Enhancement of ferromagnetism by p-wave Cooper pairing in superconducting ferromagnets

Xiaoling Jian, Jingchuan Zhang, and Qiang Gu
Department of Physics, University of Science and Technology Beijing, Beijing 100083, China

Richard A. Klemm
Department of Physics, University of Central Florida, Orlando, Florida 32816, USA
(Dated: October 20, 2009)

In superconducting ferromagnets for which the Curie temperature \( T_c \) exceeds the superconducting transition temperature \( T_c \), it was suggested that ferromagnetic spin fluctuations could lead to superconductivity with p-wave spin triplet Cooper pairing. Using the Stoner model of itinerant ferromagnetism, we study the feedback effect of the p-wave superconductivity on the ferromagnetism. Below \( T_c \), the ferromagnetism is enhanced by the p-wave superconductivity. At zero temperature, the critical Stoner value for itinerant ferromagnetism is reduced by the strength of the p-wave pairing potential, and the magnetization increases correspondingly. More important, our results suggest that once Stoner ferromagnetism is established, \( T_m \) is unlikely to ever be below \( T_c \). For strong and weak ferromagnetism, three and two peaks in the temperature dependence of the specific heat are respectively predicted, the upper peak in the latter case corresponding to a first-order transition.

PACS numbers: 71.10.-w, 71.27.+a, 75.10.LP

Due to the strong interplay between conventional superconducting (SC) and ferromagnetic (FM) states, the exploration of their possible coexistence in the same crystal might have seemed fruitless, but has nevertheless attracted a great deal of interest recently. This possible coexistence was first proposed by Ginzburg more than 50 years ago. Several years later, Larkin and Ovchinnikov and Fulde and Ferrell independently developed a microscopic theory of this coexistence in the presence of a strong magnetic field, based upon a spatially inhomogeneous SC order parameter, presently referred to as the FFLO state. Meanwhile, Berk and Schrieffer suggested that conventional s-wave superconductivity in the paramagnetic phase above the Curie temperature \( T_m \) is suppressed by critical ferromagnetic fluctuations near to \( T_m \). However, more recent calculations showed that conventional s-wave superconductivity can form in the weakly FM regime close to a quantum phase transition. In addition, Fay and Appel predicted that p-wave superconductivity could arise in itinerant ferromagnets. Their pioneering work indicated that longitudinal ferromagnetic spin fluctuations could result in a p-wave “equal-spin-pairing” SC state within the FM phase.

Experimentally, a major development occurred with the observation by Saxena et al. that UGe\(_2\), nominally an itinerant FM compound, undergoes an SC transition at low \( T_c \) values under high pressure. An SC state was also found in other itinerant ferromagnets such as ZrZn\(_2\) and URhGe\(_2\). In each case, the regime of the SC phase appears completely within that of the FM phase, suggesting a cooperative effect between the SC and FM states.

These experimental achievements have stimulated renewed theoretical interest in the subject. Recently, a large effort has been devoted to the understanding of the underlying physics of the coexisting SC and FM states, with a focus upon the SC pairing mechanism and the orbital symmetry of the SC order parameter. Although earlier works by Suhl and Abrikosov suggested that an s-wave pairing interaction between conduction electrons could be mediated by ferromagnetically-ordered localized spins, such as by impurities, recent studies of these SC ferromagnets have assumed that the itinerant electrons involved in both the FM and SC states are within the same band. Some of these studies assumed conventional s-wave pairing. For example, Karchev et al. studied an itinerant electron model in which the same electrons are responsible for both the FM and SC states. In that study, the Cooper pairs were assumed to be in a spin-singlet state, and the ferromagnetism was described within the Stoner model. However, the resulting SC ferromagnetic state was shown to be energetically unfavorable when compared to the conventional, nonmagnetic SC state. A possible exception to this incompatibility could occur if the magnetic instability were to arise from a dynamic spin exchange interaction, as discussed by Cuoco et al. On the other hand, a number of other workers avoided the likely incompatibility of the SC and FM states by assuming a spin-triplet SC order parameter with p-wave orbital symmetry, for simplicity. Kirkpatrick et al. indicated that a p-wave SC state meditated by ferromagnetic spin fluctuations is more likely to coexist within the Heisenberg FM phase regime than within the paramagnetic phase regime. Machida and Ohmi studied the properties of a p-wave SC ferromagnet phenomenologically. More recently, a microscopic model of the coexistence of a nonunitary spin-triplet SC state with a weakly itinerant FM state was developed by Nevidomskyy. The present nature of the SC coexistent with the FM state in these ferromagnetic superconductors is still somewhat controversial, although increasingly, additional experiments on the U-based materials have provided increasing

\( T_c \)

\( T_m \)
support for a spin-triplet state rather than a spin-singlet one.\textsuperscript{18,19,20,21}

Most theoretical studies have focused primarily on the
effect of the established ferromagnetism upon the
ature of the coexistent superconductivity, as summarized
above. However, to fully understand the interplay be-
tween the SC and FM states when they coexist, one
should also study the feedback effect of the superconduc-
tivity upon the ferromagnetism itself, as has been done
in only one study to date\textsuperscript{17}.

Here we study explicitly the effects of the p-wave pair-
ing on the FM ordering, using the Stoner model of itin-
erant ferromagnetism as the starting point. We cal-
culate the critical Stoner parameter $U_c$, the magneti-
ization $m$, and the two parallel-spin p-wave gap func-
tion magnitudes, $\Delta_{\pm}$, respectively, as functions of the
pair-interaction strength $V$. We also discuss finite-
temperature properties, including the $T$-dependencies of
these order parameters and the specific heat $C(T)$.

We take the Hamiltonian for the ferromagnetic super-
conductor to have the form

$$H_{FM+SC} = \sum_{k,\sigma} (\epsilon_k - \mu - \sigma M) c_{k\sigma}^{\dagger} c_{k\sigma} + \frac{1}{2V} \sum_{k,k',\sigma,\sigma'} V_{SC}(k,k') c_{k,\sigma}^{\dagger} c_{-k',\sigma'} c_{-k',\sigma} c_{k\sigma},$$

where $\sigma = \pm$ represent the single-particle spin states,
and the single-quasiparticle part of $H$ comprises the
Stoner model for itinerant electrons, where $\epsilon_k$ is the
non-magnetic part of the quasiparticle dispersion, $\mu$ is the
chemical potential and $M = U(\langle n_+ \rangle - \langle n_- \rangle)/2$ is the
magnetic molecular-field with $U$ the Stoner exchange in-
teraction, and $V$ is the sample volume. The pairing
potential is taken to have the $p$-wave form\textsuperscript{22}, $V_{SC}(k,k') = -V \mathbf{k} \cdot \mathbf{k}'$. In weak coupling theory, $V$ is non-zero and
assumed to be constant only within the narrow energy
region $|\epsilon - \epsilon_F| \leq \omega_c$ near to the Fermi energy $\epsilon_F$, where
$\omega_c$ is the energy cut-off.

Because of the pair-breaking effects of the strong
exchange field in ferromagnets, we assume that only
parallel-spin Cooper pairs can survive. Thus we set the
p-wave antiparallel-spin gap function $\Delta_0 = 0$ and retain
the two gap functions with parallel-spin states $m_S = \pm 1$,
$\Delta_{\pm}$. The SC order parameter is assumed to have the
following p-wave symmetry\textsuperscript{22}, $\Delta_{\pm}(k) = (\mathbf{k} \times \pm \mathbf{i} \mathbf{k}_y) \Delta_{\pm}$.

The Hamiltonian is treated via the Green function
method within the mean-field theory framework. In ad-
tion to the normal Green function $G_{\pm}(k, \tau - \tau') = -\langle T_{\tau'} c_{k\sigma}(\tau) c_{-k\sigma}^{\dagger}(\tau') \rangle$, the anomalous Green function
describing the pairing of electrons should be introduced,
$\tilde{G}_{\pm}(k, \tau - \tau') = \langle T_{\tau'} c_{k\sigma}(\tau) c_{-k\sigma}(\tau') \rangle$. Using the standard
equation of motion approach, the Green functions are

derived to be

$$G_{\pm}(k, ip_n) = \frac{-ip_n + \epsilon_k + M}{p_n^2 + (\epsilon_k + M)^2 + |\Delta_{\pm}(k)|^2},$$

$$\tilde{G}_{\pm}(k, ip_n) = \frac{\Delta_{\pm}(k)}{p_n^2 + (\epsilon_k + M)^2 + |\Delta_{\pm}(k)|^2},$$

where the $p_n$ are the Matsubara frequencies, and the FM
and SC order parameters are respectively defined as

$$M = \frac{U}{2V} \sum_k \langle \langle n_{k+} \rangle - \langle n_{k-} \rangle \rangle,$$

$$\Delta_{\pm}(k) = -\frac{1}{V} \sum_{k'} V_{SC}(k,k') \tilde{G}_{\pm}(k', \tau = 0).$$

All of the order parameters can be calculated using the
above Green functions. They are found to satisfy

$$M = \frac{U}{2V} \sum_k \left\{ \frac{\epsilon^\uparrow_k [1 - 2f(E_{-k})]}{2E_{-k}(k)} - \frac{\epsilon^\downarrow_k [1 - 2f(E_{+k})]}{2E_{+k}(k)} \right\},$$

$$\Delta_{\pm}(k) = \frac{-1}{V} \sum_{k'} V_{SC}(k,k') \frac{1 - 2f(E_{\pm k}(k'))}{2E_{\pm k}(k')} \Delta_{\pm}(k'),$$

where $\epsilon^{\uparrow,\downarrow}_k = \epsilon_k - \mu \pm M$, $E_{\pm}(k) = \sqrt{(\epsilon^{\uparrow,\downarrow}_k)^2 + |\Delta_{\pm}(k)|^2}$, and $f(E)$ is the Fermi function. The chemical potential
$\mu$ is determined from the equation for the number of
electrons per unit volume, or particle density,

$$n = \frac{1}{V} \sum_k \left\{ 1 - \frac{\epsilon^\downarrow_k [1 - 2f(E_{-k})]}{2E_{-k}(k)} - \frac{\epsilon^\uparrow_k [1 - 2f(E_{+k})]}{2E_{+k}(k)} \right\},$$

which is equal to unity at half filling.

Equations (4), (5) and (6) with $n = 1$ comprise the self-
consistent equations for the ferromagnetic superconduct-
ning system. We solve the equations for the simple case of
a spherical Fermi surface at half filling. It is convenient
to solve these equations by converting the summations
over $k$-space to continuum integrals over energy,

$$M = \frac{U}{32\pi^2} \int_0^\infty d\epsilon \int_0^\pi d\theta \sin \theta \sqrt{\epsilon} \left\{ \frac{\epsilon^\uparrow \tanh \frac{\epsilon^\uparrow}{2T}}{E_{-}} - \frac{\epsilon^\downarrow \tanh \frac{\epsilon^\downarrow}{2T}}{E_{+}} \right\},$$

$$1 = \frac{V}{32\pi^2} \int_{\pi_{\pm}}^{\pi} d\epsilon \int_0^\pi d\theta \times \left\{ \frac{\epsilon^\uparrow \sin^3 \theta}{E_{-}} \tanh \frac{E_{\pm}}{2T} \right\},$$

$$n = \frac{1}{16\pi^2} \int_0^\infty d\epsilon \int_0^\pi d\theta \sin \theta \sqrt{\epsilon} \times \left\{ 2 - \frac{\epsilon^\uparrow \tanh \frac{\epsilon^\uparrow}{2T}}{E_{-}} - \frac{\epsilon^\downarrow \tanh \frac{\epsilon^\downarrow}{2T}}{E_{+}} \right\}.$$
where $\tau_{F\pm} = \overline{\tau} \pm \overline{M}$, $\overline{\tau}^{\uparrow\downarrow} = \overline{\tau} - \tau_{F\uparrow}$, and $\overline{E}_{\pm} = \sqrt{(\overline{\tau}^{\uparrow\downarrow})^2 + \sin^2 \theta \overline{\Delta}_K^2}$. In the above equations, the unit of energy is rescaled by the factor $\frac{\hbar^2 n^{2/3}}{2m}$. The dimensionless interactions $\overline{U}$ and $\overline{V}$ are thus defined by $\overline{U} = U(\frac{\hbar^2 n^{2/3}}{2m})^{-1}$ and $\overline{V} = V(\frac{\hbar^2 n^{2/3}}{2m})^{-1}$, and the dimensionless energies $\tau_{F\pm}$, $\overline{\tau}$, $\overline{E}_{\pm}$, $\overline{\Delta}_\pm$ and $\overline{\tau}$ are defined analogously. The dimensionless temperature is defined by $\overline{T} = k_B T(\frac{\hbar^2 n^{2/3}}{2m})^{-1}$. We choose $\overline{\tau}_c = 0.01 \overline{\tau}_F$, where $\overline{\tau}_F$ is the dimensionless Fermi energy at $\overline{M} = \overline{T} = 0$.

By solving the equations self-consistently, we can investigate the interplay between the magnetism and the superconductivity in the coexisting state. This issue was discussed previously based on a similar framework, with the emphasis placed on the effects on the SC pairing due to the critical spin fluctuations in FM compounds[15]. The present work focuses on the reciprocal action, i.e., the influence of the SC on the FM.

According to Stoner theory, a Fermi gas can exhibit ferromagnetism only when the effective FM exchange is larger than the critical Stoner point. For a system described by Eq. (1), $U$ represents the effective exchange interaction. In the absence of the $p$-wave SC interaction, $\overline{V} = 0$, the dimensionless Stoner point $\overline{U}_c(0) = 12.76104$. For $\overline{V} \neq 0$, we calculate $\overline{U}_c(\overline{V})$. As shown in Fig. 1 the $\overline{T} = 0$ Stoner point $\overline{U}_c(\overline{V})$ decreases as $\overline{V}$ increases, which implies that the $p$-wave Cooper pairing reduces the barrier to the onset of the magnetization of the Fermi gas. We note that $\overline{V}$ might be very small in a real system, so the enhancement effect of the superconductivity on the ferromagnetism may be very weak. The inset of Fig. 1 shows the details of $\overline{U}_c(\overline{V})$ in the region of small $\overline{V}$, where the decreasing tendency of $\overline{U}_c(\overline{V})$ with increasing $\overline{V}$ still can be seen clearly.

To further demonstrate the influence of the SC on the FM, we discuss the magnetization $m \equiv \langle n_+ \rangle - \langle n_- \rangle$ as a function of $\overline{V}$ at $\overline{T} = 0$. Here we use $m = 2M/U$ instead of $M$ to eliminate the dependence of $\overline{U}_c$ upon $\overline{V}$. As shown in Fig. 2 $m(\overline{V})$ increases with $\overline{V}$ for each given value of $\overline{U}$. For $\overline{U} > \overline{U}_c(0)$, $m(0)$ is finite, since the system is spontaneously magnetized, and $m(\overline{V})$ increases monotonically from $m(0)$, eventually reaching unity at a finite $\overline{V} \approx 2300$. For $\overline{U} < \overline{U}_c(0)$, however, $m(\overline{V}) = 0$ for $\overline{V} < \overline{U}_c(\overline{V})$, and then $m(\overline{V}) \neq 0$ increases sharply with $\overline{V}$ for $\overline{V} \geq \overline{U}_c(\overline{V})$, eventually reaching unity at $\overline{V} \approx 2300$. The critical value $\overline{V}_c(\overline{U})$ corresponds to the reduction in the Stoner point $\overline{U}_c(\overline{V})$ at which the onset of the ferromagnetism is induced, as pictured in Fig. 1. This is a second way in which the $p$-wave superconductivity can enhance the ferromagnetism.

A similar effect was found in the ferromagnetic spin-1 Bose gas which exhibits two phase transitions, the FM transition and Bose-Einstein condensation (BEC). The BEC temperature increases with FM couplings and, on the other hand, the FM transition is significantly en-

![FIG. 1: The Stoner point $\overline{U}_c(\overline{V})$ as a function of the $p$-wave interaction strength $\overline{V}$ at $\overline{T} = 0$. Inset: Enlargement of the region $0 \leq \overline{V} \leq 500$.](image1)

![FIG. 2: Plots of the electronic magnetization $m \equiv \langle n_+ \rangle - \langle n_- \rangle$ as a function of the $p$-wave interaction strength $\overline{V}$ at $\overline{T} = 0$ for fixed values of $\overline{U}$. From larger to smaller $m$ at fixed $\overline{V}$, $\overline{U} = 12.8$ (short dotted), 12.77 (dashed), 12.761 (solid), 12.743 (dotted), 12.7 (dash-dotted) and 12.495 (short dashed). Inset: Enlargement of the region $0 \leq \overline{V} \leq 500$.](image2)

![FIG. 3: Plots of $\overline{\Delta}_\pm$ (dashed) and $\overline{\Delta}_-\pm$ (solid) as functions of $\overline{V}$ at $\overline{U} = 12.77$ and $\overline{T} = 0$. $\overline{V}_A$ is the value of $\overline{V}$ at which $\Delta_\pm$ has a maximum, and $\overline{V}_A \to 0$ at $\overline{V} \to \sim 2300$, the point at which $m \to 1$ in Fig. 2](image3)
FIG. 4: Shown are plots of the order parameters $\overline{M}$ (dotted), $\overline{\Delta}_+$ (dashed), and $\overline{\Delta}_-$ (solid) as functions of $T$ in the coexistence state for $\overline{\nabla} = 300$. $\overline{M}$ (dash-dotted) is the magnetic order parameter when $\overline{\nabla} = 0$. (a) $\overline{U} = 12.79 > \overline{U}_e(0)$ and $\overline{T}_m > \overline{T}_{c+}$. (b) $\overline{U} = 12.77 > \overline{U}_e(0)$ but $0 < \overline{T}_m < \overline{T}_{c+}$. (c) $\overline{U} = 12.76 < \overline{U}_e(0)$ but $\overline{U} > \overline{U}_e(\overline{\nabla})$. The ferromagnetism is induced due to the $p$-wave pairing ($\overline{M} \neq 0$) even though $\overline{M} = 0$.

hanced due to the onset of the BEC$^a$. Considering that triplet Cooper pairs behave somewhat like spin-1 bosons, a FM superconductor is analogous to a FM Bose gas.

Figure 4 displays plots of the $p$-wave SC order parameters, $\overline{\Delta}_\pm$ as functions of $\overline{\nabla}$ at $\overline{U} = 0$ and $\overline{\nabla} = 12.77$, just above the $\overline{U} = 0$ Stoner point $\overline{U}_e(0)$. Although with increasing $\overline{\nabla}$, $\overline{\Delta}_+$ rises monotonically, $\overline{\Delta}_-$ initially rises, reaches a maximum at $\overline{U}_A$, and then decreases at an increasing rate until it vanishes discontinuously when $m(\overline{\nabla}) = 1$. For $\overline{U} = 12.77$, $m(\overline{\nabla}) > 0$, which is shown by the dashed curve in Fig. 2 so that $\overline{\Delta}_+ > \overline{\Delta}_-$ for all $\overline{\nabla}$. Since $m$ also grows with $\overline{\nabla}$, the mean number of spin-down electrons decreases with increasing $\overline{\nabla}$, vanishing when $m \to 1$ at $\overline{\nabla} \approx 2300$, at and beyond which $\overline{\Delta}_- \to 0$.

We now discuss the finite temperature properties of the system. We define $\overline{M}$ to be the magnetic order parameter when $\overline{\nabla} = 0$, for which $\overline{\Delta}_- = 0$. The $\overline{T}$ dependencies of the order parameters $\overline{\Delta}_\pm$, $\overline{\Delta}_\pm$, and $\overline{M}$ are obtained numerically and shown for $\overline{\nabla} = 300$ and three different $\overline{U}$ cases in Fig. 4. The order parameters become non-vanishing below their respective dimensionless transition temperatures $\overline{T}_{c+}$, $\overline{T}_m$, and $\overline{T}_m'$. In each case, the SC order parameters $\overline{\Delta}_\pm$ increase monotonically with decreasing $\overline{T}$ below $\overline{T}_{c+}$, respectively. In the FM superconductor, $\overline{T}_{c-} < \overline{T}_{c+}$ and $\overline{\Delta}_-(\overline{T}) < \overline{\Delta}_+(\overline{T})$, as shown in Figs. 4(a), 4(b), and 4(c). In addition, $\overline{M}(\overline{T})$ also increases monotonically with decreasing $\overline{T}$ for the ferromagnet in the absence of any superconductivity, as depicted in Figs. 4(a) and 4(b) for the respective cases $\overline{U} > \overline{U}_e(0)$ and $\overline{T}_m' > \overline{T}_{c+}$ and $0 < \overline{T}_m < \overline{T}_{c+}$. However, the $\overline{T}$-dependence of $\overline{M}$ is non-trivial when $p$-wave superconductivity is present. In the first case pictured in Fig. 4(a), $\overline{M}(\overline{T}) = \overline{M}(\overline{T})$ for $\overline{T}_m > \overline{T}_{c+}$, as in the absence of superconductivity. However, $\overline{M}(\overline{T})$ exhibits an upward kink at $\overline{T}_{c+}$ below which $\overline{\Delta}_+ \neq 0$. Then, for $\overline{T}_{c-} < \overline{T} < \overline{T}_{c+}$, $\overline{M}$ increases sharply with decreasing $\overline{T}$, and exhibits a downward kink at $\overline{T}_{c-}$ below which $\overline{\Delta}_- \neq 0$. Below $\overline{T}_{c-}$, $\overline{M}(\overline{T})$ then decreases monotonically with $\overline{T}$. This case was discussed previously in a similar scenario$^b$.

The case $\overline{T}_m < \overline{T}_{c+}$ not previously discussed is more interesting. Two examples of this case with $\overline{\nabla} = 300$ are shown in Figs. 4(b) and 4(c). In Fig. 4(b), the magnetization $\overline{M}$ for $\overline{\nabla} = 0$ and $\overline{\Delta}_\pm = 0$ is so weak that $0 < \overline{T}_m < \overline{T}_{c-}$, but a non-vanishing $\overline{\nabla}$ enhances the magnetization, $\overline{M}$, causing the actual dimensionless Curie temperature $\overline{U}_m$ to equal $\overline{T}_{c+}$, below which both $\overline{\Delta}_+(\overline{T})$ and $\overline{M}(\overline{T})$ become continuously non-vanishing, signaling a first-order transition. Their behaviors for $\overline{T} < \overline{T}_{c+} = \overline{T}_m$ are then qualitatively similar to that shown in Fig. 4(a), with $\overline{\Delta}_-(\overline{T}) \neq 0$ for $\overline{T} < \overline{T}_{c-}$, causing a downward kink in $\overline{M}(\overline{T})$ at $\overline{T}_{c-}$, below which $\overline{M}(\overline{T})$ decreases monotonically with $\overline{T}$. For the more extreme case when $\overline{U} < \overline{U}_e(0)$ and $\overline{T}_m = 0$ but $\overline{U} > \overline{U}_e(\overline{\nabla})$ depicted in Fig. 4(c), the behaviors of the three order parameters are very similar to that shown in Fig. 4(b).

Considering that $\overline{\nabla}$ is usually small in real systems, a case with $\overline{\nabla} = 20$ is checked, as shown in Fig. 5 where $\overline{U}$ is taken to be 12.761, slightly lower than $\overline{U}_e(0)$ but larger than $\overline{U}_e(20) \approx 12.7608$. Fig. 5 looks very similar.
to Fig. 4(c).

Although we did not investigate the limit $\mathbf{V} \to 0+$, the examples with $\mathbf{V} = 300$ and $\mathbf{V} = 20$ of the case $\mathbf{T}_{m} < \mathbf{T}_{c+}$ pictured in Figs. 4(b), 4(c) and Fig. 5 suggest that in FM superconductors, the actual Curie temperature $\mathbf{T}_{m}$ is unlikely to ever be lower than the upper SC transition temperature $\mathbf{T}_{c+}$, even if the FM order were extremely weak. In other words, these examples argue against the possibility of a FM $\mathbf{T}$ regime inside the $p$-wave triplet SC regime, with an actual $\mathbf{T}_{m} < \mathbf{T}_{c+}$. Analogously, it was shown that the ferromagnetic transition never occurs below the Bose-Einstein condensation in the FM spin-1 Bose gas. Moreover, the present results are to some extent consistent with the observed phase diagrams of UGe$_2$ and ZrZn$_2$, and with the theoretical discussion of Walker and Samokhin, who argued that the superconductivity only occurs within the FM region. In addition, this scenario is consistent with de Haas van Alphen experiments under pressure on UGe$_2$.

However, very recent experiments on UCoGe under pressure were interpreted as potentially having such a FM regime inside the SC regime near to the FM quantum critical point. However, the $dc$ resistance and $ac$ susceptibility measurements of $T_m$ and $T_{c+}$ could not determine if there was a FM regime inside the SC one for pressures just below their extrapolated quantum critical pressure $p_{c}$, allowing for a first-order phase transition at the point when $T_m = T_{c+}$, beyond which only a parallel-spin triplet state exists. Further experiments are encouraged to determine if the FM and SC phases regimes with $0 < T_m < T_{c+}$ at fixed pressure actually exist in UCoGe.

As suggested by the results for the temperature dependencies of the order parameters, the FM superconducting system shows multiple phase transitions, which can be determined experimentally from measurements of the specific heat. The specific heat at constant volume for our model can be calculated from

$$C(\mathbf{T}) = \mathbf{T}\frac{\partial S}{\partial \mathbf{T}},$$

where the electronic contribution to the entropy $S$ can be derived from

$$S = -\sum_{k,\sigma=\pm} \{f(E_{\sigma})\ln f(E_{\sigma})
+ [1 - f(E_{\sigma})]\ln[1 - f(E_{\sigma})]\}.$$ 

The specific heat was calculated previously based on a model of $s$-wave superconductivity coexisting with ferromagnetism. For $s$-wave superconductors, there is only one SC transition temperature $\mathbf{T}_{c}$, at which there is a jump in the specific heat at the second order transition. However, the case of a $p$-wave superconductor coexisting with ferromagnetism is more interesting. In Figs. 6(a) and 6(b), the results for the specific heat corresponding to the cases pictured in Figs. 4(a) and 4(b) for the order parameters are shown. For the case $\mathbf{U} > \mathbf{U}_{c}$ pictured in Figs. 6(a) and 6(b), there are three phase transitions at the temperatures $\mathbf{T}_{c-} < \mathbf{T}_{c+} < \mathbf{T}_{m}$. In Fig. 6(b), an example of the case $\mathbf{T}_{m} < \mathbf{T}_{c+}$ when $\mathbf{V} = 0$ pictured in Fig. 4(b) is shown. In this case with $\mathbf{V} = 300$, there is a first-order phase transition at $\mathbf{T}_{m} = \mathbf{T}_{c+}$, and a second-order phase transition at $\mathbf{T}_{c-}$.

In conclusion, it is shown that $p$-wave triplet Cooper pairing can enhance the ferromagnetism in superconducting ferromagnets. This enhancement is most prominent for the magnetic exchange interaction $U$ very near to the Stoner point $\mathbf{U}_{c}(-\mathbf{0})$, the critical value for the strength of the exchange interaction required for the onset of ferromagnetism in the absence of the $p$-wave pairing interaction $V$. With finite $V$, $\mathbf{U}_{c}(V)$ is reduced and the ferromagnetic order parameter increases in magnitude with increasing $V$. The temperature dependencies of the magnetic and parallel-spin superconducting order parameters and of the specific heat are calculated. The results show that the Curie temperature is unlikely to ever be lower than the upper SC transition temperature, in agreement with pressure measurements on UGe$_2$. This feature also may be relevant to recent experiments on UCoGe. The temperature dependence of the specific heat exhibits two peaks for weak ferromagnetism in the coexistence state, with a first-order transition at the combined ferromagnetic and upper $p$-wave SC transition, and a lower second-order $p$-wave SC transition for strong ferromag-
netism, the specific heat exhibits three second-order transitions. Our results support the possible coexistence of $p$-wave superconductivity with a ferromagnetic state.

This work was supported by the Key Project of the Chinese Ministry of Education (No. 109011), the Fok Yin-Tung Education Foundation, China (No. 101008), and the NCET program of Chinese Ministry of Education (NCET-05-0098). X. J. would like to thank Jihong Qin for helpful discussions.

* Electronic address: qgu@sas.ustb.edu.cn

1. V. L. Ginzburg, Zh. Eksp. Teor. Fiz. 31, 202 (1956)[Sov. Phys. JETP 4, 153 (1957)].
2. A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47, 1136 (1964) [Sov. Phys. JETP 20, 762 (1965)].
3. P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964).
4. N. K. Berk and J. R. Schrieffer, Phys. Rev. Lett. 17, 433 (1966).
5. K. B. Blagoev, J. R. Engelbrecht and K. S. Bedell, Phys. Rev. Lett. 82, 133 (1999).
6. D. Fay and J. Appel, Phys. Rev. B 22, 3173 (1980).
7. S. S. Saxena, P. Agarwal, K. Ahilan, F. M. Grosche, R. K. W. Haselwimmer, M. J. Steiner, E. Pugh, I. R. Walker, S. R. Julian, P. Monthoux, G. G. Lonzarich, A. Huxley, I. Sheikin, D. Braithwaite and J. Flouquet, Nature (London) 406, 587 (2000).
8. C. Pfleiderer, M. Uhlarz, S. M. Hayden, R. Vollmer, H. v. Löhneysen, N. R. Bernhoeft and G. G. Lonzarich, Nature (London) 412, 58 (2001).
9. D. Aoki, A. Huxley, E. Ressouche, D. Braithwaite, J. Flouquet, J.-P. Brison, E. Lhotel and C. Paulsen, Nature (London) 413, 613 (2001).
10. H. Suhl, Phys. Rev. Lett. 87, 167007 (2001).
11. A. Abrikosov, J. Phys.: Condens. Matter 13, L943 (2001).
12. N. I. Karchev, K. B. Blagoev, K. S. Bedell and P. B. Littlewood, Phys. Rev. Lett. 86, 846 (2001).
13. R. Shen, Z. M. Zheng, S. Liu and D. Y. Xing, Phys. Rev. B 67, 024514 (2003).
14. M. Cuoco, P. Gentile, and C. Noce, Phys. Rev. Lett. 91, 197003 (2003).
15. T. R. Kirkpatrick, D. Belitz, T. Vojta, and R. Narayanan, Phys. Rev. Lett. 87, 127003 (2001); T. R. Kirkpatrick and D. Belitz, *ibid*, 92, 037001 (2004).
16. K. Machida and T. Ohmi, Phys. Rev. Lett. 86, 850 (2001).
17. A. H. Nevidomskyy, Phys. Rev. Lett. 94, 097003 (2005).
18. A. Huxley, I. Sheikin, E. Ressouche, N. Kernanbnos, D. Braithwaite, R. Calemczuk, and J. Flouquet, Phys. Rev. B 63, 144519 (2001).
19. F. Hardy and A. D. Huxley, Phys. Rev. Lett. 94, 247006 (2005).
20. A. Harada, S. Kawasaki, H. Mukuda, Y. Kitaoka, Y. Haga, E. Yamamoto, Y. Onuki, K. M. Itoh, E. E. Haller, and H. Harima, Phys. Rev. B 75, 140502 (2007).
21. N. T. Huy, A. Gasparini, D. E. de Nijs, Y. Huang, J. C. P. Klaasse, T. Gortenmulder, A. de Visser, A. Hamann, T. Görlach, and H. v. Löhneysen, Phys. Rev. Lett. 99, 067006 (2007).
22. G. D. Mahan, many-particle physics (Kluwer Academic/Plenum Publishers, New York, 1990), Chap.10.
23. Q. Gu and R. A. Klemm, Phys. Rev. A 68, 031604 (2003); Q. Gu, K. Bongs, and K. Sengstock, *ibid*, 70, 063609 (2004).
24. H. P. Dahal, J. Jackiewicz and K. S. Bedell, Phys. Rev. B 72, 172506 (2005).
25. M.B. Walker and K.V. Samokhin, Phys. Rev. Lett. 88, 207001 (2002).
26. J. Jackiewicz, K. B. Blagoev and K. S. Bedell, Philos. Mag. 83, 3247 (2003).
27. T. Terashima, T. Matsumoto, C. Terakura, S. Uji, N. Kimura, M. Endo, T. Komatsubara, and H. Aoki, Phys. Rev. Lett. 87, 166401 (2001).
28. E. Slooten, T. Naka, A. Gasparini, Y. K. Huang, and A. de Visser, Phys. Rev. Lett. 103, 097003 (2009).