Time-domain Simulation and Nonlinear Analysis on Ride Performance of Four-wheel Vehicles

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Abstract. A nonlinear dynamic model with eight DOFs of a four-wheel vehicle is established in this paper. After detaching the nonlinear characteristics of the leaf springs and shock absorbers, the multi-step linearizing method is used to simulate the vehicle vibration in time domain, under a correlated four-wheel road roughness model. Experimental verifications suggest that the newly built vehicle model and simulation procedure are reasonable and feasible to be used in vehicle vibration analysis. Furthermore, some nonlinear factors of the leaf springs and shock absorbers, which affect the vehicle ride performance (or comfort), are investigated under different vehicle running speeds. Some substantial rules of the nonlinear vehicle vibrations are revealed in this paper.

1. Introduction
As the standard of living has been improving, people are more concerned about the ride comfort of their cars. To improve the ride comfort, a great deal of research on vehicle vibration systems including road roughness was issued from theory and experiment fields in the past few decades. Most of these research works were based on linear or nonlinear models such as SDOF and MDOF with road excitations in terms of PSD or time course [1-3]. Some research involved the parameter optimization of passive suspension, and moreover, the active and semi-active suspension studies [4,5]. It can be seen that most of the research mentioned above assumed that the vehicles were running at constant speeds and oscillating in small amplitudes, therefore, the vehicle vibrations were regarded as linear and stationary random processes. In more usual cases such as starting, accelerating and braking, vehicle vibration systems working at variable running speeds show their nonlinear and nonstationary characteristics accordingly [6]. Because of theory restriction, however, there are only a few of efforts to vehicle nonlinear vibrations were performed. Recently, nonlinear theories have been developed and applied in some engineering fields [7-10]. In this paper, some nonlinear factors of vehicle suspensions are introduced into a vehicle model with eight DOFs, time-domain simulations and experimental verifications are performed for vibration evaluations. Furthermore, the nonlinear factors of the vehicle suspension contribute to ride comfort of the sample vehicle are studied.

2. System modelling
2.1. Dynamic modelling of a vehicle

As known, a real vehicle is a very complex nonlinear system with MDOF. It is almost impossible to build an exact model to describe the vehicle vibration processes. In this paper, based on the SY6480 light bus, a nonlinear dynamic model with eight degrees of freedom is established, shown in figure 1, on the assumption that the vehicle has symmetrical weight with respect to \(X\) axis, and road inputs are isotropic ergodic processes, ignoring tire damping and any other vibration sources except for road roughness. In figure 1, the origin of the coordinate system is at the center of gravity of the vehicle body, the \(X\), \(Y\), and \(Z\) axes point to the running direction, the left and the upper of the vehicle, respectively. The eight degrees of freedom are: the vertical displacement of the vehicle body (\(Z_B\)), the roll angle of the vehicle body (\(\theta_B\)), the pitch angle of the vehicle body (\(\phi_B\)), the vertical displacements of the left- and right-unsprung masses of the front axle (\(Z_{fl}\) and \(Z_{fr}\)), the vertical and angular displacements of the rear axle (\(Z_r\) and \(\theta_r\)), and the vertical displacement of the driver seat (\(Z_s\)), respectively.

Mathematically, differential equations at a moment \(t\) of the model in figure 1 can be derived from the Lagrange equation as,

\[
[M] \ddot{\mathbf{z}}_t + [C(t)] \dot{\mathbf{z}}_t + [K(t)] \mathbf{z}_t = [P] \mathbf{I}(t)
\]  

(2.1)

where, \([M]\), \([C(t)]\) and \([K(t)]\) are the mass, damping and stiffness matrices, \([P]\) is the transfer matrix from road displacement \(\{I(t)\}\) to road force vector, \(\{\mathbf{z}_t\}\) are the system responses. Note that \([C(t)]\) and \([K(t)]\) are constant matrices within a small time interval \([t, t + \Delta t]\), but are variable with the time changing, their values depend on the system vibration velocity and displacement, respectively.

![Figure 1. Nonlinear dynamic model of a vehicle with eight DOFs](image)

2.2. Nonlinear modelling of the leaf springs

Leaf springs, as nonlinear components with sluggish damping in vehicle suspensions, are difficult to be modeled. An in-situ nonlinear model was adopted in this paper [11]. It can be shown as,

\[
F_s = k_{s1}x + k_{s2}x^3
\]  

(2.2)

where, \(F_s\) is the elastic force, \(x\) is the spring distortion, \(k_{s1}\) and \(k_{s2}\) are the stiffness coefficients of the leaf spring. Figure 2 shows the relationship between \(F_s\) and \(x\). As seen that, \(x(t)\) leads to \(F_s(x(t))\) at the moment \(t\), and after a time increment \(\Delta t\), the increments of \(x\) and \(F_s\) are shown as \(\Delta x(t)\) and \(\Delta F_s(x(t))\). If \(\Delta t\) is small enough, let \(k(t) = \frac{dF_s}{dx}\), the \(\Delta F_s\) at the moment \(t\) can be expressed as,

\[
\Delta F_s(t) = \Delta F_s(x(t)) = k(t) \Delta x(t)
\]  

(2.3)
Therefore, the fact that the leaf spring is linear in a small time interval can be taken into account.

2.3. Nonlinear modelling of the shock absorbers
In this work, a nonlinear mathematical model of the shock absorbers of SY6480 bus is described by a polynomial expression, which was derived by fitting a set of tested data from a shock absorber test-bed, using the least square method. The polynomial expression can be written into three sections as,

\[ F_D = \begin{cases} 
  c_{dm} \dot{x}^m & -1.0 \leq \dot{x} \leq 0 \\
  c_{dm} \dot{x}^m & 0 < \dot{x} \leq 1.0 \\
  c \dot{x} & \text{others} 
\end{cases} \] (2.4)

where, \( F_D \) is the damping force, \( \dot{x} \) is the piston velocity, \( c_{dm} \), \( c_{du} \) and \( c \) are the damping coefficients in the corresponding model orders \( m \) and \( n \) of the shock absorber, here \( m \) and \( n \) equal to 1, 2, 3, 4, 5, respectively. The velocity characteristic of the shock absorber is shown in figure 3. Analogy with the above detaching procedure of the leaf springs, detaching results of the shock absorber suggest that, in a small \( \Delta t \), let \( c(t) = dF_D/d\dot{x} \), the damping force increment at the moment \( t \) can be expressed as that in equation (2.5),

\[ \Delta F_D(t) = \Delta F_D(\dot{x}(t)) = c(t) \Delta \dot{x}(t) \] (2.5)

A conclusion may be drawn that the relationship between the damping force and the piston velocity of the shock absorber is linear within a small time interval.

![Figure 2. Nonlinear characteristic of the leaf spring](image)

![Figure 3. Nonlinear characteristic of the shock absorber](image)

3. System simulation

3.1. Correlated four-wheel road model
A road model, so-called the correlated four-wheel road roughness derived in the past literature [12], was used in this paper. Mathematically, it was written as follows,

\[ \{ \dot{f}(t) \} = [F_o] \{ f(t) \} + 2\pi \sqrt{G_o V} [B_o] \{ x(t) \} + 2\pi \sqrt{G_o V} [B_o] W(t) \] (3.1a)

\[ \{ \dot{x}(t) \} = [A_o] \{ x(t) \} + [B_o] W(t) \] (3.1b)

where, \( \{ f(t) \} \) is the road roughnesses of four wheels; \( \{ x(t) \} \) is the state transfer vector; \( W(t) \) is a row vector that consists of a white-noise series with a zero mean value each; \( [F_o], [B_o], [A_o] \) and \( [B_o] \) are the coefficient matrices; \( V \) is the vehicle running speed; and \( G_o \) is the road coefficient, representing different grades of road.
3.2. Simulation of nonlinear vehicle vibration

The Runge-Kutta method is used to solve the system equations in the present work. Thus, it is necessary to rewrite equation (2.1) into a form of state equations. Multiply both sides of equation (2.1) by \([M]^{-1}\), and let 
\[
\begin{bmatrix}
\dot{z}_t \\
\dot{x}_t
\end{bmatrix} =
\begin{bmatrix}
[M]^{-1}z_t \\
[M]^{-1}x_t
\end{bmatrix},
\begin{bmatrix}
d(t) \\
I(t)
\end{bmatrix} =
\begin{bmatrix}
-M^{-1}C(t) \\
-M^{-1}K(t)
\end{bmatrix} \begin{bmatrix}
\dot{z}_t \\
\dot{x}_t
\end{bmatrix},
\begin{bmatrix}
B(t) \\
0
\end{bmatrix} = 0,
\begin{bmatrix}
I(t)
\end{bmatrix} = \begin{bmatrix}
P[I(t)]
\end{bmatrix},
\]
the system state equations can be expressed as,
\[
\begin{align*}
\dot{x}_t &= \begin{bmatrix}
0
\end{bmatrix}_{8 \times 1},
\begin{bmatrix}
E
\end{bmatrix}_{8 \times 8} \begin{bmatrix}
B(t)
\end{bmatrix}_{8 \times 1} + \begin{bmatrix}
A(t)
\end{bmatrix}_{8 \times 8} \begin{bmatrix}
\dot{x}_t
\end{bmatrix}_{16 \times 1} + \begin{bmatrix}
0
\end{bmatrix}_{8 \times 1},
\begin{bmatrix}
I(t)
\end{bmatrix} = \begin{bmatrix}
0
\end{bmatrix}_{8 \times 1},
\end{align*}
\]
Here \([x_t]_{16 \times 1}\) is the state transfer vector with 16 dimension, \([E]\) is an 8-dimensional unit matrix, and \([I(t)]\) is the vector of force excitation.

The multi-step linearizing approach is considered to obtain the vehicle responses, and the following steps are performed in this paper. Taking an enough length of time \([0,T]\) and dividing it into \(m\) sections, we get a time series \(\{t_0, t_1, \ldots, t_m\}\) with a small interval \(\Delta t\). Then, using the Runge-Kutta method, the system responses in \([t_i, t_{i+1}]\) can be easily calculated by solving equation (3.2) in each time interval \(\Delta t\), and therefore the responses in the studied time length \([0, T]\). Figures 4 and 5 show the calculated accelerations and their spectra at the eight degrees of freedom of interest. Note that the equilibrium position of the suspension is assumed as the vehicle initial state in the simulation procedure.

4. Experimental verification

Following the international standard ISO2631, experimental verifications are performed by setting a SY6480 light bus with good working conditions running on a straight gravel road at speeds of 30, 40, 50, and 60km/h, respectively. The selected parameters in signal processing are: the cutoff frequency \(f_c = 100Hz\), the sampling interval \(\Delta t = 5ms\), the frequency resolution \(\Delta f = 0.1935Hz\), and the Hanning window. The acceleration sensors are fixed on the midpoints of the driver seat, the rear axle

![Figure 4.
Calculated accelerations at the eight DOFs of the vibration system, where x-axis denotes time \(t\) (s) and y-axis denotes accelerations \(z_x\) ((m/s^2) or (rad/s^2)))](image)
5. Effect of Nonlinear Factors

As known that responses of a nonlinear system are abnormal and variable with time changing. Assuming the sample vehicle is running on a gravel road at different speeds, the nonlinear vehicle responses are simulated in this paper. Moreover, the averaged values of stiffnesses of the leaf springs, damping coefficients of the shock absorbers, partial frequencies of the rear suspension, the relative damping ratios of the front and rear suspensions are calculated and listed in table 1. Here the averaged values are defined by averaging the corresponding values at all the time points \( \{ t_0, t_1, t_2, \ldots, t_m \} \) in the time interval \([0, T]\). As seen in Table 1, when the vehicle speed increases from 10km/h to 70km/h, the averaged stiffness of the leaf spring and averaged damping coefficients of the shock absorbers changed so much that the averaged relative damping ratios of the front and the rear suspensions increased from 0.1638 to 0.3131 and from 0.1706 to 0.2520, respectively. As a result, the averaged
Figure 7. Comparison of the acceleration spectra of the driver seat

Table 1. The parameters changing with vehicle speed of the leaf spring and shock absorber

| Vehicle running speed $V$ (km/h) | 10    | 20    | 30    | 50    | 70    |
|----------------------------------|-------|-------|-------|-------|-------|
| Averaged stiffness of the leaf spring $k$ (N/m) | 154620| 119200| 97950 | 90890 | 90920 |
| Averaged damping coefficients of the shock absorber $c$ (N·s/m) | Front 1921.9 | 2435 | 2921.9 | 3658.3 | 3673.9 |
| | Rear 6585.8 | 6654.4 | 7082.9 | 7645.8 | 7460.2 |
| Averaged partial frequency of the rear suspension $f_r$ (Hz) | 3.2904 | 2.8891 | 2.6190 | 2.5228 | 2.5232 |
| Averaged relative damping ratios of the suspensions $\zeta$ | Front 0.1638 | 0.2075 | 0.2490 | 0.3117 | 0.3131 |
| | Rear 0.1706 | 0.1962 | 0.2304 | 0.2578 | 0.2520 |

partial frequencies of the rear suspension decreased from 3.29Hz to 2.52Hz. We draw a conclusion that the leaf springs and the shock absorbers in vehicle suspensions have biggish nonlinear characteristics, cannot be dealt with as linear elements in the vehicle vibration studies.

Nonlinearity of suspension elements impacting on vehicle’s ride comfort depends on the nonlinear parameters in the vehicle suspensions. In the models of the leaf springs and shock absorbers, see equations (2.2) and (2.4), the nonlinearity of leaf springs relies on the three-order stiffness $k_{s2}$. Figures 8 and 9 show the relationships between $k_{s2}$ and the partial frequencies and the relative damping ratios of the rear suspension. Generally say, with the $k_{s2}$ increasing, the partial frequencies tend to increase but the relative damping ratios change inversely. The $k_{s2}$ effects are also influenced by the running speeds of the vehicle, as such that, with the vehicle speed increasing, the nonlinear effects of $k_{s2}$ on the partial frequencies and relative damping ratios are getting slighter. When the vehicle speed exceeds 50km/h on the gravel road, the averaged partial frequencies change to a constant of 2.52Hz, see figure 8. The nonlinearity of shock absorbers behaves as the orders and values of the nonlinear damping coefficients in the polynomial expression in equation (2.4). This equation reveals that the lower the expression order and the bigger the nonlinear damping coefficients are, the severer the nonlinear factors influence vehicle vibrations.

6. Conclusions

This paper presented a set of approaches for solving the nonlinear vibration problem of road vehicles.
Based on the SY6480 light bus, a nonlinear vehicle model with eight DOFs was established, considering the nonlinear characteristics of the leaf springs and shock absorbers. Under a road model called correlated four-wheel road roughness, the vehicle vibration at different working cases was simulated in time domain by using the multi-step linearizing method. Experimental verification suggested that the simulation approaches proposed in this work are reliable and feasible. The analysis of the nonlinear factors effect on the ride comfort implied that it is necessary to consider the nonlinearities of leaf springs and shock absorbers. It can be also concluded that the lower the road excitation strength and vehicle running speed are, the severer the suspension nonlinear factors affect ride comfort of vehicles.

It should be mentioned that the work done in this paper may be extended to simulate any other four-wheel vehicle running on any grade of road. Based on the proposed approaches, further research may be assumed to match and optimize nonlinear parameters of vehicle suspensions, and therefore improve ride performance of vehicles.

Acknowledgements
This work has been supported by the Research Foundations of the Ministry of Education of China, the State Administration of Foreign Experts Affairs of China, and partially supported by the Research Fund of the Educational Department of Liaoning Province, China.

References
[1] Dodds C J and Robson J D 1973 J. Sound Vib. 31 175-183
[2] Ge Z M, Hsiao C L and Chen Y S 2007 Int. J. Nonlinear Sci. 8(1) 89-100
[3] Barnett W A, Serletis A and Serletis D 2006 Int. J. Nonlinear Sci. 7(2) 191-196
[4] Hac A 1985 J. Sound Vib. 100(3) 343-357
[5] Haday M B A and Crolla D A 1989 Proc. Ins. Eng. J. Automobile Engineering. D 203 125-135
[6] Zhang L J, Lee C M and Wang Y S 2002 Int. J. Automotive Technology. 3(3) 101-109
[7] Zhang W, Chen Y and Cao D X 2006 Int. J. Nonlinear Sci. 7(1) 35-58
[8] Li S J and Liu Y X 2006 Int. J. Nonlinear Sci. 7(2) 177-182
[9] Liu C X and Liu T L 2006 Int. J. Nonlinear Sci. 7(3) 345-352
[10] Meng Z Q, Meng G and Li H G 2007 Int. J. Nonlinear Sci. 8(1) 21-30
[11] Liu C and Wang Z F 1996 J. Wuhan Automotive Engineering University. 18(2) 1-5
[12] Wang Y S, Geng A L and Zhang L J 2004 Chinese J. Automotive Engineering. 26(2) 177-182