Nearly Insulating Strongly Correlated Systems: Gossamer Superconductors and Metals

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Recently a new phenomenological Hamiltonian was introduced to describe the superconducting cuprates in which correlations and on-site Coulomb repulsion are introduced by partial Gutzwiller projection. This Gossamer Hamiltonian has an exact ground state and differs from the t-J and Hubbard Hamiltonians in possessing a powerful attractive interaction among electrons responsible for Cooper pairing in a d-wave channel. It is a faithful description for a superconductor with strong on-site electronic repulsion. The superconducting tunnelling gap remains intact and despite on-site repulsion. Near half-filling the Gossamer superconductor with strong repulsion has suppressed photoemission intensities and superfluid density, is unstable toward an antiferromagnetic insulator and possesses an incipient Mott-Hubbard gap. The Gossamer technique can be applied to metallic ground states thus possibly serving as an apt description of strongly correlated metals. Such a Gossamer metallic phase, just as the Gossamer superconducting one, becomes arbitrarily hard to differentiate from an insulator as one turns the Coulomb correlations up near half-filling. Both the metallic and superconducting states undergo a quantum phase transition to an antiferromagnetic insulator as one increases the on-site Coulomb repulsion. In the Gossamer model we reach the critical point at half-filling by fully projecting double occupancy. Such a critical point might be the Anderson Resonating Valence bond state.

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I. INTRODUCTION

The high temperature superconducting cuprates are unusual in large part because they develop from correlated antiferromagnetic insulators when doped. The antiferromagnetism and insolation are caused by the strong on-site Coulomb repulsion among the Copper d-electrons. Although these electron correlations have been postulated to be the key ingredient to the superconductivity in the cuprates, Laughlin and some of us have recently suggested that correlation effects compete and are detrimental to the superconductivity. A specific model was proposed, the Gossamer superconductor, in which the spectral function will evolve with decreasing doping toward that of an insulator with two Hubbard bands, a Hubbard gap and an ever fainter band of mid-gap states corresponding to the collapsing superfluid density. Despite this behavior the model has an exact d-wave superconducting ground state for all dopings up to the half-filled undoped state. This is achieved through partial Gutzwiller projection of the superconductor.

Underdoped cuprate superconductors have superfluid density and transition temperature that are proportional to each other and vanish linearly with doping. The proportionality is consistent with the transition being an order-parameter phase instability: the superconducting transition temperature, $T_c$, is lower than the pairing temperature because the superconductor is becoming increasingly unstable to loss of phase coherence due to the small superfluid density. We thus expect the measured pseudogap in underdoped cuprates to be caused by pre-existing Cooper pair correlations. This idea is supported by optical sum rule studies, the giant proximity, and the recent heat-transport measurements showing superconducting vortex-like effects above the transition temperature.

Although the motivation for the Gossamer model is the superconducting cuprates for which it provides a very apt description, the behavior should be general to all superconductors with strong on-site repulsion and even to strongly correlated metals. Such strongly correlated superconductors will be spectroscopically identical to doped Mott-insulators close to half-filling, except for a small amount of conducting fluid corresponding to the dephased superconductor. This naturally accommodates experiments that hint at conduction in the supposedly antiferromagnetic insulating phase with carrier density proportional to doping, the existence of a d-wave node deep in the underdoped regime detached from the lower Hubbard band and simply materializing at mid-gap with increasing doping is increased from zero, and a d-wave gap in the quasiparticle spectrum that grows monotonically as the doping decreases and saturates at a value of about 0.3 eV.

The idea that the “insulator” might actually be a tenuous superconductor is implicit in the early ideas of the Anderson on the resonating valence bond (RVB) state and in more recent work. This idea has always run counter to the basic premise of RVB theory that superconductivity should be a universal aspect of quantum antiferromagnetism. The conventional spin density wave ground state, which contains no superconductivity, is a faithful prototype for a quantum
antiferromagnet contradicting this basic premise. Thus not all antiferromagnets are superconductors but some of them are and they constitute a separate class of antiferromagnets: they contain a strong attractive interaction giving rise to superconductivity and a tiny background superfluid density due to strong electron correlations. The Gossamer Hamiltonian stabilizes superconductivity in the latter antiferromagnets over the spin density wave state. This suggests that Coulomb interactions are not sufficient to explain cuprate superconductivity.

Finally, we will discuss the application of the Gossamer technique to correlated metallic systems\textsuperscript{12,13}. The phase diagram of Vanadium sesquioxide, V\textsubscript{2}O\textsubscript{3}, and other Mott insulators is such that the antiferromagnetism and insulator disappear with pressure at $T = 0^\circ$ and/or doping. With increasing temperature, the antiferromagnetic order melts and one is left with a disordered phase. Applying pressure to this insulating-like phase it undergoes a first order phase transition into a metal. The line of first order transitions between the metal and the insulator-like phase terminates at a critical temperature absent localization effects. We propose that this is happening for the disordered insulating-like phase of V\textsubscript{2}O\textsubscript{3}. This suggests that the metal and the insulator-like phase cannot be fundamentally different as one can go continuously from one into the other above the critical temperature.

The Gossamer metal will have properties similar to the Gossamer superconductor. The number of conducting electron states and with it, the density of states at the Fermi level collapses to zero near half-filling when the correlations are strong. The missing spectral weight comes from quasiparticle excitations. Here providing the energy of quasiparticle excitations. The quantum fugacity, $z_0$, in the projector is the extra probability of having an electron at site $j$ after projecting and is necessary in order to keep the total number of particles constant at $(1 - \delta)N$ as one varies $\alpha_0$, where $\delta$ is the doping. The charge states of a site are statistically independent and characterized by a fugacity $\delta$. The condition that the total charge on the site be $1 - \delta$ is

$$\frac{2z + 2\delta^2}{1 + 2z + \delta^2} = 1 - \delta$$

before projecting. After projecting the condition becomes

$$\frac{2zz_0 + 2(1 - \alpha)(zz_0)^2}{1 + 2zz_0 + (1 - \alpha)(zz_0)^2} = 1 - \delta$$

where $1 - \alpha = (1 - \alpha_0)^2$, giving

$$z = \frac{\sqrt{1 - \alpha(1 - \delta^2)} - \delta}{(1 - \alpha)(1 + \delta)} = \frac{1 - \delta}{1 + \delta} z_0 .$$

The parameter $z_0$ is the factor by which $z$ exceeds $(1 - \delta)/(1 + \delta)$, its value for $\alpha_0 = 0$:

$$z_0 = \sqrt{1 - \alpha(1 - \delta^2)} - \delta \frac{(1 - \alpha)(1 + \delta)}{(1 - \alpha)(1 - \delta)} .$$

The Gossamer superconducting ground state is

$$|\Psi\rangle = \Pi_\alpha |\Phi\rangle$$

where $|\Phi\rangle$ is the BCS ground state:

$$|\Phi\rangle = \prod_{\mathbf{k}} (u_{\mathbf{k}} + v_{\mathbf{k}} c_{\mathbf{k} \uparrow}^\dagger c_{\mathbf{k} \downarrow}^\dagger)|0\rangle .$$

where $u_{\mathbf{k}}$, $v_{\mathbf{k}}$ are the BCS coherence factors given by

$$u_{\mathbf{k}} = \sqrt{\frac{E_{\mathbf{k}} + \epsilon_{\mathbf{k}} - \mu}{2E_{\mathbf{k}}}}$$

$$v_{\mathbf{k}} = \sqrt{\frac{E_{\mathbf{k}} - (\epsilon_{\mathbf{k}} - \mu)}{2E_{\mathbf{k}}}}$$

with dispersion

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + \Delta_{\mathbf{k}}^2}$$

providing the energy of quasiparticle excitations. Here

$$\epsilon_{\mathbf{k}} = 2t(\cos(k_x a) + \cos(k_y a))$$

II. GOSSAMER SUPERCONDUCTOR

The Gossamer superconductor is defined as a superconducting ground state which contains Coulomb correlations introduced by a partial Gutzwiller projection which suppresses the probability of having two electrons on the same site:

$$\Pi_\alpha = \prod_j z_0^{(n_j^\uparrow + n_j^\downarrow)/2} (1 - \alpha_0 n_j^\uparrow n_j^\downarrow) .$$

$0 \leq \alpha_0 < 1$ is a measure of how effective the “projector” hinders double occupancy and in a real material it will
is the kinetic energy for a square lattice with spacing \( a \), \( \mu \) is the chemical potential and

\[
\Delta_k = \Delta_0 (\cos(k_x a) - \cos(k_y a))
\]  

is the d-wave superconducting gap as measured for the superconducting cuprates. In superconductors the coherence factors \( u_k \) and \( v_k \) are normalized, \( u_k^2 + v_k^2 = 1 \), and related to the number of carriers in order to set the value of the chemical potential. For doped cuprates with doping \( \delta \), we have

\[
\frac{1}{N} \sum_k u_k^2 = 1 - \frac{1}{N} \sum_k u_k^2 = \frac{1 - \delta}{2}.
\]

(12)

For the Gossamer ground state\(^6\) we stay away from full projection \((\alpha_0 < 1 \text{ always})\) in order for the partial projector to have an inverse:

\[
\Pi^{-1}_\alpha = \prod_j z_0^{-(n_{j\uparrow} + n_{j\downarrow})/2}(1 + \beta_0 n_{j\uparrow} n_{j\downarrow})
\]

with \( \beta_0 = \alpha_0/(1 - \alpha_0) \). This invertibility enables us to define the Hamiltonian:

\[
\mathcal{H} = \sum_{k\sigma} E_k B_{k\sigma}^\dagger B_{k\sigma}, \quad B_{k\sigma} |\Psi\rangle = 0.
\]

(14)

where

\[
B_{k\uparrow\downarrow} = \Pi_\alpha b_{k\uparrow\downarrow} \Pi^{-1}_\alpha = \frac{1}{\sqrt{N}} \sum_j e^{i k \cdot r_j} u_k \{ 1 - \beta_0 n_{j\downarrow} n_{j\downarrow}(1 - \alpha_0 n_{j\uparrow} n_{j\downarrow}) \} c_{j\downarrow}^\dagger
\]

\[
\times z_0^{1/2} u_k (1 + \beta_0 n_{j\downarrow} n_{j\downarrow}) c_{j\uparrow\downarrow} + z_0^{1/2} v_k (1 - \alpha_0 n_{j\uparrow} n_{j\downarrow}) c_{j\downarrow\uparrow}^\dagger
\]

(15)

with

\[
b_{k\uparrow\downarrow} = u_k c_{j\uparrow\downarrow} \pm v_k c_{j\downarrow\uparrow}^\dagger
\]

(16)

the Bogoliubov quasiparticle operators.

The Gossamer ground state\(^6\) is an exact lowest energy eigenstate of the Gossamer Hamiltonian\(^6\) by virtue of its zero eigenvalue and the positivity of the Hamiltonian:

\[
\langle \chi | \mathcal{H} | \chi \rangle = \sum_{k\sigma} E_k |B_{k\sigma} \chi |B_{k\sigma} \chi \rangle \geq 0
\]

(17)

for any wavefunction \( |\chi\rangle \). The Gossamer ground state is adiabatically continuous to the BCS ground state by continuously decreasing \( \alpha_0 \) to zero. Since it does not cross a phase boundary in the process, its uniqueness follows from the uniqueness of the BCS ground state up to a phase. Therefore the Gossamer superconductor describes the same phase of matter as the BCS superconductor. The Gossamer superconductor ground state and its low energy excitations map to the the ground state and low-lying excitations of a BCS superconductor in a one to one manner.

### III. QUASIPARTICLE DISPERSION AND SPECTROSCOPY OF THE GOSSAMER SUPERCONDUCTOR

Let us now consider the quasiparticle excitations of this superconductor. Since the operators \( B_{k\uparrow\downarrow} \) no longer have fermionic anticommutation relations with their hermitian adjoints, they cannot be used to create eigenstates of the Hamiltonian. The physical meaning of this is that the quasiparticles interact. We will approximate the quasiparticle-like eigenstates with the variational wavefunctions

\[
|\tilde{k}\sigma\rangle = \Pi_\alpha b_{k\sigma}^\dagger |\Phi\rangle.
\]

(18)

The quasiparticle energy is approximated by the expected value

\[
\frac{\langle \tilde{k}\sigma | \mathcal{H} | \tilde{k}\sigma \rangle}{\langle \tilde{k}\sigma | \tilde{k}\sigma \rangle} = E_k \frac{\langle \Psi | \mathcal{H} | \Psi \rangle}{\langle \Psi | \tilde{k}\sigma \rangle \langle \tilde{k}\sigma | \Psi \rangle} \approx E_k.
\]

(19)

This is an almost miraculous result, the dispersion is unchanged from the dispersion of the unprojected superconductor. This follows by evaluating the relevant norms, which can be done by hand only approximately\(^22\,^27\) as we reproduce here. From

\[
b_{k\uparrow\downarrow}^\dagger |\Phi\rangle = \frac{1}{u_k} c_{k\uparrow\downarrow}^\dagger |\Phi\rangle = \frac{1}{v_k} c_{-k\downarrow\uparrow} |\Phi\rangle
\]

(20)

we get

\[
\frac{1}{N} \sum_k u_k^2 \frac{\langle \Psi | B_{k\uparrow\downarrow} \Pi^2 b_{k\uparrow\downarrow}^\dagger | \Phi \rangle}{\langle \Psi | \Pi_\alpha^2 | \Phi \rangle} = \frac{\langle \Phi | c_{j\uparrow\downarrow} \Pi_\alpha^2 c_{j\uparrow\downarrow}^\dagger | \Phi \rangle}{\langle \Phi | \Pi_\alpha^2 | \Phi \rangle} = z_0 \frac{1 + (1 - \alpha) z_0}{1 + 2 z_0 (1 - \alpha) (z_0)^2} = 1 + \frac{\delta}{2}
\]

(21)

and

\[
\frac{1}{N} \sum_k v_k^2 \frac{\langle \Psi | b_{k\uparrow\downarrow} \Pi^2 b_{k\uparrow\downarrow}^\dagger | \Phi \rangle}{\langle \Psi | \Pi_\alpha^2 | \Phi \rangle} = \frac{\langle \Phi | c_{j\downarrow\uparrow} \Pi_\alpha^2 c_{j\downarrow\uparrow}^\dagger | \Phi \rangle}{\langle \Phi | \Pi_\alpha^2 | \Phi \rangle}
\]
inverse photoemission to be

$$
\frac{\langle \Psi | \Psi \rangle}{\langle k\sigma | k\sigma \rangle} \sim 1
$$

We now consider the low-energy spectroscopic properties of the Gossamer superconductor. We will repeatedly use the matrix elements for \( j \neq j' \). In order to evaluate them, we assume that the amplitude for a given configuration is weighted by the square root of its corresponding probability, and that these weights add equally. We get

$$
\frac{\langle \Phi | c_{j\uparrow} \Pi_{\alpha}^2 c_{j'\downarrow} | \Phi \rangle}{\langle \Phi | \Pi_{\alpha}^2 | \Phi \rangle} \times \frac{\langle \Phi | \Phi \rangle}{\langle \Phi | c_{j\uparrow} \Pi_{\alpha}^2 c_{j'\downarrow} | \Phi \rangle}
$$

and

$$
\frac{\langle \Phi | c_{j\uparrow} \Pi_{\alpha}^2 c_{j'\downarrow} | \Phi \rangle}{\langle \Phi | c_{j\uparrow} \Pi_{\alpha}^2 c_{j'\downarrow} | \Phi \rangle} \times \frac{\langle \Phi | \Phi \rangle}{\langle \Phi | \Phi \rangle}
$$

Thus under strong projection near half-filling this model exhibits the pseudogap phenomenon: The quasiparticle dispersion remains unchanged from its unperturbed value \( E_{\tilde{c}} \) as \( \alpha_0 \) increases from 0 to 1, but the superfluid density decreases from 1 to \( 2|\delta|/(1 + |\delta|) \). The strong projector collapses the superfluid density with doping and introduces correlations intrinsic to an antiferromagnetic insulator as Hubbard band-lik lobes grow as we shall show next.

### IV. MAGNETIC CORRELATIONS OF THE GOSSAMER SUPERCONDUCTOR

In the present and the following section we consider how the Gossamer superconductor under strong projection and near half-filling develops behavior that is practically impossible to differentiate from that of a correlated insulator. We now study the Gossamer magnetic behavior.

In order to determine the electronic correlations arising in the Gossamer Hamiltonian Eq. (27), we expand it and analyze its terms:

\[
\mathcal{H} = \sum_{k} E_{\tilde{c}} B_{k\sigma} B_{k\sigma} = A + B + C
\]

where \( A, B, C \) are, explicitly:

\[
A = \sum_{\tilde{k}} \sum_{\sigma} e^{-ik(\vec{r}_{i} - \vec{r}_{j})} \left( z_0^{-1} u_k^2 (1 + \beta_0 n_{i\downarrow})(1 + \beta_0 n_{j\downarrow}) c_{j\uparrow}^\dagger c_{j\downarrow} + \right.
\]

\[
\left. z_0 v_k^2 (1 - \alpha_0 n_{i\uparrow})(1 - \alpha_0 n_{j\uparrow}) c_{j\uparrow}^\dagger c_{j\uparrow} \right\} \{\uparrow \downarrow \uparrow \downarrow \}
\]

\[
B = \sum_{\tilde{k}} \sum_{\sigma} e^{-ik(\vec{r}_{i} - \vec{r}_{j})} u_k v_k \left( (1 + \beta_0 n_{i\downarrow})(1 - \alpha_0 n_{j\uparrow}) c_{j\uparrow}^\dagger c_{j\downarrow} + \right.
\]

\[
\left. (1 + \beta_0 n_{j\downarrow})(1 - \alpha_0 n_{i\downarrow}) c_{i\downarrow} c_{i\uparrow} \right\} \{\uparrow \downarrow \uparrow \downarrow \}
\]

\[
C = \sum_{\tilde{k}} \sum_{\sigma} \sum_{\sigma} E_{\tilde{c}} u_k v_k \left\{ \alpha_0 (1 + \beta_0 n_{j\downarrow}) c_{j\uparrow}^\dagger c_{j\downarrow} + \right.
\]

\[
\left. \beta_0 (1 - \alpha_0 n_{j\uparrow}) c_{j\downarrow} c_{j\uparrow} \right\} \{\uparrow \downarrow \uparrow \downarrow \}
\]

The term \( C \) vanishes the identity

\[
E_{\tilde{c}} u_k v_k = \frac{\Delta_k}{2}
\]
and the relation:
\[
\sum_{\vec{k}} e^{-i\vec{k}(\vec{r}_i-\vec{r}_j)} \Delta_{\vec{k}} = \begin{cases} 
0 & \vec{r}_i \neq \vec{r}_j + \vec{a} \\
\Delta_0 & \vec{a} \parallel \vec{x} \\
-\Delta_0 & \vec{a} \parallel \vec{y}
\end{cases} \quad (34)
\]
(true for a d-wave gap) where \(\vec{a}\) is vector pointing toward a nearest neighbor in the lattice. This term would be nonzero for an s-wave superconductor and it might be interesting to study what happens in such a case. The \(A\) term is responsible for the chemical potential, kinetic energy, as well as a Hubbard U terms. \(B\) is responsible for the super-conducting part of the Gossamer Hamiltonian. The d-wave form of the gap makes \(B\) have no on-site contributions, but it may be of interest to look at an s-wave superconductor. \(A\) contains on-site and off-site contributions:
\[
A = A_{\text{on site}} + A_{\text{off site}}
\]
\[
A_{\text{on site}} = \sum_{\vec{k}} \frac{E_{\vec{k}}}{N} \sum_j^N \left\{ z_0^{-1} v_k^2 (1 + \beta_0 n_{j\downarrow}) c_{j\uparrow}^\dagger c_{j\uparrow} + 1 \right\}
\]
\[
+ z_0 v_k^2 (1 - \alpha_0 n_{j\downarrow})^2 c_{j\uparrow}^\dagger c_{j\uparrow} \right\}
\]
\[
A_{\text{off site}} = \sum_{\vec{k}} \frac{E_{\vec{k}}}{N} \sum_{i \neq j} e^{-i\vec{k}(\vec{r}_i-\vec{r}_j)}
\]
\[
\times \left\{ z_0^{-1} v_k^2 (1 + \beta_0 n_{i\downarrow})(1 + \beta_0 n_{j\downarrow}) c_{j\uparrow}^\dagger c_{j\uparrow} + 1 \right\}
\]
\[
+ z_0 v_k^2 (1 - \alpha_0 n_{i\downarrow})(1 - \alpha_0 n_{j\downarrow}) c_{i\downarrow}^\dagger c_{i\downarrow} \right\}
\]
\[
(35)
\]
Thus the Gossamer Hamiltonian has a Hubbard \(U\) term which arises from the “projector”...

At almost full projection \(\alpha_0 \to 1^-\), \(U\) grow arbitrarily large.

The off-site contributions of \(A\) provide the hopping (kinetic energy) term in the Hamiltonian. Partial projection, particularizing to zero doping and imposing the mean field values \(\langle n_{i\uparrow} \rangle = \langle n_{i\downarrow} \rangle = 1/2\), the off-site contribution becomes:
\[
A_{\text{off site}} = \frac{1}{4} \frac{(2 - \alpha_0)^2}{1 - \alpha_0} \sum_{i \neq j}^N (\epsilon_k - \mu) c_{i\sigma}^\dagger c_{j\sigma}
\]
\[
(40)
\]
At half filling and imposing the mean field condition \(\langle n_{i\uparrow} \rangle = \langle n_{i\downarrow} \rangle = 1/2\), and keeping in mind that \(E_{\vec{k}}\) and \(\Delta_{\vec{k}}\) are even in \(\vec{k}\) we thus obtain
\[
B = \frac{1}{4} \frac{(2 - \alpha_0)^2}{1 - \alpha_0} \sum_{i \neq j} \Delta_{\vec{k}} (c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + c_{i\downarrow} c_{j\uparrow})
\]
\[
(42)
\]
The superconducting gap survives upon projection, but is renormalized by the same constant as the kinetic energy. Upon strong projection, the physically relevant ratio, \(U/\Delta_0\), is a number of order unity or greater which provides the right physics for antiferromagnetism and insulation.

The superconducting part of the Gossamer Hamiltonian, \(B\) when unprojected, just the Cooper pairing attraction term
\[
\sum_{\vec{k}} \Delta_{\vec{k}} (c_{\downarrow\uparrow}^\dagger c_{\downarrow\downarrow} + c_{\uparrow\downarrow} c_{\uparrow\uparrow})
\]
\[
(41)
\]
Hamiltonian with a d-wave pairing interaction added to it. We now define the “spinors”

$$\Psi_k \equiv \begin{bmatrix} c_{k\uparrow} \\ c_{k\downarrow}^\dagger \end{bmatrix},$$  \hspace{1cm} (43)

so that the “noninteracting” part of the Gossamer Hamiltonian, i.e. the part with the $U$ term disregarded, can be written

$$\mathcal{H} = \frac{1}{4} \frac{(2 - \alpha_0)^2}{1 - \alpha_0} \begin{bmatrix} \epsilon_k & \Delta_k \\ -\Delta_k & -\epsilon_k \end{bmatrix}$$  \hspace{1cm} (44)

where $\mu$ has been omitted because we are at half-filling. The bare Green function, $G_k(E) = 1/(E - \mathcal{H})$, is then given by

$$G_k(E) = \frac{1}{E^2 - \gamma^2(\epsilon_k^2 + \Delta_k^2)} \begin{bmatrix} E + \gamma \epsilon_k & -\gamma \Delta_k \\ -\gamma \Delta_k & E - \gamma \epsilon_k \end{bmatrix}$$  \hspace{1cm} (45)

with $\gamma \equiv (2 - \alpha_0)^2/4(1 - \alpha_0)$.

In order to show the magnetic ordering properties of the Gossamer Hamiltonian at half-filling, we will compute the magnetic susceptibility and tune it through the transition. The bare susceptibility is given by

$$\chi_q^0(\omega) = \frac{1}{(2\pi)^3} \int \int \text{Tr}[G_k(E)G_{k+q}(E+\omega)] \, dEdk.$$  \hspace{1cm} (46)

We calculate the effects of $U$ by the the ladder approximation for the the spin susceptibility $\chi_q(\omega) = \chi_q^0(\omega)/[1 + U\chi_q^0(\omega)]$. The numerical evaluation of the spin susceptibility is shown in the figure. We see that beyond at critical value for $U$ of order of $t$ and or$\Delta_0$, the system goes to the critical point becoming infinitely susceptible to going over into an antiferromagnetic insulator as signaled by the diverging susceptibility at the critical value. The ladder technique cannot provide the correct critical value of $U/t$, nor the correct critical exponents, but it will, however, provide a faithful qualitative picture of the transition, and of the divergence of the spin susceptibility at the critical point for the development of antiferromagnetic order.

V. MOTT-HUBBARD INSULATING-LIKE BEHAVIOR OF THE GOSSAMER SUPERCONDUCTOR

Consistent with the approach to antiferromagnetic order at half-filling, we will see the formation of the Mott-Hubbard gap. Quasiparticle photoemission can only account for a small part of the sum rule

$$\frac{\langle \Psi | c_{k\sigma}^\dagger c_{\bar{k}\sigma} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \approx g^2 \frac{\gamma^2}{\nu_k^2} + \frac{1 - g^2}{2}.$$  \hspace{1cm} (47)

The rest must occur at a higher energy scale, the value of which may be estimated by computing the expected energy of a hole. Using the anticommutators

$$\{B_{k\sigma}, c_{j\uparrow}\} = \frac{1}{\sqrt{N}} e^{i\vec{k} \cdot \vec{r}_j} \left[ -z_0^{-1/2} \alpha_0 v_k c_{j\uparrow} c_{k\downarrow}^\dagger \right]$$  \hspace{1cm} (48)

$$\{B_{k\sigma}, c_{j\downarrow}\} = \frac{1}{\sqrt{N}} e^{i\vec{k} \cdot \vec{r}_j} \left[ -z_0^{-1/2} \beta_0 u_k c_{j\downarrow} c_{k\uparrow}^\dagger \right] + z_0^{1/2} v_k (1 - \alpha_0 n_{j\downarrow})$$  \hspace{1cm} (49)

it follows that

$$\langle \Psi | c_{k\sigma}^\dagger \mathcal{H} c_{\bar{k}\sigma} | \Psi \rangle = z_0 \left[ -\alpha_0 \frac{1}{2} \right] \nu_k^2 E_k$$

$$+ \frac{1}{N} \sum_{\bar{q}} \gamma \Delta_k^{1/2} E_{k+\bar{q}}^2 [z_0 \alpha_0^2 A_q^2 \nu_k^2 + \frac{\beta_0^2}{\alpha_0} B_q^2 \nu_k^2]$$  \hspace{1cm} (50)

with

$$A_q = \frac{1}{N} \sum_j \langle \Psi | \hat{S}_0 \cdot \hat{S}_j | \Psi \rangle$$

$$+ \frac{1}{4} \frac{\langle \Psi | n_0 n_j | \Psi \rangle}{\langle \Psi | \Psi \rangle} \left( 1 - \delta \right)^2 \frac{e^{i\vec{q} \cdot \vec{r}_j}}{4}$$  \hspace{1cm} (51)
If we go through similar gymnastics we determine that the energy to add an electron is the exact particle-hole conjugate of this expression, produced from it by interchanging $v_k$ and $u_k$, negating $\delta$, and substituting of $1 - n_j$ for $n_j$. Therefore the electron spectral function is symmetric at half-filling. We see that, in the Gossamer superconductor, the spectral function consists of Mott-Hubbard “lobes” at high energies with a faint band of states at mid-gap associated with the Gossamer quasiparticles. The chemical potential stays pinned at midgap with doping.

We have seen that, under strong projection ($\alpha_0 \to 1$), the Gossamer superconductor has a superfluid density that collapses with doping and projection. This collapsing superfluid density leads to a temperature order parameter phase instability consistent with the transition out of the superconducting state in underdoped cuprates. Even without the development of antiferromagnetism, such a superconductor would be insulating since it would dephase due to the small superfluid density perhaps thorough lattice-mediated crystallization of the gossamer superconductor which would also provide a ready explanation for why the static stripes occur at $\delta = 1/8$ in some cuprates and not others, why stripes are destroyed by moderate pressure, and why the materials insulate when subjected to strong magnetic fields.

VI. METAL

The Gossamer technique should be applied with more care to a metallic Fermi sea ground state than to a superconducting BCS ground state. The metal has the abundance of low energy degrees of freedom which might make the projection uncontrolled in the infrared leading to unphysical results. On the other hand, the superconductor does not have such a plethora of low energy degrees of freedom and the calculation is assured to have no infrared problems: it is regularized by the superconducting order. After obtaining the results, we collapse the gap to zero to study the physics of Gossamer metals.

Since the Gossamer superconducting ground state is adiabatically continuable to the BCS ground state by continuously varying $\alpha_0$ to zero, in a similar fashion once we collapse the gap to obtain the Gossamer metal ground state, it will be adiabatically deformable to a regular Fermi sea metallic ground state and, hence they will be the same zero temperature phase of matter.

In order to study quasiparticle dispersion in the Gossamer metal, we remember that, for the superconductor, the wavefunction

\[ |\vec{k}\sigma\rangle = \Pi_{\alpha} \delta_{\vec{k}\sigma} |\Phi\rangle \]  \hspace{1cm} (53)

represents an appropriate approximation to the low energy quasiparticle excitations with dispersion

\[ \frac{\langle \vec{k}\sigma | H | \vec{k}\sigma \rangle}{\langle \vec{k}\sigma | \vec{k}\sigma \rangle} = E_{\vec{k}} \frac{\langle \Psi | \Psi \rangle}{\langle \vec{k}\sigma | k\sigma \rangle} \simeq E_k \]  \hspace{1cm} (54)

that, after collapsing the superconducting gap, is unchanged from the dispersion from that in the regular metal. This is contrary to usual thinking in which correlation effects are believed to make the charge carriers heavy. In the Gossamer metal, the carriers are not getting arbitrarily heavy as we approach the transition, but the metallic band is becoming thinner and the missing spectral weight goes to energies far from the Fermi sea to forming Hubbard bands. The carriers are just as fast, there are just less of them which degrades the conductivity.

The photoemission amplitudes were calculated for the Gossamer superconductor. Collapsing the gap, we obtain them for the Gossamer metal:

\[ \frac{\langle -\vec{k} \downarrow | c_{\vec{k}\uparrow} | \Psi \rangle}{\sqrt{\langle -\vec{k} \downarrow | -\vec{k} \downarrow \rangle \langle \Psi | \Psi \rangle}} = 0 \] ,

\[ \frac{\langle -\vec{k} \downarrow | c_{\vec{k}\uparrow} | \Psi \rangle}{\sqrt{\langle -\vec{k} \downarrow | -\vec{k} \downarrow \rangle \langle \Psi | \Psi \rangle}} = g \] ,  \hspace{1cm} (55)

with

\[ g^2 \simeq \frac{2\alpha_0}{\alpha} \left\{ 1 - \frac{\alpha_0}{\alpha} \left[ 1 - \sqrt{1 - \alpha(1 - \delta^2)} \right] \right\} \] \hspace{1cm} (56)

The suppression of photoemission amplitude is caused by the smaller number of metallic electrons whose number is diminished from that in the unprojected metal by a factor $g^2$ which goes to $2|\delta|/(1 + |\delta|)$ as $\alpha_0 \to 1$. This is consistent with the superfluid density suppression in the Gossamer superconductor which goes like $g^2$ as there is a sum rule for the superconductor making the density of conducting electrons equal to the superfluid density.

The existence of the growing Hubbard U term means that as we go to half-filling and full projection magnetic correlations will get enhanced leading to a diverging magnetic susceptibility in the exact same way as for the Gossamer superconductor after we collapse the gap. The spectral weight will consist of Mott-Hubbard bands at high energies with the chemical potential pinned at midgap at an ever fainter band from which the Gossamer quasiparticles are excited with a dispersion unchanged from the noninteracting metal.
VII. DISCUSSION

We have reviewed the ideas of the Gossamer technique for superconductors and the recent extensions to metals. The technique has the advantage of introducing strong correlations, but yet prohibits the ground state for undergoing a transition to the insulating state. The best we can do is get to the critical point at half-filling by projecting fully.

For the superconductor we saw that the superfluid density and photoemission amplitude becomes suppressed at half-filling and the magnetic correlations become enhanced as we tune the correlations up. Thus the superconducting state with strong on-site repulsion is unstable toward insulation and antiferromagnetism close to half-filling, being arbitrarily close to a continuous zero temperature phase transition into an antiferromagnetic insulator. The critical point is at half-filling when fully projected and it is the Anderson RVB ground state.

For the Gossamer superconductor the instability is exactly at half-filling while for a different Hamiltonian the instability can occur at nonzero doping. Since the Gossamer superconductor is adiabatically continuable to a completely regular BCS superconductor our correlation effects are generic to the superconducting state.

The proximity to the antiferromagnetic transition found here under strong projection will make the spectroscopic properties of the material be very much like those of an antiferromagnetic insulator near half-filling. The superfluid density will be so low that it would be almost impossible to tell that the system is not an insulator except at extremely long wavelengths or low energy scales. An antiferromagnet with a small interpenetrating density of dephased superfluid provides a possible explanation for the recent measurements of metallic transport below the Néel temperature in underdoped LSCO. That the charge mobility in these measurements is equal to that in the optimally doped material suggests a common origin, possibly the dephased Gossamer superconductor. Moreover, adding by hand an extra Hubbard term, an insulating static stripe phase would be stabilized with a possible coexistence of dephased superfluid. Coupling of the coexisting dephased superfluid to the stripe phase would lead to anisotropic Copper-Oxygen plane charge transport.

Phase fluctuations have not been included in the Gossamer superconducting Hamiltonian because they are irrelevant to the fermi spectrum, which is characterized by an energy scale much higher than the superconducting Tc. However, they are essential for accounting for both the Uemura plot and strange-metal transport above Tc. The transport of a dephased fermion fluid with Cooper pair correlations formed when the order parameter dephases would not exhibit any traditional metallic behavior.

We also review the application of the Gossamer technique to metals. The Gossamer metal describes strongly correlated bad metal behavior that is very hard to distinguish from true insulating behavior, the implication being that the magnetically disordered insulating-like phases in some systems might really be a Gossamer metal with very much degraded conductivity. The degraded conductivity arises from a depletion of spectral weight of metallic electrons and not by an ever growing effective mass of the carriers.

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