Design of a magnetically driven current cloak

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Abstract
An inhomogeneity into a conductive matrix deforms the flow pattern of an applied electric current. A usual current cloak can be defined as a permanent modification of the matrix properties around the inhomogeneity guaranteeing that the current flow pattern is similar before and after passing by the modified zone, so it implies the ‘electrical invisibility’ of the inhomogeneous region. Here we introduce the concept of a current cloak that can be tuned by means on an external field. We demonstrate analytically and using finite elements simulations that a current cloak can be constructed and manipulated by an external magnetic field for a concrete system consisting in a magneto-resistive matrix with an inclusion of larger conductivity.

Supplementary material for this article is available online

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(Some figures may appear in colour only in the online journal)

1. Introduction

The lack of homogeneity associated with uncontrolled defects such as pores, second phase inclusions and grain boundaries may cause multiple undesirable effects in materials, ranging from poor mechanical properties to a decrease in electrical conductivity [1–4]. These defects are typically avoided by carefully modifying the ‘macroscopic’ procedure for obtaining the material in question. In particular, a point-like inclusion of a second phase displaying an electrical conductivity different from that of the matrix may produce a subtler phenomenon: the deformation of current lines in its surroundings. This problem can be tackled by fine-tuning the properties of the matrix near the defect, producing a ‘cloaking’ effect.

Cloaks can be defined as materials modified in such a way that a non-homogeneous region inside them is made invisible (i.e. indistinguishable from its surroundings) to electromagnetic fields or sound waves, for example. The subject was kicked off by the pioneering theoretical work of Pendry and Leohardt in optical cloaks [5, 6]: if a region of space has optical properties different from the rest of the material, it is possible to engineer a change of properties in the vicinity of the inhomogeneity so the rays of light detected from a point far from the region follow the same trajectories they had before entering the region in question. The term metamaterial is commonly used to describe a material engineered in such way [7]. Over the following years, several types of cloaks were reported in the optical [8–13] and microwave [14–18] frequency ranges.

Later on, cloaks invaded the scenario of stationary (or near-stationary) fields. Wood and Pendry were the first to propose a cloak to conceal a defect in the presence of a low-frequency magnetic field [19]. The idea was experimentally
proven a few years later by a combination of two materials with ‘opposite’ magnetic behaviours: a superconductor and a ferromagnet [20]. The boundary was immediately pushed further to include stationary fields [21]. Yungui Ma et al designed and obtained experimentally a ‘bifunctional cloak’ able to conceal the lines of electrical current (current cloak) or heat flux lines (heat cloak) around a cavity in a metal, and an inclusion with conductivity different from that of its surroundings [22]. Importantly, in all of these cloaks, the material properties must be permanently modified around the inhomogeneity in order to achieve the desired effect. Although there are several works that report ‘remote cloaks’ not involving those permanent modifications [23–25], they commonly rely on active elements and elaborated inductor-capacitor networks.

Here, we propose a different way to achieve the cloaking effect: an external field is used to ‘drive’ the material properties around the inhomogeneity. We illustrate the idea with a current cloak able to suppress the deformation of the current lines around an inclusion of larger conductivity within a magneto-resistive matrix. Firstly, the feasibility of achieving the cloaking effect is demonstrated analytically. Then, we show it by means of finite elements simulations for a realistic set of materials and magnetic fields. We call the new metamaterial Externally Driven Current Cloak or, for our particular case, Magnetically Driven Current Cloak.

2. The magnetically driven current cloak

2.1. General description

Figure 1(a) sketches a possible realization of a magnetically driven current cloak where the magnetic field is provided by a pair of super-magnets. Figure 1(b) is a top view of figure 1(a) indicating all the parameters involved in the calculation of the cloaking condition. The inner circle of radius $R_c$ is the inclusion (or core), whose conductivity $\sigma_c$ is larger than that of the magneto-resistive matrix at zero applied field, $\sigma_m$.

![Figure 1](https://example.com/figure1.png)

Figure 1. Current cloak driven by an external magnetic field. (a) A possible realization of a magnetically driven current cloak. (b) Parameters used to derive the cloaking condition (see text). (c) Magneto-resistive matrix with inclusion at zero magnetic field. (d) Matrix alone at cloaking magnetic field, $B_{cloak}$. (e) Matrix with inclusion at $B_{cloak}$. In (c–e), darker blue indicates larger conductivity, black lines represent the electric current (or electric field) lines. In (d) and (e), the applied magnetic field is represented by ‘x’s.
compensates that described in (c). Figure 1(e) illustrates the effect of $B_{\text{cloak}}$ on the matrix-core composite, so the flow lines are the overlapping between the patterns sketched in (c and d): the result is that the current flow becomes homogeneous immediately outside the region containing the magnetic field—the current cloak effect has been achieved. Now, we calculate the magnetic field needed to produce the cloaking, $B_{\text{cloak}}$, as a function of the radii and conductivities.

### 2.2. An analytical expression for the cloaking condition

The continuity equation for an electric current density $\vec{J}$, represented by the black lines in figure 1 reads $\vec{\nabla} \cdot \vec{J} = 0$. If $\vec{E}$, $V$ and $\sigma$ are the electric field, the potential and the conductivity, one can write $\vec{J} = \sigma \vec{E}$ (where $\vec{E} = -\nabla V$). By combining the previous expressions, we get a Laplace equation for the potential $V$ in cylindrical coordinates. If we solve it for the system illustrated in figures 1(a, b) and impose the cloaking condition, i.e. $\vec{E}$ is uniform outside the area where the magnetic field is applied, we get (see supporting online material (available online at stacks.iop.org/JPD/54/325301/mmedia)):

$$\sigma_m = \frac{R_b^2(\sigma_c + \sigma_B) + R_c^2(\sigma_c - \sigma_B)}{R_b^2(\sigma_c + \sigma_B) + R_c^2(\sigma_B - \sigma_c)}. \quad (1)$$

It is worth noting that equation (1) is a natural development of previous works on current and other kinds of cloaks [21, 22].

If we follow the thread of previous work in the field, the cloaking condition implies inserting a ring of conductivity $\sigma_B$ and inner and outer radii $R_c$ and $R_b$, respectively, so all the parameters fulfill equation (1). Our approach is different: since $\sigma_B$ is a function of the magnetic field, we propose applying a magnetic field $B_{\text{cloak}}$ on the ring-shaped region in such a way that the resulting conductivity matches the value given by equation (1). Naturally, $B_{\text{cloak}}$ depends on the magnetoresistive behavior of the matrix.

### 2.3. A realistic magnetically driven current cloak

#### 2.3.1. The case of a homogeneous magnetic field.

Now, we analyze the case of a composite made of a bismuth matrix shaped as a $(25 \times 25 \times 1)$ mm³ sheet, whose magnetoresistive behavior is illustrated in figure 2 (8.55 $\times$ 10⁶ S m⁻¹ at zero field [26]). A cylindrical tin inclusion of radius 1.6 mm is located at the center of the matrix. The conductivity of the inclusion—which is very weakly dependent on the magnetic field—is taken as 7.94 $\times$ 10⁶ S m⁻¹ [26]. The external field is confined to a region of radius 3.2 mm around the inclusion, which can be easily achieved by using a couple of cylindrical super-magnets of appropriate radii (figure 1(a)). We will assume that the resulting magnetic field is homogeneous within $r \leq R_B$.

Figure 3 shows FEM simulations of the electric current density and equipotential lines along the sample at different magnetic fields perpendicular to the large face of the matrix. A current of 5.5 A is injected along the left lateral $(25 \times 1)$ mm² vertical face of the matrix and flows towards ground at the opposite right face with a uniform density $J_m = 2.2 \times 10^5$ A m⁻² in the case of no inclusion and zero applied magnetic field. For zero applied field (figure 3(a)) the lines are deformed towards the inclusion, corresponding to the case sketched in figure 1(c). For a field $B = 1.25$ T (figure 3(b)), the deformation of the current lines towards the highly conductive core is compensated by the ring of depleted conductivity due to the effect of the applied magnetic field, so they are homogeneous outside the zone where the magnetic field is confined. This situation, analogous to the one sketched in figure 1(e) implies that $B_{\text{cloak}} \approx 1.25$ T for our composite. In fact, this is the field value that satisfies equation (1), taken for the specific geometry and conductivities of the cloak, as well as the curve shown in figure 2.

For a magnetic field of 2.00 T (figure 3(c)), the conductivity of the ring around the inclusion has decreased to a point that it ‘over-compensates’ the original deformation of the current lines, so there is no longer cloaking. The same information can be deduced from the electric potential (figures 3(d–f)) which can be argued a quite accessible quantity, from the experimental point of view.

In order to quantify the deformation of the current density near the cloak, and properly estimate $B_{\text{cloak}}$, a more detailed analysis is needed. We propose to do it as illustrated in figure 4. It shows how the electric current density changes along a vertical line running from the top to the bottom of the sample, which is tangent to the region where the magnetic field is applied (see inset of figure 4). The two vertical dotted lines in the main graph correspond to the upper and lower edges of the region where the magnetic field is applied, respectively. For $B = 0$, the current density above and below that region decreases from its value when there is no inclusion nor magnetic field applied, since the current lines ‘bend towards’...
the higher conductivity core. The bending is smaller for $B = 0.60$ T and virtually non-existent for $B = 1.25$ T. The lines bend in the opposite direction for fields of 1.70 and 2.00 T. In the region where the magnetic field is applied, along the same vertical line, the deformation of the originally uniform current density field is more evident, even so, for $B = 1.25$ T there is no tangible sign of deformation. This magnetic field was chosen according to equation (1) for the cloaking condition to be fulfilled. It should be pointed out that $B_{\text{cloak}}$ does not change if the sample is larger in the $x$ direction (see reference frame of figure 1(b)); as a matter of fact, the current density profiles of figure 4 also remain the same as expected.

A number of ways have been reported to quantify cloaking [28, 29]. Here, we introduce a simple approach to the matter, which allows easy comparison with experimental measurements of the current (or of the electric potential) along an imaginary line on the sample, and saves computational time. Based on the curves shown in figure 4, the deviations of the

Figure 3. Magnetically driven cloaking for a realistic sample. FEM simulations of the electric current density (left column) and equipotential lines (right column) for a 1 mm thickness polycrystalline Bi matrix with a tin inclusion of $R_c = 1.6$ mm radius (inner circle), and a magnetic field applied within a region of radius $R_B = 3.2$ mm (outer circle). (a, d), (b, e) and (c, f) correspond to magnetic fields of 0, 1.25 and 2.00 T, respectively. Here, $B_{\text{cloak}} \approx 1.25$ T.
current density from its uniform value when there is no inclusion nor magnetic field applied can be quantitatively evaluated by defining a cloaking parameter:

$$P = 1 - \frac{\text{Max}(|J(x) - J_{in}|)}{J_{in}},$$

(2)

where $J_{in}$ is the uniform current density for no inclusion or magnetic field applied and $J(x)$ is the current density dependence along the vertical line displayed in the inset of figure 4 for any given values of the magnetic field and $R_B$. This allows to quantify how far we are from achieving the cloaking condition. If the applied magnetic field is different from $R_{cloak}$, then $P < 1$, suggesting that the device is not in the cloaking state. On the other hand, if the applied field coincides with $R_{cloak}$, then $P = P_{cloak} = 1$ and the device will behave as a genuine current cloak. Notice that equation (2) can be easily adapted to consider equipotential lines as a reference to quantify the cloaking effect.

By varying the area of the zone where the external magnetic field is applied (i.e. by modifying $R_B$) and then varying the value of the applied magnetic field in each geometry, it is possible to build a ‘parameter phase diagram’ using the definition of $P$ given by equation (2). Figure 5 illustrates the behavior of $P$ as a function of $R_B/R_c$ and the applied magnetic field $B$. These magnitudes were chosen because they can be externally modified to achieve the cloaking condition for a given inclusion into a given matrix with fixed $\sigma_c$ and $R_c$. The cloaking and non-cloaking states are visible in figure 5 in terms of $P$: the red ridge corresponds to the sets of external parameters that make possible the cloaking ($P_{cloak} = 1$). The smaller the radius $R_B$ the strongest the magnetic field we have to apply to achieve the current cloaking condition. The set of parameters, for which $P = P_{cloak} = 1$, matches that calculated using equation (1) within negligible uncertainty ($\approx0.1\%$).

Figure 5 also illustrates that the cloaking condition is achievable even for $R_B$ values comparable to the size of the matrix, suggesting that its external boundaries do not affect considerably the cloaking effect. This is shown in the supporting online material.

Up to now we have only considered the modification of the current flow pattern by the presence of one inhomogeneity in the magneto-resistive matrix. If two identical inclusions are deployed symmetrically side-by-side along the $y$-axis, the cloaking effect is again achieved for the same value of the magnetic field (see supporting online material). This is presumably the case for a larger number of inclusions (even in the case they are randomly distributed), provided the matrix is long enough along the $y$-axis.

2.3.2. The case of a non-homogeneous magnetic field.

Although the analytical results as well as the previous simulations demonstrate the proposed cloaking effect for realistic materials, we have assumed a perfectly homogeneous magnetic field within the region $R \leq R_B$. However, when two cylindrical magnets are used, the resulting field is actually non-homogeneous within the gap between the magnets.

In order to include such inhomogeneity in our simulations, we have computed the magnetic field within a 1 mm gap between two cylindrical magnets with B-H characteristics resembling those of commercial FeNbB magnets [30]. Figure 6 shows a comparison of the electric current density along the red vertical line presented in the inset of figure 4 for the same system described at the beginning of section 2.3. As a result of the application of a uniform field (already seen in the previous subsection) and the application of the non-uniform magnetic field produced by realistic facing magnets. The electric current density due to the non-uniform field (red curve in figure 6) slightly deforms from the straight line we had in figure 4 (black curve); the blue curve in figure 6 stands for the current density at zero magnetic field (i.e. where cloaking is not possible). The top left inset in figure 6 shows the magnetic field profile in an axial cut of the cylindrical magnets. The top right inset presents the $B/B_{max}$ dependance with the
In summary, we have proposed the idea that current cloaks can be tuned by an applied magnetic field, at least for certain composite materials. We demonstrate that the cloaking condition can be achieved in the case of a circular inclusion embedded in a magneto-resistive plate, by applying a perpendicular magnetic field of appropriate intensity. We provide enough quantitative information to experimentally test the idea by using realistic materials, and magnetic fields attainable, within a certain range, by standard super-magnets.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

3. Conclusions

Figure 6. Comparison of the electric current density for the system described at the beginning of section 2.3 along the same probing line, as a result of the application of a uniform field (black curve) and the application of the non-uniform magnetic field produced by real facing magnets (red curve). The blue curve corresponds to the current density at zero magnetic field (i.e. where cloaking is not possible). The top insets show the slow decrease of the magnetic field as we move radially out of the inclusion. The bottom ones display the behavior of \(P_{\text{cloak}}\) and \(B_{\text{cloak}}\) in terms of \(R_{\text{g}}/R_{\text{i}}\) (black and red data points stand for homogeneous and non-homogeneous fields respectively). All quantities related to the cloak were calculated maximizing \(P\) in order to focus in the cloaking state.

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radial coordinate, \(r\), defined in figure 1(b); \(B_{\text{max}}\) is the maximum value of magnetic field in the gap, located at \(r = 0\). The slow decrease of the magnetic field as we move radially out of the inclusion (shown in the top insets of figure 6) makes the perfect cloak no longer achievable in terms of equation (1). However, we will demonstrate in the following that it is possible to get very near it. Notice that the red and black curves in the main graph of figure 6, as well as the quantities presented for the cloaking state in the bottom insets, were calculated maximizing \(P\) after varying \(R_{\text{g}}\) and \(B\) in the same fashion it was done for obtaining figure 5.

In the bottom insets of figure 6, \(P_{\text{cloak}}\) as well as \(B_{\text{cloak}}\) in terms of \(R_{\text{g}}/R_{\text{i}}\) for \(R_{\text{i}} = 1.6\) mm are displayed. The black data points correspond to the homogeneous field case and the red ones correspond to the more realistic, non-homogeneous case. It can be seen that the smaller the area of application of the magnetic field, the farther the system is from the perfect cloaking state. The improvement of the cloak as \(R_{\text{g}}/R_{\text{i}}\) increases is in part associated with the fact that the core is farther from the probing line where \(P\) is being computed. However, it should be noticed that the maximum deviation of \(J(x)\) from \(J_{\text{max}}\) is less than 2%, which still makes our setup an excellent approximation of a perfect cloak. Moreover, the \(B_{\text{cloak}}\) dependencies on \(R_{\text{g}}/R_{\text{i}}\) for homogeneous and non-homogeneous magnetic fields basically overlap, as seen from the right bottom inset of figure 6: it illustrates the robustness of our cloaking proposal in spite of the lack of homogeneity of the magnetic field in the gap between realistic cylindrical magnets.

3. Conclusions

In summary, we have proposed the idea that current cloaks can be tuned by an applied magnetic field, at least for certain composite materials. We demonstrate that the cloaking condition can be achieved in the case of a circular inclusion embedded in a magneto-resistive plate, by applying a perpendicular magnetic field of appropriate intensity. We provide enough quantitative information to experimentally test the idea by using realistic materials, and magnetic fields attainable, within a certain range, by standard super-magnets.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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