We study the dynamical formation of disoriented chiral condensates in very high energy nucleus-nucleus collisions using Bjorken hydrodynamics and relativistic nucleation theory. It is the dynamics of the first order confinement phase transition which controls the evolution of the system. Every bubble or fluctuation of the new, hadronic, phase obtains its own chiral condensate with a probability determined by the Boltzmann weight of the finite temperature effective potential of the linear sigma model. We evaluate domain size and chiral angle distributions, which can be used as initial conditions for the solution of semiclassical field equations.
1 Introduction

It is expected that quark–gluon plasma will be created in high energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and at the Large Hadron Collider (LHC) at CERN. The experiments should allow us to study the chiral and/or confinement phase transition/crossover. One possible consequence of plasma formation and disassembly is the creation of misaligned chiral domains (disoriented chiral condensates or DCC) which are regions of space–time where the average value of the chiral field does not point in the same direction as the surrounding true vacuum. For a brief review see [1].

This possibility was really emphasized and studied by Rajagopal and Wilczek [2] in the context of heavy ion collisions within a version of the linear $\sigma$–model. It was found that if the system evolves close to thermal equilibrium the average domain size is of order $1/T_c$, the inverse of the phase transition temperature. The resulting domain sizes are therefore too small to have many observable consequences; enhanced baryon–antibaryon production may, however, be one of them [3]. Subsequent studies of DCC concentrated on nonequilibrium scenarios like quenching [4] where the system is suddenly relaxed from an initial thermal state above $T_c$ to zero temperature. The resulting field configuration is unstable and decays on time scales of order $1/m_\sigma$. Again one obtains domain sizes which are too small to have many experimental consequences. Finally, in an annealing scenario [5], medium modifications of the $\sigma$ were taken into account, allowing for a reduction of $m_\sigma$ and therefore an increase of domain size of up to 3 fm.

All aforementioned approaches assume that the phase transition is, or at least close to, second order. In this paper we would like to investigate the consequences of a first order phase transition. The disoriented chiral domains are created via statistical fluctuations. We assume that a relativistic heavy ion collision results in the formation of an extended volume of quark gluon plasma which transforms itself into a hadronic resonance gas through homogeneous nucleation of hadronic bubbles [7]–[9]. Bubbles of the new hadronic phase are created by statistical fluctuations which grow due to their lower free energy. The phase transition is completed when the entire plasma has been transformed into the hadronic resonance gas. This dynamics implements the confinement characteristics of the phase transition. The dynamical information is contained in the free energy difference between the plasma phase and the resonance gas phase and in their transport properties.

We assume that the dynamical evolution will be dominated by this color confinement at temperatures close to the critical temperature $T_c$. In principle we could imagine that every hadronic bubble nucleated in this way starts out with the chiral field pointing in a random direction in internal symmetry space. The bubbles grow with time until the whole plasma phase is transformed into the hadronic phase, resulting in a distribution of domains with different sizes and different chiral angles. For definiteness we use the linear $\sigma$–model to evaluate the part of the hadronic free energy dependent on the chiral field.
The effect of this contribution on the dynamical evolution is perturbative.

The domains formed in this approach will be larger or of order $R_c$, the critical size radius to nucleate an hadronic bubble. This is a dynamical quantity which diverges at the critical temperature and approaches the hadronic length scale $1/T_c$ at small temperatures. We calculate a domain size and chiral angle distribution. This distribution may be taken as the initial condition for solving the semiclassical equations of motion for the subsequent time evolution of the system [6].

Our approach will neglect the possibility that a bubble nucleated with a very large radius might contain several chiral domains. This is not a serious concern since the average nucleation bubble is not so large. Furthermore, the resulting domain size and angle distribution should also depend on the rapidity. For given rapidity the nucleated bubbles will obtain a certain eccentricity in the beam direction. In this first study we neglect this complication.

In section 2 we start out by outlining the implementation of the dynamical evolution of the system. The free energies we are using in this section are at zero chiral angle. In section 3 we evaluate the perturbative chiral contribution to the hadronic free energy at nonzero chiral angle and show how to obtain the domain size and angle distribution. In section 5 we discuss and summarize our results.

2 Dynamics of the Confinement Phase Transition

In this section we briefly review the dynamics of the hot matter as it passes through a first order confinement phase transition. The overall picture is that quark–gluon plasma is formed at high temperature and subsequently expands and cools. The plasma must cool below the critical temperature before bubbles of the hadronic phase can be nucleated. When enough bubbles have been nucleated their growth causes a reheating, turning off further bubble nucleation. Eventually all matter is converted to the hadronic form, and cooling begins again. Eventually the hadrons lose thermal contact with each other and they free–stream to the detectors. It is within this scenario that we consider the formation of DCC. The works [7]–[9] should be consulted for more details about this scenario.

2.1 Review of nucleation dynamics

The rate for the nucleation of the hadron phase out of the plasma phase can be written as

$$I = I_0 e^{-\Delta F_*/T},$$

where $\Delta F_*$ is the change in the free energy of the system with the formation of a critical size hadronic bubble and $I_0$ is the prefactor with dimensions of inverse volume inverse time. In general, statistical fluctuations at $T < T_c$ will produce bubbles with associated
free energy
\[ \Delta F = \frac{4\pi}{3} [p_q(T) - p_h(T)]R^3 + 4\pi R^2\sigma. \] (2)

Here \( p \) is the pressure of the quark or hadron phase at temperature \( T \), and \( \sigma \) is the surface free energy of the quark-gluon/hadron interface. Since \( p_q < p_h \) it follows that there is a bubble of critical radius
\[ R_*(T) = \frac{2\sigma}{|p_h(T) - p_q(T)|}. \] (3)

Smaller bubbles tend to shrink because the surface energy is too large relative to volume energy, and larger bubbles tend to grow. The free energy of the critical size bubble is therefore
\[ \Delta F_* = \frac{4}{3}\pi \sigma R_*^2. \] (4)

The prefactor has been computed in a coarse-grained effective field theory approximation to QCD to be
\[ I_0 = \frac{16}{3\pi} \left( \frac{\sigma}{3T} \right)^{3/2} \frac{\sigma \eta_q R_*}{\xi_q^4 (\Delta w)^2}, \] (5)

where \( \eta_q \) is the shear viscosity in the plasma phase, \( \xi_q \) is a correlation length in the plasma phase, and \( \Delta w \) is the difference in the enthalpy densities of the two phases. At the critical temperature, \( R_* \rightarrow \infty \), and the rate vanishes.

Given the nucleation rate one would like to know the (volume) fraction of space \( h(t) \) which has been converted from QCD plasma to hadronic gas at the proper time \( t \), which is the time as measured in the local comoving frame of an expanding system. This requires a kinetic equation which uses \( I \) as an input. If the system cools to \( T_c \) at time \( t_c \) then at some later time \( t \) the fraction of space which has been converted to hadronic gas is
\[ h(t) = \int_{t_c}^t dt' I(T(t'))[1 - h(t')]V_{\text{bub}}(t', t). \] (6)

Here \( V_{\text{bub}}(t', t) \) is the volume of a bubble at time \( t \) which had been nucleated at the earlier time \( t' \).

Once formed, bubbles will grow, as it is favorable from the point of view of free energy. We assume the growth law
\[ v(T) = v_0 [1 - T/T_c]^{3/2}, \] (7)
where \( v_0 \) is a model-dependent constant. The simple illustrative model for bubble growth is
\[ V_{\text{bub}}(t', t) = \frac{4\pi}{3} \left( R_*(T(t')) + \int_{t'}^t dt'' v(T(t'')) \right)^3. \] (8)

This expression assumes that the interface between the inside and outside of the bubble is created at rest in the local comoving frame.
A dynamical equation is needed to describe how the system expands. We will use Bjorken’s longitudinal scaling hydrodynamics \[10\]. The derivative of the energy density is related to the enthalpy density as

$$\frac{de}{dt} = -\frac{w}{t}. \quad (9)$$

This assumes kinetic equilibrium among the particles but not phase equilibrium. It is a statement of energy conservation. The energy density is

$$e(T) = h(t)e_h(T) + [1 - h(t)]e_q(T), \quad (10)$$

where \(e_h(T)\) and \(e_q(T)\) are the energy densities in the two phases at temperature \(T\), and similarly for \(w\).

We model the plasma phase by a gas of gluons and massless quarks of either two or three flavors with a bag constant \(B\) to simulate confinement. The pressure is

$$p_q = g_q \frac{\pi^2}{90} T^4 - B, \quad (11)$$

where \(g_q\) is chosen appropriately. We model the pressure in the hadronic phase according to the formula

$$p_h = g_h \frac{\pi^2}{90} T^4 \quad (12)$$

where \(g_h\) is an effective number of degrees of freedom relevant for the temperature range of interest, namely, \(130 < T < 170\) MeV. Including noninteracting pions alone, for example, gives \(g_h \approx 3\). Here we will consider two parameter sets. Set A includes only \(u\) and \(d\) quarks. In the hadron phase we include the particles \(\pi, \eta, \omega, \rho\). In the aforementioned temperature range \(g_h \approx 4.6 \ [11]\). Set B includes \(u, d\) and \(s\) quarks. In the hadron phase we include all states in the Particle Data Book with a repulsive interaction among them. This equation of state was studied in \[12\]. A good parametrization is obtained with \(g_h \approx 7.5\).

The critical temperature is obtained by equating the pressures of the two phase. We adjust the bag constant so that \(T_c = 160\) MeV. In set A one gets \(B^{1/4} = 220\) MeV and in set B one gets 232 MeV. The other parameters are as in \[8\], namely: \(\sigma = 50\) MeV/fm\(^2\), \(\xi_q = 0.7\) fm, \(\eta_q = 14.4T^3\) and \(v_0 = 3\).

### 2.2 Numerical results

The evolution equations may be solved as in \[8\]. Figure 1 shows the temperature as a function of time. It is assumed that the system cools to \(T_c\) at a local comoving time of 3 fm/c. The system must supercool to about \(0.95T_c\) before noticeable nucleation begins. The system continues to expand and cool due to inertia, but is slowed down by bubble nucleation and growth. The temperature reaches a local minimum at around \(0.80T_c\), at
which point the release of latent heat is sufficient to begin reheating. As the temperature goes up, the nucleation rate decreases, and basically shuts off at about $0.95 T_c$. Thereafter there is essentially no new bubble creation; the transition completes due to the growth of existing bubbles. When the fraction of space $h$ occupied by hadrons reaches 100% the transition is complete and the temperature will again fall. This is not shown on the figure. It is the state of the system at this moment which will determine the initial conditions for solving the equations of motion for the chiral fields.

Figure 2 shows the volume of a hadronic bubble at the end of the phase transition as a function of the time at which it was nucleated. The volume of a critical size bubble is a monotonically decreasing function of temperature. This means that bubbles which were nucleated early or late in the supercooling phase should have the largest volume, as is seen to be the case. The smallest bubbles should be those which were nucleated near the minimum of the temperature curve, which is also seen to be the case. The minimum of the volume curve is offset to somewhat later times than the minimum of the temperature curve due to the subsequent growth of bubbles.

Figure 3 shows the density of bubbles as a function of their volume at the end of the transition. This is obtained by keeping track of how many bubbles were nucleated between times $t$ and $t + dt$ and then following their growth to the end of the transition at time $t_f$. The density distribution is

$$\frac{dn}{dV} = \int_{t_c}^{t_f} \frac{dt}{t} I(t) [1 - h(t)] \delta (V - V_{\text{bub}}(t, t_f)),$$

where the factor $t/t_f$ takes into account the fact that the density decreases due to the expansion of the system. In general there will be two times contributing to the same final bubble volume, as already seen in figure 2. The distribution plotted in figure 3 is made dimensionless by multiplying by the minimum volume squared and is plotted against the linear size of the bubble to better display the results. For set A the minimum final volume is 22 fm$^3$ while for set B it is 10 fm$^3$. It should be remarked that at the end of the transition the bubbles will not be spherical due to the longitudinal, Bjorken, expansion along the beam axis. They will be elongated along the beam direction with a typical eccentricity of $t_f/t_{sc} \approx 3 - 4$ where $t_{sc}$ is the time at which the temperature has dropped to its minimum value along the supercooling curve.

Some important numbers from the supercooling and reheating cycle are listed in Table 1. For each parameter set we list: the time $t_{sc}$ and temperature $T_{sc}$ when supercooling ends and reheating begins; the completion time $t_f$ and temperature $T_f$ of the phase transition; the minimum volume of a bubble $V_{\text{min}}$ at the end of the transition and the time $t_{\text{min}}$ at which it was nucleated.

The distribution $dn/dV$ at the end of the transition diverges at its lower limit like $1/\sqrt{1 - V_{\text{min}}/V}$. This is a rather weak, integrable, singularity which may, however, have important consequences for the observation of DCC in heavy ion experiments.

One remarkable feature of this dynamics is that the contribution to large bubbles
comes mainly from early times, \( t < t_{sc} \). Bubbles which are nucleated near the bottom of the temperature curve do not have sufficient time to grow very much before reheating causes the growth velocity to decrease substantially. Bubbles nucleated at later times have a larger (critical) volume to begin with, but their growth is also suppressed by a decreasing growth velocity. In addition, their rate of production is suppressed relative to early times due to the factor \( 1 - h \) in eq. (13); there is insufficient space available between existing bubbles to nucleate new ones. The distribution for volumes larger than about 100 fm\(^3\) is almost completely determined by the initial stages of the phase transition. Even if we would shut–off nucleation by hand at time \( t_{sc} \) we would essentially obtain the same size distribution at large volumes. The reheating phase of the transition does not influence the size distribution at large volumes!

The behavior of \( dn/dV \) at large volumes is well fitted to the form \( \exp[-a \left( V/V_{\text{min}} \right)^{2/3}] \). The values of the coefficient \( a \) are given in Table 1. The exponential dependence on the surface area is straightforward. Large bubbles at the end of the transition arise from large bubbles nucleated early on. The probability to nucleate one is proportional to the rate obtained from eqs. (1) and (4), which is exponential in the surface area of a critical bubble. Allowing for growth due to longitudinal expansion we can estimate roughly that

\[
a = \left( \frac{4 \pi}{3} \right)^{1/3} \frac{\sigma}{T_c} \left( \frac{t_0}{t_f} \right)^{2/3} V_{\text{min}}^{2/3}. \tag{14}
\]

If we compare this result with the fitted values given in Table 1 we get agreement to better than a factor of 2.

## 3 DCC Within a Nucleated Bubble

In the previous section we discussed how a first order phase transition from the deconfined quark–gluon phase to the confined hadronic phase may occur via supercooling, nucleation, bubble growth, and reheating. We did not allow for the possibility that a disoriented chiral condensate could appear spontaneously with the nucleation of the bubble. In this section we will. For simplicity and definiteness we will use the linear \( \sigma \)–model to implement approximate chiral symmetry. The essence of this approach is that the free energy difference \( \Delta F(R) \) in eq. (2) contains all hadronic degrees of freedom, including the pions and the \( \sigma \)–meson, but at zero chiral angle. To describe now chiral condensates at high temperatures we use the ansatz

\[
\Delta F(R, \theta) = \Delta F(R, 0) + \Delta F_{\text{chiral}}(R, \theta), \tag{15}
\]

where \( \Delta F_{\text{chiral}} \) is a perturbative contribution to the free energy difference between hadronic and quark–gluon phases for nonzero chiral angle; it is normalized such that \( \Delta F_{\text{chiral}}(R, 0) = 0 \). For low temperatures the system will tend to orient itself towards \( \theta = 0 \). On the other hand, if the temperature is larger than a certain chiral critical temperature \( T_{\text{ch}} \) all
angles should be approximately equally likely. The aim of this section is to construct \( \Delta F_{\text{chiral}}(R, \theta) \) and to show how to obtain, perturbatively, a distribution in the chiral angle over an ensemble of differently-sized disoriented chiral domains.

### 3.1 The effective potential for chiral angles

We will evaluate the finite temperature effective potential representing the degrees of freedom of the chiral condensate by using the linear sigma model.

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \pi)^2 - \frac{\lambda}{4} \left( \sigma^2 + \pi^2 - c^2 / \lambda \right)^2 + H \sigma.
\]

The parameters of the Lagrangian \( \mathcal{L} \) are fixed at tree level by imposing the existence of spontaneous symmetry breaking, PCAC and the meson masses in vacuo. This leads to the condensates \( \langle \sigma \rangle = v \) and \( \langle \pi_i \rangle = 0 \) where

\[
\begin{align*}
v (\lambda v^2 - c^2) - H &= 0, \\
H &= f_\pi m_\pi^2, \\
m_\pi^2 &= \lambda v^2 - c^2, \\
m_\sigma^2 &= m_\pi^2 + 2 \lambda v^2.
\end{align*}
\]

(17)

Numerical values used here are: \( f_\pi = 94.5 \) MeV, \( m_\pi = 140 \) MeV, and \( m_\sigma = 1 \) GeV. The value of the \( \sigma \)-mass is chosen in accordance with the analysis of reference [15]. The evaluation of the system of equations in (17) yields \( v = f_\pi, H = 47.6 \) MeV/fm\(^2\), \( \lambda = 54.9 \) and \( c = 686 \) MeV.

The finite temperature effective potential for the linear sigma model is discussed in many places; for example, in [16]. Since eq. (16) has a remnant O(3) symmetry in the isovector sector we can simplify this task by letting the condensate point in the third direction of the isovector space. To evaluate the thermal masses we expand the fields around an arbitrary point as:

\[
\begin{align*}
\sigma(x) &= v \cos \theta + \sigma'(x), \\
\pi_3(x) &= v \sin \theta + \pi_3'(x).
\end{align*}
\]

(18)

We find

\[
\begin{align*}
m_1^2 &= m_2^2 = m_3^2 = \lambda v^2 - c^2, \\
m_0^2 &= 3 \lambda v^2 - c^2.
\end{align*}
\]

(19)

(20)

In the high temperature limit of the one-loop approximation one keeps terms that are of order \( T^4 \) and \( m^2 T^2 \) only. Then the effective potential is:

\[
V_{\text{eff}}(T; v, \theta) = \frac{\lambda}{4} \left[ v^4 - (T_{\text{ch}}^2 - T^2) v^2 \right] - H v \cos \theta.
\]

(21)
The chiral critical temperature is defined by $T_{\text{ch}} = \frac{\sqrt{2c^2/\lambda}}{\lambda}$. It is the temperature where the curvature term in the effective potential vanishes. When chiral symmetry is exact, $m_\pi = 0$, and there is a second order phase transition associated with chiral symmetry restoration. For our choice of parameters this temperature is 130.9 MeV. It is useful to recognize that for a fixed chiral angle $\theta$ the effective potential is minimized when $v$ satisfies the cubic equation

$$v^3 - \frac{1}{2} \left( T_{\text{ch}}^2 - T^2 \right) v - \frac{H}{\lambda} \cos \theta = 0.$$  \hfill (22)

We shall label solutions to this equation as $v_\theta(T)$.

To develop an intuitive understanding of the dynamics of the chiral contribution we first plot, in figure 4, the value of the chiral condensate at the global minimum of the effective potential as a function of the temperature. The global minimum is always at zero chiral angle when chiral symmetry is dynamically broken by nonzero pion mass. When chiral symmetry is exact the potential is independent of angle. The dashed curve in the figure represents the chiral limit $m_\pi = 0$ in which we recover a second order chiral phase transition; the chiral critical temperature approaches 133.6 MeV when keeping the $\sigma$–mass and pion decay constant fixed. With the physical value of the pion mass the chiral condensate never goes to zero; the minimum of the effective potential is always away from the origin and chiral symmetry cannot be restored. Nevertheless the chiral condensate $v_0(T)$ becomes very small at high temperatures, being about 7.9 MeV at the confinement temperature of 160 MeV. For high temperatures it decreases like $2/\lambda T^2$ according to equation (22). The minimum of the effective potential itself in (21) decreases like $2H^2/\lambda T^2$. We can now compare the magnitude of the pressure difference of the quark–gluon and hadronic phases in eqs. (11)-(12) to the magnitude of the effective potential for the chiral angle $\theta$. For more than 2% supercooling the chiral contribution is perturbative. There is essentially no nucleation for such small supercooling so the chiral contribution is safely in the perturbative region.

In figure 5 we plot the isotherms of the function $v_\theta(T)$ in the $\pi_3$–$\sigma$ plane. At low temperatures the isotherms are approximately circles with radii $\leq f_\pi$. The radius shrinks and the circle becomes more distorted as the temperature increases. For temperatures less than

$$T_{\text{saddle}} = \sqrt{T_{\text{ch}}^2 - 6 \left( \frac{H}{2\lambda} \right)^{2/3}}$$  \hfill (23)

there is a saddle point in the $\theta = \pi$ direction. This temperature is $T_{\text{saddle}} = 114.9$ MeV when the pion mass has its physical value. Above this temperature there is no solution to the cubic equation at $\theta = 0$. Rather, the saddle point splits into two and circles back around towards the origin with increasing temperature. At a temperature $T_{\text{ch}} = 130.9$ MeV it reaches the origin. Between $T_{\text{saddle}}$ and $T_{\text{ch}}$ we have a whole range of backward angles without a minimum in the radial direction. Above the chiral critical temperature we have once again minima in all directions, but they are all centered away from the
origin, corresponding to $\theta \leq \pi$. Chiral symmetry is approximately restored in the sense that $v_\theta(T)$ is very small compared to $f_\pi$ and $T$.

The behavior depicted in figure 5 must be dependant on the use of the linear $\sigma$–model to some extent. Nevertheless it is interesting to see that there is a range of temperatures where the effective potential does not have minima in the radial direction for parts of the backward chiral plane. This would lead to a strong dynamical enhancement of the forward direction in the chiral angle distribution. We consider this a typical example of the general statement that the inclusion of both dynamical and thermal effects on the evolution of distributions tends to smooth out transitions between states and, in this sense, suppresses large angle contributions.

So far we have only discussed the volume term of the chiral angle effective potential. Generally one expands the free energy of a finite system in terms of volume, surface, curvature and logarithmic contributions. The isotherms of figure 5 are valleys with only one minimum located at $\theta = 0$. Therefore no surface contribution can be defined in principle. Only if we include a higher order term in $\sigma$ in the symmetry breaking potential, like a quadratic one, can we generate a metastable configuration and therefore define a surface free energy. A quadratic term, for example, would generate a barrier at $\theta = \pm \pi/2$ and in this way assure the metastability of the condensate at $\theta = \pi$ [16].

To construct a surface free energy requires two phases with equal pressures. There are, in fact, two phases under consideration, the quark–gluon and the hadronic phases. Between these there is a surface free energy (always accepting a first order phase transition). However, the $\theta$ dependence of it is not calculable within the linear $\sigma$–model, nor within any model which does not incorporate the quark and gluon degrees of freedom. We take the point of view that the plasma does not care in which direction the chiral field points. The $\theta$ dependence of the surface free energy is taken to be zero in this paper.

### 3.2 Domain size distributions

Consider a bubble which was nucleated at a temperature $T$ with a volume $V$. The probability that this bubble was nucleated with a particular chiral angle $\theta$ (constant throughout its interior) is

$$\frac{dP}{d\theta} \propto \exp \left[ -\Delta F_{\text{chiral}}(V, T, \theta)/T \right], \quad (24)$$

where $F_{\text{chiral}}(V, T, \theta)/V = V_{\text{eff}}(T, v_\theta(T), \theta) - V_{\text{eff}}(T, v_0(T), 0)$. If it is assumed that the direction of the chiral condensate does not change from the time the bubble was nucleated to the end of the phase transition then the joint distribution in chiral angle and domain volume at the end of the transition can be computed. From eqs. (13) and (23):

$$\frac{d^2n}{dVd\theta} = \int_{t_c}^{t_f} dt \frac{dt}{t_f} I(t)[1 - h(t)]\delta(V - V_{\text{bub}}(t), t_f) \frac{dP}{d\theta}(V_{\text{bub}}(t), T(t), \theta). \quad (25)$$
On the other hand the direction of the chiral condensate may relax according to the probability distribution appropriate to the current size and temperature of the bubble. If this is the case then the joint distribution at the end of the phase transition would be

\[
\frac{dn}{dV d\theta} = \int_{t_c}^{t_f} \frac{dt}{t_f} I(t)[1 - h(t)] \delta (V - V_{bub}(t, t_f)) \frac{dP}{d\theta}(V_{bub}(t, t_f), T(t_f), \theta)
\]

\[
= \frac{dP}{d\theta}(V, T(t_f), \theta) \int_{t_c}^{t_f} \frac{dt}{t_f} I(t)[1 - h(t)] \delta (V - V_{bub}(t, t_f))
\]

\[
= \frac{dP}{d\theta}(V, T(t_f), \theta) \frac{dn}{dV}(V)
\]

(26)

where \(dn/dV\) is from eq. (13). It is difficult to know which limit more closely approximates reality. The question is: What is the relaxation time for the chiral condensate within a bubble? We make an estimate of the critical damping constant which separates the limits of weak and strong damping, analogous to a damped harmonic oscillator, in the appendix. For the purpose of this paper we compute both limits which will provide bounds on what can happen when relaxation effects are included.

To get a feel for the difference between the extreme limits of underdamping and overdamping we plot, in figure 6, the effective potential for the chiral angle divided by the temperature versus angle. The Boltzmann/Gibbs factor is obtained by multiplying by a volume and exponentiating. The solid curve is for the temperature at the bottom of the supercooling curve where the nucleation rate is the highest. The dashed curve is for the final temperature at the end of the phase transition. From these curves one can already predict that the final distribution in chiral angle will be much broader if there is overdamping; that is, if the chiral angle can relax fast enough to follow the temperature as it goes up.

The full double differential distributions in chiral angle and domain size is shown in figure 7. Panels (a) and (b) represent parameter sets A and B, respectively, with the chiral angle determined at the moment the bubble was nucleated. The “angular distribution” is strongly peaked in the forward direction, falling by one to two orders of magnitude as \(\theta\) increases from 0 to \(\pi/2\). This is because the dominating nucleation temperature is around 0.80\(T_c\) which is comparable to the chiral symmetry breaking scales of \(\sqrt{2}f_\pi\) (spontaneous) and \(m_\pi\) (dynamical). Panels (c) and (d) represent parameter sets A and B, respectively, with the chiral angle determined at the end of the phase transition. Since the temperature then is greater, about 0.99\(T_c\), the angular distribution is much flatter for domain sizes less than about 30\(V_{min}\). Thus a large relaxation rate is beneficial to the formation of DCC at this point in the heavy ion collision.

4 Conclusion

We discussed the dynamical formation of disoriented chiral condensates in ultrarelativistic heavy ion collisions. Our basic idea is that a first order confinement phase transition
governs the dynamical evolution of the system. Once a hot plasma region is formed it will cool according to Bjorken hydrodynamics. Bubbles or fluctuations of the low temperature hadronic phase can form and grow after the system supercools below the critical temperature. This process can be described by homogeneous nucleation theory. Every bubble or fluctuation can have its own chiral condensate. The orientation of a condensate is determined by the Boltzmann/Gibbs weight of the finite temperature effective potential of the linear sigma model. We evaluate domain size and chiral angle distributions.

This model is the first attempt to use the dynamics of a first order phase transition to generate disoriented chiral condensates. Many of the approximations we made in this model can be relaxed in more elaborate calculations. For example, the influence of the chiral contribution to the free energy is treated perturbatively to lowest order. The dependence on rapidity, the eccentricity of the domains, is neglected. The nucleation description we apply is insufficient both at the very early stages of the supercooling process, where very large bubbles containing several different domains might form, as well as in the final stages of the phase transition, where surface and/or topological effects between overlapping domains might become important. An improved description of the thermodynamic potential for the chiral degrees of freedom as well as an investigation of the importance of dissipation in the evolution of a chiral domain embedded in a hadronic heat bath is desirable.

It is currently accepted that both the confining character of QCD as well as the properties imposed by an approximate chiral symmetry are crucial in determining the properties of the QCD phase transition. In the model presented we intended to incorporate both properties in a sensible description of the phase transition dynamics and the resulting collective observables. We are of the opinion that modifications and improvements of our model as well as different approaches in the same class of models will be crucial in not only addressing issues like the formation of DCCs but also in addressing all issues related to the late stages of the quark–hadron phase transition.

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**References**

[1] For a short review, see: S. Gavin, Nucl. Phys. A 590, (1995) 163c, and references therein.
[2] K. Rajagopal and F. Wilczek, Nucl. Phys. B 379 (1993) 395.

[3] J. I. Kapusta and A. M. Srivastava, Phys. Rev. D 52 (1995) 2977.

[4] K. Rajagopal and F. Wilczek, Nucl. Phys. B 404 (1993) 577.

[5] S. Gavin and B. Müller, Phys. Lett. B 329 (1994) 486.

[6] L.P. Csernai and I.N. Mishustin, Phys. Rev. Lett. 74 (1995) 5005, L.P. Csernai, I.N. Mishustin and A. Mocsy, Heavy Ion Phys. (1996) in press.

[7] L. P. Csernai and J. I. Kapusta, Phys. Rev. D 46 (1992) 1379.

[8] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. 69 (1992) 737.

[9] L. P. Csernai, J. I. Kapusta, Gy. Kluge and E. E. Zabrodin, Z. Phys. C 58 (1993) 453.

[10] J. D. Bjorken, Phys. Rev. D 27 (1983) 140.

[11] D. K. Srivastava and B. Sinha, J. Phys. G: Nucl. Part. Phys. 18 (1992) 1467; J. Alam, D. K. Srivastava, B. Sinha and D. N. Basu, Phys. Rev. D 48 (1993) 1117.

[12] J. Kapusta and K. Olive, Nucl. Phys. A 408 (1983) 478; Phys. Lett. B 209 (1988) 295; and K. Olive, private communication.

[13] D. K. Srivastava, J. Pan, V. Emel’yanov and C. Gale, Phys. Lett. B 329 (1994) 157.

[14] J. I. Kapusta, A. P. Vischer and R. Venugopalan, Phys. Rev. C 51 (1995) 901; A. P. Vischer, Nucl. Phys. A 590 (1995) 585c.

[15] W. Lin and B. D. Serot, Phys. Lett. B 233 (1989) 23; Nucl. Phys. A 512 (1990) 637.

[16] J. I. Kapusta and A. M. Srivastava, Phys. Rev. D 50 (1994) 5379.
Appendix

In this appendix we study small amplitude oscillations of the chiral angle about the absolute minimum of the effective potential to get an estimate of the time scales involved relative to the time evolution of the temperature. Replacing the potential in the Lagrangian (16) with the effective potential in (21) results in the equation of motion for $\theta$.

$$v^2 \partial^2 \theta - H v \sin \theta = 0.$$  \hspace{1cm} (27)

This is the same equation one would have obtained by neglecting the thermal one–loop corrections. However, it does not include thermal effects on the derivative terms. The thermal effects are incorporated via the radial field $v$ which will be constrained to satisfy $\partial V_{\text{eff}}/\partial v = 0$ in equation (22). Hence we replace $v$ in eq. (27) with $v_0(T)$ thus eliminating its thermal fluctuations.

We would like to solve eq. (27) deep inside a hadronic bubble where we can neglect any spatial gradients. The equation of motion conserves energy and therefore cannot describe the decay of $\theta$ into the absolute minimumum, which must happen if the temperature decreases to zero. To obtain a simple order of magnitude estimate of the time scale in the decay process we introduce a dissipative term of the form $\eta v^2 \partial \theta/\partial t$. In this ansatz $\eta$ has the dimension of energy and corresponds to a Langevin type of friction term. For small oscillations around zero, relevant at low temperatures:

$$v_0^2(T) \ddot{\theta} + \eta v_0^2(T) \dot{\theta} + H v_0(T) \theta = 0.$$ \hspace{1cm} (28)

The oscillation frequencies $\omega_\pm$ are

$$\omega_\pm = \frac{i}{2} \eta \pm \sqrt{-\frac{1}{4} \eta^2 + \frac{H}{v_0(T)}}.$$ \hspace{1cm} (29)

The oscillator is critically damped for the value

$$\eta_c = 2 \frac{H}{v_0(T)} \rightarrow 2m_\pi, \quad T \ll T_{\text{ch}}.$$ \hspace{1cm} (30)

Since this is a typical hadronic scale it is not clear whether the chiral angle will be under– or over–damped. A stronger damping than this allows the system to relax very rapidly on time scales of less than $\eta_c^{-1}$.

For temperatures larger than $T_{\text{ch}}$ we see from figure 5 that the isotherms are approximately circles in the forward half–plane with radius $1/2v_0(T)$ centered at $(1/2v_0(T), 0)$. Expanding eq. (27) around the center of this circle we retain eqs. (24) and (30) but with $v_0$ and $\eta^{-1}$ scaled by 1/2. Using the results from section 2 about the high temperature behavior we find that

$$\eta_c = \sqrt{\lambda} T, \quad T \gg T_{\text{ch}}.$$ \hspace{1cm} (31)
The friction constant would have to increase linearly with the temperature at high temperatures to remain over-damped. This result can be understood as a consequence of approximate chiral restoration at high temperatures forcing $v_0(T)$ to become small. Actually, our small angle approximation becomes ill-defined then.
|                   | Set A | Set B |
|-------------------|-------|-------|
| $g_h$             | 4.6   | 7.5   |
| $g_q$             | 37    | 47.5  |
| $B^+$ (MeV)       | 220   | 232   |
| $t_{sc}$ (fm/c)   | 6.7   | 7.2   |
| $T_{sc}/T_c$      | 0.81  | 0.79  |
| $t_f$ (fm/c)      | 30.9  | 25.4  |
| $T_f/T_c$         | 0.991 | 0.987 |
| $t_{min}$ (fm/c)  | 8.7   | 9.7   |
| $V_{min}$ (fm$^3$)| 21.8  | 10.1  |
| $a$               | 0.67  | 0.41  |

**Table 1.** Summary of observables and scales for the two different parameter sets.
Figure Captions

**Figure 1.** The time evolution of the temperature.

**Figure 2.** The bubble volume at the end of the phase transition as a function of the time it was nucleated.

**Figure 3.** The domain size distribution at the end of the phase transition.

**Figure 4.** The thermal average value of the $\sigma$–field.

**Figure 5.** Polar plot of the radial minimum of the finite temperature effective potential in the $\pi_3$–$\sigma$ plane.

**Figure 6.** The cost in free energy per unit volume divided by the temperature as a function of the chiral angle. The solid curve is evaluated at the moment the bubble is nucleated while the dashed curve is evaluated at the end of the phase transition.

**Figure 7.** The double distribution $V_{\min}^2\frac{d^2n}{dV\,d\theta}$ in chiral domain size and direction at the end of the phase transition. In panels (a) and (b) the direction is fixed at the moment of bubble nucleation and in panels (c) and (d) the direction is fixed by the temperature at the end of the phase transition.