Emergence of strongly correlated electronic states driven by the Andreev bound state 
in d-wave superconductors

Shun Matsubara and Hiroshi Kontani
Department of Physics, Nagoya University, Nagoya 464-8602, Japan
(Dated: October 22, 2019)

As well known, the surface Andreev bound state (ABS) forms at the open (1, 1) edge of a $d_{x^2−y^2}$-wave superconductor. Although large local density of states (LDOS) in the ABS can lead to the emergence of exotic strongly correlated electronic states, theoretical studies on this issue has been limited. To understand important effects of ABS on the electronic correlation, we study the cluster Hubbard model with an open (1, 1) edge in the presence of a bulk d-wave gap. We calculate the site-dependent spin susceptibility by performing random phase approximation (RPA) and fluctuation exchange (FLEX) approximation in the real space. We find that at the (1, 1) edge, drastic ferromagnetic (FM) fluctuations occur owing to the ABS. In addition, as the temperature decreases, the system rapidly approaches a magnetic-order phase slightly below the transition temperature of the bulk d-wave superconductivity (SC). In this case, the FM fluctuations are expected to induce interesting phenomena such as edge-induced triplet SC and quantum critical phenomena.

I. INTRODUCTION

In bulk cuprate superconductors, the spin fluctuations cause interesting phenomena. For example, d-wave SC [1–6] and non-Fermi liquid phenomena in the normal state [7–10]. Moreover, both the Hall coefficient and magnetoresistance are strongly enlarged due to the spin fluctuation-driven quasiparticle scattering [11–13]. In recent years, the axial and uniform charge-density-wave (CDW) is observed in various optimally- and under-doped cuprate superconductors [14–17]. The discovery of CDW has activated the study of the present field. To explain the CDW mechanism, spin-fluctuation-driven CDW formation mechanisms have been proposed [18–22].

In many previous studies, electronic states in bulk systems with translational symmetry have been analyzed. On the other hand, real space structures such as surfaces, interfaces, and impurities break the translational symmetry of a system, and they can induce interesting phenomena that cannot be realized in the bulk systems. In the normal states of cuprate superconductors, YBa$_2$Cu$_3$O$_{7−x}$ (YBCO) and La$_{2−δ}$Sr$_δ$CuO$_4$ (LSCO), non-magnetic impurities induce a local magnetic moment around them, and the uniform spin susceptibility exhibits the Curie-Weiss behavior [23–25]. In theoretical studies, various analyses are performed using the Heisenberg and Hubbard models containing a non-local impurity, and the enhancement in the spin fluctuations is obtained [30–33]. In case of a local impurity, the enhancement in the local spin susceptibility is reproduced by the improved fluctuation-exchange (FLEX) approximation performed in the real space [34]. Because these analyses are performed in the real space, the site-dependence of the spin susceptibility is satisfactorily explained.

Recently, the present authors predicted theoretically that ferromagnetic (FM) fluctuations develop at the open (1, 1) edge of the two-dimensional cluster Hubbard model. In addition, as the temperature decreases, the local mass-enhancement factor and quasi-particle damping increase strongly at the (1, 1) edge, and the system approaches the magnetic critical point. The above are edge-induced quantum critical phenomena [35]. These impurity or edge-induced magnetic criticalities originate from the high local density of states (LDOS) sites caused by the Friedel oscillation. Moreover, the enhanced spin fluctuations may cause interesting phenomena such as edge-induced spin triplet SC.

On the other hand, surfaces or interfaces also cause various interesting phenomena in the superconducting state (SC state). At the (1, 1) edge or interface of $d_{x^2−y^2}$-wave superconductors, the Andreev bound state (ABS) is formed, and the LDOS increases at the Fermi level [36–41]. This originates from the sign change in the bulk d-wave superconducting gap (SC gap). The ABS is observed by scanning tunneling spectroscopy (STS) experiments as the zero-bias conductance peak [42–45]. The surface ABS is also regarded as the odd-frequency pairing amplitude induced at the surface of an even-frequency superconductor [46, 47]. Owing to the increase in the LDOS caused by the ABS, a strong electron correlation is expected to emerge near the edge. However, theoretical studies on the effects of an ABS on this electron correlation have been limited. Furthermore, a surface or an interface can induce a time-reversal symmetry breaking (TRSB) SC state. For example, it is proposed that the (1, 1) edge of a d-wave superconductor exhibits $d+is$-wave SC [48–50]. In this case, the relative phase between the s- and d-wave gaps is $\pi/2$. The emergence of TRSB SC state has been discussed in polycrystalline YBCO [51] or twinned iron-based superconductor FeSe in the nematic phase [52]. To understand such interesting SC at a surface or an interface, we have to clarify the effect of the ABS on the spin fluctuations, which can mediate surface-induced SC.

In this study, we investigate the prominent effects of the ABS on the surface electron correlation. For this purpose, we construct the two-dimensional cluster Hubbard model with the (1, 1) edge in the bulk d-wave SC state, and calculate the site-dependent spin susceptibility...
by performing random-phase-approximation (RPA) and FLEX approximation in the real space. We find that the strong FM fluctuations at the (1,1) edge are enhanced much more drastically in the bulk d-wave SC state than in the normal state. The strong FM fluctuations induced by the surface ABS may drive interesting emerging phenomena, such as edge-induced SC.

II. MODEL AND THEORETICAL METHOD

In this study, we analyze the square-lattice cluster Hubbard model with a d-wave SC gap:

\[ H = \sum_{i,j,\sigma} t_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

\[ + \sum_{i,j} \Delta^d_{i,j} \left( c_{i\uparrow}^\dagger c_{j\downarrow} + c_{i\downarrow} c_{j\uparrow} \right), \]

where \( t_{i,j} \) is the hopping integral between sites \( i \) and \( j \). We set the nearest, next nearest, and third-nearest hopping integrals as \( (t, t', t'') = (-1, 1/6, -1/5) \), which correspond to the YBCO tight-binding (TB) model. \( c_{i\sigma}^\dagger \) and \( c_{i\sigma} \) are the creation and annihilation operators of an electron with spin \( \sigma \), respectively. \( U \) is the on-site Coulomb interaction, and \( \Delta^d_{i,j} \equiv \Delta^d_{i,j} \) is the d-wave SC gap between sites \( i \) and \( j \). Figure 1(a) shows the Fermi surface of the periodic Hubbard model at filling \( n = 0.95 \). Then, antiferromagnetic (AFM) fluctuations develop owing to the \( Q = (\pi, \pi) \) nesting.

In this study, we investigate a cluster Hubbard model with an open (1,1) edge. Figure 1(b) shows the square lattice with the (1,1) edge. \( Y = 1 \) corresponds to the edge layer. For convenience, in this study, we analyze the one-site unit cell structure shown in Figure 1(c). This model is periodic along the \( x \)-direction, whereas the translational symmetry is violated along the \( y \)-direction. By performing a Fourier transformation on the \( x \)-direction, the first term of (1) is expressed as

\[ H^0 = \sum_{k_x,y,y',\sigma} H^0_{y,y', y}(k_x) c_{k_x,y,\sigma}^\dagger c_{k_x,y',\sigma}. \]

We also perform a Fourier transformation on the \( x \)-direction of the d-wave gap \( \Delta_{i,j} = \Delta^d(\delta_{x-1,y-1} - \delta_{x,y+1} - \delta_{x,y-1}) \). Here, we assume that \( \Delta^d_{i,j} \) is real and non-zero only between the nearest sites. Its \((k_x, y, y')\) representation is given as

\[ \Delta^d_{y,y'}(k_x, T) = \Delta^d(T) \left\{ \frac{e^{-ik_x}}{2} \delta_{y,y'+1} + \frac{e^{ik_x}}{2} \delta_{y,y'-1} \right\}, \]

where \( \Delta^d(T) \) is the temperature-dependence of the d-wave gap function. We suppose that \( \Delta^d(T) \) obeys the BCS-like \( T \)-dependence:

\[ \Delta^d(T) = \Delta^d_0 \tanh \left( \frac{1.74 \sqrt{T_c d}}{T} - 1 \right), \]

where \( \Delta^d_0 = \Delta^d_0(T = 0) \). Now, we denote the number of sites along \( y \)-direction as \( N_y \). The \( N_y \times N_y \) Green functions in the d-wave SC state, \( G, F \) and \( F^\dagger \), are given as

\[ (\hat{G}(k_x, \varepsilon_n) F(k_x, \varepsilon_n) F^\dagger(k_x, \varepsilon_n) \hat{G}(k_x, -\varepsilon_n) ) \]

\[ = (\varepsilon_n \mathbb{1} - \hat{H}^0(k_x) - \hat{\Sigma}(k_x, \varepsilon_n) - \Delta^d(k_x) - \Delta^d(k_x) )^{-1} \]

\[ - \varepsilon_n \mathbb{1} + \hat{H}^0(k_x) + \hat{\Sigma}(k_x, \varepsilon_n) \]

where \( \varepsilon_n = (2n + 1)\pi T \) is the fermion Matsubara frequency. Here, \( (\hat{H}^0)_{y,y'} = (\hat{H}^0)_{y',y}, \hat{\Sigma}(k_x, \varepsilon_n) \) is the normal self-energy of the FLEX approximation. In the RPA, we set \( \hat{\Sigma} = 0 \) in the Green function.

To demonstrate the emergence of the ABS at the (1,1) edge of the TB model in the bulk d-wave SC state, we calculate the LDOS given by

\[ D_y(\varepsilon) = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} dk_x \text{Im} G_{y,y}(k_x, \varepsilon - i\delta). \]

Figure 2 displays the obtained LDOS for \( \Delta^d(T) = 0.08 \) by setting \( \delta = 0.01 \). At the edge \( (y = 1) \), \( D_y(\varepsilon) \) has a large peak at the Fermi level, \( \varepsilon = 0 \), owing to the ABS. The LDOS at \( y = 300 = N_y/2 \) exhibits a V-shape \( \varepsilon \)-dependence, which corresponds to the bulk LDOS in the d-wave SC state. Note that the height of the peak is proportional to the size of the bulk d-wave gap.

Next, we calculate the spin susceptibility of the (1,1) edge of the TB model using the RPA or FLEX approximation in the \((k_x, y, y')\) representation. Figure 3 shows the diagrams of the irreducible susceptibilities, \( \chi^0 \) and \( \phi^0 \). They are given by the Green functions, \( G, F \) and \( F^\dagger \), as

\[ \chi^0_{y,y'}(q_x, \omega_l) = \frac{T}{|k_x|} \sum_{k_x, n} G_{y,y'}(q_x + k_y, \omega_l + \varepsilon_n) G_{y',y}(k_x, \varepsilon_n), \]

\[ \phi^0_{y,y'}(q_x, \omega_l) = \frac{T}{|k_x|} \sum_{k_x, n} F_{y,y'}(q_x + k_x, \omega_l + \varepsilon_n) F_{y',y}(k_x, \varepsilon_n). \]
edge and bulk, respectively. For convenience, we set δ = 0.01.

FIG. 3. Diagram of the irreducible susceptibility, $\chi^{0}(y,\omega_{l})$. The line with an arrow is Green function $G$. The line with two arrows is $F$ or $F^\dagger$.

$$\chi^{0}(y,\omega_{l}) = y \times F^{\dagger}_{y',y}(k_{x},\varepsilon_{n}),$$

(8)

where $\omega_{l} = 2l\pi i T$ is the boson Matsubara frequency. $\phi^{0}$ is finite only in the SC state. The $N_{y} \times N_{y}$ matrix of the spin (charge) susceptibility $\tilde{\chi}^{(c)}(s)$ is calculated using the irreducible susceptibility, $\chi^{0}$ or $\phi^{0}$, in the SC state.

$$\tilde{\chi}^{(c)}(s) = \tilde{\phi}^{(c)}(s) = \frac{1}{1 - (+)U\tilde{\phi}^{(c)}(s,\omega_{l})}^{-1},$$

(10)

The spin (charge) Stoner factor, $\alpha_{S}$ ($\alpha_{C}$), is obtained as the largest eigenvalue of $U\tilde{\phi}^{(c)}(s,\omega_{l})$ at $\omega_{l} = 0$. In this formulation, $\alpha_{S} > \alpha_{C}$ is always satisfied. The magnetic order is realized when $\alpha_{S} \geq 1$.

Figure 4 shows the Feynman diagram of the normal self-energy $\Sigma$. The wavy line, $V$, is composed of $\chi^{s}$. The straight line with an arrow from $y'$ to $y$ is the normal Green function, $G$.

FIG. 4. Feynman diagram of the normal self-energy $\Sigma$. The wavy line, $V$, is composed of $\chi^{s}$. The straight line with an arrow from $y'$ to $y$ is the normal Green function, $G$.

III. NUMERICAL RESULT OF $\tilde{\chi}^{s}$ AND $\alpha_{S}$ IN REAL SPACE

Next, we perform the RPA and FLEX analyses for the cluster Hubbard model with the bulk $d$-wave SC gap, with the translational symmetry along the $x$-direction. We set the number of $k_{x}$-meshes as $N_{x} = 64$ (RPA) or 32 (FLEX), that of sites along the $y$-direction as $N_{y} = 64$ (RPA) or 40 (FLEX), and that of Matsubara frequencies as $N_{\omega} = 1024$. We set the electron filling, $n = 0.95$; the transition temperature for the $d$-wave is $T_{c,d} = 0.04$. The Coulomb interaction is $U = 2.25$ in the RPA, and $U = 3.0$ in the FLEX. Here, the unit of energy is $|t|$, which corresponds to $\sim 0.4$eV in cuprate superconductors. By performing this analysis, we show that the ABS drastically enhances the FM fluctuations at the $(1,1)$ edge, and the system rapidly approaches a magnetic-order phase.

A. Result of the RPA study

First, we study the site-dependent static spin susceptibility, $\tilde{\chi}^{s}(q_{x},\omega_{l} = 0)$, in the $d$-wave SC state using the RPA. Hereafter, we refer to the spin susceptibility in the $d$-wave SC state and normal state as $\tilde{\chi}$ and $\tilde{\chi}^{(n)}$, respectively. We also introduce the following susceptibilities in the SC state to clarify the origin of the enhancement in the FM fluctuations:

$$\tilde{\chi}^{s} = \tilde{\phi}^{(s)}(1 - U\tilde{\phi}')^{-1} \tilde{\phi}' = \chi^{0}$$

(12)

$$\tilde{\chi}'' = \tilde{\phi}''(1 - U\tilde{\phi}'')^{-1} \tilde{\phi}'' = \chi^{0(n)} + \tilde{\phi}^{0}$$

(13)

Here, $\chi^{0}$ and $\chi^{0(n)}$ are the irreducible susceptibilities in the bulk $d$-wave SC and normal states, respectively. In susceptibility $\tilde{\chi}''$ ($\tilde{\chi}''$), the effect of $d$-wave gap through $\tilde{\phi}^{0}$ ($\tilde{\chi}^{0}$) is subtracted.

Figure 5 shows the obtained RPA susceptibilities $\chi_{y,y'}(q_{x})$ for $\Delta_{0}^{d} = 0.16$ at $T = 0.0388$. The site diagonal component, $\chi_{y,y'}(q_{x})$, represents the correlation of the spins in the same layer $y$. In the edge layer ($y = 1$),
\( \chi_{y,y}(q_x) \) has a large peak at \( q_x = 0 \). This result means that strong FM fluctuations develop in the \((1,1)\) edge layer. The FM correlation along the edge layer is consistent with the AFM correlation in the periodic Hubbard model. This strong enhancement occurs only for \( y = 1 \) and \( y = 2 \). In fact, the Stoner factor is \( \alpha_S = 0.990 \) with the edge, whereas \( \alpha_S = 0.673 \) in the periodic model. Therefore, the present model with the bulk \( d \)-wave SC gap approaches the magnetic quantum critical point with introduction of the edge.

Next, we compare the \( d \)-wave SC and normal state. Figure 5 shows \( \chi_{1,1} \) and \( \chi_{1,1}^{(n)} \) in the model with edge. The enhancement in the FM fluctuations is much more drastic in the \( d \)-wave SC state compared to that in the normal state discussed in Ref. [35]. Therefore, this strong enhancement cannot be explained only by the existence of edge.

Furthermore, we examine the contribution from \( \hat{\phi}^0 \) and \( \hat{\chi}^0 \) to the enhancement of total spin susceptibility. In Figure 6, we present \( \chi_{1,1}'(q_x) \) and \( \chi_{1,1}'(q_x) \). The height of the peak of \( \chi' \) is much smaller than that of \( \hat{\chi} \). On the other hand, the height of the peak of \( \chi'' \) is enlarged whereas slightly lower than that of \( \hat{\chi} \). Therefore, \( \hat{\phi}^0 \) due to anomalous Green functions give the dominant contribution for the increment of \( \hat{\chi}' \) whereas \( \hat{\chi}^0 - \hat{\chi}^{0(n)} \) also gives minor contribution.

Figure 7(a) shows the \( q_x \)-dependence of irreducible susceptibility \( \phi^0 \). In the bulk, \( \phi^0_{32,32} \) is zero because \( x \)-axis is the direction of the \( d \)-wave gap node. Interestingly, \( \phi^0_{1,1} \) is finite and has a peak at \( q_x = 0 \). This is explained as an effect of the ABS, which corresponds to the odd-frequency SC induced at the \((1,1)\) edge as discussed in Refs. [40, 47]. We give brief discussion on this issue in Appendix A. In Figure 7(b), we show the \( q_x \)-dependence of the irreducible susceptibility, \( \chi^0 \), with \( \chi^{0(n)} \). At the edge, \( \chi_{1,1}^0 \) is slightly larger than \( \chi_{1,1}^{0(n)} \) owing to the peak of LDOS due to the ABS. In summary, the ABS enhances the FM fluctuations at the \((1,1)\) edge mainly through the development of \( \hat{\phi}^0 \). Figure 8 shows the \( T \)-dependence of \( \alpha_S \) in the RPA. The inset shows the \( T \)-dependence of the size of the \( d \)-wave gap, which is given in Eq. (4). The transition temperature of the bulk \( d \)-wave SC is set as \( T_{cd} = 0.04 \). Then, \( 2\Delta^d_0/T_c = 4.8 \) for \( \Delta^d_0 = 0.08 \) (0.16). \( \alpha_S \) in the SC state increases sharply as \( T \) decreases compared to that in the normal state, due to the development of the ABS. The increase for \( \Delta^d_0 = 0.16 \) is sharper than that for \( \Delta^d_0 = 0.08 \) because the height of the ABS is proportional to \( \Delta^d_0 \). \( \alpha_S \) reaches unity at \( T \approx 0.038 \) for \( \Delta^d_0 = 0.16 \), and the edge FM order is realized. To summarize, we predict the emergence of FM order at \((1,1)\) edge of \( d_{x^2-y^2} \)-wave superconductors.

### B. Results of the FLEX study

In this paper, we have studied \( \chi'(q_x) \) using the FLEX approximation. In this approximation, the negative feed-
back effect due to the site-dependent self-energy is considered, and the size of the d-wave gap is renormalized by the self-energy. The renormalized d-wave gap in bulk, \( \Delta_0^{d*} \), is evaluated by \( \Delta_0^{d*} = \Delta_0^d / Z_{\text{bulk}} \), where \( Z_{\text{bulk}} \) is the on-site mass-enhancement factor in the bulk given by

\[
Z_{\text{bulk}} = 1 - \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_x \frac{\partial}{\partial \varepsilon} \text{Re} \Sigma_{y=y'=0}(k_x, \varepsilon - i\delta)|_{\varepsilon=0}.
\]

Equation (14)

Figure 9 shows the \( q_x \)-dependence of \( \chi_{y,y}(q_x) \) in the FLEX approximation for \( \Delta_0^d = 0.08 \) at \( T = 0.02 \). With this parameter, we obtain \( \Delta_0^{d*} \approx 0.05 \) and \( 2\Delta_0^{d*} / T_{cd} \approx 2.5 \). At the (1,1) edge (\( y = 1 \)), \( \chi_{1,1}^{*}(q_x) \) has a large peak at \( q_x = 0 \). The Stoner factor is \( \alpha_S = 0.986 \), whereas \( \alpha_S = 0.896 \) in the periodic model without edge. Therefore, the enhancement in the FM fluctuations at the edge is confirmed by both the RPA and FLEX approximation, irrespective of the presence or absence of self-energy.

Figure 10 shows the \( T \)-dependence of \( \alpha_S \) in the FLEX approximation. We set the transition temperature of the d-wave SC as \( T_{cd} = 0.04 \). We obtained the renormalized gaps, \( \Delta_0^{d*} \approx 0.1 \) and \( 2\Delta_0^{d*} / T_e \approx 5 \) for \( \Delta_0^d = 0.16 \).

**IV. EFFECT OF THE FINITE d-WAVE COHERENCE LENGTH ON THE EDGE-INDUCED SPIN FLUCTUATIONS**

In this section, we study the enhancement in the FM fluctuations when the d-wave gap is suppressed near the edge for a finite range, \( 1 \leq y \leq \xi_d \), where \( \xi_d \) is the coherence length of the d-wave SC. We set the \( y \)-dependence of the d-wave gap as

\[
\Delta_{y,y'}^{d}(k_x, T) \times \left( 1 - \exp \left( \frac{y + y' - 2}{2\xi_d} \right) \right) .
\]

Equation (15)

The \( y \)-dependence of given \( |\Delta_{x=0,y+1,x=0,y'}^d| \) is shown in Figure 11(a). The inset shows the corresponding nearest neighbor bonds in the real space. Figure 11(b) shows the LDOS at the edge. Although the height of the peak of
the ABS is reduced, the peak structure remains for the finite $\xi_d$ ($\lesssim 10$).

FIG. 11. (color online) (a) Site-dependence of the $d$-wave gap suppressed near the edge over $\xi_d$. We set $\Delta^d_0 = 0.08$, and plot it at $T = 0.032$. $\xi_d = 0$ corresponds to the site-independent $d$-wave gap. The inset shows the nearest neighbor bonds corresponding to $\Delta^d_{x=0,y=0,1,z=0,1}$. (b) LDOS at the $(1,1)$ edge for the finite $\xi_d$.

Next, we calculate the $T$-dependence of $\alpha_S$ using the RPA, and Figure 12 shows the result for (a) $\Delta^d_0 = 0.08$ and (b) $0.16$. The increase in $\alpha_S$ for $\xi_d = 3, 10$ is moderate compared to that for $\xi_d = 0$, owing to the suppression of the ABS. For $\Delta^d_0 = 0.16$, $\alpha_S$ reaches unity at $T \approx 0.037$ even for $\xi_d = 10$. Therefore, we still obtain the drastic enhancement in the FM fluctuations for the more realistic $d$-wave gap with finite $\xi_d$ ($\lesssim 10$).

FIG. 12. (color online) $T$-dependence of $\alpha_S$ by the RPA for (a) $\Delta^d_0 = 0.08$ or (b) $0.16$ with finite $\xi_d$. The red dashed line represents $\alpha_S$ for the site-independent $d$-wave gap. The black solid line represents $\alpha_S$ in the normal state.

V. SUMMARY

In this study, we revealed that the ABS drastically enhances the FM fluctuations at the $(1,1)$ edge of the $d$-wave superconductor. For this purpose, we construct the two-dimensional square lattice Hubbard model with the edge in the presence of the bulk $d$-wave SC gap. Then we perform the site-dependent RPA calculation in the real space. By detailed analysis, we found that edge-induced FM fluctuations are mainly caused by the increment of $\phi^0$ due to the ABS. Furthermore, the Stoner factor $\alpha_S$ exhibits drastic increase just below the bulk $d$-wave $T_c$, and edge-induced FM order or fluctuations is expected to emerge. Such ABS-induced magnetic critical phenomena have been confirmed by using the FLEX approximation, which includes the negative feedback effect of the normal self-energy. Finally, we verified that the enhancement in FM fluctuations are still prominent even if the effect of finite coherence length $\xi_d$ ($\lesssim 10$) is taken into account. Therefore, we conclude that the ABS-induced FM order or strong FM fluctuations appears in various $d$-wave superconductors such as cuprate.

The result of the present study indicates the emergence of interesting FM order and induced phenomena. For example, the FM order will induce the splitting of the ABS peak, which may be observed by STM/STS study. Figure 13 shows the LDOS for up and down spins at the edge with the magnetization $(M_0 = 0.10)$. The magnetization is given by the Zeeman term $H_M = M_0/2 \sum_{k_x,\sigma} \sigma c_{k_x,\sigma}^\dagger c_{k_x,\sigma}$. In addition, an edge-induced triplet SC is expected to be realized theoretically. In this case, the bulk $d$-wave SC and edge-induced triplet SC may coexist at the $(1,1)$ edge, similar to the $d + is$-wave state discussed in Ref. [48–50]. This presents an important problem for the future, to understand the edge-induced SC state in strongly correlated electron systems.

FIG. 13. (color online) LDOS at the edge of $d$-wave superconductor ($\Delta^d_0 = 0.20$) when the magnetization ($M_0 = 0.10$) emerges. The red solid line and blue dotted line represent the LDOS for up and down spins, respectively. For convenience, we set $\delta = 0.01$.

ACKNOWLEDGMENTS

We are grateful to Y. Tanaka, S. Onari, and Y. Yamakawa for valuable comments and discussions. This work was supported by the JSPS KAKENHI (No.
Appendix A: Relation between enhanced FM fluctuations and odd-frequency superconductivity

Here, we examine the Green function, $F$, by which the irreducible susceptibility $\phi^0$ is composed. Figure 14 (a) shows the $\varepsilon_n$-dependence of $\text{Re} F_{y,y}(\pi/4, \varepsilon_n)$. In the bulk, $\text{Re} F_{y,y}(\pi/4, \varepsilon_n) = 0$ because the $x$-direction is the node direction of $d$-wave gap. However, at the edge, $\text{Re} F_{x,1}(\pi/4, \varepsilon_{n})$ is finite, and it shows an odd-frequency-dependence. This odd-frequency pair amplitude can be understood as another physical picture of the ABS. Figure 14 (b) shows the $k_x$-dependence of $\text{Re} F_{y,y}(k_x, i\pi T)$. At the edge, $\text{Re} F_{x,1}(k_x, i\pi T)$ is finite and has peaks at $k_x \approx 4\pi/5$ and $k_x \approx 6\pi/5$, whereas $\text{Re} F_{y,y}(k_x, i\pi T) = 0$ in the bulk. These peaks generate the enhancement of $\phi^0_{y,1}$ at $q_{\parallel} = 0$. Therefore, the enhancement in the FM fluctuations by $\phi^0$ can be explained as the direct effect of the odd-frequency pairing, which is an aspect of the ABS.

![Green function](image)

**FIG. 14.** (color online) Green function $F$ calculated for $\Delta_d^0 = 0.16$ at $T = 0.0388$. (a) $\varepsilon_n$-dependence of $\text{Re} F_{y,y}(k_x = \pi/4, \varepsilon_n)$. The red and green points represent the component in the edge ($y = 1$) and bulk (periodic system), respectively. (b) $k_x$-dependence of $\text{Re} F_{y,y}(k_x, i\pi T)$. The red solid line and green dotted line represents the component in the edge and bulk, respectively.

[1] N. E. Bickers and S. R. White, Phys. Rev. B 43 8044 (1991).
[2] P. Monthoux and D. J. Scalapino, Phys. Rev. Lett. 72 1874 (1994).
[3] S. Koikegami, S. Fujimoto, and K. Yamada, J. Phy. Soc. Jpn. 66 1438 (1997).
[4] T. Takimoto and T. Moriya, J. Phy. Soc. Jpn. 66 2459 (1997).
[5] T. Dahm, D. Manske, and L. Tewordt, Europhys. Lett. 55 93 (2001).
[6] D. Manske, I. Eremin, and K. H. Bennemann, Phys. Rev. B 67 134520 (2003).
[7] T. Moriya and K. Ueda, Adv. Phys. 49 555 (2000).
[8] T. Moriya and K. Ueda, Rep. Prog. Phys. 66 1299 (1993).
[9] P. Monthoux and D. Pines, Phys. Rev. B 47 6069 (1993).
[10] H. Kontani, Rep. Prog. Phys. 71 026501 (2008).
[11] H. Kontani, K. Kanki, and K. Ueda, Phys. Rev. B 59 14723 (1999).
[12] H. Kontani, J. Phys. Soc.Jpn, 70 2840 (2001); H. Kontani, Phys. Rev. Lett. 89 237003 (2002).
[13] H. Kontani, Phys. Rev. B 64 054413 (2001).
[14] G. Ghiringhelli, M. L. Tacon, M. Minola, S. Blanco-Canosa, C. Mazzoli, N. B. Brookes, G. M. D. Luca, A. Frano, D. G. Hawthorn, F. He, T. Loew, M. M. Sala, D. C. Peets, M. Salluzzo, E. Schierle, R. Sutarto, G. A. Sawatzky, E. Wescbe, B. Keimer, and L. Braicovich, Science 337, 821 (2012).
[15] J. Chang, E. Blackburn, A. T. Holmes, N. B. Christensen, J. Larsen, J. Mesot, R. Liang, D. A. Bonn, W. N. Hardy, A. Waterphnul, M. von Zimmermann, E. M. Forgan, and S. M. Hayden, Nat. Phys. 8, 871 (2012).
[16] K. Fujita, M. H. Hamidian, S. D. Edkins, C. K. Kim, Y. Kohsaka, M. Amini, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E. A. Kim, S. Sachdev, and J. C. Davis, Proc. Natl. Acad. Sci. U.S.A. 111, E3026 (2014).
[33] N. Bulut, Phys. Rev. B 61 9051 (2000).

[34] H. Kontani and M. Ohno, Phys. Rev. B 74 014406 (2006); H. Kontani and M. Ohno, J. Magn. Magn. Mat. 310 483 (2007).

[35] S. Matsubara, Y. Yamakawa, and H. Kontani, J. Phys. Soc. Jpn 87 073705 (2018).

[36] C. R. Hu, Phys. Rev. Lett. 72 1526 (1994).

[37] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. 74 3451 (1995).

[38] S. Kashiwaya, Y. Tanaka, M. Koyanagi, and K. Kajimura, Phys. Rev. B. 53 2667 (1996).

[39] M. Matsumoto and H. Shiba, J. Phys. Soc. Jpn. 64 14703 (1995).

[40] Y. Nagato and K. Nagai, Phys. Rev. B 51 16254 (1995).

[41] S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. 63 1641 (2000).

[42] S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima, and K. Kajimura, Phys. Rev. B 51 1350 (1995).

[43] I. Iguchi, W. Wang, M. Yamazaki, Y. Tanaka, and S. Kashiwaya, Phys. Rev. B 62 R6131 (2000).

[44] J. Y. T. Wei, N. -C. Yeh, D. F. Garrigus, and M. Strasik, Phys. Rev. Lett. 81 2542 (1998).

[45] J. Geek, X. X. Xi, and G. Linker, Z. Phys. B 73 2542 (1988).

[46] Y. Tanaka, M. Sato, and N. Nagaosa, J. Phys. Soc. Jpn. 81 011013 (2012).

[47] Y. Tanaka and A. A. Golubov, Phys. Rev. Lett. 98 037003 (2007).

[48] M. Matsumoto and H. Shiba, J. Phys. Soc. Jpn. 64 3384 (1995).

[49] M. Matsumoto and H. Shiba, J. Phys. Soc. Jpn. 64 4867 (1995).

[50] M. Matsumoto and H. Shiba, J. Phys. Soc. Jpn. 65 2194 (1995).

[51] M. Sigrist, K. Kuboki, P. A. Lee, A. J. Millis, and T. M. Rice, Phys. Rev. B 53 2835 (1996).

[52] T. Watashige, Y. Tsutsumi, T. Hanaguri, Y. Kohsaka, S. Kasahara, A. Furusaki, M. Sigrist, C. Meingast, T. Wolf, H. v. Lheyesen, T. Shibauchi, and Y. Matsuda, Phys. Rev. X 5 031022 (2015).

[53] S. Matsubara and H. Kontani, unpublished