Y. C., Peng, P., Zhi, Y. & Xiao, Y. F. Cooling of macroscopic mechanical resonators in hybrid atom-optomechanical systems. Phys. Rev. A 92, 033841 (2015).
10. Guo, Y., Li, K., Nie, W. & Li, Y. Electromagnetically-induced-transparency-like ground-state cooling in a double-cavity optomechanical system. Phys. Rev. A 90, 053841 (2014).
11. Marquardt, F., Chen, J. P., Clerk, A. A. & Girvin, S. M. Quantum Theory of Cavity-Assisted Sideband Cooling of Mechanical Motion. Phys. Rev. Lett. 99, 093902 (2007).
12. Yin, Z. Q., Li, T. & Feng, M. Three-dimensional cooling and detection of a nanosphere with a single cavity. Phys. Rev. A 83, 013816 (2011).
13. Clark, J. B., Lecocq, F., Simmonds, R. W., Aumentado, J. & Teufel, J. D. Sideband Cooling Beyond the Quantum Limit with Squeezed Light. arXiv: 1605.08795 (2016).
14. Asjad, M., Zippli, S. & Vitali, D. Suppression of Stokes scattering and improved optomechanical cooling with squeezed light. arXiv: 1606.09007 (2016).
15. Zhang, W. Z., Cheng, J., Li, W. D. & Zhou, L. Optomechanical cooling in the non-Markovian regime. Phys. Rev. A 93, 063853 (2016).
16. Tian, L. Cavity cooling of a mechanical resonator in the presence of a two-level-system defect. Phys. Rev. B 84, 035417 (2011).
17. Wang, Y. D. & Clerk, A. A. Using Interference for High Fidelity Quantum State Transfer in Optomechanics. Phys. Rev. Lett. 108, 153603 (2012).
18. Dobrindt, J. M., Wilson-Rae, I. & Kippenberg, T. J. Parametric Normal-Mode Splitting in Cavity Optomechanics. Phys. Rev. Lett. 101, 263602 (2008).
19. Gröblacher, S., Hammerer, K., Vanner, M. R. & Aspelmeyer, M. Observation of strong coupling between a micromechanical resonator and an optical cavity field. Nature (London) 460, 724 (2009).
20. Yin, Z. Q., Yang, W. L., Sun, L. & Duan, L. M. Quantum network of superconducting qubits through an optomechanical interface. Phys. Rev. A 91, 012333 (2015).
21. Clerk, A. A., Marquardt, F. & Jacobs, K. Back-action evasion and squeezing of a mechanical resonator using a cavity detector. New J. Phys. 10, 095010 (2008).
22. Vitali, D. et al. Optomechanical Entanglement between a Movable Mirror and a Cavity Field. Phys. Rev. Lett. 98, 030405 (2007).
23. Genes, C., Vitali, D. & Tombesi, P. Emergence of atom-light-mirror entanglement inside an optical cavity. Phys. Rev. A 77, 050307(R) (2008).
24. Bai, C. H., Wang, D. Y., Wang, H. F., Zhu, A. D. & Zhang, S. Robust entanglement between a movable mirror and atomic ensemble and entanglement transfer in coupled optomechanical system. Sci. Rep. 6, 33404 (2016).
25. Nie, W. J., Lan, Y. H., Li, Y. & Zhu, S. Y. Generating large steady-state optomechanical entanglement by the action of Casimir force. Sci. China-Phys. Mech. Astron. 57, 2276 (2014).
26. Asjad, M., Zippli, S. & Vitali, D. Mechanical Einstein-Podolsky-Rosen entanglement beyond the resolved sideband regime with a finite-bandwidth squeezed reservoir. Phys. Rev. A 93, 062307 (2016).
27. Wu, Q., Zhang, J. Q., Wu, J. H., Feng, M. & Zhang, Z. M. Tunable multi-channel inverse optomechanically induced transparency and its applications. Opt. Express 23, 18334 (2015).
28. Li, W. L., Jiang, Y. F., Li, C. & Song, H. S. Parity-time-symmetry enhanced optomechanically-induced transparency. Sci. Rep. 6, 31095 (2016).
29. Liao, J. Q. & Tian, L. Macroscopic Quantum Superposition in Cavity Optomechanics. Phys. Rev. Lett. 116, 163602 (2016).
30. Jähne, K. et al. Cavity-assisted squeezing of a mechanical oscillator. Phys. Rev. A 79, 063819 (2009).
31. Purdy, T. P., Yu, P. L., Peterson, R. W., Kampel, N. S. & Regal, C. A. Strong Optomechanical Squeezing of Light. Phys. Rev. X 3, 031012 (2013).
32. Kronwald, A., Marquardt, F. & Clerk, A. A. Dissipative optomechanical squeezing of light. New J. Phys. 16, 063058 (2014).
33. Mari, A. & Eisert, J. Gently Modulating Optomechanical Systems. Phys. Rev. Lett. 103, 213603 (2009).
34. Gu, W. J., Li, G. X. & Yang, Y. P. Generation of squeezed states in a movable mirror via dissipative optomechanical coupling. Phys. Rev. A 88, 013835 (2013).
35. Tan, H. T., Li, G. X. & Meystre, P. Dissipation-driven two-mode mechanical squeezed states in a movable mirror. Phys. Rev. A 87, 033829 (2013).
36. Asjad, M. et al. Robust stationary mechanical squeezing in a kicked quadratic optomechanical system. Phys. Rev. A 89, 023848 (2014).
37. Wang, D. Y., Bai, C. H., Wang, H. F., Zhu, A. D. & Zhang, S. Steady-state mechanical squeezing in a hybrid atom-optomechanical system with a highly dissipative cavity. Sci. Rep. 6, 24421 (2016).
38. Zhang, L., Liu, Y. X. & Nori, F. Cooling and squeezing the fluctuations of a nanomechanical beam by indirect quantum feedback control. Phys. Rev. A 79, 052102 (2009).
39. Blencowe, M. & Wybourne, M. Quantum squeezing of mechanical motion for micron-sized cantilevers. Phys. B (Amsterdam, Neth.) 280, 553 (2000).
40. Rabl, P., Shinnim, A. & Zoller, P. Generation of squeezed states of nanomechanical resonators by reservoir engineering. Phys. Rev. B 70, 205304 (2004).
41. Nunnenkamp, A., Berktje, K., Harris, J. G. E. & Girvin, S. M. Cooling and squeezing via quadratic optomechanical coupling. Phys. Rev. A 82, 021806 (2010).
42. Liao, J. Q. & Law, C. K. Parametric generation of quadrature squeezing of mirrors in cavity optomechanics. Phys. Rev. A 83, 033820 (2011).
43. Kronwald, A., Marquardt, F. & Clerk, A. A. Arbitrarily large steady-state bosonic squeezing via dissipation. Phys. Rev. A 88, 063835 (2013).
44. Lü, X. Y., Liao, J. Q., Tian, L. & Nori, F. Steady-state mechanical squeezing in an optomechanical system via Duffing nonlinearity. Phys. Rev. A 91, 013834 (2015).
45. Agarwal, G. S. & Huang, S. Strong mechanical squeezing and its detection. Phys. Rev. A 93, 043844 (2016).
46. Braginsky, V. B. & Khalili, F. Y. Quantum Measurement (Cambridge University Press, Cambridge, England, 1992).
47. Wu, L. A., Kimble, H. J., Clerk, A. A. Robust squeezed states of nanomechanical resonators by reservoir engineering. Phys. Rev. Lett. 108, 153603 (2012).
48. Slusher, R. E., Hollberg, L. W., Yurke, B., Mertz, J. C. & Valley, J. Observation of Squeezed States Generated by Four-Wave Mixing in an Optical Cavity. Phys. Rev. Lett. 55, 2409 (1985).
49. Ma, S. L., Li, P. B., Fang, A. P., Gao, S. Y. & Li, F. L. Dissipation-assisted generation of steady-state single-mode squeezing of collective excitations in a solid-state spin ensemble. Phys. Rev. A 88, 013837 (2013).
50. Mancini, S. & Tombesi, P. Quantum noise reduction by radiation pressure. Phys. Rev. A 49, 4055 (1994).
51. Safavi-Naeini, A. H. et al. Squeezed light from a silicon micromechanical resonator. Nature (London) 500, 185 (2013).
52. Brooks, D. W. et al. Non-classical light generated by quantum-noise-driven cavity optomechanics. Nature (London) 488, 476 (2012).
53. Wollman, E. et al. Quantum squeezing of motion in a mechanical resonator. Science 349, 952 (2015).
54. Su, S. L., Guo, Q., Wang, H. F. & Zhang, S. Simplified scheme for entanglement preparation with Rydberg pumping via dissipation. Phys. Rev. A 92, 022328 (2015).
55. Su, S. L., Shao, X. Q., Wang, H. F. & Zhang, S. Preparation of three-dimensional entanglement for distant atoms in coupled cavities via atomic spontaneous emission and cavity decay. Sci. Rep. 4, 7566 (2014).
56. Aldridge, J. S. & Cleland, A. N. Noise-Enabled Precision Measurements of a Duffing Nanomechanical Resonator. Phys. Rev. Lett. 94, 156403 (2005).
57. Katz, I., Retzker, A., Straub, R. & Lifshitz, R. Signatures for a Classical to Quantum Transition of a Driven Nonlinear Nanomechanical Resonator. Phys. Rev. Lett. 99, 040404 (2007).
58. Jacobs, K. & Landahl, A. J. Engineering Giant Nonlinearities in Quantum Nanosystems. Phys. Rev. Lett. 103, 067201 (2009).
59. Bowcn, W. P. & Mølmer, G. J. Quantum Optomechanics (CRC Press, 2013).
60. Yan, D. et al. Duality and bistability in an optomechanical cavity coupled to a Rydberg superatom. Phys. Rev. A 91, 023813 (2015).
61. Xiong, W., Jin, D. Y., Qiu, Y., Lam, C. H. & You, J. Q. Cross-Kerr effect on an optomechanical system. Phys. Rev. A 93, 023844 (2016).
62. Jing, H. et al. PT-Symmetric Phonon Laser. Phys. Rev. Lett. 113, 053604 (2014).
63. Peng, B. et al. Nonreciprocal light transmission in parity-time-symmetric whispering-gallery microcavities. Nat. Phys. 10, 394 (2014).
64. Grudinin, I. S., Lee, H., Painter, O. & Vahala, K. J. Phonon Laser Action in a Tunable Two-Level System. Phys. Rev. Lett. 104, 083901 (2010).
65. Chang, L. et al. Parity-time symmetry and variable optical isolation in active-passive-coupled microresonators. Nat. Photon. 8, 524 (2014).
66. Huang, P. et al. Generating giant and tunable nonlinearity in a macroscopic mechanical resonator from a single chemical bond. Nat. Commun. 7, 11517 (2016).
67. Vanner, M. R. et al. Pulsed quantum optomechanics. Proc. Natl. Acad. Sci. USA 108, 16182 (2011).
68. Vanner, M. R., Hofer, J., Cole, G. D. & Aspelmeyer, M. Cooling-by-measurement and mechanical state tomography via pulsed optomechanics. Nat. Commun. 4, 2295 (2013).

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Author Contributions
D.Y.W. designed the scheme under the guidance of H.F.W., A.D.Z. and S.Z. D.Y.W. and C.H.B. carried out the theoretical analysis. All authors contributed to the interpretation of the work and the writing of the manuscript. All authors reviewed the manuscript.

Additional Information
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