EXOTIC BOUND STATE PRODUCTION
AT HADRON COLLIDERS

J.H. Kühn and E. Mirkes

Institut für Theoretische Teilchenphysik
Universität Karlsruhe
Kaiserstr. 12, Postfach 6980
7500 Karlsruhe 1, Germany

Abstract

Hadronic production of nonrelativistic boundstates of $b'$ or isosinglet quarks $D$ with suppressed weak decays is investigated for LHC and SSC energies. QCD corrections to production and decay rates are incorporated. Large rates for final states from $\eta_\nu \rightarrow HZ$ are predicted.

1Supported by BMFT Contract 055KA94P
The theoretical and experimental aspects of hadronic quarkonium production have attracted a great deal of interest after the original discovery of \(J/\Psi\) and \(\Upsilon\). Gluon fusion seemingly dominates the cross section with a small admixture of quark-antiquark annihilation in \(p\bar{p}\) and \(\pi^-p\) collisions at lower energies. Given sufficiently high center of mass energy and luminosity, the same parton processes could also lead to toponium production at a future hadron collider. The production rate of \(1^-\) bound states through the reaction \(g + g \rightarrow 1^- + g\) is tiny. The cross section for gluon-gluon fusion into the \(1^S0\) state \(\eta_t\), however, would be sufficiently large to produce a large number of resonances. The experimental signal that might conceivably be accessible under realistic assumptions is the \(\gamma\gamma\) decay mode of \(\eta_t\). However, this mode is severely reduced by the tiny branching ratio of order \(10^{-4}\) down to \(10^{-5}\) — a consequence of the large single quark decay rate of \(\eta_t\). Detailed investigations which include higher order corrections for the production rate and a realistic QCD potential show that the window for the discovery of \(\eta_t\) is extremely narrow. Fairly optimistic assumptions have to be employed to access quark masses even in the range of 100 to 110 GeV [2, 3].

However, the situation changes drastically if the single quark decay mode of the \(1^S0\) bound state is absent or at least strongly suppressed, as is the case for a \(b'\) with suppressed couplings to the lighter \(u, c\), and \(t\) quarks, or for isosinglet quarks denoted in the following by \(D\). The branching ratio for decay modes of these bound states with truly characteristic final states such as \(\gamma\gamma\), \(\gamma Z\), \(ZZ\), \(WW\), \(HZ\) are no longer drastically suppressed and may even become dominant, as is the case for the \(HZ\) mode in the high mass region. These would be easily accessible in future experiments at LHC or SSC, for quark masses even up to 1000 GeV. Predictions for these reactions in Born approximation have been derived in references [4, 5, 6].

In a recent paper [2, 3], QCD corrections were evaluated for the production of nonrelativistic \(1^S0\) bound states in hadronic collisions. These corrections are positive and fairly large, justifying the renewed investigation of exotic bound state production. The corrections can be also applied to the production of bound states composed of a \(b'\) with \(I_3 = -1/2\) and \(e_Q = -1/3\) or of a \(D\) quark with \(I_3 = 0\) and \(e_Q = -1/3\).

In the following we specify the general structure of the cross section for \(\eta_{b',D}\) production in hadronic collisions

\[
h_1(P_1) + h_2(P_2) \rightarrow \eta_{b',D} + X
\]

in the framework of perturbative QCD. In (1) \(h_1\) and \(h_2\) denotes unpolarized hadrons with momenta \(P_i\). The hadronic cross section in NLO is thus given by

\[
\sigma^H(S) = \int dx_1 dx_2 f_{a_1}^a(x_1, Q_F^2) f_{b_2}^b(x_2, Q_F^2) \delta^{ab}(s = x_1 x_2 S, \alpha_s(\mu^2), \mu^2, Q_F^2)
\]
where one sums over \( a, b = q, \bar{q}, g \). \( f_a^h(x, Q_F^2) \) is the probability density to find parton \( a \) with fraction \( x \) in hadron \( h \) if it is probed at scale \( Q_F^2 \) and \( \hat{\sigma}^{ab} \) denotes the parton cross section for the process

\[
a(x_1 P_1) + b(x_2 P_2) \rightarrow \eta_{b'} D + X
\] (3)

from which collinear initial state singularities have been factorized out at a scale \( Q_F^2 \) and implicitly included in the scale-dependent parton densities \( f_a^h(x, Q_F^2) \).

The partonic cross sections \( \hat{\sigma}^{ab} \) in NLO for the \( gg, qg \) and \( q\bar{q} \) initiated reactions are given in [3]. They are proportional to the square of the bound state wave function at the origin \( R(0) \) and hence dependent on the QCD potential. For the evaluation we employ the two-loop QCD potential \( V_J \). The numerical results depend critically on the value of the QCD parameter \( \Lambda^{(4)}_{\overline{MS}} \). The results for the dimensionless quantity \( |R(0)|^2/M_\eta^3 \) are displayed in fig. 1 for different values of \( \Lambda^{(4)}_{\overline{MS}} \) (200, 300 and 500 MeV) to indicate the characteristic uncertainty in the prediction. These \( \Lambda \) values correspond to \( \alpha^{(5)}_{s\overline{MS}} = 0.109, 0.1165 \) and 0.127 respectively (and to \( \Lambda^{(5)}_{\overline{MS}} = 0.132, 0.207 \) and 0.366 MeV). For the parton distribution functions we employ MT set B1 with \( \Lambda^{(4)}_{\overline{MS}} = 194 \text{MeV} \) and work in the DIS factorization scheme. The renormalization and factorization scales are fixed at \( M = M_\eta \). We use the \( \overline{MS} \) definition of \( \alpha_s \) at two-loop accuracy with six flavours

\[
\alpha_{s\overline{MS}}(Q) = \frac{12\pi}{(33 - 2n_f) \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{6(153 - 19n_f) \ln \ln(Q^2/\Lambda^2)}{(33 - 2n_f)^2 \ln(Q^2/\Lambda^2)} \right]
\] (4)

and \( \Lambda^{(6)}_{\overline{MS}} = 58 \text{MeV} \) to be consistent with the \( \Lambda^{(4)}_{\overline{MS}} \) value from the parton distributions.

The \( K \)-factor and the production cross section are shown in figs. 2 and 3a respectively as functions of the bound state mass for three different \( \Lambda \) values in the potential. The uncertainty from the choice of the parton distribution functions is shown in fig. 3b. The additional uncertainty from the choice of the renormalization and factorization scales is estimated to be less than 10% [3].

To obtain predictions for the interesting reactions these production cross sections have to be multiplied by the various possible branching ratios. It is convenient to normalize the decay rates relative to the two photon decay rate \( \Gamma_{\gamma\gamma} \) with

\[
\Gamma_{\gamma\gamma} = 12e_\gamma^4 \alpha^2 |R(0)|^2 / M^2
\] (5)

With \( R_{ab} := \Gamma_{ab}/\Gamma_{\gamma\gamma} \ (ab = \gamma Z, HZ, ZZ, WW, gg, t\bar{t}) \) one finds
\[ R_{\gamma Z} = 2 \frac{v_Q^2}{e_Q y^2} \left( \frac{2p}{M} \right) \]  
\[ R_{HZ} = \frac{a_Q^2}{e_Q} \frac{1}{\rho_Z^2 y^2} \left( \frac{2p}{M} \right)^3 \]  
\[ R_{ZZ} = \frac{(v_Q^2 + a_Q^2)^2}{e_Q^4 y^4} \frac{1}{(1 - 2\rho_Z)^2} \left( \frac{2p}{M} \right)^3 \]  
\[ R_{WW} = \frac{1}{2e_Q (4\sin^2 \theta_W)^2} \frac{1}{(1 - \rho)^2} \left( \frac{2p}{M} \right)^3 \]  
\[ R_{gg} = 2 \frac{a_\alpha^2(M)}{9} \frac{e_Q^4 \alpha^2}{e_Q^4 y^4} \]  
\[ R_{t\bar{t}} = 6 \frac{a_Q^2 a_t^2 m_t^2 M^2}{e_Q^4 y^4} \frac{2p}{M^2} \]  

where
\[ \rho_i \equiv M_i^2/M^2, \quad \rho \equiv 2 \left( 1/4 - \rho_Q + \rho_W \right) \]  
\[ v_Q \equiv 2 I_{3Q} - 4 e_Q \sin^2 \theta_W, \quad a_Q = 2 I_{3Q} \]  \[ y \equiv 2 \sin 2\theta_W \]  

QCD corrections to the decay rates will be discussed below. In eq. (12) \( I_{3Q} \) denotes the third component of the weak isospin of the \( Q^- \)-type quark and \( \theta_W \) is the Weinberg angle. \( p \) denotes the three-momentum of the final state particles.

The single quark decay mode which dominates for \( \eta_t \) proceeds through the decay of the heavy quark into \( b + W \). For a heavy sequential quark \( b' \) of isospin \( I_3 = -1/2 \) the analog decay \( b' \rightarrow t' + W \) is presumably strongly suppressed by a small mixing angle \( \theta' \) if the mass of the isospin partner \( t' \) is larger than \( m_{b'} \). Once \( \theta'^2 < 10^{-3} \) the annihilation decays dominate and single quark decays can be ignored for most practical purposes. Similar considerations apply to decays of isosinglet quarks \( D \) into \( b + Z, t + W \) or \( c + W \) which are also inhibited by small mixing angles. For the subsequent discussion these single quark modes will be ignored. The dominant decays are thus all proportional to the wave function squared, the branching ratios \( \Gamma_{ab} / \sum \Gamma_{ab} \) are independent of the wave function, and as a result can be predicted unambiguously.

The WW mode needs further specification: The decay of \( \eta_{b'} \) into \( WW \) proceeds through the virtual isospin partner \( t' \) and hence depends on the mass of \( t' \). Present limits on the deviation of the \( \rho \)-parameter from one as deduced from electroweak precision measurements limit the mass splitting \( \Delta m \) between \( m_{t'} \) and \( m_{b'} \) in a drastic way and \( \Delta m < 200 \) GeV can be considered a generous upper bound. For simplicity we shall assume equal \( b' \) and \( t' \) masses \( \rho_Q = 1/4 \) in eqs. (12) in the subsequent numerical evaluation.
For bound states $\eta_D$ from isosinglet quarks the WW decay may in principle proceed through virtual quarks of charge $2/3$. However, the $D$-$t$-$W$-coupling is suppressed by small mixing angles and the WW mode can therefore safely be neglected. The $HZ$ mode is absent as a consequence of $a_D = 0$.

QCD corrections to annihilation decays are at present available for hadronic $(gg-)$, for $\gamma\gamma-$ and for the $tt-$decay modes.

\[
\Gamma_{\gamma\gamma} = \Gamma_{\gamma\gamma}^{Born} \left[ 1 + \frac{\alpha_s(\mu)}{\pi} \left( \frac{\pi^2}{3} - \frac{20}{3} \right) \right]
\]

\[
\Gamma_{had} = \Gamma_{had}^{Born} \left[ 1 + \frac{\alpha_s(\mu)}{\pi} \left( (11 - \frac{2}{3} n_f) \ln \frac{\mu}{M} + \frac{159}{6} - \frac{31}{9} n_f \right) \right]
\]

\[
\Gamma_{tt} = \Gamma_{tt}^{Born} \left( 1 - 4 \frac{\alpha_s(\mu)}{\pi} \right) \left[ 1 + 4 \frac{\alpha_s(\mu)}{\pi} \frac{1}{\beta_t} \left[ A(\beta_t) + P(\beta_t) \ln \frac{1 + \beta_t}{1 - \beta_t} + Q(\beta_t) \right] \right]
\]

where $n_f = 6$ denotes the number of light flavours (including top). In the last equation of (13) the functions $A(\beta_t)$, $P(\beta_t)$ and $Q(\beta_t)$ are given by

\[
A(\beta_t) = (1 + \beta_t^2) \left[ \frac{\pi^2}{6} + \ln \frac{1 + \beta_t}{1 - \beta_t} \ln \frac{1 + \beta_t}{2} + 2 \text{Li}_2 \left( \frac{1 - \beta_t}{1 + \beta_t} \right) + 2 \text{Li}_2 \left( \frac{1 + \beta_t}{2} \right) - 2 \text{Li}_2 \left( \frac{1 - \beta_t}{2} \right) - 4 \text{Li}_2(\beta_t) + \text{Li}_2(\beta_t^2) \right] + 3 \beta_t \ln \frac{1 - \beta_t^2}{4 \beta_t} - \beta_t \ln \beta_t.
\]

\[
P(\beta_t) = \frac{19}{16} + \frac{2}{16} \beta_t^2 + \frac{3}{16} \beta_t^4
\]

\[
Q(\beta_t) = \frac{21}{8} \beta_t - \frac{3}{8} \beta_t^3
\]

where $\beta_t = \sqrt{1 - 4m_t^2/M^2}$. The first factor in the corrections for $tt$ has its origin in the $\eta_Q$ vertex correction, the second factor is given in [3]. Finally, the scale $\mu$ in $\alpha_s$ is adopted to M in our numerical calculation. The QCD corrections to the $\gamma\gamma$ decay rate are negative at the level of about -10% whereas the corrections to the hadronic decay increases the decay rate by about +35%. The QCD corrections to $\Gamma_{tt}$ depend on the mass of the boundstate and vary from +40% for $M_\eta = 300$ GeV to -20% for $M_\eta = 2000$ GeV.

Since $\gamma\gamma$, $\gamma Z$, $ZZ$ and $WW$ can be identified in the limit $M_Z, M_W << 2m_{W,D}$ after appropriate adjustment of the gauge coupling, the QCD corrections can be assumed to cancel to a large extent in the ratios listed in eqs. (6, 8, 9). Corrections for the HZ mode are missing and we shall simply adopt eq. (7) for the present purpose. The normalized decay rates $R_{ab}$ are shown in figs. 4a and b for $\eta_V$ and $\eta_D$ respectively, the resulting branching ratios $\Gamma_{ab}/\sum \Gamma_{ab}$ are displayed in figs. 4c and d. The pattern is significantly different for the two cases:
The branching ratio of $\eta_{b'}$ into $HZ$ is sizable and dominant for a large mass range. Also the $WW$ mode is fairly important, amounting to about 1% of the hadronic mode, which is presumably difficult to detect experimentally. The $\gamma\gamma$, $ZZ$ and $\gamma Z$ modes are all relatively small with their ratio basically governed by their relative couplings and the statistical factor 1/2 for $\gamma\gamma$ and $ZZ$.

$$\Gamma_{WW} : \Gamma_{ZZ} : \Gamma_{\gamma Z} : \Gamma_{\gamma\gamma} \approx 1 : 0.462 : 0.064 : 0.021$$

This expectation for asymptotic quark mass values is well met above $m_{b'} > 150$ GeV. One thus anticipates a large production rate for $HZ$ and even $WW$ will become fairly important.

This is in contrast to the situation for $\eta_D$ decays: $HZ$ and $WW$ are absent, and the asymptotic relation reads

$$\Gamma_{\gamma\gamma} : \Gamma_{\gamma Z} : \Gamma_{ZZ} \approx 1 : 0.60 : 0.091$$

In this case $\gamma\gamma$, $\gamma Z$ and $ZZ$ constitute the dominant visible modes with a branching ratio of about $10^{-4}$.

The resulting product of production cross sections times branching ratio for SSC and LHC energies are displayed in figs. 5a and b for several interesting $\eta_{b'}$ channels, indicating large rates in particular for $HZ$. The production and decay of $\eta_D$ is shown in figs. 5c and d indicating that $\eta_D$ could be observed in the $\gamma\gamma$, $\gamma Z$ and $ZZ$ modes with characteristic signals.

To summarize:

Production and decay rates for bound states of exotic quarks can be predicted unambiguously. The uncertainty from the QCD potential, $\alpha_s$ and the structure functions appears to be well under control. The rates for $pp \to \eta_{b'}(\to HZ) + X$ are particularly promising as a consequence of the large branching ratio. The branching ratios into $WW$, $ZZ$, $\gamma Z$, or $\gamma\gamma$ are significantly smaller but in any case larger than the corresponding branching ratios of a toponium resonance.

Throughout this paper the SQD mode has been ignored completely, a clearly oversimplifying assumption. The mixing of both $b'$ and $D$ with other quarks is presumably small and it seems difficult to make any hypothesis. Instead we compare the total annihilation rate (for $V_J$ with $\Lambda_{MS}^{(4)} = 300$ MeV) with the single quark decay rate (through the charged current) in fig. 6 for an assumed mixing angle of $10^{-2}$. This illustrates that annihilation decays would be clearly dominant for $b'$ under this assumption. For a complete treatment of $\eta_D$ we would have to include FCNC single quark decays $D \to qZ$. In view of the completely unknown mixing angle we refrain from a detailed analysis. It is evident from fig. 6 that for mixing angles of $O(10^{-2})$ or less and $m_D$ below $\sim 500$ GeV the main conclusions remain unaffected.
References

[1] G. Pancheri, J.-P. Revol and C. Rubbia, *Phys. Lett.* B **277** (518) 1992.

[2] J.H. Kühn and E. Mirkes, *Phys. Lett.* B **296** (1992) 425.

[3] J.H. Kühn and E. Mirkes, Karlsruhe preprint TTP92-32, to appear in *Phys. Rev.* D (1993).

[4] J.H. Kühn, *Act. Phys. Pol.* B **12** (1981) 347.

J.H. Kühn, *Act. Phys. Austr.* Suppl. **XXIV** (1982) 203.

[5] V. Barger et al., *Phys. Rev.* D **35** (1987) 3366.

[6] J.H. Kühn and P. Zerwas, *Phys. Reports* **167** (1988) 323.

[7] K. Igi and S. Ono, *Phys. Rev.* D **33** (1986) 3349.

[8] J. Morfin and Wu-Ki Tung, *Z. Phys.* C **52** (1991) 13.

[9] L.J. Reinders, H. Rubinstein and S. Yazaki, *Phys. Reports* **127** (1985) 1.

[10] A. Martin, R. Roberts and J. Stirling, *Phys. Rev.* D **43** (1991) 3645.

[11] J. Kwieciński, A. Martin, R. Roberts and J. Stirling, *Phys. Rev.* D **42** (1990) 3645.

[12] M. Glück, E. Reya and A. Vogt, DO-TH 91/07: see also *Z. Phys.* C **53** (1992) 127.

Figure captions

**Fig. 1** Predictions for $R(0)^2/M^3$ of the S-wave ground state as a function of $M$ for the potential $V_J$ with $\Lambda_{\overline{MS}}^{(4)}=200, 300$ and 500 MeV (dotted, solid and dashed lines).

**Fig. 2** Ratio between the radiatively corrected production cross section for $pp \to \eta_{b,D} + X$ and the lowest order result for $\sqrt{S}=16$ and 40 TeV. We use MT set B1 parton distributions with $\Lambda_{\overline{MS}}^{(4)}=194$ MeV and work in the DIS factorization scheme.

**Fig. 3** a) Production cross section for $pp \to \eta_{b,D} + X$ in NLO for $\sqrt{S}=16$ and 40 TeV. We use MT set B1 parton distributions with $\Lambda_{\overline{MS}}^{(4)}=194$ MeV and work in the DIS factorization scheme (potentials as in fig. 1).
b) Dependence of the cross section on the parton distribution functions: MT set B1 [8] (solid), MT set SN [8] (dotted), MRS set B200 [10] (short-dash-dotted), KMRS set B [11] (long-dash-dotted) and GRV [12] (dashed).

Fig. 4 Ratios $R_{ab} = \Gamma_{ab}/\Gamma_{\gamma\gamma}$ ($ab = HZ, t\bar{t}, gg, WW, ZZ, \gamma Z$) of $\eta_{\nu}$ (fig. a) and $\eta_D$ (fig. b) as functions of $M$ and corresponding branchings ratios (figs. c and d) for a top mass of 140 GeV in $\Gamma_{t\bar{t}}$.

Fig 5. Cross section for $\eta_{\nu}$ (fig. a,b) and $\eta_D$ (fig. c,d) production including NLO corrections multiplied by the branching ratios at $\sqrt{S} = 40$ TeV and 16 TeV. We use MT set B1 parton distributions with $\Lambda_{MS}^{(4)} = 194$ MeV and work in the DIS factorization scheme (potentials as in fig. 1).

Fig 6. Total widths for $\eta_{\nu,D}$ from annihilation (see fig. 4) compared with the widths from single quark decays into $b + W$. 