Thermal Hadronization, Hawking-Unruh Radiation and Event Horizon in QCD

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Abstract

Because of colour confinement, the physical vacuum forms an event horizon for quarks and gluons; this can be crossed only by quantum tunneling, i.e., through the QCD counterpart of Hawking radiation by black holes. Since such radiation cannot transmit information to the outside, it must be thermal, of a temperature determined by the strong force at the confinement surface, and it must maintain colour neutrality. The resulting process provides a common mechanism for thermal hadron production in high energy interactions, from $e^+e^-$ annihilation to heavy ion collisions. The analogy with black-hole event horizon suggests a dependence of the hadronization temperature on the baryon density.

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INTRODUCTION

Over the years, hadron production studies in a variety of high energy collision experiments have shown a remarkably universal feature. From $e^+e^-$ annihilation to $p-p$ and $p-\bar{p}$ interactions and further to collisions of heavy nuclei, covering an energy range from a few GeV up to the TeV range, the production pattern always shows striking thermal aspects, connected to an apparently quite universal temperature around $T_H \simeq 160 - 190$ MeV \cite{1,2}.

What is the origin of this thermal behaviour? While high energy heavy ion collisions involve large numbers of incident partons and thus could allow invoking some “thermalisation” scheme through rescattering, in $e^+e^-$ annihilation the predominant initial state is one energetic $q\bar{q}$ pair, and the number of hadronic secondaries per unit rapidity is too small to consider statistical averages.

A further piece in this puzzle is the observation that the value of the temperature determined in the mentioned collision studies is quite similar to the confinement/deconfinement transition temperature found in lattice studies of strong interaction thermodynamics \cite{3}. While hadronization in high energy collisions deals with a dynamical situation, the energy loss of fast colour charges “traversing” the physical vacuum, lattice QCD addresses the equilibrium thermodynamics of unbound vs. bound colour charges. Why should the resulting critical temperatures be similar or even identical?

In ref.\cite{4}, which is summarized in this contribution (see also ref.\cite{5}), these hadronization phenomena are considered as the QCD counterpart of the Hawking radiation emitted by black holes (BH) \cite{6}. BHs provide a gravitational form of confinement that was quite soon compared to that of colour confinement in QCD \cite{2,3}, where coloured constituents are confined to “white holes” (colourless from the outside, but coloured inside).

The main results in ref.\cite{4} are:

- Colour confinement and the instability of the physical vacuum under pair production form an event horizon for quarks, allowing a transition only through quantum tunnelling; this leads to thermal radiation of a temperature $T_H$ determined by the string tension.

- Hadron production in high energy collisions occurs through a succession of such tunnelling processes. The resulting cascade is a realization of the same partition process which leads to a limiting temperature in the statistical bootstrap and dual resonance models.

- In kinetic thermalization, the initial state information is successively lost through collisions, converging to a time-independent equilibrium state. In contrast, the stochastic QCD Hawking radiation is “born in equilibrium”, since quantum tunnelling a priori does not allow information transfer.

- The temperature $T_H$ of QCD Hawking radiation depends only on the baryon number and the angular momentum of the deconfined system. The former provides the dependence of $T_H$ on the baryochemical potential $\mu$, while the angular momentum pattern of the radiation allows a centrality-dependence of $T_H$ and elliptic flow. In particular the $\mu$ dependence of $T_H$, will be discussed in more details in sec. 4.

THERMAL PRODUCTION PATTERN

Let us first summarize the thermal production pattern in elementary collisions, $e^+e^-, pp, \bar{p}p, ..$ and in nucleus-nucleus scattering.
The partition function of an ideal resonance gas is given by
\[ \ln Z(T) = V \sum_i \frac{d_i}{(2\pi)^3} \phi(m_i, T) \] (1)
where \( d_i \) is the degeneracy factor and \( \phi(m_i, T) \) is the Boltzmann factor
\[ \phi(m_i, T) = 4\pi m_i^2 T K_2(m_i/T) \] (2)

Therefore the relative abundances of the species \( i \) and \( j \) turns out
\[ \frac{N_i}{N_j} = \frac{d_i \phi(m_i, T)}{d_j \phi(m_j, T)} \] (3)

and for transverse energy larger than \( T \)
\[ \frac{dN}{dp_T^2} \sim \exp -\frac{1}{T} \sqrt{m_i^2 + p_T^2} \] (4)

In elementary collisions the statistical hadronization model [9] fits the data on the species abundances by two parameters: \( T \) and \( \gamma_s \), that describes the strangness suppression.

For LEP data at \( \sqrt{s} = 91.2 \) Gev [2], \( T = 170 \pm 3 \pm 6 \) Mev and \( \gamma_s = 0.691 \pm 0.053 \) where the systematic error is obtained by varying the resonanse gas scheme and the contributing resonances. The PEP-PETRA data at different energies , \( 14 < \sqrt{s} < 45 \) can be fitted [2] with an average temperature \( T = 165 \pm 6 \) Mev and \( \gamma_s \approx 0.7 \pm 0.05 \). The pp SPS data at energies \( \sqrt{s} = 19, 23.8, 26 \) Gev give \( T = 162.4 \pm 1.6 \) Mev and \( \gamma_s \approx 0. \pm 0.036 \) [2]. The other data for \( K^+p \) and \( \pi^+p \) scattering at energies close to SPS one and for \( \bar{p}p \) at larger energy can be fitted by similar values.

The fitted values of the temperature are depicted in fig. 1 [10].

Therefore there is an universal hadronization temperature \( T_H = 170 \pm 10 - 20 \) Mev which is independent on the species, on \( \sqrt{s} \) and on the incident configuration. Moreover, also the transverse momentum spectra in elementary collisions can be fitted by the same value \( T_H \) [2].

In heavy ion collisions there is a new parameter which describes the finite baryon density, i.e. the baryon chemical potential \( \mu_B \). The fits of the species abundances at high energy ( peak SPS and RHIC) give [2] :
- \( T_H = 168 \pm 2.4 \pm 10 \) Mev; \( \mu_B = 266 \pm 5 \pm 30 \) Mev at \( \sqrt{s} = 17 \) Gev for (SPS) Pb-Pb,
- \( T_H = 168 \pm 7 \) Mev; \( \mu_B = 38 \pm 11 \pm 5 \) Mev at \( \sqrt{s} = 130 \) Gev for (RHIC) Au-Au at \( y=0 \),
- \( T_H = 161 \pm 2 \) Mev; \( \mu_B = 20 \pm 4 \) Mev at \( \sqrt{s} = 200 \) Gev for (RHIC) Au-Au.

In conclusion, hadron abundances in all high energy collisions ( \( e^+e^- \) annihilation,hadron-hadron and heavy ion collisions) are those of an ideal resonance gas at universal temperature \( T_H \approx 170 \pm 10 - 20 \) Mev.

**EVENT HORIZON AND HADRONIZATION**

The idea that a thermal medium, with a kinetic thermalization by multiple partonic interaction, has been produced in the collisions could explain the previous phenomenon for a nucleus-nucleus scattering but does not work for \( e^+e^- \) and hadron-hadron scattering.

One has to look for an universal, “non-kinetic” thermalization mechanism.

Indeed, in gravitation there is a well known example: the BH Hawking radiation has a thermal radiation spectrum due to tunnelling through the event horizon and the Hawking temperature is given by \( T_{BH} = 1/8\pi GM \), where \( M \) is the (Schwarzschild) BH mass and \( G \) is the Newton constant [6].
Therefore the conjecture [4] is that colour confinement and hadronization mechanism are in strong analogy with BH physics and event horizon.

There are many reasons to believe that color confinement can be described by a color horizon in QCD because the theory is non linear and therefore it has an effective curved geometry [11, 12]. However, since we discuss the hadronization mechanism, is better to consider the Unruh effect.

As shown by Unruh [13], systems with uniform acceleration $a$, have an event horizon and see a thermal bath with temperature $T_U = a/2\pi$. For a particle of mass $m$, in uniform acceleration the equation of motion is solved by the parametric form

$$x = \frac{1}{a} \cosh a\tau, \quad t = \frac{1}{a} \sinh a\tau,$$

where $a = F/m$ denotes the acceleration in the instantaneous rest frame of $m$, and $\tau$ the proper time, with $d\tau = \sqrt{1 - v^2} dt$. The resulting world line is shown in fig.2 with the event horizon beyond which $m$ classically cannot pass. The only signal the observer can detect as consequence of the passage of $m$ is thermal quantum radiation of temperature

$$T_U = \frac{a}{2\pi}.$$  

In the case of gravity, $a$ is the “surface gravity” (i.e. the acceleration at the horizon), $a = 1/(4GM)$, and hence one recovers the Hawking temperature.

In summary, the acceleration leads to a classical turning point and hence to an event horizon, which can be surpassed only by quantum tunnelling and at the expense of complete information loss, leading to thermal radiation as the only allowed signal.
On the other hand, at quantum level, it is well known that in a strong field the vacuum is unstable against pair production \[14\]. For example, in $e^+e^-$ annihilation a $\bar{q}q$ pair is initially produced and when the linear potential is such that $\sigma x > \sigma x_Q \equiv 2m$ the string connecting $\bar{q}q$ breaks and the color neutralization induces an effective quantum event horizon (see figs 3, 4).

The $\bar{q}q$ flux tube has a thickness given by \[3\]

$$ r_T \simeq \sqrt{\frac{2}{\pi \sigma}} \quad (7) $$

and the $\bar{q}_1q_1$ is produced at rest in $e^+e^-$ cms but with a transverse momentum

$$ k_T \simeq \frac{1}{r_T} \simeq \sqrt{\frac{\pi \sigma}{2}}. \quad (8) $$

The acceleration (or deceleration) associated with the string breaking and color neutralization mechanism turns out to be \[3\]

$$ a \simeq 2k_T \simeq \sqrt{2\pi \sigma} \simeq 1.1 \text{ GeV}, \quad (9) $$
which leads to

\[ T_q = \frac{a}{2\pi} \simeq \sqrt{\frac{\sigma}{2\pi}} \simeq 180 \text{ MeV} \quad (10) \]

for the hadronic Unruh temperature. It governs the momentum distribution and the relative species abundances of the emitted hadrons.

Notice that the previous hadronization mechanism can be described by saying that \( \bar{q}_1 \) reaches the \( q_1 \bar{q}_1 \) event horizon and tunnels to become \( \bar{q}_2 \). The emission of hadron \( \bar{q}_1 q_2 \) can be considered as Hawking radiation.

**VACUUM PRESSURE AND BARYON DENSITY**

It is interesting to consider the extension of the previous mechanism in the case of systems with a net baryon number, i.e. with a new “charge” which can modify the tunnelling process and the Hawking-Unruh hadronization temperature.

In the BH case the effect of a total charge \( Q \) changes the Hawking temperature according to the formula (see for example [15])

\[ T_{BH}(M,Q) = T_{BH}(M,0) \left\{ \frac{4\sqrt{1 - Q^2/GM^2}}{(1 + \sqrt{1 - Q^2/GM^2})^2} \right\}; \quad (11) \]

Note that with increasing charge, the Coulomb repulsion weakens the gravitational field at the event horizon and hence decreases the temperature of the corresponding quantum excitations. As \( Q^2 \rightarrow GM^2 \), the gravitational force is fully compensated. The crucial quantity here is the ratio \( Q^2/GM^2 \) of the overall Coulomb energy, \( Q^2/R \), to the overall gravitational energy, \( GM^2/R \). Equivalently, \( Q^2/GM^2 = P_Q/P_G \) measures the ratio of inward gravitational pressure \( P_G \) at the event horizon to the repulsive outward Coulomb pressure \( P_Q \).

In QCD, we have a “white” hole containing coloured quarks, confined by chromodynamic forces or, equivalently, by the pressure \( B \) of the physical vacuum. If the system has a non-vanishing overall baryon number, there will be a Fermi repulsion between the corresponding quarks, and this repulsion will provide a pressure \( P(\mu) \) acting against \( B \), with \( \mu \) denoting the corresponding quark baryochemical potential. We thus expect a similar reduction of the hadronization temperature as function of \( \mu \). To quantify this aspect let us consider, at \( T = 0 \), the Fermi pressure, \( P = d_g \mu^4/24\pi^2 \), where \( d_g \) is the degeneracy factor, versus the QCD vacuum pressure due to the gluon condensate \( B = < (\alpha_s/\pi) G_{\mu\nu}^2 > \) which is a decreasing (largely unknown) function of the baryon density. By following the analysis of ref. [16] for the dependence of the gluon condensate on the baryon number, the critical density, where \( B \) balances the Fermi pressure, turns out about \( n_c = 5 - 6n_0 \), with \( n_0 \) the nuclear saturation density.

A slightly different result is obtained in the description of deconfinement by percolation [17] : \( n_c \simeq 4n_0 \).

In the intermediate region, where both \( T \) and \( \mu \) are finite, we want to compare the effect of the Fermi repulsion to the vacuum pressure through the Hawking-Unruh form, i.e., we replace \( Q^2/GM^2 \) in eq.(11) by \( (\mu/\mu_0)^4 \), giving

\[ T(\mu)/T_0 = \frac{4\sqrt{1 - (\mu/\mu_0)^4}}{(1 + \sqrt{1 - (\mu/\mu_0)^4})^2}. \quad (12) \]

The resulting behaviour of \( T(\mu) \) is shown in Fig.5 for \( \mu_0 \) corresponding to the previous \( n_c \simeq 4 - 6n_0 \). In the same figure is shown the results obtained by using \( (\mu/\mu_0)^2 \) rather than \( (\mu/\mu_0)^4 \) in eq.(12). In both cases the function remains rather flat up to large value of \( \mu \).
FIG. 5: T dependence on $\mu$ by eq.(12) with $(\mu/\mu_0)^n$ for $n = 2, 4$

Clearly this approach is overly simplistic, since it reduces the effect of the additional quarks to only their Fermi repulsion. A more general way of addressing the problem would be to introduce an effective $\mu$-dependence of the string tension.

[1] R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147; Nuovo Cim. A 56 (1968) 1027.
[2] F. Becattini, Z.Phys. C69 (1996) 485 ($e^+e^-$);
F. Becattini and U. Heinz, Z.Phys. C76 (1997) 268 ($pp/p\bar{p}$);
J. Cleymans and H. Satz, Z.Phys. C57 (1993) 135 (heavy ions);
F. Becattini et al., Phys. Rev. C64 (2001) 024901 (heavy ions);
F. Becattini and G.Passaleva Eur. Phys. J. C23(2002)551;
P. Braun-Munziger, K. Redlich and J. Stachel, (heavy ions).
[3] See e.g., M.Cheng et al. Phys. Rev. D74 (2006) 054507 for the latest state and references to earlier works.
[4] P.Castorina, D.Kharzeev and H.Satz, Eur. Phys. J. C52 (2007) 187.
[5] H.Satz, *Thermal Hadron Production and Hawking-Unruh Radiation in QCD* CERN Particle Physics Seminar, May 22 2007.
[6] S. W. Hawking, Comm. Math. Phys. 43 (1975) 199.
[7] E. Recami and P. Castorina, Lett. Nuovo Cim. 15 (1976) 347.
[8] A. Salam and J. Strathdee, Phys. Rev. D18 (1978).
[9] E.Fermi, Prog. Theor. Phys.5 (1950) 570; L.D.Landau, Izv. Akad. SSSR , Ser. Fiz. 17 (1953) 51; R.Hagedorn , Nuovo Cim. 15 (1960) 434.
[10] F. Becattini, Nucl. Phys. A702 (2002) 336, proceeding of the Bielefeld Symposium , “Statistical QCD” 26-30 August 2001.
[11] M. Novello et al., Phys. Rev. D61 (2000) 045001.
[12] D.Kharzeev, E.Levin and K.Tuchin , Phys. Lett. B547 (2002) 21; Phys. Rev. D70 (2004) 054005.
[13] W. G. Unruh, Phys. Rev. D14 (1976) 870.
[14] J. Schwinger, Phys. Rev. 82 (1951) 664.
[15] See e.g., Li Zhi Fang and R. Ruffini, *Basic Concepts in Relativistic Astrophysics*, World Scientific, Singapore 1983.
[16] M.Baldo, P.Castorina and D.Zappala’ , Nucl. Phys. A743 (2004) 13.
[17] V. Magas and H. Satz, Eur. Phys. J. C32 (2003) 115.