SEARCHING FOR DARK MATTER

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ABSTRACT

The observational evidence for dark matter on progressively larger cosmic scales is reviewed in a rather pedagogical fashion. Although the emphasis is on dark matter in galaxies and in clusters of galaxies, its cosmological evidence as well as its physical nature are also discussed.

1. Introduction

Almost all of our information about the Universe comes from electromagnetic radiation at different wavelengths. Because we obviously miss a lot of photons, the existence of dark matter should hardly be surprising. Clearly, the only way to infer its presence rests upon the resulting gravitational effects on luminous matter. This strategy has a long tradition in astronomy. For instance, in 1846 it led to the discovery of Neptune from unexplained residuals in the motion of Uranus. In a similar way, in 1933 Zwicky pointed out that the very existence of the Coma cluster of galaxies would be impossible unless its dynamics were dominated by dark matter. Regretfully, it took nearly four decades before Zwicky’s suggestion became a respectable research topic. Today, we know that dark matter largely outweights luminous mass. Besides, we know that most of the dark matter differs drastically from ordinary stuff. Ironically, while still waiting to know what dark matter really is, we know for sure that it is responsible for the formation of all structure in the Universe, and so ultimately for our existence as well.

This talk describes in a rather pedagogical manner the observational evidence for dark matter on progressively larger cosmic scales. Emphasis is given to dark matter in galaxies and in clusters of galaxies. The cosmological evidence and the physical nature of dark matter are discussed more briefly (these topics are addressed also by other speakers at this conference). Throughout, the conservative attitude is taken that the Universe is described by known physical laws. Considerably more attention is paid to basic physical principles than to observing techniques. In order to keep the number of references under control, we largely quote review papers, from which the original references can easily be traced.

2. Cosmological perspective

According to the hot big bang cosmology, the Universe is described by the Friedmann-Robertson-Walker model, which emerges from general relativity through
the Cosmological Principle. In terms of a set of comoving coordinates, the space-time line element can be written as
\[
ds^2 = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 - k r^2} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right],
\]
where \( R(t) \) is the cosmic scale factor and the constant \( k \) is proportional to the gaussian curvature of tridimensional space. Then it follows that tridimensional space is open for \( k < 0 \), flat (euclidean) for \( k = 0 \) and closed for \( k > 0 \).

The dynamics of the Universe is parametrized by the cosmic scale factor, which obeys the equations that arise by inserting the metric dictated by eq. (1) into Einstein equations, namely
\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{R^2}, \tag{2}
\]
\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right), \tag{3}
\]
where \( \rho \) and \( p \) denote the cosmic mass density and pressure (respectively). Observe that by combining eqs. (2) and (3) together, the following conservation equation arises
\[
\frac{d}{dt} \left( \rho R^3 \right) = -\frac{3p}{c^2} R^2 \dot{R}. \tag{4}
\]
What remains to be specified at this point is the equation of state of the Universe, namely the relationship between \( \rho \) and \( p \). It is useful to write such a relation in the form
\[
p = w \rho c^2, \tag{5}
\]
where \( w \) – which can be supposed constant – takes the values 0 for nonrelativistic matter, 1/3 for radiation and \(-1\) in the presence of a nonvanishing vacuum energy described by a cosmological constant. Notice that in the latter case, \( \rho \) and \( p \) have opposite signs. By inserting eq. (5) into eq. (4), we get
\[
\rho \sim R^{-3(1+w)}. \tag{6}
\]
Hence, \( \rho \sim R^{-3} \) for nonrelativistic matter, \( \rho \sim R^{-4} \) for radiation and \( \rho = \text{constant} \) for a cosmological constant.

Now, it is very convenient to define the Hubble parameter
\[
H \equiv \frac{\dot{R}}{R}, \tag{7}
\]
the critical density
\[
\rho_c \equiv \frac{3H^2}{8\pi G}, \tag{8}
\]
\footnote{More generally, \( \rho \) is the cosmic energy density divided by \( c^2 \).}
and the cosmic density parameter
\[ \Omega \equiv \frac{\rho}{\rho_c}, \tag{9} \]
in terms of which eq. (2) can be rewritten as
\[ \Omega = 1 + \frac{kc^2}{H^2R^2}. \tag{10} \]
Consequently, the knowledge of \( \Omega \) at a particular cosmic time – which may be taken to be just the present – is crucial to determine the geometry of the Universe. Indeed, recalling the connection between \( k \) and geometry, we find that the Universe is open for \( \Omega < 1 \), flat (euclidean) for \( \Omega = 1 \) and closed for \( \Omega > 1 \).

So far, no assumption has been made about \( \rho \) and \( p \). Suppose now that they are both positive, meaning that the Universe only contains ordinary matter and radiation (a cosmological constant is accordingly ruled out). Then the knowledge of \( \Omega \) also determines the evolution of the Universe uniquely. For, in such a situation \( \rho \) decreases faster than \( R^{-2} \) as \( R \) increases. So, by eq. (2), \( \dot{R} \) never vanishes as long as \( k \leq 0 \), whereas it does vanish at some cosmic time for \( k > 0 \). Therefore, an open or flat Universe (\( \Omega \leq 1 \)) expands forever, while a closed Universe (\( \Omega > 1 \)) eventually recollapses. Put in a slightly different fashion, geometry is destiny as long as the cosmological constant vanishes. Another crucial feature of a Universe without cosmological constant is that the evolution is necessarily decelerated. This immediately follows from eq. (3) and can be traced to the fact that ordinary gravity is always attractive.

Things are different in the presence of a cosmological constant and the above connection between geometry and evolution gets lost. In fact, both eternal expansion and recollapse can occur for any kind of geometry, depending on the actual value of the vacuum energy density \( \rho_\Lambda \) and of the energy density \( \rho_M \) of ordinary stuff (matter and radiation) \( \text{[2]} \). A striking manifestation of the cosmological constant is that the cosmic evolution can be accelerated. This comes about today \( b \) for \( \rho_\Lambda > 0 \) and \( \rho_M < 2 \rho_\Lambda \), owing to eq. (3). Accordingly, gravity becomes effectively repulsive on cosmic scales when the negative pressure of the vacuum dominates.

In the following, we will take for the present value of the Hubble parameter – the Hubble constant – \( H_0 \simeq 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Accordingly, the critical density is \( \rho_c \simeq 0.92 \cdot 10^{-29} \text{ g cm}^{-3} \simeq 1.36 \cdot 10^{11} \text{ M}_\odot \text{ Mpc}^{-3} \text{ c} \). Furthermore – for future needs – we introduce the notations
\[ \Omega_M \equiv \frac{\rho_M}{\rho_c}, \tag{11} \]
\[ \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_c}. \tag{12} \]

\( b \)In the present Universe, the energy density of radiation is negligible with respect to that of nonrelativistic matter.

\( c \)The subfix \( \odot \) denotes solar quantities. We recall that \( 1 \text{ pc} \simeq 3.1 \cdot 10^{18} \text{ cm} \).
It goes without saying that $\Omega_M$ and $\Omega_\Lambda$ represent the contributions to the cosmic density parameter from ordinary stuff and from the vacuum (respectively), and obviously
\[ \Omega = \Omega_M + \Omega_\Lambda. \]  
(13)

Finally, all contributions to the $\Omega$ parameter considered below refer to the present.

The main goal of the subsequent analysis is the observational determination of the contributions to the $\Omega$ parameter from the various cosmic structures.

A very useful concept for that purpose is the mass-to-light ratio $\Upsilon$ of a given astronomical object, having total mass $M$ and absolute optical luminosity $L$. More precisely, $\Upsilon$ is defined in solar units as
\[ \Upsilon \equiv \frac{M}{M_\odot} \left( \frac{L}{L_\odot} \right). \]  
(14)

Hence, $\Upsilon$ quantifies the total mass in terms of the emitted light. Occasionally, we will also be concerned with a similar quantity pertaining however to luminous mass and denoted by $\Upsilon_*$, but we shall always carefully state which mass is being referred to. Of course, the Sun has $\Upsilon = 1$ by definition. Because main-sequence stars are characterized by the relation $L \sim M^{3.5}$, bright $O$, $B$ stars have $\Upsilon \ll 1$, whereas red dwarfs have $\Upsilon \gg 1$. Models of stellar evolution in galaxies allow for the determination of the mass-to-light ratio $\Upsilon_{*,X}$ corresponding to luminous matter in galaxies of various types $X$ (ellipticals $E$, lenticulars $S0$, spirals $Sa$, $Sb$, $Sc$ and irregulars $Irr$). The resulting mean values are displayed in Table 1.

| $X$ | $\Upsilon_{*,X}$ |
|-----|-----------------|
| $E$  | 6.5             |
| $S0$ | 5               |
| $Sa$ | 3               |
| $Sb$ | 2               |
| $Sc$ | 1               |
| $Irr$ | 1              |

As an important preliminary step in the search for dark matter, we estimate the contribution to the $\Omega$ parameter from luminous matter in galaxies. Actually, it turns out that the whole optical luminosity of the Universe is produced by galaxies, and hence such a contribution pertains to all the luminous matter in the Universe. Let us begin by recalling that galaxy surveys allow for the determination of the luminosity function $\Phi(L)$, which gives the mean number density of galaxies per unit luminosity.

$d$Throughout, optical luminosity generally refers to the blue band.

*eWe warn the reader that – unless otherwise stated – luminous refers to optical luminosity.
Its analytic expression is

$$\Phi(L) = \frac{\Phi_*}{L_*} \left( \frac{L}{L_*} \right)^\alpha e^{-L/L_*},$$

(15)

where $\Phi_* \simeq 0.41 \cdot 10^{-3} \, Mpc^{-3}$, $L_* \simeq 2.53 \cdot 10^{10} \, L_\odot$ and $\alpha \simeq -1.25$. Because the whole optical luminosity of the Universe arises from galaxies, the mean cosmic luminosity density is

$$\mathcal{L} = \int_0^\infty dL \, L \, \Phi(L) \simeq 1.4 \cdot 10^8 \, L_\odot \, Mpc^{-3}.$$  

(16)

Observations also provide the fraction $\mathcal{F}_X$ of the cosmic luminosity produced by the galaxy population of type $X$. Their values are reported in Table 2.

| Type | $\mathcal{F}_X$ |
|------|----------------|
| $E$  | 0.11           |
| $S_0$| 0.21           |
| $S_a$| 0.28           |
| $S_b$| 0.29           |
| $S_c$| 0.05           |
| $Irr$| 0.06           |

Hence, the corresponding mean luminosity density is

$$\mathcal{L}_X = \mathcal{F}_X \, \mathcal{L} \simeq 1.4 \cdot 10^8 \, \mathcal{F}_X \, L_\odot \, Mpc^{-3}.$$  

(17)

We can convert $\mathcal{L}_X$ into the mean cosmic density $\rho_{*,X}$ of luminous mass contained in the galaxy population $X$ by the mass-to-light ratio $\Upsilon_{*,X}$, thereby getting

$$\rho_{*,X} = \mathcal{L}_X \, \Upsilon_{*,X} \, M_\odot \, Mpc^{-3} \simeq 1.4 \cdot 10^8 \, \mathcal{F}_X \, \Upsilon_{*,X} \, L_\odot \, Mpc^{-3}.$$  

(18)

Consequently, the resulting contribution to $\Omega$ is

$$\Omega_{*,X} \equiv \frac{\rho_{*,X}}{\rho_c} \simeq 1.03 \cdot 10^{-3} \, \mathcal{F}_X \, \Upsilon_{*,X},$$  

(19)

whose values – for the various galaxy populations – are listed in Table 3.

| Type | $\Omega_{*,X}$ |
|------|---------------|
| $E$  | 0.73 $\cdot$ 10$^{-3}$ |
| $S_0$| 1.08 $\cdot$ 10$^{-3}$ |
| $S_a$| 0.87 $\cdot$ 10$^{-3}$ |
| $S_b$| 0.59 $\cdot$ 10$^{-3}$ |
| $S_c$| 0.05 $\cdot$ 10$^{-3}$ |
| $Irr$| 0.06 $\cdot$ 10$^{-3}$ |
Thus, we come to the conclusion that the contribution from all the luminous mass in the Universe is

\[ \Omega_\ast = \sum_X \Omega_{\ast,X} \simeq 0.004 \, . \]  

(20)

As a matter of fact, the above argument – relating the mass-to-light ratio of a galaxy population \( \Upsilon_{\ast,X} \) to the corresponding contribution to the cosmic density parameter \( \Omega_{\ast,X} \) – remains true even if the mass-to-light ratio \( \Upsilon_X \) is considered, which refers to all galactic mass and not just to the luminous component. Accordingly, the counterpart of eq. (19) yields the contribution \( \Omega_X \) from the galaxy population \( X \).

Explicitly

\[ \Omega_X \simeq 1.03 \cdot 10^{-3} \mathcal{F}_X \Upsilon_X \, . \]  

(21)

Of course, at this stage \( \Upsilon_X \) is unknown, owing to the existence of dark matter in galaxies. Nevertheless – thanks to eq. (21) – the determination of \( \Upsilon_X \) (to be discussed in Sect. 3) will allow us to quantify the cosmological relevance of a given galaxy population.

3. Astrophysical evidence of dark matter

This Section is the core of the present review and addresses the observational evidence for dark matter in galaxies and in clusters of galaxies.

3.1. Spiral galaxies

Stars in spiral galaxies are mainly contained – along with cold neutral hydrogen HI clouds – in a thin disk characterized by an exponential surface brightness profile

\[ I(R) \sim e^{-R/R_d} \, , \]  

(22)

where \( R \) is the galactocentric distance and \( R_d \) denotes the disk scale length. Typically, one finds \( R_d \simeq 2 - 4 \) kpc and the disk optical radius turns out to be \( R_{\text{opt}} \simeq 4 \) \( R_d \). Stars and cold HI clouds travel on nearly circular orbits around the galactic centre with velocity \( v_c(R) \), and so their centripetal acceleration equals the gravitational one

\[ \frac{v_c^2(R)}{R} = - g_R(R,0) \, . \]  

(23)

Hence, the rotation curve – namely the graph of \( v_c(R) \) versus \( R \) – traces the gravitational acceleration in the disk \( g_R(R,0) \). This fact lies at the basis of the best strategy to discover dark matter in spiral galaxies.

Basically, a rotation curve is constructed by measuring the circular velocity – at different values of the radius – by the Doppler shift of certain spectral lines in either the optical or the radio band of the emitted galactic radiation. The observed rotation
curve of a given spiral is then compared with the one produced by luminous matter alone: a discrepancy would be a clear signal of dark matter.

Let us consider the main steps of this procedure in some detail. In spite of the fact that it is virtually impossible to find identical rotation curves, it turns out that they all share the same qualitative behaviour: observations show that they rise linearly in the inner region until a maximum is reached near \( R \simeq 2 R_d \), beyond which they stay flat out to the last measured point. Schematically

\[
v_c(R) \sim \begin{cases} 
  R, & R < R_d \\
  \text{constant}, & R > 3 R_d .
\end{cases}
\]  

(24)

As we said, such a rotation curve has to be compared with the one arising solely from luminous matter, whose evaluation proceeds as follows. In the first place, the surface brightness profile – as given by eq. (22) with \( R_d \) fixed by a photometric fit – has to be converted into the disk surface density profile \( \Sigma(R) \). Because colour and luminosity gradients in spiral disks are generally modest, it makes sense to suppose that the disk mass-to-light ratio \( \Upsilon_d \) is constant. Accordingly, one gets \( \Sigma(R) \sim \Upsilon_d I(R) \), and so eq. (22) entails

\[
\Sigma(R) \sim e^{-R/R_d} ,
\]  

(25)

It can next be shown that this mass distribution produces the following rotation curve

\[
v_c(R) \sim \left[ I_0 \left( \frac{R}{2 R_d} \right) K_0 \left( \frac{R}{2 R_d} \right) - I_1 \left( \frac{R}{2 R_d} \right) K_1 \left( \frac{R}{2 R_d} \right) \right]^{1/2} R ,
\]  

(26)

where \( I_0(\cdot), I_1(\cdot), K_0(\cdot) \) and \( K_1(\cdot) \) are modified Bessel functions [5]. Although eq. (26) looks complicated, its qualitative behaviour is very simple: a linear rise in the inner region continues until a maximum is reached near \( R \simeq 2 R_d \), which is followed by a keplerian fall-off at larger galactocentric distances. To a good approximation, eq. (26) can be rewritten as

\[
v_c(R) \sim \begin{cases} 
  R, & R < R_d \\
  R^{-1/2}, & R > 3 R_d .
\end{cases}
\]  

(27)

Let us now compare eqs. (24) and (27). As far as the inner region \( R < R_d \) is concerned, the agreement is good, thereby implying that luminous matter is the whole story. But in the outer region \( R > 3 R_d \) the disagreement is dramatic, with the circular velocity systematically larger than expected on the basis of luminous matter alone. Actually, a larger \( v_c \) (for fixed \( R \)) implies by eq. (23) a larger \( g_R \), which entails in turn by Poisson equation (see below) a larger mass density \( \rho \). Consequently, dark matter must lurk at galactocentric distances \( R > 3 R_d \).

Clearly, the flat behaviour of the observed rotation curves provides solid evidence that the outer region of spiral galaxies is dominated by dark matter. This turns out to be a universal properties of spiral galaxies.
Yet, the actual shape of the dark matter distribution cannot be unambiguously determined from the rotation curve alone. This is true even under the simplifying assumption that such a distribution – just like the one of luminous matter – is axisymmetric about the galaxy rotation axis. Employing cylindrical coordinates \((R, \phi, z)\), the gravitational acceleration in a generic point \(g(R, z)\) is related to the mass density \(\rho(R, z)\) by Poisson equation

\[
\frac{1}{R} \frac{\partial}{\partial R} \left( R g_R(R, z) \right) + \frac{\partial g_z(R, z)}{\partial z} = -4\pi G \rho(R, z) .
\] (28)

Owing to eq. (23), the rotation curve merely fixes \(g_R(R, 0)\), and so the lack of knowledge about \(g_R(R, z)\) with \(z \neq 0\) and \(g_z(R, z)\) prevents the unique determination of \(\rho(R, z)\). Only by assuming that the dark matter distribution has spherical symmetry (about the galaxy centre) can the dark matter density profile \(\rho(r)\) be uniquely derived from the rotation curve. Indeed, now the Poisson equation reads

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 g_r(r) \right) = -4\pi G \rho .
\] (29)

So, upon integration we get

\[
g_r(r) = -\frac{G M(< r)}{r^2} ,
\] (30)

where

\[
M(< r) \equiv 4\pi \int_0^r dr' r'^2 \rho(r')
\] (31)

denotes the integrated mass profile, namely the total mass inside the sphere of radius \(r\). Combining eqs. (23) and (30) together, we find

\[
M(< r) \sim v_c^2(r) r .
\] (32)

Still, in the region where the observed rotation curve is flat eq. (32) becomes

\[
M(< r) \sim r ,
\] (33)

and so eq. (31) implies

\[
\rho(r) \sim r^{-2} ,
\] (34)

which is the density profile of a singular isothermal sphere (SIS). Thus, we come to the conclusion that spiral galaxies are surrounded by a SIS halo dominated by dark matter.

Notationally, we denote by \(r\) or \(R\) the radius of a sphere, while \(R\) stands for the radius of a circle.

A SIS is a self-gravitating spherical model with diagonal pressure tensor and velocity dispersion independent of position.

However, one should not forget that such a conclusion rests upon the simple but unproved assumption of spherical symmetry.
The systematic analysis of spiral galaxy rotation curves started nearly twenty years ago, both in the optical and in the radio band, where the 21 cm emission line of HI is used.

Early optical studies found the mass-to-light ratio $\Upsilon_{\text{opt}}$ pertaining to a sphere with radius $R_{\text{opt}}$ systematically exceeds by roughly a factor of 2 the mass-to-light ratio $\Upsilon_*$ of the luminous mass $M_*$. Because $\Upsilon_{\text{opt}}/\Upsilon_* = M(< R_{\text{opt}})/M_*$, it follows

$$M_{\text{dark}}(< R_{\text{opt}}) \simeq M_* ,$$

meaning that the optical region of a spiral contains roughly equal amounts of luminous and dark matter.

However, such a conclusion should be understood more like a suggestion than a real proof. For, the optical method is (just by definition) bound to probe the region $R < 4 R_d$ (recall that $R_{\text{opt}} \simeq 4 R_d$), and in fact the above analysis has been carried out up to $R \simeq 3.5 R_d$. Still, eq. (26) implies that at $R \simeq 3.5 R_d$ the circular velocity of luminous matter has decreased only by 8% relative to its maximum at $R \simeq 2 R_d$. So, it is very difficult to rule out the keplerian fall-off by restricting the attention to such a narrow range of galactocentric distances.

Remarkably enough, radio observations resolve the issue. Indeed, HI clouds typically extend out to twice the optical radius, thereby allowing for the determination of $v_c(R)$ up to $R \simeq 8 R_d$. In this way, the existence of dark matter can be established even outside the optical region, where it actually dominates the mass distribution. A beautiful example concerns the Sc spiral galaxy NGC3198, whose rotation curve has been mapped out to $R \simeq 10 R_d \simeq 30 \text{ kpc}$. The resulting mass-to-light ratio is $\Upsilon \simeq 18$. We know from Table 1 that for an Sc spiral the mass-to-light ratio of luminous matter is $\Upsilon_* ,\text{Sc} \simeq 1$, and so the following result emerges

$$M_{\text{dark}}(< 30 \text{ kpc}) \simeq 17 M_* ,$$

with $M_*$ denoting the luminous mass.

At still larger galactocentric distances, no stars or cold HI clouds are present. Therefore, tracers of a different kind have to be identified in order to probe the mass profile of dark halos. Several bright spirals happen to possess fainter – and presumably less massive – satellite galaxies (similarly to the case of our Milky Way, which has the two Magellanic Clouds as satellite galaxies). This fact offers the possibility to investigate the gravitational field of the primary galaxy through the dynamical behaviour of the satellites. Of course, their orbital period is by far too long to observe a significant portion of the orbit, and so the ensemble of satellites has to be handled in a statistical fashion. This strategy has been applied to a sample of 115 satellites

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1. This line arises from the hyperfine transitions in the ground state. Notice that the higher level is populated because the temperature of the interstellar medium is definitely larger than the energy gap between the hyperfine levels.
around 69 primaries, having mean luminosity $L \simeq 2 \cdot 10^{10} L_\odot$. We stress that the underlying philosophy is to suppose that the primaries are sufficiently similar that the satellites can be treated as orbiting a *single* (typical) galaxy, thereby significantly enhancing the statistical relevance of the satellite sample. This analysis entails that dark halos extend beyond $R \simeq 200 \, \text{kpc}$, with

$$M(<200 \, \text{kpc}) \simeq 2 \cdot 10^{12} M_\odot.$$ (37)

Then the corresponding mass-to-light ratio is $\Upsilon \simeq 100$. Because this relation can be regarded as *typical* for spiral galaxies, we can state that the mean mass-to-light ratio of these galaxies is

$$\Upsilon_S \simeq 100.$$ (38)

Recalling the values quoted in Table 1, it follows that any spiral obeys the condition

$$M_{\text{dark}} > 30 \, M_*,$$ (39)

thereby implying that all spiral galaxies are totally *dominated* by dark matter.

A comparison among observations at different galactocentric distances of the same spiral and for different spirals yields

$$\Upsilon_S(r) \simeq 60 \left( \frac{r}{100 \, \text{kpc}} \right).$$ (40)

Let us finally address the *cosmological relevance* of spiral galaxies. We consider first the contribution to the $\Omega$ parameter from *luminous* matter alone. Recalling the relevant values listed in Table 3, we get

$$\Omega_{*,S} \simeq 1.5 \cdot 10^{-3}. $$ (41)

Because of the presence of dark matter, the total contribution to $\Omega$ turns out to be much larger. By combining eq. (21) and (38) together and using the relevant values reported in Table 2, we find

$$\Omega_S \simeq 1.03 \cdot 10^{-1} \left( \mathcal{F}_{Sa} + \mathcal{F}_{Sb} + \mathcal{F}_{Sc} \right) \simeq 6.4 \cdot 10^{-2}. $$ (42)

### 3.2. Elliptical galaxies

Luminous matter in elliptical galaxies has a *spheroidal* distribution, well described by the *De Vaucouleurs* surface brightness profile

$$I(R) \sim \exp \left\{ -7.67 \left[ (R/R_e)^{1/4} - 1 \right] \right\},$$ (43)

where $R$ is the galactocentric distance and $R_e$ is the *effective radius* (typically $R_e \simeq 3 - 5 \, \text{kpc}$). Moreover, the star motion in ellipticals is highly chaotic, with velocity...
dispersions usually as large as velocities themselves. Manifestly, in such a situation a rotation curve provides no information about the galactic gravitational field, and so different techniques have to be devised to look for dark matter in elliptical galaxies.

A classic approach rests upon the dynamical analysis of stellar motion and can be summarized as follows. Any specific stellar population of ellipticals can be thought of as a collisionless fluid in a steady state, resulting from the balance between the kinetic pressure – brought about by the above-mentioned chaotic motion – and the overall gravitational field. Assuming spherical symmetry and denoting by \( r \) the galactocentric distance, it can be shown that the star number density profile \( n_s(r) \) obeys the following equation:

\[
\frac{d}{dr} \left( n_s \sigma_r^2 \right) + \frac{2 n_s A \sigma_r^2}{r} + \frac{GM(<r) n_s}{r^2} = 0 .
\] (44)

This is just Euler equation for a fluid with nondiagonal pressure tensor parametrized by the radial velocity dispersion \( \sigma_r \) and the anisotropy function \( A(r) \). We emphasize that \( M(<r) \) – as defined by eq. (31) – pertains to the total galactic mass (responsible for the overall gravitational field). It is very easy to see that eq. (44) can be rewritten in the form:

\[
M(<r) = -\frac{\sigma_r^2}{G} \left( \frac{d \ln n_s}{d \ln r} + \frac{d \ln \sigma_r^2}{d \ln r} + 2 A \right) r .
\] (45)

So, we can find the overall integrated mass profile of the elliptical in question provided that we succeed in determining the three functions \( n_s(r) \), \( \sigma_r(r) \) and \( A(r) \). From a conceptual point of view, \( M(<r) \) plays here the same rôle as \( v_c(R) \) did for spiral galaxies: a discrepancy between \( M(<r) \) and the integrated mass profile of luminous matter alone would provide positive evidence for dark matter. Unfortunately, only the surface brightness profile on the sky \( I(R) \) and the velocity dispersion profile along the line-of-sight \( \sigma_{||}(R) \) are the available observables, and so there is not enough information to uniquely fix the unknown functions \( n_s(r) \), \( \sigma_r(r) \) and \( A(r) \). As a result, \( M(<r) \) can be determined only by making some assumption on the functional form of \( A(r) \), as suggested by models of galaxy formation. For a long time, instrumental limitations prevented the application of such a dynamical analysis to a tracer population having a sufficiently large galactocentric distance, thereby severely hindering its effectiveness. But in the last few years the situation has considerably improved and today globular clusters and planetary nebulae can be mapped out to \( r \simeq 6 R_e \). As far as dark matter is concerned, the above dynamical analysis leads to a result which strongly depends on the specific elliptical that is being considered. In some cases, there is no evidence for dark matter out to \( r \simeq 4 R_e \), whereas is other cases one typically gets \( \Upsilon_E \simeq 10 - 15 \). Recalling from Table 1 that \( \Upsilon_{*E} \simeq 6.5 \), it follows

\[
M_{dark}(<4 R_e) \simeq M_* ,
\] (46)

where again \( M_* \) denotes the luminous mass. Quite recently, this conclusion has been confirmed by a totally different technique, namely by the observation of strong
gravitational lensing of a background galaxy produced by the elliptical MG2016 + 112. Thus – analogously to what happens for spiral galaxies – also the optical region of ellipticals contains roughly equal amounts of luminous and dark matter. However, there are several exceptions to this statement (which is not the case for spiral galaxies).

Bright elliptical galaxies generally contain a sizable amount of hot ionized gas at temperature $T_g \simeq 3 \cdot 10^6 - 1 \cdot 10^7$ K, which gives rise – by thermal Bremsstrahlung (free-free transitions) – to an X-ray emission with luminosity $L_X \simeq 10^{39} - 10^{42}$ erg s$^{-1}$. Such an X-ray emission is considerably more diffuse than optical light, implying that the gas distribution extends out to $r \simeq 20 - 80$ kpc. Quite remarkably, the previous method can be applied – without the above-mentioned shortcomings – to this gas, whose pressure tensor is necessarily diagonal (because of the Pascal law). Consequently, the anisotropy function $A(r)$ vanishes identically and eq. (45) becomes

$$M(<r) = - \frac{k_B T_g}{G m} \left( \frac{d \ln n_g}{d \ln r} + \frac{d \ln T_g}{d \ln r} \right) r,$$

where $n_g(r)$ and $T_g(r)$ are the gas density and temperature profiles (respectively), while $m \simeq 0.6 m_p$ denotes the mean particle mass of the gas. Of course, eq. (47) rests upon the assumption – which will be justified later on – that the gas is in hydrostatic equilibrium.

The present strategy can be implemented as follows. X-ray observations – performed by satellite-borne detectors – provide the surface brightness profile of a given bright elliptical. An excellent fit to the data is achieved by the analytic expression

$$I_X(R) \simeq \left[ 1 + \left( \frac{R}{a_X} \right)^2 \right]^{-3 \beta + 1/2},$$

where typically $\beta \simeq 0.4 - 1.0$ and $a_X \simeq 1 - 9$ kpc. Upon deprojection, eq. (48) yields the X-ray luminosity density

$$j_X(r) \sim \left[ 1 + \left( \frac{r}{a_X} \right)^2 \right]^{-3 \beta}.$$

Determining the gas temperature profile is a much more difficult job, but to a good approximation it can be assumed that the gas distribution is isothermal. Because the X-ray emission is due to Bremsstrahlung, the luminosity density $j_X(r)$ goes like the square of the gas number density $n_g(r)$, and so from eq. (49) we get

$$n_g(r) \sim \left[ 1 + \left( \frac{r}{a_X} \right)^2 \right]^{-3 \beta/2}.$$

\[ j \]This phenomenon will be discussed later on.
which – upon substitution into eq. (47) – gives

\[ M(< r) = \frac{3\beta k_B T_g}{G m} \frac{(r/a_X)^2}{1 + (r/a_X)^2} r . \]  

(51)

What is the physical meaning of this result? We pointed out that typically one finds \( a_X < 10 \text{ kpc} \). Thus, in the region \( r > 10 \text{ kpc} \) – which corresponds to the galactic halo – eq. (51) takes the form

\[ M(< r) \sim r , \]  

(52)

from which – thanks to eq. (31) – the corresponding overall density profile is immediately deduced and reads

\[ \rho(r) \sim r^{-2} . \]  

(53)

We already encountered this expression and we know that it describes a singular isothermal sphere (SIS). So, we see that bright ellipticals possess a SIS halo\(^k\).

What is such an halo made of? In order to settle this issue, the integrated mass profile of the gas \( M_g(< r) \) has to be computed and compared with \( M(< r) \) as given by eq. (51), for the observed mean values of the relevant parameters \( T_g \simeq 7.8 \cdot 10^6 \text{ K} \), \( \beta \simeq 0.5 \) and \( a_X \simeq 5 \text{ kpc} \). Once again, a discrepancy between \( M_g(< r) \) and \( M(< r) \) would signal the presence of dark matter. An explicit calculation yields

\[ M_g(< r) \simeq 6.3 \cdot 10^7 \left( \frac{r}{\text{kpc}} \right)^{1.5} M_\odot , \]  

(54)

and

\[ M(< r) \simeq 3.6 \cdot 10^{10} \left( \frac{r}{\text{kpc}} \right) M_\odot . \]  

(55)

Thus, we conclude that bright elliptical galaxies are surrounded by a SIS halo dominated by dark matter. We stress that – in spite of the great difference in their optical properties – bright ellipticals and spirals are qualitatively identical as far as dark matter is concerned. Moreover, assuming \( L \simeq 2 \cdot 10^{10} L_\odot \) for the mean optical luminosity of bright ellipticals, their mean mass-to-light ratio resulting from eq. (55) is

\[ \Upsilon_E(r) \simeq 180 \left( \frac{r}{100 \text{ kpc}} \right) . \]  

(56)

Comparing eqs. (51) and (56), we see that – for equal values of radius and luminosity – bright ellipticals contain roughly 3 times more dark matter than spirals.

\(^k\)This conclusion rests on the hydrostatic equilibrium assumption, which can be justified as follows. Basically, the dynamical behaviour of the hot gas depends on its ability to get rid of its thermal energy and is determined by the competition between the cooling time \( t_{\text{cool}} \) and the free-fall time \( t_{\text{ff}} \). When \( t_{\text{cool}} < t_{\text{ff}} \), cooling occurs efficiently and the gas collapses toward the galactic centre. On the other hand, for \( t_{\text{cool}} > t_{\text{ff}} \) the gas stays in hydrostatic equilibrium. It can be shown that – for the observed values of the relevant parameters (see below) – just the latter situation is actually realized outside the central region of a bright elliptical.
How big are the halos of elliptical galaxies? Preliminary observations based on statistical gravitational lensing show – though with large uncertainties – that bright ellipticals have halos similar to those of spirals, which seem to be consistent with eq. (56). Hence, the resulting mean value of the mass-to-light ratio for these galaxies is

$$\Upsilon_{E,b} \simeq 300 .$$ (57)

Combining this result with $\Upsilon_{*,E} \simeq 6.5$ (from Table 1), we get

$$M_{dark} \simeq 45 M_*$$ (58)

which implies that bright elliptical galaxies are totally dominated by dark matter. As already emphasized, a similar clear-cut statement cannot be made for fainter ellipticals.

Let us finally consider the cosmological relevance of elliptical galaxies. As far as their luminous matter is concerned, we recall from Table 3 that the contribution to the $\Omega$ parameter is

$$\Omega_{*,E} \simeq 0.73 \cdot 10^{-3} .$$ (59)

Obviously, the presence of dark matter makes the total contribution to the $\Omega$ parameter considerably larger, but the lack of knowledge about the relevance of dark matter in fainter ellipticals prevents a reliable estimate. The best we can do is to derive the upper bound

$$\Omega_E < 6.5 \cdot 10^{-2} ,$$ (60)

which rests upon eqs. (21) and (57).

3.3. Clusters of galaxies

Galaxies are not randomly distributed throughout the Universe, but tend to aggregate on the Megaparsec scale. Rich clusters contain $30 - 300$ galaxies inside a sphere of Abell radius, conventionally defined as $R_A \simeq 2.1$ Mpc. The resulting optical luminosity is $L(< R_A) \simeq 1.2 \cdot 10^{13} - 1.2 \cdot 10^{14} L_\odot$. Regular clusters typically show a centrally condensed region and are characterized by spherical symmetry, whereas irregular clusters have no characteristic shape.

As already pointed out, the first evidence for a large amount of dark matter in the Universe came from the virial analysis of the Coma cluster by Zwicky in 1933. This strategy has since been applied to many regular clusters and can be summarized as follows. Any isolated self-gravitating system reaches an equilibrium state, in which gravity is balanced by kinetic pressure. Accordingly, the potential energy $U$ is related to the kinetic energy $K$ by the virial theorem

$$2K + U = 0 .$$ (61)
Assuming spherical symmetry, the potential energy can be represented as

$$U = -\frac{\alpha GM^2}{R}, \quad (62)$$

where $M$ is the total mass, $R$ is the radius of the system and $\alpha$ is a constant which reflects the actual density profile. Because the kinetic energy can be written – in terms of the mean-square velocity $\langle v^2 \rangle$ – as

$$K = \frac{1}{2} M \langle v^2 \rangle, \quad (63)$$

eq. (61) yields

$$M = \frac{\langle v^2 \rangle R}{\alpha G}. \quad (64)$$

However, the application of eq. (64) to regular clusters is not as straightforward as it might seem. For, the cluster density profile is unknown, and so the constant $\alpha$ in eq. (64) gives rise to an uncertainty in the mass determination. Another source of uncertainty is due to the fact that real clusters do not possess a sharp edge, making $R$ an ill-defined quantity. A way out of both difficulties is provided by the following alternative version of eq. (64)

$$M = \frac{\langle v^2 \rangle R_M}{\alpha G}, \quad (65)$$

where $R_M$ is the mean effective radius, a quantity which is observationally well-defined in terms of galaxy counts. Notice that $\alpha$ has disappeared from eq. (65). Still, an additional uncertainty in the estimated mass arises through the determination of $\langle v^2 \rangle$ for the cluster galaxies. For, what is really measured is the one-dimensional galaxy velocity dispersion $\sigma$ along the line of sight, but there is no way to tell how $\sigma$ is related to $\langle v^2 \rangle$ in the general case. So, it is usually assumed that the galaxy velocity distribution in regular clusters is isotropic, in which case $\langle v^2 \rangle = 3\sigma^2$. In spite of this and other uncertainties, it is widely believed that the virial mass estimates for regular clusters are fairly good. A sample of values of the mass-to-light ratio $\Upsilon_{RC}$ for regular clusters derived by the virial theorem is reported in Table 4.

Table 4
Clusters of galaxies contain a large amount of hot ionized gas at temperature \( T_g \simeq 1 \cdot 10^7 - 1.5 \cdot 10^8 \) K, which produces an X-ray emission with luminosity \( L_X \simeq 6 \cdot 10^{42} - 2 \cdot 10^{45} \text{ erg s}^{-1} \) by thermal Bremsstrahlung (free-free transitions)\(^{20}\). Moreover, the X-ray emission is more diffuse than optical luminosity, entailing that the gas distribution extends well beyond the Abell radius. Manifestly, the situation is largely analogous to what happens in bright elliptical galaxies, and so an analysis quite similar to the one considered in the previous Subsection can be carried out for regular clusters as well. Even in this case the hydrostatic equilibrium assumption outside the central region turns out to be justified. Furthermore, isothermality is a good first approximation. Therefore, by just repeating the same steps as before the resulting cluster integrated mass profile is

\[
M(< r) = \frac{3\beta k_B T_g}{GM} \frac{(r/a_X)^2}{1 + (r/a_X)^2} r, \tag{66}
\]

where the observed mean values of the relevant parameters are now \( T_g \simeq 5 \cdot 10^7 \) K, \( \beta \simeq 0.7 \) and \( a_X \simeq 0.3 \text{ Mpc} \). Because one typically finds \( a_X < 0.5 \text{ Mpc} \), in the region \( r > 0.5 \text{ Mpc} \) eq. (66) becomes

\[
M(< r) \sim r. \tag{67}
\]

We know that the corresponding overall density profile is

\[
\rho(r) \sim r^{-2}, \tag{68}
\]

which describes a singular isothermal sphere (SIS). Quite remarkably, observations show that not only spiral and bright elliptical galaxies but also regular clusters of galaxies are surrounded by a SIS halo.
While the gas is obviously a constituent of the SIS cluster halo, a nontrivial contribution from dark matter is present. Indeed, we know that dark matter dominates galaxies, and so it necessarily lurks inside regular clusters. Moreover, some further dark matter can exist in the intracluster space. So, the real question is whether hot gas or dark matter dominates the cluster mass budget. It goes without saying that this issue can be resolved by evaluating the gas mass fraction

\[ f_g(r) \equiv \frac{M_g(< r)}{M(< r)} \quad (\text{69}) \]

for the above mean values of the parameters involved. It turns out that \( f_g \) is independent of \( r \) (this is simply due to \( \beta \approx 0.7 \)) and we get

\[ f_g \approx 0.12 \quad (\text{70}) \]

In addition, the total gas mass comes out invariably larger than the luminous mass of the cluster galaxies. Thus, we conclude that also regular clusters of galaxies are dominated by dark matter.

Besides leading to values of the mass-to-light ratio for regular clusters which are consistent with those previously found from the virial analysis, this method also has a different implication. We will discuss in the next Section the nature of the dark matter and for now we merely allow for the existence of nonbaryonic dark matter. Because regular clusters are so extended and practically nothing escapes from them during their evolution, it seems natural to suppose that they just reflect the mean composition of the whole Universe \[ (\text{19}) \]. As a consequence, the cluster baryon fraction \( f_B \) should equal the cosmic baryon fraction

\[ f_B \approx \frac{\Omega_B}{\Omega_M} \quad (\text{71}) \]

where \( \Omega_B \) denotes the baryonic contribution to the cosmic density parameter. Once luminous matter in galaxies as well as possible baryonic dark matter are taken into account, eq. (70) implies

\[ f_B \geq 0.17 \quad (\text{72}) \]

Combining eqs. (71) and (72) together, we find

\[ \Omega_M \leq 5.9 \Omega_B \quad (\text{73}) \]

We will use this result in Sect. 4.

Coming back to clusters of galaxies, another powerful tool to discover the presence of dark matter is gravitational lensing \[ (\text{21}) \], namely the distortion of light rays when they pass close to a mass clump. Clusters act as gravitational lenses, which can magnify, distort and multiply the images of background galaxies. A careful study
of the properties of these images provides informations about the mass distribution inside the lens, and so ultimately on the existence of dark matter in clusters.

We begin by stressing that in a typical situation the distances background galaxy-cluster and cluster-observer are much larger than the size of the cluster itself. Accordingly, clusters behave as thin lenses, which means that lensing effects do not depend on the tridimensional mass distribution but rather on the surface density $\Sigma(R)$ on the sky. So, it is this quantity that can be derived from observations.

**Strong lensing** – This phenomenon concerns only regular clusters and is characterized by the existence of giant arcs inside the image of the cluster. Let us now discuss how these arcs arise and how they can be used to estimate the cluster mass.

In the first place, we recall that regular clusters possess spherical symmetry, namely axial symmetry about the line of sight to their centre. In such a situation, it can be shown that the *lens equation* takes the form

$$Y \sim \left(1 - \frac{m(< R)}{\pi R^2 \Sigma_{cr}}\right) R,$$

where $m(< R)$ denotes the mass inside the cylinder of radius $R$ about the line of sight

$$m(< R) \equiv 2\pi \int_0^R dR' R' \Sigma(R'),$$

while $\Sigma_{cr}$ stands for a reference value for the lens surface density which is completely fixed by the lens (cluster) and source (background galaxy) distances. In order to understand the meaning of eq. (74), consider two planes orthogonal to the line of sight to the cluster centre: the *lens plane* and the *source plane*, whose meaning is clear from their names. Accordingly, $Y$ denotes a generic point $S$ in the source plane, whereas $R$ represents the *image* $I$ of $S$ in the lens plane. Because of gravitational lensing, $S$ and $I$ are misaligned and $I$ is the point actually seen by the observer. It can also be shown that the *magnification* of the image is

$$\mu \sim \left(1 - \frac{m(< R)}{\pi R^2 \Sigma_{cr}}\right)^{-1}.$$

What is the image of that particular point $Y = 0$ in which the line of sight intersects the source plane? By eq. (74), we find that such an image is a whole circle – the *Einstein ring* – defined by the condition

$$m(< R) = \pi R^2 \Sigma_{cr}.$$  

Moreover – owing to eq. (76) – this image has infinite magnification. Although such a divergence is merely an artifact of the geometric-optics approximation upon which

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1Both $Y$ and $R$ are measured from the point in which the line of sight intersects the corresponding plane.

2Obviously, the no-lensing solution $R = 0$ is here unphysical and has to be discarded.
eq. (76) is based, the real magnification is nevertheless quite large. This circumstance qualifies the point \( Y = 0 \) as a (degenerate) caustic. Hence, the image of a point source located just on the caustic is the Einstein ring. For an extended source close to the caustic, the situation is somewhat similar: the highly magnified image consists of two elongated arcs, which are sectors of the Einstein ring located on opposite sides. But even a tiny perturbation of the axial symmetry leads to a strong demagnification of one of these arcs, and so a single arc becomes the observational signature.

We are now in position to come back to the giant arcs seen in clusters of galaxies. It has been shown that their only physically consistent explanation is the above-discussed phenomenon, arising when a background galaxy lies occasionally close to the caustic, namely when it happens to be almost aligned with the cluster centre. Because it is easy to determine observationally both the radius of an arc and the distances of cluster and background galaxy \(^a\), such an interpretation leads – by eq. (77) – to an estimate of the cluster mass enclosed by the corresponding Einstein ring.

Let us address the main limitations of this technique. First of all, the formation of giant arcs requires the existence of the caustic, which is a characteristic feature of the strong lensing regime. A necessary and usually sufficient condition for this to occur is that in at least one point of the lens the surface density exceeds \( \Sigma_{cr} \) and it turns out that only regular clusters are dense enough to meet this constraint \(^b\). In addition, an almost perfect alignment observer-cluster-background galaxy is required, which is clearly a rather unlikely situation. Finally, only the cluster mass inside the Einstein ring can be estimated in this way.

Weak lensing \(^c\) – Every cluster produces weakly distorted images of all background galaxies that lie sufficiently close to its position on the sky. Because lensing compresses the image in one direction while stretching it in the orthogonal direction, the observed lensed images of background galaxies are called arclets. Were galaxies perfectly round, the ellipticities of their arclets would tell us how strong the gravitational field is at every arclet position, from which the cluster mass can be derived. In reality, the unlensed image a generic galaxy has a nonvanishing ellipticity, which depends on the unknown galaxy orientation. So, the knowledge of the ellipticity of the corresponding arclet is useless. Still, suppose to contemplate many background galaxies at once. Then the average source ellipticity vanishes, since the individual ellipticities are manifestly randomly oriented and uncorrelated. Therefore, the average ellipticity of the corresponding arclets quantifies the weak lensing effect. What makes this technique effective is the existence of the so-called Tyson population of faint blue galaxies with a surprisingly high surface number density, so that even a small patch

\(^a\)In reality, background galaxies are too faint for their redshifts to be measured, but these quantities can be measured for their highly magnified images. Because gravitational lensing does not cause any frequency shift, the redshift of an arc yields the distance of the source through Hubble expansion law.

\(^b\)Equivalently – once \( \Sigma(R) \) and \( \Sigma_{cr} \) are given – eq. (77) can be viewed as a condition for the formation of giant arcs. Only regular clusters fulfil such a condition.
of the sky – over which $\Sigma(R)$ can be taken as constant – is densely filled by them. Consequently, a map of the surface density $\Sigma(R)$ on the sky can be derived from arclet observations. In spite of its conceptual simplicity, weak lensing is technically rather complex and will not be considered here any further.

We stress that – at variance with the previous methods – the mass determination based on gravitational lensing does not require any assumption about the dynamical state of the cluster. It is gratifying that even this strategy yields values of the mass-to-light ratio which generally agree with those derived by the virial analysis and $X$-ray studies.

Specifically, the application of the above-discussed techniques to regular clusters gives values lying around the mean

$$\Upsilon_{RC} \simeq 210.$$ (78)

Because regular clusters contain a mix of spiral and elliptical galaxies of similar luminosity, we expect the resulting $\Upsilon_{RC}$ to lie between $\Upsilon_S$ and $\Upsilon_E$. Recalling eqs. (38) and (57), this is just what eq. (78) tells us.

Clusters obviously contain the dark matter present in the cluster galaxies, but it might well happen that additional dark matter lurks in the intracluster space. However, an explicit analysis shows that the total galactic matter plus the hot gas fully account for the total cluster mass, thereby ruling out such a possibility.

4. Cosmological evidence and nature of dark matter

Cosmology not only provides additional and dramatic evidence for the existence of a large amount of dark matter in the Universe, but also offers crucial informations about its physical nature. Cosmological implications for dark matter are nowadays rather well known in the particle physics community and are discussed in other talks at this conference. Moreover, a thorough account would be fairly complex. Because of these reasons, our analysis will be brief and rather schematic. Three specific items will be addressed, from which informations about the physical nature of dark matter will emerge.

4.1. Primordial nucleosynthesis

A crucial implication of the hot big bang cosmological model is that light elements – like deuterium $D$, helium $^{3}He$, $^{4}He$ and lithium $Li^{7}$ – must have formed during the first few minutes in the life of the Universe. Because the temperature monotonically decreases during the cosmic expansion, atomic nuclei can form when the energy of background photons becomes comparable to the nuclear binding energy. With the number of light neutrinos fixed to 3, the predicted light element abundances depend

\footnote{They also contain lenticular galaxies, but this fact is unimportant for the present argument.}
on a single free parameter, the cosmic baryon density $\Omega_B$. In fact, calculations show that an increase of $\Omega_B$ leads to slightly more $^{4}\text{He}$, but the resulting amounts of $D$ and $^{3}\text{He}$ drop dramatically. So, a comparison between the predicted and observed light element abundances unambiguously fixes $\Omega_B$ \cite{25}. Indeed, the agreement is achieved for $\Omega_B$ within a narrow range

$$\Omega_B \simeq 0.04 - 0.05 .$$  \hspace{1cm} (79)

A remark is in order. No astrophysical process is known in which $D$ is produced, and so all the deuterium present in the Universe should be cosmological. Consequently, the comparison between theory and observation is particularly clean. In addition, local estimates of $D$ abundance are in good agreement with measurements in high-redshift clouds along the line of sight to a distant quasar.

Regardless of big bang nucleosynthesis, an independent estimate of $\Omega_B$ – which turns out to agree with eq. (79) – arises from the features of high-redshift Lyman-α forest absorption lines of neutral hydrogen observed in the spectra of background quasars \cite{26}.

Before turning to a different argument, we stress that – owing to eq. (79) – the upper bound (73) becomes

$$\Omega_M \leq 0.30 .$$  \hspace{1cm} (80)

### 4.2. Accelerated cosmic expansion

Long ago, Hubble realized that informations about the geometry of the Universe can be obtained by observing standard candles – astronomical objects of known absolute luminosity – located at cosmological distances. Basically, the idea is as follows. Once we know the absolute luminosity of a source, the measurement of its apparent luminosity (radiative flux) yields the distance $D$. Moreover, it can be shown that $D$ depends – in a known way – on the source redshift $z$ through the parameters $\Omega_M$ and $\Omega_\Lambda$ \footnote{Of course, for $z \ll 1$ one has $D \sim z$, which is the famous Hubble expansion law.}. In practice, a curve in the plot of apparent luminosity versus redshift – the so-called Hubble diagram – is labelled by the pair $(\Omega_M, \Omega_\Lambda)$. Suppose now that both apparent luminosity and redshift are measured for a sample of identical standard candles. Accordingly, a curve in the Hubble diagram gets singled out, and so a functional relationship between $\Omega_M$ and $\Omega_\Lambda$ emerges.

A few years ago, this strategy has been applied to a sample of distant Type-Ia supernovae, which are believed to be good standard candles \cite{27}. A best-fit to the data yields

$$\Omega_\Lambda \simeq 1.33 \, \Omega_M + 0.33 .$$  \hspace{1cm} (81)

Because eq. (81) entails $\rho_\Lambda > 0$ and $\rho_M < 2 \, \rho_\Lambda$, we are forced to conclude that the present cosmic expansion is accelerated.
Observe that – by eq. (81) – the upper bound (80) yields
\[ \Omega_A \leq 0.73. \] (82)

4.3. Cosmic Microwave Background

Another fundamental implication of big bang cosmology is the existence of the Cosmic Microwave Background (CMB), which is just primordial light redshifted by the cosmic expansion. Besides providing a wonderful confirmation of the big bang paradigm, the CMB yields a wealth of informations about dark matter.

Much in the same way as nuclei form once the temperature drops below the nuclear binding energy, atoms come into existence when the temperature becomes comparable to the atomic binding energy. When this process – named recombination – takes place, matter becomes neutral and decouples from radiation, which accordingly streams freely throughout the Universe. As a result, the CMB brings to us a snapshot of the Universe just at recombination, namely at redshift \( z_{\text{rec}} \approx 1100 \) (corresponding to \( t_{\text{rec}} \approx 3 \cdot 10^5 \) years after the big bang).

Soon after its discovery in 1965, it was established that the CMB has a black body spectrum peaked at \( T_0 \approx 2.72 \) K. Early analyses showed that the CMB is highly isotropic, once our peculiar motion – producing a systematic anisotropy with \( \Delta T/T_0 \sim 10^{-3} \) – is corrected for. However, in 1992 the COBE mission discovered anisotropies in the CMB spectrum, corresponding to temperature fluctuations \( \Delta T/T_0 \sim 10^{-5} \) on the angular scale of \( 7^\circ \). Remarkable progress has been made in the last few years, with the BOOMERANG, MAXIMA and DASI missions detecting similar temperature fluctuations down to \( 1^\circ \). Quite recently, the WMAP mission has succeeded in discovering temperature fluctuations on the angular scale of \( 0.2^\circ \).

What is the physical meaning of the CMB fluctuations? Because the post-recombination Universe is essentially transparent to the CMB photons, those which we detect now had their last interaction with matter on a virtual sphere – centered at our position – named last scattering surface (LSS). At the time of recombination, a generic point of the LSS had an horizon of size \( d_h \approx c t_{\text{rec}} \), which we see today under an angle \( \theta_1 \). Therefore, only events lying within \( \theta_1 \) – about a given direction in the sky – were causally connected at recombination. As a consequence, only CMB fluctuations on angular scales \( \theta < \theta_1 \) yield informations about physical processes occurring during recombination. However, recombination was not an instantaneous process and this fact implies that the LSS is a shell of finite thickness, corresponding to an observed angular scale \( \theta_2 \) (obviously \( \theta_2 < \theta_1 \)). As a result, CMB fluctuations get smeared out over angular scales \( \theta < \theta_2 \). Thus, we conclude that recombination physics shows up in CMB fluctuations on angular scales in the range \( \theta_2 < \theta < \theta_1 \). These fluctuations are the imprint on the CMB of acoustic oscillations in the matter-radiation fluid just before decoupling, with gravity providing the driving force while radiation pressure causes the restoring one.
A quantitative description of the CMB fluctuations emerges from a statistical treatment based on the harmonic analysis of $\Delta T/T_0$. Were measurements be performed on a plane, $\Delta T/T_0$ would depend on $x, y$ and we would represent $\Delta T/T_0(x, y)$ as a Fourier series. But in reality $\Delta T/T_0$ is measured on the celestial sphere and so it depends on $\theta, \varphi$. Accordingly, the following multipole expansion naturally arises

$$\frac{\Delta T}{T_0}(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \varphi) ,$$

(83)

where $Y_{lm}(\theta, \varphi)$ are spherical harmonics and the coefficients $a_{lm}$ are *gaussian random variables* defined by

$$\langle a_{lm} \rangle = 0 ,$$

(84)

$$\langle a_{lm} a_{lm'}^* \rangle = c_l \delta_{ll'} \delta_{mm'} ,$$

(85)

with $\langle \cdots \rangle$ representing the average over the whole sky. Denoting by $\alpha$ the angle between two arbitrary directions $\hat{n} \equiv (\theta, \varphi)$ and $\hat{n'} \equiv (\theta', \varphi')$ ($\cos \alpha = \hat{n} \cdot \hat{n'}$), the CMB autocorrelation function is

$$C(\alpha) \equiv \left\langle \frac{\Delta T}{T}(\theta, \varphi) \frac{\Delta T}{T}(\theta', \varphi') \right\rangle .$$

(86)

It can be shown that the autocorrelation function can be represented in terms of the multipole moments $c_l$ as

$$C(\alpha) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1)c_l P_l(\cos \theta) ,$$

(87)

where $P_l(\cos \theta)$ are Lagendre polynomials. It turns out that each term in eq. (87) corresponds to a well-defined angular scale, given by

$$\theta \sim \frac{180^0}{l} .$$

(88)

Hence, fluctuations on small angular scales correspond to large multipole orders (and vice-versa). Consider now the CMB power spectrum, namely the graph of $l(l+1)c_l$ versus $l$, and denote by $l_1$ and $l_2$ the multipole orders corresponding – by eq. (88) – to $\theta_1$ and $\theta_2$ (respectively). Then the above-discussed acoustic oscillations show up as *acoustic peaks* in the CMB power spectrum within the interval $l_1 < l < l_2$. It is precisely these peaks that tell us much about dark matter.

Specifically, the *position* of the first peak is controlled by $\theta_1$. Being an angle, $\theta_1$ is very sensitive to the geometry of the Universe, that is to say to $\Omega$. So, the actual position of the first peak yields the specific value of $\Omega$. The result is

$$\Omega \simeq 1 .$$

(89)
Thanks to eq. (13), eq. (89) entails that $\Omega_M$ and $\Omega_\Lambda$ have to meet the constraint
\[ \Omega_M + \Omega_\Lambda \simeq 1 . \]  
(90)
Moreover, it can be shown that the ratio of the heights of the first to the second peak gives
\[ \Omega_B \simeq 0.045 . \]  
(91)

### 4.4. Discussion

Let us now try to combine the various pieces of information obtained above into a coherent cosmological setting.

Perhaps, the most dramatic result is expressed by eq. (89). For, it implies that the Universe is spatially flat – namely euclidean – in agreement with the natural expectation based on cosmic inflation.

Another remarkable fact is that the value of $\Omega_B$ – as given by eq. (91) – nicely fits within the range (79). Hence, we see that cosmology provides a solid prediction about the total amount of baryons in the Universe.

Combine next eq. (81) with eq. (90). Then it follows
\[ \Omega_M \simeq 0.29 , \]  
(92)
\[ \Omega_\Lambda \simeq 0.71 . \]  
(93)

In the first place, observe that eqs. (92) and (93) are consistent with the upper bounds (80) and (82) (respectively). As far as $\Omega_M$ is concerned, several other independent estimates exist, which are based mainly on galaxy surveys and observations of large-scale structure. They cluster around eq. (92), however with considerable scatter. Once allowance for experimental errors is made, the resulting individual values of $\Omega_M$ tend to agree with eq. (92) and the upper bound (80) is (marginally) satisfied.

So, we see that all the various contributions to the $\Omega$ parameter turn out to fit within a consistent cosmological scenario, the so-called concordance cosmology.

### 4.5. Nature of dark matter

Finally, we briefly address the nature of dark matter as implied by the above analysis.

Baryonic dark matter – We estimated in Sect. 2 the contribution $\Omega_\ast$ to the $\Omega$ parameter from luminous matter – see eq. (20) – and we found that $\Omega_\ast$ is smaller than $\Omega_B$ – as fixed by eq. (91) – by a factor of 10. Hence, about 90% of the baryons do not emit photons in the optical band. What about the rest of the baryons? A recent inventory of the baryonic content of the Universe – obtained by combining available observational data with the whole body of theoretical knowledge – shows...
that the observed baryon budget today is dominated by X-ray emitting hot gas in groups and clusters of galaxies and accounts for almost 40% of the $\Omega_B$ value dictated by eq. (91). Thus, nearly half of the existing baryons are invisible and make up baryonic dark matter. We do not discuss this topic any further and simply refer to our review paper 30.

Nonbaryonic dark matter – As the reader will certainly have noticed, $\Omega_M$ turns out to largely exceed $\Omega_B$. Therefore, dark matter – as clumped in galaxies – is dominated by elementary particles carrying no baryon number, the so-called WIMPs (weakly interacting massive particles) 7. In this way, a deep connection emerges between cosmology and particle physics 8. Besides, it should be emphasized that the existence of nonbaryonic dark matter is also required by an independent argument, namely galaxy formation. Indeed, it would be impossible to explain the existence of structure in the present Universe with baryons alone 3.

Dark energy – This is the stuff responsible for the accelerated cosmic expansion and described by $\Omega_\Lambda$. In Sect. 2 we assumed that $\Omega_\Lambda$ arises from the vacuum energy represented by a cosmological constant, but this view is too restricted. In fact, dark energy can be anything that emits no light, has negative pressure and does not significantly cluster on the Megaparsec scale. Incidentally, the latter fact can be viewed as a natural consequence of the negative pressure 7.

Thus, finding the dark baryons, detecting WIMPs and discovering the specific properties of the dark energy are crucial observational challenges for contemporary cosmology.

5. Acknowledgements

We thank professor Milla Baldo Ceolin for her kind invitation to talk at this splendid conference.

6. References

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\(^7\)See the contribution of R. Bernabei to these proceedings.
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\(^9\)See the contribution of S. Matarrese to these proceedings.
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