General Parton Distributions and Counting of Helicity-Flip Nucleon Form Factors

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Abstract

We give a brief overview of the General Parton Distributions (GPDs), including their properties, connections to conventional quantities, and relationships to physical observables. We also perform a systematic analysis on the non-forward matrix elements of twist-two quark and gluon helicity-flip operators and explain their relationships to the GPDs. We systematically count the number of independent nucleon form factors in non-forward scattering (of matrix elements of these operators), by matching the allowed quantum numbers with their crossing channel counterparts and time reversal symmetry considerations (a method developed in [2]). We finally analyze and write down the form factor expansion of the quark operator matrix elements.

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I. INTRODUCTION

One of the most important frontiers in strong interaction physics is the study of the structure of the nucleon. Many unanswered questions still exist due to the non-perturbative nature of the bound state problem in Quantum ChromoDynamics (QCD). However, recent theoretical and experimental efforts on the so-called General Parton Distributions (GPDs) have shed new light on the problem, especially on the possibilities of a three-dimensional view and a complete description of the nucleon (eg, see [3, 4, 5] and many others).

The GPDs generalize and interpolate between the ordinary Parton Distribution Functions (PDFs) and elastic form factors—both having been studied extensively for many years—and therefore contain rich structural information. They can be accessed in high-energy (exclusive) diffractive processes in which the nucleon recoils elastically after receiving a non-zero momentum transfer in the so-called deeply virtual limit, eg, Deeply Virtual Compton Scattering (DVCS) and diffractive electro-production of vector mesons. From a more theoretical point of view, GPDs are closely related to the matrix elements of quark and gluon operators in QCD, through generalized nucleon form factors, the knowledge of which is essential to fully describe the nucleon. In particular, the matrix elements of those operators of twist-two are of the most importance and interest, because they usually give the leading contribution (hence the synonym leading-twist) in appropriate hard processes, have clear physical interpretation (eg, corresponding to the energy-momentum tensor), and are often more accessible to experimental measurement and relatively simple.

The enumeration of independent nucleon form factors of twist-two operators and the expansion into form factors of non-forward matrix elements of these operators are among the essential understandings of the problem. We extend such studies, following a method based on the partial wave formalism and crossing symmetry (first developed in [2]), to helicity-flip twist-two quark and gluon operators [6], by systematically enumerate the number of independent hadronic form factors of both and write down the form factor expansion of the general quark operators [1].

II. GENERAL PARTON DISTRIBUTIONS

Generalized Parton Distributions represent the low-energy (soft) internal structure of the nucleon, in three dimensions. They connect and are generalizations of Feynman PDFs and elastic electromagnetic form factors. The PDFs usually result from the overlap of hadron wave functions and contain information on the longitudinal momenta and polarizations carried by various partons in a fast moving hadron. On the other hand, the traditional form factors contain information on the transverse momenta of partons, often in the form of sum rules related to, for example, charges, local currents and the energy-momentum tensor (of QCD). The GPDs, on the other hand, in general have their physical interpretations given in terms of probability amplitudes and/or parton correlation functions, and have simple physical significance in light-cone coordinates (or the infinite momentum frame).

A. Forward and Non-forward Scenarios

In the forward scenario, eg, in deep inelastic scattering (DIS), one relates the imaginary part of a forward (Compton) scattering amplitude to the cross section via the optical the-
orem. The factorization of the forward amplitude into the hard scattering part and the soft physics is achieved through the formal approach of operator product expansion (OPE) [7]. The hard scattering is calculated order by order in perturbation theory and the soft part is parameterized as the PDFs, which are essentially matrix elements of light-cone bilocal quark and gluon operators between equal momentum states. A renormalization group equation governs the factorization scale dependence of the PDFs/matrix elements, while the cross section is eventually related to (usually a linear combination of) the PDFs.

The GPDs arise mostly in non-forward scattering processes, and among such processes DVCS is the cleanest of all. Various studies [8] have shown that even in such cases, with both longitudinal and transverse momentum transfer, factorization is still valid and non-forward OPE can be similarly performed. That is, the non-forward amplitude can be factorized into the hard scattering part, calculable order by order in perturbation theory, and the soft part, now parameterized as GPDs (with proper insertions of operators), as shown in figure 1.

In particular, the lower parts of the above graph give rise to four general quark distributions (from two different operators): \( (H_q(x, \xi, t), E_q(x, \xi, t)) \) (quark spin sum) and \( (\tilde{H}_q(x, \xi, t), \tilde{E}_q(x, \xi, t)) \) (spin difference), where, the now standard kinematical variables used in discussions of GPDs (through for example DVCS) are defined as (see, eg, [3, 6])

\[
\begin{align*}
\mathcal{T}^\mu &= (P' + P)^\mu/2 = p^\mu + (\bar{M}^2/2)n^\mu, \\
\Delta^\mu &= P'^\mu - P^\mu = -2\xi(p^\mu - (\bar{M}^2/2)n^\mu) + \Delta^\mu, \\
\bar{M}^2 &= M^2 - \Delta^2/4, \\
\end{align*}
\]

and \( x \) and \( \xi \) are longitudinal momentum fractions referring to \( \mathcal{P} \) (The initial nucleon and parton have longitudinal momentum fractions \( 1 + \xi \) and \( x + \xi \), respectively.).

### B. Connection to Other Observables

GPDs display characteristics of both the forward parton distributions and nucleon form factors. They reduce to the DIS structure functions in the forward limit of \( (P = P', \xi = t = 0) \)

eg, \( H_q(x, 0, 0) = q(x), \quad \tilde{H}_q(x, 0, 0) = \Delta q(x) \)

where \( q(x) \) and \( \Delta q(x) \) are forward quark (spin independent) and quark helicity (spin dependent) distributions.
GDPs are also closely related to the elastic form factors. Their first moments (integration over \( x \) of the GDPs) are related to the nucleon form factors by the sum rules:

\[
\int_{-1}^{1} dx H_q(x, \xi, t) = F_1(t), \quad \int_{-1}^{1} dx E_q(x, \xi, t) = F_2(t), \\
\int_{-1}^{1} dx \tilde{H}_q(x, \xi, t) = G_A(t), \quad \int_{-1}^{1} dx \tilde{E}_q(x, \xi, t) = G_P(t).
\]

where \( F_1 \) and \( F_2 \) are quark Dirac and Pauli form factors (for electromagnetic current), respectively, and \( G_A \) and \( G_P \) the quark axial and pseudoscalar form factors (for axial current), respectively. Higher moments of GDPs are related to (usually linear combinations of) general nucleon form factors of general parton (local) operators \([3, 4]\).

Most interestingly, the GDPs contain rich information on the spin structure of the nucleon, in particular, about the orbital motion of partons in a (polarized) nucleon. Nucleon spin flip needs orbital angular momentum, and GDPs (through DVCS) is the only place so far probing it—through Ji’s Sum Rule \([9]\):

\[
\frac{1}{2} \int_{-1}^{1} dx \ (H_q + E_q) = J_q(t)
\]

where \( J_q(0) \) is the Total Angular Momentum of quarks (spin + orbital). A similar result holds for the gluons and one finds

\[
J_q(0) + J_g(0) = \frac{1}{2}.
\]

Indeed, it is believed that GDPs offer an opportunity of a more complete description of the nucleon wave-function.

C. General Parton Operators and GDPs

In the forward case, the irreducible matrix elements of the parton operators, obtained after factorizing out the tensorial/Lorentz structure and kinematic factors, are combinations of moments of the conventional PDFs, and can be used to define the PDFs \([2]\). In the non-forward case, similarly, matrix elements of (twist-two) parton operators are expanded into general nucleon form factors (and kinematic/Lorentz factors). Moments of GDPs are linear combinations of these form factors, that is, they are polynomials in the general nucleon form factors in powers of \( \xi \). These general “sum rules” can be used to define the GDPs. From the point view of the low-energy nucleon structure, the GDPs can be considered as the generating functions for the form factors of the twist-two operators. The following is the list of twist-two quark and gluon operators and their corresponding GDPs \([1, 3, 4]\):

\[
\begin{align*}
\mathcal{O}^{\mu_1 \cdots \mu_n}_q &= \bar{\psi}_q i \bar{D}^{(\mu_1} \cdots i \bar{D}^{\mu_{n-1})} \gamma_\mu \psi_q \quad \leftrightarrow \quad \left( H_q(x, \xi, t), \ E_q(x, \xi, t) \right) \\
\tilde{\mathcal{O}}^{\mu_1 \cdots \mu_n}_q &= \bar{\psi}_q i \tilde{D}^{(\mu_1} \cdots i \tilde{D}^{\mu_{n-1})} \gamma_5 \psi_q \quad \leftrightarrow \quad \left( \tilde{H}_q(x, \xi, t), \ \tilde{E}_q(x, \xi, t) \right) \\
\mathcal{O}^{\mu_1 \cdots \mu_n}_g &= F^{(\mu_1 \alpha i} \tilde{D}^{\mu_2} \cdots i \tilde{D}^{\mu_{n-1})} F_{\alpha}^{\mu_n)} \quad \leftrightarrow \quad \left( H_g(x, \xi, t), \ E_g(x, \xi, t) \right)
\end{align*}
\]
The lowest spin, the helicity-flip GPDs are defined explicitly by [4, 6, 10]:

\[ O_{g}^{\mu_{1}...\mu_{n}} = F(\mu_{1}\alpha_{1} D \cdots i D^{\mu_{n-1}} iF_{\alpha_{n}}) \quad \leftrightarrow \quad \left( H_{g}(x, \xi, t), \bar{E}_{g}(x, \xi, t) \right) \]

\[ O_{q}^{\mu_{1}...\mu_{n}\alpha} = \bar{\psi}_{q} i^{\mu_{1}} D \cdots i D^{\mu_{n-1}} \sigma^{\mu_{n}\alpha} \psi_{q} \quad \leftrightarrow \quad \left( H_{Tq}(x, \xi, t), E_{Tq}(x, \xi, t), \bar{H}_{Tq}(x, \xi, t), \bar{E}_{Tq}(x, \xi, t) \right) \]

\[ O_{g}^{\mu_{1}...\mu_{n}\alpha\beta} = F(\mu_{1}\alpha_{1} D \cdots i D^{\mu_{n-1}} F^{\mu_{n}\beta}) \quad \leftrightarrow \quad \left( H_{Tg}(x, \xi, t), E_{Tg}(x, \xi, t), \bar{H}_{Tg}(x, \xi, t), \bar{E}_{Tg}(x, \xi, t) \right) \]



We concentrate on the last two–each of which corresponds to 4 (helicity-flip) GPDs. For the lowest spin, the helicity-flip GPDs are defined explicitly by [4, 6, 10]:

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'S'|\bar{\psi}_{q}(-\frac{1}{2}\lambda n)\sigma^{\mu\nu}\psi_{q}(\frac{1}{2}\lambda n)|PS\rangle = H_{Tq}(x, \xi) U(P'S')\sigma^{\mu\nu} U(PS) \\
+ \bar{H}_{Tq}(x, \xi) \bar{U}(P'S') \frac{F^{[\mu i \Delta^\alpha]}_{\nu} M^2}{M} U(PS) + E_{Tq}(x, \xi) \bar{U}(P'S') \frac{\gamma^{[\mu i \Delta^\nu]}_{\nu} M}{M} U(PS) \\
+ \bar{E}_{Tq}(x, \xi) \bar{U}(P'S') \frac{\gamma^{[\mu i \Delta^\nu]}_{\nu} M}{M} U(PS) + ...
\]

\[
1 \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'S'|F^{(\mu\alpha)}(-\frac{\lambda}{2} n)F^{(\nu\beta)}(\frac{\lambda}{2} n)|PS\rangle = H_{Tg}(x, \xi) \bar{U}(P'S') \frac{\sigma^{[\mu i \Delta^\alpha]}_{\nu} M}{M} U(PS) \\
+ \bar{H}_{Tg}(x, \xi) \bar{U}(P'S') \frac{\gamma^{[\mu i \Delta^\alpha]}_{\nu} M^2}{M} U(PS) + E_{Tg}(x, \xi) \bar{U}(P'S') \frac{\gamma^{[\mu i \Delta^\nu]}_{\nu} M}{M} U(PS) \\
+ \bar{E}_{Tg}(x, \xi) \bar{U}(P'S') \frac{\gamma^{[\mu i \Delta^\nu]}_{\nu} M}{M} U(PS) + ...
\]

(2)

where each distribution is also implicitly dependent on \( Q^2 \) and \( t = \Delta^2 \). In the first equation, \([\mu\nu]\) denotes anti-symmetrization of the two indices and the ellipses represent higher twist structures. The gauge link between the quark fields is not explicitly shown. In the second equation \([\mu\alpha]\) and \([\nu\beta]\) are antisymmetric pairs and \((\cdots)\) signifies symmetrization of the two and removal of the trace.

### III. COUNTING OF TWIST-2 HELICITY-FLIP NUCLEON FORM FACTORS

#### A. The General Counting Method

A general method was developed by Ji and Lebed [2] to count the number of independent nucleon form factors using overall CPT invariance and crossing symmetry. It is a basic property of relativistic quantum field theory that the number of independent amplitudes is the same in all crossed channels. Therefore, in the direct channel one uses parity and time reversal invariance for non-forward matrix elements of the parton operators \( \langle P|O|P\rangle \), while in the crossed channel one uses parity and charge conjugation invariance for creation of a particle-antiparticle pair \( \langle P|\bar{O}|0\rangle \). The two resulting types of constraints are equivalent due to overall CPT invariance, leading to same structure for allowed form factor decompositions. Thus by matching the allowed quantum numbers of the two channels one can systematically count the number of independent nucleon form factors. We apply this method to the helicity-flip operators/nucleon form factors (in eqn. 2).
B. Counting of Independent Nucleon Form Factors

The quark helicity-flip operator $\mathcal{O}_{T_{\mu_1 \ldots \mu_n}}$ transforms as representations of Lorentz group $(\frac{n-1}{2}, \frac{n+1}{2})$ and $(\frac{n+1}{2}, \frac{n-1}{2})$. As a tensor of $(n, 1)$ (with $n$ symmetric and 1 pair of antisymmetric indices), the generic tensorial counting gives its number of independent elements as $2 \times n(n + 2)$ (for details on notation and tensorial counting, see [1, 11, 12]).

For the matrix elements in the crossed channel, $\langle P | \mathcal{O}_{T_{\mu_1 \ldots \mu_n}} | 0 \rangle$, the particle and antiparticle pairs $J^{PC}(L)$ values are $J = L + S$, $P = (-1)^{L+1}$, $C = (-1)^{L+S}$, $S = 0, 1$. In terms of $J = n$, we have $S = 0$: $L = J$, $P = (-1)^{J+1}$, $C = (-1)^{J}$, $(-1)^{J+1}$, while $S = 1$: $L = J+1$, $P = (-1)^{L}$, $C = (-1)^{L}$. On the other hand, for the matrix elements in the direct channel, $\langle P | \mathcal{O}_{T_{\mu_1 \ldots \mu_n}} | P \rangle$, the representation $(A, B)$ and $(B, A)$ have angular momentum $J = |A - B|, |A - B| + 1, \ldots, A + B$ (since $\bar{J} = \bar{A} + \bar{B}$). Since parity transforms $A \leftrightarrow B$, for each $J$, both $J^\pm$ are allowed. The charge conjugation $C$ of $\gamma^\mu$, (each) $i \mathcal{D}^\mu$, and $\sigma^{\mu\nu}$ are all $-1$. Thus we have $J = 1, 2, \ldots, n$, $P = \pm$, $C = (-1)^n$ (with $L = J \pm 1$).

From the above analysis, we obtain the results in Table I including lists of allowed quantum numbers in both channels, the allowed quantum number structure from matching between the two channels, and the enumeration of independent form factors for a rank $r = n + 1$ twist-two quark helicity-flip operator.

| $J_{\text{max}}$ | $P \bar{P}$ ($J^{PC}(L)$) | $\mathcal{O}_{T_{\mu_1 \ldots \mu_n}}$ ($J^{PC}$) | Matched $J^{PC}$ | Enumeration |
|------------------|-----------------------------|---------------------------------|-----------------|-------------|
| 0                | 0++(1), 0−−(0)              | N/A                             | N/A             | N/A         |
| 1                | 1++(1), 1−−(1), 1−−(0), 1−−(2) | 1+++, 1−−−                            | 1−−−, 1−−−(×2) | (1 + 1) = 3 |
| 2                | 2++(1), 2++(3), 2−−(2) | 3+++, 3−−−, 2+−−, 2−+−, 3++−, 3−−−, 2−−−, 3−−−(×2) | 1+++, 2++(×2), 2−−− | (1 + 1) = 4 |
| 3                | 3++(3), 3−−(3), 3−−(4) | 3+++−, 3−−−, 3−−−                          | 3++−, 3−−−(×2) | (1 + 1) + (1 + 1) = 7 |
| 4                | 4++(5), 4−−(4) | 4+++−, 4−−−, 4−−−                         | 4++−, 4−−−(×2) | (1 + 1) + 1 + (1) = 6 |
| ...              | ...                         | ...                                      | ...            | ...         |
| odd              | $n^{−−}(n \pm 1)$, $n^{++}(n)$, $n^{+−}(n)$ | $1++−$, 1−−−, ..., 1−−−, 1−−−(×2) | 1−−−, 1−−−(×2) | (1 + 1) + ... + (1 + 1) = $\frac{3n+1}{2} + \frac{n-1}{2} = 2n + 1 (=2r-1)$ |
| even             | $n^{+−}(n \pm 1)$, $n^{−−}(n)$, $n^{++}(n)$ | $1++, 1−−−, ..., n++$, $n^{−−}$ | $1++, 2++(×2)$, $2−−−$ | (1 + 1) + ... + (1 + 1) = $\frac{n}{2} \times 3 + \frac{n}{2} = 2n (=2r-2)$ |
In the case of the gluon helicity-flip operators $O_{gT}^{\mu_0\nu_0\mu_1\ldots\mu_n} (= F^\mu_{\alpha i} D^{\nu_0}_i \cdots \tilde{D}^{\nu_0}_{i+n} F_{\nu_0\nu_0})$, they are $(n+2, 2)$ tensors and transform under representations $(\frac{n}{2}, \frac{n+4}{2})$ and $(\frac{n+4}{2}, \frac{n}{2})$, with angular momentum values $J = 2, 3, \ldots, n+2$. At the same time, both parity values are allowed, and the bilinear gluon fields $FF$ have positive charge conjugation. Following similar steps as in the quark case, the enumeration in Table II is obtained ($r = n + 4$ is the rank of the tensor operator).

**TABLE II: The enumeration of independent form factors of helicity-flip gluon operators.**

| $n$ | $O^{\mu_0\nu_0\mu_1\ldots\mu_n}$ | Matched ($J^{PC}(L)$) | Enumeration |
|-----|----------------------------------|------------------------|-------------|
| 0   | $2^{++}, 2^{-+}$              | $2^{++}(1), 2^{++}(3), 2^{-+}(2)$ | 3           |
| 1   | $2^{+-}, 2^{--}, 3^{+-}, 3^{--}$ | $2^{--}(2), 3^{--}(3), 3^{--}(2), 3^{--}(4)$ | $1 + 3 = 4$ |
| 2   | $2^{++}, 2^{-+}, 3^{+-}, 4^{+-}$ | $2^{++}(1), 2^{++}(3), 2^{-+}(2), 3^{+-}(3), 4^{+-}(3)$ | $3 + 1 + 3$ |
|     |                                  | $4^{+-}(3), 4^{+-}(5), 4^{+-}(4)$ | = 7         |
| 3   | $2^{+-}, 2^{--}, 3^{--}, 4^{--}$ | $2^{--}(2), 3^{--}(3), 3^{--}(2), 4^{--}(4)$ | $1 + 3 + 1$ |
|     | $4^{--}, 5^{--}, 5^{--}$        | $4^{--}(4), 5^{--}(5), 5^{--}(4), 5^{--}(6)$ | +3 = 8      |
|     |                                  |                         |             |
| odd $n$ | $2^{+-}, 2^{--}, \ldots, \ [n+2]^{+-}, [n+2]^{--}$ | $2^{--}(2), 3^{--}(3), 3^{--}\times 2, \ldots, \ [n+2]^{+-}(n+2), [n+2]^{--}\times 2$ | $(1 + 3) \times \frac{n+1}{2}$ |
|     |                                  |                         | $= 2(n+1) = 2r - 6$ |
| even $n$ | $2^{++}, 2^{-+}, \ldots, \ [n+2]^{++}, [n+2]^{--}$ | $2^{++}\times 2, 2^{-+}(2), \ldots, \ [n+2]^{++}\times 2, [n+2]^{--}(n+2)$ | $3 + (1 + 3)\times \frac{n}{2}$ |
|     |                                  |                         | $= 2n+3 = 2r - 5$ |

**C. Form Factor Decomposition of Quark Operators**

Before we can write down the form factor decomposition of the quark operators, we need to discuss whether time-reversal invariance (Hermiticity requirement) would impose further constraints on the form factors.

The higher rank quark operators will have factors of $\mathcal{F}$ and $\Delta$ after taking matrix elements between $P'$ and $P$, coming from the covariant derivatives $[3, 13]$. From Hermiticity requirements, each factor of $\Delta^\mu$ will introduce a factor of $-1$ while $P^\mu$ factor will not. The overall sign factor from the covariant derivatives in the matrix element, will be $(-1)^l$ with $l$ the number of factors of $\Delta$. On the other hand, the Lorentz structures have the following behavior

$$\mathcal{U}[\gamma^\alpha, \gamma^\mu_1]U \sim \sigma^{\mu\nu}; \quad \mathcal{U}[\gamma^\alpha, \mathcal{F}^\mu]U \sim \gamma^\mu; \quad \mathcal{U}[\gamma^\alpha, \Delta^\mu_1]U \sim \gamma^\mu; \quad \mathcal{U}[\mathcal{F}, \Delta^\mu_1]U \sim \mathcal{U}U$$
where the \( \sim \) sign means having the same properties under time reversal (hermitian, complex conjugate) and the first type of terms is odd ("-"), while the rest three are even ("+"). These four are the only ones we need to consider in this case \( \{1, 3, 4\} \). The operator \( \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n} \) is odd ("-") overall. Therefore to give the matrix elements the proper signs under time reversal/Hermitian, namely overall odd ("-"), we have the enumeration in Table III from these potentially additional constraints for the matrix element \( \langle P'| \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n} | P \rangle \).

| Term \( \mathcal{U}[\gamma, \mu] \) | \( \hat{\tau} \) | Sign allowed from Factors of \( \bar{P} \) and \( \Delta \) |
|---|---|---|
| \( n=1 \) | \( \cdots \) | \( n=2k+1 \) | \( n=2k \) |
| \( \mathcal{U}[\gamma, \mu] U \) | - | + | \((-1)^l\) with \( l = 0, 2, \ldots, 2k (= n-1) \) | \((-1)^l\) with \( l = 0, 2, \ldots, 2k-2 (= n-2) \) | \((-1)^l\) with \( l = 0, 2, \ldots, 2k-1 (= n-1) \) | \((-1)^0\) with \( l = 0, 2, \ldots, 2k-2 (= n-2) \) |
| (# allowed) | (1) | | \( k + 1 = \frac{n+1}{2} \) | \( k = \frac{n}{2} \) | \( k = \frac{n}{2} \) |
| \( \mathcal{U}[\gamma, P^\mu] U \) | + | + | \((-1)^l\) with \( l = 1, 3, \ldots, 2k-1 (= n-2) \) | \((-1)^l\) with \( l = 1, 3, \ldots, 2k-1 (= n-1) \) | \((-1)^l\) with \( l = 1, 3, \ldots, 2k-1 (= n-2) \) | \((-1)^0\) with \( l = 1, 3, \ldots, 2k-2 (= n-2) \) |
| (# allowed) | (0) | | \( k = \frac{n+1}{2} \) | \( k = \frac{n+1}{2} \) | \( k = \frac{n+1}{2} \) |
| \( \mathcal{U}[\gamma, \Delta^\mu] U \) | + | - | \((-1)^l\) with \( l = 0, 2, \ldots, 2k (= n-1) \) | \((-1)^l\) with \( l = 0, 2, \ldots, 2k-2 (= n-2) \) | \((-1)^l\) with \( l = 0, 2, \ldots, 2k-1 (= n-1) \) | \((-1)^0\) with \( l = 0, 2, \ldots, 2k-2 (= n-2) \) |
| (# allowed) | (1) | | \( k + 1 = \frac{n+1}{2} \) | \( k = \frac{n}{2} \) | \( k = \frac{n}{2} \) |
| \( \mathcal{U}[P^\gamma, \Delta^\mu] U \) | + | - | \((-1)^l\) with \( l = 0, 2, \ldots, 2k (= n-1) \) | \((-1)^l\) with \( l = 0, 2, \ldots, 2k-2 (= n-2) \) | \((-1)^l\) with \( l = 0, 2, \ldots, 2k-1 (= n-1) \) | \((-1)^0\) with \( l = 0, 2, \ldots, 2k-2 (= n-2) \) |
| (# allowed) | (1) | | \( k + 1 = \frac{n+1}{2} \) | \( k = \frac{n}{2} \) | \( k = \frac{n}{2} \) |
| (Total #) | 3 | | \( 4k+3 = 2n+1 \) | \( 4k = 2n \) |

It is obvious that the two enumerations are consistent with each other, that is, time reversal invariance does not further limit the number of independent form factors. Therefore, the explicit decomposition of the non-forward matrix elements of quark helicity-flip operators into independent general nucleon form factors, Lorentz structures, and kinematical factors is \( \mathcal{U} \).

\[
\langle P'| \mathcal{O}^{\mu_1 \mu_2 \cdots \mu_n} | P \rangle = \mathcal{U}(P') \sigma^{\alpha \mu_1} U(P) \sum_{i=0}^{[n+1]} A_{n,2i-1} \Delta^{\mu_2} \Delta^{\mu_3} \cdots \Delta^{\mu_{2i-1}} \bar{P}^{i_{2i}} \cdots \bar{P}^{i_n}
+ \mathcal{U}(P') [\gamma^\alpha, \bar{P}^\mu] U(P) \sum_{i=0}^{[\frac{n}{2}]} B_{n,2i} \Delta^{\mu_2} \Delta^{\mu_3} \cdots \Delta^{\mu_{2i+1}} \bar{P}^{i_{2i+1}} \cdots \bar{P}^{i_n}
+ \mathcal{U}(P') [\gamma^\alpha, \Delta^\mu] U(P) \sum_{i=0}^{[\frac{n+1}{2}]} iC_{n,2i-1} \Delta^{\mu_2} \Delta^{\mu_3} \cdots \Delta^{\mu_{2i-1}} \bar{P}^{i_{2i}} \cdots \bar{P}^{i_n}
+ \mathcal{U}(P') [\bar{P}^\alpha, \Delta^\mu] U(P) \sum_{i=0}^{[\frac{n+1}{2}]} iD_{n,2i-1} \Delta^{\mu_2} \Delta^{\mu_3} \cdots \Delta^{\mu_{2i-1}} \bar{P}^{i_{2i}} \cdots \bar{P}^{i_n}.
\]
IV. SUMMARY AND OUTLOOK

The generalized parton distributions generalize both the Feynmen PDFs and form factors and offer the possibility of explore at the parton level the three-dimensional structure, and unravel the spin structure, of hadrons.

By using CPT invariance and crossing symmetry we obtained the enumeration of independent general nucleon form factors of twist-two helicity-flip quark and gluon operators. The numbers of independent form factors are: for spin-$J$ quark operator (rank $r = J$), $2J - 1$ (odd $J$) and $2J - 2$ (even $J$), while for spin-$J$ gluon operator (rank $r = J + 2$), $2J - 2$ (odd $J$) and $2J - 1$ (even $J$). The form factor decomposition of the quark operators is also explicitly written out, in terms of the independent form factors, kinematical factors and Lorentz structures. Immediate further work would be the explicit decomposition of the gluon helicity-flip operators.

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