Localized plumes in three-dimensional compressible magnetoconvection

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Accepted 2010 October 27. Received 2010 September 29; in original form 2010 July 13

ABSTRACT

Within the umbrae of sunspots, convection is generally inhibited by the presence of strong vertical magnetic fields. However, convection is not completely suppressed in these regions: bright features, known as umbral dots, are probably associated with weak, isolated convective plumes. Motivated by observations of umbral dots, we carry out numerical simulations of three-dimensional, compressible magnetoconvective motions in an electrically conducting fluid (Chandrasekhar 1961). When the dynamics are dominated by magnetic fields, convection takes the form of weak, narrow plumes. In an idealized model of magnetoconvection, Weiss, Proctor & Brownjohn (2002) found a steady, almost hexagonal pattern of convection in the magnetic features on the solar surface. A typical sunspot consists of a central umbral region, surrounded by a complex filamentary penumbra. Umbral regions appear dark because their surface temperatures are (typically) only 70–85 per cent of the mean surface temperature of the non-magnetic photosphere (see, for example, Thomas & Weiss 2008). This reduction in temperature is due to the fact that the convective transport of heat is impeded within sunspot umbrae by the presence of strong, near-vertical magnetic fields (which can often exceed 3000 G).

Detailed observations of sunspot umbrae have shown that they are not uniformly dark. In almost all sunspots, bright point-like structures can be observed – these are known as umbral dots (Danielson 1964). These bright features are warmer than their immediate surroundings, but are (generally) cooler than the surrounding photosphere (see, for example, Sobotka & Hanslmeier 2005; Kitai et al. 2007). It is difficult to determine the characteristic size of an umbral dot, although these features are always small compared to the umbral diameter. In a recent study, Kitai et al. (2007) found that the umbral dots in one particular sunspot had typical diameters of approximately 220–350 km, although a significant number appeared to be much smaller than this (possibly below the resolution limit of 220–350 km, although a significant number appeared to be much smaller than this (possibly below the resolution limit). Umbral dots are also short-lived features. Kitai et al. (2007) found that most of the umbral dots in their survey had lifetimes of between 5 and 20 min. In an earlier study, Sobotka, Brandt & Simon (1997) found a much broader range of lifetimes for umbral dots (with a small percentage lasting longer than 2 h), although, like Kitai et al. (2007), they found a mean lifetime of approximately 15 min. Most umbral dots exhibit no systematic proper motions. However, those that appear to form at the umbral/penumbral boundary (which are often associated with penumbral grains) tend to migrate radially inwards towards the centre of the umbra (Sobotka et al. 1995; Kitai et al. 2007). There is some observational evidence for weak upflows within umbral dots (Socas-Navarro et al. 2004; Bharti, Jain & Jaaffrey 2007) as well as downflows around their edges (Bharti, Jain & Jaaffrey 2007; Ortiz, Rubio & van der Voort 2010). Clearly, the observations indicate that umbral dots correspond to convective plumes within sunspot umbrae. Further theoretical support for this conclusion comes from the work of Deinzer (1965), who determined that convective motions must be present within the umbra, as radiative processes alone could not transport sufficient energy to the surface.

Theoretical studies of umbral convection tend to be based upon local models of magnetoconvection in a Cartesian domain. It is well known that a strong vertical magnetic field tends to inhibit convection motions in an electrically conducting fluid (Chandrasekhar 1961). When the dynamics are dominated by magnetic fields, convection takes the form of weak, narrow plumes. In an idealized model of magnetoconvection, Weiss, Proctor & Brownjohn (2002) found a steady, almost hexagonal pattern of convection in the magnetically dominated regime. More recently, Schüssler & Vögler (2006; see also Bharti, Beeck & Schüssler 2010) have carried out a more realistic set of calculations, including the effects of partial ionization and radiative transfer. These simulations produced a...
time-dependent pattern of individual convective plumes, the properties of which compare very favourably to observations of umbral dots. In the calculations of Weiss et al. (2002) and Schüssler & Vögler (2006), convective features tend to be distributed across the whole computational domain. It is worth noting that observations indicate that the distribution of umbral dots tends to be rather non-uniform (Sobotka & Hanslmeier 2005). This could simply be explained by variations in intensity (as seen in the calculations of Schüssler & Vögler 2006), but some degree of localization in the distribution of convective plumes could also help to explain the observed distribution of umbral dots.

In a two-dimensional model of incompressible magnetoconvection, Blanchflower (1999) found strongly localized, steady convective states (see also Dawes 2007). These were named convectons. Restricting attention to a simplified model, in which the governing equations were projected on to a minimal set of Fourier modes in the vertical direction, Blanchflower & Weiss (2002) were also able to find oscillatory localized states in three spatial dimensions. In both these cases, these localized states were found in the subcritical parameter regime (in which the static, purely conducting state is linearly stable to convective perturbations). These localized states are an extreme example of a phenomenon that is known as flux separation (Tao et al. 1998). Convective plumes tend to expel magnetic flux (Weiss 1966), causing it to accumulate in the surrounding fluid. In this subcritical parameter regime, the magnetic field that surrounds the plume becomes sufficiently strong that convection in the surrounding fluid is completely inhibited, giving rise to a truly localized convective state. If conditions are appropriate for subcritical convection within sunspots (something that is certainly plausible), these results suggest that it may be possible for truly localized umbral dots to form within sunspot umbrae.

In this paper, we demonstrate the existence of steady localized convective plumes in three-dimensional compressible magnetoconvection. Unlike Blanchflower & Weiss (2002), we make no simplifying assumptions regarding the vertical structure of the convective flows. The set-up of the model is described in detail in the next section of the paper. Numerical results from this model are presented in Section 3. In the final section, we relate our findings (in qualitative terms) to observations of umbral dots.

## 2 Problem Description and Set-up

We consider the evolution of a layer of compressible, electrically conducting fluid, heated from below, in the presence of an imposed magnetic field. Various properties of the fluid, including the thermal conductivity, \( K \), the shear viscosity, \( \mu \), the magnetic diffusivity, \( \eta \), the magnetic permeability, \( \mu_0 \), and the specific heat capacities at constant pressure and density (\( c_P \) and \( c_V \), respectively) are assumed to be constant. At a position \( x \) and time \( t \), we define \( \rho(x, t), T(x, t) \) and \( \mathbf{u}(x, t) \) to be the fluid density, temperature and velocity field (respectively), whilst \( \mathbf{B}(x, t) \) represents the magnetic field.

This fluid occupies a three-dimensional Cartesian domain with \( 0 \leq z \leq d \) and \( 0 \leq x, y \leq 8d \). The axes of this coordinate system are orientated so that the \( z \)-axis points vertically downwards, parallel to the constant gravitational acceleration, \( g = g \hat{z} \). For this model problem, periodic boundary conditions are imposed in the \( x \)- and \( y \)-directions, whilst the upper and lower boundaries (at \( z = 0 \) and \( z = d \)) are assumed to be impermeable and stress free. Furthermore, fixed temperature boundary conditions are applied at the upper and lower boundaries with \( T = T_0 \) at \( z = 0 \) and \( T = T_0 + \Delta T \) at \( z = d \) (\( \Delta T > 0 \)). It is also assumed that the horizontal components of any magnetic fields that are present vanish at \( z = 0 \) and \( z = d \). When the layer is static, the imposed magnetic field is uniform and vertical, i.e. \( \mathbf{B} = B_0 \hat{z} \).

Before writing down the governing equations for this system, we can express these in non-dimensional form. More details of this procedure can be found in Matthews, Proctor & Weiss (1995) and Bushby & Houghton (2005). Very briefly, all lengths are scaled by the layer depth, \( d \), whilst an acoustic time-scale, \( \tau = (R_\star T_0)^{1/2} \) (where \( R_\star \) is the gas constant) is used to rescale time. After rescaling \( \rho, T, \mathbf{u} \) and \( \mathbf{B} \) in an appropriate way (see Bushby & Houghton 2005 for more details), the governing equations for this system can be written in the following form:

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}),
\]

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} = -\nabla (\rho T + F|\mathbf{B}|^2/2) + \theta(m + 1)\rho \mathbf{\hat{z}} + \nabla \cdot (\mathbf{FBB} - \rho \mathbf{uu} + \kappa \mathbf{\tau}),
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \kappa \xi_0 \nabla \times \mathbf{B}),
\]

\[
\frac{\partial T}{\partial t} = -\mathbf{u} \cdot \nabla T - (\gamma - 1) \nabla \cdot \mathbf{u} + \frac{\kappa}{\rho} \nabla^2 T + \frac{\kappa(\gamma - 1)}{\rho} \left( \sigma \tau^2/2 + F \xi_0 |\nabla \times \mathbf{B}|^2 \right),
\]

where the components of the stress-tensor, \( \mathbf{\tau} \), are given by

\[
\tau_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij},
\]

whilst \( \mathbf{B} \) satisfies the standard constraint that \( \nabla \cdot \mathbf{B} = 0 \). The pressure \( P \) satisfies

\[
P = \rho T.
\]

Equation (1) describes the conservation of mass, whilst equation (2) is simply the momentum equation, written in conservative form. Note that the \( \theta(m + 1)\rho \mathbf{\hat{z}} \) term in equation (2) represents the effects of gravity whilst the two terms that are quadratic in \( \mathbf{B} \) correspond to the Lorentz force. The evolution of the magnetic field is governed by the standard magnetic induction equation (3). The final two terms in the thermal equation (4) represent the effects of viscous and ohmic heating. The non-dimensional parameters that appear in the governing equations are defined in Table 1. For later convenience, we also introduce the Chandrasekhar number,

\[
Q = F/\kappa^2 \xi_0 \sigma.
\]

### Table 1. The non-dimensional parameters in the governing equations for compressible magnetoconvection. Note that \( \rho_0 \) corresponds to the unperturbed density at the upper surface of the layer. All other parameters are as defined in the text.

| Parameter | Definition | Values used |
|-----------|------------|-------------|
| \( \gamma \) | \( c_P/c_V \) | 5/3 |
| \( m \) | \( gdR/\Delta T - 1 \) | 1.0 |
| \( \theta \) | \( \Delta T/T_0 \) | 10.0 |
| \( \kappa \) | \( K/\rho_0 \mu_0(\kappa R\xi_0)^{3/2} \) | 0.2 |
| \( \xi_0 \) | \( \eta_0 c_P/\kappa \) | 0.1 |
| \( \sigma \) | \( \mu c_P/\kappa \) | 1.0 |
| \( F \) | \( B_0^2/\mu_\Omega \mu R T_0 \) | Variable |
and the Rayleigh number,

\[ Ra = (m + 1 - my)(1 + \theta / 2)2^m \gamma \left( \frac{m + 1}{\gamma \sigma} \right)^{m-1} \]  

(8)

The Chandrasekhar number is a measure of the strength of the imposed magnetic field, whilst the Rayleigh number measures the destabilizing influence of the temperature gradient that is imposed across the layer. If all other parameters are fixed, varying \( Ra \) and \( Q \) is equivalent to varying \( F \) and \( \kappa \).

There is a non-trivial equilibrium solution to these governing equations, corresponding to a static polytropic layer with a uniform, vertical magnetic field:

\[ u = 0, \quad T(z) = 1 + \theta z, \quad \rho(z) = (1 + \theta z)^m, \quad B = \mathbf{\hat{z}}. \]  

(9)

In this equilibrium state, the parameters that are given in Table 1 imply that the layer of fluid is highly stratified, with the temperature and density both varying by a factor of 11 across the depth of the domain. When there is no magnetic field present (i.e. when \( Q = 0 \) or, equivalently, \( F = 0 \)), the critical Rayleigh number for the onset of convection in this layer is approximately \( Ra \approx 1189 \).

For the parameters given in Table 1, \( Ra = 6000 \), hence this layer is convectively unstable in the absence of an imposed magnetic field. As in Boussinesq magnetoconvection (Chandrasekhar 1961), magnetic fields tend to inhibit convection. Hence, non-zero values of \( Q \) lead to an increase in the critical Rayleigh number. Table 1 also gives values for two diffusivity ratios, \( \sigma \) and \( \zeta_0 \). For simplicity, we set \( \sigma = 1.0 \). The value of \( \zeta_0 \) plays a crucial role in determining the near-onset bifurcation structure. As was the case in the truncated Boussinesq model of Blanchflower (1999), a choice of \( \zeta_0 = 0.1 \) ensures that the equilibrium state can be unstable to oscillatory modes of convection as well as stationary modes.

3 NUMERICAL RESULTS

Given the complexity of the governing equations, it is necessary to solve these numerically. We cover the Cartesian domain with a computational mesh, typically consisting of \( 128 \times 128 \times 96 \) grid points. Using standard fast Fourier transform (FFT) libraries, all horizontal derivatives are evaluated in Fourier space. Fourth-order finite differences are used to calculate the vertical derivatives. The temporal evolution of this system is determined by an explicit third-order Adams–Bashforth scheme. This code is parallelized using MPI.

3.1 Supercritical convection

As described in Section 2, one simple solution to these governing equations corresponds to a static polytropic layer in the presence of a uniform, vertical magnetic field (equation 9). We use this static solution (plus a small thermal perturbation) as an initial condition for the code. For the parameter values that are given in Table 1, this initial state is convectively unstable provided that the Chandrasekhar number does not exceed a value of (approximately) \( Q \approx 160 \). Since this equilibrium solution is unstable when \( Q < 160 \), we refer to this region of parameter space as the ‘supercritical’ parameter regime.

Fig. 1 shows snapshots of the system for two different values of \( Q \) (\( Q = 120 \) and 150), once a statistically steady state has been reached. Each plot shows the temperature distribution across a horizontal layer, just below the upper surface of the domain. When \( Q = 120 \), the solution is characterized by several time-dependent plumes from which most of the surface magnetic field has been expelled by the convective motions. Elsewhere the flow is dominated by magnetic effects, so much so that the Lorentz force is strong enough to (almost) completely inhibit convection. Solutions of this form are reminiscent of the ‘flux-separated’ states that were found by Weiss et al. (2002) although, in that study, small-scale convective cells tended to be found in the magnetically dominated regions. Here the magnetic suppression of convection is much more pronounced in this flux-separated state. A very different form of solution is found when \( Q = 150 \) (lower part of Fig. 1). In this simulation, the convection forms an oscillatory pattern that is distributed across the whole of the domain.

Linear theory (see, for example, Chandrasekhar 1961) predicts oscillatory convection in the magnetically dominated regime, at least for low values of \( \zeta_0 \). Hence the oscillatory behaviour of the simulation at \( Q = 150 \) is unsurprising. The convection at \( Q = 120 \) is also time dependent, as noted above, but this flux-separated state is representative of a different solution branch. Although the present model is much more complicated than the two-dimensional Boussinesq model that was considered by Blanchflower (1999), it is clearly of interest to relate the two studies. The flux-separated state at \( Q = 120 \) would correspond to one of the multiple roll states that are...
Figure 2. Like the lower part of Fig. 1, this shows the temperature distribution, for $Q = 150$, across a horizontal layer just below the upper surface of the domain. This solution has been generated by following the flux-separated solution branch.

shown in the bifurcation diagram in fig. 3 of Blanchflower (1999). The existence of different solution branches raises the possibility that a given set of parameter values could be associated with more than one stable state. Following the procedure that is described by Blanchflower (1999), solution branches can be followed by starting a simulation with fully developed convection, rather than evolving it from a static state. Having adjusted the value of $Q$, the simulation can then be evolved again until a statistically steady state has been found. For example, if the $Q = 120$ solution is taken as an initial condition, the flux-separated solution branch can be followed by gradually increasing the strength of the magnetic field (by increasing the value of $Q$). The outcome of such a procedure is illustrated in Fig. 2, which shows the temperature distribution for a flux-separated solution at $Q = 150$. Comparing this plot with the lower part of Fig. 1, it is clear that (at least) two distinct solutions exist for this set of parameter values. This flux-separated solution has smaller convective plumes than the $Q = 120$ case. This is a consequence of the fact that magnetic fields are now strong enough to inhibit convection in a larger proportion of the computational domain. The typical scale of convection is reduced by increasing the strength of the imposed magnetic field. Additionally the rate at which convective plumes merge together, and split apart, reduces as the strength of the imposed magnetic field is increased.

3.2 The subcritical parameter regime

As noted above, the static polytropic layer is (linearly) stable to convective perturbations for values of the Chandrasekhar number in excess of approximately $Q = 160$. Hence, we refer to this range of parameter space as the ‘subcritical’ regime. In order to find non-trivial behaviour in this parameter regime, it is clearly necessary to adopt a non-static initial condition for any simulations that are carried out. Given the results that were presented in the previous section, it is natural to try to track the flux-separated solution branch into this subcritical regime by incrementally increasing the value of $Q$ (as before, looking for a statistically steady state before each increment). By following an analogous procedure, Blanchflower (1999) found localized states in a two-dimensional model, so this would appear to be the most sensible approach.

As the value of $Q$ is gradually increased, following the flux-separated branch, the dynamical influence of the magnetic field becomes greater. This leads to a reduction in both the number and scale of the field-free convective plumes. This process continues until a single steady plume remains. This plume is the three-dimensional analogue of the two-dimensional ‘convecton’ solutions that were found by Blanchflower (1999). This three-dimensional convecton is illustrated in Figs 3 and 4. At $Q = 215$, this localized convective plume is almost axisymmetric, being slightly elongated in the $x$-direction, with a broad central upwelling, surrounded by a narrow downflow region. Convection is completely suppressed everywhere else by a strong, uniform, vertical magnetic field. The magnetic field distribution within the convecton is more complicated. At the surface, the field is almost completely expelled by the diverging convective flows. Towards the base of the plume, converging convective flows lead to an accumulation of vertical magnetic flux at the base of the convective upflow. The stratification of the layer clearly does

Figure 3. Steady, localized convection at $Q = 215$. This shows the temperature distribution in a horizontal plane just below the upper surface of the computational domain. Taking into account the periodic boundary conditions, this solution corresponds to a single convective plume. The convection is close to axisymmetric, but slightly elongated in the $x$-direction.

Figure 4. As Fig. 3, these plots show snapshots of a convecton at $Q = 215$. Top panel: the perturbation to the background temperature distribution along a vertical slice through $x = 7.5$. Contours range from $-0.11$ (black) to 0.87 (white). Bottom panel: the vertical component of the magnetic field in the same vertical slice. Contours range from 0.22 (black) to 3.67 (red).
play a role in determining the structure of this localized convective feature: slightly larger temperature perturbations are found near the top of the layer. It should be stressed that this convection is steady and is therefore not simply a transient phenomenon. It is also worth noting that, although the horizontally averaged convective flux is small, the local perturbations to the thermodynamic variables are of the order of unity within the convection itself. Therefore this is also a dynamically significant feature, albeit a highly localized one.

It should be noted that this convection is not truly localized rather, by virtue of the horizontal boundary conditions, it is part of a periodic array of such convective structures. As seen in Fig. 4 each structure is separated by approximately six non-dimensional units which is approximately three times the width of the convective structure. Therefore, any influence from neighbouring convection will be small. An initial investigation in larger computational domains confirms this to be the case. Similar localized states can be found, although the range of $Q$ for which they exist does change with domain size. Attempts to find an analytic description of a perfectly axisymmetric convection are ongoing.

Having found this convection, this solution branch can also be tracked to determine its range of stability. In fact, the solution that is shown in Figs 3 and 4 corresponds to the largest value of $Q$ at which the convection is found to be stable. This is also the value of $Q$ at which the greatest degree of localization is found. As this solution branch is followed into the weaker field regime (by gradually decreasing the value of $Q$), the convection grows, becoming increasingly asymmetric. For example, at $Q = 190$ the solution is still highly localized and steady, but the convective cell is about 25 per cent wider than the convector at $Q = 215$ and is more elongated in the $x$-direction. For this larger localized state, it is plausible that the finite size of the computational domain is (weakly) influencing the symmetry of the convective plume. Reducing $Q$ still further, we find that the solution is still mostly localized, although there are now weak (but significant) fluctuations elsewhere in the domain. More importantly, the convection is no longer steady. Instead, the plume ‘wobbles’ periodically about its central axis (as illustrated in Fig. 5). This is not a true oscillatory state, and the amplitude of the fluctuations is small, but this behaviour is certainly interesting. Similar vacillation has been seen as a way for steady convection patterns to lose stability via a Hopf bifurcation (Rucklidge et al. 1993). It is also possible that the observed oscillations are a consequence of the finite geometry, in the sense that the convection could be interacting with periodic copies of itself. Calculations in larger domains to investigate this further are currently underway.

### 4 DISCUSSION AND CONCLUSIONS

In these numerical simulations we have found strongly localized convective plumes in the subcritical regime of compressible magnetoconvection. These states are not embedded within a background of diffuse weak convection, but rather a static fluid (see Fig. 3). It is believed that strongly localized, steady states of this type have not previously been observed in three spatial dimensions. Whether umbral dots ever exhibit such an extreme degree of localization is unclear, but these results do suggest that highly localized plumes could occur if the magnetic fields within sunspot umbrae are strong enough (locally) to produce subcritical conditions for magnetoconvection.

The strongly localized states found in this work are found by continuation of flux-separated states to higher values of $Q$. A similar procedure was carried out by Weiss et al. (2002); however, in their work it was found that for sufficiently strong imposed magnetic fields the flux-separated solution was lost in favour of small-scale regular convection. Weiss et al. (2002) used the parameter value $\zeta_0 = 0.2$, whereas in this work $\zeta_0 = 0.1$. It is well known that the value of $\zeta_0$ controls the near-onset bifurcation structure. Comparison of this work with the findings of Weiss et al. (2002) indicate that it also controls whether or not the solution branch corresponding to flux-separated states continues into the subcritical regime.

Although it is appealing to relate these highly localized convective plumes to umbral dots, one aspect of these solutions that does not compare favourably with umbral dots is the fact that these localized states are steady (or, at best, weakly time dependent, as shown in Fig. 5). So, unlike the observed umbral dots, these solutions do not have finite lifetimes. This may be associated with the simplified nature of the model, which assumes a uniform background state: spatiotemporal inhomogeneities in the umbral background may play an important role in determining the lifetime of an umbral dot. Alternatively, time-dependent behaviour may be the result of interacting plumes. Perhaps the time-dependent state that is shown in Fig. 2 is a more realistic representation of the distribution of convective plumes within sunspot umbrae. Work is ongoing to establish whether or not it is possible to find truly localized states that exhibit significant time dependence. The simplified model of Blanchflower & Weiss (2002) suggests that oscillatory solutions should exist; however, while we cannot rule out this possibility, oscillatory localized states have not yet been observed in the full system of equations.

This model is clearly a highly idealized representation of magnetoconvection within sunspot umbrae. Truly localized states have not yet been found in more realistic models of photospheric magnetoconvection. It would be of great interest to establish whether or not processes such as radiative transfer promote (or inhibit) the formation of strongly localized plumes. It should also be stressed that the range of parameter values that can be considered in numerical models of this type bear little resemblance to true photospheric values (although this is true of all numerical magnetoconvection calculations, regardless of the level of physical complexity).
Having said that, we believe that results from models of this type do provide some useful insights into photospheric magnetoconvection. Magnetoconvection is not the only branch of physics in which strongly localized states are relevant. Similar states have now been found in a variety of other systems, including binary fluid convection (Bergeon & Knobloch 2008), buckling rod problems, nonlinear optics (Vladimirov et al. 2002) and experiments on a ferrofluid in an applied magnetic field (Richter & Barashenkov 2005). In a more abstract setting, localized states have also been found in the one-dimensional bistable Swift–Hohenberg equation (Burge & Knobloch 2006, 2007a,b). The results presented in this paper share many similarities with the behaviour of the Swift–Hohenberg equation in one dimension, as well as the other physical systems mentioned above. There is now a good theoretical understanding of localized states in one extended dimension; however, in higher dimensions the problem is not well understood, and has recently been posed as an open problem (Knobloch 2008). Having said that, some recent progress has been made. For example, Lloyd et al. (2008) have investigated the properties of spatially localized states in the two-dimensional Swift–Hohenberg equation. In future work, we intend to further explore the relationship between the behaviour of this simple pattern forming system and the magnetoconvection equations.

ACKNOWLEDGMENTS

This work has been supported by EPSRC grant EP/D032334/1 (SMH) and STFC grant ST/H002332/1. PJB would like to acknowledge the support of STFC. Large-scale computations were performed on the UKMHD Consortium machine based in St Andrews, UK.

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