The design of mathematics learning using didactical engineering to develop the mathematical comprehension ability and self-confidence of elementary students

R A Silo\textsuperscript{1}, T Herman\textsuperscript{2} and A Jupri\textsuperscript{3}

\textsuperscript{1}SD Negeri 5 Melayu, Muara Teweh, Indonesia
\textsuperscript{2,3}Universitas Pendidikan Indonesia (Departement of Mathematics Education), Bandung, Indonesia

E-mail: ropal@student.upi.edu

Abstract. This research was based learning obstacles experienced by the students of elementary school. The obstacles were categorized into three learning obstacles namely epistemological, ontological, and didactical. The focuses of this research were to analyze the learning obstacles in addition, arrange the hypothetical learning trajectory, and design a learning material suitable to the students’ characteristics using didactical engineering. The design was considered to be able to set a learning environment setting which would minimize learning obstacles. The research used qualitative approach with descriptive analysis method. The subject was 27 first-grade students of an elementary school in Bandung. The data were collected using test instrument which was essay problems and non-test instruments which were observation sheets, interview, documents study, and both audio and audio-visual documentations. The research results showed that there were development of mathematical comprehension ability and self confidence in the majority of students following learning design implementation based on the learning environment setting involving situation of action, formulation, validation, and institutionalization.

1. Introduction
In general, mathematics learned by students since 1\textsuperscript{st} grade of elementary school. It means that students in elementary school are already prepared to enter the world of mathematics that has its own difficulties at each education level that they will pass. Early elementary school students learning experience could determine whether they would like or dislike mathematics lessons. The lack of the students’ understanding ability at early level will have an impact on concept mastery in the next material, thus emphasize the importance of mathematical understanding skills as one of the things that students must master. The low ability of students’ understanding in mathematics will have an impact on students' attitudes toward mathematics and vice versa.

White’s research obtained information that around 70% of errors that students at age 7 made in working out a math problem is on the level of understanding [1]. Student’s failure to comprehend mathematics could result in inability to complete daily tasks. This is interconnected with results of preliminary observation and interview towards first grade students in a primary school in Bandung. The results of these observations informed that the students were confused when doing addition of numbers two times in a row. On another question, students were confused when adding up the natural number
one digit with zero. Students assumed that the numbers 0 (zero) has a value of 1. Interview of the homeroom teacher revealed that in the process of delivering the material about the number zero, students are only given an explanation in the form of illustrations that leads to the definition numbers of zero without concrete activities carried out by the students.

The above findings informed that the importance of an activity which can provide experience for the students. This activity will encourage and give instruction of situation to students in doing their activities that provide hands on activity. Therefore, before designing a teaching program that reflects the student, the researcher firstly should analyse the source of the problems that exist in the form of learning obstacle to the ability of understanding mathematical students. The analysis result of the learning obstacle becomes the basis for drawing up a learning trajectory which is suitable to the characteristics of students and then plan the learning design by using didactical engineering [2].

The use of didactical engineering is intended to develop the ability of mathematical understanding students based on the results of learning obstacle analysis that found previously. Didactical engineering is a learning design that was developed specifically based on Brousseau, which is useful for building and assessing a situation and its effectiveness [3].

Based on the problems, this research is focused on investigating the hypothetical learning trajectory design in the addition numbers up to 99 in 1st grade of elementary school and the implementation results of learning design using didactical engineering in developing the ability of mathematical understanding students.

2. Literature Review

2.1. The ability of mathematical understanding

The ability of understanding according to Uno and Koni is a person's ability in deciphering, interpreting, translating, or representing something in its own way according to the knowledge he had received [4]. Understanding depends on the idea that has been owned and depends on the creation of new relationships between ideas. The ability of understanding is one of the ability mathematically to the students. With the ability of mathematical understanding then it will facilitate students to master mathematical concepts on the next material [5].

To measure the ability of mathematical understanding, these are the indicators that can be used, namely: (a) identify and create examples and not examples; (b) translate and interpret the meaning of symbols, tables, diagrams, pictures, charts, as well as the sentence mathematically; (c) understand and apply mathematical ideas; and (d) make an estimation [6].

2.2. Didactical engineering (DE)

According to the Theory of Didactical Situations all mathematical knowledge can be taught by positioning students in the right situation, where there is interaction among the three elements, namely students, teachers, and milieu [7]. Milieu is all material objects, knowledge, and interaction with other people during the activity of learning [2]. Based on the Theory of Didactical Situations, the teacher can develop the right design at the bottom of the situation which is controlled in accordance with the conditions of the existing classes. The design here is didactical engineering. Douady defines didactical engineering as a teacher-engineer associated with an action in a didactic and ensure about the implementation of project learning with a group of students [8]. Other definitions are also expressed by Artigue which states that didactical engineering is an engineering didactic for action-based research on systems of education with the metaphor the teacher is seen as an engineer [9].

Didactical engineering is composed of three stages, namely: 1) analysing the introduction, where the theoretical framework and the research objectives are set; 2) designing and setting the analysis a priori in a learning design; and 3) implementing the design that has been designed, make the a posteriori analysis of the design that has been implemented, then draw a conclusion overall of all the stages [10]. Didactical engineering is based on the findings of the learning obstacle. Brousseau suggests three factors that can cause obstacles or learning difficulties experienced by students during the learning process [2], namely:
Ontogenical learning obstacle is the difficulty of learning in terms of psychological where students are experiencing learning difficulties due to the factor of mental readiness, in this way of thinking students who have not entered due to age factor and level of development [11-13].

Didactical learning obstacle is a learning difficulty that occurs because of mistakes presentation of teaching material so that lead to misconceptions and can be also derived from the provision of false concepts or teaching concepts that are not suitable to the readiness of the child [14].

Epistemological learning obstacle is the difficulty of learning that occurs because of incompleteness student understanding about a concept, only seeing the origins. The presence of obstacle epistemological is to identify the learning process that needs some settings related to productivity on the expected students ability [15].

2.3. Hypothetical Learning Trajectory (HLT)
In a lesson planning, teachers needs to make a prediction about how the possibility of students mathematics learning in particular. Prediction is concerned with how thinking skills and the ability understanding students will develop on the learning activities [16]. The prediction of the process of student learning is called the Hypothetical Learning Trajectory (HLT). Wijaya mention that HLT is the hypothesis of the learning process students which is useful for designing action or alternative strategies to cope with the various problems that may be faced by students in the learning process [17]. HLT compiled should be based on the design learning that can facilitate the ability of the students in according to their learning stages [18, 19].

3. Research Methods

3.1. Method and research design
The method of research is the qualitative method. The use of this method is intended to explore the phenomena that can not be quantified and descriptive. Research design has used didactical engineering. The design of didactical engineering is designed in such a way as to build and assess a situation and effectiveness [3].

3.2. Participants and research areas
The participants in this research were students of 1st grade at the elementary school in the Bandung city. Preliminary research was conducted in October-December 2016. Advanced research was conducted on odd and even semester of the academic year 2016/2017.

3.3. Research instrument
This research used both instrument test and non-test. The instrument test is used to measure the ability of mathematical understanding students on the concept understanding aspects and the understanding of the mechanical. This instrument was in the form of a description in the addition numbers up to 99. The non-test instrument was observation, interviews to teachers and students, the documentation study on the device learning (analysis of teaching materials and documents of lesson plan), and documentation in audio or audio-visual to record information unnoticed by researcher observation and also to document the implementation design that was designed.

3.4. Data analysis
Data analysis was performed since data collection was held and after completed data collection within a certain period. Data analysis was done from the answer sheets of students which contained mathematical understanding test on theory of the addition numbers up to 99. Each student was interviewed to confirm the answers. The interview result was made on the transcripts to facilitate the categorization of learning obstacle that emerge.
4. Results

4.1. The ability of mathematical understanding preliminary review
In the beginning of the review on the aspect of mathematical understanding ability, the formation of students’ understanding of new concepts seemed very diverse. It was influenced by learning activities of students outside of school. The results of interview with the homeroom teacher informed that most students took ‘les calistung’ (red: read, write, and count) in kindergarten. On the few occasions when students were exposed to concepts that are new, previous knowledge as if it were ‘not useful’. Whereas the initial capital (knowledge) owned by students can be used as a foothold in gaining new knowledge.

The first part of the research still used the traditional approach such as the process of drill and practice in learning activities. In addition, the evaluation of learning which emphasizes learning outcomes and less attentive to the process of the learning activity itself. This is what ultimately makes learning math become less meaningful or less ‘stuck’ for students. In addition, the learning process also only rely on text books and less related to everyday life. Students become less understand the mathematical concepts and the difficulty of applying or relating in everyday life. Level of thinking elementary students are still concrete, so it is so contrast with the characteristics of mathematics itself, where the object is abstract. This abstract is seen as a ‘gap’ that causes many students experiencing difficulty in mathematics.

4.2. Curriculum and teaching materials review
The analysis results showed a difference in some theories in the teaching materials with the curriculum used. This difference can trigger the emerge of learning obstacle. The differences such as seen in figure 1 below.

![Figure 1. The emergence of learning obstacle on the teaching materials](image)

Figure 1 above showed that in question number five the results of the addition are more than 100. If we look at the 2013 curriculum on the knowledge competency (KI-3) and skill competency (KI-4) in point 3.2 it is mentioned that “get to know the original number until 99 by using the objects around home, school, or playground” and on point 4.1 mentioned that “parse a whole number up to 99 by using existing objects around the home, school, or a place to play” [20]. This indicates there were a mismatch between the teaching materials and the content material in the curriculum, so it could trigger the emergence of didactical learning obstacle. Therefore, the teacher roles are to sort out the material according to the standard competency and basic competency as well as the characteristics of children is required.

4.3. The respondent ability tests analysis
Respondent Ability Test (RAT) consisted seven grains of questions including addition of numbers up to 99. The analysis results of the students response to the RAT questions showed that students generally encounter errors in completing the RAT questions it. Students response of AMR is shown in figure 2 below.
Figure 2. Student responses to AMR

The fulfilment process that occurred by the AMR students (see figure 2) is described as in the transcript extract of the interview in below.

Researcher : “43 plus 38 of these, how is the result?” (pointed the addition in the answer sheet of the students)
AMR : “Six two one, Sir.”
Researcher : “How did you get the six two one?”
AMR : “3 plus 8 is not enough, so 3 borrow 1 by 4 so that it became 13. Then 13 is added to 8 the result became 21, Sir”.
Researcher : “So, Is it 21 directly written at the bottom?”
AMR : “Yes”.
Researcher : “And next, how can you get 6?” (point to the number 6 which is the result of addition)
AMR : “This was 4 borrowed 1 so that it became 3, Sir. Then 3 added by 3 became 6, Sir!”

Based on students response in general on RAT’s questions, we understood there’s a need to plant early concepts summation with the use of concrete objects and invite students to do mathematics through a hands on activity. Activities through concrete objects will help students to understand the abstract concept of the addition in the form of numbers or symbols by concretized through abstract objects [21]. Once students began to understand the summation with the use of concrete objects, teachers changed over to the semi-concrete objects such as the use of the emblem, symbol, or image.

4.4. Hypothetical Learning Trajectory (HLT) plan

Hypothetical Learning Trajectory (HLT) in the addition of numbers up to 99 is designed based on the analysis results of the emergence learning obstacle seen in the preliminary research. The HLT design is addition of numbers up to 10 and sum to zero; the addition of numbers up to 20 and the addition in everyday life; the addition of numbers up to 50; the addition of numbers up to 99 and the addition of numbers by the saving technique; and the addition of numbers up to 99 in everyday life.

4.5. Instructional design plan

Instructional design packed into five times meeting. Every instructional design consists of learning purposes had different competencies that are expected from each design. In instructional design-1 the learning purposes are to the addition of numbers up to 10 and addition of numbers with a zero. In instructional design-1 this didactic situation is divided to three steps and each step contained a priori analysis or prediction of the students’ response as well as the didactic anticipation.

In instructional design-2 the learning purposes to be achieved is the addition of numbers up to 20 and the addition of numbers in everyday life. The activities in the instructional design-2 this consists of four didactic situations. Every didactic situation are contained a priori analysis and the didactic anticipation. On the instructional design-3 the purpose of learning to be achieved is the addition of numbers up to 50. In the instructional design-3 there are four didactic situations that will be developed. Every didactic situation designed contained a priori analysis and the didactic anticipation. On the instructional design-
4 the purpose of learning would be achieved successfully if the student can do the addition of numbers up to 99 and students can do the addition by the saving technique. To develop the ability of mathematical understanding student, there are four didactic situations that will be developed based on the learning purposes to achieved. Every didactic situation designed contained a priori analysis and the pedagogical didactic anticipation.

In instructional design-5 of learning purposes to be achieved is the student able to do the addition of numbers up to 99 in everyday life. To develop the students’ ability of mathematical understanding, in instructional design-5 there are three didactic situations that will be developed. The development of the situation didactic is based on the learning purposes to be achieved. Every didactic situation contained a priori analysis and the didactic anticipation.

4.6. A posteriori analysis and validation
After the design plan is implemented, the next step is a posteriori analysis to the implementation of all the design was reviewed from the ability aspect of mathematical understanding. The analysis results informed that the ability of mathematical understanding students began to develop when compared by the review in a preliminary research on the same aspect. Indicators the students’ ability of mathematical understanding and self-confidence in general have been achieved. This is seen by the student responses that occur during learning, than written responses form of the group answers results on the group worksheet or student answers on the student worksheet and student responses orally if they expressed the process or the results that they have done in each didactic situation. Furthermore, the development of mathematical understanding is also seen by the observations results made by the teacher and the other two observers.

In the implementation process of the learning design still appear the obstacle to epistemological on the stage of action, formulation, and institutionalization on a small portion of students joined in the group. At the stage of validation, the results of the a priori analysis is compared by the results of the a posteriori analysis. If the item is on the a priori analysis suitability by the a posteriori analysis then it would be otherwise predictable. On the contrary if the item is on the a priori analysis contradictory with the a posteriori analysis it will be expressed is not predictable.

The validation results shown in the lesson plan-1 and lesson plan-2 there were some situations didactic that was not predictable, especially in the second, third, and fourth didactic situation. This is seen by the students response was different from the prediction that were at this stage a priori. The results are seen in post implementation of the lesson plan-1 and lesson plan-2 were analyzed and used as material considerations to designed the next lesson plan, so that the things that can trigger the emerge of learning obstacle can be avoided. The analysis results also to see the effectiveness of didactic situation that has been designed.

5. Conclusion and Suggestion

5.1 Conclusion
Hypothetical learning trajectory design in addition of numbers up to 99 are prepared based on the analysis results of findings the learning obstacle that later became the basis in the preparation of instructional design. The implementation results of didactical engineering design showed that there are development of mathematical understanding on majority of students, both at the stage of action, formulation, validation, and institutionalization. The ability of mathematical understanding on students is seen to support in each other towards the acquisition of knowledge on the concept further.

5.2 Suggestion
The didactical engineering design is an engineering didactic that highly effected by the characteristics of the students and the learning environment (milieu) that are typical of each school, so for the teachers/education practitioners who want to develop this design should be able to adapt to the characteristics of students in the didactical engineering design will be implemented.
6. References

[1] White A L 2010 Numeracy, literacy, and newman's error analysis J. of Sci. and Math. Edu. in Southeast Asia 33 2

[2] Brousseau G 2002 Math. Edu. Library 19

[3] González-Martín A S, Bloch I, Durand-Guerrier V and Maschietto M 2014 Didactic situations and didactical engineering in university mathematics: cases from the study of calculus and proof Research in Mathematics Education 16 2

[4] Uno H B and Koni S 2012 Assessment Pembelajaran: salah satu bagian penting dari pelaksanaan pembelajaran yang tidak dapat diabaikan adalah pelaksanaan pembelajaran (Jakarta: Bumi Aksara)

[5] Hiebert J and Carpenter T P 1992 Handbook of Research on Mathematics Teaching and Learning ed D A Grouws (Reston: NCTM)

[6] Lestari K E and Yudhanegara M R 2015 Mathematical education research (Bandung: PT. Refika Aditama)

[7] Artigue M 2009 Didactical Design in Mathematics Education ed C Winslow (Rotterdam: Sense Publishers) pp 7-16

[8] Canu M, Duque M and Hosson C D 2016 Active learning session based on didactical engineering framework for conceptual change in students' equilibrium and stability understanding European J. of Engineering Edu 10

[9] Balacheff N 1990 Towards a problematique for research on mathematics teaching J. for Research in Math. Edu. 21 4 pp. 258-72

[10] Daro P and Mosher F 2011 Learning Trajectories in Mathematics: A Foundation for Standards; Curriculum, Assessment and Instruction (New York: CPRE)

[11] Wijaya A 2009 Proc. of National Conference of Mathematics (Yogyakarta: Universitas Negeri Yogyakarta)

[12] Clements D H and Sarama J 2009 Learning Trajectory in Early Mathematics-Sequences of Acquisition and Teaching (Canada: Canadian Language and Research Network)

[13] Clements D H and Sarama J 2004 Learning trajectory in mathematics Education Mathematical Thinking and Learning 6 2 pp. 81-9

[14] Dayal A and Hendrata H 2014 Matematika 1 (Bogor: PT. Quanda Inti Solusi)

[15] Georges C, Hoffmann D and Schiltz C 2017 Mathematical abilities in elementary school: do they relate to number-space associations? J. of Experimental Child Psyc. 161 pp 126-147