MAGNETIC HELICITY OF THE GLOBAL FIELD IN SOLAR CYCLES 23 AND 24

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ABSTRACT

For the first time we reconstruct the magnetic helicity density of the global axisymmetric field of the Sun using the method proposed by Brandenburg et al. and Pipin et al. To determine the components of the vector potential, we apply a gauge which is typically employed in mean-field dynamo models. This allows for a direct comparison of the reconstructed helicity with the predictions from the mean-field dynamo models. We apply this method to two different data sets: the synoptic maps of the line-of-sight magnetic field from the Michelson Doppler Imager (MDI) on board the Solar and Heliospheric Observatory (SOHO) and vector magnetic field measurements from the Vector Spectromagnetograph (VSM) on the Synoptic Optical Long-term Investigations of the Sun (SOLIS) system. Based on the analysis of the MDI/SOHO data, we find that in solar cycle 23 the global magnetic field had positive (negative) magnetic helicity in the northern (southern) hemisphere. This hemispheric sign asymmetry is opposite to the helicity of the solar active regions, but it is in agreement with the predictions of mean-field dynamo models. The data also suggest that the hemispheric helicity rule may have reversed its sign during the early and late phases of cycle 23. Furthermore, the data indicate an imbalance in magnetic helicity between the northern and southern hemispheres. This imbalance seems to correlate with the total level of activity in each hemisphere in cycle 23. The magnetic helicity for the rising phase of cycle 24 is derived from SOLIS/VSM data, and qualitatively its latitudinal pattern is similar to the pattern derived from SOHO/MDI data for cycle 23.

Key words: magnetohydrodynamics (MHD) – Sun: magnetic fields

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1. INTRODUCTION

Generation of the magnetic field in the Sun is tightly related with convective helical motions. In the framework of axisymmetric dynamos, the magnetic field is typically decomposed into toroidal and poloidal components. Parker (1955) suggested that the solar dynamo can be presented as a periodic transformation of the poloidal magnetic field, $B_p = B_\phi e_\phi + B_r e_r$, into a toroidal field, $B_t = \bar{B}_r e_\phi - \bar{B}_\phi e_r$ (via the differential rotation), and the reverse transformation of $\bar{B}_t$ to $\bar{B}_p$ by helical convective motions. Further development of the dynamo theory showed that the two processes produce helical magnetic fields on both small and large spatial scales (Frisch et al. 1975; Pouquet et al. 1975), and that the conservation of magnetic helicity is an important factor for the dynamical quenching of large-scale magnetic field generation (Kleeorin & Ruzmaikin 1982; Cattaneo & Vainshtein 1991; Vainshtein & Cattaneo 1992; Kleeorin & Rogachevskii 1999; Kleeorin et al. 2000; Brandenburg & Subramanian 2005).

Early observations of various proxies of magnetic/current helicity established what is now known as the hemispheric helicity rule: magnetic fields of active regions exhibit preferentially negative (positive) helicity in the northern (southern) hemisphere (Seehafer 1990; Pevtsov et al. 1995; Kuzanyan et al. 2000; Zhang et al. 2010 and references therein).

On the other hand, some researchers (e.g., Brandenburg et al. 2003; Blackman & Brandenburg 2003; Warnecke et al. 2011; Pipin et al. 2013a, 2013b) argued that the magnetic helicity of a large-scale (global) axisymmetric field should be positive/negative in the northern/southern hemisphere. Furthermore, the mean-field dynamo models predict reversals of the sign of helicity in association with the propagation of a dynamo wave inside the convection zone (e.g., Warnecke et al. 2011; Pipin et al. 2013b). Reversals of the hemispheric helicity rule have been reported in observations, but the results seem inconclusive, with some researchers reporting the presence of such reversals in early/late phases of the solar cycle (Bao et al. 2000; Hagino & Sakurai 2005) while others question their existence (Pevtsov et al. 2001, 2008; Gosain et al. 2013). Such apparent controversy may be resolved via direct comparison of observations with model predictions. Since dynamo models can provide detailed information about the distribution of the magnetic helicity of the global axisymmetric field in the convection zone and near the photosphere, it is highly desirable to directly compare these model estimates with the observations.

Theory suggests that magnetic helicity on small and large scales should have opposite signs (e.g., Pouquet et al. 1976; Seehafer 1996; Blackman & Brandenburg 2003; Brandenburg & Subramanian 2005; Pipin et al. 2013a, 2013b). Therefore, small-scale helicity may dissipate on small spatial scales subject to Ohmic dissipation (e.g., Pouquet et al. 1976), or the helicity of both signs could emerge through the solar photosphere. Early measurements of the vertical component of large-scale current helicity density (e.g., Pevtsov & Latsushko 2000; Wang & Zhang 2010) found that in its sign, the large-scale magnetic fields follow the same hemispheric helicity rule as the active regions. These early studies concentrated on spatial scales larger than the active regions but smaller than the solar hemisphere. One should note that in the framework of a mean-field dynamo, the magnetic fields of active regions represent the “small-scale” fields, while the large-scale fields refer to spatial scales that are comparable in size to the solar hemisphere. In this article, we address the helicity determination for large-scale magnetic fields as defined by the mean-field dynamo theory. To avoid confusion with previous studies, we use the terms “global” and “large-scale” to refer to magnetic fields on spatial scales much larger than active regions. We reconstruct the magnetic...
and in Section 5 we discuss our findings.

2. THE FORMALISM BEHIND THE COMPUTATION OF THE HELICITY OF THE GLOBAL AXISYMMETRIC FIELD

Let us represent the axisymmetric magnetic field \( \mathbf{B} \) as
\[
\mathbf{B} = \mathbf{e}_\phi \mathbf{B}_\phi + \nabla \times (\mathbf{A}_\phi \mathbf{e}_\phi) = \nabla \times \mathbf{A},
\]
(1)
\[
\mathbf{A} = r \mathbf{A}_r + \mathbf{A}_\phi \mathbf{e}_\phi = r \mathbf{T} + \nabla \times (r \mathbf{S}),
\]
(2)
where \( \mathbf{r} = r \mathbf{e}_r, \nabla \times (r \mathbf{A}_r) = \mathbf{e}_\phi \mathbf{B}_\phi \). The representation of the vector potential in Equation (2) is often employed in mean-field dynamo models. In the spherical coordinates, the scalar functions \( S \) (poloidal potential) and \( T \) (toroidal potential), which are functions of \( t \) (time), \( r \) (radius), \( \theta \) (polar angle), and \( \phi \) (azimuth), are uniquely determined with a gauge (e.g., Krause & Rädler 1980; Bigazzi & Ruzmaikin 2004):
\[
\int_0^{2\pi} \int_{-1}^1 S d\mu d\phi = \int_0^{2\pi} \int_{-1}^1 T d\mu d\phi = 0,
\]
(3)
where \( \mu = \cos \theta \). Equation (3) is time-dependent and it is applicable to arbitrary \( r \), including the solar surface at \( r = R_\odot = R \). The magnetic helicity density is given by \( \mathbf{A} \cdot \mathbf{B} \). In this study, we concentrate on the axisymmetric magnetic field, and thus we ignore the dependence of magnetic field components on azimuth \( \phi \).

Let us suppose that we have information about the axisymmetric components of the toroidal field, \( \mathbf{B}_\phi \), and the poloidal field, \( \mathbf{B}_r \), from the observations. Then, decomposing the \( r \) and \( \phi \) components of the magnetic field and its vector potential on a series of Legendre polynomials \( P_n \) and \( P_n^1 \),
\[
\mathbf{A}_\phi (t, \theta) = \sum_{n=1}^N a^{(n)}_\phi (t) P_n^1 (\cos \theta),
\]
(4)
\[
\mathbf{B}_r (t, \theta) = \sum_{n=1}^N b^{(n)}_r (t) P_n (\cos \theta),
\]
(5)
\[
\mathbf{B}_\phi (t, \theta) = \sum_{n=1}^N b^{(n)}_\phi (t) P_n^1 (\cos \theta),
\]
(6)
\[
\mathbf{A}_r (t, \theta) = \sum_{n=1}^N a^{(n)}_r (t) P_n (\cos \theta),
\]
(7)
and using a known relation between \( P_n \) and \( P_n^1 \),
\[
P_n^1 = - \sin \theta \frac{\partial P_n}{\partial \mu}, \quad - \frac{\partial P_n^1}{\partial \mu} = n (n + 1) P_n,
\]
one can derive the following algebraic relations between the coefficients for the magnetic field and the vector potential Legendre polynomial series:
\[
a^{(n)}_\phi (t) = - R b^{(n)}_\phi (t) \frac{1}{n (n + 1)},
\]
(8)
where \( R \) is the radius of the Sun and \( n \) is some finite (reasonably large) number. In the study, we adopt \( n \geq 48 \).

The derivations of the vector potential and helicity in Brandenburg et al. (2003) are based on odd modes of Equation (8). Here, we take into account both the radial and the toroidal magnetic fields, and we use information from both the even and odd modes of the coefficients in Equations (4)–(7). This approach allows us to study both the symmetric (relative to the solar equator) and antisymmetric components of the magnetic helicity. To calculate a proxy for the poloidal component of the large-scale current helicity density, \( \mathbf{B}_r (\nabla \times \mathbf{B})_r \), we employ the following identities:
\[
(\nabla \times \mathbf{B})_r (t, \theta) = \sum_{n=1}^N (\nabla \times \mathbf{B})^{(n)}_r (t) P_n (\cos \theta),
\]
(10)
\[
(\nabla \times \mathbf{B})^{(n)}_r (t) = - \frac{n (n + 1) b^{(n)}_\phi (t)}{R}.
\]
(11)

The scalar function \( S \) (poloidal potential; see Equation (2)) can be determined within the uncertainty of a gauge (constant), which does not affect the vector potential of the poloidal field (the toroidal part of the vector potential \( \mathbf{A}_\phi \)). The gauge in Equation (3) only affects the vector potential of the toroidal field (the poloidal component of the vector potential \( \mathbf{A}_r \)). This component of the vector potentials is determined via Equations (7) and (9). In the reconstruction of the vector potential, Equation (3) can be satisfied numerically by redefining \( \mathbf{A}_r (t, \mu) = \tilde{A}^{(0)}_r (t, \mu) - \pi C (t) \), where \( \tilde{A}^{(0)}_r (t, \mu) = \sum_{n=1}^N a^{(n)}_r (t) P_n (\mu) \) and \( C \) can be determined numerically from the integration
\[
\int_0^{2\pi} \int_{-1}^1 \tilde{A}^{(0)}_r (t, \mu) d\mu d\phi = 2\pi \int_{-1}^1 \tilde{A}^{(0)}_r (t, \mu) d\mu = C (t).
\]
(12)

In the course of the reconstruction, we found that the amplitude of \( C (t) \) is rather small in comparison with \( \tilde{A}^{(0)}_r (t, \mu) \).

Equations (1)–(3) and (12) ensure that \( \int_{-1}^1 \tilde{A}_r B_\phi d\mu = \int_{-1}^1 A_r B_\phi d\mu \), which should be expected from the topological considerations (see Section 4 and Brandenburg et al. 2003). The formalism employed in the computation of the helicity of the global field was tested using the results of the mean-field dynamo models given in Pipin et al. (2013b).

3. DATA REDUCTION

3.1. Construction of the Maps

Next, we employed the synoptic maps of the line-of-sight (LOS) magnetic fields from the Solar and Heliospheric Observatory/Michelson Doppler Imager (SOHO/MDI) data set (Scherrer et al. 1995; Liu et al. 2004; Hoeksema et al. 2010; Sun et al. 2011). The \( \mathbf{B}_r \) and \( \mathbf{B}_\phi \) components of the magnetic field are derived from a set of synoptic maps constructed using 10 degree-wide longitudinal segments of the solar disk image centered at the following longitudes relative to the central meridian: \( \phi_0 = 0, \pm (15^\circ, 30^\circ, 45^\circ, \text{and} 60^\circ) \).
The poloidal and toroidal components of the magnetic field can be determined following the approach of Duvall et al. (1979), under the assumption that the observed changes in the magnetic fields over several days are entirely due to the change in the projection of the same magnetic field vector. Ideally, the method requires a comparison of the same feature on all of the images (see Duvall et al. 1979; Grigoryev et al. 1986; Pevtsov & Latushko 2000; Wang & Zhang 2010). Instead, we employ a more simplified approach by comparing areas with the same latitudes and longitudes in the synoptic maps constructed for different longitudinal offsets.

For each pixel on these synoptic maps of the LOS field,

$$B_t(t, \phi_i, \theta) = \bar{B}'_r(t, \theta) \sin \theta \cos \phi_i + \bar{B}'_\phi(t, \theta) \sin \phi_i,$$

where $B_t$ is the LOS component of the magnetic field and $\bar{B}'_r$ and $\bar{B}'_\phi$ are its radial (poloidal) and toroidal components of the global magnetic field (here we introduce the $\bar{B}'$ notation to distinguish from $B_t$ computed by the SOHO/MDI team using a different approach). One can see that a simple addition/subtraction of the synoptic maps taken with symmetric longitudinal offsets relative to the central meridian (e.g., $\pm 30^\circ$) allows us to determine $\bar{B}'_r$ and $\bar{B}'_\phi$ from Equation (13). To lessen the effects of magnetic field evolution, we smooth each synoptic map by convolving it with a symmetric two-dimensional (2D) Gaussian function with an FWHM of four solar degrees. Next, we determine the toroidal and poloidal components by fitting Equation (13) for each Carrington longitude using the data taken with different longitudinal offsets. Finally, we average the obtained components of the magnetic field vector over each Carrington rotation to derive the latitudinal profiles of $\bar{B}'_r$ and $\bar{B}'_\phi$. As a test, we compared $\bar{B}'_r$ with $B_t$ from the synoptic charts of the radial solar magnetic field provided by the MDI team (Sun et al. 2011). We found that the latitudinal profiles of $\bar{B}'_r$ derived by us agree well with the $B_t$ profiles derived by the SOHO/MDI team. Some minor deviations were found at very high latitudes near the polar regions. The latter could be explained by the fact that the SOHO/MDI synoptic maps of the radial ($B_t$) field employ polar field filling, while we employ $B_t$ synoptic charts without pole filling. In further computations, we use the latitudinal profiles of $\bar{B}_r$. As another test, for three solar rotations (CR1913, 1979, 2058) we compared the latitudinal profiles of radial and toroidal fields derived by us with those from Wang & Zhang (2010). We found a good agreement between the two independent derivations.

Figure 1 demonstrates the derived distribution of the radial and toroidal components of the magnetic field for the entire SOHO/MDI data set. One can see several well-known patterns. For example, at high latitudes, the radial flux (Figure 1(a)) shows weak polar fields of correct sign, as well as polar field reversals shortly after the maximum of cycle 23. In mid-latitudes, the prevailing polarity field is positive/negative in the northern/southern hemisphere, in agreement with the leading polarity of active region fields for cycle 23. As the leading polarity flux in active regions is more compact (and hence may last longer) in comparison with the following polarity flux, average synoptic charts, such as Figure 1, tend to emphasize the leading fields. The toroidal field (Figure 1(b)) in active region belts is negative/eastward in the northern hemisphere (and it is positive/westward in the southern hemisphere) in agreement with the prevailing polarity orientation of active regions in cycle 23 (i.e., the Hale polarity rule). The data also show a weak (but persistent) pattern of east–west inclination of the solar magnetic field, which is in agreement with the findings of Lo et al. (2010) and Sun et al. (2011).

Figure 1(b) shows that at high latitude (near polar) regions there is a weak but persistent toroidal field oriented in the direction opposite to the field of the active regions. Thus, for example, weak fields in the declining phase of cycle 23 are oriented westward (positive)/eastward (negative) in the northern/southern hemisphere. In combination with the polarity of the polar field in cycle 23, this implies that the weak fields outside of active regions are inclined (pointing towards) up-eastward in the southern hemisphere and down-westward in the northern hemisphere. Such tilt was previously noted by Duvall et al. (1979) and Pevtsov & Latushko (2000). One could also note that at high latitudes, the sign of the toroidal component of the weak field corresponds to the orientation of the active region magnetic fields in the next cycle, cycle 24, as if these
weak fields herald cycle 24 starting at high latitudes well before the first active region of this cycle emerges. Tlatov et al. (2010, 2013) found signs of the extended solar cycle in the orientation of ephemeral active regions several years prior to the beginning of a sunspot cycle, which qualitatively agrees with the high latitude patterns shown in Figure 1.

3.2. Mitigation of Orbital Periodicity and the Reduction of Noise

Toroidal flux shows the effects related to a one year orbital periodicity and the presence of a noise component. To mitigate the effects of orbital periodicity, we employ the following strategy. Since the toroidal magnetic field and the toroidal vector potential should be zero at the poles, we restricted the strategy. Since the toroidal magnetic field and the toroidal vector potential should be zero at the poles, we restricted the computation of $\hat{B}_\phi$ to $\pm 70^\circ$ latitudes. For latitudes between $70^\circ$ and $90^\circ$ the toroidal field was extrapolated linearly from lower latitudes.

Noise in the toroidal flux was reduced by convolving the data (Figure 1) with a 2D Gaussian function:

$$G(\mu, t) = \exp \left( -\frac{\mu^2}{2b^2} - \exp \left( -\frac{t^2}{2a^2} \right) - e^{-2} \left( 3 - \frac{t^2}{2a^2} \right) \right),$$

where $t$ is a discrete time given in units of Carrington rotations (CR). The data are defined at the homogeneous mesh in $\mu$. For the spatial $\mu$ coordinate we employ the filter with the FWHM to be equal to 20 points of mesh, which corresponds to $b \approx 0.04$ in Equation (14). For the time coordinate, the FWHM is equal to 24 CR, i.e., $a = 12$ CR. In addition, we apply reflection conditions at the boundaries (Hathaway 2009). For the spatial component of the filter and the vanishing first derivative at the end-points for the time component of the filter (second term in square brackets in Equation (14). A similar filter is usually applied for sunspot number analysis.

3.3. The Derivation of Vector Potentials

Figure 2 shows the derived symmetric and asymmetric (relative to equator) components of the radial and toroidal magnetic fields. (One may note that the amplitude of solar cycle variations in the symmetric components is smaller compared to the asymmetric component. We postpone the discussion of this until Section 4.) Following Zhang et al. (2010), the window sizes of the temporal and spatial scales involved in Equation (14) are chosen to correspond to the scales of turbulent diffusion (we thanks K. M. Kuzmany for pointing out this idea), which is about $10^{12}$ cm$^2$ s$^{-1}$ (Abramenko et al. 2011).

Next, we interpolate the data to the collocation points of the Legendre polynomials, $\mu_j = \cos \theta_j$, which are taken at zeros of $P_n(\mu)$. The order of the polynomial approximation, $n$, should be sufficiently high. We found that the results do not change significantly for $n \geq 48$, which was the basis for selecting $N = 48$ as upper limit for summation in Equation (15).

The coefficients $a_n^{(\mu_j)}(t)$ in Equation (4) can be found using the Equation (1) and properties of the Legendre polynomials. The matrix equation for $a_n^{(\mu_j)}(t)$ becomes

$$\bar{R}_\phi(t, \mu_j) = - \sum_{n=1}^{N} a_n^{(\mu_j)}(n+1)P_n(\mu_j),$$

$$\bar{B}_\phi(t, \mu_j) = \sum_{n=1}^{N} b_n^{(\mu_j)}(t)P_n^1(\mu_j).$$

By solving the matrix equations at the collocation points, one can find the coefficients for the vector potential components and restore the distribution of magnetic helicity density. The validity of this reconstruction procedure was tested using the output of a mean-field dynamo model of Pipin et al. (2013b).

The main conclusions of this paper are drawn from the analysis based on LOS magnetic field synoptic maps. These maps cover solar cycle 23 and the beginning of cycle 24. In addition, for the rising phase of solar cycle 24 we employed the vector synoptic maps from the Vector Spectromagnetograph (VSM) on board the Synoptic Optical Long-term Investigations of the Sun (SOLIS) system (Gosain et al. 2013). The maps cover 20 consecutive solar rotations starting from CR2109; this data set covers the period from 2011 March to 2012 December. The time-latitude distribution of the radial and toroidal components of the vector magnetic field are shown in Figures 3(a) and (b). Figure 3(c) shows the average latitudinal profiles of two components of the axisymmetric large-scale magnetic field. The profiles were obtained by averaging over Carrington rotations 2109–2128 and applying the Gaussian filter with an FWHM equal to 30 pixels in sine of the latitude.

While there is no overlap between the MDI and VSM data sets to allow for a more direct comparison, we note that the
distributions of the radial and toroidal fields from the two data sets exhibit somewhat similar behavior. For example, similar to Figure 1, the mean toroidal field derived from the vector data is mostly negative in the northern hemisphere, and it is mostly positive in the southern hemisphere. The polarity of the radial field in the main peak (negative in the northern hemisphere and positive in the southern hemisphere) corresponds to the leading and following polarity fields of the dissipating active regions. MDI data (Figure 1(a)) show similar patterns in some parts of cycles 23 and 24 (e.g., see the “tip” of the cycle 24 “butterfly” in the northern hemisphere). These general similarities provide some level of confidence for our method of deriving the radial and toroidal components of the large-scale magnetic field from MDI synoptic maps of LOS flux.

4. RESULTS

Figures 4(a) and (b) present the evolution of the power spectra \( \sqrt{b_n(t)}^2 \) and \( \sqrt{a_n(t)}^2 \). The coefficients \( b_n(t) \) and \( a_n(t) \) decay rapidly with the increasing number of modes. Furthermore, we found that the asymmetric component of the magnetic field exhibits a faster decay, which we interpret as this component being more global in its nature as compared with the symmetric component. In hindsight, we note that one can draw a similar conclusion using the results of the Stenflo & Guedel (1988) study.

Figure 5 shows the reconstructed components of the vector potential which have been computed for two cases: (1) taking into account odd and even modes of the spectral harmonics and (2) including only the even modes (associated with the asymmetric part of the global magnetic field). Here, we used the first 11 modes in Equations (4) and (7). Restricting the expansion to 11 modes is well justified by a rapid decay of \( a_n(t) \) for higher modes (Figure 4(b)).

The pattern of the symmetric (relative to the equator) component of \( \bar{A}_\varphi \) is similar in appearance to the reconstruction made by Brandenburg et al. (2003) based on the Stenflo & Guedel (1988) data. Note that in our case, we do not restrict the study to a particular symmetry of the global field about the equator. We find that the poloidal component of the vector potential (Figure 5(a)) exhibits a break in the equatorial symmetry (see the change in sign of \( \bar{A}_\varphi \) around the year 2004). The asymmetry between the northern and southern hemispheres is also present at high latitudes prior to the year 1998. On the other hand, the pattern of the \( \bar{A}_\varphi \) (Figure 5(b)) exhibits no significant changes over solar cycle 23.

Figure 6 shows the components of the large-scale magnetic field. Here, again, we use the first 11 modes in Equations (5) and (6). The obtained evolution of \( \bar{B}_{r} \) is in agreement with the results of Ulrich & Tran (2013). The pattern of \( \bar{B}_{\varphi} \) in Figure 5(b) (even modes) closely resembles Figure 2(a). We also find that the phase relation \( \bar{B}_r, \bar{B}_\varphi < 0 \) holds in the equatorial region (see also Stix 1976; Yoshimura 1976). Brandenburg et al. (2003) argued that this relation is tightly related to the sign of the magnetic helicity density. Figure 7 supports this conjecture for the asymmetric (relative to solar equator) part of the magnetic helicity density.

Now that we have the radial and toroidal components of the magnetic field and vector potential, we are able to compute the corresponding contributions to magnetic helicity density \( \bar{A} \cdot \bar{B} = \bar{A}_r \bar{B}_r + \bar{A}_\varphi \bar{B}_\varphi \). The distribution of magnetic helicity density in cycle 23 (Figure 7(a)) shows a strong hemispheric asymmetry, with positive/negative helicity in the northern/ southern hemispheres. This hemispheric asymmetry is opposite in sign to the hemispheric helicity rule found in active regions. There is no contradiction here. In the dynamo theory, the active regions are thought to represent the “small-scale” magnetic fields (see Brandenburg et al. 2003 for further discussion), while in this paper we derive the helicity of large-scale fields (in mean-field dynamo terminology). The fact that the large-scale helicity derived by us has a sign opposite to that of the helicity of active regions is in agreement with the notion that the dynamo produces the helicity of two opposite signs segregated by their spatial scales.

The patterns of the toroidal (\( \bar{A}_\varphi \bar{B}_\varphi \)) and radial (\( \bar{A}_r \bar{B}_r \)) components of magnetic helicity density are quite different (Figure 7(b)). Comparing Figure 7 and Figure 8(a), we conclude that the total helicity in the polar regions is defined by the \( A_r B_r \) contribution. In equatorial regions, both \( A_r B_r \) and \( A_\varphi B_\varphi \) have
Modern measurements of solar vector magnetic fields are normally restricted to a single layer in the solar atmosphere (typically, the photosphere). These observations are insufficient to derive the true magnetic helicity. Instead, various proxies of helicity are used. Figure 8(c) shows the evolution of one of these helicity proxies, the radial component of the current helicity density, $\tilde{B}_r (\nabla \times \tilde{B})_r$. In comparison with true magnetic helicity density (Figure 8(a)), $\tilde{B}_r (\nabla \times \tilde{B})_r$, shows a more complex pattern. While, on average, the current helicity density follows the same
hemispheric sign asymmetry as the magnetic helicity density, during the maximum of solar cycle 23 the $B_r (\nabla \times \vec{B})$, exhibits a distinct “zebra” pattern with opposite helicity bands present in both hemispheres. Whether these bands persist through the minimum of cycle 23 is not clear as our data are insufficient to make a definite conclusion about this. Similar “zebra” patterns in $B_t (\nabla \times \vec{B})$, were found in the past (e.g., Pevtsov & Latushko 2000; Pevtsov & Balasubramaniam 2003; Gosain et al. 2013). Pevtsov & Balasubramaniam (2003) speculated about a possible relation between the latitudinal bands of current helicity density and the subphotospheric pattern of torsional oscillations.

Figure 9 compares latitudinal profiles of $\vec{A} \cdot \vec{B}$ and $B_t (\nabla \times \vec{B})$, computed using data from SOHO/MDI (about the year 2011) and SOLIS/VSM (Figure 3(c), in the year 2012). The 90% confidence interval was computed in the same manner as for data shown in Figures 1(c) and 3(c). The residual contribution from modes higher than $n = 11$ (Equations (4)–(7)) is an order of magnitude smaller than the contribution of the first 11 modes. While both the magnetic helicity density and $B_t (\nabla \times \vec{B})$, exhibit the hemispheric helicity rule in both data sets, there are some differences. For example, the latitudinal profiles of helicity can differ because of evolutionary changes (MDI and VSM data included in this comparison correspond to periods, which are about one year apart). Other sources of difference could include a difference in sensitivity to the magnetic field and the noise levels, as well as the treatment of polar fields.

### 5. DISCUSSION AND CONCLUSIONS

Using synoptic charts from SOHO/MDI, for the first time we reconstruct the magnetic helicity density of the global axisymmetric field of the Sun. In solar cycle 23, the global axisymmetric magnetic field exhibits positive magnetic helicity in the northern hemisphere, and negative in the southern. In general, such reconstructions require a knowledge of the $B_r$ and $B_t$ components of the axisymmetric magnetic field. In the past, vector global magnetic field components were reconstructed via various approaches (Pevtsov & Latushko 2000; Ulrich & Boyden 2005; Lo et al. 2010; Mordvinov et al. 2012). Here we used synoptic charts of the LOS magnetic field corresponding to different longitudinal offsets relative to the central meridian to compute $B_r$ and $B_t$, see Equation (13). The derived $B_r$ agrees well with the $B_r$ provided by the MDI team, which we see as an indirect validation of our method.

Based on the analysis of dynamo equations, Brandenburg et al. (2003) suggested that the magnetic helicity of the global magnetic field in solar cycle 23 should be positive/negative in the northern/southern hemisphere. Pipin et al. (2013b) analyzed the distributions of magnetic helicity for large- and small-scale magnetic fields in the axisymmetric mean-field dynamo, taking into account the conservation of the total magnetic helicity in the dynamo processes. They concluded (see their Figures 2(d), (e), and 5) that the magnetic helicity density of the large-scale field should have a positive sign in the northern hemisphere and a negative sign in the southern hemisphere during most of the magnetic cycle. Our present results provide observational support to these early theoretical predictions. We find that during most of cycle 23, global magnetic fields exhibited a persistent pattern of positive/negative helicity in the northern/southern hemispheres.

Regarding the helicity of the active region magnetic fields (small scale in the framework of this discussion), the hemispheric helicity rule is negative/positive in the northern/southern hemispheres (Seehafer 1990; Pevtsov et al. ...)
Two results support the notion that the solar dynamo creates helicity of two opposite signs, as suggested in early papers. However, the helicity of both signs seem to cross the solar photosphere. We further found that the hemispheric helicity rule for global magnetic fields exhibits sign-reversals in the early and late phases of cycle 23. If the helicities of small- and large-scale fields are tied together, then this should imply a need for similar reversals in the hemispheric helicity rule for active regions. A summary of some researchers claimed to have observed reversals in the helicities of active region magnetic fields. We further found that the helicity of both signs seem to cross the solar photosphere. We further found that the hemispheric helicity rule for global magnetic fields exhibits sign-reversals in the early and late phases of cycle 23. If the helicities of small- and large-scale fields are tied together, then this should imply a need for similar reversals in the hemispheric helicity rule for active regions. Alas, while some researchers claimed to have observed reversals in the helicities of active region magnetic fields, other researchers were not able to find them (Pevtsov et al. 2001, 2008; Gosain et al. 2013). Clearly, this question about possible reversals in the helicities of active region magnetic fields needs to be re-examined. Although the predictions of the model by Pipin et al. (2013b) agree qualitatively with the results reported in our paper, there are some differences related to the shape of the helicity density patterns. For example, our present results suggest that a magnetic helicity density pattern of the same sign can extend from the equator to the poles, which is not seen in the model. Similarly, the pattern of $\mathbf{A} \cdot \mathbf{B}$ of reverse sign penetrates to equatorial regions during the minima of the cycle around the years 1997 and 2009. If the reversals of the hemispheric helicity rule are real, then this will pose a challenge for some of the proposed mechanisms of helicity generation (e.g., helicity generation by differential rotation; Berger & Ruzmaikin 2000).

As an alternative explanation, our results could be interpreted in the framework of the helicity of the axisymmetric and non-axisymmetric parts of global magnetic fields. In that model, the axisymmetric component of the helicity (derived in this article) follows the hemispheric helicity rule opposite in sign to the non-axisymmetric component (associated with active regions). Such a possibility was raised by Zhang (2006), who used Berger & Ruzmaikin (2000) data to show that the helicity flux of non-axisymmetric modes was opposite in sign to the helicity of the axisymmetric ($m = 0$) mode.

One may also question the importance of vector magnetic field measurements for studies of global helicity when the LOS data seem to provide reasonable results. Here, we presented the first ever derivations of the magnetic helicity density of the global field based on vector synoptic maps. While we see similarities in the distribution of the global helicity derived from LOS and vector data, there are also some differences. For example, synoptic maps of the toroidal field derived from LOS data show a more or less uniform distribution of the magnetic field polarity of one sign suggested by the Hale polarity law. In addition to that pattern, the vector field maps show a mix of two different polarities: one is more concentrated and the other is somewhat diffuse (Figure 3(b)). The diffuse component (of toroidal field) seems to correspond to a trailing polarity field. Such a component of the large-scale field is not present in the maps of the toroidal flux derived from the LOS magnetic fields. The vector field data are limited to cycle 24, and thus are insufficient to conclude whether or not there are any changes in this diffuse component of the toroidal field with solar cycle. This and other differences between derivations based on LOS or vector field data require further investigation.

Our findings indicate that the helicity of the large-scale magnetic fields is imbalanced between the northern and the southern hemispheres in different phases of the solar cycle. However, when taken over the entire cycle, the positive and negative helicity of the large-scale magnetic field is well-balanced. Indirectly, this is in agreement with Georgoulis et al. (2009), who found that helicity injection through the solar photosphere associated with active region magnetic fields is well balanced over solar cycle 23. On the other hand, Yang & Zhang (2012) reported significant imbalance between the helicity fluxes of the northern and southern hemispheres. Our findings (Figure 8(c)) allow us to reconcile the conclusions of Georgoulis et al. (2009) and Yang & Zhang (2012).

Due to the limitations of the existing data sets, observational studies of helicity often refer to proxies of the current helicity density. It is usually assumed that these proxies represent magnetic helicity sufficiently well. Contrary to that, we find that while the general tendencies are similar in the magnetic and current helicity densities, there are differences, for example, in small-scale patterns, which may be present in one helicity proxy but are absent in the other. For example, the proxy of current helicity, $B_x (\nabla \times \mathbf{B})_x$, exhibits a distinct “zebra” pattern, but no such pattern is present in the distribution of the magnetic helicity. Previously, Pevtsov & Latushko (2000) and Gosain et al. (2013) reported a similar pattern in the current helicity density of large-scale magnetic fields. The pattern could also be expected from the spatial structure of the dynamo wave of the large-scale magnetic field components $\mathbf{B}_x$ and $\mathbf{B}_y$, which are illustrated in Figure 5(a). We note that in the equatorial regions, the inequality $\mathbf{B}_x \cdot \mathbf{B}_y < 0$ holds for most of the sunspot cycle (see Figure 5(a)). We also found that the modes $b^{(3)}$ and $b^{(2)}$ dominate, which means that $\mathbf{B}_x (\nabla \times \mathbf{B})_y \sim b^{(2)} b^{(3)} P_2 P_3$, where the sign of the $b^{(3)}$ defines the hemispheric sign rule and the product $P_2 P_3$ defines the zebra pattern as illustrated in Figure 9.
Figures 8(c) and 9. Thus, the results shown in Figure 8(c) are expected for any dynamo model that qualitatively reproduces Figure 5(a).

Finally, keeping in mind the approximations which were used in the reconstruction of the components of the global magnetic field of the Sun, our results should be considered as preliminary. Further development in this direction is likely to shed more light on the role of magnetic helicity in global solar and astrophysical dynamos.

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APPENDIX

The components of the large-scale axisymmetric field and the components of its vector potential are related with the following equations:

\[ \tilde{B}_r = - \frac{1}{r} \frac{\partial (\sin \theta \tilde{A}_\phi)}{\partial \mu}, \quad (A1) \]

\[ \tilde{B}_\phi = \frac{\sin \theta}{r} \frac{\partial \tilde{A}_r}{\partial \mu}, \quad (A2) \]

where \( \mu = \cos \theta \). Then, using integration by part, we obtain

\[
\begin{align*}
\int_{-1}^{1} \tilde{A}_\phi \tilde{B}_\phi d\mu &= \int_{-1}^{1} \frac{\sin \theta \tilde{A}_\phi}{r} \frac{\partial \tilde{A}_r}{\partial \mu} d\mu \\
&= \sin \theta \frac{\tilde{A}_\phi}{r} \bigg|_{-1}^{1} - \int_{-1}^{1} \frac{\tilde{A}_r}{r} \frac{\partial (\sin \theta \tilde{A}_\phi)}{\partial \mu} d\mu \\
&= \int_{-1}^{1} \frac{\tilde{A}_r}{r} \tilde{B}_r d\mu. \quad (A3)
\end{align*}
\]