Output Entanglement Saturation and Optimization in Cavity Optomechanics

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Output entanglement is a key element in quantum information processing. Here, we show how output entanglement saturation emerges, and how to obtain optimal output entanglement between two filtered output fields in a three-mode optomechanical system. First, we obtain the expression of optimal time delay between the two filtered output fields, from which we obtain the optimal coupling for output entanglement in the case of without time delay. In this case, we find the optimal output entanglement will saturate if the driving strength is strong enough, and we give the expression of saturation value. Furthermore, we obtain the optimal output entanglement with optimal time delay. These results can be used to realize quantum information processing in cavity optomechanics.

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Introduction.—Entanglement is the distinguishing feature of quantum mechanics because it is responsible for nonlocal correlations between observables and now it has become a basic resource for many quantum information processing schemes [1]. For example, entanglement is required in quantum teleportation in which quantum information can be transmitted from one location to another with security. Extending entanglement into macroscopic systems has become a prominent experimental objective, and would allow us to explore the quantum-classical boundary. So far, a number of theoretical and experimental works on entanglement between macroscopic objects have been studied, such as between atomic ensembles [2, 3], between superconducting qubits [4–7], and between mechanical oscillator and microwave fields [8]. Recently, quantum entanglement in cavity optomechanics has received increasing attention for the potential to use radiation pressure to generate various entanglement between subsystems [9–38]. Due to the small mechanical decay rate in optomechanical systems, the information can be stored in mechanical modes for long time. Hence, entangled optomechanical systems could be profitably used for the realization of quantum communication networks, in which the mechanical modes play the role of local nodes where quantum information can be stored and retrieved, and optical modes carry this information between the nodes [9–12].

In fact, any quantum communication application involves traveling output modes rather than intracavity ones. Therefore, it is very important to study how to obtain optimal output entanglement in cavity optomechanics. Here, we use a three-mode optomechanical system (see Fig. 1) to generate output entanglement between two filtered output optical fields. This setup has been realized in several recent experiments [39–41]. Because in such a system the parametric-amplifier interaction and the beam-splitter interaction can entangle the two intracavity modes, the output cavity ones are also entangled with each other. In previous theory works [34–38], the output entanglement in this model has been studied. But in most of them, the output entanglement were studied with equal-coupling, zero filter bandwidth or without time delay between the two output fields.

In this paper, we mainly focus on how output entanglement saturation emerges, and how to obtain optimal output entanglement with optimal coupling and optimal time delay for large bandwidth. First, we obtain the important expression of optimal time delay between the two filtered output fields. With large bandwidth and no time delay, the optimal output entanglement appears at the point where the optimal time delay equals zero, from which we obtain the expression of optimal coupling. It
is very interesting that the optimal output entanglement will saturate if the driving field is strong enough, and the saturation value is sensitive to filter bandwidth. As far as we know, the phenomenon of entanglement saturation has not been studied before. Finally, we obtain the optimal output entanglement with optimal time delay and optimal couplings. We believe the results of this paper are very important to experimental and theoretical physicists who work on entanglement or quantum information processing.

**System model.**—We consider a three-mode optomechanical system in which two cavities are coupled to a common mechanical resonator (see Fig. 1). The Hamiltonian of the system reads

$$H = \omega_m \hat{b}^\dagger \hat{b} + \sum_{i=1,2} [\omega_i \hat{a}_i^\dagger \hat{a}_i + g_i (\hat{b}^\dagger + \hat{b}) \hat{a}_i^\dagger \hat{a}_i].$$  \tag{1}$$

Here, $\hat{a}_i$ is the annihilation operator for cavity $i$ with frequency $\omega_i$ and damping rate $\kappa_i$, $\hat{b}$ is the annihilation operator for mechanics resonator with frequency $\omega_m$ and damping rate $\gamma$, and $g_i$ is the optomechanical coupling strength. In order to generate the steady entanglement between the two output fields, we drive cavity 1 (2) at the red (blue) sideband with respect to mechanical resonator: $\omega_{d1} = \omega_1 - \omega_m$ and $\omega_{d2} = \omega_2 + \omega_m$. If we work in a rotating frame with respect to the free Hamiltonian, following the standard linearization procedure, and making the rotating-wave approximation (in this paper, we focus on the resolved-sideband regime $\omega_m \gg \kappa_1, \kappa_2$), hence, the Hamiltonian of the system can be written as

$$\hat{H}_{\text{int}} = G_1 \hat{b}^\dagger \hat{d}_1 + G_2 \hat{b} \hat{d}_2 + \text{H.c.}$$  \tag{2}$$

Here, $\hat{d}_i = \hat{a}_i - \hat{\bar{a}}_i$, $\hat{\bar{a}}_i$ being the classical cavity amplitude. $G_i$ is the effective coupling strength which can be easily controlled by adjusting the strength of driving fields. Here, without loss of generality, we take $G_i$ as real number.

Based on Eq. (2), the dynamics of the system is described by the following quantum Langevin equations for relevant operators of mechanical and optical modes

$$\frac{d}{dt} \hat{b} = -\frac{\gamma}{2} \hat{b} - i(G_1 \hat{d}_1 + G_2 \hat{d}_2) - \sqrt{\gamma} \hat{b}^{\text{in}},$$

$$\frac{d}{dt} \hat{d}_1 = -\frac{\kappa_1}{2} \hat{d}_1 - iG_1 \hat{b} - \sqrt{\kappa_1} \hat{d}^{\text{in}},$$

$$\frac{d}{dt} \hat{d}_2 = -\frac{\kappa_2}{2} \hat{d}_2 + iG_2 \hat{b} - \sqrt{\kappa_2} \hat{d}^{\text{in}}.$$

Here, $\hat{b}^{\text{in}}, \hat{d}^{\text{in}}$ are the input noise operators of mechanical resonator and cavity $i (i = 1, 2)$, whose correlation functions are $\langle \hat{b}^{\text{in}}(t) \hat{b}^{\text{in}}(t') \rangle = N_m \delta(t - t')$ and $\langle \hat{d}^{\text{in}}(t) \hat{d}^{\text{in}}(t') \rangle = N_i \delta(t - t')$ respectively. $N_m$ and $N_i$ are the average thermal populations of mechanical mode and cavity $i$, respectively. In the following discussion, we mainly study how output entanglement saturation emerges, so we assume these average thermal populations are zero (zero temperature). According to the Routh-Hurwitz stability conditions [12] and we focus on the regime of strong cooperativities $C_i \equiv 4G_i^2/\gamma_1 \gg 1$ and $\kappa_i \gg \gamma$ in this paper, the stability condition of our system can be obtained as $G_2^2 / G_1^2 > \max(\kappa_1 / \kappa_2, \kappa_2 / \kappa_1)$ for $\kappa_1 \neq \kappa_2$, and the system is always stable if $\kappa_1 = \kappa_2$ and $G_2 \leq G_1$ [24].

**Output fields and optimal time delay.**—In a quantum network, entangled output photon pairs are a useful resource for quantum information processing. Here, the output entanglement can be generated when the output optical fields pass through the filter functions $f(\omega)$ which satisfy $\int_{-\infty}^{\infty} |f(\omega)|^2 d\omega = 1$. For simplicity, we adopt a rectangle filter with a bandwidth $\sigma$ centered about the frequency $\omega$ to generate the output temporal modes, i.e., $f(\omega') = \{\theta[\omega' - (\omega - \frac{\sigma}{2})] - \theta[\omega' - (\omega + \frac{\sigma}{2})]\}/\sqrt{\sigma}$ with $\theta[\omega]$ the Heaviside step function. Then, the filtered optical output fields can be written as

$$D_{\text{out}}^{\text{opt}}[\omega, \sigma, \tau] = \frac{1}{\sqrt{\sigma}} \int_{-\omega}^{\omega} d\omega' e^{-i\omega' \tau} \hat{d}_{\text{out}}^{\text{opt}}(\omega').$$  \tag{4}$$

Here, $\omega_\pm = \omega \pm \frac{\sigma}{2}$, and $\tau_1$ is the absolute time at which the wave packet of interest is emitted from cavity $i$. The time delay between this two output fields is defined as $\tau = \tau_1 - \tau_2$. Without loss of generality, we set $\tau_1 = \tau$ and $\tau_2 = 0$. Using the Langevin equation Eq. (3) and the input-output relation [43], we can derive the output operators in terms of the input operators. And we use the logarithmic negativity [44, 45] to quantify the entanglement between the filtered output cavity modes $D_{\text{out}}^{\text{opt}}[\omega, \sigma, \tau]$ and $D_{\text{out}}^{\text{opt}}[-\omega, \sigma, 0]$. We write $D_{\text{out}}^{\text{opt}}[\omega, \sigma, \tau]$ as $\hat{D}_i$ and set equal cavity damping rate $\kappa_1 = \kappa_2 = \kappa$ for simplicity, and the mechanical damping rate $\gamma$ is much less than other parameters.

It can be proven that our system is equivalent with a two-mode squeezed thermal state [36]. From the equivalent relation, it can be concluded that the optimal time delay for output entanglement between the two filtered output fields is the time delay that makes the modulus of the correlator $\langle \hat{D}_1 \hat{D}_2 \rangle$ reach a maximum [36]. Through complex calculation, we obtain the expression of the optimal time delay for resonant frequency ($\omega = 0$ in the rotating frame) as follows

$$\tau_{\text{opt}} = \frac{20(G_2^2 - G_1^2) + 5\kappa^2 + 3\sigma^2}{10(G_1^2 + G_2^2)\kappa}.$$  \tag{5}$$

For the special case of equal coupling ($G_1 = G_2$) and $\sigma \ll \kappa$, the expression of $\tau_{\text{opt}}$ will become $\kappa / 4G_1^2$ which is consistent with the result in Ref. [36, 57]. It can be seen from Eq. (5) the optimal time delay $\tau_{\text{opt}}$ will be positive with the increase of $G_2$. We will see that the coupling $G_2$ at which the optimal time delay equals zero is a very special coupling. In the following, we study the output entanglement in two cases, without and with time delay respectively.

**Optimal couplings and output entanglement saturation.**—In this part, we study how to obtain the optimal
coupling and the output entanglement saturation in the case of without time delay ($\tau = 0$). The entangling interaction $G_2(\hat{b}\hat{d}_2 + \hat{b}^\dagger\hat{d}_2^\dagger)$ in $H_{\text{int}}$ entangles the mechanical resonator $\hat{b}$ and cavity mode $\hat{d}_2$, and the beam splitter interaction $G_1(\hat{b}^\dagger\hat{d}_1 + \hat{b}\hat{d}_1^\dagger)$ swaps the $\hat{b}$ and $\hat{d}_1$ states, these two combined interactions yield the net entanglement between $\hat{d}_1$ and $\hat{d}_2$. The entanglement generated within the cavities can be transferred to the filtered output fields. It is obvious that the output entanglement will disappear as coupling $G_2 = 0$. Moreover, the output entanglement for resonant frequency almost equals zero in the case of equal-coupling $G_2 = G_1$. Therefore, it is certain that there will be an optimal coupling $G_2^{\text{opt}}$ that makes the output entanglement $E_N(\omega = 0)$ reach a maximum.

Before getting the expression of optimal coupling $G_2^{\text{opt}}$, we do some analysis first. It is certain that the output entanglement with optimal time delay will be larger than that without time delay. In addition, it can be seen from Eq. (5) that the optimal time delay may equal zero. It means that the curve of output entanglement with optimal time delay will be tangent to that without time delay at the point where the optimal time delay equals zero. Hence, we guess the output entanglement without time delay. The other parameters are $\gamma = 1, \kappa = 10^3, G_1 = 10\kappa$.

![Image](FIG. 2: (Color online) (a) The output entanglement $E_N(\omega = 0)$ are plotted with large bandwidth $\sigma = \kappa/10$ (black line), $\sigma = \kappa$ (red line), and the two dashed lines are their corresponding ones with numerical optimal time delay. (b) The output entanglement are plotted with small bandwidth $\sigma = \kappa/10^4$ (red line), $\sigma = 2\kappa/10^3$ (yellow line), the boundary bandwidth $\sigma_b = \sqrt{4\kappa^3}/4G_1$ (violet line), and their common line with numerical optimal time delay (blue dashed line). (c) The output entanglement $E_N(\omega = 0)$ vs $G_1/\kappa$ with optimal coupling Eq. (6) for bandwidth $\sigma = \kappa/10$ (black line), $\sigma = \kappa/2$ (blue line), $\sigma = \kappa$ (red line), and the saturation values are plotted according to Eq. (8) (green dashed lines).)

From Eq. (6) and Eq. (7), we obtain the boundary between small bandwidth and large bandwidth as $\sigma_b = \sqrt{3\kappa^3}/4G_1^2$. We also plot the output entanglement $E_N(\omega = 0)$ with boundary bandwidth $\sigma = \sigma_b$ (violet line) in Fig. 2(b) from which it can be seen the maximum value of output entanglement will not appear at the tangent point anymore as $\sigma < \sigma_b$.

Based on Eq. (6) and Eq. (7), we can obtain the expressions about maximum value of output entanglement. But most of the expressions are lengthy. Here, we just give an interesting result for large bandwidth, i.e. output entanglement saturation. In general, entanglement will increase with the increase of driving strength. But we find the optimal output entanglement $E_N(\omega = 0)$ (with optimal coupling Eq. (6)) will saturate as $G_1 \gg \sqrt{\frac{\kappa^3}{\gamma_0^{\text{opt}}}}$. In Fig. 2(c), we plot optimal output entanglement $E_N(\omega = 0)$ vs $G_1/\kappa$ for bandwidth $\sigma = \kappa$ (red line), $\sigma = \kappa/2$ (blue line), $\sigma = \kappa/10$ (black line). It can be seen clearly from Fig. 2(c) that the optimal output entanglement will approach a constant (saturation value) with the increase of coupling $G_1$. The saturation value can be obtained as

$$E_N^{\text{sat}} = -\ln\sqrt{\frac{(\kappa^2 + \sigma^2)(15\kappa^2\sigma + 4\sigma^3 - 3\sigma^2\beta^2)}{9\kappa^2\sigma^2\alpha^2}}$$
with $\alpha = 5\kappa^2 + 3\sigma^2, \beta = \kappa \arctan \left( \frac{\omega}{\kappa} \right)$. The saturation value plotted according to Eq. (8) in Fig. 2(c) (three green dashed lines) perfectly fits the numerical result. Due to entanglement saturation, the coupling $G_1$ is not the stronger the better. It can also be seen from Fig. 2(c) that for larger bandwidth, such as $\sigma = \kappa$ (red line), the coupling $G_1$ slightly greater than $\kappa$ can make output entanglement saturation occur. For the case of $\sigma \ll \kappa$, the saturation value Eq. (8) can be simplified as $\ln \left[ \frac{175\kappa^5}{4\sigma^2} \right]$ which is sensitive to filter bandwidth for high power.

**Optimal output entanglement**—Now, we study how to find the optimal output entanglement with optimal time delay ($\tau = \tau_{\text{opt}}$). First, we plot the optimal time delay vs. $G_2/G_1$ according to Eq. (5) (blue solid line) and the numerical result (red dashed line) in Fig. 3(a). The optimal time delay will become positive with the increase of $G_2$ (see the inset of Fig. 3(a)). In Fig. 3(b), we plot the corresponding output entanglement with these two optimal time delay with parameters $\gamma = 1, \kappa = 10^5, \sigma = \kappa, G_1 = 10\kappa$. The two curves of output entanglement fit very well except a very small area (see the inset of Fig. 3(b)). It can be seen from Fig. 3(b) that the maximum of output entanglement does not appear at the point where the optimal time delay equals zero. If $G_1 \gg \kappa, \sigma$, we can obtain the optimal coupling and the optimal output entanglement as

$$G_2^{\text{opt}} = G_1 - \left( \frac{\alpha (15\kappa^2\sigma + 4\sigma^3 - 3\alpha\beta)}{400(6\sigma - 3\beta)} \right)^{1/4} \quad (9)$$

and

$$E_N^{\text{opt}} = -\ln \left[ \frac{\alpha (2\sigma - \beta) (15\kappa^2\sigma + 4\sigma^3 - 3\alpha\beta)}{4800G_1\sigma^2} \right]. \quad (10)$$

We plot the highest point according to Eq. (9) and Eq. (10) in Fig. 3(b) (see the green dot), and it fits the numerical result very well.

Until now, we study the output entanglement just for resonant frequency ($\omega = 0$). In Fig. 3(c), we plot output entanglement $E_N(\omega)$ vs. $\omega/\kappa$ with optimal coupling Eq. (9) and with numerical optimal time delay (red line). It is obviously seen from Fig. 3(c) that the optimal output entanglement Eq. (10) is just the optimal one in the whole center frequency domain of output fields. For contrast, we also plot the output entanglement with equal-coupling ($G_2 = G_1$) and without time delay ($\tau = 0$) (blue dashed line). It can be seen from Fig. 3(c) the output entanglement in the vicinity of resonant frequency can be largely enhanced by adopting optimal coupling Eq. (9) and optimal time delay. In experiment, it just need to adjust the coupling strength $G_2$ according to the parameters of the systems to obtain the optimal output entanglement.

**Conclusions.**—In summary, we have studied theoretically the output entanglement between two filtered output fields in a three-mode cavity optomechanical system. We obtain the important expression of optimal time delay Eq. (5) between the two filtered output fields. Based on the analysis about this expression, we obtain the optimal coupling Eq. (6) for output entanglement in the case of without time delay. We give a reasonable boundary between large bandwidth and small bandwidth. For large bandwidth, the optimal output entanglement will saturate if the coupling strength satisfies $G_1 \gg \sqrt{\frac{\kappa^2}{4\sigma^2}}$, and the saturation value is obtained as Eq. (8). In addition, using the optimal coupling Eq. (9) and optimal time delay, we find the optimal output entanglement Eq. (10) for resonant frequency ($\omega = 0$) is just the optimal one in the whole center frequency domain of output fields. Our results can also be applied to other parametrically coupled three-mode bosonic systems, and may be useful to experimentalists to obtain large entanglement.

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[1] S. L. Braunstein, P. van Loock, Rev. Mod. Phys. 77, 513C577 (2005).
[2] B. Julsgaard, A. Kozhekin, and E. S. Polzik, Nature (London) 413, 400-403 (2001).
[3] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, I. C. Cirac, and E. S. Polzik, Phys. Rev. Lett. 107, 080503 (2011).
[4] A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch, P. R. Johnson, J. R. Anderson, A. J. Dragt, C. J. Lobb, F. C. Wellstood, Science 300, 1548-1550 (2010).
[5] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, I. C. Cirac, and E. S. Polzik, Phys. Rev. Lett. 107, 080503 (2011).
[6] L. DiCarlo, M. Reed, L. Sun, B. L. Johnson, J. M. Gambetta, L. Frunzio, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Nature (London) 467, 574 (2010).
[7] E. Flurin, N. Roch, F. Mallet, M. H. Devoret, and B. Huard, Phys. Rev. Lett. 109, 183901 (2012).
[8] T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Science 342, 710 (2013).
[9] S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. Lett. 90, 137901 (2003).
[10] S. Pirandola, S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. A 68, 062317 (2003).
[11] S. Pirandola, S. Mancini, D. Vitali, and P. Tombesi, J. Mod. Opt 51, 901 (2004).
[12] S. Pirandola, D. Vitali, P. Tombesi, and S. Lloyd, Phys. Rev. Lett. 97, 183901 (2012).
[13] S. Kiesewetter, Q. Y. He, P. D. Drummond, and M. D. Reid, Phys. Rev. A 90, 043805 (2014).
[14] M. Bhattacharya, P.-L. Giscard, and P. Meystre, Phys. Rev. A 77, 030303(R) (2008).
[15] R. X. Chen, L. T. Shen, Z. B. Yang, H. Z. Wu, and S. B. Zheng, Phys. Rev. A 89, 023843 (2014).
[16] J. Q. Liao, Q. Q. Wu, and F. Nori, Phys. Rev. A 89, 014302 (2014).
[17] C. J. Yang, J. H. An, W. Yang, and Y. Li, Phys. Rev. A 92, 062311 (2015).
[18] M. Paternostro, D. Vitali, S. Gigan, M. S. Kim, C. Brukner, J. Eisert, and M. Aspelmeyer, Phys. Rev. Lett. 99, 250401 (2007).
[19] C. Wipf, T. Corbitt, Y. Chen, and N. Mavalvala, New J. Phys. 10, 095017 (2008).
[20] C. Genes, A. Mari, P. Tombesi, and D. Vitali, Phys. Rev. A 78, 032316 (2008).
[21] Sh. Barzanjeh, D. Vitali, P. Tombesi, and G. J. Milburn, Phys. Rev. A 84, 042342 (2011).
[22] Sh. Barzanjeh, M. Abdi, G. J. Milburn, P. Tombesi, and D. Vitali, Phys. Rev. Lett. 109, 130503 (2012).
[23] Sh. Barzanjeh, S. Pirandola, and C. Weedbrook, Phys. Rev. A 88, 042331 (2013).
[24] Y.-D. Wang and A. A. Clerk, Phys. Rev. Lett. 110, 253601 (2013).
[25] M. C. Kuzyk, S. J. van Enk, and H. Wang, Phys. Rev. A 88, 062341 (2013).
[26] D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Phys. Rev. Lett. 98, 030405 (2007).
[27] S. G. Hofer, W. Wieczorek, M. Aspelmeyer, and K. Hammerer, Phys. Rev. A 84, 052327 (2011).
[28] U. Akram, W. Munro, K. Nemoto, and G. J. Milburn, Phys. Rev. A 86, 042306 (2012).
[29] K. Sinha, S. Y. Lin, and B. L. Hu, Phys. Rev. A 92, 023852 (2015).
[30] J. Li, I. Moaddel Haghighi, N. Malossi, S. Zippilli, and D. Vitali, New J. Phys. 17, 103037 (2015).
[31] J. Li, G. Li, S. Zippilli, D. Vitali, and T. Zhang, Phys. Rev. A 95, 043819 (2017).
[32] M. Asjad, P. Tombesi, and D. Vitali, Phys. Rev. A 94, 052312 (2016).
[33] Q. Liu, B. He, L. Yang, M. Xiao, arXiv:1704.05445.
[34] L. Tian, Phys. Rev. Lett. 110, 233602 (2013).
[35] Z. J. Deng, S. J. M. Habraken, and F. Marquardt, New J. Phys. 18, 063022 (2016).
[36] Z. J. Deng, X. B. Yan, Y. D. Wang, and C. W. Wu, Phys. Rev. A 93, 033842 (2016).
[37] Y.-D. Wang, S. Chesi, and A. A. Clerk, Phys. Rev. A 91, 013807 (2015).
[38] X. B. Yan, Phys. Rev. A 96, 053831 (2017).
[39] C. Dong, V. Fiore, M. C. Kuzyk, and H. Wang, Science 338, 1609 (2012).
[40] J. T. Hill, A. H. Safavi-Naeini, J. Chan, and O. Painter, Nat. Commun. 3, 1196 (2012).
[41] R. Andrews, R. W. Peterson, T. P. Purdy, K. Cicak, R. W. Simmonds, C. A. Regal, and K. W. Lehnert, Nat. Phys. 10, 321 (2014).
[42] E. X. DeJesus and C. Kaufman, Phys. Rev. A 35, 5288 (1987).
[43] C. Gardiner and P. Zoller, Quantum Noise, 3rd ed. (Springer, New York, 2004).
[44] G. Vidal and R. F. Werner, Phys. Rev. A 65, 032314 (2002).
[45] M. B. Plenio, Phys. Rev. Lett. 95, 090503 (2005).