The Generalized Polarizabilities of the Nucleon: Status Report

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Abstract. The Generalized Polarizabilities of the nucleon are fundamental observables describing the complex non-perturbative physics of the nucleon at low energies. Measuring all the six polarizabilities requires challenging double polarization experiments, while cross-section measurements give access to two linear combinations of them. The status of the present knowledge on the Generalized Polarizabilities is reviewed with focus on the experimental aspects.

1. Introduction
The Generalized Polarizabilities (GPs) are six fundamental quantities characterizing the electromagnetic structure of the nucleon at low energies. GPs are measured via Virtual Compton Scattering (VCS): a virtual photon scatters on the nucleon and a real photon is emitted in the final state: $\gamma^* N \rightarrow N \gamma$.

Experimentally, this means measuring the photon-electroproduction reaction $ep \rightarrow ep\gamma$. GPs were first introduced in the context of nuclear physics [1] and later the idea was applied to nucleons [2].

Experiments have been performed at Bates [3], MAMI [4, 5] and JLab [6] for measuring the reaction cross-section: such a measurement permits to extract two combinations of GPs, while only double-polarization experiments are able to disentangle all of them.

Since now a double-polarization experiment was performed only at MAMI [7].

2. Virtual Compton scattering at low energies
Real Compton scattering (RCS, $\gamma N \rightarrow N \gamma$) gives access to the electric and magnetic polarizabilities, which are a measure of how the nucleon internal structure is deformed when an external electromagnetic field is applied.

It turns out that the nucleon is a very stiff object, since the polarizabilities are small: $\alpha_E = (12.1 \pm 0.3_{stat} \pm 0.5_{syst}) \times 10^4$ fm$^3$, $\beta_M = (1.6 \pm 0.4_{stat} \pm 0.6_{syst}) \times 10^3$ fm$^3$ [8]. As reference, the electric polarizability of an atom is approximately proportional to its volume, while $\alpha_E$ is much smaller than the nucleon’s volume. This is a clear sign that the internal constituents of the nucleon are strongly bound.

GPs are a generalization of the polarizability concept. In VCS, the momentum of the virtual photon can be varied and the polarizabilities $\alpha_E$, $\beta_M$ become momentum-dependent quantities: $\alpha_E(Q^2)$, $\beta_M(Q^2)$. The VCS reaction at low energies is fully parameterized by six GPs, two of them being $\alpha_E(Q^2)$, $\beta_M(Q^2)$. The additional four GPs are called “Spin GPs” because a
spin-flip transition is involved. GPs can provide information about the spatial distribution of polarization inside the nucleon volume, in analogy to the information about charge and magnetization provided by the (Fourier-transformed) form factors $G_E(Q^2)$ and $G_M(Q^2)$.

VCS on the proton is performed measuring the exclusive $ep \rightarrow ep\gamma$ photon-electroproduction reaction. The reaction is fully identified by five independent kinematical variables. The photon-electroproduction amplitude is composed by the coherent sum of three contributions: the Bethe-Heitler (BH), Born (B), and non-Born (nB) processes. BH and B (BH+B) can be fully calculated within QED, given the knowledge of the proton for factors $G_E^p(Q^2)$ and $G_M^p(Q^2)$, while nB is parameterized with the GPs and it represents the unknown part of the amplitude.

3. Overview of the theoretical models

The GPs were calculated within different theoretical models: constituent quark model [9], linear sigma model [10], effective Lagrangian model [11], heavy barion chiral perturbation theory (HBχPT) [12], covariant chiral perturbation theory (χPT) and dispersion relations (DR) [13].

The DR model has the advantage to be applicable also beyond the pion production threshold, while chiral perturbation theories are limited to low momenta but represent a more fundamental model, closer to QCD and its symmetries.

In general, the theoretical models predict different GPs as a function of $Q^2$. In the following, we will discuss chiral and DR models in more details, since they represent the main contribution to the field.

The first chiral perturbation theory calculation performed is [12], where all the GPs were calculated at $O(p^3)$ in HBχPT. Three GPs were also calculated at $O(p^4)$ in [14] without adding
Figure 2. Measured component of the beam-recoil polarization of the proton. The data shows a deviation from the BH+B calculation due to the GPs. A new GPs combination $P^\perp_{LT}$ can be extracted from the data. The first (second) quoted errors are statistical (systematical).

any new low-energy constant. The result of [14] indicates that the full convergence is still not achieved at this order. A calculation at $O(p^4)$ in fully covariant $\chi$PT is also available [15]. In this case, the GPs are closer to the DR result, but the applicability remains confined at relatively low momenta and likely the addition of more degrees of freedom (e.g. the $\Delta$-resonance) is needed.

The DR model [13] predicts the four spin GPs and the electric and magnetic GPs are parameterized with a dipole shape (in analogy to the form factors) which introduces two parameters $\Lambda_\alpha$ and $\Lambda_\beta$ which have to be adjusted to the data.

4. Experimental activity
VCS was measured at Bates, MAMI and JLab with a similar technique: coincidence detection of the recoil proton and scattered electron, while the photon is reconstructed by missing mass. The $ep \rightarrow ep\gamma$ reaction cross section is measured and the extraction of the GPs information can be done using two different techniques. The first method is based on a low-energy expansion (LEX) of the amplitudes in the outgoing photon momentum $q_t$, which permits to write the cross-section as a BH+B part and a nB part linear in $q_t$. This method can be applied if higher-order contributions in the LEX are not relevant. Once the cross-section is measured, the BH+B part is subtracted and the remaining nB term will contain two linear combinations of the GPs: $P_{LL} - (1/\epsilon)P_{TT}$ and $P_{LT}$. $P_{LL}, P_{TT}$ and $P_{LT}$ are $Q^2$-dependent structure functions while $\epsilon$ is the virtual photon polarization (see ??). The two combinations can be separated with a Rosenbluth-like linear fit.

The second method is based on the DR model where no LEX expansion is needed and the data can be directly compared to the model fixing the two constants $\Lambda_\alpha$ and $\Lambda_\beta$. Both techniques have been applied in the analysis of VCS data. The world data for the generalized electric and magnetic GPs are summarized in fig. 1. The LEX approach measures combinations of GPs, therefore the contribution of the spin-GPs has to be subtracted in a model-dependent way. In the figure, $\alpha_E(Q^2), \beta_M(Q^2)$ are obtained subtracting the DR prediction for the spin-GPs.

From the figure, it is clear that the DR model is not able to capture the behavior of all the data points in the $Q^2$ range and the data might suggest a non-trivial fall-off of $\alpha_E(Q^2)$.
Table 1. Results on the structure functions from VCS cross section and double-polarization measurements.

in the 0.3 GeV$^2$ region. The turning point suggested by the data in $\beta_M(Q^2)$ at low $Q^2$ is another interesting feature to investigate, since it is predicted by many nucleon models. The physical interpretation of the turning point is a cancellation effect between paramagnetic (due to resonances) and diamagnetic (due to the pion cloud) effects. At MAMI, experiments at three additional kinematical points ($Q^2 = 0.1, 0.2, 0.5$ GeV$^2$) are planned, in order to clarify the evolution of these observables.

For disentangling all the GPs, a double polarization experiment is required together with the unpolarized cross-section measurement. The first experiment in this regard was performed at MAMI [7]. A 70% polarized electron beam was used, together with a proton polarimeter. The polarimeter is able to measure the three polarization components of the recoiling proton after the spin precession in the spectrometer magnetic field. Due to the limited statistics and the small effect of the GPs, it was possible to extract a new GPs combination ($P_{LT}^\perp$) from one proton polarization component. The result is showed in fig. 3: a clear departure from the BH+B calculation is visible, which is the sign of the GPs contribution. For the analysis of the double polarization data, a LEX approach was used.

All the available experimental results are summarized in tab. 4

5. Future plans
In the last decade, different experiments performed VCS measurements for the extraction of the GPs and also the theoretical activity was intense. A picture of the GPs is emerging, but still work should be done. The data suggest a non-trivial behavior as a function of the $Q^2$, which if confirmed will be a challenge for the nucleon models and an interesting new insight in the proton structure at the same time. The long term goal is the disentanglement of all the six GPs. and this program requires very challenging double polarization experiments. At MAMI, new unpolarized experiments are already planned for measuring the $Q^2$ evolution of $\alpha_E(Q^2)$ and $\beta_M(Q^2)$.

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