Assessment of the turbulent energy paths from the origin to dissipation in wall-turbulence

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Abstract. The present study is devoted to the description of the energy fluxes from production to dissipation in the augmented space (3-dimensional space of scales plus wall-distance) of wall-turbulent flows. As already shown in Cimarelli \textit{et al.} (2010), an interesting behavior of the energy fluxes comes out from this analysis consisting of spiral-like paths in the combined physical/scale space where the controversial reverse energy cascade plays a central role. The observed behaviour conflicts with the classical notion of the Richardson/Kolmogorov energy cascade and may have strong repercussions on both theoretical and modeling approaches to wall-turbulence. Two dynamical processes are identified as driving mechanisms for the fluxes, one in the near wall region and a second one further away from the wall. The former, stronger one is related to the dynamics involved in the near-wall cycle. The second suggests an outer self-sustaining mechanism. Here we extend these results to larger Reynolds number using LES data of a turbulent channel flow at $Re_\tau = 970$ confirming the presence of an outer regeneration cycle which seems to be composed by systems of attached eddies.

1. Introduction

One of the most important aspect of wall-turbulent flows is the anisotropy induced by the mean gradient. This anisotropy is reflected by the presence of a turbulent energy production process embedded in the system. Moreover, the presence of the wall induces inhomogeneity which leads to a spatial redistribution of turbulent kinetic energy. Thanks to the inhomogeneity of the flow, the problem of wall-turbulent flows has been classically studied by dividing the flow domain into well characterized regions depending on wall-distance. In this context, the classical view of wall bounded flows is based on a production region close to the wall (buffer layer) and two energy sink regions one at the wall and another one in the core flow which is eventually anticipated by an equilibrium layer for large Reynolds number flows. However, the description in physical space alone is insufficient to capture the complete dynamics of wall-turbulence since the turbulent processes take place into an hierarchy of scales of motion. Indeed, it is well known that turbulent flows are characterized by fluctuations which range in size from the characteristic width of the flow to much smaller scales, which become progressively smaller as the Reynolds number increases. In this view, turbulent flows are characterized by different dynamics distributed among the various scales of motion. The classical idea regarding the physical processes occurring on these scales concerns the scale-space distribution of the turbulent kinetic energy. In particular, for homogeneous turbulence, Richardson (1922) introduced the idea that kinetic energy enters
the turbulence through a production mechanism at the largest scales of motion. This energy is then transferred by inviscid processes to smaller scales until, at the smallest one, the energy is dissipated by viscous action.

Hence, in wall-turbulent flow, such as channel flow, two energy transfer take place, one in the wall-normal direction due to inhomogeneity and one through eddies of different size. In this scenario, it is, therefore, necessary to consider a general approach, able to analyze the scale-dependent dynamics in inhomogeneous conditions and the balance equation for the second order structure function actually matters for it. The multidimensional description given by this equation has been shown crucial to understand the formation and sustainment of the turbulent fluctuations fed by the energy fluxes coming from the near-wall production region (Cimarelli et al., 2010). Here, in the present work we extend these results at higher Reynolds number using LES data. In particular, in section 2 we will briefly discuss the use of LES simulation as a research-tool for the analysis of large Reynolds number flows and we will present the LES data set used in this work. The results of the generalized Kolmogorov equation applied to the LES data of a turbulent channel flow at $Re_\tau = 970$ will be shown in section 3. The conclusions will conclude the paper.

2. Large Eddy Simulation approach as a tool for the extension of DNS results to higher Reynolds number turbulence

It is well established that in turbulent flows the energy carrying structures are directly affected by the boundary conditions, hence they are highly non universal and generally anisotropic, while the small scales tend to be more homogenous and isotropic than the large ones. Therefore, it is thought that relatively simple and universal models can be used to describe the last part of the energy spectrum, when, for example, a Large Eddy Simulation (LES) approach is considered for the computation of moderately large Reynolds number flows.

In LES, a low-pass filtering operation is used to decompose the velocity field, $u^*_i$, into the sum of a filtered (or resolved) component, $\bar{u}^*_i$, and a residual unresolved component (or subgrid scale motion, SGS), $u^{*\text{sgs}}_i$, so that the resulting filtered velocity field can be adequately resolved on a relatively coarse grid and the total velocity field has the decomposition,

$$u^*_i(x, t) = \bar{u}^*_i(x, t) + u^{*\text{sgs}}_i(x, t)$$

Once defined the filtering operation, the evolution equation of the filtered velocity field, $\bar{u}^*_i$, can be obtained by applying this operation to the Navier-Stokes equations yielding to

$$\frac{\partial \bar{u}^*_i}{\partial t} = 0$$

$$\frac{\partial \bar{u}^*_i}{\partial t} + \frac{\partial \bar{u}^*_i \bar{u}^*_j}{\partial x_j} = \frac{-1}{\rho} \frac{\partial p^*}{\partial x_i} + \nu \frac{\partial^2 \bar{u}^*_i}{\partial x^2_j} - \frac{\partial \tau^*_ij}{\partial x_j}$$  \hspace{1cm} (1)

where the effects of the small unresolved scales appears in the subgrid stress tensor $\tau^*_ij = u^*_i u^*_j - \bar{u}^*_i \bar{u}^*_j$ which must be modeled. Arguably, the most important effect of the subgrid scales on the large ones, is the resolved energy drain/source that results from the interaction between resolved and subgrid motion. In this context, most of the commonly used LES models assume that the main role of the subgrid scales is to remove energy from the large resolved motion and dissipate it through the action of a diffusion mechanism analogous to the viscous forces, see Kraichnan (1976), leading to the concept of eddy-viscosity where $\tau_{ij} = -\nu_T \bar{S}_{ij}$ with $\bar{S}_{ij} = \frac{1}{2}(\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i)$. These assumptions are based on the idea of an inertial range in the spectrum of scales. Indeed, as asserted by the 4/5 law, Kolmogorov (1991), in the inertial
Figure 1. Left: mean velocity profile $U^+(y^+)$ for the present LES (circle) compared with the DNS from Hoyas & Jiménez (2008) (solid line). The linear and logarithmic regions are indicated by dashed lines corresponding to $U^+(y^+) = y^+$ and $U^+(y^+) = 2.46 \log(y^+) + 5.1$ respectively.

Right: root mean square profile of the turbulent velocity fluctuations, $\sqrt{\langle u'^2 \rangle}$, $\sqrt{\langle v'^2 \rangle}$ and $\sqrt{\langle w'^2 \rangle}$ for the present LES (circle) compared with the DNS from Hoyas & Jiménez (2008) (solid lines).

range the energy flux is independent of the scale under consideration, is from large to small scales and it is proportional to the viscous energy dissipation.

This kind of approach has given good results in homogeneous and in unbounded shear flows but less in wall turbulence. In fact, in such flows, the anisotropic turbulent production affects most of the turbulent eddies and the shear is sufficiently strong to hinder isotropy recovery even at small inertial scales, see Casciola et al. (2005) and Jacob et al. (2008). Beside these observations, the single most striking phenomenon is the complete modification of the Richardson scenario up to a reverse energy cascade in the form of energy fluxes loops shown in Cimarelli et al. (2010) and in the following section. The reason for this modification is the focusing of the turbulent generation mechanisms at small scales near the wall which feed larger motion. This leads to overwhelming difficulties for LES, since energy should emerge from nowhere in the subgrid scales to drive the coherent dynamics of the resolved scales.

These issues have been rationalized in Cimarelli & De Angelis (2011) and Cimarelli & De Angelis (2010) via a multidimensional analysis of the filtered dynamics in wall-flows. From these works emerge that for large filter lengths, $l_F$, the subgrid dynamics strongly affect the resolved motion through a nonlinear displacement and source of resolved energy. In this condition the success of the simulations is entirely demanded to the quality of the LES model which must be able to capture backward energy transfer and nonlinear energy distribution processes. Otherwise, for filter scales which satisfy the constraint, $l_F^+ < 100$ and $l_F^+ < 20$, the main physical processes of wall-turbulence are directly resolved and the subgrid scales play only a minor role of energy draining at small resolved scales. In this condition the LES technique can be seen as a poorly resolved direct numerical simulation which need a supplement linear term for the governing equations of dissipative nature.

Considering these limits we have performed a large eddy simulation of a turbulent channel flow at $Re_\tau = 970$ in a computational domain $8\pi h \times 2h \times 3\pi h$ with $640 \times 257 \times 640$ grid points respectively corresponding to a resolution in wall-units in the homogeneous directions of $\Delta x^+ = 38$ and $\Delta z^+ = 14$. The grid points are non-equidistantly distributed in the wall-normal directions, with a maximum resolution at the wall of $\Delta y^+ = 0.7$ while in the channel center
of $\Delta y^+=12$. The simulation is performed using a fully spectral method, Fourier series in the wall-parallel directions and Chebyshev polynomials in the wall-normal direction, and the time is advanced with a standard mixed Crank-Nicholson/Runge-Kutta scheme. The unresolved turbulent fluctuations are treated via the ADM model (Stolz et al., 2001). This model is a deconvolution method which reconstructs the resolved part of the subgrid stresses via an approximate inversion of the filtering operator, whereas the contributions of the unresolved stresses are modelled via a purely dissipative relaxation term acting at the smallest resolved scales. We would like to point out that for the present simulation the grid resolution has been chosen to be very fine for an LES. As shown in Gualtieri et al. (2007) for homogeneous shear flows and in Schlatter et al. (2010) for a turbulent boundary layer with a slightly different model and considering the constraint on the filter lengths mentioned above, accurate channel flows statistics could be obtained with the present approach. As an example, the mean velocity profile and the turbulent fluctuation intensities scaled in viscous units are shown in figure 1 in comparison with the DNS data from Hoyas & Jiménez (2008) relative to a channel flow at $Re_\tau = 950$ with the same domain extension. The agreement between the LES and DNS data is good, meaning that the mean and fluctuating quantities are well captured even by the lower resolution of the LES simulation.

We would like to stress that the aim of the present work is not the validation of the present LES approach or the reduction of the resolution as much possible, but the main goal is to obtain accurate data at moderate Reynolds number in order to appreciate the Reynolds number effects on the findings reachable with DNS techniques at lower Reynolds number flows. Indeed, the present LES data will be used in the next section for the analysis of the generalized Kolmogorov equation extending to larger Reynolds number the results obtained in Cimarelli et al. (2010) with a DNS of a turbulent channel flow at $Re_\tau = 550$.

3. The paths of energy in a turbulent channel flow at $Re_\tau = 970$

The most important contribution to the description of the energy transfer mechanisms in turbulence is the Kolmogorov theory. Under the assumption of a statistical isotropic condition, this theory is an exact quantitative result obtained by the balance of the second order structure function, $\langle \delta u^2 \rangle$, where $\delta u = u(x_s + r_s) - u(x_s)$ is the fluctuating velocity increment and $\langle \cdot \rangle$ denotes ensemble average. Although this is a well known result it is useful to go back over its assumptions. The balance of $\langle \delta u^2 \rangle$, for small scales but sufficiently large so that the viscous diffusion processes may be neglected, reduces to the $4/5$ law,

$$\langle \delta u^3 \rangle = -\frac{4}{5} \langle \epsilon \rangle r$$

where $||$ denotes longitudinal velocity increments and $\epsilon = \nu (\partial u_i / \partial x_j) (\partial u_i / \partial x_j)$ is the pseudo-dissipation. This relation establishes that the turbulent energy is transferred through the inertial range from large to small scales independently on the scale under consideration and with a constant rate proportional to the energy input/dissipation, $\langle \epsilon \rangle$. There is no direct energy injection and no direct energy extraction. This picture is believed to universally occur independently of the large-scale processes which feeds the turbulence, but fails in wall-turbulence where the interaction between anisotropic production and inhomogeneous spatial fluxes strongly modifies the energy cascade up to a reverse cascade in the near-wall region (Marati et al., 2004).

Wall-bounded turbulence is characterized by several processes which maybe thought as belonging to two different classes: phenomena which occur in physical space and phenomena which take place in the space of scales. As a consequence, a full understanding of these phenomena requires a detailed description of the processes occurring simultaneously in physical
and scale space. A tool for the study of these phenomena is the generalized form of the Kolmogorov equation (Hill, 2002). This equation, specialized for a channel flow with a longitudinal mean velocity $U(y)$, reads,

$$\begin{align*}
\frac{\partial \langle \delta u^2 \delta u_i \rangle}{\partial r_i} + \frac{\partial \langle \delta u_y^2 \delta U \rangle}{\partial r_x} + 2 \langle \delta u_i \delta v \rangle \left(\frac{dU}{dy}\right)^\ast + \frac{\partial \langle \nu^* \delta u_i^2 \rangle}{\partial Y_c} = \\
-4\langle \epsilon^\ast \rangle + 2\nu \frac{\partial^2 \langle \delta u^2 \rangle}{\partial r_i \partial r_i} - \frac{2}{\rho} \frac{\partial \langle \delta p \delta v \rangle}{\partial Y_c} + \frac{\nu}{2} \frac{\partial^2 \langle \delta u^2 \rangle}{\partial Y_c^2}
\end{align*}$$

(3)

where $\ast$ denotes a mid-point average, i.e. $u_i^\ast = (u_i(x_i^0) + u_i(x_i^s))/2$ and $\langle \rangle$ denotes now average in the homogeneous directions. Equation 3 is written in a four dimensional space, $(r_x, r_y, r_z, Y_c)$ hereafter denoted as augmented space, and involves a four dimensional energy fluxes vector field $\mathbf{\Phi} = (\Phi_{r_x}, \Phi_{r_y}, \Phi_{r_z}, \Phi_c)$,

$$\nabla \cdot \mathbf{\Phi}(r, Y_c) = \xi(r, Y_c)$$

(4)

where $\nabla \cdot$ is a four dimensional divergence, $\Phi_r = (\Phi_{r_x}, \Phi_{r_y}, \Phi_{r_z}) = \langle \delta u^2 \delta u \rangle - 2\nu \nabla \cdot \langle \delta u^2 \rangle$, $\Phi_c = \langle \nu^* \delta u^2 \rangle + 2\langle \delta p \delta v \rangle / \rho - \nu \delta v \langle \delta u^2 \rangle / 2dY_c$, and $\xi = 2\langle \delta u \delta v \rangle \left(\frac{dU}{dy}\right)^\ast - 4\langle \epsilon^\ast \rangle$. This form allows us to appreciate the two scale-energy fluxes occurring in wall-flows, namely $\Phi_c$ and $\Phi_{r_z}$ in physical space. These fluxes assembled in the vector $\mathbf{\Phi}$ balance with a source term $\xi$ which accounts for the energy production and dissipation. When this term reaches a positive value, $\xi(r, Y_c) > 0$, the energy injection via turbulent production exceeds the rate of energy dissipation. Therefore, the regions of the augmented space where $\xi > 0$ can be thought as characterized by a scale-energy excess.

The phenomenon of scale-energy excess is a peculiar aspect which characterizes wall-turbulent flows with respect to homogenous flows where the source term satisfies the constrain $\xi_{\text{hom}}(r) \leq 0$. In homogeneous flows an excess of scale-energy cannot be observed. The energy transfer is initialized at the largest scales by production whose amount equals the energy dissipation, $\xi_{\text{hom}}(r) = 0$ for $r \to \infty$. Then, out of the limit of large scales, the source term becomes negative, $\xi_{\text{hom}}(r) < 0$, due to the monotonic decrease of the production moving to small scales, see Casciola et al. (2003). Whereas, in wall-turbulence there is not a balance between energy injection and dissipation due to the presence of the inhomogeneous spatial fluxes. Indeed, it is well known that turbulent production exceeds dissipation in the buffer layer leading to an excess of scale-energy $\xi(r, Y_c) > 0$ at least for larger scales where the scale-energy processes approach the single-point terms of the turbulent kinetic energy balance (Marati et al., 2004). This is a very important phenomenon which strongly modifies the energy fluxes pattern of wall-turbulence from those usually observed in homogeneous flows. Equation 4 describes a vector field $\mathbf{\Phi}(r, Y_c)$ where are present both energy source ($\xi(r, Y_c) < 0$) and sink ($\xi(r, Y_c) > 0$) regions in the augmented space of wall-turbulence.

As shown in the top figure 2 which is a cut of the augmented space at $Y_c^+ = 20$ for $r_y = 0$, the energy source region and, therefore, the peak of energy production, take place deep inside the spectrum of scales. The energy is not introduced at the top of the spectrum as the classical paradigm of turbulence leads to believe, i.e. for large $r_x$ and $r_z$, but amid the spectrum of scales and, therefore, there is not an isotropic recovery. While, the energy sink region occurs at the smallest separations close to the origin $r_x = r_z = 0$. Given this topology of the energy source/sink, the energy fluxes follow a sort of loop in the space of scales. The fluxes first diverge from the energy source region feeding longer and wider turbulent fluctuations through a reverse cascade. Then, the fluxes converge to a classical forward cascade reaching the region of energy sink at the smallest dissipative scales. From the present view, the reverse energy cascade is due to the energy source which initializes the energy transfer at small scales. The location of the energy source scale-range in the buffer layer, shown in top figure 2, appears closely related to the action of the coherent structures involved in the near-wall cycle (Jimenez & Pinelli, 1999). In particular
Figure 2. Projections of the energy fluxes vector field $\Phi$ (inertial component) and isocontours of the energy source $\xi$ in the $Y_c^+=20$ (top) and $Y_c^+=120$ (bottom) plane of the augmented space of a turbulent channel at $Re_\tau = 970$ obtained with an LES approach. Note that the contour colors are scaled for each wall-distance.

The spacing of this region suggest that this is presumably the imprint of the quasi-streamwise vortices. In this view, the near-wall cycle corresponds to the energy fluxes loop shown in top figure 2. In particular, the energy source can be thought as the scale-energy extracted from the mean flow to generate the quasi-streamwise vortices. A fraction of this energy is directly cascading down to small scales and dissipated as a result of the bursting of these structures due to instability. The remaining fraction of energy is transferred to feed, through a reverse cascade, the streamwise velocity streaks. This phenomenon can be thought as the result of the interaction of the streamwise vortices with the mean shear. In the end, the energy associated to the streaks is cascading down to the smallest scales through a classical forward cascade and dissipated as a result of the bursting of the streaks due to instability. In this view, forward and reverse cascade coexist in the space of scales highlighting the cyclic nature of the turbulent self-sustaining mechanisms.

Let us now consider inhomogeneity, and, hence, the combination of these energy fluxes loops in the $(r_x, r_z)$-space with the spatial flux through different wall-distances $Y_c$. The trajectories of the energy fluxes in the $(r_x, r_z, Y_c)$-space are shown in figure 3. The energy diverge from the energy source at the small scales of the buffer layer, see top figure 2, and following a spiraling behavior feed longer and wider turbulent structures reaching the outer regions of the flow. Note the combined nature of the paths taken by the scale energy, which loops in the three-dimensional space moving from small scales towards larger $r_x$ and $r_z$ while ascending towards increasing $Y_c$. A further distinguishing feature of Fig. 3 is the eventual convergence of the trajectories towards the zero-separation dissipative scales at $r_x = r_z = 0$. The overall picture conforms to a system of ascending spiral-like curves which end up in the small-scale range at different wall-normal
Figure 3. Trajectories of the energy fluxes vector $\Phi$ (inertial component) in the $(r_x, r_z, Y_c)$-space ($r_y = 0$) of a turbulent channel at $Re_\tau = 970$ obtained with an LES approach. The colors encode the strength of the flux.

positions to be understood as a $Y_c$-distributed dissipative range, sink of scale-energy. Overall, the energy transfer systematically takes place towards larger scales (reverse energy cascade) and in the upward direction before eventually bending towards the dissipative range (direct energy cascade). We like to stress that a multidimensional description is crucial to understand the formation and sustainment of larger fluctuating structures fed by the energy excess in the near wall production region.

All these results conform with those obtained in Cimarelli et al. (2010) but with DNS data of a turbulent channel flow at a lower Reynolds number $Re_\tau = 550$. An important feature emerging from this work, was the presence of a second peak of energy source whose wall-normal positions and scales made it suspect of a result of an outer cycle composed by attached eddies. Still the energy fluxes loop in the space of scales moving away from the wall, see bottom figure 2, suggesting dynamics pretty similar to that described for $Y_c^+ = 20$. At a given distance from the wall, the maximum energy source occurs for $r_x = 0$, see both figures 2. Its location in the $(Y_c^+, r_z^+)$-plane, reported in figure 4, defines the typical spanwise scale of the energy source $\xi$. Near the wall, $Y_c^+ < 80$, this length scale increases quadratically, $r_z^+ \approx 35 + 0.02Y_c^+2$. Actually, within the buffer layer, $Y_c^+ < 30$, the length scale stays almost constant. This behaviour would have been described in Townsend’s terms as detached, i.e. independent on the wall-normal position, as opposed to attached meaning increasing linearly with wall-distance. Moving away from the wall, a second behaviour takes over where the spanwise scale of the energy source is to a very good degree linear with $Y_c$. This scaling motivated Cimarelli et al. (2010) to assert that the second outer peak of energy source, shown in bottom figure 2, is a result of turbulent generation mechanisms involving systems of attached eddies whose action should increase as the extent of the log-layer with the Reynolds number. Now, with the help of LES data at larger Reynolds number we can establish that it scales in external units, see figure 4. Indeed, the linear scaling takes place in a wider region for the channel flow case at $Re_\tau = 970$ (as the extent of the logarithmic layer is expected to be increased) involving a range of wall-distances $90 < Y_c^+ < 0.2Re_\tau$. 


Figure 4. Locus of the energy source $\xi$ maxima in the $r_x = 0$ plane. The solid line is the DNS simulation at $Re_\tau = 550$ while the circles are the LES at $Re_\tau = 970$. The dashed lines represent a quadratic behaviour, $r^+_z = 35 + 0.02Y^+_c$, and two linear profile, $r^+_z = (5/4)Y^+_c + 55$ and $r^+_z = (5/4)Y^+_c + 110$.

4. Conclusions

The present work is devoted to the assessment of the energy fluxes in wall-turbulent flows via the analysis of the generalized Kolmogorov equation applied to LES data of a turbulent channel flow at $Re_\tau = 970$. The analysis confirm the results obtained in Cimarelli et al. (2010) highlighting that the energy fluxes still follow a spiral-like path in the combined scale/physical space also at higher Reynolds number. From the present view of wall-turbulence, the reverse cascade is a basic element of the energy fluxes dynamics. It takes place systematically at different wall-distances and not only in the near-wall production region. At the base of this phenomenon is the presence of a peak of energy production which causes the divergence of the energy fluxes amid the spectrum of scales. In the buffer layer, this energy source scale-range appears to be closely related to the dynamics of the quasi-streamwise vortices of the near-wall cycle. Whereas, thanks to the larger Reynolds number reachable with the LES approach, in the so-called logarithmic layer the energy source seems to be related to regeneration mechanisms involving systems of attached eddies. As highlighted by the present data, this process should increase its action with the Reynolds number together with the extent of the logarithmic layer.

Acknowledgments

The authors wish to acknowledge the support of the Italian Ministry of Research MIUR under PRIN08 grant and of the Region Emilia Romagna under the CIPE grant. The DNS was performed on a computing time grant provided by CASPUR.
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