INVESTMENT SUBSIDIES AND REGIONAL WELFARE: A DYNAMIC FRAMEWORK

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Investment Subsidies and Regional Welfare:  
A Dynamic Framework*

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Abstract
Subsidising investment in lagging regions is an important regional policy instrument in many countries. Some argue that this instrument is not specific enough to concentrate the aid towards the regions that are lagging behind most, because investment subsidies benefit capital owners who might reside elsewhere, possibly in very rich places. Checking under which conditions this is true is thus highly policy relevant. The present paper studies regional investment subsidies in a multiregional neoclassical dynamic framework. We set up a model with trade in heterogeneous goods, with a perfectly integrated financial capital market and sluggish adjustment of regional capital stocks. Consumers and investors act under perfect foresight. We derive the equilibrium system, show how to solve it, and simulate actual European regional subsidies in computational applications. We find that the size of the welfare gains depends on the portfolio distribution held by the households. If households own diversified asset portfolios, we find that the supported regions gain roughly the amounts that are allocated to them in the form of investment subsidies. If they only own local capital stocks, a part of the money is lost through the drop in share prices. From the point of view of total welfare, the subsidy is not efficient. It can lead to a welfare loss for the EU as a whole and definitely leads to welfare losses in the rest of the world, from where investment ows to the supported EU regions.

JEL classification: C31, F12, F15, J31

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1. Introduction

Subsidising investment in lagging regions is the most important regional policy instrument in the EU as well as within European countries (European Union, 2017). Some argue that this instrument is not specific enough to concentrate the aid towards the regions that are lagging behind most, because the benefits trickle down to other places and little is left to the regions to be supported themselves (Dupont and Martin, 2006). There are potentially two channels that could transmit the benefits to places that were not intended to be supported in the first place: trade and the capital market. Through the trade channel the demand impulse generated by a subsidy partly spreads out to the rest of the world. A repercussion on the capital market comes from the fact that subsidies affect stock prices. They make capital in the supported regions more abundant and thus press its market value down. If people in the supported regions happen to own these capital stocks they suffer from an asset loss. Eventually the lagging regions might benefit less than if they got the money directly in a lump sum fashion. The policy issue to be answered in this paper is to check under which conditions this statement holds true.

From the theory point of view, investment subsidies are mainly seen as an instrument to support lagging regions and raise economic growth (Barro and Sala–i–Martin, 1995). As a support instrument, they can be a more efficient policy than alternatives, for example employment subsidies (Fuest and Huber, 2000). At the same time, investment subsidies can play a role in the international tax competition by increasing the incentives for firms to invest and to settle in certain locations (Baldwin et al., 2003). It is in this context of capital mobility between regions that questions about capital ownership, interregional distribution of income and resulting welfare effects of investment subsidies can arise.

In a static model incorporating agglomeration externalities and firm mobility, Dupont and Martin (2006) show that regional capital subsidies may increase inequality, hurting the poor regions. Their framework, however, misses several important features of the actual policies. First, in practice subsidies are supporting new investment projects and not the existing capital stock. Second, investment is a dynamic phenomenon resulting from intertemporal trade-offs, which cannot be reasonably reflected in a static framework. A dynamic approach is therefore needed to study the impact of investment subsidies in an appropriate way.

Regarding the first point, it has to be stressed that subsidising investment and subsidising the user cost of capital is not the same. The impact is very different. The main point is that subsidising the user cost of capital favours both, owners of the existing stock as well as asset owners everywhere in the world earning a higher interest on their asset. Subsidising investment potentially may harm rather than benefit owners of the existing stock. The stock becomes less scarce, when investments get cheaper, and thus its market value drops. If the subsidy is small in comparison to the world stock of assets such that its impact on the world real interest rate is negligible, the loss of stock value may dominate.

The effects of regional subsidies (in particular, the EU Structural Funds) have been widely studied empirically. This literature includes the results of simulation models (Capello and Fratesi, 2012; Brandsma et al., 2015; Garau and Lecca, 2015; Ribeiro et al., 2018), case studies (Huggins, 1998; Lolos, 1998) and econometric models (Dall’erba and Le Gallo, 2008; Mohl and Hagen, 2010; de Castris and Pellegrini, 2012; Cerqua and Pellegrini, 2014). The findings from econometric studies and case studies are in general mixed, due to the use of different methods, varying sample sizes and time periods, and range from no significant effect of regional funds to large and positive impacts on the supported regions (Dall’erba and Le Gallo, 2008).

Given these uncertainties, it is worthwhile to study the effects of the investment subsidies with
the help of simulation models, where the results can be clearly traced back to the shocks and the model assumptions. A modern approach suitable to study such questions is the dynamic spatial computable general equilibrium modelling framework (Bröcker and Korzhenevych, 2013). Dynamic spatial CGE models are rare and usually recursive, meaning that static solutions in each period are connected with each other using ad-hoc dynamic transition rules. In contrast, the approach in this paper is to apply a fully forward-looking model, which allows consistent saving and investment decisions and a proper welfare analysis.

The present paper thus studies regional investment subsidies in a multiregional neoclassical dynamic framework. We set up a model with trade in heterogeneous goods, with a perfectly integrated financial capital market and sluggish adjustment of regional capital stocks. Consumers and investors act under perfect foresight. We derive the equilibrium system, show how to solve it, and simulate regional policies in a computational application. We compare the welfare gain generated by an investment subsidy with a hypothetical welfare gain that households would obtain if they got the subsidy as a lump sum transfer.

The aims of the paper are the following. First, we show that a spatial forward-looking CGE model is an operational tool that can be used in policy applications. Second, in terms of policy application we are able to answer several questions about the investment subsidies: Is this instrument effective (does is produce a positive net welfare effect)? Do target regions gain more through a subsidy than through a a lump-sum transfer? What are the costs of the subsidy in terms of global welfare?

The paper starts with a non-technical introduction into the model in Section 2, followed by a technical explanation in Section 3. Section 4 introduces investment subsidies, Section 5 is a brief outline of the solution technique (leaving lots of technical details aside). Section 6 explains the calibration, Section 7 presents the simulation results, and Section 8 concludes.

2. Non-technical model description

The model to be presented is a dynamic spatial computable general equilibrium (DSpCGE) model for a closed system of regions. This system covers the whole world in our empirical application, while the policy under study is executed by just a small part of the world, the EU. By embedding it into the world economy, open economy repercussions are fully taken account of. The specification of the production and household sectors as well as of the goods markets is close to an earlier static model (Bröcker, 1998) that has been widely applied under the brand name CGEurope in transport policy evaluation (Bröcker et al., 2010; Korzhenevych and Bröcker, 2009).

Agents of the economy are firms and households. The state is not modelled as an own sector. State production is subsumed under the production sector; state consumption is subsumed under the household sector. The basic setup is an open-economy version of the Ramsey optimal savings model, combined with an investment adjustment costs framework (Abel and Blanchard, 1986). Thus, both, households and firms make intertemporal decisions and have perfect foresight.

Two types of goods are distinguished in the model: local and tradable. Local goods can only be sold within the region of production, while tradables are sold everywhere in the world, including the own region. Identical firms located in the region produce output by combining capital, effective amount of labour service, local goods, and a composite of tradable goods (coming from all regions) using a Cobb-Douglas (CD) technology. The composite of tradables is defined as a constant elasticity of substitution (CES) index good. Homogeneous gross output serves a double purpose: first, it is one-to-one transformed into the local good (without a price mark-up), and secondly, it is used as the only input in the production of a variety of tradable goods under increasing returns to scale. The market for local goods is perfectly competitive, while monopolistic competition in
Dixit and Stiglitz (1977) style prevails in the tradables market. Firms produce not one single, but a whole bundle of product varieties. The number of varieties supplied by the region is endogenously determined by the free entry condition requiring market entry or exit until firms can just cover their respective fixed cost by the monopolistic price mark-up.

Firms decide at any moment about both, production and investment. Investment needs a composite investment good as an input, which is assumed to be a CD composite of two types of goods: locals and the composite of tradables. The investment good can however not be transformed into gross investment one-to-one. There are adjustment costs of investment, in addition to the investment itself; these costs are assumed to increase, if the capital growth rate increases. Following the literature (see e.g. Barro and Sala–i–Martin (1995)) we assume quadratic adjustment costs. By introducing this assumption, we want to rule out an implausible outcome of the basic open-economy version of the Ramsey model, where the adjustment of capital stocks is done through an instantaneous jump to the new locations. The existence of adjustment costs implies that the stock of capital in a region has a stock price that in general differs from the price of the investment good, by which the stock is built up. This leads to a regional investment function, known as Tobin’s q-theory, making regional investment depend on the regional stock price of capital.

Households in a region are regarded to represent the present as well as all future generations. Hence, they maximize utility over an infinite time horizon subject to the constraint, that the present value of their consumption expenditures does not exceed the present value of their wage income plus the value of their actual shares in the world’s stock of capital. Utility is given by a an intertemporal CES function. The consumption composite is constructed using a nested CD-CES function, which is the same as the one used to construct the composite investment good. The income share not used for consumption is saving, that is the per unit of time increase in the value of assets held by the households. The asset total held by all households in the economy equals the total stock value in the economy at any moment.

Households take the wage rate paid per unit of labour service as given. Each individual is assumed to provide one unit of labour service inelastically. The population is assumed to be immobile and constant. The effective labour is assumed to grow at a constant rate. This is the assumption of Harrod neutral technical progress. However, as we will show, the steady-state growth rate is scaled up by a term that reflects the presence of an additional source of productivity growth on the dynamic equilibrium path, the expanding variety of tradables. This allows us to link our model to the literature on semi-endogenous growth pioneered by Jones (1998).

In addition to the distinction of goods, factors, firms, and households by location, the spatial dimension comes in through the costs for goods movement depending on geography. The total trade costs for goods to be delivered from one region to another is assumed to amount to a share in the traded value. The way trade costs are modelled resembles — but is not identical with — the “iceberg” approach (Samuelson, 1954). Following an earlier static model (Bröcker, 1998), we assume that trade costs for goods arriving in a region are paid to a zero-profit “transport service”, doing the job by consuming composite tradables, composed in the same way as the composite tradables used by households and firms.

Regional policy is introduced by assuming that in a lagging region the government rebates to the investor a certain share of the investment cost. This subsidy is financed by collecting a proportional labour income tax with a uniform rate raised in all regions of a jurisdiction executing the policy (the EU in our application).
3. The formal model

The model is set up in continuous time \( t \) in years, running from zero to infinity. To avoid notational clutter, we usually suppress both, the time argument and the regional subscript. At time \( t = 0 \) the policy intervention is announced and its implementation starts at the same point in time. In the calibration, \( t = 0 \) corresponds to the year 2011 that the calibration data are collected for. As the model is non-monetary, the unit of account is arbitrary. We choose \( € \) as unit of account. At \( t = 0 \) one \( € \) in the model coincides with one \( € \) in the data. The arbitrary diachronic movement of the unit of account is chosen in such a way that the nominal interest rate is at all times equal to the households’ time preference rate \( \rho \), a model parameter that is equal across regions and constant over time. This is just for notational convenience. The real interest rate and thus also the inflation rate are endogenous.

3.1. Firms

Due to the CD technology and perfect competition on the input markets, firms with sales value \( M \) spend \( \alpha M \) for labour, \( \gamma M \) for non-tradables and \( \eta M \) for composites of tradables. The remaining part \( \beta M \) goes as remuneration for the service of capital in goods production to the shareholders \((\alpha + \beta + \gamma + \eta = 1)\).

The mill price is

\[
p^m = w^\alpha (\beta M / K)^\beta (p^m)^\gamma (p^d)^\eta,
\]

where \( w \) denotes the wage rate, \( p^d \) is the composite tradables price and \( K \) is the real capital stock. Firms can use part of the output to produce varieties of tradables under Dixit-Stiglitz monopolistic competition, to sell them at sales price \( p^s \), that customers in the world (including the region itself) are willing to pay. By choice of unit the mill price of tradables is also \( p^m \). The mill price either equals the sales price or exceeds it. If the firm sells tradables, then the mill price and sales price must be equal. If the mill price exceeds the sales price sales of tradables are zero. The firm would produce only for the non-tradables market. This leads to the complementarity (with tradables supply denoted \( S \))

\[
S \geq 0, \quad p^m \geq p^s, \quad S(p^m - p^s) = 0.
\]

The regional population is assumed to be immobile and constant at \( \bar{L} \). The effective amount of labour input is assumed to grow at an exogenous rate of technological progress, \( \tilde{\xi} \), i.e.

\[
L_r(t) = \bar{L} \exp(\tilde{\xi} t)
\]

Firms do not only produce, they also invest. For adding the gross investment \( I \) to the existing capital stock they need \( I(1 + (\zeta/2)(I/K)) \) units of investment goods. The “first unit” of investment goods is transformed to installed capital one-to-one. If capital grows, however, more than one unit of the investment good per unit of new capital installed is required, and the input requirement per unit of new capital gets larger with a larger growth rate. This increase is the stronger, the larger the adjustment cost parameter \( \zeta \). For the sake of simplicity both, the consumption bundle and the investment good are the same, a CD composite of non-tradables and tradables, with expenditure shares \( \epsilon \) and \( 1 - \epsilon \), respectively, and with price \( p^c \). We thus have investment costs as

\[
J = p^c I \left(1 + \frac{\zeta I}{2 K} \right).
\]
Capital depreciates at a rate $\delta$ p.a. Thus
\[ \dot{K} = I - \delta K. \] (5)

The shares in the capital stock can be traded at unit price $q$ on the market, and firms choose investment $I$ maximising $qI - J$. The first order condition is
\[ \frac{I}{K} = \frac{q/p^c - 1}{\zeta}. \] (6)

$q/p^c$ is called “Tobin’s $q$” in the literature. According to (6) real capital cannot “jump” between regions as in static NEG models with mobile capital. It’s growth rate is finite. Capital grows the faster, the larger is Tobin’s $q$ and the smaller is the adjustment cost parameter.

For using the capital stock, firms have to pay a rental rate to their shareholders that is equal to the marginal value product of capital. The marginal value product has two components, one is the marginal value product in production of goods already mentioned. Per unit of capital it is $\beta M/K$. The other is the marginal investment cost reduction brought about by an extra unit of capital installed. According to equation (4) it amounts to
\[ -\frac{\partial J}{\partial K} = \frac{p_c \zeta}{2} \left( \frac{I}{K} \right)^2. \]

Thus the rental rate is
\[ v = \beta M/K + \frac{p_c \zeta}{2} \left( \frac{I}{K} \right)^2. \] (7)

### 3.2. Consumers

Consumers maximise discounted utility
\[ U = \int_0^\infty u(C(t)) \exp(-\rho t) \, dt \]
subject to the flow budget constraint
\[ \dot{A} = \alpha M + \rho A - C, \] (8)

with asset value $A$, real consumption $C$ and nominal consumption $C = C_p^c$. Real consumption $C$ is a CD composite of local and tradable goods with respective expenditure shares $\epsilon$ and $1 - \epsilon$. Its composite price is thus
\[ p_c = (p_m^c)^\epsilon (p_d^c)^{1-\epsilon}. \] (9)

The present value utility $U$ is characterized by a constant intertemporal elasticity of substitution $\theta$:
\[ u(C) = \frac{C^{1-1/\theta} - 1}{1 - 1/\theta}. \]

This is a concave optimisation problem. The first order condition requires the marginal utility as of today to be equal — up to a constant factor — to the price as of today, i.e. the future price discounted to today. Thus $C(t)^{-1/\theta} \exp(-\rho t) \propto p_c^c(t) \exp(-\rho t)$. Solving for $C$ yields
\[ C = m(p_c^c)^{1-\theta}, \] (10)
with $m$ varying across regions, but being constant over time. $m$ is an endogenous variable to be determined in the solution of the system. It must be such that, when starting with a certain asset $A(0)$ in $t = 0$, the asset trajectory stays within certain bounds.

### 3.3. Trade

Consumers and firms buy a CES composite (with an elasticity of substitution $\sigma$) of tradable varieties produced everywhere and sold under conditions of Dixit-Stiglitz monopolistic competition. The composite is consumed, used as a production input and as a component of the composite investment good. It is also used to produce the transport service. Transport cost is added to the sales price $p^s_r$ leading to the inclusive price $p^s_r\Theta_{rs}$ in destination $s$ for a good coming from origin $r$. Here it is assumed that nominal transport cost for a given origin-destination pair is a fixed share of the nominal value of the good, valued at mill price. We call this the “modified iceberg assumption”. It differs from the standard iceberg assumption in that we assume the composite — not the variety itself — to be used for the transport service of an individual variety. This is more plausible than the often criticised iceberg assumption, though the results differ only slightly.

From these considerations follows the trade equation (with explicit regional subscripts, but the time argument still suppressed)

$$T_{rs} = \frac{S_r(p^s_r\Theta_{rs})^{-\sigma} D_s}{\sum_{r'} S_{r'}(p^s_{r'}\Theta_{r's})^{-\sigma}}. \quad (11)$$

$D$ is demand including demand for the transport service. It is in other words demand for tradables valued at c.i.f. prices, which is

$$D = (1 - \epsilon)(C + J) + \eta M. \quad (12)$$

The CES form of demand implies a composite price of tradables in the destination region $s$

$$p^d_s = \psi \left( \sum_r S_r(p^s_r\Theta_{rs})^{1-\sigma} \right)^{1/(1-\sigma)}. \quad (13)$$

$\psi$ is an arbitrary scaler. The choice does not affect any result, but it offers a degree of freedom to choose the average level of prices.

### 3.4. Equilibrium

Labour market equilibrium requires

$$\alpha M = wL. \quad (14)$$

Equilibrium on the market for non-tradables requires the value of non-tradables $(M - S)$ to equal the demand value of non-tradables, which is intermediate $(\gamma M)$ plus final $(\epsilon(C + J))$ demand. Thus we must have

$$M - S = \epsilon(C + J) + \gamma M. \quad (15)$$

Equilibrium in the tradables market requires the value of supply $S_r$ in the region to equal the value of demand of all regions for tradables from region $r$, i.e.

$$S_r = \sum_s T_{rs}. \quad (16)$$

Finally, equilibrium on the market for shares in capital stocks requires shares in all stocks to earn
the common interest rate \( \rho \) implying \( \rho q = v - \delta q + \dot{q} \), or solving for \( \dot{q} \),

\[
\dot{q} = (\rho + \delta)q - v. \tag{17}
\]

This is the non-arbitrage condition implied by a perfect frictionless asset market. In equilibrium it must also be guaranteed that the asset total \( \sum_r A_r \) in the entire economy equals the total value of capital stocks \( \sum_r q_r K_r \). One can show that this condition automatically holds for all times, if it holds for one point in time. This is Walras’ law.

Equations (1) to (17) give us 17 equations to determine the 17 unknowns \( p^m, p^s, J, \dot{K}, I, v, L, p^c, A, C, T, D, p^d, w, M, S \) and \( \dot{q} \). Three of these equations ((5), (8) and (17)) are differential equations to determine \( \dot{K} \), \( \dot{A} \) and \( \dot{q} \), respectively, the others are algebraic to determine the remaining 14 unknowns. We thus have a differential algebraic equation (DAE) system to find the time path of all endogenous variables. It is however not yet complete. Two points are open: first, what are the boundary conditions for the three dynamic variables, and second, how to determine \( m \).

As to the first point: capital stock is inherited and thus given at \( t = 0 \),

\[
K(0) = K_0. \tag{18}
\]

Its level has to be calibrated by benchmark year observations. Households also inherit their respective assets, giving the boundary conditions for \( A \),

\[
A_r(0) = \sum_s \Lambda_{rs} q_s(0) K_s(0), \tag{19}
\]

where parameter \( \Lambda_{rs} \) gives the share of region \( r \) in the property of capital stock in region \( s \) at \( t = 0 \). The parameter restrictions \( \sum_s \Lambda_{rs} = 1 \ \forall r \) guarantee that, at \( t = 0 \), the asset total in the entire economy equals the total value of capital stocks.

The benchmark equilibrium is assumed to be a perfect foresight equilibrium. In such an equilibrium only the values of the assets matter, not the composition of the portfolios. This is because any kind of asset a household can hold earns the same nominal interest. We have to be careful, however, when we allow for unexpected shocks. Such a shock leads to revaluations of the capital stocks, and this affects different households differently, if they do not all hold assets of identical composition. If we compare time paths after a shock with a non-shock time path in welfare terms, start conditions in real terms must be held unchanged between the paths. Otherwise we would allow wealth to rein from heaven or disappear to nowhere. As prices, including the stock price \( q \), change from one path to the other, the same must in general hold for the assets held by the households. After a shock it is not the nominal value of the asset, that has to be held unchanged, but the real stocks making up the respective portfolio.

A third boundary condition is needed for \( q \). It is given by the transversality condition of the dynamic optimisation of firms: in the long run, the market value of a firm’s capital stock must converge to zero, in present values,

\[
\lim_{t \to \infty} K(t) q(t) \exp(-\rho t) = 0. \tag{20}
\]

As to the second point: for determining the vector \( m \) controlling the level of consumption we exploit the transversality condition of the households’ optimisation problem saying that a household’s
asset must have a present value zero in the long run, as already noted above,

\[
\lim_{t \to \infty} A(t) \exp(-\rho t) = 0.
\] (21)

This is intuitive: the level of consumption must be such that its present value just equals the asset value at \( t = 0 \) plus the present value of all future labour income.

We now have just the right number of equations and boundary conditions to determine the equilibrium. To be sure, this is just a hint at a good chance to be able to solve the model, no existence proof. In particular, it is not obvious whether the transversality conditions make sure that only one, not many trajectories would converge in the way required. It is known that under perfect competition without externalities saddle path stability holds (Farmer, 1999), which is a kind of local uniqueness condition, saying that any trajectory fulfilling the constraint has no other one in its neighbourhood doing so as well, if the neighbourhood is chosen small enough. This result can fail under increasing returns, but local properties around the steady state found in our numerical experiments never show any such pathology. We highly trust in saddle path stability of the solution of our model.

4. Evaluating investment subsidies

As mentioned we study two intervention scenarios: i) a subsidy for private investment costs (“Subs”) and ii) a lump sum transfer (“Lump”). The lump sum transfer is not a regional policy actually observed. It serves as a reference to see whether the subsidies exert a stronger influence than what one achieves by transferring the money directly to the households in the supported regions.

The interventions come as unexpected shocks. The regional impact is evaluated by looking at the Relative Equivalent Variation (REV) caused by this shock for the representative regional household. REV is a welfare measure expressed as a percentage of consumption, based on an intuitive idea: assume the shock did not happen, but you wanted to make the household as well off as it would be with the shock in place, by increasing its consumption by a constant percentage for the entire future. REV is the percentage doing it. Multiplied with the benchmark value of consumption, it is easily converted into monetary terms.

Subsidising private investment means that, in the supported regions, the government rebates to the investor a certain share of the investment cost. If the tax payer bears the share \( \Gamma \) of the investment costs, investors maximise \( qI - (1 - \Gamma)J \) rather than \( qI - J \). This leads to a replacement of (6) with

\[
\frac{I}{K} = \frac{q/(1 - \Gamma)p^c}{\zeta} - 1.
\] (22)

The expression \( q/(1 - \Gamma)p^c \) can be understood as a subsidy-corrected Tobin’s \( q \). The rental rate in (7) becomes

\[
v = \beta M/K + p^c(1 - \Gamma)\zeta \left( \frac{I}{K} \right)^2.
\] (23)

Clearly, investments are the higher, the more they are subsidised, other things equal. It is important to note, however, that the regional stock price drops down, because investors foresee the subsidy to trigger more investment and thus to depress rental rates. To the best of our knowledge, the literature is not aware of this effect.
As a reference, we introduce a lump sum transfer $\Omega_r$, changing (8) to

$$\dot{A} = \Omega + (1 - t)\alpha M + \rho A - C,$$

(24)

where $t$ is the labour tax rate to be explained below.

All policy interventions are financed by the EU. We let its budget be balanced at any time. Public debt is not admitted. Therefore at any time the subsidies or transfers must be paid by taxes. We assume a labour tax with a uniform rate within the EU. Elsewhere it is zero. The state budget constraint then reads

$$\sum_r t_r (\alpha M_r + \rho A_r) = \sum_r \Gamma_r J_r,$$

(25)

or in the transfer case

$$\sum_r t_r (\alpha M_r + \rho A_r) = \sum_r \Omega_r,$$

(26)

with tax rate $t_r$. $\Gamma_r$ and $\Omega_r$ follow an exogenous trajectory, while the tax rate is endogenous.

5. Solution

We start by showing constructively that there is a steady state where all variables grow at a constant rate, though not all at the same rate. Assuming all variables to grow at a constant rate leads to a linear equation system that can be solved for all growth rates. One gets in particular

$$\dot{K} = \dot{I} = \xi$$

and $\dot{q} = \dot{p}^c = \dot{v} = -\xi / \theta$ with

$$\xi = \frac{\alpha (\sigma - \epsilon)}{\beta \epsilon + \alpha \sigma + \gamma - 1} \tilde{z}.$$  

(27)

$\xi / \theta$ is in other words the rate of deflation. Thus the real interest rate is $\xi / \theta + \rho$. All nominal values, $M$, $C$, $J$, $S$, $D$, $T$, $C$ and $A$ grow at the rate $(1 - 1/\theta)\xi$. The nominal wage per worker (not per effective worker) grows at this rate as well. One may easily check that, given these rates, the prices $p^m$, $p^s$ and $p^d$ also grow at constant rates. $\xi$ is the rate of real growth of consumption, capital and investment. The fraction in (27) can be shown to be larger than one, if its denominator is positive, what we assume. The inequalities

$$\alpha \sigma + \beta \epsilon + \gamma - 1 < \alpha \sigma + \beta + \gamma - 1 = \alpha \sigma - (\alpha + \eta) < \alpha \sigma - \alpha \epsilon$$

prove the denominator to be less than the nominator, thus the fraction is larger than one. Positivity of the nominator is guaranteed with any sensible choice of parameters. $\alpha \sigma \geq 1$ is already sufficient. The labour share in the output value $\alpha$ is usually in the order of 1/3 and $\sigma$ is suggested to be considerably larger than 3, so that the condition is satisfied. If the denominator happened to approach zero, any positive HARROD neutral technical progress, how small ever it may be, would let growth explode. The condition that the denominator be strictly positive can be interpreted as the “no black hole condition” in our context.

Real growth is faster in this economy than the rate of HARROD neutral technical progress, unlike the standard SOLOW model, where both are the same. The deviation is due to the fact that in our model there is an aggregate economies of scale effect. If the economy grows, product diversity increases, which makes production and investment more productive and consumers more satisfied.
The factor amplifying the rate of Harrod neutral technical progress gets larger, if $\sigma$ gets smaller or the share of tradables in production or consumption and investment gets larger.

To solve the model, we must at any point in time, with given dynamic state variables $K$, $q$ and $A$, solve a non-linear system to obtain the other 14 unknowns. Inserting the solution into the differential equations (5), (8) and (17) then gives us a system of non-autonomous ordinary differential equations in the state variables $K$, $q$ and $A$ of dimension $3n$, that we can integrate over time. But there is an obvious difficulty: $m$ is in the beginning unknown, and one does not know where to start with regard to the stock price vector $q$. The respective degrees of freedom are closed by the transversality conditions (20) and (21).

An intuitive strategy is trying “forward shooting”. Though we do not apply it, its description helps understanding the problem. One would start with a guess for $q(0)$ and $m$ and integrate forward in time. Typically $q$ and $A$ would however diverge rapidly and the transversality conditions would fail to hold. Then one would revise the initial guesses and would try again to hit the target. If e.g. for some region $q_r(0)$ is chosen too high and thus escapes to heaven, one would reduce the initial guess. Similarly, if the initial guess of $m_r$ is too high such that the household runs into a rapidly increasing deficit, one would reduce the initial guess of $m_r$.

Though intuitive, such a procedure does not work in practice. Variables escape extremely rapidly from a stable path, even if they are chosen only slightly different from their correct (though unknown) start values. Two fundamental tricks help to solve the system. First, the system is transformed in such a way that it becomes autonomous (i.e. time does not appear as an argument in the DAE) and converges to a stationary state. Second, the transversality conditions, that are difficult to handle because of the limit operator, are replaced with boundary conditions for a point in finite time far enough in the future, 50 years, say.

We make use of the fact that the system converges to a steady state. Though one cannot force it to reach this steady state already in finite time, we can make sure by the boundary conditions that it reaches the so-called saddle path of a system that linearly approximates the system to be solved around the steady state. It thus comes close — in fact very close — to the true solution path in finite time; for details see Bröcker and Korzhenevych (2013).

The mathematical problem to be solved is of the general form

$$\dot{x} = f(x, m),$$

where $x \in \mathbb{R}^{4n}$ is the state vector with components $A$, $K$, $q$ and $L$, each of length $n$. As $m$ has also length $n$ there are $5n$ degrees of freedom. We have boundary conditions at $t = 0$ fixing initial values for $A$, $K$ and $L$, thus closing $3n$ degrees of freedom. $2n$ degrees of freedom are left to be closed by transversality conditions (20) and (21), that are transformed to linear constraints in finite time. For the non-linear solution we use a collocation method to solve two-point boundary value problems. We use the MATLAB code bvp6c (Hale and Moore, 2008), a refinement of MATLAB’s original code bvp4c (Shampine et al., 2005). The details of the solution procedure are described in Bröcker and Korzhenevych (2013).

6. Data and calibration

6.1. Regional setup

The following calculations have been performed for two alternative aggregations. In the first aggregation (Aggregation 1), the focus is on 16 NUTS2 regions in Poland, as well as Baltic states Estonia, Latvia and Lithuania. These regions receive a substantial amount of EU regional funding.
(see Table 3 below). Further regions include Germany, as well as aggregate regions for the other areas of the EU and the rest of Europe and one “Rest of the world” region. In the second aggregation (Aggregation 2), Poland is a single region and instead Bulgaria and Romania are disaggregated. The two schemes cover 26 and 25 regions, respectively. For computational reasons we have to keep the total number of regions limited. The comparison of the results from the two aggregations should illustrate the robustness of the model with regard to changes in regional setting. Illustrating time paths are plotted for a high subsidy case (Latvia), a low subsidy case (Germany) and for the rest of world (ROW).

6.2. Data sources

Data on the amounts of money from the ERDF and the Cohesion Fund used to subsidise regional private investment in the years 2007-2015 stem from DG Regio. These data are provided for NUTS2 regions. All subsidies to the private sector in each region have been added up to arrive at the total amount of regional private investment subsidies.

Two pieces of information that are needed to identify the initial steady state are the values of regional GDP and trade deficit in the benchmark (see below). The source of GDP data for NUTS2 European regions is Eurostat (2017). United Nations Statistics Division (2017b) data is used for other regions in the model. International trade data stem from United Nations Statistics Division (2017a). The base year for GDP and trade data is 2011.

The matrix of distance cost mark-ups is the aggregated version (with regional GDPs used as weights) of the full matrix computed for the whole world. The parameters of the distance function are estimated from a gravity model for international trade as in Bröcker et al. (2010). Land distances were calculated using a global road network from OpenStreetMap (OpenStreetMap contributors (2017)). Travel distances over see were purchased from AtoBviaC data provider. See to land distance conversion rates were taken from Hummels (1999).

6.3. Calibration

Table 1 lists the assumed values of elasticities and share parameters. Cobb-Douglas parameters are calculated from the aggregate GTAP data (Narayanan et al., 2012). The adjustment cost parameter ζ is chosen taking account of the implications for the value of Tobin’s q and the convergence speed. The econometric estimates of Tobin’s q (e.g. Blanchard et al. (1993)) usually do not exceed 1.5, while the plausible speed of convergence for the capital stock should not be higher than 0.05 per year. With the chosen parametrization, both criteria are fulfilled for our model.

| Parameter | α   | β   | γ   | ε   | δ   | σ   | θ   | ζ   | ξ   | ρ   |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Value     | 0.24| 0.19| 0.29| 0.28| 0.60| 0.05| 12.0| 0.80| 6.0  | 0.02| 0.025|

Table 1: Preference and technology parameters

The scaling factor ψ is set such that the initial price level of the local goods in the rest of the world is equal to 1. The depreciation rate is set at the value 0.05 per year. The elasticity of substitution among brands of tradables is set at 12. The elasticity of intertemporal substitution is commonly assumed to lie between 0 and 1. We choose the value of 0.8. The rate of growth of the efficient labour stock ˜ξ is calibrated according to (27), where the real growth rate of consumption ξ is set at 2% per year. The rate of time preference, ρ, is calibrated assuming the real interest rate of 5% per year.

In addition to distance costs, the iceberg costs in the model include international trade barriers. These are calibrated such that international trade flows for any pair of countries in the benchmark
equilibrium coincide with the observations. Interregional trade flows are calibrated based on the steady-state solution of the model as in Bröcker et al. (2010).

It remains to specify the initial values for effective labour, assets and capital. This is done by introducing an appropriate number of constraints that have to hold at $t = 0$, as shown by the following equations. All variables in the equations have to be thought of as evaluated at the initial steady state. For assets and effective labour, the constraints force the solution to reproduce certain data. First, $L$ is calibrated such that the observed GDP $\bar{Y}$ is reproduced in the model. Given the model’s GDP $Y = (\alpha + \beta)M$ and $wL = \alpha M$ this gives the calibration equation

$$\bar{Y} = \frac{\alpha + \beta}{\alpha} wL$$

for $L$. Note that $L$ is effective labour incorporating regional labour productivity. The wage is also wage per unit of effective labour. As long as we are not interested in wage per worker, no data on labour input is required.

As to assets, the steady state growth rate of $A$ means $\dot{A} = A(1 - 1/\theta)\xi$. Inserting this into (8) and rearranging yields

$$A = Y \frac{1}{\rho - (1 - 1/\theta)\xi} \left( \frac{C}{Y} - \frac{\alpha}{\alpha + \beta} \right).$$

We can infer on $C/Y$ from the trade surplus fulfilling the familiar national accounts identity $S - D = Y - C - J$ following from adding (12) plus (15). Dividing by $Y$ and substituting trade data $\bar{S}$ and $\bar{D}$ for $S$ and $D$ yields the calibration equation for $C/Y$,

$$\frac{\bar{S} - \bar{D}}{Y} = 1 - \frac{C}{Y} - \frac{J}{Y},$$

which then is inserted into (28) to obtain $A$. On the sub-national level we do not have the trade data. We therefore impose the constraint (29) on the national level for countries with regional subdivision and apply uniform ratios $C/Y$ across regions of the same nation.

Finally, no extra data is needed for calibrating capital stocks. Instead, capital stocks follow from the assumption that the benchmark solution is a steady state. Using equations (5) to (7) and (17) as well as the steady state growth rates from Section 5, one can derive the calibration equation

$$Y = \phi p^\rho K$$

with a positive constant $\phi$ only depending on parameters that are uniform across regions and constant over time (i.e. the small type Greek letters). We omit the rather unwieldy expression. With data on capital stocks one could dispense with the steady state assumption and calibrate a non-steady state start point instead. But we neither have such data, nor would we feel the resulting time paths to be terribly convincing. Regions would converge fairly rapidly towards their respective steady states, a pattern that we actually do not see in the data. We simply assume that these adjustments already took place before $t = 0$.

Subsidies and transfers are assumed to stay in place forever. This schedule is common knowledge for all agents in the model, once the subsidy plan is announced, and it is taken as 100% credible by the public. To make the subsidy and the transfer scenarios comparable we design them in a way that makes the nominal amount that the central state pays to a supported region equal under both settings. The point of departure are the amounts of funds available for subsidies, which are known from the actual data (as a total amount for a period of 8 years, see Section 6.2). These nominal
amounts are first distributed along the steady-state time path of the model such that the steady-state growth rate of transfer value is fulfilled and are then further extrapolated into the future. This gives the exogenous levels of \( Z_{rt} \). As a consequence, if the amount of transfers for each year is divided by the corresponding value of gross investment, the resulting ratio is constant along the steady-state time path. This constant ratio represents the subsidy rate \( \Gamma_r \), constant over time. Thus \( \Omega_{rt} = \Gamma_r J_{rt} \), where bar indicates the benchmark level.

7. Simulation results

Our benchmark is a world without any policy intervention. At \( t = 0 \) the introduction of a subsidy or a transfer comes as a full surprise. Subsidy announcement leads to updating of the present value of capital installed, quantified by the stock price \( q \), and thus also of assets held by the households. Thus \( A \) and \( q \) jump at the day of announcement (Figure 1).

The time path of the capital stock illustrates that the subsidy pushes investments in the subsidised regions up, and that this push is largest in the beginning. Capital stock accumulates above the initial benchmark level and converges to a new steady state level (Figure 1, upper panel). If we had lower adjustment cost we would observe a stronger concentration of high investments in the subsidised regions immediately after the shock. When investment jumps up due to support, the trade surplus in the supported regions goes down or trade deficit up (Figure 1, bottom panel).

In the case of the lump sum transfer, there is virtually no reaction on the capital market; prices are largely unaffected. However, the incomes of the households go up in the supported regions, and thus their consumption and savings go up. Additional assets are accumulated and the trade deficit is permanently larger (Figure 2).

As the true initial distribution of the assets is unknown, Table 2 shows regional welfare effects for two extreme variations of these policy scenarios, labelled “global portfolio” and “local portfolio”. “Global portfolio” means that portfolio compositions of all households are identical. Thus each household owns a perfectly diversified portfolio of all assets available worldwide. The perfectly diversified portfolio is the best one to be held by risk averse individuals, if future shocks are unpredictable, but it is likely not what we would observe in practice. “Local portfolio” is the other extreme, where households mainly own the stock of the region they live in. The regions that have a surplus of initial assets over their respective capital stock value are assumed to have invested this difference into a perfectly diversified portfolio.

First of all, Table 2 shows that the positive welfare effects in the supported regions are stronger under the subsidy scenarios than under the transfer scenarios. One source of these positive effects, compared to the transfer scenarios, are the diversity gains (due to the Dixit-Stiglitz structure of the demand for tradable goods). Another channel is the labour market, where the wages are driven up as a reaction to more production activity using the subsidised capital. However, the subsidy is more expensive for the donor countries like Germany. In general, the donor regions, which have low subsidy rates but co-finance subsidies to other regions through taxes, show negative welfare effects.

An interesting observation is that under the conditions of a global portfolio the effects for the supported regions are stronger than under the conditions of a local portfolio (Table 2), on average, 15% higher. Why is that? Figure 1 (middle right panel) nicely reveals the mechanism. The subsidy lets the market value of capital drop. This is an effect that matters in quantitative terms. It is important here to distinguish investment subsidies from subsidising user costs of capital. The latter benefit all capital owners, those owning the existing stock as well as those investing in new stocks. The former harms owners of existing stock who are facing new competition. While regional workers own the complementary factor, capital owners own a competing factor which becomes more abundant
due to lower cost of investment. Thus, when households own the local capital stock which loses value due to the subsidy, their welfare gains are reduced. Also, the welfare losses in the donor regions are lower in the case of global asset ownership.

In Table 3, the relative welfare effects are converted into Euro values. The results suggest that under the conditions of global asset ownership, the investment subsidy has a positive net effect on the EU as a whole. However, based on the sensitivity analysis employing alternative subsidy schemes, we found that this effect only holds true if the economic size of the supported regions is small enough. For the subsidy under the conditions of a local portfolio the net effect on the EU is negative, and for both transfer scenarios, it is close to zero.

The rest of the world experiences negative effects under the subsidy scenarios (which are though small in relative terms, but are large in absolute terms, as Table 3 illustrates). The reason for this is the relocation of investment towards the supported regions in the EU. Under the assumption of global capital markets, the new investments in the rest of the world go down, there is a reduction in capital stock and output in comparison to the benchmark steady state. Temporarily, consumption reduces (in favour of investment in the EU) and the trade surplus increases (due to exports to the EU). Overall, this leads to a cost in terms of global efficiency. In contrast, the transfer scenarios are welfare-neutral for the rest of the world.

The impact of the diversity gains is demonstrated by including results from an alternative model setup, in which the endogenous variety effect à la Dixit-Stiglitz is switched off. This is done by fixing product diversity of tradables supply in each region at their respective benchmark levels in the equations (11) and (13). This then resembles a classical Armington (1969) assumption for trade in heterogeneous goods. Table 4 shows that in the subsidy scenarios the welfare gains for the supported regions shrink slightly. The diversity gains for the supported regions are thus visible, but amount to only 0.1-0.2 per mill in terms of the relative equivalent variation in consumption. Under the conditions of a local portfolio, this small change however makes households better off with a transfer than with a subsidy (last two columns of Table 4). For the donor regions, diversity gains seem to compensate a small part of the welfare costs of the subsidy. For the transfer scenarios, the diversity gains virtually do not play a role.

What can be inferred on the effectiveness of the investment subsidy as a policy instrument from our results? Table 3 compares the nominal values of the welfare gains and the respective EU funds in the first year of the simulation period. Overall, one can see that the regional welfare effects of the subsidy scenario with a global portfolio (column 3) come very close to the size of the funds (column 2). In some cases, the welfare impacts are even somewhat higher (e.g. in Bulgaria and Romania). For the other scenarios, the welfare effects are clearly below the value of the funds. The latter result is consistent with the classical argument on the dead-weight loss imposed by a market intervention.

A final point refers to the robustness of the modelling results with regard to alternative aggregation schemes. Table 5 shows the results for Aggregation 2 (with disaggregated regions in Bulgaria and Romania). We can now compare the results from different model runs. In the bottom part of Table 3, we report the sum over the absolute effects for the Polish regions. The values only negligibly deviate from the results for Poland as a single region in Table 5 (region 19). The same is true for the disaggregated (Table 5) and aggregated (Table 3) versions of Bulgaria and Romania. The results for single-region countries like Latvia or Germany or for the whole world in both simulations are also virtually the same. This demonstrates the robustness of the model.
8. Conclusions

This paper applied a multiregional dynamic model to study the regional welfare impacts of investment subsidies. First, regarding the methodology, the paper shows that a fully consistent neoclassical model with forward looking agents can well be implemented and solved, and that it is a useful tool in policy analysis in general and in the study of investment subsidies in particular. We show that this model is robust to various regional aggregation settings, although the maximum number of regions is currently limited.

Second, as to the policy issue raised in the beginning, we see that investment subsidies are effective in that the supported regions enjoy welfare gains. The size of the welfare effects however depends on the portfolio distribution held by the households. If households own diversified portfolios, we find that the supported regions gain roughly the amounts that are allocated to them in the form of investment subsidies. If they only own local stocks and are the only owners of the local stocks, a part of the money is lost through the drop in share prices. This capital devaluation effect of the investment subsidy has not been described in the literature so far. The diversity gains generated by the model structure are small, but they are enough to make households in the supported regions better off under the subsidy than under the lump sum transfer.

Third, from the point of view of total welfare, the subsidy is not efficient. It can lead to a welfare loss for the EU as a whole and definitely leads to welfare losses in the rest of the world, from where investment flows to the supported EU regions. The inefficiency is higher in the setting without the diversity effects.
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Figure 1: Time paths of key variables in the subsidy scenario (with a local portfolio)
Figure 2: Trade balance and consumption in the transfer scenario (with a local portfolio)
| Region                  | Subsidy rate $\Gamma_r$, per mill | Global portfolio | Local portfolio |
|-------------------------|-----------------------------------|------------------|----------------|
|                         |                                   | Subs  | Lump | Subs  | Lump |
| 1. Łódzkie              | 8.64                              | 3.38  | 2.88 | 2.95  | 2.89 |
| 2. Mazowieckie          | 4.90                              | 1.69  | 1.43 | 1.47  | 1.44 |
| 3. Małopolskie          | 9.62                              | 3.83  | 3.25 | 3.34  | 3.25 |
| 4. Śląskie              | 6.38                              | 2.36  | 2.00 | 2.07  | 2.01 |
| 5. Lubelskie            | 9.50                              | 3.79  | 3.23 | 3.30  | 3.22 |
| 6. Podkarpackie         | 15.14                             | 6.31  | 5.39 | 5.57  | 5.41 |
| 7. Świętokrzyskie       | 10.19                             | 4.08  | 3.47 | 3.59  | 3.49 |
| 8. Podlaskie            | 14.03                             | 5.81  | 4.97 | 5.10  | 4.97 |
| 9. Wielkopolskie        | 6.23                              | 2.30  | 1.95 | 1.99  | 1.95 |
| 10. Zachodniopomorskie  | 7.81                              | 2.98  | 2.56 | 2.57  | 2.56 |
| 11. Lubuskie            | 7.41                              | 2.80  | 2.40 | 2.43  | 2.41 |
| 12. Dolnoslaskie        | 6.00                              | 2.19  | 1.86 | 1.91  | 1.86 |
| 13. Opolskie            | 12.14                             | 4.94  | 4.23 | 4.32  | 4.25 |
| 14. Kujawsko-Pomorskie  | 7.83                              | 3.01  | 2.57 | 2.62  | 2.57 |
| 15. Warminsko-Mazurskie | 11.01                             | 4.43  | 3.80 | 3.88  | 3.80 |
| 16. Pomorskie           | 5.65                              | 2.02  | 1.72 | 1.75  | 1.72 |
| 17. Estonia             | 8.42                              | 3.11  | 2.67 | 2.71  | 2.67 |
| 18. Latvia              | 14.68                             | 5.68  | 4.86 | 4.99  | 4.87 |
| 19. Lithuania           | 7.48                              | 2.46  | 2.09 | 2.15  | 2.09 |
| 20. Germany             | 0.70                              | -0.23 | -0.22| -0.24 | -0.22|
| 21. Rest East EU        | 5.34                              | 2.01  | 1.70 | 1.73  | 1.71 |
| 22. Bulgaria, Romania   | 4.76                              | 1.58  | 1.33 | 1.37  | 1.33 |
| 23. Rest of the EU      | 0.79                              | -0.16 | -0.17| -0.18 | -0.17|
| 24. Balkan states and Turkey | -0.04                        | 0.00  | -0.01| 0.00  | 0.00 |
| 25. Rest of Europe      | -0.07                             | 0.00  | -0.06| 0.00  | 0.00 |
| 26. Rest of the world   | -0.05                             | 0.00  | -0.03| 0.00  | 0.00 |

Table 2: Relative equivalent variation in consumption and investment subsidy rate, per mill (Aggregation 1)
| Region                | Transfer value, $\Omega_{rt}$ | Global portfolio | Local portfolio |
|-----------------------|--------------------------------|------------------|-----------------|
|                       | Subs | Lump | Subs | Lump |
| 1. Lódzkie            | 58   | 60   | 52   | 51   |
| 2. Mazowieckie        | 121  | 109  | 95   | 93   |
| 3. Małopolskie        | 79   | 83   | 72   | 70   |
| 4. Slaskie            | 91   | 89   | 78   | 76   |
| 5. Lubelskie          | 40   | 42   | 37   | 36   |
| 6. Podkarpackie       | 62   | 68   | 60   | 58   |
| 7. Swietokrzyskie     | 28   | 29   | 26   | 25   |
| 8. Podlaskie          | 35   | 38   | 33   | 32   |
| 9. Wielkopolskie      | 64   | 62   | 54   | 53   |
| 10. Zachodniopomorskie| 33   | 33   | 28   | 28   |
| 11. Lubuskie          | 18   | 18   | 15   | 15   |
| 12. Dolnoslaskie      | 57   | 54   | 47   | 46   |
| 13. Opolskie          | 28   | 30   | 26   | 26   |
| 14. Kujawsko-Pomorskie| 39   | 39   | 34   | 33   |
| 15. Warminsko-Mazurskie| 33  | 35   | 31   | 30   |
| 16. Pomorskie         | 35   | 33   | 29   | 28   |
| 17. Estonia           | 41   | 42   | 36   | 36   |
| 18. Latvia            | 88   | 96   | 84   | 82   |
| 19. Lithuania         | 68   | 69   | 60   | 58   |
| 20. Germany           | 546  | -404 | -399 | -432 | -399 |
| 21. Rest East EU      | 645  | 599  | 508  | 517  | 509  |
| 22. Bulgaria, Romania | 239  | 215  | 181  | 187  | 182  |
| 23. Rest of the EU    | 2162 | -1139| -1161| -1282| -1164|
| 24. Balkan states and Turkey | -22 | -1  | -8   | 0   |
| 25. Rest of Europe    | -92  | 0    | -85  | 0   |
| 26. Rest of the world | -1241| 2    | -849 | 0   |

| Poland total          | 820  | 821  | 699  | 717  | 701   |
| East EU               | 1902 | 1842 | 1565 | 1601 | 1568  |
| West EU               | 2708 | -1544| -1560| -1715| -1563 |
| All EU                | 4610 | 299  | 5    | -114 | 5     |
| World total           | 4610 | -1056| 6    | -1055| 6     |

Table 3: EV results of Table 2 transformed to absolute amounts at $t = 0$ and transfers at $t = 0$, million Euro (Aggregation 1)
| Region               | Global portfolio | | Local portfolio | |
|----------------------|------------------|------------------|------------------|------------------|
|                      | Subs | Lump | Subs | Lump |                      | Subs | Lump | Subs | Lump |                      | Subs | Lump | Subs | Lump |                      | Subs | Lump | Subs | Lump |                      | Subs | Lump | Subs | Lump |                      |
| 1. Łódzkie           | 58   | 51   | 50   | 52   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 2. Mazowieckie       | 105  | 93   | 91   | 94   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 3. Małopolskie       | 80   | 71   | 70   | 72   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 4. Ślaskie           | 86   | 76   | 74   | 76   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 5. Lubelskie         | 40   | 36   | 35   | 36   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 6. Podkarpackie      | 66   | 59   | 58   | 59   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 7. Świętokrzyskie    | 28   | 25   | 25   | 25   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 8. Podlaskie         | 36   | 32   | 32   | 33   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 9. Wielkopolskie     | 60   | 53   | 51   | 53   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 10. Zachodniopomorskie | 31   | 28   | 27   | 28   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 11. Lubuskie         | 17   | 15   | 15   | 15   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 12. Dolnoslaskie     | 52   | 47   | 45   | 47   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 13. Opolskie         | 29   | 26   | 25   | 26   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 14. Kujawsko-Pomorskie | 38   | 34   | 32   | 34   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 15. Warminsko-Mazurskie | 34   | 30   | 29   | 30   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 16. Pomorskie        | 32   | 28   | 27   | 28   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 17. Estonia          | 39   | 36   | 35   | 35   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 18. Latvia           | 93   | 83   | 81   | 83   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 19. Lithuania        | 66   | 59   | 57   | 59   |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 20. Germany          | -428 | -403 | -458 | -404 |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 21. Rest East EU     | 576  | 513  | 492  | 515  |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 22. Bulgaria, Romania | 207  | 183  | 177  | 184  |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 23. Rest of the EU   | -1235| -1172| -1386| -1178|                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 24. Balkan states and Turkey | -21  | 0    | -7   | 0    |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 25. Rest of Europe   | -87  | 0    | -80  | 0    |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| 26. Rest of the world | -1190| 1    | -785 | 0    |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| East EU              | 1773 | 1580 | 1529 | 1586 |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| West EU              | -1664| -1575| -1844| -1581|                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| All EU               | 110  | 4    | -314 | 5    |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |
| World                | -1188| 5    | -1187| 5    |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |       |      |      |      |                      |

Table 4: Equivalent variation results transformed to absolute amounts at $t = 0$, million Euro (Aggregation 1, no variety effect)
| Region                  | Global portfolio |          | Local portfolio |          |
|-------------------------|------------------|----------|-----------------|----------|
|                         | Subs  | Lump | Subs  | Lump |
| 1. Severozapaden        | 4     | 3    | 3     | 3    |
| 2. Severen tsentralen   | 7     | 6    | 6     | 6    |
| 3. Severoiztochen       | 3     | 3    | 3     | 3    |
| 4. Yugoiztochen         | 7     | 6    | 6     | 6    |
| 5. Yugozapaden          | 16    | 13   | 13    | 13   |
| 6. Yuzhen tsentralen    | 14    | 12   | 13    | 12   |
| 7. Nord-Vest            | 30    | 25   | 26    | 26   |
| 8. Centru               | 28    | 24   | 25    | 24   |
| 9. Nord-Est             | 23    | 19   | 20    | 19   |
| 10. Sud-Est             | 9     | 7    | 7     | 7    |
| 11. Sud - Muntenia      | 26    | 22   | 22    | 22   |
| 12. Bucuresti - Ilfov   | 20    | 16   | 17    | 17   |
| 13. Sud-Vest Oltenia    | 17    | 15   | 15    | 15   |
| 14. Vest                | 12    | 10   | 10    | 10   |
| 15. Greece, Cyprus      | 247   | 208  | 216   | 210  |
| 16. Estonia             | 42    | 36   | 36    | 36   |
| 17. Latvia              | 96    | 82   | 84    | 82   |
| 18. Lithuania           | 68    | 58   | 60    | 58   |
| 19. Poland              | 823   | 697  | 719   | 699  |
| 20. Germany             | -404  | -399 | -433  | -399 |
| 21. Rest East EU        | 509   | 508  | 516   | 508  |
| 22. Rest West EU        | -1390 | -1370| -1499 | -1374|
| 23. Balkan and Turkey   | -22   | 0    | -8    | 0    |
| 24. Rest of Europe      | -92   | 0    | -85   | 0    |
| 25. Rest of the world   | -1240 | 2    | -849  | 1    |
| Bulgaria, Romania       | 215   | 182  | 187   | 182  |
| East and South-East EU  | 2091  | 1771 | 1818  | 1776 |
| West EU                 | -1794 | -1770| -1932 | -1773|
| All EU                  | 297   | 2    | -114  | 2    |
| World                   | -1057 | 3    | -1056 | 3    |

Table 5: Equivalent variation results transformed to absolute amounts at $t = 0$, million Euro (Aggregation 2)