Abstract

We propose a general scheme for constructing models in which the Standard Model (SM) gauge interactions are the mediators of supersymmetry breaking to the fields in the supersymmetric SM, but where the SM gauge groups couple directly to the sector which breaks supersymmetry dynamically. Despite the direct coupling, the models preserve perturbative unification of the SM gauge coupling constants. Furthermore, the supergravity contributions to the squark and slepton masses can be naturally small, typically being much less than 1% of the gauge mediated (GM) contributions. Both of these goals can be achieved without need of a fine-tuning or a very small coupling constant. This scheme requires run-away directions at the renormalizable level which are only lifted by non-renormalizable terms in the superpotential. To study the proposed scheme in practice, we develop a modified class of models based on $SU(N) \times SU(N - 1)$ which allows us to gauge a $SU(N - 2)$ global symmetry. However, we point out a new problem which can exist in models where the dynamical supersymmetry breaking sector and the ordinary sector are directly coupled – the two-loop
renormalization group has contributions which can induce negative $(\text{mass})^2$ for the squarks and sleptons. We clarify the origin of the problem and argue that it is likely to be surmountable. We give a recipe for a successful model.
1 Introduction

Low-energy supersymmetry is widely regarded as the most promising stabilization mechanism for the hierarchy between the Planck scale and the weak scale (for a review see [1]). However, supersymmetry by itself does not necessarily generate a hierarchy. If the hierarchy is to be explained, it most likely has to result from the breaking of supersymmetry by non-perturbative dynamics so that the existence of a small scale is naturally understood by dimensional transmutation [2]. This motivates the study of dynamical supersymmetry breaking in supersymmetric gauge theories [3].

The scheme most commonly considered in the literature assumes that supersymmetry is broken dynamically in a hidden sector whose coupling to the Standard Model (SM) is solely due to Planck-scale suppressed interactions [4]. The motivation for such ideas is increased since gravity automatically provides such a coupling, and, furthermore it seems that string theory naturally accommodates hidden sectors. However, there has been a growing realization that string theory, or supergravity alone, does not automatically possess a phenomenologically viable mechanism to maintain the degeneracy between sfermions of the same gauge quantum numbers. This degeneracy is the simplest way in which the stringent constraints from flavor-changing neutral currents and other rare processes can be met [5].

However, if supersymmetry breaking is dominantly communicated to the supersymmetric standard model (SSM) by the SM gauge interactions themselves, then the required degeneracy is automatic, given that the ever-present supergravity contributions are negligible. This highly appealing mechanism of gauge mediation (GM) guarantees enough suppression of flavor-changing neutral currents [6].

Explicit, and quite complete, models based on gauge mediation have been constructed in the past few years [7, 8, 9]. These models achieve the gauge mediation of supersymmetry breaking in the following way. The sector which breaks supersymmetry dynamically (the DSB sector) has a non-anomalous global symmetry which is weakly gauged (the messenger gauge group). The exchange of the messenger gauge multiplet induces supersymmetry breaking effects in a second sector (the messenger sector). Concretely, this messenger sector contains at minimum a gauge singlet chiral superfield $S$, and a set of fields which transform under the standard model gauge groups in a vector-like manner (messenger fields). The supersymmetry breaking effects induce expectation values for both the $A$- and $F$-components of $S$. The coupling of $S$ to the vector-like messenger fields then in turn induces, via these messenger fields, the SSM gaugino and soft scalar masses via one- and two-loop diagrams, respectively. We will refer to this class of models as Original Gauge Mediation (OGM) Models.

Even though the final stage of SM gauge mediation is itself quite appealing, the OGM models seem rather contrived in their use of a cascade of different interactions to communicate supersymmetry breaking to the SSM. In fact, the authors of Refs. [7, 8, 9] were forced to introduce the messenger sector to “insulate” the standard model from
the dynamical sector so as to maintain the perturbative SM gauge coupling unification as indicated by the data from the LEP/SLC experiments. If one couples the DSB to the SSM sector directly via the standard model gauge interactions, there tends to be a large multiplicity of new fields charged under the SM, so that the SM gauge coupling constants reach their Landau poles well before unification. Moreover, if one wishes to avoid a color-breaking minimum of the potential [10, 11], the messenger sector has to be even more complicated than originally thought. Also as we will discuss in Section 2, the OGM models suffer from a number of phenomenologically uncomfortable features. Thus, given the recent progress in understanding supersymmetric gauge theories (for a review see [12]), it is natural to ask whether one can find a more elegant scheme based on the new DSB models recently constructed [9, 13, 14, 15, 17, 18, 19, 20, 21, 22].

In this paper, we study a general scheme in which the DSB sector is coupled directly to the SM gauge interactions, while maintaining all the attractive features of the basic GM idea. Beyond the simplicity of the direct coupling, we aim to achieve three other ambitious goals: (1) the preservation of perturbative gauge unification, (2) the natural suppression of supergravity contributions to the sfermion masses so as to maintain degeneracy, (3) no fine-tuning of parameters or very small coupling constants. The basic idea is to find a model which has a classical flat direction, $X$, not lifted at the renormalizable level, but which allows a non-renormalizable operator in the superpotential that does lift $X$. The direction $X$ then acquires a large expectation value while the theory maintains only a small vacuum energy. This results in a natural hierarchy $\langle F_X \rangle \ll \langle X \rangle^2$. If it is arranged that the fields charged under the SM acquire masses due to $\langle X \rangle$, their contribution to the running of the gauge coupling constants appears only above $\langle X \rangle$ and hence perturbative gauge unification can be preserved. On the other hand, the superparticle masses generated by GM are proportional to the ratio $\langle F_X \rangle / \langle X \rangle$ and can be kept at the phenomenologically desired value $\langle F_X \rangle / \langle X \rangle \sim 10^4$ GeV. As an added bonus, we point out that such a scheme overcomes many of the phenomenologically undesirable characteristics of the OGM models. A discussion of these features is contained in Section 6.

In fact, Poppitz and Trivedi [23] have recently built a model along the general lines stated above in which the dynamical sector and the standard model groups are directly coupled, while keeping the gauge coupling constants perturbative up to the GUT-scale. Unfortunately their model suffers from two severe problems. The first is that supergravity mediated soft masses are of the same order of magnitude as the gauge-mediated soft terms, and thus sfermion degeneracy is not naturally guaranteed, spoiling the original motivation for GM.

We demonstrate that this problem can be overcome by developing a modified class of DSB models based on a $SU(N) \times SU(N - 1)$ gauge group, that allows a $SU(N - 2)$ global symmetry to be weakly gauged. To achieve this it is necessary to add new fields to the models of Poppitz, Shadmi and Trivedi [15]. It is non-trivial that at the quantum level the resulting additional classical flat-directions are lifted, and DSB is
maintained. We perform detailed analyses of the potential and the particle spectrum, and it is demonstrated that this class of models achieves the three goals listed above. In particular, even though $\langle X \rangle$ is close to the GUT-scale, the modified models which we present naturally suppress the supergravity contributions to the sfermion masses.

However, as a consequence of our analysis, we discover a new problem that unfortunately afflicts both our models and those of Poppitz and Trivedi. Explicitly, there exist at two-loops, contributions to the renormalization groups equations of SSM scalar (mass)$^2$ that drive them negative at the weak scale. This problem appears to be model-dependent, and hinges upon the details of exact numerical coefficients. It occurs only in models where there are light ($\sim 10^4$ GeV) fields charged under the standard model gauge groups which directly originate in the DSB sector, and which thereby acquire a large soft-SUSY-breaking scalar mass of order $(10^4$ GeV)$^2$. We believe that this is not a generic consequence of our proposed scheme, and given the rapidly growing list of DSB models, theories fulfilling the three goals without generating particles charged under the SM gauge groups with large SUSY-breaking masses will be found.

The organization of the paper is as follows. In Section 2 we review the basics of the original gauge mediation models, and discuss a number of their phenomenologically less desirable features, some of which do not seem to be widely appreciated. In Section 3 we outline previous attempts at simplifying the structure of the OGM models. Section 4 explains in greater detail the structure of our proposed scheme, readers who are only interested in the requirements on DSB models which allow the direct coupling to the ordinary sector should go directly to this section. Section 5 contains the $SU(N) \times SU(N-1)$ model and its analysis, and demonstrates that it fulfills our three goals. In Section 6 we address the phenomenologically favorable aspects of this general scheme. Section 7 discusses the problem that our $SU(N) \times SU(N-1)$ models, as well as those of Poppitz and Trivedi, have with large negative contributions to the SSM sfermion (mass)$^2$. We conclude in Section 8, while a long appendix contains many details of the modified $SU(N) \times SU(N-1)$ models.

2 Original Gauge Mediation Models

In this section we review the OGM models of Dine, Nelson and collaborators, and then go on to discuss various phenomenological problems of these models. Let us first review the mechanism of gauge mediation itself. Consider $N$ vector-like multiplets $q_i$ and $\bar{q}_i$ all transforming as $3 + 3^*$ under color $SU(3)$. Suppose that they acquire an invariant supersymmetric mass

$$W = \langle S \rangle \bar{q}_i q_i$$  \hspace{1cm} (2.1)

due to the $A$-component of a chiral superfield $S$ gaining an expectation value, as well as a supersymmetry-breaking bilinear mass term

$$V = \langle F_S \rangle (\bar{q}_i \bar{q}_i + c.c.)$$  \hspace{1cm} (2.2)
in the potential, due the $F$-component of $S$ also gaining an expectation value as a result of some dynamics. Here $\tilde{q}$ and $\tilde{q}$ represent the scalar components of the corresponding chiral superfields. Integrating out these vector-like multiplets generates gluino and squark masses,

$$M_{\tilde{g}} = \frac{N\alpha_s}{4\pi}B,$$

$$m_{\tilde{q}}^2 = 2NC_2 \left(\frac{\alpha_s}{4\pi}\right)^2 B^2,$$

where $B \equiv \langle F_S \rangle / \langle S \rangle$. Here $C_2 = 4/3$ is the second-order Casimir invariant for the relevant representation. Similar results hold for the SM states charged under $SU(2)$ and $U(1)$ once vector-like messengers carrying these quantum numbers are similarly included.

There are two important points to note about this mechanism. First, for $N \simeq 1$, the gaugino and scalar masses are generated with comparable magnitudes, which is phenomenologically desirable. Second, the scalar masses are flavor-blind, i.e. the same for scalars of the same gauge quantum numbers in different generations. This is vital if one wishes to suppress the phenomenologically dangerous flavor-changing effects mediated by sfermion loops. Finally, the typical size of the supersymmetry breaking for the messengers is given by $\langle F_S \rangle$ in such models. It is noteworthy that this size of supersymmetry breaking is itself not important in estimating the squark, slepton or gaugino masses; rather, they are functions of the ratio $B$ only. For phenomenologically desirable gaugino and sfermion masses we therefore require $B \simeq 60$ TeV, for $N = 1$, with $B$ correspondingly reduced for larger values of $N$.

On the other hand, the gravitino mass is sensitive to the fundamental size of supersymmetry breaking. In the simplest and most successful OGM models, the DSB sector has a $U(1)$ global symmetry which can be weakly gauged (messenger $U(1)$). If extra fields are introduced which are charged under this $U(1)$ and which also couple in the superpotential to $S$, then it is possible at two-loops for the scalar components of these charged fields to pick up soft (mass)$^2$ terms, which in turn lead to the $A$ and $F$ components of $S$ gaining vacuum expectation values. Thus, even if we assume large values for the messenger $U(1)$ gauge coupling, the fundamental scale of supersymmetry breaking must be $\Lambda \simeq 5,000$ TeV, or larger, in these models. Thus the gravitino mass, which is given by $m_{3/2} \simeq \Lambda^2/M_s$, where $M_s = M_{\text{Planck}}/\sqrt{8\pi}$ is the reduced Planck mass, is not simply a function of $B$.

Before going on to attempts to simplify this structure, we wish to review some of the physics problems of the OGM models, not just related to aesthetics.

**µ-problem.** The so-called $\mu$-term, the invariant mass term for the Higgs doublets in the superpotential, has to be of the same order as the supersymmetry breaking parameters if we are to naturally explain the stability of the weak scale, even though it is allowed by supersymmetry. Now, dynamical supersymmetry breaking can potentially
explain why \( \mu \) is of order the weak scale if the \( \mu \)-term is also generated dynamically. However, the particular implementations of this idea discussed so far look even more contrived than the OGM models [9, 24]. This is due to a combination of different problems. The most naive attempts tend to generate too large a supersymmetry breaking \( \mu \)-term (sometimes called \( m_3^2 \) or \( B\mu \)) for an appropriate size of \( \mu \). A coupling to a singlet Higgs field appears to be the next thing to try; however it usually results in a light axion-like pseudo-Nambu-Goldstone boson in the Higgs spectrum and is excluded by the \( Z \)-decay experiments [7]. The reason for the existence of this light pseudo-scalar is an approximate \( R \)-symmetry in the Higgs sector superpotential which exists for any strictly trilinear couplings. The \( R \)-symmetry is explicitly broken by trilinear supersymmetry breaking terms, which are induced by gaugino masses at one-loop; however their magnitudes turn to be too small because of the limited amount of running between the messenger scale and the weak scale in models with messengers at the \( 10^4 - 10^5 \) GeV scale.

**Electroweak symmetry breaking.** It is an interesting fact that radiative electroweak symmetry breaking can work within the OGM models. Even though the logarithm of the messenger scale to the weak scale is not large, the Higgs boson mass squared \( m_2^2 \) is driven negative because the squarks acquire much larger masses than the Higgs bosons in the gauge mediated scenarios. However, the resulting mass is too negative [11], of the order of \( m_2^2 \approx -(500 \text{ GeV}^2) \). This is to be compared to the value required by electroweak symmetry breaking: \( \mu^2 + m_2^2 = -m_2^2/2 = -(70 \text{ GeV}^2) \) for a moderately large \( \tan \beta \). It is possible to fine-tune the negative mass-squared of the Higgs with a large positive \( \mu^2 \), but a displeasing fine-tuning at the level of a percent seems unavoidable. (And clearly this question is then linked with the first question of how to generated the \( \mu \)-term.)

**Global minima of the messenger potential.** The superpotential of the messenger sector in OGM models requires us to sit at a local, rather than a global, minimum of the potential [10, 11]. Specifically, the vector-like messenger fields have a \( D \)-flat direction under the standard model gauge group, which has a much lower energy than the desired minimum. One can complicate the messenger sector by introducing another singlet or other fields so as to make the desired minimum the global one, but this certainly makes the model even more baroque.

**ll\( \gamma \gamma \) events.** It has been claimed that within the OGM-like models the lightest neutralino or sleptons can decay into the gravitino and a photon/lepton (possibly involving a cascade of decays) within the Fermilab collider detectors, leaving a distinct experimental signature (of photon(s) plus missing \( E_T \)) as compared to hidden sector supergravity mediated models. In particular the presence of two \( ll\gamma \gamma \) events within Run I of the Tevatron data has generated understandable interest. Unfortunately, we are not aware of any consistent models which actually give a low enough gravitino decay constant so as to allow the lightest neutralino or slepton to decay inside the detector.
The only candidate is a vector-like model directly coupled to the messengers [17], which relies on a local minimum and a dynamical assumption.

**Exotic stable particles.** The DSB sector and messenger sectors tend to possess stable particles, some of which are charged. Even though it is just possible that they might be dark matter candidates, they tend to overclose the Universe as their mass scale is quite high [25]. Moreover, there are very strong constraints on the abundance of stable charged particles [26].

**R-axion.** Most of models which break supersymmetry dynamically have an exact but spontaneously broken $U(1)_R$ symmetry. They therefore produce a massless Nambu–Goldston boson. A combination of light meson decays, beam dump experiments, quarkonium decay, the population of red giants or white dwarfs in globular clusters, and the duration of the supernova 1987A neutrino burst put lower bounds on the scale of $U(1)_R$ symmetry breaking of around $10^{8}$ GeV. However, it was argued that the cosmological constant must be cancelled by introducing a constant term in the superpotential which thereby breaks the $U(1)_R$ symmetry explicitly. This explicit breaking can generate an $R$-axion mass which avoids astrophysical constraints [27]. It is, however, somewhat displeasing that the solution depends on this mechanism of cancelling the cosmological constant, given that we understand so little about why it is so small. Other solutions to the cosmological constant problem, such as no-scale supergravity, may not solve the R-axion problem [3].

**Cosmology.** Even though it is correct that the OGM models do not suffer from the Polonyi-problem [28], one of the major cosmological problems of the supergravity-mediated hidden sector scenarios, there exist other cosmological difficulties in these models. Specifically, a gravitino with mass of order 100 keV is expected in the OGM models. This is in the most disfavored mass range from the cosmological point of view [29]. If the scenario is further regarded as a low-energy limit of superstring theory, the string dilaton/moduli also have very light masses of order 100 keV. For such masses they are stable for cosmological time scales and their coherent oscillations vastly overclose the Universe by 15 orders of magnitude [30], unless a period of keV-scale inflation is invoked with its attendant problems.

## 3 Previous Attempts

In this section we review some previous attempts to simplify the OGM models discussed in Section 2. The most interesting of these in our opinion is due to Poppitz and Trivedi [23], which we discuss in the next section.

Only a few attempts to couple the standard model gauge group directly to the DSB sector have been made because of the following problems. In order for the dynamical sector to have a large enough global symmetry, a subgroup of which is identified as the
SM gauge group, the gauge group in the dynamical sector tends to become very large. This results in a large number of extra matter fields transforming non-trivially under the SM, leading to a Landau pole for the SM gauge couplings only slightly above the messenger scale. Also the supergravity contribution to the squark and slepton masses must be small enough compared to the contribution from GM to guarantee sufficient sfermion degeneracy – the main motivation for GM of supersymmetry breaking.

The existence of a class of models in which supersymmetry is dynamically broken and which accommodate relatively large global symmetries has long been known. The \(SU(2k + 1)\) models with an anti-symmetric tensor \(A^{\alpha\beta}\) and \((2k - 3)\) anti-fundamentals \(\bar{F}_i^\alpha\), are the classic examples constructed by Affleck, Dine, and Seiberg \[3\] which break supersymmetry dynamically.\[\ast\] The superpotential which lifts all classical flat directions, \(W = \lambda_{ij} A^{\alpha\beta} \bar{F}_i^\alpha \bar{F}_j^\beta\) can preserve an \(SP(k - 2)\) symmetry if the \(\lambda_{ij}\) coupling constant matrix is proportional to the symplectic matrix \(J\), \(J^2 = -1\). In particular, the \(SU(k - 2)\) subgroup of the \(SP(k - 2)\) global symmetry is anomaly free and can be identified with a part of the standard model gauge group.\[\dagger\] If we wish to embed color \(SU(3)\) into the global symmetry, the minimal size of the DSB gauge group is \(SU(11)\). If we wish to embed the full \(SU(5)\) extension of the SM gauge group, we need \(SU(15)\). Such large gauge groups result in an addition of 11 or 15 vector-like color-triplet quarks to the standard model, respectively. Note, on the other hand, that the size of the supersymmetry breaking effects in the mass spectrum is expected to be comparable to the scale of all masses in these models because there is only one scale in the problem: the scale parameter \(\Lambda\) of the DSB gauge group. Then this scale must be around \(10^4\) GeV in order to generate squark and slepton masses of the desired size \(\sim 10^2 - 10^3\) GeV, and as a consequence the color gauge coupling constant blows up at scales \(5 \times 10^7\) GeV, or \(3 \times 10^6\) GeV, respectively.\[\ddagger\]

In principle one can solve this problem by employing a small coupling constant: if one coupling constant in the superpotential is much less than unity, the situation is similar to models with multiple scales. Even though there may be various ways in which a small coupling constant might be obtained in a natural way \[3\], these models are not truly satisfactory at this stage.

\[\ast\] This class of models is non-calculable because they do not have classical flat directions along which one can analyze the theory semi-classically. However, the Witten index of these models can be shown to vanish by adding a massive vector-like field and so the theories are likely to break supersymmetry.\[\dagger\] Various subgroups of the models also allow models of supersymmetry breaking, e.g., \(SU(2k) \times U(1)\), \(SU(2k - 4) \times SU(5) \times U(1)\), or \(SU(2k - 3) \times SU(4) \times U(1)\).\[\ddagger\] The full \(SP(k - 2)\) can be gauged only with the inclusion of additional fields in the fundamental of \(SP(k - 2)\) so as to avoid Witten’s global anomaly.

\[\ddagger\] The apparent blow-up of SM gauge coupling constants may not necessarily be a disaster, if one can regard such models as low-energy effective descriptions of other theories which are ultraviolet safe. Such a situation may arise if the SM gauge groups are dual to asymptotically free gauge groups, or if they are embedded into much larger groups which are asymptotically free. Unfortunately, no realistic example of this is known.
There have also been other attempts to simplify the structure of the OGM models. The idea has been to couple the DSB sector and vector-like messenger fields directly in the superpotential, thereby eliminating the messenger gauge field. Unfortunately, this direction has not been successful either. One possibility was discussed by Intriligator and Thomas [17], based on the vector-like DSB models [16, 17]. The vector-like messenger fields couple to the O’Raifeartaigh-like singlet field in the DSB sector in the superpotential, and obtain a supersymmetry breaking soft mass term. However, this model has a color-breaking supersymmetric minimum of the potential, and the desired supersymmetry breaking minimum is not absolutely stable. Even if one accepts such an unstable local minimum, a dynamical assumption is needed to ensure that the singlet field develops an expectation value in its $A$-component, necessary to generate the invariant mass for the messengers. This region of field space is strongly coupled, and there is no control over the Kähler potential, so that it is impossible to tell whether or not such an expectation value develops. Another proposal by Hotta, Izawa and Yanagida [32] also tried to eliminate the messenger gauge field by coupling the messenger fields to the vector-like model in the superpotential. However, in order to break supersymmetry, they had to assume a non-generic superpotential not justified by any symmetry, as well as making some other dynamical assumptions.

4 The scheme

Here we describe our simple DSB scheme which maintains perturbative gauge coupling unification by naturally generating a large invariant mass for the fields that act as messengers, while keeping $B \simeq 10^4$ GeV using non-renormalizable terms in the superpotential. The scheme we propose has three rather ambitious goals: (1) the preservation of perturbative gauge unification, (2) the natural suppression of supergravity contributions to the sfermion masses so as to maintain degeneracy, (3) no fine-tuning of parameters or very small coupling constants.

In most models which break supersymmetry dynamically, there is a non-perturbative superpotential which forces fields to move away from the origin. By lifting all flat directions at the classical level, by adding suitable (renormalizable) terms to the superpotential, the balance between the non-perturbative and tree-level terms determines a stable minimum of the potential with a finite vacuum energy. In contrast, suppose that the model has some flat directions $X$ which are not lifted by the superpotential at the renormalizable level. Then the vacuum runs away from the origin along such directions. This runaway behavior can be stopped by the possible addition of non-renormalizable terms. Therefore, the size of the field expectation values along such flat directions, $\langle X \rangle$, can be large while the vacuum energy, $\langle F_X \rangle^2$, stays small. On the other hand, such a large expectation value can give large masses to other fields if they have renormalizable couplings to $X$. In particular it is possible that
the fields which acquire masses from $X$ act effectively as messengers.

As mentioned above, non-renormalizable models have already been utilized by Pope-pitz and Trivedi [23] to couple the DSB sector directly to the SM. For example, they considered a $SU(13) \times SU(11)$ model, which has an $SP(5)$ global symmetry into which a weakly gauged $SU(5)$ can be embedded. They found that this model allows a direct coupling between the DSB sector and the SM gauge groups, while maintaining the perturbative SM gauge coupling unification. Unfortunately the models suffered from the problem that both $\langle X \rangle$ and $\langle F_X \rangle$ were too large, allowing the supergravity contribution to the sfermion masses to dominate.

Concretely, in their model, vector-like fields are generated at a mass scale of $\sim \langle F_X \rangle / \langle X \rangle$, which have both invariant mass and supersymmetry breaking bilinear mass terms. The gauge-mediated masses for sfermions due to loops of $N$ vector-like messenger fields scale as $\sim \sqrt{N} (\alpha/4\pi) \langle F_X \rangle / \langle X \rangle$, and need to be around 100 to 1000 GeV. A large $\langle X \rangle$ thus implies a large $\langle F_X \rangle$. On the other hand, a large $\langle F_X \rangle$ generates supergravity contributions to the scalar masses of order the gravitino mass, $m_{3/2} \sim \langle F_X \rangle / M_*$. To retain squark and slepton degeneracy, which is the primary motivation for gauge mediation, one needs to suppress $m_{3/2}$ to be at most 10% of the gauge-mediated contribution. This leads to an upper bound on $\langle X \rangle$,

$$\langle X \rangle \lesssim 0.1 \sqrt{N} \frac{\alpha}{4\pi} M_* \simeq 2 \times 10^{15} \text{ GeV}$$

if we take $N \simeq 10$ and $\alpha \simeq 1/25$. Their model gives $\langle X \rangle \sim 6 \times 10^{16} \text{ GeV}$. As we will show in the following sections, our models satisfy this bound.

A general scheme for constructing a simple model of gauge mediation is clear after these considerations. Find a model where not all of the classical flat directions are lifted at the renormalizable level. The model must have a relatively large global symmetry which can be gauged. Add suitable non-renormalizable terms to the superpotential such that all flat directions are lifted. Then the supersymmetry is broken with a natural suppression of $\langle F_X \rangle \ll \langle X \rangle^2$ so that the masses of extra fields coupled to the standard model are heavy $\sim \langle X \rangle$ while keeping the size of induced supersymmetry breaking small enough, at $\langle F_X \rangle / \langle X \rangle \sim 10^4 \text{ GeV}$. A constraint here is that the non-renormalizable operators should not be of too high a dimension. This was the reason the $SU(13) \times SU(11)$ model failed in generating too large a supergravity contribution to the scalar masses. We need to keep the size of $A$-component expectation value $\langle X \rangle$ to be within the range Eq. (4.1).

In fact, the phenomenological requirements can be conveniently summarized in terms of the dimensionality of the operator which lifts the relevant flat direction. Sup-
pose the flat direction $X$ is lifted by an operator of mass dimension $n$, such that the non-renormalizable operator in the superpotential is $W \sim \phi^n / M_*^{n-3}$ with $\phi$ standing generically for chiral superfields. Then we obtain the following crude estimate of the scales by requiring $B \equiv \langle F_X \rangle / \langle X \rangle \simeq 10^4$ GeV,

\[ \langle X \rangle \sim M_* \left( \frac{B}{M_*} \right)^{1/(n-2)}, \quad (4.2) \]

\[ \langle F_X \rangle \sim B M_* \left( \frac{B}{M_*} \right)^{1/(n-2)}, \quad (4.3) \]

and

\[ m_{3/2} \sim \frac{\langle F_X \rangle}{M_*} \sim B \left( \frac{B}{M_*} \right)^{1/(n-2)}. \quad (4.4) \]

We would like to keep $\langle X \rangle$ large enough to maintain perturbative unification. In the one-loop renormalization group analysis, one obtains $\langle X \rangle \gtrsim 10^{16-60/N}$ GeV for $N$ extra $5 + 5^*$ pairs. The constraint from a two-loop analysis tends to be somewhat stronger than this. On the other hand, we need to keep $m_{3/2} \lesssim 100$ GeV or so to have TeV-squarks naturally degenerate at a percent level. Therefore the dimensionality of the operator should satisfy

\[ 2 + \frac{14.4}{2 + 60/N} \lesssim n \lesssim 9.2. \quad (4.5) \]

It should be noted that the precise constraints depend on details of the models (such as an accidental cancellations, existence of many operators with different dimensionalities, etc) and the upper and lower bounds above are only ball-park numbers. Furthermore, the upper bound depends on the assumption that the non-renormalizable operators are suppressed by the reduced Planck scale $M_*$. If the suppression is by a lower scale $\bar{M}$, the upper bound becomes weaker, $n \lesssim 2 + (14.4 - \log_{10}(M_*/\bar{M})) / (2 - \log_{10}(M_*/\bar{M}))$.

However, as we will discuss at length later, the particular model which we will present in Section 5, as well as the models of Poppitz and Trivedi, suffer from a serious problem. The squark and slepton masses are driven negative due to two-loop contributions to their renormalization group evolution. The origin of this problem is clear. Specifically, these contributions are due to fields charged under the SM arising in the DSB sector, with masses of around 10 TeV, and which in addition acquire large soft supersymmetry breaking masses also at the 10 TeV scale. We do not consider this a problem of the scheme itself. If we could find a model which does not produce light charged fields, the problem can be trivially circumvented. We believe that the scheme we discussed can be realized while avoiding the problem of negative squark, slepton masses after more exploration for models which break supersymmetry dynamically.

The recipe for the construction of a successful model can therefore be easily summarized. The DSB sector should ideally have the following features.
1. It must accommodate a large global symmetry, such as $SU(5)$ or at the very least
$SU(3)$.\footnote{Even with a model $M$ which allows only an $SU(3)$ global symmetry, gauge unification may be achieved through triplicating the model as $M^3/Z_3$ and using trinification, where the SM gauge group is embedded in $SU(3)^3/Z_3$.}

2. It leaves some flat directions unlifted at the renormalizable level.

3. The addition of non-renormalizable operators lifts all flat directions and the model breaks supersymmetry dynamically. The dimensionality of the non-renormalizable operators should satisfy the constraint Eq. (4.5).

4. All directions with non-trivial quantum numbers under the standard model gauge group are lifted at the renormalizable level to avoid light charged fields which drive the sfermion masses negative via two-loop running effects.

Once one finds a model of DSB which satisfies the above criteria, a direct coupling of the DSB sector and the SM gauge groups is possible and achieves the three goals we desire. Moreover, the model would not have a problem of negative sfermion masses as will be described in Section 6 and hence is phenomenologically viable. Finally, the model would have many phenomenologically desirable features compared to the OGM models as discussed in Section 6.

5 A Model

In this section, we describe an example of a model along the lines of the scheme proposed in Section 4. The model is based on an $SU(7) \times SU(6)$ gauge group. It generates a large $\langle X \rangle$ while naturally keeping $\langle F_X \rangle$ small, allowing perturbative gauge coupling constants up to the Planck scale. In fact, the model has an $SU(5)$ global symmetry which can incorporate the SM gauge group, so that the perturbative gauge unification in the SSM is kept intact. The model has certain aesthetically appealing features. Its particle content is completely chiral, i.e., none of the fields are allowed to have mass terms. Even though $M$ is high, the supergravity contribution can be kept small enough for an appropriate range of a coupling constant in the superpotential so that the squark degeneracy is a natural consequence of the model. However, we will show later in Section 6 that when studied in detail the model suffers from a fatal problem. Although not phenomenologically viable, the model in this section can be regarded as a demonstration that the following requirements are simultaneously achievable: (1) direct coupling of the DSB sector and the standard model gauge group while maintaining the perturbative unification of gauge coupling constants, (2) enough suppression of the supergravity contribution, (3) no very small coupling constant. We now describe this model in detail.
Table 1: The charge assignments of the fields in the $SU(7) \times SU(6)$ model under the non-anomalous $U(1)$, $U(1)_R$, and $SU(5)$ symmetries.

|     | $Q$ | $L_i$ | $L_6$ | $R_i$ | $R_6$ | $R_7$ | $\phi$ |
|-----|-----|-------|-------|-------|-------|-------|-------|
| $U(1)$ | +5  | -12   | +30   | +7    | -35   | -35   | +42   |
| $U(1)_R$ | -2   | +4    | -10   | 0     | +14   | +2    | -14   |
| $SU(5)$ | 1    | 5     | 1     | 5     | 1     | 1     | 5     |

The particle content of the model is quite simple. It is the same as the $SU(N) \times SU(N-1)$ models proposed in Ref. [18] except for the addition of a field $\phi$. Under the $SU(7) \times SU(6)$ gauge group, we introduce three sets of fields, $Q(7,6)$, $L_I(7,1)$ for $I = 1, \cdots, 6$, and $R^I(1,6)$ for $I = 1, \cdots, 7$. We distinguish the first five of $L_i$ and $R_i$ ($i = 1, \cdots, 5$) from the rest ($L_6$, $R_6$, and $R_7$ – in the general case $L_6, \ldots, L_{N-1}$ and $R_6, \ldots, R_N$) because we would like to impose an $SU(5)$ global symmetry which can be gauged. We also need a field $\phi^i$ ($i = 1, \cdots, 5$) which is a singlet under the $SU(7) \times SU(6)$ gauge group, but transforms as a fundamental under $SU(5)$, in order to cancel the $SU(5)^2$ anomaly. The addition of this field implies that the flat-direction analysis of [20] must be redone, and it is a non-trivial fact that quantum mechanically this model still dynamically breaks supersymmetry, as we will see explicitly below.

The most general superpotential compatible with the $U(1)$ and $U(1)_R$ symmetry listed in Table 1 (these two $U(1)$’s are non-anomalous) is given by

$$W = \lambda L_i Q R^i + \lambda' L_6 Q R_6 + \frac{g}{M_*} L_i Q R_6 \phi^i + \frac{\alpha}{M_*^2} b_6 + \frac{h}{M_*^4} b_i \phi^i.$$  
(5.1)

Here and below, the “baryon” operators $b_I$ are defined by

$$b_I = \frac{1}{6!} \epsilon_{i_1 \cdots i_6} \epsilon^{\alpha_1 \cdots \alpha_6} R_{i_1}^{\alpha_1} \cdots R_{i_6}^{\alpha_6}.$$  
(5.2)

We chose the scale of non-renormalizable operators to be the reduced Planck scale, $M_*$. This is the worst choice from the point of view of suppressing the supergravity contribution as we will see later. Still, the model suppresses the supergravity contribution enough because the dimensionality of the operator is $n = 6$ satisfying the constraint Eq. (5.3) discussed in Section 4.

Here we summarize the main points of the analysis, whose details can be found in the appendix. The $D$ flat directions of the theory are parameterized by the gauge invariant operators $b_I, Y_{ij} = L' Q R' l$, and $B = \text{det}(LQ)$ with a constraint: $\epsilon_{i_1 i_2 \cdots i_6} Y_{i_1 i_3} \cdots Y_{i_5 i_6} \propto B \epsilon_{i_1 i_2 \cdots i_6} b_{i_7}$. In the appendix we show that our superpotential forces all these operators to vanish classically due to the conditions for a supersymmetric vacuum, and hence all these $D$ flat directions are lifted at the classical level. It is non-trivial that even
after adding the singlet field $\phi$, all classical flat directions except for $\phi$ are lifted. We will see later that the quantum effects lift the origin. Since the $b_I$ directions are lifted only by non-renormalizable operators, the fields roll down along one of these directions. Therefore, it is useful for later purposes to analyze the classical Lagrangian along these directions in the absence of non-renormalizable terms in the superpotential.

Along the $b_7$ direction, $R^1$ to $R^6$ acquire the same expectation values

$$\langle R^1, R^2, R^3, R^4, R^5, R^6, R^7 \rangle = \begin{pmatrix}
\rho & 0 & 0 & 0 & 0 & 0 \\
0 & \rho & 0 & 0 & 0 & 0 \\
0 & 0 & \rho & 0 & 0 & 0 \\
0 & 0 & 0 & \rho & 0 & 0 \\
0 & 0 & 0 & 0 & \rho & 0 \\
0 & 0 & 0 & 0 & 0 & \rho \\
\end{pmatrix}. \quad (5.3)$$

This configuration breaks the $SU(6)$ gauge group completely. All components of $Q$ and $L$ fields are massive along $b_7$ direction, and can be integrated out, leaving an unbroken pure $SU(7)$ theory. It is this group which generates a non-perturbative superpotential in quantum analysis. The situation is the same for arbitrary $b_I$ direction as long as the determinant of $Q$, $L$ mass matrix is non-vanishing. The Kähler potential for $b_I$ at the classical level can be obtained using the method of Poppitz and Randall \[33\]. The calculation is described in the appendix, and we find

$$K = 6(b^*J)_{IJ}^{1/6}. \quad (5.4)$$

At the quantum level, the effective pure $SU(7)$ gauge coupling depends on $\langle R \rangle$ through matching at the masses of the $Q$, $L$ fields. Gaugino condensation in the $SU(7)$ gauge group generates a non-perturbative superpotential for $R$ which prefers larger $\langle R \rangle$. The balance between this non-perturbative term and tree-level term $b_6$ determines the value of $\langle R \rangle$ as well as its $F$-component. Now, a non-vanishing $F$-component of $R$ implies that the massive $Q$ and $L$ fields also have supersymmetry breaking bilinear mass terms. We will find that only $b_{6,7}$ acquire vacuum expectation values, so an $SU(5)$ symmetry is left unbroken by the dynamics, corresponding to a vacuum expectation value for the $7 \times 6$ $R$ matrix which is proportional to the identity in the upper $5 \times 5$ block. The unbroken $SU(5)$ is the diagonal subgroup of the original global $SU(5)$ and the gauged $SU(6)$ symmetry. The $Q, L$ fields then contain 7 pairs of $(5 + 5^*)$ under the $SU(5)$, with supersymmetric masses $\langle R \rangle \equiv \langle R^3 \rangle$ and supersymmetry breaking bilinear mass terms $\langle F_R \rangle \equiv \langle F_R^3 \rangle$. The $Q, L$ fields, then, mediate supersymmetry breaking effects to the standard model at order $7(\alpha/4\pi)\langle F_R \rangle/\langle R \rangle$. Below we briefly describe the analysis of the model; details are given in the appendix.

Explicitly, the $SU(7)$ gaugino condensate generates a non-perturbative superpotential,

$$W_{\text{non-pert}} = (\Lambda^{15}(\det' \lambda)b_7)^{1/7}. \quad (5.5)$$
which prefers larger $b_I$. Here, det$'$ refers to the determinant of the $\lambda_{ij}$ matrix for $I = 1, \cdots, 6$. Therefore, it is consistent to analyze the model along $b_I$ flat directions where $SU(6)$ is completely broken, and a perturbative Kähler potential for $b_I$ is valid.

The complete Lagrangian far along the $b_I$ flat directions is then given by

$$K = 6(b^* J b_J)^{1/6} + \phi^*_i \phi^j, \quad (5.6)$$

$$W = (\Lambda^{15}(\text{det}' \lambda)b_7)^{1/7} + \frac{\alpha}{M^3} b_6 + \frac{h}{M^4} b_i \phi^i. \quad (5.7)$$

Following the analysis presented in appendix, a numerical minimization of the potential for the case $N = 7$ gives the location of the minimum

$$b_{N-1} = -0.0702 \alpha^{-7/6} \Lambda^{5/2} M^{7/2}, \quad (5.8)$$

$$b_N = 0.0791 \alpha^{-7/6} \Lambda^{5/2} M^{7/2}. \quad (5.9)$$

Here and below, we absorb the unimportant factor det$'$ into the definition of $\Lambda$.

Now we can discuss the various mass scales in the model and give numerical results. We will show that the supergravity contribution can be suppressed enough compared to the gauge-mediation contribution if $\alpha$ is not too small. We find that the mass of the anomaly-cancelling field $\phi$ tends to be light. However it can be beyond the experimental lower bound with a somewhat small but not unnatural value of $\alpha$. There is a parameter region which is consistent with both requirements.

First of all, the scale of the vacuum expectation value of $A$- and $F$-components of the $R^k_i$ field, which generates the masses and supersymmetry breaking for the $Q, L$ messenger fields charged under $SU(5)$, is (see appendix)

$$\langle R \rangle = 0.688 \alpha^{-7/36} \Lambda^{5/12} M^{7/12}, \quad (5.10)$$

$$\langle F_R \rangle = 0.0425 \alpha^{1/36} \Lambda^2 \left( \frac{\Lambda}{M} \right)^{1/12} \quad (5.11)$$

In order to get correct mass scales for the gaugino masses, we require

$$B \equiv \frac{\langle F_R \rangle}{\langle R \rangle} \simeq 10^4 \text{ GeV}, \quad (5.12)$$

which generates the gluino mass of 600 GeV. By setting $M_* = 2.4 \times 10^{18}$ GeV, we obtain

$$\Lambda = \alpha^{-2/15} 3 \times 10^{10} \text{ GeV} \quad (5.13)$$

$$\langle F_R \rangle = \alpha^{-1/4} (3 \times 10^9 \text{ GeV})^2 \quad (5.14)$$

$$\langle R \rangle = \alpha^{-1/4} 8 \times 10^{14} \text{ GeV} \quad (5.15)$$

Note that $\langle R \rangle$ is large enough so that the gauge coupling constants stay completely perturbative up to the reduced Planck scale even though we have added effectively seven $5+5^*$ pairs.
There is a light $b_k - \phi^k$ fermion since its mass is generated by the highest dimension operator in the model. Its mass is given by

$$m_\phi^2 = 0.0237 V_0 \frac{h^2}{\alpha^2 M_*^2} = (\alpha^{-5/4} h \times 13 \text{ GeV})^2.$$ \hspace{1cm} (5.16)

Phenomenology requires such extra vector-like matter must be heavier than $\gtrsim 200 \text{ GeV}$ for quark-like and $\gtrsim 80 \text{ GeV}$ for lepton-like fields. Barring the fact that the invariant masses are enhanced due to the renormalization group running, we require the original $m_\phi$ to be larger than 100 GeV which is actually a too strong requirement. This bound translates into $\alpha < 0.19$ for $h = 1$. Therefore, the phenomenological constraint is satisfied without taking an unnaturally small coupling constant.

The supergravity effect scales as $m_{3/2}^2 = \langle V \rangle / 3 M_*^2 = 0.0234 \alpha^{1/18} \Lambda^2 (\Lambda / M_*)^{13/6}$ where $\langle V \rangle$ is the vacuum energy after supersymmetry breaking. Numerically, we find this is $m_{3/2} = \alpha^{-1/4} \times 13 \text{ GeV}$. In order to keep the squark degeneracy at the level of 1%, we need $m_{3/2} \lesssim 10^2 \text{ GeV}$ for squarks, or $\alpha > 3 \times 10^{-4}$. Even with a more stringent constraint $m_{3/2} \lesssim 30 \text{ GeV}$, still $\alpha$ as small as 0.04 is allowed. We should also emphasize that choosing $M_*$ to be the reduced Planck scale makes the supergravity effect as competitive as possible to the gauge mediated effect. Any smaller value for $M_*$ makes the supergravity contribution further negligible. Therefore, for a natural choice of $3 \times 10^{-4} \lesssim \alpha \lesssim 0.19$, we obtain a heavy enough $b-\phi$ fermion while suppressing the supergravity contribution not to spoil the squark degeneracy more than a percent level.

In summary, we have demonstrated that we can simultaneously achieve our three goals in this model. However, as we have mentioned already, the model faces a new problem, which is directly tied to the existence of the relatively light $b - \phi$ multiplets. Before discussing this serious problem, however, we wish to outline, in a general way, the phenomenological advantages of a scheme in which both the mass scale of the messengers, and the fundamental scale of supersymmetry breaking are higher than in the OGM models.

6 Phenomenology

We now discuss general phenomenological features distinguishing our scheme from the original models. The only ingredient used in this section is a high $\langle X \rangle$ with $\langle F_X \rangle / \langle X \rangle \sim 10^4 \text{ GeV}$. Specifics could easily depend on details of individual models, but the features we discuss in this section are generic to any models which follow our scheme. Most of the problems mentioned in section 2 are improved upon. The only point which is somewhat worse than in the OGM models is the cosmological problem associated with Polonyi-like fields; however it is not as serious as in the hidden sector models.

*These numbers are by no means precise.
**Superparticle mass spectrum and electroweak symmetry breaking.**

In our scheme, the mass scale of effective messengers is rather high in order to keep the gauge coupling constants perturbative. This simple fact provides us with a distinct superparticle mass spectrum differing from the OGM models [7, 8, 9]. The masses tend to be much closer to each other. In particular, the splitting between squarks and sleptons are much smaller [34].

This point has two phenomenological consequences. One is that one can tell two schemes apart by measuring superparticle masses precisely. In fact, future colliders will have the capability of measuring superparticle masses quite well; to the 10% level for squarks and gluino and to the 1% level for the mass splitting between the first and second neutralinos at the LHC, depending upon certain theoretical assumptions [35], and to the 1% level for any superparticles without any assumptions at an $e^+e^-$ linear collider [36, 37, 38, 39]. Another point is that the fine-tuning in electroweak symmetry breaking becomes mild. One of the problems in the original scheme [7, 8, 9] is that the squarks are much heavier than the sleptons, and the negative stop contribution to the Higgs mass squared is too large, forcing an order 1% fine-tuning so as to correctly achieve electroweak symmetry breaking [10]. On the other hand, the mass splitting between the squarks and sleptons is much smaller in our scheme and hence the fine-tuning is less severe; we do not expect a fine-tuning at more than the 10% level.

**µ-problem**

The $\mu$-problem can be solved in a simple way by extending the minimal particle content to include a singlet (NMSSM) with the following superpotential,

$$W = \lambda_1 H_u H_d S + \lambda_2 S^3.$$  \hspace{1cm} (6.1)

This simple model does not work in the OGM models because it posses an approximate $U(1)_R$ symmetry which is broken only by small soft supersymmetry breaking trilinear couplings of order 10 GeV. A spontaneous breaking of such an approximate global symmetry leads to an unacceptably light axion-like pseudoscalar Higgs boson which appears in $Z$-decay in combination with a light neutral Higgs boson [11]. In our scheme, however, the trilinear couplings are induced from gaugino masses with a large logarithm and are much larger.

It is known that the coupling of a singlet to $H_u H_d$ may destabilize the hierarchy in the hidden sector models. If $H_u$ and $H_d$ are embedded into multiplets unifed with massive color-triplets, their loops may induce a tadpole for the singlet of order $V \sim (g^2/16\pi^2)(\ln M_{H_C}^2/m_Z^2)M_{H_C}m_{3/2}^2S$. In our case the problem is somewhat improved because the soft supersymmetry breaking is power-suppressed for the color-triplet Higgs. However it can still be too large. The soft supersymmetry breaking for the color-triplets is of order $\sim (g^2/16\pi^2)^2(F_X/M_{H_C})^2$, and the
tadpole is given by

\[ V \sim \left( \frac{g^2}{16\pi^2} \right)^3 \left( \ln \frac{M_{H_C}^2}{m_Z^2} \right) M_{H_C} \left( \frac{F_X}{M_{H_C}} \right)^2 S. \] (6.2)

Since we fix \( F_X/X \sim 10^4 \) GeV, this is problematic if \( X \gtrsim 10^9 \) GeV. Fortunately, the existence of this tadpole is model-dependent and certain theories do not produce it. In general, the models which naturally avoid proton decay via \( H_C \) exchange do not produce this tadpole. Examples are the flipped SU(5) model \([40]\), and the models with Babu–Barr mechanism \([41]\). The tadpole problem certainly puts constraints on GUT-model building but appears surmountable.

- **Exotic stable particles**

Possible stable particles from the DSB sector are mostly much heavier \( \sim \langle X \rangle \) than in the OGM models. Therefore, primordial inflation could well dilute them away. There typically are particles around \( 10^4 \) GeV scale which correspond to flat directions lifted only by higher dimension operators. As will be discussed later in Section \([4]\), we would like such “light” particles to transform trivially under the standard model not to drive squark and slepton masses negative at the weak scale. Also by definition, these fields have expectation values and hence there are presumably no conserved quantum numbers associated with them. Therefore, they are likely to decay and there is no problem with the closure limit. However, their scalar components act similarly as the Polonyi field in the hidden sector scenario, and hence might have a coherent oscillation. We will come back to this question shortly.

- **R-axion**

The decay constant is much higher than in the OGM models. It is interesting that one may have a decay constant in an interesting range, \( \langle X \rangle \sim 10^8–10^13 \) GeV, such that the \( R \)-axion may be a viable candidate for the QCD axion. As discussed in Section \([4]\), the decay constant has to be less than about \( \langle X \rangle \lesssim 2 \times 10^{15} \) GeV in order to suppress the supergravity contributions to the squark and slepton masses. For a decay constant above \( 10^{15} \) GeV, the coherent oscillation of the axion may overclose the Universe. However, a decay of long-lived particle may dilute the axion coherent oscillation, and the axion decay constant up to \( 10^{15} \) GeV may be allowed \([12]\). It is noteworthy that the same range of \( \langle X \rangle \lesssim 10^{15} \) GeV is preferred both by viable cosmology and the suppression of supergravity contributions to the squark and slepton masses.

In order for the \( R \)-axion to be the QCD axion, one needs to suppress the possible higher dimension operators which could break \( R \)-symmetry explicitly \([13]\). This still remains as a significant constraint on Planck-scale physics. It is worth pointing out that the \( R \)-axion originates from gauge non-singlet fields in these models.
so that it is somewhat easier to forbid operators up to certain dimensionalities as an accidental consequence of gauge symmetries which are stable against quantum gravitational effects.

- **Gravitino problem.**
  In the OGM models, the gravitino mass is expected to be order 100 keV. In this mass range, the decay of usual LSP’s into the gravitino overcloses the Universe \cite{29,30}. One should suppress the temperature so that most of the SUSY particles were never created. In our scheme, the gravitino mass is higher, and the upper bound on the reheating temperature is much weaker. However, a too large gravitino mass \( m_{3/2} \geq 10 \text{ GeV} \) is forbidden because the decay of LSP’s into gravitino occurs too late, and destroys the success of nucleosynthesis. It is interesting that this constraint is similar to the other constraint to suppress the flavor-non-universal SUGRA contribution to the scalar masses. They are consistent with each other.

- **\( ll\gamma\gamma \) events.**
  The gravitino decay constant in our scheme is much higher and it is impossible to have the lightest superparticle in the SSM decay into the gravitino inside a typical collider detector. We do not except \( ll\gamma\gamma \)-type events arising from pair production of sleptons each decaying into a gravitino, a lepton and a photon. Overall the collider phenomenology is somewhat similar to the hidden sector case except the following point. It is not a cosmological problem for the lightest superparticle in the SSM to be a charged particle because it decays into a gravitino well before nucleosynthesis. Therefore, a light charged superparticle, such as a slepton or a chargino, may be a viable lightest SSM superparticle and charged tracks inside the detector may be a signal for this scheme of mediation.

- **String Moduli.**
  The moduli in the string theory acquire masses only through supersymmetry breaking, and therefore their masses are expected to be of the same order as the gravitino mass \( m_{3/2} \). In the OGM models, this is as small as 100 keV, and hence the moduli are stable. Their coherent oscillations, with initial values of order Planck scale as expected generically for moduli fields, overclose the Universe by 15 orders of magnitude \cite{30}. Since the mass scale is so low, it is quite difficult to eliminate them. In our scheme, the moduli masses are much higher. Even though their decays would occur after nucleosynthesis and would cause disastrous effects if there weren’t any dilutions, a thermal inflation \cite{44} could easily eliminate them.

- **Pseudo-Polonyi Problem.**
Models in our scheme generically produce particles around the $10^4$ GeV scale. The particles correspond to directions in the field space which are not lifted by the renormalizable superpotential, but are lifted by non-renormalizable terms. Since the size of the potential energy is $\sim \langle F_X \rangle^2$ while the natural size of the field variation is $\sim \langle X \rangle$, the mass generically turns out to be order $m_X^2 \sim \langle F_X \rangle^2 / \langle X \rangle^2 \sim (10^4 \text{ GeV})^2$. Because of its large vev, however, the field interacts with other fields via interactions suppressed by the scale $\langle X \rangle$. As a result, its decay rate is quite suppressed. The precise expression for its decay rate is model-dependent. For instance, the baryon-fields $b_6$ and $b_7$ in the $SU(7) \times SU(6)$ model are the Polonyi-like fields in this case. Their decay proceeds via a loop diagram of the heavy $Q$-$L$ multiplets into the SM gauge multiplets, e.g., two gluons or gluinos. For both of them, the decay rate is suppressed further by the loop factor with 7 color-triplets, $(7\alpha_s/\pi)^2$. In general, we expect the decay rate for a Polonyi-like field to be

$$\Gamma_X \sim \frac{1}{8\pi} \left( \frac{N\alpha_s}{\pi} \right)^2 \frac{m_X^3}{\langle X \rangle^2} \sim (10^{-2} \text{ sec})^{-1} \left( \frac{m_X}{10^4 \text{ GeV}} \right)^3 \left( \frac{\langle X \rangle}{10^{15} \text{ GeV}} \right)^{-2} N^2. \quad (6.3)$$

Here, $N$ is the number of multiplets which contribute to the loop diagram.

If such a long-lived scalar particle has an initial field amplitude of order $\langle X \rangle$, its coherent oscillation acquires a large energy density and its subsequent decay produces an enormous energy and entropy. In the hidden sector scenario of supersymmetry breaking, this decay tends to occur after nucleosynthesis, and destroys the success of the big-bang nucleosynthesis predictions for the light element abundances. Fortunately in our case, we need $\langle X \rangle \lesssim 2 \times 10^{15}$ GeV (4.1) to suppress supergravity contribution to the scalar masses, and hence the decay of the field $X$ is likely to occur before the nucleosynthesis time, $\tau \sim 1$ sec, even for the worst case, $N = 1$. This difference is due to a less suppressed coupling ($\langle X \rangle$ vs $M_*$) and a larger mass ($m_X \sim 10^4$ GeV vs $m_{3/2} \sim 100$ GeV).

The entropy production due to the decay of the coherent oscillation may still be a concern in general. The dilution factor is given by $D \sim \sqrt{8\pi \langle X \rangle^3 / (N\alpha_s m_X M_*^2)} \sim 3 \times 10^{16} (\langle X \rangle / M_*)^3$, and hence not very important for $\langle X \rangle \lesssim 10^{13}$ GeV. For a higher $\langle X \rangle$, the entropy production may become more significant: for a possibly maximum $\langle X \rangle \sim 2 \times 10^{15}$ GeV required from suppressing supergravity contribution, $D \lesssim 10^7$. It is never as bad as in the hidden sector case $D \sim \sqrt{8\pi M_* / m_{3/2}} \sim 10^{16}$. An efficient baryogenesis such as in Affleck–Dine scenario [45] could well be sufficient.
7 A New Problem

We have seen in Section 5 that one can achieve the three goals of perturbative unification despite the direct coupling, a natural suppression of supergravity contribution, and no need for a fine-tuning or a very small coupling constant, simultaneously along the lines of the scheme we described in Section 4. However, the model presented in the Section 5 suffers from a fatal flaw: the standard model gauge group gets broken in running down from the high scale to the weak scale. This problem has not been discussed in the literature, and we will describe it in detail in this section. It has to be emphasized that the problem is model-dependent and may not exist in certain other models with directly coupled DSB sector. We are currently not aware of any DSB models which achieve all three goals without the problem described in this section. Nonetheless it seems likely that a viable model in our scheme can be found.

The problem is that the model in Section 5 has light chiral multiplets $b_i$, charged under the standard model, with supersymmetry breaking in the form of soft scalar masses of order $m^2_{b_i} \sim \left(\langle F_R \rangle / \langle R \rangle \right)^2$. While these multiplets only communicate to the ordinary sector via gauge interactions, there is a negative, logarithmically divergent two loop contribution to the running of the ordinary sfermion soft masses due to the soft mass of $b$, which wins over the positive contribution from the $Q,L$ messengers and the positive contribution from 1 loop running due to the standard model gaugino masses. In the absence of the $\phi$ field, the fermionic components of the $b_i$ superfields are the massless fermions required by 't Hooft anomaly matching, and they pick up a mass of order the weak scale after coupling to $\phi$. The scalar components of the $b_i$ superfield, however, have a soft scalar mass of order $m^2_{b_i} = \left(\langle F_R \rangle / \langle R \rangle \right)^2$. Numerically, we find that $\left(\langle F_R \rangle / \langle R \rangle \right)^2 = 0.0038 \Lambda^{10/3} M^{-4/3}$ while $m^2_{b_i} = 0.104 \Lambda^{10/3} M^{-4/3}$, so $m^2_{b_i} \sim 27(\langle F_R \rangle / \langle R \rangle)^2$. Consider now the renormalization of the squark masses from the scale $v$ to lower scales. The relevant RGE keeping only the strong gauge coupling is [\[ \frac{d}{dt} m^2_{\tilde{q}} = -\frac{1}{3} \frac{32}{(16\pi^2)^2} g_3^2 M_3^2 + \frac{1}{3} \frac{16}{(16\pi^2)^2} g_3^4 m^2_{b_i} \] where the first term is the positive 1-loop contribution from the gluino mass $M_3$, the second is the negative two loop contribution from $m^2_{b_i}$, and we have neglected the two loop gluino mass contributions. We use the notation $t = \ln \mu$ here and below. We can treat $m^2_{b_i}$ as constant since all the contributions to its running from the gluino and sfermion masses are negligible compared to $m^2_{b_i}$. Also, we can treat $M_3/g_3^2$ as fixed at its initial value coming from the $Q,L$ messengers $M_3/g_3^2 = N/(16\pi^2)/(\langle F_R \rangle / \langle R \rangle)$ with $N = 7$. Then, we find

\[ \frac{d}{dt} m^2_{\tilde{q}} = -\frac{16}{3} \frac{1}{(16\pi^2)^2} g_3^4 \left(\frac{\langle F_R \rangle}{\langle R \rangle}\right)^2 \left(\frac{2N^2 g_3^2}{16\pi^2} - 27\right) \] (7.1)

*The 1-loop hypercharge $D$ term contribution from $m^2_{b_i}$ vanishes since the $b_i$ scalar masses respect $SU(5)$ invariance and so $\text{Tr} Y m^2_{b_i} = 0$. 
For $N = 7$, the quantity in the brackets gives a negative contribution to the squark masses, and for running from $\langle R \rangle \sim 10^{14}$ GeV to the weak scale, we have

$$\Delta m^2_\tilde{q} \sim -0.1 \left( \frac{\langle F_R \rangle}{\langle R \rangle} \right)^2$$  \hspace{1cm} (7.2)

which dominates over the positive contribution from the $Q,L$ messengers

$$m^2_\tilde{q}(\langle R \rangle) = \frac{16N}{3} \frac{1}{(16\pi^2)^2} \left( \frac{\langle F_R \rangle}{\langle R \rangle} \right)^2 \sim .002 \left( \frac{\langle F_R \rangle}{\langle R \rangle} \right)^2.$$  \hspace{1cm} (7.3)

The most stringent constraints on the relative size of $(\langle F_R \rangle/\langle R \rangle)^2$ and $m^2_b$ actually arises from the renormalization group flow of the right-handed and left-handed SSM sleptons. The renormalization group equations, including the dangerous two-loop contributions of the light $b_k$ scalars, are:

$$\frac{d}{dt} m^2_L = -\frac{1}{(16\pi^2)} \left( 6g_2^2|M_2|^2 + \frac{6}{5}g_1^2|M_1|^2 \right)$$
$$+ \frac{1}{(16\pi^2)^2} \left( 3g_2^4m_b^2 + \frac{9}{5}g_1^4m_b^2 \pm \frac{6}{5}g_1^2m_b^2 \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{6}g_1^2 \right)$$  \hspace{1cm} (7.4)

for the left-handed slepton doublet, and

$$\frac{d}{dt} m^2_i = -\frac{1}{(16\pi^2)} \left( \frac{24}{5}g_1^2|M_1|^2 \right)$$
$$+ \frac{1}{(16\pi^2)^2} \left( \frac{12}{5}g_1^4m_b^2 + \frac{12}{5}g_1^2m_b^2 \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{6}g_1^2 \right)$$  \hspace{1cm} (7.5)

for the right-handed sleptons. The ± signs in the two-loop terms of Eqs. (7.4,7.5) correspond to the assignment of $b_k$ to either a 5 or $\bar{5}$ of SU(5) respectively, and arise from $D$-term interactions. (Note that we are neglecting two-loop contributions to these RG equations proportional to the soft mass-squared of $\phi$, since these soft masses are suppressed relative to those of the $b_k$ states.) The renormalization group equation for the right-handed slepton mass potentially provides the severest constraint on the relative size of $(\langle F_R \rangle/\langle R \rangle)^2$ and $m^2_b$, since the term of $\mathcal{O}(g_1^2g_3^2m_b^2)$ dominates the gaugino contribution (which varies as $\mathcal{O}(g_1^4N^2(\langle F_R \rangle/\langle R \rangle)^2)$ to the greatest extent. However, if we use the freedom in assigning the $b_k$ states to either the 5 or $\bar{5}$, to choose the negative sign in Eq. (7.3) – the 5 assignment – then the right-handed slepton mass-squared is actually increased by the potentially dangerous two-loop terms. The sign of the two-loop terms for the left-handed slepton doublet is then fixed, and this state is the first to have its mass-squared driven negative by RG flow.

The difficulty above is not specific to the $SU(7) \times SU(6)$ model but is quite generic: we always expect some of the moduli to gain soft masses of order $(\langle F_R \rangle/\langle R \rangle)^2$ in
any theory with a single scale where some of the flat directions are lifted by high
dimension operators. If these fields are charged under the standard model, the two loop
contribution to the ordinary sfermion masses are problematically large and negative.
Note, however, that the sign of the contribution to ordinary sfermion masses depends

\[ \frac{\langle F_R \rangle}{\langle R \rangle} \]

crucially on \( N \) and the relative enhancement or suppression of the charged modulus field
mass relative to \( \frac{\langle F_R \rangle}{\langle R \rangle} \). For instance, the situation is worsened in the \( SU(7) \times SU(6) \) model, since \( m_b^2 \) was enhanced by a factor \( \sim 27 \) relative to \( \frac{\langle F_R \rangle}{\langle R \rangle} \). If there

was a suppression rather than an enhancement, the positive gaugino contribution could

have dominated and there would be no problem.

Despite the impression from Eq. (7.1), a larger \( N \) or a lower scale \( M < M_* \) of the

non-renormalizable operators does not solve the problem. To see whether such a flaw

occurs, we need to calculate the exact expressions for \( \frac{\langle F_R \rangle}{\langle R \rangle} \) and \( m^2 \) in terms of

\( \Lambda, M \) (the scale of the non-renormalizable operators, no longer restricted to be \( M_* \)),

and \( N \), for a general model. As discussed in the appendix, the formula for \( \frac{\langle F_R \rangle}{\langle R \rangle} \) is given by

\[
\frac{\langle F_R \rangle}{\langle R \rangle} = m \left( \frac{\beta_{N-1} + \beta_{1/N}^{1/N}}{\langle \beta \rangle^{1/(N-1)}} \right),
\]

with

\[
m = \alpha^{(N+1)/(N-1)^2} \Lambda^{(2N+1)(N-3)/(N-1)^2} M^{-(N-4)(N+1)/(N-1)^2},
\]

while the mass of the \( b \)-scalars are given by

\[
m^2_b = m^2 \left( \beta^1 \beta \right)^{(N-3)/(N-1)} \frac{N-2}{N-1} \left( 1 + \frac{1}{N^2} (\beta_{N-1}^2 \beta_N)^{-1/(N-1)/N} - \frac{1}{\beta \beta_N} \beta_{N-1} + \beta_{1/N}^{N/|N|^2} \right).
\]

The subleading contributions to \( m_b^2 \) arise from the diagonalization of the \( b - \phi \) system,

and the last \( \lambda \)-dependent term of the full scalar potential. Neither of these additional

complications change our conclusions.

Equations (7.6) and (7.8) show that both \( m_b^2 \) and \( \frac{\langle F_R \rangle}{\langle R \rangle} \) depend on the quantity \( m^2 \), and thus their dependence on all dimensionful parameters (\( \Lambda \) and \( M \), and therefore the coupling \( \alpha \) as well) is identical. Thus we gain nothing by working at fixed

\( N \) but allowing \( \Lambda \) and \( M \) to vary. However, since the SSM gaugino masses vary as

\( N \alpha_i (\langle F_R \rangle/\langle R \rangle)/4\pi \), it seems advantageous to consider models with larger \( N \). To see

that this is not the case one can study the behavior of \( \beta_{N-1} \) and \( \beta_N \) as functions of \( N \)
in the large-\( N \) limit by explicitly minimizing the scalar potential. A simple analysis

leads to the result that

\[
\beta_{N-1} \sim - \frac{1}{N + 1 + (N \beta_N)^{-2}} \quad \text{with} \quad \beta_N \sim O(1/N).
\]

This implies in turn that \( \beta_{N-1} \sim O(1/N) \), as well as

\[
\beta_{N-1} + (\beta_N)^{1/N} / N \sim O(1/N^2).
\]
These equations show that both expressions that enter the RG equations, $N^2 (F_R/R)^2$ and $m_b^2$, have the same $N$-scaling,

$$m_b^2 \sim N^2 \left( \frac{F_R}{R} \right)^2 \sim O(1/N^2),$$

and thus no advantage is gained by going to the large-$N$ limit.

Therefore we find that all models in the $SU(N) \times SU(N - 1)$ class, suffer from the problem that the SSM squark and slepton (mass)$^2$ driven negative. Finally, note that the $SU(N) \times SU(N - 2)$ models of Poppitz and Trivedi also suffer from the flaw that the two-loop contribution of light $b$-like states drive SSM sfermion (mass)$^2$ negative.

It is desired, therefore, to have a model of supersymmetry breaking where none of the light degrees of freedom are charged under the (weakly gauged) global symmetries, thus avoiding the dangers of this section completely. We explored all existing DSB models available in literature and none of them appear to achieve the three goals while satisfying this requirement. However, the list of DSB models is growing rapidly recently, and non-renormalizable models have not been explored extensively in literature. We believe that a continued effort along the scheme we propose will result in a simple and phenomenologically viable model of gauge mediation.

8 Conclusion

We proposed a new scheme for the construction of simpler models of gauge mediation. The new scheme aims to simultaneously achieve the following ambitious goals: (1) a much simpler structure by the direct coupling of the standard model gauge groups to the DSB sector while maintaining perturbative unification, (2) a natural suppression of the supergravity contribution despite the high scale of supersymmetry breaking, and (3) no fine-tuning of parameters or very small coupling constant. We found a modified class of DSB models based on $SU(N) \times SU(N - 1)$ which have classical flat directions lifted quantum mechanically, and which allow the gauging of an $SU(N - 2)$ global symmetry. Based on this new class of models, we demonstrated that all the above goals can be achieved.

The basic idea for a successful direct coupling is to employ models where at least one of the classical flat directions $X$ is unlifted at the renormalizable level, but is lifted after adding suitable non-renormalizable terms to the superpotential. The direction $X$ which

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†The situation improves if we embed many 5’s or larger representations into the global $SU(N - 2)$ symmetry. For instance, by embedding 75 into the global symmetry, which requires $N = 77$, we can enhance the gaugino contribution by a factor of 50 relative to that of the $b$-scalars, which might be barely enough. However perturbativity of the SM gauge couplings is then lost.

‡This has been independently realized by the authors of Ref. [46].
is not lifted at the renormalizable level then acquires a large expectation value with small vacuum energy. Therefore a natural hierarchy $\langle F_X \rangle \gg \langle X \rangle^2$ is achieved. If the fields charged under the standard model gauge groups acquire masses due to $\langle X \rangle$, their contribution to the running of the SM gauge coupling constants appears only above $\langle X \rangle$ and hence perturbative gauge unification is relatively easy to preserve. Even though the scales are much higher than in the OGM models, the theories which we studied still naturally suppress the supergravity contributions to the sfermion masses, and hence squark degeneracy, the primary motivation of the gauge mediation mechanism, is automatic.

Our requirements for a successful DSB model were given in Section 4 and are repeated here:

1. It must accommodate a large global symmetry, such as $SU(5)$ or $SU(3)$.

2. Some of the flat directions are unlifted at the renormalizable level.

3. The addition of non-renormalizable operators lifts the flat directions and the model breaks supersymmetry. The dimensionality of the non-renormalizable operators should satisfy the constraint $2 + \frac{144}{2+40/N} \lesssim n \lesssim 9.2$.

However, we found that there is a new type of problem which has not been discussed before in literature. The particular models which we employed generate supermultiplets below $10^5$ GeV charged under the standard model gauge interactions, and their scalar components have large soft-SUSY breaking masses of order $(10^4 \text{ GeV})^2$. They contribute to the renormalization group evolution of squark and slepton masses at the two-loop level, and drive them negative at low energies. This problem unfortunately cannot be avoided by varying the size of the gauge groups within the class of models we considered.

It is clear that the problem is rather specific to models which produce light multiplets charged under the standard model gauge group. It is likely that there are models which do not produce such a spectrum. This consideration leads to a fourth requirement:

4. Directions with non-trivial quantum numbers under the standard model gauge group are lifted at the renormalizable level to avoid light charged fields.

While none of the existing DSB models available in literature appear to satisfy the above four requirements, there has been great recent progress in the construction of DSB models, and non-renormalizable models are just beginning to be extensively explored. We strongly believe that a continued effort along the lines of our proposed scheme will result in simple and phenomenologically viable model of gauge mediation.
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A Details of the $SU(N) \times SU(N - 1)$ models

In this appendix we present some of the details of the analysis of the model of section 5. We first show that the superpotential Eq. (5.1) lifts all classical flat directions except for $\phi \neq 0$ direction. The complete set of gauge invariant operators is $Y_i^I = L_i Q R^I$, $b_I = (RRRRRR)_I$, and $B = \det(LQ)$ with a constraint: $\epsilon_{i_1 i_2 \cdots i_6} Y^{i_1} Y^{i_2} \cdots Y^{i_6} b_I \propto B e^{i_1 i_2 \cdots i_6} b_I$. We will show that all operators vanish classically because of the conditions for a supersymmetric vacuum. The general $SU(N) \times SU(N - 1)$ case can be discussed in exact parallel.

The conditions for a supersymmetric vacuum are given by

\[
\begin{align*}
\frac{\partial W}{\partial L_i} &= \lambda Q R^i + \frac{g}{M} Q R^6 \phi^i = 0, \quad (A.1) \\
\frac{\partial W}{\partial L_6} &= \lambda ' Q R^6 = 0, \quad (A.2) \\
\frac{\partial W}{\partial \phi^i} &= \frac{g}{M} L_i Q R^6 + \frac{h}{M^4} b_i = 0, \quad (A.3) \\
\frac{\partial W}{\partial R^i} &= \lambda L_i Q + \frac{\alpha}{M^3} (R^5)_{6,i} + \frac{h}{M^4} (R^5)_{j,i} \phi^j = 0, \quad (A.4) \\
\frac{\partial W}{\partial R^6} &= \lambda ' L_6 Q + \frac{g}{M} L_i Q \phi^i + \frac{h}{M^4} (R^5)_{j,6} \phi^j = 0, \quad (A.5) \\
\frac{\partial W}{\partial b_7} &= \frac{\alpha}{M^3} (R^5)_{6,7} + \frac{h}{M^4} (R^5)_{j,6} \phi^j = 0, \quad (A.6) \\
\frac{\partial W}{\partial Q} &= \lambda L_i R^i + \lambda ' L_6 R^6 + \frac{g}{M} L_i R^6 \phi^i = 0. \quad (A.7)
\end{align*}
\]

Eq. (A.2) requires $QR^6 = 0$, and therefore, $Y_i^6 = 0$ for all $i$ Then Eq. (A.1) simplifies to $\lambda Q R^i = 0$, and hence $Y_j^i = 0$ for all $i, j = 1, \cdots, 6$. Once $Y_j^6$ are known to vanish, Eq. (A.3) gives $b_i = 0$ for $i = 1, \cdots, 5$. Now we multiply Eq. (A.6) by $R^6$. The second term vanishes, and we obtain $b_7 = 0$. By multiplying the same equation by $R^7$, we obtain $\alpha b_6 + h b_j \phi^j / M = 0$, but we know already $b_j = 0$ and conclude $b_6 = 0$. Therefore, all $b_i, b_6$, and $b_7$ vanish. Using Eq. (A.4) multiplied by $R^7$, the second and third terms vanish, and we find $Y_i^7 = 0$. Finally with Eq. (A.7) multiplied by $R^7$, combined with vanishing of $Y_i^7$, we obtain $Y_j^7 = 0$. Therefore, all $Y$’s vanish. The last step is to multiply Eq. (A.4) by $\phi^j$. We find $\lambda L_i Q \phi^i + \alpha (R^5)_{6,i} \phi^i / M^3 = 0$. By using
Eq. \((A.3)\), one can rewrite the condition as \(\lambda L_i Q^i + (\alpha/h)(M\chi L_\theta Q + g L_i Q^i) = 0\), and hence, \((\lambda + \alpha g/h)L_i Q^i + (\alpha\chi/h)ML_\theta Q = 0\). If \(\phi_i = 0\), \(L_\theta Q = 0\) and hence \(\mathcal{B} = \det(L_{i,\theta}) = 0\). If \(\phi_i \neq 0\), we have a linear relation between \(L_i Q\) and \(L_\theta Q\) and hence again \(\mathcal{B} = 0\). Therefore, \(\mathcal{B}\) always vanishes. This concludes the proof that all gauge invariant polynomials vanish classically except \(\phi^i\). As argued later, the \(\phi\) flat direction gets lifted at the quantum level.

The Kähler potential for \(b_I\) at the classical level can be obtained for any \(N\) using the method of Poppitz and Randall [33], where the heavy SU\((N-1)\) vector multiplet \(V\) is integrated out classically by setting it to its classical equation motion, leaving the effective Kähler potential for the light moduli only. One starts with the Kähler potential for \(R^i\) fields \(K = R^i e^V R = \text{Tr}(e^V R R^i)\), and requires the stationary condition of \(K\) with respect to arbitrary variation of \(V\)(the \(T^a\) are SU\((N-1)\) generators):

\[
0 = \frac{d}{dV^a} \text{Tr}^a R R^i e^{(1-t)V} \int_0^1 \text{d}te^V R R^i e^{(1-t)V} \tag{A.8}
\]

so

\[
\int_0^1 \text{d}te^V R R^i e^{(1-t)V} = c I. \tag{A.9}
\]

It is easy to show that the above can only be satisfied if \(U \equiv e^V\) commutes with \(R R^i\). Then, we must have \(R R^i U = c I \rightarrow \det RR^i = c^{N-1}\), since \(\det U = e^{\text{Tr} V} = 1\). Therefore, the Kähler potential is

\[
K = \text{Tr} R R^i U = (N-1)c = (N-1)(\det RR^i)^{-1/(N-1)} \tag{A.10}
\]

By writing down \(\det(R R^i)\) explicitly, we obtain

\[
\det(R R^i) = \frac{1}{(N-1)!} \epsilon^{\alpha_1 \alpha_2 \cdots \alpha_{(N-1)}} \epsilon_{\beta_1 \beta_2 \cdots \beta_{(N-1)}} (R_{\alpha_1 I_1}^I R^{I_1 \beta_1})(R_{\alpha_2 I_2}^I R^{I_2 \beta_2}) \cdots (R_{\alpha_{(N-1)} I_{(N-1)}}^I R^{I_{(N-1)} \beta_{(N-1)}}) \tag{A.11}
\]

where we use the normalization \(b_J = \frac{1}{(N-1)!} \epsilon_{J I_1 \cdots I_{(N-1)}} R^{I_1} \cdots R^{I_{(N-1)}}\). Therefore, we finally obtain

\[
K = (N-1)(b^* b_J)^{1/(N-1)}. \tag{A.12}
\]

In order to discuss the mass spectrum, we need to explicitly minimize the potential and expand the theory around the vacuum. Let us first determine the vacuum in the general SU\((N)\)×SU\((N-1)\) model [13] without the \(\phi\) field. We will show later that the classical flat direction \(\phi \neq 0\) is actually lifted and justify this treatment. The superpotential is

\[
W = (\lambda^{2N+1}(\det' \lambda)b_N)^{1/N} + \frac{\alpha}{M^{N-4}} b_{N-1}. \tag{A.13}
\]
We redefine \( \Lambda \) to absorb the unimportant factor \( \det' \lambda \) hereafter. It is useful to rescale the fields as
\[
b_I = \alpha^{-N/(N-1)} \Lambda^{(2N+1)/(N-1)} M^{N(N-4)/(N-1)} b_I. \tag{A.14}
\]

By using the rescaled field \( b_I \), the Lagrangian is given by
\[
K = \left[ \alpha^{-N} \Lambda^{2N+1} M^{N(N-4)} \right]^{2/(N-1)^2} (N-1)(\beta^* \beta_j)^{(1/(N-1)}, \tag{A.15}
\]
\[
W = \left[ \alpha^{-1} \Lambda^{2N+1} M^{N(N-4)} \right]^{1/(N-1)} (\beta_N^{1/N} + \beta_{N-1}). \tag{A.16}
\]

The matrix for the kinetic term and its inverse are given by
\[
K^I_J = \left[ \alpha^{-N} \Lambda^{2N+1} M^{N(N-4)} \right]^{2/(N-1)^2} (\beta^I \beta)^{-(N-2)/(N-1)} \left( \delta^I_J - \frac{N-2}{N-1} \frac{\beta^I \beta_J}{\beta^I \beta} \right) \tag{A.17}
\]
\[
K^{-1I}_J = \left[ \alpha^{-N} \Lambda^{2N+1} M^{N(N-4)} \right]^{-2/(N-1)^2} (\beta^I \beta)^{(N-2)/(N-1)} \left( \delta^I_J + \frac{N-2}{N-1} \frac{\beta^I \beta_J}{\beta^I \beta} \right) \tag{A.18}
\]

Therefore, the potential is given by
\[
V = V_0(\beta^I \beta)^{(N-2)/(N-1)} \left( 1 + \frac{1}{N^2} (\beta^* \beta_N)^{-(N-1)/N} + \frac{N-1}{\beta^I \beta} |\beta_{N-1} + \frac{1}{N} \beta^{1/N}_N|^2 \right), \tag{A.19}
\]

where the overall scale of the potential is
\[
V_0 = \left[ \alpha^{1/(N-1)^2} \Lambda^2 \left( \frac{\Lambda}{M} \right)^{(N-4)/(N-1)^2} \right]^2. \tag{A.20}
\]

It is amusing to derive the above potential starting with the original fields \( R \); in the process, we will directly determine \( K^{-1} \). Restricting \( R \) to lie on the \( D \) flat space, the potential is
\[
V = \frac{\partial W}{\partial R^K} \frac{\partial W^*}{\partial R^K_{\alpha}} = \frac{\partial W}{\partial b^I} \frac{\partial W^*}{\partial b^I} K^{-1I}_J \tag{A.21}
\]

where
\[
K^{-1I}_J = \left( \frac{\partial b^I}{\partial R^K} \frac{\partial b^I}{\partial R^K_{\alpha}} \right) \tag{A.22}
\]

A point on the \( D \) flat space is specified by values for \( b_1,..,b_N \). We can always make a global rotation to go to a basis where only \( b_N \) is non-vanishing, and the \( D \) flat direction is \( R_i = \rho \) for \( i = 1,..,N-1 \) with all other \( R's \) vanishing, and with \( \rho^{(N-1)} = b_N \). Now, if \( R \) is taken to be an \( N \times (N-1) \) matrix, \( b_I \) is the determinant of the matrix with the \( I' \)th column removed, and \( \frac{\partial b^I}{\partial R^K} \) is the determinant with the \( J, K' \)th columns and the \( \alpha \)'th row removed. It is easy to compute \( K^{-1} \) in this basis, and we find
\[
K^{-1I}_J = (\rho^* \rho)^{(N-2)} (\delta^I_J + (N-2) \delta^I_N \delta^N_J). \tag{A.23}
\]
But it is trivial to write this in a basis independent way:

$$ (\rho^* \rho) = (b^\dagger b)^{\frac{N-3}{N-1}}, \quad \delta J^I \delta J^J \text{ is invariant and } \delta J^I \delta J^J \text{ is the projection operator onto the } b \text{ direction } \frac{bb^\dagger}{b^\dagger b}, \text{ so}$$

$$ K^{-1}_{II} = (b^\dagger b)^{\frac{N-3}{N-1}} \left( \delta J^I + (N - 2) \frac{bb^\dagger}{b^\dagger b} \right) \quad (A.24) $$

exactly as before.

By expanding the above potential with respect to $\beta_k$ for $k = 1, \ldots, N - 2$, and normalizing it correctly by the coefficient of their kinetic term, the masses for these fields are given by

$$ m_k^2 = m^2 (\beta^\dagger \beta)^{(N-3)/(N-1)} \frac{N - 2}{N - 1} \left( 1 + \frac{1}{N^2} (\beta_N^* \beta_N)^{(N-1)/N} - \frac{1}{\beta^\dagger \beta} |\beta_{N-1} + \beta_N^{1/N} / N|^2 \right), \quad (A.25) $$

where the overall scale is given by

$$ m \equiv \alpha^{(N+1)/(N-1)^2} \Lambda^{(2N+1)(N-3)/(N-1)^2} M^{-(N-4)(N+1)/(N-1)^2}. \quad (A.26) $$

Further coupling to the $\phi$ field, the potential reads

$$ V = V_0 (\beta^\dagger \beta)^{(N-2)/(N-1)} \left( 1 + \frac{1}{N^2} (\beta^* \beta_N)^{(N-1)/N} + \frac{h^2}{\alpha^2 M^2} \phi^\dagger \phi + \frac{N - 2}{\beta^\dagger \beta} \right) \left| \beta_{N-1} + \frac{1}{N} \beta_N^{1/N} + \frac{h}{\alpha M} \phi^k \beta_k \right|^2 + \left( \alpha^{-1} \Lambda^{2N+1} M^{N-4} \right)^{2/(N-1)} \frac{h^2}{\alpha^2 M^2} (\beta^* \beta_k), \quad (A.27) $$

for $k = 1, \ldots, N - 2$.

Numerically minimizing the potential for the case $N = 7$ gives the location of the minimum

$$ \beta_6 = -0.0702, \quad (A.28) $$

$$ \beta_7 = 0.0791. \quad (A.29) $$

By expanding the potential around the minimum up to second order in $\beta_k$ and $\phi^k$, and writing $\lambda \equiv h/\alpha M$ for simplicity, we obtain

$$ V = V_0 (0.070 + 4.409 \beta^* \beta_k + 0.309 \lambda (\beta_k \phi^k + \text{c.c.}) + 0.0236 \lambda^2 \phi^* \phi^k) $$

$$ + \left( \alpha^{-1} \Lambda^{2N+1} M^{N-4} \right)^{2/(N-1)} \lambda^2 (\beta^* \beta_k), \quad (A.30) $$

For the purpose of proving that the $\beta_k$ and $\phi_k$ have positive definite mass eigenvalues, we do not need to further rescale $\beta_k$ to make the Kähler potential canonical. We can also drop the last term since it is always an additional positive contribu tion to the $\beta_k$.
mass squared. By taking the determinant of the mass matrix on \((\beta_k, \phi_k^*)\) space, it is straightforward to prove that \(\phi^k = \beta_k = 0\) is a stable minimum for any values of \(\lambda\) and \(\alpha\). Thus, the model keeps an SU(5) symmetry intact at the minimum of the potential. Since only \(b_6, b_7\) are non-zero at the minimum, the corresponding \(R\) on \(D\) flat space has the form \(R_i^I = R\) for \(i = 1, \ldots, 5, R_6^I = \psi, R_7^I = \chi\) with \(|\psi|^2 + |\chi|^2 = |R|^2\), and all other \(R\)’s vanishing. The unbroken SU(5) is the diagonal subgroup of the original global SU(5) and the gauged SU(6) symmetries. The \(Q, L\) fields then contain 7 pairs of \((5 + 5^*)\) under SU(5), with supersymmetric mass \(R\) and supersymmetry breaking bilinear \(F_R \equiv F_{R_i^I}\), and they can mediate supersymmetry breaking to the ordinary sector.

We now determine \(R\) and \(F_R\) in terms of \(b_{N,N-1}\) for general \(N\):

\[
R = (b_{N-1}^2 + b_N^2)^{1/2(N-1)},
\]

\[
F_R = \frac{\partial W}{\partial R_i^I} = \Lambda^{(2N+1)/N} \frac{1}{N} b_N^{-(N-1)/N} \frac{\partial b_N}{\partial R_i^I} + \frac{\alpha}{M^3} \frac{\partial b_{N-1}}{\partial R_i^I}. \tag{A.32}
\]

But \(\frac{\partial b_N}{\partial R_i^I} = b_N/R, \frac{\partial b_{N-1}}{\partial R_i^I} = b_{N-1}/R\), so

\[
F_R = 1/R \left( \Lambda^{(2N+1)/N} \frac{1}{N} b_N^{1/N} + \frac{\alpha}{M^3} b_{N-1} \right), \tag{A.33}
\]

and we find easily

\[
\frac{F_R}{R} = m \left( \frac{\beta_{N-1} + \beta_N^{1/N}}{(\beta^1 \beta)^{1/(N-1)}} \right) \tag{A.34}
\]

with \(m\) as given in (A.22).

Finally, we demonstrate that the classical flat direction \(\phi \neq 0\) is lifted quantum mechanically. As clear from the analysis above, the vacuum energy increase as \(\alpha^{2N/(N-1)^2}\). In the presence of \(\phi \neq 0\), one can perform a rotation in the flavor space so that the \(N - 1\)-dimensional vector \((\lambda \phi^I, \alpha)\) has a value only in the \(N - 1\)-th component. Note that such a rotation keeps the non-perturbative superpotential \(\propto b_N^{1/N}\) intact. It effectively increases the value of \(\alpha\) to \(\alpha_{\text{new}} = \sqrt{|\alpha|^2 + |\lambda|^2 \phi^I \phi} \geq |\alpha|\). Therefore, the classical flat direction \(\phi\) is lifted quantum mechanically and it develops a stable minimum when \(\phi\) is driven back to the origin.
References

[1] H. Murayama, [hep-ph/9410285], Invited talk presented at the 22nd INS International Symposium on Physics with High Energy Colliders, Tokyo, Japan, March 8–10, 1994. Published in Proceedings of INS Symposium, World Scientific, 1994.

[2] E. Witten, Nucl. Phys. B188, 513 (1981).

[3] I. Affleck, M. Dine and N. Seiberg, Phys. Lett. 137B, 187 (1984); Phys. Rev. Lett. 52, 1677 (1984); Phys. Lett. 140B, 59 (1984); Nucl. Phys. B256, 557 (1985); Y. Meurice and G. Veneziano, Phys. Lett. 141B, 69 (1984); K. Konishi and G. Veneziano, Phys. Lett. 187B, 106 (1987).

[4] A. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982); R. Barbieri, S. Ferrarra and C. Savoy, Phys. Lett. 119B, 343 (1982); L. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D27, 2359 (1983).

[5] S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981).

[6] M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. B189, 575 (1981); C. Nappi and B. Ovrut, Phys. Lett. B113, 175 (1982); M. Dine and W. Fischler, Nucl. Phys. B204, 346 (1982); L. Alvarez-Gaume, M. Claudson and M. Wise, Nucl. Phys. B207 96 (1982).

[7] M. Dine and A. Nelson, Phys. Rev. D48, 1277 (1993).

[8] M. Dine, A. Nelson, and Y. Shirman, Phys. Rev. D51, 1362 (1995).

[9] M. Dine, A. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D53, 2658 (1996).

[10] N. Arkani-Hamed, C. Carone, L. Hall, and H. Murayama, LBL Report LBL-38911, [hep-ph/9607298], Phys. Rev. D54 7032 (1996)

[11] I. Dasgupta, B. Dobrescu, and L. Randall, Boston Univ. Report BUHEP-96-25, [hep-ph/9607487].

[12] K. Intriligator and N. Seiberg, Nucl. Phys. Proc. Suppl. 45BC, 1 (1996).

[13] H. Murayama, Phys. Lett. B355, 187 (1995).

[14] E. Poppitz and S. Trivedi, Phys. Lett. B365, 125 (1996).

[15] P. Pouliot and M. Strassler, Phys. Lett. B375, 175 (1996).

[16] K.-I. Izawa and T. Yanagida, Prog. Theor. Phys. 95, 829 (1996).

[17] K. Intriligator and S. Thomas, Nucl. Phys. B473, 121 (1996).
[18] E. Poppitz, Y. Shadmi, and S. Trivedi, Fermilab Report EFI-96-15, hep-th/9605113.

[19] C. Csaki, L. Randall, and W. Skiba, MIT Report MIT-CTP-2532, hep-th/9605108.

[20] E. Poppitz, Y. Shadmi, and S. Trivedi, Fermilab Report EFI-96-24, hep-th/9606184.

[21] C. Csaki, L. Randall, W. Skiba, and R. Leigh, Phys. Lett. B387, 791 (1996).

[22] K. Intriligator and S. Thomas, SLAC Report SLAC-PUB-7143, hep-th/9608046.

[23] E. Poppitz and S. Trivedi, Fermilab Report EFI-96-35, hep-ph/9609529.

[24] G. Dvali, G. Giudice, A. Pomarol, Nucl. Phys. B478, 31 (1996).

[25] S. Dimopoulos, G. Giudice, and A. Pomarol, CERN Report CERN-TH-96-171, July 1996, hep-ph/9607227.

[26] S. Dimopoulos, D. Eichler, R. Esmailzadeh, and G. D. Starkman, Phys. Rev. D41, 2388 (1990);

[27] J. Bagger, E. Poppitz, and L. Randall, Nucl. Phys. B426, 3 (1994).

[28] T. Banks, D. Kaplan, and A. Nelson, Phys. Rev. D49, 779 (1994).

[29] T. Moroi, H. Murayama, and M. Yamaguchi, Phys. Lett. B303, 289 (1993).

[30] A. de Gouvêa, T. Moroi, and H. Murayama, LBNL-39753, UCB-PTH-96/63, hep-ph/9701244.

[31] N. Arkani-Hamed, J. March-Russell, and H. Murayama, in progress.

[32] T. Hotta, K.-I. Izawa, and T. Yanagida, Report UT-752, hep-ph/9606203.

[33] E. Poppitz and L. Randall, Phys. Lett. B336, 402 (1994).

[34] M. Peskin, SLAC Report SLAC-PUB-7133, hep-ph/9604333. To be published in the proceedings of Yukawa International Seminar 1995: From the Standard Model to Grand Unified Theories, Kyoto, Japan, 21-25 Aug 1995, in Prog. Theor. Phys.; S. Dimopoulos and G. Giudice, CERN Report CERN-TH-96-255, hep-ph/9609344; S. Dimopoulos, S. Thomas, and J. Wells, SLAC Report SLAC-PUB-7237, hep-ph/9609434.

[35] I. Hinchliffe, F. Paige, M. Shapiro, and J. Soderqvist, LBL Report LBL-39412, hep-ph/9610544, October 1996.
[36] T. Tsukamoto, K. Fujii, H. Murayama, M. Yamaguchi, and Y. Okada, Phys. Rev. D51, 3153 (1995).

[37] J. Feng and D. Finnell, Phys. Rev. D49, 2369 (1994).

[38] J. Feng, M. Peskin, H. Murayama, and X. Tata, Phys. Rev. D52, 1418 (1995).

[39] NLC ZDR Design Group and the NLC Physics Working Group (S. Kuhlman et al.). SLAC Report SLAC-R-0485, hep-ex/9605011, Jun 1996.

[40] I. Antoniadis, J. Ellis, J.S. Hagelin, and D.V. Nanopoulos, Phys. Lett. 194B, 231 (1987).

[41] K.S. Babu, S.M. Barr, Phys. Rev. D48 5354 (1993).

[42] M. Kawasaki, T. Moroi, and T. Yanagida, Phys. Lett. B383, 313 (1996).

[43] M. Kamionkowski and J. March-Russell, Phys. Lett. B282, 137 (1992); R. Holman et al., Phys. Lett. B282, 132 (1992).

[44] D. Lyth and E. Stewart, Phys. Rev. Lett. 75, 201 (1995); Phys. Rev. D53, 1784 (1996).

[45] I. Affleck and M. Dine, Nucl. Phys. B249, 361 (1985).

[46] G. Anderson, E. Poppitz, and S. Trivedi, Chicago report EFI-96-50, in progress.