Statistical experiments using the multiple regression research for prediction of proper hardness in areas of phosphorus cast–iron brake shoes manufacturing

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Abstract. Multivariate research is important in areas of cast–iron brake shoes manufacturing, because many variables interact with each other simultaneously. This article focuses on expressing the multiple linear regression model related to the hardness assurance by the chemical composition of the phosphorous cast irons destined to the brake shoes, having in view that the regression coefficients will illustrate the unrelated contributions of each independent variable towards predicting the dependent variable. In order to settle the multiple correlations between the hardness of the cast–iron brake shoes, and their chemical compositions several regression equations has been proposed. Is searched a mathematical solution which can determine the optimum chemical composition for the hardness desirable values. Starting from the above–mentioned affirmations two new statistical experiments are effectuated related to the values of Phosphorus [P], Manganese [Mn] and Silicon [Si]. Therefore, the regression equations, which describe the mathematical dependency between the above–mentioned elements and the hardness, are determined. As result, several correlation charts will be revealed.

1. Introductory notes
The best way for brake makers to achieve better shoes is to ensure that better materials and improved manufacturing processes are used and that shoes users take account of friction conditions and improved braking processes. The main problem in shoe’s manufacturing processes is the optimal chemical composition assurance [1–5]. Therefore, the cast–iron brake shoes – with lamellar graphite and with a high content of phosphorus (0.8–1.1%) – need a special investigation [1–4], [6–8]. In order to establish the optimal condition for the cast–iron brake shoes we proposed a mathematical modelling study by using the statistical analysis and multiple regression equations [1–4]. Multivariate research is important in areas of cast–iron brake shoes manufacturing, because many variables interact with each other simultaneously [1–4], [8].

The properties of grey iron are primarily dependent on its composition, being influenced by the elements present in irons. The lower strength grades can be consistently produced by simply selecting the proper melting stock, but the higher strength grades require close control of their processing as well as their composition. Within a class of grey iron, hardness is a good indicator of its engineering properties and many of these properties are directly related to its hardness. The hardness is affected by
the processing of the grey iron as well as the composition because these factors influence the microstructure [1–4].

The statistical analysis is useful for controlling the industrial production of braking shoes, due to the shoe’s chemical composition is not specified by railway customers. Due to industrial data analysis, the improving of the braking shoes technological process of manufacturing, according to the specifications, is needed. Statistical evaluations are helpful for the brake shoe–makers in determining their actions to improve the shoe’s quality [1–5]. Our studies on the multivariate analysis combine the statistical analysis techniques and the multivariate mapping for easier data interpretation.

Several correlations between the chemical components of the grey phosphorus cast–iron destined to the brake shoes – Carbon [C], Sulphur [S], Manganese [Mn], Silicon [Si] and Phosphorus [P] – and the obtained brake shoes hardness [HB] was presented in recent works related with [1–4]. Using the Matlab area, we proposed a multivariate statistical analysis related to:

- the general behavior of Carbon [C] content in relation with the rest of chemical elements, which have influence on the hardness of brake shoes [1], [3]. As result, correlation charts are determined (HB = f([C], [S]); HB = f([C], [Mn]), HB = f([C], [Si]) and HB = f([C], [P])).
- the combined behavior of all chemical elements of grey phosphorus cast iron on the hardness of brake shoes, in several bi–component correlations, with focus on the general behavior of Phosphorus [P] content in relation with the rest of chemical elements [1], [4]. As result, correlation charts are determined (HB = f([S], [P]); HB = f([Si], [P]), HB = f([C], [P]) and HB = f([Mn], [P]))
- the combined behavior of all chemical elements of grey phosphorus cast iron on the hardness of brake shoes, in several bi–component correlations, with focus on the general behavior of Manganese [Mn] content in relation with the rest of chemical elements [1], [4]. As result, correlation charts are determined (HB = f([Mn], [P]), HB = f([Mn], [C]), HB = f([Mn], [Si]) and HB = f([Mn], [S]))
- the relationship between the equivalent carbon content value (which is mathematically calculated on basis of the chemical composition) and the hardness of cast iron brake shoes [1], [4]. As result, correlation charts are determined for case HB = f((Ceh))
- the cumulative effect of Carbon [C], Manganese [Mn] and Sulphur [S] upon the hardness of cast iron brake shoes [1], [2]. As result, correlation charts are determined for case HB = f([C], [S], [Mn])
- the cumulative effect of Carbon [C], Phosphorous [P] and Sulphur [S] upon the hardness of cast iron brake shoes [1], [2]. As result, correlation charts are determined for case HB = f([C], [S], [P])

The current performed research upon a number of 100 charges of phosphorous cast–iron brake shoes had in view to obtain multivariate three–dimensional correlations between the cast–iron brake shoes’ hardness and its chemical composition, defined by several representative elements – Phosphorus [P], Manganese [Mn] and Silicon [Si] –, which determine the iron’s microstructure. As result, several regression surfaces and correlation charts will be revealed and a comparative study will be made.

2. Methodology

The mathematical modeling establishes a methodology was described in several previous works [1–4]. In these studies we disposed industrial data from cast–iron brake shoes’ manufacturing, therefore, the optimization model is based on these data [1–4]. The main goal of the used multiple regression analysis is to observe whether two or more variables co vary, and to quantify the strength of the relationship between these. The regression model expresses the relationship in the form of several regression equations, which predict the brake shoes behavior [1–4].

In the current statistical experiments we use the regression analyses for predict of proper hardness, having in view the interest related to:

- the cause–and–effect relationships, commonly used for investigating the relationship between two independent variables of the chemical composition and the hardness value. In this case, we try to determine the relationships – in form of several regression equations – between Manganese [Mn]
and Silicon [Si], Manganese [Mn] and Phosphorus [P], respectively Silicon [Si] and Phosphorus [P], related to the hardness value, [HB]. The general technological scope is to identify if the cumulative cause–and–effect relationships, commonly used for investigating the relationship between three independent variables of the chemical composition and the hardness value. In this case, we try to determine the relationship between three variables – Manganese [Mn], Silicon [Si] and Phosphorus [P], and the hardness value, [HB] [1–4];

- the associated variables (if exist), without necessarily inferring a cause–and–effect relationship, knowing that a strong association between two or more components results in an increase in the accuracy of the prediction of proper hardness [1–4].

Therefore, is searched a mathematical solution which can determine the proper chemical composition for phosphorous cast–iron brake shoes for the hardness desirable values [1–4]. Starting from the above–mentioned affirmations two new statistical experiments are effectuated related to the values of Phosphorus [P], Manganese [Mn] and Silicon [Si]. In fact, the obtained regression equations will describe the mathematical dependency between the above–mentioned elements and the hardness. Thus the optimal additions can be determined in these elements to assure the proper hardness [1–4].

3. Statistical experiments and results

3.1. The first statistical experiment

In the first statistical experiment, the cumulative effect on hardness of two elements – Manganese [Mn] and Silicon [Si] – is analyzed. The optimal form of modeling in the case of [HB] = f([Mn], [Si]) is given by the regression equation (1), where the correlation coefficient is rf(1) = 0.7372.

\[
[HB] = a_1[Mn]^2 + a_2[Si]^2 + a_3[Mn][Si] + a_4[Mn] + a_5[Si] + a_6
\]  

(1)

where the statistically determined values of the correlation coefficients are: \( a_1 = 36.9005; \)
\( a_2 = -189.9378; \)
\( a_3 = -107.6425; \)
\( a_4 = -70.7314; \)
\( a_5 = 427.5707; \)
\( a_6 = 158.5458; \)

By the same methodology another two cumulative effects on hardness are analyzed, using the influence of Manganese [Mn] and Phosphorus [P], respectively the Silicon [Si] and Phosphorus [P] values. The regression equations are presented in (2)–(3), where the correlation coefficient are \( rf(2) = 0.8441 \) and \( rf(3) = 0.8449 \), representing the cause–and–effect relationships in the cases of \( [HB] = f([Mn], [P]) \) and \( [HB] = f([Si], [P]). \)

\[
[HB] = a_1[Mn]^2 + a_2[P]^2 + a_3[Mn][P] + a_4[Mn] + a_5[P] + a_6
\]  

(2)

where the statistically determined values of the correlation coefficients are: \( a_1 = -101.5218; \)
\( a_2 = -57.3992; \)
\( a_3 = -225.2918; \)
\( a_4 = 313.5547; \)
\( a_5 = 256.3256; \)
\( a_6 = 14.3819; \)

\[
[HB] = a_1[Si]^2 + a_2[P]^2 + a_3[Si][P] + a_4[Si] + a_5[P] + a_6
\]  

(3)

where the statistically determined values of the correlation coefficients are: \( a_1 = -67.2629; \)
\( a_2 = 25.9138; \)
\( a_3 = -70.6481; \)
\( a_4 = 251.1173; \)
\( a_5 = -29.5826; \)
\( a_6 = 135.0779; \)

3.2. The second statistical experiment

In the second statistical experiment, the cumulative effect of all three elements (Phosphorus [P], Manganese [Mn] and Silicon [Si]) upon the obtained hardness is analyzed. The optimal form of modeling in the case of \( [HB] = f([Mn], [Si], [P]) \) is given by the regression equation (4), where the correlation coefficient is \( rf(4) = 0.8024. \)

\[
[HB] = a_1[Si]^2 + a_2[Mn]^2 + a_3[P]^2 + a_4[Si][Mn] + a_5[Mn][P] + a_6[P][Si] + a_7[Si] + a_8[Mn] + a_9[P] + a_{10}
\]  

(4)
where the statistically determined values of the correlation coefficients are: $a_1 = 33.4696$; $a_2 = -125.8125$; $a_3 = -79.4532$; $a_4 = -72.4779$; $a_5 = 46.2496$; $a_6 = 521.8024$; $a_7 = 59.1273$

Because these areas, described by the equation (4), cannot be represented in the three–dimensional space, the independent variables ([$Mn$], [Si] and [P]) were replaced with their average values ([Si]$_{med}$, [Mn]$_{med}$ and [P]$_{med}$), successively. Therefore, in the case of correlation HB= f([Mn], [Si], [P]), the equations (5)–(7) were obtained, in which the correlation coefficient are $r_f(5) = 0.7356$, $r_f(6) = 0.8011$ and $r_f(7) = 0.7305$.

$$HB = b_1[Mn]^2 + b_2[P]^2 + b_3[Mn][P] + b_4[Mn] + b_5[P] + b_6$$

(5)

where the correlation coefficients have the following statistically determined values: $b_1 = -125.8125$; $b_2 = -79.4531$; $b_3 = -280.5296$; $b_4 = 391.1682$; $b_5 = 316.5751$; $b_6 = -33.3034$

$$HB = c_1[Si]^2 + c_2[P]^2 + c_3[Si][P] + c_4[Si] + c_5[P] + c_6$$

(6)

where the correlation coefficients have the following statistically determined values: $c_1 = 33.4696$; $c_2 = -79.4531$; $c_3 = 46.2496$; $c_4 = -116.3266$; $c_5 = 60.1281$; $c_6 = 333.1839$

$$HB = d_1[Si]^2 + d_2[Mn]^2 + d_3[Si][Mn] + d_4[Si] + d_5[Mn] + d_6$$

(7)

where the correlation coefficients have the following statistically determined values: $d_1 = 33.4696$; $d_2 = -125.8125$; $d_3 = -72.4779$; $d_4 = -74.0382$; $d_5 = 293.9225$; $d_6 = 196.1442$

4. Generation the variation domains

The equations (1)–(3) and (5)–(7), belonging to the three–dimensional space, are represented graphically, and several variation domains results, which can be interpreted as correlation charts [1–4]. In this way, the correlation of the values of the two independent variables can be made, so that the dependent variables [HB] can be obtained between the requested limits [1–4].

The cumulative effect on hardness of Manganese [Mn] and Silicon [Si], corresponding to the first statistical experiment, is presented in Figure 1 and Figure 2. The cumulative effect on hardness of Manganese [Mn] and Phosphorus [P], respectively the cumulative effect on hardness of Silicon [Si] and Phosphorus [P] are presented in Figures 3–6, corresponding to the same statistical experiment.

Figure 1. The regression surface of hardness in the [Mn]–[Si] correlation (according to the first statistical experiment)
Figure 2. The hardness diagram in the [Mn]–[Si] correlation (according to the first statistical experiment)

Figure 3. The regression surface of hardness in the [Mn]–[P] correlation (according to the first statistical experiment)

Figure 4. The hardness diagram in the [Mn]–[P] correlation (according to the first statistical experiment)
Figure 5. The regression surface of hardness in the [P]–[Si] correlation (according to the first statistical experiment)

Figure 6. The hardness diagram in the [P]–[Si] correlation (according to the first statistical experiment)

Figure 7. The regression surface of hardness in the [P]–[Si]–[Mn] correlation when [P] = [P]_{med} (corresponding to the second statistical experiment)
Figure 8. The hardness diagram in the $[P]–[Si]–[Mn]$ correlation when $[P] = [P]_{med}$ (corresponding to the second statistical experiment).

Figure 9. The regression surface of hardness in the $[P]–[Si]–[Mn]$ correlation when $[Si] = [Si]_{med}$ (corresponding to the second statistical experiment).

Figure 10. The hardness diagram in the $[P]–[Si]–[Mn]$ correlation when $[Si] = [Si]_{med}$ (corresponding to the second statistical experiment).
The cumulative effects of Phosphorus [P], Manganese [Mn] and Silicon [Si] upon the obtained hardness, corresponding to the second statistical experiment, in the Figures 7–12 are presented. We make the mention that the analyzed statistical values were successively replaced with their average values ([Si]med, [Mn]med and [P]med), resulting new correlation diagrams.

5. Discussions
5.1. Discussion on the mathematical model
The mathematical model quantifies the influences of the independent variables ([Mn], [Si] and [P]) on the dependent variable ([HB]), and it was established by a statistical analysis. The real values of the process variables were chosen statistically, their limits of variation being given by the industrial data from the phosphorous cast–iron brake shoes manufacturing [1–4].

In order to settle the correlation between the [HB], and the above–mentioned parameters ([Mn], [Si] and [P]) the following model of polynomial equation type has been proposed:

\[
Y = a_1[X_1]^2 + a_2[X_2]^2 + a_3[X_3]^2 + a_4[X_1][X_2] + a_5[X_1][X_3] + a_6[X_2][X_3] + a_7[X_1] + a_8[X_2] + a_9[X_3] + a_{10}
\] (9)
where $a_1, ..., a_{10}$ are the regression coefficients.

5.2. Discussion on the regression analysis
In order to estimate the independent variables overall influence on hardness [HB], the multiple correlation coefficients were calculated. The obtained values are: $r_{f1} = 0.5773$ and $r_{f2} = 0.6301$, respectively. Analyzing the values of regression coefficients one can conclude that the process performance it is noticeably influenced by all three parameters, in both statistical experiments. From the value of the determination coefficient resulted that the proposed mathematical model is adequate for a confidence interval of 57–63%. In other words, in the chosen intervals the independent variables influence the hardness in an extent of approximately 60%. The rest of approximately 40% can be attributed to the effect of other chemical elements or manufacturing factors ignored during these statistical experiments.

5.3. Discussion on the graphically addenda
For visualization of the regression surfaces and for the calculation correlation coefficients the software Matlab was used. The spatial representations of the response surfaces [HB] for constant values of variables reveal that all data are located inside the considered surface.

The three–dimensional regression surfaces are a graphical representation of the regression equations (1)–(3), according to the first statistical experiment, respectively equations (5)–(7), corresponding to the second statistical experiment. It is plotted to explain interaction of the variables and locate the optimal level of each variable for maximal response. Each regression surface represents the different combinations of two variables at one time while maintaining the other variable at the average value.

These regression surfaces obtained in the experiments (Figure 1, Figure 3 and Figure 5, respectively Figure 7, Figure 9 and Figure 11) and its respective correlation charts (Figure 2, Figure 4 and Figure 6, respectively Figure 8, Figure 10 and Figure 12) provide a visual interpretation of the interaction between two factors (in the bi–component analysis), and between three factors (when the third factor have an constant value). These regression surfaces facilitate the determination of proper experimental conditions. The point for which the response is optimized is the point called the stationary point. The stationary point may be a point of maximum response, minimum response or a saddle point (point of inflexion).

5.4. Comparative study
Comparative study can be made between:
- Figure 2 and Figure 8, which represents, in fact, the hardness diagram in the [Mn]–[Si] correlation.
- Figure 4 – Figure 10, which represents, in fact, the hardness diagram in the [Mn]–[P] correlation.
- Figure 6 – Figure 12, which represents, in fact, the hardness diagram in the [P]–[Si] correlation.

Table 1 present the comparative study’s results.

| Table 1. Comparative study (at [HB] = 220) |
|-----------------------------------------|
| The [Mn]–[Si] correlation | The [Mn]–[P] correlation | The [P]–[Si] correlation |
| The first statistical experiment | The second statistical experiment | The first statistical experiment | The second statistical experiment |
| [Mn] | [Si] | [P] | [Si] |
| 1.62–2.04 | 1.68–1.96 | 0.7–0.9 | 0.5–0.85 | 0.75–1.6 | 0.65–1.1 |
| [Mn] | [P] | [Si] | [Si] |
| 0.5–0.7 | 0.48–0.8 | 0.5–0.8 | 0.72–0.8 | 1.7–2.2 | 1.4–2.1 |
Therefore, the comparative study is an optimisation process, in which we can determine the proper hardness [HB] that depends by the analysed variables – Phosphorus [P], Manganese [Mn] and Silicon [Si]. The technological domains for the analysed variables according to the correlation diagrams (at [HB] = 220), are presented in Table 2, as recommendation.

### Table 2. Technological domains according to the correlation diagrams (at [HB] = 220)

| [Si]    | [P]    | [Mn]  |
|---------|--------|-------|
| 1.7–1.96 | 0.75–0.85 | 0.5–0.7 |

6. Conclusions

One of the main chapters of the statistics referring to the ability to predict. Although it is not find the perfect relations, by means of regression, can make statements of a variable, depending on the other values. The present researches are going to establish the influence of the Phosphorus [P], Manganese [Mn] and Silicon [Si] upon the hardness [HB] of the braking shoe’s material (i.e. gray phosphorous cast iron with lamellar graphite).

Therefore, the realization of a proper chemical composition – Phosphorus [P], Manganese [Mn] and Silicon [Si], in this case – can constitute a technical efficient mode to assure the exploitation properties of the brake shoes. Having in view the previous works [1–4], presented as partial results in the complex statistical analyses related to the brake shoes manufacturing, we can conclude that the exploitation properties can be achieved by the manufacturing process, through a proper conduct of the iron’s making processes.

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