In this paper we begin the development of a formalism for the description of high energy neutrino interactions. It is based upon field theory quantized on a null plane. We set up the general formalism as well as some techniques needed to perform phenomenological calculations. We show that the formalism developed by Wolfenstein is recovered at the cost of making two approximations: one has to treat the charged lepton fields in the Hartree–Fock approximation and one has to take the short distance limit of the Hartree–Fock correlation function. As an example, we discuss the resonant interaction of electron neutrinos in an electron gas.

1 Introduction

The theory of neutrino interactions at high energies will play an increasingly important role in the future. High energy neutrino oscillation and interaction experiments will be performed by means of accelerator generated neutrino beams as well as in experiments relying upon extraterrestrial sources of neutrinos such as active galactic nuclei (AGN) and binary systems of stars. In the latter setup, the center of mass energies in a neutrino interaction with matter are likely to reach a few TeV, corresponding to laboratory energies of the incident neutrinos of the order of a few PeV. Traditionally, neutrino interactions have been either described by the single gauge boson exchange approximation (as in the case of the theory of deep inelastic scattering of neutrinos on quarks) or in a contact interaction approximation (for instance, in the theory of neutrino oscillations in matter or in vacuum as developed by Wolfenstein [1]). While there exist several derivations of the Wolfenstein formalism [2, 3], it is clear that a more general formalism is needed at the highest energies. To quote but one example, if electron antineutrinos of an energy of about 6.4 PeV pass through an electron gas, they excite the $W^-$ resonance (the “Glashow resonance”). Such a situation may occur, for instance, when electron antineutrinos generated deep in an AGN pass the electron plasma on their way out. Clearly, neither the contact interaction approximation, nor a gauge boson exchange approximation are adequate; the resonance is excited in the s–channel; in this case, one has to be able to treat the presence...
of matter adequately. Due to the fact that neutrinos are nearly massless and we are interested in their interactions at high energies, the null plane or front form formulation of the theory is the most convenient one for the purpose. In the next section we briefly review the formalism. Thereafter, we outline the procedure for obtaining an effective theory of neutrino interactions in matter; as an application, we show how Wolfenstein’s equation is recovered form the general theory. In section 4 we illustrate the use of the theory by describing the interaction of electron antineutrinos ($\bar{\nu}_e$) with an electron gas in the energy region where the $W$ boson is excited as an s channel resonance. Sec. 5 contains a discussion of the results.

2 A Review of the Null Plane Formalism

The null plane or front form of quantum field theory is, in essence, a constrained Hamiltonian formulation of a field theory given by its Lagrangian and either operator quantization rules or path integral prescription. Instead of a spacelike surface, however, as it is the practice in setting up a traditional Hamiltonian formalism, one prescribes Cauchy data on a plane with a null normal vector. Such planes always contain characteristic lines of a relativistic wave equation. As a consequence, the number of independent Cauchy data is smaller than the ones one can prescribe on a spacelike surface, see, e.g. [4]. This is expressed in the form of constraints obeyed by the fields entering the theory. As in any Hamiltonian formulation of a relativistically invariant field theory, manifest Lorentz invariance is lost. In what follows, we describe the formulation of the theory of a Dirac fermion interacting with an external gauge field. The Dirac field may carry a representation of an internal symmetry group. The generalization to the case of a Yang–Mills theory or a theory containing scalar fields as well is straightforward and it has been described in a number of articles on the subject; a sampling of some articles is given in ref. [5]. Recently, an attempt has been made to restore some of the symmetries (parity invariance in particular) in a null plane formulation, see [6]. Due to the fact that we are interested in a parity violating theory, those developments are not needed here.

We begin with introducing a coordinate transformation in Minkowski space. We always work with a metric chosen as $g_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ in the usual Cartesian basis. Now introduce the coordinates,

$$t = \frac{1}{\sqrt{2}} \left(x^0 - x^3\right) \quad \text{and} \quad z = \frac{1}{\sqrt{2}} \left(x^0 + x^3\right).$$

Correspondingly, the components of the metric tensor become,

$$g_{zt} = g_{tz} = 1, \quad g_{AB} = -\delta_{AB}$$

and all other components vanish. Here and in the following, capital Latin subscripts and superscripts refer to the directions perpendicular to the chosen conjugate null directions given by eq. (1). Correspondingly, the generators of the
Dirac algebra are, $\gamma^t = (\gamma^0 + \gamma^3)/\sqrt{2}$, $\gamma^z = (\gamma^0 - \gamma^3)/\sqrt{2}$ and $\gamma^A$. The Dirac Lagrangian is of the usual form,

$$\mathcal{L} = \bar{\psi} \left( \frac{i}{2} \partial - ig_{(+)A_{(+)}} - ig_{(-)A_{(-)}} \right) \psi + \bar{\psi} m \psi$$

(3)

Here, $\partial := 1/2 \gamma^\mu \partial_\mu$, $A_\pm := \gamma^\mu (1 \pm \gamma^5) A_\mu^{(\pm)}$. Internal symmetry indices are suppressed.

We now introduce spinor projectors corresponding to the two null directions given in eq. (1), viz.

$$P_t = \gamma^t \gamma^t$$
$$P_z = \gamma^z \gamma^z$$

(4)

For the sake of brevity, we also introduce the notation,

$$P_t \psi = \phi$$
$$P_z \psi = \chi$$

A straightforward manipulation leads to the useful relation:

$$\gamma_5 = -i \gamma^1 \gamma^2 (P_t - P_z)$$

(5)

Consequently, the chiral projectors, $(1 \pm \gamma_5)/2$ act as helicity projectors upon the spinors projected to the conjugate null directions given by eq. (1). We define the helicity projectors,

$$H^\pm = \frac{1 \mp i \gamma^1 \gamma^2}{2}$$

(6)

With this, eq. (3) becomes:

$$\sqrt{2} \mathcal{L} = i \phi^\dagger \partial_t \phi + i \chi^\dagger \partial_z \chi + \phi^\dagger \gamma^z m \chi + \chi^\dagger \gamma^t m \phi - i \phi^\dagger \left( g_{(+)A_{(+)}} H^+ + g_{(-)A_{(-)}} H^- \right) \phi$$
$$- i \chi^\dagger \left( g_{(+)A_{(+)}} H^- + g_{(-)A_{(-)}} H^+ \right) \chi$$
$$+ \phi^\dagger \gamma^5 \gamma^A \left( \frac{i}{2} \partial_A - ig_{(+)} A^A_{(+)} H^- - ig_{(-)} A^A_{(-)} H^+ \right) \chi$$
$$+ \chi^\dagger \gamma^5 \gamma^A \left( \frac{i}{2} \partial_A - ig_{(+)} A^A_{(+)} H^+ - ig_{(-)} A^A_{(-)} H^- \right) \phi$$

(7)

It is to be emphasized that eq. (7) is completely symmetric under the interchange of the two conjugate null directions. However, the symmetry is destroyed if we decide to solve an initial value problem by specifying the Cauchy data on one of the null planes. For the sake of definiteness, we describe the procedure by specifying initial data on a plane $t = 0$; the procedure for Cauchy data specified on $z = 0$ is completely analogous to the one described here and it can be obtained by interchanging $\phi$ with $\chi$.

We recognize that if $t$ is regarded as the “time” variable, only $\phi$ obeys an equation of motion; there is no time derivative in the equation obeyed by $\chi$. Consequently, the equation obeyed by $\chi$ is a constraint and thus $\chi$ can be eliminated altogether from the equations of motion. Given the the Lagrangian, eq. (7),
the constraint can be solved at least formally. The resulting equations of motion obeyed by \( \phi \) are, in general, non–local; nevertheless, they are legitimate dynamical equations.

In the case of eq. (7), we proceed by choosing the gauge, \( A^\pm_z = 0 \). In this gauge the solution of the constraint is trivial, since one just has to invert the operator \( \partial_z \). In order to present the result, we introduce the covariant derivatives,

\[
\nabla^{(\pm)}_k = \partial_k - g_{(\pm)} A^\pm_k \quad (k = A, t)
\]

After elimination of the constraints, the end result reads:

\[
\frac{1}{\sqrt{2L}} = \phi^i i \nabla_i^{(-)} H^- \phi + \phi^i i \nabla_i^{(+)} H^+ \phi - \left( \left( m - i \nabla_i^{(-)A} \right) H^- \phi \right)^\dagger \frac{P_k}{k} \left( m - i \nabla_i^{(-)A} \right) H^- \phi - \left( \left( m - i \nabla_i^{(+)} A \right) H^+ \phi \right)^\dagger \frac{P_k}{k} \left( m - i \nabla_i^{(+)} A \right) H^- \phi
\]

(9)

In this equation, we denoted \( k = i \partial_z \) and, indeed, in the gauge chosen the easiest way of eliminating the constraint is by means of a Fourier transformation in \( z \). The singularity at \( k = 0 \) has to be eliminated by taking the principal value, due to Hermiticity requirements.

We notice that eq. (9) is of the canonical form,

\[
\mathcal{L} = \pi \partial_t \phi - \mathcal{H},
\]

(10)
as described recently by Jackiw [6]. In fact, the entire procedure of elimination of the constraints follows the pattern described in that reference.

Were there no mass terms and no mixing between left and right–handed components, there would be a one to one correspondence between particles (antiparticles) and negative (positive) helicities, respectively. Therefore a two component theory as described by eq. (9) would be exact. However, at sufficiently high energies one expects that the amplitudes of the “wrong” helicity components are suppressed by factors of the \( O(m\alpha/E) \), where \( m\alpha \) is some of the eigenvalues of the mass matrix appearing in the previous equations, for instance, in eq. (9). Therefore, the two component theory is expected to be a good effective theory. Parity conservation (see ref. [7]) is not an issue since we are dealing with electroweak interactions. (We recall that under parity, \( t \leftrightarrow z \), \( x^A \rightarrow -x^A \) and \( \phi \leftrightarrow \chi \). Thus, specifying Caucy data on a \( t = \text{const.} \) (\( z = \text{const.} \), resp.) surface, parity is, by necessity, violated.)

The Feynman rules can be easily read off from eq. (9). One notices that they have the appearance of the rules for a nonrelativistic Schrödinger theory in 2+1 dimensions. This is not an accident: it is due to the fact that the stability group of a null direction in Minkowski space is \( E(2) \), the Euclidean group in two dimensions.
3 Neutrino Interactions in Matter

In this section we begin the development of the formalism needed to describe neutrino interactions in matter. For the sake of simplicity, we discuss explicitly the case of neutrino interactions in a uniform electron gas. As we proceed, it will become obvious that the formalism can be generalized in a straightforward manner to describe other physical situations of interest.

Our starting point is the effective action, see, e.g., ref. [8], \( \Gamma \), viewed as a functional of classical neutrino and electron fields, denoted by \( \psi \) and \( \Psi \), respectively. Both \( \psi \) and \( \Psi \) are regarded as Dirac fields. Internal symmetry indices are suppressed as before.

The effective action can be expanded in a functional power series (Volterra series). We argue that in many cases of physical interest, the Volterra series of \( \Gamma \) can be broken off after the first few terms. Clearly, the the second derivatives with respect to \( \psi \) and \( \Psi \) give the free actions for these fields, with the appropriate mass terms. Fourth and higher derivatives give the effective interaction kernels. Both from a dimensional argument and from the explicit calculation that follows, one realizes that terms proportional to \( (\Psi \Psi)^k \) (with any arrangement of the space–time arguments of the fields) are proportional to \( n_e^k \), \( n_e \) being the density of the electron gas. As a consequence, in most cases, terms proportional to higher powers of the density can be dropped. On a similar basis we omit derivatives higher than second in \( \psi \), because we assume that the neutrino beam considered is sufficiently dilute so that self interactions of neutrinos can be neglected. (In addition, in the electroweak theory terms containing higher powers of the electron density also contain higher powers of the fine structure constant and, perhaps, of inverse gauge boson masses.)

With this in mind, we now write the effective action:

\[
\Gamma[\psi, \Psi] = S_0[\psi] + S_0[\Psi] + \int \Psi(1)\Psi(2)\mathcal{H}(1, 2; 3, 4)\Psi(3)\Psi(4) + \int \psi(1)\Psi(2)\mathcal{K}(1, 2; 3, 4)\Psi(3)\psi(4) + \ldots \tag{11}
\]

In eq. (11), the space–time points have been denoted simply by Arabic numerals; the integrations extend over points occurring twice under the integral sign. Clearly, eq. (11) defines an interacting system of electrons and neutrinos. Assuming that \( \mathcal{H} \) and \( \mathcal{K} \) have been computed in some approximation, one can solve this classical field theory in order to represent the neutrino electron interaction. The relevant Green functions can be represented as a sum of tree diagrams, with interaction vertices given by the kernels \( \mathcal{H} \) and \( \mathcal{K} \). An easy way of generating these diagrams is to introduce an auxiliary second quantization of the electron and neutrino fields and then compute the tree diagrams in this theory.

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2 This is, in essence, the classical statistical argument for the dominance of binary collisions in a gas of moderate density.
As an application, let us show how the Wolfenstein formalism is recovered in this framework.

The crucial point is to make a Hartree–Fock approximation in eq. (11). This means that one has to replace bilinear products of the form $\Psi(1)\Psi(2)$ in the terms proportional to $H$ and $K$ (and in the higher order terms) in the effective action by their expectation values in the electron gas. In this way, the effective action reduces to a quadratic functional. On performing the above-mentioned truncation on the term proportional to $H$, one can absorb the resulting functional into $S_0[\Psi]$ to a good approximation. In fact, it follows from straightforward invariance arguments that the expectation value (denote it by $H$) must be of the form,

$$H = \int \overline{\Psi} h_0 \Psi + \overline{\Psi} h_1 \partial \Psi + \partial_\mu \overline{\Psi} h_2 \partial^\mu \Psi + \ldots,$$

(12)

where all fields are to be taken at the same point.

If the electron gas is non–relativistic, as we assume, the higher derivative terms may be omitted and we see that the effect of the self interaction within the electron gas is just a mass shift and a wave function renormalization. (In standard many body theory, see e.g. ref. [9], one often introduces an energy dependent effective mass in order to take some of the higher derivative terms into account. While this can be done within the framework of the present formalism, it is unimportant from the point of view of the argument that follows.)

Let us now concentrate on the electron–neutrino interaction term. On replacing the bilinear electron operator by its expectation value, the resulting expression is of the form:

$$\int \overline{\psi}(1)K(1,2)\psi(2),$$

(13)

where the kernel is given by

$$K(1,2) = \int K(1,3;2,4)C(3,4).$$

(14)

The quantity denoted by $C(3,4)$ is the two point correlation function of the electron gas. Due to the homogeneity of the electron gas, it depends only on the difference of its two arguments. On making a non–relativistic approximation to the kinematics for quantities referring to the electron gas, one obtains the well known expression:

$$C(x,t) = n_e \frac{3}{2} \exp(\frac{i\tau m_e / p_F}{2}) \int_{-1}^{1} du u \sin u \rho \exp(iu^2 \tau p_F / m_e)$$

(15)

In eq. (15), $p_F$ is the Fermi momentum; $\tau$ and $\rho$ are the time and radial distance measured in units of the Fermi wavelength, i.e. $\tau = t p_F$, $\rho = r p_F$. (In the usual nonrelativistic treatment, the exponential factor before the integral is absent; this is merely a question of the definition of the chemical potential.) Finally, $m_e$ stands for the value of the effective electron mass in the gas.
One can now proceed to decompose the Dirac spinor describing the neutrino according to its projections onto the two conjugate null directions as described in the previous Section and eliminate the constraint. In order to present the result, we use an operator notation, such that, for instance:

\[ K\phi := \int K(1, 2)\phi(2) \]

In this way we obtain the effective Lagrangian:

\[
\sqrt{2} L_{\text{eff}} = i\phi^\dagger \partial_t \phi - \phi^\dagger (-i\nabla - m + K) \left( i\partial_z + \gamma^t K \right)^{-1} (i\nabla - m + K) \phi, \tag{16}
\]

where \( \nabla = \gamma^A \partial_A \).

In essence, the variation of eq. (16) yields the Wolfenstein equation describing the propagation of a neutrino in a medium. In order to obtain the form usually quoted in the literature, \textit{e.g.} in ref. [10], the following steps are needed.

1. One assumes that the propagation is one dimensional; hence, \( \partial_A \phi \) can be chosen to be zero by an appropriate choice of the coordinate system.

2. The interaction in the medium is given by the standard model of electroweak interactions, \textit{i.e.} in the rest frame of the gas,

\[ K = K_v \gamma^0 + K_a \gamma^0 \gamma^5 \tag{17} \]

3. One neglects the “wrong helicity” components, \textit{i.e.} one approximates,

\[ H^- \phi \approx \phi, \quad H^+ \phi \approx 0 \]

and (\textit{mutatis mutandis}) similarly for the conjugate problem in which \( z \) is regarded the time and \( \chi \) the dynamical variable.

4. One realizes that in eq. (15) the characteristic momentum scale is given by \( p_F \), while in \( K \) it is the gauge boson mass, \( M \). In any environment of interest (with the possible exception of the very early Universe) \( M \gg p_F \).

Hence, one can safely approximate,

\[ C(x, t) \approx C(0, 0) = n_e \]

As a result, considerable simplifications occur. In particular, after neglecting the “wrong” helicities and replacing the the electron correlation function by the electron density, one realizes that the coefficients \( K_v \) and \( K_a \) occur only in the combination:

\[ \kappa = n_e (K_v - K_a) \tag{18} \]
By eliminating the constraint, one identifies the the effective Hamiltonian for the field $\phi$:

$$H_{\text{eff}} = \phi^\dagger \left[ \frac{m^2 + \nabla^2}{2(k - \kappa)} + \kappa \right] \phi$$

(19)

If the denominator in eq. (19) were just equal to $k$, this would be identical to the Hamiltonian for the Wolfenstein equation: assuming one dimensional propagation and choosing the coordinate system appropriately, one would obtain the equation discussed e.g. in ref. [10]. Under certain circumstances, one is indeed justified in approximating $k - \kappa \approx k$. However, the correct formula is given by eq. (19) and care is needed in cases when the denominator may become small.

The exercise just described is useful because it makes the limitations of the Wolfenstein formalism explicit. Within the framework where it is usually applied, i.e. the treatment of the solar neutrino problem, the limitations are totally insignificant. However, often one finds applications of the formalism to problems involving high energy neutrino interactions, where it is, at best, of limited validity. In particular, one notices that the Hartree–Fock approximation one has to make in order to reproduce Wolfenstein’s results describes the interaction of neutrinos with an electron gas entirely in terms of the creation and subsequent annihilation of a hole in the Fermi sea. At energies of interest in high energy neutrino interactions, with $\sqrt{s}$ perhaps a few hundred GeV, large momentum transfer processes are important; in particular, electrons can be ejected into the continuum instead of annihilating with a hole.

4 Neutrino Interactions at the W Resonance

As an illustration of the formalism developed in the preceding Section, we consider $\overline{\nu}_e$ of a laboratory energy approximately equal to 6.4 PeV, incident upon an electron gas. In a previous paper, [12] we discussed the physical circumstances under which this process leads to somewhat unanticipated results. However, there we used the Wolfenstein formalism uncritically. Here we show that the present formalism reproduces the the results of ref. [12].

We notice that at the energy mentioned, $W$ is excited as an $s$–channel resonance. As a consequence, it is reasonable to retain only those diagrams which resonate at the $W$ mass. For the sake of simplicity, we also make the customary approximations, viz. we evaluate the self energy part of the $W$ propagator and the vertices at the the mass of the $W$. (For most purposes, this is an adequate approximation and, in essence, it is equivalent to using a Breit–Wigner formula to describe the resonance.)

In this approximation the effective electron neutrino interaction can be written in the form:

$$\Gamma_{\text{int}} = g^2 \int \overline{\Psi}(1) \gamma_\mu \frac{1 - \gamma_5}{2} \psi(1) \Delta \left( 1 - 2 \right) \overline{\psi}(2) \gamma_\mu \frac{1 - \gamma_5}{2} \Psi(2) + (\text{conj.}),$$

(20)
where \( g^2 \) is the coupling evaluated at the resonance and \( \Delta \) stands for the propagator of the \( W \): it is the Fourier transform of the quantity,

\[
\frac{1}{M^2 - k^2 - iM\gamma}
\]

As usual, we denoted the self energy part evaluated at resonance by \( M^2 - iM\gamma \). Hence, \( M \) is the physical mass and \( \gamma \) is the total width. By performing a Fierz transformation, the last equation can be brought to the more convenient form:

\[
\Gamma_{\text{int}} = g^2 \int \Psi(1) \gamma^\mu \frac{1 - \gamma^5}{2} \Psi(2) \Delta (1 - 2) \psi(2) \gamma^\mu \frac{1 - \gamma^5}{2} \psi(1) + (\text{conj.}) \quad (21)
\]

In this form one can easily make a Hartree–Fock approximation, by replacing the bilinear quantity in the electron operators by its expectation value. On assuming that the electron correlation function is evaluated in the rest frame of the gas and replacing the correlation function by its value at the origin, the contribution is of the form as in eq. (18), with \( K_v = -K_o \). At this point, we are now ready to write down the effective Hamiltonian with the resonant interaction. Due to the fact that the effective action was reduced to a quadratic functional and we consider one dimensional propagation only, it is convenient to Fourier transform the effective action. In this manner, the computation of the inverses of the operators entering the constraints becomes trivial. We write the Hamiltonian in the form (cf. eq. (19)):

\[
H = \phi^\dagger m^2 + \frac{2(p - \kappa)\kappa}{2(p - \kappa)} \phi.
\]

Here, \( p \) stands for the variable conjugate to \( z \); we suppressed the argument of \( \phi \); integration over \( p \) is understood.

The question arises whether this Hamiltonian can be replaced by a conventional one, i.e.

\[
H = \phi^\dagger m^2 + \frac{2p\kappa}{2p} \phi.
\]

We now proceed to show that in the case of the resonant interaction discussed here, such a replacement is well justified. First of all, it is convenient to rewrite the expression of \( \kappa \) in terms of the elastic and total widths of the resonance. This is easily accomplished by noticing that the usual invariant variable \( s \) can be written as \( s = 2pm_e \), \( M_e \) being the mass of the electron. Straightforward manipulations then lead to the expression near \( s = M^2 \):

\[
2p\kappa \approx m_e^2 \frac{M\gamma_e}{s - M^2 + iM\gamma} \quad (22)
\]

In eq. (22), \( m_e \) is a mass scale characterizing the electron gas; it is defined by the relation \( m_e^2 = n_e/m_e \). The following Table, taken from ref. [12], contains the values of the characteristic mass for some environments of interest.
Electron densities and characteristic masses for some environments

| Environment          | \(n_e \text{[cm}^{-3}\)] | \(m_e^2 \text{[eV}^2\)] |
|---------------------|---------------------------|--------------------------|
| stellar interior (sun) | \(10^{27}\)             | \(2 \times 10^4\)       |
| Earth               | \(1.6 \times 10^{24}\)   | \(3 \times 10^4\)       |
| water               | \(3 \times 10^{23}\)     | \(5 \times 10^3\)       |

We now notice that there are no vanishing denominators in the Hamiltonian, since \(\kappa\) is complex. Furthermore, the correction to the conventional Hamiltonian arising from eq. (19) is of the order of magnitude \(2\kappa/p\). Near resonance this is

\[
\left| \frac{2\kappa}{p} \right| \approx \frac{\gamma_e n_e^2 m_e^2}{4\gamma M^2}.
\]

Even for an environment like a stellar interior, the corrections are minuscule. There may be substantial corrections to the naive form of the Wolfenstein Hamiltonian in very dense environments, e.g. in the interior of a neutron star or in the early Universe. However, in all probability, the Hartree–Fock approximation breaks down before these corrections become truly significant.

One can include flavor degrees of freedom without any difficulty and discuss neutrino mixing near the resonance as it has been done in ref [12]. None of the conclusions about the validity of the Wolfenstein formalism is altered by such a generalization and therefore we shall not dwell on this topic any further.

5 Discussion

The basic purpose of this work has been to establish a formalism leading to an effective theory of neutrino interactions in matter. It became clear that the use of the null plane formalism is suitable for the description of these interactions. Clearly, however, what is needed is a reliable calculation of the effective action. There are several possible approaches to this problem with an increasing degree of complexity. In the approach discussed here, one puts the classical fields corresponding to all but a few species equal to zero. (Here we discussed the simplest case: only the fields corresponding to neutrinos and the target medium are kept.) Even at this level, some interesting generalizations are possible; in particular, at high neutrino densities, such as in the early Universe, one can no longer neglect the interaction between the various flavors of neutrinos; consequently, one has to deal with a rather non-trivial multichannel problem. In some recent articles, Kostelecky, Pantaleone and Samuel, ref. [11] addressed this problem entirely within the framework of the Wolfenstein formalism. However, those authors, in essence, make a dilute gas approximation by neglecting the effects of the Fermi sea, while keeping the interaction terms between neutrinos. The internal consistency of such a procedure is not quite obvious and further studies are needed in order to clarify the issues involved.
More importantly, in discussing high energy neutrino interactions, one has to go beyond the Hartree–Fock approximation as already discussed, since in that approximation a substantial amount of physical information is lost. Furthermore, in order to obtain a reliable theory, one will have to follow the evolution of several components of the system even if ultimately only the transition probabilities for the stable particles are kept.

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