Strategy to detect the gravitational radiation counterpart of \(\gamma\)-ray bursts

Silvano Bonazzola\(^1\), Eric Gourgoulhon\(^1\)

\(^1\) D.A.R.C. (UPR 176 du CNRS), Observatoire Paris-Meudon, 92195 Meudon Cedex, France

Abstract. Both observational and theoretical rates of binary neutron star coalescence give low prospects for detection of a single event by the initial LIGO/VIRGO interferometers. However, by utilizing at the best all the a priori information on the expected signal, a positive detection can be achieved. This relies on the hypothesis that \(\gamma\)-ray bursts are the electromagnetic signature of neutron star coalescences. The information about the direction of the source can then be used to add in phase the signals from different detectors in order (i) to increase the signal-to-noise ratio and (ii) to make the noise more Gaussian. Besides, the information about the time of arrival can be used to drastically decrease the observation time and thereby the false alarm rate. Moreover, the fluence of the \(\gamma\)-ray emission gives some information about the amplitude of the gravitational signal. One can then add the signals from \(\sim 10^4\) observation boxes (\(\sim\) number of \(\gamma\)-ray bursts during 10 years) to yield a positive detection. Such a detection, based on the Maximum a Posteriori Probability Criterium, is a minimal one, in the sense that no information on the position and time of the events, nor on any parameter of the model, is collected. The advantage is that this detection requires an improvement of the detector sensitivity by a factor of only \(\sim 1.5\) with respect to the initial LIGO/VIRGO interferometers, and that, if positive, it will confirm the \(\gamma\)-ray burst model.

A widely spread out gamma-ray burst model is related with the coalescence of two neutron stars (N.S.) at cosmological distance \[\text{(1)}\]. In this model, the gravitational energy liberated during the disruption of the less massive star of a binary system of N.S. is transformed into electromagnetic energy and radiated in the X,\(\gamma\) range \[\text{(1)}\].

No direct evidence of the validity of this model exits until now. Only energy budget considerations and an estimated rate of coalescences of N.S. computed in the frame of stellar evolution theory, confort the idea that at least a large fraction of \(\gamma\)-ray bursts are generated by the above mechanism \[\text{(1)}\].

In this communication, we want to show that the class of a typical noise of \(10^{-23}/\sqrt{\text{Hz}}\) G.W. detectors is sensitive enough to detect, after a few years of observation, the associated G.W. emission of the coalescence. The strategy

\[^1\] to appear in the Proceedings of the Second Workshop on Gravitational Wave Data Analysis (Orsay, 13-15 November 1997).
Figure 1: Fraction of the sky covered when phasing the signal of different detectors (L1 = LIGO 1, L2 = LIGO 2, V = VIRGO), in the case where the direction of the source is known with an accuracy of 5°. A portion of the sky is said to be covered if the phase lag between different detectors resulting from the inaccurate knowledge of the source position is lower than $\pi/2$.

(a) proposed here consists in using at the best all the a priori information and hypothesis of the model and to “sum” the signals of many events.

Historically, coalescing N.S. were considered as the most promising source of detectable G.W. A detection rate of few events/year was estimated in an (to much) optimistic case (for a review on the subject see [1]). Since then, there is widely spread out consensus that the coalescing rate of N.S. is one per year in a sphere of 200 Mpc of radius. $\gamma$-ray burst have a repetition rate 10 times lower, and a focusing mechanism must be invoked in order to explain this discrepancy [9].

The predicted sensitivity of the first VIRGO detector will allow us to detect a coalescence of N.S. at a distance of 27 Mpc with a signal/noise (S/N) ratio of 7 [2] (the figure 7 was choosen in view to have one false alarm per year under the hypothesis that the noise is Gaussian). Therefore the detection rate of coalescing N.S. will result to be 0.5 event per century (!) or 10 times lower if no beaming mechanism exist in the $\gamma$-ray burst emission.

In view to increase the detection rate it is convenient to add in phase the signal of different detectors. In fact if $N$ is the number of detectors, the S/N increases as the $\sqrt{N}$ and the detection rate as $N^{3/2}$. Moreover, adding different
Figure 2: Same as Fig. 1 when the direction of the source is known with an accuracy of 1° instead of 5°.

Adding in phase the outputs of different detectors is easy if the position of the source in the sky is known with a sufficient accuracy. This is the case of an optical or radio detection of the source (pulsars or supernovae).

For γ-ray bursts, the error box of the recent BATSE experiment aboard the Compton-GRO satellite is about 5° × 5°. This accuracy in the determination of the position of the source in the sky will allow the phasing of the signals from 3 detectors (VIRGO, LIGO 1 and LIGO 2) for signals whose frequencies are lower than 200 Hz (See Fig 1). Figure 2 shows that reducing the error boxes by a factor of 5 will be enough to phase the output of 3 detectors. It is quite possible that the new generation of X and γ-ray detectors will improve the localization accuracy. In our opinion, the Gravitational Wave Community can play an important role in convincing the γ-ray Community of the importance of improving the accuracy of positioning the sources.

The hypothesis that γ-ray bursts are generated by coalescing N.S. allows us to suppose that the frequency of the gravitational radiation sweeps the frequency window of the detectors a few seconds before the beginning of the γ-ray emission; moreover, we suppose that the strength of the gravitational signal and the fluence of the electromagnetic one are correlated with the distance of the source. These quite natural hypotheses will be considered as a priori informations and will be used in computing the a posteriori probability density.
of detection of the signal. Other a priori informations will be enumerated (and used) in what follows.

The Maximum a Posteriori Probability Criterium (M.P.P.C.) \[7\] will be the tool used in view to utilize at the best all the a prior information given by the model. Let us consider a simple example to show how the M.P.P.C. works.

Consider a sinusoidal signal of length $T$: $y(t) = a \times \cos(\omega t + \phi)$ in presence of a white Gaussian noise $n(t)$. The sampled signal is:

$$y_i = a \times \cos(\omega t_i + \phi) + n_i, \quad i = 1, 2, ..., N,$$

where $N$ is the number of sampling points: $N = T \times \nu_s$, $\nu_s$ being the sampling frequency. The density probability of having $N$ $y_i$ values is

$$P(y_1, y_2, ... y_N | a, \phi) = \frac{1}{(\sigma \sqrt{2\pi})^N} \exp \left( -\frac{1}{2} \sum_{i=1}^{N} (y_i - a \cos(\omega t_i + \phi))^2 / 2\sigma^2 \right).$$

Let us suppose that the phase $\phi$ is known. In this case the maximum likelihood criterium allows us to estimate the amplitude of the signal $a$.

The best estimation of $a$ is given by solving the equation $\partial_a P(y_1, y_2, ... y_N | a, \phi) = 0$, from which we obtain the well known result $a = 2/N \sum_{i=1}^{N} y_i \cos(\omega t_i + \phi)$ and the Signal/Noise ratio reads

$$S/N = aT/B,$$

where $T$ is the time length of the signal (sec.) and $B$ is the instrumental noise per \( \sqrt{Hz} \).

Consider now the little more complicated case, in which the phase $\phi$ is not known. We have two possible strategies: the first one consists in estimating simultaneously $a$ and $\phi$ by solving the system $\partial_a P(y_1, y_2, ... y_N | a, \phi) = 0$, $\partial_\phi P(y_1, y_2, ... y_N | a, \phi) = 0$. In this way we obtain one value for $a$ and a value for $\phi$. This gives too much information because we do not need to know $\phi$. The alternative strategy consists in maximizing the posteriori probability density defined by $\int_0^{2\pi} P(\phi)P(y_1, y_2, ... y_N | a, \phi)d\phi$ where $P(\phi)$ is the a priori probability density of the phase. Because we have no information on the phase (except that it spans the interval $[0, 2\pi]$) we apply the so-called Equipartition of the Ignorance Principle \[7\],\[4\],\[8\] to set $P(\phi) = \text{const.} = 1/2\pi$. After integration, the amplitude $a$ is found by the maximum of the function $\exp a \times I_0(a)$ where $I_0$ is the Bessel function of 0 order \[7\]. The asymptotic value of $a$ for $S/N \to \infty$ is $a = \sqrt{c^2 + s^2}$ where $c = 1/N \sum_{i=1}^{N} y_i \cos(\omega t_i)$ and $s = 1/N \sum_{i=1}^{N} y_i \sin(\omega t_i)$, i.e. the value of $a$ is given by the power spectrum of the signal. The $S/N$ is

$$S/N = aT/(\sqrt{2} \times B),$$

\( \Box \)
i.e. \(1/\sqrt{2}\) times worse than in the previous case in which the phase was known. The factor \(\sqrt{2}\) is the price to pay for our ignorance. If more then one “box” exist, say \(J\) boxes, then the M.P.P.C. tell us that we have to maximize the a posteriori probability given by the product of the density probability of each box:

\[
P_1(y_1, y_2, \ldots, y_N | a, \phi)P_2(y_2, y_2, \ldots, y_N | a, \phi)\ldots P_J(y_1, y_2, \ldots, y_N | a, \phi) \quad (5)
\]

It is easy to show that, roughly speaking, the signal-to-noise ratio is increased by a factor \(J^{1/4}\).

In the real case, i.e. the detection of the signal from coalescing N.S., the probability density distribution \(P(y_1, y_2, \ldots, y_N | a, \phi, m_1, m_2, i., .)\) depends on more variables than \(a\) and \(\phi\), namely the masses \(m_1\) and \(m_2\) of the two N.S., the angle \(i\) between the orbital plane and the line of sight, etc... The a priori informations that we have about these parameters must be used (for example, we know that the masses of the two N.S. cannot be arbitrary, but lie between, say, \(M_1 = 0.2M_\odot\) and \(M_2 = 3M_\odot\), and so on). Therefore, the a posteriori probability density reads

\[
\int_0^{2\pi} \int_{M_1}^{M_2} \int_{M_1}^{M_2} \int_0^{\pi/2} \ldots P(m_1)P(m_2)P(i)P(\phi) \ldots \times \times P(y_1, y_2, \ldots, y_N | a, \phi, m_1, m_2, i., .) \ d\phi \ dm_1 \ dm_2 \ d(i)
\]

Under the conservative hypothesis that gravitational radiation observations must start, say, one minute before the electromagnetic counterpart of the \(\gamma\)-ray burst (the time at which the two N.S. merge is not precisely known either for observational reasons and for the lack of a reliable model), the total observation time is decreased by a factor of \(\sim 2 \times 10^{-3}\) (1 minute/(number of minutes per day)\times(number of \(\gamma\)-ray bursts per day)). Therefore the false alarm probability is reduced of the same factor (vice-versa \(S/N\) is reduced to 6 for the same false alarm probability). Note that the idea of using the information given by the timing and direction of the \(\gamma\)-ray bursts to increase the detection rate has been already developed by Kochanek & Piran.

With five detectors (2 + 2 for LIGO 1 and LIGO 2 and 1 for VIRGO) the distance at which a N.S. coalescence can be detected with a \(S/N\) of 7 is increased by factor \(\sqrt{5}\), i.e. 60 Mpc or 420 Mpc for \(S/N = 1\). If the coalescence happens at a distance of, say, 4.2 Gpc, the \(S/N\) ratio will be 0.1. Taking into account that in 10 years of observation time, \(10^4\) \(\gamma\)-ray bursts will be observed (The \(\gamma\)-ray bursts rate is about 3 per day), we shall dispose of \(10^4\) “boxes” with a gravitational \(S/N\) ratio of \(\sim 0.1\) in each box.

By “adding” the signal of the \(10^4\) boxes, as explained in the above example, the \(S/N\) ratio will be increased by factor of 10 \((= (10^4)^{1/4})\), i.e. \(S/N = 1\). Note that this is a pessimistic estimation: in fact not all \(\gamma\)-ray bursts are situated at
4.2 Gpc. Under the hypothesis of an uniform spatial distribution, the average 
γ-burst distance is reduced by a factor $\sqrt{3}$ and therefore the signal-to-noise ratio 
increases by a factor of $\sqrt{3}$: $S/N = 1.73$. This is not the end of the story: in fact 
we have not used yet the hypothesis that the intensity of the gravitational signal 
is correlated with the electromagnetic one (more precisely with its fluence).

The operation of “adding” the different boxes must be done by weighing 
the boxes with a weight proportional to the fluence of the γ-ray bursts. The 
final result is $S/N = 5.5$. This is not enough to have an acceptable false alarm 
probability; in fact a $S/N \geq 7$ is required to have a false alarm probability 
$\leq 10^{-3}$.

It appears that an improvement by a factor $\geq 1.3$ of the sensibility of the 
detectors will be enough to test the hypothesis that γ-ray bursts are generated 
by coalescing N.S.

The above results must be considered as preliminary ones. In fact we have 
supposed that the detectors are aligned (optimistic hypothesis); in real calculation 
the directionality of the detectors should be taken into account and used 
to weigh the signal of each box. The $S/N$ ratio should be improved a little bit.

**CONCLUSIONS**

There is a general agreement that N.S. coalescence rate is $\sim 1$ per year 
within a sphere of 200 Mpc. Taking into account the incertitude of the above 
estimation (a factor of 2 - 3, L. Yungelson private communication), this value is 
in a good agreement with the observed rate of γ-ray bursts in the same volume 
(0.1 per year). A possible beaming of a factor 10 is plausible and is invoked 
in view to eliminate the discrepancy between the theoretical model and the 
observational data.

A more important beaming of the γ-ray emission during the merging of two 
N.S. would increase the coalescence rate. It is however quite difficult to imagine 
which physical mechanism would decrease the isotropy of the electromagnetic 
radiation: the velocity of the two merging N.S. is indeed about half the velocity 
of light and therefore the aberration effects are not strong.

Recently, Wilson et al. [10] have found by numerical simulations that the 
more massive of the two N.S. can collapse and form a black hole before the 
merging if its mass is close to the critical one. Even in this unlikely particular 
case the electromagnetic radiation is not suppressed: in fact the less massive 
star will be destroyed by the tidal forces of the just born black hole.

An improvement of the detectors sensitivity by a factor of at list 10 is re-
quired in order to detect a few individual events per year with a reasonable 
false alarm probability. Nobody knows when and how the thermal noise of the 
mirrors will be reduced by such an important factor (one of us (S.B.) is indebted 
to Prof. A. Giazotto for helpful discussions on this point). These (quite pes-
simistic) conclusions hold only for the coalescence of N.S. Let us recall that a coalescence of massive black holes (100 $M_\odot$) can be detected up to $\sim 3$ Gpc.

In this paper we have showed that by phasing the signals of different detectors and by using all the a priori informations given by the $\gamma$-ray burst model, a positive detection can be achieved in a few years of observation time with a little improvement (less than a factor of 2) of the first generation of the VIRGO class detectors. The price to pay is the loss of information: at the end of the observation only a positive detection of gravitational radiation will be achieved and a model for $\gamma$-ray burst confirmed, but the time and the position of the events will be lost.

References

[1] Bonazzola S., & Marck J.A., 1994, Ann. Rev. Nucl. Part. Sci. 45, 655
[2] Hello P., this Conference
[3] Kochanek C.S., Piran T., 1993, Astrophys. J. 417, L17
[4] Levine B., 1973, Fondements Théoriques de la Radiotechnique Statistique, Vol. 2, Ed. MIR, Moscou, p. 272
[5] loc. cit., p. 285, Eq. (5.59)
[6] Moschkovitch R., this Conference
[7] Stuart A. & Ord J.K., 1991, Kendall’s Advanced Theory of Statistics, Arrold, 5$^{th}$ edition, Vol. 1, p. 281, Section 8.5
[8] Stuart A. & Ord J.K., 1991, Kendall’s Advanced Theory of Statistics, Arrold, 5$^{th}$ edition, Vol. 2, p. 1231, Section 31.75
[9] Yungelson L., this Conference
[10] Wilson J.R. Mathews G.J., & Marronetti P., 1996, Phys. Rev. D 54, 1317