Development of linear algebra learning material based on mathematical understanding and representation

C D Rosita*, T Nopriana and I L K Dewi
Department of Mathematics Education, Universitas Swadaya Gunung Djati, Cirebon, Indonesia

*citadwirosita@unswagati.ac.id

Abstract. This research aims to design a Linear Algebra learning material that can facilitate the enhancing of students’ mathematical understanding and representation. The research method is research and development (R&D) which consists of three main stage, namely the preliminary, development, and dissemination. The research was limited to the development stage. The results concluded that the assessment of the experts (validator) on learning materials is in the category of valid with a small revision of the exercise questions part. The result of a practical test of learning material is 87.81% (very practices). Meanwhile, the result of limited trials indicates that learning material can be completed students’ mathematical understanding classically and individually. Besides, the mathematical representation of student has not reached the mastery both in classically and individual.

1. Introduction
The aim of giving mathematics at every level of education is to train students as a problem solver. Certain cognitive abilities need to be possessed by a problem solver so that he can maximize the knowledge he has acquired and can use it optimally. A problem solver must have the cognitive abilities needed to understand and represent a mathematical situation, create algorithms on certain problems, process various types of information, and run computing, and must also be able to identify and manage a set of appropriate resolution strategies to solve problems [1]. The ability of students to understand the prerequisite concepts and mathematical concepts being studied is the key to success in obtaining certain mathematical knowledge and solving problems encountered during learning. When students solve mathematical problems, students indirectly adapt and expand existing knowledge by connecting or linking new information obtained with prior knowledge. A series of mental activities will make new information related to the structure of student knowledge. Students who have good problem-solving abilities are also supported by their ability to manipulate verbal language, graphics, images, and mathematical symbols [2]. The ability of mathematical representation is needed by students when faced with a problem. The representative ability can help students in simplifying a mathematical concept that is presented in several forms of representation [3]. Appropriate representation can lead students to obtain the right knowledge, while the wrong representation will lead to ill problem-solving process. Therefore, the ability of mathematical understanding and representation together is important to be possessed by students to achieve better quality learning including Linear Algebra lectures.

The idea of students' mathematical understanding and representation abilities in learning has been the topic of several studies lately. The study conducted by Fatqurhohman resulted in the finding that
training students to do procedural understanding will strengthen and develop their understanding of concepts [4]. Through the use of algorithms in identifying problems and connecting concepts with various representations can facilitate educators to achieve the development of the ability to understand the concept of students during learning, even though the level of understanding of students varies. Activities during learning are very important in building an understanding of both procedural and conceptual [5]. Another study about efforts to develop mathematical understanding ability through the design of instructional materials concluded that based on the results of the limited test of teaching materials for a group of mathematics education students, the results of the students' mathematical comprehension were both classical and individual [6]. The mathematical representation was studied by several previous researchers associated with mathematical topics at the school and college level. Most students are weak in carrying out mathematical representation activities regarding connecting procedures and processes to various representations of relevant concepts seen when students are faced with one of the questions related to Linear Algebra material [7]. The availability of teaching materials based on certain mathematical abilities would facilitate the development of mathematical abilities that are the goal of learning by paying attention to the level of diversity of students' initial abilities [8].

The purpose of this study was to describe the design process of teaching materials and the ability to test comprehension and representation skills, as well as to analyze the effectiveness of learning that utilizes the teaching materials that have been developed. The effectiveness of the use of teaching materials is measured through classical and individual completeness tests on students' comprehension and mathematical ability test results after using teaching materials. The mathematical comprehension ability studied is limited to students' understanding of the mathematical concepts that are relational, namely the ability to correctly associate something with other things and realize the process they are doing. Meanwhile, the mathematical representation ability that is measured is the ability to represent mathematical concepts into symbols and expressions. The selection of the two types of capabilities along with their nature and type is strongly suspected to be relevant to the topics of Linear Algebra developed. Furthermore, these two capabilities are translated into learning indicators and learning outcomes that are relevant to these topics.

2. Method
This study is a Research and Development method that consisting of three stages, namely preliminary, development, and dissemination. The step research on this scheme up to the development stage (experimentation on the limited learning material of Linear Algebra).

As a preliminary stage, in this research activity conducted background analysis of students who viewed from the initial math ability (IMA) and student learning motivation before utilizing learning materials that have been developed. Also, the researchers conducted concept analysis, task analysis, and formulation of Linear Algebraic material indicator. At the development stage, the researcher conducts the preparation of learning material, a questionnaire of practicality, and also conducts validation activities by the expert. Data analysis performed on each validation sheet of the research instrument. The validator writes the assessment with 5 (five) degrees of assessment scale, i.e., invalid (value 1); less valid (value 2); valid enough (value 3), valid (value 4); and very valid (value 5). After performing a validation test and practice test of learning materials to experts and students, with due observation of validation and practicality then after the learning materials revised according to the validator suggestion. Then, the researcher conducted a limited trial of utilization of learning materials Algebra Linear. This limited trial is the final stage of the development stage. The purpose of experimenting with limited learning materials is to know the effectiveness of the use of learning materials in learning.

3. Result and discussion

3.1. Preliminary stage
The IMA grouping criteria are based on the mean (x̄) and standard deviation (s). Researchers obtained IMA data based on test results about linear algebra course prerequisite materials, namely Matrices
Algebra courses. Based on the data obtained, it is known that the average ($\bar{x}$) IMA student is 70.37 with a standard deviation value ($s$) of 9.78. Based on these two data, the criteria of IMA grouping and percentage of students are as presented in Table 1 below.

| IMA             | Group of Student     | Percentage of students |
|-----------------|----------------------|------------------------|
| $IMA \geq 80.14$| High group students  | 13.5%                  |
| $60.59 \leq IMA < 80.14$| Medium group students | 78.4%                  |
| $IMA < 60.59$   | Low group student    | 8.1%                   |

Researchers also analyzed student learning motivation. Motivation is an important element in learning and learning. Some research on student achievement shows motivation as a factor that affects the process and student learning outcomes. In other words, that high student learning motivation, will lead to quality learning activities that will result in high learning achievement. For example, in a study by Mc Clelland shows that achievement motivation contributes up to 64% of learning processes and outcomes [9]. Based on the above findings teachers may consider intervening regarding improving student learning motivation.

There is a set of motivational principles that can be applied in the learning process, called the ARCS model [9]. In the model presented here are four categories of motivational conditions that must be considered by the teacher to produce lectures interesting, meaningful and provide challenges for students. The four motivational conditions are Attention, Relevance, Confidence, and Satisfaction. Based on the above description, in this research, the researcher took the data of student motivation by using student motivation questionnaire which includes student ARCS condition. Based on the calculation of the average score obtained student motivation criteria as follows table 2

| Classification | Average number of classification | Interpretation |
|----------------|---------------------------------|----------------|
| Attention      | 3.65                            | Good           |
| Relevance      | 3.73                            | Good           |
| Confidence     | 3.28                            | Pretty Good    |
| Satisfaction   | 3.92                            | Good           |

In the preliminary activities also carried out the concept analysis of the subject matter developed. Analysis of the concept aims to identify, elaborate and systematically compile relevant concepts to be taught. Most of the concepts in Linear Algebra material can be constructed through concepts previously accepted by students.

At the stage of task analysis, identification of the main academic skills that need to be developed in learning materials. This analysis is based on competency standards and indicators of achievement of learning outcomes. Based on the task analysis for Linear Algebra material it is expected that students can have the ability of understanding and mathematical representation with indicators presented in Table 3 below.

| Mathematical Understanding | Mathematical Representation |
|----------------------------|----------------------------|
| Associate between concepts or mathematical topic. | can present mathematical problems into symbols and mathematical expressions. |
| Apply mathematical concepts to solving a mathematical problem. | Solve mathematical problems using verbal representation. |
3.2. Development stage

The criterion to certify that the validated material has a good validation degree, if at least the validation level reached is a valid level. The validators provide the opinion that linear algebra learning materials can be used with small revisions in the exercise section of the question and have been corrected according to the validator's suggestion.

**Table 4. Recapitulation of the results of the validity of learning materials**

| Interval                                      | Criteria | Level of Validation |
|-----------------------------------------------|----------|---------------------|
| Principles of consistency, relevance, and adequacy | 4,9      | Valid               |
| The principle of constructivism               | 4,3      | Valid               |
| The content of learning materials             | 4,4      | Valid               |

To see the student's response to Linear Algebra material, researchers gave questionnaires practicalities of learning materials. Data obtained from the sheet of practicality filled by students then calculated the percentage. Practical sheets are given in the form of a questionnaire with a statement item requiring approval (Strongly Agree, Agree, Disagree and Strongly Disagree). The statements in the questionnaire are presented in Table 5 below.

**Table 5. Questionnaire of practical materials.**

| A. Ease of Use                                                                                   | B. Time Efficiency                                                                 | C. Benefits                                                                                                      |
|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| The language used is everyday language in the classroom, so it makes me easier to learn learning materials. | It does not take long to understand the contents of the learning materials as there are instructions for using the module. | This learning material can encourage me to build knowledge independently in understanding the material presented. |
| With guidance on the use of learning materials help me understand the contents of the learning materials. | It does not take a long time to work on the concepts based on conceptual understanding abilities in this learning material. | Exercises are easy to do as I understand the materials that have been taught in the learning materials          |
| The materials presented in the materials are easy to understand.                              | It does not take a long time to work on mathematical representation skills on this learning material. | I can see the level of mastery I achieved for every material presented because of the degree of mastery of this material.|
|                                                                                                 | The time spent in studying this material is more effective than learning using textbooks available in the library. |                                                                                                                                 |

Based on the results of a practical test of learning materials given to 10 students, each student responded more than 81.25% and entered the category very practical. So the results of calculations using the combined practice formula obtained a percentage of 87.812% with a very practical interpretation.

After performing validation test and practice test of learning materials to experts and students, with due observation of validation and practicality then after the learning materials revised according to the validator suggestion. Then, the researcher conducted a limited trial of utilization of learning materials Algebra Linear. This limited trial is the final stage of the development stage. The purpose of experimenting with limited learning materials is to know the effectiveness of the use of learning materials in learning. Effectiveness here is defined by the mastery of mathematical ability of students on aspects of understanding and mathematical representation either classical or individual.
Classical and individual mastery learning of students on mathematical understanding and representation is described in the table 6 below.

| Test Type | Mathematical Understanding | Mathematical Representation |
|-----------|----------------------------|-----------------------------|
| One side average test. Significance 5% | df= 51, Sig. (SPSS 16) = 0.083 = 8.3% > 5% | df= 51, Sig. (SPSS 16) = 0.015 = 1.5% < 5% |
| H₀ (average test of students' mathematical understanding and representation ability is greater or equal to 65) | H₀ accepted. average test of students' mathematical understanding and representation ability is greater or equal to 65 | H₀ rejected. The average test of mathematical representation of students is less than 65. |
| Calculation of Average Score | 69.83 | 56.65 |

**Conclusion:** The ability of mathematical understanding of students achieve mastery of classical, but not on the ability of the mathematical representation

**Table 7. Individual mastery learning.**

| Test Type | Mathematical Understanding | Mathematical Representation |
|-----------|----------------------------|-----------------------------|
| H₀ (Proportion of students who score more than or equal to 65= 75%) | Two-party proportion test Individual mastery learning is 75% | From a total of 52 students into the sample, 35 students have a value of more than 65. The proportion of the sample is 35/52 |
| The formula to calculate individual mastery learning [14] | Zhitung = -1.28 | Zhitung = -5.45 |
| Information: | Z_Table, 5% = -1.64 | Z_Table, 5% = -1.64 |
| x/n: Sample proportion | It was obtained that -1.64 < Zhitung = -1.28 < 1.64, then H₀ accepted. | It was obtained that Zhitung = -5.45 < 1.64, then H₀ rejected. |
| π₀: Proportion value = 0.75 | N | |
| n: Number of sample | |
| Then the result is compared to the z table value using 5% significance. With criteria accepted H₀ if, −Z_Table,5% < Z_hitung < Z_Table,5% | Conclusion: Students' mathematical understanding ability utilizing learning material reaches individual mastery. | Conclusion: The mathematical representation skills of students who utilize learning material do not achieve individual mastery. |

The development of learning materials is one of the efforts to improve the learning process in the classroom. A design study that develops lecture learning materials Numeric methods with software help
can improve students' ability to solve problems related to numeric, graphics, and symbols. Other research emphasizes on aspects of the development process in which the results of the development of teaching materials use a contextual approach to junior high school mathematics materials, are declared valid in developing aspects of student character. The effectiveness of instructional materials developed through the MEAs approach (model-eliciting activities) is based on the classical and individual completeness test of students on the test of mathematical problem-solving ability shows that the teaching material is in the valid, practical, and effective category in improving mathematical problem-solving abilities of high school students [10-12].

The results of this study indicate that Linear Algebra learning materials are designed effectively in improving students' mathematical understanding abilities. It can be seen from the achievement of the students' ability to comprehend the mathematical understanding of both classical and individual. In addition to being guided by the principle of developing teaching materials in general, the instructional material developed also contains four specific characteristics regarding constructivism, namely paying attention to prior knowledge, emphasis on social activities, emphasis on the principle of the zone of proximal development, scaffolding, and cognitive apprenticeship. Students' understanding ability will develop when students are in a learning community with the same goal. Therefore, learning materials are designed in such a way that the exercises and concepts learned can be completed and understood only when students collaborate. It is in line with the opinion that states that if the principles of constructivism are implemented through the creation of a learning environment so that students act as learners [13]. In this study, it was also found that the students' mathematical representation abilities were not complete when viewed on an individual basis. The aspects of mathematical representation developed in teaching materials have not been fully effective in improving the overall ability of mathematical representation. The weakness of students in mathematical representation ability is caused by difficulties in connecting various types of representations and changing a mathematical idea into different representations [14]. Therefore, students need to be trained in the ability to interpret a mathematical idea into the right representation.

4. Conclusion
This study is development of Linear Algebra learning material research. The learning material is in a valid category with a small revision on the part of the exercise questions. The result of the practice of learning materials is very practical. Meanwhile, the limited trial results show that learning materials can solve students' mathematical understanding both in classical and individual. The ability of mathematical representation of students has not reached mastery in classical or individual.

Acknowledgments
Our thanks go to the Directorate of Research and Devotion To Community (DRPM) Directorate General of Research and Research Development of Kemenristek Dikti Republik Indonesia which has financed the activity this research.

References
[1] Zhu Z 2007 Gender differences in mathematical problem-solving patterns: A review of literature. *International Education Journal* 8 (2) 187-203
[2] Hwang W Y, Chen N S, Dung J J, and Yang Y L 2007 Multiple Representation Skill and Creativity Effects on Mathematical Problem Solving using a Multimedia Whiteboard System. *Educational Technology and Society* 10(2)
[3] Rosengrant D, Etkina E, and Heuvelen A V 2006 An Overview of Recent Research on Multiple Representations. *AIP Conference Proceedings*. doi:10.1063/1.2508714
[4] Fatqurrohman 2016 Transition Process of Procedural to Conceptual Understanding in Solving Mathematical Problems. *International Education Studies* 9(9) 188-189
[5] Hasnida N C G and Zakaria E 2011 Students’ Procedural and Conceptual Understanding of Mathematics. *Australian Journal of Basic and Applied Sciences* 5(7) 684-691.
[6] Rosita C D, Nopriana T, and Dewi I L K 2018 Bahan Ajar Aljabar Linear Berbasis Kemampuan Pemahaman Matematis. *Unnes Journal of Mathematics Education Research* 6(2) 266-272.

[7] Rosita C D, Noto M S, and Laelasari 2014 Analisis Kemampuan Pemahaman Matematis pada Mata Kuliah Aljabar Linear 1. *Euclid, Jurnal Pendidikan Matematika Unswagati* 1(2) 60-69.

[8] Rosita C D 2016 The Development Of Courseware Based On Mathematical Representations And Arguments In Number Theory Courses. *Journal of Mathematics Education* 5(2) 138-139.

[9] John Keller 1987 Development and use of the ARCS model of motivational design *Journal of Instructional Development*, 10(3), 2-10

[10] A D Handayani 2017 *J. Phys.: Conf. Ser.* 895 012019

[11] R Johar 2018 *J. Phys.: Conf. Ser.* 1088 012039

[12] Irwan 2018 *J. Phys.: Conf. Ser.* 1040 012028

[13] Bada, Olusegun S 2015 Constructivism Learning Theory: A Paradigm for Teaching and Learning. *IOSR Journal of Research & Method in Education (IOSR-JRME)* 5(6) 66-70

[14] Rahmawati D, Purwanto, Subanji, Hidayanto E, and Anwar R B 2017 Process of Mathematical Representation Translation from Verbal into Graphic. *International Electronic Journal Of Mathematics Education* 12(3) 367-381