Effect of latent mass in inhomogeneous cosmological model with perfect fluid and self-acting scalar field.

V.M. Zhuravlev\textsuperscript{1}, D.A. Kornilov\textsuperscript{2}
Ulyanovsk State University
Institute for Theoretical Physics
\textsuperscript{1} zhuravl@sv.uven.ru
\textsuperscript{2} kda@sv.uven.ru

Abstract. The inhomogeneous cosmological model with matter in the form self-acting scalar field and perfect fluid is considered. On the basis of exact solutions is considered the evolution of density distribution of a matter in space on a background cosmological expansion by the Universe. Is shown, the first, in such model the equation of a matter state is variable in time and is closely connected to character cosmological expansions. Secondly, it is shown with point of view of the observer the Universe looks as space flat, but with effect of latent mass. This effect consists in that the mass of a perfect fluid by dynamic measurements surpasses the own mass perfect fluid that is explained by presence scalar field.

PACS-94: 04.20.-q

The usual approach to study cosmological models consists from two stages. The first stage consists in construction of global homogeneous and isotropic model of the Universe of FRW-type with a scalar field and (or) matter in the form perfect fluid. At the second stage it is studied evolution perturbation of matter density on a background of the homogeneous metrics constructed at the first stage. Thus the inhomogeneity of the Universe is taken into account only in the first order of the perturbation theory, that is connected with linearization of the dynamics equations. However, finding out structure of the Universe on the large scales (voids, strings) show that the process of their formation can be essentially non-linear and for simulation of their occurrence it is necessary to use some other approaches. In the present work the attempt is undertaken to investigate properties of the self-coordinated model of a special scalar field and perfect fluid inducing a gravitational field on the basis of the exact solutions of the Einstein equations, describing Universe with cosmological evolution. In this case the distribution of matter in the Universe are described exact, instead of by the approximate solutions of the equations. Therefore in such model it is possible to study all effects of interaction of three fields in an exact form in formation various spatial structures of a matter in space.

The most suitable material for construction this models is well-known in the General Relativity theory the Majumdar-Papapetrou metrics [1, 2] originally used for construction self-gravitating electrostatic or magnitoststic fields.

Basis of suggested in the present work models is the generalization of the Majumdar-Papapetrou metrics to describe not static effects connected with cosmological expansion. Thus an electrostatic or magnitoststic field is replaced by some generalized scalar field. Nevertheless this scalar field has a similar energy-momentum tensor to electrostatic field distinguished from them only by presence of its self-action. Self-action of this field describes
by potential function which one in many respects determines cosmological properties of model. The second kind of material field in this model is the perfect fluid. In result the researched model represents inhomogeneous cosmological model with self-acting scalar field and matter with state equation generally varies in due course. Last fact is important from the point of view modern representations about existence of various epochs in evolution of the Universe during the substance filling the Universe had various properties. At an inflationary stage it is a matter close to quasi-vacuum state, i.e. it prevails of a field component, in the subsequent epoch it is the isotropic radiation \( p = \varepsilon/3 \) and now it is a matter close to a dust \( p \sim 0 \). In the present work the attempt is done(made) to analyse such inhomogeneous models from the point of view of evolution including all these stages of development by the Universe.

1. General view of the metrics researched in the present work is following

\[
ds^2 = e^{A(x,y,z)+b(t)}dt^2 - e^{-A(x,y,z)+a(t)} \left(dx^2 + dy^2 + dz^2\right),
\]

\[
g_{ik} = \text{diag}\{e^{A+b}, -e^{-A+a}, -e^{-A+a}, -e^{-A+a}\}, \quad i, k = 0, 1, 2, 3,
\]

where \( A = A(x, y, z) \) - function of coordinates \( x^1 = x, x^2 = y, x^3 = z \) and not dependent from \( x^0 = t \), and \( a(t), b(t) \) - some functions of time. The metric of such form with \( a(t) = 0, b(t) = 0 \) is known in the classical theory of GR as the Majumdar-Papapetrou metrics. In a case \( a(t) \neq 0, b(t) \neq 0 \) these not static metrics also describe some global cosmological dynamics of the Universe with local inhomogeneity of the space it was connected with function \( A \).

There is one more important underclass of the metrics of this type having the following form

\[
ds^2 = e^{A(x,y,t)+a(z)} \left(dt^2 - dx^2 - dy^2\right) - e^{-A(x,y,t)+b(z)}dz^2.
\]

It describes non-static gravitational processes in space with coordinate \( z \). Dependence from \( z \) of functions \( a, b \) in this case is connected to some inhomogeneous and anisotropy space lengthways the select axis \( z \).

Let’s consider the matter inducing metric property space - time of the kind \( \{1\} \) the mix from perfect fluid with energy-momentum tensor (TEM)

\[
T^{(m)0}_0 = \varepsilon(x, y, z, t), \quad T^{(m)1}_1 = T^{(m)2}_2 = T^{(m)3}_3 = -p(x, y, z, t), \quad T^{(m)i}_k = 0, \quad i \neq k
\]

(\( \varepsilon \) and \( p \) - density of energy and pressure of a fluid) and scalar field with energy-momentum tensor

\[
T^{(\phi)}_{ik} = -\nabla_i \phi \nabla_k \phi + \frac{1}{2} g_{ik} g^{lm} \nabla_l \phi \nabla_m \phi + g_{ik} V(\phi, t)
\]

(\( V(\phi, t) \) - potential of self-action of a scalar field). As follows from \( \{1\} \) TEM of a scalar field considered in this work differs from TEM of usual scalar field, for example Higg’s scalar field, by opposites sign, but coincides with sign of TEM of an electrostatic field with potential \( \phi \). However, equation \( \{4\} \) differs from TEM of an electrostatic field by presence of self-action potential.

The Einstein equations

\[
G_{ik} = \kappa T_{ik}
\]
\( \varepsilon = 8\pi G/c^4 \) - Einstein’s gravitational constant for the metrics (II) with TEM

\[ T_{ik} = T_{ik}^{(\phi)} + g_{ij} T_{k}^{(m)j} \]

are reduced to simple set from two equations, which is possible to write down in the following form

\[ p(x, y, z, t) = V(\phi) + g(t)e^{-b-A}, \]
\[ \varepsilon(x, y, z, t) = c^2\rho(x, y, z, t) = \frac{1}{\varepsilon}e^{A-a}\Delta A - V(\phi) + \frac{\dot{a}^2}{\varepsilon}e^{-b-A}, \]

and one statement

\[ \phi(x, y, z, t) = \frac{a(t) - A(x, y, z)}{\sqrt{2\varepsilon}}, \]

identifying a scalar field with characteristic of the metrics. To these equations it is necessary obviously to add the equation arising the ambassador variations of a Lagrangian density of a matter by \( \phi \). This equation has the following form

\[ \Delta A = \sqrt{2\varepsilon}e^{-A+a}\frac{\partial V(\phi)}{\partial \phi} + \left( \frac{\dot{b} - 3\dot{a}}{2} - \ddot{a} \right) e^{a-b-2A} \]

Further, for function \( A \) the equation (II) should not on the right contain obvious dependence from coordinate \( t \), as function \( A \) is obvious from \( t \) does not depend. It imposes specific conditions on potential \( V(\phi, t) \). By the elementary kind \( V(\phi, t) \) it is possible to satisfy to condition of independence of the right part in (II) from \( t \), is potential of a kind

\[ V(\phi) = V_0 \exp\{\sqrt{2\varepsilon}\phi\}. \]

In this case we have

\[ \Delta A = \sigma e^{-2A}, \]
\[ p = g(t)e^{-b-A} + V(\phi) = p_0(t)e^{-A}, \]
\[ \varepsilon = c^2\rho = g(t)e^{-b-A} - V(\phi) + \sqrt{2\frac{\partial V}{\partial \phi}} = p_0(t)e^{-A}, \]

where

\[ p_0(t) = g(t)e^{-b} + V_0e^a, \]
\[ \sigma = 2\varepsilon V_0e^{2a} + \left( \frac{\dot{b} - 3\dot{a}}{2} - \ddot{a} \right) e^{a-b} = \text{const.} \]

Last statement is equation connecting \( a \) and \( b \), at any meanings of parameters \( \sigma \) and \( V_0 \). From here for (II) we come to the extreme rigid state matter equation

\[ p = \varepsilon, \]
or to absence of a matter: at \( p = 0 \) is automatically received \( \varepsilon = 0 \).

That the state matter equation would have more general view, for example, \( p = \gamma(t) \varepsilon \), it is necessary to require performance of the following equation for \( V(\phi, t) \):

\[
V(\phi, t) + q(t) \exp\{\sqrt{2\alpha_{\phi}\phi}\} = \gamma(t) \left( -V(\phi, t) + \sqrt{\frac{2}{\alpha_{\phi}}} \frac{\partial V}{\partial \phi} + q(t) \exp\{\sqrt{2\alpha_{\phi}\phi}\} \right).
\]

Here

\[
q(t) = g(t) e^{-b(t)-a(t)}.
\]

The general solution of this equation rather \( V(\phi, t) \) at parametrical dependence from \( t \) has the following form

\[
V(\phi, t) = V_1(t) \exp\left\{ \frac{\gamma+1}{2\gamma} \sqrt{2\alpha_{\phi}\phi} \right\} - q(t) \exp\{\sqrt{2\alpha_{\phi}\phi}\}, \tag{15}
\]

where \( V_1(t) \) - any function \( t \). The parameter \( \gamma \) can be thus function of time, and can and to not be. However in all cases, when \( \gamma \neq 1 \) and \( \gamma \neq 0 \) the self-action potential will be obvious function of time. For example, for the case, when the substance represents by itself isotropic radiation with \( \gamma = 1/3 \) potential looks like

\[
V(\phi, t) = V_1(t) \exp\{2\sqrt{2\alpha_{\phi}\phi}\} - q(t) \exp\{\sqrt{2\alpha_{\phi}\phi}\}. \tag{16}
\]

In general case equation for \( A(x, y, z) \) will look like

\[
\Delta A = A_1 e^{-(1+3\gamma)A/(2\gamma)} + A_2 e^{-2A}. \tag{17}
\]

Because the right member of the equation (17) did not contain a time \( t \) dependence, it is necessary that the functions \( a, b, V_1 \) satisfied to following requirements:

\[
A_1 = \text{const} = 4\alpha V_1(t)e^{3a}, \tag{18}
\]

\[
A_2 = \text{const} = -\left[ \frac{\dot{a}(\dot{b} + a)}{2} - \ddot{a} \right] e^{a-b}. \tag{19}
\]

At once from (18) it is possible to obtain a kind of function \( V_1(t) \):

\[
V_1(t) = \frac{A_1\gamma}{\alpha(1+\gamma)} e^{(1+3\gamma)a/(2\gamma)}. \tag{20}
\]

Equation (19) contains two unknown functions, one of them \( b(t) \) remains uncertain and connected with selection of time variable. For simplicity let’s assume \( b(t) = 0 \). It is means that variable \( t \) is the phisical time. In this case differential equation

\[
\ddot{a} - \frac{\dot{a}^2}{2} = A_2 e^{-a},
\]

defines form of function \( a(t) \), and it has the solution in an explicit form in only elliptic functions of the first kind [7].
Figure 1: Evolution of the scale factor in case $A_2 = -1$. Initial condition is follows: $R(0) = 0$, $\left. \frac{dR}{dt} \right|_{t=0} = 1$. 
Last equation determines evolution of the scale factor \( R(t) = \exp \{-a(t)/2\} \) too. In figure 1 the results of a numerical analysis of the differential equation

\[
\frac{d^2 R}{dt^2} = -\frac{A_2}{2} R^3,
\]

that defines evolution of the scale factor.

From the equation (21) it is visible that the changes of evolution of the scale factor are determined by the sign of the constant \( A_2 \). In particular, in a case \( A_2 < 0 \), the second derivative on time from the scale factor is positive (positive acceleration) and the expansion of the Universe will be accelerated. For more best estimate of velocity of expansion of the Universe in a fig. 1 the diagrams power and exponential functions are given. It is visible, that the expansion at a small times is a very good approximated by a linear function, then the very fast growth of the scale factor follows.

2. To find out properties of a field \( \phi \) and its role in represented theory it is important to study dynamics of a trial particle in a gravitational field described by the metrics (1). Let’s assume, that the field \( \phi \) does not interact directly with a usual matter. Then at absence of direct interaction trial particle with a field \( \phi \) the equations of its dynamics will look like Geodetic

\[
\frac{d}{dt} u^i + \Gamma^i_{kj} u^k u^j = 0, \quad \frac{dx^k}{dt} = u^k, \quad i, j, k = 0, 1, 2, 3,
\]

where \( u^k \) - 4-speed of a particle and \( \Gamma^i_{kj} \) is the Christoffel symbols. For the metrics (1) 4-speed of a particle are normed by a condition

\[
u^i u_i = g_{ik} u^i u^k = e^A \left(e^{-2A}(u^0)^2 - (u^1)^2 - (u^2)^2 - (u^3)^2\right) = c^2.
\]

where \( c \) - speed of light.

Substituting in (22) Christoffel symbols for the metrics (1) we receive

\[
\frac{d}{dt} u^\alpha = -\frac{1}{2} \left(e^{2A}(u^0)^2 + (u^1)^2 + (u^2)^2 + (u^3)^2\right) A_{\alpha\beta} + u^\alpha (u^\beta A_{\beta\gamma}), \quad \alpha, \beta = 0, 1, 2;
\]

\[
\frac{d}{dt} u^0 = -u^0 (u^\beta A_{\beta\gamma}).
\]

Having copied these equations for covariant components of 4-speed of a particle using statement

\[
u^0 = e^{-A} u_0, \quad u^1 = -e^A u_1, \quad u^2 = -e^A u_2, \quad u^3 = -e^A u_3,
\]

we receive

\[
\frac{d}{dt} u_0 = 0,
\]

\[
\frac{d}{dt} u_\alpha = -\frac{1}{2} \left(2(u_0)^2 - c^2 e^A\right) A_{\alpha\beta}, \quad \alpha = 0, 1, 2.
\]

Thus covariant component \( u_0 \) of 4-speeds of a particle is constant and last three equations are equivalent to equations of movement of a particle in a Newtonian field of gravitation:

\[
\frac{d}{dt} u_\alpha = -\Psi_{\alpha}, \quad \alpha = 0, 1, 2,
\]
with gravitation potential

$$\Psi = (u_0)^2 A - \frac{c^2}{2} e^A.$$  (29)

Actually first member in this expression proportional to a square of kinetic energy of a trial particle is the item connected with potential forces of inertia and only second can be interpreted as Newtonian potential of a field of gravitation. For interpretation of movement the particles in this case are necessary for considering the equations (24). This equations is possible to write down as

$$\frac{d}{dt} u^\alpha = - \frac{\partial \Phi}{\partial x^\alpha} + F^\alpha,$$  (30)

where $\Phi$ now true potential of a field of gravitation:

$$\Phi = \frac{c^2}{2} e^A + \Phi_0,$$  (31)

$\Phi_0$-any constant, and $F$ is the gyroscopic force of inertia arising at the expense of local rotation of reference system. It will easily be convinced as $F$ looks like

$$F = [u \times [u \times \nabla A]],$$

where on the right there is a double vector product on flat contravariant card of co-ordinates. As the force $F$ is gyroscopic, it does not make work.

Repeating the calculations have been carried out above for the metrics (1) it is possible to receive similar to (30) equations for contravariant and covariant component of 4-speeds in the metrics (2).

3. We shall consider now the distribution of a matter corresponds to potential fields of gravitation (31). Using (7),(10) we come to the following equation:

$$\Delta \Phi \equiv \frac{c^2}{2} e^A \left( (\nabla A)^2 + \Delta A \right)$$  (32)

Last member in the right part, square-law on a gradient of a field, will be equal to energy enstity $\varepsilon_\phi$ of a scalar field $\phi$. Using (11) the equation (32) it is possible to write down in a standard form

$$\Delta \Phi = 4\pi G \varrho,$$

where

$$\varrho = \rho_d + \frac{c^2}{2} V(\phi) + c^2(\nabla \phi)^2 e^{-\sqrt{2} \phi}$$

is complete density of all kinds of a matter in space: $\rho_d$ - density usual matter, and the other items represent energy density of a field $\phi$ at the expense of self-action of a field $\phi$.

Let’s notice this result describe the presence of effect of latent mass. Scalar field in this model not detected by not dynamic measurements, as is actually connected by virtue of equality (8) with the metrics of space - time and itself does not interact with usual matter. But at dynamic measurements will be present additional mass. For any volume $V$ of space the additional mass $M_l$ is equal to following

$$M_l = \int_{\Omega} \left[ \frac{c^2}{2} V(\phi) + c^2(\nabla \phi)^2 e^{-\sqrt{2} \phi} \right] d\Omega.$$
This formula show that in considering model the effect of latent mass exists. The value of latent mass defined by function of $A(x, y, z)$ and dependent from cosmological variation of scale factor that equal to $R(t) = \exp -a(t)/2$.

**Acknowledgements.** Work is supported by Russian Fund of Basic Researches (grant 01-98-18040).

**References**

[1] D. Kramer, H. Shtepahny, M. Mc-Callumn, E. Heralt. *The exact solutions of the Einstein equations*, M.: Energoizdat, 172 (1982).

[2] A. Papapetrou, Proc. Roy. Irish Acad. A 51, 191 (1947); Majumdar S.D., Phys. Rev. 72, 390 (1947)

[3] H. Yilmaz, *Phys. Rev.* 111, 1417 (1958)

[4] H. Yilmaz, *Ann. Phys.* (N.Y.) 101, 413 (1976)

[5] S. Kaniel and Y. Itin, *Nuov. Cim.*, 113B (1998), gr-qc/9707008 (1998)

[6] Y. Itin, gr-qc/9806110 (1998)

[7] Prudnikov A.P. Brichkov Yu.A. Marichev O.I. *Integral and series*. M, Nauka, 1981, 800s.