We examine the validity of the Fermi-liquid description of the dilute 2D hole gas in the crossover from 'metallic'-to-'insulating' behaviour of $\rho(T)$. It has been established that, at $r_s$ as large as 29, negative magnetoresistance does exist and is well described by weak localisation. The dephasing time extracted from the magnetoresistance is dominated by the $T^2$-term due to Landau scattering in the clean limit. The effect of hole-hole interactions, however, is suppressed when compared with the theory for small $r_s$.

There has recently been much attention drawn to the unusual crossover in the temperature dependence of resistivity with varying carrier concentration from 'metallic' to 'insulating' ($d\rho/dT > 0$ to $d\rho/dT < 0$) behaviour, which has been seen in some low-density 2D systems. With decreasing carrier density $p$ the ratio of the Coulomb to Fermi energy $r_s = U/E_F \propto m^*/p^{1/2}$ increases, so it was suggested that in these systems the Fermi-liquid description is not valid and new approaches are needed. In the insulating state at low hole densities, $p \simeq 7.7 \times 10^9$ cm$^{-2}$ and interaction parameter $r_s \simeq 35$ (with $m^* = 0.37 m_0$), was attributed to Wigner crystallisation. At the same time, it was shown that even at $r_s$ \( \sim \) 10 to 14 the 2D hole gas can manifest itself as a normal Fermi liquid, as far as weak localisation (WL) and weak hole-hole interaction (HHI) effects are concerned.

As conventional theories of weak localisation and electron interactions are derived for $r_s \ll 1$, their applicability to $r_s \sim 10 - 14$ in (3) is already a surprising fact. One can argue though that in the structures studied the carrier density was not low enough and mobility not high enough for the deviations from the Fermi liquid description to be seen ($p \simeq 4.6 \times 10^{10}$ cm$^{-2}$ and peak mobility $\mu_p = 2.5 \times 10^5$ cm$^2$V$^{-1}$s$^{-1}$ in (3)). In this work, we examine the existence of WL and HHI in a 2D hole gas with lower density down to $p = 1.17 \times 10^{10}$ cm$^{-2}$. In our high mobility structures, with peak mobility $\mu_p = 6.5 \times 10^5$ cm$^2$V$^{-1}$s$^{-1}$, the experimental conditions approach those in (3), where the Wigner crystal formation has been claimed. We show that even at $r_s = 29$ the strong Coulomb interaction does not affect the WL description of the negative magnetoresistance (3)(4), although it suppresses the contributions to the phase-breaking time and Hall coefficient of the weak hole interactions expected in a disordered system with small $r_s$.

The experiments have been performed on a high mobility heterostructure formed on a (311)A GaAs substrate, where the 2D hole gas at the GaAs/AlGaAs interface is separated from the Si-modulation doped layer by a 500Å AlGaAs undoped spacer. A standard four-terminal low-frequency lock-in technique has been used for resistivity measurements at temperatures down to 45 mK, with currents of 1-10 nA to avoid electron heating. The hole density $p$ is varied by the front gate voltage to provide the range of $r_s$ from 10 to 29 (with effective mass $m^*$ taken as 0.38 $m_0$).

Fig. 1a shows a typical temperature dependence of the longitudinal resistivity in our samples. The lower part of the plot, corresponding to higher densities, has a 'metallic' behaviour with $d\rho/dT > 0$. As the hole density is decreased, $\rho(T)$ becomes nonmonotonic, and further decreasing $p$ leads to an 'insulating' dependence with $d\rho/dT < 0$. In the 'metal' to 'insulator' crossover, where $r_s$ varies from 23 to 29, we have observed negative perpendicular magnetoresistance, Fig. 1b, which increases with lowering the hole density. It is natural to ascribe this effect to WL which occurs due to quantum interference of elastically scattered carriers on closed phase-coherent paths. However, great care should be taken in analysing WL in high-mobility structures.

Firstly, the application of the conventional theory of WL is based on the diffusion approximations and is restricted by the range of magnetic fields $B < B_T$, where $B_T = h/4D\tau$ is the 'transport' magnetic field, $D$ is diffusion coefficient and \( \tau \) is momentum relaxation time. Physically, this means that the magnetic length $L_B$ has to be larger than the mean free path $l$. Within this approach, no negative magnetoresistance is expected at $B > B_T$ when $L_B < l$. In our high mobility samples the value of $B_T$ is very small, ranging from 0.003 to 0.08 T for the densities studied. At the same time, the NMR is observed up to \( \sim 0.2 \) T, where Shubnikov-de Haas oscillations start. This means that even at $B > B_T$ there is a phase-breaking effect of magnetic field, which acts on the trajectories which are still smaller than $L_B$ (and smaller than $l$). The theory of WL in such a regime has been considered in several papers (1)(2), although no experimental tests of the theories have yet been performed.

Secondly, all theories of WL discuss the positive mag-
netoconductivity $\Delta \sigma_{xx}(B) = \delta \sigma_{xx}(T, B) - \delta \sigma_{xx}(T, 0)$, which is due to the decrease of the negative correction $\delta \sigma_{xx}(T, B)$ to the longitudinal classical (Drude) conductivity. Usually, the phase-breaking effect of magnetic field is seen at small fields where its effect on the Drude conductivity is negligible. In a high mobility system, however, the phase-breaking effect will coexist with the magnetic field dependence of the Drude conductivity itself:

$$\sigma_{xx}^D(B) = \frac{\sigma_0}{1 + (\mu B)^2},$$

where one cannot neglect the parameter $\mu B$. To analyse $\Delta \sigma_{xx}(B)$ due to WL, the classical (negative) magnetoconductivity has to be first subtracted.

Fig. 2 shows the total conductivity as a function of magnetic field, obtained by inversion of the resistivity tensor, $\sigma_{xx}^\text{tot}(B) = (1/\rho_{xx})/(1 + \rho_{xy}^2/\rho_{xx}^2)$, where $\rho_{xx}(B)$ and $\rho_{xy}(B)$ are measured simultaneously. The dotted line is the classical expression Eq. (1), with $\mu B = \rho_{xy}(B)\sigma_0$. The zero field conductivity $\sigma_0$ is used as an adjustable parameter to make the best fit in the higher field region where WL is expected to be totally suppressed. For $\rho_{xy}(B)$ we use the expression $\rho_{xy} = B/\epsilon_p$, where the density $\rho$ is determined from Shubnikov-de Haas oscillations in the higher field regime. The difference between the solid and dotted lines then gives the WL correction $\delta \sigma_{xx}(T, B)$, which decreases to zero at high fields where the classical field effect dominates. From this difference the zero-field value $\delta \sigma_{xx}(T, 0)$ is then subtracted to obtain the required dependence $\Delta \sigma_{xx}(B)$.

To analyse our experimental data, we use the WL theory developed beyond the diffusion approximation. It gives the magnetoconductivity at an arbitrary ratio $B/B_{tr}$, provided $\tau < \tau_\varphi$ is satisfied:

$$\Delta \sigma(B) = \frac{-e^2}{\pi h(1 + \gamma)^2} \left[ \sum_{n=0}^{N} \left( \frac{b \cdot \psi_n(b)}{1 + \gamma - \psi_n(b)} \right) - \ln \frac{1 + \gamma}{\gamma} \right],$$

where $\gamma = \tau/\tau_\varphi$, $\tau_\varphi$ is the dephasing time, $b = \frac{1}{1+\gamma^2} \frac{B}{B_{tr}}$, $\psi_n(b) = \int_0^\infty d\xi \cdot e^{-\xi - b \xi^2/4} L_n(b \xi^2/2)$, and $L_n$ are the Laguerre polynomials. In Fig. 3a we show representative data at different densities in the middle of the temperature range studied, plotted against dimensionless magnetic field $B/B_{tr}$ ($B_{tr}$ is found as $(4\pi h \mu_0 \sigma_0/e^2)^{-1}$). Solid lines in Fig. 3 are obtained from Eq. (2), where $\gamma$ is used as an adjustable parameter. At lowest $p$ and $T$ the error in determining $\gamma$ is 10%, but as the density or/and temperature increases it steeply drops to 5%. The obtained $\gamma$-values range from 0.04 to 0.43 and satisfy the condition $\tau < \tau_\varphi$.

The apparent agreement with WL theory suggests that, surprisingly, even at $\tau_\varphi \sim 23 - 29$ the Fermi-liquid description of the system remains valid. The further evidence of this has been obtained from the analysis of the temperature dependence $\tau_\varphi^{-1}(T)$ of the dephasing rate. Estimations show that the contribution to $\tau_\varphi^{-1}(T)$ of electron-phonon scattering is negligible in the studied temperature range. According to the Fermi-liquid theory, the dephasing rate due to electron-electron scattering is dominated either by a linear or quadratic term, dependent on the parameter $\tau k_B T/h$:

$$\tau_\varphi^{-1}(T) = \frac{\alpha (k_B T)^2}{\hbar E_F} \ln \frac{4 E_F}{k_B T},$$

when $k_B T \tau/h \gg 1$ (3)

$$\tau_\varphi^{-1}(T) = \frac{k_B T}{2 E_F \tau} \ln \left( \frac{2 E_F \tau}{\hbar} \right),$$

when $k_B T \tau/h \ll 1$ (4)

where $\alpha = \pi/8$. The quadratic term in Eq. (3) is due to Landau-Babar scattering associated with collisions in a clean Fermi-liquid with large momentum transfer, and the linear term in Eq. (4) corresponds to particle-particle interactions with small energy transfer in disordered conductors. In the experiment, the parameter $k_B T \tau/h$ varies from 0.06 to 0.8 for the lowest studied density and from 0.1 to 0.9 for the highest density, so that we need to examine the applicability of both expressions to the dependence $\tau_\varphi^{-1}(T)$ extracted from the analysis of the magnetoconductance data, Fig. 4a.

It is interesting to note that in the whole range of $p$, including the lowest densities with 'insulating' dependence $\rho(T)$, we have seen Shubnikov-de Haas oscillations. In the studied sample a shift of the Shubnikov-de Haas minima was seen with increasing temperature from 45 mK to 600 mK, indicating a weak ($\sim 10\%$) increase of the hole density. Thus, it was convenient to analyse the dephasing rate as the product $\tau_p^{-1} \cdot p$, with density $p$ directly measured at each temperature by the Shubnikov-de Haas effect. In this case Eqs. (3-4) are re-written as

$$\tau_p^{-1} \cdot p = \frac{m^*}{\pi \hbar} \left[ \alpha k_B T^2 \ln \frac{4 E_F}{k_B T} \right],$$

$$\tau_p^{-1} \cdot p = \frac{m^*}{\pi \hbar^3} \left[ k_B T \ln \left( \frac{2 E_F \tau}{\hbar} \right) \right]$$

The experimental curves in Fig. 4a show two distinct features: a non-linear form and a saturation at low temperatures. We have established that in the entire range of hole densities and temperatures, the data are well described by the quadratic term, Eq. (3), with a zero-temperature saturation value $1/\tau_p(T = 0) = 1/\tau_p^\text{sat}$ added to it. In the analysis of $\tau_p(T)$ we used values $p, E_F$ and $\tau$ experimentally determined at each $T$. Coefficient $\alpha$, found as an adjustable parameter, agrees within 20% accuracy with the value $\pi/8$ for all hole densities, inset to Fig. 4a. At the same time, the data show that the linear term in the dephasing rate is suppressed by more than an order of magnitude compared with the value estimated using Eq. 6 [8]. The expected contribution to the dephasing rate for the middle density $p = 1.3 \times 10^{10}$ cm$^{-2}$ is shown in Fig. 4a as a solid line (which is practically a straight line due to the weak ($\sim 10\%$) temperature dependence of $\tau$ and $E_F$).
We suggest two possible reasons for the suppression of the linear term. According to the conventional theories, the $T$-term originates from electron-electron scattering in systems with a diffusive character of transport, when the interaction potential between electrons is weak ($r_s << 1$). The interactions in our system are strong. In addition, in our high mobility system, the coherent paths of size $L_c$ contain only a small number of scatterers ($\sim 10$), thus the diffusion approximation may not be valid. The non-diffusive transport does not affect, however, the $T^2$ term which is an intrinsic property of clean systems.

Let us briefly discuss the saturation of $\tau^{-1}_s$. The problem of saturation of the dephasing rate at low $T$ has been known for many years \[14\][16], with several explanations of this effect suggested. A characteristic feature of the saturation in our case is that it becomes more pronounced with increasing density. Then a possible origin of the saturation can be a nonequilibrium external noise which does not disturb the temperature dependence of $\tau^{-1}_s$ in the Fermi liquid and manifests itself simply as an additive to the dephasing rate \[4\]. According to \[14\] the saturation due to noise is $1/\tau^{-1}_s \sim D^{1/5}(\Omega E_{ac})^{1/5}$, where $\Omega$ is the radiation frequency and $E_{ac}$ is amplitude of the ac electric field. In Fig. 4b the value $1/\tau^{-1}_s$ is plotted as a function of $D$ and shows agreement with the $D^{1/5}$ dependence.

In a conventional Fermi liquid, WL is usually accompanied by electron-electron interaction effects, which are seen as quantum corrections to the conductivity and Hall coefficient. At small $r_s$, the two corrections are related as $\delta R_H(T)/R_H = -2\delta \sigma(T)/\sigma$ \[2\]. In \[2\] it was argued that in p-GaAs heterostructures the weak hole-hole interaction effect persists up to $r_s \sim 10 - 14$. Now we have measured the temperature dependence of the Hall coefficient at much larger $r_s$. In Fig. 4c we plot the Hall coefficient as $(R_H T)^{-1}$ for different temperatures in the range from 45 to 400 mK at the density $p = 1.45 \times 10^{10}$ cm$^{-2}$ (solid squares). The decrease of $R_H$ with increasing $T$ appears to be of the same order of magnitude as that estimated from theory \[2\]. However, the observed shift of Shubnikov-de Haas minima has indicated an increase of the hole density with temperature in this experiment (circles in Fig. 4c), such that it agrees well, within 2%, with the change in $R_H$. The inset in Fig.4c shows good agreement between the densities measured by Hall and Shubnikov-de Haas effects at different $V_g$ for $T = 45$ mK. One can then conclude that the interaction effects in the Hall coefficient appear to be much weaker than expected from the perturbation theory derived at small $r_s$. We believe that this suppression of the temperature dependence in the Hall coefficient has the same origin as the absence of the linear term in the dephasing rate, namely strong interactions at large $r_s$ and, possibly, breakdown of the diffusion approximation.

To summarise, we have investigated the applicability of the Fermi-liquid description to a high mobility, low density 2D hole gas with large $r_s$, approaching the conditions of expected Wigner crystallisation. We have found that the negative magnetoresistance in the crossover region from 'metal' to 'insulator' persists up to $r_s \sim 29$, and is caused by weak localisation. The dephasing rate $\tau^{-1}_s(T)$ is dominated by a $T^2$-contribution due to Landau-Baber scattering, which is a characteristic property of a Fermi-liquid in the clean limit. At the same time, the linear in $T$-term in the scattering rate, due to scattering with small energy transfer, is suppressed. Its decrease is accompanied by absence of the temperature dependence in the Hall coefficient. This demonstrates directly that conventional understanding of interaction effects, developed at $r_s \ll 1$, has to be modified for a high-mobility Fermi liquid with strong interactions.

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Figure captions:

Fig. 1 (a) Temperature dependence of the resistivity at different hole densities. The dashed box encloses the domain of the negative magnetoresistance (NMR) study. (b) NMR at $T = 45$ mK, for different hole densities.

Fig. 2 (a,b) Experimental (total) conductivity $\sigma_{xx}(B)$ shown by solid lines in the figures and circles in the insets (the insets present zoomed-in regions). Dashed lines in the insets represent the classical magnetoconductivity Eq. (1).

Fig. 3 Magnetoconductivity $\Delta \sigma_{xx}^{WL}$ as a function of dimensionless magnetic field ($B_{tr} = \hbar/4De\tau$). (a) $p = 1.17; 1.21; 1.3; 1.45; 1.7 \times 10^{10}$ cm$^{-2}$ at $T = 200$ mK; (b) $p = 1.21 \times 10^{10}$ cm$^{-2}$ at $T = 45, 120, 200, 300, 400, 500$ mK. Solid lines are fit to Eq. (2).

Fig. 4 (a) Temperature dependence of the dephasing rate at different gate voltages. The low-temperature densities from bottom to top are: $p = 1.17; 1.21; 1.3; 1.45; 1.7 \times 10^{10}$ cm$^{-2}$. Solid lines are fits to Eq. (3) with the values of $\alpha$ shown in the inset. The straight line is plotted using equation Eq. (4).

(b) The saturation value of the dephasing rate at $T = 0$ against diffusion coefficient.

(c) The Hall coefficient at different temperatures, presented as $(eR_H)^{-1}$ (solid squares) for $p = 1.45 \times 10^{10}$ cm$^{-2}$ at $T = 45$ mK. The density determined from the Shubnikov-de Haas effect is shown by open circles. Inset: hole density measured by the Hall and Shubnikov-de Haas effects at different $V_g$, $T = 45$ mK.
For (a), with $p = 10^{10}$ cm$^{-2}$, the value of $\rho_{xx}$ (kΩ) changes with $T$ (K). For (b), with $T = 45$ mK, the value of $\rho_{xx}$ (kΩ) also changes with $B$ (T).
(a) $T = 45 \text{ mK}$
\[ p = 1.17 \times 10^{10} \text{ cm}^{-2} \]

(b) $T = 45 \text{ mK}$
\[ p = 1.45 \times 10^{10} \text{ cm}^{-2} \]
(a) $\Delta \sigma_{\text{WL}} = 1.7 \times 10^{10}$

$T = 200 \text{ mK}$

$p_h = 1.17 \times 10^{10} \text{ cm}^{-2}$

(b) $\Delta \sigma_{\text{WL}}$

$T = 45 \text{ mK}$

$p_h = 1.21 \times 10^{10} \text{ cm}^{-2}$

$T = 500 \text{ mK}$
\( \frac{1}{\tau_{\phi}} \) (10^{11} s^{-1}) \cdot p \left( 10^{10} \text{ cm}^{-2} \right)

\( D^{1/5} \) (cm\(^2\)/s\(^{1/5}\))

\( T \) (K)

\( (eR_H)^{-1}, p \left( 10^{10} \text{ cm}^{-2} \right) \)