ARE THERE GOODWIN EMPLOYMENT-DISTRIBUTION CYCLES? THEORETICAL AND EMPIRICAL EVIDENCE

Mario García Molina
Eleonora Herrera

Resumen

El modelo predador-presa de Goodwin predice ciclos en el espacio empleo-distribución. Después de las controversias del capital de Cambridge, este modelo en particular no puede ser visto como teóricamente consistente. No obstante, se brinda evidencia a favor de comportamiento dinámico no lineal para una muestra de 67 países, algunos de los cuales presentan ciclos similares a los predichos por el modelo.

Palabras claves: modelo predador-presa, comportamiento dinámico no lineal, ciclos de demanda. JEL: E32, E19, O47.

Abstract

Goodwin’s predator prey model predicts cycles in the employment distribution space. After the Cambridge capital controversies this particular model can not be seen as theoretically consistent. However, evidence is provided in favour of non-linear dynamic behaviour for a sample of 67 countries, some of which have cycles similar to those predicted by the model.

Key words: predator prey model, non-linear dynamic behavior, demand cycles. JEL: E32, E19, O47.

1 Mario García es Doctor en Economía, se desempeña como Profesor Asociado de la Escuela de Economía (Facultad de Ciencias Económicas - Universidad Nacional de Colombia). E-mail: mgarciamo@unal.edu.co.
Eleonora Herrera es Magister en Economía y actualmente es la Directora del Centro de Estudios y Análisis Económico de la Universidad Antonio Nariño. E-mail: director.ceae@uan.edu.co
La serie Documentos FCE considera para publicación manuscritos originales de estudiantes de maestría o doctorado, de docentes y de investigadores de la Facultad de Ciencias Económicas de la Universidad Nacional de Colombia; resultado del trabajo colectivo o individual y que hayan sido propuestos, programados, producidos y evaluados en una asignatura, en un grupo de investigación o en otra instancia académica.

Documentos FCE
Escuela de Economía
ISSN 2011-6322

La serie Documentos FCE puede ser consultada en el portal virtual: http://www.fce.unal.edu.co/publicaciones/

Coordinador de Publicaciones
Carlos Andrés Álvarez Gallo
Profesor Asociado - FCE

Equipo de publicaciones - FCE
Jenny Paola Lis Gutiérrez
David Alejandro Bautista Cabrera
Juan Carlos García Sáenz
Manfred Acero Gómez

Administrador portal publicaciones
Miguel Benjamín Ibañez Solís

Diseño
Andrea Paola Parra Martínez

Contacto: Unidad de Divulgación y Publicaciones. Oficina 116 edificio 310 Facultad de Ciencias Económicas.

Correo electrónico:
comunica_fcebog@unal.edu.co

Periodicidad: Trimestral

Este documento puede ser reproducido citando la fuente. El contenido y la forma del presente material es responsabilidad exclusiva de sus autores y no compromete de ninguna manera a la Escuela de Economía, ni a la Facultad de Ciencias Económicas, ni a la Universidad Nacional de Colombia.
Richard Goodwin's (1967, 1972) model of the growth cycle attempted to show how accumulation takes a cyclical form due to the interaction of capitalists and workers in a Marxian way. This simple theoretical model of endogenous cycles has been further explored in several directions including the realism of its assumptions, its stability and possible generalizations (Goodwin et al 1984); there has been less interest in performing empirical studies, although exceptions are Atkinson (1969), Desai (1984) Solow (1990), Harvie (2000) and Moreno (2002). This literature has provided evidence on single countries or on a small number of them, the largest sample so far being 10 OECD countries in the case of Harvie (2000).

The aim of the article is to provide evidence about employment distribution cycles á la Goodwin for a larger number of countries, including several degrees of development.

The model's central dynamics is a cyclical relationship between distributed shares and employment. This behaviour should underlie the dynamics of developed capitalist economies but it should also at least partially explain the behaviour of capitalist developing countries. If this relationship holds, it should appear in the scatterplots of the worker's share against the employment rate. On a quantitative level, the model's estimated centers can be compared with the actual centers of the cycles. Finally, Goodwin's assumptions can be tested to check for their validity. The procedure of looking for qualitative and quantitative evidence and testing the assumptions was performed by Harvie (2000), who found a quarter of a cycle for 10 OECD countries during the period between the late 1960’s and the mid 1990’s, which qualitatively supported the idea of a cycle. However, the estimated centers lied outside the actual cycles, while the assumptions were not justified, which meant that, at a quantitative level, the model was not adequate. Harvie’s methodology is used here for a wider sample of countries in order to have a better grasp of the extent to which the model holds empirically and the directions for future developments.

However, before providing empirical evidence, care should be taken as this is an aggregate capital model and after the Cambridge capital controversies this is no lesser issue. This matter is discussed in section 2. Even if the model has theoretical problems, the question remains whether there are cycles in
the employment-distribution space. Section 3 provides evidence in this regard from a sample of 67 countries with different levels of development. Section 4 discusses the evidence and concludes with suggestions for future study.

**Goodwin's model**

Goodwin (1967, 1972) models the cyclical behaviour in the workers share of national income and the employment rate by means of the Lotka–Volterra predator–prey model (Lotka, 1956; Volterra, 1931A, 1931B, 1937). The model attempts to formally present Marx's idea that the interaction between distribution and employment was at the root of capitalism's booms and crises.

Assuming a constant relation of constant (the value of the means of production) to variable (wages) capital, both constant and variable capital will grow until full employment of labour is reached. In the vicinity of full employment, real wages will rise (i.e. a real Phillips curve), but rising real wages dampen accumulation, and thus the downturn begins.

Higher real wages diminish profits and, as a result, there is a lesser accumulation rate. However, the lower rate of accumulation will create unemployment, removing the disproportion between capital and exploitable labour-power. Real wages fall and accumulation starts again.

Goodwin assumes two factors of production: capital and labour. All quantities are real and net. Labour productivity \( a \) and the labour force \( n \) grow at constant rates (equations 1 and 2). Sigma is the fixed capital output ratio (equation 3), which determines the employment level, \( l \) (equation 4). \( k \) is the total stock of capital, \( q \) is real output, \( l \) is employment, \( w \) is the real wage, \( u \), the workers’ share of national income (equation 5); and \( v \), the employment rate (equation 6).

\[
a = a_0 e^{\alpha t}; \quad \alpha > 0
\]

\[
n = n_0 e^{\beta t}; \quad \beta > 0
\]

\[
\sigma = k/q
\]
ARE THERE GOODWIN EMPLOYMENT-DISTRIBUTION CYCLES? THEORETICAL AND EMPIRICAL EVIDENCE

\[ l = \frac{q}{a} \]  \hspace{1cm} (4)

\[ u = \frac{wl}{q} = \frac{w}{a} \]  \hspace{1cm} (5)

\[ v = \frac{l}{n} \]  \hspace{1cm} (6)

\[ \kappa = (1-u)q \]  \hspace{1cm} (7)

\[ \frac{\kappa}{w} = -\gamma + \rho v, \gamma > 0, \rho > 0 \]  \hspace{1cm} (8)

Capitalists are assumed to save and invest all their profits and workers to consume all their wages (equation 7). Finally, a linear real Phillips curve is assumed, i.e. real wages rise as employment increases (equation 8).

From equations (1)–(8) a pair of differential equations in the state variables \( u \) and \( v \) can be obtained:

\[ \frac{\kappa}{\alpha} = \left[ -(\alpha + \gamma) + \rho v \right] u \]  \hspace{1cm} (9)

\[ \frac{\kappa}{\beta} = \left[ \frac{1-u}{\sigma} - (\alpha + \beta) \right] v \]  \hspace{1cm} (10)

The solution of the model is a family of closed cycles around a centre, i.e., the economy comes back to the initial point and starts the cycle again.

**Capital theory problems**

As it is clear in the equations, the model assumes aggregate capital. It should be noted that the model is inspired in an explanation given by Marx in Chapter 25 of volume I of *The Capital*. (Goodwin, 1972, p. 442). (Marx, [1887] 1974, p. 575). Here Marx assumes a given rate of constant to variable capital, which is equivalent to Goodwin’s constant ratio of output to capital.
However, it should also be noticed that, as this is Volume I, Marx assumes equal organic composition in all sectors, i.e. a single capital good (Pasinetti). This would not be a problem for Marx, as he is concerned with the behaviour of social capital and in Volume III he would consider the transformation problem and the existence of several organic compositions of capital among sectors, but Goodwin's model remains in a one commodity world.

After the Cambridge controversies (Harcourt, 1975) and Sraffa's (1960) contribution, it is clear that any measure of capital as an aggregate will be dependent on distribution, hence, dealing with aggregate capital would not be a big problem, provided distribution remained constant or its effect on the value of capital were taken into account.

Nevertheless, this is precisely Goodwin's weak point. For what he tries to explain is the cyclical behaviour of distribution. Therefore, capital cannot be taken as independent variable. Assuming a constant technology and a constant rate of growth, the value of capital could vary in the same or the opposite direction of the rate of profits depending on the concavity of the wage profits curve, i.e. on the price Wicksell effects. If several techniques are available, real Wicksell effects should also be taken into account. This amounts to say that empirical measures of capital would be biased in any direction along a Goodwin cycle, which might explain why the estimated centers lye outside of the observed cycle (Figure 1). But this is not the only problem as, during an estimation period of several decades, technology is bound to change, and therefore new potential Wicksell effects will appear and others disappear. Hence, quantitative evidence based upon the calculation of the centre using the amount of capital would only be valid if positive or negative Wicksell effects were shown to be not important during the period. Hence, the centers lying outside the circle is not really empirical evidence against the model. However, the dependence on a given path for aggregate capital at the same time as endogenous changes in distribution speaks against the consistency of this particular model.

---

1 Figure 1 is on the employment distribution plane. Actual centres tend to be closer to the x axis than estimated centres and the distance is large among the two point for countries with actual cyclical behaviour.
It should be said in favour of Goodwin that he was trying to make a first didactical approximation to the problem of endogenously explaining cycles. However, this would not be a solid explanation to explain real world phenomena unless we take this problem seriously. Otherwise, we would be accepting Goodwin's model as a *parable* in the same way Samuelson accepted the neoclassical production function.

Several ways ahead are possible. One would be the improvement of non-linear estimation methods appropriate for the model, such as that of Dibeh et al (2007). This would account for better estimates although it would not solve in principle the theoretical problem of treating capital as measured independently on a changing distribution.

A second possibility would be to work on heterogeneous capital versions of the model. This avenue would take some time to develop due to the complications of non-linear multiple commodity modeling.
A third possibility would be to expand qualitative assessments to wider groups of countries in the hope of eventually finding stylized facts that can be introduced in more realistic versions of the model. A first step in this direction will be given in the next section.

**Are there distribution-employment cycles?**

The scatterplots of the share of wages against the employment rate were made for 67 countries. The results were widely diverse. A group of twenty-six countries (Australia, Austria, Belgium, France, Germany, Italy, New Zealand, United Kingdom, United Sates, Bahrain, Estonia, India, Ireland, Jamaica, Japan, Kazakhstan, Kenya, Kyrgyz Republic, Moldova, Netherlands, Panama, Slovakia, Sri Lanka, Thailand, Tunisia and Zimbabwe) behaves in the way predicted by the model.

Figure 2: GOODWIN CYCLES
A second group comprised of nine countries (Costa Rica, Honduras, Turkey, Armenia, Belarus, Iceland, Ecuador, Mexico and Paraguay), have a cycle in the opposite direction, that is, they do not show evidence of profit squeeze but rather of Keynesian-like or demand-pushed cycles such as the one found by Barbosa-Filho and Taylor (2003) for the United States. For a third group of thirty-two countries (Algeria, Bolivia, Botswana, Brazil, Canada, Chile, Colombia, Czech Republic, Denmark, Finland, Greece, Hong Kong, Jordan, Kuwait, Luxembourg, Malta, Mauritius, Namibia, Norway, Peru, Poland, Portugal, Russian Federation, South Africa, South Korea, Spain, Sweden,
Switzerland, Tanzania, Trinidad and Tobago Ukraine and Venezuela) there is no evidence of a cycle.

Figure 3: DEMAND CYCLES
A third group of countries has no clear tendency but behaviour is too diverse to be catalogued here.

Figure 4: Atypical Behaviour
ARE THERE GOODWIN EMPLOYMENT-DISTRIBUTION CYCLES? THEORETICAL AND EMPIRICAL EVIDENCE

![Graphs showing the employment-distribution cycles for Denmark, Finland, Greece, Hong Kong, Jordan, and Korea.](image-url)
ARE THERE GOODWIN EMPLOYMENT-DISTRIBUTION CYCLES? THEORETICAL AND EMPIRICAL EVIDENCE

Portugal

1970

2003

Peru

1970

2005

Poland

1991

2005

Russian Federation

1989

2006

Spain

1970

2006

South Africa

1950

2006
It should be noticed that the results are not clearly related to the degree of development as measured by the income per capita. However, high income countries behave either in the way predicted by Goodwin or in an atypical fashion, not in a demand cycle way; middle income countries show all dynamics, either Goodwin, demand or atypical, although African, European and American countries tend to be atypical; and low income countries tend to show Goodwin cycles. This can be seen in table 1.
### TABLE 1: Type of Cycle vs. Income per Country

| Type of Cycle | INCOME |  |
|---------------|--------|----------------|
| Goodwin       | High   | Middle | Low |
| America       | America| Africa   |     |
| United States | Jamaica| Kenya    |     |
| Asia          | Panamawe | Asia   |     |
| Bahrain       | Africa  | Asia    |     |
| Japan         | Tunisia | India    |     |
| Europe        | Asia    | Kyrgyz Republic |     |
| Austria       | Kazakhstan| Europe |   |
| Belgium       | Sri Lanka| Moldova | |
| France        | Thailand |     |     |
| Germany       | Europe  |     |     |
| Ireland       | Estonia |     |     |
| Italy         | Slovak Republic |     |     |
| Netherlands   | Oceania |     |     |
| United Kingdom| New Zealand  |     |     |
| Oceania       | Australia|     |     |
| Demand        | Europe  | America | America |
| Iceland       | Costa Rica | Honduras | |
| Ecuador       | Asia    | Armenia | |
| Mexico        | Paraguay | Asia    |     |
| Asia          | Turkey  | Europe  |     |
| Europe        | Belarus |     |     |
| Atypical Behaviour | America | Africa | Africa |
| Canada        | Algeria | Tanzania | |
| Asia          | Botswana |     |     |
| Hong Kong, China | Mauritius |     |     |
| Kuwait        | Namibia |     |     |
| Europe        | South Africa |     |     |
| Denmark       | America |     |     |
| Finland       | Bolivia |     |     |
| Greece        | Brazil  |     |     |
| Luxembourg    | Chile   |     |     |
| Norway        | Colombia|     |     |
| Spain         | Peru    |     |     |
| Sweden        | Trinidad and Tobago |     |     |
| Switzerland   | Venezuela, RB |     |     |
| Asia          | Jordan  |     |     |
| Korea, Rep.   |         |     |     |
| Europe        | Czech Republic |     |     |
| Malta         | Poland  |     |     |
| Portugal      | Russian Federation |     |     |
| Ukraine       |         |     |     |

Source: The information for this table was obtained based on the estimation on GDP per capita made by the World Bank for 2005.
One possible explanation for this diversity is that capitalism has evolved in several ways in different countries according to different kinds of institutions, particularly labour market institutions. As noted by Arrighi and more recently by Robinson, institutional development may be linked to the pattern of population and migration.

The institutions regulating capital also differ between countries. A few countries (mainly Anglo-Saxon ones) rely on more atomistic shareholders and individual firms. In contrast, most other countries have business groups with controlling shareholders being much more important than the typical textbook would believe (La Porta). In general, institutions might help to classify the different kinds of capitalism and help to understand diversity in the behaviour between distribution and employment.

With regard to quantitative evidence, the theoretical centers of the cycles lied outside of the cycles in all cases, as it happened in Harvie (2000), while the Phillips curve only hold for thirty nine countries.

Note that twenty-one countries had the expected qualitative behaviour despite the Phillips curve not holding. A particular problem of the real Phillips curve is that Goodwin assumes a linear version of it in order to obtain the Predator Prey equations. However, it would be more reasonable to believe that wages would only increase near full employment, not before, which would suggest a non-linear version. However, a more realistic version would make model building far too complicated.
### Table 2: Significance of the Phillips curve

| Country                  | Significance of the Phillips Curve variables | \( \gamma \) | \( \rho \) |
|--------------------------|---------------------------------------------|-------------|----------|
| Algeria                  |                                             | *           | *        |
| Australia                |                                             | *           | *        |
| Belgium                  |                                             | ***         | ***      |
| Bolivia                  |                                             |             |          |
| Brazil                   |                                             |             |          |
| Canada                   |                                             | **          | **       |
| Chile                    |                                             | **          | **       |
| Colombia                 |                                             |             |          |
| Costa Rica               |                                             | *           | **       |
| Denmark                  |                                             | **          | **       |
| Estonia                  |                                             |             |          |
| Finland                  |                                             |             |          |
| France                   |                                             | ***         | ***      |
| Greece                   |                                             |             |          |
| Ireland                  |                                             | *           |          |
| Jamaica                  |                                             |             |          |
| Japan                    |                                             | *           |          |
| Kazakhstan               |                                             |             |          |
| Kyrgyz Republic          |                                             | ***         | ***      |
| Luxembourg               |                                             | **          | **       |
| Malta                    |                                             | *           | **       |
| Mauritius                |                                             | ***         | ***      |
| Mexico                   |                                             | ***         | ***      |
| Moldova                  |                                             |             |          |
| Namibia                  |                                             |             |          |
| Norway                   |                                             |             |          |
| Poland                   |                                             |             |          |
| Portugal                 |                                             | *           | **       |
| Russian Federation       |                                             | **          | **       |
| South Korea              |                                             | ***         | ***      |
| Switzerland              |                                             |             |          |
| Thailand                 |                                             | **          | **       |
| Trinidad & Tobago        |                                             | ***         | ***      |
| Turkey                   |                                             |             |          |
| Ukraine                  |                                             | ***         | ***      |
| United Kingdom           |                                             | *           |          |
| United States            |                                             | *           |          |
| Venezuela                |                                             |             |          |

Note: *** (**) (*) indicates parameters test statistics is significance at 1 %,( 5%) (10%) 2 tail test
**Conclusions**

Goodwin’s simplified mathematical model is subject to the Cambridge criticism because it assumes the amount of capital to change independently from endogenously determined distribution. This is not to deny its value because it implies that a one commodity world would present endogenous cycles, which is a relevant proposition, given that the mainstream of the profession deals with one commodity exogenous cycles.

However, when trying to understand real-world problems the evidence is mixed. For the sample of 67 countries, evidence of cycles similar to those predicted by Goodwin was found for twenty six countries. Evidence of demand led cycles was found for nine countries. And no clear evidence of cyclic behaviour was found for the remaining (32 countries). The diversity is not readily related to the degree of development and it was suggested that explaining it would require a better understanding of the different ways of capitalism evolved around the world.

**References**

Acemoglu, D; Johnson, S; Robinson, J A. 2001. “The colonial origins of comparative development: an empirical investigation”. American Economic Review, 91: 1369-1401.

Arrighi, Giovanni. 1978. La geometría del imperialismo. México: Siglo XXI.

Atkinson, A B. 1969. “The timescale of economic models: how long is the long run?”. The Review of Economic Studies, 36(2): 137-152.

Barbosa-Filho, Nelson H. and Taylor, Lance. (2006). “Distributive And Demand Cycles In The Us Economy—A Structuralist Goodwin Model” Metroeconomica 57:3 (2006) 389–411.

Desai M. 1984 Goodwin, R. M., Kruger, M. and Vercelli, A (ed.). (1984) Nonlinear Models of Fluctuations and Growth, an international symposium, Siena, Italy, 24-27, March 1983. Lecture Notes in Economic and Mathematical Systems Vol 228. Berlin, New York: Springer-Verlag.

Desai, Meghnad, et al. (2004)” A Clarification of the Goodwin Model of The Growth Cycle”, Discussion papers in economics, Department of Economics, University College London, Discussion paper 04 – 04, pp. 1– 15.

Ghassan Dibeh Dmitry G. Luchinsky Daria D. Luchinskaya Vadim N. Smelyanskiv. 2007. “A Bayesian estimation of a stochastic predator prey model of economic fluctuations. SPIE proceedings vol 6601.
Harvie, David. (2000). “Testing Goodwin: growth cycles in ten OECD countries” Cambridge Journal of Economics 2000, 24, 349–376.
Kinsella, Stephen. (2005). “Search and Selection in the Goodwin Growth Model” August 22, 2005.
McAnulty, J.C., Naines J.B. and Strotz, R.H. (1953). “Goodwin’s Nonlinear Theory of the Business Cycle: An Electro-Analog Solution” Cowles Fundation Paper 74 Reprinted from Econometria 21(3). July 1983.
Moreno, Álvaro M. (2002) “El Modelo de Ciclo y Crecimiento de Richard Goodwin. Una evaluación empírica para Colombia”. Cuadernos de Economía, Vol. XXI, No. 37, Bogotá.
Solow, R. 1990. “Goodwin’s Growth Cycle: Reminiscence and Rumination”, publicado en Velupillai, J. (ed.) (1990): Nonlinear Multisectoral Macrodynamics, Londres, McMillan, pgs. 31-41.
Velupillai, K. Vela. (1998). “Richard Goodwin 1919-1996)” The Economic Journal, Vol 108, No. 450. (Sep 1998) pp 1436-1449.
Appendix

Table 3: Average growth rate for productivity and population

| Country          | Productivity $\alpha$ | Population $\beta$ |
|------------------|------------------------|--------------------|
| Algeria          | 1.6%                   | 3.4%               |
| Australia        | 2.2%                   | 2.0%               |
| Belgium          | 2.6%                   | 0.4%               |
| Bolivia          | 0.8%                   | 2.8%               |
| Brazil           | 2.4%                   | 3.0%               |
| Canada           | 2.2%                   | 1.9%               |
| Chile            | 1.8%                   | 2.4%               |
| Colombia         | 1.9%                   | 3.2%               |
| Costa Rica       | 2.1%                   | 3.6%               |
| Denmark          | 2.3%                   | 0.6%               |
| Estonia*         | -0.3%                  | -1.3%              |
| Finland          | 2.9%                   | 0.3%               |
| France           | 2.6%                   | 0.7%               |
| Greece           | 3.4%                   | 1.3%               |
| India            | 2.3%                   | 1.7%               |
| Ireland          | 3.7%                   | 1.5%               |
| Jamaica          | 1.3%                   | 1.4%               |
| Japan            | 4.3%                   | 0.9%               |
| Kazakhstan*      | 1.0%                   | -0.1%              |
| Kyrgyz Republic* | -1.9%                  | 1.3%               |
| Luxembourg       | 2.8%                   | 2.1%               |
| Malta            | 1.0%                   | 1.2%               |
| Mauritius        | 7.9%                   | 2.1%               |
| Mexico           | 3.1%                   | 3.5%               |
| Moldova*         | -5.8%                  | -2.7%              |
| Namibia          | 3.0%                   | -2.4%              |
| Norway           | 3.0%                   | 1.1%               |
| Poland           | 1.9%                   | -0.2%              |
| Portugal         | 4.1%                   | 0.8%               |
| Russian Federation* | 0.7%                | 0.2%               |
| South Korea      | 6.0%                   | 2.5%               |
| Switzerland      | 1.5%                   | 1.3%               |
| Thailand         | 4.5%                   | 1.5%               |
| Trinidad & Tobago| 2.3%                   | 1.6%               |
| Turkey           | 2.6%                   | 1.5%               |
| Ukraine*         | -3.2%                  | -1.2%              |
| United Kingdom   | 2.1%                   | 0.4%               |
| United States    | 2.1%                   | 1.8%               |
| Venezuela        | -0.1%                  | 3.8%               |

* The data for this countries is only available since 1989.
Econometric estimation

For the econometric estimation we follow Harvie’s methodology for all variables of the model, in addition the state variables, i.e. u, v were passed through the Hodrick – Prescott Filter in order to release the tendency. For the productivity growth, labor force growth and the Phillips curve estimation the parameters were estimated using ordinary least squares (OLS) regressions. For countries with more than 17 data observations, we run a unit root test and a cointegration test.

Parameter estimation

Productivity growth $a$

It assumes like an exponential productivity growth function $a = a_0e^{\alpha t}$, where $\alpha$ is the constant growth rate. This is the parameter estimate for each country using (OLS), by the following equation:

$$\ln a_t = \ln \hat{a}_0 + \hat{\alpha} + \epsilon_{1t} \quad (11)$$

Labour force growth $\beta$

Using a similar exponential function for the labour force growth $n = n_0e^{\beta t}$, where $\beta$ is the constant growth rate and using the same estimation methodology with (OLS) by the following equation:

$$\ln n_t = \ln \hat{n}_0 + \hat{\beta} + \epsilon_{2t} \quad (12)$$

Capital-output ratio $\sigma$

In the model, Goodwin assumes a constant capital-output ratio $\sigma = k/q$ so a simple estimation of the mean for the variable was calculated for each country.

Phillips curve $\gamma$ and $\rho$

For the mayor purpose of the model, we calculated a long run Phillips curve by the following equation:

$$w_t = -\gamma + \rho v_t + \phi w_{t-1} \quad (13)$$
DATA SOURCES

- United Nation Statistics Division [http://unstats.un.org/](http://unstats.un.org/)
- International Monetary Fund [www.imf.org/](http://www.imf.org/)
- World Bank [www.worldbank.org/](http://www.worldbank.org/)
- Comisión Económica para Latinoamérica y el Caribe [www.cepal.org/](http://www.cepal.org/)
- Organisation for Economic Co-operation and Development [www.oecd.org/](http://www.oecd.org/)
- International Labour Organization [www.ilo.org/](http://www.ilo.org/)
- Asian Development Bank (ADB) [http://www.adb.org/](http://www.adb.org/)
- University of Kiel [www.uni-kiel.de/](http://www.uni-kiel.de/)
- "The Conference Board and Groningen Growth and Development Centre Total Economy Database, January 2007, [http://www.ggdc.net](http://www.ggdc.net)"
- Comunidad Andina de Naciones [www.comunidadandina.org](http://www.comunidadandina.org)
- MERCOSUR [http://www.mercosur.int](http://www.mercosur.int)
- International Institute for Applied Systems Analysis [www.iiasa.ac.at](http://www.iiasa.ac.at)
- Organisation for Economic Co-Operation and Development. "Labour Force Statistics 1978 - 2005". OECD Publications Service, 1999-2006 Edition.
- Organisation for Economic Co-Operation and Development. (1998) “National Accounts Main Aggregates 1960 - 2004”. OECD, 1998-2006 Edition.
- Organisation for Economic Co-Operation and Development.(2002) “Historical Statistics 1970-2000”. OECD, 2001 Edition.
- International Labour Organization. “Yearbook of Labour Statistics 1950-2006”. ILO 1950-2006.
- Marcos Souza y Aumara Feu (2005) " Capital Stock in Latin America: 1950-2000 " Economy and energy, Periodical

Algeria

- Bank of Algeria [http://www.bank-of-algeria.dz/](http://www.bank-of-algeria.dz/)
- National Office of Statistics (N.O.S) [http://www.ons.dz/](http://www.ons.dz/)

Armenia

- Central Bank of Armenia [http://www.cba.am/](http://www.cba.am/)
- National Statistical Service of the Republic of Armenia [http://www.armstat.am/](http://www.armstat.am/)

Australia

- Australian Bureau of Statistics [http://www.abs.gov.au/](http://www.abs.gov.au/)
- Reserve Bank of Australia [http://www.rba.gov.au/](http://www.rba.gov.au/)
Are there Goodwin employment-distribution cycles? Theoretical and empirical evidence

Austria
- Austrian National Bank: http://www.oenb.at/
- Statistics Austria: http://www.statistik.at/

Bahrain
- Central Bank of Bahrain: http://www.cbb.gov.bh/
- Ministry of Finance: http://www.mofne.gov.bh/

Belarus
- National Bank of the Republic of Belarus: http://www.nbrb.by/
- The Ministry of Statistics and Analysis: http://belstat.gov.by

Belgium
- Statistics Belgium: http://www.statbel.fgov.be/
- Banque Nationale de Belgique: http://www.nbb.be/

Bolivia
- Instituto Nacional de Estadística: http://www.ine.gov.bo/
- Banco Central de Bolivia: http://www.bcb.gov.bo/
- CEPAL: www.cepal.org/

Botswana
- Central Statistical Office: http://www.cso.gov.bw/
- Bank of Botswana: http://www.bankofbotswana.bw/

Brazil
- Instituto Brasileiro de Geografia e Estatística (IBGE): http://www.ibge.gov.br/
- Banco Central Do Brasil: http://www.bcb.gov.br/

Canada
- Statistics Canada: http://www.statcan.ca/
- Bank of Canada: http://www.bank-banque-canada.ca/

Chile
- Instituto Nacional de Estadísticas: http://www.ine.cl/
- Banco Central de Chile: http://www.bcentral.cl/

Colombia
- Departamento Administrativo Nacional de Estadística: http://www.dane.gov.co/
- Departamento Nacional de Planeación: www.dnp.gov.co/
- Banco de la República de Colombia: http://www.banrep.gov.co/

Czech Republic
- Czech Statistical Office: http://www.czso.cz/
- Czech National Bank: http://www.cnb.cz/

Costa Rica
- Instituto Nacional de Estadística y Censos: http://www.inec.go.cr/
- Banco Central de Costa Rica: http://www.bccr.fi.cr/
Denmark
- Statistics Denmark [http://www.dst.dk/]
- Danmarks National Bank [http://www.nationalbanken.dk/]

Ecuador
- Instituto Nacional de Estadísticas y Censos [http://www.inec.gov.ec/]
- Banco Central de Ecuador [http://www.bce.fin.ec/]

Estonia
- Statistics Estonia [http://www.stat.ee/]
- Bank of Estonia [http://www.bankofestonia.ee/]

Finland
- Statistics Finland [http://www.stat.fi/]
- Bank of Finland [http://www.bof.fi/en]

France
- Institut National de la Statistique et des Études Économiques [http://www.insee.fr/]
- Le Portail de la statistique publique française [http://www.statistique-publique.fr/]
- Bank of France [http://www.banque-france.fr/]

Germany
- Federal Statistical Office [http://www.destatis.de/]
- Deutsche Bundesbank [http://www.bundesbank.de/]

Greece
- National Statistical Service of Greece [http://www.statistics.gr/]
- Bank of Greece [http://www.bankofgreece.gr/]

Honduras
- Instituto Nacional de Estadística [http://www.ine-hn.org/]
- Banco Central de Honduras [http://www.bch.hn/]

Hong Kong
- Census and Statistics Department [http://www.censtatd.gov.hk/]
- Hong Kong Monetary Authority [http://www.info.gov.hk/]

Iceland
- Statistics Iceland [http://www.statice.is/]
- Central Bank of Iceland [http://www.sedlabanki.is/]

India
- Ministry of Statistics and Programme Implementation [http://www.mospi.gov.in/]
- Reserve Bank of India [http://www.rbi.org.in/]

Ireland
- Central Statistics Office Ireland [http://www.cso.ie/]
• Central Bank and Financial Services Authority of Ireland  
  http://www.centralbank.ie/

Italy
• National Institute of Statistics  http://www.istat.it/
• Bank of Italy  http://www.bancaditalia.it/

Jamaica
• Statistical Institute of Jamaica  http://www.statinja.com/
• Bank of Jamaica  http://www.boj.org.jm/

Japan
• Statistics Bureau  http://www.stat.go.jp/
• Bank of Japan  http://www.boj.or.jp/

Jordan
• Department of Statistics  http://www.dos.gov.jo/
• Central Bank of Jordan  http://www.cbj.gov.jo/

Kazakhstan
• Agency of Statistics of the Republic of Kazakhstan  http://www.stat.kz/
• National Bank of Kazakhstan  http://www.nationalbank.kz/

Kenya
• Central Bureau of Statistics  http://www.cbs.go.ke/
• Central Bank of Kenya  http://www.centralbank.go.ke/

Korea
• National Statistical Office  http://www.nso.go.kr/
• Bank of Korea  http://www.bok.or.kr/

Kuwait
• Statistical and Information Sector, Ministry of Planning  
  http://www.mop.gov.kw/
• Central Bank of Kuwait  http://www.cbk.gov.kw/

Kyrgyzstan
• National Statistical Committee of Kyrgyz Republic  
  http://www.stat.kg/
• National Bank of the Kyrgyz Republic  http://www.nbkr.kg/

Luxembourg
• STATEC  http://www.statec.public.lu/fr/
• Central Bank of Luxembourg  http://www.bcl.lu/

Malta
• National Statistics Office  http://www.nso.gov.mt/
• Central Bank of Malta  http://www.centralbankmalta.com/

Mauritius
• Central Statistical Office  http://www.gov.mu/
• Bank of Mauritius  http://bom.intnet.mv/
Mexico
- Instituto Nacional de Estadística, Geografía e Informática [http://www.inegi.gob.mx/]
- Banco de Mexico [http://www.banxico.org.mx/]

Moldova
- National Bureau of Statistics [http://www.statistica.md/]

Namibia
- Central Bureau of Statistics [http://www.npc.gov.na/]
- Bank of Namibia [http://www.bon.com.na/]

Netherlands
- Statistics Netherlands [http://www.cbs.nl/]
- Netherlands Bank [http://www.dnb.nl/]

New Zealand
- Statistics New Zealand [http://www.stats.govt.nz/]
- Reserve Bank of New Zealand [http://www.rbnz.govt.nz/]

Norway
- Statistics Norway [http://www.ssb.no/]
- Central Bank of Norway [http://www.norges-bank.no/]

Panama
- Contraloría General de la Republica de Panamá [http://www.contraloria.gob.pa/]

Paraguay
- Dirección General de Estadística, Encuestas y Censos [http://www.dgeec.gov.py/]
- Banco Central de Paraguay [http://www.rbnz.govt.nz/]

Peru
- Instituto Nacional de Estadística e Informática (INEI) [http://www.inei.gob.pe/]
- Banco Central de Reserva de Peru [http://www.bcrp.gob.pe/]

Poland
- Central Statistical Office [http://www.stat.gov.pl/]
- National Bank of Poland [http://www.nbp.pl/]

Portugal
- Instituto Nacional de Estatística [http://www.ine.pt/]
- Banco de Portugal [http://www.bportugal.pt/]

Russian Federation
- Federal State Statistics Service [http://www.gks.ru/]
- Bank of Russia [http://www.cbr.ru/]
Slovakia
- Statistical Office of the Slovak Republic [http://www.statistics.sk/]
- National Bank of Slovakia [http://www.nbs.sk/]

South Africa
- Statistics South Africa [http://www.statssa.gov.za/]
- The South Africa Reserve Bank [http://www.reservebank.co.za/]

Spain
- Instituto Nacional de Estadistica [http://www.ine.es/]
- Banco de España [http://www.bde.es/]

Sri Lanka
- Department of Census and Statistics [http://www.statistics.gov.lk/]
- Central Bank of Sri Lanka [http://www.lanka.net/centralbank/]

Sweden
- Statistics Sweden [http://www.scb.se/]
- Central of Sweden [http://www.riksbank.com/]

Switzerland
- Swiss National Bank [http://www.snb.ch/]

Tanzania
- National Bureau of Statistics [http://www.nbs.go.tz/]
- Bank of Tanzania [http://www.bot-tz.org/]

Thailand
- National Statistical Office [http://web.nso.go.th/]
- Central Bank of Thailand [http://www.bot.or.th/]

Trinidad y Tobago
- Central Statistical Office [http://www.cso.gov.tt/]
- Central Bank of Trinidad and Tobago [http://www.central-bank.org.tt/]

Tunisia
- Institut national de la statistique [http://www.ins.nat.tn/]
- Central Bank of Tunisia [http://www.bct.gov.tn/]

Turkey
- Turkish Statistical Institute [http://www.turkstat.gov.tr/]
- Central Bank of Turkey [http://www.tcmb.gov.tr/]

Ukraine
- The State Committee of Statistics of Ukraine [http://www.ukrstat.gov.ua/]
- National Bank of Ukraine [http://www.bank.gov.ua/]

United Kingdom
- Office for National Statistics [http://www.statistics.gov.uk/]
- Bank of England [http://www.bankofengland.co.uk/]
United States of America
- Bureau of Economic Analysis http://www.bea.gov/
- Board of Governors of the Federal Reserve http://www.federalreserve.gov/

Venezuela
- Instituto Nacional de Estadística http://www.ine.gov.ve/
- Banco Central de Venezuela http://www.bcv.org.ve/

Zimbabwe
- Central Statistical Office http://www.zimstat.co.zw/
- Reserve Bank of Zimbabwe http://www.rbz.co.zw/

Results

The rest of the countries cannot be analyzed because the variables of the Phillips curve was positive which means that the model cannot be run under this circumstance.

There are two capital/output ratio parameter estimated $\hat{\sigma}$ was calculated from a different database including The International Institute for Applied Systems Analysis, Christophe Kamps, Kiel Institute for World Economics and Marcos Souza y Aumara Feu (2005). The other capital/output ratio $\sigma^*$ was taken from Nehru - Dhareshwar (1993) for some countries, the aim of this methodology was compare this different way of estimation and its impact on the results. Note that there is no a big difference between the two estimations despite of the change on the capital/output ratio.

The variables U and V are the central points of the closed cycle and $T$ are the period, $\alpha$ Productivity growth, $\beta$ Labour force growth, $\sigma$ Capital-output ratio and $\gamma, \rho$ are the Phillips curve parameters, where

$$u^* = 1 - (\alpha + \beta)\sigma \quad (14)$$

$$v^* = (\alpha + \beta)l \rho \quad (15)$$

$$T = \frac{2\pi}{\left[ (\alpha + \gamma)((1/\sigma) - (\alpha + \beta)) \right]^{\frac{3}{2}}} \quad (16)$$
Table 4: Estimation of cycle’s centers and periods

| COUNTRY       | Capital/output ratio | Center and period estimate with capital/output ratio | Center and period estimate with capital/output ratio |
|---------------|-----------------------|-----------------------------------------------------|-----------------------------------------------------|
|               | \( \sigma^* \) | \( \hat{\sigma} \) | \( U^* \) | \( V^* \) | \( T^* \) | \( \hat{U} \) | \( \hat{V} \) | \( \hat{T} \) |
| ALGERIA       | 2.4 2.4             | 0.863 4.324 4.018 | 0.883 4.324 4.018 |
| AUSTRALIA    | 3.5 4.5             | 0.855 1.056 36.349 | 0.812 1.051 42.492 |
| BELGIUM      | 2.8 3.2             | 0.913 1.035 35.759 | 0.902 1.035 38.329 |
| BOLIVIA      | 4.3 1.6             | 0.846 0.736 0.120 | 0.944 0.736 0.068 |
| BRAZIL       | 1.9 2.6             | 0.896 0.789 1.183 | 0.861 0.789 1.398 |
| CANADA       | 2.6 3.4             | 0.894 0.783 30.690 | 0.863 0.783 35.535 |
| CHILE        | 2.3 2.1             | 0.902 0.873 0.072 | 0.911 0.873 0.069 |
| COLOMBIA     | 1.7 1.6             | 0.914 0.661 4.556 | 0.918 0.661 4.436 |
| COSTARICA    | 1.8 1.5             | 0.897 0.817 3.668 | 0.913 0.817 3.332 |
| DENMARK      | 2.9 3.3             | 0.914 0.962 20.034 | 0.904 0.962 21.206 |
| ESTONIA      | 4.5 4.6             | 1.081 1.001 22.910 | 1.083 1.001 23.166 |
| FINLAND      | 3.8 4.3             | 0.878 1.564 51.404 | 0.860 1.564 55.687 |
| FRANCE       | 3.2 3.2             | 0.897 1.247 41.352 | 0.895 1.247 41.643 |
| GREECE       | 2.7 2.9             | 0.874 9.267 58.762 | 0.865 9.267 60.991 |
| INDIA        | 2.6 2.6             | 0.898 0.769 5.096 | 0.898 0.769 5.096 |
| UNITED KINGDOM | 2.7 1.7         | 0.931 1.927 50.995 | 0.956 1.930 40.212 |
| IRELAND      | 2.9 2.3             | 0.850 2.052 48.211 | 0.880 2.052 42.360 |
| JAMAICA      | 5.3 5.3             | 0.856 0.484 45.965 | 0.856 0.484 45.965 |
| JAPAN        | 3.3 5.8             | 0.829 0.861 2.938 | 0.696 0.861 4.278 |
| KAZAKHSTAN   | 5.5 5.6             | 0.950 0.872 0.756 | 0.949 0.872 0.766 |
| KOREA        | 2.3 2.3             | 0.807 0.951 0.793 | 0.807 0.951 0.793 |
| KYRGYZSTAN   | 4.4 4.5             | 1.024 0.851 2.354 | 1.025 0.852 2.390 |
| LUXEMBURGO   | 3.3 3.3             | 0.841 1.055 31.124 | 0.841 1.056 31.124 |
| MALTA        | 3.0 3.0             | 0.933 0.923 2.480 | 0.933 0.923 2.480 |
| MAURITIUS    | 2.6 2.6             | 0.737 0.982 19.617 | 0.737 0.986 19.617 |
| MÉXICO       | 2.0 2.1             | 0.889 0.957 8.543 | 0.886 0.957 8.638 |
| MOLDOVA      | 4.1 4.4             | 1.348 0.752 1.481 | 1.379 0.752 1.528 |
| NAMIBIA      | 3.9 3.9             | 0.975 2.530 47.100 | 0.975 2.530 47.100 |
| NORWAY       | 4.3 3.8             | 0.825 1.229 24.300 | 0.846 1.229 22.547 |
| POLAND       | 2.1 2.2             | 0.963 0.809 4.247 | 0.963 0.809 4.286 |
| PORTUGAL     | 3.5 1.7             | 0.828 1.532 47.744 | 0.917 1.535 31.532 |
| RUSSIAN FEDERATION | 4.6 4.8 | 1.044 0.897 2.385 | 1.046 0.897 2.439 |
| SWITZERLAND  | 3.6 0.8             | 0.897 0.976 49.405 | 0.978 1.013 21.673 |
| THAILAND     | 2.4 2.4             | 0.857 0.996 15.260 | 0.857 0.995 15.260 |
| TRINIDAD Y T. | 1.7 1.7         | 0.935 0.845 11.589 | 0.935 0.845 11.589 |
| TURKEY       | 2.3 2.3             | 0.906 2.067 47.506 | 0.906 2.067 47.506 |
| UKRAINE      | 4.5 4.9             | 1.199 0.893 0.309 | 1.215 0.893 0.319 |
| UNITED STATES | 2.7 2.9         | 0.895 1.057 36.678 | 0.889 1.057 37.724 |
| VENEZUELA    | 2.5 1.8             | 0.908 0.713 3.713 | 0.934 0.713 3.105 |