Effect of Height and Surface Roughness of a Broad Crested Weir on the Discharge Coefficient: Experimental Study

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Abstract. Weir is usually incorporated as control or regulation devices in hydraulic systems, with flow measurement as their secondary. It is normally intended for use in the field and thus to regulate broad discharges. Broad-Crested weir is among the oldest common weir types. In this paper, the effect of height and surface roughness for different Board Crested weirs models were studied on discharge coefficient (Cd) in a horizontal open channel. In the crest of the weir, certain materials may be combined with concrete (e.g., boulders) or may be used as cladding to minimize the effect of water overflow (e.g. stone). The weir surface should not be considered smooth in this case, and the discharge coefficient (Cd) must be re-estimated. For these purposes, laboratory flume was used to study the effect of height and surface roughness on the discharge coefficients with four of the different weir models dimensions of the concrete blocks. In this study, the flow conditions were considered to be free water flow and the viscosity effect was neglected. In all cases, the weir height effect was directly proportional to the discharge coefficient while the surface roughness effect was found to be inversely proportional to the coefficient Cd of the case study.

1. Introduction
A weir is a small dam constructed across a valley or river channel and also used to form an impoundment reservoir, sometimes called an overflow dam. To identify the crest of an overflow spillway on a large dam, the term weir is also often used. Typically, weir is built using concrete, masonry of stone or gabions. The majority of weirs are used for measuring flows [1]. They are also used to provide drinking water or to control floods and help keep a river navigable. The broad-crested weir (see figure 1) was chosen for this study to show the effects of the height and the roughness of its crest on the coefficient of discharge because it is simple to model this form of weir in the laboratory. Classified under the term, Broad-Crested Weir is this structure over which the streamlines run parallel to each other a minimum of for a short distance, so as that a hydrostatic pressure distribution could even be assumed at the control section [2, 3]. To induce this condition, the length within the direction of the flow of the weir crest (L) is restricted to the total upstream energy head over the crest (H+P) as shown in figure 1.
The hydraulic properties of broad crested weirs are generally expressed by the coefficient of discharge \( C_d \) that characterizes the linear relationship between the flow rate \( Q \) and the overflowing head \( H \). Broad-crested weirs with the \( H / L \) ratio ranging from 0.02 to 1.9 and \( H / P \) from 0.1 to 0.9 were studied by Govinda Rao and Muralidhar in 1963, and then discharge coefficient \( C_d \) values were found. They seem to be the first to propose the division of finite crest length weirs into four groups of long-crested, broad-crested, short-crested, and sharp-crested weirs [4]. Horton in 1907 proposed that \( C_d \) relies mainly on the ratio of the upstream head to the weir length in the direction of flow \( H / L \) and that if \( H \) is greater than 3 cm, the effects of viscosity and surface tension may be ignored [5]. A classification of flow over weirs of finite crest length based on \( H / L \) was proposed by Singer in 1964 [6]. He considered the range of \( H / L \) from 0.08 to 0.33 and found that \( C_d \) is constant with a value of 0.85 for the \( H / (H+P) \) range from 0.18 to 0.36, where \( H / (H+P) \) accounts for the velocity of the flow entering the weir. Azimi and Rajaratnam in 2009, ignored the correction of the discharge equation velocity head and established a new \( C_d \) equation through regression analysis with \( C_d \) experimental observation from previous studies. New \( C_d \) equations have been derived for square and round edged upstream weirs. They divided the flat topped weir into three groups on the basis of the \( H / L \) ratio, such as long crested, broad crested and short crested weir [7].

Most researchers defined weir as having a smooth surface made of concrete and, in experimental studies, it can be made of other comparatively smooth materials such as plywood or fiberglass. However, Parílkova et al in 2009 showed that, in the case of low overflow head, the roughness of the surface of the broad crested weir had a significant impact on the discharge coefficient [8], [9].

This paper focuses on study different models of Broad Crested Weir in the laboratory and the effect of height and surface roughness on the discharge coefficients (\( C_d \)) with free flow conditions.

Different formulas have been developed to measure discharges over weirs. In the case of free flow conditions, most formulas can be expressed in the following general form [10, 11]:

\[
Q_f = C \cdot B \cdot H^{1.5}
\]

(1)

Where, \( C \) is the weir coefficient while \( B \) and \( H \) are the width of weir and height of water respectively (see figure 1). The weir coefficient \( C \) is expressed in the function of the discharge coefficient \( C_d \) from the definition of the Bernoulli equation as follows: [7, 12]:

\[
C = \frac{2}{3} C_d \sqrt{\frac{2g}{3}}
\]

(2)

In a rectangular channel, for different \( H / (H + P) \) ratio values ranging from 0.1 to 1.0, Ranga and Asawa proposed the formula that shown below (equation 3) to find different values of discharge coefficient (\( C_d \)) and for experimental studies in laboratory by the try and the error method [13, 14]:

![Figure 1. Broad Crested Weir.](image-url)
Viscosity and surface tension effects are ignored if the water height is greater than 3 cm above the weir surface \([7, 15]\). The discharge coefficient \(C_d\) is introduced to account for head loss due to weir surface roughness and the depth of the water above the weir crest is taken into account instead of the energy head i.e. the velocity head is ignored \([12]\).

The Statistical Software for Social Sciences (SPSS) is one of the statistical applications used in all scientific research methods, in which it is distinguished by its high capacity to process data on the state of the study through a collection of lists and tools by which the data can be entered for the program, in which it defines and statistically analyses variables and generalizes them \([16, 17]\).

The data were statistically analysed in the manner of analysis regression - curve estimation using the Statistical Analysis Program (SPSS).

2. Experimental Work

Concrete blocks of different dimensions by the mixing ratio \((1: 2: 4)\) which are ideal for laboratory simulation of this type of weir. Four blocks of ordinary concrete were poured in the following sizes (B, P, L): \((48\text{cm} \times 12\text{cm} \times 12\text{cm})\), \((48\text{cm} \times 10\text{cm} \times 10\text{cm})\) and \((48\text{cm} \times 8\text{cm} \times 8\text{cm})\) (figure 2-a). A single model with dimensions \((48\text{cm} \times 12\text{cm} \times 12\text{cm})\) was created to observe the effect of the surface roughness of the crest on the discharge coefficient by substituting the gravel in the concrete mixture (cement, sand, gravel) for the boulder (figure 2-b). Boulders of irregular form, with lengths varying from 5 to 10 cm, were used and distributed uniformly on the surface of the model. The concrete models of the weirs (four blocks of concrete) were put in the treatment basins for 7 days to harden before they were placed in the laboratory channel shown in figure 3. The laboratory channel used is 50 cm wide and 200 cm long and is rounded by a 30 cm high glass wall. To represent the Weir, the sample is positioned in the middle of the channel. The electric water pump was used to supply the channel with water flow.

\[
\frac{H}{H+P} = \sqrt[2/3]{\frac{C_d^{2/3} - 2}{C_d}} \sqrt{3} \tag{3}
\]

Figure 2. Concrete blocks of different dimensions to simulate the weir: a. blocks of ordinary concrete (cement, sand and gravel). b. concrete block (cement, sand and boulder).
3. Results and Discussion

It is possible to change the discharge several times by setting the L-length and measuring the value of \( H \) multiple times and applying the equation (3) to find different values of discharge coefficient (Cd) and for experimental studies in laboratory by the try and the error method [10]. Equations (1), (2) and (3) were solved for different values of \( (H/ (H + P)) \), and the result are shown in tables (1), (2), (3), and (4).
Table 1. Experimental results for block of ordinary concrete (cement + sand + gravel) With (48cmx12cmx12cm) and P = 12cm (case 1).

| Sequencing | H(cm) | H/ (H + P) | Cd   | C   | Q (m³/sec) |
|------------|-------|------------|------|-----|------------|
| 1          | 3     | 0.20       | 0.59 | 1.01| 2.52x10⁻³  |
| 2          | 4     | 0.25       | 0.61 | 1.04| 3.994x10⁻³ |
| 3          | 5     | 0.29       | 0.63 | 1.07| 5.74x10⁻³  |
| 4          | 6     | 0.33       | 0.65 | 1.11| 7.83x10⁻³  |

Table 2. Experimental results for block of ordinary concrete (cement + sand + gravel) With (48cmx10cmx10cm) and P = 10cm (case 2).

| Sequencing | H(cm) | H/ (H + P) | Cd   | C   | Q(m³/sec) |
|------------|-------|------------|------|-----|-----------|
| 1          | 4     | 0.28       | 0.61 | 1.04| 3.994x10⁻³ |
| 2          | 5.5   | 0.35       | 0.63 | 1.07| 5.78x10⁻³  |
| 3          | 7     | 0.41       | 0.64 | 1.093| 9.72x10⁻³  |
| 4          | 8     | 0.44       | 0.65 | 1.105| 12x10⁻³    |

Table 3. Experimental results for block of ordinary concrete (cement + sand + gravel) With (48cmx8cmx8cm) and P = 8cm (case 3).

| Sequencing | H(cm) | H/ (H + P) | Cd   | C   | Q(m³/sec) |
|------------|-------|------------|------|-----|-----------|
| 1          | 2     | 0.20       | 0.62 | 1.07| 1.45x10⁻³  |
| 2          | 5     | 0.39       | 0.66 | 1.14| 6.12x10⁻³  |
| 3          | 6     | 0.43       | 0.67 | 1.15| 8.113x10⁻³ |
| 4          | 7     | 0.47       | 0.68 | 1.163| 10.34x10⁻³ |

Table 4. Experimental results for block of ordinary concrete (cement + sand + boulder) With (48cmx12cmx12cm) and P = 12cm (case 4).

| SEQUENCING | H(cm) | H/ (H + P) | Cd   | C   | Q(m³/sec) |
|------------|-------|------------|------|-----|-----------|
| 1          | 5     | 0.29       | 0.61 | 1.04| 5.58x10⁻³  |
| 2          | 7     | 0.37       | 0.62 | 1.064| 9.46x10⁻³  |
| 3          | 8     | 0.40       | 0.67 | 1.14| 12.38x10⁻³ |
| 4          | 8.5   | 0.42       | 0.68 | 1.16| 13.8x10⁻³  |

As previously stated, equation (3) is true for values of (H/ (H + P)) ratio ranging from (0.1 to 1). All values of (H/ (H + P)) ratio in this analysis, which ranged from 0.2 to 0.5. In other terms, they were within the framework of the application of the equation.

To measure the strength of the relationship between discharge coefficient Cd and the water head in upstream H/(H+P), non-linear regression analysis can be used. The square of the correlation coefficient R² is an indicator of fitness for non-linear regression models. The value of R² would be close to 1 if the strength of the relationship between discharge coefficient Cd and the water head in upstream H/(H+P). The results statistically analysed using the (SPSS).

This regression analysis was demonstrated through the curves shown in figures ((5), (6), (7) and (8)). In all cases, the discharge coefficient (Cd) remained directly proportional to the H/(H+P). The regression analysis revealed a high correlation between the discharge coefficient Cd and the ratio H/(H+P), where the square values of the correlation coefficient R² in the four cases were more than 0.99 (so close to 1). As a result, the relationship between the discharge coefficient and the water head upstream of the broad crested weir was significant. Furthermore, equations were obtained for the estimation of the discharge coefficient for each of the four cases (See figures (5), (6), (7) and (8)).
As mentioned previously, a single model (case 4) has been made to assess the effect of the surface roughness of the crest on the discharge coefficient. By matching its results with those of (case 1), which have the same dimensions, more specifically at the same flow conditions when the overflow height $H$ in each of these cases is the same ($H=5$ cm), the discharge coefficient was decreased by increasing the roughness of the weir crest surface. In sum, the value of the discharge coefficient was 0.63 in the case of a smooth surface (case 1) and 0.61 in the case of a rough surface (case 4) i.e., a decrease of around 3% (see table 1 & table 3).

![Figure 5. Cd-(H/(H+P)) Relation for the case 1.](image1)

\[ R^2 = 99.9 \]
\[ Cd = 0.544 + 0.218(H/(H+P)) - 0.351(H/(H+P))^2 + 2.084(H/(H+P))^3 \]

![Figure 6. Cd-(H/(H+P)) Relation for the case 2.](image2)

\[ R^2 = 99.9 \]
\[ Cd = 0.544 + 0.217(H/(H+P)) + 0.043(H/(H+P))^2 \]
Figure 7. Cd-(H/ H+P) Relation for the case 3.

\[ R^2 = 99 \]
\[ Cd = 0.544 + 0.482(H/H+P) - 0.542 (H/H+P)^2 + 0.302(H/H+P)^3 \]

Figure 8. Cd-(H/ H+P) Relation for the case 4.

\[ R^2 = 99 \]
\[ Cd = 0.544 + 0.204(H/H+P) + 0.184 (H/H+P)^2 \]
4. Conclusion
In this study, the discharge coefficient $C_d$ is the correction factor for the calculated discharge values due to the effect of the change in the water head over the weir, friction losses and the neglect of the velocity head. All results of discharge coefficient $(C_d)$ were ranged between 0.61 and 0.68. The non-linear regression analysis showed a high correlation between the discharge coefficient $C_d$ and the $H/(H+P)$ ratio. In other words, we can conclude that all the resulting values had good agreements between $C_d$ and $(H / H+P)$. For all the cases in this study, there was a direct relationship between the discharge coefficient $C_d$ and the ratio $(H/(H+P))$, or there is also a direct relationship with the water head upstream. The effect of surface roughness on the discharge coefficient was reported.

5. References

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