Deformation of the Planetary Orbits
Caused by the Time Dependent Gravitational Potential in the Universe

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Abstract

In the recent paper [4], assuming a linear change of a gravitational potential \( V \) in the universe, i.e. \( \Delta V = -c^2 H \Delta t \), are explained both the Hubble red shift and the anomalous acceleration \( a_P \) from the spacecraft Pioneer 10 and 11 [1]. The change of the potential \( V \) causes an accelerated time which is easily seen by the Hubble red shift. But the change of the potential causes also change of the distances in the galaxy, and hence modification on the planetary orbits. In this paper it is shown that the planetary orbits are not axially symmetric, neglecting the relativistic corrections. The angle from the perihelion to the aphelion is \( \pi - \frac{H \Theta}{3} \), while the angle from the aphelion to the perihelion is \( \pi + \frac{H \Theta}{3} \cdot e \pi \), where \( \Theta \) is the orbit period, \( e \) is the eccentricity of the elliptic trajectory and \( H \) is the Hubble constant. There is no perihelion precession caused by the time dependent gravitational potential \( V \). The quotient \( \Theta_2 : \Theta_1 \) of two consecutive orbit periods \( \Theta_1 \) and \( \Theta_2 \) is equal to \( \Theta_2 : \Theta_1 = 1 + \frac{H \Theta}{3} \). This formula is tested for the orbit period of the pulsars B1534+12 and 1855+09 which have very good timing, and the results are satisfactory.

In the recent paper [4] is given an assumption that in the universe there is a gravitational potential \( V \), which decreases linearly (or almost linearly), such that \( \Delta V = -c^2 H \Delta t \), where \( H \) is the Hubble constant. This change of the potential just causes the Hubble red shift \( \nu = \nu_0 \left( 1 - \frac{H H}{c} \right) \) and the large velocities among the galaxies are apparent. The change of the potential is probably caused by the dark energy in the universe, which is about 67% [2].

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Note that the gravitational potential is assumed to be larger near the gravitational bodies, than the potential where there is no gravitation. For example, near the spherical body we accept that the potential is \(\frac{GM}{r}\). If we accept that this potential is \(-\frac{GM}{r}\), then for the time dependent potential it should be \(\Delta V = c^2 H \Delta t\).

The change of the gravitational potential causes nonuniform space-time. Let us denote by \(X, Y, Z, T\) our natural coordinate system in the space-time deformed by the gravitational potential \(V\) and let us denote by \(x, y, z, t\) the normed coordinates of an imagine coordinate system, where the space-time is "uniform". This physically occurs for example, if we travel with a lift away from the ground with slow velocity \(v = 2.44\, \text{cm/s}\). Then we have the same time dependent change of the gravitational potential like in the universe, but in the lift we have also a space dependent gravitational potential, which causes the acceleration toward the Earth.

According to the general relativity we have the following equalities

\[
\begin{align*}
\frac{dx}{dt} &= \left(1 + \frac{V}{c^2}\right)^{-1} dX = \left(1 + tH\right) dX, \\
\frac{dy}{dt} &= \left(1 + \frac{V}{c^2}\right)^{-1} dY = \left(1 + tH\right) dY, \\
\frac{dz}{dt} &= \left(1 + \frac{V}{c^2}\right)^{-1} dZ = \left(1 + tH\right) dZ, \\
\frac{dt}{dX} &= \left(1 - \frac{V}{c^2}\right)^{-1} dT = \left(1 - tH\right) dT.
\end{align*}
\]

Hence we obtain

\[
\left(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\right) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) (1 - 2tH)
\]

and by differentiating this equality we get

\[
\left(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\right) = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\right) - 3HT \left(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\right) - 2H \left(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\right)
\]

(5)

where \(\left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\right)\) is the Newtonian acceleration. In normed coordinates \(x, y, z, t\) there is no any acceleration caused by the time dependent gravitational potential, because the gradient of \(V\) vanishes. Thus, according to the coordinates \(X, Y, Z, T\) appears an additional acceleration

\[
-3HT \left(\frac{d^2X}{dT^2}, \frac{d^2Y}{dT^2}, \frac{d^2Z}{dT^2}\right) - 2H \left(\frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT}\right).
\]

(6)
Using the acceleration (6) in [4] is explained the anomalous acceleration $a_p \approx Hc$, which causes an anomalous frequency shift of the radio signals which are sent to the spacecraft Pioneer 10 and 11 and re-transmitted to the Earth [1].

The acceleration (6) appears also in the planetary orbits. In order to study that, first we shall consider two special cases of deformations, like basic generators of deformations. Further we shall neglect the relativistic deformations (for example relativistic perihelion precession), in order to emphasize the deformations caused by the time dependent gravitational potential. We shall consider three parameters of the planetary orbits: 1. the angle between the radii given by the perihelion and aphelion, i.e. its departure $\Delta \varphi$ from $\pi$; 2. the perihelion precession; and 3. the quotient $\Theta_2 : \Theta_1$, where $\Theta_1$ and $\Theta_2$ are two consecutive orbit periods of the planet.

I. Assume that instead of (1), (2), (3) and (4) we have the following coordinate transformations

$$x = (1 + \lambda tH)X, \ y = (1 + \lambda tH)Y, \ z = (1 + \lambda tH)Z, \ t = T, \ (\lambda = \text{const.}), \ (7)$$

which means that we have only change in the space coordinates, and there is no change in the time coordinate. According to the normed coordinates $x, y, z, t$ the trajectory in the plane of motion is given by

$$\frac{1}{r} = \frac{\rho_1 + \rho_2}{2} + \frac{\rho_1 - \rho_2}{2} \cos \varphi, \ (8)$$

where $\rho_1 = 1/r_1$ and $\rho_2 = 1/r_2$ are constants, just like in the Newtonian theory.

According to (7), the equality (8) becomes

$$\frac{1}{R} = (1 + \lambda tH)\left[\frac{\rho_1 + \rho_2}{2} + \frac{\rho_1 - \rho_2}{2} \cos \varphi\right]. \ (9)$$

The angle $\varphi$ is a common parameter for both coordinate systems. The equation $\frac{d \varphi}{d \varphi} = 0$ for the extreme values of $1/R$, according to (9), yields to

$$\frac{d}{d \varphi} \left[\frac{\rho_1 + \rho_2}{2} (1 + \lambda tH) + \frac{\rho_1 - \rho_2}{2} (1 + \lambda tH) \cos \varphi\right] = 0,$$

$$-\frac{\rho_1 - \rho_2}{2} (1 + \lambda tH) \sin \varphi + \left[\frac{\rho_1 + \rho_2}{2} + \frac{\rho_1 - \rho_2}{2} \cos \varphi\right] \frac{dt}{d \varphi} \lambda H = 0,$$

$$\sin \varphi = \left[\frac{\rho_1 + \rho_2}{\rho_1 - \rho_2} + \cos \varphi\right] \frac{\lambda H}{d \varphi}.$$

The solution of this equation of $\varphi$ when $\varphi \approx 0$ can be obtained approximately by putting $\cos \varphi = 1$ and hence

$$\varphi_1 = \frac{2 \rho_1}{\rho_1 - \rho_2} \frac{\lambda H}{(\frac{d \varphi}{d t})_{\text{per.}}} = \frac{2r_1 \lambda H}{(\rho_1 - \rho_2)C}, \quad (10)$$
where \( C = r^2 \frac{d^2 \varphi}{dt^2} = \text{const.} \) according to the second Kepler’s law. The solution of \( \varphi \) when \( \varphi \approx \pi \) can be obtained approximately by putting \( \cos \varphi = -1 \), i.e.

\[
\varphi_2 = \pi - \frac{2\rho_2}{\rho_1 - \rho_2} \frac{\lambda H}{(\rho_1 - \rho_2)C} = \pi - \frac{2r_2\lambda H}{(\rho_1 - \rho_2)C},
\]

(11)

Hence

\[
\varphi_2 - \varphi_1 = \pi - \frac{2(r_2 + r_1)\lambda H}{r_1 - r_2} = \pi - \frac{2r_1r_2\lambda H}{Ce},
\]

where \( e \) is the eccentricity. Assuming that the eccentricity of the ellipse is small, then \( \frac{r_1}{r_1 - r_2} \approx \frac{d^2 \varphi}{dt^2} \approx \frac{2\pi}{\Theta} \), where \( \Theta \) is the orbit period of the planet, and hence

\[
\Delta \varphi = \varphi_2 - \varphi_1 - \pi \approx -\frac{\lambda H\Theta}{\pi e}.
\]

(12)

Further, for the precession of the perihelion we obtain the angle

\[
\frac{2\rho_1}{\rho_1 - \rho_2} \left[ \frac{\lambda H}{(\rho_1 - \rho_2)\Theta} - \frac{\lambda H}{(\rho_1 - \rho_2)_0} \right] = 0,
\]

(13)

neglecting the terms of order \( H^2 \). Hence the angle from the aphelion to the perihelion is equal to

\[
\pi + \frac{2r_1r_2\lambda H}{Ce} \approx \pi + \frac{\lambda H\Theta}{\pi e},
\]

(14)

and the planetary orbit is not axially symmetric.

Note that in the previous considerations it was assumed that \( H \) does not change with the time. If \( H \) changes with the time, then will appear a slight departure of order \( H^2 \) and it is negligible.

Since \( t = T \) in this case, it is

\[
\Theta_2 : \Theta_1 = 1.
\]

(15)

II. Assume that instead of (1), (2), (3), and (4) we have the following coordinate transformations

\[
x = X, \quad y = Y, \quad z = Z, \quad dt = (1 - \mu t) dT, \quad (\mu = \text{const.}),
\]

(17)

which means that we have only change in the time coordinate. Thus, the planetary orbit in the \((R, \varphi)\)-plane is an ellipse like according to the \( x, y, z, t \) coordinate system.
Using that \( T = t + \frac{\mu}{2} H t^2 \), for the quotient \( \Theta_2 : \Theta_1 \) which corresponds to \( t_2 = t_1 \) for the orbit period we obtain

\[
\Theta_2 : \Theta_1 = \left( \left[ 2\Theta + \frac{\mu}{2} H (2\Theta)^2 \right] - \left[ \Theta + \frac{\mu}{2} H \Theta^2 \right] \right) : \left( \Theta + \frac{\mu}{2} H \Theta^2 \right) = 1 + \mu \Theta H,
\]
i.e.

\[
\Theta_2 : \Theta_1 = 1 + \mu \Theta H. \tag{18}
\]

Finally, note that the equations (17) imply that

\[
\frac{dX}{dT} = \frac{dx}{dt} \left( 1 - \mu H t \right),
\]

\[
\frac{d^2 X}{dT^2} = \frac{d}{(1 + \mu H t) dt} \left( \frac{dx}{dt} (1 - \mu H) \right) = (1 - \mu H)^2 \frac{d^2 x}{dt^2} - \mu H \frac{dx}{dT},
\]
and analogously is true for \( Y \) and \( Z \) coordinates. Indeed,

\[
\left( \frac{d^2 X}{dT^2}, \frac{d^2 Y}{dT^2}, \frac{d^2 Z}{dT^2} \right) = (1 - 2\mu TH) \left( \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2}, \frac{d^2 z}{dt^2} \right) - \mu H \left( \frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT} \right). \tag{19}
\]

Combining both special cases I. and II. we obtain the following conclusion:

If the acceleration is given by

\[
\left( \frac{d^2 X}{dT^2}, \frac{d^2 Y}{dT^2}, \frac{d^2 Z}{dT^2} \right) = (1 - (\lambda + 2\mu) TH) \left( \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2}, \frac{d^2 z}{dt^2} \right) - (2\lambda + \mu) H \left( \frac{dX}{dT}, \frac{dY}{dT}, \frac{dZ}{dT} \right), \tag{20}
\]
then the angle \( \Delta \varphi \) is given by (12), there is no precession of the perihelion and the quotient \( \Theta_2 : \Theta_1 \) is given by (18).

Let us consider the space-time in the universe, where we accepted the equalities (1), (2), (3) and (4). If we put \( \lambda = \frac{1}{3} \) and \( \mu = \frac{1}{4} \), then the equation (20) becomes just the equation (5). Thus, for the parameter \( \Delta \varphi \) we obtain

\[
\Delta \varphi = \frac{H \Theta}{3 \pi e}. \tag{21}
\]

According to the formula (21) the angle \( \Delta \varphi \) for Mercury is equal to \(-9 \times 10^{-12}\) radians. For a comparison, the perihelion precession according to the general relativity is about \(4.8 \times 10^{-7}\) radians. From the formula (21) we see that the angle \( \Delta \varphi \) can easier be measured if we consider a planet with small eccentricity and long period \( \Theta \). For example, if we consider the orbit of the Earth and use that \( e \approx 1/60 \), and for \( H \) take the value \((14 \times 10^9 \text{ years})^{-1}\), then we obtain that \( \Delta \varphi \approx -0.45 \times 10^{-9}\) radians, i.e. about \((-10^{-4})^\circ\). Hence the aphelion is displaced for \(1\text{AU} \times \Delta \varphi \approx -67\text{m} \). The value of \( \Delta \varphi \) for Neptune is about 300 times larger than the value for the Earth and can easily be measured,
but the half of the orbit period is 83 years and it is too long. The value of $\Delta \varphi$ for Venus is 1.5 times larger than the value for the Earth.

According to (??), the quotient $\Theta_2 : \Theta_1$ of two consecutive orbit periods is $1 + \mu \Theta H = 1 + \frac{4}{3} \Theta H$, and it should be "normed" with $1 + \Theta H$ according to (4), because in the coordinate system $X, Y, Z, T$ after each orbit period the time is faster $1 + \Theta H$ times. Hence the required quotient is equal to

$$\Theta_2 : \Theta_1 = \frac{1 + \frac{4}{3} \Theta H}{1 + \Theta H} = 1 + \frac{1}{3} \Theta H.$$  \hspace{1cm} (22)

If we consider the planet Earth, i.e. $\Theta$ is one year, then

$$\Theta_2 - \Theta_1 = \frac{1}{3} \Theta^2 H \approx 0.001 \text{ s}.$$  

The difference $\Theta_2 - \Theta_1$ for Pluto is about one minute and it is easy to be detected, but two orbit periods is too long time.

Formula (22) can be applied for double stars, for example the binary pulsars. From (22) we get

$$\dot{P}_b = \frac{1}{3} PH,$$  \hspace{1cm} (23)

where $P_b = \Theta$ is the orbit period of the binary pulsar. While the gravitational radiation decreases the orbit period, (23) increases. This increment very often is much smaller than the decay caused by the gravitational radiation. For example, for the Hulse-Taylor binary pulsar PSR B1913+16 [6] the increment is 1% of the decay of the orbit period and it is difficult to be detected. So we choose a binary pulsar with longer orbit period, and/or small decay caused by the gravitational radiation. Such examples are the pulsars PSR B1534+12 [5] and PSR B1885+09 [3]. Moreover, both binaries have very good timings and hence are convenient for precise tests. In case of PSR B1534+12 the decay of $\dot{P}_b$ caused by the gravitational radiation is about $-0.1924 \times 10^{-12}$, while the increment caused by (23) is about $0.027 \times 10^{-12}$, and hence together we have decay about $-0.1654 \times 10^{-12}$. On the other side, the measured value of $\dot{P}_b$ is $(-0.174 \pm 0.011) \times 10^{-12}$, where the galactic corrections are included. Hence there is agreement of both results. The agreement is much better for the pulsar PSR B1885+09 [3]. In this case it is found that (formulae (18) and (20) in [3])

$$-\frac{1}{2P_b}(\dot{P}_b^{\text{obs}} - \dot{P}_b^{\text{exp}}) = (-9 \pm 18) \times 10^{-12} \frac{1}{\text{yr}},$$  \hspace{1cm} (24)

where $\dot{P}_b^{\text{obs}}$ and $\dot{P}_b^{\text{exp}}$ are the observed and expected values respectively. According to (23), $\dot{P}_b^{\text{obs}} - \dot{P}_b^{\text{exp}} = \dot{P}_b H/3$, and the left side of (24) becomes $-11.9 \times 10^{-12} \frac{1}{\text{yr}}$. Note that here it is taken $\dot{P}_b^{\text{exp}} = 0$, because the influence from the gravitational radiation is negligible. Moreover, the kinematic galactic corrections are one order smaller than (24). Thus, in case of PSR B1885+09 the significant part of the measured value of $\dot{P}_b$ is caused by the formula (23).

This example well confirms the presented theory.


References

[1] J. D. Anderson P. A. Liang, E. L. Lau, A. S. Liu, M. M. Nieto and S. G. Turyshev, Study of anomalous acceleration of Pioneer 10 and 11, Phys. Rev. D 65 082004 (2002), eprint: gr-qc/0104064.

[2] W. L. Freedman, M. S. Turner, Measuring and understanding of the universe, Rev. of Modern Phys. 75, 1433-1447 (2003).

[3] V. M. Kaspi, J. H. Taylor, and M. F. Ryba, High-precision timing of millisecond pulsars. III. Long-term monitoring of PSRs B1885+09 and B1937+21, Astrophys. J., 428 (1994), 713.

[4] K. Trenčevski, Hubble red shift and the anomalous acceleration of Pioneer 10 and 11, submitted for publication, eprint: gr-qc/0402024.

[5] I. H. Stairs, S. E. Thorsett, J. H. Taylor, and A. Wolszczan, Studies of the Relativistic Binary Pulsar PSR B1534+12: I. Timing Analysis, The Astrophysical J., eprint: astro-ph/0208357.

[6] J. M. Weisberg and J. H. Taylor, The Relativistic Binary Pulsar B1913+16, 2003, in ASP Conf. Ser. 302: Radio Pulsars, p. 93, eprint: astro-ph/0211217.