The conception of photons

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Part I

Planck, Einstein, and key events in the early history

Abstract: In the year 1900 Max Planck was led by experimental observations to propose a strange formula for the intensity as a function of frequency for light emitted by a cavity made in a hot substance such as a metal. Planck provided a derivation based on peculiar properties to be obeyed by the emitters and absorbers in the cavity. I attempt to point out some nuts and bolts reasoning that could have provided a clue to the physical reasoning.

In 1905, Einstein made the bold hypothesis that under certain circumstances, radiation could be absorbed and emitted as packets of energy and also propagated without spreading out like waves. Einstein was able to predict the formula for the photoelectric effect based on his hypothesis. While the formula was experimentally verified by 1913, his peers seem to have rejected its interpretation in terms of light quanta. Einstein himself was aware of its inherent contradictions. The first part of this article goes over this period of struggle with the photon concept, and sets the stage for the entry of S N Bose’s critical contribution in 1923.

1 Prologue

1.1 From an embarrassment to a paradigm shift

If ideas could speak they would tell us very strange tales. Firstly how the idea arose is itself an interesting question. And it has any life of its own only if the contemporary people accept it and propagate it as interesting. There are many ideas for which it is said their time had come, and in this case several different people independently think of the same thing within a short period of time. Einstein’s was an exceptional case to have come up with not one but several different profound ideas within just a few years’ time and to have articulated them within a single year, 1905, and to have received a quick acceptance for them. There is some evidence that Special Relativity was being contemplated by several others, some of them stalwarts, at the same time. But the conviction and clarity with which Einstein expounded its fundamental nature probably got it instant fame.

However another of Einstein’s profound ideas from the same year, contained in a paper “On a heuristic point of view concerning the production and transformation of light” had a rather different fate. This is the paper that introduced the idea of what we now call a photon. It was the only paper to take forward the rather heterodox ideas put forward by Max Planck about the behaviour of light in his early papers in the year 1900. This too was written with the same clarity and conviction that characterised Einstein’s papers. Not only was this idea not accepted, it was in fact considered to be an embarrassment by his contemporaries. And even while he became reputed for his other papers, which earned him a full professorship at Berlin, there was pressure on him to withdraw this particular paper. In practical terms, the patently ludicrous nature of the idea delayed his admission into the Prussian Academy by several years, and therefore also probably delayed his Nobel prize. It was not until S. N. Bose from India provided him with a “missing link” derivation in
1924 that Einstein himself fully, and the rest of the world for the first time, became convinced of the correctness of the idea. It is ironic indeed that Einstein received his Nobel Prize for none of his other astounding ideas but precisely this one which was causing so much embarrassment. However, the Prize was not given for the profundity of its core concept, but merely for it being a correct phenomenological prediction. And the award of the Prize, however circumspect, happened in 1921, before the clinching proof provided by Bose.

The goal of this article is to put these events in perspective, while also discussing the science involved, along with the evidence it is based upon. We begin with a discussion of what the challenge of Black Body radiation was, followed by a possible reasoning that may make plausible Planck’s path to the leap into the unknown. Next I discuss Einstein’s paper, his argument for the core new concept it advanced and the possible reasons for the conviction Einstein carried for an apparently irreconcilable stance which came to be eschewed by all his colleagues. I also try to conjecture why it fell to S. N. Bose in the distant colonial Indian university of Dhaka and with a gap of two decades, to provide the systematic derivation. In the later parts of the article I take up the aftermath of the revolution started by this paper, viz., the emergence of Quantum Mechanics. We shall be concerned mainly with Einstein’s own response to the later developments, in that he came to disapprove of the schema of the new Mechanics that his path breaking paper had helped to unravel. I also briefly give the follow up story of the circumstances that led to the development of the complete definitive theory of light in the hands of Glauber and Sudarshan. The great debate of whether light is a particle as Newton proposed, or a wave, as per Huygens’ sophisticated constructions, finds a culmination in this modern description. It does not endorse either side as “true”, but shows both descriptions as merely two facets of a multifaceted, subtle phenomenon!

While the new Mechanics opened up the gates to new phenomena, new materials and new forces of nature, its originator seems to have remained unconvinced of its additional conceptual foundations. In this sense, this is the story of an idea which was ardviously protected by its proposer for decades under attack from the contemporaries, but whose subsequent implications were rejected by the same originator just after the idea received a resounding confirmation by numerous experiments and an enthusiastic acceptance by the rest of the world.

2 The antecedents

2.1 The quiet before the storm

The end of the nineteenth century appeared to mark an epoch of triumphs in the science of Physics. Heat, light and electricity, independent subjects under Physics in any school textbook, were suddenly coming closer. The science of Thermodynamics had been put on sound and consistent footing. Maxwell had put together a comprehensive mathematical and conceptual framework of Electromagnetism. The several laws due to Coulomb, Ampère, Faraday and others painstakingly put together over a century were beautifully united in a common conceptual framework. An added wonder of this accom-
plishment was that light for the first time could be clearly understood to be an Electromagnetic phenomenon. Finally, radiated heat could be understood and a form of light.

It may be these developments that led Lord Kelvin to announce that all the major discoveries of Physics had already been made and future explorations will only help to improve the preciseness of the value of “this or that constant”. To be sure, Newton’s schema of Mechanics was the conceptual and mathematical framework to which all motion should conform. Euler, Bernoulli, Poisson and others had extended the Newtonian framework to deal with continuum systems like fluids and solids. Heat had been recognised as a form of energy and interconvertible with mechanical energy. Energy was also getting a place in the scheme of Maxwell’s Electromagnetism which in turn was also unifying the phenomenon of light with the rest of Physics. These developments comprehensively embraced all the domains of phenomena that Physics as a mathematical science could embrace, placing them on a platform of universal concepts and frameworks.

2.2 The chinks in the armour

Yet, there were developments that needed reckoning, some happening soon after Lord Kelvin’s declaration. The biggest development that was to eventually challenge all of Physics was the discoveries of Chemistry. Dalton’s theory of atoms and valence were very crucial developments towards atomistic and unified theory of the world. Combined with Avogadro’s observation about a standardised number of atoms that would occur in any material under specified standard conditions, was putting an atomistic view of the world on a sound footing.

A dichotomy immediately becomes apparent, since the continuum mechanics of solids and fluids assumed substances to be ideal continua, while all the materials were slowly being revealed as an agglomeration of only a fixed basic list of “atoms”. To be sure, atomistic view of the world had long been proposed in many schools of thought around the world. The reason is somewhat obvious, with hindsight. Iron or copper from any mine in the world had the same properties. Wood or oil always burned; water in any water body was more or less the same substance, with some differences. Thus it was easy to guess that there were some primary substances, with a basic unit that carried these special qualities. If all materials were indefinitely divisible, it would be difficult to distinguish between them in the ultimate limit of subdivision. On the other hand the presence of a basic unit, the so called atom could be a candidate for encoding the few basic properties like colour, smell taste etc that are characteristic of the substance. Thus continuum mechanics could be understood as a good effective description on a larger scale, while atomic picture as the more fundamental one.

There was another direction where new phenomena demanded understanding. Kirchhoff and Bunsen had been carrying out experiments with metals heated to very high temperatures. This was made possible by that humble equipment available to every school laboratory today, the Bunsen burner. But the efficiency with which it burned gas to produce complete com-
bustion and a blue flame made possible experiments that were previously difficult to perform. One of the great observations of Kirchhoff and Bunsen was that the spectra observed in light from heavenly bodies like stars were exactly the same as those that could be obtained from substances found on the Earth, such as Hydrogen, Calcium, Sodium etc. We may consider this a next step in unification since Newton. Newton had shown that curved paths of planetary motion were just a generalisation of the fall of the apple on the earth. The heavenly bodies obeyed exactly the same law as the bodies on Earth. Now Kirchhoff and Bunsen were showing that the constitution of the substances in the distant stars was the same as that of the substances on the Earth. Together with the atomistic theory, this was showing that the Earth was just one among a large number of bodies made up of the same materials obeying the same laws.

One more development that began to unfold in the 1890’s was radioactivity. Fluorescence was a known phenomenon and could be understood as an excited state of an atom or a molecule. When Henri Becquerel first observed a radioactive substance he mistook it for a more peculiar kind of fluorescent substance. Henri Poincaré was the one to point out that the properties that Becquerel had observed suggested that this peculiar radiation arose spontaneously from deep inside the atom, and not merely from the excitation and de-excitation of the same atom. As we now know, these were the first clues to new fundamental force laws of nature, the Weak and the Strong nuclear forces. The stupendously high energy outputs and the longevity of stars could be understood only after the nuclear forces were understood.

3 The stage is set

3.1 A challenge to theorists

Kirchhoff and Bunsen made a salient observation other than the universality of substances mentioned above. It is a common observation that all metals when hot begin to glow. They glow in a specific colour that is characteristic of the metal that is being heated. However, Kirchhoff and Bunsen observed that as the temperature becomes really high, the characteristics specific to the substance grew gradually less important, and all the metals glowed in a universal manner. They had made measurements of the intensity of the glow in different wavelength ranges, i.e., at different colours of the spectrum. Kirchhoff came to conclude that the rate of energy emission in a given frequency range depends only on the temperature of the emitting metal, and independent of the specific properties of the metal. As early as 1859 Kirchhoff wrote this as a paper challenging theorists to find an explanation.

In what follows we are not going to adhere to the original chain of development of ideas. We shall adopt an ahistoric vain and pose ourselves “what if” questions such that if we could be selective about the sequence of discoveries, what would be a good sequence in which to explore the facts so that a coherent picture of the physical laws emerges inductively. The history of the personalities and of the main events is nevertheless interesting and we shall use that as the main framework for exposition, but in the interest of keeping the emerging physical principles clear we may skip some false starts and topical intermediate developments. In this vain we may proceed by recapitulating Kirchhoff’s observation as
a) the spectrum of emitted energy is independent of the substance emitting it, and

b) the emitted total intensity depends only on temperature.

Considering the fact that individual substances are known to have their characteristic frequencies in which they absorb or emit, we may interpret a) to mean that the spectrum was somehow a property of light itself. Whereas b) means that the only characteristic that could change from one situation to another was the temperature. This in turn can be meant to read that light must be subject to thermodynamic laws in a manner similar to atoms and molecules.

From the ordinary substances, standard gas laws were already known to exist. Further, a microscopic picture existed from which to derive the bulk laws. The distribution of kinetic energy among the atoms or molecules in an ideal gas was known in the form of the Maxwell-Boltzmann formula,

\[
f(v) = \sqrt{\frac{m}{2\pi kT}}^3 \frac{4\pi v^2 e^{-mv^2/2kT}}{c^3}
\]

Here \( v \) is a possible speed for the molecule, \( m \) is the mass of a molecule, \( k \) is Boltzmann’s constant appearing in all of Thermodynamics, and \( T \) is the temperature expressed in Kelvin’s absolute scale. The expression \( mv^2/2 \) is the kinetic energy of an atom or molecule. For the purpose of comparison with what follows we can write down the kinetic energy density in an interval of speeds \( v \) to \( v + dv \) as

\[
\rho_{MB}(v) = \frac{1}{2}mv^2 f(v)
\]

In view of point a) of Kirchhoff’s challenge, the effect needed to be understood in its essentials, without interference from other incidental effects. As is usually done in theoretical physics, to get to such a situation some idealisation and simplification were introduced. It could be shown that the property Kirchhoff was highlighting could be observed without other encumbrance if one dug a small hole in the surface of a metal. Then the cavity acted like a “perfect emitter”, as well as a “perfect absorber”, also called a “Black Body”. Kirchhoff’s proposal of the frequency distribution being completely determined by the temperature applied accurately to Black Body radiation. The stated problem thus also came to be known as the Black Body radiation problem.

To keep the physics issues clear we now jump ahead here to the answer to this challenge, whose history will be dealt with later in Sec. 3.3. The distribution of electromagnetic energy into frequencies as determined by experiment, and matching Kirchhoff’s hypothesis was discovered correctly by Max Planck in the year 1900. It looks like this

\[
\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{e^{-h\nu/kT}}{1 - e^{-h\nu/kT}}
\]

Here \( \nu \) (greek letter nu, distinct from the letter \( v \) used above for velocity) is the frequency of the light, \( \rho \) (greek letter rho) is the intensity of emitted radiation per unit volume per unit frequency interval, \( c \) is the speed of light and finally, \( h \) is a new constant of nature, called Planck’s constant. We have written out the formula in the modern notation. While Planck was quick to realise that he had identified a new constant of nature, many aspects of the formula were going to remain rather unclear for two decades.

Formulas (2) and (3) are complicated even for a college student. The main point to focus on is
that the factor
\[ e^{-\frac{\text{Energy}}{kT}} \]
(4)

occurring in formula (2), an exponential, is known as the Boltzmann factor, and a similar factor occurs in Eq. (3). The Boltzmann factor is the intrinsic probability that a particle will be found to have that particular value of energy in the hot medium at temperature \( T \). The product \( kT \) which has the dimension of energy is determined from the bulk measurement of temperature by say a thermometer, while “energy” in the numerator is intrinsic energy of a microscopic member of the gas. Hence, if we associated \( h\nu \) with energy of light, then by analogy with formula (2), one could think of formula (3) as relating to the “gas of light” or “light gas”. In fact before Planck’s complete formula there was an “almost” correct formula of Wien, which read

\[ \rho_{\text{Wien}}(\nu) = \frac{8\pi h\nu^3}{c^3}e^{-h\nu/kT} \]  
(5)

Wien’s formula had found considerable success in the short wavelength or high frequency regime, but had begun to show deviation at long wavelengths and low frequencies. But from theoretical point of view, it was a completely unfathomable formula at that stage of knowledge. Neither the Boltzmann exponential, nor the front factor of \( \nu^3 \) could be understood.

Based on Maxwell theory, light was known to be waves. The energy of a wave was determined by its amplitude and not its frequency, i.e., by the extent of vibration, not by the speed of vibration. Thus if a fundamental principle of Thermodynamics called the “equipartition of energy” was applied, all the modes of light would oscillate with the same amplitude to absorb the same amounts of energy, in other words, no energy dependence in the form of the Boltzmann factor would appear in the energy distribution function. This observation was made by Lord Rayleigh in 1898 but seems to have escaped attention. Further, Lord Rayleigh observed that from classical point of view, a factor \( \nu^2 \) was to be expected for modes of oscillation in three space dimensions. He also noted that at sufficiently small frequencies, the exponential in Wien’s formula approaches unity, and then the classical assumption of a distribution independent of frequency would become reliable. Based on this, he went on to propose that at low frequencies where the usual thermodynamics begins to be applicable, the front factor should be \( kT\nu^2 \) instead of \( \nu^3 \). This is exactly what Rubens found two years later and communicated to Planck prior to publication, and which spurred Planck to arrive at the correct formula. Planck’s 1900 papers however do not refer to Lord Rayleigh’s remarks which had appeared in the British journal Philosophical Magazine in 1898. It is an interesting question whether Planck’s fortuitous foray could have been made two years earlier had he known of and relied upon Lord Rayleigh’s remark. The question whether it was the language barrier or a cultural difference in professional circles that kept the efforts across the English channel sequestered from each other is also an interesting one, because as we shall later argue, this may also have been the reason why the discovery of the full law concerning photons awaited S. N. Bose for two whole decades, and who also was far away from the flourishing centres of science.

If we were very bold, (and willing to be ostracised by the community of Physicists of that time), we could argue something like this. Since light was known to be waves, this had to be a gas
of waves, and Wien’s formula should be taken to mean that the energy of individual microscopic unit of this gas has energy $h\nu$ as suggested by the Boltzmann factor, an association never made before for waves. Further, combined with Lord Rayleigh’s argument that the front factor should have a $\nu^2$ just from the counting of modes, the additional $\nu$ in the $\nu^3$ factor could be correctly understood as being proportional to the energy. Now in analogy with $mv^2/2$ as the energy of a molecule in formula (2), and comparing Eq. (5), we could begin to identify $h\nu$ in formula (3) with the energy of light. Thus the spectrum would correctly display the Boltzmann like distribution of energy. There would still be a major discrepancy in the denominator, but we would be set substantially on the right track.

As it turned out, the correct formula (3) was arrived at after further experimental detail of the spectrum was known, and Planck would have been greatly assisted in the process of trying to explain it, had he adopted such a bold hypothesis. Planck however was a very conservative physicist, and in any case, bold hypotheses without foundation are not the way of science. He therefore first made sure the formula fitted the experimental data, and in a later paper, to explain the origin of the formula, adopted what seemed like a thermodynamic argument applied to absorbers and emitters in the walls of the cavity. He did not focus attention on light itself, because that would have led him immediately to the dead end suggested by Equipartition Principle mentioned above. Planck’s thermodynamic argument was circuitous and puzzling to many physicists, but that was closest to anything like an explanation one could advance with that state of knowledge.

Planck’s caution in avoiding application of the thermodynamic argument to light gas was justified. There was a fundamental difference in conception between the two substances. Maxwell’s theory relied on the fact that the Electromagnetic field phenomena were continuum phenomena in the ideal sense. There was no limit to how much you subdivide the space to be observed, there would be newer degrees of freedom of the Electromagnetic field. This was unlike other substances, where the atomic properties would become observable beyond some extent of subdivision, and attempts to subdivide the space into regions smaller than the atomic dimensions would not bring in any new degrees of freedom.

Despite the difficulties of understanding formula (3), it was good news. If we deviate from Planck and adopt the view that some kind of thermodynamic argument could in fact be applied to light then thinking along the lines of the bold hypothesis would be fruitful. Light had been understood as a wave phenomenon, but here its collective system was cast in a form similar to a gas of particles. Heat had been clearly understood as a form of energy of molecular motion only half a century earlier, and the far infra red light was understood as heat waves. Now it was being found that the energy contained in light, when confined to a cavity, also had the properties of heat, and had a temperature characterising its thermodynamics. Diverse phenomena were coming closer and appearing to obey the same framework of laws. We may think of this as the meta-principle that may have guided Einstein in making the hypothesis even bolder than ours, to be discussed in Sec. 4. However, a further mysterious difference was the modified denominator of formula (3). In subsection 3.3 we shall see how Planck could deduce the pres-
ence of the extra \( -e^{-\nu/kT} \) in the denominator, but its full derivation awaited S. N. Bose.

### 3.2 Statistical Mechanics

To further understand and appreciate Kirchhoff’s challenge we need a diversion into Thermodynamics and its basis in Mechanics, as provided by Statistical Mechanics. Thermodynamics had made great strides, with the recognition that heat was a form of energy and that pressure could be understood as the collective average force exerted on the walls of the container by the motion of myriads molecules. A major intellectual challenge to full understanding of Thermodynamics as collective mechanical behaviour of molecules was the fact that heat could only be dissipated, and if channelised to produce usable mechanical energy, there were theoretical limits on what fraction of it could be so converted.

A formula had been found by Nicolas Sadi Carnot, relating heat considered as energy and its capacity to do work. This formula when examined showed that there would always be some heat which will be wasted and cannot be converted to usable work. Based on this fact, Rudolf Clausius developed the concept of “entropy” suggesting the sense of “wasted” form of energy ( trope meaning turned away). For an amount of heat \( \Delta Q \) being exchanged with a heat bath at temperature \( T \), the associated entropy \( \Delta S \), quantifying the irrecoverable part of the total energy, is given as

\[
\Delta S = \frac{\Delta Q}{T}
\]

(6)

Here \( \Delta \) (upper case of Greek letter delta) is meant to suggest a small change in, and a small amount of, the respective quantities \( S \) or \( Q \). This profound and very subtle identification balanced the Thermodynamic equations and accounted for all observations. However a microscopic explanation of entropy at molecular level was lacking.

This gap in the understanding was sought to be fulfilled by Ludwig Boltzmann. He introduced the concept of “ensembles”, to mean the set of all possible states the mechanical system could assume in principle. He then set about proposing the rules of computation that would explain how the observed Thermodynamics would emerge from this fundamental counting of all possible configurations. He could obtain both the distribution law (4), and the weightage factor (4) as a universal feature, and also identify a fundamental quantity associated with ensembles as the entropy of gases as quantified in eq. (6) from his fundamental postulates of Statistical Mechanics. His famous formula reads

\[
S = k \log W
\]

(7)

where \( k \) as before is the Boltzmann constant, and \( W \) is the number of all the possible states that the system can attain consistent with general conservation laws. In a sense then, the challenge thrown up by Kirchhoff’s observation was to identify and enumerate the list of possible microscopic states of light treated as some kind of substance. Everybody took the Maxwell theory as the basis of their computation and enumerated the fundamental microscopic states accordingly. And the results were a disaster. They obtained an answer that suggested indefinitely large contribution to energy at shorter wavelengths\(^1\). This classical expectation elaborated

\(^1\)This is true for example if we take Lord Rayleigh’s
in 1904 by Jeans to counter Planck’s 1900 hypothesis, came to be known as the “ultraviolet catastrophe” of Black Body radiation. In a sense this indefinite growth in energy was to be expected, since the electromagnetic field was assumed to be a continuum in the Newtonian sense, consisting of newer degrees of freedom down to indefinitely smaller scales of distance.

### 3.3 An act of desperation

We now return to the historical development. One person to face up to Kirchhoff’s challenge seriously was Max Planck. He was appointed a professor at Berlin in 1889 and became full professor in 1892, to occupy the chair previously occupied by Kirchhoff. He was well established but keen on making a mark. He was the only theorist in this Department but it proved fortuitous for him, because in another laboratory PTR in Berlin (later to become PTB, the German bureau of standards), Pringsheim and Lummer were working on experimental determination of the spectrum, or the frequency dependence of intensity of the Black Body radiation. Ironically while the theoretical spectrum failed in the short wavelength limit (newer degrees of freedom emerging at ever smaller distances in the wave picture), Planck got a hint about the correct answer from the knowledge of the long wavelength limit.

Interestingly, the spectrum of Black Body radiation was much more difficult to measure at long wavelengths. At long wavelengths, Electromagnetic radiation is essentially what we would call heat waves. These are very difficult to channel and measure accurately. However Pringsheim and Lummer of Berlin had painstakingly developed techniques that allowed them to measure the Black Body radiation in the infrared, i.e., long wavelength region of the spectrum.

From Kirchhoff’s time gatherings among professors’ families were common in Berlin, and Planck continued the tradition. One Sunday afternoon Rubens an experimentalist visitor at PTR, was on a family visit to Planck’s place for tea. During this he revealed what he had begun to see, namely at long wavelengths, the spectrum was proportional to temperature, unlike at short wavelengths. We can paraphrase it as per our discussion above to mean that at low frequencies, the formula took the form

\[
\rho_{\text{Rubens}} \propto \nu^2 kT
\]  

and no Boltzmann factor. As per the reasoning of Lord Rayleigh discussed below Wien’s formula [5], this relation was in keeping with Maxwell theory as well as Equipartition Principle. One has to associate the same energy with all the modes of the electromagnetic field as one would for classical waves, and by equipartition theorem this would be \(kT/2\) for every independent degree of freedom. The front factor \(\nu^2\) is simply related to the density of the possible modes at that frequency.

Thus one required a clever interpolation, matching Wien’s enigmatic but working formula Eq. (5) at high frequencies, and matching the classical expectation (8) at low frequencies. Based on this hint, Planck could quickly work out a possible form of the correct spectrum. Let us see how this could have been done. The expo-
ential function (4) which occurs in both formulas (2) and (3) has an infinite but simple power series expansion
\[ e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} \ldots \]  
(9)

Now if \( x \) is a number small compared to 1, say 0.1, then \( x^2, x^3 \) etc are much much smaller and can be ignored in a first approximation. So we see that if we consider the expression \( 1 - e^{-h\nu/kT} \), then when \( h\nu/kT \) is small, it is approximated by just \( h\nu/kT \). Since this appears in the denominator of (4), we get \( \nu^2T \) instead of \( \nu^3 \) in the numerator of the formula (2). This happens for small values of \( \nu \), i.e., for large values of the wavelength \( \lambda = c/\nu \). This linear dependence on temperature was exactly what Rubens was reporting to Planck, and which was being confirmed by others such as Kurlbaum, Pringsheim and Lummer.

Having confirmed the correctness of the spectrum he was predicting, Planck would have set about working out a physical explanation or pinpointing the assumptions that would underlie such an answer. Note that the law for the molecules (1) has no such \( -e^{-h\nu/kT} \) in the denominator. Such a modification could also not come from any accidental specific properties of light. It would have to emerge through some reasoning to be on the same footing as the fundamental weightage factor (4) of Boltzmann. Try as he would he could not find a convincing reason. In later recollections Planck referred to his difficult efforts to prove the formula as “an act of desperation” (or a desperate attempt, sometimes translated as an act of despair by the fact that none of the known theories worked). And he did somehow arrive at an explanation.

We now attempt a “back of the envelope” or nuts and bolts reasoning for how he may have arrived at such an explanation. Let us now use another fact of algebra: when a quantity \( y \) is small, (actually just less than 1 in magnitude is enough) we have the formula for the sum of the infinite series
\[ 1 + y + y^2 + y^3 + \ldots = \frac{1}{1-y} \]  
(10)

Now \( e^{-h\nu/kT} \) is always less than 1 if \( h\nu/kT \) is positive, and which it is on the physical grounds of positivity of \( h\nu \) interpreted as energy. So the required condition for the expansion (10) to be applicable is satisfied. We rearrange the second fraction on the right hand side of eq. (3) to read

\[
\frac{e^{-h\nu/kT}}{1 - e^{-h\nu/kT}} = e^{-h\nu/kT} \left(1 + e^{-h\nu/kT} + e^{-2h\nu/kT} + e^{-3h\nu/kT} + \ldots\right) = e^{-h\nu/kT} + e^{-2h\nu/kT} + e^{-3h\nu/kT} + e^{-4h\nu/kT} \ldots
\]

Here we used the rules about exponentials viz., for any integer \( n \), \( (e^y)^n = e^{ny} \). We now see a summation of Boltzmann factors of the type of eq. (4). Planck may well have set about giving a microscopic argument for this particular summation. Planck used the concept of entropy and Boltzmann’s method to derive his answer. In doing so however, it became necessary for him to ascribe some peculiar properties to the emissions and absorptions by atoms and molecules in the walls of the cavity treated as oscillators in interaction with the radiation. Considering that Eq. 4
contains a string of Boltzmann factors containing integer multiples of a basic frequency $\nu$, he was led to the following hypothesis. This was that an oscillator of frequency $\nu$, could emit or absorb radiation only in integer multiples of the basic unit $h\nu$. There was no reason in classical radiation theory for the absorptions and emissions to be “quantised” in integer units, nor was the energy of oscillations ever found to be proportional to frequency $\nu$. Of course, nor had anyone previously encountered a physical constant $h$ of the given magnitude and dimensions.

What Planck needed to assume to make sense of this formula was very startling. Given an oscillator of frequency $\nu$, it could absorb and emit energy only in integer multiples of the basic energy unit $h\nu$. This was the only way the string of Boltzmann factors in the summation made sense. In arriving at this explanation he came very close to the explanation S. N. Bose finally provided to his formula. In the present section let us only note this particular assumption as implemented by Planck, viz., given a total number say $N$ of “energy units” $h\nu$, one should consider all possible assignments of occupation numbers $n_1, n_2$, to the various excited levels, such that $n_1 + n_2 + \ldots = N$. This is the crucial assumption. Then by Boltzmann’s methods one could prove that when one asks for the maximum entropy configuration among such assignments, letting $N$ become large, one automatically comes to the answer eq. (11), implicitly, the summation of eq. (11).

4 The light quantum proposal

4.1 A heuristic viewpoint

Einstein saw the problems with understanding Planck’s formula to be of a fundamental nature. He begins in his paper by pointing out that a profound difference exists in our conception of material particles on the one hand and radiation on the other. Both carry energy and several common attributes, but radiation was presumed to possess physical degrees of freedom down to infinitesimally small length scales, while material particles being discrete did not possess any new physical degrees of freedom smaller than their size. According to him the solution resided in giving up the simplistic view of radiation as continuum. While all the macroscopic phenomena connected with radiation like reflection and refraction were well explained by the continuum wave character, he believed this behaviour of light did not persist under all circumstances. Particularly, he noted, the usual phenomena occur under conditions of high intensity and long wavelengths. He noted three new experimental results that had emerged in the late 1800’s which also seemed to be demanding an explanation. These were photoluminescence, photoelectric effect and photoionisation. He seized on the possibility that the problems faced in understanding “light gas” had not so much to do with the gaseous phase, the presence of the cavity and so on, but to do with some fundamental properties of radiation itself, which were shared in situations other than in gaseous phase. Thus Einstein begins with a bold announcement in the introductory section of his paper,

“As according to the assumption con-
sidered here, in the propagation of a light ray emitted from a point source, the energy is not distributed continuously over ever increasing volumes of space, but consists of finite number of energy quanta localised at points of space that move without dividing, and can be absorbed or generated only as complete units.”

It is a revolutionary idea when we think about it even today. Electromagnetic theory works so well in explaining all the engineering phenomena based on the assumption that the fields are a continuum. Did just the one or two new facts demand so radical a change in thinking to lead one to say “... energy is not distributed continuously over ever increasing volumes of space, but consists of finite number of energy quanta ...”?\[1\]

The subject of theoretical physics is about achieving simplicity and economy of thought. It hinges on identifying core concepts and their relationship to various possible environments in which they occur. The challenge is to identify these core concepts in a way that they remain applicable to all phenomena and also in a way that their relationship to all possible environments is of a universal nature. Here by “environment” we mean processes such as emission and absorption where the radiation encounters other systems or gets transformed. What Einstein was pointing out was that at least three phenomena, photoluminescence, photoelectric effect and cavity radiation could not be reconciled with the conception of light as a continuum wave phenomenon. Of course if one were so bold as to propose an alternative conception, there was a need to reconcile it with the standard one under standard situations. Here Einstein faced a difficulty. He did not know how the wave picture would yield to his new bold particulate picture in terms of “energy quanta” whose “energy is not distributed continuously over ever increasing volumes of space”. But his stipulations for the new concept are laid out with legal precision in the quoted paragraph. Indeed Einstein had to face this enigma of irreconcilability for two more decades.

However, Einstein could have had his authority drawn from another, even broader consideration, viz., the need to reconcile the corpuscular conception of matter with the continuum conception of radiation\[3\]. His stance seems to suggest that the unity of core concepts for matter and radiation was more important to him than reconciling the two alternative descriptions of radiation. This could be the vision that gave him strength to hold on to his rather radical concept.

4.2 Entropy as a hint to discretisation

We now proceed to give the original argument according to Einstein why the Planck formula must be read as a formula applied to Thermodynamics of “quanta of light”. Here the notion of quanta, or or equivalently, discreteness enters because the energy of radiation with frequency $\nu$ turn out to be integer multiple of the basic unit $h\nu$. As we shall explain in the next sub-

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\[1\] It is reported that Einstein was fascinated both by Maxwell’s grand synthesis as well as by corpuscular conception of matter. Interestingly, Avogadro’s hypothesis regarding the universality of the number of corpuscles in a gas under standard conditions had taken a whole century to be accepted. It had a crucial role in Boltzmann’s theses on microscopic origins of Thermodynamics. Einstein’s 1902 doctoral thesis concerned an experimental determination of Avogadro Number.
section 4.3 we find that Einstein uses the Statistical Mechanics definition of entropy and its relationship to the number of gas molecules, but never the detailed derivation along the lines of Boltzmann. Accordingly, Einstein first observes that the Planck formula and its ultraviolet limit known as the Wien law give a relation between entropy $S$ and volume $V$ of the subsystem of the radiation of frequency $\nu$ with energy $E_{\nu}$ as

$$S - S_0 = \frac{E_{\nu}}{\beta \nu} \ln \left( \frac{V}{V_0} \right)$$  \hspace{1cm} (12)

where the quantities $S_0$, $V_0$ are the values at some convenient reference point and $\beta$ is a constant. Next he notes that according to Boltzmann's definition of entropy, the entropy depends on the number of particles $n$ with a reference value $N$, and the volume occupied, according to the formula

$$S - S_0 = R \left( \frac{n}{N} \right) \ln \left( \frac{V}{V_0} \right) = \left( \frac{R}{N} \right) \ln \left( \frac{V}{V_0} \right)^n$$  \hspace{1cm} (13)

where $R$ is the gas constant appearing ideal gas equation of state. Thus, using the rules of logarithm, if we rewrite (12) as

$$S - S_0 = \frac{R}{N} \ln \left( \frac{V}{V_0} \right)^{(NE_{\nu}/R\beta \nu)}$$  \hspace{1cm} (14)

then we have the correspondence that there is a number $n$ associated with radiation of energy $E_{\nu}$,

$$n = \left( \frac{NE_{\nu}}{R\beta \nu} \right)$$  \hspace{1cm} (15)

so that $n$ can be identified with “the number of quanta”, a concept which is devoid of meaning in wave theory of Maxwell. Needless to say the phenomenological constants in equation (15), which is written here in its historical form, actually work out to just reproduce the Planck constant $h$. In other words, eq. (15) is nothing but the famous law $E_{\nu} = nh\nu$ with $n$ a natural number. We may think of making such an association as in eq. (15) also as a theorist's opportunism. Armed with this interpretation of Planck formula, Einstein proceeds to apply the concept of quanta to the then yet unexplained results of some controlled experiments.

It is worth emphasising at this point that Einstein deduced the photon hypothesis through Wien's formula. He did not make use of the full Planck formula, and for that enigmatic formula he advanced no explanation. He merely proceeded by conviction to make further predictions based on his conception of photons. In hindsight we know that it is the additional $e^{-h\nu/kT}$ in the denominator which is the characteristic of a quantum gas. Ironically this was not the hint that led him to the quantum proposal. His reasoning was more along the lines of our “bold hypothesis”, discussed in sec. 3.1. This was surely revolutionary in proposing packets of energy of value proportional to frequency and propagating undivided in a specific direction from source to receiver, where Maxwell theory assumed waves. Yet, the real bombshell of how uncannily different quanta are from classical conception of particles was not to arrive until S. N. Bose's 1924 derivation would be further dissected.

### 4.3 Photoelectric effect

Sparkling and colourful objects like prisms, precious stones and so on were one impetus to research on light. The end of the nineteenth century saw another class of phenomena, glowing
substances, that attracted researchers’ attention. It may be noted that often such research was not as restrictively channelled as a “Physics” experiments but rather a part of study of natural environment, with the aim of collating, classifying and archiving along the lines one would do with rocks, minerals, or even birds, plants and insects. Geophysical exploration in fact appears to have been a big hobby enterprise of people of leisure, with there being learned societies waiting to hear eagerly of new discoveries from ever newer habitats and unexplored corners of the world. This was within the general ethos of nineteenth century Europe. We may mention in passing that the discovery of radioactivity by Henri Becquerel was somewhat of an accidental discovery made in the course of such explorations.

In this list of things being explored was photoluminescence. There were phosphorescent and fluorescence substances that after being exposed to sunlight, seemed to retain the energy and continued to glow in the absence of the Sun. Such glow eventually faded within a few days depending on the substance. Despite the variety of substances, the phenomenon seemed to be governed by Stokes law which stated that the frequency of light emitted was always less than the largest frequency within the absorbed light. In classical electromagnetism, it was always the amplitude of light, i.e., the extent of undulations of the fields (actually the square of the latter, viz., the intensity) that was a characteristic of the energy the light wave carried. However, according to the Planck relation \( E = h \nu \) it is the frequency of undulations that determines the energy. Thus the photoluminescence law could not be suspected to be essentially an energy conservation law in classical theory. But from the Planck relation one can see that the limitation on frequency is the statement that emitted light cannot be of greater energy than the absorbed light. This law would then make common sense, because if a substance began to emit more energy than it absorbed it could not remain a stable substance for long. On the other hand the lower energy emissions were only dissipating the energy stored during the day, with the substance returning to its normal stable state after the emissions.

For Einstein, who had grasped the generality of the law \( E = h \nu \) as valid intrinsic relation for light quanta, the above logic must have been immediate. But the world was still believing along with Planck that his relation somehow came into force only for the energy exchange between the “oscillators” in the walls of a cavity and the radiation trapped within the cavity. They had not associated it with a property of light itself.

The law regarding photoluminescence was already an empirically observed fact, and Einstein gave an explanation for it. The more radical proposal to be made by Einstein was regarding photoelectric effect. Here where the data were still somewhat unclear, he made a radical hypothesis based on some general clues, and then proposed a specific law, to be verified by experiments. As mentioned earlier, Einstein had come to the realisation that the light quantum or photon picture gained validity in the limit of very low intensity but short wavelengths. In the case of photoelectric effect, the salient features to emerge were again of a cut off value of energy and a linear dependence of a measurable quantity on frequency.

In photoelectric effect, impinging radiation was found to eject electrons from alkali metals. Using vacuum tube techniques and electrostatic plates, it was possible to channel the ejected electrons and observe them as current. The first
Salient feature to be discovered by Heinrich Hertz and by Hallwachs was that there seemed to be a maximum value to the kinetic energy of the ejected electrons. There was a “stopping potential” which could be applied to the electrons, which would balance the maximum kinetic energy of the electrons. Further, this maximum kinetic energy seemed to depend on frequency of the radiation. Increasing the intensity of the radiation did not impart greater kinetic energy to individual electrons. The situation has an analogy to the photoluminescence case, if we remember that energy balance of individual emission processes is determined by frequency and not by intensity. Finally, if the stopping potential was made large enough, no electrons would reach the anode gathering the current. The effect was first discovered by Heinrich Hertz in 1880’s followed by investigations in the case of alkali metals by Hallwachs and later around 1903, the effect studied by Hungarian physicist Philipp Lenard for gases which underwent ionisation. Lenard also reported a dependence of the energy of ejected electrons on the frequency rather than intensity of the radiation.

Putting these things together, while they were still considered somewhat controversial, Einstein advanced a clear cut equation for the maximum kinetic energy of the ejected electrons,

\[ KE_{\text{max}} = h\nu - W \]  \hspace{1cm} (16)

Here the quantity \( W \) is the largest value of the stopping potential that would extinguish the current, and it depended on the specific substance under study. However, the first term was universal, independent of the system being investigated, whether metal or gas. It depended only on the frequency of the radiation being sent in and the new constant of nature, \( h \). As in the case of photoluminescence, if you are tuned to the idea of packets of energy, and frequency as the determinant of their energy, then this is simply an equation of energy conservation. \( W \) is simply some threshold energy the electron must acquire, by deducting it from the impinging energy \( h\nu \), to come free of the substance from which it is being ejected.

This concludes the first part of our journey into the origins of the uncanny idea of photons. In the next and concluding part we shall see how it contained the seeds of Quantum Mechanics, and the path from here that led to the full quantum theory of light.
Abstract: In this second part of the article we present how S N Bose’s 1924 paper provided a systematic derivation of Planck formula using the conception of photons, filling the major lacuna that was preventing the acceptance of the photon concept by the Physics community. This derivation further widened the chasm between classical conceptions and the actual behaviour of the microscopic world as already heralded by the photon proposal. In particular, the very concept of a quantum as an “independent” entity even when not interacting with other entities is rendered invalid. Classical intuition was subverted in Bose’s derivation by a new rule, regarding counting of independent states of the system rather than counting individual quanta. We discuss the implications of the Quantum Mechanics that eventually emerged, showing that the seeds of some of its uncanny conceptual content were already foreshadowed in Einstein’s proposal. While he was instrumental in setting off the revolution, the full implications of the revolution became unpalatable to him. We may expect that as experiments make the quantum world more familiar, the currently projected enigmas will gradually disappear.

5 Towards the birth of the quantum

5.1 Seeds of the dreaded rules of the Quantum?

In the first part we saw how Einstein arrived at the famous formula applicable to photoelectric effect,

$$KE_{max} = h\nu - W$$

According to him, once light in this setting was understood to behave like packets of energy, the above formula was simply energy conservation formula. The proposal that the energy of the light quantum should be proportional to its frequency ran against the grain of Maxwell’s electromagnetism where light could be shown to be a phenomenon similar to waves in any medium.

But the surprise of this proposal is not restricted to this little paradox. The import of the ideas we have now covered – and presumably accepted by you dear reader, as valid, – is truly stupendous. Sometimes one wonders whether Einstein fully grasped the extent of damage his proposal was doing to some of the well established classical notions. Let us assume as clearly articulated by him, that the emitted light was going to proceed without spreading out, as an undivided packet of energy into a specific direction. Considering that the emitting body was a point like object such as an atom or a molecule, one is immediately faced with the question, “which” direction will the emitted quantum proceed in? Even if the emitting body had a size, it could well be very simple, say the Hydrogen atom, which can be presumed to be spherically symmetric. Then simple classical reasoning would suggest that the radiation should emerge as a
spherical wave respecting the symmetry of the emitter. But according to the photon hypothesis, the emission process must choose a preferred direction of emission. What fundamental principles govern the choice of this direction are not spelt out by the new stipulation. Yet, with a century of experience of Quantum Mechanics we know that this is indeed how the emission of a photon occurs and in a sense typifies the nature of all quantum processes. Only under repeated identical observations can we establish the overall isotropy of the emission phenomenon, while in an individual event, the symmetry will not be respected. We may also cite another example of the often studied “particle in a box”. Consider an electron confined within a box but with no other interactions. From quantum mechanics we find that its location is not evenly distributed within the box. Depending upon the state it is in, it will be found preferentially at selected locations, violating the homogeneity of the container. However observations of a large number of such boxes will indeed restore the homogeneity of locations. To repeat, the earliest hypothesis made by Einstein already encodes the principle now used by all practitioners of quantum mechanics, viz., isolated quantum processes have to occur with one specific eigenvalue of the concerned observable revealed in a given experiment.

Thus, in a sense, the seeds of the dreaded Quantum Mechanics were already sown in Einstein’s original proposal when he generalised Planck’s law originally proposed for a radiation gas to individual events of emission and absorption. But an equally drastic phenomenon of nature had not yet been articulated, and it awaited the correct Boltzmann ensemble of photons as conceived by S. N. Bose two decades later. And this phenomenon is the intrinsic indistinguishability of quanta which makes us realise that quanta are not at all “particles” such as billiard balls we are familiar with, but profoundly novel entities.

5.2 Opportunism of theorists

We may view the bold attempts of both Planck and Einstein as an opportunism of sorts, the readiness to jump into the unknown, abandoning the comfortable territory, for the possibility of obtaining a correct answer. As we noted earlier, Planck later came to consider his effort as “an act of desperation”.

Einstein on the other hand, faced a stigma. While he became famous for his Special Relativity, the famous relationship between rest mass and energy etc., he was under pressure from senior colleagues to retract his radical ideas about discrete nature of electromagnetic phenomena. Specifically, it was creating difficulty in getting him elevated as a fellow of the Prussian Academy. Despite being nominated several times, the committee examining his case seemed to choke at this particular paper. It is reported that in a subsequent nomination his proponents even attempted an apology on his behalf, something to the effect that occasionally in his eagerness to explicate very difficult phenomena he is led to rather radical proposals, but this need not be held against him etc.

After this went on for several years, he was forced to issue some public clarification about his rather radical and unsavoury paper at the Solvay conference in 1911. But he stood up to the stalwarts, asserting that “I must insist on the validity of the new concept at least within the domain of phenomena for which it furnishes
an explanation.” His statement made in German was however so carefully worded that it was interpreted as him having reservations about this concept, at least in the English speaking world. In fact Robert Millikan who confirmed the photoelectric effect experimentally, in his 1916 paper refers to Einsteins quantum proposal as “bold, not to say reckless”, considers it to have been “generally abandoned”, and in his conclusions states that the proposal “… is found so untenable that Einstein himself, I believe, no longer holds to it.”

By 1908 Einstein became preoccupied with General Theory of Relativity and seems to have not returned to the question of light quanta. It is notable that he also proposed an explanation for the behaviour of specific heats of solids based on quantum vibrations in 1907 which met again instant success as a general idea though not quite correct in detail. But his photoelectric effect explanation was shunned by all experts. This caused difficulty in his becoming a member of the Prussian Academy, and since membership of national academy is a natural step towards the Nobel Prize, also a delay in his getting that coveted Prize. Einstein was quite a celebrity based on his Special Relativity and was becoming the next genius after Isaac Newton with his formulation in 1915 of the General Theory of Relativity. But the stupendous intellectual achievement of Special Relativity did not meet the criteria of new phenomenological content required by the Nobel committee, while the discovery of phenomena that would conclusively establish General Relativity remained far in the future.

In 1914 Robert Millikan confirmed the formula proposed by Einstein, yet nobody including Millikan seemed to believe the conceptual basis of the formula. Thus it was that with much struggle the well wishers of Einstein and no doubt well wishers of the subject of Physics managed to convince the Nobel committee to award the 1921 Prize to Einstein for his discovery of the photoelectric formula. And thus the Prize was awarded to him, taking care to state in the citation that it was ”for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect”. Note that it is not for the correct conceptual basis or theoretical explanation of the effect, it is merely for the correct “discovery” of the law, the prediction of the equation verified by Millikan.

While talking about opportunism, let us jump ahead a little and refer to sec. 6.2 where we discuss the core new concept underlying the contribution of S. N. Bose. It is common to note in critical assessments that this derivation is technical, brief and while it proves the formula, does not sufficiently explicate the new assumptions involved. One has to note however that while Einstein was bold enough to move ahead to making a new prediction, he made no attempt to explain the additional $-e^{-\frac{h\nu}{kT}}$ term in the denominator Eq. (3). While Planck gleaned the formula, and Einstein could grasp the quantum nature of the phenomenon, it was Bose who for the first time clearly derived the whole expression from Boltzmann’s ensembles, also incorporating a revolutionary counting for photons.

5.3 A curious case of inadequate diffusion of scientific knowledge?

It is important at this point to note a few ironical quirks of history and personalities. Einstein in this paper of 1905 is somewhat circumspect
of the methods used by Boltzmann. He does use the microscopic picture of entropy as proposed by Boltzmann. But he carefully avoids using the method of ensembles. For one he seems to think that listing all the members of an ensemble – all the possible states a system can possibly attain consistent with energy conservation – is still no guarantee that one has enumerated all possible dynamical effects which occur when a system is undergoing time evolution. This scepticism is similar to what has come to be called the question of ergodicity, but the way Einstein states his objection it seems to be even stronger than the question of ergodicity. Secondly we are told by the editor John Stachel of the collection of Einstein’s papers during that “miraculous year”, that there were more reasons for which Boltzmann’s contemporaries did not agree with him in detail though many agreed in principle. And this was because Boltzmann had a verbose writing style and the definitions of the concepts he would propose were not sharply defined and would seem to change even within the course of the same long essay. As we now know there were also a few errors of normalisation in his formulae.

All the issues associated with Statistical Mechanics had been adequately addressed by Josiah Willard Gibbs in the USA by the turn of the 1900’s. Had Einstein accessed that treatise, his doubts would probably have been addressed and he would have proceeded to give a detailed derivation of the Planck formula starting from his fundamental conception of photons, using the techniques of Boltzmann, the same way the latter had provided microscopic explanation of classical Thermodynamics. But this was not to be. Probably because Einstein did not read English back then and also perhaps because the centre of gravity of science and intellectual discourse was Europe and Gibbs’s treatise was slow in gaining acceptance there. We may then summarise the impasse in the progress towards full understanding of the Planck formula on two ironical circumstances: Albert Einstein firmly believed in photons but would not produce a proof using the Statistical Mechanics, while the rest of the world refused to believe in photons but certainly had many experts who knew the latest reliable methods in Statistical Mechanics but who probably did not bother to apply them to a gas of photons.

5.4 Confirmation from far away, far later

It thus fell upon Satyendra Nath Bose, a professor in Dhaka (or Dakka) University in 1924 to produce the required proof. Bose as a younger man venerated Albert Einstein, and being far away from the European crucible of science was perhaps immune to the prejudice prevailing against the notion of photons. Further, as a brilliant scholar he had no doubt mastered the methods of Statistical Mechanics, again without too much prejudice because being from colonial India he had ease of access to the English source material, probably including Gibb’s treatise. Thus it was that he set about ascribing a discretised character to the phase space of radiation. In Mechanics, where both positions and momenta of particles need to be considered as independent variables, the word phase space refers to the abstract space labelled by this combined set of coordinates. He made the assumption that in line with quantum principles, the phase space needs to be divided into discrete cells of size $h^3$, a quantity whose unit dimensions match those of a volume element in phase space. To this author’s
knowledge this was also the first calculation of density of states for quantised bosons. The previous calculations had introduced frequency dependent volume factors in phase space within the wave picture. Bose’s partitioning of the phase space is what we now call box normalisation, and it is clear from his paper that this was very important conceptually to Bose as the full package of the quantum hypothesis.

He then proceeded to list the possible states of the ensemble of photons and inadvertently distributed the photons in available phase space boxes without any discrimination among them. He then applied Boltzmann’s method to identify the equilibrium distribution which would dominate. It yielded exactly the formula due to Planck.

It is not possible to go into the details of S. N. Bose’s all too brief but paradigm setting paper. But he had the full answer. There was some imagination and then there was precise logic and a computation. Neither desperate nor opportunistic, this derivation had the entire formula of Planck proved from first principles of Statistical Mechanics and the conception of radiation as photons. It is said that he sent his paper in English first to the Philosophical Magazine in 1923. But it was rejected. He then sent his paper to Einstein addressing him as Respected Master. Einstein immediately grasped the significance of this paper. This was the calculation he had sought for the previous two whole decades. He translated it and communicated it to the Zeitschrift für Physik. He then proceeded as a follow up to work out the consequences of the new method of calculation applied to massive particles. The combined general formulation is called Bose-Einstein Statistics to distinguish it from the classical Maxwell-Boltzmann Statistics.

Bose’s one off contribution has evoked puzzlement and its far reaching implications also perhaps jealousy. True, he did not have a consistent output of scientific contributions within that subject area like a European scientist. Being sensitive to the condition of his country he shifted his attention to practical problems of semiconductor devices. But nobody denies the brilliance and scholarship of Bose. It appears that he himself did not grasp the novel assumption he had inadvertently made in his derivation. While the discrete partitioning of the phase space is an important step, the success of the derivation relies on an additional crucial assumption. If we read the wording of how Planck finally convinced himself of his derivation in the year 1900, we see a parallel. Planck was thinking of energy as a generic quantity to be distributed among those oscillators in the walls of the cavity. And he spoke of distributing “energy units” into the available excited states of the oscillators. Of course with hindsight we know the oscillators were a completely unnecessary scaffolding. Bose on the other hand had to contend with the same energy units, now conceived as photons, themselves the objects of Statistical Mechanics to be handled by set rules. The scaffolding of cavity oscillators was abandoned once and for all. And he implicitly distributed photons among their own available energy levels according to the same indistinguishability approach as Planck. As we elaborate below, this is the key novel assumption, which naturally produces the denominator of Eq. (3) of part I, viz.,

$$\rho(\nu) = \frac{8\pi\hbar^3}{c^3} \frac{e^{-\hbar\nu/kT}}{1 - e^{-\hbar\nu/kT}}$$

without any reference to any oscillators. Bose
himself missed this particular fact, and there is nothing to indicate that even Einstein understood it at the time of communicating his paper. It was indeed something very very subtle. Thus to Bose we may attribute the credit for arriving at the “light gas” distribution formula by applying the principles of statistical mechanics directly to photons considered as fundamental entities.

6 Quantum Mechanics

6.1 Ideas whose time had come

Between 1905 and 1924 Einstein returned to the physics of light a few times, but the issue of validation of the photon hypothesis remained unresolved. In 1917 as the general relativity revolution was catching on, he devoted attention to radiation again, and wrote his famous insightful paper on the so called A and B coefficients, concerning emission and absorption of light in atomic sources. These observations went on to become the underlying framework for developing the laser.

But the proof of the Planck formula still evaded Einstein. In 1921 he got his Nobel prize. But the really eventful year for the story we are pursuing was 1924. It was in this year that Louis-Victor-Pierre-Raymond, 7th duc de Broglie submitted a thesis to the French Academy for a doctorate degree. In it he proposed that if as per Einstein, electromagnetic waves have a particle like character, conversely the electron must have a wave character. He proposed the equally preposterous formula associating a wave of wavelength \( \lambda \) with an electron of momentum \( p \).

\[
\lambda = \frac{h}{p} \quad (17)
\]

This is analogous to the relation \( \lambda = c/\nu \) for electromagnetic waves if we recognise the Planck relation \( E = h\nu \), and the Special Relativistic relation for photons, \( p = E/c \). It appears that the members of the Academy were flummoxed by this hypothesis, and after some discussion sent it off outside France, to Albert Einstein himself for examination. Even Einstein must have been suitably puzzled. However, he had received the letter from Bose just a few months earlier. He had now been fully convinced of his hypothesis of waves behaving as quanta. Much to the Academy’s surprise, Einstein approved De Broglie’s thesis proposing material particles behaving like waves. I would now like to refer back to subsec. 4.1 of part I. There we considered the possibility that the reason why Einstein could withstand the pressure from the stalwarts to withdraw his light quantum paper was perhaps the fact that the unity of the core concepts for matter and radiation was more important to him than reconciling the two alternative descriptions of radiation. The reason why he would readily accept de Broglie hypothesis can be ascribed to this line of thinking. Until specific writings or records can be uncovered to support this, it is a matter of conjecture whether Einstein’s ready acceptance of de Broglie’s thesis had anything to do with him having seen a closure to his 1905 hypothesis in the paper of Bose.

de Broglie had an illustrious career as a philosopher scientist. To begin with he was a duke by inheritance. He quickly became a member of the French Academy. His thesis of 1924 proposed that electrons have waves associated
with them, which he called pilot waves. The waves were supposed to escort the particle like pilot vehicles in front of the car of a dignitary. de Broglie had also conjectured that “the electron has an internal clock that constitutes part of the mechanism by which a pilot wave guides a particle”. With the development of quantum mechanics, and detailed consideration of its implications, it has been recognised that there are no waves that pilot the particle. The particle description and the wave description are complementary to each other, and mutually exclusive. They are not valid simultaneously. This contradicted the original metaphysical motivations of the wave hypothesis. As such de Broglie remained vehemently opposed to the subsequent development of his wave ideas into wave mechanics. Unlike Bose about whose contribution questions continued to be asked, de Broglie was awarded the Nobel prize in 1929. 

To the credit of de Broglie hypothesis is the fact that in the hands of Erwin Schrödinger it bloomed into the landmark new mathematical formulation of Quantum Mechanics in 1926. Equally importantly, in 1927 the results of the Davisson and Germer experiments at Bell Labs, scattering of slow electrons from crystalline Nickel target matched de Broglie’s wavelength formula remarkably. As for the development of Quantum Mechanics, Heisenberg was the first in making the radical proposal that one must abandon the notion of a trajectory in Quantum Mechanics, and went on to propose the principles of matrix mechanics. In 1925, this version of the theory was difficult to digest by many as matrices were foreign to physicists, and the palpable picture that waves offered, and in terms of which Schrödinger’s 1926 theory was formulated, rapidly gained acceptance. Although both formulations are equivalent, the wave formulation holds sway in most of non-relativistic problems of quantum mechanics. This is somewhat unfortunate as there are no “waves” in the ordinary sense of waves in water or strings, but only a method to implement the Principle of Linear Superposition as we explain later.

6.2 States and quanta: the essence of quantum physics

We finally explain the core new conceptualisation of nature that the Bose-Einstein statistics offers to us. The novel counting that enabled Bose to arrive correctly at the Planck-Einstein formula was that in his counting, the states containing several quanta received equal weightage regardless of how they were assembled from states of single quanta. To understand this, we consider the example of two coins. Suppose we have two identical coins. And we toss them both independently. Now we try to anticipate the number of times the various possible configurations will show up. There are only three possibilities, both heads, both tails and a third possibility, one head and one tail. We may list these as HH, TT and HT. Since the coins are indistinguishable, TH is same as HT and count as the same configuration. However we know very well from classical experience that out of the total number of possible configurations, the HT (or TH) is going to occur in two different ways, and hence twice as often compared to the HH and TT configurations each. The weightage we associate in Boltzmann statistics with the states of such a system are indicated in the second column of Table 1 under the heading Classical.

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4Wikipedia page on Louis de Broglie.
Table 1: The weightage factors associated with possible states of two indistinguishable coins in Classical, Bose-Einstein and Fermi-Dirac statistics.

|        | Classical | Q:B-E | Q:F-D |
|--------|-----------|-------|-------|
| H H    | $\frac{1}{4}$ | $\frac{1}{4}$ | 0     |
| HT or TH | $\frac{1}{2}$ | $\frac{1}{3}$ | 1     |
| TT     | $\frac{1}{4}$ | $\frac{1}{4}$ | 0     |

However, the Bose-Einstein case is different in a very subtle way. In the quantum Bose-Einstein counting, the coins are so completely identical that we are not able to assign twice as much weightage to the HT situation. It has exactly the same weightage as the HH and TT situations. This is indicated in the table in the column with heading $Q : B - E$. This kind of counting applies to particles of spin values in integer multiples of $\hbar$, i.e., 0, $\hbar$, 2$\hbar$, etc. For completeness we have included the last column which corresponds to Fermi-Dirac statistics obeyed by all particles of half-integral spin, i.e. values $\hbar/2$, 3$\hbar/2$, .. etc. examples of which are the electron, the proton and the neutron. If such a species has no quantum numbers other than the one distinguishing H from T, then the Pauli exclusion principle forbids HH as well TT. There is only one state admissible as per quantum principles, HT, with weightage unity!

This state of affairs is called quantum “indistinguishability” versus classical indistinguishability. But as we shall argue, the label of “indistinguishability” is predicated on a classical prejudice. And this has resulted in enduring confusion and also false hopes of somehow circumventing the unpalatable non-classical content of Quantum Mechanics! The suggestion in the adjective “indistinguishable” is that there are two distinct entities to begin with. The quantum logic is however taciturn, and less revealing of its secrets. Let us assign a value +1 to H and −1 to T in some units. Now the quantum logic allows an observable called the “number”, and the value of this number is 2 in this example, as we have considered two quanta. However, this system has only one unique state corresponding to total value 0 of the H/T quantum number. The availability of the observable “number” whose value is 2, should not be confused with there being “two particles” in the classical sense. So the question of “distinguishing” between them does not arise. There is only one quantum state containing two particles, the H/T quantum number of the state being zero, and the weightage of this state is exactly the same as that of the other states which have H/T quantum number +2 or −2.

At the heart of Quantum Mechanics is the principle of linear superposition. What we consider classically to be distinct configurations can be “added” in a precise mathematical sense in quantum mechanics. And the weightages of the superposed states have to be such that the sum of their squares must add up to unity. As we descend from the macroscopic level to the microscopic, there are two ways that the quantum rules set in. One is as the systems become simple, such as small molecules and atoms, systems which are described by a small number of observables. The other important way quantum principles manifest themselves is through the Bose-Einstein or Fermi-Dirac enumeration of states. But the transition to the fully quantum domain
most often is not sharply defined. For example, at standard room temperature and pressure, the molecules of hydrogen or carbon dioxide gas obey quantum rules of emission and absorption of radiation, but as a collective system they obey Maxwell-Boltzmann or classical statistics. What this means is that these behave as independent quantum systems. For each molecule, its internal states would be a superposition of its various standard states (in technical language, eigenstates) but the collective state is not found to be a superposition of some standard states, but rather just like the states of a small macroscopic particle. This happens because the gas is very dilute, viz., the average separation between the molecules is about 50 times larger than their intrinsic size. Typically the collective states of a system display quantum mechanical superposition only when the system is densely populated with quanta of a given species.

In the domain where the quantum rules apply, the counting of the possible states becomes different and defies classical common sense. For a variety of systems even when dense, an approximate picture which allows thinking in terms of the original isolated quanta works, especially when the quanta have weak mutual interaction. In such situations one constructs the general states of the system as products of single particle states. This is purely a mathematical convenience. Unfortunately this leaves behind the feeling that two distinct quanta have been put together, even though the symmetrisation or anti-symmetrisation are applied correctly. Quite a few paradoxes arise simply from this naive thinking. The “indistinguishability” of quantum systems, more correctly, the appropriate statistics has to be treated as an integral part of the Quantum principles and not as an added rule.

When this is done consistently, no paradoxes remain, though the rules may continue to intrigue us.

7 Conclusion and outlook

7.1 Final story of light

The novel description of light that began with Planck’s formula and was properly recognised as quantum behaviour of light by Einstein, reached maturity with the development of Bose-Einstein statistics. While much of the attention got diverted to condensed matter and nuclear physics, developments in optics continued separately. One of these was the inelastic scattering of light due to internal structure of molecules and crystals. This effect, discovered by C. V. Raman earned him the 1930 Nobel prize. While Quantum Electrodynamics, as a dynamical theory of photons and electrons led to profound developments, the physics of photons by themselves had entered quantum era in the 1950’s when the maser was developed, soon leading to the invention of the laser.

In the late 1950’s, in the course of using photomultiplier tubes for the study of stars, Hanbury Brown and Twiss developed intensity interferometry, whose quantum principles at first seemed to be unclear. By 1963 Glauber and independently E. C. G. Sudarshan explicated the formalism that applied to the quantised Maxwell field in all possible settings. Glauber received the Nobel prize for this development in 2005. In the treatment given by Sudarshan it was emphasised that the quantum mechanical formula given there accounts for all the possible states
of light, which subsume the classical states. Put another way, what we usually think of as classical Maxwell waves is actually a state of the quantised Maxwell field, a special state of the photons. Here the classical description applies exactly and no modifications are needed when quantum mechanics is taken into account. This formulation of Sudarshan can be considered to be the final closure on the theories of light originating with Newton and Huygens, evolved through the historical path of Planck, Einstein and Bose.

It is intriguing to note that photons have two very special properties, one is zero rest mass and the second is zero charge, (or the absence of mutual interactions). These properties have provided us entries, respectively, into the realms of special relativity and quantum mechanics. The zero mass property means that they are always moving at the largest limiting speed permissible in nature, “the speed of light”, and this property has thus provided us a key to Special Relativity. On the other hand, zero mass and zero charge properties have facilitated the observation of the peculiar properties of a quantum gas that we are discussing here. Masslessness means that there is no intrinsic “size” to a photon such as the Compton wavelength for massive particles. Thus there is no limiting dilution in which this gas begins to obey Maxwell-Boltzmann statistics, unlike in the case of molecular gases as we noted in sec. 2.2. And the absence of interactions ensures that it remains a gas of free quanta to which Bose-Einstein Statistics can be applied in all laboratory situations. Massive particles such as atoms also display quantum superposition and enter a collective state called a Bose-Einstein condensate. But to see this we need to prepare their collections with extreme care. Providing sufficient density may trigger interactions; instead, extremely low temperatures are used. For photons, the quantum characteristics are readily visible because they constitute a non-interacting quantum gas, which enabled the revolution in the hands of Planck.

The theorem of Sudarshan shows that photons provide one more access to the quantum world. Dirac has emphasised in his textbook “Principles of Quantum Mechanics”, that the new content of Quantum Mechanics is the principle of linear superposition. In Classical Mechanics, there is no meaning to a plus sign between two possible trajectories of a particle. It cannot be following both. In Quantum Mechanics, it is valid to superpose via a plus sign two states of the system which yields a new possible state of the system. The above theorem of Sudarshan then has another intriguing implication. Recall that the classical states of radiation appear without modification in the complete quantum description. As such, the linear superposition principle of electromagnetic fields that is taught at undergraduate level is actually nothing but the linear superposition principle of Quantum Mechanics!

7.2 The enigmatic Quantum

Very soon after the basic rules of the new Quantum Mechanics were understood, it became apparent that the outcomes of experiments could be predicted only statistically or on the average. Heisenberg had proposed his matrix mechanics with a clear call that the notion of trajectories must be abandoned. He then backed up his abstract formalism operationally through a thought experiment, by showing that attempts to measure one property of the trajectory, say
the position, would necessarily mess up the complementary property, the momentum. This consequence was natural because the measuring technique itself had to rely on sending one quantum system, a photon, to “view” another, the electron. It was impossible to improvise any apparatus that was capable of yielding information of the quantum domain without at the same time obeying quantum principles. Ergo, it was impossible to beat the uncertainty in measuring the attributes of a trajectory below a limit set by the new constant of nature, \( h \), or in modern usage, the quantity \( \hbar = h/2\pi \).

Such a probabilistic outcome given by a fundamental framework was an anathema to Einstein and to many others of that generation. Needless to say, the debates continue to rage and are also current. Further, there was the puzzling property that a quantum state was intrinsically non-local; the wavefunction was always spread over a space like domain. This seemed to intuitively contradict Special Theory of Relativity. The enigma of this situation was formulated by Einstein and his collaborators Podolsky and Rosen with characteristic clarity and has come to be called the EPR paradox. From a pragmatic point of view, paradoxical the situation is, but inconsistent it is not; and no attempts at arriving at an inconsistency with the basic tenets of Special Relativity, even in thought experiment, have succeeded, nor has a clever experiment been designed that would force an extension of Quantum Mechanics.

The other puzzling aspect of Quantum Observation is that only specific eigenvalues are returned as the outcome of measurement. For instance the average spin of an electron in a beam may be 0.35\( \hbar \), but that only means that if you made measurements on many electrons in that beam, that would be the average outcome. In any one specific measurement that manages to capture only one electron, the answer will be precisely either \(+h/2\) or \(-h/2\).

This fact has been well verified. But it leads to the following paradoxical situation called Wigner’s Friend or Schrödinger Cat depending on how amicable or macabre your inclination is.\[\text{[5]}\] Once measured, the system will go on being in that eigenstate, say spin \(+h/2\). But now if you sit quietly after that measurement, your friend who walks in has no way to decide whether it is already in an eigenstate or not without actually making the measurement herself. The dilemma at hand can be seen to result from the rule of quantum mechanics, that once an attribute is measured, the net effect of the measurement process is to leave the system in one of the eigenstates of the particular observable. But this rule creates an unequal situation for different categories of observers, those who first carried out such an observation on a generically prepared system, and those that come later, without knowing whether the measurement has been made by some other party. Thus the well established notion objectivity even in the classical world of observers seems to be endangered.

Nobody wants to kill a cat ever, let alone twice, so such a paradox jumps out to challenge common sense. But the resolution is very simple. If the first observer has already made the measurement, then the system can be considered to

\[\text{[5] Actually Wigner’s Friend is a paradox which is a step beyond the more direct paradox presented by Schrödinger’s Cat. Due to brevity of the presentation here, and presuming that many readers are already familiar with these paradoxes, I have taken the liberty to speak of them together.}\]
have been prepared in that specific eigenstate for the next observer. No contradictions arise but the challenge to common sense persists. Also one hopes that a refinement of the formalism would make measurement a more intrinsic and organic part of the formalism than a drastic “collapse” into specific eigenstates. Constructing such a formalism is an active area of research. But we would expect that such a formalism will only be an extension, without modifying any of the core tenets of Quantum Mechanics.

Since the Quantum is maligned so much in common discourse, it is worth emphasising that there is a lot that is counter intuitive in Physics. As we know, understanding the phenomenology of classical motion was itself a great intellectual enterprise, culminating with the discourses published by Galileo. Its final refined version we accept with equanimity is due to Newton. Yet, there is much that is conceptually unsatisfactory about Newtonian framework which we have come to take for granted. The foremost among them is the notion of limits as needed for the infinitesimal calculus. Through the formal concept of instantaneous velocity, we are convinced by Newton that a particle can be at a point and also moving while still being at that point! In fact it is supposed to possess all orders of time derivatives while still just being at its original point. In the bygone era of theology this would have remained an active area of debate, but not so in modern engineering. While the high level of refinements in real analysis ensure that there is no logical contradiction, the point remains that this is a mind game. Operationally it is impossible to make your stop watch measure vanishingly small duration. Indeed, now we know that Quantum Mechanics will kick in and will show that the Newtonian process of a limit is a figment of our imagination.

It is time we accepted that our intuition is based on the cognitive faculties tuned to the classical experience. And that physical science requires a kind of sophistication that may yield counter-intuitive theorems. And some of the puzzles will fade from common discourse, much as theology of yester years, as highschool students begin to interact with quantum systems and the quantum framework earlier in their physics syllabus.

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