P and T odd nucleon-nucleon interaction in
a two higgs doublet model

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Abstract
A two higgs doublet model with tree level natural flavor conservation can lead to P and T odd nucleon-nucleon (NN) interaction through tree level Feynman diagrams. In this article we estimate the magnitudes of the parameters that characterize the strength of the S-PS type of NN interaction. Stringent constraints on the parameters can be derived from the measurement of EDM’s of Xe, Cs and Hg atoms in the ground state. We show that the predicted values of the parameters lie close to such constraints provided $\frac{|v_2|}{|v_1|}$ is large.
Although in the SM [1] only one higgs doublet was used to give mass to the weak
gauge bosons and fermions, there is no good theoretical justification for using only one
doublet. In fact some of the attractive features of the SM like the tree level custodial
symmetry and natural flavor conservation [NFC] are preserved in suitably chosen versions
of the two higgs doublet model [THDM]. The minimal supersymmetric standard model [2]
which has been extremely successful in explaining all the experimental data so far, requires
two higgs doublets with Y=±1 to preserve supersymmetry.

In this article we shall consider a general THDM in which tree level natural flavor
conservation is guaranteed by requiring that φ_1 couples only to RH I_3 = −1/2 fermions and
φ_2 couples only to RH I_3 = 1/2 fermions [3]. We shall show that such a model can give rise
to four fermionic S-PS nucleon-nucleon interaction which are strong enough to produce
observable P and T odd effects in high precision atomic and molecular experiments. The
coupling of the neutral higgs bosons φ_0^1 and φ_0^2 to the Q = ±2/3 and Q = −1/3 quarks are given
by

\[ L_y = -[\frac{1}{v_1} \bar{D}_i m_i D_i^R \phi_0^1 + \frac{1}{v_2} \bar{U}_j m_j U_j^R \phi_0^2 + h.c.]. \] (1)

where we have denoted the Q = ±2/3 and Q = −1/3 quarks generically by U and D respectively.
i and j are the generation indices. Consider first the contribution of Q = ±2/3 quarks to the
P and T odd NN interaction. We have

\[ L_{UU}^1 = \frac{m_i U m_j U}{8 v_2^2} \bar{U}_i (1 − \gamma_5) U_i \bar{U}_j (1 − \gamma_5) U_j \ i < \phi_0^1 \phi_0^1 >. \] (2a)

and

\[ L_{UU}^2 = \frac{m_i U m_j U}{8 v_2^*} \bar{U}_i (1 + \gamma_5) U_i \bar{U}_j (1 + \gamma_5) U_j \ i < \phi_0^2 \phi_0^2 >. \] (2b)

The P and T odd S-PS mixing term \( \bar{U}_i U_i \bar{U}_j \gamma_5 U_j \) will arise from the Im parts of the
propagators \( i < \phi_0^2 \phi_0^2 > \) and \( i < \phi_0^2 \phi_0^2 >. \) Note that \( i < \phi_0^2 \phi_0^2 > \) is real and therefore it
does not contribute to \( L_{P T V}^{UU}. \) In unitary gauge [4] we have \( \phi_0^1 = \frac{v_1}{\sqrt{2} |v_1|} [\Phi_1 − i \frac{|v_1|}{v} \Phi_3] \) and
\( \phi_0^2 = \frac{v_2}{\sqrt{2} |v_2|} [\Phi_2 − i \frac{|v_2|}{v} \Phi_3] \) where \( \Phi_1, \Phi_2 \) and \( \Phi_3 \) are real scalar fields. It then follows that
\[
Im \frac{\phi_0^0 \phi_2^0}{v_2^2} = -Im \frac{\phi_2^{0*} \phi_0^{0*}}{v_2^2} = \frac{|v_1|}{|v_2|^2} \Im \Phi \Phi_3 >. \tag{3}
\]

and
\[
L_{PTV}^{UU} = - \frac{|v_1|}{2|v_2|^2} \Im \Phi \Phi_3 > \bar{U}_i m_i U_i \bar{U}_j m_j U_j r_\gamma U_j. \tag{4}
\]

where a summation over generation indices i and j is implied. In order to convert the above P and T odd interaction between a pair of U quarks into P and T odd NN interaction we have to find the matrix elements of \(\sum_i \bar{U}_i m_i U_i\) and \(\sum_j \bar{U}_j m_j U_j r_\gamma U_j\) between nucleon states [5]. The light quarks u and d, which are the valence quarks for nucleon and whose masses are much small compared to the QCD scale \(\Lambda_c\), contribute very little to the scalar matrix element. The heavy quarks s, c, b and t on the other hand exist inside the nucleon only as virtual states and therefore their contribution to the scalar and pseudoscalar matrix elements can arise only from closed loops. It can be shown that the contribution of each heavy quark to the scalar matrix element is given by
\[
< N|\bar{Q}_h m_h Q_h |N > = \zeta m_N \bar{N} N
\]
where \(Q_h\) stands for a heavy quark and 
\[
\zeta = \frac{2}{29}. \tag{5}
\]

For the \(Q = \frac{2}{3}\) quarks the net contribution to the pseudoscalar matrix element is given by
\[
< N|\bar{u} r_\gamma u |N > + 2 < N|\bar{Q}_h r_\gamma Q_h |N > = \frac{2m_u - m_d}{m_u + m_d} m_N (-g_A) \bar{N} r_\gamma \tau_3 N - \frac{1}{2} m_N (-g_A^0) \bar{N} r_\gamma N. \tag{6a}
\]
Whereas for the $Q = -\frac{1}{3}$ quarks the net contribution is given by [5]

$$< N|\bar{d}\gamma_5 m_d|N> + 2< N|\bar{Q}_h\gamma_5 Q_h|N> = \frac{m_u - 2m_d}{m_u + m_d} m_N (-g_A)\bar{N}\gamma_5 \tau_3 N - m_N (-g_A^0)\bar{N}\gamma_5 N.$$  

(6b)

Here $g_A$ and $g_A^0$ are the axial vector coupling constants associated with $< N|\partial_\mu J_5^\mu|N>$ and $< N|\partial_\mu J_5^\mu|N>$ respectively. We then get

$$< N|L_{UU}^{PTV}|N> = \zeta m_N^2 (-g_A^0) \frac{|v_1|}{|v_2|^2 v} \bar{N}N\bar{N}\gamma_5 N - \zeta m_N^2 (-g_A) \frac{2m_u - m_d}{m_u + m_d} \frac{|v_1|}{|v_2|^2 v} \bar{N}N\bar{N}\gamma_5 \tau_3 N.$$  

(7)

Similarly the P and T odd NN interactions arising from the exchange of neutral higgs boson between a pair of D quarks and an U and a D quark are given by

$$< N|L_{DD}^{PTV}|N> = \zeta m_N^2 (-g_A) \frac{|v_2|}{|v_1|^2 v} \bar{N}N\bar{N}\gamma_5 N - \zeta m_N^2 (-g_A) \frac{m_u - 2m_d}{m_u + m_d} \frac{|v_2|}{|v_1|^2 v} \bar{N}N\bar{N}\gamma_5 \tau_3 N.$$  

(8)

and

$$< N|L_{UU}^{PTV}|N> = \zeta m_N^2 (-g_A^0) \frac{|v_1|}{|v_2|^2 v} \bar{N}N\bar{N}\gamma_5 N + \zeta m_N^2 (-g_A) \frac{2m_u - m_d}{m_u + m_d} \frac{|v_1|}{|v_2|^2 v} \bar{N}N\bar{N}\gamma_5 \tau_3 N.$$  

(9)

For $q^2 << m_h^2$ the propagator factors appearing above can be written as $< \Phi_1 \Phi_3 > \approx -i \sum_n \frac{C_{1n} C_{3n}}{m_n}$ and $< \Phi_2 \Phi_3 > \approx -i \sum_n \frac{C_{2n} C_{3n}}{m_n}$. Here the index $n$ stands for the neutral higgs boson mass eigenstates. The orthogonal $3 \times 3$ matrix $C_{in}$ connects the gauge eigenstates $\Phi_i$ with the mass eigenstates $\tilde{\Phi}_n$. We shall assume that only the lightest neutral higgs (h) contributes dominantly to the propagators. In that case we get
by NN interactions [6]. For the two higgs doublet model under consideration they are given

$$L_{P\bar{T}V}^{NN} = \left[ \frac{C_{1h}C_{3h}v^3}{m_h^2|v_1||v_2|^2} + \frac{C_{2h}C_{3h}v^3}{m_H^2|v_1||v_2|^2} \right] \sqrt{2}G_F\zeta m_N^2(-g_A)\bar{N}N\bar{N}h\gamma_5N$$

$$- \left[ \frac{C_{1h}C_{3h}v}{(m_u + m_d)m_h^2} \frac{1}{|v_2|^2} (2m_u - m_d) + \frac{|v_2|}{|v_1|^2} (m_u - 2m_d) \right]$$

$$+ \left[ \frac{C_{2h}C_{3h}v}{(m_u + m_d)m_h^2} \frac{1}{|v_1|^2} (m_u - 2m_d) + \frac{|v_1|}{|v_2|^2} (2m_u - m_d) \right] \sqrt{2}G_F\zeta m_N^2(-g_A) \times$$

$$\bar{N}N\bar{N}h\gamma_5\tau_3N.$$  \hspace{1cm} (10)

Phenomenologically one can write

$$L_{P\bar{T}V}^{NN} \equiv -C_{S-PS}^0 \frac{G_F}{\sqrt{2}} \bar{N}N\bar{N}h\gamma_5N - C_{S'PS}^1 \frac{G_F}{\sqrt{2}} \bar{N}N\bar{N}h\gamma_5\tau_3N.$$  \hspace{1cm} (11)

where $C_{S-PS}^0$ and $C_{S-PS}^1$ are the parameters characterizing the strenghts of P and T odd NN interactions [6]. For the two higgs doublet model under consideration they are given by

$$C_{S-PS}^0 = -2\zeta(-g_A) \frac{m_N^2v^3}{m_h^2|v_1||v_2|^2} \left[ \frac{C_{1h}C_{3h}}{|v_1|} + \frac{C_{2h}C_{3h}}{|v_2|} \right].$$  \hspace{1cm} (12)

and

$$C_{S-PS}^1 = 2\zeta(-g_A) \frac{m_N^2v}{m_h^2} \left[ \frac{C_{1h}C_{3h}}{(m_u + m_d)} \frac{1}{|v_2|^2} \left( 2m_u - m_d \right) + \frac{|v_2|}{|v_1|^2} (m_u - 2m_d) \right]$$

$$+ \left[ \frac{C_{2h}C_{3h}}{(m_u + m_d)} \frac{1}{|v_1|^2} (m_u - 2m_d) + \frac{|v_1|}{|v_2|^2} (2m_u - m_d) \right].$$  \hspace{1cm} (13)

Note that the isovector coupling constant $C_{S-PS}^1$ vanishes if we set $|v_1| = |v_2|$ and $m_u = m_d$ as required by isospin symmetry. We now turn to the values of the parameters that appear on the rhs of the phenomenological constants $C_{S-PS}^0$ and $C_{S-PS}^1$. For an orthogonal matrix $|C_{1h}C_{3h}| \leq .5$ and $|C_{2h}C_{3h}| \leq .5$. Chiral symmetry breaking exhibited in the light pseudoscalar mass spectrum implies that $m_u = 5$ Mev and $m_d = 7$ Mev.

The direct search for the neutral higgs boson by LEP1 has produced the lower bound $m_h > 58.4$ Gev [7] for the three generation SM with minimal Higgs sector and $m_{h_1} > 44$ Gev ($m_{h_1} < m_{h_2}$) for the minimal supersymmetric SM. The CDF group associated with
the Fermilab tevatron has reported having observed the top quark with a mass of around 175 Gev [8]. The CDF result implies a large mass hierarchy between the top quark and the bottom quark which can naturally arise from a large value of \( \frac{|v_2|}{|v_1|} \) if \( g_t(m_t) \approx g_b(m_b) \). In the later case we get \( |v_1| \approx 5 \text{ Gev} \) and \( |v_2| \approx 175 \text{ Gev} \). The axial vector coupling constants \( g_A^0 \) and \( g_A \) can be assumed to be approximately equal, i.e. \( g_A^0 \approx g_A \approx -1.24 \). For the above values of the parameters we find that \( |C_{S-P}^0| \leq 3.08 \times 10^{-2} \) and \( |C_{S-P}^1| \leq 2.30 \times 10^{-2} \).

The question that arises is how does the predicted values of the parameters compare with the limits derived from experiments searching for P and T violating effects in atomic and molecular systems. The P and T odd NN interaction given by eqn. [11] can give rise to various T odd nuclear multipoles [6], the simplest of which are the electric dipole moment (EDM), Schiff moment (SM) and magnetic quadrupole moment (MQM). The EDM of the nucleus does not lead to an overall atomic or molecular EDM because of the well known Schiff’s theorem [9]. Further for atoms in the ground state with closed electronic shells the MQM does not induce an atomic EDM. So the nuclear SM is the only T odd nuclear multipole that leads to an atomic EDM. The SM’s of \(^{129,131}\text{Xe}, \quad ^{133}\text{Cs} \) and \(^{199,201}\text{Hg} \) nuclei has been calculated [6] for the P and T odd NN interaction given by eqn. [11]. The EDM’s of Xe, Cs and Hg atoms have also been related [6] to their respective SM’s using the relativistic Hartree-Fock technique. The theoretical calculations when combined with the measurement of linear Stark effect in the ground state of Xe, Cs and Hg atoms produces the following bounds on the isoscalar parameter \( C_{S-P}^0 \) [6]:

\[
C_{S-P}^0(Xe) = -0.07 \pm 0.24,
\]

\[
C_{S-P}^0(Cs) = 0.6 \pm 2.4 \pm 0.6 \quad \text{and} \quad C_{S-P}^0(Hg) = 0.01 \pm 0.03.
\]

The experimental limit obtained from Hg is the most stringent. The reason being due to larger Z one gains an order of magnitude in the EDM by going from Xe or Cs to Hg. Experiments with TlF molecule also lead to a bound on \( C_{S-P}^0 \), but it is only an order of magnitude estimate because of theoretical uncertainties involved in estimating the SM of Tl nucleus. We find that the predicted values of \( C_{S-P}^0 \) in our model lie surprisingly close to the experimental limits derived from measurements with Hg for which the statistical error is smallest. It is
therefore extremely important to improve the accuracy for measuring the EDM of atoms and molecules in order to discriminate the different models of T violation on the basis of their closeness to the experimental bound and obtain a better physical understanding of its origin.

In conclusion in this article we have estimated the magnitude of the parameters that characterize the strength of P and T odd NN interaction in the context of a THDM. The model satisfies the conditions of tree level NFC. We find that if the ratio of vev’s $\frac{|v_2|}{|v_1|} \approx \frac{m_t}{m_b}$ then the predicted values of the parameters are close to the best experimental limits obtained from measurements with Hg. A large value of $\tan \beta = \frac{|v_2|}{|v_1|}$ might naturally explain the large t-b mass splitting. However the same could also have other phenomenological effects which should be carefully analysed before accepting such a scenario. Present experiments however do not rule out this possibility and that makes the values of $C_{S-P}^0$ and $C_{S-P}^1$ presented in this article somewhat interesting.

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