On The Uplink Throughput of Zero-Forcing in Cell-Free Massive MIMO with Coarse Quantization

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Abstract—The recently proposed Cell-Free massive MIMO architecture is studied for the uplink. In contrast to previous works, joint detection is performed using global CSI. Therefore, we study strategies for transferring CSI to the CPU taking into account the fronthaul capacity which limits CSI quantization. Two strategies for pilot-based CSI acquisition are considered: estimate-and-quantize and quantize-and-estimate. These are analysed using the Bussgang decomposition. For a given quantization constraint for the data and CSI the achievable rate per user with Zero-Forcing is determined. Numerical results show that quantize-and-estimate (the simpler strategy) is similar to or better than estimate-and-quantize, especially for 1-bit resolution.

Index Terms—Cell-Free Massive MIMO, Fronthaul, Quantization, Bussgang, Channel Estimation.

I. INTRODUCTION

The next generation of wireless networks (including 5G) will be required to provide a high capacity per user and per unit area due to the increasing number of users and the variety of applications expected in the near future. Cell-free massive MIMO has been gaining more attention recently as it has the potential to meet this demand [1]. It can be regarded as a form of network MIMO which makes use of a large number of distributed antennas, referred to Access Points (AP), spread over a large coverage area. The term "cell-free" was motivated by the notion of blurring the role of cells so that all users can be served by all APs over the same resources using network MIMO techniques to avoid mutual interference. Because a large number of APs serve a smaller number of users, it benefits also from channel hardening as in co-located massive MIMO [2].

Nevertheless, the joint transmission/detection in the current cell-free system is based only on local Channel State Information (CSI). We identify this a limitation, since relying on local CSI at the APs restricts the feasible choice of processing to conjugate beamforming or Maximum Ratio Combining (MRC). Other forms of processing such as Zero Forcing (ZF) could be performed at Central Processing Unit (CPU), but would require additional CSI transfer via the fronthaul. However, CF-massive MIMO already faces the problem of high fronthaul load requirement. To address these issues, we study in this letter joint detection with global CSI at the CPU and the strategy of acquiring the required CSI. To deal with the growth of fronthaul load we assume a coarse quantization constraint, which is also of interest for the low-cost implementation of APs. We show that using appropriate CSI acquisition strategies, much improved detection techniques can be applied at the CPU resulting in a significant rate improvement.

After a brief description of our system model we investigate two strategies of CSI acquisition. The first is called estimate-and-quantize (EQ) where channel estimation is carried out at the AP. The channel estimate is quantized and then the quantized form is sent to the CPU. As alternative we consider quantize-and-estimate (QE), where the APs quantize the received pilot and send it to the CPU. From these quantized received pilots the CPU performs the channel estimation. Further, we compare their performance and their corresponding throughput for ZF detection. Surprisingly enough, the QE strategy, which is simpler for the implementation at the AP, has good performance and a significant performance improvement over EQ for 1-bit fronthaul resolution. Overall, the superiority of utilizing global CSI, even with coarse quantization, is shown to be significant compared to utilizing only local CSI with infinite resolution.

II. SYSTEM MODEL

We consider the uplink transmission of a cell-free system [1], where we have $K$ single-antenna users (UEs) and $M$ single-antenna Access Points (APs) connected to a Central Processing Unit (CPU) by $M$ error-free fronthaul links. The main processing for these $M$ APs are virtualized at the CPU, where the communication between them occurs in baseband form. We assume that the fronthaul link connecting the $m$-th AP with the CPU can in practice transmit reliably at a maximum rate of $R_m$.

A. Channel Model

The channel between the $k$-th user and the $m$-th AP is specified (as in [1]) by

$$g_{mk} = h_{mk}b_{mk}^{1/2},$$

where the coefficient $h_{mk}$ models the small-scale fading between the $k$-th user and the $m$-th AP with the assumption that it is i.i.d. $\sim CN(0,1)$. The large-scale fading is denoted by $b_{mk}$, which is likely to be different for each user $k$ and each AP $m$ due to the distributed configuration. The channel from all $K$ users to all $M$ APs can then be expressed as the element-wise product of small-scale and large-scale fading matrix given by

$$G = H \odot D^{1/2}, \quad H \in \mathbb{C}^{M \times K} \quad \text{and} \quad D \in \mathbb{R}^{M \times K}. \quad (2)$$

B. Quantization Scheme

To simplify our analysis, we consider fronthaul links with $R_m = R, \forall m \in \{1, \ldots, M\}$, corresponding to the quantization level $L = 2^R$. Therefore, we apply an $L$-level scalar quantizer $Q$ at each AP as an interface to the fronthaul with

$$Q(x) = \sum_{l=0}^{L-1} q_lT_l(x), \quad (3)$$

where $T_l(x)$ is equal 1 for $x_l < x \leq x_{l+1}$ and 0 otherwise. We consider $Q$ as a uniform quantizer with a fixed step size $\Delta = x_{l+1} - x_l$ and a reconstruction value $q_l = (l + \frac{1}{2})\Delta$. For a complex-valued signal we quantize separately the real

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and imaginary part. We assume that the large scale fading $\beta_{mk}$ is relatively constant over a long period and known at the APs. Thus, we can scale the input-output signal of the quantizer according to $\beta_{mk}$ and approximate the normalised input as normally distributed.

The function $Q$ is the scalar quantization process, which is particularly nonlinear for small $L$. To analyse it, we can use the Bussgang decomposition [3]. Accordingly, for a nonlinear function $Q(x)$ we can write it as

$$y = Q(x) = \alpha_q x + d.$$  \hspace{1cm} (4)

The distortion term $d$ is then uncorrelated to the input signal $x$. The linear factor $\alpha_q$ depends on the characteristic of the quantizer $Q$ and the distribution $f(x)$ of the input signal $x$ [3, 4]. As given in [5], for normally distributed input and uniform quantizer the Bussgang decomposition [3] is then uncorrelated to the input signal $x$.

Further, we define the power ratio of input $x$ and output $y$ as

$$\lambda_q = \frac{\mathbb{E}\{|y|^2\}}{\mathbb{E}\{|x|^2\}} = \frac{\Delta^2}{\frac{1}{4} + 4\sum_{l=1}^{L/2-1} (1 - \Phi(l\Delta))},$$  \hspace{1cm} (5)

where $\Phi$ is the Gaussian cumulative distribution function. We choose here the step size $\Delta$ that maximizes the Signal to Distortion Noise Ratio (SDNR) at the output of the quantizer defined as

$$\text{SDNR} = \frac{\mathbb{E}\{|y|^2\}}{\mathbb{E}\{|d|^2\}}.$$  \hspace{1cm} (6)

From (4) and (6) the power of the distortion is given by

$$\mathbb{E}\{|d|^2\} = \mathbb{E}\{|y - \alpha_q x|^2\} = (\lambda_q - \alpha_q^2)\mathbb{E}\{|x|^2\}.$$  \hspace{1cm} (7)

III. CSI ACQUISITION STRATEGIES

The CSI is acquired based on the estimation of known pilots transmitted by the users. In this case, the $k$-th user transmits $\sqrt{\tau} \varphi_k$ as its pilot, where a specific random sequence $\varphi_k \in \mathbb{C}^{\tau \times 1}$ is taken from an orthonormal basis with $|\langle \varphi_k, \varphi'_k \rangle| = \delta_{kk'}$ and $\|\varphi_k\|^2 = 1$. The sequence length $\tau$ is assumed to be less than or equal to the coherence interval $\tau_c$. The $m$-th AP observes the received pilot $y_{p,m}$ from all $K$ users as

$$y_{p,m} = \sqrt{\tau \rho_p} \sum_{k=1}^{K} g_{mk} \varphi_k + w,$$  \hspace{1cm} (9)

where $\rho_p$ is the transmit SNR of the pilot and $w \sim \mathcal{CN}(0, I_K)$ is an additive noise vector with zero mean and unit variance. To ensure that all pilots are orthogonal for all $K$ users, one should only allow $K \leq \tau$ users who transmit their pilots simultaneously. In this case, the transmitted pilots satisfy

$$\Theta \Theta^H = \tau \rho_p I_K,$$  \hspace{1cm} (10)

where $\Theta = \sqrt{\tau \rho_p} [\varphi_1, \ldots, \varphi_K]$.

In the ideal case of perfect fronthaul [11] the channel $g_{mk}$ can be estimated at the AP and sent to the CPU which then has the global CSI. In this case, the received pilot $y_{p,m}$ at the $m$-th AP is projected onto $\varphi_k^H$ giving:

$$r_{p,mk} = \varphi_k^H y_{p,m} = \sqrt{\tau \rho_p g_{mk}} + \sqrt{\tau \rho_p} \sum_{k' \neq k}^{K} g_{mk'} \varphi_k^H \varphi_{k'} + \varphi_k^H w.$$  \hspace{1cm} (11)

To obtain the estimate of $g_{mk}$ we use the LMMSE estimator given by

$$\hat{g}_{mk} = e_{mk} r_{p,mk}.$$  \hspace{1cm} (12)

We choose $e_{mk}$ that minimizes the mean squared error

$$e_{mk} = \mathbb{E}\{|g_{mk} - \hat{g}_{mk}|^2\}.$$  \hspace{1cm} (13)

The unique minimum is obtained by taking the derivative of $e_{mk}$ and setting it equal to zero giving

$$e_{mk} = \mathbb{E}\{|r_{p,mk} g_{mk}|^2\} = \frac{\sqrt{\tau \rho_p} \beta_{mk}}{\mathbb{E}\{|r_{p,mk}|^2\}} = \frac{\sqrt{\tau \rho_p} \beta_{mk}}{\mathbb{E}\{|r_{p,mk} g_{mk}|^2\} + 1},$$  \hspace{1cm} (14)

where the last equation follows from (11). With the optimal coefficient $e_{mk}$ the minimum mean squared error is then given by

$$e_{mk} = \mathbb{E}\{|g_{mk}|^2\} - \mathbb{E}\{|r_{p,mk}|^2\} e_{mk} = \beta_{mk} - \gamma_{mk},$$  \hspace{1cm} (15)

where we denote the mean squared of the channel estimate of $g_{mk}$ by $\gamma_{mk} \triangleq \mathbb{E}\{|\hat{g}_{mk}|^2\}$.

A. Estimate-and-Quantize

In this scheme we estimate the channel coefficient $g_{mk}$ first as given in (12). So that it may be sent via limited fronthaul to the CPU, the estimated channel $\hat{g}_{mk}$ is quantized at each AP where we assume all $M$ APs have the same quantizer. The CPU receives $\hat{g}_{mk}^{qc}$, which can be decomposed by Bussgang as

$$\hat{g}_{mk}^{qc} = Q(\hat{g}_{mk}) = \alpha_{qc} \hat{g}_{mk} + d_{qc}.$$  \hspace{1cm} (16)

The mean squared error after quantization is given by

$$e_{mk}^{qc} = \mathbb{E}\{|g_{mk} - \hat{g}_{mk}^{qc}|^2\} = \mathbb{E}\{|g_{mk}|^2\} + \mathbb{E}\{|\hat{g}_{mk}^{qc}|^2\} - 2\mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}^{qc}\}.$$  \hspace{1cm} (17)

We can apply (16) to express the third term as

$$2\mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}^{qc}\} = 2\alpha_{qc} \mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}\} + 2\mathbb{E}\{g_{mk}^{*} d_{qc}\} = 2\alpha_{qc} \mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}\},$$  \hspace{1cm} (18)

where the second term vanishes due to uncorrelation. We then obtain

$$e_{mk}^{qc} = \mathbb{E}\{|g_{mk}|^2\} + \mathbb{E}\{|\hat{g}_{mk}^{qc}|^2\} - 2\mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}\} = \mathbb{E}\{|g_{mk}|^2\} + \mathbb{E}\{|\hat{g}_{mk}^{qc}|^2\} - 2\alpha_{qc} \mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}\} = \beta_{mk} - 2\alpha_{qc} \mathbb{E}\{g_{mk}^{*} \hat{g}_{mk}\}.$$  \hspace{1cm} (19)

This scheme is possible in practice by deploying a very high resolution Quantizer for estimating $g_{mk}$ which is followed by a low resolution quantizer to fit the fronthaul requirement.

B. Quantize-and-Estimate

Unlike the previous scheme, here we quantize the pilot first and send it to the CPU to estimate $g_{mk}$. In this case, at the CPU we have the quantized received pilots which once again
may be decomposed using the Bussgang decomposition as
\[ y_{p,m}^q = Q(y_{p,m}) = \alpha_{qp}y_{p,m} + d_{qp}. \] (21)

The noisy quantized observation at the CPU is given as
\[ y_{p,m}^q = \varphi^H_k y_{p,m} = \alpha_{qp}\varphi^H_k y_{p,m} + \varphi^H_k d_{qp}. \]

We then apply the LMMSE estimator
\[ \hat{y}_{mk}^q = \mathcal{E}(y_{mk}^q) \],
with
\[ c_{mk}^q = \frac{\alpha_{qp}^2 a_{mk}}{(\lambda_{qp} - \alpha_{qp}^2)b_m}, \] (24)
where
\[ a_{mk} = \tau_{rp} \sum_{k'=1}^{K} \beta_{mk'} |\varphi^H_k \varphi^H_{k'}|^2 + 1, \quad \text{and} \]
\[ b_m = \rho_p \sum_{k=1}^{K} \beta_{mk} + 1. \] (27)

We then also obtain (see Appendix A)
\[ c_{mk}^q = \mathbb{E}(|y_{mk}^q - \hat{y}_{mk}^q|^2) \]
\[ = \beta_{mk} - \left( \frac{\alpha_{qp}^2 a_{mk}}{(\lambda_{qp} - \alpha_{qp}^2)b_m} \right) \gamma_{mk}. \] (28)

IV. THE ACHIEVABLE RATES WITH COARSE QUANTIZATION

The uplink data received at all \( M \) APs may be described by
\[ y = \sqrt{\rho_u} G x + n. \] (29)

After quantization and transmission via the fronthaul the CPU obtains the data signal \( r \), which can also be decomposed as
\[ r = Q(y) = \alpha_{qd} y + d_{qd} \]
\[ = \sqrt{\rho_u} \alpha_{qd} \tilde{G} x + \alpha_{qd} m + d_{qd}, \] (30)
where \( \tilde{G} \) is the channel estimation error including the quantization error. Further, we can treat \( \tilde{G} \) as the true channel and treat the second term and so forth as the effective noise written as
\[ r = \sqrt{\rho_u} \alpha_{qd} \tilde{G} x + z. \] (31)

Due to the nature of the matrix \( G \) in the case of distributed massive MIMO, which tends to have independent large scale fading coefficients, the closed form expression of SINR for ZF is intractable. To obtain the SINR expression for our quantized CF massive MIMO we follow the approximation derived in [6].

We apply \( \tilde{G} \) in our zero forcing detector to detect the data from (31) and apply a filter \( \Lambda^{-1/2} \) to \( r \) to whiten \( x \), such that the ZF detector matrix
\[ A^H = (\tilde{G}^H \Lambda \tilde{G})^{-1} \tilde{G}^H \Lambda^{-1/2}, \] (32)
\[ \Lambda = \mathbb{E}\{zz^H\} \quad \text{and} \quad A^H \Lambda^{-1/2} \tilde{G} = I_K. \] (33)

After detection we obtain
\[ \hat{x} = \sqrt{\rho_u} \alpha_{qd} x + (\tilde{G}^H \Lambda \tilde{G})^{-1} \tilde{G}^H \Lambda^{-1} z. \] (34)

The SINR for the \( k \)-th user can then be expressed as
\[ \text{SINR}^Z_F = \frac{\rho_u \alpha_{qd}^2}{\left[(\tilde{G}^H \Lambda \tilde{G})^{-1} \tilde{G}^H \Lambda^{-1} zz^H \Lambda^{-1} \tilde{G}^H \Lambda \tilde{G})^{-1}\right]_{kk}}. \] (35)

In this way, we can approximate the \( \text{SINR}^Z_F \) as [6]
\[ \text{SINR}^Z_F = \rho_u \alpha_{qd}^2 \left( \frac{M - K + 1}{M} \right) \tilde{G}^H \Lambda^{-1} \tilde{g}_k, \] (36)
where for \( M \) APs the matrix \( \Lambda \) is given by
\[ \Lambda = \text{diag}\{\Lambda_1, \ldots, \Lambda_M\}, \quad \text{and} \]
\[ \Lambda_m = \sigma_{\alpha_{qd}}^2 + \sigma_{\alpha_{qd}}^2 \alpha_n + \alpha_{qd}^2 \sum_{k=1}^{K} \epsilon_{mk}, \] (38)
where \( \sigma_{\alpha_{qd}}^2 \) is the distortion variance resulted from quantizing data, \( \sigma_n^2 \) is the noise variance and \( \epsilon_{mk} \in \{\epsilon_{mk}^c, \epsilon_{mk}^q\} \) is the estimation error from (20) or (28) depending on the scheme. The achievable rate per user in the uplink is then given by
\[ R^Z_F = \log_2 (1 + \text{SINR}^Z_F). \] (39)

V. NUMERICAL RESULTS

In the following, we provide some numerical results for the considered schemes above. We do simulations with system parameters similar to [11] where there are \( M = 100 \) APs and \( K = 20 \) users distributed uniformly in an area of \( 1 \times 1 \) km². For the channel \( y_{mk} \) given in (11) we model the large scale fading \( \beta_{mk} \) as
\[ \beta_{mk} = \text{PL}_{mk} \cdot 10^{\frac{z_{mk}}{10}}, \] (40)
where the factor \( 10^{\frac{z_{mk}}{10}} \) is the uncorrelated shadowing with the standard deviation \( \sigma_{sh} = 8 \) dB and \( z_{mk} \sim \mathcal{N}(0, 1) \). The path loss coefficient follows the three-slope model according to
\[ \text{PL}_{mk} = \begin{cases} -L - 35 \log_{10}(d_{mk}), & d_{mk} > d_1 \\ -L - 15 \log_{10}(d_1) - 20 \log_{10}(d_{mk}), & d_0 < d_{mk} \leq d_1 \\ -L - 15 \log_{10}(d_1) - 20 \log_{10}(d_0), & d_{mk} \leq d_0, \end{cases} \] (41)
where \( d_{mk} \) is the distance between the \( m \)-th AP and the \( k \)-th user, \( d_0 = 0.01 \) km, \( d_1 = 0.05 \) km, and
\[ L \triangleq 46.3 + 33.9 \log_{10}(f) - 13.83 \log_{10}(h_{AP}) - (1.1 \log_{10}(f) - 0.7) h_u + (1.56 \log_{10}(f) - 0.8). \] (42)

We choose the carrier frequency \( f = 1.9 \times 10^3 \) GHz, the AP antenna height \( h_{AP} = 15 \) m and the user antenna height \( h_u = 1.65 \) m. In our simulation the normalized transmit SNRs \( \rho_u \) and \( \rho_p \) are defined as the transmit power divided by the noise power which is \( B \times k_b \times T_0 \times \text{noise figure} \). We suppose that the bandwidth \( B = 20 \) MHz, the Boltzmann constant \( k_b = 1.381 \times 10^{-23} \), the noise temperature \( T_0 = 290 \) Kelvin and the noise figure = 9 dB. For simplicity we observe the case of orthogonal pilot transmission with \( \tau = K \), where we allocate \( \tau = 20 \) symbols for the pilot from the overall \( \tau_c = 200 \).

In Fig. 1, we first validate our analytical approximations in (20) and (28) with simulations in terms of their MSE. It is shown that our analyses for both strategies are quite close to
simulations. In at least 80% of cases (those with the lower MSE) the QE scheme gives a poorer MSE than EQ and for L > 2 this proportion increases. However, it is the larger channel estimate errors that have stronger influence on the rate. Using the corresponding channel estimation errors we then evaluate the average achievable rates per user given in (39). In this case, we compare their performance in terms of their throughput defined as

\[ S_{u,k}^{ZF} = \frac{B}{2} \frac{1 - \tau / \tau_c}{R_{u,k}^{ZF}}. \]  

(43)

As shown in Fig. 2, the QE scheme achieves higher throughput than the EQ scheme for L = 2 and coincides for L = 4 over the whole range of transmit power. It can also clearly be observed that ZF with low quantization level L = 4 can already outperform MRC even with infinite quantization precision. This demonstrates the great improvement resulting from having global CSI available at the CPU. With 5-bits we are about 5 dB away from ZF with ideal fronthaul to reach 50 Mb/s/Hz per user average throughput.

VI. CONCLUSION

This letter shows the benefit of having global CSI at the CPU for the uplink of cell-free massive MIMO. We have established the MSE expression of CSI-acquisition strategies and compared their performance. We have presented their corresponding average throughput for Zero-Forcing detection. In this case, the ZF-QE scheme outperforms Zero-Forcing detection.

APPENDIX A

DERIVATION OF EQUATION (28)

For the estimator in (23) we have \( c_{mk}^{qp} \) that minimizes the MSE given by

\[ c_{mk}^{qp} = \frac{\mathbb{E}\{r_{p,mk}^q g_{mk}\}}{\mathbb{E}\{|r_{p,mk}|^2\}}. \]  

(44)

From (22) we can express the numerator of \( c_{mk}^{qp} \) as

\[ \mathbb{E}\{r_{p,mk}^q g_{mk}\} = \alpha_{qp} \mathbb{E}\{\ast_{p,mk} g_{mk}\} + \mathbb{E}\{\varphi_k^H d_{qp} g_{mk}\} = \alpha_{qp} \sqrt{\tau_p \beta_{mk}}, \]  

(45)

where the second term vanishes due to uncorrelation. Likewise we can express the denominator as

\[ \mathbb{E}\{|r_{p,mk}|^2\} = \alpha_{qp}^2 \mathbb{E}\{|r_{p,mk}|^2\} + \mathbb{E}\{|\varphi_k^H d_{qp}|^2\}, \]  

(46)

where the first term is given by

\[ \alpha_{qp}^2 \mathbb{E}\{|r_{p,mk}|^2\} = \alpha_{qp}^2 \left( \tau_p \rho_p \sum_{k'=1}^{K} \beta_{mk'} |\varphi_k \varphi_{k'}|^2 + 1 \right) \]  

(47)

and the second term is given by

\[ \mathbb{E}\{|\varphi_k^H d_{qp}|^2\} = \|\varphi_k^H\|^2 \mathbb{E}\{|d_{qp}|^2\} = (\lambda_{qp} - \alpha_{qp}^2) \mathbb{E}\{|y_{p,m}|^2\} = (\lambda_{qp} - \alpha_{qp}^2) \left( \rho_p \sum_{k=1}^{K} \beta_{mk} + 1 \right). \]  

(48)

With \( \alpha_{mk} \) and \( b_m \) given in (26) and (27) then

\[ c_{mk}^{qp} = \frac{\alpha_{qp} \sqrt{\tau_p \beta_{mk}}}{\alpha_{qp} \alpha_{mk} + (\lambda_{qp} - \alpha_{qp}^2) b_m}, \]  

(49)

We have the MSE given by

\[ \epsilon_{mk}^{qp} = \mathbb{E}\{|g_{mk}|^2\} = \frac{\mathbb{E}\{s_{p,mk}^q g_{mk}\}^2}{\mathbb{E}\{|r_{p,mk}|^2\}^2}, \]  

(50)

where the second term can also be expressed as

\[ \gamma_{mk}^{qp} = \frac{\alpha_{qp}^2 \tau_p \rho_p \beta_{mk}^2}{\alpha_{qp}^2 \alpha_{mk}^2 + (\lambda_{qp} - \alpha_{qp}^2) b_m} = \frac{\gamma_{mk}}{\alpha_{qp}^2 \alpha_{mk}^2 + (\lambda_{qp} - \alpha_{qp}^2) b_m}. \]  

(51)

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