Josephson effect through magnetic skyrmion

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We discover that the multiple degrees of freedom associated with magnetic skyrmions: size, position, and chirality, can all be used to control the Josephson effect and \(0\)–\(\pi\) transitions occurring in superconductor/magnetic skyrmion/superconductor junctions. In the presence of two skyrmions, the Josephson effect depends strongly on their relative chirality and leads to the possibility of a chirality-transistor effect for the supercurrent where the magnitude of the critical current is changed by several orders of magnitude simply by reversing the chirality of a magnetic skyrmion. These findings demonstrate the rich physics that emerges when combining topological magnetic objects with superconductors and could lead to new perspectives in superconducting spintronics.

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The interplay between superconductivity and ferromagnetism in hybrid structures has received much attention in recent years \([1,2]\), due to its allure from a fundamental physics viewpoint and also because of improved and new functionality brought about by using superconductors in spintronics \([3]\). Due to the proximity effect, the Cooper pairs induced in the ferromagnet acquire a finite center of mass momentum. Therefore, the pair amplitude oscillates in space which may result in a sign change of the Josephson current in ferromagnetic Josephson junctions: the \(0\)–\(\pi\) transition \([4–7]\). The \(0\)–\(\pi\) transition was originally observed in Josephson junctions through weak ferromagnets. \([8,9]\) Also, in the presence of inhomogeneity of the magnetic order, triplet pairing with spin aligned with the local exchange field is generated in the ferromagnet due to spin flip scattering \([10,11]\). Experiments have successfully demonstrated the presence of such spin-triplet pairing by observing a Josephson current through strong ferromagnets \([12–14]\), which can be explained via the concepts of spin-mixing and spin-rotation taking place near the superconductor/ferromagnet interface \([15]\). Using equal spin triplet pairings, the possibility arises to enhance existing effects or discover new ones in spintronics \([3,14,20]\). Recently, it has been also proposed that inhomogeneous ferromagnet/superconductor junctions can create topological superconductivity. \([21,23]\)

Currently, much interest is garnered by magnetic skyrmions in chiral magnets \([24–26]\). Such objects are characterized by a topologically protected spin configuration. Due to their peculiar magnetic structure, several intriguing phenomena have been discovered such as topological and skyrmion Hall effects \([27,29]\) and current-driven motion of skyrmion with ultralow current density \([30–34]\). It has been shown that magnetic skyrmions can be also driven by a temperature gradient \([35–38]\). A thermal gradient is predicted to induce a skyrmion motion towards the high temperature region accompanied by a skyrmion Hall effect \([35]\). Skyrmions are accompanied by a degree of freedom known as their chirality, which is determined by their spin swirling direction. It has been experimentally demonstrated that the chirality of skyrmions can be changed both via a small external magnetic field \([39]\) and spin-orbit interactions \([40]\). This opens the exciting prospect that any physical quantity that responds to a change in the skyrmion chirality degree of freedom will be controllable via an external field.

In this Letter, we investigate how the new degrees of freedom of magnetic skyrmions, such as chirality, can influence the supercurrent-response and quantum ground state of Josephson junctions including skyrmions. We find that the supercurrent is strongly influenced by the (i) size, (ii) position, and (iii) chirality of magnetic skyrmions. We discover that the \(0\)–\(\pi\) transition can in fact be triggered by changing any of these three skyrmion properties, which in turn have been confirmed to be experimentally tunable via external magnetic fields \([39]\), spin-orbit interactions \([40]\), and electric currents \([41,42]\). This offers a new and dynamical way of manipulating the quantum ground state of a superconducting system via magnetic skyrmions. We also show that the strong dependence on the chirality creates a chirality-transistor effect for supercurrents, where the critical current is changed by several orders of magnitude upon reversing the chirality of a skyrmion. In what follows, we will demonstrate these properties in junctions featuring both single and two skyrmions.

We consider a 2D superconductor / magnetic skyrmion / superconductor junction as shown in Fig. 1. By assuming that the proximity effect is weak, we utilize the linearized Usadel equation:

\[
D \nabla^2 f_s - 2i \omega_n f_s - 2i f_t \cdot h = 0, \quad (1)
\]

\[
D \nabla^2 f_t - 2i \omega_n f_t - 2i f_s h = 0. \quad (2)
\]

Here, \(D\) and \(\omega_n\) are the diffusion constant in the magnet and Matsubara frequency, respectively. \(f_s\) is the singlet anomalous Green’s function while \(f_t\) represents the triplet anomalous Green’s functions. \(h\) is the exchange field representing a magnetic structure with two skyrmions:

\[
h = \frac{\hbar}{1 + |u|^2} \left( \frac{2 \text{Re} u}{2 \text{Im} u} \right) \left( \frac{2 \text{Re} u}{1 - |u|^2} \right) \quad (3)
\]
with
\[ u = \frac{i\lambda}{x - x_c - i(y - y_c)} + \frac{i\lambda'}{x - x'_c - i(y - y'_c)}. \] (4)

Here, \((x_c, y_c)\) and \((x'_c, y'_c)\) determine the centers of the two skyrmions. \(\lambda\) and \(\lambda'\) are the characteristic sizes of the skyrmions. The signs of \(\lambda\) and \(\lambda'\) determine the chiralities of the skyrmions. \(h\) is the magnitude of the exchange field. By setting \(\lambda' = 0\), the above exchange field represents a single skyrmion texture. We consider the magnetic region in \(-L/2 \leq x, y \leq L/2\). The interfaces are located at \(x = \pm L/2\).

The boundary condition at \(x = -L/2\) reads \([45]\)
\[ -\gamma_B\xi \frac{\partial f_s}{\partial x} + G_s f_s = F_s, \quad -\gamma_B\xi \frac{\partial f_i}{\partial x} + G_s f_i = 0 \] (5)
where \(i = 1, 2, 3\) and \(G_s\) and \(F_s\) are bulk Green’s functions in the superconductor given by
\[ G_s = \frac{\omega_n}{\sqrt{\omega_n^2 + \Delta^2}}, \quad F_s = \frac{\Delta \exp(-i\varphi/2)}{\sqrt{\omega_n^2 + \Delta^2}}. \] (6)

Here, \(\gamma_B\) describes the interface barrier strength, \(\xi\) is the superconducting coherence length, \(\Delta\) is the gap function, and \(\varphi\) is the phase difference between the superconductors. The boundary condition at \(x = L/2\) is given by changing the signs of the derivative and \(\varphi\) in the above boundary condition. The boundary condition at \(y = \pm L/2\) reads
\[ \frac{\partial f_\alpha}{\partial y} = 0 \] (7)
with \(\alpha = s, 1, 2, 3\). The Josephson current is calculated as
\[ \frac{eI_sR}{2\pi T_C} = -\frac{T}{T_C} \sum_{n \geq 0} \text{Im}(f^*_s \partial_x f_s - f^*_i \partial_x f_i) \] (8)
with the (transition) temperature \(T(T_C)\) and resistance of the magnet per length \(R\). We define the total current as \(I_X = \frac{1}{2} \int_{-L/2}^{L/2} I_x dy\) at \(x = -L/2\). The critical current and that including the sign of the current are denoted by \(I_{XC}\) and \(I'_{XC}\), respectively: \(I_{XC} = |I'_{XC}|\). Below, we fix the parameters as \(\gamma_B = 10, T/T_C = 0.9\) and \(h/\Delta_0 = 1.5\) where \(\Delta_0\) denotes the gap energy at zero temperature. Calculation of the Josephson current requires a solution of the 2D Usadel equation. We have solved the Usadel equation numerically by using an iterative method.

We begin by considering junctions with a single skyrmion \((\lambda' = 0)\) as shown in Fig. 1(a). In Fig. 2(a), we show the critical current as a function of the length of the magnetic region \(L\) for several sizes of the skyrmion \(\lambda\). The skyrmion is assumed to be positioned in the center of the junction, \(x_c = y_c = 0\). We find a 0-\(\pi\) transition as a function of \(L\). It is also seen that the transition point can be controlled by altering the size of the skyrmion, \(\lambda\). In Fig. 2(b), we show the critical current as a function of \(\lambda\) for several \(L\) and \(x_c = y_c = 0\). For

\(L/\xi = 4\), the 0-\(\pi\) transition occurs around \(\lambda/\xi = 0.42\). These results indicate that the 0-\(\pi\) transition is tunable by changing magnetic field and also by applying electric field since electric field breaks inversion symmetry and hence can modify the Dzyaloshinskii-Moriya interaction. The tunable size of skyrmions in helimagnetic alloys via spin-orbit coupling has been also experimentally verified in Ref. \([40]\), indicating that the 0-\(\pi\) transition predicted here can be manipulated via changing the skyrmion size according this route.

The next aspect we consider is how the skyrmion position influences the supercurrent response of the system. A unique feature of skyrmions is that the ultralow current density \((\sim 10^2 \text{ A/cm}^2)\) can induce their translational and/or rotational motions, which is typically 5 orders of magnitude smaller than the required density in conventional domain wall ferromagnets. This has been experimentally demonstrated in the helimagnet MnSi \([41]\) and FeGe \([42]\). Motivated by this, in Fig. 3(a), we show the critical current as a function of the position of the skyrmion \(x_c\) and \(y_c\) for \(\lambda/\xi = 0.5\) and \(L/\xi = 4.2\). It is found that the critical current becomes minimum when the skyrmion is located on a circle. The critical current becomes large when the skyrmion is near the corner of the magnetic region. Figure 3(b) shows the critical current including the sign of the current for \(\lambda/\xi = 0.5\) and \(L/\xi = 4.2\). We see that a 0-\(\pi\) transition occurs by changing the position of the skyrmion. Since the position of the skyrmion can be manipulated by current or temperature gradient, this offers a way to control the 0-\(\pi\) transition in this junction.

Now, let us consider junctions with two skyrmions (see Fig. 4(b)) and focus on the effect of the chiralities. The presence of multiple skyrmions in the Josephson junction is particularly relevant in light of the experimental demonstration of multiple skyrmion configuration featuring skyrmions with both types of chiralities \([39, 40]\). The chirality was shown to be
FIG. 2: (Color online) (a) The critical current as a function of the length of the magnetic region $L$ for several sizes of the skyrmion $\lambda$. (b) The critical current as a function of $\lambda$ for several $L$. We set $x_c = y_c = 0$ and $\lambda' = 0$.

FIG. 3: (Color online) (a) The critical current $eI_{XC}/2\pi T_C$ as a function of the position of the skyrmion $x_c$ and $y_c$. (b) The critical current including the sign of the current $eI_{XC}'/2\pi T_C$ as a function of $x_c$ and $y_c$. $\lambda/\xi = 0.5$, $\lambda' = 0$, and $L/\xi = 4.2$. In (b), the magnitude of the critical current is restricted in a region for visibility.

For chiral magnet MnSi, the material parameters are esti-
mated as $h \sim 3\text{meV}$ and $\lambda \sim 10\text{nm}$ \cite{29}, and hence the size of the magnetic interlayer of $L \sim 100\text{nm}$ is required to realize the $0-\pi$ transition predicted in this paper. Due to the complexity of the non-linear partial coupled differential equations, we have here used the linearized Usadel equation which is known to capture well the basic physics of superconductor/ferromagnet hybrid structures. It would be of interest to relax the requirement of a weak proximity effect in order to access, e.g., shorter junction lengths, which might reveal $0-\pi$ transitions at even more accessible experimental conditions. Also, we have here considered junctions with a single and two skyrmions. Skyrmions can also form a hexagonal lattice, and the application of our work to such a skyrmion configuration would be also informative. We leave this for future explorations.

In summary, we have investigated the Josephson effect in superconductor/magnetic skyrmion/superconductor junction. It is found that the degrees of freedom associated with the skyrmions (size, position, and chirality), which recently have been demonstrated experimentally to be tunable via different routes, lead to a new dynamical way to control the $0-\pi$ transition, offering the tantalizing prospect of a chirality-transistor for supercurrents.

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