NON-STANDARD MODEL OF THE NUCLEON ELECTROMAGNETIC STRUCTURE AND ITS PREDICTABILITY

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Abstract

Unitary and analytic ten-resonance model of the nucleon electromagnetic (e.m.) structure with canonical normalizations and asymptotics is constructed on a four-sheeted Riemann surface. It describes well all existing experimental space-like and time-like data on the nucleon e.m. form factors (ff’s), including also FENICE (Frascati) results on the neutron, for the first time. This is achieved without any external constraints on the isovector spectral functions following from the \( \pi N \)-scattering data and pion e.m. ff behaviour through the unitarity condition. Just opposite, the model itself predicts a pronounced effect of the two-pion continuum on the isovector spectral functions revealing the strong enhancement of the left wing of the \( \rho(770) \)-resonance close to two-pion threshold.

The existence of the fourth excited state of the \( \rho(770) \) meson with parameters \( m_{\rho^{(4)}} = 2506 \pm 38 \text{ MeV}, \Gamma_{\rho^{(4)}} = 700 \pm 179 \text{ MeV} \), the large values of \( f_{\phi NN}^{(1,2)} \) coupling constants, indicating a violation of the OZI rule and also the isoscalar spectral function behaviours are predicted by the presented model.

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1 Introduction

The electromagnetic structure of (e.m.) the nucleons, as revealed in elastic electron-nucleon scattering, is completely described by four independent scalar functions, called form factors (ff’s) and dependent on the square momentum transfer $t = -Q^2$ of the virtual photon. They can be chosen in a different way, e.g. as the Dirac and Pauli ff’s, $F_1^D(t)$, $F_1^P(t)$ and $F_2^P(t)$, $F_2^P(t)$, or the Sachs electric and magnetic ff’s, $G_E^p(t)$, $G_E^n(t)$ and $G_M^p(t)$, $G_M^n(t)$, or isoscalar and isovector Dirac and Pauli ff’s, $F_1^D(t)$, $F_1^P(t)$ and $F_2^P(t)$, $F_2^P(t)$ and isoscalar and isovector electric and magnetic ff’s, $G_E^p(t)$, $G_E^n(t)$ and $G_M^p(t)$, $G_M^n(t)$, respectively.

The Dirac and Pauli ff’s are naturally obtained in a decomposition of the nucleon matrix element of the e.m. current into maximally linearly independent covariants constructed from the four-momenta, $\gamma$-matrices and Dirac bispinors of nucleons as follows

$$\langle N| J_{\mu}^{e.m.}|N \rangle = e\bar{u}(p')\left\{\gamma_\mu F_1^N(t) + i\frac{2}{m_N}\sigma_{\mu\nu}(p' - p)\nu F_2^N(t)\right\}u(p)$$

(1)

with $m_N$ to be nucleon mass.

On the other hand, the electric and magnetic ff’s are very suitable in an extraction of the experimental information on the nucleon e.m. structure from the measured cross sections

$$\frac{d\sigma^{lab}(e^- N \rightarrow e^- N)}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{1}{1 + (\frac{2E}{m_N})^2 \sin^2(\theta/2)} \left[ G_E^2 - \frac{t}{4m_N^2} G_M^2 \right]$$

(2)

(\(\alpha = 1/137, E\)-the incident electron energy)

and

$$\sigma^{c.m.}_{tot}(e^+ e^- \rightarrow NN) = \frac{4\pi\alpha^2 \beta_N}{3t} \left[ |G_M(t)|^2 + \frac{2m_N^2}{t} |G_E(t)|^2 \right], \quad \beta_N = \sqrt{1 - \frac{4m_N^2}{t}}$$

(3)

or

$$\sigma^{c.m.}_{tot}(\bar{p}p \rightarrow e^+ e^-) = \frac{2\pi\alpha^2}{3p_{c.m.}\sqrt{t}} \left[ |G_M(t)|^2 + \frac{2m_N^2}{t} |G_E(t)|^2 \right],$$

(4)

($p_{c.m.}$-antiproton momentum in c.m. system)

as there are no interference terms between them.

The isoscalar and isovector Dirac and Pauli ff’s are the most suitable for a construction of various phenomenological models of the nucleon e.m. structure.
In recent years abundant and very accurate data on the nucleon e.m. ff’s appeared. All references concerning the nucleon space-like data can be found in [1], besides the recent precise measurements [2-10]. In the time-like region see [11-19]. Here in particular, the FENICE experiment in Frascati has measured, besides the proton e.m. ff’s [18], the magnetic neutron ff in the time-like region [19] for the first time. There are also valuable results on the magnetic proton ff [14,15] at higher energies to be measured at FERMILAB.

The present space-like region data will be even considerably improved in the few GeV region when experiments at TJNAF will be completed. There are also other experiments under way at MAMI, ELSA, MIT-Bates involving polarized beams and/or targets in order to give better data in the space-like region, in particular for the electric ff of the neutron, but also the magnetic proton and neutron ones.

This all stimulated a new dispersion theoretical analysis [20,21] of the nucleon e.m. ff data in the space-like region and in the time-like region too [22]. The latter works are an update and extension of historically the most competent nucleon ff analysis carried out by Höhler and collaborators [23]. However, the model does not allow to describe all the time-like data consistently, while still giving a good description of the data in the space-like region.

The work presented here was incentivated just by the results in [20,22] and also by predictions of the spectral function behaviours in the framework of the chiral perturbation theory [24].

In the analysis [20] external constraints on the isovector spectral functions were used, which consist in the two-pion continuum effect on the left wing of the $\rho(770)$-resonance following from the $\pi N$-scattering data and pion e.m. ff behaviour through the unitarity condition. Further one can see that our ten-resonance unitary and analytic model, which is just an improvement of [1,25] and an extension of [26], contains an explicit two-pion continuum contribution given by the unitary cut starting from $t = 4m^2_{\pi}$. Then despite of the fact that here the unstable $\rho$-meson is taken into account only as complex conjugate pairs of poles on the second and third Riemann sheets of the four sheeted Riemann surface, the model itself predicts the strong enhancement of the left wing of the $\rho(770)$ resonance in the isovector spectral functions and moreover, it is consistent with results of [20,27].
Another success of the presented model is the automatic prediction of isoscalar nucleon spectral function behaviours to be consistent with chiral perturbation theory results [24]. Naturally, a description of all existing space-like and time-like nucleon e.m. ff data, including also FENICE (Frascati) results from $e^+e^- \to n\bar{n}$, is achieved for the first time.

The paper is organized as follows. In section 2. the unitary and analytic ten-resonance model of the nucleon e.m. structure with canonical normalizations and asymptotics as predicted by the quark model of hadrons is constructed. An evaluation of all free parameters of the model (however, with clear physical meaning) by a fit of all existing data is carried out in section 3. In section 4. we predict the isovector and isoscalar nucleon spectral function behaviours. The last section is devoted to conclusions and discussion.

2 Ten resonance unitary and analytic model of nucleon e.m. structure

The all four sets of nucleon e.m. ff’s discussed in the introduction are related by means of the expressions

\[ G_E^p(t) = G_E^s(t) + G_E^v(t) = F_1^p(t) + \frac{t}{4m_p^2}F_2^p(t) = [F_1^s(t)] + [F_1^v(t)] + \frac{t}{4m_p^2}[F_2^s(t) + F_2^v(t)]; \]
\[ G_E^n(t) = G_E^s(t) - G_E^v(t) = F_1^n(t) + \frac{t}{4m_n^2}F_2^n(t) = [F_1^s(t) - F_1^v(t)] + \frac{t}{4m_n^2}[F_2^s(t) - F_2^v(t)]; \]
\[ G_M^p(t) = G_M^s(t) + G_M^v(t) = F_1^p(t) + F_2^p(t) = [F_1^s(t) + F_1^v(t)] + [F_2^s(t) + F_2^v(t)]; \]
\[ G_M^n(t) = G_M^s(t) - G_M^v(t) = F_1^n(t) + F_2^n(t) = [F_1^s(t) - F_1^v(t)] + [F_2^s(t) - F_2^v(t)], \]

and for the value $t = 0$ normalized as follows

\[ (i) \quad G_E^p(0) = 1; \quad G_E^s(0) = 1 + \mu_p; \quad G_E^v(0) = 0; \quad G_M^s(0) = 1/2(1 + \mu_p + \mu_n); \quad G_M^v(0) = 1/2(1 - \mu_p - \mu_n); \]
\[ (ii) \quad G_E^p(0) = G_E^v(0) = 1/2; \quad G_M^s(0) = 1/2(1 + \mu_p + \mu_n); \quad G_M^v(0) = 1/2(1 - \mu_p - \mu_n); \]
\[ (iii) \quad F_1^s(0) = 1; \quad F_1^v(0) = \mu_p; \quad F_1^n(0) = 0; \quad F_2^n(0) = \mu_n; \]
\[ (iv) \quad F_1^s(0) = F_1^v(0) = 1/2; \quad F_2^n(0) = \mu_p + \mu_n; \quad F_2^n(0) = 1/2(\mu_p - \mu_n), \]

where $\mu_p$ and $\mu_n$ are the proton and neutron anomalous magnetic moments, respectively.
Our ten-resonance unitary and analytic model represents a consistent unification of the following three fundamental knowledges about the e.m. ff’s:

1. The experimental fact of a creation of unstable vector-meson resonances in the $e^+e^-$-annihilation processes into hadrons.

2. The hypothetical analytic properties of the nucleon e.m. ff’s.

3. The asymptotic behaviour of nucleon e.m. ff’s as predicated [28] by the quark model of hadrons.

The most suitable set of ff’s for a construction of the model are the isoscalar and isovector parts of the Dirac and Pauli ff’s to be, in the first place, saturated by the isoscalar and isovector vector mesons possessing the quantum numbers of the photon. Here we stand up for a view that as there are no data on the nucleon e.m. ff’s at the region $0 < t < 4m_N^2$ of a manifestation of the majority of resonances under consideration, the resonance parameters have to be always fixed at the world averaged values and then investigated their consistency with existing ff data in other regions and also with principles on the base of which the considered model is constructed.

In Review of Particle Physics [29] we find just 5 isoscalar resonances $\omega(782), \phi(1020), \omega'(1420), \omega''(1600), \phi'(1680)$ with required properties. However, one finds only 3 isovector resonances $\rho(770), \rho'(1450), \rho''(1700)$ with quantum numbers of the photon there. On the other hand, we have obtained an experience in [1,25,26,30] that the most stable description of existing data is obtained if equal number of isoscalar and isovector resonances in the investigated model is taken into account. Therefore in the isovector Dirac and Pauli ff’s we consider also the third excited state of the $\rho$-meson, $\rho'''(2150)$, revealed in [31], and moreover, we also introduce hypothetically the fourth excited state of the $\rho$-meson $\rho''''(?)$, the mass and width of which are free parameters of the model. As one can see further in a comparison of the model with all existing data, those resonance parameters will be found to be quite reasonable and thus they provide simultaneous perfect description of the space-like and time-like nucleon ff data, including also the FENICE (Frascati) results on the neutron.

Now, in order to take into account the experimental fact of a creation of vector-meson
resonances in $e^+e^-$ annihilation into hadrons, we start with the vector-meson-dominance
(VMD) parametrization of the isoscalar and isovector parts of the Dirac and Pauli ff’s

$$F^s_1(t) = \sum_{\omega, \phi, \omega'} \frac{m_s^2}{m_s^2 - t} \left(f^{(1)}_{sNN}/f_s\right); \quad F^v_1(t) = \sum_{\theta, \theta', \theta''} \frac{m_v^2}{m_v^2 - t} \left(f^{(1)}_{vNN}/f_v\right);$$

$$F^s_2(t) = \sum_{\omega, \phi, \omega'} \frac{m_s^2}{m_s^2 - t} \left(f^{(2)}_{sNN}/f_s\right); \quad F^v_2(t) = \sum_{\theta, \theta', \theta''} \frac{m_v^2}{m_v^2 - t} \left(f^{(2)}_{vNN}/f_v\right), \quad \text{(7)}$$

where $m_s$ and $m_v$ are isoscalar and isovector vector-meson masses, $f^{(1)}_{sNN}$, $f^{(1)}_{vNN}$ and $f^{(2)}_{sNN}$, $f^{(2)}_{vNN}$ are vector and tensor vector-meson-nucleon coupling constants and $f_s$, $f_v$ are the universal vector-meson coupling constants to be determined in a vector-meson decay into two charged leptons. Here in the isoscalar ff’s also $\phi(1020)$ and $\phi(1680)$ meson contributions are considered as there are clear indications on the strange quark content in the nucleon and the OZI rule violation as well.

The expressions (7) do not govern neither the normalization conditions (3), nor the asymptomatic behaviour

$$t^{i+1} F^{s,v}_{i}(t)|_{t \rightarrow \infty} \sim \text{constant}, \quad i = 1, 2 \quad \text{(8)}$$

to be consistent up to logarithmic correction with results as predicted [28] by the quark model of hadrons. However, any serious attempt to describe the present experimental data on the nucleon e.m. ff’s has to account for these constraints. Their explicit requirement in (7) leads to four systems of algebraic equations

I. \[ \sum_{\omega, \phi, \omega'} (f^{(1)}_{sNN}/f_s) = \frac{1}{2} \]
\[ \sum_{\omega, \phi, \omega'} (f^{(1)}_{sNN}/f_s)m_s^2 = 0 \] \quad \text{(9)}

II. \[ \sum_{\theta, \theta', \theta''} (f^{(1)}_{vNN}/f_v) = \frac{1}{2} \]
\[ \sum_{\theta, \theta', \theta''} (f^{(1)}_{vNN}/f_v)m_v^2 = 0 \] \quad \text{(10)}
III. \[
\sum_{\omega, \phi, \omega', \phi'} (f_{sNN}^{(2)}/f_s) = \frac{1}{2}(\mu_p + \mu_n)
\]
\[
\sum_{\omega, \phi, \omega', \phi'} (f_{sNN}^{(2)}/f_s)m^2_s = 0
\]
\[
(f_{\omega NN}/f_{\omega})m^2_\omega (m^2_\omega + m^2_\omega + m^2_\omega + m^2_\omega) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\omega + m^2_\phi + m^2_\phi) +
(f_{\omega NN}/f_{\omega})m^2_\omega (m^2_\omega + m^2_\omega + m^2_\omega + m^2_\omega) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\omega + m^2_\phi + m^2_\phi) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\omega + m^2_\phi + m^2_\phi) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\omega + m^2_\phi + m^2_\phi) = 0
\]  

IV. \[
\sum_{\epsilon, \phi', \epsilon', \phi''} (f_{vNN}^{(2)}/f_v) = \frac{1}{2}(\mu_p - \mu_n)
\]
\[
\sum_{\epsilon, \phi', \epsilon', \phi''} (f_{vNN}^{(2)}/f_v)m^2_v = 0
\]
\[
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\phi + m^2_\phi + m^2_\phi) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\phi + m^2_\phi + m^2_\phi) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\phi + m^2_\phi + m^2_\phi) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\phi + m^2_\phi + m^2_\phi) +
(f_{\phi NN}/f_{\phi})m^2_\phi (m^2_\phi + m^2_\phi + m^2_\phi + m^2_\phi) = 0
\]

for \((f_{sNN}^{(1)}/f_s), (f_{vNN}^{(1)}/f_v), (f_{sNN}^{(2)}/f_s), \) and \((f_{vNN}^{(2)}/f_v), \) which reduce a number of free parameters of the constructed model remarkably.

Solutions of the (11)-(12) can be chosen in the following form

I. \[
(f_{\omega NN}/f_{\omega}) = \frac{1}{2}\frac{m^2_\omega}{m^2_\omega - m^2_\omega} - (f_{\omega NN}/f_{\omega})\frac{m^2_\omega}{m^2_\omega - m^2_\omega} - \]
\[
(f_{\phi NN}/f_{\phi})\frac{m^2_\phi}{m^2_\phi - m^2_\phi} + (f_{\phi NN}/f_{\phi})\frac{m^2_\phi}{m^2_\phi - m^2_\phi} = 0
\]
\[
(f_{\phi NN}/f_{\phi}) = -\frac{1}{2}\frac{m^2_\omega}{m^2_\omega - m^2_\omega} + (f_{\omega NN}/f_{\omega})\frac{m^2_\omega}{m^2_\omega - m^2_\omega} +
(f_{\phi NN}/f_{\phi})\frac{m^2_\phi}{m^2_\phi - m^2_\phi} - (f_{\phi NN}/f_{\phi})\frac{m^2_\phi}{m^2_\phi - m^2_\phi}
\]
\[ \text{II. } (f^{(1)}_{\nu NN}/f_{\theta}) = \frac{1}{2} \frac{m_{\nu}^2 - m_{\theta}^2}{m_{\nu}^2 - m_{\theta}^2} - (f^{(1)}_{\nu NN}/f_{\theta}) \frac{m_{\nu}^2 - m_{\theta}^2}{m_{\nu}^2 - m_{\theta}^2} + \\
+ (f^{(1)}_{\nu NN}/f_{\theta}) \frac{m_{\nu}^2 - m_{\theta}^2}{m_{\nu}^2 - m_{\theta}^2} + (f^{(1)}_{\nu NN}/f_{\theta}) \frac{m_{\nu}^2 - m_{\theta}^2}{m_{\nu}^2 - m_{\theta}^2} (14) \]

\[ \text{III. } (f^{(2)}_{\omega NN}/f_{\omega}) = \frac{1}{2} (\mu_p + \mu_n) \frac{m_{\omega}^2 m_{\omega}^2}{(m_{\omega}^2 - m_{\nu}^2)(m_{\omega}^2 - m_{\theta}^2)} - \\
- (f^{(2)}_{\phi NN}/f_{\phi}) \frac{m_{\omega}^2 m_{\omega}^2}{(m_{\omega}^2 - m_{\nu}^2)(m_{\omega}^2 - m_{\theta}^2)} + \\
- (f^{(2)}_{\phi NN}/f_{\phi}) \frac{m_{\omega}^2 m_{\omega}^2}{(m_{\omega}^2 - m_{\nu}^2)(m_{\omega}^2 - m_{\theta}^2)} (15) \]

\[ \text{IV. } (f^{(2)}_{\phi NN}/f_{\phi}) = \frac{1}{2} (\mu_p - \mu_n) \frac{m_{\phi}^2 m_{\phi}^2}{(m_{\phi}^2 - m_{\nu}^2)(m_{\phi}^2 - m_{\theta}^2)} - \\
- (f^{(2)}_{\phi NN}/f_{\phi}) \frac{m_{\phi}^2 m_{\phi}^2}{(m_{\phi}^2 - m_{\nu}^2)(m_{\phi}^2 - m_{\theta}^2)} - \\
- (f^{(2)}_{\phi NN}/f_{\phi}) \frac{m_{\phi}^2 m_{\phi}^2}{(m_{\phi}^2 - m_{\nu}^2)(m_{\phi}^2 - m_{\theta}^2)} (15) \]
Pauli nucleon ff’s still into the zero-width VMD expressions which transform the original parametrizations (\(\text{(1)}\)) of the isoscalar and isovector Dirac and Pauli nucleon ff’s still into the zero-width VMD expressions

\[
F^n_1(t) = \frac{1}{2} \frac{m^2_\omega, m^2_\omega}{(m^2_\omega, -t)(m^2_\omega, -t)} + \\
+ \left\{ \frac{m^2_\omega, m^2_\omega}{(m^2_\omega, -t)(m^2_\omega, -t)} \frac{m^2_\omega, -m^2_\omega}{m^2_\omega, -m^2_\omega} - \frac{m^2_\omega, m^2_\omega}{(m^2_\omega, -t)(m^2_\omega, -t)} \frac{m^2_\omega, -m^2_\omega}{m^2_\omega, -m^2_\omega} \right\} (f^{(1)}_{\omega NN}/f_\omega) + \\
+ \left\{ \frac{m^2_\omega, m^2_\phi}{(m^2_\omega, -t)(m^2_\phi, -t)} \frac{m^2_\omega, -m^2_\phi}{m^2_\omega, -m^2_\phi} - \frac{m^2_\omega, m^2_\phi}{(m^2_\omega, -t)(m^2_\phi, -t)} \frac{m^2_\omega, -m^2_\phi}{m^2_\omega, -m^2_\phi} \right\} (f^{(1)}_{\phi NN}/f_\phi) - \\
+ \left\{ \frac{m^2_\phi, m^2_\omega}{(m^2_\phi, -t)(m^2_\omega, -t)} \frac{m^2_\phi, -m_\omega}{m^2_\phi, -m_\omega} - \frac{m^2_\phi, m^2_\omega}{(m^2_\phi, -t)(m^2_\omega, -t)} \frac{m^2_\phi, -m_\omega}{m^2_\phi, -m_\omega} + \\
+ \frac{m^2_\phi, m^2_\omega}{(m^2_\phi, -t)(m^2_\omega, -t)} \right\} (f^{(1)}_{\phi NN}/f_\phi), \\
\]

\[
F^\nu_1(t) = \frac{1}{2} \frac{m^2_\nu, m^2_\nu}{(m^2_\nu, -t)(m^2_\nu, -t)} + \\
+ \left\{ \frac{m^2_\nu, m^2_\nu}{(m^2_\nu, -t)(m^2_\nu, -t)} \frac{m^2_\nu, -m^2_\nu}{m^2_\nu, -m^2_\nu} - \frac{m^2_\nu, m^2_\nu}{(m^2_\nu, -t)(m^2_\nu, -t)} \frac{m^2_\nu, -m^2_\nu}{m^2_\nu, -m^2_\nu} \right\} (f^{(1)}_{\nu NN}/f_\nu) + \\
+ \left\{ \frac{m^2_\nu, m^2_\nu}{(m^2_\nu, -t)(m^2_\nu, -t)} \frac{m^2_\nu, -m^2_\nu}{m^2_\nu, -m^2_\nu} - \frac{m^2_\nu, m^2_\nu}{(m^2_\nu, -t)(m^2_\nu, -t)} \frac{m^2_\nu, -m^2_\nu}{m^2_\nu, -m^2_\nu} + \\
+ \frac{m^2_\nu, m^2_\nu}{(m^2_\nu, -t)(m^2_\nu, -t)} \right\} (f^{(1)}_{\nu NN}/f_\nu), \\
\]
\[\begin{align*}
&\left\{ m^2_{\nu^+}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( f^{(1)}_{\nu^+NN} / f_{\nu^-} \right) - \\
&\left\{ m^2_{\nu^+}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( f^{(1)}_{\nu^+NN} / f_{\nu^-} \right) + \\
&\left\{ m^2_{\nu^+}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( f^{(1)}_{\nu^+NN} / f_{\nu^-} \right).
\end{align*}\]

\[F^2_\nu(t) = \frac{1}{2} (\mu_\nu + \mu_\nu) \frac{m^2_{\nu^+}m^2_{\nu^-}m^2_{\nu^-}}{(m^2_{\nu^+} - t)(m^2_{\nu^-} - t)} + \]

\[\begin{align*}
&\left\{ m^2_{\nu^+}m^2_{\nu^-}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( m^2_{\nu^+} - t)(m^2_{\nu^-} - t)(m^2_{\nu^-} - t) \right\} \left( f^{(2)}_{\nu^+NN} / f_{\nu^+} \right) + \\
&\left\{ m^2_{\nu^+}m^2_{\nu^-}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( m^2_{\nu^+} - t)(m^2_{\nu^-} - t)(m^2_{\nu^-} - t) \right\} \left( f^{(2)}_{\nu^+NN} / f_{\nu^+} \right),
\end{align*}\]

\[\frac{1}{2} (\mu_\nu - \mu_\nu) \frac{m^2_{\nu^+}m^2_{\nu^-}m^2_{\nu^-}}{(m^2_{\nu^+} - t)(m^2_{\nu^-} - t)} + \]

\[\begin{align*}
&\left\{ m^2_{\nu^+}m^2_{\nu^-}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( m^2_{\nu^-} - t)(m^2_{\nu^-} - t)(m^2_{\nu^-} - t) \right\} \left( f^{(2)}_{\nu^+NN} / f_{\nu^+} \right) + \\
&\left\{ m^2_{\nu^+}m^2_{\nu^-}m^2_{\nu^-} - m^2_{\nu^+} \right\} \left( m^2_{\nu^-} - t)(m^2_{\nu^-} - t)(m^2_{\nu^-} - t) \right\} \left( f^{(2)}_{\nu^+NN} / f_{\nu^+} \right).
\end{align*}\]
There are also another expressions utilized for the vector meson masses squared
respectively, and a subsequent incorporation of the non-zero values of vector meson widths.

cation of the following special non-linear transformations predicts its smoothly varying behaviour (see e.g. [20], [27]).

and identities

\[ 0 = t_0^s - \frac{4(t_{1n}^s - t_0^s)}{[1/V_{s0} - V_{s0}]^2}, \quad 0 = t_0^s - \frac{4(t_{1n}^s - t_0^s)}{[1/U_{s0} - U_{s0}]^2}, \]
\[ 0 = t_0^v - \frac{4(t_{1n}^v - t_0^v)}{[1/W_{s0} - W_{s0}]^2}, \quad 0 = t_0^v - \frac{4(t_{1n}^v - t_0^v)}{[1/X_{s0} - X_{s0}]^2}. \]
following from \( \{21\} \), where \( V_{s0}, W_{t0}, U_{s0}, X_{t0} \) are the zero-width (therefore they have a subindex \( 0 \)) VMD poles and \( V_N, W_N, U_N, X_N \) are the normalization points (corresponding to \( t = 0 \)) in the \( V, W, U, X \) planes, respectively, and \( t_0^s = 9m_N^2, t_0^t = 4m_N^2, t_{in}^1, t_{in}^2, t_{in}^{1V}, t_{in}^{2V} \) are square-root branch points as it is transparent from the inverse transformations to \( \{21\} \), e.g.

\[
V(t) = \sqrt{\left(\frac{t_{1in} - t_0^s}{t_0^s}\right)^{1/2} + \left(\frac{t_{1in} - t_0^t}{t_0^t}\right)^{1/2}} - \sqrt{\left(\frac{t_{1in}^s - t_0^s}{t_0^s}\right)^{1/2} - \left(\frac{t_{1in}^t - t_0^t}{t_0^t}\right)^{1/2}}
\]

(24)

and similarly for \( W(t), U(t) \) and \( X(t) \).

Really, the relations \( \{21\}-\{23\} \) first transform every \( t \)-dependent term and every constant term consisting of a ratio of mass differences in \( \{17\}-\{20\} \) into a new form as follows. For instance the term \( m_{\omega}^2/(m_{\omega}^2 - t) \) in \( \{17\} \) is transformed into the following factorized form:

\[
\frac{m_{\omega}^2}{m_{\omega}^2 - t} = \frac{m_{\omega}^2 - 0}{m_{\omega}^2 - t} = \frac{(1 - V^2)}{1 - V_N^2} \frac{(V_N - V_{\omega}) (V_N + V_{\omega}) (V_N - 1/V_{\omega}) (V_N + 1/V_{\omega})}{(V - V_{\omega}) (V + V_{\omega}) (V - 1/V_{\omega}) (V + 1/V_{\omega})}.
\]

(25)

The constant mass terms, e.g. \( (m_{\omega}^2 - m_{\omega'}^2)/(m_{\omega}^2 - m_{\omega'}^2) \) also from \( \{17\} \), become as follows

\[
\frac{m_{\omega}^2 - m_{\omega'}^2}{m_{\omega}^2 - m_{\omega'}^2} = \frac{(m_{\omega}^2 - 0) - (m_{\omega'}^2 - 0)}{(m_{\omega}^2 - 0) - (m_{\omega'}^2 - 0)} = \frac{(V_N - V_{\omega}) (V_N + V_{\omega}) (V_N - 1/V_{\omega}) (V_N + 1/V_{\omega})}{(V_{\omega} - 1/V_{\omega})^2} - \frac{(V_{\omega} - 1/V_{\omega})^2}{(V_N - V_{\omega}) (V_N + V_{\omega}) (V_N - 1/V_{\omega}) (V_N + 1/V_{\omega})}
\]

\[
\left[ \frac{(V_{\omega} - 1/V_{\omega})^2}{(V_N - V_{\omega}) (V_N + V_{\omega}) (V_N - 1/V_{\omega}) (V_N + 1/V_{\omega})} \right] / \left[ \frac{(V_{\omega} - 1/V_{\omega})^2}{(V_N - V_{\omega}) (V_N + V_{\omega}) (V_N - 1/V_{\omega}) (V_N + 1/V_{\omega})} \right] - \frac{(V_N - V_{\omega}) (V_N + V_{\omega}) (V_N - 1/V_{\omega}) (V_N + 1/V_{\omega})}{(V_{\omega} - 1/V_{\omega})^2}
\]

\[
= C_{\omega}^{1s} - C_{\omega'}^{1s}.
\]

Then by utilization of the relations between complex and complex conjugate values of the corresponding zero-width VMD pole positions in the \( V, W, U, X \) planes

\[
V_{\omega0} = -V^*_{\omega0}; V_{\phi0} = -V^*_{\phi0}; V_{\omega} = -V^*_{\omega}; V_{\omega} = 1/V_{\omega}; V_{\phi0} = 1/V_{\phi0}
\]

\[
W_{\phi0} = -W^*_{\phi0}; W_{\phi0} = -W^*_{\phi0}; W_{\phi0} = -W^*_{\phi0}; W_{\phi0} = 1/W_{\phi0}; W_{\phi0} = 1/W_{\phi0}
\]

(27)
following from the fact that in a fitting procedure we find

\begin{align}
  m_{\omega}^2 - \Gamma_{\omega}/4 &< t_{in}^{1s}; \quad m_{\phi}^2 - \Gamma_{\phi}/4 < t_{in}^{1s}; \quad m_{\omega}^2 - \Gamma_{\omega}/4 < t_{in}^{1s}, \\
  m_{\omega}^2 - \Gamma_{\omega}/4 &> t_{in}^{1s}; \quad m_{\phi}^2 - \Gamma_{\phi}/4 > t_{in}^{1s}, \\
  m_{\omega}^2 - \Gamma_{\omega}/4 &< t_{in}^{2s}; \quad m_{\phi}^2 - \Gamma_{\phi}/4 < t_{in}^{2s}; \quad m_{\omega}^2 - \Gamma_{\omega}/4 < t_{in}^{2s}, \\
  m_{\omega}^2 - \Gamma_{\omega}/4 &> t_{in}^{2s}; \quad m_{\phi}^2 - \Gamma_{\phi}/4 > t_{in}^{2s}, \quad (28)
\end{align}

and subsequent introduction of the non-zero values of vector-meson widths \( \Gamma \neq 0 \) by the substitutions

\begin{align}
  m_{s}^2 &\rightarrow (m_{s} - i \frac{\Gamma_{s}}{2})^2; \quad m_{v}^2 \rightarrow (m_{v} - i \frac{\Gamma_{v}}{2})^2, \quad (29)
\end{align}

one gets, for every isoscalar and isovector Dirac and Pauli ff, one analytic function in the whole complex \( t \)-plane besides two right-hand cuts of the following forms

\begin{align}
  F_{1s}^{1s} [V(t)] &= \frac{1}{1 - V_{\omega}^{2}} \left[ \frac{1}{1 - V_{\omega}^{2}} \left( V_{N} - V_{\omega} \right) \right. \\
  &\quad \cdot \left. \frac{(V_{N} - V_{\omega})(V_{N} - V_{\omega}^*)}{(V - V_{\omega})(V - V_{\omega}^*)} \right] \\
  &\quad + \left[ \frac{(V_{N} - V_{\omega})(V_{N} - V_{\omega}^*)}{(V - V_{\omega})(V - V_{\omega}^*)} \right] \\
  &\quad \cdot \left. \frac{(V_{N} - V_{\omega})(V_{N} - V_{\omega}^*)}{(V - V_{\omega})(V - V_{\omega}^*)} \right] \\
  &\quad - \left[ \frac{(V_{N} - V_{\omega})(V_{N} - V_{\omega}^*)}{(V - V_{\omega})(V - V_{\omega}^*)} \right] \\
  &\quad \cdot \left. \frac{(V_{N} - V_{\omega})(V_{N} - V_{\omega}^*)}{(V - V_{\omega})(V - V_{\omega}^*)} \right]
\end{align}
\[
\begin{align*}
& (V_N - V_{\omega})(V_N - V_{\omega}^*)(V_N - 1/V_{\omega})(V_N - 1/V_{\omega}^*) \\
& \cdot \left( f_{\omega NN}/f_{\omega} \right) + \\
& + \left[ (V_N - V_{\omega})(V_N - V_{\omega}^*) (V_{N} + V_{\omega} - V_{\omega}^*) (V_{N} + V_{\omega}) \right] \\
& (V_N - V_{\omega})(V_N - V_{\omega}^*)(V_N - 1/V_{\omega})(V_N - 1/V_{\omega}^*) \\
& \cdot \left[ (V_{\omega} - V_{\omega}^*)(V_{\omega} + V_{\omega} - V_{\omega}^*) (V_{\omega} + V_{\omega}) \right] \\
& - \left[ (V_N - V_{\omega})(V_N - V_{\omega}^*)(V_N - 1/V_{\omega})(V_N - 1/V_{\omega}^*) \right] \\
& \cdot \left[ (V_{\omega}^2 - V_{\omega}^2)(V_{\omega}^2 - V_{\omega}^2)(V_{\omega}^2 - V_{\omega}^2) \right] \\
& (V_N - V_{\omega})(V_N - V_{\omega}^*)(V_N - 1/V_{\omega})(V_N - 1/V_{\omega}^*) \\
& \cdot \left[ (V_{\omega}^2 - V_{\omega}^2)(V_{\omega}^2 - V_{\omega}^2)(V_{\omega}^2 - V_{\omega}^2) \right] \\
& (V_N - V_{\omega})(V_N - V_{\omega}^*)(V_N - 1/V_{\omega})(V_N - 1/V_{\omega}^*) \\
& \cdot \left[ (V_{\omega}^2 - V_{\omega}^2)(V_{\omega}^2 - V_{\omega}^2)(V_{\omega}^2 - V_{\omega}^2) \right]
\end{align*}
\]

\[
F_1^t[W(t)] = \frac{1 - W^2}{1 - W_N^2} \left\{ \begin{array}{l}
\frac{1}{2} (W_N - W_{\omega})(W_N - W_{\omega}^*)(W_N - 1/W_{\omega})(W_N - 1/W_{\omega}^*) \\
(W_N - W_{\omega})(W_N - W_{\omega}^*)(W_N - 1/W_{\omega})(W_N - 1/W_{\omega}^*) \\
\end{array} \right\} + \\
\left[ (W_N - W_{\omega})(W_N - W_{\omega}^*)(W_N - 1/W_{\omega})(W_N - 1/W_{\omega}^*) \right] \\
\left[ (W_N - W_{\omega})(W_N - W_{\omega}^*)(W_N - 1/W_{\omega})(W_N - 1/W_{\omega}^*) \right] \\
\left[ (W_N - W_{\omega})(W_N - W_{\omega}^*)(W_N - 1/W_{\omega})(W_N - 1/W_{\omega}^*) \right] \\
\left[ (W_N - W_{\omega})(W_N - W_{\omega}^*)(W_N - 1/W_{\omega})(W_N - 1/W_{\omega}^*) \right] \\
\end{align*}
\]
\[
\begin{align*}
F_2^s[U(t)] &= \left(1 - \frac{U_\omega^2}{U_N^2}\right)^6 \left\{ \frac{1}{2}(\mu_p + \mu_n) \frac{(U_N - U_\omega)(U_N - U_{\omega^*})(U_N + U_{\omega^*})(U_N + U_{\omega^*})}{(U - U_{\omega^*})(U - U_{\omega^*})(U + U_{\omega^*})(U + U_{\omega^*})} \cdot \frac{(U_N - U_\omega)(U_N - U_{\omega^*})(U_N - 1/U_\omega)(U_N - 1/U_{\omega^*})}{(U - U_{\omega^*})(U - U_{\omega^*})(U - 1/U_\omega)(U - 1/U_{\omega^*})} \right\} \\
&\cdot \left[ \frac{(W_N - W_{\omega^*})(W_N - W_{\omega^*}^*)(W_N - 1/W_{\omega^*})(W_N - 1/W_{\omega^*}^*)}{(W - W_{\omega^*})(W - W_{\omega^*}^*)(W - 1/W_{\omega^*})(W - 1/W_{\omega^*}^*)} \cdot \frac{C_{\omega^*}^{1v} - C_{\omega^*}^{1v}}{C_{\omega^*}^{1v} - C_{\omega^*}^{1v}} \right] \\
&\cdot \left[ (W_N - W_{\omega^*})(W_N - W_{\omega^*}^*)(W_N - 1/W_{\omega^*})(W_N - 1/W_{\omega^*}^*) \\
&\cdot \left[ (W_N - W_{\omega^*})(W_N - W_{\omega^*}^*)(W_N - 1/W_{\omega^*})(W_N - 1/W_{\omega^*}^*) \right] \left\{ f_{\omega^*NN}/f_{\omega^*} \right\} + \\
&\cdot \left[ (W_N - W_{\omega^*})(W_N - W_{\omega^*}^*)(W_N + W_{\omega^*})(W_N + W_{\omega^*}^*) \right] \left\{ (f_{\omega^*NN}/f_{\omega^*}) - \right\}
\end{align*}
\]
\[
\begin{align*}
&\frac{(U_N - U_\omega)(U_N - U_\omega^*)(U_N - 1/U_\omega)(U_N - 1/U_\omega^*)}{(U - U_\omega)(U - U_\omega^*)(U - 1/U_\omega)(U - 1/U_\omega^*)} + \\
&\left[ \frac{(U_N - U_\omega^*)(U_N - U_\omega)(U_N + U_\omega)(U_N + U_\omega^*)}{(U - U_\omega^*)(U - U_\omega)(U + U_\omega)(U + U_\omega^*)} \right] \\
&\left[ \frac{(U_N - U_\omega^*)(U_N - U_\omega^*)(U_N + U_\omega^*)(U_N + U_\omega)}{(U - U_\omega^*)(U - U_\omega^*)(U - 1/U_\omega^*)(U - 1/U_\omega)} \right] \\
&\frac{(U_N - U_\omega)(U_N - U_\omega^*)(U_N + 1/U_\omega)(U_N + 1/U_\omega^*)}{(U - U_\omega)(U - U_\omega^*)(U + 1/U_\omega)(U + 1/U_\omega^*)} C_{2s}^{\phi} - C_{2s}^{\phi^*} \frac{C_{2s}^{\phi} - C_{2s}^{\phi^*}}{C_{2s}^{\phi^*} - C_{2s}^{\phi}} + \\
&\frac{(U_N - U_\omega)(U_N - U_\omega^*)(U_N + 1/U_\omega)(U_N + 1/U_\omega^*)}{(U - U_\omega)(U - U_\omega^*)(U + 1/U_\omega)(U + 1/U_\omega^*)} C_{2s}^{\phi^*} - C_{2s}^{\phi} \frac{C_{2s}^{\phi^*} - C_{2s}^{\phi}}{C_{2s}^{\phi} - C_{2s}^{\phi^*}} \\
&\left[ \frac{(U_N - U_\omega^*)(U_N - U_\omega)(U_N + U_\omega)(U_N + U_\omega^*)}{(U - U_\omega^*)(U - U_\omega)(U + U_\omega)(U + U_\omega^*)} \right] \\
&\left[ \frac{(U_N - U_\omega^*)(U_N - U_\omega^*)(U_N + U_\omega^*)(U_N + U_\omega)}{(U - U_\omega^*)(U - U_\omega^*)(U - 1/U_\omega^*)(U - 1/U_\omega)} \right] \left( f_{\phi N N}^{(2)} f_{\phi} \right) + \\
\end{align*}
\]
\[
F_2^v[X(t)] = \left( \frac{1 - X^2}{1 - X_N^2} \right)^6 \left\{ \frac{1}{2} (\mu_p - \mu_a) \frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi'})^2(X_N - 1/X_{\phi'})^2}{(X - X_{\phi'})^2(X - X_{\phi'})^*(X - 1/X_{\phi'})^2(X - 1/X_{\phi'})^2} \right. \\
+ \frac{(X_N - X_{\phi})(X_N - X_{\phi}^*)(X_N - 1/X_{\phi})(X_N - 1/X_{\phi}^*)}{(X - X_{\phi})^2(X - X_{\phi})^*(X - 1/X_{\phi})^2(X - 1/X_{\phi})^2} + \\
\left. \frac{(X_N - X_{\phi})(X_N - X_{\phi}^*)(X_N - 1/X_{\phi})(X_N - 1/X_{\phi}^*)}{(X - X_{\phi})^2(X - X_{\phi})^*(X - 1/X_{\phi})^2(X - 1/X_{\phi})^2} \right\}
\]
\[
\begin{align*}
&= \frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X - 1/X_{\phi^*})(X - 1/X_{\phi^*}^*)} \cdot \\
&+ \frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X - 1/X_{\phi^*})(X - 1/X_{\phi^*}^*)} \cdot \left( f_{\phi^*}^{(2)}/f_{\phi^*} \right) + \\
&\quad \left\{ \begin{array}{l}
\frac{(X_N - X_{\phi^*})(X_N + X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N + X_{\phi^*}^*)}{(X - X_{\phi^*})(X + X_{\phi^*}^*)(X + X_{\phi^*}^*)(X - X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} - \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X - 1/X_{\phi^*})(X - 1/X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} + \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N + X_{\phi^*}^*)(X_N + X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X + X_{\phi^*}^*)(X + X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} + \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X - 1/X_{\phi^*})(X - 1/X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} - \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N + X_{\phi^*}^*)(X_N - 1/X_{\phi^*}^*)(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X + X_{\phi^*}^*)(X - 1/X_{\phi^*}^*)(X - 1/X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} - \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X - 1/X_{\phi^*})(X - 1/X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} - \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N + X_{\phi^*}^*)(X_N + X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X + X_{\phi^*}^*)(X + X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} + \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N - 1/X_{\phi^*})(X_N - 1/X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X - 1/X_{\phi^*})(X - 1/X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} + \\
\frac{(X_N - X_{\phi^*})(X_N - X_{\phi^*}^*)(X_N + X_{\phi^*}^*)(X_N + X_{\phi^*}^*)}{(X - X_{\phi^*})(X - X_{\phi^*}^*)(X + X_{\phi^*}^*)(X + X_{\phi^*}^*)} \cdot C_{\phi^*}^{2v} - C_{\phi^*}^{2v} \cdot C_{\phi^*}^{2v} + \}
\end{align*}
\]

where

\[
C_{r}^{1s} = \frac{(V_N - V_{r})(V_N - V_{r}^*)(V_N - 1/V_{r})(V_N - 1/V_{r}^*)}{-(V_{r} - 1/V_{r})(V_{r}^* - 1/V_{r}^*)}; \quad r = \omega, \phi, \omega^* \\
C_{l}^{1s} = \frac{(V_N - V_{l})(V_N - V_{l}^*)(V_N + V_{l})(V_N + V_{l}^*)}{-(V_{l} - 1/V_{l})(V_{l}^* - 1/V_{l}^*)}; \quad l = \omega^*, \phi^* \quad \text{(34)}
\]
Our ten-resonance unitary and analytic model of the nucleon e.m. structure depends after all subsequent predictions. In the next sections this model is used to analyze all existing nucleon e.m. ff data and to obtain and the correct asymptotic behaviours as predicted by the quark model of hadrons. In the unitary and analytic model of the nucleon e.m. structure with canonical normalizations (6) corresponding to vector-meson resonances placed on unphysical sheets.

As a result each ff is defined on a four-sheeted Riemann surface in $t$-variable with poles corresponding to vector-meson resonances placed on unphysical sheets.

The expressions (30)-(33), together with the relations (5), represent just a ten-resonance unitary and analytic model of the nucleon e.m. structure with canonical normalizations (6) and the correct asymptotic behaviours as predicted by the quark model of hadrons. In the next sections this model is used to analyze all existing nucleon e.m. ff data and to obtain subsequent predictions.

### 3 Analysis of all existing space-like and time-like data

Our ten-resonance unitary and analytic model of the nucleon e.m. structure depends after all on the following parameters $t_{1n}^{1s}$, $t_{1n}^{1v}$, $t_{2n}^{1v}$, $t_{2n}^{1s}$, $m_\omega$, $\Gamma_\omega$, $m_\phi$, $\Gamma_\phi$, $m_\rho$, $\Gamma_\rho$, $m_{\rho'}$, $\Gamma_{\rho'}$, $m_{\rho''}$, $\Gamma_{\rho''}$, $m_{\rho'''}$, $\Gamma_{\rho'''}$, $f_{\omega NN}^{(1)}$, $f_{\omega NN}^{(2)}$, $f_{\phi NN}^{(1)}$, $f_{\phi NN}^{(2)}$, $f_{\rho NN}^{(1)}$, $f_{\rho NN}^{(2)}$, $f_{\rho'' NN}^{(1)}$, $f_{\rho'' NN}^{(2)}$, $f_{\rho'''} NN$,$f_{\rho'''} NN$, however, with the clear physical meaning. Not all of them are free. For instance, owing to a simple reason that almost all (except for $\rho''$ and $\rho'''$) considered resonances are situated in the region $t_0 < t < 4m_N^2$, where no experimental informations on the nucleon e.m. ff’s exists up to now, one can not expect in a fitting procedure to be able to determine their correct masses and widths. Therefore, for $\omega$, $\phi$, $\omega'$, $\omega''$, $\phi'$, $\rho$, $\rho'$, and $\rho''$ they are fixed at the reliable
world averaged values given by Review of Particle Physics [29] and then investigated in the framework of constructed model to be consistent with existing experimental information on nucleon e.m. ff’s. The parameters of $\rho^{\pm\pm}$ are taken from [31].

Thus we are left finally only with the following 16 free parameters $t_{in}^{1s}$, $t_{in}^{1v}$, $t_{in}^{2s}$, $t_{in}^{2v}$, $m_{\rho^{\pm\pm}}$, $\Gamma_{\rho^{\pm\pm}}$, $f_{\omega NN}/f_\omega$, $f_{\phi NN}/f_\phi$, $f_{\rho NN}/f_\rho$, $f_{\phi\rho NN}/f_{\phi\rho}$, $f_{\rho^{\pm\pm}NN}/f_{\rho^{\pm\pm}}$, $f_{\phi^{\pm\pm}NN}/f_{\phi^{\pm\pm}}$, as the four effective inelastic thresholds are specific quantities in the constructed model, the fourth excited state of the $\rho$-meson is not identified experimentally up to now and at present there are no model independent and consistent values of the $f_{\omega NN}/f_\omega$, $f_{\phi NN}/f_\phi$, $f_{\rho NN}/f_\rho$, $f_{\phi\rho NN}/f_{\phi\rho}$, $f_{\rho^{\pm\pm}NN}/f_{\rho^{\pm\pm}}$, coupling constant ratios.

For their numerical evaluations we have collected 476 experimental points, mostly for the proton in the space-like region up to $t = -33 \ GeV^2$, however, including also new time like data of the proton [13-18] and the neutron [19] above the $N\bar{N}$ threshold. The data have been analyzed by means of the relations (5) and (30)-(33) by using the CERN program MINUIT. The best description of them was achieved with $\chi^2/ndf = 1.43$ and the following values of free parameters

\begin{align*}
  t_{in}^{1s} &= 2.6012 \pm 0.6391 \ GeV^2 \quad t_{in}^{1v} = 3.5220 \pm 0.0059 \ GeV^2 \\
  t_{in}^{2s} &= 2.7200 \pm 0.6271 \ GeV^2 \quad t_{in}^{2v} = 3.6316 \pm 0.6235 \ GeV^2 \\
  (f_{\omega NN}/f_\omega) &= 1.1112 \pm 0.0030 \quad (f_{\rho NN}/f_\rho) = 0.3843 \pm 0.0043 \\
  (f_{\phi NN}/f_\phi) &= -0.9389 \pm 0.0056 \quad (f_{\phi^{\pm\pm}NN}/f_{\phi^{\pm\pm}}) = -0.0840 \pm 0.0008 \\
  (f_{\phi\rho NN}/f_{\phi\rho}) &= -0.3255 \pm 0.0047 \quad (f_{\rho^{\pm\pm}NN}/f_{\rho^{\pm\pm}}) = 0.0299 \pm 0.0003 \\
  (f_{\rho^{\pm\pm}NN}/f_{\rho^{\pm\pm}}) &= -0.2659 \pm 0.0287 \quad m_{\rho^{\pm\pm}} = 2506 \pm 38 \ MeV \\
  (f_{\phi^{\pm\pm}NN}/f_{\phi^{\pm\pm}}) &= 0.1190 \pm 0.0032 \quad \Gamma_{\phi^{\pm\pm}} = 700 \pm 179 \ MeV \\
  (f_{\rho^{\pm\pm}NN}/f_{\rho^{\pm\pm}}) &= 0.0549 \pm 0.0005 \\
  (f_{\phi^{\pm\pm}NN}/f_{\phi^{\pm\pm}}) &= -0.0103 \pm 0.0001.
\end{align*}

A compilation of the world nucleon ff data and their description by our ten-resonance unitary and analytic model is graphically presented in Figs. 1-4. One can see from Fig. 4b that unlike the authors of the paper [22] we are able to describe FENICE time-like data on
neutron [19] quite well. The same is valid also for the FERMILAB proton time-like data [14,15] (see Fig. 2b). The latter was possible to achieve by an introduction of a hypothetical fourth excited state of the \(\rho(770)\)-meson, the parameters of which were found in a fitting procedure of all existing data to be quite reasonable. Its existence, however, has to be proved by an identification also in other processes than only in \(e^+e^- \to N\bar{N}\).

Of particular interest is a determination of the radii of the isoscalar and isovector parts of the Dirac and Pauli ff’s. They are given in Table 1, where for a comparison results of the papers [20] and [23] are presented too. The corresponding proton and neutron radii are given in Table 2. Here we would like to stress that we do not use in our model the neutron charge radius to be determined very accurately by measuring the neutron-atom scattering length [32] as a constraint like in [20] and it is a prediction of the model to be \(\langle r_{E_n}^2 \rangle = -0.097 \text{fm}^2\).

In order to demonstrate explicitly substantial deviations from the dipole fit in all channels and at the same time a violation of the nucleon ff scaling, particularly at large momentum transfer, we show in Figs. 5-8 ratios of appropriately normalized electric and magnetic proton and neutron ff’s in the space-like region to the dipole formula \(G_D(t) = (1 - t/0.71)^{-2}\).

4 Predictions of our unitary and analytic model of the nucleon e.m. structure

The unitary and analytic ten-resonance model of the nucleon e.m. structure constructed in this paper represents a harmonious unification of all known nucleon ff properties always into one analytic function, i.e. one smooth function on the whole real axis, for every nucleon e.m. ff. As a result one can believe then the predicted behaviours of these nucleon e.m. ff’s to be realistic also outside the regions of existing experimental data.

Valuable is the predicted existence of the fourth excited state of the \(\rho(770)\)-meson with resonance parameters \(m_{\rho^{**}} = 2500 \text{ MeV}\) and \(\Gamma_{\rho^{**}} = 700 \text{ MeV}\) without of which one could not achieve a satisfactory description of the FENICE time-like neutron data [19] (see Fig. 4b) and also eight FERMILAB proton points [14,15] (Fig. 2b) at higher energies.

Taking into account the numerical results (88) of the parameters and the transformed
relations (13)-(16) by means of the expressions (34)-(37) into the forms

I. \[
(f^{(1)}_{\omega; NN}/f_{\omega}) = \frac{1}{2} \frac{C^{1s}_{\omega}}{C^{1s}_{\omega} - C^{1s}_{\omega}} - (f^{(1)}_{\omega; NN}/f_{\omega}) \frac{C^{1s}_{\omega} - C^{1s}_{\omega}}{C^{1s}_{\omega} - C^{1s}_{\omega}} - \]
\[ - (f^{(1)}_{\phi NN}/f_{\phi}) \frac{C^{1s}_{\phi} - C^{1s}_{\phi}}{C^{1s}_{\omega} - C^{1s}_{\omega}} + (f^{(1)}_{\phi NN}/f_{\phi}) \frac{C^{1s}_{\phi} - C^{1s}_{\phi}}{C^{1s}_{\omega} - C^{1s}_{\omega}} + \]
\[ + (f^{(1)}_{\phi NN}/f_{\phi}) \frac{C^{1s}_{\phi} - C^{1s}_{\phi}}{C^{1s}_{\omega} - C^{1s}_{\omega}} - (f^{(1)}_{\phi NN}/f_{\phi}) \frac{C^{1s}_{\phi} - C^{1s}_{\phi}}{C^{1s}_{\omega} - C^{1s}_{\omega}} \]

II. \[
(f^{(1)}_{\rho; NN}/f_{\rho}) = \frac{1}{2} \frac{C^{1v}_{\rho}}{C^{1v}_{\rho} - C^{1v}_{\rho}} - (f^{(1)}_{\rho; NN}/f_{\rho}) \frac{C^{1v}_{\rho} - C^{1v}_{\rho}}{C^{1v}_{\rho} - C^{1v}_{\rho}} + \]
\[ + (f^{(1)}_{\rho; NN}/f_{\rho}) \frac{C^{1v}_{\rho} - C^{1v}_{\rho}}{C^{1v}_{\rho} - C^{1v}_{\rho}} + (f^{(1)}_{\rho; NN}/f_{\rho}) \frac{C^{1v}_{\rho} - C^{1v}_{\rho}}{C^{1v}_{\rho} - C^{1v}_{\rho}} - \]
\[ - (f^{(1)}_{\rho; NN}/f_{\rho}) \frac{C^{1v}_{\rho} - C^{1v}_{\rho}}{C^{1v}_{\rho} - C^{1v}_{\rho}} - (f^{(1)}_{\rho; NN}/f_{\rho}) \frac{C^{1v}_{\rho} - C^{1v}_{\rho}}{C^{1v}_{\rho} - C^{1v}_{\rho}} \]

III. \[
(f^{(2)}_{\omega; NN}/f_{\omega}) = \frac{1}{2} (\mu_{p} + \mu_{n}) \frac{C^{2s}_{\omega}}{C^{2s}_{\omega} - C^{2s}_{\omega}} - \]
\[ - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} - \]
\[ - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} + \]
\[ + (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} \]

\[
(f^{(2)}_{\omega; NN}/f_{\omega}) = \frac{1}{2} (\mu_{p} + \mu_{n}) \frac{C^{2s}_{\omega}}{C^{2s}_{\omega} - C^{2s}_{\omega}} + \]
\[ - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} + \]
\[ + (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} \]

\[
(f^{(2)}_{\omega; NN}/f_{\omega}) = \frac{1}{2} (\mu_{p} + \mu_{n}) \frac{C^{2s}_{\omega}}{C^{2s}_{\omega} - C^{2s}_{\omega}} + \]
\[ - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} + \]
\[ + (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} - (f^{(2)}_{\phi NN}/f_{\phi}) \frac{C^{2s}_{\phi}}{C^{2s}_{\phi} - C^{2s}_{\phi}} \]
Taking into account the strict VMD model giving $K_\rho$ and evaluations of other authors [20,23] $K_\omega$ coupling constant ratios. In particular for $\rho$ the following additional coupling constant ratio values are predicted

\begin{align*}
K_\rho &= \frac{f_{\rho NN}^{(2)}/f_\rho}{f_{\rho NN}^{(1)}/f_\rho} = 7.945 \\
K_\omega &= \frac{f_{\omega NN}/f_\omega}{f_{\omega NN}^{(1)}/f_\omega} = 0.154.
\end{align*}

(43)

Taking into account the strict VMD model giving $K_\rho = \mu_\rho - \mu_n = 3.71$, $K_\omega = \mu_\rho + \mu_n = -0.12$ and evaluations of other authors [20,23] $K_\rho$ to be large ($\sim 6$) and $K_\omega$ to be small ($\sim 0$), our
results seem to be quite reasonable. Other tensor-to-vector coupling ratios are:

\[ K_\phi = 0.283, \ K_\omega = -0.049, \ K_{\omega'} = -0.404, \ K_{\rho'} = -0.366, \]
\[ K_\rho = -2.196, \ K_{\rho'} = -1.620, \ K_{\rho''} = -0.356, \ K_{\rho'''} = -0.188. \] (45)

The universal vector meson coupling constants \( f_s \) and \( f_v \) are determined from the widths of the leptonic decays, i.e.

\[ \frac{f_v^2}{4\pi} = \frac{\alpha^2}{3} \frac{m_\nu}{\Gamma(\nu \to e^+e^-)}. \] (46)

Then numerical values

\[ f_\rho = 5.0320 \pm 0.1089; \ f_\omega = 17.0499 \pm 0.2990; \ f_\phi = -12.8832 \pm 0.0824 \] (47)

are obtained from the corresponding world averaged lepton widths [29] and the universal \( \omega', \omega^{''}, \omega^{'''} \) and \( \rho', \rho^{''}, \rho^{'''} \) meson coupling constants

\[ f_{\omega'} = 47.6022 \pm 7.5026; \ f_{\omega^{''}} = 48.3778 \pm 7.5026 \] (48)

and

\[ f_{\rho'} = 13.6491 \pm 0.9521; \ f_{\rho^{''}} = 22.4020 \pm 2.2728 \] (49)

have been found from the lepton widths estimated by Donnachie and Clegg [33].

As a result, the following numerical values of the corresponding coupling constants are predicted

\[ f_{\omega NN}^{(1)} = 18.9527; \ f_{\rho NN}^{(1)} = 1.9335; \]
\[ f_{\phi NN}^{(1)} = 12.0956; \ f_{\rho NN}^{(1)} = 10.4375; \]
\[ f_{\omega NN}^{(1)} = 24.0153; \ f_{\rho NN}^{(1)} = -13.8870; \]
\[ f_{\omega^{''} NN}^{(1)} = 7.1696; \]
\[ f_{\omega NN}^{(2)} = 2.9189; \ f_{\rho NN}^{(2)} = 15.3627; \]
\[ f_{\phi NN}^{(2)} = 3.4251; \ f_{\rho NN}^{(2)} = -22.9168; \]
\[ f_{\omega NN}^{(2)} = -1.1686; \ f_{\rho NN}^{(2)} = 22.4916; \]
\[ f_{\omega^{''} NN}^{(2)} = -2.8988. \] (50)
Their squares divided by 4\(\pi\) are presented in Table 3 and Table 4, where for a comparison also values obtained by other authors are shown.

One can immediately notice large value of \(f^{(1,2)}_{\phi NN}\) coupling constants which may indicate a violation of OZI rule [34].

By using the numerical values (50) one can predict the \(\omega - \phi\) mixing angle, employing the relation

\[
\frac{\sqrt{3}}{\cos \vartheta} \frac{f^{(1)}_{\rho NN}}{f^{(1)}_{\omega NN}} = \frac{f^{(1)}_{\phi NN}}{f^{(1)}_{\omega NN}}.
\]

(51)

It takes the value \(\vartheta = 0.7175\), which is very near to the ideal mixing.

Our model predicts the neutron charge radius \(r^n_E\) to be negative automatically and we are not in need to secure this phenomenon by a constraint following from the low-energy neutron-atom scattering results like in [20].

Nevertheless, the most important predictions of our unitary and analytic model of the nucleon e.m. structure are the isovector spectral function behaviours (see Fig.9) to be consistent with predictions of Höhler and Pietarinen [27] and Mergel, Meißner and Drechsel [20], which have been carried out on the base of the Frazer and Fulco [35] unitarity relation by using the pion e.m. \(F_\pi(t)\) and the \(P\)-wave \(\pi\pi \rightarrow N\bar{N}\) partial wave amplitudes obtained by an analytic continuation of the experimental information on \(\pi N\)-scattering into the unphysical region.

The method of our prediction of the latter consist in the following. The ten-resonance unitary and analytic model of nucleon e.m. structure constructed in this paper contains an explicit two-pion continuum contribution given by the unitary cut starting from \(t = 4m^2_\pi\), from where just isovector spectral function start to be different from zero. Then despite of the fact that the unstable \(\rho\)-meson is taken into account as complex conjugate pairs of poles shifted from the real axis into the complex plane on the second and third Riemann sheet of the four-sheeted Riemann surface, the model predicts the strong enhancement on the left wing of the \(\rho(770)\) resonance in the isovector spectral functions automatically. And just agreement of our predictions with those obtained by means of the Frazer and Fulco unitarity relation convinces us that our model constructed in this paper is really unitary.

Another success of the presented model is a prediction of isoscalar nucleon spectral func-
tion behaviours (see Fig.10) for the first time as the model contains an explicit three-pion continuum contribution given by the unitary cut starting from $t = 9m_\pi^2$, from where just isoscalar spectral functions start to be different from zero.

5 Conclusions

We have constructed unitary and analytic ten-resonance (5 isoscalars and 5 isovectors) model of the nucleon e.m. structure which represents a harmonious unification of all known nucleon ff properties, like analyticity, reality condition, experimental fact of creation of vector-meson resonances in electron-positron annihilation processes, normalization and the asymptotic behaviour as predicted for nucleon e.m. ff’s by the quark model of hadrons. It depends only on parameters with clear physical meaning. They are four effective square-root branch points, representing contribution of all other higher thresholds given by the unitarity condition, the mass and width of the hypothetical fourth excited state of $\rho(770)$-meson and coupling constants of some resonances under consideration. They all are numerically evaluated by an analysis of all existing space-like and time-like nucleon ff data.

We would like to note that by means of our model presented in this paper we have described all existing nucleon ff data for the first time, including also FENICE neutron time-like data and FERMILAB proton eight points at higher energies. In the latter an existence of the $\rho^{\ast \ast}(2500)$ resonance with parameters $m_{\rho^{\ast \ast}} = 2500\text{ MeV}$ and $\Gamma_{\rho^{\ast \ast}} = 700\text{ MeV}$ is crucial. So, there is challenge to experimental physicists to confirm an existence of this resonance also in other processes than $e^+e^- \rightarrow N\bar{N}$.

Our unitary and analytic ten-resonance nucleon ff model gives a lot of reasonable predictions. However, the most important among them are isoscalar and isovector spectral function behaviours, which coincide also with predictions obtained in the framework of heavy baryon chiral perturbation theory [24].

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Figure captions

Figs. 1a-2b A simultaneous optimal fit of all existing data on proton e.m. ff’s in the space-like and time-like regions by the unitary and analytic ten resonance model of the proton e.m. structure, represented by expressions (5) and (30)-(33).

Figs. 3a-4b A simultaneous optimal fit of all existing data on neutron e.m. ff’s in the space-like and time-like regions by the unitary and analytic ten resonance model of the neutron e.m. structure, represented by expressions (5) and (30)-(33).

Figs. 5-8 Ratios of appropriately normalized electric and magnetic proton and neutron ff’s in the space-like region to the dipole formula.

Fig. 9 Predicted behaviours of the isovector spectral functions by the ten-resonance unitary and analytic model of the nucleon e.m. structure.

Fig. 10 : Predicted behaviours of the corresponding isoscalar spectral functions.
Table 1: Dirac and Pauli isoscalar and isovector radii of nucleons

|                | \( r_1^{(s)} \) [fm] | \( r_2^{(s)} \) [fm] | \( r_1^{(v)} \) [fm] | \( r_2^{(v)} \) [fm] |
|----------------|-----------------------|-----------------------|-----------------------|-----------------------|
| our results    | 0.771                 | 0.283                 | 0.733                 | 0.898                 |
| Ref. [20]      | 0.782                 | 0.845                 | 0.765                 | 0.893                 |
| Ref. [23]      | 0.767                 | 0.837                 | 0.759                 | 0.863                 |

Table 2: Electric and magnetic, and also Dirac and Pauli radii of the proton and neutron

|                | \( r_p^{E} \) [fm] | \( r_p^{M} \) [fm] | \( r_n^{n} \) [fm] | \( r_p^{p} \) [fm] | \( r_n^{n} \) [fm] |
|----------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| our results    | 0.827               | 0.860               | 0.891               | 0.752               | 0.914               | 0.883               |
| Ref. [20]      | 0.847               | 0.836               | 0.889               | 0.774               | 0.894               | 0.893               |
| Ref. [23]      | 0.836               | 0.843               | 0.840               | 0.761               | 0.883               | 0.876               |

Table 3: Coupling constants of the isoscalar vector mesons to nucleons

|                | \( f_{\omega NN}^{(1)}/4\pi \) | \( f_{\phi NN}^{(1)}/4\pi \) | \( f_{\omega NN}^{(2)}/4\pi \) | \( f_{\phi NN}^{(2)}/4\pi \) |
|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| our results    | 28.58                           | 11.64                           | 45.89                           | 4.09                            |
| Ref.[20]       | 34.6                            | 6.7                             | –                               | –                               |
| Ref. [23]      | 24.0                            | 5.1                             | –                               | –                               |

|                | \( f_{\omega NN}^{(2)}/4\pi \) | \( f_{\phi NN}^{(2)}/4\pi \) | \( f_{\omega NN}^{(2)}/4\pi \) | \( f_{\phi NN}^{(2)}/4\pi \) |
|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| our results    | 0.67                            | 0.93                            | 0.11                            | 0.67                            |
| Ref.[20]       | 0.9                             | 0.3                             | –                               | –                               |
| Ref. [23]      | –                               | 0.2                             | –                               | –                               |
Table 4: Coupling constants of the isovector vector mesons to nucleons

|                | $f_{\rho NN}^{(1)2}/4\pi$ | $f_{\rho' NN}^{(1)2}/4\pi$ | $f_{\rho'' NN}^{(1)2}/4\pi$ |
|----------------|-----------------------------|-----------------------------|-----------------------------|
| **our results** | 0.30                        | 8.67                        | 15.35                       |
| Ref. [20]      | –                           | 40.27                       | 793.53                      |
| Ref. [23]      | 0.55                        | –                           | –                           |
|                | $f_{\rho NN}^{(2)2}/4\pi$ | $f_{\rho' NN}^{(2)2}/4\pi$ | $f_{\rho'' NN}^{(2)2}/4\pi$ |
| **our results** | 18.78                       | 41.79                       | 40.26                       |
| Ref. [20]      | –                           | 143.97                      | 304.07                      |
| Ref. [23]      | 24.0                        | 11.5                        | –                           |
