The Research on Path Fusion of Telemetry and Trajectory

Zebo Zhu
China Satellite Maritime Tracking and Controlling Department, Jiangyin 214431, China

Abstract. Aimed to solve the problem of path fusion of telemetry and trajectory, one method is put forward, which uses the result of weighting fusion by calculating the function of supporting degree among data to replace the input measured data of the filter and carry on Kalman filtering, and then the dynamic data fusion tracking of multiple group data is obtained. The result of simulation shows that this method can acquire more accurate fusion data.

1. Introduction
The measuring device of the measuring ship has its base fixed on the hull deck. Due to the influence of the waves, the measuring ship is in a changing dynamic environment during the measurement. The position and attitude of the ship change at the time, so it can conduct measurement of the ship. In addition to measuring the measurement error of the device itself, the data also includes measurement errors of the hull, which makes it difficult to measure the trajectory of the target. In the target measurement process, in addition to the measurement equipment data obtained by the survey ship [1], the measurement target can obtain its own telemetry trajectory data through its own inertial navigation device or GPS device. For these two types of data, the error between the two types can be considered to be independent, and although there are multiple devices in the measuring device, the measurement error between them cannot be assumed to be distributed independent due to the influence of the hull.

Due to the influence of factors such as the accuracy, system error and measurement error of a single measuring device, the measured data often has incomplete or large uncertainty, which includes several measured wild values. The data fusion technology is based on the measurement data of multiple measurement devices. After appropriate comprehensive processing and analysis, it can obtain higher measurement accuracy than a single device.

At present, there are extensive and in-depth researches on data fusion technologies and algorithms at home and abroad. Mean method, least squares method, weighted least squares estimation, maximum likelihood estimation, median method, etc. are all applied to data fusion. For Bayesian, maximum likelihood estimation and other methods, more prior knowledge is needed, and the calculation is complicated. The prior probability is often obtained by experience, and the subjective factor is large. In addition, considering various error factors, the measured value is not always strict service normal distribution.

In this paper, the cubic B-spline curve is used to smooth the measurement data, complete the time alignment, and then use the data support matrix to perform the weighted fusion of the data of the remote measurement data, and then the fusion result is Kalman filter. The dynamic fusion data of the remote measurement data is obtained. Finally, the LMS adaptive transversal filter is used to complete the remote trajectory data fusion to obtain the final trajectory data.
2. Research on Trajectory Fusion Method

The algorithm firstly preprocesses the original measurement data by means of data combination and cubic B-spline curve, which complements the blank value elimination and smoothing the measurement data, and performs time calibration and alignment. Then according to the different types of remote measurement data respectively, it conducts the calculation of the support function matrix between data. Since the inter-data support matrix fully utilizes the information contained in the plurality of related measurement data, it does not require statistical properties such as strict service normal distribution, which is easy to calculate in real time. But the inter-data support matrix only considers the similarity between data at the same time. Degree does not consider the temporal correlation of the data, so the inter-data support matrix is used as the input of the Kalman filter to obtain temporal correlation and smoothing. Due to the independence of the measurement equipment, the measurement error of the remote measurement data is independent. The data obtained by Kalman filtering can be regarded as independent high-precision measurement data between remote measurements, so the adaptive lateral direction is utilized by LMS. The filter completes the final data fusion and obtains the data with the error signal tending to be the smallest. The algorithm processing block diagram is as shown:

![Algorithm processing block diagram](image)

3. Trajectory Fusion Method Analysis

3.1. Curve Fitting

In order to fit the target motion trajectory independently measured by each measurement data source, the B-spline curve is used to fit the measurement point data.

The B-spline curve equation is

\[ p(u) = \sum_{i=0}^{n} V_i B_{i,k}(u) \]

In which, \( V_i \) \((i=0,1,\ldots,n)\) is the control vertices, and the lines connected in order are called B-spline control polygons. \( B_{i,k}(u) \) \((i=0,1,\ldots,n)\) is called the k-th order B-spline basis function.
The B-spline curve is defined by the control vertices. In order to fit the trajectory of the target, the vertices of the control polygon must be calculated first. In order to simplify the calculation, the target trajectory is fitted using a cubic B-spline curve. Wherein, the matrix of the cubic B-spline curve is expressed as:

\[
\begin{bmatrix}
-1 & 3 & -3 & 1 \\
3 & -6 & 3 & 0 \\
-3 & 0 & 3 & 0 \\
1 & 4 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
V_i \\
V_{i+1} \\
V_{i+2} \\
V_{i+3} \\
\end{bmatrix}
\]

in which \((i=1,2,\ldots,n-1)\).

According to the mathematical definition of the spline function, there may be multiple existences of cubic B-spline curves through \(m\) data points. In order to determine a unique curve passing \(m\) data points, it is first necessary to determine the control points of the B-spline curve. The initial tracking measurement point data is used as the number of unknown control vertices, the control point is selected by the least square method, and the curve is adjusted by the Hausdorff distance measurement approximation deviation.

3.2. Inter-data Support Matrix Estimation

The inter-data support matrix estimation algorithm makes full use of the information contained in the measurement data itself, which does not require true value data as a fusion standard nor require the measurement data to strictly obey the probability distribution. For the inter-data support matrix estimation fusion method, there is the reference for details.

At the same time, there are \(n\) measuring devices to measure a certain research object, and \(x_1, x_2, \ldots, x_n\) \(n\) measured values are obtained. The authenticity of the measured data is determined by the data itself, \(x_1, x_2, \ldots, x_n\) the higher the authenticity \(x_i\), the higher the degree of support \(x_i\) is supported by the rest of the data.

Define \(d_{ij}\) the relative distance between the measured data as the expression:

\[
d_{ij} = |x_i - x_j| (i, j=1,2,\ldots,n)
\]

The larger the value \(d_{ij}\), the greater the difference between the two data, that is, the smaller the mutual support between the two data. Defining support function:

\[
r_{ij} = -\frac{d_{ij}}{\max(d_{ij})} + 1
\]

So if \(d_{ij}\) is bigger, the value of \(r_{ij}\) is smaller. When \(d_{ij}\) takes the maximum value, its support function \(r_{ij}\) is 0. It can be seen from the reasoning that the value of the support function of the data to itself \(r_{ij}\) is 1. \(r_{ij}\) Can be seen that the value \(d_{ij}\) on the top is \([0, 1]\). A support matrix \(R\) is established, in which each data in \(R\) represents the degree of mutual support between the two data. Now, according to \(R\), the degree of comprehensive support of a certain data by other data is obtained, that is, the weight coefficient of the \(i\)-th measurement data in the whole measurement data \(\bar{w}_i\) itself. According to the information sharing principle, the sum of the information amounts of the optimal fusion estimation can be equivalently decomposed. The sum of the amount of information for several measurement data, there is
\[ \sum_{i=1}^{n} \bar{w}_i = 1 \]

Non-negative number \( v_1, v_2, \ldots, v_n \)

\[ \bar{w}_i = v_1 w_{i1} + v_2 w_{i2} + \cdots + v_n w_{in} \]

Is

\[ W = RV, \]

in which \( W = [\bar{w}_1 \cdots \bar{w}_n]^T, V = [v_1 \cdots v_n]^T \) Because \( R \) has the maximum modulus characteristic value \( \lambda \geq 0 \), and \( \lambda V = RV \) which can get the eigenvector \( V = [v_1 \cdots v_n]^T \). If

\[ \bar{w}_i = \frac{v_i}{v_1 + v_2 + \cdots + v_n} \]

the weight coefficient of the \( i \)-th measurement data \( x_i \) can be obtained, and the data fusion result can be obtained as follows:

\[ x = \bar{w}_1 x_1 + \bar{w}_2 x_2 + \cdots + \bar{w}_n x_n. \]

3.3. Kalman Filter

Kalman filter [2] is an effective means to solve the problem of state estimation. Assume that the mathematical model of the stochastic dynamic system is:

\[
\begin{align*}
x(k + 1) &= \Phi(k + 1, k) \cdot x(k) + \Gamma(k + 1, k) \cdot w(k) \\
z(k + 1) &= H(k + 1) \cdot x(k + 1) + v(k + 1)
\end{align*}
\]

\( x \) is the system state vector of the dimension \( m_x \); the system state transition matrix \( \Phi \) is the dimension \( m_x \times m_x \); the system noise matrix \( \Gamma \) is the dimension \( m_w \); the system noise vector \( z \) is the dimension \( m_z \); the system observation vector is the dimension, \( H \) is the system observation matrix of the dimension \( m_z \times m_x \); \( v \) is the observation of the dimension Noise vector \( m_v \). Assuming that the sums of the systems \( w(k) \) are uncorrelated with \( v(k + 1) \) and each has a zero mean Gaussian white noise, the initial state \( x(0) \) is a Gaussian random vector.

According to the estimation criterion of the Kalman filter, the estimator \( \hat{x}(k) \) is an unbiased estimation \( x(k) \) and a minimum variance estimation. When Kalman filtering is performed, the measured value at time \( k \) is the real-time measurement data of the measuring device, and if the measured value at time \( k \) is replaced by the estimated value of the inter-data support matrix of the plurality of measuring devices, \( x(k) = \bar{w}_1 x_1(k) + \cdots + \bar{w}_n x_n(k) \)

In fact, the equation \( \bar{w}_i \) is the measurement weighting factor of each measuring device at the current moment.

3.4. LMS Adaptive Transversal Filter

The LMS adaptive transversal filter is a special Wiener filter that minimizes the output data error signal by adaptively adjusting the filter weight coefficients. Since this filter can automatically optimize the weight system by gradually estimating the statistical characteristics of the input signal, the output can be optimized.
After the measurement information of different measuring devices is time-aligned, after the coordinate transformation, the discrete measurement data at the same time in the same coordinate system is obtained. After the data is simultaneously input into the filter, the position vectors in each direction are respectively filtered and calculated. We can get the scattered fusion vector value [3].

Taking the target height value as an example, the measured values of M times at the same time are simultaneously input into the filter, and the input vector can be expressed as:

\[ X(n) = [x_1(n) \cdots x_M(n)]^T \]

The weight vector at this moment is expressed as

\[ W(n) = [w_1(n) \cdots w_M(n)]^T \]

The output of the filter is

\[ y(n) = \sum_{i=1}^{M} w_i(n)x_i(n) = W^T \cdot X(n) = X^T \cdot W(n) \]

\( y(n) \) is the data fusion value at time \( n \). In solving, the gradient method and the Levinson-Durbin algorithm can be used.

4. Testing and Verification

In order to verify the correctness of the algorithm model, taking a transit target tracking as an example, there are two sets of measurement equipment tracking, which are external measurement track 1 and external measurement track 2. The telemetry device accepts two different sources of telemetry track data respectively. For telemetry track 1 and telemetry track 2, the track interval is 1 second. Taking the trajectory height component as an example, the inter-data support matrix is estimated for the remote trajectory source trajectory. The Kalman filter is used to filter the telemetry trajectory data, and finally adaptive lateral filtering is performed as the picture shows:

![Diagram](image-url)
The red curve is the curve of the measured trajectory after Kalman filtering, the black is the curve of the telemetry trajectory after Kalman filtering, and the blue is the trajectory fusion curve of the telemetry. According to the target refined track data obtained afterwards, we analyze the residuals as shown below:

It can be seen from the figure that the trajectory residual after fusion is better than the residual of the single used measurement trajectory or telemetry trajectory, and red is the final fusion trajectory residual.

5. Summary
In this paper, a trajectory fusion algorithm is proposed for single-source multi-source measurement data. A functionally complete data fusion calculation model is designed. The implementation algorithm of each functional module is studied and applied to the trajectory data processing. From the perspective of engineering application, the algorithm is easy to establish mathematical models, and the calculation is not large. It avoids the introduction of a priori statistical probability and other uncertain factors, which has a strong reference for improving the real-time performance of trajectory fusion. Under the premise of satisfying the real-time nature, the accuracy of the data after fusion is greatly improved.
References
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