The importance of $\ln(1/x)$ resummation: a new QCD analysis of HERA data

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Fits to the final combined HERA deep-inelastic scattering cross-section data within the conventional DGLAP framework of QCD have shown some tension at low-$x$ and low-$Q^2$. A resolution of this tension incorporating $\ln(1/x)$-resummation terms into the HERAPDF fits is investigated using the xFitter program. The kinematic region where this resummation is important is delineated. Such high-energy resummation not only gives a better description of the data, particularly of the longitudinal structure function $F_L$, it also results in a gluon PDF which is steeply rising at low $x$ for low scales, $Q^2 \simeq 2.7$ GeV$^2$, contrary to the fixed-order (FO) NLO and NNLO gluon PDF. This contribution is based on the results presented in Ref. 1.

1 Input data sets

The input datasets in use are the final combined $e^\pm p$ cross-section measurements of H1 and ZEUS (both from neutral-current (NC) and charged-current (CC) processes and for $e^+p$ and $e^-p$ scattering) and the HERA combined charm from ZEUS and H1. The inclusion of charm data in the fit is useful to determine the optimal charm pole mass. Additionally, since they extend to rather small values of $x$, they may be sensitive to $\ln(1/x)$ resummation effects.

2 Fit strategy

The present QCD analysis uses the xFitter program and is based on the HERAPDF2.0 setup. The quark distributions at the initial scale $Q_0^2$ were represented by the generic form:

$$xq_i(x,Q_0) = A_i x^{B_i} (1 - x)^{C_i} P_i(x),$$

where $P_i(x) = 1 + O(x)$ defines a polynomial in powers of $x$. The parametrised quark distributions $q_i$ were chosen to be the valence quark distributions ($xu_v, xd_v$) and the light anti-quark distributions ($x\bar{U} = x\bar{u}, xD = x\bar{d} + x\bar{s}$). The gluon distribution was parametrised with the more flexible form:

$$xg(x) = A_g x^{B_g} (1 - x)^{C_g} P_g(x) - A'_g x^{B'_g} (1 - x)^{C'_g}.$$

The normalisation parameters $A_{u_v}$ and $A_{d_v}$ were fixed using the quark counting rules and $A_g$ using the momentum sum rule. The normalisation and slope parameters, $A$ and $B$, of $\bar{u}$ and $\bar{d}$ were set equal such that $x\bar{u} = x\bar{d}$ at very small $x$. The strange PDFs $xs$ and $x\bar{s}$ were parametrised as $xs = x\bar{s} = 0.4x\bar{D}$, representing a suppression of strangeness with respect to the light down-type sea quarks, but the input data are not sensitive to the fraction of strangeness.

The $\ln(1/x)$ resummation corrections are available in the HELL code, which is a standalone code that implements the resummation corrections to the DGLAP splitting functions $P$ and to the DIS coefficient functions $C$ (both massless and massive) up to next-to-leading-log accuracy in $\ln(1/x)$ (NLLx). The scale at which PDFs are parameterised have been chosen to be $Q_0^2 = 2.56$ GeV$^2$ as compared to 1.9 GeV$^2$ of HERAPDF2.0. The reason is that the numerical computation of $\ln(1/x)$-resummation corrections may become unreliable at low scales due to the large value of the strong coupling $\alpha_S$.

3 Results

The effect of $\ln(1/x)$ resummation on splitting functions and DIS coefficient functions is more dramatic at NNLO than at NLO. In fact, the full calculation with NNLO+NLLx resummation is closer to the NLO result than it is to the NNLO result. This is not accidental and is mostly
due to the perturbative instability of the NNLO correction to the splitting functions generated by small-x logarithms. Thus, to better assess the impact of the ln(1/x) resummation on the original HERAPDF analysis, we only focus on NNLO fits.

As well as evaluating uncertainties due to the experimental statistical and systematic errors we have performed an exploration of model and parametrisation uncertainties as follows. We have varied the charm mass \((m_c = 1.41, 1.51 \text{ GeV})\), the bottom mass \((m_b = 4.25, 4.75 \text{ GeV})\), the strong coupling \(\alpha_S(m_Z^2)\) \((\Delta\alpha_S = \pm 0.002)\), the strangeness fraction \((f_s = 0.3, f_0.5)\), the initial scale \((Q_0^2 = 2.88 \text{ GeV}^2)\), and the \(Q^2\) cut on the data \((Q^2_{\text{min}} = 2.7 \text{ GeV}^2, 5 \text{ GeV}^2)\). Furthermore, parametrisation uncertainties have been explored by adding extra terms to the polynomials \(P_i(x)\) of Eq. 1. The only noticeable difference comes from the addition of a linear term to the polynomial \(P_{uv}(x)\) of the valence up quark PDF. The largest contribution to the uncertainty on the gluon distribution arises from the variation of the \(Q^2_{\text{min}}\) cut to 5 GeV. Interestingly, this uncertainty is reduced for the fit with ln(1/x) resummation, due to reduced tensions with the data. Fig. 1 shows a comparison of PDFs with and without ln(1/x) resummation at \(Q^2 = 3 \text{ GeV}^2\). This figure displays also the full uncertainty bands. When resummation is included, both the gluon and the total singlet PDFs rise towards low \(x\), in contrast to the behaviour of the gluon when resummation is not included.

The \(\chi^2\) values for the fits are summarised in Tab. 1. There is a decrease of 73 units in \(\chi^2\) when the ln(1/x) resummation is used. Most of this difference is coming from the highly accurate NC \(E_p = 920 \text{ GeV}\) data which probe the low-\(x\) and low-\(Q^2\) region and are thus most sensitive to ln(1/x) resummation (413/377 to be compared to 446/377). As expected, a decrease in \(\chi^2\) has been also observed in the NC \(E_p = 820 \text{ GeV}\) (65/70 for the NNLO+NLLx fit vs. 70/70 for the FO NNLO fit) and in the charm data (49/47 vs. 48/47), which are also expected to have some sensitivity. Other data sets entering the fit probe higher \(x\) and \(Q^2\) and their \(\chi^2\) are not significantly changed. In Fig. 2 the fit results are compared to the NC \(E_p = 920 \text{ GeV}\) inclusive reduced cross-section data in the lowest \(Q^2\) bins included in the fits. It is evident that for the fit including ln(1/x)-resummation effects, not only the initial description of the data is better,
makes reduction of the $\chi^2$ better reproduced by the fit that includes $\ln(1/x)$ significant smaller. In particular, it is evident that the low-$x$ but also the correlated shifts are smaller and this is one of the reasons why the $\chi^2$ of the fit is significantly smaller. In particular, it is evident that the low-$x$ turn-over of the measurements is better reproduced by the fit that includes $\ln(1/x)$ resummation, which in turn explains the big reduction of the $\chi^2$. This is a direct consequence of the steeper gluon at low $x$ which makes $F_L$ larger at low $x$ causing a more pronounced turn-over of the reduced cross section, defined as follows:

$$\sigma_{\text{red}} = F_2 - \frac{y^2}{Y_+} F_L,$$

where $F_2$ and $F_L$ are the structure functions related to the parton distributions, $Y_+ = 1 + (1 - y)^2$ and $y = Q^2/(sx)$. This point is also illustrated in Fig. 3 where the H1 $F_L$ measurement is compared to the theoretical predictions of $F_L$ with and without $\ln(1/x)$. It is clearly visible that the description of this data set is improved in the former case thanks to the fact that $\ln(1/x)$-resummed predictions for $F_L$ are larger at low $x$.

The results presented so far indicate that the improvement of the description of the HERA data when including $\ln(1/x)$ resummation is driven by the low-$x$ and low-$Q^2$ data. We can also delineate the kinematic region responsible for the improvement more precisely. To do so, we have performed $\chi^2$ scans in $Q^2_{\text{min}}$ with no cut in $x$, and in $x_{\text{min}}$ (where $x_{\text{min}}$ is the minimum value of Bjorken $x$ allowed in the fit) fixing $Q^2_{\text{min}} = 2.7 \text{ GeV}^2$. Furthermore, an additional $\chi^2$ scan in $y_{\text{max}}$ has been done, excluding from the fit data with $y > y_{\text{max}}$. The $\chi^2$ scans as a function of $Q_{\text{min}}$, $x_{\text{min}}$ and $y_{\text{max}}$ allow us to delineate the region of the ($x,Q^2$)-plane in which

![Figure 2](image1.png)

Figure 2 – The HERA NC $E_p = 920$ GeV data compared to the fits with and without $\ln(1/x)$ resummation for the $Q^2 = 3.5$, $Q^2 = 3.5$ and $4.5$ GeV$^2$ bins.

![Figure 3](image2.png)

Figure 3 – Left: The H1 measurement of $F_L$ compared to the predictions with and without $\ln(1/x)$ resummation; Right: Scatter plot of the low-$x$ and low-Q$^2$ kinematic region covered by the HERA1+2 inclusive data and charm data at $E_p = 920$ GeV. The green shaded area indicates the region in which $\ln(1/x)$ resummation has a significant effect.
\[ \ln\left(\frac{1}{x}\right) \] resummation is important. Fig. 3 displays a zoom of the low-\(x\) and low-\(Q^2\) kinematic region covered by the HERA1+2 inclusive and charm data at \(E_p = 920 \text{ GeV}\). The green shaded area indicates the region such that \(x < 5 \times 10^{-4}, 2.7 \text{ GeV}^2 < Q^2 < 15 \text{ GeV}^2,\) and \(0.4 < y < 1\) (assuming \(\sqrt{s} = 318 \text{ GeV}\)) determined by combining the results of the scans discussed above. This provides an estimate of the region where \(\ln(1/x)\) resummation provides a significantly better description of the HERA data as compared to FO predictions.

4 Conclusion

In conclusion, \(\ln(1/x)\) resummation provides a substantial improvement in the description of the precise HERA1+2 combined data and it overcomes a major disadvantage of the FO analyses, namely a decreasing gluon PDF at low \(x\) and \(Q^2\). It represents an alternative to the addition of higher-twist terms \(^9,^{10},^{11}\) and does not suffer from the pathological features of some of these analyses \(^9\).

References

1. Abdolmaleki, H. et al, Impact of low-\(x\) resummation on QCD analysis of HERA data, arXiv:1802.00064 (2018).
2. Abramowicz, H. et al, Combination of measurements of inclusive deep inelastic \(e^\pm p\) scattering cross sections and QCD analysis of HERA data, Eur. Phys. J. C75, 580 (2015).
3. Abramowicz, H. et al, Combination and QCD Analysis of Charm Production Cross Section Measurements in Deep-Inelastic \(e p\) Scattering at HERA, Eur. Phys. J. C73, 2311 (2013).
4. Alekhin, S. et al, HERAFitter, Eur. Phys. J. C75, 304 (2015).
5. Aaron, F.D. et al, Combined Measurement and QCD Analysis of the Inclusive \(e^\pm p\) Scattering Cross Sections at HERA, JHEP 1001, 109 (2010).
6. Bonvini, M. et al, Towards parton distribution functions with small-\(x\) resummation: HELL 2.0, JHEP 12, 117 (2017).
7. Ball, R. et al, Parton distributions with small-\(x\) resummation: evidence for BFKL dynamics in HERA data, arXiv:1710.05935 (2017).
8. Gao, J. et al, The Structure of the Proton in the LHC Precision Era, arXiv:1709.04922 (2017).
9. Abt, I. et al, Study of HERA \(e p\) data at low \(Q^2\) and low \(x_{Bj}\) and the need for higher-twist corrections to standard perturbative QCD fits, Phys. Rev. D94 3, 034032 (2016).
10. Harland-Lang, L. A. et al, The impact of the final HERA combined data on PDFs obtained from a global fit, Eur. Phys. J. C76, 186 (2016).
11. Motyka, L. et al, Evidence of quasi-partonic higher-twist effects in deep inelastic scattering at HERA at moderate \(Q^2\), arXiv:1707.05992 (2017).

\(^\text{a}\)The actual plane over which the constraint acts is the \((x, Q^2/s)\)-plane. However, for simplicity in the following we will only consider the \(E_p = 920 \text{ GeV}\) inclusive and the charm datasets that were both taken at \(\sqrt{s} = 318 \text{ GeV}\).

\(^\text{b}\)In fact, given the range in \(y\), the constraint on \(x\) has no effect on the shaded area.