Reheating in Constant-roll $F(R)$ Gravity

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In this work we address the reheating issue in the context of $F(R)$ gravity, for theories that the inflationary era does not obey the slow-roll condition but the constant-roll condition is assumed. As it is known, the reheating era takes place after the end of the inflationary era, so we investigate the implications of a constant-roll inflation era on the reheating era. We quantify our considerations by calculating the reheating temperature for the constant-roll $R^2$ model and we compare to the standard reheating temperature in the context of $F(R)$ gravity. As we demonstrate, the new reheating temperature may differ from the standard one, and in addition we show how the reheating era may restrict the constant-roll era by constraining the constant-roll parameter.

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I. INTRODUCTION

One of the challenges in modern cosmology is to describe the era which connects the end of the inflationary period with the subsequent radiation and matter domination eras, an era known as reheating era. The Universe after the inflationary era is very cold, due to the abrupt nearly de Sitter expansion, and the matter contained in the Universe needs somehow to be thermalized. It is conceivable that such a task is quite complicated and there exist various proposals in the literature, see for example the recent review [1]. The reheating era invokes many procedures that are involved in what is now known as Big Bang Nucleosynthesis, and during this era the energy density that drove the quasi de Sitter expansion during the inflationary era, will thermalize the matter content of the Universe. In most well-known approaches in reheating, the inflaton field transfers its energy to the Standard Model particles via direct couplings to these fields [2–4]. However, in this context there are some drawbacks of having to fine-tune the couplings significantly, in order to avoid large couplings during the inflationary era. An alternative approach for describing the reheating era is offered by modified gravity, and particularly from $F(R)$ gravity [5], in which case the reheating effects take place once the inflationary era ends. In the modified gravity description, gravity has an effect of the effective equation of state of the matter fields, and in effect, these produce a non-trivial effect on the field equations, which in turn thermalize the matter content of the Universe, see Ref. [5] for the $R^2$ model description of the reheating era.

In most approaches on the description of the reheating era, a slow-roll era is assumed for the preceding inflationary era. Recently however, another interesting research stream described an alternative evolutionary possibility for the inflationary period, know as constant-roll inflation [6–21], see also [22, 23] for an alternative perspective on this issue, and also see Ref. [24] for the $F(R)$ gravity generalization. The constant-roll inflationary models have the interesting property of predicting non-Gaussianities [24], even in the context of the single scalar field models [6–14], so this makes these models conceptually appealing and robust towards future observations of non-Gaussianities in the power spectrum of primordial curvature perturbations.

In a recent work we provided a generalization of the constant-roll inflationary era in the context of $F(R)$ gravity [24], see also [26] for an alternative approach. The focus in this letter is to investigate what are the effects of a constant-roll inflationary era on the reheating process, in the context of $F(R)$ gravity. We shall use the $R^2$ model [24] and we shall calculate the reheating temperature for the case that the preceding inflationary era was a constant-roll one, and we will compare the resulting reheating temperature to the one corresponding to the case that the inflationary era was a usual slow-roll one. As we demonstrate, the ratio of the two reheating temperatures can be quite large, depending strongly on the $F(R)$ gravity model, and also we show that the reheating era can constrain the constant-roll era, since the parameters that quantify the constant-roll era must be constrained for consistency.

This paper is organized as follows: in section II we provide some essential information for the $F(R)$ constant-roll inflationary era. We focus on the $R^2$ inflation case, and as we show, the constant-roll condition affects the rate of the quasi-de Sitter expansion. In section III we investigate the effects of the constant-roll quasi-de Sitter expansion on the reheating era, and we calculate the ratio of the reheating temperatures corresponding to the constant-roll and slow-roll preceding inflationary eras. As we show, the ratio depends on the constant-roll parameter that quantifies the constant-roll era. Finally, the conclusions follow at the end of the article.
II. THE CONSTANT-ROLL INFLATION CONDITION WITH $R^2$ GRAVITY

The constant-roll $F(R)$ gravity inflationary era was introduced in [24] (see also [26] for an alternative viewpoint), and the main assumption was that the constant-roll condition becomes as follows,

$$\frac{\ddot{H}}{2H\dot{H}} \simeq \beta,$$

with $\beta$ being a real parameter. The condition (11) is a natural generalization of the constant-roll condition for a canonical scalar field, since the expression in (11) is the second slow-roll index in the $F(R)$ gravity frame. For the purposes of this paper, we shall consider a vacuum $F(R)$ gravity (for reviews see [29–31]), with the gravitational action being of the following form,

$$S_{F(R)} = \int d^4x\sqrt{-g} \left( \frac{F(R)}{2\kappa^2} \right),$$

with $g$ being the trace of the background metric, which we shall assume to be a flat Friedmann-Robertson-Walker (FRW) metric with line element,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2,$$

where $a(t)$ is the scale factor. Upon variation of the action (2) with respect to the metric tensor, the gravitational equations of motion are,

$$3F_R H^2 = \frac{F_R R - F}{2} - 3H\dot{F}_R,$$

$$-2F_R \dot{H} = \ddot{F} - H\dot{F},$$

where the expression $F_R$ stands for $F_R = \frac{\partial F}{\partial R}$ and the “dot” indicates differentiation of the corresponding quantity with respect to the cosmic time. In Ref. [24], we explored the inflationary dynamics of constant-roll inflation in the context of $F(R)$ gravity, and as we showed it is possible to obtain observational indices of inflation compatible with the Planck observational data. We used two well-known $F(R)$ gravity models in order to exemplify our results, and particularly the $R^2$ model and a power-law $F(R)$ gravity model, and in this paper we shall use the $R^2$ inflation model in order to investigate how the reheating era is modified if a constant-roll inflationary era precedes the reheating era.

The $F(R)$ gravity function in the case of the $R^2$ model [27] has the following form,

$$F(R) = R + \frac{1}{36H_i} R^2,$$

with $H_i$ being a phenomenological parameter with dimensions of mass$^2$, and we assume that $H_i \gg 1$. During the inflationary era we shall assume that the first slow-roll index $\epsilon_1 = -\frac{\dot{H}}{H^2}$ satisfies $\epsilon_1 \ll 1$, and also that the constant-roll condition (11) holds true. Hence, during the era for which the conditions $\dot{H} \ll H^2$ and also $\ddot{H} \sim 2\beta H \dot{H}$ hold true, the gravitational equations (4) and (5) can be written as follows,

$$\ddot{H} - \frac{\dot{H}^2}{2H} + 3H\dot{H} = -3H\ddot{H}, \quad \ddot{R} + 3HR + 6H_i R = 0.$$

In view of the constant-roll condition $\ddot{H} \sim 2\beta H \dot{H}$, the first differential equation appearing in Eq. (7), can be written as follows

$$\ddot{H} \ddot{H} \left( 2\beta + \frac{\epsilon_1}{2} + 3 \right) \dot{H} = -3H_i,$$

and due to the fact that $\epsilon_1 \ll 1$, by eliminating the $\epsilon_1$-dependent term in Eq. (8), and by solving the resulting differential equation, we obtain the following solution at leading order,

$$H(t) = H_0 - H_i (t - t_k),$$

with the parameter $H_0$ being arithmetically of the order $O(H_i)$. Also, the parameter $H_i$ appearing in Eq. (9) is equal to,

$$H_i = \frac{3H_i}{2\beta + 3}.$$
and in addition, the time instance \( t = t_r \) corresponds to the horizon crossing time instance. Clearly, the cosmological evolution \(^9\) is a quasi-de Sitter evolution, just as in the ordinary slow-roll \( R^2 \) inflation model, with the difference being that in the ordinary \( R^2 \) model case, \( \beta = 0 \) and hence \( H_I \to H_I \). As it was shown in \(^{24}\), the constant-roll inflationary era for the \( R^2 \) model comes to an end, due to the production of curvature fluctuation, which ends the inflationary era. Then, when the constant-roll era ends, the constant-roll condition does not hold true and due to the presence of the term \( \dot{H} \) in the gravitational equations, the oscillating reheating era commences, as we evince in the next section.

### III. REHEATING IN CONSTANT-ROLL \( R^2 \) GRAVITY

After the graceful exit from the inflationary era, the Universe enters an intermediate era, which should make a connection between the inflationary era and the radiation and matter domination eras. During the reheating era, the Standard Model particles are thermalized by the Universe, and this is an important feature of any viable cosmological, since after the inflationary era the Universe is cold due to the abrupt nearly exponential expansion that the Universe underwent during the inflationary era. As we mentioned, we shall investigate how the reheating era is affected due to a constant-roll \( F(R) \) gravity era, and we directly compare the resulting picture with the ordinary slow-roll \( F(R) \) gravity model. We shall focus on the \( R^2 \) model of Eq. \(^3\), although it is expected that similar results can be obtained for any \( F(R) \) gravity. In the following we adopt the approach and notation of Ref. \(^5\). As we show, the effects of a constant-roll quasi-de Sitter evolution can be found directly on the reheating temperature, so we shall make a comparison of the ordinary slow-roll case and the constant-roll case. The reheating era brings new cosmological features into play since the term \( \ddot{H} \) in the corresponding differential equation in Eq. \(^7\), cannot be omitted. In this case, the scalar curvature evolves as a damped oscillation, with a restoring force being of the form \( \sim 3H_I \). During the reheating era, the Hubble rate can be found by solving the following differential equation,

\[
\ddot{H} - \frac{H^2}{2H} + 3H_I H = -3H \dot{H}.
\]

During the reheating era the terms \( \sim \dot{H} \) and \( \sim \frac{H^2}{2H} \) start to dominate the evolution, however the term \( \sim \ddot{H} \) is comparably negligible. Consider that \( t = t_r \) is the time instance that the reheating era commences, so for \( t < t_r \), the Hubble rate is given in Eq. \(^9\), and for \( t > t_r \), by solving the differential equation \(^{11}\), we get the following solution,

\[
H(t) \simeq \frac{3}{\omega} \left[ r + \frac{4(1 - r)}{3(t - r)} + \frac{4}{8}\omega \sin 2\omega(t - t_r) \right] .
\]

The corresponding scale factor during the reheating is,

\[
a(t) = a_r \left( 1 + \frac{\omega(t - t_r)}{4} \right)^{2/3} ,
\]

with \( a_r \) being equal to \( a_r = a_0 e^{\frac{H_I - H_0}{\omega}} \), and \( a_0 \) is the scale factor at the beginning of inflation. In Eq. \(^{12}\), the parameter \( \omega \) is affected by the transition from constant-roll to reheating at \( t = t_r \), and can be determined by using the following condition,

\[
\left| \frac{H^2}{2H} \right| = \left| 3H \dot{H} \right| .
\]

The condition \(^{14}\) combined with the quasi-de Sitter evolution \(^{11}\) and with the reheating evolution \(^{12}\), yields the following form of \( \omega \) and \( t_r \),

\[
\omega = \sqrt{\frac{3H_I}{2}} \quad t_r \simeq H_I H_0 .
\]

An approximate form of the scalar curvature can also be determined, since during the reheating era, the Ricci scalar is approximately equal to \( R \simeq 6\dot{H} \), thus we approximately have,

\[
R(t) \simeq -\frac{6\omega \sin 2\omega(t - t_r)}{\left( \frac{3}{\omega} + \frac{4(1 - r)}{3(t - r)} + \frac{4}{8}\omega \sin 2\omega(t - t_r) \right)} .
\]
At this point recall that the parameter $H_I$ contains a hidden $\beta$-dependence, as it can be seen in Eq. (10), and recall that the parameter $\beta$ is the constant-roll condition parameter of Eq. (1). Essentially the parameter $\beta$ determines the shape and the size of the reheating phase.

Now let us quantify the effects of the constant-roll on the $R^2$ gravity reheating by comparing the reheating temperature for the constant-roll and slow-roll $R^2$ model. In order to calculate the reheating temperature, it is assumed that the matter content consists of a scalar field $\phi$ with gravitational equation $g^{\mu\nu}\phi_{,\mu\nu} = 0$. As it was shown in [3], the effect of the matter content is connected to the square average of the scalar curvature $R$, and the $(t, t)$ component of the field equations yields,

$$H^2 + \frac{1}{18H_I}\left(RH - \frac{R^2}{12}RH^2\right) = \frac{8\pi}{3}G\rho_c,$$

where energy-density term $\rho_c$ is defined to be,

$$\rho_c = \frac{N}{a^3} \int_t^{t_r} \frac{\omega}{1152\pi} \bar{R}^2a^4 dt,$$

and it refers to the constant-roll case. By assuming that during the reheating era, the energy density $\rho_c$ is totally converted to radiation energy $\rho_r = \frac{N\pi^2}{30}T_r^4$, where $N$ is the total number of relativistic particles, while $T_r$ is the reheating temperature corresponding to the case that a constant-roll case precedes the reheating epoch. By denoting $\rho_s$ and $T_s$, the energy density and the reheating temperature corresponding to the case that a slow-roll inflationary era precedes the reheating epoch, by using Eq. (18) and the corresponding formula for the slow-roll pre-reheating era, and also by assuming that the final time instance in the integral appearing in Eq. (18), is $t \simeq t_r + 10\omega$, we obtain the following relations,

$$\frac{\rho_c}{\rho_s} = \frac{\omega_s}{\omega}, \quad \frac{T_r}{T_s} = \frac{\omega}{\omega_s},$$

where the parameter $\omega_s$ for the slow-roll era quasi-de Sitter evolution reads,

$$\omega_s = \sqrt{\frac{3H_i}{2}}.$$

By combining Eqs. (10), (15) and (20), we finally obtain the ratio of the constant-roll to slow-roll reheating temperatures,

$$\frac{T_{rc}}{T_{rs}} = \sqrt{\frac{3}{2\beta + 3}},$$

in which it can clearly be seen that the parameter $\beta$ affects the reheating temperature, so the constant-roll era preceding the reheating era, can affect the reheating era. In order to have a quantitative picture of how the constant-roll parameter $\beta$ affects the reheating temperature, we shall plot the ratio $\frac{T_{rc}}{T_{rs}}$, as a function of $\beta$. In Fig. 1 we plot the behavior of the ratio $\frac{T_{rc}}{T_{rs}}$ as a function of the constant-roll parameter $\beta$. As it can be seen in the left plot, the
ratio $\frac{T_{rc}}{T_{rs}}$ takes large values as $\beta \to -3/2$, since the ratio is singular at $\beta = -3/2$. This means that for $\beta \to -3/2$, the constant-roll reheating temperature may differ significantly from the corresponding slow-roll reheating temperature. However, as $\beta$ takes larger values, the ratio $\frac{T_{rc}}{T_{rs}}$ tends to zero, which means that the reheating temperature in the two cases is almost the same. Moreover, the equation (21) can be considered as a constraint on the parameter $\beta$ for the constant-roll $R^2$ model, since $\beta$ must be $\beta > -3/2$, so the reheating era imposes some constraints on the constant-roll inflationary era.

IV. CONCLUSIONS

In this paper we discussed the implications of a constant-roll inflationary era on the dynamics of reheating, in the context of $F(R)$ gravity. After discussing how the constant-roll condition modifies the dynamics of inflation in $F(R)$ gravity, we focused on the $R^2$ model and we demonstrated that the constant-roll condition predicts a quasi-de Sitter evolution which governs the inflationary era. As we demonstrated this quasi-de Sitter evolution affects the reheating era, and it modifies directly the reheating temperature. We compared the reheating temperature corresponding to a constant-roll inflationary era preceding the reheating era, to the corresponding slow-roll case for the $R^2$ model, and we found that when the constant-roll parameter approaches $\beta \to -3/2$, the two reheating temperatures may differ significantly. Finally, we demonstrated that the reheating era may restrict the constant-roll parameter and in effect the whole constant-roll inflationary era, but this feature is strictly model dependent. For example, in the case of the $R^2$ model, the constant-roll parameter must be $\beta > -3/2$ for physical consistency reasons. In principle the results we found in this work may apply to other $F(R)$ gravities as well, and it would be interesting to investigate this issue in the context of string inspired modified gravities, such Gauss-Bonnet scalar theories, a task we hope to address in a future work.

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