Always on non-nearest-neighbour coupling in scalable quantum computing

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New Journal of Physics 9 (2007) 27
Received 17 May 2006
Published 13 February 2007
Online at http://www.njp.org/
doi:10.1088/1367-2630/9/2/027

Abstract. In many previous ‘always-on’ quantum computing (QC) schemes the non-nearest-neighbour couplings are often omitted because of their weak strength compared with the nearest-neighbour coupling. In this paper, we consider the problem of residual long-range interaction. We propose a QC scheme in which by the method of generalized ‘barrier spin’ encoding, the influence of the long-range coupling can be significantly suppressed. The universal quantum gates could be deterministically established. We also study the application of this scheme in a Josephson junction system.

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1. Introduction

One of the crucial problems in realizing scalable quantum computation (QC) is performing two qubit gates. In many of the proposed QC schemes, the couplings between qubits are set as variable functions subject to external control. However, in many physical systems this requirement is not easy to satisfy. If the inter-qubit interaction Hamiltonian is diagonal in the computational basis it is then possible to suppress them by repeated refocusing pulses like those used in NMR. This refocusing technique requires injecting pulses onto every qubit of the quantum system. Recently various alternative proposals based on encoding techniques have been developed to solve the always-on coupling problem. Zhou et al [1, 2] suggest encoding logical qubits in interaction free subspace (IFS), while Benjamin et al [3] suggest changing interqubit interaction from non-diagonal Heisenberg type to diagonal Ising type by tuning the Zeeman energy splits of individual qubits. Very recently ‘global control’ ideas have also been proposed [4, 5]. Schemes implementing these ideas into antiferromagnetic spin rings and optical systems have already been delivered [6, 7].

These elegant proposals mainly consider a model of a 1D spin-1/2 chain with perpetual couplings between neighbouring qubits. However in many realistic systems there is not only the nearest-neighbour interaction, but also the next-nearest-neighbour, or even longer range interactions. The long-range coupling terms may originate from residual wavefunction overlap or long-range Coulomb interaction. Since their strengths are often much smaller than that of the nearest-neighbour coupling, the non-nearest-neighbour interaction terms were often ignored in previous schemes. This omission thus induces systematical errors to QC process. In this paper, we aim at the residual long-range interaction problem. Our main idea is that we can take some lower order terms of the long-range coupling into account, while leaving the higher order terms as random noise. Then for the lowest order terms we try to negate their influence by proper encoding methods. Here, we propose a QC scheme in which by the method of generalized ‘blockade spin’, the influence of the next-nearest-neighbour interaction is effectively neutralized, hence the precision of quantum gates is improved. Deterministic single qubit rotations and inter-qubit control phase (CPHASE) gates could be performed based on the generalized ‘blockade spin’ architecture. In addition, compared with previous schemes in which the long-range couplings were ignored, our scheme does not cause the speed of quantum gates to slow significantly. The feasibility of this scheme in a 1D Josephson charge qubit array system is discussed before conclusion.

2. Generalized ‘blockade spin’ method

We consider a 1D spin-1/2 chain consisting of 2n + 1 spins, with tunable XXZ interaction between neighbouring spins [8]. The Hamiltonian reads:

\[ H_{\text{Ideal}} = H_S + H_I, \]  

where

\[ H_S = \sum_{i=1}^{2n+1} H_i^S = \sum_{i=1}^{2n+1} B_i^x \sigma_i^x + B_i^z \sigma_i^z, \]  

\[ H_I = \sum_{i=1}^{2n+1} B_i^x \sigma_i^x + B_i^z \sigma_i^z, \]  

where

\[ B_i^x, B_i^z \]

are suitable parameters for encoding and decoding.
Spin 1 2 3 4 5 6 7 8 9 10
0 A 1 B 0 C 1 D 0 E

Figure 1. Schematic diagram of previous ‘always-on’ QC schemes. For the always-on $\sum_{i=1}^{2n} J_{1} \sigma_i^z \sigma_{i+1}^z$ type inter-spin coupling, qubits will suffer continuous phase gates with their neighbours. We may only use the even spins (the hexagonal ones) to encode information while we use the odd spins (the rounded ones) as blockade spin. When we perform a single qubit operation, the blockade spins are ‘frozen’ in definite states $|0\rangle$ or $|1\rangle$ in order to negate the influence of the perpetual Ising interaction.

$$H_I = \sum_{i=1}^{2n} H_{i,i+1}^{i+1} = \sum_{i=1}^{2n} J_{i,i+1}(\sigma_i^z \sigma_{i+1}^z + \sigma_i^y \sigma_{i+1}^y) + J_1 \sigma_i^z \sigma_{i+1}^z.$$  \hspace{1cm} (3)

Here, we assume that the values of $B_i^z$, $B_i^y$, and $J_{i,i+1}$ are tunable, while $J_1$ remains constant. This model has been studied extensively in previous papers. Various systems including optical lattice, quantum dots, and Josephson junction array could be described by this ‘popular’ Hamiltonian. In addition, methods developed for the XXZ interaction can be easily generalized to XY and Heisenberg type interactions \[3\].

The previously proposed ‘always-on’ QC schemes based on $H_{\text{ideal}}$ can be briefly described as follows \[2, 3\]. For the untunable $\sum_{i=1}^{2n} J_{1} \sigma_i^z \sigma_{i+1}^z$ term, the ‘blockade spin’ methods have been developed to negate its influence on single qubit operations. As shown in figure 1, only the even spins (the hexagonal ones) are chosen as logical qubits while the odd spins (the rounded ones) are used as ‘blockade’. When we perform single qubit operations, the blockade spins are ‘frozen’ at states $|0\rangle$ or $|1\rangle$ in an interlaced manner, so the $n$ even spins feel no coupling from their neighbours. The single qubit gates are realized by tuning the magnetic fields on the corresponding encoded spins, while two qubits gates are established by effectively tuning on the inter-spin exchange interaction.

We now consider the long-range couplings, which are omitted in $H_{\text{ideal}}$ but do exist in realistic systems. Without loss of generality, we may consider an omitted permanent next-nearest-neighbour interaction term having the form

$$H_L = J_2 \sum_{i=1}^{2n-1} \sigma_i^z \sigma_{i+2}^z.$$  \hspace{1cm} (4)

When we perform quantum gates following previous schemes considering nearest-neighbour interaction only, deviation between the realistic evolution and the ideal expectation is induced by $H_L$. To solve this problem we incorporate $H_L$ into $H_{\text{ideal}}$ and consider the evolution of the spin chain governed by the generalized model Hamiltonian

$$H_M = H_S + H_I + H_L.$$  \hspace{1cm} (5)

where $H_S$, $H_I$, and $H_L$ are described by equations (2)–(4). Here, we further set $B_i^y = 0$ for any $i$ in the whole quantum information process, i.e. there are no $\sigma^y$ terms in $H_S$. 

New Journal of Physics 9 (2007) 27 (http://www.njp.org/)
We use two spins to encode one logical qubit, that is, the two spins 3 and 4 as one qubit and spins 7 and 8 as another. Two ‘blockade spins’ both in |0⟩ state are placed between two logical qubits in order to negate the influence of next-nearest-neighbour interactions.

Similar to the ‘blockade spin’ methods used in [3], our intuitive idea is freezing two ‘blockade spins’ between each two logical qubits to negate the influence of the nearest-neighbour and next-nearest-neighbour always-on Ising interaction. As sketched in figure 2, we encode one logical qubit by two physical spins, using the spin states |0⟩ and |1⟩ as logical |0⟩ and |1⟩. The hexagonal ones are spins used to encode information while the circular ones are used as blockades. We use the spins 3 and 4 as one logical qubit while spins 7 and 8 as another. When performing single logical qubit operations, we would make the blockade spins (spins 1, 2, 5, 6, 9 and 10) all ‘frozen’ in state |0⟩, hence the influence of permanent Ising interactions on single logical qubit operations can be effectively negated.

3. QC scheme based on generalized ‘blockade spin’ architecture

Now we show how we realize deterministic universal quantum gates, that is, the single qubit $\sigma^x$ rotations, the single qubit $\sigma^z$ rotations, and interqubit CPHASE gates on the generalized ‘blockade spin’ architecture. The single qubit $\sigma^x$ rotation on the qubit encoded by spins 3 and 4 is easy by tuning $J_{3,4}$. In addition, we note a trivial fact [9] that the single qubit $\sigma^z$ rotation can be constructed by the CPHASE gate and single qubit $\sigma^z$ rotation. Therefore the central problem becomes implementing the interqubit CPHASE gate. Our main idea is using the exchange interaction $J_{4,5}$ and $J_{6,7}$, we can swap the states of spins 4 and 7 onto spins 5 and 6, then a controllable phase can be added on one of the four logical qubit states while the other three states remain unchanged.

We separate the spins 3, 4 and 5 as one party and spins 6, 7 and 8 as the other. We set $B_i = 0$ for any $i$ in the whole process of performing the CPHASE gate. Initially we set $J_{i,i+1} = 0$ for any $i$. In this situation the four possible logical qubits states \{ |00⟩, |01⟩ \}_3,4,5 \otimes \{ |00⟩, |01⟩ \}_6,7,8 have equal static energy, which is defined as the energy zero point from now on. As mentioned above, in the whole process of realizing a two-qubit gate, what we control is just $J_{4,5}$ and $J_{6,7}$, so the Hamiltonian $H_M$ in equation (5) can be reduced into a 9D Hilbert space

$$S = \text{span}\{ |00⟩, |01⟩, |10⟩ \}_3,4,5 \otimes \{ |00⟩, |01⟩, |10⟩ \}_6,7,8 \},$$

and $H_M$ can be viewed as a function of $J_{4,5}$ and $J_{6,7}$: $H_M = H_M(J_{4,5}, J_{6,7})$. Below we use the label |abcdef⟩ to label the quantum states of the six spins from spin 3 to spin 8, the first a for spin 3, the second b for spin 4 . . . the last f for spin 8. For example, |100010⟩ labels the state in which the spin 3 and spin 7 are in state |1⟩ while the spin 4, 5, 6 and 8 are in state |0⟩.

Figure 2. Generalized ‘blockade spin’ encoding method. We use two spins to encode one logical qubit, that is, the two spins 3 and 4 as one qubit and spins 7 and 8 as another. Two ‘blockade spins’ both in |0⟩ state are placed between two logical qubits in order to negate the influence of next-nearest-neighbour interactions.
Indeed, $S$ can be reduced to four subspaces. The first one is a trivial subspace $S_1$ spanned by a single state $|100001\rangle$. The second is 2D

$$ S_2 = \text{span}\{|[100]\rangle_{3,4,5} \otimes |010\rangle, |100\rangle\rangle_{6,7,8}. \tag{7} $$

Under basis $\{|100010\rangle, |100100\rangle\}$ Hamiltonian $H_M(J_{4,5}, J_{6,7})$ can be reduced to

$$ H_2(J_{4,5}, J_{6,7}) = \begin{bmatrix} 0 & 2J_{6,7} \\ 2J_{6,7} & 0 \end{bmatrix}. $$

The third is similar to the second

$$ S_3 = \text{span}\{|[010]\rangle, |001\rangle\rangle_{3,4,5} \otimes |001\rangle\rangle_{6,7,8}. \tag{8} $$

Under basis $\{|010001\rangle, |001001\rangle\}$ the reduced Hamiltonian is

$$ H_3(J_{4,5}, J_{6,7}) = \begin{bmatrix} 0 & 2J_{4,5} \\ 2J_{4,5} & 0 \end{bmatrix}. $$

The fourth is 4D

$$ S_4 = \{|010\rangle, |001\rangle\rangle_{3,4,5} \otimes |010\rangle, |100\rangle\rangle_{6,7,8}. \tag{9} $$

Under basis $\{|010010\rangle, |010100\rangle, |001010\rangle, |001100\rangle\}$ the reduced Hamiltonian is

$$ H_4(J_{4,5}, J_{6,7}) = \begin{bmatrix} 0 & 2J_{6,7} & 2J_{4,5} & 0 \\ 2J_{6,7} & -4J_2 & 0 & 2J_{4,5} \\ 2J_{4,5} & 0 & -4J_2 & 2J_{6,7} \\ 0 & 2J_{4,5} & 2J_{6,7} & -4J_1 \end{bmatrix}. \tag{10} $$

$H_4(J_{4,5}, J_{6,7})$ is quite similar to the NMR type Hamiltonian: if we set span $\{|010\rangle, |001\rangle\rangle_{3,4,5}$ as the ‘left qubit’ and span $\{|010\rangle, |100\rangle\rangle_{6,7,8}$ as the ‘right’, we can see that $J_{4,5}$ and $J_{6,7}$ play the role of tunable local $X$ operation, while the perpetual $\sum J_i \sigma_i^\uparrow \sigma_{i+1}^\uparrow + J_2 \sigma_i^\uparrow \sigma_{i+2}^\uparrow$ terms induce an untunable $a(\sigma_i^\uparrow + \sigma_k^\uparrow) + b \sigma_i^\uparrow \sigma_k^\uparrow$ interaction. However, techniques developed for NMR such as refocusing usually can hardly be employed in other systems (especially solid state systems including quantum dot and superconducting circuits) because the requirement for a fast, strong pulse is not easy to satisfy, so we present an alternative way to perform the CPHASE gate.

In the first step, we tune only $J_{4,5}$ while setting $J_{6,7} = 0$. The $XXZ$ interaction between spin 4 and 5 keeps states $|100001\rangle$ and $|100010\rangle$ unchanged. In space $S_3$, taking the Bloch sphere representation we see the $J_{4,5}$-induced transformation is a rotation about the $X$-axis. In space $S_4$ things become more complex. Since our initial state in $S_4$ is only $|010010\rangle$, the $J_{4,5}$-induced transformation in $S_4$ is finally reduced to a 2D subspace $S'_4 = \text{span}\{|[010]\rangle, |001\rangle\rangle_{3,4,5} \otimes |[010]\rangle\rangle_{6,7,8}\}. Under basis $\{|010010\rangle, |001010\rangle\}$ the reduced Hamiltonian is

$$ H'_4(J_{4,5}, 0) = \begin{bmatrix} 0 & 2J_{4,5} \\ 2J_{4,5} & -4J_2 \end{bmatrix}. \tag{11} $$
\(H'_4(J_{4,5}, 0)\) described a rotation about an axis on the \(X-Z\)-plane because the static energy of \(|010010\) and \(|001010\) are slightly different due to the long-range interaction \(H_L\).

In \(S'_4\), with tunable \(J_{4,5}\) acting as \(\sigma^z\)-operation, we can perform an exact \(\pi\) rotation around the \(X\)-axis in the Bloch sphere representation: we assume that the maximum value of \(J_{4,5}\) we can tune to is \(X_1/2\). We set parameter \(\theta\) and \(X_2\) as \(\cos \theta = \frac{x_1}{\sqrt{x_1^2 + (2J_5)^2}}\), \(\sin \theta = \frac{2J_5}{\sqrt{x_1^2 + (2J_5)^2}}\),

\[
\begin{align*}
\cos 2\theta &= \frac{x_1}{\sqrt{x_1^2 + (2J_5)^2}}, \\
\sin 2\theta &= \frac{2J_5}{\sqrt{x_1^2 + (2J_5)^2}}.
\end{align*}
\]

We then define the rotation (we set \(\hbar = 1\))

\[
R(X) = \exp \left[ iH'_4(X, 0)\pi/2\sqrt{X^2 + (2J_5)^2} \right].
\]

Thus we have

\[
R(X_1) = i \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix},
\]

\[
R(X_2) = i \begin{bmatrix} \sin 2\theta & \cos 2\theta \\ \cos 2\theta & -\sin 2\theta \end{bmatrix}.
\]

The combined rotation is what we want

\[
R(X_1)R(X_2)R(X_1) = \exp\{-i\pi\sigma^z/2\}.
\]

This above tuning of \(J_{4,5}\) at last forms a unitary transformation \(U_1\) which is only nontrivial in space \(S_3\) and \(S'_4\): \(|100001\) and \(|010010\) remain unchanged; \(|001010\) is transformed into \(|001001\); \(|010001\) is transformed into a superposition of \(|010001\) and \(|001001\). As shown in equations (12)–(15), the required time for this step is

\[
T_1 = \pi/\sqrt{X_1^2 + (2J_5)^2 + \pi/2\sqrt{X_2^2 + (2J_5)^2}}.
\]

We can see that \(T_1\) mainly depends on the maximal \(XY\) interaction strength we could have. We further note an important fact that if we reperform the tuning of \(J_{4,5}\) with inverse strength, i.e. \(X_1\) to \(-X_1\), \(X_2\) to \(-X_2\), we can get the inverse operation of \(U_1\) in \(S\).

In the second step we tune \(J_{4,5}\) to zero but begin to control \(J_{6,7}\). Quite similar to the previous step, this exchange interaction is only nontrivial in \(S_2\) and \(S_4\). In \(S_4\), under the Bloch sphere representation the \(J_{6,7}\)-induced transformation is a rotation about the \(X\)-axis. In \(S_4\), since the initial state \(|010010\) is transformed into \(|001010\) in the first step, the \(J_{6,7}\)-induced transformation is restricted to another 2D subspace \(S''_4 = |001\rangle_{3,4,5} \otimes |010\rangle_{6,7,8}\). Under basis \(|001010\rangle, |001100\rangle\) the reduced Hamiltonian in this subspace is

\[
H''_4(0, J_{6,7}) = \begin{bmatrix} -4J_2 & 2J_{6,7} \\ 2J_{6,7} & -4J_1 \end{bmatrix}.
\]

The form of \(H''_4(0, J_{6,7})\) is quite similar to that of the previous \(H'_4(J_{4,5}, 0)\), so we can perform another unitary transformation \(U_2\) similar to \(U_1\) which implements a \(\pi\) rotation around the \(X\)-axis in space \(S''_4\), transforming state \(|001010\rangle\) to \(|001100\rangle\).

After the above two steps, we review the intermediate states we get: \(|100001\rangle\) remains unchanged; \(|100010\rangle\) is changed into the superposition of \(|100010\rangle\) and \(|100100\rangle\); \(|010010\rangle\) is
changed into the superposition of $|010001\rangle$ and $|001001\rangle$; $|010010\rangle$ is changed into $|001100\rangle$. The previous three intermediate states are degenerate under $H_{M}(0,0)$ but the last state has nonzero static energy $-4J_{1}$. Therefore, in the third step we tune off all exchange coupling for a time interval $\tau$. In this period the state $|001100\rangle$ experiences an additional phase $\varphi$ due to its nonzero static energy. In the last step we perform the inverse operation of $U_{2}$ and $U_{1}$ to transform the four intermediate states back to the initial four states.

At the end of all the four steps we have performed a CPHASE gate $U(\varphi)$ between two logical qubits, adding a controllable phase $\varphi$ on the state $|010010\rangle$ while the other three states remain unchanged. Under the basis $\{|100\rangle, |010\rangle\}_{3,4,5} \otimes \{|010\rangle, |001\rangle\}_{6,7,8}$ the matrix form of $U(\varphi)$ is

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & e^{i\varphi} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(18)

Immediately after implementation of the CPHASE gate $U$, we note that $(\sigma_{z}^{+} \times U(\varphi)^{2} = e^{i\varphi} e^{i\sigma_{z}^{+} \varphi}$ [9], i.e. with the single qubit $\sigma_{z}^{+}$ rotation of the second qubit and controllable CPHASE gate between the first and the second qubit, the single qubit $\sigma_{z}^{+}$ rotation of the first qubit can be realized.

In summary of this section, we have demonstrated all the required universal gates for QC. In this scheme, by using an appropriate encoding method, the influence of the next-nearest-neighbour interaction is effectively ruled out. The quantum gates we have proposed are neither approximative nor probabilistic but deterministic. Besides, the speed of the CPHASE gate mainly depends on the strength of the nearest-neighbour exchange interaction. The quantum gate speed of this scheme is of the same level as that of previous schemes in which the next-nearest-neighbour interaction was neglected.

4. A potential physical application: Josephson junction charge qubit array

In this section, we consider the application of our scheme to realistic systems. We take the long-range interaction in a Josephson charge qubit system as an example. The typical Josephson-junction charge qubit is shown in figure 3 [10]; it consists of a small superconducting island
with \( n \) excess Cooper pairs, connected by a tunnel junction with capacitance \( C_J \) and Josephson coupling energy \( E_J \) to a superconducting electrode. A control gate voltage \( V_g \) is coupled to the system via the gate capacitor \( C_g \). The Hamiltonian of the Cooper pairs box (CPB) is

\[
H_{\text{Josephson}} = E_c(n - n_g)^2 + E_J \cos \Phi,
\]

where \( E_c = (2e)^2/2(C_g + C_J) \) is the charge energy, \( n_g = C_g V_g/2 \) is the gate charge, and \( \Phi \) is the conjugate variable to \( n \). When \( E_c \gg E_J \), if we tune \( n_g \) close to the degeneracy point \( n_g = 1/2 \), only the states with zero and one Cooper pairs play a role while all other states in much higher energy levels can be ignored. In this case the CPB can be reduced to a two-state quantum system.

A further step is replacing the single Josephson junction by two identical junctions in a loop configuration in order to gain tunable tunnel amplitude [11]. The effective Hamiltonian can be written in the spin-1/2 notation as

\[
H_{\text{Josephson}} = -B_z \sigma_z - B_x \sigma_x,
\]

where the state with zero Cooper pairs corresponds to the spin state \(| \downarrow \rangle \) and that with one Cooper pair to \(| \uparrow \rangle \); \( B_z \) and \( B_x \) are the effective magnetic fields which are controlled by the biased gate voltage and frustrated magnetic flux.

For coupling two CPBs, the direct capacitance coupling [12, 13] is mostly intrinsic and it has been realized in experiments. However its drawback is also obvious, that is, the couplings induced by connective capacitors is untunable. A schematic plot of an array of capacitively coupled CPBs is shown in figure 4. For an array of \( M \) CPBs with identical junction capacitance \( C_J \) coupled to the previous and next CPBs by coupling capacitors \( C_c \), the static charge energy can be written as [14, 15]

\[
H_C = \frac{(2e)^2}{2} \vec{n} C^{-1} \vec{n}^\dagger,
\]

where \( \vec{n} = (n_1 - n_{g1}, n_2 - n_{g2}, n_3 - n_{g3}, \ldots n_M - n_{gM}) \) is the charge number vector of the \( M \) CPBs, and \( C \) is the capacitance matrix of the system whose diagonal term \( C_{i,i} \) equals the sum of capacitance around the \( i \)th CPB, and non diagonal term \( C_{i,j} \) corresponds to the connective capacitance between CPB \( i \) and \( j \). For the intermediate qubits of the array

\[
C_{i,j} = C_0[\delta_{i,j}(1 + 2\epsilon) - \delta_{i,j\pm 1}\epsilon], \quad 1 < i, \quad j < N,
\]
Figure 5. A schematic plot of two CPBs coupled by a SQUID.

where \( C_0 = C_g + C_1 \) and \( \epsilon = C_c / C_0 \). For qubits on the edge of the array a small correction is needed: \( C_{1,1} = C_{M,M} = C_0(1 + \epsilon) \).

Since \( C \) is a tridiagonal matrix, \( C^{-1} \) has nonzero matrix elements on the second, third and other diagonals which characterize the capacitor induced Coulomb interaction between distant CPBs. If \( \epsilon \ll 1 \), the off-diagonal elements of \( C^{-1} \) decay exponentially as \( C^{-1}_{i,j} \sim C^{-1}_{i,i} \epsilon^{\lvert i-j \rvert} \).

Making the replacement \( n_i = (1 + \sigma^z_i) / 2 \), we get that the interaction term \( C_{i,j}^{-1}(n_i - n_{g_i})(n_j - n_{g_j}) \) provides always-on Ising type interaction between distant CPBs \( i \) and \( j \).

If we go further, replacing the coupling capacitance by a superconducting quantum interference device (SQUID), we can have tunable \( XY \) interaction between neighbouring qubits in addition to perpetual Ising interaction (figure 5) \([16]–[18]\). Due to flux quantization the phase across the coupling SQUID is \( \Phi_1 - \Phi_2 + \alpha \), \( \alpha \) being a constant controlled by the frustrated flux. If we tune \( \alpha \) to zero, the coupling term \( \cos(\Phi_1 - \Phi_2) \) becomes the \( XY \) interaction \( \sigma^x_i \sigma^x_j + \sigma^y_i \sigma^y_j \).

Now we can see the correspondence between the theoretical Hamiltonian in equation (5) and the realistic physical system: the tunable SQUIDs of single qubits induce tunable \( \sigma^x \) terms in the single qubit Hamiltonian; in addition, all the bias voltages are biased on the degeneracy point \( n_{g} = 1/2 \) in order to protect the qubits from the \( 1/f \) noise effect \([19]\) corresponding to the fact that there is no \( \sigma^z \) term in the single qubit Hamiltonian; the exponential decay capacitive couplings correspond to the nearest Ising part in \( H_I \) and the next-nearest-neighbour Ising part \( H_L \), while the tunable SQUID coupling corresponds to the tunable \( XY \) part in \( H_I \). We also see that the required performances in the proposed scheme correspond to tuning the external magnetic field frustrated in the SQUID loops.

As we have mentioned before, the Ising type long-range interaction in a Josephson array system falls exponentially with distance in a manner of \( C_{i,j}^{-1} \sim C_{i,i}^{-1} \epsilon^{\lvert i-j \rvert} \). Therefore a straightforward idea is that we can make \( \epsilon \ll 1 \) so that the effect of the long-range interaction in the Josephson charge qubit array system can be reduced. However, the speed of the two qubits operation depends on \( \epsilon \), decreasing \( \epsilon \) would slow the speed of the two qubits gate. Besides, making \( \epsilon \) smaller and smaller may be highly challenging in experimental realization. Thus we may choose an alternative way. The main idea is, based on the decay property of the long-range interaction, we can take some lower order terms of long-range interaction into account...
while leaving the higher order terms as random noise. For the lowest order we may use the encoding methods and QC schemes discussed before to negate the influence of the few lowest order couplings. In addition, a natural generalization of the proposed scheme is that we may use 3 spins as one ‘blockade’ to negate the influence of the third order long-range interaction, or even \( m \) spins as one ‘blockade’ to negate the influence of the \( m \)th order long-range interaction. Therefore we could apply the generalization of our QC scheme to suppress the speed of deviation induced by higher orders of long-range interaction into some tolerable domain.

Some schemes of performing QC in capacitively coupled Josephson arrays have been proposed [15]–[20]. In these schemes the realization of quantum gates requires tuning the qubit out of the optimal operation point \( n_{gi} = 1/2 \), which may reduce the coherence time of qubits by a few orders of magnitude. This problem does not occur in our scheme because in our scheme the qubits are always frozen at the optimal bias point, hence decoherence from the \( 1/f \) noise effect is greatly reduced. In addition, the only parameter requiring tuning is the flux frustrated in SQUID loops, i.e. only one type of operation is needed to implement the universal quantum gates, while in the previous schemes mentioned both voltage type and flux type tuning are required. We could also translate our idea into Heisenberg systems such as quantum dots and optical lattices.

5. Conclusion

In conclusion, in this paper we have studied the non-nearest-neighbour interaction effect in the 1D spin-1/2 chain model. With a generalized ‘blockade spin’ method, we propose a QC scheme in order to suppress the influence of long-range interaction. In this scheme we effectively neutralize the influence of the next-nearest-neighbour interaction, thus the precision of quantum gates is updated. Deterministic universal quantum gates are implemented in this architecture. The quantum information speed of this scheme is of the same level as that of previous schemes in which the long-range interaction strength was ignored. We also discuss the feasibility of the scheme in a 1D Josephson charge qubit array system. This scheme may offer improvements in dealing with systematic errors in quantum computing.

Acknowledgments

Y H thanks J M Cai, M Y Ye, Y F Xiao, X F Zhou and Y J Han for fruitful discussions. This study was funded by the National Fundamental Research Program, Innovation funds from Chinese Academy of Sciences, NCET-04-0587, and the National Natural Science Foundation of China (grant nos. 60121503 and 10574126).

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