ABSTRACT

The analogy between intermittency and scaling in statistical physics is extended to the case of more variables. It is shown that the inclusive densities predicted by the perturbative QCD obey generalized homogeneity principle which leads to many consequences in statistical physics. In particular the hyperscaling may be a prototype for the hyperintermittency in multiparticle production at high energies.

1. Introduction

During the last few years the phenomenon of intermittency has been intensively studied theoretically and experimentally. These studies have originated from the attempt to distinguish dynamical and statistical fluctuations [1]. Simple models with nontrivial dynamical fluctuations were proposed. They showed characteristic power dependence of multiplicity moments on the volume of the phase space. Therefore intermittency is often identified with such a power behaviour. Also the terms self-similarity and fractality were used in this context. It was later realized, that the above power dependence is equivalent to the power behaviour of the correlation functions in the corresponding range of variables [2].

It turned out that a good laboratory to study intermittency is provided by statistical physics. In the latter intermittency is analogous to scaling, or equivalently, to the existence of the nontrivial anomalous dimensions of various field-theoretical operators. In statistical systems this power behaviour occurs in the vicinity of the second order phase transition and is specified by a finite set of phenomenologically defined critical exponents. Confirming the original motivation such a behaviour is driven by the (thermal) fluctuations of arbitrary sizes. However in addition to the simple scaling also the phenomenon of hyperscaling occurs in statistical systems. This offers an interesting possibility to extend the analogy between the intermittency in multiparticle production and scaling in statistical systems to yet another level, namely to attempt to define the counterpart of the hyperscaling in multiparticle production. To this end we first review shortly the idea of the

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1 To be published in the Proceedings of the XXIV International Symposium on Multiparticle Dynamics, Eds. A. Giovannini, S. Lupia and R. Ugoccioni, World Scientific, Singapore.
hyperscaling on the example of the two-dimensional Ising model.

2. Hyperscaling in statistical physics

The vanishing of magnetization at the Curie point defines the critical exponent \( \beta \)
\[
m(t) \equiv < s > \sim t^\beta, \quad t = (T_c - T)/T_c > 0.
\]
(1)
Other exponents control divergence of the specific heat \( c \), susceptibility \( \chi \) and of the correlation length \( \xi \)
\[
e(t) \sim |t|^{-\alpha},
\]
(2)
\[
\chi(t) \sim |t|^{-\gamma}
\]
(3)
\[
\xi(t) \sim |t|^{-\nu}.
\]
(4)
Connected correlation function also scales with the distance at the critical point
\[
\Gamma(r_1, r_2) \equiv < s_1 s_2 > - < s >^2 \sim \frac{1}{|r_1 - r_2|^{d-2+\eta}},
\]
(5)
where \( d \) is the dimension of the system. In general all observables depend on two variables: reduced temperature \( t \) and external magnetic field \( h \), e.g. \( < s > = m(t, h) \).
All previous formulae apply for \( h = 0 \). When considered as the functions of the magnetic field, \( h \), magnetization also shows power behaviour (at \( T = T_c \)) albeit with different exponent \( \delta \).
\[
m(0, h) \sim h^{1/\delta},
\]
(6)
and similarly for other observables, Eqs. (2-4), but with different exponents. Discussed above analogy with multiparticle production regards power behaviour of the normalized factorial moments
\[
M_q \equiv \frac{< n^{(q)} >_\Delta}{< n >_\Delta} \sim \Delta^{-\alpha_q},
\]
(7)
as the counterpart of these scaling laws in statistical physics. In this equation \( <>_\Delta \) denotes averaging over the phase space region of size \( \Delta \).

It was observed \cite{3} that the variety of scaling laws, Eqs. (1 - 4, 6) follow from the single scaling law of the free energy density in two variables
\[
f(\lambda^y t, \lambda^y h) = \lambda^d f(t, h),
\]
(8)
which is refered to as the generalized homogeneity principle \cite{4}. Similar twodimensional scaling was also postulated for the correlation functions. The generalized homogeneity was later justified theoretically using the renormalization group approach \cite{5, 6}. Note that there are only two independent exponents \( y_t, y_h \) in Eq. (8).
All phenomenological exponents \( \beta, \gamma, \nu \) can be derived from these two and consequently there are relations between them \cite{4, 7}. There is an important class of

\footnote{In the particular case of the two-dimensional Ising model \( \alpha = 0 \) and \( c \) diverges logarithmically.}
relations between phenomenological exponents of statistical systems which contains
the dimensionality of the space $d$. Such relations are called hyperscaling relations
and their derivation requires additional assumptions. The most widely known is
the Josephson relation which reads for the two-dimensional Ising model

$$\nu d = 2 - \alpha.$$  \hspace{0.5cm} (9)

Hyperscaling, Eq. (9), has a simple interpretation proposed by Pippard and Ginsberg.
According to Eq. (2) the singular part of the density of the free energy scales as

$$f(t) \sim |t|^{2-\alpha},$$  \hspace{0.5cm} (10)

which, using Eqs. (9) and (4) can be rewritten as

$$f(t) \sim \frac{1}{\xi(t)^d}.$$  \hspace{0.5cm} (11)

This means that the change of the free energy due to the fluctuations of the size
$\sim \xi$ is of the order 1 which may be intuitively expected [4].

Our aim is to explore a possibility of extending the analogy between the
scaling and the intermittency, and to attempt to define the "hyperintermittency"
in multiparticle spectra as the counterpart of the hyperscaling in statistical sys-
tems. In this lecture we shall restrict ourselves to the first step necessary in such
a construction, namely we will look for the analog of the generalized homogeneity
principle, Eq. (8), in the spectra of particles produced at high energies.

3. Generalized homogeneity in perturbative QCD

It turns out that the predictions of the perturbative QCD for the inter-
mittency in partonic cascade, have interesting properties which are similar to the
consequences of Eq. (8). Recently intermittency in QCD jets was studied by several
groups [8],[9],[10]. Together with W. Ochs we have calculated QCD predictions for
the variety of partonic observables. They reveal rather interesting universality and
the new type of scaling behaviour (see also the lecture by Wolfgang Ochs in these
Proceedings).

$$\rho(Q_1, Q_2) \sim Q_1^{a(Q_1)\omega(\epsilon)},$$  \hspace{0.5cm} (12)

$$\epsilon = \frac{\ln Q_2}{\ln Q_1}.$$  \hspace{0.5cm} (13)

where $\rho$ is the generic partonic density which depends essentially on the two vari-
bles, (or scales) which are relevant to the particular process (see Table I for the
list of studied reactions and for the definitions of variables in various cases). $a(q)$
is related to the coupling constant $a(q)^2 = 6\alpha_s(q)/\pi$ and $\omega(\epsilon)$ denotes the calculable
scaling function which is largely process independent.

It is readily seen that, similarly to the free energy, Eq. (8), the generic density
(12) shows the power behaviour \[ along the special family of curves

$$Q_2 = Q_1^d,$$  \hspace{0.5cm} (14)

3Violated only by the running coupling constant.
in the plane \((Q_1, Q_2)\). In contrast to Eq.\((8)\) the exponent \(\epsilon\) is not fixed \((0 < \epsilon < 1)\) in QCD, while it is a single number, \(\epsilon = y_h/y_t\), in the Ising model. This is the consequence of more complex behaviour of QCD under the renormalization group transformations. Also, the simple \((\sim \lambda^d)\) scaling along the curve \([14]\) in \([8]\) is replaced by more complicated \(\lambda^{\alpha(Q_1)}\) one. Nevertheless, the basic analogy between Eqs.\((8)\) and \((12)\) is established.

To go further and to define hyperintermittency in close analogy to the hyperscaling relations, Eqs.\((9,11)\), would require more detailed analysis of the additional assumptions leading to the latter in statistical models. This is beyond the scope of this lecture. Since however the generalized homogeneity, Eq.\((8)\), implies a class of relations between critical exponents, one can foresee that similar class would follow from Eq.\((12)\) for partonic counterparts of the magnetization, specific, heat, etc. Naturally, they follow only for fixed \(\epsilon\), therefore they will be less restrictive in the QCD case.

4. Outlook

To conclude, we have shown that extending the intermittency study to more variables, leads to more complete analogy with statistical physics, which in turn could provide better understanding of multiparticle production. QCD results \([4]\) exhibit generalized homogeneity well known in statistical physics. Therefore the existence of the hyperintermittency seems plausible and one may start searching for its implications.

\(^4\)Derived in the double logarithmic approximation.

| \(\alpha_s\) | inclusive process | observable | \(Q_1\) | \(Q_2\) |
|----------------|-------------------|------------|-------|-------|
| constant       | 1-parton          | \(\rho^{(1)}_{(P,\Theta)}(k)\) | \(\frac{P\Theta}{Q_0}\) | \(\frac{k\Theta}{Q_0}\) |
| running        | 2-parton          | \(\rho^{(2)}_{(P,\Theta)}(\theta_{12})\) | \(\frac{P\Theta}{X}\) | \(\frac{P\Theta_{12}}{X}\) |
| running        | n-parton          | \(h^{(n)}_{P'}(\theta, \delta)\) | \(\frac{P\Theta}{X}\) | \(\frac{P\Theta}{X}\) |

Table 1: Table I. Observables and corresponding kinematical variables for the three processes studied: momentum distribution, distribution of the relative angle, and the cumulant moment of n-th order respectively.
It is interesting to point out one practical consequence of hyperintermittency. The relation of type (9) allows to determine the correlation length in the scaling regime. Usually this parameter controls the exponential (or gaussian) fall-off of the correlation functions. Near the critical point however $\xi$ is very large and, consequently it is hard to extract it from the very slow spatial decrease of the correlation functions. Fortunately, hyperscaling provides a simpler way, since, according to Eq.(11), $\xi$ can be directly determined from the temperature dependence of some observables.

5. Acknowledgements

We would like to mention the talk of Wu Yuanfang at this Symposium [11] where the study of more complex scaling patterns in many variables is proposed on more geometrical basis.

This work is partly supported by the KBN grants No PB 0488/P3/93 and 2P30Z2520G.

6. References

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