Mixed RF/FSO Relaying Systems with Hardware Impairments

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Abstract—In this work, we provide a detailed analysis of a dual-hop fixed gain (FG) amplify-and-forward relaying system, consisting of a hybrid radio frequency (RF) and free-space optical (FSO) channels. We introduce an impairment model which is the soft envelope limiter (SEL). Additionally, we propose the partial relay selection (PRS) protocol with outdated channel state information (CSI) based on the knowledge of the RF channels in order to select one relay for the communication. Moreover, the RF channels of the first hop experience Rayleigh fading while we propose a unified fading model for the FSO channels, called the unified Gamma Gamma (GG), taking into account the atmospheric turbulence, the path loss and the misalignment between the transmitter and the receiver aperture also called the pointing error. Novel closed-forms of the outage probability (OP), the bit error probability (BEP) and the average ergodic capacity (clear, light/moderate fog, moderate/heavy rain and hazy). The system and channel models while the outage probability, the bit error probability and the ergodic capacity analysis are given in Section III. Numerical and analytical results are reported in Section V.

Keywords—Partial relay selection, outdated CSI, amplify-and-forward, pointing error, path loss, hardware impairments.

I. INTRODUCTION

With the rapid increase of Internet mobile stations and the high demands for bandwidth, RF cellular networks have reached a bottleneck due to the limited access to spectrum resources. In addition, existing backhaul network infrastructure, which connects the core network to the edges, cannot support the massive flows of data traffic. Recently, researchers have proposed to use optical fibers (OF) as a solution for alleviating the backhaul load congestion. However, as the number of cells becomes very large (e.g., in ultra dense cellular networks), networks can still suffer from the limited use of the OF, where cable installations are very costly and sometimes even restricted [1]. To support the large number of users, FSO (free-space optical) technology emerges as an alternative or complementary solution to the RF and OF links, since it is more flexible, license-free, power efficient, cost effective, no installation restriction and most importantly it increases the capacity of cellular networks [2]. Because of these advantages, FSO became a timely research topic and are classified into many types, depending on the amplifier nature [8]. It has been shown previously in [9] that the High Power Amplifier (HPA) non-linearities create non-linear distortion which creates not only irreducible floors for the outage probability and the bit error probability but also saturates the system capacity by a destructive ceiling. Although it is very challenging to consider non-ideal hardware, the work mentioned here considered impaired hardware over full RF systems with single relay [10].

Our contribution is to quantify the impacts of the HPA non-linearities on the mixed RF/FSO system with multiple relays. Although Balti et. al [3], [4] introduced the hardware impairments to the mixed RF/FSO system, they considered a general model of hardware impairments. In this work, we introduce a more specific impairment model which is the soft envelope limiter (SEL) HPA non-linearities to the system. Additionally, we assume the intensity modulation and direct detection (IM/DD) for signal reception and since the RF channels are time-varying, the partial relay selection (PRS) with outdated channel state information (CSI) is adopted to select one relay for the signal forwarding. Furthermore, we suggest the unified Gamma Gamma (GG) for the FSO channels model considering the pointing error, the path loss attenuation and the atmospheric turbulence related to the weather state (clear, light/moderate fog, moderate/heavy rain and hazy)

The rest of this paper is structured as follows: Section II describes the system and channel models while the outage probability, the bit error probability and the ergodic capacity analysis are given in Section III. Numerical and analytical results are discussed in Section IV. Finally, the concluding remarks are reported in Section V.

II. SYSTEM AND CHANNEL MODELS

A. System Model

In this system, $S$ communicates with $D$ by selecting one relay among an intermediate set of $N$ relays. To achieve this...
selection, the PRS protocol states that $S$ receives the CSIs from the relays, sorts them in an increasing order and then selects the relay/channel with the highest CSI. Given that the RF channels are time-varying, the incoming CSIs instantaneously fluctuate before and after the relay selection. In this case, the highest SNR link before the selection does not remain the same after the selection and thereby the selection will be achieved based on outdated CSI. Moreover, since the relays operate at the half-duplex mode, the best relay may not be always available for the transmission. In this case, $S$ will select the next best available relay and so on so forth. To model the relation between the outdated and the updated CSIs, we associate the correlation coefficient $\rho$ as follows:

$$h_{1(m)} = \sqrt{\rho} \hat{h}_{1(m)} + \sqrt{1 - \rho} w_m$$  \hspace{1cm} (1)

where $w_m$ is a random variable that follows the circularly complex Gaussian distribution with the same variance of the channel gain $\hat{h}_{1(m)}$. The correlation coefficient $\rho$ is given by the Jakes’ autocorrelation model as follows:

$$\rho = J_0(2\pi f_d T_d)$$  \hspace{1cm} (2)

where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind, $T_d$ is the time delay between the current and the delayed CSI versions and $f_d$ is the maximum Doppler frequency of the channels.

Supposing that $S$ selects the relay with rank $m$, the amplification gain can be given by:

$$G = \sqrt{\frac{\sigma_t^2}{\mathbb{E}[|h_m|^2] P_s + \sigma_1^2}}$$  \hspace{1cm} (3)

where $\mathbb{E}[\cdot]$ is the expectation operator, $P_s$ is the average transmitted power from $S$, $\sigma_t^2$ is the noise power and $\sigma_1^2$ is the mean power of the signal at the output of the relay block. For a given saturation level $A_{sat}$, the amplifier operates at a certain input back-off (IBO), which is given by:

$$\text{IBO} = \frac{A_{sat}^2}{\sigma_p^2}$$  \hspace{1cm} (4)

Since the HPA creates a non-linear distortion, we refer to the Bussgang linearization theory to linearize the distortion \cite{[11]}. Thus, the output of the nonlinear circuit can be given by \cite{[5] Eq. (9)):

$$\Omega_m = \nu x + b$$  \hspace{1cm} (5)

where $\nu$ is the scale of the input signal and $b$ is an uncorrelated non-linear distortion with the input signal that follows the Gaussian distribution $b \sim \mathcal{CN}(0, \sigma_b^2)$. For the SEL model, the parameters $\nu$, $\sigma_b^2$ and the clipping factor $\mu$ are given by:

$$\nu = 1 - \exp\left(-\frac{A_{sat}^2}{\sigma_p^2}\right) + \frac{\pi \sigma_{sat} x}{2 \sigma_p} \text{erfc}\left(-\frac{A_{sat}}{\sigma_p}\right)$$

$$\sigma_b^2 = \sigma_p^2 \left[1 - \exp\left(-\frac{A_{sat}^2}{\sigma_p^2}\right) - \nu^2\right]$$

$$\mu = 1 - \exp\left(-\frac{A_{sat}^2}{\sigma_p^2}\right)$$  \hspace{1cm} (6)

where erfc(·) is the complementary error function. The average transmitted power at the relay can be given in terms of the clipping factor as follows:

$$P_t = \mu \sigma_p^2$$  \hspace{1cm} (7)

The relays employ the subcarrier intensity modulation (SIM) for the electrical to optical conversion. The overall signal-to-noise plus distortion ratio (SNDR) is given by:

$$\gamma_{ni} = \frac{\gamma_{1(m)} \gamma_{2(m)}}{\kappa \gamma_{2(m)} + \mathbb{E}[\gamma_{1(m)}] + \kappa}$$  \hspace{1cm} (8)

where $\kappa$ is defined by:

$$\kappa = 1 + \frac{\sigma_b^2}{\mu^2 G^2 \sigma_1^2}$$  \hspace{1cm} (9)

Note that for the case of ideal relays ($\kappa = 1$), Eq. (8) is reduced to:

$$\gamma_{id} = \frac{\gamma_{1(m)} \gamma_{2(m)}}{\gamma_{2(m)} + \mathbb{E}[\gamma_{1(m)}] + 1}$$  \hspace{1cm} (10)

which is the expression of the overall SNR of an ideal relaying system.

The instantaneous SNR of the first hop can be given by:

$$\gamma_{1(m)} = \frac{|h_m|^2 P_s}{\sigma_1^2}$$  \hspace{1cm} (11)

The average SNR of the first hop can be expressed as follows:

$$\overline{\gamma}_1 = \frac{P_s}{\sigma_1^2}$$  \hspace{1cm} (12)

The instantaneous SNR of the $m$-th optical channel can be obtained by:

$$\gamma_{2(m)} = \frac{|I_m|^2 \eta^2 P_s^2}{\sigma_2^2}$$  \hspace{1cm} (13)

where $I_m$, $\eta$ and $\sigma_2^2$ are the $m$-th channel gain, the optical-electrical conversion and the noise power of the optical channel, respectively.

B. Channels Model

Since the RF channels of the first hop are subject to the Rayleigh fading, the instantaneous SNRs $\gamma_{1(m)}$ and $\gamma_{2(m)}$ are two jointly exponential random variables. The joint probability density function (PDF) of $\gamma_{1(m)}$ and $\gamma_{2(m)}$ is expressed as follows:

$$f_{\gamma_{1(m)}, \gamma_{2(m)}}(x, y) = \frac{1}{(1 - \rho) \overline{\gamma}_1} e^{\frac{x + y}{(1 - \rho) \overline{\gamma}_1}} I_0\left(\frac{2 \sqrt{\rho \mu y}}{(1 - \rho) \overline{\gamma}_1}\right)$$  \hspace{1cm} (14)

where $I_\nu(\cdot)$ is the $\nu$-th order modified Bessel function of the first kind.

After some mathematical manipulations, the cumulative distribution function (CDF) of $\gamma_{1(m)}$ is given by:

$$F_{\gamma_{1(m)}}(x) = 1 - m \left(\frac{N}{m}\right)^{m-1} \sum_{n=0}^{m-1} \frac{(-1)^n}{N - m + n + 1} \binom{m - 1}{n}$$

$$\times \exp\left(-\frac{(N - m + n + 1) x}{((N - m + n)(1 - \rho) + 1) \overline{\gamma}_1}\right)$$  \hspace{1cm} (15)

The expectation $\mathbb{E}[\gamma_{1(m)}]$ can be expressed as follows:

$$\mathbb{E}[\gamma_{1(m)}] = m \left(\frac{N}{m}\right)^{m-1} \sum_{n=0}^{m-1} \binom{m - 1}{n} (-1)^n$$

$$\times \frac{[(N - m + n)(1 - \rho) + 1] \overline{\gamma}_1}{(N - m + n + 1)^2}$$  \hspace{1cm} (16)
Regarding the channels of the second hop, the channel gain \( I_m \) can be expressed as follows:
\[
I_m = I_a I_l I_p
\]
(17)
where \( I_a, I_l \) and \( I_p \) are the atmospheric turbulence, the path loss and the pointing error, respectively. The table below summarizes the main parameters of the optical channel.

| Parameter | Symbol |
|-----------|--------|
| Weather attenuation | \( \sigma \) |
| Jitter variance | \( \sigma_j^2 \) |
| Rylov variance | \( \sigma_R^2 \) |
| Wave number | \( k \) |
| Wavelength | \( \lambda \) |
| Pointing error coefficient | \( \xi \) |
| Beam waist at the relay | \( w_0 \) |
| Beam waist | \( w_L \) |
| Equivalent beam waist | \( w_{Leq} \) |
| Length of the optical link | \( L \) |
| Radius of the receiver aperture | \( \alpha \) |
| Fraction of the collected power at \( L = 0 \) | \( A_0 \) |
| Radius of curvature | \( F_0 \) |
| Refractive index of the medium | \( C_n \) |
| Small scale turbulence | \( \alpha \) |
| Large scale turbulence | \( \beta \) |
| Radial displacement of the beam at the receiver | \( R \) |

Using the Beers-Lambert law, the path loss can be expressed as follows:
\[
I_l = \exp(-\sigma L) \tag{18}
\]
The pointing error \( I_p \) made by Jitter can be given as:
\[
I_p = A_0 \exp\left(\frac{-2R^2}{w_{Leq}^2}\right) \tag{19}
\]
Assuming that the radial displacement of the beam at the detector follows the Rayleigh distribution, the PDF of the pointing error can be expressed as follows:
\[
f_{I_p}(I_p) = \frac{\xi^2}{A_0^2} I_p e^{-1} , \quad 0 \leq I_p \leq A_0 \tag{20}
\]
The small and large atmospheric scales can be determined by:
\[
\alpha = \left(\exp\left[\frac{0.49\sigma_R^2}{(1 + 1.11\sigma_R^2)}\right] - 1\right)^{-1} \tag{21}
\]
\[
\beta = \left(\exp\left[\frac{0.51\sigma_R^2}{(1 + 0.69\sigma_R^2)}\right] - 1\right)^{-1}
\]
where the Rylov variance is given by:
\[
\sigma_R^2 = 1.23 C_n^2 1^{7/6} L^{11/6} \tag{22}
\]
The pointing error coefficient can be expressed in terms of the Jitter standard deviation and the equivalent beam waist as follows:
\[
\xi = \frac{w_{Leq}}{2\sigma_s} \tag{23}
\]
We can also relate \( w_{Leq} \) with the beam width \( w_L \) of the Gaussian laser beam at the distance \( L \) as follows:
\[
w_{Leq}^2 = \frac{w_L^2 \sqrt{\pi} \text{erf}(v)}{2v \exp(-v^2)} , \quad v = \frac{\sqrt{2\xi}}{\sqrt{2w_L}} \tag{24}
\]
where \( \text{erf}(\cdot) \) is the error function. The fraction of the collected power at the relay \( A_0 \) is given by
\[
A_0 = |\text{erf}(v)|^2 \tag{25}
\]
The Gaussian beam waist itself can be defined as:
\[
w_L = w_0 \sqrt{(\Theta_0 + A_0)(1 + 1.63 \sigma_R^{12/5} \Lambda_1)} \tag{26}
\]
\[
\Theta_0 = 1 - \frac{L}{F_0} , \quad A_0 = 2L k w_0^\alpha , \quad \Lambda_1 = \frac{\Lambda_0}{\Theta_0^5 + \Lambda_0} \tag{26}
\]

After some mathematical manipulations, the average electrical SNR of the optical channel can be obtained by:
\[
\gamma_2 = \frac{P^2 I^2}{\sigma_n^2} h^2 A_0 I^2 , \quad h = \frac{\xi^2}{\xi^2 + 1} \tag{27}
\]
Since the atmospheric turbulence \( I_a \) is modeled as GG, the PDF of the optical irradiance of the \( m \)-th channel can be given by \([12, Eq. (13)]\):
\[
f_{I_m}(I_m) = \frac{\xi^2 \alpha \beta}{A_0 I_l \Gamma(\alpha) \Gamma(\beta)} G_1^{3.0} \left(\frac{\alpha \beta I_m}{A_0 I_l} \right) |\xi^2 - 1, \alpha - 1, \beta - 1 \tag{28}
\]
After some algebraic transformations, the PDF of the instantaneous optical SNR can be derived as follows:
\[
f_{\gamma_{2,m}}(x) = \frac{\xi^2}{2^{1/2} \Gamma(\alpha) \Gamma(\beta)} G_{1,1.3}^{3.0} \left(x, \frac{\xi^2}{\xi^2 + 1}, \xi^2, \alpha, \beta \right) \tag{29}
\]
where \( G_{p,q}^{m,n}(\cdot) \) is the Meijer’s G-function.

III. PERFORMANCE ANALYSIS

A. Outage Probability Analysis

The outage probability (OP) is interpreted as the probability that the end-to-end SNDR \( \gamma_{mi} \) falls below a certain outage threshold \( \gamma_{th} \). It can be defined as:
\[
P_{\text{out}}(\gamma_{th}) \triangleq \text{Pr}[\gamma_{mi} < \gamma_{th}] \tag{30}
\]
where \( \text{Pr}[\cdot] \) is the probability measure notation.

Substituting the expression of the overall SNDR (8) in Eq. (30) and after some mathematical manipulations, the OP can be derived as follows:
\[
P_{\text{out}}(\gamma_{th}) = 1 - \frac{2^{\alpha + \beta - 2\xi^2 m}}{\pi \Gamma(\alpha) \Gamma(\beta)} \left(\frac{N}{m}\right) \sum_{n=0}^{N} \left(-1\right)^n \left(\frac{m}{n}\right) \times \exp\left(-\frac{(N - m + n + 1)\kappa_{\gamma_{th}}}{((N - m + n)(1 - \rho) + 1)\gamma_1}\right) \times \frac{\gamma_1^{\gamma_0}}{16((N - m + n)(1 - \rho) + 1)\gamma_1^{\gamma_0}} \left|\frac{\kappa_{\gamma_{th}}}{\gamma_2}\right| \tag{31}
\]
where $\zeta$, $\kappa_1$ and $\kappa_2$ are given by:

$$\zeta = \mathbb{E} [\gamma_{(m)}] + \kappa, \quad \kappa_1 = \Delta(2 : \xi^2 + 1)$$

$$\kappa_2 = \Delta(2 : \xi^2 + 1), \Delta(2 : \alpha), \Delta(2 : \beta), 0$$

$$\Delta(j : x) \triangleq x/j, \ldots, (x + j - 1)/j$$

(32)

To get further insight into the behavior of the outage system at high SNR, we derive a simpler form of an asymptotic expression using the expansion of the Meijer’s G-function as follows:

$$P_{out}^{\xi}(\gamma_{th}) \approx 1 - \frac{2^{\alpha+\beta-3}m^2}{\pi \Gamma(\alpha) \Gamma(\beta)} \frac{N}{m} \sum_{n=0}^{m-1} \frac{(-1)^n (m-n)}{N - m + n + 1}$$

$$\times \exp \left( - \frac{(N-m+n+1)\kappa_{th}}{((N-m+n)(1-\rho)+1)} \right)$$

$$\times \sum_{r=1}^{\gamma} \prod_{j=1,j \neq r}^{\gamma} \Gamma(\kappa_{2,j} - \kappa_{2,r})$$

$$\times \left( \frac{\alpha \beta h^2}{16((N-m+n)(1-\rho)+1)} \right)^{\kappa_{2,r}}$$

(33)

B. Bit Error Probability Analysis

For most binary modulations, BEP can be defined as:

$$P_e = \frac{\delta \tau}{2 \Gamma(\tau)} \int_0^\infty \gamma^{\tau-1} e^{-\delta \gamma} F_\gamma(\gamma) d\gamma$$

(34)

Replacing the expression of the OP (31) in Eq. (34) and after some algebraic manipulations, BEP can be written as:

$$P_e = \frac{1}{2} - \frac{2^{\alpha+\beta-4}m\zeta^2}{\pi \Gamma(\alpha) \Gamma(\beta) \Gamma(\tau)} \frac{N}{m} \sum_{n=0}^{m-1} \frac{(-1)^n (m-n)}{N - m + n + 1}$$

$$\times \left( \frac{\delta}{\rho \kappa + \delta} \right) \sum_{r=1}^{\gamma} \prod_{j=1,j \neq r}^{\gamma} \Gamma(\kappa_{2,j} - \kappa_{2,r}) \left( \frac{(\alpha \beta h^2) \zeta^2}{16(\rho \kappa + \delta)^{\gamma_2}} \right)$$

(35)

Note that $\varrho$ is given by:

$$\varrho = \frac{(N-m+n+1)}{((N-m+n)(1-\rho)+1)}$$

(36)

The high SNR asymptotic expression of the BEP can be derived using the expansion of the Meijer’s G-function as follows:

$$P_e^\infty \approx \frac{1}{2} - \frac{2^{\alpha+\beta-4}m\zeta^2}{\pi \Gamma(\alpha) \Gamma(\beta) \Gamma(\tau)} \frac{N}{m} \sum_{n=0}^{m-1} \frac{(-1)^n (m-n)}{N - m + n + 1}$$

$$\times \sum_{r=1}^{\gamma} \prod_{j=1,j \neq r}^{\gamma} \Gamma(\kappa_{2,j} - \kappa_{2,r}) \left( \frac{\delta}{\rho \kappa + \delta} \right) \left( \frac{(\alpha \beta h^2) \zeta^2}{16(\rho \kappa + \delta)^{\gamma_2}} \right)$$

(37)

C. Ergodic Capacity Analysis

The average ergodic capacity, expressed in bps/Hz, can be defined as the maximum error-free data rate transmitted by the overall system channels. Considering IM/DD detection, the ergodic capacity can be obtained by:

$$C_e = \mathbb{E} \left[ \log_2 \left( 1 + \frac{e\varrho^2}{2\pi} \right) \right]$$

(38)

Using the integration by part, the ergodic capacity can be formulated as follows:

$$C_e = \frac{e}{2 \pi \log(2)} \int_0^\infty F_\gamma(\gamma) \frac{d\gamma}{1 + \frac{e\varrho^2}{2\pi}}$$

(39)

where $F_\gamma(\gamma)$ is the complementary CDF (CCDF). Substituting the CCDF of (31) in Eq. (39) and after some mathematical manipulations, the ergodic capacity can be derived as follows:

$$C_e = \frac{2^{\alpha+\beta-4}m\zeta^2}{\pi \Gamma(\alpha) \Gamma(\beta) \log(2)} \frac{N}{m} \sum_{n=0}^{m-1} \frac{(-1)^n (m-n)}{N - m + n + 1}$$

$$\times \left( \frac{(\alpha \beta h^2) \zeta^2}{16((N-m+n)(1-\rho)+1)} \right)$$

$$\times H^{\varrho^2}_{(0,1),1,1,2,7,0} \left( \frac{1}{(0,1,1)} \bigg| \begin{array}{c} \kappa_1, 1 \gamma_1 \\ \kappa_2, 1 \gamma_2 \end{array} \right) \left( \frac{\varrho^2}{e} \right)^{\kappa_{2,r}}$$

(40)

where $H^{m_1,n_1,m_2,n_2,m_3,n_3}_{p_1,q_1,p_2,q_2,p_3,q_3}(-[\cdot,\cdot])$ is the bivariate Fox H-function and $|x_j|$ is the vector containing $j$ elements equal to $x$.

Since the closed-form formula provides limited engineering insights, we derive a high SNR expression of the ergodic capacity. Substituting the CCDF of (33) in Eq. (39), the asymptotic channel capacity can be expressed as follows:

$$C_e^\infty \approx \frac{2^{\alpha+\beta-4}m\zeta^2}{\pi \Gamma(\alpha) \Gamma(\beta) \log(2)} \frac{N}{m} \sum_{n=0}^{m-1} \frac{(-1)^n (m-n)}{N - m + n + 1}$$

$$\times \sum_{r=1}^{\gamma} \prod_{j=1,j \neq r}^{\gamma} \Gamma(\kappa_{2,j} - \kappa_{2,r})$$

$$\times \left( \frac{\delta}{\rho \kappa + \delta} \right) \left( \frac{(\alpha \beta h^2) \zeta^2}{16(\rho \kappa + \delta)^{\gamma_2}} \right)$$

(41)

where $\Gamma(\cdot,\cdot)$ is the incomplete upper gamma function and $\omega$ is given by:

$$\omega = \frac{(\alpha \beta h^2) \zeta^2 (N - m + n + 1) \zeta}{16((N - m + n)(1 - \rho) + 1)^{\gamma_2}}$$

(42)

Since the relays are impaired, we can also derive a ceiling in terms of the impairment components that limits the capacity as the impairment becomes more severe. This ceiling is given by [3 Eq. (37)]:

$$C_c = \log_2 \left( 1 + \frac{e\varrho^2}{2\pi(\mu - \nu^2)} \right)$$

(43)
IV. NUMERICAL RESULTS AND DISCUSSION

In this section, the derived analytical expressions are compared with the numerical results using Monte Carlo simulations. Correlated Rayleigh channel coefficients are generated using (1). The atmospheric turbulence $I_a$ is generated using the expression $I_a = I_{ax} \times I_{ay}$, where the two independent random variables $I_{ax}$ and $I_{ay}$ follow the Gamma distribution. In addition, the pointing error is simulated by generating the radial displacement $R$ following the Rayleigh distribution and then by applying Eq. (19). Since the path loss is deterministic, it can be generated using the relation (18). The table below presents the simulation parameters.

| Parameter  | Value     |
|------------|-----------|
| $L$        | 1 km      |
| $\lambda$  | 1550 nm   |
| $P_0$      | -10 m     |
| $u_0$      | 5 cm      |
| $w_0$      | 5 mm      |
| $\rho$     | 0.9       |
| Modulation | CBFSK     |

Fig. 1: Outage probability vs. average SNR for different weather conditions and for $C_n^2 = 5 \times 10^{-14}$, $N = m = 5$, $\sigma_s = 3.75$ cm, IBO = 30 dB.

Fig. 2: Bit error probability versus the average SNR for weak and moderate atmospheric turbulences and for ($N = m = 2$, $N = m = 8$, clear air ($\sigma = 0.43$ dB/km), $C_n^2 = 5 \times 10^{-14}$, $\sigma_s = 3.75$ cm, IBO = 30 dB).

Fig. 3: Ergodic capacity versus the average SNR for different values of the pointing error coefficients and for ($N = m = 3$, clear air ($\sigma = 0.43$ dB/km), $C_n^2 = 5 \times 10^{-14}$, IBO = 30 dB).

TABLE II: Simulation Parameters

Fig. 1 shows the variations of the outage probability with respect to the average SNR for various weather conditions. Clearly, we see that the system works better for clear air. However, as the weather changes from a hazy to a rainy situation, the outage performance deteriorates. For an average SNR of 40 dB, the system achieves the following outage values $10^{-5}, 5 \times 10^{-4}, 3 \times 10^{-3}$ and 0.2 for clear air, hazy, moderate and heavy rain, respectively. In fact, the weather attenuation loss comes from the scattering of the signal due to the atmospheric particles. For clear air, the weather is quiet and the scattering loss is negligible or small. Given that the high frequency signals are greatly disturbed by the fog, clouds and dust particles, FSO signal depends not only on the rain which is the major attenuating factor but also on the rate of the rainfall as shown by the figure. In fact, the rain droplets cause a substantial scattering in different directions that mainly attenuate the signal power during the propagation and this phenomena can be explained in more details according to the Rayleigh model of scattering.
under the moderate atmospheric turbulences. Fig. 3 provides the variations of the ergodic capacity against the average SNR for different values of the pointing error coefficients. We observe that the system works better as the pointing error coefficient decreases. In fact, as this coefficient \( \xi \) decreases, the pointing error effect becomes more severe. For a given average SNR of 30 dB, the system capacity achieves the following rates 1, 3.9, 7 and 8 bps/Hz for the pointing error coefficients equal to 0.2, 0.4, 0.7 and 0.9, respectively. Thereby, the ergodic capacity gets better as the pointing error coefficient becomes higher.

Fig. 4 shows the variations of the ergodic capacity versus the average SNR for different values of IBO and for \((N = m = 3, \text{clear air} \ (\sigma = 0.43 \text{ db/km}), \ C_n^2 = 5 \ 10^{-14}, \ \sigma_s = 3.75 \text{ cm})\).

Fig. 4: Ergodic capacity versus the average SNR for different values of IBO and for \((N = m = 3, \text{clear air} \ (\sigma = 0.43 \text{ db/km}), \ C_n^2 = 5 \ 10^{-14}, \ \sigma_s = 3.75 \text{ cm})\).

V. Conclusion

In this work, we investigate a mixed RF/FSO system with multiple relays under the hardware impairments. We derive novel closed-forms of the outage probability, bit error probability, ergodic capacity and high SNR approximation. We evaluate the outage performance in various weather conditions from clear air to more severe state such as heavy rain. We conclude that this performance deteriorates as the weather becomes more severe since the scattering loss increases as the weather worsens. The bit error probability is studied under weak and moderate turbulences for different numbers of relays and it turns out that the system works better especially for large numbers of relays under weak turbulences. Additionally, the system capacity is very sensitive to the pointing error as the coefficient becomes very low. We also quantify the impact of the hardware impairments on the ergodic capacity in terms of the IBO values. We conclude that the capacity saturates more by the impairment ceilings as the IBO decreases.

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REFERENCES

[1] P. V. Trinh, T. C. Thang, and A. T. Pham, “Mixed mmWave RF/FSO Relaying Systems Over Generalized Fading Channels With Pointing Errors,” IEEE Photonics Journal, vol. 9, no. 1, pp. 1–14, Feb 2017.

[2] C. Hoymann, W. Chen, J. Montoto, A. Golitschek, C. Koutsiminis, and X. Shen, “Relaying operation in 3GPP LTE: challenges and solutions,” IEEE Communications Magazine, vol. 50, no. 2, pp. 156–162, February 2012.

[3] E. Balti, M. Guizani, B. Hamdaoui, and Y. Maalej, “Partial Relay Selection For Hybrid RF/FSO Systems with Hardware Impairments,” in 2016 IEEE Global Communications Conference, Washington, USA, Dec. 2016.

[4] E. Balti, M. Guizani, and B. Hamdaoui, “Hybrid Rayleigh and Double-Weibull over Impaired RF/FSO System with Outdated CSI,” in IEEE ICC ’2017 Mobile and Wireless Networking (ICC’17 MWN), Paris, France, May 2017, pp. 4771–4776.

[5] N. Maletic, M. Cabarkapa, and N. Neskovic, “Performance of fixed-gain amplify-and-forward nonlinear relaying with hardware impairments,” International Journal of Communication Systems, 2015.

[6] N. Varshney and P. Puri, “Optimal power allocation for decode-and-forward based mixed MIMO-RF/FSO cooperative systems,” in 2016 International Conference on Signal Processing and Communications (SPCOM), June 2016, pp. 1–5.

[7] C. Studer, M. Wenk, and A. Burg, “MIMO transmission with residual transmit-RF impairments,” in 2010 International ITG Workshop on Smart Antennas (WSA), Feb 2010, pp. 189–196.

[8] H. Bouhadda, H. Shaiel, D. Roviras, R. Zayani, Y. Medjahdi, and R. Bouallegue, “Theoretical analysis of BER performance of nonlinearly amplified FBMC/OQAM and OFDM signals,” EURASIP Journal on Advances in Signal Processing, vol. 2014, no. 1, p. 60, 2014.

[9] D. Dardari, V. Tralli, and A. Vaccari, “A theoretical characterization of nonlinear distortion effects in OFDM systems,” IEEE Transactions on Communications, vol. 48, no. 10, pp. 1755–1764, Oct 2000.

[10] E. Bjornson, M. Matthaiou, and M. Debbah, “A New Look at Dual-RF/FSO Hop Relaying: Performance Limits with Hardware Impairments,” IEEE Transactions on Communications, vol. 61, no. 11, pp. 4512–4525, November 2013.

[11] N. Y. Ermolova and S. G. Haggman, “An extension of Bussgang’s theory to complex-valued signals,” in Proceedings of the 6th Nordic Signal Processing Symposium, 2004. NORSIG 2004., June 2004, pp. 45–48.

[12] G. T. Djordjevic, M. I. Petkovic, A. M. Cvetkovic, and G. K. Karagiannidis, “Mixed RF/FSO Relaying With Outdated Channel State Information,” IEEE Journal on Selected Areas in Communications, vol. 33, no. 9, pp. 1935–1948, Sept 2015.