Polarized proton acceleration in ultra-intense laser interaction with near critical density plasmas

X. F. Li\textsuperscript{1,2,3}, P. Gibbon\textsuperscript{1,4} \textsuperscript{a} A. Hützen\textsuperscript{5,6}, M. Büscher\textsuperscript{5,6}, S. M. Weng\textsuperscript{2,3} \textsuperscript{b} M. Chen\textsuperscript{2,3}, and Z. M. Sheng\textsuperscript{2,3,7,8}

\textsuperscript{1} Institute for Advanced Simulation, Jülich Supercomputing Centre, Forschungszentrum Jülich, 52425 Jülich, Germany
\textsuperscript{2} Key Laboratory for Laser Plasmas (MoE), School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{3} Collaborative Innovation Center of IFSA, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{4} Centre for Mathematical Plasma Astrophysics, Katholieke Universiteit Leuven, 3000 Leuven, Belgium
\textsuperscript{5} Peter Grünberg Institut (PGI-6), Forschungszentrum Jülich, 52425 Jülich, Germany
\textsuperscript{6} Institut für Laser- und Plasmaphysik, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany
\textsuperscript{7} SUPA, Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK
\textsuperscript{8} Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China

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The production of polarized proton beams with multi-GeV energies in ultra-intense laser interaction with targets is studied with three-dimensional Particle-In-Cell simulations. A near-critical density plasma target with pre-polarized proton and tritium ions is considered for the proton acceleration. The pre-polarized protons are initially accelerated by laser radiation pressure before injection and further acceleration in a bubble-like wakefield. The temporal dynamics of proton polarization is tracked via the T-BMT equation, and it is found that the proton polarization state can be altered both by the laser field and the magnetic component of the wakefield. The dependence of the proton acceleration and polarization on the ratio of the ion species is determined, and it is found that the protons can be efficiently accelerated as long as their relative fraction is less than 20\%, in which case the bubble size is large enough for the protons to obtain sufficient energy to overcome the bubble injection threshold.

I. INTRODUCTION

With the development of laser technology, especially chirped pulse amplification (CPA)\textsuperscript{1}, remarkable progress has been achieved in the field of laser-plasma acceleration\textsuperscript{2,3}. Since laser-driven wakefield acceleration (LWFA) was first proposed by Tajima and Dawson in 1979\textsuperscript{4}, electron beams with quasi-monoenergetic peaks up to 7.8 GeV have been generated using a peak laser power of 850 TW interacting with a 20 cm capillary charge waveguide in 2019\textsuperscript{5}. The maximum energy for laser-driven ion acceleration is around 85 MeV, via a high energy laser incident on micrometer thick plastic targets\textsuperscript{6}. Near-100 MeV protons were obtained through a hybrid scheme of radiation pressure and sheath acceleration\textsuperscript{7}. These plasma-based accelerators have prompted a new class of diagnostic techniques different from those common to traditional accelerators\textsuperscript{8}. In order to effectively utilize and develop laser plasma acceleration, many characteristics of particle beams, \textit{e.g.} energy spread, charge, pulse duration and emittance, have been steadily improved. However, the polarization of particles has only rarely been studied in plasma-based acceleration\textsuperscript{9}.

Polarization is defined as the collective spin state of a particle beam. It is commonly employed in nuclear physics\textsuperscript{10}, high energy physics\textsuperscript{11} and material physics\textsuperscript{12}. For laser-driven accelerators, there are two approaches leading to polarized particle beams: a \textit{de novo} polarization build-up from an unpolarized target in the interaction with laser, or polarization preservation of pre-aligned spins during the acceleration. In the first instance, the spin polarization of an ultra-relativistic electron beam has been investigated through colliding with an intense laser pulse in the quantum radiation-dominated regime\textsuperscript{13}. A possible way for producing highly polarized positron beams was also proposed via interaction of an ultra-relativistic electron beam with counter-propagating two-color intense laser pulses\textsuperscript{14}. For a pre-polarized target, it has been recently demonstrated that nuclear and electron spin-polarized H and D densities of at least 10\(^{19}\)\textpercm\(^{-3}\) with 10\textmu s lifetimes can be produced by photo-dissociation of HBr and DI with circularly polarized UV light pulses\textsuperscript{15}. Using this kind of pre-polarized target, M. Wen \textit{et al} demonstrated that kiloampere polarized electron beams can be produced through LWFA in the bubble regime\textsuperscript{16}. Furthermore, a vortex laser interacting with pre-polarized plasma was proposed to produce energetic electrons with high polarization and beam charge\textsuperscript{17}.

Because the proton is 1836 times heavier than the electron or positron, it is much more difficult to align the spin of protons via laser-solid interaction, as demonstrated at the ARCTurus laser facility in Düsseldorf\textsuperscript{18–20}. For this reason, acceleration of polarized proton beams has been investigated using a pre-polarized target composed of two different ion species close to the critical density for a Ti:Sapphire laser. By irradiating pre-polarized monatomic gases from photo-dissociated hy-
dron halide molecules with a petawatt laser, proton beams with nearly 100 MeV energy and 80% polarization via magnetic vortex acceleration (MVA) mechanism were predicted. In order to obtain several GeV protons from near-critical plasma targets, wakefield acceleration offers a good option. The work of B. F. Shen et al showed that the protons can be trapped and accelerated efficiently in the bubble regime, where the plasma consists a small proportion of protons and a larger proportion of heavier tritium ions. It should be noted that the density ratio of protons and heavier ions is different in the above two studies. Besides, the protons need to be pre-accelerated to a sufficient energy in order to be injected into the wakefield. In the research of M. Liu et al, with pre-accelerated by the radiation pressure acceleration (RPA) in a thin solid foil, the protons were captured by the LWFA in an underdense gas.

In this paper, pre-acceleration of protons via direct laser acceleration (DLA) with an ultra-intense, circularly polarized laser is proposed. This initial phase is followed by proton acceleration in a wakefield, where the polarization dynamics are studied in detail. Finally, the influence of the proton:ion ratio on the acceleration and polarization is investigated.

II. SIMULATION METHOD

A series of simulations were performed to study the polarization dynamics during the acceleration of pre-polarized protons with a modified version of the 3D particle-in-cell (PIC) code EPOCH. A laser pulse with \( \lambda = 0.8\mu m \) wavelength and clockwise polarization is propagated along the \( x \)-direction with a focused transverse Gaussian profile. The spot size is \( w_0 = 10\lambda \) and the pulse duration is \( \tau = 20fs \). The normalized laser amplitude is \( a_0 = eE_0/m_ec = 316/\sqrt{2} \), corresponding to a laser electric field \( E_0 = 8.0 \times 10^{14} V/m \), as adopted in Ref. where \( e,m_e \) are the electron mass and charge, respectively. The simulation mesh resolution is \( dx = 0.03125\lambda \) and \( dy = dz = 0.5\lambda \), in longitudinal and transverse directions respectively. There are 4 macro-particles per species per cell. The moving window simulation box is \( 50\lambda \times 90\lambda \times 90\lambda \) with open boundary conditions in each direction. The proton polarization is defined by its spin vector \( \mathbf{s} \), which has an absolute value of 1 and a direction calculated from the T-BMT equation

\[
\Omega = \frac{q}{mc^2} ([ap+\frac{1}{\gamma}B-\frac{ap\gamma}{\gamma+1}cB\mathbf{v}c^{-1} ap+\frac{1}{\gamma+1}c\mathbf{v}c^{-1} E)]
\]

(1)

where \( m,q \) are the proton mass and charge, respectively; \( ap = 1.87 \) is the dimensionless anomalous magnetic moment of the proton, \( \gamma \) the Lorentz factor of the proton velocity, \( c \) the light speed in vacuum, \( B \) the magnetic field and \( E \) the electric field in the laboratory frame. Equation (1) can be evaluated using an adapted version of the standard Boris operator splitting method commonly used for the momentum integration in PIC codes. Although the particle spin can in principle be altered by the Stern-Gerlach and Sokolov-Ternov effects, these have been neglected in our study based on prior work of Ref. The simulation geometry is shown in Figure 1, where the initial electron, proton and tritium densities are \( 1.5 \times 10^{22} cm^{-3} \), \( 1.0 \times 10^{20} cm^{-3} \) and \( 1.4 \times 10^{21} cm^{-3} \) respectively. The plasma density is uniform and the vacuum length at left edge is \( 10\lambda \). The proton:tritium density ratio is 1:14. The target is initially pre-polarized in the \( x \)-direction, as depicted by the red arrows in Fig. 1.

III. RESULT AND DISCUSSION

An intense laser pulse injected into an underdense plasma can result in a blowout state in which the electrons are expelled by laser while the ions remain immobile. Owing to the charge separation field, the electrons return back to the axis forming a positively charged “bubble”, which has a phase velocity equal to the group velocity of the driver laser. Increasing the plasma density to near critical density results in a lower bubble velocity and an increase of the electromagnetic field in the bubble. Because of its longer exposure to a more intense accelerating field, the motion of protons cannot be ignored, and some protons may be captured by the wakefield. As shown in Fig. 2(a), the electron bubble is formed at 100fs and some electrons are injected into the tail of bubble. There are also some protons distributed near the driving laser, which located at the head of bubble. The proton density forms a vortex microstructure. The rotation direction is same as the laser polarization. For the laser amplitude \( a_0 = 316/\sqrt{2} \), the quiver velocity of protons would be \( v_p/c = a_0/1836 = 0.172 \), which means that the protons can move in the laser field. During the first stage of pre-acceleration, the proton motion is therefore directly affected by the ultra-intense laser field. This can be demonstrated by examining the dependence of the resultant kinetic energy distribution on the laser polarization in the first stage. Figs. 2(b) and 2(c) show the distri-
The original motivation of this study was to analyse the time evolution of the proton spin. The polarization of selected protons is investigated first. The distribution of the proton spin at 100fs is plotted in Fig. 3(a). At this time, the $s_x$ values do not change significantly, remaining close to their initial value of $s_x = 1$. At the same time, the spins exhibit a periodic structure at the laser wavelength. The corresponding proton positions and a map of $B_z$ are shown in Fig. 3(c). The protons are located in the laser field near the axis, where the $B_z$ of wakefield could be ignored. Here, the laser electric intensity is $\sim 10^{14}$ V/m, the correspondence of magnetic field nearly $10^6$ T. At 100fs, the kinetic energy of proton is not very large $\gamma \approx 1$. From Eq. (1), $\Omega \approx \frac{2}{m_e} (\alpha_0 + \frac{1}{\gamma}) B = 2.67 \times 10^{13}$ Hz, where the cycle of rotation is nearly 23fs, implying that the polarization could also be affected by the laser field directly.

At later times, the protons with kinetic energy $E_k > 4$ GeV are still located at regime with $E_x > 0$, as shown in Fig. 2(d). It means that the high energy protons can catch up with the electron “bubble” and continue to gain energy in the wakefield. To track the acceleration dynamics further, nearly 2000 protons are selected with $E_k > 4$ GeV at 400fs. Their positions at 100fs are shown as black dots in Fig. 2(d); their subsequent positions relative to the longitudinal electric field profile at later times are displayed in Fig. 2(f). At 200fs, these protons (purple dots) are still located within the accelerating region of the wakefield with $E_x > 0$, although they undergo some slippage before regaining some ground between 200fs and 400fs. In other words, the protons are trapped in the wakefield at nearly 400fs, and are accelerated continuously. On the other hand, their transverse position increases with time, implying that the bunch experiences a defocusing radial field.

A further series of simulations show that the protons always slip out from the bubble regime and cannot be further accelerated if a laser with linear polarization is used. Compared to the circular polarization case, the bubble size is smaller and the longitudinal filed $E_x$ is weaker in the case of linear polarization owning to the oscillating term of ponderomotive force $\alpha_0$. The protons cannot obtain enough energy to catch up the acceleration field in the bubble.

The distribution of proton density at same time with anti-clockwise and linear polarization, respectively. Compared with the case of clockwise polarization, the vortex structure follows the laser polarization such that the microstructure is imprinted directly by the laser. When the laser is linearly polarized, the proton density takes on a multilayer instead. These features are also reflected in the proton kinetic energy ($E_k$) phase space – Fig. 2(d) for the clockwise polarized laser. Here, the proton energy has a periodic variation with the laser wavelength, another strong indication that the protons are directly modulated by the laser.

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By contrast, later on at 400fs (in Fig. 3(b)), the polar-
The average and maximum energies are also displayed here as the red and blue lines respectively. The maximum energy is nearly 8 GeV and the average energy is 6.5 GeV at 400fs. Over time, the protons are accelerated while their polarization decreases. It should be noted that the polarization $P$ increases near 300fs according to this definition.

After protons become trapped in the wakefield, they are accelerated continuously, yielding a final energy spectrum of all protons at 800fs shown in Fig. 4(b). The corresponding distribution of $\langle s_x \rangle$ is given in Fig. 4(c). Although the maximum energy is nearly 14.63 GeV, this is recorded by a single simulation particle, which means that $\langle s_i \rangle = s_i$ and its polarization $P$ is always 1. In order to study the polarization with meaningful statistics, it is necessary to use $\langle s_x \rangle$ instead of $P$. As revealed in Fig. 4(c), the $\langle s_x \rangle$ becomes negative, which causes the protons’ polarization to change rapidly via this acceleration mechanism. The number of protons with $E_k > 4$ GeV was nearly 80.70 pC. Here, $E_k > 4$ GeV was used because of the proton energy cannot be higher than 4 GeV without acceleration in the wakefield. In this case, the average energy is 7.46 GeV and the polarization is $P = 0.57$.

The proton dynamics is sensitive to the number ratio of hydrogen to tritium, as shown in Fig. 5(a). Here, the position of $E_x = 0$ is denoted as a dashed blue line. The proton positions with maximum $E_k$ for different ratios are shown by different colored lines. A position below zero means that the protons slip out from the acceleration region, and can no longer be accelerated by the wakefield. This phenomenon occurs for the proton fraction above 20%, a case illustrated in Fig. 5(b), where the protons with energy higher than 3 GeV at 400 fs are located in the $E_x < 0$ region of the wakefield and thus they cannot be accelerated further.

The $E_x(x)$ profiles at 120fs are shown in Fig. 6(a) for different ion ratios. In the case of electron acceleration in bubble regime, the ion or proton is always considered immobile, as denoted as dashed black line. For near-critical densities, the ion motion cannot be ignored. Here, we can see that the $E_x$ profile is clearly altered by the ion composition, especially its slope $\Delta E_x/\Delta x$. With increasing...
FIG. 6. (a) The distribution of $E_x$ with $x$ for different ratios. (b) The longitudinal radius of $E_x$ and the velocity of bubble, where $\gamma_b = 1/\sqrt{1 - (v_b/c)^2}$. (c) The average energy of protons whose kinetic energy larger than 4 GeV and their polarization $P$.

proportion of tritium, the distribution of $E_x$ approaches the fixed ion case. The bubble radius is defined as the distance from the head of the bubble to its center, where $E_x = 0$ (dashed blue line). This parameter increases with the amount of tritium, whereas the velocity of the bubble decreases, as shown in Fig. 6(b). After being pre-accelerated directly by laser at the head of bubble, it takes longer for protons to slip back to the center of bubble with larger radius, where the proportion of protons is smaller. This helps the proton to obtain enough energy to catch up the wakefield. Although our study has been restricted to hydrogen-tritium mixtures, our results should be equally applicable to HCl gas targets, where the density ratio of proton and electron is 1:18. The density ratio of proton and electron is more convenient, considering the species of ion has been changed and the motion of heavier ions, either Tritium or Chlorine can be ignored.

Finally, we have studied that the dependence of average energy and polarization on the ion ratio, – Fig. 6(c). Although the average $E_k$ of protons with $E_k > 4$ GeV increases with lower proton fraction, their polarization decreases. This difference of polarization can be accounted for by transverse defocusing of protons in the bubble regime, which could feel more intense magnetic field.

**IV. SUMMARY**

To summarize, we have studied the generation of high-energy proton beams including polarization properties in the interaction of ultra-high-intensity lasers with near-critical density plasmas. After pre-acceleration by a circularly polarized laser, the protons are trapped in the front region of a wakefield bubble and further accelerated by the wakefield, where the acceleration gradient is nearly $10^{14}$ V/m. The protons can gain 10 GeV in this field within 100$\mu$m. Because the laser electric field is nearly $10^{14}$ V/m, the proton polarization can be affected by laser. As the transverse size of the proton bunch increases, it experiences the full bubble magnetic field up to $0.1 \times 10^6$ T, so its polarization is also modified by the wakefield. Finally, the relative proportion of hydrogen and tritium in the gas has a strong influence on the proton acceleration in this regime. The radius and the velocity of the accelerating field structure depend critically on the ion ratio. For sufficiently large ratios, a polarized proton beam can be trapped by the wakefield and accelerated to multi-GeV energies.

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