A Modified Theory Of Newtonian Mechanics

Amir H. Abbassi* & Amir M. Abbasi†, ‡

Department of Physics, School of Sciences,
Tarbiat Modarres University, P.O.Box 14155-4838
Tehran, Iran

Abstract

A specific model of the inertial law is presented by which we can have some deeper insight into the essence of mass and inertia. In this modified theory there is no need to keep the concept of absolute space and the third law as a principle. By introducing a convenient form for gravitational law the coupling constant G becomes a function of inertial parameters of the universe.

Keywords: Absolute Space, Classical Theories of Gravity, Cosmology, Inertia.

According to the Newton’s second law of motion the relation \( \vec{F} = m\vec{a} \) holds between the applied force on a particle and its acceleration in absolute space and any other frames which are related to it by Galilean transformations. These reference frames are called inertial frames and \( m \) is the inertial mass of the particle. The relation between coordinates of two inertial systems \( S \) and \( S' \) which are moving with constant velocity \( \vec{v} \) with respect to each other are:

\[
\begin{align*}
  t' &= t \\
  \vec{a}' &= \vec{a} \\
  \vec{u}' &= \vec{u} - \vec{v} \\
  \vec{x}' &= \vec{x} - \vec{v}t + \vec{x}_0
\end{align*}
\]

Despite its simple appearance and practical applications Newton’s concept of absolute space which is the basis of Newtonian mechanics (NM) and in

*E-mail: ahabbasi@net1cs.modares.ac.ir
†Permanent add: Department of Physics, Faculty of Sciences, Tehran University
‡E-mail: amabasi@khayam.ut.ac.ir
turn the basis of mass and energy is not well defined and has been criticized. Cogent arguments against absolute space are [1]:

a - There is no unique way of locating Newton’s absolute space within the finite class of inertial frames.

b - It conflicts with one’s understanding to conceive of a thing which acts but cannot be acted upon.

In his critique of Newton’s conceptions in “The Science of Mechanics”, Mach suggested a set of pioneering implicit ideas then addressed by A. Einstein as Mach’s Principle (MP) [2]. According to MP, every motion is only comprehensive as a relative motion, inertia is due to an interaction with average mass of the universe [3] and the distribution of matter in the universe determines the inertial frame at each point. There have been many attempts to find Machianized reformulations of mechanics [4, 5, 6]. Einstein’s main aim in the general theory of relativity (GR) was to do this through giving an equation for gravitation and inertia together. But it came out that his theory shows some non-Machian aspects and do not fulfill the MP. For instance it has solution for empty space. Also there are many alternative theories of gravitation motivated by Mach’s ideas [4, 5, 6]. Since in GR as our standard theory of gravitation we are still faced with unresolved questions such as singularity problems and lack of well defined source for inertia, then it is reasonable to have more discussions about foundations. It seems with closer scrutiny of Newton’s second law we may find some insight.

Actually in this work our aim is to present a simple method for Machianization of NM. We propose that inertial effect is a mutual interaction between two particles which in any non-rotating arbitrary reference frame $S$ is proportional to the difference between their accelerations and to the inertial charges of each individual particles as follows

$$\vec{F}_{\text{inertia}} = \mu . c_1 . c_2 (\vec{a}_1 - \vec{a}_2)$$

(2)

where $\vec{a}_1$, $\vec{a}_2$ are accelerations of particles 1 and 2 with respect to $S$, their inertial charges are denoted by $c_1$, $c_2$ respectively, and $\mu$ is an inertial coupling constant. This can be easily extended to systems consisting of $N$ particles. Again in any non-rotating arbitrary reference frame $S$ we have

$$\vec{F}_i = \mu . c_i \sum_{j=1}^{N} c_j (\vec{a}_i - \vec{a}_j)$$

(3)

where $\vec{F}_i$ is the applied force on particle $i$ and the summation is done over all particles.
In the real world the inertial charge and Newtonian inertial mass of a particle are related so that

\[ m_i = \mu c_i \sum_{j=1}^{\text{all}} c_j \]  

where summation is done over all particles in the universe. Since local inhomogeneities have no observed effects on the inertial mass then it is accepted that the inertial mass is determined by the global structure of the universe and the relation (4) for the inertial mass is in accordance with this general belief. This can be accounted as a simple formulation of Mach’s idea about inertia. In terms of inertial mass, equation (3) can be rewritten in the following form

\[ \vec{F}_i = m_i \left( \vec{a}_i - \frac{\sum_{j=1}^{\text{all}} m_j \vec{a}_j}{\sum_{j=1}^{\text{all}} m_j} \right) \]  

These new forms of the second law i.e. equations (3) and (5) are invariant under transformation to a more general group of reference frames \( S' \) than Galilean ones which we may call as generalized Galilean transformations.

\[
\begin{align*}
    t' &= t \\
    \vec{a}' &= \vec{a} - \vec{b} \\
    \vec{u}' &= \vec{u} - \vec{b}t - \vec{v} \\
    \vec{x}' &= \vec{x} - \frac{1}{2} \vec{b}t^2 - \vec{v}t + \vec{x}_0
\end{align*}
\]  

Here \( \vec{b} \), \( \vec{v} \) and \( \vec{x}_0 \) are the constant acceleration, velocity and position of \( S' \) with respect to \( S \) at \( t = 0 \) respectively.

Within the set of reference frames \( S' \) with different values of \( \vec{b} \) and \( \vec{v} \) there is a unique subset call it \( S'' \) moving with acceleration \( \vec{b}_0 \)

\[ \vec{b}_0 = \frac{\sum_{j=1}^{\text{all}} m_j \vec{a}_j}{\sum_{j=1}^{\text{all}} m_j} \]  

with respect to \( S \). In \( S'' \) the form of Newton’s second law of inertia is recovered, \( \vec{F}_i = m\vec{a}'' \). Therefore, inertial frames are a unique set of frames which are moving with acceleration \( \vec{b}_0 \) with respect to \( S \) with \( \vec{v} \) and \( \vec{x}_0 \) be any arbitrary value. This again may be accounted for as a feasible formulation of Mach’s idea about inertial frames. Since \( \vec{b}_0 \) is determined globally then local inhomogeneities have a negligible effect on it and inertial frames.
It is evident that equations (3) and (5) also satisfy Newton’s third law. For a two particle system we have
\[ \vec{F}_1 = -\vec{F}_2 = \mu_c.c_1.c_2(\vec{a}_1 - \vec{a}_2) \]  
(8)
and for a system with N particles it makes
\[ \sum_{j=1}^{all} \vec{F}_i = 0 \]  
(9)
So there is no need to introduce Newton’s third law as an extra principle.

We can extend this model to Newton’s law of gravitation. Equivalence principle here means that the source of inertia and gravitation is the same. Then if we define the gravitational force between two particles of inertial charges \( c_1 \) and \( c_2 \) as
\[ |\vec{F}_G| = \frac{\mu^2.c_1.c_2}{|\vec{r}_{12}|^2} \]  
(10)
where \(|\vec{r}_{12}| = |\vec{r}_1 - \vec{r}_2|\) is the relative separation between particles 1 and 2. Then we can express gravitational constant \( G \) in terms of inertial charges \( c_i \) as
\[ G = \left( \sum_{j=1}^{all} c_j \right)^{-\frac{1}{2}} \]  
(11)
which may be accounted for as another aspect of Machian implications that the so-called physical constants of nature (like \( G \)) are to be determined with some global features of the universe. Since the inertial mass and the gravitational constant are finite quantities, this means that \( \left( \sum_{j=1}^{all} c_j \right) \) is finite too, and the universe cannot be infinitely extended.

The Lagrangean formalism based on this model of inertial law is the same as the one for NM except that the kinematic energy of system \( T \) which is equal to \( \sum_i \frac{1}{2}m_i v_i^2 \) is replaced by
\[ T = \sum_i \frac{1}{2}m_i v_i^2 - \frac{[\sum_i m_i \vec{v}_i]^2}{2 \sum_i m_i} \]  
(12)
\[ = \frac{1}{4} \sum_i \sum_j m_i m_j (\vec{v}_i - \vec{v}_j)^2 \sum_j m_j \]
This is an invariant scalar. Then total energy of the system i.e. the sum of kinetic energy \( T \) and potential energy \( V(r_{ij}) \), a function of relative separation \( r_{ij} = \vec{r}_i - \vec{r}_j \), is invariant in all non-rotating reference frames.
Remarks

(i) - It may seem that choosing non-rotating frames is some kind of restriction which reduces the generality of the chosen frames. This is however, not the case, since rotating frames with respect to the whole universe are distinguishable and can be fixed.

(ii) - Our results show that $G$ and $m$ are not constants of nature but may change anytime the total inertial charge of the universe changes significantly. This can happen in the early stage of evolution of the universe, i.e. in the epochs of pair production.

(iii) - The concept of energy introduced with this theory is independent of the reference frame.

(iv) - There is no inertial structure for empty space.

These may show us how something should change in a modified theory of relativity.

References

[1] Rindler, W. (1977). *Essential Relativity*, (2nd ed.), Springer-Verlag.

[2] Mach, Ernst (1960). *The Science of Mechanics: A Critical and Historical Account of Its Development*. LaSalle: Open Court.

[3] Einstein, Albert (1912). “Gibt es eine Gravitationswirkung, die der elektrodynamischen Induktionswirkung analog ist?” *Vierteljahrsschrift für gerichtliche Medizin und öffentliches Sanitätswesen*. 44: 37-40.

[4] Barbour, Julian B. (1974). *Relative-Distance Machian Theories*, Nature 249: 328-329.

[5] Barbour, Julian B., Bertotti, Bruno (1977). *Gravity and Inertia in a Machian Framework*, Nuovo Cimento 38B: 1-27.

[6] Barbour, J., Pfister, H. (Eds.)(1995). *Mach’s Principle: From Newton’s Bucket to Quantum Gravity*, Birkhauser, Boston.

[7] Sciama, Dennis W. (1953). *The Origin of Inertia*, Monthly Notices of the Royal Astronomical Society 113:34-42.
[8] Brans, C., Dicke, R.H. (1961). *Mach’s Principle and a Relativistic Theory of Gravitation*, Phy. rev. 124: 925-935.

[9] Hoyle, F., Narlikar, J.V. (1964). *A New Theory of Gravitation*, Proceedings of the Royal Society, Ser. A 282: 191-207.