Measurement and Analysis of Deformation Behavior of Polyethylene Tube Subjected to Linear Loading and Unloading Stress Paths with Large Strain

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Keywords: biaxial test, tensile stresses, plastic, polymer, polyethylene tube, deformation behavior.

Abstract. A material testing apparatus for measuring the biaxial deformation behavior of a polymer tube has been developed to quantitatively evaluate the deformation behavior of polymeric materials. The testing apparatus can apply axial force and internal pressure to a tubular specimen. A noncontact strain measurement system was also developed, and the biaxial strain components and the radius of curvature in the axial direction of the bulging specimen are continuously measured to control the stress path applied to the specimen. Polyethylene tube with an outer diameter of 17 mm and a thickness of 2 mm are used as a test sample. The tubular specimens were subjected to linear stress paths with stress ratios of $\sigma_\phi: \sigma_\theta = \text{1:0, 4:1, 2:1, 4:3, 1:1, 3:4, 1:2, 1:4, and 0:1}$, where $\sigma_\phi$ and $\sigma_\theta$ are the axial and circumferential stress components, respectively, applied to the central area of the bulging specimen. Loading and unloading tests were performed to determine the biaxial true stress-logarithmic plastic strain curves. The strain rate was $1 \times 10^{-3}\text{s}^{-1}$. From these test results, contours of plastic work and the directions of the plastic strain rates were measured to identify a proper material model for the test sample using the Yld2000-2d yield function (Barlat et al., 2003).

Introduction

Polyethylene (PE) tubes have been used in water distribution lines and have built up a lot of trust and achievements such as earthquake resistance of lifelines [1]. However, it may receive a sudden external force during use, and its safety is required to be evaluated in advance by numerical simulations. In order to guarantee the accuracy of the numerical simulation, it is necessary to guarantee the accuracy of the material model used in the calculation [2]. Therefore, the deformation behavior of PE tube is required to be measured under multiaxial stress states [3]. The deformation behavior of resin materials is affected by strain rate; therefore, it should be measured and evaluated with the strain rate being kept constant.

The authors have developed a multiaxial tube expansion test (MTET) apparatus that can apply precisely controlled linear stress paths to a circular tube test piece made of resin. We could measure the biaxial stress-strain curves of a PE test piece subjected to many linear stress paths in a large strain range exceeding 0.71 [4]. In order to evaluate the plastic deformation characteristics of the test sample, it was necessary to extract the plastic strain by subtracting the elastic strain from the measured total strain. However, it could not accurately determine the elastic strain as it was difficult to identify the Young’s modulus of the test sample in the elastic range.

In this study, we developed a new experimental method for determining elastic strain. We applied a linear stress path to a PE circular tube test piece, then unloaded it with the same stress ratio as the loading, to measure the permanent (plastic) strain. Then, the relationship between the true stress (the stress value immediately before the unloading) and the logarithmic plastic strain were determined. Moreover, from the data of the true stress—logarithmic plastic strain curve, we measured the contours...
of plastic work and the directions of the plastic strain rates to identify a proper material model (yield function) for reproducing these measured data.

**Experimental Methods**

The test material was PE extruded circular tube with a nominal wall thickness $t_0$ of 2 mm, an outer diameter $D_0$ of 17 mm, and a length of 110 mm, see Figure 1. Hereinafter, the axial and circumferential directions of the specimen are denoted as $\phi$ and $\theta$, respectively.

For the biaxial stress test of the test sample the multiaxial tube expansion (MTET) apparatus developed in [4] was used. Figure 2 shows the schematic diagram of the testing machine. Axial force $T$ and internal pressure $P$ are applied to the specimen with both ends being clamped. The true stress components, $\sigma_\phi$ and $\sigma_\theta$, are generated in the center of the bulging specimen. Water was used as the working fluid. The pump can discharge a specified amount of pulsating current regardless of the discharge pressure within the pressure range of $P \leq 20$ MPa.

Figure 3 shows the apparatus for measuring the outer diameter of the bulging specimen. The frame shown in the figure is fixed at the center of the testing machine. It has an area camera (KEYENCE CV-HV035M) on the top, a pair of transmission type laser displacement meter (IG-028 manufactured by KEYENCE), and three reflective laser displacement meters (KEYENCE IL-065) placed every 120° in the $\theta$ direction. The transmission type laser displacement meter measures the initial outer diameter of the specimen attached to the testing machine. As the outer diameter of the specimen increases, it eventually exceeds the measurable range of the displacement meter. Therefore, the outer diameter, $D$, during the test is measured using the reflective laser displacement meters.

**Fig. 1** Geometry of specimen for the multiaxial tube expansion test. (Dimensions are in mm.)

The outer diameter is 17mm. Inner diameter is 13mm.

**Fig. 2** Schematic of multiaxial tube expansion testing method.
Fig. 3 Noncontact diameter measurement apparatus.

Nine linear stress paths, $\sigma_\phi: \sigma_\theta = 1:0, 4:1, 2:1, 4:3, 1:1, 3:4, 1:2, 1:4, \text{and } 0:1$, were applied to the specimen, and the $\sigma_\phi - \varepsilon_\phi$ and $\sigma_\theta - \varepsilon_\theta$ curves were measured. Two types of tests were performed: monotonic tensile (MT) test and loading and unloading (LU) test. LU tests were performed to measure the biaxial true stress-logarithmic plastic strain (SS) curves. In the LU test, the maximum principal stress $\sigma_1$ was increased by 1 MPa and unloaded each time. The strain rate was controlled to be kept constant at approximately $1 \times 10^{-3} \text{s}^{-1}$.

Experimental Results

Figure 4 shows the variation of the strain rate with time during the LU tests with $\sigma_\phi: \sigma_\theta = 0:1$ and 1:0. It was confirmed that the strain rate was successfully controlled to be approximately $10^{-3}/\text{s}$ during loading for both tests.

![Fig. 4 Maximum principal strain rate for $\sigma_\phi: \sigma_\theta = 1:0$ and 0:1](image)

Figure 5 compares the measured stress paths with the reference. The number of tests per stress path was two, and the one that could measure the stress-strain curve up to a larger strain was adopted as the experimental value. For $\sigma_\phi: \sigma_\theta = 1:2$ and 1:4 the maximum principal stress of $\sigma_\theta = 18$ and 33 MPa could be measured, respectively, while for $\sigma_\phi: \sigma_\theta = 1:1, 4:3,$ and 2:1 the maximum stress achieved was limited to $\sigma_\phi = 12$ MPa as the specimen fractured near the clamping jigs.
**Fig. 5** Reference and experimental stress paths.

**Figure 6** shows the true stress – logarithmic strain (ss) curves measured for ML and LU tests. The thick solid lines are for ML tests and the thin solid lines are for LU tests. It is interesting to note that the ss curves obtained from the ML tests envelops those measured form the LU tests.

![Graph showing true stress vs. logarithmic strain for ML and LU tests](image)

**(a)** $\sigma_{\phi}:\sigma_{\theta} = 1:0$

**(b)** $\sigma_{\phi}:\sigma_{\theta} = 2:1$

**(c)** $\sigma_{\phi}:\sigma_{\theta} = 1:1$

**(d)** $\sigma_{\phi}:\sigma_{\theta} = 1:2$

**(e)** $\sigma_{\phi}:\sigma_{\theta} = 0:1$

**Fig. 6** True stress – logarithmic strain curves measured for monotonic and loading-unloading tests.

From the LU curve, the relationship between the residual (plastic) strain after unloading, $\varepsilon^P$, and the associated true stress, $\sigma$, immediately before the unloading was measured as the $\sigma - \varepsilon^P$ curves; the discrete measurement points were linearly interpolated to obtain a continuous true stress-logarithmic strain diagram. For the $\sigma_0 - \varepsilon_0^P$ diagram measured from the uniaxial tensile test with $\sigma_{\phi}:\sigma_{\theta} = 1:0$, the plastic work per unit volume, $W_0$, consumed until the specific value of $\varepsilon_0^P$ was reached was measured. For other stress paths, the true stress components, $\sigma_{\phi}$ and $\sigma_{\theta}$, at an instant when the same amount of plastic work as $W_0$ was consumed was measured and plotted in the principal stress space. The group of stress points associated with $W_0$ give a contour of plastic work associated with $\varepsilon_0^P$. **Figure 7(a)** shows the contours of plastic work. We succeeded in measuring the
work contours up to $\varepsilon_0^P = 0.07$ for the nine stress paths. Figure 7(b) shows the work contours, the stress values of which were normalized by $\sigma_0$ associated with each work contour. The flow stresses are larger for $\sigma_\phi$: $\sigma_\theta = 3:4$ to 1:4 than those for $\sigma_\phi$: $\sigma_\theta = 4:1$ to 4:3; a significant anisotropy was observed.

![Graph](image)

**Fig. 7** Measured results of the stress points forming the contours of plastic work for different values of $\varepsilon_0^P$. (a) Raw data. (b) Normalized by the uniaxial axial tensile flow stress, $\sigma_0$, belonging to the same group of each work contour.

### Identification of the Yield Function

A material model (yield function) was identified using the Yld2000-2d yield function [5] (see Appendix) with the experimental data of the work contours and the directions of the plastic strain rates for $\varepsilon_0^P = 0.07$. The exponent $M$ and the eight coefficients $\alpha_i$ ($i = 1 \sim 8$) of the Yld2000-2d yield function were determined so that the objective function $F$ shown below was minimized [6]

$$
F = \sum_{j=1}^{N} w_{j,\sigma}(l_{j,C} - l_{j,M})^2 + \sum_{j=1}^{N} w_{j,\beta}(\beta_{j,C} - \beta_{j,M})^2.
$$

Here, $N (= 9)$ is the number of stress points forming a work contour, $l_{j,C}$ is the distance between the origin of the principal stress space and the calculated yield locus along the stress path to the $j^{th}$ stress point, $l_{j,M}$ is the distance between the origin of the principal stress space and the $j^{th}$ stress point, $\beta_{j,C}$ is the direction of the plastic strain rate calculated using the yield function and the associated flow rule for the $j$-th stress path and $\beta_{j,M}$ is the direction of the plastic strain rate measured for the $j$-th stress path, see Figure 8. $w_{j,\sigma}$ and $w_{j,\beta}$ are weight coefficients. By changing the values of $w_{j,\sigma}$ and $w_{j,\beta}$, three identification methods were performed: (A) the reproducibility of the contours of plastic work was emphasized, (B) the reproducibility of both the contours of plastic work and the directions of the plastic strain rates, $\beta$ (degree), was emphasized, and (C) the reproducibility of $\beta$ (degree) was emphasized. Table 1 shows the values of $w_{j,\sigma}$ and $w_{j,\beta}$ used in the three types of identification.
Fig. 8 Schematic illustrations for identifying the parameter of the Yld2000-2d yield function.

Table 1 Values of $w_{\sigma,\varphi}$ and $w_{\beta,\varphi}$.

| Stress ratio $\sigma_x : \sigma_y$ | Method A | Method B | Method C |
|----------------------------------|----------|----------|----------|
| $w_{\sigma,\varphi}$ | $w_{\beta,\varphi}$ | $w_{\sigma,\varphi}$ | $w_{\beta,\varphi}$ | $w_{\sigma,\varphi}$ | $w_{\beta,\varphi}$ |
| 1:0 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 4:1 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 2:1 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 4:3 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 1:1 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 3:4 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 1:2 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 1:4 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |
| 0:1 | 1 | 0.1 | 1 | 0.4 | 1 | 1 |

Fig. 9 Comparison of the experimental data at $\varepsilon^p_0 = 0.07$ for (a) the stress points forming the contour of plastic work and (b) the directions of the plastic strain rates with those calculated using the von Mises and the Yld2000-2d yield functions.

Figure 9 compares the experimental data of (a) the contour of plastic work and (b) the directions of the plastic strain rates with those calculated using the von Mises and the Yld2000-2d yield functions. The experimental data are for $\varepsilon^p_0 = 0.07$. Any method could express the anisotropy of the material. Method A has a better agreement with the work contour than the others. Method B does not reproduce the stress point for $\sigma_\varphi : \sigma_\theta = 1:0$, but it fits well with the other stress points forming the work contour. Method C has the best agreement with the directions of the plastic strain rates while it still has a large deviation from the experimental data of the work contour for a range of $45^\circ < \varphi < 90^\circ$. To identify a material model that can accurately reproduce both the work contour and the directions of plastic strain rates is a challenge for future work.
Conclusions

The MTET was applied to the PE circular tube. The work hardening behavior and the directions of the plastic strain rates under biaxial tension were precisely measured to identify a proper material model. The following findings were obtained.

1) A novel material testing method using LU procedure was developed to measure the true stress–logarithmic plastic strain curves of the test sample subjected to biaxial stress states.

2) The contours of plastic work showed that the flow stresses are larger for $\sigma_\phi: \sigma_\theta = 3:4$ to $1:4$ than those for $\sigma_\phi: \sigma_\theta = 4:1$ to $4:3$; a significant anisotropy was observed.

3) To identify a material model that can accurately reproduce both work contours and the directions of plastic strain rates is a challenge for future work.

Appendix

Barlat and coworkers generalized the isotropic, non-quadratic yield function proposed by Hershey [7] to propose an anisotropic, non-quadratic yield function, $\Phi$, for the plane stress case [5]:

$$\Phi \equiv \frac{1}{3^M} \left[ \left( (2\alpha_1 + \alpha_2)\sigma_\phi + (-\alpha_1 - 2\alpha_2)\sigma_\theta \right)^M + \left( (2\alpha_3 - 2\alpha_4)\sigma_\phi + (-\alpha_3 + 4\alpha_4)\sigma_\theta \right)^M + \left( (4\alpha_5 - \alpha_6)\sigma_\phi + (-2\alpha_5 + 2\alpha_6)\sigma_\theta \right)^M \right] = 2(\sigma_\text{R})^M \quad (A1)$$

Here, $\alpha_i \ (i = 1 - 6)$ is material parameters and $\sigma_\text{R}$ is a reference flow stress, which depends on a measure of the accumulated plastic strain, $\varepsilon_0^p$. It is noted that $\alpha_7$ and $\alpha_8$ were not determined in this study as the material tests were performed in the principal stress space ($\sigma_\phi\theta = 0$).

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