Simplified mathematical model of realizing the residual energy resource when decommissioning a high-power channel-type reactor

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Abstract. We consider the problem of realization of the residual energy resource of fuel during decommissioning of the High-Power Channel-Type reactor. A mathematical model of the fuel assemblies spectrum for energy production is provided. A possible way of decommissioning is formation of a two-zone core charge: fresh fuel assemblies are put in the centre, burnt out on the periphery. The results are obtained on the point model of the fuel assemblies spectrum dynamics in the linear approximation of the with linear dependences of the power and the multiplication factor of FAs from energy production and indicate only the possibility of optimizing the fuel use in the process of removing the RBMK from service.

1. Introduction

The problem of nuclear power plant decommission is a complex task, and one of the aspects of which is optimal utilization of the fuel assemblies (FA) of decommissioned reactors. In relation to High-Power Channel-Type reactors (HPCTR), the problem of optimal use of shutdown reactors residual energy resources was considered in [1]. It has been shown that the re-use of fuel from a shutdown reactor in other nuclear power units can save a significant amount of fresh fuel, from tens to hundreds and even thousands of FAs. At the same time, the process of decommissioning of nuclear power plants from several power units was optimized. However, it is possible to use the fuel resource inside each power unit of nuclear power plants through internal permutations.

The physical and structural features of the HPCTR [2] (possibility of FA rearrangements during operation) make it possible to implement various approaches to the formation of a critical charge without fuelling, for example, removing the most burnt fuel ("collapsing" reactor) or forming a multi-zone (in particular, two-zone) charge, when the burnt fuel is placed on the periphery. That is, there is a principal possibility to prolong the operation of the decommission reactor being due to the appropriate structuring of core charge and thereby use the energy resource of unburnt fuel.

The purpose of this article is to use a simple model to pay attention to the physical possibility and efficiency of using the residual nuclear fuel resource due to internal core restructuring with a decrease in power during decommissioning.
2. Mathematical model of the FA spectrum dynamics in energy production.

To conduct research on the use of energy resources of the decommissioned reactor, the following mathematical model of the FA spectrum dynamics for energy production is proposed:

Let us introduce the legend: $E$ - energy production of FAs, MW · day / FA, $n(E, t)dE$ - the number of FAs with energy production in $[E; E + dE]$ at time $t$; $S(E, t)dE$ is the number of FAs that have energy production in $[E; E + dE]$ discharged from the reactor at time $t$; $F(E, t)dE$ is the number of FAs with energy production in $[E; E + dE]$, loaded into the reactor at time $t$; $q(E, t)$ is the number of FAs increasing their energy production per level $E$ per unit time at time $t$.

The rate of change of FAs number with energy production in $[E; E + dE]$ at time $t$, is determined by the rate of decrease due to burn up $q(E + dE, t) - q(E, t) = \frac{\partial q}{\partial E} dE$, the unloading rate $S(E, t)dE$, and the loading rate due to possible loading other reactors $F(E, t)dE$.

$$\frac{\partial n(E, t)}{\partial t} = - \frac{\partial q(E, t)}{\partial E} - S(E, t) + F(E, t)$$

(1)

The connection between the fuel burnup flow $q(E, t)$ and the FA number $n(E, t)$ is determined from the following considerations: for a unit of time, the FA energy production changes by the value $W(E) \cdot 1$, where $W(E)$ is the power of the FA with energy production $E$. Thus, all FAs with energy production ranging $E - W(E) \cdot 1$ to $E$ will cross the energy production level $E$, that is,

$$q(E, t) = n(E, t) \cdot W(E)$$

(2)

Then we can reduce equation (1) to the first-order partial differential equation for the function $q(E, t)$:

$$\frac{1}{W(E)} \frac{\partial q(E, t)}{\partial t} = - \frac{\partial q(E, t)}{\partial E} - S(E, t) + F(E, t)$$

(3)

The initial condition is:

$$q(E, 0) = n(E, 0)W(E)$$

(4)

The boundary condition:

$$q(0, t) = q_0(t)$$

(5)

Equation (3) is defined on the interval $(0, E_m)$. The physical formulation of the problem assumes that when the maximum depth of energy production is reached, $E_m$ FA is extracted from the core. In the "ideal" mode of steady overloads, when only fresh FAs are loaded and only FAs with maximum energy output $E_m$ are unloaded, the mathematical model (3–5) looks like:

$$\left\{ \begin{array}{l} 0 = - \frac{\partial q(E, t)}{\partial E} \\ q(0) = q_0 \end{array} \right.$$

(6)

Then the fuel distribution in the energy production is:

$$n(E) = \frac{q_0}{W(E)}$$

(7)

Power of the FA has a linear dependency on energy production and neutron flux density

$$W(E, r) = \frac{\phi(r)}{\phi_{max}} (W_0 - B \cdot E)$$

(8)

The distribution of FAs for energy production $n(E)$ does not depend on the neutron flux density. Let us consider equation of FAs rate change in energy production interval $E \Delta E + dE$ at time $t$ without fuelling:

$$\frac{\partial n(E, t)}{\partial t} = - \frac{\partial q(E, t)}{\partial E}$$

(9)
Taking in account (2), (8) we can obtain this partial differential equation with boundary condition:

\[
\begin{cases}
\frac{\partial q(E,t)}{\partial t} + (W_0 - B \cdot E) \cdot \frac{\partial q(E,t)}{\partial E} \\
q(0,t) = q_0 \cdot \sigma(t)
\end{cases}
\]

(10)

Where \( \sigma(t) = (1 - sgn(t))/2 \).

This partial differential equation may be solved using characteristic analysis [3].

With regard to this equation (10), the characteristics can be written as:

\[
dt = \frac{dE}{W_0 - B \cdot E}
\]

(11)

Solving (11) as ordinary differential equation for \( t(E) \), we obtain:

\[
t = -\frac{1}{B} \ln(W_0 - BE) + c_1
\]

(12)

Now from equation (12) we can obtain constant \( c_1 \). This constant is actually a function of parameters \( t, E \), and solving equation (10) comes down to searching such function \( \Phi \), that \( \Phi(c_1) = 0 \) for any \( t \in (0, \infty) \).

\[
\Phi\left(t + \frac{1}{B} \ln(W_0 - BE)\right) = 0
\]

(13)

We can search this function using boundary condition (10). First of all, we have to express constant \( c_1 \) from (12) and substitute it to boundary condition (10) using \( E = 0 \):

\[
c_1 = t + \frac{1}{B} \ln(W_0) \rightarrow t = c_1 - \frac{1}{B} \ln(W_0)
\]

\[
\Phi\left(t + \frac{1}{B} \ln(W_0)\right) = q_0 \cdot \sigma\left(c_1 - \frac{1}{B} \ln(W_0)\right)
\]

(14)

After (14), we can finally obtain burn up \( q(E,t) \):

\[
\Phi(c_1) = q_0 \cdot \sigma\left(c_1 - \frac{1}{B} \ln(W_0)\right) = q_0 \left(t + \frac{1}{B} \ln(W_0 - BE) - \frac{1}{B} \ln(W_0)\right)
\]

\[
q(E,t) = q_0 \cdot \sigma\left(t + \frac{1}{B} \ln\left(\frac{W_0 - BE}{W_0}\right)\right)
\]

(15)

We also may find numerical solution for equation (10) using T-pattern [4] on a grid and difference equations. This part might be really important for future works, where boundary and initial conditions are suggested to be more complicated, and so analytic solution for functions \( q(E,t), n(E,t) \) might not exist.

\[
\begin{cases}
-\frac{q_{i+1,j-1}}{2h} + \frac{1}{W_0 - BE_j} \cdot \frac{q_{i+1,j}}{\tau} - \frac{q_{i+1,j+1}}{2h} = \frac{1}{W_0 - BE_j} \cdot \frac{q_{i,j}}{\tau} \\
q_{0,j} = q_0 \\
q_{i,0} = q_0 \Delta_t
\end{cases}
\]

(16)

On grid edges we use right-handed difference to approximate \( \partial q/\partial E \).

To compare two solutions – analytic and numeric, and to make sure that this difference scheme can be used with satisfaction precise, we calculated the average relative error [5] for two solutions obtained for \( t \in [0, 400] \). This error reaches 1%, and as we mostly use average value of \( n(E,t) \) or integrate it, we take it as a good result. However, we have to admit, that maximum error is much greater, mostly along the line \( E_0(t) \).

Thus, we have obtained that under the initial conditions in case of stationary operation mode of the reactor, the distribution of FAs by energy producing coincides with the initial one (7). This means that with constant fuelling, the FAs dynamics spectrum is constant.

The dependence of energy generation on time in the core can be determined from the equation of characteristics (12):
\[ t = - \frac{1}{B} \ln (c(W_0 - BE)) \]  \hspace{1cm} (17)

We can determine the value of the constant \( c \) from the initial conditions: at \( t = 0 \), the minimum energy producing equals 0 (because fueling was still existing):

\[ E_0(t) = \frac{W_0}{B} (1 - e^{-Bt}) \]  \hspace{1cm} (18)

To prove (18), it is necessary to show that count of FAs at moment \( t \) equals to count of FAs at moment \( t + \Delta t \):

\[ \int_{E_1(t)}^{E_2(t)} n(E) dE \equiv \int_{E_1(t+\Delta t)}^{E_2(t+\Delta t)} n(E) dE \]  \hspace{1cm} (19)

Identity (19) can be proofed if we get from (17) time of FAs existence in charge, express \( E_1(t+\Delta t) \) and \( E_2(t+\Delta t) \) through \( E_1(t) \) and \( E_2(t) \) and then obtain right side of identity.

So, we have obtained that in the decommissioned reactor the FAs spectrum is numerically kept the same as in the operating reactor, however the boundary of the spectrum beginning is shifted and depends on the time past after fueling stopped.

A model was constructed with parameters close to those of the HPCTR [2].

\[ K(E) = K_0 - A E, K_{\infty} = 1.02, K_0 = 1.2, A = 1.1535 \cdot 10^{-4} \text{ FA} / (\text{MWt} \cdot \text{day}) \]

\[ W(E) = \frac{\phi(r)}{\phi_{\text{max}}}(W_0 - B \cdot E); W_0 = 3 \cdot \frac{\text{MWt}}{FA}, B = 5.3571 \cdot 10^{-4} \text{ day}^{-1} \]

The following physical values are used in calculations: the number of FAs in the core \( N = 1600 \); the square of the migration length \( M^2 = 0.04 \text{ m}^2 \); the area of the cell with FA \( a^2 = 0.06 \text{ m}^2 \).

3. Two-zone reactor

It is proposed to consider a model of a two-zone fuel loading reactor. In the central zone is fuel with energy production from \( E_0(t) \) to \( E_x(t) \) and in the peripheral from \( E_x(t) \) to \( E_m(t) \). As the fuel burns out, the left and right boundaries of the spectrum \( E_0(t) \) shift to the right, and the boundary between the zones \( E_x(t) \) to the left. The critical state of the reactor is described by a system of equations:

\[
\begin{align*}
\Delta \phi_1 + \frac{K_1}{M^2} \phi_1 &= 0, \\
\Delta \phi_2 + \frac{K_2}{M^2} \phi_2 &= 0
\end{align*}
\]  \hspace{1cm} (20)

where for the inner zone:

\[
\begin{align*}
K_1 &= \frac{\int_{E_0}^{E_x} n(E)K(E) dE}{\int_{E_0}^{E_x} n(E) dE}, \\
N_1 &= \frac{\int_{E_0}^{E_x} n(E) dE}{\pi r_1^2} = a^2 N_1
\end{align*}
\]  \hspace{1cm} (21)

And for the outer zone:
\[
\begin{align*}
K_2 &= \frac{\int_{E_x}^{E_{m}} n(E) K(E) dE}{\int_{E_x}^{E_{m}} n(E) dE}, \\
N_2 &= \int_{E_x}^{E_{m}} N(E) dE, \\
\pi R^2 &= a^2 (N_1 + N_2) 
\end{align*}
\]

(22)

For identical diffusion coefficients in the zones, the equations of uniqueness [6] have the form:

\[
\begin{align*}
\phi'_1 (0) &= \phi_2 (R) = 0, \\
\phi_1 (r_1) &= \phi_2 (r_1), \\
\phi'_1 (r_1) &= \phi'_2 (r_1) 
\end{align*}
\]

(23)

The solution of the two-zone problem [6] allows to obtain a critical condition that will contain in addition to \( K_{\text{eff}} \), the parameters \( r_1, R, K_1, K_2 \), which in turn depend on \( E_0, E_x, E_{\text{max}} \). At the same time, the concept of a two-zone reactor does not involve the extraction of FAs, but only their rearrangement to the periphery.

4. Results

The simulation of the decommissioning of a HPCTR was carried out using the two strategies described above. The minimum permissible effective multiplication factor was assumed equal to 1.02, (considering the margin for axial leakage and the operational reserve of reactivity).

Two-zone charged the reactor can provide energy up to 400 GW by the time of total loss of criticality, but by the time it stops, power is reduced to 14% of the nominal. The amount of unprocessed resource decreases in comparison with the "collapsing" reactor - 311 FAs in terms of fresh fuel. This is 56% less than in the case of a simple stop.

These estimates are limiting until the complete exhaustion of the reactivity reserve and are obtained on the basis of a simplified model of a HPCTR with linear dependences of the power and the multiplication factor of FAs from energy production. In this case, the long operation of the reactor is explained by a significant decrease in power. There may be reasons limiting the reactor’s decommissioning - technological and economic constraints. In general, it has been shown that by organizing the appropriate core charge, it is possible to effectively use energy resources in a HPCTR being decommissioned, using such simple operations as the FA rearrangements in the core.

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