Cosmological Magnetic Field: a fossil of density perturbations in the early universe

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The origin of the substantial magnetic fields that are found in galaxies and on even larger scales, such as in clusters of galaxies, is yet unclear. If the second-order couplings between photons and electrons are considered, then cosmological density fluctuations, which explain the large scale structure of the universe, can also produce magnetic fields on cosmological scales before the epoch of recombination. By evaluating the power spectrum of these cosmological magnetic fields on a range of scales, we show here that magnetic fields of $10^{-18.1}$ gauss are generated at a 1 megaparsec scale and can be even stronger at smaller scales ($10^{-14.1}$ gauss at 10 kiloparsec). These fields are large enough to seed magnetic fields in galaxies and may therefore have af-
fected primordial star formation in the early universe.

Conventional models for the generation of large-scale magnetic fields are mostly classified into two categories: astrophysical and cosmological mechanisms. Astrophysical mechanisms—often involving the Biermann battery effect in which magnetic fields are generated from an electric current driven by the rotation of the system (1)—can explain the small-scale amplification of fields, such as in stars or in supernova explosions (2,3). However, these mechanisms do not fully explain fields on larger cosmological scales, such as those known to exist in galaxies and clusters of galaxies. Reconnecting the magnetic field lines might increase the coherence length from the size of stars to that of galaxies (tens of kpc) although there is still no convincing evidence to favor this scenario (reviewed in [4]). Or perhaps large-scale magnetic fields were directly generated in the early universe, several 100 million years after the Big Bang, when the universe was reionized [5] or protogalaxies were formed [6,7]. However the models still remain uncertain because of the lack of observations of a high redshift universe.

On the other hand, cosmological mechanisms based on inflation [8,9] have no difficulty in accounting for the length of coherence; the accelerating expansion of the universe stretches small-scale quantum fluctuations to scales that can exceed the causal horizon. However, because standard electromagnetic fields are conformally coupled to gravity, magnetic fields simply dilute away as the universe expands. Eventually the amplitudes of the magnetic fields become negligibly small at the end of inflation. To produce substantial primordial magnetic fields during inflation, new coupling—such as exotic coupling of electromagnetic fields to non-standard particles [10,11,12] or gravity [13]—must be introduced. This new coupling must amplify magnetic fields against cosmological expansion. Therefore, the nature of generated magnetic fields, such as their amplitude or spectrum, depends strongly on the assumptions built into standard cosmology or particle physics. Moreover, it is argued that almost all the models that generate magnetic fields at the inflationary epoch are ruled out, because they would produce
a large amount of gravitational waves before the Big Bang nucleosynthesis, which then make cosmic expansion faster to bring an overproduction of helium nuclei in the universe (14).

In addition to the astrophysical and cosmological mechanisms, there is a third category for the generation of large-scale magnetic fields. Small density fluctuations during the cosmological recombination of hydrogen atoms inevitably induce magnetic fields. Compton and Coulomb scatterings are so efficient that photons, protons and electrons are approximated to a tightly coupled fluid. If these three kinds of fluids moved in exactly the same way, magnetic fields could not be generated. However, because photons scatter off electrons preferentially compared with protons, small differences in velocity between protons and electrons are generated, which yields an electric current (15, 16). Moreover, we show that the anisotropic pressure of photons pushes the electrons in a different way from the protons (eq. S2). The rotation of the electric current thus generates magnetic fields. However, the rotation (or vector) mode of perturbations in the linear order is known to be damped away in the expanding universe. Therefore it is essential to consider the second-order couplings in the Compton scattering term (17, 18, 19). The magnetic fields generated through this process are correlated with temperature fluctuations at the recombination epoch because the electric current is associated with the density perturbations of photons (Fig. 1).

There are three main contributions to the generation of magnetic fields (eq. S2): (i) the baryon-photon slip term, (ii) the vorticity difference term, and (iii) the anisotropic pressure term. These terms are derived from the fact that electrons are pushed by photons through Compton scattering when velocity differences exist between them or when there is anisotropic pressure from photons. We derive here the power spectrum of magnetic fields (eq. S19), and then perform a numerical calculation to evaluate it. The power spectrum of magnetic fields $S(k)$ is defined by the expected variance of the Fourier component of magnetic fields $\vec{B}(\vec{k})$ as $S(k) \equiv \left\langle |\vec{B}(\vec{k})|^2 \right\rangle$, where $\vec{k}$ is the wave vector. The component of the field with characteristic
wavelength scale $\lambda$ can then be derived through $B_\lambda \approx \sqrt[k^3S(k)/(2\pi^2)]$ with $\lambda = 2\pi/k$. We consider a standard cosmological model \[20\] which consists of photons, baryons, cold dark matter, neutrinos, and the cosmological constant, and we fix all the cosmological parameters to the standard values (eq. S26). The density perturbations of these parameters were solved numerically for a range of scales from 10 kpc up to 10 Gpc, and they were then integrated to obtain $S(k)$ (eq. S19). We found that the field strength of generated magnetic fields at the time of cosmological recombination can be as large as $10^{-18.1}$ G at 1Mpc comoving scale, and it becomes even larger at smaller scales ($10^{-14.1}$ G at 10 kpc) (Fig. 2). After cosmological recombination, no magnetic fields would be generated, because most of the electrons were combined into hydrogen atoms and Compton scattering was no longer efficient. This means that the fields presently have an amplitude of $10^{-24.1}$ G at 1Mpc ($10^{-20.1}$ G at 10kpc) because magnetic fields decay adiabatically as the universe expands after their generation. The field strength is large enough to seed the galactic magnetic fields required by the dynamo mechanism, which is typically of the order of $10^{-20}$ to $10^{-30}$ G at around the 10 kpc scale [4][27].

Over the range of scales calculated, the generated magnetic fields increase monotonically with decreasing scale. We found that the field has a spectrum $S(k) \propto k^4$ at scales larger than $\sim 10^{2.5}$ Mpc, which corresponds to super-horizon scales at recombination; $S(k) \propto k^0$ at intermediate scales ($10^{2.5}$ Mpc $< \lambda < 10^{1.5}$ Mpc); and $S(k) \propto k^1$ at scales smaller than $\sim 10$ Mpc, where the contribution from the anisotropic stress of photons dominates. This means that the field strength $B$ is proportional to $k^2$ at scales smaller than 1Mpc. If the primordial power spectrum of density fluctuations is given by a simple power law as predicted by inflation \[22\], our result implies that magnetic fields with strength $B \approx 10^{-12.8}$ G arise on a 100 pc comoving scale at $z \approx 10$ (where $z$ is the cosmological redshift). This value helps us to understand the evolution of structures in the high-redshift universe, because those magnetic fields would be strong enough to trigger a magneto-rotational instability in the accretion disks surrounding
very first stars (population III stars), and it affect the transport of their angular momentum \(^{(23)}\).
The transport of angular momentum plays an important role for the accretion of matter onto protostars. A typical mass scale of population III stars is key \(^{(23)}\) for the early reionization and chemical evolution of the universe; therefore, cosmologically generated magnetic fields should be one of the essential ingredients in the model of structure formation in the high-redshift universe.

The behavior of the power spectrum \(S(k)\) can be understood by considering the spectra of source terms (eq. S3) at each redshift from the deep radiation-dominated era to cosmological recombination (Fig. 3). We found that at recombination, both baryon-photon slip and anisotropic stress contribute almost at the same order of magnitude around horizon scales. The contributions from the earlier epochs are dominated by anisotropic stress of photons, and they give rise to a larger amount of magnetic fields at smaller scales. Because velocity differences between electrons and photons are suppressed when energy density of radiation dominated in the early universe, anisotropic stress of photons could not be negligible at small scales. We also found that magnetic fields are mainly generated when the baryon-photon fluids undergo acoustic oscillation after crossing the horizon [Fig. 3, left panel, (2)] until photon diffusion processes \(^{(24)}\) erase the perturbations [Fig. 3, left, (3) and (4)]. The generated spectrum of magnetic fields was obtained by the non-linear convolution and time integration of these spectra of source terms (eq. S19), but approximately given by the superposition of them at each redshift.

Because the creation of magnetic fields mainly occurs when the modes of density perturbations with the corresponding scale enter the cosmic horizon and become causally connected (Fig. 3), the magnetic fields should exist at small scales below \(~ 10\) Mpc even where the Silk damping effect by diffusion of photons has swept away the density perturbations at the last scattering epoch. Thus, in principle, the detection of magnetic fields below the \(~ 10\) Mpc scales calculated here would tell us about density perturbations in photons (and baryons) in the
early universe even at scales smaller than the diffusion scale at recombination. In this sense, the magnetic field generated by this mechanism can be regarded as a fossil of density perturbations in the early universe, whose signature in photons and baryons has been lost. Therefore, this result provides the possibility of probing observations on how density perturbations in photons have evolved and been swept away at these small scales where no one can, in principle, probe directly through photons.

The amplitude of the cosmologically generated magnetic fields is too small to be observed directly through polarization effects or synchrotron emission. However, magnetic fields with such small amplitude may be detected by gamma-ray burst observations through the delay of the arrival time of gamma-ray photons due to the magnetic deflection of high energy electrons responsible for such gamma-ray photons. We suggest that, because the weak magnetic fields should inevitably be generated from cosmological perturbations as presented here, and because they should exist all over the universe even in the intercluster fields, then the weak fields should be detectable by future high-energy gamma-ray experiments, such as GLAST (the Gamma Ray Large Area Space Telescope).

Although the power of $B$ increases as $k^2$ on small scales (Fig. 2), the diffusion due to Coulomb scattering between electrons and protons damps the magnetic fields around $k \sim 10^{12} \text{Mpc}^{-1} \left[ \frac{1+z}{10^4} \right]^{7/4}$. Therefore, the energy density in magnetic fields remains finite. Because magnetic fields at smaller scales result from density perturbations in the early universe, one needs to take into consideration the high-energy effects neglected in the collision term: e.g., relativistic corrections for the energy of electrons and weak interactions along with the Compton scattering. These effects would become important when the temperature of the universe was above $\sim 1 \text{ MeV}$, which corresponds to the comoving wave number $\sim 10^5 \text{ Mpc}^{-1}$. These effects can be safely neglected at the scales considered here.
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Figure 1: All sky map (top) of cosmological microwave background anisotropy obtained by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite (20) and schematic picture (bottom) of cosmological magnetic fields generated from density fluctuations (0.5° × 0.5° sky field, which corresponds to 130 Mpc × 130 Mpc comoving scale). Red regions are hot spots and blue regions are cold spots, with a range of temperatures $\sim 2.725 \pm 200 \mu$K. The magnetic field vectors are shown together with the map. Strong magnetic fields are generated by currents where the gradient of density perturbation in photons is large.
Figure 2: Spectrum of magnetic fields $S(k)$ generated from cosmological perturbations at cosmological recombination. We plotted $\sqrt{k^3 S(k)}$ instead of $S(k)$ to measure in units of gauss. Blue dashed and red dot-dashed lines show contributions from the baryon-photon slip and photon’s anisotropic stress (the first and third terms in eq.(S2)), respectively. The spectrum decays as $k^4$ at scales larger than that of the cosmic horizon at cosmological recombination. At small scales, the contribution from the anisotropic stress of photons dominates and the spectrum has a slope proportional to $k$. 
Figure 3: Plots of source terms in Fourier space per $d\log(1 + z)$, $k^3 \rho_{\gamma} \delta_{\gamma} (v_e - v_\gamma)$ (baryon-photon slip contribution; left) and $k^3 \rho_{\gamma} \Pi_{\gamma} v_e$ (anisotropic stress contribution; right), at different redshifts. Here, $\rho_{\gamma}$ is the energy density of photons, $H$ is the Hubble parameter, $\delta_{\gamma}$ is the density fluctuation of photons, $v_e$ is the bulk velocity of electrons, $v_\gamma$ is the bulk velocity of photons, and $\Pi_{\gamma}$ is the anisotropic stress of photons. The magnetic field spectrum (Fig.2) is obtained by time and $k$-space convolution integrals of these spectra (eq.(S19)). The spectrum at each redshift is divided into four parts from large scales to small ones (blue line in the left panel): (1) featureless primordial power law spectrum at super horizon scales, (2) acoustic oscillation spectrum at sub-horizon scales, (3) damping spectrum at diffusion scales, and (4) power law phase after the diffusion damping before recombination (29). These spectra indicate that magnetic fields are created when the modes of perturbations come across the cosmic horizon and undergo acoustic oscillations. The redshifts in the figure are chosen only for illustration.