Baryons and Baryonic Matter in Holographic QCD from Superstring

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We study baryons and baryonic matter in holographic QCD using a D4/D8/D8 multi-D-brane system in the superstring theory. We obtain the chiral soliton solution for baryons in the four-dimensional meson theory derived from the multi-D-brane system. For the analysis of finite baryon-density matter, we investigate the chiral soliton on $S^3$ in holographic QCD, and find the delocalization of the soliton, i.e., the swelling of baryons in dense matter.

1. The Yang-Mills theory from D-branes

To understand the nonperturbative properties of QCD is one of the most important and difficult problems remaining in theoretical physics. As a recent progress, a new method called holographic QCD has been developed to analyze nonperturbative QCD using the superstring theory.

The superstring theory is well-defined in ten-dimensional space-time, and includes D$p$-branes as $(p+1)$-dimensional soliton-like objects of fundamental strings. On the surface of $N$ D-branes, there exists $U(N)$ gauge symmetry and $U(N)$ gauge field appears from the open string. In the space around the massive D-brane, a supergravity field is created. Since the gravity field depends on the distance from the D-brane, one more coordinate appears in the gravity side.

In general, D-branes lead to SUSY theories, reflecting superstring nature. Following Witten, to get non-SUSY gauge theories, we break SUSY explicitly by spatial $S^1$-compactification of D-branes with the periodic/anti-periodic boundary condition for bosons/fermions. The inverse radius of the $S^1$ is called as the Kaluza-Klein mass $M_{KK}$, and the gaugino mass becomes $O(M_{KK})$. Then, non-SUSY gauge theories are formed on the compactified D-brane at larger scale than $1/M_{KK}$.

The four-dimensional non-SUSY $U(N_c)$ Yang-Mills theory is realized on $S^1$-compactified $N_c$ D4-branes, where only the gauge field $A^\mu$ remains to be massless. On the other hand, the effect of the D4-brane can be also described by the gravity field around it, under the hypothesis of gauge/gravity correspondence. In fact, the Yang-Mills theory or the gauge sector of QCD is constructed using the $S^1$-compactified $N_c$ D4-branes, and it is transferred into a higher-dimensional gravity theory in the holographic framework. Due to the strong-weak coupling duality, nonperturbative quantities of large-$N_c$ QCD are calculable with the classical gravity theory.

2. Construction of holographic QCD

Four-dimensional massless QCD can be constructed with the D4/D8/D8 multi-D-brane system consisting of spatially $S^1$-compactified $N_c$ D4-branes attached with $N_f$ D8-D8 pairs, as shown in Fig.1(a). From this multi-D-brane system, the higher-dimensional theory of holographic QCD is formulated.

In holographic QCD, “color” and “flavor” are described as different physical objects, i.e., different D-branes: the D4 gives color and the D8 gives flavor. Here, gluons appear on the D4-brane, and quarks, which have color and flavor, appear at the cross point between D4 and D8/D8.

Usually, holographic QCD is based on large-$N_c$ argument, where $N_c$ D4-branes have a large mass proportional to $N_c$. With the gauge/gravity duality, $N_c$ D4-branes can be replaced by the gravity field, and the system becomes $N_f$ D8-branes in the presence of the gravity background of the D4-
brane, as shown in Fig. 1(b). In large $N_c \gg N_f$, the gravitational contribution from D8/DS can be neglected, which corresponds to the quenched approximation [5].

In large $N_c$ and large 't Hooft's coupling $\lambda$, $N_f$ D8-branes in the D4 gravity background lead to the Dirac-Born-Infeld (DBI) action in nine-dimensional space-time. Here, only flavor degrees of freedom remain, since the D4-brane with color dimensional space-time. Here, only flavor degrees of freedom remains, since the D4-brane with color degrees of freedom remains, since the D4-brane with color dimensionality is already replaced by the gravity background.

The DBI action reduces to a five-dimensional flavored Yang-Mills theory with a curved metric on extra fifth-coordinate, in the leading order. The 5-dimensional flavored gauge field $A^\mu(x_\nu, z)$ appears from 8-8 strings, and the fifth-coordinate $z$ appears from the distance from the D4-brane. Through the mode expansion of $A^\mu(x_\nu, z)$ in the fifth $z$-direction, a four-dimensional effective theory of pions and (axial) vector mesons can be obtained from the holographic construction of QCD. This meson effective theory has only two independent parameters, and describes many phenomenological features of mesons [6].

3. Baryons as brane-induced Skyrmions

In large $N_c$, QCD is reduced to weakly interacting theory of mesons and glueballs [6], and baryons do not appear as direct degrees of freedom. Instead, the baryon appears as a topological chiral soliton [7,8] of mesons in large-$N_c$ QCD.

In Ref. [3], we performed the first study of the baryon as the chiral soliton in holographic QCD [5]. As a remarkable fact, the topological chiral soliton picture for baryons can be directly derived from QCD in the holographic framework: the Skyrme term appears from holographic QCD without appearance of other four or more derivative terms in the leading order of $1/N_c$ and $1/\lambda$.

For the argument of QCD, the Kaluza-Klein mass $M_{KK}$ plays a role of ultra-violet cutoff scale. Considering the consistency with the cutoff scale of $M_{KK} \sim 1$GeV in the holographic approach, we take only pions and $\rho$-mesons, and derive their four-dimensional effective action [5] without use of small-amplitude expansion, to describe the nonlinear configuration of solitons.

In the chiral soliton picture, the baryon is described with the hedgehog configuration of Nambu-Goldstone bosons. For the chiral field $U(x_\nu) \equiv e^{i\pi(\langle x_\nu \rangle / f_\pi)} \equiv \{\sigma(x_\nu) + i \tau^a \Pi^a(x_\nu)\}/f_\pi$ and the $\rho$-meson field $\rho^\mu(x_\nu)$, the hedgehog configuration can be expressed as

$$U^*(x) = e^{i(x^o \hat{x}^o F(r))}, \quad \rho_1^*(x) = \varepsilon_{iab} \tau^a \hat{x}^b \tilde{G}(r), \quad \rho_0^*(x) = 0,$$

with $x^o = (t, \mathbf{x})$, $r \equiv |\mathbf{x}|$ and $\hat{x}^o \equiv x^o/r$. For the baryon with unit topological charge, the pion profile $F(r)$ has the topological boundary conditions, $F(0) = \pi$ and $F(\infty) = 0$.

We derive the Euler-Lagrange equations, which are coupled nonlinear differential equations of pion and $\rho$-meson profiles, $F(r)$ and $\tilde{G}(r)$ [6]. Under the topological boundary conditions, we solve the field equations and obtain the stable chiral soliton solution, which we call “brane-induced Skyrmion” [3]. With the experimental inputs of the pion decay constant $f_\pi = 92.4$MeV and the $\rho$-meson mass $m_\rho = 776$MeV (i.e., $M_{KK} \approx 948$MeV, $\varepsilon \approx 7.315$ for the Skyrme parameter), we estimate the mass and the radius of the hedgehog baryon as $M_B \approx 834$MeV and $\sqrt{r^2} \approx 0.37$fm.

Figure 1. (a) The multi-D-brane configuration corresponding to massless QCD: spatially $S^1$-compactified $N_c$ D4-branes attached with $N_f$ D8-DS pairs. Gluons appear from 4-4 strings, and quarks appear from 4-8 and 4-$\bar{8}$ strings. (b) D8-branes with D4 supergravity background. Mesons appear from 8-8 strings.
4. Baryonic matter in holographic QCD

It is interesting to apply holographic QCD to finite-density QCD [9,10], which is difficult to study with lattice QCD. Here, we consider the baryonic matter in large $N_c$, since the holographic QCD is formulated in large $N_c$.

In the large-$N_c$ scheme of the baryonic matter, the kinetic energy, the N-$\Delta$ mass splitting and quantum fluctuations (apart from $O(1)$ zero-point quantum fluctuations) are $O(1/N_c)$, so that they are suppressed relative to the static baryon mass of $O(N_c)$, and the baryonic matter reduces to static solid-like soliton matter, i.e., static brane-induced Skyrme matter in holographic QCD [10].

To analyze such static Skyrme matter, we take a mathematical trick proposed by Manton and Ruback [11]. To simulate the many Skyrmion system, one unit cell shared by a Skyrmion in a three-dimensional closed manifold $S^3$ with finite radius $R$, as shown in Fig.2. The single Skyrmion placed on the surface volume $2\pi^2R^3$ of the manifold $S^3$ corresponds to the finite-density baryonic matter with $\rho_B = 1/(2\pi^2R^3)$.

Then, we investigate the single brane-induced Skyrmion on $S^3$ with the radius $R$, which simulates the baryonic matter in large-$N_c$ holographic QCD [10]. We derive and solve the field equations of $F(r)$ and $G(r)$ for the hedgehog soliton on $S^3$, with $0 \leq r \leq \pi R$. Here, the topological boundary conditions are $F(0) = \pi$ and $F(\pi R) = 0$.

![Figure 2](image)

Figure 2. Schematic figure of the Skyrme matter on a flat physical space $\mathbb{R}^3$, and the single Skyrmion on $S^3$ with a finite radius $R$. The single Skyrmion on $S^3$ simulates the Skyrme matter on $\mathbb{R}^3$.

Figure 3. (a) The pion profile $F(r)$ and (b) the $\rho$-meson profile $G(r)$ in the brane-induced Skyrmion on $S^3$ with $R=4.0, 2^{1/2}, 1.19 [1/ef_\pi]$. Figure 4 shows the energy density $\varepsilon(r)$ of the single brane-induced Skyrmion on $S^3$ with various values of $R$. As $R$ decreases, the baryonic soliton is gradually delocalized. This delocalization of the chiral soliton on $S^3$ indicates the “swelling of baryons” in dense matter [10], which leads to the reduction of N-$\Delta$ mass splitting in the framework of the chiral soliton. For $R \leq R_c$, the system becomes homogeneous, and there occurs the “delocalization phase transition” for the baryonic soliton [10], which would suggest deconfinement.
investigated the chiral soliton on for the analysis of baryonic matter, we have inho- 
gobaryon for the first time in holographic QCD.

obtained the chiral soliton solution of the hedge-
brane system in the superstring theory. W e have
onic matter in holographic QCD using a multi-D-
and chiral symmetry restoration.

of about 7 calization phase transition at the critical density
of the baryonic soliton. W e have found the delo-
QCD, and found the delocalization or the swelling
of the critical radius
R

the spatially-averaged chiral condensate on
symmetry in the nonlinear representation, using
also investigate the manifestation of the chiral
deconfinement and chiral symmetry restoration
and globally chiral restored, which would indicate
deconfined and chiral restored, zero, that is, the system becomes homogeneous
for
R
= 1.19

(1/
ε
f
π
)

Figure 4. The energy density
ε
(r)

of the branch-
induced Skyrmion on
S
3

with various values of
the radius
R
[1/
ε
f
π
]. The system becomes homog-
eneous for
R
≤
R
c
(≃ 1.19).

For the quantitative description of delocalization of the baryonic soliton, we define the “local-
ization order parameter”
Φ(R)

from the spatial fluctuation of the energy density
ε(x)

as

Φ(R) = \frac{1}{2} \int_{S^3} d^3 x |\bar{\varepsilon}(x) - \bar{\varepsilon}_{id}|, \quad \bar{\varepsilon}(x) = \frac{\varepsilon(x)}{E} \quad (2)

with
E = \int_{S^3} d^3 x \varepsilon(x)

and
\bar{\varepsilon}_{id} = 1/(2\pi^2 R^3).

We also investigate the manifestation of the chiral symmetry in the nonlinear representation, using
the spatially-averaged chiral condensate on
S
3
,

⟨\sigma(x_\mu)⟩_{S^3}

As
R
 decreases, both order parameters
Φ(R)

and

⟨\sigma(x_\mu)⟩_{S^3}

decrease, as shown in Fig.5. At the critical radius
R_c ≃ 1.19 [1/\epsilon f_\pi], they go to zero, that is, the system becomes homogen-
eous and globally chiral restored, which would indicate deconfinement and chiral symmetry restoration
at high density of
ρ_c ≃ 7.12\rho_0.

In summary, we have studied baryons and bary-
onic matter in holographic QCD using a multi-D-
brane system in the superstring theory. We have
obtained the chiral soliton solution of the hedge-
hog baryon for the first time in holographic QCD.
For the analysis of baryonic matter, we have in-
vestigated the chiral soliton on
S
3

in holographic QCD, and found the delocalization or the swelling
of the baryonic soliton. We have found the delo-
calization phase transition at the critical density
of about 7\rho_0, which would indicate deconfinement
and chiral symmetry restoration.

Figure 5. (a) The localization order parameter
Φ

and (b) the spatially-averaged chiral condensate

⟨\sigma(x_\mu)⟩_{S^3}

as the function of the radius
R

of
S
3
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