On high $Q^2$ behavior of the pion form factor for transitions $\gamma^*\gamma \to \pi^0$ and $\gamma^*\gamma^* \to \pi^0$ within the nonlocal quark-pion model.

I.V. Anikin$^1$, A. E. Dorokhov$^{1,2}$, Lauro Tomio$^2$

$^1$ Bogoliubov Laboratory for Theoretical Physics, Joint Institute for Nuclear Research, 141980, Dubna, Russia
$^2$ Instituto de Física Teórica, UNESP, Rua Pamplona, 145, 01405-900, São Paulo, Brazil

(March 20, 2018)

The behavior of the transition pion form factor for processes $\gamma^*\gamma \to \pi^0$ and $\gamma^*\gamma^* \to \pi^0$ at large values of space-like photon momenta is estimated within the nonlocal covariant quark-pion model. It is shown that, in general, the coefficient of the leading asymptotic term depends dynamically on the ratio of photon virtualities. The kinematic dependence of the transition form factor allows us to obtain the relation between the pion light-cone distribution amplitude and the quark-pion vertex function. The dynamic dependence indicates that the transition form factor $\gamma^*\gamma \to \pi^0$ at high momentum transfers is very sensitive to the nonlocality size of nonperturbative fluctuations in the QCD vacuum.

Keywords: Nonperturbative calculations, pion form factors, nonlocal theories, and models

I. INTRODUCTION

The interest in the form factor $T_{\pi^0}(q_1^2, q_2^2)$ for transition processes $\gamma^*(q_1)\gamma(q_2) \to \pi^0(p)$ and $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$, where $q_1$ and $q_2$ are photon momenta, has again increased recently. Experimentally, the data on the form factor $T_{\pi^0}$ for small virtuality of one of the photons, $q_1^2 \approx 0$, with the virtuality of the other photon being scanned up to 8 GeV$^2$, are known from CELLO [1] and CLEO [2] Collaborations. Theoretically, at zero virtualities, the form factor $T_{\pi^0}(0, 0)$ is related to the axial anomaly. At asymptotically large photon virtualities, the behavior is predicted by perturbative QCD (pQCD) [3, 4] (see [5] for recent discussions) and depends crucially on the internal pion dynamics that is parametrized by the nonperturbative pion distribution amplitude (DA), $\varphi^A_s(x)$, with $x$ being the fraction of the pion momentum, $p$, carried by a quark. Moreover, the knowledge of the off-shell structure of the form factor enables one to significantly reduce the uncertainty in the evaluation of the hadronic light-by-light scattering contribution to the muon $g-2$ [6], which is relevant for the current experiment E821 at BNL [7].

In the following, it is convenient to parametrize photon virtualities as $q_1^2 = -(1 + \omega)Q^2/2$, $q_2^2 = -(1 - \omega)Q^2/2$, where $Q^2$ and $\omega$ are, respectively, the total virtuality of the photons and the asymmetry in their distribution:

$$Q^2 = -(q_1^2 + q_2^2) \geq 0, \quad \omega = (q_1^2 - q_2^2)/(q_1^2 + q_2^2).$$

The experimental data from CLEO [2] for the process $\gamma^*\gamma \to \pi^0$ ($|\omega| = 1$) can be fitted by a monopole form factor:

$$T_{\pi^0}(q_1^2 = -Q^2, q_2^2 = 0)|_{fit} = \frac{g_{\pi\gamma\gamma}}{1 + Q^2/\Lambda^2_{\pi}}, \quad \Lambda_{\pi} \simeq 0.77 \text{ GeV},$$

where $g_{\pi\gamma\gamma}$ is the two-photon pion decay constant. In the lowest order of pQCD, by using the light-cone Operator Product Expansion (OPE), the high $Q^2$ behavior of the form factor is predicted [3, 4] as

$$T_{\pi^0}(q_1^2, q_2^2)|_{Q^2 \to \infty} = J(\omega) \frac{f_\pi}{Q^2} + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{Q^4}\right),$$

with the asymptotic coefficient given by

$$J(\omega) = \frac{4}{3} \int_0^1 \frac{dx}{1 - \omega^2(2x - 1)^2} \varphi^A_s(x),$$

where $f_\pi = 93$ MeV is the weak pion decay constant and the leading-twist pion light-cone DA is normalized by $\int_0^1 dx \varphi^A_s(x) = 1$. Since the pion DA reflects the internal nonperturbative pion dynamics, the prediction of the value of $J(\omega)$ is a rather nontrivial task, and its accurate measurement would provide quite valuable information.
It is important to note that for the transition process considered, the leading asymptotic term of pQCD expansion \( \phi_\pi^{A,\text{asympt}}(x) \) is not suppressed by the strong coupling constant \( \alpha_s \). Hence, the pQCD prediction \( \phi_\pi^{A,\text{asympt}}(x) \) may become reasonable at the highest of the presently accessible momenta \( Q^2 \sim 10 \text{ GeV}^2 \). At asymptotically high \( Q^2 \), the DA evolves to \( \phi_\pi^{A,\text{asympt}}(x) \approx 6x(1-x) \) and \( J_{\text{asympt}}(\omega = 1) = 2 \). The fit of CLEO data \( \phi_\pi^{A,\text{asympt}}(x) \) corresponds to \( J_{\text{CLEO}}(\omega \approx 1) \approx 1.6 \) indicating that already at moderately high momenta this value is not too far from its asymptotic limit.

However, since the pQCD evolution of the DA reaches the asymptotic regime very slowly, its exact form at moderately high \( Q^2 \) may not coincide with \( \phi_\pi^{A,\text{asympt}}(x) \). At lower \( Q^2 \), the power corrections to the form factor become important. Thus, to study the behavior of the transition form factor, at experimentally accessible \( Q^2 \), is the subject of nonperturbative dynamics, where the same type of the leading high \( Q^2 \) behavior, as in eq. (3), was obtained by different methods. So, the theoretical determination of the transition form factor is still challenging, and it is desirable to obtain direct calculations of \( T_{\gamma\gamma}^\mu(q_1^2, q_2^2) \), without any a priori assumptions about the shape of the pion DA.

The transition form factor in the symmetric kinematics, \( \phi_\pi^2 = q_2^2 \), at high virtualities was considered in \([10]\) by using the local OPE with the result \( J_{\text{OPE}}(\omega = 0) = 4/3 \) for the asymptotic coefficient, which is in agreement with prediction from \( \phi_\pi \) at \( \omega = 0 \). Within the local OPE, one can represent \( J_{\text{OPE}}(\omega) \) as an expansion in powers of \( \omega^2 \), with the coefficients of expansion given by the moments of the pion DA: \( \int_0^1 dx (2x-1)^{2n} \phi_\pi^{A}(x) \). In \([11] \) (see also \([12]\)), it was shown that the local OPE is well convergent in the kinematic region, when the photon virtualities are close to each other: \( |\omega| \lesssim 1/2 \). In this kinematics, the result for the asymptotic coefficient is still close to 4/3. However, in these references it was pointed out that for \( |\omega| \gtrsim 1/2 \) potentially large corrections to the first term of the local OPE \([11,12]\) and also to the light-cone pQCD \([3]\) predictions exist at any finite \( Q^2 \). With increasing \( |\omega| \), the number of terms of OPE with higher-dimension vacuum expectation values grows rapidly, but it is practically a hopeless task to find more than few terms of the local expansion. Therefore, much more detailed information about the nonperturbative QCD vacuum is necessary to have control over the operator expansion.

In ref. \([13]\), some progress was achieved, by using a refined technique based on the OPE with nonlocal condensates \([14]\) which is equivalent to inclusion of the whole series of power corrections. By using the QCD sum rules with nonlocal condensates, it was shown that this approach works in almost the whole kinematic region \( |\omega| \lesssim 1 \), and that for high values of the asymmetry parameter \( |\omega| \gtrsim 0.8 \), the pion transition form factor is very sensitive to the nonlocal structure of the QCD vacuum. The latter is characterized by the quark virtuality in the vacuum \([13,14]\), \( \lambda^2_{\text{c}} \), and, within the instanton model \([13]\), may be expressed through the average instanton size, \( \rho_c \), as \( \lambda^2_{\text{c}} \approx 2\rho_c^{-2} \) \([15,16]\). In \([13]\), the form factor \( \gamma^*\gamma \rightarrow \pi^0 \) was directly calculated from a QCD sum rule for the three-point function, leading to the estimate \( J_{\text{QCD}^{\text{asym}}}(\omega = 1) \approx 1.6 \pm 0.3 \).

The covariant nonlocal low-energy models (see, e.g., \([13,20]\)), based on the Schwinger-Dyson (SD) approach to dynamics of quarks and gluons, have many attractive features, as the approach is consistent with the low-energy theorems. In particular, the Abelian axial anomaly is within this approach, and the standard result for \( T_{\gamma\gamma}(0,0) \equiv g_{\gamma\gamma}^\gamma = (4\pi^2 f_\pi)^{-1} \) is reproduced exactly. Within this nonperturbative model of quark-pion interaction, both the small mass and composite structure of the pion are realistically described. Furthermore, the intrinsic nonlocal structure of the model may be motivated by fundamental QCD processes like the instanton and gluon exchanges. In \([20]\) the transition form factor \( \gamma^*\gamma \rightarrow \pi^0 \) was considered at low \( Q^2 \) and agreement with data was obtained. There, it was observed that the results are very sensitive to the value of constituent quark mass.

In this letter, within the covariant nonlocal low-energy model of the quark-pion interaction we study the high \( Q^2 \) behavior of the pion transition form factor \( \gamma^*\gamma \rightarrow \pi^0 \) in general kinematics. We show that the asymptotic coefficient \( J(\omega) \), as demonstrated in QCD sum rules \([13,18]\), depends on the kinematics of the transition process and on the internal pion dynamics induced by the nonlocal structure of the QCD vacuum. The dynamic dependence of \( J \) is governed by the so-called diluteness parameter \( M_q/\lambda_q \), where \( M_q \) is the constituent quark mass. When considering the model dependence of the asymptotic coefficient \( J \), experimental data can be very useful to distinguish between different assumptions made on nonperturbative dynamics of the QCD vacuum. Within the nonlocal quark-pion model the expression for the asymptotic coefficient \( J \) is found in the whole kinematic region of \( \omega \). Moreover, from this dependence, the pion DA is reconstructed in terms of the quark-pion vertex function.

II. EFFECTIVE QUARK-PION MODEL AND PION TRANSITION \( \gamma^*\gamma \rightarrow \pi^0 \) FORM FACTOR

The effective quark-pion dynamics can be summarized in the covariant nonlocal action given by

---

1 This is in contrast with the case of electromagnetic form factors and the wide-angle Compton scattering process (e.g., see Ref. \([1]\), where the soft overlap contributions are important at moderately high \( Q^2 \).
where the dynamic vertex $F[\frac{x+y}{2}, x-y/2; \Lambda^{-2}]$ with nonlocality size $\Lambda^{-1}$ depends on the coordinates of the quark and antiquark; $q(x)$ and $\pi(x)$ are, respectively, the quark and pion fields. The nonlocal vertex characterizes the coordinate dependence of order parameter for spontaneous chiral-symmetry breaking and can be expressed in terms of the nonlocal quark condensates.

In the following calculations, we restrict ourselves to the approximation (see, e.g. [20])

$$F[\frac{x+y}{2}, x-y/2; \Lambda^{-2}] \rightarrow F(y^2, \Lambda^{-2}),$$

when the dynamic quark-pion vertex depends only on the relative coordinate of the quark and antiquark squared, $y^2$, if neglecting the dependence of the vertex on angular variable $(\pi x)$. The Fourier transform of the vertex function in the Minkowski space is defined as $\tilde{F}(k^2; \Lambda^2) = \int d^4 x F(x^2; \Lambda^{-2}) \exp(-ikx)$ with normalization $\tilde{F}(0; \Lambda^2) = 1$, and we assume that it rapidly decreases in the Euclidean region ($k^2 = -k_E^2 \equiv -u$). We also approximate the momentum-dependent quark self-energy in the quark propagator $S^{-1}(k) = \tilde{k} - M_q$ by a constant mass $\Lambda$ and neglect small effects of the pion mass. We have to note that the approximations used are not fully consistent. In particular, due to neglecting the momentum dependence of the quark mass, some low-energy theorems are violated. Further, as we show below, the choice of the model for the quark-pion vertex (11) depending only on the relative coordinate induces a certain artifact in the $x$ behavior of DA (see below). However, these deficiencies of the approximation chosen are not essential for the present purposes and do not lead to essential numeric errors.

The quark-pion coupling is given by the compositeness condition [20]

$$g_{\pi\bar{q}q}^2 = \frac{N_c}{8\pi^2} \int_0^\infty du \tilde{F}^2(-u; \chi^{-2}) \frac{3 + 2u}{(1 + u)^3},$$

and the pion weak decay constant is expressed by

$$f_\pi = \frac{N_c g_{\pi\bar{q}q}}{4\pi^2} M_q \int_0^\infty du \tilde{F}(-u; \chi^{-2}) \frac{1}{(1 + u)^2}.$$  

We have rescaled the integration variable by the quark mass squared and introduced the parameter $\chi = M_q/\Lambda$ that characterizes the diluteness of the QCD vacuum. Within the instanton vacuum model, the size of nonlocality of the nonperturbative gluon field, $\rho_c \sim \Lambda^{-1}$, is much smaller than the quark Compton length $M_q^{-1}$; thus, $\chi$ is a small parameter [13].

Let us consider the contribution to the $\gamma^*\gamma^*\pi^0$ invariant amplitude as calculated from the triangle diagrams:

$$M(\gamma^*(q_1, e_1)\gamma^*(q_2, e_2) \rightarrow \pi^0(p)) = m_{\pi\gamma\gamma}(q_1, e_1; q_2, e_2) + m_{\pi\gamma\gamma}(q_2, e_2; q_1, e_1)$$

where $e_i (i = 1, 2)$ are the photon polarization vectors, and

$$m_{\pi\gamma\gamma}(q_1, e_1; q_2, e_2) = -\frac{N_c}{3} g_{\pi\bar{q}q} \int \frac{d^4k}{(2\pi)^4} \tilde{F}(k^2; \Lambda^2) tr\{i\gamma_5 S(k-p/2)\bar{e}_2 S[k-(q_1-q_2)/2]\bar{e}_1 S(k+p/2)\}.\hspace{1cm} (9)$$

If the tensor $\epsilon_{\mu\nu\rho\sigma}e'_1^\mu e'_2^\nu e_2^\rho e_1^\sigma$ is factorized from this amplitude, the form factor can be expressed as

$$T_{\pi\pi}(q_1^2, q_2^2) = \frac{g_{\pi\bar{q}q}}{2\pi^2} M_q I_{\pi\gamma\gamma}(q_1^2, q_2^2, p^2),$$

where the Feynman integral $I_{\pi\gamma\gamma}(q_1^2, q_2^2, p^2)$ is given by

$$I_{\pi\gamma\gamma}(q_1^2, q_2^2, p^2) = \int \frac{d^4k}{i\pi^2} \int \frac{d^4l}{i\pi^2} \tilde{F}(k^2; \Lambda^2) \bar{q}(\bar{x}p) + \bar{q}(xp)$$

with $q(x)$ being the fraction of the pion momentum $p$ carried by the quark produced at the $q_1$ ($q_2$) photon vertex. The relevant diagram is similar to the handbag diagram for hard exclusive processes, with the main difference that one should use, as a nonperturbative input, the quark-pion vertex instead of the pion DA. As we see below, this similarity allows one to translate the form of the quark-pion vertex into a specific shape of the pion DA.
III. PION TRANSITION \( \gamma^*\gamma^* \to \pi^0 \) FORM FACTOR AT MODERATELY HIGH \( Q^2 \)

In this section, we estimate the asymptotics of the transition form factor. To this end, we rewrite the expression for integral \( [14] \) in the form that is obtained after rotating to the Euclidean space \( [k^2 \to -u, -id^4k \to \pi^2udu, F(k^2; \Lambda^2_2) \to F(-u; \Lambda^2)] \), by using the Feynman \( \alpha \)–parameterization for the denominators and integrating over the angular variables. Then, the corresponding integral \( I_{\pi,\gamma,\gamma} \) is given by

\[
I_{\pi,\gamma,\gamma}(q_1^2, q_2^2, p^2) = \int_0^\infty duu \bar{F}(-u; \Lambda^2) \int_0^1 d\alpha \left[ \frac{1}{\sqrt{b^4 - a_+^4}} \left( b^2 + \sqrt{b^4 - a_+^4} \right) + \frac{1}{\sqrt{b^4 - a_-^4}} \left( b^2 + \sqrt{b^4 - a_-^4} \right) \right],
\]

(12)

where

\[
b^2 = M_\pi^2 + u - \frac{1}{2} \alpha Q^2 - \frac{1}{4} (1 - 2\alpha) p^2, \quad a_\pm^4 = 2\alpha Q^2 (\alpha \pm \omega (1 - \alpha)) - (1 - 2\alpha) u p^2.
\]

(13)

In this way, the expression \( [12] \) can be safely analyzed in the asymptotic limit of high total virtuality of the photons \( Q^2 \to \infty \). Moreover, the integral over \( \alpha \) can be taken analytically, leading, in the chiral limit \( m_\pi = 0 \), to the asymptotic expression given by eq. \( [3] \), where

\[
J(\omega) \equiv J_{np}(\omega) = \frac{2N_\pi}{3\omega} \left\{ \int_0^\infty du \frac{\bar{F}(-u; \chi^{-2})}{1 + u} \ln \left[ \frac{1 + u (1 + \omega)}{1 + u (1 - \omega)} \right] \right\}
\]

(14)

\[
N_\pi = \left[ \int_0^\infty duu \frac{\bar{F}(-u; \chi^{-2})}{(1 + u)^2} \right]^{-1}.
\]

The integrand in the numerator of \( [14] \) is quite different from the integrand defining the decay constant \( f_\pi \), given in eq. \( [8] \). From eq. \( [14] \) it is clear, that the prediction of the nonperturbative approach about the asymptotic coefficient is rather sensitive to the product \( \chi \) of the value of the constituent mass \( M_q \) and the size of nonlocality \( \Lambda^{-1} \) of the vertex \( F(x^2; \Lambda^{-2}) \) and to the relative distribution of the total virtuality among photons, \( \omega \). In particular, for the off-shell process \( \gamma^*\gamma^* \to \pi^0 \) in the kinematic case of symmetric distribution of photon virtualities, \( q_1^2 = q_2^2 \to -\infty \) \( (\omega \to 0) \), the result obtained from eq. \( [14] \) is \( J(\omega = 0) = 4/3 \) in agreement with the OPE prediction \( [10] \).

Let us note, that we use an approximation to the model with constant constituent quark masses for all three quark lines in the diagram of the process. However, the asymptotic result \( [14] \) is independent of the value of the mass parameter in the quark propagator with hard momentum flow, as it should be. The other two quark lines remain soft during the process; thus, the mass parameter \( M_q \) may be considered as given on a certain characteristic soft scale in the momentum-dependent case \( M_q(\lambda_q^2) \). It means that the dynamic and kinematic dependence of \( J_{np}(\omega) \) found in \( [14] \) will be unchanged, even if one includes the momentum dependence of the quark mass and considers the dressed quark-photon vertex which goes into the bare one, \( \gamma^\mu \), as one of the squared quark momenta becomes infinite.

Both the expressions for \( J \) derived within the nonlocal quark-pion model \( [4] \) and from the light-cone OPE \( [3] \) can be put into the common form

\[
J(\omega) = \frac{2}{3\omega} \int_0^1 d\xi R(\xi) \ln \left[ \frac{1 + \xi \omega}{1 - \xi \omega} \right]
\]

(15)

with

\[
R_{pQCD}(\xi) = -\frac{d}{d\xi} \varphi_\pi^A \left( \frac{1 + \xi}{2} \right) \quad \text{and} \quad R_{np}(\xi) = N_\pi \bar{F} \left( \frac{-\xi}{1 - \xi}; \chi^{-2} \right) \frac{1}{1 - \xi}, \quad 0 \leq \xi \equiv (2x - 1) \leq 1
\]

(16)

and similar expressions for \(-1 \leq \xi \leq 0\). Equating both contributions, we find the pion DA in terms of the vertex function on a certain low-energy scale \( \mu_0^2 \sim \Lambda^2 \)

\[
\varphi_\pi^A(x) = N_\pi \int_{|2x - 1|}^1 \frac{dy}{1 - y} \bar{F} \left( \frac{-y}{1 - y}; \chi^{-2} \right).
\]

(17)

Thus, we show that eq. \( [14] \) obtained within the nonlocal quark-pion model is equivalent to the standard lowest-order pQCD result \( [4] \), with the only difference that the nonperturbative information accumulated in the pion DA \( \varphi_\pi^A(x) \) is represented by the quark-pion vertex function \( \bar{F}(-u; \chi^{-2}) \).
We have to note that an explicit form of the asymptotic coefficient \( \lambda_0^2 \) and the relation between the DA and the vertex function depend on the model of quark-pion interaction \( \chi \). In particular, the expression \( \lambda_0^2 \approx 0.5 \pm 0.1 \text{GeV}^2 \) is obtained within the approximation \( \chi \), when the quark-pion vertex depends only on the relative coordinate. This approximation results in the artificial dependence of DA on the modulo function of \( x \) and leads to the nonsmooth behavior of the distribution at \( x = 1/2 \) (see Fig. 1). These peculiarities disappear if the angular dependence of the vertex motivated by, e.g., the instanton model is recovered \( \chi \).

Let us estimate a realistic value for the diluteness parameter \( \chi \) and check if the model under consideration is consistent with CLEO data. The vertex function \( \tilde{F}(k^2; \Lambda^2) \) phenomenologically describes the nonlocal structure of the nonperturbative QCD vacuum and may be modeled within the instanton vacuum model \( \chi \). For the present purpose, the vertex function can be well approximated by the Gaussian form \( \tilde{F}(k^2; \Lambda^2) = \exp(k^2/\Lambda^2) \). The inverse size of the vertex nonlocality, \( \Lambda \), is naturally related to the average virtuality of quarks that flow through the vacuum, \( \Lambda^2 = 2 \times 10^{-1} \text{MeV} \). Varying the model parameters within the intervals \( \Lambda^2 \approx 5 \text{MeV} \), we find that the DA is suppressed at the edges of the kinematic interval \( 1 - |1 - 2x| < 2x_0 \), where quarks are soft. As it was pointed out earlier, the incorporation of nonperturbative effects results in the intrinsic transverse structure of hadronic wave functions \( \chi \), as well as the Sudakov perturbative factor \( \chi \) modifies the hard scattering picture of exclusive reactions and essentially improves it. As a result, perturbative QCD calculations of the hadron form factors extend the kinematic region of self - consistency from asymptotic values of \( Q \) to the region starting from \( Q \sim O(1 \text{GeV}) \). In the opposite extreme case of a very dense medium (heavy quark limit, \( M_q >> \Lambda \)), \( J_{\text{dense}}(\omega = 1) = 4/3 \), as it is predicted by the first term in the OPE result \( \chi \). In that case, the limit \( y_1 \) is small, and the integrand in \( \chi \) is concentrated in the vicinity of \( x = 1/2 \). Thus, the DA becomes \( \phi^4(x) \propto \delta(x - 1/2) \), as it is expected. As we shown above, a realistic situation is in-between these two extremes.

These different situations are illustrated in terms of the pion DA, \( \chi \), in Fig.1. As it is clear from the figure, the model pion DA, under the realistic choice of the parameter \( \chi \approx 0.4 \), is close to the asymptotic DA. As noticed in the introduction, by considering the actual accessible data, the nonperturbative dynamics may dominate. Therefore, the data turn out to be quite restrictive and uniquely indicate that the dilute regime is realized in the QCD vacuum. In Fig. 2, for the process \( \gamma \gamma \rightarrow \pi^0 \) (\( \omega = 1 \)), we plot the asymptotic coefficient \( J_{\text{np}}(\omega = 1) \) as a function of the dynamical diluteness parameter squared \( \chi^2 \). In this figure, we indicate the values of \( J_{\text{np}}(\omega = 1) \) obtained from CLEO data and model predictions. In Fig. 3, the asymptotic coefficient \( J_{\text{np}}(\omega) \) is plotted as a function of the kinematic asymmetry parameter \( \omega \), at \( \chi^2 = 0.15 \) and \( \chi^2 = 0.35 \). The first value of \( \chi \) corresponds to the model estimate; and the second one, to the central point of the CLEO data fit.

---

\(^2\) In \( \chi \), emerging of a similar cusp for the pion distribution function and its disappearance, if the angular dependence in the vertex is taken into account, were demonstrated.
To get further interpretation of eq. (17), we can express the DA as the transverse momentum integral of the pion light-cone wave function

\[ \varphi^A_\pi(x) = \int_0^\infty d\vec{k}_\perp^2 \Psi^A_\pi(x, \vec{k}_\perp^2). \]  

(20)

Rewriting the r.h.s. of (17) via the original variable \( u = y/(1-y) \) and then substituting \( u \) by the light-cone combination \( \vec{k}_\perp^2/(x\bar{x}) \), that is the invariant mass of the \( q\bar{q} \) pair squared, we identify the pion wave function as

\[ \Psi^A_\pi(x, \vec{k}_\perp^2) = \frac{N_\pi}{x \bar{x} M_q^2 + \vec{k}_\perp^2} \Theta \left( \frac{1 - 2x|x\bar{x}|}{1 - |1 - 2x| M_q^2} \right). \]

(21)

The vertex function \( \bar{F} \) in our model of the pion wave function plays a similar role as the sharp \( \Theta \)-function in the “local duality” wave function \[ \Psi^{A,LD}_\pi(x, \vec{k}_\perp^2) \sim \Theta(\vec{k}_\perp^2 \leq x\bar{x} s_0) \], with \( s_0 = 8\pi^2 f_\pi^2 \) being the duality interval. Note that numerically \( s_0 \approx 0.67 \text{ GeV}^2 \) is close to the value of the nonlocality parameter \( \lambda_q^2 \). As in the case of (17), the pion light-cone wave function (21) displays non-analytic dependence on \( x \) that disappears if a more realistic quark-pion vertex with the angular dependence is considered.

IV. DISCUSSION AND CONCLUSIONS.

Recently, in [23], it was claimed that the Schwinger-Dyson approach predicts the same asymptotic coefficient \( J(\omega) = 4/3 \) for all nonlocal quark-photon vertices \( \Gamma^{\mu} \{q(k)\bar{q}(k')\gamma_\mu(q)\} \) which go into the bare ones, \( \gamma^\mu \), as soon as one of the squared momenta (\( k^2 \) or \( k'^2 \)) becomes infinite (in the Curtis-Pennington [24] form of the vertex). In [23], the quark propagator that depends on the photon momenta was approximated, at large \( Q^2 \), by its asymptotic form \[ \left[ M_q^2 - (k - (q_1 - q_2)/2)^2 \right]^{-1} \rightarrow -2/Q^2. \] After this change, the integral (11) attains the same form as the integral in (6) defining \( f_\pi \). By taking into account the coefficients in front of the integrals of equations (8) and (6), one immediately reaches the conclusion of [23] (see also [27]) about the asymptotic coefficient \( J = 4/3 \). As we show above, such a quick asymptotic estimation is rather naive and does not lead to the accurate result. The approximation made in [23] is justified only in the formal limit \( M_q >> \Lambda \).

Our analysis is based on the consideration of a triangle diagram in which the quark propagator and quark-pion vertex are determined nonperturbatively. In this respect, our approach is close to earlier work [23]. However, in [23], the approximations in the calculation of the triangle diagram were used that simplify the dynamics of the process. It turns out that these approximations are not justified in the kinematic region of large \( |\omega| \). As a result the expression was obtained for the asymptotic coefficient that is independent of the internal nonlocal structure of the pion.

In conclusion, within the covariant nonlocal model describing the quark-pion dynamics, we obtain the \( \pi\gamma^*\gamma^* \) transition form factor at moderately high momentum transfers squared, where the perturbative QCD evolution does not yet reach the asymptotic regime. From the model calculations it is possible to find the absolute normalization of the asymptotic \( Q^{-2} \) term. The asymptotic normalization coefficient \( J(\omega) \), given in (4), depends on the ratio of the constituent quark mass on a certain soft scale to the characteristic size of QCD vacuum fluctuations and also on the kinematics of the process. This result does not confirm the statement about the universality of the asymptotic coefficient given in [22,23,24]. When considering the dependence of the asymptotic coefficient on the internal dynamics, the CLEO data are consistent with a small value of the diluteness parameter, which confirms the hypothesis about the small density of the instanton liquid vacuum [15]. From the comparison of the kinematic dependence of the asymptotic coefficient of the transition pion form factor, given by pQCD and the nonperturbative model, the new relation eq. (4) between the pion distribution amplitude and the dynamical quark-pion vertex function is derived. In the specific case of symmetric kinematics, our result agrees with the one obtained by OPE [10]. The present results are in accordance with the conclusions made in [13,18,29] within the QCD sum rules. A more complete analysis of the light pseudo-scalar meson transition form factors will be done later, where effects of the finite hadron masses, nonlocality of the quark-photon vertex, etc., will be considered.

Acknowledgments

We are thankful to N.I. Kochelev for discussions on the subject of the paper. Our special thanks are to S.V. Mikhailov for stimulating and helpful discussions. The referee’s remarks were highly useful. One of us (A.E.D.) thanks the colleagues from Instituto de Física Teórica, UNESP, (São Paulo) for their hospitality and interest in the work. A.E.D. was partially supported by St. - Petersburg center for fundamental research grant: 97-0-6.2-28. I.T. thanks partial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico do Brasil (CNPq) and,
in particular, to the “Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP)” to provide the essential support for this collaboration.

[1] CELLO Collaboration (H.-J. Behrend et. al.), Z. Phys. C 49 (1991) 401.
[2] CLEO Collaboration (J. Gronberg et. al., Phys. Rev. D 57 (1998) 33.
[3] G. P. Lepage and S. J. Brodsky, Phys. Lett. B 87 (1979) 359; Phys. Rev. D 22 (1980) 2157.
[4] M.K. Chase, Nucl.Phys. B167 (1980) 125.
[5] F. Del Aguila, M.K. Chase, Nucl.Phys. B193 (1981) 517; E. Braaten, Phys. Rev., D 28 (1983) 524; E.P. Kadantseva, S.V. Mikhailov, A.V. Radyushkin, Sov. J. Nucl. Phys. 44 (1986) 326; P. Gospdinsky, N. Kivel, Nucl.Phys. B521 (1998) 274.
[6] I. V. Musatov and A. V. Radyushkin, Phys. Rev. D 56 (1997) 2713 and references therein.
[7] M. Hayakawa and T. Kinoshita, Phys. Rev. D 57 (1998) 465.
[8] R.M. Carey et. al., Phys. Rev. Lett. 82 (1999) 253.
[9] N. Isgur, C.H. Llewellyn - Smith Nucl. Phys. B 317 (1989) 526; Phys. Rev. Lett. 52 1080 (1984); A.V. Radyushkin, Nucl. Phys. A 532 (1991) 141; V.A. Nesterenko, A.V. Radyushkin, Phys.Lett. B115 410 (1982); ibid. B128 439 (1983); A.V. Radyushkin, Phys.Rev. D58 114008 (1998); M. Diehl, T. Feldmann, R. Jacob and P. Kroll, Eur. Phys. J. C 8 (1999) 409.
[10] V.A. Novikov et.al., Nucl.Phys., B 237 (1984) 525.
[11] A. Manohar, Phys. Lett. B244 (1990) 101.
[12] A.S. Gorsky, Sov. J. Nucl. Phys. 46 (1987) 537.
[13] S.V. Mikhailov and A. V. Radyushkin, Sov. J. Nucl. Phys. 52 (1990) 697.
[14] S.V. Mikhailov and A.V. Radyushkin, JETP Lett. 43(1986) 712; Sov.J.Nucl. Phys. 49 (1989) 494; Phys. Rev. D45 (1992) 1754.
[15] T. Schäfer and E.V. Shuryak, Rev. of Mod. Phys. 70 (1998) 323, and references therein.
[16] A.E. Dorokhov, S.V. Esaibegyan, and S.V. Mikhailov, Phys. Rev. D 56 (1997) 4062.
[17] A.E. Dorokhov, Lauro Tomio, Quark distribution in the pion within the instanton model, preprint IFT-P.071/98 (1998), hep-ph/9803329.
[18] A. V. Radyushkin and R. T. Ruskov, Nucl. Phys. B 481 (1996) 625; The asymptotics of the transition form factor $\gamma^\gamma \to \pi^0$ and QCD sum rules, hep-ph/9706518.
[19] B. Holdom, J. Terning, K. Verbeek, Phys. Lett. B 232 (1989) 351; Phys. Lett. B 245 (1989) 612.
[20] H. Ito, W.W. Buck, F. Gross, Phys. Rev. C45 (1992) 1918; Phys. Lett. B287, 23 (1992); I. Anikin, M. Ivanov, N. Kulimandova, V. Lyubovitskii, Phys. Atom. Nucl. 57 (1994) 1082.
[21] R. Jakob, P. Kroll, Phys. Lett. B315 (1993) 463.
[22] A.E. Dorokhov, Nuovo Cim., 109A (1996) 391.
[23] H.-N. Li, G. Sterman, Nucl. Phys. B381 (1992) 129.
[24] V. M. Belyaev and B.L. Ioffe, Sov. Phys. JETP 56 (1982) 493 [Zh. Eksp. Teor. Fiz. 83 (1982) 876]; A. A. Ovchinnikov, A. A. Pivovarov, Yad. Fiz. 48 (1988) 1135.
[25] D. Kekez and D. Klabučar, Phys. Lett. 457B (1999) 359.
[26] D.C. Curtis, M.R. Pennington, Phys. Rev. D42 (1990) 4165.
[27] D. Klabučar and D. Kekez, Fizika B (Zagreb) 8 (1999) 303 [hep-ph/9905251]; P. Tandy, Fizika B (Zagreb) 8 (1999) 295 [hep-ph/9902545]; C. D. Roberts, Fizika B (Zagreb) 8 (1999) 285 [hep-ph/9901094].
[28] A. Anselm, A. Johansen, E. Leader, L. Lukaszuk, Zeit. Phys., A359 (1997) 457.
[29] A.P. Bakulev, S.V. Mikhailov, Phys. Lett. B436 (1998) 351.
FIG. 1. The pion distribution amplitude as a function of fraction variable $x$ as given by (17) at different values of the diluteness parameter: $\chi^2 = 0.15$ (solid line), $\chi^2 = 0.0001$ (short-dashed line), $\chi^2 = 4$ (long-dashed line). The asymptotic distribution amplitude is marked by point line.
FIG. 2. The asymptotic coefficient $J_{np}$ as function of the diluteness parameter squared $\chi^2$, for the process $\gamma\gamma^* \to \pi^0$ ($\omega = 1$). It is indicated the values of $J_{np}$ obtained from the fit of CLEO data (central point), and the predictions obtained from nonperturbative covariant model ($\chi^2 = 0.15$), pQCD [3]; the right arrow points to the limiting value of $J = 4/3$ at $\chi^2 \to \infty$. 
FIG. 3. The asymptotic coefficient $J_{np}$ as a function of the kinematic parameter $\omega$. Solid line corresponds to $\chi^2 = 0.15$, giving $J_{np}(\omega = 1) = 1.8$. Short-dashed line is for $\chi^2 = 0.35$, giving $J_{np}(\omega = 1) = 1.6$. Long-dashed line corresponds to asymptotic pQCD prediction given by (4) with $J_{asympt}(\omega = 1) = 2$. It is also indicated $J_{np}$ for symmetric kinematics $q_1^2 = q_2^2$. 