Are crude oil and natural gas extreme prices interdependent?

Fernanda Fuentes¹,³ and Rodrigo Herrera²

¹Facultad de Ingeniería, Universidad de Talca, km 1 Los Niches, Curicó, Chile
²Facultad de Economía y Negocios, Universidad de Talca, Lircay s/n, Talca, Chile
³Email: fefuentes@utalca.cl

Abstract. We investigated the relationship between extreme prices of crude oil and natural gas. We found evidence that extreme events in these markets are interdependent and exhibit a self-excited dynamic behavior, where the intensity of extreme events in the oil market influences the occurrence rate and magnitude of extreme events in the natural gas market. In addition, we have determined that it is possible to better manage the Value-at-Risk, when both markets interact.

1. Introduction

With the advanced development of society and the growth of new technologies, energy consumption has become a research topic of great importance. Among the most commercialized energy commodity are oil and natural gas. Numerous studies have focused on analyzing the relationship between the prices of these markets, using commonly co-integration and error correction models, methodologies based on volatility and copula models [1-7].

Recently some researchers have examined the behavior of these energy commodities focusing only in extreme prices, using POT models, Maximum blocks, Conditional-EVT models and Extreme copulas [8-10]. The analysis of the extreme events allows to obtain more complete information about these markets in periods of financial crisis. However, stylized facts typically present in the financial series, cannot be captured by methodologies that assume independent and identically distributed data, as the models currently do.

Unlike previous studies, we propose a novel specification for the multivariate extension of the Hawkes-POT univariate model [11]. This conditional methodology based on self-excited point processes uses the history of past events to capture the dynamic behavior of extreme events in the tail distribution. Besides, we work only with extreme observations, avoiding that mean values biasing estimates.

We analyze how the intensity of extreme events in the natural gas market is influenced by the dynamic of the occurrence and magnitude of extreme events in the oil market, and vice versa. Also, we analyze if it is possible better manage the value at risk (VaR) in these markets, if the way in which they interact is known. The contribution of this paper is twofold, on the one hand, we present an empirical analysis of the relationship between crude oil and natural gas prices in periods of extreme loss relaxation of the assumption of iid data. On the other hand, we present a methodological proposal, introducing a new specification for the multivariate extension of the Hawkes POT model.

This paper is organized as follows: In Section 2, we introduce the proposed methodology. Section 3 includes the empirical analysis with the description of the models, estimation results and analysis of the risk measures. Finally, Section 4 presents the conclusions.
2. Methodology

We define \( \{X^m_t\}_{t \geq 1} \) as the negative returns of the oil (\( m = 1 \)) and the natural gas (\( m = 2 \)) markets, with \( m = 1, \ldots, M \). In addition, we define a sufficiently high threshold \( u^m > 0 \), about which all the observations are considered extreme. A marked point process (MPP) can be obtained for each marginal \( m \) as a set of observations \( \{t^m_i, y^m_i\} \), where \( t^m_i (0 = t^m_0 \leq t^m_1 \leq \cdots \leq t) \), corresponds to the occurrence time of the high return, whose magnitude or mark \( y^m_i \) is defined by means of a high threshold \( u^m \), i.e., \( y^m_i = X^m_{t_i} - u^m \). Thus, the stochastic process accounting for the number of extreme events up to time \( t \) is defined as

\[
N^m(t) \equiv \sum \mathbb{1}_{t^m_i < t} = \sum \mathbb{1}_{t^m_i < t} \mathbb{1}_{y^m_i = y}
\]  

(1)

The internal history of the process is generated by arrival times of events and mark sequence \( \mathcal{H}_t = \{(t^m_i, y^m_i), \forall i, m: t^m_i < t\} \), which may be interpreted as the available information set up to but not including time \( t \).

In terms of its conditional intensity, the MPP \( N^m(t) \) can be characterized through two stochastic processes: a ground intensity \( \lambda^m_y(t|\mathcal{H}_t) \), which determines the dynamic of the occurrence of extreme events, and a probability density function \( g^m(y|\mathcal{H}_t, t) \), which describes the distribution of the marks.

\[
\lambda^m(t, y|\mathcal{H}_t) = \lambda^m_y(t|\mathcal{H}_t) g^m(y|\mathcal{H}_t, t).
\]  

(2)

The ground intensity follows a Hawkes structure

\[
\lambda^m_y(t|\mathcal{H}_t) = \mu_m + \sum_{k=1}^M \eta_{mk} \cdot \sum_{i:t^m_i < t} h_{mk}(y, t)
\]  

(3)

where \( \mu_m \) is the baseline rate independent of the self-exciting process (exogenous intensity) and \( \eta_{mk} \) determines the influence of the component \( m \) through the intensity of events in the component \( k \) (endogenous process). In addition, an exponential kernel \( h_{mk}(y, t) = \alpha_k \exp (\delta_{mk} y^m_i - \alpha_k (t - t^m_i)) \) is used to characterize the instant memory of the conditional intensity. In this expression, \( \alpha_k \) is the rate of decay of the instant intensity and \( \delta_{mk} \) is the parameter that captures the influence that the size of the extreme events \( k \) has on the intensity of the market \( m \). When \( m = k \) this is a self-exciting effect, otherwise it represents cross-excitation.

Following the central idea of the POT models, the marks follow a generalized Pareto distribution

\[
g^m(y|\mathcal{H}_t, t) = \frac{1}{\beta^m_y(y|\mathcal{H}_t)} \left( 1 + \xi^m \frac{y^m}{\beta^m_y(y|\mathcal{H}_t)} \right)^{-1/m - 1}
\]  

(4)

where \( \xi^m > 0 \) denotes the shape parameter, while the scale parameter is defined as follows

\[
\beta^m_y(y|\mathcal{H}_t) = \beta^m_y + \sum_{k=1}^M \beta^m_k \cdot \sum_{i:t^m_i < t} h_{mk}(y, t)
\]  

(5)

and seeks to relate the instant intensity of the self-exciting process and the size of the extreme events. Finally, assume that the events \( \{t^m_i, y^m_i\} \) in each marginal were observed during an interval of time \( (0, T] \), the estimation of the model is through the maximization of its log-likelihood

\[
\ln L = \sum_{m=1}^M \sum_{j=1}^{N^m(t)} \left[ \ln g(y|\mathcal{H}_t, t) + \ln \lambda^m_y(t|\mathcal{H}_t) \right] - \sum_{m=1}^M \int_0^T \lambda^m_y(s|\mathcal{H}_s)ds.
\]  

(6)

The VaR in terms of a Hawkes-POT model corresponds to

\[
\text{VaR}^{t+1}_\alpha = u + \frac{\beta(y|\mathcal{H}_t)}{\xi} \left( \frac{1 - \alpha}{\lambda^m_y(t|\mathcal{H}_t)} \right)^{-\xi} - 1
\]  

(7)

where the sub-index \( m \) has been eliminated to facilitate the exposure [12].

The likelihood ratio test of unconditional coverage (\( LRuc \)), the likelihood ratio test of independence (\( LRind \)) and the likelihood ratio test of conditional coverage (\( LRcc \)) are used to evaluate the VaR accuracy [13]. All tests are approved with p-values greater than 0.05, and evaluated considering three levels of trust: 0.95, 0.99, and 0.999.

3. Empirical analysis

The data used correspond to daily prices from WTI crude oil and Henry Hub natural gas were obtained from the Federal Reserve Economic Data. We consider the position of an investor and used the
negative log-returns of these prices, see Figure 1. The in-sample period used for the estimation includes observations from January 7, 1997 to December 31, 2014, whereas observations from January 2, 2015 to December 31, 2016 are used for backtesting.

![Figure 1. Prices in dollars and returns of oil and natural gas. Time series of indices start January 7, 1997 and end December 31, 2016.](image)

### 3.1. Threshold selection

One of the most complex tasks in this area is to choose a threshold that allows to establish the minimum value in which all the observations will be considered extreme events. This selection plays an important role in the estimation of model parameters. The main difficulty is in the trade-off between bias and variance. If we choose a low threshold, the number of observations increases and the estimate becomes more accurate. However, with this choice we also introduce observations from the center and the estimation becomes biased. In this research, we follow the study proposed by [14] and use the mean residual life plot technique introduced by [15]. This technique consists of plotting the mean excess on the threshold in relation to different thresholds. According to this graph if the assumption that the excesses on the threshold have Generalized Pareto distribution, then the graph should be linear in $u$ with slope $\xi/(1-\xi)$. Figure 2 shows the results of this plot for the analyzed markets. A good compromise between bias and variance according to this graph would be a threshold between the 0.90 and 0.92 quantile for both financial series. Additionally, we evaluate the technique proposed by [16]. The idea is to choose the threshold for which allows us to obtain the most accurate tests for the in-sample VaR. We consider thresholds between 0.9-0.92, according to the estimates, better results are obtained with thresholds of 0.9 and 0.91. While lower accuracy of the in-sample VaR is obtained when assessing the quantile 0.92. Results are available upon request. We decided therefore to work with the maximum possible, 10% of the most extreme events for both series, which is also consistent with [17].
Figure 2. Residual life plot to select the threshold of events in each marginal (natural gas and WTI oil).

3.2. Interdependence and dynamic linkages

We estimated four Hawkes-POT models. Each of these models raises hypotheses of how extreme events interact. Model 1 is the most general specification where both markets are completely related, allowing cross-excitement through their intensities as well as through the magnitudes of the extreme returns. On the opposite end is Model 2, which fully restricts the interaction between the intensities and magnitudes in both markets (i.e., $\eta_{12} = \eta_{21} = \beta^1 = \beta^2 = \delta_{12} = \delta_{21} = 0$), indicating independence in the intensity of occurrence and magnitudes of the extreme events in both markets. According to the results obtained for the two above models and in line with previous literature [5] we proposed two unidirectional models to capture whether extreme events in the natural gas market are indeed affected by those occurring in the oil market. Model 3 considers that there can only be influence from the oil market to the natural gas market, i.e., there is no influence from the natural gas to the oil market through either its intensity, marks, or in the magnitude of the extreme events within of the self-exciting process (i.e., $\eta_{12} = \beta^1 = \delta_{12} = 0$). Finally, Model 4, in addition to including the restrictions of Model 3, limits the possibility that the intensity of the extreme events at the oil market has any influence on the magnitude of the extreme events in the natural gas market ($\beta^2 = 0$).

Table 1 shows the results of the estimations. The best fit according to the AIC is Model 4; therefore, there is evidence that extreme events in both markets are interdependent, but unidirectionally from the oil to the natural gas market. In the case of Model 1, although it has a good fit, many of its interdependence parameters have values near zero. On the other hand, Model 2, considering that both markets are completely independent, is too restrictive to represent their behavior. Finally, although Model 3 can restrict the direction of interdependence, this does not fully capture the way in which the two markets interact.

The dynamics of conditional intensity for Model 4 is illustrated in Figure 3. The first two panels show the marks or magnitudes of the excesses above the threshold, and the two following panels show the intensity of the estimated conditional occurrence. Both markets show a simultaneous increase in the intensity and magnitude of the extreme events driven by episodes like the Asian Crisis and slightly during the Subprime Crisis, consistent with that described by [18-20]. The barcode plot in the lower panel validates our results: light grey colors indicate independent events and dark colors simultaneous occurrences.

Analyzing Model 4 in detail, in both markets the endogenous behavior of the extreme events ($\eta_{11}$, $\eta_{22}$ and $\eta_{21}$) is higher than the exogenous one ($\mu_1$ and $\mu_2$). In particular, the rate of self-excitement is around 0.30 for both markets, whereas the rate of cross-excitement from the oil market towards the natural gas market is 0.15. A similar result is found for the positive influence of the magnitude of extreme events ($\delta_{11}$ and $\delta_{22}$) on the intensity, as well as the intensity of the extreme events on their magnitude ($\beta^1$ and $\beta^2$). Finally, the response of a previous extreme event on the
intensity function in both markets decays exponentially towards their baseline intensities, with a faster rate for natural gas than for the oil market ($\alpha_1$ and $\alpha_2$).

**Table 1.** The fit of the parameters for the four models evaluated and their respective standard errors, as well as log-likelihood and Akaike information criterion.

|                | Model 1     | Model 2     | Model 3     | Model 4     |
|----------------|-------------|-------------|-------------|-------------|
| **Intensity**  |             |             |             |             |
| Oil $\mu_1$    | 0.056       | 0.057       | 0.056       | 0.056       |
|                | (0.008)     | (0.008)     | (0.008)     | (0.008)     |
| Oil $\eta_1$   | 0.218       | 0.215       | 0.217       | 0.217       |
|                | (0.057)     | (0.056)     | (0.056)     | (0.056)     |
| Oil $\eta_2$   | 0.000       |             |             |             |
| Natural Gas $\alpha_1$ | 0.038       | 0.041       | 0.039       | 0.039       |
|                | (0.008)     | (0.008)     | (0.008)     | (0.008)     |
| Natural Gas $\delta_{11}$ | 0.633       | 0.638       | 0.637       | 0.637       |
|                | (0.071)     | (0.069)     | (0.070)     | (0.070)     |
| Natural Gas $\delta_{12}$ | 1.591       | 0.036       | 0.037       | 0.037       |
|                | (1.765)     | (0.048)     | (0.037)     | (0.037)     |
| Natural Gas $\mu_2$ | 0.059       | 0.386       | 0.357       | 0.357       |
|                | (0.005)     | (0.006)     | (0.005)     | (0.005)     |
| Natural Gas $\eta_2$ | 0.152       | 0.151       | 0.151       | 0.151       |
|                | (0.070)     | (0.069)     | (0.069)     | (0.069)     |
| Natural Gas $\eta_{21}$ | 0.083       | 0.081       | 0.084       | 0.084       |
|                | (0.010)     | (0.010)     | (0.011)     | (0.011)     |
| Natural Gas $\delta_{22}$ | 0.307       | 0.302       | 0.308       | 0.308       |
|                | (0.026)     | (0.025)     | (0.026)     | (0.026)     |
| Natural Gas $\delta_{21}$ | 0.000       |             | 0.001       |             |
|                | (0.002)     |             | (0.002)     |             |
| Natural Gas $\rho_0^1$ | 0.349       | 0.353       | 0.351       | 0.351       |
|                | (0.044)     | (0.044)     | (0.044)     | (0.044)     |
| Natural Gas $\rho_1^1$ | 1.236       | 1.222       | 1.233       | 1.233       |
|                | (0.261)     | (0.255)     | (0.258)     | (0.258)     |
| Natural Gas $\beta_2^1$ | 0.000       |             |             |             |
|                | (0.000)     |             |             |             |
| **Magnitude**  |             |             |             |             |
| Oil $\xi_{11}$ | 0.024       | 0.024       | 0.024       | 0.024       |
|                | (0.044)     | (0.044)     | (0.044)     | (0.044)     |
| Oil $\beta_0^2$ | 0.196       | 0.194       | 0.197       | 0.197       |
|                | (0.030)     | (0.030)     | (0.030)     | (0.030)     |
| Oil $\beta_2^2$ | 2.340       | 2.369       | 2.333       | 2.333       |
|                | (0.265)     | (0.266)     | (0.265)     | (0.265)     |
| Oil $\beta_1^2$ | 0.000       |             | 0.000       |             |
|                | (0.002)     |             | (0.003)     |             |
| Oil $\xi_{12}$ | 0.021       | 0.023       | 0.021       | 0.021       |
|                | (0.048)     | (0.048)     | (0.048)     | (0.048)     |
| Log Lik        | -3341.04    | -3345.00    | -3342.33    | -3342.33    |
| AIC            | 6722.07     | 6718.00     | 6718.66     | 6714.66     |

In a final exercise we determine if there is any advantage in the accuracy of the estimation of VaR forecasting, considering the synergies existing between the two markets. The backtesting results are in table 2. First, we investigated the fit of the VaR at different levels for two out-of-sample year (January 2, 2015 - December 31, 2016). The four models showed only two exceptions to the 0.999 in natural gas in the test of conditional ($\text{LR}_{\text{cc}}$) and non-conditional coverage ($\text{LR}_{\text{uc}}$), respectively. From a statistical point of view, the tests do not seem to have enough power to reject the given sample sizes. Therefore, we decided to re-estimate the models, increasing the sample period, for which we consider the last three years as prediction period (January 2, 2014 - December 31, 2016), whereas the rest of the sample from January 7, 1997 is used for the estimation of the model. In this case, general Model 1 and unidirectional Model 3 and Model 4 show a better fit where only the independence hypothesis ($\text{LR}_{\text{ind}}$) was rejected in the exceptions of 0.95 VaR in the natural gas market. Whilst, Model 2 also presents a
rejection in the $LR_{uc}$ test at VaR 0.99 in the natural gas market, showing evidence of the advantage that entails considering the interaction between both markets.

![Figure 3](image_url)

**Figure 3.** The first and second panel display the negative price returns of oil and natural gas and the estimations of the VaR at 99% (black line on returns). The third and fourth panel show the intensity of occurrence of these events. The last panel is a barcode illustrating the extreme events that occurred simultaneously.

**Table 2.** Number of errors in the VaR adjustments and P value of the statistical tests LRuc, LRind and LRcc at 95%, 99% and 99.9%.

| Model 1 | Oil | Natural Gas |
|---------|-----|-------------|
| 2015-2016 | 0.95 | 0.99 0.999 | 0.99 0.99 0.999 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |
| 2014-2016 | 0.99 0.99 0.99 | 0.99 0.99 0.99 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |

**Model 2**

| Oil | Natural Gas |
|-----|-------------|
| 2015-2016 | 0.95 | 0.99 0.999 | 0.99 0.99 0.999 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |
| 2014-2016 | 0.99 0.99 0.99 | 0.99 0.99 0.99 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |

**Model 3**

| Oil | Natural Gas |
|-----|-------------|
| 2015-2016 | 0.95 | 0.99 0.999 | 0.99 0.99 0.999 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |
| 2014-2016 | 0.99 0.99 0.99 | 0.99 0.99 0.99 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |

**Model 4**

| Oil | Natural Gas |
|-----|-------------|
| 2015-2016 | 0.95 | 0.99 0.999 | 0.99 0.99 0.999 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |
| 2014-2016 | 0.99 0.99 0.99 | 0.99 0.99 0.99 | 0.99 10 0.05 0.02 0.02 0.007 | 0.02 0.14 0.61 0.39 0.39 0.14 |

4. Conclusions

In this research we analyzed the degree of interdependence between the oil and natural gas markets from the point of view of the dynamics of negative extreme returns through a bivariate Hawkes-POT model. The estimates show that the interdependence is unidirectional, where the intensity of extreme
events in the oil market influences the dynamics of the occurrence and magnitude of extreme events in the natural gas market. In these markets we observed an endogenous behavior of extreme events, responsible for the formation of clusters. The results from the VaR forecasting show that it is possible to improve the predictions if we consider the way in which the two markets interact from the point of view of their co-movements at extreme levels.

Acknowledgments
CONICYT-PFCHA/Doctorado Nacional/2017-21171244

References
[1] Bildirici M E and Bakirtas T 2014 The relationship among oil, natural gas and coal consumption and economic growth in BRICTS (Brazil, Russian, India, China, Turkey and South Africa) countries Energy 65 134–144
[2] Efimova O and Serletis A 2014 Energy markets volatility modelling using GARCH Energy Economics 43 264–273
[3] Ewing B T, Malik F and Ozfidan O 2002 Volatility transmission in the oil and natural gas markets Energy Economics 24 525–538
[4] Grégoire V, Genest C and Gendron M 2008 Using copulas to model price dependence in energy markets Energy Risk 5 62–68
[5] Hartley P, III K B M and Rosthal J 2007 The relationship between crude oil and natural gas prices Information Administration, Office of Oil and Gas 37
[6] Krichene N 2002 World crude oil and natural gas: A demand and supply model Energy Economics 24 557–576
[7] Pindyck R S 2004 Volatility in natural gas and oil markets The Journal of Energy and Development 30 1–19
[8] Aloui R, Aissa M S Ben, Hammoudeh S and Nguyen D K 2014 Dependence and extreme dependence of crude oil and natural gas prices with applications to risk management Energy Economics 42 332–342
[9] Ghorbel A and Trabelsi A 2014 Energy portfolio risk management using time-varying extreme value copula methods Economic Modelling 38 470–485
[10] Gulpinar N and Katata K 2013 Modelling oil and gas supply disruption risks using extreme-value theory and copula Journal of Applied Statistics 41 37–41
[11] Chavez-Demoulin V, Davison A C and McNeil A J 2005 Estimating value-at-risk: a point process approach Quantitative Finance 5 227–234
[12] Herrera R and Schipp B 2013 Value at risk forecasts by extreme value models in a conditional duration framework Journal of Empirical Finance 23 33–47
[13] Christoffersen P F 1998 Evaluating interval forecasts International economic review 39 841–862
[14] Chavez-Demoulin V and McGill A J 2012 High-frequency financial data modeling using Hawkes processes Journal of Banking and Finance 36 3415–3426
[15] Davison A C and Smith R L 1990 Models for Exceedances over High Thresholds Journal of the Royal Statistical Society Series B (Methodological) 52 1–4
[16] Herrera R 2013 Energy risk management through self-exciting marked point process Energy Economics 38 64–76
[17] Smith R L 1987 Estimating tails of probability distributions The Annals of Statistics 15 1174–1207
[18] Atil A, Lahiani A and Nguyen D K 2014 Asymmetric and nonlinear pass-through of crude oil prices to gasoline and natural gas prices Energy Policy 65 567–573
[19] Ramberg D J 2010 The Relationship between Crude Oil and Natural Gas Spot Prices and its Stability Over Time (Doctoral dissertation, Massachusetts Institute of Technology)
[20] Villar J A and Joutz F L 2006 The relationship between crude oil and natural gas prices Energy Information Administration Office of Oil and Gas. Washington DC, October 2006