Flux flow noise and braided rivers of superconducting vortices

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Current-voltage measurements of type-II superconductors are described by a coarse-grained model of superconducting vortex dynamics. We find that the power spectra of the voltage fluctuations, and the noise power, are related to the large scale morphology of the plastic flux flow. At currents corresponding to the peak in differential resistance, the flux flow forms a braided river, the noise power is maximal, and the power spectra has a $1/f^\alpha$ form with $\alpha \approx 1.8$ over a wide frequency range. This agrees with recent experiments on NbSe$_2$. The observed variation of $\alpha$ with applied current is a crossover phenomenon.

Temporally correlated fluctuations with a power spectrum characterized by a frequency distribution of the form $1/f^\alpha$ appear ubiquitously in nature. These correlations are often observed in time series of nonequilibrium systems such as fluvial rivers [1], nerve cells [2], quasars [3], and Barkhausen emission [4], to name a few. Despite attempts at a general explanation [5,6], the concrete sources of these fractal temporal fluctuations are usually unknown. Here we show, using a cellular model of vortex dynamics, that the $1/f^\alpha$ spectrum observed in voltage noise measurements of type II superconductors is related to the complex spatio-temporal pattern of the plasticly flowing magnetic flux through the system. This suggests a general link between long range spatial correlations observed in driven dissipative systems and long range temporal correlations obtained in time series measurements of them. This link was first proposed in the context of sandpile models [5], but the original models did not display nontrivial $1/f^\alpha$ noise [6]. The sandpile type model presented here to describe plastic flux flow, however, clearly demonstrates this link.

Marley, Higgins, and Bhattacharya (MHB) found in current-voltage (IV) measurements on the type-II superconductor NbSe$_2$ that the voltage fluctuations show a $1/f^\alpha$ decay in the power spectrum with an exponent $\alpha$ that varied with the applied transport current $I_c$. Holding the applied magnetic field constant and increasing the transport current, they found that $\alpha$ varied, increasing from its initial value near 1 to about 1.8, and then falling again. The low frequency noise power $fS(f)$ was also measured. At a constant applied field, $fS(f)$ has a pronounced maximum at the transport current for which the largest value of $\alpha$ was measured, suggesting that both phenomena are related.

Those experiments were performed in the so-called “peak-effect regime” where the soft flux line lattice formed by the superconducting vortices is thought to deform by tearing, giving plastic flow [7,8]. At magnetic fields $H_{pl} < H < H_p$, the critical current, $I_c$, is an increasing function of magnetic field, and the $IV$ relation is highly nonlinear. As $H$ is increased, the size of the peak in the differential resistance, $dI/dV$, grows, reaching a maximum value at $H_c \approx 1.88$ $T$ which is significantly different from $H_{pl} \approx 1.81$ $T$ and $H_p \approx 1.95$ $T$. The magnitude of the differential resistance peak shrinks and it occurs at increasingly larger $I$, with increasing $H > H_c$. At $H_c$ itself, the peak in the differential resistance occurs at $I_c$, which marks the onset of the flow of superconducting vortices where the voltage, $V$, becomes non-zero.

In this letter, we describe the $IV$ noise experiments of MHB using a coarse-grained cellular model of superconducting vortex dynamics [9], reminiscent of a sandpile model. We show that a fractal power spectrum exists only at the maximum of the differential resistance while for other values of the applied transport current, $I$, significant deviations from the power-law behavior occur. This can be easily understood in terms of the underlying morphology of the vortex flow, which has been shown to form a braided river at the current corresponding to the peak in the differential resistance [10]. The braided structure is also responsible for the maximum of the low frequency noise power. In particular, our results support claims [4] that complex patterns of the vortex flow (which were not experimentally observed themselves) are responsible for the observed noise in the $IV$ measurements.

The cellular vortex model [9] used here is defined as follows. Consider a square lattice of size $L \times L$ in which each site $i$ is occupied by an integer number of vortices, $m_i$. The total energy of the vortex system includes the short-ranged repulsive pairwise interaction between the vortices and the attractive interaction of vortices with the pinning potential, $\tilde{V}$. For a given configuration of vortex number \{m\}, the total energy of the system is:

$$H\{m_j\} = \sum_{i,j} J_{ij} m_i m_j - \sum_i \tilde{V}_i m_i ,$$

where $J_{ij}$ includes an on-site interaction, and a weaker nearest-neighbor interaction. The model describes a vortex system coarse-grained to the scale of the London length.
The motion of the vortices is assumed to be highly overdamped, described by the equation of motion $\vec{\dot{v}} \propto \vec{f}$. The force to move a unit vortex from $x$ to $y$ is calculated by taking a discrete gradient of Eqn. (1),

$$F_{x\rightarrow y} = \hat{V}_y - \hat{V}_x + [m_x - m_y - 1]
+ r[m_{x1} + m_{x2} + m_{x3} - m_{y1} - m_{y2} - m_{y3}], \quad (2)$$

where the nearest neighbors of $x$ are $y, x1, x2,$ and $x3$, and the nearest neighbors cells of $y$ are $x, y1, y2,$ and $y3$. The onsite vortex interactions have unit strength, while $r$ describes the strength of the nearest neighbor vortex interactions. The normalized pinning potential $\hat{V}_x$ is a random number taken from a uniform distribution in the interval between zero and $V_{\text{max}}$. In each time step, all cells of the lattice are updated in parallel. A single vortex moves from a cell to a neighboring cell if the force in that direction is positive, or equivalently if the total energy of the system is lowered.

Many alternatives exist to handle the situation when more than one unstable direction appears for a vortex to move. In this work, we use two of them. In the first version of the model, the unstable direction that has the largest force is chosen and the vortex moves in that direction. The fluctuation spectrum in this case is generated solely by a deterministic dynamics. The second version of the model is stochastic. If more than one unstable direction exists, one of them is chosen randomly and the vortex moves in that direction.

Since the parameter $r$ controls the strength of the vortex repulsions, increasing $r$ decreases the steady state slope of the vortex pile. Thus, $r$ controls the critical current, which in the simulations is the slope of the vortex pile where vortices begin to continuously move. In the experiments, the critical current can be controlled by the applied magnetic field. Therefore, increasing the applied field $H_{\text{applied}} < H < H_c$ in the experiments is represented by decreasing $r$ in the simulations. Periodic boundary conditions apply at the top and bottom boundaries.

Initially, the system was prepared with a V-shaped flux density profile characteristic of the Bean state. This describes the application of a magnetic field, and was accomplished by starting with an empty lattice and then raising the left and right boundaries to equal heights, letting vortices enter and penetrate into the system until a stable state was reached. Then, a transport current $I$ was applied. According to Ampere’s law, a transport current will induce a magnetic field that raises the magnetic field on one boundary and lowers it on the opposite side. Thus, a transport current is modeled by shifting the height of, say, the left boundary versus the right boundary. Those changes were made in half integer steps. At small $I$, the shift between the left and right boundary conditions is small, and the system reaches a pinned state with no vortex flow. However, at larger $I$, the average slope is steep enough that vortices continue to flow from the high (left) boundary to the low (right) boundary, where they are removed from the system. The $IV$ characteristic then is the relation between the magnitude of the shift (representing the applied transport current) and the average flow rate of vortices (representing the voltage) when the critical current is exceeded.

![FIG. 1. Simulation results for the differential resistance, for five different value of the vortex interaction strength, $r$.](image)

The differential resistance $(dI/dV)$ curves for a number of different values of $r$ are shown in Fig. 1 for the deterministic model. For $r = 0.13$, the $IV$ curve is entirely convex, so the corresponding differential resistance curve has a large peak at the onset of vortex motion. This value of $r$ thus corresponds to $H_c$ in the experiments of MHB. As $r$ decreases, the $IV$ curve becomes S-shaped, and the peak in the differential resistance gets smaller and occurs at increasing $I$. These results are entirely consistent with the experimental $IV$ measurements in the range $H_c < H < H_p$. As shown in [10], the shape of the $IV$ curve is related to the large scale morphology of the vortex flow. In particular, the peak of the differential resistance, or equivalently the inflection point of the $IV$ curve, occurs where the vortex flow has a braided river morphology which is characterized by a self-affine (multi)fractal structure. At small $r$ the curve is quite noisy, especially near onset. This phenomena has been seen in experiments where it has been called “IV fingerprints”. As will be discussed below, this noise is due to glassy channel switching of the vortex flow.

The power spectra for a few values of the applied current at $r = 0.13$ are shown in Fig. 2a. The current $I = 0.185$ is just above onset, where for this value of $r$ the vortex flow is a braided river. The power spectrum for this value of $I$ shows $1/f^\alpha$ type behavior with $\alpha \approx 1.8$ over a range of frequencies from about $5.0 \times 10^{-4}$ to about $5.0 \times 10^{-2}$. At higher frequencies, there is a crossover to behavior that decays less rapidly with $f$, presumably due to effects associated with the discrete lattice used in the simulations. At lower frequencies, there is a crossover to
white noise which is related to the longest time scale over which correlations are retained in the system.

As $I$ increases, the range of frequency where $1/f^\alpha$ noise occurs shrinks and the white noise roll off at low frequencies sets in at a higher frequency. This indicates that the long range correlations in the system fade away. That happens presumably because, as the braided river floods, which occurs as $I$ increases, the spatial correlation length shrinks. The frequency at which the high frequency crossover occurs is independent of $I$, but the magnitude of the power spectrum at high frequency grows with $I$ because of the larger number of vortices moving. If one were to fit a value for the exponent $\alpha$ as a function of $I$, a varying exponent would be measured due to crossover. The exponent $\alpha$ would range from about 1.8 near the peak in the differential resistance to near 0 at large $I$. This range of $\alpha$ is precisely what was reported by MHB close to $H_c$. Agreement is also found for the low frequency noise power which is obviously largest for $I = 0.185$, and decreases with increasing $I$. In particular, the crossover picture is consistent with the very limited range ($\approx 1$ decade) of power-law behavior found in experiments [13]. Note that for $r = 0.13$, both the deterministic and stochastic versions of the model give essentially the same results.

Mostly similar results are also found for the power spectra at $r = 0.01$, which, for the deterministic version of the model, are shown in Fig. 2b. However, there are also some differences. At this value of $r$, the peak in the differential resistance does not occur at onset, which is at $I_c = 0.310$, but rather at $I = 0.375$. Near $I = 0.375$, the power spectrum has a $1/f^\alpha$ form with $\alpha \approx 1.8$ over a broad range of frequencies. There are again crossovers at both large and small frequencies to regions where the power spectrum decays less rapidly with $f$. The frequency at which the low frequency crossover occurs gets larger with increasing $I$, as the braided river floods and the spatial correlation length decreases.

FIG. 2. Voltage noise power spectra for (a) $r = 0.13$ and (b) $r = 0.01$. In both graphs, the solid line corresponds to the current at the peak of the differential, and the straight portion of both of those curves has a slope of approximately -1.8.

The low frequency noise power as a function of $I$ for $r = 0.01$ is shown in Fig. 3. It is calculated by adding all of the points of $S(f)$, with $f$ between $10^{-3}$ and $10^{-4}$. The result rises many orders of magnitude and peaks where the river flow is braided, at the peak in the differential resistance. This result is also seen in the experiments.

At transport currents $I$ closer to $I_c$, where the S-shaped IV curve is concave, and the vortices flow through isolated filamentary channels [14], the voltage fluctuations are periodic in the deterministic model. Also, in this regime, long-time scale transient and glassy behavior occurs. This is due to channel switching. At $I$ close to $I_c$, the system typically switches between a number of different sets of channels before settling into the steady-state. The system can continue switching for $10^6$ timesteps, or more, before settling. It does finally settle into a stationary configuration, but only after many different sets of channels are probed. As the current increases, the transient time becomes longer than we can measure, possibly infinite, near the peak in the differential resistance.

For the stochastic model, the results at $r = 0.01$ differ

FIG. 3. Low frequency noise power for $r = 0.01$. This can be compared with measurements described in Fig. 4 of Ref. [7].
somewhat from those of the deterministic model. First, the critical current $I_c$ is larger in the stochastic case. Apparently, the stochastic lattice updating can frustrate flow through isolated filamentary channels. Second, in the stochastic model for $I > I_c$ the system never settles into a stationary river configuration. Instead, channel switching continues to occur for at least $10^7$ lattice updates, and presumably continues indefinitely. However, $1/f^\alpha$ noise with $\alpha \approx 1.8$ also occurs in this version of the model near the peak in the differential resistance.

Additionally, it is interesting to note that, in the braided regime, the change in the total number of vortices contained within the system also show $1/f^\alpha$ fluctuations with $\alpha \approx 1.15$. This agrees quantitatively with the fall-off power spectrum measured in the field ramping experiments of Field and co-workers [14].

Other theoretical studies of vortex dynamics have also found $1/f^\alpha$ behavior in the power spectrum of the voltage noise. Driving the system in a different fashion designed to maintain a critical state, Mohler and Stroud [15] using the cellular model [12] and Olson, et al. [17], using molecular dynamics simulations found fractal behavior in the noise spectrum. Those results are consistent with the fact that in both cases the vortex flow has a braided river morphology [11,12]. However, the value of $\alpha$ they measured deviates from that measured here due to the different driving. Another approach was taken by Domínguez et al. [8]. Integrating a set of equations that results from a discretization of the London model, he found two regimes of vortex flow, filamentary motion near onset, and homogeneous motion at higher currents. In the filamentary channel regime, he discovered $1/f$ behavior of the voltage noise power spectrum. Using molecular dynamics simulations, Kolton et al. [10], also found the same two regimes and a peak in the low frequency noise power at the transition between them. They also suggested that the transition is from plastic flow in the filamentary regime to smectic flow in the homogeneous regime, thereby associating the transition with a reordering of the flux line lattice. The cellular model used here cannot describe any possible structural transitions in the flux line lattice. Nevertheless, we find remarkable quantitative similarity with experimental voltage noise measurements involving the transition from filamentary, through braided, to homogeneous motion of the vortices.

In summary, voltage noise in IV measurements has been studied in a simple, cellular model of vortex dynamics. The model captures the essential features of the plastic flow of the flux line lattice formed by the superconducting vortices: $1/f^\alpha$ behavior of the power spectrum was observed where the differential resistance has a peak, and the vortex flow has been shown to form a braided river. This result is robust, occurring over all values of the vortex interaction strength $r$, and for two different versions of the model, one completely deterministic and the other stochastic. The results model experiments conducted on NbSe$_2$, in the peak-regime where the FLL moves plasticly. Most notably, quantitative agreement is found in the behavior of the exponent $\alpha$ between simulations and experiments, both of which find $\alpha \approx 1.8$ at the peak of the differential resistance. Both also find that the low frequency noise power is maximal at the peak in the differential resistance. Therefore, we conclude that is the self-organized spatio-temporal structure of the braided river flow that is responsible for the observed $1/f^\alpha$ behavior of the IV power spectra in superconductors.

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