1. INTRODUCTION

Large-amplitude X-ray and γ-ray variability from active galactic nuclei (AGNs), black hole binaries (BHBs), and gamma-ray bursts (GRBs) is thought to be the signature of hot (relativistic) plasma (Blandford 1990; Mushotzky, Done, & Pounds 1993, hereafter MDP93; Fishman & Meegan 1995). Many of these sources exhibit rapid large-amplitude variability (MDP93; Green, McHardy, & Lehto 1993). The combination of rapid large-amplitude variability and high emissivity is thought to be evidence for compact sources (Fabian 1992; Blandford 1990; MDP93; Fishman & Meegan 1995).

Roughly half of AGNs and BHBs exhibit 1/fα decline in their power density spectrum (PDS), where 1 < α < 2 (Press 1978; MDP93; McHardy 1988; Green et al. 1993; Ulrich, Maraschi, & Urry 1997). Some GRBs also exhibit nonlinear oscillations superposed on the general power-law decay (Meredith, Ryan, & Young 1995). Recently, several time series analysis methods were developed to distinguish between PDSs with linear and nonlinear origins (e.g., Kaplan & Glass 1995; Vio et al. 1992; Scargle 1990). By using these methods, evidence of nonlinearity in some AGNs and GRBs was found (Vio et al. 1992; Boller et. al. 1997; Leighly & Obrian 1997; Meredith et al. 1995; Yuan et al. 1995). Some of the light curves were also found to be self-similar (Scargle, Steinman-Cameron, & Young 1993; McHardy & Czerny 1993).

Several authors have borrowed cellular automaton (CA) models from other scientific fields to simulate the nonlinear behavior of AGNs, BHBs, and GRBs. CAs are tables of rules that describe how to model a nonlinear system with identical elements, commonly used when the partial differential equations for a nonlinear system are not easily solvable (Jackson 1989; Kaplan & Glass 1995). Using a CA, Mineshige, Tak-euchi, & Nishimori (1994) assumed that the accreting material in BHBs is in the form of an accretion disk and that the avalanches are analogues to those in the self-organized criticality CAs for sand piles (Bak, Tang, & Wiesenfeld 1988, hereafter BTW88). Scargle et al. (1993) assumed that the material in BHBs is in the form of a ring, as in the dripping hand model of Crutchfield & Kaneko (1988). Stern & Svensson (1997) suggested that a pulse-avalanche CA may be appropriate for an expanding “fireball” in GRBs if there is magnetic turbulence due to an unknown instability. These models are motivated by similarity to simulations from other fields and by the assumed geometry of some astrophysical point sources. The aim of this Letter is to construct an independent CA directly from the accepted physical conditions in the plasma.

The physical conditions in compact plasma with perturbations and the CA table of rules are described in § 2. In § 3 it is shown that the plasma indeed evolves to a self-organized critical state for a wide range of emission rates and that the emerging power-law spectrum is of the nonlinear 1/fα variety. In § 4, the last section, the expected observational ramifications and conclusions are discussed.

2. CA MODEL FOR COMPACT PLASMAS WITH PERTURBATIONS

In compact plasmas with nonlinear processes, the energy lost to emission or added to plasmas on a doubling timescale is a significant fraction of the total energy (Sivron 1995; Piran 1995). The introduction of a large energy density perturbation into the plasma should result in an increase of emission if the energy is efficiently radiated. For example, emission will rise due to a pair-runaway process in which the increased number of leptons results in efficient Compton cooling. Postshock pair cascades are readily produced in compact sources with strong perturbations (Sivron, Caditz & Tsuruta 1996, hereafter SCT96).

Strong perturbations of excess density of the order of the average plasma density form shocks (Landau & Lifshitz 1987). This holds true for relativistic fluids and hot collisional plasma (Taub 1949; Iwamoto 1989, hereafter I89). The timescale for perturbation steepening to shocks is small in accreting sources (Papaloizou & Pringle 1984; Narayan 1991) and GRBs (Piran 1995). All strong perturbations may be modeled as effective radiative shocks that can dominate the light curve (SCT96). The perturbations are nonlinear in the sense that a strong perturbation significantly changes the plasma temperature and speed of sound that determine the output due to the next perturbation.

The input for the simulations in this Letter is a strong nonlinear perturbation moving with input velocity U. The output luminosity is L*, the number of intervals is N, and the running index is 1, 2, ..., i, ..., N. The simulations follow the CA rules...
of Table 1. The simulation is nonlinear in the sense that parameters in cells \( i + 1, i + 2, \text{etc.} \) depend on the output \( L_i \).

For demonstrative purposes, numbers appropriate for a typical source—the active nucleus of a Seyfert I galaxy—are used. The emitting source of plasma, not including rest mass, is \( \text{X} = 5R_{\text{Sch}} = 1.5 \times 10^{13} \text{ cm} \), which is appropriate for a central black hole of mass \( M = 10^7 M_{\odot} \), and the accretion rate relative to the Eddington accretion rate is \( \dot{m} = 0.1, 0.5, 1.0, 1.5 \). Here \( R_{\text{Sch}} = 2GM/c^2 \) is the Schwarzschild radius of the central black hole with mass \( M \), \( \dot{m} = \dot{M}/\dot{M}_{\text{Edd}} \) is the accretion rate, \( \dot{M}_{\text{Edd}} = 4\pi GMm_p(c^2/\sigma T) \) is the Eddington accretion rate, and \( m_H \) and \( \sigma_T \) are the hydrogen mass and the Thompson cross section, respectively (Blandford 1990).

The parameters for the simulation are as follows: the time interval is \( \Delta_t \), and \( \Delta = 1 \text{ s} \) for the case of Seyfert I galaxies. The total mass of the plasma, \( m = 3 \times 10^{24} \text{ g} \), is chosen so that, using the plasma deflection length with electron temperature \( \sim 3 \times 10^9 \text{ K} \), the plasma deflection length is smaller than \( X \) and the plasma is collisional (I89). The total energy of the plasma, not including rest mass, is \( E_c \). The total temperature of the plasma is \( T = E_c/(1.5n_k) \), where \( n_k = 3m/(4\pi X^3) \), \( X \) is the Boltzmann constant, and \( m_H \) is the average mass of an atom. The mass of each perturbation is \( m_i = m_\Delta \). The kinetic energy of a perturbation is \( \delta E = \delta E_{\text{max}} \) with equal probability.\(^1\) \( \delta E_{\text{max}} \) is the Keplerian energy of perturbation with mass \( m \), at \( 5R_{\text{Sch}} \). Using the usual special relativistic expression, the input speed of each perturbation is \( U_i = c[1 - (m_i/c^2\delta E)^2]^{1/2} \). The output luminosity is \( L_i \), and the efficiency of converting gravitational to radiative energy is \( c \).\(^2\) The initial conditions were set at multiples of \( E_i = 0.05mc^2 \). Energy perturbations with Gaussian distributions were also tried with similar results (see §3).

The effective shocks are selected in the following way: The parameters \( T, n_k, \) and \( U_i \) are sent to a subroutine that uses the Ranking-Hugoniot relations for a hot collisional plasma to find the postshock conditions (I89; SCT96). If these conditions are sufficient for pair-cascades and if the perturbation moves faster than \( c \), the shock is considered effective (Svensson 1982, 1984; Björnsson & Svensson 1991; SCT96). The speed of sound in the plasma \( c \), depends on \( T \) and is typically slightly larger than \( c/3 \). There are two alternating CAs: if there are no effective shocks, the CA is the second row in the table; if there is an effective shock, the third row is the CA. The number of columns in that row is \( J = (XU_i)/\Delta_t + 1 \), an integer that determines the number of subsequent cells still affected by effective shocks. In Table 1, the width of row B was selected to be \( J = 3 \) for demonstrative purposes.

The following scenario demonstrates the nonlinearity of the simulation: If initially there is no effective shock, the energy \( E_{\text{i,n}} \) is usually smaller than \( E_c \) and \( L_i \Delta_e \) is much smaller than \( \delta E_{\text{max}} \), because the small nonshock emission is diffusion dominated (see Table 1). As a result, the energy and temperature of the plasma in the subsequent intervals increase as does the speed of sound. In the following interval, the probability of exceeding the speed of sound is therefore smaller. If, on the other hand, there is an effective shock, the energy decreases according to columns (2)–(4) in the third row, lowering the speed of sound and increasing the probability of shocking the plasma in the next interval.

3. RESULTS

The light curve, PDS, phase, and autocorrelation function (ACF) of the output from the model in Figures 1a, 1b, and 1c resemble the results of BTW88, Mineshige et al. (1994), Schargle et al. (1993), and Stern & Svensson (1997). As in BTW88, self-organization is achieved independently of initial conditions because the system quickly evolves to a state in which perturbations of average energy shock the plasma. Self-organization is not obtained, however, for accretion rates lower than \( \dot{m} = 0.1 \) or larger than \( \dot{m} = 1.0 \). At \( \dot{m} < 0.1 \), the plasma temperature grows without bound because there are too few large perturbations at supersonic speeds.\(^3\) For \( \dot{m} > 1.0 \), our model was not stable.

In Figure 1a, the light curve corresponding with the higher \( \dot{m} \) takes longer to reach self-organized criticality because while increasing its temperature, the denser plasma loses more energy through small shocks. The minimum doubling timescale is smaller for the higher \( \dot{m} \) objects because \( \dot{m} \) was increased by lowering the size of the plasma. The shock therefore extracts energy from the plasma over a shorter period, resulting in stronger flares. For accreting sources, this corresponds with a smaller compact object.

The PDS exponent in Figure 1b is \( \alpha = 0.85 \pm 0.03 \) for \( \dot{m} = 1 \) and \( \alpha = 0.79 \pm 0.03 \) for \( \dot{m} = 0.5 \) (fit not shown). In both cases, the PDS includes emission from the time after the critical state has been established and the subsequent 900 s. Energy perturbations with Gaussian distribution of width \( 0.4\delta E_{\text{max}} \) yielded \( \alpha = 0.84 \pm 0.03 \) and \( 0.79 \pm 0.03 \). As expected, for increasing \( \dot{m} \), small high-frequency shocks are increasingly suppressed as the temperature and associated speed of sound increase just before the more frequent large shocks. With decreasing \( \dot{m} \), we get \( \alpha = 0 \) because as the time interval between perturbations grows, the system responds linearly to the random perturbations. As in BTW88, a flat PDS is changed to \( 1/f^{\alpha} \) due to suppression of short-scale “avalanches” in do-

\(^1\) The distribution is the result of virial radiative pressure and gravity that produce super- and sub-Keplerian speeds for perturbation of large enough size (Sivron 1995; SCT96).

\(^2\) In the case of accretion sources, \( c = 0.06 \) for a Schwarzschild black hole.

\(^3\) In such cases, modest magnetic fields are needed for cooling with super-Alfvénic perturbations and reconnection events.
mins of increasing sizes that were flattened by larger avalanches. In the model presented here, the $1/f^{\alpha}$ is due to the suppression of weak radiative shocks just before stronger shocks. Here the “critical slope” of BTW88, the speed of sound, depends on the previous temperature in the plasma. This model is therefore analogous to a one-dimensional BTW88 sand pile with critical slope that depends on the speed and location of previous avalanches.

Correlation on short timescales (in the first few seconds and at around 50–80 s) for $m = 1$ can be seen in the ACF in Figure 1c. The enhanced ACF at low timescales makes sense because of the anticipated correlation of perturbations. The peaks are due to the total sum of different average delays due to various conditions in the plasma. The effect of the initial perturbation is lost over timescales $t > X/[\mathcal{M}(c_s)]$, where $\mathcal{M} = U_l/c_s$ is the Mach number. For $m = 0.5$ there is less overall correlation but more correlation at times $\sim 80$ s (not shown).

Another method by which the nonlinear dependence is demonstrated is shown in Figure 2a in which the phase-space diagram of the outputs $L_N$ is compared with $L_{N+1}$. With no correlation the path should randomly fill the correlation space, as seen in Figure 2b for a 100 s delay between $L_N$ with $L_{N+100}$.

4. DISCUSSION AND CONCLUDING REMARKS

The model shows that $1/f^{\alpha}$ nonlinear PDS can be created without relying on a specific geometry. One problem with this model, that in accreting sources the predicted $\alpha$ is too low, can be corrected by adding parametric dimensions. Adding parametric dimensions usually results in larger $\alpha$ (BTW88). Simulation with added parameters, one of which is related to the two-dimensional angular momentum transfer for accreting sources, is presented in our subsequent work (Sivron & Leighly 1998, hereafter SL98). For low $m$ and flat geometry, the added
Fig. 2.—Left: Phase-space diagram with 1 s delay shows a very clear sign of nonlinear dependence. The order of points is such that for a point $N$ on the lower accumulation line, the point is on the upper accumulation line. Right: Same phase relation with 100 s delay shows no dependence at that time interval, corresponding with loss of information over longer times than $\Delta t_{\text{U}}$.

parameter is expected to yield results similar to those of Mineshige et al. (1994), since the random input and output for each disk cell are correlated. A second parameter, analogous to the parameter associated with the time profile of shots (Takeuchi, Mineshige, & Negoro 1995), is related to the profile of radiation events from each shock (SL98). Contrary to observations, there is no correlation on a timescale longer than $10^{-3}$ s for BHBs in the model (Negoro et al. 1994). This is a problem correctable with angular momentum transfer because $m$ will decrease with the less effective angular momentum transfer associated with small nonshocking perturbations, making the subsequent shocks less effective (SL98).

For accreting sources, the simulation yields average output temperatures that are lower for high $m$. This is because of an increase in emission efficiency with increasing $m$. However, when $m$ is high, the cooling of the postshock plasma is effective and a large portion of the postshock material is cooled to a “cold phase” of temperature $\sim 10^6$–$10^7$ K. The effect on observations is an enhancement of the soft X-ray emission (Guilbert & Rees 1988; Celotti, Fabian, & Rees 1992; Sivron & Tsuruta 1993; SCT96; Kuncic, Celotti, & Rees 1997), a result already consistent with observations of narrow-line Seyfert type I galaxies and “regular” Seyfert I galaxies. Narrow-line Seyfert type I galaxies that have the same emission rate of regular Seyfert galaxies have a steeper X-ray spectrum, a slightly larger $\alpha$, and probably smaller black holes and larger $m$ (Leighly 1997; SL98).

The correlation of $m$, $\alpha$, and enhanced thermal emission is general. The model therefore predicts that energy density perturbations in the initial fireball of GRBs can produce nonlinear temporal variations that can then give rise to winds with varying Lorenz factor $\Gamma$ (Sivron 1995; SL98). The emission can be the result of low $\Gamma$ winds loaded with baryonic matter overtaking the high $\Gamma$ initial winds that slow down as the fireball sweeps up external matter (Rees & Mészáros 1997). The nonlinear variations in this scenario are frozen in and reveal themselves when the initial fireball expands and becomes optically thin. This scenario is consistent with GRBs’ observed light curves.

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