Higgs decay to light scalars in the semi-constrained NMSSM

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The next-to minimal supersymmetric standard model (NMSSM) with non-universal Higgs masses, or the semi-constrained NMSSM (scNMSSM), extend the minimal supersymmetric standard model (MSSM) by a singlet superfield and assume universal conditions except for the Higgs sector. It can not only keep the simpleness and grace of the fully constrained MSSM and NMSSM, and relax the tension that they face after the 125-GeV Higgs boson discovered, but also predict an exotic phenomenon that Higgs decay to a pair of light singlet-dominated scalars (10 ∼ 60 GeV). This condition can be classified to three scenarios according to the identities of the SM-like Higgs and the light scalar: (i) the light scalar is CP-odd, and the SM-like Higgs is $h_2$; (ii) the light scalar is CP-odd, and the SM-like Higgs is $h_1$; (iii) the light scalar is CP-even, and the SM-like Higgs is $h_2$. In this work, we compare the three scenarios, checking the interesting parameter schemes that lead to the scenarios, the mixing levels of the doublets and singlets, the tri-scalar coupling between the SM-like Higgs and a pair of light scalars, the branching ratio of Higgs decay to the light scalars, and sensitivities in hunting for the exotic decay at the HL-LHC and the future lepton colliders such as CEPC, FCC-ee, and ILC.

I. INTRODUCTION

In 2012 a new boson of about 125 GeV was discovered at the LHC [1, 2], and in later years it was verified as the SM-like Higgs boson with more and more data [3–7]. But some other questions still exist, e.g., whether another scalar survives in the low mass region, and whether there is exotic Higgs decay to light scalars. Before the LHC, for the low integrated luminosity (IL) the LEP did not exclude a light scalar with a smaller production rate than the SM-like Higgs [8]. The CMS (ATLAS) collaboration searched for resonances directly in $b\bar{b}\mu\mu$ channel in the $10 \sim 60$ (20 ∼ 70) GeV [9, 10]. The two collaborations also searched for the exotic Higgs decay to light resonances in final states with $b\bar{b}\tau^+\tau^-$ [11], $bb\mu^+\mu^-$ [12, 13], $\mu^+\mu^-\tau^+\tau^-$ [14–16], $4\tau$ [16, 17], $4\mu$ [18–20], $4b$ [21], $\gamma\gamma gg$ [22], $4\gamma$ [23]. But there is still sufficient space left of physics on the exotic decay. For example, in the $b\bar{b}\tau^+\tau^-$ channel reported by CMS collaboration [11], the 95% exclusion limit is 3% at least in the 20 ∼ 60 GeV region. But according to simulations, the future limits can be 0.3% at the High-Luminosity program of the Large Hadron Collider (HL-LHC) [24], 0.04% at the Circular Electron Positron Collider (CEPC), and 0.02% at the Future Circular Colliders in $e^+e^-$ collisions (FCC-ee) [25, 26].

This exotic Higgs decay to light scalars can be motivated in many theories beyond the Standard Model (BSM) [27], e.g., the next-to minimal supersymmetric standard model (NMSSM), the simplest little Higgs model, the minimal dilataton model, the two-Higgs-doublet model, the next-to two-Higgs-doublet model, the singlet extension of the SM, etc. Several phenomenological studies on the exotic decay exist in these models [28–42].

The NMSSM extend the MSSM by a singlet superfield $\hat{S}$, solving the $\mu$-problem of it, and relax its fine-tuning tension after Higgs discovered in 2012 [43–49]. However, as supersymmetric (SUSY) models, the MSSM and NMSSM both suffer from a huge parameter space of over 100 dimensions. In most studies, some parameters are assumed equal at low-energy scale manually, leaving only about 10 free ones, and without considering the Renormalization Group Equations (RGEs) running from high scales [43–49]. In Ref.[33] a Higgs boson of 125 GeV decay to light scalars were studied in the NMSSM with parameters set in this way. While in constrained models, congeneric parameters are assumed universal at the Grand Unified Theoretical (GUT) scale, leaving only four free parameters in the fully-constrained MSSM (CMSSM) and four or five in the fully-constrained NMSSM (CNMSSM) [50–57]. However, it was found that CMSSM and CNMSSM were nearly excluded considering the 125 GeV Higgs data, high mass bounds of gluino and squarks in the first two generations, muon $g-2$, dark matter relic density and detections [56–62].

The semi-constrained NMSSM (scNMSSM) relaxes the unified conditions of the Higgs sector at the GUT scale, thus it is also called NMSSM with non-universal Higgs mass (NUHM) [63–66]. It not only keeps the simpleness and grace of the CMSM and CNMSSM, but also relax the tension that they facing after the SM-like Higgs discovered [67], and also predicts interesting light particles such as a singlino-like neutralino [68], and light Higgsino-dominated NLSPs [69–71], etc. In this work, we study the scenarios in the scNMSSM with a light scalar of 10 ∼ 60 GeV, and the detections of exotic Higgs decay to a pair of it.

The main point of this paper is listed as follows. In Sec. II, we introduce the model briefly and give some related analytic formulas. In Sec. III we present in detail the numerical calculations and discussions. Finally, we draw our conclusions in Sec. IV.
II. THE MODEL AND ANALYTIC CALCULATIONS

The superpotential of NMSSM, with $Z_3$ symmetry, is written as [72]

$$W = W_{Yuk} + \lambda S \tilde{H}_u \cdot \tilde{H}_d + \frac{1}{3} \kappa \tilde{S}^3,$$

from which the so-called F-terms of the Higgs potential can be derived as

$$V_F = |\lambda S|^2(|H_u|^2 + |H_d|^2) + |\lambda H_u \cdot H_d + \kappa S|^2.$$

The D-terms is the same as in the MSSM

$$V_D = \frac{1}{8} (g_1^2 + g_2^2) (|H_d|^2 - |H_u|^2)^2 + \frac{1}{2} g_3^2 |H_u \cdot H_d|^2,$$

where $g_1$ and $g_2$ are the gauge couplings of $U(1)_Y$ and $SU(2)_L$ respectively. Without considering the SUSY-breaking mechanism, at a low-energy scale the soft-breaking terms can be imposed manually to the Lagrangian. In the Higgs sector these terms corresponding to the superpotential are

$$V_{soft} = M_{H_u}^2 |H_u|^2 + M_{H_d}^2 |H_d|^2 + M_S^2 |S|^2 + \left(\lambda A_s SH_u \cdot H_d + \frac{1}{3} \kappa A_s S^3 + h.c.\right),$$

where $M_{H_u}, M_{H_d}, M_S^2$ are the soft masses of Higgs fields $H_u, H_d, S$ and $A_s, A_k$ are the trilinear couplings at $M_{SUSY}$ scale respectively. However, in the scNMSSM the SUSY breaking is mediated by gravity, thus the soft-parameters at $M_{SUSY}$ scale are running naturally from the GUT scale complying with the RGEs.

At electroweak symmetry breaking, $H_u, H_d$ and $S$ get their vacuum expectation values (VEVs) $v_u, v_d$ and $v_s$ respectively, with $\tan \beta = v_u/v_d$, $\sqrt{v_u^2 + v_d^2} \approx 173$ GeV, and $\mu_{eff} = \lambda v_s$. Then they can be written as

$$H_u = \left(\begin{array}{c}H_u^+ \\ v_u + \frac{\phi_1 + i \phi_3}{\sqrt{2}}\end{array}\right), \quad H_d = \left(\begin{array}{c}v_d + \frac{\phi_2 + i \phi_3}{\sqrt{2}} \\ H_d^+\end{array}\right),$$

$$S = v_s + \frac{\phi_3 + i \phi_1}{\sqrt{2}}. \quad (5)$$

The Lagrangian is consist of the F-terms, D-terms, and soft-breaking terms, so with the above equations one can get the tree-level squared-mass matrix of CP-even Higgses in the base $\{\phi_1, \phi_2, \phi_3\}$ and CP-odd Higgses in the base $\{\varphi_1, \varphi_2, \varphi_3\}$ [72]. After diagonalizing the mass squared matrices including loop corrections [73], one can get the mass-eigenstate Higgses (three CP-even ones $h_{1,2,3}$ and two CP-odd ones $a_{1,2}$, in mass order) from the gauge-eigenstate ones ($\phi_{1,2,3}, \varphi_{1,2,3}$):

$$h_i = S_{ik} \phi_k, \quad a_j = P_{jk} \varphi_k, \quad (6)$$

where $S_{ik}, P_{jk}$ are the corresponding components of $\phi_k$ in $h_i$ and $\varphi_k$ in $a_j$ respectively, with $i, k = 1, 2, 3$ and $j = 1, 2$.

In the scNMSSM, the SM-like Higgses (hereafter denoted as $h$ uniformly) can be CP-even $h_1$ or $h_2$, and the light scalar (hereafter denoted as $s$ uniformly) can be CP-odd $a_1$ or CP-even $h_1$. Then the couplings between the SM-like Higgs and a pair of light scalars $C_{hss}$ can be written at tree level as [74]

$$C_{h_2 h_1 h_1} = \frac{\lambda^2}{\sqrt{2}} \left[ v_u (\Pi_{211}^{122} + \Pi_{211}^{133}) + \frac{v_d (\Pi_{211}^{121} + \Pi_{211}^{233}) + v_s (\Pi_{211}^{311} + \Pi_{211}^{222})} + \frac{\lambda v_s (\Pi_{211}^{311} + \Pi_{211}^{222})}{\sqrt{2}} \right] \right]$$

or

$$C_{h_{a1} h_{a1}} = \frac{\lambda^2}{\sqrt{2}} \left[ v_u (\Pi_{211}^{122} + \Pi_{211}^{133}) + \frac{v_d (\Pi_{211}^{121} + \Pi_{211}^{233}) + v_s (\Pi_{211}^{311} + \Pi_{211}^{222})}{\sqrt{2}} \right] \right]$$

where $\Pi_{211}^{ijk} = 2 S_{2i} S_{1j} S_{1k} + 2 S_{1i} S_{2j} S_{1k} + 2 S_{1i} S_{1j} S_{2k}$.

Then the light scalars continually decay to light SM particles, such as a pair of light quarks or leptons, or gluons or photons though loops. The widths of light scalar decay to quarks and charged leptons at tree level are given by

$$\Gamma(h \rightarrow s s) = \frac{1}{32 \pi m_h} C_{h s s}^2 \left(1 - \frac{4 m_s^2}{m_h^2}\right)^{1/2}. \quad (9)$$

Then the light scalars continually decay to light SM particles, such as a pair of light quarks or leptons, or gluons or photons though loops. The widths of light scalar decay to quarks and charged leptons at tree level are given by

$$\Gamma(s \rightarrow l^+ l^-) = \frac{\sqrt{2} G_F}{8 \pi} m_s m_l^2 \left(1 - \frac{4 m_s^2}{m_l^2}\right)^{p/2}, \quad (10)$$

$$\Gamma(s \rightarrow q \bar{q}) = \frac{N_c G_F}{4 \sqrt{2} \pi} C_{sqq}^2 m_s m_q^2 \left(1 - \frac{4 m_q^2}{m_s^2}\right)^{p/2}, \quad (11)$$

where $p = 1$ for CP-odd $s$, and $p = 3$ for CP-even $s$. And the couplings between light scalar and up-type or down-type
quarks are given by
\[ C_{h_1t_Lt_R} = \frac{m_t}{\sqrt{2v\sin\beta}} S_{11}, \]  
\[ C_{h_1b_Lb_R} = \frac{m_b}{\sqrt{2v\cos\beta}} S_{12}, \]  
\[ C_{a_1t_Lt_R} = i \frac{m_t}{\sqrt{2v\sin\beta}} P_{11}, \]  
\[ C_{a_1b_Lb_R} = i \frac{m_b}{\sqrt{2v\cos\beta}} P_{12}. \]

### III. NUMERICAL CALCULATIONS AND DISCUSSIONS

In this work, we first scan the following parameter space with NMSSMTools-5.5.2 [74, 75].

\[ 0 < \lambda < 0.7, \quad 0 < \kappa < 0.7, \quad 1 < \tan\beta < 30, \]
\[ 100 < \mu_{\text{eff}} < 200 \text{ GeV}, \quad 0 < M_0 < 500 \text{ GeV}, \]
\[ 0.5 < M_{1/2} < 2 \text{ TeV}, \quad |A_0|, |A_{\lambda}|, |A_{\kappa}| < 10 \text{ TeV}. \]

The constraints we imposed in our scan including: (i) An SM-like Higgs of 123 ~ 127 GeV, with signal strengths and couplings satisfying the current Higgs data [3–7]. (ii) Search results for exotic and invisible decay of the SM-like Higgs, and Higgs-like resonances in other mass regions, with HIGGSBOUNDS-5.7.1 [76–78]. (iii) The muon g–2 constraint, like in Ref. [68]. (iv) The mass bounds of gluino and the first-two-generation squark over 2 TeV, and search results for electroweakinos in multilepton channels [79]. (v) The dark matter relic density \( \Omega h^2 \) below 0.131 [80], and the dark matter and nucleon scattering cross section below the upper limits in direct searches [81, 82]. (vii) The theoretical constraints of vacuum stability and Landau pole. After these constraints, the surviving samples can be categorized into three scenarios:

- **Scenario I:** \( h_2 \) is the SM-like Higgs, and the light scalar \( a_1 \) is CP-odd;
- **Scenario II:** \( h_1 \) is the SM-like Higgs, and the light scalar \( a_1 \) is CP-odd;
- **Scenario III:** \( h_2 \) is the SM-like Higgs, and the light scalar \( h_1 \) is CP-even.

In Tab. I, we list the ranges of parameters and light particle masses in the three scenarios. From the table, one can see that the parameter ranges are nearly the same expect for \( \lambda, \kappa, \) and \( A_\kappa, \) but the mass spectrums for light particles are totally different.

To study the different mechanisms of Higgs decay to light scalars in different scenarios, we recombine relevant parameters, and show them in Fig.1. From this figure one can find that:

- For Scenarios I and III, \( \lambda A_\kappa S_{22} \approx \lambda^2 v_s, \) where \( 0.03 \lesssim S_{22} \lesssim 0.07 \) is at the same order with \( 1 / \tan\beta, \) for the mass scale of the CP-odd doublet scalar \( M_A \sim 2\mu_{\text{eff}} / \sin2\beta \sim A_\lambda \gg \kappa v_s \) and \( \tan\beta \gg 1 \) [33]. Thus the SM-like Higgs is up-type-doublet dominated.

| Scenario I | Scenario II | Scenario III |
|------------|-------------|--------------|
| \( \lambda \) | 0 ~ 0.58 | 0 ~ 0.24 | 0 ~ 0.57 |
| \( \kappa \) | 0 ~ 0.21 | 0 ~ 0.67 | 0 ~ 0.36 |
| \( \tan\beta \) | 14 ~ 27 | 10 ~ 28 | 13 ~ 28 |
| \( \mu_{\text{eff}} \) [GeV] | 103 ~ 200 | 102 ~ 200 | 102 ~ 200 |
| \( M_0 \) [GeV] | 0 ~ 500 | 0 ~ 500 | 0 ~ 500 |
| \( M_{1/2} \) [TeV] | 1.06 ~ 1.47 | 1.04 ~ 1.44 | 1.05 ~ 1.47 |
| \( A_0 \) [TeV] | -2.8 ~ 0.2 | -3.2 ~ -1.0 | -2.8 ~ 0.6 |
| \( A_\lambda(M_{\text{GUT}}) \) [TeV] | 1.3 ~ 9.4 | 0.1 ~ 10 | 1.1 ~ 9.8 |
| \( A_\kappa(M_{\text{GUT}}) \) [TeV] | -0.02 ~ 5.4 | -0.02 ~ 0.9 | -0.7 ~ 5.7 |
| \( A_\kappa(M_{\text{SUSY}}) \) [TeV] | 2.0 ~ 10.1 | 0.8 ~ 10.9 | 1.6 ~ 10.2 |
| \( A_\kappa(M_{\text{SUSY}}) \) [GeV] | -51 ~ 42 | -17 ~ 7 | -803 ~ 11 |
| \( m_{h_1} \) [GeV] | 3 ~ 129 | 98 ~ 198 | 3 ~ 190 |
| \( m_{h_1} \) [GeV] | 4 ~ 123 | 123 ~ 127 | 4 ~ 60 |
| \( m_{\kappa} \) [GeV] | 123 ~ 127 | 127 ~ 5058 | 123 ~ 127 |
| \( m_{\kappa} \) [GeV] | 4 ~ 60 | 0.5 ~ 60 | 3 ~ 697 |

- For Scenario I, \( \kappa A_\kappa, k^2 v_s, \) and \( \lambda \kappa v_s \) are at the same level of a few GeV; but for Scenario II, \( k^2 v_s \) can be as large as a few TeV for small \( \lambda \) and large \( \kappa. \)

- Specially, for Scenario III, \( \kappa A_\kappa \approx -4\kappa^2 v_s, \) or \( A_\kappa \approx -4\kappa v_s. \)

According to the large data of the 125 GeV Higgs, and current null results searching for non-SM Higgs, the 125 GeV Higgs should be doublet dominated and the light scalar should be singlet dominated. Therefore, both the singlet component in the SM-like Higgs and the doublet component in the light Higgs should be a small quantity generally. We show how small they can be, and their relative scale in Fig.2. From this figure, we can see as following for the three scenarios:

- **Scenario I:** The up-type-doublet component of the light scalar \( -0.0015 \lesssim P_{11} < 0 \) is and proportional to the parameter \( \lambda \), thus the total doublet component of the light scalar \( P_{1D} \equiv \sqrt{P_{11}^2 + P_{12}^2} \approx P_{11} \tan\beta \ll 0.04 \); while the singlet component of the SM-like Higgs \( |S_{23}| \lesssim 0.3 \).

- **Scenario II:** The up-type-doublet component of the light scalar \( -0.0006 \lesssim P_{11} < 0 \) is and proportional to the parameter \( \lambda \), thus total doublet component of the light scalar \( 0 < P_{1D} \lesssim 0.013 \); while the singlet component in the SM-like Higgs \( |S_{13}| \lesssim 0.3 \).

- **Scenario III:** The up-type-doublet component of the light scalar and the singlet component of the SM-like Higgs are anticorrelated \( S_{11} \approx -S_{23} \), and the range of them is \( -0.15 \lesssim S_{11} \lesssim 0.2 \), with the sign related...
FIG. 1. Surviving samples for the three scenarios in the $\lambda A_s S_{22}$ versus $\lambda^2 v_u$ (upper), where $S_{22}$ (left and right) and $S_{12}$ (middle) are the down-type-doublet component coefficient in the SM-like Higgs, and $\kappa A_\kappa$ versus $\kappa^2 v_u$ (lower) planes respectively. Colors indicate $\lambda^2 v_u$ (upper) and $\lambda \kappa v_u$ (lower) respectively.

FIG. 2. Surviving samples for the three scenarios in the $P_{11}$ versus $S_{23}$ (left), $P_{11}$ versus $S_{13}$ (middle), and $S_{11}$ versus $S_{23}$ (right) planes respectively, where $S_{23}$ (left and right) and $S_{13}$ (middle) are the singlet component in the SM-like Higgs, and $P_{11}$ (left and middle) and $S_{11}$ (right) are the up-type-doublet component of the light scalar respectively. Colors indicate the parameter $\lambda$. 
Considering the values of and correlations among parameters and component coefficients, the couplings between the SM-like Higgs and a pair of light scalars can be simplified as:

\[
C_{h_2, a_1} \simeq \sqrt{2} \lambda_{v} u + \sqrt{2} \lambda A_1 P_{11} \tan \beta ,
\]

\[
C_{h_1, a_1} \simeq \sqrt{2} \lambda_{v} u + \sqrt{2} \lambda A_1 P_{11} \tan \beta + 2 \sqrt{2} \lambda \kappa v S_{13} ,
\]

\[
C_{h_2, h_1} \simeq \sqrt{2} \lambda_{v} u - \sqrt{2} \lambda A_1 S_{12} + \sqrt{2} \lambda \kappa v S_{11} + 2 \sqrt{2} \lambda \kappa v S_{23} + \frac{3 \kappa^2}{2} v S_{11} S_{11} - 2 \sqrt{2} \lambda \kappa v S_{12} .
\]

In Fig. 3 we show the exotic branching ratio \( Br(h \rightarrow ss) \) including one-loop correction correlated with the mass of the light scalar, and the coupling between the SM-like Higgs and a pair of the light scalars at tree level. Since the 125 GeV Higgs is constrained to be very SM-like, its decay widths and branching ratios to SM particles cannot vary much. Thus combined with Eq.(9), it is natural that the branching ratios to light scalars are proportional to the square of the tri-scalar couplings. The significant deviations for the negative-coupling samples in Scenario III are because of the one-loop correction of the stop loops,

\[
\Delta C_{h_2, h_1} \simeq S_{21} S_{11}^2 \frac{3 \sqrt{2} m_4^4}{16 \pi^2 v_{u}^2} \ln \left( \frac{m_{t} m_{h_2}}{m_{h_1}^2} \right) ,
\]

which can be as large as 5 GeV. While for Scenario I and II, they are

\[
\Delta C_{h_2, a_1} \simeq S_{21} P_{11}^2 \frac{3 \sqrt{2} m_4^4}{16 \pi^2 v_{u}^2} \ln \left( \frac{m_{t} m_{h_2}}{m_{a_1}^2} \right) ,
\]

\[
\Delta C_{h_1, a_1} \simeq S_{11} P_{11}^2 \frac{3 \sqrt{2} m_4^4}{16 \pi^2 v_{u}^2} \ln \left( \frac{m_{t} m_{h_1}}{m_{a_1}^2} \right) .
\]

Since \( P_{11} \ll S_{11} \) as seen from Fig. 2 the loop correction in Scenario I and II is much smaller than that in Scenario III. In the following figures and discussions, we refer to the coupling \( C_{hss} \) as including the one-loop correction \( \Delta C_{hss} \) if without special instructions.

### A. Detections at the HL-LHC

At the LHC, the SM-like Higgs first can produce in gluon fusion (ggF), vector boson fusion (VBF), associated with vector boson (Wh, Zh), or associated with \( t \bar{t} \) processes, where cross section in the ggF process is much larger than that of others. Then the SM-like Higgs can decay to a pair of light scalars, and each scalar can then decay to a pair of fermions, or gluons, or photons. The ATLAS and CMS collaborations have searched for these exotic decay mode in final states of \( b \bar{b} \tau^+ \tau^- \) [11], \( b \mu^+ \mu^- \) [12, 13], \( \mu^+ \mu^- \tau^+ \tau^- \) [14–16], \( 4 \tau \) [16, 17], \( 4 \mu \) [18–20], \( 4 b \) [21], \( \gamma \gamma gg \) [22], \( 4 \gamma \) [23], etc. These results are included in the constraints we considered.

As we checked, the main decay mode of the light scalar is usually to \( b \bar{b} \) when \( m_s \gtrsim 2 m_t \). However, the color backgrounds at the LHC are very large, thus minor Zh production process is used in detecting \( h \rightarrow 2s \rightarrow 4b \), as well VBF used for \( h \rightarrow 2s \rightarrow \gamma \gamma gg \). For the other decay mode, the main production processes ggF can be used. Considering the cross sections of production and branching ratios of decay, and the precisions of detection, we found the detections in \( 4b, 2b2\tau, \) and \( 2\tau 2\mu \) channels are important for the scN MSSM.
FIG. 4. Surviving samples for the three scenarios in the signal rate \( \mu_{ggF} \times Br(h \rightarrow ss \rightarrow 4b) \) versus the mass of light Higgs \( m_s \) planes respectively, with colors indicate the tri-scalar coupling \( C_{hss} \) including one-loop correction, where \( h \) denote the SM-like Higgs \( h_2 \) (left and right) and \( h_1 \) (middle), and \( s \) denote the light scalar \( \alpha_1 \) (left and middle) and \( h_1 \) (right) respectively. The solid curves are the simulation result of the 95% exclusion limit in the corresponding channel at the HL-LHC with 300 fb\(^{-1} \) [33].

FIG. 5. Same as in Fig.4, but show the signal rate \( \mu_{ggF} \times Br(h \rightarrow ss \rightarrow 2\tau 2b) \), and 95% exclusion limits in the corresponding channel at the HL-LHC with 3000 fb\(^{-1} \) [24].

FIG. 6. Same as in Fig.4, but show the signal rate \( \mu_{ggF} \times Br(h \rightarrow ss \rightarrow 2\tau 2\mu) \), and 95% exclusion limits in the corresponding channel at the HL-LHC with 3000 fb\(^{-1} \) [24].
And the signal rates are \( \mu_{Zh} \times Br(h \rightarrow ss \rightarrow 4b) \), \( \mu_{ggF} \times Br(h \rightarrow ss \rightarrow 2b2\tau) \), and \( \mu_{ggF} \times Br(h \rightarrow ss \rightarrow 2\tau2\mu) \) respectively, where \( \mu_{ggF} \) and \( \mu_{Zh} \) are the ggF and Zh production rate normalized to their SM value respectively.

For detections of the exotic decay at the HL-LHC, we use the simulation results of 95% exclusion limit in Refs.\[24, 33\]. Suppose with integrated luminosity of \( L_0 \), the 95% exclusion limit for branching ratio in some channel is \( Br_0 \) in the simulation result, then for a sample in the model if the signal rate is \( \mu_i \times Br \) (\( i \) denote the production channel), the signal significance with integrated luminosity of \( L \) will be

\[
ss = 2 \frac{\mu_i \times Br}{Br_0} \sqrt{\frac{L}{L_0}}, \tag{23}
\]

and the integrated luminosity needed to exclude the sample in the channel at 95% confidence level (with \( ss = 2 \)) will be

\[
L_e = L_0 \left( \frac{Br_0}{\mu_i \times Br} \right)^2, \tag{24}
\]

and the integrated luminosity needed to discover the sample in the channel (with \( ss = 5 \)) will be

\[
L_d = L_0 \left( \frac{5}{2} \right)^2 \left( \frac{Br_0}{\mu_i \times Br} \right)^2. \tag{25}
\]

In Fig.4, 5, and 6, we show the signal rates for surviving samples in the three scenarios, and the 95% exclusion limits \[24, 33\] in the 4b, 2b2\( \tau \), and 2\( \tau \)2\( \mu \) channels respectively. From these figures one can see that

- With the light scalar heavier than 30 GeV, the easiest way to discover the exotic decay is in the 4b channel, and the minimal integrated luminosity needed to discover the decay in this channel can be 650 fb\(^{-1}\) for Scenario II.
- With the light scalar lighter than 20 GeV, the 2\( \tau \)2\( \mu \) channel can be important, especially for samples in the Scenario II, and the minimal integrated luminosity needed to discover the decay in this channel can be 1000 fb\(^{-1}\).
- With the light scalar heavier than 2\( m_\mu \), chance all exist to discover the decay in the 2b2\( \tau \) channel, and the minimal integrated luminosity needed to discover the decay in this channel can be 1500 fb\(^{-1}\) for Scenario II.

### B. Detections at the future lepton colliders

In future lepton colliders such as CEPC, FCC-ee, and International Linear Collider (ILC), the main production process of the SM-like Higgs is Zh, and the color backgrounds are very little, thus these lepton colliders are powerful in detecting the exotic decay. There have been simulation results in many channels, such as 4b, 4j, 2b2\( \tau \), 4\( \tau \), etc. \[26\]. With the same method as in the last subsection, one can do similar analyses.

In Fig.7, 8, 9, and 10, we show the signal rates for surviving samples in the three scenarios, and the 95% exclusion limits at the CEPC, FCC-ee, and ILC, and in the 4b, 4j, 2b2\( \tau \), and 4\( \tau \) channels respectively \[26\]. From these figures one can see that:

- As in Fig.7, when the light scalar is heavier than about 15 GeV and the tri-scalar coupling is large enough, the branching ratio of 4b channel is significant. The minimal integrated luminosity needed to discover the decay in this channel can be 0.31 fb\(^{-1}\) for Scenario II and III at the ILC.
- As in Fig.8, for Scenario I and II, the exotic Higgs decay can be expected to be observed in the 4j channel when its mass is lighter than 11 GeV. While for Scenario III, the light scalar available by CEPC can be as heavy as 40 GeV. And the minimal integrated luminosity needed to discover the exotic decay in this channel can be 18 fb\(^{-1}\) for Scenario II at the ILC.
- As in Fig.9 and 10, the signal rates in 2b2\( \tau \) and 4\( \tau \) channel are in similar trends. The branching ratios are tiny before the light scalar reaches the mass threshold, the maximum of branching ratios occur around \( m_\eta = 12 \) GeV, and the minimal integrated luminosity needed to discover the decay in 2b2\( \tau \) channel can be 3.6 fb\(^{-1}\) for Scenario II at the ILC, in 4\( \tau \) channel can be 0.22 fb\(^{-1}\) for Scenario III at the ILC.

### IV. CONCLUSIONS

In this work, we have discussed the exotic Higgs decay to a pair of light scalars in the scNMSM, or the NMSSM with NUHM. First, we did a general scan over the nine-dimension parameter space of the scNMSM, considering the theoretical constraints of vacuum stability and Landau pole, and experimental constraints of Higgs data, non-SM Higgs searches, muon g-2, particle searches, relic density and direct searches for dark matter, etc. Then we found three scenarios with a light scalar of 10 \( \sim \) 60 GeV: (i) the light scalar is CP-odd, and the SM-like Higgs is \( h_2 \); (ii) the light scalar is CP-odd, and the SM-like Higgs is \( h_1 \); (iii) the light scalar is CP-even, and the SM-like Higgs is \( h_2 \). For the three scenarios, we check the parameter schemes that lead to the scenarios, the mixing levels of the doublets and singlets, the tri-scalar coupling between the SM-like Higgs and a pair of light scalars, the branching ratio of Higgs decay to the light scalars, and the detections at the hadron colliders and future lepton colliders.

In this work, we compare the three scenarios, checking the interesting parameter schemes that lead to the scenarios, the mixing levels of the doublets and singlets, the tri-scalar coupling between the SM-like Higgs and a pair of light scalars, the branching ratio of Higgs decay to the light scalars, and the detections at the hadron colliders and future lepton colliders.
FIG. 7. Surviving samples for the three scenarios in the signal rate $\mu_{Zh} \times Br(h \rightarrow ss \rightarrow 4b)$ versus the mass of light Higgs $m_s$ planes respectively, with colors indicate the tri-scalar coupling $C_{hss}$ including one-loop correction, where $h$ denote the SM-like Higgs $h_2$ (left and right) and $h_1$ (middle), and $s$ denote the light scalar $a_1$ (left and middle) and $h_1$ (right) respectively. The solid, dashed, and dotted lines are the simulating result of 95% exclusion limit in the corresponding channel at the CEPC with 5 ab$^{-1}$, FCC-ee with 30 ab$^{-1}$, and ILC with 2 ab$^{-1}$ respectively [26].

FIG. 8. Same as in Fig.7, but show the signal rate $\mu_{Zh} \times Br(h \rightarrow ss \rightarrow 4j)$, and 95% exclusion limits in the corresponding channel [26]. The “4j” denotes four jets, including gluon and light quarks except for $b$.

FIG. 9. Same as in Fig.7, but show the signal rate $\mu_{Zh} \times Br(h \rightarrow ss \rightarrow 2b2\tau)$, and 95% exclusion limits in the corresponding channel [26].
Finally, we draw following conclusions regarding a light scalar, and the exotic Higgs decay to a pair of it in the scNMSSM:

- There are interesting different mechanisms in the three scenarios to tune parameters to get the small tri-scalar couplings.
- The singlet component of the SM-like Higgs in the three scenarios are at the same level of $\lesssim 0.3$, and is roughly one-order larger than the doublet component of the light scalar in Scenario I and II.
- The coupling between the SM-like Higgs and a pair of light scalars at tree level is $-3 \sim 5$, $-1 \sim 6$ and $-10 \sim 5$ GeV for Scenario I, II, and III respectively.
- The stop-loop correction to the tri-scalar coupling in Scenario III can be a few GeV, much larger than that in Scenario I and II.
- The most effective way to discover the exotic decay at the future lepton collider is in the $4\tau$ channel; while that at the HL-LHC is $4b$ for the light scalar heavier than 30 GeV, or $2b2\tau$ and $2\tau2\mu$ for a lighter scalar.

In details, the minimal integrated luminosity needed to discover the exotic Higgs decay at the HL-LHC, CEPC, FCC-ee, and ILC are summarized in Tab.II, and the tuning mechanisms in the three scenarios to get the small tri-scalar coupling can be seen from Figs. 1, 2 and Eqs. (17), (18), (19).

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TABLE II. The minimum integrated luminosity for discovering the exotic Higgs decay at the future colliders, where the “@I, II, III” means the three different scenarios.

| Deacy Mode | HL-LHC | CEPC | FCC-ee | ILC |
|------------|--------|------|--------|-----|
| ($b\bar{b})(b\bar{b})$ | 650 fb$^{-1}$(@II) | 0.42 fb$^{-1}$(@III) | 0.41 fb$^{-1}$(@III) | 0.31 fb$^{-1}$(@II) |
| ($j\bar{j})(j\bar{j})$ | - | 21 fb$^{-1}$(@II) | 18 fb$^{-1}$(@II) | 25 fb$^{-1}$(@II) |
| ($\tau^+\tau^-)(\tau^+\tau^-)$ | - | 0.26 fb$^{-1}$(@III) | 0.22 fb$^{-1}$(@III) | 0.31 fb$^{-1}$(@III) |
| ($b\bar{b})(\tau^+\tau^-)$ | 1500 fb$^{-1}$(@II) | 4.6 fb$^{-1}$(@II) | 3.6 fb$^{-1}$(@II) | 4.4 fb$^{-1}$(@II) |
| ($\mu^+\mu^-)(\tau^+\tau^-)$ | 1000 fb$^{-1}$(@II) | - | - | - |

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