Elastic depinning transition for superconductor vortices

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Abstract. We present 2D numerical simulation results of vortex lattices driven over a random disorder. We study the vortex dynamics at the depinning threshold \( F_c \) for weak disorder at zero temperature. Elastic regimes are analysed in the framework of a second order phase transition where the velocity response \( v \) to the driving force \( F \) behaves like \( v \sim (F - F_c)^\beta \). The exponent \( \beta = 0.27 \pm 0.04 \) is extracted from the critical region of several large lattice sizes.

1. Introduction
Driven ordered objects in disordered media display a threshold force \( F_c \) across which the system goes from a pinned phase to a sliding state. These systems may be interfaces like magnetic domain walls, contact lines of liquid menisci or crack propagation in solids, or periodic structures like charge-density waves (CDWs), vortex lattices in type II superconductors, colloïds or Wigner crystals. When the pinning of the underlying substrate is weak the statics and dynamics are dominated by the system elasticity which favors order. In the context of CDWs the depinning at the threshold force was analysed some time ago \cite{1} as a standard equilibrium critical phenomenon where the velocity \( v \) is the order parameter and the reduced force \( f = (F - F_c)/F_c \) is the control parameter. Further analytical and numerical works developed this analogy to define universality classes and critical exponents (see e.g. Refs. \cite{2} and references therein). In the second order transition analogy the velocity of the elastic object at zero temperature vanishes at the critical force \( F_c \) as \( v \sim f^\beta \) with \( \beta < 1 \). Numerical simulations for interfaces in a space dimension \( d = 2 \) show \( \beta \approx 1/3 \) \cite{3} while for periodic objects in \( d = 2 \) various exponents have been found, \( \beta \approx 2/3 \) is found for CDWs \cite{4}, Wigner crystals \cite{5}, and colloïds \cite{6}, while other works found \( \beta \approx 0.5 \) \cite{7} and \( \beta = 0.92 \pm 0.01 \) \cite{8} on colloïds, and \( \beta = 0.35 \) \cite{9} in stripe systems.
We report in this paper zero temperature numerical simulation results on 2D superconductor vortex lattices driven over weak random disorder. Several lattice sizes have been investigated close to the elastic depinning threshold and the results agree with a second order phase transition. The velocity depinning exponent \( \beta \) is measured and compared to similar systems.

2. Numerical model
We study \( N_v \) Abrikosov vortices interacting with \( N_p \) pins randomly placed in the \((x, y)\) plane. We consider the London limit \( \lambda_L \gg \xi \), where \( \lambda_L \) is the penetration length and \( \xi \) is the coherence length, \textit{i.e.} we treat superconductor vortices as point particles. At zero temperature we integrate the second order Newton’s equations of motion of each vortex \( i \) at position \( \mathbf{r}_i \).
The total force on each vortex includes a viscous damping term \( \eta \frac{dv}{dt} \) with \( \eta \) the viscosity coefficient, the Lorentz driving force \( F^L = F \hat{x} \) in the \( x \) direction due to an applied current, and the conservative vortex-vortex \( U^v(r_{ij}) \) and vortex-pin \( U^{vp}(r_{ip}) \) interactions where \( r_{ip} \) is the distance between the vortex \( i \) and the pinning site located at \( r_p \), and \( r_{ij} \) is the distance between the vortices \( i \) and \( j \) located at \( r_i \) and \( r_j \). The vortex-vortex pairwise repulsive interaction is given by a modified Bessel function \( U^v(r_{ij}) = \alpha_v K_0(r_{ij}/\lambda_L) \) and the attractive pinning potential is given by \( U^{vp}(r_{ip}) = -\alpha_p e^{-(r_{ip}/R_p)^2} \) where \( R_p \) is the radius of the pins, and \( \alpha_v \) and \( \alpha_p \) are tunable parameters. We fix the strength of the vortex-vortex interaction by setting \( \alpha_v = 2.83 \times 10^{-3} \lambda_L \epsilon_0 \) where \( \epsilon_0 \) is an energy per unit length. Several values of the relative disorder strength \( \alpha_p/\alpha_v \) have been investigated. The LAMMPS classical molecular dynamics code [10] with a velocity Verlet algorithm is used for several system sizes, from \( N_v = 270 \) up to \( N_v = 12000 \) vortices. Various rectangular shaped basic cells of size \( (L_x, L_y) \) have been investigated, from the almost square \( (L_x, L_y) = (5, 6\sqrt{3}/2)n\lambda_L \) with \( n = 3, 8, 20 \), up to very elongated strips \( (L_x, L_y) = (400, 20\sqrt{3}/2)\lambda_L \). The strip geometry elongated in the driving force direction allowed the study of the critical depinning properties for large system sizes. Care has been taken to check that the basic cell anisotropy induced by the strip geometry does not alter the critical properties, and in particular the determination of the velocity critical exponent \( \beta \). Periodic boundary conditions are used in both \( x \) and \( y \) directions. The vortex-vortex interaction is dealt with using a neighbor list method with a cutoff radius \( r_c = 6.5 \lambda_L \). The number of pins is set to \( N_p = N_v \), and their radius is \( R_p = 0.22 \lambda_L \). The average vortex distance is \( \epsilon_0 = \lambda_L \). We use a unit system in which \( \lambda_L = 1 \), \( \epsilon_0 = 1 \) and \( k_B = 1 \). We fix \( \eta/m = 0.1 \) where \( m \) is the vortex mass, which ensures that the second order Newton’s vortex dynamics is identical to the overdamped dynamics limit usually computed for superconductor vortices.

3. Results
For several disorder realizations, we start from a triangular lattice at high velocity wherefrom the driving force is slowly decreased down to the so-called critical depinning force \( F_c \) below which the system is permanently pinned. We show in Fig. 1a the evolution of the mean critical depinning force with respect to the relative disorder strength \( \alpha_p/\alpha_v \) for \( N_v = 8000 \) vortices in a basic cell of size \( (L_x, L_y) = (400, 20\sqrt{3}/2)\lambda_L \). As already seen in previous works of similar systems (see for example Ref. [6]), the rapid increase in the depinning force indicates a crossover from elastic dynamics dominated by elasticity to plastic dynamics dominated by disorder. After the study of the 2D critical behavior of plastic depinning in superconductor vortices [11], we now study the 2D critical behavior of elastic depinning. In the following we choose the relative disorder strength \( \alpha_p/\alpha_v \approx 5 \times 10^{-3} \) for the elastic critical dynamics study. Fig. 1b shows the typical trajectories of the vortices at the elastic depinning threshold. All vortices depin together and with the same mean velocity, which means that all vortices keep the same neighbors as they move. The structure is topologically ordered and vortices flow in elastically coupled rough static channels. The dynamics is jerky and the velocity of the center of mass is periodic in time where the period corresponds to the time for each vortex to replace its preceding neighbor in the same channel.

We plot in Fig. 2 the average longitudinal velocity \( v \) of the center of mass of the vortices with respect to the reduced force \( f = (F - F_c)/F_c \) for several lattice sizes, where \( F_c \) is the critical depinning force measured for each disorder realization. Three regions appear in Fig. 2. Region I is the manifestation of the finite size effects in the system whose signature is the single particle regime [11] where \( v \sim f^{1/2} \) as shown by the lines of slope 1/2. In this region, possible hysteretic depinning may be measured with different values \( F_c^{up} \) and \( F_c^{down} \) of the threshold force when increasing or decreasing the force. However, the size \( F_c^{up} - F_c^{down} \) of the hysteresis decreases.
Figure 1. a) Transition from elastic to plastic dynamical regimes shown by the evolution of the mean critical depinning force $F_c$ with respect to the relative disorder strength $\alpha_p/\alpha_v$ for $N_v = 8000$ vortices in a basic cell of size $(L_x, L_y) = (400, 20\sqrt{3}/2)\lambda_L$. b) Typical trajectories of vortices at the elastic depinning threshold obtained for $N_v = 12000$ vortices. A snapshot of the vortex positions (filled circles) at a given time is superimposed to the trajectories. For clarity only a small part of the basic cell of size $(L_x, L_y) = (100, 120\sqrt{3}/2)\lambda_L$ is shown.

Figure 2. Vortex velocity $v$ versus the reduced force $f$ for several lattice sizes: a) $N_v = 5000$ in a basic cell $(L_x, L_y) = (100, 50\sqrt{3}/2)\lambda_L$ b) $N_v = 8000$ in a basic cell $(L_x, L_y) = (400, 20\sqrt{3}/2)\lambda_L$

when the system size increases, which therefore confirms that such hysteresis phenomenon is just a finite size effect. Region II is the region where a power law regime $v \sim f^\beta$ with $\beta < 1$ is measured which we identify with the critical regime of the continuous depinning transition. Finally, in region III the system is far above the critical depinning threshold and approaches the asymptotic linear behavior $v \sim f$ obtained in the absence of disorder.

To reinforce the second order elastic depinning transition scenario we compute several lattice sizes and for each size we compute several disorder realizations wherefrom we extract a mean value of the depinning exponent $\beta$. In Fig. 3a we show the evolution of $\beta$ with respect to the transverse size $L_y$ for a fixed longitudinal size $L_x = 100\lambda_L$ which shows that the values of $\beta$ that we measure for $L_y \geq 18\lambda_L$ become independant of the transverse size $L_y$. In particular they do not depend on the basic cell anisotropy since we measure identical values in square basic cells $(L_x, L_y) = (100, 120\sqrt{3}/2)\lambda_L$. In Fig. 3b we show the evolution of $\beta$ with respect to the longitudinal size $L_x$ for various transverse sizes $L_y \geq 18\lambda_L$. It shows that $\beta$ has reached a constant value for $L_x \geq 100\lambda_L$. Taking the mean value of $\beta$ over 68 realizations of disorder obtained for $L_x \geq 100\lambda_L$, we obtain the result $\beta = 0.27 \pm 0.04$. 


4. Conclusion
We have shown numerical simulation results on 2D superconductor vortex dynamics in random media. A crossover from elastic dynamics dominated by elasticity to plastic dynamics dominated by disorder is found. We investigated the depinning transition in the elastic regime. Our results are consistent with a second order phase transition with a velocity exponent $\beta = 0.27 \pm 0.04$.

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References
[1] Fisher D S 1985 Phys. Rev. B 31 1396
[2] Bustingorry S, Kolton A B and Giamarchi T 2010 Phys. Rev. B 82 094202
[3] Duenmer O and Krauth W 2005 Phys. Rev. E 71 061601
[4] Myers C R and Sethna J P 1993 Phys. Rev. B 47 11171
[5] Piacente G and Peeters F M 2005 Phys. Rev. B 72 205208
[6] Reichhardt C and Olson C J 2002 Phys. Rev. Lett. 89 078301
[7] Pertsinidis A and Ling X S 2001 Bull. Am. Phys. Soc. 46 181
[8] Chen J, Cao Y and Jiao Z 2004 Phys. Rev. E 69 041403
[9] Olson Reichhardt C J, Reichhardt C and Bishop A R 2011 Phys. Rev. E 83 041501
[10] Plimpton S 1995 J. Comput. Phys. 117 1
[11] Fily Y, Olive E, Di Scala N and Soret J C 2010 Phys. Rev. B 82 134519
[12] Luo M B and Hu X 2007 Phys. Rev. Lett. 98 267002