Open strange meson $K^{\pm}_1$ in hot and dense nuclear matter

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Abstract

Using the unification of the chiral SU(3) model and QCD sum rules, we deduce the in-medium properties of $K^{\pm}_1$ meson. Within the chiral SU(3) model, medium modified gluon and quark condensates are evaluated through their interactions with the scalar fields ($\sigma$, $\zeta$, $\delta$, and $\chi$). These condensates are further used as input in the Borel transformed equations of QCD sum rules to evaluate the in-medium mass of strange $K^{\pm}_1$ meson. The in-medium property of the above meson can be used to study the restoration of chiral symmetry in nuclear matter.

1 Introduction

The quark condensates act as an order parameter of the QCD chiral symmetry [1]. In non-perturbative QCD, the non-zero condensates lead to the spontaneously broken chiral symmetry which also explains the mass difference between the vector and the partner axial-vector meson. This implies that the mass of these mesons is the same in the chiral restored phase [1]. The axial-vector $K_1(1270)$ meson is the partner of vector $K^*(892)$ meson and in this article, we are focusing on the medium modified mass-shift of $K^+_1(\bar{u}s)$ and $K^-_1(\bar{s}u)$ mesons in the hot symmetric nuclear medium. To compute the mass-shift, we employ the union of QCD sum rules [1] and chiral hadronic model [2]. The effective hadronic mean-field model incorporates the fundamental QCD features, for example non-linear realization of the chiral symmetry and trace anomaly [2]. It has been used in literature to investigate the medium induced mass splitting across charged partners of pseudoscalar $K$ (self-consistent chiral SU(3) model) [3] and $D$ meson (chiral SU(3) model + QCD sum rules) [2]. In this paper, the medium (density and temperature) dependent quark (up and strange) and gluon condensates are computed from the chiral hadronic model, which is used further in QCD sum rules to calculate the in-medium mass of $K_1$ mesons. Within the chiral hadronic model, the above mentioned condensates are evaluated using the in-medium scalar fields [4]. In the next segment, we show the brief methodology used in the current article.
2 Formalism

Using operator product expansion (OPE), the current-current correlator up to dimension 6 can be written as [1]

\[ \Pi(q^2) = B_0 Q^2 \ln \frac{Q^2}{\mu^2} + B_2 \ln \frac{Q^2}{\mu^2} - \frac{B_4}{Q^2} - \frac{B_5}{Q^4}, \]  

(1)

with \( Q^2 \equiv -q^2 \), \( \mu = 1 \) GeV as renormalization scale and \( B_n \) as Borel coefficients [1]. In the nuclear medium, the degeneracy of \( K_1^+ \) and \( K_1^- \) do not hold, consequently, the charge symmetry breaking leads to even and odd contributions from the correlator, given as

\[ \Pi(q^2) = \Pi^e(q^2) + q_0 \Pi^o(q^2). \]  

(2)

For small nuclear density (\( \rho_N \)), i.e., keeping only the linear terms in \( \rho_N \), using Borel transformation the correlator can be expressed as [1]

\[ O(M^2)O'_\pm + M(M^2)m'_\pm + S(M^2)s'_{0\pm} = C_{\pm}(M^2), \]  

(3)

with

\[ O(M^2) = -m_{K^1}^2 e^{-m_{K^1}^2/M^2}, \quad M(M^2) = O_{K^1} \frac{m_{K^1}}{2 + \frac{2m_{K^1}^2}{M^2}} e^{-m_{K^1}^2/M^2}, \]

\[ S(M^2) = \frac{1}{2} \left( 1 + \frac{m_{K^1}}{\sqrt{\pi}} \right) (B_0 s_0 - B_2) e^{-s_0/M^2}, \quad C_{\pm}(M^2) = -m_{(\bar{u}u)} N + \frac{\alpha}{12\pi} (G^2) N \]

\[ + \frac{m_{N}}{2} (A^u_2 + A^s_2) \pm \frac{m_{K^1}}{3} (A^u_1 - A^s_1) \pm \frac{32\pi\alpha_s}{9M^2} \left\{ \langle \bar{u}u \rangle_N \langle \bar{s}s \rangle_0 + \langle \bar{u}u \rangle_0 \langle \bar{s}s \rangle_N + \frac{2}{9} \langle \bar{u}u \rangle_0 \langle \bar{s}s \rangle_0 \right\} \]

\[ + \langle \bar{s}s \rangle_N \langle \bar{s}s \rangle_0 \right\} - \frac{5m_{N}}{6M^2} (A^u_4 + A^s_4) \pm \frac{2m_{K^1} m_{N}}{3M^2} (A^u_3 - A^s_3) + \frac{m_{K^1} m_{K^1}}{2} \left\{ \langle \bar{u}u \rangle_N \langle \bar{u}u \rangle_0 \right\}. \]  

(4)

The in-medium mass (\( m_{K^1}^\pm \)), overlapping strength (\( O'_{K^1} \)) and threshold parameter (\( s'_{0\pm} \)) of \( K_1^\pm \) mesons can be expressed via relations [1]

\[ O'_{K^1} = O_{K^1} + \Delta O'_{K^1} = O_{K^1} + O'_{K^1} \rho_N, \]

\[ m'_{K^1} = O_{K^1} + \Delta m'_{K^1} = m_{K^1} + m'_{K^1} \rho_N, \]

\[ s'_{0\pm} = s_0 + \Delta s'_{0\pm} = s_0 + s'_{0\pm} \rho_N, \]  

(5)

where \( O_{K^1} \), \( m_{K^1} \) and \( s_0 \) denote the vacuum values of overlapping strength, mass and threshold parameter for \( K_1^\pm \) meson. In Eq. 4, \( A_i^\pm \) parameters are calculated using the MSTW parton distribution function at \( \mu = 1 \) GeV [1]. Also, in the same equation, the in-medium nucleon expectation values of up \( \langle \bar{u}u \rangle_N \), strange quark \( \langle \bar{s}s \rangle_N \) and gluon condensates \( \langle G^2 \rangle_N \) are computed using the definition \( \langle O \rangle_N = \frac{2m_N}{\rho_N} \langle \langle O \rangle_{\rho_N} - \langle O \rangle_{\text{vac}} \rangle \) through the density dependent condensates from the chiral SU(3) model [4]. The quark condensates can be associated to explicit symmetry breaking [2]

\[ \sum_i m_i \langle \bar{q}_i q_i \rangle_{\rho_N} = -\mathcal{L}_{\text{ESB}}, \]  

(6)

with

\[ \mathcal{L}_{\text{ESB}} = -\frac{2}{\lambda_0^2} \left[ \frac{1}{2} m_{K^1}^2 f_{\pi}(\sigma + \delta) + \frac{1}{2} m_{\rho}^2 f_{\pi}(\sigma - \delta) + \left( \sqrt{2} m_{K^1} f_{K} - \frac{1}{\sqrt{2}} m_{\rho}^2 f_{\pi} \right) \right]. \]  

(7)
Using Eq. 6, the light quark condensates can be expressed as

\[
\langle \bar{u}u \rangle_{\rho_N} = \frac{1}{m_u} \left( \frac{X}{\chi_0} \right)^2 \left[ \frac{1}{2} m_{\pi}^2 f_\pi (\sigma + \delta) + \left( \frac{X}{\chi_0} \right)^2 (\sqrt{2} m_{\pi}^2 f_\pi - m_{\pi}^2 f_\pi) \right],
\]

\[
\langle \bar{s}s \rangle_{\rho_N} = \frac{X}{\chi_0 m_s} (\sqrt{2} m_{\pi}^2 f_\pi - m_{\pi}^2 f_\pi) \zeta.
\]

(8)

In above \( m_u \) and \( m_s \) denote the mass of up, and strange quark. Furthermore, the scalar gluon condensate \( \langle G^2 \rangle_{\rho_N} \), given by

\[
\langle G^2 \rangle_{\rho_N} = \frac{8}{9} \left( 1 - d \right) X^4 + \left( \frac{X}{\chi_0} \right)^2 \left( m_{\pi}^2 f_\pi \sigma + \left( \sqrt{2} m_{\pi}^2 f_\pi - \frac{1}{\sqrt{2}} m_{\pi}^2 f_\pi \right) \zeta \right),
\]

(9)
is extracted using scalar fields of chiral hadronic model [2]. Further, from Eq. (3), we define the function \( F_{\pm}(O_{\pm}', m_{\pm}', s_{\pm}'_0) \) as

\[
F_{\pm}(O_{\pm}', m_{\pm}', s_{\pm}'_0) \equiv \int_{M_{\min}^2}^{M_{\max}^2} \left\{ O(M^2)O_{\pm} + M(M^2)m_{\pm}' + S(M^2)s_{\pm}' - C_{\pm}(M^2) \right\}^2 dM^2,
\]

(10)

with \( M_{\min}^2 \) and \( M_{\max}^2 \), as the lower and upper limit of the Borel window [1]. The following six simultaneous linear equation for \( O_{\pm}', m_{\pm}', s_{\pm}'_0 \) are obtained

\[
\begin{align*}
O_{\pm}' &\int dM^2 F^2(M^2) + m_{\pm}' \int dM^2 O(M^2)M(M^2) + s_{\pm}'_0 \int dM^2 O(M^2)S(M^2) \\
&= \int dM^2 O(M^2)C_{\pm}(M^2),
\end{align*}
\]

\[
\begin{align*}
O_{\pm}' &\int dM^2 O(M^2)M(M^2) + m_{\pm}' \int dM^2 M^2(M^2) + s_{\pm}'_0 \int dM^2 M^2(M^2) \\
&= \int dM^2 M^2(M^2)C_{\pm}(M^2),
\end{align*}
\]

\[
\begin{align*}
O_{\pm}' &\int dM^2 O(M^2)S(M^2) + m_{\pm}' \int dM^2 M^2(M^2)S(M^2) + s_{\pm}'_0 \int dM^2 S^2(M^2) \\
&= \int dM^2 S^2(M^2)C_{\pm}(M^2),
\end{align*}
\]

(11)

using the minimization of function \( F_{\pm} \), i.e.,

\[
\frac{\partial F_{\pm}}{\partial O_{\pm}'} = \frac{\partial F_{\pm}}{\partial m_{\pm}'} = \frac{\partial F_{\pm}}{\partial s_{\pm}'_0} = 0.
\]

(12)
The in-medium mass of \( K_1^+ \) and \( K_1^- \) along with \( O_{\pm}' \) and \( s_{\pm}'_0 \) are solved simultaneously using Eq. (11) under the influence of density-dependent quark and gluon condensates.

3 Numerical Results and Discussions

In this segment, the in-medium mass difference across the charged \( K_1^+ (u\bar{s}) \) and \( K_1^- (\bar{u}s) \) is discussed. In the present work, the medium modified \( K_1 \) mass is computed from the Borel transformed current correlator in the QCD sum rules at zero width approximation [1]. The density
Table 1: Various parameters used in the present work [1].

| $m_{K_1}$ (GeV) | $O_{K_1}$ (GeV$^2$) | $s_0$ (GeV$^2$) | $m_u$ (GeV) | $m_s$ (GeV) | $m_N$ (GeV) | $\alpha_s$ |
|----------------|-------------------|----------------|-----------|-----------|-----------|---------|
| 1.270          | 0.048             | 2.4           | 0.0426    | 0.117     | 0.940     | 0.5     |

Figure 1: In-medium mass shift of $K_1^+$ and $K_1^-$. 

dependent up quark, strange quark and gluon condensates are calculated in chiral hadronic model [4], and are plugged in the calculations of QCD sum rules [1]. In the calculations of Borel sum rules, the minimum $M_{\text{min}}^2$ and maximum $M_{\text{max}}^2$ limit of Borel window is taken to be 1.06 and 2.17 GeV$^2$, respectively [1]. Also, the different parameters utilized in the current investigation are tabulated in table 1.

In fig. 1, we illustrate the in-medium mass of $K_1^+$ and $K_1^-$ mesons with nuclear density at zero temperature. We observe a huge mass splitting between charged partners of $K_1$ meson in symmetric nuclear medium, which increases with the rise in nuclear density. The attractive mass shift of $K_1^-$ meson grow significantly in the medium whilst the mass shift of $K_1^+$ modifies feebly. Under the effect of finite temperature ($T=0.150$ GeV), we observed the change in mass to be very less for both charged partners. The results obtained here are in good agreement with the observations of QCD sum rules [1]. For better comparison, the observations from the chiral hadronic model model + QCD sum rules (present investigation) and QCD sum rules alone are compared in table 2. In the solo QCD sum rules approach [1], the quark and gluon condensates are computed under the linear density approximation. The mass shift of $K_1^-$ can be experimentally anticipated from the hadronic decays $K_1 \to K\rho$ and $K_1 \to K^*\pi$ as well as excitation function. The in-medium $K_1N$ properties can also be studied through the $K_1^-$ interactions with several nuclei at J-PARC [1,5].
Table 2: In-medium values of shift in overlapping strength $\Delta O_{K_1^\pm}^*$ (GeV$^2$), mass $\Delta m_{K_1^\pm}^*$ (GeV), and, threshold parameter $\Delta s_0^\pm$ (GeV$^2$) at $\rho_N=0.16$ fm$^{-3}$, $T=0$ MeV.

|                   | $\Delta O_{K_1^-}^*$ | $\Delta m_{K_1^-}^*$ | $\Delta s_0^-$ | $\Delta O_{K_1^+}^*$ | $\Delta m_{K_1^+}^*$ | $\Delta s_0^+$ |
|-------------------|----------------------|----------------------|----------------|----------------------|----------------------|----------------|
| This work         | -2.88×10^{-2}        | -0.256               | -1.28         | -2.8×10^{-3}        | -0.0374              | -0.247         |
| QCD sum rules [1] | -3.09×10^{-2}        | -0.249               | -1.25         | -2.72×10^{-3}       | -0.0348              | -0.234         |

4 Conclusion

In the present investigation, using the union of the QCD sum rules and chiral hadronic model, we evaluated the medium modified $K_1$ mass in the symmetric nuclear medium. We find that the charge symmetry does not hold in the nuclear medium for $K_1^-$ meson as the medium influenced mass of $K_1^-(\bar{u}s)$ meson modifies significantly at nuclear saturation density whereas in-medium mass $K_1^+(u\bar{s})$ meson modifies less.

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