The polarized TMDs in the covariant parton model approach

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Abstract. We derive relations between polarized transverse momentum dependent distribution functions (TMDs) and the usual parton distribution functions (PDFs) in the 3D covariant parton model, which follow from Lorentz invariance and the assumption of a rotationally symmetric distribution of parton momenta in the nucleon rest frame. Using the known PDF $g_1(x)$ as input we predict the $x$- and $p_T$-dependence of all polarized twist-2 naively time-reversal even (T-even) TMDs.

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TMDs [1, 2] open a new way to a more complete understanding of the quark-gluon structure of the nucleon. Indeed, some experimental observations can hardly be explained without a more accurate and realistic 3D picture of the nucleon, which naturally includes transverse motion. The azimuthal asymmetry in the distribution of hadrons produced in deep-inelastic lepton-nucleon scattering (DIS), known as the Cahn effect [3], is a classical example. The intrinsic (transversal) parton motion is also crucial for the explanation of some spin effects [4]–[16].

In previous studies we discussed the covariant parton model, which is based on the 3D picture of parton momenta with rotational symmetry in the nucleon rest frame [17]–[26]. In this model we studied all T-even TMDs and derived a set of relations among them [23]. It should be remarked that some of the relations among different TMDs were found (sometimes before) also in other models [27]–[34].

In the recent paper [35] we further develop and broadly extend our studies [24]–[25] of the relations between TMDs and PDFs. The formulation of the model in terms of the light-cone formalism [23] allows us to compute the leading-twist TMDs by means of the light-front correlators $\phi(x, p_T)_{ij}$ [2] as:

$$\frac{1}{2} \text{tr} \left[ \gamma^+ \phi(x, p_T) \right] = f^q_1(x, p_T) - \frac{\varepsilon^{jk} p^j_T S^k_T}{M} f^l_1(x, p_T),$$

$$\frac{1}{2} \text{tr} \left[ \gamma^+ \gamma_5 \phi(x, p_T) \right] = S_L g^q_1(x, p_T) + \frac{p_T S_T}{M} g^l_1(x, p_T),$$

$$\frac{1}{2} \text{tr} \left[ i\sigma^{ij} \gamma_5 \phi(x, p_T) \right] = S_T^j h^q_1(x, p_T) + S_L \frac{p_T^j}{M} h^{l,q}_1(x, p_T) + \frac{(p_T^j p_T^k - \frac{1}{2} p_T^2 \delta^{jk}) S^k_T}{M^2} h^l_1(x, p_T) + \frac{\varepsilon^{jk} p^j_T}{M} h^{l,q}_1(x, p_T).$$
In the present contribution we report about new results related to the polarized distributions [35].

In our approach all polarized leading-twist T-even TMDs are described in terms of the same polarized covariant 3D distribution \( H(p^0) \). This follows from the compliance of the approach with relations following from QCD equations of motion [23]. As a consequence all polarized TMDs can be expressed in terms a single “generating function” \( K^q(x, p_T) \) as follows

\[
g^q_1(x, p_T) = \frac{1}{2x} \left( x + \frac{m}{M} \right)^2 \frac{-p_T^2}{M^2} \times K^q(x, p_T),
\]

\[
h^q_1(x, p_T) = \frac{1}{2x} \left( x + \frac{m}{M} \right)^2 \times K^q(x, p_T),
\]

\[
g^{\perp q}_{1T}(x, p_T) = \frac{1}{x} \left( x + \frac{m}{M} \right) \times K^q(x, p_T),
\]

\[
h^{\perp q}_{1T}(x, p_T) = -\frac{1}{x} \times K^q(x, p_T),
\]

with the “generating function” \( K^q(x, p_T) \) defined (in the compact notation of [23]) by

\[
K^q(x, p_T) = M^2 x \int d\{p^1\} \equiv d\{p^1\} \equiv \frac{d}{dp^0} \frac{H^q(p^0)}{p^0 + m} \delta \left( \frac{p^0 - p^1}{M} - x \right).
\]

We have shown that due to rotational symmetry the following relations hold:

\[
K^q(x, p_T) = M^2 \left( 1 + \frac{p_T^2 + m^2}{x^2 M^2} \right),
\]

\[
\pi x^2 M^4 H^q \left( \frac{M}{2x} \right) = 2 \int_x^1 \frac{dy}{y} g^q_1(y) + 3 g^q_1(x) - \frac{d g^q_1(x)}{dx},
\]

where we took the limit \( m \to 0 \) in (7). In that limit we obtain for the generating function (6) the result

\[
K^q(x, p_T) = 2 x - \xi \left( 1 + \frac{p_T^2}{x^2 M^2} \right),
\]

and from (4) we obtain

\[
g^{\perp q}_{1T}(x, p_T) = \frac{2 x - \xi}{\xi^3 M^2} \left( 2 \int_\xi^1 \frac{dy}{y} g^q_1(y) + 3 g^q_1(\xi) - \xi \frac{d g^q_1(\xi)}{d\xi} \right).
\]

This relation yields for \( g^{\perp q}_{1T}(x, p_T) \), with the LO parameterization of [36] for \( g^{\perp q}_1(x) \) at 4 GeV\(^2\), the results shown in Fig. 1.

The remarkable observation is that \( g^{\perp q}_1(x, p_T) \) changes sign at the point \( p_T = M x \), which is due to the prefactor (this is the definition of the variable \( \bar{p}^1 \) in the limit \( m \to 0 \))

\[
2 x - \xi = x \left( 1 - \frac{(p_T / M x)^2}{2} \right) = -2 \bar{p}_0 / M \]
in (9). The expression in (10) is proportional to the quark longitudinal momentum $\bar{p}^1$ in the proton rest frame, which is determined by $x$ and $p_T$ [35]. This means, that the sign of $g_1^q(x, p_T)$ is controlled by sign of $\bar{p}^1$. To observe these dramatic sign changes one may look for multi-hadron jet-like final states in SIDIS. Performing the cutoff for transverse momenta from below and from above, respectively, should affect the sign of asymmetry.

There is some similarity to $g_2^q(x)$ which also changes sign, and is given in the model by [21]

$$g_2^q(x) = \frac{1}{2} \int H^q(p^0) \left( p^1 - \frac{(p^1)^2 - p_T^2/2}{p^0 + m} \right) \delta \left( \frac{p^0 - p^1}{M} - x \right) \frac{d^3p}{p^0}. \quad (11)$$

The $\delta-$function implies that, for our choice of the light-cone direction, large $x$ are correlated with large and negative $p^1$, while low $x$ are correlated with large and positive $p^1$. Thus, $g_2(x)$ changes sign, because the integrand in (11) changes sign between the extreme values of $p^1$. Let us remark, that the calculation of $g_2(x)$ based on the relation (11) well agrees [19] with the experimental data.

The other TMDs (4) can be calculated similarly and differ, in the limit $m \to 0$, by simple $x$-dependent prefactors

$$h_1^{qL}(x, p_T) = \frac{x}{2} K^q(x, p_T), \quad g_{1T}^{qL}(x, p_T) = K^q(x, p_T), \quad h_{1T}^{qT}(x, p_T) = -\frac{1}{x} K^q(x, p_T). \quad (12)$$

The resulting plots are shown in Fig. 2. We do not plot $h_{1L}^{qT}$ since this TMD is equal to $-g_{1T}^{qL}$ in our approach [23]. Let us remark, that $g_{1T}^q(x, p_T)$ is the only TMD which can change sign. The other TMDs have all definite signs, which follows from (4, 12). Note also that pretzelosity $h_{1T}^{qT}(x, p_T)$, due to the prefactor $1/x$, has the largest absolute value among all TMDs. Noteworthy, pretzelosity is related to quark orbital angular momentum in some quark models [32, 33], including the present approach [26].
Figure 2. The TMDs $h_1^q(x, p_T)$, $g_1^q(x, p_T)$, $h_1^q(x, p_T)$ for u- and d-quarks. Left panel: The TMDs as functions of $x$ for $p_T/M = 0.10$ (dashed), 0.13 (dotted), 0.20(dash-dotted lines). Right panel: The TMDs as functions of $p_T/M$ for $x = 0.15$ (solid), 0.18 (dashed), 0.22 (dotted), 0.30 (dash-dotted lines).
To conclude, let us remark that an experimental check of the predicted TMDs requires care. In fact, TMDs are not directly measurable quantities unlike structure functions. What one can measure for instance in semi-inclusive DIS is a convolution with a quark fragmentation function. This naturally “dilutes” the effects of TMDs, and makes it difficult to observe for instance the prominent sign change in the helicity distribution, see Fig. 1. A dedicated study of the phenomenological implications of our results is in progress.

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References
[1] Collins J C 2003 Acta Phys. Polon. B 34 3103, Collins J C, Rogers T C and Stasto A M 2008 Phys. Rev. D 77 085009, Collins J C and Hautmann F 2000 Phys. Lett. B 472 129, 2001 J. High Energy Phys. 03 016, Hautmann F 2007 Phys. Lett. B 655 26
[2] Mulders P J and Tangerman R D 1996 Nucl. Phys. B 461, 197 [1997 Erratum-ibid. B 484 538], Bacchetta A, Diehl M, Goeke K, Metz A, Mulders P J and Schlegel M 2007 JHEP 0702 093
[3] Cahn R N, 1978 Phys. Lett. B 78 269, 1989 Phys. Rev. D 40 3107
[4] Airapetian A et al. [HERMES Collaboration] 2000 Phys. Rev. Lett. 84 4047, 2001 Phys. Rev. D 64, 097101
[5] Avakian H et al. [CLAS Collaboration] 2004 Phys. Rev. D 69 112004
[6] Airapetian A et al. [HERMES Collaboration] 2005 Phys. Rev. Lett. 94 012002
[7] Alexakhin V Y et al. [COMPASS Collaboration] 2005 Phys. Rev. Lett. 94 202002
[8] Ageev E S et al. [COMPASS Collaboration] 2007 Nucl. Phys. B 765 31
[9] Alekseev M et al. [COMPASS Collaboration] 2009 Phys. Lett. B673 127
[10] Airapetian A et al. [HERMES Collaboration] 2009 Phys. Rev. Lett. 103 152002, 2010 Phys. Lett. B693 11
[11] Avakian H et al. [CLAS Collaboration] arXiv:1003.4549 [hep-ex]
[12] Alekseev M G et al. [COMPASS Collaboration] arXiv:1007.1562 [hep-ex]
[13] Adams D L et al. [E581 Collaboration and E704 Collaboration] 1991 Phys. Lett. B 261 201
[14] Adams D L et al. [FNAL-E704 Collaboration] 1991 Phys. Lett. B 264 462
[15] Bravar A et al. [Fermilab E704 Collaboration] 1996 Phys. Rev. Lett. 77 2626
[16] Adams J et al. [STAR Collaboration] 2004 Phys. Rev. Lett. 92 171801
[17] Zavada P 1997 Phys. Rev. D 55 4290
[18] Zavada P 2002 Phys. Rev. D 65 054040
[19] Zavada P 2003 Phys. Rev. D 67 014019
[20] Efremov A V, Teryaev O V and Zavada P 2004 Phys. Rev. D 70 054018
[21] Zavada P 2007 Eur. Phys. J. C 52 121
[22] Efremov A V, Schweitzer P, Teryaev O V and Zavada P 2009 AIP Conf. Proc. 1149 547
[23] Efremov A V, Schweitzer P, Teryaev O V and Zavada P 2009 Phys. Rev. D 80 014021
[24] Zavada P 2011 Phys. Rev. D 83 014022
[25] Efremov A V, Schweitzer P, Teryaev O V and Zavada P arXiv:0912.3380 [hep-ph]
[26] Efremov A V, Schweitzer P, Teryaev O V and Zavada P 2010 PoS DIS2010 253 arXiv:1008.3827 [hep-ph], Avakian H, Efremov A V, Schweitzer P, Teryaev O V and Zavada P arXiv:1008.1921 [hep-ph]
[27] Jakob R, Mulders P J and Rodrigues J 1997 Nucl. Phys. A 626 937
[28] Meissner S, Metz A and Goeke K 2007 Phys. Rev. D 76 034002
[29] Avakian H, Efremov A V, Goeke K, Metz A, Schweitzer P and Teckentrup T 2008 Phys. Rev. D 77 014023
[30] Avakian H, Efremov A V, Schweitzer P and Yuan F 2008 Phys. Rev. D 78 114024, Avakian H, Efremov A V, Schweitzer P, Teryaev O V, Yuan F and Zavada P 2009 Mod. Phys. Lett. A 24 2995
[31] Pasquini B, Cazzaniga S and Boffi S 2008 Phys. Rev. D 78 034025
[32] She J, Zhu J and Ma B Q 2009 Phys. Rev. D 79 054008
[33] Avakian H, Efremov A V, Schweitzer P and Yuan F 2010 Phys. Rev. D 81 074035
[34] Pasquini B and Lorcé C arXiv:1008.0945 [hep-ph]
[35] Efremov A V, Schweitzer P, Teryaev O V and Zavada P arXiv:1012.5296 [hep-ph]
[36] Leader E, Sidorov A V and Stamenov D B 2006 Phys. Rev. D 73 034023