Super-Resolution Channel Estimation for MmWave Massive MIMO

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Abstract—Millimeter-wave massive MIMO with hybrid precoding can significantly reduce the number of required radio frequency (RF) chains. However, due to the limited number of RF chains, the requirement of large pilot overhead will be used to estimate the high-dimensional mmWave massive MIMO channel. To solve this problem, we propose a super-resolution channel estimation scheme based on modified two-dimensional (2D) unitary ESPRIT algorithm in this paper, which can jointly estimate the continuously distributed angle of arrivals/departures (AoAs/AoDs) with high accuracy. Specifically, we first estimate a low-dimensional effective channel by designing the uplink training signals. Then, since the low-dimensional effective channel has the same shift-invariance of array response as the high-dimensional mmWave MIMO channel to be estimated, we jointly estimate the super-resolution estimates of AoAs and AoDs by exploiting the modified 2D unitary ESPRIT algorithm. Furthermore, we can obtain the associated path gains based on the least squares (LS) criterion. Finally, the high-dimensional mmWave MIMO channel will be reconstructed according to the obtained parameters. Simulation results verify that the proposed scheme is more accuracy than conventional schemes, even with a much lower pilot overhead.

I. INTRODUCTION

Millimeter-wave (mmWave) massive MIMO has been considered as a key technology for the next generation wireless communications [1]. To combat the severe path loss of mmWave channels, the base station (BS) and mobile station (MS) need to achieve the large array gain by deploying a large number of antennas in the massive MIMO systems. However, one main problem is that each antenna requires the associated expensive radio frequency (RF) chain and high power-consuming analog-to-digital converters (ADCs) [1], [2], which is impractical for the conventional full digital precoding MIMO systems. The hybrid precoding with a limited number of RF chains has been proposed to reduce the hardware cost and power consumption as well as achieve the spatial multiplexing [3], [4].

However, during the period of channel estimation, the acquisition of accurate channel state information (CSI) by using a limited number of RF chains is challenging in practice [1]. To this end, several channel estimation schemes have been developed for mmWave massive MIMO with hybrid precoding systems recently [5]–[7]. Specifically, in [5], by exploiting the Arnoldi iteration of Krylov subspace method, the main singular values and corresponding singular subspaces of mmWave MIMO channels can be obtained. However, during the estimation period, multiple echoing operations between the BS and MS will introduce too much noise to accurately estimate the high-dimensional mmWave MIMO channel. In [6] and [7], the compressed sensing (CS)-based schemes have been proposed to reduce pilot overhead by exploiting the inherent angular sparsity of mmWave channels. Nevertheless, the assumption discrete arrivals/departures (AoAs/AoDs) in existing CS-based channel estimation schemes compared with the practical continuously distributed AoAs/AoDs will cause a certain performance loss [8].

In order to estimate the continuous AoAs/AoDs of channels with high accuracy in practice, we will introduce the classical spatial spectrum estimation algorithms, such as the ESPRIT-type algorithms [9], [10], which are widely used in the full digital array. However, the shift-invariance of array response caused by different subarrays must be maintained to utilize the ESPRIT-type algorithms in mmWave MIMO systems with hybrid precoding [9]–[11]. With this in mind, a modified 2D unitary ESPRIT based super-resolution channel estimation scheme is proposed in this paper. The training signals at both BS and MS for channel estimation will be first designed to obtain a low-dimensional effective channel with the shift-invariance of array response. Then, the super-resolution estimates of the AoAs and AoDs can be jointly obtained by exploiting the modified 2D unitary ESPRIT algorithm with the help of this low-dimensional effective channel matrix. In addition, by applying the least squares (LS) estimator, the path gains are immediately estimated. After the AoDs, AoDs, and the corresponding path gains are acquired, the high-dimensional mmWave MIMO channel will be directly reconstructed. Simulation results have verified that the proposed super-resolution channel estimation scheme outperforms conventional schemes.

Notation: The italic boldface lower and upper-case symbols denote column vectors and matrices, respectively. Superscripts $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$, $(\cdot)^{\dagger}$ denote the conjugate, transpose, conjugate transpose, inversion, and Moore-Penrose inversion operators, respectively. $\|a\|_2$ is the $l_2$ norm of $a$. $\otimes$ and $\odot$ denote the Kronecker product and Khatri-Rao product (i.e., the column-wised Kronecker product), respectively. diag$(a)$ is a diagonal matrix with elements of $a$ on its diagonal and diag$(A)$ denotes a block diagonal matrix with subblocks of $A$ on its block diagonal if $A$ is made up of multiple subblocks. vec$(A)$ denotes the vectorization operation of $A$. $I_n$ ($O_{m \times n}$) denotes an identity (null) matrix with size $n \times n$ ($m \times n$), and
The mmWave channel matrix $H$ denotes the real part and imaginary part of the corresponding arguments, respectively. $J_n$ is an exchange matrix with size $n \times n$ that reverses the order of rows of $I_n$.

II. SYSTEM MODEL

In this paper, we consider a time division duplexing (TDD) mmWave massive MIMO system with hybrid precoding, where the BS and MS are equipped with $N_{BS}$ and $N_{MS}$ antennas but only with $N_{RF}^{BS}$ and $N_{RF}^{MS}$ RF chains, respectively [1], [3], [5]–[7]. For the uplink transmission, the received signal $y \in \mathbb{C}^{N_b \times 1}$ with $N_b$ independent data streams at the BS can be expressed as

$$y = WHFS + WHn,$$

where the hybrid combiner $W = W_{RF}W_{BB} \in \mathbb{C}^{N_{BS} \times N_b}$ with the analog combiner $W_{RF} \in \mathbb{C}^{N_{BS} \times N_{RF}^{BS}}$ and the digital combiner $W_{BB} \in \mathbb{C}^{N_{RF}^{BS} \times N_b}$. $H$ is the high-dimensional mmWave channel matrix. The hybrid precoder $F = F_{RF}F_{BB} \in \mathbb{C}^{N_{MS} \times N_b}$ with the analog precoder $F_{RF} \in \mathbb{C}^{N_{MS} \times N_{RF}^{MS}}$ and the digital precoder $F_{BB} \in \mathbb{C}^{N_{RF}^{MS} \times N_b}$. In addition, since both $F_{RF}$ are realized by the analog RF phase shifters, every element of $F_{RF}$ should satisfy the constraint of constant modulus, i.e.,

$$\|F_{RF}[m,n]\|_2 = 1/\sqrt{N_{MS}}$$

for the $(m,n)$th elements of $F_{RF}$, and so does $W_{RF}$. Furthermore, the digital precoder $F_{BB}$ will be normalized as $\|F_{RF}F_{BB}\|_F^2 = N_{MS}$ to guarantee the total transmit power constraint. $s \in \mathbb{C}^{N_{BS} \times 1}$ is the transmitted signal from the BS to MS, and $n \in \mathbb{C}^{N_{BS} \times 1}$ is the complex additive white Gaussian noise (AWGN), which follows $\mathcal{CN}(0,\sigma_n^2)$.

Due to the inherent sparsity of mmWave channels, we will next consider a channel matrix $H$ with only $L$ dominant paths corresponding to $L$ different scatterers, which can be written as

$$H = \sqrt{\frac{N_{BS}N_{MS}}{L}} \sum_{l=1}^{L} \alpha_l a_{BS}(\theta_l)a_{MS}^H(\varphi_l),$$

where $\alpha_l \sim \mathcal{CN}(0,\sigma_\alpha^2)$ and $\sigma_\alpha^2 = 1$ is the complex gain of the $l$th path, $\theta_l$ and $\varphi_l$ are the azimuth angles of AoA and AoD pair of the $l$th path, respectively, and $a_{BS}(\theta_l) \in \mathbb{C}^{N_{BS} \times 1}$ and $a_{MS}(\varphi_l) \in \mathbb{C}^{N_{MS} \times 1}$ are the corresponding steering vectors. Here, for the typical uniform linear array (ULA) with $N$ antennas and angle $\psi$ [1], [5]–[7], we have

$$a(\psi) = \frac{1}{\sqrt{N}}\begin{bmatrix} e^{j2\pi\Delta\sin(\psi)} & \cdots & e^{j2\pi(N-1)\Delta\sin(\psi)} \end{bmatrix}^T,$$

where $\Delta = d/\lambda$ denotes the normalized spacing of adjacent antennas (typically $\Delta = 1/2$), $\lambda$ is the wavelength, and $d$ is the spacing of adjacent antennas. Furthermore, we can rewrite the mmWave channel matrix $H$ as follows

$$H = A_{BS}DA_{MS}^H,$$

where $A_{BS} = [a_{BS}(\theta_1), \cdots, a_{BS}(\theta_L)] \in \mathbb{C}^{N_{BS} \times L}$, $A_{MS} = [a_{MS}(\varphi_1), \cdots, a_{MS}(\varphi_L)] \in \mathbb{C}^{N_{MS} \times L}$, and $D = \text{diag}(d)$ is a diagonal matrix with $d = \sqrt{N_{BS}N_{MS}/L} [\alpha_1, \cdots, \alpha_L]^T$.

III. PROPOSED CHANNEL ESTIMATION SCHEME

In this section, we first design the uplink training signals to estimate a low-dimensional effective channel with the same shift-invariance of array response as the high-dimensional mmWave MIMO channel matrix. After that, based on the shift-invariance of array response, the super-resolution estimates of paired AoAs and AoDs are jointly obtained by exploiting the modified 2D unitary ESPRIT algorithm. Furthermore, the path gains are estimated by applying the LS estimator. Finally, according to the acquired AoAs, AoDs, and path gains, we can reconstruct the high-dimensional mmWave MIMO channel.

A. Design of Training Signals

To estimate the high-dimensional mmWave MIMO channel, we will first estimate the AoAs and AoDs with high accuracy. It demands that we should design the appropriate uplink training signals to guarantee the low-dimensional effective channel having the same shift-invariance of array response as the high-dimensional mmWave channel can be acquired. This training signals consist of the analog RF part and digital baseband part at both BS and MS. Specifically, we begin by considering the uplink channel estimation in $T_{MS}$ time slots or one time block, where the received signal $Y = [y_1, \cdots, y_{T_{MS}}] \in \mathbb{C}^{N_b \times T_{MS}}$ can be written as

$$Y = WHFS + WHN,$$

where $S = [s_1, \cdots, s_{T_{MS}}] \in \mathbb{C}^{N_b \times T_{MS}}$ is the transmitted pilot signal block in $T_{MS}$ time slots, $N = [n_1, \cdots, n_{T_{MS}}] \in \mathbb{C}^{N_b \times T_{MS}}$ is the AWGN, and we consider the channel matrix is static in the stage of channel estimation here. Furthermore, we will jointly consider $N_{b}^{R}N_{b}^{T}$ time blocks, so the aggregated received signal $\tilde{Y} \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times N_{b}^{R}N_{b}^{T}T_{MS}}$ can be expressed as

$$\tilde{Y} = \begin{bmatrix} Y_{1,1} & \cdots & Y_{1,N_{b}^{T}} \; \vdots & \ddots & \vdots \; Y_{N_{b}^{R}}^{R,1} & \cdots & Y_{N_{b}^{R}}^{R,N_{b}^{T}} \end{bmatrix} = \tilde{W}^{H}H\tilde{F}\tilde{S} + \tilde{W}^{H}\tilde{N},$$

where $Y_{i,j} \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times T_{MS}}$, for $i = 1, \cdots, N_{b}^{R}$ and $j = 1, \cdots, N_{b}^{T}$, is the received signal in the $((i-1)N_{b}^{T} + j)$th time block, and the aggregated hybrid combiner $\tilde{W} = [W_{1}, \cdots, W_{N_{b}^{R}}] \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times N_{b}^{R}}$ and precoder $\tilde{F} = [F_{1}, \cdots, F_{N_{b}^{R}}] \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times N_{b}^{R}}$ will be designed later. $\tilde{S} = \text{diag}[S, \cdots, S] \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times N_{b}^{R}N_{b}^{T}T_{MS}}$ is the aggregated pilot signal with $N_{b}^{T}$ identical pilot signal blocks $S$ on the block diagonal, $\tilde{W} = \text{diag}[\tilde{W}] \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times N_{b}^{R}N_{b}^{T}N_{b}^{T}}$ and $\tilde{N} \in \mathbb{C}^{N_{b}^{R}N_{b}^{T} \times N_{b}^{R}N_{b}^{T}T_{MS}}$ is the corresponding AWGN matrix. Thus, the total pilot overhead required for channel estimation is $T = T_{MS}N_{b}^{R}N_{b}^{T}$.

To directly make use of ESPRIT-type algorithms to estimate the AoAs/AoDs, we will next design the aggregated precoder $\tilde{F}$ and combiner $\tilde{W}$. Particularly, we make a appropriate assumption that the aggregated precoder $\tilde{F}$ and combiner $\tilde{W}$ have the following forms

$$\tilde{F} = \alpha_f \begin{bmatrix} I_{N_{b}^{R}N_{b}^{T}} & \; \cdots \; & I_{N_{b}^{R}N_{b}^{T}} \end{bmatrix},$$

$$\tilde{W} = O_{N_{b}^{R}N_{b}^{T}N_{b}^{T}} \begin{bmatrix} O_{N_{b}^{R}N_{b}^{T}N_{b}^{T}} & \cdots & O_{N_{b}^{R}N_{b}^{T}N_{b}^{T}} \end{bmatrix},$$
\[ \tilde{W} = \alpha_w \begin{bmatrix} I_{N_b^N S} & O_{(N_{BS} - N_{b^R} N_b) \times N_{b^R} N_b} \end{bmatrix}, \]

where \( \alpha_f \) and \( \alpha_w \) are the scale factors to guarantee the constraints of constant modulus and power. Therefore, the low-dimensional effective channel matrix \( \tilde{H} \in \mathbb{C}^{N_b^T \times N_b^R \times N_b N_s} \) can be presented as

\[ \tilde{H} = \tilde{W}^H H \tilde{F} = \alpha_f \alpha_w \begin{bmatrix} H_{1,1} & \cdots & H_{1,N_b^T N_s} \\ \vdots & \ddots & \vdots \\ H_{N_b^R N_b,1} & \cdots & H_{N_b^R N_b, N_b^T N_s} \end{bmatrix}, \]

where \( H_{m,n} \) represents the \((m, n)\)th element of \( H \). From (7), it’s obvious that elements of first \( N_b^T N_s \) rows and first \( N_b^R N_s \) columns of the high-dimensional channel matrix \( H \) constitute the low-dimensional effective channel matrix \( \tilde{H} \). Hence, \( \tilde{H} \) and \( H \) share the same shift-invariance of array response. To reduce pilot overhead, it stands to reason that we can use the low-dimensional effective channel matrix \( \tilde{H} \) to acquire the super-resolution estimates of AoAs and AoDs.

Next, according to aforementioned analysis, the analog precoders \( \{F_{RF,j}\}_{j=1}^{N_b^R} \) and digital precoders \( \{F_{BB,j}\}_{j=1}^{N_b^T} \) as well as the analog combiners \( \{W_{RF,j}\}_{j=1}^{N_b^R} \) and digital combiners \( \{W_{BB,j}\}_{j=1}^{N_b^T} \) should be well designed to guarantee (5) and (6). Note that the scale factors \( \alpha_f \) and \( \alpha_w \) will be neglected for clarity. Specifically, we can consider a unitary \( U_{N_{RF}^N} = [u_1, \cdots, u_{N_{RF}^N}] \in \mathbb{C}^{N_{RF}^N \times N_{RF}^N} \) as the set of the uplink training signals. In this paper, we take a discrete Fourier transform (DFT) matrix as the candidate, since it shares the orthogonality among different columns, i.e. \( u_m u_m^H = N_{MS}^N \) for \( m = 1, \cdots, N_{RF}^N \), while \( u_m^H u_n = 0 \) for \( m \neq n \). Then, we further consider the \( j \)th digital precoder \( F_{BB,j} \) come from the first \( N_s \) columns of \( U_{N_{MS}^N} \), i.e. \( F_{BB,j} = [u_1, \cdots, u_{N_s}] \), where \( N_s = N_{MS}^N - 1 \) is considered. Meanwhile, for the \( j \)th analog precoder \( F_{RF,j} \in \mathbb{C}^{N_{MS}^N \times N_{RF}^N} \), we consider the following expression \( F_{RF,j} = [F_{RF,j}^1, \cdots, F_{RF,j}^{N_R}]^H \), where \( F_{RF,j}^1 = [u_{N_{RF}^N,1}, \cdots, u_{N_{RF}^N,N_s}] \) and \( F_{RF,j}^2 = [u_{N_{RF}^N,N_s+1}, \cdots, u_{N_{RF}^N,N_{MS}^N-j_Ns}] \), composed of \((j - 1) N_s\) and \((N_{MS} - j_Ns)\) identical \( u_{N_{RF}^N,m} \), respectively. According to the designed digital and analog precoders \( F_{BB,j} \) and \( F_{RF,j} \), we have \( F_{BB} = [F_{BB,1}, \cdots, F_{BB,N_{RF}^N}] \) and \( F_{RF} \) combining matrices \( \{W_{RF,j}\}_{j=1}^{N_{RF}^N} \) can constitute the aggregated precoder \( \tilde{F} = [F_1, \cdots, F_{N_{RF}^N}] \) and the aggregated combiner \( \tilde{W} = [W_1, \cdots, W_{N_{RF}^N}] \), respectively. As a result, (4) can be further written as

\[ \tilde{Y} = \alpha_w \alpha_f \begin{bmatrix} I_{N_b^R N_s} & O_{(N_{BS} - N_{b^R} N_b) \times N_{b^R} N_b} \end{bmatrix} H \times \begin{bmatrix} I_{N_b^T N_s} \\ O_{(N_{MS} - N_{b^T} N_b) \times N_{b^T} N_b} \end{bmatrix} S + \tilde{W}^H \tilde{N} = \tilde{H} S + \tilde{W}^H \tilde{N}. \]

To accurately acquire the low-dimensional effective channel matrix \( \tilde{H} \) from the aggregated received signal \( \tilde{Y} \) in (8), we will utilize the LS estimator to obtain its estimate, that is \( \hat{H} = \tilde{Y} S^H (S S^H)^{-1} \). For convenience, here we assume that the transmit pilot signal block \( S \) is a unitary matrix, since it has the perfect autocorrelation property (i.e. \( S S^H = N_s I_{N_s} \) with \( T_{MS} = N_s \)). In this way, the low-dimensional effective channel matrix can be obtained by \( \hat{H} = \tilde{Y} S^H / N_s \).

Based on above design, we can acquire \( \hat{H} \) preserving the shift-invariance of array response as \( \hat{H} \). This motivates us to exploit the ESPRIT-type algorithms to estimate the AoAs/AoDs, which will be elaborated in the following subsection.

B. Modified 2D Unitary ESPRIT Algorithm

In this subsection, we propose a modified 2D unitary ESPRIT algorithm at the receiver to jointly obtain the super-resolution estimates of AoAs and AoDs.

1) Spatial Smoothing Preprocessing: Without loss of generality, we consider the low-dimensional effective channel matrix \( \tilde{H} \) in (7) with the size of \( N_{TR} \times N_T \). We first introduce the spatial smoothing technique in [11] to alleviate the influence of coherent signals caused by multiple AoAs or AoDs close to each other. Specifically, we consider integers \( m_1 \) and \( m_2 \) as the smoothing parameters, where \( 2 \leq m_1 \leq N_T \) and \( 1 \leq m_2 \leq N_T - 1 \). Meanwhile, for \( 1 \leq i \leq m_2 \) and \( 1 \leq j \leq m_1 \), we define a submatrix of \( \tilde{H} \) as the shifted matrix \( \tilde{H}^{(i,j)} \in \mathbb{C}^{R \times T} \) (for simplicity, let \( R = N_{TR} - m_2 + 1 \) and \( T = N_T - m_1 + 1 \), whose elements are that of the submatrix composed by \( i \)th to \((R-1+i)\)th rows and \( j \)th to \((T-1+j)\)th columns of \( \tilde{H} \). Then, we can construct the smoothed data matrix \( \mathcal{H} \in \mathbb{C}^{m_1 R \times m_2 T} \) as

\[ \mathcal{H} = \begin{bmatrix} \tilde{H}^{(1,1)} & \cdots & \tilde{H}^{(m_2,1)} \\ \vdots & \ddots & \vdots \\ \tilde{H}^{(1,m_1)} & \cdots & \tilde{H}^{(m_2,m_1)} \end{bmatrix}. \]

2) Real Processing: To take full advantage of obtained data and reduce the computational complexity, the complex-valued data matrix \( \mathcal{H} \) will be further transformed into the real-valued matrix \( \mathcal{H}_R \in \mathbb{C}^{m_1 R \times 2 m_2 T} \) by utilizing the forward backward averaging technique [10], i.e. \( \mathcal{H}_R = (Q_{m_1}^H \otimes Q_{m_2}^H) \left[ \begin{bmatrix} \mathcal{H} & \mathcal{H} \end{bmatrix}_R \right] Q_{2 m_2 T} \), where \( Q \) is a sparse and unitary matrix, satisfying \( JQ^* = Q \) as shown in [10].

3) Rank Reduction: To mitigate the influence of noise, we apply the singular value decomposition (SVD) of \( \mathcal{H}_R \), i.e. \( \mathcal{H}_R = U \Sigma V^H \), to distinguish the signal subspace and noise subspace. In order to extract the information of AoAs and AoDs in the real matrix \( \mathcal{H}_R \), we subsequently take the first \( L \) columns of the left singular matrix \( U \), denoted as \( \tilde{U}_R \in \mathbb{R}^{m_1 R \times L} \), to approximate the dominant \( L \)-dimensional column span of \( \mathcal{H}_R \).
4) Joint Diagonalization: According to [10], for a certain non-singular matrix \( T \in \mathbb{R}^{L \times L} \), we can obtain the shift-invariance equations

\[
E_{\theta,R} \hat{U}_R T \Theta T^{-1} = \hat{E}_{\theta,1} \hat{U}_R,
\]

\[
E_{\varphi,R} \hat{U}_R T \Phi T^{-1} = \hat{E}_{\varphi,1} \hat{U}_R,
\]

where \( E_{\theta,R} = \text{Re} \{ E_{\theta} \} \), \( E_{\theta,1} = \text{Im} \{ E_{\theta} \} \), \( E_{\varphi,R} = \text{Re} \{ E_{\varphi} \} \), and \( E_{\varphi,1} = \text{Im} \{ E_{\varphi} \} \) with

\[
E_{\theta} = I_{m_1} \otimes (Q_{H}^{H} - 1 \times 0 \times Q_{R})
\]

\[
E_{\varphi} = (Q_{m_1 - 1}^{H} \times 0 \times I_{m_1 - 1}) \otimes \hat{R},
\]

respectively. In (10), \( \Theta = \text{diag} \{ \hat{\theta}_1, \ldots, \hat{\theta}_L \} \in \mathbb{R}^{L \times L} \) and \( \Phi = \text{diag} \{ \hat{\varphi}_1, \ldots, \hat{\varphi}_L \} \in \mathbb{R}^{L \times L} \) are diagonal matrices with the diagonal elements \( \hat{\theta}_l = \tan(\pi \Delta \sin(\hat{\theta}_l)) \) and \( \hat{\varphi}_l = \tan(\pi \Delta \sin(\hat{\varphi}_l)) \), for \( l = 1, \ldots, L \). The shift-invariance equations in (10) can be solved separately via least squares (LS) or total least squares (TLS), yielding real-valued matrices \( T \Theta T^{-1} \) and \( T \Phi T^{-1} \). Then, these two matrices can be jointly diagonalized as \( \Psi = (T \Phi T^{-1})^j + j (T \Theta T^{-1})^j \).

C. Reconstruct High-Dimensional mmWave MIMO Channel

According to the obtained AoAs \( \{ \hat{\theta}_l \}_{l=1}^L \) and AoDs \( \{ \hat{\varphi}_l \}_{l=1}^L \) above, the high-dimensional mmWave MIMO channel can be reconstructed. We first reconstruct the matrices \( \hat{A}_{\text{BS}} \) and \( \hat{A}_{\text{MS}} \) according to the steering vectors \( a_{\text{BS}}(\hat{\theta}_l) \) and \( a_{\text{MS}}(\hat{\varphi}_l) \) in (1). Then, based on (2) and (7), we have the low-dimensional effective channel matrix \( \hat{H} = \hat{W}^H \hat{A}_{\text{BS}} \hat{A}_{\text{MS}} \hat{F} \). We further vectorize \( \hat{H} \) as

\[
\hat{h} = \left( \hat{H}^H \hat{F} \right)^T (\hat{W}^H \hat{A}_{\text{BS}}) d = Zd,
\]

where \( \hat{h} = \text{vec} (\hat{H}) \), \( Z = \left( \hat{A}_{\text{MS}} \hat{F} \right)^T (\hat{W}^H \hat{A}_{\text{BS}}) \), and we use the identity \( \text{vec}(ABC) = (C^T \otimes A)b \) with \( B = \text{diag}(b) \). Using the LS estimator, we can obtain the LS solution of associated path gain \( \hat{d} \), i.e.

\[
\hat{d} = \arg \min_{\hat{d}} \| \hat{h} - Zd \|_2^2 = (Z^H Z)^{-1} Z^H \hat{h}. \quad (12)
\]

Finally, according to the obtained steering vector matrices \( \hat{A}_{\text{BS}}, \hat{A}_{\text{MS}} \), and the gain of paths \( \hat{d} \) above, we can reconstruct the high-dimensional mmWave MIMO channel as \( \hat{H} = \hat{A}_{\text{BS}} \hat{d} \hat{A}_{\text{MS}}^H \).

IV. Simulation Results

In this section, we consider a typical mmWave massive MIMO system with hybrid precoding with perfect synchronization [13], and the simulation parameters are shown as follows. The number of antennas at the BS and MS \( N_{\text{BS}} = N_{\text{MS}} = 64 \) with associated \( N_{\text{RF}}^{\text{BS}} = N_{\text{RF}}^{\text{MS}} = 4 \) RF chains, the number of paths \( L = 5 \), \( T_{\text{MS}} = 3 \), \( N_{b} = 10 \), the stacking parameters \( m_1 = m_2 = 13 \), and the AoAs \( \{ \hat{\theta}_l \}_{l=1}^L \) and AoDs \( \{ \hat{\varphi}_l \}_{l=1}^L \) follow the uniform distribution \( [-\pi/3, \pi/3] \). Here, we investigate the performance of proposed scheme, including the normalized mean square error (NMSE) and the average spectral efficiency (ASE), by comparing it with the ACS-based scheme [6] and the OMP-based scheme [7].

Fig. 1 compares the NMSE performance of different channel estimation schemes versus SNRs, where the pilot overheads required for them are different, i.e. \( T_{\text{ACS}} = 1500 \) for the ACS-based scheme, \( T_{\text{OMP}} = 576 \) for the OMP-based scheme, and \( T_{\text{Proposed}} = 300 \) for the proposed scheme. Note that SNRs in terms of transmitter are the ratio of pilot signal power to noise power. From Fig. 1, it can be observed that the NMSE performance of our proposed scheme significantly outperforms the other two schemes, even with a much reduced pilot overhead, and the performance gap between the proposed scheme and its counterparts becomes larger when SNR increases, especially SNR is more than 10 dB. This is because the proposed channel estimation scheme can acquire the super-resolution estimates of the AoAs and AoDs with high accuracy. By contrast, the ACS-based and the OMP-based channel estimation schemes suffer from the obvious performance floor when SNR becomes large. From the NMSE performance and a much reduced pilot overhead, we can conclude that the proposed scheme is more attractive for the mmWave massive MIMO systems with hybrid precoding.

Fig. 2. compares the NMSE performance of different channel estimation schemes versus SNRs, where \( N_{\text{RF}} = 4, N_{\text{RF}} = 8, N_{\text{RF}} = 16 \), and the same pilot overhead for all schemes are considered. From Fig. 2, we can observe that the NMSE performance of the proposed scheme improve considerably when the number of RF chains increases from \( N_{\text{RF}} = 4 \) to \( N_{\text{RF}} = 16 \). This is because the larger number of RF chains can enlarge the effective observation dimension and thus improve the NMSE performance. By contrast, when \( N_{\text{RF}} \) increases, the NMSE performance of both the ACS-based and the OMP-based channel estimation schemes improves slightly due to
the floor effect. This phenomenon further confirms the fact that quantizing the continuously distributed AoAs and AoDs for channel estimation will lead to an inevitable quantization error and thus a non-negligible performance loss.

For channel estimation will lead to an inevitable quantization that quantizing the continuously distributed AoAs and AoDs the floor effect. This phenomenon further confirms the fact that quantizing the continuously distributed AoAs and AoDs for channel estimation will lead to an inevitable quantization error and thus a non-negligible performance loss.

Fig. 3 compares the ASE performance of different channel estimation schemes against SNRs, where the optimal performance with the perfect CSI known at both BS and MS are considered as the upper bound [5], [7]. Note that SNRs here are the ratio of transmitted signal power to noise power and we assume that transmitted signal power is equal to pilot signal power. From Fig. 3, it can be observed that our proposed scheme is superior to the ACS-based and OMP-based channel estimation schemes, and its performance approaches the optimal performance when SNR is larger than -5 dB, and the performance gap between the proposed scheme and the ACS-based scheme is considerably distinct. It is worthy pointing out that although the performance gap between the proposed scheme and the OMP-based scheme becomes smaller as SNR increases, the computational complexity of the proposed scheme is much smaller than that of the OMP-based scheme. The specific complexity analysis can be found in [12].

V. CONCLUSIONS

This paper proposes a modified 2D unitary ESPRIT based super-resolution channel estimation scheme for the mmWave massive MIMO systems with hybrid precoding. Specifically, we first obtain a low-dimensional effective channel having the same shift-invariance of array response as the mmWave MIMO channel to be estimated by designing an efficient uplink training signals. Then, by exploiting the modified 2D unitary ESPRIT algorithm, the super-resolution estimates of AoAs and AoDs can be jointly acquired. Furthermore, the associated path gains can be acquired by using the LS estimator. Finally, the mmWave MIMO channel can be reconstructed according to the obtained parameters of channel. Simulation results verify that the proposed super-resolution channel estimation scheme can achieve the better performance than conventional schemes even with a much lower pilot overhead.

Fig. 2. NMSE performance comparison of different schemes versus SNRs, where $N_{RF} = 4$, $N_{RF} = 8$ and $N_{RF} = 16$ are considered, respectively.

Fig. 3. Comparison of ASE performance among different channel estimation schemes versus SNRs.

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