Numerical Modal Analysis of Kinked Bars –Triangle Case of Study

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Abstract. The paper deals with numerical modal analysis of a kinked bars similar with an open triangle, with different sizes, angles and diameters of bar. Using SIMCENTER software, the first step was to design the geometry of each type of structure, then discretization of model in finite elements and assigning properties of material (in this case, Al alloy). The results in terms of eigenvalues and eigenmodes of each structure revealed that the angles of kinked bars influenced the value of natural frequency: with increase the tip angle from 30° to 60° and 90°, the eigenvalue decreased in ratio of 1:1.39:1.611. Also, the length of each side of the kinked bar influences the modal response of the structure: increasing the length from 200 mm to 450 mm, leads to decreases of eigenvalue of almost 5.6 times. The applicability of this study is linked to percussion instruments used in musical orchestra and also to explain the energy transfer and acoustic effect of different types of triangle instruments.

1. Introduction

The triangle is a percussion musical instrument that is obtained by bending a metal bar with a circular section in the shape of an open triangle. In fact, the triangle is a metallic bar bent into the shape of an equilateral or isosceles triangle, with rounded vertices [1, 2]. The triangle has effectively only two vertices, the third is an open apex to allow the free vibration of the bar. Commonly, the transverse section of the triangle is circular. Some models have a variable section at the end of the free bars.

This instrument is classified as an idiophone, ie the production of sound is done only by itself vibration, without the intervention of an auxiliary part. The triangle is obtained in several geometric configurations from different types of material: stainless steel, aluminum alloys, brass, and the section of the bar can be constant along its entire length or variable. From a mechanical point of view, the triangle is a combination between the right beam and the kinked beam, having an axis of constructive symmetry with respect to the open top which differs from the axis of loading and boundary conditions. The modern triangle comes in several sizes with an edge commonly being 20cm or 30 cm and could be up to a maximum of 40 cm or 45 cm. The triangles are made of a bent rod of circular cross section, produced by folding the metal around a jig. The shape of the triangle could be equilateral or isosceles. The orchestral triangle is in fact a combination of straight and curved bars, and therefore is an instrument of relatively complex structure. The triangle is struck
with a metallic mallet, and is supported with a thread around the top bend of the hanging instrument. The struck triangle generates longitudinal and flexural modes of vibration, which are excited perpendicular or parallel to the plane of the triangle.

For longitudinal vibrations it can be assumed that the frequency of a bar of circular cross section and of length \( L \) is calculated as in equation (1) [3]:

\[
f_n = \frac{n}{2} (2n + 1)^2 \frac{\pi}{8} \frac{E}{\rho} L^2 \sqrt{\frac{E}{\rho}}
\]

with frequencies in the ratios \( 3^2: 5^2: 7^2: 9^2: 11^2: 13^2 \).

For vibrations perpendicular to the plane of the triangle, the displacement is small. As noted by [3 – 5] for the transverse modes, the modes of vibration of a triangle are almost identical to those of the straight bar from which the triangle is bent. This statement argued in favor of the hypothesis that the shape of the triangle has no big effect on its vibration modes. However, [6, 7] noted that the curvature of a bar influences the stress distribution which is determined by the inertia moments applying at the end of a bar. For flexural vibrations in the plane, there is an important discontinuity in impedance at each bend angle. The transverse vibration within one edge couples to the longitudinal vibration in the adjacent edge. Therefore, each edge of the triangle will support its own vibration modes and will be coupled to another edge, to form sets of normal mode triplets, for an edge of the same length.

The aim of this paper is to determine the eigenmodes and eigenfrequencies of different types of triangles in order to analyze the dynamic behavior of them, using finite element analysis (FEA).

2. Materials and methods

2.1. Materials
Within these researches, the geometries of three triangles were made, named in the study as cases: case A, in which the bar is bent in the form of an equilateral triangle, with the subcases in which the length of the side and the diameter differ; case B - the shape of an isosceles triangle, with the subcases in which the side length differs in a row and case C - isosceles right triangle, with the subcases in which the side length differs in a row (figure 1).

The geometric characteristics of the studied structures are presented in table 1. The material from which these triangles are made is C17200 beryllium copper alloys, having the physical and elastic characteristics in table 1.

![Figure 1. The types of triangles studied: a) equilateral triangle, coded A; b) isosceles triangle, coded B; c) isosceles right triangle coded C.](image-url)
Table 1. The physical and geometrical features of triangles.

| Cases   | A – B - C |
|---------|-----------|
| Subcases | 1.1. | 1.2. | 1.3. | 2.1. | 2.2. | 2.3. |
| Length L (mm) | 200 | 300 | 450 | 200 | 300 | 450 |
| Diameter d (mm) | 12 | 12 | 12 | 8 | 8 | 8 |
| Density (kg/m3) | | | 8250 | | | |
| Elasticity Modulus (MPa) | | | 125000 | | | |
| Poisson Coefficient | | | 0.3 | | | |

2.2. Simulation of modal analysis

Finite element modeling of the proposed structures was done with Simcenter 3D software. In the first stage, the 18 constructive variants of the triangles were made, each structure being then discretized with finite elements. In terms of boundary conditions, the triangles were considered free structure. Having a relatively simple structure, in the simulation we choose for finite elements of Thetraedral type (TET) with a size of 3 mm (figure 2). The material characteristics were introduced and then the FEA analysis was run, obtaining the proper modes of all the sub-cases considered.

![Figure 2.](image)

3. Results and discussions

After the FEA analysis, the modal shapes and fundamental frequencies resulted. It was found that the first six modes are structural modes, with very low eigenfrequencies, which is why eigenvalues were recorded and analysed starting with the seventh mode. Also, geometrical characteristics of the triangle have a determinant effect on the modes of vibration as shown in table 2 and 3. With increasing the diameter of rod, the increases of eigenfrequencies it’s observed (figure 3). The biggest difference due to the increase of the diameter by 50%, is obtained in the case of the isosceles triangle, when the first eigenfrequency increases by 57%. With the increase of the bar length, of 2.25 times, a decrease of the first frequency is observed regardless of the diameter and angles.

The low frequency modes are mainly flexural modes and are related to the size, and the mechanical properties of the material of which the triangle is made. At constant length, the frequency increases with the vibration modes, so that the increasing length of the edge determines the decrease in frequency. This means that the triangles of shorter edges vibrate at higher frequencies than the
triangles of larger edges. As regards the triangle type, in general, the eigenfrequencies of the isosceles triangle of L = 20 cm, are higher than for other types of triangle.

Figures 4, 5 and 6 shows the first modal shape of vibration of the equilateral, isosceles and rectangular isosceles triangles, with a diameter of 8 mm.

Table 2. The eigenfrequencies of analysed triangles with diameter of 12 mm.

| Mode/Cases | A1.1 | A1.2 | A1.3 | B1.1 | B1.2 | B1.3 | C1.1 | C1.2 | C1.3 |
|------------|------|------|------|------|------|------|------|------|------|
| 1          | 132  | 53   | 22   | 191  | 74   | 30   | 84   | 34   | 15   |
| 2          | 133  | 54   | 23   | 258  | 97   | 39   | 92   | 37   | 16   |
| 3          | 217  | 88   | 37   | 536  | 208  | 84   | 195  | 81   | 34   |
| 4          | 623  | 274  | 120  | 837  | 366  | 160  | 527  | 223  | 96   |
| 5          | 787  | 336  | 145  | 1142 | 474  | 202  | 567  | 237  | 101  |
| 6          | 1192 | 492  | 208  | 1205 | 507  | 216  | 706  | 309  | 135  |
| 7          | 1225 | 508  | 213  | 1456 | 573  | 235  | 826  | 349  | 150  |
| 8          | 1243 | 516  | 221  | 2041 | 976  | 448  | 1228 | 516  | 221  |
| 9          | 1325 | 544  | 229  | 2694 | 1163 | 508  | 1293 | 539  | 230  |
| 10         | 2066 | 964  | 434  | 2842 | 1309 | 579  | 1503 | 655  | 283  |

Table 3. The eigenfrequencies of analysed triangles diameter of 8 mm.

| Mode/Cases | A2.1 | A2.2 | A2.3 | B2.1 | B2.2 | B2.3 | C2.1 | C2.2 | C2.3 |
|------------|------|------|------|------|------|------|------|------|------|
| 1          | 87   | 35   | 15   | 121  | 48   | 20   | 55   | 23   | 10   |
| 2          | 88   | 36   | 15   | 168  | 64   | 26   | 61   | 25   | 10   |
| 3          | 141  | 58   | 24   | 351  | 137  | 56   | 129  | 54   | 23   |
| 4          | 422  | 184  | 81   | 566  | 245  | 107  | 353  | 149  | 64   |
| 5          | 525  | 224  | 97   | 762  | 316  | 135  | 377  | 158  | 67   |
| 6          | 802  | 329  | 139  | 826  | 341  | 144  | 478  | 208  | 91   |
| 7          | 829  | 338  | 142  | 928  | 370  | 153  | 545  | 231  | 100  |
| 8          | 831  | 347  | 148  | 1483 | 677  | 303  | 830  | 346  | 147  |
| 9          | 869  | 357  | 151  | 1797 | 776  | 338  | 861  | 358  | 153  |
| 10         | 1471 | 659  | 293  | 2104 | 915  | 392  | 1038 | 441  | 190  |

Figure 3. Comparison of first eigenfrequencies in all studied cases.
Figure 4. The first eigenmodes of equilateral triangle – case A.
Figure 5. The first eigenmodes of isosceles triangle – case B.
Figure 6. The first eigenmodes of isosceles right triangle – case C.
The triangles having shorter edges have higher frequency modes. i.e. the frequency of mode 1 of the triangle having a perimeter of 665 cm is about 542 Hz and of the triangle having a perimeter of 1350 mm the frequency is only 134 Hz. At higher modes, the difference is more important, such as for mode 4, the frequencies are respectively 489 Hz and 1993 Hz. The effect of the bar length or of the perimeter on the modal frequencies of three isosceles triangles of about 40 o and having a perimeter varying from 665 mm to 1350 mm is given in table 4 [7]. In figure 7 it can be noticed the modal shapes of equilateral triangle. In all cases, the flexural modes are presented with increasing the number of nodes and antinodes with increasing the frequency.

In table 4, a comparison between modal analysis of a straight rod and kinked rod performed by Bucur, 2020 [8], is presented. Generally, the triangles recorded a lower frequency compare to straight rod, for the flexural mode of vibration which is considered the main mode for beam.
9

525 Hz

801 Hz

869 Hz
Figure 7. The modes of vibration of the equilateral triangle, having equal edges of length 200 mm, and a diameter of 8 mm.

Table 4. Vibrations modes of a triangle compared to a straight rod, made of steel [7].

| Vibrator                  | Specification     | Mode 1 | Mode 2 |
|---------------------------|-------------------|--------|--------|
|                          |                   | In plane | Out of plane | Frequency (Hz) | In plane | Frequency (Hz) | In plane |
| Rod                       | Straight          | 61      | -       | 166      |          |               |          |
| Triangle isosceles        | Angle at base 30° | 68      | 66      |          | 88       |               |          |
| Triangle right angle      | Angle at base 45° | 55      | 55      |          | 88       |               |          |
| Triangle equilateral      | Angle at base 60° | 48      | 49      |          | 89       |               |          |
| Differences to rod        |                   | %       | %       | +11.5%   | %        | +46.98%       |          |
| Triangle isosceles        | Angle at base 30° | -11.5%  | -7.5%   | -4.98%   |          |               |          |
| Triangle right angle      | Angle at base 45° | +9.8%   | +9.8%   |          |          |               |          |
| Triangle equilateral      | Angle at base 60° | +21.3%  | +19.6%  |          |          |               |          |
4. Conclusions
The paper analysed the dynamic comparison of different types of triangles from a geometric point of view in order to establish the causal relationships between the modes of vibration, their eigenfrequency and their dimensions. The bending angle of the bars, the length of the sides of the triangle and the diameter play a decisive role in the vibrations of the triangle and implicitly in the emission of sounds, this being a necessary percussion instrument in musical orchestras.

Future research aims at experimental analysis on the frequency spectrum, damping and dominant frequencies of commercial triangles, in correlation with the elastic properties of the materials from which they are used.

5. References
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