Influence of the stiffness and the speed on the stick-slip process

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Abstract. In this paper the authors experimentally investigated the influence of the linear speed and the stiffness of a single degree-of-freedom mass-spring system on the evolution of the stick-slip process. Using the UMT-2 Tribometer a steel mass was sliding on glass surface with a linear speed varied between 0.02 mm/s and 1 mm/s and spring stiffness varied between 40 N/m and 442 N/m. The experimental results evidenced that increasing both the linear speed and the spring stiffness can reduce the stick-slip process. Thus, the authors proposed an original friction model to simulate the stick-slip process by considering the dependence of the dynamic coefficient of friction with the sliding speed in the “slip” phase. The theoretical model is based on dynamic equilibrium of all the forces acting on the mass in transient phases. The proposed theoretical model was validated with experiments by simulating the stick-slip process for given linear speed and spring stiffness.

1. Introduction
In many sliding tribological systems operating in dry conditions, the friction forces have a nonlinear behavior as result of the alternately between stick and slip processes on the contact surface. The differences between static and dynamic friction coefficients, correlated both with the system rigidity and sliding speed generate the stick-slip motion. Generally, in mechanical applications the stick – slip motion is generate in low speed conditions (as in slide guide for machine tools). In other tribological systems (as in violins) the elasticity of the violin strings is essentially for the strings vibrations caused by the stick-slip processes. In many mechanical applications the values for static and kinetic friction coefficients are considered as constant values depending on the pair of materials in contact. To obtain a rigorous dynamic model in stick-slip conditions cannot be use the classical Coulomb’s friction coefficient. Also, considering constant values for both static and dynamic friction coefficient cannot explain the appearance of the stick-slip processes.

Elmer [1] suggests that two important aspects reer to the static and dynamic friction coefficient must be considered. First consideration consists in a dependence of the static friction coefficient with the sticking time. So, by increasing of the sticking time it can be observed the increasing of the static friction coefficient.

Nordhagen [2] propose for static friction coefficient \( \mu_s \), following dependence: \( \mu_s \propto a + b \cdot t^{0.1} \), where \( a \) and \( b \) are material constants and \( t \) is the sticking time. It can be supposed that increasing of the sticking time a plastic relaxation on the top of roughness can appears with increasing of the real contact area and, as result an increasing of the static friction coefficient.
The second problem is refer to the influence of the sliding speed on dynamic friction coefficient \( \mu_k \). Some mathematical models indicate the variation of the dynamic friction coefficient with sliding speed. For small sliding speeds Rao [3] proposed a linear dependence between dynamic friction coefficient and sliding speed given by following relation:

\[
\mu_k = \mu_s - \frac{a}{W} \cdot v,
\]

where \( W \) is normal load and \( v \) is the sliding speed.

In the presence of the lubricant in a sliding tribological system Zuleeg [4] propose to be analyzed the stick-slip process by using for friction coefficient the Stribeck curve described by following equation:

\[
\mu_s = \text{sgn}(v) \cdot \left| v \right| \cdot a_1 + a_2 + a_3 \cdot \exp\left(-\frac{|v|}{a_4}\right)
\]

(1)

where \( v \) is sliding speed and \( a_1, a_2, a_3, a_4 \) are parameters depending of the materials’ pair. For \( a_1=1, a_2=0.1, a_3=0.3 \) and \( a_4=0.1 \), it can obtain a classical Stribeck curve having a static friction coefficient \( \mu_s=0.4 \) (for \( v=0 \)).

Majdoub et al. [5] studied the dynamic responses of a single degree-of-freedom mass-spring system having sliding contact by including a velocity-dependence friction coefficient by a pseudo-polynomial friction model.

The stick-slip motion in micro systems has been analyzed by Liu et al. [6] considering five friction models as Coulomb model, Stribeck model, Dahl model, LuGre model, and the elastoplastic friction model. The experiments realized by authors demonstrated that the best model for stick-slip in micro systems is the LuGre model. Other complex stick-slip models have been presented in [7,8]. Stoica et al. [9] studied experimentally the stick-slip processes in the disc brake operating in the range of small and very small speeds.

In the present paper the authors experimentally investigated the variation of the friction forces as function of the sliding speed and stiffness of a single degrees of freedom mass-spring system and based on the theoretical friction model solved the nonlinear dynamic equation for stick-slip motion.

2. Experimental equipment and procedure

The experiments were performed on a classical oscillator as mass-spring system. Figure 1 illustrates the equipment and details of the experimental configuration used to investigate the stick-slip process. The mechanical oscillator consists in a steel mass (0.242 kg) connected by an elastic spring to a fixed support attached to the force sensor from UMT-2 Tribometer. The mass is placed on a glass plane surface that moves with a constant speed \( v \). In the beginning, the mass moves together with the glass plate due to the adherence between the two surfaces and the elastic force in the spring increases to a maximum value \( F_s \). Once the spring is largely deformed the mass is sliding on the glass surface in opposite direction for a short time when elastic force is equal to a minim friction force \( F_k \). The “stick” and “slip” phases are evidenced by variation of the elastic force measured by the sensor and displayed on the computer, as function of time and moving distance.

The experiments were performed at sliding speeds between 0.02 to 1 mm/s and spring stiffness of 40 N/m to 442 N/m in dry conditions. Previously, the glass surface and the contact mass surface were cleaned with grease and oil dissolvent and the laboratory’s temperature and humidity varied between (22 - 24°C) and (40 - 50%), respectively.

The friction surface of the mass has been finished with a roughness Ra=(0.6 - 0.8) \( \mu m \) and the contact pressure between mass and glass plate was 0.006 MPa. The maximum sliding distance for the tests was 25 mm and the time varied as function of sliding speed.
Figure 1. Experimental setup for stick-slip tests: (a) general view of UMT-2 Tribometer and (b) details of the mass-spring system.

In figures 2 and 3 are presented the variation of the measured force $F_x$ as function of time and variation of measured force $F_x$ as function of sliding distance, respectively for a sliding speed of 0.1 mm/s and spring stiffness $k=48.6$ N/m. The force $F_x$ measured by the sensor is in fact the friction force between the mass and glass surface if neglecting the inertial force which appears in the “slip” phase of the stick-slip process. The two phases are very evident: adherence between the mass and the glass plate with a relatively long period (“stick” phase) and sliding of the mass on the glass plate in a very short time (“slip” phase). The variation of the friction forces, the static friction coefficient and the time were investigated for every slip phase during the tests and further used in the proposed mathematical friction model.

Figure 2. Variation of the friction force in time.
3. Influence of the speed on the stick-slip process

To evidence the influence of the linear speed on the stick-slip process a lot of tests with variation of the linear speed between 0.02 mm/s and 1 mm/s have been realized. All the tests were realized with a stiffness having value of 40 N/m. For every linear speed of the table three tests were realized on a distance of 20 mm. The representative registration curves for each test have been superposing in figure 4. Following remarks can be made:

- The stick-slip process is maintained between (0.02 and 0.5) mm/s linear speed values. To the linear speed of 0.5 mm/s important decreasing of the variations between stick and slip phases can be observed. Increasing of the linear speed to 1 mm/s leads to complete reducing of the stick–slip process.
- In the absence of the stick-slip process the friction force have a constant minimum value comparable to the minimum values of the friction forces obtained in the stick-slip processes.
- The values of the static friction coefficient definite as the ratio between maximum friction force and the weight force of the mass varied between 0.15 and 0.2. The minimum values of the friction coefficients obtained in stick-slip process varied between 0.095 and 0.11 while the dynamic friction coefficient obtained for linear speed of 1 mm/s has a mean value of 0.092.
4. Influence of the stiffness on the stick-slip process
In all friction process the stiffness of the tribological system can be an important parameter to develop or to reduce the stick-slip process. To evidence the importance of the stiffness in stick-slip process the authors realized the tests with modified stiffness of the spring, from 46.76 N/m to 442 N/m. The experiments were realized in dry conditions for a linear speed of 0.1 mm/s and a distance of 25 mm.
The results are presented in figure 5. Following remarks can be made:
- At low stiffness the stick-slip process is characterized by high differences between maximum and minimum friction forces and a minimum number of oscillations.
- By increasing of the stiffness increases the number of the oscillations and decreases the differences between maximum and minimum friction forces.
- For k=442 N/m the stick-slip process is considerably reduced and the friction force has minimum value. The dynamic friction coefficient corresponding to this stiffness have values between 0.06 and 0.07.

![Figure 5. Influence of the stiffness on the stick-slip process.](image)

5. Mathematical model
In figure 6 are presented the forces acting on the mass \( m \) in the “slip” process during the stick-slip process. Following forces act on the mass: elastic force \( F_e \), friction force \( F_f \) and inertial force \( F_i \). The direction of these three forces correspond to the figure 6. The table of the Tribometer have a constant linear speed \( v \). In a first adherence phase (“stick” phase) the mass is displaced to the distance \( x_0 \) in relation to a fixed mark. In the second phase (“slip” phase) the mass have a fast displacement in opposite direction at a speed \( \dot{x} = \frac{dx}{dt} \). Also, the variation of the mass speed for a very short time leads to an acceleration \( \ddot{x} = \frac{d^2x}{dt^2} \). In each moment the force recorded by the sensor, \( F_x \) corresponds to the variation of the elastic force \( F_e \).
The maximum elastic force in “stick” phase is:

\[ F_{es} = k \cdot x_n \]  

(2)

In the “slip” phase, the dynamic equilibrium of the mass \( m \) leads to following equation:

\[ F_e - F_i - F_f = 0 \]  

(3)

Elastic force \( F_e \) is given by equation:

\[ F_e = k \cdot (x_n - x) \]  

(4)

Inertial force \( F_i \) is given by equation:

\[ F_i = m \cdot \frac{d^2x}{dt^2} \]  

(5)

Friction force \( F_f \) is given by equation:

\[ F_f = \mu_s \cdot m \cdot g \]  

(6)

For dynamic friction coefficient it was considered the Rao [3] model:

\[ \mu_s = \mu_s + c \cdot \left( \frac{dx}{dt} - v \right) \]  

(7)

where \( c \) is a constant determined according to the experimental results.

By including Equations (4) - (7) in (3) following differential equation results:

\[ \frac{d^2x}{dt^2} - 2 \cdot \alpha \cdot \frac{dx}{dt} + \omega^2 \cdot x = A \]  

(8)

where:

\[ 2 \cdot \alpha = c \cdot g \cdot \omega^2 = \frac{k}{m} \]; \( A = \omega^2 \cdot x_n - \mu_s \cdot g - c \cdot g \cdot v \), \( g \) being the gravitational acceleration.

Imposing initial conditions: \( t=0, x=0 \) and \( \frac{dx}{dt} = v \), the differential equation (8) leads to the following relation for sliding distance \( x \) as function of time:

\[ x(t) = \frac{A}{r_1 \cdot r_2} + B_1 \cdot e^{r_1 \cdot t} + B_2 \cdot e^{r_2 \cdot t} \]  

(9)

where:

\[ r_1 = \alpha + \sqrt{\alpha^2 - \omega^2} ; \quad r_2 = \alpha - \sqrt{\alpha^2 - \omega^2} \]  

(10)
Equation (9) can be solved in hypothesis of \( r_1 \) and \( r_2 \) to be real values. Means that the constant \( c \) satisfied following condition:

\[
c > \pm \frac{2}{g} \sqrt{\frac{k}{m}}
\]  

(12)

The dynamic friction coefficient must be lower than static friction coefficient \( (\mu_d < \mu_s) \). So according to equation (7) the constant parameter \( c \) must have negative value for given stick–slip conditions.

6. Stick-slip simulation

The simulation program for stick-slip process has been organized by a succession of adherence and slip phases in time in accord with the experimental results. The simulating program has been based on the experimental results obtained for linear speed \( v=0.1 \text{ mm/s} \) and spring stiffness \( k=48.6 \text{ N/m} \). In figure 7(a) are indicated the points of maximum forces \((A_i)\) and minimum forces \((B_i)\), respectively for the above mentioned experiment. Corresponding to the points \( A_i \) were determined the values of the static friction coefficients \( \mu_{si} \) \( (\mu_{si} = \frac{F_{di}}{m \cdot g}) \) and the values for minimum friction coefficients corresponding to points \( B_i \) \( (\mu_{min,i} = \frac{F_{bi}}{m \cdot g}) \). In figure 7(b) is presented a detailed sequence of the “slip” process. The time of the “slip” process, \( t_{si} \), is determined by relation \( t_{si} = t_{Bi} - t_{Ai} \). The maximum slip distances for each “slip” phases were determined with relations:

\[
x_{si} = \frac{(F_{di} - F_{bi})}{k}
\]  

(13)

Also, the initial for starting the “slip” phases \( x_{oi} \) were determined according to relations:

\[
x_{oi} = \frac{F_{di}}{k}
\]  

(14)
According to the diagrams presented in figure 7, following limits for experimental parameters have been obtained: $\mu_{si} = (0.15 - 0.2)$, $\mu_{min,i} = (0.08 - 0.09)$, $t_{si} = (0.5-0.6)$ seconds, $x_{si} = (3.5 - 5.6)$ mm, $x_{oi} = (7.8-9.9)$ mm. The simulation has been focused on the test presented in figure 7.

In the first step of simulation we intended to respect the values of the sliding distances $x_{si}$ and sliding time $t_{si}$ for all the four “slip” phases using the given values for maximum and minimum forces. So, equation (8) has been used for every “slip” phase to determine the constant parameter $c$ imposing following condition:

To determine the values of the constant parameter $c$ following nonlinear equations has been solved:

$$x(t_{si}, c) - x_{si} = 0$$

(15)

where $x(t_{si}, c)$ are the solutions of Eq. (8) determined for the slip time $t_{si}$, and $x_{si}$ are the sliding distances realized in “slip” phases.

For all four “slip” phases the values of the parameter $c$ varied between -10 s/m and -15 s/m. Using an average value for parameter $c = -12$ s/m has been simulated the stick –slip process, step by step according to following relations:

- for $0 < t < t_{A1}$, $F_x(t) = k \cdot v \cdot t$;
- for $t_{A1} < t < t_{B1}$, $F_x(t) = k \cdot [x_{oi} - x(t - t_{A1})]$;
- for $t_{B1} < t < t_{A2}$, $F_x(t) = F_{A1} - F_{B1} + k \cdot v \cdot (t - t_{B1})$.

Step (1) corresponds to the first “stick” phase, step (2) corresponds to the first “slip” phase, step (3) corresponds to the second “stick” phase and the next phases have similar succession.

In figure 8 is presented the distribution of the simulated forces $F_x$. A comparison between the variation of the registered (figure 7(a)) and the simulated forces (figure 8) highlights some differences caused, especially by the use of an average value for the parameter $c$. 

Figure 7. Variation of the maximum and minimum forces (a) and variation of the slip time (b) for linear speed $v=0.1$ mm/s and stiffness $k=48.6$ N/m.
Figure 8. Variation of the simulated forces $F_x$ for linear speed $v=0.1$ mm/s and stiffness $k=48.6$ N/m.

Also, the dynamic friction coefficient described by Eq.7 has a Stribeck curve as is presented in figure 9. It can be observed in a first short time an important decreasing of the dynamic friction coefficient to a minimum value followed by slower growth.

Figure 9. Variation of the dynamic friction coefficient during the “slip” phase for linear speed $v=0.1$ mm/s and stiffness $k=48.6$ N/m

7. Conclusions
The influences of the speed and stiffness on the stick-slip process have been experimentally evidenced by a single degree-of-freedom mass-spring system. The experiments were realized for a dry steel surface on a glass surface at a contact pressure of 0.006 MPa. The linear speed of the glass plate varied between 0.02 mm/s and 1 mm/s. Also, the spring stiffness varied between 40 N/m to 442 N/m.

All the tests were realized in dry conditions. Following important results have been obtained:

- For an imposed stiffness of 40 N/m the stick-slip process is maintained between (0.02 and 0.5) mm/s linear speed values. Increasing of the linear speed to 1 mm/s leads to complete reducing of the stick–slip process.
The values of the static friction coefficient varied between 0.15 and 0.2 and have not a constant value during the test. The dynamic friction coefficient obtained for linear speed of 1 mm/s has a mean value of 0.092.

At low stiffness the stick-slip process is characterized by high differences between maximum and minimum friction forces and a minimum number of oscillations for a given distance.

By increasing of the stiffness increases the number of the oscillations and decreases the differences between maximum and minimum friction forces. For a stiffness of 442 N/m the stick-slip process is considerably reduced and the friction force has minimum value. The dynamic friction coefficients corresponding to this stiffness have values between 0.06 and 0.07.

A mathematical model based on the variation of the dynamic friction coefficient with the sliding speed has been developed.

The simulation was realized for a linear speed of 0.1 mm/s and spring stiffness of 48.6 N/m. Based on the registered diagram have been determined the sliding distances and sliding times during the “slip” phases as basic parameters for determining the variation of dynamic friction coefficient.

A comparison between the variation of the registered and simulated forces highlights some differences caused, especially by the use of an average value for the parameter c included in dynamic friction coefficient.

Finally it can be concluded that the stick-slip processes are depending of a lot of variable parameters as static friction coefficient, adequate model for dynamic friction coefficient, stiffness of the system in correlation with sliding speed, roughness of the contacting surfaces.

8. References

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