The Research of Markov Chain Application under Two Common Real World Examples

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Abstract. Markov chain is a random process with Markov characteristics, which exists in the discrete index set and state space in probability theory and mathematical statistics. Based on probability theory, the Markov chain model is a quantitative prediction model for stationary random phenomena using autoregressive process methods. This article first introduces the Markov Chain and its related principles, then in order to study the applicability of Markov Chain, two common life situations are used in practical applications, and the conclusion that Markov Chain can accurately predict the probability is drawn; finally evaluated the Markov chain model and advocated for its wide application.

1. Introduction
As an important branch of theory of probability, stochastic process hold up half the probability theory, nowadays, it is widely used in statistical physics, such as weather forecast astrophysics strategy decision population theory reliability of safety science and technology of economic mathematics [1, 2, 3]. Computer science and other fields is widely existing in the nature of random process, using the theory of random process of modeling, always starts with the Markov chain such as we know the Brownian motion of particles in the liquid do commercial activities for the research of the daily sales situation in speech signal video signal in digital communication, and so on [4]. It can describe the irregular movement with mathematics, which has great guiding significance to the real production and life [5].

“The future is independent of the past given the present.” -- Andrey Andreyevich Markov. Tsarist Russian mathematician Andrey Markov once said a philosophical sentence: “The future is independent of the past given the present”. The research output of Markov -- Markov Chain, completely reflected his ideology. Markov Chain is a statistical model of a random process, using this statistical model people can predict and process some events in nature. The most important property of Markov Chain is “Memorylessness”, which the probability distribution of one status only depends on its previous one and the past few are irrelevant [6]. Such a typical example is the Weather forecast. Using the Markov chain, only recent or current dynamic data can be used to predict the future so that the purpose of predicting weather changes can be easily achieved. Other wide use of Markov Chain could be found in the financial sector to forecast the stock price. The stock price of a company depends only on the company's current overall situation and market expectations of the stock price and has nothing to do with the company's stock price a month ago. Markov chains are also commonly used in the biological field to estimate the activity of molecules and the reproduction of cells [7, 8, 9, 10].
This article will simulate two simple events by introducing several properties of Markov chain and its calculation process to express its usage. This article selects the application of Markov chain in weather forecasting and elections, among which Markov Chain is often used in the field of weather forecasting, but it is rarely used in elections. Through the application comparison of these two fields, the practicability of Markov Chain can be verified.

2. Application of Markov Chain in Election

In a simple situation of electing a popular politician named AP for the US Congress:

- If AP has never been elected, then the probability that they will be elected is 1/2.
- If they lose the ballot, they can run again in the following election next year.
- If AP has already been elected and is currently in office, then their probability of being re-elected is 9/10.
- If AP loses the ballot, they retire from politics.

In order to describe this situation in the language of Markov Chain, indicating all the statuses and transitions and stating the appearance of Markov property, the 4 conditions are fitted into 4 statuses:

- Never been elected
- Being elected
- Currently in office
- Retire

2.1. Retire

Then connect each statuses using shift curves with the corresponding probability to get the figure above (Figure 1). In this case, every AP starts from never been elected and end their career at most retire. Additionally, this type of Markov Chain is called Absorbing Markov Chain [11]. Absorptivity is a state where the one cannot leave once he enter. Absorbent Markov chain refers to a Markov chain in which each state can reach an absorptive state [12, 13]. As never been elected (N) is the origin and retire (R) is the terminus, the absorbing statuses of this Markov Chain are N and R, and the transition statuses are E (being elected) and C (currently in office). To get into the office (C), the only way is get elected in the last election; no matter how many times have the AP joined the election, maybe the second time, maybe five times, or maybe never joined, the probability for an elected (E) AP to be the statuses of currently in office (C) is still 100 percent. It can be concluded that the probability distribution of every status only depends on the statuses next to it, saying this here is the Markov property in this situation.

Except for the way drawn in Figure 1, there is more ways to express the simulation as a diagram of Markov Chain. Because the probability of getting elected (E) turned to be currently in office (C) is 1, E and C can be combined as C1, then the diagram can be drawn as Figure 2. To describe Figure 2, there are only 3 statuses, the absorbing statuses are still N and R, but the transition statuses shorten to be C1; it contains the Markov property because of the frequency the AP runs for the election would not affect the probability distribution of retire.

![Figure 1. Markov Chain of voting Simulation within 4 status.](image-url)
The combination of two statuses seems workable, then thinking about divide one statuses to more. Considering the real situation, the AP may be the first time in the election, maybe the second time, or maybe the last time before they end their career. So the statuses of “Never been elected” (C) can be expanded as “The first year in the election” (Y1), “The second year in the election” (Y2), … “Nth year in the election” (Yn) from Figure 3. In the same way, “Currently in office” can be expanded to “The first year in office” (I1), “The second year in office” (I2), … and to “Nth year in office” (In) (Figure 3). It is already mentioned above that when describing the Markov property of the situation, the frequency the AP run for will not influence the probability distribution, so each year (Yi & Ii) the probability of winning the election is the same. Figure 1 and Figure 3 essentially calculate the same thing, but the number of their statuses is different, thus the upper bound for the number of statuses in this simulation is infinite.

Now is clear that there are a few ways to graph this situation, so can the diagram be simpler again? Here is a try of using only two statuses in graphing. In words, never been elected and retire can have the same property in which they are both out of the office, so the situation could be generally separate as “Out of office” (O) and “In-office” (C1) as Figure 4. However, it is not available to show the two different probability within one curve. If the AP is out of the office because they have retired, then the probability of they still out of the office next term is absolutely 1 and they will never back to the office; but if the reason of the AP out of the office is that they have never been elected, then there is some chance for them to win the election. Therefore, the two statuses containing the same condition but different probability to do the same thing cannot be combined as one status, therefore, the least number of the statuses to describe the voting simulation is 3, not 2.
At this point, some readers may wonder, why the infinite statuses can combine to one but the two cannot, and what defined the lower bound of numbers of statuses?

First, the infinite statuses shares the same feature -- probability. All the probability distribution from the expanded statuses to “election” (E) is the same and this is the reason that they can compose together. Now it is clear that the upper bound and lower bound of the number of the status do not exactly depend on the number of event but more likely to depend on the number of different probability from other statuses to the status which is to predict.

3. Application of Markov Chain in Example Weather Forecast

Here is a further proof:

In a common situation of weather forecast, knowing that today is a sunny day (S),

- the probability that tomorrow is still sunny day is 0.4
- the probability that tomorrow is rainy day (R) or cloudy day (C) is 0.3
- rainy day (R) and cloudy day (C) will not continued more than 1 day
- the probability that rainy day (R) followed by cloudy day (C) is 0.4
- the probability that rainy day (R) followed by sunny day (S) is 0.6
- the probability that cloudy day (C) followed by rainy day (R) or sunny day (S) is 0.5

As it have been mentioned above that the weather forecast is a model of Markov Chain, then the probability distribution of weather tomorrow is only depends on the weather today. A tree diagram is drawn below (Figure 5).

It seems like a complex model and appears with 3 events. In fact, based on the Markov property, Figure 6 can be simplified as Figure 7.
However, if now saying that knowing that a day is a rainy day (R) in order to predict the likely day of a rainy day in the week, the simulation can be represent within only 2 statuses, namely "Rainy Day (R)" and "No Rain" "(NR) (Figure 8), which means that sunny day and cloudy day have been amalgamated to 1 status. Since the probability of S followed by R or C is the same, and the probability of C followed by S or R is the same, then the diagram can be drawn as Figure .

Obvious sunny day and cloudy day are not identical weathers but in this problem they can composed together, and this is because of the same probability, which proved the inference stated above.

Looking back to the first circumstance and apply it in to real word problems, assuming that AP is running for the first time, in how many years should they expect to retire? And what is the probability that they will retire within ten years?

Matrix is the significant tool people use to solve Markov Chain problems, and the size of the Matrix is based on the number of status. So as Figure 2 is the shortest Markov Chain for the situation, then its transfer matrix will be easier to calculate.

The state distribution can be written as a random row vector x, which satisfies the relationship x^{(n+1)} = x^n P \ [14,15]. Therefore, if the system is in state x(n) at time n, after three time periods, the distribution at time n + 3 should be:

\[
\begin{align*}
x^{n+i} &= (x^{n+i-2})P \\
x^{i-1} &= (x^{i-2})P \\
x^n &= x^i P^i
\end{align*}
\]
To find the years the AP expected to retire, the only step is find the i. Knowing that this is the first the AP run the election, so \( x = [1\ 0\ 0] \). Keep multiplying \( x \) with \( P \), then when \( i = 46 \), \( x_i = [0.009\ 0.991] \) (Figure 9).

So at the 47th year, the AP have 99% to be retired.

Same way to solve the second question (within 10 years means that including this year \( x_{10} \)): \( x_{10} = [0.002\ 0.4818\ 0.5162] \) (Figure 10), so the probability that the AP will retire within 10 years is 51.62%.

\[
x^{i+i} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}^i \\
x^{i+9} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.9 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}^9
\]

**Figure 9.** Matrix Calculation Process for Expect Retire Year.  
**Figure 10.** Matrix Calculation Process for Probability of Retire in 10 Years

### 4. Conclusion

After solving the problems, the importance of the reason for choosing these examples is essential to discuss. First of all, these two simulations are the epitome of real events and are common in life, but people rarely associate them with probability distribution. Through these two examples, people can better realize the universality and convenience of probability distribution, and at the same time, using a commonly used model Markov Chain as an entry point, they can show in detail the practicality of statistical models [16]. All events with random nature can be modeled by Markov Chain. In addition to common financial stock problems and biological reproduction problems, many practical systems, such as queuing systems, manufacturing systems and inventory systems, can also be expressed by Markov Chain. Even in gambling, some people pray for gods and worship Buddha and some people have already started to use Markov Chain modeling to obtain absolute returns [17]. People use the Markov model to calculate the state transition probability matrix of an event [18].

The example of Election shows that Markov chain can construct different models for the same event according to the calculation purpose, which demonstrates the advantage of Markov Chain with high flexibility [19,20]. In addition, the example of weather forecast reflects the easy collection of Markov Chain. Different from the massive data base required by predictive models such as unary linear regression and multiple linear regression, the application of Markov Chain does not require repeated and long-term observation, only a specific sample set needs to be collected. As long as the random process has Markov properties, Markov chains are indispensable. In today's era of advanced technology and information flooding, individual subjective judgments are no longer sufficient to defeat technology. In the seemingly irregular changes, the Markov chain reveals the mystery of random processes for us. From simple weather forecasts to complex financial models, its applications everywhere.

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