This paper studies the fundamental tradeoff between storage and latency in a general wireless interference network with caches equipped at all transmitters and receivers. The tradeoff is characterized by an information-theoretic metric, fractional delivery time (FDT), which is defined as the delivery time of the actual load normalized by the total requested bits at a transmission rate specified by degrees of freedom (DoF) of the given channel. We obtain achievable upper bounds of the minimum FDT for the considered network with $N_T(\geq 2)$ transmitters and $N_R=2$ or $N_R=3$ receivers. We also obtain a theoretical lower bound of the minimum FDT for any $N_T$ transmitters and $N_R$ receivers. Both upper and lower bounds are convex and piece-wise linear decreasing functions of transmitter and receiver cache sizes. The achievable bound is partially optimal and is at most $\frac{N_T+1}{N_T}$ and $\frac{N_T+2}{N_T}$ times of the lower bound in the considered $N_T \times 2$ and $N_T \times 3$ network, respectively. To show the achievability, we propose a novel cooperative transmitter/receiver coded caching strategy. It offers the freedom to adjust file splitting ratios for FDT minimization. Based on this caching strategy, we then design the delivery phase carefully to turn the considered network opportunistically into more favorable channels, including X channel, broadcast channel, multicast channel, or a hybrid form of these channels. Receiver local caching gain, coded multicasting gain, and transmitter cooperation gain (interference alignment and interference neutralization) are thus leveraged in different cache size regions and different channels.

**Index Terms**

Wireless cache network, coded caching, content delivery, multicast, and interference management.
I. Introduction

Over the last decades, mobile data traffic has been shifting from connection-centric services, such as voice, e-mails, and web browsing, to emerging content-centric services, such as video streaming, push media, application download/updates, and mobile TV [1]–[3]. These contents are typically produced well ahead of transmission and can be requested by multiple users though at possibly different times. This allows us to cache the contents at the edge of networks, e.g., base stations and user devices, during periods of low network load. The local availability of contents at the network edge has significant potential of reducing user access latency and alleviating wireless traffic. Recently, there have been increasing interests from both academia and industry in characterizing the impact of caching on wireless networks [4]–[7].

Caching in a shared link with one server and multiple cache-enabled users is first studied by Maddah-Ali and Niesen in [8]. It is shown that caching at user ends brings not only local caching gain but also global caching gain. The latter is achieved by a carefully designed cache placement and coded delivery strategy, which can create multicast chances for content delivery even if users demand different files. The idea of coded caching in [8] is then extended to the decentralized coded caching in a large network in [9], which achieves a rate close to the optimal centralized scheme. Taking file popularity into consideration, the authors in [10]–[12] showed that the peak traffic load is close to optimal within a constant gap by partitioning users into two groups and then using the decentralized scheme in [9] for content delivery. In [13], the authors considered the wireless broadcast channel with imperfect channel state information at the transmitter (CSIT) and showed that the gain of coded multicasting scheme can offset the loss due to the imperfect CSIT. In specific, if the user cache size is large enough, the content delivery latency approaches to the case with perfect CSIT and no cache. Besides the shared link or broadcast channels mentioned above, coded multicasting is also investigated in other network topologies, such as hierarchical cache networks [14], device-to-device cache networks [15], and multi-server networks [16].

Caching at transmitters is studied in [17]–[19] to exploit the opportunities for transmitter cooperation and interference management. In specific, the authors in [17] exploited the MIMO cooperation gain via joint beamforming by caching the same erasure-coded packets at all edge nodes in a backhaul-limited multi-cell network. The authors in [18] studied the degrees of
freedom (DoF) and clustered cooperative beamforming in cellular networks with edge caching via a hypergraph coloring problem. The authors in [19] studied the transmitter cache strategy in the cache-aided interference channel under an information-theoretical framework. It is shown that splitting contents into different parts and caching each part in different transmitters can turn the interference channel into X channel, broadcast channel, or hybrid channel and hence increase the system throughput via interference management. The authors in [20] presented a lower bound of delivery latency in a general interference network with transmitter cache and showed that the scheme in [19] is optimal in certain region of cache size.

The above literature reveals that caching at the receiver side can bring local caching gain and coded multicasting gain, and that caching at the transmitter side can induce transmitter cooperation for interference management and load balancing. It is thus of theoretical importance and practical interest to investigate the impact of caching at both transmitter and receiver sides.

In this paper, we aim to study the fundamental limits of caching in a general wireless interference network with caches equipped at all transmitters and receivers as shown in Fig. 1. The performance metric to characterize the gains of caching varies in the existing works. For the broadcast channel with receiver cache, the authors in [8] characterized the gain by memory-rate tradeoff, where the rate is defined as the load of the shared link normalized by the file size in the delivery phase. For the interference channel with transmitter cache, the authors in [19] characterized the gain by the standard DoF from the information-theoretic studies in the delivery phase. In [20], the authors proposed the storage-latency tradeoff, where the latency is defined as the relative delivery time with respect to an ideal baseline system with unlimited cache and no interference in the high signal-to-noise ratio (SNR) region. In our considered wireless interference network with both transmitter and receiver caches, the standard DoF is unable to capture the potential reduction in the traffic load due to receiver caching, and the rate is unable to capture the potential DoF enhancement due to cache-induced transmitter cooperation. Interestingly, the latency-oriented performance metric introduced in [20] can reflect not only the load reduction due to receiver cache but also the DoF enhancement due to transmitter cache, since it evaluates the delivery time of the actual load at a transmission rate specified by the given DoF. As such, we adopt the storage-latency tradeoff to characterize the fundamental limits of caching in this work. In specific, we measure the performance by the fractional delivery time (FDT), denoted as $\tau(\mu_R, \mu_T)$, which is a function of the normalized receiver cache size $\mu_R$ and the normalized
transmitter cache size \( \mu_T \).

Our preliminary results on the latency-storage tradeoff study in the special case with \( N_T = 3 \) transmitters and \( N_R = 3 \) receivers are presented in [21]. Note that an independent work on the similar problem with both transmitter and receiver caches is studied in [22]. We shall discuss the differences with [22] at appropriate places throughout the paper presentation. The main contributions and results of this work are as follows:

- **A novel file splitting and caching strategy**: We propose a novel file splitting and caching strategy for any transmitter and receiver numbers and at any feasible normalized cache sizes. This strategy is more general than the existing file splitting and caching strategy in [8], [19], [22]. It offers the freedom to adjust the file splitting ratios for caching gain optimization.

- **Achievable storage-latency tradeoff for \( N_T \times 2 \) and \( N_T \times 3 \) networks**: Based on the proposed file splitting and caching strategy, we obtain an upper bound of the minimum FDT for the interference networks with \( N_T \) transmitters and \( N_R = 2 \) or 3 receivers. The main idea is to design the delivery phase carefully so that the network topology can be opportunistically changed to more favorable ones, including X channel, multicast channel, broadcast channel, or a hybrid form of these channels. Interference neutralization, interference alignment, and real interference alignment are used in different channels to increase the system DoF. The obtained FDT is a convex and piece-wise linear decreasing function of the transmitter and receiver cache sizes. Our analysis shows that the transmitter cooperation gain, local caching gain, and coded multicasting gain can be leveraged in different cache size regions. Our analysis also shows that the optimal file splitting ratios are not unique when the transmitter and receiver cache storage exceed a threshold.

- **Lower bound of storage-latency tradeoff**: We also obtain a lower bound of the minimum FDT for the general \( N_T \times N_R \) interference network by using cut-set bound and genie-message approach. With this lower bound, we find that the achievable FDT upper bound in the \( N_T \times 2 \) and \( N_T \times 3 \) networks is optimal at certain regions. We also show that the maximum multiplicative gap between the upper bound and the lower bound is at most \( \frac{N_T+1}{N_T} \) and \( \frac{N_T+2}{N_T} \), respectively, for \( N_R = 2 \) and \( N_R = 3 \).

The remainder of this paper is organized as follows. Section II introduces our system model and performance metric. Section III presents the main results of this paper. Section IV describes the cache placement strategy. Section V and VI illustrate the content delivery strategy for the \( N_T \times 2 \) and \( N_T \times 3 \) interference network, respectively. Section VII presents some discussions of
our results. Section VIII proves the lower bound of FDT, and Section IX concludes this paper.

Notations: $(\cdot)^T$ denotes the transpose. $[K]$ denotes set $\{1, 2, \cdots, K\}$. $\lceil x \rceil$ denotes the largest integer no greater than $x$. $(x_j)_{j=1}^K$ denotes vector $(x_1, x_2, \cdots, x_K)^T$. $(x)^+$ denotes the maximum of $x$ and 0. $(x)^+ = \max\{0, x\}$. $A_{1\sim S}$ denotes set $\{A_1, A_2, \cdots, A_S\}$. $\mathcal{CN}(m, \sigma^2)$ denotes the complex Gaussian distribution with mean of $m$ and variance of $\sigma^2$.

**II. System Description And Performance Metrics**

A. System Description

Consider a general cache-aided wireless interference network with $N_T (\geq 2)$ transmitters and $N_R (\geq 2)$ receivers as illustrated in Fig. 1, where each node is equipped with a cache memory of finite size. Each node is assumed to have single antenna. The communication link between each transmitter and each receiver experiences channel fading and is corrupted with additive white Gaussian noise. The communication at each time slot $t$ over this network is modeled by

$$Y_j(t) = \sum_{p=1}^{N_T} h_{jp}(t)X_p(t) + Z_j(t), j = 1, 2, \cdots, N_R,$$

where $Y_j(t) \in \mathbb{C}$ denotes the received signal at receiver $j$, $X_p(t) \in \mathbb{C}$ denotes the transmitted signal at transmitter $p$, $h_{jp}(t) \in \mathbb{C}$ denotes the channel coefficient from transmitter $p$ to receiver $j$ which is identically and independently (i.i.d.) distributed as some continuous distribution, and $Z_j(t)$ denotes the noise at receiver $j$ distributed as $\mathcal{CN}(0, 1)$. The channel is assumed to be quasi-static fading so that it remains constant during a transmission frame that contains many time slots but varies independently from one frame to another.

Consider a database consisting of $L$ files ($L >> N_R$), denoted by $\{W_1, W_2, \cdots, W_L\}$. Each file is chosen independently and uniformly from $[2^F] = \{1, 2, \cdots, 2^F\}$ randomly, where $F$ is the file size in bits. Each transmitter has a local cache able to store $M TF$ bits and each receiver has a local cache able to store $M RF$ bits. The normalized cache sizes at each transmitter and receiver are defined, respectively, as

$$\mu_T \triangleq \frac{M_T}{L}, \quad \mu_R \triangleq \frac{M_R}{L}.$$

The network operates in two phases, cache placement phase and content delivery phase. During the cache placement phase, each transmitter $p$ designs a caching function

$$\phi_p : [2^F]^L \rightarrow [2^{FM_T}] ,$$
Fig. 1: Cache-aided wireless interference network with $N_T$ transmitters and $N_R$ receivers.

mapping the $L$ files in the database to its local cached content $U_p \triangleq \phi_p(W_1, W_2, \ldots, W_L)$. Each receiver $j$ also designs a caching function

$$\psi_j : [2^F]^L \rightarrow [2^{FM_R}],$$

mapping the $L$ files to its local cached content $V_j \triangleq \psi_j(W_1, W_2, \ldots, W_L)$. The caching functions $\{\phi_p, \psi_j\}$ are assumed to be known globally at all nodes. Note that in this paper, we restrict our study to the caching functions that allow for arbitrary coding within each file, but do not allow for inter-file coding. Similar assumptions have been made in [23].

In the delivery phase, each receiver $j$ requests a file $W_{d_j}$ from the database. We denote $d \triangleq (d_j)_{j=1}^{N_R} \in [L]^{N_R}$ as the demand vector. Each transmitter $p$ has an encoding function

$$\Lambda_p : [2^{FM_T}] \times [L]^{N_R} \times \mathbb{C}^{N_T \times N_R} \rightarrow \mathbb{C}^T.$$ 

Transmitter $p$ uses $\Lambda_p$ to map its cached content $U_p$, receiver demands $d$ and channel realization $H$ to the signal $(X_p[t])_{t=1}^T \triangleq \Lambda_p(U_p, d, H)$, where $T$ is the block length of the code. Note that $T$ depends on the receiver demand $d$ and channel realization $H$, and thus can also be denoted as $T_d, H$ (With a slight abuse of notation, we will use $T$ again to denote the average worst-case delivery time in Definition 1). Each codeword $(X_p[t])_{t=1}^T$ has an average transmit power constraint $P$. Each receiver $j$ has a decoding function

$$\Gamma_j : [2^{FM_R}] \times \mathbb{C}^T \times \mathbb{C}^{N_T \times N_R} \times [L]^{N_R} \rightarrow [2^F].$$

We denote $(Y_j[t])_{t=1}^T$ as the signal received at receiver $j$. Upon receiving $(Y_j[t])_{t=1}^T$, each receiver $j$ uses $\Gamma_j$ to decode $\hat{W}_{d_j} \triangleq \Gamma_j(V_j, (Y_j[t])_{t=1}^T, H, d)$ of its desired file $W_{d_j}$ using its cached content.
$V_j$ and the channel realization $H$ as side information. The worst-case error probability is

$$P_e = \max_{d \in [L]^{NR}} \max_{j \in [NR]} \mathbb{P}(\hat{W}_{d,j} \neq W_{d,j}).$$

The given caching and coding scheme $\{\phi_p, \psi_j, \Lambda_p, \Gamma_j\}$ is said to be feasible if the worst-case error probability $P_e \to 0$ when $F \to \infty$.

Note that the cache placement phase and the content delivery phase take place on different timescales. In general, cache placement is in much large timescale (e.g. on a daily or hourly basis) while content delivery is in a much shorter timescale. As such, the caching functions designed in the cache placement phase are unaware of the future content requests, but the coding functions during the content delivery phase are dependent on the caching functions.

### B. Performance Metrics

In this work, we adopt the following latency-oriented performance metrics originally proposed in [20].

**Definition 1**: The delivery time (DT) for a given feasible caching and coding scheme is defined as

$$T = \lim_{P \to \infty} \lim_{F \to \infty} \max_d \mathbb{E}_H(T_d^d, H).$$

**Definition 2**: The fractional delivery time (FDT) for a given feasible caching and coding scheme is defined as

$$\tau(\mu_R, \mu_T) = \lim_{P \to \infty} \lim_{F \to \infty} \sup_d \frac{\max \mathbb{E}_H(T_d^d, H)}{N_R F \cdot 1/ \log P}.$$ (2)

Moreover, the minimum FDT at given normalized cache sizes $\mu_T$ and $\mu_R$ is defined as

$$\tau^*(\mu_R, \mu_T) = \inf \{\tau(\mu_R, \mu_T) : \tau(\mu_R, \mu_T) \text{ is achievable}\}.$$ (3)

The above performance metrics are defined in the asymptomatic sense when $P \to \infty$ and $F \to \infty$. It is clear that the FDT and DT are related by $\tau = \frac{T \log P}{N_R F}$. The FDT $\tau(\mu_R, \mu_T)$ can be regarded as the worst-case relative time with respect to delivering the total $N_R F$ requested bits in an interference-free baseline system with transmission rate $\log P$ in the high SNR region.

**Remark 1**: Our definition of FDT $\tau$ is slightly different from the normalized delivery time (NDT) $\delta$ in [20] in that our FDT is further normalized by $N_R$, the number of receivers (content requesters). That is, $\tau = \delta / N_R$. With such normalization, the FDT is defined for the total $N_R F$
bits requested in the network rather than the $F$ bits requested by a single receiver as in [20]. As a result, the range of minimum FDT is $0 \leq \tau^* \leq 1$, which is truly normalized.

**Remark 2:** The FDT can also be expressed as $\tau = \frac{R}{N_{R-DoF}}$, where $R$ is the traffic load as defined in [8] and DoF is the sum DoF of the considered channel. In the special case with transmitter cache only as in [19], we have $\tau(\mu_R = 0, \mu_T) = 1/\text{DoF}$. As a result, the FDT evaluates the delivery time of the actual load at a transmission rate specified by DoF of the given channel, and hence is particularly suitable to characterize the performance of the wireless network with both transmitter and receiver caches.

**Remark 3** (Feasible domain of FDT): The FDT introduced above is able to measure the fundamental tradeoff between the cache storage and content delivery latency. However, not all normalized cache sizes are feasible. Given fixed $L$ and $M_T$, all the transmitters together can store at most $N_T M_T F$ bits of files, which leaves $LF - N_T M_T F$ bits of files to be stored in all receivers. Thus we must have $M_R F \geq LF - N_T M_T F$. This gives the feasible region for the normalized cache sizes as:

$$\begin{cases} 0 \leq \mu_R, \mu_T \leq 1 \\ \mu_R + N_T \mu_T \geq 1 \end{cases}.$$  

(4)

Throughout this paper, we study the FDT only in the feasible domain (4).

**III. MAIN RESULTS**

In this section, we present our main results on the fundamental storage-latency tradeoff for the cache-aided wireless interference network. The first two theorems present achievable upper bounds of the minimum FDT for $N_R = 2$ and 3 receivers, respectively. The last theorem presents the lower bound of the minimum FDT for any $N_T$ and any $N_R$.

**Theorem 1** (Achievable FDT for the $N_T \times 2$ network): For the cache-aided interference network with $N_T \geq 2$ transmitters and $N_R = 2$ receivers, the minimum FDT is upper-bounded by

$$\tau^*(\mu_R, \mu_T) \leq \begin{cases} \frac{1}{2} - \frac{1}{2} \mu_R, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T 2}^1 \\ \frac{N_T + 2}{2N_T} - \frac{N_T + 2}{2N_T} \mu_R - \frac{1}{2} \mu_T, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T 2}^2 \end{cases},$$  

(5)

where $\mathcal{R}_{N_T 2}^i$ are given below and sketched in Fig. 2.

$$\mathcal{R}_{N_T 2}^1 = \{(\mu_R, \mu_T) : 2 \mu_R + N_T \mu_T \geq 2, 0 \leq \mu_R \leq 1, \mu_T \leq 1\}$$

$$\mathcal{R}_{N_T 2}^2 = \{(\mu_R, \mu_T) : 2 \mu_R + N_T \mu_T \geq 2, 0 \leq \mu_R \leq 1\}.$$
Theorem 2 (Achievable FDT for the $N_T \times 3$ network): For the cache-aided interference network with $N_T \geq 2$ transmitters and $N_R = 3$ receivers, the minimum FDT is upper-bounded by

1) When $N_T = 2$,

$$\tau^* (\mu_R, \mu_T) \leq \begin{cases} 
\frac{1}{3} - \frac{1}{3} \mu_R, & (\mu_R, \mu_T) \in R^1_{23} \\
\frac{1}{2} - \frac{1}{6} \mu_R - \frac{1}{3} \mu_T, & (\mu_R, \mu_T) \in R^2_{23} \\
\frac{11}{18} - \frac{5}{9} \mu_R - \frac{1}{9} \mu_T, & (\mu_R, \mu_T) \in R^3_{23}, \\
\frac{5}{6} - \frac{7}{6} \mu_R - \frac{1}{3} \mu_T, & (\mu_R, \mu_T) \in R^4_{23} \\
\frac{7}{6} - \frac{7}{6} \mu_R - \mu_T, & (\mu_R, \mu_T) \in R^5_{23} 
\end{cases}$$

(6)

where $\{R^i_{23}\}_{i=1}^5$ are given below and sketched in Fig. 3(a)

$$R^1_{23} = \{(\mu_R, \mu_T) : 3 \mu_R + 2 \mu_T \geq 3, \mu_R \leq 1, \mu_T \leq 1\}$$

$$R^2_{23} = \{(\mu_R, \mu_T) : 3 \mu_R + 2 \mu_T < 3, 3 \mu_R \geq 1, 3 \mu_R + 4 \mu_T > 3\}$$

$$R^3_{23} = \{(\mu_R, \mu_T) : 3 \mu_R + 2 \mu_T \geq 2, \mu_T \leq 1, 3 \mu_R < 1\}$$

$$R^4_{23} = \{(\mu_R, \mu_T) : 3 \mu_R + 2 \mu_T < 2, \mu_R \geq 0, 2 \mu_T > 1\}$$

$$R^5_{23} = \{(\mu_R, \mu_T) : 2 \mu_T \leq 1, 3 \mu_R + 4 \mu_T \leq 3, \mu_R + 2 \mu_T \geq 1\}$$

2) When $N_T = 3$,

$$\tau^* (\mu_R, \mu_T) \leq \begin{cases} 
\frac{1}{3} - \frac{1}{3} \mu_R, & (\mu_R, \mu_T) \in R^1_{33} \\
\frac{4}{9} - \frac{4}{9} \mu_R - \frac{1}{3} \mu_T, & (\mu_R, \mu_T) \in R^2_{33} \\
\frac{1}{2} - \frac{5}{9} \mu_R - \frac{1}{3} \mu_T, & (\mu_R, \mu_T) \in R^3_{33}, \\
\frac{13}{18} - \frac{8}{9} \mu_R - \frac{1}{2} \mu_T, & (\mu_R, \mu_T) \in R^4_{33} \\
\frac{8}{9} - \frac{8}{9} \mu_R - \mu_T, & (\mu_R, \mu_T) \in R^5_{33} 
\end{cases}$$

(7)

Fig. 2: Cache size regions in the $N_T \times 2$ network.
where \( \{ \mathcal{R}_i \}_{i=1}^5 \) are given below and sketched in Fig. 3(b)

\[
\begin{align*}
\mathcal{R}_{33}^1 &= \{ (\mu_R, \mu_T) : \mu_R + \mu_T \geq 1, \mu_R \leq 1, \mu_T \leq 1 \} \\
\mathcal{R}_{33}^2 &= \{ (\mu_R, \mu_T) : \mu_R + \mu_T < 1, 2\mu_R + \mu_T \geq 1, \mu_R + 2\mu_T > 1 \} \\
\mathcal{R}_{33}^3 &= \{ (\mu_R, \mu_T) : 3\mu_R + 3\mu_T \geq 2, 2\mu_R + \mu_T < 1, \mu_R \geq 0 \} \\
\mathcal{R}_{33}^4 &= \{ (\mu_R, \mu_T) : 3\mu_R + 3\mu_T < 2, \mu_R \geq 0, 3\mu_T > 1 \} \\
\mathcal{R}_{33}^5 &= \{ (\mu_R, \mu_T) : 3\mu_R \leq 1, \mu_R + 2\mu_T \leq 1, \mu_R + 3\mu_T \geq 1 \}
\end{align*}
\]

3) When \( N_T \geq 4 \),

\[
\tau^*(\mu_R, \mu_T) \leq \begin{cases} 
\frac{1}{3} - \frac{1}{3} \mu_R, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T}^1 \\
\frac{1}{3} + \frac{1}{N_T(N_T-1)} - \left( \frac{1}{3} + \frac{1}{N_T(N_T-1)} \right) \mu_R - \frac{1}{3(N_T-1)} \mu_T, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T}^2 \\
\frac{1}{3} + \frac{1}{3N_T(N_T-1)} - \left( \frac{1}{3} + \frac{1}{3N_T(N_T-1)} \right) \mu_R - \frac{N_T^2 - 2}{3(N_T-1)} \mu_T, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T}^3 \\
\frac{1}{3} + \frac{1}{3N_T(N_T-1)} - \frac{1}{3(3N_T)(N_T-1)} \mu_R - \frac{1}{3} + \frac{N_T^2 - 2}{3(N_T-1)} \mu_T, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T}^4 \\
\frac{1}{3} + \frac{1}{3N_T(N_T-1)} - \frac{1}{3} + \frac{5}{3N_T} \mu_R - \mu_T, & (\mu_R, \mu_T) \in \mathcal{R}_{N_T}^5
\end{cases}
\]

where \( \{ \mathcal{R}_i \}_{i=1}^5 \) are given below and sketched in Fig. 3(c)

\[
\begin{align*}
\mathcal{R}_{N_T}^1 &= \{ (\mu_R, \mu_T) : 3\mu_R + N_T\mu_T \geq 3, 0 \leq \mu_R \leq 1, \mu_T \leq 1 \} \\
\mathcal{R}_{N_T}^2 &= \{ (\mu_R, \mu_T) : \mu_R \geq 0, 3\mu_R + N_T\mu_T < 3, 2\mu_R + N_T\mu_T \geq 2 \} \\
\mathcal{R}_{N_T}^3 &= \{ (\mu_R, \mu_T) : 2\mu_R + N_T\mu_T < 2, 3\mu_R + N_T\mu_T \geq 2, 3\mu_R + 2N_T\mu_T \geq 3 \} \\
\mathcal{R}_{N_T}^4 &= \{ (\mu_R, \mu_T) : N_T\mu_T > 1, \mu_R \geq 0, 3\mu_R + N_T\mu_T \leq 2 \} \\
\mathcal{R}_{N_T}^5 &= \{ (\mu_R, \mu_T) : N_T\mu_T \leq 1, 3\mu_R + 2N_T\mu_T \leq 3, \mu_R + N_T\mu_T \geq 1 \}
\end{align*}
\]

**Theorem 3** (Lower bound of FDT for the \( N_T \times N_R \) networks): For the cache-aided wireless interference network with \( N_T \geq 2 \) transmitters and \( N_R \geq 2 \) receivers, the minimum FDT is lower bounded by

\[
\tau^*(\mu_R, \mu_T) \geq \max \left\{ \frac{1}{N_R}(1 - \mu_R), \tau_l \right\},
\]

where

\[
\tau_l = \max_{l=1,2,\ldots,\min\{N_T,N_R\}} \frac{1}{N_R} \left\{ N_R - (N_T - l)(N_R - l)\mu_T - \left( \frac{2N_R - l + 1}{2} \cdot l + (N_R - l)^2 \right) \mu_R \right\}.
\]

In the special case when intra-file coding is not allowed in the caching functions, we have

\[
\tau_l = \max_{l=1,2,\ldots,\min\{N_T,N_R\}} \frac{1}{N_R} \left\{ N_R - (N_T - l)(N_R - l)\mu_T - \left( \frac{2N_R - l + 1}{2} \cdot l + (N_R - l)^2 \right) \mu_R \\
+ \left( \frac{2N_R - l}{2}(l - 1) + (N_R - l)^2 \right) (1 - N_T\mu_T)^+ \right\}.
\]
Fig. 3: Cache size regions in the $N_T \times 3$ network. (a) $N_T = 2$, (b) $N_T = 3$, (c) $N_T \geq 4$.

The proofs of Theorem 1 and Theorem 2 will be given in Section V and VI, respectively. The proof of Theorem 3 will be given in Section VIII.

Remark 4 (Optimality): It can be seen from the above theorems that both the upper and lower bounds of the minimum FDT are convex and piece-wise linearly decreasing functions of normalized cache sizes $\mu_R$ and $\mu_T$. It is also seen that the achievable FDT is optimal (i.e., coincide with the lower bound) when $(\mu_R, \mu_T)$ lies in region $\mathcal{R}_1$ ($\mathcal{R}_{N_T 2}$, $\mathcal{R}_{23}^1$, $\mathcal{R}_{33}^1$ and $\mathcal{R}_{N_T 3}^1$ in Theorem 1, 2, respectively). When there is no intra-file coding in the caching functions, the achievable FDT is also optimal when $(\mu_R, \mu_T)$ lies on the bottom boundary line $\mu_R + N_T \mu_T = 1$ (see in Figs. 2, 3). In other cache size regions, the maximum multiplicative gap between the
achievable upper bound and the theoretical lower bound is given in the following Corollary, the proof of which is shown in Section VIII.

**Corollary 4** (Gap of FDT for the $N_T \times 2$ and $N_T \times 3$ networks): For the cache-aided wireless interference network with $N_T \geq 2$ transmitters and $N_R = 2, 3$ receivers, the multiplicative gap between the upper bound and the lower bound of the minimum FDT is at most $\frac{N_T+1}{N_T}$ and $\frac{N_T+2}{N_T}$, respectively.

Below we discuss some special cases and the connections with existing works.

1) **Transmitter cache only ($\mu_R = 0$):**

In the special case when $\mu_R = 0$ (transmitter cache only), the achievable FDT for the $3 \times 3$ network in Theorem 2 reduces to

$$\tau^*(0, \mu_T) \leq \begin{cases} 13/18 - \mu_T/2, & 1/3 \leq \mu_T \leq 2/3 \\ 1/2 - \mu_T/6, & 2/3 < \mu_T \leq 1 \end{cases},$$

which is the same as the upper bound of 1/DoF in [19]. Also, when $\mu_R = 0$, given that $(1 - N_T \mu_T)^+ = 0$ from the feasible domain, $\tau_l$ in Theorem 3 reduces to

$$\tau_l = \max_{l=1,2,\ldots,\min\{N_T,N_R\}} \frac{1}{N_R} (N_R - (N_T - l)(N_R - l)\mu_T).$$

Thus, Theorem 3 reduces to the same lower bound $\delta^*(\mu)$ in [20] as a special case.

2) **Full transmitter cache ($\mu_T = 1$):**

When $\mu_T = 1$, each transmitter can cache all the files and hence the network can be viewed as a virtual broadcast channel as in [8] except that the server (transmitter) has $N_T$ distributed antennas. From Theorem 1 and Theorem 3, we obtain that the optimal FDT of the $N_T \times 2$ network is $\tau^* = \frac{1-\mu_R}{2}$. Comparing to the result in [8], i.e., $\tau = \frac{1-\mu_R}{1+2\mu_R}$ at $\mu_R \in \{0, 1/2, 1\}$, we can see that our FDT is better when $0 \leq \mu_R < \frac{1}{2}$ and they are same when $\frac{1}{2} \leq \mu_R \leq 1$. From Theorem 2 and 3, we obtain that the achievable FDT of the $2 \times 3$ network is

$$\tau = \begin{cases} \frac{1}{2} - \frac{5}{6} \mu_R, & 0 \leq \mu_R \leq 1/3 \\ \frac{1}{3} - \frac{1}{3} \mu_R, & 1/3 < \mu_R \leq 1 \end{cases},$$

and that the optimal FDT of the $N_T(\geq 3) \times 3$ network is $\tau^* = \frac{1-\mu_R}{3}$. Comparing to the result in [8], i.e., $\tau = \frac{1-\mu_R}{1+3\mu_R}$ at $\mu_R \in \{0, 1/3, 2/3, 1\}$, we can see that our FDT is better when $0 \leq \mu_R < \frac{2}{3}$ and they are same when $\frac{2}{3} \leq \mu_R \leq 1$ for $N_T(\geq 2) \times 3$ network. The above performance improvements are all due to transmitter cooperation gain.
IV. FILE SPLITTING AND CACHE PLACEMENT

In this section, we propose a novel file splitting and cache placement scheme for any given normalized cache sizes $\mu_R$ and $\mu_T$ and any transmitter and receiver node numbers $N_T$ and $N_R$. This scheme is the basis of the proofs of all the achievable FDTs.

In this work, we treat all the files equally without taking file popularity into account. Thus, each file will be split and cached in the same manner. Without loss of generality, we focus on the splitting and caching of file $W_i$ for any $1 \leq i \leq L$. Since each bit of the file is either cached or not cached in every node, there are $2^{N_T+N_R}$ possible cache states for each bit. Not every cache state is, however, legitimate. In specific, every bit of the file must be cached in at least one node. In addition, every bit that is not cached simultaneously in all receivers must be cached in at least one transmitter. As such, the total number of legitimate cache states for each bit is given by $2^{N_T+N_R} - \binom{N_T}{N_R} = \sum_{j=0}^{N_R-1} \binom{N_R}{j} \binom{N_T}{N_R-j} + 1$. Now with possibly different lengths, we can partition each $W_i$ into $\sum_{j=0}^{N_R} \sum_{p=1}^{N_T} \binom{N_R}{j} \binom{N_T}{p} + 1$ subfiles exclusively, each associated with one unique cache state.

Define receiver subset $\Phi \subseteq [N_R]$ and transmitter subset $\Psi \subseteq [N_T]$. Then, denote $W_{ir_1t_2}$ as the subfile of $W_i$ cached in receiver subset $\Phi$ and transmitter subset $\Psi$. For example, $W_{ir_1t_2}$ is the subfile cached in receivers 1 and 2 and transmitters 1 and 2, and $W_{ir_0t_1t_2}$ is the subfile cached in none of the receivers but in transmitters 1, 2 and 3. Similarly, we denote $W_{ir_1}$ as the collection of the subfiles from $W_i$ that are cached in receiver subset $\Phi$, i.e., $W_{ir_1} = \bigcup_{\Psi \subseteq [N_T]} W_{ir_1t_2}$. We assume that the subfiles that are cached in the same number of transmitters and the same number of receivers have the same size. Due to the symmetry of all the nodes as well as the independence of all files, this assumption is valid and does not lose any generality. Thus, we denote the size of $W_{ir_1t_2}$ by $a_{|\Phi||\Psi|}F$, where $|\Psi|$ and $|\Phi|$ are the cardinalities of $\Psi$ and $\Phi$, respectively, and $a_{|\Phi||\Psi|}$ is the file splitting ratio to be optimized later. For example, the size of $W_{ir_1t_2}$ is $a_{22}F$ and the size of $W_{ir_0t_1t_2}$ is $a_{03}F$. Here, the file splitting ratios $\{a_{|\Phi||\Psi|}\}$ should satisfy the following constraints:

\footnote{This is because we do not allow receiver cooperation and all the messages must be sent from the transmitters.}
integer point \( \text{(12)} \) comes from the constraint of file size. This is because for each file, the number of its subfiles cached in \(|\Phi|\) out of \(N_R\) receivers and \(|\Psi|\) out of \(N_T\) transmitters is given by \({N_R \choose |\Phi|}{N_T \choose |\Psi|}\) and they all have same length of \(a_{|\Phi||\Psi|}\) bits, for \(|\Phi| = 0, 1, \ldots, N_R\) and \(|\Psi| = 1, 2, \ldots, N_T\) or \((|\Phi| = N_R, |\Psi| = 0)\). Constraint \( \text{(13)} \) comes from the receiver cache size limit. This is because for each receiver, the total number of subfiles it caches is given by \(\sum_{|\Phi|=0}^{N_R} \sum_{|\Psi|=1}^{N_T} \left( \frac{N_R}{|\Phi|} \frac{N_T - 1}{|\Psi| - 1} \right) a_{|\Phi||\Psi|} \leq \mu_R \). Among them, there are \(\frac{N_R - 1}{|\Phi| - 1}\left( \frac{N_T}{|\Psi|} \right)\) subfiles with length of \(a_{|\Phi||\Psi|}F\) bits, for \(|\Phi| = 1, \ldots, N_R\) and \(|\Psi| = 1, 2, \ldots, N_T\), and there is only one subfile with length of \(a_{N_R0}F\) bits. Likewise, constraint \( \text{(14)} \) comes from the transmitter cache size limit. This is because for each transmitter, the total number of subfiles it caches is given by \(\sum_{|\Phi|=0}^{N_R} \sum_{|\Psi|=1}^{N_T} \left( \frac{N_R - 1}{|\Phi|} \frac{N_T}{|\Psi| - 1} \right) a_{|\Phi||\Psi|} \leq \mu_T \). Among them, there are \(\frac{N_R}{|\Phi|}\left( \frac{N_T - 1}{|\Psi| - 1} \right)\) subfiles with length of \(a_{|\Phi||\Psi|}F\) bits for \(|\Phi| = 0, \ldots, N_R\) and \(|\Psi| = 1, 2, \ldots, N_T\).

Remark 5 (Integer points): Consider the special case where \((\mu_R, \mu_T)\) satisfies \(N_R\mu_R = m\) and \(N_T\mu_T = n\) with \(m\) and \(n\) being any integers. These normalized cache size values are referred to as integer points, where every bit of each file can be cached simultaneously at \(m\) receivers and \(n\) transmitters on average. The authors in [22] proposed to split each file so that each bit is cached exactly at \(m\) receivers and \(n\) transmitters. For example, in a \(3 \times 3\) interference network at integer point \((\mu_R = 1/3, \mu_T = 2/3)\), they proposed to partition each file equally into 9 disjoint subfiles, each of fractional size \(a_{12} = 1/9\), and place each subfile at exactly 2 transmitters and 1 receiver. Such equal file splitting and cache placement is, however, not unique. Alternatively, we can partition each file into 2 subfiles, one cached at all 3 transmitters but not any receiver with fractional size \(a_{03} = 2/3\) and the other cached at all 3 receivers but not any transmitter with fractional size \(a_{30} = 1/3\). As we will show in Section VII, these two file splitting and caching strategies achieve the same FDT. Through this example, it can be seen that our proposed file splitting and cache placement strategy is more general. It offers the freedom to adjust the file splitting ratios for caching gain optimization as will be discussed in Section V and VI.
V. ACHIEVABLE SCHEME IN THE $N_T \times 2$ INTERFERENCE NETWORK

In this section, we prove the achievability of the FDT for the cache-aided $N_T \times 2$ interference network in Theorem 1. The main idea of the proof is to design the delivery phase given the file splitting and caching strategy presented in Section IV and then to compute and minimize the achievable FDT by optimizing the file splitting ratios. Without loss of generality, we assume that receiver 1 desires $W_1$ and receiver 2 desires $W_2$ in the delivery phase. In specific, receiver 1 wants subfiles $\{W_{1r_0}, W_{1r_2}\}$ and receiver 2 wants subfiles $\{W_{2r_0}, W_{2r_1}\}$. We divide these subfiles into three groups according to their cache states and present the delivery scheme for each group individually. As will be clear in this section, the cache states of the three group of subfiles transform the original interference network into three different network topologies, namely, X channel, broadcast channel, and multicast channels, and therefore exploit interference alignment gain, transmitter cooperation gain, and coded multicasting gain, respectively.

A. Content Delivery

1) Subfiles cached in zero receiver and one transmitter ($\{W_{1r_0t_\Psi}, W_{2r_0t_\Psi}\}|\Psi|=1$): Each of these subfiles is cached in one transmitter and desired by one receiver. The message flow pattern is shown in Fig. 4(a). Clearly, the network topology becomes an $N_T \times 2$ X channel. In the general $N_T \times N_R$ X channel, the maximum sum DoF is $\frac{N_RN_T}{N_T+N_R-1}$ by using interference alignment [24]. Thus, Given that the total amount of bits to deliver is $2N_Ta_{01}F$, the delivery time for these subfiles over the $N_T \times 2$ X channel is $T = \frac{2N_Ta_{01}F}{2N_T\log P/(N_T+1)}$ and the corresponding FDT is $\tau = \frac{(N_T+1)a_{01}}{2}$.

2) Subfiles cached in zero receiver and two or more transmitters ($\{W_{1r_\Psi}, W_{2r_\Psi}\}|\Psi|\geq2$): Since each of these subfiles is cached in at least two transmitters, transmitter cooperation gain can be exploited. To illustrate the delivery scheme, we take the set of subfiles with $|\Psi| = 2$ for example. The message flow pattern is shown in Fig. 4(b). We use time division multiple access (TDMA) technique to let each transmitter cooperation set deliver their messages sequentially. In specific, we divide the transmission time into $\left(\frac{N_T}{2}\right)$ time slots. In each time slot, we select a transmitter subset $\Psi$ (e.g. $\Psi = \{1, 2\}$), and let transmitters in this subset to cooperatively transmit $W_{1r_\Psi}$ and $W_{2r_\Psi}$ simultaneously to receiver 1 and 2, respectively. Clearly, the network topology in each slot becomes a MISO broadcast channel with two transmit antennas and two single-antenna receivers. In the general MISO broadcast channel with $N_T$ transmit antennas...
and $N_R$ single-antenna receivers, the optimal sum DoF is $\min\{N_T, N_R\}$ \cite{25}. Thus, the DoF in our considered MISO broadcast channel with $N_T = N_R = 2$ is 2, and the delivery time is $T = \frac{(N_T)a_{02}F}{\log P}$. In the general case with $|\Psi| = i \geq 2$, we also use the TDMA technique so that all the $\binom{N_T}{i}$ possible transmitter cooperation sets take turns to transmit. The delivery time is $T = \frac{(N_T)a_{0i}F}{\log P}$. Thus, the total FDT for the delivery of subfiles $\{W_{jr_\Psi}\}_{j\in[2]}$ for all $\Psi$'s with $|\Psi| \geq 2$ is $\tau = \frac{1}{2} \sum_{i=2}^{N_T} \binom{N_T}{i}a_{0i}$.

3) Subfiles cached in one receiver and one or more transmitters ($\{W_{1r_\Psi}, W_{2r_\Psi}\}_{|\Psi|\geq1}$): Since each subfile requested by one receiver is already cached in the other receiver, coded multicasting gain can be exploited. In specific, consider the subfiles $W_{1r_\Psi}, W_{2r_\Psi}$ for any transmitter subset $\Psi$. Transmitters in each subset $\Psi$ can generate a new message $W_{12r_\Psi} \triangleq W_{1r_\Psi} \oplus W_{2r_\Psi}$, where $\oplus$ denotes the bit-wise XOR. Each coded message is needed by both receivers. To illustrate the delivery scheme, we take the set of messages with $|\Psi| = 2$ for example. The message flow pattern is shown in Fig. 4(c). We adopt TDMA technique again so that all the $\binom{N_T}{2}$ possible transmitter cooperation sets take turns to transmit. In specific, we divide the transmission time into $\binom{N_T}{2}$ time slots. In each time slot, we select one transmitter subset $\Psi$ (e.g. $\Psi = \{1, 2\}$) and let transmitters in this subset cooperatively transmit $W_{12r_\Psi}$ to receiver 1 and 2. The network topology in each slot now becomes a broadcast channel with common information only, which we refer to as multicast channel. The maximum sum DoF of the multicast channel is 1, no matter the transmitters cooperate or not. Its converse can be proved easily using cut-set bound on each receiver. Thus the delivery time $T = \frac{(N_T)a_{12}F}{\log P}$ is achieved. In the general case with $|\Psi| = i$, we also use the TDMA technique so that all the $\binom{N_T}{i}$ possible transmitter cooperation sets take turns to transmit. The delivery time is $T = \frac{(N_T)a_{1i}F}{\log P}$. Thus, the total FDT for the delivery of subfiles $\{W_{jr_{k\Psi}}\}_{j\neq k}$ for all $\Psi$’s is $\tau = \frac{1}{2} \sum_{i=1}^{N_T} \binom{N_T}{i}a_{1i}$.

B. Optimization Of Splitting Ratios

Summing up all the FDT obtained in the previous subsections, we obtain the total FDT in the delivery phase as

$$\tau = \frac{1}{2} \left( (N_T + 1)a_{01} + \sum_{i=2}^{N_T} \binom{N_T}{i}a_{0i} + \sum_{i=1}^{N_T} \binom{N_T}{i}a_{1i} \right). \tag{15}$$
Fig. 4: Message flow patterns in the $N_T \times 2$ network. (a) subfiles $\{W_{jr^{t}}\}_{|\psi|=1}$, (b) subfiles $\{W_{j_{r^{t}}}\}_{|\psi|=2}$, (c) messages $\{W_{12r^{t}}\}_{|\psi|=2}$. Subfiles are listed beside the channel which carries them. Dashed circle denotes that the transmitters inside it cooperate with each other in delivery. Solid circle denotes that the channels inside it carry the same subfile.
Our goal is to minimize the FDT subject to the file slitting ratio constraints in (12)(13)(14). This is formulated as:

\[
\min \; \tau(\mu_R, \mu_T) \quad (16)
\]

\[
s.t. \quad (12)(13)(14),
\]

which is a standard linear programming problem. Using linear equation substitution and other manipulations, we can obtain the optimal solutions in closed form as follows. Here, all the regions are defined in Theorem 1.

1) Region \( R_{NT}^1 \): The optimal FDT in this region is \( \tau^* = \frac{1}{2} - \frac{1}{2} \mu_R \). The optimal file splitting ratios are not unique but must satisfy

\[
a^*_{01} = 0 \quad \text{and} \quad \sum_{i=1}^{N_T} \binom{N_T}{i} a^*_{1i} + \sum_{i=0}^{N_T} \binom{N_T}{i} a^*_{2i} = \mu_R.
\]

When \( \mu_T \geq 1 - \mu_R \), one feasible solution is

\[
a^*_{20} = \mu_R, \quad a^*_{0N_T} = 1 - \mu_R, \quad (17)
\]

and other ratios are 0. When \( \mu_T < 1 - \mu_R \), one feasible solution is

\[
a^*_{20} = \mu_R, \quad a^*_{02} = \frac{1 - \mu_R - \mu_T}{(N_T - 1)(N_T/2 - 1)}, \quad a^*_{0N_T} = \frac{N_T \mu_T - 2(1 - \mu_R)}{N_T - 2}, \quad (18)
\]

and other ratios are 0.

2) Region \( R_{NT}^2 \): The optimal FDT in this region is \( \tau^* = \frac{N_T-1}{2^{N_T}} - \frac{N_T-1}{2^{N_T}} \mu_R - \frac{1}{2} \mu_T \). The optimal file splitting ratios are not unique but must satisfy

\[
a^*_{01} = \frac{2}{N_T}(1 - \mu_R) - \mu_T, \quad a^*_{0n_1} = 0(\forall n_1 \geq 3), \quad a^*_{1n_2} = 0(\forall n_2 \geq 2), \quad a^*_{2n_3} = 0(\forall n_3 \geq 1),
\]

\[
N_T a^*_{11} + a^*_{20} = \mu_R,
\]

\[
(N_T - 1)a^*_{02} + 2a^*_{11} = 2\mu_T - \frac{2}{N_T}(1 - \mu_R).
\]

One feasible solution is

\[
a^*_{01} = \frac{2}{N_T}(1 - \mu_R) - \mu_T, \quad a^*_{20} = \mu_R, \quad a^*_{02} = \frac{2}{N_T - 1} \left( \mu_T - \frac{1}{N_T}(1 - \mu_R) \right), \quad (19)
\]

and other ratios are 0.

Thus, Theorem 1 is proved.
VI. ACHIEVABLE SCHEME IN THE $N_T \times 3$ INTERFERENCE NETWORK

In this section, we prove the achievability of the FDT for the cache-aided $N_T \times 3$ interference network in Theorem 2. Similar to Section V, the main idea of the proof is to design the delivery phase given the proposed file splitting and cache placement scheme and then to compute and minimize the achievable FDT by optimizing the file splitting ratios. However, compared to the previous section, the message flow patterns and the corresponding transmission schemes in the delivery phase are more complex due to more number of receivers. We shall elaborate the details in this section. Without loss of generality, we assume that receiver 1, 2, 3 desires files $W_1$, $W_2$, $W_3$, respectively. Therefore, each receiver $j$, for $j \in [3]$, only needs the subfiles $W_{jrg}$, $W_{jrk}$ and $W_{jrk_{kl}} (k, l \neq j)$. We divide these subfiles into six groups according to their cache states, and present the delivery scheme for each group individually. Similar to the previous section, the cache states of the six groups of subfiles transform the original interference network into six different network topologies, including X channel, broadcast channel, multicast channels and hybrid form of these channels, and thus exploits various types of caching gains.

A. Content Delivery

1) Subfiles cached in zero receiver and one transmitter ($\{W_{jrgt_\Psi}\}_{j \in [3], |\Psi|=1}$): Each message is desired by one receiver and available at one transmitter. The message flow pattern is shown in Fig. 5(a), and the network topology becomes an $N_T \times 3$ X channel, for which the maximum sum DoF is $\frac{3N_T}{N_T+2}$ [24]. Given that the total amount of bits to deliver is $3N_Ta_01F$, the delivery time of these subfiles is $T = \frac{(N_T+2)a_01F}{\log P}$ and the corresponding FDT is $\tau = \frac{N_T+2}{3}a_01$.

2) Subfiles cached in zero receiver and two transmitters ($\{W_{jrgt_\Psi}\}_{j \in [3], |\Psi|=2}$): Each message is desired by one receiver and available at two different transmitters. The message flow pattern is shown in Fig. 5(b), and the network topology can be seen as a partially cooperative X channel. In the following lemma, we obtain the sum DoF of this channel by using interference neutralization and real interference alignment. The proof is given in Appendix A.

Lemma 1: The achievable sum DoF of the $N_T \times 3$ partially cooperative X channel is $\frac{3N_T(N_T-1)}{N_T(N_T-1)+1}$.

Using Lemma 1 and given that the total amount of bits to deliver is $3\left(\frac{N_T}{2}\right)a_02F$, we obtain the FDT as $\tau = \frac{N_T(N_T-1)+1}{6}a_02$. 
3) Subfiles cached in zero receiver and three or more transmitters ($\{W_{jr\cap t_\Psi}\}_{j\in[3],|\Psi|\geq3}$): To illustrate the delivery scheme, we take the set of subfiles with $|\Psi|=3$ for example. The message flow pattern is shown in Fig. 5(c). Similar to the delivery of subfiles $\{W_{jr\cap t_\Psi}\}_{j\in[2]}$ with $|\Psi|=2$ in Section V-A, we divide the transmission time into $\left(\binom{N_T}{3}\right)$ time slots and use TDMA technique to let each transmitter cooperation set deliver their messages sequentially. The network topology in each slot thus becomes a MISO broadcast channel with three transmit antennas and three single-antenna receivers whose sum DoF is $3$ [25]. When $|\Psi|=i \geq 3$, we also use the TDMA technique so that all the $\left(\binom{N_T}{i}\right)$ possible transmitter cooperation sets take turns to transmit. Thus the total FDT of subfiles $\{W_{jr\cap t_\Psi}\}_{j\in[3]}$ for all $\Psi$’s with $|\Psi| \geq 3$ is $\tau = \frac{1}{3} \sum_{i=3}^{N_T} \binom{N_T}{i} a_{1i}$.

4) Subfiles cached in one receiver and one transmitter ($\{W_{jr\cap k t_\Psi}\}_{j,k\in[3],j\neq k,|\Psi|=1}$): Similar to the delivery of subfiles in Section V-A-2, coded multicasting gain can be exploited when delivering these subfiles. In specific, each transmitter $p$ can generate a new message $W_{jkt_p}^\oplus \triangleq W_{jrkt_p} \oplus W_{krt_p}$ needed by receiver $j$ and $k$, where $j,k \in [3], j < k$. In the delivery phase, there are $3N_T$ independent messages in total. Each message is desired by two different receivers, and is available at one transmitter. The message flow pattern is shown in Fig. 5(d), and the network topology can be seen as a hybrid X-multicast channel. Lemma 2 below presents the sum DoF of this channel. The proof is based on interference alignment and is shown in Appendix B.

**Lemma 2**: The achievable sum DoF of the $N_T \times 3$ hybrid X-multicast channel is $\frac{3N_T}{2N_T+1}$.

Using Lemma 2 and given that the total amount of bits to deliver is $3N_T a_{11} F$, we obtain $\tau = \frac{1}{3} (2N_T + 1) a_{11}$.

5) Subfiles cached in one receiver and two or more transmitters ($\{W_{jr\cap k t_\Psi}\}_{j,k\in[3],j\neq k,|\Psi|\geq2}$): Similar to the previous case, coded multicasting gain can also be exploited. The difference is that the network topology becomes a partially cooperative hybrid X-multicast channel, whose message flow pattern is shown in Fig. 5(e) with $|\Psi|=2$ as an example. Lemma 3 below presents the sum DoF of this channel. Its proof is based on interference neutralization and is detailed in Appendix C.

**Lemma 3**: The achievable sum DoF of the $N_T \times 3$ partially cooperative hybrid X-multicast channel is $\frac{3}{2}$.

As such, the total FDT of subfiles $\{W_{jr\cap k t_\Psi}\}_{j,k\in[3],j<k}$ for all $\Psi$’s with $|\Psi| \geq 2$ is $\tau = \frac{2}{3} \sum_{i=2}^{N_T} \binom{N_T}{i} a_{1i}$. 


6) Subfiles cached in two receivers and one or more transmitters ($\{W_{jrt_{\Psi}}\}_{j,r\in[3],k,l\neq j,|\Psi|\geq1}$): In this final group of subfiles, each subfile requested by one receiver is already cached in the other two receivers. Coded multicasting opportunities can thus be similarly exploited. In specific, for every transmitter subset $\Psi$, we can generate a new message $W_{123t_{\Psi}} \triangleq W_{1r2t_{\Psi}} \oplus W_{2r1t_{\Psi}} \oplus W_{3r1t_{\Psi}}$ needed by all three receivers. The message flow pattern of the example with $|\Psi| = 2$ is shown in Fig. 5(f). As before, we divide the transmission time into $(N_T^2)$ time slots and use TDMA technique so that all the $(N_T^2)$ transmitter cooperation sets take turns to deliver their messages. The network topology in each slot thus becomes the multicast channel, for which the maximum sum DoF is 1. Thus, the total FDT of subfiles $\{W_{jrt_{\Psi}}\}_{k,l\neq j}$ for all $\Psi$’s is $\tau = \frac{1}{3} \sum_{i=1}^{N_T} (\binom{N_T}{i}) a_{2i}$.

B. Optimization Of Splitting Ratios

Summing up all the FDT obtained in the previous subsection, we obtain the total FDT in the delivery phase as

$$\tau = \frac{1}{3} \left( (N_T + 2)a_{01} + \frac{N_T(N_T - 1) + 1}{2} a_{02} + \sum_{i=3}^{N_T} \binom{N_T}{i} a_{0i} + (2N_T + 1)a_{11} ight. + \left. 2 \sum_{i=2}^{N_T} \binom{N_T}{i} a_{1i} + \sum_{i=1}^{N_T} \binom{N_T}{i} a_{2i} \right). \quad (20)$$

Substituting (20) into (16) and solving this linear programming problem, we obtain the optimal solutions in closed form as follows. Here all the regions are defined in Theorem 2.

1) When $N_T = 2$:

Region $R_{23}^1$: The optimal FDT in this region is $\tau^* = \frac{1}{3} - \frac{1}{3}\mu_R$. The file splitting ratios are not unique but must satisfy

$$a_{01}^* = a_{02}^* = a_{11}^* = 0,$$

$$a_{12}^* + 4a_{21}^* + 2a_{22}^* + a_{30}^* + 2a_{31}^* + a_{32}^* = \mu_R.$$  

One feasible solution is

$$a_{12}^* = \frac{1 - \mu_R}{2}, a_{30}^* = \frac{3}{2}\mu_R - \frac{1}{2}, \quad (21)$$

and other ratios are 0.
Fig. 5: Message flow patterns in the $N_T \times 3$ network.
Region $R^2_{23}$: The optimal FDT in this region is $\tau^* = \frac{1}{2} - \frac{1}{2} \mu_R - \frac{1}{5} \mu_T$. The file splitting ratios are not unique but must satisfy

\[
\begin{align*}
a_{11}^* &= \frac{1}{2} (1 - \mu_R) - \frac{1}{3} \mu_T, \ a_{01}^* = a_{02}^* = a_{22}^* = a_{31}^* = a_{32}^* = 0, \\
a_{12}^* + 4a_{21}^* + a_{30}^* &= 2\mu_R + \frac{2}{3} \mu_T - 1, \\
3a_{12}^* + 3a_{21}^* &= \frac{3}{2} \mu_R + 2\mu_T - \frac{3}{2}.
\end{align*}
\]

One feasible solution is

\[
a_{11}^* = \frac{1}{2} (1 - \mu_R) - \frac{1}{3} \mu_T, \ a_{12}^* = \mu_R + \frac{2}{3} \mu_T - \frac{2}{3}, \ a_{02}^* = 1 - 3\mu_R, \quad (22)
\]

and other ratios are 0.

Region $R^3_{23}$: The optimal FDT in this region is $\tau^* = \frac{11}{18} - \frac{2}{9} \mu_R - \frac{1}{5} \mu_T$. The file splitting ratios are unique and given by

\[
\begin{align*}
a_{11}^* &= \frac{1 - \mu_T}{3}, \ a_{12}^* = \mu_R + \frac{2}{3} \mu_T - \frac{2}{3}, \ a_{02}^* = 1 - 3\mu_R, \quad (23)
\end{align*}
\]

and other ratios are 0.

Region $R^4_{23}$: The optimal FDT in this region is $\tau^* = \frac{5}{6} - \frac{7}{6} \mu_R - \frac{1}{3} \mu_T$. The file splitting ratios are unique and given by

\[
\begin{align*}
a_{11}^* &= \frac{\mu_R}{2}, \ a_{01}^* = 1 - \frac{3}{2} \mu_R - \mu_T, \ a_{02}^* = 2\mu_T - 1, \quad (24)
\end{align*}
\]

and other ratios are 0.

Region $R^5_{23}$: The optimal FDT in this region is $\tau^* = \frac{7}{6} - \frac{7}{6} \mu_R - \mu_T$. The file splitting ratios are unique and given by

\[
\begin{align*}
a_{01}^* &= \frac{3}{2} (1 - \mu_R) - 2\mu_T, \ a_{11}^* = \mu_T - \frac{1}{2} (1 - \mu_R), \ a_{30}^* = 1 - 2\mu_T, \quad (25)
\end{align*}
\]

and other ratios are 0.

2) When $N_T = 3$:

Region $R^1_{33}$: The optimal FDT in this region is $\tau^* = \frac{1}{3} - \frac{4\mu_R}{3}$. The optimal splitting ratios are not unique but must satisfy

\[
\begin{align*}
a_{11}^* &= a_{01}^* = a_{02}^* = 0, \quad (26) \\
a_{30}^* + 3a_{31}^* + 3a_{32}^* + a_{33}^* + 6a_{21}^* + 6a_{22}^* + 2a_{23}^* + 3a_{12}^* + a_{13}^* &= \mu_R, \quad (27)
\end{align*}
\]
One feasible solution is
\[ a_{30}^* = \mu_R, a_{03}^* = 1 - \mu_R, \]
and other ratios are 0.

Region \( R_{33}^2 \): The optimal FDT is \( \tau^* = \frac{4}{9} - \frac{4\mu_R}{9} - \frac{\mu_T}{9} \). The optimal splitting ratios are not unique but must satisfy
\[
a_{01}^* = a_{02}^* = a_{31}^* = a_{32}^* = a_{33}^* = a_{22}^* = a_{23}^* = a_{13}^* = 0, \\
a_{11}^* = \frac{1}{3} - \frac{\mu_R}{3} - \frac{\mu_T}{3}, \\
a_{30}^* + 6a_{21}^* + 3a_{12}^* = 2\mu_R + \mu_T - 1, \\
3a_{21}^* + 6a_{12}^* = \mu_R + 2\mu_T - 1.
\]

One feasible solution is
\[ a_{11}^* = \frac{1}{3} - \frac{\mu_R}{3} - \frac{\mu_T}{3}, a_{30}^* = 2\mu_R + \mu_T - 1, a_{03}^* = \mu_R + 2\mu_T - 1, \]
and other ratios are 0.

Region \( R_{33}^3 \): The optimal FDT is \( \tau^* = \frac{1}{2} - \frac{5}{9}\mu_R - \frac{1}{9}\mu_T \). The optimal splitting ratios are unique and given by
\[ a_{11}^* = \frac{\mu_R}{3}, a_{02}^* = 1 - 2\mu_R - \mu_T, a_{03}^* = 3\mu_R + 3\mu_T - 2, \]
and other ratios being 0.

Region \( R_{33}^4 \): The optimal FDT is \( \tau^* = \frac{13}{18} - \frac{8}{9}\mu_R - \frac{1}{2}\mu_T \). The optimal splitting ratios are unique and given by
\[ a_{11}^* = \frac{\mu_R}{3}, a_{01}^* = \frac{2}{3} - \mu_R - \mu_T, a_{02}^* = \mu_T - \frac{1}{3}, \]
and other ratios being 0.

Region \( R_{33}^5 \): The optimal FDT is \( \tau^* = \frac{8}{9} - \frac{8}{9}\mu_R - \mu_T \). The optimal splitting ratios are unique and given by
\[ a_{11}^* = \frac{\mu_R}{3} + \mu_T - \frac{1}{3}, a_{01}^* = 1 - \mu_R - 2\mu_T, a_{30}^* = 1 - 3\mu_T, \]
and other ratios being 0.
3) When \( N_T \geq 4 \):

Region \( R^1_{N_T} \): The optimal FDT in this region is \( \tau^* = \frac{1}{3} - \frac{1}{3} \mu_R \). The optimal splitting ratios are not unique but must satisfy
\[
\sum_{i=2}^{N_T} \binom{N_T}{i} a_{1i}^* + 2 \sum_{i=1}^{N_T} \binom{N_T}{i} a_{2i}^* + \sum_{i=0}^{N_T} \binom{N_T}{i} a_{3i}^* = \mu_R.
\]
When \( \mu_T \geq 1 - \mu_R \), one feasible solution is
\[
a_{30}^* = \mu_R, a_{0N_T}^* = 1 - \mu_R,
\]
and other ratios are 0. When \( \mu_T < 1 - \mu_R \), one feasible solution is
\[
a_{30}^* = \mu_R, a_{03}^* = \frac{1 - \mu_R - \mu_T}{\binom{N_T}{3} - \binom{N_T-1}{2}}, a_{0N_T}^* = \frac{N_T \mu_T - 3(1 - \mu_R)}{N_T - 3},
\]
and other ratios are 0.

Region \( R^2_{N_T} \): The optimal FDT is \( \tau^* = \frac{1}{3} + \frac{1}{N_T(N_T-1)} - (\frac{1}{3} + \frac{1}{N_T(N_T-1)}) \mu_R - \frac{1}{3(N_T-1)} \mu_T \). The optimal splitting ratios are not unique but must satisfy
\[
a_{02}^* = \frac{6}{N_T(N_T-1)} (1 - \mu_R) - \frac{2 \mu_T}{N_T - 1}, a_{01}^* = a_{11}^* = 0,
\]
\[
a_{0n_1}^* = 0 (\forall n_1 \geq 4), a_{1n_2}^* = 0 (\forall n_2 \geq 3), a_{2n_3}^* = 0 (\forall n_3 \geq 2), a_{3n_4}^* = 0 (\forall n_4 \geq 1),
\]
\[
\binom{N_T}{2} a_{12}^* + 2 N_T a_{21}^* + a_{30}^* = \mu_R,
\]
\[
\binom{N_T - 1}{2} a_{03}^* + 3(N_T - 1) a_{12}^* + 3 a_{21}^* = 3 \mu_T - \frac{6}{N_T} (1 - \mu_R).
\]
One feasible solution is
\[
a_{02}^* = \frac{6}{N_T(N_T-1)} (1 - \mu_R) - \frac{2 \mu_T}{N_T - 1}, a_{03}^* = \frac{2}{(N_T-1)(N_T-2)} \left( 3 \mu_T - \frac{6}{N_T} (1 - \mu_R) \right),
\]
\[
a_{30}^* = \mu_R,
\]
\[
(35)
\]
\[
(36)
\]

Region \( R^3_{N_T} \): The optimal FDT is \( \tau^* = \frac{1}{3} + \frac{2N_T-5}{3N_T(N_T-1)} - (\frac{1}{3} + \frac{2N_T-5}{3N_T(N_T-1)}) \mu_R - \frac{N_T-3}{3(N_T-1)} \mu_T \).
The optimal splitting ratios are unique and given by
\[
a_{02}^* = \frac{2}{N_T - 1} \left( 2 \mu_T - \frac{3}{N_T} (1 - \mu_R) \right), a_{11}^* = \frac{2}{N_T} (1 - \mu_R) - \mu_T, a_{30}^* = N_T \mu_T + 3 \mu_R - 2,
\]
\[
(37)
\]
and other ratios being 0.

Region $\mathcal{R}_{N_T3}^4$: The optimal FDT is $\tau^* = \frac{1}{3} + \frac{1}{N_T} + \frac{N_T-2}{3N_T} - \left( \frac{1}{3} + \frac{5}{3N_T} \right) \mu_R - \left( \frac{1}{3} + \frac{N_T-2}{3(N_T-1)} \right) \mu_T$.

The optimal splitting ratios are unique and given by
\[
a_{11}^* = \frac{\mu_R}{N_T}, \quad a_{01}^* = \frac{2}{N_T} - \mu_T - \frac{3}{N_T} \mu_R, \quad a_{02}^* = \frac{2\mu_T - \frac{2}{N_T}}{N_T - 1},
\]
and other ratios being 0.

Region $\mathcal{R}_{N_T3}^5$: The optimal FDT is $\tau^* = \frac{1}{3} + \frac{5}{3N_T} - \left( \frac{1}{3} + \frac{5}{3N_T} \right) \mu_R - \mu_T$. The optimal splitting ratios are unique and given by
\[
a_{11}^* = \mu_T - \frac{1}{N_T}(1 - \mu_R), \quad a_{01}^* = \frac{3}{N_T}(1 - \mu_R) - 2\mu_T, \quad a_{30}^* = 1 - N_T \mu_T,
\]
and other ratios being 0.

Theorem 2 is thus proved.

VII. DISCUSSION

In this section, we provide some discussions on our proposed caching and delivery schemes, and offer some important insights into the impact of caching in the considered interference networks.

A. On Caching at integer points

It is important to realize from Theorem 1 and Theorem 2 that the achievable FDT $\tau(\mu_R, \mu_T)$ at the feasible cache size region (4) is a lower convex envelope of the FDTs at integer points $\mu_R \in \{0, \frac{1}{N_R}, \frac{2}{N_R}, \ldots, 1\}$ and $\mu_T \in \{0, \frac{1}{N_T}, \frac{2}{N_T}, \ldots, 1\}$. Therefore, it suffices to discuss the optimal caching at integer points since the optimal caching at non-integer points can be obtained by memory sharing and file partitioning.

We find from Section V-B and Section VI-B, surprisingly, that the equal file splitting strategy is optimal (though may not always be unique) at the integer points. By the equal file splitting strategy, each file is split into $\binom{N_R}{N_R \mu_R} \binom{N_T}{N_T \mu_T}$ equal-sized subfiles, each cached in $N_R \mu_R$ receivers and $N_T \mu_T$ transmitters, and with file splitting ratios given by $a_{N_R \mu_R, N_T \mu_T} = \frac{1}{\binom{N_R}{N_R \mu_R} \binom{N_T}{N_T \mu_T}}$ and all the rest $a_{ij} = 0$. The proposed delivery scheme introduced before then transforms the network topology to a generalized hybrid X-multicast channel with multicast size $1 + N_R \mu_R \in$.

\footnote{See Remark 5 in Section IV for the definition of integer points.}
\{1, 2, \ldots, N_R\}$ and transmitter cooperation size $N_T \mu_T \in \{1, 2, \ldots, N_T\}$. Therefore, the FDT can be expressed in a unified form as

$$\tau = \frac{(1 - \mu_R)}{(1 + N_R \mu_R) \cdot \text{DoF}_{N_T, N_R \mu_R}}$$

(40)

for both $N_T \times 2$ and $N_T \times 3$ networks. Here, $\text{DoF}_{N_T \mu_T, N_R \mu_R}$ is the DoF of the generalized X-multicast channel whose value may vary for different $N_T \mu_T$ and $N_R \mu_R$. The exact values of DoF and the corresponding FDT at all possible integer points are summarized in Table I and Table II for the $N_T \times 2$ and $N_T \times 3$ networks, respectively.

The unified expression of FDT in (40) reveals the gains of caching more explicitly. The term $(1 - \mu_R)$ denotes the receiver local caching gain, that is, each receiver only needs a fraction $(1 - \mu_R)$ of its desired file. The term $(1 + N_R \mu_R)$ denotes the coded multicasting gain, that is, each transmitted message is needed by $1 + N_R \mu_R$ different receivers. This gain derives from our caching scheme that we cache different messages at different receivers, and from our delivery scheme that multicasting opportunities are created by coding among the transmitted messages. The DoF term reflects the cache-induced transmitter cooperation gain via interference neutralization or interference alignment.

From Table I and Table II, it can be seen that at given transmitter cache size, increasing the receiver cache storage decreases the DoF of the network. This is because more coded multicasting opportunities are exploited in the delivery phase when the receiver cache storage increases, thereby, limiting the DoF due to the nature of multicast channels. But on the other hand, multicast transmission reduces the independent message flows to be sent over the channel. As a result, the overall FDT still decreases as the receiver cache size increases. It is also seen that when transmitter cache storage or transmitter number increases, the FDT decreases along with the increase of DoF. This is because more transmitter cooperation gain can be exploited in the delivery phase. However, notice that both the DoF and the FDT have a limit when transmitter cache storage or transmitter number is large enough. This is because receiver cache storage and receiver number now become the bottlenecks of the network. In the next subsection, we show in detail that when transmitter cache storage exceeds a threshold, the FDT is optimal. Thus, no more transmitter cooperation gain can be exploited when further increasing the cache storage.
TABLE I: Channels and DoFs at Integer Points in the $N_T \times 2$ Interference Network

| $\mu_R = 0$ | $\mu_R = \frac{1}{3}$ |
|-------------|---------------------|
| $\mu_T = \frac{1}{N_T}$ | $\mu_T = \frac{2}{N_T}$ | $\mu_T \geq \frac{3}{N_T}$ |
| channel | DoF | $\tau$ | channel | DoF | $\tau$ | channel | DoF | $\tau$ |
| X channel | $\frac{2N_T}{N_T + 1}$ | $\frac{N_T + 1}{2N_T}$ | Multicast channel | 1 | $\frac{1}{9}$ | Broadcast channel | 2 | $\frac{1}{9}$ | Multicast channel | 1 | $\frac{1}{9}$ |

TABLE II: Channels and DoFs at Integer Points in the $N_T \times 3$ Interference Network

| $\mu_R = 0$ | $\mu_R = \frac{1}{3}$ | $\mu_R = \frac{2}{3}$ |
|-------------|---------------------|---------------------|
| $\mu_T = \frac{1}{N_T}$ | $\mu_T = \frac{2}{N_T}$ | $\mu_T \geq \frac{3}{N_T}$ |
| Channel | DoF | $\tau$ | Channel | DoF | $\tau$ | Channel | DoF | $\tau$ |
| X channel | $\frac{3N_T}{N_T + 2}$ | $\frac{N_T + 1}{3N_T}$ | X-multicast channel | $\frac{3N_T}{2N_T + 1}$ | $\frac{2N_T + 1}{9N_T}$ | Multicast channel | 1 | $\frac{1}{9}$ | Partially cooperative X channel | $\frac{3N_T}{N_T(N_T - 1) + 1}$ | $\frac{N_T(N_T - 1) + 1}{3N_T(N_T - 1)}$ | Partially cooperative X-multicast channel | $\frac{3}{9}$ | $\frac{2}{9}$ | Multicast channel | 1 | $\frac{1}{9}$ |
| Partially cooperative X channel | $\frac{3N_T}{N_T(N_T - 1) + 1}$ | $\frac{N_T(N_T - 1) + 1}{3N_T(N_T - 1)}$ | Partially cooperative X-multicast channel | $\frac{3}{9}$ | $\frac{2}{9}$ | Multicast channel | 1 | $\frac{1}{9}$ |
| Broadcast channel | $\frac{3}{9}$ | $\frac{2}{9}$ | Multicast channel | $\frac{3}{9}$ | $\frac{2}{9}$ | Multicast channel | 1 | $\frac{1}{9}$ |

B. On the non-uniqueness of optimal file splitting ratios

It is important to realize from the previous two sections, that the optimal file splitting ratios for FDT minimization at certain cache size regions are not unique. Mathematically, this is quite expected since a linear programming problem like (16) in general does not have unique solutions. However, the physical meaning of each solution can vary dramatically. Let us take the integer point $(\mu_R = \frac{1}{3}, \mu_T = \frac{2}{3})$ in the $3 \times 3$ network for example. According to Section VI-B, there are two optimal solutions for the file splitting ratios. One is $a_{12} = \frac{1}{9}$ with all the rest $a_{ij} = 0$, which is the equal file splitting strategy as discussed before. It can exploit local caching gain, coded-multicasting gain, and transmitter cooperation gain. Another optimal solution is $a_{03} = \frac{2}{3}$, $a_{30} = \frac{1}{3}$ with the rest $a_{ij} = 0$. This solution means that each file is split into two subfiles, one has fractional size $a_{03} = \frac{2}{3}$ and is cached simultaneously at all 3 transmitters but none of the receivers, the other subfile has fractional size $a_{30} = \frac{1}{3}$ and is cached simultaneously at all
3 receivers but none of the transmitters. From the proposed delivery scheme in Section VI-A, it is seen that this solution exploits both the receiver local caching gain and the transmitter cooperation gain. In particular, the transmitter cooperation turns the interference network into a MISO broadcast channel with sum DoF = 3.

In general, we find that when \( N_T \mu_T + N_R \mu_R \geq N_R \) (i.e., Region \( \mathcal{R}_1 \) in Figs. 2-3), the optimal file splitting ratios and the delivery scheme are not unique. When \( N_T \geq N_R \), besides the equal file splitting strategy, the optimal ratios can also be \( a_{N_T,0} = \mu_R, a_{0,N_R} = \frac{1-\mu_R}{N_R} \). This solution means that each file is split into \( 1 + \left( \frac{N_T}{N_R} \right) \) subfiles, one has fractional size \( a_{N_T,0} = \mu_R \) and is cached simultaneously at all receivers but none of the transmitters, and each of the other subfiles has fractional size \( a_{0,N_R} = \frac{1-\mu_R}{N_R} \) and is cached simultaneously at \( N_R \) out of \( N_T \) different transmitters but none of the receivers. According to the delivery schemes proposed in Section V-A and VI-A, each set of \( N_R \) transmitters that cache a same subfile can cooperate and form the MISO broadcast channel with \( N_R \) transmit antennas and \( N_R \) single-antenna receivers, whose DoF is \( N_R \). Thus, we can write the FDT as

\[
\tau = \frac{1-\mu_R}{\text{DoF}_{BC}} = \frac{1-\mu_R}{N_R}.
\]  

(41)

Note that this scheme transforms the network into the MISO broadcast channel, and thus does not exploit the coded multicasting gain. Comparing (41) with (40), it can be seen that the transmitter cooperation gain in the broadcast channel has the same contribution as the combined coded-multicasting and transmitter cooperation gain in the equal file splitting strategy.

When \( N_T < N_R \), we can not construct the broadcast channel again, because the limited number of transmitters becomes a bottleneck. In this case, the optimal solution can be \( a_{N_T,0} = 1 - \frac{N_R}{N_T} (1-\mu_R), a_{N_R-N_T,N_T} = \frac{N_R(1-\mu_R)}{(N_R-N_T)} \). This solution means that each file is split into \( 1 + \left( \frac{N_R}{N_R-N_T} \right) \) subfiles, one has fractional size \( a_{N_T,0} = 1 - \frac{N_R}{N_T} (1-\mu_R) \) and is cached simultaneously at all receivers but none of the transmitters, and each of the other subfiles has fractional size \( a_{N_R-N_T,N_T} = \frac{N_R(1-\mu_R)}{(N_R-N_T)} \) and is cached simultaneously at all transmitters and \( N_R - N_T \) out of \( N_R \) different receivers. In the delivery phase, only the subfiles with fraction size \( a_{N_R-N_T,N_T} \) is transmitted, and the local caching gain, coded-multicasting gain and transmitter cooperation gain are still exploited.

The multiple choices of file splitting ratios offer more freedoms to choose appropriate caching and delivery scheme according to different limitations in practical systems, such as transmitter or receiver computation complexity or file splitting constraints.
C. On The Differences With [22]

Although the similar caching problem is considered in [22], their performance metric, caching scheme, and conclusion are significantly different from ours. First, we adopt the FDT as the performance metric while [22] used the standard DoF. As we noted in Remark 2 in Section II, FDT is particularly suitable for the considered network because it reflects not only the load reduction due to receiver cache but also the DoF enhancement due to transmitter cache. In specific, we can express the FDT as \( \tau = \frac{R}{N_R \times \text{DoF}} \), where \( R \) is the traffic load normalized by each file size. To illustrate this in detail, we consider the integer points \((\mu_R = \frac{1}{3}, \mu_T = \frac{2}{3})\) and \((\mu_R = \frac{2}{3}, \mu_T = \frac{1}{3})\) in the \(3 \times 3\) interference network. In [22], they use the equal file splitting strategy in the content placement phase and have DoF = 3 in the delivery phase at both points. However, the actual delivery time at these two points is different. At point \((\mu_R = \frac{1}{3}, \mu_T = \frac{2}{3})\), they split each file into 9 equal-sized subfiles, each cached in one receiver and two transmitters, i.e. \(a_{12} = \frac{1}{9}\) and other ratios are 0. Thus, each receiver caches 3 out of 9 subfiles of its desired file and only needs the other 6 subfiles in the delivery phase. The corresponding FDT is \( \tau = \frac{3 \times 6 \times a_{12}}{3 \times 3} = \frac{2}{9} \).

On the other hand, at point \((\mu_R = \frac{2}{3}, \mu_T = \frac{1}{3})\), they also split each file into 9 equal-sized subfiles, but each cached in two receivers and one transmitter, i.e. \(a_{21} = \frac{1}{9}\) and other ratios are 0. Thus, each receiver caches 6 out of 9 subfiles of its desired file and only needs the other 3 subfiles in the delivery phase, yielding the corresponding FDT \( \tau = \frac{3 \times 3 \times a_{21}}{3 \times 3} = \frac{1}{9} \). Clearly, the DoF alone is unable to fully capture the gains of joint transmitter and receiver caching as FDT does. Due to such inefficiency, the achievable DoF at non-integer points cannot simply be obtained as a linear combination of the DoFs at integer points through memory and file partitions as in [22].

Second, the file splitting ratios in [22] are pre-determined at each given cache size point \((\mu_R, \mu_T)\) as noted in Remark 5 of Section IV. However, our file splitting ratios are obtained by solving a linear programming problem at each given cache size point and thus are probably optimal under the given caching strategy.

Another difference between our work and [22] is that the transmission scheme in [22] is restricted to one-shot linear processing, while we allow interference alignment which may require infinite symbol extension. As such, our FDT analysis is more complete than the DoF analysis in [22].
VIII. LOWER BOUND OF THE MINIMUM FDT FOR $N_T \times N_R$ NETWORKS.

In this section, we present the proof of the lower bound of the minimum FDT for any $N_T$ and $N_R$ in Theorem 3, and the proof of the maximum multiplicative gap in Corollary 4 in Section III.

A. Lower Bound of minimum FDT (Proof of Theorem 3)

The main idea of the proof is based on cut-set bound and genie-message approach. Without loss of generality, we assume that receiver $1, 2, \ldots, N_R$ request files $W_1, W_2 \ldots W_{N_R}$, respectively.

By the converse assumption, each receiver $j$ can decode its intended message $W_j$ with the received signal $Y_j$ and its local cache content $V_j$ as side information. Therefore, we have the following bound, where $\varepsilon_F$ is a function of $F$ and satisfies $\varepsilon_F \to 0$ when $F \to \infty$.

$$F \varepsilon_F = H(W_j|Y_j, V_j)$$

$$= H(W_j|V_jW_j) - I(W_j; Y_j, V_jW_1\sim jW_L|V_jW_j)$$

$$= H(W_j) - H(V_jW_j) + H(V_jW_j|W_j) - I(W_j; Y_j, V_jW_1\sim jW_L|V_jW_j)$$

$\geq (1 - \mu_R)F - I(W_j; Y_j, V_jW_1\sim jW_L|V_jW_j)$

$$= (1 - \mu_R)F - (H(Y_j, V_jW_1\sim jW_L|V_jW_j) - H(Y_j, V_jW_1\sim jW_L|V_jW_j|W_j))$$

$\geq (1 - \mu_R)F - (H(V_jW_1\sim jW_L|V_jW_j) + H(Y_j|V_j) - H(V_jW_1\sim jW_L|V_jW_j|W_j))$

$$= (1 - \mu_R)F - H(Y_j|V_j)$$

$\geq (1 - \mu_R)F - T \log(2\pi e(c \cdot P + 1))$,

where $V_jW_s$ denotes the content of file $W_s$ cached in receiver $j$, and $c$ is only related to channel coefficients. Here, (42a) follows from the Fano’s inequality; (42b) follows from the chain rule; (42d) follows from the fact that $H(V_jW_j|W_j) = 0$, and each receiver can cache $\mu_R F$ bits of each file $W_j$ on average since it has $M_R F$ bits of cache storage; (42e) follows from the chain rule; (42g) follows from conditioning reducing entropy and (42i) follows from Lemma 1 in [20]. Therefore, by letting $F \to \infty$ and $P \to \infty$, we have $T \log P \geq (1 - \mu_R)F$ from (42), or equivalently

$$\tau^* \geq \frac{1 - \mu_R}{N_R}.$$

(43)
Fig. 6: Given received signals $Y_{1\sim l}$ from receivers $1, 2, \cdots, l$, transmitter caches $U_{l+1 \sim N_T}$ from transmitters $l + 1, l + 2, \cdots, N_T$ and receiver caches $V_{1 \sim N_R}$ at all receivers, the genie can successfully decode desired files for all receivers.

Thus, the first term on the right hand side of (9) is proved.

To prove $\tau^* \geq \tau_l$ in (10) which allows for arbitrary intra-file coding in the caching functions, we use the genie-message approach similar to [20] except that we take the receiver cache content into account in our method. By the converse assumption, we can make the following argument. Given received signals $Y$ from any $l$ receivers, local caches $U$ from any $N_T - l$ transmitters, and all the local caches $V_{1 \sim N_R}$ at receivers, the desired files for all receivers can be successfully decoded in the high-SNR regime by the genie. More specifically, for example, we consider the case when the genie has the received signals $Y_{1\sim l}$ from receivers $1, 2, \cdots, l$, transmitter caches $U_{l+1 \sim N_T}$ from transmitters $l + 1, l + 2, \cdots, N_T$ and receiver caches $V_{1 \sim N_R}$ at all receivers. With transmitter caches $U_{l+1 \sim N_T}$, the genie can reconstruct the transmitted signals $X_{l+1 \sim N_T}$ at transmitters $l + 1, l + 2, \cdots, N_T$. Given transmitted signals $X_{l+1 \sim N_T}$ and received signals $Y_{1\sim l}$, it can further reconstruct the remaining unknown transmitted signals $X_{1\sim l}$ almost surely as stated.
in Lemma 3 in [20]. With all the transmitted signals $X_{1\sim N_T}$, it can reconstruct all the received signals $Y_{1\sim N_R}$ and can further decode all the desired files successfully together with local caches $V_{1\sim N_R}$. The details of our method is illustrated in Fig. 6. Using Lemma 2 in [20] and from the analysis above, we have

$$H(W_{1\sim N_R}|Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}, W_{N_R+1\sim L}) = F \varepsilon_F + T \varepsilon_P \log P,$$

where $\varepsilon_P$ is a function of power $P$ and satisfies $\varepsilon_P \to 0$ when $P \to \infty$. Then, the entropy of desired files $W_1, W_2, \cdots, W_{N_R}$ can be expressed as

$$N_R F = H(W_{1\sim N_R}|W_{N_R+1\sim L})$$

$$= H(W_{1\sim N_R}|Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}, W_{N_R+1\sim L}) + I(W_{1\sim N_R}; Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}|W_{N_R+1\sim L})$$

$$= I(W_{1\sim N_R}; Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}|W_{N_R+1\sim L}) + F \varepsilon_F + T \varepsilon_P \log P. \quad (45)$$

The first term in (45) is upper-bounded by:

$$I(W_{1\sim N_R}; Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}|W_{N_R+1\sim L})$$

$$= H(Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}|W_{N_R+1\sim L}) - H(Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}|W_{1\sim l}) \quad (46a)$$

$$= H(Y_{1\sim l}, U_{l+1\sim N_T}, V_{1\sim N_R}|W_{N_R+1\sim L}) - T \varepsilon_P \log P \quad (46b)$$

$$\leq H(Y_{1\sim l}|W_{N_R+1\sim L}) + H(U_{l+1\sim N_T}, V_{1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}) \quad (46c)$$

$$\leq lT \log(2\pi e(c \cdot P + 1)) + H(V_{1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}) + H(U_{l+1\sim N_T}|V_{1\sim N_R}, Y_{1\sim l}, W_{N_R+1\sim L}) \quad (46d)$$

$$\leq lT \log(2\pi e(c \cdot P + 1)) + H(V_{1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}) + H(U_{l+1\sim N_T}|W_{1\sim l}, W_{N_R+1\sim L}) + F \varepsilon_F \quad (46e)$$

$$\leq lT \log(2\pi e(c \cdot P + 1)) + H(V_{1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}) + (N_T - l)(N_R - l) \mu_T F + F \varepsilon_F. \quad (46f)$$

Here, the second term in (46b) follows from the noise of received signal $Y$; (46d) follows from the chain rule and Lemma 1 in [20]; (46e) follows from the Fano’s inequality; (46f) follows from the fact that each transmitter can cache $\mu_T F$ bits of each file on average since it has $M_T F$ bits of caching storage.
Then, we bound the term \( H(V_{1 \sim N_R} | Y_{1 \sim l}, W_{N_R+1 \sim L}) \) as follows

\[
H(V_{1 \sim N_R} | Y_{1 \sim l}, W_{N_R+1 \sim L}) = H(V_1 | Y_{1 \sim l}, W_{N_R+1 \sim L}) + H(V_2 \sim N_R | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_1)
\]

\[
\leq N_R \mu_R F + H(V_2 | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_1) + \sum_{i=3}^{l} H(V_i | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_{i-1}, W_{i-1})
\]

\[
+ H(V_{l+1 \sim N_R} | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_{l-1}, W_{l-1}) + F \varepsilon_F
\]

\[
\leq N_R \mu_R F + (N_R - 1) \mu_R F + \sum_{i=3}^{l} H(V_i | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_{i-1}, W_{i-1})
\]

\[
+ H(V_{l+1 \sim N_R} | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_{l-1}, W_{l-1}) + F \varepsilon_F
\]

\[
H(V_{l+1 \sim N_R} | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_{l-1}, W_{l-1}) + F \varepsilon_F
\]

\[
\leq N_R \mu_R F + (N_R - 1) \mu_R F + \sum_{i=3}^{l} (N_R - i + 1) \mu_R F + \sum_{i=l+1}^{N_R} H(V_i | Y_{1 \sim l}, W_{N_R+1 \sim L}, V_{l-1}, W_{l-1})
\]

\[
+ F \varepsilon_F
\]

\[
\leq N_R \mu_R F + \sum_{i=2}^{l} (N_R - i + 1) \mu_R F + (N_R - l)(N_R - l) \mu_R F + F \varepsilon_F
\]

\[
= \left( \frac{2N_R - l + 1}{2} \right) \mu_R F + F \varepsilon_F.
\]

Here, (47b) follows from the Fano’s inequality and the fact that given receiver 1 has \( M_R F \) cache storage, it can cache \( \mu_R F \) bits of each file on average; (47c)-(47d) follows from the fact that each \( V_i \) \((i \leq l)\) has \( \mu_R F \) unknown bits of each file in \( W_{i \sim N_R} \) on average since the files \( W_{i-1} \) and \( W_{N_R+1 \sim L} \) are known; (47e) follows from the fact that each \( V_i \) \((i > l)\) has \( \mu_R F \) unknown bits of each file in \( W_{l+1 \sim N_R} \) on average since the files \( W_{1 \sim l} \) and \( W_{N_R+1 \sim L} \) are known.

Combining (45)-(46)-(47f), we have

\[
N_R F \leq lT \log(2\pi e(c \cdot P + 1)) + (N_T - l)(N_R - l) \mu_T F + \left( \frac{2N_R - l + 1}{2} \right) \mu_R F
\]

\[
+ F \varepsilon_F + T \varepsilon_P \log P.
\]

Dividing \( N_R F \) on both sides of (48) and letting \( F \to \infty \) and \( P \to \infty \), we have

\[
\tau = \lim_{F \to \infty} \lim_{P \to \infty} \frac{T \log P}{N_R F}
\]

\[
\geq \frac{1}{N_R l} \left\{ N_R - (N_T - l)(N_R - l) \mu_T - \left( \frac{2N_R - l + 1}{2} \right) \mu_R \right\}.
\]

(49)
Optimizing the bound in (49) over all possible choices of \( l = 1, 2, \cdots, \min\{N_T, N_R\} \), (10) is proved.

Next, we prove \( \tau^* \geq \tau_I \) in (11) where the caching functions \( \{\phi_p\} \) at each transmitter \( p \), and \( \{\psi_j\} \) at each receiver \( j \) do not allow for intra-file coding or the inter-file coding. In specific, \( H(V_{1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}) \) in (46f) can be upper-bounded by

\[
H(V_{1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}) = H(V_{1\sim l}, W_{N_R+1\sim L}) + H(V_{2\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1})
\]

\[
\leq N_R \mu RF + H(V_{2\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1}) + \sum_{i=3}^{l} H(V_{i}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1\sim i-1}, W_{1\sim i-1})
\]

\[
+ H(V_{l+1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1\sim l}, W_{1\sim l}) + \epsilon_F \epsilon_F
\]

\[
\leq N_R \mu RF + (N_R - 1) (\mu_R - (1 - N_T \mu_T)^+) F + \sum_{i=3}^{l} H(V_{i}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1\sim i-1}, W_{1\sim i-1})
\]

\[
+ H(V_{l+1\sim N_R}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1\sim l}, W_{1\sim l}) + \epsilon_F \epsilon_F
\]

\[
\leq N_R \mu RF + (N_R - 1) (\mu_R - (1 - N_T \mu_T)^+) F + \sum_{i=3}^{l} (N_R - i + 1) (\mu_R - (1 - N_T \mu_T)^+) F
\]

\[
+ \sum_{i=l+1}^{N_R} H(V_{i}|Y_{1\sim l}, W_{N_R+1\sim L}, V_{1\sim l}, W_{1\sim l}) + \epsilon_F \epsilon_F
\]

\[
\leq \sum_{i=2}^{l} (N_R - i + 1) (\mu_R - (1 - N_T \mu_T)^+) F + (N_R - l)(N_R - l) (\mu_R - (1 - N_T \mu_T)^+) F
\]

\[
+ N_R \mu RF + \epsilon_F \epsilon_F
\]

\[
= \left(\frac{2N_R - l + 1}{2} l + (N_R - l)^2\right) \mu RF - \left(\frac{2N_R - l}{2} (l - 1) + (N_R - l)^2\right) (1 - N_T \mu_T)^+ F
\]

\[
+ \epsilon_F \epsilon_F.
\]

Here, (50b) follows from the Fano’s inequality and the fact that given receiver 1 has \( M_R F \) cache storage, it can cache \( \mu_R F \) bits of each file on average; (50c) follows from the fact that all receivers must cache at least the same \( (1 - N_T \mu_T)F \) bits of each file on average to guarantee the feasibility of the scheme if \( N_T M_T F \leq LF \); (50d) follows from the fact that each \( V_i \ (i \leq l) \) has \( (\mu_R - (1 - N_T \mu_T)^+) F \) unknown bits of each file in \( W_{1\sim N_R} \) on average since we already know files \( W_{1\sim i-1} \) and \( W_{N_R+1\sim L} \); (50e) follows from the fact that each \( V_i \ (i > l) \) has \( (\mu_R - (1 - N_T \mu_T)^+) F \)
unknown bits of each file in $W_{l+1 \sim N_R}$ on average since we already know files $W_{1 \sim l}$ and $W_{N_R+1 \sim L}$.

Note that the inequalities (50) and (47) are the main novelty of the proof of the converse.

Combining (45)(46f)(50f), we have

$$N_{RF} \leq lT \log(2\pi e(c \cdot P + 1)) + (N_T - l)(N_R - l)\mu_T F + \left(\frac{2N_R - l + 1}{2} \cdot l + (N_R - l)^2\right) \mu_R F$$

$$- \left(\frac{2N_R - l}{2} \cdot (l - 1) + (N_R - l)^2\right) (1 - N_T \mu_T)^+ F + F \varepsilon_F + T \varepsilon_P \log P.$$ 

(51)

Dividing $N_{RF}$ on both sides of (51) and letting $F \rightarrow \infty$ and $P \rightarrow \infty$, we have

$$\tau = \lim_{P \rightarrow \infty} \lim_{F \rightarrow \infty} \frac{T \log P}{N_{RF}}$$

$$\geq \frac{1}{N_R l} \left\{ N_R - (N_T - l)(N_R - l)\mu_T - \left(\frac{2N_R - l + 1}{2} \cdot l + (N_R - l)^2\right) \mu_R \right.$$ 

$$+ \left(\frac{2N_R - l}{2} \cdot (l - 1) + (N_R - l)^2\right) (1 - N_T \mu_T)^+ \right\}.$$ 

(52)

Optimizing the bound in (52) over all possible choices of $l = 1, 2, \cdots, \min\{N_T, N_R\}$, (11) is proved. Combining (43)(49)(52), Theorem 3 is proved.

**B. Maximum Multiplicative Gap (Proof of Corrolary 4)**

Given the fact that the achievable FDTs (upper bound) are convex and piece-wise linearly decreasing functions of $\mu_R$ and $\mu_T$, we can rewrite the achievable FDT as the maximum of these linear functions at different regions. Let us first consider the $N_T \times 2$ interference network. The achievable FDT in (5) can be rewritten as

$$\tau_{\text{achievable}} = \frac{1}{2} \max \left\{ 1 - \mu_R, \left(\frac{N_T + 2}{N_T} (1 - \mu_R) - \mu_T \right) \right\}.$$ 

(53)
We thus have:

\[ g = \frac{\tau_{\text{achievable}}}{\tau_{\text{lowerbound}}} = \frac{\max \left\{ 1 - \mu_R \frac{(N_T+2)(1-\mu_R)}{N_T} - \mu_T \right\}}{\max \left\{ 1 - \mu_R, \tau_l \right\}} \]  
(54a)

\[ \leq \frac{\max \left\{ 1 - \mu_R \frac{(N_T+2)(1-\mu_R)}{N_T} - 1 + \frac{\mu_R}{N_T} \right\}}{1 - \mu_R} \]  
(54b)

\[ \leq \frac{\max \left\{ 1 - \mu_R \frac{(N_T+1)(1-\mu_R)}{N_T} \right\}}{1 - \mu_R} \]  
(54c)

\[ = \frac{N_T + 1}{N_T} \]  
(54d)

Here, (54c) follows from (4). Using the similar method, we can prove that for the \( N_T \times 3 \) interference network, we have \( g \leq \frac{N_T+2}{N_T} \) and the details are ignored. Thus, Corollary 4 is proved.

IX. CONCLUSION

In this paper, we have studied the storage-latency tradeoff for the cache-aided interference network with \( N_T \) transmitters and \( N_R \) receivers. We characterized the tradeoff by the fractional delivery time. We have obtained the achievable upper bounds of the minimum FDT for the \( N_T \times 2 \) and \( N_T \times 3 \) interference networks. We have also obtained the theoretical lower bound for the general \( N_T \times N_R \) interference network. Both upper and lower bounds are convex and piece-wise linear decreasing functions of normalized cache sizes \( \mu_R \) and \( \mu_T \). The achievable bound is optimal at certain cache size regions and its maximum multiplicative gap to the lower bound is \( \frac{N_T+1}{N_T} \) and \( \frac{N_T+2}{N_T} \) for \( N_R = 2 \) and \( N_R = 3 \) receivers, respectively. The proposed caching strategy is general for any transmitter and receiver numbers and at any feasible cache size regions. The optimal file splitting ratios are obtained by solving a linear programming problem for FDT minimization under the given caching strategy. Analysis shows that the optimal ratios are not unique, unlike existing works where the ratios are all pre-determined. This offers freedoms to choose appropriate caching strategy in practical systems with different constraints. As a byproduct, the achievable DoFs of three new types of channels are obtained, namely, the partially cooperative X channel, the X-multicast channel, and the partially cooperative X-multicast channel.
The extension of the achievable FDT to the cache-aided interference network with \( N_R > 3 \) receivers is challenging due to that the channels formed by the delivery of different subfiles are much more complicated.

**Appendix A: Proof of Lemma 1**

Consider the \( N_T \times 3 \) partially cooperative X channel in Section VI-A. When \( N_T = 3 \), the channel is shown in Fig. 5(b). Note that in the case with \( N_T = 3 \), the achievable DoF is already studied in [26]. Here, we extend the proof to the general case for arbitrary \( N_T \) using the similar method.

In this channel, each receiver wants \( \binom{N_T}{2} \) independent messages, each of which is available at two different transmitters. Denote \( A_{pq}, B_{pq}, C_{pq} \) as the message available at transmitters \( p, q \) (We let \( p < q \) to avoid the overlap of messages, i.e., \( A_{12} \) and \( A_{21} \) are the same message.) and needed by receiver 1, 2, 3, respectively. Since each message is available at two transmitters, transmitter cooperation can be applied to neutralize interference at receivers. In specific, each message needed by receiver \( i \) is split into two equal-sized submessages. The first part is neutralized at receiver \( j \), \( j \in \{1, 2, 3\}\{i\} \), and the second part is neutralized at receiver \( k \), \( k \in \{1, 2, 3\}\{i, j\} \). Note that we use the superscript of each submessage to denote the receiver index at which the submessage will be neutralized, e.g. submessages \( A_{2pq}^2 \) and \( A_{pq}^3 \) are neutralized at receiver 2 and 3, respectively.

In what follows, we use the method similar in [26] to design the precoding factors so that each submessage is neutralized at a specified receiver. For example, for the submessage \( A_{pq}^2 \), we design the precoding factor \( h_{2q} \) at transmitter \( p \) and \(-h_{2p} \) at transmitter \( q \). With this precoding operation, a single virtual source instead of two transmitters is transmitting the submessage \( A_{pq}^2 \). We refer to this virtual source as the virtual transmitter for the submessage \( A_{pq}^2 \). It is seen that the equivalent channel coefficients from the virtual transmitter for \( A_{pq}^2 \) to receiver 1, 2, 3 can be expressed as \((h_{1p}h_{2q} - h_{1q}h_{2p}), 0, (h_{3p}h_{2q} - h_{3q}h_{2p})\), respectively, and \( A_{pq}^2 \) is exactly neutralized at receiver 2. Similarly, we can obtain the equivalent channel coefficients from the virtual transmitter for each submessage to receiver 1, 2, 3, respectively, as shown in TABLE III. Here, all the complex scaling factors \( \alpha \) are independent and \( \gamma \) is related to the power constraint, which will be explained later.

From TABLE III it is worth mentioning that there are totally \( 3N_T(N_T - 1) \) submessages transmitted from the virtual transmitters. For each receiver, there are \( N_T(N_T - 1) \) desired submessages and \( 2N_T(N_T - 1) \) undesired submessages as interference. From the aforementioned precoding factor design method, \( N_T(N_T - 1) \) undesired submessages are neutralized at each receiver. There are additional \( N_T(N_T - 1) \) undesired submessages remaining at each receiver. In what follows, we aim to align these \( N_T(N_T - 1) \) additional undesired submessages using real interference alignment [27].

To use real interference alignment, we encode each submessage into \( \Omega N_T(N_T - 1) \) independent data streams where \( \Omega \) is an integer and to be determined later. Each data stream is modulated over integer constellation \( \mathbb{Z}_Q \triangleq \mathbb{Z} \cap [-Q, Q] \) and scaled with a unique monomial from set \( V_\Omega \). The monomial set is defined as follows:
TABLE III: Equivalent channel coefficients for submessages

| Submessage ($\forall p, \forall q > p$) | Equivalent Channel Coefficient |
|--------------------------------------|-------------------------------|
|                                      | at Receiver 1 | at Receiver 2 | at Receiver 3 |
| $A_{pq}^2$                           | $\gamma a_{pq}^2(h_{1p}h_{2q} - h_{1q}h_{2p})$ | 0 | $\gamma a_{pq}^2(h_{3p}h_{2q} - h_{3q}h_{2p})$ |
| $A_{pq}^3$                           | $\gamma a_{pq}^3(h_{1p}h_{3q} - h_{1q}h_{3p})$ | $\gamma a_{pq}^3(h_{2p}h_{3q} - h_{2q}h_{3p})$ | 0 |
| $B_{pq}^1$                           | 0 | $\gamma a_{pq}^1(h_{2p}h_{1q} - h_{2q}h_{1p})$ | $\gamma a_{pq}^1(h_{3p}h_{1q} - h_{3q}h_{1p})$ |
| $B_{pq}^3$                           | $\gamma a_{pq}^3(h_{1p}h_{3q} - h_{1q}h_{3p})$ | $\gamma a_{pq}^3(h_{2p}h_{3q} - h_{2q}h_{3p})$ | 0 |
| $C_{pq}^1$                           | 0 | $\gamma a_{pq}^1(h_{2p}h_{1q} - h_{2q}h_{1p})$ | $\gamma a_{pq}^1(h_{3p}h_{1q} - h_{3q}h_{1p})$ |
| $C_{pq}^2$                           | $\gamma a_{pq}^2(h_{1p}h_{2q} - h_{1q}h_{2p})$ | 0 | $\gamma a_{pq}^2(h_{3p}h_{2q} - h_{3q}h_{2p})$ |

**Definition 3:** For a complex set $\mathcal{X}$ with $s$ complex elements, i.e., $\mathcal{X} = \{x_1, x_2, \cdots, x_s\}$, its monomial set is

$$\mathcal{V}_1(\mathcal{X}) \triangleq \{ x_1^{\omega_1} x_2^{\omega_2} \cdots x_s^{\omega_s} : \forall \omega_1, \omega_2, \cdots, \omega_s \in \mathbb{Z} \cap [1, \Omega] \},$$

where $\omega_1, \omega_2, \cdots, \omega_s$ are all integers between 1 and $\Omega$, and the cardinality of $\mathcal{V}_1(\mathcal{X})$ is $|\mathcal{V}_1(\mathcal{X})| = \Omega^s$.

In specific, each data stream in $B_{pq}^3$ and $C_{pq}^2$ ($\forall p, \forall q > p$) is scaled with a unique monomial in set $\mathcal{V}_1(\mathcal{X}_1)$ with

$$\mathcal{X}_1 = \{ \alpha_{Bpq}^3(h_{1p}h_{3q} - h_{1q}h_{3p}), \alpha_{Cpq}^2(h_{1p}h_{2q} - h_{1q}h_{2p}) : \forall p, \forall q > p \}.$$

Each data stream in $A_{pq}^3$ and $C_{pq}^1$ ($\forall p, \forall q > p$) is scaled with a unique monomial in set $\mathcal{V}_1(\mathcal{X}_2)$ with

$$\mathcal{X}_2 = \{ \alpha_{Apq}^3(h_{2p}h_{3q} - h_{2q}h_{3p}), \alpha_{Cpq}^1(h_{2p}h_{1q} - h_{2q}h_{1p}) : \forall p, \forall q > p \}.$$

Each data stream in $A_{pq}^2$ and $B_{pq}^1$ ($\forall p, \forall q > p$) is scaled with a unique monomial in set $\mathcal{V}_1(\mathcal{X}_3)$ with

$$\mathcal{X}_3 = \{ \alpha_{Apq}^2(h_{3p}h_{2q} - h_{3q}h_{2p}), \alpha_{Bpq}^1(h_{3p}h_{1q} - h_{3q}h_{1p}) : \forall p, \forall q > p \}.$$

Thus, for all $p$ and all $q > p$, submessages $B_{pq}^3$ and $C_{pq}^2$ are both encoded to the points in the signal constellation $\sum_{v \in \mathcal{V}_1(\mathcal{X}_1)}(vZQ)$, submessages $A_{pq}^3$ and $C_{pq}^1$ are both encoded to the points in the signal constellation $\sum_{v \in \mathcal{V}_1(\mathcal{X}_2)}(vZQ)$, and submessages $A_{pq}^2$ and $B_{pq}^1$ are both encoded to the points in the signal constellation $\sum_{v \in \mathcal{V}_1(\mathcal{X}_3)}(vZQ)$.

Now we consider the received signal constellation at receiver 1, which can be expressed as

$$y = c_1 + c_2 + c_3,$$
The received constellations $C_1$ and $C_2$ are the desired data streams encoded from desired submessages $A_{pq}^2$ and $A_{pq}^3$, respectively, and the received constellation $C_3$ are the interference encoded from $B_{pq}^3$ and $C_{pq}^2$ $(\forall p, \forall q > p)$. Using similar arguments in \cite{26}, we have

$$C_3 \subset \gamma \sum_{v \in V_{N_T}(N_T-1)Q} (vZ_{N_T}(N_T-1)Q),$$

which implies the interference $B_{pq}^3$ and $C_{pq}^2$ $(\forall p < q)$ are aligned into a constellation of cardinality much smaller than the multiplication of the transmitted constellation cardinalities. The received signal constellation at receiver 2 and 3 can be obtained similarly and thus omitted here.

Next, we need to assure the monomials forming the received constellation are linearly independent as functions of $h, \alpha$ and $\gamma$, so that Theorem 3 in \cite{26} can be applied to prove the decodeability of this signal constellation.

It can be seen that the monomials forming $C_1$ are functions of $\alpha_{A_{pq}}^2$ and $\alpha_{B_{pq}}^1$ $(\forall p, \forall q > p)$, the monomials forming $C_2$ are functions of $\alpha_{A_{pq}}^3$ and $\alpha_{C_{pq}}^1$, and the monomials forming $C_3$ are functions of $\alpha_{B_{pq}}^3$ and $\alpha_{C_{pq}}^2$. Since none of them are overlapped, we can guarantee the independence across the monomials forming constellations $C_1, C_2$ and $C_3$.

Then, it remains to prove the linear independence of the monomials within each constellation $C_i$. Without loss of generality, we take $C_1$ as an example. Note that monomials forming $C_1$ are $\{\gamma \alpha_{A_{pq}}^2(h_{1p}h_{2q} - h_{1q}h_{2p})\} : \forall p, \forall q > p\}$. In what follows, we use contradiction to prove the linear independence of the monomials across different $p, q$. We assume that a monomial $\alpha_{A_{12}}^2(h_{11}h_{22} - h_{12}h_{21}) \cdot v_1$ in the constellation of $A_{12}^2$ can be linearly expressed by other monomials in the constellation of $A_{pq}^2$ with different $p, q$, where

$$v_1 = \gamma \prod_{p=1}^{N_T-1} \prod_{q=p+1}^{N_T} (\alpha_{A_{pq}}^2(h_{3p}h_{2q} - h_{3q}h_{2p}))^{\omega^2_{pq}} (\alpha_{B_{pq}}^1(h_{3p}h_{1q} - h_{3q}h_{1p}))^{\omega^1_{pq}}.$$

Then, to linearly express $\alpha_{A_{12}}^2(h_{11}h_{22} - h_{12}h_{21}) \cdot v_1$, the monomials of the constellations of $A_{pq}^2$ with different $p, q$ should have the same power of each scaling factor $\alpha$ in the monomial $\alpha_{A_{12}}^2(h_{11}h_{22} - h_{12}h_{21}) \cdot v_1$ due to the fact that each $\alpha$ is independent, i.e., these monomials should have the following power form of the scaling factor $\alpha$:

$$\alpha_{A_{12}}^2 \prod_{p=1}^{N_T-1} \prod_{q=p+1}^{N_T} (\alpha_{A_{pq}}^2)^{\omega^2_{pq}} (\alpha_{B_{pq}}^1)^{\omega^1_{pq}}.$$

The received constellations $C_1$ and $C_2$ are expressed by other monomials in the constellation of $C_{pq}^2$ forming $C_3$ where $\forall q > p$ with different $p, q$, we can guarantee the independence across the monomials forming constellations $C_1, C_2$ and $C_3$. In what follows, we use contradiction to prove the linear independence of the monomials across different $p, q$. We assume that a monomial $\alpha_{A_{12}}^2(h_{11}h_{22} - h_{12}h_{21}) \cdot v_1$ in the constellation of $A_{12}^2$ can be linearly expressed by other monomials in the constellation of $A_{pq}^2$ with different $p, q$, where

$$v_1 = \gamma \prod_{p=1}^{N_T-1} \prod_{q=p+1}^{N_T} (\alpha_{A_{pq}}^2(h_{3p}h_{2q} - h_{3q}h_{2p}))^{\omega^2_{pq}} (\alpha_{B_{pq}}^1(h_{3p}h_{1q} - h_{3q}h_{1p}))^{\omega^1_{pq}}.$$
Equation (56) implies that for any constellation of $A^2_{pq}$ with $(p, q) \neq (1, 2)$, it only remains monomial function of all channel coefficients $\alpha_{A_{pq}}(h_{33}h_{22} - h_{32}h_{21})$ which can contribute to the linear expression. Thus, the linear expression can be written as

$$\alpha_{A_{12}}^2(h_{11}h_{22} - h_{12}h_{21}) \cdot v_1$$

which can be linearly expressed as

$$\frac{N_T-1}{p+1} \sum_{q=p+1}^{N_T} c_{pq} \alpha_{A_{pq}}^2(h_{1p}h_{2q} - h_{1q}h_{2p}) \alpha_{A_{12}}^2(h_{33}h_{22} - h_{32}h_{21}) \cdot v_1, \tag{57}$$

where $c_{pq}$ is the scalar. Dividing $v_1$ and all $\alpha$ in (57), it is equivalent to say that $h_{11}h_{22} - h_{12}h_{21}$ can be linearly expressed as

$$\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{33}h_{22} - h_{32}h_{21}} = \sum_{p=1}^{N_T-1} c_{pq} h_{1p}h_{2q} - h_{1q}h_{2p}, \tag{58}$$

Note that both sides of (58) are functions of all channel coefficients $\{h_{pq} : p \neq q\}$. Doubling $h_{11}$ and $h_{12}$ while fixing other channel coefficients in (58), given the assumed linear dependence, we can have

$$2(h_{11}h_{22} - h_{12}h_{21}) = \sum_{q=3}^{N_T} c_{1q} h_{11}h_{2q} - h_{1q}h_{21} + \sum_{q=3}^{N_T} c_{2q} h_{12}h_{2q} - h_{1q}h_{22} + \sum_{p=3}^{N_T-1} \sum_{q=p+1}^{N_T} c_{pq} h_{1p}h_{2q} - h_{1q}h_{2p}. \tag{59}$$

Subtracting (58) from (59), we have

$$\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{33}h_{22} - h_{32}h_{21}} = \sum_{q=3}^{N_T} c_{1q} h_{11}h_{2q} - h_{1q}h_{21} + \sum_{q=3}^{N_T} c_{2q} h_{12}h_{2q} - h_{1q}h_{22}. \tag{60}$$

By changing $h_{23}$ only and fixing other channel coefficients, it is easy to see that

$$c_{13} \frac{h_{11}h_{23}}{h_{33}h_{23} - h_{32}h_{21}} + c_{23} \frac{h_{12}h_{23}}{h_{33}h_{23} - h_{32}h_{21}}$$

remains constant as a function of $h_{23}$, which implies $c_{13} = c_{23} = 0$. Thus, similarly, for all $q > 2$, we can have $c_{1q} = c_{2q} = 0$ which is contradicted with the assumption of linear dependence. Therefore, we proved the linear independence of the monomials across constellations of $A^2_{pq}$ with different $p, q$. Similar arguments can be applied to constellations of $C_2$ and $C_3$. Given linear independence of monomials in each monomial set $\mathcal{V}_1(A_1)$, $\mathcal{V}_1(A_3)$ and $\mathcal{V}_3(A_3)$, the linear independence of the monomials of the received constellations, as functions of $h, \alpha$ and $\gamma$, is thus proved.

By setting

$$I \triangleq N_T(N_T - 1)\Omega^N_T(N_T - 1) + (\Omega + 1)N_T(N_T - 1),$$

$$Q \triangleq \frac{1}{N_T(N_T - 1)}p^{\frac{l-1}{l-2}},$$

$$\zeta \triangleq \frac{I}{2} - 1 + \varepsilon,$$

$$\gamma \triangleq c_1P^{\frac{l-3+\varepsilon}{l-4+\varepsilon}},$$

we have
where \(c_1, \varepsilon\) are constants and \(c_1, \varepsilon > 0\), and using Theorem 3 in [26] and results in [28], we can have the minimum distance

\[
\Delta \geq c_2 \gamma (N_T(N_T - 1)Q)^{-\varepsilon} = c_2 c_1 P^{\varepsilon/2}
\]  

(61)

for almost all channel coefficients \(h\) and scaling factors \(\alpha\). Here \(c_2\) is a constant independent with \(P\) and is positive for almost all \(h\). From (61), the decodability of the signal constellation is guaranteed for almost all channel coefficients \(h\) as \(\Delta \to \infty\) when \(P \to \infty\). The average transmit power is upper-bounded by

\[
c_3 \gamma^2 Q^2 = \frac{c_3 c_1^2}{(N_T(N_T - 1))^\varepsilon} P.
\]  

(62)

Then, the DoF for one receiver under constellation modulation can be calculated similarly in [27] as follows:

\[
\lim_{P \to \infty} \frac{1 - \exp(-\Delta^2/8)}{\log P} \log |\text{desired signal cardinality}| = \lim_{P \to \infty} \frac{\log(2Q + 1)}{\log P} \left( \frac{N_T(N_T - 1)\Omega^{N_T(N_T - 1)}}{N_T(N_T - 1)\Omega^{N_T(N_T - 1)} + (\Omega + 1)N_T(N_T - 1) + 2\varepsilon} \right)
\]

(63)

Letting \(\Omega \to \infty\) and \(\epsilon \to 0\), we obtain that the sum DoF is \(3 \cdot \frac{N_T(N_T - 1)}{N_T(N_T - 1) + 1}\). Thus, Lemma 1 is proved.

**Appendix B: Proof of Lemma 2**

Consider the \(N_T \times 3\) hybrid X-multicast channel in Section VI-A. When \(N_T = 3\), the channel is shown in Fig. 5(d). We denote \(A_p, B_p, C_p\) as the message available at transmitter \(p\) and needed by receivers \(\{2, 3\}\), receivers \(\{1, 3\}\) and receivers \(\{1, 2\}\), respectively. By using a \((2N_T + 1)\)-symbol extension, the effective channel between transmitter \(p\) and receiver \(j\) over \((2N_T + 1)\) channel uses, denoted by \(H_{jp}\), becomes a \((2N_T + 1)\times(2N_T + 1)\) diagonal matrix with diagonal entries chosen i.i.d from some continuous distribution. We further denote \(v_{A_p}, v_{B_p}, v_{C_p}\) as the \((2N_T + 1)\times1\) precoding vector for the transmitted symbol \(x_{A_p}, x_{B_p}, x_{C_p}\), which are encoded from \(A_p, B_p, C_p\), respectively. Then, the received signal at receiver 1 can be expressed as

\[
y_1 = \sum_{p=1}^{N_T} H_{1p} \left( v_{A_p} x_{A_p} + v_{B_p} x_{B_p} + v_{C_p} x_{C_p} \right) + n_1,
\]

where \(n_1\) denotes the \((2N_T + 1)\times1\) additive white Gaussian noise (AWGN) vector at receiver 1. Note that \(x_{B_p}\) and \(x_{C_p}\) are the desired signals and \(x_{A_p}\) is the interference at receiver 1. To this end, we need to design the precoding vectors \(\{v_{A_p}\}_{p=1}^{N_T}\) as

\[
H_{1p}v_{A_p} = v, \forall p \in [N_T] \Rightarrow v_{A_p} = H_{1p}^{-1}v,
\]

(64)

where \(v\) is a \((2N_T + 1)\times1\) vector to be determined later, so that all the interference can be aligned in a same subspace at receiver 1. Similarly, for receiver 2 and 3, we need to design \(\{v_{B_p}\}_{p=1}^{N_T}\) and \(\{v_{C_p}\}_{p=1}^{N_T}\) such that

\[
H_{2p}v_{B_p} = v, \forall p \in [N_T] \Rightarrow v_{B_p} = H_{2p}^{-1}v,
\]

(65)

\[
H_{3p}v_{C_p} = v, \forall p \in [N_T] \Rightarrow v_{C_p} = H_{3p}^{-1}v.
\]

(66)
Let \( \mathbf{v} \) be the \((2N_T + 1) \times 1\) vector with each element non-zero, for example \( \mathbf{v} = (1, 1, \cdots, 1)^T \), we can construct all the precoding vectors according to (64)-(66). Next, we need to ensure the space spanned by the desired signals is linearly independent of the space spanned by the interference at each receiver. Without loss of generality, we take receiver 1 as an example. At receiver 1, the span space of the \( 2N_T \) desired data streams can be expressed as

\[
\text{span}(\mathcal{H}_D) \triangleq \text{span}\left(\begin{bmatrix}
H_{11} & H_{12} & \cdots & H_{1N_T} \\
H_{21} & H_{22} & \cdots & H_{2N_T} \\
\vdots & \vdots & \ddots & \vdots \\
H_{31} & H_{32} & \cdots & H_{3N_T}
\end{bmatrix}\mathbf{v}\right),
\]

and the span space of the interference can be expressed as

\[
\text{span}(\mathcal{H}_I) \triangleq \text{span}\left(\begin{bmatrix}
\mathbf{v} \\
\mathbf{v} \\
\vdots \\
\mathbf{v}
\end{bmatrix}\right).
\]

Since each channel matrix is chosen independently of each other, \([\mathcal{H}_D \mathcal{H}_I]\) is a full-rank matrix with probability 1, which guarantees the decodability of the desired data streams. Similarly, receiver 2, 3 can also decode its desired signals. As a result, \(3N_T\) data streams are transmitted over \((2N_T + 1)\) channel uses, which indicates that the sum DoF is \(\frac{3N_T}{2N_T + 1}\). Thus, Lemma 2 is proved.

**APPENDIX C: PROOF OF LEMMA 3**

Consider the \(N_T \times 3\) partially cooperative hybrid X-multicast channel in Section VI-A, where each transmitter subset \(\Psi (|\Psi| = i \geq 2)\) has three independent messages, each of which is needed by two receivers. The special case for \(N_T = 3, |\Psi| = 2\) is shown in Fig. 5(e) and we will prove the general case that \(|\Psi| = i \geq 2\) here. We denote \(A_\Psi, B_\Psi, C_\Psi\) as the message available at transmitters in \(\Psi\) needed by receivers \(\{2, 3\}\), receivers \(\{1, 3\}\) and receivers \(\{1, 2\}\), respectively. By using a \(2^{(N_T)}\)-symbol extension, the effective channel between transmitter \(p\) and receiver \(j\) over \(2^{(N_T)}\) channel uses, denoted by \(\mathbf{H}_{jp}\), becomes a \(2^{(N_T)} \times 2^{(N_T)}\) diagonal matrix with diagonal entries chosen i.i.d from some continuous distribution. We further denote \(\mathbf{v}_{A_\Psi}^p, \mathbf{v}_{B_\Psi}^p, \mathbf{v}_{C_\Psi}^p\) as the \(2^{(N_T)} \times 1\) precoding vector for the transmitted symbol \(x_{A_\Psi}, x_{B_\Psi}, x_{C_\Psi}\), which are encoded from message \(A_\Psi, B_\Psi, C_\Psi\), respectively, at transmitter \(p \in \Psi\). Then, the signal received at receiver 1 can be expressed as

\[
\mathbf{y}_1 = \sum_{\Psi} \left( \sum_{p \in \Psi} \mathbf{H}_{1p} \left( \mathbf{v}_{A_\Psi}^p x_{A_\Psi} + \mathbf{v}_{B_\Psi}^p x_{B_\Psi} + \mathbf{v}_{C_\Psi}^p x_{C_\Psi} \right) \right) + \mathbf{n}_1.
\]

Note that \(x_{B_\Psi}\) and \(x_{C_\Psi}\) are the desired data streams and \(x_{A_\Psi}\) are the interference at receiver 1. To this end, we need to design the precoding vectors as

\[
\sum_{p \in \Psi} \mathbf{H}_{1p} \mathbf{v}_{A_\Psi}^p = 0,
\]

so that all the interference are neutralized. Similarly, for receiver 2 and 3, we need to design the precoding vectors such that

\[
\sum_{p \in \Psi} \mathbf{H}_{2p} \mathbf{v}_{B_\Psi}^p = 0,
\]

\[
\sum_{p \in \Psi} \mathbf{H}_{3p} \mathbf{v}_{C_\Psi}^p = 0.
\]
Besides the three constraints above, we need to further design $v^p_{A\Psi_n}$, $v^p_{B\Psi_n}$, $v^p_{C\Psi_n}$ so that receivers can successfully decode its desired message. Define $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)^T$ as the $2\binom{N_T}{i}$ \times 1 vector with its $i$-th element being 1 and other elements being 0. Due to the fact that there are $\binom{N_T}{i}$ different choices of $\Psi$, we should design the precoding vector $v^p_{A\Psi_n}$, $v^p_{B\Psi_n}$, $v^p_{C\Psi_n}$ for transmitter $p$ at set $\Psi_n \ (n \in \binom{N_T}{i})$ as

$$v^p_{A\Psi_n} = w^p_{A\Psi_n} e_n, \quad (70)$$

$$v^p_{B\Psi_n} = w^p_{B\Psi_n} e_{n+(N_T)}, \quad (71)$$

$$v^p_{C\Psi_n} = w^p_{C\Psi_n} e_n + w^p_{C\Psi_n} e_{n+(N_T)}. \quad (72)$$

where factor $w$ is designed to satisfy interference neutralization $(67)$–$(69)$, respectively.

Then, we consider received signal at receivers. Without loss of generality, we take receiver 1 as an example. The signal received at receiver 1 can be expressed as

$$y_1 = \sum_{n=1}^{\binom{N_T}{i}} \left( \sum_{p \in \Psi_n} H_1 p \left( v^p_{A\Psi_n} x_{A\Psi_n} + v^p_{B\Psi_n} x_{B\Psi_n} + v^p_{C\Psi_n} x_{C\Psi_n} \right) \right) + n_1$$

$$= \sum_{n=1}^{\binom{N_T}{i}} \left( \sum_{p \in \Psi_n} H_1 p \left( v^p_{B\Psi_n} x_{B\Psi_n} + v^p_{C\Psi_n} x_{C\Psi_n} \right) \right) + n_1$$

$$= \sum_{n=1}^{\binom{N_T}{i}} \left( \sum_{p \in \Psi_n} H_1 p \left( w^p_{B\Psi_n} e_{n+(N_T)} x_{B\Psi_n} + \left( w^p_{C\Psi_n} e_n + w^p_{C\Psi_n} e_{n+(N_T)} \right) x_{C\Psi_n} \right) \right) + n_1. \quad (73)$$

From $(68)$ and $(69)$, we can see $w^p_{B\Psi_n}$, $w^p_{C\Psi_n}$ and $w^p_{C\Psi_n}$ is independent with $H_1 p$, thus none of the received factors of desired signals $x_{B\Psi_n}$ and $x_{C\Psi_n}$ degenerates to zero with probability one. Therefore, receiver 1 can first decode $x_{C\Psi_n}$ using zero-forcing along vector $e_n$, and then decode $x_{B\Psi_n}$ along $e_{n+(N_T)}$ $(\forall n \in \binom{N_T}{i})$ with $x_{C\Psi_n}$ as side information. Similarly, receiver 2, 3 can also successfully decode its desired signals. As a result, $3\binom{N_T}{i}$ data streams are transmitted over $2\binom{N_T}{i}$ channel uses, which indicates that the sum DoF is $\frac{3}{2}$. Thus, Lemma 3 is proved.

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