Crinkles in the last scattering surface: Non-Gaussianity from inhomogeneous recombination.

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Abstract

The perturbations in the electron number density during recombination contributes to the cosmic microwave background bispectrum through second order terms. Perturbations in the electron density can be a factor of $\sim 5$ larger than the baryon density fluctuations on large scales as shown in the calculations by Novosyadlyj. This raises the possibility that the contribution to the bispectrum arising from perturbations in the optical depth may be non-negligible. We calculate this bispectrum and find it to peak for squeezed triangles and of peak amplitude of the order of primordial non-Gaussianity of local type with $f_{NL} \approx 0.05 \sim -1$ depending on the $\ell$-modes being considered. This is because the shape of the bispectrum is different from the primordial one although it peaks for squeezed configurations, similar to the local type primordial non-Gaussianity.

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I. INTRODUCTION

First order perturbation theory has been of sufficient accuracy for analysis of the Cosmic Microwave Background (CMB) observations so far. However future CMB experiments will have high enough precision that second order effects would need to be taken into account for theory to have similar accuracy. The second order contributions will in particular be important for the higher order statistics like the three point correlation or the bispectrum. Second order effects in CMB have been studied previously [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Bartolo et al. have derived the Boltzmann equations at second order and also the analytic solutions for the CMB transfer function at second order with some simplifying assumptions [14, 15], see also [16].

All numerical and analytic calculations at second order so far have ignored the contribution arising from the perturbations in the electron number density, \( \delta_e \). These contributions are expected to be small compared to other second order terms since \( \delta_e \) multiplies the collision term which has contributions from the difference of first order radiation and electron dipoles, radiation quadrupole and higher order moments of the radiation transfer function. These terms are small during recombination compared to the monopole terms. Recombination however depends on matter and radiation densities and perturbations in the electron number density can be quite different from the perturbations in the matter and radiation densities. This was calculated by Novosyadlyj [17] who showed that this is indeed the case and perturbations in the electron number density can be \( \sim 5 \) times the baryon number density perturbations.

We calculate the CMB bispectrum on all scales arising due to the perturbations in electron number density and compare it with the bispectrum expected from a primordial non-Gaussianity of the local type. This is a full numerical calculation without any other approximation except that we only consider terms involving \( \delta_e \). Although this bispectrum turns out to be below the detection levels of future experiments like Planck [18], there are some important general implications which are discussed in the conclusions section. We use the following cosmological parameters for our calculations (values at redshift \( z = 0 \) unless specified): baryon density \( \Omega_b = 0.0418 \), cold dark matter density \( \Omega_c = 0.19647 \), cosmological constant \( \Omega_\Lambda = 0.76173 \), number of massless neutrinos \( N_\nu = 3.04 \), Hubble constant \( H_0 = 73 \), CMB temperature \( T_{CMB} = 2.725 \), primordial Helium fraction \( y_{He} = 0.24 \), redshift
of reionization $z_{ri} = 10$, primordial gravitational potential power spectrum $P(k) = 2\pi^2/k^3$

II. INHOMOGENEOUS RECOMBINATION

We use the code DRECFAST [19] by Novosyadlyj [17], which is a modification of the recombination code RECFAST [20] to calculate the perturbations in the electron number density $\delta_e = (n_e - \bar{n}_e)/\bar{n}_e$ during recombination. $n_e$ is the local electron number density and $\bar{n}_e$ is the mean electron density. Perturbations in baryon ($\delta_b$) and photon density ($\delta_\gamma$) result in perturbations in the electron density with an amplitude that is amplified or suppressed depending on which terms in the evolution equations prevail. Specifically photo-ionization prevails on superhorizon scales resulting in $\delta_e \sim 5 \times \delta_b$ during recombination. We refer the reader to [17] for further details.

We will not consider the full second order Boltzmann equation [14] but only the terms involving the perturbed electron density. This is given by:

$$\frac{\partial \Theta^{(2)}}{\partial \tau} + \hat{n}.\nabla_x \Theta^{(2)} - \dot{\kappa} \Theta^{(2)} = -\dot{\kappa} \delta_e \left[ \Theta^{(1)}_0 - \Theta^{(1)} + \hat{n}.V_b - \frac{1}{2} P_2(V_b, \hat{n}) \Pi^{(1)} \right], \quad (1)$$

where $\tau$ is conformal time and $\tau_0$ its value today. $\Theta = \Delta T/T = \Theta^{(1)} + \Theta^{(2)} +$ higher order terms is the fractional perturbation of CMB temperature, superscripts indicate the order of perturbation while subscripts denote the multipole moment. All other perturbations are of first order and we will omit the superscript for them. Vector quantities are in bold face and their magnitudes in normal face with $\hat{\ }$ denoting unit vectors. We have omitted the factor of $1/2$ usually multiplied with the second order term [14] for convenience. $\hat{n}$ is the unit vector along the line of sight, $\dot{\kappa} \equiv d\kappa/d\tau = -\bar{n}_e \sigma_T a$ is the mean differential optical depth due to Compton scattering, $\sigma_T$ is the Thomson scattering cross section and $a$ is the scale factor. We take the electron velocity to be equal to the baryon velocity $V_b$. $P_2$ is the Legendre polynomial of order 2 and $\Pi^{(1)} = \Theta_2^{(1)} + \Theta_{P0}^{(1)} + \Theta_{P2}^{(1)}$ is the polarization term, subscript $P$ denoting the polarization field [21]. We must caution that this partial equation is gauge dependent because $\delta_e$ depends on the gauge. We will be using conformal Newtonian gauge for $\delta_e$. The combinations of terms multiplying $\delta_e$ is gauge invariant. All perturbed quantities are functions of $\tau$ and coordinates on spatial slice $x$. $\Theta$ is in addition a function of line of sight angle $\hat{n}$.

Following standard procedure [14, 22], we take the Fourier transform of Equation (1) and
integrate formally along the line of sight.

\[ \Theta^{(2)}(\mathbf{k}, \hat{n}, \tau_0) = \int_0^{\tau_0} d\tau e^{i k \mu (\tau - \tau_0)} g(\tau) \int \frac{d^3 k'}{(2\pi)^3} \delta_e(\mathbf{k} - \mathbf{k'}, \tau) \]

\[ \times \left[ \Theta_0^{(1)}(\mathbf{k'}, \tau) - \Theta^{(1)}(\mathbf{k'}, \hat{n}, \tau) + \hat{n} \cdot \hat{k'} V_b(\mathbf{k'}, \tau) - \frac{1}{2} P_2(\hat{k'} \cdot \hat{n}) \right]. \]  

(2)

where \( g(\tau) = - \dot{k}(\tau) e^{-\kappa(\tau)} \) is the visibility function and \( \kappa(\tau) \equiv \int_0^{\tau} d\tau' n_e(\tau') \sigma_T a(\tau') \). Also\( V_b(\mathbf{k'}, \tau) = \hat{k'} V_b(\mathbf{k'}, \tau) \). We now take the spherical harmonic transform of Equation 2 to get the multipole moments, \( \Theta^{(2)}_{\ell m} \).

\[ \Theta^{(2)}_{\ell m}(\mathbf{k}, \tau_0) = \int \Theta^{(2)}(\mathbf{k}, \hat{n}, \tau_0) Y^*_{\ell m}(\hat{n}) d\hat{n} \]

This integral can be performed after decomposing \( \Theta^{(1)}(\mathbf{k}, \hat{n}, \tau) = \sum_{\ell''} (-i)^{\ell''} (2\ell'' + 1) P_{\ell''}(\hat{n}) \Theta^{(1)}_{\ell''}(\mathbf{k}, \tau) \) and using relations between exponential, spherical harmonics, spherical Bessel functions and Legendre polynomials [23]. Note that \( \Theta_0^{(1)} \), which is the dominant term in the multipole expansion of \( \Theta^{(1)} \), will cancel out. We will see later that the dipole term partially cancels the effect of \((V_b)\), the Vishniac term. So only \( \ell \geq 2 \) modes in \( \Theta^{(1)} \), which are expected to be small compared to monopole, will contribute to the bispectrum. The result is:

\[ \Theta^{(2)}_{\ell m}(\mathbf{k}, \tau_0) = \int_0^{\tau_0} d\tau g(\tau) \int \frac{d^3 k'}{(2\pi)^3} \delta_e(\mathbf{k} - \mathbf{k'}, \tau) \]

\[ \times \left[ (4\pi)^2 \sum_{\ell' m' m''} i^{\ell'} (-i)^{\ell''} \frac{(2\ell' + 1)(2\ell'' + 1)}{4\pi(2\ell + 1)} C^{(0)\ell' 0}_{\ell' 0} C^{(0)\ell'' m'}_{\ell'' m' m''} Y^*_{\ell' m'}(\hat{k}) Y^*_{\ell'' m''}(\hat{k'}) \Theta^{(1)\ell''}(\mathbf{k'}, \tau) \right] \]

\[ \equiv \int_0^{\tau_0} d\tau g(\tau) \int \frac{d^3 k'}{(2\pi)^3} \delta_e(\mathbf{k} - \mathbf{k'}, \tau) S^{\ell m}(\mathbf{k}, \hat{k'}, \mathbf{k'}, \tau) \]

\( C^{(0)\ell' 0}_{\ell' 0} \) are Clebsch-Gordon coefficients, \( j_\ell \) are spherical Bessel functions. The sums are over all allowed values of \( \ell m \) with the exceptions explicitly specified. The last line defines the function \( S^{\ell m} \). Its arguments are written so that we can keep track of the part, \( \mathbf{k'} \), that statistical variables like temperature anisotropy depend on from the part that deterministic functions depend on, \( \hat{k'} \), separately.
III. BISPECTRUM

We can now use Equation \(3\) to calculate the bispectrum. This is defined as:

\[
B_{m_1m_2m_3}^{\ell_1\ell_2\ell_3} = \langle a_{\ell_1m_1}(x, \tau_0) a_{\ell_2m_2}(x, \tau_0) a_{\ell_3m_3}(x, \tau_0) \rangle + 2 \text{ permutations},
\]

where \(a_{\ell m}(x, \tau_0)\) are the coefficients in the spherical harmonic expansion of the corresponding temperature anisotropy. \(\langle \rangle\) denotes the ensemble average. At second order they are just the Fourier transform of \(\Theta^{(2)}_{\ell m}(k, \tau_0)\) while at first order they can be computed from \(\Theta^{(1)}_{\ell m}(k, \tau_0)\).

\[
a_{\ell m}^{(2)}(x, \tau_0) = \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} \Theta^{(2)}_{\ell m}(k, \tau_0)
\]

\[
a_{\ell m}^{(1)}(x, \tau_0) = 4\pi \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} ( -i)^{\ell_1 + \ell_2} \Theta^{(1)}_{\ell m}(k, \tau_0) Y_{\ell m}^*(\hat k)
\]

We can now calculate the first term of the bispectrum in Equation \(4\)

\[
\langle 1, 1, 2 \rangle \equiv \langle a_{\ell_1m_1}(x, \tau_0) a_{\ell_2m_2}(x, \tau_0) a_{\ell_3m_3}(x, \tau_0) \rangle
\]

\[
= (4\pi)^2 \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} e^{i(k_1 + k_2 + k_3) \cdot x} ( -i)^{\ell_1 + \ell_2} Y_{\ell_1m_1}^*(\hat k_1) Y_{\ell_2m_2}^*(\hat k_2)
\]

\[
\int_0^{\tau_0} d\tau g(\tau) \int \frac{d^3k'}{(2\pi)^3} \langle \delta_\epsilon(k_3 - k', \tau) S_{\ell_3m_3}(k_3, \hat k', k', \tau) \Theta^{(1)}_{\ell_1}(k_1, \tau_0) \Theta^{(1)}_{\ell_2}(k_2, \tau_0) \rangle
\]

We can write each term in the ensemble average as a transfer function times initial gravitational potential perturbation. Thus,

\[
\delta_\epsilon(k_3 - k', \tau) = \Phi_1(k_3 - k') \delta_\epsilon(|k_3 - k'|, \tau)
\]

\[
\Theta^{(1)}_{\ell_1}(k_1, \tau_0) = \Phi_1(k_1) \Theta^{(1)}_{\ell_1}(k_1, \tau_0)
\]

\[
S_{\ell_3m_3}(k_3, \hat k', k', \tau) = \Phi_1(k') S_{\ell_3m_3}(k_3, \hat k', k', \tau)
\]

\[
\langle \Phi_1(k_1) \Phi_1(k_2) \rangle = (2\pi)^3 \delta^3(k_1 + k_2) P(k_1)
\]

We are using same symbols for statistical variables and their deterministic transfer function counterparts, with arguments determining which one we mean. \(\delta^3\) is the three dimensional Dirac delta distribution and \(P(k) = 2\pi^2 / k^3\) is the initial power spectrum. Since we assume the initial perturbation to be Gaussian, the 4-point ensemble average can be decomposed into 2-point ensemble averages.

\[
\langle \Phi_1(k_3 - k') \Phi_1(k') \Phi_1(k_1) \Phi_1(k_2) \rangle = \langle \Phi_1(k_3 - k') \Phi_1(k') \rangle \langle \Phi_1(k_1) \Phi_1(k_2) \rangle
\]
integrals is: summed using, for example, formulas tabulated in [23]. The result after performing these integrals involving spherical harmonics result in Wigner 3jm symbols which can then be used. Using Equations 8 in 7 we can perform all angular integrals and all radial integrals except in the last step we have used one of the Dirac delta distributions to integrate over

$$\left\langle \Phi_i(k_3 - k')\Phi_i(k_2) \right\rangle \right| \Phi_i(k')\Phi_i(k_2)\rangle$$

\[= (2\pi)^6 \delta^3(k_3)P(k')\delta^3(k_1 + k_2)P(k_1) + (2\pi)^6 \delta^3(k_3 + k_1 - k')P(k_1)\delta^3(k_2 + k')P(k_2)\]

\[+ (2\pi)^6 \delta^3(k_3 + k_2 - k')P(k_2)\delta^3(k_1 + k')P(k_1)\]

(6)

First term in Equation 6 contributes only for \(k_3 = 0\), it is a product of monopole and power spectrum and is unobservable. The other two terms are identical with \(k_1, k_2\) terms interchanged. So we need consider only one of these. Denoting the two terms by superscript \((1, 2)\) and \((2, 1)\) we can write the first term of the bispectrum as:

\[
\langle 1, 1, 2 \rangle^{(1, 2)} = \langle 1, 1, 2 \rangle^{(1, 2)} + \langle 1, 1, 2 \rangle^{(2, 1)},
\]

\[
\langle 1, 1, 2 \rangle^{(1, 2)} = (4\pi)^2 \int _0 ^\tau _0 d\tau g(\tau) \int \frac{d^3 k'}{(2\pi)^3} \delta_3(k_3, k', k_2, \tau) \Theta_1^{(1)}(k_1, \tau_0) \Theta_1^{(1)}(k_1, \tau_0) \delta^3(k_1 + k_2 + k_3)
\]

(7)

In the last step we have used one of the Dirac delta distributions to integrate over \(k'\). To proceed further we use the representation of Dirac delta distribution as Fourier transform of unity and the expansion of exponential function in spherical harmonics.

\[
\delta^3(k_1 + k_2 + k_3) = \int \frac{d^3 r}{(2\pi)^3} e^{i(k_1 + k_2 + k_3).r}
\]

\[
e^{i(k.r)} = 4\pi \sum _{\ell, m} \ell \ell (kr) Y_{\ell m}(\hat{k}) Y_{\ell m}(\hat{r})
\]

(8)

Using Equations 8 in [7] we can perform all angular integrals and all radial integrals except two which involve transfer functions of perturbations and the line of sight integral. The integrals involving spherical harmonics result in Wigner 3jm symbols which can then be summed using, for example, formulas tabulated in [23]. The result after performing these integrals is:

\[
\langle 1, 1, 2 \rangle^{(1, 2)} = \sqrt{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)} \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) \int _0 ^\tau _0 d\tau g(\tau) B_{\ell_1}^{\ell_1}(\tau) B_{\ell_2}^{\ell_2}(\tau)
\]

(9)
\[ B_{\delta \Theta}^{\ell_1}(\tau) = \frac{2}{\pi} \int k_1^2 dk_1 P(k_1) \Theta_{\ell_1}^{(1)}(k_1, \tau_0) \delta_e(k_1, \tau) j_{\ell_1}[k_1(\tau_0 - \tau)] \]

\[ B_{\Theta \Theta}^{\ell_2}(\tau) = \frac{2}{\pi} \int k_2^2 dk_2 P(k_2) \Theta_{\ell_2}^{(1)}(k_2, \tau_0) \]

\[ - \sum_{\ell'' \geq 1, \ell_2} i^{\ell'' + \ell_2 + \ell_2'} (-1)^{\ell_2'} (2 \ell'' + 1)(2 \ell_2 + 1) \begin{pmatrix} \ell_2' & \ell_2 & \ell'' \\ 0 & 0 & 0 \end{pmatrix}^2 \Theta_{\ell''}^{(1)}(k_2, \tau) j_{\ell_2'}[k_2(\tau_0 - \tau)] \]

\[ + i V_b(k_2, \tau) j_{\ell_2}[k_2(\tau_0 - \tau)] + \frac{1}{4} \Pi^{(1)}(k_2, \tau) \left\{ 3 j_{\ell_2'}[k_2(\tau_0 - \tau)] + j_{\ell_2}[k_2(\tau_0 - \tau)] \right\} \]

\[ = \frac{2}{\pi} \int k_2^2 dk_2 P(k_2) \Theta_{\ell_2}^{(1)}(k_2, \tau_0) \]

\[ - \sum_{\ell'' \geq 2, \ell_2} i^{\ell'' + \ell_2 + \ell_2'} (-1)^{\ell_2'} (2 \ell'' + 1)(2 \ell_2 + 1) \begin{pmatrix} \ell_2' & \ell_2 & \ell'' \\ 0 & 0 & 0 \end{pmatrix}^2 \Theta_{\ell''}^{(1)}(k_2, \tau) j_{\ell_2'}[k_2(\tau_0 - \tau)] \]

\[ + [\theta_b(k_2, \tau) - \theta_\gamma(k_2, \tau)] \frac{j_{\ell_2'}[k_2(\tau_0 - \tau)]}{k_2} \]

\[ + \frac{1}{4} \Pi^{(1)}(k_2, \tau) \left\{ 3 j_{\ell_2'}[k_2(\tau_0 - \tau)] + j_{\ell_2}[k_2(\tau_0 - \tau)] \right\} \overset{(10)}{=} \]

In the last step we have defined \( i V_b = \theta_b/k \) and \( \theta_\gamma = 3k \Theta_1 \) and evaluated the sum over \( \ell_2' \) explicitly for \( \Theta_1 \). It can be seen from this expression that the effect of Vishniac term \( \theta_b \) is partly cancelled out by \( \theta_\gamma \). In this form the gauge invariance of \( B_{\Theta \Theta}^{\ell_2} \) is also apparent. In arriving at these expressions we have also used the identity \( j_{\ell}(-x) = (-1)^{\ell} j_{\ell}(x) \). The prime on the Bessel functions denotes the derivative with respect to the argument.

We can now write down the final expression for the angular averaged bispectrum defined by:

\[ B_{\delta \Theta \Theta}^{\ell_2 \ell_3} = \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\delta \Theta \Theta}^{\ell_1 \ell_2 \ell_3} \]

\[ = \sqrt{(2 \ell_1 + 1)(2 \ell_2 + 1)(2 \ell_3 + 1) \frac{4}{4\pi}} \int_{\tau_0} d\tau g(\tau) \left[ B_{\delta \Theta}^{\ell_1}(\tau) B_{\delta \Theta}^{\ell_2}(\tau) \right] \]

\[ + B_{\delta \Theta}^{\ell_2}(\tau) B_{\delta \Theta}^{\ell_3}(\tau) + B_{\delta \Theta}^{\ell_3}(\tau) B_{\delta \Theta}^{\ell_2}(\tau) + B_{\delta \Theta}^{\ell_2}(\tau) B_{\delta \Theta}^{\ell_3}(\tau) + B_{\delta \Theta}^{\ell_3}(\tau) B_{\delta \Theta}^{\ell_2}(\tau) + B_{\delta \Theta}^{\ell_2}(\tau) B_{\delta \Theta}^{\ell_3}(\tau) \overset{(11)}{=} \]
IV. PRIMORDIAL NON-GAUSSIANITY OF LOCAL TYPE

We will compare our results with the bispectrum from primordial non-Gaussianity of local type. This is given by [24, 25]:

$$B_{\text{prim}}^{\ell_1 \ell_2 \ell_3} = 2\sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)}{4\pi}} \int_0^{\tau_0} d\tau (\tau_0 - \tau)^2 \left[ \beta_{\ell_1}(\tau) \beta_{\ell_2}(\tau) \alpha_{\ell_3}(\tau) + \beta_{\ell_2}(\tau) \beta_{\ell_3}(\tau) \alpha_{\ell_1}(\tau) + \beta_{\ell_3}(\tau) \beta_{\ell_1}(\tau) \alpha_{\ell_2}(\tau) \right]$$

$$\beta_\ell(\tau) = \frac{2}{\pi} \int k^2 dk P(k) \Theta_\ell(k, \tau_0) j_\ell[k(\tau_0 - \tau)],$$

$$\alpha_\ell(\tau) = \frac{2}{\pi} \int k^2 dk f_{NL} \Theta_\ell(k, \tau_0) j_\ell[k(\tau_0 - \tau)],$$

where $f_{NL}$ is the non-Gaussianity parameter defined by the following form for the primordial potential, $\Phi_i(x) = \Phi_L(x) + f_{NL}(\Phi_L^2(x) - \langle \Phi_L^2(x) \rangle)$ with $\Phi_L(x)$ Gaussian. Note that the expression for $\beta_\ell$ is similar to $B_{\beta \delta}^\ell$ and $B_{\beta \Theta}^\ell$, the difference being the additional modulation by the terms at recombination in the later case. As we will see later, $\alpha_\ell$ is similar in shape to the visibility function $g(\tau)$ but peaks at an earlier time. All plots and results are for $f_{NL} = 1$.

V. NUMERICAL CALCULATION AND RESULTS

We calculate $\delta_\epsilon$ in conformal Newtonian gauge using DRECFAST [17]. All other first order terms are calculated using CMBFAST [22]. In particular $\Theta_\ell^{(1)}(k, \tau)$ is given by the line of sight integral:

$$\Theta_\ell^{(1)}(k, \tau) = e^{\kappa(\tau)} \int_0^\tau d\tau' S^{(1)}(k, \tau') j_\ell[k(\tau - \tau')]$$

Here $S^{(1)}(k, \tau')$ is the usual first order source term. Since we are evaluating the transfer function at $\tau < \tau_0$, we get an extra factor of $e^{\kappa(\tau)}$, otherwise this is same as the standard line of sight formula [22]. $\Theta_\ell^{(1)}$ becomes smaller with increasing $\ell''$ and we cut off the sum in Equation (10) at $\ell'' = 30$. This is accurate for $\tau < 1000\text{Mpc}$ which is sufficient for the present calculation since the visibility $g(\tau)$ is non-negligible only for $240\text{Mpc} \lesssim \tau \lesssim 800\text{Mpc}$ (Figure 2). Wigner 3jm symbols are calculated using the code by Gordon and Schulten [26] which is publicly available at SLATEC common mathematical library [27].

Figure 1 shows a comparison of $\beta_\ell(\tau)$ and $B_{\beta \Theta}^\ell(\tau)$. The modulation by $\delta_\epsilon$ results in shifting the peak to later times. Also visibility $g(\tau)$ can be compared to primordial term
FIG. 1: $\beta_\ell(\tau)$ and $B_{\phi\phi}^\ell(\tau)$ is shown as a function of $\tau$ for several values of $\ell$.

FIG. 2: $(\tau_0 - \tau)^2 \alpha_\ell(\tau)$ for several values of $\ell$ and the visibility function $g(\tau)$ as a function of conformal time $\tau$. $(\tau_0 - \tau)^2 \alpha_\ell(\tau)$ peaks earlier than $g(\tau)$.
FIG. 3: $B_{\Theta}^\ell(\tau)$ is shown for several values of $\ell$. Also shown are contributions from the polarization term $\Pi$, slip term $\theta_b - \theta_g$ and from all the other terms $\sum_{\ell \geq 2} \Theta^{(1)}_\ell$.

FIG. 4: $0.1 \times \ell(\ell + 1)\beta_\ell(\tau_s), 0.01 \times \ell(\ell + 1)B_{\Theta}^\ell(\tau_s), \ell(\ell + 1)B_{\Theta}^\ell(\tau_s)$ and contributions to it from polarization, slip and rest of the terms is shown as a function of multipole moments $\ell$. Some of the functions have been scaled as specified above.
FIG. 5: Absolute value of $B^{\ell_1\ell_2\ell_3}_{\text{prim}}$ labeled “Primordial” and $B^{\ell_1\ell_2\ell_3}_{\text{rec}}$ labeled “Recombination” for $\ell_3 = 10$. Z axis is on linear scale while color plot shows the same on log scale.

$\alpha_\ell$. They are similar in magnitude but have a different shape (Figure 2). $B^x_{\Theta \Theta}$ is however much smaller in magnitude than the other terms, $B_{\delta \Theta}$ and $\beta_\ell$, as can be seen from Figures 3 and 4 at low $\ell$ but become comparable at high $\ell$. This results in a much smaller bispectrum from recombination at low $\ell$ compared to the primordial one. Figures 5, 6, 7 and 8 show the absolute value of the bispectrum from the primordial non-Gaussianity with $f_{NL} = 1$ and that due to inhomogeneous recombination for $\ell_3 = 10, 200, 1000, 2000$ as a function of $\ell_1, \ell_2$. Z-axis is on linear scale while the color map is on log scale. They are almost identical at the peaks but differ considerably away from the peaks which occur when either $\ell_1$ or $\ell_2$ is equal to $\ell_3$ and the other is small, a signature of the local nature of the non-Gaussianity. At low $\ell$ the amplitude $B^{\ell_1\ell_2\ell_3}_{\text{rec}}$ is much smaller than $B^{\ell_1\ell_2\ell_3}_{\text{prim}}$ but they become comparable at high $\ell$. Their signs are however different and this will become apparent when we estimate the confusion in $f_{NL}$ due to $B^{\ell_1\ell_2\ell_3}$.

To estimate the confusion to the estimate of $f_{NL}$ we follow [28] and define the statistic

$$S_{\text{rec}} = \sum_{\ell_1 \leq \ell_2 \leq \ell_3} \frac{B^{\ell_1\ell_2\ell_3}_{\text{rec}}B^{\ell_1\ell_2\ell_3}_{\text{prim}}}{C_{\ell_1}C_{\ell_2}C_{\ell_3}} \approx f_{NL} \sum_{\ell_1 \leq \ell_2 \leq \ell_3} \frac{(B^{\ell_1\ell_2\ell_3}_{\text{prim}})^2}{C_{\ell_1}C_{\ell_2}C_{\ell_3}} \quad (14)$$

The result of solving Equation 14 for $f_{NL}$ is shown in Figure 9 as a function of $\ell_{\text{max}}$, where $\ell_{\text{max}}$ is the maximum value of $\ell$ included in the sum in Equation 14. As expected from the examination of bispectra, $f_{NL}$ is small and positive at low $\ell_{\text{max}}$ but $\sim -1$ at high $\ell_{\text{max}}$. 
VI. CONCLUSIONS

We have calculated the CMB bispectrum due to inhomogeneous recombination. This was expected to be small because the combination of terms multiplying $\delta_e$ is small. However calculations by Novosyadlyj [17] showed that $\delta_e$ could be large and this suggested that the CMB bispectrum could be non-negligible. Although it turns out to be small it is still larger than what one might have expected from making an estimate based on tight coupling or instantaneous recombination approximation [15] and ignoring the perturbations due to inhomogeneous recombination. This is especially evident at high $\ell_{\text{max}}$.
FIG. 8: Absolute value of $B_{\ell_1 \ell_2 \ell_3}^{\text{prim}}$ labeled “Primordial” and $B_{\ell_1 \ell_2 \ell_3}^{\text{recomb}}$ labeled “Recombination” for $\ell_3 = 2000$. Z axis is on linear scale while color plot shows the same on log scale.

FIG. 9: Comparison of primordial bispectrum from local type non-Gaussianity with bispectrum due to inhomogeneous recombination in terms of parameter $f_{NL}$ as a function of $\ell_{\text{max}}$, the maximum $\ell$ mode considered.

from recombination looks remarkably like the local type primordial bispectrum, which is not entirely unexpected since both arise due to product of two first order terms. Since the other second order terms in the Boltzmann equation [14] are expected to be larger than the ones we considered, our calculation motivates a full second order numerical calculation of these terms in order to assess their effect on future experiments such as Planck [18] and the level to which they cause confusion when probing for primordial non-Gaussianity.
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