The impact of the magnetic field and viscous dissipation on the thin film unsteady flow of GO-EG/GO-W nanofluids

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Abstract. The unsteady flow of nanoliquid film over a flexible surface has been inspected. The water and ethylene glycol are used as the base liquids for the graphene oxide platelets. The comparison of the two sorts of the nanoliquids has been used for the heat transfer enhancement applications. The thickness of the nanoliquid film kept variable under the influence of applied magnetic field and viscous dissipation. The governing equations for the flow problem have been altered into the set of nonlinear differential equations. The BVP 2.0 package has been used for the solution of the problem. The sum of the square residual error has been calculated up to the 10th order approximations. It has been observed that the graphene oxide ethylene glycol based nanofluid (GO-EG) has more efficient for the heat transfer enhancement as compared to the graphene oxide water based nanofluid (GO-W). The impact of the physical parameters has been plotted and discussed. The problem has been solved through homotopy analysis method (HAM) as described by Liao et al. [26]. This method is frequently used for the solution of nonlinear problems and show that this method is quickly convergent to the approximate solution. This method gives us series solution in the form of function and all the physical parameter of the problem involved in this method.

1. Introduction
The nanofluids are the effective agents for the energy resources. These fluids have a vibrant part in the enhancement of hotness transmission devices utilizing in various industries and engineering fields. The gaining of energy is not sufficient, but also to regulate the consumptions of energy and this is possible only to adopt the advance heat transfer liquids to control the wastages of energy and to gain the most heat transmission which is the demand of the industry and other relevant scientific fields. Prior to the utilization of nanotechnology, analysts and engineers have confronted such huge numbers of issues, identifying with heat transfer fluids, however, with the advancement of nanometer sized particles and its uses in the heat transfer fluids have altogether enhanced thermal conductivity. The mixture of small sized metal atoms up to 5% of the base liquids known as nanofluids. Nanofluids are composed to guarantee operative thermal conductivity developments and to achieve the uprising weights of
cooling/heating and other requirements. Nanofluid is a spreading consisting of nanometer-sized particles, called nanoparticles. Nanoliquids are scatterings expressed as a powerful meeting of moving nanoparticles in a base liquid. Haq et al. [1] research nanofluid thermal supervision, over a trapezoidal cavity using finite element method. They observe that to heat the domain in part the temperature will be rises. In the presence of magnetic field, Rudraswamy et al. [2] studied three dimensional nanofluid flow over two dimensional moving plate. Soomro et al. [3] analyse nanoparticle movement over a stretching plate using passive method. Gul et al. [4] explored the experimental study to examine the stable dispersion of the graphene nanoparticle and to look at the GO-H2O nanofluid by using two rotating discs. Yu et al. [5] find the thermal conductivity and viscosity to discuss the thermal carrying properties of ethylene glycol-base nanofluid. In the year 1995, a Chinese researcher exhibits his primary document about nanofluid in the field of nanotechnology [30]. He projected another category of liquids with increasing thermal properties. These nanofluids including dense particles (1-100Nm) of different metals like (Alumina, copper, gold and so forth.) and the base fluids like (GO-W, EG, Engine oil, and so on). After the advent of this technology, numerous engineering issues have been tackled, particularly; problems relating to low thermal conductivity of ordinary liquids were overcome because of the pioneering work in the field of nanotechnology. Due to these properties of nanofluid many analysts started investigating the nanofluid flow in different geometries. Sandeep et al. [31] examined the thermal improvement in the time dependent nanoliquid including aluminum alloy nanoparticles AA7072 and AA7075 and magnetic field is applied. They examined the characteristics of these cited nanoparticles alloys on the momentum and thermal boundary layers. Xie and Chen [6] investigated that the thermal conductivity of base fluid is much less than that of graphene oxide nanosheet. To fulfill this space, Xiao et al. [7] tried to model analytical expression for thermal conductivity of nanofluid by using the result of heat convection. Buongiorno [8] studied a complete analysis about the nanofluid. Ellahi et al. [9] investigated the temperature dependent viscosity and MHD properties of non-Newtonian nanofluid in a pipe. Khan and Pop [10] investigated the motion of nanofluid by using stretching plate. Gul et al. [11] investigated magneto hydrodynamics nanofluid by using movable and heat transfer cylinder. Shah et al. [12] study the effects of hall current on steady three dimensional non-Newtonian nanofluid in a rotating frame with Brownian motion and thermophoresis properties. The thermal conductivity and stability of carbon based nanofluids is very high visual, properties dynamic surface area environmental and mechanical stability of carbon based fluid are satisfactory. Due to the electronic hybridization $SP$, $SP^2$, $SP^3$ Carbon is an arresting component, graphene is defined as the single coated 2D sheets of graphite. Graphene is mostly used as a nanofluid due to the organized potential wall the solubility of graphene is very poor. So we used graphene in the form of graphene oxide (GO), as it charge result and form sure dispersition in water due to its high oxidize structure (GO) which is amusing dispensability in oils. The dynamic research area of (GO) piratical is the rotating disk modal of nanofluid. The most important application of (GO) is for industrial machinery and many engineering apparatus, for example turbine system centrifugal pumps rotating blades, turbo machinery hard disk jet motors and computer storage systems. The central purpose of this flow is to stability of centrifugal forces by engaging the circular pressure gradient. Heat transfer is one of the important properties in chemical processes. The heat transfer properties of the base fluid such as water, mineral oil are very less different method are used to increase heat transfer, for example reduced heat transfer time, and heat exchangers size can be minimized. Ethylene glycol (EG) can be used as a cooling fluid and anti freezing agent to improve thermal properties because the thermal conductivity of the metallic, nonmetallic and carbon structures are very higher than the base fluids. Von Karman et al. [13] observed the motion of viscous liquids by a rotating disk. Fang (2007) have used the Von Karman idea to find the exact solution of the Navier-Stokes equations. Turkyilmazoglu and Senel (2013) studied impacts of heat in viscous fluid flow over a rotating disk. Rashidi et al. [14] analyzed MHD flow due to a turning of the disk. Lance and Rogers (1962) investigated the fluid flow between two infinite rotating plates. Turkyilmazoglu [15] presented nanofluid flow by a rotating disk. Hatami et al. (2014) examined the laminar flow of the nanofluid due to rotation and shrinkage of disks.
Sheikholeslami et al. [16] have studied the nanoliquid flow over an inclined rotating disk. Shah et al. [17] have examined the rotating flow of micropolar nanofluid under the influence of the electrical MHD and Hall current. Attia et al. [18] has studied viscous flow between parallel plates with magneto hydrodynamics. Vajravelua and Kumar [19] have examined magneto hydrodynamics viscous fluid flow between two horizontal and parallel plates in a rotating system, in which one plate is strained and the second one is porous. They got numerical solution and studied the result of physical parameters. Sheikholeslami et al. [20] have examined the flow of viscous nanofluid fluids among parallel plates with revolving systems in three dimensions under the magneto hydrodynamics (MHD) effects. For the solution of the modelled problems, they used numerical methods and labelled the properties of attaining parameters in notified. Mahmoodi and Kandelousi [21] studied the hydro magnetic effects of kerosene-alumina nanofluid flow in the existence of heat transfer study. The differential transformation method is used in their work. Ganguly et al. [32] examined the forced convection heat transmission augmentation using a magnetic fluid under the effect of line dipole. They correlate the rise in heat transmission due to thermomagnetic convection with those properties of nanofluid under the effect of the applied magnetic field. Malvandi and Ganji [33], theoretically studied the effect of nanoparticle movement in a perpendicular conduit under the effect of magnetic dipole. Haghshenas increased heat transfer coefficient. Qasim et al. [22] investigate mathematical model for nanoliquid film subject to Brownian motion and thermophoresis, they addressed the analysis in the presence of viscous dissipation and transverse magnetic field. Aziz et al. [23] investigate the inner heat production of thin film flow by using time depending moving sheet. Tawade et al. [24] discuss the thermal radioactivity of thin film flow in the existence of magnetic field. They use Newton Raphson and Runge-Kutta method to solve nonlinear differential equation. Khan et al. [25] investigate unsteady three non-Newtonian thin film fluids flow over a moving surface with variable fluid properties. Gul et al. [29] explored the heat transmission in ferrofluid in the existence of a magnetic field. In this research work, we investigated the impact of the magnetic field and viscous dissipation of the thin film unsteady flow of GO-EG and GO-W nanofluid. The problem has been solved through homotopy analysis method (HAM) as described by Liao et al. [26]. This method is frequently used for the solution of nonlinear problems and show that this method is quickly convergent to the approximate solution. This method gives us series solution in the form of function and all the physical parameter of the problem involved in this method. The researchers [27-28] used this method for the various nonlinear problems occurring in the field of science and engineering.

Figure 1. Geometry of the problem.
2. Mathematical formulation

Consider the unsteady flow of graphene oxide water and ethylene glycol (GO-W/GO-EG) based nanofluids over a flexible sheet. \( U_\omega = \frac{bx}{(1-\gamma t)} \) is the unsteady flexible velocity of the sheet in the \( x \) direction such that \( b, \gamma \) are positive constants and \( t \) is the term used for time. The uniform magnetic field \( B_0 \) adopted from the Faradays law applied in upright direction to the flow pattern. The time dependent transverse magnetic field \( B(t) = B_0 \left( 1 - \gamma t \right)^{-\frac{1}{2}} \). The temperature distribution \( T_\omega(x,t) = T_0 - T_T \left( \frac{bx^2}{2y} \right) \left( 1 - \gamma t \right)^{-\frac{3}{2}} \) on the surface is supposed to vary with the distance \( x \) from the slit.

The continuity, momentum and thermal boundary layer equations are settled [23]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_f}{\rho_f} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf}}{\rho_{nf}} \frac{B^2(t)}{\rho_{nf}} u \quad (2)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_{nf}}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_f}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 \quad (3)
\]

where \( u \) and \( v \) are the velocity component along \( x \) and \( y \) direction respectively. The thermal diffusivity of the base fluid is

\[
\alpha_{nf} = \frac{k_{nf}}{\rho C_p} . \quad (4)
\]

The associated boundary condition is

\[
u = U_\omega, v = 0, T = T_\omega \quad \text{at} \quad y = 0 \quad (5)
\]

\[
\frac{\partial u}{\partial y} = \frac{\partial h}{\partial t} = 0, v = \frac{\partial h}{\partial t} \quad \text{at} \quad y = \delta, \quad (6)
\]

where \( \delta \) is the film thickness. The similarity transformation is defined as

\[
\eta = \left( \frac{b}{\nu_f (1 - \gamma t)} \right)^{\frac{1}{2}} \, y, \psi(x,y,t) = \left( \frac{\nu_f b}{1 - \gamma t} \right)^{\frac{1}{2}} \, x f(\eta), T(x,y,t) = T_0 - T_T \left( \frac{bx^2}{2y} \right) \left( 1 - \gamma t \right)^{-\frac{3}{2}} \theta(\eta). \quad (7)
\]

The stream function \( \psi(x,y) \) is defined as \( u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \) where \( \beta \) is the dimensionless film thickness and is defined as

\[
\beta = \left( \frac{b}{\nu_f (1 - \gamma t)} \right)^{\frac{1}{2}} h(t). \quad (8)
\]

From equation (8)

\[
\frac{dh}{dt} = -\gamma \beta \left( \frac{\nu}{h} \right)^{\frac{1}{2}} \left( 1 - \gamma t \right)^{-\frac{1}{2}}. \quad (9)
\]

Putting equation (7) into equations (2), (3), (4), (5), we obtain the following coupled system of ordinary differential equation.

\[
f''' + (1 - \phi)^{2.5} \left[ (1 - \phi) + \phi \frac{\rho_s}{\rho_f} \right] \left[ f f''' - (f')^2 - S \left( f' + \frac{\eta}{2} f'' \right) \right] - (1 - \phi)^{2.5} M f' = 0 \quad (10)
\]

\[
k_{nf} \frac{\theta''}{k_f} + Pr \left[ (1 - \phi) + \phi \frac{(\rho C_p)_{nf}}{(\rho C_p)_{sf}} \right] \left[ f \theta' - 2 f' \theta - S \left( 3 \theta + \eta \theta' \right) \right] + Ec(f'')^2 = 0. \quad (11)
\]
Here the prime denote the differentiation with respect to similarity variable $\eta$ and dimensionless parameters $S, M, Pr$ and $Ec$ are unsteadiness parameters and defined as

$$ S = \frac{\gamma}{b}, \quad M = \frac{\sigma_n B_0^2}{\rho B}, \quad Pr = \frac{\nu_{nf}}{\alpha_f}, \quad Ec = \frac{U^2_{\infty}}{C_p(T_\infty - T_0)}. $$  

The transform boundary condition is

$$ f(0) = 0, \quad f'(0) = 1, \quad f'(\beta) = 0, \quad \theta(0) = 1, \quad \theta'(\beta) = 0. \quad (13) $$

The physical quantities of interest are the skin friction coefficient $C_{nf}$, the local Nusselt number $Nu$ and the local Sherwood number $sh$ which is defined as

$$ C_{nf} = \frac{\tau_\infty}{\rho U^2_{\infty}}, \quad Nu = \frac{q_\infty}{k(T_\infty - T_0)} \times $$

where $\tau_\infty = \mu \left( \frac{\partial u}{\partial y} \right)_y$ = 0, $q_\infty = -k \left( \frac{\partial T}{\partial y} \right)_y = 0$ and $q_m = -D_B \left( \frac{\partial c}{\partial y} \right)_y$ = 0, are the shear stress heat and mass fluxes at the surface respectively using the variables in equation (7), the associated expressions for dimensionless skin friction coefficient $C_{nf}$ reduced the Nusselt number $-\theta'(0)$ and reduced Sherwood number $-\phi'(0)$ are defined as

$$ R_{Re} \frac{1}{2} C_{fx} = -f''(0), \quad R_{Re} \frac{1}{2} Nu_x = -\theta'(0) \quad (15) $$

where $Re_x = \frac{u_{\infty} x}{v}$ is a local Reynolds number based on stretching velocity.

3. Method of solution

Using the initial guessed values and auxiliary linear operators from equations (10-11)

$$ f_0(\eta) = \frac{3}{2} \left( \frac{\eta^3}{6} - \frac{\beta \eta^2}{2} + \left( \frac{\beta - 1}{2} \right) \eta + \left( \frac{1}{3} - \frac{\beta}{2} \right) \right) \left( \frac{-2\beta A^*(1-\beta)^2}{(1-\beta)^3} + \frac{A^*}{2} (\eta^2 + 1) + (1 - A^*) \eta \right), \quad (16) $$

$$ \theta_0(\eta) = 1, \quad \text{since} \quad A^* = \frac{M}{\sqrt{\beta}} \theta(\beta), \quad (17) $$

$$ L_f = \frac{d^4 f}{d \eta^4}, \quad L_\theta = \frac{d^2 \theta}{d \eta^2}, \quad (18) $$

with constant properties

$$ L_f(C_1 + C_2 \eta + C_3 \eta^2 + C_4 \eta^3) = 0 \quad \text{and} \quad L_\theta(C_5 + C_6 \eta) = 0, \quad (19) $$

where $C_i (i = 1, 2, \ldots, 6)$ are arbitrary constant which is include in general solution. The average squared residuals error is presented and can be written as

$$ \varepsilon_m^f = \frac{1}{n+1} \sum_{j=1}^n \left[ \kappa_f \left( \sum_{j=1}^n f(\eta)_{j=d\eta} \right) \right], \quad (20) $$

$$ \varepsilon_m^\theta = \frac{1}{n+1} \sum_{j=1}^n \left[ \kappa_\theta \left( \sum_{j=1}^n \theta(\eta)_{j=d\eta} \right) \right], \quad (21) $$

$$ \varepsilon_m^t = \varepsilon_m^f + \varepsilon_m^\theta. \quad (22) $$
Table 1. Exhibits the numerical values for the skin friction coefficient for different physical parameters when $h = -0.1, Pr = 0.7, Ec = 0.1, \beta = 0.1$.

| $M$ | $S$ | $f''(0)$ | $f''(0)$ | $f''(0)$ | $f''(0)$ |
|-----|-----|-----------|-----------|-----------|-----------|
|     |     | GO-W      | GO-W      | GO-EG     | GO-EG     |
| 0.1 | 0.1 | 20.3264   | 20.3335   | 28.7696   | 30.4545   |
| 0.2 | 0.2 | 20.3340   | 20.3412   | 30.3040   | 30.6264   |
| 0.3 | 0.3 | 20.7430   | 20.7777   | 30.4609   | 30.7778   |

Table 1 shows the numerical values of skin friction coefficient for different physical parameters. Here by increasing the value of magnetic field $M$ and unsteady parameter $S$, the values of skin friction also increase. If the value of unsteady parameter $S$ is increasing, then the skin friction coefficient value is also increasing in both GO-W and GO-EG.

Table 2. Exhibits the numerical values of a local Nusselt number of different physical parameters, when $h = -0.1, M = 0.7, S = 0.1, \beta = 0.1$.

| $Ec$ | $Pr$ | $\theta'(0)$ | $\theta'(0)$ | $\theta'(0)$ | $\theta'(0)$ |
|------|------|--------------|--------------|--------------|--------------|
|      |      | GO-W         | GO-W         | GO-EG        | GO-EG        |
| 0.1  | 0.5  | -0.1437      | -0.4409      | -0.62116     | -0.4411      |
| 0.2  | -1.2456 | -1.2571      | -1.2458      | -1.2576      |
| 0.3  | -1.8697 | -1.8869      | -1.8700      | -1.8875      |
| 0.6  | 0.6026 | -1.8337      | -0.6027      | -0.6098      |
| 0.7  | 0.5870 | -1.8875      | -0.5871      | -0.5952      |

Table 2 shows the numerical values of local Nusselt number for various physical parameters. Here for increasing the values of Prandtl number $Pr$, and unsteady parameter $Ec$ Eckert number, the value of local Nusselt number $Nu$ also increases.

Table 3. Individual averaged squared residual errors for GO-W when $h = -0.1, M = 0.7, S = 0.1, \beta = 0.1, \phi = 0.1, Pr = 6.5, Ec = 0.5$.

| $m$  | $\varepsilon^f_m$ GO-W | $\varepsilon^\theta_m$ GO-W |
|------|-------------------------|--------------------------|
| 6    | $3.7784 \times 10^{-2}$ | $1.87671 \times 10^{-1}$ |
| 12   | $5.14122 \times 10^{-4}$ | $2.37131 \times 10^{-3}$ |
| 18   | $7.97126 \times 10^{-5}$ | $5.07647 \times 10^{-4}$ |
| 24   | $6.44141 \times 10^{-6}$ | $3.44806 \times 10^{-5}$ |
| 30   | $8.50496 \times 10^{-7}$ | $5.46253 \times 10^{-6}$ |
Table 3 and Table 4 represent the individual’s average square residuals error of water-graphene oxide and ethylene glycol-graphene oxide executed for different order of approximation. We also noticed that average square residual error value can be reduced by increasing the order of approximation.

**Table 4.** Individual averaged squared residual errors for GO-EG when $Pr = 6.5, Ec = 0.5, M = 0.7, S = 0.4, \beta = 0.3, \phi = 0.1$.

| $m$ | $\varepsilon_m^{\epsilon}$ GO-EG | $\varepsilon_m^{\theta}$ GO-EG |
|-----|---------------------------------|--------------------------------|
| 6   | $1.11762 \times 10^{-2}$        | $2.91773 \times 10^{-1}$      |
| 12  | $5.55814 \times 10^{-4}$        | $7.77139 \times 10^{-3}$      |
| 18  | $6.39644 \times 10^{-5}$        | $6.4508 \times 10^{-4}$       |
| 24  | $6.55799 \times 10^{-6}$        | $4.79449 \times 10^{-5}$      |
| 30  | $8.1187 \times 10^{-7}$         | $3.77827 \times 10^{-6}$      |

**Figure 2.** Individual average squared residual errors for GO-W.
Figures 2 and 3 indicate the sum of the square residual errors for the velocity pitch and temperature distribution up to the 30th order approximation. These figures show that the convergence of the obtained results are very strong for the proposed problem.

From Figure 4, we observe that if amount of the thin film thickness parameter $\beta$ is increasing, the fluid motion will be decrease initially and then the fluid motion is increasing. Physically, the resistance force are increasing with the increasing value of $\beta$ and consequently the velocity profile decay. The decay effect is comparatively larger in the GO-EG.
Figure 5. Effect of dimensional less nanoparticle volume fraction on velocity profile.

From Figure 5, we observe that increasing values of the nanoparticle volume fraction raises the velocity near the sheet surface and the velocity field delayed after the point of inflection. This effect is comparatively strong in the GO-EG.

Figure 6. Effect of magnetic field on velocity profile.

From Figure 6, by increasing magnetic field the velocity profile will decrease because magnetic field produce Lorentz force which oppose the motion and hence decrease the motion, so the velocity decrease by increasing magnetic field.
Figure 7. Effect on unsteadiness parameter on the velocity profile.

Figure 7 shows the effect of unsteadiness parameter $S$ on the velocity profile in the presence of magnetic field. The dimensional axial velocity is decreasing with increasing the unsteadiness parameter $S$ because with the increasing of $S$. This impact is identical for the both sorts of nanofluids.

Figure 8. Effect of Prandtl number versus temperature profile.

Figure 8 indicates that increase in the Prandtl number, decreasing the temperature profile. In fact, the thickness of the momentum boundary layer to be larger than that of the thermal boundary layer, or that the viscous diffusion is larger than the thermal diffusion and therefore, the larger amount of the Prandtl number reduces the thermal boundary layer.
Figure 9. Effect of dimensionless nanoparticle volume fraction on temperature profile.

Figure 9 indicates the increasing values of the nanoparticle volume fraction raises temperature. This effect is comparatively strong in the GO-EG.

Figure 10. Effect of Eckert number on the temperature profile.

From Figure 10, the relation between Eckert number and temperature profile is direct relationship of the physical studies of this result, by increasing the Eckert number. It will enhance the kinetic energy due to this intermolecular collision is increasing, so the temperature profile is increasing.

4. Conclusion
The thin film of nanofluids over an unstable and flexible sheet has been considered. The (GO-W) and (GO-EG) nanofluids have been compared for the heat transfer enhancement applications. The (GO-EG) nanofluid have stronger thermal conductivity and therefore, it is concluded that (GO-EG) nanofluid is more efficient for the heat transfer enhancement as compared to the (GO-W) nanofluid. The OHAM
technique has been used for the solution of the problem and the strong convergence has been achieved through the residual analysis. The impact of the physical constraints versus the velocity and temperature profiles have been plotted and discussed. The main points are pointed out as:

- Increasing the magnetic field opposing the fluid motion, so the velocity field is decreasing.
- By increasing the unsteadiness parameter $S$, the velocity profile decreases and this effect is comparatively strong in the (GO-EG) nanofluid.
- Increasing Eckert number rises the kinetic energy to enhance the temperature.

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