SPIKE-TIMING-DEPENDENT BACK PROPAGATION IN DEEP SPIKING NEURAL NETWORKS

A PREPRINT

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March 27, 2020

ABSTRACT

The success of Deep Neural Networks (DNNs) can be attributed to its deep structure, that learns invariant feature representation at multiple levels of abstraction. Brain-inspired Spiking Neural Networks (SNNs) use spatiotemporal spike patterns to encode and transmit information, which is biologically realistic, and suitable for ultra-low-power event-driven neuromorphic implementation. Therefore, Deep Spiking Neural Networks (DSNNs) represent a promising direction in artificial intelligence, with the potential to benefit from the best of both worlds. However, the training of DSNNs is challenging because standard error back-propagation (BP) algorithms are not directly applicable. In this paper, we first establish an understanding of why error back-propagation does not work well in DSNNs. To address this problem, we propose a simple yet efficient Rectified Linear Postsynaptic Potential function (ReL-PSP) for spiking neurons and propose a Spike-Timing-Dependent Back-Propagation (STDBP) learning algorithm for DSNNs. In the proposed learning algorithm, the timing of individual spikes is used to carry information (temporal coding), and learning (back-propagation) is performed based on spike timing in an event-driven manner. Experimental results demonstrate that the proposed learning algorithm achieves state-of-the-art performance in spike time based learning algorithms of SNNs. This work investigates the contribution of dynamics in spike timing to information encoding, synaptic plasticity and decision making, providing a new perspective to design of future DSNNs.

Keywords Deep neural networks · Spiking neural networks · Spike-timing-dependent · Back-propagation · Event-driven

1 Introduction

The success of Deep Neural Networks (DNNs) is attributed to the deep hierarchy structure, that learns the representations of big data with multiple levels of abstraction [1]. Research in DNNs has advanced the state-of-art in many machine learning tasks, such as image recognition [2], speech recognition [3,4], natural language processing [5,6], and medical diagnosis [7]. However, training of DNNs generally requires high computing resources (e.g., GPUs and computing clusters). Therefore, in power-critical computing platform, such as edge computing, implementation of DNNs is greatly limited [8,9]. Spiking Neural Networks (SNNs) provide a low-power alternative to neural network implementation. It is designed to emulate brain computing, that has the potential to provide computing capabilities equivalent to that of DNNs on an ultra-low-power spike-driven neuromorphic hardware [10,12]. However, due to the relatively shallow network structures, SNNs have yet to match the performance of DNNs in pattern classification tasks on standard benchmarks [13,14].

1This research is supported by Programmatic Grant No. A1687b0033 from the Singapore Government’s Research, Innovation and Enterprise 2020 plan (Advanced Manufacturing and Engineering domain), the National Natural Science Foundation of China (Grant No. 61976043 and 61573081), the National Key Research and Development Program of China (2018AAA0100202), the Zhejiang Lab (Grant No. 2019KC0AB02). Email: maluzhang@nus.edu.sg
There has been a growing interest in the implementation deep structures for SNNs (DSNNs) [9,12,14–20]. Unfortunately, the training of DSNNs is not straightforward as the well-studied error back-propagation (BP) learning algorithm is not applicable due to the complex temporal dynamics and the non-differentiable spike function. Addressing the issue of DSNN training, there have been many successful implementations, that can be classified into three categories.

The first category is ANN-to-SNN conversion methods. They train an equivalent ANN, and then approximately convert the pre-trained ANN into SNN version [21–30]. The goal of ANN-to-SNN methods is to leverage the state-of-the-art ANN training algorithms, so that the converted SNN version can reach the competitive classification performance of its off-line trained ANN counterparts. Due to the approximation, the conversion suffers from some loss of accuracy. Despite many studies, such as weight/activation normalization [26–28], and adding noise to the model [24, 25], the solutions are far from perfect. Furthermore, the conversion SNNs use the spike rate of a spiking neuron to encode analog activity of an ANN neuron. Thus, a high spike rate is required to simulate the stronger analog, which would then mask the discrete nature of the spike activity, and is energy-intensive [31]. Also, the inference time of the spike rate based encoding scheme is another problem [19].

The second category is the membrane potential driven learning algorithms, treating the neuron’s membrane potential as differentiable signals to solve the non-differential problems of spikes with the surrogate derivatives [32]. For example, [33–35] back-propagate errors based on the membrane potential at a single time step, which ignore the temporal dependency, but only use signal values at the current time instance. To resolve this problem, SLAYER [9] and STBP [16,17] train deep SNNs with surrogate derivatives based on the idea of Back-propagation Through Time (BPTT) algorithm. While competitive accuracies are reported on the MNIST and CIFAR10 datasets [17], the computational and memory demands of these algorithms are high for BPTT because the entire sequence must be stored to compute the gradients exactly.

The third category is the spike-driven learning algorithms, which uses the timing of spikes as the relevant signals for controlling synaptic changes. The typical examples include SpikeProp [36] and its derivatives [37–40]. These methods apply a linear assumption that the neuron’s membrane potential increases linearly in the infinitesimal time around the spike time, then the derivative of spike function can be calculated and the back-propagation can be implemented in multi-layer SNNs. Recently, Mostafa [31] applied non-leaky integrate-and-fire neurons to avoid the problem of the non-differentiable spike function, and the work showed competitive performance on MNIST dataset. The performance of SNN with spike-driven learning algorithm is further improved by [41] and [42]. However, the existing spike-driven learning algorithms suffer from certain limitations, such as the problems of dead neuron and gradient exploding, which require several complicated skills to relieve these problems [31]. For example, in [31], some constraints are imposed on synaptic weights to overcome dead neuron problem, and gradient normalization strategy is used to overcome the problem of gradient exploding. These complex training strategies limit the scalability of the learning algorithms.

Among the existing learning algorithms, the spike-driven learning algorithms perform the SNNs training in a strictly event-driven manner, and are compatible with the temporal coding in which the information is carried by the timing of individual spikes in a very sparse manner; Hence, spike-driven learning algorithms hold the potential of enabling ultra-low-power event-driven neuromorphic hardware. With these considerations, we focus on developing more effective spike-driven learning algorithms for deep SNNs, and make the following contributions in this paper:

1) We thoroughly analyze the issues that make the well-studied BP algorithm incompatible for training SNNs, including the problems of non-differentiable spike generation function, gradient exploding and dead neuron. Building on such an understanding, we put forward a Rectified Linear Postsynaptic Potential function (ReL-PSP) for spiking neurons to resolve these problems.

2) Based on the proposed ReL-PSP, we derive a new spike-timing-dependent BP algorithm (STDBP) for DSNN. In this algorithm, the timing of spikes is used as the information-carrying quantities, and learning happens only at the spike times in a totally event-driven manner.

3) Due to the good scalability of the proposed algorithm, we extend it to convolutional spiking neural network (CSNN), and achieve an accuracy of 99.2% on MNIST dataset. To our best knowledge, this is the first implementation of a CSNN structure based on the spike-timing-based supervised learning algorithm.

Experimental results demonstrate that the proposed learning algorithm achieves the state-of-the-art performance in spike time based learning algorithms of SNNs. This work provides a new perspective to investigate the significance of spike timing dynamics in information coding, synaptic plasticity, and decision making in SNNs-based computing paradigm.
2 Problem Description

Error back-propagation (specifically stochastic gradient descent) is the workhorse for the remarkable success of DNNs. However, as shown in Fig. 1, the dynamics of a typical artificial neuron in DNNs and that in SNNs is rather different, and the well-studied BP algorithm cannot be directly applied to deep SNNs due to issues of non-differentiable spike function, exploding gradients and dead neurons. In the following, we will discuss these issues in depth.

Consider a fully connected DSNN. For simplicity, each neuron is assumed to emit at most one spike. In general, the membrane potential $V^l_j(t)$ of neuron $j$ in layer $l$ can be expressed as

$$V^l_j(t) = \sum_{i} \omega^l_{ij} \varepsilon(t - t_{i}^{l-1}) - \eta(t - t_{j}^{l})$$  \hspace{1cm} (1)

where $t_{i}^{l-1}$ is the spike of the $i$th neuron in layer $l-1$, and $\omega^l_{ij}$ is the synaptic weight of the connection from neuron $i$ (in $l-1$ layer) to neuron $j$ (in $l$ layer). Each incoming spike from neuron $i$ will induce a postsynaptic potential (PSP) at neuron $j$, and the kernel $\varepsilon(t - t_{i}^{l-1})$ is used to describe the PSP generated by the spike $t_{i}^{l-1}$. Hence each input spike makes a contribution to the membrane potential of the neuron as described by $\omega^l_{ij} \varepsilon(t - t_{i}^{l-1})$ in Eq. (1). There are several PSP functions, and a commonly used one is the alpha function which is defined as

$$\varepsilon(t) = \frac{t}{\tau} \exp\left(1 - \frac{t}{\tau}\right), \quad t > 0$$  \hspace{1cm} (2)

Fig. 2(a) shows the waveform of the alpha PSP function. As shown in Fig. 2(b), integrating the weighted PSPs gives the dynamics of the membrane potential $V^l_j(t)$. The neuron $j$ will emit a spike when its membrane potential $V^l_j(t)$ reaches the firing threshold $\vartheta$, as mathematically defined in the spike generation function $F$:

$$t_j^l = F\{t|V^l_j(t) = \vartheta, t \geq 0\}$$  \hspace{1cm} (3)

Once a spike is emitted, the refractory kernel $\eta(t - t_{j}^{l})$ is used to reset the membrane potential to resting.

To train SNNs using BP, we need to compute the derivative of the postsynaptic spike time $t_j^l$ with respect to a presynaptic spike time $t_i^{l-1}$ and synaptic weight $\omega_{ij}$ of the corresponding connection:

$$\frac{\partial t_j^l}{\partial \omega_{ij}} = \frac{\partial t_j^l}{\partial V^l_j(t_j)} \frac{\partial V^l_j(t_j)}{\partial \omega_{ij}} \quad \text{if} \quad t_j^l > t_i^{l-1}$$  \hspace{1cm} (4)

$$\frac{\partial t_j^l}{\partial t_i^{l-1}} = \frac{\partial t_j^l}{\partial V^l_j(t_j)} \frac{\partial V^l_j(t_j)}{\partial t_i^{l-1}} \quad \text{if} \quad t_j^l > t_i^{l-1}$$  \hspace{1cm} (5)
As presented in Section 2, BP cannot be directly applied in DSNNs due to problems of non-differentiable spike function, whereby $K$ with $\frac{\partial V}{\partial t}$ the problem of gradient exploding happens. (c) Spiking neurons suffer from the problem of dead neuron.

Figure 2: (a) Alpha shape PSP function. (b) The membrane potential barely reaches the firing threshold, and exploding gradient occurs. (c) The alpha-PSP neuron with weak synaptic weights is susceptible to be a dead neuron.

Due to the discrete nature of the spike generation function (Eq. 3), the difficulty of Eq. 4 lies in solving the partial derivative $\frac{\partial t_j}{\partial V_j(t)}$, which we referred to as the problem of non-differentiable spike function. Existing spike-driven learning algorithms [36,43] assume that the membrane potential $V_j(t)$ increases linearly in the infinitesimal time interval before spike time $t_j$. Then, $\partial t_j/V_j(t)$ can be expressed as

$$\frac{\partial t_j}{\partial V_j(t)} = \frac{-1}{\partial V_j(t)/\partial t_j} = \frac{-1}{\sum_i N \omega_{ij} \frac{\partial \varepsilon(t_j - t_i^{-1})}{\partial t_j}}$$

with

$$\frac{\partial \varepsilon(t_j - t_i^{-1})}{\partial t_j} = \frac{\exp(1 - (t_j - t_i^{-1})/\tau)}{\tau^2} (\tau + t_i^{-1} - t_j)$$

The exploding gradient problem occurs when $\partial V_j(t)/\partial t_j \approx 0$ i.e. the membrane potential just reaches the firing threshold, emitting a spike (Fig. 2). Since $\partial V_j(t)/\partial t_j$ is the denominator in Eq. 6, this causes Eq. 6 to explode with large weight updates. Although various strategies have been proposed to alleviate this problem, such as adaptive learning rate [44] and dynamic firing threshold [40], it has not been fully resolved.

From Eq. 5, when the presynaptic neuron does not emit a spike, the error cannot be back propagated through $\partial V_j(t)/\partial t_j$. This is the dead neuron problem. This problem is also common in DNNs with ReLU activation function. However, due to the leaky nature of the PSP kernel and spike generate mechanism, the problem of dead neuron is more severe in SNNs. As shown in Fig 2c, there are three input spikes, and the neuron emits a spike with large synaptic weights (blue). With slightly reduced synaptic weights, the membrane potential stays sub-threshold and the neuron becomes a dead neuron (green). When the neuron does not spike, no errors can back-propagate through it. The problem of dead neuron is fatal in spike-driven learning algorithms.

### 3 Method

In this section, we describe how the above challenges maybe overcome and a DSNN may still be trained using BP. To this end, we introduce the Rectified Linear Postsynaptic Potential function (ReL-PSP) for the spiking neuron model. In Section 3.2 the proposed spike-timing-dependent back propagation (STDBP) learning algorithm (based on ReL-PSP) is presented.

#### 3.1 ReL-PSP Based Spiking Neuron Model

As presented in Section 2, BP cannot be directly applied in DSNNs due to problems of non-differentiable spike function, exploding gradient and dead neuron. To overcome the above-mentioned problems, we propose a simple yet efficient Rectified Linear Postsynaptic Potential (ReL-PSP) based spiking neuron model, and the dynamics of the proposed neuron model is defined as

$$V_j(t) = \sum_i N \omega_{ij} K(t - t_i^{-1})$$

whereby $K(t - t_i^{-1})$ is the kernel of the PSP function, which is defined as

$$K(t - t_i^{-1}) = \begin{cases} t - t_i^{-1} & \text{if } t > t_i^{-1} \\ 0 & \text{otherwise} \end{cases}$$

Figure 2: (a) Alpha shape PSP function. (b) The membrane potential barely reaches the firing threshold, and exploding gradient occurs. (c) The alpha-PSP neuron with weak synaptic weights is susceptible to be a dead neuron.
As shown in Fig. 3(a), given an input spike at \( t_{i}^{l-1} \), the membrane potential after \( t_{i}^{l-1} \) is a linear function of time \( t \). Since the shape of the proposed PSP function resembles that of a rectified linear function, we name it the ReL-PSP function. In the following, we will analyze how the proposed neuron model solves the above-mentioned problems.

![Figure 3](image)

**Figure 3:** There are three input spikes denoted as \( t_1, t_2, t_3 \). The blue and green lines show the membrane potential with large and small synaptic weights, respectively. (a) ReL-PSP function. (b) Trace of the neuron membrane potential during threshold crossing. (c) The ReL-PSP neuron generates spikes at \( t_1^l \) and \( t_2^l \) with large and small synaptic weights, respectively (\( t_2^l < t_3^l \)).

### 3.1.1 Non-differentiable spike function

As shown in Fig. 3, due to the linearity of the ReL-PSP, the membrane potential \( V_{j}^{l}(t) \) increases linearly prior to spike time \( t_{j}^{l} \). In this case, there is no need to assume linearity, and we can directly use Eq. 10 to compute \( \frac{\partial t_{j}^{l}}{\partial V_{j}^{l}(t_{j}^{l})} \). This resolves the problem of non-differentiable spike generation.

\[
\frac{\partial t_{j}^{l}}{\partial V_{j}^{l}(t_{j}^{l})} = \frac{1}{\partial V_{j}^{l}(t_{j}^{l})/\partial t_{j}^{l}} = \frac{-1}{\frac{\sum_{i}^{N} \omega_{ij}^{l} \frac{\partial K(t_{i}^{l-1} - t_{j}^{l-1})}{\partial t_{j}^{l}}}{\sum_{i}^{N} \omega_{ij}^{l}}} = \frac{-1}{\sum_{i}^{N} \omega_{ij}^{l}} \]  \( \text{if } t_{j}^{l} > t_{i}^{l-1} \)  \( (10) \)

The precise gradients in BP provide the necessary information for optimization, and is key to the high accuracy of DNNs. Without having to assume linearity, we use the precise value of \( \frac{\partial t_{j}^{l}}{\partial V_{j}^{l}(t_{j}^{l})} \) instead of approximating it, and avoid accumulating errors across multiple layers.

### 3.1.2 Gradient explosion

Exploding gradient occurs when the denominator in Eq. 6 approaches 0. In this case, the membrane potential just reaches the firing threshold at spike time, and is caused by the combined effect of \( \omega_{ij}^{l} \) and partial derivative of the PSP function. Compared to the Alpha-PSP function, which has zero gradient at its peak, there is less chance for such a scenario to occur in the ReL-PSP function. As the partial derivative of ReL-PSP, \( \frac{\partial K(t_{j}^{l} - t_{i}^{l-1})}{\partial t_{j}^{l}} \), is always equal to 1, Eq. 10 can be expressed as \( -1/\sum_{i}^{N} \omega_{ij}^{l} \).

As \( \sum_{i}^{N} \omega_{ij}^{l} \) may still be close to 0, the exploding gradient problem may not be completely solved. However, as from Eqs. 3 and 8, we obtain the spike time \( t_{j}^{l} \) as a function of input spikes and synaptic weights \( \sum_{i}^{N} \omega_{ij}^{l} \):

\[
\vartheta = \sum_{i}^{N} \omega_{ij}^{l} K(t - t_{i}^{l-1}) \]  \( (11) \)

Re-arranging, the spike time \( t_{j}^{l} \) can be calculated as

\[
t_{j}^{l} = \frac{\vartheta + \sum_{i}^{N} \omega_{ij}^{l} t_{i}^{l-1}}{\sum_{i}^{N} \omega_{ij}^{l}} \]  \( (12) \)

Should the \( \sum_{i}^{N} \omega_{ij}^{l} \) be close to 0, the spike \( t_{j}^{l} \) will be emitted late, and may not contribute to the spike \( t_{j}^{l+1} \) in the next layer. Therefore, the neuron \( j \) in the \( l \) layer does not participate in error BP, and does not result in exploding gradient.
3.1.3 Dead neuron

In neural networks, sparse representation (few activated neurons) has many advantages, such as information disentangling, efficient variable-size representation, linear separability etc. However, sparsity may also hurt predictive performance, as given the same number of neurons, it reduces effective capacity of the model [35]. Unfortunately, as shown in Fig. 3(c), due to the leaky nature of the alpha-shape PSP and the spike generation mechanism, such a spiking neuron is more likely to suffer from the dead neuron problem. However, as shown in Fig. 3(c), with the ReL-PSP kernel, the PSP increases over time. Hence the neuron with a more positive sum of weights fires earlier than one with a less positive sum, with lower probability of becoming a dead neuron. Overall, the proposed ReL-PSP greatly alleviates the dead neuron problem as the PSP does not decay over time, while maintaining a sparse representation to the same extent of the ReLU activation function.

3.2 Error Backpropagation

Given a classification task with \( n \) categories, each neuron in the output layer is assigned to a category. When a training sample is presented to the neural network, the corresponding output neuron should fire the earliest. There are several loss functions that can be constructed to achieve this goal [31][41][42]. In this work, the cross-entropy loss function is used. To minimise the spike time of the target neuron and maximises the spike time of non-target neurons, we use the softmax function on the negative values of the spike times in the output layer: \( p_j = \exp(-t_j)/\sum_i^n \exp(-t_i) \). Then, the loss function is given by

\[
L(g, t^o) = -\ln \frac{\exp(-t^o[g])}{\sum_i^n \exp(-t^o[i])}
\]

where \( t^o \) is the vector of the spike times in the output layer and \( g \) is the target class index [31].

The loss function is minimised by updating the synaptic weights across the network. This has the effect of delaying or advancing spike times across the network. The derivatives of the first spike time \( t_j^l \) with respect to synaptic weights \( \omega_{ij}^l \) and input spike times \( t_i^{l-1} \) are given by

\[
\frac{\partial t_j^l}{\partial \omega_{ij}^l} = \frac{\partial t_j^l}{\partial V_j^l(t_j^l)} \cdot \frac{\partial V_j^l(t_j^l)}{\partial \omega_{ij}^l} = \frac{t_i^{l-1} - t_j^l}{\sum_i^n \omega_{ij}^l} \quad \text{if} \quad t_j^l > t_i^{l-1}
\]

\[
\frac{\partial t_j^l}{\partial t_i^{l-1}} = \frac{\partial t_j^l}{\partial V_j^l(t_j^l)} \cdot \frac{\partial V_j^l(t_j^l)}{\partial t_i^{l-1}} = \frac{\omega_{ij}^l}{\sum_i^n \omega_{ij}^l} \quad \text{if} \quad t_j^l > t_i^{l-1}
\]

Based on Eq 14 and Eq 15, standard BP can be applied to train a DSNN.

4 Experiments

In this section, we investigate two SNNs: the fully connected SNN and convolutional SNN on image classification task based on the MNIST dataset [36] so as to benchmark their learning capabilities with existing spike-driven learning algorithms.

4.1 Temporal Coding

The MNIST dataset comprises of 60,000 \( 28 \times 28 \) grayscale images for training and 10,000 \( 28 \times 28 \) grayscale images for testing. We first convert the images into spike trains. There are many encoding strategies. Rate-coding assumes that a higher sensory variable corresponds to a higher firing rate [37] and requires a large number of encoding spikes to be transmitted. However, this is highly inefficient with minimal information content in each spike. In this work, a more efficient temporal coding scheme is used, that encodes information in individual spike time, with the assumption that strongly activated neurons tend to fire earlier [48].

As shown in Fig. 3, the input information is encoded in spike timing of neurons, with each neuron firing only once. More salient information is encoded as an earlier spike in the corresponding input neuron. The encoding spikes then propagate to the subsequent layers in a temporal fashion. Each neuron in the hidden and output layer receives the spikes from its presynaptic neurons, and it emits a spike when the membrane potential reaches the threshold. Similar to the input layer, the neurons in the hidden and output layer that are strongly activated will fire first. Therefore, temporal coding is maintained throughout the DSNN, and the output neuron that fires earliest categorizes the input stimulus.
4.2 Experimental Results

Table 1 shows the classification accuracies of the two SNNs, and other spike-driven learning algorithms on the MNIST dataset. The proposed STDBP learning algorithm could reach accuracies of 98.1% and 98.4% with network structures of 784-400-10 and 784-800-10, respectively. They outperform previously reported results with same network structure. For example, with the structure of 784-400-10, the classification accuracy of our method is 98.4%, while the accuracy achieved by Mostafa [31] is 97.5%. Another advantage of our algorithm is that it does not need additional training strategies which are widely used in previous works to improve their performance. This facilitates large-scale implementation of STDBP. Moreover, to our best knowledge, this is the first implementation of a SCNN based on spike-driven learning algorithms. The model achieves an accuracy of 99.2%, higher than the fully connected SNN.

Table 1: The classification accuracies of existing spike-driven learning algorithms on the MNIST dataset. We use the following notation to indicate the SNN architecture. Layers are separated by - and spatial dimensions are separated by ×. The convolution layer and pooling layer are represented by C and P, respectively.

| Model            | Coding  | Network Architecture | Additional Strategy                      | Acc. % |
|------------------|---------|----------------------|------------------------------------------|--------|
| Mostafa [31]     | Temporal| 784-800-10           | Weight and Gradient Constraint           | 97.5   |
| Tavanaei et al [49] | Rate   | 784-1000-10          | None                                     | 96.6   |
| Comsa et al [42] | Temporal| 784-340-10           | Weight and Gradient Constraint           | 97.4   |
| Kheradpisheh et al [41] | Temporal| 784-400-10          | Weight constraint                        | 97.4   |
| STDBP (This work) | Temporal| 784-400-10          | None                                     | 98.1   |
| STDBP (This work) | Temporal| 784-800-10          | None                                     | 98.4   |
| STDBP (This work) | Temporal| 28×28-6C5-P2-16C5-128-10 | None                                     | 99.2   |

Fig. 5 shows the distribution of spike timing in the hidden layers and of the earliest spike time in the output layer across 10000 test images for two SNNs, namely 784-400-10 and 784-800-10. For both architectures, the SNN makes a decision after only a fraction of the hidden layer neurons. For the 784-400-10 topology, an output neuron spikes (a class is selected) after only 48.6% of the hidden neurons have spiked. The network is thus able to make very rapid decisions about the input class. In addition, during the simulation time, only 66.3% of the hidden neurons have spiked. Therefore, the experimental results demonstrate that the proposed learning algorithm works in a accurate, fast and sparse manner.

5 Discussion and Conclusion

In this work, we analysed the problems that BP faces in a DSNN, namely, non-differentiable spike function, exploding gradient, and dead neuron problem. To address these problems, we propose the Rectified Linear Postsynaptic Potential function (ReL-PSP) for spiking neurons and the STDBP learning algorithm for DSNNs. We evaluate the proposed method on both multi-layer fully connected SNN and CSNN. Our experiments on MNIST reach an accuracy of 98.4%.
Figure 5: Histograms of spike times in the hidden layers and the output layer across 10000 test images for the two SNNs (784-400-10 and 784-800-10).

with the fully connected SNN and 99.2% with the CSNN, which is the state-of-art in spike-driven learning algorithms for SNNs.

Many studies have been proposed to train DSNNs, such as conversion methods [21][30], and surrogate gradients methods [16][17][22][35]. These methods are not compatible with temporal coding and spike-based learning mechanism, and the advantages of SNNs have not been fully exploited, especially spike timing. Due to the non-differentiability of spike function, BP in SNNs using spike timing remains an open question. To perform BP using spike timing, many methods have been proposed [41][56][43]. Two common drawback of these methods are exploding gradients and dead neurons, which have been partially addressed using techniques such as constraints on weights and gradient normalization. These techniques affect learning efficiency and limit application of these learning algorithms in large-scale networks. The proposed STDBP learning algorithm with ReL-PSP spiking neuron model can train DSNNs directly without any additional technique, hence allowing the DSNN to scale, as shown in the high accuracy of the SCNN.

In addition to being fast, sparse and more accurate, the proposed ReL-PSP neuron model and STDBP have some other features that might make it more energy-efficient and (neuromorphic) hardware friendly. Firstly, compared to the alpha-shape PSP function, the linear ReL-PSP function is simpler for hardware implementation. Secondly, unlike rate-based encoding methods that require more time to generate enough output spikes for classification, our method takes advantage of temporal coding and uses a single spike, which is more sparse and energy-efficient, given energy is mainly consumed during spike generation and transmission. Thirdly, without additional training techniques, on-chip training in neuromorphic chips would be much easier to realize.

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