Site index and height-growth curves for *Eucalyptus urograndis*

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**Abstract.** Site index curves were adjusted for *Eucalyptus urograndis* stands located in the municipality of Nova Canaã, MT, using the guide curve method. Six mathematical models were tested to estimate the dominant height of the trees according to age and to verify their accuracy for the definition of site indices. The models tested presented similar values with $R^2$ values around 0.40 and errors close to 15%, although none presented expressive superiority in relation to the others. The Schumacher model was used for the construction of curves with four classes of site and with amplitude of three meters each. The model was chosen by statistics presented, for the simplicity and adequate estimation of the heights of the trees with only two parameters, presenting the best graphical distribution of the residues.

**Keywords:** Curve-Guide; Anamorphism, productivity

**Introduction**

In the forest sciences the site is used to designate the influence of the environment on the production of wood, or other products obtained from a forest stand (Batista & Couto, 1986). In forest plantations whose purpose is logging the quality of the site is defined as its potential for the production of wood of a species or of a forest considering that better qualities had higher production (Clutter et al., 1983). In the management of stands and in the timber production planning of a forestry company, determining the productivity of forest sites is a basic factor.

According to Campos & Leite (2013), the quality of the site or productive capacity of the site can be defined as the potential that a given place has for the production of wood or another product, considering a particular species or clone. To quantify the production capacity of a stand and to define a site index, a numerical expression of the quality of the site is generally used taking the dominant height of the forest stands.

Site index (SI) is a measurement commonly used by foresters to describe the productivity of a site. Typically this measurement is used to describe sites growing well-stocked even-aged forests. Site index is the average height of the dominant and codominant trees on the site, at a given age (base age). The higher the SI, the higher the site productivity (trees will grow faster than on a site with a lower SI). (Clutter et al., 1983).

For the definition of this site index, equations are used that describe the functional relationships between dominant height and age. With these equations one can then construct the site index curves, where each curve represents a site index, which corresponds to the average height of the dominant trees at a reference age, called the index age. The definition of this index age is arbitrary, but in general it must be close to the cutoff shift.

The classification of the productive capacity of a stand is an important task to be done since there are several attributes that depend on the greater or lesser productivity of the site. These attributes can be the dimension of forest products at different ages; the viability or otherwise of forestry projects; the different responses of certain forest crops to the conditions of the site; The identification of the productivity potential of forest property provides consistent information for the diagnosis and prescription of soil management and conservation.

To quantify the productive capacity of stands of the same age, the most usual method in forest management is the determination of site indices, estimated as a function of height and age of the trees. This method has been widely used mainly because it quantifies the productive capacity of the site, through a numerical expression (Husch et al., 1993).

The most appropriate way to express this index is by relating the dominant height of the trees with their age (Campos et al., 1985). To construct the productivity curve, it is necessary to define a mathematical model that represents the relationship of height with age, as well as defining the behavior of the family of curves constructed from the model.
Different mathematical models have been tried and used to represent the evolution of dominant height with age. The guide curve method allows to generate only anamorphic curves. One of the most used models with this method is the Schumacher model, which represents a family of height / age curves. In this way, this work aims to define classes of site index for stands of *Eucalyptus urograndis*, located outside the municipality of Nova Canaã del Norte in the state of Mato Grosso.

Methods

The study area consists of a 15-hectare plantation of *Eucalyptus urograndis* located on the Gamada estate, municipality of Nova Canaã del Norte, Mato Grosso state.

The climate of the region, according to the Köppen classification, is tropical rainy with a clear dry season, with average annual temperatures of 26°C, and average annual precipitation of 2175 mm, with intensities of greater in January, February and March (Alvares et al., 2013). The topography of the area is flat, the soil classified as Latosol Red Yellow (Oliveira et al., 2010).

*Eucalyptus urograndis* was planted in January 2009, with spacing of 4 x 3 meters, with a total area of 15 hectares, in the preparation of the soil was applied fertilizer coverage 20-00-10, plus 2% Boron and 1% Zin. at 10 and 14 months I apply NPK (01-18-18) in a dose of 200g per plant.

Twenty permanent plots of rectangular format (20 by 30 meters) with area of 600 m² each, randomly distributed were established. The database includes measurements of height and diameter at 1.30 m in the ages of 19, 25, 33, 39 and 52 months. We used pairs of dominant heights and ages of the data obtained in the 20 permanent plots. Thus, the pairs of ages and heights are comprised between the age of 19 and 52 months.

With the data of the carried out inventory different traditional mathematical models, existing in the literature, that express the relation between average height of the dominant trees and the age (Table 1) were tested.

### Table 1. Models adjusted by the guide curve method to express the local productive capacity of *Eucalyptus urograndis* stands in the northern region of Mato Grosso, Brazil.

| Identification | Model | Author |
|----------------|-------|--------|
| Model 1        | \( \ln(H_{dom}) = \beta_0 + \beta_1 \frac{1}{ld} \) | Schumacher | (1) |
| Model 2        | \( H_{dom} = \beta_0 + \beta_1 \frac{id}{1+\beta_2 \cdot id} \) | Prodan | (2) |
| Model 3        | \( H_{dom} = \beta_0 + \beta_1 \left[ 1 - \exp(\beta_2 \cdot id) \right] \) | Spillman | (3) |
| Model 4        | \( H_{dom} = \beta_0 + \beta_1 \frac{1}{ld} \) | Curtis | (4) |
| Model 5        | \( \log(H_{dom}) = \beta_0 + \beta_1 \left( \log10 \cdot (id) \right) \) | Smalley & Bower | (5) |
| Model 6        | \( H_{dom} = \beta_0 + \beta_1 \frac{1}{\log(id)} \) | Larson & Moehring | (6) |

Where: \( \ln = \) natural logarithm; \( \log = \) Logarithm; \( H_{dom} = \) Height of dominant trees (m); \( ld = \) Age of stands; \( \beta_0; \beta_1; \beta_2 = \) Coefficients to be obtained.

The adjustment of the models was carried out using the statistical program SPSS and the selection of the best adjusted equation considered the following measures of precision: Graphic analysis of waste expressed as a percentage; stability; Absolute Residual Standard Error (Syx) and percentage (Syx%); Coefficient of adjusted determination (R²aj). The adjustment of the non-linear models was performed by means of non-linear regression analysis.

The anamorphism test is applied to verify the existence of a linear relationship between site indices and the mean dominant heights at different ages. However, before verifying that linear relationship through simple linear regression, it was necessary to find the value of the site index corresponding to each Hdom value, by means of the following mathematical expression.

\[
I_{ref} = \ln\left( \frac{H_{dom}}{\exp(b_0) \cdot H_{dom}^{b_2}} \right) \tag{1}
\]

Where: \( I_{ref} = \) reference age; \( H_{dom} = \) dominant height; \( b_0 = \) coefficient of the mathematical model adjusted for the classification of sites.

Subsequently, the reliability of the anamorphic curves was checked by verifying the existence of the linear relationship between the sitio indices and the average dominant heights and the different ages, by means of the adjustment of the following statistical mode

\[
I_{S} = b_0 + b_1 \cdot H_{dom} \tag{2}
\]

Where: \( I_{S} = \) site index; \( H_{dom} = \) mean dominant height at different ages; \( b_0 \) and \( b_1 \) = coefficients estimated by the fit of the model.

The simple linear model was adjusted to confirm the hypothesis that there is a strong linear relationship between the analyzed variables, expressing that the site index does not depend on age, but on the productive capacity of the premises.

The average height of the dominant trees at 19 and 52 months was 7.41 to 19.49 meters, respectively. The mean height values of the dominant trees are distributed in the different
classes of site, in the different ages. With the average height of the dominant trees in the age-index they have the lower and upper limit. The difference of the lower and upper limit results in the total amplitude, which divided by the total amplitude (four), results in the amplitude between classes.

The site curves were constructed for each model tested, with amplitude of 3 m. This amplitude was defined as a function of the distribution of the observed values of $h_{dom} \times \text{Age}$ at various ages and at the index age of 72 months. Thus, four site classes were defined for each model and the site indices defined were 20, 23, 26 and 29 m at the index age of 72 months. This procedure allowed the curves to encompass all data and with integer values, facilitating their practical application.

### Results and discussion

At 52 months of age, the stands of *Eucalyptus urograndis* had the following characteristics: $Dap = 16.71$ cm ($Syx = 0.32$), $Ht = 19.49$ m ($Syx = 0.29$), $g = 0.022$ m$^2$ (Syx = 0.00084), $G = 18.99$m$^2$ ha$^{-1}$, and $Vt = 178.97$m$^3$ ha$^{-1}$.

#### Adjusted model

The statistical parameters obtained allowed the selection of the model that best fit the data, observing the values of the coefficient of determination adjustment ($R^2$ adjust), the standard error of estimation ($Syx$) Table 2. The model by Schumacher, Prodan and Larson and Moehring presented adjusted precision coefficients greater than 0.90, indicating that more than 90% of the total variation of the independent variable (Hdom) was explained by the regression. These models presented very close adjustment and precision statistics as can be observed by the distribution of the residues (Figure 1).

#### Table 2. Coefficients and adjustment statistics of different models for classification of site indices for *Eucalyptus urograndis* in the northern region of Mato Grosso.

| Model | $b_0$  | $b_1$  | $b_2$  | $R^2$ | $R^2_{\text{adj}}$ | (Syx) |
|-------|--------|--------|--------|-------|---------------------|-------|
| 1     | 3.62   | -31.82 | -0.008 | 0.93  | 0.93                | 0.09  |
| 2     | -8.73  | 0.97   | -0.008 | 0.91  | 0.91                | 1.46  |
| 3     | 0.36   | 7.48   |        | 0.87  | 0.87                | 1.741 |
| 4     | 27.11  | -395.5 |        | 0.89  | 0.88                | 1.64  |
| 5     | 0.20   | 0.15   |        | 0.88  | 0.88                | 0.05  |
| 6     | -32.43 | 30.83  |        | 0.90  | 0.90                | 1.50  |

Table 2 shows the results of the adjustments of the linear model for the anamorphism test, which relates the site index with the dominant height in the different ages.

The arbitrarily selected reference age was 72 months, in which the dominant height of the different sites was compared. For the selection of this parameter, the rotation of the cutting shift was taken into account for the species in Brazil that considers the age when the species is managed in total density.

It is observed, however, that the studied stands have not yet reached this age of rotation and the latest data were collected at 52 months of age.

From the estimated site index values for each plot, a linear equation was adjusted between the site indices and the average dominant heights in the different ages considered. Therefore, for each position studied, an equation of the site index was obtained as a function of the dominant height.

The Spillman model was the one that presented the worst statistics of fit and precision, in addition to the distribution of residuals with biased estimation and overestimation for the intermediate values of the estimated dependent variable (Figure 1). Among the models with the best performance are the Schumacher, Prodan and Larson & Moehring models.

The site index curves were constructed from the Schumacher model (Figure 2), and the upper and lower limits of the dominant heights are presented, in meters, for each of the four room classes and for all the stages (Table 3). With these data (Table 3) and knowing the age and dominant height it is possible to make the classification of site for plantations of *E. urograndis* for the region where the plantation is established.

When comparing the models of Schumacher and Prodan, we find that the Schumacher model has advantages, this model better describes the variation of dominant height as a function of age using only two coefficients.

The Schumacher model for the construction of the site index curves selected in this study was tested by other authors for different coniferous stands and broadleaf stands in different geographical areas, using Santos (2012) for *Eucalyptus spp.* in Araripe, Pernambuco, by Téo et al. (2011) for *Pinus taeda* in the region of Cazador-SC, and Miranda et al. (2015) for *Eucalyptus* in the state of Bahia. It was also used by Crechi et al. (2011) for *Eucalyptus grandis* in Mesopotamia Argentina, by Jerez-Rico et al., (2011) for Teak plantations in Venezuela.

Thus, Schumacher’s model was selected for the statistics presented, for the simplicity and for estimating well the heights of the trees with only two parameters that present the best graphic distributions of the waste.

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Figure 1. Residual dispersion of the models tested with the variables $H_{dom}$, Age

Figure 2. Site index curves for *Eucalyptus urograndis* stands in the northern region of Mato Grosso, Brazil.

| Age (months) | Site Index |
|--------------|------------|------------|------------|------------|
|              | I (20)     | II (23)    | III (26)   | IV (29)    |
| 19           | 5,83       | 6,70       | 7,58       | 8,45       |
| 25           | 8,71       | 10,02      | 11,33      | 12,63      |
| 33           | 11,86      | 13,64      | 15,42      | 17,20      |
| 39           | 13,76      | 15,82      | 17,89      | 19,95      |
| 52           | 16,87      | 19,40      | 21,94      | 42,47      |
| 60           | 18,31      | 21,05      | 23,80      | 26,55      |
| 72           | 20,00      | 23,00      | 26,00      | 29,00      |
| 84           | 21,30      | 24,50      | 27,69      | 30,89      |
Classification of the site by the guide curve method.

Taking into account that the site index curves present an anamorphic pattern in which the growth trend in height is the same for all sites and the slope is constant and common, varying only the point of intersection, we used the method of the guide curve to generate the site index curves. The data of the permanent plots were adjusted to the Schumacher model, being the age measured in months; the guide curve was established with their respective dominant heights in different ages.

The guide curve method is considered a relatively simple method used to generate equations for anamorphic site index curves. The use of a greater or lesser number of site classes depends on the details desired for site classification, values between 2 and 7 site classes being common in forestry. This method was used by Miguel et al. (2011) for plantations of *Eucalyptus urophylla*, by Silva et al. (2015) in young plantation of *Tectona grandis*.

The methods of the guide curve for the classification of productive capacity for stands not intervened by Eucalyptus clear were tested by Castro et al. (2015) indicating the authors efficiency of method, also Retslaff et al., (2015), use several models to construct site curves by the guide curve method. Other examples of construction of site curves for Eucalyptus spp. can be found in Miguel & Leal (2012). Our study is in concurrence with what was pointed out by Souza: (2006) that the classification in homogeneous areas of productivity of each zone or class of site is fundamental for the elaboration and execution of the sustainable management plans.

Knowing the dominant height of any stand of the same age is possible to define in which kind of site is classified and it is also possible to obtain the average dominant height that a stand can reach in a certain place at a certain age.

It should be noted that because the reference age has been simulated, site index curves should be considered provisional. Another fact that must be considered is the procedure adopted to generate the site index curves, that is, the guide curve method.

Although widely used in forestry science, most companies, due to the ease of application and the most reliable results, may present some errors, considering the same rate of growth in height for different sites in different ages. The concept of a same height growth rate for different sites that make up the same family of curves demonstrated in some cases does not accurately represent the growth of the forest stand.

After establishing the linear relationship between the index of site and dominant height ($S = b_0 + b_1 \times H_{dom}$) in the ages under study, it was observed that in all ages there was a strong correlation between the site index and the dominant height, being that at these ages the correlation coefficients (R) were always above 0.99. The intersection $b_0$ was approximately equal to zero. The slope $b_1$ was equal to 1 in the reference age, being greater than 1 for smaller ages and less than 1 for older ages. This behavior indicates that the site index does not depend on the age, but on the productive capacity of the site, which indicates that the site index curves for *E. urograndis* in the northern region of Mato Grosso have an anamorphic pattern. Anamorphic curves were designed by several authors, among them Téo et al. (2015) in stands of *Pinus taeda* L., in the region of Caçador, SC, Brazil, Araújo Júnior et al. (2016) in *Eucalyptus* stands.

Conclusion

The Schumacher model is suitable for the construction of site index curves by the guide curve method, due to the superiority of the plot stability test, although its adjustment statistics are close to the other models.

The guide curve method is suitable for the studied database, although no model has achieved total stability.

The definition of five site classes and the amplitude of 3 meters for each class is sufficient to cover the variability in dominant height of the stand. The constructed site curves are adequate for the classification of *Eucalyptus urograndis* sites in the northern region of Mato Grosso.

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