On Quasinormal modes and Quantization of Area of Black Holes

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Abstract. Perturbations of black holes give very important tools of understanding physical properties of black holes. The relaxation of the perturbation is described by quasinormal modes with complex frequency. Using this, there has been some proposal of Hod and later by Maggiore that one can obtain quantization of the area of black holes. These ideas, when applied to rotating black holes were less than controversial. In this work we address how the area can be quantized with equal spacing. We also comment on application of this to other black holes in three dimensional spacetime.

1. Introduction

Black holes are one of the most fascinating objects in physics which have been confirmed to exist in our Universe. The term black hole was first used in public by J.A. Wheeler at the Sigma Xi - Phi Beta Kappa lecture in 1967 held at New York Hilton hotel [1]. Classical behavior of a black hole is rather simple, and depend only on a few parameters, due to the well known ‘No Hair Theorem’. Again Wheeler was involved coining this [2]. Whether these black hole solutions are stable under linear perturbation was the question raised by Regge and Wheeler in 1957, when they wrote down the wave equation of linear fluctuation around a spherically symmetric Schwarzschild black hole of mass $M$ [3]. The ‘Regge-Wheeler’ equation has a form of Schrödinger equation, with an effective potential

$$V_{\text{eff}} = \left(1 - \frac{2GM}{r}\right) \left[\frac{l(l+1)}{r^2} + \frac{2GM}{r^3}\right],$$

(1)

where $l$ is the angular momentum mode number, and $G$ is the Newton constant. This seemingly easy Schrödinger equation still requires numerical solution. It was Chandrasekhar and Detweiler [4] who have done some numerical calculations and obtained the first few modes. The wave should vanish at the black hole horizon and there is no incoming wave from the asymptotic infinity. These boundary conditions allow only specific values of the frequency, like normal modes. Due to the perfect absorption at the horizon, the wave decays away, so there is an imaginary term in the frequency. So we call these modes quasinormal modes.

In particular the quasinormal mode becomes, for large imaginary value, as follows:

$$\omega_n M \sim 0.0437 \frac{i}{4} \left(n + \frac{1}{2}\right) + O\left(\frac{1}{\sqrt{n+1}}\right).$$

(2)
This behavior is independent of the value of the angular momentum to leading order. The real part of the quasinormal mode become a universal number.

When people performed some analytic calculations \[5\], it was shown that the real part behaves as

\[
0.0437 \approx \frac{\log 3}{8\pi}.
\] (3)

The imaginary part can be easily computed for the large damping case. One can use the Born approximation. When perform the Fourier transformation of the Regge-Wheeler potential for the approximation, we obtain some Gamma functions. The poles of the Gamma functions occur when we impose the boundary conditions. These lead to the discrete values of the imaginary part of the quasinormal frequency.

For the Reissner-Nordstrom black hole, the linear wave equation around it was studied by Zerilli \[6\]. The Kerr black hole case was much more involved and was first written down by Teukolsky \[7,8\], who used the Newman-Penrose formalism.

These quasinormal modes are important probes of black holes. Just like we can get some idea of the property of wine inside a casket by tapping on it, quasinormal modes capture some of the properties of classical black holes.

2. Black hole area quantization

It is believed that the quantum behavior of black holes could play a significant role as a testing ground for the quantum theory of gravity. Sometime ago Bekenstein \[9\] proposed that since the black hole horizon area is an adiabatic invariant, it should be quantized. The argument was through Ehrenfest principle \[10\] which says that only an adiabatically invariant quantities are quantizable. When we consider the minimum change of the horizon area by an absorption of a test particle into the black hole, it is reasonable that this minimum area should be proportional to the Planck area \(l_P^2 = \frac{\hbar G}{c^3}\). Therefore intuitively we have

\[
A_n = \gamma n l_P^2 = \gamma n \hbar, \quad n = 0, 1, 2, \ldots,
\] (4)

where \(\gamma\) is an undetermined dimensionless constant. (We are using the units \(c = G = 1\)). There has been a lot of work in relation to this observation \[11\].

An important development on this line of though was done by Hod \[12\] who suggested that the dimensionless parameter \(\gamma\) can be determined by the quasinormal mode of a black hole. He argued that the real part of the asymptotic quasinormal mode \(\omega_R\), can be regarded as the transition frequency in the semiclassical limit and therefore the when there is a quantum emitted there will be change in the black hole mass, \(\Delta M = \hbar \omega_R\). Since \(A = 16\pi M^2\), we have

\[
\Delta A = 32\pi M \Delta M = 32\pi M \frac{\hbar \log 3}{8\pi M} = 4 \log \hbar.
\] (5)

Therefore he could determine the dimensionless parameter \(\gamma\) of Bekenstein to be \(4 \log 3\). Some people in loop quantum gravity found some significance in this number. Furthermore we have an equally spaced area spectrum.

A different view on this problem was provided by Kunstatter \[13\]. By viewing as the quasinormal mode as a mechanical system with frequency \(\omega_R\), then noticed that the quantity

\[
I = \int \frac{dE}{\omega}
\] (6)

is an adiabatic invariant, at least for a vibrational system with frequency \(\omega\). Once we have an adiabatic invariant we can perform Bohr-Sommerfeld quantization. For example when we plug
in the ADM mass for the energy of Schwarzschild black hole, and use $\omega_R$ for the frequency in the denominator of the integrand, we get $M^2 = \frac{\log 3}{4\pi n \bar{\hbar}}$, in agreement with Hod’s result.

More recently, Maggiore [14] argued that we should view the quasinormal mode as a damped harmonic oscillator, especially for the highly excited modes. Actually it is these highly excited modes that we can safely use the Bohr-Sommerfeld quantization. Furthermore the transition frequency should be the characteristic classical frequency $\omega_c$ where

$$\omega_c \approx \left( |\omega_I| \right)_k - \left( |\omega_I| \right)_{k-1}. \tag{7}$$

With this new identification, the change in the mass is $\Delta M = \bar{\hbar} \omega_c$, and for highly damped quasinormal modes, it is not dependent on the real part but rather the imaginary part of the quasinormal mode spectrum. The result is that we now have $A_n = 8\pi \bar{\hbar}$.

When these methods were applied to rotating Kerr black hole, the result was that the area spectrum is no longer equally spaced [15-18]. This lead to some confusion. In these works, the area was taken to be the adiabatic invariant. By using the area law for the entropy, $S = A / 4$, we then have $I \sim \int \frac{T_H dS}{T_H}$, where we have multiplied the Hawking temperature $T_H$. The reason for this is that quasinormal modes is proportional to the Hawking temperature in the asymptotic region. If we use the first law of black hole thermodynamics we the get

$$I \sim \int \frac{dM - \Omega dJ}{\omega} = n \bar{\hbar} \tag{8}$$

where $\Omega$ is the angular velocity at the horizon and $j$ is the angular momentum of the black hole. It turns out that thus obtained area of black hole has a very complicated area spectrum, which is not equally spaced.

To us this is not satisfactory and we would like to have an equally spaced area spectrum, because Bekenstein’s argument was rather general and also intuitively it is correct. Why was the adiabatic invariant was quantized? As mentioned before, Ehrenfest principle says that any classical adiabatic invariant corresponds to a quantum object with discrete spectrum. However this spectrum need not be equally spaced. There is a particular subclass of adiabatic invariants are quantized with equally spaced spectrum. It is the action variable

$$I = \frac{1}{2\pi} \oint pdq. \tag{9}$$

When we do Bohr-Sommerfeld quantization, we have $I = n \bar{\hbar}$. I.e. an action variable is an adiabatic invariant but the reverse is not necessarily true. So instead of considering the area itself as the adiabatic invariant, we need to identify the action variable. It is the integration of the energy divided by the transition frequency! The change in energy is the change in the mass $M$.

3. Area Spectrum of a Rotating BTZ Black Hole

To demonstrate our suggestion that we have to quantize the action variable, and that we do indeed get an equally spaced area spectrum even for a rotating case, we would like to consider the rotating BTZ black hole solution in 2+1 dimensional spacetime [19].

$$ds^2 = - \left( -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} \right) dt^2 + \left( -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} \right) \frac{dr^2}{r^2} + r^2 \left( d\phi - \frac{J}{2r^2} dt \right)^2, \tag{10}$$

where the cosmological constant is given by $\Lambda = -\frac{l^2}{r^4}$. The mass $M$ and angular momentum $J$ of the black hole can be expressed in terms of the outer and inner horizons, $r_{\pm}$, as follows:

$$M = \frac{r_+^2 + r_-^2}{l^2}, \quad J = \frac{2r_+ r_-}{l}. \tag{11}$$
The two families of the quasinormal modes of the rotating BTZ black hole for a massive scalar field are given by \[20, 21\]

\[
\omega_R = -\frac{m}{l} - i \frac{(r_+ + r_-)}{l^2} \left( 2k + 1 + \sqrt{1 + \mu} \right), \quad \omega_L = \frac{m}{l} - i \frac{(r_+ - r_-)}{l^2} \left( 2k + 1 + \sqrt{1 + \mu} \right),
\]

where \(m \in \mathbb{Z}\) and \(k = 0, 1, 2, \ldots\). The \(m\) and \(k\) are the angular quantum number and the overtone quantum number respectively, and \(\mu\) is the mass parameter, \(\mu \equiv u^2 l^2/\hbar^2\), where \(u\) is the mass of the scalar field. At large \(k\) for a fixed \(|m|\), in particular for \(k \gg |m|\), the two families of quasinormal modes give two possible transition frequencies, \(\omega_{Rc}\) and \(\omega_{Lc}\). We find the two transition frequencies corresponding to each quasinormal mode:

\[
\omega_{Rc} = \frac{2(r_+ + r_-)}{l^2} = \frac{2\sqrt{M + J/l}}{l}, \quad \omega_{Lc} = \frac{2(r_+ - r_-)}{l^2} = \frac{2\sqrt{M - J/l}}{l}.
\]

We consider the two action variables corresponding to each possible transition frequency. We obtain the two quantization conditions:

\[
\mathcal{I}_R = \int \frac{dM}{\omega_{Rc}} = l\sqrt{M + J/l} = n_R \bar{\hbar}, \quad \mathcal{I}_L = \int \frac{dM}{\omega_{Lc}} = l\sqrt{M - J/l} = n_L \bar{\hbar},
\]

where \(n_R, n_L = 0, 1, 2, \ldots\). Notice that we have \(n_R \geq n_L\).

Because the total horizon area is quantized and equally spaced:

\[
A_{\text{tot}} = A_{\text{out}} + A_{\text{in}} = 2\pi n_R \bar{\hbar}, \quad n_R = 0, 1, 2, \ldots
\]

Interestingly, the difference of the two horizon areas is also quantized

\[
A_{\text{sub}} \equiv A_{\text{out}} - A_{\text{in}} = 2\pi l \sqrt{M - J/l} = 2\pi n_L \bar{\hbar}, \quad n_L = 0, 1, 2, \ldots
\]

We therefore find the quantizations of the outer and inner horizon areas:

\[
A_{\text{out}} = \pi (n_R + n_L) \bar{\hbar}, \quad A_{\text{in}} = \pi (n_R - n_L) \bar{\hbar},
\]

where \(n_R \geq n_L\). Therefore we find that both the outer and inner horizon areas are equally spaced with the same spacing; \(\Delta A_{\text{out}} = \pi \bar{\hbar}, \quad \Delta A_{\text{in}} = \pi \bar{\hbar}\). By Bekenstein-Hawking area law, the entropy spectrum is given by

\[
S = \frac{4\pi r_+}{\bar{\hbar}} = \frac{A_{\text{tot}} + A_{\text{sub}}}{\bar{\hbar}} = 2\pi (n_R + n_L).\]

Hence it also has equal spacing, \(\Delta S = 2\pi\) consistent with Bekenstein’s proposal; Notice that the spacing of the entropy spectrum is half of the non-rotating BTZ black hole case and same as for the Schwarzschild black hole. So this simple exercise shows that even for rotating black holes, albeit in three dimensions, has equal spacing \[23\].

4. The BTZ black hole with the gravitational Chern-Simons term

Let us try to do a slightly more There is a small debate in the gravity community arguing which is more fundamental? Geometry or information? Or as Wheeler has put, "It or Bit?". This issue cannot be resolved in cases where the entropy is proportional to the horizon area. This is the case for black hole solutions in Einstein gravity, where the action is the Einstein-Hilbert action. However, for the cases where there are additional terms in the action, entropy is no longer proportional to the horizon area of the black hole. For these cases comparison of the area
and the entropy spectra becomes very interesting as we will see below. The question is, which of these has equal spacing, if any? For this purpose we will consider black holes which arises when there is gravitational Chern-Simons term in the action:

\[
S = \frac{1}{16\pi G} \int_M d^3x \sqrt{-g} \left( R + \frac{2}{l^2} \right) - \frac{l}{96\pi G\nu} \int_M d^3x \sqrt{-g} \varepsilon^{\lambda\mu\nu} \Gamma^{\tau}_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\nu\tau} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right) .
\] (19)

It again has the rotating BTZ black hole as a solution. The metric of the rotating BTZ black hole is exactly the metric given in the previous section. Also, the quasinormal modes are same as for the rotating BTZ black hole without the Chern-Simons term. However we have to use ADT mass rather than ADM mass. ADT mass of a BTZ black hole is given as follows:

\[
M = M + \frac{J}{3\nu l} ,
\] (20)

where \(M\) and \(J\) are the mass and the angular momentum for the BTZ black hole in the absence of the Chern-Simons term. So, the change of the energy of the black hole should be considered as the change of the ADT mass \(M\), i.e. \(dE = dM\). The action variable \(I\) is given by

\[
I = \int \frac{dE}{\omega_c} = \int \frac{dM}{\omega_c} = \int \frac{dM}{\omega_c} + \frac{1}{3\nu l} \int \frac{dJ}{\omega_c} .
\] (21)

The two action variables, \(I_+\) and \(I_-\), are obtained and quantized as follows:

\[
I_\pm = \frac{(3\nu \pm 1)l}{24G\nu} \sqrt{8GM \pm 8GJ/l} = n_R \hbar .
\] (22)

We find that the total horizon area spectrum is given by

\[
A_{\text{tot}} \equiv A_{\text{out}} + A_{\text{in}} = 2\pi l \sqrt{8GM + 8GJ/l} = \frac{48\pi G\nu}{(3\nu + 1)} n_R \hbar ,
\] (23)

where \(n_R = 0, 1, 2, \ldots\) This spectrum is equally spaced but the spacing is dependent on the coupling constant \(\nu\). The spectra of the outer and inner horizon areas are obtained as follows:

\[
A_\pm = \frac{24G\pi \nu}{9\nu^2 - 1} \left[(3\nu - 1) n_R \pm (3\nu + 1) n_L \right] \hbar ,
\] (24)

where \((3\nu - 1) n_R \geq (3\nu + 1) n_L\). \(A_+ = A_{\text{out}}\) and \(A_- = A_{\text{in}}\). These area spectra are not equally spaced and dependent on the coupling constant \(\nu\), so that they are affected by the Chern-Simons term.

Now let us find the entropy spectrum. There is \(\nu\) dependence in the relation between horizon areas and the entropy as follows:

\[
S = \frac{1}{4G \hbar} \left( A_{\text{out}} + \frac{1}{3\nu} A_{\text{in}} \right) .
\] (25)

However, in the entropy spectrum \(\nu\) dependence drops out completely and we have and equally spaced spectrum:

\[
S = 2\pi (n_R + n_L) .
\] (26)

The entropy spectrum (26) is independent of the coupling constant \(\nu\) and exactly same as for the rotating BTZ black hole without the Chern-Simons term obtained in the previous section. Therefore the Chern-Simons term does not affect the entropy spectrum of the BTZ black hole, even though it affects the area spectra. This suggests that the entropy is more fundamental than the horizon area. We will consider another black hole solution in the next section which will enforce this point.
5. Area spectrum vs Entropy Spectrum

Again the action we will consider will have the gravitational Chern-Simons term together with the usual Einstein Hilbert action. There is a new solution called the warped AdS black holes with a deformation parameter $\nu \geq 1$. The case $\nu = 1$ is related to the BTZ black hole considered in the previous section. The metric is given by [24, 25]

$$ds^2 = l^2 \left[ dt^2 + \frac{dr^2}{(\nu^2 + 3) (r - r_+)(r - r_-)} + \left(2 \nu r - \sqrt{(\nu^2 + 3) r_+ r_-}\right) dt d\theta \right.\left. + \frac{r}{4} \left(3 (\nu^2 - 1) r + (\nu^2 + 3) (r_+ + r_-) - 4 \nu \sqrt{(\nu^2 + 3) r_+ r_-}\right) d\theta^2 \right],$$

where $r_+$ and $r_-$ are the outer and inner horizons, respectively. The coordinates $t$ and $r$ are dimensionless. The entropy of the warped AdS black hole is given by [25, 26]

$$S = \frac{\pi l}{24 G \nu \hbar} \left(9 \nu^2 + 3 \right) r_+ - \left(\nu^2 + 3 \right) r_- - 4 \nu \sqrt{(\nu^2 + 3) r_+ r_-}. \quad (28)$$

As we can see it has rather complicated $\nu$ dependence. The conserved charges such as the ADT mass $M$ and angular momentum $J$ are given by [25, 26]

$$M = \frac{(\nu^2 + 3)}{24 G} \left( r_+ + r_- - \frac{1}{\nu} \sqrt{(\nu^2 + 3) r_+ r_-} \right), \quad \quad (29)$$

$$J = \frac{(\nu^2 + 3) \nu l}{96 G} \left[ (r_+ + r_- - \frac{1}{\nu} \sqrt{(\nu^2 + 3) r_+ r_-})^2 - \frac{4}{\nu^2} \left( r_+ r_- \right)^2 \right], \quad \quad (30)$$

which satisfy the first law of the black hole thermodynamics, $dM = \tilde{T} dS + \tilde{\Omega} dJ$, with the Hawking temperature $\tilde{T}$ and the angular velocity of the horizon $\tilde{\Omega}$ given by [25]

$$\tilde{T} \equiv \frac{T}{l} = \frac{(\nu^2 + 3)}{4 \pi l} \left(2 \nu r_+ - \sqrt{(\nu^2 + 3) r_+ r_-}\right), \quad \quad (31)$$

$$\tilde{\Omega} \equiv \frac{\Omega}{l} = -\frac{2}{(2 \nu r_+ - \sqrt{(\nu^2 + 3) r_+ r_-}) l}. \quad \quad (32)$$

From the metric we can read off the inner and the outer horizon areas,

$$A_{out} = \frac{\pi l}{2 \nu r_+ - \sqrt{(\nu^2 + 3) r_+ r_-}}, \quad (33)$$

$$A_{in} = \frac{\pi l}{2 \nu r_- - \sqrt{(\nu^2 + 3) r_+ r_-}}. \quad (34)$$

When we calculate the spectra of the outer and inner horizon areas we get [namkwon2]:

$$A_{out} = \frac{48 \pi G \nu^2}{5 \nu^2 + 3} \left[ n_R + \frac{5 \nu^2 + 3}{4 \nu^2} n_L \right] \hbar, \quad \quad (35)$$

$$A_{in} = \frac{48 \pi G \nu^2}{5 \nu^2 + 3} \left[ n_R - \frac{5 \nu^2 + 3}{4 \nu^2} n_L \right] \hbar. \quad \quad (36)$$

These area spectra are not equally spaced and dependent on the coupling constant $\nu$. The entropy spectrum is obtained by rewriting the entropy expression (28) in terms of the horizon areas as follows:

$$S = \frac{(9 \nu^2 + 3) A_{out} \pm (\nu^2 + 3) A_{in}}{48 G \nu^2 \hbar}. \quad \quad (37)$$
Because of the Chern-Simons term, the entropy is proportional to not only the outer horizon area but also the inner horizon area, and again there is $\nu$ dependence. Nevertheless, when we plug in the area spectra, miraculous canceling of $\nu$ happens and the entropy spectrum is as follows:

$$S = 2\pi (n_R + n_L).$$  \hspace{1cm} (38)

This entropy spectrum is independent of the coupling constant $\nu$ and the spacing is given by $\Delta S = 2\pi$ [27]. Furthermore this is the exactly same as for the rotating BTZ black holes with and without the Chern-Simons term. It implies that the entropy spectra of the black holes have a universal behavior regardless of the presence of the gravitational Chern-Simons term. Therefore we would like to claim that the entropy spectrum rather than the area spectrum of a black hole should be equally spaced. Furthermore it strongly suggest the entropy which has a universal behavior is more fundamental than the area. Wheeler towards the end of his career was leaning towards information rather than the geometry as the fundamental entity of gravitation. We pleasantly find some examples which agrees with his deep insights.

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