Comparing two experimental procedures for multi-position calibration of a MEMS-type IMU

J S Eger, M C Porath
Federal University of Santa Catarina (UFSC), Joinville, Brazil
E-mail: james.eger@ufsc.br

Abstract. The proper calibration of a transducer has direct influence on its measurement accuracy. Procedures for calibrating MEMS-type IMUs generally require sophisticated and expensive equipment. An alternative procedure called multi-position calibration has shown to be efficient and only demands that the transducer be moved in different orientations. We investigate the influence of the repeatability of these orientations by comparing two different experimental procedures – robotic-motion and hand-motion of the IMU sensor. Statistical analysis of the results makes it clear that there are no significant differences for either variances or means of calibrated parameters between both experimental procedures.

1. Introduction
According to Chatfield [1], calibration of a transducer is the process of comparing instrument outputs with known reference information and determining the coefficients that force the output to agree with the reference information over a range of output values. In other words, it is the process of identifying the quantities in the transducer’s measurement model, such as scale factor, cross-axis sensitivities and biases. Hence, proper calibration of a transducer has direct influence on its measurement accuracy.

In the case of MEMS-type inertial sensors, it is known that bias, scale-factor and misalignment of transducer’s axes are the dominant deterministic error elements [2]. For example, an uncompensated accelerometer bias will introduce an error proportional to the square of the elapsed time when calculating position.

Procedures for the calibration of MEMS-type IMUs are well described in the literature. Conventionally, it is done under controlled environment and using sophisticated equipment such as rate tables and “perfect” cube-shaped mounting frames, as in [2, 3, 4]. In search for low-cost and in-field calibration, alternative procedures have been developed. Tedaldi et al. [5] presented a method based on multi-position scheme, providing scale factor, misalignment parameters and biases for both accelerometers and gyroscopes triads. It does not need any external equipment and only requires the IMU to be moved by hand in a set of different static orientations. Results achieved with both synthetic and real data show effectiveness of the method.

Qureshi and Golnaraghi [6] presented a similar method, with slight differences in the mathematical model and in the gyroscopes’ calibration procedure. The experimental results obtained through a custom-built IMU and a commercial IMU against reference data confirm the validity of the method.
Both aforementioned multi-position methods use hand motion of the IMU to a number of different orientations. A question may arise whether the variation of the orientations has significant influence on the calibration results. In this work we compare the results of multi-position calibration for the accelerometer triad of a commercial IMU in two situations: i) using a parallel robot to move the IMU to 30 different but highly repeatable static orientations; ii) moving the IMU by hand to 30 different but less repeatable static orientations.

2. Mathematical Background of Multi-position Calibration

The measurement model of a triad of MEMS accelerometers is given by equation 1 [5, 6]

\[ f_m = (KT)(f + b + e) \] (1)

Where: \( f_m \) is the vector of transducer’s outputs \((m/s^2)\); \( K \) is the scale factor matrix; \( T \) is the misalignment matrix; \( f \) is the vector of true specific forces; \( b \) is the vector of transducer’s biases; \( e \) stands for noise terms.

In the multi-position method, noise is neglected because averaging of the signals is applied over each static interval [5]. The measurement model is therefore simplified as in equation 2:

\[ f_m = (KT)(f + b) \] (2)

In more detail,

\[
K = \begin{bmatrix}
s_x & 0 & 0 \\
0 & s_y & 0 \\
0 & 0 & s_z
\end{bmatrix}
\] (3)

\[
T = \begin{bmatrix}
1 & -\alpha_{yz} & \alpha_{zy} \\
0 & 1 & -\alpha_{zx} \\
0 & 0 & 1
\end{bmatrix}
\] (4)

\[
b = \begin{bmatrix}
b_x \\
b_y \\
b_z
\end{bmatrix}
\] (5)

The calibration parameters to be found are collected to form the vector \( X \) (equation 6):

\[ X = [s_x, s_y, s_z, \alpha_{yz}, \alpha_{zy}, \alpha_{zx}, b_x, b_y, b_z] \] (6)

Where: \( s_x, s_y, s_z \) are the scale factors for \( x, y \) and \( z \) axes, respectively; \( \alpha_{ij} \) stands for the misalignment between real axis \( i \) and nominal axis \( j \); \( b_x, b_y, b_z \) are bias terms.

The function \( h \) is defined (equation 7):

\[ f = h(f_m, X) = (KT)^{-1}(f_m - b) \] (7)

For the accelerometer triad, the total specific force in any static orientation should be equal to the magnitude of local gravity. In the multi-position calibration method, the IMU is moved to a set of different and temporarily static orientations. From this, we can derive the cost function \( G(X) \) (equation 8):

\[ G(X) = \sum_{k=1}^{N} \left( \|h(f_m, AV, X)\|^2 - \|g_l\|^2 \right)^2 \] (8)
Where: $f_{m,AV}$ is the average of measured specific force during each static interval; $N$ is the number of different orientations (static intervals); $\|g\|$ is the magnitude of the local gravity vector.

The unknown parameters are found by minimizing the cost function. In this work we employ the Levenberg-Marquardt minimization algorithm.

3. Experimental procedures
The experimental procedures are presented next. The transducer is the commercial IMU Xsens MTi-G-700. Each experiment has been run 3 times, for the assessment of variability in the results. Warm-up and cool-down periods were observed in order to include turn-on/turn-off variations. The initial guess for vector $X$ of equation 6 is $X = [-1.0006; -0.9992; -0.9972; 0; 0; 0; 0.0270; 0.0075; -0.0060]$, experimentally obtained based on recommendations from [2].

3.1. Hand motion
In the first situation, we moved the IMU by hand. Sets of data with 2 seconds of duration were measured in each of the 30 different static orientations. Figure 1 shows the IMU and the fixture used to generate different and stable orientations.

3.2. Robotic motion
In the second situation, a parallel robot (Stewart Platform) was used to move the IMU to 30 different and highly repeatable static orientations – our robot’s expanded uncertainty (95%) in angle measurement is estimated to be less than 0.1° around each axis. Sets of data with 2 seconds of duration were also measured in each of the 30 different static orientations. Figure 2 shows the IMU mounted on the platform.

![Figure 1. Fixture for hand-motion experiment.](image1)

![Figure 2. IMU mounted on Stewart Platform for robotic-motion experiment.](image2)

4. Results
Table 1 shows the results of average, standard deviation and repeatability (95%) for bias parameters $b_x$, $b_y$ and $b_z$ from both situations:
Table 1. Results for bias parameters from both hand-motion and robotic-motion experiments.

|                  | Hand        | Robot       | Hand        | Robot       | Hand         | Robotic      |
|------------------|-------------|-------------|-------------|-------------|--------------|--------------|
| Average          | 0.008467    | 0.009867    | 0.008260    | 0.01019     | 0.01930      | 0.01943      |
| Standard-deviation| 0.002040    | 0.002519    | 0.000432    | 0.002719    | 0.000662     | 0.002434     |
| Repeatability (95%) | 0.0088      | 0.0108      | 0.0019      | 0.0117      | 0.0029       | 0.0105       |

One can notice that repeatability values are in general low in relation to the average values. Visual analysis suggests that the results from both situations are not significantly different. This also applies to all other estimated parameters. We carried out statistical tests for a more rigorous assessment. First we compared the variances through an F-test where the alternative hypothesis states that there is difference in true variances from each experiment. Then we compared the averages through a t-test where the alternative hypothesis states that there is difference in true means. Table 2 summarizes these tests:

Table 2. Results of hypothesis tests for variances and means of both experiments.

| Calibration Parameter | Variance Tests (H₁ : σ_hand ≠ σ_robot) | Mean Tests (H₁ : μ_hand ≠ μ_robot) |
|-----------------------|----------------------------------------|-------------------------------------|
|                       | P-value                               | P-value                             |
| bₓ                    | 0.80                                  | 0.50                                |
| bᵧ                    | 0.05                                  | 0.30                                |
| bᶻ                    | 0.10                                  | 0.90                                |
| α_yz                  | 0.20                                  | 0.50                                |
| α_zy                  | 0.30                                  | 0.50                                |
| α_yz                  | 0.05                                  | 0.05                                |
| sₓ                    | 0.30                                  | 0.70                                |
| sᵧ                    | -                                     | -                                   |
| sᶻ                    | -                                     | -                                   |

Considering a typical significance level of 5%, none of the tests rejects the null hypothesis – in fact, most tests present P-values much higher. Results for sᵧ and sᶻ are omitted because there were absolutely no differences in the samples, leading to unrealistic outputs of the tests. On the whole, no significant differences between the results from both experiments were detected.

5. Conclusion
We compared two ways of generating different and stable orientations for multi-position calibration of a MEMS-type IMU: highly repeatable robotic motion against less precise hand motion of the IMU. Statistical tests did not detect any significant differences for either variances or means between both cases. Hence, repeatability of the orientations is not important for the multi-position calibration method.
References
[1] Chatfield A B 1997 Fundamentals of High Accuracy Inertial Navigation (Washington DC: AIAA)
[2] Syed Z F et al 2007 A new multi-position calibration method for MEMS inertial navigation systems Meas. Sci. Technol. 18 1897-1907
[3] Titterton D H and Weston J L 2004 Strapdown Inertial Navigation Technology (Washington DC: AIAA)
[4] Aydemir G A and Saranli A 2012. Characterization and calibration of MEMS inertial sensors for state and parameter estimation applications Measurement 45 1210-1225
[5] Tedaldi D Pretto A and Menegatti E 2014. A robust and easy to implement method for IMU calibration without external equipments IEEE Int. Conf. on Robotics and Automation (ICRA) 3042-3049
[6] Qureshi U and Golnaraghi F 2017 An Algorithm for the In-Field Calibration of a MEMS IMU IEEE Sensors Journal 17 7479-7486