Four-dimensional understanding of quantum mechanics and Bell violation

Jarek Duda
Jagiellonian University, Golebia 24, 31-007 Krakow, Poland, Email: dudajar@gmail.com

Abstract—While our natural intuition suggests us that we live in 3D space evolving in time, modern physics presents fundamentally different picture: 4D spacetime, Einstein’s block universe, in which we travel in thermodynamically emphasized direction: arrow of time. Arguments for such nonintuitive and nonlocal living in kind of ’4D jello” come among others from: Lagrangian mechanics we use from QFT to GR saying that history between fixed past and future situation is the one optimizing action, special relativity saying that different velocity observers have different own time directions, general relativity deforming shape of the entire spacetime up to switching time and space below the black hole event horizon, or the CPT theorem concluding fundamental symmetry between past and future for example in the Feynman-Stueckelberg interpretation of antiparticles as propagating back in time.

Accepting this nonintuitive living in 4D spacetime: with present moment being in equilibrium between past and future - minimizing tension as action of Lagrangian, leads to crucial surprising differences from intuitive ”evolving 3D” picture - allowing to conclude Bell inequalities, violated by the real physics. Specifically, particle in spacetime becomes own trajectory: 1D submanifold of 4D, making that statistical physics should consider ensembles like Boltzmann distribution among entire paths (like in Ising model), what leads to quantum behavior as we know from Feynman’s Euclidean path integrals or similar Maximal Entropy Random Walk (MERW). It results for example in Anderson localization, or the Born rule with squares - allowing for violation of Bell inequalities. As e.g. for S-matrix, quantum amplitude turns out to describe probability at the end of half-spacetime from a given moment: toward past or future, to randomly get some value of measurement we need to ”draw it” from both time directions - getting the squares of Born rules. Tension from both time directions is also suggested in quantum experiments like Wheeler’s delayed choice experiment, it will be argued that it is also crucial for example in Wheeler’s delayed choice experiment, the CPT theorem concluding fundamental symmetry between past and future for example in the Feynman-Stueckelberg interpretation of antiparticles as propagating back in time.

Keywords: quantum mechanics, nature of time, spacetime, Einstein’s block universe, Born rule, Bell inequalities, Shör algorithm, Euclidean path integrals, statistical physics, Ising model, Maximal Entropy Random Walk (MERW)

I. INTRODUCTION

Starting with special relativity (SR) a century ago, modern physics uses 4D spacetime view of our world - Einstein’s block universe, in which we travel in time direction. Also a century ago quantum mechanics (QM) was born, bringing many nonintuitive consequences, like violation of Bell inequalities. As briefly presented in Fig. 1, 2, 4 and 3, this article argues that these two revolutions of our understanding - violating our natural intuitions, are in fact deeply connected: that living in spacetime has surprising microscopic consequences seen in QM formalism. Let us start with reminding well known arguments and consequences of the spacetime view of modern physics.

Figure 1. Three philosophies for finding probability distribution of a walker, for example electron. Left: ”static 3D” situation. Center: example of ”evolving 3D” philosophy of time, assuming Boltzmann distribution among trajectories from the past to the current moment. As it will be discussed, it leads to probability distribution proportional to the first power of quantum ground state amplitude. Right: ”4D spacetime” situation where particle becomes its full trajectory. Like in Feynman’s Euclidean path integrals (or Ising model), Boltzmann distribution among such full trajectories leads to Born rule: focusing on a fixed-time cut, to randomly get a given position, we need ”to draw it” from both past and future half-trajectories. Hence, probability is proportional to the product of two (identical here) ”evolving 3D” probabilities, getting the Born rule. Bottom: stationary probability distribution predicted by them for [0, 1] infinite potential well, only the ”spacetime” consideration agrees with the QM ground state.

Figure 2. Top: schematic proof of simple Bell-like inequality ([1], [2]): assuming any hidden probability distribution among 2³ = 8 possibilities {0, 1}³ for 3 binary variables ABC, we get inequality written at the top. Surprisingly, QM allows to violate it. Bottom: example of its violation using Born rules like in MERW e.g. in Ising model (Boltzmann vs Feynman path ensemble): as in Fig. 1, amplitude corresponds to probability toward one of two directions (spatial in Ising, temporal in QM), to randomly get some value we need ”to draw it” twice, getting probability as normalized square of sum of amplitudes. To realize it with Ising model, as shown we would need width 3 lattice of spins with interaction preventing ↑↑↑ and ↓↓↓ triples, and measure 2 out of 3 spins in a triple. Details in Section III.


Our natural human intuition has evolved for past-future reason-result chains of consequences: initiated in our Big Bang, leading to us through creation of our planet, evolution, our development. However, since the special relativity we know that "situation in a given moment", more formally called the hypersurface of the present, in fact depends on the observer’s frame of reference: it changes with his velocity accordingly to Lorentz boost, which also modifies the direction of time. The general relativity it even further, modifying the entire spacetime accordingly to local mass/energy concentrations, up to extreme situation below the black hole event horizon, where time and space directions literally switch places, making "situation in a given moment" very far from our biological intuition.

This ambiguity of time direction is also seen in the Lagrangian formalism we successfully use to describe reality in all scales: from quantum field theories to the general relativity. It has multiple equivalent formulations, starting with our intuitive "evolving 3D" picture: Euler-Lagrange equation allows to evolve the situation forward in time, from a situation (as values and the first derivatives) in a given moment. However, mathematically it also allows to use these equations to evolve situation backward in time, as Lagrangian mechanics is usually time (or CPT) symmetric.

Quite different formulation is through action optimization: fixing situation (only values, without the first derivatives) in two moments in time, the history between them is the one optimizing action. While we can translate between such solutions, those originally found with each of them have subtle differences, visualized in Fig. 3, e.g. action optimization has different locality than the one assumed in Bell theorem.

Lagrangian mechanics for field theories can additionally be

\[
\mathcal{L} = \frac{1}{2} \sum_{i=0,1,2,3} \left( \partial_\tau \phi_i \right)^2 + V(\phi)
\]

Surprisingly, energy density (Hamiltonian) is often completely 4D symmetric like here: does not emphasize any time direction in 4D. Choosing a frame of reference, it determines time direction '0', for which we can find the Lagrangian which Legendre transform is the given Hamiltonian...
Two containments with gas
Classical: returning situations
\textbf{Poincaré recurrence theorem}

\textit{~}mean field, „Stosszahlansatz” approximate, describe

\textbf{effective, statistical physics:}
\( p \)- percentage of particles up

\( \left( \frac{n}{p(n)} \right) \approx 2^{n/2(p)} \)

\textbf{entropy } \( h: \)
\textbf{zero vs growth}

\( n \rightarrow \infty \)
\( p=1/2 \) dominates

\( h \): minimizing tension as Hamiltonian. Choosing some

\( n \) of colors of balls before the marked positions - this assumption allows to

\( \phi \rightarrow \infty \) with

\( \pi = \partial \phi, \mathcal{L} = \pi \partial \phi - \mathcal{H} \). This Lagrangian chooses

\( \mathcal{H} \) minimizing tension as Hamiltonian. Choosing some
time direction and situation in its hypersurface of the present,
we can transform Hamiltonian to Lagrangian, find Euler-Lagrange equation for it, and use it to evolve situation from
this arbitrary hyperplane in a chosen direction of time.

\textbf{Fundamental similarity of past and future is also the base of quantum field theories requiring CPT symmetry due to the Schwinger’s CPT theorem [3]. Feynman diagrams represent antiparticles as propagating backward in time (Feynman-Süeckelberg interpretation) [4].}

In contrast, against e.g. SR and fundamental CPT symmetry, 2nd law of thermodynamics emphasizes some “arrow of time”. However, thermodynamics is not

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Scattering matrix: from minus and plus infinity, what is analogous to seeing the present moment as a result of two propagators: by Aharonov [11] in so-called two-state vector formalism: by Aharonov [11] in so-called two-state vector formalism.

John Cramer’s transactional interpretation of QM [10] is based on this inherent time symmetry of quantum unitary evolution, suggesting propagation of information in both time directions.

Time symmetric formulation of QM is also advocated by Aharonov [11] in so-called two-state vector formalism: seeing the present moment as a result of two propagators: from minus and plus infinity, what is analogous to scattering matrix: \( \langle \psi_f | U(t_f, t_i) | \psi_i \rangle \), and the view presented here. Article “Asking photons where they have been” [9] presents its very nice experimental confirmation: vibrating mirrors allows to conclude from the final light beam which mirrors have been visited. However, in properly chosen setting they obtain also signal from mirrors which naively should not be visited, unless we focus exactly on mirrors visited by photons propagating in both time directions.

In this article we will focus on statistical consequences of given moment (hypersurface of the present) being in equilibrium between past and future in spacetime as kind of “4D jello”. In this picture particles are no longer just “moving points”, but rather their trajectories: one-dimensional submanifolds of the spacetime. From statistical physics perspective, it brings the question of what objects should we use in the considered ensembles, e.g. while assuming Boltzmann distribution like presented in Fig. 1 - only the last one: considering Boltzmann distribution among full trajectories, like in Feynman’s Euclidean path integrals or related Maximal Entropy Random Walk (MERW) [13, 14], has thermodynamical agreement with predictions of QM: leading to stationary probability distribution of the quantum ground state, with crucial differences like avoiding the boundaries for \([0, 1]\) infinite potential well. Boltzmann path ensemble is also at heart of Ising model - MERW can be also used as random walk along Ising sequence.

In the next Section we will focus on MERW philosophy, briefly presented in Fig. 8 [12] as a reparation of standard diffusion models - which for example wrongly predict that semiconductor should have nearly uniform probability distribution of electrons, making it a conductor against experiment (tiny electric field would cause electron flow). This fundamental disagreement was repaired by QM showing strong localization property for these electrons (e.g. Anderson), preventing the conductance. This crucial problem of stochastic modelling has caused that it is currently seen as a completely different realm than QM. However, electrons are indivisible quants of electric charge, what should prevent them from being objectively blurred. Heisenberg uncertainty principle limits abilities of measurement, which are sophisticated destructive processes idealized for example by the Stern-Gerlach experiment - having also reversible interpretations like in Figure 9. In contrast, this principle is commonly seen as limitation for example for objective position of electron in atom. However, modern techniques like field-emission electron microscopy already allow to get resolution below the size of atomic orbital: strip electrons from single atom, use EM field shaped to act as a lens for magnification, and measure positions of theses single electron in detector matrix [16]. This way they literally obtained photography of orbitals: densities by averaging over positions of single electrons. Anyway, using Heisenberg principle as an excuse for ignoring questions about objective dynamics is no longer valid when increasing the scale, for example while asking question about local currents in a lattice: what is the...
Figure 10. While MERW evolution leads to stationary probability distribution exactly as in the quantum ground state, the excited quantum states are also seen in this evolution. Specifically, the coefficient corresponding to $M\psi_i = \lambda_i \psi_i$ eigenfunction/eigenvector, drops $\approx (\lambda_i/\lambda_0)^t$ times during $t$ steps, where $\lambda_0$ is the dominant (largest) eigenvalue. As energy turns out to correspond to $-\lambda$, we get exponential weakening of contributions of excited states - in contrast to standard QM they are not stable, however, excited states in nature are also unstable. The top row of above diagram shows example of such evolution: as initial contribution of the ground state is very small, the evolution first gets close to the first excited state (subdiffusion), then it kind of tunnels to the ground state (superdiffusion). This simple model neglects conservation laws e.g. of energy, which might prevent the walker from going to the ground state - its perturbed averaged trajectory should instead smoothen toward some close excited probability distribution. Also for multiple repelling walkers (like electrons), they should choose densities of successive eigenstates here (like in Pauli exclusion principle).

probability distribution for electron jumping to neighboring parts of this lattice - getting a stochastic model for its dynamics (conductance). There are also examples of larger objects for which we should expect objective positions and so stochastic models, but in some situations their quantum description works surprisingly good, for example the nuclear shell model for baryons - MERW shows that such success of QM formalism does not disqualify stochastic description, in contrast it is also supported from this perspective, there is universality of quantum predictions.

MERW allows to understand and repair this problem for example of seeing electron conductance as some statistical flow of charges - also where standard diffusion models had to give up, like defected lattice of semiconductors. The reason for this disagreement of standard stochastic models is that what looked as a natural choice for transition probabilities (like GRW) or stochastic propagator, often turns out only approximation of what is expected by statistical physics: entropy maximization. By repairing this approximation, MERW turns as close QM as we could expect from a diffusion model, like recreating equilibrium probability distribution exactly as the quantum ground state density. Also probability densities of excited states appear there as preferred, but can diffuse further (unless adding some constraint) like in Fig. [10]

Hence MERW can be seen as quantum correction to diffusion models. However, this is still only diffusion, not a complete QM - it ignores interference, which requires e.g. some internal clock (de Broglie’s $E = mc^2 = \hbar \omega$, zittebewegung) of particle. Beside providing clear intuitions for looking problematic properties of QM (like squares in formalism leading to violation of Bell’s inequalities), like in Fig. [11] such quantum corrections to diffusion can be also useful especially as practical approximations of extremely demanding complete quantum modeling of conductance: in semiconductor, microscopic scale, or single molecule electronics.

In the third Section there will be explained MERW’s analogue of measurement, especially for violation of Bell’s inequalities. Fourth Section focuses on quantum computation - we will argue that Shor’s algorithm also exploits the fact that we live in a spacetime, suggest a general approach for designing quantum algorithms. There is also discussed hypothetical approach for time-loop computers. Finally the last Section briefly discusses a possible ways to expand this simple but surprisingly successful effective model: just Boltzmann distribution among possible paths, into a more complete picture of physics, effectively described by quantum field theories - in perturbative approximation using ensemble of scenarios with varying number of particle: Feynman diagrams.
II. MAXIMAL ENTROPY RANDOM WALK
AS QUANTUM CORRECTIONS TO DIFFUSION

Let us start with the common problem of choosing a random walk (as Markov process) on a graph defining the space of interest - which later will be chosen for example as a lattice, where we can introduce inhomogeneity (defects) like in Fig. 8 or perform infinitesimal limit to get diffusion as continuous random walk. This section contains a condensed informal introduction, more complete description can be found as PhD Thesis of the author [14], here is light introduction.

For simplicity assume here that the graph is indirected and defined by its (symmetric) adjacency matrix: \( M_{ij} = M_{ji} \) = 1 if there is edge between vertex \( i \) and \( j \), 0 otherwise. From the perspective of physics, this adjacency matrix can be seen as simplified (zero potential) Bose-Hubbard Hamiltonian for a particle travelling between a set of sites connected as in this graph, jumping for \( i \) to \( j \) is first annihilation \( a_i \) then creation \( a_j^+ \):

\[
\mathcal{H} = - \sum_{\text{edge} \ (ij)} a_j^+ a_i \equiv -M
\]

### A. Standard random walk (GRW) and its suboptimality

We would like to choose a stochastic matrix \( S \) for this graph: \( S_{ij} = \Pr(\gamma_{t+1} = j | \gamma_t = i) \), which is nonzero only for graph edges, outgoing probabilities for each vertex have to sum to 1:

\[
0 \leq S_{ij} \leq M_{ij} \leq 1, \quad \forall i \sum_j S_{ij} = 1. \tag{2}
\]

The standard way to choose random walk, referred as Generic Random Walk (GRW), is assigning equal probability to each outgoing edge, what for indirected graph leads to stationary probability distribution \( (\sum_i \rho_i S_{ij} = \rho_j) \) with probability of vertex being proportional to its degree \( d \):

\[
S_{ij}^G = \frac{M_{ij}}{d_i} \quad \rho_i^G = \frac{d_i}{\sum_j d_j} \quad \text{for} \quad d_i = \sum_j M_{ij} \tag{3}
\]

Before commenting the above choice, let us remind the Principle of maximum entropy of Jaynes [19]. Imagine a length \( n \) sequence of ‘0’ and ‘1’, the number of such sequences is \( 2^n \). Now focus on subspace of possibilities with density \( p \in [0, 1] \) of value ‘1’: with approximately \( pn \) of ‘1’. Using Stirling formula \( (k! \approx \sqrt{2\pi k} (k/e)^k) \) we can find asymptotic number of such combinations, plotted in Fig. 12:

\[
\left( \frac{n}{pn} \right)^{\approx 2^n h(p)} \quad \text{for} \quad h(p) = -p \log(p) - (1-p) \log(1-p)
\]

being the Shannon entropy (\( \log \equiv \log_2 \)), which has single maximum \( h(1/2) = 1 \). Hence splitting the set of all 0/1 sequence into disjoint subsets with \( p \) density, asymptotically \( (n \to \infty) \) the \( p = 1/2 \) uniform probability case will combinatorially completely dominate all the other subsets.

Generally, like in the famous Boltzmann’s formula, entropy is just (normalized) logarithm of the number of possibilities. Hence focusing on subset described by parameters (like density), maximizing entropy means focusing asymptotically on nearly all possibilities - contribution of subsets corresponding to suboptimal parameters asymptotically vanishes in exponential way with the size of the system. It can be summarized in the Principle of maximum entropy: probability distribution which best represents the current knowledge is the one with largest entropy. Without additional knowledge, entropy is maximized for uniform probability distribution on a given set. Assigning energy to objects/possibilities and fixing total energy, we get Boltzmann distribution instead. These two distributions are the base of statistical physics.

Returning to random walk on a graph, GRW clearly maximizes entropy for every vertex - is kind of local maximization. The question is if it maximizes average entropy per step: averaged over probability distribution of being in a given vertex. This measure is also called entropy rate:

\[
H(S) = \sum_i \rho_i \sum_j S_{ij} \log(1/S_{ij}) \tag{4}
\]
It turns out to be equal to normalized entropy in the space of sequences generated by such Markov process \( S \):

\[
H(S) = \lim_{n \to \infty} -\frac{1}{n} \sum_{\gamma_0 \ldots \gamma_n} \Pr(\gamma) \log(1/\Pr(\gamma))
\]

where

\[
\Pr(\gamma_0 \ldots \gamma_n) = \rho_0 S_{\gamma_0 \gamma_1} \ldots S_{\gamma_n-1 \gamma_n}
\]

is probability of obtaining sequence \( \gamma \).

By Maximal Entropy Random Walk (MERW) we will refer to the choice of \( S \) matrix which maximizes \( H(S) \) over all random walks on a given graph: fulfilling conditions (2). As this maximization involves corresponding stationary probability distribution: dominant eigenvector of \( S \) matrix to eigenvalue \( 1 \): \( \rho S = \rho \), for maximization it is more convenient to use formula (5), which reaches maximum for uniform probability distribution among (infinite) paths generated by a given Markov process.

In many cases GRW already maximizes \( H(S) \) making it equal with MERW, for example for regular graphs (all vertices have the same degree), like regular lattice and so standard diffusion in empty homogeneous space obtained as continuous limit of the lattice. The simplest example of nonoptimality of GRW is Fibonacci coding case, presented in Fig. 15. More physical examples are defected lattices, for example representing a semiconductor, or its continuous limit: diffusion in inhomogeneous space. In contrast to standard diffusion which leads to nearly uniform stationary probability distribution, MERW leads to very strong localization properties - exactly as the quantum ground state (for e.g. Bose-Hubbard or Schrödinger Hamiltonian), what we would from QM consideration and for example prevents semiconductor from being a conductor by poisoning electrons in entopic wells as mentioned in Fig. 8.

The GRW nearly uniform stationary probability distribution can be seen as maximizing entropy in spatial "static 3D" picture like in Fig. 11 for a fixed time cut of spacetime, leading e.g. to uniform density in \([0,1]\) potential well there. In contrast, MERW philosophy maximizes entropy in 4D spacetime picture: where particles become their trajectories, leading to \( \rho(x) \propto \sin^2(\pi x) \) there, exactly as predicted also by QM.

**B. MERW formulas and Born rule**

While GRW assumes uniform probability distribution among outgoing edges: paths of length one, let as analogously define GRW\(_{t}\) to assume uniform probability distribution among length \( t \) paths (GRW \( \equiv \text{GRW}_1 \)). The number of length \( t \) paths from vertex \( i \) to \( k \) for which the first step is to \( j \) is \( M_{ij}(M^{t-1})_{jk} \), hence the stochastic matrix of GRW\(_{t}\) is \( S_{ij} \propto M_{ij} \sum_k (M^{t-1})_{jk} \).

MERW assumes uniform probability distribution among possible infinite paths, what allows to see it as \( t \to \infty \) limit of GRW\(_{t}\) like in Fig. 9. To derive formula for \( t \to \infty \) limit, for simplicity let us assume that our indirected graph \( M = M^T \) is connected and aperiodic, where the Frobenius-Perron theorem says that \( M \) has non-degenerated (single) dominant eigenvalue \( \lambda > 0 \) and its corresponding eigenvector (left and right are equal for symmetric \( M \)) has real nonnegative coordinates:

\[
\psi M = \lambda \psi \quad \text{maximizing } \lambda \in \mathbb{R}^+, \quad \psi \in \mathbb{R}^+
\]

Non-degenerated dominant eigenvalue makes that in the \( t \to \infty \) limit we have \( M^t \to \lambda^t \psi (\psi^T \text{in bra-ket formalism), getting MERW } S_{ij} \propto M_{ij} \psi_j \text{ as limit of GRW}_t \).

For normalization, as \( \sum_j M_{ij} \psi_j = \lambda \psi_i \), we get the final formula for MERW stochastic matrix:

\[
S_{ij} = \frac{M_{ij} \psi_j}{\lambda} \quad \rho_i \propto \psi_i^2
\]

where the above formula for stationary probability distribution, \( \rho_i = \psi_i^2 / \sum_j \psi_j^2 \) after probabilistic normalization, can be easily verified:

\[
\sum_i \psi_i^2 \cdot \frac{M_{ij} \psi_j}{\lambda} = \sum_i \psi_i M_{ij} \frac{\psi_j}{\lambda} = \lambda \psi_j \frac{\psi_j}{\lambda} = \psi_j^2
\]

The above derivation of MERW stochastic formula has used ensemble of infinite half-paths going forward in time, with (normalized) \( \psi \) describing probability distribution at the beginning of such ensemble. To analogously derive the formula for its stationary probability distribution, we can fix a position \( i \) and use ensemble of infinite half-paths toward both past in future. Using bra-ket formalism, both derivation can be informally written as:

\[
|i\rangle\langle i|, |j\rangle (M^t-1) \to \lambda^t \psi_j \text{ imple } S_{ij} \propto M_{ij} \psi_j
\]

\[
M^t |i\rangle\langle i| \to \lambda^t \psi_i^2 \text{ imple } \rho_i \propto \psi_i^2
\]

This way we get a natural intuition for \( \rho_i \propto \psi_i^2 \) containing Born rule as in Fig. 4 the quantum amplitude \( \psi \) describes situation at the beginning of past or future half-spacetime (usually equal). If we want to measure a position or some observable in a given moment, we need to "draw" this random value from both past and future directions, getting final probability being product of both original (identical) probabilities, getting squares known from the quantum formalism, which as we know for example lead to violation of Bell inequalities wrongly expected by our natural "evolving 3D" intuition.
C. Boltzmann path distribution

Observe that taking a power of MERW stochastic matrix \( (S^t)^{ij} \), or calculating probability distribution of a path \( \gamma \), the intermediate \( \psi_j/\psi_i \) terms cancel, getting:

\[
(S^t)^{ij} = \frac{(M^t)^{ij} \psi_j}{\lambda^t} \psi_i
\]

Pr(\( \gamma_0 \ldots \gamma_t \)) = \frac{\psi_{\gamma_0} M_{\gamma_0 \gamma_1} \cdots M_{\gamma_{t-1} \gamma_t} \psi_{\gamma_t}}{\lambda^t}

(8)

This way we get another "local" equivalent condition for MERW (written in Fig. 8), for all two vertices, each path of given length between them is equally probable.

In statistical physics we emphasize some possibilities by introducing energy, going from uniform to Boltzmann distribution \( p_\gamma \propto e^{-\beta E_\gamma} \), which is obtained from the Jaynes Principle of Maximum entropy by maximizing entropy under constraint of fixed average energy.

To take Boltzmann distribution to the MERW philosophy we need first to define energy of paths. A simple way is through choosing energy (potential) corresponding to each edge: \( V_{ij} \), then define energy of a path as sum over all its edges:

energy of path \( (\gamma_0 \ldots \gamma_t) \) is \( V_{\gamma_0 \gamma_t} + \ldots + V_{\gamma_{t-1} \gamma_t} \)

In equation (9) we can change from uniform to such Boltzmann distribution among paths by just using more general \( M \) matrix: still real nonnegative, but carrying weights corresponding to related potential:

\[
M_{ij} = A_{ij} e^{-\beta V_{ij}} \quad \text{for} \quad A_{ij} \in \{0, 1\}
\]

(9)

being the adjacency matrix.

To use vertex potential \( V_i \) instead, we can take e.g. \( V_{ij} = \frac{1}{2} (V_i + V_j) \).

D. Lattice and continuous limit to Schrödinger equation

As discussed, adjacency matrix can be seen as minus identity matrix: 

\[
M = A - \delta
\]

where \( \delta \) is a diagonal matrix with \( \delta_{ii} = 1 \) and \( \delta_{ij} = 0 \) for \( i \neq j \). Lattices are basic graphs used in physics, continuous situation can be realized as infinitesimal limit of a lattice. Let us start with 1D lattice from Fig. 13: MERW with potential barriers realized using self-loops (1 at diagonal of adjacency matrix, possibility to remain in the vertex), here removed in defects: \( V_x = 0 \) if vertex \( x \) contains self-loop, 1 otherwise.

While for GRW stationary probability distribution is proportional to degree of a vertex, getting very weak localization, for MERW we first need to find the dominant eigenvector of adjacency matrix \( (\lambda \psi = M \psi) \):

\[
(\lambda \psi)_x = \psi_{x-1} + (1 - V_x)\psi_x + \psi_{x+1} / -3\psi_x \quad \text{if} \quad \psi_{x-1} - 2\psi_x + \psi_{x+1} + V_x\psi_x = 0
\]

where maximization of \( \lambda \) has become minimization of energy \( E := 3 - \lambda \) due to change of sign.

The \( \psi_{x-1} - 2\psi_x + \psi_{x+1} \) term is discrete Laplacian, making formula (10) discrete analogue of stationary Schrödinger equation. Hence MERW predicts going to exactly the same stationary probability distribution \( p_x \propto \psi_x^2 \) as predicted by quantum mechanics here - with very strong localization properties, for example preventing semiconductor from being a conductor.

To get the standard 1D continuous Schrödinger equation, let us take a regular lattice with \( \epsilon \) time step and \( \delta \) lattice constant. To consider real potential \( V \), we can assume Boltzmann distribution among paths as in (9). The \( \lambda \psi_x = (M\epsilon \psi)_x \) eigenequation becomes for example:

\[
\lambda \psi_x = e^{-\beta \epsilon V_{x-1}} \psi_{x-1} + e^{-\beta \epsilon V_x} \psi_x + e^{-\beta \epsilon V_{x+1}} \psi_{x+1}
\]

As we are interested in \( \epsilon, \delta \to 0 \) limit, let us use approximations: \( e^{-\beta \epsilon V} \approx 1 - \beta \epsilon V \) and \( V_{x-1} \approx V_x \approx V_{x+1} \). After simple transformations and multiplying by \(-1\) as previously (to change from maximization of \( \lambda \) to minimization of \( E \)), we get:

\[
3 - \lambda \epsilon \psi_x \approx - \frac{1}{\beta \epsilon} \frac{\psi_{x-1} + \psi_x + \psi_{x+1}}{\epsilon} + V_x \psi_x
\]

Defining energy and choosing relation between time and space steps: with square required for getting from discrete to continuous random walk (as standard deviation grows with square root of time):

\[
E_\epsilon = \frac{3 - \lambda \epsilon}{3\beta \epsilon} \quad \epsilon = \frac{\delta^2}{3\alpha}
\]

for some \( \alpha \) parameter, in the \( \delta \to 0 \) limit we get standard 1D stationary Schrödinger equation:

\[
E\Psi = \left(-\frac{\alpha}{\beta} \Delta + V\right) \Psi = \left(-\frac{h^2}{2m} \Delta + V\right) \Psi
\]

(11)

assuming \( \alpha/\beta = h^2/2m \). Considering time dependant situation and comparing the continuity equations [14], suggests to choose: \( \alpha = h/2m, \beta = 1/h \). For such generalized MERW there are also appearing other QM properties like Heisenberg principle.

The nature and values of such constants describing "fundamental noise" are crucial but not well understood. A natural source might be intrinsic periodic process of particles, so called zitterbewegung or de Broglie’s clock \( \epsilon = \hbar \omega \), which has been directly observed for electrons [20], [21]. The MERW behavior also sees excited states, as presented in Fig. 10 suggesting that random thermodynamical distortion of a classical trajectory should make it smoothen toward probability cloud of close (overlapping) potentially excited quantum state.

What is surprising in the above lattice derivations is that Laplacian, which in standard QM is related with momentum, here appears only from spatial structure of the lattice as these corrected diffusion models do not consider velocity of particle. To add kinetic energy into considerations, we could perform MERW in phase space: (space, velocity), like in the Langevin equation, however, it becomes much more complicated.

The fact that Boltzmann distribution among paths leads to quantum thermodynamical predictions is not surprising as this MERW philosophy seems close to Feynman’s Euclidean path integrals (EPI), however, there are some essential differences:

- Philosophy: EPI starts with assuming the axioms of QM, then performs "Wick rotation" to imaginary time
covering 4 out of 8 possibilities - as shown in Fig. 2. Right: ergodic picture usually assuming a complex deterministic evolution (neglecting e.g. thermodynamical fluctuations), density is obtained by averaging over e.g. a single trajectory. Center: the discussed MERW-like philosophy based the principle of maximum entropy required by thermodynamical model, using Boltzmann distribution among possible trajectories, leading to quantum-like densities. Local transition probabilities (stochastic propagator) are calculated from ensemble of trajectories. The object does not directly use these probabilities (nonlocal: depending on the entire system), just performs some complex dynamics - only we use probabilities of this thermodynamical model to predict the safest evolution of our knowledge.

- both having questionable clarity. In contrast, MERW just repairs diffusion: accordingly to the principle of maximum entropy, repairing known disagreements with reality of standard diffusion.
  - Formula: standard EPI propagator lacks stochastic normalizetion, especially the crucial $\psi_j/\psi_t$ term, which modifies the behavior with position in a very nonlocal way (dependent on the entire system).
  - Statistics: standard EPI assumes Boltzmann distribution among paths in a given time period, like in the GRW philosophy - conditioning the behavior on this arbitrarily chosen time period. In contrast, MERW uses ensemble of paths infinite in both time directions.
  - Complexity: EPI starts with the continuous case, which path integration is mathematically problematic. In contrast, MERW philosophy starts with well understood discrete case.

Mathematically closer to MERW is Zambrini’s Euclidean quantum mechanics [22], but like for Nelson’s stochastic quantum mechanics [23], the motivation is fitting the expected behavior of QM, instead of MERW’s just concluding from required fundamental mathematical principle: of entropy maximization.

III. MEASUREMENT AND BELL INEQUALITIES

While our intuition of living in 3D space evolving in time requires satisfaction of Bell-like inequalities, they can be violated in real world or QM - let us understand it from perspective of living in 4D spacetime instead: where the basic objects are trajectories and we should consider their perspective of living in 4D spacetime instead: where the choice of maximum entropy required by thermodynamical model, using Boltzmann distribution among possible trajectories, leading to quantum-like densities. Local transition probabilities (stochastic propagator) are calculated from ensemble of trajectories. The object does not directly use these probabilities (nonlocal: depending on the entire system), just performs some complex dynamics - only we use probabilities of this thermodynamical model to predict the safest evolution of our knowledge.

- Kolmogorov 3rd axiom: probability of alternative of disjoint events is sum of individual probabilities: $P_{AB} = P_{A0} + P_{AB1}$, leading to the inequality (12).
- Born rule: probability of alternative of disjoint events is proportional to square of sum of their amplitudes: $P_{AB} \propto (\psi_{AB0} + \psi_{AB1})^2$.

As in Fig. 2 this Born rule assumption allows to violate inequality (12), for example taking $\psi_{000} = \psi_{111} = 0$, $\psi_{001} = \psi_{010} = \psi_{011} = \psi_{100} = \psi_{101} = \psi_{110} > 0$ we get $Pr(A = B) = Pr(A = C) = Pr(B = C) = 1/5$ and so violation of the inequality to $3/5 < 1$.

Assuming as in Euclidean path integrals or MERW: uniform probability distribution among paths, we got the squares like in Born rules by multiplying amplitudes from both time directions. To formalize it we need to define MERW measurement, which in QM is destructive process: transforms usually continuous initial state into a discrete set of possibilities: eigenvectors of measurement operator.

To understand destructiveness of measurements, adapt it to a simple model like MERW, let us look at Stern-Gerlach experiment which is used as idealization of measurement: its applies strong magnetic field to transform initially continuous space of spin directions into one of two possibilities: parallel or anti-parallel alignment (only these two do not have Larmor precession hence minimize kinetic energy), which can be later separated using field gradient.

We can imagine that after aligning in strong magnetic field, spin can no longer flip during this flight in strong magnetic field - it leads to idealized condition which can be adapted for MERW and turns out sufficient: during measurement its outcome cannot change.

A. Realization of Bell violation example

Let us take “during measurement its outcome cannot change” rule to MERW like in Figure. 17. In all but time 0-1 step there are allowed steps accordingly to the assumed graph: blue edges in cube on the left (can be also used simpler, e.g. width 3 Ising lattice with interaction forbidding $\downarrow\downarrow\downarrow$ and $\uparrow\uparrow\uparrow$, we measure 2 out of 3 spins). In the remaining time 0-1 step we measure $AB$: first two out of three variables. This step is governed by the measurement rule: it cannot change the measured coordinates. However, it can change the third (unmeasured) coordinate - which is volatile in this measurement, otherwise inequality (12) would be satisfied.

For this spacetime diagram presented in the right part of Fig. 17 let us assume MERW rule that all possible paths (using blue edges) are equally probable - asking what
percentage of them goes through the four boxes corresponding to measurement outcomes, we will correspondingly get 1/10, 4/10, 4/10, 1/10 probabilities, which violate the inequality.

Specifically, let us calculate the number of past paths from time $t = -l$ to $t = 0$ in Fig. 17. For the 000 and 111 final vertices there is only a single such path. For the remaining 6 vertices there are 2 possibilities for each time step, hence there are $2^l$ paths ending in each of these vertices. Analogously for future paths: from time $t = 1$ to $t = l + 1$, their number is 1 for the 000 and 111, and $2^l$ for the remaining 6 vertices.

Now let us count bidirectional paths: from $t = -l$ to $t = l + 1$ going through each of 4 measurement boxes. For the top and bottom box, corresponding to measuring 00 or 11 of $AB$ coordinates, the number of paths is $(2^l + 1)^2$. For the remaining two boxes, the number of paths is $(2^l + 2^l)^2$. Asymptotically ($l \to \infty$) we get:

$$\Pr(A = 0, B = 0) = \lim_{l \to \infty} \frac{(2^l + 1)^2}{2(2^l + 1)^2 + 2(2^l + 2^l)^2} = \frac{1}{10}$$

and analogously for the remaining three measurement outcomes, getting probability of going through each of the four boxes being correspondingly: 1/10, 4/10, 4/10, 1/10.

Hence, in this scenario $\Pr(A = B) = 2/10$ when measuring the first two coordinates. Analogously measuring the remaining two pairs (different grouping into pairs of vertices), we get violation of the inequality:

$$\Pr(A = B) + \Pr(A = C) + \Pr(B = C) = 6/10 < 1$$

B. Born rules in general MERW measurement

To generalize this Born rule to adjacency matrix $M$, assume that measurement chooses between disjoint subsets of possibilities (4 red squares in Fig. 17), splitting the set of vertices $X$ into disjoint subsets (components) distinguished by the measurement:

$$X = \bigcup_i X_i \quad \text{and we need} \quad P(X_i) \propto \left( \sum_{j \in X_i} \psi_j \right)^2$$

Using the rule that two neighboring steps are in the same component during measurement as before:

**Definition:** MERW measurement in time $t = 0$ for split $X = \bigcup_i X_i$ modifies the original uniform ensemble among paths $(\gamma_t)_{t \in \mathbb{Z}}$ to all paths satisfying: $\exists_i \gamma_0, \gamma_1 \in X_i$ (beside usual $M_{\gamma_t \gamma_{t+1}} = 1$ for $t \neq 0$).

The number of paths from $t = -l$ to $t = l + 1$ going through $X_i$ in time 0 and 1 is:

$$\sum_{ab} \sum_{jk \in X_i} (M^t)_{aj}(M^t)_{kb} \rightarrow \lambda^{2l} \sum_{jk \in X_i} \psi_j \psi_k = \lambda^{2l} \left( \sum_{j \in X_j} \psi_j \right)^2$$

where $\psi, \lambda$ are dominant eigenvector/eigenvalue ($M \psi = \lambda \psi$, assume it is unique), asymptotically ($l \to \infty$) getting general Born rule: $\Pr(X_i) \propto \left( \sum_{j \in X_j} \psi_j \right)^2$.

IV. FOUR DIMENSIONAL UNDERSTANDING OF QUANTUM COMPUTATION

While violation of Bell inequalities is rather only an interesting fact regarding consequences of QM, much deeper and applied exploitation of quantum strangeness is proposed for quantum computers, especially the Shor’s algorithm shifting the factorization problem from exponential to polynomial complexity. This believed exponential classical cost is crucial for safeness of widely used asymmetric cryptography like RSA, which could be endangered by quantum computers if overcoming technical (or deeper?) difficulties of their implementation.

Such possibility of shifting from classical exponential to quantum polynomial complexity suggests some computational superiority coming with the unintuitive properties of quantum mechanics - understanding of which might help us designing new quantum algorithms, especially to understand the question of existence of polynomial quantum algorithms for NP-complete problems, for which positive answer could, among others, endanger all kind of used cryptography.

One characteristic property of quantum algorithms is the requirement to use only reversible operations (gates)
Figure 18. **Top:** Schematic diagram of quantum subroutine of Shor’s algorithm \[24\] for finding prime factors of natural number \(N\). For a random natural number \(y < N\), it searches for period \(r\) of \(f(a) = y^a \text{ mod } N\). This period can be concluded from measurement of value \(c\) after quantum Fourier Transform (QFT\(^{-1}\)) and with some large probability \(O(1)\) allows to find a nontrivial factor \(r\) of \(N\). The Hadamard gates produce state being superposition of all possible values of \(a\). Then classical function \(f(a)\) is applied, getting superposition of \(|a⟩|f(a)⟩\). Due to necessary reversibility of applied operations, this calculation of \(f(a)\) requires use of auxiliary qubits, initially prepared as \(|0⟩\). Now measuring the value of \(f(a)\) returns some random value \(m\), and restricts the original superposition to only \(a\) fulfilling \(f(a) = m\). Mathematics ensures that \(\{a: f(a) = m\}\) set has to be periodic here \(|g^r (y) \equiv 1 \text{ mod } N\), this period \(r\) is concluded from the value of Fourier Transform (QFT\(^{-1}\)). Seeing the above process as a situation in 4D spacetime, qubits become trajectories, state preparation mounts their values (chosen) in the past (beginning), measurement mounts their values (random) in the future (end). Superiority of this quantum subroutine comes from future-past propagation of information (tension) by restricting the original ensemble in the first measurement. **Bottom:** NP complete problem (3SAT) approach with Ising model \[25\] - restricting Boltzmann path ensemble to corresponding to solutions of given instance of problem. It brings fundamental question if Boltzmann (or Feynman in quantum computers) ensemble of e.g. \(2^{1000}\) paths is more than an idealization?

as quantum evolution is unitary. Observe that \((x, y, z) \rightarrow (x, y, z \text{ XOR } g(x, y))\) is its own reverse and allows to realize any boolean function like AND, OR and XOR if using prepared auxiliary bit \(z = 0\). While we could classically reverse such gates and their sequences realizing some function, the requirement of a large number of prepared auxiliary bits prevents such use of reversible operations to actually reverse a difficult function, like the discrete logarithm - it would require fixing on both ends of the process: of final values of the function and initial values of the auxiliary bits.

Hence the question is if we could influence some complex (reversible/time symmetric) computational process on its both ends (initialization and output/measurement) in order to obtain a somehow more superior computation capabilities, e.g. shifting some problem from exponential to polynomial complexity? We could fix a system of rubber bands on its both ends, like for anyons forming braids in Kitaev’s hypothetical topological quantum computers \[26\]. However, it seems technically difficult to realize logic gates on such rubber bands in 3D. Even if we could realize basic gates for them, minimizing the tension of such rubber band system might be physically very difficult to stabilize for solving our computational problem, especially that for hard problems the number of local energy minima grows exponentially with problem size \[27\]. Another way to fix values in both boundaries is replacing Feynman with Boltzmann path ensemble: going to Ising model as in bottom of Fig. [18] - bringing question if such ensemble (also Feynman) of e.g. \(2^{1000}\) paths is more than an idealization?

The situation seems more optimistic if this “rubber band setting” is in 4D spacetime: is a system of trajectories of some qubit carriers. One reason is that realizing logic gates is simpler in 4D than in 3D thanks to more freedom. More importantly, optimization of such system to solve our problem is no longer a continuous process, but from perspective of action optimization formulation of Lagrangian mechanics: we can imagine that nature has already solved the problems we are planning to ask.

Figure [18] contains such schematic picture for Shor’s algorithm. Fixing situation in the past is easy: just prepare the qubits in some chosen states. Additionally, quantum measurement gives some possibility to affect the system also from the future direction: in case of Shor’s algorithm it restricts the original ensemble to only possibilities having the same (randomly chosen) value of calculated function. As emphasized in this diagram, the consequence of this restriction (tension) seems to propagate backward in time here, like in Wheeler’s or delayed choice quantum erasure QM experiments, or in action optimizing formulation of Lagrangian mechanics.

Hence the suggested general approach to exploit the quantum superiority e.g. to search for polynomial algorithm for some NP-complete problem is:

1) Use Hadamard gates to get superposition of exponentially large set of possibilities, for example of all inputs to the problem among which we search for the satisfying one,
2) Perform some chosen classical function on these inputs, getting superposition like \(\sum_a |a⟩|f(a)⟩\),
3) Measure value of this function, restricting the ensemble to \(\sum_{a: f(a) = m} |a⟩|m⟩\),
4) Ask a question about this final ensemble, for example about its periodicity using QFT. Another basic question we can realize is if the size of resulting superposition is larger than one, what can be done by first producing multiple copies of bits of the inputs (e.g. using \((x, y) \rightarrow (x, y \text{ XOR } x)\) for auxiliary \(y = |0⟩\)), then measuring them: values of their measurements will vary if the superposition contains more than one possibility.

It seems tempting to implement with quantum gates e.g. verifier for an instance of NP-complete problem, and try to restrict ensemble to inputs satisfying this verifier, but in standard approach such restriction does not seem realizable. However, if being able to realize CPT analogue of state preparation, it might influence measurement outcomes, allowing to solve NP problems with quantum computers. Such hypothetical possibility is suggested in the next section using e.g. ring laser, which might be able to intuitively pull photons of chosen polarity from measured objects, hopefully changing probability distribution of measurement outcomes.
V. HYPOTHETICAL NEGATIVE PHOTON PRESSURE AND ITS POTENTIAL APPLICATIONS

While it is natural to push objects with photons, turns out there are also lots of successful approaches for optical pulling (e.g. [28]) starting with optical tweezers [29] awarded the 2018 Nobel prize. Generally EM radiation pressure is a vector $\vec{p} = (E \times H)/c$, allowing for negative radiation pressure to pull e.g. solitons ([30], [31]). While there might also exist other approaches, here we propose a few laser settings, which assuming CPT symmetry should allow to create analogous negative photon pressure in narrow spectrum, and discuss their potential applications.

Lasers are believed to be governed by two basic equations for population of ground $N_1$ and excited $N_2$ state, for

\[ \frac{\partial N_2}{\partial t} = -\frac{\partial N_1}{\partial t} = -B_{21} \rho(\nu)N_2 \]

and absorption:

\[ \frac{\partial N_2}{\partial t} = B_{12} \rho(\nu)N_1 \]

(13) governed by $B_{12} = B_{21}$ Einstein coefficients [32]. Both these equations act on e.g. pumped laser crystal, often with population inversion $N_2 > N_1$. Additionally, produced photons are often absorbed by some external target, stimulating its excitation. Looking at it from perspective after CPT symmetry, stimulation equation should also apply to some external target, what means stimulated emission equation should act on it in standard perspective (no CPT). To exploit it, these should be different targets, also initially excited ($N_2 > 0$ e.g. lamp) - what requires some asymmetry, available for a few settings discussed further as in Fig. 20 e.g. Free Electron Laser, synchrotron, ring laser.

A. Proposed realizations as lasAr (emission ↔ Absorption)

CPT theorem, originally proven by Julian Schwinger [3], says that CPT symmetry (of charge conjugation + parity transformation + time reversal) holds for all physical phenomena. It allows to transform Feynman diagrams into their still valid CPT analogues: replacing particles with antiparticles, also reversing time. While it has lots of microscopic confirmations [33], the big question is if it is still valid for macroscopic settings - if their CPT analogues should also work as supposed? In other words, if preparing CPT analogue of a scenario (e.g. initial conditions), should it lead to CPT symmetry of behavior, evolution? In theory, decomposing a setting into Feynman diagrams, and CPT transforming each of them, they should build CPT analogue of the original setting - which should work as in standard physics. Including causality: preparing CPT analogue of a scenario with clear causality, its time direction should be reversed.

Building CPT analogue of a scenario seems technically extremely challenging, however, there might be found situations where it is technologically reachable. Free Electron Laser (FEL) seems to be such an example (also just synchrotron radiation). It is just electrons travelling in properly shaped magnetic field emit photons as synchrotron radiation, causing later excitation of target. Performing its CPT transform, positrons travel in the opposite direction, causing earlier deexcitation of target - getting lasAr replacing stimulated emission with stimulated absorption.

b) Proposed setting e.g. in synchrotron, wiggler, undulator.

c) Macroscopic CPT symmetry test using ring laser:

Figure 20. Potential consequences of hypothesis that CPT theorem is still valid in macroscopic physics. a) In free electron laser (FEL) electrons travelling in properly shaped magnetic field emit photons as synchrotron radiation, causing later excitation of target.

d) In contrast to stimulated emission/absorption, spontaneous processes are time asymmetric due to solution we live in: it is easy for photons to escape to the future, it is difficult to produce them in the past.
travelling in the opposite direction.

From causality perspective, FEL through stimulated emission causes later excitation of target, while in its CPT transformed analogue: stimulated absorption causes earlier deexcitation of target. In other words, at least naively, laser should become lasAr - replacing stimulated emission with stimulated absorption: laser ↔ lasAr. If the target is gas-discharge lamp (of overlapping spectrum), it would naturally deexcite in isotropic way - the question to test is if such lasAr could additionally slightly increase probability of deexcitation of these atoms in its direction and spectrum? As in Fig. [19] stimulation of target deexcitation seems also allowed by Einstein's general relativity.

The use of positrons does not seem to matter, suggesting to just use electrons instead - going back to FEL. It also brings question why such effects are not already observed there? The answer seems simple: because it would require target previously excited in the corresponding spectrum, what seems usually not satisfied, and generally not searched for - but should be relatively easy to test in a dedicated experiment e.g. in synchrotron or FEL.

While such CPT analogue of laser was proposed by the author in 2009 (e.g. [https://groups.google.com/forum/#!topic/sci.physics.foundations/xhUfe8akaS0]), it seems still remain untested experimentally, hopefully to be repaired in a near future e.g. in synchrotron like Solaris in Cracow. As b) in Figure [20] it seems to just require placing a gas-discharge lamp behind a segment (necessary transparent window, alternatively behind the bending magnet/main synchrotron photon source), surrounded by detectors (with hole toward lasAr) combined with electronics monitoring input/output power balance of lamp-detectors. Activating the wiggler/undulator/FEL targeting this lamp, naively should increase probability of deexcitation in this direction (in addition to isotropic radiation), what should be seen as lowered energy in the monitored energy balance.

Such test has turned out technically challenging (far ultraviolet, no FEL), however, a colleague has pointed existence of ring lasers, suggesting more accessible alternative as in Figure [20]. They used closed photon trajectories, a single direction can be made dominating inserting optical isolator. From perspective after CPT transform, this direction would be reversed (also switching written two equations) - causing excitation of the shown target "behind the laser", what in the original perspective (no CPT) would mean causing deexcitation of this target - seen as darkening by detectors around (e.g. a camera with filter). From equations perspective (N2 is number of excited atoms), ∂N2/∂t = -B21ρ(ν)N2 should act not only on the central target, but (symmetrically to the second equation) also on the target on the left, if only it is initially excited: N2 > 0, the more the stronger the effect. Performing such test seems much more accessible, hopefully to be conducted in a near future. However, in contrast to FEL/synchrotron, the asymmetry is imperfect here - there is a percentage of photons travelling in the opposite direction, which would excite the target - against the hypothetical stimulated deexcitation, making it more difficult to detect. In this case, e.g. fast rotation of target might allow to distinguish the two effects.

**Photonic chip application: two-way quantum computers**

![Diagram of quantum computer](image)

**Figure 21.** Some potential computational applications of proposed lasAr as CPT analogue of laser (e.g. free electron laser, synchrotron, ring laser), hopefully creating negative radiation pressure ([10], [11]) for photons in narrow spectrum. Top: while Shor-like quantum computers are one-way mounted only in the past by state preparation, e.g. Ising model based computer like in Fig. [18] is two-way: mounted in both directions. lasAr e.g. ring laser might allow to mount in the future, replacing measurement by detectors, especially for photonic computers [14]. This way hopefully increase probability of producing photons of chosen e.g. polarity. For example implementing verifier of NP problem, and pulling photons corresponding to its satisfactory results (e.g. 3-SAT split into alternatives), hopefully to restrict initial entanglement of all inputs to the satisfying ones. Center: potential applications for time-loop computers: making that action optimization finds fixed point of a (time) loop, designed to solve given instance of NP problem this way. To send multiple bits with such lasAr, one could use spectral, temporal or spatial division, e.g. lattice of shutters (or mirrors like in DLP projectors). There is also shown alternative sending single bit - creating contradictory NOT time loop, unless HRNG generates satisfying input - hopefully preferred for action optimization. Bottom: Constructed time loops might be self-contradictory, requiring imperfections: possibility to lie, breaking such causal loop, e.g. inside the used electronics or such channel back in time. The proposed XOR game allows to compare strength of such hypothetical channels back in time: player A chooses bit a, player B chooses bit b, and they have opposite goals. They use own channels to send the final values, optimizing own goals - the one with stronger channel should win.

**B. Potential applications of negative photon pressure**

If such test will turn out successful, or there are found other realisations of negative photon pressure, applications could start e.g. with increasing rate of some chemical (e.g. photolithography) or nuclear transitions producing characteristic photons - “pulling photons” of this energy by placing such target behind e.g. ring or free electron laser tuned to this energy. Much more ambitious future application might be stimulated proton decay (if it is possible, energy density > 100 × than fusion, from any matter) by some optimized sequence of photon pushing and pulling.

The most crucial direction seems potential information/computation applications, summarized in Fig. [21]. The basic one is hypothetical, hopefully allowing to solve especially NP complete problems, two-way quantum computer enhancement - mounted in both directions as e.g. Ising computer in Fig. [18] by some CPT analogue of state preparation. For example photonic quantum computer with detectors replaced e.g. with ring laser pulling photons of chosen polarity - in which there are encoded e.g. results of alternatives for 3-SAT problem, adding preference to
choose a satisfying input by their initial entanglement from some Hadamard gates. If it is possible, the effect might be small change of measurement probability distributions, which could be strengthen e.g. by some error correction.

Adding shutter to lasAr, CPT arguments suggest the stimulated emission in target naively should happen earlier than opening shutter by optical path length: distance divided by speed of light (≈ 3.3 nanoseconds per meter). If it will turn out true, and overcoming delays of the setting/electronics, it might allow for information channel back in time between shutter and electronics monitoring energy balance of the lamp. Adding some delay line, e.g. with mirrors, optical fibers, techniques for slowing down light, might allow to overcome the delays - to literally send one bit of information slightly earlier. To extend it to multiple bits, one could e.g. use spatial division (like lattice of shutters or mirrors as in DLP projector), frequency division, maybe also temporal division (more complicated here).

Achieving nanosecond-scale time difference this way would be sufficient to put a simple ASIC chip in time loop (e.g. 3-SAT verifier): send its output back, and use as its earlier input. Building such ASIC chip for a given instance of NP-complete problem (e.g. 3-SAT), as in Fig 2[1] output the input if it is correct, or input+1 (cyclically) otherwise, would transform this problem into search of fixed point of this loop. Closing this loop with cables, if there is clock it would check one input per cycle until finding a correct one. It is interesting open question what would happen without clock - some complex electron hydrodynamics which should stabilize by finding the fixed point of the loop (solving our problem). Closing this loop in time e.g. by lasAr, action optimization of Lagrangian mechanics governing our world should find this fixed point - solving the given computational problem ... unless there is a simpler way to break such loop, e.g. lie in such channel or electronics.

Self-contradictory loops are also possible, e.g. with just NOT gate - leading to observed oscillations if being closed in space. Being able to close NOT into a loop in time, action optimizing physics would have to make one of them (channel or gate) to lie. Therefore, such channels cannot be perfect, always need to have a way to lie - breaking the weakest link (for action optimization) of given self-contradictory causal loop. For this purpose action optimization could e.g. exploit imperfections of the used electronics, however, such hypothetical channel seems the most likely weakest link. To test, compare strength of such channels as capability to avoid lying in difficult cases, Fig. 2[1] also suggests XOR game: putting them in situation in which exactly one is right, the stronger channel should win (alternatively e.g. NOT gate might lie). For such evaluation there could be alternatively used some standardized chips with easy to break causal links.

This way to avoid contradictory NOT time loop, action optimization should make this generator already choose a satisfying input. Generally making choices based on good HRNG could give action optimization freedom to optimize its generated values based on their later consequences, also in potential macroscopic applications.

Assuming CPT remains valid for macroscopic physics, still construction of such time-loop computer would be technically extremely challenging (strong channels, error resistant electronics), but it might be reachable - should be at least taken into considerations. Especially that it might allow to break currently used cryptography (if reaching sufficient delay and strength): with verifier checking if a given key leads to decoded file not being just a noise. Protection against such attacks might be done by adding computationally costly initialization: necessary before application of a new cryptographic key - to require much longer delays and stronger channels. However, it might finally lead to post-cryptography world: with safe only basic techniques like one-time pad. Further improvements of such channels could allow to use verifier e.g. testing molecules for desired properties for drug design, testing possible algorithms/methodologies/procedures for desired outcomes, etc. - to find satisfying choices with one of such time-loop approaches for problems testable in practical time, as physical world analogies of NP problems.

Finally, being able to send information back for macroscopic time differences (e.g. using systems of satellites for delay lines), could allow to prevent currently unpredictable unwanted events: there would be attempt to send back such missing information - creating inconstancy, hence action optimization should modify the weakest links of such reason-result chain (e.g. in quantum measurement level) to get self-consistent time loop (Novikov self-consistency principle) - for example with satisfying outcome: not requiring to use such channel. In other words, just having access to such channel, we would enforce physics to make its best to prevent our bad choices, optimize randomness for outcomes preferred by us - like mentioned creation of contradictory NOT time loop for non-satisfying outcomes (easier to optimize if leaving freedom by making choices based on good HRNG). While there would be many dangers on the way, if well balancing strengths of such channels worldwide (between players of different/opposite goals), it might lead to a much more harmonic world based on trust and common goals, potentially without crime and various types of gambling (economy based on objective values), with choices made optimizing their actual future consequences - maybe also avoiding wars, suboptimal politicians, dangers of new technologies, etc.

VI. CONCLUSIONS AND FURTHER PERSPECTIVES

While our natural intuition suggests us "evolving 3D" picture of the world; which e.g. allows to conclude Bell inequalities for resulting correlations, quantum mechanics has Born rules instead: squares relating amplitudes and probabilities - leading to violation of such inequalities. Living in 4D spacetime, what is nonintuitive but required by modern understanding of physics like special relativity, Lagrangian mechanics or QFT, requires to consider paths/scenarios (Feynman diagrams) as the basic objects - what leads to quantum-like behavior, starting with Anderson localization,
Born rules and Bell violation. Specifically, its consequence is the present moment being in equilibrium between past and future, tension from both these directions is described by the asymptotic behavior for example of $M^t = (-\mathcal{H})^t$ propagator from infinity as in (7) (left/right in Ising model), which is quantum amplitude of the ground state. Finally to randomly get some value of a measurement, we intuitively need to draw it from both time directions, getting probability being the square of amplitude. Consequences of living in 4D spacetime are seen in many quantum experiments like Wheeler’s, delayed choice quantum erasure, as discussed also in Shor’s algorithm, maybe to be extended to even stronger time-loop computers.

Another conclusion from MERW is that the stochastic and quantum realms of physics, which have historically split their ways for example due to disagreement in predictions for semiconductor due to Anderson localization, can reunite if repairing the subtle approximation in entropy maximization, using ensembles of full paths. Especially interesting and important are situations in intersection of both worlds, like good understanding of electron flow in microscopic systems, what is crucial in modern electronics reaching level of single atoms and molecules, where standard Ohm’s law is not satisfied: a fixed potential difference for identical local situations can lead to different currents, behaving in a nonlocal way: depending on the entire system like in Fig. 11. MERW-based modelling can be used as a practical approximation of extremely costly complete quantum calculations e.g. of p-n junction - discussed in [17].

As discussed, Boltzmann distribution among paths is a simple model suggested by many perspectives, like being successfully used in Ising model, the principle of maximum entropy of statistical physics, living in 4D spacetime, or agreement with expected quantum predictions and confirming experiments - making it a promising direction for understanding of physics governing our world. However, it obviously also contains essential simplifications, some of which are visualized in Fig. 22. One of them is varying number of particles in real physical scenarios, what is repaired in perturbative quantum field theories by considering ensemble among more sophisticated scenarios: Feynman diagrams. A real scenario represented by such simplified diagram contains additionally configuration of fields of interactions, for example electromagnetic, suggesting field theories for more fundamental description. Particles having a charge maintain nearly singular configuration of electric field - robust configurations of fields are technically called solitons, topology brings a natural mathematical tool to explain their charge quantization like in Fig. 23.

One of essential properties ignored by Boltzmann distribution among paths is the requirement for interference: some particle’s internal periodic process, called de Broglie’s clock ($E = mc^2 = \hbar \omega$) or zitterbewegung, which has been directly observed for electron for example as increased absorption when synchronizing period of such clock with lattice constant of silicon crystal ([20], [21]). It causes coupled “pilot waves” of the surrounding field, confirmed as de Broglie-Bohm interpretation of QM for example by experiment measuring average trajectories of interfering photons in double-slit experiment [35]. While principle of complementarity forbids measuring both corpuscular and wave natures simultaneously, it does not exclude that particle has objectively both natures at a time, especially that no conditions for choosing one of them are specified (e.g. in which moment meeting electron and proton become hydrogen?), or mechanisms for such change of nature, and there are experiments successfully exploiting both natures at a time like Afshar’s [36].

There are also lots of experimental hydrodynamical analogs of QM, especially started by Yves Couder, which show that such classical objects coupled with waves they create (droplet on a vibrating liquid surface) allow to recreate many quantum phenomena like: interference [39] in double-slit experiment (particle goes one trajectory, interacting with waves it created - going through all trajectories), tunneling [40] (depending on practically random hidden parameters - highly complex state of the field), orbit quantization [41] in-

![Figure 22. Boltzmann distribution among trajectories is a simple effective model recreating some basic properties of physics, quantum mechanics. One expansion toward fundamental physics is adding possibility to combine or decay particles, requiring to extend the ensemble of objects to consider from paths to more complex scenarios like Feynman diagrams. This way we would go to universal picture of perturbative quantum field theories. Further steps toward fundamental physics will eventually require asking for field configurations behind such single diagrams, including gluing of EM fields in the center of charge, avoiding energy infinity obtained by assuming that charge is a perfect point. Such localized field configurations for particles are technically called solitons, for which quantization of charge has well known mathematical analogue: topological charge, discussed in Fig. 23.](image)

![Figure 23. Top: examples of 2D topological charges of vector field - looking at a loop around such singularity, charge is the (integer) number of complete field rotations. If “removing arrows of vectors” we can also get charges being multiplicity of 1/2 this way (like spin). Well known example in nature of such field configuration stabilized by topology (topological soliton) is fluxon (Abrikosov vortex) in superconductor, carrying quan of magnetic field. Going to 3D analogs, e.g. in liquid crystals there are observed also Coulomb-like interactions [37] for such topological charges. Mathematically, defining EM field as curvature of vector field, Gauss-Bonnet theorem acts exactly as the Gauss law: integrating curvature over a closed surface, we get the total topological charge inside this surface - leading to electrodynamics with Maxwell’s equations governing dynamics of such quantized charges.](image)
Figure 24. Topological soliton particle model approach [48] (gathered materials: https://github.com/JarekDuda/liquid-crystals-particle-models) exploring resemblance of topological defects in liquid crystals, e.g. experimentally allowing to obtain Coulomb-like interaction [37]. Higgs-like potential enforces e.g. vector field to of unitary vectors far from particles, also allowing to regularize singularity (charge) to finite energy. Interpreting field curvature as EM field, Gauss law returns topological charge - getting Maxwell equations with built-in charge quantization (Faber's approach [38]). In biaxial nematic, governed by modification of Landau-de Gennes model, we have 3 distinguishable axes, leading to 3 types of hedgehog: the same charge, but different mass - resembling 3 leptons, also requiring magnetic dipole moment due to the hairy ball theorem. Extending further to field of 4 distinguishable axes in 4D ($\text{SO}(3) \to \text{SO}(1,3)$ vacuum), represented by real symmetric tensor field ($\mathbf{M}$), leads to additional second set of Maxwell equations - as in gravitoelectromagnetism approximation of the general relativity.

Including double quantization [42], of both radius and angular momentum like in Bohr-Sommerfeld (particle has to find a resonance” with the field - its internal phase has to return to initial state during full orbit), Zeeman splitting analogue [43] (using Coriolis as Zeeman force), like in MERW; recreating quantum eigenstates with statistics of trajectory [44], also Bell violation [45].

The universality of quantum formalism is also recreated in other hydrodynamical analogues, e.g. Casimir effect: two plates in vibrating water tank also were experimentally shown to attract [46] as wave energy between them is lower due to restriction by the plates. There is also hydrodynamical analogue of Aharonov-Bohm effect suggested by Berry [47], using vortex, vorticity and Casimir force analogues of solenoid, magnetic field and Lorentz force.

To summarize, while there is unsuccessful belief that we need to find a boundary between classical and quantum world, this boundary blurs e.g. with hydrodynamical analogues or MERW - it might turn out nonexistent: they can be just different perspectives/descriptions of the same system. For example coupled oscillators can be described by...
evolution of their positions ("classical"), or in the base of their normal modes - where this evolution becomes literally unitary ("quantum"). Lattices of such oscillators are used to model crystals: classically, or equivalently in Fourier basis: using phonons as normal modes - which are treated as (quasi)particles in Feynman diagrams. In continuous limit of such lattice we get field theories - which can be modelled with hydrodynamical analogues. Solitons are localized particle-like configurations of fields, effectively described by QFT. Using topological solitons we get charge/spin quantization, pair creation/annihilation, and electromagnetism-like interaction for them. Finally Couder’s quantization suggests how to understand atoms: Schrödinger equations describes coupled wave, which to minimize energy needs to become how to understand atoms: Schrödinger equations describes interaction for them. Finally Couder’s quantization suggests of such lattice we get field theories - which can be mod-

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