On the Intriguing Connections of Regularization, Input Gradients and Transferability of Evasion and Poisoning Attacks

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ABSTRACT

Transferability captures the ability of an attack against a machine-learning model to be effective against a different, potentially unknown, model. Studying transferability of attacks has gained interest in the last years due to deployment of cyber-attack detection services based on machine learning. For these applications of machine learning, service providers avoid disclosing information about their machine-learning algorithms. As a result, attackers trying to bypass detection are forced to craft their attacks against a surrogate model instead of the actual target model used by the service. While previous work has shown that finding test-time transferable attack samples is possible, it is not well understood how an attacker may construct adversarial examples that are likely to transfer against different models, in particular in the case of training-time poisoning attacks. In this paper, we present the first empirical analysis aimed to investigate the transferability of both test-time evasion and training-time poisoning attacks. We provide a unifying, formal definition of transferability of such attacks and show how it relates to the input gradients of the surrogate and of the target classification models. We assess to which extent some of the most well-known machine-learning systems are vulnerable to transfer attacks, and explain why such attacks succeed (or not) across different models. To this end, we leverage some interesting connections highlighted in this work among the adversarial vulnerability of machine-learning models, their regularization hyperparameters and input gradients.

1 INTRODUCTION

The wide adoption of machine learning and deep learning algorithms in many critical applications introduces strong incentives for motivated adversaries to manipulate the results and models generated by these algorithms. For instance, attackers can deliberately influence the training dataset to manipulate the results of a predictive model (in poisoning attacks [11, 30, 38–40, 45, 47, 59]), or cause misclassification of new data in the testing phase (in evasion attacks [7, 16, 18, 24, 42, 53, 57, 60]).

Creating poisoning and evasion attack data points is not a trivial task, particularly when many online services avoid disclosing information about the machine learning algorithm in use. As a result, attackers are forced to craft their attacks against a surrogate model instead of the real model used by the service, hoping that the attack will be effective on the real model. The transferability property of an attack is satisfied when an attack developed for a particular machine learning model (i.e., a surrogate model) is also effective against another model (i.e., the target model). Studying attack transferability has gained interest in recent years due to deployment of cyber-attack detection services based on machine learning.

In this paper we focus on understanding what makes attacks transferable. In particular, we focus on evasion and poisoning attacks that are crafted with gradient-based optimization techniques, a popular mechanism used to create attack data points. The first gradient-based attacks against machine learning were demonstrated by Biggio et al. in [7] for test-time evasion attacks, and in [11] for training-time poisoning attacks. Then, Szegedy et al. [53], while aiming to interpret decisions of deep neural networks, independently discovered the phenomenon of adversarial examples against deep neural networks, i.e., that deep nets were also vulnerable to small changes in the input data crafted with a gradient-based attack algorithm (see, e.g., [14] for an historical perspective on the evolution of attacks against machine learning). Attack data points are commonly referred to as adversarial examples in the case of test-time evasion attacks, and poisoning points in the case of training-time poisoning attacks, although it is not uncommon to refer to adversarial examples as a general synonym for both types of attack.

While transferability of evasion attacks has been widely investigated [7, 21, 24, 31, 36, 42, 43, 53, 54, 58], transferability of poisoning attacks is still largely unexplored, the work in [37] being a notable exception. In spite of these efforts, little is understood about what are the factors that make some attacks more transferable than others, both for evasion and poisoning attacks.

Contributions. In this work, we present the first comprehensive evaluation of transferability of both poisoning and evasion attacks. We consider a wide range of classifiers, including deep neural networks, support vector machines with both linear and RBF kernels, logistic regression, ridge regression, k-nearest neighbors, and random forest. Our evaluation relies on a formal definition of transferability for evasion and poisoning attacks and an approximate relaxation of this definition, giving empirical metrics connected to
We discuss background on threat modeling against machine learning with \( x \) and \( Y \in \{-1, +1\} \), respectively, and the training data with \( D \) on \( X \) and \( Y \). The input gradients, which in turn arise from high-dimensional problems and/or low regularization/smoothness of the decision function. This clearly hinders transferability across models with different levels of regularization and smoothness of the decision function.

We discuss background on threat modeling against machine learning and how to craft evasion and poisoning attacks in Sect. 2. We then formally define transferability for both evasion and poisoning attacks, and show its approximate connection with the input gradients used to craft the corresponding attack samples (Sect. 3). Experiments are reported in Sect. 4, highlighting connections among regularization hyperparameters, the size of input gradients, and transferability of attacks, on two case studies involving the classification of handwritten digits and Android malware. We discuss related work in Sect. 5. While our analysis is restricted to two-class classification problems, we believe that our conclusions can be easily generalized to multiclass settings, as discussed in the concluding remarks of this work (Sect. 6).

\section{THREAT MODEL AND ATTACKS}

In this paper, we consider a range of adversarial models against machine learning systems. Attackers are defined by: (i) their goal or objective in attacking the system; (ii) their knowledge of the system; (iii) their capabilities in influencing the system through manipulation of the input data. Before we detail each of these, we introduce our notation, and point out that the threat model and attacks considered in this work are suited to two-class classification algorithms. We nevertheless refer the reader to [14, 35, 37] for the corresponding extensions to multiclass settings.

**Notation.** In the following, we denote the sample and label spaces with \( X \) and \( Y \in \{-1, +1\} \), respectively, and the training data with \( D = (x_i, y_i)_{i=1}^n \), where \( n \) is the training set size. We use \( L(D, w) \) to denote the loss incurred by the classifier \( f : X \mapsto Y \) (parameterized by \( w \)) on \( D \). Typically, this is computed by averaging a loss function \( \ell(y, x, w) \) computed on each data point, i.e., \( L(D, w) = \frac{1}{n} \sum_{i=1}^n \ell(y_i, x_i, w) \). We assume that the classification function \( f \) is learned by minimizing an objective function \( L(D, w) \) on the training data. Typically, this is an estimate of the generalization error, obtained by the sum of the empirical loss \( L \) on training data \( D \) and a regularization term.

\subsection{Attacker’s Goal}

We define the attacker’s goal based on the desired security violation. In particular, the attacker may aim to cause: an integrity violation, to evade detection without compromising normal system operation; an availability violation, to compromise the normal system functionalities available to legitimate users; or a privacy violation,

to obtain private information about the system, its users or data by reverse-engineering the learning algorithm [4, 5, 10, 14, 27, 37].

\subsection{Attacker’s Knowledge}

We characterize the attacker’s knowledge \( \kappa \) as a tuple in an abstract knowledge space \( K \) consisting of four main dimensions, respectively representing knowledge of: (k.i) the training data \( D \); (k.ii) the feature set \( X \); (k.iii) the learning algorithm \( f \), along with the objective function \( L \) minimized during training; and (k.iv) the parameters \( w \) learned after training the model. This categorization enables the definition of many different kinds of attacks, ranging from white-box attacks with full knowledge of the target classifier to black-box attacks in which the attacker knows almost nothing about the target system. While we refer the reader to [14] for a more detailed categorization of such attacks, including the definition of gray-box attack scenarios, in this paper we consider a simplified setting only involving white-box and black-box (transfer) attacks, as detailed below.

**Perfect-Knowledge (PK) White-Box Attacks.** We assume here that the attacker has full knowledge of the target classifier, i.e., \( \kappa = (D, X, f, w) \). This setting allows one to perform a worst-case evaluation of the security of machine-learning algorithms, providing empirical upper bounds on the performance degradation that may be incurred by the system under attack.

**Limited-Knowledge (LK) Black-Box Attacks.** We assume here that the feature representation \( X \) is known, while the training data \( D \) and the type of classifier \( f \) are not known to the attacker. We nevertheless assume that the attacker can collect a surrogate dataset \( \hat{D} \), ideally sampled from the same underlying data distribution as \( D \), and train a surrogate model \( \hat{f} \) on such data to approximate the target function \( f \) (potentially using feedback from \( f \) to relabel \( \hat{D} \)). Then, the attacker can craft the attacks against \( \hat{f} \), and then check whether they successfully transfer to the target classifier \( f \). By denoting limited knowledge of a given component with the hat symbol, such black-box attacks can be denoted with \( \kappa = (D, X, \hat{f}, \hat{w}) \). They have been widely used to evaluate the transferability of attacks across learning algorithms, as firstly shown in [7] and then in [42, 43]. It is finally worth remarking that surrogate models have been largely used in the field of black-box mathematical optimization to find optima of functions which are not differentiable or analytically tractable. In these cases, gradient information from a (differentiable) surrogate function \( \hat{f} \) (that resembles the target \( f \)) can be used to speed up the optimization process.

\footnote{Even though we do not consider privacy attacks in this work, we refer the reader to some examples of privacy attacks based on iteratively querying the target system. They include model-extraction attacks, aimed to steal machine-learning models provided as a service; and model-inversion and hill-climbing attacks, aimed to steal sensitive information like the face and fingerprint templates of users of biometric authentication systems [1, 8, 22, 23, 33, 55].}

\footnote{With the term feature representation, we do not mean the internal representations built by learning algorithms like kernel methods and deep networks, but rather the set of input features. For images, this means that we do consider pixels as the input features, consistently with other recent work on black-box attacks against machine learning [42, 43].}
2.3 Attacker’s Capability

This attack characteristic defines how the attacker can influence the system, and how data can be manipulated based on application-specific constraints. If the attacker can manipulate both training and test data, the attack is said to be causalive. It is instead referred to as exploratory, if the attacker can only manipulate test data. These scenarios are more commonly known as poisoning [11, 28, 34, 37, 59] and evasion attacks [7, 9, 10, 16, 24, 53].

Another aspect related to the attacker’s capability depends on the presence of application-specific constraints on data manipulation. For instance, to evade malware detection, malicious code has to be modified without compromising its intrusive functionality. This may be done against systems leveraging static code analysis, by injecting instructions or code that will never be executed [7, 19, 25, 57]. These constraints can be generally accounted for in the definition of the optimal attack strategy by assuming that the initial attack sample \(x\) can only be modified according to a space of possible modifications \(\Phi(x)\). In some cases, this space can also be mapped in terms of constraints on the feature values of the attack samples, e.g., by imposing that feature values corresponding to occurrences of some instructions in static malware detectors can only be incremented [7, 19, 57].

2.4 Gradient-based Attacks

Given the attacker’s knowledge \(x \in \mathcal{K}\) and an attack sample \(x' \in \Phi(x)\) along with its label \(y\), the attacker’s goal can be defined in terms of an objective function \(\mathcal{A}(y, x', \kappa) \in \mathbb{R}\) (e.g., a loss function) which measures how effective the attack sample \(x'\) is. The optimal attack strategy can be thus given as:

\[
  x^* \in \arg \max_{x' \in \Phi(x)} \mathcal{A}(y, x', \kappa)
\]

Note that, for the sake of clarity, we consider here the optimization of a single attack sample, but this formulation can easily account for multiple attack points, e.g., by iteratively optimizing one attack point at a time as in [14, 59].

We show how this general formulation encompasses both evasion and poisoning attacks against supervised learning algorithms, even though it has also been used to attack clustering [6, 12, 13] and feature selection algorithms [59, 61].

**Attack Algorithm.** Before detailing the specific instances of the optimization problem given in Eq. (1) for evasion and poisoning attacks, we describe a standard gradient-ascent algorithm that can be used to solve the aforementioned problem in both cases. It is given as Algorithm 1. It iteratively updates the attack sample along the gradient of the objective function, ensuring the resulting point to be within the feasible domain through a projection operator \(\Pi_\kappa\). The gradient step size \(\eta\) is determined in each update step with a simple line-search method based on bisection, to reduce the number of iterations required to reach a local or global optimum (e.g., depending on whether the objective function and the constraints are concave).

Notably, gradient-based algorithms for the generation of evasion and poisoning attacks against machine learning have been first proposed by Biggio et al. [7, 11], and re-discovered independently by Szegedy et al. [53] and follow-up work [16, 24], though only in the context of test-time evasion, under the name of adversarial examples.

We finally remark that non-differentiable learning algorithms, like decision trees and random forests, can be attacked with more complex strategies [29, 41] or using the same algorithm against a differentiable surrogate learner [48].

### Algorithm 1 Gradient-based Evasion and Poisoning Attacks

**Input:** \(x, y\): the input sample and its label; \(\mathcal{A}(y, x, \kappa)\): the attacker’s objective; \(\kappa = (D, X, f, w)\): the attacker’s knowledge parameter vector; \(\Phi(x)\): the feasible set of manipulations that can be made on \(x\); \(\epsilon > 0\): a small number.

**Output:** \(x'\): the adversarial example.

1. Initialize the attack sample: \(x' \leftarrow x\)
2. repeat
3. Store attack from previous iteration: \(x \leftarrow x'\)
4. Update step: \(x' \leftarrow \Pi_\kappa(x + \eta \nabla_x \mathcal{A}(y, x, \kappa))\), where the step size \(\eta\) is chosen with line search (bisection method), and \(\Pi_\kappa\) ensures projection on the feasible domain \(\Phi\).
5. until \(|\mathcal{A}(y, x', \kappa) - \mathcal{A}(y, x, \kappa)| \leq \epsilon\)
6. return \(x'\)

where \(\|u\|_p\) is the \(\ell_p\) norm of \(u\), and we assume that the classifier parameters \(w\) are known. For the black-box case, it suffices to use the parameters \(\hat{w}\) of the surrogate classifier \(\hat{f}\). The loss function \(\ell(y, x', w)\) considered in this work is simply \(-y\hat{f}(x')\), as in [7]. We refer the reader to [14, 35] for the extension of evasion attacks to the multiclass setting (where the attacker may additionally specify which class the attack sample should be assigned to, among the available ones, by properly defining the objective function).

The manipulation constraints \(\Phi(x)\) are given in terms of: (i) a distance constraint \(|x' - x|_p \leq \epsilon\), which sets a bound on the maximum input perturbation between \(x\) (i.e., the input sample) and the corresponding modified adversarial example \(x'\); and (ii) a box constraint \(x_{lb} \leq x' \leq x_{ub}\) (where \(u \leq v\) means that each element of \(u\) has to be not greater than the corresponding element in \(v\)), which bounds the values of the attack sample \(x'\).

For images, the former constraint is used to implement either dense or sparse evasion attacks [20, 35, 48]. Normally, the \(\ell_2\) and the \(\ell_\infty\) distances between pixel values are used to cause an indistinguishable image blurring effect (by slightly manipulating all pixels). Conversely, the \(\ell_1\) distance corresponds to a sparse attack in which only few pixels are significantly manipulated, yielding a salt-and-pepper noise effect on the image [20, 48]. In the image domain, the
box constraint can be used to bound each pixel value between 0 and 255, or to ensure manipulation of only a specific region of the image. For example, if some pixels should not be manipulated, one can set the corresponding values of $x_{lb}$ and $x_{ub}$ equal to those of $x$. This is of interest to create real-world adversarial examples, as it avoids the manipulation of background pixels which do not belong to the object of interest [35, 49]. Similar constraints have been applied also for evading learning-based malware detectors [7, 19, 20, 48, 57].

**Maximum-confidence vs. minimum-distance evasion.** The formulation of evasion attacks given in Eqs. (2)–(4), as in [7], aims to produce adversarial examples that are misclassified with maximum confidence by the classifier, within the given space of feasible modifications. This is substantially different from crafting minimum-distance adversarial examples, as formulated in [53] and in follow-up work (e.g., [43]). This difference is conceptually depicted in Fig. 1. In particular, in terms of transferability, it is now widely acknowledged that higher-confidence attacks have better chances of successfully transfer to the target classifier (and even of bypassing countermeasures based on gradient masking) [3, 16, 21]. For this reason, in this work we consider evasion attacks that aim to craft adversarial examples misclassified with maximum confidence.

**Initialization and Smoothing.** There are two other factors that are known to improve transferability of evasion attacks, as well as their effectiveness in the white-box setting. The first one consists of running the attack starting from different initialization points to mitigate the problem of getting stuck in poor local optima (i.e., points misclassified with lower confidence) [7, 21, 61]. The second one is known in the literature of mathematical optimization as smoothing, and consists of averaging gradients nearby the point of interest to reduce the impact of noise [21, 58]. This may be very helpful when the objective function changes very quickly around the point of interest, and gradients at specific locations are thus unreliable and noisy, hindering the optimization process.

In this work we do not consider smoothing (although it may be easily accounted for in our algorithm), but we do consider additional initialization points when running our evasion attacks, to improve their effectiveness against nonlinear algorithms. In addition to starting the gradient ascent from the initial point $x$, we also consider starting the gradient ascent from the projection of a randomly-chosen point of the opposite class onto the feasible domain. This helps finding better local optima, through the identification of more promising paths towards evasion, as also discussed in [7, 21, 58, 61].

**2.6 Poisoning Attacks**

Poisoning attacks consist of manipulating training data (mainly by injecting adversarial points into the training set) to either favor intrusions without affecting normal system operation, or to purposely compromise normal system operation to cause a denial of service. The former are referred to as poisoning integrity attacks, while the latter are known as poisoning availability attacks [14, 59]. In this work we focus on the latter, as their transferability properties have not yet been widely investigated [14, 37], conversely to those exhibited by recent backdoor and trojaning attacks [17, 26], which belong to the category of poisoning integrity attacks [14]. Nevertheless, (i) crafting transferable poisoning availability attacks is much more challenging than crafting transferable poisoning integrity attacks, as the latter have a much more modest goal; and (ii) the following formulation can also be used to craft poisoning integrity attacks, as we will detail in the remainder of this section.

As for the evasion case, we formulate poisoning in a white-box setting, given that the extension to black-box attacks is immediate through the use of surrogate learners. Poisoning is formulated as a bilevel optimization problem in which the outer optimization maximizes the attacker’s objective $A$ (typically, a loss function $L$ computed on untainted data), while the inner optimization amounts to learning the classifier on the poisoned training data [11, 34, 59]. This can be made explicit by rewriting Eq. (1) as:

$$
\max_{x'} L(D_{\text{val}}, w^*) = \sum_{j=1}^{m} \ell(y_j, x_j, w^*)
$$

$$
s.t. \quad w^* \in \arg \min_w L(D_{\text{tr}} \cup (x', y), w)
$$

where $D_{\text{tr}}$ and $D_{\text{val}}$ are two data sets available to the attacker. The former, along with the poisoning point $x'$, is used to train the learner on poisoned data, while the latter is used to evaluate its performance on untainted data, through the loss function $L(D_{\text{val}}, w^*)$. Notably, the objective function implicitly depends on $x'$ through the parameters $w^*$ of the poisoned classifier.

In poisoning availability attacks, the untainted validation set $D_{\text{val}}$ contains a set of representative points of the test data, and the attacker aims to have misclassified as many of them as possible. In the integrity case, the set $D_{\text{val}}$ may just contain few well-crafted intrusive samples that the attacker aims to have misclassified at test time. Accordingly, while both attacks share the same formulation, poisoning integrity attacks are much easier to craft.

Although the given formulation considers a single attack point, multiple-point poisoning attacks can be staged by solving the aforementioned problem iteratively, optimizing one attack point at a time [37, 59]. While poisoning attacks do not typically have restrictions on the manipulation of the poisoning points, the attacker’s capability is limited by assuming that the attacker can inject only a small fraction of poisoning points into the training set.

Poisoning points can be optimized via gradient-ascent procedures, as that given in Algorithm 1. Provided that the attacker function is differentiable w.r.t. $w$ and $x$, the required gradient can
be computed using the chain rule [11, 14, 37, 59]:

\[
\nabla_x A = \nabla_x L + \frac{\partial w}{\partial x} \nabla_y L. 
\]

(7)

The term \( \frac{\partial w}{\partial x} \) captures the implicit dependency of the parameters \( w \) on the poisoning point \( x \). Under some regularity conditions, this derivative can be computed by replacing the inner optimization problem with its stationarity (Karush-Kuhn-Tucker, KKT) conditions, i.e., with its implicit equation \( \nabla_w L(D_\theta \cup (x', y), w) = 0 \).

By differentiating this expression w.r.t. the poisoning point \( x \), one obtains:

\[
\nabla_x \nabla_w L + \frac{\partial w}{\partial x} \nabla_y L = 0.
\]

(8)

Solving for \( \frac{\partial w}{\partial x} \), we obtain

\[
\frac{\partial w}{\partial x} = -\left( \nabla_x \nabla_w L \right) \left( \nabla^2_w L \right)^{-1} \nabla_y L.
\]

(9)

While we refer the reader to [37] for a more detailed derivation of the aforementioned gradient, we report here its compact expression for SVM poisoning, with \( L \) corresponding to the dual SVM learning problem, and \( L \) to the hinge loss (in the outer optimization):

\[
\nabla_x A = -\alpha c \frac{\partial k_{x'}}{\partial x} y_k + \alpha c \left( \frac{\partial k_{x'}}{\partial x} \right) y_k.
\]

(10)

We use \( c, s \) and \( k \) here to respectively index the attack point, the support vectors, and the validation points for which \( \ell(y, x, w) > 0 \) (corresponding to a non-null derivative of the hinge loss). The coefficient \( \alpha c \) is the dual variable assigned to the poisoning point by the learning algorithm, and \( k \) and \( K \) contain kernel values between the corresponding indexed sets of points. We refer the reader to [11] for further details on the derivation of poisoning attacks against SVMs. Poisoning attacks targeting other classifiers can be derived similarly [30, 37, 59].

3 TRANSFERABILITY, INPUT GRADIENTS AND REGULARIZATION

We discuss here the main contribution of this work, which highlights an intriguing connection among transferability of both evasion and poisoning attacks, input gradients and regularization.

We start by formally defining transferability of an attack point as the loss attained by the target classifier \( f \) (parametrized by \( w \)) under the influence of the given attack point \( x' = x + \delta \):

\[
T = \ell(y, x + \delta, w),
\]

(11)

where, for each given point \( x, y \), the adversarial perturbation \( \delta \) is crafted against the surrogate classifier \( \tilde{f} \) (parametrized by \( \tilde{w} \)):

\[
\delta \in \arg \max_{\|\delta\|_p \leq \varepsilon} \ell(y, x + \delta, \tilde{w}),
\]

(12)

and the \( \ell_p \) norm of the perturbation \( \delta \) is upper bounded by \( \varepsilon \). This is consistent with the \( \ell_p \)-norm constraint used to craft evasion attacks. Although poisoning attacks do not necessarily require this specific constraint, they are included in this formulation if we consider \( x \) as the initial poisoning point (with its label \( y \) flipped) prior to run the gradient-ascent attack algorithm.

The given definition of transferability, suited to both evasion and poisoning attacks, can be simplified through a linear approximation of the loss function, which reasonably holds for sufficiently-small input perturbations:

\[
T = \ell(y, x + \delta, w) \approx \ell(y, x, w) + \nabla_y \ell(y, x, w)^\top \delta
\]

(13)

Rewriting Eq. (12) using the same linear approximation, one yields the maximization of an inner product over an \( \varepsilon \)-sized ball:

\[
\max_{\|\delta\|_p \leq \varepsilon} \delta^\top \nabla_y \ell(y, x, w) = \varepsilon \|\nabla_y \ell(y, x, w)\|_q,
\]

(14)

where \( \ell_q \) is the dual norm of \( \ell_p \). It is not difficult to see that the above problem is maximized, for \( p = 2 \), by \( \delta = \hat{\varepsilon} \nabla_y \ell(y, x, w) \|\nabla_y \ell(y, x, w)\|_q \); for \( p = \infty \), by \( \delta \in \varepsilon \text{sign}(\nabla_y \ell(y, x, w)) \); and for \( p = 1 \), by setting the values of \( \delta \) that correspond to the maximum absolute values of \( \nabla_y \ell(y, x, w) \) to their sign, i.e., \( \pm 1 \), and 0 otherwise. Substituting the optimal value of \( \delta \) into Eq. (13), we can compute the increase of the loss function \( \Delta T = \delta^\top \nabla_y \ell(y, x, w) \) under a transfer attack in closed form. For example, for \( p = 2 \), it is given as:

\[
\Delta T = \varepsilon \|\nabla_y \ell(y, x, w)\|_q^2 \|\nabla_y \ell(y, x, w)\|_2 \leq \varepsilon \|\nabla_y \ell(y, x, w)\|_2.
\]

(15)

where the upper bound is obtained when the surrogate classifier is equal to the target (white-box attacks), and similar results hold for \( p = 1 \) and \( p = \infty \) (using the dual norm in the right-hand side).

Intriguing Connections. The above finding reveals three interesting connections among transferability of attacks, regularization and size of input gradients, detailed below.

(1) Transferability depends on the size of the gradient of the target classifier, regardless of the surrogate: the larger this gradient is, the larger the attack impact may be. Note that this is a general result related to the adversarial vulnerability of classifiers, not only to transferability. Adversarial vulnerability has already been shown to depend on the size of the input gradients [50], although this has been only discussed for evasion attacks and in relationship to the increase of the dimensionality of the input space. Here, we confirm this result also for poisoning attacks and, more interestingly, we highlight it in the context of transferability.

(2) The size of input gradients also depends on the level of regularization. Classifiers which are highly regularized tend to have smaller input gradients (i.e., they learn smoother functions that are more robust to attacks), and vice-versa. Notably, this holds for both evasion and poisoning attacks (e.g., the poisoning gradient in Eq. 10 is proportional to \( \alpha c \), which is larger when the SVM is weakly regularized). This result has also another interesting consequence: if a classifier has large input gradients (e.g., due to high-dimensionality of the input space and low level of regularization), for an attack to succeed it suffices to apply only tiny, imperceptible perturbations. As we will see in the experimental section, this explains why adversarial examples against deep neural networks can often only be slightly perturbed to mislead detection, while when attacking strongly-regularized classifiers in low dimensions, modifications become more evident.

(3) If we compare the increase in the loss function in the black-box case (the left-hand side of Eq. 12) against that corresponding to white-box attacks (the right-hand side), we find that the relative increase in loss is given by the cosine of the angle between the gradient of the surrogate and that of the target classifier. This is a very
We report the complete security evaluation curves. We start by reporting our experiments on evasion attacks. We well in predicting transferability between pairs of classifiers. which is in turn reduced when classification functions are smoother with two hidden layers (NN) and hyperbolic tangent as activation function. 

We run 5 independent repetitions to average the results on different training-test splits. In each repetition, we run white-box and black-box attacks, using 3,000 samples to train the target classifier and 1,000 distinct samples to train the surrogate classifier (without even relabeling the surrogate data with labels predicted by the target classifier; i.e., we do not perform any query on the target). We modified test digits in both classes using an $\ell_2$-constrained attack in this case, with $\varepsilon \in [0, 5]$. 

We consider the following classifiers from scikit-learn [44]: (i) SVM with the linear kernel and $C \in \{0.01, 1, 100\}$ (SVM); (ii) SVM with the RBF kernel ($\gamma = 0.01$) and $C \in \{0.01, 1, 100\}$ (SVM RBF); (iii) logistic classifier with $C \in \{1, 10\}$ (LOGISTIC); (iv) ridge regressor with $\alpha \in \{1, 10\}$ (RIDGE); (v) a fully-connected neural network with two hidden layers (NN) and hyperbolic tangent as activation function. We additionally consider as target classifier a Random Forest (RF) consisting of 30 base decision trees. These configurations are chosen to evaluate the robustness of classifiers that exhibit similar test accuracies but different levels of regularization. 

The results for white-box evasion attacks are reported in Fig. 2. We report the complete security evaluation curves, showing the mean test error (over the 5 runs) against an increasing maximum admissible distortion $\varepsilon$. The mean test error values are further averaged over all the considered values of $\varepsilon$. This value is referred to as $err$ in the legend, and we will use it as a synthetic measure to compactly denote the success of the attack. In other words, the higher $err$ is, the higher the classification error (or evasion rate) is.

The results clearly show that strongly-regularized classifiers are less vulnerable against evasion attacks. The underlying reason is that classifier vulnerability depends on the size of the input gradients, which is in turn reduced when classification functions are smoother and more regularized. This can be seen in Fig. 3 by comparing the value of $err$ for both white- and black-box attacks with the size of the input gradients of target classifiers. Note how this behavior is consistent within each family of classifiers, as the regularization hyperparameter changes.

Interestingly, nonlinear classifiers tend to be in general less vulnerable than linear ones in this case. Moreover, note that strongly-regularized linear and nonlinear classifiers provide better surrogate models on average. The reason is that they learn smoother (and) stabler functions that are capable of better approximating the target function (even when weakly regularized, and thus more prone to overfit a specific training set). Comparing the results of the black-box evasion attack transferability in Fig. 3 with the gradient alignment between surrogate and target classifiers reported in Fig. 4, it is clear that the latter measure provides a good indication of which classifier can be a better surrogate for a given target classifier. Worth remarking, this measure is extremely fast to evaluate, as it does not require simulating any attack. Nevertheless, this is only a relative measure of the attack transferability, as its final impact depends on how much the target classifier is regularized; i.e., on the size of the input gradients of the target classifier.

We finally report the images of some manipulated digits in Fig. 5. Such images highlight that imperceptible modifications suffice to evade weakly-regularized classifiers, while larger modifications are required to evade strongly-regularized classifiers. The reason is that weakly-regularized classifiers exhibit large input gradients and, thus, very small input changes cause large variations in the output function, whereas strongly-regularized classifiers are smoother and, thus, much less sensitive to small input changes.

4.1.2 Drebin. The Drebin data [2] consists of around 120,000 legitimate and around 5000 malicious Android applications, labeled using the VirusTotal service. A sample is labeled as malicious (or positive, with $y = +1$) if it is classified as such from at least five out of ten anti-virus scanners, while it is flagged as legitimate (or negative, with $y = -1$) otherwise. The structure and the source code of each application is encoded as a sparse feature vector consisting of around a million binary features denoting the presence or absence of permissions, suspicious URLs and other relevant information that can be extracted by statically analyzing Android applications. We adopt the same experimental setting as in the previous case, using 30,000 samples to learn surrogate and target classifiers, and 30,000 samples for testing.

We perform feature selection to retain only 5,000 features, chosen by maximizing information gain, i.e., $|p(x_k = 1|y = +1)−p(x_k = 1|y = −1)|$ (estimated on the training data), where $x_k$ denotes the $k$th feature. While this feature selection process does not significantly affect the classification performance (the detection rate is only reduced by 2%, on average, at 1% false alarm rate), it drastically reduces the computational complexity of the corresponding classification algorithms.

In each repetition, we run white-box and black-box evasion attacks on 1,000 distinct malware samples (selected from the test data) against an increasing number of modified features in each malware $\varepsilon \in \{0, 1, 2, \ldots, 30\}$. This is achieved by imposing the $\ell_1$ constraint $\|x'−x\|_1 ≤ \varepsilon$. As in previous work, we further restrict the attacker to only inject features into each malware sample, to avoid compromising its intrusive functionality [7, 19].

To evaluate the impact of the aforementioned evasion attack, we measure the evasion rate (i.e., the fraction of malware samples
The reported security evaluation curves show how the test error varies against an increasing maximum admissible perturbation $\epsilon \in [0, 5]$. The mean test error computed over the whole security evaluation curve is reported as $\text{err}$ in the legend for linear (left) and nonlinear (right) classifiers.

For poisoning attacks, we report experiments on the MNIST89 models (depending on the specific target classifier). The mean test error computed over the whole security evaluation curve is reported as $\text{err}$ in the legend for linear (left) and nonlinear (right) classifiers.

Figure 3: Transferability of black-box evasion attacks on MNIST89 (bottom matrix). Each cell contains the value $\text{err}$ denoting the test error of the target classifiers (in columns) on the attack samples crafted against the surrogate learners (in rows), averaged over the entire security evaluation curve for $\epsilon \in [0, \ldots, 5]$. The higher $\text{err}$ is, the more the attack transfers to the target classifier. The value of $\text{err}$ for white-box attacks (first row) and the average norm of the input gradients of target classifiers (second row) are also reported.

misclassified as legitimate) at 1% false alarm rate (i.e., when only 1% of the legitimate samples are misclassified as malware). As in the previous experiment, we report the complete security evaluation curve for the white-box attack case, whereas we report only the value of $\text{err}$ for the black-box case. The results, reported in Figs. 6-8, confirm the main findings of the previous experiments, which can be summarized as follows:

1. strongly-regularized (linear and nonlinear) classifiers are less vulnerable to evasion;
2. they often provide better surrogate functions than their weakly-regularized counterparts; and
3. the gradient alignment between surrogate and target classifiers provides a reliable metric to identify good surrogate models (depending on the specific target classifier).

Figure 4: Transferability of black-box evasion attacks on MNIST89, estimated as the cosine of the angle between the input gradients of the surrogate (in rows) and of the target (in columns), averaged on the unmodified test samples.

4.2 Transferability of Poisoning Attacks

For poisoning attacks, we report experiments on the MNIST89 dataset.
We consider the following surrogate classifiers: linear and RBF SVMs (with different regularization parameters).

As target classifiers, in addition to the aforementioned SVMs, we consider the logistic classifier logistic classifier with $C ∈ \{1, 100\}$ (LOGISTIC), $k$-nearest neighbors with $\ell_1$ ($k$NN-l1) and $\ell_2$ ($k$NN-l2) distances, RF with 100 base decision trees, and the Convolutional Neural Network (CNN) used on MNIST data by Carlini et al. [15].

We consider 500 training samples, 1,000 validation samples to compute the attack, and a further 1,000 samples to evaluate the test error. The test error is computed against an increasing number of poisoning points into the training set, from 0% to 20% (corresponding to 125 poisoning points). The reported results are averaged on 10 independent, randomly-drawn data splits.

The results for white-box poisoning are reported in Fig. 9. Similarly to the evasion case, weakly-regularized classifiers are more vulnerable to poisoning attacks as their input (poisoning) gradients are larger (see Fig. 10).

The results for black-box poisoning are reported in Fig. 10. Worth remarking, here the best surrogate classifiers are those matching the regularization level of the target. This is due to the inherent geometry of the optimization landscape, which exhibits local optima for poisoning attacks in very different regions of the feature space when the regularization hyperparameter of the target classifier changes significantly. This behavior is anyway captured by our transferability measure, i.e., the average cosine angle (gradient alignment) between the surrogate and the target classifiers, reported in Fig. 11.

Finally, a visual inspection of the adversarially-crafted poisoning digits, shown in Fig. 12, reveals that the poisoning points crafted against weakly-regularized classifiers are only minimally perturbed, while the ones computed against strongly-regularized classifiers exhibit larger, visible perturbations. This is due to the fact that the norm of the input gradients of weakly-regularized classifiers is much larger, therefore even little perturbations are heavily amplified and turn out to be sufficient to significantly increase the classification error.

5 RELATED WORK

Transferability. Transferability of evasion attacks has been studied in previous work [7, 21, 31, 36, 42, 43, 53, 54, 58]. In particular,
Liu et al. [31] and Tramer et al. [54] have introduced some transferability measures, including gradient alignment, based on reasonable, heuristic motivations. In this work, we have shown that the implicit, underlying assumption behind this metric is the linearity of the loss function with respect to the input sample. Furthermore, starting from a clear, formal definition of transferability, we have demonstrated that gradient alignment is not the only important factor influencing the success of an attack, but that it also depends on the size of the input gradients of the target classifier. This is another relevant factor which has not been highlighted in [31, 54], due to the lack of a proper formalization of the notion of attack transferability. In the context of poisoning where there is very little work on transferability, the exception being a preliminary investigation for neural networks [37]. That work indicates that poisoning examples are transferable among very simple network architectures (logistic regression, MLP, and Adaline). Another exception is given by the work in [52], which however neither analyzes the transferability between different learning models nor provides a mathematical formulation of transferability.

**Input gradient regularization.** Prior work has investigated the role of input gradients and Jacobians. Some of these works have considered training in order to decrease the magnitude of input gradients to defend against evasion attacks [32, 46] or improve classification accuracy [51, 56]. In [50], the magnitude of the input gradient is identified as a cause for vulnerability to evasion attacks. They all identify a smaller input gradient magnitude as a more desirable property for a model in response to evasion attacks. Nevertheless, to the best of our knowledge, our work is the first to show a similar property in the context of poisoning attacks.

### 6 CONCLUDING REMARKS

In this paper we have conducted an analysis and experimental evaluation of the transferability of evasion and poisoning attacks on several machine-learning models and settings, showing that such attacks exhibit similar transferability properties. We have defined evasion and poisoning attack transferability under the same formalization, and highlighted its connections with input gradients via a linear approximation. This has in turn revealed that not only the alignment of the input gradients between surrogate and target classifiers plays a role, but also that the transferability of an attack significantly depends on the size of the input gradients of the target classifier. Our experiments have highlighted for the first time novel factors that significantly affect attack transferability. First, we have shown that in both white- and black-box settings, weakly-regularized target classifiers have larger input gradients and are thus much more vulnerable. Second, attacks against surrogate classifiers with gradients more aligned to those of the target do transfer better, but the overall impact on the target depends on the gradient size (regularization level) of the target classifier. Finally, we have shown that attacks crafted against weakly-regularized classifiers (with large input gradients) require much less modifications than those required to attack strongly-regularized classifiers to succeed, as shown by the images of the manipulated digits we have reported for both evasion and poisoning attacks. Despite some preliminary studies on the transferability of evasion attacks, to the best of our

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**Figure 7:** Transferability of black-box evasion attacks on Drebin (bottom matrix). Each cell contains the value $err$ denoting the test error of the target classifiers (in columns) on the attack samples crafted against the surrogate learners (in rows), averaged over the entire security evaluation curve for $\epsilon \in [0, \ldots, 30]$. The higher $err$ is, the more the attack transfers to the target classifier. The value of $err$ for white-box attacks (first row) and the average norm of the input gradient of target classifiers (second row) are also reported.

**Figure 8:** Transferability of black-box evasion attacks on Drebin, estimated as the cosine of the angle between the input gradients of the surrogate (in rows) and of the target (in columns), averaged on the unmodified test samples.
knowledge, the connections among input gradients, regularization and attack transferability studied in our paper have never been investigated in such detail. Most importantly, our work is the first one investigating such connections for poisoning attacks, for which transferability has been largely unstudied so far. Although our analysis is limited to binary classifiers, we firmly believe that our results also hold for multiclass classification problems. We will evaluate this aspect in more detail in future work.

Figure 9: White-box poisoning attacks on MNIST89. The reported security evaluation curves show how the test error varies against an increasing fraction of poisoning points injected into the training set (from 0% to 20%). The mean test error computed over the whole security evaluation curve is reported as err in the legend for linear (left) and nonlinear (right) classifiers.

Figure 10: Transferability of black-box poisoning attacks on MNIST89 (bottom matrix). Each cell contains the value err denoting the test error of the target classifiers (in rows) on the attack samples crafted against the surrogate learners (in columns), averaged over the entire security evaluation curve. This curve reports the classification error against an increasing fraction of poisoning points injected into the training set, from 0% to 20% (corresponding to 125 poisoning points). The higher err is, the more the attack transfers to the target classifier. The value of err for white-box attacks (first row) and the average norm of the input gradient of target classifiers (second row) are also reported.

Figure 11: Transferability of black-box poisoning attacks on MNIST89, estimated as the cosine of the angle between the input gradients of the surrogate (in rows) and of the target (in columns), averaged on the unmodified test samples.

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Figure 12: Poisoning digits crafted against linear and RBF SVMs, for decreasing level of regularization, i.e., increasing value of $C$ ($C = 0.01, 1, 100$ for the linear SVM, and $C = 1, 10, 100$ for the RBF SVM with $\gamma = 0.01$). Large modifications are required to have significant impact on strongly-regularized classifiers (smaller $C$ values within each family of classifiers), while minimal changes are very effective on weakly-regularized SVMs (larger $C$ values). The reason is that the size of input gradients increases with $C$, as regularization decreases; thus, when $C$ is large (regularization is low), even very small changes can largely affect the decision function. This makes (minimally-modified) attacks crafted against weakly-regularized classifiers less effective against strongly-regularized ones.

$C_{01} = 100$, for the RBF SVM with $\gamma = 0.01$, $C_{10} = 10$, and $C_{100} = 100$. Large modifications are required to change the decision of the classifier. For the linear SVM, even very small changes can largely affect the decision function. This makes (minimally-modified) attacks crafted against weakly-regularized classifiers less effective against strongly-regularized ones.

$C_{01}$ is large (regularization is low), even very small changes can largely affect the decision function. This makes (minimally-modified) attacks crafted against weakly-regularized classifiers less effective against strongly-regularized ones.

$C_{01}$ is large (regularization is low), even very small changes can largely affect the decision function. This makes (minimally-modified) attacks crafted against weakly-regularized classifiers less effective against strongly-regularized ones.
