Forces transmission in the case of SSR structural group

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Abstract. This paper contains the SSR spatial group analysis, following to determinate the forces transmission index. It comes to conclusion, this index is a two variable function, defined by discrete values and graphically represented by a surface. Knowing this function it has been studied a linkage containing the SSR group – the spatial four bar linkage of orthogonal type. To avoid very high values of reactions, the forces transmission index must be inferior limited. Verifying of this limitation is done by the above mentioned surfaces level curves. This method is performed by authors, and it had been used previously for planar linkages only.

1. Introduction

The SSR structural group (figure 1) is probably the most used structural group in spatial linkages design. It is found at four bars linkages and at the more complex ones too. It is a group of Assure type, with degree of freedom \( L = 0 \). Although, by calculus, it results \( L = 1 \), it exists a passive relationship - rotation of link 1 around \( AB \) axe (figure 1), which make to lose one degree of freedom.

In a series of previous papers [1-15] it has been approached the forces transmission problem in mechanisms, applied to planar linkages and we enunciated some ideas, which will be resumed as following.

- The criterion according to which we appreciate the forces transmission quality is the size of reactions in kinematic pairs. In other words, as reactions have more reduced size, for the same technological forces, as transmission quality is better. We specify this aspect because there are other criteria to appreciate forces transmission quality too [16-17].
- The entity under analysis is not the mechanism taken as a whole, but it is the structural group. This option is justified by the fact that equation system deserving static analysis, associated to each group, is determined and can be separately solved. The solutions of this system are just the reactions in pairs, and its size depends only on group configuration and not on the whole mechanism. We mention that by configuration we understand the structural group pose, reported to one of its elements.
- The reactions size depends on the determinant of the linear equation system, associated to the group and intervening in static analysis without friction. This determinant is found at the denominator, when reactions are calculated using the rule of Cramer, thus this dependence is more emphasized at the singular positions vicinity, where the determinant tends to zero and reactions tend to infinity. The forces transmission index defines with formula:

\[
T = \frac{|D|}{D_r} 
\]
where, \( D \) is before named determinant and \( D_r \) is a referential value which equals maximal value of \( |D| \), reported to the structural group configuration. The index \( T \) belongs to \([0,1]\) interval. When \( T = 0 \), a singular position has been noticed, and forces transmission quality is as favourable as \( T \) is far away from zero. In order to assure avoidance of the reactions with unacceptable bigger values, \( T \) must satisfy condition:

\[
T \leq T_a
\]  

(2)

where \( T_a \) is an admissible value. The index \( T \) expresses as a function of configuration through some geometric variable parameters (angles or lengths). The analysis of a group, regarding forces transmission, consists in establishing of this function. In the same time, it emphasizes the singular configurations, fact of great importance for knowing the group working. The number of independent parameters determining the configuration is given by its degree of freedom,

\[
M = 6(n - 1) - 5C_{3i} - 4C_{4i} - 3C_{5i},
\]

(3)

Where \( C_{3i}, C_{4i}, \) and \( C_{5i} \) are the numbers of interior pairs of each class.

To perform a mechanism behaviour analysis, it uses results given by component groups study. Thus, it calculates \( T \) indices for each group, aided by configuration parameters, obtained from mechanisms positions analysis considered as a whole. Its values must satisfy condition (2) for each component group. We can say that mechanism has a forces transmission index of value equal to the smallest index occurring to the structural groups.

2. Analysis of SSR structural group

We will perform static analysis of SSR structural group, reported to a referential system, located as in figure 1. The link 2 is supposed to be unloaded because it has an undetermined movement, and link 3 is loaded by a known torque only. The unknowns of the problem are reaction \( \vec{R}_{23}(R_{23x}, R_{23y}, R_{23z}) \) and reaction torque \( \vec{M}_{r43}(M_{r43x}, M_{r43y}) \).

Figure 1. The spatial SSR structural group \( l_2 = 10, l_3 = 3 \) (dimensionless).

It writes torque equation for link 3, reported to point \( D \):

\[
\vec{R}_{23} \times \vec{CD} + \vec{M}_{r43} + \vec{M}_3 = 0.
\]

(4)

From torques equation reported to point \( B \), written for link 2,
it results that direction of reaction \( \vec{R}_{23} = -\vec{R}_{23} \) is the line \( BC \), and \( \vec{R}_{23} \) can be expressed like this:

\[
\vec{R}_{23} = T_{23} \cos \varphi_{2x} + \vec{j}R_{23} \cos \varphi_{2y} + \vec{k}R_{23} \cos \varphi_{2z},
\]

where, \( \cos \varphi_{2x} \), \( \cos \varphi_{2y} \), \( \cos \varphi_{2z} \) are directory cosines of the direction \( BC \). Vector equation (4) projects on the coordinate axes, obtaining the following scalar equation system:

\[
\begin{align*}
R_{23} (y_c - y_D) \cos \varphi_{2x} + M_{43x} &= -M_{3x} \\
-R_{23} (x_c - x_D) \cos \varphi_{2y} + M_{43y} &= -M_{3y} \\
R_{23} (x_c - x_D) \cos \varphi_{2z} - (y_c - y_D) \cos \varphi_{2z} &= -M_{3z}.
\end{align*}
\]

Taking into account that \( y_c - y_D = 0 \), \( x_c - x_D = -l_3 \), it results,

\[
M_{43x} = -M_{3x},
\]

and system (7) reduces to the following two equation:

\[
\begin{align*}
R_{23} l_3 \cos \varphi_{2x} + M_{43y} &= -M_{3y} \\
-R_{23} l_3 \cos \varphi_{2y} &= -M_{3z}.
\end{align*}
\]

The system solutions are:

\[
\begin{align*}
R_{23} &= \frac{-M_{3x}}{D} \\
M_{43y} &= \frac{-l_3 (M_{3x} \cos \varphi_{2x} + M_{3y} \cos \varphi_{2y})}{D},
\end{align*}
\]

and the coefficients determinant is:

\[
D = \begin{vmatrix} l_3 \cos \varphi_{2x} & 1 \\ -l_3 \cos \varphi_{2y} & 0 \end{vmatrix}.
\]

According to definition (1), the index \( T \) expresses by following function:

\[
T = \left| \cos \varphi_{2y} \right|.
\]

By reasons depending on notations correlation from other papers \[6, 7, 13\], angle between line \( BC \) and axis \( Cy \) is written down \( \varphi_{2y} = \alpha \), and its complement, formed by \( BC \) with plane \( Cxz \) is written down \( \gamma \).

Then it can write:

\[
T = \left| \cos \alpha \right| = \left| \sin \gamma \right|.
\]

From formula (14) it results that the group has an infinity of singular configurations \( T = 0 \), when \( \alpha = \frac{\pi}{2} \) and \( \gamma = 0 \). In other words, singular configurations are noticed when projection \( CB_y \) of the link 2 \( (BC) \) on the plane inside link 3 moves, is collinear with link 3 \( (CD) \). The index \( T \) has
maximum value, equals 1, at $\alpha = 0$ and $\gamma = \frac{\pi}{2}$, therefore, when link 2 is perpendicular on the plane formed by revolute axis $D$ and centre of the spherical joint $C$.

Configuration of this group is determined by two independent parameters. This fact results from the configuration degree of freedom calculus (3), taking into account link number, $n = 2$ and number of interior joints $C = 1$. It results $M = 3$, which decreases with an unit because of a passive mobility, thus, the real degree of freedom is $M = 2$. This means that $T$ expresses as a function of two parameters which can be satisfactory chosen. It is interesting the case, when the two parameters are the distances $d_2 = z_h$ and $d_3 = DB_{xy}$. The relationship between these parameters and T index is given by solving the triangle $DCB_{xy}$, where $DB_{xy} = d_3$, $DC = l_3$ and $CB_{xy} = l_{2b}$ - projection of the segment $BC$ on the plane $Cxy$, results from triangle $CBB_{xy}$,

\[ l_{2b} = \sqrt{l_2^2 - d_2^2}. \]  

(15)

Then it calculates angle $C$, the length $B,B_{xy} = CB_y$ and finally angle $\alpha$:

\[ \hat{C} = \arccos \frac{l_{2b}^2 + l_2^2 - d_3^2}{2l_{2b}l_3} \]  

(16)

\[ CB_y = l_{2b} \sin C \]  

(17)

\[ \alpha = \arccos \frac{CB_y}{l_2}. \]  

(18)

Based on this algorithm it plots the surface $T(d_2, d_3)$ - (figure 2) and level curves for different values of $T$ - (figure 3).

**Figure 2.** 3D representation of the $T(d_2, d_3)$ surface.
Figure 3. The level curves of \( T(d_2,d_3) \) surface.

It is useful to represent the domain from the plane \( d_2,d_3 \), for which structural group has real configurations, i.e. behind described algorithm gets real solutions. The problem reduces to write the existence conditions of the triangle \( DCB_{y_1} \), from which result maximum value - \( d_{3\text{max}} \) and minimum value \( d_{3\text{min}} \):

\[
d_{3\text{max}} = t_{2h} + l_1, \tag{19}
\]
\[
d_{3\text{min}} = |t_{2h} - l_1|. \tag{20}
\]
Figure 4. The searched domain, characteristic points and segments.

The curves $d_{3\min}(d_z)$ and $d_{3\max}(d_z)$ limit the searched domain, and points located on these curves lead to singular configurations ($T = 0$) - (figure 4). In the same referential system it plots the curve $l_{2h}(d_z)$ - formula (15), and curve $d_{3M}$ - the $d_3$ value for which $T = T_{\max}$, calculated with formula:

$$d_{3M} = \sqrt{l_3^2 + l_{2h}^2}.$$  \hspace{1cm} (21)

The contour from figure 4 contains some characteristic points and segments that lead to configurations with certain properties, following from formulae (15-21).

$P_1(0, l_z - l_v)$: $BC \in Cx$, $\alpha = \frac{\pi}{2}$, $\gamma = 0$, $T = 0$,

$P_2(0, l_z + l_v)$: $BC \in Cx$, $\alpha = \frac{\pi}{2}$, $\gamma = 0$, $T = 0$,

$P_3\left(0, \sqrt{l_z^2 + l_v^2}\right)$: $BC \in Cy$, $\alpha = 0$, $\gamma = \frac{\pi}{2}$, $T = 1$. 
The segment $PP_3$ : $BC \in Cxy$, $T \in [0,1]$,  

$P_1\left(\sqrt{l_2^2-l_3^2},0\right)$ : $B \equiv D$, $\alpha = \frac{\pi}{2}$, $\gamma = 0$, $T = 0$,  

$P_1(l_2,l_3)$ : $BC \perp Cxy$, $\alpha = \frac{\pi}{2}$, $\gamma = 0$, $T = 0$,  

$P_1(0,l_2+l_3)$ : $BC \in Cx$, $\alpha = \frac{\pi}{2}$, $\gamma = 0$, $T = 0$,  

The segment $PP_2$ : $BC \in Cxy$, $T \in [0,1]$, The points from this segment have $T$ of maximum value, reported with the set of points with the same $d_2$.

3. Application

We shall apply the previous results in order to analyse forces transmission at the spatial four bar linkage of orthogonal type (figure 5), which contains the SSR group. As we have been shown, we shall use the function $T(d_2,d_3)$ for calculating the index $T$.

Figure 5. The spatial orthogonal four bar linkage containing a SSR group.

For this purpose we shall determine the functions $d_2(\varphi)$ and $d_3(\varphi)$, where $\varphi$ is the independent parameter of mechanism – the driving link position angle.

$$
\begin{align*}
  d_2 &= z_b = z_a + l_1 \sin \varphi \\
  d_3 &= B_y D = \sqrt{x_D^2 + (l_1 \cos \varphi - y_D)^2}.
\end{align*}
$$

Introducing these functions into algorithm given by formulae (16-18), it obtains the function $T(\varphi)$ graphically represented in figure 6. If we adopt for $T_\alpha$ usual value $T_\alpha = 0.7$, it observes that condition

$$
(2) is respected. This verification can be made aided the function \( \gamma(\phi) \) too, taking \( \gamma_a = \arcsin(T_a) \). For the same purpose it can use level curves from Figure 3.

![Graphical representation of \( T(\phi) \)](image)

It determines the function \( d_4(d_z) \) proper to the four bar spatial linkage, based on parametric equations (20). The graph of this function superposes on the level curves (figure 3) of the surface \( T(d_z, d_z) \). It notices that in our case, the closed-loop curve \( d_4(d_z) \) is located inside the level curve \( T = 0.7 \) (yellow colour), which shows that condition (2) is respected.

4. Conclusion

The SSR structural group, treated in this paper, shows that forces transmission can be also approached using the previous described by us procedure, for planar linkages. This procedure is based on an index, defined and characteristic for each group, which is proportional with coefficients determinant, found to static analysis.

The SSR group configuration has two degrees of freedom, therefore transmission index expresses by a function of two variables, which can be represented by a surface. This fact is not characteristic to planar groups and leads to an original approach, regarding the verifying of this index aided by level curves.

The results of SSR group analysis allow an easy approach of every linkage, containing this group. This fact results from the contained in paper application, representing a spatial pour bar linkage of orthogonal type.
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