Dynamic response research of projectile-based device with the load-reliving subassembly’s material properties nonlinearity

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Abstract. According to the characteristics of the structure of the projectile-based device, designed Hopkinson bar experiment, established the nonlinear dynamic model of projectile-based device with load-reliving subassembly, solved by theory of stress wave, comparing to the result of Hopkinson bar experiment, they are the same, it is useful for engineering design and analysis of projectile-based device.

1. Introduction
The instantaneous high overload generated during the projectile launching process will cause impacts on the missile-borne devices, which is easy to make the missile-borne devices fail. In order to ensure the effective operation of the missile-borne devices, special protection must be made for the carried missile-borne devices [1-2].

The theory and experiment method are used to study the dynamic response mechanism of missile-borne devices during launch. Through the Hopkinson bar impact experiment on the missile-borne device, the dynamic response experimental data is obtained; the dynamic equivalent theoretical model of the missile-borne device is established, and the numerical results obtained by solving the equivalent theoretical model are compared with the experimental data to analyze the launch of the missile-borne device Dynamic response in the process.

2. Design of Hopkinson Bar Impact Experiment
The Hopkinson bar experiment is used to determine the dynamic mechanical behavior of the test piece under high strain rate and the dynamic constitutive relationship of the material. This experiment is widely used in the study of impact dynamics. The biggest advantage of the Hopkinson bar experiment is to decouple the stress wave effect and the strain rate effect separately. For long and uniform input and output rods, only the stress wave effect is considered, and for the short and small test piece, only the strain rate effect is considered. But the premise is that the test piece must be short enough, and the wave impedance of the test piece must be equivalent to the wave impedance of the input and output rods, otherwise the output signal will be very weak, which will affect the accuracy of the experiment.

There are many components in the missile-borne device, and it is difficult to measure the dynamic constitutive relationship separately; the wave impedance of the load shedding component is much lower than the wave impedance of the input and output rods, and the independent measurement will also affect the experimental accuracy because the output signal is too weak. Considering that both the bomb-borne device and the load reduction component are packaged in one piece, the two can be regarded as a whole:
the destruction limit of the bomb-borne device is used as the overall yield limit. The specific measurement results and the integrity of the lens are shown in Tab. 1.

**Tab 1.** The experiment results of projectile-based device without load-reliving subassembly

|          | Input peak stress \( MPa \) | Output peak stress \( MPa \) | Lens condition |
|----------|-----------------------------|-----------------------------|----------------|
| Experiment One | 73.0                         | 62.3                        | Intact         |
| Experiment Two  | 71.4                        | 62.8                         | Intact         |
| Experiment Three | 96.5                       | 74.3                        | Intact         |
| Experiment Four  | 96.8                        | 77.8                        | Intact         |
| Experiment Five | 108.2                      | 80.8                        | Cracked        |
| Experiment Six  | 107.8                      | 86.4                        | Broken         |

From the above experimental results, it can be found that when the impact test is performed on the bomb-loaded device alone, when the input stress reaches 108.2 MPa, the peak output stress reaches 80.8 MPa, and the lens on the bomb-loaded device is cracked. The glass on the lens is mostly fragile. Materials, after consulting the engineering manual, it is found that the failure limit of glass is generally 70-100MPa. Therefore, it can be determined that the ultimate load that the selected missile-borne device can withstand is 80MPa. Then, the Hopkinson bar impact test was carried out on the bomb-borne device with load reduction component. The experimental results are shown in Tab. 2.

**Tab 2.** The experiment results of projectile-based device with load-reliving subassembly

|          | Input peak stress \( MPa \) | Output peak stress \( MPa \) | Maximum compression of load shedding component \( \mu m \) | Lens condition |
|----------|-----------------------------|-----------------------------|---------------------------------|----------------|
| Experiment One | 165.3                      | 49.9                        | 6.5                             | Intact         |
| Experiment Two  | 179.5                      | 50.7                        | 6.6                             | Intact         |
| Experiment Three | 153.6                      | 48.7                        | 6.3                             | Intact         |
| Experiment Four  | 149.9                      | 38.8                        | 5.8                             | Intact         |
| Experiment Five | 166.1                      | 46.7                        | 6.1                             | Intact         |
| Experiment Six  | 194.6                      | 65.0                        | 7.2                             | Intact         |

3. Constitutive models of missile-borne devices and their load-reducing components

3.1. Establishment of equivalent model

With reference to related literature and theoretical research results [3-5], the constitutive model of the nonlinear viscoelastic material studied in this paper can be replaced by the model shown in Fig. 1. It consists of a nonlinear spring \( A_0 \) and a Maxwell body \( A_1 \) in parallel. This model can not only reflect the stress wave propagation problem in the impact process, but also the strain rate problem with time parameters.

![Fig 1. Nonlinear constitutive model of viscoelastic material](image-url)
As shown in Fig. 1, the constitutive equation of a nonlinear spring can be written as:

\[ \sigma_0 = E_0\varepsilon + \alpha\varepsilon^2 + \beta\varepsilon^3 \]  \hspace{1cm} (1)

In the formula, \( E_0, \alpha, \beta \) are the property parameters of the material itself, \( \sigma_0 \) are the stress on the nonlinear spring, and \( \varepsilon \) are the strain. The Maxwell constitutive equation can be expressed as:

\[ \frac{\partial \varepsilon}{\partial t} = \frac{1}{E_1} \frac{\partial \sigma_1}{\partial t} + \frac{\partial \sigma_1}{\partial \eta_i} \]  \hspace{1cm} (2)

In the formula, \( E_1 \) is the Maxwell elastic constant, \( \eta_i \) is the Maxwell viscosity constant. In this way, the overall constitutive equation can be expressed as:

\[ \frac{\partial \sigma}{\partial t} = \frac{\partial \sigma_0}{\partial t} + \frac{\partial \sigma_1}{\partial t} \]  \hspace{1cm} (3)

Among them, we can see \( \sigma = \sigma_0 + \sigma_1 \) from the force relationship. Substitute (1) and (2) into (3) to get:

\[ \frac{\partial \sigma}{\partial t} + \frac{E_1 \sigma}{\eta_i} = \frac{\partial \varepsilon}{\partial t} \cdot E_1 + \frac{E_0 \sigma_0}{\eta_i} + \frac{\partial \sigma_0}{\partial t} \]  \hspace{1cm} (4)

3.2. Solution of equivalent model
The propagation of stress waves in one-dimensional nonlinear viscoelastic materials is mainly determined by three equations.

Continuous equation

\[ \frac{\partial v}{\partial x} = \frac{\partial \varepsilon}{\partial t} \]  \hspace{1cm} (5)

Equation of motion

\[ \rho_0 \frac{\partial v}{\partial x} = \frac{\partial \sigma}{\partial x} \]  \hspace{1cm} (6)

Constitutive equation

\[ \frac{\partial \sigma}{\partial t} + \frac{E_1 \sigma}{\eta_i} = \frac{\partial \varepsilon}{\partial t} \cdot E_1 + \frac{E_0 \sigma_0}{\eta_i} + \frac{\partial \sigma_0}{\partial t} \]  \hspace{1cm} (7)

In order to solve the above equations, the continuous equations can be discretized. Under the known boundary conditions and initial value loading conditions, the appropriate time and space steps are selected and the recursive calculation is performed to obtain accurate numerical solutions.
3.2.1. The establishment of the difference equation of the nonlinear compatibility relation. From the physical meaning of the characteristic line, select a certain space step and time step, and the calculation domain can be divided into several small areas by the characteristic line, as shown in Fig. 2. In these regions, all stress states are equal, and the stress states of adjacent regions satisfy the compatibility relationship of stress waves. When the applied load and boundary conditions are known, the stress state of each small area can be recursively calculated by the difference format of the compatibility relationship. For the $ij$ area, its stress state is completely determined by the number of $ij - 1$, $i - 1j + 1$ and $i - 1j$ areas the compatibility relationship with the area obtained, the specific equation is as follows:

$$v_{ij} - v_{ij-1} = \frac{1}{\rho C_{V}} (\sigma_{ij} - \sigma_{ij-1}) -$$

$$\frac{\sigma_{ij-1} - \sigma_{j}}{E_{0} + E_{1} + 2\alpha \varepsilon_{ij-1} + 3\beta \varepsilon_{ij-1}^{2}} \left( X_{ij} - X_{ij-1} \right) \theta_{i}$$

(8)

$$v_{i} - v_{i+1} = \frac{1}{\rho C_{V}} (\sigma_{i} - \sigma_{i+1}) +$$

$$\frac{\sigma_{i} - \sigma_{i}}{E_{0} + E_{i} + 2\alpha \varepsilon_{i+1} + 3\beta \varepsilon_{i+1}^{2}} \left( X_{ij} - X_{i+1} \right) \theta_{i}$$

(9)

$$\varepsilon_{ij} - \varepsilon_{ij-1} = \frac{\sigma_{ij} - \sigma_{ij}}{d \sigma_{ij}} + \frac{\varepsilon_{ij-1} - \varepsilon_{ij}}{d \varepsilon_{ij}} \left( t_{ij} - t_{ij-1} \right) \theta_{i}$$

(10)

3.2.2. Solving nonlinear difference equations. Combining the actual assembly and size of this type of missile-borne device and its load-reducing components, it can be known that the length of the calculation area is 77.5mm, the calculation time is 50 $\mu$s. The space step is 0.5mm, and the time step is 0.5 $\mu$s.

Incorporate equations (8), (9), (10), as shown in Tab. 3:
Tab 3. The experiment results and the theoretical results

|                | Output stress (experimental) $\text{MPa}$ | Output stress (calculated) $\text{MPa}$ | The absolute value of the error |
|----------------|-------------------------------------------|-----------------------------------------|--------------------------------|
| Experiment One | 49.9                                      | 51.7                                    | 3.6%                           |
| Experiment Two | 50.7                                      | 52.8                                    | 4.1%                           |
| Experiment Three | 48.7                                       | 51.1                                    | 4.9%                           |
| Experiment Four | 38.8                                       | 40.7                                    | 4.9%                           |
| Experiment Five | 46.7                                       | 48.9                                    | 4.7%                           |
| Experiment Six  | 65.0                                       | 68.1                                    | 4.8%                           |

The error between the calculated output stress and the experimentally measured output stress is within 5%.

4. Conclusion

(1) The Hopkinson bar impact experiment of the missile-borne device and its load-reducing component is designed, the missile-borne device and its load-reducing component are regarded as a whole, and the experimental data of its dynamic response is obtained, which provides an experimental basis for theoretical calculation.

(2) The shock dynamic response characteristic line equation of the missile-borne device is constructed, and the dynamic state of the entire calculation domain is solved step by step through the known initial value conditions and boundary value conditions. Through the comparison with the experimental data of Hopkinson bar, the numerical value the difference between the solution and the experimental result is no more than 5%, and it can be considered that the theoretical model basically conforms to the actual situation.

(3) Analyze and study the impact dynamic response of missile-borne components and load shedding components in essence, explain the load shedding mechanism of load shedding components, and have certain guiding significance for projectile design, material selection, and engineering research.

References

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