Kerr-Schild solutions of the Einstein-Maxwell field equations, containing semi-infinite axial singular lines, are investigated. It is shown that axial singularities break up the black hole, forming holes in the horizon. As a result, a tube-like region appears which allows matter to escape from the interior without crossing the horizon. It is argued that axial singularities of this kind, leading to very narrow beams, can be created in black holes by external electromagnetic or gravitational sources. In the work of Debney, Kerr and Schild [1], a broad class of stationary solutions of the Einstein-Maxwell field equations was considered, among which are the solutions containing axial semi-infinite singular lines, to be investigated in what follows. Our aim is, in particular, to investigate in detail the structure of horizons for these solutions, to discuss some of the effects which the appearance of the axial singularities may cause in the rotating astrophysical sources, and to consider the physical consequences which may originate from these singularities in initially stable black holes. We show that axial singularities “break up” the Kerr black hole, forming a hole in the horizon which connects the internal with the external regions. As a result, although a “horizon” is still present, it does not isolate the Kerr singularity from the exterior any more, and it turns out to be “half dressed”. These results add this class of solutions to the (rather restricted) family of solutions which have a clear physical meaning, and, therefore, we believe they are of interest per se. Moreover, we conjecture here that the formation of axial singularities may result in the production of jets [4], thus providing an alternative model for important astrophysical phenomena.

1. Introduction. Most of the applications of General Relativity to the astrophysics of compact objects are based on the Kerr solution of the Einstein equations, which describes the rotating stationary black hole. Of course, although the Kerr solution brilliantly describes stationary phases of these objects, it can hardly trace nonstationary behaviors, like bursts and jet formation. However, the Kerr metric is only one member of a wider class of Einstein-Maxwell solutions, the Kerr-Schild class. Here we will consider the rotating Kerr-Schild solutions containing axial semi-infinite singular lines. To the best of our knowledge, these solutions have, so far, not received enough attention in astrophysics, and it seems that they have never been analyzed in detail from the physical point of view.

One of the first mentions of axial singular lines in the exact solutions of the Einstein-Maxwell field equations can be found in the paper by Robinson and Trautman [1]. Axial singularities, which are analogs of the Dirac monopole, appear in the NUT and Kerr-NUT solutions [2] and are related to magnetic charge and to some “magnetic type” of mass which does not have, so far, a clear (astro)physical interpretation, at least for isolated sources. In the work of Debney, Kerr and Schild [3], a broad class of stationary solutions of the Einstein-Maxwell field equations was considered, among which are the solutions containing axial semi-infinite singular lines, to be investigated in what follows. Our aim is, in particular, to investigate in detail the structure of horizons for these solutions, to discuss some of the effects which the appearance of the axial singularities may cause in the rotating astrophysical sources, and to consider the physical consequences which may originate from these singularities in initially stable black holes. We show that axial singularities “break up” the Kerr black hole, forming a hole in the horizon which connects the internal with the external regions. As a result, although a “horizon” is still present, it does not isolate the Kerr singularity from the exterior any more, and it turns out to be “half dressed”. These results add this class of solutions to the (rather restricted) family of solutions which have a clear physical meaning, and, therefore, we believe they are of interest per se. Moreover, we conjecture here that the formation of axial singularities may result in the production of jets [4], thus providing an alternative model for important astrophysical phenomena.

2. Structure of the Kerr solution. The Kerr-Newman metric in Kerr-Schild form can be written as

\[ g^{\mu\nu} = \eta^{\mu\nu} - 2hk^\mu k^\nu, \]  

where \( k^\mu \) is a null vector field \( k_\mu k^\mu = 0 \) which is tangent to the Kerr principal null congruence. The function \( h \) has the form \( h = \frac{M r - c^2 t - 2 M \cos \theta}{c^2 r - c^2 a \cos \theta} \), where the oblate coordinates \( r, \theta \) are used on the flat Minkowski background \( \eta^{\mu\nu} \). In the case of rotating Kerr solutions, the Schwarzschild horizon splits into four surfaces: two surfaces of the staticity limit, \( r_{s+} \) and \( r_{s-} \), which are determined by the condition \( g_{00} = 0 \), and two surfaces of the event horizons, \( S'(x^\mu) = \text{const} \), which are the null surfaces determined by the condition \( g^{\mu\nu}(\partial_\nu S)(\partial_\mu S) = 0 \). In
the case $e^2 + a^2 > M^2$, the horizons of the Kerr-Newman solution disappear and the Kerr singular ring turns out to be naked. To avoid the problems with a two-sheet topology, this singularity may be covered by a smooth disklike source [5,6].

3. Rotating solutions with axial singularities.  
– The Kerr metric above is actually only a special case of a much more general class of exact solutions of the Einstein-Maxwell field equations with axial singularities, discovered by Debney, Kerr and Schild in the seminal paper [3]. These solutions are obtained considering the Kerr-Schild ansatz $\eta^{\mu\nu} = n^{\nu} - 2hk^\mu k^\nu$, with a null vector field $k^\mu (k_\mu k^\mu = 0)$, which is tangent to a geodesic and shear-free principal null congruence (PNC). As a result, the spacetimes are algebraically special, and many tetrad Ricci components vanish, which leads to a strong restriction on the tetrad components of the electromagnetic field as well. We skip here the details of the derivation, to mention only the fact that the corresponding solutions turn out to be aligned with the Kerr PNC, in the sense that

$$F^{\mu\nu} k_\mu = 0.$$  

The Debney-Kerr-Schild solutions arise when one more restriction on the electromagnetic field is imposed, which leads to stationary solutions without electromagnetic waves. The unique non-zero component of the field tensor in this case is $F_{31} = -(AZ)_{,1}$ where $Z$ is the (complex) expansion of the PNC. The function $A$ has the general form

$$A = \psi(Y)/P^2,$$

where $P = 2^{-1/2}(1 + Y\bar{Y})$, and $\psi$ is an arbitrary holomorphic function of $Y$. The resulting metric has the Kerr-Schild form (1), where the function $h$ is given by [3]

$$h = M(Z + \bar{Z})/(2P^3) - A\bar{A}Z\bar{Z}/2.$$  

In terms of spherical coordinates on the flat background one has $Y(x) = e^{i\theta} \tan \frac{\theta}{2}$, which is singular at $\theta = \pi$. This singularity will be present in any holomorphic function $\psi(Y)$, and, consequently, in $A$ and $h$. Therefore, all of the solutions of this class — with the exclusion of the case $\psi = \text{const}$ which corresponds to the Kerr-Newman solution — will be singular at some angular direction $\theta$.

The simplest cases are $\psi = q/(Y + c)$ and $\psi = q(Y + b)/(Y + c)$, which correspond to an arbitrary direction of the axial singularity. However, the sum of singularities in different directions is also admissible $\psi(y) = \sum_i q_i (Y + b_i)/(Y + c_i)$, as well as polynomials of higher degree.

We should stress here that the above solutions do not contradict the assertion of the “no hair theorem” of Carter and Robinson, which states uniqueness of the Kerr and Kerr-Newman solutions [7] under certain regularity hypotheses, since these are here not satisfied here. Indeed, the spacetime has to be asymptotically flat and regular out of the horizon, conditions that are not fulfilled, since the axial singularity extends to infinity. Also, the assumptions on the structure of the horizon are not fulfilled. Indeed, the horizon of a black hole should be smooth and convex. But here the horizon is not smooth at the bifurcation points, and does not form a convex surface. In this connection it may be noted that, recently, multiparticle Kerr-Schild solutions [8] have been obtained. In the case of the two-particle solution, the Kerr-Newman solution exhibits two semi-infinite axial singularities which are caused by interaction with an external particle and are oriented along the line connecting the two particles.

4. Structure of the horizons and diagrams of the maximal analytic extension (MAE).  
– Let us now analyze the structure of the horizons for the solutions containing axial singular lines. It is convenient to use the metric in the Kerr coordinates $(t, r, \theta, \phi)$, [3]. The metric can be represented in the form

$$ds^2 = (2H - 1)dt^2 + \Sigma(d\theta^2 + \sin^2 \theta d\phi^2) + 2(dr - a \sin^2 \theta d\phi)^2 - 4Hdt(dr - a \sin^2 \theta d\phi),$$

where $\Sigma = r^2 + a^2 \cos^2 \theta$, $H = Mr/\Sigma$ for vacuum metric and $H = (Mr - e^2/2)/\Sigma$ for the charged Kerr-Newman solution.

The simplest axial singularity is the pole $\psi = q/Y$. In this case the function $H$ takes the form

$$H = [Mr - 1/2(q/\tan \frac{\theta}{2})^2]/\Sigma.$$  

The boundaries of the ergosphere, $r_{s+}$ and $r_{s-}$ (which are determined by the condition $g_{tt} = 0$ or $2H - 1 = 0$), are the solutions of the algebraic equation

$$2Mr - r^2 - (q/\tan \frac{\theta}{2})^2 - a^2 \cos^2 \theta = 0.$$  

This solution acquires a new feature: the surfaces $r_{s+}$ and $r_{s-}$ turn out to be joined by a tube, forming a simply connected surface (see Figs. 1-3) which has the topology of a sphere.

The surfaces for the event horizons are null and obey the differential equation

$$(\partial_r S)^2 [r^2 + a^2 + (q/\tan \frac{\theta}{2})^2 - 2Mr] - (\partial_\theta S)^2 = 0.$$  

One can ignore the time and $\phi$ dependencies of the surface $S$. However, the coefficients do depend on $\theta$, and $(\partial_\theta S)^2 \neq 0$, which does not allow us to get an explicit solution. Fortunately, $(\partial_\theta S)^2$ is very small for most of the horizon surface, with the exclusion of a small $\theta$–vicinity of the axial singularity. We represent the surface $S$ in the form $r = r(\theta)$ and obtain an approximate solution which is very close to the exact one for the regions where $(\partial_\theta r)^2$ is small. Neglecting the term $(\partial_\theta r)^2$, one obtains
that an approximate form of this surface is described by the equation $r^2 + a^2 + \left(\frac{q}{\tan \frac{\theta}{2}}\right)^2 - 2Mr = 0$. The resulting surfaces are shown in Figs. 1-3 for different values of the parameters $M$, $a$ and $q$. The axis of symmetry is the horizontal $z$-axis, and the axial singular beam is directed along the direction of angular momentum.

![Fig. 1](image1.png)

**FIG. 1.** Near extremal black hole with a hole in the horizon, for $M = 10$, $a = 9.98$, $q = 0.1$. Represented is the axial section. The singular ring is placed at the boundary of the disk $r = 0$. The event horizon is a closed connected surface surrounded by the closed connected surface $g_{00} = 0$.

![Fig. 2](image2.png)

**FIG. 2.** Large hole in the horizon, for $M = 10$, $a = 9$, $q = 0.3$. Axial section. The singular ring is placed at the boundary of the disk $r = 0$. The event horizon is a closed connected surface surrounded by the closed connected surface $g_{00} = 0$.

![Fig. 3](image3.png)

**FIG. 3.** Narrow hole in the horizon, for $M = 10$, $a = 9.5$, $q = 0.03$. Axial section. The singular ring is placed at the boundary of the disk $r = 0$. The event horizon is surrounded by the closed surface of boundary of the ergosphere, $g_{00} = 0$.

Note that, for physical reasons, the event horizon has to lie inside the boundaries of the ergosphere. We clearly see that the resulting surfaces satisfy this requirement. Similar to the boundary of the ergosphere, the two event horizons are joined into one connected surface with spherical topology, and the surface of the event horizon lies inside the boundary of the ergosphere. As it is seen from the figures, the axial singularities lead to the formation of the holes in the black hole horizon, which opens up the interior of the “black hole” to external space.

The structure of the diagrams of the maximal analytic extension (MAE) [9] in these solutions depends on the section considered. If the section is chosen away from the axial singularity and corresponding tube-like region, the diagram of the MAE will be just the same as for the usual solution for a rotating black hole. If the section goes through the axial singularity, the tube-like hole in the horizon leaves a trace on all patches of the MAE. The $r_+$ and $r_-$ surfaces are deformed and approach towards each other, joining at some distance from the axial singularity and forming an access duct to the interior of the former black hole. Therefore, tube-like channels connecting the interior and the exterior at some angular direction will appear on all patches of the diagram.

These black holes with holes in the horizon have thus preferred directions along which the causal structure differs from that of ‘true’ black holes. Their singularity is, therefore, naked, but the nakedness is of a very peculiar type, since it manifests itself in specific directions only. A similar situation occurs with other non-spherical exact solutions, like e.g. the so called Gamma metric [10].

5. **Possible astrophysical applications.** – Two main questions appear in confronting astrophysical applications of these solutions: (i) which consequences will follow from the existence of the holes in the black hole horizon, and (ii) which kind of mechanism could lead to the appearance of the axial singularity and corresponding holes in the horizon?

Note that axial singularities may carry travelling elec-
tromagnetic and gravitational waves which propagate along them as along waveguides, a phenomenon described by exact singular pp-wave solutions of the Einstein-Maxwell field equations [2]. The appearance of the axial singularities in rotating astrophysical sources may be related to their *excitations* by gravitational and/or electromagnetic waves, and has to be necessarily caused by some nonstationary process. It was argued in [11,12] that electromagnetic excitation of black holes leads inevitably to the appearance of axial singularities. The motivation for this statement is based on the treatment of the exact aligned wave solutions of the Maxwell equations on the Kerr background, satisfying (2), since only the aligned wave solutions are consistent with the geodesic and shear-free principal null congruence of the Kerr geometry, and only those may be used as candidates for the exact consistent solutions of the Einstein-Maxwell field equations.

Similar to the stationary case considered in [3], the general aligned solutions are described by two self-dual tetrad components $F_{12} = AZ^2$ and $F_{11} = \gamma Z - (AZ)_{11}$, where the function \( \tilde{A} \) acquires an extra parameter \( \tau \) which is a complex retarded-time [12] (see the appendix for details). The simplest wave modes

$$\psi_n = q Y^n \exp i \omega_n \tau \equiv q (\tan \frac{\theta}{2})^n \exp i (n \phi + \omega_n \tau)$$

(9)

can be labeled by the index \( n = \pm 1, \pm 2, \ldots \), which corresponds to the winding number for the phase wrapped around the axial singularity. The leading wave terms have the form $F_{\text{wave}} = f_R \, d\zeta \wedge du + f_L \, d\zeta \wedge dv$, where $f_R = (AZ)_{11}$ and $f_L = 2 Y \psi (Z/P)^2 + Y^2 (AZ)_{11}$ are the factors describing the “left” and “right” waves propagating, correspondingly, along the \( z^- \) and \( z^+ \) semi-axes. Near the \( z^+ \) axis, \( |Y| \to 0 \), and for \( r \to \infty \), we have $Y \simeq e^{i \theta} \frac{P}{\rho}$, where $\rho$ is the distance from the \( z^+ \) axis. Similarly, near the \( z^- \) axis $Y \simeq e^{i \theta} \frac{P}{\rho}$ and $|Y| \to \infty$. For $|n| > 1$ the solutions contain axial singularities which do not fall off asymptotically, but are increasing, denoting instability. The leading wave for $n = -1$,

$$F_{-1}^{+} = -4 q e^{i 2 \phi + i \omega_1 \tau_+} \frac{\rho^2}{\rho^2} d\zeta \wedge du,$$

(10)

is singular at the \( z^+ \) semi-axis and propagates to $z = +\infty$, while for $n = 1$,

$$F_{1}^{+} = 4 q e^{i 2 \phi + i \omega_1 \tau_-} \frac{\rho^2}{\rho^2} d\zeta \wedge dv,$$

(11)

the singularity is at the \( z^- \) semi-axis and the wave propagates to $z = -\infty$.

By considering the limiting fields near the singular axis, one can find the corresponding self-consistent solutions of the Einstein-Maxwell field equations [12]. They are singular pp−waves [2] having the Kerr-Schild form of the metric (1) with a constant vector $k^\mu$. In particular, the wave propagating along the \( z^+ \) axis has $k_\mu dx^\mu = -2^{1/2} du$. Therefore, wave excitations of the Kerr geometry lead to the appearance of singular pp−waves which propagate outward along the axial singularities. In real situations, axial singularities cannot be stable and they will presumably correspond to some type of jet or burst, hence it is natural to conjecture that the related holes in the horizon will also be at the origin of jet formation.

Observational evidence shows a preference for two-jetlike sources, as e.g., in the field of radio loud sources [13,14]. These jets are emitted in opposite directions along the same axis. This scenario corresponds to our treatment of the sum of two singular modes with $n \pm 1$ and to two oppositely positioned holes in the horizon. The results are basically the ones obtained for the single beam case, applied to $0 \leq \theta \leq \pi/2$ and $\pi/2 \leq \theta \leq \pi$.

It is known that the Kerr solution has a repulsive gravitational force acting on the axis of symmetry for $r < a$. It can be described by the Newton potential $\Phi(r, \theta) = -2 h = -2 M r / (r^2 + a^2)$. In the small vicinity of the axial singularity, a gravitational repulsion from the singularity will appear too, since the potential acquires the form

$$\Phi(r, \theta) = -2 h = (-2 M r / (r^2 + a^2)^2),$$

(12)

where $|\psi|^2 \sim q^2 \tan^2 \frac{\theta}{2}$. Quantum processes of pair creation near the axial singularity will also take place, and presumably be responsible for its regularization [15].

On the other hand, electromagnetic pp−waves along the singularity will cause a strong longitudinal pressure pointed outwards from the hole. It can be easily estimated for the modes of the pp−waves with $n = \pm 1$ [12]. For example, the corresponding energy-momentum tensor is $T^\mu_\nu = \frac{1}{4 \pi} (F_{-1}^{+})^2 k^\mu k^n$, and the wave beam with mode $n = -1$, propagating along the \( z^+ \) half-axis, will exert the pressure

$$p_{z^+} = \frac{2 q_{\rho} e^{-2 \omega_{-1}}}{\pi \rho^4}$$

(13)

where $p$ is an axial distance from the singularity and $\omega_{-1}$ the frequency of this mode. For the exact stationary Kerr-Schild solutions, one can use this expression in the limit $\omega_{-1} = 0$.

**Conclusions.** From the analysis above, we conclude that the aligned excitations of the rotating black hole (or naked rotating source) lead, unavoidably, to the appearance of axial singularities accompanied by outgoing traveling waves and also to the formation of holes at the horizon, which can lead on its turn to the production of astrophysical jets [4].

Multiparticle Kerr-Schild solutions [8] suggest that axial singularities are to be bi-directional and oriented along the line connecting the interacting particles. Thus, it will be interesting to analyze in further detail the observed jets in order to check the conjecture that they may be indeed triggered by radiation coming from remote active objects.
Finally, one may suspect this effect to be related to the known phenomenon of superradiance, although the usual treatment of the latter does not take into account the condition (2), which specifically leads to the formation of narrow beams.

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Appendix: Aligned e.m. solutions on the Kerr-Schild background. The aligned field equations for the Einstein-Maxwell system in the Kerr-Schild class were obtained in [3]. The electromagnetic field is given by tetrad components of the self-dual tensor \( F_{12} = AZ^2 \), \( F_{31} = \gamma Z - (AZ)_{,1} \), where commas denote the directional derivatives with respect to the chosen null tetrad vectors. The equations for the electromagnetic field are

\[
A_{,2} - 2Z^{-1}\dot{Z}Y_{,3} A = 0, \tag{14}
\]

\[
DA + \dot{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3} \gamma = 0. \tag{15}
\]

where \( D = \partial_1 - Z^{-1}Y_{,3} \partial_1 - \dot{Z}^{-1}Y_{,3} \partial_2 \). Solutions of this system were given in [3] only for the stationary case for \( \gamma = 0 \), while the oscillating e.m. solutions correspond to the case \( \gamma \neq 0 \).

For the sake of simplicity we consider the gravitationally Kerr-Schild field as stationary, although in the resulting e.m. solutions the axial symmetry is broken, which has to lead to oscillating backgrounds if the back reaction will be taken into account. Define a complex retarded time parameter \( \tau = t_0 + i\sigma = \tau|_L \) [16], which satisfies the relations

\[
(\tau),_2 = (\tau),_4 = 0 . \tag{16}
\]

Eq. (14) becomes \((AP^2),_2 = 0\), which can be integrated, yielding \( A = \psi(Y,\tau)/P^2\). It has the form obtained in [3]. The only difference is in the extra dependence of the function \( \psi \) from the retarded-time parameter \( \tau \). This means, that the stationary solutions obtained in [3] may be considered as a low-frequency limit of these solutions.

One can easily check that the action of the operator \( D \) on the variables \( Y, \dot{Y} \) and \( \rho \) is

\[
DY = D\dot{Y} = 0, \quad D\rho = 1 , \tag{17}
\]

and therefore \( D\rho = \partial_\rho/\partial t_0 D t_0 = P D t_0 = 1 \), which yields

\[
D t_0 = P^{-1}. \tag{18}
\]

As a result, Eq. (15) takes the form

\[
\dot{A} = -(\gamma P),Y , \tag{19}
\]

where \( (\cdot) = \partial_\rho \).

For the stationary background considered here, \( P = 2^{-1/2}(1 + YY) \), and \( \rho = 0 \). The coordinates \( Y, \tau \) are independent from \( \dot{Y} \), which allows us to integrate Eq. (19). We obtain the following general solution

\[
\gamma = -P^{-1}\int \dot{A}d\dot{Y} = \frac{2^{1/2}\dot{Y}}{PY} + \phi(Y,\tau)/P, \tag{20}
\]

where \( \phi \) is an arbitrary analytic function of \( Y \) and \( \tau \). We set \( \phi(Y,\tau) = 0 \). The term \( \gamma \) in \( F_{31} = \gamma Z - (AZ)_{,1} \) describes a part of the null electromagnetic radiation which falls off asymptotically as \( 1/r \) and propagates along the Kerr principal null congruence \( e^3 \). As it was discussed in [16], it describes a loss of mass by radiation with the stress-energy tensor \( T^{(\gamma)}_{\mu\nu} = \frac{1}{2} \gamma e^3_{\mu} e^3_{\nu} \).

We now evaluate the term \((AZ)_{,1}\). For the stationary case we have the relations \( Z_{,1} = 2ia\dot{Y}(Z/P)^3 \) and \( \tau_{,1} = -2ia\dot{Y}Z/P^2 \). This yields

\[
(AZ)_{,1} = \frac{Z}{P^2}(\psi - 2i\dot{\psi}/P) - 2\dot{\psi}P/P + 2ia\dot{Y} Z/P^3. \tag{21}
\]

Since \( Z/P = 1/(r + ia \cos \theta) \), this expression contains the terms which fall off like \( r^{-2} \) and \( r^{-3} \). However, it contains also the factors which depend on the coordinate \( Y = e^{\rho/2}\tan \bar{\phi} \) and can be singular at the \( z^\pm \)-axis, forming the narrow beams, i.e. the half-infinite lines of singularity. In particular, it can be the \( z^+ \) or \( z^- \) axis, which correspond to \( \theta = 0 \) and \( \theta = \pi \) (cases \( n = \pm 1 \), respectively).

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