Study on Transverse Load Distribution of Hinged Hollow Beam

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Abstract. The bridge is a kind of space structure, when the car load on a part of the bridge, the impact of its load will be transmitted to the surrounding. In this paper, the hinge plate method is used to calculate and analyze the simply supported hollow slab of a certain arch bridge. Considering the hinge plate mounting method is suitable for pouring concrete bridge connecting the longitudinal tongue and groove joints, horizontal beams fabricated separate beam only in the middle between the free flaps or reinforced with steel connection. Therefore, the transverse analysis and calculation of the superstructure of box girder are carried out by using hinge plate method. And mechanical analysis of the transverse beam with finite element software MIDAS Civil grillage method.

1. Introduction

The horizontal distribution of loads refers to how the vehicle load acting on the bridge is distributed among the main girders, or how the girders share the vehicle live load. For a beam composed of multiple beams, and through the transverse beams, bridges and other horizontal connections to form a whole beam bridge [1]. When the bridge has a load, because the lateral stiffness of the structure will inevitably make the main beam of different load to share the load. For the determination of the internal force value S of a certain section of a main beam, the concept of influencing line is introduced in the vertical and horizontal directions of the bridge, simplify the problem of space as a plane problem. Which is:

\[ S = P \cdot \eta(x, y) \approx P \cdot \eta_2(y) \cdot \eta_1(x) \]  

(1)

In the formula: \( \eta(x, y) \) —— Internal force influence surface of a beam in space calculation. 
\( \eta_1(x) \) —— Single beam in the x-axis direction of a section of the internal force of the line. 
\( \eta_2(y) \) ——When the unit load acts in different positions along the transverse direction (y-axis direction) of the deck, the variation ratio of the load ratio assigned by a beam is also referred to as the influence of the lateral distribution of load on a beam.

“\( P \cdot \eta_2(y) \)” is the load distributed to a beam in the transverse direction when the load “\( P \)” acts on point \( A(x, y) \). Temporarily expressed as “\( P \)” . The maximum load “\( P_{\text{max}} \)” can be obtained according to the loading at the most unfavorable position. We define that “\( P \)” is the axle weight, then m is called the lateral distribution of the load, which means that the maximum load assumed by a root beam is a multiple of the axle (usually less than 1).
2. Articulated plate method
For the cast-in-place concrete longitudinal seam joints of the assembly-type bridge and a welded bridge with no intermediate beams connected between wings or welded crossheads. As the block between the horizontal with a certain connection structure, its connection is very rigid and weak. The kind of structure is practically closing to a plurality of slit plates that parallels to each other. For this type of force structure, the lateral hinge plate theory can be used to calculate the lateral distribution of the load.

The bridge consists of 9 hollow slab beams, the cross-section of the bridge and the typical cross-section of the hollow plate as shown in figure 1 and figure 2.

![Figure 1. Full bridge cross section (unit: mm).](image1)

![Figure 2. Hollow plate typical cross section (unit: mm).](image2)

2.1. The calculation process
Calculate the stiffness factor.

\[
\gamma = \frac{b \varphi}{2 \omega} = \frac{\pi^2 E I}{4 G I_T} \left( \frac{b}{l} \right)^2 \approx 5.8 \left( \frac{b}{l} \right)^2 = 0.01247
\]

\[
\left( \omega_1 + a_1 \varphi_1 \right)/ \omega = \frac{I}{I_1} + 23.2 \frac{I}{I_{T1}} \left( \frac{a_1}{l} \right) = 0.840786
\]

When the unit load is applied to any beam, the vertical load assigned to each beam is solved. The vertical load of a beam calculated by the different load on the board is connected into a curve. And the horizontal impact line of the load of a sheet is formed.

2.2. The calculation result
According to the calculation, the hinge force values of each hinge point and the vertical load values of each beam are obtained under the unit force. The hinge values of the nodes are shown in table 1, and the vertical load values for each plate are listed in table 2.
Table 1. Hinge of hinge points.

| Hinge number | \( g_1 \)  | \( g_2 \)  | \( g_3 \)  | \( g_4 \)  | \( g_5 \)  | \( g_6 \)  | \( g_7 \)  | \( g_8 \)  | \( g_9 \)  |
|--------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1            | 0.7755     | -0.1974    | -0.1606    | -0.1318    | -0.1004    | -0.0930    | -0.0810    | -0.0731    | 0.0696     |
| 2            | 0.6108     | 0.6421     | -0.3013    | -0.2473    | -0.1943    | -0.1745    | -0.1520    | -0.1372    | 0.1306     |
| 3            | 0.4769     | 0.5014     | 0.5554     | -0.3752    | -0.3021    | -0.2648    | -0.2307    | -0.2083    | 0.1981     |
| 4            | 0.3671     | 0.3858     | 0.4274     | 0.4905     | -0.4316    | -0.3684    | -0.3210    | -0.2898    | 0.2757     |
| 5            | 0.2757     | 0.2898     | 0.3210     | 0.3684     | 0.4316     | -0.4905    | -0.4274    | -0.3858    | 0.3671     |
| 6            | 0.1981     | 0.2083     | 0.2307     | 0.2648     | 0.3021     | 0.3752     | -0.5554    | -0.5014    | 0.4769     |
| 7            | 0.1306     | 0.1372     | 0.1520     | 0.1745     | 0.1943     | 0.2473     | 0.3013     | -0.6421    | 0.6108     |
| 8            | 0.0696     | 0.0731     | 0.0810     | 0.0930     | 0.1004     | 0.1318     | 0.1606     | 0.1974     | 0.7755     |

Table 2. The vertical load values assigned to each beam.

| Beam number | \( p_{i1} \) | \( p_{i2} \) | \( p_{i3} \) | \( p_{i4} \) | \( p_{i5} \) | \( p_{i6} \) | \( p_{i7} \) | \( p_{i8} \) | \( p_{i9} \) |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1           | 0.2245      | 0.1974      | 0.1606      | 0.1318      | 0.1004      | 0.0930      | 0.0810      | 0.0731      | 0.0696      |
| 2           | 0.1646      | 0.1605      | 0.1407      | 0.1155      | 0.0939      | 0.0815      | 0.0710      | 0.0641      | 0.0610      |
| 3           | 0.1339      | 0.1407      | 0.1433      | 0.1280      | 0.1078      | 0.0903      | 0.0787      | 0.0710      | 0.0676      |
| 4           | 0.1099      | 0.1155      | 0.1280      | 0.1343      | 0.1295      | 0.1036      | 0.0903      | 0.0815      | 0.0775      |
| 5           | 0.0914      | 0.0961      | 0.1064      | 0.1221      | 0.1368      | 0.1221      | 0.1064      | 0.0961      | 0.0914      |
| 6           | 0.0775      | 0.0815      | 0.0903      | 0.1036      | 0.1295      | 0.1343      | 0.1280      | 0.1155      | 0.1099      |
| 7           | 0.0676      | 0.0710      | 0.0787      | 0.0903      | 0.1078      | 0.1280      | 0.1433      | 0.1407      | 0.1339      |
| 8           | 0.0610      | 0.0641      | 0.0710      | 0.0815      | 0.0939      | 0.1155      | 0.1407      | 0.1605      | 0.1646      |
| 9           | 0.0696      | 0.0731      | 0.0810      | 0.0930      | 0.1004      | 0.1318      | 0.1606      | 0.1974      | 0.2245      |

3. Finite element space calculation
The beam stiffness method is used to replace the bridge superstructure with the equivalent beam grid. The bending stiffness and torsional rigidity of each section distributed on the space board (beam) are concentrated in the nearest equivalent grating. The longitudinal stiffness of the actual structure is concentrated in the longitudinal beam element, and the lateral stiffness is concentrated in the transverse beam.

3.1. Modeling introduction
The finite element program Midas Civil was used to simulate the bridge, and the model was established by using the beam grid method. Because the hinge only transmits the shear force and does not transmit the bending moment. The two ends of the beam are released when the beam is simulated. At the same time, the bending moment of the beam is similar to the bending moment of the hollow plate beam for the accuracy of the calculation [2]. When the beam grid method is used, the simulation of the lateral connection is very important, which is directly related to the accuracy of the result. In this paper, the stiffness of the virtual beam is less than the rigidity of the stringer [3-5].

Figure 3. Finite element analysis model.
3.2. Calculation results

According to the concept of horizontal distribution of load, the following relationship should be established in any section of the bridge across the longitudinal direction:

\[
\frac{\omega_i(x)}{\omega_j(x)} = \frac{M_i(x)}{M_j(x)} = \frac{Q_i(x)}{Q_j(x)} = \frac{P_i(x)}{P_j(x)} = C
\]

In the formula, \( x(x), M(x), Q(x), P(x) \) are the deflection, bending moment, shear force and the assigned force and load of section \( i \) and \( j \), respectively, and \( C \) is constant.

According to equation (2), the vertical distribution of the influence of the horizontal distribution of the load can be expressed as

\[
\eta_x = \frac{\sum_{i=1}^{n} \omega_i}{\sum_{i=1}^{n} \omega_i}, \quad \eta_M = \frac{\sum_{i=1}^{n} M_i}{\sum_{i=1}^{n} M_i}, \quad \eta_Q = \frac{\sum_{i=1}^{n} Q_i}{\sum_{i=1}^{n} Q_i}
\]

In the finite element model, the amplitude of the beam is obtained by loading the unit sine load, and the influence of the horizontal distribution of the bridge load and the variation law of the beam are obtained by the formula (3). The figure 4 is the magnitude of the moment value of the load when the sine load is applied to the beam No. 1. The influence line coordinates are summarized in table 3.

**Table 3.** Finite element method to calculate the vertical load value.

| Beam number | \( p_{i1} \) | \( p_{i2} \) | \( p_{i3} \) | \( p_{i4} \) | \( p_{i5} \) | \( p_{i6} \) | \( p_{i7} \) | \( p_{i8} \) | \( p_{i9} \) |
|-------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1           | 0.1722        | 0.1518        | 0.1297        | 0.1133        | 0.1004        | 0.0905        | 0.0833        | 0.0785        | 0.0761        |
| 2           | 0.1520        | 0.1508        | 0.1356        | 0.1171        | 0.1037        | 0.0935        | 0.0861        | 0.0812        | 0.0788        |
| 3           | 0.1305        | 0.1359        | 0.1380        | 0.1259        | 0.1102        | 0.0993        | 0.0914        | 0.0863        | 0.0838        |
| 4           | 0.1141        | 0.1175        | 0.1261        | 0.1310        | 0.1214        | 0.1080        | 0.0994        | 0.0939        | 0.0912        |
| 5           | 0.1012        | 0.1042        | 0.1104        | 0.1215        | 0.1287        | 0.1215        | 0.1104        | 0.1042        | 0.1012        |
| 6           | 0.0912        | 0.0939        | 0.0994        | 0.1080        | 0.1214        | 0.1310        | 0.1261        | 0.1175        | 0.1141        |
| 7           | 0.0838        | 0.0863        | 0.0914        | 0.0993        | 0.1102        | 0.1259        | 0.1380        | 0.1359        | 0.1305        |
| 8           | 0.0788        | 0.0812        | 0.0861        | 0.0935        | 0.1037        | 0.1171        | 0.1356        | 0.1508        | 0.1520        |
| 9           | 0.0761        | 0.0785        | 0.0833        | 0.0905        | 0.1004        | 0.1133        | 0.1297        | 0.1518        | 0.1722        |

3.3. Compare results

The results of the hinge plate method are compared with the finite element results in the same figure.
(a) The load acts on beam 1.

(b) The load acts on beam 1.

(c) The load acts on beam 3.

(d) The load acts on beam 4.

(e) The load acts on beam 5.

(f) The load acts on beam 6.

(g) The load acts on beam 7.

(h) The load acts on beam 8.
Figure 5. Results contrast curve.

4. Conclusion
The results show that the stiffness of the simulated beam is close to that of the longitudinal beam, and the calculated results are more accurate. In this paper, the stiffness of the simulated beam is slightly smaller than that of the rigid beam. In the beam stiffness, did not consider the side beam and the beam of the bending stiffness of the gap, so the edge beam effect line calculation results are low.

The lateral load distribution than the load transverse distribution of beam calculated on the hinge plate is calculated by the method is more uniform, this is because that along the length of plate can transfer shear hinged plate, but only a finite element model of shear transfer in virtual beam position. The beam grid method can calculate the bending moment and shear transverse distribution of the full length of the bridge, and the hinge plate method can only reflect the horizontal distribution of the bending moment.

5. Reference
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