Multi-Color Model for the Protoplanetary Disks HL Tau and HD142527

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Abstract. Protoplanetary disks are circumstellar disks of gas and dust, from which planets may eventually form or be in the process of forming. Recent direct imaging of them has enabled us to derive the density and temperature distributions. Interestingly they often show quite different features depending on the wavelength observed. The near-infrared emission is dominated by scattering of stellar light while the mm- and submm- wave emissions are dominated by thermal dust emission. Thus, the near-infrared emission traces a low density surface layer where stellar light is scattered toward us. The mm- and submm- wave emission trace the high density part of the disk near the mid plane. In order to explain the wavelength-dependent images, we have constructed a passive disk model for HL Tau and HD142527. The former shows concentric rings in the ALMA image while the latter shows a highly asymmetric arc. Our models are based on the multi-color radiation transfer calculation. It takes account of radiation ranging from 100 nm to 3.16 mm. We used the M1 model to solve the radiative equilibrium. Our model gives some constraints on the optical properties of the dust.

1. Introduction
Newly formed stars are often associated with rotating disks consisting of gas and dust. They are named protoplanetary disks since planets are thought to be formed therefrom. They are extended more than a hundred AU from the host stars and their structures are directly observed with large ground based telescopes in the infrared and with ALMA in the radio. The brightness distributions enable us to derive the density and temperature of the dust therein if the dust opacity is given. The dust distribution is valuable for assessing planet formation in the disks. If they are measured at multiple wavelengths, we can get some constraints on the dust opacity. Thus the multi-color images are important probes into the disks.

Main heating source is radiation from the host stars in the protoplanetary disks. The disk surface layers absorb irradiation from the host star while scattering a part of it. They reemit mid-infrared radiation to heat up inner layers of the disk. The inner dense layers are responsible for brightness in the mm- and sub-mm images taken with ALMA. In short, each layer absorbs short wavelength radiation and reemits long wavelength radiation to achieve thermal equilibrium. The absorption, emission, and scattering are dominated by dust particles. Thus we need to take account of the radiative processes ranging from optical to radio in modeling the protoplanetary disks.
With these in mind, we have constructed a multi-color model for protoplanetary disks. Our model takes account of the radiative processes by solving the M1 model[1] in which only the zeroth and first moments of radiation, i.e., the energy density and flux, are the unknowns. It covers wide range of radiation from 0.1 µm to 3.16 mm with 226 colors. We summarize the methods of modeling and application to two interesting objects, HL Tau and HD142527. The former disk consists of multiple rings while the latter has a banana shape. The methods are given in §2. Application to HD142527 based on Muto et al.[2] is shown in §3 while that to HL Tau is given in §4. Short summary is given in §5.

2. Numerical Methods

We use the cylindrical coordinates, \((r, z, \varphi)\), in which the host star is located at the origin and the disk mid plane coincides with \(z = 0\). For simplicity we assume the disk is symmetric with respect to the rotation axis \((\partial/\partial \varphi = 0)\) and the mid plane \((z = 0)\). This assumption is valid for HL Tau since it consists of concentric rings as seen in the ALMA image [3]. It is also reasonable for HD142527 since the disk has a small radial extent and the azimuthal variation is mild.

In our model the radiative energy density \((E_\nu)\) and flux \((F_\nu)\) at frequency \(\nu\) are expressed as

\[
E_\nu(r, z) = E_{\nu,\text{star}}(r, z) + E_{\nu,\text{disk}}(r, z),
\]

\[
F_\nu(r, z) = F_{\nu,\text{star}}(r, z) + F_{\nu,\text{disk}}(r, z),
\]

respectively, where the subscripts star and disk denote those emitted from the star and disk, respectively. They are evaluated in the frequency range of \(3.00 \times 10^{15} \text{ Hz} \leq \nu \leq 9.48 \times 10^{10} \text{ Hz}\), i.e., in the wavelength range \(0.1 \mu m \leq \lambda \leq 3.16 \text{ mm}\) with the frequency resolution of \(\Delta \log \nu = 0.02\). The former is evaluated to be

\[
E_{\nu,\text{star}}(r, z) = \frac{R_{\text{star}}^2}{c (r^2 + z^2)} B_\nu (T_{\text{star}}) \exp \left[ -\tau_\nu(r, z) \right],
\]

\[
F_{\nu,\text{star}}(r, z) = \frac{c}{\sqrt{r^2 + z^2}} \left( \frac{r}{z} \right) E_{\nu,\text{star}}(r, z),
\]

\[
\tau_\nu(r, z) = \int_0^r (\kappa_{\nu,\text{abs}} + \kappa_{\nu,\text{sca}}) \rho \left( \frac{r'}{r} \right) z^2 \frac{1 + \left( \frac{z}{r} \right)^2}{r} dr',
\]

where \(\kappa_{\nu,\text{abs}}\) and \(\kappa_{\nu,\text{sca}}\) denote absorption and scattering opacity at frequency \(\nu\), respectively. The symbols, \(B_\nu\) and \(c\), denote the Planck function and the speed of light, respectively. The symbols, \(R_{\text{star}}\) and \(T_{\text{star}}\), denote the radius and effective temperature of the host star, respectively. The scattering opacity is defined as

\[
\kappa_{\nu,\text{sca}} = \int \frac{d\sigma_{\text{sca}}}{d\Omega} (1 - \cos \theta) d\Omega,
\]

where \(d\sigma_{\text{sca}}/d\Omega\) and \(\theta\) denote the differential scattering cross section and the angle between the incident and scattered photons, respectively. We use this definition to take account of anisotropy scattering in the frame work of moment equations.

The radiation from the disk is computed by the M1 model equations,

\[
\frac{\partial E_{\nu,\text{disk}}}{\partial t} + \nabla \cdot F_{\nu,\text{disk}} = -\kappa_{\nu,\text{abs}} \rho \left[ E_{\nu,\text{disk}} - \frac{4\pi B_\nu(T)}{c} \right] + \kappa_{\nu,\text{sca}} E_{\nu,\text{star}},
\]

\[
\frac{\partial F_{\nu,\text{disk}}}{\partial t} + c^2 \nabla P_{\nu,\text{disk}} = -\left( \kappa_{\nu,\text{abs}} + \kappa_{\nu,\text{sca}} \right) \rho F_{\nu,\text{disk}},
\]
Figure 1. Left panel shows model opacity in which the dust is assumed to have the power law size distribution with $a_{\text{max}} = 1 \text{ mm}$. The blue, red, and black curves denote $\kappa_{\text{abs}}$, $\kappa_{\text{sca}}$, and $\kappa_{\text{tot}} = \kappa_{\text{abs}} + \kappa_{\text{sca}}$, respectively, as a function of the wavelength, $\lambda$. Right panel is the same but for model opacity with $a_{\text{max}} = 1 \mu\text{m}$.

where

$$P_{\nu,\text{disk}} = \left( 1 - \chi_{\nu,\text{disk}} \frac{I}{2} + \frac{3\chi_{\nu,\text{disk}} - 1}{2} n_{\nu,\text{disk}} n_{\nu,\text{disk}} \right), \quad (9)$$

$$\chi_{\nu,\text{disk}} = \frac{3 + 4 (f_{\nu,\text{disk}})^2}{5 + \sqrt{4 - 3 (f_{\nu,\text{disk}})^2}}, \quad (10)$$

$$f_{\nu,\text{disk}} = \frac{|F_{\nu,\text{disk}}|}{cE_{\nu,\text{disk}}}, \quad (11)$$

$$n_{\nu,\text{disk}} = \frac{F_{\nu,\text{disk}}}{|F_{\nu,\text{disk}}|}. \quad (12)$$

The dust temperature, $T$, is chosen to satisfy the condition for thermal equilibrium,

$$\int \kappa_{\nu,\text{abs}} \left\{ E_{\nu}(r, z) - \frac{4\pi B_{\nu} [T(r, z)]}{c} \right\} d\nu = 0, \quad (13)$$

at each location.

In this paper, we assume that the gas to dust ratio is constant at $\rho/\rho_{\text{dust}} = 100$ for simplicity. In other words, we ignore dust sedimentation. The density distribution is given so that the gas is in hydrostatic equilibrium in the vertical direction,

$$\frac{\partial P}{\partial z} = -\frac{GM_{\text{star}} z}{(r^2 + z^2)^{3/2}} \rho(r, z), \quad (14)$$

where $G$ and $M_{\text{star}}$ denote the gravitational constant and the mass of the host star, respectively. The gas pressure is expressed as

$$P(r, z) = \frac{k}{\mu m_{\text{H}}} \rho(r, z) T(r, z), \quad (15)$$
where $k$ and $m_H$ denote the Boltzmann constant and the mass of H atom, respectively. The mean molecular weight is assume to be $\mu = 2.339$.

We use the model opacity shown in Figure 1. Left panel shows the model opacity in which the dust is assumed to have the power law size distribution, $a^{-3.5}$ with $a_{\text{max}} = 1$ mm, where $a$ denotes the radius of a dust particle. The dust is assumed to consist of silicate, carbonaceous grains, and water ice having mass fractional abundances of $\zeta_{\text{sil}} = 0.0043$, $\zeta_{\text{carbon}} = 0.0030$, and $\zeta_{\text{ice}} = 0.0094$, [4] respectively. We reduce the scattering opacity artificially by a factor of 10 (the red dashed curve) in some models in order to evaluate the effects of scattering. We also use the model opacity shown in the right panel of Figure 1 in some other models. The latter is obtained by assuming $a_{\text{max}} = 1$ $\mu$m. The methods for computation of the dust opacity are given in Numura & Millar[5].

We have obtained radiative equilibria by following the time evolution of the M1 model equations in an explicit scheme given in Kanno et al. [6]. We have evaluated the expected brightness by solving the equation of radiative transfer,

$$\frac{dI_\nu}{ds} = -\kappa_{\nu,\text{abs}} [I_\nu - B_\nu(T)] - \kappa_{\nu,\text{sca}} \left( I_\nu - \frac{cE_\nu}{4\pi} \right),$$

along the line of sight, $s$, where the values of $T$ and $E_\nu$ are taken from an equilibrium model.

3. HD 142527

The protoplanetary disk of HD142527 is thought to be in the transitional phase from gas-rich one to gas-poor debris one. It has an inner hole of dust emission and shows strong azimuthal asymmetry in the radio continuum (see, e.g., Fukagawa et al.[7] and references therin). The northern part of the disk is more than 20 times brighter at 336 GHz than the southern part. This means that the dust is concentrated strongly in the northern part.

We have made models to evaluate the dust surface density in the northern and southern parts of the disk. Although the brightness has a large contrast, it varies smoothly and more slowly in the azimuthal direction than in the radial direction. Hence we apply our 2D symmetric model to each of the northern and southern narrow sectors of the disk. We define the northern sector as the region of $11^\circ < PA < 31^\circ$ and the southern one as that of $211^\circ < PA < 231^\circ$. 336 GHz continuum emission is strongest in the northern sector and weakest in the southern sector. The peak brightness is 1.2 Jy arcsec$^{-2}$ and 0.050 Jy arcsec$^{-2}$ in the northern and southern sectors, respectively[2].

We use the opacity model of $a_{\text{max}} = 1$ mm, since the model opacity of $a_{\text{max}} = 1$ $\mu$m cannot reproduce the brightness observed in the northern sector. This is because the opacity is very small at 336 GHz when $a_{\text{max}} = 1$ $\mu$m. The central star of HD 142527 is assumed to have the model parameters shown in Table 1.

| Star       | $T_{\text{star}}$ | $R_{\text{star}}$ | $M_{\text{star}}$ |
|------------|-------------------|-------------------|-------------------|
| HD 142527  | 6250 K            | 3.8 $R_\odot$    | 2.2 $M_\odot$    |
| HL Tau     | 4000 K            | 6.9 $R_\odot$    | 0.5 $M_\odot$    |

Figure 2 shows the density and temperature distributions in the best fit models for the northern and southern sectors. The models covers the rectangular region of $30 \text{ AU} \leq r \leq 410 \text{ AU}$ and $|z| \leq 120 \text{ AU}$ with the resolution of $\Delta r = \Delta z = 2 \text{ AU}$. The dust surface density distribution...
Figure 2. Dust density and temperature distributions in the best fit models for HD142527. The upper and lower panels are for PA = 11° - 31° and 221° - 23°, respectively. Reproduction of Fig. 8 of Muto et al.[2].

is assumed to be

\[ \Sigma_d(r) = \Sigma_{d,0} \exp \left\{ -\max \left[ \left( \frac{r - r_0}{w} \right)^2, 10 \right] \right\}, \]  

(17)

where the values of \( \Sigma_0, r_0 \), and \( w \) are given in Table 2. The temperature decreases with increase in the radius. It is lower in the northern sector than in the southern, since the surface density is higher and hence the optical depth is larger.

Table 2. Best fit models for HD 142527

| Sector    | \( \Sigma_0 \)     | \( r_0 \)    | \( w \)    |
|-----------|---------------------|--------------|-------------|
| North     | 0.6 g cm\(^{-2}\)   | 173 AU       | 26 AU       |
| South     | 8.45 \times 10^{-3} g cm\(^{-2}\) | 196 AU | 34 AU       |

Note that the contrast in \( \Sigma_0 \) is much larger than that in the brightness. This larger contrast is explained as follows. The brightness is roughly proportional to \( T (1 - e^{-\tau}) \), where \( \tau \) denotes the optical depth. The brightness is not proportional to the surface density when the optical depth is appreciable. The northern sector has a lower temperature and is semi-transparent even at 336 GHz. Thus the contrast in the surface density (\( \sim 70 \)) is higher than that in the brightness (\( \sim 24 \)).
It should be also noted that the location of the peak surface density is about 20 AU further from the host star than that of peak brightness. This is due to the radial temperature gradient. The inner disk is radiating more energy per unit mass because it has a higher temperature, thus the brightness peak is shifted radially inwards from the peak surface density as the increase in brightness per unit mass dominates the radial decrease in surface density from the peak.

More detailed comparison with observation is given in Muto et al[2]. They have computed expected line emission from the above mentioned model. Gas is distributed over a wider region from several tens AU to ~ 400 AU in the radial direction, although the line emission has a peak inside the dust disk. In other words, the dust is more concentrated in the radial direction.

4. HL Tau

HL Tau is a very young star located in a molecular ridge and its central star has not been detected in the optical wavelength. The optical and near infrared images depict conical reflection nebula extending up to several hundreds AU and bipolar jets in the center of the nebula. HL Tau is also associated with disk and infalling envelope observed in the radio. Recently Atacama Large Millimeter/submillimeter Array (ALMA) has clarified that the disk is geometrically thin and consists of concentric rings when observed at wavelengths of 2.9, 1.3, and 0.87 mm[3]. The disk inclination and position angle are derived to be $i = 46.72 \pm 0.05$ and $PA = 138.0 \pm 0.7$, respectively.

![Figure 3](image_url)

**Figure 3.** An illustration for jets, disk and envelope around HL Tau.

Figure 3 illustrates the geometry of jets, disk and envelope around HL Tau. Dust particles should be concentrated near the mid plane, since the ALMA images show clear gaps in the disk.\(^1\) On the other hand, the optical and near-infrared images indicate that an appreciable amount of dust particles are distributed well above the midplane. With these constraints in mind we have constructed models for the mid-IR emitting disk and flared atmosphere.

\(^1\) After this conference we have learned that the same argument is given by Pinte et al[9].
In this work we use the surface density model of Kwon et al. [8],

$$\Sigma(r) = \Sigma_0 \left( \frac{r}{R_c} \right)^{-0.22} \exp \left[ 1 - \left( \frac{r}{R_c} \right)^{2.22} \right], \quad (18)$$

where $R_c$ and $\Sigma_0$ denote the typical disk size and the surface density at $r = R_c$, respectively. The former is fixed at their best fit value, $R_c = 78.9$ AU, while the latter is treated as a free parameter. We denote $\Sigma_0$ in unit of the best fit value of Kwon et al. [8], $\Sigma_{Kwon} = 26.3$ g cm$^{-2}$.

Equation (18) does not take account of gaps since it is derived before the discovery of the gaps. We also ignore the gaps in our modeling for simplicity. This is because our interest is focused on the choice of the dust model. We have removed the complexity in the dust temperature due to the gaps.

![Image](image-url)

**Figure 4.** The solid curves denote the Planck temperature for the model opacity of $a_{max} = 1 \mu$m, while the dashed curves do the mid plane temperature. The red, blue and black curves denote the models of $\Sigma_0/\Sigma_{Kwon} = 0.4$, 0.6, and 0.8, respectively.

First we examine the model opacity in which the radius of the largest dust particle is assumed to be $a_{max} = 1 \mu$m. The computation box covers the rectangular region of $40$ AU $\leq r \leq 112$ AU and $|z| \leq 72$ AU with resolution of $\Delta r = \Delta z = 0.6$ AU. Figure 5 shows the mid plane temperature ($T_m$) and the Planck temperature ($T_P$) as a function of the radial distance from the host star, $r$. The Planck temperature is defined to be

$$T_P = \frac{h\nu}{k} \left[ \ln \left( \frac{2h\nu^2}{c^2 I_\nu} + 1 \right) \right]^{-1}, \quad (19)$$

where $h$ and $k$ denote the Planck and Boltzmann constants, respectively. We obtained the Planck temperature by substituting the expected intensity on the disk major axis, $I_\nu$, into Equation (19). The red, blue and black dashed curves denote the mid plane temperature in the models of $\Sigma_0/\Sigma_{Kwon} = 0.4$, 0.6, and 0.8, respectively. It is much lower than the observed one. It increases with increase in $\Sigma_0$. However, it cannot be high enough since the mid plane temperature (solid curves in Figure) is also low and decreases with increase in $\Sigma_0$. The Planck temperature coincides with the dust temperature in the limit of large optical depth. Thus we cannot expect that the observed brightness is reproduced by the model opacity of $a_{max} = 1 \mu$m.

We have included gaps in our modeling after the conference. We have relaxed the assumption that the gas to dust ratio is constant in the model.
Next we examine the model opacity in which the radius of the largest dust particle is assumed to be $a_{\text{max}} = 1$ mm. The left panel of Figure 5 is the same as Figure 4 except for $a_{\text{max}} = 1$ mm and $\Sigma_0$. The surface density is taken to be $\Sigma_0/\Sigma_{K\text{won}} = 0.6, 1.0, \text{and } 1.5$. The Planck temperature (i.e., the brightness) depends little on $\Sigma_0$ and much lower than the mid plane temperature. The large difference between $T_p$ and $T_m$ is due to the large scattering opacity in the mm-wave. Figure 1 shows that the scattering opacity is 10 times larger than the absorption one at $\lambda = 1$ mm.

In order to confirm this idea we reduced the scattering opacity by a factor 10, while the absorption opacity remains unchanged. The right panel of Figure 5 shows $T_p$ and $T_m$ in the models in which only the scattering opacity is reduced artificially. The mid plane temperature is affected little by the reduction in the scattering opacity. However the Planck temperature is increased by the reduction in the scattering opacity.

Figure 6 compares our model of $\Sigma_0/\Sigma_{K\text{won}} = 1.5$ with the observation with ALMA. The scattering opacity is reduced by a factor of 10 artificially from the model opacity of $a_{\text{max}} = 1$ mm. The computation box covers the rectangular region of 30 AU $\leq r \leq 126$ AU and $|z| \leq 96$ AU with resolution of $\Delta r = \Delta z = 0.4$ AU. The black solid, dotted and dashed curves denote the Planck temperature at Bands 7, 6 and 3 shown in Zhang et al.[10], respectively. The red solid, dotted and dashed curves denote the Planck temperature in our model at 343.5, 233.0 and 101.9 GHz, respectively. They are roughly consistent with the observed ones, although the gaps are not taken into account. After the conference, we have confirmed that the gaps can be reproduced by modifying the surface density profile.

5. Summary and Discussion
We have applied our multi-color model to interesting objects, HD 142527 and HL Tau. They can evaluate the temperature distribution one the model opacity is given. We can confirm the validity by molecular line emissions as has been done in Muto et al[2]. They provide some constraints on the opacity model as shown in the modeling of HL Tau. The scattering opacity of mm-sized dust grains are much larger than the absorption one in the current model. When the scattering opacity is dominant, the Planck temperature is appreciably lower than the dust temperature even when the layer is optically thick. ALMA observations at bands 6 and 7 suggest...
Figure 6. Comparison of our model with the observed Planck temperature at Bands 3, 6 and 7 of ALMA [10].

that the Planck temperature is saturated at $\sim 55$ K around $r = 30$ AU. This suggests that the dust temperature should be appreciably higher than $\sim 55$ K, if the dust opacity is dominant. We wonder whether the dust temperature could be so high.

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References
[1] González, M., Audit, E., Huyynh, P. 2007, A&A, 464, 429
[2] Muto, T., Tsukagoshi, T., Momose, M. et al. 2015, PASJ in press
[3] Partnership A., Brogan C. L., Pérez L. M. et al 2015 ApJL, 808, L3
[4] Anders, E., Grevesse, N. 1989, Geochim, Cosmochim. Acta, 53, 197
[5] Nomura, H., Miller, T.J. 2005, A&A, 438, 923
[6] Kanno, Y., Harada, T., Hanawa, T. 2013, PASJ, 65, 72
[7] Fukagawa, M., Tsukagoshi, T., Momose, M. et al. 2013, PASJ, 65, L14
[8] Kwon W., Looney L. W. and Mundy L. G. 2011 ApJ, 741, 3
[9] Pinte, C., Dent, W.R.F., Menard, F. et al. 2015, arXiv:1508.00584
[10] Zhang, K., Blake, G.A., and Bergin, E.A. 2015, Ap/JL, 806, L7

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