Load Distribution on Small-world Networks

Huijie Yang, Tao Zhou, Wenxu Wang, and Binghong Wang
Department of Modern Physics and Nonlinear Science Center,
University of Science and Technology of China,
Hefei Anhui 230026, China

Fangcui Zhao
College of Life Science and Bioengineering,
Beijing University of Technology, Beijing 100022, China

(Dated: March 23, 2022)

Mapping a complex network to an atomic cluster, the Anderson localization theory is used to obtain the load distribution on a complex network. Based upon an intelligence-limited model we consider the load distribution and the congestion and cascade failures due to attacks and occasional damages. It is found that the eigenvector centrality (EC) is an effective measure to find key nodes for traffic flow processes. The influence of structure of a WS small-world network is investigated in detail.

PACS numbers: 89.75.Hc, 89.20.Hh, 05.10.-a

Congestion is a fatal problem in many real world complex systems such as the Internet, the power grid and the transportation networks, etc. The dynamical mechanism of the occurrence and the possible ways to alleviate the congestion attract special attentions in recent years. It is a complex phenomenon that depends on a large amount of variables, including the dynamical processes of traffic flows based upon protocols, the topology structures of the complex networks, the tolerance of the elements under attacks or occasional damages.

In the previous works in modeling traffic flows on complex networks, a basic assumption is that the networks possess regular (or random) and homogenous structures [1-14]. However, statistical measurements of structures tell us that the real world networks deviate from the Erdos-Renyi and regular networks significantly. The influence of topology on traffic flow becomes the new focus at present time [15-17]. And the development of complex network theory makes it possible for us to deal with this problem in detail.

In the extensive works, models of traffic flows on computer networks have been studied, where one common feature for all the models is that the routers route the data packets to their destinations in a complete intelligent way, that is, a router finds for a packet the shortest path between the host and the destination and forward it along this path step by step. These models can be called intelligence models (IMs). For an actual network with a large amount of routers, it is hard for all the routers to synchronize their route tables due to the exponential increase of source consumption and the dynamics of networks (adding and/or deleting of nodes and edges).

And finding a shortest path way based upon complete route tables is clearly a non-trivial task from the perspectives of source consumptions and algorithms. Recently, new models with limited intelligence, called intelligence-limited models (ILMs), are proposed to make the considerations much close to reality [18,19]. This feature of protocols determines directly the load distribution of the traffic flows on complex networks and the consequent congestion and cascade failures.

When the load of traffic flow is light enough, the created packets can be processed and delivered in time, leading a steady state being reached after a short transient time. With the increase of load, congestion may occur for some nodes, which may induce cascade failures in a complex network. Hence, obtaining the load distribution at the steady state is one of the basic problems in understanding the dynamical processes of traffic flow. In this Letter, mapping the traffic flow to a statistical feature of a large amount of particles in an atomic cluster, the molecular-orbitt theory is used to obtain the load distribution at the steady state on a complex network. This approach can describe in a unified way the how the network structure and transferring protocols affect the traffic behaviors.

Consider a complex network, the routing algorithms of each node can be illustrated as,

1. Each node can generate \( \alpha \) packets per time step.
2. Once a packet reaches its destination, it is removed from the traffic.
3. The destinations are distributed homogenously on the complex network.
4. At each time step, the probability for node \( i \)'s delivering a packet to node \( j \) is \( D(i, j) \). It is determined by the protocols and the structures of complex networks.
5. Map a complex network to an atomic cluster, the nodes and edges to atoms and bonds, respectively. The packet
flow on a complex network can be regarded as the statistical features of a large amount of particles in a large cluster of atoms. Denote the adjacent matrix of the complex network with $A$, the element $A_{ij}$ is 1 and 0 if the nodes $i$ and $j$ are connected and disconnected, respectively. The the coupling Hamilton of this cluster reads,

$$H = H_0 + V,$$

$$(H_0)_{ij} = \varepsilon_0 \delta_{ij},$$

$$V_{ij} = A_{ij} \cdot v_{ij}, \quad (1)$$

where $\varepsilon_0$ is the site energy of each node and $V_{ij}$ the coupling between the nodes $i$ and $j$.

The probability of a packet’s jumping from node $i$ to node $j$ should be,

$$D(i, j) \propto |V_{ij}|^2. \quad (2)$$

The values of $v_{ij}$ can be determined from the transferring strategies in protocols. As an example we consider the intelligence-limited model presented in reference [18], named partial intelligence-limited model (PILM) in this Letter. In that model the probability for a packet’s jumping from node $i$ to node $j$ is proportional to a power function of the node $j$’s degree, e.g., $k(j)^{2\alpha}$. The coupling between node $i$ and node $j$ should be,

$$V^{PILM}_{ij} \propto A_{ij} \cdot [k(i) \cdot k(j)]^\alpha \quad (3)$$

Consider a special condition that the route table of each router contains only the information that whether it is the destination of a packet or not. Because of this intelligence limitation, the packets at a node are delivered forward in a random way to the connected nodes if it is not the destinations. This model is called complete intelligence-limited model (CILM) in the present Letter. For this CILM model, the coupling between node $i$ and node $j$ should be,

$$V^{CILM}_{ij} \propto A_{ij} \quad (4)$$

At each time step, a node $i$ delivers $C(i)$ packets to the nodes connecting with it. In literature this deliver capacity, $C = \{C(i) \mid i = 1, 2, 3, \cdots, N\}$, is designed according to the assumption that the packets are delivered to the destinations along the shortest paths, i.e., the complete intelligence of routers (IMs). In the ILMs, this capacity should be re-designed according to the load distribution.

From the coupling Hamilton depicted in Eq. (1), the Anderson’s localization theory [20] can be employed to investigate the load distribution in complex networks. Consider conditions with small values of the packet creation rate $\alpha$, steady states can be reached after a short transient time. For a regular network the distribution of traffic packets are homogenous on the whole network. For a general complex network, the periodic symmetry is broken, which can induce the localization of the distribution of traffic packets. Denote the eigenvector corresponding to the maximal eigenvalue $E_{\text{max}}$ of the coupling matrix $H$ with,

$$V_{\text{max}} = \{V_{\text{max}}(i) \mid i = 1, 2, 3, \cdots, N\}. \quad (5)$$

At a steady state, the load distribution reads,

$$p_{\text{steady}}(i) = \frac{|V_{\text{max}}(i)|^2}{\sum_{s=1}^{N} |V_{\text{max}}(s)|^2}, \quad i = 1, 2, 3, \cdots, N. \quad (6)$$

This load distribution can reflect directly the structure effects and protocol effects on the traffic flows.

It should be noted that the packets can aggregate theoretically at a node without limits. The packets can be regarded as bosons, which can stay in a same molecular orbit simultaneously. In all the possible molecular orbits, the principal eigenvector is the only candidate at which the packets can reach their randomly selected destinations. For the other molecular orbits, there exist some special nodes with zero values of the occurrence probabilities. In these states some packets may stay always in a local area and can not arrive at their destinations. Hence, the packet current will not reach a steady state. Setting a life time to each packet can guarantee the number of the packets in the network tends to a certain constant, but this local-based steady state can not realize the communications we expected. The steady state under the random selection of destinations should be the principal eigenvector.

Hence, the deliver capacity can be designed as,

$$C = \{C(i) = (1 + \delta) \cdot p_{\text{steady}}(i) \cdot \alpha_0 N \mid i = 1, 2, 3, \cdots, N\}, \quad (7)$$

where $\delta > 0$ is the redundant capacity designed for each router, $\alpha_0$ the maximal value of the designed packet creation rate.

With the increase of $\alpha$, congestion becomes possible due to the increase of loads for all the nodes. Theoretically, we can obtain the critical point as $\alpha_{\text{critical}} = (1 + \delta)\alpha_0$, at which the load of each node reaches its designed deliver capacity. But before this critical point occasional congestions may occur randomly on the network due to the fluctuation of the loads. As one of the interesting measurements we consider the re-distribution of the loads and the possible cascade failures due to occasional congestions. At each time step, reckon the number of the nodes whose loads overcome their deliver capacities as the number of new congestions.

The nodes with heavy loads should be key nodes in considering the traffic flow on a complex network. Hence, the eigenvector centrality measure [21,22] is suitable to identify key nodes in a complex network. By this way we can investigate the attack effects.
Consider the CILM model on WS small-world networks with the rewiring probability \( p_r \in [0, 0.2] \). The WS small-world networks are generated using the model proposed in Ref. [23]. The size of a network is \( N = 3000 \) and the right-handed number of nodes joined with each node is \( k = 2 \).

Fig. 1 shows that the probability distribution function of load obeys a power-law, the exponent approximates to \( \sim 2.16 \). There exist some nodes with heavy loads, which should be key nodes for traffic flows in these complex networks.

Fig. 2 presents the loads of all the nodes in a WS small-world network with \( p_r = 0.12 \), from which we can find that the nodes labelled 160, 564, 1092, 735 are the most important nodes.

Randomly selected 30 nodes are removed from this network to simulate the occasional damages. For each time step we obtain the re-distributed loads. The nodes whose loads overcome the designed capacities are regarded as new congestions and removed from the original complex network. Fig. 3 presents the possible cascade failures due to occasional damages. When the redundant capacity \( \delta \) is large enough, these occasional damages cannot spread all over the network and the traffic flow is free and uncongested. There is a critical value of \( \delta^{occ} = 0.43 \), under which the global cascade failures will occur.

Fig. 4 gives the possible cascade failures due to attacks. The key node labelled 160 with the heaviest load is removed from the original network. If the redundant capacity is not larger than the critical value \( \delta^{attack} = 0.54 \), this removal will induce the global cascade failures. This critical value is significantly larger than \( \delta^{occ} \).

In summary, mapping the traffic flow of packets to a statistical feature of a large amount of electrons in an atomic cluster, we propose the coupling Hamilton to describe the jumping processes between the nodes of a complex network. The Anderson localization theory is used to obtain the load distribution on a complex network at steady state.

This method to determine the load distribution on a complex network makes it possible to design the delivering capacities of nodes according to the conditions such as the structure of a complex network, the packet transferring protocol, the tolerance performance of each node and even the bandwidths of the edges, etc. Consequently, from the coupling Hamilton, we can detect the key nodes in the dynamical process of traffic flow.

The CILM model is used to consider the traffic processes on a WS small-world network. For this complex network there exist some key nodes, removal of which may induce cascade failures. Occasional damages can also induce cascade failures. There are two critical points, \( \delta^{occ} \) and \( \delta^{attack} \), under which cascade failures can occur due to occasional damages and attacking the most important key node, respectively. These simulations show that the method proposed in this paper is powerful and universal to determine the load distribution on a complex network at steady state and to detect the key nodes in the dynamical process of traffic flow.

Acknowledgments

This work has been partially supported by the National Natural Science Foundation of China under Grant No. 70471033, 10472116 and No.70271070. It is also supported by the Specialized Research Fund for the Doctoral Program of Higher Education (SRFD No. 20020358009).
FIG. 3: Cascade failures due to occasional damages on the WS small-world network $(p_r = 0.12)$. $\alpha_0 = 10$. Randomly selected 30 nodes are removed from this complex network to simulate the occasional damages. For each time step we obtain the re-distributed loads. The nodes whose loads overcome the designed capacities are regarded as new congestions and removed from the original complex network. There is a critical value of $\delta_{occ} = 0.43$, under which cascade failures may occur. In the inset we present the number of congestions at 30 unit time for different value of redundant $\delta$.

One of the authors (H. Yang) would like to thank Prof. Y. Zhuo, Prof. J. Gu in China Institute of Atomic Energy and Prof. S. Yan in Beijing Normal University for stimulating discussions.

1 H. Li and M. Maresca, IEEE Trans. Comput. 38, 1345(1989).
2 W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, Comput. Commun. Rev. 23, 183(1993).
3 M. S. Taqqu, W. Willinger, and R. Sherman, Comput. Commun. Rev. 27, 5 (1997).
4 M. E. Crovella and A. Bestavros, IEEE/ACM Trans. Netw. 5, 835(1997).
5 T. Ohira and R. Sawatari, Phys. Rev. E 58, 193 (1998).
6 M. Faloutsos, P. Faloutsos, and C. Faloutsos, Comput. Commun. Rev. 29, 251 (1999).
7 H. Fiks and A. T. Lawniczak, Math. Comput. Simul. 51, 101(1999).
8 R. V. Solé and S. Valverde, Physica A 289, 595(2001).
9 A. Arenas, A. Díaz-Guilera, and R. Guimerà, Phys. Rev. Lett. 86, 3196(2001).
10 R. Guimerà, A. Arenas, and A. Díaz-Guilera, Physica A 299, 247(2001).
11 R. Guimerà, A. Arenas, A. Díaz-Guilera, and F. Giralt, Phys. Rev. E 66, 026704 (2002).
12 R. Guimerà, A. Díaz-Guilera, F. Vega-Redondo, A. Cabrales, and A. Arenas, Phys. Rev. Lett. 89, 248701 (2002).
13 M. Woolf, D. K. Arrowsmith, R. J. Mondragón-C, and J. M. Pitts, Phys. Rev. E 66, 046106(2002).
14 S. Valverde and R. V. Solé, Physica A 312, 636(2002).
15 Liang Zhao, Ying-Cheng Lai, Kwangho Park, and Nong Ye, Phys. Rev. E 71, 026125(2005).
16 A. E. Motter, A. P. S. de Moura, Y.-C. Lai, and P. Das-gupta, Phys. Rev. E 65, 065102(R) (2002).
17 M. di Bernardo, F. Garofalo, S. Manfredi and F. Sorrentino, arXiv:cond-mat/0504302.
18 B. Tadic, S. Thurner, G. J. Rodgers, Phys. Rev. E 69, 036102(2004).
19 Beom Jun Kim, Cang No Yoon, Seung Kee Han, Hawoong Jeong, Phys. Rev. E 65, 027103 (2002).
20 P. A. Lee and T. V. Ramakerishnan, Rev. Mod. Phys. 57, 287(1985).
21 P. Bonacich, J. Math. Sociol., 2, 113(1972).
22 M. E. J. Newman, Phys. Rev. E 70, 056131(2004).
23 D.J. Watts and S.H. Strogatz, Nature (London) 393, 440(1998).
FIG. 4: Cascade failures due to attacks. $\alpha_0 = 10$. The key node No.160 with the heaviest load is removed from the original complex network. This removal can induce cascade failures. The critical value is $\delta^{\text{attack}}_c = 0.54$, which is larger than $\delta^{\text{occ}}_c = 0.43$ significantly. In the inset we present the number of congestions at 30 unit time for different value of redundant $\delta$. 