GECAM Localization of High-energy Transients and the Systematic Error

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Abstract

The Gravitational Wave High-energy Electromagnetic Counterpart All-sky Monitor (GECAM) is a pair of microsatellites (i.e., GECAM-A and GECAM-B) dedicated to monitoring gamma-ray transients including the high-energy electromagnetic counterparts of gravitational waves, such as gamma-ray bursts, soft gamma-ray repeaters, solar flares, and terrestrial gamma-ray flashes. Since launch in 2020 December, GECAM-B has detected hundreds of astronomical and terrestrial events. For these bursts, localization is the key for burst identification and classification as well as follow-up observations in multiple wavelengths. Here, we propose a Bayesian localization method with Poisson data with Gaussian background prior. We apply this method to identify GECAM burst candidates as well as follow-up observations in multiple wavelengths. We validate this method by Monte Carlo simulations, and then apply it to a burst sample with accurate location and find that the mean value of the systematic error of GECAM-B localization is ~2°.5. By considering this systematic error, we can obtain a reliable localization probability map for GECAM bursts. Our methods can be applied to other gamma-ray monitors.

Unified Astronomy Thesaurus concepts: Astronomical methods (1043); Observational astronomy (1145); Gamma-ray astronomy (628); Time domain astronomy (2109)

Supporting material: figure set

1. Introduction

The Gravitational Wave High-energy Electromagnetic Counterpart All-sky Monitor (GECAM, Li et al. 2022b) is a space-based instrument dedicated to the detection of gamma-ray electromagnetic counterparts of gravitational waves. The GECAM Localization of High-energy Transients (Li et al. 2022b) is a pair of spacecrafts (i.e., GECAM-A and GECAM-B) dedicated to detecting gamma-ray transients including the high-energy electromagnetic counterparts of gravitational waves, such as gamma-ray bursts, soft gamma-ray repeaters, solar flares, and terrestrial gamma-ray flashes. Since launch in 2020 December, GECAM-B has detected hundreds of astronomical and terrestrial events. For these bursts, localization is the key for burst identification and classification as well as follow-up observations in multiple wavelengths. Here, we propose a Bayesian localization method with Poisson data with Gaussian background prior to identify GECAM burst candidates as well as follow-up observations in multiple wavelengths. We validate this method by Monte Carlo simulations, and then apply it to a burst sample with accurate location and find that the mean value of the systematic error of GECAM-B localization is ~2°.5. By considering this systematic error, we can obtain a reliable localization probability map for GECAM bursts. Our methods can be applied to other gamma-ray monitors.

Abbott et al. 2017a, 2017b; Goldstein et al. 2017; Savchenko et al. 2017; Li et al. 2018) and fast radio bursts (Lorimer et al. 2007; Li et al. 2021a), as well as other high-energy astrophysical and terrestrial transient sources, such as gamma-ray bursts (GRBs, Klebesadel et al. 1973), soft gamma-ray repeaters (SGRs, Woods & Thompson 2004), solar flares (SFLs, Parks & Winckler 1969), terrestrial gamma-ray flashes (TGFs, Fishman et al. 1994), and terrestrial electron beams (Dwyer et al. 2008; Xiong et al. 2012).
Launched in 2020 December, GECAM has been operating in low Earth orbit (600 km altitude and 29° inclination angle, Han et al. 2020). GECAM consists of twin microsatellites (i.e., GECAM-A and GECAM-B) and each comprises 25 gamma-ray detectors (GRDs, Lv et al. 2018; Zhang et al. 2019; An et al. 2022) and eight charged particle detectors (CPDs, Li et al. 2022a; Xu et al. 2022; Zhang et al. 2022a). With the LaBr₃ crystal read out by a silicon photomultiplier array, GRDs could monitor the entire unoccupied sky in the energy range of 15 keV to 5 MeV (Zhang et al. 2022b, 2022d). The CPD is designed to detect charged particles in the orbit, which is powerful for observing terrestrial electron beams and helping the GRDs to distinguish between gamma-ray burst events and charged particle burst events (Zhao et al. 2021).

Since launch, GECAM-B has detected many GRBs (Wang et al. 2021a; Song et al. 2022), SGRs (Xie et al. 2022), SFLs, and TGFs, while GECAM-A has not been able to observe yet due to power supply issues. The main scientific goal of GECAM is to detect high-energy electromagnetic counterparts associated with gravitational waves and fast radio bursts: analysis of such association usually requires the coincidence of timing and localization. For all kinds of bursts, localization is the crucial information for joint observations with other instruments. Therefore, the capability of localization is of fundamental importance for the GECAM mission.

As shown in Table 1, a series of localization algorithms have been developed by the GECAM team for different stages after the trigger of a burst (Liao et al. 2020; Zhao et al. 2021; C. Cai et al. 2023, in preparation; Y. Huang et al. 2023, in preparation):

1. For onboard triggered bursts, the flight software computes their locations and provides a preliminary classification of the bursts within ~20 s (Zhao et al. 2021). The flight software employs a $\chi^2$ minimization localization method considering the very limited onboard resources of computing and storage.

2. To allow rapid follow-up observations with GECAM triggers, the GECAM alert data are downlinked in real time by the global short message communication service (Li et al. 2021b) of the Beidou navigation satellite system (Yuang et al. 2019). With the alert data, an automatic on-ground analysis pipeline is used to provide a refined localization result, which is based on the $\chi^2$ minimization localization method. Moreover, a one-time spectrum-location iteration is used to optimize the localization spectral template. Thus this localization is expected to be more reliable than the in-flight ones.

3. After the full science data (including time-tagged event data and binned data) are downloaded to the ground, both automatic and human-in-the-loop analyses for the burst will be initiated, including detailed temporal, positional, and spectral analyses. Among them, the localization is based on a Bayesian localization method with Poisson data with Gaussian background (PGSTAT) profile likelihood, which is the main topic of the present paper.

4. If the burst is jointly observed by other instruments, such as Insight-HXMT/HE, Fermi/GBM, and Swift/BAT, the triangulation (time-delay) localization within our ETJASMIN pipeline would also be implemented to improve the location (Xiao et al. 2021, 2022b).

In the present paper, we focus on the ground localization with full science data, rather than flight location or ground location with alert data. The Bayesian localization methods with fixed spectral templates have been discussed in previous work (Zhao et al. 2022). Here we extend this method to real observational data by replacing the Poisson likelihood with the PGSTAT profile likelihood. We also propose a new method to estimate the systematic error of location.

We note that in this work only GECAM-B data are used since GECAM-A has not been able to observe yet (Li et al. 2022b).

This paper is structured as follows. The GECAM instruments, detectors, and data used in localization analysis are presented in Section 2. In Section 3, a Bayesian localization method with PGSTAT profile likelihood is proposed for GECAM. In Section 4, we present the localization analysis for 23 bright bursts (including GRBs, SGRs, and SFLs) and three extremely short bursts (TGFs). We discuss the systematic error and demonstrate the estimation of systematic uncertainty using GECAM localizations in Section 5. Finally, a summary is given in Section 6.

### 2. The GECAM Instrument

The GECAM payload mainly consists of 25 GRDs (for 15 keV–5 MeV X/$\gamma$ rays), eight CPDs (for 150 keV–5 MeV charged particles), and the Electronic Box (EBOX, An et al. 2022; Li et al. 2022b; Xu et al. 2022). As illustrated in Figure 1, 25 GRDs and six CPDs are placed with different orientations in the detector dome of the GECAM satellite to monitor most of the sky region, while two CPDs are installed on the +X side of the payload EBOX. The flight localization...
and alert data are generated by the in-flight trigger and localization software, which is executed on the data management board of the EBOX (Li et al. 2022b). Since the CPDs have low detection efficiency to gamma rays (Li et al. 2022a; Xu et al. 2022; Zhang et al. 2022a), we only use 25 GRDs to compute burst localization. The pointing directions of the GRDs in GECAM payload coordinates are listed in Table 2.

The event-by-event data from GRDs are employed to conduct localization analysis. GECAM features the highest time resolution among instruments of its kind, i.e., 0.1 μs (Xiao et al. 2022a).

The dead time is 4 μs for a normal event and >69 μs for an overflow event (i.e., event with higher energy deposition than the maximum measurable energy) (Liu et al. 2021).

Each GRD detector has two read-out channels: a high-gain channel and a low-gain channel (Liu et al. 2021). The normal events of GRDs (i.e., within the detection energy range) are registered in 4096 raw analog-to-digital convertor channels by data acquisition electronics and converted to 447 pulse-invariant (PI) energy channels for high gain (∼15 keV to ∼300 keV) and low gain (∼300 keV to ∼5 MeV). For simplicity, only the counts from ∼30 keV to ∼200 keV (i.e., PI channels from 45 to 150) of the high-gain channel are adopted in the localization of the present work, because most counts from detected bursts are recorded in this energy range. The detector response is constructed based on a series of ground tests (Zhang et al. 2019; An et al. 2022; Li et al. 2022b) before launch and comprehensive simulations incorporating the GECAM spacecraft mass model into GEANT4 (Guo et al. 2020), and has been further calibrated in-flight with characteristic lines and known bursts (Zhang et al. 2022c; Zheng et al. 2022; R. Qiao et al. 2023, in preparation; Y. Q. Zhang et al. 2023, in preparation).

### 3. Localization Methodology

#### 3.1. Localization Method

Zhao et al. (2022) proposed a Bayesian localization method with Poisson likelihood for the fixed spectral template, which was validated through comprehensive tests. But that method is designed for the ideal case where the expected background is known.

| Source Intensity Type | Medium Bright Burst |
|----------------------|---------------------|
| Spectral model       | Comptonized         |
| Spectral index       | 1.50                |
| $E_{\text{peak}}$ (keV) | 500                 |
| Duration (s)         | 10.0                |
| Fluence (erg cm$^{-2}$) | $2.17 \times 10^{-5}$ |

**Note.** The incident angle is zenith = 5°85, azimuth = 22°50 in GECAM payload coordinates, which corresponds to R.A. = 119°63, decl. = +65°40 (true position) at 2021-07-22T01:00:00 UTC. The background level is set to 1000 counts s$^{-1}$ for each GRD, which is a similar level to that measured in orbit. Fluence is calculated in 10–1000 keV.
and error bars are the measured counts and 3σ deviation.

- Moderate Comptonized function
- Soft Comptonized function
- and the expected counts

**Template Model Low-energy Index**

| Template | Model         | Low-energy Index | $E_{\text{peak}}$ (keV) |
|----------|---------------|------------------|--------------------------|
| Soft     | Comptonized function | −1.95          | 50                       |
| Moderate | Comptonized function | −1.15          | 350                      |
| Hard     | Comptonized function | −0.25          | 1000                     |

For real observations, the expected background is unknown and we can only use the estimated background and have to take into account its uncertainty. Therefore, here we propose the Bayesian localization method with Poisson data with Gaussian background assumptions made:

1. the background is known precisely, thus the PGSTAT profile likelihood is reduced to the simple Poisson likelihood (Equation (A2));
2. the utilized localization template is derived from the input spectrum of the simulated bursts. The simulated counts in each detector are derived from the Poisson fluctuation of the expectation of total counts, which is the sum of the burst counts (the input burst spectrum convolved with the detector response) and the background. The true position for simulation and more detailed setting can be found in Table 3.

The localization result for simulated bursts is shown in Figure 8(a). To validate the location probability map and credible region, we check the distribution of the highest posterior density (HPD) cumulative probability of the true position in the location maps for simulated bursts. These results indicate that this localization method could give reliable and correct location results for simulated bursts, and no systematic uncertainties exist in the samples.

### 3.2. Localization Procedures

To present the procedure of localization analysis in detail, here we take GRB 220511A as an example.

**Step 1.**

The source and background intervals are manually selected, and polynomial fitting for the background is implemented, as shown in Figure 2(a). We utilize GECAMtools to implement the polynomial fitting of the background, which uses a weighted least-squares ($\chi^2$) statistic to fit the polynomial coefficients through a two-pass approach, as used in the GBM data analysis in RMFIT and GBM Data Tools. To select the time interval for a source, we tried several different ranges,
and choose the best one from the localization results, i.e., the size of statistical error. Then, for the source interval, the estimated background and its uncertainty, the exposure time, and total observed counts corrected for dead time are obtained.

Step 2.
As shown in Figure 2(b), a fixed template localization with all 25 GRDs is performed. We use three fixed spectral templates (Table 4) (see also new spectral templates of Table 1 in Goldstein et al. 2020) through the Bayesian localization method with PGSTAT profile likelihood. In analogy to GBM DoL (Connaughton et al. 2015), the maxima of the maximum PGSTAT profile likelihood of the localization of three spectral templates is regarded as the location result. This localization is termed FIX hereafter.

Step 3.
To derive a refined spectral template for localization, we implement a one-time spectrum-location iteration. A spectral fitting to the location center of the FIX

Figure 3. GECAM localization for GRB 220511A (Step 4 in Section 3). (a) The spectral fitting for the accurate position based on the reference location. (b) The consistency check of the measured data and the expected counts (the sum of estimated background and expected source counts). The source contribution is derived from this spectral fitting results. (c) The location’s HPD credible region with this spectral template for selected detectors. (d) To illustrate that using all detectors will introduce localization derivation, the location’s credible region with this spectral parameter for all 25 GRDs is shown. The GECAM localization result of 23 bright GRBs, SGRs, and SFLs is shown in Table 6.

Figure 4. The distribution of the incidence zenith/azimuth angle for the reference locations of 23 bright bursts (red crosses) in GECAM payload coordinates.
| Burst Name | Trigger Time (UT) | Flight Location | BD Ground Location | Reference Location | References |
|------------|------------------|----------------|-------------------|-------------------|------------|
|            |                  | R.A. (deg)     | Decl. (deg)       | ERR (deg)         | α (deg)    | R.A. (deg) | Decl. (deg) | ERR (deg) | α (deg)    | R.A. (deg) | Decl. (deg) | θ (deg) | φ (deg) | |
| GRB 210121A | 2021-01-21T18:41:48.800 | 22.1 | −49.5 | 1.3 | 4.6 | 23.3 | −47.7 | 2.0 | 4.5 | 17.0 | −46.4 | 40.9 | 267.2 | IPN\textsuperscript{a} |
| GRB 210511B | 2021-05-11T11:26:40.600 | 318.0 | +59.5 | 3.2 | 2.9 | 320.3 | +60.1 | 3.7 | 4.1 | 312.9 | +58.4 | 120.4 | 38.6 | IPN\textsuperscript{b} |
| GRB 210606B | 2021-06-06T22:01:08.100 | 85.5 | −16.5 | 1.0 | 2.4 | 87.8 | −18.3 | 1.4 | 1.6 | 88.0 | −16.7 | 15.9 | 312.7 | IPN\textsuperscript{c} |
| GRB 210619B | 2021-06-20T00:00:00.950 | 334.7 | +28.6 | 31.0 | 13.9 | 318.8 | +29.2 | 7.4 | 4.8 | 319.7 | +33.9 | 136.4 | 70.0 | Swift-XRT\textsuperscript{d} |
| GRB 210822A | 2021-08-22T09:18:18.000 | 310.3 | +4.5 | 1.0 | 5.9 | 298.3 | +1.0 | 1.2 | 7.5 | 304.4 | +5.3 | 106.2 | 207.8 | Swift-XRT\textsuperscript{e} |
| GRB 210927B | 2021-09-27T23:34:45.600 | 240.3 | +69.5 | 9.9 | 8.3 | 249.6 | +70.4 | 3.3 | 5.3 | 263.0 | +73.8 | 42.5 | 213.4 | IPN\textsuperscript{f} |
| GRB 211120A | 2021-11-20T23:05:20.600 | 311.3 | +41.3 | 1.0 | 3.2 | 305.0 | +41.4 | 1.3 | 7.6 | 315.1 | +42.9 | 53.9 | 141.2 | IPN\textsuperscript{g} |
| GRB 220514A | 2022-05-14T12:24:32.950 | 139.8 | +19.1 | 3.5 | 9.7 | 143.4 | +12.6 | 4.8 | 4.2 | 147.7 | +13.1 | 86.2 | 98.7 | INTEGRAL-IBAS\textsuperscript{h} |
| SFL 210508 | 2021-05-08T18:30:39.950 | 31.1 | +45.9 | 7.3 | 9.7 | 24.0 | +55.6 | 7.4 | 10.9 | 17.2 | +45.6 | 50.6 | 2.0 | ... |
| SFL 211028 | 2021-10-28T17:37:24.000 | 213.6 | −28.8 | 3.4 | 15.5 | 218.7 | −16.3 | 3.5 | 6.4 | 212.9 | −13.3 | 50.3 | 3.6 | ... |
| SGR 1935+2154(a) | 2021-07-07T00:33:31.700 | 292.7 | +23.8 | 1.0 | 2.1 | 292.0 | +24.3 | 3.1 | 2.9 | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{i} |
| SGR 1935+2154(b) | 2021-09-11T17:01:10.301 | 292.5 | +22.7 | 1.0 | 1.4 | 289.8 | +23.9 | 1.5 | 4.1 | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{j} |
| SGR 1935+2154(c) | 2021-09-12T00:34:37.450 | none | none | none | none | none | none | none | none | none | none | none | none | Swift-BAT\textsuperscript{k} |
| SGR 1935+2154(d) | 2021-09-14T11:10:36.250 | none | none | none | none | none | none | none | none | none | none | none | none | Swift-BAT\textsuperscript{l} |
| SGR 1935+2154(e) | 2021-09-22T20:12:16.502 | 291.2 | 22.2 | 5.6 | 2.3 | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{m} |
| SGR 1935+2154(f) | 2022-01-12T08:39:25.450 | 293.6 | 20.0 | 1.0 | 1.9 | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{n} |
| SGR 1935+2154(g) | 2022-01-14T20:07:03.050 | none | none | none | none | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{o} |
| SGR 1935+2154(h) | 2022-01-14T20:15:54.401 | none | none | none | none | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{p} |
| SGR 1935+2154(i) | 2022-01-14T20:11:05.151 | 295.3 | 21.0 | 1.0 | 1.7 | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{q} |
| SGR 1935+2154(j) | 2022-01-14T20:29:07.250 | 297.3 | 16.6 | 1.2 | 6.3 | 296.8 | +19.7 | 2.0 | 3.6 | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{r} |
| SGR 1935+2154(k) | 2022-01-15T17:21:59.304 | 295.5 | 21.1 | 2.7 | 1.9 | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{s} |
| SGR 1935+2154(l) | 2022-01-23T20:06:38.751 | 300.4 | 19.0 | 1.0 | 6.9 | none | none | none | none | none | 293.7 | +21.9 | 26.6 | 64.3 | Swift-BAT\textsuperscript{t} |

Notes. Due to not being triggered on board or software failure, the flight location and Beidou ground location for some bursts are lacking.

\textsuperscript{a} IPN, Svinkin et al. (2021).
\textsuperscript{b} IPN, Hurley et al. (2021a).
\textsuperscript{c} IPN, Hurley et al. (2021b).
\textsuperscript{d} Swift-XRT, D’Avanzo et al. (2021).
\textsuperscript{e} Swift-XRT, Page et al. (2021).
\textsuperscript{f} INTEGRAL-IBAS, Mereghetti et al. (2022).
\textsuperscript{g} Swift-BAT, Ambrosi et al. (2019).
| Burst Name | R.A. (deg) | Decl. (deg) | ERR (deg) | α (deg) | CSa (%) |
|------------|------------|-------------|-----------|---------|---------|
| GRB 210121A | 27.0       | −47.8       | 2.0       | 6.9     | 100.0   |
| GRB 210511B | 308.4      | +59.3       | 3.2       | 2.5     | 55.0    |
| GRB 210606B | 85.6       | −20.1       | 1.0       | 4.1     | 100.0   |
| GRB 210619B | 321.1      | +34.0       | 0.6       | 1.2     | 100.0   |
| GRB 210822A | 300.6      | −0.1        | 2.2       | 4.2     | 97.6    |
| GRB 211120A | 305.9      | +40.7       | 1.1       | 7.2     | 100.0   |
| GRB 220511A | 288.8      | +18.4       | 2.3       | 1.4     | 12.3    |
| SFL 210508  | 47.3       | +19.0       | 11.0      | 2.4     | 4.3     |
| SFL 211028  | 224.6      | −7.3        | 7.9       | 13.0    | 88.5    |
| SGR 1935    | 289.0      | +20.9       | 0.6       | 4.5     | 100.0   |
| SGR 1935    | 294.7      | +21.3       | 0.6       | 1.0     | 78.8    |
| SGR 1935    | 297.9      | +21.5       | 2.6       | 3.9     | 99.6    |
| SGR 1935    | 295.5      | +19.7       | 2.2       | 2.7     | 63.3    |
| SGR 1935    | 293.0      | +20.0       | 1.1       | 2.1     | 99.3    |
| SGR 1935    | 291.3      | +25.0       | 0.6       | 3.8     | 100.0   |
| SGR 1935    | 291.0      | +24.2       | 0.8       | 3.4     | 100.0   |
| SGR 1935    | 290.1      | +23.8       | 1.3       | 3.9     | 99.9    |
| SGR 1935    | 293.9      | +22.7       | 0.6       | 0.8     | 100.0   |
| SGR 1935    | 296.1      | +20.3       | 1.0       | 2.7     | 100.0   |
| SGR 1935    | 295.0      | +21.9       | 1.0       | 1.2     | 50.7    |
| SGR 1935    | 298.8      | +21.5       | 0.6       | 4.7     | 100.0   |

Notes. We do not characterize error by using the circle region, but instead by the area of the error region. However, to compare with other methods, we list the equivalent radius in this table. CS: The cumulative sum probability of the reference location.
localization is first implemented. After the consistency check of the observational and expected counts, we redo the localization, for which the spectral template is reconstructed by the spectral fitting result. This localization is termed RFD hereafter.

(a) As shown in Figure 2(c), to derive a refined spectrum for localization, a spectral fitting to the location center of the FIX localization is implemented. These three GRDs (GRD #11, 12, and 21) within 60° from the location center and with detection significance >5σ are used for this spectral fitting.

(b) Check the consistency of the expected counts (the sum of estimated background and expected source contribution) and measured counts registered in detectors, as shown in Figure 2(d). Although excess counts in some detectors are significant, their expected counts are inconsistent with the measured data due to the imperfection of the off-axis response. Therefore, only detectors within 3σ deviation between the measured and expected counts are adopted for localization in the following refined template localization. The source contribution is derived from the refined spectrum.

(c) As shown in Figure 2(e), using the refined spectral fitting results and the selected detectors, we construct the spectral template and localize the burst again.

Step 4.

To check the best scenario of the one-time iteration localization, similar to the RFD localization, we do a one-time spectrum-location iteration, but the accurate position based on the reference location21 is adopted for spectral fitting. This localization is termed APR hereafter.

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21 Reference location: the accurate position provided by instruments such as Swift and IPN, etc.

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### Table 7

| Burst Name | Model | Low-energy Index | High-energy Index | $E_{\text{cut}}$ | Model | Low-energy Index | High-energy Index | $E_{\text{cut}}$ |
|------------|-------|-----------------|-----------------|-------------|-------|-----------------|-----------------|-------------|
| GRB 210121A | cutofffl | 0.47 | none | 287.69 | cutofffl | 0.43 | none | 276.58 |
| GRB 210619B | band | −0.45 | −10.00 | 165.60 | band | −0.64 | −1.67 | 157.46 |
| GRB 210822A | cutofffl | 0.57 | none | 216.15 | cutofffl | 0.58 | none | 219.03 |
| GRB 210927B | cutofffl | 0.47 | none | 179.92 | cutofffl | 0.35 | none | 137.03 |
| GRB 211120A | cutofffl | 0.55 | none | 144.89 | cutofffl | 0.67 | none | 187.95 |
| GRB 210511B | band | −1.49 | −2.01 | 42.92 | band | −1.06 | −1.66 | 45.57 |
| GRB 210606B | band | −1.22 | −1.51 | 459.39 | band | −1.23 | −1.51 | 502.56 |
| GRB 220511A | band | −1.22 | none | 333.28 | cutofffl | −1.35 | −10.00 | 535.58 |
| SGR 211028L | powerlaw | 7.69 | none | 56.53 | powerlaw | 7.72 | none | 50.44 |
| SGR 211028L | powerlaw | 6.11 | none | 6.45 | none | 6.45 | none | 6.45 |
| SGR 1935+2154(a) | cutofffl | 0.44 | none | 17.82 | cutofffl | 0.43 | none | 17.82 |
| SGR 1935+2154(b) | cutofffl | 0.33 | none | 14.86 | cutofffl | 0.55 | none | 16.35 |
| SGR 1935+2154(c) | cutofffl | −0.48 | none | 12.58 | cutofffl | −0.48 | none | 12.59 |
| SGR 1935+2154(d) | cutofffl | −1.55 | none | 9.40 | cutofffl | −1.55 | none | 9.40 |
| SGR 1935+2154(e) | cutofffl | −0.08 | none | 14.27 | cutofffl | −0.08 | none | 14.27 |
| SGR 1935+2154(f) | powerlaw | 3.62 | none | 3.62 | powerlaw | 3.62 | none | 3.62 |
| SGR 1935+2154(g) | cutofffl | 0.61 | none | 21.32 | cutofffl | 0.69 | none | 22.02 |
| SGR 1935+2154(h) | cutofffl | 1.06 | none | 25.16 | cutofffl | 1.06 | none | 25.17 |
| SGR 1935+2154(i) | cutofffl | 0.57 | none | 25.78 | cutofffl | 0.57 | none | 25.78 |
| SGR 1935+2154(j) | cutofffl | −0.07 | none | 16.59 | cutofffl | −0.08 | none | 16.58 |
| SGR 1935+2154(k) | cutofffl | 0.50 | none | 21.73 | cutofffl | 0.50 | none | 21.75 |
| SGR 1935+2154(l) | cutofffl | −0.06 | none | 14.03 | cutofffl | −0.06 | none | 14.04 |

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Figure 5. The distribution of the angular separation between the reference positions and the location centers of FIX (green), RFD (blue), and APR (red) localization.

(a) First conduct the spectral fitting to the reference location, as shown in Figure 3(a).
(b) As shown in Figure 3(b), do the consistency check for detectors.
(c) Then reconstruct the spectral template and redo localization with the spectral fitting parameters and selected detectors. The location’s credible region is shown in Figure 3(c).

For the burst localization of Fermi/GBM, Burgess et al. (2017) noted that localization with all detectors could introduce a systematic deviation, which is also seen in GECAM
localization. As shown in Figure 3(d), APR localization with all 25 GRDs deviates further from the reference location than the case that only uses the selected detectors.

We should note that the RFD localization is what we do for normal observations of bursts. For those bursts with an accurate position, we just do the APR localization for the study of localization methodology.

For RFD localization, it should be noted that the spectrum-location iteration should be used for localization. However, the automatic iteration localization procedure is challenging for the observational data, because it includes many steps, such as the selection of background and source intervals, selection of spectral model, evaluation of spectral fitting results, consistency check, detector selection, etc. These steps may influence the final localization result. The automatic iteration localization procedure is still under development and testing.

4. Localization Results

4.1. GRBs, SGRs, and SFLs

By the end of 2022 May, hundreds of bursts had been detected by GECAM-B (Zhao et al. 2021; C. Cai et al. 2023, in preparation; Y. Huang et al. 2023, in preparation), including 23 bright bursts (nine GRBs, 12 SGRs, and two SFLs), which are accurately localized by external instruments (e.g., Swift, Interplanetary Network (IPN), and INTEGRAL-IBAS). The reference locations of the 23 bright bursts and the results of their flight and ground location with alert data (see Table 1) are listed in Table 5. These bursts could be used to estimate the systematic error of localization (see the next section). The FIX, RFD, and APR location results of these bursts can be found in Table 6 and Appendix C. The spectral parameters for RFD and APR localization are listed in Table 7.

The incident angles of these bursts in GECAM payload coordinates are shown in Figure 4. The distributions of the angular separation between the location centers and reference locations for different localizations (FIX, RFD, and APR) are shown in Figure 5. As expected, the RFD localization (all bursts have offset <8°) is closer to the reference location than the FIX localization (all bursts have offset <13°), while the APR one (all bursts have offset <8°) is closer than the RFD one. The influence of spectral template and detector selection will be further discussed in Section 5. We note that the 1σ statistical error of GRBs with fluence ∼1 × 10^{-5} erg cm^{-2} (10–1000 keV) (e.g., GRB 210927B, GRB 220511A, and GRB 220514A) is ∼2°, which is generally consistent with the design of the GECAM mission.

4.2. TGFs

TGFs are very short events with a few counts (Fishman et al. 1994; Grefenstette et al. 2009; Østgaard et al. 2015, 2019; Ursi et al. 2017; Roberts et al. 2018; Lindanger et al. 2020; Maiorana et al. 2020). Generally, a luminous TGF contains ∼100 observed counts in several hundred microseconds, which means there are usually only ∼10 counts registered in a detector. Locating these submillisecond-duration weak bursts is very challenging, for the following reasons:

1. The limited counting statistics of measured counts registered in each detector challenge the accuracy and precision of localization.
2. Most of the time, there fewer than 10 GRDs are facing the Earth simultaneously. A constrained location cannot be obtained for these cases because of the imperfect response of off-axis detectors.
3. Compton scattering between the TGFs and the atmosphere may occur. Thus the incident direction of TGF photons is likely dispersed, rather than a single direction for a point source.

TGFs detected by the Burst and Transient Source Experiment (BATSE) aboard the Compton Gamma Ray Observatory (Fishman et al. 1994) are localized with the BATSE Burst Location Algorithm (LOCBURST), which uses the χ^2 method (Pendleton et al. 1999). However, Zhao et al. (2022) showed that the χ^2 method is likely problematic for this kind of weak burst. Besides, the background uncertainties are not considered in LOCBURST.

Here, we use TGFs to further test the performance of our localization algorithm in the case of limited counting statistics. We adopt three luminous TGFs detected by GECAM-B (Y. Zhao et al. 2023, in preparation) with ∼100 counts detected by ∼11 GRDs facing the Earth. The geographic positions of the GECAM-B nadir when these three TGFs were detected are shown in Figure 6.

Since a reasonably good spectral fitting for such weak bursts is nearly impossible due to the very limited number of counts, we choose a typical TGF photon spectrum (Sarria et al. 2021) to generate the localization template:

$$f(E) = A \times E^{-1} \times \exp(-E/E_{\text{cut}}),$$

where A is amplitude, E is energy, and E_{\text{cut}} is cutoff energy, which is set to 7 MeV.

As shown in Figure 7(a), there are ∼100 counts measured in a short duration of ∼200 μs for a TGF observed by GECAM, which is remarkably bright, thus the effect of dead time should be considered. We perform dead-time correction every 30 μs considering the extremely short duration.

Figure 6. Geographic distribution of the GECAM nadir points of three bright TGFs (red crosses), which are used for localization analysis.
TGF. However, most TGFs have no associated lightning for various reasons (Dwyer et al. 2012), thus their precise locations are unknown. But a good estimation of a TGF’s location is that it should come from a region within ~800 km of the GECAM-B nadir point when the TGF is detected, which is usually narrow on the sky map (see Figure 7(c)) and thus considered to be the most likely area of TGF origin (Grefenstette et al. 2009; Roberts et al. 2018; Lindanger et al. 2020). Therefore, the geocentric HPD cumulative probability could be used to evaluate the correctness of TGF localization.

As shown in Figure 7(c) and Appendix D, although the error region is large on the sky map, the location error region could cover most of the region near the nadir, and the geocentric HPD cumulative probabilities for all of these TGFs are <95.45% (corresponding to 2σ), which is consistent with the terrestrial origin of these bursts. These results also demonstrate that the TGF localization given by our Bayesian localization method with PGSTAT profile likelihood is reasonable.

5. Systematic Error of Localization

In this section, we investigate the approach to estimating systematic error for localization and analyze the location systematic error for GECAM. Many factors could lead to the systematic error in localization:

1. The inaccuracies of detector responses. Generally, the source counts come from three components: (a) the direct response to a burst; (b) the response of Compton scattering of the spacecraft and other payloads; (c) the response of atmospheric scattering (Guo et al. 2020). Due to the complexity of Compton scattering and atmospheric scattering, which relies heavily on the assumption and the Monte Carlo simulation process, the response of detectors with large off-axis angles is usually imperfect. These effects are still under study for GECAM.

2. The localization method and parameter settings, i.e., the spectral parameters as well as the selection of detectors and energy channels. As noted by Zhao et al. (2022), the χ² localization method for weak bursts could give an incorrect probability error region. Burgess et al. (2017) and Berlato et al. (2019) reported that the localization of fixed spectral templates will introduce systematic uncertainties. As we will discuss in the following, the detector selection will also affect the results.

5.1. Estimation Method for Systematic Error of Localization

In the previous works, a Bayesian approach was proposed and adopted to estimate the systematic uncertainties of localization (Briggs et al. 1999; Connaughton et al. 2015; Wang et al. 2021b), which is summarized in Appendix B. This method assumes that the localization error region is symmetric and that the uncertainty regions are circular. However, these two assumptions are usually not satisfied in the localization of bursts. As shown in Figures 2 and 3, the localization error region is irregular. To illustrate the limitation of this approach to estimation, we derive the systematic error with this method for simulated bursts that only contain statistical error, as shown by the black line in Figure 8(b). A nonzero systematic error can be seen with the peak around ∼0.5°, due to the shape of the noncircular error region. We find that this systematic
uncertainty will increase as the noncircular tendency of the error region increases. Here, we propose to derive the systematic error based on confidence level (CL) tests, described as follows:

Step 1: Total localization map. (1) For a given assumed systematic error $\sigma_{SYS}$, calculate the total localization error map by convolving the statistical probability map with $\sigma_{SYS}$ for each burst; (2) then derive the HPD cumulative probability for the true position of the simulated burst (see Appendix A); (3) divide the confidence level evenly into $M_s$ bins, and calculate the number of bursts $U(t)$ in each confidence level bin $t$.

Step 2: Confidence level test. For a reasonable $\sigma_{SYS}$, the expectation of the number of bursts in each confidence level bin should be $N_s/M_s$, where $N_s$ is the total number of simulated bursts. Thus, we can construct the likelihood that the distribution of the number of bursts $U(t)$ in a confidence level bin $t$ is observed for each assumed systematic error $\sigma_{SYS}$:

$$P_{CL} = \prod_{t=1}^{M_s} \left(\frac{N_s}{M_s}\right)^{U(t)} \times \exp \left(\frac{-N_s}{M_s}\right) \frac{U(t)!}{N_s!},$$

Then we can normalize this likelihood to obtain the probability of systematic error:

$$P_{CL} = \frac{P_{CL}'}{\sum_{t}^{} P_{CL}'}.$$

As shown by the red line in Figure 8(b), for the sample of bursts with statistical error only, our method for estimating systematic error could give the correct zero systematic error (i.e., peaks at $\sim 0^\circ$), rather than the nonzero results given by the existing method in the literature.

To further test our method, the true position of the burst sample is artificially moved to a fake position, which deviates by $6^\circ$ from the true one. As shown in Figure 8(c), our method can correctly give the systematic error whose probability distribution peaks at $6^\circ$. The simulated burst distribution in each confidence level bin for typical systematic uncertainties is shown in Figure 8(d). An individual inspection of simulated bursts is shown in Figure 8(e).
5.2. Systematic Error of GECAM Location

To quantitatively investigate what factors contribute to systematic error, we conduct the systematic estimation for (1) FIX localization, (2) RFD localization, and (3) APR localization. Due to the limited number of bursts, we choose five confidence level bins with a bin width of 20% for our estimation of systematic error. As shown in Figure 9, the maximum probability of systematic uncertainties of FIX [RFD] [APR] peaks at 4.0° [2.5°] [2.5°]. For FIX and RFD locations, the Gaussian fitting to their distribution results in peaks at 4.18° and 2.62°, respectively.

From the comparison between the FIX and RFD locations, we note that an extra systematic error of $2.5°$ is introduced in the FIX location. The reason for this could be: (1) FIX location makes use of fixed spectral parameters; (2) FIX location uses all detectors. The systematic errors of RFD and APR locations are similar since the absolute offset of the location (which is given by the FIX localization) used by RFD is quite close to the accurate APR location, thus the spectral fitting results and the localization templates are very similar for RFD and APR localizations.

In previous work, detectors with an offset of $\leq 80°$ between the location center and the pointing direction of the detector and with satisfactory significance are generally used for localization and spectral analysis. This is because the off-axis detectors’ response is imperfect in most cases. However, all detectors should be used if the data obey Poisson fluctuations (Burgess et al. 2017). Although no significant counts are registered in some detectors, they still help to reject some location regions. This tendency can be found in the simulated result. Therefore, the previous detector selection scheme (offset of $\leq 80°$ between the location center and the pointing direction of the detector and with satisfactory significance) may introduce extra deviation and reduce the statistics.

6. Summary

In this work, we propose a Bayesian localization method with PGSTAT profile likelihood for GECAM bursts. Compared to the method in Zhao et al. (2022), this method takes the background estimation and uncertainties into account, making it applicable for real observational data.

This localization method is applied to a sample of 23 bright sources (nine GRBs, 12 SGRs and two SFLs) with external accurate locations. We found that the $1\sigma$ statistical error of the three bright GRBs (fluence $\sim 1 \times 10^{-5}$ erg cm$^{-2}$) is $\sim 2°$, which is generally consistent with the original design of the GECAM mission. We also find that GECAM can reasonably locate millisecond-duration weak bursts (TGFs) with our method.

Regarding the estimation of systematic error, we find that the existing Bayesian approach to estimating systematic uncertainties in the literature will give an incorrect result because it assumes a circular statistical error region. We propose an approach based on confidence level validation, which is validated with simulations. We applied our method to a burst sample with accurate location and found that the mean value of the systematic error of GECAM localization is $\sim 2.5°$. We notice that by considering this systematic error, we can obtain a
final localization probability map that can pass the confidence level test. Since our methods of burst localization and estimation of systematic error are universal for gamma-ray monitors, they can be applied to other missions, such as Fermi/GBM (Meegan et al. 2009), SVOM/GRM (Yu et al. 2020), POLAR (Produit et al. 2018), POLAR-2 (Hulsman 2020), and GRID (Wen et al. 2019).

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Appendix A

The Bayesian Localization Method with PGSTAT Profile Likelihood

The measured counts (i.e., counts in a given detector or a channel of a detector) follow a simple Poisson distribution if the background is known exactly:

\[
P_{\text{Poisson}}(S|B, M) = \frac{(B + M)^S \exp(-(B + M))}{S!},
\]

where \(B\), \(M\), and \(S\) are the expected background level, the expected source counts, and the measured counts. The total of the background \(B\) and source contribution \(M\) gives the expected measured counts.

Based on this simple Poisson distribution, a Poisson likelihood and its logarithmic form for the localization method of fixed spectral templates can be written as

\[
L_p(i) = \prod_{j=1}^{n} \frac{(b_j + f_j \cdot m_{j,i})^s_j \cdot \exp(-(b_j + f_j \cdot m_{j,i}))}{s_j!},
\]

\[
\ln L_p(i) = \sum_{j=1}^{n} [s_j \cdot \ln(b_j + f_j \cdot m_{j,i}) - (b_j + f_j \cdot m_{j,i}) - \ln s_j!],
\]

where \(s_j\) is the total measured counts registered in detector \(j\), and \(b_j\) is the expectation value of the background. We regard the expected source contribution as \(f_j \cdot m_{j,i}\), where \(m_{j,i}\) is the specific spectral template for localization, which is a matrix of counts of each detector \(j\) for each incident direction \(i\) (the whole sky is pixeled by HEALPix), and \(f_j\) is the normalization factor to account for the fluence ratio between the real burst and the preset spectral shape of the fixed burst used to construct the template \(m_{j,i}\).

For measured data, the expected value of the background is unknown. Generally, we can obtain the estimated background \(\bar{B}\) and its uncertainty \(\sigma\) from background analysis (e.g., the polynomial fitting to the background intervals), and these background uncertainties should be considered in the likelihood of Poisson data. In this case, the Poisson data with Gaussian background (PGSTAT) statistic can be utilized, e.g., XSPEC (Arnaud et al. 2022) and the BALROG algorithm (see also Equations (9) and (10) in Burgess et al. 2017).

In practice, the estimated background \(\bar{B}\) and associated uncertainty \(\sigma\) come from the polynomial fitting to the background intervals. Therefore, the background distribution is taken to be a truncated normal form:

\[
P(B|\bar{B}, \sigma) = \frac{\Theta(B)}{\sqrt{2\pi}\sigma} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{B - \bar{B}}{2\sigma^2}\right),
\]

where \(\Theta(B)\) is the standard error function and \(\Theta\) is the Heaviside step function, which imposes the absolute prior constraint that the background cannot be negative. Thus the background’s truncated normal distribution should be considered in the likelihood of Poisson data:

\[
P(S|M, \bar{B}, \sigma) = \int_{0}^{\infty} dB \frac{(B + M)^S \exp(-(B + M))}{S!} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{B - \bar{B}}{2\sigma^2}\right).
\]

Due to the time-consuming task of numerical estimation for the background marginalization (Equation (A5)), here it is replaced by a profile likelihood. For a given number of observed counts, the integral is taken to be the value of the integrand evaluated at its peak (i.e., for the value of the background \(\bar{B}\), that maximizes the integrand). This is the approach implemented by Arnaud et al. (2022) to obtain the Poisson data with Gaussian background (PGSTAT) statistic that is an option in XSPEC. This is equivalent to assuming that

\[
P(S|M, \bar{B}, \sigma) \propto \exp\left(-\left(\bar{B} + M - \frac{\sigma^2}{2}\right)\right), S = 0
\]

\[
(B + M)^S \cdot \exp\left(-\left(\bar{B} + M + \frac{(\bar{B} - \bar{B})^2}{2\sigma^2}\right)\right), S > 0.
\]

Its logarithmic form can be written as

\[
\ln P(S|M, \bar{B}, \sigma) \propto -\left(\bar{B} + M - \frac{\sigma^2}{2}\right), S = 0
\]

\[
S \cdot \ln(\bar{B} + M) - \left(\bar{B} + M + \frac{(\bar{B} - \bar{B})^2}{2\sigma^2}\right), S > 0
\]

where in the \(S > 0\) case,

\[
\bar{B} = \frac{1}{2}(\bar{B} - M - \sigma^2)
\]

\[
+ \sqrt{(M + \sigma^2 - \bar{B})^2 - 4(M\sigma^2 - S\sigma^2 - B\bar{B})},
\]

where \(\bar{B}\) maximizes the integrand evaluated at its peak for given observational data \(S\).

Then, a logarithmic PGSTAT profile likelihood for spectral template localization can be constructed from
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Equations (A6)–(A8):

\[ \ln \mathcal{L}_{PG} = \sum_{j=1}^{n} \left( \hat{b}_j + f_i \cdot m_{ij} - \frac{\sigma_j^2}{2} \right), \quad S = 0 \]

\[ s_j \cdot \ln(\hat{b}_j + f_i \cdot m_{ij}) = \left( \hat{b}_j + f_i \cdot m_{ij} + \frac{(\hat{b}_j - \bar{b}_j)^2}{2\sigma_j^2} \right), \quad S > 0, \]

(A9)

and

\[ \hat{b}_j = \frac{1}{2}(\hat{b}_j - f_i \cdot m_{ij} - \sigma_j^2) + \frac{1}{2}(f_i \cdot m_{ij} + \sigma_j^2 - \hat{b}_j)^2 - 4(f_i \cdot m_{ij} \cdot \sigma_j^2 - s_j \cdot \sigma_j^2 - \hat{b}_j \cdot f_i \cdot m_{ij}), \]

(A10)

where \( s_j \) is the total number of observed counts in detector \( j \), \( n \) is the total number of detectors, and \( \hat{b}_j \) is the expectation value of the background.

During the localization process with the fixed templates, \( f_i \) could be derived from the maximization for each direction (\( i \)), thus the burst position (i.e., direction \( i \)) is the only parameter of interest, whose prior could be assumed to be uniform over the whole celestial sphere: \( P_{\text{prior}}(i) = 1/N \), where \( N \) is the total number of HEALPix pixels of the whole sky. With the parameter prior and likelihood as shown in Equation (A3), we derive the location results (location center, probability map, and credible region) through the Bayesian inference.

We summarize this Bayesian localization method based on PGSTAT profile likelihood as follows:

Step 1: For each incident direction \( i \), maximize the likelihood \((\ln \mathcal{L}(i))\) by adjusting the normalization factor \( f_i \). The mathematical form of likelihood and logarithmic likelihood is equivalent to the maximization process.

Step 2: Calculate the posterior probability through Bayesian inference. Thus the posterior distribution, \( P(i|s) \), could be derived from the prior probability \( P_{\text{prior}}(i) \), the conditional probability for a given direction \( i \) to obtain the observed counts \( s \), and evidence \( P(s) \):

\[ P(i|s) = \frac{P_{\text{prior}}(i) \cdot P(s|i)}{P(s)} = \frac{1}{N} \cdot \frac{P(s|i)}{\sum_i \frac{1}{N} \cdot P(s|i')} = \frac{P(s|i)}{\sum_i P(s|i')}. \]

(A11)

By substituting the conditional probability \( P(s|i) \) by the likelihood \((\mathcal{L}(i))\), one can get the posterior probability for each direction \( i \), which is also the localization probability map:

\[ P(i) = \frac{\mathcal{L}(i)}{\sum_i \mathcal{L}(i')} \]

(A12)

Step 3: For simplicity, we take the direction with the maximum \( P(i) \) as the location center and the Bayesian credible region with \( N \% \) HPD as the \( N \% \) confidence interval of the burst position.

**Appendix B**

**The Bayesian Approach to Estimating Systematic Uncertainties**

A Bayesian approach is usually adopted to estimate the systematic uncertainties of localization (Briggs et al. 1999; Connaughton et al. 2015; Wang et al. 2021b). The models are based on the Fisher probability density function, which has been called the Gaussian distribution on the sphere (Fisher et al. 1993):

\[ P(\alpha)d\Omega = \frac{k \cdot \exp(k \cdot \cos \alpha)}{2\pi \cdot (\exp(k) - \exp(-k))}d\Omega, \]

(B1)

where \( \alpha \) is the angular separation between the measured and true positions, \( k \) is termed the concentration parameter, and \( d\Omega \) is the solid angle. Considering \( \sigma_{\text{TOTAL}} \) to be the radius of the circle containing 68.27% of the total probability, integrating Equation (B1) relates \( k \) and \( \sigma_{\text{TOTAL}} \) in radians (Briggs et al. 1999):

\[ k = \frac{1}{(0.66 \cdot \sigma_{\text{TOTAL}})^2}. \]

(B2)

The localization uncertainties \((\sigma_{\text{TOTAL}})\) consist of statistical uncertainties \((\sigma_{\text{STAT}})\) and systematic uncertainties \((\sigma_{\text{SYS}})\):

\[ \sigma_{\text{TOTAL}}^2 = \sigma_{\text{STAT}}^2 + \sigma_{\text{SYS}}^2. \]

(B3)

During the calculation, the solid angle \( d\Omega \) is replaced with angular separation \( d\alpha \) (\( d\Omega = 2\pi \sin \alpha d\alpha \)):

\[ P(\alpha)d\alpha = \frac{k \cdot \exp(k \cdot \cos \alpha) \cdot 2\pi \sin \alpha}{2\pi \cdot (\exp(k) - \exp(-k))}d\alpha. \]

(B4)

Then integrating \( d\alpha \):

\[ \int P(\alpha)d\alpha = \int \frac{k \cdot \exp(k \cdot \cos \alpha) \cdot 2\pi \sin \alpha}{2\pi \cdot (\exp(k) - \exp(-k))}d\alpha, \]

(B5)

and

\[ P(\alpha) = \frac{k \cdot \exp(-1 + \cos \alpha) \cdot \sin \alpha}{1 - \exp(-2k)}. \]

(B6)

For each burst, the probability \( P_{q,r} \) could be calculated from Equations (B2), (B3), and (B6) for a given assumed systematic error \( r \), where \( q = 1, 2, ..., Q \) (\( Q \) is the number of bursts). Then successively multiplying \( P_{q,r} \) and normalizing:

\[ P_{n}' = \prod_{q} P_{q,r}(\alpha), \]

(B7)

and

\[ P_n = \frac{P_n'}{\sum_r P_n'}. \]

(B8)

The normalized probability \( P_n \) represents the systematic error's probability distribution.
Appendix C
GECAM Localization Results for GRBs, SGRs, and SFLs

The GECAM localization result for 22 bright GRBs, SGRs, and SFLs are presented in Figure 10 (one is shown in Figures 2 and 3), as listed in Table 6.

Figure 10. GECAM localization results of GRB 210121A. (a) The light curve of GRD # 03 high gain, which contains the majority of net (burst) counts. (b) The RFD spectral fitting result. (c) The APR spectral fitting result. The location credible region of (d) FIX, (e) RFD, and (f) APR localization. The captions are the same as Figure 2.

(The complete figure set (22 images) is available.)

Appendix D
GECAM Localization Results for TGFs

The GECAM localization result of two bright TGFs are presented in Figures 11 and 12 (one is shown in Figure 7).
Figure 11. GECAM localization results of TGF 210520. (a) The light curve and time–energy scatter plot of all 25 detectors. (b) The light curve and time–energy scatter plot of GRD #16, which contains the majority of net counts in the discovery bin. (c) Sky map of the location. The captions are the same as for Figure 7.

Figure 12. GECAM localization results of TGF 220112. (a) The light curve and time–energy scatter plot of all 25 detectors. (b) The light curve and time–energy scatter plot of GRD #25, which contains the majority of net counts in the discovery bin. (c) Sky map of the location. The captions are the same as for Figure 7.
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