Inflation with Holographic Dark Energy

Bin Chen\textsuperscript{1}, Miao Li\textsuperscript{2,3}, Yi Wang\textsuperscript{3}

\textsuperscript{1} School of Physics, Peking University, Beijing 100871, P.R.China
\textsuperscript{2} The Interdisciplinary Center for Theoretical Study of China (USTC), Hefei, Anhui 230027, P.R.China
\textsuperscript{3} Institute of Theoretical Physics, Academia Sinica, Beijing 100080, P.R.China

Abstract

We investigate the corrections of the holographic dark energy to inflation paradigm. We study the evolution of the holographic dark energy in the inflationary universe in detail, and carry out a model-independent analysis on the holographic dark energy corrections to the primordial scalar power spectrum. It turns out that the corrections generically make the spectrum redder. To be consistent with the experimental data, there must be an upper bound on the reheating temperature. We also discuss the corrections due to different choices of the infrared cutoff.

1 Introduction

It was pointed out in \cite{1} that an ultraviolet (UV) cutoff of a field theory should be set by an infrared (IR) cutoff in order that the quantum zero-point energy of a system should not exceed the mass of a black hole of the same size. Recent studies on the so-called holographic dark energy are based on this observation. In the original paper \cite{1}, using the Hubble scale as the IR cutoff, one obtains a vacuum energy density comparable to the present day dark energy. Unfortunately it was later pointed out in \cite{2} that the resulting equation of state for dark energy does not match the data. However, one of the present authors (Li) \cite{3} suggested that in cosmological
applications, the IR cutoff should be set by the future event horizon. This suggestion, combined with the simplest assumption that dark energy is given by the vacuum energy, provides a possible solution of the three problems on cosmological constant [4], namely, why the cosmological constant is not large, nonzero, and comparable to the matter energy density at the present time.

Following [3], if one choose the future event horizon as the IR cutoff, the vacuum energy density is given by

$$\rho_\Lambda = 3c^2 M_p^2 R_h^{-2}, \quad (1)$$

where $c$ is a constant to be determined by data fitting of the experiments. Both theoretical reasoning and data fitting suggest $c$ is of order 1. $R_h$ is the future event horizon, the boundary of the volume a fixed observer may eventually observe:

$$R_h \equiv a \int_t^\infty \frac{dt}{a} = a \int_x^\infty \frac{dx}{aH}, \quad (2)$$

where $x \equiv \ln a$.

In cosmological applications, this vacuum energy density (1) should couple with gravity and be added to the energy density term in the Friedmann equation. It can cause the late time acceleration of the universe as a component of dark energy.

As have pointed out by Li [3], if the holographic dark energy drives the late time acceleration of the universe, then it should also affect inflation [5]. It is shown that at roughly 60 e-folds before the end of inflation, the holographic dark energy becomes comparable with the inflaton energy in energy density. So the holographic dark energy should induces corrections to the standard inflation at long wave lengths, these may be observable at the largest scales of the CMB spectrum. To study how the holographic dark energy affects inflation and the CMB spectrum is the main aim of this paper.

This paper is organized as follows. In section 2, we discuss in more detail the dynamics of the holographic dark energy during inflation and during the post-inflationary epoch. In section 3, we study the perturbations and the scalar power spectrum. We perform a model-independent analysis of the effect of the holographic dark energy. We find that it leads to a upper bound on the reheating temperature. Some inflation models such as the $\lambda\varphi^4$ monomial model satisfy this requirement, while other models such as the $m^2\varphi^2$ monomial model and the small field inflation models should be
modified to coexist with the holographic dark energy.

The IR cutoff set by the future event horizon is physically natural and agrees empirically with the late time acceleration. However, it is still unknown in first principle by what mechanism the IR cutoff sets the UV cutoff and the vacuum energy. Some other kinds of IR cutoff as well as the linear combinations of them have also been considered for cosmological purpose[6]. In section 4, we consider other IR cutoffs other than the future event horizon, namely, IR cutoffs set by the particle horizon and the Hubble scale. We find that for the case of particle horizon, the corrections during inflation is almost the same as the future event horizon case. While the IR cutoff set by the Hubble scale seems problematic since the corrections fail to decay. We end with the conclusions and discussions in section 5.

2 The Evolution of Holographic Dark Energy

For simplicity, we consider inflation driven by a single minimally coupled inflaton field. The Friedman equation is

$$3M_p^2H^2 = \frac{1}{2} \dot{\varphi}^2 + V + 3c^2M_p^2R_h^{-2}, \quad (3)$$

We assume that the holographic dark energy does not couple to the inflaton. Then the equation of motion of the inflaton is not affected by the existence of the holographic dark energy,

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi = 0. \quad (4)$$

From (4) and the time derivative of (3), we derive another equation

$$-2M_p^2\dot{H} = \dot{\varphi}^2 + 2c^2M_p^2R_h^{-2} \left(1 - \frac{1}{R_hH}\right) \quad (5)$$

And we impose the usual slow-roll conditions,

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad |\epsilon| \ll 1, \quad (6)$$

$$\delta \equiv -\frac{\ddot{\varphi}}{H\dot{\varphi}}, \quad |\delta| \ll 1, \quad (7)$$

The validity of these slow-roll conditions will be discussed later in this section.

In the inflation models without holographic dark energy, the background equations turns into pure algebraic equations by using slow roll approximation. While when we
take holographic dark energy into consideration, as \(R_h\) is non-local in the scale factor, we still have to solve a system of differential equations. In \(\textit{[3]}\), a convenient method to solve the equations is carried out by taking \(\Omega_\Lambda \equiv \rho_\Lambda/(3M_p^2H^2)\) as the unknown function. The Friedman equation \(\textit{[3]}\) is rewritten as

\[
\frac{1}{aH} = \frac{1}{a} \sqrt{1 - \Omega_\Lambda} \sqrt{\frac{3M_p^2}{V}}.
\]  

Combining the definition of \(R_h\) and \(\Omega_\Lambda\) with \(\textit{[8]}\), we have the differential equation

\[
\Omega'_\Lambda = -2\Omega_\Lambda(1 - \Omega_\Lambda) \left(1 - \frac{\sqrt{\Omega_\Lambda}}{c}\right),
\]

where the prime denotes the derivative with respect to \(x \equiv \ln a\).

The solution of \(\textit{[9]}\) can be written as

\[
-2x + \text{constant} = \frac{1}{1 - \sqrt{\Omega_\Lambda}} + \ln \Omega_\Lambda - \frac{3}{2} \ln(1 - \sqrt{\Omega_\Lambda}) - \frac{1}{2} \ln(1 + \sqrt{\Omega_\Lambda}),
\]

when \(c = 1\). In the \(\Omega_\Lambda \to 0\) limit, the above equation \(\textit{[10]}\) reduces to \(\Omega_\Lambda \sim a^{-2}\), in agreement with \(\textit{[3]}\).

For the \(c \neq 1\) case,

\[
-2x + \text{constant} = -\frac{1}{1 - c^2} \left(-c^2 \ln(1 - \Omega_\Lambda) + c^2 \ln \Omega_\Lambda + c \ln \frac{1 + \sqrt{\Omega_\Lambda}}{1 - \sqrt{\Omega_\Lambda}}\right) + 2 \ln \left(c - \sqrt{\Omega_\Lambda}\right) - \ln \Omega_\Lambda
\]

The \(c \neq 1\) case has a smooth \(c = 1\) limit, which agrees with \(\textit{[10]}\). A divergence occurs at \(\Omega_\Lambda = c^2\). It is due to the big rip singularity in the phantom-like cosmology when \(c < 1\). It can be shown that when \(\Omega_\Lambda \to 0\), it also scales as \(\Omega_\Lambda \sim a^{-2}\), the same as in the \(c = 1\) case.

We present the numerical solutions in Fig\(\textit{[1]}\) where we give one example in each class \((c = 1, c < 1\) and \(c > 1\)). Note that \(c = 1\) is a natural value of \(c\), \(c = 0.8\) is the best fit value for SN+CMB+BAO\(\textit{[7]}\), and \(c = 1.2\) is taken for comparison. In \(\textit{[7]}\), there are best fit values from other experiments such as \(c = 0.6\) from X-ray gas mass fraction of rich clusters, the solution behaves qualitatively the same in each class. We observe that when \(c \geq 1\), the universe will undergo a long holographic dark energy dominated period given a dark energy dominated initial condition. While once the inflaton takes a significant part in the energy density, it will quickly become
Figure 1: The evolution of $\Omega_\Lambda$ as a function of the logarithm of the scale factor $a$. Here $a$ is not normalized to $a_0 = 1$, the normalization is chosen for numerical convenience. We neglected the change of the inflaton energy density in time for simplicity.

In the $c < 1$ case, the critical point lies at $\Omega_\Lambda = c^2$ instead of $\Omega_\Lambda = 1$. It is due to the divergence at $\Omega_\Lambda = c^2$ in (11). Note that in this figure we have assumed the inflaton energy stays constant. A more rigorous calculation should take the time variation of $\varphi$ into account.

Other background quantities such as $H$ and $\varphi$ during inflation can be calculated using $\Omega_\Lambda$ by (8) and (4).

Since it seems difficult to give an initial condition of $\Omega_\Lambda$ from first principle, we determine $\Omega_\Lambda$ by experiments. From the WMAP experiment[8], $\Omega_\Lambda \simeq 0.76$ in the present time. This can help to fix an initial condition for $\Omega_\Lambda$ during inflation.

It can be shown that in the radiation dominated and matter dominated epochs, the scaling property of $\rho_\Lambda$ is $\rho_\Lambda \sim a^{-2}$. Then, from the number of e-folds[9],

$$N_{\text{COBE}} = 62 - \ln(10^{16}\text{GeV}/V_{\text{end}}^{1/4}) - \frac{1}{3} \ln(V_{\text{end}}^{1/4}/\rho_{\text{reh}}^{1/4}),$$

when we assume that the reheating happens immediately after the inflation, and the reheating period is short enough, we get the initial value of $\rho_\Lambda$,

$$\rho_{\text{COBE}} = 1.2 \times 10^{-9} \left(\frac{T_{\text{reh}}}{10^{16}\text{GeV}}\right)^4 M_p^4 = 4.2 T_{\text{reh}}^4$$

Translated into $R_h$, this is

$$R_h = 5c \times 10^4 \left(\frac{10^{16}\text{GeV}}{T_{\text{reh}}}\right)^2 M_p^{-1}. $$

5
It is very sensitive to $T_{\text{reh}}$, the reheating temperature. We will see that the above relation provides a constraint on the reheating temperature, after combined with the effect of the holographic dark energy on the spectral index.

Before calculating the perturbations, let us pause to discuss the validity of the slow-roll conditions. From (9) we get

$$\dot{\Omega}_\Lambda = 2H\Omega_\Lambda \left( \frac{\sqrt{\Omega_\Lambda}}{c} - 1 \right) (1 - \Omega_\Lambda).$$

So $\Omega_\Lambda$ is a slow-roll quantity only when $\Omega_\Lambda \to 1$. In the $c < 1$ case, $\Omega_\Lambda \to c^2$ can satisfy the slow-roll condition as well. Then from Friedman equation in flat space

$$\Omega_\varphi + \Omega_\Lambda = 1,$$

we conclude that $\Omega_\varphi$ is a slow-roll quantity only when $\Omega_\Lambda \to 0$ or $\Omega_\Lambda \to 1$ (and $\Omega_\Lambda \to c^2$ when $c < 1$). For a intermediate value of $\Omega_\Lambda$, either $\epsilon$ or $\delta$ slow-roll condition (or both of them) is broken. In the following, we mainly consider the limit when $\Omega_\Lambda$ is small enough not to spoil the slow-roll conditions. Although the slow-roll regime does not take a large part in the whole parameter space, it is interesting enough from the experimental point of view. It is because as we will see in the following sections, the existence of the holographic dark energy produces a rather red spectrum. A slow-roll inflation with a small part of holographic dark energy already conflicts with the CMB experiment if this regime lies in the observable part of inflation. What we are interested in is to give a constraint of the upper bound of holographic dark energy at the beginning of the observable inflation. This constraint can be translated into a constraint on the reheating temperature. So to study the slow-roll regime is enough for our purpose.

3 Perturbations and the Spectrum

Since the holographic dark energy depends only on the background quantities, it does not seem to produce its own perturbations. So the standard perturbation equations stay unchanged. From these equations, and use the equation (14), we can get the diagonized equation for $\phi$ in the longitudinal gauge,

$$\ddot{\phi} + \left( H - \frac{2\ddot{\varphi}}{\dot{\varphi}} \right) \dot{\phi} + \left( 4\dot{H} - H \frac{2\ddot{\varphi}}{\dot{\varphi}} + \frac{\dot{\varphi}^2}{M_p^2} \right) \phi - \frac{\nabla^2}{a^2} \phi = 0,$$
where \( \phi \) is the perturbation of the metric, \( ds^2 = a^2 (- (1 + 2\phi) d\tau^2 + (1 - 2\phi) dx^i dx^i) \).

Let \( u \equiv \phi/\dot{\phi} \), then the equation for \( u \) reads

\[
\frac{d^2}{d\tau^2} u_k + \frac{1}{\tau^2} (-4\epsilon + \delta + \sigma) u_k + k^2 u_k = 0,
\]

where \( \tau \) is the conformal time and

\[
\sigma \equiv \frac{\dot{\phi}^2}{H^2 M_p^2}.
\]

It takes the form of a Bessel equation. As usual, we can write down its solution and determine the normalization constants by the usual procedure.

In order to compare with the CMB experiments, it is useful to single out a conserved quantity after horizon crossing. The usual comoving curvature perturbation is not conserved when the holographic dark energy is present during inflation. It is because the inflaton field fluctuates while the holographic dark energy does not, then the perturbation is not adiabatic. In this case, we can use a nearly conserved quantity instead (for a derivation of the general nearly conserved quantity in a slow-roll inflation, see [11]),

\[
\mathcal{R} = \frac{2M_p^2 H^2}{\dot{\phi}^2} \left( \frac{\dot{\phi}}{H} + \phi \right) \exp \left( 2c^2 \int_t^{t_{LS}} \frac{1}{R_h^2 H} \left( 1 - \frac{1}{R_h H} \right) dt \right),
\]

with the power spectrum

\[
\mathcal{P}_R = \frac{H^4}{4\pi^2 \dot{\phi}^2} \exp \left( 4c^2 \int_t^{t_{LS}} \frac{1}{R_h^2 H} \left( 1 - \frac{1}{R_h H} \right) dt \right),
\]

where \( t_{LS} \) is the time of the last scattering surface. Since the holographic dark energy decays fast, where to put the upper bound of the integration is not important.

The spectral index takes the form

\[
n_s - 1 = -8\epsilon + 2\delta + 2\sigma = -4\epsilon + 2\delta - \frac{4c^2}{R_h^2 H^2} \left( 1 - \frac{1}{R_h H} \right).
\]

Now let us derive a model-independent formula for the corrections to the spectral index produced by the holographic dark energy. Since the slow roll parameter

\[
\eta \equiv \epsilon + \delta
\]

depends only on the form of the inflaton potential and the Hubble constant, it is not influenced by the holographic dark energy. The inflaton potential is set by the
model, so is not affected by the existence of the holographic dark energy. The Hubble constant is calculated backwards \( N_{\text{COBE}} \) e-folds from the final condition of inflation. As we will see, at the time later than that of \( N_{\text{COBE}} \) during inflation, the holographic dark energy should not affect the evolution of the universe. So the Hubble constant at \( N_{\text{COBE}} \) is also not changed whether the holographic dark energy exists or not. When \( \Omega_\Lambda \) is small, the slow roll parameter \( \epsilon \) can be decomposed into

\[
\epsilon = -\frac{1}{2} \sqrt{\frac{3M_p^2}{V^3}} V\dot{\varphi} + \frac{c^2}{R_h^2 H^2} \left( 1 - \frac{1}{R_h H} \right) \equiv \epsilon_0 + \frac{c^2}{R_h^2 H^2} \left( 1 - \frac{1}{R_h H} \right) \quad (24)
\]

where \( \epsilon_0 \) is the original contribution in the inflation models without the holographic dark energy, and \( \frac{c^2}{R_h^2 H^2} \left( 1 - \frac{1}{R_h H} \right) \) is a correction term.

The spectral index can be written as

\[
n_s - 1 = -6\epsilon_0 + 2\eta - \frac{10c^2}{R_h^2 H^2} \left( 1 - \frac{1}{R_h H} \right) \quad (25)
\]

where \(-6\epsilon_0 + 2\eta\) is the standard contribution from the single field inflaton models without the holographic dark energy. Therefore, we see that the effect of the holographic dark energy is to make the spectrum redder. Note that from experiment we do not expect too red a spectrum, there is a constraint from the experiment on \( R_h H \). The constraint can be translated into a constraint on the Hubble constant during the inflation and the reheating temperature by \( (14) \). A constraint \(-\frac{10c^2}{R_h^2 H^2} \left( 1 - \frac{1}{R_h H} \right) > -0.05\) is shown in Fig.2. If we suggest further that \( T_{\text{reh}} \simeq V^{1/4}_{\text{end}} \), then the constraint on the reheating temperature is

\[
T_{\text{reh}}^4 < 1.2 \times 10^{-3} V_{\text{COBE}} \quad (26)
\]

The equation \( (26) \) tells us that the energy scale during the last \( N_{\text{COBE}} \) e-folds of inflation should drop three orders of magnitude. This provides a constraint on the inflation models flavored by the holographic dark energy.

For the monomial potential \( V = \lambda M_p^{4-p} \varphi^p \), if we assume inflation ends at roughly \( \epsilon = 1 \), \( \varphi \) can be expressed as a function of the e-folding number as

\[
\varphi_N = \sqrt{2p N + \frac{p^2}{2} M_p} \quad (27)
\]

so the relation between \( T_{\text{reh}} \) and \( V_{\text{COBE}} \) can be expressed as

\[
T_{\text{reh}}^4 = \left( \frac{4N_{\text{COBE}}}{p} + 1 \right)^{-\frac{2}{p}} V_{\text{COBE}} \quad (28)
\]
Figure 2: Constraint on the Hubble scale during inflation and the reheating temperature. The lower half of the figure is not favored by experiments because it produces too red a spectrum (the correction to $n_s - 1$ is smaller than $-0.05$). The correction is not sensitive to $c$.

For example, the $m^2\varphi^2$ model gives $T_{\text{reh}}^4 \simeq 8 \times 10^{-3}V_{\text{COBE}}$, so for this model other mechanism is needed to be consistent with the existence of the holographic dark energy. While the $\lambda\varphi^4$ model has $T_{\text{reh}}^4 \simeq 3 \times 10^{-4}V_{\text{COBE}}$, which is consistent with the existence of the holographic dark energy.

For the small field inflation models, $V = V_0 \left(1 - \left(\frac{\varphi}{M_P}\right)^p\right), p \geq 2$, the energy scales do not change much during inflation. So the small field inflation models are not favored by the existence of the holographic dark energy.

We do not need to worry about the late time entropy perturbation, because the entropy perturbation decays fast with the decay of the holographic dark energy.

It is also an interesting issue whether and how the IR cutoff set by the future event horizon affect the observed CMB spectrum. In [12], it has been studied in a simple toy model how the finiteness of the future event horizon influences the angular power spectrum, especially the low $l$ behavior. Since the boundary conditions studied in [12] are not respected during the expansion of the universe, we take a slightly different treatment: we simply cut off the primordial spectrum and set the primordial spectrum to zero when $\pi/k$ is larger than the present future event horizon. When $\pi/k$ is smaller than the present future event horizon, the primordial spectrum is not modified.

For illustrative purpose, we show in Fig.3 the CMB power spectrum with and
Figure 3: The CMB temperature angular power spectrum. In this figure, we considered the spectrum with and without primordial holographic dark energy corrections, as well as with and without late time direct IR cutoff. If we take the late time IR cutoff into consideration, the curves with and without holographic dark energy nearly coincide. So we see that the late time IR cutoff can hide the corrections given by the primordial holographic dark energy. And without the late time IR cutoff, the power spectrum can become extremely red in some parameter region. This together with Fig. 2 provide a constraint on the Hubble scale during inflation and the reheating temperature.
without the contribution of holographic dark energy, as well as with and without the late time IR cutoff. The parameter for the holographic dark energy sector is chosen as the best fit value of SN+CMB+BAO\([7]\). In Fig.3, we take a large correction from holographic dark energy as an example. Such a large correction conflicts with the experimental data, and as shown in Fig.2, it is in the “Not Acceptable” region.

Now we have seen that even a small amount of holographic dark energy can produce a rather red spectrum, which can give a constraint on the reheating temperature \(T_{\text{reh}}\). To have a better understanding of the whole picture, here we also present a simple (and crude) estimate for the regime where the holographic dark energy is dominate or of the same order as the energy density of the inflaton. As discussed in the last section, in this regime, the slow-roll approximation breaks down. According to \([13]\), even for the fast rolling we can still take

\[
\delta \varphi \sim \frac{H}{2\pi},
\]

just like the slow-roll inflation case, where \(\delta \varphi\) is the frozen amplitude of the inflaton field fluctuations outside the horizon. The existence of the holographic dark energy results in a larger Hubble constant, so the fluctuations generally becomes larger in this regime. This implies that the power spectrum was made redder by a large holographic dark energy in the early stage of the inflation. Therefore, the observed fluctuations of the inflaton should be created during the slow-roll regime.

4 Other Realizations of Holographic Principle

Although the IR cutoff set by the future event horizon is theoretically natural and experimentally interesting, there are other candidates of IR cutoffs. The arbitrariness comes from the lack of knowledge of the holographic principle. So in this section, we consider as well two other kinds of IR cutoffs, namely, the particle horizon and the Hubble scale, even though for these two cases the equation of state for dark energy does not agree with data. Note that the perturbation equations \((17)\) and \((18)\) still hold for these two cases.

For the particle horizon case, the formulae are similar to the case with the future
event horizon. The particle horizon is defined as

$$R_H = a \int_0^t \frac{dt}{a}, \quad (30)$$

and the vacuum energy density is given by

$$\rho_\Lambda = 3c^2 M_p^2 R_H^{-2}. \quad (31)$$

The Friedman equation takes the form

$$3M_p^2 H^2 = \frac{1}{2} \dot{\phi}^2 + V + 3c^2 M_p^2 R_H^{-2}, \quad (32)$$

Take derivative respect to the equation (32), and use (4), we can get

$$-2M_p^2 \dot{H} = \dot{\phi}^2 + 2c^2 M_p^2 R_H^{-2} \left( 1 + \frac{1}{R_H H} \right) \quad (33)$$

For the background equations expressed by $\Omega_\Lambda$, we have

$$\Omega_\Lambda' = -2\Omega_\Lambda (1 - \Omega_\Lambda) \left( 1 + \sqrt{\Omega_\Lambda} c \right) \quad (34)$$

where $\Omega_\Lambda$ is the ratio of the vacuum energy density to the critical density in the particle horizon case.

As in the future event horizon case, we can get the nearly conserved quantity,

$$\mathcal{R} = \frac{2M_p^2 H^2}{\dot{\phi}^2} \left( \frac{\dot{\phi}}{H} + \phi \right) \exp \left( 2c^2 \int_{t_0}^{t_L} \frac{1}{R_H H} \left( 1 + \frac{1}{R_H H} \right) dt \right), \quad (35)$$

with the power spectrum

$$\mathcal{P}_\mathcal{R} = \frac{H^4}{4\pi^2 \dot{\phi}^2} \exp \left( 4c^2 \int_{t_0}^{t_L} \frac{1}{R_H H} \left( 1 + \frac{1}{R_H H} \right) dt \right), \quad (36)$$

and the spectral index

$$n_s - 1 = -8\epsilon + 2\delta + 2\sigma = -4\epsilon + 2\delta - \frac{4c^2}{R_H^2 H^2} \left( 1 + \frac{1}{R_H H} \right). \quad (37)$$

The spectral index can be split into two parts each related to the usual slow-roll inflation and to the holographic dark energy

$$n_s - 1 = -6\epsilon_0 + 2\eta - \frac{10c^2}{R_H^2 H^2} \left( 1 + \frac{1}{R_H H} \right) \quad (38)$$

Obviously the presence of the holographic dark energy makes the spectrum redder.
The vacuum energy density during inflation can be calculated similarly to the future event horizon case. If we assume that the vacuum energy related to the particle horizon is of the same order compared with the matter density in the present time, then we have at the e-folds $N_{\text{COBE}}$ before the end of inflation, $R_H \sim R_h$. So the IR cutoff set by $R_H$ gives a similar, but slightly redder spectrum than the future event horizon case.

Finally, let’s consider the IR cutoff set by the Hubble scale. The vacuum energy density is set to

$$\rho_\Lambda = 3c^2 M_p^2 H^2. \quad (39)$$

The Friedman equation can be written as,

$$3M_p^2(1 - c^2)H^2 = \frac{1}{2} \dot{\varphi}^2 + V \quad (40)$$

and as usual we can get the following equation on $\dot{\varphi}^2$

$$-2M_p^2(1 - c^2)\dot{H} = \dot{\varphi}^2 \quad (41)$$

Although $c = 1$ may be the most natural value of $c$, we note that $c = 1$ forces both the kinetic and potential energy of $\varphi$ vanish. A small departure from $c = 1$ leads to a smaller required value of $\varphi$ for a given Hubble constant during inflation. It is not surprising since the IR cutoff set by $H$ can be think of as a redefinition of the Planck mass, and a smaller effective Planck mass (i.e. a stronger gravitational coupling) leads to a smaller required value of the inflaton to drive inflation. However, when we consider the perturbations and calculate the spectrum, the difference is not simply a rescaling of Planck mass. This is because just as in the future event horizon case, the vacuum energy produced by holographic principle depends nonlocally on the Hubble constant, so the vacuum energy do not produce perturbation by itself.

The nearly conserved quantity takes the form

$$\mathcal{R} = \frac{2M_p^2 H^{2+2c^2}}{M^{-2c^2} \dot{\varphi}^2} \left( \frac{\dot{\varphi}}{H} + \varphi \right), \quad (42)$$

with the power spectrum

$$P_\mathcal{R} = \frac{H^{4+4c^2}}{4\pi^2 M^{4c^2} \dot{\varphi}^2}, \quad (43)$$

where $M$ is a constant of the dimension of energy.
The spectral index can be written as
\[ n_s - 1 = -8\epsilon + 2\delta + 2\sigma = -(4 + 4c^2)\epsilon + 2\delta. \] (44)

Note that the power spectrum is suppressed from the usual one by a factor \((H/M)^2c^2\). If we take \(M\) as the Planck scale or the string scale, the problem of a hierarchy of scales during inflation can be solved or eased by this kind of IR cutoff. But \(M\) cannot be determined as usual by the final condition of inflation. Because the models with this kind of IR cutoff have a non-decaying correction from the holographic dark energy. As a toy model, it cannot come back to the standard inflation models.

Therefore, to take the Hubble scale as the IR cutoff is problematic. The long-lived entropy perturbation is produced. We need extra mechanism for the entropy perturbation to decay. And furthermore, the post inflationary period is problematic because the non-decaying large correction to the standard picture may modify some exactly known results such as the nucleosynthesis. One possible solution is to assume that the holographic bound for the vacuum energy is not saturated for a large portion of the history of the universe. In [14], such a possibility is investigated.

5 Conclusions

In conclusion, in this paper, a detailed study on the effects of the holographic dark energy to the inflation is carried out.

We find that the corrections on the spectral index generally make the scalar primordial power spectrum redder than that without the holographic dark energy. In order that the perturbations do not become too large to conflict with the experiments, a constraint on the reheating temperature is given. The constraint is that the energy scale should drop three orders of magnitude during the last \(N_{\text{COBE}}\) e-folds of inflation. As we take the monomial potential for inflation for example, the \(\lambda\varphi^4\) model satisfies the requirement while the monomial \(m^2\varphi^2\) model and the small field inflation models have to add extra mechanism to live with the holographic dark energy. In our discussion, we assume that the reheating happens right after the end of inflation, it could be interesting to consider the case of late-time reheating.
We compared the IR cutoff set by the particle horizon and the Hubble scale during inflation with that by the future event horizon. We find that the cutoff set by the particle horizon provide a similar but slightly redder power spectrum than the one produced in the future event horizon case. The cutoff set by the Hubble scale seems problematic since the correction does not decay during the whole evolutionary history of the universe, which may spoil some exactly known results.

Acknowledgments

This work was supported by grants of NSFC. BC was also supported by the Key Grant Project of Chinese Ministry of Education (NO. 305001). We thank Xin Zhang for discussions.

References

[1] A. Cohen, D. Kaplan and A. Nelson, [hep-th/9803132], Phys. Rev. Lett. 82 (1999) 4971.

[2] S.D.H. Hsu, Phys. Lett. B 594, 13(2004).

[3] M. Li, Phys.Lett. B603 (2004) 1, [hep-th/0403127]; Q.-G. Huang and M. Li, JCAP 0408 (2004) 013, [astro-ph/0404229]. Qing-Guo Huang, Miao Li, [hep-th/0410095]. JCAP 0503 (2005) 001.

[4] J. Polchinski, [hep-th/0603249]. Rapporteur talk at the 23rd Solvay Conference in Physics, December, 2005.

[5] A. H. Guth, Phys. Rev. D 23, 347 (1981); A. D. Linde, Phys. Lett. B 108, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982)

[6] Shin’ichi Nojiri, Sergei D. Odintsov, [hep-th/0506212]. Gen.Rel.Grav. 38 (2006) 1285-1304.

[7] Qing-Guo Huang, Yungui Gong, [astro-ph/0403590]. JCAP 0408 (2004) 006. Hsien-Chung Kao, Wo-Lung Lee, Feng-Li Lin, [astro-ph/0501487]. Phys.Rev. D71
(2005) 123518. Xin Zhang, astro-ph/0504586, Int. J. Mod. Phys. D14 (2005) 1597-1606; Xin Zhang, Feng-Quan Wu, astro-ph/0506310, Phys. Rev. D72 (2005) 043524; Zhe Chang, Feng-Quan Wu, Xin Zhang, astro-ph/0509531, Phys. Lett. B633 (2006) 14-18; Xin Zhang, astro-ph/0609699, Phys. Rev. D 74 (2006) 103505.

Xin Zhang, Feng-Quan Wu, astro-ph/0701405

[8] D. N. Spergel et al., astro-ph/0603449.

[9] David H. Lyth, Antonio Riotto, hep-ph/9807278, Phys.Rept. 314 (1999) 1-146.

[10] Viatcheslav F. Mukhanov, H. A. Feldman and Robert H. Brandenberger, Phys. Rept. 215, (1992) 203

[11] Bin Chen, Miao Li, Tower Wang and Yi Wang, astro-ph/0610514

[12] Kari Enqvist and Martin S. Sloth, hep-th/0406019, Phys.Rev.Lett. 93 (2004) 221302; Kari Enqvist, Steen Hannestad and Martin S. Sloth, astro-ph/0409275 JCAP 0502 (2005) 004.

[13] Andrei Linde, hep-th/0110195, JHEP 0111 (2001) 052.

[14] B. Guberina, R. Horvat, H. Nikolic, astro-ph/0611299