Decoupling of Anomalous Top-Quark-Decay Vertices in Angular Distribution of Secondary Particles

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ABSTRACT

Angular distribution of a secondary particle from top-quark decays is studied in a simple and general manner, paying careful attention to how relevant the top-quark production mechanism is. The conditions that lead to the distribution free from any possible anomalous top-quark decay interactions are specified. Some approximations adopted in earlier papers are relaxed and their relevance is discussed.

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Top-quark interactions could provide relevant information on physics beyond the Standard Model (SM) because of its huge mass. For instance the top-quark Yukawa couplings are expected to be enhanced comparing to those for lighter fermions and therefore precise tests of top-quark interactions could either reveal new scalar degrees of freedom or limit possible extensions of the SM Higgs sector. Since future high-energy accelerators like NLC/LHC will operate as factories of top quarks, a lot of attention has been paid to study their production mechanisms (for a review, see [1] and the reference list there).

Only anomalous $t\bar{t}\gamma$, $t\bar{t}Z$ and $t\bar{t}g$ couplings have usually been considered in those studies, however there is a priori no good reason to assume that the top-quark decay is properly described by the SM couplings. Therefore in a series of papers (see e.g. [2, 3, 4]) we have performed analysis of top-quark decay products assuming the most general couplings both for the production and the decay.

In Ref.[3] we found that the angular distribution of the final leptons in $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^+ \cdots$ is not sensitive to modification of the SM $V-A$ decay vertex. The same conclusion was also reached by Rindani [5] through an independent calculation. We usually suffer from too many parameters to be determined while testing top-quark couplings in a general model-independent way. Therefore, a distribution insensitive to a certain class of non-standard form factors is obviously a big advantage as it increases expected precision for the determination of other remaining relevant couplings [4]. Furthermore in Ref.[6] we have noticed that this phenomenon appears not only in the process $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^+ \cdots$, but also in any $t\bar{t}$ production process.

There, however, we have limited ourselves to semileptonic SM-like decays $t \rightarrow bW \rightarrow b\ell\nu$ from $t\bar{t}$ pair production and used the explicit decay-width formula. The aim of this report is to complete full generalization of those preceding works. That is, we intend to relax these conditions studying general top-quark productions and its decays as model-independently as possible. Eventually we will specify the necessary conditions for decoupling of the anomalous top-quark-decay effects in the angular distributions of secondary particles from the top-quark decays.

Let us consider a general top-quark production process $1 + 2 \rightarrow t + \cdots$ followed by its decay $t \rightarrow f + \cdots$ where $f$ denotes the secondary particle that we are going
Following our preceding works, we assume $\sqrt{s}$ to be much higher than the threshold energy. Since the ratio of the top-quark width $\Gamma_t$ to its mass $m_t$ is of the order of $10^{-2}$ we can adopt the narrow-width approximation in the open-top region. Then one can apply the Kawasaki-Shirafuji-Tsai formula [7] in order to determine the $f$ distribution:

$$\frac{d\sigma}{d\tilde{p}_f} \equiv \frac{d\sigma}{d\tilde{p}_f} (1 + 2 \to t + \cdots \to f + \cdots)$$

$$= 2 \int d\tilde{p}_t \frac{d\sigma}{d\tilde{p}_t} (s_t = n) \frac{1}{\Gamma_t} \frac{d\Gamma_t}{d\tilde{p}_f} = 2B_f \int d\tilde{p}_t \frac{d\sigma}{d\tilde{p}_t} (s_t = n) \frac{1}{\Gamma} \frac{d\Gamma}{d\tilde{p}_f}. \quad (1)$$

Here $d\tilde{p}$ denotes the Lorentz-invariant phase-space element $d\mathbf{p}/[(2\pi)^3 2\mathbf{p}^0]$, $d\Gamma/d\tilde{p}_f$ is the spin-averaged top-quark width

$$\frac{d\Gamma}{d\tilde{p}_f} \equiv \frac{d\Gamma}{d\tilde{p}_f} (t \to f + \cdots),$$

$B_f \equiv \Gamma/\Gamma_t$, and $d\sigma(s_t = n)/d\tilde{p}_t$ is the single-top-quark inclusive cross section

$$\frac{d\sigma}{d\tilde{p}_t} (s_t = n) \equiv \frac{d\sigma}{d\tilde{p}_t} (1 + 2 \to t + \cdots ; s_t = n)$$

with the polarization vector $s_t$ being replaced with the so-called “effective polarization vector” $n$

$$n_\mu = -[g_\mu\nu - \frac{p_t^\mu p_t^\nu}{m_t^2}] \sum_{\text{spin}} \frac{\int d\Phi \overline{B} \Lambda_+ \gamma_5 \gamma^\nu B}{\sum_{\text{spin}} \int d\Phi \overline{B} \Lambda_+ B}, \quad (2)$$

where the spinor $B$ is defined such that the matrix element for $t(s_t) \to f + \cdots$ is expressed as $\overline{B} u_t(p_t, s_t)$, $\Lambda_+ \equiv p_t + m_t$, $d\Phi$ is the relevant final-state phase-space element, and $\sum_{\text{spin}}$ denotes the appropriate spin summation.

The angular distribution of $f$ shall be calculated based on eq.(1). Since $d\Gamma/d\tilde{p}_f$ is a Lorentz-invariant quantity depending only on $p_t$ and $p_f$, the distribution is a function of $p_t p_f$ alone

$$d\Gamma/d\tilde{p}_f = F(\xi), \quad (3)$$

Note that we are not limiting ourselves here to the SM-like decays with $f = \ell^+, b$ which we have considered in earlier papers.

In the threshold region, this formula might be no longer valid due to large corrections. For example, non-factorizable QCD corrections appear at the level of 10% in $e\bar{e} \to t\bar{t} \to \ell^+ X$ [8] (see also [9] and references therein for studies in the threshold region).
where \( \xi \equiv p_t p_f = E_t E_f (1 - \beta \cos \theta_{tf}) \), \( \beta = |p_t|/E_t \) and we have neglected the mass of \( f \). Integrating over \( p_f \) we find:

\[
\Gamma = \frac{1}{2(2\pi)^2 \beta E_t} \int dE_f d\xi F(\xi) = \frac{1}{(2\pi)^2 m_t^2} \int d\xi \xi F(\xi),
\]

where we used the following constraint on \( E_f \) for a fixed \( \xi \) in the \( E_f \) integration:

\[
\frac{\xi}{E_t(1 + \beta)} \leq E_f \leq \frac{\xi}{E_t(1 - \beta)}.
\]

On the other hand, the \( f \) angular distribution is

\[
\frac{d\sigma}{d\Omega_f} = \frac{B_f}{(2\pi)^3} \int dE_f d\sigma \left[ \frac{d\sigma}{d\tilde{p}_t} \right] (s_t = n) \frac{1}{\Gamma} F(\xi).
\]

Here the polarization-vector \( n \) can in general depend on \( E_f \) and the integration over \( E_f \) cannot be performed before explicit calculation of \( d\sigma(s_t = n)/d\tilde{p}_t \).

However, if the vector \( n \) is free from \( E_f \), we can perform the \( E_f \) integration independently of the production mechanism since \( d\sigma/d\tilde{p}_t \) can depend on \( E_f \) only through \( n \) inserted instead of \( s_t \). Let us briefly discuss possible dependence of \( n \) on \( E_f \). From the definition (2), \( n \) is a linear combination of \( p_t \) and \( p_f \): \( n = a p_t + b p_f \).

Since \( n p_t = 0 \) we obtain

\[
n = \alpha^f \left( \frac{m_t}{p_t p_f} p_f - \frac{p_t}{m_t} \right),
\]

where we have introduced the depolarization factor \( \alpha^f \equiv -a m_t \). We also have the following inequality from eq.(2)

\[
-1 \leq n^2 \leq 0 \implies -1 \leq \alpha^f \leq 1,
\]

which can be easily proven in the top-quark rest frame via a generalized triangle inequality. Since we assumed that \( f \) is massless, \( E_f \) drops out in \( p_f/(p_t p_f) \) in eq.(7) and the above inequality constrains the \( E_f \) dependence of \( \alpha^f \). This discussion is never a proof that \( n \) is always \( E_f \)-independent, but eq.(8) is a strong constraint and it will not be unreasonable to consider an \( E_f \)-independent \( n \) vector.

Therefore, let us now temporarily assume that \( n \) is not a function of \( E_f \). Then we can carry out the \( E_f \) integration in eq.(6):

\[
\frac{d\sigma}{d\Omega_f} = \frac{B_f}{(2\pi)^3} \int d\tilde{p}_t \frac{d\sigma}{d\tilde{p}_t} (s_t = n) \frac{1}{\Gamma} \int dE_f E_f F(\xi).
\]
Since eq.(4) gives
\[
\int dE_f E_f F(\xi) = \frac{1}{E_i^2 (1 - \beta \cos \theta_{ff})^2} \int d\xi \xi F(\xi) = \frac{(2\pi)^2 (1 - \beta^2)}{(1 - \beta \cos \theta_{ff})^2} \Gamma,
\]
eventually we obtain
\[
\frac{d\sigma}{d\Omega_f} = \frac{1}{2\pi B_f} \int d\tilde{p}_t \frac{d\sigma}{d\tilde{p}_t} (s_t = n) \frac{1 - \beta^2}{(1 - \beta \cos \theta_{ff})^2}.
\] (10)

Note that there are only two possible ways that the structure of the top-quark decay vertex could influence the distribution:

i) through the width \(d\Gamma/d\tilde{p}_f\), ii) through the effective polarization vector \(n\).

Therefore we conclude that if the polarization vector \(n\) (i.e. \(\alpha^f\)) depends neither on \(E_f\) nor on anomalous top-quark-decay vertices, the angular distribution \(d\sigma/d\Omega_f\) is not altered by those anomalous vertices except for possible trivial modification of the branching ratio \(B_f\). Furthermore, if we focus on the single standard-decay channel \(t \to bW^+ \to b\ell^+\nu_\ell\), even that dependence disappears. This result is never a trivial matter since the lepton-energy distribution, e.g., does depend on the anomalous decay parameters even when \(n\) satisfies the above conditions [2].

Let us stress here the difference between this conclusion and those in [3, 6]: If we focus only on the decay mode \(t \to bW^+ \to b\ell^+\nu_\ell\), the calculations in [3, 6] can of course give the same result. However we cannot say thereby anything on other decay modes since we fully used the explicit decay-width formula for that mode there to get the results. On the other hand, the present formalism can do. Indeed, there could be infinite other decay patterns (although \(t \to bW^+ \to b\ell^+\nu_\ell\) will be the leading channel). That is, if we extend the Higgs sector, the top-quark could decay via charged Higgs exchange. If we introduce effective four-fermion interactions, then it would decay through a contact \([tb\ell\nu]\) coupling. Even within the standard model, \(t \to bW \to b\ell\nu + \text{arbitrary number of photons}\) could occur. The present formula is applicable to any of them. This comparison clearly shows that our present formalism is the full important generalization of our previous works.

Finally, we consider the structure of \(n\) for the main-decay mode \(t \to b\ell^+\nu_\ell\) focusing on \(f = \ell^+\) and \(b\). Within the SM, it was found in Ref.[10] that
\[
\alpha^\ell^+ = 1 \quad \text{and} \quad \alpha^b = \frac{(2M_W^2 - m_\ell^2)}{(2M_W^2 + m_\ell^2)}.
\] (11)
Using the most general covariant $tbW$ coupling

$$
\Gamma^\mu \propto \bar{u}(p_b) \left[ \gamma^\mu (f_1^LP_L + f_1^RP_R) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W} (f_2^LP_L + f_2^RP_R) \right] u(p_t),
$$

(12)

where $P_{L/R} = (1 \mp \gamma_5)/2$ and $k$ is the momentum of $W$, we have confirmed in [3] that $\alpha^{\ell^+}$ remains unchanged while $\alpha^b$ receives corrections proportional to $\text{Re}(f_2^R)$. In those calculations, all the fermions except $t$ were treated as massless, the narrow-width approximation was adopted also for the decaying $W$, and only the [SM]-[non-SM] interference terms were taken into account. Thus, $n$ was indeed $E_\ell$-independent for those cases within our approximations, and the leptonic angular distribution cannot depend on the decay interactions even for the general $tbW$ vertex. Since the depolarization factor for $f = b$ was found to be sensitive to non-SM interactions already for $m_b = 0$, hereafter we shall focus only on the leptonic distribution.

In the above calculations, we have used the following spinor $B$, defined below eq.(2), for the interaction specified in (12):

$$
B \propto \left[ \gamma^\mu (f_1^LP_L + f_1^RP_R) + \frac{i\sigma^{\mu\nu}k_\nu}{M_W} (f_2^LP_L + f_2^RP_R) \right] u(p_b) \times \bar{v}(p_\ell) \gamma_\mu P_L u(p_\nu) \delta(k^2 - M_W^2),
$$

(13)

while the phase-space element $d\Phi$ needed in (2) is

$$
d\Phi = d\tilde{p}_b d\tilde{p}_\nu \delta^4(p_t - p_b - p_\ell - p_\nu).
$$

(14)

Let us now test the relevance of the approximations so far adopted. First the $b$-quark mass: It is seen from the form of $B$ that if $m_b = 0$ then only $f_2^R$ term can interfere with the leading $V - A$ from factor. However, for $m_b \neq 0$ all the form factors do contribute. Nevertheless, we have found via explicit analytical calculations that the depolarization factor for $f = \ell^+$ still remains unchanged: $\alpha^{\ell^+} = 1$ even when $m_b$ is not neglected. So, when only interference terms are kept, $\alpha^{\ell^+}$ depends neither on $E_{\ell^+}$ nor on the anomalous couplings regardless what bottom-quark mass was employed! On the other hand, however, we have noticed that $\alpha^{\ell^+}$ does receive non-SM corrections of the order of $[\text{non-SM}]^2$ not only when eq.(12) is used for the $tbW$ vertex but also when similar anomalous terms were
introduced into the $\ell\nu W$ vertex. The narrow-width approximation for the decaying $W$ was also found to be inevitable for $\alpha^{\ell^+} = 1$. That is, if we replace $\delta(k^2 - M_W^2)$ in eq.(13) by the squared $W$ propagator with a finite width, $\alpha^{\ell^+}$ does not stay unchanged even if only interference terms are kept.

There are a few remarks here in order:

- The angular dependence seen in eq.(10) cannot have any dynamical origin as we have never specified top-quark interactions. The angular distribution $d\Gamma/d\tilde{p}_f$ entering eq.(1) is isotropic in the top-quark rest frame as top quark is unpolarized. Therefore the dependence on $\theta_{tf}$ of the integrand (10) is just a remnant of the Lorentz transformation from the top-quark rest frame to the LAB frame:

$$\frac{d\Gamma}{d\cos\theta_{tf}} = \frac{1 - \beta^2}{(1 - \beta \cos \theta_{tf})^2} \frac{d\Gamma^*}{d\cos\theta^*}, \quad (15)$$

where $d\Gamma^*/d\cos\theta^*$ is the constant distribution defined in the top-quark rest frame.

- Our derivation of the angular distribution (10) applies to any top-quark production process, including pair and single productions at both $e^+e^-$ and hadronic machines (or $\gamma\gamma$ collisions enabled by laser-electron/positron backward scatterings) in the open-top region. For $e^+e^-$ collisions the absolute value of top-quark momentum is fixed by $\beta^2 = 1 - 4m_t^2/s$ and eq.(10) reduces to

$$\frac{d\sigma}{d\Omega_f} = \frac{2m_t^2}{\pi s} B_f \int d\Omega_t \frac{d\sigma}{d\Omega_t}(s_t = n) \frac{1}{(1 - \beta \cos \theta_{tf})^2}, \quad (16)$$

which is what we have derived in [6] including beyond-the-SM interactions.\(^\text{\#3}\)

On the other hand, the distribution in the CM frame of hadron-hadron collisions has some additional factors since the hadron CM frame and the parton CM frame are different from each other and they are connected through Lorentz transformation. However, any Lorentz boost can never produce anomalous-decay-parameter dependence. So, if $d\sigma/d\cos\theta$ in the parton-CM frame is free from the non-SM form factors, then the one in the hadron-CM frame is free from the non-SM form factors.

\(^{\#3}\text{The analogous SM result has been found in Ref.[10, 11].}\)
frame is also free from them. Consequently, our decoupling theorem holds in hadron-hadron collisions, too.

In conclusion, we have investigated the angular distribution of a secondary particle $f$ (without concentrating only on $f = \ell^+, b$) in processes like $1+2 \to t+\cdots$ followed by $t \to f + \cdots$ neglecting the $f$ mass and applying the narrow-width approximation for the decaying top quark. It has been shown that if the effective polarization vector $n$ contains neither non-SM top-quark couplings nor $E_f$ (as is the case for $f = \ell^+$ within our approximation) then the whole angular distribution of $f$ has no non-SM top-quark-decay contributions. Non-standard corrections can enter the leptonic distribution only through modification of $n$ as corrections to the narrow-width approximation, $m_t \neq 0$ terms or contributions quadratic in non-SM vertices. However, if the polarization vector $n$ does contain non-SM contributions (as is the case for $f = b$), then the angular distribution of $f$ receives extra correction from the top-quark-decay vertices only through the production cross section of the polarized top quark.

We emphasize that our conclusions concerning the decoupling holds for any possible top-quark production mechanism even if the bottom-quark mass is not neglected. Therefore unknown top-quark couplings that parameterize the angular distribution of $f$ in the case of SM polarization vector (e.g. $f = \ell^+$) are reduced to those that influence the production process. Since fewer unknown parameters lead to higher precision for their determination, we believe that the angular distribution will be useful while testing top-quark couplings at future colliders.

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REFERENCES

[1] D. Atwood, S. Bar-Shalom, G. Eilam and A. Soni, Phys. Rept. 347 (2001), 1 (hep-ph/0006032).

[2] B. Grzadkowski and Z. Hioki, Nucl. Phys. B484 (1997), 17 (hep-ph/9604301).

[3] B. Grzadkowski and Z. Hioki, Phys. Lett. B476 (2000), 87 (hep-ph/9911505).

[4] B. Grzadkowski and Z. Hioki, Nucl. Phys. B585 (2000), 3 (hep-ph/0004223).

[5] S.D. Rindani, Pramana 54 (2000), 791 (hep-ph/0002006).

[6] B. Grzadkowski and Z. Hioki, Phys. Lett. B529 (2002), 82 (hep-ph/0112361).

[7] Y.S. Tsai, Phys. Rev. D4 (1971), 2821; ibid. D13 (1976), 771 (Erratum);
    S. Kawasaki, T. Shirafuji and S.Y. Tsai, Prog. Theor. Phys. 49 (1973), 1656.

[8] M. Peter and Y. Sumino, Phys. Rev. D57 (1998), 6912 (hep-ph/9708223).

[9] A. H. Hoang et al., Eur. Phys. J. C3 (2000), 1 (hep-ph/0001286);

[10] T. Arens and L.M. Sehgal, Nucl. Phys. B393 (1993), 46.

[11] T. Arens and L.M. Sehgal, Phys. Lett. B302 (1993), 501.