Progress in the experimental and computational methods of work function evaluation of materials: A review

O.C. Olawole a,*, D.K. De b, O.F. Olawole c, R. Lamba d, E.S. Joel e, S.O. Oyedepo f, A.A. Ajayi g, O.A. Adegbite h, F.I. Ezema i, S. Naghdi k, T.D. Olawole l, O.O. Obembe m,n, K.O. Oguniran e

a Department of Physics, Covenant University, Ota, Ogun State, Nigeria
b Sustainable Green Power Technologies, Mansfield, TX, 76063, USA
c Department of Physics, Mountain Top University, Ibafo, Ogun State, Nigeria
d Department of Electrical Engineering, Malaviya National Institute of Technology Jaipur, Malaviya Nagar, Jaipur-302017, Rajasthan, India
e Department of Earth Sciences, Faculty of Natural, Applied and Health Sciences, Anchor University, Lagos State, Nigeria
f Department of Mechanical Engineering, Covenant University, Ota, Ogun State, Nigeria
g Department of Mathematical and Physical Sciences, Abe Babalola University, Ado-Ekiti, Nigeria
h Department of Chemical Engineering, Covenant University, Ota, Ogun State, Nigeria
i Department of Physics and Astronomy, University of Nigeria, Nsukka, Enugu State, Nigeria
j Africa Centre of Excellence for Sustainable Power and Energy Development, University of Nigeria, Nsukka, Enugu State, Nigeria
k Physikalisch-Technische Bundesanstalt, Abbeest, 2-12, 10587 Berlin, Germany
l Department of Biochemistry, Covenant University, Ota, Ogun State, Nigeria
m Department of Biological Sciences, Covenant University, Ota, Ogun State, Nigeria
n UNESCO Chair on Plant Biotechnology, Plant Science Research Cluster, Department of Biological Sciences, Covenant University, Ota, Ogun State, Nigeria

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A B S T R A C T

The work function, which determines the behaviour of electrons in a material, remains a crucial factor in surface science to understand the corrosion rates and interfacial engineering in making photosensitive and electron-emitting devices. The present article reviews the various experimental methods and theoretical models employed for work function measurement along with their merits and demerits are discussed. Reports from the existing methods of work function measurements that Kelvin probe force microscopy (KPFM) is the most suitable measurement technique over other experimental methods. It has been observed from the literature that the computational methods that are capable of predicting the work functions of different metals have a higher computational cost. However, the stabilized Jellium model (SJ) has the potential to predict the work function of transition metals, simple metals, rare-earth metals and inner transition metals. The metallic plasma model (MPM) can predict polycrystalline metals, while the density functional theory (DFT) is a versatile tool for predicting the lowest and highest work function of the material with higher computational cost. The high-throughput density functional theory and machine learning (HTDTML) tools are suitable for predicting the lowest and highest work functions of extreme material surfaces with cheaper computational cost. The combined Bayesian machine learning and first principle (CBMLFP) is suitable for predicting the lowest and highest work functions of the materials with a very low computational cost. Conclusively, HTDTML and CBMLFP should be used to explore the work functions and surface energy in complex materials.

1. Introduction

The work function, according to the scientists of the twentieth century (fundamental property of material’s surface) is the minimum energy required to extract an electron from a material surface [1, 2, 3, 4, 5, 6]. Its dependence on physical factors such as temperature, surface dipole, doping, and electric field cannot be underestimated as investigated by the authors of the twentieth century [7, 8, 9, 10] that
image potential contributes to the work function of materials and this work function depends on temperature [11, 12, 13, 14, 15, 16]. Also, experimental and theoretical findings revealed that work function is dependent on electronic heat capacity [17], thermal expansion of the lattice [17], Young’s modulus [17, 18, 19], friction [20], adhesion [21], and surface energy [22], fracture toughness [19, 20, 23], corrosion [15, 21, 24], and yield strength and hardness [26].

This work function remains a governing factor of the band alignment in semiconductors [27] and an essential property of metal-semiconductor contact quality [28, 29] in the development of new carbon/graphene-based devices such as graphene transistors [30, 31], carbon-nanotubes [31, 32], gas nanosensors [33], and electronic metallurgy [35]. Also, a twenty-first-century scholar [34] revealed that the behaviour of ions, atoms and molecules inside solid materials are governed by work function (φ) [11, 26] and some other key factors such as vacuum level (E_{vac}), Fermi level (E_F), energy bandgap (E_g), electron affinity (E_A) and ionization energy (I_E), as shown in Fig. 1(a-b). Consequently, E_g is defined as the energy difference between the conduction band and valence band [36] while E_{vac} is the local vacuum level [37]. In addition, E_A is the amount of energy released when an additional electron is added to a neutral atom or molecule [25, 27]. However, I_E in Fig. 1 depicts the minimum energy required for removing an electron out of a neutral atom or molecule in its ground state [34]. E_F defines the energy up to which electrons occupy all energy states of metal at absolute zero temperature [36]. Also, the work function in Eq. (1) typifies the difference between the Fermi level and the vacuum level, as depicted in Fig. 1b. In Fig. 1 [34], CBM denotes the conduction band minimum (CBM), and VBM denotes the valence-band maximum (VBM) through which electron and hole transit in an inorganic semiconductor. LUMO is the lowest unoccupied molecular orbitals (LUMO), and HOMO is the highest occupied molecular orbitals (HOMO) which are energy levels through which the electron and hole migrate in a semiconductor.

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\phi = E_{\text{vac}} - E_F
\]  

Arguably, materials with a lower work function that operate at a lower temperature are noted to release more electrons [29, 30]. Consequently, the work function is of tremendous technological importance in many applications such as thermionic energy converters [31, 32], printed and organic electronics [42, 43, 44], emerging perovskite solar cells technology fluorescent light tube [45], electron emission gadgets (terahertz sources) [46, 47], scientific instruments that use electron sources [30, 39, 40, 41, 50], catalysis [42, 43, 51, 52], electrochemistry [53, 54, 55], corrosion resistance [25, 56, 57] and as an interfacial diagnostic tool in material [58]. These novel applications are realizable if the materials are mechanically, chemically and thermally stable (towards tungsten and lanthanum hexaboride), preferably caesiated metals of low work function [48, 49, 59, 60], as applied in thermionic energy converter [38, 61, 62, 63, 64].

Furthermore, unusual result happens when adsorption and desorption kinetics [65, 66] takes place on the host (material) surface as recorded in the desorption of xenon on the homogeneous surfaces of palladium Pd(110), Pd(100), and Pd(111) [67]. Interestingly, Pd(110), Pd(100) have the same packing density of 5.73 x 10^{14} \text{Xe/cm}^2 as against Pd(111) with different packing density of 5.09 x 10^{14} \text{Xe/cm}^2. Also, the sticking coefficient differs in the three Pd surfaces such that Pd(110) has one dimensional sticking coefficient with Xenon atom found on the lattice trough. Pd(100) face has a zero sticking coefficient, while Xe atoms have a total sticking coefficient (sitting in the threefold hollow sites) on Pd(111) face [67]. Another study of desorption of Tellurium (Te) on W(100) resulted in three distinct binding states on the homogeneous surface [68]. Since the work function of a material is extremely sensitive to the quality of the surface (environmental factor) that causes it. Also, an in-depth understanding of the effects of the environment on the quality of work function measurements is required. Moreover, high spatial resolution measurements are needed for the modern technology-based smaller devices to characterise the complex heterogeneities and homogeneities in the materials used in the microelectronics nanoscience scales.

2. Homogeneous and heterogeneous surfaces

Homogeneous surfaces are formed by interacting monocrystalline films and monocrystalline solids such that its work function contrast Φ° between the work function of positive ion, Φ^+ and negative ion, Φ^- must be equal to zero electron-volt (ΔΦ° = Φ^+ − Φ^- = 0.0 eV). Kawano [69] opined that the entire surface of a monocrystalline specimen W(1 1 0) is homogeneous when it is free of systemic errors which means its degree of monocrystallization (φ_m) = 100% and ΔΦ° = 0.00 eV. Heterogeneous surfaces are formed by surface patches (bumpy and defective) that are interacting electrostatically when their neighbouring surfaces of different work functions are next to each other. Cerofolini and Fondi [70] proposed a model that describes the electron emission from heterogeneous surfaces (single-crystal comprises numerous emitting faces, polycrystalline films, and polycrystalline solids) of metals that are created by a group of isolating homogeneous zones. Each homogeneous patch has a constant work function. The analytical models
representing the work function distribution on the surface of heterogeneous surfaces for thermionic emission, photoemission and field emission phenomena were developed by Cerofolini and Fondi [70]. The work function of heterogeneous surfaces varies significantly from one face to another. The work function (in eV) of different faces of tungsten (b.c.c. lattice) is varying greatly with crystal faces-(100), (110), (011), (112) and (116) is given as 4.64, 5.05, 4.53, 4.39, 4.73 and 4.39 eV respectively [71, 72, 73]. The emission from heterogeneous solid metal surfaces can be categorized as photoemission, field emission, and thermionic emission depending upon the energy source, which are briefly defined as follows:

**Photoemission**: In photoemission, the input energy is the photons/light energy. The photons, incident on the metal surface having energy greater than the work function of the metal causes the electrons to be emitted out from the metal surface.

**Thermionic emission**: In thermionic emission, the input energy source is the heat/thermal energy. When a metal is heated up to a sufficiently high temperature of order of 2000 °C or more, the electrons are emitted out from the metal surface. The electron emission depends on the temperature of the input heat.

**Field emission**: In field emission, the input energy source is the strong electric field. When a sufficiently high electric field is applied to a metal surface, the electrons are emitted out from the metal surface. The electron emission is proportional to applied electric field.

The mathematical models describing the work function distribution on the metal surfaces for different emission phenomena are given in Table 1. The detailed derivations of finding the expressions of work functions of heterogeneous surfaces for these three different emission phenomena are given in Appendix A [62]. It is found that a same law cannot be applied to both a heterogeneous surface and a homogeneous surface.

### 3. Determination of work function

The measurement of the work function of materials with spatially resolved and good reliability is the main barrier in surface characterization for advanced technological applications [74]. The work functions measurement techniques of materials can be broadly classified into two categories. The first category is based on the electron emission process, and the work function is measured on an absolute scale. The material’s surface is simulated using different methods such as thermal emission (thermionic emission), photoemission (ultraviolet photoelectron spectroscopy (UPS), and photoelectric effect), field emission (applied electric field) or a combination of these methods. The second category measures the work function differences between various metals (Kelvin probe force microscopy (KPFM)) or during surface modifications similar to adsorption processes. The measurement of work functions using these techniques is relative and generally requires the standard reference work function calculation. Therefore, experimental protocols, including techniques from both categories, are usually needed for measuring work function with good reliability. This study divides the determination of a material’s work function into experimental and computational methods.

### 4. Experimental method

The commonly used experimental techniques for work function determination are photoelectric measurements, ultraviolet photoelectron spectroscopy (UPS), and thermionic emission (Fig. 2), while the work function differences-based techniques for work function determination are contact potential difference (Kelvin probe force microscopy (KPFM), Fig. 3), vibrating capacitor (Zissman method), and diode methods.

Among the various characterization techniques for work function measurements of different materials, KPFM and x-ray photo electron emission microscopy (XPEEM) are the most promising and powerful work function measurements with high spatial resolution and reliability. However, Table 2 shows the experimental findings of various authors on the work function of materials.

### 5. Computational methods

Table 3 reviews some of the scholarly models adopted by theorists in fitting the work functions of reliable experimental data.

### 6. Conclusion

The review summarizes the various experimental techniques and computational models for measuring the work function of different materials. Although, desorption methods, both as a single method and as a combined technique with other procedures such as flash desorption and infrared spectroscopy, have been adopted to interpret the mechanism of materials’ surface reaction. The other most widely used electron emission-based techniques (photoelectric measurements, UPS, thermionic emission) and the work function differences-based techniques (contact potential difference [the KPFM], vibrating capacitor [Zissman method] and diode methods) for measurement of work function have been discussed. The work function distribution of homogeneous and heterogeneous surfaces has also been discussed in brief. The review also reveals that KPFM and XPEEM are the most promising, accurate, reliable, and powerful techniques for work functions measurement. Moreover, it has been observed that the metallic plasma model can predict the work functions of simple transition, inner transition and rare-earth metals as compared to the other models. However, the stabilized Jellium model (SJM) has the potential to predict the work functions of transition metals, simple metals, rare-earth metals and inner transition metals. The metallic plasma model (MPM) can predict the work functions of the polycrystalline metals. Also, the study showed that the density functional theory (DFT) is suitable modelling method for predicting the lowest and highest work functions of the materials, but with a higher computational cost. The high-throughput density functional theory and machine learning (HTDFTML) tools are suitable for predicting the lowest and highest work functions of extreme materials surfaces with cheaper computational cost.

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**Table 1. The work functions of heterogeneous solid metal surfaces for different emission phenomena.**

| Emission Phenomena | Work Function Expression |
|--------------------|--------------------------|
| Photoemission      | \( \psi(E) = \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \frac{\psi(\phi)}{\phi} \) |
| Thermionic emission| \( \theta(\phi) = \frac{1}{\pi} \int_{-\infty}^{\infty} \exp(\phi(\phi)) \phi(\phi) d\phi \) |
| Field emission     | \( \psi(\xi) = \int_{-\infty}^{\infty} \exp(\xi(\xi)) \psi(\xi) d\xi \) where \( \psi(\xi) = 2\pi (\xi^{3/2}) / 3, \xi = h/\phi \) |

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**Fig. 2.** Schematic diagram of the thermionic electron emission from a thermionic energy converter. \( T_c \) is cathode temperature, and \( T_a \) is anode temperature \( J_c \) and \( J_a \) are the current flow from cathode to anode and from anode to cathode, respectively.
| Technique                         | Principle/Governing Equation                                                                 | Measurement                                                                 | Defect                                                                 | Merit                                                                 | Demerits                                                 | Findings                                                                 |
|----------------------------------|-----------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|------------------------------------------------------------------------|----------------------------------------------------------------------|------------------------------------------------------------------------|--------------------------------------------------------------------------|
| KPFM                             | Atomic force microscopy $W_F = W_{AO} - eV_{AO}$.                                                | Work function (WF) is obtained from the potential difference between the tip of the cantilever and the sample. | It is highly suitable for 2D transition metal dichalcogenide (TMD) materials [76]. | It is helpful to obtain the electronic state, composition, and homogeneity of the sample. It has excellent spatial resolution and energy sensitivity. | KPFM is affected by humidity (environmental factor).                     | It can measure the work function of metals, semiconductors, and nanomaterials with all accuracy up to 10 meV with a spatial resolution of around 10 nm. |
| Ultra-violet photoelectron spectroscopy (UPS) | Photon-excited charge emission process $W_F = h\nu - (E_F - E_{\text{exdd}})$. | UPS measures the kinetic energy spectra of the emitted photoelectrons. | partially suitable UPS gives relatively results of the WF. | UPS is prone to surface contamination. It only perform in a vacuum (At ambient conditions there is a lack of information of the material’s work function) [76]. | Work function is obtained from the subtraction of the width of the photoelectron spectrum from the photon energy of the excitation light. |
| Thermionic emission (TE)         | Thermally excited charge emission process $W_F = k_BT \log\left(\frac{J}{A \gamma T^2}\right)$ | TE determines the WF from Richard- | Not suitable Removal of sample surface’s adsorbent at high temperature. | TE is affected by space charge, a higher work function of the cathode and a few materials that can withstand the temperature of 2000 K. | Existence of a linear relationship between current density and work function of the sample. |
| Field electron Emission (FE)     | Electrostatic-excited charge emission $W_F = k_BT \log\left(\frac{J}{A \gamma T^2}\right)$ | FE determines the WF from Fowler-Nordheim of Equation (4). | Not suitable FE gives more detailed information about the bulk than the surface of the materials. | FE measures the bulk materials. Susceptible to contamination of the emitter surface. | A linear relationship between current density and work function of the sample. |
| Photoemission electron microscopy (PEEM) | Generation of image contrast through local variation in electron emission. | WF is determined from secondary electron emission spectra. | partially suitable for material PEEM enhances lateral resolution but eliminates topographic effects in samples. | PEEM is a suitable method for flat and smooth surface sample with a spatial resolution of around 1 μm and energy resolution of around 400 meV [78]. | PEEM is a suitable method for flat and smooth surface sample with a spatial resolution of around 1 μm and energy resolution of around 400 meV [78]. |
| Internal photoemission spectroscopy (IPES) | Modification of Fowler’s theory | WF is found from the spectral onset of electron or hole photoemission from one solid into another Barrier and bandgap measurement. | IPES is reliable and straightforward to use. | An error occurs when the sample is not perpendicular to the electron detector. Disagreement in literature over function due to IPES Sensitivity nature to any subtle changes in the experimental conditions. | IPES is suitable to determine the work function of samples such as insulators, metals semiconductors and nanomaterials [79, 80]. |
| Low energy electron microscopy (LEEM) | Strong elastic scattering of low energy electrons. It uses the combination of high resolutions imaging and high sensitivity from local electrostatic potentials. | Quantification of local WF differences. | It provides a high lateral resolution. It uses low energy to image the surface of the sample. | Difficulty in interpreting samples that contain different areas of work function. | A suitable method to probe the work function of a sample. |
| Scanning tunnelling microscopy (STM) | Vacuum tunnelling | Measuring the current through the tip of the scanning tunnelling microscope as a function of the distance. | Partially suitable Determination of work function on the atomic scale. | STM has low throughput and limited field of view. | It has a great potential to study the local work function of graphene on Ir(111) [81]. |
| Capacitance-voltage (C-V)        | Sweeping the applied voltage from accumulation to inversion. Also, from inversion to accumulation. | Both capacitance-voltage on terraced oxide structure and current-current (J-V) on barrier height was used to examine the effective work functions of the structures. | Both C-V and J-V are suitable for predicting the practical work function of the material. | C-V is not suitable for the actual work function of the material. | C-V is suitable for measuring the effective work function of terraced oxide structure, while J-V is suitable for measuring the actual work function of the system [82]. |
| Contact potential difference (CPD) (Kelvin method and Diode method) | Usually gold is used as a reference electrode. | Examine the adsorption of molecule surfaces and modify mono crystal surface. | Applicable to insulator as in the case of Kelvin probe method [83]. Concise experimental set-up (diode method [84, 85]). | Work function significantly relies on tip condition. It does not work effectively for immersed specimen. | Applicable in corrosion science electrochemical use for electrodes just covered by ultra-thin electrolyte layers [86]. |
Table 3. Various models for predicting the work function of metals.

| Number of Metals | Model                                                                 | Contribution                                                                 | Merit                                                                 | Demerit                                                                 | Error (%) |
|------------------|----------------------------------------------------------------------|------------------------------------------------------------------------------|----------------------------------------------------------------------|------------------------------------------------------------------------|-----------|
| 14               | Adoption of uniform background charge (instead of the lattice of ions) with exchange and correlations effects. | Only nine elements out of fourteen slightly modelled the experimental values correctly [26]. | Simple pseudopotential model is suitable for simple metals. | Model not suitable for metals in s-d bands. | -         |
| 26               | The authors applied the hydrodynamic model of Bloch and Tomas-Fermi approximation. | They were reasonable agreement among numerical values, experimental results, and theoretical prediction for the examined metals [87]. | It is suitable for the numerical value of the integral at 0.4. | It is not suitable for the numerical value of the integral at 0.5. The model only considered the contribution of free electrons to the work function. | -         |
| 32               | Utilization of a purely electrostatic approach                      | They used the Mathesis construct (overlapping spherical atomic charge-density approximation and chemical potential) to fit the experimental data [88]. | It models relationship between dipole barrier of non-polar crystal and the moment of the spherical densities of metals. | The model is not suitable for metals surface calculations. | 80%       |
| 5                | Thomas-Fermi Von Weizsäcker method to solve the slab geometry jellium model for five alkali metals and Pseudojellium model for four metals. | The Jellium model fitted the experimental data (Smith [89]) well than the Pseudojellium model (Lang and Kohn [90]). | It is useful for calculating the electronic properties of alkali-metal slabs. | The model is not useful for the electronic properties of other metal slabs. | 6% (Jellium model) and 15% (Pseudojellium model). |
| 1                | Modelling work function as a function of electron density parameter within the local-density approximation. | Model predicted the low, metallic and high bulk electron densities [91]. | Model is useful for predicting work functions of low, and metallic bulk electron densities. | Model is not suitable for higher bulk electron densities. It noted the collapsed of Jellium model for highly-non-uniform bulk densities. | -         |
| 12               | Structure-less pseudo-potential model                               | They modelled work function dependence on electron density parameters. The study also corrected the anomalies of Jellium model [92]. | The model is appropriate to study the interfaces, metallic clusters, vacancies, and the electromagnetic response of materials. | It is not suitable for high density metals. | -         |
| 17               | Improved ideal electron gas model                                  | They found that the surface energies of the model reduce by 25%, and the work functions were more significant by 15% compared with the experiment. | It can be used to predict the work functions of simple metals. | It is not suitable for complex metals. | -         |
| 40               | Green's function technique is based on the linear muffin-tin orbitals together with tight-binding. | They found a model that could physical. Described the surface phenomena of the materials [94]. | Abinitio model accurately predicted the monovalent, divalent, polycrystalline metals and trivalent of non-transition metals with experimental values. | It is not suitable for 5d elements. | 15%       |
| 9                | Density functional theory                                         | The study disapproved relationship between surface energy and cohesive force in that contributions of orbital structure, polarization, and free atom spin are needless in surface phenomena [95]. | It is suitable for calculating the surface energy and work function of low index surfaces of 4d transition metals. | Computationally expensive | -         |
| 2                | Jellium slab model                                                | The study found how the variations of the Fermi wavelength are dependent on the thickness of a crystalline slab [96]. | It is appropriate for the calculation of nanomaterial work function. | It is not applicable to the work function of macro material. | -         |
| 1                | Thin slab/Jellium slab model                                      | The model determined precisely the work function of Aluminium (Al) and also showed how quantum side effect could be reduced [97]. | It is suitable for Al. | The model only applicable to one element and not all metals. | -         |

(continued on next page)
Table 3 (continued)

| Number of Metals | Model                                      | Contribution                                                                 | Merit                                                                 | Demerit                                                                 | Error (%) |
|------------------|--------------------------------------------|------------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|-----------|
| 59               | Abinitio models                            | The relationship among the work function, the Fermi energy and the electron density made the model to establish better agreement with experimental for metals and semimetals [98]. | It is suitable for metals and semimetals.                              | The models are not suitable for semiconductors.                         | 8.12%     |
| 29               | Classical electrodynamics model            | Study found emission process of microscopic structure surface of metals, and provided justification for continuous decoherence from quantum to classical states [99]. | It is used to model the work function of metals and the optimum work functions for single layers of electrolytic elements on metal substrates. | The model cannot predict the effective mass of the escaping electron with orientation within the crystal. | -         |
| 3                | Phenomenological model                     | Classical electrodynamics models fitted the experimental data far better than Abinitio [198]. | The models can probe the work function of metals, transition metals, lanthanides, and actinides accurately. | It cannot be used to upgrade the Abinitio model.                       | -         |
| 3                | Application of repeated charged slabs in density functional theory (DFT) | Charged slab model was in good agreement with the conventional density functional theory for Al(100), Si(111) and TiO2(110) [101]. | It provides total energies of charged surface systems.                  | The model is only applicable to three elements.                         | 2.3%      |
| 20               | DFT                                        | The study showed that the work function of noble metals is dependent on surface orientation [182]. | The model is only useful for rough prediction of noble metals.          | It breaks down in Smoluchowski law.                                    | -         |
| 3                | DFT                                        | The authors found out that work function is dependent on surface termination, surface reconstructions, oxygen vacancies, and Hetero-structuring [104]. | It is used to search for a novel, low work function material specifically in electron emission applications. | High computational cost.                                              | -         |
| 73               | DFT                                        | The applied local density approximation (LDA) and the Perdew-Burke-Ernzerhof parametrization of the generalized gradient approximation (PBE-GGA) functional to probe the surface energy and work function of materials [105]. | Both LDA and PBE-GGA functional are good for the prediction of the material work function. LDA can predict surface energy correctly. | PBE-GGA can not be used to predict surface energy of a material. Higher computational cost. | <3 eV     |
| 10               | DFT                                        | Study found out that stable Ba-O adsorbates less than unity are suitable surfaces for thermionic cathode. Each passivated dangling bond of tungsten was thermodynamically favoured with a single O atom, and Ba that resulted into BaO1.12O or BaO1.32O [106]. | It is used as Cs-coated metal for thermionic energy converter.          | It is not applicable to one polycrystalline metal. Adsorption of Ba and O are not considered for pits, ledges, and surface steps polycrystalline W. High computational cost. | -         |
| 9                | DFT used in Vienna ab initio Simulation Package (VASP) with PBE functional | The study found a strong correlation between the 2p band centres and work functions that depicted the volcano plot [107]. | It is used to benchmark the lowest work function a material can attain. | It is only applicable to 2d materials.                                  | -         |
| 5                | Semi-empirical equation (modern use of Schrodinger equation) | Semi-empirical equation predicted the work functions Li, Na, K, Rb very well with the experimental. Experimental value of Caesium 2.14 eV was doubted [108]. | It gave higher accuracy in the absence of computer programme and extra parameters. | The method was not tested for all the metals.                          | 2.3%      |
| 7                | Unconventional quantum model              | The study found the analytical equations for the permitted single- electron energy states in the real metal and the energy of the system N interacting electrons. It also applied mechanical parameters of metals to modify the function of the density of states [109]. | It is suitable for alkali metals. It also used to calculate the electron chemical potential of alkali metals. | The method is not suitable to calculate the chemical potentials of metals because of negative results. | -         |
Table 3 (continued)

| Number of Metals | Model                        | Contribution                                                                 | Merit                                                                                                           | Demerit                                                                                                                   | Error (%) |
|------------------|------------------------------|-----------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|-----------|
| 1                | DFT                          | [110]                                                                        | The Scandium (Sc) is a suitable material for tuning O chemical potential in high-emission Sc-containing cathode. | -                                                                                                                          |           |
| 1                | First Principles             | They observed the absolute work functions and electron affinities of bulk $SrTiO_3$, $SrO$, and $TiO_2$ terminated along the $TiO_2$ and $SrO$ planes [111]. | It reveals how defects increase the work function of the materials. Increment of negative charge density on the surface. | Limited to two terminations.                                                                                           | -         |
| 72               | Wulff shape model for weighted work function | The authors built a largest database for anisotropic work functions. Also, they examined the simple bond breaking rules for metallic systems with numerous maximum Miller index [112]. | It is useful for probing a more generalized work function anisotropy.                                          | The database is only for polycrystalline specimens.                                                                        | 0.2 eV    |
| 1                | DFT                          | It validated the result of STEM for 2D TMDs [76].                           | It connects the work function variance and the defect density of heterogeneity materials with high spatial resolution and energy sensitivity. | -                                                                                                                          |           |
| 25               | Double Dipole Model          | The study discovered that work function remained unchanged because the chemical potential is not dependent on temperature. It also found out that work function experienced changes because dipole layer of the metal was temperature dependent [113]. | It helps in determining the work function of twenty-five metals.                                                    | It is not applicable to all metals.                                                                                         |           |
| 1                | DFT with Heyd-Scuseria–Ernzerhof (HSE) | They model when compare with available experimental data could predict accurately the work function values of clean and reconstructed $SrTiO_3$ surfaces [114]. | It models oxide material of $SrTiO_3$ very well.                                                                 | The model has not been tested on other oxides materials.                                                                   | 0.2 eV    |
| 10               | Machine Learning Approach (Combination of Bayesian Machine Learning and First-Principles Calculation) | This model successfully predicts the work functions of the top lowest and top highest metals [115]. | The model can predict higher or lower work functions of numerous metals including a lanthanide alkali, and alkaline earth metal. The model used bulk calculation. Low computational cost. | The model ignored the work functions relationship with their surfaces.                                                        |           |
| 59               | Metallic plasma model (MPM) and stabilised jellium model (SJM) | Results of MPM [116] were compared with stabilised jellium model (SJM), ab initio model and experimental values [116]. MPM predicted the experimental well SJM [92] was in good agreement with experimental data for the transition metals, simple metals, rare-earth metals and inner transition metals [116]. | MPM is suitable for pure metals and polycrystalline metals. MPM is suitable for non-metals. | Abinitio model only fitted the experimental values of $Ba$, $Ca$ and Sr well and failed to predict all other metals. SJM is not suitable for non-metals. | -         |
| 29220            | high-throughput density functional theory and machine learning | The model achieve the same accuracy as DFT but with a greater computational speed that is 10^5 times faster than DFT [117]. | The model can predict material surfaces with both extreme low and high work functions. Low computational cost. | The model has not been tested for materials with relaxed surfaces. The model has not considered surface energy as to which surface termination is most stable. | 0.19 eV   |
| 15               | DFT                          | The study found $(La, Ba)_6$ $B_6$ suitable because of its lower work function over $LaB_6$ and $BaB_6$ [118]. | It suitable to find the work function of hexaboride, tetaborides, and transition metal nitrides. | The model will not be suitable at the presence of vacancy defects.                                                            |           |
This study suggests that the HTDFTML and CBMLFP models should be explored in more detail in future to determine the work functions and surface energy (finding surface termination that provides higher stability) of the complex metals in relation to their surface phenomena in terms of grain boundaries formation, catalytic behaviour, surface segregation, adsorption, growth rate, sintering and the formation of the crystallites. Further, the surface relaxation should be included in both HTDFTML and CBMLFP computations in order to get work functions of relaxed surfaces. Lastly, temperature-dependent work function models may be helpful in exploring new materials.

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Appendix A

The heterogeneous surface is formed by a set of homogeneous patches having their respective work function as \( \Phi_i \). The average current density emitted from heterogeneous surface is given as [70]:

\[
J(X) = \sum_i J(X, \Phi_i) \varphi(\Phi_i)
\]  

where, \( J(X, \Phi_i) \) is the current density emitted from the patch with work function, \( \Phi_i \) and \( X \) is the photon energy \( (E) \), temperature, \( (T) \) and electric field, \( (F) \) in the photoemission, thermo-emission and field emission respectively. Eq. (1) is true when each patch is emitting independently and there is no lateral interaction among the patches. However, for the heterogeneous surface, the continuum limit of Eq. (1a) can be considered as:

\[
J(X) = \int_0^{\infty} J(X, \Phi) \varphi(\Phi) d\Phi
\]  

where, \( \varphi(\Phi) d\Phi \) is the surface fraction having work function between \( \Phi \) and \( \Phi + d\Phi \).

Eq. (2a) is valid when the following hypotheses are strictly valid: the surface is formed by a set of homogeneous patches, each having a work function, \( \Phi_i \) and there is no laterally interaction in the patches.

Since \( J(X, \Phi_i) \) is theoretically known using the different emission laws and \( J(X) \) is experimentally known, then the distribution function, \( \varphi(\Phi) \) can be determined. The function, \( J(X) \) is different for a homogeneous and heterogeneous surfaces.

Now, by considering, \( J(X) = J(X, \overline{\Phi}) \), Eq. (2a) can be written as:

\[
J(X, \overline{\Phi}) = \int_0^{\infty} J(X, \Phi) \varphi(\Phi) d\Phi
\]  

Eq. (3a) is satisfied if \( \varphi(\Phi) = \delta(\Phi - \overline{\Phi}) \) which means the surface must be homogeneous. The function, \( \varphi(\Phi) \) must satisfy the following normalized condition:

\[
\int_0^{\infty} \varphi(\Phi) d\Phi = 1
\]

A.1. Photoemission

The Fowler [119] theory of photoemission is modified by the Du Bridge [120] and it is called as Fowler-Du Bridge theory which is stated as:

\[
J(E, \Phi) = B I k^2 R T^2 \left( \frac{E - \Phi}{k_b T} \right)
\]  

where \( I \) the light intensity, \( B \) is a characteristic constant, independent of \( E \) and \( \Phi \), and \( k_b \) is the Boltzmann constant.

The average current density emitted from heterogeneous surface (given in Eq. (2a)) in case of photoemission phenomena is given as:
\[ j(E) = \int_0^\infty B I k_0^2 T^2 F \left( E - \Phi \right) \varphi(\Phi) d\Phi \quad (5a) \]

Eq. (4a) can also be expressed as conditionally:
\[ J_0(E, \Phi) = \begin{cases} 0 & E < \Phi, \\ \frac{1}{2} B I (E - \Phi)^2 & E \geq \Phi 
\end{cases} \quad (6a) \]

Substituting Eq. (6a) into (5a), the average current density emitted from heterogeneous surface in photoemission is given as:
\[ j(E) = \frac{1}{2} B I \int_0^E (E - \Phi)^2 \varphi_\alpha(\Phi) d\Phi \quad (7a) \]

The solution of Eq. (7a) which gives the approximate work function distribution is given as:
\[ \varphi_\alpha(E) = \frac{1}{B I} \frac{d^3 j(E)}{dE^3} \quad (8a) \]

Now, to check accuracy of Eq. (8a), the differentiation of Eq. (5a) three times with respect to E is given as:
\[ \frac{d^3 j(E)}{dE^3} = B I \int_0^\infty k_0^2 T^2 \frac{d^3}{dE^3} F \left( E - \Phi \right) \varphi(\Phi) d\Phi \quad (9a) \]

Substituting Eq. (9a) into Eq. (8a) results in:
\[ \varphi_\alpha(E) = \int_0^\infty k_0^2 T^2 \frac{d^3}{dE^3} F \left( E - \Phi \right) \varphi(\Phi) d\Phi \quad (10a) \]

where, subscript ‘u’ represents the approximation value.

The identity is given as
\[ \Phi(E) = \int_0^E \delta(E - \Phi) \varphi(\Phi) d\Phi \quad (11a) \]

The comparison of Eq. (10a) with the identity given by Eq. (11a) represents that the approximate work function distribution, \( \varphi_u(E) \) is exactly similar to the exact one \( \Phi(E) \). In photoemission phenomena, the method used for evaluating the quality of approximation has a general validity and the relation given by Eq. (12a) must be found for evaluating the validity of the method.
\[ \varphi_u(E) = \int_0^E H(E, \Phi) \varphi(\Phi) d\Phi \quad (12a) \]

where, \( H(E, \Phi) \) is a kernel that depends on the used approximation. Comparison of Eq. (12a) and Eq. (11a) reflects that the more \( H(E, \Phi) \) resembles \( \delta(E - \Phi) \), the more the method is correct.

**A.2. Thermionic emission**

According to the Richardson law, the thermionic emission from each homogeneous patch at temperature, \( T \) is given as:
\[ J(T, \Phi) = AT^2 \exp \left( -\frac{\Phi}{k_0 T} \right) \quad (13a) \]

where \( A \) is a universal constant or it may depend on the potential barrier form. It is assumed here that \( A \) does not depend on \( E \) and \( \Phi \).

The average current density emitted from heterogeneous surface (given in Eq. (2a)) in case of thermionic emission phenomena is given as:
\[ j(T) = AT^2 \int_0^\infty \exp \left( -\frac{\Phi}{k_0 T} \right) \varphi(\Phi) d\Phi \quad (14a) \]

By substituting, \( \beta = \frac{1}{k_0 T} \), \( \hat{y}(\beta) = k_0^2 \beta^2 j(\beta)/A \) and \( j(\beta) = j(T(\beta)) \) in Eq. (14a) results in the following expression:
\[ \hat{y}(\beta) = \int_0^\infty \exp(-\beta \Phi) \varphi(\Phi) d\Phi \quad (15a) \]

It is clear from Eq. (15a) that the left hand side function, \( \hat{y}(\beta) \) is the Laplace transform of the work function distribution, \( \varphi(\Phi) \) with respect to \( \beta \) and it is strictly related to the experimentally known function, \( j(T) \).

The inverse Laplace transform of Eq. (15a) gives the exact solution which is given as:
\[ \varphi(\Phi) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \exp(\beta \Phi) \hat{y}(\beta) d\beta \quad (16a) \]

The Laplace transform converges at a point, \( \epsilon \) which is point of the real positive semi-axis.

**A.3. Field emission**

The field emitted current density for a homogeneous surface emitting at 0 K, is function of field strength, \( F \) and work function \( \Phi \) which is given by the Fowler-Nordheim theory as follows:
\[ J(F, \Phi) = a F^2 \exp \left( -\frac{\Phi^{3/2}}{F} \right) \quad (17a) \]

where \( a \) and \( b \) are considered constants which are very slowly varying functions of \( F^{1/2}/\Phi \).

The average current density emitted from heterogeneous surface (given in Eq. (2a)) in case of field emission phenomena is given as:
\[ j(F) = \int_0^\infty \frac{F^2}{\Phi} \exp \left( -\frac{\Phi^{3/2}}{F} \right) \varphi(\Phi) d\Phi \quad (18a) \]

After substituting \( \xi = b/F \) and \( \zeta = \Phi^{3/2} \) in Eq. (18a) and simplifying it results in:
\[ \chi(\xi) = \int_{\infty}^{\infty} \exp(-\zeta \xi) \varphi(\zeta) d\zeta \quad (19a) \]

where, \( \chi(\xi) = \xi^{2/3} j(F(\zeta))/a^2 b^2 \) and \( \psi(\zeta) = 2\pi \zeta^{1/3}/3 \zeta \). It is clear from Eq. (19a) that the function \( \chi(\xi) \) or subsequently \( j(F) \) is the Laplace transform of the function \( \psi(\zeta) \) or subsequently \( \varphi(\Phi) \). The inverse Laplace transform of Eq. (19a) gives the exact solution of \( \psi(\zeta) \) or subsequently \( \varphi(\Phi) \) which is given as:
\[ \psi(\zeta) = \int_{\infty}^{\infty} \exp(\zeta \xi) \chi(\xi) d\xi \quad (20a) \]

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