Supersymmetric dark energy

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Abstract

We study a noninteracting supersymmetric model in de Sitter spacetime. A soft supersymmetry breaking induces a nonzero vacuum energy density. A short distance cut-off of the order of Planck length provides a matching between the vacuum energy density and the cosmological constant related to the de Sitter expansion parameter.

1 Introduction

It is generally accepted that the cosmological constant term which was introduced ad-hoc in the Einstein-Hilbert action is actually related to the vacuum energy density of matter fields. The vacuum energy density estimated in a simple quantum field theory is by about 120 orders of magnitude larger than the value required by astrophysical and cosmological observations. On the other hand, in a field theory with exact supersymmetry the vacuum energy, and hence the cosmological constant (CC), is equal to zero as the contributions of fermions and bosons to the vacuum energy precisely cancel [1].

A nonzero CC implies the de Sitter symmetry group of spacetime rather than the Poincaré group which is the spacetime symmetry group of an exact supersymmetry. Based on observational evidence for an accelerating expansion [2, 3, 4], the vacuum energy density dominates the total energy density today. Hence, the spacetime today is close to de Sitter approaching asymptotically a de Sitter universe with metric

\[ ds^2 = dt^2 - a(t)^2 d\vec{x}^2, \]  

where \( a = e^{Ht} \) and \( H \) is a constant. This metric describes empty space with cosmological constant \( \Lambda = 8\pi G \rho_\Lambda \), where the vacuum energy density \( \rho_\Lambda \) is related to \( H \) by the Friedman equation:

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_\Lambda. \]  

The structure of de Sitter spacetime automatically breaks the supersymmetry [5]. Conversely, a low energy supersymmetry breaking could in principle generate a nonzero CC.

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of an acceptable magnitude. Unfortunately, the scale of supersymmetry breaking required by the particle physics phenomenology must be of the order of 1 TeV or larger implying a CC too large by about 60 orders of magnitude. However, some nonsupersymmetric models with equal number of boson and fermion degrees of freedom have been constructed \[6\] so that all the divergent contributions to the vacuum energy density cancel and a small finite contribution can be made comparable with the observed value of the CC.

In this paper we investigate the fate of vacuum energy when an unbroken supersymmetric model is embedded in asymptotically de Sitter spacetime. We do not claim that our model describes a realistic scenario but it is tempting to speculate along the following lines. Large scale observations reveal that dark and baryonic matter gravitationally cluster occupying relatively small volume of space in the form of poor and reach clusters connected by filaments and sheets \[7\]. Most of the volume is occupied by large scale voids almost empty of both baryon and dark matter.

Our working assumption is that voids contain no matter apart from fluctuations of a supersymmetric vacuum as a relict of symmetry breaking in the early universe. The early universe with exact supersymmetry underwent a set of symmetry breaking phase transitions. Before the supersymmetry breaking, domains with different vacua may have been formed with domain walls separating the domains \[8\]. Then, at a later time, supersymmetry breaking took place in some if the domains, which thereafter remained populated by dark and baryonic matter. Our main assumption is that supersymmetry remains unbroken in the voids. However, since the global geometry is de Sitter, the lack of Poincare symmetry will lift the Fermi-Bose degeneracy and the energy density of vacuum fluctuations will be nonzero. This type of “soft” supersymmetry breaking is similar to the supersymmetry breaking at finite temperature where the Fermi-Bose degeneracy is lifted by quantum statistics \[9, 10\].

The remainder of the paper is organized as follows. In section 2 we describe our model. The calculations and results are presented in section 3 and concluding remarks in section 4.

2 The model

Here we consider a noninteracting Wess-Zumino supersymmetric model with \(N\) species and calculate the energy density of vacuum fluctuations in de Sitter spacetime. In general, the supersymmetric Lagrangian \(\mathcal{L}\) for \(N\) chiral superfields has the form \[11\]

\[
\mathcal{L} = \sum_i \Phi_i^\dagger \Phi_i |_{D} + W(\Phi)|_{F} + \text{h.c.} ,
\]

where the index \(i\) distinguishes the various left chiral superfields \(\Phi_i\) and \(W(\Phi)\) denotes the superpotential for which we take

\[
W(\Phi) = \frac{1}{2} \sum_i m_i \Phi_i \Phi_i .
\]

Eliminating auxiliary fields by equations of motion the Lagrangian \(\mathcal{L}\) may be recast in the form

\[
\mathcal{L} = \partial_\mu \phi_i^\dagger \partial^\mu \phi_i - m_i^2 |\phi_i|^2 + \frac{i}{2} \bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i - \frac{1}{2} m_i \bar{\Psi}_i \Psi_i ,
\]
where $\phi_i$ are the complex scalar and $\Psi_i$ the Majorana spinor fields. For simplicity, from now on we suppress the dependence on the species index $i$.

Next we assume a curved background spacetime geometry with metric $g_{\mu\nu}$. Spinors in curved spacetime are conveniently treated using the so called vierbein formalism. The metric is decomposed as

$$g_{\mu\nu}(x) = e^a_\mu e^b_\nu \eta_{ab},$$

where the set of coefficients $e^a_\mu$ is called the vierbein. The action may be written as

$$S = \int d^4x \sqrt{-g} (L_B + L_F),$$

where $L_B$ and $L_F$ are the boson and fermion Lagrangians, respectively. The Lagrangian for a complex scalar field may be expressed as the sum of the Lagrangians for two real fields

$$L_B = \frac{1}{2} \sum_{i=1}^2 \left( g^{\mu\nu} \phi_i^\mu \phi_i^\nu - m^2 \phi_i^2 \right).$$

The fermion part is given by

$$L_F = \frac{i}{4} \left( \bar{\Psi} \tilde{\gamma}^\mu \Psi \mu - \bar{\Psi} \gamma^\mu \Psi \right) - \frac{1}{2} m \bar{\Psi} \Psi,$$

where $\tilde{\gamma}^\mu$ are the curved spacetime gamma matrices

$$\tilde{\gamma}^\mu = e^a_\mu \gamma^a,$$

with ordinary Dirac gamma matrices denoted by $\gamma^a$, and $e^a_\mu$ is the inverse of the vierbein. The covariant derivatives of the spinor are defined as

$$\Psi_{;\mu} = \Psi_{,\mu} - \Gamma_{\mu} \Psi,$$

$$\bar{\Psi}_{;\mu} = \bar{\Psi}_{,\mu} + \bar{\Psi} \Gamma_{\mu},$$

where

$$\Gamma_{\mu} = \frac{1}{8} \omega_{\mu}^{ab} [\gamma^a, \gamma^b],$$

with the spin connection

$$\omega_{\mu}^{ab} = -\eta^{bc} e_c^\nu (e^a_\nu,_{\mu} - \Gamma^\lambda_{\mu\nu} e^a_\lambda).$$

In FRW metric the vierbein is diagonal and in spatially flat FRW spacetime takes a simple form

$$e^a_\mu = \text{diag}(1, a, a, a)$$

where $a = a(t)$ is the cosmological expansion scale.
3 Calculation of the vacuum energy density

It is convenient to work in the conformal frame with metric

\[ ds^2 = a(\eta)^2 (d\eta^2 - d\vec{x}^2). \]  

where the proper time \( t \) of the isotropic observers is related to the conformal time \( \eta \) as

\[ dt = a(\eta) d\eta. \]  

In particular, we will be interested in de Sitter spacetime with

\[ a = e^H t = -\frac{1}{H \eta}. \]  

In order to calculate the energy density of the vacuum fluctuations we need the vacuum expectation value of the Hamiltonian. The Hamiltonian may be expressed as the sum of the boson and fermion parts

\[ H = H_B + H_F. \]  

From (7-9) with metric (16) we obtain

\[ H_B = \sum_{i=1}^{2} \left( \frac{1}{2a^2} (\partial_\eta \varphi^i)^2 + \frac{1}{2a^2} (\nabla \varphi^i)^2 + m^2 \varphi^i \right), \]  

\[ H_F = -i \frac{1}{4a^4} \left( \bar{\psi} \gamma^j \partial_j \psi - (\partial_j \bar{\psi}) \gamma^j \psi \right) + \frac{1}{2a^3} m \bar{\psi} \psi. \]  

Consider first the contribution of the scalar fields. Each real scalar field operator is decomposed as

\[ \varphi(\eta, \vec{x}) = \sum_k a^{-1} \left( \chi_k(\eta) e^{i\vec{k} \vec{x}} a_k + \chi_k^*(\eta) e^{-i\vec{k} \vec{x}} a_k^\dagger \right), \]  

where \( a_k \) and \( a_k^\dagger \) are the annihilation and creation operators, respectively. The function \( \chi_k \) satisfies the field equation

\[ \chi_k'' + (m^2 a^2 + k^2 - a''/a) \chi_k = 0, \]  

where \( ' \) denotes a derivative with respect to the conformal time \( \eta \). In massless case the exact solutions to this equation may easily be found \[13\]. In particular, in de Sitter spacetime \( a''/a = 1/\eta^2 \), and one finds positive frequency solutions \[14\]

\[ \chi_k = \frac{1}{\sqrt{2V_k}} e^{-ik\eta} \left( 1 - \frac{i}{k\eta} \right). \]  

The operators \( a_k \) associated to these solutions annihilate the adiabatic vacuum in the asymptotic past (Bunch-Davies vacuum) \[13\] \[15\].

If \( m \neq 0 \) solutions to (23) may be constructed by making use of the WKB ansatz

\[ \chi_k(\eta) = \frac{1}{\sqrt{2V aW_k(\eta)}} e^{-i \int^\eta aW_k(\tau) d\tau}, \]  

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where the function $W_k$ may be found by solving (23) iteratively up to an arbitrary order in adiabatic expansion \[12\]. For our purpose we need the solution up to the 2nd order only which reads
\[
W_k = \omega_k + \omega^{(2)},
\]
where
\[
\omega_k = \sqrt{m^2 + k^2/a^2}.
\]
The general expression for the second order term is \[12\]
\[
\omega^{(2)} = \frac{3}{8} \frac{\dot{a}^2}{\omega_k a^2} - \frac{3}{4} \frac{\ddot{a}}{\omega_k} - \frac{3}{4} \frac{k^2 \dot{a}^2}{\omega_k^2 a^2} + \frac{1}{4} \frac{k^2 \ddot{a}}{\omega_k^3 a} + \frac{5}{8} \frac{k^4 \dot{a}^2}{\omega_k^4 a^2},
\]
where overdot denotes a derivative with respect to $t$. For de Sitter spacetime, using \[18\] we obtain
\[
W_k = \omega_k - \frac{H^2}{\omega_k} \left[1 + \mathcal{O}(m^2/\omega_k^2)\right],
\]
We can calculate now the vacuum expectation value of the boson Hamiltonian. Using the properties of $a$ and $a^\dagger$ and replacing the sum over momenta by an integral in the usual way
\[
\sum_k = V \int \frac{d^3 k}{(2\pi)^3},
\]
from \[22\] we find
\[
<\mathcal{H}_B> = \frac{V}{a^4} \int \frac{d^3 k}{(2\pi)^3} \left(|\chi_k'|^2 + a^2 \omega_k^2 |\chi_k|^2\right).
\]
Using \[25\] with \[29\] we obtain
\[
<\mathcal{H}_B> = \frac{1}{a^3} \int \frac{d^3 k}{(2\pi)^3} \left[\omega_k + \frac{1}{2} \frac{H^2}{\omega_k} \left(1 + \frac{m^2}{\omega_k^2} - \frac{H^2}{\omega_k^2} + \mathcal{O}(\omega_k^{-4})\right)\right].
\]
The first term in square brackets is identical to the flat spacetime result. The second term is a quadratically divergent contribution due to de Sitter geometry, the next two terms are logarithmically divergent, and the rest is finite.

Next we proceed to quantize the fermions. The Dirac equation in curved spacetime may be derived from \[9\]. Specifically for a spatially flat FRW metric we obtain
\[
i\gamma^0 \left(\partial_0 + \frac{3}{2} \frac{\dot{a}}{a}\right) \Psi + i \frac{1}{a} \gamma^j \partial_j \Psi - m \Psi = 0.
\]
It is convenient to rescale the Majorana fermion field $\Psi$ as
\[
\Psi = a^{-3/2} \psi.
\]
and introducing the conformal time we obtain for $\psi$ the usual flat spacetime Dirac equation
\[
i\gamma^0 \partial_\eta \psi + i \gamma^j \partial_j \psi - am \psi = 0,
\]
with time dependent effective mass $am$. The quantization of $\psi$ is now straightforward \[16, 17\]. The Majorana field $\psi$ may be decomposed as usual

$$
\psi(\eta, \vec{x}) = \sum_{\vec{k}, s} (u_{ks}(\eta)e^{i\vec{k}\vec{x}}b_{ks} + v_{ks}(\eta)e^{-i\vec{k}\vec{x}}b_{ks}^\dagger),
$$

(36)

where the spinor $u_{ks}$ may be expressed as

$$
u_{ks} = \frac{1}{\sqrt{V}} \left( (i\zeta_k' + am\zeta_k)\phi_s \right). 
$$

(37)

Here, the two-spinors $\phi_s$ are the helicity eigenstates which may be chosen as

$$
\phi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \phi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
$$

(38)

The spinor $v_{ks}$ is related to $u_{ks}$ by charge conjugation

$$
v_{ks} = i\gamma^0\gamma^2(\bar{u}_{ks})^T.
$$

(39)

The norm of the spinors may be easily calculated

$$
\bar{u}_{ks}u_{ks} = -\bar{v}_{ks}v_{ks} = \frac{1}{V}(am\zeta_k^* - i\zeta_k')(am\zeta_k + i\zeta_k') - \frac{1}{V}k^2|\zeta_k|^2.
$$

(40)

The mode functions $\zeta_k$ satisfy the equation

$$
\zeta_k'' + (m^2a^2 + k^2 - ima')\zeta_k = 0.
$$

(41)

In addition, the functions $\zeta_k$ satisfy the condition \[17\]

$$
k^2|\zeta_k|^2 + (am\zeta_k^* - i\zeta_k')(am\zeta_k + i\zeta_k') = C_1^2.
$$

(42)

It may be easily verified that the left-hand side of this equation is a constant of motion of equation \[11\]. The constant $C_1$ is fixed by the normalization of the spinors and by the initial conditions. A natural assumption is that at $t = 0$ ($\eta = -1/H, a = 1$) the solution behaves as a plane wave $\zeta_k = C_2e^{-iEt}$, where $E = \sqrt{k^2 + m^2}$. This gives $\zeta_k(0) = C_2$, $\zeta_k'(0) = -iC_2E$, and hence $C_1^2 = 2C_2^2E(m + E)$. From (40) and (42) we obtain

$$
\bar{u}_{ks}u_{ks} = -\bar{v}_{ks}v_{ks} = \frac{1}{V}(C_1^2 - 2k^2|\zeta_k|^2),
$$

(43)

which at $t = 0$ reads

$$
\bar{u}_{ks}u_{ks} = -\bar{v}_{ks}v_{ks} = C_1^2 \frac{m}{VE}.
$$

(44)

For $C_1^2 = 1$ this coincides with the standard flat spacetime normalization \[13\].

In massless case the solutions to \[11\] are plane waves. For $m \neq 0$ two methods have been used to solve \[11\] for a general spatially flat FRW spacetime: a) expanding in negative powers of $\sqrt{m^2 + k^2}$ and solving a recursive set of differential equations \[16\] b) using a WKB ansatz similar to \[25\] and the adiabatic expansion \[17\].
By making use of the decomposition (36) and the standard anti-commuting properties of the creation and annihilation operators, the vacuum expectation value of the fermion Hamiltonian (21) may be written as

\[< \mathcal{H}_F > = \frac{1}{2a^4} \sum_{\vec{k},s} \bar{v}_{ks} (am - \vec{k} \gamma) v_{ks}.\]  

(45)

Evaluating the expression under the sum and replacing the sum with an integral as in (30) we obtain

\[< \mathcal{H}_F > = \frac{1}{a^4} \int \frac{d^3 k}{(2\pi)^3} [i k^2 (\zeta_k \zeta_k^* - \zeta_k^* \zeta_k) - am].\]  

(46)

The expression in square brackets was calculated in [16] for a spatially flat FRW metric. We quote their result for the divergent contribution:

\[< \mathcal{H}_F >_{\text{div}} = \frac{1}{a^4} \int \frac{d^3 k}{(2\pi)^3} \left[ -E \left( \frac{(a^2 - 1)m^2}{2E} \right) + \frac{(a^2 - 1)^2 m^4}{8E^3} + \frac{(a')^2 m^2}{8E^3} \right].\]  

(47)

Note that the first three terms in square brackets are identical to the first three terms in the expansion of \(a\omega_k = \sqrt{E^2 + a^2 m^2 - m^2}\) in powers of \(E^{-2}\). Hence we can write

\[< \mathcal{H}_F >_{\text{div}} = \frac{1}{a^3} \int \frac{d^3 k}{(2\pi)^3} \left[ -\omega_k + \frac{(a')^2 m^2}{8a^2 \omega_k^3} + \mathcal{O}(\omega_k^{-5}) \right].\]  

(48)

The first term in square brackets is precisely the flat spacetime vacuum energy of the fermion field. The second term is a logarithmically divergent contribution due to the FRW geometry and the last term is finite and vanishes in the flat-spacetime limit \(a' \to 0\). Note that, as opposed to bosons, there is no quadratic divergence of the type \(H^2/\omega_k\).

Assembling the boson and fermion contributions, the final expression for the vacuum energy density of each chiral supermultiplet is

\[\rho_{\text{vac}} = < \mathcal{H}_F + \mathcal{H}_B > = \frac{1}{a^3} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2 \omega_k} \left[ H^2 \left( 1 + \frac{5 m^2}{4 \omega_k^2} \right) - \frac{H^2}{\omega_k^2} + \mathcal{O}(\omega_k^{-4}) \right].\]  

(49)

The dominant contribution comes from the first term which diverges quadratically. To make the result finite we change the integration variable to the physical momentum \(p = k/a\) and introduce a cutoff of the order of the Planck mass \(\Lambda_{\text{cut}} \sim m_{\text{Pl}}\). The leading term yields the total energy density of the vacuum fluctuations

\[\rho_{\text{vac}} = \frac{NH^2}{4\pi^2} \int_0^{\Lambda_{\text{cut}}} p \phi \left( 1 + \mathcal{O}(p^{-2}) \right) \approx \frac{NH^2 \Lambda_{\text{cut}}^2}{8\pi^2} \left( 1 + \mathcal{O}(\Lambda_{\text{cut}}^{-2} \ln \Lambda_{\text{cut}}) \right),\]  

(50)

where \(N\) is the number of chiral species. Note that the leading term in (49) is due to bosons; fermions only provide a precise cancellation of the quartically divergent flat spacetime vacuum term.

If the Einstein cosmological term is precisely zero then the only source of the vacuum energy density \(\rho_{\Lambda}\) are the vacuum fluctuations of matter fields. Hence we identify \(\rho_{\text{vac}} = \rho_{\Lambda}\) and if we compare (50) with the Friedman equation (2) we find that our cutoff should satisfy

\[\Lambda_{\text{cut}} \approx \sqrt{\frac{3\pi}{N}} m_{\text{Pl}}.\]  

(51)
It is worthwhile to note that several approaches \cite{18, 19, 20, 21, 22, 23, 24, 25} with substantially different underlying philosophy lead to results similar to \cite{50}. Fluctuations in the energy density of a scalar field were estimated \cite{21} yielding the same order of magnitude as in \cite{50}. There, a cosmological horizon radius $R_H = 1/H$ was employed as a long distance cutoff. This idea is similar in spirit with \cite{22} where an upper bound

$$\rho_{\text{vac}} \approx \Lambda_{\text{UV}}^4 \leq \frac{3}{8\pi} \frac{m_{\text{Pl}}^2}{L^2}$$

was proposed from a holographic principle. Here, $\Lambda_{\text{UV}}$ and $L$ denote the ultraviolet and long distance cutoffs, respectively. Our result would saturate the holographic bound \cite{52} provided $L = 1/H$.

In recent papers \cite{23, 24} a residual quadratic contribution of the form $H^2 \Lambda_{\text{cut}}^2$ has been found after canceling the flat spacetime contribution. In particular, the work \cite{23} presents a calculation of the zero-point energy using only a massless boson field and obtains two types of contributions: the quadratic and the quartic type $\Lambda_{\text{cut}}^4$. Then, the quartic contributions to CC was simply canceled by hand on the basis of the procedure used previously in the literature with the so-called ADM mass. As we have demonstrated here (see also \cite{25}), in a supersymmetric world such a cancellation by fiat is unnecessary because the cancellation between bosons and fermions of all (not only quartically divergent) flat-spacetime contributions is naturally provided by supersymmetry.

Another important point \cite{23, 25} is that the vacuum fluctuations cannot be interpreted as a part of CC because the vacuum fluctuations do not yield the equation of state $p = -\rho$, as a consequence of the energy momentum tensor not having a CC form. This behavior was already observed in flat space time if a three dimensional cutoff regularization was employed \cite{26}. A covariant regularization in flat space time should yield the vacuum energy momentum tensor of the form $T_{\mu\nu} = \rho g_{\mu\nu}$. Naively, in curved spacetime one would generalize this to the CC form $T_{\mu\nu} = \rho g_{\mu\nu}$. However, since a curved geometry involves the Riemann tensor and its covariant derivatives we may expect the energy momentum tensor at linear curvature order to be of the form $T_{\mu\nu} = (\alpha + \beta R)g_{\mu\nu} + \gamma R_{\mu\nu}$ where $\alpha$, $\beta$, and $\gamma$ are constants that do not depend on curvature. In reference \cite{27} the vacuum contribution to the energy momentum tensor has been investigated using an explicitly covariant regularization scheme for an unbroken supersymmetric model embedded in a general curved spacetime. One loop contributions to the effective potential were calculated using a covariant UV cutoff in an approach similar to Sobreira et al \cite{28}.

4 Conclusion

We have found that the leading term in the energy density of vacuum fluctuations is of the order $H^2 m_{\text{Pl}}^2$ if we impose a short distance cutoff of the order $m_{\text{Pl}}^{-1}$. In this way, if we require that the de Sitter expansion parameter $H$ equals the Hubble parameter today, the model provides a phenomenologically acceptable value of the vacuum energy. We have also found that a consistency with the Friedman equation implies that a natural cutoff must be inversely proportional to $\sqrt{N}$. A similar natural cutoff has been recently proposed in order to resolve the so called species problem of black-hole entropy \cite{29}. 
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