Lagrangian for the Majorana-Ahluwalia Construct

Valeri V. Dvoeglazov†
Escuela de Física, Universidad Autónoma de Zacatecas
Antonio Dovalí Jaime s/n, Zacatecas 98000, ZAC., México
Internet address: VALERI@CANTERA.REDUAZ.MX
(May 31, 1995)

Abstract

The equations describing self/anti-self charge conjugate states, recently proposed by Ahluwalia, are re-written to covariant form. The corresponding Lagrangian for the neutral particle theory is proposed. From a group-theoretical viewpoint the construct is an example of the Nigam-Foldy-Bargmann-Wightman-Wigner-type quantum field theory based on the doubled representations of the extended Lorentz group. Relations with the Sachs-Schwebel and Ziino-Barut concepts of relativistic quantum theory are discussed.

PACS numbers: 11.10.Ef, 11.30.Cp

*Submitted to “Nuovo Cim. A”.

†On leave of absence from Dept. Theor. & Nucl. Phys., Saratov State University, Astrakhanskaya ul., 83, Saratov RUSSIA. Internet address: dvoeglazov@main1.jinr.dubna.su
Recently, the Majorana-McLennan-Case construct for neutrino and photon \[1,2\] got a substantial development in the works of Ahluwalia et al. [3]. In connection with observation of candidate events for neutrino \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) oscillations at LSND LAMPF [4], that are not predicted and are not explained by gauge theories of the Glashow-Weinberg-Salam (GWS) type [5], an alternative insight in neutral particle physics has some reasons. While thirty years passed since the proposal of the GWS model, we are still far from understanding many its essential theoretical ingredients; first of all, the fundamental origins of “parity violation” effect [6], the Kobayashi-Maskawa mixing [7] and Higgs phenomenon [8]. Experimental neutrino physics and astrophysics provided us by new puzzles [9], that until now did not find adequate explanation.

In the mean time, the Majorana-McLennan-Case-Ahluwalia construct is based on a very natural principle for describing neutral particles: a principle of self/anti-self charge conjugacy of the states that correspond to neutrinos, \(j = 1\) and higher spin neutral particles. The kinematical framework of the theory has been given in ref. [3]. Unusual properties of the construct, such as “incompatibility of simultaneous existence of self/anti-self charge conjugacy and helicity eigenstates”, impossibility of the “standard fashion” gauge interaction, bi-orthonormality of physical states and the remarkable \(\lambda^{S,A} \leftrightarrow \rho^{A,S}\) property with respect to space reflection, have been revealed. The 4-spinors used there (that describe self/anti-self charge conjugate states) are the following:

\[\lambda(p^\mu) \equiv \left( \begin{array}{c} \zeta_\lambda \Theta_{[j]} \phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{array} \right), \quad \rho(p^\mu) \equiv \left( \begin{array}{c} \phi_R(p^\mu) \\ \zeta_\rho \Theta_{[j]} \phi_R^*(p^\mu) \end{array} \right) \, . \] (1)

\(\zeta_\lambda\) and \(\zeta_\rho\) are the phase factors that are fixed by the conditions of self/anti-self conjugacy, \(\Theta_{[j]}\) is the Wigner time-reversal operator for spin \(j\). They are called usually the Majorana-(like) spinors.

Next, in the papers [3] the following equation for \(\lambda^{S,A}(p^\mu)\) has been presented:

\[\left( \begin{array}{c} \zeta_\lambda \exp (-J \cdot \varphi) \Xi_{[j]}^{-1} \Theta_{[j]} \exp (-J \cdot \varphi) \\ -\Xi_{[j]} \end{array} \right) \lambda(p^\mu) = 0. \] (2)

The analogous equation for \(\rho^{S,A}(p^\mu)\) is:

\[\left( \begin{array}{c} \zeta_\rho^* \exp (-J \cdot \varphi) \Xi_{[j]} \Theta_{[j]} \exp (J \cdot \varphi) \\ -\Xi_{[j]} \end{array} \right) \rho(p^\mu) = 0, \] (3)

provided that the overall phase factors of \(\phi_R^h(\hat{p}^\mu)\) are chosen to be equal to the ones of \(\phi_L^h(\hat{p}^\mu)\).

\(\Xi_{[j]}\) is the matrix connecting at-rest 2-spinors with their complex conjugates:

\[\left[ \phi_{L,R}^h(\hat{p}^\mu) \right]^* = \Xi_{[j]} \phi_{L,R}^h(\hat{p}^\mu) \, ; \] (4)

\(J\) are the spin-\(j\) matrices; \(\hat{p}^\mu\) denotes the at-rest 4-momentum, \(\varphi\) are the Lorentz boost parameters.

The philosophy of a Lagrangian field theory is based on realization of the Principle of Least Action, that is assumed to be valid for all physical systems. “The dynamics of

\(^a\)In the paper we try to keep the notation of ref. [3]. Some important results of the cited paper will be used below without reference.
the system is specified once the Lagrangian is given” [10]. Unfortunately, in ref. [3] field
dynamics has not been presented in details. The main goal of this Letter is to re-write
the “instant-form” equations for the 4-spinors $\lambda_{S,A}(p^\mu)$ and $\rho_{S,A}(p^\mu)$, Eqs. (2) and (3), to
covariant forms and to deduce the corresponding Lagrangian with the aim to construct the
dynamics of “truly” neutral particles in the $(1/2, 0) \oplus (0, 1/2)$ representation space.

First of all, we note that

$$\Theta_{1/2} \Xi_{1/2} = \Xi_{1/2}^{-1} \Theta_{1/2} = i \frac{\sigma_1 p_2 - \sigma_2 p_1}{\sqrt{(p + p_3)(p - p_3)}} =$$

$$= U_+(p^\mu)U_-(p^\mu) = -U_{+1}^{-1}(p^\mu)U_{-1}(p^\mu) = -U_{-1}^{-1}(p^\mu)U_{+1}(p^\mu) = U_{-1}^{-1}(\tilde{p}^\mu)U_{+1}(p^\mu) \ ,$$

where $U_{\pm}(p^\mu)$ are the matrices of the $2 \times 2$ unitary transformation to the helicity represen-
tation [11a,Eq.(3.1)] and ref. [12, p.71]:

$$U_+(p^\mu)\sigma_3 U_+^{-1}(p^\mu) = \left(\frac{\sigma \cdot p}{p}\right) \ , \quad U_-^{-1}(p^\mu)\sigma_3 U_-(p^\mu) = -\left(\frac{\sigma \cdot p}{p}\right) \ ;$$

$p = |p| = \sqrt{E^2 - m^2}$ and $\tilde{p}^\mu$ is the parity-conjugated momentum. Using Eq. (3) one
can obtain the following equations for 4-spinors of the second kind $\lambda_+(p^\mu) = \mathcal{U}\lambda(p^\mu)$ and $\rho_+(p^\mu) = \mathcal{U}\rho(p^\mu)$, $\tilde{\lambda}_+(p^\mu) = \mathcal{U}\lambda(p^\mu)$ and $\tilde{\rho}_+(p^\mu) = \tilde{\mathcal{U}}\rho(p^\mu)$ in the new representations:

$$\left[\zeta_\lambda(\cosh^2 \frac{\rho}{2} - \sinh^2 \frac{\rho}{2})\gamma_5 \gamma_0 - \mathbb{I}\right] \lambda_+(p^\mu) = 0 \ ,$$

$$\left[\zeta_\lambda(\cosh^2 \frac{\rho}{2} - \sinh^2 \frac{\rho}{2})\gamma_5 \gamma_0 + \mathbb{I}\right] \tilde{\lambda}_+(p^\mu) = 0 \ .$$

The unitary matrices $\mathcal{U}$ are implied to be of the following set:

$$\mathcal{U}_{\pm} = \begin{pmatrix} U_{\pm}(\tilde{p}^\mu) & 0 \\ 0 & U_+(p^\mu) \end{pmatrix} \quad , \quad \tilde{\mathcal{U}}_{\pm} = \begin{pmatrix} U_{\pm}(p^\mu) & 0 \\ 0 & U_{\pm}(\tilde{p}^\mu) \end{pmatrix} \ .$$

The transformation to the helicity representation is a rotation, indeed, that, according to
the ordinary viewpoint, can not have influence physical results. Let me mention that the
equations similar to Eqs. (7a-b) could also be obtained after calculations with the transfor-
mation matrix $\Omega_{\lambda}^{(1/2)}$, or $\Omega_{\lambda}^{(1/2)}$, the matrix of a transfer to the light-front representation. For
instance, one can re-write Eq. (2) to the following non-dynamical form ($h$ is the helicity;
$\sigma \cdot \tilde{p})\phi_{L,R}(p^\mu) = h\phi_{L,R}$):

$$\begin{pmatrix} -\mathbb{I} & -\zeta_\lambda(h\sigma_3 p - p_3)/\sqrt{p \cdot \tilde{p}} \\ +\zeta_\lambda(h\sigma_3 p + p_3)/\sqrt{p \cdot \tilde{p}} & -\mathbb{I} \end{pmatrix} \lambda(p^\mu) = 0 \ .$$

It is not surprising since the famous Melosh transformation, ref. [13,14] (see also [15]),

---

We have used that

$$\exp(\pm \frac{\sigma}{2} \cdot \rho) = \cosh \frac{\rho}{2} \pm (\sigma \cdot \rho) \sinh \frac{\rho}{2} \ .$$

The equations for $\rho_H(p^\mu)$ and $\tilde{\rho}_H(p^\mu)$ are obtained from (7a,b) after the substitutions $\zeta_\lambda \rightarrow \zeta_{\tilde{\rho}}$. 

---

3
\[ \Omega(\frac{1}{2}) = \frac{1}{[2(E + m)p^+]^{1/2}} \left( \begin{array}{cc} \beta(\frac{1}{2}) & 0 \\ 0 & \beta(\frac{1}{2}) \end{array} \right) , \quad \beta(\frac{1}{2}) = \left( \begin{array}{cc} p^+ + m & -p_r \\ p_t & p^+ + m \end{array} \right) , \] (10)

is shown in ref. \[16\] to be a rotation too. As a matter of fact, these results hint that neutral particle states can “live” on the light cones only.

Nevertheless, one can still deduce dynamical equations for \( \lambda^{S,A}(p^\mu) \). From the analysis of the rest spinors, Eqs. (22a,b) of ref. \[3d\] one can find another form of the Ryder-Burgard relation for the \( j = 1/2 \) case:

\[ \left[ \phi^h_L (p^\mu) \right]^* = (-1)^{1/2-h} e^{-i(\theta_1 + \theta_2)} \Theta^{[1/2]}_L \phi^-h (p^\mu) . \] (11)

Provided that the overall phase factors of at-rest spinors are chosen to be \( \theta_1 + \theta_2 = 0 \) we come to the equations:

\[ \left[ \frac{i}{m} \gamma_5 \hat{p} - 1 \right] \Upsilon_\pm (p^\mu) = 0 , \] (12a)
\[ \left[ \frac{i}{m} \gamma_5 \hat{p} + 1 \right] \mathcal{B}_\pm (p^\mu) = 0 . \] (12b)

Here we defined 4-spinors, that are in helicity eigenstates \[20,19\]:

\[ \Upsilon_\pm (p^\mu) = \left( \pm i \Theta^{1/2}_1 \left[ \phi_L^{\mp 1/2}(p^\mu) \right]^* \right) , \quad \mathcal{B}_\pm (p^\mu) = \left( \mp i \Theta^{1/2}_1 \left[ \phi_L^{\pm 1/2}(p^\mu) \right]^* \right) . \] (13)

Of course, we could start from the equation \[8\] and obtain the equivalent set:

\[ \tilde{\Upsilon}_\pm (p^\mu) = \left( \mp i \Theta^{1/2}_1 \left[ \phi_R^{\mp 1/2}(p^\mu) \right]^* \right) , \quad \tilde{\mathcal{B}}_\pm (p^\mu) = \left( \pm i \Theta^{1/2}_1 \left[ \phi_R^{\mp 1/2}(p^\mu) \right]^* \right) . \] (14)

The latter can differ from the former only by a phase factor \( e^{if_\pm} \) provided that we keep the ordinary normalization of the Pauli \( \phi_{L,R} \) spinors.

One can then use the relations between the 4-spinors \( \Upsilon(p^\mu), \mathcal{B}(p^\mu) \) and \( \lambda(p^\mu) \). \[\text{footnote}^d\]

\[^d\]Let me note that definitions of Melosh and Ahluwalia for spin-1/2 are connected in the following way: \( S_{\text{ref.}} \equiv \beta_{\text{ref.}} \), within an accuracy of normalization and with the Pauli matrices \( \sigma \) are in the standard representation. The definitions of Kondratyuk and Terent’ev, as follows: \( U_{\text{ref.}} \equiv \beta^{*_{\text{ref.}}} \). \( T \) stands for transpose operation, the asterisk, for complex conjugation.

\[^d\]Different generalizations of the Ryder-Burgard relation for left- and right- at-rest spinors \( \phi_R(\hat{p}^\mu) = \pm \phi_L(\hat{p}^\mu) \), so called by Ahluwalia, have also been discussed in refs. \[17,13\].

\[^d\]In ref. \[19\] we have used the slightly different notation: \( \Upsilon_\pm (p^\mu) \equiv \pm \mathcal{B}_\pm^{(2)} (p^\mu) \), \( \mathcal{B}_\pm (p^\mu) \equiv \pm \Upsilon_\pm^{(1)} (p^\mu) \); and \( \tilde{\Upsilon}_\pm (p^\mu) \equiv \pm \tilde{\mathcal{B}}_\pm^{(1)} (p^\mu) \), \( \tilde{\mathcal{B}}_\pm (p^\mu) \equiv \pm \tilde{\Upsilon}_\pm^{(1)} (p^\mu) \).

\[^f\]The arrows \( \uparrow \downarrow \) should be referred to ‘chiral helicity’ introduced in ref. \[2\].
\[ \Upsilon_+ (p^\mu) = \pm \frac{1 + \gamma_5}{2} \lambda_+^{S,A} + \frac{1 - \gamma_5}{2} \lambda^-_{A}, \]  
\[ \Upsilon_- (p^\mu) = \mp \frac{1 + \gamma_5}{2} \lambda^-_{S,A} - \frac{1 - \gamma_5}{2} \lambda^A_{S}, \]  
\[ \mathcal{B}_+ (p^\mu) = \pm \frac{1 + \gamma_5}{2} \lambda^S_{+} + \frac{1 - \gamma_5}{2} \lambda^-_{S,A}, \]  
\[ \mathcal{B}_- (p^\mu) = \pm \frac{1 + \gamma_5}{2} \lambda^S_{-} + \frac{1 - \gamma_5}{2} \lambda^A_{S}, \]  

and between the 4-spinors \( \tilde{\Upsilon} (p^\mu), \tilde{\mathcal{B}} (p^\mu) \) and \( \rho (p^\mu) \):

\[ \tilde{\Upsilon}_+ (p^\mu) = \left[ \frac{1 + \gamma_5}{2} \rho^S_{+} \pm \frac{1 - \gamma_5}{2} \rho^-_{S,A} \right], \]  
\[ \tilde{\Upsilon}_- (p^\mu) = \left[ \frac{1 + \gamma_5}{2} \rho^S_{-} \mp \frac{1 - \gamma_5}{2} \rho^A_{S} \right], \]  
\[ \tilde{\mathcal{B}}_+ (p^\mu) = \left[ \frac{1 + \gamma_5}{2} \rho^S_{+} \mp \frac{1 - \gamma_5}{2} \rho^A_{S} \right], \]  
\[ \tilde{\mathcal{B}}_- (p^\mu) = \left[ \frac{1 + \gamma_5}{2} \rho^S_{-} \pm \frac{1 - \gamma_5}{2} \rho^A_{S} \right]. \]  

Furthermore, we assume that the parity violation is not explicit in the meaning of ref. [21] and one can use Eqs. (40a,b) of ref. [3c]. Finally, if imply like ref. [3d] that the \( \lambda^S (p^\mu) \) (and, therefore, \( \rho^A (p^\mu) \), see Eqs. (23)) are the solutions corresponding to positive frequencies; and \( \lambda^A (p^\mu) \) and \( \rho^S (p^\mu) \), to negative frequencies, the equations for 4-spinors of the same ‘chiral helicity’ take the following “mad” forms comparing with the ordinary Dirac case [22]:

\[ i \gamma^\mu \partial_\mu \lambda^S (x) - m \rho^A (x) = 0, \]  
\[ i \gamma^\mu \partial_\mu \rho^A (x) - m \lambda^S (x) = 0; \]  

and

\[ i \gamma^\mu \partial_\mu \lambda^A (x) + m \rho^S (x) = 0, \]  
\[ i \gamma^\mu \partial_\mu \rho^S (x) + m \lambda^A (x) = 0. \]  

They can be written in the 8-component form as follows:

\[ [i \Gamma^\mu \partial_\mu - m] \Psi (+) (x) = 0, \]  
\[ [i \Gamma^\mu \partial_\mu + m] \Psi (-) (x) = 0, \]  

where we defined the Weinberg *dibispinors*:9


9Of course, one can re-write the obtained equations to two-component form for \( \phi_R (p^\mu) \), \( \phi_L (p^\mu) \), \( i \theta [1/2] \phi^R_R (p^\mu) \) and \( i \theta [1/2] \phi^L_L (p^\mu) \). Cf. with the equations (11a,b) of ref. [2b].

hThis name has been used in my previous works for the set of \( F_{\mu\nu} \) and its dual \( \tilde{F}_{\mu\nu} \), the wave functions (operators) of the antisymmetric tensor field. I take a liberty to apply it for the 8-component wave functions of the \( (1/2,0) \oplus (0,1/2) \) representation space too, taking into account the significant contribution of Dr. Weinberg to modern physics in his pioneer works of 1964-69, ref. [23].
\( \Psi_+(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix} \), \( \Psi_-(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix} \). \hspace{1cm} (20)

The set of 8 \( \times \) 8-component \( \Gamma^- \) and \( T \)-matrices is written as

\[
\begin{align*}
\Gamma^\mu &= \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}, \quad 
\Gamma^5 &= \begin{pmatrix} \gamma^5 & 0 \\ 0 & \gamma^5 \end{pmatrix}, \quad 
L^5 &= \begin{pmatrix} \gamma^5 & 0 \\ 0 & -\gamma^5 \end{pmatrix}, \quad (21)
\end{align*}
\]

\[
T_{11} = \pm i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad 
T_{01} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad 
T_{10} = \pm i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (22)
\]

Next, one can propose, e.g., the following Lagrangian \( \overline{\Psi}(\pm) \equiv \Psi^\dagger(\pm) \Gamma_0 \): \( \mathcal{L}^{(1)} = \frac{i}{2} \left[ \overline{\Psi}_+(\pm) \Gamma^\mu \partial_\mu \Psi_+ + \overline{\Psi}_-(\pm) \Gamma^\mu \partial_\mu \Psi_- \right] - m \left( \overline{\Psi}_+(\pm) \Psi_+ - \overline{\Psi}_-(\pm) \Psi_- \right) \) - \hspace{1cm} (24)

It is useful to note that one can introduce the following gradient transformations of the first kind for \( \lambda^{S,A}(x) \) and \( \rho^{S,A}(x) \) spinors:

\[
\begin{align*}
\lambda'(x) &\rightarrow (\cos \alpha - i\gamma^5 \sin \alpha) \lambda(x), \quad (25a) \\
\overline{\lambda}'(x) &\rightarrow \overline{\lambda}(x)(\cos \alpha - i\gamma^5 \sin \alpha), \quad (25b) \\
\rho'(x) &\rightarrow (\cos \alpha + i\gamma^5 \sin \alpha) \rho(x), \quad (25c) \\
\overline{\rho}'(x) &\rightarrow \overline{\rho}(x)(\cos \alpha + i\gamma^5 \sin \alpha). \quad (25d)
\end{align*}
\]

In terms of the field functions \( \Psi_\pm(x) \) they are written

\[
\begin{align*}
\Psi'_\pm(x) &\rightarrow \left( \cos \alpha + iL^5 \sin \alpha \right) \Psi_\pm(x), \quad (26a) \\
\overline{\Psi}'_\pm(x) &\rightarrow \overline{\Psi}_\pm(x) \left( \cos \alpha - iL^5 \sin \alpha \right). \quad (26b)
\end{align*}
\]

\(^i\)Similar set of matrices has been defined in ref. [24,25c].

\(^j\)This form of the Lagrangian for neutral particles supports the remark made after Eq. (10). See also the remark after Eq. (15) in ref. [2a].

\(^k\)In general, phase factors in the gradient transformations of 4-spinors \( \lambda \) and \( \rho \) could be different. (see forthcoming papers).
Due to the commutation relation (23) the Lagrangian (24) is invariant with respect to (26a-26b). Using the analogy with ordinary quantum electrodynamics the local gradient transformations (gauge transformations) could also be defined. Like the ordinary case we are forced to introduce the compensating field of the vector potential, but in the case under consideration the covariant derivative is introduced in a slightly different fashion:

\[ \partial_\mu \rightarrow \nabla_\mu = \partial_\mu - igL^5 A_\mu \quad , \]

\[ A'_\mu(x) \rightarrow A_\mu + \frac{1}{g} \partial_\mu \alpha \quad . \]

This tells us that self/anti-self conjugate states (more precisely, the Weinberg dibispinor) can possess the axial charge.

We would like to point out connection of the construct proposed in ref. [3] with the similar formulations met in the literature [25,26], see also [27], and, in particular, with the formulation of refs. [28,21]. The asymptotically “chiral” massive fermions proposed by Dr. Ziino, are essentially the fields \( \lambda^A \sim \psi^\rho \), and \( \rho^S \sim \psi^\rho \). It is possible to find the corresponding relations between \( \lambda^S, \rho^A \) of ref. [3] and Ziino’s fields of the opposite frequencies. Barut and Ziino noted on the needed modifications of our understanding [21c] the concept of the quantization space: “Such a Fock space should have the manifestly covariant structure

\[ \mathcal{F} \equiv \mathcal{F}^0 \otimes S_{in} \quad , \]

where \( \mathcal{F}^0 \) is an ordinary Fock space for one indistinct type of positive- and negative-energy identical spin-\( \frac{1}{2} \) particles (without regard to the proper-mass sign) and \( S_{in} \) is a two-dimensional internal space spanned by the proper-mass eigenstates \( |+m> \), \( |-m> \) thus doubling \( \mathcal{F}^0 \). This allows [the Dirac-like fields] to be mixed if a rotation is performed in \( S_{in} \).” As a matter of fact, this phenomenon called doubling has been discovered by Wigner [29], who enumerated the irreducible projective representations of the full Poincaré group (including reflections). The ordinary Dirac construct [22] reflects only a simplest case of the general theory.

Then, curiously, an attempt to set up the Majorana-like anzatzen in the form:

\[ \phi_L = e^{i\vartheta} \Theta_{[1/2]} \phi_R^* \quad , \]

where \( \vartheta = 0, \pm \frac{\pi}{2}, \pi \), on the resulting equations (17a-18b) leads to tachyonic solutions.

Next, if we start to built the neutral particle theory from the 4-spinors of the first kind we would obtain the Case equations [2]. They coincide with a local limit of the Sachs equation [33,34]:

\[ q_\mu \partial_\mu \phi_\alpha - \lambda \Theta_{[1/2]} \phi_\alpha^* = 0 \quad , \]

and its conjugate; the latter follow from his treatment of quantum theory on the ground of the Einstein’s interpretation, which is an alternative to the Bohr-Heisenberg viewpoint.

\[ ^1 \text{The faults of the standard approach (first of all, to the interaction problem) seemed to be realized by Dr. Dirac himself [30]. See also [31] for an explicit construct of the charge conjugation representation for Dirac fields. The example of the FNBWW-type quantum field theory in the (1,0) \oplus (0,1) has been given in [32].} \]

\[ ^m \text{The physical content still depends on the choice of the phase factor } \vartheta. \]
$q_\mu$ is the quaternion, $\lambda$ is a complex parameter with the dimension of mass. According to the Sachs viewpoint “the field equation (31) represents the inertial mass appearing as a continuous function, $m = \lambda \hbar / c$, rather than a constant parameter”. As mentioned in ref. [34c] such an interpretation predicts an indefinite spectrum of neutrino masses. From the other hand, in the recent preprint Prof. Bilenky with collaborators [35] indicated that “if the LSND signal is confirmed, it would mean that... there is no natural hierarchy of coupling among generations in the lepton sector... If future experiments confirm the existence of [the atmospheric] neutrino anomaly and the result of the LSND experiment also... it would be necessary to assume the existence of an additional... neutrino state besides the three active flavor neutrino states”. Bilenky’s statement provides some phenomenological grounds to the Sachs theoretical construct.

Next, let us mention that for the first time the analysis of the dynamics, that follow from the “doubled” representation of the extended Lorentz group has been undertaken in ref. [25c]. The Markov’s set of equations for fermion-antifermion 8-component wave function turns out to be invariant with respect to the transformation with $\Gamma^5 T_{01}$ matrix, provided that fermion and its antifermion masses are assumed equal. “Only such kinds of interactions which are not invariant with respect to these transformations can remove the degeneracy over the bare particle masses.” As a matter of fact, in that preprint Prof. Markov proposed such a type of interaction that can solve the hierarchy problem [26]. Finally, the reader can wish to reveal transparent connections with the physical content based on the fundamental principle of indistinguishability of identical particles, discussed in the paper [37, p.195].

The main result of this Letter is the Lagrangian for the Majorana-McLennan-Case-Ahluwalia construct. The wave functions (field operators) present themselves 8-component dibispinors. From a group-theoretical viewpoint the construct is an example of the Nigam-Foldy-Bargmann-Wightman-Wigner-type theory based on the doubled representations of the extended Lorentz group [31,29]. In the approaching papers we are going to find dynamical invariants following from the proposed Lagrangian, propagators and to built the Feynman diagram technique for this type of Poincaré invariant theories. Probably, the quantum-field particle dynamics should be constructed on the base of the kinematical postulates of Faustov, Ryder, Burgard and Ahluwalia, and with taking into account the ideas worked out by Sachs, Schwebel, Markov, Ziino, Barut and Pashkov [38].

ACKNOWLEDGMENTS

This paper is a continuation of considerable efforts undertaken by Profs. D. V. Ahluwalia and A. F. Pashkov “to prove well-founded”. Especially, I thank Prof. A. F. Pashkov for drawing my attention to refs. [11,33,34] and Prof. D. V. Ahluwalia, to ref. [14].

I am grateful to Zacatecas University for professorship.

Recently, a mathematical treatment of the another model with fields transforming in accordance with the double representation of the extended Lorentz group has been undertaken [13]. It leads to very interesting physical consequences, such as: in the framework one can obtain both charged and neutral particles; the formalism admits both commutation and anticommutation relations for describing one or another states, there is the “puzzled” state with zero energy-momentum, zero charge and the Pauli-Lyuban’sky operator.
REFERENCES

[1] E. Majorana, Nuovo Cim. 14 (1937) 171 [English translation: D. A. Sinclair, Tech. Trans. TT-542, National Research Council of Canada]

[2] J. A. McLennan, Phys. Rev. 106 (1957) 821; K. M. Case, Phys. Rev. 107 (1957) 307

[3] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Mod. Phys. Lett. A9 (1994) 439; Acta Phys. Polon. B25 (1994) 1267; D. V. Ahluwalia, “Incompatibility of self-charge conjugation with helicity eigenstates and gauge interactions.” Preprint LA-UR-94-1252 [hep-th/9404100], Los Alamos, Apr. 1994; “McLennan-Case Construct for Neutrino, its Generalization, and a Fundamentally New Wave Equation. Preprint LA-UR-94-3118 [hep-th/9409134], Los Alamos, Sept. 1994

[4] C. Athanassopoulos et al., Candidate Events in a Search for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ Oscillations. Preprint LANL LA-UR-95-1238 [nucl-ex/9504002], Los Alamos, Apr. 1995, submitted to “Phys. Rev. Lett.”

[5] S. L. Glashow, Nucl. Phys. 22 (1961) 579; S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; A. Salam, in: Proc. of the VIII Nobel Symposium. Stockholm, 1968. Ed. N. Svartholm, p. 367

[6] T. D. Lee and C. N. Yang, Phys. Rev. 104 (1956) 254; C. S. Wu et al., Phys. Rev. 105 (1957) 1413

[7] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; A. Salam, in: Proc. of the VIII Nobel Symposium. Stockholm, 1968. Ed. N. Svartholm, p. 367

[8] T. D. Lee and C. N. Yang, Phys. Rev. 104 (1956) 254; C. S. Wu et al., Phys. Rev. 105 (1957) 1413

[9] N. Cabbibo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49 (1975) 652

[10] P. W. Higgs, Phys. Lett. 12 (1964) 132; Phys. Rev. Lett. 13 (1964) 508; Phys. Rev. 145 (1966) 1156

[11] For recent review see: G. Gelmini and E. Roulet, Neutrino Masses. Preprint UCLA/94/TEP/36 [hep-ph/9412278] Nov. 1994, to appear in “Repts. Prog. Phys.”

[12] A. O. Barut, Electrodynamics and Classical Theory of Fields and Particles. (Dover Pub., Inc., New York, 1980)

[13] R. A. Berg, Nuovo Cim. 42A (1966) 148; M. Sachs, Nuovo Cim. 43A (1966) 1175

[14] Yu. V. Novozhilov, “Vvedenie v teoriyu elementarnykh chastits.” (Nauka, Moscow, 1971) [English translation: “Introduction to Elementary Particle Theory.” (Pergamon Press, Oxford, UK, 1975)]

[15] H. J. Melosh, Phys. Rev. D9 (1974) 1095

[16] Yu. V. Novozhilov and E. V. Prokhatyuk, Teor. Mat. Fiz. 1 (1969) 101 [English translation: Theor. Math. Phys. 1 (1969) 78]

[17] L. A. Kondratyuk and M. V. Terent’ev, Yadern. Fiz. 31 (1980) 1087 [English translation: Sov. J. Nucl. Phys. 31 (1980) 561]

[18] R. N. Faustov, Relativistskie preobrazovaniya odnochastnychch volnovkh funktsii. – Relativistic Transformations of One-Particle Wave Functions. Preprint ITF-71-117P, Kiev, Sept. 1971, in Russian

[19] V. V. Dvoeglazov, Significance of the Spinorial Basis In Relativistic Quantum Mechanics. Preprint EFUAZ FT-95-12 [hep-th/9504134], Mar. 1995, submitted to “Nuovo Cim. B”

[20] V. V. Dvoeglazov, Extra Dirac Equations. Preprint EFUAZ FT-95-13 [hep-th/9504150], Apr. 1995, submitted to “J. Math. Phys.”

[21] V. V. Dvoeglazov, Neutral Particles in Light of the Majorana-Ahluwalia Ideas. Preprint EFUAZ FT-95-11 [hep-th/9504158], Feb. 1995, submitted to “Int. J. Theor. Phys.”

[22] G. Ziino, Ann. Fond. L. de Broglie 14 (1989) 427; ibid 16 (1991) 343; A. O. Barut and G. Ziino, Mod. Phys. Lett. A8 (1993) 1011

[23] P. A. M. Dirac, Proc. Roy. Soc. A117 (1928) 610; ibid 118 (1928) 351

[24] M. Gelfand and M. L. Tsetlin, ZhETF 31 (1956) 1107 [English translation: Sov. Phys. JETP 4
(1957) 947]; G. A. Sokolik, ZhETF 33 (1957) 1515 [English translation: Sov. Phys. JETP 6 (1958) 1170]

[25] M. A. Markov, ZhETF 7 (1937) 579, 603; On the Difference Between Muon and Electron Masses (On Two Types of Dirac Fields). Preprint JINR D-1345, Dubna, 1963

[26] J. Brana and K. Ljolje, Fizika 12 (1980) 217, 287

[27] F. J. Belinfante and W. Pauli, Physica 7 (1940) 177

[28] A. O. Barut, Phys. Rev. Lett. 20 (1968) 893; Physica A142 (1987) 467, 488

[29] E. P. Wigner, in “Group theoretical concepts and methods in elementary particle physics – Lectures of the Istanbul Summer School of Theoretical Physics, 1962”. Ed. F. Gürsey

[30] P. A. M. Dirac, in Mathematical Foundations of Quantum Theory. (Academic Press, Inc., 1978), p. 1; Proc. Cambridge Phil. Soc. 30 (1934) 150; Proc. Roy. Soc. A322 (1971) 435; ibid A328 (1972) 1

[31] B. P. Nigam and L. L. Foldy, Phys. Rev. 102 (1956) 1410

[32] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Phys. Lett. B316 (1993) 102

[33] M. Sachs, Ann. Phys. 6 (1959) 244; Nuovo Cim. Suppl. 21, X (1961) 197; J. Math. Phys. 3 (1962) 843; Nuovo Cim. 34 (1964) 81; Found. Phys. 10 (1980) 921; ibid 11 (1981) 329

[34] M. Sachs, Lett. Nuovo Cim. 32 (1981) 307; Nuovo Cim. 37 (1965) 888; ibid 107A (1994) 1709

[35] S. M. Bilenky et al., Neutrino Oscillations in the Framework of Three-Generation Mixing With Mass Hierarchy. Preprint DFTT 25/95, JHU-TIPAC 95013 [hep-ph/9504405]. Apr. 1995

[36] A. O. Barut, Phys. Lett. 73B (1978) 310; Phys. Rev. Lett. 42 (1979) 1251;

[37] I. G. Kaplan, J. Mol. Structure 272 (1992) 187

[38] A. F. Pashkov, private communications (1984-95)