Rare Kaon Decay to Missing Energy: Implications of the NA62 Result for a $Z'$ Model

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Meson decays offer a good opportunity to probe new physics. The rare kaon decay $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ is one of the cleanest of them and, for this reason, is rather sensitive to new physics, in particular, vector mediators. NA62 collaboration, running a fixed-target experiment at CERN, recently reported an unprecedented sensitivity to this decay, namely a branching fraction of $BR(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = (11^{+4.0}_{-3.5}) \times 10^{-11}$ at 68% C.L. Vector mediators that couple to neutrinos may yield a sizeable contribution to this decay. Motivated by the new measurement, we interpret this result in the context of a concrete $Z'$ model, and put our findings into perspective with the correlated $K_L \rightarrow \pi^0\nu\bar{\nu}$ decay measured by KOTO collaboration, current, and future colliders, namely the High-Luminosity and High-Energy LHC.

I. INTRODUCTION

Mesons have played a key role in the understanding of properties of elementary particles. The introduction of strangeness along with isospin lead us to the eight-fold way, based on the SU(3) flavor symmetry. These theoretical insights have contributed to the discovery of quantum chromodynamics as we know today. Another good example is the famous $\theta - \tau$ puzzle. Two different decays were found for charged strange mesons known at the time as $\theta^+$ and $\tau^+$. The decay modes had different parities, but the particles were shown to have the same mass and lifetime. It was indeed a puzzling observation. Later, it was realized that weak interactions violate parity, and these two particles were actually the same $K^+$-meson. Additionally, the Glashow-Iliopoulos-Maiani (GIM) mechanism and quark charm surfaced as an explanation of the absence of weak flavor changing neutral currents in the processes such as $K^+ \rightarrow \pi^+ \nu\bar{\nu}$. The discovery of CP violation in the $K^0 - \bar{K}^0$ system further proved the importance of meson physics for our understanding of nature. Furthermore, meson systems are able to access possible new sources of CP violation that are of paramount importance for explaining the observed baryon-antibaryon asymmetry in our universe [1]. Lastly, the $K^+$ rare decay into neutrinos can efficiently probe the presence of heavy vector mediators, beyond the Standard Model (SM), [2–9] via the Feynman diagrams displayed in Fig.1.

Figure 1. Feynman diagrams that illustrate how vector mediators can contribute to the $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ decay. The first diagram requires $Z'$ coupling to neutrinos, whereas the second further requires couplings to the top quark.

The meson decay $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ is a flavor changing neutral current process that occurs in the SM via the box and penguin diagrams, with the latter being dominated by the top quark contribution. Due to the GIM and loop suppression, the SM contribution to this decay is very small, reading $BR(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11}$ [10], while the NA62 currently imposes $BR(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = 11.0^{+4.0}_{-3.5} \times 10^{-11}$ [11, 12].

1 KOTO collaboration has also recently reported the observation of three anomalous events in the $K_L \rightarrow \pi^0\nu\bar{\nu}$ decay. This anomaly requires the branching ratio for this decay mode to be about two orders of magnitude larger than the SM one [4], indicating the presence of a new light and long-lived particle with mass of the order of 100 MeV. There is no such light particle in our model. Hence the KOTO anomaly will be regarded as a statistical fluke.

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and nicely hosting dark matter candidates [14–21], addressing neutrino masses [22–25], among other interesting phenomena [3, 26–33]. As a result of the enlarged gauge group, the 3-3-1 model have several new gauge bosons, such as \( W' \) and \( Z' \) bosons, which are subject to restrictive constraints rising from collider physics [34–37], muon anomalous magnetic moment [38–41], and lepton flavor violation [42, 43]. We will investigate the role of the \( Z' \) gauge boson in the rate \( K^+ \) decay and then use this to draw bounds on the \( Z' \) mass using the \( K_L \rightarrow \pi^0 \nu \bar{\nu}, K^+ \rightarrow \pi^+ \nu \bar{\nu} \) decays.

Our work is structured as follows: In Section II, we review the 3-3-1 model and compute the \( Z' \) couplings necessary to our analyses; in Section III, we discuss the computation of the branching fractions of the kaons in our model; in Section IV, we discuss our results and, finally in Section VI, we draw our conclusions.

\[ Q = \frac{1}{2} \left( \lambda_3 + \beta \lambda_8 \right) N I = \left( \begin{array}{c} 1 \frac{1}{2} + N \frac{1}{2} + N \frac{3}{2} + N \end{array} \right), \]

where \( \lambda_3 = \text{diag}(1, -1, 0), \lambda_8 = \text{diag}(1, 1, -2)/\sqrt{3}, \) and \( I \) are the diagonal Gell-Mann matrices and the identity matrix, respectively. We took \( \beta = -\frac{1}{\sqrt{3}} \) because in our work we choose the model known as 3-3-1 with right-handed neutrinos (3-3-1 r.h.n) [44, 45]. However, we highlight that our conclusions are also applicable to the 3-3-1 model with neutral fermions proposed in [14]. The hypercharge in this model is given as,

\[ Y = 2Q - \lambda_3 = 2N - \frac{\sqrt{3} \lambda_8}{3}, \]

which is identical to the standard model one. The left (L) and right (R)-handed fermionic fields of this model are represented as follows,

\[ f_L^a = \left( \begin{array}{c} \nu_L^a \\ \ell_L^a \\ (\nu_R^a) \end{array} \right) \sim (1, 3, -1/3), \]

\[ \ell_R^a \sim (1, 1, -1), \]

\[ Q_{ijL} = \left( \begin{array}{c} d_{iL} \\ -u_{iL} \\ d'_{iL} \end{array} \right) \sim (3, 3, 0), \]

\[ u_{iR} \sim (3, 1, 2/3), \]

\[ d_{iR} \sim (3, 1, -1/3), \]

\[ d'_{iR} \sim (3, 1, -1/3), \]

\[ Q_{3L} = \left( \begin{array}{c} u_{3L} \\ d_{3L} \\ T_L \end{array} \right) \sim (3, 3, 1/3), \]

\[ u_{3R} \sim (3, 1, 2/3), \]

\[ d_{3R} \sim (3, 1, -1/3), \]

\[ T_R \sim (3, 1, 2/3), \]

where \( a = 1, 2, 3 \) and \( i = 1, 2 \) are the generation indexes, \( f_L^a \) and \( Q_{ijL} \) represent the left-handed leptonic and quark triplets, respectively. These fields encompass the SM spectrum like neutrinos (\( \nu^a = \nu_e, \nu_\mu, \nu_\tau \)), charged leptons \( \ell^a = e, \mu, \tau \), and quarks \( u_i = \pi, \varphi, d_i = \varphi, \pi, \) \( u_3 = t \) and \( d_3 = b \). Besides, there are other particles additional to the SM: the right-handed neutrino \( (\nu_R^a) \) and three new heavy exotic quarks \( d''_{iL} \) and \( T_R \). In Eqs. (3), (4), we have specified the field assignments indicating how they transform under the symmetries \( \text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_N \), respectively. The values of their electric charge and hypercharge can be found from Eqs. (1) and (2).

Furthermore, the 3-3-1 r.h.n model contains three scalar fields \( \chi, \eta \) and \( \rho \) in the following representations

\[ \chi = \left( \begin{array}{c} \chi^0 \\ \chi^+ \\ \chi^- \end{array} \right) \sim (1, 3, -1/3), \]

\[ \rho = \left( \begin{array}{c} \rho^+ \\ \rho^0 \\ \rho^- \end{array} \right) \sim (1, 3, 2/3), \]

\[ \eta = \left( \begin{array}{c} \eta^0 \\ \eta^- \\ \eta^+ \end{array} \right) \sim (1, 3, -1/3). \]

These scalar triplets in Eq. (6) are responsible for the spontaneous symmetry breaking (SSB) in the model, with the following vacuum (vev) structure,

\[ \langle \chi \rangle = \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right), \langle \rho \rangle = \left( \begin{array}{c} 0 \\ v_\rho \\ 0 \end{array} \right), \langle \eta \rangle = \left( \begin{array}{c} 0 \\ v_\eta \\ 0 \end{array} \right). \]

We will assume that \( v_\chi \gg v_\eta, v_\rho \), leading to the two-step SSB,

\[ \text{SU}(3)_L \times \text{U}(1)_N \xrightarrow{\langle \chi \rangle} \text{SU}(2)_L \times \text{U}(1)_Y \xrightarrow{\langle \eta \rangle, \langle \rho \rangle} \text{U}(1)_Q \]

with \( U(1)_Q \) being the \( U(1) \) from electrodynamics.
The fermion masses rise from the Yukawa Lagrangian that gives its connection to the Z' boson, we need to extract flavor-changing neutral currents. To do so, we start by reviewing how the quark-changing terms arise. The quarks fields follow the standard rotation relation are, 

Table I. Z' coupling to neutrinos and the left-handed down-type quarks. 

In order to study meson physics in our model, and investigate the connection to the Z' boson, we need to extract flavor-changing neutral currents. To do so, we start by reviewing how the quark-changing terms arise. The quarks fields follow the standard rotation relation.

The fourth term leads to a $3 \times 3$ antisymmetric neutrino mass matrix $\mathbf{V}$ which means that the model has one massless and two degenerate neutrino mass eigenstates. Moreover, the flavor-changing terms arise as well. In addition to the SM fields, there is a new massive gauge boson, $Z'$, which is predicted by the model as a result of the enlarged (CC) and neutral (NC) currents are found to be. 

$$M_{\nu} = \frac{1}{2} g_{\nu} \left( M_1 + M_2 \right) + \left( M_1 - M_2 \right) \phi_{\nu} = \sin \theta_{\nu} \text{ and } C_{\nu} = \cos \theta_{\nu}.$$ 

Changing the basis we get: 

$$\Delta L \sim \frac{1}{\sqrt{2}} \Delta L_{\nu} \equiv \Delta L_{\nu}.$$ 

where $D_L = (d, l, s)$, and $D_R = (u, c, t).$ Using the unitarity of the $L_{\nu}$ matrix we finally find the coupling between the quarks $d$ and $s$, which is given by,

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where \(i, j = \sin \tilde{\theta}_{ij}, \tilde{c}_{ij} = \cos \tilde{\theta}_{ij}\) and \(\delta_j\) are the phases, with \(i, j = 1, 2, 3\). For our purposes, the following entries will be important,

\[
V_{LL}^d = -\tilde{s}_{13} e^{-i\delta_1} \\
V_{LL}^d = -\tilde{c}_{13} \tilde{s}_{23} e^{-i\delta_2}
\]

then, the product which appears in the \(\Delta L^d\) coupling is

\[
V_{LL}^d V_{LL}^d = -\tilde{s}_{13} \tilde{c}_{13} \tilde{s}_{23} \tilde{s}_{23} e^{-i(\delta_1 - \delta_2)} \equiv |V_{LL}^d V_{LL}^d| e^{-i\delta} \quad (19)
\]

where we leave the product \(|V_{LL}^d V_{LL}^d|\) and the phase \(\delta\) as free parameters.

### III. KAON DECAYS

The rare Kaon decay modes \(K^+ \to \pi^+ \nu \bar{\nu}\) and \(K_L \to \pi^0 \nu \bar{\nu}\) are considered good modes in flavor physics, as they are very well understood theoretically and are sensitive to new physics contributions. In the SM both decays occur only at loop level and are dominated by \(\pi\) physics contributions. In the SM both decays occur only at loop level and are dominated by \(\pi\) physics contributions. In the SM both decays occur only at loop level and are dominated by \(\pi\) physics contributions.

The decay \(K^+ \to \pi^+ \nu \bar{\nu}\) is CP conserving, whereas \(K_L \to \pi^0 \nu \bar{\nu}\) is CP violating. In the 331 model, the new sources of flavor Violating Kaon decays on the parameter space of the 331 model regarding meson FCNC processes, we consider are the phase interdependence leads to the Grossman-Nir limit \([53]\),

\[
\eta_X X_0(x_t) + \frac{\Delta L^L}{g_{SM} m Z'} + \frac{\Delta L^d}{V_{ts} V_{td}} \quad (22)
\]

and,\(g_{SM}^2 = 4 \frac{m_\ell^2 G_F^2}{2 \pi^2} = 1.78137 \times 10^{-7} \text{GeV}^{-2}\).

The first term in Eq. (22) represents the SM contribution, which is dominated by \(Z\)-penguin and box diagrams, and includes QCD and electroweak corrections. The factor \(\eta_X\) is close to unity, \(\eta_X = 0.994,\) and

\[
X_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \ln x_t \right]
\]

with \(x_t = \frac{m_t^2}{m_\ell^2}\). The 331 contribution is enclosed in the second term of Eq. (22), with \(\Delta L^L\) and \(\Delta L^d\) given in the Eqs. (14) and (15), respectively. If \(Z'\) is absent we have \(\Delta L^d = \Delta L^L = 0\) and the SM result is recovered.

The decays \(K^+ \to \pi^+ \nu \bar{\nu}\) and \(K_L \to \pi^0 \nu \bar{\nu}\) are related to each other in the SM, via isospin symmetry, since both transitions are ruled by the same short distance operator. This interdependence leads to the Grossman-Nir limit \([53]\),

\[
BR(K_L \to \pi^0 \nu \bar{\nu}) \leq 4.3 BR(K^+ \to \pi^+ \nu \bar{\nu}) \quad (23)
\]

For a fixed value of \(BR(K^+ \to \pi^+ \nu \bar{\nu})\), this theoretical bound provides an upper limit for \(BR(K_L \to \pi^0 \nu \bar{\nu})\), which is typically still stronger than the current experimental limits. The bound remains valid in SM extensions in which the new physics is much heavier than the Kaon mass. In particular, the 331 model obeys the bound, as we will show in the next section.

Having presented the formulas that summarizes the predictions of the 331 model regarding meson FCNC processes, we can now discuss the implications of the experimental searches for the flavor violating Kaon decays on the parameter space of the model.
In this section, we present our results for the FCNC processes using Eqs. (20) and (21), for the 331 r.h.n model, but we emphasize that our findings are also applicable to the 3-3-1-ILHN [14], which is a 3-3-1 version that features heavy neutral leptons instead of right-handed neutrinos. Anyway, we compare our results with the last limits from KOTO [54] and, NA62 Run1 (2016 + 2017 + 2018)² [11, 12]. In Fig. 2, we display the branching for $K_L$ (right panel) and $K^+$ (left panel) versus the $Z'$ mass for the 331 r.h.n model, namely combining the standard model contribution and the new one provided by the new $Z'$ boson, as mentioned above. We compute both branching varying the $Z'$ mass, the phase $\delta$ (color bar) and we fix the product $|V_{L32}^{d*}V_{L31}^d| = 10^{-3}$. From the left-panel of Fig. 2 one can conclude that the KOTO experiment excludes at most $Z'$ masses around 400 GeV, which occurs for $\delta > 1.4$. However, from the right-panel of Fig. 2, we find that $Z'$ masses below $3 \times 10^9$ GeV might be excluded depending on the value adopted for the phase $\delta$. For $\delta \to \pi/2$, the NA62 sensitivity to heavy $Z'$ mediators severely weakens. Hence, NA62 yields complementary limits to other existing probes [35, 55, 56]. We can also see the presence of a dip due to destructive interference between the $Z'$ and the SM contributions, where the branching ratio lies below the SM predicted value. In the plot it occurs for $Z'$ masses in the 600 – 900 GeV range, but in general its depth and location depends on the combination of $|V_{L32}^{d*}V_{L31}^d|$ and $\delta$. Notice that there is no dip when $\delta = \pi/2$, while it reaches its maximum depth for $\delta = 0$.

We have drawn quantitative conclusions based on a particular value for $V_{L32}^{d*}V_{L31}^d$. To assess how our bounds change for different choices of this product we examine the range $10^{-4} < V_{L32}^{d*}V_{L31}^d < 10^{-1}$. In Fig. 3 we show exclusion plots in the plane $V_{L32}^{d*}V_{L31}^d$ versus $m_{Z'}$, for fixed $\delta = 0$ (left-panel) and $\delta = \pi/2$ (right-panel). Here we combined limits from NA62, KOTO, and LHC, High-Luminosity LHC (HL-LHC) and High-Energy-LHC (HE-LHC). The strongest collider bounds on the $Z'$ mass stems from the resonant production of $Z'$ decaying into dileptons [40].

There is an interesting interplay between collider and flavor physics. While collider bounds rely mostly on the interaction strength between the $Z'$ boson and fermions, which is fixed in 3–3–1 models, the flavor bounds weaken with $|V_{L32}^{d*}V_{L31}^d|$. The collider bounds displayed in Fig. 3 are conservative, as they take into account the presence of $Z'$ decays into right-handed neutrinos and exotic quarks, which can be light enough so that the decays are not kinematically forbidden. The original lower mass bound reads $m_{Z'} > 4$ TeV [55], but those exotic decays were ignored in [55]. If more decay channels become available, then the bound weakens. Here, in order to take into account this uncertainty on the $Z'$ decay modes, we assume conservatively a branching fraction of 50% into charged leptons, which leads to the grey region in Fig. 3 in agreement with [41]. We also show the prospects for the HL-LHC, with 3 ab$^{-1}$ of integrated luminosity, and the HE-LHC, which corresponds to an integrated luminosity of 15 ab$^{-1}$ at a center-of-mass energy of 27 TeV. These projected collider limits were obtained using the code described in [57], which can be used to forecast lower mass limits for resonance searches, which is precisely our case.

We found that the NA62 bounds from the decay $K^+ \to \pi^+ \nu \bar{\nu}$ are rather restrictive, providing stronger limits than the ones from colliders over a significant region of the parameter space. Looking at left-panel of Fig.3, for instance, where $\delta = 0$, NA62 can exclude $Z'$ masses as high as 10 TeV,
if \( |V_{L32}^{d*}V_{L31}^d|\) is of the order \(10^{-2}\) or larger. In the same vein, NA62 enforces the product \( |V_{L32}^{d*}V_{L31}^d|\) to remain below \(\sim 10^{-3}\), when \(m_{Z'} \sim 2\) TeV.

On the other hand, these parameters are completely unconstrained by KOTO when \(\delta = 0\), since the contribution from \(Z'\) to the decay \(K_L \rightarrow \pi^0\nu\bar{\nu}\) vanishes in this case. In the absence of new CP violating sources, the \(BR(K_L \rightarrow \pi^0\nu\bar{\nu})\) takes the same value as in the SM, since this process will occur only through the contribution to CP violation from the SM via CKM matrix.

This can be easily understood with Eq. (21), from which we see that \(BR(K_L \rightarrow \pi^0\nu\bar{\nu})\) depends only on the imaginary part of \(X_{eff}\), in particular, on \(Im(V_{ts}^V V_{td}^d)\) when \(\delta = 0\). Still in the left-panel of Fig. 3 we notice that in the range shown, the Grossman-Nir limit does not appear, because the suppressed values of \(BR(K_L \rightarrow \pi^0\nu\bar{\nu})\) makes this bound easily satisfied for practically any reasonable value of \( |V_{L32}^{d*}V_{L31}^d|\). However, as \(\delta\) increases from 0 to \(\pi/2\), this bound becomes more relevant, likewise the KOTO exclusion region gradually grows, reflecting the enhancement in the \(K_L \rightarrow \pi^0\nu\bar{\nu}\) decay, while the NA62 exclusion region slightly decreases. Nevertheless, even with maximum enhancement at \(\delta = \pi/2\) (right plot), the KOTO bounds remain less constraining compared to NA62 and the Grossman-Nir limit.

V. DISCUSSION

Our conclusions relied on the presence of flavor changing interactions involving the \(Z'\) boson, but as the model features a large scalar sector, that could be new sources of flavor changing interactions rising from the heavy scalar fields. These new contributions have been shown to be subdominant. Thus can be safely ignored. Moreover, the entire reasoning was based on the 3-3-1 model with right-handed neutrinos, but our results are also applicable to the 3-3-1 model where the right-handed neutrinos are replaced by heavy neutral fields. This occurs because these models have the same neutral current. In summary, our finding are relevant for two different models and solid irrespective of the presence of scalar fields.

VI. CONCLUSION

In this work we explored the FCNC processes mediated by a \(Z'\) gauge boson featuring both in the 3-3-1 r.h.n and 3-3-1LHN models. We computed the \(K^+\) and \(K_L\) decay rates to missing energy, considering the extra contributions from the \(Z'\) in addition to the SM contribution, leaving the quark mixing matrix and the \(Z'\) mass as free parameters. We performed a complementary analysis using the results from NA62, KOTO, and the LHC (current and prospects) to set bounds on the 331 r.h.n parameters. We found that the last result of the NA62 experiment was able to constrain a large region of the parameter space, setting lower limits on the \(Z'\) mass that can be more stringent than those from dilepton searches at the LHC. For example, we can impose \(m_{Z'} > 10\) TeV for \( |V_{L32}^{d*}V_{L31}^d| \sim 10^{-3}\), while \( |V_{L32}^{d*}V_{L31}^d| \lesssim 2 \times 10^{-3}\) for \(m_{Z'} = 3\) TeV. These results apply for \(\delta = 0\), where the sensitivity of NA62 is maximum. In the case when the new CP violation effects are large our constraints weaken.

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