Generalized Chaplygin gas and CMBR constraints

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We study the dependence of the location of the Cosmic Microwave Background Radiation (CMBR) peaks on the parameters of the generalized Chaplygin gas model, whose equation of state is given by \( p = -A/\rho^\alpha \), where \( A \) is a positive constant and \( 0 < \alpha \leq 1 \). We find, in particular, that observational data arising from Archeops for the location of the first peak, BOOMERANG for the location of the third peak, supernova and high-redshift observations allow constraining significantly the parameter space of the model. Our analysis indicates that the emerging model is clearly distinguishable from the \( \alpha = 1 \) Chaplygin case and the \( \Lambda \)CDM model.

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I. INTRODUCTION

It has been recently suggested that the change of behaviour of the so-called dark energy density might be controlled by the change in the equation of state of the background fluid \( \Pi \) instead of the form of the potential, thereby avoiding well known fine-tuning problems of quintessence models. This is achieved via the introduction, within the framework of Friedmann-Robertson-Walker cosmology, of an exotic background fluid, the generalized Chaplygin gas, described by the equation of state

\[
p_{\text{ch}} = \frac{-A}{\rho^\alpha_{\text{ch}}},
\]

where \( \alpha \) is a constant in the range \( 0 < \alpha \leq 1 \) (the Chaplygin gas corresponds to the case \( \alpha = 1 \)) and \( A \) a positive constant. Inserting this equation of state into the relativistic energy conservation equation, leads to a density evolving as

\[
\rho_{\text{ch}} = \left( A + \frac{B}{a^{3(1+\alpha)}} \right)^{1/\alpha},
\]

where \( a \) is the scale factor of the Universe and \( B \) an integration constant. Remarkably, the model interpolates between a universe dominated by dust and a De Sitter universe via a phase described by a “soft” matter equation of state phase, the velocity of light. Actually, as discussed in Ref. \[6\], it is only for values in the range \( 0 < \alpha \leq 1 \) that the analysis of the evolution of energy density fluctuations makes sense.

It was also shown in Ref. \[6\] that the model can be described by a complex scalar field whose action can be written as a generalized Born-Infeld action corresponding to a “perturbed” \( d \)-brane in a \( (d+1,1) \) spacetime. It is clear that this model has a bearing on the observed accelerated expansion of the Universe \[3\] as it automatically leads to an asymptotic phase where the equation of state is dominated by a cosmological constant, \( 8\pi G A^{1/(1+\alpha)} \). It was also shown that the model admits, under conditions, an inhomogeneous generalization which can be regarded as a unification of dark matter and dark energy \[2,3\] and that it can be accommodated within the standard structure formation scenarios \[2,3,4\]. Therefore, the generalized Chaplygin gas model seems to be a viable alternative to models where the accelerated expansion of the Universe is explained through an uncancelled cosmological constant (see \[4\] and references therein) or through quintessence models with one \[7,8,11,12,13,14,15,16\] or two scalar fields \[7,8,9\].

These promising results have led, quite recently, to a wave of interest aiming to constrain the generalized Chaplygin model using observational data, particularly those arising from SNe Ia \[21,22,23,24,25\].

In this work, we shall consider the constraints arising from the positions of the first three CMBR peaks on the parameter space of the generalized Chaplygin gas, applying the same method that has been used recently to constrain quintessence models (see e.g. Refs. \[27,28,29\]). We find, in particular, that the positions of first and third peaks lead to fairly strong constraints although a sizeable portion of the parameter space of the model is still compatible with BOOMERANG and Archeops data. Further correlating the resulting region with the observations of supernova and high-redshift objects leads to quite tight constraint on the parameter space of the generalized Chaplygin model. It is important to stress that the generalized Chaplygin gas differs, as discussed in Ref. \[23\], from quintessence and tracker models in what concerns the so-called “statefinder” parameters \((r, s)\) \[24\].
and spectral index for the initial energy density perturbations, $n = 1$.

We start by computing $l_A$ for the case of the generalized Chaplygin gas. Rewriting Eq. (2) in the form

$$\rho_{ch} = \rho_{ch0} \left( A_s + \frac{1 - A_s}{a^{3(1+\alpha)}} \right)^{1/1+\alpha},$$

(5)

where $A_s \equiv A/\rho_{ch0}^{1+\alpha}$ and $\rho_{ch0} = (A + B)^{1/1+\alpha}$, the Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3} \left[ \frac{\rho_{r0}}{a^4} + \frac{\rho_{b0}}{a^3} + \rho_{ch0} \left( A_s + \frac{1 - A_s}{a^{3(1+\alpha)}} \right)^{1/1+\alpha} \right],$$

(6)

where we have included the contribution of radiation and baryons as this is not accounted for by the generalized Chaplygin gas equation of state.

Several important features of Eq. (5) are worth remarking. First of all, $a_s$ must lie in the interval $0 \leq a_s \leq 1$ as otherwise $\rho_{ch}$ will be undefined at some $a$. Secondly, for $a_s = 0$, the Chaplygin gas behaves as dust and, for $a_s = 1$, it behaves like a cosmological constant. Notice that only for $\alpha = 0$, the Chaplygin gas corresponds to a $\Lambda$CDM model. Hence, for the chosen range of $\alpha$, the generalized Chaplygin gas is clearly different from $\Lambda$CDM. Another relevant issue is that the sound velocity of the fluid is given, at present, by $\alpha A_s$ and thus $\alpha A_s \leq 1$.

Using

$$\frac{\rho_{r0}}{\rho_{ch0}} = \frac{\Omega_{r0}}{\Omega_{ch0}} = \frac{\Omega_{r0}}{1 - \Omega_{r0} - \Omega_{b0}},$$

(7)

and

$$\frac{\rho_{b0}}{\rho_{ch0}} = \frac{\Omega_{b0}}{\Omega_{ch0}} = \frac{\Omega_{b0}}{1 - \Omega_{r0} - \Omega_{b0}},$$

(8)

we obtain

$$H^2 = \Omega_{ch0} H_0^2 a^{-4} X^2(a),$$

(9)

with

$$X(a) = \frac{\Omega_{r0}}{1 - \Omega_{r0} - \Omega_{b0}} + \frac{\Omega_{b0} a}{1 - \Omega_{r0} - \Omega_{b0}}$$

$$+ a^4 \left( A_s + \frac{1 - A_s}{a^{3(1+\alpha)}} \right)^{1/1+\alpha}.$$  

(10)

Using the fact that $H^2 = a^{-4} \left( \frac{da}{dt} \right)^2$, we get

$$d\tau = \frac{da}{\Omega_{ch0} H_0 X(a)},$$

(11)

so that

FIG. 1: Dependence of the position of the CMBR first peak, $l_1$, as a function of $\alpha$ for different values of $A_s$. Also shown are the observational bounds on $l_1$ from BOOMERANG (dashed lines), see Eq. (17), and Archeops (full lines), see Eq. (18).
dust or non-relativistic matter at last scattering. Since, according to our dark matter-energy unification hypothesis, \(\rho_{ch}\) will behave as dust or non-relativistic matter at last scattering

\[
\rho_{ch} \approx \frac{\rho_{ch0}}{\alpha^3} (1 - A_s)^{1/1+\alpha},
\]

we get

\[
\varphi_1 \approx 0.267 \left( \frac{r_{ls}}{0.3} \right)^{0.1},
\]

where \(r_{ls} = \rho_r(z_{ls})/\rho_m(z_{ls})\) is the ratio of radiation to matter at last scattering. Since, according to our dark matter-energy unification hypothesis, \(\rho_{ch}\) will behave as dust or non-relativistic matter at last scattering

\[
l_m \equiv l_A (m - \varphi_m) .
\]

In an idealised model of the primeval plasma, there is a simple relation between the location of the \(m\)-th peak and the acoustic scale, namely \(l_m \approx ml_A\). However, the location of the peaks is slightly shifted by driving effects and this can be compensated by parameterising the location of the \(m\)-th peak, \(l_m\), as in \(\varphi_m\)

\[
l_m = l_A (m - \varphi_m) .
\]

It is not in general possible to derive analytically a relationship between the cosmological parameters and the peak shifts, but one can use fitting formulae that describe their dependence on these parameters; in particular, we have for the spectral index of scalar perturbations \(n = 1\) and for the amount of baryons \(\Omega_0 h^2 = 0.02\)

\[
\varphi_1 \approx 0.267 \left( \frac{r_{ls}}{0.3} \right)^{0.1},
\]

where \(r_{ls} = \rho_r(z_{ls})/\rho_m(z_{ls})\) is the ratio of radiation to matter at last scattering. Since, according to our dark matter-energy unification hypothesis, \(\rho_{ch}\) will behave as dust or non-relativistic matter at last scattering

\[
\rho_{ch} \approx \frac{\rho_{ch0}}{\alpha^3} (1 - A_s)^{1/1+\alpha},
\]

we get

\[
r_{ls} = \frac{\Omega_r0}{\Omega_{ch0}(1 - A_s)^{1/1+\alpha}} \approx \frac{\Omega_{r0}a_{ls}^{-1}}{1 - \Omega_{r0} - \Omega_{ch0}(1 - A_s)^{1/1+\alpha}} .
\]

Using Eqs. (12) and (13)-(16), we have plotted in Figure 1, \(l_1\) as a function of \(\alpha\) for different values of \(A_s\), where we have also drawn lines corresponding to the observational bounds on \(l_1\) as derived from BOOMERANG \(\[31\] (dashed lines)

\[
l_1 = 221 \pm 14 .
\]

and Archeops data \([32]\) (full lines)

\[
l_1 = 220 \pm 6 .
\]

Notice that, since \(\alpha A_s \leq 1\), for a specific value of \(A_s\) curves end where this relation gets saturated, \(\alpha A_s = 1\).

It is very difficult to extract any constraints from the position of the second peak since it depends on too many parameters, hence we shall disregard it hereafter.

As for the shift of the third peak, it turns out to be a relatively insensitive quantity \([28]\)

\[
\varphi_3 \approx 0.341 .
\]

Figure 2 shows \(l_3\) as a function of \(\alpha\) for different values of \(A_s\), where the dashed lines are the current lower and upper bounds on \(l_3\) as derived from BOOMERANG data \([31]\)

\[
l_3 = 845^{+12}_{-25} .
\]

We see that \(l_3\) puts rather tight constraints on the parameters of the model, \(\alpha\) and \(A_s\).

Figure 3 shows the constraints on the parameter space of the generalized Chaplygin gas model, the \((A_s, \alpha)\) plane, that are obtained from the observational bounds on the location of the first (full contour) and third (dashed contour) CMBR peaks. Hence, from the CMBR point of view the allowed region of the model parameters lies in the intersection between these two contours.

III. DISCUSSION AND CONCLUSIONS

In this paper we have shown that the location of the CMBR peaks, as determined via Archeops and BOOMERANG data, allows constraining a sizeable portion of the parameter space of the generalized Chaplygin gas model. Our results indicate that the constraints

\[
\rho_{ch} \approx \frac{\rho_{ch0}}{\alpha^3} (1 - A_s)^{1/1+\alpha},
\]

we get

\[
r_{ls} = \frac{\Omega_r0}{\Omega_{ch0}(1 - A_s)^{1/1+\alpha}} \approx \frac{\Omega_{r0}a_{ls}^{-1}}{1 - \Omega_{r0} - \Omega_{ch0}(1 - A_s)^{1/1+\alpha}} .
\]
arising from the position of the first peak, as recently announced by the Archeops collaboration, imply, for \( \alpha \leq 1 \), that \( 0.57 \lesssim A_s \lesssim 0.91 \).

On the other hand, the location of the third acoustic peak arising from the BOOMERANG collaboration provides strong constraints on the parameter space of the model, as indicated in Figure 3 (dashed contour region). Notice that compatibility with data requires that only the fairly small intersecting region is allowed, that is \( 0.74 \lesssim A_s \lesssim 0.90 \); consistency with SNe Ia data suggests on its hand that \( 0.6 \lesssim A_s \lesssim 0.85 \), and this together with the bound arising from the APM 08279 + 5255 source, \( A_s \geq 0.81 \), lead us to obtain a fairly tight constraint \( 0.81 \lesssim A_s \lesssim 0.85 \) and \( 0.2 \leq \alpha \lesssim 0.6 \). Furthermore, we stress that the allowed region in Figure 3 is clearly distinct from the Chaplygin gas (\( \alpha = 1 \)) and the \( \Lambda \)CDM model.

Clearly, with future high precision measurements of the MAP and PLANCK satellites, we expect that the position of the first three peaks will be determined to high accuracy, thus allowing further constraints on the parameter space of the generalized Chaplygin gas model. Correlating the resulting constraints with SNe Ia, redshift objects and, for instance, gravitational lensing data may uniquely determine these parameters.

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