XY Z: the case of $Z_c(3885)/Z_c(3900)$

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Abstract. The resonant structure $Z_c(3885)/Z_c(3900)$ is studied in a unitary, $J/\psi\pi-D^*\bar{D}$ coupled channel $T$-matrix formalism. The two experimental spectra ($D^*\bar{D}$, $J/\psi\pi$) in which this state is seen are reproduced in two different scenarios. In both scenarios the two observations, $Z^+_c(3900)$ and $Z^+_c(3885)$, are shown to have the same common origin. In the first one, the $Z_c$ originates from a resonance with a mass close to and above the $D\bar{D}$ threshold. In the second one, the $Z_c$ peak stems from a virtual state. Precise measurements of the line shapes around the $D\bar{D}^*$ threshold are called for in order to understand the nature of this state.

1. Introduction

In this talk we report on our recent work [1] about the nature of the $Z_c(3900)/Z_c(3885)$ state(s). The resonant-like structure $Z_c(3900)^\pm$ was first seen simultaneously by the BESIII and Belle collaborations [2, 3] in the $J/\psi\pi$ spectrum produced in the $e^+e^-\rightarrow Y(4260)\rightarrow J/\psi\pi\pi^-$ reaction. It was confirmed in an analysis [4] based on CLEO-c data. A similar structure, $Z_c(3885)^\pm$, with quantum numbers favored to be $J^P = 1^+$, has also been reported by the BESIII collaboration [5, 6] in the $D^*\bar{D}$ spectrum of $e^+e^-\rightarrow D^*\bar{D}\pi$. It is generally accepted that the two structures have indeed the same common origin. We generically denote it here as $Z_c$.

The state $Z_c(3900)$ is an exotic one, since it couples strongly to charmonium and yet it is charged, thus its minimal constituent quark content should be four quarks, $c\bar{c}ud$ (for $Z_c^+$). Many different interpretations of its nature have been given, most of which can be traced back from references in Ref. [1]. It has also been studied in Lattice QCD (see Ref. [7] for references). A simultaneous study of the two reactions in which the $Z_c$ structure has been seen was performed in Ref. [1].1 From this analysis, information about this seemingly resonant intriguing structure is extracted. In Sec. 2 a $D^*\bar{D}$, $J/\psi\pi$ coupled channel formalism is settled. The resulting $T$-matrix is used in the calculation of the amplitudes for the reactions $Y(4260)\rightarrow J/\psi\pi\pi, D^*\bar{D}\pi$, assuming that the $Y(4260)$ state is dominantly a $D_1(2420)\bar{D}$ + c.c. bound state [9, 10]. The results and conclusions are presented in Sec. 3.

1 In Ref. [8], both reactions were considered and used to fix parameters at the one-loop level. In that work, it is shown that the narrow near threshold states like the $Z_c$ cannot be simply kinematical effects.
2. Formalism

Let us denote with 1 and 2 the $J/\psi\pi$ and $\bar{D}^*D$ channels, respectively, with $I = 1$ and $J^{PC} = 1^{+-}$ (in what follows, the $C$-parity denotes that of the neutral member of the isospin triplet). The coupled-channel $T$-matrix is written as:

$$T = (1 - V \cdot G)^{-1} \cdot V,$$

where $G$ is a loop function diagonal matrix, and the matrix elements of the potential $V$ read:

$$V_{ij} = 4\sqrt{m_1m_2}\sqrt{m_1m_2}e^{-\frac{q_i^2}{\Lambda_2}}e^{-\frac{q_j^2}{\Lambda_2}}C_{ij}.$$  \hspace{1cm} (2)

The c.m. momentum squared of the channel $i$ is denoted by $q_i^2$. The details of Eq. (2) can be found in Ref. [1].

The $J/\psi\pi \to J/\psi\pi$ can be neglected [11, 12], and we take $C_{11} = 0$. For the $\bar{D}^*D \to J/\psi\pi$ S-wave transition, we make the simplest possible assumption, taking it as a constant, $C_{12} \equiv \tilde{C}$. In a momentum expansion, the lowest order contact potential for the $\bar{D}^*D \to \bar{D}^*D$ transition is simply a constant as well, denoted by $C_{22} \equiv C_{1Z}$ [13]. However, even with two coupled channels, no resonance above the highest threshold can be generated in the complex plane with only constant potentials. To allow some energy dependence for the $V_{22}$ term, we introduce a new parameter $b$, and write:

$$C_{22}(E) = C_{1Z} + b(\bar{E} - m_D - m_{D^*}).$$  \hspace{1cm} (3)

To regularize the interactions we employ a standard gaussian regulator, $e^{-\frac{q_i^2}{\Lambda_2}}$. \hspace{1cm} (2) We take cutoff values $\Lambda_2 = 0.5 - 1$ GeV [13]. At the $Z_c$ energy, the c.m. momentum of the $J/\psi\pi$ channel is $q_1 \approx 0.7$ GeV, and hence we use $\Lambda_1 = 1.5$ GeV. The loop functions in the matrix $G$ can be found in Ref. [1].

For the $e^+e^-$ annihilations at the $Y(4260)$ mass, both BESIII and Belle have seen the $Z_c$ structure in the $J/\psi\pi$ final state [2, 3], but only BESIII reports data for the $\bar{D}^*D$ channel [5, 6]. Hence, for consistency, we will only study the BESIII data. In particular, we will consider the most recent double-$D$-tag data of Ref. [6]. For definiteness, we will consider the reported spectra of the $D^{*-}D^0$ and $J/\psi\pi^-$ final states.

We denote with $\mathcal{M}_1$ ($\mathcal{M}_2$) the amplitude for the $Y \to J/\psi\pi^+\pi^-$ ($Y \to D^{*-}D^0\pi^+$) decay, and we respectively call $s$ and $t$ to the invariant masses squared of $J/\psi\pi^-$ and $J/\psi\pi^+$ ($D^{*-}D^0$ and $D^{*-}\pi^+$) in the first (second) decay. Up to some common irrelevant constant, both amplitudes can be written as:

$$|\mathcal{M}_1(s,t)|^2 = |\tau(s)|^2 q_\pi^4(s) + |\tau(t)|^2 q_\pi^4(t) + \frac{3\cos^2\theta - 1}{4}(\tau(s)\tau(t)^* + \tau(s)^*\tau(t))q_\pi^2(s)q_\pi^2(t),$$  \hspace{1cm} (4)

$$\tau(s) = \sqrt{2}I_3(s)T_{12}(s) + \alpha,$$  \hspace{1cm} (5)

$$|\mathcal{M}_2(s,t)|^2 = \left|\frac{1}{t - m_{D_1}^2} + I_3(s)T_{22}(s)\right|^2 q_\pi^2(s) + |\beta(1 + T_{22}(s)G_2(s))|^2,$$  \hspace{1cm} (6)

where $q_\pi^2(s) = \lambda(m_{Y}^2, s, m_{D}^2)/(4M_\pi^2)$, and $\theta$ denotes the relative angle between the two pions in the $Y(4260)$ rest frame. Further, $I_3(s)$ is the scalar three-meson non-relativistic loop function, and $m_{D_1}$ is the mass of the $D_1$ meson (details can be found in Ref. [1]). The parameters $\alpha$ and $\beta$ in Eqs. (5) and (6) are unknown. Details on the different contributions in Eqs. (4)–(6) can be found in Ref. [1].

\hspace{0.5cm} 2 In principle, one expects the predictions of physical (observables) quantities to be independent of the chosen regularization procedure. In a context similar to the one presented in this work, this has been studied in Ref. [14].
Besides a contribution from the amplitudes \( (\mathcal{A}_i) \), the spectrum for both reactions have some background \( (\mathcal{B}_i) \), and thus we write them as:

\[
\mathcal{N}_i(s) = K_i (\mathcal{A}_i(s) + \mathcal{B}_i(s)) ,
\]

\[
\mathcal{A}_i(s) = \int_{t_i-}^{t_i+} dt |\mathcal{M}_i(s,t)|^2 ,
\]

where \( t_{i,\pm}(s) \) are the limits of the \( t \) Mandelstam variables for the decay mode \( i \). To obtain a reasonable estimate of the coupling \( \tilde{C} \) and the two global constants \( K_i \), we consider a further experimental input from Ref. [5],

\[
R_{\text{exp}} = \frac{\Gamma (Z_c(3885) \rightarrow DD^*)}{\Gamma (Z_c(3900) \rightarrow J/\psi\pi^0)} = 6.2 \pm 1.1 \pm 2.7 ,
\]

and estimate this ratio as

\[
R_{\text{th}} = \frac{\int ds \mathcal{A}_2(s)}{\int ds \mathcal{A}_1(s)} ,
\]

that is, as the ratio of the background subtracted areas of each physical spectrum around the \( Z_c \) mass, namely in the range \( \sqrt{s} = (3900 \pm 35) \) MeV. An inspection of Fig. 1 shows that the tail of the \( DD^* \) spectrum is small, and we set \( B_2 = 0 \). For the \( J/\psi\pi \) spectrum, \( B_1 \) is parameterized with a symmetric smooth threshold function as used in the experimental work of Ref. [2]:

\[
B_1(s) = B_1 \left[ (\sqrt{s} - m_{1-})(m_+ - \sqrt{s}) \right]^{d_1} ,
\]

with \( m_{1-} = m_{J/\psi} + m_\pi \) and \( m_+ = m_Y - m_\pi \). The parameters \( B_1 \) and \( d_1 \) are free.

3. Results

We have three free parameters directly related to our \( T \)-matrix \( (C_{1Z}, \tilde{C}, \text{and } b) \), and six \( (B_1, d_1, \alpha, \beta \text{ and } K_{1,2}) \) related to the background and the overall normalization. These nine free parameters are adjusted to reproduce the data of Refs. [2,6] (a total of 104 data points).

We perform four different fits, corresponding to the two cases of keeping the parameter \( b \) that carries the energy dependence of the \( DD^* \) potential, free or set to zero, and for each of these, we choose \( \Lambda_2 \) to be 0.5 or 1 GeV [13]. The specific values of the parameters in each of these fits can be found in Ref. [1]. The description of the experimental spectra is very good in all cases, as can be seen in the top panels of Fig. 1, where the results from one of the fits (\( b \) free and \( \Lambda_2 = 0.5 \) GeV) are shown together with the data. The other fits lead to results similar to those shown in Fig. 1. In any case, we see that we are able to simultaneously reproduce the two available BESIII data sets related to the \( Z_c^\pm(3900)/Z_c^\pm(3885) \) state with a single \( DD^* \) structure for the very first time.

We next study the pole structure of the \( T \)-matrix. Poles can be found in different Riemann sheets of the \( T \)-matrix, which are reached through analytical continuation of the loop functions. The specific definitions of the Riemann sheets can be found in Ref. [1]. We define the mass and the width of the \( Z_c \) from its pole position, \( \sqrt{s} = M_{Z_c} - i\Gamma_{Z_c}/2 \). For the case \( b \neq 0 \), we find poles on the (11) Riemann sheet, which is connected to the physical one above the \( DD^* \) threshold. In Fig. 2, we compare the pole position obtained in this work for the \( Z_c \) resonance with the experimental determinations of Refs. [2–6]. The specific values for the poles positions can be seen in Ref. [1]. As seen in Fig. 2, good agreement within errors is found. The real part of these energies is clearly above threshold, so they correspond to a resonance, which really (physically) exists as an unstable particle.
For the case $b = 0$, however, the situation is quite different. While the description of the experimental data is still quite good, the pole in this case is located below threshold, with a small imaginary part (around 8 MeV), and in the (01) Riemann sheet. If the $J/\psi\pi^-$ channel were now switched off ($\tilde{C} = 0$), this pole would move into the real axis in the unphysical Riemann sheet of the elastic amplitude $T_{22}$. In this sense, the obtained pole does not qualify as a resonance, and we see it as a virtual or anti-bound $D\bar{D}^*$ state. It does not correspond to a particle, but it produces observable effects at the $D\bar{D}^*$ threshold similar to those produced by a near threshold resonance or bound state [16]. The line shapes of a virtual state and a near-threshold resonance are different since the former is peaked exactly at the threshold while the latter, in principle, is above. This can be seen in the left bottom panel of Fig. 1 where the $J/\psi\pi^-$ spectrum for the two fits $b = 0$ and $b \neq 0$ are shown (for the case $\Lambda_2 = 0.5$ GeV). Although the two curves are different, each one would approximately lie within the error band of the other. Very precise data

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Invariant mass distributions for $J/\psi\pi^-$ in the decay $Y(4260) \rightarrow J/\psi\pi\pi$ (left panels) [2] and for $D^{*-}D^0$ in the decay $Y(4260) \rightarrow D^{*}D\pi$ (right panels) [6]. The top panels show the results for the fit $b \neq 0$, $\Lambda_2 = 0.5$ GeV. In the bottom panels, the two fits $b = 0$ and $b \neq 0$ are compared (without error bands) for the case $\Lambda_2 = 0.5$ GeV. In the $J/\psi\pi^-$ spectrum, the $\bar{D}D^*$ threshold is marked with a vertical black line.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Comparison of the $Z_c$ resonance pole positions determined in this work for two values of the cutoff $\Lambda_2$ with the experimental determinations of Refs. [2–6]. The shaded areas take into account our statistical and systematic uncertainties (added in quadratures). The numerical values can be found in Ref. [1].}
\end{figure}

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
Experiment & $\Lambda_2$ (GeV) & $\Gamma$ (MeV) \\
\hline
BESIII & 0.5 GeV & 1.0 GeV \\
CLEO & & \\
Belle & & \\
\hline
\end{tabular}
\caption{Summary of the resonance pole positions.}
\end{table}
with a good energy resolution and small bin size are necessary to distinguish among them. Since both natures for the $Z_c$ structure (resonance or virtual state) arise in fits of good quality, it must be stated that the experimental information available at this time cannot fully discriminate between both scenarios. Thus, claims about the $Z_c$ structure should be made with caution. If it is finally shown to be a virtual state, then it cannot be a tetraquark, since it does not correspond to a normal particle, and it can only have a hadronic molecular nature, in the sense that it appears only because of the $DD^*$ interaction.

4. Summary

Summarizing, we have reported on our recent work [1] which presents the first simultaneous study of the invariant mass distributions of the $J/\psi\pi$ and $D^*D$ channels (where the $Z_c(3900)$ is seen) with fully unitarized amplitudes. We find that these data sets are well reproduced in two different scenarios. In the first one, in which there is an energy dependence in the $D^*D \rightarrow D^*D$ potential, the $Z_c$ appears as a dynamically generated $D^*D$ resonance. In the second one, where the aforementioned energy dependence is not allowed, it appears as a virtual state, the pole being located below the $DD^*$ threshold. In both scenarios, it is demonstrated that both data sets can be reproduced with a single $Z_c$ state, so that the two experimentally observed structures $Z_c^+(3900)$ and $Z_c^-(3885)$ are proven to correspond to the same state seen in different channels. It is really important to discriminate between these two scenarios, for which purpose a very precise measurement of the line shapes around (specially above) the $DD^*$ threshold is needed.

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