Ultra-Light Pion and Baryon WIMPzillas Dark Matter

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We consider a dark confining gauge theory with millicharged Ultra-Light Pions (ULP) and heavy baryons as dark matter candidates. The model simultaneously realizes the ultra-light (STStrongly-interacting Ultralight Millicharged Particle or “STUMP”) and superheavy (“WIMPzilla”) dark matter paradigms, connected by the confinement scale of the dark QCD. It is a realization of millicharged ULDM, unlike conventional axions, and exhibits a mass splitting between the charged and neutral pions. ULPs can easily provide the observed density of the dark matter, and be cosmologically stable, for a broad range of dark QCD scales and quark masses. The dark baryons, produced via gravitational particle production or via freeze-in, provide an additional contribution to the dark matter density. Dark matter halos and boson stars in this context are generically an admixture of the three pions and heavy baryons, leading to a diversity of density profiles. That opens up the accessible parameter space of the model compared with the standard millicharged DM scenarios and can be probed by future experiments. We briefly discuss additional interesting phenomenology, such as ULP electrodynamics, and Cosmic ULP Backgrounds.

CONTENTS

I. Introduction 1

II. Effective Field Theory and Cosmological History of Dark QCD 2
   A. Portals to the Standard Model 3
   B. The Spectrum 4
      1. Pions 4
      2. \(\eta'\) Meson 5
      3. Glueballs 5
      4. Dark Baryons 5

III. Dark Matter 6
   A. Pion Dark Matter: Light and Ultra-Light 6
      1. Ultra-Light Pions (ULPs) 6
      2. Light Pions 7
   B. Baryon WIMPzillas 7
      1. Higgs portal 7
      2. Inflaton portal 8
      3. Millicharged QED 8
      4. Gravitational production 9

IV. ULP Halos and Boson Stars 9

V. Current Constraints and Avenues for Detection 11

VI. Discussion 12
   A. Number density of heavy dark baryons 13
   References 14

I. INTRODUCTION

Few things are known with the degree of certainty that it is known dark matter exists: Cosmic Microwave Background (CMB) data, in the context of the \(\Lambda CDM\) cosmological model, indicates a non-zero abundance of dark matter to a statistical significance greater than 70\sigma [1]. However, beyond its gravitational influence, little is known about the identity of dark matter. Emblematic of our ignorance of dark matter is the mass of its constituent degrees of freedom, which could range from \(10^{-22}\) eV to \(10^{15}\) grams.

Dark matter candidates in the sub-eV mass range are collectively known as Ultra-Light Dark Matter (ULDM) [2]. In the decades since the advent of axion dark matter [3–5], this class of models has been populated with Fuzzy [6, 7], Bose-Einstein condensate [8], bosonic superfluid, [9, 10] fermionic superfluid [11, 12], and superconducting, e.g., STStrongly-interacting Ultralight Millicharged Particle (STUMP) [13], dark matter candidates, amongst others. Axion dark matter is but the tip of the ULDM iceberg.

These models are in part compelling due to their potential for resolving small-scale tensions in the \(\Lambda CDM\) model, such as the core-cusp problem, missing satellites, and galactic rotation curves [14–17]. However the modern science case is broader and deeper than this alone (see [18] for a recent overview). ULDM exhibit a wide array of interesting phenomenology, such as vortices [19–22] and their imprints [23, 24], electromagnetic signatures (e.g., [25], see [26] for a review), gravitational waves [27–31], and in varied particle physics contexts, such as muon \(g−2\) experiments [32] and neutrino oscillation experiments [33]. This wide array of potential observables for both current and next generation experimental efforts motivates a thorough study of the ULDM model space.

In this paper we study the emergence of millicharged
of the dark matter
the confinement scale and quark mass, constituting a fraction

FIG. 1. Taxonomy of the ULP-WIMPzilla model. Dark matter is comprised of ultra-light pions (ULPs) and superheavy dark baryons, with masses \( m_\pi \) and \( m_b \) jointly determined by the confinement scale and quark mass, constituting a fraction of the dark matter \( f_\nu \) and \( f_e = 1 - f_\nu \) respectively. The ULPS and WIMPzillas interact with each other through the dark strong force (DQCD) and visible electromagnetism by their millicharges \( e_e \) (mQED). They also interact with the Standard Model electromagnetically as well as gravitationally, through the Higgs, or via the inflaton.

ULPs in the context of a confining gauge theory. We find that the pions of this theory, namely, ultra-light pions (ULPs), are an excellent dark matter candidate. These are analogous to the Standard Model pions (Sec. IIIA) and superheavy baryons (Sec. IIIB) as the cold dark matter. Light and ultra-light pions are produced via the misalignment mechanism in the early universe. The baryon WIMPzillas can be generated gravitationally or via freeze-in mechanism by an inflaton portal, Higgs portal, and millicharged QED interactions.

ULPs and WIMPzillas interact with each other through the dark strong force (DQCD) and visible electromagnetism by their millicharges \( e_e \) (mQED). They also interact with the Standard Model electromagnetically as well as gravitationally, through the Higgs, or via the inflaton.

FIG. 2. Summary of dark matter production mechanisms with respect to mass. This setup predicts an admixture of light dark pions (Sec. IIIA) and superheavy baryons (Sec. IIIB) as the cold dark matter. Light and ultra-light pions are produced via the misalignment mechanism in the early universe. The baryon WIMPzillas can be generated gravitationally or via freeze-in mechanism by an inflaton portal, Higgs portal, and millicharged QED interactions.

Notation: the dark \( SU(N)_x \) gauge field and its quarks are denoted as \( X_\mu \) and \( \chi \) respectively. The SM Higgs doublet is denoted as \( H \) and \( B_h \) is the SM's \( U(1)_Y \) hypercharge. Throughout this work, unless otherwise specified, by “quark”, “pion”, “baryon”, “eta-prime”, and “glueball”, refer to dark sector components. Here \( (\pi^0, \pi^+, \pi^-) \) are the dark pions, \( \eta' \) is the dark eta-prime, and \( (b, n) \) denote the dark baryons. The dark baryon number is shown as \( B' \). The Hubble parameter is shown as \( H \). Finally, to avoid confusion with the SM baryon density, we used \( \Omega_\nu \), and \( f_\nu \) for the relic density and fraction of energy in the dark baryons.

II. EFFECTIVE FIELD THEORY AND COSMOLOGICAL HISTORY OF DARK QCD

We consider a confining dark gauge symmetry \( SU(N)_x \) coupled to two dark quarks ("up" and "down") in the fundamental representation of the dark color \( SU(N)_x \) as

\[
\mathcal{L}_X = i \bar{\chi} \partial_\mu \chi - m_\chi \bar{\chi} \chi - \frac{1}{2} \mathrm{Tr} X_{\mu\nu} X^{\mu\nu},
\]
where $\mathcal{D} = \nabla - ig_A \mathbf{X}$, in which $\mathbf{m}_\chi = \text{diag}(m_u, m_d)$ is a quark mass matrix. The quarks $\chi_{u,d}$ carry dark baryon numbers as $b'_u = b'_d = \frac{1}{3}$. We are interested in the limit that $m_\chi$ is negligible compared to the confinement scale of the $SU(N)_\chi$, i.e., $m_\chi \ll \Lambda_\chi$. The theory Eq. (1) then has an approximate $SU(2)_L \times SU(2)_R \times U(1)_Y \times U(1)_A$ chiral symmetry, under which $\chi_{L,R}$ transform as a doublet.

At energy scales below the confinement scale of $SU(N)_\chi$, the dark QCD ("DQCD") vacuum spontaneously breaks the $SU(2)_L \times SU(2)_R$ global symmetry down to the diagonal subgroup $SU(2)_V$, via the x-quark-antiquark condensate as

$$\langle \chi_i^L \chi_j^R \rangle = V^3 \delta_{ij}, \quad i, j \in \{u, d\},$$

where $V \sim \Lambda_\chi^3$. The quarks are then confined into mesons and hadrons. We have two composite pseudoscalar states,

$$\pi = \pi^0 \tau_3, \quad \eta',$$

where $\tau_a$ are the generators of the $SU(2)$ algebra, the $SU(2)$ triplet $\pi$ are Nambu–Goldstone bosons (NGBs) corresponding to spontaneous symmetry breaking of $SU(2)_A$, and $\eta'$ is a massive singlet, which are all their own antiparticles. There is additionally a spectrum of mesons and baryons, just as in standard visible QCD. However, unlike QCD, here there is no analog of the electroweak interactions, which makes the pions stable.

The mass spectrum of the lowest-lying mesonic and hadronic states is shown in Fig. 3. Before studying the spectrum of the theory in detail, we first consider ways in which dark QCD may be coupled to the visible Standard Model.

A second possibility is a Higgs portal to the Standard Model. For example, the dark quarks can be directly coupled to the SM Higgs via a dimension-5 operator $\mathcal{L}_{\chi-H} = \sum_{i=u,d} \frac{\mu_i}{\Lambda_H} H^\dagger H \tilde{\chi}_i \chi_i$, where $H$ is the SM Higgs doublet, and $\Lambda_H$ is the mass of the heavy mediator between DM and Higgs. In the confining phase, this interaction generates effective couplings of the Higgs to both the mesons and the baryons. A variation on this is to consider portals of the confined phase, and in particular consider dark-baryon-philic couplings of the Standard Model, analogous to leptophilic dark matter [49] and leptophobic dark matter [50]. Dark-baryon-philic interactions can emerge from simple UV completions, e.g., in models such as [51, 52] wherein the global $U(1)$ dark baryon number is promoted to a spontaneously broken gauge symmetry, as proposed in [13].

An additional possibility is to couple the dark sector to the SM indirectly, i.e. through an inflaton portal. This could again take the form of a baryon-philic coupling, or couplings to the full spectrum of the confined phase.
B. The Spectrum

We now consider the spectrum of composite states in the confining phase of dark QCD. Among the composite states, we will see that the pions and baryons are stable while η’ and glueballs are unstable and short-lived.

1. Pions

The dynamics of the pions in the confined phase is well described by chiral perturbation theory [53]. In this framework, confinement can be described as a spontaneous symmetry breaking of a composite field $\Sigma_{ij}$, with additional terms which explicitly break the SU(2)$_A$ global symmetry. The action can be expressed as,

$$S_\Sigma = \int dx^4 \sqrt{-g} \left( \text{Tr}[D_\mu \Sigma]^2 + m_\Sigma^2 \Sigma^2 - \frac{\lambda}{4} \Sigma^4 \right) + \mathcal{L}_{\chi\text{PT}},$$

where $D_\mu \Sigma$ is the covariant derivative defined as

$$D_\mu \Sigma = \partial_\mu \Sigma - ieA_\mu(Q \Sigma - \Sigma Q), \quad Q = \text{diag}(\varepsilon_u, \varepsilon_d),$$

and $\mathcal{L}_{\chi\text{PT}}$ denotes terms that explicitly break the chiral symmetry.

In the symmetry-broken phase, $\Sigma_{ij}$ can be expanded around its vacuum as

$$\Sigma_{ij} = \frac{F_\pi + \sigma(x)}{\sqrt{2}} \exp \left( \frac{2i\mu_\pi(x)\tau_3}{F_\pi} \right),$$

where $\langle \Sigma \rangle = \frac{F_\pi}{\sqrt{2}} \text{diag}(1,1)$ is the vacuum solution. The effective action of the pions is then determined by $\mathcal{L}_{\chi\text{PT}}$, which, at leading order, is given by,

$$\mathcal{L}_{\chi\text{PT}} = \text{Tr}[\Sigma^\dagger m_\chi + m_\chi^\dagger \Sigma] + \ldots.$$  

Moreover, the most generic $\chi$PT can have a term as [54, 55]

$$\mathcal{L}_{\chi\text{PT}} = i\bar{f}(\eta')\text{Tr}[\Sigma^\dagger (m_\chi - m_\chi^\dagger \Sigma)],$$

where $f(\eta')$ is an odd function of $\eta'$. Although $\eta'$ is not a true Goldstone boson due to the axial $U(1)_A$ anomaly of the strong interactions, it combines with the pions via the above effective interaction.

In the case of neutral and degenerate-mass quarks, the effective action of the pions takes an exceptionally simple form as

$$\mathcal{L}_\pi|_{m_u = m_d} = \frac{1}{2} \left( \text{Tr}[\partial_\mu \pi \partial^\mu \pi + m_u V^3 \pi \text{Tr} \cos \left( \frac{\pi}{F_\pi} \right) \right),$$

in strong resemblance to a conventional axion.

More generally, expanding to quadratic order in the pions, we find

$$\mathcal{L}_\pi = \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + D_\mu \pi^+ D^\mu \pi^- - \frac{m_\pi^2}{2} \pi^2 - m_\pi^0 \pi^0 - m_\pi^\pm \pi^\pm - \mathcal{L}_{\pi\gamma},$$

where the mass of pions is given by the Gell-Mann-Oakes-Renner relation

$$m_\pi^2 = \frac{V^3}{F_\pi^2} (m_u + m_d),$$

for the neutral pion, where $F_\pi \sim V \sim \Lambda_x$, and

$$m_{\pi^\pm}^2 = m_{\pi^0}^2 + 2\xi e^2 F_\pi (\varepsilon_u - \varepsilon_d)^2,$$

for the charged pions, where the last term is the electromagnetic contribution to the mass splitting of $\pi^\pm - \pi^0$, and $\xi$ is an order one parameter which should be fit by data. Therefore, given that $\varepsilon_u \sim -\varepsilon_d = \varepsilon$, $\xi = O(1)$ and $F_\pi \sim V$, we have the mass splitting

$$\frac{m_{\pi^0}^2 - m_{\pi^\pm}^2}{m_{\pi^+}^2} \sim e^2 \frac{\varepsilon F_\pi}{m_u + m_d},$$

which, depending on $\varepsilon$, $F_\pi$, and the quark mass $m_\chi$, can range from negligibly small to very large.

The pions are also coupled to the SM photon. The pion-photon interaction $\mathcal{L}_{\pi\gamma}$ in Eq. (11) is the chiral anomaly of the $U(1)_A$ via the triangle diagram of the massive charged baryons as [56]

$$\mathcal{L}_{\pi\gamma} = \sum_{b} \bar{Q}_b Q_b \frac{N_c}{8\pi^2 F_\pi} \varepsilon^0 FF,$$

where $N_c$ is the number of colors and $\sum_b Q_b^2 = A\varepsilon^2 e^2$ is the charge of baryons inside the triangle loop where $A = 1/2$ ($A = 1$ ) in type-I (type-II). The pions also inherit quartic self-interactions and couplings coming both from the expansion of the cosine potential and from higher-order terms in chiral perturbation theory.

The coupling to the photon allows the neutral pion to decay as $\pi^0 \rightarrow \gamma\gamma$. Taking the $m_{\pi^0} \ll m_b$ limit, this decay rate can be written as

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{A\alpha_2^2 \varepsilon^4 \lambda^2 m_{\pi^0}^3}{64\pi^3 m_b^2} = \frac{A\alpha_2^2 \varepsilon^4 m_{\pi^0}^3}{64\pi^3 F_\pi^2}.$$  

Therefore, the lifetime of $\pi^0$ in type-I is twice the lifetime of $\pi^0$ in type-II. Assuming $m_u \sim m_d = m_\chi$ and given that $m_{\pi^0}^2 \sim m_\chi \Lambda_x$, we find

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) \sim \frac{2A\alpha_2^2}{64\pi^3} \varepsilon^4 m_\chi \left( \frac{m_\chi}{\Lambda_x} \right)^{\frac{3}{2}}.$$  

In order for the $\pi^0$ to be long lived enough and a DM candidate, its lifetime should be longer that the age of Universe, $\Gamma = h/\tau_U < h/(4 \times 10^{17} \text{ s})$. This can be translated to the condition,

$$\left( \frac{\varepsilon}{10^{-8}} \right)^4 \left( \frac{100 \text{ MeV}}{\Lambda_x} \right)^{\frac{3}{2}} \left( \frac{m_\chi}{0.1 \text{ MeV}} \right)^{\frac{3}{2}} < \frac{3}{2}.$$  

Therefore, the pions can be made stable through any or all of a high confinement scale $\Lambda$, a small quark mass $m_\chi$, or a small millicharge $\varepsilon$. 


2. $\eta'$ Meson

Similar to SM QCD, the global $U(1)_A$ symmetry of our dark QCD is broken by instantons. The mode associated to the $U(1)_A$ is the $\eta'$ meson, which receives its mass due to the explicit breaking of $U(1)_A$ by the $SU(N_c)$ instantons as

$$V(\eta') = \Lambda'^4 \cos\left(\frac{\eta'}{F_{\eta'}}\right),$$

where $F_{\eta'} \simeq \Lambda_x$ is the decay constant of $\eta'$. This endows the $\eta'$ with a mass $m_{\eta'} \sim \Lambda_x$, which is parametrically heavier than the pions. The effective Lagrangian of the $\eta'$ can be written as

$$\mathcal{L}_{\eta'} = \frac{1}{2} \partial_\mu \eta' \partial^\mu \eta' + \Lambda'^4 \cos\left(\frac{\eta'}{F_{\eta'}}\right) - \frac{\alpha_{\eta'\pi}}{4\pi} \frac{\eta'}{F_{\eta'}} F_{\mu\nu} \tilde{F}^{\mu\nu} + \alpha_{\eta'\pi} (m_u - m_d) |\eta'| \pi^0 \vec{\pi},$$

where $\alpha_{\eta'\pi}$ and $\alpha_{\eta'\pi}$ are two constants that should be given by the simulation. Note that the last term is isospin symmetry violating and hence is proportional to $|m_u - m_d|$.

The dark $\eta'$ is unstable, just as in SM QCD. The decay channels of the $\eta'$ are the decay to two photons, i.e. $\eta' \rightarrow \gamma\gamma$, as well as decay to three pions, i.e. $\eta' \rightarrow \pi^+ \pi^- \pi^0$ and $\eta' \rightarrow \pi^0 \pi^0 \pi^0$. The decay rate of $\eta'$ to two photons can be estimated in terms of the same process in the SM as

$$\Gamma(\eta' \rightarrow \gamma\gamma) \approx \varepsilon^4 \left(\frac{m_{\eta'}}{m_{\eta'_SM}}\right)^3 \Gamma(\eta'_{SM} \rightarrow \gamma\gamma).$$

The $\eta'_{SM} \rightarrow \gamma\gamma$ is the dominate decay channel of $\eta'_{SM}$ which specifies its life-time as short as $\tau_{\eta'_{SM}} \approx 3 \times 10^{-21}$ s.

In the $\eta'$ case, we have

$$\Gamma(\eta' \rightarrow \gamma\gamma) \approx \frac{1}{10^{-2} s} \left(\frac{\varepsilon}{10^{-13}}\right)^4 \left(\frac{m_{\eta'}}{10^{10} \text{GeV}}\right)^3 \frac{1}{\Lambda'^4},$$

making the $\eta'$ is unstable on cosmological time-scales. Meanwhile the decay rate of $\eta'$ to three pions is given as

$$\Gamma(\eta' \rightarrow \pi\pi\pi) = \frac{\alpha_{\eta'\pi}^2 (m_u - m_d)^2}{64(2\pi)^3} m_{\eta'},$$

which can be cosmologically fast or slow, depending on the mass splitting of the quarks, the $\eta'$-pion coupling, and the mass of the $\eta'$. Given these two decay channels, the generic expectation is that any initial $\eta'$ population should be unstable and decay to photons and pions.

3. Glueballs

In addition to the mesons and baryons, one expects that gluons can also form colourless states, called glueballs [57]. In that case, dark Glueballs (DG) can form below the confinement scale as pure gluonic states. The properties of the glueballs in Yang-Mills have been studied in lattice gauge theory [58–62]. Their spectrum can entirely be parameterised by the confinement scale of the theory, or equivalently lightest glueball mass, $m_G \sim 6\Lambda_x$.

The full QCD (including quarks), however, is more complicated and an active experimental and theoretical area of research. The lightest DG (denoted as $G$) is a scalar with quantum numbers $J^{PC} = 0^{++}$, and its effective Lagrangian can be written as

$$\mathcal{L}_G = \frac{1}{2} \partial_\mu G \partial^\mu G + \frac{1}{2} m_G^2 G^2 + f_3 G^3 + f_4 G^4,$$

where $m_G$ is the mass, $f_3$ and $f_4$ are the self-couplings. The glueballs can be stable or decay through a variety of portals to the mesons. Given that $G$ is a singlet under chiral symmetry, the DG-meson interactions is [63]

$$\mathcal{L}_{G\Sigma} = (g_1 F_\pi G + g_2 G^2) \text{Tr}[\Sigma \Sigma],$$

which leads to a $G$ and pions interaction term as

$$\mathcal{L}_{G\pi} \approx \frac{\theta m_G^2}{2F_\pi} G \pi^2,$$

where $\theta = \frac{g_1 F_\pi}{(m_G^2 - m_\pi^2)}$ which its value should be given by the data. Therefore, the scalar DG decays into to pions with the decay rate

$$\Gamma(G \rightarrow \pi^0 \pi^0) = \frac{3 \theta^2}{32\pi} \left(\frac{m_G}{F_\pi}\right)^2 \sqrt{m_G^2 - m_\pi^2}.$$

Assuming that the $\theta$ is $O(1)$, we have

$$\Gamma(G \rightarrow \pi^+ \pi^-) \approx 10\Lambda_x,$$

which implies that DG are unstable and very short-lived.

As a result, the dark glueballs have no cosmological effect in this setup. For a similar framework in a different part of parameter space with cosmological relevant glueballs see [37] and [64].

4. Dark Baryons

The lightest dark baryons are $p = uud$ and $n = ddu$ with masses $m_p \sim m_n \approx N_c \Lambda_x$. Both baryons are stable, even in the charged case, since there are no electrons or neutrinos in the model, and dark baryon number is conserved. The dark baryons therefore serve as a potential dark matter candidate. For $\Lambda_x \gg \text{GeV}$, this falls in the category of superheavy dark matter (recently reviewed in [42]), and in particular, superheavy dark fermions. These baryons are feebly coupled to the SM and can only get generated via the freeze-in mechanism in the early universe, or else gravitationally. We defer a discussion of the primordial production of baryons to Sec. III.
III. DARK MATTER

We now focus our attention on the dark matter problem, namely, obtaining the observed relic density of cosmologically stable cold collisionless dark matter. As discussed above, the stable composite states in our model are the pions and the baryons. Here we consider these individually.

A. Pion Dark Matter: Light and Ultra-Light

As a simple cosmological history, we consider that confinement occurs at a high scale $\Lambda_x$, above the scale of cosmic inflation. Similar to the conventional axion vacuum misalignment mechanism, the pion fields are each initialized with an initial value comparable to the pion decay constant. The phase of cosmic inflation serves to both homogenize the pion fields, and redshift away any thermal relics from the confining phase transition.

In order to constitute the observed dark matter, we demand the dark pions satisfy the following requirements:

1. Ultra-Light and Light: We are interested in ultra-light pions (ULPs), $m_\pi < 1\text{ eV}$, and light pions $m_\pi = (1\text{ eV} - 10\text{ keV})$. They are generated via the misalignment mechanism in the early universe.

2. Stability: In our dark QCD, with neutral quarks, the pions are stable, since there are no electroweak interactions. In the millicharged case, this requires $\theta_{\pi^a} \sim \pi$, this leads to an upper bound on the scale of inflation, as

$$H_{\text{inf}} \lesssim 8.8 \times 10^{-5} \frac{\Omega_{DM}}{\Omega_\pi} \frac{F_\pi}{M_{pl}}. \quad (30)$$

Finally the requirement that the dark matter be collisionless is satisfied due to the fact that the pion self-interactions and couplings are suppressed by the decay constant $F_\pi$, and additionally by the requirement that the millicharge be small enough to extend the pion lifetime to greater than the age of the universe. In the following, we discuss the parameter space of dark pions. (See the left panel of Fig. 5).

3. Relic Density: The three pions together constitute the observed dark matter density. The present day pion abundances is given by,

$$\Omega_\pi = \frac{1}{6} (9\Omega_\pi)^{3/4} \frac{F_\pi^2}{M_{pl}^2} \sum_{a=0,\pm} \left( \frac{m_\pi}{H_0} \right)^{1/2} \theta_{\pi^a}^2 \quad (29)$$

in analogy to the usual axion case [65].

4. Isocurvature Modes: Light pions pick up isocurvature perturbations during inflation which is strongly constrained by the CMB [66]. Assuming $\theta_{\pi^a} \sim \pi$, this leads to an upper bound on the scale of inflation, as

$$\Omega_{\pi a} \lesssim \frac{2\Lambda_\pi}{\Omega_{DM} \Omega_\pi F_\pi M_{pl}}. \quad (31)$$

FIG. 5. The admixture of dark pions-WIMPzilla baryons with respect to parameters of the model, for freeze-in production of dark baryons (see Fig. 7 for gravitational production). [Left Panel] Accessible parameter space for the pions produced by misalignment mechanism where the misalignment angle are $\theta_i \approx \pi$. The shaded region shows parameters corresponds to $\Omega_\pi/\Omega_{DM} \lesssim 1$ and the dashed line marked shows $\Omega_\pi/\Omega_{DM} = 10^{-2}$. [Middle and Right Panels] Freeze-in production of baryons (by Higgs portal Eq. (39), Inflaton portal Eq. (45), and QED portal Fig. 6) for the preheating phase with $w = 0$ (middle panel), and $w = 1$ (right panel). The parameter $\Lambda$ denotes $\Lambda y$ for the Higgs portal case, $\sqrt{2}\Lambda / \lambda$ in the inflaton portal case, and $13m_\pi / \epsilon$ for the freeze-in via millicharged case. Each shaded region represents part of the parameter space with $\Omega_b/\Omega_{DM} \lesssim 1$ for the given value of $m_b$. The dashed lines inside each shaded area makes $\Omega_\pi/\Omega_{DM} = 10^{-2}$. 

The dark pions with masses $m_\pi < 1\text{ eV}$ evolve as a coherent scalar field. Similar to Axion-like particles.
(ALPs), this implies the ULPs evolve as cold pressureless dark matter in the late universe, when each ULP field oscillates in the minimum of its potential. Demanding that the pions constitute an $O(1)$ fraction of the observed dark matter density enforces a lower bound on the pion decay constant, and hence the dark QCD scale, given by $F_\pi > 8.8 \times 10^{10}$ GeV. From these considerations, and the relation $m_\pi^2 \sim m_\chi^2 A_\chi$, we may deduce the mass range for the dark quarks, as

$$m_\chi < 5 \times 10^{-20} \text{eV}.$$  

(31)

Further demanding that the charged pions, with mass given by Eq. (13), remain in the ultra-light regime, $m_\pm < 1 \text{eV}$, puts an upper bound on the charge of the dark quarks as

$$\varepsilon e < 1 \text{ eV} / F_\pi.$$  

(32)

Curiously, the mass bound Eq. (31) is within a few orders of magnitude of the benchmark Fuzzy Dark Matter mass. For the purposes of this work, we do not consider the fine-tuning of the dark quark mass to be any more concerning of a problem then it already is for bosonic ultra-light DM (e.g. axion or fuzzy DM).

2. Light Pions

The light dark pions have masses in the interval $m_\pi^\pm = (1 \text{ eV} - 10 \text{ keV})$. Using $H_0 \approx 2 \times 10^{-33} \text{eV}$ in Eq. (29), we have

$$\Omega_\pi \approx 0.25 \left( \frac{F_\pi}{8.8 \times 10^{10} \text{ GeV}} \right)^2 \left( \frac{m_\pi^\pm}{1 \text{ eV}} \right)^2 \left( \frac{\theta_a}{\pi} \right)^2.$$  

(33)

Demanding that the light dark pions made all the dark matter today gives

$$F_\pi \geq 8.8 \times 10^9 \text{GeV}.$$  

(34)

Therefore, our dark sector with light pions has heavy baryons with mass,

$$m_b \gtrsim 10^{10} \text{ GeV}.$$  

(35)

Moreover, the mass and millicharge of the quarks can be on one of the following ranges:

$$1.1 \times 10^{-20} \text{eV} \leq m_\chi \leq 1.1 \times 10^{-11} \text{eV} \quad \text{and} \quad \varepsilon e < \frac{1 \text{ eV}}{F_\pi},$$

which put an extremely tight upper bound on the charge as $\varepsilon e < 10^{-19}$, or

$$m_\chi \leq 1.1 \times 10^{-20} \text{eV} \quad \text{and} \quad \frac{1 \text{ eV}}{F_\pi} < \varepsilon e < \frac{10^4 \text{ eV}}{F_\pi},$$

which demands $\varepsilon e < 10^{-14}$.

B. Baryon WIMPzillas

The Dark QCD we consider also allows for stable dark baryons. These may be produced gravitationally at the end of inflation, or else through freeze-in production via their interactions with the SM. Here we consider and study three different freeze-in mechanisms, i.e. inflaton portal freeze-in, Higgs portal freeze-in, and millicharged QED freeze-in. We summarize these different possibilities in Fig. 2. Gravitational production of the WIMPzillas is shown in Fig. 7.

At this point we need to further specify the thermal evolution of the universe and in particular the reheating phase that follows cosmic inflation. For the sake of generality, we consider the following phenomenological reheating model

$$\rho_{\text{reh}} = \delta_{\text{reh}} \left( \frac{a_{\text{inf}}}{a_{\text{reh}}} \right)^4 \rho_{\text{inf}},$$  

(36)

where $\rho_{\text{inf}} = 3M_{\text{pl}}^2 H_{\text{inf}}^2$ is the inflation energy density, $\delta_{\text{reh}} \approx (a_{\text{reh}}/a_{\text{inf}})^{(3w-1)}$ is the efficiency of the reheating process, and $w$ is the effective equation of state in the intermediate period between the end of inflation and the formation of the thermal bath. In case that the inflaton oscillates coherently about the minimum of the potential, its energy density redshifts as matter, i.e. $w = 0$ with a $\delta_{\text{reh}} > 1$. On the other hand, if inflation ends with domination of the kinetic term, i.e. $w = 1$, we have $\delta_{\text{reh}} < 1$. The reheating energy density is $\rho_{\text{reh}} = \frac{3}{8} a_0 T_{\text{reh}}^4$. The temperature after reheating scales as $T \propto 1/a$, while between the end of inflation until reheating it scales differently and at the beginning it scales as $[67]

$$T(a) \approx \left( \frac{a_{\text{inf}}}{a} \right)^{(1+w)} T_{\text{max}}$$  

(37)

where $a_{\text{inf}} < a < a_{\text{reh}}$ and $T_{\text{max}}$ is

$$T_{\text{max}} \approx 0.2 \left( \frac{100}{g_{\text{eff}}} \right)^{\frac{1}{2}} \frac{1}{\rho_{\text{inf}} T_{\text{reh}}^2}.$$  

(38)

1. Higgs portal

The dark baryons can be coupled to the SM Higgs as

$$\mathcal{L}_H = \frac{y}{\Lambda_H} H^H H \sum_{i=p,n} \bar{b}_i b_i,$$  

(39)

where we approximately take $m_p \approx m_n$ and $y = y_b \approx y_n$. This interaction can arise either from SM Higgs-dark quark interaction $\mathcal{L} \propto |H|^2 \bar{\chi} \chi$, or by dark-baryon-phlic interactions, e.g. in a scenario with gauged baryon number [51]. The latter case leaves the pion mass unchanged and only contributes to the baryon mass. In contrast, in the former case, after the electroweak phase transition, the coupling Higgs makes generates an effective mass
for the dark pions as $m_{\pi^0} \sim 246 \left( \frac{m_b}{\Lambda_H} \right)^{1/2} \text{GeV}$, which, in
the region of interest for freeze-in production, generically lifts the ULPs out of the ultra-light regime. For simplicity,
here we treat the pion mass independently from the
Higgs-baryon coupling.

At temperatures above the Electroweak (EW) symmetry
breaking scale and $\Lambda_x > T_{\text{reh}}$, the thermally averaged
annihilation cross section of dark fermions with mass $m_b$

\[ \langle \sigma v \rangle \approx \frac{1}{8 \pi} \frac{G_F^2 m_b^2}{\Lambda_H^2} \text{[GeV]} \]  

is

\[ \langle \gamma \rangle \equiv \frac{1}{8 \pi} \frac{G_F^2 m_b^2}{\Lambda_H^2} \text{[GeV]} \]  

The decay rate of the dark baryons associated with the Higgs interaction is

\[ \Gamma_H = \frac{3T}{(2\pi)^2} \left( \frac{yT}{\Lambda_H} \right)^2 \left( \frac{m_b}{T} \right)^{-1/2} e^{-m_b/T}. \]  

(40)

Demanding that the WIMPzillas are never in thermal equilibrium with the SM, we find

\[ y^{-1} \Lambda_H \gg \frac{1}{(2\pi)^2} \left( \frac{m_b}{T_{\text{max}}} \right)^{-1/2} e^{m_b/M_{\text{pl}} T_{\text{max}}}. \]  

(41)

For typical values of $m_b \sim 10^{10}$ GeV, $m_b/T_{\text{max}} \sim 10$, and $y \sim 0.1$, it gives $\Lambda \sim 10^{14}$ GeV.

This interaction generates a relic density of dark
baryons via the freeze-in mechanism. Details of the cal-
y\[ \text{culations are provided in App. A, and here we only report the final results. Concretely, the relic density of the
dark baryons produced through the Higgs portal is (see Eq. (A9))

\[ \Omega_{b'} h^2 \simeq A \left( \frac{y m_b}{\Lambda_H} \right)^2 \frac{\exp \left[ 10 \left( 3 - \alpha \beta^2 \right) \right]}{\beta^4/(1+w)^{-1/2}} \], \]  

(42)

where $\alpha$ and $\beta$ are

\[ \alpha = \frac{m_b}{\sqrt{H_{\text{inf}} M_{\text{pl}}}} \quad \text{and} \quad \beta = \frac{\sqrt{H_{\text{inf}} M_{\text{pl}}}}{T_{\text{reh}}} \]  

(43)

which are both more than one and $A = \frac{10^{10} w^{-1/2}}{2(1+w)}$. We have $A = 1/2$ for $w = 0$ and $A = 3/2 \times 10^{-2}$ for $w = 1$.

The abundance of dark baryons as a function of model parameters is shown in Fig. 5, in the plane of $m_b/T_{\text{reh}}$
and $m_b/\sqrt{M_{\text{pl}} H_{\text{inf}}}$, with the value of coupling $y$ indicated by background color. Dashed lines indicate 1% of
the dark matter in dark baryons while solid lines indicate 100%. Regions below the dashed and above the solid lines
correspond to intermediate fractions between 1 – 100%.

From this one may appreciate that, given values for any
two of these parameter combination, one may tune the
dark baryon abundance by varying the third. We define
the fraction of dark matter that this process produces as

\[ f_{b'} \equiv \frac{\Omega_{b'}}{\Omega_{DM}} \]  

(44)

which may easily range from 0-1. This is illustrated in
Fig. 5.

\[ \Gamma_{\gamma} \equiv \frac{3 \varepsilon^2 \alpha_{\text{em}}^2}{(2\pi)^2} \left( \frac{T}{m_b} \right)^{1/2} e^{-m_b/T}. \]  

(50)
We demand that the WIMPzillas are never in thermal equilibrium with the SM. This puts an upper bound on the millicharge as

\[ \varepsilon \ll 4 \times 10^2 \left( \frac{m_b}{M_{pl}} \right)^{3/2} \left( \frac{m_b}{T_{max}} \right)^{1/2} e^{2m_b/T_{max}}. \] (51)

Given that \( m_b \gtrsim 10^{10} \text{ GeV} \) and \( m_b > T_{max} \), this condition is satisfied throughout the parameter space for \( \varepsilon \lesssim 10^{-2} \).

The relic density of the heavy dark baryons produced today through freeze-in by QED processes is

\[ \Omega_{b'} h^2 \approx 10^{-2} A \varepsilon^2 \frac{\alpha_{em}}{10^{-2}} \frac{\exp \left[ 10(3 - \alpha \beta^2) \right]}{\beta^4/(1+w)-1/2}, \] (52)

where \( \alpha_{em} = e^2/4\pi \), and \( A, \alpha \) and \( \beta \) are similar to (52) (See Fig. 5). Fig. 5 implies that generating significant fraction of dark baryons from the mQED interactions requires \( \varepsilon \gtrsim 10^{-4} \). As we discuss later in Sec. V, this possibility is ruled out by constraints on the millicharge the light pions.

4. **Gravitational production**

Finally, dark baryons may be produced gravitationally, without relying on any direct coupling to the Standard Model or the inflaton. So-called “gravitational particle production” [38–41, 68] is generated by the non-adiabatic expansion of spacetime that occurs at the end of inflation. See [42] for a recent review.

In our work we assume that the dark QCD theory is in the confining phase during inflation and reheating, and hence \( m_{b'} \gg \mathcal{H} \). Gravitational production in this parameter range has been extensively studied in recent years [69–73]. In particular, the dark matter density in the regime \( \mathcal{H} \ll m_{b'} \), with \( m_{b'} \lesssim m_{o_\phi} \), with \( m_{o_\phi} \) the mass of the inflaton, is given by [73]

\[ \frac{\rho_{b'}}{s} \approx 4 \times 10^{-10} \text{GeV}^2 \mathcal{C} \left( \frac{m_{b'}}{10^9 \text{GeV}} \right) \frac{\mathcal{H}}{10^9 \text{GeV}} \left( \frac{T_{re}}{10^{10} \text{GeV}} \right)^2 \left( \frac{m_{b'}}{m_{o_\phi}} \right)^2. \] (53)

where \( \mathcal{C} = 10^{-2} - 10^{-3} \) is a numerical constant, and \( s \) is the entropy density. This may be compared to the observed DM abundance \( \rho_{DM}/s \sim 4 \times 10^{-10} \text{ GeV} \). As a fiducial example, we fix \( m_{o_\phi} = 10^{13} \text{ GeV} \) and \( T_{re} = 10^{11} \text{GeV} \). The dark matter abundance, as a function of \( \mathcal{H} \) and the ratio \( m_{b'}/\mathcal{H} \) is shown in Fig. 7. In this figure the black line denotes parameters such that \( \Omega_{b'} \) gives 100% of the observed dark matter abundance, while the gray and light gray lines correspond to 10% and 1% of the dark matter respectively.

**FIG. 7.** Gravitational Production of Dark Baryons WIMPzillas. The black, gray, and light gray, lines correspond to 100%, 10%, and 1% of the observed dark matter in gravitationally produced dark baryons.

### IV. ULP HALOS AND BOSON STARS

We now consider cosmological and astrophysical implications of our model. A striking feature of ultra-light bosonic dark matter candidates is the existence of self-gravitating soliton solutions. In the context of fuzzy dark matter, with \( m \sim 10^{-22} \text{ eV} \), these solitons are on cosmological scales, forming the core of dark matter halos. For larger masses, such as the benchmark QCD axion mass \( m \sim 10^{-5} \text{ eV} \), the soliton solutions instead correspond to boson stars, which have their own signatures, such as gravitational waves. In both cases, the soliton solutions can be described in terms of wavefunction, corresponding to the non-relativistic limit of the scalar field. The resulting equations of motion are Schrödinger and Poisson equations. For a discussion of the allowed mass range and constraints, see [74].

We seek to construct such soliton solutions in the context of ULP dark matter. To do so, we first decompose the charged pions \( \pi_{\pm} \) into real components, as

\[ \pi_{\pm} = \pi^1 \pm i\pi^2 \] (55)

and take the non-relativistic limit of \( \pi_{0,1,2} \) as

\[ \pi^i = \frac{1}{\sqrt{2m_i}} [\psi_i e^{-im_i t} + \psi_i^* e^{im_i t}] \], (56)

with \( i = 0, 1, 2 \). The equations of motion of the system are then given by 3 copies of the Schrödinger equation,

\[ i\hbar \frac{\partial \psi_i}{\partial t} = -\frac{\hbar^2}{2m_i} \nabla^2 \psi_i + m_i \Phi \psi_i \] (57)

coupled via the Poisson equation,

\[ \nabla^2 \Phi = 4\pi G \sum_{i=0,1,2} m_i |\psi_i|^2. \] (58)

where \( m_{1,2} \equiv m_{\pm} \). For simplicity we have neglected non-gravitational interactions of the pions, which are suppressed by the pion decay constant. We leave this interesting aspect to future work.
To construct soliton solutions, we follow the references [75], [76], and [77]. Assuming an ansatz for the spatial dependence of the pion wavefunctions [75]

$$\psi_i(x) = \sqrt{\frac{3N_i}{\pi^3 R_i^3}} \operatorname{sech} \left( \frac{r}{R_i} \right)$$  \hspace{1cm} (59)$$

the Hamiltonian of the system may be straightforwardly derived as [77]

$$H = \sum_{i=1}^{3} \left[ a \frac{N_i}{m_i R_i^2} + b \frac{m_i^2 N_i^2}{R_i^2} \right] + \sqrt{2} \hbar \sum_{i,j=1}^{3} \frac{m_i m_j N_i N_j}{\sqrt{R_i^2 + R_j^2}}$$  \hspace{1cm} (60)$$

which generalizes the expression of [77] from two to three axions, but specializes to the sech ansatz of [75]. The constants $a$ and $b$ are ansatz dependent, and for the sech ansatz are given by [75]

$$a = \frac{12 + \pi^2}{6\pi^2}, \quad b = \frac{6}{\pi^4} \left( 12\zeta(3) - \pi^2 \right).$$  \hspace{1cm} (61)$$

Finally, following [75] we work in dimensionless variables, defined by the rescalings given in [75].

We note that a general solution for the pion wavefunctions will correspond to a soliton with a net electric charge. For simplicity, we focus on solutions that are electrically neutral. The charge of a soliton may be expressed as,

$$Q = 4\epsilon \epsilon \int d^3 x \left( \dot{\pi}_1 \pi_2 - \dot{\pi}_2 \pi_1 \right),$$  \hspace{1cm} (62)$$

where we have decomposed the standard relation for a charged complex scalar field into the real scalar components. An electrically neutral soliton corresponds to one of three cases: $\pi_1 = \pi_2$, $\pi_1 = 0$, or $\pi_2 = 0$. We consider the case $\pi_1 = \pi_2$, and hence $\psi_1 = \psi_2$.

To find an approximate ground state solution, building on the procedure of [77], we vary the Hamiltonian with respect to the radii $R_i$, while holding fixed both the total mass of the halo and the relative fraction of the mass contained within each dark matter component. We fix the total mass,

$$M_{\text{tot}} = \sum_{i=1}^{3} M_i,$$  \hspace{1cm} (63)$$

where

$$M_i \equiv m_i N_i = m_i \int d^3 r |\psi_i(r)|^2.$$  \hspace{1cm} (64)$$

We also fix the relative fraction of the mass in each component,

$$f_i = \frac{N_i}{N_{\text{tot}}},$$  \hspace{1cm} (65)$$

One may reasonably expect that $f_i$ depends on local environment of dark matter solitons, and on the primordial abundances of the ULPs. In this sense, varying $f_i$ corresponds to accounting for the diversity of dark matter halos.

Soliton solutions for varying $f_i$ are given in the left panel of Fig. 8, where we consider the simple case that $m_\pm = 3m_0$. In all cases we explicitly confirm that the Hessian of the Hamiltonian is positive definite, and thus the solutions are stable. We consider four fiducial example solitons, corresponding a purely $\pi^0$ halo, and halos with 5%, 20%, and 100% of the mass contained in $\pi^\pm$. One may easily appreciate that by adjusting the fraction of dark matter in the charged vs. neutral pions, the central density becomes higher and the core radius smaller. This effect becomes more dramatic as the ratio of pion masses is made larger. While we restrict ourselves here to a mild mass hierarchy, we note this is made
partly for illustrative purposes, and additionally so that
the non-relativistic limit may be consistently applied to
both fields.

The impact of raising the charged pion mass is shown
in the right panel of Fig. 8, where we fix the charged
pions to be 5% of the mass of the halo, and adjust the
charged pion mass. For larger values of the charged pion
mass, a distinct soliton is present at small radii, with a
density that is orders of magnitude larger than the outer
dges of the density profile.

The diversity of ULP soliton density profiles is par-
ticularly interesting given the mild tension between the
diversity of observed halos [78] and the predicted uni-
versal properties of Fuzzy dark matter solitons, see, e.g.,
[74, 79, 80] for a discussion and constraints. In the ULP
model, a diversity of halos naturally emerges from adjust-
ing the mixture of pions that make up the dark matter
halo. In the case of a large high hierarchy, e.g. if \( \pi^0 \)
is in the fuzzy range while \( \pi^\pm \) is in the conventional QCD
axion range, dark matter halos can be expected to range
from an the Navarro–Frenk–White (NFW) profile to the
cored halos familiar from fuzzy dark matter.

\section{Current Constraints and Avenues for Detection}

In this section, we discuss the current constraints and
the possibility of detection of the millicharged particles
(MCP), as well as the dark QCD phase transition, and
ULPs more generally.

**Bounds on millicharge:** The constraints on light
MCPs are very strong. In Table II we summarize the cur-
rent astrophysical, cosmological, and laboratory bounds
on MCPs with \( m_{\text{MCP}} \lesssim 1 \text{ MeV} \). Some of these constraints
can be avoided in our setup, as indicated in the last col-
umn of the table. The strongest bound on MCP dark
matter comes from CMB, leading to the constraint [88–
90]

\[
\varepsilon < 10^{-15} \left( \frac{m}{\text{eV}} \right).
\]

This constraint can be avoided in our setup once the
fraction of the DM density in the charged pions is less
than 0.4\%, i.e. \( f_{\pi^\pm} < 0.1 \), or if the charged pion mass
is in the ultralight regime (it is noteworthy to mention
that CMB constraints on millicharged coherent scalars,
e.g., \( m < 1 \text{ eV} \), have never been worked out, and we
deer to future work.) Note this still leaves open the
possibility of ULPs comprising 100\% of the dark matter,
with \( f_{\pi^0} \gtrsim 0.996 \). As a result, the allowed parameter
space for our charged pions is greatly enlarged compared
to standard scenario of MCP dark matter. The strongest
bound on the millicharge, that cannot be avoided, comes
from the stellar cooling that gives [81, 82]

\[
\varepsilon < 1.7 \times 10^{-14}.
\]

Several scenarios are proposed in the literature to evade
the stellar bounds. For details see [98] and the references
therein. The millichargened dark pions also interact with
the SM baryons. The Rutherford-type scattering cross-
section of the pions off of a SM proton through a photon is

\[
\sigma_{\pi^\pm b} \sim \frac{\alpha_{\text{em}}^2 \varepsilon^2}{m_{\text{MCP}}^2 v_{\text{rel}}^2},
\]

where \( v_{\text{rel}} \) is its relative velocity, and \( \mu \approx m_{\pi^\pm} \) is the
SM proton and dark pions reduced mass. That gives an
upper bound on the charge as \( \varepsilon < 10^{-15} \left( \frac{m_{\pi^\pm}}{\text{eV}} \right) \).

**Detection of dark QCD phase transition:** If the
dark QCD phase transition is first order, it can produce
gravitational waves background with the frequency peak
\( f \gtrsim 10^4 \text{Hz} \). Such frequencies are above the LIGO/Virgo
band and in the range of ultra high-frequency gravita-
tional waves. For a recent review on several detector
concepts which have been proposed to detect this ultra-
high frequency signals see [99].

**Other directions for ULP discovery:** Analogous to
axon electrodynamics, the ULP model exhibits *ULP-
electrodynamics*. This is a described by the Lagrangian,

\[
\mathcal{L}_{\text{ULP-EM}} = \mathcal{L}_\pi + \frac{1}{4} F^2 + \varepsilon^2 \varepsilon \pi^+ \pi^- A_\mu A^\mu + g \frac{\pi^0}{2 F} F \tilde{F},
\]

where \( \mathcal{L}_\pi \) corresponds to the mass and kinetic terms of
the ULPs. From this one may derive the modified vac-
uum Maxwell equations as,

\[
\nabla \cdot \vec{E} = \frac{g}{F_\pi} \vec{B} \cdot \nabla \pi^0 - e^2 \varepsilon \pi^+ \pi^- V,
\]

\[
\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \frac{g}{F_\pi} \left( \vec{E} \times \nabla \pi^0 - \vec{B} \frac{\partial \pi^0}{\partial t} \right) - e^2 \varepsilon \pi^+ \pi^- \vec{A}.
\]
We defer to future work an exploration of the ULP signal for axion detection experiments such as ADMX [100]. One might also search for these interactions in form of “Cosmic ULP backgrounds”, analogous to the recently proposed Cosmic Axion Background [101], but with one cosmic background for each ULP. Also in a cosmological context, the coupling of photons to the charged pions may be an additional source of resonant production of the former. The equation of motion for a Fourier mode of the photon field $A^\mu$, with wavenumber $k$ and polarization $\pm$, in an Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime, is given by,

$$A^\mu_{k\pm} + (k^2 + e^2\varepsilon^2\pi^+\pi^- \pm \frac{g}{f_\pi} k^0 \gamma^\mu) A_{k\pm} = 0,$$

where $'\gamma^\mu$ denotes a derivative with respect to conformal time. This above differs from the more conventional axion case (see e.g. [102]) by the mass-like coupling to the charged pions. The latter provides an additional mechanism for parametric resonance production of photons, as has been studied in an axion context in e.g. [103].

### VI. DISCUSSION

In this work we have developed the theory of Ultra-Light Pion (ULP) dark matter and dark baryon WIMPZillas, which together comprise the ULP-WIMPZilla model. In this model, ultra-light dark matter arises as composite states of a confining gauge theory, namely the Goldstone bosons of chiral symmetry breaking, analogous to the Standard Model pions. In the limit of very small dark quark masses, $m_\chi \lesssim 10^{-18}\text{eV}$, and high confinement scale $\Lambda_\chi \gtrsim 10^{10}\text{GeV}$, the dark pions enjoy an axion-like cosmological history and can provide the observed abundance of dark matter. The mass spectrum of pions encodes the charge and confinement scale of the dark QCD-like theory, and is in turn encoded in the density profile of dark matter halos (or boson stars, depending on the mass of the pions).

As the name would suggest, the pions themselves are only part of the ULP-WIMPZilla model. There are additional degrees of freedom in the theory which may exhibit interesting dynamics, such as the dark baryons. Due to the high confinement scale, the dark baryons naturally realize the WIMPZilla paradigm. Produced either gravitationally or via freeze-in, the dark baryons can constitute a small to significant fraction of the dark matter for a wide range of parameters.

The are a few things worth highlighting before closing this article:

1. The ULP-WIMPZilla model is the first scenario of ultra-light ($m < eV$) dark matter in a confining gauge theory, and the first example of an electrically millicharged ultra-light ($m < eV$) dark matter candidate. The related but distinct mass range $m = [\text{eV}, 10\text{keV}]$ is studied in [34].

2. ULPs are the first model of 3 ultralight scalars, building on previous work on two ultralight scalars [76, 77]. The ULP model predicts two of the three scalars are degenerate in mass, and predicts a mass splitting with the third set by the millicharge.

3. The ULP-WIMPZilla model is the first scenario to unify ultralight and fermionic WIMPZilla dark matter, with the two components unavoidable connected by common underlying parameters. Dark matter is generically an admixture of neutral pions, charged pions, and baryon WIMPZillas. This leads to a diversity of dark matter halos.

| Method                        | Constraint on $\varepsilon$ | Reference | Avoidance |
|-------------------------------|-----------------------------|-----------|-----------|
| Supernova cooling             | $10^{-9} < \varepsilon < 10^{-7}$ | [81]     |           |
| Stellar cooling               | $\varepsilon < 1.7 \times 10^{-14}$ | [81, 82] |           |
| Solar cooling                 | $\varepsilon < 10^{-13.6}$ | [83]     |           |
| Magnets                       | $\varepsilon^2 \frac{k^0}{k^2} < 10^{-12}$ | [84]     |           |
| Milky Way satellites          | $\varepsilon < 10^{-15} \frac{k^0}{k^2}$ | [34, 85] | $f_\varepsilon < 10\%$ |
| BBN                           | $\varepsilon < 2.1 \times 10^{-9}$ | [81, 86, 87] |           |
| CMB                           | $\varepsilon < 2 \times 10^{-12} \frac{k^0}{k^2}$ | [88–90] |           |
| SZ effect                     | $\varepsilon < 2 \times 10^{-9}$ | [91]     | $m_\varepsilon \gtrsim 10^{-7} \text{eV}$ |
| SN dimming                    | $\varepsilon < 4 \times 10^{-9}$ | [92]     | $m_\varepsilon \gtrsim 10^{-7} \text{eV}$ |
| Pulsar timing and FRBs       | $\varepsilon \frac{k^0}{k^2} < 10^{-8}$ | [93]     | $f_\varepsilon < 10\%$ |
| Laboratory Cosmology          |                             |           |           |
| Laser experiments             | $\varepsilon < 3 \times 10^{-6}$ | [92]     |           |
| Lamb shift                    | $\varepsilon < 10^{-4}$ | [94]     |           |
| Positronium                   | $\varepsilon < 3.4 \times 10^{-5}$ | [95]     |           |
| Coulomb’s law deviations      | $\varepsilon \lesssim 5 \times 10^{-6}$ | [96]     | $m_\varepsilon \gtrsim 1 \text{eV}$ |
| Schwinger effect in cavities  | $\varepsilon \lesssim 10^{-6}$ | [97]     | $m_\varepsilon \gtrsim 1 \text{eV}$ |

TABLE II. The current (astrophysical, cosmological, and laboratory) constraints on MCPs with $m_\text{MCP} \lesssim 1 \text{MeV}$, such as light and ultra-light pions. (For possible proposed mechanisms to evade stellar bounds see [98] and the references therein.) The column “avoidance” refers to the region of parameter space where the constraint of that row does not apply. Here $f_\varepsilon$ denotes the fraction of dark matter, not the decay constant.
4. Depending on the millicharge, the quark mass, and confinement scale, the charged pions may have a mass comparable to the neutral pion or may be much heavier. In the latter case, the neutral pion can be wave-like (or ‘fuzzy’) while the charged pions may be wave-like or particle-like. Charged pions with $m_{\pi^\pm} > 10$ keV, exhibit their own phenomenology, which we defer to future work.

5. The strongest constraint on the light millicharge by CMB, is alleviated if the charged pions are a sub-dominant component of the dark matter, $f_{\pi^\pm} < 0.4\%$. This is independent of the fraction of DM in the neutral pions or dark baryons. This significantly opens up the parameter space of the model, while being testable at future experiments. The ultra-light scalar with $m < 1$ eV behaves as a coherent state and the CMB constraints on that case have never been worked out. We leave this effect to future work.

Finally, we address an important question: how can ULPs, and their UV completion in dark QCD, be distinguished experimentally from a conventional axion model, with a UV completion in scalar fields, and which is devoid of strong interactions? The suggestion presented in this work is to perform a dedicated search for the whole structure of the theory, namely, the ULP-WIMPzilla model, its pattern of charges and masses, and the associated phenomenology (such as gravitational waves from the dark QCD phase transition). This proposal sidesteps a related but distinct question: Given a detection at ADMX or related axion-search – how may we test the nature of the axion as fundamental or composite? We leave this interesting question to future work.

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**Appendix A: Number density of heavy dark baryons**

This appendix presents the calculations of heavy fermion generation by a Higgs portal of the form Eq. (39). Here we followed the analysis performed in [67]. At temperatures above the EW symmetry breaking ($T > 100$ GeV), the Higgs field includes two states i.e. $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$, and hence there are two annihilation channels to dark baryons

\[ H^0H^0 \rightarrow \bar{b}_s b_s \quad \text{and} \quad H^+H^- \rightarrow \bar{b}_s b_e. \tag{A1} \]

Due to the isospin symmetry, the matrix elements of these two channels are equivalent, and the final thermally averaged cross section is doubled. The thermally averaged annihilation cross section is

\[
\langle \sigma_H v \rangle = \frac{4^2}{n_{\text{eq}}n_{\text{eq}}} \int \frac{d^3 \vec{p}_1}{(2\pi)^3} \int \frac{d^3 \vec{p}_2}{(2\pi)^3} \bar{\sigma}_{H^+H^- \rightarrow \bar{b}_s b_s} v_{\text{Møll}}(p_1, p_2) \times \exp\left(-\frac{(E_1 + E_2)}{T}\right), \tag{A2} \]

where $\bar{\sigma}_{H^+H^- \rightarrow \bar{b}_s b_s}$ is the spin-averaged annihilation cross section and $v_{\text{Møll}}(p_1, p_2)$ is the Møller velocity

\[
v_{\text{Møll}}(p_1, p_2) = \sqrt{\vec{v}_1 \cdot \vec{v}_2} - |\vec{v}_1 \times \vec{v}_2|, \tag{A3} \]

here $\vec{v} = \vec{p}/E$. Since $\bar{\sigma}_{H^+H^- \rightarrow \bar{b}_s b_s}$ is a function of $s$ only, Eq. (A4) can be further simplified as [67]

\[
\langle \sigma_H v \rangle = \frac{4^2}{n_{\text{eq}}n_{\text{eq}}} \frac{T}{32\pi^4} \int_0^{\infty} ds \sqrt{s - 4m_b^2} K_1(\sqrt{s}/T) \times \bar{\sigma}_{H^+H^- \rightarrow \bar{b}_s b_s}(s), \tag{A4} \]

where $s = (p_1 + p_2)^2$, $K_1$ is the modified Bessel function of the second kind of order 1, and $\bar{\sigma}_{H^+H^- \rightarrow \bar{b}_s b_s}$ is the cross section associated to the Higgs annihilation

\[
\bar{\sigma}_{H^+H^- \rightarrow \bar{b}_s b_s}(s) = \frac{1}{32\pi^4} \frac{\lambda^2}{\Lambda_H^3} \frac{1}{8} \sqrt{s - 4m_b^2} \sqrt{s - 4m_H^2}. \tag{A5} \]

In the limit that $m_b \gg T$, the thermally averaged annihilation cross section of dark fermions with mass $m$ is [67]

\[
\langle \sigma_H v \rangle \approx \frac{1}{32\pi^4} \frac{\lambda^2}{\Lambda_H^3} \frac{3T}{m_b}. \tag{A6} \]

The number density of dark baryons generated by the SM Higgs is

\[
n_b(t) = a^{-3}(t) \int_{\Delta a(t)} \frac{d\ln a}{\Delta H} a^3(\Gamma_H)n_{\text{eq}} \approx \frac{4}{(2\pi)^4} \left( \frac{a_{\text{inf}}}{a(t)} \right)^{\frac{3}{2}} \left( \frac{\lambda}{\Lambda_H} \right)^2 \frac{m_b T_{\text{max}}^3}{H_{\text{inf}}} \exp\left[-\frac{2m_b}{T_{\text{max}}} \right]. \tag{A7} \]

where $n_{\text{eq}} = 4(\frac{m_b T}{2\pi})^\frac{3}{2} e^{-\frac{m_b}{T}}$ we used $m_b \gg T_{\text{reh}}$ limit and we have

\[
\frac{a_{\text{inf}}}{a(t_0)} = \frac{a_{\text{reh}}}{a(t_0)} = \left( \frac{\rho_{\text{rer}}}{\rho_{\text{inf}}} \right)^{\frac{3}{2}} \left( \frac{g_{0,s}}{g_{\text{reh},s}} \right)^{\frac{1}{2}} T_0. \tag{A8} \]

Here $T_0$ is the temperature of the Universe today and $g_{\text{reh},s}$ and $g_{0,s} = 3.91$ are the effective number of relativistic degrees of freedom contributing to the total entropy at reheating and today respectively. The relic density of the dark baryons produced today through the Higgs
portal is

\[ \Omega'_b = \frac{m_b \rho(t_0)}{\rho_{\text{crit}}} \]

\[ \approx \frac{5 \times 10^{10}/(2\pi)^4}{1 + w} g_{r_{\text{reh}}} \left( \frac{T_{\text{reh}}}{T_{\text{reh}}} \right)^3 \left( \frac{\Lambda}{\Lambda_H} \right)^2 \left( \frac{m_b^2}{T_{\text{reh}}} \right)^2 \]

\[ \times \left( \frac{0.2}{10^5} \right)^{5.3} \frac{3.91}{100} \left( \frac{10^{\pi^2} - (1/w)^{1/2}}{(1/w)^{1/2}} \right)^{1/(w+1) - 13/8} \]

\[ \times \frac{T_{\text{reh}}^{4/(1+w) + 5/2}}{(M_{\text{pl}} H_{\text{inf}})^{2/(1+w) + 7/4}} M_{\text{pl}} \exp\left[-\frac{2 m_b}{T_{\text{max}}} \right] eV^{-4}, \]

where \( \rho_{\text{crit}} = 0.8 \times 10^{-10} eV^4 \) is the total energy density of the universe today. In case that \( T_{\text{reh}} \gg T_{\text{EW}} \) and using the fact that \( T_0 = 0.24 \times 10^{-3} eV \) and \( M_{\text{pl}} = 2.4 \times 10^{18} GeV \), we have

\[ \Omega'_b \approx \frac{5.6 \times 10^{15}}{(2\pi)^2} (1+w) \left( \frac{\Lambda}{\Lambda_H} \right)^2 \left( \frac{M_{\text{pl}} H_{\text{inf}}}{T_{\text{reh}}^{1 + w/3}} \right)^{1 + w/3} \]

\[ \times \exp\left[ -10 \left( \frac{m_b^2}{M_{\text{pl}} H_{\text{inf}}} \right)^{1/2} \left( \frac{T_{\text{reh}}^2}{T_{\text{inf}}^{1 + w/3}} \right)^{1/2} \right]. \] (A9)

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