Measurement of the angle $\alpha$ at BABAR

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Abstract. We present recent measurements of the CKM angle $\alpha$ using data collected by the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider at the SLAC National Accelerator Laboratory, operating at the $\Upsilon(4S)$ resonance. We present constraints on $\alpha$ from $B \to \pi\pi$, $B \to \rho\rho$ and $B \to \rho\pi$ decays.

1. Introduction
The measurements of the angles $\alpha$, $\beta$ and $\gamma$ of the Unitarity Triangle (UT) at the B-factories are providing precision tests of the Standard Model (SM) description of CP violation. This description is provided by the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing [1, 2]. We summarize the experimental constraints on $\alpha$ UT angle obtained from $B$-meson decays to $\pi\pi$, $\rho\rho$ and $\rho\pi$ with the BABAR experiment at the SLAC National Accelerator Laboratory. The BABAR detector and the PEP-II accelerator are described elsewhere [3].

2. Analysis Method
2.1. General formula
The decay of a neutral $B$-meson into a pair of $\pi$ or $\rho$ mesons, $B \to hh$ ($h = \pi, \rho$), occurs via two topologies: a tree-level process and a one-loop penguin diagram. The $CP$ parameter $\lambda_{hh}$, defined by $\lambda_{hh} = \frac{p A}{q A}$, where $q$ and $p$ are the complex coefficient that link the mass and the flavor eigenstates in the $B$ system, and $A$ ($\overline{A}$) is the $B^0$ ($\overline{B}^0$) decay amplitude, can be expressed in terms of $\alpha$ as

$$\lambda_{hh} = e^{2i\alpha} \frac{1 - (|V_{td}^* V_{tb}|/|V_{ud}^* V_{ub}|)P/T e^{-i\alpha}}{1 - (|V_{td}^* V_{tb}|/|V_{ud}^* V_{ub}|)P/T e^{i\alpha}},$$

where $T$ and $P$ are complex amplitudes dominated by tree and penguin topologies, respectively.

The quantity experimentally measured is the time-dependent decay rate

$$f_{Q_{tag}} = e^{-|\Delta t/\tau}|1 - Q_{tag} C_{hh} \cos(\Delta m_d \Delta t) + Q_{tag} S_{hh} \sin(\Delta m_d \Delta t)|,$$

where $\tau$ is the neutral $B$ lifetime and $\Delta m_d$ is the $B^0\overline{B}^0$ oscillation frequency. $\Delta t$ is the proper time difference between decays of the $B$ to $hh$ (B$_{\text{rec}}$), and the second $B$ in the event, denoted by B$_{\text{tag}}$. The $Q_{tag}$ parameter is related to the flavor of the B$_{\text{tag}}$: $Q_{tag} = +1(-1)$ if the B$_{\text{tag}}$ is a $B^0$ ($\overline{B}^0$). The $CP$-violating asymmetries $C_{hh}$ and $S_{hh}$ are related to the $\lambda_{hh}$ parameter by

$$S_{hh} = 2Im(\lambda_{hh})/(1 + |\lambda_{hh}|^2), \quad C_{hh} = (1 - |\lambda_{hh}|^2)/(1 + |\lambda_{hh}|^2).$$
$S_{hh}$ reflects the $CP$-violation induced by the interference between the mixing and decay processes; $C_{hh}$ is the direct $CP$-violating asymmetry which comes from the interference between different decay topologies. In the absence of penguin contributions ($P = 0$), $C_{hh}$ vanishes and $S_{hh}$ is simply related to the CKM angle $\alpha$ by $S_{hh} = \sin(2\alpha)$.

In the more general case of the $B^0(B^\prime)$ → $\rho^\pm \pi^\mp$ decays, the time-dependent decay rate is given by

$$f_{Qtag}^{\rho^\pm \pi^\mp} = (1 \pm A_{\rho\pi}) e^{-|\Delta t|/\tau} \left[ 1 - Q_{tag}(C_{\rho\pi} \pm \Delta C_{\rho\pi}) \cos(\Delta m_d \Delta t) + Q_{tag}(S_{\rho\pi} \pm \Delta S_{\rho\pi}) \sin(\Delta m_d \Delta t) \right],$$

where, the $\pm$ sign depends on whether the $\rho$ meson is emitted by the $W$ boson or comes from the spectator quark. $A_{\rho\pi}$ is the direct $CP$ violation parameter measuring the asymmetry between the $\rho^+\pi^-$ and $\rho^-\pi^+$ final states, while $\Delta S_{\rho\pi}$ and $\Delta C_{\rho\pi}$, which arise from the fact that two production modes of the $\rho$ are possible, are dilution terms and have no $CP$ content.

2.2. The isospin analysis

Using the strong isospin symmetry, the angle $\alpha$ can be extracted up to discrete ambiguities from the $CP$-violating asymmetries defined above \cite{3}. The decay amplitudes of the isospin-related final states obey the pentagonal relations

$$\sqrt{2}(A_{\rho\pi}^+ + A_{\rho\pi}^0) = 2A_{\rho\pi}^{00} + A_{\rho\pi}^{+-} + A_{\rho\pi}^{++}, \quad \sqrt{2}(A_{\rho\pi}^- + A_{\rho\pi}^{0+}) = 2A_{\rho\pi}^{00} + A_{\rho\pi}^{++} + A_{\rho\pi}^{--};$$

where $A_{\rho\pi}^{ij} = A(B^0$ or $B^+ \rightarrow \rho^i \pi^j)$ and $\overline{A}_{\rho\pi}^{ij} = A(B^0$ or $B^- \rightarrow \rho^j \pi^i)$, $i, j = +, -, 0$. With the use of these relations, 12 unknowns (6 complex amplitudes with one unphysical phase, and the CKM angle $\alpha$) are to be determined while 13 observables are available: $S_{\rho\pi}$, $C_{\rho\pi}$, $\Delta S_{\rho\pi}$, $\Delta C_{\rho\pi}$, $A_{\rho\pi}$; four average branching fractions $B(B \rightarrow \rho \pi)$; two time-dependent $CP$-violating asymmetries in the $B^0 \rightarrow \rho^0 \pi^0$ decay ($S_{\rho\pi}^{00}$, $C_{\rho\pi}^{00}$) and two direct $CP$ asymmetries in $B^+ \rightarrow \rho^+ \pi^0$ and $B^- \rightarrow \rho^0 \pi^+$ decays.

In the case of $B \rightarrow hh$ ($h = \pi, \rho$), Eq. 5 simplify to triangular relations

$$\sqrt{2}A_{hh}^{+0} = A_{hh}^{+-} + A_{hh}^{00}, \quad \sqrt{2}A_{hh}^{0+} = 2A_{hh}^{00} + A_{hh}^{++} + A_{hh}^{--}.$$  

The information counting leads then to 6 unknowns and 7 observables: 3 branching fractions $B(B \rightarrow hh)$, $C_{hh}$, $S_{hh}$, and $C_{hh}^{00}$, $S_{hh}^{00}$. In the $\pi\pi$ system $S_{\pi\pi}^{00}$ is impossible to measure (as the $\pi^0$ is reconstructed from two-photons decay, there is no way to measure the decay vertex), then one is left with 6 observables: $\alpha$ can be extracted with an 8-fold ambiguity within $[0, \pi]$ \cite{4}.

3. Experimental Results

3.1. $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$

The various branching fractions and $CP$-asymmetries measured in $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$ decays are summarized in Table 1. In the case of charged decays the charge asymmetry is defined as $A_{CP}(B \rightarrow hh) = -C_{hh}$. The measurements are sufficiently well established to perform an isospin analysis.

The present measurement for the $\pi^+\pi^-$ mode excludes the absence of $CP$ violation ($C_{\pi\pi}, S_{\pi\pi} = (0, 0)$ at a C.L. of 6.7$\sigma$. The relatively high branching fraction of the $\pi^0\pi^0$ mode tends to separate the 8-fold ambiguities in the $\alpha$ extraction, which only allows a weak constraint on $\alpha$ to be set. With the current experimental measurements two of the eight ambiguities are nearly merged. The range $[23^\circ, 67^\circ]$ in $\alpha$ is excluded at the 90% C.L. \cite{6}. The solution is in agreement with the global CKM fit \cite{13} \cite{14} which gives the range $[71^\circ, 109^\circ]$ at 68% C.L.
Observable Value

The analysis of $B \rightarrow \rho \rho$ is potentially complicated due to the possible presence of three helicity states for the decay. The helicity zero state, which corresponds to longitudinal polarization of the decay, is $CP$-even but the helicity ±1 states are not $CP$ eigenstates. Fortunately, this complication is avoided by the experimental finding that the dominant polarization is longitudinal, $f_L(\rho^+\rho^-) = 0.992 \pm 0.024^{+0.028}_{-0.013}$ [9], $f_L(\rho^0\rho^0) = 0.75 \pm 0.14 \pm 0.05$ [10] and $f_L(\rho^+\rho^0) = 0.950 \pm 0.015 \pm 0.006$ [11] ($f_L \equiv \Gamma_L/\Gamma$, where $\Gamma$ is the total decay rate and $\Gamma_L$ is the rate of the longitudinally-polarized mode). The $B^0 \rightarrow \rho^0\rho^0$ branching fraction is small compared with that of the $B^+ \rightarrow \rho^+\rho^0$ mode, which indicates that the penguin to three ratio ($P/T$, cf. Eq. 1) is small compared with that of the $B \rightarrow \pi\pi$ system [1]. This has the effect of merging the different ambiguities in the extraction of $\alpha$. The latest $B^0 \rightarrow \rho^0\rho^0$ BABAR results present the first measurement of the time-dependent $CP$ asymmetries $C^{00}_L$ and $S^{00}_L$. The inclusion of these measurements has the effect of raising the 8-fold degeneracy on $\alpha$: the data only favors two solutions out of eight [10] [11]. These two effects allow to set a strong constraint on $\alpha$, where only two solutions are seen, corresponding to $\alpha = (92.4^{+6.0}_{-6.3})^o$ at 68% C.L. [11] for the one in agreement with the global CKM fit [13] [14].

| Mode $\rightarrow$ | $B(10^{-10})$ | $C$ | $S$ |
|------------------|-------------|-----|-----|
| $\pi^+\pi^-$    | $5.5 \pm 0.4 \pm 0.3$ [7] | $-0.68 \pm 0.10 \pm 0.03$ [6] | $-0.25 \pm 0.08 \pm 0.02$ [6] |
| $\pi^+\pi^0$    | $1.83 \pm 0.21 \pm 0.13$ [6] | $-0.43 \pm 0.26 \pm 0.05$ [6] |             |
| $\rho^+\rho^-$  | $25.5 \pm 2.1^{+8.6}_{-1.9}$ [9] | $0.01 \pm 0.15 \pm 0.06$ [9] | $-0.17 \pm 0.2^{+0.06}_{-0.06}$ [9] |
| $\rho^0\rho^0$  | $0.92 \pm 0.32 \pm 0.14$ [10] | $0.2 \pm 0.8 \pm 0.3$ [10] | $0.3 \pm 0.7 \pm 0.2$ [10] |

Table 1. Summary of BABAR measurements of $B \rightarrow \pi\pi$ and $B \rightarrow \rho\rho$ decays. The measurements for the $\rho\rho$ system corresponds to the longitudinal component of the decay rate. The errors quoted are statistical and systematic, respectively.

3.2. $B \rightarrow \rho\pi$

The $B \rightarrow \rho\pi$ measurement reported here is a time-dependent amplitude analysis of $B^0 \rightarrow (\rho\pi)^0$. The interferences between the intersecting $\rho$ resonance bands are modeled over the whole Dalitz Plot using the isobar model [15]. This allows determination of the strong phase differences from the interference pattern, which permits direct extraction of the angle $\alpha$ with reduced ambiguities. The Dalitz amplitudes and time-dependence are contained in the 26 coefficients of the bilinear form-factor terms occurring in the time-dependent decay rate, which are determined from a likelihood fit. The values obtained for these coefficients are converted back into the quasi-two-body $CP$ observables (c.f. Eq. 1), which are more intuitive in their interpretation. Table 2 reports the experimental findings on these observables [12].

| Observable | Value |
|------------|-------|
| $C_{\rho\pi}$ | $0.15 \pm 0.09 \pm 0.05$ |
| $\Delta C_{\rho\pi}$ | $0.39 \pm 0.09 \pm 0.09$ |
| $C^{\rho\pi}_{\rho\rho}$ | $-0.10 \pm 0.40 \pm 0.53$ |
| $A^{\rho\pi}_{\rho\rho}$ | $-0.14 \pm 0.05 \pm 0.02$ |

| Observable | Value |
|------------|-------|
| $S_{\rho\pi}$ | $-0.03 \pm 0.11 \pm 0.04$ |
| $\Delta S_{\rho\pi}$ | $-0.01 \pm 0.14 \pm 0.06$ |

Table 2. Summary of BABAR measurements from the time-dependent amplitude analysis of $B^0 \rightarrow (\rho\pi)^0$ decays. The errors quoted are statistical and systematic, respectively.

These measurements allow the determination of the limit $\alpha = (87^{+45}_{-13})^o$ at 68% C.L., with almost no constraint at 95% C.L. This result is particularly interesting as there is an unique
solution in the $[0, 180]^\circ$ range, which helps to break the ambiguities obtained from the $\pi\pi$ and $\rho\rho$ results. A hint of CP-violation is obtained at the level of $3\sigma$.

4. Summary
Several analyses have been conducted in $\overline{B}$ABAR to extract the angle $\alpha$ of the UT. In the last few years the measurements of this angle have become increasingly precise. The measurements provided from the $B \to \pi\pi/\rho\rho/\rho\pi$ modes give complementary constraints on $\alpha$. For the $B \to \rho\rho$ system, the inclusion of the $S_{\rho\pi}^0$ observable allows to favor two of the 8-fold ambiguities on $\alpha$, and the relatively large $B(B^+ \to \rho^+\pi^0)$, with respect to $B(B^0 \to \rho^0\pi^0)$, causes the ambiguities to degenerate in two peaks, improving the precision of the constraint. The measurements from the $B^0 \to (\rho\pi)^0$ time-dependent amplitude analysis give a direct access to $\alpha$, disfavoring the ambiguities. The combined constraint averaging all the $\pi\pi$, $\rho\rho$ and $\rho\pi$ measurements from $\overline{B}$ABAR and Belle gives $\alpha = (89.0^{+4.4}_{-4.2})^\circ$ at 68% C.L. (see Fig. 1), which is in good agreement with the global CKM fit [13, 14].

![Figure 1](http://example.com/figure1.png)

**Figure 1.** Constraints on $\alpha$, provided by the CKMfitter group [13], expressed as one minus the confidence level as a function of angle. The constraints are constructed averaging the $\overline{B}$ABAR and Belle measurements for the $\pi\pi$ (dotted green curve), $\rho\rho$ (dash-dotted blue curve) and $\rho\pi$ (dashed red curve) systems. The solid filled green curve represents the combined constraint using all the systems.

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6. References
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