In-plane motion of wind turbine blade under the combined action of parametric and forced excitations

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Abstract. This paper presents an analysis for the nonlinear in-plane motion of wind turbine blade under the combined action of parametric and forced excitations. The Bernoulli-Euler beam theory is adopted to describe blade motion. Effects including gravity, structural damping and geometric nonlinearity are included in the mathematical model. The rotating speed is considered as a nominal speed with a harmonic perturbation. The periodic oscillation of rotating speed brings about parametric excitations in stiffness and damping terms and forced excitations. The harmonic balance method is applied to get the approximate solution of dynamic response. Numerical integral of original system is used to verify the analytical solution. Influences of structural damping, perturbed amplitude and frequency of rotating speed on blade behaviour are discussed.

1. Introduction
Hub motion can cause the oscillation of rotating speed that will bring about parametric and forced excitations for a wind turbine blade. In studying parametric excitation of wind turbine blade, Chopra and Dugundji [1] studied principal parametric resonance and self-excited flutter of the coupled feather-flap-lead/lag vibration, while the blade was considered as a rigid body with hinges; effects of design parameters on dynamic responses were analyzed. Larsen et al. [2–4] adopted Bernoulli-Euler beam theory to describe the coupled vibration of slender blade and investigated nonlinear response, parametric instability and stochastic stability of blade in 1:2 internal resonance associated with parametric excitation; influences of aerodynamic damping, perturbed amplitude, external excitation, linear and nonlinear parametric terms on dynamic responses were discussed. Ramakrishnan and Feeny [5] introduced a mathematical model for blade in-plane motion with parametric excitation caused by gravitational load by employing extended Hamiltonian principle, and presented an analysis for dynamic responses of blade in super-harmonic resonance; effects of ratio between rotating angular frequency and natural frequency, external excitation, parametric forcing term and nonlinearity on blade behaviours were studied. Except these literature for wind turbine blades, there are lots of works on parametric excitations, e.g. [6–17], for beams, plates, strings, cables and rigid bodies.

Previous researches are mainly on parametric excitations in resonance cases. In this work, the nonlinear in-plane motion (the most unstable motion for a blade due to small aerodynamic and

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structural damping) of wind turbine blade under the combined actions of parametric and forced excitations in general case is studied. The structural damping which has been studied scarcely in previous researches about nonlinear dynamics of blade is added to the model via the Kelvin-Voigt principle. The harmonic balance method is adopted to obtain the approximate solution of dynamic responses. Influences of structural damping, perturbed frequency and perturbed amplitude on blade behaviours are discussed.

2. Mathematical model

A sketch of an isolated blade is illustrated in Figure 1. The in-plane bend is called lead/lag. The Cartesian coordinate system (xyz) is a body coordinate system which is rigidly attached to the blade root such that the x-axis corresponds to the undeformed elastic axis, the y-axis is along the lead/lag direction, and the z-axis is perpendicular to the plane of rotation. The Euler-Bernoulli beam theory is adopted to describe the vibration of slender blade driven by wind flow. The structural damping is introduced to the model through the Kelvin-Voigt principle \((\sigma = \varepsilon + \eta \dot{\varepsilon})\); a detailed deduction process of structural damping terms is given in [18] and [19]. The governing equation is derived as:

\[
\left( Ev''\right)^\prime - \left( F_a v'\right)' - m \Omega^2 \left( e_x + v\right) + m \ddot{v} - \left( \Phi A v'^2 \dot{v}'\right) + \left( \Phi iv^2 \right)^\prime = F_y
\]  

(1)

Where \(E\) is the elastic modulus, \(I\) is the area moment inertia, \(v\) is the bending displacement, \(F_T = \int_0^L \sigma dA = EA (v'^2/2)\) is the axial force whose steady part \(N_p = \int_0^L m(y) \Omega^2 (e^y \cos \beta_p) dy\) is the centrifugal force, \(m\) is the mass per unit length, \(\Omega\) is the rotating angular speed, \((e_x, e_y)\) is the blade root offset from hub center in \((x, y)\)-direction due to inclined installation, \(\Omega\) is the viscosity factor, \(A\) is the section area, \(F_y\) is the resultant distributed force in y-direction. Gravity and aerodynamic force are considered as distributed forces acting on blade sections. Gravity has the following form:

\[
F_y^{(G)} (x) = -mg \sin(\Omega t)
\]  

(2)

The following unsteady aerodynamic force given in Li et al. [20] that is derived from Greenberg’s expressions is adopted in this paper:

\[
F_y^{(a)} = \left( \frac{\rho ac}{2} \right) \Omega^2 \left[ \lambda^2 R^2 \cos \theta - \lambda \left(1 - \beta_p^2\right) R_x \sin \theta - \lambda R e_x \sin \theta - (C_{\infty} / a) x^2 \right] \\
- \left( \frac{\rho ac}{2} \right) \left[ \lambda R \sin \theta + (2C_{\infty} / a) x \right] \dot{v} + (c / 4) \dot{\ddot{v}} \sin^2 \theta
\]  

(3)
where $\rho$ is the air density, $a$ is the section lift curve slope, $c$ is the chord length, $\lambda = \lambda_0 + \lambda_1 (r/R)^2 + \lambda_2 (r/R) \cos(\Omega t)$ is the rotor inflow ratio (1/$\lambda$ is the blade tip speed ratio), $\lambda_0$ is the inflow ratio at hub, $\lambda_1 = \left[ \frac{p(p-1)}{4} \right] \left( \frac{R}{h_0} \right)^2 (\lambda_0 + \sigma a/8)$, $p$ is the velocity gradient constant, $R$ is the rotor radius, $h_0$ is the hub height from ground level, $\sigma = N_c / (\pi R)$ is the solid ratio, $N$ is the blade number in a wind turbine, $\lambda_2 = (pR/h_0) (\lambda_0 + \sigma a/8)$, $\theta = \theta_0 + \theta_1 (x)$ is the twist angle from chordwise direction to lead/lag direction, $\theta_0$ is the setting angle, $\theta_1$ is the pre-twist angle, $\beta_p$ is the coning angle between blade and vertical plane, $\text{CDO}$ is the drag coefficient.

Due to hub motion, there are oscillations for the nominal speed $\Omega_0$ of a rotor blade in operating process. The practical speed can be expressed in Fourier-series:

$$\Omega(t) = \Omega_0 + \sum_{n=1}^\infty a_n \cos(n \Omega_0 t) + \sum_{n=1}^\infty b_n \sin(n \Omega_0 t)$$

To discussed the effects of speed oscillation more distinctly, the speed $\Omega$ is denoted as

$$\Omega = \Omega_0 + \varepsilon \cos(\Omega_0 t + \varphi_\epsilon)$$

where the perturbed amplitude $\varepsilon$ is a small amount of $\Omega_0$. Galerkin’s projection is employed to discrete the continuous model. The displacement is expressed as

$$v = \gamma(x) q(t)$$

where $\gamma(x)$ is the first-order undamped modal function; it is obtained by using the numerical integral basing on Green function [21]. $q(t)$ is the modal coordinate that satisfies the following equation:

$$\ddot{q} + \omega^2 q - 2m\varepsilon \Omega q + \Omega^2 c_d \dot{q} + c_1 q + c_2 \dot{q}^2$$

$$-\Omega^2 a + b q^3 - \Omega^2 \dot{b} q = \Omega^2 f \cos(\Omega t) + \dot{g} \sin(\Omega t)$$

Where $\omega$ is the linear frequency, the third term is the time-dependent elastic restoring force, $c_d$-term is the aerodynamic damping force, $c_1$- and $c_2$-terms are structural damping forces, $a$-term is the linear static displacement, the last term preceding equals sign is the dynamic part of axial force, $f$-term is the aerodynamic excitation, the last term is gravity. Expressions of coefficients are given in Appendix A.

The generalized displacement is disintegrated into static and dynamic parts: $q(t)=q_s+q_d(t)$. Equations for two parts are obtained as:

$$\omega^2 q_s - \Omega^2 a_s + b q^3_s - \Omega^2 \dot{b} q_s = 0$$

$$\ddot{q}_d + \omega^2 q_d - 2m\varepsilon \Omega q_d + \Omega^2 c_d \dot{q}_d + \left[ \Omega_0 + \varepsilon \cos(\Omega_0 t + \varphi_\epsilon) \right] c_s q_d$$

$$+c_1 q_d + c_2 \dot{q}_d \left( q_d + q_s \right)^2 - 2m\varepsilon \Omega \cos(\Omega_0 t + \varphi_\epsilon) a_d + b \left( 3q_s q_d^2 + q_s^3 \right)$$

$$= \left[ \Omega_0 + \varepsilon \cos(\Omega_0 t + \varphi_\epsilon) \right]^2 f \cos(\Omega t) + \dot{g} \sin(\Omega t)$$

where $\dot{\omega} = \sqrt{\omega^2 + 3bq^2_0}$ is the nonlinear frequency due to geometric nonlinearity. In equation (9), $(2\pi/\Omega_0)$-periodic stiffness, aerodynamic damping and external excitations appear due to rotating speed oscillation. The equation of static displacement can be solved by numerical method directly. The equation of dynamic displacement is solved by the harmonic balance method in this paper.

3. Analytical solution

$(2\pi/\Omega_0)$- and $(2\pi/\Omega_p)$-periodic terms in dynamic response are emphasized by neglecting small higher-order-harmonic-terms. The solution is assumed to be:
Substituting equations (5) and (10) into equation (9) and equating the same harmonic terms, the following algebraic equations are obtained:

\[
\begin{align*}
\left( \dot{\omega}^2 - \Omega_0^2 \right) a_j & - \left( \Omega_0^2 c_s + \Omega_4 c_{s1} + \Omega_4 c_{s2} q^2 \right) b_j - \frac{1}{4} \Omega_4 c_{s2} b_j \left( a_j^2 + b_j^2 + 2 c_j^2 + 2 d_j^2 \right) \\
+ \frac{3}{4} b a_j \left( a_j^2 + b_j^2 + 2 c_j^2 + 2 d_j^2 \right) &= \ddot{g} \\
\left( \Omega_0^2 c_s + \Omega_4 c_{s1} + \Omega_4 c_{s2} q^2 \right) a_j + \left( \dot{\omega}^2 - \Omega_0^2 \right) b_j + \frac{1}{4} \Omega_4 c_{s2} a_j \left( a_j^2 + b_j^2 + 2 c_j^2 + 2 d_j^2 \right) \\
+ \frac{3}{4} b b_j \left( a_j^2 + b_j^2 + 2 c_j^2 + 2 d_j^2 \right) &= \Omega_0^2 f \\
\left( \dot{\omega}^2 - \Omega_p^2 \right) c_j - \left( \Omega_0^2 c_s + \Omega_4 c_{s1} + \Omega_4 c_{s2} q^2 \right) d_j - \frac{1}{4} \Omega_4 c_{s2} d_j \left( 2 a_j^2 + 2 b_j^2 + c_j^2 + d_j^2 \right) \\
+ \frac{3}{4} b c_j \left( 2 a_j^2 + 2 b_j^2 + c_j^2 + d_j^2 \right) &= -2 \dot{\omega} Q g_i \sin \varphi_p - 2 \dot{\omega} Q a_i \sin \varphi_p \\
\left( \Omega_0^2 \Omega_p c_s + \Omega_4 c_{s1} + \Omega_4 c_{s2} q^2 \right) c_j + \left( \dot{\omega}^2 - \Omega_0^2 \right) d_j + \frac{1}{4} \Omega_4 c_{s2} c_j \left( 2 a_j^2 + 2 b_j^2 + c_j^2 + d_j^2 \right) \\
+ \frac{3}{4} b d_j \left( 2 a_j^2 + 2 b_j^2 + c_j^2 + d_j^2 \right) &= 2 \dot{\omega} Q g_i \cos \varphi_p + 2 \dot{\omega} Q a_i \cos \varphi_p
\end{align*}
\]

(11)

(12)

(13)

(14)

Four harmonic amplitudes can be derived from equations (11)–(14) through a numerical computation.

4. Numerical results and discussion

The data for a three-bladed MW wind turbine are taken in analysis. Blade length \( L = 48 \text{ m} \), hinge offset \( e = 0.035L \), elastic modulus \( E = 3 \times 10^{10} \text{ Pa} \), blade density \( \rho_b = 1.8 \times 10^3 \text{ kg/m}^3 \), nominal speed \( \Omega_0 = 25 \text{ r/min} \), section lift curve slope \( c_{\text{L}} = 2 \pi \), drag coefficient \( C_{\text{D}} = 0.018 \), hub height \( h_0 = 2L \), air density \( \rho = 1.25 \text{ kg/m}^3 \), velocity gradient constant \( p = 0.167 \), gravitational acceleration \( g = 9.8 \text{ m/s}^2 \).

Changes of amplitudes with structural damping ratio \( \zeta = \epsilon / (2 \dot{\omega}) \), perturbed amplitude \( \epsilon \) and frequency ratio \( \gamma = \Omega_p / \dot{\omega} \) for \( \varphi_p = 0 \) are illustrated in figure 2. \( \varphi_p = 0 \) represents the cosine oscillation \( \Omega = \Omega_0 + \epsilon \cos(\Omega_0 t) \). It shows that four amplitudes all tend to constants with increasing \( \gamma \), especially that \( c_j \) and \( d_j \) approach 0 for large \( \gamma \); this implies that the steady state response is a \((2\pi/\Omega_0)-\text{periodic motion}\) for large \( \gamma \). There’s a noticeable change near \( \gamma = 1 \) for each amplitude due to the primary resonance arising from \((2\pi/\Omega_p)\)-harmonic forced excitations; this change becomes dramatic with increasing \( \pm \epsilon \) and decreasing \( \zeta \). For small \( \gamma \), the harmonic amplitude \( c_j \) is near zero; so the \((2\pi/\Omega_0)\)-periodic term is mainly \( c_j \), \( c_j \) generates same amplitudes.

To reveal the effects of the sine oscillation \( \Omega = \Omega_0 + \epsilon \sin(\Omega_0 t) \), \( \varphi_p = -\pi / 2 \) is taken in analysis and curves of four harmonic amplitudes with respect to \( \epsilon \), \( \gamma \) and \( \zeta \) are presented in figure 3. It is the same with conclusions for cosine oscillation that \((2\pi/\Omega_0)\)-periodic terms are very tiny for large \( \gamma \), and there is a obvious change for each amplitude near \( \gamma = 1 \) that increases with increasing \( \pm \epsilon \) and decreasing \( \zeta \). For small \( \gamma \), the harmonic amplitude \( d_j \) is almost zero; so the \((2\pi/\Omega_p)\)-periodic term is mainly \( \sin(\Omega_0 t) \)-term arising from forced excitations, and the absolute value of the amplitude \( c_j \) of the \((2\pi/\Omega_p)\)-periodic term increases with increasing \( \pm \epsilon \) and decreasing \( \zeta \). \( \epsilon \) and \(-\epsilon \) generate same amplitudes.
Figure 2. Variations of four harmonic amplitudes with respect to structural damping ratio $\zeta$, perturbed amplitude $\varepsilon$ and frequency ratio $\gamma$, where $\phi_p=0$.

Figure 3. Variations of four harmonic amplitudes with respect to structural damping ratio $\zeta$,
perturbed amplitude $\varepsilon$ and frequency ratio $\gamma$, where $\phi_p=-\pi/2$.

Figure 4. Time history of dynamic displacement for $\varepsilon=0.10$, $\zeta=0.005$, $\gamma=0.8$, $\phi_p=0$.

Figure 5. Time history of dynamic displacement for $\varepsilon=0.10$, $\zeta=0.005$, $\gamma=1$, $\phi_p=-\pi/2$.

Figure 6. Time history of dynamic displacement for $\varepsilon=0.10$, $\zeta=0.005$, $\gamma=3$, $\phi_p=0$. 
To validate analytical solutions, time histories of the dynamic displacement for three different frequency ratios, $\gamma = 0.8, 1$ and $3$, are shown in figures 4–6. The numerical results are derived from the direct numerical integral of the system (9). Figure 4 illustrates the dynamic response of the in-plane motion for $\varepsilon = 0.10$, $\zeta = 0.005$, $\gamma = 0.8$ and $\varphi_p = 0$. $\gamma = 0.8$ represents the case that the vibration is near the primary resonance for $(2\pi / \Omega_p)$-periodic forced excitations. The time interval in this figure is $2 \times (2\pi / \Omega_0)$. One finds that the analytical solution coincides with the numerical result for linear vibration very well. There is a difference between analytical solution and numerical result of nonlinear vibration because of the nonlinear effects. The dynamic response is not a periodic motion obviously.

The primary resonant response ($\gamma = 1$) is given in figure 5. The parameters are taken to be $\varepsilon = 0.10$, $\zeta = 0.005$ and $\varphi_p = -\pi/2$. The time interval is $(2\pi / \Omega_0)$. It shows that analytical solution and two numerical results don’t coincide with each other well. The periodic property of analytical solution is the same with that of linear vibration. However, due to nonlinearities, the periodic property and amplitude of analytical result do not coincide with those of nonlinear vibration very well.

Dynamic response for a large perturbed frequency is presented in figure 6. The time interval is taken to be $(2\pi / \Omega_0)$. The parameters are selected as $\varepsilon = 0.10$, $\zeta = 0.005$, $\gamma = 3$ and $\varphi_p = 0$. One finds that the analytical solution coincides with linear vibration very well. However, there is a difference in amplitudes between the analytical result and nonlinear vibration that arises from nonlinear effects. The period of blade motion is $(2\pi / \Omega_0)$. So $(2\pi / \Omega_p)$-harmonic terms are very tiny in dynamic displacement for large perturbed frequency.

5. Conclusion

In this work, the nonlinear in-plane motion of a wind turbine blade under the combined actions of parametric and forced excitations was analyzed. The Bernoulli-Euler beam theory was adopted to describe the vibration of the slender blade. Gravity, structural damping and large deformation were included in mathematical model. The rotating speed was assumed to be a nominal speed with a cosine oscillation. The harmonic balance method was used to get the approximate solution of the dynamic response by selecting the fundamental harmonic terms of nominal rotating angular frequency and perturbed frequency. Influences of structural damping, perturbed frequency and perturbed amplitude on blade motion were discussed. The following conclusions were derived: (1) the harmonic oscillation of rotating speed brings about time-dependent stiffness, periodic-changing aerodynamic damping, and forced excitations; (2) for small perturbed frequency, the influence of rotating speed oscillation is mainly in generating forced excitations; harmonic amplitude increases with increasing perturbed amplitude and decreasing structural damping; (3) when perturbed frequency is near natural frequency, the speed oscillation can introduce the primary resonance; in this case harmonic amplitudes depend on structural damping and perturbed amplitude sensitively; (4) for large perturbed frequency, harmonic terms corresponding to rotating speed oscillation are very tiny; effects of structural damping and perturbed amplitude on blade motion are little.

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Appendix A. Coefficients of discrete model

\[
\begin{align*}
    d_1 &= \left[ p (p - 1) \sigma a / 32 \right] \left( r_i / h_0 \right)^2 & d_2 &= -\left( p \sigma a / 8 \right) \left( r_i / h_0 \right) \\
    d_3 &= 1 + \left[ p (p - 1) / 4 \right] \left( r_i / h_0 \right)^2 & d_4 &= -p \left( r_i / h_0 \right) \\
    \bar{m} &= \int_0^L \left( m + \rho a c^2 \sin^2 \theta / 8 \right) \gamma^2 dx & \bar{m} &= \frac{1}{m} \int_0^L m y^2 dx
\end{align*}
\]
\[
c_a = \frac{1}{m} \int_{0}^{L} \rho acx(C_{do}/a)\gamma^2dx + \frac{1}{m} \int_{0}^{L} \left(\rho ac/2\right) R \sin(\theta_i + \lambda_o d_\alpha)\gamma^2dx
\]
\[
c_{ai} = \frac{\pi}{m} \int_{0}^{L} \left( I_{\gamma}^a \right)^* \gamma dx
\]
\[
c_{sz} = -\frac{\pi}{m} \int_{0}^{L} \left( A_{\gamma}^{sz} \right)^* \gamma dx
\]

\[
b = -\frac{1}{2m} \int_{0}^{L} \left( EA_{\gamma}^{sz} \right)^* \gamma dx
\]
\[
b = -\frac{1}{m} \int_{0}^{L} \left[ \gamma \int_{0}^{L} m(e + y \cos \beta) dy \right] \gamma dx
\]
\[
g = -\frac{1}{m} \int_{0}^{L} mg \gamma dx
\]

\[
a_s = \frac{1}{m} \int_{0}^{L} me_{x\gamma} dx - \frac{1}{m} \int_{0}^{L} \left( \rho ac/2 \right) (C_{do}/a) x^2 \gamma dx + \frac{1}{m} \int_{0}^{L} \left( \rho ac/2 \right) R^2 \cos \theta_0^2 \left( d_i^2 + \frac{d_\alpha^2}{2} \right) \gamma dx
\]
\[
- \frac{1}{m} \int_{0}^{L} \left( \rho ac/2 \right) \left[ (1-\beta^2) R x \sin \theta + Re_x \sin \theta \right] (d_i + \lambda_o d_\alpha) \gamma dx
\]
\[
+ \frac{1}{m} \int_{0}^{L} \left( \rho ac/2 \right) R^2 \cos \theta \left[ (d_i^2 + \frac{d_\alpha^2}{2}) + \lambda_o (2d_i d_\alpha + d_\alpha d_\alpha) \right] \gamma dx
\]

\[
f = \frac{1}{m} \int_{0}^{L} \rho ac R^2 \cos \theta \left[ d_i d_\alpha + \lambda_o (d_i d_\alpha + d_\alpha d_\alpha) + \lambda_o^2 d_\alpha d_\alpha \right] \gamma dx
\]
\[
- \frac{1}{m} \int_{0}^{L} \frac{\rho ac}{2} \left[ (1-\beta^2) x^2 + e^2 \right] R \sin \theta (d_i + \lambda_o d_\alpha) \gamma dx
\]

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