ON THE DEFICIENCY OF 8–10 DAY GALACTIC CEPHEIDS

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ABSTRACT

The Galactic Cepheid period histogram has a strong dip between 8 and 10 days that has defied an explanation based on evolutionary and linear pulsation studies. We show here that this deficiency is caused by the instability of the nonlinear fundamental pulsation cycle in this period range. The strong metallicity dependence of this instability is consistent with the absence of a corresponding minimum in the Magellanic Cloud data. Our results also suggest that the Galactic Cepheids must have a large spread in metallicity.

Subject headings: Cepheids — hydrodynamics — instabilities — Magellanic Clouds — stars: oscillations (including pulsations)

It is a well-known fact that the observed period distribution of the Galactic Cepheids has a pronounced minimum in the 8–10 day period range, as shown in Figure 1, which displays the histogram for the period distribution of the Galactic Cepheids constructed from the Galactic Cepheid database of Ferron et al. (1995). Since it is difficult to separate overtone and fundamental pulsators for the Galaxy above 5 days, our histogram necessarily contains both the fundamental and the overtone Cepheids.

Becker, Iben, & Tuggle (1977) combined the location and duration of Cepheid model crossings of the instability strip from evolutionary calculations with a birthrate function to infer a theoretical period histogram. One of their conclusions was that a minimum in the distribution was not compatible with a standard birthrate function, and could only be explained if an ad hoc two-component birthrate function were adopted (cf., however, Chiosi 1989). Their evolutionary computations were performed with the now superseded Los Alamos opacities, which are now known to be considerably too weak. However, while the new opacities will produce a different period distribution, it is difficult to see how they might cause a two-humped one. It also seems unlikely to us that an observational bias due to slightly lower amplitudes (factor of 2) in stars as bright as the Cepheids could explain such a pronounced deficiency, which therefore has to be real.

In this Letter we show that the minimum in the period distribution of Cepheids is a natural result of the nonlinear dynamics associated with the fundamental pulsations of the Cepheids, and that it has therefore nothing to do with evolutionary calculations.

The proper mass-luminosity (ML) relations for the Cepheid model sequences would normally have to be obtained from evolution calculations. At the present time, however, there is enough uncertainty and disagreement (Buchler et al. 1996; Beaulieu et al. 1997) to lead us to determine these ML relations differently. From the structure of the Fourier decomposition parameters $\phi_{21}$ and $R_{21}$ for the Galaxy and for the Magellanic Clouds, one infers that the 2:1 resonance occurs in the vicinity of a period of 10 days. Based on this fact, we construct the ML relation as follows: For each metallicity $Z$ we determine the mass $M$ and luminosity $L$ for which the $P_{o} = 10$ day equilibrium model is resonant, viz., $P_{2}/P_{o} = \frac{1}{2}$ (bump Cepheid) with an effective temperature $T_{eff}$ that lies $\Delta T = 100$ K to the right of the fundamental blue edge. [Mathematically, for given composition parameters $X$ and $Z$ and a given $\Delta T$, we solve for $P_{2}(M, L, T_{eff}) = 10$. $P_{2}(M, L, T_{eff}) = 5$ with $T_{eff} = T_{BE} - \Delta T$, where $T_{BE} = T_{BE}(M, L)$.] From these anchor values ($M, L$) we then derive a ML relation with a slope chosen to be 3.56 that is close to what evolutionary calculations indicate.

We use the Livermore OPAL95 opacities (Iglesias & Rogers 1996) combined with the molecular opacities of Alexander & Ferguson (1994) to compute the nonlinear fundamental pulsations with the relaxation method of Stellingwerf (1974). We put a temperature anchor at 11,000 K with 30 constant-mass shells in the surface, and with 90 zones increasing geometrically inward. The pseudoviscosity parameters are $C_{O} = 4$ and $\alpha = 0.01$. Convection is ignored, and we caution that the models lose their validity far from the blue edge. Concomitantly with the relaxation to the periodic pulsation, the code performs a Floquet analysis of the limit cycles (i.e., the periodic finite-amplitude pulsations; e.g., Buchler 1990). We recall that the Floquet exponents measure the linear stability of the limit cycle to perturbations with respect to all the possible modes (e.g., Ince 1944): For a limit cycle to be stable, and thus to be observable, all Floquet exponents, $\lambda_{i}$, have to be negative. This stability analysis is an extremely useful by-product of the relaxation method. In fact, without the Floquet analysis we might have to integrate thousands of cycles to ascertain the stability of a limit cycle, and, in the case of instability, we would not be able to compute the limit cycles at all and determine how unstable they are. The relaxation code, on the other hand, is robust enough to converge on a limit cycle even when the latter is mildly unstable.

In Figure 2 we exhibit the behavior of the Floquet stability exponent, $\lambda_{1}$, of the fundamental limit cycle as a function of its (nonlinear) period $P_{o}$. (In this period region it is the perturbation with the first overtone that is the least stable.) The six sequences have metallicities with $Z$ ranging from 0.013 to 0.035. In order to avoid cluttering, we have split the figure into two subfigures, the top showing the metallicities $Z = 0.013$, ...
0.015, and 0.020, and the bottom the values 0.020, 0.025, 0.030, and 0.035. The hydrogen mass fraction has been chosen at $X = 0.70$. All model sequences run $\Delta T = 100$ K to the right of the fundamental blue edge. For metallicities in the range $Z \approx 0.014$–0.035 these fundamental limit cycles are thus unstable to a perturbation in the first overtone.

In order to see the effect of the location of the Cepheids with respect to the blue edge, we display in Figure 3 results obtained for a sequence with $\Delta T = 400$ K (with the same ML relation as for the 100 K sequences described above). The width of the period region of unstable limit cycles thus depends somewhat on location with respect to the instability strip, and shrinks with $\Delta T$.

Older nonlinear Cepheid pulsations that were performed with the now obsolete Los Alamos opacities (Moskalik & Buchler 1991) indicated that the fundamental Cepheid pulsations were stable in the vicinity of 10 days (except for an absolutely minute range, as their Fig. 1 shows). It is quite clearly the overall increase in the opacities that is responsible for the instability. One can get a good feeling for the sensitivity of the Floquet exponents to opacity from Figure 3 of Buchler (1996), where the results of calculations with the OPAL95, the OPAL93, and the Los Alamos opacities are compared. Roughly speaking, the change from OPAL93 to OPAL95 is equivalent to an increase in $Z$ from 0.02 to 0.03, for example (Buchler et al. 1996).

We caution the reader, however, that there is some sensitivity of the Floquet exponents to the zoning and to other numerical parameters that are used in the nonlinear calculations (Kovács 1990; Yecko, Kolláth, & Buchler 1997), as well as to convection whose effects we have ignored. The precise values of the metallicity $Z$ for the onset of instability, and of the period range thereof, should therefore not be taken too literally, although the existence of an instability in the broad 8–10 day range seems to be essentially independent of such numerical parameters.

What is the reason behind the instability of the fundamental limit cycles? We show now that the resonance between the fundamental mode of oscillation and the second overtone ($P_1/P_0 = \frac{1}{2}$) cause an overall decrease of the pulsation amplitude that in turn destabilizes the fundamental limit cycle. We note that this is the same resonance that causes the familiar Hertzsprung bump progression.

It is a well-known observational fact that the pulsation amplitudes of the classical Cepheids exhibit a drop for periods in the vicinity of 10 days. The same feature is found in numerical modeling. The amplitude equation formalism (e.g., Buchler 1993) explains how nonlinear effects associated with the resonance $P_1/P_0 = \frac{1}{2}$ between the fundamental and the second overtone are responsible for the drop in the overall pulsation amplitude. Furthermore, large amplitudes increase the stability of a limit-cycle pulsation, as the same formalism shows. Indeed, the Floquet exponent $\lambda_1$ behaves

![Figure 1](image1.png)

**Fig. 1.** Period distribution of fundamental and overtone Galactic Cepheids (from the data of Fernie et al. 1995).

![Figure 2](image2.png)

**Fig. 2.** Floquet stability exponent $\lambda_1$ for the nonlinear fundamental Cepheid pulsations as a function of period and metallicity $Z$ (with $X = 0.70$) for models 100 K to the right of the fundamental blue edge. Bottom: Square of the radius Fourier amplitude of the limit cycle scaled by the period, $a_F$; see text.

![Figure 3](image3.png)

**Fig. 3.** Floquet stability exponent $\lambda_1$ for the nonlinear fundamental Cepheid pulsations as a function of period for two sequences with $\Delta T = 100$ and 400 K and with $Z = 0.020$ and $X = 0.70$. 
as (Buchler, Moskalik, & Kovács 1991)

\[ \lambda_i = (\kappa_i - q_{10} A_i^2 - q_{12} A_i^4) P_0, \]

where \( \kappa_i \) is the linear growth rate of mode 1, the \( q_{ij} \) are nonlinear coupling coefficients that depend on the structure of the star, and the \( A_i \) are the pulsation amplitudes of the excited modes, i.e., the fundamental and the resonant second overtone here.

For simplicity, let us make a few approximations. First we ignore the smaller amplitude of the second overtone, \( A_2 \), compared to \( A_0 \). Second, with good reasons we assume that the cubic coupling coefficients \( q \) are positive and that they scale as \( q_{10} = c_{10} P_0 \) (Kovács & Buchler 1989). Third, we disregard the small variation of \( \kappa_i P_0 \) over the plotted range of periods. In equation (1) the amplitudes are relative, i.e., they refer to \( \delta R(t)/R \). Thus we associate \( A_0 \), with the lowest Fourier amplitude \( a_0 \) of the absolute computed radius variations, scaled by the stellar radius \( R_* \), i.e., \( a_0 = a_0/R_*. \) How the radius scales with period can be found by the following rough estimate. Using the mass-luminosity relation \( L \sim M^{3.56} \), the period–mean density relation \( R_0 \sim \rho^{-1/2} \sim R^{1/2} M^{-1/2} \), the surface luminosity equation \( L \sim R^2 T^4 \), and the shape of the blue edge \( T \sim L^a \), with \( a = -0.05 \), one finds

\[ P_0 \sim R^\beta, \quad \beta = \frac{3}{2} - \frac{1}{3.56(1 - 4\alpha)} \sim 1.27. \]

With \( \beta \approx 1 \), equation (1) is reduced to

\[ \lambda_i \approx \kappa_i P_0 - c_{10} a_0^2. \]  

A comparison of the bottom panel of Figure 2 with the upper panels confirms the correlation between pulsation amplitude and stability.

On the basis of the stability properties of the fundamental limit cycles exhibited in Figures 2 and 3, we thus reach the following conclusions. First, for metallicities in the range \( Z \approx 0.014–0.035 \) the fundamental limit cycle is unstable to a perturbation in the first overtone. Second, the window of instability shifts to higher period with increasing \( Z \). Third, the window narrows as one moves from the blue edge into the instability strip.

Turning now to the astronomical implications, we note that the Galactic period histogram has a dip, but not an actual gap in the 8–10 day range. If the minimum of the histogram is now interpreted as only due to a dispersion in metallicity, the observed 8–10 day fundamental Cepheids, approximately a third (from Fig. 1), must therefore have lower metallicity, \( Z < 0.013 \) (or else an unlikely \( Z > 0.035 \)). We note, though, that Becker et al. (1977), albeit on totally different grounds, also suggested that Galactic metallicity dispersion is large (a range \( Z_{\text{max}}/Z_{\text{min}} \) of 3–5). More recently, Fry & Carney (1997) also found a large dispersion (\( Z_{\text{max}}/Z_{\text{min}} \approx 2.5 \)).

However, it may be objected that since the stability of the fundamental limit cycles depends also on the location with respect to the instability strip, a wide strip could similarly reduce the Cepheid deficiency, especially on the lower period side. There are evolutionary arguments (Becker et al. 1977) that the Cepheid instability strip may be much narrower than is generally assumed. An estimation of the instability strip from the Fourier decomposition parameters also suggested a narrower rather than a larger instability strip (Buchler, Moskalik, & Kovács 1990). A weak dispersion in effective temperatures is probably not sufficient by itself to account for the observed survival of 8–10 day Cepheids.

It is likely that both effects, dispersion in metallicity and dispersion in effective temperature, contribute to this survival. Whether it is dominated by the dispersion in metallicity or by the width of the instability strip can be tested with an observational measurement of the metallicities of the individual Cepheids. Galactic Cepheids with solar metallicity, for instance, and periods of 8.000669 days for DL Cas, or 10.15073 days for \( \xi \) Gem (Fry & Carney 1997), that lie at the edges of the expected period interval of instability, could indicate that this interval is slightly narrower than predicted here or imply that the stars are relatively cool (respectively \( \sim 200 \text{ K} \) and \( \sim 400 \text{ K} \) from the blue edge).

While there is a deficiency of fundamental Cepheid pulsators in the 8–10 day range, there is absolutely no reason to believe that there is a deficiency of the corresponding stars. As expected, hydrodynamical calculations show that these stars pulsate in stable first-overtone limit cycles. They should therefore show up as an excess in the histogram at lower periods (lower by a factor \( \approx 0.7 \), the typical period ratio \( P_1/P_0 \)). Indeed, the histogram of Figure 1 is certainly compatible with an excess in the 5.6–7.0 day range. (The left overbar shows the first-overtone period range corresponding to the fundamental range shown on the right.)

Turning now to other galaxies, we recall that Becker et al. (1977) also present a period histogram for M31, a galaxy that also has a relatively high metallicity. Again the histogram shows the dip in the 8–10 day range, consistent with our results.

In contrast, the Magellanic Clouds are known to have considerably lower average metallicities than the Galaxy, and indeed, in agreement with our stability analysis, the period histograms for the Magellanic Clouds (Becker et al. 1977) do not show much indication of a minimum. The small hollow near 9 days, if statistically significant, may again be an indication of a spread in metallicity putting some Magellanic Cepheids above the threshold value of \( Z \approx 0.013 \).

If the 2:1 resonance at 10 days plays such an important role, one may wonder whether other resonances might make themselves felt, especially because the Galactic Cepheid period distribution (Fig. 1) seems to indicate a deficiency of Cepheids in two other places.

First, if the gap near \( P \approx 25 \text{ days} \) is indeed significant, it has an interesting implication. Numerical hydrodynamic calculations (Moskalik & Buchler 1991 and Moskalik, Buchler, & Marom 1992 for the new opacities) show that the \( P_1/P_0 = \frac{3}{2} \) resonance can destabilize the fundamental Cepheid pulsation and lead to a periodic pulsation with double period, in which alternating cycles, however, differ only slightly. We note, though, that Ferri (1996) has not found any evidence of alternations in his Cepheid data. However, in support of this scenario, Antonello & Morelli (1996) have suggested that the star CC Lyr does exhibit alternations. The observational data of CC Lyr are very limited, however, and additional observations of this star are necessary to confirm the existence of alternations. Is it possible that the dip in Figure 1 could arise because such stars with alternating cycles may not have been classified as Cepheids? If this resonance is indeed strong enough to cause instability to alternations, then this would add an additional observational constraint for Cepheid models.

Second, Figure 1 perhaps also suggests a minimum in the vicinity of 19 days. One notes that this is the period region
where the fundamental mode is in a 4:1 resonance with the fifth overtone \( (P_4/P_5 = \frac{4}{5}) \). However, the linear stability of the fourth overtone is quite large, and a hydrodynamical survey is necessary to verify whether it can play a sufficiently large dynamical role to destabilize the fundamental limit cycle.

In conclusion, the simple and natural explanation for the deficiency of \( \approx 8 \pm 10 \) day Cepheid variables is that the fundamental mode limit cycles are unstable. The corresponding stars pulsate in the first overtone with period \( P_1 \approx 0.7P_0 \), giving rise to a relative excess of overtone Cepheids in the corresponding range of \( \approx 5.6 \pm 7.0 \) day periods. It is thus not necessary to invoke an ad hoc two-component birthrate function to explain the Cepheid period distribution.

Our calculations and the agreement they provide with the minimum in the observed Galactic Cepheid distribution, and the lack of a minimum in the lower metallicity Large Magellanic Cloud data, also provide a further confirmation that the new opacities are in the right ballpark. Indeed, if they were much weaker, the 8–10 day fundamental Cepheid pulsations would all be stable, thus not giving the observed minimum. On the other hand, if they were much stronger, then they would predict a minimum for the Large Magellanic Cloud as well, and a smaller one, or none at all, for the Galactic Cepheids, in contrast with observation.

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