An Epistemic Approach to Coercion-Resistance for Electronic Voting Protocols

Ralf Küsters and Tomasz Truderung
University of Trier
Germany
Email: {kuesters, truderun}@uni-trier.de

February 18, 2013

Abstract

Coercion resistance is an important and one of the most intricate security requirements of electronic voting protocols. Several definitions of coercion resistance have been proposed in the literature, including definitions based on symbolic models. However, existing definitions in such models are rather restricted in their scope and quite complex.

In this paper, we therefore propose a new definition of coercion resistance in a symbolic setting, based on an epistemic approach. Our definition is relatively simple and intuitive. It allows for a fine-grained formulation of coercion resistance and can be stated independently of a specific, symbolic protocol and adversary model. As a proof of concept, we apply our definition to three voting protocols. In particular, we carry out the first rigorous analysis of the recently proposed Civitas system. We precisely identify those conditions under which this system guarantees coercion resistance or fails to be coercion resistant. We also analyze protocols proposed by Lee et al. and Okamoto.

1 Introduction

Coercion resistance is one of the most important and intricate security requirements of voting protocols [23,33]. Intuitively, a voting protocol is coercion resistant if it prevents voter coercion and vote buying. In other words, a coercer should not be able to influence the behavior of a voter. A notion closely related to coercion resistance, but somewhat weaker is receipt freeness, first proposed in [8].

Most voting schemes and systems that aim to achieve coercion resistance or receipt freeness come without a rigorous security proof. Maybe not surprisingly, some of these protocols have been found to be flawed (see, e.g., discussions in [32] and [19]). The lack of proofs is partly due to the fact that only recently first formal definitions of coercion resistance and receipt freeness have been proposed in the literature, both based on cryptographic and symbolic models [4,13,17,21–23,31]. With “cryptographic models” we mean models in which messages are modeled as bit strings and adversaries are probabilistic polynomial time Turing machines. In contrast, symbolic models take a more abstract view on cryptography. In this paper, our focus will be on symbolic models. While security guarantees in cryptographic models are typically stronger than in symbolic models, security proofs in cryptographic models are usually very involved, and as a result, often omitted or only sketched. For electronic voting protocols, which are among the most complex security protocols, this is even more so (see, e.g., [8,14,19,23,28,33,34]). Conversely, security proofs in symbolic models are easier to carry out and they are more amenable to tool support. Research on security protocol analysis has demonstrated that, while not all, but many attacks on security protocols can be uncovered and prevented by means of symbolic protocol analysis (see, e.g., [3,7,9,10,12,26,30]). In some cases, security guarantees established in symbolic models even imply security in cryptographic models (see, e.g., [2,15,29]). Hence, symbolic models certainly have their merits for security protocol analysis, including the analysis of voting protocols.

However, the definitions of coercion resistance in symbolic models proposed in the literature thus far are rather restricted in scope, yet quite complex and not always intuitive (see Section 7 for a detailed discussion).

Contribution of this paper. One of the main contributions of this paper is to provide a general, yet intuitive and simple definition of coercion resistance. Our definition follows an epistemic approach. It is formulated in a model-independent way. In particular, it can be instantiated by different symbolic models. While the focus of this work is on voting protocols, our definition may be applicable beyond this domain.

In order to analyze concrete voting protocols, we instantiate our framework by a rather standard symbolic model. Within our model, we prove several general statements, which underline the adequacy of our model and which have not been proven in other symbolic models. Among others, we show that coercion resistance w.r.t. a single coerced voter implies coercion resistance w.r.t. multiple coerced voters.

*This paper is an extended version of [27].
As a proof of concept, we analyze coercion resistance of three voting protocols: the recently proposed voting system Civitas [14], a voting protocol by Lee et al. [28] and one by Okamoto [33]. As to the best of our knowledge, Civitas and the scheme by Okamoto have not been rigorously analyzed before. Our modeling, in particular of Civitas, is quite detailed and goes beyond the level of detail considered in other works. For example, for Civitas we model dishonest authorities and the zero-knowledge proofs authorities have to provide to prove their compliance with the protocol. We precisely identify those conditions under which coercion resistance is guaranteed and point out situations in which the protocols do not provide coercion resistance, thereby relativizing previous claims and providing new insights into and improvements of the protocols. The analyzes of the example protocols illustrate that our definition of coercion resistance allows to specify various degrees of coercion resistance in a fine-grained way. Without this flexibility of our definition, no reasonable statement about the coercion resistance of voting protocols would be possible as every protocol builds on its own assumptions and provides specific security guarantees.

Structure of this paper. In the following section, we present our definition of coercion resistance. A concrete instantiation of this definition is provided in Section 3 with general properties given in Section 4. The analyzes of the three mentioned voting protocols are then presented in Sections 5, 6 and Appendix D. Related work is discussed in Section 7. We conclude in Section 8. More details and proofs can be found in the appendix.

2 Defining Coercion Resistance

In this section, we present our definition of coercion resistance in an epistemic framework, independent of a specific, symbolic protocol or adversary model. A concrete instantiation will be considered in Section 3.

Our definition of coercion resistance is based on what we call a coercion system. A coercion system will be induced by a voting protocol (see Section 3). It emphasizes in an abstract way those parts relevant for defining coercion resistance, without the need to consider details of a protocol and adversary model. More intuition is provided following the next definition.

Definition 1. A coercion system is a tuple $S = (R, V, C, E, r, \sim)$, where $R$ is a set of runs, $V$, $C$, and $E$ are sets of possible programs of coerced voters, the coercer, and the environment, respectively, $r$ is a mapping which assigns a set $r(v, c, e) \subseteq R$ of runs induced by $(v, c, e)$ to each tuple $(v, c, e) \in V \times C \times E$, and $\sim$ is an equivalence relation on the set $R$, which determines the view of a coercer on a run.

A coercion system determines the possible behaviors of coerced voters, the coercer, and the environment. The environment is the part of the system controlled neither by the coercer nor by the coerced voter. The environment typically describes the possible behaviors of honest entities, such as honest voters and authorities; dishonest voters and authorities will be subsumed by the coercer. The programs carried out by these honest entities will be determined by the voting protocol under consideration. However, the environment typically does not fix up front how and if certain honest voters vote. It may also leave open the number of voters as well as how many of them and which voters are honest or dishonest. The set $r(v, c, e)$ describes the possible runs obtained when the programs $v$, $c$, and $e$ of the coerced voter, the coercer, and the environment, respectively, run together. A run is typically a sequence of configurations induced by the interaction of $v$, $c$, and $e$. However, for a general definition of coercion resistance it is not necessary to fix such details at this point. The reason that we do not define $r(v, c, e)$ to be a single run is that a run of $v$, $c$, and $e$ might involve some non-deterministic choices, e.g., non-deterministic scheduling of messages. The equivalence relation $\sim$ defines the view of the coercer. The intuition is that if two runs $\rho$ and $\rho'$ are equivalent w.r.t. $\sim$, i.e., $\rho \sim \rho'$, then the coercer has the same view in both runs. In other words, these runs look the same from the coercer’s point of view.

We can now turn to the definition of coercion resistance. For the following discussion, we concentrate on the case that only a single voter is coerced. The case of multi-voter coercion resistance is discussed later.

Given a coercion system $S = (R, V, C, E, r, \sim)$, the idea behind our definition of coercion resistance is as follows:

Our definition assumes that the coerced voter has a certain goal $\gamma$ that he/she would try to achieve in absence of coercion. Formally, $\gamma$ is a subset of $R$, the set of runs of $S$. If, for example, $\gamma$ is supposed to express that the coerced voter wants to vote for a certain candidate, then $\gamma$ would contain all runs in which the coerced voter voted for this candidate and this vote is in fact counted. Jumping ahead, as we will see in the analysis of concrete protocols, often such a goal cannot be achieved. This is, for example, the case if ballots are sent over an unreliable channel or an election authority misbehaves in an observable way and as a result the election process is stopped. A more realistic goal $\gamma$ would then be that the coerced voter successfully votes for a certain candidate, provided the voters ballot is delivered in time and the election authority did not misbehave in an observable way.

Now, in the definition of coercion resistance we imagine that the coercer provides the coerced voter with a program $v \in V$ (the coercion strategy), which the coercer wants the coerced voter to run, instead of the program the coerced voter would carry out when following the voting protocol. The program $v$ might determine the candidate for which the coercer wants the coerced voter to vote for or might dictate the coerced voter not to vote (abstention attack). The choice of the candidate or whether or not the coerced voter should abstain from voting might even depend on the
course of the election process and the information that the coercer has gathered thus far. Such information can be gathered by the program \( v \) or might be given to the program by the coercer; in the most general setting, one assumes that the coercer can freely communicate with the program \( v \), and by this, further influence and control the behavior of the coerced voter. Rather than directly manipulating the outcome of the election, the purpose of \( v \) might as well be to merely test whether the coerced voter follows the prescribed program \( v \); for example, to find out whether this voter is “reliable”, and hence, is a good candidate for coercion in later elections. This illustrates that the intentions of the coercer are manifold and hard to predict. The set \( V \) should therefore contain all programs that a coercer could possibly give to a coerced voter. However, as shown in Section 41 in a concrete communication model, it often suffices to consider just one program that simply forwards all messages from/to the coercer. Nevertheless, taking the set \( V \) into account only makes our definition more flexible since different classes of coercion strategies can be specified.

Our definition of coercion resistance requires that for all \( v \in V \), there exists a program \( v' \in V \), the counter strategy, that the coerced voter can run instead of \( v \), such that (i) the voter always achieves his/her own goal \( \gamma \) by running \( v' \) and (ii) the coercer does not know whether the coerced voter run \( v \) or \( v' \). In other words, in every run in which the coerced voter run \( v \), the coercer thinks, given his/her view of the run, that it is possible that the coerced voter run \( v' \). Conversely, in every run in which the coerced voter run \( v' \), the coercer thinks that it is possible that the coerced voter run \( v \). So, the coercer cannot know whether the coerced voter followed the coercer’s instructions (i.e., run \( v \)) or just tried to achieve his/her own goal (by running \( v' \)). If in some situations the coercer knew that the coerced voter run either \( v \) or \( v' \), then the voter could be influenced: The coercer could give positive and/or negative incentives for running \( v/v' \), e.g., by offering money and/or threatening the coerced voter.

The above leads to the following definition. The meaning of \( \alpha \) is explained below.

**Definition 2.** Let \( S = (R, V, C, E, r, \sim) \) be a coercion system and \( \alpha, \gamma \subseteq R \). The system \( S \) is coercion resistant in \( \alpha \) w.r.t. \( \gamma \), if for each \( v \in V \) there exists \( v' \in V \) such that the following conditions are satisfied.

(i) For every \( c \in C \), \( e \in E \), and \( \rho \in r(v, c, e) \cap \alpha \), there exists \( \rho' \in r(v', c, e') \cap \alpha \) such that \( \rho \sim \rho' \).

(ii) For every \( c \in C \), \( e \in E \), and \( \rho \in r(v', c, e) \cap \alpha \), there exists \( \rho' \in r(v, c, e') \cap \alpha \) such that \( \rho \sim \rho' \).

(iii) For every \( c \in C \) and \( e \in E \), we have \( r(v', c, e) \subseteq \alpha \).

Condition (iii) in the above definition directly captures that if the coerced voter runs the counter strategy \( v' \), then independently of the actions of the coercer \( c \) and the environment \( e \), the coercer achieves his/her goal. To explain the conditions (i) and (ii), let us first ignore the set \( \alpha \). Then (i) says that, for every run \( \rho \) in which the coerced voter carries out \( v \), there exists another run \( \rho' \) in which the coerced voter carries out \( v' \) such that the view of the coercer, who runs \( c \) in both runs, is the same. In other words, even though the coerced voter carried out \( v \), from the coercer’s point of view it is possible that the coerced voter carried out \( v' \). The programs \( e \) and \( e' \) in (i) might, for example, differ in the way honest voters voted. So even though the coerced voter might not have voted in the way intended by the coercer, the coercer can not tell from the outcome of the election, as the coercer does not have complete knowledge about how everybody voted. Analogously, condition (ii) says that in every run in which the coerced voter run \( v' \), the coercer thinks that it is possible that the coerced voter run \( v \). Altogether (i) and (ii) say that the coercer never knows whether the coerced voter run \( v \) or \( v' \).

Now, let us discuss the purpose of \( \alpha \). The intuition is that \( \alpha \) describes a property of the environment (which, as mentioned, includes the honest voters) in terms of a set of words, even though the coerced voter carried out \( v \), the coercer thinks, given his/her view of the run, that it is possible that the coerced voter run \( v' \). Conversely, in every run in which the coerced voter run \( v' \), the coercer thinks that it is possible that the coerced voter run \( v \). So, the coercer cannot know whether the coerced voter followed the coercer’s instructions (i.e., run \( v \)) or just tried to achieve his/her own goal (by running \( v' \)). If in some situations the coercer knew that the coerced voter run either \( v \) or \( v' \), then the voter could be influenced: The coercer could give positive and/or negative incentives for running \( v/v' \), e.g., by offering money and/or threatening the coerced voter.

The example protocols analyzed in Sections 5, 6, and Appendix 13 will further illustrate the usefulness and necessity of the parameters \( \alpha \) and \( \gamma \) of our definition of coercion resistance. These parameters allow to precisely capture under what conditions a protocol is coercion resistant, making for a quite fine-grained and general notion of coercion resistance.

Definition 2 only stipulates the existence of a counter strategy \( v' \), given a coercion strategy \( v \). However, it might in general not be easy to come up with \( v' \) given \( v \). Fortunately, as already mentioned above, we can show that it is often sufficient to come up with a counter strategy only for what we call a dummy coercion strategy, which merely forwards messages to/from the coercer. Given such a counter strategy, one can, in a generic way, construct a counter strategy for any given coercion strategy (see Section 41). We believe that the construction of a counter strategy from
a (dummy) coercion strategy should be part of the protocol specification, so that a voter knows how to defend against coercion (see also [31]).

We note that Definition 2 captures coercion resistance in a possibilistic way. We do not consider probabilities. While Definition 2 requires that from the coerced voter’s point of view it is always possible that the coerced voter run \( v \), say, the definition does not talk about the probability for this to be the case. If this probability was low, the coercer could tend to believe that the coerced voter run \( v' \). We leave a probabilistic/cryptographic version of our definition as future work. The analysis carried out in this work for the three voting protocols shows that already in a possibilistic setting non-trivial security guarantees can be proved and subtle vulnerabilities can be uncovered.

While in Definition 2 only one goal of the coerced voter is considered, a protocol should of course be coercion resistant no matter what goal the coerced voter would like to achieve; for example, no matter which candidate the coerced voter would like to vote for. This is captured by the following generalization of Definition 2.

**Definition 3.** Let \( S = (R, V, C, E, r, \sim) \) be a coercion system and \( \Gamma \) be a set of subsets of \( R \). Then \( S \) is coercion resistant in \( \alpha \) w.r.t. \( \Gamma \), if \( S \) is coercion resistant in \( \alpha \) w.r.t. \( \gamma \), for each \( \gamma \in \Gamma \).

**Multi-voter coercion.** So far, we had in mind that \( v \) and \( v' \) stand for programs carried out by a single coerced voter. Nevertheless, we can just as well think of \( v \) and \( v' \) as tuples of programs carried out by multiple coerced voters, where the tuples may be of varying length, depending on how many voters are coerced. In other words, our definition of coercion resistance directly carries over to the case of multi-voter coercion resistance, where multiple voters are coerced at the same time. However, the requirement “for all \( v \) there exists a \( v' \) such that . . .” in the definition of coercion resistance then only means that a coerced voter can pick a counter strategy depending on all the programs in \( v \). This is too weak. A coerced voter should be able to pick his/her counter strategy independently of other coerced voters; a coerced voter may in general not know who else is coerced and with whom he/she can (safely) collaborate. Therefore, for multi-voter coercion resistance, we replace the requirement “for all \( v \) there exists a \( v' \) such that . . .” by “there exists a function \( f \) which maps a coercion strategy for one voter to a counter strategy for one voter such that, for every tuple \( v \) of programs, \( v' = f(v) \) is a counter strategy such that . . .”, where \( f(v) \) means that \( f \) is applied to every single program in the tuple \( v \).

In Section 4.3 we show that (a slight extension of) coercion resistance w.r.t. a single coerced voter implies multi-voter coercion resistance. So, to obtain multi-voter coercion resistance it suffices to consider the case of a single coerced voter.

## 3 A Concrete Protocol and Adversary Model

In this section, we instantiate the framework presented in the previous section by a concrete protocol and adversary model. Several instantiations are possible, including, for example, one based on I/O automata or process calculus. For the sake of brevity, we pick a quite abstract one, in which computations are described by certain functions, called atomic processes. However, the results presented in the subsequent sections also carry over to other models. We note that these sections should be intelligible without the concrete protocol and adversary model presented in this section.

### 3.1 Terms and messages

Let \( \Sigma \) be some signature for cryptographic primitives (including a possibly infinite set of constants for representing participant names, etc.), \( X = \{x_1, x_2, \ldots\} \) be a set of variables, and \( \mathcal{X} \) an infinite set of nonces, where the sets \( \Sigma, X, \mathcal{X} \) are pairwise disjoint. For \( N \subseteq \mathcal{X} \), the set \( T_N \) of terms over \( \Sigma \cup N \) and \( X \) is defined as usual. Ground terms, i.e., terms without variables, represent messages. We assume some fixed equational theory associated with \( \Sigma \) and denote by \( \equiv \) the congruence relation on terms induced by this theory. The exact definition of \( \Sigma \) and the equational theory will depend on the cryptographic primitives used in the voting protocol under consideration. For the voting protocols we analyze in Section 6 and Appendix 1 quite involved signatures and equational theories will be considered, which, among others, allow to model homomorphic encryption and various kinds of zero knowledge proofs (designated-verifier reencryption proofs, distributed plaintext equivalence tests, etc.). A simple example of a signature \( \Sigma_{ex} \) and its associated equational theory is provided in Figure 1. A term of the form \( \text{sig}_{\{m\}} \) represents a message \( m \) signed using the (private) key \( k \). Checking validity of such a signature is modeled by equation (1). The fact that signatures do not necessarily hide the signed message is taken care of by equation (2). A term of the form \( \text{x}{\text{pub}(k)} \) represents the ciphertext obtained by encrypting \( x \) under the public key \( \text{pub}(k) \) using randomness \( r \). Decryption of such a term using the corresponding private key \( k \) is modeled by equation (3). A term of the form \( \langle x, y \rangle \) models the pairing of terms \( x \) and \( y \). The components \( x \) and \( y \) of \( \langle x, y \rangle \) can be extracted by applying the operators \( \text{first}(\cdot) \) and \( \text{sec}(\cdot) \), respectively, as modeled by the equations (4). Let \( \equiv_{ex} \) denote the congruence relation induced by the equational theory in Figure 1 then we have that \( \text{dec}([\{a\}]_{\text{pub}(k)}, \text{first}([k, b])) \equiv_{ex} a \).

### 3.2 Event sequences and views

Let \( \mathcal{C} \) be a set of channels (channel names). An input/output event is of the form \((c : m)\) and \((\bar{c} : m)\), respectively, for \( c \in \mathcal{C} \) and a message \( m \) (note that \( \bar{c} \notin \mathcal{C} \)).
of input events, we denote by $\text{chan}(\rho)$ for every $C$ of $\rho$ a finite or infinite sequence of events is called an event sequence. For an event sequence $\rho = (c_1 : m_1), (c_2 : m_2), \ldots$ of input events, we denote by $\text{chan}(\rho)$ the sequence $c_1, c_2, \ldots$ of channels. For $C \subseteq \text{Ch}$, we denote by $\rho|_C$ the subsequence of $\rho$ containing only the events $(c : m)$ with $c \in C$.

Let $\tau \in T_N$ be a term. Then, with $\rho$ as above, we denote by $\tau[\rho]$ the message $\tau[m_1/x_1, m_2/x_2, \ldots]$, where $x_i$ is replaced by $m_i$. (Recall that the set of variables is $X = \{x_1, x_2, \ldots\}$.) For example, assume that $\tau_{ex} = \text{dec}(x_1, \text{first}(x_2))$ and $\rho_{ex} = (c_1 : \{a\}|_{\text{pub}(k)}), (c_2 : \{k, b\})$. Then $\tau_{ex}[\rho_{ex}] = \text{dec}(\tau\{a\}|_{\text{pub}(k)}, \text{first}(\{k, b\})) \equiv_{ex} a$.

Borrowing the notion of static equivalence from [1], we call two event sequences $\rho$ and $\rho'$ of input events statically equivalent w.r.t. a set $C \subseteq \text{Ch}$ of channels and a set $N \subseteq \mathcal{N}$ of nonces, written $\rho \equiv_N^C \rho'$, if (i) $\text{chan}(\rho|_C) = \text{chan}(\rho'|_C)$ and (ii) for every $\tau_1, \tau_2 \in T_N$ we have that $\tau_1[\rho|_C] \equiv \tau_2[\rho|_C]$ iff $\tau_1[\rho'|_C] \equiv \tau_2[\rho'|_C]$. Intuitively, a party listening on channels $C$ and a priori knowing the nonces in $N$, cannot distinguish between the inputs received according to $\rho$ and those received according to $\rho'$. We call the equivalence class of $\rho$ w.r.t. $C$ and $N$ the $(C,N)$-view on $\rho$. For example, if $k$, $k'$, $a$, and $b$ are different constants, $r$ and $r'$ are nonces, $C = \{c_1, c_2\}$, and $N = \emptyset$, then it is easy to see that $\rho^1_{ex} = (c_1 : \{a\}|_{\text{pub}(k)}), (c_2 : \{k, b\}), (c_3 : k)$ and $\rho^2_{ex} = (c_1 : \{b\}|_{\text{pub}(k)}), (c_2 : \{k', b\})$ yield the same $(C,N)$-view w.r.t. $\equiv_{ex}$.

3.3 Processes

Processes are built from atomic processes. An atomic process is basically a function that given a sequence of input events (representing the history so far) produces a sequence of output events. We require that an atomic process behaves the same on inputs on which it has the same view. Formally, atomic processes are defined as follows.

**Definition 4.** An atomic process is a tuple $p = (I, O, N, f)$ where

(i) $I, O \subseteq \text{Ch}$ are finite sets of input and output channels, respectively,

(ii) $N \subseteq \mathcal{N}$ is a set of nonces used by $p$,

(iii) $f$ is a mapping which assigns a sequence $f(U) = (c_1 : \tau_1) \cdots (c_n : \tau_n)$ with $c_i \in O$ and $\tau_i \in T_N$ to each $(I, O)$-view $U$.

We refer to $I$, $O$ and $N$ by $I_p$, $O_p$, and $N_p$, respectively. We note that the sets $I_p$ and $O_p$ do not have to be disjoint (which means that $p$ can send messages to itself).

We note that (iii) guarantees that $p$ performs the same computation on event sequences that are equivalent according to $\equiv_N^C$, and hence, on which $p$ has the same view. This is why $f$ is defined on $(I, N)$-views rather than on sequences of input events.

For an event sequence $\rho$, we write $p(\rho)$ for the output produced by $p$ on input $\rho$. This output is $(c_1 : \tau_1[\rho']) \cdots (c_n : \tau_n[\rho_n])$, where $\rho' = \rho|_I$ and $(c_1 : \tau_1) \cdots (c_n : \tau_n) = f(U)$ for the equivalence class $U$ of $\rho'$ w.r.t. $\equiv_N^C$. For example, let $I = \{c_1, c_2\}$, $N = \emptyset$, $U$ be the equivalence class of $\rho^1_{ex}$, and assume that $f(U) = (c_1 : (x_1, \text{first}(x_2)))$. Then, $p(\rho^1_{ex}) = (c_1 : \{a\}|_{\text{pub}(k)}, \text{first}(\{k', b\}))$, which modulo $\equiv_{ex}$ can be equivalently written as $c_1 : \{a\}|_{\text{pub}(k)}, k'$ and $p(\rho^2_{ex}) = (c_1 : \{b\}|_{\text{pub}(k)}, \text{first}(\{k', b\}))$, which modulo $\equiv_{ex}$ can be equivalently written as $c_1 : \{b\}|_{\text{pub}(k)}, k'$. Note that since $\rho^1_{ex}$ and $\rho^2_{ex}$ yield the same $(I, N)$-view w.r.t. $\equiv_{ex}$, $p$ performs the same transformation on $\rho^1_{ex}$ and $\rho^2_{ex}$.

For atomic processes $p$ and $p'$, we write $p \simeq p'$, if $p$ and $p'$ perform the same computation up to renaming of nonces. Formally, for atomic processes $p = (I, O, N, f)$, $p' = (I, O, N', f')$, we write $p \simeq p'$, if there exists a bijection $h : N \rightarrow N'$ such that $h(f(U)) = f'(h(U))$ for every $(I, N)$-view $U$. This is extended to processes (see below) in the obvious way.

A process $P$ is a finite set of atomic processes with disjoint sets of input channels and sets of nonces, i.e., $I_p \cap I_{p'} = \emptyset$ and $N_p \cap N_{p'} = \emptyset$, for distinct $p, p' \in P$. The set of input/output channels and the set of nonces of $P$ is $I_P = \bigcup_{p \in P} I_p$, $O_P = \bigcup_{p \in P} O_p$, and $N_P = \bigcup_{p \in P} N_p$, respectively.

We say that $P$ is a process over $(I, O, N)$, if $I_P \subseteq I$, $O_P \subseteq O$, and $N_P \subseteq N$. By $\Pi(I, O, N)$ we denote the set of all processes over $(I, O, N)$, for some $N \subseteq \mathcal{N}$.

For a finite event sequence $\rho$ with the last event of the form $(c : m)$, we write $P(\rho)$ for $p(\rho)$, where $p$ is the (unique) element of $P$ such that $c \in I_p$ (if such a $p$ does not exists, then $P(\rho)$ is undefined).

Given a process $P$ and a finite sequence $s_0$ of output events over $O_P$, a run $\rho$ of a process $P$ initiated by $s_0$ is a finite or infinite sequence of input and output events which evolves from $s_0$ in a natural way: An output event is chosen non-deterministically (initial from $s_0$). Once an output event has been chosen, it will not be chosen anymore later on. By definition of processes, there exists at most one atomic process, say $p$, in $P$ with an input channel corresponding to the output event. Now, $p$ (if any) is given the input event corresponding to the chosen output event, along with all previous input events on channels of $p$. Then, $p$ produces a sequence of output events as described above. Now, from these or older output events an output event is chosen non-deterministically, and the computation continues as before. The notion of a run is formally defined below.
Definition 5. Let $P$ be a process and $s_0$ be a finite sequence of events. A run induced by $P$ initiated by $s_0$ is a sequence of events $s = e_1 e_2 \ldots$ such that

(i) $s$ begins with $s_0$,

(ii) There exists a bijective function $f$ from non-negative integers to non-negative integers such that for each $i$, if $e_i$ is an input event $(c : m)$, then $f(i) < i$ and $e_{f(i)} = (\hat{c} : m)$, and, moreover, for each $i < j$ with $e_i = (c : m)$ and $e_j = (c : m')$ we have $f(i) < f(j)$,

(iii) If $\rho$ can be splitted into $\rho_1 \rho_2 \rho_3$, where $\rho_1$ ends with an input event, $\rho_2$ contains output events only, and $\rho_3$ either is empty or begins with an input event, then $\rho_2 = P(\rho_1)$.

An run is fair if it is finite or, in case it is infinite, each message sent is eventually delivered (i.e., for each output event $e_i$ there exists $j$ such that $f(j) = i$).

A run is finite if all output events were chosen at some point and there is no new output event left that has not yet been chosen; otherwise a run is infinite. We emphasize that $s_0$ can induce many runs, due to the non-deterministic delivery of messages.

We call two processes $P$ and $P'$ non-conflicting if $I_P \cap I_{P'} = \emptyset$ and $N_P \cap N_{P'} = \emptyset$. In this case, we will write $P_1 \equiv P_2$ instead of $P_1 \parallel P_2$.

If $P \subseteq P'$, we call process $P$ a subprocess of process $P'$. For such a $P$, we define an equivalence relation $\equiv_P$ on runs induced by $P'$ as follows: $P_1 \equiv_P P_2$ iff $P'_1 \equiv_{N_P} P'_2$. Hence, $P_1 \equiv_P P_2$ means that from the point of view of $P$, the runs $P_1$ and $P_2$ look the same. In particular, $P$ behaves the same on these runs.

3.4 Protocols and Their Induced Coercion Systems

Definition 6. A protocol is a tuple $S = (A, in, out, s_0, P)$, where

(i) $A$ is a finite set of agent names, with access to input and output channels $in(a), out(a) \subseteq CH$, respectively, such that $in(a) \cap in(a') = \emptyset$ for $a \neq a'$;

(ii) $s_0$ is a finite sequence of output events, the initial output sequence, for initializing parties,

(iii) for every $a \in A$, $P(a) \subseteq \Pi(out(a), in(a))$ is the set of programs or processes of $a$; this set is assumed to be closed under $\circ$.

For example, if $a$ is an honest voter, then $P(a)$ would typically contain a program for each way $a$ could vote, possibly including abstention of voting. We note that the set $A$ typically contains the coercer and coerced parties, i.e., these entities are part of the protocol specification.

If $A = \{a_1, \ldots, a_n\}$ and $p_i \in P(a_i)$, then $(p_1 \| \cdots \| p_n)$ is an instance of $S$, where the $p_1, \ldots, p_n$ are non-conflicting.

A run of $S$ is a fair run of the process $p_1 \| \cdots \| p_n$ initiated by $s_0$, where $p_1 \| \cdots \| p_n$ is some instance of $S$. For a protocol $S = (A, in, out, s_0, P)$ and $a \in A$, a channel $c$ is said to be private channel of $a$, if $c \in in(a) \cap out(a)$ and $c \not\in in(a') \cup out(a')$ for all $a' \neq a$.

Now, let $S = (A, in, out, s_0, P)$ be a protocol with $A = \{v, c, e\}$. Typically, $e$ subsumes all honest principals and processes in $P(e)$ are of the form $p_1 \| \cdots \| p_n$, where $p_i$ are programs of honest voters and authorities. Dishonest voters and authorities are subsumed by the coercer $c$ and coerced voters by $v$. For such a protocol we can define a coercion system, as follows.

Definition 7. Let $S = (A, in, out, s_0, P)$ be a protocol with $A = \{v, c, e\}$. The coercion system induced by $S$ is $(R, V, C, E, r, \sim)$, where

(i) $V = P(v)$, $C = P(c)$, and $E = P(e)$,

(ii) $R$ is a set of tuples of the form $(v, c, e, \pi)$, with non-conflicting $v \in V$, $c \in C$, $e \in E$ and $\pi$ is a run induced by $(v \parallel c \parallel e)$,

(iii) for every $v \in V$, $c \in C$, $e \in E$, $r(v, c, e) = \{(\hat{v}, \hat{c}, \hat{e}, \pi) \mid \hat{v} \simeq v, \hat{c} \simeq c, \hat{e} \simeq e, \pi \text{ is a run of } S \text{ by } (\hat{v} \parallel \hat{c} \parallel \hat{e}) \}$ is the set of runs of the process formed by $v, c, e$, closed under renaming of nonces,

(iv) for all $(v, c, e, \pi), (v', c', e', \pi') \in R$, we have $(v, c, e, \pi) \sim (v', c', e', \pi')$ iff $e = e'$ and $\pi \equiv_e \pi'$.

Hence, the relation $\sim$ models the view of the coercer $c$ on runs of $S$.

4 General Properties

In this section, we state general properties of coercion systems induced by protocols, as introduced in the previous section. On the one hand, these properties facilitate proofs of coercion resistance of voting protocols. On the other hand, they demonstrate the adequacy of our model. In Section 4.1, we show that, under reasonable assumptions, to prove coercion resistance it is not necessary to consider all coercion strategies, i.e., all programs $v \in V$, but rather suffices to consider a single coercion strategy, the dummy strategy. In Section 4.2, we briefly discuss the notion of receipt freeness and show that it is implied by our notion of coercion resistance. We also show, in Section 4.3, that multi-voter coercion resistance, where multiple voters are coerced, is implied by a slight extension of single-voter coercion resistance, where only one voter is coerced. Except for the second statement, the other statements have not been proven in other works on the symbolic analysis of voting protocols.

4.1 Dummy Theorem

The theorem that we want to prove, requires normal protocols. In these protocols the coerced voter and the coercer can freely communicate (there are input and output channels in both directions) and the set of programs of both entities contains all processes, with appropriate input and
output channels. For general coercion resistance, protocols are typically defined in this way.

We define the dummy coercion strategy $v_0$ to be the process which simply forwards to the coercer all the messages it receives from the environment and, conversely, forwards to the environment all the messages it receives from the coercer.

Now, we call a coercion system for a protocol dummy coercion resistant if it is coercion resistant in case a counter strategy is demanded only for the dummy coercion strategy.

To state our dummy theorem, we need to define a relation $\sim$ on runs which defines a certain view of the environment. Let $S = (A, in, out, s_0, P)$ be a protocol with $A = \{v, c, e\}$ and let $\pi$ be a run of $P$. Then, by $\text{env}(\pi)$ we denote the subsequence of $\pi$ which only contains input and outputs events for channels of $v$, i.e., events of the form $(c : m)$ and $(\bar{c}, m)$ with $c \in in(e) \cup out(e)$. Now, for runs $\pi$ and $\pi'$ we write $\pi \sim \pi'$ iff $\text{env}(\pi) = \text{env}(\pi')$. We extend this relation to the set of runs of the coercion system of $S$: $(v, c, e, \pi) \sim (v', c', e', \pi')$ iff $e = e'$ and $\pi \sim \pi'$. We say that a set $H$ of runs is closed under $\sim$, if $\pi \in H$ and $\pi \sim \pi'$ implies $\pi' \in H$.

In the following theorem, we assume that $\alpha$ and $\gamma$ are closed under $\sim$. As $\alpha$ and $\gamma$ are typically defined based on the view of the environment, the assumption is satisfied in most applications, including the protocols that we analyzed.

**Theorem 1.** Let $S = (V, C, E, r, \sim)$ be a coercion system for a normal protocol and $\alpha, \gamma$ be sets of runs of $S$ closed under $\sim$. Then dummy coercion resistance implies (full) coercion resistance.

**Proof sketch (see Appendix A.4 for the full proof).** Assume that $v_0'$ is the counter strategy for the dummy strategy $v_0$. Let $v \in V$ be any coercion strategy. Then we show that the parallel composition of $v_0'$ and $v$, i.e., the process $v' \parallel v$, with a proper renaming of channels, is a counter strategy for $v$.

We note that theorems of a similar flavor as the one above are also considered in cryptographic, simulation-based settings (see, e.g., [11, 24, 25]).

### 4.2 Receipt Freeness

We define receipt freeness similarly to coercion resistance, but with the assumption that the coercer cannot send any messages directly to the coerced voter. Hence, only the coerced voter can send messages to the coercer. These messages can be considered as receipts. This intuition is shared with many other works. One could further weaken the following definition by fixing a certain class of coercion strategies, where, for example, the coerced voter basically follows the protocol but provides the coercer with all the information obtained during the run of the protocol.

**Definition 8.** A coercion system $S = (V, C, E, r, \sim)$ is receipt-free in $\alpha$ w.r.t. $\gamma$, if the system $S' = (V, C', E, r, \sim)$, where $C'$ consists of all the programs in $C$ which do not directly send messages to the coerced voter, is coercion-resistant in $\alpha$ w.r.t. $\gamma$.

Alternatively to restricting the coercer, one could require the coerced voter not to accept messages from the coercer. As an immediate consequence of the above definition, we obtain the following theorem.

**Theorem 2.** If a coercion system is coercion-resistant, then it is receipt-free.

### 4.3 Multi-voter Coercion Resistance

In this section, we show that multi-voter coercion resistance is implied by a slight extension of single-voter coercion resistance. The main idea is that in case of multiple coerced voters, all coerced voters, except for one, can be considered to be dishonest, and hence, their behavior can be subsumed by the coercer, leaving the case of a single coerced voter.

In what follows, let $S = (A, in, out, s_0, P)$ be a protocol with $A = \{v, c, e\}$. According to the definition of multi-voter coercion resistance (see Section 2), we assume that the programs of $v$ are processes of the form $(p_1 \parallel \cdots \parallel p_n)$, where $p_i$ represents a process of the coerced voter $v_i$, with its own set $I_i$ and $O_i$ of input and output channels, respectively. We have that $in(v) = I_1 \cup \cdots \cup I_n$ and $out(v) = O_1 \cup \cdots \cup O_n$.

Given $S$, we define for every coerced voter $v_i$ a new protocol $S_i$, where $v_i$ is the only coerced voter and every other coerced voter is considered to be dishonest, and hence, subsumed by the coercer. The environment $e$ in $S_i$ is the same as in $S$.

We let $T$ denote the coercion system for $S$ and $T_1, \ldots, T_n$ the coercion systems for $S_1, \ldots, S_n$, respectively.

Now, we slightly extend the notion of (single-voter) coercion resistance, as mentioned before. An explanation follows the definition.

**Definition 9.** A system $S = (R, V, C, E, r, \sim)$ is coercion resistant for $(\alpha_0, \ldots, \alpha_n)$ w.r.t. $\gamma$, where $\alpha_0, \ldots, \alpha_n, \gamma \subseteq R$, if for each $v \in V$ there exists $v' \in V$ such that the following conditions are satisfied.

(i) For every $k \in \{1, \ldots, n\}$, $c \in C$, $e \in E$, and $\rho \in r(v, c, e) \cap \alpha_k$, there exists $e' \in E$ and $\rho' \in r(v', c, e') \cap \alpha_{k-1}$ such that $\rho \sim \rho'$.

(ii) For every $k \in \{1, \ldots, n\}$, $c \in C$, $e \in E$, and $\rho \in r(v', c, e) \cap \alpha_k$, there exists $e' \in E$ and $\rho' \in r(v, c, e') \cap \alpha_{k-1}$ such that $\rho \sim \rho'$.

(iii) For every $c \in C$ and $e \in E$, we have that $r(v', c, e) \subseteq \gamma$.

First note that condition (iii) of the definition is the same as the corresponding condition in Definition 2. Also, for $n = 1$ and $(\alpha_0, \alpha_1) = (R, \alpha)$ the rest of the conditions coincide with Definition 2 as well. A property $\alpha_i$ contains, for example, all runs in which there are at least $i$ votes for all candidates by honest voters. Now, when going from a run where the coerced voter carries out $v$ to a run where
he/she carries out $v'$, then in the latter runs honest voters might have to vote in different ways in order to balance the behavior of $v'$. The above definition requires that in the run with $v'$ still $a_{i-1}$ is satisfied, and hence, in the example, there are still at least $i-1$ votes for all candidates by honest voters.

We obtain the following theorem, which says that to prove multi-voter coercion resistance, it suffices to show single-voter coercion resistance in the sense of Definition 4. Despite the quantification over $i$ in the following theorem, it typically suffices to prove (single-voter) coercion resistance for one $T_i$, due to symmetry.

**Theorem 3.** Let $S, S_1, \ldots, S_n$ and $T, T_1, \ldots, T_n$ be defined as above. Let $\alpha_0, \ldots, \alpha_n$ and $\gamma_1, \ldots, \gamma_n$ be properties of $T$, i.e., sets of runs of $T$. If, for each $i \in \{1, \ldots, n\}$, we have that $T_i$ is coercion-resistant for $(\alpha_0, \ldots, \alpha_n)$ w.r.t. $\gamma_i$, then $T$ is multi-voter coercion resistant in $\alpha_n$ w.r.t. $\gamma_1 \cap \cdots \cap \gamma_n$.

The proof of this theorem is postponed to Appendix A.2. As mentioned before, the proof of this theorem relies on the fact that coerced voters, except for one, can be considered to be dishonest voters, and hence, can be subsumed by the cooperator. Our analysis on the protocol by Okamoto [33] show that if dishonest voters are not considered, then single-voter coercion does in fact not imply multi-voter coercion. One can show that the Okamoto protocol is coercion resistant in the case of a single coerced voter without any dishonest voters. But the protocol is not coercion resistant with two coerced voters and still no dishonest voters (see Appendix D).

5 Civitas

In this section, we briefly recall the Civitas system [14], discuss how this system is modeled in our framework, and present positive and negative results of our analysis of Civitas, i.e., we state conditions under which Civitas does not guarantee coercion resistance and conditions under which coercion resistance is achieved. This is the first rigorous analysis of Civitas and our analysis brings out subtleties that have not been observed before. A detailed treatment can be found in Appendix B.

5.1 Protocol Description

We now briefly describe the Civitas system. A more detailed specification of this system in our framework is provided in the appendix. We start with a short description of the various cryptographic primitives employed in Civitas. *Cryptographic primitives. Civitas uses, among others, encryption schemes that allow for homomorphic encryption, random reencryption, and/or distributed decryption. In an encryption scheme with distributed decryption, a public key is generated by multiple parties. This public key can be used for encryption as usual. However, the participation of all parties involved in generating the public key is necessary to decrypt a message encrypted under the public key. Civitas also uses a distributed plaintext equivalence test (PET), where multiple parties participate in determining whether two different ciphertexts contain the same plaintext. Finally, Civitas employs a number of zero-knowledge proofs and a mix network.*

**Protocol participants.** The Civitas system assumes the following protocol participants: the supervisor $S$, voters $v_0, \ldots, v_m$, the bulletin board $B$ (which is a kind of write-only, publicly accessible memory), registration tellers $R_0, \ldots, R_k$, ballot boxes $B_0, \ldots, B_k$, and tabulation tellers $T_0, \ldots, T_k$. As in [14], we make the following assumptions: $S, B, R_0, X_0,$ and $T_0$ are honest, the remaining voting authorities may be dishonest. An arbitrary number of voters are dishonest, they are subsumed by the cooperator. The channel between the coerced voter and the honest registration teller is untappable. Channels from voters to the ballot boxes are anonymous, but not untappable (the cooperator can see whether ballots are sent to a ballot box).

For now, we consider one coerced voter, say $v_0$. We note that in [14], it is assumed that $v_0$ knows which one of the registration tellers is honest. It is in fact easy to see that Civitas is not coercion resistant otherwise. We discuss the case of multi-voter coercion at the end of this section.

**Phases of the protocol.** The protocol has three phases: the setup, voting, and tabulation phase.

In the setup phase the following steps are performed. The tabulation tellers collectively generate a public key $K_T$ and post it on the bulletin board; messages encrypted under $K_T$ are decrypted in a distributed manner by the tabulation tellers. Next, each registration teller $R_j$ randomly generates, for each voter $v_i$, a private credential share $s_{ij}$ and posts the corresponding public share $S_{ij} = \{s_{ij}\}^{K_T}$ on the bulletin board, where $r_{ij}$ represents the random coins used in the encryption of $s_{ij}$. The public credential $S_i$ of $v_i$ is publicly computable as $S_i = (S_{i0} \times \cdots \times S_{ik})$. Now, a voter $v_i$ registers at each $R_j$ to acquire his/her private credential shares $s_{ij}$, which comes with a designated verifier reencryption proof (DVRP) that $s_{ij}$ corresponds to the public share $S_{ij}$ posted on the bulletin board (such a proof is built using the public key of the voter; a voter, or any party who knows the corresponding private key, is able to forge such a proof, which is crucial for coercion resistance). The voter then computes his/her private credential $s_i = s_{i1} \times \cdots \times s_{ik}$.

In the voting phase, a voter $v_i$ posts his ballot $b_i$ on all the ballot boxes (it is enough, if the ballot is published on only one such a box to be taken into account in the tabulation phase). A ballot consists of an encrypted vote $\{v\}^{K_T}$, the encrypted credential $\{s_i\}^{K_T}$, a zero-knowledge proof showing that $v$ is a valid vote, and a zero knowledge-proof showing that the submitter simultaneously knows $s_i$ and $v_i$.

In the tabulation phase, tabulation tellers collectively tally the election by performing the following steps: (1) They retrieve the ballots from ballot boxes and the public credentials from the bulletin board. (2) They check the
proofs of the ballots, eliminating those ballots with invalid proofs. (3) Using PETs, duplicate ballots, i.e., ballots with the same encrypted credential, are eliminated according to some fixed policy. (4) First the ballots and then the credentials are mixed by each tabulation teller, by applying a permutation and using reencryption. (5) Ballots without valid credentials are eliminated, again using PETs. (6) The votes of the remaining ballots are decrypted in a distributed manner by the tabulation tellers and published. In steps (3)-(6) zero-knowledge proofs are posted to ensure that these steps are performed correctly.

5.2 Negative Results

Clarkson et al. [14] claim that under the assumptions mentioned before, Civitas is coercion resistant. Just as in the protocol by Juels et al. [23], the idea behind the counterstrategy of the coerced voter is to provide the coercer with a fake credential, which prevents the coerced voter from voting. Clarkson et al. briefly mention that a voter might not be able to vote if a registration teller refuses to provide a credential share to the voter and propose to use an additional voting authority, which attests the misbehavior of the registration teller. However, in the course of trying to prove that Civitas is coercion resistant, we found further problems that make clear that, under the mentioned conditions, Civitas is coercion resistant. We found further problems that make clear that, under the mentioned conditions, Civitas is coercion resistant. Just as in the protocol by Juels et al. [23], the idea behind the counterstrategy of the coerced voter is to provide the coercer with a fake credential, which prevents the coerced voter from voting. Clarkson et al. briefly mention that a voter might not be able to vote if a registration teller refuses to provide a credential share to the voter and propose to use an additional voting authority, which attests the misbehavior of the registration teller. However, in the course of trying to prove that Civitas is coercion resistant, we found further problems that make clear that, under the mentioned conditions, Civitas is coercion resistant.

The first problem is the following. We may well assume that all dishonest registration tellers provide credential shares to all voters. But they might in addition inform the coerced voter who has registered. Now, if the coercion strategy dictates the coerced voter not to register, there is no way that the coerced voter can register, as the coercer would be informed. In particular, there is no counter strategy that would allow the coerced voter to vote for \( z \), as the coerced voter cannot register in the first place, and hence, does not know all credential shares required for casting a valid ballot.

There is also another more subtle coercion strategy, which instructs the coerced voter to reveal his/her private key to the coercer before the registration phase. Now, a dishonest registration teller collaborating with the coercer, can use this private key to forge the DVRP. As a result, the coerced voter cannot be sure to have obtained a valid credential share. Hence, even if this voter obtained a credential share from every registration teller, he/she might still not be able to vote.

5.3 Positive Results

We found that Civitas is coercion resistant in all of the following three settings:

1. All registration tellers are honest and the goal of the coerced voter is to successfully vote for the candidate of his/her choice.

2. The goal of the coerced voter is to prevent the coercer from casting a valid ballot, where otherwise the assumptions about channels and honest and dishonest authorities are as in [14] and discussed above.

3. The goal of the coerced voter is to successfully vote for the candidate of his/her choice, but the coercion strategies are restricted in that they first dictate the coerced voter to register as prescribed by the protocol and only then follow some arbitrary coercion strategy. Otherwise, the assumptions are as in [14] and discussed before.

The assumptions in the first setting appear to be too strong, given that the main difference of Civitas compared to the Juels et al. protocol, on which Civitas is based, was to replace a single trusted registration teller by a group of possibly dishonest registration tellers. The second setting does not provide the coerced voter with much guarantees. The last setting, which we refer to by Civitas with restricted coercion strategies, seems to be the most interesting and certainly the most challenging to prove. We will therefore concentrate on this setting in the rest of the section. One can imagine that the registration is performed long before the election and that in this phase the coercer does not yet try to influence the voter.

We note that in case of Civitas with restricted coercion strategies, the coercer can still ask the voter to reveal his/her private key, but only after the registration of the voter. Hence, the voter can check whether he/she has obtained a valid credential share. Also note that registration tellers might be dishonest.

The main theorem of this section states that Civitas with restricted coercion strategies is coercion resistant in \( \alpha \) w.r.t. \( \gamma_z \) for any candidate \( z \), in the sense of Definition 2.

We now formulate \( \alpha \) and \( \gamma_z \).

We first introduce some terminology. We say that a ballot posted by a voter is posted successfully, if this ballot is delivered to the honest ballot box before the voting phase ends. A run \( \rho \) is fair w.r.t. the coerced voter \( v_0 \), if, in this run, (1) all the registration and tabulation tellers follow the protocol, i.e. post all messages and correct zero-knowledge proofs, as required, (2) \( v_0 \) obtains his credentials before the voting phase ends, and (3) if \( v_0 \) posts a valid ballot before the voting phase ends, then this ballot is posted successfully.

The properties \( \gamma_z \) and \( \alpha \) defined next, will be discussed below.

For every candidate (or valid vote) \( z \), the goal \( \gamma_z \) of the coerced voter \( v_0 \) is defined to be the set of all runs satisfying the following conditions: If a run is fair w.r.t. \( v_0 \), then the coerced voter successfully votes for \( z \).

The set \( \alpha \) of runs contains all runs satisfying the following conditions: (1) For each possible candidate (or valid vote), there is at least one honest voter who successfully casts this vote. (2) There is at least one honest voter who obtains his credential before \( v_0 \) finishes registration and abstinenc from
voting. (3) There is at least one honest voter who obtains his credential, but posts successfully a ballot with an invalid credential. (4) There is at least one honest voter who posts a ballot after $v_0$ finishes registration.

Let us first discuss $\gamma_z$. By Definition 2 (iii) $\gamma_z$ means that the counter strategy of $v_0$ must be such that $v_0$ votes successfully for $z$ in every fair run. In runs that are not fair w.r.t. $v_0$, it is clear that the vote of the coerced voter will not be counted, either because a tabulation teller misbehaved in an observable way, making the election invalid, or the ballot did not reach any ballot box in time, and as a result is not decrypted and published by a tabulation teller. The latter can happen if messages on the network are delayed for too long, possibly caused by the coercer. These are obvious reasons why a vote might not be counted. Hence, $\gamma_z$ is a very strong goal.

Now, consider the conditions (1) to (4) for $\alpha$: Condition (1) was already motivated in Section 2. Condition (2) is needed because if no honest voter abstains from voting, the coerer could tell that the coerced voter does not abstain from voting, even though he/she was supposed to abstain, just by counting the published votes. Moreover, if $v_0$ completed registration before everybody else (the coerer can even force this to happen when cooperating with a dishonest registration teller), then if some ballot is posted, the coerer knows that this must have been $v_0$. (We assume that honest voters do not post ballots without completing registration.) In this way, the coerer could again force $v_0$ to abstain from voting. Condition (3) is also necessary. If the coerer posts a ballot with the fake credential provided by $v_0$, and if all honest voters only post valid credentials, then the coerer can tell that he/she was fooled, and hence, the counter strategy of the coerced voter fails. Finally, condition (4) is needed for similar reasons as condition (2).

Conditions (1) and (4) arguably exclude runs that are unlikely to happen anyway. However, this is debatable for condition (3) (maybe also for (2)). There is no reason to assume that an honest voter would use an invalid credential, even if he/she has a valid one (such a voter would have to deviate from the protocol). To avoid condition (3), we suggest that Civitas contains some authority which randomly casts some ballots with invalid credentials. Similar “noise” can also help to avoid condition (2).

**Theorem 4.** The coercion system induced by Civitas with restricted coercion strategies is coercion resistant in $\alpha$ w.r.t. $\gamma_z$, for any valid vote $z$.

The proof of this theorem is given in the appendix. Let us note that the theorem holds for any number of honest and dishonest voters and authorities. We also note that the proof of this theorem does not depend on the policy used to remove duplicates. In particular, it does not matter whether re-voting is allowed or not.

**Multi-voter coercion.** Theorem 4 can easily be generalized to multi-voter coercion resistance. Suppose that a number $k$ of voters is being coerced. Suppose that the goal of voter $v_k$ is $\gamma_k$. By Theorem 3 to prove multi-voter coercion resistance in $\alpha$ w.r.t. $\gamma = (\gamma_1 \cap \cdots \cap \gamma_n)$, it is enough to prove (*) : a system with only one coerced voter $v_i$ is coercion resistant for $(\alpha_0, \ldots, \alpha_n)$ w.r.t. $\gamma_i$, with $\alpha_n = \alpha$.

We define $\alpha_k$ as the set of runs where (1) for each possible vote, there are at least $k$ honest voters who successfully cast this vote, (2) there are at least $k$ honest voters who obtain their credentials, before any of the coerced voters finishes registration, and abstain from voting, (3) there are at least $k$ honest voters who obtain their credential, but post ballots with invalid credentials, (4) there are at least $k$ honest voters who post a ballot after the coerced voters finish registration.

The proof of (*) is very similar to the one for Theorem 4. Hence, multi-voter coercion resistance follows.

### 6 Lee et al. Protocol

In this section, we analyze a protocol proposed by Lee et al. [28] within our framework. We show that the protocol is not coercion resistant in general, but propose an extension of the protocol for which we can show coercion resistance.

#### 6.1 Protocol Description

The Lee et al. protocol assumes that every voter owns a tamper-resistant device, called a randomizer.

In the setup phase, the tallying tellers $T_1, \ldots, T_k$ generate and publish their common public key $K_T$ for threshold decryption.

In the voting phase, a voter prepares his/her ballot, containing a vote encrypted under $K_T$, and gives it to his/her randomizer which reencrypts the ballot and signs it, and sends the result $s$ back to the voter along with a designated verifier reencryption proof (DVRP) (such a DVRP can be forged by anyone who knows the private key of the voter). This part of the communication is assumed to be entirely private. Then the voter checks the proof, computes his/her own signature on $s$ and posts it on the bulletin board.

In the tallying phase, the following is done: (1) the double signatures of voters and their randomizers on the posted ballots are verified and invalid ballots are eliminated. (2) the remaining ballots are shuffled and reencrypted and the result is posted on the bulletin board, (3) talliers jointly decrypt shuffled ballots and publish the tally result. Correctness of all these steps is assured by posting appropriate non-interactive zero-knowledge proofs.

#### 6.2 Negative Results

Assuming that the goal of the coerced voter is to vote for a particular candidate, it is easy to see that this protocol is not coercion resistant: There is a simple abstention attack where the coerer disallows the coerced voter to put a ballot signed by this voter on the bulletin board. So, one can at most hope to prove that if a ballot signed by the coerced
voter and his/her randomizer has been put on the bulletin board, then the vote of the coerced voter is counted. However, even this weaker form of coercion resistance cannot be shown: A coercer could prepare a ballot with some invalid vote which is unlikely to occur otherwise and then ask the coerced voter to give this ballot to his/her randomizer, sign the result and put it on the bulletin board. The coercer can check whether his/her vote is decrypted, assuming a dishonest tallying teller collaborating with the coercer. (The Lee et al. protocol is designed to deal with dishonest tallying tellers.) Therefore, a counter strategy is forced to use the ballot prepared by the coercer, and hence, the goal of the coerced voter cannot be achieved.

6.3 Positive Results

To prove coercion resistance, one could assume that all tallying tellers are honest, but this is not the point of the Lee et al. protocol. We instead propose a slight extension of the protocol, where the randomizer expects in addition to the ballot a zero-knowledge proof which shows that the vote in the ballot is well-formed (just as in Civitas). The randomizer then checks the proof before replying. With this extension of the protocol, we obtain coercion resistance for a natural $\gamma_z$ and $\alpha$: $\gamma_z$ contains all runs where the coerced voter successfully votes for $z$, if some ballot signed by this voter and his/her randomizer appears on the bulletin board (within the voting phase) and all zero-knowledge proofs that have to be provided by the authorities are valid. Note that this goal does not exclude abstention attacks. For the same reason explained above, these attacks are still possible in the extended version of the Lee et al. protocol. The set $\alpha$ is simply the set of runs where for each possible vote there is at least one honest voter who successfully casts this vote.

Theorem 5. The coercion system induced by the extended version of the Lee et al. protocol is coercion resistant in $\alpha$ w.r.t. $\gamma_z$, for any valid vote $z$.

The proof of this theorem is sketched in the appendix.

7 Related Work

Coercion resistance in a symbolic model was first formulated by Deaume et al. [16–18]. This work was then further developed by Backes et al. [4]. Both the work by Deaume et al. and Backes et al. were motivated by the desire to use ProVerif [10], a tool for security protocol analysis, for the automatic analysis of voting protocols. Due to the focus on automation, the notions of coercion resistance studied in these works are more restricted than the one considered here. For example, the notion of coercion resistance introduced by Deaume et al. does not apply to Civitas or the protocol by Juels et al. [23], as the class of coercion strategies and counter strategies they consider are too restricted.

To show coercion resistance of the Lee et al. protocol, Deaume et al. study a variant of this protocol which is different to the one studied here. One of the abstention attacks that we point out still works for their variant. However, this attack is out of the scope of their notion of coercion resistance. Conversely, the notion of coercion resistance by Backes et al. is inspired by the one of Juels et al., which in turn is especially tailored to the specific protocol structure of the protocol by Juels et al. and the specific forms of coercion strategies. In order to facilitate automation, the protocol models that Deaume et al. and Backes et al. consider are much coarser than ours. For example, the way votes are tallied is simplified and mix networks and proofs of compliance are not modeled.

There is also a more fundamental difference between the work by Deaume et al. and Backes et al. on the one hand, and our work on the other hand. The symbolic model by Deaume et al. and Backes et al. is the applied pi calculus [1], with its notion of observational equivalence for comparing systems/processes. Observational equivalence is a bisimulation relation which demands that every step of one system is matched by a similar step of the other system. In particular, in the works by Deaume et al. and Backes et al. the two systems in which the coerced voter runs the coercion strategy and the counter strategy, respectively, are related using the notion of observational equivalence. This is fundamentally different to the approach taken here: In our epistemic approach, we relate traces of systems and say that for every trace of one system, there exists a trace of the other system such that the coercer has the same view on both traces. In the two traces, honest voters may vote in different ways. By this, votes (including abstention) can be balanced in case coerced voters vote in different ways in the two systems and this balancing may be based on the traces as a whole. Conversely, observational equivalence, with its strict stepwise correspondence between systems, prohibits a simple balancing of votes. As a result, the formulations of coercion resistance proposed by Deaume et al. and Backes et al. are very complex and less intuitive. In Deaume et al., the balancing problem is tackled by restricting the set of coercers and coercion strategies. It is assumed that the coercer’s goal is to vote for a particular party and that coercion strategies only slightly deviate from the prescribed protocol. Altogether this leads to a rather weak notion of coercion resistance, excluding, for example, abstention attacks and other natural coercion strategies, e.g., those relevant for Civitas. Backes et al. introduce what they call an extractor to solve the balancing problem, which makes the definition of coercion resistance quite complex and hard to understand.

In [21, 22], Jonker et al. also follow an epistemic approach to model properties of voting protocols. However, they do not consider coercion resistance, only receipt freeness. Receipt freeness is modeled w.r.t. a message that a voter could use as a receipt. This is only a very rough approximation of the intuition behind receipt freeness. Also, Jonker et
al. do not model any cryptographic operators. A more recent work on receipt freeness by Jonker et al. is [20].

The work by Baskar et al. [6] focuses on the decidability of knowledge-based properties of voting protocols. However, they only study a very simplistic notion of receipt-freeness, which resembles privacy of votes; coercion resistance is not considered.

As already mentioned in the introduction, there also exist several cryptographic definitions of coercion resistance and receipt freeness (see, e.g., [13, 23, 31, 33, 35]). On the one hand, compared to the cryptographic definitions, our symbolic approach abstracts from many cryptographic details, including details of cryptographic primitives and probabilistic aspects. This leads to weaker security guarantees. On the other hand, the simplicity of the symbolic approach in general, and our definition in particular, facilitates the analysis of protocols and is more amenable to automation, which, given the complexity of voting protocols, is a crucial advantage.

8 Conclusion

In this paper, we presented a general, yet simple and intuitive definition of coercion resistance of voting protocols in an epistemic setting, which does not depend on any specific, symbolic protocol or adversary model. We applied our definition to three different voting protocols, two of which, namely Civitas and the protocol by Okamoto, have not been rigorously analyzed before. For all three protocols, we identified conditions under which these protocols are coercion resistant or fail to be coercion resistant. To obtain these results it was vital that our definition of coercion resistance allows to specify various degrees of coercion resistance in a way more fine-grained than in previous proposals. Our analyzes brought out several insights about the three protocols that have not been observed before and that led us to propose improvements of the protocols.

We believe that our definition of coercion resistance provides a good basis for automated analysis of coercion resistance, in particular since the definition can be instantiated with different protocol and adversary models. However, carrying out tool supported analysis was out of the scope of the present work.

References

[1] M. Abadi and C. Fournet. Mobile Values, New Names, and Secure Communication. In Proceedings of the 28th ACM Symposium on Principles of Programming Languages (POPL 2001), pages 104–115. ACM Press, 2001.

[2] M. Abadi and P. Rogaway. Reconciling two views of cryptography (the computational soundness of formal encryption). In J. van Leeuwen, O. Watanabe, M. Hagiya, P.D. Mosses, and T. Ito, editors, Theoretical Computer Science, Exploring New Frontiers of Theoretical Informatics, International Conference (IFIPTCS 2000), volume 1872 of Lecture Notes in Computer Science, pages 3–22. Springer-Verlag, 2000.

[3] A. Armando, D.A. Basin, Y. Boichut, Y. Chevalier, L. Compagna, J. Cuéllar, P.H. Drielsma, P.-C. Héam, O. Koucharenko, J. Mantovani, S. Mödersheim, D. von Oheimb, M. Rusinowitch, J. Santiago, M. Turunni, L. Vigano, and L. Vigneron. The AVISPA Tool for the Automated Validation of Internet Security Protocols and Applications. In K. Etessami and S.K. Rajamani, editors, Computer Aided Verification, 17th International Conference (CAV 2005), volume 3576 of Lecture Notes in Computer Science, pages 281–285. Springer-Verlag, 2005.

[4] M. Backes, C. Hritcu, and M. Maffei. Automated Verification of Remote Electronic Voting Protocols in the Applied Pi-Calculus. In Proceedings of the 21st IEEE Computer Security Foundations Symposium (CSF 2008), pages 195–209. IEEE Computer Society, 2008.

[5] M. Backes, M. Maffei, and D. Uruhu. Zero-Knowledge in the Applied Pi-calculus and Automated Verification of the Direct Anonymous Attestation Protocol. In 2008 IEEE Symposium on Security and Privacy (S& P 2008), pages 202–215. IEEE Computer Society, 2008.

[6] A. Baskar, R. Ramanujam, and S. P. Suresh. Knowledge-based modelling of voting protocols. In Dov Samet, editor, Proceedings of the 11th Conference on Theoretical Aspects of Rationality and Knowledge (TARK-2007), pages 62–71, 2007.

[7] G. Bella, F. Massacci, and L.C. Paulson. An overview of the verification of SET. International Journal of Information Security, 4:17–28, 2005.

[8] J. C. Benaloh and D. Tuinstra. Receipt-free secret-ballot elections (extended abstract). In Proceedings of the Twenty-Sixth Annual ACM Symposium on Theory of Computing (STOC 1994), pages 544–553. ACM Press, 1994.

[9] K. Bhargavan, C. Fournet, A. D. Gordon, and S. Tse. Verified Interoperable Implementations of Security Protocols. In Proceedings of the 19th IEEE Computer Security Foundations Workshop (CSFW-19 2006), pages 139–152. IEEE Computer Society, 2006.

[10] B. Blanchet. An Efficient Cryptographic Protocol Verifier Based on Prolog Rules. In Proceedings of the 14th IEEE Computer Security Foundations Workshop (CSFW-14), pages 82–96. IEEE Computer Society, 2001.

[11] R. Canetti. Universally Composable Security: A New Paradigm for Cryptographic Protocols. Technical report, Cryptology ePrint Archive, December 2005. Online available at http://eprint.iacr.org/2000/067.ps.

[12] I. Cervesato, A.D. Jaggard, A. Scedrov, J.-K. Tsay, and C. Walstad. Breaking and fixing public-key Kerberos. In International Workshop on Issues in the Theory of Security (WITS 2006), 2006.

[13] B. Chevallier-Mames, P.-A. Fouque, D. Pointcheval, J. Stern, and J. Traoré. On Some Incompatible Properties of Voting Schemes. In JAvoS Workshop On Trustworthy Elections (WOTE 2006), 2006.

[14] M. R. Clarkson, S. Chong, and A. C. Myers. Civitas: Toward a Secure Voting System. In 2008 IEEE Symposium on Security and Privacy (S& P 2008), pages 354–368. IEEE Computer Society, 2008.
[15] V. Cortier, S. Kremer, R. Küsters, and B. Warinschi. Computationally Sound Symbolic Secrecy in the Presence of Hash Functions. In Proceedings of the 26th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS 2006), volume 4337 of Lecture Notes in Computer Science, pages 176–187. Springer, 2006.

[16] S. Deleune, S. Kremer, and M. D. Ryan. Verifying Privacy-type Properties of Electronic Voting Protocols. Journal of Computer Security, 2009. To appear.

[17] S. Deleune, S. Kremer, and M. D. Ryan. Coercion-Resistance and Receipt-Freeeness in Electronic Voting. In Proceedings of the 19th IEEE Computer Security Foundations Workshop (CSFW'06), pages 28–39. IEEE Computer Society Press, 2006.

[18] S. Deleune, S. Kremer, and M. D. Ryan. Verifying properties of electronic voting protocols. In Proceedings of the IAVoSS Workshop On Trustworthy Elections (WOTE’06), pages 45–52, 2006.

[19] M. Hirt and K. Sako. Efficient receipt-free voting based on homomorphic encryption. In B. Preneel, editor, Advances in Cryptology – EUROCRYPT 2000, volume 1807 of Lecture Notes in Computer Science, pages 539–556. Springer, 2000.

[20] H. Jonker, S. Mauw, and J. Pang. Measuring voter-controlled privacy. In Proceedings of the 4th Conference on Availability, Reliability and Security – ARES’09. IEEE Computer Society, 2009. To appear.

[21] H. L. Jonker and W. Pieters. Receipt-Freeeness as a special case of Anonymity in Epistemic Logic. In IAVoSS Workshop On Trustworthy Elections (WOTE 2006), 2006.

[22] A. Juels, D. Catalano, and M. Jakobsson. Coercion-resistant electronic elections. In Proceedings of Workshop on Privacy in the Electronic Society (WPES 2005). ACM Press, 2005.

[23] R. Küsters. Simulation-Based Security with Inexhaustible Interactive Turing Machines. In Proceedings of the 19th IEEE Computer Security Foundations Workshop (CSFW-19 2006), pages 309–320. IEEE Computer Society, 2006.

[24] R. Küsters, A. Datta, J. C. Mitchell, and A. Ramanathan. On the Relationships Between Notions of Simulation-Based Security. Journal of Cryptology, 21(4):492–546, 2008.

[25] R. Küsters and T. Truderung. Reducing Protocol Analysis with XOR to the XOR-free Case in the Horn Theory Based Approach. In P. Syverson, S. Jha, and X. Zhang, editors, Proceedings of the 15th ACM Conference on Computer and Communications Security (CCS 2008), pages 129–138. ACM Press, 2008.

[26] R. Küsters and T. Truderung. An Epistemic Approach to Coercion-Resistance for Electronic Voting Protocols. In 2009 IEEE Symposium on Security and Privacy (S&P 2009). IEEE Computer Society, 2009. To appear.

[27] R. Küsters and T. Truderung. An Epistemic Approach to Coercion-Resistance for Electronic Voting Protocols. In Proceedings of Information Security and Cryptology (ICISC 2003), volume 2971 of Lecture Notes in Computer Science, pages 245–258. Springer, 2003.

[28] B. Lee, C. Boyd, E. Dawson, K. Kim, J. Yang, and S. Yoo. Providing receipt-freeness in mixnet-based voting protocols. In Proceedings of Information Security and Cryptology (ICISC 2003), volume 2971 of Lecture Notes in Computer Science, pages 245–258. Springer, 2003.

[29] A. Juels, D. Catalano, and M. Jakobsson. Soundness of Formal Encryption in the Presence of Active Adversaries. In M. Naor, editor, First Theory of Cryptography Conference (TCC 2004), volume 2951 of Lecture Notes in Computer Science, pages 133–151. Springer, 2004.

[30] D. Micciancio and B. Warinschi. Simulation-Based Security with Inexhaustible Interactive Turing Machines. In P. Syverson, S. Jha, and X. Zhang, editors, Proceedings of Information Security and Cryptology (ICISC 2003), volume 2971 of Lecture Notes in Computer Science, pages 245–258. Springer, 2003.

[31] J. C. Mitchell, V. Shmatikov, and U. Stern. Fine-State Analysis of SSL 3.0. In Seventh USENIX Security Symposium, pages 201–216, 1998.

[32] D. Micciancio and B. Warinschi. Soundness of Formal Encryption in the Presence of Active Adversaries. In M. Naor, editor, First Theory of Cryptography Conference (TCC 2004), volume 2951 of Lecture Notes in Computer Science, pages 133–151. Springer, 2004.

[33] T. Moran and M. Naor. Receipt-Free Universally-Verifiable Voting With Everlasting Privacy. In C. Dwork, editor, Advances in Cryptology - CRYPTO 2006, 26th Annual International Cryptology Conference, Proceedings, volume 4117 of Lecture Notes in Computer Science, pages 373–392. Springer, 2006.

[34] T. Moran and M. Naor. Split-ballot voting: everlasting privacy with distributed trust. In P. Ning, S. De Capitani di Vimercati, and P. F. Syverson, editors, Proceedings of the 2007 ACM Conference on Computer and Communications Security, CCS 2007, pages 246–255. ACM, 2007.

[35] T. Okamoto. Receipt-Free Electronic Voting Schemes for Large Scale Elections. In B. Christianson, B. Crispo, T. M. A. Lomas, and M. Roe, editors, Proceedings of the 5th International Workshop on Security Protocols, volume 1361 of Lecture Notes in Computer Science, pages 25–35. Springer, 1997.

[36] K. Sako and J. Kilian. Receipt-Free Mix-Type Voting Scheme — A practical solution to the implementation of a voting booth. In Advances in Cryptology — EUROCRYPT ’95, International Conference on the Theory and Application of Cryptographic Techniques, volume 921 of Lecture Notes in Computer Science, pages 393–403. Springer-Verlag, 1995.

[37] V. Teague, K. Ramchen, and L. Naish. Coercion-Resistant tallying for STV voting. In IAVoSS Workshop On Trustworthy Elections (WOTE 2008), 2008.

A General Properties

A.1 Proof of Theorem 1

Before we present the proof of Theorem 1 we define normal protocols precisely.

Definition 10. A protocol \( \Pi = (A, in, out, s_0, P) \) with \( A = \{ v, c, e \} \) is normal, if (i) \( v \) and \( c \) are connected by some input and output channels (in both directions), (ii) both \( v \) and \( c \) have an unbounded number of private channels (see the paragraph after Definition 9), (iii) \( P(v) = \Pi(in(v), out(v)) \) and \( P(c) = \Pi(in(c), out(c)) \).

Proof of Theorem 1. We first introduce some terminology and prove general lemmas about processes for forwarding messages between channels.
Let $p = (I, O, N, f)$ be an atomic process and $h$ be a channel renaming, i.e. a injection from $Ch$ to $Ch$. We define an atomic process $h(p)$ as $(I', O', N', f')$ with $I' = h(I)$ and $O' = h(O)$, where, for each $(I', N)$-view $U = (\lambda, W)$, we put $f'(U) = h(f(h^{-1}(\lambda), W))$. We extend the domain of a channel renaming to arbitrary (non necessarily atomic) processes in a natural way.

Now, for processes $P_1, P_2$ and a channel renaming $h$, we will write $P_1 \sqsubseteq_h P_2$, if for each run $\pi$ induced by $P_1$, the run $h(\pi)$ is induced by $P_2$. The following lemma is easy to prove.

**Lemma 1.** For a process $P$ and a channel renaming $h$, we have $P \sqsubseteq_h h(P)$.

For $c, d \in Ch$, we denote by $\uparrow^d_c$ the process which simply forwards on channel $d$ every message received on $c$. By $\uparrow^d_c$ we will denote $(\uparrow^d_c || \uparrow^d_c').$ For a process $P[0, c_1]$ which uses channels $c_0$ and $c_1$, we will write $P[a_0, a_1]$ for the process which uses $a_1$ instead of $c_1$ and otherwise behaves like $P[0, c_1]$. Let $P[0, c_1]$ for $P[a_0, a_1]$ for $h(\pi) = h(x_0, x_1), c, d \in \$.$

Let $P$ be a subprocess of some process $P'$. We define an equivalence relation $\equiv_P$ on runs induced by $P'$. Let $P'$ be a process, and $x_0, x_1$ be channels not used neither by $P[0, c_1]$ nor $P$. Let $P_1 = (P' || P[0, c_1])$ and $P_2 = (P' || P[x_0, x_1])$. Then:

1. For each run $\pi$ induced by $P_1$ there exists a run $\pi'$ induced by $P_2$ with $\pi \equiv_P \pi'$.
2. For each run $\pi$ induced by $P_2$ there exists a run $\pi'$ induced by $P_1$ with $\pi \equiv_P \pi'$.

**Proof.** Let $f$ be the process $\uparrow^d_c$. To prove (1), suppose that $\pi$ is a run induced by $P_1$. We construct $\pi'$ in the following way. Whenever it happens in $\pi$ that $(c_0 : m)$ is delivered and, in consequence, a reply of $P[0, c_1]$ is sent, then two steps are performed in $\pi'$: first, $(c_0 : m)$ is delivered and, in consequence, the message $(x_0 : m)$ of $f$ is sent; and second, the message $(x_0 : m)$ sent in the first step is immediately delivered and, in consequence, the reply of $P[x_0, x_1]$ is sent. Furthermore, whenever it happens in $\pi$ that $(c_1 : m)$ is sent by $P[0, c_1]$, then two steps are performed in $\pi'$: the corresponding message $(x_1 : m)$ is sent by $P[x_0, x_1]$ and then this message is immediately delivered and, in consequence, $(c_1 : m)$ is sent by $f$. It is easy to show that $\pi'$ obtained in this way is a run induced by $P_2$ and $\pi \equiv_P \pi'$.

To prove (2), suppose that $\pi$ is a run induced by $P_2$. We construct $\pi'$ in the following way. Whenever it happens in $\pi$ that $(x_1 : m)$ is sent by $P[x_0, x_1]$, then, in $\pi'$, the corresponding message $(c_1 : m)$ is sent by $P[0, c_1]$. When, in $\pi$, such a message $(x_1 : m)$ is delivered and, in consequence, the reply $(c_1 : m)$ of $f$ is sent, no corresponding step is performed in $\pi'$, so, in particular, $(c_1 : m)$ is kept as a message to be delivered. Furthermore, whenever it happens in $\pi$ that a message $(c_0 : m)$ is delivered and, in consequence, the reply $(x_0 : m)$ of $f$ is sent, then no corresponding step is taken in $\pi'$, so, $(c_0 : m)$ is kept as a message to be delivered. When, in $\pi$, a message $(x_0 : m)$ is delivered to $P[x_0, x_1]$, then, in $\pi'$, we can deliver $(c_0 : m)$ to $P[0, c_1]$. It is easy to show that $\pi'$ obtained in this way is a run induced by $P_1$ and $\pi \equiv_P \pi'$. (Note, however, that one cannot prove $\pi \equiv_P \pi'$.) This completes the proof of Lemma 2.

To make the proof of Theorem 1 simpler, we assume that $d = (a_0, a_1)$ with $a_0 \notin \$ and $a_1 \notin \$ for each $c$. Similarly, we assume that $d = (d_0, d_1)$ with $d_0 \notin \$ and $d_1 \notin \$ for each $c$. We stress, that these assumptions make the proof simpler, but are by no mean crucial and can be easily dropped.

For channels $d = (x_0, x_1)$, let $v_0[\vec{y}] = (\uparrow^d_{a_0, a_1})$. So, $v_0[\vec{y}]$ simply forwards on channel $x_0$ each message received on $d_0$ and forwards on $d_1$ each message on channel $x_1$. Now, $v_0$ is just $v_0[\vec{y}]$.

To prove Theorem 4, suppose that $v_0$ is not a coercion strategy in $\gamma$ w.r.t. $\gamma$ and $v_0$ is a counter-strategy for $v_0$. Let $v$ be a strategy in $V$. We will construct a counter-strategy $v'$ for $v$.

We will write $v_0'[\vec{y}]$ instead of $v_0'$, as channels $\vec{y}$ are used by $v_0'$. Similarly, we will write $v[\vec{d}, \vec{d}, \vec{d}]$, $a$ for each $c$. Let $\vec{x} = (x_0, x_1)$ be some private channels of $v$ not used in $v_0$ nor $v_0'$. Such channels exist due to Condition (ii) of Definition 10. Similarly, for a given $c \in C$, let $\vec{y} = (y_0, y_1)$ be some internal channels of $c$ not used in $c$.

We define $v'$ as $(v_0'[\vec{x}] || v[\vec{x}, \vec{d}])$. We will show that $v'$ is a counter-strategy for $v$. Let $\sigma = \{\vec{x} \mapsto \vec{a}, \vec{a} \mapsto \vec{y}, \vec{y} \mapsto \vec{x}\}$.

The following lemma holds true, because none of $\vec{a}, \vec{x}, \vec{y}$ is used by any $c \in E$.

**Lemma 3.** Let $\rho = (\vec{v}, \vec{c}, \vec{e}, \pi)$ be a run of $S$. We have that $\pi \equiv \sigma(\pi)$.

**Proof.** Let $\pi = (\vec{v}, \vec{c}, \vec{e}, \pi)$ be a run of $S$. We have that $\pi \equiv \sigma(\pi)$.

**Lemma 4.** Let $\pi_1, \pi_2$ be runs induced by some $(\vec{v} || \vec{c} || \vec{e})$ such that channels $\vec{x}$ do not occur in $\pi_1, \pi_2$. If $\pi_1 \equiv \pi_2$, then $\pi_1 \equiv \pi_2$.

**Sketch of proof.** The lemma follows from the observation, that, for each channel $z$ occurring in $\pi_1$ or $\pi_2$ (note that $z \neq x$), if $\sigma^{-1}(z) \in I_{e_0}$, then $z \in I_e$.
and e ∈ E. So, let ρ1 ∈ r(v′, c, e), which means that

ρ1 = ((i0, x | x, a), c, a, e, π),

for some i0 ∈ r0, v ∼ v, e ∼ e, and some π induced by (i′0, x | x, a, c | c, e). So, by Lemma 1, σ(π) is a run induced by (i0, x | x, a, c | c, e) and thus

ρ2 = ((i0, x | x, a, c | c, e, σ(π)),

is in r(v0′, v′, a, c | c, e | e, π), (note that v′, a, c | c, e | e, π) is in P(c), by condition (iii) of Definition 11. Because v0′ is a counter-strategy for v0 | a, we have ρ2 ∈ α. By Lemma 3, π ∼ σ(π), which implies ρ1 ∼ ρ2. Because γ is closed under ⊳, we obtain ρ1 ∈ γ.

Finally, we will show that Item (ii) of the definition of coercion resistance holds for v and v′, i.e. for each c, e, and ρ ∈ r(v, c, e) ∩ α, there exist e′ ∈ E and ρ′ ∈ r(v, c, e) such that ρ ∼ ρ′. For Item (i) one can proceed similarly. This completes the proof of the theorem.

So, let ρ1 ∈ r(v′, c, e) ∩ α. We proceed, as above, and so, ρ1 is like above and, for ρ2 defined as above, ρ1 ∼ ρ2 holds. Because α is closed under ⊳, we have ρ2 ∈ α. As v0 | a is a counter-strategy for v0 | a, there exists e′ ∈ E and a run ρ′ ∈ r(v0 | a, v′ | x, a, c | c | e′) with ρ2 ∼ ρ′. This means that

ρ2 = (v0 | a, v′ | x, a, c′ | e′, π′),

for some e′ ∼ e′, and some π′ induced by (v0 | a, v′ | x, a, c′ | e′ | e′) such that π′ ≡ e′ | x, a, c′ | e′ | e′ | e′). By Lemma 1, σ−1(π′) is a run induced by (v0 | x, v′ | x, a, c | c | e′) and thus

ρ1 = ((v0 | x, v′ | x, a, c | c, e′, π′))

is in r(v0 | x, v′ | x, a, c | c, e′, π′) (note that v0 | x, v′ | x, a, c | c, e′) is in P(v), by condition of coercion resistance (iii) of Definition 11. Since π′ ≡ e′ | x, a, c′ | e′ | e′ | e′). By Lemma 2, there exists a run π′ such that (v′ | x, a, c′ | e′ | e′) with π′ ≡ e′ | x, a, c′ | e′ | e′ | e′). Hence, π′ ≡ e′ | x, a, c′ | e′ | e′ | e′), and so, finally, we obtain a run (v′, v, e′, π′) ∼ ρ1 in r(v, c, e).

A.2 Proof of Theorem 3

Before we prove the theorem, we state some definitions only sketched or omitted in Section 4.3.

Let S be a protocol as in Section 4.3. We define St = (A, inA, outA, s0, P), where v now represents voter v1 only, e is unchanged, and c gets direct access to the channels of the coerced voters v1, ..., vn, i.e., inA(v) = In, outA(v) = OIn, inA(c) = inA(0) ∪ ∪nE, and outA(c) = outA(c) ∪ ∪nE. Moreover, Pi(v) = Pi(v) = Pi inA(v), outA(v), and P1(c) = Pi inA(c), outA(c).

For the proof of Theorem 3, we define a mapping from runs ρ of T to runs ρ(1) of T1 and from properties β of T to properties β(1) of T1: Recall that each v ∈ P(v) is of the form (v1 | v2, v3 | v4) with v1 ∈ Π(1, O1). For a run ρ = ((v1 | v2, v3 | v4), c, e, π), we define ρ(1) as (v1, (v1 | v2, v3 | v4), c, e, π). For a property β of T, we define β(1) to be (β | c, e, π). When it is clear from the context, we will write β instead of β(1), treating β as a property of T1.

We can now turn to the proof of Theorem 3. We define a function f which maps a coercion strategy v1 of the i-th voter to a counter strategy v′ = f(v1), by defining v′ as some (arbitrarily chosen) counter strategy for v1 in T1 (such a counter strategy exists, since T1 is coercion resistant).

Now, for any v ∈ P(v) which, as we know, must be of the form (v1 | v2, v3 | v4) with v1 ∈ P1(v), and for v′ = (v′ | v′, c, e), where v′ = f(v1), we will show that T, along with v and v′, meets the conditions of the definition of multi-voter coercion resistance.

First, let us show that condition (ii) holds. Let c ∈ P(c), e ∈ P(e) and ρ ∈ r(v, c, e). So, ρ is of the form ((v′ | v′ | v′ | v′), c, e, π). For each i ∈ {1,...,n} we have that ρ(i) ∈ ri(v′, v′, c, e), where ci = (v′ | v′ | v′ | v′ | v′, c, e). So, ρ(i) ∈ γi and so ρ(i) ∈ γi. Hence, ρ ∈ γi ∩ ∩n=1 γn.

Now, let us show that condition (i) holds. The proof for condition (ii) is very similar. Let ρ ∈ r(v, c, e) ∩ α, for some c and e. Let uk denote (v′ | v′ | v′ | v′) ∩ ∩n=1 v′. Note that u0 = v and u0 = v′. We will show, by induction, that for each k ∈ {0,...,n} there exists ek and rk such that ek ∈ r(uk, c, ek) ∩ αn−k and rk ∼ ρ. Note that, for k = 0, we can simply take e0 = c and ρ0 = ρ. So, let us assume that the above holds for k − 1. We will show that it also holds for k. So, we have some ek−1 and rk−1 ∼ ρ such that rk−1 ∈ r(uk−1, c, ek−1) ∩ αn−k−1. It follows that ρk−1 ∈ α(k−1) and ρk−1 ∈ ri(vk, c, ek−1), where ek′ = (v′ | v′ | v′ | v′ | v′, c, e, π). By coercion resistance of Tk, there exists ek and rk ∈ r(ek′, c, ek) ∩ αn−k such that rk ∼ ek. Let rk be such that rk = rk(k). Hence, ρk ∼ ρk−1 (as the coherer can see more in Tk than in T). By transitivity of ∼, we have ρk ∼ ρ. We also have that ρk ∈ r(uk, c, ek) and ρk ∈ αn−k.

B Civitas

In this section we provide a detailed modeling of Civitas in our framework and present the proof of coercion resistance of this system.

B.1 Cryptographic Primitives

We use a term of the form (m, m′) to represent a pair of messages m and m′ with first(p) and sec(p) yielding, respectively, the first and the second component of a pair p. A term sigm(k) represents the signature on a message m under (a private) key k. Such a signature can be verified using pub(k), the public key corresponding to k. We also assume that such a signature reveals m.
We use the following terms to represent randomized encryption with reencryption and homomorphic property: \{m\}_k represents a term \(m\) encrypted under a (public) key \(k\) using a randomness \(r\); \text{dec}(c, k) represents a decryption of a ciphertext \(c\) with a key \(k\) (\(k\) is intended to be a private key corresponding to the public key under which \(c\) is encrypted); \text{reenc}(c, k, r) represents a reencryption of a ciphertext \(c\) under a (public) key \(k\) with randomness \(r\) (we have \text{reenc}({m_1}_k, k, r') = {m_1}_k^{r+r'}\). We also use symbols + and \(\times\), equipped with the appropriate equational theory, to express the \textit{homomorphic} property of the encryption: \{m_1\}_k^{r_1} \times \{m_2\}_k^{r_2} = \{m_1 \times m_2\}_k^{r_1+r_2}.

\textit{Distributed decryption} is modelled as follows. Suppose that \(x_1, \ldots, x_n\) are private key shares of some agents \(a_1, \ldots, a_n\). Then, \(\text{pub}(x_1), \ldots, \text{pub}(x_n)\) are the corresponding \textit{public key shares} (which are intended to be published). The \textit{distributed public key} of \(a_1, \ldots, a_n\) is now \(K = \text{pub}(x_1) \times \cdots \times \text{pub}(x_n)\). To decrypt a ciphertext \(c = \{m\}_K\) that is a message \(m\) encrypted under this key, the cooperation of all \(a_1, \ldots, a_n\) is necessary: each \(a_i\) posts his \textit{public decryption share} \(p_i = \text{dshare}(c, x_i)\). Now, the result of decryption (that is the message \(m\)) can be computed from these shares: \(m = \text{distdec}(p_1, \ldots, p_n)\).

In a very similar way we model \textit{distributed plaintext equivalence test} (PET), which can be used to determine, whether, for two ciphertext \(c\) and \(c'\), the plaintext of \(c\) and \(c'\) are the same, without revealing anything more about these plaintexts (in particular, without decrypting \(c\) and \(c'\)). Suppose, again that \(x_1, \ldots, x_n\) are private key shares of \(a_1, \ldots, a_n\) and \(K = \text{pub}(x_1) \times \cdots \times \text{pub}(x_n)\) is their distributed public key. To perform a PET on ciphertexts \(c = \{m\}_K\) and \(c' = \{m'\}_K\) (that is to check whether \(m\) and \(m'\) are the same), each \(a_i\) posts his \textit{public PET share} \(p_i = \text{petshare}(c, c', x_i)\). Now, the result of the PET can be computed from these shares: \(\text{distpet}(p_1, \ldots, p_n) = T\) iff the \(m = m'\).

The equational theory for modeling these primitives is given in the appendix (Fig. 2). We assume additionally that + and \(\times\) are equipped with equations for associativity and commutativity property (we could consider more complex equational theory for there operators, which however makes the proof more complicated). This theory will be denoted by \(E\).

\subsection*{B.2 Zero-knowledge Proofs}

We will model the zero-knowledge proofs used in the protocol following the approach of [5]. A zero-knowledge proof will be represented by a term \(P = \text{ZK}^{n, k}_\alpha(t_1, \ldots, t_n; s_1, \ldots, s_k)\) where \(t_1, \ldots, t_n\) are terms called the \textit{private component} (the proof will keep these terms secret), terms \(s_1, \ldots, s_k\) are called the \textit{public component} (the proof reveals these terms), and \(\alpha\) is a term built upon variables \(x_1, \ldots, x_n, y_1, \ldots, y_k\) (no other variables and no nonces can occur in this term; \(x_i\) is intended to refer to \(t_i\), while \(y_i\) is intended to refer to \(s_i\)), called the \textit{formula} of \(P\).

We have the following equalities associated to zero-knowledge proofs. The first group of equations reveals the public components (also the formula) of a proof. The second one allows one to check validity of a proof.

\[
\begin{align*}
\text{public}(\text{ZK}^{n, k}_\alpha(t_1, \ldots, t_n; s_1, \ldots, s_k)) = \langle \varphi, s_1, \ldots, s_k \rangle \\
\text{check}(\text{ZK}^{n, k}_\alpha(t_1, \ldots, t_n; s_1, \ldots, s_k)) = T
\end{align*}
\]

if \(\varphi\) is a formula build upon \(x_1, \ldots, x_n, y_1, \ldots, y_k\), and \(\varphi[x_i/x, y_i] \equiv_k T\)

To model Civitas, we will use zero-knowledge proofs formally defined in Fig. 3. We use semicolons only to enhance legibility, as a mean of separating private and public components. The meaning of these proofs is as follows:

\text{KnowPriv}(x; y) represents a proof of knowledge of the private key \(x\) associated with the given public key \(y\) (i.e. \(y = \text{pub}(x)\)).

\text{DVRP}(\alpha; x; m, m', k, k_v) represents a designated-verifier reencryption proof which shows that \(m'\) is a reencryption of \(m\) under \(k\); \(k_v\) is the public key of the designated verifier who, having the corresponding private key, is able to forge a faked proof; \(\alpha\) is an additional randomness used to construct the proof. The proof is valid if either (a) \(m' = \text{reenc}(m, k, x)\) or (b) \(k_v = \text{pub}(x)\), i.e. \(x\) is a private key associated with public key \(k_v\) of the designated verifier.

\text{ProofDShare}(x; p, y, c) represents a proof that \(p\) is the public share for distributed decryption of \(c\) w.r.t. \(y\), i.e. \(p = \text{dshare}(c, x)\) and \(y = \text{pub}(x)\).

\text{ProofPETShare}(x; p, y, c, c') represents a proof that \(p\) is the public share for distributed PET of ciphertexts \(c\) and \(c'\) w.r.t. \(y\), i.e. \(p = \text{petshare}(c, c', x)\) and \(y = \text{pub}(x)\).

\text{OneOf}(r; m, k, b) represents a proof that \(m\) is an encryption under \(k\) of one of the values in \(b = \{b_1, \ldots, b_l\}\) (\(m = \{b\}_K\), where \(b\) is an element of \(b\)).

\text{MutKnow}(m, m', r, r'; c, c', k) represents a proof of mutual knowledge of the plaintexts contained in ciphertexts \(c\) and \(c'\) (\(c = \{m\}_K\) and \(c' = \{m'\}_K\)).

\text{ProofMix}(\vec{r}, \vec{c}_1, \vec{c}_2, k) where \(\vec{r}, \vec{c}_1, \vec{c}_2\) are tuples of length \(l\), represents a proof that \(\vec{c}_2\) is obtained from ciphertexts \(\vec{c}_1\) by mixing (i.e. applying some permutation) and reencryption (\(\vec{r}\) is the collection of random values used in reencryption), i.e. \(\vec{c}_2[\pi(i)] = \text{reenc}(\vec{c}_1[i], k, r_i[i])\), for some permutation \(\pi\) of \(\{1, \ldots, l\}\).

\subsection*{B.3 Protocol Description}

\textbf{The participants.} The participants of the protocol are: the voters \(v_0, \ldots, v_m\), the supervisor \(S\), the bulletin board \(B\) registration tellers \(R_0, \ldots, R_k\), ballot boxes \(X_0, \ldots, X_k\), and tabulation tellers \(T_0, \ldots, T_k\). We will assume that \(B, R_0, \ldots,
checksig(\text{sig}_k \{m\}, \text{pub}(k)) = T
extractmsg(\text{sig}_k \{m\}) = m
\text{dec}(\{x\}^r_{\text{pub}(k)}) = x
\text{reenc}(\{x\}^r_{k}, k, r') = \{x\}^{r+r'}
\text{reenc}(\text{reenc}(x, k, r), k, r') = \text{reenc}(x, k, r + r')
\{m_1\}^{r_1}_{k} \times \{m_2\}^{r_2}_{k} = \{m_1 \times m_2\}^{r_1+r_2}_{k}
\text{distdec}(p_1, \ldots, p_k) = m \text{ where } p_i = \text{dshare}(\{m\}^r_Y, x_i)
\text{with } Y = (\text{pub}(x_1) \times \cdots \times \text{pub}(x_k))
\text{distpet}(p_1, \ldots, p_k) = T \text{ where } p_i = \text{petshare}(\{m\}^r_Y, \{m\}^{r'}_Y, x_i)
\text{with } Y = (\text{pub}(x_1) \times \cdots \times \text{pub}(x_k))
\text{first}(x, y) = x \quad \text{sec}(x, y) = y
x \oplus x = T \quad T \lor x = T
T \land T = T \quad x \lor T = T

Figure 2: Theory E — equational theory for modeling Civitas.

KnowPriv stands for ZK_{\varphi}^{1,1}
DVRP stands for ZK_{\varphi}^{2,4}
ProofDSHare stands for ZK_{\varphi}^{1,3}
ProofPETShare stands for ZK_{\varphi}^{1,4}
OneOfI stands for ZK_{\varphi}^{1,3}
MutKnow stands for ZK_{\varphi}^{2,5}
ProofMixI stands for ZK_{\varphi}^{1,3}

with \varphi = (y_1 \equiv \text{pub}(x_1))
with \varphi = (y_2 \equiv \text{reenc}(y_1, y_3, x_2) \lor y_4 \equiv \text{pub}(x_2))
with \varphi = (y_1 \equiv \text{dshare}(y_3, x_1) \land y_2 \equiv \text{pub}(x_1))
with \varphi = (y_1 \equiv \text{petshare}(y_3, y_4, x_1) \land y_2 = \text{pub}(x_1))
with \varphi = \bigvee_{i=1}^l (y_1 = \{x_1\}^{-1}_{y_2})
with \varphi = (y_1 = \{x_1\}^{-1}_{y_2} \land y_2 = \{x_2\}^{-1}_{y_2})
with \varphi = \bigvee_{i \in P_1} \bigwedge_{i=1}^l (y_2[\pi(i)] \equiv \text{reenc}(y_1[i], y_3, x_1[i]))

Figure 3: Shortcuts for zero-knowledge proofs. In the equations \(t[i]\) denotes the \(i\)-th element of a tuple \(t\) (obtained by appropriately applying destructors to \(t\)), and \(P_1\) denotes the set of all permutations of \(\{1, \ldots, l\}\).

\(X_0\) and \(T_0\) are honest. The remaining voting authorities may be dishonest. We will also assume that some of voters are dishonest and cooperate with the coercer. We assume that the channel from the voter’s trusted registration teller is untappable.

In what follows, we assume that \(i\) ranges over the set \(\{0, \ldots, m\}\) and \(j\) ranges over \(\{0, \ldots, k\}\). For a participant \(a\), we will write \text{sig}_a \{m\} instead of \text{sig}_{\text{pub}(k_a)} \{m\}\. We will also write \text{pub}(a) instead of \text{pub}(k_a).

Setup phase. We do not model here the first part of the setup phase, where the supervisor posts the ballot design (the set of valid votes), identifies the tellers by posting their public keys, and posts the electoral roll (the set of authorized voters). Instead, we assume that the public keys of the voting authorities, the ballot design, and the electoral roll are fixed. Below, we describe the remaining steps of this phase.

Tabulation tellers collectively generate a public key for a distributed encryption scheme and post it on the bulletin board (decryption of messages encrypted under this key requires the participation of all tabulation tellers):

\begin{align*}
\text{(KGen1)} & \quad T_j \rightarrow B : \text{sig}_{T_j} \{h(y_j)\} \\
\text{(KGen2)} & \quad T_j \rightarrow B : \text{sig}_{T_j} \{y_j, \text{KnowPriv}(x_j; y_j)\}
\end{align*}

where \(y_i = \text{pub}(x_i)\) and \(x_i\) is random value, the private key share of \(T_j\). After the first step, all the tellers wait until all commitments are available. After the second step, they check proofs (by “checking a proof \(p\)” we mean verifying that its public components are as required and that check(p) = T). Now, \((y_1 \times \cdots \times y_k)\) is the distributed public key of \(T_1, \ldots, T_k\). We will refer to this key by \(K_T\).

Next, each registration teller \(R_j\) randomly generates credential shares \(s_{ij}\) (for each voter \(v_i\)) and post these shares on the bulletin board:

\begin{align*}
\text{(Cred)} & \quad R_j \rightarrow B : \text{sig}_{R_j} \{i, S_{ij}\} \quad \text{(for each } i, j)\)
\end{align*}

where \(S_{ij} = \{s_{ij}\}^{r_{ij}}_{K_T}\) and \(r_{ij}\) are random. The public credential of \(v_i\) is now publicly computable as \(S_i = (S_{i1} \times \cdots \times S_{ik})\).
Policy) is performed, by running PET-encrypted ballot credentials known information.

The policy is enforced and, for each ballot, a designated-verifier reencryption proof which shows that the encrypted vote of voter is a reencryption of the encrypted vote of voter. The voter verifies this proof. Now, his private credential is $s_i = (s_i0 \times \cdots \times s_i k)$.

Voting phase. Each voter sends his ballot $b_i$ containing his vote along with his credential to all ballot boxes:

\[(\text{Vote}) \quad v_i \to X_j : b_i = \langle \{s_i \}_k, \{v_i \}_k, P_i, P_K \rangle\]

where $r_i, r'_i$ are random, $v_i$ is the vote chosen by $v_i$, $P_i = \text{OneOf}(r_i'; \{v_i \}_k, K_T, b_1, \ldots, b_l)$, and $P_K = \text{MutKnow}(s_i, v_i, r_i, r'_i; \{s_i \}_k, \{v_i \}_k, K_T)$. The value $v_i$ will be called the vote of $b_i$. $s_i$ is ballot credential of $b_i$. $\{v_i \}_k$ will be called the encrypted vote of $b_i$, and $\{s_i \}_k$ will be called the encrypted credential of $b_i$. $P_i$ is a zero-knowledge proof which shows that the voter is well-formed with respect to the ballot design ($v_i$ is one of the valid votes $z_1, \ldots, z_l$), and $P_K$ is a zero-knowledge-proof which shows that the voter submits simultaneously only $s_i$.

We will some times write $b_i(v', s')$ for the message like $b_i$ but with $v'$ and $s'$ instead of $v_i$ and $s_i$.

Tabulation phase. Before the tabulation phase, each ballot box posts a commitment to all ballot boxes on the bulletin board:

\[(\text{Comm1}) \quad X_j \to B : \text{sig}_{X_j}(j, C_j)\]

where $C_j = h(\text{contents}(X_j))$. The supervisor then posts his own signatures on all these commitments, defining the set of votes to be tabulated:

\[(\text{Comm2}) \quad S \to B : \text{sig}_S(j, C_j)\]

Then, the tabulation tellers collectively tally the election: All tabulation tellers (1) retrieve the ballots from all ballot boxes and the public credentials from the bulletin board. They also verify that the content of ballot boxes corresponds to the commitments posted in (Comm2). Then, they (2) check proofs in retrieved ballots and eliminate any ballot with an invalid proof. Note that these steps are performed by each teller independently, and the resulting set of votes, let us denote it by $B$, is determined by the publicly known information.

Next, (3) duplicate elimination (according to some fixed policy) is performed, by running PET$(c, c')$, for all encrypted ballot credentials $c, c'$ from distinct ballots in $B$:

\[(\text{PET1}) \quad T_j \to B : \text{sig}_{T_j}(\alpha_j(c, c'), P_{\alpha_j}(c, c'))\]

where $\alpha_j(c, c') = \text{petshare}(c, c', x_j)$ and $P_{\alpha_j}(c, c') = \text{ProofPETShare}(x_j; \alpha_j(c, c'), y_j, c, c')$. Now, each teller waits until all the tellers post their share and verifies the proofs. The result of PET for $c, c'$ is $\text{distpet}(\alpha_0, \ldots, \alpha_k)$ (it evaluates to $T$ if the PET passes) and is publicly computable. For each two ballots for which PET holds true, only one is kept (according to the mentioned policy).

Next, (4) mixing ballots is performed on the list of remaining ballots $\bar{u}_0$. Each tabulation teller in turn applies its own random permutation $\bar{r}^+_j$ with reencryption. We assume that $\bar{u}_j$ is the input for $j$-th teller:

\[(\text{Mix1}) \quad T_j \to B : \text{sig}_{T_j}(\bar{u}_{j+1}, P_{u_j})\]

where $\bar{r}^+_j$ is a vector of random values, $\bar{u}_{j+1}[\bar{r}^+_j(i)] = \text{reenc}(\bar{u}_j[i], K_T, \bar{r}_j[i])$, and $P_{\bar{u}_j} = \text{ProofMix}(\bar{r}^+_j; \bar{u}_j, \bar{u}_{j+1}, K_T)$. The result of mixing is $\bar{w}_{k+1}$. Similarly, mixing credentials is performed on the list $\bar{w}_0 = (S_0, \ldots, S_m)$ of public credentials. Each tabulation teller in turn applies its own random permutation $\bar{r}^+_j$ with reencryption. We assume that $\bar{w}_j$ is the input for $j$-th teller:

\[(\text{Mix2}) \quad T_j \to B : \text{sig}_{T_j}(\bar{w}_{j+1}, P_{\bar{w}_j})\]

where $\bar{r}^+_j$ is a sequence of random values, $\bar{w}_{j+1}[\bar{r}^+_j(i)] = \text{reenc}(\bar{w}_j[i], K_T, \bar{r}_j[i])$, and $P_{\bar{w}_j} = \text{ProofMix}(\bar{r}^+_j; \bar{w}_j, \bar{w}_{j+1}, K_T)$. The result of mixing is $\bar{w}_{k+1}$.

The next step is invalid ballots elimination where ballots without valid credentials are eliminated. For each ballot with the encrypted credential $c$, PET$(c, c')$ is performed against every public credential $c'$:

\[(\text{PET2}) \quad T_j \to B : \text{sig}_{T_j}(\beta_j(c, c'), P_{\beta_j}(c, c'))\]

where $\beta_j(c, c') = \text{petshare}(c, c', x_j)$ and $P_{\beta_j}(c, c') = \text{ProofPETShare}(x_j; \beta_j(c, c'), y_j, c, c')$. Now, each teller waits until all the tellers post their share and verifies the proofs. The result of PET for $c, c'$ is $\text{distpet}(\beta_0, \ldots, \beta_k)$ and is publicly computable. If this test fails for all $c'$, the ballot is removed.

Finally, decrypt step is performed, for each of the remaining ballots. Decryption is applied to the encrypted vote $c$ of each of the remaining ballots (but not to the encrypted credentials):

\[(\text{Decr}) \quad T_j \to B : \text{sig}_{T_j}(\gamma_j(c), P_{\gamma_j}(c))\]

where $\gamma_j(c) = \text{dshare}(c, x_j)$ and $P_{\gamma_j}(c) = \text{ProofDSHare}(x_j; \gamma_j(c), y_j, c)$. Each teller waits until the remaining tellers submit their shares and verifies the proofs. Now, the decrypted vote is $v = \text{distdec}(\gamma_0, \ldots, \gamma_k)$. At this point the result of the voting process is publicly computable.

B.4 Modelling of the Protocol

In addition to the participants enumerated in Section [1.3], we assume that the coercer $c$ and a key issuer $K$ also participate in the protocol. The role of the key issuer is to
generate private and public keys to each participant and, to provide these keys on request.

We assume that \( v_0 \) is the coerced voter. The voters \( v_1, \ldots, v_n \), for some \( n < m \), are honest, while \( v_{n+1}, \ldots, v_m \) are dishonest and will not be modelled directly, but, instead, will be subsumed by the coercer. As we mentioned before, we assume that \( S, B, R_0, X_0, \) and \( T_0 \) are honest. The remaining authorities, that is \( X_i, R_i, T_i \), for \( i \in \{1, \ldots, k\} \), are assumed to be dishonest and will be subsumed by the coercer. Additionally, to model anonymous channels, we introduce agents \( C_0, \ldots, C_\ell \). The role of \( C_k \) is to simply forward messages to \( X_j \) (and so, \( X_j \) cannot associate a sender to a received message).

The set \( Ch \) of the channels used in the protocol consists of:

- \( ch_{[x->z]} \) for each protocol participants \( x \) (including \( S \)),
- \( ch_{[x->K]} \) and \( ch_{[x->\ell]} \) for each protocol participant \( x \neq K \),
- \( ch_{[B->z]} \) and \( ch_{[x->B]} \) for each protocol participant \( x \neq B, x \neq C_j, \) and \( x \neq X_j \),
- \( ch_{[C_j->X]} \) and \( ch_{[x->C_j]} \) for each protocol participant \( x \),
- \( ch_{[X_j->B]} \)
- \( ch_{[v_i->R_j]} \) and \( ch_{[R_j->v_i]} \),
- \( ch_{[v_i->e]} \) and \( ch_{[e->v_i]} \),
- \( ch_{[init->S]} \) is a channels used to initiate \( S \).

We will use the following notation: for a set \( B \), we will write \( ch_{[B->c]} \) for the set of all \( c \in Ch \) of the form \( ch_{[x->y]} \) with \( x \in B \). Similarly, we will write \( ch_{[x->B]} \) for the set of all \( c \in Ch \) of the form \( ch_{[x->y]} \) with \( y \in B \).

We assume that \( \Sigma \) contains, in addition, constants \( init, request, \) and \( done \) and also constants representing the participant names.

The protocol we take is \( S = (A, in, out, \sigma_0, P) \), where \( A = \{v, c, e\} \) and \( in, out, \sigma_0 \), and \( P \) are defined as follows: \( in \) and \( out \) are the functions:

\[
\begin{align*}
in(v) &= ch_{[e->v_0]} \quad out(v) = ch_{[v_0->e]} \\
in(c) &= ch_{[e->D]} \quad out(c) = ch_{[D->e]} \\
in(e) &= ch_{[e->H]} \quad out(e) = ch_{[H->e]}
\end{align*}
\]

where \( H = \{S, B, R_0, X_0, T_0, v_1, \ldots, v_n\} \) is the set of honest participants and \( D = \{c, v_{n+1}, \ldots, v_m, R_1, \ldots, R_k, X_1, \ldots, X_k, T_1, \ldots, T_k\} \) is the set of dishonest participants. Additionally, both \( v_0 \) and \( c \) have an infinite number of private channels (i.e. channels that occur only in \( in(v) \cap out(v) \) or \( in(c) \cap out(c) \), respectively). In particular, let \( c_v \) be some private channel of \( v \).

The initial sequence \( \sigma_0 = (ch_{[init->S]} : \text{init}) \). For each participant \( a \), we define the set \( P(a) \) of programs of this participant as follows.

**Key-Issuer.** \( P(K) \) consists of programs which assign a distinct nonce \( k_a \) (the private key of \( a \)) to each participant \( a \) and, in response to \( ch_{[x->K]} : \text{request} \), send on channel \( ch_{[K->x]} \) the tuple containing the public keys \( pub(k_a) \) of all the participants and, additionally, the private key \( k_x \) of \( x \).

**Bulletin board.** The set \( P(B) \) contains one program which immediately forwards all received messages to all participants (except for \( X_j \)).

**Ballot box.** The set \( P(X_0) \) contains one program which, after obtaining the message \( done \) from \( S \), posts the commitment of its content (i.e. the list of the messages received so far) on the bulletin board and publishes this content, i.e. sends the content to all participants (except for \( B, C_j, \) and \( X_j \)). In addition, this program immediately forwards to the coercer each message he receives (this model the fact that the coercer is able to intercept messages sent to the ballot box, but cannot block them).

**Supervisor.** The set \( P(S) \) consists of a program which, in response to the message \( init \) initiates \( T_j \) and \( R_j \) by sending them \( init \) message and waits until the setup phase is completed (all the necessary commitments and key shares are posted). Then it sends \( init \) to all voters and message \( done \) to itself (this models “waiting” for the voting phase to end). When this message is delivered, it sends \( done \) to all the ballot boxes and waits for their commitments. After it obtains these commitments, he signs them and posts on the bulletin board.

**Registrars.** The set \( P(R_0) \) consists of programs which, in response to the message \( init \) sent by \( S \), request for keys and, after obtaining them, pick a distinct nonces \( s_{ij} \) and \( r_{ij} \), and post \( \text{sig}_{R_j}(v_i, S_j) \) to the bulletin board, as defined in (Cred), for each \( i \in \{0, \ldots, m\} \). Then, on request sent by \( v_i \), it replies with (Reg2).

**Honest voters.** The set \( P(v_i) \), for \( i \in \{1, \ldots, n\} \), consists of (a) programs \( \xi_z \), for each valid vote \( z \), which after receiving message \( \text{init} \) from \( S \), take the keys from \( K \), request for credentials (Reg1) and, after obtaining them all (Reg2), post their ballots \( b_i \) (Vote), with \( v_i = z \) and fresh nonces \( r_i \) and \( r'_i \), to all \( C_j \); (b) programs which register like \( \xi_z \), but do not post any ballot (abstain from voting); (c) programs \( \xi_1 \), which is defined like \( \xi_z \), but instead of posting a valid ballot, posts a ballot with an invalid credential (some fresh nonce);

**Anonymous channel.** The set \( P(C_j) \) consists of one program which forwards to \( X_j \) every message it receives.

**Tallier.** The set \( P(T_0) \) consists of the following programs: a program, after receiving \( init \) from \( S \), participates in the procedure of public key generation: it picks a nonce \( x_j \) (its private key) and posts (KGen1) and then, when it sees that all the tellers have posted their messages (note that he can see it, because the bulletin board forwards all the messages to every participant), it post (KGen2), waits for the corresponding messages of the remaining tellers and checks the proofs (it these tests fail, it halts).
Then, after it obtains (forwarded by the bulletin board) the commitment of $S$ on the contents of all ballot boxes (sent in step (Comm2)), it participates in the tabulation procedure: it post messages as defined in steps (PET1)–(Decr). After each step, it waits for the remaining tellers to post their messages and verifies whether these messages have an appropriate form and the zero-knowledge proofs are correct. If these tests fail, it halts.

Coerced voter. The set $P(v_0)$ consists of program of the form $(v_{\text{reg}} \parallel v)$, where $v \in \Pi(I_0, O_0)$, for $O_0 = out(v_0)$ and $I_0 = in(v_0) \setminus \{ch_{S_{\text{reg}}}, ch_{K_{\text{reg}}}, ch_{R_{\text{reg}}}, \}$. Hence, we first show that coercion-resistance of the protocols is equivalent to coercion resistance of some normal protocol $\hat{S}$. Let $\hat{S}$ be defined like protocol $S$ with only one difference: the subprocess $v_{\text{reg}}$ of any program of the coerced voter will be now run by a distinct agent $v'_0$, which will be a part of the environment.

Let $\hat{R}$ be the coercion system induced by $\hat{S}$, and $\hat{R}$ be the coercion system induced by $S$. Note that $\hat{R}$ is equivalent to the coercion system induced by $\hat{S}$. Let $\hat{R}$ be the coe of a run of the system induced by $S$, which means that $\hat{R} = ((v_{\text{reg}} \parallel v)\cdot c\cdot e\cdot \pi)$, where $\pi$ is induced by $(v_{\text{reg}} \parallel v \cdot c\cdot e\cdot \pi)$. By $\hat{R}$ we denote $(v, c, v_{\text{reg}} \parallel c, \pi)$. Note that $\hat{R}$ is a run of $\hat{S}$. We extend the operator $\hat{\cdot}$ to properties of $\hat{S}$ in a natural way: $\hat{\pi} = \{\hat{\rho} : \hat{\rho} \in \beta\}$. It is easy to show that the following lemma holds.

**Lemma 5.** $\hat{R}$ is coercion-resistant in $\alpha$ w.r.t. $\gamma_2$ iff $\hat{R}$ is coercion-resistant in $\alpha$ w.r.t. $\gamma_2$.

One can show that protocol $\hat{S}$ is normal and both $\alpha$ and $\gamma$ are now closed under $\hat{\cdot}$. Hence, we can use Theorem 1. So, it is enough to provide a counter-strategy $v'$ for a strategy $v$ which simply forwards to the coercer all the messages obtained from the remaining participants and forwards to these participants all the messages obtained from the coercer.

Let $v'$ be the process which after obtaining the registration data on $c_v$ post the ballot $b_0 = b[z, s_0]$ and, in the same time, behaves like the forwarder $v$ with the following exception. When he obtains the registration data on $c_v$, he changes it before forwarding: he replaces $s_0$ by a fresh nonce $\tilde{s}_0$ (a faked credential) and $D_0$ by $\tilde{D}_0 = DVRP(\tilde{\delta}_0, k_{v_0}; S_0, D_0, \pi_{\text{reg}}(v_0))$ (a faked proof) with random $\delta_0$ and $\tilde{S}_0 = \{\tilde{s}_0\}_{K_{v_0}}$ (recall that $pub(v_0)$ stands for $pub(k_{v_0})$). We will show that $v'$ is a counter-strategy for $v$.

**First, we show that condition (iii) of Definition 2 holds** for $v'$. Let $\rho$ be a run of the system induced by $v'$, i.e. $\rho$ is induced by $(v' \parallel c \parallel e)$, for some $c \in C, e \in E$. If $\rho$ is not fair, then there is nothing to prove. So, suppose that $\rho$ is fair. First, note that, by the fairness assumption, all $R_0, \ldots, R_k$ post all messages and zero-knowledge proofs as required. Since $v_{\text{reg}}$ does not send out his private key, the DVRP-s he gets cannot be faked, and thus the private credential he obtains is valid. Second, $v'$ obtains his registration data before the voting phase ends.

Since $v'$ posts then a valid ballot $b_0$ right away (still before the voting phase ends), by the fairness assumption, this ballot is posted successfully and so $b_0$ is in the initial pool of votes to be tabulated.

Now, it is easy to show that $b_0$ will be successfully processed by tabulation tellers, using the fact that $v'$ never reveals his private credential (so it is not used in any other ballot) and the assumption that the run is fair (and so all the tabulation tellers have to correctly perform all the expected step, because otherwise they would not be able to construct valid zero-knowledge proofs).

**Now, we will show that condition (i) of Definition 2 holds** for $v$ and $v'$. So, let $\rho \in \alpha$ be a run induced by $(v \parallel c \parallel e)$, for some $c \in C$ and $e \in E$.

Since $\rho$ is in $\alpha$, there is some honest voter, say $v_1$, who successfully posts a ballot $b_1[z, s_1]$ (that is a ballot with vote $z$), some honest voter, say $v_2$, who obtains his credential and successfully posts a ballot $b_2[z_2, s_2]$ with an invalid credential (a fresh nonce $s_2^*$), and some honest voter, say $v_3$, who posts his ballot $b_3$ after $v_0$ finishes registration.

We take $e' \in E$ which is like $e$ with the following exceptions: (a) $v_3$ abstains from voting, (b) if $v_3$ in $\rho$ posted his ballot successfully, then $v_1$ votes like $v_3$ voted in $\rho$; and (c) moreover, if at least one proper ballot with $s_0$ is in $\rho$ successfully posted and $z_3$ is the vote in the ballot with $s_0$ that is kept after duplicate elimination (note that $z_3$ must be a valid vote), then $v_0$ posts a valid ballot with $z_3$ instead of the invalid one. Also, instead of using permutations $\pi_0$ and $\pi_0^*$ in steps (Mix1) and (Mix2), To uses slightly different permutations (see Sect. 5.3).

The run $\rho'$ of $(v' \parallel c \parallel e')$ is constructed from $\rho$ in the following way. The messages in $\rho'$ are delivered in the same order like the corresponding messages in $\rho$ with the following exceptions: first, the message sent by $v_{\text{reg}}$ on $c_v$ is delivered immediately and, second, the ballot sent by $v_0$ in $\rho'$ is delivered at the same step, when the ballot sent by $v_3$ is delivered in $\rho$ (because $v_0$ sends his ballot, just when he gets the registration data on $c_v$, and $v_3$ posts his ballot after...
it, the ballot of \( v_0 \) is ready to be delivered at the mentioned step).

Now, one can show that \( \rho \sim \rho' \). The rough idea is as follows: \( v_2 \) is used to hide the fact that valid ballots possibly posted by the coercer in \( \rho \) become invalid in \( \rho' \); \( v_3 \) is used to hide the fact that \( v_0 \) posts his ballot in \( \rho' \), but abstains from voting in \( \rho \), and finally \( v_1 \) is used to balance the outcome of the voting. Due to the fact that the coercer cannot tell any difference between an original DVRP and a faked one and the fact that the messages posted on the bulletin board are mixed and reencrypted before decryption, the frames are indistinguishable to the coercer.

Details of the proof depend on (a) whether or not the coercer successfully posts at least one valid ballot with \( s_0 \) and (b) whether or not \( v_3 \) posts his ballot successfully. In next subsection, we present a detailed proof for one of these cases: when the coercer successfully posts one proper ballot with \( s_0 \), and \( v_3 \) also posts his ballot successfully.

Now, we will show that condition (ii) of Definition 2 holds for \( v \) and \( v' \). So, let \( \rho \in \alpha \) be a run induced by \((v', \rho, e, c, \epsilon)\), for some \( c \in C \) and \( e \in E \). We define the vote \( z_c \), as follows: if the coercer, in \( \rho \), successfully posts at least one ballot with \( s_0 \) (where \( s_0 \) is computed like \( s_0 \), but using \( s_{00} \) instead of \( s_{00} \)), then let \( z_c \) be the vote in the ballot containing \( s_0 \) which is left after duplicate elimination phase; otherwise let \( z_c \) be any vote. Note that, since a valid ballot has to contain a proof that a vote in it is valid, \( z_c \) must be a valid vote.

Since \( \rho \) is in \( \alpha \), there is some honest voter, say \( v_1 \), who obtains his credential, before \( v_{reg} \) finishes registration, but abstains from voting, and some honest voter, say \( v_2 \), who successfully votes for \( z_c \).

We take \( e' \in E \) which is as \( e \) with the following exceptions: \( v_1 \) votes for \( z \) and, moreover, if at least one ballot with \( s_0 \) is, in \( \rho \), successfully posted, then \( v_2 \) posts \( b[z_c, s_2] \), for some unused nonce \( s_2 \), instead of \( b[z, s_2] \). We also need to slightly change the permutations used by \( T_0 \) in (Mix1) and (Mix2).

The run \( \rho' \) of \((v, \rho, e, e')\) is constructed from \( \rho \) in the following way. The messages in \( \rho' \) are delivered in the same order like the corresponding messages in \( \rho \) with the following exceptions: The ballot sent by \( v_1 \) is delivered in the same step, when the ballot sent by \( v_0 \) was delivered in \( \rho \) (it is possible because this ballot is posted before \( v_{reg} \) finishes registration).

Now, one can show that \( \rho \sim \rho' \). The rough idea is as follows: \( v_2 \) is used to hide the fact that the ballots involving the credential given by the coerced voter and possibly posted by the coercer are invalid in \( \rho \) but valid in \( \rho' \); \( v_1 \) is used to hide the fact that \( v_0 \) posts his ballot in \( \rho' \), but not in \( \rho' \). Moreover, due to the fact that the coercer cannot distinguish an original DVRP and the faked one, and the fact that the messages posted on the bulletin board are mixed and reencrypted, the frames are indistinguishable to the coercer.

B.6 Detailed Case Analysis

In this subsection we give a detailed proof that the runs \( \rho \) and \( \rho' \), as constructed in the proof for condition (i) above, are indistinguishable to the coercer in the case the coercer, in \( \rho \), successfully posts exactly one proper ballot \( b[c[z, s_0]] \), and \( v_3 \) also successfully posts his ballot \( b_3[z_3, s_3] \).

Formally, we have to show the following. The run \( \rho \) is of the form \((\hat{v}, \hat{c}, \hat{c}, \pi)\), where \( \hat{v} \equiv v, \hat{c} \equiv c, \hat{c} \equiv \epsilon, \) and \( \pi \) is a run induced by \((\hat{v} || \hat{c} || \hat{e})\). Similarly, \( \rho' \) is of the form \((\hat{v}', \hat{c}', \hat{c}', \pi')\), where \( \hat{v}' \equiv v', \hat{c}' \equiv c', \hat{e}' \equiv \epsilon', \) and \( \pi' \) is a run induced by \((\hat{v}' || \hat{c}' || \hat{e}')\). By the definition of \( \sim \), we need to prove that \( \pi \equiv \pi' \), which means that \( \pi \equiv \pi' \). Since, \( I_c \subseteq in(c) \) it is enough to show that \( \pi \equiv N \) where \( N = N_c \) and \( I = in(c) \). This, by the definition of \( \equiv_N \), is equivalent to the following statement, where \( \varphi = \pi_I \) and \( \varphi' = \pi'_I \): (i) \( chan(\varphi) = chan(\varphi') \) and (ii) for each \( \tau_1, \tau_2 \in T_N \), we have that \( \tau_1[\varphi] \equiv \tau_2[\varphi] \iff \tau_1[\varphi'] \equiv \tau_2[\varphi'] \).

The proof goes as follows. First we describe \( \varphi \) and \( \varphi' \) (which contain exactly those messages that are seen by the coercer) and show that condition (i) holds. Then we will show that condition (ii) holds as well.

Let us first informally point out the differences in view of the coercer on \( \rho \) and \( \rho' \). These views are very similar, in particular, the lists of votes published by the tallying tellers in both cases are exactly the same. The main differences are summarized in the table below, where \( s_0 \) denotes \( s_{00} \times s_{01} \times \cdots \times s_{0k} \) (i.e. the faked private credential of \( v_0 \)).

| the (faked) credential sent to \( c \) | \( s_0 \) | \( s_1 \) |
|--------------------------------------|-------|-------|
| the ballot posted by \( c \)          | \( b_1[z, s_0] \) | \( b_1[z, s_0] \) |
| the ballot posted by \( v_3 \) / \( v_0 \) | \( b_1[z_3, s_3] \) | \( b_0[z, s_0] \) |
| the ballot posted by \( v_1 \)        | \( b_1[z, s_1] \) | \( b_1[z, s_1] \) |
| the ballot posted by \( v_2 \)        | \( b_2[z_2, s_2] \) | \( b_2[z_2, s_2] \) |

The messages placed in the same raw of the table are seen by the coercer, in \( \rho \) and \( \rho' \), respectively, at the same channel and the same step.

We need also to specify the mentioned permutations \( \pi_0 \) and \( \pi_0' \), used by \( T_0 \) in \( \rho \), instead of \( \pi_0 \) and \( \pi_0' \), to mix votes and credentials. So, \( \pi_0' \) is like \( \pi_0 \) but places the reencryptions of \( b_2[z_2, s_2] \), \( b_1[z_3, s_1] \), \( b_0[z, s_0] \), \( b_1[z, s_0] \) in the place where \( \pi_0 \) places the reencryptions of \( b_1[z, s_0] \), \( b_1[z_3, s_3] \), \( b_1[z, s_1] \), and \( b_2[z_2, s_2] \), respectively. The permutation \( \pi_0' \) is like \( \pi_0' \) but places the permutations of \( S_2, S_1, S_0, S_3 \) in the place where \( \pi_0 \) places the reencryptions of \( S_0, S_3, S_1, S_2 \), respectively, where \( S_i \) are reencryptions public credentials produced in step (Mix2).

Detailed description of \( \varphi \) and \( \varphi' \), which is the sequence of messages received by the coercer in \( \rho \) and \( \rho' \), respectively, is given in Fig. 4. Only messages which are not produced by the coercer (i.e. neither constructed nor randomly generated by him, like for instance messages posted by him on the bulletin board and forwarded to him back) are presented, as the messages produced by him are not essential to the proof (instead of using them in \( \tau_i \), one can use the corresponding terms that were used to construct them). Also, we
Figure 4: Messages in $\varphi$ and $\varphi'$. $B_x$ denotes the sequence of encrypted ballot credentials in $B$. $C_x$ ($\tilde{C}_x$) denotes the sequence of encrypted ballot credentials in $C$ ($\tilde{C}$), and $C_x$ ($\tilde{C}_x$) is the sequence of encrypted votes in $C$ ($\tilde{C}$). By $b[x,y]$ and $\tilde{S}_i$ we denote reencryptions of $b[x,y]$ and $S_i$ made by $T_0$. $P_{\alpha_0}$ ($P_{\beta_0}$) and $P_{\gamma_0}$ ($\tilde{P}_{\gamma_0}$) are the zero-knowledge proofs posted by $T_0$ in the mixing ballots and mixing credentials phase, respectively. Messages that occur in $\varphi$ and $\varphi'$ at the same positions, are placed at corresponding positions in the table (for example $\tilde{S}_3$ and $\tilde{S}_1$).

Messages (1) and (2) are posted by $T_0$ in steps (KGen1) and (KGen2). Messages (3) are posted by $R_0$ in step (Cred). (4) comprises messages sent by $R_0$ in step (Reg2) to dishonest voters who have requested for credentials. Note that up to this point messages in both $\varphi$ and $\varphi'$ are exactly the same. (5) contains the messages sent by $R_0$ to $v_0$ in step (Reg2) and forwarded to the coercer (in $\varphi$) or a faked version of these messages (in $\varphi'$). (6) comprises votes posted by voters on ballot boxes. Messages (7) are the commitments on the content of ballot boxes signed by these boxes and by the supervisor and posted on the bulletin board in steps (Comm1) and (Comm2). (8) are PET shares and proofs posted by $T_0$ in the duplicate elimination phase of tabulation. (9) and (10) are the results of mixing with reencryption of ballots posted by $T_0$ in the mixing ballot phase and the mixing credential phase, respectively, along with the appropriate proofs. By $b[x,y]$ and $\tilde{S}_i$ we denote reencryptions of $b[x,y]$ and $S_i$ made by $T_0$. (11) contains the PET shares and proofs posted by $T_0$ in the invalid ballots elimination phase. Finally, (12) are the distributed decryption shares and corresponding zero-knowledge proofs posted by $T_0$ in (Decr).

One can check that condition (i) holds (i.e. that the chan($\varphi$) = chan($\varphi'$)). Hence, to complete the proof, it is enough to prove that the condition (ii) also holds, which is stated by the lemma bellow.

Lemma 6. For each $\tau_1, \tau_2 \in T_N$, we have that $\tau_1[\varphi] \equiv \tau_2[\varphi']$ iff $\tau_1[\varphi'] \equiv \tau_2[\varphi']$.

The remainder of this section is devoted to sketch the proof of this lemma.

A destructor is any of the following symbols: first, second, unsig, checksig, dec, distdec, distpet, public, and check. The

| Messages in frame $\varphi$ | Messages in frame $\varphi'$ |
|-----------------------------|-----------------------------|
| (1) (KGen1)                | $h(y_0)$                   |
| (2) (KGen2)                | $y_0, \text{KnowPriv}(x_0; y_0)$ |
| (3) (Cred)                 | $i, S_{i0}$                |
| (4) (Reg2)                 | $s_{i0}, \tilde{r}_{i0}, D_{i0}$ |
| (5) (Reg2)                 | $s_{i0}, \tilde{r}_{i0}, D_{i0}$ |
| (6) (Vote)                 | $B = \left\{ b_i[z_3, s_3], b_i[z, s_1], b_i[z_2, s_2^*] \right\}$ |
| (7) (Comm)                 | $\text{sig}_{\alpha}(j, C_j)$, $\text{sig}_{\beta}(j, C_j)$ |
| (8) (PET1)                 | $\alpha_0(c, c'), P_{\alpha_0}(c, c')$ (for $c \neq c'$ in $B_x$) |
| (9) (Mix1)                 | $C = \left\{ \tilde{b}_i[z_3, s_3], \tilde{b}_i[z, s_1], \tilde{b}_i[z_2, s_2^*] \right\}$ |
| (10) (Mix2)                | $S = \left\{ \tilde{S}_0, \tilde{S}_3, \tilde{S}_1, \tilde{S}_2 \right\}$ |
| (11) (PET2)                | $\beta_0(c, c'), P_{\beta_0}(c, c')$ (for $c \in C_x$ and $c' \in S$) |
| (12) (Decr)                | $\gamma_0(c), P_{\gamma_0}(c)$ (for $c \in C_x$) |
remaining symbols of $\Sigma$ are constructors. We will consider equations associated with destructors as rewriting rules, read from left to right (note that there is exactly one rule associated with each destructor). Moreover, the equations associated with reenc($\cdot$, $\cdot$) will be also considered as rewriting rules. A term is said to be reduced, if all the mentioned above equations, regarded as rewriting rules, are applied.

We will call $\varphi$ and $\varphi'$ frames. We will sometimes write $\varphi(x_i)$ instead of $x_i[\varphi]$ (for the $i$-the element of $\varphi$). A frame is closed under applying destructors, if whenever a term of the form $g(x_1, t_1, \ldots, t_n)[\varphi]$, with some destructor $g$, reduces at the top (i.e. a reduction can be applied at the top level of the term), the result of total reduction of this term is also an element of the frame. We stress that such a result is a reduced term.

Now, we define $\varphi_0$ as a closure under applying destructors of $\varphi$ and $\varphi_1$ as the corresponding closure of $\varphi'$, where “corresponding” means that the results obtained by applying the same terms (of the form $g(x_1, t_1, \ldots, t_n)$) to both frames are, in both frames, at the same position. We will show that, for each $\tau_1, \tau_2 \in T_N$, we have that $\tau_1\{\varphi_0\} \equiv \tau_2\{\varphi_0\}$ iff $\tau_1\{\varphi_1\} \equiv \tau_2\{\varphi_1\}$, which immediately implies Lemma 6.

A test is an expression of the form $\tau_1 = \tau_2$. We will say that a test $\tau_1 = \tau_2$ holds in a frame $\varphi$, if $\tau_1[\varphi] \equiv \tau_2[\varphi]$. A test is basic, if it is either of the form (a) $x_i = \tau$, where $\tau$ is a term with no destructor in the head, (b) $x_i = x_j$, or (c) $x_i = g(x_1, \ldots, t_n)$, where $g$ is destructor and $t_1, \ldots, t_n$ are some terms.

We define the size of a term in the usual way, but in case of terms representing zero-knowledge proofs ($ZK^\varphi_{\vec{t}}(i)$), the size of the formula $\varphi$ is taken into account too.

The following lemma says that the frames $\varphi_0$ and $\varphi_1$ are indistinguishable w.r.t. basic tests.

**Lemma 7.** For a basic test $\tau_1 = \tau_2$, the following is true: $\tau_1\{\varphi_0\} \equiv \tau_2\{\varphi_0\}$ iff $\tau_1\{\varphi_1\} \equiv \tau_2\{\varphi_1\}$.

**Sketch of Proof.** (a) First, let us consider the case where the test is of the form $x_i = \tau$, where $x_i$ is one of $x_1, x_2, \ldots$ and $\tau$ has no destructor in its head. We consider all such cases by case. For instance, let $\varphi_0(x) = \{x_i\}_{i \in K}$, and $\varphi_1(x) = \{x\}^{\varphi}_{K_1}$ (these messages come from $b_3[z_3, s_3]$ and $b_0[z, s_0]$ posted by the voters on the bulletin boxes). Suppose that $x = \tau$ holds in $\varphi_0$. $\tau$ cannot have a constructor in its head, because, there is no $\tau'$ such that $\tau'[\varphi_0] \equiv \tau'_3$ ($\tau'_3$ is never revealed). So, $\tau$ has to be a variable. However, no other variable gives a term equivalent to $\varphi_1(x)$. Hence, $\tau$ must be $x$ and the test under consideration also holds in $\varphi_1$.

(b) If a test is of the form $x_i = x_j$, one can easily see, considering again case by case, that it holds in $\varphi_1$ iff it holds in $\varphi_2$.

(c) If a test is of the form $x_j = g(x_i, t_1, \ldots, t_n)$, where $g$ is destructor and $t_1, \ldots, t_n$ are some terms, one should, again, consider all possible $x_i$ case by case. For instance, if $g = \text{distdec}$ (i.e. a distributed decryption is applied), then $x_i$ must be decrypted, and the test reduces in $\varphi_1$, if $\tau$ is reduced in $\varphi_2$.

Finally, we prove the following fact which completes the proof of Lemma 6.

**Lemma 8.** For each $\tau_1, \tau_2 \in T_N$, we have that $\tau_1\{\varphi_1\} \equiv \tau_2\{\varphi_2\}$ iff $\tau_1\{\varphi_2\} \equiv \tau_2\{\varphi_1\}$.

**Sketch of Proof.** For the sake of contradiction, let us suppose that $\tau_1 = \tau_2$ is a test which distinguishes these frames, i.e. it holds in $\varphi_1$ and does not hold in $\varphi_2$. Then the test is minimal (w.r.t. the size of terms) in both frames. So, in both frames the reduction is applied. Moreover, in case, when the destructor in the head of $\tau_j$ is $\text{distpet}$, we use some particular properties of the frames under consideration and the fact that the arguments of these destructors can be freely rearranged.)
By Lemma 5 we only need to consider three cases, depending of whether the top symbol of \( \tau_0 \) is (a) \( \text{reenc} \), (b) +, or (c) ×. In each case, one obtains a contradiction. For instance, let us consider the case (a). So, \( \tau_0 \varphi_i \) is of the form \( \text{reenc}(\text{reenc}(m, k, r), k, r') \) or \( \text{reenc}(\{m\}_k^r, k, r') \). However, since \( \tau_0 \) is assumed to be reduced, \( \tau_0 \) has to be of the form \( \text{reenc}(x_i, k, r') \), for some variable \( x_i \). Now, since the frames are reduced, \( \varphi_i(x_i) \) cannot be of the form \( \text{reenc}(m, k, r) \), so it must be of the form \( \{m\}_k^r \). Hence, \( \tau_0 \varphi_i \) is of the form \( \text{reenc}(\{m\}_k^r, k, r') \) and it reduces to \( \{m\}_k^{r' + r} \). Note that there is no term \( \sigma \) such that \( \sigma[\varphi_i] \equiv r \), as \( r \) is never revealed. It implies that \( \tau_1 \) has to be of the form \( \text{reenc}(x_i, k, r') \) with \( \varphi_i(x_i) = \{m\}_k^r \) (there is no other way of obtaining \( \{m\}_k^{r' + r} \)). So, \( x_i = x_i \) holds in \( \varphi_i \) and, by Lemma 7 also holds in \( \varphi_{1-i} \). It, however, means that the test \( \tau_0 = \tau_1 \) holds in \( \varphi_{1-i} \), which contradicts the assumption that it distinguishes the frames.

\[\square\]

C. Lee et al. Protocol

In this section we describe the protocol [28] in more details and sketch the proof of coercion-resistance of this protocol. We can model cryptographic primitives used in this protocol, like in case of Civitas (see Fig. 2), with some small modifications toward the threshold decryption scheme.

C.1 Description of the Protocol

The set of agents we take is \( A = \{v_0, \ldots, v_n, S, r_0, \ldots, r_m, c, B, T_1, \ldots, T_k, M_1, \ldots, M_k\} \), where \( v_0, \ldots, v_n \) are voters, \( S \) is the supervisor, \( r_0, \ldots, r_m \) are tamper-resistant randomisers, \( B \) is the bulletin board, \( T_1, \ldots, T_k \) are the tallying authorities, \( M_1, \ldots, M_k \) are mixers, and \( c \) is the coercer. We assume that the coerced voter is \( v_0 \).

All the messages posted on \( B \) are publicly available. The communication channel between a voter and his tamper-resistant randomiser is assumed to be untappable (i.e. it cannot be observed by the coercer). The remaining channels are public (can be observed by the coercer).

In the setup phase the tallying tellers \( T_1, \ldots, T_k \) generate and publish his common public key \( K_T \) for the threshold decryption. Then, from the point of view of a voter \( v_i \), the protocol execution consists of three steps:

\[
\text{(P1)} \quad v_i \rightarrow r_i : m_i
\]

\[
\text{(P2)} \quad r_i \rightarrow v_i : \text{sig}_{k_{0i}}(m_i^r), \text{DVRP}(\beta; m_i, m_i^r, K_T, \text{pub}(v_0))
\]

\[
\text{(P3)} \quad v_i \rightarrow B : \text{sig}_{\text{pub}(m_0)}(\text{sig}_{k_{0i}}(m_i^r))
\]

where \( m_i = \{v_i\}_n, m_i^r = \text{reenc}(m_i, K_T, \beta) \), and \( v_i \) denotes the vote chosen by \( v_i \); \( \alpha_i \) is a random value generated by this voter, and \( \beta \) is a random value generated by \( r_i \).

In the second phase of the protocol, the following steps are performed: (1) \( S \) verifies the double signatures of voters and their randomisers on the posted ballots, and publishes valid ballots on the bulletin board. (2) The, mixers \( M_1, \ldots, M_k \), in turn, shuffle and reencrypt these ballots, and post the result on the bulletin board. (3) Tallyers jointly decrypt shuffled ballots using the \((t, k)\)-threshold ElGamal decryption protocol, and finally, (4) \( S \) publishes the tally result.

We assume that the correctness of all these steps is assured by posting appropriate non-interactive zero-knowledge proofs. This guarantees that only decryptions allowed by the protocol are performed, provided only a small fraction of the entities is dishonest.

C.2 Proof of Theorem 5

Recall that we want to prove coercion-resistance for the extended version of the protocol. The extension described in Section 3 can be formalised as follows. The voter, instead of step (P1), performs the following step.

\[
\text{(P1a)} \quad T \rightarrow B : m_i, P_i
\]

where \( P_i \) is is a zero-knowledge proof which shows that the vote is well-formed with respect to the ballot design, i.e. \( v_i \) is one of the valid votes (one can do it like in Civitas). Then, \( r_i \), before replying with (P2), checks this proof.

First, one can show that the protocol is normal and both \( \alpha \) and \( \gamma \) are closed under \( \oplus \). Hence, we can use Theorem 11. So, it is enough to provide a counter-strategy \( v' \) for a strategy \( v \) which simply forwards to the coercer all the messages obtained from the remaining participants and forwards to these participants all the messages obtained from the coercer.

Let \( z \) be a choice of \( v_0 \). Let \( v' \) be the process which behaves like the forwarder \( v \) with the following exception. When he is instructed to send a message \((m_c, P_c)\), then, instead, he sends \((m_0, P_0)\) as in specified in (P1a), and, instead of forwarding the answer of \( r_0 \) to the coercer, he sends him \( m_0' \) signed by \( r_0 \) along with a faked DVRP for \( m_c \) and \( m_0' \). We will show that \( v' \) is a counter-strategy for \( v \).

First, we show that condition (iii) of Definition 2 holds for \( v' \). Let \( \rho \) be a run of the system induced by \( v' \), i.e. \( \rho \) is induced by \((v' \parallel c \parallel e)\), for some \( c \in C, e \in E \). If no message of the form \( m^* \), as defined above, is posted on bulletin board and tallied, then there is nothing to prove. So, suppose that some \( \text{sig}_{v_0}(\text{sig}_{v_0}(m)) \) is posted and tallied. The only message signed by \( r_0 \) in \( m_0' \), which is a reencryption of the ballot containing the vote \( z \), so \( m = m_0' \). Hence, as the tabulation phase has to be done correctly (because otherwise the authorities would not be able to construct valid zero-knowledge proofs), this vote is published.

Now, we will show that condition (i) of Definition 2 holds for \( v \) and \( v' \). So, let \( \rho \in \alpha \) be a run induced by \((v \parallel c \parallel e)\), for some \( c \in C \) and \( e \in E \). Since \( \rho \) is in \( \alpha \), there is some honest voter, say \( v_1 \), who successfully votes for \( z \).
chosen vote

blind signatures and trapdoor commitment. The equational

D Okamoto Protocol

D.1 Cryptographic Primitives

BC(v', f'(α, v, r, v'), f(α)) = BC(v, r, f(α))

splitVerif(\{f(sum(x_1, \ldots, x_n), f(x_1), \ldots, f(x_n)\}) = T

Figure 5: Equational Theory for Okamoto Protocol.

We take \(e' \in E\) which is as \(e\) with the following exceptions: if, in \(ρ\), a message of the form \(\text{sig}_{\text{re}} \{\text{sig}_{\text{re}} \{m\}\}\) is posted on the bulletin board, where \(m\) is a ballot with some vote \(z_c\) (note that \(z_c\) has to be a valid vote), then \(v_1\), in \(e'\), votes for \(z_c\) instead of \(z\).

The run \(ρ'\) of \((v' || e || e')\) is constructed from \(ρ\) in a natural way: the messages in \(ρ'\) are delivered in the same order like the corresponding messages in \(ρ\). One can show that \(ρ \sim ρ'\). The rough idea is as follows: \(v_1\) is used to balance the outcome of the election. Due to the fact that the coercer cannot tell any difference between an original DVRP and a faked one, and the fact that the messages posted on the bulletin board are mixed and reencrypted before decryption, the runs are indistinguishable to the coercer.

Now, we will show that condition (i) of Definition 2 holds for \(v\) and \(e'\). So, let \(ρ = α\) be a run induced by \((v' || e || e)\), for some \(c \in C\) and \(e \in E\). We define the vote of \(z_c\): if the coercer voter, in \(ρ\), is instructed to use his randomiser to reencrypt some ballot \(m_c\) with a valid proof that \(m_c\) contains a valid vote \(v\), then \(z_c = v\); otherwise let \(z_c\) by any vote. Since \(ρ\) is in \(α\), there is some honest voter, say \(v_1\), who successfully votes for \(z_c\).

We take \(e' \in E\) which is as \(e\) with the following exceptions: if, in \(ρ\), a message of the form \(\text{sig}_{\text{re}} \{\text{sig}_{\text{re}} \{m\}\}\) is post on the bulletin board (note that \(m\) must be \(m_c\) as defined above) then \(v_1\), in \(e'\), votes for \(z_c\) instead of \(z\).

The run \(ρ'\) of \((v' || e || e')\) is constructed from \(ρ\), again, in a natural way: the messages in \(ρ'\) are delivered in the same order like the corresponding messages in \(ρ\). One can show that \(ρ \sim ρ'\), for the same reasons as previously.

D Okamoto Protocol

In this section we describe the protocol [33] and discuss its properties.

D.1 Cryptographic Primitives

In addition to the common cryptographic properties (which can be modelled like in Figure 2, the protocol makes use of blind signatures and trapdoor commitment. The equational theory associated with these primitives is given in Fig. 5.

These primitives are used in the following way. For a chosen vote \(v\) and random values \(r\) and \(α\), a voter can compute trapdoor-commitment for \(v\), denoted by \(BC(v, r, f(α))\).

As it is only \(f(α)\), not \(α\) itself, what is used to compute this expression, the commitment can be checked (re-computed) using \(v\), \(r\), and \(f(α)\). However, the voter, who also knows \(α\), can, for any vote \(v'\), forge a value \(f'(α, v, r, v')\) which gives the same commitment value, i.e. \(BC(v, r, f(α)) = BC(v', r', f(α))\).

D.2 Description of the Protocol

The set of agents is \(\{v_0, \ldots, v_n, A, B, T, R_1, \ldots, R_N\}\), where \(v_0, \ldots, v_n\) are voters, \(A\) is an administrator, \(B\) is a bulletin board, \(T\) is a timeliness commission member, and \(R_1, \ldots, R_N\) are PRC members.

Channels between \(v_i\) and \(A\) are network channels (the Internet). Messages posted on the bulletin board are sent through an anonymous channel. The voter \(v_i\) send messages to \(T\) and \(R_j\) using untappable, anonymous channel.

From the point of view of \(v_i\), the protocol execution consists of the following steps: First, \(v_i\) randomly generates \(α_i^1, \ldots, α_i^N\) and computes \(α_i = \sum(α_i^1, \ldots, α_i^N)\). Then he computes \(G_i = f(α_i)\) and \(G_i^1 = f(α_i^1)\). Next, he randomly chooses \(r_i\) and \(t_i\). Let

\[
\begin{align*}
    m_i &= BC(v_i, r_i, G_i), \\
    m_i' &= (m_i, G_i, G_i^1, \ldots, G_i^N), \\
    x_i &= \text{blind}(m_i', t_i, \text{pub}(A)), \\
    z_i &= \text{sig}_{\text{re}} \{x_i\}
\end{align*}
\]

Now, the following messages are exchanged:

(P1) \(v_i \rightarrow A: \{x_i, z_i, v_i\}_{\text{pub}(A)}\)
(P2) \(A \rightarrow v_i: y_i = \text{sig}_{\text{re}} \{x_i\}\)

As we mentioned, the communication channel between \(v_i\) and \(A\) is a public channel. Before executing (P2), \(A\) checks the signature \(z_i\) on \(x_i\) and verifies that \(v_i\) has the right to vote and he has not applied yet. After step (P2) is performed, \(v_i\) takes \(s_i = \text{unblind}(y_i, t_i)\) (which is equivalent to \(\text{sig}_{\text{re}} \{m_i'\}\)). The successive steps are:

(P3) \(v_i \rightarrow B: (m_i', s_i)\)
(P4) \(v_i \rightarrow T: (v_i, r_i, m_i)\)
(P5) \(v_i \rightarrow R_j: (α_i^j, G_i)\)
(P6) \(R_j \rightarrow B: u_i^j = (f(α_i^j), G_i), \text{ sig}_{\text{re}} \{u_i^j\}\)

In the counting stage, \(T\), using messages from the bulletin board, checks whether the message obtained from \(v_i\), in step (P4) is a valid ballot, as is explained below. Then \(T\) publishes valid votes in random order.

To check whether to accept a message (P4), \(T\) does the following: He looks for the matching message \((m_i', s_i)\) published on the bulletin board \((m_i' \text{ has to contain } m_i \text{ as the first component}) \) and verifies that \(s_i = \text{sig}_{\text{re}} \{m_i'\}\). Then he verifies that \(\forall G_i\) taken from \(m_i'\) it is true that \(m_i\) is in fact equal to \(BC(b_i, r_i, G_i)\). He also checks whether, for all \(G_i^1\) taken from \(m_i'\), the corresponding message \((G_i^1, G_i)\) was published on the bulletin board by \(R_j\)
and that \( \operatorname{splitVerif}(G_i, G_i^1, \ldots, G_i^N) = T \). If all these tests pass, the vote is accepted. \( T \) also provides a zero-knowledge proof that he has honestly published valid votes.

In [33], it is mentioned that the above voting scheme is not coercion-resistant, if one of the PRC’s is not honest (cooperates with the coercer). So, a more complicated variant of the voting scheme (Scheme B) is also proposed. In this variant, the relation between \( \alpha_i \) and \( A = \{\alpha_i^1, \ldots, \alpha_i^N\} \) is that it is enough to know some number \( K < N \) of elements in \( A \) to be able to compute \( \alpha_i \) (as opposed to the variant presented above, where all \( N \) elements of \( A \) are necessary to compute \( \alpha_i \)).

D.3 Properties of the Protocol

In short, the Okamoto protocol does not provide coercion resistance even under strong assumptions. However, the protocol is interesting in that it highlights the difference between single-voter coercion and multi-voter coercion, in absence of dishonest voters.

In [33], the proof of coercion-resistance is based on the observation that the only way to make a ballot accepted is to send valid \( \alpha_i^j \) to all \( R_j \). So because the channels between the voter and PRC’s are untappable, it can be only the voter who sends these values and, in consequence, he has to know them. Thus, he is able to compute \( \alpha_i \) and make up a value \( r’ \) which enables him to vote for the vote of his choice. This reasoning misses, however, the fact that different \( \alpha_i^j \) can be sent by different voters or even by the coercer, if he is also a voter.

If we assume that the coercer is an entitled voter or there is some dishonest voter, the protocol is clearly not coercion-resistant: The coercer prepares a ballot, ask the coerced voter to obtain a blind signature on this ballot and then completes the process by himself, using the anonymous untappable channels he has access to.

Event if we assume that the coercer is not an entitled voter and there is no dishonest voter, then still the protocol is not coercion-resistant, provided that more than one voter is coerced at the same time. In this case the coercion strategy is as follows. All the coerced voters are supposed to obtain a blind signature of the appropriate voting authority on messages provided by the coercer. Then, the coercer distributes the private credential shares to the voters in such a way that no coerced voter has a complete collection of private credential shares, i.e., the shares for one vote are distributed among different coerced voters. As a result, no coerced voter can open his/her commitment in an arbitrary way. This suffices for the ballots of the coercer to be accepted. To the best of our knowledge, this attack has not been observed before. A more detailed description of this attack follows.

Suppose that there are \( N \) voters that are coerced (recall that \( N \) is the number of PRC’s; we chose this number for simplicity of the proof). The coercion strategy is as follows. All the coerced voters are supposed to obtain a signature on messages provided by the coercer. The \( i \)-th message is build, as \( m_i^j \) in the protocol description, using \( \alpha_i^1, \ldots, \alpha_i^N \). Each \( v_i \) is then supposed to send \( x_i \), like in the protocol description, for \( v_i \) chosen by the coercer. Furthermore, \( v_i \) is supposed, for each \( j \in \{1, \ldots, N\} \), to forward \( \alpha_i^j \) to \( R_j \), where \( a = (i + j) \mod N \). These shares are the only ones that the voter learns. So, he is not able to compute any of \( \alpha_i \) (it is also true in scheme B), because he knows only one private share for each \( \alpha_i \). Thus, the only valid vote \( v_i \) can send to \( T \) is \( v_i \), as demanded by the coercer. Because \( T \) provides a zero-knowledge proof that the submitted votes are accounted for, the coercer can verify, that this vote has been really posted by the voter.

While the above attack allows the coercer to vote as he wishes, an abstention attack is possible even if only one voter is coerced, the coercer is not entitled to vote and there are not dishonest voters.

The only setting in which we could prove coercion resistance of the Okamoto protocol is in the setting just described where \( \alpha \) is defined similarly to the Lee et al. protocol and the goal \( \gamma \) is merely that if the coerced voter posts message (P3) on the bulletin board, then his/her successfully votes for the candidate of his/her choice.

Note that for this result to hold it is essential that only one voter is coerced.