The distribution of extreme share return in different Malaysian economic circumstances

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Abstract

This paper presents a study on the performance of probability distribution in various financial periods by investigating the effect of economic cycle on extreme stock return activity. Malaysian stock price KLCI data from 1994-2008 were split into three economy periods corresponding to the growth, financial crisis, and recovery. Four prevalent distributions, specifically generalized lambda distribution (GLD), generalized extreme value (GEV), generalized logistic (GLO), and generalized paretian (GPA) had been employed to model weekly and monthly maximum and minimum share returns of Kuala Lumpur Composite Index (KLCI). L-moment approach had been used to estimate the parameter, while k-sample Anderson darling (k-ad) test had been applied to measure the goodness of fit estimation. In conclusion, GLD is the most appropriate distribution to represent weekly maximum and minimum returns for overall three economic scenarios in Malaysia.

Keywords: Value-at-risk (VaR), extreme share returns, Bursa Malaysia, Kuala Lumpur composite index (KLCI), generalized lambda distribution (GLD)

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INTRODUCTION

Investment risk has been studied extensively since the 1960s. Investment risk can be explained as the probability of lower return from what is expected in the market. Investors have long struggled with this risk uncertainty as part of their aim is to minimize investment deficit. Even so, movement in the stock market is unpredictable, apart from being influential to economic report (Chen et al., 1986). Knowledge on extreme stock market distribution is crucial to investment analysts to execute proper projection and develop risk management.

Share return distribution dispersion is defined as unrestricted volatility model by comparing the standard deviation of stock returns. Previous studies majorly highlighted the function of share return distribution dispersion in the economic cycle. Finance practitioners use distribution dispersion to assess return volatility (Garcia et al., 2014), microeconomic ambiguity (Bloom et al., 2018), trends correlations in the worldwide stock market (Solnik and Roulet, 2000), nation countercyclical factor (Gomes et al., 2003), and as the sign for potential active risk (De Silva et al., 2001). From distribution perspective, Longin (1996, 2000) emphasized the importance of distribution information in risk boundary prediction, in policing principal requirements for security and the prospects markets. It has been reported that unsuitable distribution assumptions may lead to erroneous calculation and shortfall to stockholders (Danielsson et al., 1998). Previous investigations on share return distribution assumption have established that the share price return is not distributed as normal distribution, since the characteristics are fat-tails and resemble the Paretoan distributions family (Fama, 1965; Gray and French, 1990; Harris and Kucukozmen, 2001; Peiro, 1994; Theodossiou, 1998; Xub, 1995). Recently, substitute distributions have been proposed as they can ascertain the characteristics of equity return data better and explain the extreme outcome using GLD, GLO, GEV, GPA, and Pearson (PE3) distribution, see (Hussain and Li, 2015; Marsani et al., 2017; Tolikas, 2014).

To understand the extreme share market behavior, it is sensible to determine whether economic situation affects the performance of the probability distribution. The motivation of the present study is to extend our knowledge on extreme share return distribution by remarking the economic scenario factor; and to the best of our information, this study is one of the first attempts to examine distribution return in different economic circumstance. This study emphasizes on evaluating the ability of famous probability distributions, namely GLD, GLO, GEV, GPA, and PE3 to fit the extreme KLCI stock returns in different economy phases. The paper is organized as follows: Section 2 describes the methodology embraced in this work, Section 3 explains the data used, including Malaysian economy phase, and elaborates on the experimental results of the investigation. Lastly, a brief discussion on the discoveries and summary are presented in Section 4.

METHODOLOGY

Price movement is usually illustrated using graphical representation. For this study, extreme return series was generated using block maxima-minima procedure, and then fitted using L-moment parameter estimation methods. To inspect distribution capabilities in forecasting extreme price return given economic
situation, K-ad and VaR graphical representation plot were applied. Detailed approaches used are as follows.

Economy period

Referring to Malaysian quarterly gross domestic product (GDP) growth report, in this research, the Malaysian economic history had been split into three sub-divisions: economic growth period from January 1994–June 1997, economic crisis period from July 1997–December 2001, and recovery period from January 2002–June 2008.

KLCI share market

In this study, 14 years of daily Kuala Lumpur Share Exchange (KLCE) data had been sourced, that is from January 1994 to June 2008. This data was obtained from Yahoo Finance. Daily share returns were computed as \( \frac{R_t}{R_{t-1}} \), where \( R_t \) is the price return index at the time \( t \), and \( R_{t-1} \) is yesterday’s price index. Fig. 1 shows the KLCI price index and Fig. 2 shows the daily returns corresponding to three different economy periods, which are economic growth, economic crisis, and recovery period.

\[ R_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \]

\[ R_{t-1} = \ln \left( \frac{P_{t-1}}{P_{t-2}} \right) \]

\[ R_{t-3} = \ln \left( \frac{P_{t-3}}{P_{t-4}} \right) \]

\[ R_{t-n} = \ln \left( \frac{P_{t-n}}{P_{t-(n-1)}} \right) \]

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Table 1 Probability density function (PDF), cumulative distribution function (CDF) and quantile function x(F).

| Generalized Lambda Distribution | Generalized Extreme Value |
|---------------------------------|---------------------------|
| Range of x: \(-\infty \leq x < \infty\) | \(\beta + \frac{\alpha}{\kappa} \leq x < \infty, \quad \kappa < 0\) |
| PDF | No explicit analytical form |
| \(f(x) = e^{\kappa x / (1 - e^{\kappa x})} \), \(y = \kappa \log (1 - \kappa (x - \beta) / \alpha), \quad \kappa \neq 0\) |
| CDF | \(\alpha \left(\kappa e^{\kappa x} + \beta (1 - F)^{\kappa - 1}\right) \geq 0\) |
| \(s(F) = \beta + \alpha (1 - F)^{\kappa - 1} \), \(\kappa \neq 0\) |
| Special cases: \(\kappa = 0\) is the Gumbel distribution; \(\kappa < 0\) is a Frechet distribution; Reverse GEV with \(\kappa > 0\), \(-x(1 - F)\) is the Weibull distribution |

Note that the derivation of GLD parameter in this study was conducted by following the procedure proposed by Asquith (2007).

K-sample Anderson darling (K-ad) test

The K-ad suggested by Scholz and Stephens (1987) is the generalization of the double-sample Anderson-Darling test. To determine the most exceptional distribution execution in assessing the price return behavior, we optimized the benefits of slight parametric theory on the K-ad test, where through this analysis, the similarity and variation between two samples could be distinguished by considering the sensitivity at the tail area. The K-ad test is given as follows:

\[
AD_k = \sum_{i=1}^{k} \left( \frac{\bar{F}_x(x) - H(x)}{|\bar{F}_x(x) - \bar{F}_y(x)|} \right)^2 H'(x)
\]

where \(n_i\) is the sample size of \(x_i\) and \(H(x)\) is the observed distribution function of the pooled sample of all \(\bar{F}_x(x)\), where \(0 \leq i \leq k - 1\). Since K-ad test statistic should show similarity between experimental and pooled samples, the smallest value of K-ad shall verify the best-fitted distribution.

RESULTS AND DISCUSSION

This section explains the descriptive statistics for each interval. Consequently, the goodness of fit tests using K-ad test is deliberated.

The Value at risk (VaR) analysis was carried out by using the probability plot representation.

**Data sample statistics**

Table 2 presents a descriptive statistic for the daily (overall), weekly, and monthly share price returns. Daily data series recorded the lowest return at -24.1534 % and the highest at 20.8174 %. Interestingly, both maximum and minimum returns were recorded in the year 1998 throughout the economic crisis. The mean average for the daily return displayed a negative value of 0.0023 % and standard deviation of 1.5817 %. Focusing on the mean average for weekly and monthly series, positive mean values denote the maximum series return and while negative denote the mean value for the minimum series. Distribution Skewness is 0.4454, indicating the tail to the right, and Kurtosis of 44.0272 supports our claim that the distribution for an overall period is fat-tail. Another exciting finding from the table is different standard deviation dispersion range values for different economy phase, which are 1.1764 –1.6060 for growth period, 2.4713–3.9031 for crisis period, and 0.6205–1.2733 for the recovery period. Also, the standard deviation of each of the economy series is different, which explains that the instability is not apparent between the minimum and maximum series. Daily Kurtosis return is 44.0272, signaling that the distribution for an overall period is fat-tail. Next, the Jarque-Bera test (JB) was performed to check series dispersals. Substantial JB value and significant p-value suggesting that the series does not follow the normal distribution. Note that the JB value reduces once the size decreases.
VaR using plot representation

VaR is applied to determine if the potential share return information is experiencing loss or gaining returns over standard atmospheres by examining the probability at the edge of the tail distribution. In this part, we investigated VaR using plot representation for GLD, GLO, GEV, GPA, PE3, and Normal (NOR) to examine which of the distributions would give reliable calculation at the distribution tailpiece.

Figs. 3, 4, and 5 exhibit cumulative density function curve plots representing GLD, GLO, GEV, GPA, PE3, and NOR for economic growth, crisis, and recovery phases for the weekly and monthly for minimum and maximum price returns, respectively. These CDF plots give clear expression to elucidate the upper and lower tail event. Note that our attention is on the upper curve area for maximum return and the lower curve area for the minimum return.

A few remarkable examinations can be made from Fig. 3. We discovered that the blue curve, which represents NOR distribution, noticeably averts from the entire observed series during growth period, indicating that this distribution is not suitable for use to calculate extreme price return efficiently.

At the same time, GPA distribution markedly miscarried while predicting extreme returns, especially during the weekly and monthly minimum when the lower tail curve diverts from reaching the utmost minimum return. Although the CDF curve for the GLD, GEV, and GLO concurred with each other by displaying a comparable pattern, GLD curve is more prominent in term of accuracy. GLD curve attains nearly to the observed series compared to GEV and GLO distributions, especially at the edge of the upper and lower tails.

Fig. 4 displays the CDF curve plot throughout crisis period. Again NOR distribution curve is detached from the observed series and other fitting curve, which boosts our claim that normal distribution cannot calculate extreme price return properly. Also, there is no sharp division between the other curves, except for GPA distribution in weekly maximum series.

In Fig. 5, the CDF curve plot within the economy recovery phase shows that only NOR and GPA curves slightly deviate from the observational series. This discrepancy could be attributed to an unclear decision of distribution fitting in extreme share return.

K-ad analysis

The purpose of using K-ad was to compare the fitting distribution performance. In this respect, to set up the goodness of fit for each of the distribution, the daily returns had been separated into three different economy phases, namely growth, crisis, and recovery periods. Our attention was to pinpoint any economic circumstance effects in extreme return while fitting the distribution. Hence, the K-ad test results presented in Tables 3, 4, and 5 had been used to inspect the goodness of fit between the observed and fitted data.

The null hypothesis for the K-ad test expressed homogeneity in observed and fitted data series, and the approximation was adequate when smaller K-ad value had been produced.

Growth period

Table 3 shows the result of K-ad test for economic growth period with the best fitting distribution sorted accordingly from the lowest to the highest K-ad values. Focusing on weekly extreme maximum return series, GLD is ranked at the first place followed by GLO, GEV, PE3, GPA, and NOR.

Among the distributions, only NOR appears to be significant with a p-value less than $\alpha=0.05$, indicating that the series does not follow normal distribution.
Table 2 Descriptive statistics

|          | n   | min (%) | average (%) | max (%) | std. deviation (%) | variance (%) | skewness | kurtosis | jarque. bera (JB) | pval   |
|----------|-----|---------|-------------|---------|--------------------|--------------|----------|----------|------------------|--------|
| Overall  | 3577| -24.153 | -0.002      | 20.817  | 1.582              | 0.025        | 0.445    | 44.027   | 250989.400       | 0.000  |
| Growth   |     |         |             |         |                    |              |          |          |                  |        |
| w.max    | 182 | -1.860  | 1.269       | 9.712   | 1.205              | 0.015        | 2.654    | 16.565   | 1609.032         | 0.000  |
| w.min    | 182 | -6.651  | -1.198      | 4.858   | 1.176              | 0.014        | -0.811   | 10.000   | 391.509          | 0.000  |
| m.max    | 42  | 0.875   | 2.395       | 9.712   | 1.606              | 0.026        | 2.549    | 11.486   | 171.512          | 0.000  |
| m.min    | 42  | -6.651  | -2.241      | -0.480  | 1.278              | 0.016        | -1.422   | 5.282    | 23.271           | 0.000  |
| Crisis   |     |         |             |         |                    |              |          |          |                  |        |
| w.max    | 237 | -2.435  | 2.129       | 20.817  | 2.566              | 0.066        | 3.892    | 25.459   | 5579.214         | 0.000  |
| w.min    | 237 | -24.153 | -2.006      | 2.384   | 2.471              | 0.061        | -4.088   | 31.971   | 8948.295         | 0.000  |
| m.max    | 54  | 1.176   | 4.327       | 20.817  | 3.903              | 0.152        | 2.910    | 11.976   | 257.460          | 0.000  |
| m.min    | 54  | -24.153 | -3.891      | -0.796  | 3.547              | 0.126        | -3.726   | 21.092   | 861.401          | 0.000  |
| Recovery |     |         |             |         |                    |              |          |          |                  |        |
| w.max    | 343 | -0.829  | 0.818       | 4.259   | 0.620              | 0.004        | 1.193    | 6.055    | 214.788          | 0.000  |
| w.min    | 343 | -9.797  | -0.739      | 2.172   | 0.882              | 0.008        | -4.155   | 39.370   | 19891.590        | 0.000  |
| m.max    | 79  | 0.513   | 1.466       | 4.259   | 0.675              | 0.005        | 1.159    | 5.468    | 37.747           | 0.000  |
| m.min    | 79  | -9.797  | -1.452      | 0.641   | 1.273              | 0.016        | -4.200   | 27.206   | 2160.977         | 0.000  |

w=weekly, m=monthly, max=maximum, min=minimum

Table 3 K-ad test for economic growth period

| Distribution | AD | pval |
|--------------|----|------|
| gld          |    |      |
| glo          |    |      |
| gev          |    |      |
| pe3          |    |      |
| gpa          |    |      |
| nor          |    |      |

Table 4 K-ad test for economic crisis period

| Distribution | AD | pval |
|--------------|----|------|
| gld          |    |      |
| glo          |    |      |
| gev          |    |      |
| pe3          |    |      |
| gpa          |    |      |
| nor          |    |      |

Table 5 K-ad test for economic recovery period

| Distribution | AD | pval |
|--------------|----|------|
| gld          |    |      |
| gev          |    |      |
| pe3          |    |      |
| glo          |    |      |
| gpa          |    |      |
| nor          |    |      |
The single most striking observation emerging from Table 3 is the GLD, where this distribution shows a remarkable outcome for weekly maximum and minimum periods, with the lowest K-ad values of 0.1360 and 0.2920, respectively. On the other hand, different results were shown for monthly maximum and minimum periods with GEV (0.1372) and PE3 (0.2232) ranked at the first place. A further exciting result from the data was that GLD, GLO, and GEV were always the three exceptional top rank distributions, except during the monthly minimum period when PE3 gave an excellent fitting. However, the K-ad values among PE3, GEV, GLO, and GLD during this period were close to each other, ranging around 0.2.

Crisis period

Table 4 displays the K-ad test result for economic crisis period. Based on each of the period, GLD gave outstanding consequences with the smallest K-ad values among each of the intervals, compared to other distributions, except for monthly maximum return when GLD appeared to be at the third place with a value of 0.2014. Once again, the top three distributions were GLD, GLO, and GEV, except during weekly minimum interval when PE3 (1.4595) led GEV (1.5341) at the third rank.

Recovery period

Table 5 presents the K-ad test result for the economic recovery period. Once again, our calculation recommends that GLD is superior than GEV, GLO, PE3, GPA, and NOR in the overall interval, namely, weekly and monthly (maximum and minimum) extreme return with evidence of lower K-ad test values. By observing weekly and monthly for minimum interval, the fitted GPA and NOR distributions were found not appropriate to estimate weekly and monthly minimum returns with evidence of significant p-value denying the null hypothesis. In summary, GLD is tremendous in clarifying the extreme weekly and monthly returns (both maximum and minimum intervals) for each of the economic phases with exceptional efficiency during the growth period where GEV and PE3 (monthly maximum and minimum) are the best and during crisis period where GLO (monthly maximum) is the finest. The earlier result from the plot analysis supports this finding, when we concluded that GLD is the most suitable distribution in fitting overall interval by considering each of the economic phases as the CDF curve near to the empirical observation series. Consequently, from the K-ad test, we deduced that GLD is adequate in valuing overall weekly for both maximum and minimum extreme return intervals by considering the economic circumstance.

CONCLUSION

This paper reflects the distribution model performance by using k-sample Anderson darling test and value at risk plot. The most excellent group performances and the series structures have been evaluated based on three different Malaysian economic circumstances which are growth, crisis, and recovery periods. This investigation verifies that by considering each of the financial situation, GLD has given consistent accomplishments in fitting the weekly returns. This distribution is indeed the best model for both upper sides (maxima) and downside risk (minimal) by consideration of each economic situation. It can be concluded that GLD is reliable in explaining the extreme outcome in share return, since this distribution has shown the best performance compared to other distributions, especially in weekly series return. The distribution precision delivered in this investigation can promise reduction of investment risks and boost profit to the shareholder. The findings in this study is hoped to provide a better interpretation of share market expansion associated with the current or expected economic situation. Furthermore, the present work conveys new comprehension investment based on refining the fitting precision extreme share return in different economic period, particularly for Malaysia share market.

ACKNOWLEDGEMENT

The authors would like to thank Universiti Sains Malaysia and the Ministry of Higher Education for the scholarship provided.

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