Discerning Signatures of Seesaw Models and Complementarity of Leptonic Colliders

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Abstract The seesaw extensions of the Standard Models are well motivated in explaining the tiny neutrino mass. In the process these models predict extra heavy Majorana neutrino, charged lepton and charged scalars depending on the scenarios. Distinguishing such scenarios at the colliders would be one of the first goals if we receive some hint of those. We show how various leptonic colliders can play a role of complementarity in segregating such scenarios and the angular distributions can be instrumental, which carry the information of the matrix element. The $\ell^-\ell^-$, $\ell^+\ell^-\ell^+\ell^-$ colliders are explored in investigating such possibility via a PYTHIA based simulation. We emphasis on muon collider and comment on the prospect of $\mu e$ collider as well.

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1 Introduction

During last few decades Standard Model (SM) has established its supremacy over all the other contemporary models. However, the outcomes from several experiments insinuate that it does not provide a complete theoretical description for all the elementary particles. One of the areas to seek for such beyond Standard Model (BSM) effects is the neutrino sector [1]. While the SM considers neutrinos to be massless, left-chiral, Weyl fermions interacting through the EW processes only, neutrino oscillation data [2], evinces their massive nature, though tiny. Therefore, they should be either Dirac fermions or Majorana fermions by nature. Nevertheless, any irrefutable signature advocating the presence of right-chiral neutrinos in nature has not been found yet. The seesaw models, where a few number of heavy BSM particles are introduced along with the SM ones, are very elegant in resolving this issue in a ingenious way. To address this problem, while heavy right-handed singlet and triplet fermions are introduced in case of Type-I and Type-III seesaw respectively, Type-II scenario considers very massive additional scalars, triplet under $SU(2)$. Again, there exist inverse seesaw mechanisms, where two sets of the light lepton flavor violating singlet or the triplet fermions with heavy mixing among the flavour eigenstates are presumed. But the price paid in all of these scenarios is that the mass eigenstates of neutrinos become Majorana fermions. Possibilities of separating these different seesaw models at the colliders like the LHC has already been studied widely [3–24]. But the viability of any of these seesaw models has yet to be verified experimentally.

Though the hadronic colliders have the advantage of reaching very high centre of mass energy, the hadronically unclean environment makes it difficult to identify the signal with high significance. This motivates us to look into leptonic colliders, where hadronic backgrounds are small in number or negligible. In this paper, we have investigated the distinctive features of several seesaw models at various leptonic colliders. Different $\ell^+\ell^-$ machines ($\ell$ denotes muon or electron) with high energy and luminosity like ILC, CLIC, FCC-ee, muon colliders are going to be build in near feature. There is also an upcoming project MUoNE which aims to measure the hadronic contribution...
to muon $g - 2$ through the elastic $\mu e$ scattering. Again, there is proposal of $\ell^+\ell^-$ collider [25–31] also which has not been built so far because of very limited attainable SM finalstates at these colliders. For instance, the only $2 \to 2$ process possible at $e^-e^-$ collider under SM is the Moller scattering. Notwithstanding, this exquisite idiosyncrasy of $e^-e^-$ collision can now be used as a tool to keep the signal very clean while hunting for minuscule traces of BSM Physics, generally imperceptible.

It has been noticed that the angular distribution acts as a great tool in separating various BSM scenarios [32–37]. In this article we have used this tool to separate all the seesaw models at leptonic colliders. We choose several benchmark points in accordance with current experimental bounds along with diagonal coupling and study the possibility of discerning different seesaw scenarios using the angular distributions. Since, the consideration of the neutrino mass compels the Yukawa coupling for the TeV scaled BSM particles to be very small for both Type-I and Type-III cases, it is very challenging to detect their trace at any collider. Therefore, we use inverse Type-I and Type-III seesaw models, where the smallness of active neutrino masses is controlled by an additional parameter $\mu_n$. For each of the models considered, we select different finalstate and eventually look at the angular distribution of the finalstate leptons or the reconstructed BSM particle in the CM frame with respect to its angle with the beam axis. We find very distinctive angular distributions for different seesaw models at individual leptonic collider.

The paper is organized in following way. In the next section, we briefly discuss the theoretical description of different seesaw models while in section 3 we present the importance of angular distribution in discerning different BSM cases. section 4 goes over the setup for collider simulation. The subsequent section (section 5) deals with differentiating seesaw models at $\ell^+\ell^-$ collider. In section 6 we scrutinize the possibility of the same at $\ell^+\ell^-$ collider. The following section (section 7) discusses the prospect of $\ell^+\ell^-$ collider in this regard. Finally, we conclude in section 8.

## 2 Different seesaw models

In this section, we brief about different seesaw scenarios and the BSM particle spectrum that we look for at the future colliders.

### 2.1 Type-I seesaw

In case of Type-I seesaw [38–42], three generations of $SU(2)_L$ singlet, colourless, heavily massive right-handed neutrinos ($N_R (1, 1, 0)$) with zero hypercharge are introduced which have the relevant interactions and mass terms in the Lagrangian as follows:

$$-\mathcal{L}^I = Y \nu \bar{H} N_R + \frac{1}{2} M_R N_R^c N_R + h.c., \quad (1)$$

where, $L$ and $H (\equiv (H^+, H^0)^T)$ are lepton and Higgs doublets in SM with $\bar{H} = \sigma_3 H^*$, $Y_\nu$ and $M_R$ are coupling and mass matrices with generation indices being suppressed. Here, the notation used for charge conjugate field is: $\psi^c_{LR} = (\psi_{LR}^c)^*$. After the electroweak symmetry breaking (EWSB) the SM neutrinos and the Majorana fermion $N_R^c$ mix, leaving one set of light and another set of heavy neutrinos. The mass-matrix in the flavour basis and the masses for light ($\nu$) and heavy ($N^0$) neutrinos are given by:

$$\mathcal{M} = \begin{pmatrix} 0 & \frac{v_0 Y_\nu}{2} & \frac{v_0 Y_\nu}{2} \\ \frac{v_0 Y_\nu}{2} & M_R^{-1} Y_\nu^T \\ \frac{v_0 Y_\nu}{2} & M_R^{-1} Y_\nu^T \end{pmatrix}, \quad m_\nu \approx \frac{v_0^2}{2} Y_\nu M_R^{-1} Y_\nu^T \quad \text{and} \quad M_{N^0} \approx M_R, \quad (2)$$

(assuming $|v_0 Y_\nu/\sqrt{2}| \ll |M_R|$) with $H$ becoming $(0, h + v_0/\sqrt{2})$ under unitary gauge, where the vev of $H$ is $v_0/\sqrt{2}$. After the production, each of these heavy neutrinos will decay through $Z\nu$, $h\nu$ and $W^\pm, l^\pm$ modes with the decay widths given by:

$$\Gamma_{N^0}^{Z\nu} \approx \Gamma_{N^0}^{h\nu} \approx \frac{1}{2} \Gamma_{N^0}^{Wl} \approx \frac{Y_\nu^2 M_R}{32\pi}. \quad (3)$$

But, it is apparent from Equation 2 that the neutrino mass of $O(10^{-4} \text{ eV})$ either restricts the couplings of the SM leptons with the TeV-scaled BSM particles to be $O(10^{-6})$ or compels the new particles to be heavier than $10^3$ TeV for the coupling higher than $10^{-3}$. This makes the signatures of usual Type-I seesaw models very difficult to be observed even at the future experimental facilities. Notwithstanding, the inverse seesaw (ISS) kind of scenarios has the prospect to be within the detectable limit of future experiments. For Type-I case [43–45], one introduces three generations of two right-handed singlet neutrino fields ($N_a, N_b$), only one ($N_\nu$) of which couples to SM leptons. In this case, though the flavour states are very light, there exists a large off-diagonal coupling between $N_a$ and $N_b$ through the mass term $M_a$. The mass and Yukawa terms of the Lagrangian for this case is given by:

$$-\mathcal{L}^I = Y \nu \bar{H} N_a + M_a \bar{N}_a N_b + \frac{1}{2} \mu_n \bar{N}_b N_b + h.c., \quad (4)$$

where the generation indices have been suppressed. Here, $Y$ is the Yukawa coupling of $N_a$ with SM leptons, large mass term $M_a$ indicates the mixing between $N_a$ and $N_b$, whereas, the tiny mass term $\mu_n$ (in general a complex symmetric matrix) signify the mass of $N_b$. After EWSB and the mixing of the flavour states, there emerges one set of light neutrinos ($\nu$) and two sets of heavy neutrinos ($N^0, \bar{N}^0$), each set consisting of three generations. The en-
tire mass matrix in the flavour basis along with masses for light and heavy neutrinos here can be expressed as:

\[
M = \begin{pmatrix}
\frac{v_0^2 Y}{\sqrt{2}} & \frac{v_0^2 Y}{\sqrt{2}} & 0 \\
\frac{v_0^2 Y}{\sqrt{2}} & 0 & M_{\mu} \\
0 & M_{\mu} & \mu_{\alpha}
\end{pmatrix} ,
\]

\[m_\nu \approx \frac{v_0^2}{2} Y (M_{\mu}^* M_{\mu})^{\frac{1}{2}} \mu_{\alpha} M_{\mu}^{-1} Y T , \quad M_{\mu}^{-1} Y T , \quad M_{\mu N_0}^{-1} \approx M_{\mu} M_{\mu}^{-1} , \quad M_{\mu N_0}^{-1} \approx M_{\mu} M_{\mu}^{-1} , \quad \mu_{\alpha}
\]

(5)

where it has been assumed that \(|v_0 Y/\sqrt{2}| \ll |M_{\mu}|\). As can be noticed from above equation, from the Yukawa coupling \(Y\) and the large mass \(M_{\mu}\), the smallness of active neutrino mass matrix in this case is controlled by an additional parameter \(\mu_{\alpha}\). Hence, with the suitable choice of \(\mu_{\alpha}\), the coupling \(sY\) could be \(O(10^{-1})\) for \(M_{\mu} \sim O(1\, \text{TeV})\), which makes this testable at present and future colliders.

The heavy leptons will finally decay to light leptons associated with Higgs boson or weak gauge bosons. The partial decay widths for each generation of \(N^0\) and \(N^0_\beta\) in different channels are given by:

\[
\Gamma^{2\nu}_{N^0/N_0^0} \approx \Gamma^{\nu \nu}_{N^0/N_0^0} \approx \frac{1}{2} \Gamma^{W_i}_{N^0/N_0^0} \approx \frac{Y^2 M_{\mu}}{6\pi^2} .
\]

(6)

It is interesting to notice that the partial decay width for each channel presented in Equation 3 is twice of that in Equation 6. This fact can easily be understood by investigating the \(h \nu\) mode. The scalar field \(H\) couples to the right handed neutrinos through the \(Y^T L \bar{H} N_0\), where only \(N_0\) field contributes. Now, \(N_0\) contains \(\sim 50\%\) of \(N_0\) as well as \(\sim 50\%\) of \(N_0\) unlike the usual Type-I seesaw, where \(N_0\) is \(\sim 100\%\) of \(N_R\). Similar thing happens for other decay channels also.

2.2 Type-II seesaw

For Type-II seesaw \([46–54]\), a colourless \(SU(2)_L\) triplet (adjoint) scalar \((\Delta(1, 3, 1))\) with hypercharge one, mass \(M_{\Delta}\), coupling with Higgs as \(\mu_{\Delta}\) and Yukawa coupling \(Y_{\Delta}\) is added to particle list. The pertinent portion of Lagrangian is given by:

\[-\mathcal{L}_{\Delta} \supset M_{\Delta}^2 \text{Tr}(\Delta^\dagger \Delta) + \left[ Y_{\Delta} \bar{T}^{(\alpha \beta \gamma) L} - \mu_{\Delta} \bar{H}^\dagger \Delta^\dagger H + h.c. \right] \]

with \(\Delta = \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix} \).

(7)

Though, there could be other gauge invariant terms in the Lagrangian involving quartic couplings of triplet and the SM doublet as well as the self-quartic coupling of the triplet, for our analysis these terms remain irrelevant and hence we do not write them down explicitly. After the symmetry breaking, the neutral component of \(\Delta\) acquires a small vev of \(v_\Delta / \sqrt{2}\) and consequently, the neutrinos pick up a Majorana mass term in the following form:

\[m_\nu = \sqrt{2} v_\Delta Y_{\Delta} \quad \text{where} \quad v_\Delta \approx \frac{\mu_{\Delta} v_0^2}{\sqrt{2} M_{\Delta}} .
\]

(8)

Now, the doubly charged scalar can decay to same sign lepton (SSD) or di-boson channels. While the decay width to di-lepton channel is proportional to \(Y_{\Delta}^2\), the same to di-boson mode varies as \(v_\Delta^2\).

As can be noticed from the above equation that the coupling \(Y_{\Delta}\) can take the value \(O(10^{-1})\) for \(M_{\Delta} \sim O(1\, \text{TeV})\), which makes this testable at present and future colliders.

The \(\rho\) parameter in this model can be expressed as:

\[\rho = \frac{1 + (2v_\Delta^2 / v_0^2)}{1 + (4v_\Delta^2 / v_0^2)} \]

(9)

which in accordance with the present electroweak precision measurement of \(\rho = 1.00037 \pm 0.00023\) \([57]\) indicates that \(v_\Delta \lesssim 2.1\, \text{GeV}\). Type-II seesaw model can contribute to lepton flavour violating decays \(\tau \to j_k l_{\ell m}\) or \(\mu \to e e e\) at tree level via doubly charged scalar. Again, it can also enhance radiative lepton flavour violating process like \(\mu \to e\gamma\) through singly and doubly charged scalars at one loop level. The decay width for these channels can be written as \([58–62]\):

\[
\Gamma(l_i \to j_k l_{\ell m}) = \frac{1}{2(1+\delta_{km})} \left( \frac{m_j^5}{(192\pi)^2} \right) \left| (Y_{\Delta})_{km} (Y_{\Delta})_{ij} \right|^2 , \]

\[
\Gamma(l_i \to j_k \gamma) = \frac{\alpha m_j^5}{(192\pi)^2} \left| (Y_{\Delta})_{ij} \right|^2 \left( \frac{1}{M_{\Delta}^2} + \frac{8}{M_{\Delta}^2} \right) ,
\]

(10)

where \(M_{\Delta^{++}}\) and \(M_{\Delta^+}\) are the masses of \(\Delta^{++}\) and \(\Delta^+\), \(\delta_{km}\) is the Kronecker-delta and \(\alpha\) is the electromagnetic coupling constant. However, for our analysis, we choose diagonal coupling \(Y_{\Delta}\) and hence, the bounds from lepton flavour violating decays can safely be ignored. Type-II seesaw contributes to muonium anti-muonium oscillation \([62–64]\) at tree level via \(\Delta^{++}\). The probability of muonium to anti-muonium transition is given by:

\[
P(\mathcal{M} \to \bar{\mathcal{M}}) = 64 \left( \frac{3\alpha^2}{G_F m_e^2} \right)^2 \left( \frac{m_e}{m_\mu} \right)^6 \left( \frac{G_F \mathcal{M}}{G_f} \right)^2
\]

(11)

with \(G_F \mathcal{M} = (Y_{\Delta})_{ce} (Y_{\Delta})_{\mu e} \mu_e / 16 \sqrt{2} M_{\Delta}^2 \).

where \(G_f\) is the Fermi constant. The upper bound on this probability, given by the PSI experiment, is \(P(\mathcal{M} \to \bar{\mathcal{M}}) \leq 8.3 \times 10^{-11}\) at 90\% confidence level \([65]\) which is expected to be improved in MACE experiment \([66]\). For our choice of benchmark points this probability becomes less than \(\sim 10^{-12}\). Models with heavy neutrinos (like Type-I, Type-III and inverse seesaw) also contribute to this transition probability, but the effects come at one loop-level \([67, 68]\).
Through the doubly and the singly charged scalars, Type-II seesaw affects \( (g - 2) \) of muon as:

\[
\Delta a_\mu(\Delta^{++}) = -\frac{1}{4\pi^2} \frac{m_\mu^2}{M_{\Delta^{++}}^2} \sum_i \left( \frac{4}{3} - \frac{m_i}{m_\mu} \right) |(Y_\Delta)_{i\mu}|^2,
\]

\[
\Delta a_\mu(\Delta^+) = -\frac{1}{48\pi^2} \frac{m_\mu^2}{M_{\Delta^+}^2} \sum_i |(Y_\Delta)_{i\mu}|^2.
\]

(12)

It is interesting to notice that if we assume diagonal coupling, then the total contribution of triplet to muon \((g - 2)\) is negative. For our choice of benchmark points, \(\Delta a_\mu \sim -10^{-12}\), which is quite smaller than the observed experimental value, \(\Delta a_\mu^{\text{exp}} = (2.5 \pm 0.59) \times 10^{-9}\) [69]. We also did not taken into neutrino oscillation data. Actually, since our main goal is to study the angular distribution of different seesaw scenarios at leptonic colliders, we refrain ourselves from finding a parameter space respecting all the low-energy bounds. However, we considered our benchmark points with \(M_\Delta > 750\) GeV at 2\(\sigma\) level, which are allowed by the most recent CMS [15] and ATLAS [13] measurements as shown in Figure 1.

### 2.3 Type-III seesaw

Likewise, the Type-III seesaw model [70–79] consists of all the SM fields in addition to the three generations of colourless SU(2)\(_L\) triplet (adjoint) fermions \((\Sigma_L (1, 3, 0))\) with hypercharge zero, mass \(M_{\Sigma}\), and Yukawa coupling \(\sqrt{2} Y_\Sigma\).

The apposite pieces of Lagrangian are the following:

\[
- \mathcal{L}_{\text{III}} \supset \frac{1}{2} \Gamma_{\Sigma_L M_\Sigma} \Sigma_L^c \Sigma_L^c + \sqrt{2} Y_\Sigma \Sigma_L^c H + \text{h.c.},
\]

with \(\Sigma_L = \begin{pmatrix} \Sigma^0 / \sqrt{2} & \Sigma^+ & -\Sigma^0 / \sqrt{2} \end{pmatrix}\).

After EWSB, the SM neutrinos mix with the neutral components of the fermionic triplet, while the charged ones mingle with the charged leptons. The mass matrices in flavour basis and the masses for all the leptons in energy basis are given by:

\[
M = \left( \begin{array}{c c c} 0 & \frac{\nu_0 Y_{\Sigma}^T}{\sqrt{2}} \Sigma & \nu_0 Y_{\Sigma}^T \end{array} \right),
\]

\[
M' = \left( \begin{array}{c c c} m_\nu & v_\Sigma & m_\nu \\
\nu_0 Y_{\Sigma} & 0 & M_{\Sigma} \\
\nu_0 Y_{\Sigma} & M_{\Sigma} & 0 \end{array} \right),
\]

\(m_\nu \approx \frac{v_0^2}{2} Y_{\Sigma} M_{\Sigma}^{-1} Y_{\Sigma}^T ,\)

\(m_\nu \approx m_\nu', \quad M_{N^0} \approx M_{N^\pm} \approx M_{\Sigma} .\)

(14)

Here, \(l'\) denotes the SM charged leptons in flavour basis while \(l\) and \(N^\pm\) symbolise the same with the light and heavy masses in the mass-basis of Type-III scenario. Now, these heavy leptons will eventually decay to the lighter particles. While the heavy neutral particle \(N^0\) will decay through \(Z\nu, h\nu\) and \(W^\pm\nu\) with the decay widths given by:

\[
\Gamma_{N^0}^{Z\nu} \approx \Gamma_{N^0}^{h\nu} \approx \frac{1}{2} \Gamma_{N^0}^{W\nu} \approx \frac{Y_{\Sigma}^2 M_{\Sigma}}{32\pi} ,
\]

the heavy charged leptons \(N^\pm\) decays to \(Zl^\pm, hl^\pm\) and \(W^\pm\nu\) with the following decay widths:

\[
\Gamma_{N^\pm}^{Zl^\pm} \approx \Gamma_{N^\pm}^{hl^\pm} \approx \frac{1}{2} \Gamma_{N^\pm}^{W\nu} \approx \frac{Y_{\Sigma}^2 M_{\Sigma}}{32\pi} ,
\]

(16)

where we have assumed that \(M_{\Sigma} \gg m_\nu\) with \(m_\nu\) being the mass of the SM Higgs boson around 125.5 GeV. At this point, it is interesting to mention that though the masses of \(N^0\) and \(N^\pm\) are same at tree level, there can emerge a mass splitting of \(|\Delta M| \approx 166\) MeV [80] while considering the loop corrections and it can open up some decay channels of \(N^\pm\) to \(N^0\) with the decay width as follows:

\[
\Gamma_{N^\pm}^{N^0} = \frac{2G_F^2 V_{ud}^2 |\Delta M|^3 f_\Sigma^2}{\pi} \sqrt{1 - \frac{m_\nu^2}{\Delta M^2}} \quad \text{and}
\]

\[
\Gamma_{N^\pm}^{N^0\nu} = \frac{25}{3} \Gamma_{N^\pm}^{N^0\nu} = \frac{2G_F^2 |\Delta M|^5}{15\pi^3} ,
\]

(17)
where \(f_p\) is the pion form factor, \(m_\pi\) is the mass of pion, \(G_F\) is the Fermi constant and \(V_{ud}\) is the CKM matrix element. One should also notice here that none of the decay modes mentioned in Equation 17 depends on mass or Yukawa coupling of the SU(2) triplet. However, for a TeV scale fermionic triplet with Yukawa greater than \(10^{-7}\), one can safely neglect these modes due to very small branching fractions with \(\lesssim 1\%\) [81].

Like the Type-I seesaw, in this case also, the demand of light neutrino mass being less than 0.1 eV pushes the Yukawa coupling of TeV-scaled fermionic triplet to \(\mathcal{O}(10^{-6})\), which is quite challenging to observe in the colliders. Therefore, we investigate for inverse Type-III scenario (iType-III), where three generations of two left handed fermionic triplets \((\Sigma_a, \Sigma_b)\) with only one of them interacting to SM leptons are introduced [82–85]. Here also there exists a large mixing between the flavour states \(\Sigma_a\) and \(\Sigma_b\). The corresponding relevant portions of the Lagrangian are given by:

\[
-\mathcal{L}_{III}^{\text{kin}} = \sqrt{2}Y \Sigma_a \tilde{H} + Tr[\sum_a M_a \Sigma_a + \frac{1}{2} \sum_b \mu_a \Sigma_b] + h.c.,
\]

(18)

where the generation indices have been suppressed. While \(Y\) denotes the Yukawa coupling of triplet \(\Sigma_a\) with SM leptons, \(M_a\) indicates the heavy mixing between \(\Sigma_a\) and \(\Sigma_b\) while small \(\mu_a\) (complex symmetric matrix) signifies the mass of \(\Sigma_b\). After the EWSB and mixing of the flavour states, there appear different mass eigenstates for the neutral as well as the charged leptons. Like the inverse Type-I scenario, here also one set of light neutrinos \((\nu)\) and two sets of heavy neutrinos \((N^0, \tilde{N}^0)\), each set containing three generations, emerges. On the other hand, there also appear one set of light charged leptons \((l)\) and two sets of charged heavy leptons \((N^\pm, \tilde{N}^\pm)\). The mass matrices in flavour basis and the masses of all these physical states are as follows:

\[
\mathcal{M} = \begin{pmatrix}
0 & \frac{v_0}{\sqrt{2}} Y & 0 \\
\frac{v_0}{\sqrt{2}} Y^T & 0 & M_n \\
0 & M_n^T & \mu_n
\end{pmatrix}, \quad \mathcal{M}_\nu = \begin{pmatrix}
0 & v_0 Y & 0 \\
v_0 Y^T & 0 & M_n^T \\
0 & M_n^T & \mu_n
\end{pmatrix},
\]

\[
m_{\nu} \approx \frac{v_0}{2} Y (M_n^{-1})^T \mu_n M_n^{-1} Y^T, \quad m_{l} \approx m_{\nu} \quad \text{and} \quad M_{N_{\tilde{N}}^2/\tilde{N}^0} \approx M_n^2 M_n^T.
\]

(19)

where \(l'\) denotes the flavour states of SM charged leptons. Similar to the Type-I case, one can easily satisfy the neutrino bounds with suitable choice of \(\mu_n\) while considering \(Y \sim \mathcal{O}(0.1)\) and \(M_n \sim \mathcal{O}(1 \text{ TeV})\).

The heavy leptons will finally decay to the light(SM) leptons associated with the Higgs boson or the weak gauge bosons. The partial decay widths of each generation of \(N^0\) and \(\tilde{N}^0\) in different channels are given by:

\[
\Gamma_{N_{\tilde{N}}^0} \approx \Gamma_{N_{\tilde{N}}^0/\tilde{N}^0} \approx \frac{1}{2} \frac{1}{\mathcal{O}(N_{\mu}/\tilde{N}^0)} \approx \frac{Y^2 M_n}{64\pi}.
\]

(20)

As already mentioned in subsection 2.1, the partial decay widths of heavy neutrinos in this case will be half of the same in usual Type-III scenario. Another interesting feature can be observed in the behaviour of the neutral and the charged components of iType-III fermion; unlike the neutral components \(N^0/\tilde{N}^0\), which couples to \(Z, h\) and \(W^\pm\) (Equation 20), one of the charged components, \(N^\pm\) couples to \(Zt\) and \(ht\) only, while the other one, \(\tilde{N}^\pm\) interacts with \(W^\pm\) (see Appendix A). This indicates the decay of \(N^\pm\) through \(ht\) and \(Zt\) modes with 50% branching in each but \(\tilde{N}^\pm\) decays to \(W\nu\) entirely. The partial decay widths for the charged components are given by:

\[
\Gamma_{N^\pm}^{W\nu} \approx \frac{1}{2} \frac{1}{\mathcal{O}(N_{\mu}/\tilde{N}^0)} \approx \frac{Y^2 M_n}{32\pi}.
\]

(21)
The ATLAS and CMS collaborations have already searched for the existence of heavy charged (neutral) leptons at LHC. Non-observation of any such state force them to put experimental upper bounds on the pair production cross section as a function of heavy lepton mass. However, the theoretical cross section, predicted there, involves one generation of heavy lepton only, whereas, in our case there are six copies of heavy charged (neutral) leptons. Therefore, we scale the theoretical estimate lines of Refs. [13, 15] by six and reproduce the plots in Figure 2, where the black dashed lines indicate expected median, the green and yellow regions signify the $1\sigma$ and $2\sigma$ expectations respectively while the red curve symbolizes the theoretical estimation of the pair production cross section with six copies of heavy charged leptons. Considering both the results from ATLAS and CMS, we can restrict the mass of fermionic triplet in inverse Type-III case (three generations) to be higher than 1100 GeV with $2\sigma$ values of expectation.

3 Angular distributions

It is known that the angular distribution in the centre of mass (CM) frame carries information about the matrix elements, and the information of spin of initial and finalstate particles, propagator as well as vertices are also encoded. It has been shown that spins of different BSM particles can be discerned via such angular distributions [32–34]. However, such distribution at the hadronic collider like LHC has certain disadvantage of not knowing the centre of mass frame due to unknown initial state boost along the beam axis, and reconstruction of the CM frame is only possible if the finalstate is fully visible. An interesting feature of such distribution with a massless gauge boson in the centre of mass frame of the leptonic collider, is denoted by the polar angle of one of the leptons in the centre of mass frame along with the difference in angular distributions can hint a new physics signal. This is illustrated in Figure 4, which shows the simulated expectation.

4 Set up for collider simulation

At first, the models are implement at SARAH-4.14.2 [87] and model files for CalcHEP [88] are prepared. The branching fractions and production cross-sections for different BSM particles are estimated through CalcHEP. Then we generate events for different relevant modes via CalcHEP and use the generated “.lhe” files as an input to PYTHIA8 [89, 90] where the events are simulated with FastJET-3.0.3 [91]. The following criteria are being maintained during the simulation:

1. Though we are interested in the angular distribution of various channels for the whole region, to avoid beam line events we restrict the calorimeter coverage for $|\eta| < 4.5$.
2. Regarding jet formation, we use:
   - The ANTI-KT algorithm with the jet radius $R = 0.5$.
   - The minimum transverse momentum for jets $p_T^{\text{jet, min}} = 20\text{ GeV}$.
3. The stable leptons are detected with following cuts:
   - The minimum transverse momentum of the leptons $p_T^{\text{min}} = 10\text{ GeV}$ with $|\eta|_{\text{max}} = 4.5$.
   - The leptons are isolated from the jet with $\Delta R_{i,j} \geq 0.4$, where $\Delta R_{i,j} = \sqrt{\Delta\eta_{i,j}^2 + \Delta\phi_{i,j}^2}$.
   - For a selection of clean lepton, we put an additional cut i.e., the total transverse momentum of the hadrons within the cone $\Delta R = 0.3$ will be $\leq 0.15 p_T^\ell$. Here $p_T^\ell$ is the transverse momentum for the leptons within that specified cone.
4. We have already denoted $l$ as all the three generations of SM charged leptons in mass basis. However, most of our upcoming discussions are based on electron and muon only. Therefore, we symbolize these two leptons as $\ell$.

5 At $\ell^- \ell^-$ collider

Type-I seesaw is very hard to detect at $\ell^- \ell^-$ collider. However, there is production of SM finalstate i.e., $\ell^- \ell^- \rightarrow W^- W^-$, where the BSM particles in principle can play a major role. This process is completely absent in SM, due to non-existence of any Majorana fermion or doubly charged particle. Therefore, a departure from SM prediction along with the difference in angular distributions can hint a new physics signal. This is illustrated in Figure 4,
where $\ell^-\ell^- \rightarrow W^-W^-$ for iType-I and iType-III seesaw are mediated by $N^0/\tilde{N}^0$ and for Type-II seesaw it is mediated by $\Delta^{--}$ in s-channel. In case of the Type-I seesaw, $\ell^-\ell^- \rightarrow W^-W^-$ cross-section is proportional to $(Y_2^2/M_R)^2$ and as mentioned earlier, $Y_2$ has to be very small or $M_R$ has to be very large in order to keep neutrino mass $O(10^{-1}\text{ eV})$, which result in a vanishingly small cross-section. In case of the inverse seesaw, the destructive interference takes place between the t-channel diagrams mediated by $N^0$ and $\tilde{N}^0$, which in turn keeps the cross-section very low. Similar situation arises for the Type-III scenario also for the same channel. In case of the Type-II scenario, the additional contribution for this mode comes from the doubly charged scalar particle in s-channel. However, the $\ell^-\ell^-\Delta^{++}$ vertex is proportional to $Y_D$, whereas the $W^-W^-\Delta^{++}$ vertex is commensurate with $v_D$ eventuating in cross-section proportional to $v_D^2Y_D^2 \sim \frac{4}{7}m_\nu^2$ (see Equation 8). Therefore, in the case of Type-II, such cross-section is vanishingly small.

![Figure 4. Feynman diagrams for $\ell^-\ell^- \rightarrow W^-W^-$ in iType-I and iType-III seesaw (first and second panel with $N^0/\tilde{N}^0$ mediator) and in Type-II seesaw (third panel with $\Delta^{--}$ mediator).](image)

It is interesting to see that, $\ell^-\ell^-$ can still be instrumental in distinguishing at least Type-II and iType-III seesaw mechanisms via the finalstates involving leptons of different flavours (see Figure 5) or involving at least one BSM heavy leptons (see Figure 10) as detailed in the following subsections.

Multi-TeV Muon collider is proposed [92–96] with the reach around 10$\text{ab}^{-1}$ for 10$\text{TeV}$ centre of mass energy. The advantage of the muon collider over the electron is that the former has much less synchrotron radiation. Generically leptonically colliders are free from any initial state QCD radiation. Due to the proposed reach in energy and luminosity, many beyond Standard Model scenarios are looked at the forthcoming muon collider [97–108]. In this case also the collisions happen in the CM frame, making it easier to construct the angular distributions for the finalstate in the CM frame. Hence, we use muon beams as the initial states for our simulation. Regarding our analysis at this collider, we choose three different centre of mass energies which are 1.5 TeV (BP1), 2.0 TeV (BP2) and 3.0 TeV (BP3), respectively as tabulated in Table 1 (Type-II) and Table 3 (iType-III), with an integrated luminosity of 1000 fb$^{-1}$. However, the masses of the BSM particles are chosen differently in different scenarios in order to obtain significant number of events.

### 5.1 Type-II seesaw

In the Type-II seesaw at $\ell^-\ell^-$ collider, the mode $\ell^-\ell^- \rightarrow \ell^-\ell^-$ that occurs through $\Delta^{--}$ mediated s-channel, as depicted in Figure 5. Unlike the $W^-W^-$ channel, the advantage of this mode is that both the vertices in this case are proportional to $Y_D$ only. However, since this mode is mediated through s-channel only, the cross-section would reduce significantly if we move away from the resonance production. The cross-section and the angular distribution with respect to $x = \cos \theta$ for this mode in CM frame are given by:

$$
\sigma(\ell^-\ell^- \rightarrow \ell^-\ell^-) = \int_0^1 \frac{d\sigma}{dx}(\ell^-\ell^- \rightarrow \ell^-\ell^-)dx = \frac{\sqrt{s} Y_D^4}{128\pi[(s-M_\Delta^2)^2+M_\Delta^2\Gamma_\Delta^2]},
$$

(22)

where, $\sqrt{s}$ is the centre of mass energy, $\Gamma_\Delta(\approx \frac{3M_\Delta Y_D^2}{4\pi^2})$ is the total decay width of $\Delta^{--}$ (branching of di-bosons are negligible, since, $v_D$ is very small for our benchmark scenarios) and $\theta$ is the angle between the beam axis and anyone of the finalstate electrons with $x \equiv \cos \theta$. It is interesting to mention here that the angle $\theta$ in this case will vary from 0 to $\pi/2$ only, since the two particles in final state are indistinguishable.

In Figure 6, we describe the variation of total cross-section for the process $\ell^-\ell^- \rightarrow \ell^-\ell^-$ as given in Figure 6, with respect to centre of mass energy and mass of the triplet in the Type-II seesaw scenario. The darker to faint colours represent from large to less cross-sections. It is interesting to notice that, as $M_\Delta$ approaches $\sqrt{s}$, the cross-section enhances largely due to the resonance production of the doubly charged scalar and the three benchmark points are chosen near the resonance production allowed by the collider bounds as tabulated in Table 1.
However, this mode is completely non-existent in Type-I, ISS cases, and also in Type-III, iTYPE-III scenarios if $Y$ and $M_n$ are assumed diagonal. Although there is a possibility for its occurrence in iTYPE-III picture under non-diagonal $Y$ or $M_n$, the cross-section is severely low due to the fact that $\ell'Z$ coupling is quadratic in the ratio of Dirac mass term to the mass of heavy neutrinos after EWSB (i.e. $M_D^2[M_n^1M_n^{-1}]M_D$, where $M_D$ is the Dirac mass term for neutrinos, is given by $\frac{\sqrt{2}}{2} Y$, in comparison with the Type-II coupling $\ell^+\ell^-\Delta^+$, proportional to $Y_\Delta$. Therefore, observation of this mode will not undermine the presence of Type-II seesaw.

Regarding our simulation, we choose muon collider as explained before with the triplet mass $M_\Delta$ to be 1.25 TeV, 1.8 TeV and 2.7 TeV, respectively for the three benchmark points. Taking the Yukawa $Y_\Delta = 0.2$ (and $\mu_\Delta = 10$ eV) the cross-sections of the process $\mu^+\mu^- \rightarrow e^+e^-$ in the CM frame for the benchmark points are 7.4 fb, 10.7 fb and 4.8 fb, respectively, as tabulated in Table 1.

![Figure 6](imageURL)

**Figure 6.** Variation of total cross-section (in fb) for the process $\ell^-\ell^- \rightarrow \ell'^-\ell'^-$, with respect to centre of mass energy and mass of the triplet in Type-II seesaw scenario ($Y_\Delta = 0.2, \mu_\Delta = 10$ eV). The three benchmark points are represented by the yellow stars.

| Benchmark Points | $M_\Delta$ in GeV | $E_{CM}$ in TeV | Cross-section (in fb) |
|------------------|-------------------|-----------------|----------------------|
| BP1              | 1250              | 1.5             | 7.4                  |
| BP2              | 1800              | 2.0             | 10.7                 |
| BP3              | 2700              | 3.0             | 4.8                  |

**Table 1.** Masses corresponding to different benchmark points, energy of collision in CM frame and the hard scattering cross-sections (in fb) for $e^+ e^-$ final states in Type-II seesaw model and $\mu^+\mu^-$ collider. ($Y_\Delta = 0.2, \mu_\Delta = 10$ eV)

Equipped with the setup for the collider simulation as described in section 4, we describe the different kinematical distributions relevant to our analysis in the following paragraphs. In Figure 7(a), we illustrate charged lepton multiplicity and their transverse momentum distribution are shown in Figure 7(b) for all the three benchmark points. Since there is no SM background as well as model background\(^2\), we observe all the events with lepton multiplicity two only. On the other hand the transverse momentum distribution of the charged leptons can be expressed as:

\[
\left| \frac{d\sigma^{obs}}{dp^\perp_{lep}} \right| \propto \tan \theta, \tag{23}
\]

where $p^\perp_{lep}(\equiv E_{CM}/2)$ is the total three momentum of a final state lepton and $(d\sigma^{obs}/d\cos \theta)$ is independent of $\theta$ which is obvious from Equation 22. At small values of $\theta$, the transverse momentum of a final state lepton becomes small and the transverse momentum distribution also remains small since it is proportional to $\tan \theta$, shown above. However, at $\theta \rightarrow \pi/2$, the transverse momentum of a final state lepton reaches its maximum value, i.e. $E_{CM}/2$, and the transverse momentum distribution blows up due to divergence in $\tan \theta$, as can be noticed from Figure 7(b).

We simulate this channel in PYTHIA8 \[^89\] for the three mentioned benchmark points with an integrated luminosity of 1000 fb\(^{-1}\) and search for same sign di-electron signature. As mentioned earlier, there is no SM background as well as model background for this process. The signal numbers for this final state with the three benchmark points are presented in Table 2, which are very heartening.

| Final state | BP1   | BP2   | BP3   |
|-------------|-------|-------|-------|
| $2e^-$      | 7368.5| 10727.8| 4767.8|

**Table 2.** Number of events for $2e^-$ final state for the benchmark points mentioned in Table 1 with the integrated luminosity of 1000 fb\(^{-1}\).

The normalised angular distribution for this channel is presented in Figure 8, where the blue, green and red lines indicate the simulated results for three benchmark points, respectively and the black straight line signifies the theoretical estimation. It can be noticed that the simulated results matches quite well with the theoretical prediction\(^1\).

\(^{1}\) The only SM process feasible at $\ell^-\ell^-$ collider is Møller scattering which also shows lepton multiplicity to be two only.

\(^{2}\) Though in principle $\ell^-\ell^- \rightarrow \Delta^-\Delta^+$ could be a possible model background, this mode will be absent due to lack of phase space for our choice of benchmark points. Even if the benchmark point is chosen in such a way that the pair production of $\Delta^-$ is possible, still the cross-section of this process is negligible.
μ-μ- → e-e-

Figure 7. (a) multiplicity ($n_{lep}$) distributions for the charged leptons and (b) the transverse momentum ($p_{Tlep}$) distribution for the benchmark points at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$. $p_{Tlep}$ distribution is same for the two leptons for the benchmark points as they are identical, which can be depicted from Figure 5.

and this flat angular distribution of finalstate electrons relative to the angle with beam axis is the distinctive signature for the presence of doubly charged scalar $\Delta^--$ in the s-channel propagator which in turn would imply the existence of Type-II seesaw.

At this point it is important to mention that one can also use Radiation Amplitude Zero (RAZ) [86] through the $\ell^-\ell^- \rightarrow \gamma \Delta^--$ channel to probe the existence of Type-II scenario. With our benchmark points the cross-sections for this process are also large. The angular distribution of the photon will look like a tub with the RAZ occurring at $\cos \theta = 0$ implying $\theta = 90^\circ$ with respect to the beam axis as shown in Figure 9.

5.2 Type-III seesaw (inverse)

In this section, we separate out the signature of inverse Type-III seesaw at $\ell^-\ell^-$ collider. The peculiarity of iType-

III seesaw model is that it contains heavy charged leptons ($N^\pm$) along with heavy neutrinos ($N^0$) and by probing these heavy charged leptons one can discriminate the iType-III scenario. For this purpose, footprints of the scattering process $\ell^-\ell^- \rightarrow \ell^- N^-$, that occurs through Z boson mediated t and u channel diagrams, as displayed in Figure 10, should be traced. In principle, it could occur through h-mediation also; but since $\ell^+\ell^- h$ coupling very small for the light leptons, one can safely ignore it. At this point it should be noted that inverse Type-III seesaw contains six copies of heavy charged leptons; however, as mentioned in subsection 2.3, three of them ($N^\pm$) couples to $Z$ and $h$, and the rest three ($\tilde{N}^\pm$) couple to $W$-boson only. As the mentioned process is predominantly $Z$-mediated, $\tilde{N}^\pm$ will never be produced through $\ell^-\ell^-$ collision. Moreover,
we have considered diagonal coupling only. Therefore, for a particular initial state, only one type of $N^-$ will get produced. The angular distribution and total cross-section this process are given by:

$$\frac{d\sigma}{dx}(\ell^-\ell^- \rightarrow \ell^-N^-) = \frac{m_Z^2Y^2\lambda^2}{4\pi\sin^2\theta W^2(\lambda^2 - \lambda^2)^2} \left[ 2\zeta^2(1 - 2S_w^2)^2 + S_w^4\{ (1 - x^2)(2\zeta^2 - \zeta^2 - x^2\lambda^2) + 2x^2(\zeta + \lambda)^2 \} \right],$$

$$\sigma(\ell^-\ell^- \rightarrow \ell^-N^-) = \frac{m_Z^2Y^2\lambda}{4\pi\sin^2\theta W^2} \left[ \left(\frac{\lambda - 2S_w^2}{\zeta^2 - \lambda^2} \right)^2 + S_w^4\left(1 + \frac{1}{\zeta - \lambda}\right) \right] \tan^{-1}\left(\frac{1}{\zeta} \right),$$

where, $S_w = \sin\theta_w$, $\lambda = 1 - \frac{M^2}{z}$, $\zeta = 1 + \frac{2}{3}(m_Z^2 - M^2)$ with $m_Z$ being the mass of Z bosons, $\theta_w$ the Weinberg angle and $x$ is the cosine of angle between final state electron and beam axis.

![Figure 11](image)

**Figure 11.** Variation of total cross-section (in fb) for the process $\ell^-\ell^- \rightarrow \ell^-N^-$, with respect to the centre of mass energy and mass of the triplet in the Type-III seesaw scenario ($Y = 0.2$, $\mu_n = 10$ eV). The three benchmark points are represented by the yellow stars.

Figure 11 shows the variation of total cross-section for the process $\ell^-\ell^- \rightarrow \ell^-N^-$, with respect to the centre of mass energy and mass of the triplet fermion in the iType-III seesaw scenario with $Y = 0.2$, $\mu_n = 10$ eV. The darker to fainter blue regions represent larger to lesser cross-sections. The cross-section decreases as the mass of the triplet fermion increases, keeping the centre of mass energy constant. On the other hand, the cross-section increases with the increase of centre of mass energy for a fixed mass. Because of unavailability of enough phase space, the cross-section is zero, if $s \leq M_Z^2$.

It should be noted that one can also look for the process $\ell^-\ell^- \rightarrow N^-N^-$ for detection of iType-III seesaw, but the cross-section is quite small$^3$ due to the fact that on both legs we need $\ell^\pm = N^\pm$ mixings proportional to the Yukawa coupling $Y$ and also due to the reduction of the the available phase space as compared to $\ell^-N^-$ mode. Now, as mentioned in subsection 2.3 and Appendix A, this heavy charged leptons ($N^-$) will eventually decay to $Z\ell^-\nu$ or $h\ell^-\nu$. Therefore, observation of any peak in the invariant mass distribution for $jj\ell/bb\ell$ around heavy lepton mass via the reconstruction of $Z/h$ frame will confirm the existence of heavy charged leptons, hence the Type-III seesaw.

For our collider analysis, we have considered three different values of $M_n$, which are 1.25 TeV, 1.5 TeV and 2.0 TeV, respectively, for three benchmark points as shown with the yellow star in Figure 11 with the Yukawa coupling $Y = 0.2$ and $\mu_n = 10$ eV. As listed in Table 3, the cross-sections for the process $\mu^-\mu^- \rightarrow \mu^-N^-$ in the CM frame for the three benchmark points are 8.8 fb, 8.6 fb and 6.1 fb, respectively.

Regarding kinematic distributions of this process, we exhibit the lepton multiplicity and jet multiplicity in the left and right panel of Figure 12, where the blue, green and red columns indicate the three benchmark points, respectively. The other charged lepton either can come from the $N^-$ decays to $Z/h$, $\ell^-$ or $W^-$, $\nu$ and the further decays of $Z,W^-$ bosons. This results in a peak of charged lepton distribution at two as can be seen from Figure 12(a). However, due to the choice of heavy mass for the $N^-$, the jets coming from the Higgs or weak gauge boson decays are often collimated resulting a peak around one as depicted in Figure 12(b).

In Figure 13, we illustrate the transverse momentum distribution for this process with three panels describing three benchmark points. There are mainly two light leptons involved in this channel: 1) the one that gets produced associated with $N^-$ (red), 2) the one which is generated from the decay of $N^-$ (green). The first kind of light leptons will have low energies since lions share of the total energy will be carried by the massive particle $N^-$. The energy carried by this kind of leptons is given by:

$$E_{lep} = s - \frac{M_n^2}{2\sqrt{s}},$$

the values of which are 230 GeV, 440 GeV and 830 GeV respectively for the three benchmark points. Due to the presence of $\tan\theta$ in the expression of lepton $p_T$ distribution (see Equation 23), $\theta$ being the angle of the light lepton with beam axis, we observe a small bump in distribution near $p_T^{lep} = E_{lep}$. The red line beyond this value of $p_T^{lep}$ arises because of misidentification of the second kind leptons as the first kind, which is obvious from the fact that they show small peaks at the same $p_T^{lep}$, where the green curves reach their maxima. Because of the typical angular distribution of this process, large number of events for this process lie around $\cos\theta \sim \pm 1$, which implies low $p_T$ for first kind of leptons. Therefore, we see a pile up of events

$^3$ for instance, $\frac{\sigma(\ell^-\ell^- \rightarrow \ell^-N^-)}{\sigma(\ell^-\ell^- \rightarrow N^-N^-)} \sim 10^{-3}$ for $\sqrt{s} = 3$ TeV and $M_n = 750$ GeV.
Table 3. Masses corresponding to different benchmark points, energy of collision in CM frame and the hard scattering cross-sections (in fb) for $\mu^- N^- \to \mu^- N^-$ final state in Type-III seesaw model and $\mu^- \mu^-$ collider. ($Y = 0.2, \mu = 10$ eV)

| Benchmark Points | $M_n$ in GeV | $E_{CM}$ in TeV | Cross-section (in fb) |
|------------------|--------------|-----------------|-----------------------|
| BP1              | 1250         | 1.5             | 8.8                   |
| BP2              | 1500         | 2.0             | 8.6                   |
| BP3              | 2000         | 3.0             | 6.1                   |

Table 3. Masses corresponding to different benchmark points, energy of collision in CM frame and the hard scattering cross-sections (in fb) for $\mu^- N^- \to \mu^- N^-$ final state in Type-III seesaw model and $\mu^- \mu^-$ collider. ($Y = 0.2, \mu = 10$ eV)

**Figure 12.** Multiplicity distributions, (a) for the charged leptons ($n_{lep}$) and (b) for the jets ($n_{jet}$) for the benchmark points at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

**Figure 13.** Transverse momentum distribution of the 1st and 2nd charged leptons ($p_T^{lep}$), (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

In the $\mu^- \mu^-$ collider, thus we should look for same sign di-lepton plus di-jet signature in order to detect Type-III scenario. In this case the $N^- \to W^- \nu$ fails to contribute any more (see subsection 2.3 and Appendix A) and the main contributions come from $N^- \to h/Z\ell^-$. The dominant SM background in this case will be $\mu^- \mu^- \to \mu^- \mu^- Z(\to jj)$. Nonetheless, constructing the invariant mass of $jj\ell$ system, one can significantly reduce the SM background. In Figure 15, we plot the invariant mass of $jj\ell$ for all the three benchmark points, where the yellow regions represent SM background and the blue portions in the low $p_T$ region of the red curves. On the other hand, the leptons coming from the decay of $N^-$ (green) possess relatively higher transverse momentum and the distribution peaks around $M_n/2$.

Similarly, in Figure 14, we display the jet $p_T$ distribution of this process with three benchmark points in three panels. As explained earlier, the two jets come from the Higgs and $Z$-boson decays, which essentially come from the decay of $N^-$. Since $N^-$ is very heavy, the jets will be very boosted and in most of the cases the two jets coming from $Z/h$ will be identified as a fatjet [81, 109–114]. Therefore, we observe peaks around $M_n/2$ for the red curve in all the panel. The distributions of second and third jets, which appear with isolation of both the jets, fall off very rapidly with larger $p_T^{jet}$.

In the $\mu^- \mu^-$ collider, thus we should look for same sign di-lepton plus di-jet signature in order to detect Type-III scenario. In this case the $N^- \to W^- \nu$ fails to contribute any more (see subsection 2.3 and Appendix A) and the main contributions come from $N^- \to h/Z\ell^-$. The dominant SM background in this case will be $\mu^- \mu^- \to \mu^- \mu^- Z(\to jj)$. Nonetheless, constructing the invariant mass of $jj\ell$ system, one can significantly reduce the SM background. In Figure 15, we plot the invariant mass of $jj\ell$ for all the three benchmark points, where the yellow regions represent SM background and the blue portions
Figure 14. Transverse momentum distribution of the 1st, 2nd and 3rd $p_T$ ordered jets ($p_T^{jet}$), (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$. 

Figure 15. Di-jet-mono-lepton invariant mass distribution ($M_{jj\ell}$) for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$. The total (signal + SM background) signature is depicted in blue and the SM background is in yellow.

| Final states     | BP1            | BP2            | BP3            |
|------------------|----------------|----------------|----------------|
| $2\ell + 2j$     | Sig, BG        | Sig, BG        | Sig, BG        |
| + $|M_{jj\ell} - M_{\mu}| \leq 10 \text{GeV}$ | 2459.0, $1.49 \times 10^5$ | 1779.0, $1.30 \times 10^5$ | 721.0, $1.01 \times 10^5$ |
| $S_{\text{sig}}(L_{\text{int}} = 1000 \text{fb}^{-1})$ | 25.7            | 19.9           | 11.0           |
| $\int L_{\sigma} [\text{fb}^{-1}]$ | 37.9            | 63.1           | 206.6          |

Table 4. Number of events for signal and background corresponding to $2\ell + 2j$ final state for the benchmark points mentioned in Table 3 with the integrated luminosity of 1000 fb$^{-1}$.

indicate signal plus background. It can be easily seen that the blue curves exhibit sharp peak around $M_{\mu}$ while the yellow curves show continuum.

The signal-background analysis for this final state with an integrated luminosity of 1000 fb$^{-1}$ is presented in Table 4. As can be noticed, for same sign di-lepton plus di-jet signature, a huge number of SM background arises from the SM background; however, with the invariant mass cut of $|M_{jj\ell} - M_{\mu}| \leq 10 \text{GeV}$, it drops off very quickly. Therefore, signal significance of more than 10$\sigma$ can be achieved in all the three benchmark points with 1000 fb$^{-1}$ of integrated luminosity. In other words, very early data of a few 100 fb$^{-1}$ can provide 5$\sigma$ significance for all the three benchmark points in this case.

Now, we plot the angular distribution of the reconstructed particle from $M_{jj\ell}$ mass (or equivalently the muon produced in association with the heavy charged lepton) for all the three benchmark points and present them in Figure 16. In each of the plots, the green line indicates the simulated result while the brown curve signifies the theoretical estimation (via parton level and tagging $N^-$) of the distribution, and both the lines are in agreement with each other. In contrast to the angular distribution of finalstate muon in Type-II scenario (which looks flat in Figure 8),
Figure 16. Angular distribution of the reconstructed heavy charged fermion from $M_{\ell\ell}$ invariant mass for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively. The green histograms depict the simulated result and the brown lines signify the theoretical estimation.

Figure 17. Feynman diagrams for dominant contribution to $\ell^+\ell^- \rightarrow \nu N^0/\tilde{N}^0$.

6 At $\ell^+\ell^-$ collider

The $\ell^+\ell^-$ colliders have great achievements in the history of Particle Physics. The LEP was very successful in providing the correctness of the SM through electroweak precision measurements. There are various proposed $e^+e^-$ colliders, like ILC [115, 116], CLIC [117, 118], FCC-ee [119–122], which are going to be built in near future. There is a very big problem with $\mu-\mu$ collider, we look for the process $\ell^+\ell^- \rightarrow \nu N^0/\tilde{N}^0$. This mode mainly occurs through $W$-boson mediated $t$- and $u$-channel diagrams as shown in Figure 17. Though there is a possibility for occurrence of this process via $h$ and $Z$ mediated $s$-channel, but due to large centre of mass energy both of the propagators will be highly off-shell and hence contributions from those two diagrams can safely be neglected. The angular distribution and total cross section for this process is given by:

$$\frac{d\sigma}{d\cos\theta}(\ell^+\ell^- \rightarrow \nu N^0/\tilde{N}^0) = \frac{m_W^4 Y^2}{\pi s \, v_0^2 M_\ell^2} \left[ \frac{\lambda^2 (\lambda + 2 \epsilon_w)^2}{(\lambda + 2 \epsilon_w)^2 - \lambda^2 e_w^2} \right].$$

$$(26)$$

$$\sigma(\ell^+\ell^- \rightarrow \nu N^0/\tilde{N}^0) = \frac{2 Y^2 \lambda m_\ell^2}{4 \pi v_0^2 M_\ell^2} \left[ \frac{\lambda}{\lambda + e_w} \frac{2 \epsilon_w \log \left( 1 + \frac{s}{\epsilon_w^2} \right)}{(\lambda + 2 \epsilon_w)} \right].$$

$$(27)$$

where, $\epsilon_w = m_W^2 / s$ and $\lambda = (1 - M_\ell^2 / s)$. The cross-sections are shown in Figure 18 via the contour plots of 3.0 fb to 150 fb in $M_\ell - \sqrt{s}$ plane. The darker to fainter blue regions show larger to lesser cross-sections, respectively. The three benchmark points are represented by the yellow stars, represent various different values of $M_\ell$, $\sqrt{s}$, where $Y = 0.2$, $\mu_0 = 10$ eV.

It is important to mention here that only one flavour of $\nu$, $N^0$ and $\tilde{N}^0$ will be produced during this process since we have assumed diagonal couplings only. However, observation of this mode does not necessarily indicate the existence of Type-I seesaw (inverse) since Type-III seesaw (inverse) also exhibits same signature due to the presence of heavy neutrinos. Therefore, detection of this mode along with non-observation of the $\ell^\pm N^\pm$ modes, described in subsection 6.3, which is a typicality of Type-III seesaw, will confirm the presence of Type-I seesaw mechanism. It is also noteworthy that although the pair production of these

6.1 Type-I seesaw

In order to investigate the signature of inverse Type-I seesaw (ISS) at $\ell^+\ell^-$ collider, we look for the process $\ell^+\ell^- \rightarrow \nu N^0/\tilde{N}^0$. This mode mainly occurs through $W$-boson mediated $t$- and $u$-channel diagrams as shown in Figure 17. Though there is a possibility for occurrence of
heavy neutrinos can also provide signatures for Type-I and Type-III seesaws, the cross section in that case will be very low due to less available phase space as explained earlier.

For the purpose of simulation, we have considered three different values of $M_n$, which are 1.2 TeV, 1.5 TeV and 2.0 TeV respectively, for the three benchmark points. Taking the Yukawa coupling $Y$ to be 0.2 (along with $\mu_n = 10$ eV) we tabulate the cross-sections of this process at muonic collider for the three benchmark points in Table 5, which are 70.7 fb, 54.0 fb and 31.3 fb, respectively.

| Benchmark Points | $M_n$ in GeV | $E_{CM}$ in TeV | Cross-section in fb |
|------------------|--------------|-----------------|-------------------|
| BP1              | 1200         | 2.5             | 70.7              |
| BP2              | 1500         | 5.0             | 54.0              |
| BP3              | 2000         | 8.0             | 31.3              |

Table 5. Masses corresponding to different benchmark points, energy of collision in CM frame and the hard scattering cross-sections in fb for $\nu_\mu N^0/\bar{N}^0$ final states in Type-I seesaw model and $\mu^+\mu^-$ collider. ($Y = 0.2, \mu = 10$ eV)

Z-boson has 69%, 21% and 10% branching to hadronic, invisible and charged leptonic modes. Similarly, the Higgs boson decays to di-quark, di-boson and di-tau channels with 67%, 26% and 7% branching respectively. Therefore, we get multi-jet and multi-lepton signature in the final-state. The lepton and jet multiplicity for this mode is displayed in left and right panels of Figure 19. If the produced heavy neutrino decays to $Z$ and $h$, and they further decay hadronically then we have two jets with no charged lepton (plus missing energy) in the finalstate. Similarly, if $N^0/\bar{N}^0$ decays to $W$ boson and it further decays hadronically, we get mono-lepton plus di-jet (with missing energy) signature at final state. If the $Z/W$-boson decays leptonically, then only we get di-lepton as can be seen from Figure 19(a). Again the two jets coming from decay of $W$ or $Z$ or $h$ will be highly boosted and hence they might appear as one fatjet which explains the jet multiplicity one finalstates in Figure 19(b).

In Figure 20, we represent the transverse momentum distribution of lepton for the process $\mu^+\mu^- \rightarrow \nu_\mu N^0/\bar{N}^0$. The left, middle and right panels illustrate this distribution for the three benchmark points, respectively. In this leptonic collider the $\nu_\mu N^0/\bar{N}^0$ are produced back to back in the CM frame, this results the light neutrino (in fb) and the heavy neutrino ($N^0/\bar{N}^0$) having equal momentum as

$$|\vec{P}_{\nu_\mu}| = |\vec{P}_{N^0/\bar{N}^0}| = \frac{s - M^2}{2\sqrt{s}}. \quad (28)$$

Since, one particle is heavy and another is lighter, energy of the two particles will be different, which can be depicted from the following equation,

$$E_{\nu_\mu} = |\vec{P}_{\nu_\mu}| \quad \text{and} \quad E_{N^0/\bar{N}^0} = \sqrt{|\vec{P}_{N^0/\bar{N}^0}|^2 + M^2}. \quad (29)$$

For the dominant on-shell decays of $N^0/\bar{N}^0 \rightarrow \ell^+W^\mp$ the momentum is shared between the $\ell^\pm$ (red) and $W^\mp$ almost equally, resulting the charged lepton $p_T$ around half of $M_n$. The long tail of the lepton $p_T$ can be attributed to the boost of $N^0/\bar{N}^0$. The source of the second lepton (green) is from $W^\pm$ and has the usual pattern.

Similarly, Figure 21 depicts the transverse momentum distribution of jets for the benchmark points. The main source of the jets are $W^\pm$, $Z$, $h$. When those jets are boosted forming a fatjet, they tend to peak around the momentum of $W^\pm$, $Z$, $h$, which is $M_n/2$. However, when the jets are isolated then they follow the usual patter of the jets coming from $W^\pm$, $Z$, $h$ bosons, having much lower $p_T$. The third jet can be attributed to the FSR effect.

Figure 22 depicts the missing energy distributions for the benchmark points. As the collisions happen in CM frame we expect the light neutrino and heavy neutrino should take the equal momentum as shown in Equation 28 and thus expect the $p_T^{\text{miss}}$ maximum at $|\vec{P}_{\nu_\mu}|$, for completely visible decay of $N^0/\bar{N}^0$. However, this is not completely true as 50% of the time $N^0/\bar{N}^0$ decay into $h\nu$, $Z\nu$, causing a missing energy cancellation resulting the peak around $M_n/2$. Whereas, the lower end of the $p_T^{\text{miss}}$ curve
occurs when $N^0/\tilde{N}^0$ decays into complete invisible sector and missing energy cancels.

Now, to reconstruct the heavy neutrino, we use its $W^\pm \ell$ decay mode since the presence of neutrino in the other decay channels will make the reconstruction impossible. Hence, we depict the invariant mass distribution of the di-jet plus mono-lepton combination for all the benchmark points in Figure 23, where the olive green regions indicate the dominant SM background and the brown portions signify the signal plus background. As can be observed, signal plus background show peaks at the corresponding values for mass of the heavy neutrinos.

For signal-background analysis, we look for $1\ell + 2j + p_T^{\text{miss}} \geq 350$ GeV finalstate at the $\mu^+\mu^-$ collider. Since the signal mode occurs through $W$-mediated t-channel diagrams, it is expected that there will be more events in
Figure 22. Missing transverse momentum distribution ($p_T^{\text{miss}}$) for the benchmark points at the centre of mass energies of 1.5, 2.0, 3.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

Figure 23. Di-jet-mono-lepton invariant mass distribution ($M_{jj\ell}$) for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$. The total (signal + SM background) signature is depicted in brown and the SM background is in olive green.

Figure 24. Angular distribution of the reconstructed heavy neutral lepton from $M_{jj\ell}$ invariant mass for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively. The blue histograms depict the simulated result and the red dashed lines signify the theoretical estimation.

the longitudinal direction (i.e. along the beam axis) than the transverse direction. But various SM backgrounds also grow vigorously near the collinear direction of the beam axis. Therefore, to reduce the background over signal, we focus mainly in the central region with low $|\eta|$ by choosing $p_T^{\text{miss}} \geq 350$ GeV. Additionally, phase space cut of $p_T^{\text{min}} \geq 350$ GeV has been implemented while generating the hard process for both signal and background. The analysis for final state, mentioned above, with integrated luminosity of 1000 fb$^{-1}$ has been displayed in Table 6. The dominant SM backgrounds in this case are $\ell^\pm W^\mp \bar{\nu}$ and di-boson,
which decrease significantly after applying the invariant mass cut of $|M_{j\ell\ell} - M_{\nu}| \leq 10$ GeV. The results for BP1 and BP2 are very encouraging since signal significances of 7.8σ and 5.5σ with the integrated luminosity of 1000 fb$^{-1}$, respectively. A 5σ signal significance can be achieved at an integrated luminosities of 400 fb$^{-1}$ and 800 fb$^{-1}$, respectively. For BP3 we obtain only 2.2σ significance with 1000 fb$^{-1}$ of luminosity, which means that luminosity of $\sim$ 5200 fb$^{-1}$ will be needed for 5σ reach. But this requirement does not lie in the unreachable region of muon collider since 10$^7$ fb$^{-1}$ of integrated luminosity can be achieved around the centre of mass energy of 10 TeV.

Finally, we reconstruct the heavy neutrino $\tilde{N}_0/\bar{N}_0$ from the combination of di-jet plus mono-lepton and depict its angular distribution (with respect to its angle with beam axis) in Figure 24. The three benchmark points have been shown in the three panels respectively where the blue lines represents the simulated results and the red dashed curves signify the theoretical estimates. We find a tub like distribution for this process indicating more number of events along the beam axis and less number of events in the transverse direction of the beam axis.

### 6.2 Type-II seesaw

Next, we scrutinize the possibility of detecting Type-II seesaw at $\ell^+\ell^-$ collider. For this purpose, we look into the channel of pair production of doubly charged scalar, which is a typicality of Type-II seesaw. The Feynman diagrams for this process are displayed in Figure 25. It can occur through photon and $Z$ mediated s-channel process or lepton mediated t-channel diagram.

The differential and the total cross-sections are given below,

$$
\frac{d\sigma}{dx}(\ell^+\ell^- \to \Delta^{++}\Delta^{--}) = 512\pi s \sum_i C_{\Delta i}^2 \left( 1 - \epsilon_z \right)^2 \left( 1 + \beta x - 2\epsilon_\Delta \right)^2 \times \left\{ 4\epsilon_\Delta^2 - 4\epsilon_\Delta (1 + \beta x + x^2) + (1 + 2\beta x + x^2) \right\} 
$$

$$
	imes \left\{ 32\epsilon_z^2 S_w^4 C_w^4 - 8\epsilon_z C_w^2 S_w^2 (1 + 2S_w^2) + (1 + 4S_w^4) \right\} 
$$

$$
- 4\epsilon_\Delta^2 Y_3^2 S_w^2 C_w^2 (1 + \beta x - 2\epsilon_\Delta) \left( 1 + 4\epsilon_z S_w^2 C_w^2 - \epsilon_z (1 + 4S_w^2 - 4S_w^4) \right) + 4Y_3^2 S_w^4 C_w^4 (1 - \epsilon_z^2) 
$$

(30)
\[
s(\ell^+\ell^- \rightarrow \Delta^{++}\Delta^{--}) = \frac{1}{384\pi s S_w^2 C_w^4 (1 - \epsilon_z)^2} \\
\left[ -\beta \left\{ 12Y^2_\Delta S_w^2 C_w^4 (1 - \epsilon_z)^2 - 4 \beta^2 (1 - 8S_w^2 \epsilon_z) \\
+ 4S_w^2 \left( 1 + 2\epsilon_z (-1 + 2S_w^2 + 4\epsilon_z C_w^4) \right) \\
+ 6e^2 Y^2_\Delta S_w^2 C_w^4 (1 - \epsilon_z) (1 - 2\epsilon_\Delta) (1 - 4\epsilon_z C_w^2 S_w^2) \right\} \\
+ 12Y^2_\Delta S_w^2 C_w^4 (1 - \epsilon_z) \left\{ Y^2_\Delta S_w^2 C_w^4 (1 - \epsilon_z) (1 - 2\epsilon_\Delta) \\
+ 2e^2 \Delta (1 - 4\epsilon_z S_w^2 C_w^2) \right\} \coth^2 \left( \frac{1 - 2\epsilon_\Delta}{\beta} \right) \right],
\]

where, \( C_w \) and \( S_w \) are the cosine and sine of the Weinberg angle, respectively, \( \epsilon_\Delta = M_\Delta^2/s \), \( \epsilon_z = m_Z^2/s \), \( \beta = \sqrt{1 - 4M_\Delta^2/s} \), \( x \) is the cosine of the angle of doubly charged particle with the beam axis and \( e \) is the electric charge of the positron.

In Figure 26 we describe the contours of the different values of cross-sections in \( M_\Delta - \sqrt{s} \) plane. The darker to lighter blue regions depict from higher to lower values of cross-sections. The benchmark points for the collider simulations as summarised in Table 7 are shown by yellow stars.

For our simulation, we considered three benchmark points with \( M_\Delta \) being 0.85 TeV, 1.2 TeV and 1.5 TeV, respectively with the centre of mass energy equals to 2.5 TeV, 5.0 TeV and 8.0 TeV, respectively. Assuming \( Y_\Delta = 0.2 \) and \( \mu_\Delta = 10 \) eV, we find the hard scattering cross-sections for the process \( \mu^+\mu^- \rightarrow \Delta^{++}\Delta^{--} \) are 4.8 fb, 2.2 fb and 1.1 fb, respectively and they have been listed in Table 7.

Each of these doubly charged scalar will ultimately decay to same sign di-lepton with 33.33\% branching fraction to each of the lepton family. Therefore, we search for \( 2\ell^+2\ell^- \) signature at the \( \ell^+\ell^- \) collider in order to identify Type-II seesaw. Here the choice of \( \epsilon_\Delta = 10 \) eV, makes the \( W^\pm W^\mp \) mode negligible. At this point it is important to mention that one can also try to detect \( \Delta^{+}\Delta^{-} \) channel for the same purpose. But, pair production cross-section for single charged scalar is sufficiently lower than \( \Delta^{\pm}\Delta^{\mp} \) mainly due to smaller electromagnetic charge. Additionally, since \( \Delta^- \) will decay to one charged lepton and one neutrino, reconstruction of this particle from the finalstate is not possible. On the other hand, pair production of \( \Delta^0 \) is also not possible as this particle does not couple to photon, Z and charged leptons. One can also search for enhancement in \( \mu^+\mu^- \rightarrow \mu^+\mu^- \) due to \( \Delta^{++} \) mediated t-channel diagram, but the SM background in this case will be very large. Thus \( \mu^+\mu^- \rightarrow \Delta^{++}\Delta^{--} \) seems to be the best mode for searching the trace of Type-II seesaw.

The doubly charged scalar will eventually decay to same sign di-leptons with equal probability in each of the generations. But if the \( \Delta^- \) decays to two \( \tau \)-leptons then we can have jets also in the finalstate from the hadronic decays of \( \tau \). In Figure 27, we depict the charged lepton multiplicity \((a)\) and jet multiplicity \((b)\) for this process. The blue, green and red columns indicate the three benchmark points, respectively. The maximum lepton multiplicity is four for all the benchmark points. However, only for BP1, we see that lepton multiplicity peaks around four and for the rest of the benchmark points, it peaks around three due to larger boost effect. Lower multiplicities are attributed to one or more \( \Delta^{\pm\mp} \) decays to \( \tau \) and the further hadronic decay of the \( \tau \). The \( \tau \)-jet-lepton isolation is also instrumental in reducing the lepton multiplicity further. Figure 27(b) depicts the corresponding the jet-multiplicity distributions, where the jets are coming from the hadronic decays of \( \tau \). However, the jets are collimated due to large boost and thus distributions peak at one for all benchmark point.

In Figure 28, we show the transverse momentum distributions of the leptons for the mentioned channel, where each panel signifies one benchmark point. Since from the lepton multiplicity distribution we found the maximum number of leptons arising for this mode to be four, we present the \( p_T^{lep} \) distributions for all the four leptons. The
Figure 27. Multiplicity distributions, (a) for the charged leptons ($n_{lep}$) and (b) for the jets ($n_{jet}$) for the benchmark points at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

Figure 28. Transverse momentum distributions of the four charged leptons ($p_{lep}^T$) coming from $\Delta^{\pm\pm}$ decay, (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

Figure 29. The transverse ($p_{lep}^{T1}$ in red) and the longitudinal ($p_{lep}^{z1}$ in green) momentum distribution of the 1st charged lepton, (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

The highest energy achievable by each of the leptons in the final state is given by:

$$E_\ell = \frac{\sqrt{s}}{4} \left( 1 + \sqrt{1 - \frac{4M_\Delta^2}{s}} \right),$$

which, for our the benchmark points, given in Table 7, becomes 1.08 TeV, 2.35 TeV and 3.85 TeV, respectively.

Thus we observe the $p_{lep}^T$ distributions for all the leptons to vanish at the three mentioned transverse momenta for the three benchmark points. However, as all the leptons appear from the decay of doubly charged scalars, we do not see any divergence near the endpoint (like Figure 7), rather the distributions gradually reach zero. The behaviour of $p_z$ is different from $p_x$ and $p_y$ at the hard scattering level and $p_z$ tends to crossover $p_x,p_y$ at certain
Figure 30. Transverse momentum distributions of the first three $p_T$ ordered jets ($p_T^{jet}$), (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

Figure 31. Same sign di-lepton invariant mass distribution ($M_{SSD}$) for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$. The total (signal + SM background scaled by 4) signature is depicted in blue and the SM background (scaled by 4) is in yellow.

| Final states | Signal | Backgrounds |
|--------------|--------|-------------|
| $2\ell^+ + 2\ell^-$ | 1530.9 | $t\bar{t}$ | $V V$ | $VVV$ | $t\bar{t}V$ | $\ell^+\ell^-Z$ | $\tau^+\tau^-Z$ |
| $+|M_{\ell\ell} - M_\Delta| \leq 10$ GeV | 1491.2 | 0.2 | 153.1 | 133.7 | 3.5 | 13855.6 | 17.4 |
| Total | 1491.2 | 0.0 | 2.8 | 2.4 | 0.0 | 160.7 | 0.3 |
| $S_{\text{sig}} (L_{\text{int}} = 1000$ fb$^{-1})$ | 36.6 | $f \mathcal{L}_{\text{exp}}$ [fb$^{-1}$] | 18.7 |
| BP2 | 2| $2\ell^+ + 2\ell^-$ | 667.1 | 0.0 | 46.6 | 66.7 | 0.6 | 8753.2 | 5.4 |
| $+|M_{\ell\ell} - M_\Delta| \leq 10$ GeV | 592.4 | 0.0 | 1.3 | 0.7 | 0.0 | 43.8 | 0.1 |
| Total | 592.4 | 0.0 | 45.9 |
| $S_{\text{sig}} (L_{\text{int}} = 1000$ fb$^{-1})$ | 23.4 | $f \mathcal{L}_{\text{exp}}$ [fb$^{-1}$] | 45.5 |
| BP3 | 2| $2\ell^+ + 2\ell^-$ | 258.3 | 0.0 | 15.9 | 44.4 | 0.1 | 5436.5 | 2.3 |
| $+|M_{\ell\ell} - M_\Delta| \leq 10$ GeV | 252.1 | 0.0 | 0.2 | 0.5 | 0.0 | 20.3 | 0.0 |
| Total | 252.1 | 0.0 | 21.0 |
| $S_{\text{sig}} (L_{\text{int}} = 1000$ fb$^{-1})$ | 15.3 | $f \mathcal{L}_{\text{exp}}$ [fb$^{-1}$] | 107.4 |

Table 8. Number of events for signal and background corresponding to $2\ell^+ + 2\ell^-$ final state for the benchmark points mentioned in Table 7 with the integrated luminosity of 1000 fb$^{-1}$.

value of momentum, towards higher values [81]. For $p_T$ such crossover happens even at lower values of momen-
Figure 32. Angular distribution of the reconstructed doubly charged scalar from $M_{ll}$ invariant mass for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively. The blue and orange histograms depict the simulated results for positively and negatively charged scalars, respectively, whereas, the blue and orange dashed lines provide their theoretical estimations.

This effect though diminished at the decay product level like the charged lepton, but it still exists as can be seen from Figure 29.

In Figure 30, we present the transverse momentum distribution of the jets for all the benchmark points. The red and green curve indicate the hardest and second hardest jets whereas the blue and brown lines signify the other jets. As can be observed, all the distributions peak at very low transverse momentum and diminish gradually with long tails. Figure 31 describes the invariant mass distribution of same sign di-lepton pair for the dominant SM background, shown in yellow, and signal plus background (scaled by four), coloured in blue. Regarding the blue regions, we see clear peaks around the mass of the doubly charged scalar for all the three benchmark points.

Now, we move to signal-background analysis at the muonic collider with an integrated luminosity of 1000 fb$^{-1}$, looking for $2\ell^+2\ell^-$ finalstate. The results are displayed in Table 8, where one can see that the dominant SM background contributing to this final state is $\ell^+\ell^-Z$. Nevertheless, implication of invariant mass cut as $|M_{ll} - M_{\Delta}| \leq 10$ GeV, i.e. the invariant mass of same sign lepton pair should lie within a window of 10 GeV around $M_{\Delta}$, reduces the background drastically. Thus we achieve 36.6$\sigma$, 23.4$\sigma$ and 15.3$\sigma$ of signal significances for the three benchmark points, respectively with the above specified luminosity, which indicates that one can obtain 5$\sigma$ of significance with very early data.

Finally, we plot the angular distributions of reconstructed $\Delta^{\pm\pm}$ for the benchmark points with respect to the cosine of their angles with beam axis (taken in the direction of $\mu^+$) in CM frame and present them in the three panels of Figure 32. The blue and orange solid histograms indicate the simulated distributions for $\Delta^{++}$ and $\Delta^{--}$, whereas the dashed lines with same colours signify the theoretical estimates for the same. As can be observed, the angular distributions show very unique asymmetric behaviour in this scenario. If we consider the distribution of $\Delta^{++}$ (blue) in BP1 case, we find that it starts with zero at $\cos \theta = -1$ and increases gradually; it reaches the maximum at $\cos \theta \sim -0.15$ and then begins to decrease; finally near $\cos \theta \sim 0.85$, it shows a local minimum and starts to grow again till $\cos \theta = 1$. The same patterns repeats for BP2 and BP3 also with only exception of very fast enhancement near $\cos \theta \sim 1$. On the other hand, taking a mirror image of these distributions about $\cos \theta = 0$, one can get the angular distributions of $\Delta^{--}$ (orange) for the three benchmark points. Actually, the $\gamma/Z$ mediated s-channel diagram contributes to the angular distribution symmetrically around $\theta = 90^\circ$, but the t-channel diagram does not. Since, the $\Delta^{++}$ particle can arise from $\ell^+$ leg only, we see a divergence in $\cos \theta \sim 1$ direction. Again, the interference terms between s- and t-channel diagrams contribute negatively and for distribution of $\Delta^{++}$ (blue), the negativity increases in $\cos \theta > 0$ region. Thus combining all these effects, we get such typical asymmetric angular distribution in this scenario.

6.3 Type-III seesaw

As we have mentioned in subsection 6.1, the observation of the mode $\ell^+\ell^- \rightarrow \nu N^0/\bar{N}$ indicates the existence of Type-I or Type-III seesaw; however, it cannot differentiate these two models. In this section, we discuss the litmus test to distinguish these two scenario at $\ell^+\ell^-$ collider. For this purpose we investigate the channel $\ell^+\ell^- \rightarrow \ell^\pm N^\mp$. Detection of this mode accompanied by $\nu N^0/\bar{N}$ channel will point out the presence of Type-III scenario, whereas, observation of only the second channel will indicate the
existence of Type-I case. The relevant Feynman diagrams for the mentioned channel, which are s- and t-channel processes mediated by Z boson, are displayed in Figure 33. However, it is important to mention that the dominant contribution to this mode comes from the t-channel only. The differential distribution and the total cross-section for this channel can be expressed as follows:

\[
d\sigma(\ell^+\ell^- \rightarrow \ell^+ N^-) = \frac{m_{\ell}^2 Y^2 \lambda^2}{32\pi s^3 m_n^2 Z_n^4} \left[ \frac{16 S_n^4}{(\lambda - \lambda x + 2\epsilon z)^2} + \frac{(1 - 2 S_n^2)^2 (1 - x^2)(1 - \lambda + \frac{1 + \epsilon}{1 - x})}{(\lambda - \lambda x + 2\epsilon z)^2} \right],
\]

(33)

\[
\sigma(\ell^+\ell^- \rightarrow \ell^+ N^-) = \frac{m_{\ell}^2 Y^2}{16\pi s^3 m_n^2 Z_n^4} \left[ (1 - 2 S_n^2)^2 \left( 2\lambda + \frac{\lambda}{\epsilon z} \right) + \frac{4\lambda^2 S_n^4}{\epsilon z(\lambda + \epsilon z)} - (1 - 2 S_n^2)^2 (1 + \lambda + 2\epsilon z) \log \left( 1 + \frac{\lambda}{\epsilon z} \right) \right],
\]

(34)

with \(\epsilon z = m_{\ell}^2/s\), and \(\lambda = (1 - M_n^2/s)\). As stated before, here also \(C_w\) and \(S_w\) are the cosine and sine of the Weinberg angle, respectively; \(x\) is the cosine of the angle of charged heavy fermion with the beam axis. By charge conjugation symmetry we see that the cross-section is same for \(\ell^+\ell^- \rightarrow \ell^+ N^+\) process also. We plot the contours of cross-section in \(M_n - \sqrt{s}\) plane in Figure 34. The darker to lighter blue regions show higher to lower cross-section regions. The benchmark points detailed in Table 9 are presented by the yellow stars.

It is also noteworthy that \(\ell^+\tilde{N}^+\) mode cannot be generated at \(\ell^+\ell^-\) collider since \(\tilde{N}^+\) does not couple to \(Z\). For our simulation, we consider three benchmark points taking \(M_n\) equals to 1.2 TeV, 1.5 TeV and 2.0 TeV along with the centre of mass energy being 2.5 TeV, 5 TeV and 8 TeV respectively at a muonic collider. Assuming \(Y = 0.2\) and \(\mu_{\ell} = 10\) eV, we find the hard scattering cross sections for this channel to be 22.8 fb, 17.5 fb and 10.2 fb respectively which have been presented in Table 9.

### Table 9. Masses corresponding to different benchmark points, energy of collision in CM frame and the hard scattering cross-sections (in fb) for \(\mu^+ N^+\) final states in Type-III seesaw model and \(\mu^+\mu^-\) collider, \((Y = 0.2, \mu_{\ell} = 10\) eV).

| Benchmark Points | \(M_n\) in GeV | \(E_{CM}\) in TeV | Cross-section (in fb) |
|------------------|----------------|----------------|----------------------|
| BP1              | 1200           | 2.5            | 22.8                 |
| BP2              | 1500           | 5.0            | 17.5                 |
| BP3              | 2000           | 8.0            | 10.2                 |

Now, we discuss the kinematic distributions of this channel. The multiplicity distributions of charged light leptons and jets are presented in left and right panels of Figure 35. The heavy charged lepton will eventually decay to \(Z\ell\) or \(h\ell\) and \(h/Z\) will disintegrate mostly into two jets. Therefore, we obtain large number of events at lepton number two. On the other hand, for jet multiplicity distribution, we get significant number of events with di-jet. But, since the two jets coming from the decay of \(W^\pm\), \(Z\), \(h\) are highly boosted \([81, 109–114]\), often they are identified as a fatjet and hence large number of events with one jet are found.

The transverse momentum distribution of leptons for the benchmark points are depicted in Figure 36. The red histogram signifies the first lepton which gets produced in association with \(N^\pm\), while the green histogram symbolizes the second lepton that comes from the decay of \(N^\pm\). As expected, the green curves peak at half of mass of heavy leptons. The red curves, on the other hand, peak at very low \(p_T = p \sin \theta\), indicating occurrence of large number of events near the beam axis i.e. \(\cos \theta \sim \pm 1\) as shown in Equation 33. On the red curve, we also notice bumps near \(M_n/2\) which signifies the misidentification of the second lepton as the first one. But, unlike Figure 13, we do not see any additional bump in the red line for the maximum transverse momenta \((0.96\) TeV, \(2.28\) TeV and \(3.75\) TeV, respectively for the benchmark points obtained from Equation 28) by the first lepton as those corresponds to central events around \(\eta = 0, \theta = \pi/2\), where the probability is lowest.

In a similar fashion, the transverse momentum distribution of jets for all the benchmark points are portrayed in Figure 37, where the red curves indicate the hardest jets and remaining green and blue lines signify the other jets. As already mentioned in previous section, the two jets coming from decay of heavy lepton are highly boosted and
Figure 35. Multiplicity distributions, (a) for the charged leptons ($n_{lep}$) and (b) for the jets ($n_{jet}$) for the benchmark points at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

Figure 36. Transverse momentum distributions of the 1st and 2nd charged leptons ($p_{T}^{lep}$), (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

Figure 37. Transverse momentum distribution of the first three $p_T$ ordered jets ($p_{T}^{jet}$), (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb$^{-1}$.

often provides one fatjet signature. Therefore, we see the red curves also to peak around $M_{\ell}/2$.

Figure 38 describes the invariant mass distributions for the combination of di-jet plus negatively charged light lepton ($jj\ell^{-}$) concerning all the benchmark points. The SM background (scaled with two) is presented in green while the signal plus background (scaled by two) is shown by red colour. Distinctive peaks in red colour near the heavy lepton mass are clearly visible across all the benchmark scenarios while the background shows more or less flat distributions.

The signal-background analysis for this channel is presented in Table 10, where we have scrutinized opposite sign di-lepton (OSD) plus di-jet final state. As can be observed, $\ell^{+}\ell^{-}Z$ provides dominant SM background for this final state. Nonetheless, implementation of invariant mass cut of 10 GeV on the $jj\ell^{-}$ combination around $M_{\ell}$ reduces the background drastically. Thus, with an integrated lumi-
Figure 38. Di-jet-mono-lepton invariant mass distribution \( (M_{jj\ell}) \) for (a) BP1, (b) BP2 and (c) BP3 at the centre of mass energies of 2.5, 5.0, 8.0 TeV, respectively with the integrated luminosity of 1000 fb\(^{-1}\). The total (signal + SM background scaled by 2) signature is depicted in brown and the SM background (scaled by 2) is in olive green.

| Final states | Signal | Backgrounds |
|--------------|--------|-------------|
|              | \( tt \) | \( VV \) | \( VVV \) | \( t\bar{t}V \) | \( \ell^+\ell^- \) | \( \tau^+\tau^- \) |
| BP1 \( +|M_{jj\ell} - M_n| \leq 10 \text{GeV} \) | 6649.02 | 132.28 | 1802.87 | 1288.43 | 19.50 | 93434.50 | 136.62 |
| \( \text{Total} \) | 2480.43 | 0.06 | 10.49 | 4.92 | 0.00 | 284.03 | 1.52 |
| BP1 \( \int \mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1} \) | 11.30 | 47.03 |
| BP2 \( +|M_{jj\ell} - M_n| \leq 10 \text{GeV} \) | 2741.55 | 5.47 | 384.51 | 638.07 | 9.58 | 59532.6 | 34.67 |
| \( \text{Total} \) | 948.52 | 0.00 | 0.65 | 1.52 | 0.00 | 61.65 | 0.19 |
| BP2 \( \int \mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1} \) | 28.14 | 29.81 |
| BP3 \( +|M_{jj\ell} - M_n| \leq 10 \text{GeV} \) | 682.49 | 0.83 | 122.73 | 500.38 | 4.95 | 37305.7 | 11.86 |
| \( \text{Total} \) | 205.51 | 0.00 | 0.00 | 0.54 | 0.00 | 20.29 | 0.05 |
| BP3 \( \int \mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1} \) | 13.66 | 134.01 |

Table 10. Number of events for signal and background corresponding to OSD+2\( \ell \) final state for the benchmark points mentioned in Table 9 with the integrated luminosity of 1000 fb\(^{-1}\).

We achieved signal significance of 47\( \sigma \), 29.8\( \sigma \) and 13.7\( \sigma \), respectively for the benchmark points. This indicates that with integrated luminosity of less than 150 fb\(^{-1}\), one can reach 5\( \sigma \) significance for this final state across all the benchmark points.

Finally, we present the angular distributions of \( N^\pm \) with respect to the beam axis (taking the +\( z \) direction along \( \mu^+ \) for the benchmark points in Figure 39. While the solid blue and red lines illustrate the simulated distributions for \( N^- \) and \( N^+ \), respectively, the dashed curves represent the same for theoretical estimations. Now, if we concentrate on the blue curves, we see that they diverge at \( \cos \theta \sim -1 \) and decrease gradually till \( \cos \theta \sim 1 \). It is expected because more \( N^- \) will get produced in the same direction of initial state \( \mu^- \) which travels in the \(-z\) direction. On the other hand, the orange curves are the mirror images of the blue one about \( \cos \theta = 0 \). Thus we get a typical asymmetric angular distribution in the Type-III scenario.

It is worth mentioning that one can also look for pair production of heavy charged leptons which are dominated by photon and \( Z \) mediated \( s \)-channel diagrams. The cross sections for these processes under the chosen benchmark points are also comparable to the numbers presented in Table 9. But the angular distribution of any of the heavy charged lepton will be symmetric about \( \cos \theta = 0 \) [37].

7 At \( \ell^+\ell^- \) collider

Now, we investigate the prospect of \( \ell^+\ell^- \) collider in discerning the seesaw models. The forthcoming project MUonE is going to look into this kind of collision where a \( \mu^\pm \) beam
of energy \(\sim 150\) GeV will be collided with atomic electrons having energy less than 120 GeV to experimentally estimate the hadronic contribution to the muon \((g - 2)\) [124, 125]. The centre of mass energy and integrated luminosity \((\sim 1.5 \times 10^7\) nb\(^{-1}\)) of this experiment are too low to search for different BSM scenarios with heavy particles. Therefore, we propose a similar kind of collider with higher centre of mass energy (less than 1 TeV) and luminosity for the purpose of our study. Since the lower bound on the mass of inverse Type-III seesaw (with three generations) particles is 1.2 TeV (see Figure 2), this model cannot be probed at this collider. For Type-I seesaw there is no such lower bound on the mass of heavy neutrinos, the same \(\nu N^0/\bar{N^0}\) mode can be observed if \(M_{\nu N^0/\bar{N^0}} < \sqrt{s}\).

The cross-section for this process is large and if the flavour of lepton in \(1\ell + 2j + p_T^{\text{miss}}\) final-state is not tagged\(^4\), the angular distribution of \(N^0/\bar{N^0}\) would also look the same as depicted in Figure 24. Therefore, we refrain ourselves from a repetitive analysis and we move to signature of Type-II scenario.

To detect Type-II case, we search the trace of the process \(\ell^+ \ell'^- \rightarrow \ell^- \ell'^+\), i.e. the charges of \(\ell\) and \(\ell'\) get swapped, which occurs through the doubly charged scalar mediated t-channel diagram presented in Figure 40. The angular distribution and the total cross-section for this process is given by

\[
\frac{d\sigma}{dx}(\ell^+ \ell'^- \rightarrow \ell^- \ell'^+) = \frac{Y_{\Delta}}{128\pi s} \frac{(1-x)^2}{(1-x + 2\epsilon_{\Delta})^2},
\]

\[
\sigma(\ell^+ \ell'^- \rightarrow \ell^- \ell'^+) = \frac{Y_{\Delta}}{64\pi s} \left[1 + 2\epsilon_{\Delta} - 2\epsilon_{\Delta} \log \left(1 + \frac{1}{\epsilon_{\Delta}}\right)\right],
\]

where, \(\epsilon_{\Delta} = M_{\Delta}/s\) and \(x\) is the cosine of the angle between \(\mu^+\) and \(\mu^-\). In Figure 41 we plots the contours of the cross-sections in \(M_{\Delta} - \sqrt{s}\) plane, where the darker to lighter blue regions depict higher to lower values of cross-sections. The benchmark points stated in Table 11 are shown by the yellow stars.

-2.0 -1.0 0.0 1.0 2.0
-1.0 -0.5 0.0 0.5 1.0
10-5
0.001
0.010
0.100
1
10
100
cosθ
1
nobs
dnobs
dcosθ

\textbf{Figure 40.} Feynman diagrams for \(\ell^+ \ell'^- \rightarrow \ell^- \ell'^+\).
Table 11. Masses corresponding to different benchmark points, energy of collision in CM frame and the hard scattering cross-sections (in fb) for $\mu^+e^-$ final states in Type-II seesaw model and $\mu^+e^-$ collider. ($Y_\Delta = 0.2, \mu_\Delta = 10\,\text{eV}$)

| Benchmark Points | $M_\Delta$ (in GeV) | $E_{CM}$ (in GeV) | Cross-section (in fb) |
|------------------|---------------------|-------------------|----------------------|
| BP1              | 850                 | 300               | 0.15                 |
| BP2              | 850                 | 1000              | 0.51                 |

For our simulation, we have used $e^-$ and $\mu^+$ beams with equal energy to collide at a centre of mass energy of 0.3 TeV and 1.0 TeV, respectively, for the two benchmark points. Considering $Y_\Delta$ to be 0.2 (with $\mu_\Delta = 10\,\text{eV}$) we get the cross-sections for the above mentioned mode to be 0.15 fb and 0.51 fb, respectively, with $M_\Delta$ being 0.85 TeV, as presented in Table 11.

In Figure 42, we depict the kinematic distributions for this mode. The left panel exhibits the lepton multiplicity which indicates that there are events with two leptons in the final state only, as expected. The right panel demonstrates transverse momentum distributions of leptons which is supposed to diverge at half of the centre of mass energy obeying Equation 23.

Now, we look for $\mu^-e^+$ final state at the collider. The results for this final state with 1000 fb$^{-1}$ of integrated luminosity are presented in Table 12, where we find 149 and 508 numbers of events for the two benchmark points. It is interesting to mention that there is no SM background.
for this final state since this is a lepton flavour violating process.

Finally, we illustrate the angular distributions for the final state leptons in the CM frame with respect to the angle with initial state muon ($\mu^-$) beam in Figure 43, where the two benchmark scenarios are shown in the two panels. While the orange curves signify angular distributions for final state $\mu^-$, the blue curves indicate the same for final state $e^+$. In both the plots the solid lines denote the simulated result whereas the dashed lines demonstrate the theoretical predictions. As can be seen, the blue curves start from zero at $\cos \theta = -1$ and then increase gradually with $\cos \theta$. On the other hand, the orange curves are the mirror image of the blue ones about $\cos \theta = 0$ line. It is interesting to notice that the curves representing the angular distributions remain convex in nature at low centre of mass energies (like in BP1); however as the energy of interaction increases, the convexity decreases and at further high energies the curves become concave (like in BP2).

8 Conclusion

In this article, we have tried to separate the signatures of different seesaw scenarios in different leptonic colliders. We show that the angular distributions of the final state leptons or reconstructed BSM particles can provide important information about the seesaw models. In our analysis, we have mainly focused on detecting the TeV-scale BSM particles with diagonal coupling of $O(0.1)$. Since the neutrino mass bound restricts the couplings for Type-I and Type-III seesaw models with TeV-scaled new particles to be very small, these models become very challenging to be probed at colliders. Hence, we choose inverse seesaw scenarios for both Type-I and Type-III cases to be investigated. A detailed PYTHIA based analysis at $\mu^-\mu^-$, $\mu^+\mu^-$ and $\mu^+e^-$ colliders are carried out with different masses and centre of mass energies taking the diagonal couplings for the BSM particles.

The angular distributions which are instrumental in distinguishing such different seesaw scenarios at different leptonic colliders are summarized in Figure 44. It can be clearly seen that $\ell^+\ell^-$ has an advantage as all the three types of seesaw can be probed and their angular distributions are also different. The $\ell^-\ell^-$ and $\ell^+\ell^-$ colliders cannot proved the inverse Type-I and inverse Type-III, respectively. However, they able to distinguish the rest of seesaw scenarios.

As high-energetic and highly luminous muon beams are going to be available in near future we have mostly used muonic colliders during our simulation. At $\mu^-\mu^-$ collider, it is not possible to probe Type-I seesaw (inverse), since lepton number violating $W^-W^-$ mode is the only channel to be considered in this case and the cross-section for this mode is very small as the lepton number violating parameter $\mu_n$ is tiny. For Type-II scenario we searched for the mode $e^-e^-$, which should show a flat angular distribution of $e^-$ in the CM frame. On the contrary, for Type-III case we examined the $\mu^-N^-$ mode which is expected to display a tub-like angular distribution for the final state muon (or the reconstructed $N^-$) in the CM frame.

At $\mu^+\mu^-$ collider, we first looked for $\nu N^0/\bar{N}^0$ mode. It presents a bowl-like angular distribution for the reconstructed $N^0/\bar{N}^0$. However, both Type-I and Type-III scenarios exhibit same feature for this particular mode. So, in addition to it, one has to investigate the presence or absence of another mode, i.e., $\mu^\pm N^\mp$, to confirm the existence of any of these two models. For Type-III seesaw, this particular mode will exist with a typical asymmetric angular distribution of $\mu^\pm$ (or $N^\pm$), whereas in Type-I seesaw, this channel does not exist. Regarding Type-II seesaw, we have scrutinized the pair production of doubly charged scalar which displays a very unique asymmetric angular distribution.

Finally, motivated by MUonE project, we investigated the possibilities at low energy muon-electron collider ($\sqrt{s} \leq 1 \text{ TeV}$) too. Since the mass bound on inverse Type-III seesaw with three generations of triplets is $\sim 1.2 \text{ TeV}$, one needs to enhance the CM energy to probe the scenario. Since this kind collider bound is not there for Type-I case, one can search for right-handed neutrinos with mass less than 1 TeV through the mode $\nu N^0/\bar{N}^0$, which will show the similar bowl-like angular distribution if the flavour of final state lepton is not tagged. The most interesting feature at this collider is observed for Type-II seesaw when we search the mode $e^-\mu^+ \rightarrow e^+\mu^-$. The angular distribution of final state electron or muon exhibit a very distinctive asymmetric behaviour in this case.

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A Decay of $N^\pm/\bar{N}^\pm$ in iType-III model with one generation

Here, we briefly discuss the typical behaviour of iType-III seesaw involving the decay of heavy charged leptons. For simplicity, we show it with one generation of lepton and right handed neutrinos. During the diagonalization of mass matrix, the rotation matrices needed for negatively and positively charged leptons are denoted as $Z^L$ and $Z^R$ respectively whereas the same for neutral leptons are termed as $U$. Assuming $M_n \gg m_0 Y \gg m_0 Y_e$, where $Y_e$
is the Yukawa coupling for SM lepton, these three rotation matrices for iType-III seesaw can be expressed as:

\[
Z^L \approx \begin{pmatrix}
1 & -\frac{m_Y}{M_n} \\
\frac{m_Y}{M_n} & 0 & 1
\end{pmatrix},
Z^R \approx \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix},
\]

\[
U \approx \begin{pmatrix}
1 & 0 & \frac{m_Y}{M_n} \\
\frac{m_Y}{M_n} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}
\end{pmatrix}.
\]

Now, the couplings for the vertices involving the decay of the heavy charged leptons are zero and the rest of the couplings involving transition of heavy charged leptons to SM leptons are non-zero. Therefore, we find the partial decay widths of $N^\pm / N^0$ as:

\[
\Gamma(N^\pm \rightarrow \nu_e W^\mp) = 0,
\]

\[
\Gamma(N^- \rightarrow Z e^-) = \Gamma(N^+ \rightarrow Z e^+) = \frac{Y^2 M_n}{32 \pi},
\]

\[
\Gamma(\tilde{N}^- \rightarrow \nu_e W^-) = \frac{Y^2 M_n}{16 \pi},
\]

\[
\Gamma(\tilde{N}^- \rightarrow Z e^-) = \Gamma(\tilde{N}^+ \rightarrow Z e^-) = 0.
\]

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