Static properties of a bose-fermi mixture of trapped potassium atoms are studied in terms of coupled Gross-Pitaevskii and Thomas-Fermi equations for both repulsive and attractive bose-fermi interatomic potentials. Qualitative estimates are given for solutions of the coupled equations, and the parameter regions are obtained analytically for the boson-density profile change and for the boson/fermion phase separation. Especially, the parameter ratio \( R_{\text{int}} \) is found that discriminates the region of the large boson-profile change. These estimates are applied for numerical results for the potassium atoms and checked their consistency. It is suggested that a small fraction of fermions could be trapped without an external potential for the system with an attractive boson-fermion interaction.

I. INTRODUCTION

Recent development in the cooling and trapping technique of atoms and the following success in achieving the Bose-Einstein condensation of alkali atoms \([1–3]\) have now opened up a new era in the progress in quantum physics. While the condensed system serves as a testing ground for a research in fundamental problems of quantum mechanics, it also offers a new example of finite quantum many-body systems such as hadrons, nuclei and microclusters. An important characteristic of the trapped alkali atoms is that it is a dilute system of weakly interacting particles and an ideal place to test genuine properties of condensed systems predicted by theories.

Along with a further progress in the study of Bose-Einstein condensed systems, a similar technique is being extended to create a degenerate gas of fermionic atoms, where a number of theoretical studies have been made \([4]\). To realize such a degenerate fermionic system that requires a still lower temperature than bose systems, the technique of sympathetic cooling has been investigated \([5]\): the cooling mechanism through the collisions with coexisting cold bose particles in a polarized boson-fermion mixture, where the fermion-fermion interaction is less effective.

The mixture of bose and fermi particles is itself an interesting system for a study: the hydrogen-deuterium system has been studied already at the early stage of these investigations \([6]\). Recently, theoretical studies have been made for the trapped boson-fermion system of alkali atoms and proposed many interesting properties: the exotic density configurations through the repulsive or attractive boson-fermion interactions \([7,8]\), the decaying processes after the removal of the confining trap \([9]\), and the metastability of the \(^7\)Li-\(^6\)Li mixture \([10]\).

In this paper, we study static properties of a mixed bose-fermi system of trapped potassium atoms, where one fermionic (\(^{40}\)K) and two bosonic isotopes (\(^{39}\)K and \(^{41}\)K) are known as candidates for a realization of such a degenerate quantum system. Trapping of the fermionic isotope \(^{40}\)K has already been reported \([11]\). These isotopes are also of an interest since the boson-fermion interaction may be repulsive or attractive depending on their choice \([12,13]\).

In sec. III, we discuss qualitative properties of solutions from the GP and TF equations analytically; especially, their dependence on the boson/fermion number is estimated.

II. FORMULATION

We consider a spin-polarized system of bosons and fermions at \( T = 0 \) described by a Hamiltonian \( H = H_b + H_f + V_{bf} \):
of the system is just given by a replacement of field operator \( \phi \) that describes the order parameter of the Bose-Einstein condensate. (We here neglect quantities of the order \( \phi \).)

\[
H_b = \int d\vec{r} \phi^\dagger(\vec{r}) \left( -\frac{\hbar^2}{2m_b} \nabla^2 - \mu_b + \frac{1}{2}m_b\omega_b^2 r^2 \right) \phi(\vec{r})
\]
\[
+ \frac{1}{2} g \int d\vec{r} d\vec{r}' \phi^\dagger(\vec{r}) \phi^\dagger(\vec{r}') \delta(\vec{r} - \vec{r}') \phi(\vec{r}') \phi(\vec{r}),
\]
\[
H_f = \int d\vec{r} \psi^\dagger(\vec{r}) \left( -\frac{\hbar^2}{2m_f} \nabla^2 - \mu_f + \frac{1}{2}m_f\omega_f^2 r^2 \right) \psi(\vec{r}),
\]
\[
V_{bf} = h \int d\vec{r} d\vec{r}' \phi^\dagger(\vec{r}) \psi^\dagger(\vec{r}') \delta(\vec{r} - \vec{r}') \psi(\vec{r}') \phi(\vec{r}),
\]

where \( \phi(\vec{r}) \) and \( \psi(\vec{r}) \) respectively denote boson and fermion field operators with masses \( m_b \) and \( m_f \). Because of diluteness, the particle interactions have been approximated by the \( s \)-wave-dominated contact potential \( v(\vec{r} - \vec{r}') = \{g \) or \( h\} \delta^3(\vec{r} - \vec{r}') \). The strengths of the coupling constants \( g \) (boson-boson) and \( h \) (boson-fermion) are given by

\[
g = \frac{4\pi \hbar^2}{m_b a_{bb}}, \quad h = \frac{2\pi \hbar^2}{m_f a_{bf}},
\]

where \( a_{bb} \) and \( a_{bf} \) are the \( s \)-wave scattering lengths for the boson-boson and boson-fermion scatterings and \( m_{bf} = m_b m_f / (m_b + m_f) \) is a reduced mass. The fermion-fermion interaction has been neglected because of diluteness and particle polarization. The chemical potentials \( \mu_b \) and \( \mu_f \) in (1a) and (1b) are determined from the condensed boson/fermion numbers \( N_b \) and \( N_f \) through the ground state expectation values:

\[
N_b = \langle \int d\vec{r} \phi^\dagger(\vec{r}) \phi(\vec{r}) \rangle, \quad N_f = \langle \int d\vec{r} \psi^\dagger(\vec{r}) \psi(\vec{r}) \rangle.
\]

In the mean-field approximation at \( T = 0 \), the bosons occupy the lowest single-particle state \( \varphi_b(\vec{r}) \), and the energy of the system is just given by a replacement of field operator \( \phi(\vec{r}) \) in the Hamiltonian with its expectation value:

\[
\Phi(\vec{r}) \equiv \langle \phi(\vec{r}) \rangle = \sqrt{N_b} \varphi_b(\vec{r})
\]

that describes the order parameter of the Bose-Einstein condensate. (We here neglect quantities of the order \( O(N_b^{-1}) \).)

In this approximation, the boson density \( n_b(\vec{r}) \) is given by \( |\Phi(\vec{r})|^2 \).

In the same approximation, the fermion wave function is given by a Slater determinant; the single-particle states are determined, e.g., by the Hartree-Fock self-consistent equation. In the present approximation of neglecting fermion-fermion interactions, we only have to solve for the single particle states under the effective potential for fermions: the trapping potential and the boson-fermion interaction. In the actual calculation, we however adopt a semiclassical (TF) description for fermion density, which is known to provide a good approximation as far as the number of atoms is sufficiently large.

We thus obtain a set of equations for \( \Phi \) and \( n_f \):

\[
[-\frac{\hbar^2}{2m_b} \nabla^2 + \frac{1}{2}m_b\omega_b^2 r^2 + g n_b(\vec{r}) + h n_f(\vec{r})] \Phi(\vec{r}) = \mu_b \Phi(\vec{r}),
\]
\[
\frac{\hbar^2}{2m_f}[6\pi^2 n_f(\vec{r})^{2/3} + \frac{1}{2}m_f\omega_f^2 r^2 + h n_b(\vec{r})] = \mu_f,
\]

where the boson and fermion densities are defined by \( n_b(\vec{r}) = \langle \phi^\dagger(\vec{r}) \phi(\vec{r}) \rangle \sim |\Phi(\vec{r})|^2 \) and \( n_f(\vec{r}) = \langle \psi^\dagger(\vec{r}) \psi(\vec{r}) \rangle \).

### III. Qualitative Study of Solutions

Before the study of numerical results for (3) and (4), we discuss about their solutions qualitatively.

1. Radius of the boson/fermion distribution

If we neglect the boson-fermion interaction, we would obtain a much broader distribution for fermions as compared with that for bosons because of the exclusion principle. To see this, we consider a free boson/fermion system trapped in a harmonic oscillator potential. The mean square radius of each system is obtained from the virial theorem:

\[
\langle r^2 \rangle_b / N_b = \frac{3}{2} \xi_b^2, \quad \langle r^2 \rangle_f / N_f = \frac{3}{4} (n_F + 2) \xi_f^2.
\]
where the harmonic-oscillator lengths $\xi_{b,f}$ are

$$\xi_{b,f} \equiv \sqrt{\frac{\hbar}{m_{b,f} \omega_{b,f}}}.$$  

(8)

and $N_{b,f}$ are the total boson/fermion numbers. The $n_F$ in (3) is the number of fermions in the last occupied shell that is obtained as a solution of $N_f = (n_F + 1)(n_F + 2)(n_F + 3)/6 \sim n_F^3$. The ratio of the root-mean-square (rms) radii $\sqrt{\langle r^2 \rangle_f/(\langle r^2 \rangle_b}$ thus increases as the power $N^{1/6}$ when $N = N_b \sim N_f$ and $\xi_b \sim \xi_f$. In fact, for a repulsive boson-boson interaction, the boson distribution become broader than that for a free system just discussed, but this effect does not change the qualitative structure as shown in sec. V. On the other hand, the strong boson-fermion interaction may give qualitative changes of the density profiles such as the phase separation and the fermion collapse as shown in [7,8]. We also discussed about them from different points of view in iii) in this section.

ii) Scaled GP and TF equations and changes of the density distribution for the different $N_{b,f}$

For qualitative estimations of the density distribution, we should use the scaled dimensionless variables [3]:

$$\bar{x} = \frac{x}{\xi_b}, \quad \rho_{b,f} = \frac{\xi_b^3}{N_{b,f}} n_{b,f}, \quad \tilde{\rho}_{b,f} = \frac{2}{\hbar \omega_b} \mu_{b,f},$$  

(9)

where $N_{b,f}$, $\xi_{b,f}$ are the total boson/fermion numbers and the harmonic-oscillator lengths defined in eqs. (3) and (8). Using these scaled variables, the GP and TF equations in eqs. (3) and (4) become

$$[-\nabla^2 + \hat{\mu}_b \rho_b(\bar{x}) + \hat{\hbar} N_f \rho_f(\bar{x})] \hat{\varphi}_b(\bar{x}) = \tilde{\mu}_b \hat{\varphi}_b(\bar{x})$$

(10)

$$\frac{1}{R_m} \left[ 6\pi^2 N_f \rho_f(\bar{x}) \right] \frac{\ddot{\varphi}_b(\bar{x})}{\bar{x}} + R_m \omega_b^2 \bar{x}^2 + \hat{\hbar} N_b \rho_b(\bar{x}) = \tilde{\mu}_f,$$

(11)

where $R_m = m_f/m_b$ and $R_\omega = \omega_f/\omega_b$. The scaled coupling constants $\tilde{g}$ and $\tilde{\hbar}$ in eqs. (10) and (11) are defined by

$$\tilde{g} = \frac{2g}{\hbar \omega_b \xi_b} = 8\pi a_{b,f}, \quad \tilde{\hbar} = \frac{2\hbar}{\hbar \omega_b \xi_b} = 4\pi a_{b,f} \left( 1 + \frac{1}{R_m} \right),$$  

(12)

where eq. (3) has been used to obtain the $a_{b,f}$-representations, and the last factor comes from $m_b/m_{bf} = 1 + R_m^{-1}$.

From eqs. (10) and (11), we should notice that the scaled equations include six independent parameters, $(\tilde{g} N_b, \tilde{\hbar} N_f, N_f, \tilde{\hbar} N_b, R_m, R_\omega)$; for two systems where these parameters take the same values, their physical properties become similar under the scaling relations (3).

Here, based on eqs. (3) and (11), we discuss about changes of the density distribution by the scaled parameters. In this subsection, we assume $R_m = R_\omega = 1$ and the large particle numbers $N_b \sim N_f \sim N \gg 1$; for the effect of $R_\omega \neq 1$, the discussion will be given in sec. V.

Let us consider the fermion TF equation (11), first. For the chemical potential $\tilde{\mu}_f$, we use the result in the system without boson-fermion interaction: $\tilde{\mu}_f \sim 2(6N_f)^{1/3} \sim N_f^{1/3}$. It should be a good approximation when $N_f \gg 1$. Using it in eq. (11), we obtain $\rho_f \sim N_f^{-1/2}$ at the central region ($x \sim 0$). On the other hand, the boson equation (10) gives $\rho_b \sim \tilde{g} N_b^{-3/5}$ under the TF approximation valid in $N_b \gg 1$. Using these estimates, we can obtain the ratio $R_{int}$ of the boson-boson/boson-fermion interaction effects in the GP equation (10):

$$R_{int} \equiv \frac{\hat{\hbar} N_f \rho_f}{\tilde{g} N_b \rho_b} \bigg|_{x=0} \sim \frac{\hat{\hbar} N_f}{\tilde{g} N_b} \left( \frac{N_f}{\tilde{g} N_b} \right)^{1/2} \sim \frac{\hat{\hbar} N_f}{\tilde{g} N_b} \left( \frac{\tilde{g} N_b}{N_f} \right)^{1/2}.$$  

(13)

The small $R_{int} (\ll 1)$ indicates the small contribution from the boson-fermion interaction for the boson part, and we obtain $\rho_b \sim \rho_b(h = 0)$. However, when $R_{int} \geq 1$, the boson-fermion interaction is not negligible, and the boson density profile can be quite different from that in non-interacting case ($h = 0$).

In $^{39}$K-$^{40}$K system discussed in sec. V, the scaled parameters are

$$(\tilde{g} N_b, \tilde{\hbar} N_f, N_f, \tilde{\hbar} N_b) = (2.68 \times 10^{-2} N_b, 1.57 \times 10^{-2} N_f, N_f, 1.57 \times 10^{-2} N_b).$$  

(14)

In $N \sim N_b \sim N_f$, $\tilde{g} N_b$ and $\tilde{\hbar} N_f$ are in the same order, but $N_f$ is a factor of $2$ larger than $\tilde{\hbar} N_f$, so eq. (13) gives $R_{int} \sim 10^{-1}$. It explains the rather small change of the boson density profile $\rho_b \sim \rho_b(h = 0)$ in $^{39}$K-$^{40}$K system, as shown in the numerical results in sec. V.
It is very interesting to apply the above estimation for the results by \([14]\) and compare them with the results in this paper. One of the parameter set with which the boson density profiles changed drastically is

\[
(g \tilde{N}_b, \tilde{h}N_f, N_f, \tilde{h}N_b) = (4.22 \times 10^6, 4.22 \times 10^6h/g, 10^6, 4.22 \times 10^6h/g),
\]

where \(h/g = \tilde{h}/g = 0 \sim 5/4\) has been taken and the qualitative changes of the boson density profile have been seen \(h/g \geq 3/4\) \([13]\). It should be noticed that the parameters in \([15]\) are in the same order when \(h/g \sim 1\), and it means \(R_{int} \sim 1\) in that case. For this reason, the boson density profile changed drastically from that of \(h = 0\) with these parameters.

To check the relation between the boson profile change and the ratio \(R_{int}\) in eq. \([14]\) a little more, we solved eqs. \([1] \) and \([3]\) numerically for a) \(N_f = 10^6\) and b) \(N_f = 10^6\) with other parameters fixed as in \([15]\), and, for \(h/g\), two values were taken \(h/g = 0, 0.6\). The resultant boson densities are shown in fig. 1: \(n_b(h \neq 0)\) has very different profile from \(n_b(h = 0)\) in case a), but the very small change is found between them in case b). Those results shows the clear relation between \(R_{int}\) and the boson density profile.

In summary, the parameter regions exist where the boson density profile changes very large or not, and these regions are discriminated by the value of \(R_{int}\).

We should comment that the results in this paper are complementary with those with \([13]\) in \([3]\) in the scale of the ratio \(R_{int}\).

iii) Boson-fermion interaction effects for the fermion distribution function

We next consider the effect of the boson-fermion interaction on the fermion distribution, especially the phase separation phenomena in the strongly repulsive boson-fermion interaction case, which has been proposed in \([2] \). Here we assume the TF approximation for bosons (neglecting the kinetic term in eq.\((6)\)). As discussed in the literature \([2] \), the effective potential for fermions receives an additional repulsive or attractive contribution from bosons within the range \(r \leq R_b \equiv \sqrt{2\mu_b/m_0^2}\) depending on the sign of the interaction.

For a strongly repulsive boson-fermion interaction \((h > 0)\), the fermion will be squeezed out from the center, so that the two kind of particles tend to make a separate phase \([2] \). Here we give analytical estimation for this interesting phenomena. As shown in \([2] \), in the phase-separated system, the fermion are almost completely pushed away outside the boson distribution, so that the critical ratio \((h/g)\), for the phase separation can be estimated from the vanishing fermion density at the center \(n_f(r = 0) = 0\). Neglecting the boson-fermion interaction in the boson TF equation, we obtain

\[
N_f = F(\alpha) \equiv \frac{1}{3\pi}\left(\frac{\mu_b}{\tilde{h} \omega_b}\right)^3 \left\{\alpha(\alpha - 2)(\alpha - 1)^{1/2} + \alpha^3 \arctan(\alpha - 1)^{1/2}\right\}
\]

where \(\alpha = h/g \geq 1\). In fig. 2, the function \(F(\alpha)\) in \([14]\) is plotted when \(N_f = 10^3\) with the parameters used in \([2]\) : \((\tilde{g}N_b, \tilde{h}N_f, N_f, \tilde{h}N_b, R_m, R_c) = (3.0 \times 10^4, 3.0 \times 10^4, 10^3, 3.0 \times 10^3, 1, 1)\) \([14]\). From the crossing point of \(f(\alpha)\) and \(N_f = 10^3\) (dashed line) in fig. 2, the critical point can be read off: \(\alpha = (h/g)|c \sim 1.3\), which is almost consistent with the result given in \([3] \) (\(\alpha \simeq 1.18\)).

For an attractive interaction \((h < 0)\), fermions tend to concentrate and increase the overlap with bosons. Thus we may expect a coherence of the two kinds of particles to occur in this case. For large attraction, another interesting phenomena, fermion collapse in mixture has been proposed \([3]\).

IV. NUMERICAL PROCEDURE

In the numerical calculation, we solved the set of equations \([3]\) and \([1]\) by reducing them into the following equation for the order parameter:

\[
\left\[\frac{d^2}{dt^2} + \left(\frac{3}{2} - t\right) \frac{d}{dt} - \frac{1}{4} \left\{3 - \tilde{\mu}_b + \tilde{g} \frac{f(t)^2}{e^t}\right\} - \frac{\tilde{h}}{24\pi^2}\left\{\tilde{\mu}_f - \frac{m_f \omega_f^2}{m_b \omega_b^2} t - \tilde{h} \frac{f(t)^2}{e^t}\right\}^{3/2}\right\] f(t) = 0,
\]

where \(t = x^2 = (r/\xi_b)^2\) and the scaled parameters \(x, \tilde{g}, \tilde{h}\) and \(\tilde{\mu}_b,f\) have been defined in \([1]\). The function \(f\) is related to the order parameter by \(\Phi(r)/r = e^{-x^2}f(x^2)\). The boundary conditions for \(f(t)\) are given by the value at \(t = 0\) and by the asymptotic condition \(f(t \to \infty) \sim \epsilon^{(\tilde{\mu}_b - 3)/4}\). We first give initial values for \(f(0)\) and \(\mu_f\) and solved the equation \([14]\) numerically by means of the relaxation method. Finally, the fermion density \(n_f(r)\) is obtained from eq.\((14)\). The details of the calculation will be given elsewhere \([3]\).
We consider the potassium atoms where there exist two bosonic ($^{39}$K, $^{41}$K) and one fermionic ($^{40}$K) isotopes. The precise values of the interatomic interaction are not known, but the recent estimate from molecular scattering suggests a repulsive interaction between $^{39}$K and $^{40}$K, and an attractive one between $^{41}$K and $^{40}$K, although the values still have rather large ambiguities. We take up the following values for the s-wave scattering lengths \( a_{bf}(^{39}$K-$^{39}$K) = 4.29 nm, \( a_{bf}(^{39}$K-$^{40}$K) = 2.51 nm, \( a_{bf}(^{41}$K-$^{41}$K) = 15.13 nm, \( a_{bf}(^{41}$K-$^{40}$K) = -8.57 nm. We take the atomic masses to be the same for all the isotopes: \( m_b = m_f = 0.649 \times 10^{-25} \text{kg}. \) The angular frequency \( \omega_b \) of the bosonic external potential is fixed at 100 Hz, while \( \omega_f \) is allowed to vary. In this case, it is corresponding to \( \tilde{g} = 2.68 \times 10^{-2} \), \( \tilde{h} = 1.57 \times 10^{-2} \) for $^{39}$K-$^{40}$K system and \( \tilde{g} = 9.44 \times 10^{-2} \), \( \tilde{h} = -5.34 \times 10^{-2} \) for $^{41}$K-$^{40}$K system.

V. RESULTS

First, we consider the $^{39}$K-$^{40}$K system where the boson-fermion interaction is repulsive. In this case, because of \( \tilde{h}N_b/\tilde{g}N_b < 1 \), phase separation do not occur. Fig. 3 shows the density-distribution functions of bosons (a) and fermions (b) for \( N_b = 10000, N_f = 1000 \) and \( \omega_f = \omega_b = 100 \text{Hz} \). The dashed line shows the result for \( a_{bf} = 0 \) \( (h = 0) \) as compared with the solid line for \( a_{bf} \neq 0 \) \( (h \neq 0) \). As seen from the figure, the fermions have a much broader distribution than bosons even for a much less number of particles. That has been discussed in sec. III-i). The robust distribution just mentioned is reflected in the almost constant values of the rms radius and other observables for the bosonic part of the mixed condensate against the switching on/off of the boson-fermion interaction. To understand this robustness, we apply the estimation in sec. III-ii): the \((1000,1000) \) case is more robust than \((10000,1000) \) case.

Is the bosonic distribution always robust against a mixture of fermions? Aside from the possibility of changing the interaction strength via Feshbach resonances, we have a possibility of enhancing the boson-fermion interaction energy to the total boson energy becomes \( E_{BF}/E_{tot}(^{41}$K) \), while it is less than 1 per cent to the total fermion energy. The change of the boson-density distribution can be estimated from the ratio of the boson-fermion interaction energy to the total boson energy becomes \( E_{BF}/E_{tot}(^{41}$K) = 0.36 \). When \( R_\omega \neq 1 \), it becomes

\[
R_{int} = \frac{\tilde{h}N_f}{\tilde{g}N_b} \left( \frac{\tilde{g}N_b}{N_f} \right)^{1/2} R_\omega^{3/2}.
\]
so that the boson-density distribution become more sensitive for large values of $R_\omega$.

The effect of the boson-fermion interaction on fermions may show up in the opposite extreme, i.e., for vanishing $R_\omega$. In this case, the fermions can keep themselves from escape only through the attractive boson-fermion interaction. This possibility can be estimated from the chemical potential: when fermions are trapped by bosons, the $\mu_f$ should take the negative value. Taking the TF approximation for the bosonic part ($\tilde{g}_N b \gg 1, \tilde{g}_N b \gg \tilde{h} N_f$) and setting $\omega_f = 0$ for the fermionic part, we obtain the total fermion number represented by $\mu_f$:

$$N_f = \frac{1}{48} \left| \frac{h}{g} \right|^2 \left\{ \tilde{\mu}_f + \frac{h}{g} \left( \frac{15}{8\pi} \tilde{g}_N b \right)^{2/5} \right\}^3. \tag{20}$$

When $\mu_f = 0$ in (20), we obtain the maximum number for the boson-trapped fermions:

$$N_f^{\text{max}} = \frac{1}{48} \left| \frac{h}{g} \right|^{3/2} \left( \frac{15}{8\pi} \tilde{g}_N b \right)^{6/5} = 2.8 \times 10^{-4} N_b^{6/5} \tag{21}$$

where we used the parameters of $^{41}\text{K}-^{40}\text{K}$ system to obtain the last term. From eq. (21), estimations can be obtained for the boson-trapped fermion number: for $N_b = 10000$, only 18 fermions are bound, while for $N_b = 10^6$ more than 4000 fermions will be trapped within the bose condensate. This qualitative estimate is verified also in the numerical calculation for $N_b = 10000$, although one should actually take the shell effect into account for such a small number of fermions [15].

VI. SUMMARY

In the present paper, we studied static properties of a mixed system of bosons ($^{39}\text{K}$ or $^{41}\text{K}$) and fermions ($^{40}\text{K}$) in the trapping potential at temperature zero. We solved a coupled GP and TF equations for a different combination of boson and fermion particle numbers and also for a repulsive or an attractive interaction of bosons and fermions depending on the combination of potassium isotopes.

The analytical estimations have been done generally for the rms radii of the boson/fermion distribution, the parameter-dependence of their profile and the phase separation, and we have found the feasible conditions for them. These analytical estimations have been applied to the numerical results for the potassium system and also compared with the discussion done in [33]: consequently, the consistency was checked for these estimations.

For similar particle numbers and external potential parameters, the fermions have a much more extended distribution and larger energies than bosons because of the exclusion principle. We studied the effect of the boson-fermion interaction on the density distribution of both kinds of particles. For a repulsive interaction, the fermions tend to be pushed out, but the effect is not very large for the parameters of $^{41}\text{K}-^{40}\text{K}$ system adopted in the present paper. In the case of the attractive interaction, a coherence of $^{41}\text{K}$ and $^{40}\text{K}$ occurs and is strongly enhanced if the trapping potential is adjusted so as to make them overlap. It was shown also that the fermions could be kept trapped without external potential due only to an attractive interaction with trapped bose particles. One may as well consider the opposite case where the bosons are trapped only via an attractive interaction with surrounding trapped fermi particles. This will be discussed in a future publication [13].

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TABLE I. Results for the $^{39}\text{K},^{40}\text{K}$ system with ($a_{bf} = 2.51$ nm) and without ($a_{bf} = 0$) the boson-fermion interaction for ($N_b, N_f$) = (1000, 1000) and (10000, 1000). Contributions to the equilibrium energy (measured in the unit of $\hbar\omega_b/2$) per particle, the central density $n(0) (x \equiv r/\xi_b)$ and the rms radii are given for each of the constituent isotopes. Other parameters are fixed as $a_{bh} = 4.29$ nm, $\omega_f = \omega_b = 100$ Hz.

| $(N_b, N_f; a_{bf})$ | atom | $\mu_b$ or $\mu_f$ | $E_{tot}/N$ | $E_{kin}/N$ | $E_{ho}/N$ | $E_{BB}/N$ | $E_{BF}/N$ | $n(x = 0)$ | $\sqrt{(x^2)}$ |
|---------------------|------|-------------------|------------|-----------|-----------|-----------|-----------|-------------|-------------|
| $(10^3, 10^5; 0.00)$ | $^{39}\text{K}$ | 4.19 | 3.66 | 1.16 | 1.96 | 0.53 | 0.00 | 98.7 | 1.40 |
|                    | $^{40}\text{K}$ | 36.3 | 27.3 | 13.6 | 13.6 | -- | -- | 3.70 | 3.69 |
| $(10^3, 10^5; 2.51)$ | $^{39}\text{K}$ | 4.24 | 3.71 | 1.16 | 1.96 | 0.53 | 0.05 | 98.7 | 1.40 |
|                    | $^{40}\text{K}$ | 36.4 | 27.3 | 13.6 | 13.7 | -- | -- | 3.47 | 3.70 |
| $(10^5, 10^7; 0.00)$ | $^{39}\text{K}$ | 8.21 | 6.26 | 0.70 | 3.62 | 1.95 | 0.00 | 291 | 1.90 |
|                    | $^{40}\text{K}$ | 36.3 | 27.3 | 13.6 | 13.6 | -- | -- | 3.70 | 3.69 |
| $(10^5, 10^7; 2.51)$ | $^{39}\text{K}$ | 8.25 | 6.31 | 0.70 | 3.62 | 1.94 | 0.05 | 291 | 1.90 |
|                    | $^{40}\text{K}$ | 36.6 | 27.7 | 13.3 | 14.0 | -- | -- | 3.07 | 3.74 |

TABLE II. Results for the $^{41}\text{K},^{40}\text{K}$ system. See Table I for the notation. The parameters are: $a_{bh} = 15.13$ nm, $\omega_f = \omega_b = 100$ Hz.

| $(N_b, N_f; a_{bf})$ | atom | $\mu_b$ or $\mu_f$ | $E_{tot}/N$ | $E_{kin}/N$ | $E_{ho}/N$ | $E_{BB}/N$ | $E_{BF}/N$ | $n(x = 0)$ | $\sqrt{(x^2)}$ |
|---------------------|------|-------------------|------------|-----------|-----------|-----------|-----------|-------------|-------------|
| $(10^3, 10^5; 0.00)$ | $^{41}\text{K}$ | 5.83 | 4.68 | 0.90 | 2.62 | 1.15 | 0.00 | 53.5 | 1.62 |
|                    | $^{40}\text{K}$ | 36.3 | 27.3 | 13.6 | 13.6 | -- | -- | 3.70 | 3.69 |
| $(10^3, 10^5; -8.57)$ | $^{41}\text{K}$ | 5.65 | 4.49 | 0.91 | 2.62 | 1.16 | -- | 53.9 | 1.62 |
|                    | $^{40}\text{K}$ | 36.2 | 27.1 | 13.8 | 13.5 | -- | -- | 4.13 | 3.68 |
| $(10^5, 10^7; 0.00)$ | $^{41}\text{K}$ | 13.0 | 9.58 | 0.59 | 5.85 | 3.43 | 0.00 | 135 | 2.38 |
|                    | $^{40}\text{K}$ | 36.3 | 27.3 | 13.6 | 13.6 | -- | -- | 3.70 | 3.69 |
| $(10^5, 10^7; -8.57)$ | $^{41}\text{K}$ | 12.9 | 9.40 | 0.59 | 5.62 | 3.46 | -- | 136 | 2.37 |
|                    | $^{40}\text{K}$ | 35.4 | 25.6 | 14.7 | 12.7 | -- | -- | 4.71 | 3.56 |

FIG. 1. Changes of the boson density distribution for different $N_f$ values: a) $N_f = 10^6$ and b) $N_f = 10^8$ with $(\tilde{g}N_b, \tilde{h}N_f, \tilde{h}N_b, R_m, R_w) = (4.22 \times 10^6, 4.22 \times 10^6 \tilde{t}, 10^6, 4.22 \times 10^6 \tilde{t}, 1, 1)$. Solid lines are for $\hbar/g = 0.6$ and dotted lines are for $\hbar/g = 0$ (no boson-fermion interaction).

FIG. 2. Critical line of component separation for the system with $(\tilde{g}N_b, \tilde{h}N_f, N_f, \tilde{h}N_b, R_m, R_w) = (3.0 \times 10^4, 3.0 \times 10^4 h/g, 10^3, 3.0 \times 10^4 h/g, 1, 1)$.

FIG. 3. Density distribution for the $(^{39}\text{K},^{40}\text{K}) = (10000, 1000)$ system: (a) for $^{39}\text{K}$ and (b) for $^{40}\text{K}$ (b) are plotted against $x \equiv r/\xi_b$. The solid line shows the full calculation with the (repulsive) boson-fermion interaction ($a_{bf} = 2.51$ nm) and the dashed line without. Both lines are indistinguishable for the bosonic part (a).

FIG. 4. Density distribution for the $(^{41}\text{K},^{40}\text{K})$: (a) for the $^{41}\text{K}$ and (b) for $^{40}\text{K}$. The boson-fermion interaction is attractive in this case ($a_{bf} = -8.57$ nm).

FIG. 5. Dependence of the density distribution on the oscillator-frequency ratio $R_w = \omega_f/\omega_b$: a) $^{39}\text{K},^{40}\text{K}$ system with $R_w = 1, 5, 10$ and b) $^{41}\text{K}-^{40}\text{K}$ system with $R_w = 1, 4, 7$, where $(N_b, N_f) = (1000, 1000)$ and $\omega_b = 100$ Hz. The solid line shows the boson density $n_b(x)$ and the dashed line shows the fermion density $n_f(x)$.
Figure 1

radial distance : $x \neq r/$

(a)

(b)
Figure 2
Figure 3

(a) boson density

(b) fermion density

radial distance: \( \chi_b \neq r/\)

\[ N_b = 10000 \]
\[ N_f = 1000 \]

\[ N_b = 10000 \]
\[ N_f = 1000 \]
Figure 4

(a) Boson density $n_b(x)$

(b) Fermion density $n_f(x)$

Radial distance: $x = r/b$.
Figure 5

\(R = 1\)

\(R = 5\)

\(R = 10\)

\(R = 4\)

\(R = 7\)

(a) \(^{39}\text{K} - ^{40}\text{K}\) system

(b) \(^{41}\text{K} - ^{40}\text{K}\) system