ASURA: Scalable and Uniform Data Distribution Algorithm for Storage Clusters

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Abstract—Large-scale storage cluster systems need to manage a vast amount of information denoting combinations of data identifiers (IDs) and corresponding nodes that store the data. Management using data distribution algorithms, rather than management using tables, has been proposed in recent research. In algorithm management, data are distributed in accordance with a data distribution algorithm that is capable of determining, on the basis of the datum ID, the node in which the required data is being stored. Among the requirements for a data distribution algorithm are short calculation times, low memory consumption, uniform data distribution in accordance with the capacity of each node and the ability to handle the addition or removal of nodes. This paper presents a data distribution algorithm called ASURA (Advanced Scalable and Uniform storage by Random number Algorithm), which satisfies these requirements. It offers roughly 0.6-μs calculation time, kilobyte-order memory consumption, less than 1% maximum variability between nodes in data distribution, data distribution in accordance with the capacity of each node and optimal data movement to maintain data distribution in accordance with node capacity when nodes are added or removed. ASURA is contrasted in this paper qualitatively and quantitatively with representative data distribution algorithms: Consistent Hashing and Straw Buckets in CRUSH. The comparison results show that ASURA can achieve the same storage cluster capacity as Consistent Hashing with dozens fewer nodes by virtue of the uniformity of its distribution with the same level calculation time. They also show that the execution time of ASURA is shorter than that of Straw Buckets in CRUSH. The results reveal that ASURA is the best algorithm for large-scale storage cluster systems.

Index Terms—Algorithm, Distributed data structure, Data storage representations

Growth in size of data managed by computers is leading to increased storage system capacity. Recent capacities cannot be achieved with one storage node or a few storage nodes. Thus, storage cluster technologies and distributed storage technologies, which manage many storage nodes as one storage system, are urgently required. Each node in a storage system knows all the nodes in the storage cluster and each node in a storage system knows some of the nodes in a distributed storage system (peer-to-peer (P2P) system). Storage clusters are the focus of this paper. Because applications need to know data-node correspondences for the purpose of access, all combinations of data identifiers (IDs) and data-storing nodes must be managed. When data need to be accessed, the node storing them is determined on the basis of the relevant datum ID.

There are three such types of combination management: table management, algorithm management and a mixture of both. In table management, combinations of data IDs and data-storing nodes are memorized in a management table. When a datum is to be accessed, the relevant datum ID is searched for in the management table, and the ID offers access to the corresponding node. Table management is easy to implement, but it requires a large table for lots of data. Sharing and synchronizing such a large table are also enormous tasks. For example, the management of 10 petabytes of data stored in 1 megabyte data units would require 10 billion entries in the management table, and the 10 billion entries would require 80 gigabytes of space to store them (assuming 1 entry required 8 bytes for storage). Such a large table would require huge amounts of memory, storage and network bandwidth. Furthermore, if only management nodes know that table, every node that accesses the data must access the management nodes in order to access the data. Thus, management nodes become a bottleneck. An example of the table management system is the Google File System [1].

In contrast to this, algorithm management does not need to manage and synchronize such a large table. However, the difficulty in algorithm management lies in algorithm design. The algorithm must be able to determine the node corresponding to any given datum ID. It must also offer low resource usage, good load balancing in accordance with capacity of each node and the ability to handle the addition or removal of nodes. Since it has no need to memorize data-node combinations, it only requires a small table that memorizes node information, and it only needs to share this small table among all the nodes that determine which nodes are data-storing nodes. Examples of algorithm management systems are PanFS [2], Dynamo [3] and Ceph [4].

The size of the table in the table management approach is too large for practical use with large-scale storage clusters. Therefore, management using an algorithm, whether totally or partially, is the only choice for huge storage.

An ideal algorithm will first offer uniform data distribution on each node to achieve both full use of all nodes and good load balancing. It is better that data are distributed among nodes in accordance with the capacity of each node. Secondly, it is better that only a small number of data movements are required to maintain appropriate data distribution among nodes when such changes are being made in order to enable efficient node additions and removals because of the frequent need for repairs and extensions. In particular, optimal data movement is the best in such a case. Thirdly, calculation time is also a significant

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consideration. Since an algorithm for storage clusters will be used in network communications, calculation times for high-speed access need to be considerably shorter than the network communication time. A final important consideration is memory consumption. Low memory consumption and a small program size are required for the algorithm because of the limitations on system resources.

This paper focuses on an ideal algorithm that satisfies uniform data distribution, optimal data movement when nodes are added and removed and the coexistence of scalability and efficiency in theory. Ideal algorithms use pseudorandom functions and they achieve pseudo-equal data distribution. The first is Consistent Hashing [5]. Its algorithm distributes data by using a number-line ring. The Consistent Hashing algorithm is used for storage clusters, but it can also be used for distributed storage. The second algorithm is Straw Buckets in CRUSH [6].

Its algorithm distributes data by using the hash numbers of node IDs and data IDs. CRUSH has four types of algorithms. However, only Straw Buckets satisfies optimal data movement when nodes are added and removed.

This paper proposes a third choice, namely ASURA, which stands for Advanced Scalable and Uniform storage by Random number Algorithm. In ASURA, data-storing nodes are determined directly by means of a special algorithm that uses pseudorandom numbers.

This paper makes two main contributions.

- It proposes ASURA and explains how it can distribute data according to the capacity of each node in any case and can maintain appropriate data distribution with minimum data movement when nodes are added or removed.
- It presents my evaluation of ASURA in comparison with Consistent Hashing and Straw Buckets in CRUSH, both qualitatively and quantitatively, under equal conditions.
- The evaluation found that:
  1. The calculation time of Consistent Hashing was under 1 μs, while that for ASURA was about 0.6 μs. Both are short enough for use in network communications. The calculation time of Straw Buckets in CRUSH was significantly longer with a large storage cluster. Therefore, Straw Buckets in CRUSH was not evaluated in the following tests.
  2. Memory consumption with ASURA was on the kilobyte order for node numbers on the order of thousands. ASURA’s memory consumption was 1/100 to 1/10,000 that of Consistent Hashing (depending on the number of virtual nodes in Consistent Hashing).
  3. Maximum variability in data distribution with ASURA was 0.32% in the best case, where its data distribution was 10 times more uniform (maximum variability: 1/10) than that with Consistent Hashing. Storage systems using ASURA for distribution algorithm can reduce the number of nodes by from 3% to 33.1% (depending on the virtual node number of Consistent Hashing) compared with a storage system using Consistent Hashing for the distribution algorithm because of the uniformity of the data distribution.

- In an easy test in a real environment, ASURA achieved 28% more uniform data distribution with the same level calculation time as Consistent Hashing. This means that a storage system using ASURA needs 28% fewer storage nodes for the same storage capacity as a system using Consistent Hashing. And the execution time of ASURA is shorter than that of Straw Buckets in CRUSH.

- Considered overall, ASURA is a better algorithm than Consistent Hashing and Straw Buckets in CRUSH for large-scale storage clusters, which means that ASURA is the best algorithm for large-scale storage cluster systems.

Section 1 of this paper presents related work and Section 2 introduces ASURA. Section 3 discusses my qualitative evaluations of the three algorithms and Section 4 discusses my quantitative evaluations of them. Section 5 is a discussion section. Section 6 provides a brief summary of the work presented here.

I. RELATED WORK

This section focuses on distribution algorithms for storage and introduces them. There are useful algorithms that achieve near-optimal data movement when nodes are added or removed [6] [14]. However, this section focuses on algorithms that achieve optimal data movement when nodes are added or removed for clarity of the paper claim.

“RUSH: Balanced, Decentralized Distribution for Replicated Data in Scalable Storage Clusters” [7] by Honicky and Miller can distribute data uniformly. And it achieves optimal data movement when nodes are added and removed. However, it requires the maximum node number to be determined in advance. Thus, if the maximum node number is small, it lacks scalability and if the maximum node number is large, it lacks efficient processing.

Karger et al.’s Consistent Hashing in “Consistent Hashing and Random Trees: Distributed Caching Protocols for Relieving Hot Spots on the World Wide Web” [5] is a hash-number-based algorithm for managing data arrangements among a large number of nodes. Nodes were arranged on a ring formed by a number line. The region extending along the ring from a given node to the nearest node to it in a given direction was owned by that given node. Each individual data item had its own hash number, which was also located on the ring. The owner of that point on the ring was the data-storing node (Fig. 1). Data distribution was not uniform in the Consistent Hashing algorithm. Thus, a virtual node technique had to be applied to that algorithm to achieve uniform distribution. Each node in Consistent Hashing without virtual nodes had one hash number. However, in Consistent Hashing with virtual nodes, each node has many hash numbers and the nodes are arranged in many places on the ring of the number line. The arranged nodes are called virtual nodes. The areas of the nodes become coextensive as the number of virtual nodes increases, in accordance with the law of large numbers. Consistent Hashing with virtual nodes achieves both uniform data arrangement and optimal data movement when nodes are added or removed. This makes it a nearly ideal algorithm. In addition, Consistent Hashing can be used for distributed storage. If a node in the Consistent Hashing algorithm knows only some of the nodes,
the node can determine a node that is better (i.e., nearer to the right node) than itself. Some algorithms [8] [9] [10] for searching for a data-storing node in a system consisting of nodes having information about some of the nodes have been presented.

Figure 1. Consistent Hashing

Weil et al.’s Straw Buckets in CRUSH (in “CRUSH: Controlled, Scalable, Decentralized Placement of Replicated Data”[6]) is also a nearly ideal algorithm. With it, each node had an individual hash number for an individual datum item, and data were stored in the node having the largest hash number for the data (Fig. 2). Data in this algorithm were distributed uniformly and optimal movement of data was achieved to maintain a uniform distribution when nodes were added or removed. CRUSH had four types of algorithms. Despite each algorithm having advantages and disadvantages, this paper focused only on Straw Buckets because it is the only one that achieved optimal data movement when nodes were added or removed.

Figure 2. Straw Buckets in CRUSH

Chawla et al.’s “Semantics of caching with SPOCA: a stateless, proportional, optimally-consistent addressing algorithm” [11] also achieves optimal data movement when nodes are added and removed. The system using SPOCA assigns nodes with segments in a line. And the system generates a hash number on this line based on a datum ID until the hash number hits the segment in the line. Then the node assigned to the segment pointed to by the hash number is a data-storing node. SPOCA can distribute data in accordance with the capacity of each node, and it achieves optimal data movement when nodes are added and removed. However, SPOCA suffers from a trade-off between scalability and efficiency because the length of the line used by SPOCA is determined in advance. ASURA is similar to SPOCA. However, ASURA supports scalability and efficiency at the same time.

All three algorithms (Consistent Hashing, Straw Buckets in CRUSH and ASURA) achieve nearly uniform data distribution and they also achieve optimal data movement to maintain appropriate data distribution when nodes are added or removed. Futhermore, they achieve unlimited scalability with efficiency in theory. This paper contrasts them both qualitatively and quantitatively.

II. ASURA

This section introduces the operations of ASURA.

The pseudocode of ASURA is given in Appendix A. It takes as input a datum ID, the maximum segment number plus 1 and the length of each segment. And it outputs the segment number assigned to a data-storing node.

Sections 2.A, 2.B and 2.C explain the algorithm implemented in this pseudocode. First, I explain the basic form of ASURA. Second, I describe how to solve the scalability problem in the basic ASURA by using ASURA random numbers. Third, I show how to generate ASURA random numbers. Section 2.D shows the treatment of node addition and removal in ASURA.

A. Basic ASURA

This section shows how to achieve appropriate data distribution in accordance with the nodes’ capacity and optimal data movement when nodes are added or removed to keep appropriate data distribution in accordance with the nodes’ capacity. The basic algorithm in this section is very similar to SPOCA [11].

ASURA’s algorithm can be divided into two steps.

STEP 1. Nodes are assigned to segments in a number line.

STEP 2. A data-storing node is determined.

STEP 1 is the initial step. This step is executed when the system starts and nodes are added or removed. In this step, nodes are assigned to segments in a number line according to the following rules.

- Node_A 1.5 TB
  - Assigned segment(s): Segment_0
  - (not less than 0.0 and less than 1.0)
- Node_B 0.7 TB
  - Assigned segment(s): Segment_2
  - (not less than 2.0 and less than 2.5)
- Node_C 1.0 TB
  - Assigned segment(s): Segment_1
  - (not less than 3.0 and less than 3.7)
  - (not less than 1.0 and less than 2.0)
1. Nodes are assigned to a segment or segments in the number line according to the nodes’ capacity.
   A) One node can be assigned to multiple segments.
2. The correspondence among the existing nodes and the segments does not change.
3. A segment starts from the point of an integer on the number line. The segment number is the number of the starting point.
4. The segment length is under 1.0.

Rules 3 and 4 are not essential, but thanks to them, it is easy to make code and the code become efficient.

Fig. 3 is an example.

STEP 2 determines a data-storing node. This step is executed when a data-storing node is decided.

The algorithm of STEP 2 is as follows.
1. A pseudorandom number generator is initialized by the datum’s ID.
2. A random number is generated.
3. If this random number does not point to a segment in the number line, go to 2.
4. If this random number points to a segment in the number line, a node assigned to the pointed segment is a data-storing node.

In this phase, random numbers are generated repeatedly until the random number points to the segment.

The pseudorandom number generator has following characteristics.
1. If the seed (datum ID) is the same, the same random number sequence is generated.
2. If the seed (datum ID) is not the same, a superficially different random number sequence is generated.
3. Random numbers in the random number sequence are nearly homogeneously distributed.

For example, data-storing nodes for data whose IDs are A and B are determined when nodes are assigned to segments in Fig. 3. If random numbers, which are generated by a pseudorandom number generator initialized by A and B, are as follows, the datum whose ID is A is stored in Node_B, which is assigned to Segment_3.

For example, data-storing nodes for data whose IDs are A and B are determined when nodes are assigned to segments in Fig. 3. If random numbers, which are generated by a pseudorandom number generator initialized by A and B, are as follows, the datum whose ID is A is stored in Node_B, which is assigned to Segment_3.

Random numbers for the datum whose ID is A: 4.2
Random numbers for the datum whose ID is B: 3.3, 0.6…

In this algorithm, the following characteristics are achieved.
1. Data are distributed to nodes, nearly in accordance with each node’s capacity.
2. When nodes are added, only data that will be stored in added nodes move. And data are distributed to nodes in accordance with each node’s capacity. This is optimal data movement in node addition.
3. When nodes are removed, only the data that were stored in the removed nodes move. And data are distributed to nodes in accordance with each node’s capacity. This is optimal data movement in node removal.

The first characteristic is obvious. A data-storing node is assigned to a segment pointed to by a random number. Thus, the probability that each node is selected as the data-storing node depends on the segment lengths assigned to each node. And the segment lengths depend on the node’s capacity. So data are distributed to nodes in accordance with each node’s capacity.

The mathematical proof is given below:
\[ R = \{r_1, r_2, \ldots, r_N\} : \text{generated random numbers} \]
\[ r_N : \text{first random number that hits a segment} \]
\[ S : \text{numbers covered by segments} \]
\[ S \cap R = r_N \]

If a new segment \((S + S) \cap R = \{r_M, r_N\}\) is added, then
\[(S + S) \cap R = \{r_M, r_N\}, \text{ then} \]
\[(S + S) \cap R = S \cap R + S \cap R = \{r_N\} + s \cap R \]
\[= \{r_N\} + s \cap R \]
\[\vdash (S + S) \cap R = \{r_M, r_N\} \text{ and } R = \{r_1, r_2, \ldots, r_N\} \]
\[\vdash s \cap R = r_M \text{ and } M < N \]

This means that the first random number that hits a
segment is changed and the datum moves to the added node.

If \((S + s) \cap R = r_N\), then
\[
(S + s) \cap R = S \cap R + s \cap R = \{r_N\} + s \cap R
\]
\[\therefore (S + s) \cap R = \{r_N\} \text{ and } S \cap s = \phi\]
\[\therefore s \cap R = \phi\]

This means that the first random number that hits a segment is not changed and the datum stays in the current node.

The third characteristic also depends on repeatedly generating random numbers until one points to a segment. In Fig. 4(b), Node_B, which is assigned to Segment_3, is removed from the case in Fig. 3. In this case, if a random number in the random number sequence pointed to Segment_3 in Fig. 3 case before a random number in the random number sequence pointed from Segment_0 to Segment_2 in STEP 2, the datum moves to another node. If not, the datum does not move. And data are distributed to nodes in accordance with each node’s capacity after Node_B has been removed. This means that only minimal data move from removed nodes in order to keep data distribution in accordance with each node’s capacity. This is optimal data movement when nodes are removed.

For example, data-storing nodes for data whose IDs are A and B are determined when nodes are assigned to segments in Fig. 4(b). Because random numbers, which are generated by a pseudorandom number generator initialized by A and B, are unchanged, the datum whose ID is A stays in Node_C and the datum whose ID is B is moved from Node_B to Node_A, which is assigned to Segment_0.

The mathematical proof is given below:

\[
R = \{r_1, r_2 \ldots r_N\} : \text{generated random numbers}
\]
\[
r_N : \text{first random number that hits a segment}
\]
\[
S : \text{numbers covered by segments}
\]
\[
S \cap R = r_N
\]

If one segment \((s)\) in \(S\) is removed,
\[
R' = \{r_1, r_2 \ldots r_M\} : \text{generated random numbers}
\]
\[
r_M : \text{first random number that hits a segment after } s \text{ has been removed (} M \geq N\}
\]
\[
(S - s) \cap R' = r_M
\]

If \(s \cap r_N = \phi\), then
\[
(S - s) \cap R' = S \cap R' - s \cap R' = \{r_N\} - s \cap r_N = \phi \text{ and } M \geq N
\]
\[\therefore r_M = r_N
\]

This means that the datum stays in the current node if the first random number hits a segment that is not assigned to the removed node before the node is removed.

If \(s \cap r_N = r_N\), then
\[
(S - s) \cap R' = S \cap R' - s \cap R' = \{r_N\} - s \cap r_N = \phi
\]
\[\therefore r_M \neq r_N
\]

This means that the datum moves to another node if the first random number hits a segment assigned to a removed node before the node is removed.

In conclusion, ASURA achieves appropriate data distribution in accordance with nodes’ capacity and optimal data movement when nodes are added or removed to keep appropriate data distribution in accordance with nodes’ capacity.

However, the basic ASURA lacks either scalability or efficiency. If the maximum random number is small, ASURA lacks scalability; if the maximum random number is large, it lacks efficiency when segments cover a narrow area. The next section mentions how to solve this problem.

B. ASURA random numbers

This section describes ASURA random numbers, which are used in ASURA. ASURA can extend or shrink the range of its random numbers using a special method. ASURA achieves both scalability and efficiency by using ASURA random numbers.

The random number sequence for ASURA has the following characteristics. Thus, ASURA random numbers also need to have same characteristics.

1. If the seed (datum ID) is the same, the same random number sequence is generated.
2. If the seed (datum ID) is not the same, a superficially different random number sequence is generated.
3. Random numbers in the random number sequence are nearly homogeneously distributed.

The first characteristic shows how to extend the random number range.

In the example given in Fig. 3, a random number generator that generates not less than 0.0 and less than 4.0 (shown as 0.0–4.0 in this paper) can be used. An example of the random number sequence is as follows.

Random number sequence: 2.7, 3.8, 1.1 ...

In this random number sequence, Node_C is a data-storing node.

If random numbers, which are 4.0–8.0 and which are generated from a certain seed based on the datum ID, are inserted into this random number sequence while keeping the same generation probability, this example becomes as follows.

Number sequence: 5.4, 2.7, 4.3, 3.8, 7.2, 1.1 ...

In this random number sequence, random numbers that are 0.0–4.0 keep their values and order. Thus, the data-storing node is not changed from Node_C by the insertion of random numbers that are 4.0–8.0 in the case of Fig. 3.

Generally, a data-storing node is not changed when random
random numbers that are outside the assignment area of segments in the number line are inserted. And the selection of a data-storing node assigned to segments in the random number area before new random numbers are inserted is unchanged.

And this number sequence inserted in random numbers that are 4.0–8.0 is also a random number sequence. This number sequence has the following characteristics.

1. If the seed (datum ID) is the same, the same random number sequence is generated.
2. If the seed (datum ID) is not the same, a superficially different random number sequence is generated.
3. Random numbers in the random number sequence are nearly homogeneously distributed.

This means that this number sequence is a random number sequence that can be used for ASURA, so these are ASURA random numbers. When a node assigned to a segment which is in 4.0–8.0 is added, there is the probability of selecting this node as a data-storing node; and the probability of selecting a node assigned to segments in 4.0–8.0 as a data-storing node is the same as the probability of selecting a node assigned to segments in 0.0–4.0 as the data-storing node if the segment length is the same. And this achieves optimal data movement when nodes are added or removed by ASURA’s feature.

The range of random number sequences can be extended by inserting random numbers whose range is outside its number range without side effects.

Shrinking the range of random numbers can be discussed in the same way. The range of random number sequences can shrink by removing numbers that are outside the new range, which covers the assignment area of segments.

The following random number sequence was used in the previous example used in Fig. 3. And the data-storing node is Node C.

Random number sequence: 5.4, 2.7, 4.3, 3.8, 7.2, 1.1 …

If random numbers in 4.0–8.0 are removed, this random number sequence becomes the following. And the data-storing node is also Node C.

Number sequence: 2.7, 3.8, 1.1 …

If random numbers outside the assignment area of segments in the number line are removed from the random number sequence, the data-storing node does not suffer any side effects. And random numbers in the new random number sequence keep the following characteristics.

1. If the seed (datum ID) is the same, the same random number sequence is generated.
2. If the seed (datum ID) is not the same, a superficially different random number sequence is generated.
3. Random numbers in the random number sequence are nearly homogeneously distributed.

This means that this number sequence is a random number sequence that can be used for ASURA. Thus, the range of random number sequences can be shrunk by removing random numbers whose range is outside the new number range.

In conclusion, the range of random number sequences used by ASURA can be extended or shrunk. I call these random numbers ASURA random numbers. If segments cover a wider area, the range of ASURA’s random numbers can be extended for scalability without any side effects. And if segments cover a narrower area, the range can be shrunk for efficiency without any side effects. Overall, ASURA can achieve both scalability and efficiency by using ASURA random numbers. The next section describes how to generate ASURA random numbers.

C. Generation of ASURA random numbers

The algorithm for generating ASURA random numbers is very simple, and it achieves high scalability and efficiency. The pseudocode uses a different method for efficiency, but both are fundamentally the same.

The algorithm for generating ASURA random numbers uses several pseudorandom number generators. Each pseudorandom number generator has the following specifications.

1. It generates a different range of random numbers.
2. The range of random numbers generated by a pseudorandom number generator having a wider range of random numbers inevitably covers the range of random numbers generated by one having a narrower range of random numbers.
3. If the seed is the same, the same random number sequence is generated.
4. If the seed is not the same, a superficially different random number sequence is generated.
5. Random numbers in the random number sequence are nearly homogeneously distributed.

Pseudorandom number generators that have the following characteristics can be used for generating ASURA random numbers.

1. Pseudorandom number generators have the same base pseudorandom number generator.
2. Each base pseudorandom number generator has its own multiplier.
3. Each base pseudorandom number generator has its own hash seed.

An example of the base random number generator is the Mersenne twister [12].

The pseudorandom number generator generates random numbers as follows, for example.

1. It joins characters of the datum ID and characters of its hash seed (the datum ID + a hash seed).
2. It hashes (the datum ID + the hash seed). The result is a seed for the base pseudorandom number generator.
3. It generates a base random number that is 0.0–1.0 by using the base pseudorandom number generator and this seed.
4. It multiplies the base random number by the multiplier.
5. The calculation result is a random number.

Then ASURA random numbers are generated by the following algorithm.

1. A pseudorandom number generator having the narrowest range of random numbers covering all of the segments is selected. This algorithm uses this pseudorandom number generator and other pseudo random number generators with narrower range.
2. It generates a random number.
3. If this random number is in the range of random numbers of the pseudorandom number generator having the
next-widest range of random numbers, the pseudorandom number generator having the next-widest range of random numbers is selected, and the algorithm goes to 2.
4. If this random number is not in the range of random numbers of the pseudorandom number generator having the next-widest range of random numbers or if no pseudorandom number generator has the next-widest range of random numbers, then the random number is an ASURA random number.

As an example, the following case is described.
1. The first range of random numbers is 0.0–4.0.
2. The extended range of random numbers is 0.0–8.0.
3. Twice the extended range of random numbers is 0.0–16.0.

Three pseudorandom number generators are used in this case. The first, which has 4.0 as its multiplier, is used to generate ASURA random numbers that are 0.0–4.0; the second, which has 8.0 as its multiplier, is added to generate ASURA random numbers that are 0.0–8.0; the third, which has 16.0 as its multiplier, is added to generate ASURA random numbers that are 0.0–16.0.

First, let us consider the generation of ASURA random numbers that are 0.0–4.0. In this case, the first pseudorandom number generator is used simply. For example,

ASURA random numbers: 2.7, 3.8, 1.1 ...

Second, let us consider the generation of ASURA random numbers that are 0.0–8.0. In this case, the first and second pseudorandom number generators are used. First, the second one, which has the narrowest range of random numbers in covering all of the segments, generates a random number. If the generated random number is not 0.0–4.0, it is an ASURA random number, and if it is 0.0–4.0, then a random number generated by the first pseudorandom number generator, which has the next-widest range of random numbers, is an ASURA random number. An example is given next.

Random numbers generated by the first pseudorandom number generator: 2.7, 3.8, 1.1 ...

Random numbers generated by the second pseudorandom number generator: 5.4, 3.2, 4.3, 2.2, 7.2 ...

ASURA random numbers: 5.4, 2.7, 4.3, 3.8, 7.2 ...

Second, when nodes are added or removed, some data must be moved from an existing node to another node. Moving data are detected by recalculating the data-storing node. However, these recalculations involve a high processing cost. The cost of this processing can be eased by the following method.

First, a rule for assigning a segment for an additional node must be decided. This rule is simply that a segment having the
smallest segment number larger than 0 is assigned to the additional node. For example, if three nodes are assigned to segments, then:

- Node_A Segment 0 (0.0–1.0)
- Node_C Segment 2 (2.0–2.7)
- Node_E Segment 4 (4.0–4.3)

In this situation, the smallest unused integer larger than 0 is 1. Thus, if Node_B is added, then segment 1, which has 1 as its segment number, is assigned for Node_B.

Next, acceleration in node addition is discussed. In the algorithm for accelerating node addition, a number that is rounded down the smallest number that lies anterior to the finally selected number in ASURA random numbers to integer is remembered as the ADDITION NUMBER. And when a node assigned to a segment having the same segment number as the ADDITION NUMBER is added, the datum moves to the added node or the ADDITION NUMBER is recalculated.

Consider the following example. There are three nodes:
- Node_C, which is assigned to segment_3 (3.0–3.8);
- Node_D, which is assigned to segment_4 (4.0–5.0);
- Node_E, which is assigned to segment_5 (5.0–5.5).

The following random numbers are generated. The datum is stored in Node_C, Node D and Node_E (here, the replication number is 3).

Random numbers: 6.2, 3.3, 1.6, 5.1, 4.9, 8.0, 7.2

The smallest number lying anterior to the finally selected number in ASURA random numbers is 1.6. Thus, 1, which is 1.6 rounded down, is remembered as the ADDITION NUMBER. In this rule, when a node is added, it is assigned to a segment having a segment number 0. In this case, the datum has no effect. Later, when a node is added, this node is assigned to a segment having a segment number 1. In this case, the datum moves to the added node or the ADDITION NUMBER is recalculated, depending on the segment length. And because 1 is the ADDITION NUMBER, the algorithm can detect it correctly.

When the following random numbers are generated and a datum is stored in the same way:

Random numbers: 3.6, 5.4, 4.9, 6.2, 2.1, 1.9, 7.5

No number lies anterior to the finally selected number in ASURA random numbers. In this case, ASURA random numbers are extended beyond their own range until there is an unused number that lies anterior to the finally selected number in ASURA random numbers.

For example, the range of the ASURA random numbers is extended and the following random numbers are generated.

Random numbers: 3.6, 5.4, 12.2, 4.9, 6.2, 13.4, 2.1

In this case, 12, which is 12.2 rounded down to an integer, is remembered. And if 9 nodes are added and these nodes are assigned to segments having segment numbers 0, 1, 2, 6, 7, 8, 9, 10 and 11, respectively, this datum has no effect. Later, if a node is added and it is assigned to a segment having a segment number 12, the datum moves to the added node or the ADDITION NUMBER is recalculated depending on the segment length. And because 12 is the ADDITION NUMBER, the algorithm can detect it correctly.

In conclusion, only one number has to be remembered in order to accelerate node addition.

Next, the acceleration of node removal is discussed. For example, the following random numbers are generated, and a datum is stored in Node_C, which is assigned to segment_3 (3.0–3.8); Node_D, which is assigned to segment_4 (4.0–5.0); and Node_E, which is assigned to segment_5 (5.0–5.5) (here, the replication number is 3).

Random numbers: 6.2, 3.3, 1.6, 5.1, 4.9, 8.0, 7.2

If Node_C (which is assigned to a segment having a segment number 3), Node_D (which has segment number 4), or Node_E (which has segment number 5) is removed, the datum must be duplicated in another node. Furthermore, if other nodes that are assigned to segments having other segment number are removed, the datum has no effect. Thus, the system must memorize three numbers—3 (which is 3.3 rounded down), 4 (which is 4.9 rounded down), and 5 (which is 5.1 rounded down)—as REMOVE NUMBERS to accelerate node removal.

In conclusion, if the replication number is N, then N numbers must be remembered to accelerate the determination of moving data when nodes are removed.

In overall conclusion, when a segment having the smallest segment number larger than 0 is assigned to the additional node, if one number is memorized for each datum, the need to move data or recalculate that number can be judged when a node is added and if N numbers are memorized for each datum, the moving data can be judged when a node is removed.

However, there is room for discussion of combining metadata for each datum. Recalculating the data-storing node is a costly process. However, its cost is small compared with that of transferring data to another node through the network. Thus, the cost of recalculating the data-storing node is rarely a problem. All nodes can recalculate the data-storing node simultaneously. Thus, if the number of nodes is high, the scalability problem hardly occurs. This shows only that the problem of recalculating the data-storing node can be eased.

III. QUALITATIVE EVALUATION

This section discusses the evaluation of Consistent Hashing, Straw Buckets in CRUSH and ASURA, which qualitatively have ideal characteristics for a data distribution algorithm.

A preliminary evaluation confirmed that data are distributed to nodes pseudo-equally by all three algorithms and optimal data movement is achieved to maintain uniform data distribution when nodes are added or removed. Their calculation times and distribution uniformity levels were within expectations in the following evaluation.

A. Premise

The evaluations in this paper focus on scalability because the algorithms are to be used for data management in large-scale storage clusters.

Distribution algorithms were evaluated in terms of four factors: calculation time, memory consumption, distribution uniformity and flexible data distribution.

The following parameters were used:

N: number of nodes
V: number of virtual nodes
B. Calculation time

I first considered Consistent Hashing. The Consistent Hashing algorithm calculates the hash numbers of nodes from node IDs in the initial stage and it sorts them using Quicksort, for example. It then calculates the hash number of a datum in the distribution stage from the datum ID and searches for the relevant data-storing node using a binary tree search, for example. Thus, the order of the calculation time in the initial stage is \( O(NV + NV \times (\log(NV))) = O(NV \times (\log(NV))) \) and that in the distribution stage is \( O(\log(NV)) \). Karger et al. presented an algorithm for which the order of calculation time in the distribution stage is \( O(1) \) [5]. However, the operations of their algorithm are complex and a high cost is involved in retaining the effect of the algorithm when nodes are added or removed. Thus, this paper does not treat it.

Straw Buckets in CRUSH calculates the hash numbers of nodes from data IDs and node IDs, and it searches for the highest hash number for a node from among the hash numbers of all nodes. It can do this search on the fly. Thus, the order of calculation time in Straw Buckets in CRUSH is \( O(N) \).

ASURA assigns segments to nodes in the initial stage. The calculation time for assigning is short enough to be regarded as negligible. It calculates seeds for random numbers from data IDs and node IDs in the distribution stage, and it generates random numbers from the seeds until it points to the segment. As the number of nodes increases, the number of repetitions by the pseudorandom number generator approaches a constant value if the initialization time of the pseudorandom number generator is very short and negligible. The constant value depends on the ratio of random numbers that do not have a corresponding segment (the proof of this is given in Appendix B). Thus, the order of calculation time in ASURA in the distribution stage is \( O(1) \).

The initial stage is executed only once until there are no changes in nodes. Thus, if the initial stage can be executed within in a reasonable period of time, the length of the calculation time will not become a problem.

The distribution stage is executed for each calculation used to determine a data-storing node, which makes the calculation time a critical factor. With Consistent Hashing, the calculation time increases logarithmically with increasing numbers of nodes and virtual nodes. With Straw Buckets in CRUSH, it increases linearly as the number of nodes increases. With ASURA, it approaches a constant value as the number of nodes increases.

With respect to calculation times, Straw Buckets in CRUSH is the worst in terms of scalability, Consistent Hashing comes in second, and ASURA is the best.

C. Memory consumption

In Consistent Hashing, the IDs of nodes and the hash numbers of nodes and virtual nodes in a table must be memorized. Thus, the order of memory consumption is \( O(N) \).

In Straw Buckets in CRUSH, node IDs must be memorized. Thus, the order of memory consumption is \( O(N) \).

In ASURA, node IDs and the segment lengths of each node in a table must be memorized. Thus, the order of memory consumption is \( O(N) \).

With respect to memory consumption, Consistent Hashing is the worst in terms of scalability; ASURA and Straw Buckets in CRUSH are better.

D. Distribution uniformity

In Consistent Hashing, the hash numbers of nodes and virtual nodes on a ring have variability, as do those of data, which means that distribution in Consistent Hashing suffers from double variability. In Straw Buckets in CRUSH, hash numbers have variability, and in ASURA, random numbers have variability, so these two suffer from only single variability. With respect to the distribution uniformity, both Straw Buckets in CRUSH and ASURA are better than Consistent Hashing.

E. Flexible data distribution

Each node has its own limited capacity. Thus, algorithms need to manage various node capacities.

Consistent Hashing can distribute data in accordance with the node’s capacity coarsely by adjusting the virtual node number. Straw buckets in CRUSH can distribute data in accordance with the node’s capacity in a limited case by adjusting the multiplying number. ASURA can distribute data in accordance with the node’s capacity flexibly by adjusting segment lengths.

For flexible data distribution, ASURA is a better algorithm than the others.

F. Qualitative evaluation

The results of my qualitative evaluation are summarized in Table I. Each algorithm was found to have its own advantages and disadvantages.

### IV. QUANTITATIVE EVALUATION

This section discusses my quantitative evaluation of Consistent Hashing, Straw Buckets in CRUSH and ASURA.

It is assumed that capacity of the nodes is fixed because flexible capacity would invite introduce too much complexity for comparison.

A. Premise

| Calculation Time | Memory Consumption | Uniformity of Distribution | Flexible Data Distribution |
|------------------|---------------------|-----------------------------|----------------------------|
| ...               | ...                  | ...                         | ...                        |
| Consistent Hashing | Poor \( O(NV \times (\log(NV))) \) | Poor \( O(NV) \) | Poor Double variability | Medium Coarsely |
| Straw Buckets in CRUSH | Good | Poor \( O(N) \) | Good Single variability | Medium In limited case |
| ASURA | Good \( O(1) \) | Good \( O(N) \) | Good Single variability | Good Flexibly |
Quantitative evaluation poses a considerable problem, i.e., the choice of an algorithm for either hashing numbers or generating random numbers. The results achieved in quantitative evaluation will depend on this choice.

Here, it is necessary to compare hash functions and pseudorandom number generators. Random numbers generated by pseudorandom number generators may be thought of as hash numbers hashed from random number seeds. There are pseudorandom number generators that are carefully optimized to minimize calculation times. These can generate hash numbers rapidly, and the generated hash numbers can be distributed uniformly. This means that pseudorandom number generators can be used for hash functions. The hash numbers and random numbers in this research were generated using the same pseudorandom number generator, which made it possible to make a fair quantitative evaluation of Consistent Hashing, Straw Buckets in CRUSH, and ASURA.

An SIMD-oriented Fast Mersenne Twister (SFMT) [13] was used for the quantitative evaluation. SFMT can generate random numbers within short calculation times, and these random numbers are pseudo-uniformly distributed. The performance of SFMT was sufficient for this evaluation. Its relevant specifications are:

- CPU: Intel Core2Quad Q9650
- Memory: DDR2-800 2 GB x 2
- OS: Ubuntu 10.04 (Kernel 2.6.32)
- Gcc: gcc 4.4.3
- SFMT: dSFMT 2.1 compiled with -O6 -msse2 -march=core2 -DDSFMT_MEXP=521 -DHAVE_SSE2=1 -fno-strict-aliasing

B. Calculation time

This evaluation focused on the practicality of calculation times and scalability. Because the algorithms were to be used in network access, small differences in calculation times were not significant.

I assumed the following:

For all three algorithms —
- The number of nodes ranged from 1 to 1200. The calculation time was evaluated on the basis of execution time over 1,000,000 loops for different inputs.
  - For Consistent Hashing —
    - The numbers of virtual nodes tested were 1, 100 and 10,000. In the initial stage, node hash numbers were random numbers generated by SFMT, and the node hash numbers were sorted using Quick-sort. A data-storing node was searched for in a table in memory in a binary tree search in the distribution stage.
  - For Straw Buckets in CRUSH —
    - Hash numbers for each nodes were generated by SFMT and were compared on the fly with the current maximum number.
  - For ASURA —
    - The random numbers output by the first pseudorandom number generator were 0.0–16.0. The maximum numbers of random numbers output by the following pseudorandom number generators were each doubled. The nodes were numbered consecutively, with no gaps between segment numbers.

The initial stage tests for Consistent Hashing ended within realistic calculation times. That is, the calculation times for the initial stage in Consistent Hashing were not a problem. The same was true for Straw Buckets in CRUSH and ASURA.

The evaluation results for the distribution stage tests are plotted in Fig. 5 (where CH indicates Consistent Hashing, and VN indicates virtual nodes). The calculation time for Consistent Hashing and ASURA was short enough to be practical, even for data transfer in network communications. The calculation time for Straw Buckets in CRUSH was 0.82 μs in one node and 1.65 μs in two nodes, and it extended linearly beyond the graph area. This evaluation demonstrated that Consistent Hashing and ASURA are realistic choices for a large-scale storage cluster and that Straw Buckets in CRUSH suits small-scale storage clusters. For this reason, only Consistent Hashing and ASURA are considered in the following discussion of the evaluation because this evaluation focuses on scalability.

In an additional evaluation, the calculation time for ASURA with 100,000,000 nodes was 0.73 μs. This result indicates that ASURA has remarkable scalability.

C. Memory consumption

It was assumed that node IDs were 4 bytes and that node hashes and node numbers were both 4 bytes.

When the number of nodes was N and the number of virtual nodes was V, Consistent Hashing consumed 8NV bytes and ASURA consumed 8N bytes. However, when the number of nodes was 10,000 and the number of virtual nodes was 100, the memory consumption of Consistent Hashing was 7.6 MB (in the same environment, the memory consumption of ASURA was 78 KB). Thus, the memory consumption of Consistent Hashing with a realistic number of nodes and a realistic number of virtual nodes is acceptable for current computer systems.
The size of the simplest program for Consistent Hashing was 16,506 bytes and the size of the simplest program for ASURA was 19,498 bytes. Both these sizes are practical. Memory consumption is summarized in Table II.

While ASURA is the only choice for a distribution algorithm to be used in an environment with a relatively small memory, Consistent Hashing can be used in current computer environments and in a system with a small number (thousands or tens of thousands) of nodes or virtual nodes.

**D. Distribution uniformity**

Distribution uniformity in Consistent Hashing depends on the number of virtual nodes, and this number was varied in the distribution uniformity tests. With ASURA, distribution uniformity depends on the number of data per node, and this number was varied in the distribution uniformity tests.

The distribution uniformity test conditions for Consistent Hashing and ASURA were as follows.

- **Number of nodes**: 100, 1000, and 10,000
- **Number of virtual nodes for each node** (for Consistent Hashing only): 100, 1000, and 10,000
- **Number of data for each node**:
  - 1000, 3,162, 10,000, 31,622, 100,000, 316,227, and 1,000,000

The test was looped 20 times. The results are plotted in Figs. 6–8. Here, CH indicates Consistent Hashing, and VN indicates virtual nodes. The figures indicate that Consistent Hashing with 100 or 1000 virtual nodes had lower uniformity. Consistent Hashing with 10,000 virtual nodes and ASURA achieved the same level of uniformity (the results indicating maximum variability for 1000 data per node and 100 nodes are considered to have arisen from accidental error). However, ASURA clearly had better uniformity than Consistent Hashing with 10,000 virtual nodes with over 100,000 data per node. If one datum is 1 MB, the 100,000 data amount to roughly 98 GB, which is a small amount of data for current storage nodes. ASURA was able to achieve 0.32% maximum variability in the best case, while Consistent Hashing was only able to achieve 3.3% maximum variability in the best case.

The number of data per node was dominant in Consistent Hashing with 10,000 virtual nodes for fewer than 10,000 data per node, and Consistent Hashing achieved the same level of uniformity as ASURA. The number of virtual nodes was dominant in Consistent Hashing with 10,000 virtual nodes for over 100,000 data per node, and uniformity was clearly less than that for ASURA. This means that there is a limit on uniformity imposed by the number of virtual nodes. While more virtual nodes resulted in better uniformity, Consistent Hashing was unable to achieve better uniformity than ASURA, and more virtual nodes consumed more memory. For 10,000 nodes, 10,000 virtual nodes, and 4-byte node IDs and node hash numbers, 763 MB of memory will be consumed for the node-ID and node-hash-number table. The results revealed that ASURA will have better uniformity than Consistent Hashing in storage.
clusters.

E. Evaluation results

The calculation time for Consistent Hashing in the distribution stage was very short (under 1 μs) in the quantitative evaluation. The increased calculation time due to increasing numbers of nodes and virtual nodes was logarithmic, which indicates that the calculation time for Consistent Hashing is acceptable.

The calculation time for Straw Buckets in CRUSH in the distribution stage was very long. The increase in calculation time due to increasing numbers of nodes was linear in the qualitative evaluation. Thus, Straw Buckets in CRUSH suits small-scale storage clusters.

The calculation time for ASURA in the distribution stage was acceptable (around 0.6 μs) in the quantitative evaluation. The calculation time did not depend on the number of nodes, which indicates that ASURA’s calculation time is acceptable.

The memory consumption for Consistent Hashing was on an acceptable level (megabyte order) in the qualitative evaluation. The increase in memory consumption due to increasing numbers of nodes and virtual nodes was linear, which indicates that Consistent Hashing’s memory consumption is acceptable only for realistic nodes and realistic virtual nodes.

ASURA’s memory consumption was very small (kilobyte order) in the qualitative evaluation. The increase in memory consumption due to increasing nodes was linear, which indicates that ASURA’s memory consumption is acceptable.

From all these results, it seems that Consistent Hashing and ASURA can be used for data distributions among practical numbers of nodes, and that Straw Buckets in CRUSH can be used for small-scale storage clusters.

Consistent Hashing and ASURA, which are scalable algorithms, are compared below.

- The calculation time for both Consistent Hashing and ASURA was clearly smaller than the time for network communication. It was within the range of error.
- The memory consumption for ASURA was significantly lower than that for Consistent Hashing in both the quantitative and qualitative evaluations.
- The data distribution uniformity for ASURA was better than that for Consistent Hashing in both the quantitative and qualitative evaluations. This characteristic of ASURA enables the required number of storage nodes to be reduced dramatically (see discussion in section 5.B).

In summary, from the evaluation of Consistent Hashing, Straw Buckets in CRUSH and ASURA, I conclude that ASURA is the best algorithm for large storage clusters.

V. DISCUSSION

A. Replication

In storage clusters, replication between nodes is a very important issue. Because storage clusters consist of many nodes, the system must consider node failure. Data replication between nodes is commonly used as a solution to node failure.

In Straw Buckets in CRUSH, the algorithm chooses a data-replicating node naturally by selecting the node that has the second highest hash number as a data-replicating node.

However, in Consistent Hashing and ASURA, if the algorithm decides that secondary selected node as a data-replicating node it cannot choose a data-replicating node directly because it might select the same node as both a data-storing node and a data-replicating node. Thus, the system needs to check for duplication between data-storing and data-replicating nodes. This process can be implemented easily.

B. Meaning of uniform distribution

Uniform data distribution reduces the number of nodes required in a storage system.

In a usual storage cluster using algorithm distribution, the storage system stores data in a node indicated by the algorithm. If the node indicated by the algorithm is full, the storage system cannot store data any more. If the storage unit stores the data in another node, the storage system becomes very complex and very inefficient.

When data are added to the storage system using a nonuniform data distribution algorithm, one node becomes full before other nodes have enough data. This means that storage using a nonuniform data distribution algorithm cannot fully utilize the storage capacity and needs more nodes to achieve the same capacity compared with a storage system using a uniform data distribution algorithm.

If the distribution algorithm has 10% maximum variability, a storage system using this distribution algorithm needs an extra 11.1% times the number of nodes compared with one using the ideal distribution algorithm.

A storage system using ASURA, which achieves uniform distribution, can reduce the required number of nodes dramatically compared with one using Consistent Hashing, which cannot achieve uniform distribution.

C. Uniform data distribution and various data sizes and various data access frequencies

Uniform data distribution is meaningful even if data has various sizes or various access frequencies. If data distribution in the storage system is uniform, the storage system suffers from single nonuniformity: of either data size or data access frequency. However, if data distribution in the storage system is nonuniform, the storage system suffers from double nonuniformity: of both data distribution and data size or data access frequency. This causes more nonuniform usage of the data-storing capacity and nonuniform data access. It means that uniform data distribution is meaningful for storage systems storing data having various sizes or data having various access frequencies.

D. Data movement when a node is added and removed

When nodes are added and removed, some data must move to other nodes. In all three algorithms treated in this paper, number of moving data is optimal for keeping uniform distribution. And each node can determine whether or not a datum moves. However, the three algorithms have different characteristics in terms of data movement checking.

In Consistent Hashing, when nodes are added, if the datum
combines with its hash number, than recalculation of the data-storing node is unnecessary. However, remembering the hash number of each datum involves a high cost for system resources. When nodes are removed, if the datum is categorized by its virtual node, recalculation of the data-storing node is unnecessary. This can be achieved easily.

In Straw Buckets in CRUSH, when nodes are added, recalculation does not need, if maximum hash number is remembered. This needs system resource. When a node is removed, the algorithm must check whether the datum is in the removed node or not. For this check, N numbers must be remembered (N is the replication number). But it also needs system resources.

In ASURA, when a node is added, the datum having the ADDITION NUMBER that is the same as the segment number of the added node moves to the added node or the ADDITION NUMBER is recalculated (as discussed in section 2.D). Then, when the node is removed, the datum having REMOVE NUMBERS that are the same as the segment number of the removed node must move to another node. For this process, \( N+1 \) numbers must be remembered. But it needs system resources.

In data movement checking when nodes are added or removed, Consistent Hashing is the easiest, Straw Buckets in CRUSH is the second and ASURA is the third. However, it is rare for data movement checking to be a problem because its required cost is less than that for transferring data through the network, and all nodes can do it simultaneously if there are many nodes.

E. Easy evaluation in actual usage

I evaluated Consistent Hashing, Straw Buckets in CRUSH and ASURA in actual usage. In this evaluation, one real machine wrote 1,000,000 data (datum length: 1 byte) to 100 memcached-1.4.13 in two real machines by using the API of libmemcached-1.0.7, which was modified to be usable in Straw Buckets in CRUSH and ASURA. The number of virtual nodes in Consistent Hashing was set to 100 through reference to an existing system design. And the execution time and data distribution uniformity were evaluated ten times. The results are given in Table III.

| Table III. Actual usage | Execution Time | Maximum Variability |
|-------------------------|----------------|---------------------|
| Consistent Hashing      | 378.04s        | 28.21%              |
| (100 virtual nodes)     |                |                     |
| Straw Buckets in CRUSH   | 492.14s        | 0.31%               |
| ASURA                   | 379.72s        | 0.29%               |

The execution time of Straw Buckets in CRUSH is obviously long. And the times of Consistent Hashing and ASURA are short and similar. The maximum variability of Consistent Hashing is admittedly large. Those of Straw Buckets in CRUSH and ASURA are better and are similar. This means that a storage cluster that uses ASURA needs 28.1% less storage than a storage cluster that uses Consistent Hashing. The results show that ASURA is a better algorithm than Consistent Hashing and Straw Buckets in CRUSH in actual usage.

VI. CONCLUSION

This paper presented ASURA, which can distribute data in accordance with node capacity and can retain uniformity with minimum data movements when nodes are added and removed. It needs limited resources for its execution and it has enormous scalability. This paper contrasted it with similar algorithms, i.e., Consistent Hashing and Straw Buckets in CRUSH. Qualitative and quantitative evaluations were performed to demonstrate their characteristics. Overall, ASURA is the best distribution algorithm for a large storage cluster.

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APPENDIX A PSEUDOCODE OF ASURA

Input:
Datum ID is ID of datum
maximum_segment_number_plus_1 is “a maximum number in segment numbers” plus 1
segment_lengths[] is array of segment_length which is each segment. Segment_lengths[X] is segment_length whose segment number is X.

Output:
a segment number which is assigned to a data-storing node.

unsigned int asura(unsigned int Datum ID, unsigned int maximum_segment_number_plus_1, float segment_lengths[])
{
    c_max = DEFAULT_MAXIMUM_RANDOM_NUMBER;
    loop_max = 0;
    while(c_max < maximum_segment_number_plus_1){
        c_max *= 2;
        loop_max++;
    }

    initialize_control_variable_for_random_number_generator (control_variable_for_initialization, Datum ID);
    for(i = 0; i <= loop_max; i++){
        control_variable_is_used[i] = 0;
        seed_for_control_variables[i] = generate_integer_random_number (control_variable_for_initialization);
    }

    do{
        c = c_max;
        loop = loop_max;
        while(1){
            if(control_variable_is_used[loop] == 0){
                initialize_control_variable_for_random_number_generator (control_variables[loop], seed_for_control_variables[loop]);
                control_variable_is_used[loop] = 1;
            }
            do{
                result = generate_float_random_number_not_less_than_0_and_less_than_1 (control_variables[loop]) * (float)c;
            }while(result >= maximum_segment_number_plus_1);
            c = c / 2;
            if(result >= c || loop == 0){
                break;
            }
            loop--;
        }
    }while(int_result + segment_lengths[int_result] < result);

    return int_result;  // an output segment number which is assigned with a data-storing node
}

APPENDIX B CALCULATION TIME ORDER OF ASURA

In ASURA, the expectation value for the number of times that random numbers are generated does not depend on the number of nodes. The proof is given below.

This proof has the following presupposition.

- Number line assigned to segments starts from 0.0.
- Segment lengths are 1.0.
- The following parameters are used:
  - S: initial maximum of random number
  - α: incremental ratio of maximum random number
  - n: maximum segment number + segment length of the segment having the maximum segment number
  - h: length of hole that does not have a corresponding node

First, the number (x) of the random number generator for generating one ASURA random number is:

\[
S\alpha^{(x-1)} < n \leq S\alpha^x
\]

\[
\log_\alpha (S\alpha^{(x-1)}) < \log_\alpha n \leq \log_\alpha (S\alpha^x)
\]

\[
x - 1 + \log_\alpha S < \log_\alpha n \leq x + \log_\alpha S
\]

\[
x - 1 < \log_\alpha \left( \frac{n}{S} \right) \leq x
\]

Because x is an integral number.

\[
x = \left\lfloor \log_\alpha \left( \frac{n}{S} \right) \right\rfloor \ldots(1)
\]

The probability of generating an ASURA random number that points to a segment is

\[
\frac{n - h}{S\alpha^x} \ldots(2)
\]

By definition,
Thus,

\[
\frac{n}{n-h} \leq \frac{S_\alpha^x}{n-h} < \frac{\alpha n}{n-h}
\]

\[
\frac{1}{n} \leq \frac{S_\alpha^x}{n-h} < \frac{\alpha}{n} \quad \text{(3)}
\]

The expected frequency with which random numbers are generated in order to generate an ASURA random number is

\[
\frac{S_\alpha^x - S_\alpha^{-1}}{S_\alpha^x} + \frac{2S_\alpha^{-1} - S_\alpha^{-2}}{S_\alpha^x} + \ldots + \frac{xS_\alpha-S}{S_\alpha^x} + (x+1) \cdot \frac{S}{S_\alpha^x}
\]

\[
= 1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + \ldots + \frac{1}{\alpha^x}
\]

\[
= \left( \frac{1}{\alpha} \right) ^{x+1} - 1
\]

\[
= \frac{1 - \alpha}{\alpha}
\]

\[
= \frac{\alpha^{x+1} - 1}{\alpha^{x+1} - \alpha^x}
\]

\[
= \frac{\alpha}{\alpha-1} - \frac{1}{\alpha^x(\alpha-1)} \quad \text{(4)}
\]

From Eqs. (2) and (4), the expectation value for the number of times that random numbers are generated to generate an ASURA random number that points to a segment is

\[
\left( \frac{n-h}{S_\alpha^x} \right)^{-1} \left( \frac{\alpha}{\alpha-1} - \frac{1}{\alpha^x(\alpha-1)} \right)
\]

\[
= \frac{S_\alpha^x}{n-h} \left( \frac{\alpha}{\alpha-1} - \frac{1}{\alpha^x(\alpha-1)} \right) \quad \text{(5)}
\]

In Eqs. (5), when \( n \) increases, \( \frac{S_\alpha^x}{n-h} \) depends only on \( \frac{h}{n} \) from Eqs. (3). \( \frac{\alpha}{\alpha-1} \) is constant, and \( \frac{1}{\alpha^x(\alpha-1)} \) becomes 0 from Eqs. (1). Thus, the maximum limitation of the expectation value for the number of times that random numbers are generated to generate an ASURA random number that points to a segment depends only on \( \frac{h}{n} \).