Analysis of Students’ Reasoning in Answering Number Stories using Realistic Mathematics Approach

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Abstract. This study aims to analyzes students’ reasoning process in working on story problems using a realistic mathematical approach. The type of research is qualitative research. To obtain data in this study, instruments in the form of descriptive questions were compiled and developed by researchers (test method). To get accurate data, the tests used in this study must meet right test criteria. The research population in this small study was comprised of grade VII students from SMP PGRI Kasihan. Three groups of students participated in this study as research subjects. Data analysis conducted in this study was based on descriptive qualitative analysis techniques. Data analysis covered data reduction, data display, and drawing a conclusion. The results showed several locations of errors, types of errors, and causes of students’ mistakes in solving story problems in class VII SMP PGRI Kasihan, i.e., students still had difficulty in solving problematic challenges that demanded the ability to reason, were still not creative enough to prepare steps or strategy toward building complete answer, and not optimal in creating argumentation to answer problems that require the ability to provide argument or argument in the form of reasoning as the answer.

1. Introduction
As many 50.5 % of students in Indonesia have mathematical literacy below level 1, meaning they are only capable of solving 1 step in math (in this situation, students can’t even use procedure, formula, and simple algorithm to answer a mathematical question) [1,2]. As many 27.6 % sit on level 1, with the ability to make use of the procedure, formula, and basic algorithm as well as interpreting numbers and engage in direct reasoning. As many 14.8 % stick to level 2, capable of applying simple problem solving, interpreting and presenting a solution. 5.5 % of the students attain level 3, which means they can effectively solve mathematical questions for concrete situations with sufficient explanation and reasoning. Meanwhile, only 1.4 % manage to arrive at the next level. PISA 2003 defines mathematical literacy as one’s ability to identify and understand the role of math in life. Literacy includes quantity, space and shape, change and relationship, and uncertainty. Based on that definition, it can be concluded that the average skill of Indonesian students to identify and comprehend mathematics’s role in life is relatively low.

Mathematical conceptions that students possess are not the sole determinant or pillars of science. [3]. One more crucial issue is how the mathematical paradigm shaped; students are expected to master...
mathematical conceptions and mathematical sound reasoning. In reality, this will provide them with the ability to create and innovate in the math field. Right mathematical reasoning paths the way for excellent mathematical prowess. Upon dealing with questions or challenges that demand reasoning, logical thinking, and rational argumentation as an answer that cannot be found in the routine procedure—students will manage. This is in line with a statement from NRC (National Research Council) in the USA that said the ability to think and to reason would be far more needed in the current as well as the future situation.

Reasoning in learning math is an essential aspect of mathematical skills because math activity is a logical process in making decisions [4]. Students’ reasoning progress at the primary level is generally low. Many come up with arguments that mathematics education in elementary schools do not adequately emphasize reasoning development or students’ logical and thinking process. Mathematical reasoning is a process that purports to obtain a conclusion based on the logical-mathematical premise, and according to facts and relevant sources considered valid [5]. In general, mathematics education is dominated by introductions of formulas and concepts verbally but without sufficient focus on students’ reasoning. Besides, learning-teaching always progresses in a one-way mechanical lecture, with the teacher acting as the focal point of all classroom activities. Students are tasked to listen, ape, or copy in the exact manner what the teacher had given without self-initiative. Students are not given space or prompted to optimize their true potential, develop their reasoning and creativity. In conceptual mastering, students’ sense should be adequately involved to comprehend independent self-learning [6].

Two things that matter as the essence or realistic mathematics is the connection between math and reality and the conception of math as a human activity. [7]. Often, mathematics education is considered separate from students’ personal development. Mathematics education is taken for granted as focusing only on the cognitive aspect, but personal development as part of life skill should ideally be treated as an integral function of all subject lessons in school. Otherwise, this may result in students having a similar pattern of reasoning to that of their teacher. When dealing with specific problems that require realistic thinking, many face trouble to construct them logically. RME is an approach that emphasizes students’ activity in learning mathematics at school to make pupils capable of enlarging their knowledge about mathematical problems relevant to daily life [8].

Putting aside problems that demand reasoning or logic, even when students are confronted with modified questions, many of them stumble and come up with less than acceptable scores. As a result, the dream of setting up a mathematical paradigm remains somewhat distant. One culprit is the low level of awareness among math educators to incite and dig at students’ reasoning skills [9]. Educators are commonly oriented toward and use scoring as a measuring device to assess their pupils’ success after tests are given and all material learned. While in ideal math learning, problem-solving should always remain in focus. Therefore, educators must embark on more experiments by presenting a further problem that can actuate students’ reasoning ability.

However, educators are not the only factor that can explain why pupils’ reasoning skill is relatively weak in math. Interest and personal motivation among some of the students play a role that results in less than optimal effort to develop mathematical reasoning. As a result, when challenged with problems that call for creative rationale and logical thinking, many give up before trying and lack the will to solve the issue.

The result from an interview with teachers also reveals that teachers are often confronted with the fact that most students stumble in solving number stories in practice during mathematical learning in the classroom. Students feel uncomfortable asking for a solution to their difficulties in learning mathematics. Story problems are scary for most; new ideas to solve-math problems are rarely found among pupils, and to many teachers designing and selecting contextual math problems remains difficult.

2. Method
The type of research conducted in this study is qualitative research. Issue under scrutiny in this study pertains students’ difficulty in solving mathematical problem in the form of number stories using realistic mathematics approach that result in error. This research is based on previous small-scale study
conducted among students of Junior High School (SMP) PGRI Kasihan academic year 2017-2018 on 3 November 2010. Population in this limited scale research comprised grade VII students from SMP PGRI Kasihan. Three groups of students were formed as research subjects. Subjects selection was to answer the need for qualitative data set i.e. mistakes done by these pupils in answering number stories. Data collection for analysis purpose was gathered by providing a number of stories to all participating students in groups. Data obtained in this research comprised results from their work.

To facilitate data collection, researchers had developed instruments in the form of story problems on their own. To warrant accurate data, test conducted in this study must meet the criteria of good test. In addition, researchers are considered as one of research instruments and in this particular study, researchers are the intended instrument. As key instrument in qualitative research, researchers can spend considerable amount of time into their field [4]. Data analysis in this research was completed through several stages i.e.: Data Reduction in which raw data were selected, focused, simplified and transformed; Data Display, in which organized and classified data were re-written as to make conclusion possible, and Drawing Conclusion/Verification, in which data were summarized based on all that taking place in data reduction and display stage.

3. Results and discussion

Data analysis is the process of organizing and lining up data into a specific pattern, category, and basic definitions so a theme can be identified and a working hypothesis can be formulated as the data suggest[10]. In this study, data are lined up according to the students’ mistakes pattern committed by the students in working with story problems. As a research subject, It divided into three groups, low, middle, and high score.

The curious gap between them does not come with any explanation whatsoever as to why such a thing might happen. From the table, the score researcher can only see the result of students’ effort on math problems and further analyze that some had difficulties in handling number stories. In line with the research question mentioned, to obtain an answer than can sufficiently explain this phenomenon, discussion and analysis on students’ responses was conducted to explore the place and type of mistakes committed by students or the difficulties of those participating as research subjects.

Analysis of students’ reasoning process in working on story problems using realistic mathematics approach in high score (group 1) can be seen in Figure 1.

![Figure 1. Students’ answer from Group 1](image-url)
From the answer above it was evident that students had tried to figure out the answer by writing several possibilities that may lead to the correct one. On the other hand, the researcher expected a more realistic response that involves them in deeper reasoning.

According to the analysis on students’ answers, the pattern in answer c revealed that students were not accurate and discerning in understanding story problems. In essence, their thinking procedure can be accepted as correct concerning the division of planting space (4 meters), i.e., the number of trees = \( \frac{60}{4} = 15 \) trees

From the above answer, students’ thinking patterns already pointed to the right answer a (64 m\(^2\)) divided by planting space (4 meters), but still, they were not entirely correct as written in answer c. Thus their calculation missed the mark. This proved that students did not read the challenge carefully and grasped that the request was to figure the outside periphery of the pool, and as such, they failed to recheck the question, which led to a false calculation.

If we associate this with Polya steps in solving math problems, the students already understood the question to a certain degree. They could see what it takes to make a correct answer. However, their ability to evaluate solutions is relatively low because they did not appear to conduct a through evaluation of the given solution. This research result is in line with what Agnesti and Amelia found in their previous study i.e., students did not completely understand the essence of the problem and what to seek in number stories. [11].

The test above revealed that research subjects committed mistakes in their interpretation of the known elements in number stories, conceptual mistakes, principal mistakes, troubled in making mathematical sentences, experienced difficulties in understanding problem, did not fully grasp precondition concept and had assumed negative sentiment toward questions with long words (number stories) even before they started working on them.

Summarized results for the type and cause of mistakes committed by research subjects in working on number stories are identified in Table 1 as follows:

| Mistake Location                  | Type of Mistake                  | Cause of Mistake                                                                 |
|-----------------------------------|----------------------------------|----------------------------------------------------------------------------------|
| Known in problem 1                | Concept and principle (point c)  | 1. Did not read problem carefully.                                               |
| Asked in problem 1                | Concept (point c)                | 1. Did not understand the essence of the problem.                                |
|                                   |                                  | 2. Tendency to hesitate/fear to tackle the long-worded problem.                   |
| Creating model/mathematical       | Concept                          | 1. Unable to understand the mathematical sentences.                              |
| sentence                          |                                  | 2. Wrong in interpreting formula and putting into a model or mathematical sentences and mistake in the previous step. |
| Finishing model/mathematical       | Principle, operation, and concept (point c) | 3. Wrong in analyzing the problem.                                               |
| sentence                          |                                  | 1. Unable to create mathematical sentences                                         |
| Figuring the final answer to the problem | Evaluating answer | 2. Did not understand the concept of plane figure.                               |
|                                   |                                  | 3. Did not carefully identify what the problem was requested.                    |
|                                   |                                  | 1. Did not carefully evaluate problem and the final answer.                      |

On the other hand, analysis of students’ reasoning process in working on number story using realistic mathematics approach in group 1 can be seen in Figure 2.
Figure 2. Students’ answer from Group 1

Looking at the answer above, one can see that students already tried to figure the solution using a different way and made the correct choice possible. Analysis of students’ responses revealed that point a differed from the first group; the second group preferred to visualize their solution first by describing the intended problem, and thus opened up space for their creativity to come up with the final answer. Mistake pattern in answer c indicated that students did not care or precisely comprehend the problem. The reasoning process was correct about the division of road’s periphery by planting space (4 meters), but what the students had in mind was the inside periphery—they failed to recheck what the problem requested, which lead to an incorrect pattern in their answer i.e., road’s boundary = 2 (10 + 20) = 60; the number of trees $60 \div 4 = 15$

If we associate this with Polya steps in solving a mathematical problem, it was evident that students had understood the question to a certain degree. Students could grasp what it took to figure the right answer. However, their ability to evaluate is relatively weak because they did not seem to reconsider the answer carefully—whether it can fulfill the request or not.

The result from the test above indicated that research subjects committed mistake in interpreting the known elements in the problem, in concept, in mathematical, had trouble with making mathematical sentences, trouble in understanding the essence of problem, did not fully grasp the concept of precondition and had assumed negative attitude toward long story problem even before they began to work on them.

Summarized results for the type and cause of mistakes committed by research subjects in working on number stories are identified in Table 2.

| Mistake Location | Type of Mistake | Cause of Mistake |
|------------------|----------------|-----------------|
| Known in problem | Concept and principle (point c) | Cann’t write the problem carefully. |
| Asked in problem | Concept (point c) | Tendency to hesitate/fear to tackle the long-worded problem. |
| Creating model/mathematical sentences | Concept | Wrong in interpreting formula and putting into a model or mathematical sentences and mistake in the previous step. |
| Finishing model/mathematical sentence | Principle, operation, and concept (point c) | Unable to create mathematical sentences 1. Did not understand the concept of plane figure. |
Figuring the final answer of problem
Evaluating answer

1. Did not carefully identify what the problem was requested.
2. Did not carefully evaluate the problem and the final answer.

Analysis of students’ reasoning process in working on number story using realistic mathematics approach in the middle score (group 2) can be seen in Figure 3.

![Students’ answer from Middle Score](image)

Figure 3. Students’ answer from Middle Score

Looking at the above answer, it was evident that students had tried to find the correct answer using different approaches, making the right solution possible. Yet analysis of students’ work revealed that answer point a, b, and c differed from that of the first and second group. This resulted in an answer pattern, which was not correct.

There was a mistake pattern in answer a, i.e., and students did not carefully read the problem in which the width of the pool is 1 meter for each periphery; the students did not notice the word periphery, which indicated that they were not careful and correct in understanding story problem as such the students worked only on the inside area. This resulted in answer b, which is correlated with answer a, and if that answer is wrong, answer b would be wrong.

Mistake pattern in answer c indicated that the students were not careful and correct in reading number stories. In essence, their thinking process was valid about dividing road periphery by planting space (4 meters), but what they meant was the inside periphery of the road, and they failed to reconsider the problems’ request, which resulted in an incorrect pattern in their answer i.e., road’s boundary = 2 (10 + 20) = 60, number of trees = 60 : 4 = 15

About Polya steps in solving a mathematical problem, said students already had a proper understanding of the problem’s request, but they suffered from problem planning. Even if they knew what that must be fulfilled to obtain the correct solution, their ability to evaluate answers was weak. As such, they did not attempt to assess the given response, whether it can fulfill the question or not. Mistakes that the students committed in their steps indicated they were not experienced enough in-plane figure concept calculation. As such, the teacher should work harder to ensure their students are well-versed in that concept by creating math problems that can drive students’ creativity in the right direction. The test above also revealed that research subjects committed mistakes in interpreting several elements that are known in the problem, in concept, in mathematical principles, experienced difficulty in forming mathematical sentences, difficulty in understanding the essence of problem, did not fully grasp the
concept of precondition and had assumed negative attitude toward long number story even before they started working on it.

Summarized results for the type and cause of mistakes committed by research subjects in working on number stories are identified in Table 3.

| Table 3. Summarized analysis for Middle Score |
|----------------------------------------------|
| Mistake location                             | Type of Mistake                  | Cause of Mistake                                                    |
| Known in problem                            | Concept and principle             | Did not read the problem carefully                                  |
| Asked in problem                            | concept (point c)                 | 1. Did not understand the problem                                   |
|                                               |                                | 2. Tendency to hesitate/dread to tackle the long-worded problem.     |
|                                               |                                | 3. Did not comprehend mathematical sentences.                       |
| Forming model/mathematical                   | concept                         | 1. Unable to understand mathematical sentences.                      |
| sentences                                     |                                | 2. Wrong in interpreting formula and applying formula into a model   |
|                                              |                                | or mathematical sentences and mistake in the previous step.         |
| Finishing model/mathematic                   | Principle, operation, and        | 3. The mistake in analyzing the problem.                            |
| sentence                                     | concept (point c)                |                                                                    |
| Deciding the final answer for problem        | Evaluating answer                | 1. Unable to form mathematical sentences                            |
|                                              |                                | 2. Did not understand the concept of plane figure.                   |
|                                              |                                | 3. Did not carefully identify what the problem was requested.        |

Analysis of students’ reasoning process in working on a number story using a realistic mathematics approach in the low score (group 3) can be seen in Figure 4.

![Figure 4. Students’ answer from Low Score](image)

Looking at the answer above, it can be concluded that students already tried to figure the right solution using a different approach that made the correct answer possible. Yet, analysis of students’ work revealed that answer points a, b, and c differed from that of the first and second group, resulting in a wrong pattern in their solution.

The mistake pattern in answer indicated that the students did not read the problem carefully, i.e., the pool’s width is 1 meter for each periphery—they did not heed the word “periphery” and merely assumed that only the width side which got longer by 1 meter. This showed that the students were inaccurate or
correct in their interpretation of the problem and thus were prompted to work only on the road’s inside area. This had resulted in answer b, which is correlated to answer a, which means that if the answer is wrong, answer b would be false.

The mistake pattern in answer c indicates that the students were not careful and accurate in reading the problem. As such, they did not calculate division by planting space as required in the situation. This had resulted in an erroneous pattern in their answer.

Suppose we correlate this with Polya steps in solving a mathematical problem. In that case, it can be said the students had some comprehension of the problem, but at the same time exhibited weakness in planning. They might have known what was required to build a correct answer. Yet, their ability to evaluate solutions was relatively low, and they did not attempt to reconsider the given answer, whether it can fulfill the request or not. The students’ mistake in this step only indicated that they were not experienced enough with the concept of calculating a plane figure.

Consequently, teachers should rehearse this concept further by creating math problems that can direct their pupils’ creativity in the right direction. The above test showed that research subjects committed mistakes about the interpretation of known elements in problem, concept, mathematical principles, difficulties in forming mathematical sentences, in understanding the essence of the problem, did not wholly grasp precondition concept and assumed negative attitude toward a problem with a lot of sentences (number story) even before they worked it. This research result is in line with the previous findings by Suherman, which indicated that students still stumbled when they must deal with problems that demand reasoning ability [12].

Summarized results for the type and cause of mistakes committed by research subjects in working on number stories are identified in Table 4.

| Mistake Location                        | Type of Mistake                        | Cause of Mistake                                      |
|-----------------------------------------|----------------------------------------|-------------------------------------------------------|
| Known in problem                        | Concept and principle (point a, b and c) | Did not read the problem carefully                     |
| Asked in problem                        | Concept (point c)                      | 1. Did not understand problem essence                  |
|                                          |                                        | 2. Tendency to hesitate/fear to deal with long number story. |
|                                          |                                        | 3. Unable to understand mathematical sentences.         |
| Forming model/mathematical sentences    | Concept                                | 1. Did not comprehend mathematical sentences.           |
|                                          |                                        | 2. Wrong in interpreting formula and applying formula into a model or mathematical sentences and mistake in the previous step. |
|                                          | Principle, operation, and concept (point c) | 3. A mistake in analyzing the problem                  |
| Finishing model/mathematical sentence.  |                                        | 1. Unable to form mathematical sentences                |
|                                          |                                        | 2. Did not understand the concept of a plane figure     |
|                                          |                                        | 3. Did not carefully identify what the problem was requested. |
| Deciding the final answer to problem.   | Evaluating answer                      | Did not carefully evaluate the problem and final answer |

4. Conclusions
According to the above review on the research result, it can be concluded where the students committed mistakes, the types of their mistakes, the causes of their mistakes in solving problem stories in class VII SMP PGRI Kasihan and how these students experienced difficulties in tackling problems that require
reasoning ability; how they remained less than creative at planning steps or strategy in problem-solving and less than optimal in forming ideas into reasons in their answers.

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