Standard Model Nucleon EDM Revisited

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Abstract

The Cabibbo-Kobayashi-Maskawa matrix in the Standard Model is currently the only experimentally-confirmed source of CP-violation. The intrinsic electric dipole moment of the nucleon induced by this CP-phase via hadronic loop and pole diagrams has been studied more than two decades ago, but the existing calculation is subject to various theoretical issues such as the breakdown of chiral power counting and uncertainties in the determination of low energy constants. We carry out an up-to-date re-analysis on both one-loop and pole diagram contributions to the nucleon electric dipole moment based on Heavy Baryon Chiral Perturbation Theory in a way that preserves power counting, and redo the determination of the low energy constants following the results of more recent articles. Combined with an estimation of higher-order contributions, we expect the long-distance contribution to the Standard Model nucleon electric dipole moment to be approximately \((1 \times 10^{-32} - 6 \times 10^{-32}) e\,\text{cm.}\)

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I. INTRODUCTION

The search for permanent electric dipole moment (EDM) of elementary and composite particles is motivated by its CP-violating nature. We live in a universe in which the amount of baryons and antibaryons are unequal. In order to explain this asymmetry CP-violating interactions are needed to fulfill one of the three Sakharov criteria. EDMs of elementary and composite particles are, in most cases, direct consequences of these interactions which can be probed in low-energy experiments. Since the first upper limit on the neutron EDM was obtained by Smith, Purcell and Ramsey in 1957, numerous experiments have been performed to improve the precision of EDM measurements in different particle systems. Currently, the most stringent bounds on EDMs are set for the electron \(8.7 \times 10^{-29} \text{e cm, 90% C.L.}\) and mercury atom \(3.1 \times 10^{-29} \text{e cm, 95% C.L.}\), while the current upper limit on neutron and proton EDMs are \(2.9 \times 10^{-26} \text{e cm (90% C.L.)}\) and \(7.9 \times 10^{-25} \text{e cm (95% C.L.)}\) respectively (the latter is deduced from the bound on the mercury EDM). Future experiments are designed (or have been considered) to push these bounds even further down. For the neutron EDM, this includes the experiment at Paul Scherrer Institut (PSI), the CryoEDM and PNPI/ILL experiment at Institut Laue-Langevin (ILL), the SNS neutron EDM experiment at Oak Ridge, the TRIUMF experiment in Canada and the Munich experiment at Germany. These experiments are designed to reach a \(10^{-28} \text{e cm precision level for the neutron EDM}\). Also, the BNL Strong Ring EDM Collaboration has proposed a storage ring experiment designed to measure the proton EDM to a level of \(10^{-29} \text{e cm precision}\).

Although numerous Beyond Standard Model (BSM) scenarios have been proposed that can give rise to measurable EDMs within current experimental precision level, so far no definitive signal of such physics has been observed. Therefore, the CP-violating phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the Standard Model (SM) remains the only source for intrinsic EDMs. Questions have been raised concerning the expected size of EDMs coming from purely SM physics. A simple dimensional analysis using constituent quark masses may even suggest that the SM-induced neutron EDM could be as large as \(10^{-29} \text{e cm,}\)

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1 There are indeed some hopeful candidates, for example the muon \(g - 2\) anomaly; but no conclusive statement can be made before one could further improve the experimental precision and reduce the theoretical uncertainty of the SM prediction.
almost on the border of the precision level for future EDM experiments. It is therefore important to have a better estimate for the SM contribution to the nucleon EDM. The quark EDM induced by the CKM matrix only begins to appear at three loops \[11\]. A detailed calculation showed that the valence-quark contribution to the neutron EDM is of order \(10^{-34} \text{e cm} \[12\]. It was also shown that long-distance contributions, namely contributions with baryons and mesons as effective degrees of freedom (DOFs), could generate a much larger hadronic EDM. For instance, the pion-loop contribution to the neutron EDM was first studied in a paper by Barton and White \[13\] which produced log-divergent results in the chiral limit indicating that the long-range contribution may dominate. On the other hand, in a series of papers, Gavela et.al studied the pole-diagram contribution with the CP-violating phase generated by \(|\Delta S| = 1\) electroweak \[14\] and gluonic penguin diagrams \[15\]. They claimed that the latter is dominant and derived a SM neutron EDM of order \(10^{-31} \text{e cm}\) \[13\]. He et.al \[16\] did a thorough chiral-loop calculation and re-analyzed the pole-diagram contribution in \[14, 15\] and argued that the two are of the same order of magnitude. Their estimate for the neutron EDM is \(1.6 \times 10^{-31} \text{e cm} - 1.4 \times 10^{-33} \text{e cm}\), which is currently the most widely accepted estimate for the SM neutron EDM.

The purpose of this paper is to revisit the previous study of both chiral-loop and the pole contributions to the nucleon EDM in order to sharpen our SM benchmark. On the theoretical side, one could improve earlier work in several ways. For instance, the chiral loop calculation in \[16\] adopted an older meson theory utilizing a pseudoscalar strong baryon-meson coupling that should be replaced by the standard axial-vector coupling. Also, their work that utilized an effective hadronic Lagrangian in computing chiral-loop diagrams faced another well-known problem in the loss of power counting similar to that happening in the relativistic Chiral Perturbation Theory (ChPT). ChPT is a non-renormalizable theory that involves infinitely many interaction terms. Its predictive power therefore relies on the fact that higher order terms are suppressed by powers of \(p/\Lambda_\chi\) where \(p\) is the typical mass scale or momentum of hadronic DOFs and \(\Lambda_\chi \sim 1\text{GeV}\). This expansion however becomes ambiguous when baryons are included because a typical baryon mass could be \(M_B \sim 1\text{GeV}\). Therefore, \(M_B/\Lambda_\chi\) is no longer a small expansion parameter. Heavy Baryon Chiral Perturbation Theory (HBchPT) \[17\] provides a convincing way to get around this issue by performing a field redefinition in the baryon field to scale out its mass-dependence. By doing this, one can split the baryon field into “light” and “heavy” components, where the
former depends only on its residual momentum which is well-below 1GeV. After integrating out the heavy component of the baryon field, the effective Lagrangian can be written as a series expansion of $1/m_N$. This eliminates the possibility of a factor $m_N$ appearing in the numerator and thus restores the power counting. The convergence of the SU(3) HBchPT is not as good as its SU(2) counterpart because $m_K/m_N$ is not very small \cite{18, 19}. However, it is still theoretically beneficial as it provides a formal classification of different contributions into leading and sub-leading orders. In this work, the chiral-loop contribution to the nucleon EDM are recalculated up to the leading-order (LO) in the heavy baryon (HB)-expansion.

Additionally, previous numerical results of loop and pole contributions face large uncertainties due to poorly-known values of physical constants in the weak sector at that time. For example, the CP-violating phase $\delta$ of the CKM matrix quoted in Ref. \cite{16} had an uncertainty than spans one order of magnitude. The fitting of certain low energy constants (LECs) such as weak baryon-meson interaction strengths, has since been updated. Also, their theoretical estimation of various CP-phases in their effective weak Lagrangian was based on older works \cite{20, 21} which had been improved by others. Furthermore, for previous works on pole contributions, their estimation on effective CP-phases was based only on a single gluonic penguin operator without considering the full analysis of operator mixing and renormalization group running. Moreover, the approximate form of their analytic expressions was based on the out-of-date assumption that $m_t \ll m_W$. In this work, we will do a more careful determination of weak LECs, taking all these issues into account. Combining our calculation and an estimate of higher-order effects, we predict a range of the long-distance SM contribution to the nucleon EDM to be around $(1-6) \times 10^{-32} e \text{cm}$. We identify the main sources of uncertainty and discuss possible steps one could take to improve from that. At the same time, we use dimensional analysis to estimate the size of possible short-distance counterterms. We find that they could be as large as $7 \times 10^{-30} e \text{cm}$.

This work is arranged as follows: in Section II we will briefly outline the main ingredients of the SU(3) HBchPT and introduce the weak Lagrangian responsible for the generation of the nucleon EDM. In Section III we will determine the LECs. In Section IV and V we derive the analytic expressions for loop and pole contributions to the nucleon EDM respectively and calculate their numerical values. In Section VI we will provide some further discussions and draw our conclusion.
II. HBCHPT: STRONG AND ELECTROWEAK INTERACTIONS

In this section, we review some basic concepts of the ChPT with the aim of setting conventions and notation. ChPT is a low-energy effective field theory (EFT) of quantum chromodynamics (QCD) with hadrons as low energy DOFs. QCD exhibits chiral symmetry in the limit of massless quarks. However this symmetry is spontaneously broken in the ground state and leads to the emergence of Goldstone bosons which are identified as pseudoscalar mesons. The corresponding EFT obeys the same symmetry. An infinite tower of operators respecting the symmetry with increasing mass dimensions is organized in the Lagrangian. However, to achieve a level of finite precision, only a finite number of operators are retained because the effect of operators with higher-dimensions is suppressed by powers of \( p/\Lambda_{\chi} \).

We use the standard non-linear representation of chiral fields \([22–24]\), wherein the pseudoscalar meson octet is incorporated in the exponential function \( U = \exp\{i\phi/F_{\pi}\} \), where

\[
\phi = \sum_{a=1}^{8} \phi_a \lambda_a = \begin{pmatrix}
\pi^0 + \frac{1}{\sqrt{3}} \eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+
\sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}} \eta_8 & \sqrt{2}K^0
\sqrt{2}K^- & \sqrt{2}K^0 & -\frac{2}{\sqrt{3}} \eta_8
\end{pmatrix}
\]

with \( F_{\pi} \approx 93\text{MeV} \). The matrix \( U \) transforms under the chiral rotation as: \( U \rightarrow LUR^\dagger \), where \( L \) and \( R \) are transformation matrices of \( SU(3)_L \) and \( SU(3)_R \) respectively. The mass term of the meson octet is introduced by the spurion method, namely if we imagine that the quark mass matrix \( M = \text{diag}\{m_u, m_d, m_s\} \) would transform as \( M \rightarrow LMR^\dagger \), then the QCD Lagrangian would have been chiral-invariant. Therefore, its low energy effective theory should also break the chiral symmetry by the same pattern. The lowest-order operator of this property is \( \text{Tr}[MU^\dagger + UM^\dagger] \). This operator gives non-zero meson masses which is isospin-symmetric.

The ground state \( J^P = (1/2)^+ \) baryon octet is assembled into the matrix:

\[
B = \begin{pmatrix}
\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\
\Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\
\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}}
\end{pmatrix}.
\]

It transforms as: \( B \rightarrow KBK^\dagger \) with \( K = K(L,R,U) \) being a unitary matrix. In order to couple baryons with the pseudoscalar octet, we define \( \xi = \sqrt{U} \) which transforms as
\[ \xi \rightarrow L\xi K^\dagger = K\xi R^\dagger \]
and introduce the Hermitian axial vector:

\[ A_\mu = \frac{i}{2} [\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi] \tag{3} \]

which transforms as \( A_\mu \rightarrow K A_\mu K^\dagger \) under the chiral rotation (we have neglected its coupling with external field because it is not needed in this work).

We now proceed with formulating the HBchPT. In order to scale out the heavy mass-dependence a baryon, we rewrite its momentum as

\[ p_\mu = m_N v_\mu + k_\mu, \tag{4} \]

where \( m_N \) is the nucleon mass, \( v_\mu \) is the velocity of the baryon which is a conserved quantity in scattering processes in the \( m_N \rightarrow \infty \) limit and \( k_\mu \) is now the residual momentum of the baryon which is well below 1GeV. We therefore rescale the baryon field and retain its “light” component:

\[ B_v(x) = e^{i m_N v \cdot x} \frac{1 + \hat{\theta}}{2} B(x) \tag{5} \]

(“light” in the sense that it only depends on the residual momentum. Also, we will omit the subscript “v” later). We integrate out the remaining component which is “heavy”. The baryon propagator thus becomes:

\[ i S_B(k) = \frac{i}{v \cdot k - \delta_B + i \epsilon} \tag{6} \]

where \( \delta_B = m_B - m_N \) is the baryon mass splitting. This procedure also reduces the Dirac structure to either 1 or \( S^\mu \) with the latter being the spin-matrix of the baryon satisfying \( S \cdot v = 0 \). In this work we concentrate only on terms that are leading order in the HB-expansion (with the exception of the baryon electromagnetic dipole transition operator that appears in pole diagrams as we will explain below).

The lowest-order strong Lagrangian involving only the \((1/2)^+\) baryons, Goldstone bosons and electromagnetic fields relevant to our work is given by:

\[ \mathcal{L} = \frac{F^2}{4} \text{Tr}[\mathcal{D}_\mu U \mathcal{D}^\mu U^\dagger] + \frac{F^2}{4} \text{Tr}[\chi_+] + \text{Tr}[\bar{B} i v \cdot \mathcal{D} B] + 2D \text{Tr}[\bar{B} S^\mu \{ A_\mu, B \}] + 2F \text{Tr}[\bar{B} S^\mu [ A_\mu, B]] + b_D \frac{D}{2B_0} \text{Tr}[\bar{B} \{ \chi_+ , B \}] + b_F \frac{D}{2B_0} \text{Tr}[\bar{B} \{ \chi_+ , B \}] + b_0 \frac{D}{2B_0} \text{Tr}[\bar{B} B] \text{Tr}[\chi_+] \tag{7} \]

where \( D = 0.80, F = 0.50 \) \cite{22} and \( \mathcal{D}_\mu U = \partial_\mu U + i e A_\mu [Q, U] \). Here \( Q = \text{diag}\{2/3, -1/3, -1/3\} \) is the quark charge matrix while \( B_0 \) is a parameter characterizing
the chiral quark condensate and \( \chi^+ = 2B_0(\xi^\dagger M \xi^d + \xi M \xi) \) introduces the quark-mass dependence. The last three terms in Eq. (7) are responsible for the mass splitting within the baryon octet [25]. Since we have scaled out the nucleon mass from the baryon field \( B \) (so in our formalism proton and neutron will appear as massless excitations) the other baryons will have an excitation energy given by the “residual” mass \( \delta_B \). This is important later during the computation of pole diagrams.

For the purpose of pole diagram contributions we need also to include the \((1/2)^-\) baryon octet. The importance of these resonances can be traced back to the observation of the unexpectedly large violation of Hara’s theorem [26] which states that the parity-violating radiative \( B \to B' \gamma \) transition amplitude should vanish in the exact SU(3) limit. The authors of ref [27] (and later improved by [28]) pointed out that this apparent puzzle could be resolved by including baryon resonances that give rise to pole diagrams which enhance the violation of Hara’s theorem. Therefore, one should naturally expect that the same kind of diagrams will also play an important role in the determination of the nucleon EDM. The resonance \((1/2)^-\) octet is denoted as \( \mathcal{R} \):

\[
\mathcal{R} = \begin{pmatrix}
\Sigma^* + \sqrt{2} \Lambda^* & \Sigma^* + \sqrt{6} \Sigma^* + n^* \\
\Sigma^- & -\sqrt{2} \Lambda^* + \sqrt{6} \Sigma^*
\end{pmatrix}.
\]

(8)

It transforms in the same way as \( B \) except that it has a negative intrinsic parity.

The part of strong and electromagnetic chiral Lagrangian involving \( \mathcal{R} \) which is relevant to our work is given by:

\[
\mathcal{L}_\mathcal{R} = \text{Tr}[\mathcal{R} i\mathbf{v} \cdot \mathbf{D}\mathcal{R}] - \tilde{\delta}_\mathcal{R} \text{Tr}[\mathcal{R}\mathcal{R}] + \frac{\tilde{b}_D}{2B_0} \text{Tr}[\mathcal{R}(\chi^+, \mathcal{R})] + \frac{\tilde{b}_F}{2B_0} \text{Tr}[\mathcal{R}[\chi^+, \mathcal{R}]] + \frac{\tilde{b}_0}{2B_0} \text{Tr}[\mathcal{R}\mathcal{R}] \text{Tr}[\chi^+]
- 2r_D \text{Tr}[\mathcal{R}(v_\mu S_\nu - v_\nu S_\mu) \{f_+^{\mu\nu}, B\}] + \text{Tr}[\bar{B}(v_\mu S_\nu - v_\nu S_\mu) \{f_+^{\mu\nu}, R\}]
- 2r_F \text{Tr}[\mathcal{R}(v_\mu S_\nu - v_\nu S_\mu) \{f_+^{\mu\nu}, B\}] + \text{Tr}[\bar{B}(v_\mu S_\nu - v_\nu S_\mu) \{f_+^{\mu\nu}, R\}].
\]

(9)

Second to fifth terms of \( \mathcal{L}_\mathcal{R} \) give the average residual mass and mass-splitting among the \((1/2)^-\) baryon octet. Constants \( r_D \) and \( r_F \) are electromagnetic coupling strengths between \( B \) and \( \mathcal{R} \) and \( f_+^{\mu\nu} \) is the chiral field strength tensor of the electromagnetic field that, in the SU(3) version of ChPT, is given by [22]:

\[
f_+^{\mu\nu} = -e [\xi^\dagger Q \xi + \xi Q \xi^\dagger] F^{\mu\nu}
\]

(10)
with $e > 0$. The reason we include $r_D$ and $r_F$ terms even though they are formally $1/m_N$-suppressed is that they will then be compensated by small denominator $\delta_B$ factors in pole diagrams.

Next we introduce the relevant weak Lagrangian that gives rise to the nucleon EDM. As the only CP-violating effect in the SM is the complex phase in the CKM matrix, the strange quark must be included. The CP-phase is attached to various $|\Delta S| = 1$ four-quark operators that are responsible for kaon decay and nonleptonic hyperon decays. It is well-known that the product of two charged weak currents could transform as $(8_L, 1_R)$ or $(27_L, 1_R)$ under the SU(3) chiral rotation. Extra $|\Delta S| = 1$ operators could be induced via gluonic or electroweak penguin diagrams. The former transforms as $(8_L, 1_R)$ while the latter may introduce a $(8_L, 8_R)$ component that is however suppressed by the smallness of the fine structure constant. Furthermore, since $(8_L, 1_R)$ operators have isospin $I = 1/2$ while $(27_L, 1_R)$ operators can have both $I = 1/2$ and $I = 3/2$ components we would naturally expect $(8_L, 1_R)$ operators to be the dominant piece due to the experimentally observed $|\Delta I| = 1/2$ dominance in non-leptonic decay processes. Hence, effective operators we introduce later should also transform as $(8_L, 1_R)$.

The pure mesonic Lagrangian that triggers the $|\Delta I| = 1/2$ kaon decay channel is given by [24]:

$$\mathcal{L}_8 = g_8 e^{i\varphi} \text{Tr}[\lambda^+_\mu D\mu UD\mu U^\dagger] + h.c$$  \hspace{1cm} (11)

where $\lambda^+_\mu = (\lambda_6 + i\lambda_7)/2$. The non-zero value of $\varphi$ introduces the CP-violating effect. Meanwhile, the corresponding baryonic operator that triggers the nonleptonic hyperon decay is given by [29]:

$$\mathcal{L}_w^{(s)} = h_D e^{i\varphi_D} \text{Tr}[\bar{B} \{ \xi^\dagger \lambda_+ \xi, B \}] + h_F e^{i\varphi_F} \text{Tr}[\bar{B} \{ \xi^\dagger \lambda_+ \xi, B \}] + h.c.$$ \hspace{1cm} (12)

Here the superscript $(s)$ indicates that these operators mediate S-wave decays. In principle there is a counterpart operator with the Dirac structure $\gamma_5$. It is time-reversal odd and must be proportional to the complex phase in the CKM matrix. We do not need to include this extra operator because it vanishes at the LO in the HB-expansion due to the non-relativistic reduction of the Dirac structure. Also, our definitions of $h_D$ and $h_F$ here are slightly different from [29] as we set $h_D, h_F$ to be real. Complex phases are moved to $\varphi_D$ and $\varphi_F$.

Finally, for the purpose of including pole-diagram contributions, we need the weak Lagrangian that triggers the $B - \mathcal{R}$ transition. The lowest order Lagrangian is given by [30]:
\[ \mathcal{L}_w^{BR} = iw_D e^{i\varphi_D} \text{Tr}[\bar{\mathcal{R}}\{h_+, B\}] + iw_F e^{i\varphi_F} \text{Tr}[\bar{\mathcal{R}}\{h_+, B\}] + h.c \] (13)

where \( h_+ \equiv \xi^\dagger \lambda_+ \xi + \xi^\dagger \lambda_- \xi \). The counterpart with a \( \gamma_5 \) structure again vanishes due to HBchPT reduction at the LO in the HB-expansion.

### III. DETERMINATION OF THE LECs

There are altogether 12 LECs that enter in the estimate for the nucleon EDM: seven interaction strengths \( \{r_D, r_F, g_8, h_D, h_F, w_D, w_F\} \) and five CP-phases \( \{\varphi, \varphi_D, \varphi_F, \bar{\varphi}_D, \bar{\varphi}_F\} \). They are either extracted from experiments or obtained by theoretical modeling \(^2\).

Pure electromagnetic \( B - \mathcal{R} \) transition coupling strengths \( r_D \) and \( r_F \) are fitted to electromagnetic decays of \((1/2)^-\) resonances. The authors of ref. \(^{28}\) obtain:

\[ er_D = 0.033\text{GeV}^{-1}, \quad er_F = -0.046\text{GeV}^{-1}. \] (14)

The constant \( g_8 \) is fitted to the \( K_0^0 \rightarrow \pi^+ \pi^- \) decay rate, ignoring the small CP-violating effect \(^{31}\). That gives:

\[ g_8 = 6.84 \times 10^{-10}\text{GeV}^2. \] (15)

The CP-phase \( \varphi \) is, up to a negative sign, the phase of the \( K_0^0 \rightarrow \pi\pi(I = 0) \) decay amplitude:

\[ \varphi = -\xi_0 = -\frac{\text{Im} A_0}{\text{Re} A_0} \] (16)

In principle one could extract \( \xi_0 \) from the measurement of the CP-violation quantity \( \epsilon' \) in the kaon decay. However, \( \epsilon' \) is a linear combination of \( \xi_0 \) and another CP-violating phase, \( \xi_2 \), of the \( I = 3/2 \) channel. Simple estimation \(^{24}\) suggests that \( \xi_2 \) is of the same order as \( \xi_0 \), making \( \xi_0 \) hard to extract directly from the experiment. We therefore refer to theoretical estimation based on the large-\( N_c \) approach \(^{32}\) which gives:

\[ \varphi = -\xi_0 \approx -\sqrt{2}\epsilon \times (-6 \times 10^{-2}) \approx 1.89 \times 10^{-4} \approx 6.4J \] (17)

where \( J = (2.96^{+0.20}_{-0.16}) \times 10^{-5} \) \(^{31}\) is the Jarlskog invariant \(^{33}\). It is worthwhile to mention that, in Ref. \(^{16}\) the uncertainty of \( J \) spanned an order of magnitude so it was the main

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\(^2\) Unfortunately, none of these LECs we found from existing literatures contain any error bar, so we would not be able to estimate the error introduced by the fitting of LECs.
contributor of the uncertainty of the neutron EDM during that time. Nowadays we have determined \( J \) far more precisely so the associated uncertainty is sub-leading compared to uncertainties due to higher-order effects in the HB-expansion and unknown short-distance counterterms, which we will discuss later.

The four remaining interaction strengths \( h_D, h_F, w_D, w_F \) were determined in [30] by simultaneously fitting them to the s and p-wave amplitudes of nonleptonic hyperon decays:

\[
h_D \approx 0.44 \times 10^{-7} \text{GeV}, \quad h_F \approx -0.50 \times 10^{-7} \text{GeV}, \quad w_D \approx -1.8 \times 10^{-7} \text{GeV}, \quad w_F \approx 2.3 \times 10^{-7} \text{GeV}.
\]

The last two constants were determined by setting \( m_R \approx 1535 \text{MeV} \).

Finally, we need to know the four remaining CP-phases \( \{\varphi_D, \varphi_F, \tilde{\varphi}_D, \tilde{\varphi}_F\} \). These phases have been considered in ref [14], but their treatments are less satisfactory due to the neglect of the operator mixing effect and a certain outdated approximation of the small top quark mass assumption. In order to improve upon that, we review a more recent work done in ref [29] that determined \( \{\varphi_D, \varphi_F\} \) and use their result to provide an estimate of \( \{\tilde{\varphi}_D, \tilde{\varphi}_F\} \). Ref [29] pointed out that after considering operator mixing and renormalization group running, the dominant operator that gives rise to the CP-violating phase in the \(|\Delta S| = 1, |\Delta I| = 1/2 \) sector is given by:

\[
\hat{Q}_6 = -2 \sum q \bar{d}(1 + \gamma_5)qq(1 - \gamma_5)s. \tag{19}
\]

Ref [29] then computed the factorizable and non-factorizable contributions to \( \varphi_D, \varphi_F \) induced by \( \hat{Q}_6 \). Here “factorizable” means to regard \( \hat{Q}_6 \) as a product of two chiral quark densities and match it to chiral operators. The matching is done by realizing that \( \bar{q}_Rq_L \sim \partial \mathcal{L}_{QCD}/\partial m_q = \partial \mathcal{L}_{chiral}/\partial m_q \). On the other hand, the “non-factorizable” contribution is obtained simply by taking the hadronic matrix element of \( \hat{Q}_6 \) using the quark model. These two contributions are distinct because the factorizable piece contains a factor of chiral quark condensate \( F^2 \sigma B_0 \) through:

\[
\langle 0| \hat{q}^i_L \hat{q}^j_R \hat{q}^k_R \hat{q}^l_L | B B' \rangle \sim \langle 0| \hat{q}^i_L \hat{q}^j_R | 0 \rangle \langle 0| \hat{q}^k_R \hat{q}^l_L | B B' \rangle = -\frac{1}{2} F^2 \sigma B_0 \delta_{ij} \langle 0| \hat{q}^k_R \hat{q}^l_L | B B' \rangle \tag{20}
\]

while the same quantity never appears in a quark model calculation. Combining the two, they found that \( \text{Im}(h_D \exp i\varphi_D) \approx -2.2, \text{Im}(h_F \exp i\varphi_F) \approx 6.1 \), both in units of \( \sqrt{2} F_\pi G_F m^2_\pi J \). This leads to:

\[
\varphi_D \approx -1.5J, \quad \varphi_F \approx -3.6J. \tag{21}
\]
It is straightforward to see that $\tilde{\varphi}_D, \tilde{\varphi}_F$ receive no factorizable contribution. This is because it would require terms like $\bar{R}m_qB$ to appear in the strong chiral Lagrangian. Such terms would violate parity and therefore cannot exist. For the non-factorizable part, our strategy is the following: first we compute the matrix element $\langle R|\hat{Q}_6|B\rangle$ and $\langle B'|\hat{Q}_6|B\rangle$ using the quark model to find their ratio. Then, we use this ratio to infer the value of the non-factorizable part of $\tilde{\varphi}_D, \tilde{\varphi}_F$ by appropriately scaling the non-factorizable part of $\varphi_D, \varphi_F$ given in ref [29].

To obtain an estimate of hadronic matrix elements we adopt the harmonic oscillator model [27]. The structure of the spin-flavor wavefunction of the baryon octet leads to the following ratio:

$$\langle n^*|\hat{Q}_6|\Sigma^0\rangle : \langle n^*|\hat{Q}_6|\Lambda\rangle : \langle p^*|\hat{Q}_6|\Sigma^+\rangle = 1 : \sqrt{3} : -\sqrt{2}$$

which requires that $w_F\tilde{\varphi}_F = (1/3)w_D\tilde{\varphi}_D$ in our chiral Lagrangian. We also obtain the ratio between $B - B'$ and $B - R$ matrix elements:

$$\frac{\langle p^*|\hat{Q}_6|\Sigma^+\rangle}{\langle p|\hat{Q}_6|\Sigma^+\rangle} = -\sqrt{\frac{2}{3\pi m R_0}} \frac{1}{m R_0}.$$  

where $m \approx 0.34 \text{GeV}, R_0 \approx 2.7 \text{GeV}^{-1}$ are harmonic oscillator parameters. With this ratio and the non-factorizable contribution to $\varphi_D, \varphi_F$ given in [29], we obtain the non-factorizable contribution to $\tilde{\varphi}_D, \tilde{\varphi}_F$:

$$\tilde{\varphi}_D \approx 0.04J, \tilde{\varphi}_F \approx -0.01J$$

These phases are about two orders of magnitude smaller than the three other CP-phases because they are not enhanced by the chiral quark condensate. Therefore, we disregard them in the rest of our calculation.

To end this section, we should point out that there is an important sign issue in the determination of LECs. Since LECs are fitted to experiments that only involve squared amplitudes, an overall undetermined sign is left ambiguous. Therefore, if two sets of LECs are fitted separately to two unrelated experiments (for example, $\{r_D, r_F\}$ are to fit to baryon electromagnetic transitions and $\{h_D, h_F, w_D, w_F\}$ are to fit to non-leptonic hyperon decays), there is no unique way to determine the relative sign between these two sets of LECs. This introduces an extra uncertainty because a change of a relative sign can turn a constructive interference to destructive and vice versa. We will discuss the impact of this uncertainty in the last section.
Figure 1: One-loop contributions to the nucleon EDM. Each round dot denotes a $|\Delta S|=1$ weak insertion. Fig. 1(a)-(c) (and reflections) contribute to both neutron and proton EDM; while Fig. 1(d) (and reflection) contributes only to proton EDM.

IV. ONE LOOP CONTRIBUTION

In this section we present analytic and numerical results of the 1-loop contribution to the proton and neutron EDM using HBchPT. The nucleon EDM $d_N$ is defined by the term linear to the photon incoming momentum $q$ in the P and T-violating $NN\gamma$ amplitude

$$iM \equiv -2d_N v \cdot \bar{u}_NS \cdot qu_N. \quad (25)$$

Here $\varepsilon^\mu$ is the photon polarization vector. Notice that the equation has been simplified by applying the on-shell condition to the nucleon: $v \cdot q = -q^2/2m_N \to 0$.

Since each weak interaction vertex has $|\Delta S|=1$, we need at least two insertions of weak interaction vertices to obtain an EDM that is flavor diagonal. Most one-loop integrals are UV-divergent and are regularized using the $\overline{\text{MS}}$ scheme of which the combination

$$L \equiv \frac{2}{4-d} - \gamma + \log(4\pi) \quad (26)$$

is subtracted. Also, since all CP-violating phases $\{\varphi_i\}$ are small, we use the small angle approximation $\sin \varphi_i \approx \varphi_i$. Finally, following the usual spirit of ChPT, during the calculation of loops we assume that the heavy DOFs could be integrated out to generate higher-order operators that can then be discarded in our lowest-order loop calculation \textsuperscript{3}. Hence what enter the loops are the lightest DOFs, in our case the pseudoscalar meson octet and the ground-state $(1/2)^+$ baryon octet.

\textsuperscript{3} The reader should anyway be alerted that this may not always be the case. For example, Ref. \textsuperscript{[36]} pointed out that one needs to include the baryon decuplet in order to reconcile with the result of the large $N_c$-expansion.
There are four distinct types of 1-loop diagrams (see Fig. 1) that give non-zero contribution to the nucleon EDM (diagrams of other kinds are all vanishing at the LO in the HB-expansion. See the Appendix for the argument). Fig. 1(a)-(c) (plus reflections) in Fig. 1 contribute to both neutron and proton EDM. For the neutron, it reads:

\[ d_{n}^{1\text{-loop}} = -\frac{e g_s (D h_D \{\varphi - \varphi_D\} + F h_F \{\varphi - \varphi_F\})}{4\pi^2 F_\pi^4 (m_\pi^2 - m_K^2)} (m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} - \{\pi \leftrightarrow K\}) \]

\[ - \frac{\delta_S e g_s (D - F) \{\varphi - \varphi_D\} + h_F \{\varphi_F - \varphi\}}{4\pi^2 F_\pi^2 (m_\pi^2 - m_K^2)} \left( m_\pi^2 \frac{\arctan \frac{m_\pi^2 - \delta_S^2}{\delta_S^2}}{\sqrt{m_\pi^2 - \delta_S^2}} - \{\pi \leftrightarrow K\} \right). \]

We found that all terms that are analytic in quark masses cancel each other. Also notice that there is no extra singularity in the limit \( m_K \to m_\pi \) or \( \delta_B \to 0 \). Numerical estimation with \( \mu = m_N \) gives

\[ |d_{n}^{1\text{-loop}}| = 1.5 \times 10^{-32} \text{e cm}. \]

(28)

Similar calculation could be done for the proton EDM. Fig. 1(a)-(c) give

\[ d_{p}^{1\text{-loop},1} = \frac{e g_s (D \{h_D [\varphi - \varphi_D] + 3 h_F [\varphi - \varphi_F]\} + 3 F \{h_D [\varphi - \varphi_D] + h_F [\varphi_F - \varphi]\})}{24\pi^2 F_\pi^4 (m_\pi^2 - m_K^2)} \times \]

\[ (m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} - \{\pi \leftrightarrow K\}) \]

\[ - \frac{\delta_S e g_s (D - F) \{h_D [\varphi - \varphi_D] + h_F [\varphi_F - \varphi]\}}{8\pi^2 F_\pi^4 (m_\pi^2 - m_K^2)} \left( m_\pi^2 \frac{\arctan \frac{m_\pi^2 - \delta_S^2}{\delta_S^2}}{\sqrt{m_\pi^2 - \delta_S^2}} - \{\pi \leftrightarrow K\} \right). \]

\[ - \frac{\delta_A e g_s (D + 3 F) \{h_D [\varphi - \varphi_D] + 3 h_F [\varphi - \varphi_F]\}}{24\pi^2 F_\pi^4 (m_\pi^2 - m_K^2)} \left( m_\pi^2 \frac{\arctan \frac{m_\pi^2 - \delta_A^2}{\delta_A^2}}{\sqrt{m_\pi^2 - \delta_A^2}} - \{\pi \leftrightarrow K\} \right). \]

(29)

There is one extra type of diagrams contributing to the proton EDM corresponding to two insertions of \( h_i \) vertices (Fig. 1(d)). The corresponding diagrams do not generate the neutron EDM simply because one cannot find any non-vanishing combination of \( B, B', \phi \). This diagram gives for the proton EDM

\[ d_{p}^{1\text{-loop},2} = -\frac{e h_D h_F (D - F) (\varphi_D - \varphi_F) (\pi - 2 \arctan \frac{\delta_S}{\sqrt{m_K^2 - \delta_S^2}})}{16\pi^2 F_\pi^2 \sqrt{m_K^2 - \delta_S^2}} \]

\[ - \frac{e h_D h_F (D + 3 F) (\varphi_D - \varphi_F) (\pi - 2 \arctan \frac{\delta_A}{\sqrt{m_K^2 - \delta_A^2}})}{48\pi^2 F_\pi^2 \sqrt{m_K^2 - \delta_A^2}}. \]

(30)
This contribution is interesting as it is UV-finite. It depends non-analytically on quark masses and hence uniquely characterizes long-distance physics. Numerical estimate gives

\[ |d_{p}^{1-\text{loop},1}| = 6.1 \times 10^{-33} \text{e cm} \]
\[ |d_{p}^{1-\text{loop},2}| = 1.1 \times 10^{-32} \text{e cm}. \]  

We choose to present numerical results of \( d_{n}^{1-\text{loop}} \) and \( d_{p}^{1-\text{loop},1} \) separately because the former is proportional to \( g_{8}h_{i} \) while the latter is proportional to \( h_{i}h_{j} \). Since the relative sign between \( g_{8} \) and \( h_{i} \) is experimentally undetermined, these two terms can either add or subtract each other.

As a short conclusion, we stress once again that under the HBchPT formalism, our analytic results of 1-loop diagrams, Eq. (27), (29) and (30) fully respect the power counting as no powers of \( m_{B} \) appear in the numerator upon carrying out the loop integral. This should be contrasted with the relativistic calculation done in ref [16], in which they include diagrams involving MDM-like coupling that should have an explicit \( 1/m_{B} \) suppression according to the power counting, but is compensated by the \( m_{B} \) appeared in the numerator coming from the loop integral.

Finally let us discuss the effect of counterterms. Since \( d_{n}^{1-\text{loop}} \) and \( d_{p}^{1-\text{loop},1} \) are UV-divergent, we need to introduce corresponding counterterms \( d_{n}^{0}, d_{p}^{0} \) to cancel the infinities. These counterterms are generated by short-distance physics therefore their precise values cannot be calculated. To estimate the size of these counterterms we perform a naive dimensional analysis (NDA). Following [31], there are ten \( \Delta S = 1 \) four-quark operators that mix under renormalization. The effective Hamiltonian can be written as:

\[ H_{\text{eff}}^{\Delta S=1} = \frac{G_{F}}{\sqrt{2}} V_{ud} V_{us}^{\ast} \sum_{i=1}^{10} C_{i}(\mu) \hat{Q}_{i}(\mu) + h.c. \]  

Under conditions that \( \Lambda_{QCD} \approx 0.2 \text{GeV}, \mu = 1 \text{GeV} \) and the top-quark mass \( m_{t} = 174 \text{GeV} \), the largest flavor-diagonal CP-violating effect comes from the product of \( \hat{Q}_{2} \) and \( \hat{Q}_{6} \) with Wilson coefficients \( C_{2} = 1.31 - 0.044\tau \) and \( C_{6} = -0.011 - 0.080\tau \) respectively where \( \tau = -V_{ud} V_{ts}^{\ast} / V_{ud} V_{us}^{\ast} \). This gives:

\[ d_{p}^{0}, d_{n}^{0} \sim \frac{G_{F}^{2}}{2} |V_{ud} V_{us}^{\ast}|^{2} \text{Im}(C_{2}C_{6}^{\ast}) \Lambda_{\chi}^{3} \approx 7 \times 10^{-30} \text{e cm}. \]

\( ^{4} \) One can show that Eq. (30) remains real even when \( \delta_{K}, \delta_{\Lambda} > m_{K} \) by using the identity \( \arctan z = \frac{1}{2} \log \frac{1+i\tau}{1-i\tau} \).
The inclusion of the factor $\Lambda_\chi^3$ is just to make the dimension correct. Choosing $\Lambda_\chi$ as the energy scale instead of other scale like $\Lambda_{QCD}$ is for the purpose of providing an upper limit for $d_p^0$ and $d_N^0$. This analysis shows that the short-distance contribution to the nucleon EDM could be two to three orders of magnitude larger than the long-distance contribution. However the NDA estimation is never quantitatively trustable and it might happen that some accidental cancelations could suppress the actual value of $d_n^0, d_p^0$ from what expected in Eq. (33). In this sense, a detailed study of the long-distance contribution is worthwhile because it sets a solid bound of which any measurable nucleon EDM below this bound could be safely regarded as being consistent with the SM prediction.

V. POLE CONTRIBUTION

Next we estimate the contribution of pole diagrams to the nucleon EDM. For baryon intermediate states, we include the flavor octet part of the $(56, 0^+)$ and $(70, 1^-)$ baryon supermultiplets. Here we adopt the standard spin-flavor $SU(6)$ notation $(\mathcal{D}, L^p)$ where $\mathcal{D}$ is the dimension of the $SU(6)$ representation, $L$ is the orbital angular momentum and $p$ is the parity. For generality, we first write down all possible pole configurations that can contribute and divide it to two classes: Class I are those of which the photon vertex involves a weak insertion and Class II are those of which the photon vertex is purely electromagnetic (see Fig 2 and 3).

We want to single out the leading pole diagrams. First, one would expect that Class I contributions are much smaller than Class II for two reasons: (1) the weak photon vertex in Class I diagrams is due to the transition quark magnetic dipole moment (MDM) that contains a $m_s + m_d$ suppression factor or the transition quark EDM that is suppressed by $m_s - m_d$ (the latter, which vanishes if $m_s \to m_d$, is an explicit demonstration of Hara’s
Figure 3: (with reflections) Class II pole diagrams.

theorem \[26\]); (2) Class II diagrams have one more pole in the denominator. With these observations we may safely discard Class I diagrams as they are sub-leading.

Within Class II Fig. 3(a)-(d) can be shown to have an extra \(1/m_N\) suppression \[35\]. These four diagrams involve MDM-like baryon radiative transition vertices that have the structure of \((1/m_B)\epsilon^{\mu\nu\alpha\beta}q_\nu q_\alpha S_\beta\) at leading order. This structure is orthogonal to the EDM structure \(v^\mu S \cdot q\) so it cannot generate an EDM. Therefore in order to obtain an EDM one needs to go one order higher in the HB-expansion and this leads to an extra \(1/m_N\) suppression, so we can discard these four diagrams. Finally, Fig. 3(e) is smaller than Fig. 3(f)-(g) due to an extra propagator of a heavy excited state \(R\). After all these considerations, we only need to evaluate Fig. 3(f)-(g). Using Feynman rules obtained from the Lagrangian in Section II, we obtain

\[
\begin{align*}
 \frac{d_{\text{pole}}}{d_p} &= \frac{4\epsilon r_D}{9\delta_A \delta_B \cdot \delta_N \cdot \delta_\Sigma \cdot \delta_\Sigma^*} (h_D \varphi_D \{3w_F[2\delta_{\Lambda^*} \delta_\Sigma^* (\delta_{\Lambda} - \delta_\Sigma) + \delta_N \cdot \{\delta_{\Lambda^*} (\delta_{\Lambda} + \delta_\Sigma)
+ \delta_\Sigma^* (\delta_\Lambda - 3\delta_\Sigma)]\} - w_D[2\delta_{\Lambda^*} \delta_\Sigma^* (3\delta_{\Lambda} + \delta_\Sigma) + \delta_N \cdot \{3\delta_{\Lambda^*} (\delta_{\Lambda} + \delta_\Sigma)
+ \delta_\Sigma^* (\delta_{\Lambda} - \delta_\Sigma)]\} + 3h_F \varphi_F \{w_D[2\delta_{\Lambda^*} \delta_\Sigma^* (\delta_{\Lambda} - \delta_\Sigma) + \delta_N \cdot \{\delta_{\Lambda^*} (\delta_{\Lambda} - 3\delta_\Sigma)
+ \delta_\Sigma^* (\delta_{\Lambda} + \delta_\Sigma)]\} + w_F[3\delta_{\Sigma^*} (3\delta_{\Lambda^*} (\delta_{\Lambda} + \delta_\Sigma) - \delta_{\Lambda^*} (\delta_{\Lambda} - 3\delta_\Sigma)]\}
- 2\delta_{\Lambda^*} \delta_\Sigma^* (\delta_{\Lambda} + 3\delta_\Sigma)]\})
\end{align*}
\]

\[
\begin{align*}
 \frac{d_{\text{pole}}}{d_p} &= \frac{8\epsilon (\delta_{N^*} - \delta_{\Sigma^*}) (r_D + 3r_F)(w_D - w_F)(h_D \varphi_D - h_F \varphi_F)}{3\delta_N \cdot \delta_\Sigma \cdot \delta_\Sigma^*}. \quad (34)
\end{align*}
\]

In the expression above we have neglected the two small phases \(\varphi_D\) and \(\varphi_F\). Notice that Eq. (34) diverges in the \(\delta \to 0\) limit. This simply indicates that the non-degenerate perturbation theory fails in this limit and one needs to switch to degenerate perturbation
Table I: Different contributions to the SM neutron and proton EDM in units of $e\text{ cm}$, assuming the sign of LECs are those given in Section III.

| Nucleon | $|d^{\text{1-loop,1}}_N|$ | $|d^{\text{1-loop,2}}_N|$ | $|d^{\text{pole}}_N|$ |
|---------|----------------|----------------|----------------|
| neutron | $1.5 \times 10^{-32}$ | $0$ | $1.4 \times 10^{-32}$ |
| proton  | $6.1 \times 10^{-33}$ | $1.1 \times 10^{-32}$ | $1.4 \times 10^{-32}$ |

Numerical results are summarized in Table I. We alert the readers that all the numbers are only indicative of the size, because we have not yet taken the sign issue of LECs properly into account. This will be done in next section.

VI. DISCUSSION AND SUMMARY

Now we consider the uncertainty due to the undetermined relative sign between different groups of LECs. Since $r_D$ and $r_F$ are fitted simultaneously to the electromagnetic decay of $(1/2)^-$ resonance they should be multiplied by a common unknown sign factor $\delta_r$ that can take the value of $\pm 1$. The constant $g_8$ is fitted to the kaon decay rate, so it should carry another sign factor $\delta_g$. Its phase $\varphi$ however is determined theoretically so it does not have a sign ambiguity. The four remaining interaction strengths $\{h_D, h_F, w_D, w_F\}$ are fit simultaneously to s and p-wave amplitudes of the hyperon non-leptonic decay, so they should carry a common sign factor $\delta_{hw}$. Their corresponding phases are determined by first calculating Im[$h_i \exp i \varphi_i$] and Im[$w_i \exp i \tilde{\varphi}_i$] theoretically and then by dividing them by the experimentally-determined $\{h_i, w_i\}$ so the four remaining phases $\{\varphi_D, \varphi_F, \tilde{\varphi}_D, \tilde{\varphi}_F\}$ should also carry the same sign factor $\delta_{hw}$. Summing up loop and pole diagram contributions and allowing $\{\delta_r, \delta_g, \delta_{hw}\}$ to freely change between 1 and -1, we obtain a range of possible $d_n$ and $d_p$: 

$$8.7 \times 10^{-34} e\text{ cm} < |d_n| < 2.8 \times 10^{-32} e\text{ cm}$$
$$3.3 \times 10^{-33} e\text{ cm} < |d_p| < 3.3 \times 10^{-32} e\text{ cm}$$

(36)

The surprisingly-small lower bounds of $|d_n|, |d_p|$ are due to an accidental cancelation between loop and pole-diagram contributions for a very specific set of $\{\delta_i\}$. There is no reason
to believe that this cancelation still retains when higher-order diagrams are included. To estimate the size of higher-order contributions, we recall that the HB-expansion parameter is of order $m_K/m_N \sim 0.5$. Therefore to be conservative, we could assign a 100% error due to the next-to-leading-order (NLO) effects in the HB-expansion. Also, by looking at Table I we see that both loop and pole diagrams are of order $10^{-32} \, e \, cm$. So if we assume no fine cancelation between these two parts after adding the NLO contributions from the HB-expansion, then one could expect the long-distance contribution to the nucleon EDM to lie within the range:

$$1 \times 10^{-32} e \, cm < |d_n|, |d_p| < 6 \times 10^{-32} e \, cm.$$  (37)

Our estimated upper bound for $d_n$ is about a half of the corresponding value predicted in [16]. Eq. (37) is 3 (4) orders of magnitude smaller than the proposed precision level of the future proton (neutron) EDM experiment.

To summarize, even though it is well-known that the nucleon EDM induced by the Standard Model CKM matrix is well below the limit of our current experimental precision, it is still worth a thorough study as it is currently the only source of intrinsic EDMs in nature whose existence is certain. We re-analyze previous works on chiral loop and pole diagram contributions to the nucleon EDM using HBchPT at the leading order in HB-expansion, with an up-to-date determination of relevant LECs that enter our calculation. Combining with the uncertainty due to unknown relative signs of LECs and an estimate of higher-order contributions, we obtain the range for the long-distance contribution to the nucleon EDM in Eq. (37). Although a nucleon EDM of order $10^{-29} e \, cm$ as suggested in Ref. [10] could be possible as we discussed at the end of Section IV, but much of it comes from the incalculable short-distance physics which appears as counterterms in our work. In this sense, the study of the long-distance contribution provides a safe borderline such that any nucleon EDM below this borderline is consistent with the SM prediction. Finally, there are several ways to improve upon the estimate done in this work. For instance, combining lattice simulations and better experimental measurements of various hadronic decay processes, one could expect better control of both the magnitudes and signs of the required LECs. Also, a complete analysis of NLO-effects in the HB-expansion would be much desired to further restrict the allowed range of $d_n$ and $d_p$. 
Figure 4: 1-loop diagrams that vanish at LO HBchPT. The weak vertices could be placed at any allowed position and therefore are not explicitly shown.

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Appendix A: Vanishing one-loop diagrams

Here we will show that all 1-loop diagrams, other than those in Fig. 1, will not give rise to the nucleon EDM, at least at the LO in the HB-expansion.

All other possible 1-loop diagrams beside those we have calculated are summarized in Fig. 4. Since the weak Lagrangian we included in our work do not involve covariant derivatives on baryon, any baryon-photon coupling term has to arise from the ordinary P and T-conserving Lagrangian.

For Fig. 4(a), the photon vertex has to be Dirac coupling since an MDM is suppressed by $(1/m_N)^2$ as pointed out in [35]. Since the Dirac coupling is independent of the photon momentum $q$, one can define the loop momenta in a way such that the dependence of $q$ only appears in the baryon propagator. However, using the on-shell condition $v \cdot q = 0$, the baryon propagator is actually $q$-independent and so is the whole diagram. Therefore it cannot generate an EDM that is linear in $q$.

For Fig. 4(b), at the LO in the HB-expansion the $BB'\phi^\gamma$ vertex is proportional to $S^\mu$, so it cannot generate an EDM because the latter is proportional to $v^\mu$ which is perpendicular
to $S^\mu$.

For Fig. 4(c), first we notice that the $BB'\phi\phi'$ vertex cannot come from the $D$ and $F$-term of the ordinary chiral Lagrangian because that would violate parity. Therefore it can only come from $L^{(s)}_{\phi\phi'}$. In this case, it can only be parity-conserving and time reversal-conserving (PCTC), or parity-conserving and time reversal-violating (PCTV). So in order to get an EDM which is PVTV, one needs to place another PVTC or PVTV vertex in the remaining part of the diagram. This cannot be done because all the $\phi\phi'$ and $\phi\phi'\gamma$ operators we have are parity-conserving.

For Fig. 4(d), one could generate an EDM by coupling the resulting complex mass term of the baryon to its MDM. But again this contribution will be suppressed by $(1/m_N)^2$ and should be discarded in the LO of the HB-expansion.

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