Electromagnetic structure of nucleon and Roper in soft-wall AdS/QCD

Thomas Gutsche, 1 Valery E. Lyubovitskij, 1, 2, 3, 4 and Ivan Schmidt 2

1 Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D-72076 Tübingen, Germany
2 Departamento de Física y Centro Científico Tecnológico de Valparaíso-CCTVal, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
3 Department of Physics, Tomsk State University, 634050 Tomsk, Russia
4 Laboratory of Particle Physics, Tomsk Polytechnic University, 634050 Tomsk, Russia

(Dated: December 27, 2017)

We present an improved study of the electromagnetic form factors of the nucleon and of the Roper-nucleon transition using an extended version of the effective action of soft-wall AdS/QCD. We include novel contribution from additional non-minimal terms, which do not renormalize the charge and do not change the normalization of the corresponding form factors, but the inclusion of these terms results in an important contribution to the momentum dependence of the form factors and helicity amplitudes.

PACS numbers: 12.38.Lg, 13.40.Gp, 14.20.Dh, 14.20.Gk
Keywords: nucleons, Roper resonance, AdS/QCD, form factors

I. INTRODUCTION

Originally the soft-wall AdS/QCD action for the nucleon was proposed in Ref. 1. It included a term describing the nucleon confining dynamics and the electromagnetic field, and their minimal and non-minimal couplings $Q_N = \text{diag}(1, 0)$ (nucleon charge matrix) and $\eta_N = \text{diag}(\eta_\mu, \eta_\nu)$ (nucleon matrix of anomalous magnetic moments), respectively. The use of the non-minimal couplings is essential to generate the Pauli spin-flip form factors. Later, in Ref. 2, this action was used for the calculation of generalized parton distributions of the nucleon. In Ref. 3 it was extended to take into account higher Fock states in the nucleon and additional couplings with the electromagnetic field in consistency with QCD constituent counting rules 4 for the power scaling of hadronic form factors at large values of the momentum transfer squared in the Euclidean region. In Ref. 5 soft-wall AdS/QCD was developed for the description of baryons with adjustable quantum numbers $n, J, L$, and $S$. In another development, in Refs. 6-8, the nucleon properties have been analyzed using a Hamiltonian formalism. However, their calculation of the nucleon electromagnetic properties ignored the contribution of the non-minimal coupling to the Dirac form factors, and therefore, the analysis done in Refs. 6-8, is in our opinion not fully consistent. In Ref. 7 the ideas of Ref. 6 have been extended by the inclusion of higher Fock states in the nucleon, in order to calculate nucleon electromagnetic form factors in light-front holographic QCD. In this paper the Pauli form factor is again introduced by hand, using the overlap of the $L = 0$ and $L = 1$ nucleon wave function. Additionally, the expression for the neutron Dirac form factor has been multiplied by hand by a free parameter $r$.

In a series of papers 9-11 we have developed a light-front quark-diquark approach for the nucleon structure, describing nucleon parton distributions and form factors from a unified point of view. In particular, in a recent paper 11 we derived nucleon light-front wave functions, analytically matching the results of global fits to the quark parton distributions in the nucleon at the initial scale $\mu \sim 1$ GeV. We also showed that the distributions obey the correct Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution 12 to high scales. Using these constrained nucleon wave functions we get a reasonable description of data on nucleon electromagnetic form factors. We also predict the transverse parton, Wigner and Husimi distributions from a unified point of view, using our light-front wave functions we get a reasonable description of data on nucleon electromagnetic form factors. We also predict the Pauli spin-flip form factors. Later, in Ref. 13, it was also important to mention a recent paper 13, where the $\gamma^* \to \rho$ transition form factor has been calculated in soft-wall AdS/QCD. Here it was shown that the form factor is consistent with quark counting rules for differential cross sections with single and double vector meson production. It scales as $1/\sqrt{Q^2}$ and, therefore, it contributes to the electromagnetic form factors of the nucleons at subleading order.

The Roper-nucleon transition form factors and helicity amplitudes can be also discussed within this same formalism. The Roper resonance was first considered in the context of AdS/QCD in Ref. 14, where the Dirac form factor for the electromagnetic nucleon-Roper transition was calculated in light-front holographic QCD. Later, in Ref. 15, the formalism for the study of nucleon resonances in soft-wall AdS/QCD has been developed, and the first application for a detailed description of Roper-nucleon transition properties (form factors, helicity amplitudes and transition charge radii) was performed. In Ref. 16, 17 the formalism proposed in 15 was used, with a different set of parameters. An overview of the application of other theoretical approaches can be found in Refs. 15, 18. This includes recent
novel ideas about considering additional degrees of freedom for this state, such as a molecular nucleon-scalar $\sigma$ meson component \cite{19, 20}, for a realistic description of current data on Roper electroproduction performed by the CLAS Collaboration at JLab \cite{21, 22}.

In the present paper we include additional non-minimal couplings of the vector field (dual to the electromagnetic field) with fermionic (dual to the nucleon and Roper). Such terms do not renormalize the charge, but gives an important contribution to the momentum dependence of the nucleon and Roper-nucleon transition form factors (helicity amplitudes). The inclusion of these terms helps to improve the description of data. The paper is organized as follows. In Sec. II we briefly discuss our formalism. In Sec. III we present the analytical calculation and the numerical analysis of electromagnetic form factors and helicity amplitudes of the nucleon and the Roper. Finally, Sec. IV contains our summary and conclusions.

II. FORMALISM

In this section we briefly review our approach. We start with the underlying action for the study of the nucleon $N = (p, n)$ and Roper $R = (R_p, R_n)$ resonance, extended by the inclusion of photons. It is constructed in terms of the 5D AdS fields $\psi^N_{\pm,r}(x, z)$ and $\psi^R_{\pm,r}(x, z)$, which are duals to the left- and right-handed chiral doublets of nucleons (Roper resonances) $\mathcal{O}_N = (B_1^L, B_2^L)^T$ and $\mathcal{O}_R = (B_1^R, B_2^R)^T$ with $B_1 = p, R_p$ and $B_2 = n, R_n$. These fields are in the fundamental representations of the chiral $SU_L(2)$ and $SU_R(2)$ subgroups and are holographic analogues of the nucleon $N$ and Roper resonance $R$, respectively. The 5D AdS fields $\psi^N_{\pm,r}(x, z)$ are products of the left/right 4D spinor fields

$$\psi^{L/R}_{n=0,1}(x) = \frac{1 + \gamma^5}{2} \psi_{n=0,1}(x),$$  \hspace{1cm} (1)

with spin 1/2 and the bulk profiles $F^{L/R}_{\tau,n=0,1}(z) = z^2 F^{L/R}_{\tau,n=0,1}(z)$ with twist $\tau$ depending on the holographic (scale) variable $z$:

$$\psi^N_{\pm,r}(x, z) = \frac{1}{\sqrt{2}} \left[ \psi^L_0(x) F^{L/R}_{\tau,0}(z) \pm \psi^R_0(x) F^{R/L}_{\tau,0}(z) \right],$$

$$\psi^R_{\pm,r}(x, z) = \frac{1}{\sqrt{2}} \left[ \psi^L_1(x) F^{L/R}_{\tau,1}(z) \pm \psi^R_1(x) F^{R/L}_{\tau,1}(z) \right],$$ \hspace{1cm} (2)

where

$$F^{L}_{\tau,0} = \sqrt{\frac{2}{\Gamma(\tau)}} \kappa^\tau z^{\tau-1/2} e^{-\kappa^2 z^2/2},$$

$$F^{R}_{\tau,0} = \sqrt{\frac{2}{\Gamma(\tau-1)}} \kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2},$$

$$F^{L}_{\tau,1} = \sqrt{\frac{2}{\Gamma(\tau+1)}} \kappa^\tau z^{\tau-1/2} (\tau - \kappa^2 z^2) e^{-\kappa^2 z^2/2},$$

$$F^{R}_{\tau,1} = \sqrt{\frac{2}{\Gamma(\tau)}} \kappa^{\tau-1} z^{\tau-3/2} (\tau - 1 - \kappa^2 z^2) e^{-\kappa^2 z^2/2}. \hspace{1cm} (3)$$

Here the nucleon is identified as the ground state with $n = 0$ and the Roper resonance as the first radially excited state with $n = 1$. We also include the vector field $V_M(x, z)$, dual to the electromagnetic field. We work in the axial gauge $V_z = 0$ and perform a Fourier transformation of the vector field $V_\mu(x, z)$ with respect to the Minkowski coordinate

$$V_\mu(x, z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} V_\mu(q) V(q, z).$$ \hspace{1cm} (4)

We derive an EOM for the vector bulk-to-boundary propagator $V(q, z)$ dual to the $q^2$-dependent electromagnetic current

$$\partial_z \left( \frac{e^{-\varphi(z)}}{z} \partial_z V(q, z) \right) + q^2 \frac{e^{-\varphi(z)}}{z} V(q, z) = 0,$$ \hspace{1cm} (5)
where $\varphi(z) = \kappa^2 z^2$ is the dilaton field with its scale parameter $\kappa$, which is varied from 380 to 500 MeV in different fits to hadron data.

The solution of this equation in terms of the gamma $\Gamma(n)$ and Tricomi $U(a, b, z)$ functions reads

$$V(q, z) = \Gamma(1 - \frac{q^2}{4\kappa^2}) U\left( -\frac{q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right).$$

(6)

In the Euclidean region ($Q^2 = -q^2 > 0$) it is convenient to use the integral representation for $V(Q, z)$

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^a e^{-\kappa^2 z^2 \frac{x}{1-x}},$$

(7)

where $x$ is the light-cone momentum fraction and $a = Q^2/(4\kappa^2)$.

The action contains a free part $S_0$, describing the confined dynamics of nucleon, Roper and the electromagnetic field in AdS space, and an electromagnetic interaction part $S_{\text{int}}$ with

$$S = S_0 + S_{\text{int}},$$

$$S_0 = \int d^4xdz \sqrt{g} e^{-\varphi(z)} \left\{ \mathcal{L}_N(x, z) + \mathcal{L}_R(x, z) + \mathcal{L}_V(x, z) \right\},$$

$$S_{\text{int}} = \int d^4xdz \sqrt{g} e^{-\varphi(z)} \left\{ \mathcal{L}_{VVN}(x, z) + \mathcal{L}_{VVR}(x, z) + \mathcal{L}_{VRN}(x, z) \right\}.$$

(8)

$\mathcal{L}_N$, $\mathcal{L}_R$, $\mathcal{L}_V(x, z)$ and $\mathcal{L}_{VVN}(x, z)$, $\mathcal{L}_{VVR}(x, z)$, $\mathcal{L}_{VRN}(x, z)$ are the free and interaction Lagrangians, respectively, and are written as

$$\mathcal{L}_B(x, z) = \sum_{i=+,-,\tau} c^B_i \bar{\psi}_{i,\tau}(x, z) \hat{D}_i(z) \psi_{i,\tau}(x, z),$$

$$\mathcal{L}_V(x, z) = -\frac{1}{4} V_{MN}(x, z) V^{MN}(x, z),$$

$$\mathcal{L}_{VBB}(x, z) = \sum_{i=+,-,\tau} c^B_i \bar{\psi}_{i,\tau}(x, z) \hat{V}_B(x, z) \psi_{i,\tau}(x, z),$$

$$\mathcal{L}_{VRN}(x, z) = \sum_{i=+,-,\tau} c^R_{\tau N} \bar{\psi}_{i,\tau}(x, z) \hat{V}_i^{\tau N}(x, z) \psi_{i,\tau}(x, z) + \text{H.c.},$$

(9)

where $B = N, R$ and

$$\hat{D}_\pm(z) = \frac{i}{2} \Gamma_M \gamma_\mu \partial_\mu - \frac{i}{8} \Gamma_M \omega_M^{ab} \gamma_{ab} [\Gamma_N, \Gamma_b] \pm (\mu + U_F(z)), $$

$$\hat{V}_i^{\tau N}(x, z) = Q \Gamma^M V_M(x, z) + \frac{i}{4} \bar{\psi}_{i,\tau}(x, z) \hat{D}_i(z) \psi_{i,\tau}(x, z),$$

$$+ \frac{1}{4} \lambda_i^{\tau H} z^2 [\Gamma^M, \Gamma_N] \partial_\mu \partial_\nu V_{MN}(x, z) + \frac{1}{4} \lambda_i^{\tau H} z^2 [\Gamma^M, \Gamma_N] \partial_\mu \partial_\nu V_{MN}(x, z) + \frac{1}{4} \lambda_i^{\tau H} z^2 [\Gamma^M, \Gamma_N] \partial_\mu \partial_\nu V_{MN}(x, z),$$

$H = N, R, RN.$

(10)

The set of parameters $c^N$, $c^R$, and $c^{RN}$ induce mixing of the contribution of AdS fields with different twist dimension. In Refs. [2, 13] we showed that the parameters $c^B$ are constrained by the condition $\sum c^B = 1$ in order to get the correct normalization of the kinetic term $\bar{\psi}_{i}(x) i \partial \psi_{i}(x)$ of the four-dimensional spinor field. This condition is also consistent with electromagnetic gauge invariance. The couplings $\eta_i^{\tau H} = \text{diag}(\eta_i^{H_1}, \eta_i^{H_2}), \lambda_i^{\tau H} = \text{diag}(\lambda_i^{H_1}, \lambda_i^{H_2}), \gamma_i^{\tau H} = \text{diag}(\gamma_i^{H_1}, \gamma_i^{H_2}), \zeta_i^{\tau H} = \text{diag}(\zeta_i^{H_1}, \zeta_i^{H_2})$, and $\xi_i^{\tau H} = \text{diag}(\xi_i^{H_1}, \xi_i^{H_2})$, where $H_1 = p, R_p, R_{pp}$ and $H_2 = n, R_n, R_{nn}$ are fixed from the magnetic moments, slopes, and form factors of both the nucleon and Roper, while the couplings $c^{RN}$ are fixed from the normalization of the Roper-nucleon helicity amplitudes. The terms proportional to the couplings $\lambda_i^{\tau H}$, $\zeta_i^{\tau H}$, and $\xi_i^{\tau H}$ express novel nonminimal couplings of the fermions with the vector field. It does not renormalize the charge and does not change the corresponding form factor normalizations, but gives an important contribution to the momentum dependence of the form factors and helicity amplitudes.

We use the conformal metric $g_{MN} x^M x^N = \epsilon_5^a \epsilon_5^b \eta_{ab} x^M x^N = (dx_\mu dx^\mu - dz^2)/z^2$; $\epsilon_5^a = \delta_5^a / z$ is the vielbein; $\sqrt{g} = 1/z^5$. Here $\mu$ is the five-dimensional mass of the spin-$\frac{1}{2}$ AdS fermion $\mu = 3/2 + L$, with $L$ being the orbital angular momentum; $U_F(z) = \varphi(z)$ is the dilaton potential; $Q = \text{diag}(1, 0)$ is the nucleon (Roper) charge matrix; $V_{MN} = \partial_M V_N - \partial_N V_M$ is the stress tensor for the vector field; $\omega_{ab} = (\delta_5^a \delta_5^b - \delta_5^b \delta_5^a)/z$ is the spin connection term; $\sigma_{MN} = [\Gamma^M, \Gamma^N]$ is the commutator of the Dirac matrices in AdS space, which are defined as $\Gamma^M = \epsilon_5^a \Gamma^a$ and $\Gamma^a = (\gamma^a, -i\gamma^5)$. 

The nucleon and Roper masses are identified with the expressions \[3,15\]

\[
M_N = 2\kappa \sum_{r} c_r^N \sqrt{-\tau}, \quad M_R = 2\kappa \sum_{r} c_r^R \sqrt{-\tau}.
\] (11)

As we mentioned the set of mixing parameters \(c_r^{N,R}\) is constrained by the correct normalization of the kinetic term of the four-dimensional spinor field and by charge conservation as (see detail in Ref. \[3\]):

\[
\sum_{r} c_r^{N,R} = 1.
\] (12)

Baryon form factors are calculated analytically using bulk profiles of fermion fields and the bulk-to-boundary propagator \(V(Q,z)\) of the vector field (see exact expressions in the next section). Calculation technique is discussed in detail in Refs. \[3,15\].

III. ELECTROMAGNETIC FORM FACTORS OF NUCLEON, ROPER AND ROPER-NUCLEON TRANSITIONS

The electromagnetic form factors of the nucleon, Roper and Roper-nucleon transitions are defined by the following matrix elements, due to Lorentz and gauge invariance,

\[
N \rightarrow N: \quad M^\mu(p_1, \lambda_1; p_2, \lambda_2) = \bar{u}_N(p_2, \lambda_2) \left[ \gamma^\mu F_1^N(q^2) - i\sigma^{\mu\nu} \frac{q^\nu}{2M_N} F_2^N(q^2) \right] u_N(p_1, \lambda_1),
\]

\[
\mathcal{R} \rightarrow \mathcal{R}: \quad M^\mu(p_1, \lambda_1; p_2, \lambda_2) = \bar{u}_R(p_2, \lambda_2) \left[ \gamma^\mu F_1^R(q^2) - i\sigma^{\mu\nu} \frac{q^\nu}{2M_R} F_2^R(q^2) \right] u_R(p_1, \lambda_1),
\]

\[
\mathcal{R} \rightarrow N: \quad M^\mu(p_1, \lambda_1; p_2, \lambda_2) = \bar{u}_N(p_2, \lambda_2) \left[ \gamma_\perp^\mu F_1^{RN}(q^2) - i\sigma^{\mu\nu} \frac{q^\nu}{M_R + M_N} F_2^{RN}(q^2) \right] u_R(p_1, \lambda_1),
\] (13)

where \(\gamma^\mu = \gamma^\mu - q^\mu q \gamma / q^2, q = p_1 - p_2, \) and \(\lambda_1, \lambda_2, \) and \(\lambda\) are the helicities of the initial, final baryon and photon, obeying the relation \(\lambda_1 = \lambda_2 - \lambda\).

We recall the definitions of the nucleon Sachs form factors \(G_E^N(Q^2)\) and the electromagnetic radii \(\langle r_E^2 \rangle^N\) in terms of the Dirac \(F_1^N(Q^2)\) and Pauli \(F_2^N(Q^2)\) form factors

\[
G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2),
\]

\[
G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2),
\]

\[
\langle r_E^2 \rangle^N = -\frac{6}{M_N^2} \frac{dG_E^N(Q^2)}{dQ^2} \bigg|_{Q^2=0},
\]

\[
\langle r_M^2 \rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2} \bigg|_{Q^2=0},
\] (14)

where \(G_M^N(0) \equiv \mu_N\) is the nucleon magnetic moment.

Now we introduce the helicity amplitudes \(H_{\lambda_0}^{\lambda}\), which in turn can be related to the invariant form factors \(F_i^{RN}\) (see details in Refs. \[25,28\]). The pertinent relation is

\[
H_{\lambda_0}^{\lambda} = M_\mu(p_1, \lambda_1; p_2, \lambda_2) \epsilon^\mu(q, \lambda),
\] (15)

where \(\epsilon^\mu(q, \lambda)\) is the polarization vector of the outgoing photon. A straightforward calculation gives \[25,28\]

\[
H_{\lambda_0}^{\lambda} = \sqrt{\frac{Q}{Q^2}} \left( F_1^{RN} M_+ - F_2^{RN} \frac{Q^2}{M_1} \right), \quad H_{\lambda_0}^{\lambda=\pm 1} = -\sqrt{2Q} \left( F_1^{RN} + F_2^{RN} \frac{M_1}{M_1} \right),
\] (16)

where \(M_\pm = M_1 \pm M_2, \) \(Q_\pm = M_\pm^2 + Q^2\).
In the case of the Roper-nucleon transition there exists the alternative set of helicity amplitudes \( (A_{1/2}, S_{1/2}) \) related to the set \( (H_{-1/2}, H_{1/2}) \) by \cite{29,33}

\[
A_{1/2} = -b H_{1/2}, \quad S_{1/2} = b \frac{|p|}{\sqrt{Q^2}} H_{-1/2},
\]

(17)

where

\[
|p| = \frac{\sqrt{Q^2 + M^2}}{2M_R}, \quad b = \sqrt{\frac{\pi \alpha}{M + M - M_N}}
\]

(18)

and \( \alpha = 1/137.036 \) is the fine-structure constant.

Expressions for the electromagnetic form factors of the nucleons, Roper, and Roper-nucleon transitions are given as follows:

nucleon-nucleon transition,

\[
F_1^p(Q^2) = C_1(Q^2) + g_\mu^p C_2(Q^2) + \eta^\mu C_3(Q^2) + \lambda^\mu C_4(Q^2) + \zeta^\mu C_5(Q^2) + \xi^\mu C_6(Q^2),
\]

\[
F_1^n(Q^2) = g_\mu^p C_2(Q^2) + \eta^\mu C_3(Q^2) + \lambda^\mu C_4(Q^2) + \zeta^\mu C_5(Q^2) + \xi^\mu C_6(Q^2),
\]

\[
F_2^n(Q^2) = \eta^\nu C_7(Q^2) + \lambda^\nu C_8(Q^2),
\]

Roper-nucleon transition,

\[
F_1^{R,p}(Q^2) = D_1(Q^2) + g_\mu^{R,p} D_2(Q^2) + \eta^\mu^{R,p} D_3(Q^2) + \lambda^\mu^{R,p} D_4(Q^2) + \zeta^\mu^{R,p} D_5(Q^2) + \xi^\mu^{R,p} D_6(Q^2),
\]

\[
F_1^{R,n}(Q^2) = g_\mu^{R,n} D_2(Q^2) + \eta^\mu^{R,n} D_3(Q^2) + \lambda^\mu^{R,n} D_4(Q^2) + \zeta^\mu^{R,n} D_5(Q^2) + \xi^\mu^{R,n} D_6(Q^2),
\]

\[
F_2^{R,p}(Q^2) = \eta^\nu^{R,p} D_7(Q^2) + \lambda^\nu^{R,p} D_8(Q^2),
\]

\[
F_2^{R,n}(Q^2) = \eta^\nu^{R,n} D_7(Q^2) + \lambda^\nu^{R,n} D_8(Q^2).
\]

(19)

Roper-Roper transition,

\[
F_1^{R,R}(Q^2) = E_1(Q^2) + g_\mu^{R,R} E_2(Q^2) + \eta^\mu^{R,R} E_3(Q^2) + \lambda^\mu^{R,R} E_4(Q^2) + \zeta^\mu^{R,R} E_5(Q^2) + \xi^\mu^{R,R} E_6(Q^2),
\]

\[
F_1^{R,R}(Q^2) = g_\mu^{R,R} E_2(Q^2) + \eta^\mu^{R,R} E_3(Q^2) + \lambda^\mu^{R,R} E_4(Q^2) + \zeta^\mu^{R,R} E_5(Q^2) + \xi^\mu^{R,R} E_6(Q^2),
\]

\[
F_2^{R,R}(Q^2) = \eta^\nu^{R,R} E_7(Q^2) + \lambda^\nu^{R,R} E_8(Q^2),
\]

\[
F_2^{R,R}(Q^2) = \eta^\nu^{R,R} E_7(Q^2) + \lambda^\nu^{R,R} E_8(Q^2).
\]

(20)

The structure integrals \( C_i(Q^2), D_i(Q^2), \) and \( E_i(Q^2) \) are given by the analytical expressions (see in Appendix). All calculated form factors are consistent with QCD constituent counting rules \cite{4} for the power scaling of hadronic form factors at large values of the momentum transfer squared in the Euclidean region.

The parameters, which will be used in the numerical evaluations, are fixed as follows: we use the universal dilaton parameter of \( \kappa = 383 \) MeV, the sets of twist mixing parameters are fixed from data on masses of nucleon \( (c_3^N = 1.800, \ k_4^N = -1.042, \ k_5^N = 0.242) \) and Roper \( (c_3^R = 0.820, \ k_4^R = -0.242, \ k_5^R = 0.422) \). At fixed \( \kappa = 383 \) MeV and baryon masses \( M_N = 938.27 \) MeV and \( M_R = 1440 \) MeV only two parameters from the set of six twist mixing parameters are free. E.g., parameters \( c_4^N,R \) and \( c_5^N,R \) can be fixed through the parameters \( c_3^N,R \) and ratios \( M_{N,R} \) using the matching conditions \cite{11} and \cite{12}. The parameters \( \eta^p_V = 0.2988 \) and \( \eta^n_V = -0.3188 \) are analytically fixed from data on nucleon magnetic moments:

\[
\eta^p_V = \left( \frac{\kappa}{M_N} \right)^2 (\mu_p - 1), \quad \eta^n_V = \left( \frac{\kappa}{M_N} \right)^2 \mu_n,
\]

(22)

where \( \mu_p = 2.793 \) n.m. and \( \mu_n = -1.913 \) n.m. \cite{34}.

The set on the nucleon parameters \( g_\mu^p = -2.001, \ g_\mu^n = 1.731, \ \zeta^p_V = -0.109, \ \zeta^p_V = 0.101, \ \xi^p_V = -0.166, \ \xi^p_V = 0.174, \ \lambda^p_V = -0.0005, \) and \( \lambda^p_V = 0.0012 \) is fixed from data on electromagnetic radii and form factors of nucleons. The set
of Roper-nucleon parameters $c_3^{\pi N} = 0.142$, $c_4^{\pi N} = -3.942$, $c_5^{\pi N} = 3.449$, $g_{V^{p p}}^\pi = -10.095$, $\eta_{V^{p p}}^\pi = -0.551$, $\lambda_{V^{p p}}^{\pi N} = 0.020$, and $\zeta_{V^{p p}}^{\pi N} = -0.770$ is fixed from data on Roper-nucleon transition data. For simplicity we put $\Lambda_{V^{p p}} = 0$. Our results for quark and nucleon electromagnetic form factors are shown in Figs. 1-9. We compare our results with data [32-38] and the dipole fit $G_D(Q^2) = 1/(1 + Q^2/\Lambda^2)^2$. As scale parameter $\Lambda$ we use two values $\Lambda = \sqrt{0.71}$ GeV and $\Lambda = \sqrt{0.66}$ GeV, corresponding to the root-mean-square (rms) radius $r_p = 0.81$ fm and $r_p = 0.84$ fm, respectively. In particular, in Fig. 1 and 2 we present our results for the Dirac and Pauli $u$ (left panel) and $d$ (right panel) quark form factors. Here data are taken from Refs. [33, 36].

In Fig. 3 we display the Dirac proton form factor multiplied by $Q^2$ (left panel) and the ratio $Q^2 F_E^p(Q^2)/F_M^p(Q^2)$ (right panel). Results for the Dirac neutron form factor multiplied by $Q^4$ (left panel) and ratio $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$ in comparison with global Fit I and Fit II (right panel) are shown in Fig. 4. We take the central values of the results for a global fit of the charge and magnetic proton form factors from Ref. [37].

**Fit I:**

\[
G_E^p(Q^2) = \frac{1 + a_E^p \tau}{1 + b_E^p \tau + c_E^p \tau^2 + d_E^p \tau^3},
\]

\[
G_M^p(Q^2) = \frac{1 + a_M^p \tau}{1 + b_M^p \tau + c_M^p \tau^2 + d_M^p \tau^3},
\]  

(23)

where

\[
a_E^p = -0.21, \quad b_E^p = 12.21, \quad c_E^p = 12.6, \quad d_E^p = 23.0,
\]

\[
a_M^p = 0.058, \quad b_M^p = 10.85, \quad c_M^p = 19.9, \quad d_M^p = 4.4,
\]  

(24)

**Fit II:**

\[
G_E^p(Q^2) = \frac{1 + a_E^p \tau}{1 + b_E^p \tau + c_E^p \tau^2 + d_E^p \tau^3},
\]

\[
G_M^p(Q^2) = \frac{1 + a_M^p \tau}{1 + b_M^p \tau + c_M^p \tau^2 + d_M^p \tau^3}.
\]  

(25)

where

\[
a_E^p = -0.01, \quad b_E^p = 12.16, \quad c_E^p = 9.7, \quad d_E^p = 37.0,
\]

\[
a_M^p = 0.093, \quad b_M^p = 11.07, \quad c_M^p = 19.1, \quad d_M^p = 5.6.
\]  

(26)

Here $\tau = Q^2/(4M_N^2)$.

In Figs. 5 and 6 we present the ratios $G_E^p(Q^2)/G_D(Q^2)$ and $G_M^p(Q^2)/(\mu_p G_D(Q^2))$ in comparison with the global Fit I and Fit II for the dipole scale parameter $\Lambda^2 = 0.71$ GeV$^2$ (left panel) and $\Lambda^2 = 0.66$ GeV$^2$ (right panel). A detailed comparison of different ratios of the nucleon Sachs form factors is shown in Fig. 7-9. Here we use the dipole function $G_D(Q^2)$ with $\Lambda^2 = 0.71$ GeV$^2$. The Roper-nucleon transition form factors and helicity amplitudes are shown in Figs. 10 and 11. Our predictions for the Roper-nucleon helicity amplitudes are compared with experimental data of the CLAS (JLab) [23] and A1 (MAMI) [85] Collaborations, and with the MAID parametrization [86].

\[
A_{1/2}^p(Q^2) = -0.0614 \text{ GeV}^{-1/2} (1 - 1.22 \text{ GeV}^{-2} Q^2 - 0.55 \text{ GeV}^{-8} Q^8) \exp[-1.51 \text{ GeV}^{-2} Q^2],
\]

\[
S_{1/2}^p(Q^2) = 0.0042 \text{ GeV}^{-1/2} (1 + 40 \text{ GeV}^{-2} Q^2 + 1.5 \text{ GeV}^{-8} Q^8) \exp[-1.75 \text{ GeV}^{-2} Q^2],
\]  

(27)

and with the parametrization proposed by us. We find that the present data on helicity amplitudes can be fitted with the use of the formulas

\[
A_{1/2}^p(Q^2) = A_{1/2}^p(0) \frac{1 + a_1 Q^2}{1 + a_2 Q^2 + a_3 Q^4 + a_4 Q^6},
\]

\[
S_{1/2}^p(Q^2) = S_{1/2}^p(0) \frac{1 + s_1 Q^2}{1 + s_2 Q^2 + s_3 Q^4 + s_4 Q^6},
\]  

(28)

where

\[
A_{1/2}^p(0) = -0.064 \text{ GeV}^{-1/2}, \quad S_{1/2}^p(0) = 0.010 \text{ GeV}^{-1/2},
\]  

(29)
and
\[
\begin{align*}
    a_1 &= -2.03556 \text{ GeV}^{-2}, \quad a_2 = 1.24891 \text{ GeV}^{-2}, \quad a_3 = -0.90673 \text{ GeV}^{-4}, \quad a_4 = 0.41896 \text{ GeV}^{-6}, \\
    s_1 &= 16.59500 \text{ GeV}^{-2}, \quad s_2 = 1.75908 \text{ GeV}^{-2}, \quad s_3 = 3.91487 \text{ GeV}^{-4}, \quad s_4 = -0.15289 \text{ GeV}^{-6}.
\end{align*}
\]

Our results for magnetic moments, slope radii and Roper-nucleon transition helicity amplitudes at \( q^2 = 0 \) are summarized in Table I.

| Quantity | Our results | Data [34] |
|----------|-------------|-----------|
| \( \mu_p \) (in n.m.) | 2.793 | 2.793 |
| \( \mu_n \) (in n.m.) | -1.913 | -1.913 |
| \( r_E^p \) (fm) | 0.832 | 0.84087 ± 0.00039 |
| \( r_E^n \) (fm) | 0.8751 ± 0.0061 |
| \( \langle r_E^p \rangle \) (fm) | -0.116 | -0.1161 ± 0.0022 |
| \( \langle r_E^n \rangle \) (fm) | 0.793 | 0.78 ± 0.04 |
| \( r_M^p \) (fm) | 0.813 | 0.864 ± 0.009 |
| \( r_M^n \) (fm) | -0.061 | -0.060 ± 0.004 |
| \( A_{p/2}^0(0) \) (GeV\(^{-1/2}\)) | 0.008 | — |
| \( S_{p/2}^0(0) \) (GeV\(^{-1/2}\)) | — | — |

### IV. SUMMARY

In the present paper we significantly improved the description of both the nucleon and the Roper structure using a soft-wall AdS/QCD approach. We included novel contributions to the AdS/QCD action from additional non-minimal terms, which do not renormalize the charge and do not change the normalization of the corresponding form factors. They give important contributions to the momentum dependence of the form factors and helicity amplitudes in reasonable agreement with data. In the future we plan to extend our formalism to the study of other nucleon resonances.

**Acknowledgments**

This work was supported by the German Bundesministerium für Bildung und Forschung (BMBF) under Project 05P2015 - ALICE at High Rate (BMBF-FSP 202): (“Jet- and fragmentation processes at ALICE and the parton structure of nuclei and structure of heavy hadrons”, by CONICYT (Chile) Research Project No. 80140097 and under Grants No. 7912010025, 1140390 and PIA/Basal FB0821, by Tomsk State University Competitiveness Improvement Program and the Russian Federation program “Nauka” (Contract No. 0.1764.GZB.2017), and by Tomsk Polytechnic University Competitiveness Enhancement Program (Grant No. VIU-FTI-72/2017).
Appendix A: The structure integrals $C_i(Q^2)$, $D_i(Q^2)$, and $E_i(Q^2)$

Functions $C_i(Q^2)$, $D_i(Q^2)$, and $E_i(Q^2)$ are given by the analytical expressions

$$C_i(Q^2) = \sum_\tau c^{\tau N}_i C^{\tau}_i(Q^2),$$

$$C^{\tau}_1(Q^2) = B(a+1, \tau) \left( \tau + \frac{a}{2} \right),$$

$$C^{\tau}_2(Q^2) = \frac{a}{2} B(a+1, \tau),$$

$$C^{\tau}_3(Q^2) = a B(a+1, \tau + 1) \frac{a(\tau - 1) - 1}{\tau},$$

$$C^{\tau}_4(Q^2) = 2a \left[ (\tau - 1) B(a+1, \tau) - 2(2\tau - 1) B(a+1, \tau + 1) + 3(\tau + 1) B(a+1, \tau + 2) + 2(\tau^2 - 1) B(a+2, \tau + 1) - 2(\tau + 1)(\tau + 2) B(a+2, \tau + 2) \right],$$

$$C^{\tau}_5(Q^2) = -a \left[ (\tau - 1) B(a+1, \tau) + \tau(2\tau - 1) B(a+1, \tau + 1) + 2\tau(\tau + 1) B(a+1, \tau + 2) \right],$$

$$C^{\tau}_6(Q^2) = -a \left[ (\tau - 1) B(a+1, \tau) + \tau(2\tau - 3) B(a+1, \tau + 1) - 2\tau(\tau + 1) B(a+1, \tau + 2) \right],$$

$$C^{\tau}_7(Q^2) = \frac{2M_N}{\kappa} (a+1 + \tau) \sqrt{\tau - 1} B(a+1, \tau + 1),$$

$$C^{\tau}_8(Q^2) = \frac{4M_N}{\kappa} a \tau \sqrt{\tau - 1} \left[ B(a+1, \tau + 1) + 2(\tau + 1) B(a+1, \tau + 2) \right],$$

(A1)
\[ D_i(Q^2) = \sum_{\tau} c_{\tau}^{R_N} D_i^r(Q^2), \]
\[ D_1^r(Q^2) = \frac{a}{2} B(a+1,\tau+1) \left[ \sqrt{\tau-1} \left(1 + \frac{a+1}{\tau}\right) + \sqrt{\tau} \right], \]
\[ D_2^r(Q^2) = \frac{a}{2} B(a+1,\tau+1) \left( \sqrt{\tau-1} \left(1 + \frac{a+1}{\tau}\right) - \sqrt{\tau} \right), \]
\[ D_3^r(Q^2) = a \left[ (\tau-1)^{3/2} B(a+1,\tau) - \tau (\sqrt{\tau} + \sqrt{\tau-1}) B(a+1,\tau+1) + (\tau+1)\sqrt{\tau} B(a+1,\tau+2) \right], \]
\[ D_4^r(Q^2) = 2a \left[ \tau(\tau-1)^{3/2} B(a+1,\tau+1) + \tau^{3/2}(\tau-2)\sqrt{\tau+1} B(a+1,\tau+1) \right. \]
\[ \left. - (\tau+1)\sqrt{\tau} (\tau+1) (2 \tau+1) B(a+1,\tau+3) \right. \]
\[ \left. + (\tau+1)(\tau+2)\sqrt{\tau} (3+4 \tau+2 \sqrt{\tau-1}) B(a+1,\tau+4) \right. \]
\[ \left. - 2(\tau+1)(\tau+2)(\tau+3)\sqrt{\tau} B(a+1,\tau+5) \right], \]
\[ D_5^r(Q^2) = -a \left[ (\tau-1)^{3/2} B(a+1,\tau) + \tau (\sqrt{\tau} + \sqrt{\tau-1}(2\tau-3)) B(a+1,\tau+1) \right. \]
\[ \left. + \sqrt{\tau} (\sqrt{\tau} - \sqrt{\tau-1})^2 (\tau+1) B(a+1,\tau+2) - 2\sqrt{\tau}(\tau+1)(\tau+2) B(a+1,\tau+3) \right], \]
\[ D_6^r(Q^2) = -a \left[ (\tau-1)^{3/2} B(a+1,\tau) - \tau (\sqrt{\tau} - \sqrt{\tau-1}(2\tau-3)) B(a+1,\tau+1) \right. \]
\[ \left. - \sqrt{\tau} (\sqrt{\tau} + \sqrt{\tau-1})^2 (\tau+1) B(a+1,\tau+2) + 2\sqrt{\tau}(\tau+1)(\tau+2) B(a+1,\tau+3) \right], \]
\[ D_7^r(Q^2) = \frac{M_N + M_R}{2\kappa} B(a+1,\tau+1) \left[ a(\tau-1) - \tau - 1 + a\sqrt{\tau}(\tau-1) \right], \]
\[ D_8^r(Q^2) = \frac{M_N + M_R}{\kappa} a \sqrt{\tau} (\sqrt{\tau} + \sqrt{\tau-1}) \left[ \sqrt{\tau}(\tau-1) B(a+1,\tau+1) \right. \]
\[ \left. + (\tau+1)(2\sqrt{\tau}(\tau-1) - 1) B(a+1,\tau+2) - 2(\tau+1)(\tau+2) B(a+1,\tau+3) \right], \]
\[ (A2)\]
\[ E_i(Q^2) = \sum_{\tau} c^2_{\tau} E_i^\tau(Q^2), \]

\[ E_1^\tau(Q^2) = \frac{1}{2} \left[ (\tau - 1)^2 B(a + 1, \tau - 1) + \tau(2 - \tau)B(a + 1, \tau) - \tau(\tau + 1)B(a + 1, \tau + 1) + (\tau + 1)(\tau + 2)B(a + 1, \tau + 2) \right], \]

\[ E_2^\tau(Q^2) = \frac{1}{2} \left[ (\tau - 1)^2 B(a + 1, \tau - 1) + \tau(2 - 3\tau)B(a + 1, \tau) + 3\tau(\tau + 1)B(a + 1, \tau + 1) - (\tau + 1)(\tau + 2)B(a + 1, \tau + 2) \right], \]

\[ E_3^\tau(Q^2) = a \left[ (\tau - 1)^2 B(a + 1, \tau) + \tau(2 - 3\tau)B(a + 1, \tau + 1) + 3\tau(\tau + 1)B(a + 1, \tau + 2) - (\tau + 1)(\tau + 2)B(a + 1, \tau + 3) \right], \]

\[ E_4^\tau(Q^2) = 2a(\tau + 1) \left[ (\tau - 1)^2 B(a + 1, \tau + 2) + \tau(2\tau^3 - 6\tau^2 - 4\tau + 5)B(a + 1, \tau + 3) - \tau(\tau + 2)(8\tau^2 - 2\tau - 13)B(a + 1, \tau + 4) + (\tau + 2)(\tau + 3)(12\tau^2 + 10\tau - 4)B(a + 1, \tau + 5) \right. \]
\[ \left. - (8\tau + 7)(\tau + 2)(\tau + 3)(\tau + 1)B(a + 1, \tau + 6) + 2(\tau + 2)(\tau + 3)(\tau + 4)(\tau + 5)B(a + 1, \tau + 7) \right], \]

\[ E_5^\tau(Q^2) = -a \left[ (\tau - 1)B(a + 1, \tau) + 2\tau(\tau - 1)B(a + 1, \tau + 1) - (\tau + 1)B(a + 1, \tau + 2) - 2(\tau + 1)(\tau + 2)B(a + 1, \tau + 3) \right], \]

\[ E_6^\tau(Q^2) = -a \left[ (\tau - 1)B(a + 1, \tau) + 2\tau(\tau - 2)B(a + 1, \tau + 1) - (\tau + 1)(4\tau - 1)B(a + 1, \tau + 2) + 2(\tau + 1)(\tau + 2)B(a + 1, \tau + 3) \right], \]

\[ E_7^\tau(Q^2) = \frac{2M}{\kappa} \sqrt{\tau} \left[ (\tau - 1)B(a + 1, \tau) - (\tau + 1)(2\tau - 1)B(a + 1, \tau + 1) + (\tau + 1)(\tau + 2)B(a + 1, \tau + 2) \right], \]

\[ E_8^\tau(Q^2) = \frac{4M}{\kappa} a \sqrt{\tau}(\tau + 1) \left[ (\tau - 1)B(a + 1, \tau + 2) + (2\tau^2 - 4\tau + 1)(\tau + 2)B(a + 1, \tau + 3) + (3 - 4\tau)(\tau + 2)(\tau + 3)B(a + 1, \tau + 4) + 2(\tau + 2)(\tau + 3)(\tau + 4)B(a + 1, \tau + 5) \right], \]

where

\[ B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)} \]
[63] D. I. Glazier et al., Eur. Phys. J. A 24, 101 (2005).
[64] B. Plaster et al. (Jefferson Laboratory E93-038 Collaboration), Phys. Rev. C 73, 025205 (2006).
[65] E. Geis et al. (BLAST Collaboration), Phys. Rev. Lett. 101, 042501 (2008).
[66] B. S. Schlimme et al., Phys. Rev. Lett. 111, 132504 (2013).
[67] T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. 142, 922 (1966).
[68] J. Litt et al., Phys. Lett. B 31, 40 (1970).
[69] W. Bartel et al., Nucl. Phys. B58, 429 (1973).
[70] G. Hohler et al., Nucl. Phys. B 114, 505 (1976).
[71] A. F. Sill et al., Phys. Rev. D 48, 29 (1993).
[72] L. Andivahis et al., Phys. Rev. D 50, 5491 (1994).
[73] R. C. Walker et al., Phys. Rev. D 49, 5671 (1994).
[74] S. Rock et al., Phys. Rev. Lett. 49, 1139 (1982).
[75] A. Lung et al., Phys. Rev. Lett. 70, 718 (1993).
[76] P. Markowitz et al., Phys. Rev. C 48, R5 (1993).
[77] H. Anklin et al., Phys. Lett. B 336, 313 (1994).
[78] H. Gao et al., Phys. Rev. C 50, R546 (1994).
[79] E. W. Bruins et al., Phys. Rev. Lett. 75, 21 (1995).
[80] H. Anklin et al., Phys. Lett. B 428, 248 (1998).
[81] W. Xu et al. Phys. Rev. Lett. 85, 2900 (2000).
[82] G. Kubon et al., Phys. Lett. B 524, 26 (2002).
[83] W. Xu et al. (Jefferson Lab E95-001 Collaboration), Phys. Rev. C 67, 012201 (2003).
[84] V. J. Lachniet et al. (CLAS Collaboration), Phys. Rev. Lett. 102, 192001 (2009).
[85] S. Stajner et al., Phys. Rev. Lett. 119, 022001 (2017).
[86] D. Drechsel, S. S. Kamalov and L. Tiator, Eur. Phys. J. A 34, 69 (2007).
FIG. 1: Dirac $u$ and $d$ quark form factors multiplied by $Q^4$.

FIG. 2: Pauli $u$ and $d$ quark form factors multiplied by $Q^4$.

FIG. 3: Dirac proton form factor multiplied by $Q^4$ and ratio $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$. 

FIG. 4: Dirac neutron form factor multiplied by $Q^4$ and ratio $\mu_p G_E^p(Q^2) / G_D^p(Q^2)$ in comparison with global Fit I and Fit II.

FIG. 5: Ratio $G_E^p(Q^2) / G_D^p(Q^2)$ in comparison with global Fit I and Fit II for dipole scale parameter $\Lambda^2 = 0.71$ GeV$^2$ (left panel) and $\Lambda^2 = 0.66$ GeV$^2$ (right panel).

FIG. 6: Ratio $G_E^p(Q^2) / (\mu_p G_D(Q^2))$ in comparison with global Fit I and Fit II for dipole scale parameter $\Lambda^2 = 0.71$ GeV$^2$ (left panel) and $\Lambda^2 = 0.66$ GeV$^2$ (right panel).
FIG. 7: Ratio $G_E^p(Q^2)/G_D(Q^2)$ and charge neutron form factor $G_E^n(Q^2)$ in comparison with data.

FIG. 8: Ratios $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$ and $\mu_n G_E^n(Q^2)/G_M^n(Q^2)$ in comparison with data.

FIG. 9: Ratios $G_M^p(Q^2)/(\mu_p G_D(Q^2))$ and $G_M^n(Q^2)/(\mu_n G_D(Q^2))$ in comparison with data.
FIG. 10: Roper-nucleon transition form factors $F_1^{Rp}(Q^2)$ and $F_2^{Rp}(Q^2)$ up to 10 GeV$^2$.

FIG. 11: Helicity amplitudes $A_{1/2}^p(Q^2)$ and $S_{1/2}^p(Q^2)$ up to 10 GeV$^2$. 