Strong constraint on large extra dimensions from cosmology

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We have studied cosmological constraints on the number and radii of possible large extra dimensions. If such dimensions exist, Kaluza-Klein (KK) modes are copiously produced at high temperatures in the early universe, and can potentially lead to unacceptable cosmological effects. We show that during reheating, large numbers of KK modes are produced. These modes are not diluted completely by the entropy production during reheating because they are produced non-relativistically. This means that the modes produced during reheating can easily be the dominant component. For instance, for two extra dimensions the bound on their radii from considering only the thermally produced KK modes is \( R \lesssim 1.1 \times 10^{-4} \) mm. If the modes produced during reheating are also accounted for, the bound is strengthened to \( R \lesssim 2.2 \times 10^{-5} \) mm. This bound is stronger than all other known astrophysical or laboratory limits.

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I. INTRODUCTION

In the past few years there has been an enormous interest in the possibility that the presence of large extra dimensions can explain the hierarchy problem [1–3], the fact that the energy scale for gravitation (the Planck scale \( \sim 10^{19} \) GeV) is so much larger than that for the standard model (100 GeV). The idea is that the standard model fields are located on a 3+1 dimensional brane embedded in a higher dimensional bulk, where only gravity is allowed to propagate.

This already puts stringent constraints on the size of the extra dimensions. Newton’s law should definitely hold for any scale which has so far been observed. At present the best experiments have probed scales down to about 1 mm. Thus, if there are extra dimensions, they can only appear at a scale smaller than that. For simplicity we make the assumption that the \( n \) new dimensions form an \( n \)-torus of the same radius \( R_n \) in each direction [1]. If there are such extra dimensions, the Planck scale of the full higher dimensional space, \( M_{P,n+4} \), can be related to the normal Planck scale, \( M_{P,4} \), by use of Gauss’ law [1]

\[
M_{P,n+4}^2 = R^n M_{P,4}^{n+2},
\]

and if \( R \) is large then \( M_{P,n+4} \) can be much smaller than \( M_P \). If this scenario is to solve the hierarchy problem then \( M_{P,n+4} \) must be close to the electroweak scale \( (M_{P,n+4} \lesssim 10 - 100 \) TeV\]), otherwise the hierarchy problem reappears. This already excludes \( n = 1 \), because \( M_{P,n+4} \approx 100 \) TeV corresponds to \( R \approx 10^8 \) cm. However, \( n \geq 2 \) is still possible, and particularly for \( n = 2 \) there is the intriguing perspective that the extra dimensions could be accessible to experiments probing gravity at scales smaller than 1 mm. From this point on we use \( M \) instead of \( M_{P,n+4} \) to simplify notation.

So far, the strongest constraints come from the observation of the neutrino emission of SN1987A [4,5]. In the standard model, a Type II supernova emits energy almost solely in the form of neutrinos. Furthermore, the observed neutrino signal fits very well with the theoretical prediction. If extra dimensions are present, then the usual 4D graviton is complemented by a tower of Kaluza-Klein states, corresponding to the new available phase space in the bulk. Emission of these KK states can potentially cool the proto-neutron star too fast to be compatible with observations. This has lead to the tight bound that \( R \lesssim 0.66 \mu m \) (\( M \gtrsim 31 \) TeV) for \( n = 2 \) and \( R \lesssim 0.8 \) nm (\( M \gtrsim 2.75 \) TeV) for \( n = 3 \) [1]. In fact, an even stronger constraint can be obtained from considering the contribution to the diffuse gamma background from decays of the KK modes produced in cosmological supernovae. A conservative estimate yields a bound of \( R \lesssim 0.09 \mu m \) (\( M \gtrsim 84 \) TeV) for \( n = 2 \) and \( R \lesssim 0.19 \) nm (\( M \gtrsim 7 \) TeV) for \( n = 3 \) [6,7].

The other obvious place in astrophysics to look for these extra dimensions is cosmology [8,10,12] (see also [13]). In the present paper we go through the possible cosmological effects from the presence of large extra dimensions. We solve the Boltzmann equation for the production of KK modes, both during the radiation dominated epoch and during the reheating phase preceding it. We show that unless the maximum temperature reached during reheating is very low, the constraints from cosmology are much stronger than the supernova bounds.

*This assumption has been made in practically all works on the subject, however see Ref. [4] for a different model.
II. BOLTZMANN EQUATIONS

The fundamental equation governing the evolution of all species in the expanding universe is the Boltzmann equation \([13]\), \(L[f] = C[f]\), where \(L = \partial f/\partial t - p\partial f/\partial p\) is the Liouville operator and \(C\) is the collision operator describing all possible interactions. \(f\) is the distribution function for the given particle species. In the present case, there are two terms contributing to the collision operator: production and decay. There are several possible production channels: gravi-Compton scattering, pair annihilation and bremsstrahlung. In a supernova, nucleon-nucleon bremsstrahlung is by far the dominant mechanism because of the very high nucleon density. However, this is not the case in the early universe, the reason being that the early universe is a high entropy environment \((n = n_B/n_e \simeq 10^{-10})\) [14]. Therefore NN bremsstrahlung is suppressed by a large numerical factor \(\simeq n_N^2/n_e^2\). The dominant processes are instead the pair annihilation reactions \(2\gamma \rightarrow KK, \nu\nu \rightarrow KK, e^+e^- \rightarrow KK\) [11]. The matrix element for each of these processes is given simply by \([4,11]\). The matrix element for each of these processes is given simply by \([4,11]\). The matrix element for each of these processes is given simply by \([4,11]\). The matrix element for each of these processes is given simply by \([4,11]\). The matrix element for each of these processes is given simply by \([4,11]\).

In order for the standard cosmological equations to apply, it is a necessary condition that \(\rho_{KK} \ll \rho_i\), where \(i\) denotes fields on the brane. This means that we can completely neglect inverse processes in our treatment. It also has the big advantage that we can neglect Pauli blocking and stimulated emission factors in the Boltzmann equation. In this case we can use the integrated Boltzmann equation which is much simpler than the full Boltzmann equation [13]

\[
\dot{n}_m = \sum_{i = \nu, e, \gamma} \langle \sigma v \rangle_i n_i^2 - 3H n_m - \Gamma_{\text{decay}, m},
\]

where \(m\) is the mass of the KK state. For the relatively low mass modes we look at, the decay lifetime is very long [1]. Therefore decays can be completely neglected at early times and the production phase can be separated from the decay phase. The production equation is then given by [11]

\[
\dot{n}_m = -3H n_m + \frac{11m^5 T}{128\pi^3 M_P^2} K_1(m/T),
\]

where \(K_1(x)\) is a modified Bessel function of the second kind and we have assumed that \(m_e = 0\). This assumption has very little influence on the results.

A. Production during the radiation dominated epoch

The universe enters the radiation dominated epoch at some temperature \(T\), which we shall refer to as the re-heating temperature, \(T_{RH}\). Production of KK modes during this epoch was studied in detail by Hall and Smith [11], and in this section their results are rederived. The present day number density can be found be integrating the Boltzmann equation

\[
n_0(m) \simeq \frac{19}{64\pi^2 g_s \rho_{RH}} \frac{1}{M_P^3} m e^{-\Gamma_{\text{decay}, m} t_0} \int_{m/T_{RH}}^{\infty} q^3 K_1(q) dq.
\]

This equation applies to the number density for one mode with mass \(m\). However, if we are interested in the total present day contribution to the mass density from all modes, then we need to sum over all modes. This sum can be replaced by an integral over \(dm\) because the mode density is very high [6]. This integration yields the result

\[
\rho_{0, \text{thermal}} \simeq 1.9 \times 10^{-22} S_{n-1} \text{GeV}^4 \left( \frac{T_{RH}}{M} \right)^{n+2} \times \int_0^\infty dq z^{n+1} e^{-\Gamma_{\text{decay}, m} t_0} \int_0^\infty dq q^3 K_1(q),
\]

where \(S_{n-1} = 2\pi^{n/2}/\Gamma(n/2)\). In Fig. 1 we show the constraints on \(M\) from demanding that \(\rho_0 \leq \rho_{\text{crit}}\), for the case of \(n = 2\). This result is identical to what was found in Ref. [11].

![Fig. 1](image_url)

**FIG. 1.** The lower bound on \(M\) as a function of \(T_{RH}\), when only modes produced during the radiation dominated epoch are considered. The calculations assume that \(h = 0.75\) and \(n = 2\).

From this it is evident that for \(T_{RH} \gtrsim 3\) MeV the bound is tighter than what is found from SN1987A. However, it was shown in Refs. [10,11] that \(T_{RH} = 0.7\) MeV can still be compatible with BBN, so if \(T_{RH}\) is sufficiently low the overproduction of KK states can be avoided.
B. Production during reheating

In the above treatment it was assumed that the universe enters the radiation dominated epoch instantaneously at the reheating temperature. However, this is not the case for any physically acceptable scenario. Plausibly, the universe enters the radiation epoch after some reheating by the decay of a massive scalar field (or by some other means of entropy production). The only reasonable alternative is that the radiation dominated epoch extended to much higher temperatures (of the order $M$).

Here, we look at the “standard” case where reheating occurs from the decay of the inflaton field (for further discussion of inflation in scenarios with large extra dimensions, see for instance Ref. [13] and references therein).

What happens is that the universe starts reheating when the inflaton enters the oscillating regime. The important parameters are the density, $\rho_{\phi,i}$, of the inflaton when reheating begins and the decay rate of the inflaton, $\Gamma_\phi$. The Boltzmann equations for this system have been solved numerous times (see e.g. Refs. [17][19]). The result is that the temperature of the produced radiation immediately increases to a maximum value which depends on $\rho_{\phi,i}$. After this, there is a period of continual entropy production during which the universe is matter dominated by the $\phi$ field and $T \propto t^{-1/4}$ (as opposed to the case where no entropy is produced, $T \propto t^{-1/2}$). At the time $t \simeq \Gamma_\phi^{-1}$ the inflaton decays rapidly and the universe becomes radiation dominated. Using this, it is easy to calculate the number of KK modes produced during the reheating phase. $\Gamma_\phi$ is directly related to $T_{RH}$ by $T_{RH} \simeq 0.5 \sqrt{\Gamma_\phi M_P}$ [17], but the additional parameter $\rho_{\phi,i}$ is introduced in the analysis. However, instead of this we use the parameter $\alpha \equiv T_{MAX}/T_{RH}$, where $T_{MAX}$ is the maximum temperature reached during reheating (the relation between $\rho_{\phi,i}$ and $T_{MAX}$ is given in Ref. [20]). From this, one gets an expression completely equivalent to Eq. (6). During reheating entropy is produced continuously. The entropy density is given by $s = g_T T^3$ at all times, where $g_T$ is the number of relativistic degrees of freedom contributing to the entropy [13]. We assume that $g_T \simeq 10.75$ at all times. Although this is not the case at very high temperatures, the assumption introduces only a modest error. Since $T \propto t^{-1/4}$ and $a \propto t^{2/3}$ (because the universe is matter dominated), the number density of a KK-mode (if one ignores production) is $n_m \propto T^6$. This yields $n_m/s \propto T^5$ during reheating. In order to solve the Boltzmann equation during reheating we introduce the variable $X_m \equiv n_m s^{-1} (T/T_{RH})^{-5}$ which is constant during reheating if there is no production. Then the Boltzmann equation can be simply written as

$$\dot{X}_m = \frac{1}{s(T/T_{RH})^{5}} \frac{m^5 T}{128 \pi^3 M_P^3} K_1(m/T).$$

By integration this gives

$$X_{m,RH} = \frac{4}{128 \pi^3 M_P^3} \frac{T_{RH} T_{RH}^{3}}{m^5 s_{RH}} \int_{m/T_{MAX}}^{m/T_{RH}} dq 10^{K_1(q)},$$

where we can approximate $t_{RH}$ by $t_{RH} \simeq 1.5 g_*^{-1/2} M_P T_{RH}^{-2}$ [1]. The present day number density is then given by $n_{m,0} = X_{m,RH} \rho_0$. The total density of all KK-modes can be found by integrating over $dm$ [1], and gives

$$\rho_{0,RH} \simeq 1.9 \times 10^{-22} S_{-1} \text{GeV}^4 \left( \frac{T_{RH}}{M_P} \right)^{n+2} \times \int_{0}^{\infty} 2dz \frac{z^n}{\Gamma_{\text{decay},m} T_{MAX}^{z}} \left( \frac{T_{RH}}{T_{MAX}} \right)^{z-1} dq 10^{K_1(q)}.$$  

The factor $e^{-\Gamma_{\text{decay},m} T_{RH}}$ comes from the fact that some of the produced KK-modes decay before the present. This is one of the main new result of the present paper. Notice that Eq. (8) is very similar to Eq. (6), except for the fact that $z^n q^3$ is changed to $z^n q^0$. This can be understood quite simply. Modes produced during reheating are diluted by entropy production relative to modes produced during the thermal epoch by the factor $(T/T_{RH})^{-5}$. Furthermore, the time-interval for production during reheating is different from what it is during the thermal epoch. During reheating $dt/dT \propto T^{-5}$, whereas during the thermal epoch $dt/dT \propto T^{-3}$. Altogether this gives a factor $(T/T_{RH})^{-7} = z^{-2} q^7$ difference, which is what is found.

Fig. 2 shows the lower bound on $M$ for $n = 2$ and 3, as a function of $T_{RH}$ and $\alpha$. It is evident from the figures that the contours quickly become vertical for increasing $\alpha$. This happens because the KK modes produced at early times during reheating have been diluted away by entropy production so that they do not contribute significantly at late times. Even so, accounting for the modes produced during reheating does strengthen the bound on $M$. For the case of $n = 2$ the bound is 6.8 TeV if only the thermal production is included, whereas it is 9.7 TeV if reheating is also accounted for (assuming $T_{MAX} \gg T_{RH}$). For $n = 3$ these figures are 0.44 TeV and 0.66 TeV respectively.

Notice that this result is different from the result found by Giudice et al. [24], that the final abundance depends only on $T_{RH}$, and not on $T_{MAX}$. The reason is that there is a dense spectrum of modes with different masses. The abundance of each mode obeys the relations found in Ref. [24], i.e. that the final abundance for any mode with $m < \text{few} \times T_{MAX}$ is independent of $T_{MAX}$. However, for higher $T_{MAX}$, many more modes are excited, and the end result is that the total production of KK modes increases with increasing $T_{MAX}$.

Nevertheless, if we only consider the bound on $M$ from the demanding that $\rho_0 < \rho_{crit}$, it is significantly weaker than the bound from SN1987a. In the next section we consider the much stronger bound from the diffuse gamma background.
even tighter constraints can be put on $M$ if reheating is included.

For $n = 2$, the contribution to the diffuse gamma background from KK decays is given by

$$
\frac{dn}{dE} \approx 345 M_{\text{TeV}}^{-4} T_{\text{RH,MeV}}^5 \left( \frac{t_0}{10^{10} y} \right) \text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}
\times \beta^{1/2} \left[ \int_{z_{\alpha}}^{z_{\text{MAX}}} dzz^{-7/2} E(z) \int_z^{\infty} q^3 K_1(q) dq + \right.
\left. \int_{z_{\alpha}}^{z_{\text{MAX}}} 2dzz^{-7/2} E(z) \int_z^{\infty} q^{10} K_1(q) dq \right],
$$

where $\beta = E/T_{\text{RH}}$ and $E(z) = \exp(-3.3 \times 10^{-7} z^{3/2} T_{\text{RH,MeV}}^3 t_0(10^{10} y)^{3/2})$. As the upper limit for the mass integral, $z_{\text{MAX}}$, we take $z_{\text{MAX}} = 2.7 \times 10^3 T_{\text{RH,MeV}}$, because KK modes above this mass decay before CMBR formation, and therefore do not contribute to the present diffuse flux. Note that for $\alpha = 1$ this equation is equivalent to what is found in Ref. [11]. Also note that there is the same factor $q^7 z^{-7}$ difference between the thermal (the first part in the bracket) and the reheating (the second part) contributions as was found between Eqs. (2) and (5). For higher $n$ one obtains expressions very similar to Eq. (1). Observationally, the diffuse gamma background in the MeV range has been measured by the EGRET [22,24] (30 – 10$^4$ MeV) and COMPTEL [23] (0.8 - 30 MeV) experiments. The flux measured by EGRET is approximately

$$
\frac{d\phi}{dE} = 2.3 \times 10^{-3} (E/\text{MeV})^{-2.67} \text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1},
$$

and the flux measured by COMPTEL is

$$
\frac{d\phi}{dE} = 6.4 \times 10^{-3} (E/\text{MeV})^{-2.3} \text{MeV}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{ster}^{-1}.
$$

Demanding that $\frac{dn}{dE,\text{KK}} \leq \frac{dn}{dE,\text{obs}}$ translates into the lower bound on $M$ shown in Fig. 3 as a function of $T_{\text{RH}}$ and $n$. Note that for the relatively low masses we study here, constraints from light nuclei abundances [25] are not very important.

From this figure it is evident that increasing $\alpha$ leads to significantly stronger bounds on $M$. If $T_{\text{RH}} = 0.7 \text{ MeV}$, then for $n = 2$ the bound goes from $M > 73 \text{ TeV}$ at $\alpha = 1$ to $M > 167 \text{ TeV}$ at $\alpha = 1400$ ($T_{\text{MAX}} = 1 \text{ GeV}$). For $n = 3$ the corresponding numbers are $M > 3.9 \text{ TeV}$ at $\alpha = 1$ to $M > 21.7 \text{ TeV}$ at $\alpha = 1400$, a factor of 5.6 increase. In most reasonable models it is difficult to obtain values of $T_{\text{MAX}}$ which are smaller than 1 GeV, rather $T_{\text{MAX}}$ will usually be much higher than 1 GeV.

Finally, it should be noted that as $n$ is increased, the difference between including reheating and only treating thermally produced modes increases. In Table I the lower bound on $M$ is shown for different values of $n$ and $T_{\text{MAX}}$ to illustrate this.

For masses which are low enough that the KK modes have not decayed away before the present, the lower bound on $M$ as a function of $T_{\text{RH}}$ and $T_{\text{MAX}}$ exhibits a quite simple behaviour. For $\alpha = 1$, $M_{\text{min}} \propto T_{\text{RH}}^{(n+5)/(n+2)}$, whereas for constant $T_{\text{RH}}$ and large $\alpha$, $M_{\text{min}} \propto T_{\text{MAX}}^{(n-2)/(n+2)}$. This means that for $n \geq 3$ the con-

III. CONSTRAINTS FROM THE DIFFUSE GAMMA BACKGROUND

Apart from the production mechanisms there is also the possibility that the massive KK states decay into particles on the brane. The decay rate for different branches has been calculated by Han, Lykken and Zhang [3]. The decay rate into particles on the brane is of the order

$$
\Gamma \sim m^3/M_Z^2,
$$

for any kinematically allowed final state. With the temperatures discussed here, the decay lifetime is on most cases of the same order of magnitude as the Hubble time. This means that there will be visible effects, especially from the decay contribution to the diffuse gamma background. It was shown by Hall and Smith [11] (see also Ref. [21]) that this leads to a very stringent constraint on $M$, even with $T_{\text{RH}} = 1 \text{ MeV}$. The modes produced during reheating have higher mass and therefore much higher decay rates. This means that

![FIG. 2. The lower bound on $M/\text{TeV}$ as a function of $T_{\text{RH}}$ and $\alpha$, from demanding that $\rho_{\text{therm}} + \rho_{\text{RH}} \leq \rho_{\text{crit}}$. The upper panel is for $n = 2$ and the lower for $n = 3$. The value $h = 0.75$ has been used.](image-url)
FIG. 3. The lower bound on $M/\text{TeV}$ as a function of $T_{\text{RH}}$ and $\alpha$, from the diffuse gamma background. The upper panel is for $n = 2$ and the lower for $n = 3$. The value $t_0 = 10^{10}\,y$ has been used.

TABLE I. The lower bound on $M$ in TeV for different values of $n$ and $T_{\text{MAX}}$. All values are for $T_{\text{RH}} = 0.7\,\text{MeV}$.

| $n$ | $T_{\text{MAX}} = 0.7\,\text{MeV}$ | 50 MeV | 100 MeV | 1 GeV |
|-----|---------------------------------|---------|---------|-------|
| 2   | 73                              | 161     | 165     | 167   |
| 3   | 3.9                             | 16.0    | 18.6    | 21.7  |
| 4   | 0.47                            | 2.96    | 3.75    | 4.75  |
| 5   | 0.10                            | 0.89    | 1.19    | 1.55  |

IV. DISCUSSION

We have discussed in detail how KK modes are produced in the early universe. First, production during the radiation dominated epoch was discussed, and the results found in Ref. [1] were rederived.

It was then shown that if reheating before the thermal epoch is taken into account, the bound on the fundamental Planck scale, $M$, is strengthened significantly and becomes stronger than the supernova bound, albeit the theoretical uncertainty is larger. This is the main result of the present paper.

We showed that if $T_{\text{MAX}}$ during reheating is 1 GeV, then the bound on $M$ is $M > 167\,\text{TeV}$ for $n = 2$ and $M > 21.7\,\text{TeV}$ for $n = 3$. These bound can be translated into upper bounds on the radii of the extra dimensions using the relation [1]

$$R_{\text{mm}} = 2 \times 10^{31/n-16} \left(\frac{1\,\text{TeV}}{M}\right)^{1+2/n}. \quad (10)$$

From this, we get $R < 2.2 \times 10^{-5}\,\text{mm}$ for $n = 2$ and $R < 2.5 \times 10^{-8}\,\text{mm}$ for $n = 3$.

Note that in the present treatment we have assumed that the inflaton only decays to matter on the brane. If gravitons are also produced at reheating, the bounds are tightened.

We finish by discussing briefly the few possibilities for avoiding the very stringent bound obtained above. In our analysis we assumed a toroidal geometry for the extra dimensions. However, other choices of geometry lead to different spectra of KK modes. As shown in Ref. [6], compact hyperbolic manifolds can lead to spectra with a lightest mode with $m \gtrsim 10\,\text{GeV}$ and large energy spacing. In this case the cosmological bounds disappear completely (as does all other astrophysical bounds). The second possibility is that there are more branes embedded in the bulk. In that case, it is possible that the KK modes have fast decay channels to light particles on other branes. This would mean that the KK modes could have decayed into radiation before the present [3].

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