Numerical study of magnetic and pairing correlation in a bilayer triangular lattice

Shuang Wu¹, Jinling Li¹, Pan Gao¹, Ying Liang¹ and Tianxing Ma¹,²

¹ Department of Physics, Beijing Normal University, Beijing 100875, People’s Republic of China
² Beijing Computational Science Research Center, Beijing 100084, People’s Republic of China

E-mail: txma@bnu.edu.cn and liang@bnu.edu.cn

Received 17 April 2013, in final form 23 July 2013
Published 21 August 2013
Online at stacks.iop.org/JPhysCM/25/375601

Abstract

By using the determinant Quantum Monte Carlo method, the magnetic and pairing correlation of the NaₓCoO₂·yH₂O system are studied within the Hubbard model on a bilayer triangular lattice. The temperature dependence of the spin correlation function and pairing susceptibility with several kinds of symmetries at different electron fillings and interlayer coupling terms are investigated. It is found that the system shows antiferromagnetic correlation around half filling, and the fn-wave pairing correlation dominates over other kinds of pairing symmetry in the low doping region. As the electron filling decreases from half filling, both ferromagnetic correlation and the f-wave pairing susceptibility are enhanced and tend to dominate. It is also shown that both the magnetic susceptibility and pairing susceptibility decrease as the interlayer coupling increases.

(Some figures may appear in colour only in the online journal)

1. Introduction

Understanding the competition between various magnetic orders and pairing symmetries is a major challenge in superconductivity now. The discovery of superconductivity in the NaₓCoO₂·yH₂O materials offers an appealing platform to investigate the interplay among pairing interactions, magnetic fluctuations, and electronic correlation [1]. Besides doped cuprates, cobaltates are another class of a layered 3d transition-metal oxide in which superconductivity has been observed. The main difference between the two systems is that Co ions form a triangular lattice with magnetically frustrated geometry in contrast to the square lattice of the CuO₂ plane. In doped cuprates, cobaltates are another class of a layered 3d transition-metal oxide in which superconductivity has been observed. The main difference between the two systems is that Co ions form a triangular lattice with magnetically frustrated geometry in contrast to the square lattice of the CuO₂ plane. In doped cuprates, it has been well established that the Cu²⁺ moments are antiferromagnetically ordered in the CuO₂ plane, and with a low level of carrier (hole or electron) doping, antiferromagnetism is suppressed drastically and the system becomes metallic, followed by the appearance of superconductivity where the dₓ²−y² pairing symmetry dominates in the optimally doped region [2–6]. In general, the doping dependence of the pairing symmetry and the issue of quantum criticality must be considered under the premise of spatial homogeneity in the pairing potential. Results of some experiments suggest triplet pairing in cobaltates [7, 8], while some other measurements have resulted in contradicting conclusions which indicate singlet pairing [9, 10].

To investigate the superconducting mechanism of NaₓCoO₂·yH₂O system, the triangular lattice has been extensively studied theoretically [11–20]. In the Hubbard type model for this frustrated system, perturbation theory shows that d-wave and p-wave superconducting states are stable in hole doped region [21], while the renormalization group approach suggests the d + id-wave pairing symmetry in the case with antiferromagnetic exchange interactions [15]. In the strong coupling Hubbard model or in its strong coupling limits, the t − J model, mean field results again support the d + id-wave superconductivity near the half filling [11–13], while it has been confirmed by a variational Monte Carlo study [22], while in the low density region where the Fermi surface is detached, f-wave pairing is proposed to be realized [12]. Since the results obtained above are still actively debated because they are very sensitive to the approximation used, exact numerical results are highly desirable for they provide unbiased information and would serve as useful benchmarks for analytical approaches. Moreover, understanding of the magnetic order and pair symmetries of frustrated system
is still missing. For example, the situation of the pairing symmetry in $\kappa - (ET)_2X$ is complicated owing to the existence of frustration [23]. In a frustrated quantum antiferromagnet, the introduction of doping with mobile charge carriers may result in the appearance of unconventional superconductivity [24]. A bilayer triangular lattice is an ideal platform to study the interplay between magnetic fluctuation and pairing correlation in frustrated system.

Again, similar to the doped cuprates, Na$_2$CoO$_2$·yH$_2$O are layered materials, where the distance and couplings between the two Co$_2$ layers depend on the H$_2$O molecules inserted [25–28], and the interlayer coupling term is also regarded as a key to understand the superconducting mechanism. Thus, in this paper, we study the magnetic and pairing correlation within the Hubbard model on a bilayer triangular lattice by using determinant Quantum Monte Carlo simulations, which is a method that does not rely on uncontrolled approximations [29–33]. Numerical calculation reported here include results for a variety of band fillings, temperatures, pairing symmetries, and interlayer coupling terms. It is found that the system shows an antiferromagnetic correlation around half filling, and the fn-wave pairing correlation dominates over other kinds of pairing symmetry in the low doping region. As electron filling decreases, both the ferromagnetic fluctuations and the f-wave pairing susceptibilities are enhanced and tend to dominate. It is also shown that the magnetic correlation and pairing susceptibility decrease as the interlayer coupling increases. These results indicate that the competition of ferromagnetic and antiferromagnetic fluctuations in different filling region is crucial to the pairing behavior, which could be understood from the shape of the density of state (DOS) distribution in the bilayer triangular lattice.

2. Model

The sketch for the bilayer triangular lattice has been shown in figures 1(a) and (b). As shown in figure 1(a), the model for each layer is set on a triangular lattice with hexagonal shape. There are $2N_s$ sites on the diagonal, and the site number of this series of the lattice is $3N_s^2$. This lattice setting preserves most geometric symmetries of the triangular lattice. Figure 1(b) indicates the sketch for interlayer hopping, and hence the total sites for such bilayer triangular lattice is $2 \times 3N_s^2$. The case of $N_s = 4$ is shown here. The data points in the first Brillouin zone (BZ) include all the high symmetry points such as $\Gamma$, $M$, and $K$, as shown in figure 1(c). For any atom, there are six nearest neighbor atoms in the same layer and three in the other layer, which could be described as

$$H = t \sum_{(i,j)_{\sigma \sigma}} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + t' \sum_{(i,j)_{\bar{\sigma} \bar{\sigma}}} (c_{i\bar{\sigma}}^\dagger c_{j\bar{\sigma}} + \text{h.c.}) + U \sum_{\bar{k} \bar{\ell}} n_{\bar{k}\bar{\sigma}} n_{\bar{l}\bar{\sigma}} - \mu \sum_{\bar{k} \bar{\ell}} n_{\bar{k}\bar{\sigma}}$$

(1)

where $c_{i\sigma}$ ($c_{i\bar{\sigma}}^\dagger$) annihilates (creates) electrons at the site $R_i$ in the $d$th layer ($d = 1, 2$) with spin $\sigma$ ($\sigma = \uparrow, \downarrow$) and $n_{\bar{k}\bar{\sigma}} = c_{\bar{k}\bar{\sigma}}^\dagger c_{\bar{k}\bar{\sigma}}$. This system has intralayer nearest neighbor hopping $t$ and the interlayer hopping term $t'$, and these two layers have the same chemical potential $\mu$, as well as the electron–electron Coulomb interaction $U$. The system was simulated using determinant quantum Monte Carlo at finite temperature, and our numerical calculations were mainly performed on $2 \times 48$ ($N_s = 4$), $2 \times 75$ ($N_s = 5$) and $2 \times 108$ ($N_s = 6$) lattices with periodic boundary conditions.

It is generally believed that magnetic excitation might play a fundamental role in the superconducting mechanism of electronic correlated systems. To study the magnetic properties, we define the spin susceptibility in the $z$ direction at zero frequency,

$$\chi(q) = \int_{0}^{\beta} d\tau \sum_{d,d'=1,2} \sum_{\bar{k} \bar{l}} e^{i \bar{q} \bar{k} \bar{d} \bar{d}'} \langle m_{\bar{k}d \bar{d}'} \cdot m_{\bar{l}d' \bar{d}'} \rangle,$$

(2)

where $m_{\bar{k}d \bar{d}'} = e^{i H t} m_{\bar{k}d}(0) e^{-i H \tau}$, $m_{\bar{k}d} = c_{\bar{k}d\uparrow}^\dagger c_{\bar{k}d\downarrow}$, and $N_s$ represents the unit number of the lattice. To understand superconductivity in Na$_2$CoO$_2$·yH$_2$O materials, the behavior of pairing is one of the key issues. The property of pairing could be governed by the pairing susceptibility at zero frequency, which is defined as

$$P_\alpha = \frac{1}{N_s} \sum_{\bar{k} \bar{l}} \int_{0}^{\beta} d\tau \langle \Delta_\alpha(\bar{k}, \tau) \Delta_\alpha(\bar{l}, 0) \rangle,$$

(3)

and

$$\Delta_\alpha(\tau) = \frac{1}{\sqrt{N_s}} \sum_{\bar{k}} \Delta_\alpha(\bar{k}, \tau) = \frac{1}{\sqrt{N_s}} \sum_{\bar{k}} f_{\bar{k}}(l) c_{\bar{k}\uparrow}(\tau) c_{\bar{k}\downarrow}(\tau) \pm c_{\bar{k}\downarrow}(\tau) c_{\bar{k}\uparrow}(\tau),$$

(4)

where $\alpha$ denotes the symmetry of the pairing function, $i$ is the lattice site, $l$ indicates the neighboring sites, and $f_{\bar{k}}(l)$ is the site-dependent form factor of electron pairs. Considering the symmetry of the triangular lattice, possible form factors include six types: $f_{\bar{k}}(l), f_{\bar{d}1}(l), f_{\bar{d}2}(l), f_{\bar{d}3}(l), f_{\bar{f}1}(l), f_{\bar{f}2}(l), f_{\bar{f}3}(l)$. The detail forms of these pairing symmetries have been discussed by Koretsune and Ogata in [17]. Following them, the possible form factors of the pairing correlation functions in the triangular lattice have been shown in figure 2.
spin susceptibility \( \chi \) in the electron filling region from \( \langle n \rangle = 0.40 \) to 1.0, especially when the system is around half filling with different interlayer coupling terms \( t' \). Figure 3(a) shows \( \chi(q) \) versus the momentum \( q \) at \( \langle n \rangle = 1.0 \) (red line with circle), \( \langle n \rangle = 0.8 \) (dark line with square), \( \langle n \rangle = 0.6 \) (blue line with triangular) and \( \langle n \rangle = 0.4 \) (pink line with diamond) for \( U = 3|t|, t' = 0.2t \) and \( T = |t|/6 \). At half filling, the peak of spin susceptibility is located at the \( K \) point. When the system is doped away from half filling, the peak of \( \chi(q) \) moves to the \( \Gamma \) point. Here, \( \chi(K) \) measures the antiferromagnetic correlation and \( \chi(\Gamma) \) measures ferromagnetic fluctuations. Hence, antiferromagnetic correlations dominate around the half filling region and ferromagnetic fluctuation dominates in the low electron filling region. Figure 3(b) shows the \( \chi(q) \) with different interlayer coupling terms \( t' \) at half filling for \( U = 3.0|t| \) and \( T = |t|/6 \). One can see that the peak of the spin susceptibility \( \chi(q) \) is located at the \( K \) point, while \( \chi(q) \) is suppressed as \( t' \) increases.

Such suppression, as well as the competition between ferromagnetic and antiferromagnetic correlation, could be understood from the property of the DOS in bilayer triangular lattices. The DOS and band fillings with different \( t' \) have been shown in figure 4 as function of energy. One can see that, for the single-layer triangular lattice (\( t' = 0 \) in current case), its DOS in the non-interacting case has one van Hove singularity as the system is 0.5 doped away from half filling. As the interlayer coupling term \( t' \) is introduced, the van Hove singularity in the DOS tends to move further away from half filling. According to the itinerant electron ferromagnetic theory, the ferromagnetic fluctuations tend to the higher DOS on the Fermi surface, so ferromagnetic correlation dominates in the low electron filling region. As a result, spin correlation at the \( \Gamma \) point is suppressed as the interlayer coupling term increases at half filling.

Regarding the ferromagnetic correlation and antiferromagnetic fluctuation shown in figure 3 at different electron fillings, the competition between them indicates that pairing properties in such system may also be dependent on electron filling. Figure 5 presents the temperature dependence of pairing susceptibility with different symmetries at (a) \( \langle n \rangle = 0.4 \), (b) \( \langle n \rangle = 0.6 \), (c) \( \langle n \rangle = 0.8 \) and (d) \( \langle n \rangle = 1.0 \) for \( U = 3|t| \) and \( t' = 0.2t \). Basically, the behavior of pairing susceptibility

3. Results and discussion

By using the determinant Quantum Monte Carlo method, one of the authors and his collaborators have studied the magnetic correlation of the bilayer triangular lattice on the basis of single-band Hubbard model, in which ferromagnetic fluctuations near the van Hove singularities were reported [29], and the doped region is 0.60–0.85, which corresponding to electron filling 0.40–0.15 in the current case. In figure 3, we present the spin susceptibility \( \chi(0) \) to the 0 point. Here, \( \chi(0) \) measures ferromagnetic fluctuations.

Figure 2. Site-dependent form factors for s-wave, d-wave, p-wave, f-wave, and fn-wave pairing correlation functions in the triangular lattice.

The former three are singlet pairing and the latter two are triplet cases. As the triangular lattice is isotropic, the \( d_{xy} \)-wave and \( d_{xz,yz} \)-wave are degenerate, and the same goes for the \( p_x \)-wave and \( p_y \)-wave; here, we denote them as the d wave and p wave respectively [17, 18].

\[ \chi(q) = \langle \hat{S}_i \cdot \hat{S}_j \rangle - \langle \hat{S}_i \rangle \langle \hat{S}_j \rangle \]

Figure 3. Spin susceptibility \( \chi(q) \) versus the momentum \( q \) at various electron filling for \( U = 3|t|, t' = 0.2t, T = |t|/6 \) and (b) at half filling for different \( t' \). Data are shown along the path \( \Gamma \rightarrow M \rightarrow K \rightarrow \Gamma \) in the hexagonal BZ.
susceptibilities tend to saturate at the wave grows fastest. The fn-wave and s-wave pairing susceptibilities keep growing; especially, the low electron filling region, as shown in figures 5(a) and (b), it the major part of the temperature region we studied. In the with all kinds of symmetry does not change qualitatively in the major part of the temperature region we studied. In the low electron filling region, as shown in figures 5(a) and (b), it is clear that the spin triplet f-wave, p-wave and the spin singlet d-wave pairing susceptibilities keep growing; especially, the f-wave grows fastest. The fn-wave and s-wave pairing susceptibilities tend to saturate at \( n = 0.4 \). At \( n = 0.8 \), figure 5(c) shows the fn and f-wave pairing susceptibility contest the ‘race’ closely in the whole temperature region. As electron filling increases up to half filling, as shown in figure 5(d), the fn-wave pairing susceptibility tends to increase fastest, and the d-wave pairing susceptibility also has the potential to increase faster than the f-wave pairing susceptibility. However, due to the limitation of the numerical tool used here, we cannot achieve arbitrarily low temperatures within the determinant Quantum Monte Carlo method, which experiences the infamous fermion sign problem, and causes exponential growth in the variance of the computed results and hence an exponential growth in computer time as the lattice size is increased and the temperature is lowered. Basically, our numerical technology works well if the electron filling is not too close to the van Hove singularity, and in the range of \( U/T \leq 36 \), the error bar could be controlled within one per cent for a \( 2 \times 48 \) lattice.

Figures 3 and 5 indicate that the competition of ferromagnetic and antiferromagnetic fluctuations in different filling regions is crucial for the pairing behavior. Around half filling, antiferromagnetic correlation dominates in the behavior of spin correlation, and the pairing susceptibility with fn-wave pairing symmetry is the most favorable. As the system is doped away from half filling, ferromagnetic correlation tends to dominate over antiferromagnetic correlation, and it is interesting to see that the f-wave pairing is the most favorable in the ferromagnetic fluctuation dominating region, which is consistent with previous work reported by Kumar and Shastry [12].

The temperature dependence of f-wave pairing susceptibility with different \( \tau' \) is presented in figure 6(a) at \( \langle n \rangle = 0.40 \) (solid lines) and \( \langle n \rangle = 1.0 \) (dotted lines). One can see that, interlayer hopping has little influence on the f-wave pairing susceptibility for \( \langle n \rangle = 0.40 \) or \( \langle n \rangle = 1.0 \). The \( \tau' \)-dependence of \( \chi(q) \) for \( \langle n \rangle = 0.4 \) is also shown in the inset of figure 6(a). \( \chi(\Gamma) \) is suppressed very slightly as \( \tau' \) increases, which is consistent with the behavior of the pairing correlation. In figure 6(b), the pairing susceptibility and spin susceptibility are shown on a \( 2 \times 48 \) lattice, a \( 2 \times 75 \) lattice and a \( 2 \times 108 \) lattice for \( \tau' = 0.2t, U = 3|t| \) and \( \langle n \rangle = 0.4 \). Both the pairing

Figure 4. DOS and band fillings are functions of energy with (a) \( \tau' = 0 \), (b) \( \tau' = 0.1t \), (c) \( \tau' = 0.2t \) and (d) \( \tau' = 0.3t \), where the red lines represent fillings \( \langle n \rangle \) and the black lines represent the DOS.

Figure 5. Pairing susceptibility for different pairing symmetries (ss: s-wave, ps: p-wave, ds: d-wave, fs: f-wave, fns: fn wave) versus the temperature \( T/U = 3|t| \) and \( \tau' = 0.2t \). Sub-figures represent the situations of \( \langle n \rangle = 0.4 \) (a), 0.6 (b), 0.8 (c), and 1.0 (d) respectively.

Figure 6. (a) f-wave pairing susceptibility at various interlayer coupling terms, \( \tau' = 0.1t \) (dark line with squares), 0.2t (red line with circles) and 0.3t (blue line with triangles) versus the temperature \( T \) at \( \langle n \rangle = 0.40 \) (solid lines) and \( \langle n \rangle = 1.0 \) (dot lines) for \( U = 3|t| \). Inset: The spin susceptibility at different \( \tau' \) for a \( 2 \times 48 \) lattice at \( T = |t|/6, U = 3|t| \) and \( \langle n \rangle = 0.40 \). (b) f-wave pairing susceptibility for a \( 2 \times 48 \) lattice, a \( 2 \times 75 \) lattice and a \( 2 \times 108 \) lattice with \( \tau' = 0.2t, U = 3|t| \) and \( \langle n \rangle = 0.4 \). Inset: The spin susceptibility for various lattices at \( T = |t|/6 \).
susceptibility and spin susceptibility decrease slightly as the lattice size increases from $2 \times 48$ to $2 \times 75$, and results for $2 \times 75$ and $2 \times 108$ are almost the same within the error bar. Hence we may argue here that the pairing and spin susceptibility are almost independent of the lattice size.

Figure 5 shows that the fn- and f-wave pairings dominate for $(n) > 0.8$ and $(n) < 0.8$, respectively. The temperature dependence of fn-wave pairing susceptibility and d-wave pairing susceptibility with different $t'$ is shown in figures 7(a) and (b) at $(n) = 1.00$. At half filling, one can see that the pairing susceptibility is suppressed slightly by the increasing $t'$. This suppression is consistent with the behavior of spin susceptibility shown in figure 3 in which $\chi(q)$ is also suppressed as the interlayer hopping term increases.

4. Conclusions

To summarize, we have studied the magnetic and pairing correlation of the single-band Hubbard model on a bilayer triangular lattice. We performed determinant Quantum Monte Carlo simulations on the magnetic correlation and pairing susceptibility for a variety of electron fillings, temperatures and pairing symmetries. Around half filling, where the peak of the spin structure factor is located at the $K$ point, the fn-wave pairing susceptibility dominates. As electron filling decreases, the peak of spin correlation moves away from $K$, and finally locates at the $\Gamma$ point [29], which indicates that ferromagnetic fluctuation is stronger than the antiferromagnetic type when the electron filling is low enough. Correspondingly, the f-wave pairing susceptibility is enhanced and the fn-wave pairing susceptibility is suppressed as electron filling decreases, especially at low temperature. As a result, the f-wave pairing susceptibility dominates when the electron filling is lower than 0.8. Moreover, both the spin correlation and pairing susceptibility are suppressed by increasing interlayer coupling $t'$. Note that our calculations only give reliable results at $T > t/6$. Therefore it is not conclusive whether the triplet f wave really diverges or not as $T \rightarrow 0$. However, it would be important that there is a possibility of triplet superconductivity in the Hubbard model on a bilayer triangular lattice.

Acknowledgments

This work is supported by NSFC Grant No. 11104014, Research Fund for the Doctoral Program of Higher Education of China 2011003120007, SRF for ROCS (SEM), and the Fundamental Research Funds for the Central Universities in China under 2011CBA00108.

References

[1] Takada K, Sakurai H, Takayama-Muromachi E, Izumi F, Dilanian R A and Sasaki T 2003 Nature 422 53
[2] Kastner M A, Birgeneau R J, Shiran G and Endoh Y 1998 Rev. Mod. Phys. 70 897
[3] Tokura Y, Takagi H and Uchida S 1989 Nature 337 345
[4] Fujimoto T, Zheng G Q, Kitaoka Y, Meng R L, Cmaidalka J 2004 Phys. Rev. B 70 11979
[5] Feng S 2003 Phys. Rev. B 68 184501
[6] Tang X, Ma T and Guo H 2006 Physica C 436 14
[7] Iijima K, Yoshida K, Koda A, Saha S R, Kadono R, Ishida K, Takada K, Sakurai H, Takayama-Muromachi E and Sasaki T 2004 Phys. Rev. B 70 134508
[8] Masáka M, Mierzejewski M, Andrzejewski B, Foo M L, Cava R J and Klimeczuk T 2004 Phys. Rev. B 70 144516
[9] Zheng G Q, Matano K, Chen D P and Lin C T 2006 Phys. Rev. B 73 180503
[10] Fujimoto T, Zheng G Q, Kitaoka Y, Meng R L, Cmaidalka J and Chu C W 2004 Phys. Rev. Lett. 92 047004
[11] Baskaran G 2003 Phys. Rev. Lett. 91 097003
[12] Kumar B and Shastry B S 2003 Phys. Rev. B 68 104508
[13] Wang Q H, Lee D H and Lee P A 2004 Phys. Rev. B 69 092504
[14] Tanaka A and Hu X 2003 Phys. Rev. Lett. 91 257006
[15] Honerkamp C 2004 Phys. Rev. B 68 104510
[16] Kuroki K, Tanaka Y and Arita R 2004 Phys. Rev. Lett. 93 077001
[17] Koressen T and Ogata M 2005 Phys. Rev. B 72 134513
[18] Su S Q, Huang Z B, Fan R and Lin H Q 2008 Phys. Rev. B 77 125114
[19] Kuroki K, Tanaka Y and Arita R 2005 Phys. Rev. B 71 024506
[20] Johannes M D, Mazin I I, Singh D J and Papaconstantopoulos D A 2004 Phys. Rev. Lett. 93 097005
[21] Nisikawa Y and Yamada K 2002 J. Phys. Soc. Japan 71 2629
[22] Ikeda H, Nisikawa Y and Yamada K 2004 J. Phys. Soc. Japan 73 17
[23] Watanabe T, Yokoyama H, Tanaka Y, Inoue J and Chu C W 2004 Phys. Rev. Lett. 93 125114
[24] Clay R T, Li H and Mazumdar S 2008 Phys. Rev. Lett. 101 166403
[25] Gan J Y, Chen Y and Zhang F C 2006 Phys. Rev. B 74 094515
[26] Lynn J W, Huang Q, Brown C M, Miller V L, Foo M L, Schaak R E, Jones C Y, Mackey E A and Cava R J 2003 Phys. Rev. B 68 214516

Figure 7. Pairing susceptibility at various interlayer coupling terms, $t' = 0.1t$ (dark line with squares), $0.2t$ (red line with circles) and $0.3t$ (blue line with triangles) versus the temperature $T$ for $U = 3|t|$ and $n = 1.0$. Sub-figures represent the situations of fn wave (a) and d wave respectively.
[26] Jorgensen J D, Avdeev M, Hinks D G, Burley J C and Short S 2003 Phys. Rev. B 68 214517
[27] Johannes M D and Singh D J 2004 Phys. Rev. B 70 014507
[28] Xiao R J, Yang H X and Li J Q 2006 Phys. Rev. B 73 092517
[29] Hu F M, Su S Q, Ma T and Lin H Q 2009 Phys. Rev. B 80 014428
[30] Blankenbecler R, Scalapino D J and Sugar R L 1981 Phys. Rev. D 24 2278
[31] Ma T, Hu F M, Huang Z B and Lin H Q 2010 Appl. Phys. Lett. 97 112504
[32] Ma T, Lin H Q and Hu J P 2013 Phys. Rev. Lett. 110 107002
[33] Ma T, Hu F M, Huang Z B and Lin H Q 2011 Horizons in World Physics vol 276 (Hauppauge, NY: Nova Science) (chapter 8)