Comparison between two and three-dimensional POD in a turbulent boundary layer using multi-plane stereoscopic PIV

Alex Liberzon\textsuperscript{1}, Roi Gurka\textsuperscript{2} and Gad Hetsroni\textsuperscript{3}

\textsuperscript{1} School of Mechanical Engineering, Tel Aviv University, Ramat Aviv 69978, Israel
\textsuperscript{2} Department of Chemical Engineering, The Ben Gurion University of the Negev, Beer Sheva, Israel
\textsuperscript{3} Faculty of Mechanical Engineering, Technion, Haifa 32000, Israel
E-mail: alexlib@eng.tau.ac.il

Abstract. A comparison between two- and three-dimensional analyses using proper orthogonal decomposition (POD) is performed. The investigated flow field is a turbulent boundary layer in a flume. The decomposition is applied to the vorticity fields measured using a multi-plane stereoscopic particle image velocimetry (PIV) measurement system. The decomposition was applied using two methods: A) two-dimensional slices of the data that were used separately in a so-called slice-POD, and B) as a volumetric dataset that provides 3D-POD modes. Linear combination of the first three modes, energy distribution and reconstruction of snapshots are compared. Both decompositions capture most of the turbulent flow patterns; yet, the lower order modes show significant discrepancies between the slice-POD and 3D-POD. Therefore, in order to characterize coherent structures in turbulent flows, it is essential to perform both two- and three-dimensional decompositions. These two methods complement each other and can provide an improved interpretation of various flow features.

1. Introduction

Turbulent flows, being a subject of utmost technological importance, have attracted an enormous body of scientific research. Despite the tremendous effort, some aspects of the flow remain unsolved, partially because of the interaction of an infinite number of spatial and temporal scales. For example, flows that were traditionally solved as quasi-two-dimensional problems, e.g. planar jets, wakes or turbulent boundary layers over flat surfaces, require the fully three dimensional description when coherent structures are of concern (Robinson, 1991). Therefore, there is a demand for experimental methods that will provide a full description of the flow. In order to fully characterize the flow field, one will have to measure the velocity and velocity gradients fields over a broad range of scales in space and time. Although 3D experimental methods are under active development, they are still limited in either temporal or spatial resolution (Tropea \textit{et al.}, 2007; Raffel \textit{et al.}, 2007; Arroyo & Hinsch, 2008). Furthermore, eduction schemes that will allow an objective assessment of the coherent patterns in a complex turbulent environment are needed.

Among the tools available to analyse multi-scale results and identify the coherent features, we choose to use proper orthogonal decomposition (POD) technique. It was proposed by Lumley...
(1970) and further developed for the analysis of coherent structures in turbulent flows by Berkooz et al. (1993); Holmes et al. (1996) among others. POD was successfully applied to experimental data using correlation tensors of either velocity or vorticity (e.g. Gurka et al., 2006; Liberzon et al., 2005). The analysis extracts from the correlation tensor the information about the most “energetic” features, i.e. those that contributed a significant part to the total variance (correlation) included in the tensor. These features are referred to as coherent structures, since they are highly correlated and contain most of the “energy” of the flow. Using the three-dimensional experimental methods it is possible to construct multiple slices of the full correlation tensor and investigate the coherent patterns in three dimensions (Liberzon et al., 2005; Gurka et al., 2006) and in some cases in time Diamessis et al. (2010). The authors in Brand et al. (2004) performed the 3D-POD analysis to the experimental data in the mixing layer, obtained using the dual-plane PIV (two single-plane PIV systems co-aligned in space), in addition to the single-plane POD (entitled by the authors “slice-POD”). The authors have not compared between the two approaches. The reader may notice that the use of slice-POD disregards the spatial correlation between the decomposed planes. Similar analysis applied to the numerical simulations results was carried by Pastur et al. (2005). The authors performed fully resolved 3D direct numerical simulation (DNS) of an open cavity flow and applied POD to 2D snapshots. The authors argued that 2D slices of the flow field are sufficient to extract significant space-time events out of the flow. This statement appears to be correct for the open cavity flow studied numerically in Pastur et al. (2005), but it raises the question whether this statement can be generalized and applied to other turbulent flows. In this manuscript, this question will be examined.

It is our intention to demonstrate the apparent discrepancies between using 2D and 3D decompositions in characterizing turbulent structures. In turbulent boundary layers, the most common types are the low-speed streaks and the bursting phenomena. The use of stereoscopic multi-plane PIV in conjunction with the POD analysis of vorticity enabled us to observe part of the three dimensional features. In the present work we apply the approach of Liberzon et al. (2005) that characterized the coherent structures through the POD of the vorticity field in DNS of the turbulent boundary layer. In Gurka et al. (2006) similar approach was applied to the experimental data in an open flume. Lately it was also applied to the stably stratified turbulent flow by Diamessis et al. (2010). In the following, a brief description of the experimental set-up and the applied POD method is presented, along with the comparison between 2D and 3D POD analysis of a multi-plane experimental data.

2. Experimental setup

The experiments were performed in a turbulent boundary layer flow with Reₜ = 2 × 10⁴ (based on the flow height, h and the mean velocity, U) in a flume with dimensions of 4.9 × 0.3 × 0.1 meters, as shown in Figure 1a. A detailed description of the flume is given elsewhere (Liberzon et al., 2004; Gurka et al., 2004), all the necessary precautions were applied to reproduce the same experimental conditions as in Hetsroni et al. (1997). For example, grids in the inlet tank, and baffles were installed in the pipe portion of the tank, the inlet to the channel was a converging channel in order to have a smooth entrance, and the 0.75 HP, 60 RPM centrifugal pump was isolated from the system by means of rubber joints fitted to the intake and discharge pipes). The flow rate was continuously monitored by an accurate flowmeter (0.5% of the measured flow rate). In order to maintain the same height of the flow along the flume, an array of cylinders were placed at the outlet portion. The measurements have been performed with treated and filtered tap water.

A detailed overview of the multi-plane stereoscopic PIV experimental method, entitled XPIV, is shown in Liberzon et al. (2004) and a brief description is given here. The method is based
on an optical system that forms three parallel laser sheets, shown in Figure 1b. The distance between the light sheets and their intensities can be varied. The PIV system is composed of a double pulsed Nd:YAG laser (170 mJ/pulse, 15 Hz) operating at 532 nm. Two CCD cross correlation cameras (8 bit, 1024 × 1024 pixels), acquire simultaneously the images of the flow field in a stereoscopic configuration, as it is presented in Figure 1a. The calibration procedure and PIV cross-correlation analysis are performed using open source PIV software (www.openpiv.net), with 64 x 64 pixels interrogation areas and 50% overlapping. The spatial resolution of the camera is 80 μm per pixel, which yields a field of view of approximately 80 x 80 mm². The analysis produced 1000 vectors in each given field of view, filtered using standard median and global outlier filters. During the post processing analysis, 5% of the vectors were found erroneous. These vectors were removed and the gaps were filled with linear interpolations of the nearest neighbouring points. The extensive image processing allows to separate images acquired from the particles as imaged in the three different measurement planes (Liberzon et al., 2004). This provides sufficient information to construct a multi-plane velocity vector field with three velocity components per each grid point.

![Figure 1. (a) Schematics of the test rig: 1) the flume, 2) water tanks, 3) pipeline, 4) double Nd:YAG laser, 5) optical table, 6) camera support frame, 7,8) CCD cameras, 9) 45 deg. mirror, 10) quarter-plate, 11) three-plane variable intensity beamsplitter, 12) cylindrical lens, 13) three parallel laser sheets. (b) Optical setup: 1) spherical lens, 2) 45 deg. mirror, 3) quarter-wave plate in a rotational mount, 4,5) cube beamsplitters, 6) half-cube mirror, 7) cylindrical lens and 8) three parallel laser sheets.](image)

### 3. Proper orthogonal decomposition

The proper orthogonal decomposition (POD) provides a basis for the modal decomposition of an ensemble of functions, such as data obtained by numerical simulations (Holmes et al., 1996). POD provides an efficient way to capture the dominant components of turbulence (in terms of energy) using the lowest number of empirical modes or eigenfunctions. We consider a given ensemble of realizations \( \{ f^k \} , k = 1 \ldots N \) of a field \( f(x) \). POD will find the most dominant features of this field using the eigenfunctions \( \{ \phi^n \} , n = 1 \ldots N \) of the two-point correlation
tensor $R(x, x') = \langle f(x) \cdot f^*(x') \rangle$ ($^*$ denotes the conjugate) in the following manner, according to Berkooz et al. (1993):

$$\int R(x, x') \phi(x') dx' = \lambda \phi(x) \quad (1)$$

Applying the full set of eigenfunctions $n = 1 \ldots N$ and their respective coefficients $a^k_n$ (which are in fact the square roots of the eigenvalues $\lambda^n$) one can reconstruct any of the realizations of the field $\{ f^k \}$, as a linear combination of modes:

$$f^k(x) = \sum_{n=1}^{N} a^k_n \phi^n(x) \quad (2)$$

Therefore, a lower-order model of the field can be reconstructed using $K$ modes. Typically the most energetic modes (with the highest $\lambda^n$) are used:

$$\hat{f}^k(x) = \sum_{n=1}^{K} a^k_n \phi^n(x) \quad (3)$$

Sirovich (1987) developed a more efficient “snapshots” method to decompose the modes. In the present work we apply this method to the vorticity fields, $f = \omega$. For the three-dimensional analysis, the $k_{th}$ snapshot is defined based on a set of $N_x \times N_y \times N_z$ grid points and expressed as a vector of the length $n = N_x \times N_y \times N_z$ as follows:

$$\omega = [\omega^1 \ldots \omega^n]^T \quad (4)$$

For this case, a symmetric matrix substitutes for the two-point correlation tensor:

$$C = \langle (\omega - \langle \omega \rangle) \cdot (\omega - \langle \omega \rangle) \rangle \quad (5)$$

The efficiency of this method increases when the number of snapshots $M$ is significantly smaller than the size of the simulation grid. In order to recover the POD modes, we use the following relation between the eigenfunctions $\psi$ of matrix $C$ and the POD modes:

$$C \cdot \psi = \lambda \cdot \psi \quad (6)$$

Thus we obtain:

$$\psi(n) = \sum_{j=1}^{M} \psi^{(n)}_j \cdot \omega^j \quad (7)$$

Here $\psi^{(n)}_j$ denotes the $j_{th}$ element of the eigenvector $\psi$, corresponding to the $j_{th}$ eigenvalue $\lambda^{(n)}$.

Using the vorticity field instead of the velocity field enables us to obtain improved spatial characterization of the dominant features, as shown by Liberzon et al. (2005); Gurka et al. (2006). The proposed method of using a linear combination of the first (most energetic) POD modes resembles the concept of the “characteristic eddy” (e.g. Lumley, 1970). A similar approach was applied to reveal the large scale structures in free shear turbulent flows in Gordeyev & Thomas (2002):

$$\hat{\omega}(x) = \sum_{n=1}^{K} \lambda^{(n)} \phi^{(n)}((x)), \quad K \ll N \quad (8)$$
Figure 2. Cumulative energy of the POD modes using the 3D-POD (solid line) and the slice-POD for separate planes and their summation (dashed-dotted line).

In the present work we perform the POD analysis using two methods: a) two- and b) three-dimensional analysis. For the two-dimensional POD analysis, using each plane separately, the snapshots are three two-dimensional slices, each on the $N_x \times N_y$ grid. The decomposition was applied to the wall-normal component of vorticity, $\omega_y$, which is straightforward to estimate and present in these planes. For the three-dimensional analysis, the snapshots of the wall-normal vorticity component were used on a grid of $N_x \times N_y \times N_z$, for consistency. The POD modes are sorted according to their “energetic” contribution which is, in the case of vorticity, the enstrophy contribution. Therefore, the deduced features are the optimal representation of the vorticity field.

4. Results and discussion

The main focus of this communication is the comparison between two-dimensional and three-dimensional analysis of POD technique, as applied to the flow in the flume.

We observe that the “energy” distribution among the modes, as shown in figure 2 of the slice-POD and the 3D-POD is different. A plausible explanation is based on the observation that the coherent structures that populate the boundary layer are three dimensional, therefore, the amount of energy in each respective mode is proportional to the footprint of the structure in the investigated cross-section. For the volumetric analysis, the relative energy will be comprised of energy contained in the resolved scales, yet proportional to the whole structure. Whilst the two-dimensional slices, once decomposed individually, can at most represent the footprints appearing at the investigated slice. Among the three planes, one can observe that the middle plane exhibit more “energy” as compared to the other two planes. This is consistent with the results shown in Gurka et al. (2006) where the middle plane is located in the buffer region of the flume boundary layer. The buffer region is the portion of the velocity profile $U(y)$ which is associated with high values of vorticity and a peak in the turbulent kinetic energy production. The summation of the “energy” from the three planes appears in Figure 2 to be higher than the “energy” included in
Figure 3. Comparison of the wall-normal enstrophy component $\omega_y^2$ obtained using a 3D POD (a) versus the field obtained three single-plane slice-POD (b).

the 3D-POD modes. Therefore, each separate plane can be decomposed into smaller number of strong modes, providing a lower-order model or less complexity as compared to the 3D dataset. A similar conclusion is evident in figure 3 representing the sum of the first three modes in the 3D-POD (left panel) and slice-POD (right panel). The middle plane presents somewhat similar patterns, however, the upper and lower planes are significantly different (note that the same color scale is used for the two panels). The footprints of large scale motions that are visible in the first modes of the separate planes do not contribute to a 3D coherent feature which has a strong representation in the middle slice only. One can infer that the POD modes obtained in the separate plans lack the necessary spatial correlation between the planes, and in consequence overestimate the local features. This might lead to misinterpretation of some important flow features.

In order to verify that the decomposition approach using two- and three-dimensional settings are valid, we have reconstructed the vorticity field of an arbitrary snapshot using both sets of modes. The comparison is shown in figure 4 for the reconstructed snapshot using 10, 50 and 150 modes. The results are in agreement with the “energy” distribution shown above, as the number of modes required to obtain an accurate reconstruction close to the original snapshot is lower for the separate plane modes, as compared to the 3D-POD case. The reconstructed snapshot exhibits strong vortical patterns in the middle plane, resembling the modes shown in figure 3. These patterns are successfully reconstructed using 50 modes in the middle plane. However, in order to reconstruct the pattern at all vertical positions, it seems that at least 150 modes are required. It is noteworthy that the lowest order model of the snapshot using 10 modes in the slice-POD analysis might lead to an overestimate of the activity in the lowest plane (closest to the boundary) with respect to the middle plane (buffer region).

5. Summary and conclusions

The multi-plane stereoscopic particle image velocimetry measurement system (XPIV) was utilized to measure the boundary layer in a flume. Three-component velocity vector fields at three parallel streamwise - spanwise planes with a fixed distance between them were obtained. The proposed analysis emphasizes the necessity of the three-dimensional measurements in turbulent boundary layers. We calculate the POD modes of the wall normal component of the vorticity fields. This procedure was performed using two methods: A) two-dimensional slices of the data were used separately in a so-called slice-POD, and B) the results are utilized...
as a volumetric dataset to obtain the 3D-POD modes. Both decompositions capture all the essential information of the underlying flow features, as shown by the snapshots reconstruction. However, we demonstrate that the flow patterns based on the combination of the lower order modes shows discrepancies between the slice- and 3D-POD. Therefore, the interpretation of the topological features deduced from the POD modes has to be approached with care. We conclude that the two-dimensional POD analysis is valid and is important in order to emphasize the patterns which are limited in span to the cross-section, but could be misleading without the comparison with the 3D-POD analysis. In the presented case of the turbulent boundary layer, it appears that the lower plane is populated by the streaky patterns which are well resolved by the lower-plane POD modes, but absent in the 3D-POD modes. This is due to the energy distribution of the boundary layer which has a maximum at the buffer region, presented here as a middle plane.

Further investigation of the characteristics of two- and three-dimensional POD analysis could shed light on the properties of coherent structures with respect to the spatial correlations. Similar comparison of the sliced and fully three-dimensional eigenmodes is the focus of our next study, applied to a large dataset based on numerical simulation results using high spatial and temporal resolution.
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