Time variation of fundamental constants: two phenomenological models

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Abstract. Time variation of fundamental constants, a strong signal of new physics, is analyzed comparing extant data with two phenomenological models. Results are discussed.

INTRODUCTION

The Standard Model of Fundamental Interactions (SM) together with General Relativity (GR) provides a consistent description of all known low energy phenomena (i.e. low compared with the Grand Unified (GU) energy scale) in good agreement with experiment. These theories depend on a set of parameters called the “fundamental constants”, which are supposed to be universal parameters; i.e.: time, position and reference frame invariant. Indeed, Einstein Equivalence Principle, on which GR is based, implies such an invariance and thus the detection of a time, position or reference system dependence of any of the fundamental constants imply the existence of “new physics”, beyond SM or GR.

Thus, variation of fundamental constants has been an active subject of research since the introduction of the Large Number Hypothesis (LNH) by Dirac [1, 2] long ago. (See also [3]). Several of these theoretical approaches are briefly discussed in the next section.

Partially inspired by these theoretical results, many attempts have been made to detect through observation or experiment a variation of fundamental constants, since this latter will produce a host of different phenomena. Reference [4] discusses critically several of the above results and modern reviews can be found in references [5, 6]. Table 1 shows a summary the most accurate results obtained from several sources, assuming that the fine-structure constant is the only one which varies in time.

The recent announcement of a detection of a nonzero variation of the fine structure constant $\alpha$ in the absorption spectra of distant quasars [15, 11, 16] has started an active experimental and theoretical research activity around this subject and we shall try to discuss a few of the current issues.

The organization of this paper is as follows: next section briefly discusses theoretical foundations of the problem and then the effective low energy model used for testing pur-
TABLE 1. Sample results on the time variation of the fine structure constant α. The columns show the phenomenon used, the time from the present of the observation, the estimated rate of change and the reference.

| Phenomenon                  | Age ($10^9$ y) | $\dot{\alpha}/\alpha$ ($y^{-1}$) | Reference |
|-----------------------------|----------------|----------------------------------|-----------|
| Rb vs. Cs Clocks            | 0              | $(4.2 \pm 6.9) \times 10^{-15}$  | [7]       |
| Oklo reactor                | 1.8            | $(-0.2 \pm 0.8) \times 10^{-17}$ | [8, 9]    |
| Age of Earth                | 4.5            | $(1.0 \pm 0.8) \times 10^{-15}$  | [4]       |
| FS in QAS                   | 12.3           | $(-0.4 \pm 1.1) \times 10^{-15}$ | [10]      |
| MM in QAS                   | 12.4           | $(-0.61 \pm 0.23) \times 10^{-15}$ | [11]      |
| HFS H vs Opt in QAS         | 11             | $(0.32 \pm 0.50) \times 10^{-15}$ | [12]      |
| CMB Fluctuations            | 15             | $(0.0 \pm 1.3) \times 10^{-12}$  | [13, 14]  |

poses is described. Modern tests are briefly reviewed and finally results and conclusions are stated.

A ZOO OF COMPETING THEORIES

Variation of fundamental constants is a common prediction of a large set of theories which can be divided in two main groups:

1. Those attempting to unify gravitation with all the other fundamental interactions through the introduction of additional dimensions (Superstrings, Kaluza-Klein).
2. Those implementing some version of Mach’s Principle.

In the following, I shall try to extract the common features of these theories leading to the prediction of time or space variation of fundamental constants.

Kaluza-Klein theories These theories, the simplest attempting a unification of gravitation and other fundamental interactions [17, 18], assume that there are $D$ additional compactified (internal) dimensions with a scale $R \sim \ell_P$. The equations of motion for gravitation and matter can be deduced from Einstein’s equations in $4 + D$ dimensions:

$$R^{(D)}_{MN} - \frac{1}{2} g^{(D)}_{MN} R^{(D)} = \frac{8\pi G_D}{c^2} T^{(D)}_{MN}$$

(1)

where the sub(super)script $D$ indicates the $4 + D$ dimensional tensors.

The fundamental constants appear, in these theories, as functionals of the additional dimensions geometry when the system is reduced to a 4-dimensional one [19, 20, 21]. One expects, in general, for any of the SM gauge-fields coupling constants:

$$\alpha_i = C \left( \frac{R_0}{R} \right)^{-2}$$

(2)

where $R_0$ is the present value of the internal space. Since generic cosmological solutions of equations (1) have time-varying $R(t)$, Kaluza-Klein theories predict time-variation of fundamental constants. Indeed, $R$ can be parametrized in the form
\[ R = \ell p e^\phi, \text{ where } \phi \text{ is called the dilaton field and is a general function of space and time.} \]

**Superstrings** Superstring theories [22] assume that there are extended fundamental objects, called strings, that move in \( 4 + D \) space and whose excitations represent (rather, would represent) the basic fields and particles. It is assumed that the low energy form of the theory is similar to a Kaluza-Klein theory. The existence of a dilaton entails a variation of fundamental constants [23, 24].

**Machian theories** The purpose of this rather large class of theories is to implement Mach’s principle [25]: *The mass \( M \) of a physical body is a functional \( M[x | \rho] \) of the mass distribution \( \rho(x) \) in the Universe.* The same ideas has been extended to other conserved quantities such as charge or even parameters like light speed.

**Variable mass theories** In these theories [26, 27, 28, 29, 30, 31, 32], the mass of a particle is proportional to a field \( m = m_0 \mu(x) \), whose source is the mass density in the Universe \( \rho(x) \).

**Variable charge theories** Particles charges are assumed to be functions of a universal field \( \varepsilon(x) \) [33, 34, 35, 36] whose source is related to the charge distribution in the Universe \( e = e_0 \varepsilon(x) \). Maxwell’s equations take their usual form, but with a modified field tensor:

\[
F^{\mu\nu} = \frac{1}{\varepsilon}\left[ \partial^{\mu}(\varepsilon A^{\nu}) - \partial^{\nu}(\varepsilon A^{\mu}) \right] \tag{3}
\]

It can be shown that gauge invariance is preserved and charge conservation holds.

**Varying speed of light theories** Here light speed \( c = c_0 \psi(x) \) is a variable field [37, 38, 39, 40, 41, 42], and so space and time derivatives do not commute. Maxwell’s equations take the form:

\[
\frac{1}{c} \partial_\mu (c F^{\mu \nu}) = 4 \pi j^\nu \tag{4}
\]

that implies charge non-conservation [43].

**EFFECTIVE THEORIES**

Both Superstring and Kaluza-Klein theories reduce, in the low energy limit, to effective theories of the Machian type. Schematically, the reduction is as follows:

\[
\text{Superstring} \Rightarrow \text{Kaluza-Klein} \Rightarrow \text{Machian} \tag{5}
\]

The latter theory is defined through the effective Lagrangian density

\[
\mathcal{L}_{\text{eff}} = \sqrt{-g} \left[ \mathcal{L}_G + \frac{\omega}{2} (\partial_\mu \phi) + f(\phi) \mathcal{L}_M \right] \tag{6}
\]

where \( \mathcal{L}_G, \mathcal{L}_M \) are the gravitational and matter lagrangians, \( \phi \) is the “Machian” field (dilaton) and \( f(\phi) \) represents a (set of) coupling function(s). This lagrangian is assumed
valid for energies below the GUT scale. The effective gauge coupling constants will have the form:

$$\alpha(x) = \frac{f(\phi(x))}{f(\phi(x_0))} \alpha(x_0)$$  \hspace{1cm} (7)

where $x_0$ is a reference point. Besides, the laboratory scale constants are related to the GUT scale constant through the renormalization group equations [20, 21].

The cosmological equations for the above effective theory have the form (for the present Universe):

$$\frac{1}{H_0^2} \left( \frac{\dot{R}}{R} \right)^2 = \frac{\Omega_0}{R^3} + \Omega_\Lambda + \Omega(\phi, \dot{\phi})$$

$$\ddot{\phi} + 3\frac{\dot{R}}{R} \dot{\phi} = -f'(\phi) T(R, \phi)$$ \hspace{1cm} (8)

and thus time variation of the fundamental constants is a generic prediction. Depending on the details of the theory, oscillations in the fundamental constants are possible [44, 45].

Local equations of motion for the effective theory are obtained linearizing them around the reference point $x_0$: $\phi = \phi_0 + \phi$. One obtains for a quasistatic field:

$$\nabla^2 \phi = 4\pi G_N \lambda_D \rho$$ \hspace{1cm} (9)

using the fact that the source of the $\phi$ field is related to the trace of the energy momentum tensor. The parameter $\lambda_D$ is related to the local value of $f'$. The local Machian field is then proportional to the newtonian potential $U_G(r)$ and the theory predicts space variation of fundamental constants:

$$\alpha_i = \alpha_i^0 \left[ 1 + \mu_i \frac{U_G(r)}{c^2} \right]$$ \hspace{1cm} (10)

which in turn implies position-dependent binding energies.

**OBSERVATIONAL TESTING**

The variation of fundamental constants produces a host of observable effects that can be used to test or search for the effect.

**Short time ($t \ll H_0^{-1}$) effects** Among the local effects that can be used to test effective theories, change in planetary parameters (such as radii and moments of inertia) and orbital evolution are important, albeit low accuracy tests [4, 46, 45]. Laboratory experiments based on the comparison of different clock rates are presently more accurate [47, 48, 49, 7]. The Oklo phenomenon (a natural reactor that operated about 2 Gy ago) provides the most accurate bounds on the local time variation of fundamental constants [50, 9, 8]. Finally, the coincidence of different determinations of the age of the Earth ($\alpha$ vs $\beta$ ages) puts useful limits on the local changes of the constants [4].
TABLE 2. Bounds on time and space variation of fundamental constants. Temporal bounds are in units of the Hubble constant $H_0$ while spatial bounds are in units of the local Newtonian acceleration $g/c^2$.

| From local data |
|------------------|
| Model | $\dot{\alpha}/\alpha/H_0$ | $\dot{G}_F/G_F/H_0$ | $\dot{\Lambda}_Q/\Lambda_Q/H_0$ | Ref. |
| Phenomenological | $2\cdot10^{-4}$ | $2\cdot10^{-1}$ | $1.7\cdot10^{-2}$ | [4] |

| From cosmological data |
|------------------------|
| Model | $\dot{\alpha}/\alpha/H_0$ | $\dot{G}_F/G_F/H_0$ | $\dot{\Lambda}_Q/\Lambda_Q/H_0$ | Ref. |
| Kaluza-Klein | $7\cdot10^{-8}$ | $3\cdot10^{-7}$ | $3\cdot10^{-6}$ | [5] |
| Bekenstein | $3.2\cdot10^{-8}$ | | | [5] |

| Spatial bounds |
|----------------|
| Model | $c^2/g |\nabla \alpha|/\alpha$ | $c^2/g |\nabla G_F|/G_F$ | $c^2/g |\nabla \Lambda_Q|/\Lambda_Q$ | Ref. |
| Phenomenological | $1.4\cdot10^{-8}$ | $3.3\cdot10^{-2}$ | $1.0\cdot10^{-8}$ | [58] |

| Conservation Laws |
|-------------------|
| Model | $\dot{c}/c/H_0$ | Ref. |
| VSL | $4\cdot10^{-16}$ | [43] |

**Spatial effects** The classical Dicke argument [51, 52, 6] shows that a spatial variation of the fundamental constants would entail the appearance of an anomalous gravitational acceleration detectable in Eötvös-like experiments. This can be easily seen from (10) since energy-dependent binding energies imply that the energy of a macroscopic body in a gravitational field will have a passive gravitational mass different from its inertial mass which in turn implies an anomalous acceleration in a gravitational field:

$$a = \left(1 + \sum_i \frac{\mu_i E_{Bi}}{c^2} \right) g$$  \hspace{1cm} (11)

which can be tested in Eötvös-like experiments.

**Cosmological effects** There have been several attempts to directly measure the time variation of fundamental constants in the absorption spectra of distant quasars, which originate in cold gaseous clouds interposed between the quasar and ourselves. There are measurements of fine structure [12, 53, 10], yielding directly $\Delta \alpha/\alpha$. The use of multiple transitions from several elements in the same clouds has improved the sensibility of the method, yielding the first positive detection of a change in $\alpha$ [15, 11, 16].

Other quantities measured are $x = \alpha^2 g_p \frac{m_e}{m_p}$ from the ratio of optical transitions to the 21 cm radiation [54, 55, 12]; $y = g_p \alpha$ from rotational transition frequencies in diatomic molecules to the 21 cm radiation [56] and $\mu = \frac{m_e}{m_p}$ from molecular hydrogen absorption [57]. This latter result is the only direct result on the Higgs sector of the SM.
FIGURE 1. Comparison of observational cosmological data with Bekenstein’s model. Data from references [9] and [16] cannot be reconciled. The “Best Fit” is a least squares one, with $\sqrt{\omega}/l_\rho = 0.15$; the other two have been forced to $\sqrt{\omega} = l_\rho$.

**Early Universe Results** Both the Cosmic Microwave Backgroud (CMB) spectrum [59, 60, 13] and primordial nucleosynthesis [61, 5] have been used to constrain time variation of fundamental constants, since well measured results are affected by their changes.

**Conservation laws** The structural changes implied by the trasformation of constant parameters into scalar fields can destroy the validity of well known conservation laws. Such is the case of the conservation of electric charge, that is violated in the VSL theories proposed in [39, 40]. Indeed, from (4) the following law for the variation of charge in a closed system can be derived [43] $\frac{\dot{Q}}{Q} = -\frac{\dot{e}}{e}$

**RESULTS**

Although limits on the time variation of fundamental constants can be obtained from individual measurements, as in table 1, an analysis of the full wealth of data is necessary in order to compare theory and experiment [4, 5].

In reference [5] such an analysis was carried out for Kaluza-Klein theories and the Bekenstein theory. This complements the model independent analysis of references [4, 46]. Table 2 shows the upper limits found for the time and space variation of fundamental constants in those and similar analysis.

Our results are consistent with no time variation of fundamental constants over cosmological times, as most of observational results indicate. Indeed, excluding the positive
data points of references [15, 11, 16], no significant change of the final results is found. Besides, the full set of Eötvös-like experiments are consistent with null results [58] and so is electric charge conservation with the constancy of $c$ [43].

**CONCLUSIONS**

The full data set analyzed shows null results for the time variation of fundamental constants in the nearby time. On the other hand, neither Kaluza-Klein theories nor Bekenstein-like ones can explain the full set of cosmological observations. The Bekenstein model is specially interesting since it is similar to the low energy limit of the dilatonic sector of superstring theories. Figure 1 shows a few attempts to explain the data of references [15, 11, 16] and [50, 9, 8] with a Bekenstein model: no combination of parameters can do it. The difficulties are even worser if the spatial limits are included in the data set (See, however, reference [34]).

We conclude that none of the available theories is able to explain the full data set on time and space variation of fundamental constants, specially the quasar results. Were these proved free of systematic errors, we should conclude that, together with the existence and nature of dark mass and energy, they herald a forthcoming crisis on our current theoretical ideas.

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