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Nucleon-to-pion transition distribution amplitudes: a challenge for ĽPANDA

Abstract

Baryon-to-meson Transition Distribution Amplitudes (TDAs) appear as building blocks in the collinear factorized description of amplitudes for a class of hard exclusive reactions, prominent examples being hard exclusive pion electroproduction off a nucleon in the backward region and baryon-antibaryon annihilation into a pion and a lepton pair or a charmonium. Baryon-to-meson TDAs extend both the concepts of generalized parton distributions (GPDs) and baryon distribution amplitudes (DAs) encoding valuable complementary information on the hadronic structure. We review the basic properties of baryon-to-meson TDAs and discuss the perspectives for the experimental access with the PANDA detector.

Keywords Hard exclusive reactions · hadron structure

1 Introduction

The PANDA experiment at GSI-FAIR [1; 2] is being built to address fundamental questions in hadronic physics. Although much progress has been achieved in the domain of the deep structure of nucleons thanks to lepton beam initiated reactions, the interior of hadrons is still a frontier domain of the current understanding of QCD dynamics. In this respect, accessing the transition distribution amplitudes in specific exclusive reactions at PANDA is an important goal to progress in our understanding of quark and gluon confinement.

The leading twist-3 baryon to meson (antibaryon to meson) TDAs are defined through baryon (antibaryon)-meson matrix elements of the nonlocal three quark (antiquark) operators on the light cone [3; 4; 5]:

\[ \hat{O}^{\rho\beta\gamma}_{\rho\sigma\tau}(\lambda_1 n, \lambda_2 n, \lambda_3 n) = \varepsilon_{c_1 c_2 c_3} \psi^{c_1 c_2}_{\rho}(\lambda_1 n) \psi^{c_3 c_2}_{\sigma}(\lambda_2 n) \psi^{c_3 c_1}_{\tau}(\lambda_3 n), \]

where \( \alpha, \beta, \gamma \) stand for quark (antiquark) flavor indices and \( \rho, \tau, \chi \) denote the Dirac spinor indices. Antisymmetrization stands over the color group indices \( c_{1,2,3} \). Gauge links in (1) are omitted by considering the light-like gauge \( A \cdot n = 0 \). These non-perturbative objects first considered in [6], share common features both with baryon DAs (that are defined as the baryon-to-vacuum matrix elements of the same operator (1)) and with GPDs since the matrix element in question depends on the longitudinal momentum transfer between a baryon and a meson characterized by the skewness variable \( \xi \).

Baryon-to-meson TDAs arise in the collinear factorized description of several hard exclusive reactions such as backward electroproduction of mesons off nucleons [7; 8] that can be studied at JLab [9] and COMPASS. Future PANDA facility at GSI-FAIR makes it possible to access different classes of reactions that may be described in terms of baryon-to-meson TDAs, such as \( NN \) annihilation into a...
lepton pair in association with a light meson $\pi N$ or into a heavy quarkonium and a meson $\eta$. This will increase the experimental support profiting from the outstanding exclusive detection capabilities of PANDA and allow to check the universality of baryon-to-meson TDAs combining information on the space-like regime (from JLab) and the time-like regime (from PANDA).

Although baryon to meson TDAs can be introduced for all types of baryons and mesons, we mostly consider here the simplest case of nucleon-to-pion ($\pi N$) TDAs. Below we summarize the fundamental requirements for $\pi N$ TDAs which follow from the symmetries of QCD established in Refs. [13,14,8].

- For a given flavor contents, the spin decomposition of the leading-twist $3\pi N$ TDA involves eight real valued invariant functions $V_{1,2}^\pi N, A_{1,2}^\pi N, T_{1,2,3,4}^\pi N$, each depending on the longitudinal momentum fractions $x_i (\sum_{i=1}^{3} x_i = 2\xi)$, skewness parameter $\xi$ and the momentum transfer squared $\Delta^2$, as well as on the factorization scale $\mu^2$.

- The support of $\pi N$ TDAs in three longitudinal momentum fractions $x_i$ is given by the intersection of the stripes $-1 + \xi \leq x_i \leq 1 + \xi (\sum_{i=1}^{3} x_i = 2\xi)$. One can distinguish the Efremov-Radyushkin-Brodsky-Lepage-like (ERBL-like) domain and two types of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi-like (DGLAP-like) domains.

- The evolution properties of $\pi N$ TDAs are described by the appropriate generalization [15] of the ERBL/DGLAP evolution equations specific for different domains in $x_i$.

- Similarly to the GPD case, the underlying Lorentz symmetry results in the polynomiality property for the Mellin moments of $\pi N$ TDAs in the longitudinal momentum fractions $x_i$. To ensure the polynomiality and the restricted support properties for $\pi N$ TDAs one can employ the spectral representation in terms of quadruple distributions [15], which generalizes for the TDA case Radyushkin’s double distribution representation for GPDs.

- Contrary to GPDs, $\pi N$ TDAs lack a comprehensible forward limit ($\xi = 0$). It is illuminating to consider the alternative limit $\xi = 1$ in which $\pi N$ TDAs are constrained by chiral dynamics and crossing due to the soft pion theorem.

- In order to satisfy the polynomiality condition in its complete form, the spectral representation for $\pi N$ TDAs should be supplemented with a $D$ term-like contribution. The simplest possible model for such a $D$ term is the contribution of the cross-channel nucleon exchange computed in [14].

2 Pion production in association with a high invariant mass lepton pair in $\bar{p}N$ annihilation

One of the important aims of the experimental program of PANDA will be the study of the nucleon electromagnetic form factor in the time-like region in nucleon-antinucleon annihilation into a lepton pair. Outside the resonance region (for high invariant mass $Q^2 \equiv Q^2$ of the lepton pair) the description of the nucleon electromagnetic form factors can be provided by the methods of perturbative QCD [4].

In Refs. [10,11,8] we argue that a tempting possibility to apply similar pQCD methods for exclusive reaction is to consider also the nucleon-antinucleon annihilation into a high invariant mass lepton pair in association with a light meson $\mathcal{M} = \{\pi, \eta, \rho, \omega, ...\}$:

$$N(p_N, s_N) + N(p_N, s_N) \rightarrow \gamma*(q) + \mathcal{M}(p_M) \rightarrow \ell^+(p_{\ell^+}) + \ell^-(p_{\ell^-}) + \mathcal{M}(p_M).$$

(2)

The factorization mechanism for (2) suggested in Ref. [10] is shown on Fig. 1. We choose the $z$ axis along the colliding $N\bar{N}$ with positive direction along the antinucleon beam. We introduce the $t$- and $u$-channel light-cone vectors $n_t^\perp$, $p_t^\perp$; $n_u^\perp$, $p_u^\perp$ and define the $t$ and $u$ channel skewness variables $\xi^t \equiv -\frac{(p_{M} - p_{N}) n^\perp}{(p_{M} + p_{N}) n^\perp}$; $\xi^u \equiv -\frac{(p_{M} - p_{N}) n^\perp}{(p_{M} + p_{N}) n^\perp}$. The amplitude of the $N\bar{N} \rightarrow \gamma*\mathcal{M}$ subprocess is presented as a convolution of the hard part computed by means of perturbative QCD with nucleon DAs and nucleon to pion TDAs encoding the soft dynamics. The factorization is supposed to be achieved in two distinct kinematical regimes:

- the near forward regime $(s = (p_{N} + p_{\pi})^2 \equiv W^2, Q^2$ large with $\xi^t$ fixed; and $|t| = |(p_{M} - p_{N})^2| \sim 0$) this corresponds to the produced pion moving nearly in the direction of initial $N$ in $N\bar{N}$ center-of-mass system(CMS).

- the near backward regime $(s = (p_{N} + p_{\pi})^2 \equiv W^2, Q^2$ large with $\xi^u$ fixed; $|u| = |(p_{M} - p_{N})^2| \sim 0$) this corresponds to the produced pion moving nearly in the direction of initial $N$ in $N\bar{N}$ CMS.
The specific behavior in cos\(^2\) of the reaction (2) reads

\[ \text{differential cross section of the reaction (2) reads} \]

The suggested reaction mechanism should manifest itself through the distinctive forward and backward peaks of the N\(\bar{N}\) \(\rightarrow \gamma^*\pi\) cross section. The charge conjugation invariance results in perfect symmetry between the two kinematical regimes.

For definiteness, we below consider the case of the meson being a pion. Within the factorized approach of [10], at leading order in \(\alpha_s\), the amplitude of N\(\bar{N}\) \(\rightarrow \gamma^*\pi\) \(\mathcal{M}^\gamma_{\pi}^{N\bar{N}}\) reads

\[ \mathcal{M}^\gamma_{\pi}^{N\bar{N}} = \frac{C}{Q^4} \left[ S^\gamma_{\pi}^{N\bar{N}} \mathcal{I}(\xi, \Delta^2) - S^\gamma_{\pi}^{N\bar{N}} \mathcal{I}'(\xi', \Delta^2) \right], \tag{3} \]

where \(C = -\frac{(4\pi \alpha_s)^2}{2M_p^2}\); with \(f_N \approx 93 MeV\) – pion weak decay constant and \(\alpha_s\) is set to \(\alpha_s = \bar{\alpha}_s = 0.3\). The spin structures of \(S^\gamma_{\pi}^{N\bar{N}}\) and \(S^\gamma_{\pi}^{N\bar{N}}\) are defined as

\[ S^\gamma_{\pi}^{N\bar{N}} \equiv V(p_N, s_N) \epsilon^*(\lambda) \gamma_5 U(p_N, s_N); \quad S^\gamma_{\pi}^{N\bar{N}} \equiv \frac{i}{M} \bar{V}(p_\pi, s_\pi) \epsilon^* (\lambda) \Delta_T \gamma_5 U(p_N, s_N), \]

where \(V\) and \(U\) are the usual nucleon Dirac spinors and the Dirac \(\hat{v}\) notation \(\hat{v} = \gamma^\mu v^\mu\) is employed. \(\epsilon(\lambda)\) stands for the polarization vector of the virtual photon. \(\mathcal{I}\) and \(\mathcal{I}'\) denote the convolution integrals of \(\pi N\) TDAs and antinucleon DAs with the hard scattering kernels computed from the set of 21 relevant scattering diagrams [3]. The hard scattering kernels for backward \(p\bar{p} \rightarrow \gamma^*\pi\) differ from those for \(\gamma^*\pi \rightarrow p\bar{p}\) by the replacement \(-ie \rightarrow ie\) in the corresponding denominators. The averaged-squared amplitude for the process (2) then reads

\[ |\mathcal{M}^{N\bar{N} \rightarrow \ell^+\ell^- \pi}|^2 = \frac{1}{4} \sum_{s_N, s_{\bar{N}}, \gamma, \lambda} \mathcal{M}^\gamma_{\pi}^{N\bar{N}} \frac{1}{Q^2} \text{Tr} \{\bar{\epsilon}(\lambda) \hat{\epsilon}(\lambda')\} \frac{1}{Q^2} (\mathcal{M}^\gamma_{\pi}^{N\bar{N}})^*. \tag{4} \]

The differential cross section of the reaction (2) reads

\[ \frac{d\sigma}{d\theta dQ^2 \cos \theta} = \frac{1}{64W^2(W^2 - 4M^2)^2} \int d\varphi_T |\mathcal{M}^{N\bar{N} \rightarrow \ell^+\ell^- \pi}|^2, \tag{5} \]

where \(\theta\) and \(\varphi_T\) are the lepton polar and azimuthal angles in \(\ell^+\ell^-\) CMS. To the leading twist accuracy, only the transverse polarization of the virtual photon is contributing. Computing the relevant traces and integrating over the lepton azimuthal angle one gets

\[ \int d\varphi_T |\mathcal{M}^{N\bar{N} \rightarrow \ell^+\ell^- \pi}|^2 |_{\text{Leading twist}} = 2\pi \lambda^2 (1 + \cos^2 \theta) \frac{1}{Q^2} |\mathcal{M}|^2 \frac{2(1 + \xi)}{\xi Q^2} \left( |\mathcal{I}|^2 - \frac{\Delta T^2}{\mathcal{I}^2} |\mathcal{I}'|^2 \right). \tag{6} \]

The specific behavior in \(\cos^2 \theta\) of the cross section (3) together with the characteristic scaling behavior in \(1/Q\) may be seen as the distinctive features of the proposed factorization mechanism.

On Fig. 1 we present our estimates for the integrated cross section

\[ \frac{d\sigma}{dQ^2}(\Delta^2_{\min}) \equiv \int_{\Delta^2_{\min}}^{\Delta^2_{\max}} d\epsilon_T \int d\theta dQ^2 \cos \theta, \tag{7} \]
for backward $\bar{p}p \rightarrow \ell^+\ell^-\pi^0$, as a function of $Q^2$ for $W^2 = 10$ GeV$^2$ and $W^2 = 20$ GeV$^2$ within the nucleon pole exchange model [14] for $\pi N$ TDAs. We have integrated the cross section [7] over the bin in $u$ in order to implement the effect of the cut $\Delta^2_{\text{min}} \leq \Delta^2_T \leq 0$ intended to focus on the backward kinematics regime. As the numerical input for the TDA model we use different phenomenological solutions for nucleon DAs: Chernyak-Ogloblin-Zhitnitsky (COS) [16], King-Sachrajda (KS) [17], Braun-Lenz-Wittmann (BLW) NLO [18] and NNLO modification [19] of BLW. For most of the choices, the magnitudes of the cross sections are sufficient to be measurable, with the luminosity foreseen at the PANDA experiment at GSI-FAIR.

3 $J/\psi$ plus pion production in $\bar{p}N$ annihilation

A major goal of the PANDA experiment is the study of the spectrum of charmonium states. Below we argue that the concept of TDAs might be useful for understanding the non resonant background for charmonium production.

The study of charmonium exclusive decays into hadrons historically has been one of the first successful applications of perturbative QCD methods for hard exclusive reactions. The annihilation of the $c\bar{c}$ pair into the minimal possible number of gluons producing quark-antiquark pairs which then form outgoing hadrons was recognized to be the dominant mechanism. In [12] we extend the same framework for the description of nucleon-antinucleon annihilation into the heavy quarkonia together with a pion:

$$N(p_N) + \bar{N}(p_{\bar{N}}) \rightarrow J/\psi(p_\psi) + \pi(p_\pi).$$

The $\bar{N}N$ center-of-mass energy squared $s = (p_N + p_{\bar{N}})^2 = W^2$ and the charmonium mass squared $M_\psi^2$ introduce the natural hard scale for the problem. Similarly to the nucleon-antinucleon annihilation into a lepton pair and a light meson, it is assumed that the reaction (8) admits a factorized description in the near forward ($t \equiv (p_{\pi} - p_N)^2 \sim 0$) and near backward ($u \equiv (p_{\pi} - p_{\bar{N}})^2 \sim 0$) kinematical regimes.

The corresponding mechanisms are presented on Fig. 3. Similarly to the case of $NN$ annihilation into a high invariant mass lepton pair in association with a pion the $C$ invariance results in perfect symmetry between the forward and backward regimes of the reaction (8). Below we consider the backward regime. The $\bar{z}$ axis is chosen along the colliding $NN$ with positive direction along the antinucleon beam. We introduce the light-cone vectors satisfying $2p^u \cdot n^u = 1$ and define the $u$-channel skewness variable $\xi \equiv -(p_{\pi} - p_N) \cdot n^u \simeq \frac{M_\psi^2}{2W^2 - M_\psi^2}$. Following [16], in our calculation we set the relevant masses to the average value $M_\psi \simeq 2m_c \simeq M = 3$ GeV. The physical kinematical domain for the reaction (8) in the backward regime is determined by the requirement $\Delta^2_T \leq 0$, where $\Delta^2_T = \frac{1}{1 - \xi} \left( \Delta^2 - 2\xi \left( \frac{m_\pi^2}{1 + \xi} - \frac{m_\psi^2}{1 - \xi} \right) \right)$.

The leading order amplitude of the reaction (8) from the mechanism presented on Fig. 3 was computed in [12]. It reads:

$$\mathcal{M}_N^{N\pi\pi} = \mathcal{C} \frac{1}{M^5} \left[ \bar{V}(p_N, s_N) \gamma^\ast(\lambda) \gamma_5 U(p_N, s_N) \right] \mathcal{J}(\xi, \Delta^2)$$
Where $\Lambda_{\pi N}$ wave function. Its value is fixed from the charmonium leptonic decay width panel: $C$ Here function of $W$ over spins of initial nucleons we get the leading twist accuracy. Summing over the transverse polarization of charmonium and averaging the suggested reaction mechanism it is the transverse polarization of charmonium that is relevant to Here $J$ Collinear factorization of the annihilation process (8). Fig. 3 GeV $\alpha$ shows strong dependence on model we use BLW NLO phenomenological solution for nucleon DAs [18]. Note that the decay width BLW NLO solution we then have to take $\alpha$ value for the charmonium decay width into proton-antiproton the strong coupling for a given phenomenological solution in a way that it reproduces the experimental strong coupling for the gluon virtualities in question. To get a rough estimate of the cross section we fix the strong coupling for a given phenomenological solution in a way that it reproduces the experimental value for the charmonium decay width into proton-antiproton $\Gamma(J/\psi \rightarrow pp)$. In particular case of the BLW NLO solution we then have to take $\alpha_s = 0.44$. The obtained values of cross sections give hope of experimental accessibility of the reaction with PANDA. Also our predictions are consistent with the recent estimates of [20] obtained within a fully non-perturbative effective hadronic theory.

4 Conclusions

Baryon-to-meson TDAs are new non-perturbative objects which have been designed to help us scrutinize the inner structure of nucleons. Experimentally, one may access TDAs both in the space-like domain with backward electroproduction of mesons at JLab and COMPASS and in the time-like domain in antiproton nucleon annihilation processes. Extracting TDAs from space-like and time-like reactions will be a stringent test of their universality [21], and hence of the factorization property of hard exclusive amplitudes. This hopefully will help us to disentangle the complex dynamics of quark and gluon confinement in hadrons.
Fig. 4 Left panel: Differential cross section \( \frac{d\sigma}{dW^2} \) for \( p\bar{p} \rightarrow J/\psi \pi^0 \) as a function of \( W^2 \) for \( \Delta_T^2 = 0 \). Right panel: Differential cross section \( \frac{d\sigma}{d\Delta^2} \) for \( p\bar{p} \rightarrow J/\psi \pi^0 \) as a function of \( \Delta_T^2 \) for \( W^2 = 15 \text{ GeV}^2 \).

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