QUANTUM CRYPTOGRAPHY

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## CONTENTS\(^1\)

| §   | CIPHERING                  | PAGE |
|-----|----------------------------|------|
| § 2 | QUANTUM KEY DISTRIBUTION   | 9    |
| § 3 | SOME OTHER DISCRETE PROTOCOLS FOR QKD | 14   |
| § 4 | EXPERIMENTS                | 18   |
| § 5 | TECHNOLOGY                 | 26   |
| § 6 | LIMITATIONS                | 34   |
| § 7 | SUPPORTING PROCEDURES      | 35   |
| § 8 | SECURITY                   | 38   |
| § 9 | PROSPECTS                  | 51   |
|     | REFERENCES                 | 52   |

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§ 1. Ciphering

1.1. INTRODUCTION, CRYPTOGRAPHIC TASKS

There is no doubt that electronic communications have become one of the main pillars of the modern society and their ongoing boom requires the development of new methods and techniques to secure data transmission and data storage. This is the goal of cryptography. Etymologically derived from Greek κρυπτός, hidden or secret, and γραφή, writing, cryptography may generally be defined as the art of writing (encryption) and deciphering (decryption) messages in code in order to ensure their confidentiality, authenticity, integrity and non-repudiation. Cryptography and cryptanalysis, the art of codebreaking, together constitute cryptology (λόγος, a word).

Nowadays many paper-based communications have already been replaced by electronic means, raising the challenge to find electronic counterparts to stamps, seals and hand-written signatures. The growing variety of applications brings many tasks that must be solved. Let us name a few. The fundamental task of cryptography is to allow two users to render their communications unintelligible to any third party, while for the two legitimate users the messages remain intelligible. The goal of identification is to verify the identities of the communicating parties. Another cryptographic task is secret sharing: A secret, e.g., a password, is split into several pieces in such a way that when a certain minimal subset of the pieces is put together, the secret is recovered. Other cryptographic applications are, for example, digital signatures, authentication of messages, zero-knowledge proofs, and so on.

At all times people have wished to have the possibility to communicate in secrecy so as to allow nobody to overhear their messages. Archeological excavations have revealed that various types of cryptography had already been used by ancient civilizations in Mesopotamia, India, or China (Kahn [1967]). Four thousand years ago, ancient Egyptians used modified hieroglyphs to conceal their messages. In the Iliad, Homer depicts how Proetus, the king of Argolis, sends Bellerophon to Lycia with “a lethal message, coded symbols inscribed on a folded tablet” (Homer [8th c. B.C.]).

In the 5th century BC, the Spartans in Greece designed the Skytale cryptodevice, based on transposition of letters (Old Spartan Facts). A stripe of parchment or leather was wound around a wooden baton, across which the message was written. When the end of line was reached, the baton was rotated. After the parchment was unwrapped, the letters looked scrambled and only the person who possessed a baton of an identical shape could recover the message.

Another favorite and easy cipher is the substitution cipher, which substitutes each letter of a message with another letter, number or a symbol. An example is the Caesar cipher (Stinson [1995]). To communicate between the Roman legions scattered over the Roman republic, Gaius Julius Caesar used a cipher, where each letter of a message was advanced by three letters in the alphabet: A was replaced by D, B was replaced by E, C by F, and so on. Similar substitution cipher is also described in Kama Sutra.

During the Middle Ages, most cryptosystems were based on transposition or substitution or a combination of both (Leary [1996]). However, neither of these ciphers is secure, because it is possible to break them exploiting various character-
istic properties of the language, such as the frequency of individual letters and their clusters.

The invention of the telegraph in the 1830s enormously facilitated communications between people. This ancestor of modern communications, however, had a serious drawback from the cryptographic point of view – the content of the transmitted message was known to the telegraph operator. As a consequence, various codebooks were designed by people and companies that wanted to keep their communications private. The codebooks translated significant words and phrases into short, nonsensical words. The codes served two purposes: first, they reduced the size of the message and thus decreased the costs because telegrams were charged per transmitted character; and second, if the codebook was kept secret, the codes became a cipher.

The two world wars of the 20th century accelerated the development of new cryptographic techniques. Cryptographers tried to design a system where the encryption and decryption algorithms could be publicly known, but the secrecy of the message would be guaranteed by some secret information, the cryptographic key, shared between the users. In 1917, Gilbert S. Vernam proposed an unbreakable cryptosystem, hence called the Vernam cipher or One-time Pad (Vernam [1926]). Its unconditional security has been proved by Claude E. Shannon (in terms of information theory) in 1949 (Shannon [1949]). The One-time Pad is a special case of the substitution cipher, where each letter is advanced by a random number of positions in the alphabet. These random numbers then form the cryptographic key that must be shared between the sender and the recipient. Even though the Vernam cipher offers unconditional security against adversaries possessing unlimited computational power and technological abilities, it faces the problem of how to securely distribute the key. That is why it did not become widespread as Vernam had hoped. On the other hand, there are many military and diplomatic applications, where the security of communications outweighs the severe key management problems. The Vernam cipher was used by the infamous spies Theodore A. Hall, Klaus Fuchs, the Rosenbergs and others, who were passing atomic secrets to Moscow. Che Guevara also encrypted his messages to Fidel Castro by means of the One-time Pad. It was employed in securing the hot line between Washington and Moscow and it is said to be used for communications between nuclear submarines and for some embassy communications. We will come back to the Vernam cipher later on, as it is this cipher that is very expedient for quantum key distribution.

In 1918, Arthur Scherbius invented an ingenious electric cipher machine, called Enigma, which was patented a year later (Deavours and Kruh [1985]). The Enigma consisted of a set of rotating wired wheels, which performed a very sophisticated substitution cipher. After various improvements, it was adopted by the German Navy in 1926, the German Army in 1928, and the Air Force in 1935, and it was used by the Germans and Italians throughout World War II. The military Enigma had incredible $159 \times 10^{18}$ possible settings (cryptographic keys). The immense number of potential keys led Alan Turing to construct the first electronic computer, which helped break the Enigma ciphers in the course of the War. Today a Pentium-based computer can unscramble an Enigma-encrypted message within minutes.
1.2. ASYMMETRICAL CIPHERS (PUBLIC-KEY CRYPTOGRAPHY)

A new surge of interest in cryptography was triggered by the upswing in electronic communications in the late 70s of the 20th century. It was essential to enable secure communication between users who have never met before and share no secret cryptographic key. The question was how to distribute the key in a secure way. The solution was found by Whitfield Diffie and Martin E. Hellman, who invented public-key cryptography in 1976 (Diffie and Hellman [1976]). The ease of use of public-key cryptography, in turn, stimulated the boom of electronic commerce during the 1990s. Notice, however, that asymmetric ciphers can provide users who have never met with a secret channel but – without the help of a Trusted Authority – it cannot prove the identity of users.

Public-key cryptography requires two keys – the public key and the private key, which form a key pair. The recipient generates two keys, makes the public key public and keeps his private key in a secret place to ensure its private possession. The algorithm is designed in such a way that anyone can encrypt a message using the public key, however, only the legitimate recipient can decrypt the message using his/her private key.

Of course, there is a problem of authenticity of the public key. Therefore public keys are distributed through Trusted Authorities in practice.

The security of public-key cryptography rests on various computational problems, which are believed to be intractable. The encryption and decryption algorithms utilize the so-called one-way functions. One-way functions are mathematical functions that are easy to compute in one direction, but their inversion is very difficult (by “difficult” it is meant that the number of the required elementary operations increases exponentially with the length of the input number). It is, e.g., very easy to multiply two prime numbers, but to factor the product of two large primes is already a difficult task. Other public-key cryptosystems are based, e.g., on the difficulty of the discrete logarithm problem in Abelian groups on elliptic curves or other finite groups. However, it is important to point out that no “one-way function” has been proved to be one-way; they are merely believed to be. Public-key cryptography cannot provide unconditional security. We speak about computational security.

Today the most widely used public-key system is the RSA cryptosystem. RSA was invented in 1977 by Ronald Rivest, Adi Shamir and Leonard Adleman (Rives et al. [1978]), whose names form the acronym. RSA exploits the difficulty of factoring large numbers. The receiver picks two large primes $p$ and $q$ and makes their product public. Further, he chooses two large natural numbers $d$ and $e$ [such that $(de - 1) \text{ is divisible by } (p - 1)(q - 1)$]. The product $pq$ together with the number $e$ constitutes the public key. Using this key, anyone can encrypt a message $P \ (P < pq)$ employing a simple algorithm: $C = P^e \mod pq$, where $C$ is the resulting cipher text. The cipher text can easily be decrypt if the private key $d$ is known: $P = C^d \mod pq$. However, in order to invert the algorithm without knowing the private key $d$ it is necessary to find the prime factors of the modulus. Although there are several other ways to attack the RSA system, the most promising one still seems to be to attempt to factor the modulus.

In 1976 Richard Guy wrote (Guy [1976]): “I shall be surprised if anyone regularly

\footnote{This believe is based on the experience that even years of effort of many experts do not proof the opposite.}
factors numbers of size $10^{80}$ without special form during the present century”. The first challenge to break a 425-bit RSA key (equivalent to 129 decimal digits) was published in Scientific American in 1977 (Gardner [1977]). Ronald Rivest calculated that to factor a 125-digit number, the product of two 63-digit primes, would take at least $40 \times 10^{15}$ years (about one million times the age of the universe) with the best factoring algorithms then known. However, 17 years later, in 1994, new factoring algorithms had been discovered and computer power had advanced to such a level that it took 1600 computers (and two fax machines!) interconnected over the Internet only 8 months. Today a single Pentium-based PC could do the same job.

While breaking 425-bit RSA required a large number of computers, in February 1999 it was only 185 machines that managed to factor a 465-bit RSA modulus in 9 weeks. At that time, 95% of e-commerce on the Internet was protected by 512-bit keys (155-digit number). A 512-bit number was factored in August 1999 by 292 machines. That means that neither 512-bit keys provide sufficient security for anything more than very short-term security needs. All these challenges have served to estimate the amount of work and the cost of breaking a key of a certain size by public efforts. It is obviously much more difficult to estimate what can be achieved by private and governmental efforts with much larger budgets.

A network of computers is not the only way to factor large integers. In 1999 Adi Shamir proposed the TWINKLE device (Shamir [1999]) – a massively parallel optoelectronic factoring device, which is about three orders of magnitude faster than a conventional fast PC and can facilitate the factoring of 512- and 768-bit keys. Today it is already recommended to move to longer key lengths and to use key sizes of 2048 bits for corporate use and 4096 bits for valuable keys.

Another menace to the security of public-key cryptography could originate from the construction of a quantum computer. The decryption using a quantum computer would take about the same time as the encryption, thereby making public-key cryptography worthless. Algorithms capable of doing so have already been developed (Shor [1994]) and first experiments with small-scale quantum computers successfully pave the way to more sophisticated devices (Vandersypen et al. [2001]).

1.3. SYMMETRICAL CIPHERS (SECRET-KEY CRYPTOGRAPHY)

In secret-key cryptography users must share a secret key beforehand. The common key is then used for both encryption and decryption. Secure key distribution is the main drawback of secret-key cryptosystems. The security of communications is reduced to the security of secret-key distribution. In order to avoid the necessity of personal meetings or courier services to exchange the secret key, some users use public-key cryptography to distribute the key, which is then used in a secret-key cryptosystem. In such a case, even if the symmetric cipher was unconditionally secure the security of the whole system will be degraded to computational security. These so-called hybrid systems have gained a widespread use, because they combine the speed of secret-key systems with the efficiency of key management of public-key systems. They have been used for electronic purchases, financial transactions,

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$^3$Secret-key cryptography can provide its users even with unconditional security if they share a sufficiently long key (using Vernam cipher). But symmetric algorithms with the key shorter than the message are not unconditionally secure.
ATM transactions and PIN encryptions, identification and authentication of cellular phone conversations, electronic signatures, and many other applications, whose number is swelling.

The most spread secret-key cryptosystem is the Data Encryption Standard (DES) and its variations. Due to its frequent use in the hybrid systems, it is the most often used cryptosystem ever. DES was developed by IBM and the U.S. government in 1975 and it was adopted as a standard two years later. DES is an example of a block cipher – an algorithm that takes a fixed-length string of plaintext and transforms it through a series of operations into another ciphertext of the same length. In the case of DES, the block size is 64 bits. The transformation depends on the key. The algorithm consists of the cascade of 16 iterations of substitutions and transpositions and can easily be implemented in hardware, where it can reach very high speeds of encryption.

DES has experienced a similar wave of attacks as public-key cryptosystems. The algorithm uses a 56-bit key, which is reused to encrypt the entire message. As a consequence, it is only computationally secure. In 1997, RSA Data Security, Inc. published their first challenge to decrypt a plaintext message scrambled by DES. It took 96 days to break it. The researchers applied “brute force” by searching the entire keyspace of \(2^{56}\) possible keys on a large number of computers (Wiener [1997]). In January 1998, a new prize was offered. The winner of the contest used the idle time of computers connected to the Internet. More than 50,000 CPUs were linked together. The key was found after 41 days (DES Cracker 1). Another group of codebreakers chose a different approach. They built a single machine, which revealed the encrypted message “It’s time for those 128-, 192-, and 256-bit keys” after only 56 hours, searching at a rate of 88 billion keys per second (DES Cracker 2).

In the challenge in January 1999, the two previous winners combined their efforts to find the key in only 22 hours and 15 minutes, testing 245 billion keys per second. In 1993, Michael Wiener designed a DES key search machine which, based on 1997’s technology, would break DES in 3.5 hours (Wiener [1997]). The same machine based on 2000’s technology would take only 100 seconds (Silverman [2000]). The exhaustive search is not the only possible attack on DES. During the 1990s, other successful attacks were proposed that exploit the internal structure of the cipher (Biham and Knudsen [1998]).

Cryptographers attempted to improve the security of DES. Triple DES, DESX and other modifications were developed. In October 2000, a four-year effort to replace the aging DES culminated in the announcement of a new standard, the Advanced Encryption Standard (AES). It uses blocks of 128 bits and key sizes of 128, 192, and 256 bits. This standard was approved in December 2001 and went into effect in May 2002. How long will it last?

In summary, the security of conventional techniques relies on the assumption of limited advancement of mathematical algorithms and computational power in the foreseeable future, and also on limited financial resources available to a potential adversary. Computationally secure cryptosystems, no matter whether public- or secret-key, will always be threatened by breakthroughs, which are difficult to predict, and even steady progress of code-breaking allows the adversary to “reach back in time” and break older, earlier captured, communications encrypted with weaker keys.
Another common problem of conventional cryptographic methods is the so-called side-channel cryptanalysis (Rosa [2001]). Side channels are undesirable ways through which information related to the activity of the cryptographic device can leak out. The attacks based on side-channel information do not assault the mathematical structure of cryptosystems, but their particular implementations. It is possible to gain information by measuring the amount of time needed to perform some operation, by measuring power consumption, heat radiation or electromagnetic emanation. The problem of side channels will be further discussed in Section 8.7.

1.4. VERNAM CIPHER, KEY DISTRIBUTION PROBLEM

Classical cryptography can provide an unbreakable cipher, which resists adversaries with unlimited computational and technological power – the Vernam cipher. The Vernam cipher was invented in 1917 by the AT&T engineer Gilbert S. Vernam (Vernam [1926]), who thought it would become widely used for automatic encryption and decryption of telegraph messages.

The Vernam cipher belongs to the symmetric secret-key ciphers, i.e., the same key is used for both, encryption and decryption. The principle of the cipher is that if a random key is added to a message, the bits of the resulting string are also random and carry no information about the message. If we use the binary logic, unlike Vernam who worked with a 26-letter alphabet, the encryption algorithm $E$ can be written as

$$E_K(M) = (M_1 + K_1, M_2 + K_2, \ldots, M_n + K_n) \mod 2, \quad (1.1)$$

where $M = (M_1, M_2, \ldots, M_n)$ is the message to be encrypted and $K = (K_1, K_2, \ldots, K_n)$ is the key consisting of random bits. The message and the key are added bitwise modulo 2, or exclusive OR without carries. The decryption $D$ of ciphertext $C = E_K(M)$ is identical to encryption, because double modulo-2 addition is the identity, therefore

$$M = D_K(C) = (C_1 + K_1, C_2 + K_2, \ldots, C_n + K_n) \mod 2. \quad (1.2)$$

For this system to be unconditionally secure, three requirements are imposed on the key: (1) The key must be as long as the message; (2) it must be purely random; (3) it may be used only once. This was shown by Claude E. Shannon (Shannon [1949]), who laid the foundations of communication theory from the cryptographic point of view and compared various cryptosystems with respect to their secrecy. Until 1949 when his paper was published, the Vernam cipher was considered unbreakable, but it was not mathematically proved. If any of these requirements is not fulfilled, the security of the system is jeopardized. A good example is the revelation of the WWII atomic spies because of repetitive use of the key incorrectly prepared by the KGB (NSA publications).

The main drawback of the Vernam cipher is the necessity to distribute a secret key as long as the message, which prevented it from wider use. The cipher has so far found applications mostly in the military and diplomatic services. It is here

\[4\] If a key $K$ is used twice to encode two different messages $M$ and $M'$ into ciphertexts $C$ and $C'$ then one can see that $(C_1 + C'_1, C_2 + C'_2, \ldots, C_n + C'_n) \mod 2 = (M_1 + M'_1, M_2 + M'_2, \ldots, M_n + M'_n) \mod 2.$
2. QUANTUM KEY DISTRIBUTION

that quantum mechanics comes in handy and readily offers a solution. Quantum mechanics gives us the power to detect eavesdropping. Taking into account the problem of authentication, that requires the communication parties to share a certain amount of secret information, quantum cryptography provides a tool for an unlimited secret-key growing.

§ 2. Quantum key distribution

2.1. THE PRINCIPLE, EAVESDROPPING CAN BE DETECTED

As mentioned above, the main problem of secret-key cryptosystems is the secure distribution of keys. While the security of classical cryptographic methods can be undermined by advances in technology and mathematical algorithms, the quantum approach can provide unconditional security. The principle of quantum cryptography consists in the use of non-orthogonal quantum states. Its security is guaranteed by the Heisenberg uncertainty principle, which does not allow us to discriminate non-orthogonal states with certainty and without disturbing the measured system.

Within the framework of classical physics, it is impossible to reveal potential eavesdropping, because information encoded into any property of a classical object can be acquired without affecting the state of the object. All classical signals can be monitored passively. In classical communications, one bit of information is encoded into two distinguishable states of billions of photons, electrons, atoms or other carriers. It is always possible to passively listen in by splitting off part of the signal and performing a measurement on it.

In quantum cryptosystems the inviolateness of the channel is constantly tested by the use of non-orthogonal quantum states as information carriers. Because information is encoded into states with non-zero overlap, it cannot be read, copied or split without introducing detectable disturbances.

It should be noted that quantum mechanics does not avert eavesdropping; it only enables us to detect the presence of an eavesdropper. Since only the cryptographic key is transmitted, no information leak can take place when someone attempts to listen in. When discrepancies are found, the key is simply discarded and the users repeat the procedure to generate a new key.

2.2. QUANTUM MEASUREMENT

Measurement in quantum physics differs substantially from the measurement in classical physics. According to quantum theory any measurement can distinguish with certainty (i.e. without errors or inconclusive results) only among specific orthogonal state vectors (that form the so called measurement basis). Non-orthogonal states cannot be distinguished perfectly. Furthermore, quantum measurement disturbs the system in general. If the system is in a state that cannot be expressed as a multiple of one of the measurement-basis vectors but only as their linear superposition then this state is changed after the measurement. The original state is “forgotten” during the measurement process and randomly changed to the state corresponding to one of the basis vectors. Right this is the key feature of the quantum world that enables to detect the eavesdropping. Eavesdropping is nothing else than a kind of measurement on the information carrier. If non-orthogonal states
are used in transmission, eavesdropping must disturb some of them, i.e. induce errors. With a suitably designed protocol, these errors can later be discovered by the legitimate users of the channel, as will be seen in Section 2.4.

2.3. QUANTUM STATES CANNOT BE CLONED

The linearity of quantum mechanics prohibits from cloning arbitrary unknown quantum states (Wooters and Zurek [1982]). A device intended to make a copy of, say, a photon with horizontal polarization $|H\rangle$, needs to perform the following operation

$$|\text{copier}_0\rangle|\text{blank}\rangle|H\rangle \rightarrow |\text{copier}_1\rangle|H\rangle|H\rangle,$$

(2.1)

and similarly for orthogonal vertical polarization $|V\rangle$

$$|\text{copier}_0\rangle|\text{blank}\rangle|V\rangle \rightarrow |\text{copier}_2\rangle|V\rangle|V\rangle,$$

(2.2)

where $|\text{copier}_0\rangle$ is the initial state of the copier, $|\text{copier}_1\rangle$ and $|\text{copier}_2\rangle$ are its final states, and $|\text{blank}\rangle$ denotes the initial “empty” state of the ancillary system (photon) to which the information (polarization state) should be copied. However, if we want to copy a linear superposition of states $|H\rangle$ and $|V\rangle$, we obtain

$$|\text{copier}_0\rangle|\text{blank}\rangle\left(\alpha |H\rangle + \beta |V\rangle\right) = \alpha |\text{copier}_0\rangle|\text{blank}\rangle|H\rangle + \beta |\text{copier}_0\rangle|\text{blank}\rangle|V\rangle$$

$$\rightarrow \alpha |\text{copier}_1\rangle|H\rangle|H\rangle + \beta |\text{copier}_2\rangle|V\rangle|V\rangle,$$

(2.3)

which is different from the required state

$$|\text{copier}_3\rangle\left(\alpha |H\rangle + \beta |V\rangle\right)\left(\alpha |H\rangle + \beta |V\rangle\right)$$

$$= |\text{copier}_3\rangle \left(\alpha^2 |H\rangle|H\rangle + \alpha \beta |H\rangle|V\rangle + \beta \alpha |V\rangle|H\rangle + \beta^2 |V\rangle|V\rangle\right),$$

(2.4)

regardless of whether states $|\text{copier}_1\rangle$ and $|\text{copier}_2\rangle$ are identical (and equal to $|\text{copier}_3\rangle$) or not. The unitarity of quantum evolution requires that

$$\langle H|V\rangle \langle \text{blank}|\text{blank}\rangle \langle \text{copier}_0|\text{copier}_0\rangle = \langle H|V\rangle \langle H|V\rangle \langle \text{copier}_1|\text{copier}_2\rangle,$$

(2.5)

what can be satisfied only when the states to be copied are orthogonal.

Thus, the general state of a quantum object cannot be copied precisely. Duplicating can be done only approximately so that any of the resulting states is not exactly equal to the original. An optimal universal machine for approximate cloning of qubits was first designed by Bužek and Hillery [1996].

2.4. PROTOCOL BB84

Quantum key distribution (QKD) was born in 1984 when Charles H. Bennett and Gilles Brassard came up with an idea of how to securely distribute a random cryptographic key with the help of quantum mechanics (Bennett and Brassard [1984]). Hence, the protocol is called BB84. Drawing upon Stephen Wiesner’s ideas about unforgeable quantum money (Wiesner [1983], original manuscript written circa 1969), Bennett and Brassard presented a protocol that allows users to establish an identical and purely random sequence of bits at two different locations, while allowing to reveal any eavesdropping with a very high probability.
2. QUANTUM KEY DISTRIBUTION

The crucial point of the BB84 protocol is the use of two conjugated bases. The
sender of the message encodes logical zeros and ones into two orthogonal states
of a quantum system. But for each bit she randomly changes this pair of states
– i.e., she chooses one of two bases. Each state vector of one basis has equal-
length projections onto all vectors of the other basis. That is, if a measurement
on a system prepared in one basis is performed in the other basis, its outcome is
to freely random and the system “loses all the memory” of its previous state. In
fact, the non-orthogonal signal states are used for testing the transmission channel
– checking it for eavesdropping.

We need not consider any particular quantum system. However, in order to pro-
vide an example let us suppose that information is encoded into polarization states
of individual quanta of light – photons. One basis can consist, e.g., of horizontal
and vertical polarization states of photons, $|H\rangle$ and $|V\rangle$, resp.; let us call this basis
rectilinear. The other basis, diagonal, would consist of states of linear polarizations
at 45° (anti-diagonal), $|A\rangle$, and 135° (diagonal), $|D\rangle$, whereas
\[
|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle),
\]
\[
|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle).
\]

These four states satisfy the following relations
\[
\langle H|V \rangle = \langle A|D \rangle = 0
\]
\[
\langle H|H \rangle = \langle V|V \rangle = \langle A|A \rangle = \langle D|D \rangle = 1,
\]
\[
|\langle H|A \rangle|^2 = |\langle H|D \rangle|^2 = |\langle V|A \rangle|^2 = |\langle V|D \rangle|^2 = 1/2.
\]

Any measurement in the rectilinear (diagonal) basis on photons prepared in the
diagonal (rectilinear) basis will yield random outcomes with equal probabilities.
On the other hand, measurements performed in the basis identical to the basis of
preparation of states will produce deterministic results.\(^5\)

At the beginning, the two parties that wish to communicate, traditionally called
Alice and Bob, agree that, e.g., $|H\rangle$ and $|A\rangle$ stand for the bit value “0”, and $|V\rangle$ and
$|D\rangle$ stand for a bit value “1”. Now Alice, the sender, generates a sequence of random
bits that she wants to transmit, and randomly and independently for each bit she
chooses her encoding basis, rectilinear or diagonal. Physically it means that she
transmits photons in the four polarization states $|H\rangle$, $|V\rangle$, $|A\rangle$, and $|D\rangle$ with equally
distributed frequencies. Bob, the receiver, randomly and independently of Alice,
chooses his measurement bases, either rectilinear or diagonal. Statistically, their
bases coincide in 50% of cases, when Bob’s measurements provide deterministic
outcomes and perfectly agree with Alice’s bits. In order to know when the outcomes
were deterministic, Alice and Bob need an auxiliary public channel to tell each
other what basis they had used for each transmitted and detected photon. This
classical channel may be tapped, because it transmits only information about the
used bases, not about the particular outcomes of the measurements. Whenever

\(^5\)We could also consider a third basis consisting of right and left circular polarizations whose
vectors satisfy relations analogous to Eqs. (2.7). Any two of these three mentioned bases suffice
for secure quantum key distribution.
Table 1: BB84 Protocol. 1st line – Alice’s random bits. 2nd line – Alice’s random polarization bases; “+” and “×” stand for the rectilinear and diagonal bases, resp. 3rd line – actual polarization of transmitted photons. 4th line – Bob’s random detection bases. 5th line – polarization of detected photons; ‘rand’ stands for a random outcome. 6th line – Bob publicly announces his measurement bases. 7th line – Alice publicly replies when Bob set the correct measurement basis. 8th line – the cryptographic key.

If Eve is present and wants to eavesdrop on the channel, she cannot passively monitor the transmissions (single quantum cannot be split and its state cannot be copied without introducing detectable disturbances, as discussed above). What Eve can do is either to intercept the photons sent by Alice, perform measurements on them and resend them to Bob or to attach some probe to the signal photon, i.e., to let interact some system in her hands with the quantum system carrying information, keep it and measure it later. To understand the effect of eavesdropping we will consider first only the intercept-resend attack. As Alice alternates her encoding bases at random, Eve does not know the basis to make a measurement in. She must choose her measurement bases at random as well. Half the time she guesses right and she resends correctly polarized photons. In 50% of cases, though, she measures in the wrong basis, which produces errors. For example, let us suppose that Alice sends a “1” in the rectilinear basis, i.e., state $|V\rangle$, Eve measures in the diagonal basis, and Bob measures in the rectilinear basis (otherwise the bit would be discarded). Now, no matter whether Eve detects $|A\rangle$ or $|D\rangle$, Bob has a 50% chance to get $|H\rangle$, i.e., a binary “0”, instead of $|V\rangle$. Thus, if we consider
a continuous intercept-resend eavesdropping, Bob finds on average errors in 25% of those bits that he successfully detects. If Alice and Bob agree to disclose part of their strings in order to compare them, they can discover these errors. When they set identical bases, their bit strings should be in perfect agreement. When discrepancies are found, Eve is suspected of tampering with the photons, and the cryptographic key is thrown away. Thus, no information leakage occurs even in the case of eavesdropping. If their strings are identical, the key is deemed secure and secret, and can be used for the above-mentioned Vernam cipher to encrypt communications. Since the bits used to test for eavesdropping are communicated over the open public channel, they must always be discarded and only the remaining bits constitute the key. An intercept-resend attack is not the optimal eavesdropping strategy. However, any interaction with the data carriers that can provide Eve with any information on the key always cause errors in transmission.

In order to leave the original states intact, Eve could try to attach a probe and let it interact with the information carrier:

\[
|a\rangle|E\rangle \rightarrow |a\rangle|E_a\rangle \quad \text{and} \\
|b\rangle|E\rangle \rightarrow |b\rangle|E_b\rangle,
\]

(2.8)

where \(|a\rangle\) and \(|b\rangle\) denote two possible states of information carrier, \(|E\rangle\) is the initial state of Eve’s probe, and \(|E_a\rangle\) and \(|E_b\rangle\) are its final states. Any unitary interaction has to conserve the following inner product

\[
\langle a|b\rangle \langle E|E \rangle = \langle a|b\rangle \langle E_a|E_b \rangle.
\]

(2.9)

If the states \(|a\rangle\) and \(|b\rangle\) are non-orthogonal, \(\langle a|b\rangle \neq 0\), the equality (2.9) can be fulfilled only if \(\langle E_a|E_b \rangle = 1\), i.e., when the final states of Eve’s probe are identical. Eve thus cannot gain any information. It is apparent that for Eve to discriminate between two nonorthogonal states she must disturb the state of the measured objects, and thereby inevitably cause errors in transmissions. A more detailed discussion of sophisticated eavesdropping strategies will be provided in Section 8.

It should be mentioned that no physical apparatus is perfect and noiseless. Alice and Bob will always find discrepancies, even in the absence of Eve. As they cannot set apart errors stemming from eavesdropping and those from the noise of the apparatus, they conservatively attribute all the errors in transmissions to Eve. From the number of errors, the amount of information that has potentially leaked to Eve can be estimated. Afterwards Alice and Bob reconcile their bit strings using an error correction technique to arrive at an identical sequence of bits. This sequence is not completely secret. Eve might have partial knowledge about it. To eliminate this knowledge, they run a procedure called privacy amplification. Privacy amplification is a method enabling them to distill a secret bit string from their data in such a way that Eve would know even a single bit of the distilled string only with an arbitrarily small probability. Both of these procedures, error correction and privacy amplification, will be described in detail in Section 7.

\footnote{The probability that eavesdropping will not be detected decreases exponentially with the increasing number of compared bits.}
§ 3. Some other discrete protocols for QKD

3.1. TWO-STATE PROTOCOL, B92

Besides BB84, other protocols were designed. In 1992, C. H. Bennett showed (Ben
nnett [1992b]) that two nonorthogonal states are already sufficient to implement
secure QKD. Let Alice choose two nonorthogonal states and send them to Bob
in random order. When Bob performs projections onto subspaces orthogonal to
the signal states, he sometimes learns Alice’s bit with certainty and sometimes he
obtains an inconclusive outcome. After the transmission, Bob tells Alice when he
detected a bit. In this case, he does not announce the used basis, because a basis
in which he detected a photon, uniquely identifies the bit Alice had sent. This
protocol is usually called B92.

However, such a scheme is secure only in lossless systems or if the losses are
very low. In the case of higher losses, an eavesdropper could sit in the middle and
make measurements on the quantum states. If she has obtained an inconclusive
result, she blocks the signal, while if she has detected the sent state, she re-sends
a correct copy to Bob, because she knows the state with certainty. To compensate
for the blocked photons, she can send a pulse of higher intensity so that Bob cannot
observe any decrease in the expected transmission rate.

3.2. B92 PROTOCOL WITH A STRONG REFERENCE PULSE

One possibility to counteract the above mentioned eavesdropping strategy against
the B92 protocol is to encode bits into a phase difference between a dim pulse (with
less than one photon in average) and a classical strong reference pulse (Bennett
[1992b]). It means the laser pulse is split into strong and weak parts on a highly
unbalanced beam splitter. Both Alice and Bob can introduce a phase shift between
these pulses. On Bob’s side both pulses are combined again on an unbalanced beam
splitter where they interfere. Bob can also monitor the presence of all strong pulses.

Now, when Eve gets an inconclusive result, she cannot suppress the strong pulse,
because Bob must receive all of them. However, when Eve blocks only the dim pulse,
interference of the bright pulse with vacuum (instead of the dim pulse) will lead to
errors. Similarly, if Eve tries to fabricate her own dim or bright pulse (or both of
them) and send it (them) to Bob she will inevitably cause detectable errors. Even
though the B92 protocol can be unconditionally secure if properly implemented,
Eve can acquire now more information on the key for a given disturbance than in
the the case of the BB84 protocol (Fuchs et al. [1997]).

3.3. SIX-STATE PROTOCOL

In the six-state protocol, three non-orthogonal bases are used (Bruss [1998], Bechmann-
Pasquinucci and Gisin [1999]) that Alice and Bob randomly alternate. If we denote
the two conjugate bases employed in the BB84 protocol as \{\ket{0}, \ket{1}\} and \{\ket{\bar{0}}, \ket{\bar{1}}\},
where
\[
\ket{\bar{0}} = \frac{1}{\sqrt{2}} (\ket{0} + \ket{1}), \quad \ket{\bar{1}} = \frac{1}{\sqrt{2}} (\ket{0} - \ket{1}),
\] (3.1)
3. SOME OTHER DISCRETE PROTOCOLS FOR QKD

then the third basis is \{\ket{\overline{0}}, \ket{\overline{1}}\} with

\[
\ket{\overline{0}} = \frac{1}{\sqrt{2}} (\ket{0} + i \ket{1}), \quad \ket{\overline{1}} = \frac{1}{\sqrt{2}} (\ket{0} - i \ket{1}).
\] (3.2)

The probability that Alice and Bob choose the same basis is now $\frac{1}{3}$. But this disadvantage against BB84 is outweigh by the fact that eavesdropping causes higher error rate. For example, a continuous intercept-resend attack induces in average $33\%$ of errors compared to $25\%$ in the case of the BB84 protocol. In general, the maximal mutual information between Eve and Alice is smaller than in the BB84 scenario. Besides, the symmetry of the signal states simplifies the security analysis.

3.4. SARG PROTOCOL

The SARG protocol (called after the names of its authors) was proposed to beat the photon-number splitting attack (PNS)\(^8\) in QKD schemes based on weak laser pulses. It relies on Eve’s inability to perfectly distinguish between two non-orthogonal states (Scarani et al. [2004], Branciard et al. [2005]). In contrast to BB84, two values of a classical bit are encoded into pairs of non-orthogonal states. However, to implement the SARG protocol one can keep the same hardware as for BB84 and modify only the classical communication between Alice and Bob. Alice prepares four quantum states and Bob makes measurements exactly as in the BB84 protocol. But Alice does not reveal the basis but the pair of non-orthogonal signal states such that one of these states is the one she has sent. Bob guesses correctly the bit if he finds a state orthogonal to one of two announced non-orthogonal states (for details see Scarani et al. [2004]). In comparison with the BB84 protocol, SARG enables to increase the secure QKD radius when the source is not a single-photon source.

3.5. DECOY-STATE PROTOCOLS

The decoy-state method represents another way for counteract the PNS attack on QKD schemes using weak laser pulses (Hwang [2003], Wang [2004a], Wang [2004b], Lo et al. [2005b], Ma [2004]). It can substantially prolong the distance to which the secure communication is possible. If this method is used with the BB84 protocol the secure-key rate is proportional to the overall transmittance even if the light source is an attenuated laser (the secure-key rate for standard BB84 is linearly dependent on transmittance only in the case of single-photon source, with weak laser pulses it is proportional to the square of the transmittance).

The idea is based on the observation that by adding some decoy states, one can estimate the behavior of vacuum, single-photon, and multi-photon states individually. Hence, Alice sends sometimes an additional, decoy, state with a different intensity than the states used for the key transmission (but with the same wavelength, timing, etc.). These decoy states serve only for testing Eve’s presence. Eve

\(^7\)Factors like $1/3$ for the six-state protocol or $1/2$ for the BB84 are not essential. In fact, the communication can proceed in only one orthogonal basis and the other non-orthogonal states can be send randomly from time to time just to test the channel for the presence of an eavesdropper. So if the probabilities of bases are “biased” in favor of one of the bases, these factors can asymptotically reach unity (Lo et al. [2005a]).

\(^8\)In the photon-number splitting attack Eve exploits multi-photon states present in weak laser pulses. See Section 8.5.4.
does not know when Alice sends the decoy states and she cannot identify them. Changes, that Eve’s PNS attack makes on these decoy states, enable Alice and Bob to detect the PNS eavesdropping.

The essence of the decoy-state method consists in the following fact: The conditional probability \( Y_n \) that Bob detects a signal – providing that Alice’s source has emitted an \( n \)-photon state – must be the same both for the signal and decoy states. When no eavesdropper is present it must be equal to the following value given by the parameters of the apparatus:

\[
Y_n^{\text{signal}} = Y_n^{\text{decoy}} = Y_n = [1 - (1 - \eta)^n](1 - p_{\text{dark}}) + p_{\text{dark}},
\]

(3.3)

where \( \eta \) is the total transmission efficiency and \( p_{\text{dark}} \) is the probability of the detector dark count. The PNS attack inevitably changes some \( Y_n \). The quantities \( Y_n \) are not directly measurable. But what Bob can directly determine is the total detection rate for a given mean photon number \( \mu \) of Alice’s pulses:

\[
Q_\mu = e^{-\mu} \sum_{n=0}^{\infty} Y_n \frac{\mu^n}{n!}.
\]

(3.4)

If Alice and Bob use decoy states with different mean photon numbers they can estimate values of \( Y_n \) for some photon numbers \( n \) and check whether they correspond to the expected values.

The security of the decoy-state method with the BB84 protocol under the “paranoid” assumptions (Gottesman et al. [2004]) has been analyzed by Lo et al. [2005b].

3.6. ENTANGLEMENT-BASED PROTOCOLS

Another class of QKD protocols is based on quantum entanglement. The security of the original proposal was ensured by checking the violation of Bell’s inequalities (Ekert [1991]). The simplified version of the protocol works in a very similar way as BB84 (Bennett et al. [1992d]).

3.6.1. Entanglement, Bell’s inequalities

Two or more quantum systems are entangled if their global state cannot be expressed as a direct product or a statistical mixture of direct products of any quantum states of individual systems. Entanglement leads to many interesting effects unknown in classical physics. It lies in the basis of quantum teleportation (Bennett et al. [1993]) and it is responsible for the effectiveness of quantum computation (Nielsen and Chuang [2000]). Asher Peres said that “Entanglement is a trick that quantum magicians use to produce phenomena that cannot be imitated by classical magicians.” (Bruss [2002]).

In 1935 Einstein, Podolsky and Rosen (Einstein et al. [1935]) formulated a gedanken experiment employing two particles prepared in an entangled state to argument against the completeness of quantum theory. They used the fact that the result of any potential measurement on one subsystem of the properly chosen entangled pair can be predicted with certainty after the proper measurement on the other subsystem. Following this fact and a few “natural” assumptions (namely the assumptions of locality and reality) they concluded that there must simultaneously exist “elements of reality” for two complementary observables.
3. SOME OTHER DISCRETE PROTOCOLS FOR QKD

However, in 1964 John Bell (Bell [1964]) has shown that there is no local realistic theory that would give the same predictions as quantum mechanics. Namely, quantum mechanics predicts different values of certain correlations of measurement results on a bipartite system in a specific entangled state. He derived his famous inequalities that must be satisfied by any local realistic theory but that may be violated by quantum theory.

Let us denote \(A(n_1)\) and \(B(n_2)\) random variables, getting discrete values \(\pm 1\), corresponding to measurement results on two separated but somehow correlated particles, where the settings of respective measurement devices are represented by unit vectors \(n_1\) and \(n_2\) (note that \(A\) depends only on \(n_1\) and \(B\) only on \(n_2\) – this reflects the locality condition). The randomness of \(A\) and \(B\) is supposed to be caused only by some random parameters \(\lambda\) that may be common for both the particles and that we do not know (the premise of reality). The Bell inequality, in the form derived by Clauser et al. [1969], states that:

\[
\left| C(n_1, n_2) + C(n_1', n_2) - C(n_1, n_2') \right| \leq 2,
\]

where \(C(n_1, n_2)\) is the correlation function:

\[
C(n_1, n_2) = \langle A(n_1)B(n_2) \rangle = \int A(n_1, \lambda)B(n_2, \lambda) \, d\rho_\lambda.
\]

Now, let us try to describe such a situation by the quantum language, assuming two spin-half particles in the following entangled state:

\[
|\psi\rangle = \frac{1}{\sqrt{2}} \left( |n, +\rangle_1 |n, -\rangle_2 - |n, -\rangle_1 |n, +\rangle_2 \right),
\]

where state vectors \(|n, \pm\rangle\) correspond to two orthogonal projections of spin to direction \(n\). Then the quantum prediction for correlation function reads:

\[
C(n_1, n_2) = \langle \psi | (n_1 \cdot \sigma_1)(n_2 \cdot \sigma_2) | \psi \rangle,
\]

where \(\sigma_1, \sigma_2\) are vectors of Pauli matrices. If we choose the settings of the measurement apparatuses in such a way that \(n_2\) with \(n_1\), \(n_1\) with \(n_2'\), and \(n_2'\) with \(n_2\) include angle 45°, while \(n_1'\) with \(n_2'\) include angle 135°, we readily find that

\[
\left| C(n_1, n_2) + C(n_1', n_2) + C(n_1, n_2') - C(n_1', n_2') \right| = 2\sqrt{2} > 2.
\]

3.6.2. Original Ekert’s protocol and its simplified form

According to Ekert’s protocol (Ekert [1991]), Alice and Bob each obtain one particle from a pair of spin-1/2 particles in the state (3.7). (In fact, it does not matter whether they share two entangled spin-1/2 particles or, e.g., two photons with entangled polarizations.) Alice and Bob perform measurements on their respective particles in three bases defined by three orientations of their measurement devices (e.g., Stern-Gerlach apparatuses). For simplicity let us suppose that they use only directions lying in the plane perpendicular to the trajectory of the particles. Alice’s bases make angles with respect to the vertical 0°, 45°, 90°, and Bob’s bases are making 45°, 90°, 135°. There are nine possible combinations. After the quantum
transmission, during which Alice and Bob randomly and independently set their measurement bases, the settings are publicly announced. When identical bases were used, the outcomes of their measurements are correlated and become the cryptographic key. The probability that Alice and Bob use the same basis is $2/9$. The outcomes of measurements in the other bases are used to verify the violation of the Clauser-Horne-Shimony-Holt inequality (3.5). An eavesdropper attempting to correlate his probe with the other two particles would disturb the purity of the singlet state (3.7), which would result in a smaller violation of the inequality or no violation at all.

A year later Bennett et al. [1992d] proposed a simpler entanglement-based protocol without invoking directly Bell’s theorem. Here, both Alice and Bob choose only from two bases corresponding to two perpendicular orientations of their spin-measurement devices in a way very similar to BB84 protocol. In fact, the only difference from BB84 is that Alice does not send particles in a chosen spin (or polarization) state but she measures her particle from the entangled pair in one of two conjugated bases. She must select bases randomly and independently from Bob. The rest is the same as in BB84: After the transmission Alice and Bob compare their bases and keeps only those results when they used the same bases.

3.6.3. Passive setup

The system for entanglement-based QKD can be designed even in such a way that it can be operated entirely in a passive regime without any extern-driven elements (e.g., polarization rotators or phase modulators; Rarity et al. [1994]). Each particle from the entangled pair “may freely decide” on a beamsplitter in which basis it will be measured. It means, both the random key bits and random measurement basis are chosen directly by the genuine randomness of the nature.

§ 4. Experiments

4.1. QKD WITH WEAK LASER PULSES

Attenuated lasers are often used as sources in practical QKD devices. If the spectral width of the laser pulses is much smaller than their mean frequency, the state of light can well be approximated by a monochromatic coherent state. The photon-number distribution of the coherent state is governed by the Poisson statistics. The multi-photon pulses can cause problems due to the PNS attack. Eve could always split off one photon and perform a measurement on it without introducing an error. This potentially leaked information must be taken into account (see Sections 8.5.3 and 8.5.4). The trick how to beat this attack appears in the decoy-state method (see Section 3.5).

4.1.1. Polarization encoding

The very first QKD experiment that took place in 1989 (Bennett et al. [1989], Bennett et al. [1992a]) was based on polarization encoding for the BB84 protocol. For the description of the protocol, we refer the reader to Section 2.

A light-emitting diode (LED) generated light pulses that were subsequently attenuated by an interference filter and polarized by a polarizer (see Fig. 1).
4. EXPERIMENTS

Figure 1: First QKD experiment (Bennett et al. [1989]).

qubits were encoded in the polarization of photons by means of Pockels cells. The quantum channel was 32 cm of free air. Bob analyzed the polarization states using a Wollaston prism, which was preceded by another Pockels cell to choose his polarization basis. The output ports of the prism were monitored by photomultipliers.

Four years later, Gisin’s group from the University of Geneva replaced the free-air optical path by a 1 km optical fiber (Müller et al. [1993], Bréguet et al. [1994]). A semiconductor laser at 800 nm was used to generate light pulses that were detected by silicon avalanche photodiodes. Since the optical fiber deforms the polarization state of light, a manually adjustable polarization controller was employed to compensate for temporal changes of polarization.

Bends and twists of the optical fiber induce birefringence, which gives rise to different velocities of the orthogonal polarization components of light that result in the change of the polarization state. Since the degree of polarization degrades slowly in fibers, the same stress-induced birefringence can, on the other hand, be used to compensate for this deformation. A fiber spool of a suitable diameter can act as a fractional wave plate.

Franson and Ilves [1994] proposed a QKD device with an active polarization-alignment feedback loop. Such a system was demonstrated to work over a distance of 1 km (Franson and Jacobs [1995]).

The first experiment with Alice and Bob being placed in different laboratories (in this case even different towns of Geneva and Nyon) was performed by the Geneva group (Müller et al. [1995], Müller et al. [1996]). Error rates of only 3–4 % were achieved between two stations, connected by a 23 km fiber deployed under Lake Geneva. In order to reduce fiber losses, a laser at 1.3 µm was used and the photons were detected by liquid-nitrogen-cooled germanium avalanche photodiodes.

Using optical fiber is not the only way to implement QKD at a distance. Another approach is to try to communicate directly through free space. Unlike fibers, the atmosphere is non-birefringent, thereby polarization encoding is very suitable. The feasibility of free-space QKD was shown by Jacobs and Franson [1996], who managed to communicate over 150 m in a fluorescent-tube-illuminated corridor and over 75 m outdoors in daylight. It was the first free-space implementation of QKD.
after the celebrated 1989 Bennett and Brassard experiment and there were more to come. The Los Alamos group first exchanged keys at 1 km by night bouncing the photons between mirrors (Buttler et al. [1998a], Buttler et al. [1998b]), then point-to-point communication over 0.5 km in daylight was performed (Hughes et al. [2000a]) and eventually over 1.6 km in daylight (Buttler et al. [2000]). The distance 1.9 km at night were covered by Gorman et al. [2001]. Hughes et al. [2002] then demonstrated free-space QKD over 10 km. Free-space QKD over the largest distance so far was performed by the Munich group of H. Weinfurter (Kurtsiefer et al. [2002a], Kurtsiefer et al. [2002b]). Unlike the other groups, they moved to the high altitudes of the Alps to take advantage of thinner air and less air turbulence. Alice was located on the summit of Zugspitze (2962 m) and Bob was on a 23.4 km distant Karwendelspitze (2244 m).

Demonstration of free-space QKD with a single-photon source based on a nitrogen-vacancy center in diamond (see Section 5.1.3) was done by Beveratos et al. [2002] (indoor experiment over 50 m) and by Alléaume et al. [2004] (this later experiment of the same group was operated outdoors over 30 m at night).

4.1.2. Phase encoding

In this method, different polarizations (used in polarization encoding) are replaced by different phase shifts between two arms of the Mach-Zehnder interferometer. Alice controls the phase shift in one arm of the interferometer, Bob controls the phase shift in the other arm. If Alice’s and Bob’s phase shifts are the same or differ by 180°, then the behavior of the photon at Bob’s beam splitter is deterministic because of constructive interference in one of the outputs and destructive interference in the other one. If the total phase shift between the arms is different from an integer multiple of 180°, photons are detected randomly at both detectors.

In the case of the BB84 protocol, Alice encodes bit values into four non-orthogonal quantum states. She sends weak light pulses to the interferometer and sets randomly phase $\phi_A$ to 0°, 90°, 180°, or 270°. Bobs sets randomly (and independently of Alice) phase $\phi_B$ to 0° or 90°. These two values correspond to the measurement in “rectilinear” and “diagonal” bases, respectively:

| $\phi_A$ | $\phi_B$ |
|---------|---------|
| +: 0°...“1”, 180°...“0” | +: 0° |
| $\times$: 90°...“1”, 270°...“0” | $\times$: 90° |

However, in practice it is impossible to keep the same and stable phase conditions in two different arms of the Mach-Zehnder interferometer over long distances.
The way how to solve this problem was proposed already by Bennett [1992b]. Two communicating parties can employ a time multiplex and use only one optical fibre to interconnect their devices (see Fig. 2). Now two unbalanced Mach-Zehnder interferometers are used. The path length difference between the longer and shorter arm of each interferometer is larger than the width of the laser pulse. But the path differences are the same for both interferometers. The case where the photon goes first through the longer (L) arm and then through the shorter (S) one is indistinguishable from the case when it first passes the shorter and then the longer arm. This path indistinguishability results in the interference at the last beam splitter. Thus for the “central peak” (see the right side of Fig. 2) the system behaves exactly in the same way as a single balanced Mach-Zehnder interferometer. This peak is selected by the proper timing of detection and the events when the photon passed either through both shorter or both longer arms are ignored.

The first system based on phase encoding was build by Townsend et al. [1993a] (see also Townsend et al. [1993b]). The signal was sent through 10 km of fiber in a spool. Later the system was modified so that the polarization in long arms was rotated by 90° in both interferometers and the time multiplex was supplemented by a polarization multiplex. That is, at the output of Alice’s interferometer and at the input of Bob’s interferometer there were polarization beam splitters. This technique suppresses the lateral non-interfering peaks (Townsend [1994]). Further the distance were prolonged to 30 km (Marand and Townsend [1995]). Townsend [1997] also tested a wavelength-division multiplex to execute both the QKD and the classical communication through the same fiber on different wavelengths. A QKD system with a double Mach-Zehnder interferometer was realized also in Los Alamos National Laboratory (Hughes et al. [1996], Hughes et al. [2000b]). They tested it in an installed optical fiber up to a distance of 48 km. Another fiber-based system (at 830 nm) was realized by Dušek et al. [1999b]. It had implemented an active stabilization of interferometers and programmed all supporting procedures for practical QKD. The system was used as a quantum identification system (for mutual identification of the users) at a distance of 500 m. The system with silica-based integrated-optic interferometers was built by Kimura et al. [2004] and tested at a distance over 150 km. Toshiba Research Europe developed an automated system at 1550 nm with a new method for active interferometer stabilization (a “stabilization” pulse goes after each signal pulse) and tested it at the distances up to 122 km (Gobby et al. [2004], Yuan and Shields [2005]).

The systems using either the polarization encoding or double Mach-Zehnder interferometer require an active stabilization to compensate drifts and fluctuations of polarizations and/or phases. Müller et al. [1997] has proposed an interesting way how to implement QKD device (using a phase encoding) where all optical and mechanical fluctuations are automatically passively compensated (the principle of this auto-compensation is based on an earlier idea of Martinelli [1989]). Two strong mutually delayed pulses of orthogonal linear polarizations go from Bob to Alice. At Alice’s side they are attenuated (a part of them is also used to synchronization purposes), the first pulse is phase shifted (this is the way Alice encodes the information), and both pulses are reflected on a Faraday mirror. The Faraday mirror,

9If the pulse width is in the order of nanoseconds then the path lengths difference is usually a few meters.
which is a Faraday rotator followed by a mirror, exchanges their vertical and horizontal polarization components. Then these two dim pulses return to Bob. Because they go back through the same line but have properly modified polarizations by the Faraday mirror, all the polarization distortions caused by birefringence experienced by the pulses in their first trip are compensated during the return trip. At the end the sent vertical polarization returns as horizontal one and vice versa. At Bob’s side the first pulse passes a longer arm of an unbalanced Mach-Zehnder interferometer while the second pulse passes its shorter arm (the pulses are separated by a polarization beam splitter and then their polarizations are made the same). In one of the arms Bob now applies his phase shift. Because the original delay between the pulses was created by the same unbalanced interferometer no stabilization of this interferometer is needed. Since no special optical adjustment is necessary to operate this set-up it is usually called “plug&play” system. However, there are also some drawbacks: The fact that pulses must go first from Bob to Alice and then back complicates the timing of the whole process and may effectively decrease the transmission rate. The problem is, especially, with a Rayleigh backscattering. To suppress its contribution to error rate the strong pulses coming form Bob should not meet with the weak pulses propagating in the opposite direction. Further, because the strong pulses must pass the whole path from Bob to Alice before they are attenuated and the information is encoded, Eve has an opportunity to change some of their properties, e.g., their photon statistics. The system is also more sensitive to a certain “Trojan horse” attacks (see Section 8.7).

The first experimental realization was done by Zbinden et al. [1997]. The key was exchanged over a 23-km-long optical fiber installed under Lake Geneva. Later the fully automated system was tested on the same fiber (Ribordy et al. [2000]). The implemented protocol was BB84. The system was operated at 1300 nm. A similar auto-compensating system operating at 1300 nm was also independently developed at IBM (Bethune and Risk [2000]). It was tested on a 10-km-long fiber in a spool. In this set-up the pulses sent by Bob had a reduced intensity to avoid Rayleigh backscattering. Synchronization was provided by classical pulses at 1550 nm using a wavelength-division multiplex. Nielsen et al. [2001] built a system working at 1310 nm and distributed a key over 20 km in fiber. Group of A. Karlsson demonstrated that the plug&play technique can be implemented in fibers also at 1550 nm (Bourennane et al. [1999]). Later the operation of an improved Geneva plug&play setup at 1550 nm was demonstrated over a 67-km-long optical-fiber link between Geneva and Lausanne (Stucki et al. [2002]).

The first experimental demonstration of the decoy-state method (see Section 3.5) was done by Zhao et al. [2005]. Their set-up used a modified commercial QKD “plug&play” system manufactured by id Quantique. The distribution was tested over the distance of 15 km. The protocol was based on the BB84 scheme together with a practical implementation of the decoy-state method with only one decoy state. The average intensities of the signal and decoy states were chosen to be 0.8 and 0.12 photons, respectively. Roughly 88% of signal states and 12% of decoy states were transmitted.

Gisin et al. [2004] proposed a new technique for practical QKD, based on a specific protocol and tailored for an implementation with weak laser pulses. The key is obtained by a simple measurement of the times of arrival of the pulses incoming to Bob. The presence of an eavesdropper is checked by an interferometer built on an
4. EXPERIMENTS

additional monitoring line. Each logical bit is encoded into a sequence of two pulses: either one empty and one non-empty or vice versa. There is a phase coherence between any two non-empty pulses because a mode-locked laser is used as a source. Some pulses are reflected at Bob’s beam-splitter and go to the unbalanced Mach-Zehnder interferometer (monitoring line). Here is where quantum coherence plays a role. If coherence is not broken, only the detector at the particular output of the interferometer may fire at certain instants. This enables to detect an eavesdropping. The first experimental realization of this protocol was done by Stucki et al. [2005].

4.2. ENTANGLEMENT-BASED PROTOCOLS

The principle of entanglement-based protocols was explained in Section 3.6. In practical realizations only the entangled states of photons are used. However, different kinds of entanglement can be employed: For example, entanglement in polarizations of photons, entanglement in energy and time, entanglement in orbital angular momentum, or so called “time-bin” entanglement which is a special case of energy-time entanglement. Experiments with QKD using photon pairs often utilized set-ups and took up on experiments examining the violation of Bell’s inequalities. Besides QKD, the distribution of entanglement between distant users can be beneficial also for other task like quantum teleportation, quantum dense coding, quantum secret sharing, etc. However, there is a problem of coupling between the property used to encode the qubits and the other properties of the carrier electromagnetic field, that rises during the propagation in a dispersive medium. This form of decoherence gradually destroys quantum correlations between the photons. \(^{10}\) For example, polarization-mode dispersion makes two values of polarization-encoded qubit distinguishable also in temporal domain and so wipes out quantum correlations between polarizations. Similarly, chromatic dispersion degrades energy-time entanglement.

4.2.1. Polarization entanglement

In this case Alice and Bob are each provided by one photon of an entangled pair of one of these forms:

\[
\frac{1}{\sqrt{2}} (|V>_A |V>_B \pm |H>_A |H>_B), \; \quad \frac{1}{\sqrt{2}} (|V>_A |H>_B \pm |H>_A |V>_B),
\]

(4.1)

where \(|V>, |H>\) denotes single-photon states with vertical and horizontal linear polarizations, respectively. The pairs are prepared by a parametric down-conversion process in nonlinear optical crystals. Polarization entanglement is created either by one crystal using the phase matching of type-II (in a proper geometrical layout) or by two crystals with type-I phase matching that are placed closely one by one but with optic axes oriented perpendicularly to each other. Alice and Bob are equipped with polarization analyzers that can rapidly change measurement polarization bases, e.g., electro-optical polarization modulators followed by polarizing beam splitters (with photon counters behind them).

The first two experiments were reported in 2000. Zeilinger’s group (Jennewein et al. [2000]) used a BBO\(^\text{11}\) crystal, cut for type-II phase matching and pumped by

\(^{10}\)This effect has also a positive aspect: It prevents unintentional information leakage in unused degrees of freedom (Mayers and Yao [1998]).

\(^{11}\)β-BaB\(_2\)O\(_4\).
an argon-ion laser, to generate photon pairs at 702 nm (both photons had the same wavelength). Their analyzers consisted of fast modulators, polarizing beam splitters, and silicon avalanche photodiode (APD) detectors. They have demonstrated QKD over 360 m in installed single-mode fibers. Kwiat’s group in Los Alamos (Naik et al. [2000]) worked with two BBO crystals of type-I phase matching pumped by an argon-ion laser and they also produced photon pairs with degenerate wavelengths at 702 nm. They implemented original Ekert protocol and have demonstrated QKD in free space at the distance of a few meters. In addition, they simulated experimentally different eavesdropping strategies. A newer experiment was done by Poppe et al. [2004] in Vienna. Secret key was distributed over 1.45-km-long installed fiber (between a bank and the City Hall). Polarization-entangled pairs at 810 nm were produced by type-II parametric down conversion in a BBO crystal pumped by a semiconductor laser. The distribution of entanglement over 13 km in free space was demonstrated by Peng et al. [2004]. It was used both to prove a space-like separated violation of Bell’s inequality and to realize QKD based BB84-like protocol. It utilized type-II parametric down-conversion in BBO crystal pumped by an argon-ion laser. Wavelengths of entangled photons were 702 nm.

4.2.2. Energy-time entanglement, phase encoding

Now the employed two-photon entangled states have the approximate form:

\[ \int d\omega \xi(\omega) |\omega\rangle_A |\omega_0 - \omega\rangle_B, \]  

(4.2)

where \( |\omega\rangle \) denotes a single-photon state at frequency \( \omega \), \( \omega_0 \) is an optical frequency of the pump laser, and \( \xi(\omega) \) expresses the distribution of individual frequency components. The pairs are again produced by parametric down conversion in nonlinear optical crystals. Photons in states close to that given by Eq. (4.2) – neglecting vacuum and multi-pair contributions – are generated when the crystal is pumped by a laser with a large coherence time. Alice and Bob obtain one photon each and they let them pass through identically unbalanced Mach-Zehnder interferometers (one interferometer at Bob’s side, one at Alice’s side). The path lengths difference between the longer and shorter arm of each interferometer must be larger than the coherence length of generated photons but shorter than the coherence length of the pump laser. The path differences must be the same for both interferometers. The instants of detections of two photons from a pair are very tightly correlated (of the order of hundreds of femtoseconds) but the particular times of these coincident detections are uncertain and random. Therefore Alice and Bob cannot distinguish between the situations when both photons went through longer arms of their interferometers and when both of them went through shorter arms (this leads to the fourth-order interference). Alice and Bob chooses their measurement bases by changing the phase shifts between the arms of their interferometers (e.g., they can randomly and independently alternate shifts 0° and 90°). When their phase difference is 0°, the measurement outcomes are deterministic. When the phase difference is \( \pm 90° \), the results are random. Events when one photon went through a shorter arm and the other one through a longer arm, are ignored. This arrangement was originally devised by Franson [1989] for another purposes. Its use for practical QKD in fibers was proposed by Ekert et al. [1992]. The set-up can be
This QKD scheme was first realized by Ribordy et al. [2001] from the University of Geneva. They used a KNbO₃ crystal pumped by a doubled Nd-YAG laser to create entangled pairs with asymmetric wavelengths 810 nm and 1550 nm. The wavelength 810 nm gave an advantage to use efficient and low-noise Si-APD photon counters at Alice’s side (the distance between the source and Alice’s analyzer was very short). The wavelength 1550 nm of the other photon fit to low-loss window of optical fibers, so this photon travelled the longer distance between the source and Bob. Bob was connected to the source by 8.5-km-long optical fiber in a spool (the dispersion-shifted fiber was used to limit the decoherence induced by chromatic dispersion). It should be noted that the passive set-up was implemented. Two measurement bases (at each terminal) were passively randomly selected using a polarizing beam splitter. One physical interferometer behaved like two interferometers with different phase settings for two different polarizations of light.

4.2.3. Time-bin entanglement, phase-time encoding

This method is similar to the phase encoding described above. But now there is one more unbalanced Mach-Zehnder interferometer placed in the pump beam and a pulsed source is used to pump the crystal. The scheme of the apparatus is shown in Fig. 3. The generated pair can be described by the following state:

$$\frac{1}{\sqrt{2}} \left( e^{i\phi} |S\rangle_A |S\rangle_B - |L\rangle_A |L\rangle_B \right),$$

with $S$ and $L$ denoting contributions from pump pulses going through a shorter and longer arm of the interferometer, respectively. The path differences of all three interferometers should be the same. Now Alice can detect a photon in three different time windows (after each laser pulse): The first corresponds to the situation when both the pump pulse and Alice’s photon went through the shorter arms ($SS$), the
second corresponds to the combination of the shorter and the longer arm or vice versa (SL or LS), and the third corresponds to the situation when both the pump pulse and Alice’s photon went through the longer arms (LL). The same holds for Bob’s detections. To establish the secret key Alice and Bob publicly agree on the events when both of them detected a photon (does not matter at which detector) either in the first or in the third time window, but do not reveal in which one, and on the events when they both registered detector clicks in the second time window, without revealing at which detector. In the first case they assign different bit values to the first and third time window (Alice and Bob must have correlated detection times). The second case (both photons detected in the second time window) is formally equivalent to the above described phase-encoding method.

This technique was proposed by Brendel et al. [1999] (who have also built the source of pairs) and the QKD experiment was performed by Tittel et al. [2000]. The system was tested only in the laboratory. The crystal K NbO$_3$ was pumped by a pulsed semiconductor laser diode. The wavelength of down-converted photons was 1310 nm. Later, the distribution of time-bin entangled qubits was demonstrated over 50 km of optical fiber (Marcikic et al. [2004]).

§ 5. Technology

5.1. LIGHT SOURCES

5.1.1. Attenuated lasers

In practical QKD systems the attenuated lasers are still the only reasonable light sources (except systems using entangled pairs). The radiation from a laser can be usually well described by a single-mode coherent state exhibiting Poissonian photon-number distribution (with $\mu$ being a mean photon number):

$$p(n) = \frac{\mu^n}{n!} e^{-\mu}.$$  \hspace{1cm} (5.1)

Clearly, a highly attenuated laser pulse with very small $\mu$ represents a good approximation of a single-photon Fock state (or rather a superposition of states $|0\rangle$ and $|1\rangle$) because the ratio $p_{\text{multi}}/p(1)$ of the probability of more than one photon, $p_{\text{multi}} = \sum_{n=2}^{\infty} p(n)$, and a single-photon probability, $p(1)$, goes to 0 as $\mu \to 0$. The only problem is the increasing fraction of vacuum states ($n = 0$). For example, if $\mu = 0.1$ then $p(0) = 0.905$, $p(1) = 0.090$, and $p_{\text{multi}} = 0.005$. Empty pulses decrease transmission rate. A more important problem arises from detector dark counts. Because detectors must be active for all pulses including empty ones the dark-count rate is constant while the rate of non-empty pulses decreases with decreasing $\mu$. This prevents the use of arbitrarily low mean photon numbers.

The mean photon number must be chosen according to several aspects. The existence of detector dark counts and the losses in the system admonish us to use the mean photon number as high as possible. On the other hand the potential leakage of information trough the multi-photon pulses forces us to use the mean photon number as low as possible. The optimal mean photon number is such that maximizes the secure-key rate for given conditions. It results from the trade-off between the value of the detection rate and the shortening of the key due to privacy amplification (because of multi-photon contributions, privacy amplification
shortens the resulting distilled key substantially if $\mu$ is too high, namely if $\mu \gtrsim \eta$ where $\eta$ is the line transmittance; Lütkenhaus [2000]).

A good measure of the quality of imperfect single-photon sources is the second-order autocorrelation function of the source, $g_2 = \langle I^2 \rangle / \langle I \rangle^2$, i.e., the correlation measured in a Hanbury-Brown-Twiss-type experiment (I means optical intensity). It can be approximately calculated as $g_2 \approx 2 p(2)/[p(1)]^2$ if $p(1) \gg p(2) \gg \sum_{n=3}^{\infty} p(n)$. The value $g_2 = 1$ corresponds to Poissonian case, $g_2 < 1$ indicates sub-Poissonian distribution.

5.1.2. Single-photon sources: Parametric down conversion

Another way how to prepare quasi-single-photon states is to use photon pairs generated by spontaneous parametric down conversion (SPDC) (Hong and Mandel [1986]). Here the crucial point is a tight time correlation between photons in the pair. In the ideal case, if one places a photon-number detector into the path of one member of the pair (say, into the idler beam) and detects one photon then in the same time (i.e., in a very short time window of the order of hundreds of femtoseconds) there must be one photon also in the other – signal – beam.

In reality, due to losses in the signal beam, caused mainly by an inefficient coupling into the fiber, and partly also due to dark counts of the trigger detector, there may be no photon in the signal beam even if the trigger detector has clicked. However, the probability of this event is relatively low – today typically about 30%.

Nearly all practically applicable detectors cannot distinguish the number of photons and their quantum efficiency is substantially lower than 100%. Therefore, there is also non-zero probability having more than one photon in the signal beam after the trigger detection. (Notice that the number of photons in one mode is thermally distributed and the total number in all modes obeys the Poissonian distribution.) On the other hand, the efficiency of the conversion of a pump photon into the pair of sub-frequency photons is very low, typically about $10^{-10}$, so the probability of generation of multi-photon states is also low.\(^{12}\) Besides, there are techniques that allow us to eliminate partly multi-photon states. They are based on the division of the idler beam, used for triggering, into several detectors. Events with more than one detector clicks are discarded. This spatial division can be substituted by time division using one detector behind a delay loop (Řeháček et al. [2003]).

The important advantage of a SPDC quasi-single-photon source in comparison with an attenuated laser is a substantial reduction of the portion of vacuum contributions, i.e., empty signals.

From the technological point of view these sources seem feasible. Diode-laser pumped SPDC sources emitting in near-infrared region can be made compact and robust (Volz et al. [2001]).

5.1.3. Single-photon sources: Color centers

A progressive direction in the research of single-photon sources is represented by color centers in diamond. Color centers are defects in a crystal lattice due to im-

\(^{12}\) Take a source that generates $10^5$ pairs per second in average and consider a 1 ns detection window, then this probability is about $10^{-4}$. 
purities and vacancies. Crystals with such defects can be relatively easily prepared and are stable. The key advantage of the sources based on color centers is that they work at room temperatures.

Particularly, nitrogen-vacancy centers in synthetic diamond were intensively studied (Kurtsiefer et al. [2000], Broui et al. [2000], Beveratos et al. [2001]). These centers consist of a substitutional nitrogen atom and a vacancy at an adjacent lattice position. The individual nitrogen atom is excited by a focused laser beam at 532 nm. Due to the fluorescence the atom consequently emits a photon with the spectrum centered around 690 nm. The strong anti-bunching is observed. The weaker point is a broad spectrum of the generated pulses (nearly 100 nm). Optical properties of the transmission medium (absorption, refractive index, etc.) change over such a large interval of wavelengths. However, recently a new kind of crystal defect was found that can emit photons at 802 nm with the spectral width only about 1 nm (at room temperature). This color center consists of a nickel ion surrounded by four nitrogen atoms in a genuine diamond (Gaebel et al. [2004]).

The main problem of single-photon sources based on color centers is a rather low collection efficiency – currently just about 0.1 % for bulk crystals. The situation is slightly better for diamond nanocrystals – currently over 2 % (Beveratos et al. [2002]). The way how to increase the collection efficiency is to put the crystal into an optical cavity that suppresses the emission to all other spatial modes except the preferred one.

There are already first experiments with quantum cryptography using single photon sources based on nitrogen-vacancy centers (Beveratos et al. [2002], Alléaume et al. [2004]). The QKD was demonstrated in free space at a distance of 50 m.

5.1.4. Single-photon sources: Quantum dots

Quantum dots are semiconductor nanostructures ("artificial atoms") (Santori et al. [2001], Moreau et al. [2001], Zwiller et al. [2001], Hours et al. [2003], Baier et al. [2004]). By a suitable preparation a two- or more-level electronic system can be obtained. Photon emission comes from recombination of an electron-hole pair. Electron-hole pairs can be created either by optical pumping by a pulsed or continuous-wave laser or by an electric current (Yuan et al. [2002]). Various techniques of preparation of quantum-dots exist. The usual materials are, e.g., GaAs, GaAlAs, or InP.

The wavelength of emitted light is determined mainly by the material used. Sources operating at telecom wavelengths are possible (Takemoto et al. [2004]). The spectral width of a generated pulse depends on the number of excited energy levels and the average number of created electron-hole pairs.

The main practical drawback of quantum-dot photon sources is the need of cooling to the liquid-helium temperature. The latest research promises shift to temperatures about 100 K (Mandin [2004]). But the photon-number distribution of such "high-temperature" sources is worse. The other problem is very low collection efficiency (usually from $10^{-4}$ to $10^{-3}$). This means that the probability of obtaining an empty pulse is rather high. The efficiency can be increased (up to about $10^{-1}$)
5. TECHNOLOGY

by placing the quantum dot into an integrated solid-state microcavity (Gérard et al. [1998]).

The first demonstration of QKD using a quantum-dot single-photon gun was done by Waks et al. [2002]. It operated in free space to a symbolic distance of one meter.

5.1.5. Single-photon sources: Single atoms and molecules

Another alternative how to generate single-photon-like states is to make use of radiative transitions between electronic levels of a single atom (ion) or molecule.

Single ions caught in a trap and placed inside (or sent into) an optical cavity where they interact both with the excitation laser beam and the vacuum field of the cavity (Kuhn et al. [2002], Keller et al. [2004]) could represent single-photon sources with good properties (with, e.g., a narrow spectrum and high collection efficiency due to the presence of the cavity). But practical feasibility of such sources is still low because of their technological complexity (among others, high vacuum is needed).

Experiments with single organic-dye molecules are simpler because the molecules are usually caught in a polymer matrix (Brunel et al. [1999], Fleury et al. [2000], Treussart et al. [2002]) or put in a solvent (Kitson et al. [1998]) and the source is operated in usual environmental conditions and room temperatures. The photon statistics of generated states is reported to be good. The advantage is also a large scope of wavelengths that can be generated. But the critical problem is a limited stability of the molecules. Due to the photobleaching even the most stable dyes survive just a few hours of continuous excitation.

5.1.6. Entanglement source: Spontaneous parametric down conversion

By spontaneous parametric down conversion (SPDC) one can prepare photons entangled in energies (wavelengths), momenta (directions), and/or polarizations. Any of these features can be used for the purposes of QKD based on Ekert-type protocols (see Section 3.6).

In SPDC process one photon from a pump laser is converted, with a certain (small) probability, into two sub-frequency photons. The total energy and momentum are conserved thereat. Since no couple of possible frequencies and wave vectors of two generated photons is preferred the resulting quantum state is given as a superposition of all allowed cases – it is an entangled state.

SPDC occurs in non-linear optical media. E.g., in crystals KNbO$_3$, LiIO$_3$, LiNbO$_3$, $\beta$-BaB$_2$O$_4$, etc. Very perspective SPDC sources are periodically poled non-linear materials, namely waveguides in periodically poled lithium niobate (Tanzilli et al. [2001]).

5.2. DETECTORS

5.2.1. Avalanche photodiodes

The most widely used detectors in QKD systems with discrete variables are undoubtedly avalanche photodiodes (APD). In APD a single photoelectron generated by an impinging photon is multiplied by a collision ionization. This is because APD single-photon detectors are operated in a so-called Geiger mode: On the junction a
reverse voltage is applied that exceeds the breakdown voltage. Thus the impinging photon triggers an avalanche of thousands of carriers. To reset the detector the avalanche must be quenched. It could be done by a passive or active way. In the passive quenching a large resistor is placed in the detector circuit. It causes the decrease of voltage on APD after the avalanche starts. In the case of the active quenching the bias voltage is lowered by an active control circuit. This solution is faster so that higher repetition rates can be reached (up to 10 MHz). Another possibility is to work in a so-called gated mode when the bias voltage is increased above the breakdown voltage only for a short, well defined period of time.

To detect photons at specific wavelengths different materials of detector chips are needed. For the visible and near infrared region (up to 1.1 µm) the silicon APD can be used. Nowadays they are well elaborated. Compact counting modules with integrated Peltier cooling and active quenching are commercially available that offer low dark-count rates (below 50 per second) high quantum efficiencies (up to about 70 %) and maximum count rates reaching 10 MHz. Cooling to temperatures of about $-20^\circ$C is necessary to keep the numbers of dark counts induced by thermal noise in a reasonable range. Note that the dark counts, i.e., events when the detector sends an impulse even if no photon has entered it, represent an important factor limiting the operation range of QKD (see Section 6).

For telecom wavelengths, 1300 nm and 1550 nm, used in fiber communications, the silicon detectors cannot be applied. For 1300 nm germanium and InGaAs/InP detectors can be used. Germanium detectors require cooling to liquid nitrogen temperatures (77 K). Typical quantum efficiencies are about 15 %, dark-count rates about $25 \cdot 10^4$ pulses per second (at 77 K). For 1550 nm even germanium detectors cannot be used any more and currently the only generally available detectors for this wavelength window are based on InGaAs (on InP substrate). These detectors are now in common use for both telecom wavelengths. InGaAs detectors must also be cooled to low temperatures. In practice it can be done either by three-stage Peltier thermoelectric coolers (down to about $-60^\circ$C, i.e., 213 K) or by compact Stirling engines (down to about $-100^\circ$C, i.e., 173 K). Today’s typical performance of InGaAs APD at 1550 nm with a Peltier cooler is as follows: Quantum efficiency about 5–10 %, dark-count rate (in gated mode) about $10^3$ s$^{-1}$, maximal repetition frequency about 100 kHz–1 MHz (i.e., dead time about 1–10 µs). And with a Stirling cooler ($-100^\circ$C): Quantum efficiency above 10 %, hundreds dark counts per second (in gated mode), and maximal repetition frequency about 100 kHz–1 MHz. It turns out that the dark-count rate increases with increasing detection efficiency. It is always necessary to find a tradeoff between these quantities. As the number of dark counts increases with temperature, better overall performance can be achieved at lower temperatures. Also increasing signal repetition frequency leads to the growth of the number of dark counts because of the increasing probability of afterpulses.\textsuperscript{14}

Let us also mention another effect that can play a negative role in quantum cryptography. When the avalanche is quenched all charge carriers recombine. It brings the diode into an insulating state again, a full photodetection cycle is finished and the diode is ready for the next event. However, some recombinations are radiative – this results in so called backflashes. These dim light pulses propagate

\textsuperscript{14}After the avalanche is quenched some charge carriers may stay trapped on impurities. Their delayed recombination can lead to so called afterpulses – unwanted output impulses of the detector.
back to the communication channel and they could reveal the information on Bob’s basis setting to an adversary. That is, they represent a serious side channel and must be carefully eliminated (blocked) by proper filters (Kurtsiefer et al. [2001]).

An interesting possibility to improve the performance of QKD with APD detectors at telecom wavelengths could be the combination of parametric frequency up-conversion with efficient silicon APDs, instead of direct use of InGaAs APDs. The up-conversion in periodically-poled lithium niobate can be rather efficient whereas it introduces only relatively low noise. The overall quantum efficiency in combination with a silicon APD detector could then be comparable with the detection efficiency of an InGaAs APD while the dark-count rate would be lower (Diamanti et al. [2005]). This fact could enlarge the operation distance of QKD.

5.2.2. Quantum dot detectors

A Quantum Dot Resonant Tunnelling Diode is a semiconductor device with a quantum dot layer encased inside a resonant tunnelling diode structure (Blakesley et al. [2005]). In the diode two n-doped GaAs layers are separated by a double-barrier insulating AlGaAs layer and followed by a InAs self-assembled quantum dot layer. The resonant tunnel current through this double-barrier structure is sensitive to the capture of a hole excited by the photon by one of the quantum dots in the adjacent dot layer. The capture of a hole by the dot can switch the magnitude of the current flowing through the device.

The maximum detection efficiency measured with the device at 550 nm was 12%. However, the reasonable dark-count rate of 4000 s$^{-1}$ was achieved with a detection efficiency of only 5%. The device was cooled to 77 K. Measured sample could detect a new photon every 150 ns. It corresponds to about 6 MHz repetition rate (Blakesley et al. [2005]). But it is mainly limited by external electronics and the improvement to about 100 MHz is expected in a near future.

Note that the detector manufactured from GaAs cannot be used in the region of telecom wavelengths. Detectors for these wavelengths have to be built from other materials like InP.

5.2.3. Visible Light Photon Counters

Visible Light Photon Counters (VLPC) are semiconductor detectors consisting of two main layers, an intrinsic silicon layer and a lightly doped arsenic gain layer (Waks et al. [2003], Kim et al. [1999]). When a single photon is absorbed a single electron-hole pair is created. Due to a small bias voltage applied across the device, the electron is accelerated towards the transparent contact on one side while the hole is accelerated towards the gain region at the opposite side. Donor electrons in this region are effectively frozen out in impurity states because the device is cooled to an operation temperature of about 6 K. However, when a hole is accelerated into the gain region it easily kicks the donor electrons into the conduction band by impact ionization. Scattered electrons can create subsequent impact ionization events resulting in avalanche multiplication.

When a photon is detected, a dead spot of several microns in diameter is formed on the detector surface, leaving the rest of the detector available for subsequent detection events. If more than one photon is incident on the detector, it will be able
to detect all the photons as long as the probability that multiple photons land on
the same location is small. Therefore these detectors could perform efficient photon
number state detection (photon number count). However, in practice they can well
discriminate only between zero, one and more photons because of multiplication
noise.

Quantum efficiency of VLPC is about 90% and dark-count rate about $2 \cdot 10^4 \text{s}^{-1}$
at 543 nm (at 6 K).

5.2.4. Superconducting detectors

To detect single photons physical processes in superconductors can also be em-
ployed. A few different principles have been proposed that are now experimentally
tested. All these detectors require cryogenic environment. The first kind of detec-
tor, usually called Superconducting Single Photon Detector, consists of thin strips
of superconducting material, as niobium nitrate, interconnected to form a mean-
der shaped “wire” (Verevkin et al. [2002]). In this “wire” the current bias below
the critical current of the material is maintained. An impinging photon breaks a
Cooper pair and generates a hotspot that forms a resistive potential. The width
of the strips is designed in such a way that the current forced around the hotspot
exceeds the critical current. This results in the increase of resistance and a voltage
signal indicating the detection of photon. The recent measurements show that at
1300–1550 nm the samples have quantum efficiency up to 10%, dark-count rate
about $0.01 \text{s}^{-1}$ and counting rate over 2 GHz (Verevkin et al. [2004]). The mea-
surements were done at temperature 2.5 K (liquid helium).

Another type of superconducting detector is a Transition Edge Sensor (Miller et
al. [2003]). These sensors consist of superconducting thin films electrically biased
in the resistive transition. Their sensitivity is a result of the strong dependence
of resistance on temperature in the transition and the low specific heat and ther-
mal conductivity of materials at typical operating temperatures near 100 mK. The
device produces an electrical signal proportional to the heat produced by the ab-
sorption of a photon. These detectors can even determine the number of impinging
photons, i.e., they can perform a photon count. Observed efficiency at temperature
125 mK is about 20%, dark-count rate about 0.001 s$^{-1}$ (Miller et al. [2003]). The
newest results show even a better performance with a quantum efficiency over 80%
at 1550 nm (Rosenberg et al. [2005]). Unfortunately, these detectors are very slow
(dead time is about 15 $\mu$s) because it is necessary to remove the heat deposited by
each photon (Miller et al. [2003]).

Next possibility is a Superconducting Tunnel Junction Detector (Fraser et al.
[2003]). It consists of two superconducting electrodes separated by an insulating
layer forming together a Josephson junction. To suppress the tunnelling current
through the junction, a magnetic field parallel to the electrodes (parallel to the
tunnel barrier) is applied. Incident photons break Cooper pairs. It changes the
tunnelling rate according to the absorbed energy. The operating temperature is of
the order of hundreds of milikelvins. These detectors are able to register photons
from infrared to ultraviolet region.
5. TECHNOLOGY

5.3. QUANTUM CHANNELS

5.3.1. Fibers

The most promising channels for terrestrial QKD are undoubtedly single-mode optical fibers. The lowest attenuations of standard telecom fibers are at 1300 nm (about 0.35 dB/km) and at 1550 nm (about 0.2 dB/km). Unfortunately, for these wavelengths standard silicon-based semiconductor photodetectors cannot be used. In principle, it is possible to use special fibers and work around 800 nm, where the efficient detectors are available. But the attenuation of fibers at these wavelengths is relatively high, about 2 dB/km, and such fibers are not used in an existing infrastructure. Therefore, the attention is paid to standard telecom fibers and there is an effort to develop low-noise and efficient detectors for wavelengths 1300 nm and 1550 nm.

The losses in fibers represent one of the two main factors (see Section 6) limiting the operation range of QKD systems (notice that attenuation 0.20 dB/km means 99% loss after 100 km). Other problems are the strong temperature dependence of some optical properties of fibers, the disturbance of polarization states of light in fibers due to the geometrical phase and the birefringence, and the dispersion.

The distortion of polarization is a crucial obstacle for the use of any kind of polarization encoding of information. Therefore in fiber-based QKD systems phase-encoding schemes are usually employed. However, even in such a case the output polarization state must be under control. Fortunately, if the fiber is fixed the polarization properties are relatively stable.

Dispersion affects the temporal width of the broad-spectrum light pulses. Therefore the sources generating broad-band signals are not well suitable for fiber QKD. Nevertheless, there is still a possibility to work near the wavelength of 1310 nm where the silica fibers have zerochromatic dispersion or to use fibers with special refractive-index profile which have zero dispersion shifted near 1550 nm.

5.3.2. Free space

Quantum key distribution can also be accomplished through free space. The advantage of this approach is that the atmosphere has very low absorption around the wavelengths 770 nm and 860 nm where relatively efficient and low-noise silicon semiconductor detectors can be used. Besides, no optical cables have to be installed. Also, the atmosphere is not birefringent at these wavelengths and is only weakly dispersive. The disadvantage is that the free-space communication can be used only in the line-of-sight distances, no obstacle may be between communicating parties. There are also other drawbacks: The performance is highly dependent on the weather, pollution and other atmospheric conditions. There are huge differences in attenuation for different kinds of weather. For instance, for wavelengths near 860 nm the attenuation of clear air can be below 0.2 dB/km, in the case of moderate rain it is about 2 – 10 dB/km, and in heavy mist it can exceed 20 dB/km. Further, up to altitudes about 15–20 km there are considerable atmospheric turbulences. The problem is also a spurious influence of the background light, especially the ambient daylight. Another issue is the beam divergence. Due to diffraction the diameter of the beam can be considerably enlarged in large distances. This effect can cause additional loss if only a part of the beam is captured by the receiver.
§ 6. Limitations

There are two main technological obstacles that inhibit the wide spread of quantum key distribution yet: limited operational range and low transmission rates.

6.1. TRANSMISSION RATE

The key factor limiting the raw-key rate is the detector's deadtime (i.e., recovery time of the detector). In the case of avalanche photodiodes (APD) immediately after the detection event the detector is not ready for other detection. First of all, the avalanche of charge carriers must be quenched. However, there is also a problem with the so called afterpulses – clicks of detector caused by spontaneous transitions from long-living traps (levels in a forbidden band) populated by the preceding avalanche. It is necessary to wait until the carriers leave the detection (depleted) region. Typical APD dead time is from about hundred nanoseconds to a microsecond.

The next factor decreasing transmission rate appears if an attenuated laser is used as a source for QKD. Due to security requirements (suppression of multi-photon pulses) the mean photon number per pulse must be fairly below one, although this leads to a high vacuum fraction of signals.

Of course, the crucial decrease of transmission rate is due to losses in the channel.

The rate of distilled key is further decreased by error-correction and privacy-amplification procedures. The higher the error rate, the shorter is the distilled key that is obtained from the same amount of raw key.

6.2. LIMIT ON THE DISTANCE

The maximal distance over which secure QKD can be established decreases with increasing losses and increasing detector noise. The detector dark-count rate is constant (for a given detector and settings). But the key-rate decreases with increasing distance due to cumulative losses. So the relative number of erroneous bits caused by dark counts grows as long as it is so high that secure QKD is impossible. Standard amplifiers cannot be used as they would affect the states of photons in a similar manner as eavesdropping. Present-day technology allows secure operation up to about 100 km.

6.3. QUANTUM REPEATERS

The use of entangled pairs for QKD (see Section 3.6) offers an important advantage. It enables to extend the radius of secure communication to practically arbitrary distance (at least in theory). This can be reached by quantum repeaters (Dür et al. [1999]). They can do “distributed error correction” without revealing any information on the key. The communication channel is divided into shorter segments each containing a source of entangled pairs. At the ends of each segment a distillation of entanglement (Bennett et al. [1996a]) is performed. It produces a smaller number of “repaired” highly entangled pairs from an originally higher number of pairs damaged during transmission. Individual segments are “connected” by means of an entanglement swapping method (Bennett et al. [1993], Żukowski et al. [1993]). So finally Alice and Bob possess highly entangled pairs.
7. SUPPORTING PROCEDURES

§ 7. Supporting procedures

7.1. ESTIMATION OF LEAKED INFORMATION

Real devices like polarizers, fibers, detectors, etc. are never perfect and noiseless. Therefore we always have to tolerate a certain amount of errors. However, we cannot be sure that these errors do not stem from Eve’s activity (Eve could, e.g., replace some noisy part of the system by better – less noisy – one) so we have to attribute all errors to Eve. Fortunately, from the observed error rate it is possible to estimate the information leaked to Eve and then “shorten” the established key in such a way that Eve’s information on the new, shorter key is arbitrarily small.

First, Alice and Bob chose randomly a certain number of transmitted bits and compare them publicly to estimate the error rate. The higher the number of compared bits is, the higher is the probability that the actual error probability does not exceed the estimated value. Assuming the most general attack allowed by the laws of quantum physics one can find the boundary of the amount of information, Eve could get on the key, in dependence on the error rate caused by the attack. For the simplest intercept-resend attack described before (assuming non-continuous eavesdropping) Eve gets an average information per bit $I = 2\epsilon$, where $\epsilon$ is the bit-error rate. Of course, this attack is not optimal. The limiting (“worst”) values of $I(\epsilon)$ depend both on the protocol and implementation. These problems will be discussed in more detail in Section 8.

7.2. ERROR CORRECTION FOR CLASSICAL BIT STRINGS

When Alice and Bob create a sifted key by sorting out signals for which Bob has used the “wrong” bases, their key sequences need not be exactly the same. This may be caused either by an eavesdropping or by “technological” noise. Therefore, Alice and Bob must correct or eliminate the erroneous bits. Here we describe a simple error-correction procedure proposed by Bennett et al. [1992a].

Alice and Bob first agree on a random permutation of the bit positions in their strings to randomize the location of errors. Then they partition the permuted strings into blocks of size $k$ such that single blocks are believed to be unlikely to contain more than one error (block size is a function of the expected bit-error rate). For each block, Alice and Bob compare the block’s parity. Blocks with matching parities are tentatively accepted as correct. If parities do not agree, the block is subjected to a bisective search, disclosing further parities of sub-blocks, until the error is found and corrected.

To remove errors that remained undetected (e.g., because they occurred in blocks or sub-blocks with an even number of errors), the random permutation and block parity disclosure is repeated several more times, with increasing block sizes. Once Alice and Bob estimate that at most a few errors remain in the data as a whole, they change the strategy (at this point, the block parity disclosure approach becomes much less efficient because it forces Alice and Bob to reveal at least one parity bit in each block). Now they publicly choose random subsets of the bit positions in their entire respective data strings and compare the parities. If disagreement is found, the bisective search is undertaken, similar to that described above. The procedure is repeated several times, each time with a new independent random subset of bit
positions, until no errors is left. Alice and Bob are now in possession of a string that is almost certainly shared but only partly secret.

The revealed parity bits represent an additional information leaked to Eve that must be taken into account. In order to avoid this leakage of information during the reconciliation process either the exchanged parity bits must be one-time-pad encrypted or the information that is additionally made available to the eavesdropper must be taken into account during the privacy amplification step.

Other error-correcting (or reconciliation) procedures are described by Brassard and Salvail [1993] (among others the procedure that leak a minimum amount of information during reconciliation) and by Sugimoto and Yamazaki [2000].

Note that the error correction shortens the bit string at least to a fraction $1 - h(\epsilon)$, where $\epsilon$ is the error rate and $h(p) = -p \log_2(p) - (1 - p) \log_2(1 - p)$ is the Shannon entropy. This is the so called Shannon limit. Practical error-correcting procedures are less efficient and shorten the bit string even more.

7.3. PRIVACY AMPLIFICATION FOR CLASSICAL BIT STRINGS

Let us suppose that both Bob and Eve have already made measurements and they have some classical information on the key bits sent by Alice.\(^\text{15}\) If Bob has higher information on the key sent by Alice than Eve [$I(B;A) > I(E;A)$\(^\text{16}\)], then Alice and Bob can establish a new secret key, such that Eve has negligible information on it, using only one-way communication. First, Alice and Bob have to carry out an error-correction procedure in order to have the exactly same bit sequences. At that point, Alice and Bob posses identical strings, but those strings are not completely private. Next, they proceed with the following algorithm, called privacy amplification (Bennett et al. [1988], Bennett et al. [1992a], Bennett et al. [1995]).

Alice, at random, picks $N$ bits, $[X_1, X_2, \ldots, X_N]$, from the sifted key and performs an exclusive OR logic operation on them (XOR; here we will denote it by $\oplus$), which finds their sum modulo 2 (in fact she calculates a parity bit): $[X_1 \oplus X_2 \oplus \ldots \oplus X_N]$. She tells Bob which bits she did the operation on, but does not share the result. Bob then carries out the same operation with his bits on the same positions: $[Y_1 \oplus Y_2 \oplus \ldots \oplus Y_N]$ and keeps the result. As we have supposed that Alice’s and Bob’s bit strings are exactly the same ($X_i = Y_i$), Bob’s result must also be the same as Alice’s one.

Bob and Alice next replace each $N$-tuple of key bits with the calculated XOR value (these values represent a new key). Meanwhile, if Eve, who has many errors in her key, tries the same operation, it only compounds her mistakes, thus her information decreases. For example, if Eve knows the correct value of each bit with a probability $p = \frac{1}{2}(1 + \epsilon)$ then she will know the parity bit with the probability $p' = \frac{1}{2}(1 + \epsilon^N) < p$ when $\epsilon < 1$.

To put it in a more formal way, Alice and Bob share an $n$-bit string $S$, and we suppose that Eve knows at most $k$ bits of $S$. Alice and Bob wish to compute an $r$-bit key $K$, where $r < n$, such that Eve’s expected information about $K$ is

\(^{15}\)If Eve has attacked the transmission using quantum probes she can wait with measurements on her probes until Alice and Bob carry out all necessary supporting procedures and she can then modify her measurements. The procedures described below are useful even in such a case. More about security issues can be seen in Section 8.

\(^{16}\)\(I(X;Y) = H(X) + H(Y) - H(X,Y)\) with $H$ being the Shannon entropy; see Section 8.2.
below some specified bound. To do so, they must choose a compression function \( g : \{0,1\}^n \rightarrow \{0,1\}^r \) and compute \( K = g(S) \). The procedure described above is an example of a good compression function. It has been shown by Bennett et al. [1995] that if Eve knows \( k \) deterministic bits of \( S \), and Alice and Bob choose their compression function \( g \) at random from the so called universal class of hash functions, \( g : \{0,1\}^n \rightarrow \{0,1\}^r \) where \( r = n - k - s \) for some safety parameter \( s \in (0, n - k) \), then Eve’s expected information about \( K = g(S) \) is less than or equal to \( 2^{-s}/\ln 2 \) bits.

It is worth noting that if even a single discrepancy is left between Alice’s and Bob’s data after the error correction procedure, then after privacy amplification their final bit strings will be nearly completely uncorrelated.

7.4. ADVANTAGE DISTILLATION FOR CLASSICAL BIT STRINGS

Even if the mutual information on the key of Bob and Alice is lower than the mutual information of Eve and Alice \( [I(B;A) \leq I(E;A)] \) it may still be possible to establish a secret shared key by means of a two-way classical communication\(^\text{17}\) (assuming a noiseless and authenticated classical public channel; Maurer [1993]).

Alice takes an \( N \)-bit block, \([X_1, X_2, \ldots, X_N]\), of the sifted-key bits, generates a random bit \( C \) and makes the following encoding (here \( \oplus \) means XOR again; note that all bits of the block are XORed with the same bit \( C \)): \([X_1 \oplus C, X_2 \oplus C, \ldots, X_N \oplus C]\). Finally she sends this encoded block to Bob. Bob then computes \([ (X_1 \oplus C) \oplus Y_1, (X_2 \oplus C) \oplus Y_2, \ldots, (X_N \oplus C) \oplus Y_N ] \), where \([Y_1, Y_2, \ldots, Y_N] \) is his block of the sifted-key bits corresponding to Alice’s block. Bob accepts only if the result consists of the equal bits, i.e. either \([0,0,\ldots,0]\) or \([1,1,\ldots,1]\). In this case he sets either \( C' = 0 \) or \( C' = 1 \), respectively, as an element of his new key [note that if \( X_i = Y_i \) then \( (X_i \oplus C) \oplus Y_i = C \)]. If Bob’s calculation results in different bits Bob rejects the block.

This procedure is repeated with the other blocks of the sifted key and other random bits \( C \). In other words, Alice and Bob make use of a repeat code of length \( N \) with only two codewords \([0,0,\ldots,0]\) or \([1,1,\ldots,1]\). The sequence of random bits \( C \) sent by Alice and accepted by Bob represents a new key generated by Alice and the sequence of bits \( C' \) accepted by Bob represents a new key received by Bob. In this way, the probability that Bob accepts erroneously bit \( C \) sent by Alice goes down with increasing \( N \) as \( \epsilon^N \), where \( \epsilon \) is a bit-error rate in the original sifted key. Eve, on her side, has to use a majority vote to guess the bit \( C \). Hence, Bob’s information on \( C \) may be larger than Eve’s information even if Bob’s information on Alice’s bits \([X_1, X_2, \ldots, X_N]\) is lower than Eve’s one. On the new key the error correction and privacy amplification may be applied subsequently.

7.5. AUTHENTICATION OF PUBLIC DISCUSSION

In practice, the “auxiliary” information transmitted through the open channel during QKD could be modified, as it is difficult to create a physically unjammable classical channel. For example, Eve can cut both the quantum and classical channels and pretend to be Bob in front of Alice. Therefore the authentication of the messages sent over the open channel is necessary (the recipient must be able to

\(^{17}\)Two-way communication is anyway necessary for basis announcement in BB84.
check that the message has come from the “proper” sender and that it has not been modified). This procedure requires additional “key” material to be stored and transmitted. For quantum cryptography to provide unconditional security, the procedure used for authentication of public discussion must also be unconditionally secure. Such authentication algorithms exist (Wegman and Carter [1981], Stinson [1995]). They are based, e.g., on the so-called orthogonal arrays. The length of the authentication password must always be greater than the length of the authenticated message, but the authentication tag (the additional information sent together with the message to verify its origin and integrity) is relatively short. This authentication tag itself is one-time pad encrypted to avoid leaking information on the authentication password to Eve. A small random sequence of the same length as the authentication tag, used for its encryption, needs to be renewed after each QKD transmission (it may be “refilled” from the established key-string). For example, if the cardinality of the set of authenticated messages is \((p^d - 1)/(p - 1)\), where \(p\) is a prime and \(d \geq 2\) an integer, an authentication code can be created with \(p^d\) keys and \(p\) authentication tags. The deception probability is then \(1/p\) (Stinson [1995], Dušek et al. [1999b]).

Clearly, the authentication requires Alice and Bob to meet each other at the beginning in order to exchange an authentication password and primary one-time-pad key for encrypting the authentication tag. After each transmission, this key is replaced by a new one, obtained from the transmitted sequence. Therefore, the QKD cryptosystem works rather as an “expander” of shared secret information: Some initial shared secret string is needed but later it can be arbitrarily expanded.

§ 8. Security

It is the goal of QKD to deliver secret keys to the users. It differs from classical key distribution schemes as in QKD we can actually prove the security of the final key under a very limited number of natural assumptions. These include, for example, that an eavesdropper cannot have access to the data inside the devices of Alice and Bob.

In an experimental implementation one cannot demonstrate directly secure quantum key distribution: security cannot be measured as such. Security is a theoretical statement and refers to specific protocols to generate a secret key from the data we obtain in an experiment. These protocols depend on observable parameters, such as the error rate, the mean photon number of the source and the loss rate of the signals. So in an experiment, one verifies the model assumptions of the theoretical security analysis and demonstrates that one can operate the device such that the observed parameters allow the generation of a secret key following the protocol. It is important that the awareness of this point increases.

Let us have a closer look at the problem of real life implementations of QKD schemes (see Section 4). All devices we are using will be imperfect to some degree. Moreover, all quantum channels show imperfections, for example in the form of a polarization mode dispersion, dephasing in interferometric schemes, and, dominantly, loss (Gisin et al. [2002]). Basic QKD protocols test for the presence of an eavesdropper by looking for changes in the quantum mechanical signals. As a result of imperfections we have to face the situation that Alice and Bob end up with data that deviate from the ideal ones. Therefore they would have to abort QKD in an
idealized simple protocol that only tests for the presence of an eavesdropper: we have to assume the worst-case scenario that the degradation of the data is not due to the channel imperfections, but might come from an active eavesdropper. The eavesdropper could be correlated with the data of Alice and Bob, thus having some information about them. Moreover, in general Alice and Bob do not even share an error-free bit-string.

It turns out that there are ways to create a secret key despite these imperfections. For this, Alice and Bob apply some postprocessing procedures by publicly communicating over a classical, authenticated channel. Typically, these procedures include error correction and privacy amplification (see Section 7). It is important to know what key rate can be achieved from the data without compromising security. The parameters for the public discussion protocols come from the security proofs. In this section we will give some background to security proofs and report on the present status for different protocols.

8.1. ATTACKS ON IDEAL PROTOCOLS

Before we start to analyze the security of QKD in more detail, let us have a look at how Eve could actually perform her eavesdropping activity. From the theory of quantum mechanical measurements we know that any eavesdropping can be thought of as an interaction between a probe and the signals. Eve can then measure the probe to obtain information about the signals.

We distinguish three main types of eavesdropping attacks:

**Individual Attack:** In the individual attack Eve lets each signal interact with a separate probe. Eve performs then a measurement on each probe separately after the interaction. This type of attack is easy to analyze since it does not introduce correlations between the signals.

**Collective Attack:** The collective attack starts as the individual attack, as each signal interacts with its own independent probe. At the measurement stage, however, Eve can perform measurements that act on all probes coherently. We know from quantum estimation theory that such measurement can in some cases give more information about the signals than the individual measurement. For the analysis it is convenient that also this attack does not introduce correlations between the signals.

**Coherent Attack:** This is the most general attack which an eavesdropper can launch on the quantum signals exchanged between Alice and Bob. Actually, one can assume the worst case scenario that Eve has access to all signals at the same time. Then the sequence of signals is described by one high-dimensional quantum state, on which Eve can perform a measurement via a single probe. This type of interaction can introduce any type of correlations, also between subsequent signals, as seen by Alice and Bob.

Further variations of these attacks can be obtained by distinguishing whether Eve has to measure her probes before Alice and Bob continue their protocol, e.g. by exchanging basis information in the BB84 protocol, or whether she can delay her measurement until the very end of the protocol executed by Alice and Bob.

Note that Eve does not necessarily have to measure the probe to extract information about the key. The secret key will be used to encrypt a secret, or be used in a different cryptographic application, which might also use quantum tools. So
Eve might use her probes from the QKD protocol to attack the subsequent cryptographic application. The problem whether we can separate the security analysis of the different steps is known as composability. This has been addressed recently by Ben-Or et al. [2004] showing that also in the quantum case the generation of secret key via QKD can be separated from the use of this key later on. This is especially important since part of this secret key will be used to authenticate the public channel of subsequent QKD exchanges.

Another question is that of the assumptions to which extent an eavesdropper can exploit imperfections of Alice’s and Bob’s devices. As an example, consider single photon detectors: they are affected by dark counts and have a non-ideal detection efficiency (see Section 5.2). In a paranoid picture, we assume that Eve can exploit even these imperfections. She might reduce or eliminate dark counts by a suitable pulse sequence inserted into the optical fiber leading to Bob’s detectors. By a change of wavelength, she might increase the detection efficiency. Clearly, a precaution against each individual known attempt can be taken, though it will be hardly possible to list exhaustively all possible attacks. In a paranoid picture, we are on a safe side even if Eve could really do all those things. Actually, it turns out that this paranoid picture is extremely helpful to provide actual security proofs.

On the other hand, we can hope to protect against eavesdropping activities that manipulate Bob’s detectors. In that case, the secure key rate will increase clearly. However, it turns out, that it is technically harder to provide unconditional security proofs in this scenario.

In the history of QKD, the individual attack played a crucial role (Fuchs et al. [1997], Ekert et al. [1994], Lütkenhaus [1996], Slutsky et al. [1998]) since it has been easy to analyze in conjunction with the generalized privacy amplification method. However, presently the individual attack scenario loses its relevance since methods have been developed to prove unconditional security, that is, security against coherent attacks. Actually, it is widely believed that for typical protocols one needs only to consider collective attacks, though only recently steps have been made to prove this (Renner [2005]).

8.2. SECURE KEY RATES FROM CLASSICAL THREE-PARTY CORRELATIONS

A typical, practical QKD protocol consists of two phases:

**Phase I**: A physical setup generates quantum mechanical signals. These are distributed and subsequently measured. As a result, Alice and Bob hold classical data describing their knowledge about the prepared signals and the obtained measurement results.

**Phase II**: Alice and Bob use their authenticated classical channel to talk about their data, for example by sifting their data, performing error correction and privacy amplification.

The important question is, how exactly to convert the data obtained in phase I into a secret key in phase II. To understand this process and its limitation, let us have a look into the classical world. Also in classical information theory unconditional security is being discussed. There the starting point are identically and independently distributed random variables with a probability distribution for data of Alice, Bob and Eve, $P(A, B, E)$. Once one assumes correlations of a given type, described by
P(A, B, E), one can investigate whether public discussion protocols can turn these data into a secret key.

There are two main results in this context. The first one is about a lower bound on the achievable rate. This has been given by Csiszár and Körner [1978]. Remember that the Shannon entropy $H(A)$ of a random variable $A$, which takes values $a$ with probability $p(a)$, is defined as $H(A) = -\sum_{a \in A} p(a) \log_2 p(a)$, and the Shannon entropy of a joint probability distribution is analogously defined as $H(A, B) = -\sum_{a \in A, b \in B} p(a, b) \log_2 p(a, b)$ (Cover and Thomas [1991]). Then the Shannon mutual information between two parties holding the random variables $A$ and $B$, respectively, with a joint probability distribution $p(a, b)$ is then given by

$$I(A; B) = H(A) + H(B) - H(A, B) .$$  

(8.1)

Then the lower bound for the maximal secure-key rate, $R$, is given (Csiszár and Körner [1978]) by

$$R \geq \max \left( I(A; B) - I(A; E), I(A; B) - I(B; E) \right) .$$  

(8.2)

This lower bound can be achieved, if positive, in the following way: Alice and Bob perform error correction (see Section 7.2) via a one-way method, either by Alice giving error correction information to Bob, or vice versa, depending on whether the first or second expression in Eq. (8.2) is bigger. If we encode the error correction information with a one-time pad to avoid leakage of additional correlations to Eve, then this reduces the effective key rate by the fraction $1 - I(A; B)$ of the original data. In the second step, Alice and Bob perform privacy amplification, shortening their key by the fraction $I(A; E)$ or $I(B; E)$, depending on the chosen communication direction. In total we find the key rate given on the right hand side of Eq. (8.2).\(^{18}\)

Surprisingly often, we find that this classical lower bound is also cited and used in a QKD scenario, where an optimization over individual attacks is performed to give bounds on Eve’s information about Alice’s or Bob’s data. Note, that the use of the Csiszár and Körner formula is restricted to the classical case of independently and identically distributed random variables. This can only be justified if we restrict Eve to individual attacks, which are not necessarily optimal compared to coherent or collective attack. Additionally, we have to assume that Eve attacks all signals in precisely the same fashion, and that she measures the probes of each signal immediately. It is clear, that the predicted key rates from this procedure can give a rough feeling of what to expect from a more detailed security analysis, but it cannot replace it.

The second important result in the classical three-party situation is due to Maurer (Maurer [1993], Maurer and Wolf [1999]). This result gives an upper bound on the extractable secret key rate for given $P(A, B, E)$. It can be expressed in terms of the conditional mutual information $I(A; B|E)$, which is defined as

$$I(A; B|E) = H(A|E) + H(B|E) - H(A, B|E) .$$  

(8.3)

\(^{18}\)Alternatively, one can send the error correction information unencoded; then the final key is shortened in privacy amplification giving the same effective secret key rate (Cachin and Maurer [1997], Lütkenhaus [1999]).

\(^{19}\)The conditional Shannon entropy is defined as $H(X|Y) = -\sum_{x \in X, y \in Y} p(y) p(x|y) \log_2 p(x|y)$ with $p(x|y)$ being a conditional probability.
The formal definition of the upper bound, the *intrinsic information* is

\[ I(A; B \downarrow E) = \min_{E \rightarrow \bar{E}} \left[ H(A|\bar{E}) + H(B|\bar{E}) - H(A,B|\bar{E}) \right] \]  

(8.4)

where we minimize over all possible mappings from the random variable \( E \) to the random variable \( \bar{E} \) [i.e., over all possible random distributions \( P(A,B,\bar{E}) \) consistent with \( P(A,B) \)]. The intrinsic information measures how much Bob learns about Alice’s data by looking at his own data after Eve announced her data (or a function of her data). The bound is then given by

\[ R \leq I(A; B \downarrow E). \]  

(8.5)

If Bob’s data depend only on Eve’s announcement, but no longer on Alice’s data, then the intrinsic information vanishes and we find that no secret key can be generated. Note that this statement is true for all possible public discussion protocols Alice and Bob might come up with (Maurer and Wolf [1999]).

By evaluating the lower and upper bounds one finds a wide gap between them. Actually, there are no protocols known to achieve the rate of the upper bound. The method of advantage distillation (see Section 7.4) taps into the gap (Maurer [1993]). There are cases where the lower bound is initially zero, but after the application of an advantage distillation step the lower bound for the new, conditional, correlations is positive.

### 8.3. BOUNDS ON QUANTUM KEY DISTRIBUTION

So far we have been talking about the classical scenario. There we had to *assume* a specific form of the joint probability distribution \( P(A,B,E) \). In quantum mechanics we can infer from the observations on Alice’s and Bob’s side something about the ways Eve might be correlated to their data, so we are in a stronger position. At the same time, we have some added complications: Eve is free to maintain her probes in a quantum mechanical state. We cannot force her to measure her probe, thus reducing her probe to classical data. So we cannot directly use quantum mechanics to consider the class of joint probability distributions \( P(A,B,E) \) that are compatible with the observations to apply the Csiszár-Körner result. Here we have to find new lines of argumentation to provide the security statements, including new lower bounds. However, in one point the classical statements can be directly applied: the result of Maurer on upper bounds on the key rates is valid for QKD. Any individual attack compatible with the observations and quantum mechanics allows us to derive a valid upper bound (Moroder et al. [2005]). We obtain this upper bound by choosing a measurement on the individual probes. This results in a classical probability distribution \( P(A,B,E) \) and subsequently we obtain an upper bound on the key rate in the quantum case according to inequality (8.5). Other bounds are given e.g. by the regularized relative entropy of entanglement (Horodecki et al. [2003a], Christandl and Renner [2004]).

This idea allows us to address a question that is important for experimental quantum key distribution: which types of correlated data generated by a set-up of Phase I can lead at all to a secret key via a suitable designed protocol in Phase II? More specifically, given a set of signals for Alice and a choice of measurement devices for Bob, and given that one finds some joint probability distribution \( P(A,B) \)
for the signals and measurement results using some quantum channel under Eve’s control: can we at all generate a secret key from these data? What would be an upper bound for the data rate we can obtain?

As a (partial) answer it turns out that it is a necessary condition for generating a secret key from these data that they cannot be explained as coming from an entanglement breaking channel (Curty et al. [2004]). Such a channel breaks the entanglement of an entangled input state by acting on that sub-system of a bi-partite state which passes through it. It has been shown by Horodecki et al. [2003b], that each entanglement breaking channel can be represented by a so-called intercept/resend attack (see Section 2.5). In this attack Eve performs some measurement on Alice’s incoming signal, transmits the measurement result over a classical channel and then feeds a new quantum state into Bob’s measurement device which depends only on Eve’s measurement result. If the data cannot be explained in this way, we say that the data contain quantum correlations. In this situation it has been shown that the intrinsic information does not vanish (Ac{ín and Gisin [2005]).

It is easy to see that from data that can be explained as coming from an entanglement breaking channel we cannot generate a secret key. Just have a look at the joint probability distribution of Alice, Bob and Eve, regarding Alice’s signals and Bob’s and Eve’s measurement results. This class of channels assures that the joint probability distribution for Alice and Bob conditioned on Eve factors as $P(A,B|E) = P(A|E)P(B|E)$. One can insert this into the definition of the intrinsic information (using $\tilde{E} = E$) and finds quickly that the intrinsic information vanishes, using $H(A,B|E) = H(A|E) + H(B|E)$. This means that the upper bound on the key rate vanishes and no secret key can be generated. This principle allows us to narrow down the parameter regimes in which QKD can be successfully performed at all for specific setups. For specific protocols, e.g. choice of signals and measurement devices, one can convert the question whether a given set of data can be explained by an entanglement breaking channel into the problem of proving the existence of entanglement of a virtual bi-partite quantum state (Curty et al. [2004], Curty et al. [2005]). This can be done e.g. using the idea of entanglement witnesses (Horodecki et al. [1996]).

Since general security proofs can be quite complicated, it makes sense for newly proposed QKD protocols to check first for which parameter regime of the channel the upper bound does not vanish. Note that once we verified the presence of quantum correlations we only satisfied a necessary condition for secure QKD, but we still need to provide a protocol of Phase II together with a security proof to achieve QKD. It is not clear whether one can always generate a secret key once we have quantum correlations.

8.4. SECURITY PROOFS

It is time to show ideas of how to construct protocols in Phase II which turn the observed correlated data into secret key. The key requirement in quantum key distribution is that at the end of such a protocol, the quantum system in Eve’s hand should be uncorrelated with the output of the protocol: the secret key.

There are several ideas how one can achieve this goal. Consider a quantum channel which transmits faithfully two non-orthogonal states. One can show that in this case Eve cannot have interacted with the signals; more precisely, starting with
a general interaction with a probe and adding the constraint that the interaction leaves two non-orthogonal signal states invariant, one can show that the output of this action is a tensor product between the probe and the signal states. This guarantees that the probe cannot be correlated with the signals or Bob’s measurement results: the state of the probe is independent of these classical data of Alice and Bob.

Clearly, in a realistic noisy channel, we cannot expect to be able to use this principle directly. However, there is an analogy in classical information transfer. As we learned from Shannon, one can use noisy classical channels to transmit classical messages perfectly. The trick is to use classical error correction codes that encode the original message as so called codewords. The encoded message is sent through the noisy channel. The effect of the noise on the codewords can be detected and the errors can be corrected. This mechanism works asymptotically perfect.

Something similar can be done by using Quantum Error Correction Codes (QECC); Calderbank and Shor [1996], Steane [1996]. Again, the basic idea is to take the non-orthogonal signals states from the source, to encode them into a longer sequence of signals that are transmitted through the channel, and then to decode the original states asymptotically error-free. This can be done in principle, though in this form it would require Alice and Bob to perform encoding and decoding operations on several signals, which is beyond our present experimental capability. Based on this idea, and using earlier results by Mayers (Mayers [1996], Mayers [2001]), Shor and Preskill [2000] showed that one can adapt the basic idea of quantum error correction codes so that the quantum protocol becomes equivalent to the standard BB84 protocol in which Alice sends a random sequence of signals and Bob measures them in a randomly selected basis. In that case, the decoding operation of the QECC turns into classical error correction and privacy amplification and no quantum manipulation capabilities are required.

Let us have a look at this method in more detail. A QECC can correct errors which are introduced by the channel. The Shor and Preskill security proof is based on the Calderbank-Shor-Steane QECC (Calderbank and Shor [1996], Steane [1996]) which divides the errors into bit and phase errors. That is, without loss of generality, the channel applies to each signal qubit either an error operator, the $\sigma_x$ or the $\sigma_z$, or it applies the identity operator. One encodes the signal qubits into quantum codewords, e.g. into a larger number of qubits, which are then sent over the channel. As long as the number of qubits affected by error operators is sufficiently low, the action of the channel can be reverted, thanks to the additional structure that is provided by the codewords. The reversion of the $\sigma_x$ corresponds to the classical bit error correction. The errors coming from $\sigma_z$ will not be corrected, as we are interested only in the bit values of the original quantum signals. Instead, one chooses the QECC structure such that, in principle, one could have corrected the errors in the quantum domain. This happens by including redundancy in the signals. Taking out this redundancy is exactly what happens in the privacy amplification procedure.

We note that one essential step is to estimate the number of phase and bit errors, since the security hinges on the fact that one could in principle correct these errors. Therefore, in fact, it is an essential task to estimate the number of errors from the observable data. From this estimation, we can then determine the parameters characterizing the classical bit error correction and privacy amplification. It is
important to reduce this estimation problem from the quantum level to the level of classical estimation theory. In the case of the BB84 protocol and the Shor-Preskill proof this is straightforward, due to the symmetry. For other protocols more advanced methods have been developed (Tamaki et al. [2003b], Koashi [2004]).

Let us come to the next principle for security proofs. The principle exploiting the QECC method uses effectively only one-way communication. This idea can be extended to two-way communication, which turns out to tolerate higher noise levels in the channel. So far, we have been using the idea that it is sufficient to create an effective perfect channel between Alice and Bob to guarantee that Eve decouples from Alice and Bob. Another way to achieve this goal is to establish maximally entangled states between Alice and Bob. Once Alice and Bob verify this property, they can be assured that Eve is decoupled from their bi-partite states. This is what is commonly referred to as monogamy of entanglement. Clearly, once we have effective perfect channels via QECC, we can achieve the distribution of maximally entangled states. For this, Alice prepares these states locally and sends one subsystem of each state to Bob via the effective, perfect, channel. This method can be generalized in the way that Alice sends the subsystems via the noisy channel to Bob. The important idea is that Alice and Bob then perform entanglement distillation to regain a reduced number of maximally entangled states (Bennett et al. [1996b]). This assures that Eve is decoupled from their states. Actually, the use of one-way QECC is one method for this, though there are two-way protocols that can tolerate a higher error threshold. In practical QKD it is important to find those entanglement distillation protocols that can be translated again in classical post-processing of data. An example of this is the protocol and security proof based on the BB84 protocol by Gottesman and Lo [2003] and Chau [2002].

For quite a while it seemed that the security of QKD can be expressed always as an underlying entanglement purification protocol. However, recently it has been shown by the Horodecki family and Oppenheim (Horodecki et al. [2003a]) that one can go even further. They showed that one can create secret keys also from states that are bound entangled, that is from states that cannot be distilled to maximally entangled states. The important idea behind their protocols is that there are certain global unitary operations acting on their systems only, which cannot actually be performed by Alice and Bob due to their spacial separation, but which would turn the bound entangled states into products of maximally entangled states and some remaining systems. Again, Eve is then decoupled from the maximally entangled system. Alice and Bob obtain their secret key by measuring the maximally entangled state in a predefined basis. The discussed global unitary operations now have the property that they leave these measurement results invariant. So the key data will be the same with or without applying the unitary operation. Since the key is secure after application of the global unitary operation of Alice and Bob, it is also secure without performing this operation. The security is therefore not based directly only on the distillability of maximally entangled states.

8.5. SPECIFIC ATTACKS

Before we turn to the security results for given protocols, we list a few specific attacks, especially those that are applicable to realistic implementations of QKD going beyond the simple qubit picture.
8.5.1. Intercept-resend attack

We understand under the intercept-resend attack any attack where Eve performs a complete measurement on the signals which Alice sends out. A special version has been introduced already in Section 2.5. Eve then transmits the classical measurement result and prepares a new quantum state close to Bob’s detection device. In this way, she cuts out all channel imperfections. As we have seen before, the resulting correlations will not allow Alice and Bob to create a secret key. The simplest example is an intercept-resend attack in the BB84 protocol: Eve performs a measurement of the BB84 signals in one of the signal bases and prepares a state which corresponds to her measurement result. For example, if she measures in the horizontal/vertical polarization basis and obtains a vertically polarized photon, she prepares such a vertical polarized photon for Bob. Actually, in the sifted key, that is for those signals where Alice’s and Bob’s polarization basis agrees, this leads to an error rate of 25%. This error rate is composed of an error rate of 0% whenever Eve used the same basis as Alice and Bob, and 50% whenever her basis differs from theirs. It follows, that for data with more than 25% average error rate QKD cannot be successfully completed.

8.5.2. Unambiguous state discrimination attack

Let us turn to an attack that is a special case of an intercept-resend attack. It applies whenever the signal states sent by Alice are linearly independent. In this case, Eve can measure the signals with an unambiguous state discrimination (USD) measurement so that with some probability she learns, without error, the exact signal, while in the remaining cases she is left without any information about the signal states (Dušek et al. [2000]). She can now selectively continue her attack. For example, she might forward a new signal to Bob only in those cases where she knows the signal for certain, while she might send no signal at all (corresponding to sending the vacuum state) in the remaining cases. With this strategy she is able to mimic a lossy channel. As a result, the data obtained by Alice and Bob show no obvious trace of eavesdropping whenever Bob obtains a signal. Despite this absence of visible disturbance of the signal degree of freedom, no secure key can be created. A typical protocol for which this problem arises is the variation of the B92 protocol (Bennett [1992b]) which uses single photons in non-orthogonal polarization states together with single-photon detections (see Section 3.1). This protocol becomes insecure once the transmissivity of the channel sinks below a threshold which depends on the non-orthogonality of the signal states. The threshold is defined as the transmissivity where the probability of success of the USD measurement equals the detection probability for Bob via the lossy channel. In our example, the success probability of the USD measurement is given as $P_{\text{USD}}^{\text{succ}} = 1 - |\langle \varphi_0 | \varphi_1 \rangle|$ and Bob obtains the fraction $\eta$ of signals, where $\eta$ is the transmissivity of the channel. Then we find for the threshold of the transmissivity the expression (Tamaki et al. [2003a])

$$\eta_{\text{thresh}} = 1 - |\langle \varphi_0 | \varphi_1 \rangle|.$$  \hspace{1cm} (8.6)
8. SECURITY

8.5.3. Beam-splitting attack

The beam-splitting attack is a very natural attack for any optical implementation of QKD. The reason is that a lossy optical transmission line is very well described by a model consisting of an ideal line in which a beam-splitter is inserted which mimics the loss of the original line. Now Eve gets hold of the signal emerging from the second output of the beam-splitter, while Bob obtains the transmitted part. In some protocols, Eve can in these cases learn a fraction of the signal deterministically (Bennett et al. [1992a], Dušek et al. [2000]). This is the case, for example, in implementations of the BB84 protocol with weak laser pulses instead of single photons. Alice prepares here weak laser pulses in the BB84 polarizations such that the signals contain also multi-photon pulses. The beam-splitter in Eve’s attack gives for some of the signals some, or even all, photons of a signal pulse to Eve. She waits until Alice and Bob publicly communicate the polarization bases of the signals and measurement results. Then she measures her photons in the correct basis and obtains deterministically Alice’s signals. If also Bob received at least one photon, then Eve knows deterministically also a bit of the sifted key (Inamori et al. [2001]). One can show that the secret key rate is therefore bounded by

\[ R \leq p_{\text{exp}} - p_{\text{split}}, \]

where \( p_{\text{exp}} \) is the probability that a signal enters the sifted key, and \( p_{\text{split}} \) is the joint probability that Eve obtains at least one photon of the signal and that this signal enters the sifted key. In the case of weak laser pulses with mean photon number \( \mu \), we find

\[ R \leq (1 - e^{-\mu \eta}) \left(1 - e^{-\mu (1 - \eta)}\right). \]

Actually, this upper bound is positive for all values of the average photon number \( \mu \) and of the total transmissivity \( \eta \). It is clear that this attack cannot be excluded by Alice and Bob by any additional test of the channel since it represents the physical model of the channel.

8.5.4. Photon-number splitting attack

In the beam-splitting attack the photons of the incoming signal states are distributed statistically to Eve and Bob. In principle, Eve could arrange a more effective method (Dušek et al. [1999a], Lütkenhaus [2000], Brassard et al. [2000]). We have seen that Eve learns an element of the sifted key whenever she and Bob obtain at least one photon. The beam-splitter, however, sometimes sends all photons of multi-photon pulses either to Eve or Bob.

The improved eavesdropping attack, called photon-number splitting attack, starts with Eve performing a quantum non-demolition measurement of the total photon number of the signals. Whenever Eve finds a multi-photon signal, she deterministically splits one photon off, sending the other photons to Bob. Additionally, whenever she finds a single photon, she either blocks the signal or she performs a standard eavesdropping method on it and sends it on to Bob. As we see, errors in the polarization of the signal arises only by the eavesdropping on the single-photon signals. Ignoring this effect for the moment, we find again an upper bound on the possible secret key rate in analogy to the formula for the beam-splitting attack as
(Brassard et al. [2000])

\[ R \leq p_{\text{exp}} - p_{\text{multi}} \]  

(8.9)

where now \( p_{\text{multi}} \) is the joint probability that Alice sent a multi-photon signal and the signal enters the sifted key, while \( p_{\text{exp}} \) is the total probability that a signal enters the sifted key. We can evaluate this bound for a Poissonian photon number distribution with average photon number \( \mu \) and a single-photon transmissivity \( \eta \) for the channel. In this case we find

\[ R \leq (1 + \mu)e^{-\mu} - e^{-\mu \eta} \]  

(8.10)

which is positive only for certain combinations of \( \mu \) and \( \eta \). Generally, for given \( \mu \) there is a cut-off transmissivity below which no secure key rate can be generated. Note that for a realization of this attack it is important that Eve can suppress signals at will (here some single-photon signals) without paying any penalty in form of an error rate (see Sections 3.4 and 3.5).

8.6. RESULTS

So far we discussed the principles of security proofs and specific attacks. Next we will summarize results of complete security analysis as they are known so far. The results are typically given only in the limit of a large number of signals, so that all statistical effects of finite sequences of signals can be neglected.

8.6.1. Bennett 92 protocol with single photons

The Bennett protocol of 1992 (B92 protocol) uses only two non-orthogonal signal states. As discussed before, this protocol is prone to the USD attack. Nevertheless, it is possible to achieve unconditional secure key distribution over lossy channels by adapting the overlap of the input signal states. This protocol has been analyzed for lossless channels (Tamaki et al. [2003b]) and for lossy channels (Tamaki and Lütkenhaus [2004]). There is no explicit closed formula for the key rate, for a detailed discussion see the original publications.

8.6.2. BB84 protocol with single photons

The security of the BB84 protocol is well studied (Mayers [1996], Mayers [2001], Shor and Preskill [2000]. Mayers proof did not make use of random permutations of the signals and resulted in a secure key rate given by

\[ R = 1 - h(\epsilon) - h(2\epsilon), \]  

(8.11)

where \( \epsilon \) is the observed error rate and \( h(x) \) is the binary entropy function given by \( h(x) = -x \log_2 x - (1 - x) \log_2 (1 - x) \). The secure rate given by Shor and Preskill is higher, as they include a random permutation of the signals, so that they obtain

\[ R = 1 - 2h(\epsilon). \]  

(8.12)

The cut-off error rate in this scenario is about 11%. However, we know that one can verify quantum correlations up to 25%. Gottesman and Lo [2003] proposed a two-way communication protocol in the public discussion part of the protocol (Phase
II) which can come closer to this upper bound. It has been improved by Chau [2002] to tolerate 20%. This is at present the highest known error rate threshold for the BB84 protocol.

For this protocol, any loss in the channel reduces the rates only by a prefactor corresponding to the single-photon transmissivity.

The key rates are given here without the prefactor $1/2$ which would be expected since only in half of the cases the signal bases of Alice and Bob match. As Lo et al. [2005a] pointed out, Alice and Bob can choose the probabilities for the two signal bases asymmetrically. In the limit, they use basically only one basis, and test only a small number of signals in the other basis. Though this requires a larger sampling size, we can nevertheless get rid of the factor $1/2$ in the rate formulas.

8.6.3. The 6-state protocol

The six state protocol can be analyzed in similar fashion to the BB84 protocol. This has been done by Lo [2001] who found the key rate

$$R = 1 + \left(1 - \frac{3\epsilon}{2}\right) \log_2(1 - \frac{3\epsilon}{2}) + \frac{3\epsilon}{2} \log_2 \left(\frac{\epsilon}{2}\right).$$

(8.13)

Again, we made use of the idea that one can use the three bases of the protocol asymmetrically so that we do not have a prefactor $1/3$.

Also for this protocol there are improved two-way protocols. The best error threshold found so far is given by Chau [2002] as 27.6%.

8.6.4. BB84 protocol with weak laser pulses

For practical realizations the BB84 using weak laser pulses has special importance. The security of this protocol has been investigated by Inamori et al. [2001]. For this case we do not only have the key rate for long sequences, but also the complete analysis for finite key sizes. It extends the results by Mayers for the single-photon BB84, and therefore does not use the random permutation of signals. This random permutation has been introduced by Gottesman et al. [2004], so that the final key rate in the long key limit is given by

$$R = (1 - \Delta) - h(\epsilon) - (1 - \Delta) h\left(\frac{\epsilon}{1 - \Delta}\right),$$

(8.14)

where $\Delta$ is the fraction of signals received by Bob which might have leaked all its signal information to Eve via a multi-photon process. This fraction is given via the multi-photon probability of the source, $p_{\text{multi}}$, and the total signal detection probability for Bob, $p_{\text{exp}}$, as

$$\Delta = \frac{p_{\text{multi}}}{p_{\text{exp}}}. $$

(8.15)

This result holds against the most general attack of Eve, the coherent attack where Eve may delay her measurements. Moreover, it allows to give reasonable secret key rates already in the paranoid picture where all of Bob’s detection imperfections (dark counts, detection efficiency) are ascribed to Eve.

Clearly one can optimize the parameters of the experimental set-up. By variation of the mean photon number $\mu$ of the signals we find that one should choose approximately $\mu \approx \eta$ so that the key rate scales as $R \sim \eta^2$; $\eta$ is the total transmissivity.
8.6.5. BB84 with weak laser pulses and decoy states

The BB84 protocol with weak laser pulses gives a rate of $R \sim \eta^2$ which is mainly given by the photon-number splitting attack. One possibility to avoid this attack is to use the so-called decoy-states (Hwang [2003], Lo et al. [2005b], Wang [2004a], Wang [2004b]). Here Alice tests the channel not only with signals having one average mean photon number. Instead, she randomly varies the mean photon number; this she might do with two, three, or many intensity settings. The idea is that Eve can now no longer complete the full PNS attack. Of course, she can still split one photon from each multi-photon pulse, but she can no longer block the correct number of single-photon signals for each subset of signals with the same average photon number. Effectively, this forces Eve back to use the beam-splitting attack only.

This basic idea is supported by the full security analysis (Lo et al. [2005b]), and one finds that the final key rate scales as $R \sim \eta$, which is a clear improvement of the performance of these schemes. Indeed now distances of more than 100 km are possible without giving up a conservative, paranoid security notion.

8.6.6. B92 with a strong phase reference pulses

Another approach to improve the rate of QKD protocols is the use of coherent states with phase reference. The idea here is, again, to make it impossible for Eve to suppress signals without paying a penalty. The ability to do just that is what makes the USD attack and the PNS attack so powerful. This scheme has been analyzed by Koashi [2004], who confirmed that in this case the secure key rate scales again as $R \sim \eta$.

8.7. SIDE CHANNELS AND OTHER IMPERFECTIONS

So far we discussed the security assuming that the signals are prepared exactly as described in the protocol. However, in physical realizations there might be many imperfections. For example, the preparation of different signal polarizations might also affect other degrees of freedom of the signals, for example the timing or the spectrum of the signals. Therefore, by monitoring other than the intended degrees of freedom Eve might obtain information about the signal which is not captured in the typical security analysis. This situation applies also to classical cryptography where measurable quantities such as power consumption might help to break classical ciphers.

Other imperfections come into play. Consider the detection process: typically, we assume that the choices of signals happen at random. What if Eve can have some information about the basis or signal choice beforehand, if the detectors show some dependence of the chosen signal basis, or if Eve could manipulate the detectors to some degree? One example is Eve’s strategy to apply a simple intercept-resend attack mimicking Bob’s measurement strategy. Then Eve forwards not only a single photon, but a strong light pulse in the polarization that corresponds to the measurement result. If Bob’s and Eve’s measurement bases agree, Bob just recovers the signal without error. When the bases disagree, with almost certainty Bob will find that both of his single-photon detectors will fire. If Bob discards these events, this would open a loophole for Eve to manipulate Bob. For this reason, Bob has
9. PROSPECTS

It is apparent that quantum cryptography is now ready to offer efficient and user-friendly systems providing an unprecedented level of security. While classical methods are still safe enough for short-lifetime encryption, quantum cryptography may prove valuable when thinking with longer prospects. The progress in the development of quantum computers can play a significant role in speeding up the increase of the need for QKD in the IT market. Quantum key distribution can also be well combined with existing infrastructure. Even QKD with very low bit rate (hundreds of bits per second) can significantly improve security of contemporary cryptosystems. It enables, e.g., to change the secret key for symmetric ciphers like AES several times per second.

The widespread use of QKD is now restrained mainly due to the limited operational range (up to about 100 km). There are three main technological challenges that can help to improve this situation: Substantial reduction of noise of detectors working at wavelengths suitable for fiber communications (1550 nm), the development of ultra-low-attenuation fibers (based, e.g., on photonic crystals), or the development of quantum repeaters.

Challenging opportunity for future global secure networks is a long distance quantum communication between Earth and satellite or between two satellites or satellite and plane (Aspelmeyer et al. [2003]). The disturbing influence of atmosphere constraints terrestrial free-space quantum cryptography to short-range communications. On the other hand in the outer space and higher levels of atmosphere (above 10 km) only losses due to beam geometry are important.

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