Duality, Compactification, and $e^{-1/\lambda}$ Effects in the Heterotic String Theory

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Two classes of stringy instanton effects, stronger than standard field theory instantons, are identified in the heterotic string theory. These contributions are established using type IIA/heterotic and type I/heterotic dualities. They provide examples for the heterotic case of the effects predicted by Shenker based on the large-order behavior of perturbation theory. The corrections vanish as the radius of the compactification goes to infinity. For appropriate amplitudes, they are computable worldsheet or worldline instanton effects on the dual side. Some potential applications are discussed.
1. Introduction

Nonperturbative effects in string theory have come under significantly better control in recent years. This progress has gone hand in hand with new understanding of the degrees of freedom in the theory. In particular, D-branes realize concretely the “stringy instantons” predicted by Shenker [1] in an expansion about weakly coupled type II and type I string theories [2,3,4,5]. Shenker argued that all closed string theories should exhibit non-perturbative effects which fall off like \( e^{-1/\lambda} \) at weak coupling \( \lambda \) in addition to those understood from the low-energy effective field theory which fall off like \( e^{-1/\lambda^2} \). Open string theories automatically have such behavior. Type II string theories turn out to contain open strings because of the presence of D-branes in the nonperturbative spectrum, and their tension indeed goes like \( 1/\lambda_{II}^2 \).

There is one closed string theory left, for which the argument in [1] applies but D-branes do not appear: the heterotic string theory. From [1] we would expect \( e^{-1/\lambda_{het}} \) effects even in the highly symmetric decompactified ten-dimensional theory. In this note, we will find effects which fall off like \( e^{-f(M)/\lambda_{het}} \), where \( f(M) \) is a function of moduli which diverges as the size of the compact space goes to infinity.

We find these effects in two different contexts, making use of heterotic/Type IIA dualities and heterotic/type I duality. In each case the basic idea is very simple. In the four-dimensional heterotic/type IIA dual pairs, the 4d heterotic coupling \( \lambda_{h,4}^2/\alpha' \) maps to \( \alpha'/A_{IIA} \), where \( A_{IIA} \) is the area of a nontrivial 2-cycle \( C \) in the compactification on the IIA side. In a by now familiar fashion, type IIA worldsheet instanton effects, which fall off like \( e^{-A_{IIA}/\alpha'} \), map to field theory instanton effects on the heterotic side, which fall off like \( e^{-1/\lambda_{h,4}^2} \). With nontrivial \( \pi_1 \) on the type IIA side, there can be additional effects which fall off like \( e^{-R_{IIA}/\sqrt{\alpha'}} \), where \( R_{IIA} \) is the length of a nontrivial 1-cycle on \( C \). These map to stringy instantons \( e^{-h(M)/\lambda_{h,4}} \) on the heterotic side, for an appropriate function of moduli \( h(M) \).

A more general origin for stringy instanton effects comes from considering the heterotic/type I duality. In ten dimensions, the map between the couplings \( \lambda_{h,10}, \lambda_{I,10} \) and between the metrics \( g_{MN}^h, g_{MN}^I \) is simple: \( \lambda_h \leftrightarrow \lambda_{I,10}^{-1} \) and \( g_{MN}^h \leftrightarrow \lambda_h g_{MN}^I \) [3]. Upon compactification, this indicates that worldsheet instantons on one side map to effects that fall off like \( e^{-A/\lambda_{h,4}} \) on the other side, where \( A \) is the area of the 2-cycle about which the worldsheet wraps. Such worldsheet instanton effects are ubiquitous upon compactification.
Considering worldline instantons in this context leads at least naively to even stronger non-perturbative effects. This will be discussed in §2.4; the interpretation is not clear in terms of large-order behavior of perturbation theory.

In both cases, the heterotic stringy non-perturbative effects for appropriate quantities can be computed in perturbation theory on the dual side. For compactifications with extended supersymmetry \( (N \geq 2 \text{ in the } 4d \text{ sense}) \), the effects can only appear in higher-derivative (non-holomorphic) terms in the effective Lagrangian. In the more generic case of \( 4d \, N = 1 \) supersymmetry, the effects can be seen in the Kähler potential. There is in fact a restricted set of quantities for which the non-perturbative effects can be computed reliably, as will be discussed in §2.

One hope for the new understanding of non-perturbative effects is that they can help with various longstanding puzzles concerning the connection to observed low-energy physics. In particular, duality between worldsheet instantons and spacetime instantons has proved useful in computing quantities (i.e. the spacetime superpotential) relevant for lifting flat directions in moduli space and breaking supersymmetry \cite{6,7,8,9,10,11,12}. The known examples of type IIA/heterotic dual pairs include a set \cite{6} which undergo gaugino condensation in a hidden sector, potentially breaking supersymmetry. These examples are rather special, in that they arise as orientifolds of known N=2 dual pairs, as in \cite{13}. However, as will be reviewed below, the examples in \cite{13} have the feature that the type IIA side has nontrivial \( \pi_1 \), so the stringy instantons will appear. As discussed in \cite{14}, it is difficult to make this scenario of supersymmetry breaking work, but the situation is improved given the possibility for stringy instanton effects in the Kähler potential (for discussion of some models see for example \cite{15} and referenced therein). In the type I/heterotic case, a nontrivial sector of the stringy instanton effects can be computed by making use of the inheritance in the type I theory of worldsheet instanton sums computed in the type IIB theory on the same space.

The paper is organized as follows. In §2, we explain the origin of the stringy instanton effects from duality in the two cases, discuss the class of amplitudes for which the analysis applies, and raise a puzzle that appears in the heterotic/type I context. In §3 we discuss a few illustrative applications.
2. Duality and Stringy Non-Perturbative Effects

2.1. Type I Worldsheet Instantons and $e^{-f(M)/\lambda_h}$ Effects

Consider the type I string and the heterotic string compactified on a D-dimensional manifold $M$. As discussed in [3], the 10d metrics $g^I_{MN}$ and $g^h_{MN}$ and couplings $\lambda_I$ and $\lambda_h$ are related by

$$g^h_{MN} = \lambda_h g^I_{MN} \quad \lambda_h = \frac{1}{\lambda_I}. \quad (2.1)$$

In $10-D$ dimensions, each string theory has worldsheet instanton effects coming from the sector in the 2d path integral where the string worldsheet wraps around non-trivial 2-cycles. In the heterotic string, on a 2-cycle of area $A_h$, these fall off like $e^{-A_h/\alpha'}$. This maps to effects in the type I string which fall off like $e^{-A_I/\alpha'\lambda_I}$. The interpretation is simple (and more or less tautologous): the heterotic string is a Dirichlet 1-brane in the type I string, and the heterotic worldsheet instanton effects come from wrapping the D-1-brane worldsheet, with tension $1/\lambda_I$, around the cycle of area $A_I$, as in [3].

In 4d (i.e. with $D = 6$), there are field-theoretic instanton effects in the heterotic string which fall off like $e^{-1/\lambda_{h,4}^2}$, where $\lambda_{h,4}$ is the 4d heterotic string coupling. These map to effects on the type I side which fall off like $e^{-V_I/\alpha'\lambda_I}$, where $V_I$ is the volume of $M$ measured in the type I metric. This also has the simple interpretation of arising from a wrapped Dirichlet 5-brane, in accord with the identification of this object with the gauge instanton 5-brane in the heterotic theory [16].

We can now go the other way. On the type I side, there are worldsheet instanton effects which arise from nontrivial maps of the worldsheet into the target space. For 2-cycles with area $A_I$, we find effects which fall off like

$$\delta L \sim e^{-A_I/\alpha'}. \quad (2.2)$$

In terms of heterotic variables, this is

$$\delta L \sim e^{-A_h/\alpha'}. \quad (2.3)$$

So each type I worldsheet instanton effect is a stringy heterotic instanton effect. Notice that we obtain such effects without a stable solitonic type I string from the heterotic point of view.

In four-dimensional terms, the effects (2.3) cannot appear in the superpotential (or any holomorphic quantity). This agrees with the type I side due to the special properties
of the Ramond-Ramond anti-symmetric tensor field $B^I_{MN}$. This field cannot appear in
non-derivative couplings, so type I worldsheet instantons (2.2) cannot correct the superpotential. They can, however, affect non-holomorphic quantities such as the Kähler potential (and they do as we will review in §3). We will argue that there is a set of quantities for which the dual description provides a reliable computation of the nonperturbative effects in §2.3.

2.2. Type IIA Worldline Instantons and Stringy Heterotic Instantons

Consider a dual string pair with the property that the inverse string coupling on one
side maps to a radius on the other side. For simplicity let us consider situations where
the compactification manifold preserves at least one covariantly constant spinor. Then in
terms of 4d $N = 1$ superfields, we have

$$S = \frac{1}{\lambda_4^2} + i\theta \leftrightarrow T$$

where $\lambda_4$ is the 4d string coupling, $\theta$ is the model-independent axion, and $T$ is a chiral
superfield made from a Kähler modulus of the compactification. As will be reviewed in the
next section, this is typical of four-dimensional type IIA-Heterotic dual pairs with $N = 1,
N = 2$, and $N = 4$ supersymmetry. $T$ measures the size of a holomorphic curve $C$ in the
compactification manifold, so that $Re(T) = \int_C J$, where $J$ is the Kähler form. In such a
situation, worldsheet instanton effects, which fall off like

$$\delta \mathcal{L} \sim e^{-T},$$

map to spacetime instanton effects, which fall off like

$$\delta \mathcal{L} \sim e^{-S} \sim e^{-1/\lambda_4^2}.$$  (2.5)

Now consider the situation where the curve $C$ has nontrivial fundamental group,$\pi_1 \neq 0$. That is, consider the case where there is a nontrivial 1-cycle on $C$ of length $L$.
Then there will be worldsheet nonperturbative effects which fall off like

$$\delta \mathcal{L} \sim e^{-L}.$$  (2.6)

In one channel this can be interpreted as arising from a wrapped string, which is stable
because of the existence of a nontrivial cycle of length $L$ (not because of any BPS condition).
Let $L = h(M) \sqrt{Re(T)}$, where $h(M)$ is a function of moduli, independent of $T$. The dual then has the anticipated stringy behavior

$$\delta \mathcal{L} \sim e^{-h(M)/\lambda^4}. \tag{2.8}$$

It is interesting that particle-like behavior (worldline instantons) maps to stringy behavior ($e^{-h(M)/\lambda^4}$ effects).

Notice that these stringy instantons disappear in the decompactified 10-dimensional theory for fixed 10-dimensional coupling $\lambda_{10}$, since

$$\frac{1}{\lambda^4} = \frac{\sqrt{V}}{\lambda_{10}}, \tag{2.9}$$

where $V$ is the volume of the compactification. So the effects (2.8) vanish as $V \to \infty$.

Let us discuss the effects (2.7) more explicitly. In a first quantized point particle framework, there is a “worldline instanton” contribution

$$\delta_m \mathcal{L} \sim \int [dX(\gamma)] \exp(-m \int_{\gamma} ds) \tag{2.10}$$

for each state of mass $m$. This $e^{-Lm}$ behavior (for a loop of length $L$) can be extracted from the 2d sigma model path integral. Consider for example the computation of the one-loop partition function for strings propagating on a circle of radius $R \ [17] \ [18]$. The partition function is

$$Z = \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2} Tr(q^{L_0} \bar{q}^{\bar{L}_0}). \tag{2.11}$$

Here $\tau$ is the modular parameter for the worldsheet torus, $\mathcal{F}$ is the fundamental domain of the torus moduli space, $q = e^{2\pi i \tau}$, and $L_0$, $\bar{L}_0$ are the left and right-moving Hamiltonians on the worldsheet. With supersymmetry the vacuum amplitude vanishes upon summing over the spin structures. Inserting vertex operators into the correlation function will in general prevent this cancellation, so let us concentrate on one sector, say the (R,R) sector. Then the normal ordering constant vanishes and

$$L_0 = \frac{p^2}{2} + \frac{p_L^2}{2} + N_L = \frac{p^2}{2} + \frac{1}{2} (\frac{k}{2R} + nR)^2 + N_L \tag{2.12}$$

$$\bar{L}_0 = \frac{p^2}{2} + \frac{p_R^2}{2} + N_R = \frac{p^2}{2} + \frac{1}{2} (\frac{k}{2R} - nR)^2 + N_R. \tag{2.13}$$

Here $N_L, N_R$ are the oscillator contributions, $p$ is the spacetime momentum, and $k, n$ are the momentum and winding numbers on the circle. After trading the sum over $n$
for extending the integral over the whole strip \(-1/2 < \tau_1 < 1/2, \tau_2 > 0\), and Poisson resumming the momentum mode sum, we find terms of the form

\[
\int_0^\infty \frac{d\tau_2}{\tau_2^6} e^{-\left(m^2 \tau_2 + \frac{R^2}{\tau_2}\right)}
\]

(2.14)

where \(m^2\) is the oscillator number \(N\) on the string worldsheet. The saddle point for the \(\tau_2\) integral is at \(\tau_2 = R/m\), leading to an effect which falls off like

\[
\delta\mathcal{L} \sim e^{-mR}.
\]

(2.15)

In the 4\(d\) context with the relation (2.4), this leads to a sum of terms in the dual theory

\[
\delta\mathcal{L} \sim \sum_N e^{-\frac{\sqrt{N}}{\lambda} a_N},
\]

(2.16)

where \(a_N\) are possibly moduli-dependent coefficients.

2.3. Regime of Validity

In our analysis so far, we have simply mapped the couplings and radii from one member of a dual pair to the other; we then identified worldsheet or worldline instanton effects on one side to effects nonperturbative in the coupling on the other side. In order for these computations to be valid, we need (a subset of) each theory to be weakly coupled in an appropriate sense, which we will now identify.

We have a compactification down to \(d\) dimensions on both sides. The \(d\)-dimensional couplings \(\lambda_d\) and \(\lambda'_d\) can both be chosen to be weak in appropriate circumstances \[3\]. But the ten-dimensional couplings \(\lambda\) and \(\lambda'\) cannot both be weak. In the low-energy \(d\)-dimensional field theory, there are loop effects which are suppressed by powers of \(\lambda_d = \lambda/\sqrt{V}\) (here \(V\) is the volume of the compactification manifold). There are in general additional loop effects which occur with powers of the ten-dimensional coupling \(\lambda\) and no volume suppression. These are the loop effects which survive in the ten-dimensional limit. At strong \(10d\) coupling, these effects are large and cannot be computed on both sides in a controlled way. So in order to compute reliably, we must restrict to quantities which do not receive string loop corrections in the decompactified \(10d\) limit. The set of such quantities includes not only holomorphic objects in the \(d\)-dimensional theory, but also terms in the effective action which receive loop corrections only due to the supersymmetry breaking provided by the compactification. For example, the kinetic terms for the massless fields
coming from the 10d gravity multiplet in 4d $N = 1$ compactifications are in this class. They are not protected from loop corrections by holomorphy in 4d, but in the 10d limit they reduce to kinetic terms for the gravity multiplet, which are fixed by 10d $N = 1$ supergravity. On the other hand, an arbitrary scattering amplitude is not protected from loop effects in any dimension. So although the full theory is not under control from both sides of the dual pair, amplitudes which are protected in the 10d limit can be computed reliably.

2.4. Type I/Heterotic Duality Revisited: A Puzzle

Considering worldline instanton effects of the type discussed in §2.3 in the type I/heterotic context suggests even stronger instanton effects on both sides. For this duality, we have so far concentrated on effects which fall off like $e^{-\frac{\text{Area}}{\alpha'}}$ on one side or the other. Consider the special case that the compactification manifold $M$ has nontrivial fundamental group. Then as in §2.2, at the perturbative level on each side there are effects which fall off like $e^{-\frac{R}{\sqrt{\alpha'}}}$, where $R$ is the radius of the nontrivial 1-cycle. These effects map to effects on the other side which fall off like $e^{-\frac{R'}{\sqrt{\alpha'(\lambda')^2}}}$. These are stronger at weak coupling than the effects predicted in [1]. Perhaps this is related to the fact that there is a factor of $R$ in the exponent, so that the effect is suppressed by large radius as well as by weak coupling. It would be very interesting to understand the interpretation of these effects (if they do not always cancel somehow in the reliably computable amplitudes) in terms of the large-order behavior of perturbation theory.

3. Some Applications

3.1. heterotic/type I

The ten-dimensional heterotic/type I duality holds upon compactification on a smooth D-dimensional manifold $M$. One can consider the effects of §2.1 on anything from the highly symmetric $T^D$ to a generic Calabi-Yau (or even a manifold of exceptional holonomy). Of particular interest are Calabi-Yau three-folds, for which $4d \ N = 1$ supersymmetry is preserved perturbatively in $\alpha'$ and $\lambda$. Worldsheet instanton effects coming from sphere amplitudes in the type I theory are inherited from those in the type IIB string theory. These can be computed exactly for some terms in the effective action using mirror symmetry, as in [19]. One such term is the Kähler potential for moduli fields of the manifold. (In the
type I context, the gauge bundle moduli as well as the charged fields are open strings and do not couple on the sphere.)

On the quintic hypersurface in \( \mathbf{CP}^4 \) for example, the worldsheet instanton sum has been computed in terms of the single Kähler modulus \( T_I \). In terms of the holomorphic prepotential \( F \) in the type IIB compactification on the Calabi-Yau, the Kähler potential for \( T_I \) is (in “special coordinates”) \[ K(T_I, \bar{T}_I) = -\log \left[ 2(F(T_I) + F(\bar{T}_I)) - (T_I - \bar{T}_I)(F_{T_I} - \bar{F}_{\bar{T}_I}) \right] \] (3.1)

where \( F_{T_I} = \frac{\partial F}{\partial T_I} \). From [19] we have

\[
\frac{\partial^3 F}{\partial T_I^3} \sim 5 + \sum_{k=1}^{\infty} \frac{n_k k^3 e^{-2\pi k T_I}}{1 - e^{-2\pi k T_I}}
\] (3.2)

where \( n_k \) is the number of degree \( k \) holomorphic curves. Integrating this and plugging into (3.1) gives the Kähler potential. Substituting \( \text{Re}(T_h)/\lambda_h \) for \( T_I \) into the instanton contributions gives the type I sphere contribution to the stringy instanton sum in the Kähler potential on the heterotic side. This correspondence of course applies to all smooth Calabi-Yau compactifications. It would be interesting to construct models with dynamical supersymmetry breaking and check whether this contribution to the Kähler potential actually satisfies the conditions for stabilizing the dilaton discussed in [14,15]. It is intriguing that this contribution to the Kähler potential takes the \( N = 2 \) form in this \( N = 1 \) theory.

3.2. heterotic/type IIA

The simplest example of the type of duality discussed in §2.2 has \( N = 4 \) supersymmetry in 4d. One side of the pair is the type IIA theory on \( K3 \times T^2 \), and the other is the heterotic theory on \( T^6 \sim T^4 \times T^2 \). As explained in [3], the dilaton \( S \) gets exchanged with the \( T^2 \) Kähler modulus \( T \) under the duality as in (2.4). Furthermore the \( T^2 \) has nontrivial \( \pi_1 \). In this case, although terms like (2.14)-(2.16) appear, they cancel by holomorphy in some one-loop amplitudes (see for example the computation of threshold corrections from the untwisted sector of orbifolds in [21], and an analogous computation in \( N = 4 \) theories in [22]). It might be interesting to look in this \( N = 4 \) context for processes which satisfy the reliability criteria discussed in §2.3, but which are not protected by holomorphy in 4d.

In any case, let us proceed to the more generic models with less supersymmetry. The 4d \( N = 2 \) supersymmetric dual string pairs of [23,24] also satisfy (2.4). These examples, as
well as others discovered subsequently, consist of the type IIA string theory compactified on a \( K3 \) fibration on one side and the heterotic theory on \( K3 \times T^2 \) on the other. The \( K3 \times T^2 \) on the heterotic side can itself be thought of as a \( T^4 \) fibration, so that the whole dual pair is a fibration of the six dimensional duality between type IIA on \( K3 \) and the heterotic theory on \( T^4 \) \cite{23, 13, 26}. The base of this fibration is a \( P^1 \) whose area is given by the real part of the Kähler modulus \( T \). All of these models have \( S \leftrightarrow T \), where \( S \) is the dilaton superfield. However, since the base \( P^1 \) is simply connected, the effects discussed above will not occur.

For 4d \( N = 1 \) supersymmetric models obtained from the \( N = 2 \) dual pairs by orientifolding \cite{13, 14}, nontrivial \( \pi_1 \) is generated on the base. The orbifold group acts on the base \( P^1 \sim S^2 \) by identifying antipodal points on the sphere. This leaves \( RP^2 \), with \( \pi_1 = \mathbb{Z}_2 \). Hence for these examples, both of the conditions in §2 are satisfied, and we expect nontrivial stringy instanton effects on the heterotic side.

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References

[1] S. Shenker, talk at Cargese workshop on Random surfaces, Quantum Gravity, and Strings, France (1990).

[2] J. Polchinski, “The Combinatorics of Boundaries in String Theory”, Phys. Rev. D50 (1994) 6041.

[3] E. Witten, “String Theory Dynamics in Various Dimensions”, Nucl. Phys. B443 (1995) 85.

[4] J. Polchinski, “Dirichlet Branes and Ramond-Ramond Charges”, Phys. Rev. Lett. 75 (1995) 4724, hep-th/9510017.

[5] K. Becker, M. Becker, and A. Strominger, “Fivebranes, Membranes, and Nonperturbative String Theory”, Nucl. Phys. B456 (1995) 130, hep-th/9507158.

[6] S. Kachru and E. Silverstein, “N=1 Dual String Pairs and Gaugino Condensation”, Nucl. Phys. B463 (1996) 369, hep-th/9511228.

[7] E. Witten, “Non-perturbative Superpotentials in String Theory” Nucl. Phys. B474 (1996) 343, hep-th/9604030.

[8] S. Kachru, N. Seiberg, and E. Silverstein, “SUSY Gauge Dynamics and Singularities of 4d $N = 1$ String Vacua”, hep-th/9605030.

[9] R. Donagi, A. Grassi, and E. Witten, “A Nonperturbative Superpotential with $E_8$ Symmetry”, Mod. Phys. Lett. A11 (1996) 2199, hep-th/9607091.

[10] S. Kachru and E. Silverstein, “Singularities, Gauge Dynamics, and Non-perturbative Superpotentials in String Theory”, hep-th/9608194.

[11] P. Mayr, “Mirror Symmetry, $N = 1$ Superpotentials, and Tensionless Strings on Calabi-Yau Fourfolds”, hep-th/9610162.

[12] S. Katz and C. Vafa, “Geometric Engineering of N=1 Quantum Field Theories”, hep-th/9611090.

[13] C. Vafa and E. Witten, “Dual String Pairs With $N = 1$ and $N = 2$ Supersymmetry in Four Dimensions”, hep-th/9507050.

[14] T. Banks and M. Dine, “Coping with Strongly Coupled String Theory”, Phys. Rev. D50 (1994) 7454, hep-th/9406132.

[15] P. Binetruy, M.K. Gaillard, and Y. Wu, “Modular Invariant Formulation of Multi-Gaugino and Matter Condensation”, hep-th/9611149.

[16] E. Witten, “Small Instantons in String Theory”, Nucl. Phys. B460 (1996) 541, hep-th/9511030.

[17] J. Polchinski, “Evaluation of the One Loop String Path Integral”, Comm. Math. Phys. 104 (1986) 37.

[18] B. McClain and B. D. B. Roth, “Modular Invariance for Interaction Bosonic Strings at Finite Temperature”, Comm. Math. Phys. 111 (1987) 539.
[19] P. Candelas, X. de la Ossa, P. Green, and L. Parkes, “A Pair of Calabi-Yau Manifolds as an Exactly Soluble Superconformal Theory”, Nucl. Phys. B359 (1991) 21.

[20] S. Ferrara and A. Strominger, in Strings ‘89, eds. R. Arnowitt, R. Bryan, M. J. Duff, D. V. Nanopoulos, and C. N. Pope (World Scientific, 1989) 245; A. Strominger, Comm. Math. Phys. 133 (1990) 163; L.J. Dixon, V.S. Kaplunovsky, and J. Louis, Nucl. Phys. B329 (1990) 27; P. Candelas and X. de la Ossa, Nucl. Phys. B355 (1991) 455; L. Castellani, R. D’Auria and S. Ferrara, Phys. Lett. B241 (1990) 57; R. D’Auria, S. Ferrara, and P. Fre, Nucl. Phys. B359 (1991) 705.

[21] L. Dixon, V. Kaplunovsky, and J. Louis, “Moduli Dependence of String Loop Corrections to Gauge Coupling Constants”, Nucl. Phys. B355 (1991) 649.

[22] J. Harvey and G. Moore, “Five-brane Instantons and $R^2$ Couplings in N=4 String Theory”, hep-th/9610237.

[23] S. Kachru and C. Vafa, “Exact Results for N=2 Compactifications of Heterotic Strings”, Nucl. Phys. B450 (1995) 69. hep-th/9505105.

[24] S. Ferrara, J. Harvey, A. Strominger, and C. Vafa, “Second Quantized Mirror Symmetry”, Phys. Lett. B361 (1995) 59. hep-th/9505162.

[25] A. Klemm, W. Lerche, and P. Mayr, “K3 Fibrations and Heterotic Type II String Duality”, Phys. Lett. B357 (1995) 313.

[26] P. Aspinwall and J. Louis, “On the Ubiquity of K3 Fibrations in String Duality”, Phys. Lett B369 (1996) 233.