One-loop on-shell and off-shell decay $H^* \to VV$ at future $e^-e^-$ collider

Anh Thu Nguyen$^{(a,b)}$, Dzung Tri Tran$^{(a,b)}$ and Khiem Hong Phan$^{(c,d)}$

$^a)$University of Science, Ho Chi Minh City 700000, Vietnam
$^b)$Vietnam National University, Ho Chi Minh City 700000, Vietnam
$^c)$Institute of Fundamental and Applied Sciences, Duy Tan University, Ho Chi Minh City 70000, Vietnam
$^d)$Faculty of Natural Sciences, Duy Tan University, Da Nang City 550000, Vietnam

E-mail: phanhongkhiem@duytan.edu.vn, nathu@hcmus.edu.vn

Abstract. One-loop on-shell and off-shell decays $H \to VV$ with $VV = \gamma\gamma, Z\gamma, ZZ$ are presented in this paper. The effects of one-loop on-shell and off-shell Higgs decays in Higgs productions at $e^-e^-$ collisions are also then examined. We find that the impacts of one-loop Higgs decays are significant and they are must be taken into account at $e^-e^-$ collision.

1. Introduction
After discovering Standard Model-like Higgs boson \cite{1,2}, the precise measurements for Higgs productions and Higgs decay channels are one of main targets at the High Luminosity Large Hadron Collider (HL-LHC) and future lepton colliders (LC). Most of the on-shell Higgs decay processes are probed at the LHC. The data are in agreement with Standard Model (SM) predictions. In order to study the Higgs sector at different energy scales, off-shell Higgs decay channels are considerable interests at future colliders. Recently, off-shell Higgs decay $H^* \to ZZ$ have been measured at the LHC in \cite{3,4,5,6,7,8}. We also can find many interesting points from the phenomenological studies for off-shell Higgs decays in \cite{9,10,11,12,13,14,15,16,17,18,19,20}. In all mentioned references, the authors have not only provided the framework for testing the Higgs scalar sector at different energy scales but also discussed the possibility to explore new physics beyond the SM (BSM).

From theoretical computations, the evaluations for one-loop and two-loop QCD corrections to the off-shell Higgs decay $H^* \to ZZ$ in both signals and backgrounds at the LHC have been found in Ref. \cite{21,22,23,24,25,26,27,28,29,30}. While one-loop electroweak corrections to Higgs boson decay into $\gamma\gamma, Z\gamma, ZZ$ have been calculated in Refs. \cite{31,32,33,34,35,36,37,38,39,40,41}. In this paper, we present one-loop on-shell and off-shell decays $H \to VV$ with $VV = \gamma\gamma, Z\gamma, ZZ$. The effects of one-loop on-shell and off-shell Higgs decays in Higgs productions at $e^-e^-$ future lepton colliders are then examined. We find that the impacts of one-loop Higgs decays are significant and they must be taken into account at $e^-e^-$ collision.

The layout of the paper is as follows: in section 2, we discuss the calculation for one-loop Higgs boson decay into $VV$. We then study the implementation of one-loop Higgs decays to $VV$ in Higgs productions at $e^-e^-$ collision. All signal productions $e^-e^- \to e^-e^- H^* \to e^-e^-(VV)$ are examined. Conclusions and outlooks are shown in section 3.
2. Calculations
In this section, the calculations for one-loop decay channels $H^* \to ZZ$ within the Standard Model framework are discussed. The computations are performed in 't Hooft-Veltman (or $R_{\xi=1}$) gauge. The decay process $H^* \to ZZ$ consists of three group Feynman diagrams. In the first group, we have all fermions exchanging in the loop. With $W$-bosons and charged goldstone bosons and ghost particles exchanging in one-loop diagrams, we have group 2. In the group 3, one has all $Z$- or $H$-boson and neutral goldstone bosons propagating in the loop. For cancelling ultraviolet divergent (UV-divergent), we consider the counter-term diagram for vertex $HZZ$ (as plotted in group 0). All Feynman one-loop diagrams are plotted in the appendix. One-loop amplitudes for $H^* \to ZZ$ can be derived directly from the results of $H^* \to ZZ$.

The general one-loop amplitude for one-shell and off-shell Higgs decays into two on-shell vector bosons $H^*(p_H) \to V_\mu(q_1)V_\nu(q_2)$ can be expressed in terms of Lorentz structure as follows:

$$A_{H^* \to VV} = \left[ F_{00}^{VV} g^\mu\nu + F_{21}^{VV} \right] \bar{\epsilon}_\mu^\ast(q_1) \epsilon_\nu(q_2).$$

(1)

Where $\epsilon_\mu^\ast(q_1)$, $\epsilon_\nu(q_2)$ are polarization vectors of two external vector bosons. The scalar functions $F_{00}^{VV} = F_{00}^{VV}(M_{V_1}^2, M_{V_2}^2)$ and $F_{21}^{VV} = F_{21}^{VV}(M_{V_1}^2, M_{V_2}^2)$ are so-called one-loop form factors. They are functions of $p_H^2 = M_{V_1}^2 - M_{V_2}^2$. In the current work, we only focus on the case of two real external vector bosons in final state. Therefore, we have only the form factors $F_{00}^{VV}$ and $F_{21}^{VV}$ contributing to the decay rates. All related kinematic variables are given by:

$$p_H^2 = (q_1 + q_2)^2 = M_{V_1}^2 - M_{V_2}^2, \quad q_1^2 = M_{V_1}^2, \quad q_2^2 = M_{V_2}^2, \quad 2(q_1 \cdot q_2) = M_{V_1}^2 - M_{V_2}^2.$$

(2)

All Feynman amplitudes for $H^* \to ZZ$ are first written down. One then performs all Dirac traces and Lorentz contractions in $d$ dimensions with the help of Package-X [44]. Subsequently, the amplitudes are then expressed in terms of tensor one-loop integrals. Following tensor reduction in [45], all tensor one-loop integrals are then decomposed into scalar one-loop Passarino-Veltman (PV-functions) functions. The PV-functions can be computed numerically by using LoopTools [46]. As a result, the decay rates can be evaluated numerically. Analytic expressions for all one-loop form factors of Higgs decay to $\gamma\gamma, Z\gamma, ZZ$ channels can be found in [32 44 45].

General one-loop decay rate formulas for Higgs decays into two on-shell vector bosons $V_\mu(q_1)V_\nu(q_2)$ are derived as follows:

$$\Gamma_{H^* \to VV} = \frac{(2\pi)^4}{2M_{V_1}} \int d\Phi_2(M_{V_1}^2, M_{V_2}^2) \sum_{\text{pol}} |A_{H^* \to VV}|^2.$$

(3)

Where the phase space of 1 $\to$ 2 is given by

$$\int d\Phi_2(M_{V_1}^2, M_{V_2}^2) = \frac{\sqrt{\lambda(M_{V_1}^2, M_{V_2}^2)}}{128\pi^5 M_{V_1}^2}.$$

(4)

Here we use Källén function which is defined by $\lambda(a, b, c) = (a - b - c)^2 - 4bc$.

In order to calculate the squared amplitude for $H^* \to VV$, we first parameterize the sum of polarization vectors for two final bosons in general form as follows:

$$\sum_{\text{pol}} \epsilon_\alpha^\ast(q_1) \epsilon_\beta(q_1) = -g_{\alpha\beta} + g_{V_1} \frac{q_1 \cdot q_2 \delta_{\alpha\beta}}{M_{V_1}^2}.$$

(5)
Here we take $\delta V = 0, 1$ for photon $\gamma$ and $Z$-boson, respectively. Thus, the squared amplitude is casted into form of

$$
\sum_{\text{pol}} |A_{H^* \rightarrow VV}|^2 = \frac{1}{16 M_{V1}^4 M_{V2}^4} \left\{ C_{00} |F_{00}^{VV}(M_{V1}^2, M_{V1}^2, M_{V2}^2)|^2 + C_{21} |F_{21}^{VV}(M_{V1}^2, M_{V1}^2, M_{V2}^2)|^2 + C_{0021} \text{Re} \left[ F_{00}^{VV}(M_{V1}^2, M_{V1}^2, M_{V2}^2) F_{21}^{VV^*}(M_{V1}^2, M_{V1}^2, M_{V2}^2) \right] \right\}.
$$

(6)

Where all related coefficients are listed as follows:

$$
C_{00} = 4 \delta V_1 \left[ \delta V_2 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right)^2 - 4 M_{V1}^2 M_{V2}^2 \right] - 16 M_{V1}^2 M_{V2}^2 (\delta V_2 - 4),
$$

(7)

$$
C_{21} = \delta V_1 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right)^2 \left[ \delta V_2 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right)^2 - 4 M_{V1}^2 M_{V2}^2 \right]
- 4 M_{V1}^2 M_{V2}^2 \left[ \delta V_2 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right)^2 - 4 M_{V1}^2 M_{V2}^2 \right],
$$

(8)

$$
C_{0021} = 16 M_{V1}^2 M_{V2}^2 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right) (\delta V_2 - 1)
- 4 \delta V_1 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right) \left[ \delta V_2 \left( M_{V1}^2 + M_{V2}^2 - M_{VV}^2 \right)^2 - 4 M_{V1}^2 M_{V2}^2 \right].
$$

(9)

As a result, one-loop decay rates for Higgs decays into on-shell $\gamma\gamma, Z\gamma, ZZ$ are expressed explicitly as follows:

$$
\Gamma_{H^* \rightarrow \gamma\gamma} = \frac{\sqrt{\lambda(M_{2\gamma}, 0, 0)}}{(16\pi) M_{2\gamma}^3} \left\{ 4 |F_{00}^{\gamma\gamma}(M_{2\gamma}^2, 0, 0)|^2 - M_{2\gamma}^4 |F_{21}^{\gamma\gamma}(M_{2\gamma}^2, 0, 0)|^2 \right\},
$$

(10)

$$
\Gamma_{H^* \rightarrow Z\gamma} = \frac{\sqrt{\lambda(M_{2\gamma}^2, M_Z^2, 0)}}{(64\pi) M_{2\gamma}^2} \left\{ 12 |F_{00}^{Z\gamma}(M_{2\gamma}^2, M_Z^2, 0)|^2 - (M_Z^2 - M_{2\gamma}^2)^2 |F_{21}^{Z\gamma}(M_{2\gamma}^2, M_Z^2, 0)|^2 \right\},
$$

(11)

$$
\Gamma_{H^* \rightarrow ZZ} = \frac{\sqrt{\lambda(M_{ZZ}^2, M_{ZZ}^2, M_{ZZ}^2)}}{(256\pi) M_{ZZ}^3} \times
\times \left\{ \left( 48 M_Z^4 - 16 M_{ZZ}^2 M_{ZZ}^2 + 4 M_{ZZ}^4 \right) |F_{00}^{ZZ}(M_{ZZ}^2, M_Z^2, M_{ZZ}^2)|^2
+ \left( 16 M_Z^4 M_{ZZ}^2 - 8 M_Z^2 M_{ZZ}^2 + M_{ZZ}^4 \right) |F_{21}^{ZZ}(M_{ZZ}^2, M_Z^2, M_{ZZ}^2)|^2
+ \left( 32 M_Z^2 M_{ZZ}^2 - 24 M_Z^2 M_{ZZ}^2 + 4 M_{ZZ}^4 \right) \times
\times \text{Re} \left[ F_{00}^{ZZ}(M_{ZZ}^2, M_Z^2, M_{ZZ}^2) \cdot F_{21}^{ZZ^*}(M_{ZZ}^2, M_Z^2, M_{ZZ}^2) \right] \right\}.
$$

(12)

In phenomenological results, we study the impacts of on-shell and off-shell Higgs decay rates $H^* \rightarrow VV$ in Higgs productions at $e^- e^-$ collision. The cross sections for the processes
$e^- e^- \rightarrow e^- e^- H^* \rightarrow e^- e^- (VV)$ are computed. Differential cross sections with respect to $M_{VV}$ are given \[35\]:

$$\frac{d\sigma}{dM_{VV}}(\sqrt{s}, M_{VV}) = \frac{(2M_{VV}) \sigma^{e^- e^- \rightarrow e^- e^- H^*}(\sqrt{s}, M_{VV}) M_{VV} \Gamma_{H^* \rightarrow VV}(M_{VV})}{\pi} \left[ (M_{VV}^2 - M_H^2)^2 + (\Gamma_H M_H)^2 \right].$$

(13)

Where analytic formulas for cross sections of $\sigma^{e^- e^- \rightarrow e^- e^- H^*}$ have been reported in Ref. \[47\]. In this paper, differential cross sections with respect to $M_{VV}$ for all the processes $e^- e^- \rightarrow e^- e^- \gamma \gamma, Z\gamma, ZZ$ are plotted at center-of-mass energies (CMS energies) $\sqrt{s} = 350, 500$ GeV.

In Fig. 1 we plot the differential cross section for processes $e^- e^- \rightarrow e^- e^- \gamma \gamma$ with respect to $M_{\gamma \gamma}$ at CMS energies $\sqrt{s} = 350$ GeV (left panel) and at 500 GeV (right panel). We find a peak of on-shell $H \rightarrow \gamma \gamma$ at $M_{\gamma \gamma} \sim 125$ GeV. In Fig. 2, the differential cross sections for the processes $e^- e^- \rightarrow e^- e^- Z\gamma$ with respect to $M_{Z\gamma}$ at CMS energies $\sqrt{s} = 350$ GeV (left panel) and at 500 GeV (right panel) are shown. We also find a peak of on-shell $H \rightarrow Z\gamma$ at $M_{Z\gamma} \sim 125$ GeV. In Fig. 3, differential cross sections with respect to $M_{ZZ}$ for process $e^- e^- \rightarrow e^- e^- H^* \rightarrow e^- e^- (ZZ)$ at $\sqrt{s} = 350$ GeV (left panel), 500 GeV (right panel) are plotted at center-of-mass energies (CMS energies) $\sqrt{s} = 350, 500$ GeV.

In these Figures, the tree-level cross sections are plotted with solid line, and the cross sections with full one-loop electroweak corrections to $H^* \rightarrow ZZ$ are shown with dashed line. We find that the one-loop off-shell effects are visible and they must be taken into account at future colliders.

3. Conclusions

We have presented one-loop on-shell and off-shell decays $H \rightarrow VV$ with $VV = \gamma \gamma, Z\gamma, ZZ$ in this paper. The effects of one-loop on-shell and off-shell Higgs decays in Higgs productions at $e^- e^-$ collision have then studied. We find that the impacts of one-loop Higgs decays $H \rightarrow VV$
Figure 2. Differential cross sections with respect to $M_{Z\gamma}$ for process $e^-e^- \rightarrow e^-e^-H^* \rightarrow e^-e^-(Z\gamma)$ at $\sqrt{s} = 350$ GeV (left panel) and 500 GeV (right panel).

Figure 3. Differential cross sections with respect to $M_{ZZ}$ for process $e^-e^- \rightarrow e^-e^-H^* \rightarrow e^-e^-(ZZ)$ at $\sqrt{s} = 350$ GeV (left panel), 500 GeV (right panel).

are significant and they are must be taken into account at $e^-e^-$ collision.

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Appendix: Feynman diagrams

Figure 4. one-loop Feynman diagrams with exchanging $f$ in the loop (Group 1).

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Figure 5. One-loop Feynman diagrams with exchanging $W, \chi$ and ghost particles in the loop (Group 2).
Figure 6. one-loop Feynman diagrams with exchanging $Z, \chi_3$ and $H$ in the loop (Group 3).

Figure 7. Group 0: counter-term Feynman diagram.
