The improvement of mathematics teachers’ Pedagogical Content Knowledge (PCK) through mentoring

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Abstract. This study aimed to examine the improvement of mathematics teachers’ Pedagogical Content Knowledge (PCK) through mentoring as the foundation to develop a mentoring model to enhance teachers’ PCK. This study employed Research and Development method consisting of three phases: the preliminary research, prototyping and assessment phase. At the preliminary research, the teachers and mentors prepared the learning. The initial observation indicated that the initial PCK of teachers was lacking and therefore mentoring was needed to improve teachers’ PCK. The instrument of PCK used was validated for its language to assist the observers in assessing teachers’ PCK. In the following phase, the prototyping, the mentoring model was designed and then implemented for two teachers in one of the senior high school in Banda Aceh, Indonesia. The results showed that the instrument used to assess teachers’ PCK was valid to be used and the mentoring model enhanced teachers’ PCK. Thus, this study should be continued in the assessment phase. In the next phase, the mentoring model should also be validated and implemented for more teachers with a more intensive observation.

1. Introduction

Many factors contribute to improving the quality of education. One of which is the role of teachers in the learning process. Teachers are demanded to create a learning environment engaging students which in turn will enhance students’ understanding of the subject matter. Teachers are expected to have a pleasant personality to understand students as well as to have the skills to present the teaching materials so that they can easily understand the teacher’s explanation [1].

Teachers will perform the most effective way of teaching when: they are equipped with excellent pedagogy knowledge of the learning materials; realize that they are not the primary source of knowledge, recognize that students’ thinking is full, rich, and varied of knowledge so that teaching means connecting the students’ existing knowledge to the current learning materials [1] as well as utilize students’ perspective and experience to create a favorable classroom climate to motivate students [2].

An excellent teacher should not assume that they are the perfect learning resource as students have background knowledge that needs to be explored and utilized during the learning [3]. Teachers should also appreciate students’ opinions and works so that students are more confident and acknowledged in solving the problem [1]. Thus, the teachers should be not only able to master the materials but also the pedagogical strategies to understand students to prepare the conditions enabling students to learn successfully. Shulman [1] defined this capability as the Pedagogical Content Knowledge (PCK).
Pedagogical knowledge involves methods and processes of teaching including the understanding of classroom management, tasks, lesson planning and students’ learning. Pedagogical knowledge should at least consist of the understanding of the insights or educational foundations, students, curriculum development/syllabus, learning design, the implementation of educational and dialogic learning, the use of technology in learning, the evaluation of learning outcomes, and the students’ development to actualize various potentials of the students [1]. When a teacher discovers the materials that students do not understand, they should be able to modify their teaching strategies including the understanding of the subject matter and its delivery or adjust the teaching activities based on students’ achievement [4].

Pedagogical content knowledge (PCK) is teachers’ ability to understand the concepts, problems, or emerging issues adapted based on students’ talents, interests, and abilities. Teachers’ mastery of concepts influences their students’ mastery of concepts. Most of the concepts grasped by students are from teachers [5]. Learning will be more effective, efficient, and well-targeted if the teacher delivers it by implementing the appropriate teaching method considering both the physics and mental of the students.

Therefore, the competencies of teachers include The Content Knowledge, Pedagogical Content Knowledge (PCK), Curriculum Knowledge, General Pedagogical Knowledge, Knowledge of Learners, Knowledge of Educational Contexts, and Knowledge of Educational Ends [1]. Among those competencies, teachers’ PCK play a crucial role as it reflects on teachers’ special skill in teaching covering both the Pedagogical Knowledge and Content Knowledge.

One of the abilities in PCK concerns the strategies to address students’ misconception indicated when students systematically use inappropriate rules or appropriate rules, yet they are not in line with the expected learning objectives. For example, the study conducted by [6] found that when students are taught how to quickly memorize multiplication of 10 by merely adding zero to the unit, it is likely that students will have a misconception when they experience multiplication of fraction or decimal in which zero is after the decimal point.

Students’ misconceptions may occur during the lesson due to the basic concepts of mathematics formed in students experiencing the misconceptions or the misconception arising from delivery of teachers that contradicts the existing cognitive structure in students. Studies revealed that teachers who have excellent PCK would be able to anticipate misconceptions emerged in students to enhance students’ learning outcomes [6, 7].

The results of a preliminary survey conducted by the researcher to two mathematics teachers in one high school in Banda Aceh showed that the PCK of teachers was generally inadequate. The average PCK of targeted teachers in the initial survey was 29 out of 60. This was indicated by teachers’ less effort in guiding the students to discover their knowledge during the learning. The teachers delivered the lesson by following these teaching procedures: (1) conveying the concept or procedure of problem-solving, (2) explaining some examples of problems, (3) asking students together with the teacher to solve problems with similar contexts to teacher's previous example, (4) asking students to solve the practice problems independently. These procedures assisted students in solving the problems successfully; however, students found it challenging when they faced problems in different contexts. The instrument used for the initial survey was adapted from Monica and Helen [8].

PCK is both the understanding of concepts and the process of teaching including the appropriate method to teach and formulate the concepts to be understood by students [9]. To be successful in teaching, teachers should combine both Pedagogical Knowledge and Content Knowledge into PCK where the component of materials presented based on the student's previous performance. Basically, PCK can be defined as the teacher's effort to teach the subject matter to the students. Having an excellent understanding of PCK, teachers will find different methods to effectively teach the subject matter based on the characteristics of the students in the classroom [10].

A teacher cannot improve PCK without the help from others. Teachers need mentoring from experts to develop their PCK during the teaching and learning, starting from the early preparation, during and after the lesson (to obtain feedback concerning the effectiveness of the lesson conducted). The mentoring is not meant to patronize the teachers instead the mentors act as partners to provide
feedback and observe the components of PCK during the lesson [11]. Mentoring is an activity to assist each other to be responsible for the development of their abilities to be assessed afterward.

There are nine fundamental principles for the effective mentoring: a) sharing knowledge, attitudes, skills, and experience are the main change expected in mentoring activities; b) the expected outcome of mentoring is the participants’ positive self-change; c) the participants specifying the sections that will be the main issues to be developed and changed; d) trainers or mentors are not only the instructors but also facilitators in the learning process and self-development of participants. e) the mentoring results consist of the self-change of the participants; f) the framework of the change process facilitates the expected direction of change; g) skills obtained include the development of knowledge, self-potential and the impact of emerging changes; h) the quality of the mentors is appropriate to support the participants to change; i) the feasibility of the mentors’ ability should be able to improve coaching and mentoring of participants [12].

The mentoring occurred when there is a mutual relationship between the mentor and the participants to improve the participant’s potential. The core of mentoring is an open relationship that will facilitate and lead to the career development. The mentoring process occurs when the experienced individuals organize regular meetings and discussions, determine the interests of the participants and support the development of the skills perceived to be lacking [12].

There are some mentoring models include: 1) The Grow model, seeking to develop the skills and self-development goal expected by the participants. 2) The Skilled Helper, assisting the participants to identify the expected results and how to achieve them. 3) The Inner Game, motivating participants to think positively. 4) Non-directive, providing the right conditions enabling the participants to become trainers for themselves. 5) Co-Active Coaching, assisting in designing the needs of participants by listening, intuition, curiosity, trying to implement and self-management. 6) Solution-focused, finding what has not been done and improve it. 7) Team coaching, creating a mentoring team with similar interests [13]. Mentoring is expected to enhance teachers’ PCK. Mentoring activities are an important element of teachers’ capacity building and the development of teachers’ strategies for self-reflective [14].

Based on the review previously mentioned, there has no mentoring model to improve teachers’ PCK. This study aimed to describe the development of mathematics teachers’ PCK through a mentoring model.

2. Method
This study employed a Research and Development method developed by Plomp [15] consisting of three phases. However, only two phases were used in this research, namely: the preliminary research and prototyping phase. The preliminary research phase involved agreeing on the mentoring model preferred by the teacher before, during and after the learning. The developed mentoring model was adapted from the stages of lesson study (i.e., plan, do, see) by involving two observers from peers in the field of mathematics. This phase also dealt with the content analysis involving the analysis of the learning materials based on the senior high school curriculum, standard competences, basic competencies, as well as the analysis of essential concepts considered to be difficult to understand by teachers or students. Besides, the literature review was also conducted to review of relevant research concerning the common misconceptions experienced by students on the topics to be taught and examine the supporting literature on teaching materials and current issues related to the workshop concerning the understanding of PCK by the mentor lecturer.

The phase prototyping is carried out following the preliminary phase. This phase involved adapting the instrument assessing the PCK of teachers presented by Monica and Helen [8] and validating the instrument to evaluate teachers’ PCK validated by three mathematics education experts. It also covered the analysis of the early PCK of mathematics teachers participated in the research as an initial step to improve teachers’ PCK through mentoring. Also, this phase also concerned on observing the classroom to examine the teacher’s PCK improvement at the first lesson; recording and producing the first lesson video and reflecting on the first lessons conducted. Finally, this phase also repeated the
preliminary research and prototyping phase for the second and third lessons as well as developed a mentoring model shown in Figure 1. This model was adapted by merging the two-phase developed by Plomp [15] and the phase of lesson study.

![Flowchart of research to improve the PCK of teachers through mentoring](image)

**Figure 1.** The flowchart of research to improve the PCK of teachers through mentoring.

The assessment instrument to evaluate the PCK of teachers used in this study adapted from Baker’s research [8] consisting of three abilities. First, the PCK includes eight indicators including a) teaching strategies, b) thinking of the students, c) thinking to overcome students’ misconception, d) the demands of the cognitive level of the students’ task, e) the conformity of presentation with the characteristics of concept, f) knowledge of learning resources, g) knowledge of curriculum and h) the relevance of teaching materials with other relevant materials and their implementation. Second, the ability to understand the concepts related to the pedagogic ability, consisting of five indicators, includes: a) excellent understanding of the fundamentals of mathematics, b) describing the content of the lesson into the core components, c) mathematical structure and connections, d) procedural knowledge and e) methods of mathematics problem-solving. Third, pedagogical skills related to conceptual understanding ability, consisting of three indicators, includes: a) learning objectives, b) striving and maintaining the focus of students, and c). classroom management.

The teachers involved in this research were those being observed for their PCK in the preliminary research. They were two mathematics teachers in one of the senior high school in Banda Aceh, Indonesia. Both teachers have been teaching for more than 20 years and experiencing teaching at different Year level. G1, a female teacher, taught in Year 10 and G2, a male teacher, taught in Year 11. Both G1 and G2 worked at the same school, and both treated equally in the study. They participated in the workshop and were mentored by two mathematics education lecturers. The mentor lecturers assisting in the research were two lecturers from the teacher training and education faculty of Syiah Kuala University (FKIP Unsyiah) as well as the authors of this paper.

The improvement of teachers’ PCK for one lesson was assessed by computing the average assessment of teachers’ PCK based on two teacher observers and one mentor lecturer. Next, the average of the score obtained and the score of teachers’ PCK based on watching the learning video of the first lesson. The same procedures were conducted for the second and third lesson. Once all data from the three lessons gathered, the analysis of the improvement of teachers’ PCK was conducted.
3. Result and discussion
Some preparation before the research was conducted in the preliminary research stage including 1) developing the lesson plan and student worksheet together with the mentor lecturers according to the curriculum and characteristics of the students in the classroom; analyzing the teaching materials enabling students to do the task independently and minimizing misconceptions as well as discussing the possibilities of difficulties experienced by students during learning as well as how to overcome them, and preparing a PowerPoint presentation to be approved by the mentor lecturers; 2) predicting misconceptions that may occur during the learning and how to overcome them, conducting a workshop concerning the understanding of PCK by mentor lecturers for teachers involved in the research, organizing workshop about the utilization of multimedia in learning as per teacher request. Multimedia has also introduced: the use of more advanced PowerPoint, and innovative learning in teaching mathematics.

In the Prototyping phase, the research instrument was adapted to assess and observe the PCK of teachers as discussed in the research findings [8] by adapting the indicators of PCK to ease the process of the assessment conducted by the observer during the study, validating the instrument to assess the PCK of teachers. The validation results showed that the instrument was eligible to be used with a minor revision in the description of the PCK indicators (2a, 2b, 2e, and 3b) which were initially less understood leading to the observer being doubtful in assessing the PCK of teachers due to the non-operational words. The revision results are presented in Table 1:

| No | Before Validation | After Validation |
|----|-------------------|-----------------|
|    |                   |                 |
| II | The ability to understand the concepts and its relation to the pedagogical ability |                 |
|    | a. Showing a high level of understanding of mathematical aspects | a. Demonstrating a deep and thorough understanding of mathematics aspects |
|    | b. Elaborating the content of learning into fundamental components so that they are easier to be understood and implemented | b. Identifying the essential elements of mathematics concepts critically so that they are easier to be understood and implemented |
|    | e. Using a method (a certain way) in solving mathematics problems | e. Mastering the steps of solving mathematics problems systematically and accurately (the ability to understand mathematics concepts is not taken into account) |

The mathematics teachers’ initial PCK was examined in the study, the PCK of G1 and G2 was 33 and 24 respectively (out of 80). The initial PCK reflected the teachers’ ability in the initial observation before the study conducted. This study confirmed that teachers’ PCK were improved in the first, second and third lessons. The improvement is presented in Table 2.

| Lesson        | PCK score |
|---------------|-----------|
|               | G1 | G2 |
| Initial observation | 33 | 24 |
| The first lesson     | 37 | 28 |
| The second lesson    | 47 | 31 |
| The third lesson     | 51 | 37 |

Even though the PCK of both teachers were improved, the PCK of G1 increased higher than the other. Some factors may contribute to the differences including the fact that G1 prepared the presentation of the concepts that were well-structured and easier to understand leading to the students
learn and independently work on the tasks without asking many questions related to the task given by the teacher (indicator 2.b). On the other hand, many students of G2 waited for the teacher to come to them and ask about the parts they did not understand. G1, at the beginning of the lesson, have been explained in detail all the students' needs to solve the problems.

When G1 gave a problem from the example, students could solve the problem provided (indicator 1.b). In contrast, the students of G2 experienced obstacles when the problems are given was slightly different from the example. G1 prepared the lesson without any significant barriers (indicator 1.a). G1 also tried to predict the possible misconception (indicator 1.c), while G2 were doubtful of the need to predict misconception in detail considering the lack of students’ enthusiasm to learn mathematics.

In each lesson, the number of students in G1’s classroom who were focused and engaged in the learning rose (indicator 3.b), and fewer students violated the rules set together (indicator 3.c). On the other hand, students in G2’s classroom did not increase or remained stable. The fact that the lesson was not scheduled at the beginning of the school hours leading to students being less focus in learning may contribute to this issue.

Among other factors supporting the improvement, the teacher's PCK was that the mentor lecturers being familiar with both teachers before the research activities carried out so that the reflection can easily be accepted by teachers and communication during the mentoring was intimate and nurturing. The school involved in this study was also one of the partner school of Syiah Kuala University, and both teachers (G1 and G2) were actively involved in some activities involving the university such as being the mentor teacher for the student teaching internship, and the teacher professional education program and therefore the relationship of mentor lecturers and teacher became familiar. Only the teachers who had a high commitment to make changes were selected to participate in the study. Thus, the mentoring process run well without any significant constraints and the teachers voluntarily spent time after the lesson to consult with mentor lecturers. The mentoring conducted benefited the teachers to prepare the lesson optimally. The indicators on the PCK instrument, the preparation for teaching, monitoring the implementation of the learning process, and providing feedback to the teacher after the lesson, were well implemented. Besides, the collaboration between teachers and lecturers run harmoniously without any significant pressure. Teachers and mentor lecturers agreed on a schedule of mentoring by respecting the individual schedule.

Although the PCK of both teachers were improved, the scores were not satisfactory. This is because teachers were accustomed to teaching using their old method and it was difficult to change in a short time (in three lessons and mentoring). The teachers involved in the study were rarely invited by the government to attend training for improving their teaching performance. According to Tjerrd and Nienke [15], one of the reason contributes to the low teaching performance of teachers is teachers’ less participation in the training leading to their pedagogical knowledge being outdated and less motivated for self-development. A professional teacher (supported by teacher career development workshops) should be concerned with students' thinking and prepare challenging problems [15]. Students taught by professional teachers usually have an integrated, more coherent, and higher abstraction of concepts understanding [16, 17].

Based on the discussion with both G1 and G2, teachers’ motivation to change was lacking because of the overcrowded non-teaching and teaching scheduled leading to teachers had less time to prepare the lesson. This finding is in line with the study conducted by Patricia and Masako [18] found that teachers' motivation is the main predictor of students' interest. Highly motivated teachers will have an impact on how well the lesson prepared and the increase in students' learning outcome.

4. Conclusion
The results of the study show that the development of mathematics teachers’ Pedagogical Content Knowledge (PCK) through the mentoring program is increased. However, in term of the indicators of PCK, some indicators showed no improvement or declining in each lesson (for the three lesson). The PCK scores of G1 for each indicator remained stable include: the cognitive level of the problems given by G1 in each lesson was classified as C2 and C3 level (indicator 1d), G1 always had an excellent
score for the knowledge of the curriculum (indicator 1g), G1 mastered the mathematics concepts of all topics taught (indicator 2a), G1 always presented the objectives of learning at the beginning of the lesson prepared on PowerPoint slide (indicator 3a).

The PCK scores of G2 that remained stable were: the learning resources being the same in each lesson, the textbook, and PowerPoint (indicator 1.f), always linking the relevant concepts (indicator 2.c), neither oral or written teaching objective presented (indicator 3.a), no change in the strategies to engage students (indicator 3.b). One indicator of G2’s PCK decreased, the complexity of the problem given to the students decreased (indicator 1d) due to time constraint and the increasing level of the subject matter difficulty towards the application of concepts in the field other than mathematics.

It can be concluded that generally the mentoring improves teachers’ PCK. Thus, this research should be continued to the assessment phase in which the mentoring model will be re-validated and implemented on a broader scope of teachers following by intensive observation.

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