Non-stationary Characteristics of the instability in a Single-mode Laser with Fiber Feedback

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Abstract

Chaotic bursts are observed in a single-mode microchip Nd:YVO₄ laser with fiber feedback. The physical characteristic of the instability is a random switching between two different dynamical states, i.e., the noise-driven relaxation oscillation and the chaotic spiking oscillation. As the feedback strength varied, transition which features the strong interplay between two states exhibits and the dynamical switching is found to be non-stationary at the transition.

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The complex dynamics in nonlinear systems with delayed feedback which possesses infinite dimensions are of current interest in various fields including physics, chemistry, biology, economy, physiology, neurology, and optical systems [1]. Especially, instabilities of nonlinear optical resonators and lasers with delayed feedback have attracted much attention in the past decade. In history, the issue of chaotic instabilities in the output of lasers which is subjected to external feedback was initiated by the pioneering work of Lang and Kobayashi in 1980 [2]. They demonstrated the dynamical instabilities in a semiconductor laser with external feedback which features the sustaining relaxation oscillations. They also confirmed theoretically that the dynamical instabilities take place in the transition process where the lasing frequency changes from one external cavity eigenmode to another in a weak-coupling regime. Thereafter, three universal transition routes to chaos, low-frequency fluctuations (LFF) and coherence collapse have been observed in semiconductor lasers with external optical feedback for different feedback strength and/or delay time regions [3]. To LFF and coherence collapse, there still exists the open question to concern the role of noise [4]. Nowadays, another promising laser system for investigating the instabilities in lasers with delayed feedback would be the laser-diode-pumped microchip solid-state lasers which have been widely used in the practical applications. They are expected to exhibit extremely high-sensitive response to the external feedback. The reason is that the cavity round-trip time $\tau_L \propto \tau_p$: photon lifetime) compared with the fluorescence lifetimes $\tau$ is extremely short as demonstrated in self-mixing laser Doppler velocimeties [5]. Generally, the lifetime ratio $K = \tau/\tau_p$ of solid lasers ranges from $10^5$ to $10^7$, while $K \approx 10^3$ in laser diodes. Furthermore, their characteristic frequencies are of sub-MHz, thus the conventional measurement techniques can be utilized easily. Therefore, it will be much easier to study the various instabilities in solid-state laser systems than in laser diode systems. In fact, in the early experiment in 1979, Otsuka observed various instabilities in a microchip LNP (LiNdP_4O_12) solid-state laser subjected to external feedback [6]. However, the instability is only observed in the region of two lasing modes. It is expected that the instability can also occur due to the external cavity modes with small mode spacing. The mechanism is that a random intensity
fluctuation in each mode will result in mode-dependent random fluctuation in phase shift \[6\]. Hence, multimode oscillation and intrinsic mode-partition noise as well as frequency dependent nonlinear refractive index are the dynamical origins to cause the chaotic bursts in the presence of fiber \[6\]. Here we report an experimental result of a diode-pumped microchip Nd:YVO\(_4\) (yttrium orthovanadate) solid state laser, in which mode-partition noise is not essential, but the intrinsic phase fluctuation may be dominant around the onset of instability. Furthermore, it will be shown that a novel non-stationary characteristic is inherent in the instability due to the strong coupling between two different dynamical states at transition. This non-stationary feature should be unique for the characterization of generic dynamical systems.

In our experiment, a diode-pumped microchip Nd:YVO\(_4\) laser operated at single-mode regime is employed and a compound cavity is formed with single-mode fiber feedback. The laser diode (LD) and Nd:YVO\(_4\) (1mm thick, 1% Nd\(^{3+}\) doped, and the output coupling is 5 ± 2% at 1064nm) are available from CASIX, Inc. The laser crystal (Nd:YVO\(_4\)) is inserted into a 2mm-thick copper mount and the temperature is controlled at 25°C by a temperature controller (ILX, LDT-5910B). The pumping beam (with wavelength at \(\lambda_p=808\)nm) from LD, which is also temperature-controlled, is focused onto the laser crystal with a GRIN lens (0.22 pitch). The pumping threshold is around 300 mA. We also use a noise filter (ILX 320) to eliminate the pumping noise caused by the LD current driver (ILX, LDC-3744) and an interference filter (60% transmission at 1064nm and zero-transmission for the rest) to reduce the influence of pumping light on detection. In the entire pumping domain a \(\pi\)-polarized TEM\(_{00}\) mode of laser output was observed. We used a 10m single-mode fiber (3M F-SY) as the feedback loop. The feedback beam is monitored and no significant polarization has been found. Because of the reduction of the lasing threshold (about 1-2 mA less) the feedback strength is estimated to be below 1%. To further control the feedback strength, before the light entering the fiber, a rotatable polarizer (New Focus 5525) is utilized. In the measurement, a multi-wavelength meter (HP 86120B) is employed to monitor the variation of lasing mode and the lasing wavelength is 1064.245 nm. The lasing eigenmode frequency
of the compound cavity is determined by the frequency arrangement of the Nd:YVO$_4$ laser cavity mode (mode spacing 0.25 nm; i.e., $\sim 60$ GHz) and the external cavity (fiber) modes for which the number of modes is very large (the mode spacing of external cavity is around 10 MHz). We also utilize low-noise detectors (New Focus 1611; bandwidth 1GHz) for the detection of laser output. Both ac and dc ports of the detectors are connected to a transient oscilloscope (HP54542C) for data acquisition in the temporal domain. Meanwhile, a rf-spectrum analyzer (HP8591E) is employed to monitor the behavior of laser output in the rf-spectrum domain.

For later identification, we first show the result of a typical ac time series for the case of free-running and its corresponding rf-spectrum in Fig.1. Relaxation oscillation occurs around 1.6 MHz and its harmonics can be easily characterized. As the strength of feedback is increased, chaotic bursting occurs and the dominant frequency was shifted to a lower value (around 1.0 MHz) with broadened-linewidth. Typical time series of the chaotic bursting is shown in Fig.2 (a). To explore the dynamics, we employ a joint-time frequency analysis. The coincidence of frequency characteristics between Fig.1 (b) and Fig.2 (b) suggests that the low-intensity level part of chaotic bursting, the regime I in Fig.2 (a), is a noise-driven relaxation oscillation while the high-intensity level part, the regime III in Fig.2 (a), can be identified to be chaos based on a singular value decomposition analysis [7]. We noted that the basic characteristics of the dynamical transition between two states is frequency-broadening in the rf-spectrum as shown in Fig.2 (c). Nevertheless, the lasing wavelength remains the same. That is no significant line-width broadening or hopping has been seen. Meanwhile, as instability occurs, the signal-to-noise level of the lasing mode decreases and features as a fast frequency-modulated (FM) laser characteristics. This suggests that there is a FM noise in the process of instability.

Next let us address the physical mechanism of coexistence of two dynamical states. As seen from the time series, the behavior of peak power is a key factor for the dynamics. Since our system is essentially a single-mode laser with weak feedback, the Lang-Kobayashi model [2] is still applicable such that the photon density $S(t)$ follows
\[
\frac{dS(t)}{dt} = K[(n(t) - 1)S(t) + \epsilon n(t)] + 2\kappa \sqrt{S(t)S(t-T)} \cos \theta(t),
\]

where \( K \) is the time ratio between the population inversion lifetime and the photon lifetime, \( n(t) \) is the population inversion density (or carrier density), \( \epsilon \) is the spontaneous emission factor, \( \kappa \) is the feedback coupling strength, \( T \) is the delay time, and \( \theta \) is the phase difference between the output and the feedback beams. The peak power \( S_p \), therefore, follows

\[
S_p = \frac{-K\epsilon n_p(t)}{K(n_p(t) - 1) + 2\kappa \sqrt{1 + \frac{\Delta S}{S_p}} \cos \theta_p},
\]

where \( \Delta S = S_p(t) - S_p(t-T) \) and the subscript \( p \) denotes the corresponding quantities evaluated at \( S = S_p(t) \). The role of \( \Delta S \) is crucial. As the feedback coupling \( \kappa \) is almost zero, the statistics \( S_p \) should simply follow the (on-off) fluctuation of the population inversion as implied by the lasing threshold factor, \( n(t) - 1 \). With a nonzero-\( \kappa \), the dynamics of \( S_p \) will be modified by the appearance of \( \Delta S \) as well as the phase term \( \theta_p \). Since the magnitude of the peak output is directly measurable, a further investigation of the peak photon intensity and the statistics of time-difference quantity (\( \Delta S \)) will be fruitful. With oscilloscope, we repeatedly accumulate the time series of laser output and pick up the discrete peak value \( S_p(n) \), \( n = 1, 2, \ldots \) where \( n \) denotes the \( n \)th peak. To have a reliable probability, total 320,000 peaks have been collected for average at any specific rotation angle of polarizer. As the polarizer is rotated, the feedback strength will be changed. Asymptotically, as \( \kappa \to 0 \), the system features a free-running laser such that the mean and the standard deviation are small. However, there is a dramatic increase in both of the mean and the standard deviation around 41-42 degree of polarizer’s angle which implies the onset of transition. To further identify the transition, we evaluate the probability distribution of peak powers at different feedback strength (equivalent to the different polarizer’s angle). In the regime of noise-driven relaxation oscillation, the probability distribution of peak power \( P(S_p) \) follows an exponential law which features a shot-noise characteristics (as shown in Fig.3 (a)). On the other hand, in the case that chaotic bursting occurred, a tailed probability distributions will be created. If we pay attention only on the part of large intensity, by which
the exponential distribution can be neglected, a Gaussian distribution can be recognized as feedback strength is further increased as shown in Fig. 3 (c). This shows that there are two dynamical states which follows different statistics. Dynamical transition from a simple exponential distribution to a mixed distribution does occur as shown in Fig. 3 (a)-(c).

What is the influence of the onset of such a mixed distribution? By a joint probability analysis similar that used in [8], it can be identified that there is still a strong overlapping between the two probability distributions. This suggests that the interplay between two states may be rather unique where the statistics of time-difference should be crucial also as discussed above. This also means that we should look at the dynamical behavior of a $k$-step difference quantity,

$$\Delta S_p(k) = S_p(k + l) - S_p(l),$$

where $S_p(l)$ is the $l$th peak power. After the summation over the whole range of $l$, the probability $P(\Delta S_p(k))$ specifies the variation distribution with $k$-step difference. Consider that when the system is with noise-driven relaxation oscillation, the variation distributions should be the same no matter what is the value of $k$ since the switching characteristic is stationary. The difference can be characterized by a $\chi^2$ statistics which is defined as

$$\chi^2(j; k) = \sum_i^M \frac{(R_i - S_i)^2}{(R_i + S_i)},$$

where $R_i$ and $S_i$ are the probabilities of the $i$th interval of $\Delta S_p$ for $P(\Delta S_p(k + j))$ and $P(\Delta S_p(k))$, respectively. The summation is carried out for all intervals except $R_i = S_i = 0$. For a fixed $j$, the $\chi^2(j; k)$ indicates the similarity of the distribution between the variations $\Delta S_p(k)$ and $\Delta S_p(k + j)$. Foremost of all, the most crucial quantity is the successive change on similarity, i.e., $j = 1$. A large value in $\chi^2(1; k)$ means that at time $k$ the dynamics has been switched to a state which follows a dramatically different statistical distribution and this suggests a strong interplay between states. Furthermore, when the value of $\chi^2(1; k)$ is wildly varied as $k$ moved a non-stationary dynamics will be intrinsic in nature. The degree of non-stationarity of the whole process can be quantified by an average quantity, i.e.
\[ \chi^2(1) = \frac{1}{L} \sum_{k=1}^{L} \chi^2(1; k). \] (5)

In principle, the correlation of variation distributions is essentially stationary as \(\chi^2(1) \approx 0\) \[4\], i.e., there is no dramatic dynamical changes on the correlation of the variation distributions in the range of average. Shortly, one can explore the dynamical characteristics of switching with this \(\chi^2\) statistics. Let us next address our experimental results. As shown in Fig.\[4] (a) where \(L = 200\), \(\chi^2(1)s\) are almost close to zero for the relaxation oscillation regime as expected. The interesting point is the appearance of high \(\chi^2(1)\) at the medium strength of feedback (around 40° of polarizer’s angle), which also corresponds to the transition indicated by the probability distribution, the mean, and the standard deviation. This shows that the transition is associated with a non-stationary characteristic and, thus, the successive change on similarity is wild. It remains a surprise that for a larger feedback strength (small polarizer’s angle), \(\chi^2(1)\) is still almost zero again. This peculiar feature is due to the observation that an increase of feedback strength causes an increase of staying times at particular state and, as a result, the successive change on similarity will become smooth. In more details, Fig.\[4] (b) presents the degree of probability association, \(\chi^2(1; k)\). For some particular feedback strength, \(\chi^2(1; k)\) never keeps near zero, which suggests the instability has a strong non-stationary switching. On the other hand, in the regime of large polarizer’s angle (noise-driven relaxation oscillation), the \(\chi^2(1; k)\) is almost zero (Fig.\[4] (c)).

Here, we would like to emphasize that the transition indicated by a high \(\chi^2(1)\) is crucial and it reveals a rather amazing characteristics: a weak-feedback induced instability can even be associated with a wild and non-stationary successive change on the similarity of variation probability distributions.

Let us discuss the origin of such chaotic bursting and even non-stationary characteristics in such an instability. Referring to Eq.\[1\] and Eq.\[2\], since the dc value of the photon density \(S(t)\) is never zero, the term \(\cos\theta(t)\) sometimes has to be zero such as to terminate the feedback process. In such a way, the noise-driven relaxation oscillation can be created with non-zero \(\kappa\). This means that a fixed phase difference \((\theta \approx \pi/2)\) during the period of
relaxation oscillation has to be established. Therefore, the persistence in fixed phase difference is essential for switching back to the relaxation oscillation and should be critical for the overlapping in joint-probability as well as the non-stationary switching. In other words, the random phase fluctuation due to the feedback effect is essential for the chaotic bursting behavior. Hence, an inclusion of fast frequency-modulation characteristics (FM noise) to the phase fluctuation is necessary in the process as experimentally suggested from the measurement of the multiwavelength meter. Indeed, under this condition, the simulation of the single-mode Lang-Kobayashi model can reproduce the chaotic bursting features reported above. Finally, we would like to note that the similar instability has also been observed with various fiber lengths (7m, 5m, 4m, and 3m) as well as multimode fiber. Moreover, in stead of polarizer, a N.D. filter is also employed and similar results are concluded. The further details as well as the analysis of $\chi^2(j)$ with large-$j$ will be reported elsewhere.

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FIGURES

FIG. 1. Typical (a) ac time series and (b) its corresponding rf-spectrum for the case of free-running.

FIG. 2. (a) The time series of chaotic bursting behavior due to the feedback effect and its corresponding spectra according to the joint-time frequency analysis on (b) the regime I, (c) the regime II, and (d) the regime III, respectively.

FIG. 3. The probability distribution of peak powers at different feedback strengths indicated by the degrees of rotation of the polarizer at (a) 54°, (b) 41°, and (c) 0°.

FIG. 4. (a) The average probability association $\chi^2(1)$ at different feedback strengths (different angles of the polarizer) and the probability association $\chi^2(1;k)$ versus different $k$ at (b) 40° and (c) 54° of polarizer’s angles.
(b)
(c) $0^\circ$
\[ \chi^2(1) \]

Polarizer Angle
\( \chi^2(1; k) \) vs. \( k \) for \( 40^\circ \)
