Shear Lag Effect and Accordion Effect on Dynamic Characteristics of Composite Box Girder Bridge with Corrugated Steel Webs

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Abstract: This study proposed a dynamic characteristic analytical method (ANM) of a composite box girder bridge with corrugated steel web (CBGCSW) by completely considering the impact of shear lag effect and accordion effect of corrugated steel webs. Based on energy principles and variational principles, a vibration differential equation and the natural boundary conditions of a CBGCSW were developed. The analytical calculation formula for solving the vibration differential equation was then obtained. The results calculated using the ANM agreed well with previous experimental results, which validated the correctness of ANM. To demonstrate the superiority of the ANM, the vibration frequencies of several abstract CBGCSWs with varying ratios of span–width, obtained using the elementary beam theory (EBT) and the finite element method (FEM), were compared with those obtained by ANM. The efficacy of the ANM was verified and some meaningful conclusions were drawn which are helpful to relevant engineering design, such as the observation that a higher natural vibration frequency and smaller span–width ratio significantly magnified the shear lag effect of CBGCSW. The first five-order natural vibration frequencies of the CBGCSW were significantly lower than those of the composite box girder bridge with general steel web (CBGGSW), which indicates that the impact of the accordion effect is significant.

Keywords: corrugated steel webs; shear lag effect; accordion effect; span–width ratios; analytical method

1. Introduction

Composite box girder bridges with corrugated steel web (CBGCSW) are widely used in China, but there have been few studies on their dynamic characteristics considering multiple factors. In comparison with the composite box girder bridge with general steel web (CBGGSW), the CBGCSW has some obvious advantages: higher prestressing efficiency, fatigue and seismic resistance, and local bearing capacity; less affected by the shrinkage and creep of the concrete slabs; lower mass and project cost; shorter construction period; and so forth. Since the completion of the first CBGCSW, this beam structure has attracted great attention from researchers and engineers [1].

Based on the early studies on the shear mode and bending mode of vertically trapezoidal corrugated webs, two significant findings were confirmed: the buckling of the web was local and global for dense and coarse corrugation, and the corrugated web could be ignored in beam-bending studies because of its insignificant influence on the load-carrying capability of the whole beam [2]. Johnson et al. [3] drew similar conclusions based on finite-element analyses and laboratory tests, and proposed the formula for effective shear modulus. These were found to be agreeable in the
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investigation by Sayed-Ahmed [4]. Ibrahim et al. [5] studied the performance of plate girders with trapezoidally corrugated web subjected to fatigue loading, and an analysis technique based on fracture mechanics used to research the fatigue life of girders with corrugated web steel was proposed. Khalid [6] investigated the behavior of mild steel structural beams with corrugated webs under three-point bending, and determined the corrugation direction and the contribution of corrugated web to the load-carrying capability of the beam. Huang et al. [7] described the bending behavior of a corrugated web as the “accordion effect”, and an important effect of the corrugation profile on the axial stress distribution in such webs was investigated. Kim et al. [8,9] proposed an approximate method to estimate the accordion effect and verified that the prestressing efficiency of steel beams with a conventional web grew less obviously than that of steel beams with a corrugated web.

Based on previous investigation, the shear buckling behavior of corrugated webs was studied in detail, and the methods proposed in these studies provide good predictions [10–13]. The stress distribution in the flanges was investigated by Kövesdi et al. [14], and an enhanced method to obtain the transverse bending moment and the additional normal stresses was proposed. The actual behavior at the connection between the flanges and corrugated webs was studied by Hassanein et al. [15,16]; the valid design of shear strengths in previous studies was improved, and an enhanced formula for interactive shear buckling strengths was proposed for the case of fixed connection. With widespread application of high-strength steels, the performance of steel beams with corrugated web plates built with high-strength steels was studied. Moon et al. [17] proposed estimated approaches for obtaining the warping constant and determining the shear center of I-girders with corrugated web subjected to uniform bending, and Ibrahim [18] investigated the influence of the corrugated web on the lateral torsional buckling strength of wavy webs. In recent years, the shear lag effect has been the subject of intense scholarly research [19], and Cheng [20] proposed a simple approach for forecasting the deflection of a CBGCSW by taking shear lag effect into consideration.

Since most of the studies on CBGCSWs have focused on their static properties, particularly the bending resistance and shear resistance, there have been few studies on their dynamic characteristics. In this study, based on energy principles and variational principles and by completely considering the impact of shear lag effect and the accordion effect of corrugated steel webs, an analytical method that can determine the natural vibration characteristics of a CBGCSW was developed. To validate the correctness and rationality of the ANM, the results of the ANM were compared with the results of experiments, the elementary beam theory (EBT), and the finite element method (FEM). The efficacy of the ANM in this study was verified by these comparisons. Further, the impact of the shear lag effect and accordion effect were investigated. The CBGCSW natural vibration characteristics formulas derived in this study are further developments of those applied in earlier theories.

2. Free Vibration Equation for CBGCSW

2.1. Strain Energy of CBGCSW Considering Shear Lag and Accordion Effect

Since the corrugated steel webs form a folding pattern in the longitudinal direction, even under a small axial load, there is significant axial deformation. This is referred to as the accordion effect, which significantly reduces the effective elastic modulus of the corrugated webs in the longitudinal direction to approximately 1/100 or 1/1000 of that of the general steel webs [21]. Thus, in order to reduce the calculation difficulty, the axial rigidity of corrugated web plates was assumed to be negligible, i.e., the effective elastic modulus of corrugated webs in longitudinal direction $E_c = 0$. The vertical compressions, transverse normal strains, and transverse bending of the concrete slabs and the corrugated steel webs were assumed to be negligible [22,23].

On a CBGCSW section, assuming that the longitudinal displacement of any point is the superposition of the longitudinal warping displacement caused by the shear lag effect and the
longitudinal displacement according to the plane-section assumption, the longitudinal displacement expression is [24]

\[ u_i(x, y, z, t) = g_i - z\theta \quad (i = 1, 2, 3, 4) \]  

(1)

where \( \theta \) is the sectional rotation angle of a CBGCSW; \( u_i(x, y, z, t) \) \((i = 1, 2, 3, 4)\), respectively, are the longitudinal displacement functions of various points on the top plate, cantilever plate, low plate, and corrugated webs of a CBGCSW; \( g_i(x, y, t) \) \((i = 1, 2, 3)\), \( g_4(x, z, t) \), respectively, are functions for the longitudinal warping displacement of various points on the top plate, cantilever plate, low plate, and corrugated webs of a CBGCSW.

Assuming that the longitudinal warping distribution patterns of various points on the top plate, cantilever plate, low plate, and corrugated webs of a CBGCSW are parabolas, they can be respectively expressed as [25,26]

\[ g_i = \psi_i(y)U(x, t) \quad (i = 1, 2, 3, 4) \]  

(2)

\[ \psi_i(y) = \alpha_i \left( y^2 / b_i^2 - 1 \right) + D_w \]  

(3)

\[ \alpha_1 = 1, \quad \alpha_2 = b_2^2 / b_1^2, \quad \alpha_3 = z_3 b_3^2 / (b_3^2 z_1), \quad \alpha_4 = 0 \]  

(4)

On the basis of self-balance of the warping stresses, Equations (2)–(4) reduce to

\[ D_w = 2(\alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3) / (3A_0) \]  

(5)

where \( i = 2, y \) is replaced by \( \bar{y} = b_1 + b_2 - y \); \( b_1, b_2, \) and \( 2b_3 \), respectively, are the widths of the top plate, cantilever plate, and low plate, as shown in Figure 1; \( b_4 \) is the height of the corrugated web plate; \( t_1, t_2, t_3, \) and \( t_4 \), respectively, are the thicknesses of the top plate, cantilever plate, low plate, and corrugated web plate; \( z_1, z_2, \) and \( z_3 \) are the coordinates of the top-plate, cantilever-plate, and low-plate centroids in the z-direction, respectively; \( U(x, t) \) is the function for longitudinal warping amplitude; \( D_w \) is the longitudinal warping displacement of the CBGCSW section; \( A_1 = 2b_1 t_1, \quad A_2 = 2b_2 t_2, \quad A_3 = 2b_3 t_3, \quad A_4 = 2b_4 t_4; \quad A_0 = A_1 + A_2 + A_3 + n_0 A_4; \quad n_0 = E_e / E_c = 0; \quad E_c \) is the steel elastic modulus; and \( E_e \) is the concrete elastic modulus.

\[ \text{Figure 1. Dimension and coordinate system of Composite box girder bridges with corrugated steel web (CBGCSW) cross-section.} \]

The shear modulus \( G_s \) of general steel webs is greater than the effective shear modulus \( G_e \) of corrugated webs because of the folding performance of corrugated webs. Thus, the effective shear modulus \( G_e \) can be approximately expressed as \( G_e = G_s (B + D) / (B + C) \) [3], where \( B, C, \) and \( D \) are, respectively, the flat-plate section length, inclined-plate section length, and the longitudinal projected length of the inclined plate section, as shown in Figure 2.
According to Equation (1), the strain of various points on a CBGCSW section can be expressed as

\[ \varepsilon_{xi} = \frac{\partial u_i}{\partial x} = -z \frac{\partial \theta}{\partial x} + \psi_i \frac{\partial U}{\partial x} \quad (i = 1, 2, 3, 4) \]  
\[ (6) \]

\[ \gamma_{xyi} = \frac{\partial u_i}{\partial y} = \frac{\psi_i}{\partial y} U \quad (i = 1, 2, 3) \]  
\[ (7) \]

\[ \gamma_{xz} = \frac{\partial w}{\partial x} - \theta \]  
\[ (8) \]

where \( \varepsilon_{xi} \) are the normal strains of the top plate, cantilever plate, low flange, and corrugated web plate, respectively; \( \gamma_{xyi} \) is the shear strain in the horizontal plane of the top plate, cantilever plate, and low flange, respectively; \( \gamma_{xz} \) is the shear strain in the vertical plane of the corrugated web plate; and \( w \) is the vertical displacement.

The stress of various points on a CBGCSW section can be expressed as

\[ \sigma_{xi} = E_c \left( z \frac{\partial \theta}{\partial x} + \psi_i \frac{\partial U}{\partial x} \right) \quad (i = 1, 2, 3) \]  
\[ (9) \]

\[ \sigma_{x4} = E_c \left( z \frac{\partial \theta}{\partial x} + \psi_4 \frac{\partial U}{\partial x} \right) = 0 \]  
\[ (10) \]

\[ \tau_{xyi} = G_c \frac{\partial \psi}{\partial y} U(x) \quad (i = 1, 2, 3) \]  
\[ (11) \]

\[ \tau_{xz} = G_c \left( \frac{\partial w}{\partial x} - \theta \right) \]  
\[ (12) \]

where \( G_c \) is the concrete shear modulus.

The strain energy of the concrete slabs can be expressed as

\[ V_c = \frac{1}{2} \sum_{i=1}^{3} \int_{l_i} \int_{A_i} \left( E_c \varepsilon_i^2 + G_c \gamma_i^2 \right) dAdx \]
\[ = \frac{1}{2} \sum_{i=1}^{3} \int_{l_i} \int_{A_i} \left[ E_c (z^2 \theta^2 + \psi^2 U^2 - 2z \theta \psi U') + G_c (\psi' U')^2 \right] dAdx \]  
\[ (13) \]

The strain energy of the corrugated webs can be expressed as

\[ V_s = \frac{1}{2} \int_{l} \int_{A_s} G_c \left( \frac{\partial w}{\partial x} - \theta \right)^2 dAdx = \frac{1}{2} \int_{0}^{l_s} G_c A_s (\omega' - \theta)^2 dx \]  
\[ (14) \]
The total strain energy of a CBGCSW is

\[ V = V_c + V_s = \frac{1}{2} \int_0^L \left[ E_c \left( \int_{A_c} \psi^2 dA \right) \theta''^2 + E_c \left( \int_{A_c} \psi \frac{\partial \psi}{\partial y} \right)^2 \theta' \right] dx \]  

where \( L \) is the calculation span of a CBGCSW; and \( A_c \) is the sum of the section areas of the concrete slabs.

2.2. Kinetic Energy of CBGCSW Considering Rotational Inertia

After taking the influence of the rotational inertia into account, the total kinetic energy of a CBGCSW becomes

\[ T = \frac{1}{2} \int_0^L \left( \rho_c \int_{A_c} z^2 dA + \rho_s \int_{A_{se}} z^2 dA \right) \dot{\theta}^2 + \left( \rho_c \int_{A_c} \psi^2 dA + \rho_s \int_{A_{se}} \psi^2 dA \right) \dot{\theta} \dot{U} - 2 \left( \rho_c \int_{A_c} \psi \dot{d} A + \rho_s \int_{A_{se}} \psi \dot{d} A \right) \ddot{U} + \left( \rho_c A_c + \rho_s \left( 2 b_d t_d (B + C) / (B + D) \right) \right) \dot{w}^2 \]  

where \( \rho_1 = \rho_2 = \rho_3 = \rho_c \), \( \rho_s \) is the concrete density; \( \rho_4 = \rho_s \), \( \rho_s \) is the steel density; \( A_{se} = 2 b_d t_d (B + C) / (B + D) \) is the equivalent sectional area of corrugated webs as determined according to the mass equivalent principle; \( \theta'' \) and \( \theta''' \) are the partial derivatives of time \( t \) and coordinate \( x \), respectively; the same notation is used hereinafter.

3. Solution of CBGCSW Governing Differential Equation

3.1. Solving Free Vibration Equation of CBGCSW

On the basis of energy principles and variational principles \( \delta \int_0^L (T - V) dt = 0 \) [27], the bending vibration differential equation and the natural boundary conditions of a CBGCSW can be obtained.

\[ G_c A_s (w'' - \theta') - A_p \ddot{w} = 0 \]  

\[ E_c F_c U'' - E_c S_c \theta'' - G_c I_c U - F_p U + S_p \ddot{U} = 0 \]  

\[ G_c A_s (w'' - \theta') + E_c I_c \theta' - E_c S_c U'' - I_p \ddot{\theta} + S_p \ddot{U} = 0 \]  

\[ \left( E_c F_c U' - E_c S_c \theta' \right) \delta U|_{L_0} = 0 \]  

\[ G_c A_s (w'' - \theta') \delta w|_{L_0} = 0 \]  

\[ \left( E_c I_c \theta' - E_c S_c U' \right) \delta \theta|_{L_0} = 0 \]  

\[ I_p = \rho_c \int_{A_c} z^2 dA + \rho_s \int_{A_{se}} z^2 dA \]  

\[ F_p = \rho_c \int_{A_c} \psi^2 dA + \rho_s \int_{A_{se}} \psi^2 dA \]  

\[ S_p = \rho_c \int_{A_c} \psi \dot{d} A + \rho_s \int_{A_{se}} \psi \dot{d} A \]  

\[ A_p = \rho_c A_c + \rho_s A_{se} \]  

\[ I_c = \int_{A_c} z^2 dA \]  

\[ F_c = \int_{A_c} \psi^2 dA \]
\[ S_c = \int_{\mathcal{A}_c} z \psi \, dA \]  
\[ I_c = \int_{\mathcal{A}_c} (\partial \psi / \partial y)^2 \, dA \]  

To solve the bending vibration differential equation, assume that

\[ U(x, t) = U_1(x) \sin(\omega t + \varphi) \]  
\[ w(x, t) = w_1(x) \sin(\omega t + \varphi) \]  
\[ \theta(x, t) = \theta_1(x) \sin(\omega t + \varphi) \]  
\[ d^k = \frac{\partial^k}{\partial x^k} \]

Substituting Equations (31)–(34) into Equations (17)–(19),

\[ (G_c A_s d^2 + A_{\rho} \omega^2)w_1 - G_c A_s d \theta_1 = 0 \]  
\[ (E_c F_c d^2 - G_c I_c)U_1 - (E_c S_c d^2 + S_{\rho} \omega^2)\theta_1 = 0 \]  
\[ (-E_c S_c d^2 - S_{\rho} \omega^2)U_1 + G_c A_s dw_1 + (E_c I_c d^2 + I_{\rho} \omega^2 - G_c A_s)\theta_1 = 0 \]

The characteristic equations of Equations (36) and (37) are

\[
\begin{vmatrix}
E_c F_c d^2 + F_{\rho} \omega^2 - G_c I_c & 0 & -E_c S_c d^2 - S_{\rho} \omega^2 \\
0 & G_c A_s d^2 + A_{\rho} \omega^2 & -G_c A_s d \\
-E_c S_c d^2 - S_{\rho} \omega^2 & G_c A_s d & E_c I_c d^2 + I_{\rho} \omega^2 - G_c A_s
\end{vmatrix} = 0
\]

where \( \lbrack \cdot \rbrack \) is the matrix determinant; the same notation is used hereinafter.

The solutions of Equations (35)–(37) can be expressed as

\[ U_1 = \sum_{i=1}^{6} \alpha_i \beta_{1i} \exp(d_i x) \]  
\[ w_1 = \sum_{i=1}^{6} \alpha_i \beta_{2i} \exp(d_i x) \]  
\[ \theta_1 = \sum_{i=1}^{6} \alpha_i \beta_{3i} \exp(d_i x) \]

\[ \beta_{1i} = \left( E_c S_c d_i^2 + S_{\rho} \omega^2 \right) / \left( E_c F_c d_i^2 + F_{\rho} \omega^2 - G_c I_c \right) \]  
\[ \beta_{2i} = \left( G_c A_s d_i \right) / \left( G_c A_s d_i^2 + A_{\rho} \omega^2 \right) \]  
\[ \beta_{3i} = 1 \]

where \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_6]^T \) is the integration constant vector; \( d_i (i = 1, 2, \ldots, 6) \) is the characteristic root of the characteristic equation Equations (17)–(19); and \( \beta_i = [\beta_{1i}, \beta_{2i}, \beta_{3i}]^T \) is the characteristic vector corresponding to the characteristic root \( d_i \).

3.2. Solving Natural Vibration Frequency of CBGCSW

Obtained from Equations (20)–(22) and (51)–(52), the common boundary conditions of a CBGCSW of Sections 3.1 and 3.2 are
(1) The simply supported:

\[
E_c F_c U_1' - E_c S_c \theta_1' = 0 \Rightarrow \sum_{i=1}^{6} F_c \alpha_i \beta_1 i \exp(d_i x) - \sum_{i=1}^{6} S_c \alpha_i \beta_3 i \exp(d_i x) = 0
\]  

\[
w_1 = 0 \Rightarrow \sum_{i=1}^{6} \alpha_i \beta_2 i \exp(d_i x) = 0
\]  

\[
E_c I_c \theta_1' - E_c S_c U_1' = 0 \Rightarrow \sum_{i=1}^{6} I_c \alpha_i \beta_2 i \exp(d_i x) - \sum_{i=1}^{6} S_c \alpha_i \beta_1 i \exp(d_i x) = 0
\]  

(2) The fixed-support:

\[
U_1 = 0 \Rightarrow \sum_{i=1}^{6} \alpha_i \beta_1 i \exp(d_i x) = 0
\]  

\[
w_1 = 0 \Rightarrow \sum_{i=1}^{6} \alpha_i \beta_2 i \exp(d_i x) = 0
\]  

\[
\theta_1 = 0 \Rightarrow \sum_{i=1}^{6} \alpha_i \beta_3 i \exp(d_i x) = 0
\]  

After substituting the boundary conditions in Equations (17)–(19), we can obtain the algebraic equations of a CBGCSW about the integration constant vector \(\{\alpha\}\):

\[
[X(\omega)]\{\alpha\} = 0
\]  

to ensure a non-zero solution to the integration constant vector \(\{\alpha\}\), it is necessary to set the coefficient determinant to zero, i.e., \(|X(\omega)| = 0\). By solving the above corresponding equations, we can obtain the all-order natural vibration frequencies \(\omega_i (i = 1, 2, \ldots)\) of a CBGCSW.

4. Correctness and Rationality of ANM

In order to validate the practicality of the analysis method (ANM), two cases were investigated in this section.

In the first case, the results of the ANM were compared with the experimental results obtained by Ji et al. [28]. The experimental specimen was a simply supported concrete composite beam with corrugated steel webs. The natural frequencies, calculated by ANM, of the experimental specimen in the literature were compared with the experimental results, as shown in Table 1. It was found that the natural frequencies obtained by ANM agreed well with the natural frequencies measured in the experiment in the literature.

| Modal Order | Experimental Values | ANM | Error/% |
|-------------|---------------------|-----|---------|
| 1           | 19.203              | 18.939 | -1.375 |
| 2           | 55.637              | 56.659 | 1.834 |
| 3           | —                   | 97.084 | —       |

In the second case, the results of ANM were compared with results of the elementary beam theory (EBT) and the finite element method (FEM). Three simply supported abstract CBGCSWs and three fixed-support abstract CBGCSWs with different span–width ratios were investigated. Table 2 shows the geometric and material mechanical parameters of the models. According to the values of span length (L) and width (2b_1), the three span–width ratios were 6.72, 8.12, and 9.24.
Table 2. Parameters used in examples.

| Symbol | Unit       | Value      |
|--------|------------|------------|
| $E_s$  | steel elastic modulus | Mpa       | $2.00 \times 10^5$ |
| $E_c$  | concrete elastic modulus | Mpa       | $3.50 \times 10^4$ |
| $G_s$  | steel shear modulus | Mpa       | $7.35 \times 10^4$ |
| $\rho_s$ | steel density | kg/m$^3$ | 7900 |
| $\rho_c$ | concrete density | kg/m$^3$ | 2400 |
| $\mu_s$ | Poisson’s ratio of steel | | 0.27 |
| $\mu_c$ | Poisson’s ratio of concrete | | 0.18 |
| $B$    | flat plate section length | mm       | 75   |
| $C$    | inclined plate section length | mm       | 75   |
| $D$    | the longitudinal projected length of inclined plate section | mm     | 65   |
| $b_1$  | half the width of top plate | mm       | 500  |
| $b_2$  | width of cantilever plate | mm       | 400  |
| $b_3$  | half the width of low plate | mm      | 525  |
| $b_4$  | height of corrugated web plate | mm      | 400  |
| $t_1$  | thickness of top plate | mm       | 25   |
| $t_2$  | thickness of cantilever plate | mm       | 25   |
| $t_3$  | thickness of low plate | mm       | 25   |
| $t_4$  | thickness of corrugated web plate | mm       | 6    |
| $L$    | span length | mm       | 6720/8120/9240 |

The elementary beam theory (EBT), which ignores the shear lag effect, was deduced by reducing the ANM. Hence, the boundary conditions and the vibration equations of EBT were

\[
G_e A_s (w'' - \theta') - A_p \ddot{w} = 0 \tag{51}
\]

\[
G_e A_s (w' - \theta) + E_c I_c \theta' - I_p \ddot{\theta} = 0 \tag{52}
\]

\[
E_c I_c \theta' \delta \theta^L|_0 = 0 \tag{53}
\]

\[
G_e A_s (w' - \theta) \delta w^L|_0 = 0 \tag{54}
\]

The solution of natural frequency was the same as that in Section 3. The finite element analysis software ANSYS was used to establish the above model to simulate the natural vibration characteristics (Figure 3). In the finite element model, the concrete slabs were simulated using the SOLID65 element; the corrugated webs were simulated using the SHELL43 element; the common nodes of shell element and solid element established the constraint to form the rigid connection; the beam-end simply supported boundary was simulated by constraining the transverse and vertical degrees of freedom of the beam-end nodes; the fixed-support boundary was simulated by constraining the vertical, transverse, and longitudinal degrees of freedom of beam-end nodes. In the modal analysis, the subspace method was used to extract the natural frequencies.

Tables 3 and 4 show the results of EBT, ANM, and the FEM. $e_{EBT} = (EBT - FEM) / FEM \times 100\%$ is the relative calculation error of the EBT and FEM, and $e_{ANM} = (ANM - FEM) / FEM \times 100\%$ is the relative calculation error of the ANM and FEM, as shown in Figure 4.

It can be seen from Tables 3 and 4 and Figure 4, for the first five-order natural vibration modes of the simply supported and fixed-support CBGCSWs, the maximum $e_{ANM}$ was less than 4%; however, the maximum $e_{EBT}$ was more than 15%, and was almost above the red upper limit. This indicated that, after completely taking the shear lag effect and accordion effect of corrugated steel webs, the results obtained by the ANM agreed well with those of the FEM and were more accurate than the EBT. Thus, the correctness and rationality of the ANM were validated.
is the natural vibration frequencies of CBGCSW without considering the shear lag effect. ANM is the natural vibration frequencies of CBGCSW considering the shear lag effect.

5.1. Shear Lag Effect

CBGCSW

The impact of shear lag effect of CBGCSW: (a) simply supported; (b) fixed-support.

Table 2. Parameters used in examples.

| Parameter                          | Unit | Value          |
|------------------------------------|------|----------------|
| E                                 | Mpa  | 2.00×10^5      |
| ρ                                 | kg/m³| 2400           |
| c                                 | %    | 3.50×10^4      |
| G                                 | Mpa  | 2.05×10^10     |
| µ                                 |      | 0.18           |
| ρ_s                               |      | 7.35×10^10     |
| ρ_c                               |      | 7.35×10^10     |
| L                                 | mm   | 6720/8120/9240 |
| t1                                | mm   | 25             |
| t2                                | mm   | 25             |
| t3                                | mm   | 25             |
| t4                                | mm   | 6             |
| B                                 | mm   | 400           |
| C                                 | mm   | 500           |
| B_t                               | mm   | 100           |
| t_b                               | mm   | 25             |
| B_t1                              | mm   | 25             |
| t_b1                              | mm   | 25             |
| B_L                               | mm   | 65            |
| t_bL                              | mm   | 25             |
| L_b                               | mm   | 75            |
| h                                 | mm   | 400           |
| L_b1                              | mm   | 25             |
| t_bL1                             | mm   | 25             |
| h_gen                            | mm   | 75            |
| b                                 | mm   | 400           |
| c                                 | %    | 2.92           |

The results obtained by the ANM agreed well with those of the FEM and were more accurate than the EBT. Thus, the correctness and rationality of the ANM were validated.

Under the same mode order and boundary condition, the shear lag effect of a CBGCSW with inclined plate section length 65 mm significantly with the increasing of the mode order. The largest shear lag effect of the six CBGCSW was more than 15%, and was almost above the red upper limit. This indicated that, after completely taking the shear lag effect and accordion effect of corrugated steel webs, the results obtained by the ANM agreed well with those of the FEM and were more accurate than the EBT. Thus, the correctness and rationality of the ANM were validated.

Table 3. Comparison of natural vibration frequencies of a CBGCSW (simply supported).

| Span–Width Ratio | Calculation Methods | Natural Vibration Frequencies (Hz) |
|------------------|---------------------|-----------------------------------|
|                  |                     | 1st | 2nd | 3rd | 4th | 5th |
| 6.72             | EBT                 | 23.71 | 86.42 | 172.52 | 270.09 | 372.26 |
|                  | ANM                 | 23.24 | 81.33 | 156.45 | 238.41 | 323.40 |
|                  | ε_EBT/ANM%          | -0.59 | 3.58 | 8.13 | 12.08 | 15.28 |
| 8.12             | EBT                 | 16.39 | 61.27 | 126.05 | 201.97 | 283.93 |
|                  | ANM                 | 16.07 | 58.57 | 116.34 | 181.44 | 250.03 |
|                  | ε_EBT/ANM%          | -1.38 | 1.68 | 5.64 | 9.04 | 12.02 |
| 9.24             | EBT                 | 12.73 | 48.22 | 100.74 | 164.09 | 233.80 |
|                  | ANM                 | 12.57 | 46.47 | 94.22 | 149.29 | 208.17 |
|                  | ε_EBT/ANM%          | -1.70 | 0.73 | 4.04 | 7.21 | 10.00 |

Cumulative impact of shear lag effect; and ANM (9.24).
Table 4. Comparison of natural vibration frequencies of a CBGCSW (fixed-support).

| Span–Width Ratio | Calculation Methods | Natural Vibration Frequencies (Hz) |
|------------------|---------------------|-----------------------------------|
|                  |                     | 1st  | 2nd  | 3rd  | 4th  | 5th  |
| 6.72             | EBT                 | 48.06| 115.71| 199.10| 290.62| 387.22|
|                  | ANM                 | 44.88| 104.56| 176.82| 255.60| 338.52|
|                  | FEM                 | 44.12| 105.03| 179.55| 260.45| 342.96|
|                  | ε_{EBT} (%)         | 8.93 | 10.17 | 10.89 | 11.58 | 12.91 |
|                  | ε_{ANM} (%)         | 1.73 | −0.44 | −1.52 | −1.86 | −1.29 |
| 8.12             | EBT                 | 34.38| 85.15  | 149.92| 222.82| 300.48|
|                  | ANM                 | 32.63| 78.30  | 134.80| 197.35| 264.04|
|                  | FEM                 | 32.07| 78.48  | 136.80| 201.48| 269.47|
|                  | ε_{EBT} (%)         | 7.20 | 8.50   | 9.39  | 10.59 | 11.51 |
|                  | ε_{ANM} (%)         | 1.73 | −0.22  | −1.46 | −2.05 | −2.02 |
| 9.24             | EBT                 | 27.06| 68.44  | 122.39| 184.14| 250.67|
|                  | ANM                 | 25.94| 63.66  | 111.09| 164.41| 221.54|
|                  | FEM                 | 25.61| 63.78  | 112.69| 167.88| 226.66|
|                  | ε_{EBT} (%)         | 5.66 | 7.31   | 8.61  | 9.69  | 10.59 |
|                  | ε_{ANM} (%)         | 1.31 | −0.18  | −1.42 | −2.07 | −2.26 |

5. Influence of Shear Lag Effect and Accordion Effect on the Dynamic Characteristics of a CBGCSW

5.1. Shear Lag Effect

In order to investigate the impact of the shear lag effect, we further carried out degenerations of the vibration equations of a CBGCSW. Figure 5 describes the impact of shear lag effect on the natural vibration frequencies of a simply supported and fixed-support CBGCSW. \( \epsilon_c = \left| f_s - f_{ns} \right| / f_s \) is the impact of shear lag effect; \( f_s \) is the natural vibration frequencies of CBGGSW considering the shear lag effect; and \( f_{ns} \) is the natural vibration frequencies of CBGCSW without considering the shear lag effect.

![Figure 5](image)

**Figure 5.** The impact of shear lag effect of CBGCSW: (a) simply supported; (b) fixed-support.

As can be seen from Figure 5, the shear lag effects of six CBGCSW models all increased significantly with the increasing of the mode order. The largest shear lag effect of the six CBGCSW models was 15.11%. Further, there was a weakening impact of the span–width ratio on the shear lag effect. Under the same mode order and boundary condition, the shear lag effect of a CBGCSW with a smaller span–width ratio was larger. This is an indication that the shear lag effect cannot be ignored in obtaining the dynamic characteristics of CBGCSWs, especially when the span–width ratio is small.
5.2. Accordion Effect

To investigate the impact of the accordion effect, the natural vibration frequencies of the CBGCSW models were compared with the natural vibration frequencies of the composite box girder bridge with general steel web (CBGGSW) models (the natural vibration frequencies of the CBGGSW were calculated according to the existing research literature [29]. The geometric and material mechanical parameters of the CBGCSW models are shown in Table 2. The impact of the accordion effect is shown in Table 5. \( g_{ct} = (f_G - f_C)/f_C \) is the impact of the accordion effect; \( f_G \) is the natural vibration frequencies of the CBGGSW; and \( f_C \) is the natural vibration frequencies of the CBGCSW.

Table 5. The impact of the accordion effect of the CBGCSW.

| Boundary Condition | Span–Width Ratio | \( g_{ct}/\% \) | 1st  | 2nd  | 3rd  | 4th  | 5th  |
|-------------------|------------------|------------------|------|------|------|------|------|
| Simply supported   | 6.72             | 6.55             | 7.39 | 8.61 | 9.60 | 10.52|
|                   | 8.12             | 6.55             | 6.94 | 7.78 | 8.52 | 9.21 |
|                   | 9.24             | 6.55             | 6.72 | 7.37 | 7.95 | 8.50 |
| Fixed-support      | 6.72             | 12.39            | 10.46| 9.28 | 8.58 | 8.46 |
|                   | 8.12             | 11.78            | 10.36| 9.32 | 8.71 | 8.36 |
|                   | 9.24             | 11.32            | 10.20| 9.29 | 8.67 | 8.29 |

As can be seen from Table 5, compared with the CBGGSW models, the first- to fifth-order natural vibration frequencies of the CBGCSW models were significantly lower. Further, the largest impact of the accordion effect in the three models was 12.9%. This is an indication that because of the impact of the accordion effect, the bending rigidities of the CBGCSW models were significantly less than those of the CBGGSW models.

6. Conclusions

In this contribution, on the basis of the energy principles and variational principles, by using the displacement superposition method and completely considering the impact of shear lag effect and accordion effect of corrugated steel webs, an analytical method that can determine the natural vibration characteristics of a CBGCSW was developed. The following conclusions were obtained.

(1) By comparing the natural vibration frequencies calculated by the ANM with those obtained by other methods, the correctness of the ANM and the rationality of the model simplifications were verified. The natural frequencies obtained by ANM agreed well with those measured by experiment. The ANM was more accurate than the elementary beam theory (EBT) in calculating natural frequencies. Compared with the finite element model (FEM), the ANM can illustrate the key factors, such as shear lag effect and accordion effect. The influences of these key factors on the dynamic response can be analyzed more conveniently and rapidly by using the ANM. Hence, the ANM provides a theoretical basis for deriving a practical formula for engineering calculation, and addresses the deficiency of numerical simulation analysis.

(2) The shear lag effect on a CBGCSW is under the influence of the order of the natural vibration frequency and span–width ratio. The shear lag effects of six CBGCSW models all increased significantly with the increasing of the mode order. Further, the shear lag effects were significantly larger for the CBGCSWs with smaller span–width ratios. Hence, for practical engineering applications, the shear lag effect on a CBGCSW with a smaller span–width ratio is significant and cannot be neglected.

(3) Compared with the natural vibration frequencies of the CBGGSW models, the natural vibration frequencies of the CBGCSW models were significantly lower, which indicates that the impact of the accordion effect is significant. In design and seismic calculations, attention should be paid to this difference between CBGGSWs and CBGCSWs.
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