The vacuum structure in a supersymmetric gauged Nambu-Jona-Lasinio model.

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Abstract
The dynamical breakdown of the $SU(2) \times U(1)$ symmetry triggered by a top-antitop condensate is studied in a supersymmetric version of the gauged Nambu-Jona-Lasinio model. An effective potential approach is used to investigate the vacuum structure and the equivalence with the minimal supersymmetric standard model. The role of the soft supersymmetry breaking terms is analyzed in detail in a version of the model where the electroweak gauge interactions are turned off.

LPTHE 93/07 (February 1993).

*Supported in part by the CEC Science project no. SC1-CT91-0729
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1 Introduction.

The idea of a dynamical symmetry breaking mechanism at work in the standard model is appealing because it requires no fundamental scalar field to exist. Technicolor [1], a priori the most attractive approach, assumes that a new QCD-type strong interaction produces bound states at a scale of the order of the electroweak breaking scale; the interaction of the $SU(2) \times U(1)$ gauge fields with these composite fields reproduces the usual mass spectrum and mixing angle in the gauge sector. Unfortunately, problems arise when one tries to give masses to the usual fermions while avoiding FCNC processes. This requires substantial modifications leading to not very economic models like Walking Technicolor [2] which, moreover, hide a yet unknown dynamics. Even taking into account recent progress [3] it is not yet clear that one has an explicit example that solves unambiguously all the problems.

One can search for other possibilities such as dynamical symmetry breaking mechanisms of the Nambu-Jona-Lasinio type which are more tractable at a computational level but are associated with non renormalisable couplings: one is forced to introduce a physical cut-off $\Lambda$ which cannot be eliminated [4]. Applied to the standard model this yields a composite Higgs particle viewed as a top-antitop bound state which reproduces the usual low energy phenomenology [5]. The infrared equivalence with the usual standard model has been proved in a $1/N_c$ expansion [6] and it has been noted that this theory lacks predictive power due to higher-dimension operators which come into play when one approaches the compositeness scale $\Lambda_c$ where the Higgs bound state dissociates [7] (see however Ref.[8]).

The central problem of giving fermions a mass while avoiding flavor changing neutral current processes, may be solved [9]. Moreover we do not have the usual tower of pseudo Goldstone bosons of technicolor models. Unfortunately, such an approach does not avoid the fine-tuning problem: to obtain a light fermion mass (compared to the cut-off scale), one must fix the four-fermion coupling with an extraordinary precision. As usual, one possibility to evade this problem is to make the model supersymmetric [10]: quadratic divergences cancel and the fermions can be made naturally light.

Of course we lose the original motivation which was to avoid elementary scalar fields. On the other hand, the scalar fields which are thus introduced are much heavier than the usual Higgs particles so that we can keep part of the motivation in saying that there are no elementary scalar degree of freedom at the presently accessible energies (i.e. electroweak unification).
One would like to be able to explicitly check the infrared equivalence with the Minimal Supersymmetric Standard Model (MSSM) which was assumed in ref. [10], where the gap equation was written in superfield language in a leading $1/N_c$ expansion.

In this respect, a first point of relevance is the role played by the soft supersymmetry breaking terms. It has been noticed for some time [11] that their presence is necessary in order to have chiral symmetry breaking. Otherwise, supersymmetry would have to be spontaneously broken and a chiral symmetry breaking ground state would be at best degenerate with the trivial vacuum. From the MSSM point of view, one soft supersymmetry breaking term plays a key role: it is the one that yields a mixing between the two Higgs doublets. It is usually written as $B\mu H_1 \cdot H_2 + h.c.$, where $\mu$ is the supersymmetric mass term present in the superpotential. If this term is not present, the theory is plagued with an unwanted axion and zero v.e.v. for $H_1$ or $H_2$. The supersymmetric parameter $\mu$ appears naturally in the linearized version of the supersymmetric Nambu-Jona-Lasinio model. On the other hand, $B$, as any other supersymmetry breaking parameter, is put by hand: one invokes for that an underlying supergravity theory, whose precise content is not specified. We will study below in detail the role played by this parameter $B$ in the chiral symmetry breaking.

To check the equivalence, it is also of importance to include the effect of electroweak gauge interactions through the auxiliary fields in the gauge sector (D-terms). Indeed, in the MSSM, the electroweak gauge couplings are necessary to get the symmetry breaking pattern of $SU(2) \times U(1)$: they give the only quartic interactions in the scalar potential. Hence, neglecting them means either vanishing Higgs vacuum expectation values or unbounded potential. These couplings must therefore be taken into account when checking the equivalence. This is most easily done in the component formalism where the relevant D-terms can readily be isolated (by contrast with the superfield approach).

Our approach consists in writing an effective potential for the composite Higgs fields in the leading $1/N_c$ expansion, in the spirit of refs. [1] and [12]. We then investigate the vacuum structure and the possible second order phase transitions in the space of the parameters appearing in the lagrangian. In order to do so, a gauged supersymmetric version of a Nambu-Jona-Lasinio model is written in a linearized form introducing two auxiliary fields $H_1$ and $H_2$. Through radiative corrections they acquire a dynamics, becoming propagating degrees of freedom. Their quantum numbers match with those of the Higgs fields in the MSSM. The effective potential for the scalar compo-
nents of $H_1$ and $H_2$, in leading $\frac{1}{N}$, is written and the saddle point equations analyzed to find the real vacuum.

Section 2 gives a short presentation of the $\frac{1}{N}$ expansion as needed in what follows. In section 3, we analyze the role of the soft supersymmetry breaking parameters, more particularly of the $B$ term which plays an important role in the MSSM. We study the dependance on $B$ of the critical parameters of the second order phase transition associated with the breaking of chiral symmetry in a non-gauged version of the model. This also allows us to test the efficiency of the method that we will use later on the gauged model. Section 4 derives the $\beta$ function and discusses the symmetries together with the scalar spectrum for this non-gauged version. Section 5 deals with the vacuum structure of the gauged model; a detailed discussion of the equivalence with the MSSM is presented. Finally, Section 6 concludes. Some computational details are also given in an appendix.

2 The $1/N$ expansion.

Non-perturbative summation methods for Feynman graphs like the semi classical development and the $\frac{1}{N}$ expansion give important information about the pattern of symmetry breaking, non trivial fixed points in the renormalisation group evolution, bound states formation, etc. The $\hbar$ semiclassical development corresponds to an expansion in the number of loops and the first one-loop order is easily written in a compact form. This is to be contrasted with the $\frac{1}{N}$ expansion, especially in the gauge field sector of an non-abelian theory where a resummation to a given order cannot be performed analytically. We may easily understand the qualitative difference by trying to give a criterion for computing the leading order in the two cases:

(a) In the semiclassical $\hbar$ development, one uses the partition function:

$$Z = \int \mathcal{D}\Phi e^{\frac{i}{\hbar} S[\Phi]}$$

where $\Phi$ is a set of quantum fields and $S[\Phi]$ is the action of the general model considered. The order of a given Feynman graph is given by:

$$\hbar^{L-V} = \hbar^{L-1}$$

where $L$ is the number of loop(s) in the Feynman graph, this formula being valid irrespective of the dynamics of the model. In the leading order,
the vacuum structure can be studied using the Coleman-Weinberg effective potential \[13\].

(b) The $1/N$ expansion is model dependent. Take, for example, a Gross-Neveu $U(N)$ theory with $N$ fermions \[12\] and study the dynamical breakdown of the chiral symmetry. Introducing an auxiliary scalar field, the Lagrangian is given by ($g_0$ is independent of $N$):

\[
\mathcal{L} = \bar{\Psi}^a \gamma^\mu \partial_\mu \Psi^a + \frac{g_0}{2N} (\bar{\Psi}^a \Psi^a)^2 - \frac{N}{2g_0} (\sigma - \frac{g_0}{N} (\bar{\Psi}^a \Psi^a))^2
\]

\[
= \bar{\Psi}^a \gamma^\mu \partial_\mu \Psi^a - \frac{N}{2g_0} \sigma^2 + \sigma \bar{\Psi}^a \Psi^a
\]

\[
= N (\bar{\Psi}^a \gamma^\mu \partial_\mu \Psi^a - \frac{1}{2g_0} \sigma^2 + \sigma \bar{\Psi}^a \Psi^a)
\]  
(1)

where in the last line we have performed the rescaling $\Psi \rightarrow \sqrt{N} \Psi'$, which modifies the partition function with an irrelevant constant. The main difference with respect to the loop $(\hbar)$ expansion is that every closed fermion loop gives an additional $N$ factor. So, for an arbitrary graph, the associated factor is

\[
N^{-I+V+L_f} = N^{1-(L-L_f)}
\]

where $L_f$ is the number of fermion loops ($L_f < L$). The leading order corresponds to $L - L_f = 0$. More precisely, in the model that we consider, it is $L = L_f = 1$. Indeed, in any one-particle irreducible diagram, the introduction of scalar internal lines make the diagram subleading. Thus, the leading diagrams correspond to a single fermion loop and external scalar fields $\sigma$ only. The effective potential for the $\sigma$ field is exactly given in the leading order by the Coleman-Weinberg expression, or equivalently in the Jackiw functional form \[13\]:

\[
e^{iS(\sigma_c)} = \int \mathcal{D}(\Psi^a, \sigma_q) \exp S_{\text{quadratic}}(\sigma_c + \sigma_q, \bar{\Psi}^a, \Psi^a).
\]

Several points actually depend on the model considered:

- The factor associated with fermion loops ($N$) depends on the group representation.
- The leading order condition $L - L_f = \text{minimum}$, has solutions which depend on the interaction lagrangian (e.g. gauge fields).
- More generally, it is not clear whether every model has a non trivial $\frac{1}{N}$ development.
- If we want to apply the effective potential computation to the ground state determination, we may find an N dependent vacuum expectation value which will change the leading order contribution. This corresponds, in terms of Feynman graphs to a multiplication of every external leg with a N dependent factor.

Being interested in the dynamical breaking of the chiral symmetry in supersymmetric models, we study in the leading order in N the linearised version of the supersymmetrized Gross-Neveu lagrangian \[10\]. This lagrangian is written in a superfield language after performing the superfield rescaling

\[
\Phi_i \rightarrow \sqrt{N} \Phi_i, \quad H_{1,2} \rightarrow \sqrt{N} H_{1,2}
\]

as

\[
\mathcal{L} = N \left\{ \int d^4 \Theta [\Phi_i^+ \Phi_i (1 - \Sigma^2 \Theta^2 \bar{\Theta}^2)] + H_1^+ H_1 (1 - \Delta^2 \Theta^2 \bar{\Theta}^2) \\
+ \int d^2 \Theta H_2 (m H_1 - g_0 \Phi_i \Phi_i) + \int d^2 \bar{\Theta} H_2^+ (m H_1^+ - g_0 \Phi_i^+ \Phi_i^+) \\
+ \int d^2 \Theta H_2 H_1 B m \Theta^2 + \int d^2 \bar{\Theta} H_2^+ H_1^+ B m \bar{\Theta}^2 \right\}
\]

where \(i\) runs from 1 to \(N\). We have included soft supersymmetry breaking terms \((\Sigma, \Delta \text{ and } B)\). The same analysis as above yields for a given supergraph a factor:

\[
N^{-I+V+L_{\Phi_i}} = N^{1-(L-L_{\Phi_i})}.
\]

The leading N contribution is given by \(L - L_{\Phi_i}\) minimum which again means \(L = L_{\Phi_i} = 1\), that is no \(\Phi_i\) on the external legs.

Consequently, writing the effective potential \(V(H_1,H_2)\) in the leading N expansion is equivalent to using the Coleman-Weinberg one-loop formula, taken care of the fact that the different v.e.v. of the different fields do not change this result. As we will explicitly verify, it does not happen to be the case here but it does occur in non-minimal supersymmetric generalisations of the Gross-Neveu model.

If we gauge the lagrangians (1) or (3), the analysis is different, due to the non-abelian structure of the symmetry group. As is well-known, the leading contribution is then given by all planar diagrams \([14]\). However, we are not so much interested here in gauging the degrees of freedom that correspond to the strong interactions as in including the effect of electroweak \(SU(2) \otimes U(1)\) gauge interactions. Therefore, when we deal with this question in section
we will introduce $SU(2) \otimes U(1)$ propagating gauge degrees of freedom whereas we will still consider QCD gauge degrees of freedom as some sort of flavor number (taking $N$ values).

For the time being, we will consider the simpler case where no gauge degrees of freedom are propagating and we will get a closer look at the role played by soft supersymmetry breaking terms.

3 Dynamical chiral symmetry breaking (DCSB) in the minimal supersymmetric Gross-Neveu model in the $1/N$ expansion.

The supersymmetric Nambu-Jona-Lasinio model is readily written in its simplest form as:

$$\mathcal{L} = \int d^4 \Theta [\Phi_i^+ \Phi_i + G \Phi_i^+ \Phi_i \Phi_j^+ \Phi_j].$$

It has been immediately realized [11] that, in order to write a linearized version of this model, one needs two chiral superfields $H_1$ and $H_2$ whose couplings are described by eq.(3) with $G = g^2/m^2$ (eq.(3) includes also, as emphasized above, soft supersymmetry breaking terms). This doubling of the auxiliary fields is a welcome feature since it also reproduces some of the couplings of the MSSM [10]. Indeed, one naturally finds in (3) a supersymmetric mass term $mH_1H_2$ mixing the two Higgs. This is reminiscent of the $\mu$ parameter of the MSSM. In the MSSM, the presence of the $\mu$ term is a problem (the so-called $\mu$ problem [13]) because $\mu$ is naturally either zero or of the order of the underlying scale, whose role is played here by our cut-off $\Lambda$. We will return to this problem shortly.

We wish to study in detail in this section the role of the soft supersymmetry breaking terms introduced in (3). As noticed earlier [11, 10] and emphasized above, such terms – in fact the $\Sigma$ mass term – are necessary in order to get a chiral symmetry breaking ground state. We will rederive this result but will be more interested in the role played by the $B$ parameter which is so important in the MSSM. We will in particular study how the second-order phase transition depends on this parameter.

\footnote{It is important to note that, despite a kinetic term for $H_1$ in (3), both superfields $H_1$ and $H_2$ are in fact composite fields which can be eliminated through their equations of motion.}
The dynamics resulting from lagrangian (3) can be studied diagrammatically in relation with DCSB [10]. Alternatively, we will compute by functional methods the effective potential for $H_1$ and $H_2$ and minimize it. A nonzero value for $< H_1 >$ or $< H_2 >$ breaks a U(1) chiral symmetry which will be precisely defined in the next section.

In components, the lagrangian (3) reads:

\[ L = F_1^* F_1 + z_1^* (\Box - \Delta^2) z_1 - i \bar{\Psi}_1 \sigma^m \partial_m \Psi_1 + F_1^* F_1 + z_2^* (\Box - \sigma^2) z_2 - i \bar{\Psi}_2 \sigma^m \partial_m \Psi_2 + m (z_1 F_2 + z_2 F_1 - \bar{\Psi}_1 \Psi_2 - \bar{\Psi}_2 \Psi_1) - g (F_2 z_1 z_2 + 2 F_1 z_2 z_1 - z_2 \bar{\Psi}_1 \Psi_1 - 2 z_1 \bar{\Psi}_2 \Psi_2 + h.c.) - B m (z_1 z_2 + z_1^* z_2^*). \] (5)

Integrating over $F_i$ (we will verify that $< F_i > = 0$) and using the formula for the effective potential in the leading one-loop order:

\[ V_{\text{eff}} = V_{\text{tree}} + \frac{1}{2} S T r \int \frac{d^d p}{(2\pi)^d} \ln(p^2 + M^2) \] (6)

where $S T r F(M^2) = Tr(M^2) - 2 Tr(M^2)$, we readily obtain

\[ V_{\text{eff}} = V_{\text{tree}} + \frac{N}{2} \int \frac{d^d p}{(2\pi)^d} \left[ \ln(p^2 + \Sigma^2 + 4g^2 |z_2|^2 + 2g \sqrt{F_2 F_2^*}) + \ln(p^2 + \Sigma^2 + 4g^2 |z_2|^2 - 2g \sqrt{F_2 F_2^*}) - 2 \ln(p^2 + 4g^2 |z_2|^2) \right]. \] (7)

As noticed earlier, there are no one-loop contribution to the $z_i$-dependent part of the potential. Minimizing with respect to $z_1^*$, $F_1^*$, $z_2^*$, $F_2^*$ and $z_2^*$ yields the following equations:

\[ z_i = 0 \] (8)

\[ F_1 + m z_2^* = 0 \] (9)

\[ \Delta^2 z_1 - m F_2^* - B m z_2^* = 0 \] (10)

\[ ^2 \text{We have undone the rescaling (2) and redefined the coupling } g \text{ in order to include the } N \text{ dependence: } g = g_0 / \sqrt{N}. \]
\[ m z_1^* + 2g^2 F_2 N \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2 + 4g^2 |z_2|^2)^2 - 4g^2 F_2 F_2^*} = 0 \quad (11) \]

\[ 4g^2 z_2 N \int \frac{d^d p}{(2\pi)^d} \left[ \frac{p^2 + \Sigma^2 + 4g^2 |z_2|^2}{(p^2 + \Sigma^2 + 4g^2 |z_2|^2)^2 - 4g^2 F_2 F_2^*} - \frac{1}{p^2 + 4g^2 |z_2|^2} \right] \]

\[ - m F_1^* - B m z_1^* = 0 \quad (12) \]

Since \( F_i^* = 2g z_2 z_1 \), we obtain the announced result \( F_i = 0 \). Combining the four non-trivial equations we find the system:

\[ B \frac{m^2}{\Delta^2} z_2 = -F_2 \left[ \frac{m^2}{\Delta^2} + 2g^2 N \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2 + 4g^2 |z_2|^2)^2 - 4g^2 F_2 F_2^*} \right] \quad (13) \]

\[ B \frac{m^2}{\Delta^2} F_2 = z_2 \left\{ 4g^2 N \int \frac{d^d p}{(2\pi)^d} \left[ \frac{p^2 + \Sigma^2 + 4g^2 |z_2|^2}{(p^2 + \Sigma^2 + 4g^2 |z_2|^2)^2 - 4g^2 F_2 F_2^*} \right. \right. \]

\[ - \left. \left. \frac{1}{p^2 + 4g^2 |z_2|^2} \right] + m^2 (1 - \frac{B^2}{\Delta^2}) \right\} \quad (14) \]

Of course, because the original theory is non-renormalisable, all integrals have to be cut off at a scale \( \Lambda \). For future use we give the explicit form in four dimensions:

\[ B \frac{m^2}{\Delta^2} z_2 = F_2 \left[ -\frac{m^2}{\Delta^2} + \frac{Ng^2}{16\pi^2} \left( \frac{(\Sigma^2 + 4g^2 |z_2|^2)^2}{\Lambda^4} - 4g^2 F_2 F_2^* \right) \right. \]

\[ + \frac{1}{2g\sqrt{F_2 F_2^*}} (\Sigma^2 + 4g^2 |z_2|^2) \ln \left( \frac{\Sigma^2 + 4g^2 |z_2|^2}{\Sigma^2 + 4g^2 |z_2|^2 - 2g\sqrt{F_2 F_2^*}} \right) \]

\[ + O\left( \frac{1}{\Lambda^2} \right) \quad (15) \]

\[ \frac{m^2}{\Delta^2} F_2 = z_2 \left\{ \frac{Ng^2}{8\pi^2} \left( \frac{(\Sigma^2 + 4g^2 |z_2|^2)^2}{\Lambda^4} - 4g^2 F_2 F_2^* \right) \right. \]

\[ - 8g^2 |z_2|^2 \ln \left( \frac{\Sigma^2 + 4g^2 |z_2|^2}{\Sigma^2 + 4g^2 |z_2|^2 - 2g\sqrt{F_2 F_2^*}} \right) \]

\[ + 2g\sqrt{F_2 F_2^*} \ln \left( \frac{\Sigma^2 + 4g^2 |z_2|^2}{\Sigma^2 + 4g^2 |z_2|^2 - 2g\sqrt{F_2 F_2^*}} \right) \]
\[ + m^2 \left( 1 - \frac{B^2}{\Delta^2} \right) \{ + \mathcal{O}(\frac{1}{\Lambda^2}) \} \] (16)

The equations can easily be recast in a form which was already obtained by Carena et al. [10] (see appendix, section iii).

The case \( B = 0 \) has been studied in detail by Clark, Love and Bardeen (CLB) in ref. [10]. In agreement with their analysis, we immediately obtain from equations (11) and (13) that \( F_2 = z_1 = 0 \); equation (14) then reads:

\[
z_2 \left[ m^2 - 4g^2 N \Sigma^2 \int_{\Lambda}^{\infty} \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2 + 4g^2 |z_2|^2)(p^2 + 4g^2 |z_2|^2)} \right] = 0 \] (17)

which has a non trivial solution \( < z_2 > \neq 0 \) for \( G \equiv g^2/m^2 > G_c \), with \( G_c \) defined by the equation:

\[
G_c^{-1} = 4N\Sigma^2 \int_{\Lambda}^{\infty} \frac{d^d p}{(2\pi)^d} \frac{1}{p^2(p^2 + \Sigma^2)} . \] (18)

This gives \( G_c^{-1} = \frac{N\Sigma^2}{4\pi} \ln \frac{\Lambda^2}{\Sigma^2} \) in \( d = 4 \) spacetime dimensions.

We can readily verify that this corresponds to the true ground state of the theory for \( G < G_c \) because, using the saddle point equations, one can derive:

\[
V( < z_2 >) - V(0) = \frac{N\Sigma^4}{32\pi^2} f(4g^2 |< z_2 >|^2) \] (19)

where, for \( d = 4 \), \( f(x) = (1 - x^2) \ln (1 + x) + x^2(\ln x - 1) < 0 \).

Before leaving this simple case, let us note the form of the nontrivial solution of (17) in the case of \( d = 4 \) spacetime dimensions and a large cut-off \( \Lambda \):

\[
G_c^{-1} = \frac{m^2}{g^2} = \frac{N}{4\pi^2} \left[ \Sigma^2 \ln \frac{\Lambda^2}{\Sigma^2 + 4g^2 |z_2|^2} - 4g^2 |z_2|^2 \ln \frac{\Sigma^2 + 4g^2 |z_2|^2}{4g^2 |z_2|^2} \right] \] (20)

Due to supersymmetry cancellations, the quadratic divergence familiar in the Nambu-Jona-Lasinio model has cancelled and we are left with logarithmic divergences only [11, 10]. This avoids an untolerable fine-tuning but has the following drawback. The original coupling \( G \) behaves typically as
\[ \Sigma \ln \left( \frac{\Lambda^2}{\Sigma^2} \right) \] which is surprisingly large for a theory whose basic scale is \( \Lambda \) (one would expect \( G = O(\frac{1}{\Lambda^2}) \)). This is in fact nothing but the old \( \mu \)-problem of the MSSM which shows up here in a different language. Indeed, \( G^{-1} \) being small is just a reflection of the fact that the parameter \( m \) is unnaturally small compared to \( \Lambda \) in this theory.

The analysis in the case where the soft symmetry breaking term \( B \) is non-zero is more involved. We first sketch the principles of the method that we use. Suppose that, for a critical value of the coupling \( G = G_c \), we have a second order (i.e. continuous) phase transition, similar to the one found when \( B = 0 \), from the trivial vacuum \( z_2 = F_2 = 0 \) to a non trivial one. In the broken phase, close to the critical point, i.e. for \( G = G_c(1 + \epsilon) \), \( 0 < \epsilon \ll 1 \), both \( z_2 \) and \( F_2 \) must be small:

\[
\frac{4g^2|z_2|^2}{\Sigma^2} \equiv \epsilon x_1, \quad \frac{g^2|F_2|^2}{\Sigma^4} \equiv \epsilon x_2
\] (21)

where \( x_1 \) and \( x_2 \) are at most of order 1. Introducing these expressions in the saddle point equations, we can solve them order by order in \( \epsilon \).

To the order \( \epsilon^0 \), we find an equation for \( G_c \) which is precisely the condition that the transition from the trivial vacuum to the nontrivial one is continuous (second order). We can solve it for \( G_c \).

To the order \( \epsilon \), we find the behaviour of the order parameter \( z_2 \) or \( F_2 \) in terms of the control parameter \( \epsilon = \frac{G - G_c}{G_c} \), from which we can extract the critical exponents.

The linearized equations (15) and (16) read, to the order \( \epsilon \),

\[
F_2 \left\{ -\frac{1}{\Delta^2} + \frac{NG_c}{16\pi^2}(1 + \epsilon)[2 - 4\ln \frac{\Lambda}{\Sigma} + \frac{16g^2|z_2|^2}{\Sigma^2} - \frac{4g^2|F_2|^2}{\Sigma^4}] \right\} = \frac{B}{\Delta^2} z_2, \quad (22)
\]

\[
z_2 \left\{ -\frac{NG_c\Sigma^2}{2\pi^2}(1 + \epsilon)(1 + \frac{4g^2|z_2|^2}{\Sigma^2})\left( -\frac{2g^2|z_2|^2}{\Sigma^2} + \frac{g^2|F_2|^2}{\Sigma^4} + \ln \frac{\Lambda}{\Sigma} \right) \right.
- \frac{2g^2|z_2|^2}{\Sigma^2} \ln \frac{\Lambda^2}{4g^2|z_2|^2} - \frac{2g^2|F_2|^2}{\Sigma^4} \left. \right] + 1 - \frac{B^2}{\Delta^2} \right\} = \frac{B}{\Delta^2} F_2. \quad (23)
\]

Of course, a consistency check on the validity of our approximation will have to be performed a posteriori.

To the order \( \epsilon^0 \), we find:

\[
\frac{B^2}{\Delta^4} = \left[ -\frac{1}{\Delta^2} + \frac{NG_c}{16\pi^2}(2 - 4\ln \frac{\Lambda}{\Sigma}) \right] \left[ 1 - \frac{B^2}{\Delta^2} - \frac{NG_c\Sigma^2}{2\pi^2} \ln \frac{\Lambda}{\Sigma} \right] \quad (24)
\]
which reduces to (18) in the limit \( B = 0 \). Solving (24) and retaining only the leading terms\(^3\) in \( \ln \frac{\Lambda}{\Sigma} \), we obtain a unique positive solution given by:

\[
\frac{N G_c}{8\pi^2} = \frac{(\Delta^2 - 2\Sigma^2 - B^2) + \sqrt{(\Delta^2 - 2\Sigma^2 - B^2)^2 + 8\Sigma^2\Delta^2}}{8\Sigma^2\Delta^2 \ln \frac{\Lambda}{\Sigma}} \tag{25}
\]

We note that \( G_c \) decreases with \( B \). Hence \( G_c(B \neq 0) < G_c(B = 0) \). The maximum value for \( B \) (beyond which the effective potential becomes unbounded) is \( B^2 = \Delta^2 \), in which case we obtain:

\[
(G_c)_{\text{min}} = \frac{1}{4\Delta^2 \ln \frac{\Lambda}{\Sigma}} \left[ \sqrt{1 + \frac{2\Delta^2}{\Sigma^2}} - 1 \right] \tag{26}
\]

The behavior of \( G_c \) as a function of \( B \) is schematically represented on Fig.1.

To study the behaviour of the condensates near the critical point, we take into account the terms of order \( \epsilon \) (and for that matter \( \epsilon \ln \epsilon \)) in equation (22) and (23).

Defining \( \xi_1 \equiv 4g^2|z|^2/\Sigma^2 \) and \( \xi_2 \equiv g^2|F|^2/\Sigma^4 \) and combining (22) and (23), one obtains an equation of the form:

\[
\xi_2 = -x\epsilon + y\xi_1 - z\xi_1 \ln \xi_1 \tag{27}
\]

where the quantities \( x, y \) and \( z \) are strictly positive. Using (23) and the fact that \( \epsilon, \xi_1, \xi_2 \ll 1 \), we get in the leading order:

\[
\xi_1[(1 - \frac{B^2}{\Delta^2}) - \frac{N G_c\Sigma^2}{2\pi^2} \ln \frac{\Lambda}{\Sigma})^2 - \frac{B^2\Sigma^2}{\Delta^4} y] + \frac{B^2\Sigma^2}{\Delta^4} z\xi_1 \ln \xi_1 = -\frac{B^2\Sigma^2}{\Delta^4} x\epsilon \tag{28}
\]

For \( \xi_1 \to 0, \xi_1 \ll \xi_1|\ln \xi_1| \). The first term of the left-hand side is negligible and one can check that (28) has a nontrivial solution \( \xi_1 \neq 0 \) for \( \epsilon > 0 \) only (\( \ln \xi_1 \) is negative). One finds explicitly:

\[
\frac{2B^2\Sigma^2}{\Delta^4} \xi_1 \ln \xi_1 = \left[ (2\ln\Delta\Sigma - 1)(1 - \frac{B^2}{\Delta^2}) - \frac{N G_c\Sigma^2}{2\pi^2} \ln \frac{\Lambda}{\Sigma})^2 + \frac{4B^2\Sigma^2}{\Delta^4} \ln \frac{\Lambda}{\Sigma}\right] \epsilon \tag{29}
\]

\(^3\)Having in mind phenomenological applications, we can take for example \( \Lambda \simeq 10^{16} \) GeV and \( \Sigma \simeq 1 \) TeV.
Figure 1: The critical coupling $G_c$ as a function of $B$. 
The non-zero v.e.v. for $z_1$ is obtained from (29) and the value of $\xi_2$, using the equation (10). One can now check a posteriori that all the terms discarded to obtain the result (31) were of higher order.

As always in the mean field approximation for a second order phase transition, the critical coefficient $\beta$ is $\frac{1}{2}$ [13]. A way to show this is to perform the computation in $d = 4 + \eta$ dimensions and to take the limit $\eta \to 0$ in the expression for $\beta$. The property that identifies a phase transition in this model at $G = G_c$, is that for $G < G_c, (\epsilon < 0)$, $\xi_1 = \xi_2 = 0$ i.e. no condensate can be formed. It is second order because the condensates appear in a continuous way from a zero value at $G_c$, as the Nambu-Jona-Lasinio model was precisely designed for.

4 The $\beta$ function, symmetries and the mass spectrum.

Our interest in this section is twofold. First, derive the $\beta$ function for the model described in the last section. Second, study its symmetries and the corresponding scalar mass spectrum. This will prove to be useful in the next section when we undertake to gauge the electroweak degrees of freedom. It will allow us to compare the vacuum structure of the gauged and non-gauged model. Since this structure is richer when $B = 0$, we restrict here our analysis of the symmetries to this case.

4.1 The $\beta$ function.

To compute the $\beta$ function, we need only to consider the renormalized parameters $g, m$ and $G = g^2/m^2$ as they appear from the one-loop effective potential. The ultraviolet divergences are summarized in four dimensions by the following counterterm:

$$\mathcal{L}_{ct} = -\frac{1}{64\pi^2}(S T r M^4) \ln \Lambda^2$$

$$S T r M^4 = N(2\Sigma^4 + 16g^2|z_2|^2\Sigma^2 + 8g^2|F_2|^2).$$

This amounts to redefining the tree level mass parameter as follows:

$$m^2 \rightarrow m^2(1 + \frac{NG}{4\pi^2}\Sigma^2 \ln \Lambda^2)$$
whereas there is no counterterm associated with $g$ to the leading order in $N$ (see section 3).

Also, using the fact that the term $|F_2|^2$ is a kinetic term for the $H_2$ auxiliary field, one infers from (30) the wave function renormalisation constant for $z_2$

$$Z = 1 - \frac{Ng^2}{8\pi^2} \ln \Lambda^2.$$  \hspace{1cm} (32)

However the redefinition $z'_2 = Z^{\frac{1}{2}} z_2$ modifies both couplings $g$ and $m$ with the same factor $Z^{-\frac{1}{2}}$, which cancels in $G = g^2/m^2$. Therefore only (31) will contribute to the renormalisation of $g$ which gives:

$$\beta(G) = \frac{\partial G}{\partial \ln \Lambda} = -\frac{N\Sigma^2}{2\pi^2} G^2.$$ \hspace{1cm} (33)

So the theory is asymptotically free. More generally, the computation of the $\beta$ function can be performed in $d = 4 + \eta$ dimensions using the fact that the B term does not affect the ultraviolet properties in the leading N order. Then, setting $B = 0$, we derive the gap equation obtained from (17) and (18) ($g^2 \equiv g^2 \Lambda^{-\eta}$)

$$G_c^{-1} - G^{-1} = 4N\Sigma^2 \int^{\Lambda} \frac{d^d p}{(2\pi)^d} \frac{4g^2 |z_2|^2 (2p^2 + \Sigma^2 + 4\hat{g}^2 |z_2|^2)}{p^2 (p^2 + 4\hat{g}^2 |z_2|^2) (p^2 + \Sigma^2 + 4\hat{g}^2 |z_2|^2)}$$ \hspace{1cm} (34)

with respect to the cut-off $\Lambda$ to obtain the $\beta$ function:

$$\beta(G) = \eta G - \frac{\eta}{G_c} G^2 = \eta G - 4N\Sigma^2 C_d G^2$$ \hspace{1cm} (35)

where we have substituted an explicit expression for the non trivial ultraviolet fixed point (18):

$$G_c = \frac{\eta}{4N\Sigma^2 C_d}, \quad C_d = \frac{2}{\Gamma(d/2)(4\pi)^{d/2}}.$$ \hspace{1cm} (36)

Obviously, this fixed point goes to the origin in four dimensions.

### 4.2 Symmetries of the SUSY Gross-Neveu model and the mass spectrum in the case $B=0$.

We briefly discuss the symmetries of the lagrangian and compute the mass spectrum in the case $B=0$ for the sake of simplicity. Let us summarize the symmetries of the problem:
The $U(1)_R$ symmetry is broken by a $B \neq 0$ term. The non-trivial vacuum has $< z_i > = 0$, $< z_2 > \neq 0$, $< F_1 > \neq 0$, $< z_1 >= < F_2 >= 0$. So, looking at the transformation laws under the three symmetry groups indicates that:

- $O(N)$ remains unbroken;
- $U(1)_H$ and $U(1)_R$ are separately broken, but the combination $U(1)_{H+R}$ ($\beta = \alpha$) remains unbroken; on the other hand the orthogonal combination $U(1)_{H-R}$ corresponding to $\beta = -\alpha$ is spontaneously broken and yields a Goldstone boson in the mass spectrum.

To determine the full spectrum, we can use the constraint (9) to eliminate $F_1$ from $V_{\text{eff}}$, obtained from (5) and (7):

$$V_{\text{eff}}(z_1, z_2, F_2(z_1, z_2)) = m^2 |z_2|^2 + \Delta^2 |z_1|^2 - m(z_1 F_2 + z_1^* F_2^*)$$

$$+ \frac{N}{2} \int \frac{d^d p}{(2\pi)^d} \left[ \ln \left( (p^2 + \Sigma^2 + 4 g^2 |z_2|^2)^2 - 4 g^2 F_2 F_2^* \right) - 2 \ln (p^2 + 4 g^2 |z_2|^2) \right] \left( \frac{p^2}{2} + \Sigma + 4 g |z_2|^2 \right)$$

The mass matrix for a function $V(x_i, F_k(x_j))$ is given by the general formula (repeated indices are summed):

$$\frac{d^2 V}{dx_i dx_j} = \frac{\partial^2 V}{\partial x_i \partial x_j} + \frac{\partial F_k}{\partial x_j} \frac{\partial^2 V}{\partial x_i \partial F_k}$$

where we have used the constraint equation for $F_k$: $\partial V/\partial F_k = 0$.

Making use of the explicit constraint equation for $F_2$:

$$m z_1^* + 2 g^2 F_2 N \int \frac{d^d p}{(2\pi)^d} \left[ \frac{1}{(p^2 + \Sigma^2 + 4 g^2 |z_2|^2)^2 - 4 g^2 F_2 F_2^*} \right] = 0$$
we obtain:

i) in the trivial vacuum.

Here, the Goldstone boson disappears since no symmetry is broken. Indeed, the mass matrix elements are:

\[
\begin{align*}
\frac{d^2V}{dz_1^2} &= \frac{d^2V}{dz_2^2} = \frac{d^2V}{dz_1^2} = 0 \\
\frac{d^2V}{dz_1^2dz_2} &= m^2[1 - 4NG \Sigma^2 ] = m^2[1 - \frac{G}{G_c}] \\
\frac{d^2V}{dz_1^2dz_1} &= \Delta^2 + \frac{m^2}{2g^2N \int \frac{d^4p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2)^2}} \tag{40}
\end{align*}
\]

So, for \( G > G_c \), the trivial vacuum is unstable and the other vacuum (see below) becomes stable.

ii) in the nontrivial vacuum (\( z_1 = F_2 = 0, z_2, F_1 \neq 0 \)).

\[
\begin{align*}
\frac{d^2V}{dz_1dz_2} &= \frac{d^2V}{dz_1^2} = \frac{d^2V}{dz_2^2} = 0 \\
\end{align*}
\]

The computation reveals two scalars of mass \( m_1 \) (a scalar and a pseudoscalar) in the \( H_1 \) sector, the expected Goldstone boson and a scalar of mass \( m_2 \) in the \( H_2 \) sector. Using the notation \( m_T \equiv \sqrt{4g^2|z_2|^2} \), we write their masses as:

\[
\begin{align*}
m_1^2 &= \Delta^2 + \frac{m^2}{2g^2N \int \frac{d^4p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2 + m_T^2)^2}} \\
m_2^2 &= 8N g^2 m_T^2 \int \frac{d^4p}{(2\pi)^d} \left[ \frac{1}{(p^2 + m_T^2)^2} - \frac{1}{(p^2 + \Sigma^2 + m_T^2)^2} \right] \tag{41}
\end{align*}
\]

In presence of the gauge interactions, we will see that \( m_2 \) tends to decrease and this solution may disappear for large gauge couplings.

5 The vacuum structure of the gauged model.

Until now, we have only studied, in the leading \( \frac{1}{N} \) approximation, global symmetries spontaneously broken by fermionic condensates. To make con-
nection with the standard model, we now include gauged degrees of freedom and consider models of gauge symmetry $SU(N) \otimes SU(2)_L \otimes U(1)_Y$ with global $SU(N)$ and local $SU(2)_L \otimes U(1)_Y$. Because we are mainly concerned with the scalar Higgs sector, we only take into account the electroweak gauge sector leaving aside the $SU(N)$ color part which introduces technical difficulties such as the handling of the planar graphs in the leading approximation [4]. In any case, the Higgs particles being color singlets, the $SU(N)$ part introduces corrections that will not change the vacuum structure of the electroweak sector.

The interest in the gauged $SU(2)_L \times U(1)_Y$ model comes mainly from the fact that, in the minimal supersymmetric standard model (MSSM) the only quartic self interactions in the Higgs potential are given by the corresponding gauge couplings which are thus essential to the vacuum structure. Hence any comparison of a supersymmetric condensate model with the MSSM must include them. We start with the following lagrangian:

$$\mathcal{L} = \int d^4 \Theta \left\{ (\Phi_+^{\alpha} e^{q_1 y_1 + q_2 y_2} \Phi_{\alpha} + \tilde{\Phi}_+^{\alpha} e^{q_1 \tilde{y}_1 + q_2 \tilde{y}_2} \tilde{\Phi}_{\alpha}) (1 - \Sigma^2 \Theta^2) \right\}$$
$$+ \int d^2 \Theta \left[m H_1 H_2 (1 + B \Theta^2) - g H_2 \Phi_\alpha \tilde{\Phi}_\alpha \right]$$
$$+ \int d^2 \bar{\Theta} \left[m H_1 H_2^+ (1 + B \Theta^2) - g H_2^+ \Phi_\alpha \tilde{\Phi}_\alpha \right]$$

(42)

where $\alpha = 1 \ldots N$ are color indices, $q_1$ and $q_2$ are the $U(1)_Y$ and the $SU(2)_L$ gauge couplings respectively. Note that $H_1$, $H_2$, and $\Phi_\alpha$ are now $SU(2)$ doublets ($\Phi_\alpha = \Phi_{\alpha i}, i = 1, 2$, etc. and $H_1 H_2 = \epsilon_{ij} H_{1i} H_{2j}$) whereas $\tilde{\Phi}_\alpha$ are $SU(2)$ singlets. Gauge invariance requires:

$$Y_1 = -Y_2 = Y + \tilde{Y}.$$  

(43)

The D components for $SU(2)$ and $U(1)$ vector superfields are given respectively by:

$$D^a = q_2 (z_{a}^+ T^a z_{a} + z_{1}^+ T^a z_{1})$$
$$D = q_1 (Y z_{a}^+ z_{a} + \tilde{Y} \tilde{z}_{a}^+ \tilde{z}_{a} + Y_{1} z_{1}^+ z_{1})$$

(44)

with $T^a$ being a representation of the $SU(2)$ generators. Writing the component lagrangian and integrating over:
\[ F^*_{\alpha j} = g \epsilon_{ij} z_2 \tilde{z}_\alpha \]
\[ \tilde{F}_\alpha = g \epsilon_{ij} \tilde{z}_2 z_\alpha, \]

we obtain:

\[ \mathcal{L} = F_i^+ F_i + z_i^+ (\Box - \Delta^2) z_i - i \tilde{\psi}_i \bar{\sigma}^m \partial_m \psi_i + z_\alpha^+ (\Box - \Sigma^2) z_\alpha \]
\[-i \tilde{\psi}_i \bar{\sigma}^m \partial_m \psi_i + \tilde{z}_\alpha^+ (\Box - \Sigma^2) \tilde{z}_\alpha - i \bar{\psi}_\alpha \bar{\sigma}^m \partial_m \psi_\alpha \]
\[ + m \epsilon_{ij} (z_1 F_{2j} + z_2 F_{1j} - \psi_{1i} \psi_{2j} + B z_1 z_2 + h.c.) \]
\[-g \epsilon_{ij} (F_{2i} z_\alpha \tilde{z}_\alpha - z_{2i} \psi_{\alpha j} \tilde{\psi}_\alpha - \tilde{z}_\alpha \psi_{2i} \psi_{\alpha j} - z_{\alpha j} \psi_{2i} \psi_{\alpha} + h.c.) \]
\[ - \frac{q_2^2}{8} + 4 q_1^2 (Y + \tilde{Y})^2 \]
\[ - z_\alpha^* \{ \delta_{ij} [g^2 z_2^+ z_2 - Q^2 z_1^+ z_1] + \frac{q_2^2}{2} z_{1i} z_{1j} - g^2 z_2 z_2 \} z_{\alpha j} \]
\[ - |\tilde{z}_\alpha|^2 [g^2 z_2^+ z_2 + \tilde{Q}^2 z_1^+ z_1] + \mathcal{O}(z_\alpha^4) \quad (46) \]

where

\[ Q^2 = \frac{q_2^2}{4} - q_1^2 Y (Y + \tilde{Y}) \]
\[ \tilde{Q}^2 = q_1^2 \tilde{Y} (Y + \tilde{Y}) \quad (47) \]

(we will see below that \( Q^2, \tilde{Q}^2 \) are positive). The last term \( \mathcal{O}(z_\alpha^4) \) in (46) plays no role in the computation of the effective potential in the \( H_1, H_2 \) Higgs sector to the leading order in \( N \). This potential reads in \( d = 4 \) spacetime dimensions (cf. (6))

\[ V_{\text{eff}} = V_{\text{tree}} + \frac{1}{32 \pi^2} \left\{ \Lambda^2 \text{ST} \tau M^2 + \frac{1}{2} \text{ST} \tau (M^4 \ln \frac{M^2}{\Lambda^2}) - \frac{1}{4} \text{ST} \tau M^4 + \mathcal{O} \left( \frac{M^2}{\Lambda^2} \right) \right\} \quad (48) \]

We wish to compute \( V_{\text{eff}} \) in the configuration:

\[ z_1 = \begin{pmatrix} h_1 \\ 0 \end{pmatrix}, \quad z_2 = \begin{pmatrix} 0 \\ h_2 \end{pmatrix}, \quad F_1 = \begin{pmatrix} f_1 \\ 0 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 \\ f_2 \end{pmatrix} \quad (49) \]

which does not break charge conservation. The eigenvalues of the scalar mass matrix are:
\[ m_1^2 = \Sigma^2 - Q^2 |h_1|^2 \]

\[ m_{2,3}^2 = \Sigma^2 + g^2 |h_2|^2 + \frac{1}{2} (-Q^2 + \tilde{Q}^2 + \frac{q_2^2}{2}) |h_1|^2 \]

\[ \pm \sqrt{\frac{1}{4} (-Q^2 + \tilde{Q}^2 + \frac{q_2^2}{2})^2 |h_1|^4 + g^2 |f_2|^2} \]  \( (50) \)

whereas for the fermions:

\[ (m_F^+ m_F)_{\alpha_i \alpha_j} = g^2 (z_2^+ z_2 \delta_{ij} - z_2^+ z_2) \]

\[ (m_F^+ m_F)_{\tilde{\alpha} \tilde{\alpha}} = g^2 |h_2|^2 \]  \( (51) \)

Then:

\[ \frac{1}{N} St^r M^2 = 6 \Sigma^2 + 2 (-2Q^2 + \tilde{Q}^2 + \frac{q_2^2}{2}) |h_1|^2 \]  \( (52) \)

and to avoid a field dependent quadratic divergence we set:

\[ -2Q^2 + \tilde{Q}^2 + \frac{q_2^2}{2} = q_1^2 (2Y + \tilde{Y})(Y + \tilde{Y}) = 0 \]  \( (53) \)

which reduces to \( 2Y + \tilde{Y} = 0 \) and is equivalent to the usual condition of the cancellation of gauge anomalies in the standard model.

Using this condition we obtain from (47) that:

\[ Q^2 = \frac{q_2^2}{2} + q_1^2 (Y + \tilde{Y})^2 \geq \frac{q_2^2}{2}, \]

\[ \tilde{Q}^2 = 2q_1^2 (Y + \tilde{Y})^2. \]  \( (54) \)

Using also:

\[ \frac{1}{N} St^r M^4 = 6 \Sigma^4 + 4 ((Q^2 - \tilde{Q})^2 + Q^2 \tilde{Q}^2) |h_1|^4 \]

\[ + 4g^2 (-Q^2 + \tilde{Q}^2 + \frac{q_2^2}{2}) |h_1|^2 |h_2|^2 \]
we obtain the one-loop (leading $\frac{1}{N}$) effective potential:

$$V_{\text{eff}} = -|f_1|^2 + \Delta^2|h_1|^2 - m(h_1 f_2 + h_2 f_1 + B h_1 h_2 + \text{h.c.}) + \frac{Q^2}{2}|h_1|^4$$

$$+ \frac{N}{32\pi^2} \left\{ 6\Sigma^2 \Lambda^2 + \sum_{i=1}^3 m_i^4 \ln \frac{m_i^2}{\Lambda^2} - 2g^4|h_2|^4 \ln \frac{g^2|h_2|^2}{\Lambda^2} \right.$$ 

$$- \frac{3}{2} \Sigma^4 - [(Q^2 - \tilde{Q}^2)^2 + Q^2 \tilde{Q}^2]|h_1|^4 - g^2 Q^2|h_1|^2|h_2|^2$$

$$- 2 \Sigma^2 g^2|h_2|^2 - g^2|f_2|^2 \} + O\left(\frac{M^2}{\Lambda^2}\right).$$

When studying the correspondence with the minimal supersymmetric standard model (MSSM), it is particularly illuminating to consider the limit $B \to 0$. Indeed when $B = 0$, it is well-known that the MSSM has a richer vacuum structure since, depending on the parameters, besides the trivial minimum $h_1 = h_2 = 0$ and the completely nontrivial minimum, we have the mixed possibilities $h_1 = 0$, $h_2 \neq 0$ (and $h_2 = 0$, $h_1 \neq 0$). It is interesting in its own sake to see if such a vacuum structure is also present in the model considered here. On the other hand, the phenomenologically relevant case corresponds to nonzero $B$ soft terms.

Let us consider first the case $B = 0$. The saddle point equations corresponding to the potential $V_{\text{eff}}$ in (56) are given by:

$$f_1 = -m h_2^*$$

$$m f_2^* = h_1 \left\{ \Delta^2 + Q^2|h_1|^2 + \frac{N}{32\pi^2} \left[ -Q^2 \left( 2m_1^2 \ln \frac{m_1^2}{\Lambda^2} - m_2^2 \ln \frac{m_2^2}{\Lambda^2} - m_3^2 \ln \frac{m_3^2}{\Lambda^2} \right) \right] \right.$$ 

$$+ (Q^2 + \tilde{Q}^2 - \frac{q_2^2}{2})^2|h_1|^2 \left. \left[ \frac{m_2^2 \ln \frac{m_2^2}{\Lambda^2} - m_3^2 \ln \frac{m_3^2}{\Lambda^2}}{m_2^2 - m_3^2} \right] \right\}$$

$$m h_1^* = \frac{N g^2}{16\pi^2 f_2} \frac{m_2^2 \ln \frac{m_2^2}{\Lambda^2} - m_3^2 \ln \frac{m_3^2}{\Lambda^2}}{m_2^2 - m_3^2}$$

\(^4\text{Some details on the computation are given in section i) of the appendix.}\)
\[ m f_1^* = \frac{Ng^2}{16\pi^2} h_2 \left( m_2^2 \ln \frac{m_2^2}{\Lambda^2} + m_3^2 \ln \frac{m_3^2}{\Lambda^2} - 2g^2|h_2|^2 \ln \frac{g^2|h_2|^2}{\Lambda^2} \right) \] (57)

Combining them, one obtains the following two equations:

\[ 0 = f_2 \left\{ m^2 - \frac{Ng^2}{16\pi^2} \left( \frac{m_3^2 \ln \frac{m_3^2}{\Lambda^2} - m_2^2 \ln \frac{m_2^2}{\Lambda^2}}{m_2^2 - m_3^2} \right) \right\} \] (58)

\[ 0 = h_2 \left\{ m^2 + \frac{Ng^2}{16\pi^2} \left( m_2^2 \ln \frac{m_2^2}{\Lambda^2} + m_3^2 \ln \frac{m_3^2}{\Lambda^2} - 2g^2|h_2|^2 \ln \frac{g^2|h_2|^2}{\Lambda^2} \right) \right\} \] (59)

where \( m_i^2 \) are given in (50) and \( Q, \bar{Q} \) in (47) or (54).

This system has possibly four different solutions:

i) The trivial one: \( h_1 = h_2 = f_1 = f_2 = 0 \).

ii) The solution \( f_2 = h_1 = 0, h_2 \neq 0 \) which is independent of the gauge couplings and gives through (59) the usual gap equation (20) (up to a factor 2 which comes from the different field content):

\[ G^{-1} = \frac{N}{8\pi^2} \left[ \Sigma^2 \ln \left( \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2} \right) - g^2|h_2|^2 \ln \frac{g^2|h_2|^2}{\Lambda^2} \right] . \] (60)

This is the vacuum which was studied by Clark, Love and Bardeen \[10\] and we will sometimes refer to it as the CLB vacuum.

iii) The solution \( f_1 = h_2 = 0 \) and \( h_1 \neq 0 \) given by (58).

iv) The completely nontrivial solution \( h_1 \neq 0, h_2 \neq 0, f_1 \neq 0 \) and \( f_2 \neq 0 \).

We thus recover the four different vacua present in the MSSM. The last two extrema are difficult to study analytically.

For the trivial extremum, the mass matrix computed in section 4.2 remains valid, up to trivial factors of 2. So, for \( G > G_c \) with \( G_c \) given by an expression similar to (18), namely

\[ G_c = \frac{8\pi^2}{N\Sigma^2 \ln \frac{\Lambda}{\Sigma^2}} . \] (61)
it becomes unstable.

In the extremum ii), the CLB vacuum, the only difference compared with the analysis of section 4.2 lies in the mass matrix element:

\[
\frac{d^2 V}{dh_1^* dh_1} = \Delta^2 + \frac{m^2}{Ng^2 \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2 + g^2|h_2|^2)}}
\]

\[
-N Q^2 g^2|h_2|^2 \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + \Sigma^2)(p^2 + \Sigma^2 + g^2|h_2|^2)},
\]

which reads in 4 dimensions

\[
\frac{d^2 V}{dh_1^* dh_1} = \Delta^2 + \left[ N \frac{G}{16\pi^2} \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2} - 1 \right]^{-1}
\]

\[
-N \frac{Q^2 g^2|h_2|^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2}}{16\pi^2} = \Sigma^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2} - \Sigma^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2} \right] (62)
\]

For every given \(G\), we have a value of \(Q\) beyond which this extremum is unstable and cannot be the true vacuum: the gap equation and the spectrum are then completely different. For example, we will see below that there is a smooth transition to the extremum iv) which has an extra Goldstone boson.

Indeed, let us rediscuss the issue of the symmetries for the case at hand. The symmetry of our lagrangian is \(SU(N) \times SU(2)_L \times U(1)_Y \times U(1)_R\). If we are interested in the neutral Higgs sector, we can restrict ourselves to the transformations associated with the quantum numbers \(T_3, Y\) and \(R\). The charge \(Q = T_3 + \frac{Y}{2}\) being conserved, we are left with the independent combinations \(T_3 - \frac{Y}{2} \pm R\). Under \(T_3 - \frac{Y}{2} + R\) the fields transform according to:

\[
\begin{align*}
h_1 &\rightarrow e^{i\alpha} h_1, & f_1 &\rightarrow f_1 \\
h_2 &\rightarrow h_2, & f_2 &\rightarrow e^{-i\alpha} f_2
\end{align*}
\]

And under \(T_3 - \frac{Y}{2} - R\) as:

\[
\begin{align*}
h_1 &\rightarrow h_1, & f_1 &\rightarrow e^{-i\beta} f_1 \\
h_2 &\rightarrow e^{i\beta} h_2, & f_2 &\rightarrow f_2
\end{align*}
\]

In the case ii), \(<h_2> \neq 0, <h_1> = 0\), so the \(T_3 - \frac{Y}{2} - R\) symmetry is spontaneously broken, giving a Goldstone boson in the neutral sector. In the third extremum iii), the broken symmetry is \(T_3 - \frac{Y}{2} + R\) being accompanied
by its corresponding Goldstone boson. Both this two symmetries are broken in the fourth, completely non trivial, extremum leading to two massless bosons. If we restore a nonzero value for the soft supersymmetry-breaking parameter $B$, $U(1)_{R}$ is no longer a symmetry of the Lagrangian; all the last three extrema break $T_{3} - \frac{Y}{2}$ and yield one Goldstone boson.

To study the smooth transitions between the four extrema, we use the same technics as in section 3. We stress that this scenario is only one of the possibilities; the complete set of transitions (including first order) are obtained by comparing the values of the energy for the four extrema as functions of the couplings. We will however consider here only possible second order phase transitions, in the spirit of the original work of Nambu and Jona-Lasinio [4].

Suppose that we start from the trivial minimum and that for $G = G_{c}$ the system undergoes a second order phase transition from this trivial extremum to one of the three nontrivial minima. Thus for $G = G_{c}(1 + \epsilon)$, $\epsilon \ll 1$, we have: $f_{1}/\Sigma^{2}$, $g^{2}|h_{2}|^{2}/\Sigma^{2}$, $4g^{2}|f_{2}|^{2}/\Sigma^{4}$, $|h_{1}|^{2}/\Sigma^{2} \ll 1$. We keep only the leading terms in the saddle point equations (57):

\[
\begin{align*}
    f_{1} & = -mh_{2}^{*} \\
    mf_{2}^{*} & = \Delta^{2}h_{1} \\
    h_{1}^{*} & = -\frac{NmG_{c}}{16\pi^{2}}(\ln \frac{A^{2}}{\Sigma^{2}} - 1)f_{2} \\
    f_{1}^{*} & = -\frac{NmG_{c}}{8\pi^{2}}\frac{\Sigma^{2}}{\Sigma^{2}} \ln \left(\frac{A^{2}}{\Sigma^{2}}\right)h_{2}
\end{align*}
\]

which, combined, give $h_{1} = 0$ and (61). Thus the only allowed second order transition from the trivial vacuum is to the CLB vacuum ii) ($h_{1} = 0, h_{2} \neq 0$). Indeed one recovers the standard value (18) for the critical coupling. Note in particular that this critical value is independent of the gauge couplings $Q$ and $\tilde{Q}$.

The next step is to study the smooth transitions at $G \sim \tilde{G}_{c}$ from the vacuum ii) to any of the other nontrivial vacua, which for that matter can only be iv). At $G = \tilde{G}_{c}(1 + \epsilon)$, we have $|h_{2}|$ close to the value given by the gap equation (60) (in the following we note $|h_{2}|_{c}$ the corresponding value): $|h_{2}|^{2} = |h_{2}|_{c}^{2} + \delta|h_{2}|^{2}, g^{2}\delta|h_{2}|^{2}/\Sigma^{2} \ll 1$. On the other hand, we have as before $|h_{1}|^{2}/\Sigma^{2}, 4g^{2}|f_{2}|^{2}/\Sigma^{4} \ll 1$. The leading terms now read:

\[
\begin{align*}
    f_{1} & = -mh_{2c}^{*}
\end{align*}
\]
\[
\begin{align*}
  m f_2^* & = \left\{ \Delta^2 - \frac{N Q^2}{16 \pi^2} (g^2 |h_2|^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} + \Sigma^2 \ln \frac{\Sigma^2 + g^2 |h_2|^2}{\Sigma^2}) \right\} h_1 \\
  h_1^* & = -\frac{N m \tilde{G}}{16 \pi^2} \left( \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} - 1 \right) f_2 \\
  f_1^* & = -\frac{N m \tilde{G}}{8 \pi^2} \left( \Sigma^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} - g^2 |h_2|^2 \ln \frac{\Sigma^2 + g^2 |h_2|^2}{g^2 |h_2|^2} \right) h_2 (66)
\end{align*}
\]

These equations can be recast into (60) (with \( G = \tilde{G}_c \), \(|h_2|^2 = |h_2|^2_c \)), as expected since we consider a smooth transition from vacuum ii). Details on the derivation are given in the appendix, section ii).

Equations (60) and (67) can be turned into an equation giving \( Q^2 \) in terms of \(|h_2|^2\) (which is itself a monotonous function of \( G \) through (60)). It can be checked that \( Q^2 \) is itself a monotonous function of \( \tilde{G}_c \) on the interval \((\tilde{G}_c, +\infty)\), decreasing from \(+\infty\) to 0. Inverting this function, we therefore obtain the critical line between the standard vacuum ii) and the completely nontrivial vacuum shown in Fig.2.

Indeed, Fig.2 summarizes the critical values of the coupling \( G \) as a function of \( Q \) for the smooth transitions (second order) between the different vacua. It is now easy to discuss the case \( B \neq 0 \) which is more relevant for phenomenology but less rich in its vacuum structure. Only the trivial \((h_1 = h_2 = 0)\) and completely nontrivial \((h_1 \neq 0, h_2 \neq 0)\) configurations are allowed minima and the critical line between the two minima is given by \( G_c \) in (61), irrespective of \( Q \). One indeed checks from the scalar mass matrix that the trivial vacuum is unstable for \( G > G_c \).

\section{Conclusions.}

We have studied the vacuum structure in supersymmetric models with scalar condensates by using the effective potential approach in the leading \( \frac{1}{\lambda} \) approximation. The resulting saddle point equations were analyzed and the phase transitions between the different extrema obtained by linearization around the critical surfaces.
Figure 2: Phase diagram in the \((G,Q^2)\) plane for the gauged model.
We have closely studied the equivalence of the supersymmetric top-antitop condensate model with the MSSM. In the case where the soft term known as $B$ is zero\footnote{This case is chosen because of the richer structure of the vacuum.}, the vacuum structure of the condensate model is the same as the one in the MSSM at tree level, provided we take into account the $SU(2)_L \times U(1)_Y$ couplings which have leading $\frac{1}{N}$ contributions. We find four extrema; the phase diagram now involves the gauge couplings as well as the original four-fermion coupling $G$.

In the case $B \neq 0$, the vacuum solutions still depend on the gauge couplings. But the critical surface which separates the trivial vacuum from the non-trivial one is determined only by the self-coupling $G$ in distinction with the case $B = 0$. This critical surface is therefore identical to the one obtained when electroweak gauge interactions are turned off. In this version of the model, we have studied in detail the role played by the soft supersymmetry breaking terms, in particular $B$, on the breaking of chiral symmetry.

The methods developed in this paper are readily applicable to other studies such as temperature dependence of the condensate solution or similar analyses of dynamical symmetry breaking in non-minimal supersymmetric models.

**Acknowledgments**

We wish to thank C.Savoy for many valuable discussions and C.Teodorescu for help in the numerical analyses.
Appendix.

i) The effective potential in the gauged model.

We can write the lagrangian in (46) as

\[ \mathcal{L} = F_1^+ F_1 + z_1^+ (\Box - \Delta^2) z_1 - i \bar{\Psi}_1 \bar{\sigma}^m \partial_m \Psi_{1i} - i \bar{\Psi}_a \bar{\sigma}^m \partial_m \Psi_{ai} - i \bar{\Psi}_i \bar{\sigma}^m \partial_m \Psi_{ai} \]

\[ - \frac{g_2^2 + 2g^2(Y + \bar{Y})(z_1^+ z_1)^2 + Bm(\epsilon_{ij} z_{1i} z_{2j} + h.c.)}{8} \]

\[ - \frac{g\epsilon_{ij}(z_2 \bar{\Psi}_{ai} - 2z_1 \Psi_{ai} + h.c.)}{i} \]

\[ - (z_{ai}^* \bar{z}_a) \left( X \delta_{ij} + \frac{g_2^2}{2} z_{1i} z_{1j} - g^2 z_{2i} z_{2j} \frac{g\epsilon_{ki} F_{2k}}{X} \right) \left( \frac{z_{ai}^*}{z_a^*} \right) \]

with the notations:

\[ X = -\Box + g^2 z_{2i}^2 z_2 + \left( q_1^2 Y(Y + \bar{Y}) - \frac{q_2^2}{4} \right) z_1^+ z_1 + \Sigma^2, \]

\[ \bar{X} = -\Box + g^2 z_{2i}^2 z_2 + q_1^2 Y(Y + \bar{Y}) z_1^+ z_1 + \Sigma^2. \]

If M and m_f are the scalar and the fermion mass matrices, then:

\[ \ln \det(p^2 + M^2) = N \ln \{ (X \bar{X} - g^2 F_2^+ F_2)(X + \frac{q_2^2}{2} z_{1i}^2 z_2 - g^2 z_{2i}^2 z_2) \]

\[ - g^4 (z_2^+ z_2 F_2^+ F_2 - z_{2i}^+ F_2^+ F_2) \]

\[ - \frac{g_2^2}{2} g^2 \bar{X}(z_1^+ z_1 z_{2i}^2 z_2 - z_1^+ z_2 z_{2i}^2 z_1) \]

\[ + \frac{g_2^2}{2} g^2 (z_1^+ z_1 F_2^+ F_2 - z_1^+ F_2^+ F_2 + z_1^+ z_2 z_{2i}^2 z_2) \} \]

(68)

where from now on we replace \( \Box \) by \(-p^2\) in X and \( \bar{X} \) above. Then

\[ \ln \det(p^2 + m_f^2) = N \ln \det \left( \begin{array}{cc} p^2 + g^2 z_{2i}^2 z_2 & 0 \\ 0 & p^2 + g^2 z_{2i}^2 z_2 \end{array} \right) \]

\[ = N \ln p^2 (p^2 + g^2 z_{2i}^2 z_2) \]

(69)

(70)

Finally,

\[ V_{eff} = -F_1^+ F_1 + \Delta^2 z_1^+ z_1 - m \epsilon_{ij} (z_{1i} F_{2j} + z_{2j} F_{1i} + h.c.) \]

\[ + \frac{q_2^2 + 2q_1^2 (Y + \bar{Y})^2}{8} (z_1^+ z_1)^2 \]
\[ + N \int \frac{d^4 p}{(2\pi)^4} \left[ \ln \{(X \hat{X} - g^2 F_2^+ F_2)(X + \frac{g_2^2}{2} z_1^+ z_1 - g^2 z_2^+ z_2) \right. \\
\left. - \frac{g_2^2}{2} g^2 \hat{X}(z_1^+ z_1 z_2^+ z_2 - z_1^+ z_2 z_1^+ z_1) \\
+ \frac{q_2^2}{2} g^2 (z_1^+ F_2^+ F_2 - z_1^+ F_2 F_2^+) - g^4 (z_2^+ z_2 F_2^+ F_2 - z_2^+ F_2 F_2^+) \right} \\
- \ln \{p^2 (p^2 + g^2 z_2^+ z_2) \} \right] \] 

(71)

which by integration in the configuration (49) gives the result written as equation (56).

ii) The phase transition from the Clark-Love-Bardeen (CLB) vacuum to the completely nontrivial one (assuming \( B = 0 \)).

We have verified in equation (65) that a second order phase transition from the trivial to the CLB vacuum ii) occurs at \( G = G_c \). We now verify that for \( G \gg G_c \), one encounters a new smooth transition to the completely nontrivial minimum. Following the method developed in section 3, we determine the corresponding critical surface \( f(G, q_1, q_2) = 0 \) by requiring that, when we are in the vicinity of this critical surface, i.e. \( (f + \epsilon g)(G, q_1, q_2) = 0, \epsilon \ll 1 \), we have \( \frac{h_1}{\Lambda_1} \ll \frac{g_2^2 |f_2|^2}{\epsilon^2} \ll 1 \) and \( g^2 |h_2|^2 = g^2 |h_2|_c^2 (1 + \delta x) \) where \( g^2 |h_2|_c^2 \) is the CLB solution and \( \delta x \ll 1 \). Fixing \( G = G_0 \) we search for a \( Q_0^2 \) beyond which we have a completely nontrivial solution \( (h_1 \neq 0, h_2 \neq 0) \). Writing \( Q^2 = Q_0^2 (1 + \epsilon) \) and making a limited Taylor series expansion in the saddle point equations (57), we obtain the following system of equations:

\[
\begin{align*}
    f_1 & = -m h_2^* \\
    m f_2^* & = \{ A_1 + B_1 |h_1|^2 \} h_1 \\
    m h_1^* & = \frac{N g_2^2}{16\pi^2} \{ A_2 + B_2 |h_1|^2 \} f_2 \\
    m f_1^* & = \frac{N g_2^2}{16\pi^2} \{ A_3 + B_3 |h_1|^2 \} h_2 
\end{align*}
\]

(72)

where the functions \( A_i \) and \( B_i \) are of the form (in the following, \( |h_2|^2 \) means the CLB solution \( |h_2|^2 \) and \( G \) and \( Q^2 \) are \( G_0 \) and \( Q_0^2 \)):

\[
\begin{align*}
    A_1 & = \Delta^2 - \frac{N Q^2 (1 + \epsilon)}{16\pi^2} (g^2 |h_2|^2) \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} + \Sigma^2 \ln \frac{\Sigma^2}{\Sigma^2 + g^2 |h_2|^2} \\
    & - \frac{N Q^2}{16\pi^2} g^2 |h_2|^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} \delta x 
\end{align*}
\]
\[ A_2 = 1 - \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2} + \frac{g^2|h_2|^2}{\Sigma^2 + g^2|h_2|^2}\delta x \]

\[ A_3 = -2\Sigma^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2} + 2g^2|h_2|^2 \ln \frac{\Sigma^2 + g^2|h_2|^2}{g^2|h_2|^2}(1 + \delta x) \]

\[ B_1 = \frac{Q^2}{32\pi^2} \left[ Q^2(3 - 2 \ln \frac{\Lambda^2}{\Sigma^2} - \ln \frac{\Lambda^2}{\Sigma^2 + g^2|h_2|^2}) \right] + (Q^2 + \bar{Q}^2 - \frac{g_2^2}{2})^2 \ln \frac{\bar{\Lambda}^2}{\Sigma^2 + g^2|h_2|^2} \]

\[ B_2 = \frac{Q^2}{\Sigma^2 + g^2|h_2|^2} \]

\[ B_3 = -Q^2 \ln \frac{\bar{\Lambda}^2}{\Sigma^2 + g^2|h_2|^2} \]

where \( \bar{\Lambda}^2 = \Lambda^2/e \).

To find the critical surface, only the \( O(1) \) terms are kept in equations (72). They are easily combined to give equation (67). The critical surface indeed coincides with the instability surface for the CLB vacuum where the matrix element \( \frac{d^2V}{dh_1 dh_1} \) becomes negative. This observation is a strong support for the above mentioned scenario: the completely nontrivial vacuum occurs for values of the couplings at which the CLB vacuum becomes unstable. To verify that the scenario is really possible, we compute \( |h_1|^2 \), \( |\frac{g^2|h_2|^2}{\Sigma^2}| \) and \( \delta x \) as functions of \( \epsilon \). Decomposing \( A_i = A_{i0} + A_i' \), where \( A_i' \) is a collection of small terms, we obtain the equations:

\[ 2g^2|h_2|^2 \ln \frac{\Sigma^2 + g^2|h_2|^2}{g^2|h_2|^2}\delta x + B_3|h_1|^2 = 0 \]

\[ A_1'A_{20} + A_{10}A_2' + (A_{10}B_2 + A_{20}B_1)|h_1|^2 = 0 \quad (73) \]

Because \( B_3 < 0 \) we find \( \delta x \) positive and proportional to \( |h_1|^2 \) and \( |h_1|^2 = C\epsilon \) where \( C \) is a rather lengthy expression:

\[ C = \frac{NQ^2}{16\pi^2} \ln \frac{\bar{\Lambda}^2}{\Sigma^2 + g^2|h_2|^2}\left[ \ln \frac{\Sigma^2 + g^2|h_2|^2}{g^2|h_2|^2} \right]^{-1} \]
\begin{align*}
+ Q^2 \ln \frac{\tilde{\Lambda}^2}{\Sigma^2 + g^2 |h_2|^2} \left[ (\Sigma^2 + g^2 |h_2|^2) \ln \frac{\Sigma^2 + g^2 |h_2|^2}{g^2 |h_2|^2} \right]^{-1} \\
(g^2 |h_2|^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} - \Sigma^2 \ln \frac{\Sigma^2 + g^2 |h_2|^2}{g^2 |h_2|^2}) \\
+ \frac{2Q^2}{\Sigma^2 + g^2 |h_2|^2} (g^2 |h_2|^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} - \Sigma^2 \ln \frac{\Sigma^2 + g^2 |h_2|^2}{g^2 |h_2|^2}) \\
- \ln \frac{\Sigma^2 + g^2 |h_2|^2}{\Lambda^2} \left\{ Q^2 + \frac{N}{32\pi^2} [Q^2 (-3 + 2 \ln \frac{\Lambda^2}{\Sigma^2} + \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2})] \\
- (Q^2 + \tilde{Q}^2 - \frac{g^2}{2})^2 \ln \frac{\Lambda^2}{\Sigma^2 + g^2 |h_2|^2} \right\} > 0
\end{align*}

Henceforth, for \( \epsilon > 0 \), we have \(|h_1|^2 > 0\), \( \delta x > 0 \) and for \( \epsilon < 0 \), \(|h_1|^2 = \delta x = 0 \) which corresponds to a usual second order phase transition.

\textbf{iii) The gap equations for the non-gauged, } B \neq 0 \textbf{ case.}

The solutions of equations (16) may be rewritten in the following form:

\begin{align*}
G^{-1} &= \frac{N}{8\pi^2} \left[ (\Sigma^2 + 4g^2 |z_2|^2)^2 - \frac{BF_2}{2z_2} \right] \ln \frac{\Lambda^4}{(\Sigma^2 + 4g^2 |z_2|^2)^2 - 4g^2 F_2 F_2^*} \\
&- \frac{1}{2g|F_2|} (\Sigma^2 + 4g^2 |z_2|^2 + 2g \sqrt{F_2 F_2^*}) \ln \frac{\Sigma^2 + 4g^2 |z_2|^2 + 2g \sqrt{F_2 F_2^*}}{\Sigma^2 + 4g^2 |z_2|^2 - 2g \sqrt{F_2 F_2^*}} \\
&- 8g^2 |z_2|^2 \ln \frac{\Lambda^2}{4g^2 |z_2|^2}, \\
\text{where } G &= \frac{2}{m^2} \text{ and} \\
\frac{F_2}{B z_2} + 1 &= - \frac{NG \Delta^2 F_2}{16\pi^2 B z_2} \ln \frac{\Lambda^4}{(\Sigma^2 + 4g^2 |z_2|^2)^2 - 4g^2 F_2 F_2^*} \\
&- \frac{1}{2g|F_2|} (\Sigma^2 + 4g^2 |z_2|^2) \ln \frac{\Sigma^2 + 4g^2 |z_2|^2 + 2g \sqrt{F_2 F_2^*}}{\Sigma^2 + 4g^2 |z_2|^2 - 2g \sqrt{F_2 F_2^*}}.
\end{align*}
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