Motif Difference Field: A Simple and Effective Image Representation of Time Series for Classification

Yadong Zhang and Xin Chen
Center of Nanomaterials for Renewable Energy,
State Key Laboratory of Electrical Insulation and Power Equipment,
School of Electrical Engineering,
Xi’an Jiaotong University, Xi’an 710054, Shaanxi, China
{zhangyadong@stu.xjtu.edu.cn, xin.chen.nj@xjtu.edu.cn}

Abstract

Time series motifs play an important role in the time series analysis. The motif-based time series clustering is used for the discovery of higher-order patterns or structures in time series data. Inspired by the convolutional neural network (CNN) classifier based on the image representations of time series, motif difference field (MDF) is proposed. Compared to other image representations of time series, MDF is simple and easy to construct. With the Fully Convolution Network (FCN) as the classifier, MDF demonstrates the superior performance on the UCR time series dataset in benchmark with other time series classification methods. It is interesting to find that the triadic time series motifs give the best result in the test. Due to the motif clustering reflected in MDF, the significant motifs are detected with the help of the Gradient-weighted Class Activation Mapping (Grad-CAM). The areas in MDF with high weight in Grad-CAM have a high contribution from the significant motifs with the desired ordinal patterns associated with the signature patterns in time series. However, the signature patterns cannot be identified with the neural network classifiers directly based on the time series.

1 Introduction

Time series forecasting and classification are important techniques in the understanding of the varieties of dynamics in Science and Engineering. The time series data exist extensively in human daily life and industrial activities, such as health care, transportation, energy, security, finance, etc. Time series analysis including prediction, anomaly detection, and classification and etc. Convolutional neural network (CNN) [Le-cun et al., 1998] has seen a lot of successes in computer vision such as image classification. Beyond CNN, the recent development in neural networks such as Fully Convolution Network (FCN) [Shelhamer et al., 2017], ResNet [He et al., 2016], ScatNet [Bruna and Mallat, 2013], AlexNet [Krizhevsky et al., 2012], and etc. has been applied in the image classification and segmentation. Motivated by the development, particularly the recent applications of deep learning neural networks for the time series classification [Wang et al., 2016] and image encoding of time series [Wang and Oates, 2015] [Eckmann et al., 1987] [Tabar and Halici, 2017]. Based on the raw time series, FCN [Wang et al., 2016] achieves very good performance as classification. The image encoding method Gramian Angular Summation/Difference Fields (GASF/GADF) and Markov Transition Fields (MTF) [Wang and Oates, 2015] are proposed. The Tiled CNN [Le et al., 2010] using GASF/GADF can classify the time series very well while FCN using the raw time series still performs better. The Recurrence Plots [Eckmann et al., 1987] are used for the reconstruction of phase spaces of time series. Also, CNN using the Recurrence Plots [Debayle et al., 2018] is proposed for the classification of time series. The classifier of CNN and the stacked auto-encoder (SAE) using the time-frequency-domain images [Tabar and Halici, 2017] are also proposed to classify EEG Motor Imagery signals.

Although the methods such as Recurrence Plots and GASF/GADF do generate distinct image patterns, it remains unclear how to locate the significant temporal motif patterns with the encoding images of time series. Including the high-order temporal structural information in the image encoding will improve the classification accuracy when the time series has the signature temporal patterns.

Time series motifs are the temporal structures in time series. Temporal shapes and motif occurrence probabilities provide the information for the high-order structure. Permutation entropy [Bandt and Pompe, 2002] based on the time series motif ordinal patterns has been successfully used for time series complexity measurement, dynamic system characterization, stock market analysis, etc. The triadic time series motif is used for the classification [Xie et al., 2019]. In addition,
Symbolic Fourier Approximation (SFA) and Bag-of-Words are the essential components to represent the time series motifs in the Bag-of-SFA-Symbols (BOSS) model [Schafer, 2015].

Having an image encoding scheme to generate a good time series representation can help us learn the structures in the time series. We propose an imaging encoding scheme called Motif Difference Field (MDF) for the time series based on the motifs of diﬀerent lengths, which is simple and easy to implement. We apply FCN to classify the MDF images on 20 datasets of UCR Archive [Chen et al., 2015] and the triadic time series motifs achieve the best performance compared with other image-based neural network classiﬁers. Diﬀerent from the known detection approaches [Ferreira et al., 2006] for the signiﬁcant motif patterns, we identify the motif patterns of time series using the Gradient-weighted Class Activation Mapping (Grad-CAM) [Selvaraju et al., 2017] based on the MDF images and FCN. This can lead to ﬁnding the signature patterns in the time series for the classiﬁcation. A case in the TwoPatterns dataset is used to demonstrate how the Grad-CAM can extract the signiﬁcant motif patterns from the time series. Furthermore, it can make us comprehend why the MDF image works for time series classiﬁcation.

2 Motif Difference Field

We introduce the simple motif-based framework for encoding time series as images. Given a time series \(X = (x_t, t = 1, 2, 3, \ldots, T)\), the sequence of the length \(n\) time series motifs is deﬁned as

\[
M^n_s = \{M^n_s, s = 1, 2, 3, \ldots, T - (n + 1)\}
\]

where \(M^n_s\) is the length \(n\) time series motif, \(1 < n \leq T\), \(s\) is the initial time index of the motif and \(M^n_s = (x_t, t = s, s + 1, s + 2, \ldots, s + (n - 1))\). Typically, \(M^n_1 = (x_s, x_{s + 1})\) is the dual motif, \(M^n_2 = (x_s, x_{s + 1}, x_{s + 2})\) the triadic motif (triad), and \(M^n_4 = (x_s, x_{s + 1}, x_{s + 2}, x_{s + 3})\) the quad motif.

In the current deﬁnition of the motifs, the displacement of the neighboring points in motifs is unit 1. The large displacements are also important in detecting the temporal structural patterns of time series in the long-range. By generalizing the motifs with arbitrary displacements, the sequence of generalized motifs is deﬁned as

\[
M^n_{d,s} = \{M^n_{d,s}, s = 1, 2, 3, \ldots, T - (n - 1)d\}
\]

where \(M^n_{d,s} = (x_t, t = s, s + d, s + 2d, \ldots, s + (n - 1)d)\), \(d\) is the integer displacement and \(1 \leq d \leq d_{\text{max}}\) where \(d_{\text{max}} = \lfloor (T - 1)/(n - 1) \rfloor\).

The ordinal patterns of motifs are used in identifying the complexity of time series based on comparing the neighboring values [Bandt and Pompe, 2002]. In addition to the ordinal patterns, the relative amplitude in motifs have more structural information. Hence, the collection of the sequences of motif diﬀerences are deﬁned as

\[
dM^n_{d,s} = \{dM^n_{d,s}, s = 1, 2, 3, \ldots, T - (n - 1)d\}
\]

where \(dM^n_{d,s} = \{x_{s + d} - x_s, x_{s + 2d} - x_{s + d}, \ldots, x_{s + (n - 1)d} - x_{s + (n - 2)d}\}\). Since the lengths of \(dM^n_{d,s}\) are diﬀerent, a new sequence \(I^n_{d,s}\) is constructed. By assigning \(I^n_{d,s}\) to be \(dM^n_{d,s}\) initially, \(I^n_{d,s}\) is appended with \(0\) recursively until \(T - (n - 1)\), where length of \(0 = \{0, 0, 0, \ldots, 0\}\) is \(n - 1\). In other words, \(I^n_{d,s} = \{I^n_{d,s}, s = 1, 2, 3, \ldots, T - (n - 1)\}\) is denoted where

\[
I^n_{d,s} = \begin{cases} dM^n_{d,s}, & \text{if } 1 \leq s \leq T - (n - 1)d \\ 0, & \text{if } T - (n - 1)d < s \leq T - (n - 1) \end{cases}
\]

Then, the motif diﬀerence ﬁeld (MDF) is deﬁned as the collection of \(I^n_{d,s}\) with diﬀerent displacement \(d\),

\[
\text{MDF}^n = \{I^n_{d,1}, I^n_{d,2}, \ldots, I^n_{d,\text{max}}\}
\]

For the motifs of length \(n\), the corresponding MDF has \(n - 1\) elements in \(I^n_{d,s}\). Therefore, we can generate \(n - 1\) channel images accordingly. For \(i\)th channel, the array of the image is deﬁned as,

\[
G^n_i = [I^n_{d,1}(i), I^n_{d,2}(i), \ldots, I^n_{d,T-n+1}(i)]^T
\]

where \(\tilde{I}^n_d(i) = [I^n_{d,1}(i), I^n_{d,2}(i), \ldots, I^n_{d,T-n+1}(i)]^T\) and \(1 \leq i \leq n - 1\). In order to ﬁx the plenty of zeros in \(G^n_i\), each channel of MDF image is deﬁned as,

\[
\text{IMG}^n_i = G^n_i + K^n \odot G^n_i
\]

where \(\odot\) is Hadamard product, \(G^n_i\) is the 180 degree rotation of \(G^n_i\). \(K^n\) is deﬁned as a masker to prevent the overlap of two array,

\[
K^n_{d,s} = \begin{cases} 0, & \text{if } 1 \leq s \leq T - (n - 1)d \\ 1, & \text{if } T - (n - 1)d < s \leq T - (n - 1) \end{cases}
\]

Figure 1 shows the procedure of encoding the time series into two triadic MDF images.

3 Classify Time Series Using MDF with FCN

We apply the Fully Convolution Network (FCN) to classify the time series using MDF images, named as MDF-FCN. Our classiﬁcation method is evaluated on 20 datasets from [Chen et al., 2015]. On pre-split testing datasets, we compare the error rate of MDF-FCN with 3 popular classiﬁcation methods and 3 competing neural network classiﬁcation methods: 1NN classiﬁer based on DTW with best wrapping.
### 3.1 Fully Convolution Network

FCN has shown strong performance and efficiency in image segmentation given the pixel-to-pixel category-wise semantic annotation [Shelhamer et al., 2017] and time series classification [Wang et al., 2016] by extracting the features from the 1-D receptive fields.

In our problem setting, FCN is structured as the 2-D features extractor of the MDF image. As shown in Figure 2, the basic block is convolutional layers followed by ReLU activation and batch normalization (BN) layer,

\[
A_{z+1} = \text{ReLU}(BN(W_z \otimes A_z + b_z))
\]

where \( \otimes \) is the convolution operator, \( 0 \leq z \leq 3 \), \( A_0 \) is the input MDF image, \( A_z \) is the feature map of the \( z \)th convolutional layer if \( z \neq 0 \). The classification output \( y \) comes from the global average pooling layer (GAP) of \( A_3 \) followed by softmax layer,

\[
h = \text{GAP}(A_3)
\]

\[
y = \text{Softmax}(W_y h + b_y)
\]

where \( y \) is a size \( C \) vector, \( C \) is the classes number of the dataset.

The three convolutional layers in FCN have 2-D receptive fields \( \{8 \times 8, 5 \times 5, 3 \times 3\} \) and the filter size \( \{128, 256, 128\} \). Moreover, the strides of convolution operators affect the efficiency and the overlap size of receptive fields, our experiment conducts the cross-validation to provide optimal strides \( \{u_1 \times u_1, u_2 \times u_2, u_3 \times u_3\} \).

### 3.2 Experiment Settings

In our experiment, we use three different MDF images for FCN with the lengths \( n \in \{2, 3, 4\} \). We test the FCN classifier using MDF images on the 20 univariate time series datasets [Chen et al., 2015]. All the datasets have been split into training and testing by default. Each dataset is preprocessed with minmax normalization with the minimum and maximum value of its training datasets. Taking categorical cross entropy as loss function, FCN is trained with Adam [Kingma and Ba, 2014] at the learning rate 0.001, \( \beta_1 = 0.9 \) and \( \beta_2 = 0.999 \). The strides of FCN, \( \{u_1, u_2, u_3\} \), are selected to be \( \{8, 5, 3\}, \{4, 2, 2\}, \{2, 2, 2\}, \{3, 2, 1\} \) according to 4-fold cross-validation on the training dataset. Finally, we
choose the best model that achieves the lowest training loss and report its performance on the test dataset in Table 1.

3.3 Results and Analysis

We benchmark MDF-FCN with other 6 classifiers. The classification error rates on testing datasets are reported in Table 1. The motifs of small lengths, such as \{2, 3, 4\} in the Table 1, can allow to see the long range patterns due to the large possible displacements. Our results show that the triadic MDF performs better than the dual and quad in general.

Benchmarking with other six classifiers, FCN based on triadic MDF gives the smallest average of error rate. Compared to 3 popular methods, the triadic MDF-FCN performs better on 11 datasets since the high-order patterns of the time series encoded in the MDF images can be not identified by the distance-based and feature-based methods. Compared to the two other image encoding classification methods, triadic MDF-FCN performs better on other 11 datasets and the image encoding in MDF doesn’t need complicated parameters and processing. The previous work [Wang et al., 2016] shows that TS-FCN classification method can achieve a nice performance only with the raw time series data. However, TS-FCN can’t extract or detect the high-order temporal patterns while MDF-FCN can identify the significant motif ordinal patterns. That’s why MDF-FCN outperforms TS-FCN. This will be demonstrated in the next section.

4 Identify Significant Motif Ordinal Patterns

The Gradient-weighted Class Activation Mapping (Grad-CAM) [Selvaraju et al., 2017] is a technique to produce a visual explanation to varieties of CNN model-families. In the paper, we will apply Grad-CAM to detect the significant motifs that are associated with the signature patterns in time series classification. The Grad-CAM of FCN using MDF images can be generated by the gradient-based weighted combinations of the feature maps of the last convolutional layer A_k in Figure 3.

\[ L^c = \text{ReLU} \left( \sum_k (\alpha_k A^k_3) \right) \] (12)

where c is the target class of time series, k is channel index of the feature map and \( \alpha_k = \text{GAP} (\partial y^c / \partial A^k_3) \).

4.1 Symmetrized Grad-CAM

Given the MDF images of the time series in the target class, we calculate the Grad-CAM, \( L^c \), of the last FCN convolutional layer based on Equation 12. The image of Grad-CAM will be up-sampled to the size of \( d_{max} \times [T - (n - 1)] \). For simplicity, the up-sampled Grad-CAM is named as \( L^c_{d,s} \). \( L^c_{d,s} \) gives the significance of the motif difference \( dM^n_{d,s} \) in the MDF images of time series of class c. Due to the symmetry in the MDF images as constructed, we enforce the same symmetry by defining the new symmetrized Grad-CAM \( L_{d,s}^c \) with \( (L^c_{d,s} + L^c_{d',s'}) / 2 \) where \( (d,s) \) and \( (d',s') \) are the two array indices according to the 180 rotation symmetry in \( G^n \) and masked \( G^{n'} \). Figure 3d shows a time series instance in TwoPatterns testing dataset. Taking the instance as an example, Figure 3a shows the two MDF channel images. Figures 3c and 3d show the original Grad-CAM and symmetrized Grad-CAM of the MDF image respectively.

4.2 Significant Motif Ordinal Patterns

It is helpful to explain the efficiency of MDF image classification with the symmetrized Grad-CAM. Based on the symmetrized Grad-CAM, we can assign significance to the motifs. We categorize the motifs of length n according to the ordinal patterns \( \pi^n_j, j = 1, 2, 3, \ldots, m \) [Traversaro et al., 2019]. For dual time series motifs, there are three ordinal patterns; for the triadic time series motifs, there are thirteen ordinal patterns; for the quad time series motifs, there are seventy-three ordinal patterns. Since the triadic MDF-FCN gives the best results in Table 1, we will focus on the thirteen ordinal patterns of \( \pi^3_j \).

\[ \pi^3_j \in \{111, 112, 113, 122, 123, 132, 211, 213, 221, 231, 311, 312, 321\} \] (13)

Accordingly, the collection of the indices of all the \( \pi^n_j \) motifs in \( MDF^n \) is defined as,

\[ MC_j = \{(d,s) | M^n_{d,s} \in \pi^n_j, 1 \leq s \leq T - (n - 1)d, 1 \leq d \leq d_{max}\} \] (14)

The contribution of \( \pi^n_j \) motif ordinal patterns can be evaluated according to \( E_j \),

\[ E_j = \frac{1}{Z_j} \sum_{MC_j} L^n_{d,s} \] (15)

where \( Z_j \) is the length of \( MC_j \). \( E_j \) can be regarded as the significance of \( \pi^n_j \) ordinal patterns in the MDF image.

Based on \( E_j \), we can get the ranking of the significance of \( \pi^n_j \) patterns,

\[ E_{v(1)} \geq E_{v(2)} \geq \cdots \geq E_{v(m)} \] (16)

where v is the mapping of the monotonic ascending index of 1 to m to the index of \{MC_{j}, j = 1, 2, 3, \ldots, m\}. We decide which ordinal patterns of the motif are the most important according to the \( E_j \) ranking. Given \( MC_j \), we can evaluate the contributions of the motifs of \( \pi^n_j \) in the symmetrized Grad-CAM.

4.3 MDF-FCN vs. TS-FCN

In Table 1 the triadic MDF images give the best results. Figure 3 show the time series instance of TwoPatterns dataset and its triadic MDF images and Grad-CAM heat-map. In the TwoPatterns dataset, all the time series have the four plateaus which serve as the unique signatures for the classification. We can rank ordinal patterns of triadic motifs according to Equation 16. Figure 4 shows the significance of thirteen \( \pi^3_j \) motif ordinal patterns accordingly. It is found that the top two significant motif ordinal patterns are \{112, 211\} corresponding to the order of the four plateaus as shown in Figure 3a. Furthermore, the two ordinal patterns correspond to the hottest peaks in the symmetrized Grad-CAM heat-map as shown in Figure 3d. At the same time, Grad-CAM for the
TS-FCN classifier [Wang et al., 2016] can be calculated, and up-sampled to the size $T$. Since TS-FCN is fed with the raw univariate time series directly, the Grad-CAM in Equation 12 is the significance of the individual data points in the time series. Figure 3 shows the significance of the data points. The TS-FCN classifier puts much more weight in the fluctuating parts instead of four signature plateaus. This explains why MDF-FCN classifier performs much better than TS-FCN classifier when the time series has the signature of a higher-order structure.

Figure 3: The Grad-CAM heat-maps of triadic MDF images of the time series instance in the TwoPatterns dataset. Plot (a) shows the time series instance, Plot (b) the triadic MDF images, Plot (c) the Grad-CAM heat-map and Plot (d) The Symmetrized Grad-CAM heat-map. The two motif patterns of 112 and 211 in Plot (a) are the two peaks in Plot (d) and are the top two significant ordinal patterns according to Equation 16.

Figure 4: Significance of triadic time series motif ordinal patterns $\pi_j$. The bottom panel shows thirteen triadic time series ordinal patterns.
Figure 5: Significance of the data points of the time series calculated by the Grad-CAM of TS-FCN. The bottom panel shows the original time series.

5 Conclusion and Future Works

We propose a framework for encoding time series into MDF images and use FCN as the 2-D feature extractor for classification. Our benchmark study shows that the triadic MDF based on FCN achieves the best results for classification compared to other classification methods. With the help of Grad-CAM, we identify the significant motif ordinal orders that associated with the signature patterns in time series. Our analysis suggests that the MDF images encoding has the following advantages:

1. MDF images represent the motif-based temporal structural patterns of time series at the long and short ranges. From this point, it is a more effective classifier than TS-FCN.

2. MDF images are much simpler than the Recurrence Plots and the GASF/GADF-MTF.

3. Grad-CAM based on MDF-FCN can identify the significant ordinal patterns in time series. It helps us comprehend and analyze the time series classification from the motif perspective.

There is more for us to do to explore how to make the MDF-FCN classifier more efficient in the classification. The future work will include investigating how the lengths of motifs affect the performance of classification. There is more work for us to test the new image classification neural networks in the classification using the image encoding of time series. In addition, how to use the MDF-FCN method for the identification of the higher-order temporal structure in the time series is a promising application. How to classify time series using the motifs of different lengths demands more understanding of the current MDF images. The MDF-FCN framework can be extended for the time series prediction and anomaly detection.

Acknowledgments

Xin Chen acknowledges the financial support from the National Natural Science Foundation of China (Grant No. 21773182 (B030103)).
References

[Bagnall et al., 2017] A. Bagnall, J. Lines, A. Bostrom, J. Large, and E. Keogh. The great time series classification bake off: a review and experimental evaluation of recent algorithmic advances. Data Mining and Knowledge Discovery, 31:606–660, 2017.

[Bandt and Pompe, 2002] Christoph Bandt and Bernd Pompe. Permutation Entropy: A Natural Complexity Measure for Time Series. Physical Review Letters, 88(17):174102, April 2002.

[Bruna and Mallat, 2013] J. Bruna and S. Mallat. Invariant scattering convolution networks. IEEE Transactions on Pattern Analysis and Machine Intelligence, 35(8):1872–1886, Aug 2013.

[Chen et al., 2015] Yanping Chen, Eamonn Keogh, Bing Hu, Nurjahan Begum, Anthony Bagnall, Abdullah Mueen, and Gustavo Batista. The ucr time series classification archive, July 2015. www.cs.ucr.edu/~eamonn/time_series_data/

[Debayle et al., 2018] Johan Debayle, Nima Hatami, and Yann Gavet. Classification of time-series images using deep convolutional neural networks. In Jianhong Zhou, Petia Radeva, Dmitry Nikolayev, and Antanas Verikas, editors, Tenth International Conference on Machine Vision (ICMV 2017), page 23, Vienna, Austria, April 2018. SPIE.

[Eckmann et al., 1987] J.-P. Eckmann, S. Oliffson Kamphorst, and D Ruelle. Recurrence plots of dynamical systems. Europhysics Letters (EPL), 4(9):973–977, nov 1987.

[Ferreira et al., 2006] Pedro G. Ferreira, Paulo J. Azevedo, Cundefinedndida G. Silva, and Rui M. M. Brito. Mining approximate motifs in time series. In Proceedings of the 9th International Conference on Discovery Science, DS’06, page 89–101, Berlin, Heidelberg, 2006. Springer-Verlag.

[He et al., 2016] K. He, X. Zhang, S. Ren, and J. Sun. Deep residual learning for image recognition. In 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 770–778, June 2016.

[Kingma and Ba, 2014] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization, 2014. cite arxiv:1412.6980Comment: Published as a conference paper at the 3rd International Conference for Learning Representations, San Diego, 2015.

[Krizhevsky et al., 2012] Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural networks. In Proceedings of the 25th International Conference on Neural Information Processing Systems - Volume 1, NIPS’12, page 1097–1105, Red Hook, NY, USA, 2012. Curran Associates Inc.

[Le et al., 2010] Quoc V. Le, Jiquan Ngiam, Zhenghao Chen, Daniel Chia, Pang Wei Koh, and Andrew Y. Ng. Tiled convolutional neural networks. In Proceedings of the 23rd International Conference on Neural Information Processing Systems - Volume 1, NIPS’10, page 1279–1287, Red Hook, NY, USA, 2010. Curran Associates Inc.

[Lecun et al., 1998] Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition. Proceedings of the IEEE, 86(11):2278–2324, Nov 1998.

[Lines et al., 2012] Jason Lines, Luke M. Davis, Jon Hills, and Anthony Bagnall. A shapelet transform for time series classification. In Proceedings of the 18th ACM SIGKDD international conference on Knowledge discovery and data mining - KDD ’12, page 289, Beijing, China, 2012. ACM Press.

[Schäfer, 2015] Patrick Schäfer. The BOSS is concerned with time series classification in the presence of noise. Data Mining and Knowledge Discovery, 29(6):1505–1530, November 2015.

[Selvaraju et al., 2017] Ramprasaath R. Selvaraju, Michael Cogswell, Abhishek Das, Ramakrishna Vedantam, Devi Parikh, and Dhruv Batra. Grad-cam: Visual explanations from deep networks via gradient-based localization. In The IEEE International Conference on Computer Vision (ICCV), Oct 2017.

[Shelhamer et al., 2017] Evan Shelhamer, Jonathan Long, and Trevor Darrell. Fully convolutional networks for semantic segmentation. IEEE Trans. Pattern Anal. Mach. Intell., 39(4):640–651, April 2017.

[Tabar and Halici, 2017] Yousef Rezaei Tabar and Ugur Halici. A novel deep learning approach for classification of EEG motor imagery signals. J. Neural Eng., page 12, 2017.

[Traversaro et al., 2019] Francisco Traversaro, Nicolás Ciarrocchi, Florencia Pollo Cattaneo, and Francisco Redelico. Comparing different approaches to compute permutation entropy with coarse time series. Physica A: Statistical Mechanics and its Applications, 513:635–643, 2019.

[Wang and Oates, 2015] Zhiguang Wang and Tim Oates. Imaging time-series to improve classification and imputation. In Proceedings of the 24th International Conference on Artificial Intelligence, IJCAI’15, page 3939–3945. AAAI Press, 2015.

[Wang et al., 2016] Zhiguang Wang, Weizhong Yan, and Tim Oates. Time Series Classification from Scratch with Deep Neural Networks: A Strong Baseline. arXiv:1611.06455 [cs, stat], November 2016. arXiv:1611.06455.

[Xie et al., 2019] Wen-Jie Xie, Rui-Qi Han, and Wei-Xing Zhou. Time series classification based on triadic time series motifs. arXiv:1901.00110 [physics], January 2019. arXiv: 1901.00110.

[Xu et al., 2014] X. Xu, S. Huang, Y. Chen, K. Browny, I. Halilovic, and W. Lu. Tsaaas: Time series analytics as a service on iot. In 2014 IEEE International Conference on Web Services, pages 249–256, June 2014.