Recently, several discussions on the possible observability of 4-vector potential have been published in literature. Furthermore, several authors recently claimed existence of the helicity=0 electromagnetic field. We re-examine the theory of antisymmetric tensor field and 4-vector potentials. We study the massless limits too. In fact, a theoretical motivation for this venture is old papers of Ogievetskiǐ and Polubarinov, Hayashi, and Kalb and Ramond, which are widely accepted by physics community.

This paper is based on two poster presentations “About the Longitudinal Nature of the Antisymmetric Tensor Field after Quantization” and “Normalization and $m \to 0$ Limit of the Proca Theory” at the Workshop “Lorentz Group, CPT and Neutrinos” (Zacatecas, México, June 23-26, 1999).

1 Introduction

Since 1984 I was concerned with the puzzle of the so-called Kalb-Ramond field and the problem of higher spins. Later, thanks to Dirac and Weinberg works I learnt that a number of theoretical aspects of electrodynamic theory deserve more attention. First of all, they are: the problem of “fictious photons of helicity other than ±j, as well as the indefinite metric that must accompany them”; the renormalization idea, which “would be sensible only if it was applied with finite renormalization factors, not infinite ones (one is not allowed to neglect [and to subtract] infinitely large quantities)”; contradictions with the Weinberg theorem “that no symmetric tensor field of rank j can be constructed from the creation and annihilation operators of massless particles of spin j”, etc. Moreover, it appears now that we do not yet understand many specific features of classical electromagnetism, first of all, the problems of longitudinal modes, of the gauge, of the Coulomb action-at-a-distance, and of the Horwitz’ additional invariant parameter, refs. In this presentation I re-examine the theory of 4-potential field and antisymmetric tensor field of the second rank. The concept of the Ogievetskiǐ-Polubarinov-Kalb-Ramond field is simplified considerably.
2 4-potentials and Antisymmetric Tensor Field

The spin-0 and spin-1 particles can be constructed by taking the direct product of the spin-1/2 field functions. Let us firstly repeat the Bargmann-Wigner procedure of obtaining the equations for bosons of spin 0 and 1. The set of basic equations for $j = 0$ and $j = 1$ are written, e.g., ref.

\begin{align}
[i\gamma^\mu \partial_\mu - m]_{\alpha\beta} \Psi_{\beta\gamma}(x) &= 0 , \\
[i\gamma^\mu \partial_\mu - m]_{\gamma\beta} \Psi_{\alpha\beta}(x) &= 0 .
\end{align}

We expand the $4 \times 4$ matrix wave function into the antisymmetric and symmetric parts in a standard way

\begin{align}
\Psi_{[\alpha\beta]} &= R_{\alpha\beta}\phi + \gamma^5 R_{\delta\beta} \tilde{\phi} + \gamma^5 \gamma^\mu_{\delta\tau} R_\tau A_\mu , \\
\Psi_{\{\alpha\beta\}} &= \gamma^\mu_{\alpha\delta} R_{\delta\beta} A_\mu + \sigma^{\mu\nu}_{\alpha\delta} R_{\delta\beta} F_{\mu\nu} ,
\end{align}

where $R = CP$ has the properties (which are necessary to make expansions (3, 4) to be possible in such a form)

\begin{align}
R^T &= -R , \\
R^\dagger &= R , \\
R^{-1} \gamma^5 R &= (\gamma^5)^T , \\
R^{-1} \gamma^\mu R &= -(\gamma^\mu)^T , \\
R^{-1} \sigma^{\mu\nu} R &= -(\sigma^{\mu\nu})^T .
\end{align}

The explicit form of this matrix can be chosen:

\begin{equation}
R = \begin{pmatrix}
i\Theta & 0 \\
0 & -i\Theta
\end{pmatrix} , \\
\Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\
1 & 0 \end{pmatrix} ,
\end{equation}

provided that $\gamma^\mu$ matrices are in the Weyl representation. The equations (3, 4) lead to the Kemmer set of the $j = 0$ equations:

\begin{align}
m\phi &= 0 , \\
m\tilde{\phi} &= -i\partial_\mu \tilde{A}_\mu , \\
m\tilde{A}_\mu &= -i\partial^\mu \phi ,
\end{align}

and to the Proca-Duffin-Kemmer set of the equations for the $j = 1$ case:

\begin{align}
\partial_\alpha F^{\alpha\mu} + \frac{m}{2} A_\mu &= 0 , \\
2mF^{\mu\nu} &= \partial_\mu A^\nu - \partial^\nu A_\mu ,
\end{align}

\begin{flushright}
\footnotesize\textsuperscript{a}We could use another symmetric matrix $\gamma^5 \sigma^{\mu\nu}$ $R$ in the expansion of the symmetric spinor of the second rank. In this case the equations will read

\begin{equation}
i\partial_\alpha F^{\alpha\mu} + \frac{m}{2} B_\mu = 0 ,
\end{equation}
\end{flushright}

\textsuperscript{178}
In the meantime, in the textbooks, the latter set is usually written as (e.g., p. 135 of ref. [2])

\[
\partial_\alpha F^{\alpha \mu} + m^2 A^\mu = 0 ,
\]

\[
F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu ,
\]

The set (17,18) is obtained from (15,16) after the normalization change \( A_\mu \rightarrow 2m A_\mu \) or \( F^{\mu \nu} \rightarrow \frac{1}{2m} F^{\mu \nu} \). Of course, one can investigate other sets of equations with different normalization of the \( F^{\mu \nu} \) and \( A_\mu \) fields. Are all these sets of equations equivalent? As we shall see, to answer this question is not trivial. Ahluwalia argued that the physical normalization is such that in the massless-limit zero-momentum field functions should vanish in the momentum representation (there are no massless particles at rest). We advocate the following approach: the massless limit can and must be taken in the end of all calculations only, i.e., for physical quantities.

Let us proceed further. In order to be able to answer the question about the behaviour of the spin operator \( J^i = \frac{1}{2} \varepsilon^{ijk} J^j k \) in the massless limit one should know the behaviour of the fields \( F^{\mu \nu} \) and/or \( A_\mu \) in the massless limit. We want to analyze the first set (15,16). If one advocates the following definitions (p. 209 of ref. [3])

\[
e^\mu(0, +1) = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}, \quad e^\mu(0, 0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad e^\mu(0, -1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix} \quad (19)
\]

and \( \tilde{p}_i = p_i / |p|, \gamma = E_p/m \), p. 68 of ref. [3]

\[
e^\mu(p, \sigma) = L^\mu_\nu(p) e^\nu(0, \sigma) , \quad (20)
\]

\[
L^0 (p) = \gamma, \quad L^i (p) = L^0_{-i} (p) = \tilde{p}_i \sqrt{\gamma^2 - 1} , \quad (21)
\]

\[
L^i_k (p) = \delta_{ik} + (\gamma - 1) \tilde{p}_i \tilde{p}_k , \quad (22)
\]

in which the dual tensor \( \tilde{F}^{\mu \nu} = \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} F_{\rho \sigma} \) presents, because we used that in the Weyl representation \( \gamma^5 \sigma_{\mu \nu} = \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} \sigma_{\rho \sigma}; B^\mu \) is the corresponding vector potential. The equation for the antisymmetric tensor field (which can be obtained from this set) does not change its form (cf. [4i,j]) but we see some “renormalization” of the field functions. In general, it is permitted to choose various relative phase factors in the expansion of the symmetric wave function (4) and also consider the matrix term of the form \( \gamma^5 \sigma_{\mu \nu} \). We shall have additional phase factors in equations relating the physical fields and the 4-vector potentials. They can be absorbed by the redefinition of the potentials/fields. The above shows that the dual tensor of the second rank can also be expanded in potentials. This discussion is intimately related to the matter of parity properties of \( j = 1 \) field and with the theoretical quest of existence of antiparticle in the \( (1, 0) \oplus (0, 1) \) and/or \( (1/2, 1/2) \) representations.
for the 4-vector potential field, p. 129 of ref. 3

\[ A^\mu(x^\nu) = \sum_{\sigma = \pm 1} \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E_p}} \left[ \epsilon^\mu(p, \sigma) a(p, \sigma) e^{-ip \cdot x} + (\epsilon^\mu(p, \sigma))^* b^\dagger(p, \sigma) e^{ip \cdot x} \right] , \]

(23)

the normalization of the wave functions in the momentum representation is thus chosen to the unit, \( \epsilon^\mu(p, \sigma) \epsilon^\mu(p, \sigma)^* = -1 \). We observe that in the massless limit all the defined polarization vectors of the momentum space do not have good behaviour; the functions describing spin-1 particles tend to infinity. This is not satisfactory. Nevertheless, after renormalizing the potentials, e.g., \( \epsilon^\mu(p, \sigma) \rightarrow u^\mu(p, \sigma) \equiv m \epsilon^\mu(p, \sigma) \) we come to the field functions in the momentum representation:

\[ u^\mu(p, +1) = -\frac{N}{\sqrt{2m}} \left( \begin{array}{c} \frac{p_r}{E_p + m} \\ \frac{p_1}{E_p + m} \\ \frac{p_3}{E_p + m} \\ \frac{p_2}{E_p + m} \end{array} \right) , \]

\[ u^\mu(p, -1) = \frac{N}{\sqrt{2m}} \left( \begin{array}{c} \frac{p_r}{E_p + m} \\ \frac{-im + p_2}{E_p + m} \\ \frac{-ip_1 + p_3}{E_p + m} \\ \frac{-ip_3 - p_1}{E_p + m} \end{array} \right) \]

\[ u^\mu(p, 0) = \frac{N}{m} \left( \begin{array}{c} \frac{p_3}{E_p + m} \\ \frac{p_1}{E_p + m} \\ \frac{p_2}{E_p + m} \\ \frac{p_1 p_2}{E_p + m} \end{array} \right) , \]

(24)

(25)

\( (N = m \text{ and } p_r, l = p_1 \pm ip_2) \) which do not diverge in the massless limit. Two of the massless functions (with \( h = \pm 1 \)) are equal to zero when a particle, described by this field, is moving along the third axis (\( p_1 = p_2 = 0, p_3 \neq 0 \)).

\( b \) Remember that the invariant integral measure over the Minkowski space for physical particles is

\[ \int d^4 p \delta(p^2 - m^2) \equiv \int \frac{d^3 p}{2E_p} , \quad E_p = \sqrt{p^2 + m^2} . \]

Therefore, we use the field operator as in (23). The coefficient \( (2\pi)^3 \) can be considered at this stage as chosen for the convenience. In ref. 3, the factor \( 1/(2E_p) \) was absorbed in creation/annihilation operators and instead of the field operator (23) the operator was used in which the \( \epsilon^\mu(p, \sigma) \) functions for a massive spin-1 particle were substituted by \( u^\mu(p, \sigma) = (2E_p)^{-1/2} \epsilon^\mu(p, \sigma) \), what may lead to confusions in the definitions of the massless limit \( m \rightarrow 0 \) for classical polarization vectors.

\( c \) The metric used in this paper \( g^{\mu\nu} = \text{diag}(1, -1, -1, -1) \) is different from that of ref. 13.

\( d \) It is interesting to remind that the authors of ref. 13 (see page 136 therein) tried to enforce the Stueckelberg’s Lagrangian in order to overcome the difficulties related with the \( m \rightarrow 0 \) limit (or the Proca theory → Quantum Electrodynamics). The Stueckelberg’s Lagrangian is well known to contain an additional term which may be put in correspondence to some scalar (longitudinal) field (cf. also).
The third one \((h = 0)\) is

\[
\begin{pmatrix}
  p_3 \\
  0 \\
  0 \\
  \frac{p_3^2}{E_p}
\end{pmatrix}
= \begin{pmatrix}
  E_p \\
  0 \\
  0 \\
  E_p
\end{pmatrix}, \quad (26)
\]

and at the rest \((E_p = p_3 \to 0)\) also vanishes. Thus, such a field operator describes the “longitudinal photons” what is in the complete accordance with the Weinberg theorem \(B - A = h\) for massless particles (we use the \(D(1/2, 1/2)\) representation). Thus, the change of the normalization can lead to the “change” of physical content described by the classical field (at least, comparing with the well-accepted one). In the quantum case one should somehow fix the form of commutation relations by some physical principles. They may be fixed by requirements of the dimensionless of the action (apart from the requirements of the translational and rotational invariances).

Furthermore, it is easy to prove that the physical fields \(F_{\mu \nu}\) (defined as in \((15, 16)\), for instance) vanish in the massless zero-momentum limit under the both definitions of normalization and field equations. It is straightforward to find

\[
\begin{align*}
B^{(+)}(p, \sigma) &= \frac{iN}{2m} p \times u(p, \sigma), \\
E^{(+)}(p, \sigma) &= \frac{1}{2m} p_0 u(p, \sigma) - \frac{i}{2m} p u^0(p, \sigma)
\end{align*}
\]

and the corresponding negative-energy strengths for the field operator (in general, complex-valued)

\[
F_{\mu \nu}(x) = \sum_{\sigma = 0, \pm 1} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} \left[ F_{\mu \nu}^{(+)}(p, \sigma) a(p, \sigma) e^{-ipx} + F_{\mu \nu}^{(-)}(p, \sigma) b^\dagger(p, \sigma) e^{+ipx} \right]
\]

Here they are\[\]

\[
\begin{align*}
B^{(+)}(p, +1) &= -\frac{iN}{2\sqrt{2m}} \begin{pmatrix}
  -ip_3 \\
  p_3 \\
  ip_r
\end{pmatrix} = +e^{-i\alpha_{+}} B^{(-)}(p, -1), \\
B^{(+)}(p, 0) &= \frac{iN}{2m} \begin{pmatrix}
  p_2 \\
  -p_1 \\
  0
\end{pmatrix} = -e^{-i\alpha_0} B^{(-)}(p, 0), \\
B^{(+)}(p, -1) &= -\frac{iN}{2\sqrt{2m}} \begin{pmatrix}
  ip_3 \\
  p_3 \\
  -ip_l
\end{pmatrix} = +e^{-i\alpha_{+}} B^{(-)}(p, +1)
\end{align*}
\]

\(a\)In this paper we assume that \([\epsilon^\mu(p, \sigma)]^c = e^{i\alpha_{+}}[\epsilon^\mu(p, \sigma)]^*, \) with \(\alpha_{+}\) being arbitrary phase factors at this stage. Thus, \(\mathcal{C} = I_{4 \times 4}\) and \(S^C = \mathcal{K}\). Taking in mind the problem of indefinite metrics it would be interesting to investigate other choices of the \(\mathcal{C}\), the charge conjugation matrix and/or consider a field operator composed of CP-conjugate states.
and

\[
\mathbf{E}^{(+)}(\mathbf{p}, +1) = -\frac{iN}{2\sqrt{2m}} \left( \frac{E_p - \frac{p_1 p_3}{E_p + m}}{\frac{p_2 p_3}{E_p + m}} \right) = +e^{-i\alpha'_+} \mathbf{E}^{(-)}(\mathbf{p}, -1) , \quad (31)
\]

\[
\mathbf{E}^{(+)}(\mathbf{p}, 0) = \frac{iN}{2m} \left( \frac{\frac{p_1 p_3}{E_p + m}}{E_p - \frac{p_2 p_3}{E_p + m}} \right) = -e^{-i\alpha'_0} \mathbf{E}^{(-)}(\mathbf{p}, 0) , \quad (32)
\]

\[
\mathbf{E}^{(+)}(\mathbf{p}, -1) = \frac{iN}{2\sqrt{2m}} \left( \frac{E_p - \frac{p_1 p_3}{E_p + m}}{\frac{p_2 p_3}{E_p + m}} \right) = +e^{-i\alpha'_+} \mathbf{E}^{(-)}(\mathbf{p}, +1) , \quad (33)
\]

where we denoted, as previously, a normalization factor appearing in the definitions of the potentials (and/or in the definitions of the physical fields through potentials) as \(N\).

For the sake of completeness let us present the fields corresponding to the “time-like” polarization:

\[
u^\mu(\mathbf{p}, 0_t) = \frac{N}{m} \left( \begin{array}{c}
\frac{E_p}{p_1} \\
\frac{E_p}{p_2} \\
\frac{E_p}{p_3}
\end{array} \right) , \quad \mathbf{B}^{(\pm)}(\mathbf{p}, 0_t) = 0 , \quad \mathbf{E}^{(\pm)}(\mathbf{p}, 0_t) = 0 . \quad (34)
\]

The polarization vector \(u^\mu(\mathbf{p}, 0_t)\) has good behaviour in \(m \to 0\), \(N = m\) (and also in the subsequent limit \(\mathbf{p} \to 0\)) and it may correspond to some field (particle). As one can see the field operator composed of the states of longitudinal (e.g., as positive-energy solution) and time-like (e.g., as negative-energy solution) polarizations may describe a situation when a particle and an antiparticle have opposite intrinsic parities. Furthermore, in the case of the normalization of potentials to the mass \(N = m\) the physical fields \(\mathbf{B}\) and \(\mathbf{E}\), which correspond to the “time-like” polarization, are equal to zero identically. The longitudinal fields (strengths) are equal to zero in this limit only when one chooses the frame with \(p_3 = |\mathbf{p}|\), cf. with the light front formulation, ref.\[14]\.

In the case \(N = 1\) and \(\{13,16\}\) the fields \(\mathbf{B}^{\pm}(\mathbf{p}, 0_t)\) and \(\mathbf{E}^{\pm}(\mathbf{p}, 0_t)\) would be undefined.
3 Lagrangian, Energy-Momentum Tensor and Angular Momentum

We begin with the Lagrangian, including, in general, mass term:

\[
\mathcal{L} = \frac{1}{4} (\partial_\mu F_{\nu \alpha})(\partial^\mu F^{\nu \alpha}) - \frac{1}{2} (\partial_\mu F^{\mu \alpha})(\partial^\nu F_{\nu \alpha}) - \frac{1}{2} (\partial_\mu F_{\nu \alpha})(\partial^\nu F^{\mu \alpha}) + \frac{1}{4} m^2 F_{\mu \nu} F^{\mu \nu}.
\]  

(36)

The Lagrangian leads to the equation of motion in the following form (provided that the appropriate antisymmetrization procedure has been taken into account):

\[
\frac{1}{2} (\Box + m^2) F_{\mu \nu} + (\partial_\mu F_{\alpha \nu} - \partial_\nu F_{\alpha \mu}) = 0,
\]  

(37)

where \(\Box = -\partial_\alpha \partial^\alpha\), cf. with the set of equations (15,16). It is this equation for antisymmetric-tensor-field components that follows from the Proca-Duffin-Kemmer-Bargmann-Wigner consideration provided that \(m \neq 0\) and in the final expression one takes into account the Klein-Gordon equation \((\Box - m^2) F_{\mu \nu} = 0\). The latter expresses relativistic dispersion relations \(E^2 - p^2 = m^2\).

Following the variation procedure one can obtain that the energy-momentum tensor is expressed:

\[
\Theta^{\lambda \beta} = \frac{1}{2} \left[ (\partial^\lambda F_{\mu \alpha})(\partial^\beta F^{\mu \alpha}) - 2(\partial_\mu F^{\mu \alpha})(\partial^\beta F^{\lambda \alpha}) - 2(\partial^\mu F^{\lambda \alpha})(\partial^\beta F_{\mu \alpha}) \right] - \mathcal{L}g^{\lambda \beta}.
\]  

(38)

One can also obtain that for rotations \(x' = x + \omega \mu \nu x_\nu\) the corresponding variation of the wave function is found from the formula:

\[
\delta F^{\alpha \beta} = \frac{1}{2} \omega^{\kappa \tau} T^{\alpha \beta, \mu \nu} F_{\mu \nu}.
\]  

(39)

\(b\)The massless limit \((m \to 0)\) of the Lagrangian is connected with the Lagrangians used in the conformal field theory and in the conformal supergravity by adding the total derivative:

\[
\mathcal{L}_{CFT} = \mathcal{L} + \frac{1}{2} \partial_\mu (F_{\alpha \beta} \partial^\mu F^{\alpha \beta} - F^{\mu \alpha} \partial^\mu F_{\alpha \beta}).
\]  

(35)

The Kalb-Ramond gauge-invariant form (with respect to “gauge” transformations \(F_{\mu \nu} \to F_{\mu \nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu\), ref.\ref{1,2} is obtained only if one uses the Fermi procedure \textit{mutatis mutandis} by removing the additional “phase” field \(\lambda(\partial_\mu F^{\mu \nu})^2\), with the appropriate coefficient \(\lambda\) from the Lagrangian. This has certain analogy with the QED, where the question, whether the Lagrangian is gauge-invariant or not, is solved depending on the presence of the term \(\lambda(\partial_\mu A^\mu)^2\). For details see ref.\ref{2}.

In general it is possible to introduce various forms of the mass term and of corresponding normalization of the field. But, the dimensionless of the action \(S\) may impose some restrictions; we know that \(F^{\mu \nu}\) in order to be able to describe long-range forces should have the dimension \([\text{energy}]^2\). In order to take this into account one should divide the Lagrangian \((36)\) by \(m^2\); calculate corresponding dynamical invariants, other observable quantities; and only then study \(m \to 0\) limit.
The generators of infinitesimal transformations are then defined as
\[ T_{\kappa\tau}^{\alpha\beta,\mu\nu} = \frac{1}{2} g^{\alpha\mu} (\delta^\beta_\kappa \delta^\nu_\tau - \delta^\beta_\tau \delta^\nu_\kappa) + \frac{1}{2} g^{\beta\mu} (\delta^\nu_\kappa \delta^\alpha_\tau - \delta^\nu_\tau \delta^\alpha_\kappa) + \frac{1}{2} g^{\alpha\nu} (\delta^\beta_\kappa \delta^\mu_\tau - \delta^\beta_\tau \delta^\mu_\kappa) + \frac{1}{2} g^{\beta\nu} (\delta^\alpha_\kappa \delta^\mu_\tau - \delta^\alpha_\tau \delta^\mu_\kappa) . \] (40)

It is \( T_{\kappa\tau}^{\alpha\beta,\mu\nu} \), the generators of infinitesimal transformations, that enter in the formula for the relativistic spin tensor:
\[ J_{\kappa\tau} = \int d^3x \left[ \frac{\partial L}{\partial (\partial F^{\alpha\beta}/\partial t)} T_{\kappa\tau}^{\alpha\beta,\mu\nu} F_{\mu\nu} \right] . \] (41)

As a result one obtains:
\[ J_{\kappa\tau} = \int d^3x \left[ (\partial_\mu F^{\mu\nu})(g_{0\kappa} F_{0\tau} - g_{0\tau} F_{0\kappa}) - (\partial_\mu F^{\mu\nu}) F_{0\kappa} + (\partial_\mu F^{\mu\nu}) F_{0\kappa} + F^{\mu\nu}(\partial_\mu F_{\kappa\mu} + \partial_\kappa F_{0\mu} + \partial_\mu F_{\kappa 0}) - F^{\mu\nu}(\partial_\mu F_{\kappa\mu} + \partial_\kappa F_{0\mu} + \partial_\mu F_{\kappa 0}) \right] . \] (42)

If one agrees that the orbital part of the angular momentum
\[ L_{\kappa\tau} = x_\kappa \Theta_{0\tau} - x_\tau \Theta_{0\kappa} , \] (43)
with \( \Theta_{\tau\lambda} \) being the energy-momentum tensor, does not contribute to the Pauli-Lubanski operator when acting on the one-particle free states (as in the Dirac \( j = 1/2 \) case), then the Pauli-Lubanski 4-vector is constructed as follows, Eq. (2-21) of \[ 12 \]
\[ W_\mu = -\frac{1}{2} \epsilon_{\mu\kappa\tau\nu} J^{\kappa\tau} P^\nu , \] (44)
with \( J^{\kappa\tau} \) defined by Eqs. (1-2). The 4-momentum operator \( P^\nu \) can be replaced by its eigenvalue when acting on the plane-wave eigenstates.

Furthermore, one should choose space-like normalized vector \( n^\mu n_\mu = -1 \), for example \( n_0 = 0, \ n = \hat{p} = p/|p| \). After lengthy calculations in a spirit of pp. 58, 147 of \[ 12 \], one can find the explicit form of the relativistic spin:
\[ (W_\mu \cdot n^\mu) = -(W \cdot n) = -\frac{1}{2} \epsilon^{ijk} n^k J^{ij} p^0 , \] (45)
\[ J^k = \frac{1}{2} \epsilon^{ijk} J^{ij} = \epsilon^{ijk} \int d^3x \left[ F^{0\mu}(\partial_\mu F^{ij}) + F^{\mu j}(\partial^0 F^{ij} + \partial^i F^{0j} + \partial^j F^{00}) \right] . \] (46)

\(^{c}\)One should remember that the helicity operator is usually connected with the Pauli-Lubanski vector in the following manner \( (J \cdot \hat{p}) = (W \cdot \hat{p})/E_p \), see ref. \[ 14 \]. The choice of ref. \[ 14 \], p. 147, \( n^\mu = (t^\mu - p^\mu \frac{m}{|p|}) \frac{m}{|p|} \), with \( t^\mu \equiv (1,0,0,0) \) being a time-like vector, is also possible but it leads to some oscurities in the procedure of taking the massless limit. These oscurities will be clarified in a separate paper.
Now it becomes obvious that the application of the generalized Lorentz conditions (which are quantum versions of free-space dual Maxwell’s equations) leads in such a formulation to the absence of electromagnetism in a conventional sense. The resulting Kalb-Ramond field is longitudinal (helicity $h = 0$). All the components of the angular momentum tensor for this case are identically equated to zero.

One can consider connections with earlier works and regard $A^\times A^*$ term as the part of antisymmetric tensor potential and $B^\times B^*$ as the part of the 4-vector field. According to 1 we proceed in the construction of the “potentials” for the notoph (Ogievetskii-Polubarinov-Kalb-Ramond field) as follows:

$$\tilde{F}_{\mu\nu}(p) = N \left[ \epsilon^{(1)}_\mu(p)\epsilon^{(2)}_\nu(p) - \epsilon^{(1)}_\nu(p)\epsilon^{(2)}_\mu(p) \right]$$

On using explicit forms for the polarization vectors in the momentum space one obtains

$$\tilde{F}^{\mu\nu}(p) = \frac{iN^2}{m} \begin{pmatrix} 0 & -p_2 & m + \frac{p_0 p_2}{p_0 + m} & 0 \\ p_2 & 0 & \frac{p_0 p_2}{p_0 + m} & -\frac{p_1 p_2}{p_0 + m} \\ -p_1 & -m - \frac{p_2 p_0}{p_0 + m} & 0 & \frac{p_1 p_2}{p_0 + m} \\ 0 & \frac{p_2 p_0}{p_0 + m} & \frac{p_1 p_2}{p_0 + m} & 0 \end{pmatrix},$$

i.e., it coincides with the longitudinal components of the antisymmetric tensor obtained in refs. and my previous works within the normalization and different forms of the spin basis (cf. formulas (29,32) above). The longitudinal states reduce to zero in the massless case under appropriate choice of the normalization and only if $j = 1$ particle moves along with the third axis $OZ$. It is also useful to compare Eq. (48) with the formula (B2) in ref. in order to realize the correct procedure for taking the massless limit.

Finally, we agree with the previous authors, e.g., ref. see Eq. (4) therein, about the gauge non-invariance of the division of the angular momentum of the electromagnetic field into the “orbital” and “spin” part (46). We proved again that for the antisymmetric tensor field $J \sim \int d^3x E \times A$. So, what people actually did (when spoken about the Ogievetskii-Polubarinov-Kalb-Ramond field) is: When $N = m$ they considered the gauge part of the 4-vector field functions. Then, equated $A$ containing the transverse modes on choosing $p_t = p_l = 0$ in the massless limit (see formulas (24)). Under this choice the $E(p,0)$ and $B(p,0)$ are equal to zero in massless limit. But, the gauge part of $u^\mu(p,0)$ is not. The spin angular momentum can still be zero. When $N = 1$ the situation is the same because of the different form of dynamical equations. So,

\[d\text{The reader, of course, can consider equating by the usual gauge transformation, } A^\mu \rightarrow A^\mu + \partial^\mu \chi.\]
for those who prefer simpler consideration it is enough to regard all possible states of 4-potentials/antisymmetric tensor field in the massless limit in the calculation of physical observables. Of course, I would like to repeat, it is not yet clear and it is not yet supported by reliable experiments whether the third state of the 4-vector potential/antisymmetric tensor field has physical significance and whether it is observable.

However, in [4] and several previous works experiments for verification of real physical significance of 4-potentials have been designed.

4 Conclusions

The achieved conclusion is: both the antisymmetric tensor field and the 4-potential field may have third helicity state in massless limit from a theoretical viewpoint. This problem is connected with the problem of the observability of the gauge. This conclusion is achieved on the basis of the analysis of the problem from a viewpoint of the normalization.

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187