Pumping at resonant transmission and transferred charge quantization

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We consider pumping through a small quantum dot separated from the leads by two point contacts, whose conductances, $G_1$ and $G_2$, serve as pumping parameters. When the dot is pinched, i.e., $G_1, G_2 \ll e^2/h$, we find that there is a “resonance line” in the parameter plane $\{G_1, G_2\}$ along which the Fermi energy in the leads aligns with the energy of the quasi-bound state in the quantum dot. When $G_1$ and $G_2$ are modulated periodically and adiabatically such that the pumping contour defined by $G_1 = G_1(t)$ and $G_2 = G_2(t)$ encircles the resonance line, the current is quantized: the charge pumped through the dot during each period of the modulation is close to a single electronic charge.

The question of current quantization was first addressed in Ref. [1]. It was shown that under certain conditions the current $J$ induced by a slowly moving periodic potential profile corresponds to an integral number of electronic charges $e$ transferred through the sample cross-section during a single temporal period $T$ of the perturbation: $J = (e/T) \times \text{integer}$, where $T = L/v$, with $L$ and $v$ being the period of the potential profile and its velocity. Current quantization was observed in a quasi-one-dimensional (1D) GaAs channel [2,3], in which the potential profile had been created by the piezoelectric potential of a surface acoustic wave. A 1D model describing charge quantization in this experiment was discussed in Ref. [3]. It was demonstrated [3] that the current induced by a moving potential profile is equivalent to pumping, a phenomenon [4] which had been first considered in Ref. [4] and has attracted recently much theoretical [5,6] and experimental [7,8] interest.

The pumping current is excited not by applying a voltage difference, but by periodically changing some properties of the system (i.e., parameters of the system Hamiltonian), for example, the confining potential in a nanostucture. The fingerprint of pumping appears when the frequency $\omega$ is smaller than any characteristic energy of the system (adiabatic pumping). Then the charge transferred during a single period, $Q = J \times (2\pi/\omega)$, is independent of $\omega$. However, this charge is not necessarily quantized.

The simplest case is pumping at zero temperature through a two-terminal device with single-channel terminals. When two parameters of the device are modulated periodically the charge transferred by spinless electrons is [8]

$$Q = \frac{e}{\pi} \int \int d\lambda_1 d\lambda_2 \Pi(\lambda_1, \lambda_2),$$

with

$$\Pi = -\text{Im} \left[ \frac{\partial s_{11}}{\partial \lambda_1} \frac{\partial s_{12}}{\partial \lambda_2} + \frac{\partial s_{12}}{\partial \lambda_1} \frac{\partial s_{11}}{\partial \lambda_2} \right].$$

Here $s_{\alpha\beta}$ is the scattering matrix of the device calculated at the Fermi energy $\epsilon_F$ in the leads and $\lambda_1 = \lambda_1(t), \lambda_2 = \lambda_2(t)$ are adiabatically modulated parameters, displaying a closed “pumping contour” (counterclockwise) in the parameter plane $\{\lambda_1, \lambda_2\}$. The integration is over the area encircled by the pumping contour. It is obvious from this result that $Q$ is quantized only under special circumstances.

The quantization of the charge pumped through a large, almost open quantum dot (QD) with vanishing level spacing was analysed in Refs. [9,10]. In this letter we consider the opposite case, of a small, strongly pinched QD, supporting resonant transmission. We show that charge quantization can be achieved at zero temperature, provided that the pumping contour is properly chosen.

Our results can be formulated in the following generic way. Consider a QD in a two-dimensional electron gas (2DEG), separated from the leads by two single-channel point contacts PC1 and PC2 with conductances $G_1$ and $G_2$ controlled by split-gate voltages $U_1$ and $U_2$. These conductances will serve as pumping parameters. (A device of this type was used, for example, in the experiments of Ref. [10]). Let the Fermi level in the 2DEG $\epsilon_F$ be close to $\epsilon_0$, a level which is formed in the isolated QD, when $G_1, G_2 = 0$. When the QD is not isolated, but is strongly pinched, $G_1, G_2 \ll 1$ (for conductances in units of $e^2/h$), the level $\epsilon_0$ turns into a resonance at $\epsilon_0 + \Delta\epsilon_0 + i\Gamma$, with $\Delta\epsilon_0 < 0$ and $\Gamma \ll |\Delta\epsilon_0|$. Then, when
with \( \epsilon_F < \epsilon_0 \), there is a “resonance line” in the parameter plane \( \{G_1,G_2\} \), where the resonance energy is aligned with the Fermi level, \( \epsilon_F = \epsilon_0 + \Delta \epsilon_0 \). Along this line the transmission through the QD is at resonance. The conductance \( G(G_1,G_2) \) of the QD is then sharply peaked (the width of the peak is determined by \( \Gamma \)) in the direction perpendicular to the resonance line, and decreases towards the ends of this line. The transmission is maximal at \( G_1 = G_2 \) for a symmetric QD. We find that the pumped charge is quantized, \( Q \approx e \), and \( |Q - e| \to 0 \) when \( \epsilon_F \) approaches \( \epsilon_0 \) from below, provided that the pumping contour encircles the resonance line.

The resonance line can be found experimentally. When PC1 is pinched and PC2 is open, the conductance of the QD, \( G \), is dominated by the conductance of PC1, i.e., \( G = G_1 \). Measuring the dependence of \( G \) on \( U \) yields the relation between \( G_1 \) and \( U_1 \). In a similar way one finds the relation between \( G_2 \) and \( U_2 \). Using these relations and measuring \( G \) as function of \( U_1 \) and \( U_2 \), the resonance line can be obtained.

To derive the above result we consider a 1D model, similar to the one used in Ref. [1] (and also in Ref. [2]), in which the QD is confined by two potential barriers, located at \( x = \mp a/2 \). The scattering matrix of the QD, 

\[
\begin{bmatrix}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{bmatrix},
\] (1 and 2 denote the terminals at \( x = -\infty \) and \( x = +\infty \), respectively) can be constructed from the scattering matrices of the two barriers:

\[
\begin{align*}
  s_{12} &= s_{21} = t_1 t_2 \sigma / D, \\
  s_{11} &= r_1 + t_1^2 r_2 \sigma^2 / D, \\
  s_{22} &= r_2 + t_2^2 r_1 \sigma^2 / D,
\end{align*}
\] (3)

where \( r_{1,2} \) and \( t_{1,2} \) are the reflection and the transmission amplitudes of the two barriers, \( \sigma = e^{ika} \), \( D = 1 - r_1 r_2 \sigma^2 \), and \( k = (2m)^{1/2} \). The phase reference points for the waves on the left and on the right of the QD are at \( x = \mp a/2 \), respectively.

Let us confine ourselves for simplicity to delta-function potential barriers, \( V_{1,2} \delta(x \pm a/2) \). In this case,

\[
\begin{align*}
  t_{1,2} &= (1 + im V_{1,2} / k)^{-1}, \\
  r_{1,2} &= t_{1,2} - 1,
\end{align*}
\] (4)

and the scattering matrix elements are

\[
\begin{align*}
  s_{11} &= (1 - i \xi_2 - (1 + i \xi_1) \sigma^2) / D', \\
  s_{22} &= (1 - i \xi_1 - (1 + i \xi_2) \sigma^2) / D', \\
  s_{12} &= \xi_1 \xi_2 \sigma / D', \\
  D' &= -(1 - i \xi_1)(1 - i \xi_2) + \sigma^2,
\end{align*}
\] (5)

with \( \xi_{1,2} = k / m V_{1,2} \).

The energies \( \epsilon_0 \) of the bound states in the isolated QD, (that is, when \( \xi_1 = \xi_2 = 0 \), are given by \( \exp(2ika) = 1 \) with \( \epsilon_0 = k_0^2 / 2m \). Below we assume the Fermi energy to lie in the vicinity of one of these levels, \( \delta \equiv (\epsilon_F - \epsilon_0) / (v_0 / 2a) \ll 1 \), where \( v_0 = k_0 / m \). Here \( \delta \) is the detuning of the Fermi energy from the bound state, measured in units of the level spacing in the isolated QD.

At near resonance Fermi energy, the smooth energy dependence of \( t_{1,2} \) and \( r_{1,2} \) can be ignored, putting \( k = k_0 \), i.e., \( \xi_{1,2} = k_0 / m V_{1,2} \). The latter will serve as the pumping parameters. Note that for a pinched QD these parameters are related to the conductances of the point contacts in a simple way: \( G_{1,2} = \xi_{1,2}^2 \). We assume that during the pumping cycle the barriers remain high. That is, the QD will support resonant transmission during the whole pumping cycle.

For finite, but high, barriers \( (\xi_1, \xi_2 \ll 1) \), the bound state of energy \( \epsilon_0 \) turns into a quasi-bound state with complex energy \( \epsilon = \epsilon_0 + \Delta \epsilon_0 + i \Gamma \), obtained when \( D' \) in Eq. (5) tends to zero. One finds

\[
\Delta \epsilon_0 = -\left(v_0 / 2a\right)(\xi_1 + \xi_2), \quad \Gamma = \left(v_0 / 2a\right)\xi_1 \xi_2.
\] (6)

The quasi-bound state lies always below the corresponding bound one and its width \( \Gamma \) is much smaller that the shift \( \Delta \epsilon_0 \). Resonant transmission through the QD occurs when \( \epsilon_F \) is aligned with the quasi-bound state energy \( \epsilon_0 + \Delta \epsilon_0 \) up to the level width \( \Gamma \). Since \( \Gamma \ll |\Delta \epsilon_0| \) this is possible only when \( \epsilon_F \) is below \( \epsilon_0 \), i.e. \( \delta < 0 \). Hence, only this case will be considered.

The alignment of the quasi-bound state energy with the Fermi level determines the resonance line in the parameter plane: \( \xi_1 + \xi_2 = \delta \). Examination of Eq. (6) reveals that when \( |\delta| \ll 1 \) the conductance of the QD \( G = |s_{12}|^2 \) is a sharp function along the direction perpendicular to this resonance line, with width \( \sigma^2 \), corresponding to the narrow resonance in the QD. Along the resonance line \( G \) has its maximum for a symmetric QD, at \( \xi_1 = \xi_2 \), decreasing towards its ends.

For the pumping parameters \( \xi_1, \xi_2 \) the function \( \Pi \) defined in Eqs. (1) and (2) is given by

\[
\Pi(\xi_1, \xi_2) = M(\xi_1, \xi_2) / N^2(\xi_1, \xi_2),
\]

\[
M(\xi_1, \xi_2) = \xi_1 \xi_2 \left[ \xi_1 \xi_2 \sin \delta + (\xi_1 + \xi_2)(1 - \cos \delta) \right],
\]

\[
N(\xi_1, \xi_2) = |D'|^2 = \xi_1^2 \xi_2^2 + (\xi_1 + \xi_2)^2 + 2(\xi_1 + \xi_2) \sin \delta + 2(1 - \xi_1 \xi_2)(1 - \cos \delta).
\] (7)
Similar to the conductance $G = |s_{12}|^2 = \xi_1^2\xi_2^2/N$, the function $\Pi$ has its maximum near the resonance line, see Fig. 2. Because of this sharp maximum, the resulting integral is not sensitive to the form of the integration contour, when the contour \cite{cf Eq. (1)} in the parameter plane $\{\xi_1, \xi_2\}$ encircles the resonance line at distances larger than $\delta^2$. The main contribution to the integral comes from that part of the area where $\xi_1, \xi_2 < 1$, i.e. where the QD is strongly pinched and the transmission through it is resonant.

To calculate the integral and to obtain the pumped charge, it is convenient to substitute $\xi_1 + \xi_2 = |\delta|p$, $\xi_1 - \xi_2 = |\delta|pz$, $(0 < p < \infty, -1 < z < +1)$. For $|\delta| < 1$ we then have

$$M = \delta^2 f(p, z), \quad N = \delta^2[(p - 1)^2 + \delta^2 g(p, z)],$$

$$f(p, z) = -\frac{1}{16}p^3(1 - z^2)[2 - p(1 - z^2)],$$

$$g(p, z) = -\frac{1}{12} + \frac{3}{4}p - \frac{1}{16}p^2(1 - z^2) + \frac{1}{16}p^4(1 - z^2)^2. \quad (8)$$

The pumped charge is now given by

$$Q = \frac{e}{2\pi}\delta^3\int pdp\int dz\frac{f(p, z)}{[(p - 1)^2 + \delta^2 g(p, z)]^2}. \quad (9)$$

The integration over $p$ is performed using the formal relation $(x^2 + \eta^2)^{-2} = (\pi/2\eta^3)\delta(x)$ for $\delta \to 0$, with the result

$$Q = e\int dz\frac{1 - z^4}{(1 + z^2)^3}. \quad (10)$$

When the contour encircles the whole resonance line the integration over $z$ is from $-1$ to $+1$, resulting in $Q = \epsilon$, i.e. the pumped charge is quantized. In fact, the contour cannot encircle the whole resonance line exactly since its ends $z = \pm 1$ correspond to infinite barriers. However, these ends contribute little to the integral \cite{[10].}

The pumping contour in Fig. 2 corresponds to the conductances $G_1$ and $G_2$ being alternatively modulated between $G_{\min} = \xi_{\min}^2$ and $G_{\max} = \xi_{\max}^2$, such that the periodic functions $G_1(t)$ and $G_2(t)$ are shifted by a quarter of a period, which corresponds to the maximal possible pumping by a small perturbation \cite{[8]}. From Fig. 2 it is seen that the accuracy of the quantization is not very sensitive to $\xi_{\max}$ provided that $\xi_{\max} > |\delta|$. It is more sensitive to the value of $\xi_{\min}$. In terms of the conductances, this means the following. The maximal resonance conductance of the pinched QD, for a given $\epsilon_F$, occurs at the center of the resonance line where $G_1 = G_2 \equiv G_0 = \frac{1}{4}|\delta|^2$. To achieve high levels of accuracy of quantization one has to choose $1 \gg G_{\max} > 4G_0$ and make $G_{\min}$ as small as possible. For example, for $|\delta| = 0.2$, which is equivalent to $G_0 = 10^{-2}$, one obtains $q = 0.8\%$, when $G_{\max} = 6 \times 10^{-2}$ and $G_{\min} = 10^{-4}$. Note that in the experiments \cite{[10]} the maximal resistance of the PC is about $100$ GΩ, which corresponds to $G_{\min} \approx 10^{-7}$, while the minimal resistance is about $1$ MΩ, which corresponds to $G_{\max} \approx 10^{-2}$. It follows that accuracy much higher than $1\%$ is accessible in the experiments.

FIG. 2. Resonance line and pumping contour

To examine the quantization accuracy, we have computed $q \equiv (1 - Q/\epsilon) \times 100\%$ in the case where the pumping contour in the $\{\xi_1, \xi_2\}$ plane is a square box, as shown in Fig. 2. Fig. 3 exhibits $q$ as function of $\xi_{\min}$ and $\xi_{\max}$ for $\delta = -0.2$.

Even though our model is simplified, it serves to demonstrate that the basic concepts of the quantization, namely the resonant transmission due to a quasi-bound level in the QD and the existence of a resonance line in the parameter plane, are not very sensitive to details of the model chosen for the barrier potentials. This strongly supports our generic formulation of the conditions for charge quantization.

It has been shown in Ref. \cite{[10]} that charge quantization is related to the topological properties of phases of the scattering matrix. The argument is based on the representation of the scattering matrix in terms of the conductance $G$ and two phases $\alpha$ and $\beta$,

$$s = e^{i\phi} \left|\begin{array}{cc} (1 - G)^{1/2} e^{i\alpha} & iG^{1/2} \\ iG^{1/2} & (1 - G)^{1/2} e^{-i\alpha} \end{array}\right|. \quad (11)$$
and their phases are resonance line the reflection amplitudes and ξvalued function since the transmission amplitude contrast, arg is large.

The conductance to encircle the resonance line, where the conductance G for charge quantization formulated in Ref. [10], that the jumps of α have to be small on the pumping contour [10] so that the transferred charge can be represented as an integral along the pumping contour [10]

\[ Q = \frac{e}{2\pi} \int ds (1 - G) \frac{\partial \alpha}{\partial s}, \]

(12)
The phase 2α = arg s₁₁ − arg s₂₂ is shown (in units of π) in Fig. 4. This phase jumps by ±2π at the resonance line, on both sides of its center, respectively, and hence cannot be expressed as a continuous, single-valued function of ξ₁ and ξ₂. The reason for this is that at the center of the resonance line the reflection amplitudes s₁₁ = s₂₂ = 0 and their phases arg s₁₁ and arg s₂₂ are not defined. By contrast, arg s₁₂ can be presented as a continuous, single-valued function since the transmission amplitude s₁₂ ≠ 0 everywhere.

The contribution to the integral Eq. (12) comes from the jumps of α at the resonance line. Alternatively, when the phase α is required to be continuous on a selected contour which encircles the resonance line, it will change by 2π upon closing the contour. It follows that the condition for charge quantization formulated in Ref. [10], that the conductance G has to be small on the pumping contour, is not sufficient. Our results imply that for the charge to be quantized and not to be zero the pumping contour has to encircle the resonance line, where the conductance G is large.

It is worthwhile to note that although the center of the resonance line may appear as a branching point, this is not so: from Eq. (11) it can be seen that the matrix elements sᵢₖ are not analytic functions of ξ₁ + iξ₂.

The quantization can be easily modified by finite temperature effects. Then the charge, Q, has to be averaged over e_F within the temperature interval. A finite temperature will affect the charge quantization when T ≳ |e_F − e₀|. To estimate this effect consider the QD in the experiments of Ref. [16]. The level spacing in the QD is 0.03 meV, which for |δ| = 0.2 corresponds to detuning |e_F − e₀| = 6 μeV = 70 mK. Temperatures of this order will destroy the quantization.

The transferred charge quantization discussed here is different from the quantization observed in turnstile-type devices [12,13,14]. First, the quantization in such devices is essentially based on the Coulomb blockade, fixing the number of electrons in the QD’s (or quantum boxes). Second, one can see from Eqs. (1) and (2), that the phases of the scattering matrix are crucial for the appearance of the transferred charge Q, while the turnstile device operation is usually described in terms of sequential transitions and their probabilities [13].

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