Astrophysical Bounds on the Photon Charge and Magnetic Moment

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Abstract

If the photon possessed an electric charge or a magnetic moment, light waves propagating through magnetic fields would acquire new quantum mechanical phases. For a charged photon, this is an Aharonov-Bohm phase, and the fact that we can resolve distant galaxies using radio interferometry indicates that this phase must be small. This in turn constrains the photon charge to be smaller than $10^{-32} \, e$ if all photons have the same charge and smaller than $10^{-46} \, e$ if there are both positively and negatively charged photons. The best bound on the magnetic moment comes from the observed absence of wavelength-independent photon birefringence. Birefringence measurements, which compare the relative phases of right- and left-circularly polarized waves, restrict the magnetic moment to be less than $10^{-24} \, e \, cm$. This is just a few orders of magnitude weaker than the experimental bounds on the electron and neutron electric dipole moments.
1 Introduction

So far as we know, photons—quanta of the electromagnetic field—have zero mass, zero charge, and zero magnetic moment. These three properties are all crucial to our understanding of electromagnetism; they imply that the purely electromagnetic sector of the standard model is scale invariant and noninteracting.

However, it is still interesting to ask how well we really know that all these quantum numbers are zero. This can be a tricky question to ask just about the photon mass, but it is quite a bit trickier for the photon charge and magnetic moment, which control the coupling of the electromagnetic field to itself. There are at least three fully dynamical theories that can explain a photon mass—the Proca [1], Higgs [2, 3, 4], and Stueckelberg [5] models—but no reasonable theory of a self-interacting photon has ever been proposed. There are, of course, theories with interacting gauge bosons—the non-Abelian gauge theories. However, the existence of a non-Abelian gauge symmetry immediately implies the existence of a multiplet of vector bosons with different charges. The charges in these theories are also quantized. The fact that there is only one known type of photon, whose charge must be many orders of magnitude smaller than the standard quantum of charge (the proton charge $e$), rules out non-Abelian gauge theories as viable theories of a self-interacting photon.

There are more immediate physical differences between a photon mass and charge as well. The mass parameter in the Lagrangian of a Proca, Higgs, or Stueckelberg theory has meaning at the classical level; it describes the the gap in the dispersion relation for classical electromagnetic waves. A photon charge or magnetic moment, on the other hand, is intrinsically quantum mechanical in nature. The electromagnetic couplings of a propagating wave are crucially tied to the decomposition of that wave as a collection of quantized photons.

Because a putative photon mass is easier to accommodate than a photon charge or dipole moment, much more attention has been paid to it. It is also a great deal easier to place bounds on a quantity (like the mass) for which a complete description of the physics exists. In 2006, the Particle Data Group [6] listed only four bounds on the photon charge, compared with fifteen on the photon mass. There were no bounds listed for the photon’s dipole moment, even though the photon has an intrinsic spin. Yet the photon charge and magnetic moment are at least sufficiently interesting that we would like to know how tightly they can really be constrained.

However, the absence of a complete theory describing electrically or magnetically interacting photons complicates the task of placing bounds on the strength of such putative interactions. Again, this is in contrast with the case of a photon mass, which can be bounded in many different ways. The dynamical stability of magnetized galaxies gives the strongest limit on the photon mass parameter [7], although there are many assumptions associated with such a bound. The simple existence of a galaxy-scale magnetic field places limits on a possible Proca mass for the photon, and a large ambient vector potential associ-
ated with such a field could give rise to laboratory-measurable torques [8, 9]. More secure bounds, which do not require any inferences about fields on galactic scales, come from observations of very low frequency magneto-hydrodynamic waves in nebulae [10], direct tests looking for the screening of Earth’s [11] or Jupiter’s [12] magnetic field, and measurements combining screening of the solar field and magneto-hydrodynamic effects [13].

These tests are possible because we can interpret the results of our observations in the context of a well-defined theory. This is usually taken to be the Proca theory; there are critical differences if the photon mass arises from spontaneous symmetry breaking [14]. Without a complete theory describing the photon charge, we must rely on rather different techniques. For example, while positing the existence of a photon charge does tell us things about how photon propagation must be affected, it does not tell us anything reliable about how static electromagnetic fields will be modified.

In fact, the question of static fields in a charged photon theory is rather problematic. A macroscopic field can be “built up” from photons; yet even in strong fields, there is no evidence that the electromagnetic field carries charge. One can get around this by supposing that photons with multiple charges exist—that there are both positively and negatively charged photons and possibly neutral ones as well. Whether this is actually an improvement in the situation, however, is a matter of opinion. Having photons with different charges makes it possible to build up macroscopic fields that obey charge neutrality, but it also means that photons with different charges must be able to interfere, since experimentally, photons have never been observed not to interfere. One cannot avoid this by claiming that only one type of photon is ever actually being observed, the others being somehow invisible and not interacting with our detectors, because what we mean by a photon is really just the electromagnetic quantum that we can observe. If there are other quanta with different charges that we do not see, they are not photons in any meaningful sense.

Having interference between particles of unequal charges violates local charge conservation and charge superselection. However, this may not be such a serious difficulty. Any new theory that contains charged photons is unlikely to preserve gauge invariance. The Proca and Higgs theories describing photon masses involve either explicit or spontaneous breaking of gauge symmetry. It is easy to believe that a charged photon theory will lose gauge invariance as well.

However, having both positive and negative photons is potentially problematical for another reason as well. Assuming that these charged particles are massless, photon pairs could be produced at no energy cost. This could lead to complete screening of charges. Such screening does occur in theories with massless charged fermions [15]; charges are confined in neutral meson-like excitations. It seems reasonable that this might be a general feature of theories featuring massless charges.

For a long time, the best laboratory bound on the photon charge came from an old experiment looking for evidence of fractional charges [16]. The experimental setup had a light beam shining on an iron spheroid and an arrangement for monitoring the charge
on the spheroid. The photon charge bound (which was largely incidental) was derived by dividing the total change in the spheroid’s charge over an extended experimental run by the number of photons absorbed by the spheroid during that time. This seems a straightforward enough procedure, but it has a serious deficiency. According to quantum electrodynamics, during any photon absorption event, a charged particle will emit a large number of extremely low frequency bremsstrahlung photons. Formally, the number emitted is infinite, limited only by an infrared regulator \cite{17}. This fact is, in itself, another problem for theories with charged photons, since our current understanding indicates that every photon interaction event increases the number of propagating photons present by an infinite number. To get around the infinity, we would expect that the bremsstrahlung should be drastically modified at very low frequencies by the presence of even a very small photon charge; at sufficiently low frequencies, the electromagnetic interaction energy is going to dominate the usual photon energy. But at higher yet still quite small frequencies, the bremsstrahlung emission is a real effect, which the experimental bounds do not take into account. This casts this particular laboratory result into serious doubt, and it illustrates the difficulty associated with trying to measure a quantity without a full theory describing it.

Careful consideration of the preceding laboratory experiment is also useful for another reason. The bound on the photon charge derived from that experiment, which was at the $10^{-16} e$ level, gives an indication of what ranges of photons charges could be considered “large” and “small.” The $10^{-16} e$ bound indicates what magnitude the photon charge would have to have if the total number of photons in the experiment were to have an aggregate charge that would be observable by macroscopic means. This is a very rough dividing line between the “large” and “small” charge regimes. Because photons are so numerous, they can have individual charges that are many orders of magnitude smaller than $e$, yet still carry a significant charge in bulk.

Any bound on the photon charge or magnetic moment is going to be somewhat uncertain, but some bounds are more robust that others. It is quite easy to show that the photon charge is very, very small. One can simply measure the change in a photon’s energy between a source and detector placed at different voltages. This is a suitable experiment for an undergraduate laboratory course, and using Mössbauer spectroscopy, students may place bounds at the $\sim 10^{-10} e$ level in only a few hours.

More sophisticated bounds on the photon charge have often been based on the observation that a charged photon would be deflected by a magnetic field. The photons and fields involved may be either astrophysical \cite{18, 19} or created in the laboratory \cite{20}, and the best bound arrived at using this strategy is at the $4 \times 10^{-31} e$ level \cite{19} if all photons have the same charge and the $3 \times 10^{-33} e$ level if multiple charges exist. Also associated with the deflection of charged photons is a frequency-dependent time delay for photons reaching Earth; less energetic photons from a given source will be deflected onto longer arcing trajectories. The best bound inferred from the sharpness of pulsar pulses is $5 \times 10^{-30} e$, reported in \cite{21} (which corrects an error in \cite{22}). All the astrophysical bounds
require information about magnetic fields in space.

If we observe photons coming from a single source over a range of energies $\Delta E$ centered around $E$, and the angular spread of these photons is $\Delta \theta$ after they have traveled a distance $L$ through a magnetic field $B$ perpendicular to their path, we can constrain the photon charge $q$ to be

$$\frac{|q|}{e} < \frac{2E^2\Delta \theta}{eBL\Delta E}. \quad (1)$$

If extragalactic field configurations are used, $BL$ would be replaced by something of order $B\sqrt{\lambda C}$, where $\lambda C$ is the correlation length of the field. When this bound on $q$ is expressed in terms of the photon energy (as opposed to frequency), the result is independent of $\hbar$.

The deflection decreases with the photon energy, since higher-energy photons possess more momentum and are thus less deflected by the energy-independent Lorentz force. The bound (1) applies to photons all of the same charge. If there are both positively and negatively charged photons, they will be deflected in opposite directions, leading to a more sensitive bound that does not depend on the photons having a finite spread in energy. Instead, the bound is

$$\frac{|q|}{e} < \frac{E\Delta \theta}{eBL}. \quad (2)$$

Indirect searches for the effects of a photon charge are also possible. Measurements of the anisotropy of the cosmic microwave background (CMB) can be used to place strong constraints on the physics of the early universe. In the radiation dominated era, a photon charge could make huge contributions to the energy density, but no signature of this is seen in the CMB [23]. This leads to bounds on the photon charge, which depend somewhat on how the conductivity of the early epoch is modeled. If there are no cancellations between different species, and a nonzero photon charge disturbed the overall charge neutrality of the early universe, the best limit on the photon charge is at the $10^{-35} e$ level [24]. Importantly, there are no CMB bounds for the case of multiple photon charges. In that case, the photon gas remains neutral, and there is no buildup of potential energy when many photons are crammed together.

The purpose of this paper is to explore new techniques for bounding the photon charge and the previously little discussed photon magnetic moment. We can place bounds on the charge using the quantum mechanical Aharonov-Bohm effect. Charged particles moving along different paths through a magnetic field pick up different phases, and the observed coherence of photons from distant astrophysical sources allows us to place bounds on this effect and hence on the photon charge [25]. Uncharged but magnetized photons traversing different paths though an inhomogeneous magnetic field will likewise acquire phase differences, leading to analogous bounds. The photon magnetic moment can also be bounded by looking at photon birefringence. Photons that have traversed cosmological distances can be very precise probes of novel phenomena in electrodynamics. The immense distances over which they travel magnify miniscule effects. After millions of parsecs, a
tiny change in how electromagnetic waves propagate can give rise to a readily observable effect.

Obviously, the notions that the photon could have charge or that it could possess a magnetic moment are related. Any particle with both charge and spin would naively be expected to possess a nonzero magnetic moment as well. However, the typical magnitude of the magnetic moment for a particle of charge $q$ and mass $m$ is $q\hbar/mc$, which is infinite for a massless particle like the photon. So while we expect that a charged photon could well also have a magnetic dipole moment, there is no natural relationship between the size of the particle’s electric and magnetic couplings. Because of this, and because we expect both couplings to be small, we shall treat them one at a time, although a combined treatment allowing for both would be straightforward. We shall discuss the interferometric bounds on the photon charge in Section 2 and on the magnetic moment in Section 3. In Section 4 we shall show that we can place significantly better bounds on the dipole moment by measuring phase differences in another way—via photon birefringence. Section 5 offers some concluding discussion.

## 2 Interferometric Bounds: Charge

We would like to place bounds that do not depend in any crucial way on the intricate details of the interacting photon dynamics. We shall assume only that there exists an effective Lagrangian $L_{\text{eff}}$ governing the propagation of a single photon. The coupling of a charge $q$ photon to an external electromagnetic field should take the form $L_q = -\frac{2}{c} q v_\mu A^\mu_{\text{ext}}$, where $v_\mu = (c, \mathbf{v})$ is the photon’s four-velocity. This Lagrangian is essentially unique, once we specify that there must be a potential energy term $-qA^0_{\text{ext}}$ and demand conventional Lorentz transformation properties. The equation of motion derived from $L_q$ is the Lorentz force law.

Associated with $L_q$ is an additional phase that a charged photon picks up as it travels, relative to a conventional uncharged photon. If we take the eikonal approximation, in which the photon’s deflection from a straight-line path is neglected, this phase is

$$\phi = \frac{1}{\hbar} \int_0^t d\tau L_q.$$  \hfill (3)

The time interval of the photon’s flight is from 0 to $t$. Assuming the main contribution to be magnetic, we neglect the effects of the electrostatic potential. Then, taking the total distance traveled to be $L$, the phase is

$$\phi = \frac{q}{\hbar c} \int_0^L d\mathbf{\ell} \cdot \mathbf{A}_{\text{ext}}.$$  \hfill (4)

The phase $\phi$ depends on the path, and we would like to compare it for two rays emanating from the same source. We can do this by observing the phase difference between
photons arriving at different points. This is the basis of astrophysical interferometry. If two telescopes, separated by a baseline $d$, collect data from a source lying approximately in the plane perpendicular to the baseline, the observed phase difference due to a possible photon charge is equal to the difference between two phases $\phi_1$ and $\phi_2$ of the form (4).

Neglecting a miniscule contribution proportional to the integral of $\mathbf{A}_{\text{ext}}$ along the baseline, the phase difference is $\Delta \phi = \Phi q/hc$, where $\Phi$ is the magnetic flux threading between the two lines of sight. This is the standard Aharonov-Bohm phase difference, which is independent of the photon energy.

To estimate the flux $\Phi$, we must know something about the relevant magnetic fields. For randomly oriented fields, with typical magnitude $B$ and correlation length $\lambda_C$, $\Phi$ is proportional to $Bd\sqrt{\lambda_C L}$. We would like to get an idea of the accompanying numerical constant. To do this, we assume that the line of sight passes though $L/\lambda_C$ magnetic field domains, each of equal size. In each domain, the field is randomly oriented along one of the six cardinal directions; therefore only one third of the domains contribute to the total flux. The average value of $\Phi$ is then determined by the statistics of a one-dimensional random walk with $L/3\lambda_C$ steps. The mean distance from the origin after $L/3\lambda_C$ steps is $\sqrt{2L/3\pi\lambda_C}$. To get $\Phi$, we multiply this by a flux $Bd\lambda_C$ and a factor of $\frac{1}{2}$ corresponding to the triangular geometry of the threaded region. The total flux is then

$$\Phi \approx \sqrt{\frac{L\lambda_C}{6\pi}} dB.$$  \hspace{1cm} (5)

The precise numerical constants are not all that important. The main point is to show that they are not such as to change drastically the order of magnitude of the result. Including cosmological effects and the expansion of the universe tends to change the flux relatively little, the shortening of the true path length being compensated for by the stronger fields [$B$ proportional to a positive power of $(1 + z)$] in the early universe [26].

If photons moving along different paths acquired Aharonov-Bohm phase differences, this would eventually interfere with interferometry. In order for interferometry to be possible, photons arriving at different telescopes must have definite phase relations. A $\Delta \phi$ of order 1 or larger would destroy these necessary relations. The decoherence that would be caused by a nonzero $\Delta \phi$ has never been seen, and this leads directly to a limit on the photon charge.

There is a possible objection that the Aharonov-Bohm phase, being essentially quantum mechanical, should not contribute to the ordinary, essentially classical, phase that we can observe with radio waves. In this view, the novel phase would represent some kind of intrinsically quantum effect, perhaps only observable if a single photon were bifurcated and recombined (whereas in radio interferometry, we observe distinct but coherent photons at different locations). This would have to represent a new kind of interference, entirely separate from the usual interference of electromagnetic waves. Without a viable theory of self-interacting photons, we cannot reject such a proposal automatically; however, it seems even more farfetched than the possibilities of photon charges and magnetic
moments. The proposal violates the usual relationship between photons and classical waves; according to the correspondence principle, the classical and photon phases are identical.

Of course, there are other phase uncertainties in real interferometry. Telescopes’ relative positions are known to limited accuracy, but position uncertainties lead to phase differences that are proportional to the photon frequency, and they have a predictable dependence on the direction of observation. This is in contrast with an Aharonov-Bohm phase difference, which is frequency independent and varies randomly with different pointing directions. To overcome the real phase uncertainties, sophisticated fringe finding algorithms are required, and individual telescopes are often calibrated by observing reference sources located close to the real sources of interest. The reference sources tend to be comparatively nearby, however, so photons arriving from them will not have the same Aharonov-Bohm phases as photons coming from the same direction but emanating from distant galaxies. Moreover, the Very Long Baseline Interferometry Space Observatory Program (VSOP) experiment, which has the longest baselines of any available interferometer, owing to its use of the Highly Advanced Laboratory for Communications and Astronomy (HALCA) satellite, had to do much of its calibration by dead reckoning, because adjusting the alignment of the satellite was too time consuming [27].

Our ability to study objects at a distance $L$ with interferometers of baseline $d$ limits the photon charge to be smaller than

$$\frac{|q|}{e} < \sqrt{\frac{6\pi}{L\lambda_C}} \frac{hc}{d\epsilon B}. \quad (6)$$

This bound, based as it is on the Aharonov-Bohm phase, cannot be expressed in an $\hbar$-independent form. Since the Aharonov-Bohm phase is independent of energy, this bound is also independent of the photon energy $E$, in contrast to (1). However, it is still most advantageous to work with low-energy photons, because their phases can be determined most accurately.

The most problematical quantities in the expression (6) are those that characterize the cosmic magnetic field—$B$ and $\lambda_C$. These are tricky to estimate separately, but fortunately, (6) depends on the specific combination $B\sqrt{\lambda_C}$. Many potentially observable effects depend on this particular combination of parameters and for essentially the same reasons as in our bound. Weak magnetic effects tend to depend on the absolute value of the time integrated field that a traveling particle interacts with, and we already saw that this was proportional to $B\sqrt{L\lambda_C}$ for a trajectory of length $L$.

The best upper bounds on extragalactic fields come from observations of the Faraday rotation of photons moving through plasmas [28, 29, 30]. The precise bounds one may derive from the Faraday observations depend on the assumptions one makes about the large scale structure of the field and obviously on $\lambda_C$. A bound of $B \lesssim 10^{-8}$ G is reasonable, while cosmic ray and high-energy photon data from the source Centaurus A suggest that $10^{-8}$ G may also be an approximate lower bound for the magnetic field
strength in the relative vicinity of our galaxy \cite{31}. We shall use a rather low estimate of the extragalactic magnetic field. Cosmic ray data suggest that $B\sqrt{\lambda_C}$ may be roughly at the $10^{-10}$ G Mpc$^{1/2}$ level \cite{32}. This estimate depends on another conservative estimate of the number of ultra-high-energy cosmic ray sources. The density of sources could quite reasonably be higher, leading to a higher value of the field.

While $B\sqrt{\lambda_C}$ is an externally prescribed (if not entirely well known) quantity, the parameters $d$ and $L$ are experimental variables. The interferometry experiment with the largest values of $d$ is VSOP, which makes use of the HALCA telescope in space as well as Earth-based observatories. The resulting baselines extend up to $d > 3 \times 10^9$ cm. Using this large interferometer, the VSOP experiment has studied active galactic nuclei out to redshifts of $z \approx 3.5$. To place a very conservative bound on $q$, we can choose the distance $L$ to be 1 Gpc, which is corresponds to a redshift less than 1. With this estimate of $L$, along with $B\sqrt{\lambda_C} = 10^{-10}$ G Mpc$^{1/2}$ and $d = 3 \times 10^9$ cm, our conservative bound on the photon charge is

$$|q| \lesssim 10^{-32},$$

which is already the best direct bound for the case in which all photons have the same charge. If we instead consider the most distant sources the VSOP experiment studied (such as the quasar PKS 2215+020 at $z = 3.57$) and make the same assumption about cosmological fields as were made in \cite{19}—that $B \propto (1 + z)^2$—the bound improves to

$$|q| e \lesssim 1.5 \times 10^{-33}.$$  

(8)

A single demonstration that interferometry is possible, using photons from just one source, places a bound at roughly the order given. The fact that interferometry is possible for photons arriving from virtually any direction indicates that these kinds of bound are quite robust.

It is also possible to use this technique to place bounds on the photon charge if both $+q$ and $-q$ photons exist. In fact, the bounds are significantly better in such a scenario. Although the phase difference for two particles of equal charge is gauge invariant, for particles with different charges it is not. Conventionally, this is not a problem, since particles with different charges can never interfere. However, if there are photons with different charges which do interfere, this raises further questions. Since the phase differences that we measure via such interference are not gauge invariant, the gauge in this scenario must be fixed. The gauge fixing condition ought to arise naturally out of the full theory describing the photon charges. This would be analogous to the way that the Lorenz gauge condition $\partial^\mu A_\mu = 0$ must be satisfied if we introduce a Proca mass term and insist on current conservation. Unfortunately however, we lack a complete theory, and the precise form of the gauge condition is unknown.

Still, it is possible to place an order of magnitude bound on the charge, assuming the large scale structure of the magnetic field is not modified. In a magnetic field domain of
size $\lambda_C$, the typical vector potential is $B\lambda_C/2$. Conservatively assuming that the vector potential falls back to zero at the edge of the domain, there is a contribution to the phase of a charge $q$ photon of $\phi = \sqrt{\frac{L}{6\pi} \frac{\lambda_C^3 q B}{\hbar c}}$. The factor of $\sqrt{2L/3\pi \lambda_C}$ is the same as before. The phase difference for photons of charges $q$ and $-q$ is then

$$\Delta \phi = \sqrt{\frac{2L}{3\pi} \frac{\lambda_C^3/2 |q| B}{\hbar c}},$$

which is independent of the baseline $d$. There is a phase difference even if two photons follow exactly the same path, because they can have opposite charges and hence pick up opposite phases.

The phase difference $\Delta \phi$ in the multiple charge case is not a systematic phase difference between the phases observed at different points but rather a phase uncertainty in the photons seen at a single observatory. If the observed photons have equal probabilities of being positively or negatively charged (or positively, negatively, or null charged), and the mean number of photons collected during a given period is $\langle N \rangle$, the signal is subject to a phase uncertainty proportional to $\Delta \phi/\sqrt{\langle N \rangle}$. This is in sharp contrast to the essentially classical behavior that would ordinarily be seen in observations of a first-order coherent photon beam. Radio interferometers routinely make measurements in the regime where $\langle N \rangle$ for a reasonable observation period is not too much larger than 1. Such measurements reveal no evidence of a phase uncertainty that falls off only as $\langle N \rangle^{-1/2}$, seeing instead a conventional $\langle N \rangle^{-1}$ uncertainty in the measured phase. This indicates that $\Delta \phi$ is small.

In the resulting expression for a bound on $q$, the baseline in (6) is replaced with a quantity proportional to $\lambda_C$, improving the constraint on the magnitude of the charge by a factor of $O(d/\lambda_C)$. $\lambda_C$ is more difficult to determine than $B^2 \lambda_C$, but choosing a relatively conservative value of 100 kpc gives an improvement of $O(10^{-14})$. Taking our most conservative estimate of the distance to the source ($L \sim 1$ Gpc) the bound on $q$ is

$$\frac{|q|}{e} \lesssim 10^{-46},$$

while using sources farther away would again improve the constraint. This is the best bound extant on a photon charge, and it applies to the multiply charged case where the next best bounds (from the CMB) do not apply.

The VSOP experiment observed photons that had frequencies of 1.6, 5, and 22 GHz. This places the energies of the photons from which our bounds on $q$ were derived in the 6–90 $\mu$eV range at the time of their absorption. We might expect that the photon charge should be independent of energy, as is the charge of other particles. However, if the photon charge arises through the breaking of Lorentz symmetry, something more exotic might be involved.
3 Interferometric Bounds: Magnetic Moment

The same kind of analysis can be applied to the possibility of the photon possessing a magnetic moment. In this case the phase involved is not an Aharonov-Bohm phase, but a slightly more conventional dynamical phase. The interaction Lagrangian is $L_{\mu} = \vec{\mu} \cdot \vec{B}_{\text{ext}}$, where the magnetic moment is $\vec{\mu} = \mu s \hat{k}$ for a photon of helicity $s$. $L_{\mu}$ does not have correct Lorentz transformation properties on its own, but the extra terms needed to fix this problem involve the electric field, which ought to have only negligible effects in deep space.

The phase that a magnetized photon acquires as it travels is

$$\phi = \frac{\mu s}{\hbar c} \int_0^L d\vec{l} \cdot \vec{B}_{\text{ext}}.$$  \hspace{1cm} (11)

It is immediately obvious that this will give rise to a bound on $\mu$ very similar to the bounds we have already derived for $q$. We need only replace $q$ with $\mu$ and $\vec{B}_{\text{ext}} = \vec{\nabla} \times \vec{A}_{\text{ext}}$ with $\vec{\nabla} \times \vec{B}_{\text{ext}}$ in the single charge result (since the HALCA telescope only collected photons of one helicity). The characteristic size of $\vec{\nabla} \times \vec{B}_{\text{ext}}$ is $B/\lambda_C$, so the bound on $\mu$ that can be inferred from the fact that $\mu$ hasn’t interfered with astrophysical interferometry at a distance $L$ is

$$|\mu| < \sqrt{\frac{6\pi \lambda_C}{L}} \frac{\hbar c}{dB}.$$  \hspace{1cm} (12)

If $\lambda_C \sim 1$ Mpc (a conservatively large estimate) and $L \sim 1$ Gpc, the corresponding bound on $\mu$ is only at the $3 \times 10^{-8}$ e cm level. This is not a very tight bound; it is more than a thousand times larger than the Bohr magneton $\mu_B$. A photon magnetic moment this large is presumably ruled out by atomic experiments, where it could give rise to a large anomalous AC Zeeman effect. So interferometry does not provide a particularly useful constraint on the magnetic moment of the photon. However, there is a better way to place bounds on $\mu$—using birefringence.

4 Birefringence

The relationship between the bound on $\mu$ derived from birefringence and the bound derived from interferometry is analogous to the relationship between the photon charge bounds in the multiple versus single charge scenarios. The interferometric bounds on $\mu$ come from comparing the phases of waves originating at the same source but following slightly different paths. Birefringence occurs when there are two photon polarization states that interact differently with the cosmic magnetic field, even while following the same path. The resulting phase difference between right- and left-circularly polarized photons can be measured directly, by looking at the change in the polarization of linearly polarized waves, which are superpositions of the two helicity states. This is analogous to
using interferometry to compare the phases of positively and negatively charged photons, which magnetic fields deflect in opposite directions.

Birefringence in vacuum has been searched for and not seen. A systematic difference between the phase speeds of positive and negative helicity photons does not exist. The most sensitive searches for this effect were done in the context of a Lorentz-violating Chern-Simons modification of ordinary electrodynamics. In that scenario, the phase speed difference between the two helicities was independent of the magnetic field in the intervening space. In contrast, if the photon possessed a nonzero magnetic moment, we would expect to see phase differences that were random (although with a well-determined characteristic size), since they would be determined by the randomly oriented magnetic fields along the line of sight. The phase difference would not depend on the direction to the source in any predictable way, and this makes translation of the known birefringence results slightly tricky.

The phase disparity between the left- and right-circularly polarized photons due to a magnetic moment term is independent of frequency. This is an important property that this kind of birefringence shares with Lorentz-violating Chern-Simons birefringence. Photons moving through space definitely do experience birefringence, but only because of the presence of free electrons. The magnitude of the conventional Faraday rotation is proportional to the photon wavelength squared, and consequently, this effect can be subtracted away.

Searches for systematic differences in the phase speed between the two helicities have looked at the radiation from quasars with resolvable jets. The key quantity was the angle between the jet direction and the plane of polarization the source’s synchrotron emission. If this angle depended on the distance to the source, that would be strong evidence of birefringence, but no such dependence seems to exist. Indeed, for high redshift sources, the observed polarizations appear to be concentrated around the directions normal to the jets in the plane of the sky, independent of sources’ distances. This kind of polarization is exactly what we would expect for sources with magnetic flux lines pointing along their jets. Synchrotron electrons revolve around these flux lines, emitting radiation that is polarized perpendicular to the magnetic field and hence the jet. The fact that this angular correlation persists even after photons have traversed cosmological distances indicates that the photon’s magnetic moment must be small. If \( \mu \) were substantial, then the two polarizations would acquire significantly different phases over the course of their propagation, and the radiation observed on Earth would be linearly polarized in an effectively random plane.

The fact that the polarization is not randomized indicates that the relative phase between the left- and right-handed photons is less than 1. A more comprehensive analysis, combining the information available from many sources, could presumably place tighter bounds on the phase shift. This kind of analysis has been carried out as part of the searches for Lorentz violation, but we shall not do it here. Instead, we shall follow the same conservative procedures as we have previously used when looking at interferometry.
That the magnetic phase shift between two oppositely polarized photons traveling along the same path is less than 1 is an indication that

\[ |\mu| < \sqrt{\frac{3\pi \hbar c}{8L \lambda C B}}. \quad (13) \]

By comparing the phases of photons moving along the same path but interacting oppositely with the magnetic field, we have again improved our bounds by a factor of \( \mathcal{O}(d/\lambda C) \). Since we used a more conservative value of 1 Mpc in evaluating (12), the improvement is now by a factor of \( \mathcal{O}(10^{-15}) \). The final result, again using a distance \( L \sim 1 \) Gpc (and the birefringence analyses have not extended out to redshifts as high as those examined in VSOP interferometry), is

\[ |\mu| \lesssim 10^{-24} \text{ e cm}. \quad (14) \]

Obviously, this is a much stronger bound. Other dipole moments which are so far as we know zero are constrained at comparatively similar levels. The neutron and electron electric dipole moments (which violate CP) are bounded at slightly better than the \( 10^{-25} \) e cm [38, 39] and \( 10^{-27} \) e cm [40] levels, respectively.

Of course, it is from studies of photon birefringence that much of the best data on astrophysical magnetic fields comes. The rotation measure (RM) of a source characterizes the change in polarization during light’s transit due to the Faraday effect. So far, measurements of RM have not provided any direct evidence for extragalactic fields. This might seem to indicate a problem with this technique for bounding \( \mu \)—using the absence of birefringence in a magnetic field too weak to be measured by birefringence to constrain an exotic effect. This certainly suggests that better knowledge of cosmic magnetic fields is important for improving the bounds on \( \mu \). We should remember, however, that if birefringence due to the Faraday effect and/or a photon magnetic moment were observed, the two would be straightforward to disentangle, because of their different frequency dependences.

5 Conclusion

Radiation coming from distant galaxies often turns out to have features that would be quite sensitive to exotic modifications of known physics. When the hugeness of cosmic distance scales can be put to use, very tight bounds of these modifications result. However, just how sensitive a photon measurement really will be may not be at all obvious until an actual calculation is performed. The interferometric contraint on \( \mu \) is actually quite poor; the bounding value is several orders of magnitude larger than the Bohr magneton. Yet the birefringence bounds are much better. The birefringence measurement looks at the phase difference between the two helicities of photons, which interact oppositely with the magnetic field. The interferometry bound comes from comparing the phases of photons with the same helicity that have traversed different paths and is worse by the large ratio
which represents the fractional change in the extragalactic magnetic field over a distance equal to the size of the interferometer. The large difference between the bounds on the charge in the single and multiple charge scenarios arises in precisely the same way. However, it was by no means obvious \textit{a priori} that allowing for multiple charges would lead to such a huge improvement in the bounds. For comparison, the corresponding improvement in the charge bounds due to light deflection is only about two orders of magnitude; the improvement in that case is related to the replacement of the potentially small parameter $\Delta E/E$ in (1) with the constant 2.

The possibility of using Aharonov-Bohm phases to constrain the photon charge was not noticed until quite recently. Since exotic and unlikely modifications of known physics such as a photon charge or magnetic moment are little studied, it is not really surprising that a promising method for placing bounds might be overlooked. In fact, there may well be other comparatively straightforward ways to constrain the self couplings of the electromagnetic field which have simply escaped researchers' attention.

It is completely coincidental that, for the single charge case, the best bounds from deflection and pulsar timing are comparable to those from the Aharonov-Bohm effect. Improvements in both types of bounds are naturally possible. The deflection bounds depend on the mean photon energy from a given source $E$ and the energy spread $\Delta E$. Less energetic photons, measured over a wider range of energies, will give stronger constraints. The measured angular spread $\Delta \theta$ is another quantity which might be improved experimentally.

The experimental variables in the interference bound are different. The largest improvements in the single charge case might come from using longer baselines. In principle, a baseline of 2 AU is available for certain types of interferometric measurements, and a baseline this long would improve the bound on $q$ by four orders of magnitude. However, doing interferometry using a single telescope and measurements separated by half a year is obviously a daunting prospect experimentally; it is not going to happen in the near future. A more reasonable possibility for improvement in the short term—and one which is relevant in both the single and multiple charge cases—involves correlating phase data from many sources. The present bounds basically assume that only a phase decoherence $\Delta \phi \sim 1$ for a small number of distant sources is ruled out by the availability of interferometric data. By combining data from multiple sources and baselines, it should be possible to tighten the overall bound on $q$ somewhat, although it is difficult to estimate quantitatively how much improvement is possible; this procedure would also allow us to assign proper confidence levels to the bounds.

The quantities $L$, $\lambda_C$, and $B$ are determined by the sources we choose to observe. A better understanding of magnetism on extragalactic scales will provide more secure (but not necessarily numerically tighter) bounds on $q$. This applies to the bounds both from photon deflection and interferometry. Longer distances $L$ also offer some possibility for improvement, but the dependence on $L$ is only as $L^{-1/2}$, so the gain to be had in this area is not great. With greater distances $L$ also comes a greater reliance on accurate
cosmological models, which may legitimately be questioned when we have introduced such an exotic modification of known physics as a photon self coupling.

The considerations with the interferometric and the more important birefringence bounds on $\mu$ are similar. A detailed study of the polarization angles of quasars’ synchrotron radiation relative to their jet directions would probably improve the constraints on the photon magnetic moment. Such combined analyses have already been performed as part of searches for Lorentz-violating vacuum birefringence, but they are not directly applicable here. The existing analyses looked for a birefringence proportional to the distance $L$ (times cosmological corrections). To place a bound on $\mu$, the analysis would need to be redone, looking for a rotation in the plane of the polarization proportional not to $L$, but with its absolute value proportional to $\sqrt{L}$ (and having a large variance). The $\sqrt{L}$ dependence is characteristic of the magnitude of the integral of the magnetic field along the line of sight, so this is the same dependence that would be expected from the Faraday effect due to propagation through magnetized plasmas. Aggregated data sets have been used to search for this effect and to place bounds on the extragalactic $\vec{B}$, but since the Faraday effect is proportional to the wavelength squared, these analyses are again not directly applicable.

Yet while some improvement in the bounds given here is certainly possible, there is not a great deal of motivation to push the best bound $q$ or $\mu$ down by one or two orders of magnitude. What should be interesting about the present work is that it offers a new way to place bounds on these exotic possibilities. There are a number of well known reasons why it is difficult to construct sensible theories of charged or magnetized photons. This work can be seen as adding new difficulties for such theories. We have introduced a new class of effects that arise naturally in self-interacting electromagnetic theories, yet for which no evidence is seen. This reinforces the idea that photon self couplings are not viable either experimentally or theoretically.

In this paper, we have presented bounds on the photon charge and magnetic moment. The charge constraints are at the $10^{-32} \ e$ level or better if all photons carry the same charge and the $10^{-46} \ e$ level if oppositely charged photons exist. These bound come from the fact that Aharonov-Bohm phases do not interfere with the interferometric imaging of distant galaxies. The best constraint on the photon magnetic moment $\mu$ is at the $10^{-24} \ e \ cm$ level and comes from the absence of wavelength-independent photon birefringence. We know of no other previously published bounds on this quantity. These results indicate that the common assumption that photons have no self interactions is extremely well justified.

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