Chapter 1

DEPHASING AND DYNAMIC LOCALIZATION IN QUANTUM DOTS.

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Abstract

The effects of dynamic localization in a solid-state system – a quantum dot – are considered. The theory of weak dynamic localization is developed for non-interacting electrons in a closed quantum dot under arbitrary time-dependent perturbation and its equivalence to the theory of weak Anderson localization is demonstrated. The dephasing due to inelastic electron scattering is shown to destroy the dynamic localization in a closed quantum dot leading to the classical energy absorption at times much greater than the inelastic scattering time. Finally a realistic case of a dot weakly connected to leads is studied and it is shown that the dynamic localization may lead to a drastic change of the shape of the Coulomb blockade peak in the dc conductance vs the gate voltage.

Keywords: Quantum dot, dynamic localization, Coulomb blockade.

1. Introduction

The process of energy absorption by a quantum system with a time-dependent Hamiltonian underlies a large part of modern physics, both fundamental and applied. A generic Hamiltonian can be written in the form:

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t),$$  \hspace{1cm} (1.1)

where we explicitly separated the time-independent part $\hat{H}_0$ and the external perturbation $\hat{V}$ with the time dependence specified by a given function $\phi(t)$.

In the textbook example of the classical Drude absorption, $\phi(t) = E(t)$ is the time-dependent electric field which is often considered to be a harmonic
function of time $E(t) = E_0 \cos(\omega t)$. Then the energy absorption rate $W_0 = \frac{dE}{dt}$ is given by the classical Joule heat formula:

$$W_0 = \mathcal{V} E_0^2 \sigma(\omega),$$  \hspace{1cm} (1.2)$$

where $\sigma(\omega)$ is the frequency-dependent conductivity and $\mathcal{V}$ is the volume of the system. In this classical picture the energy $\mathcal{E}$ of a closed electron system grows linearly with time. Assuming the model of non-interacting electrons with the density of states $\nu$ related by $\nu \mathcal{V} = 1/\delta$ with the mean separation $\delta$ between discrete one-particle levels we find:

$$\mathcal{E} = \text{const} + \int \varepsilon \left[ f(\varepsilon) - f_0(\varepsilon) \right] \frac{d\varepsilon}{\delta} \propto \frac{T_{\text{eff}}^2}{\delta} + \text{const},$$  \hspace{1cm} (1.3)$$

where $f(\varepsilon)$ is the (non-equilibrium) electron energy distribution function, $f_0(\varepsilon) = \theta(-\varepsilon)$ is the zero-temperature Fermi-step, and $T_{\text{eff}}$ is the effective temperature of electron system. One concludes from Eq.(1.3) that the classical absorption picture corresponds to diffusion in the energy space: the width of the electron energy distribution $T_{\text{eff}}$ increases with time according to the diffusion law:

$$T_{\text{eff}}^2 = D_0 t,$$  \hspace{1cm} (1.4)$$

where $D_0 = W_0 \delta$ is the energy diffusion coefficient.

Apparently, this corresponds to the Markovian random walk over the spectrum of non-perturbed system caused by absorption and emission of energy quanta $\hbar \omega$, each absorption and emission events being independent of other ones. Note that in this random walk picture the only limitation on the validity of Eqs.(1.2,1.4) is the finite width of the quasi-continuous energy band that leads to a saturation in the effective temperature or absorbed energy. The trivial example of such a saturation are Rabi oscillations in a two-level system where the averaged energy does not change with time.

In the past two decades attention of the scientific community was drawn to a different and much less trivial example of saturation in the time-dependent energy of closed driven systems called dynamic localization (DL). In this case the spectrum of $H_0$ is essentially unlimited in the energy space, yet, after a certain time the absorption rate $\dot{W}(t) = D(t)/\delta$ or the energy diffusion coefficient $D(t)$ in Eq.(1.4) vanishes. The DL in the energy space was observed in numerical simulations on the kicked quantum rotor (KQR) – particle on a circle with $\hat{H}_0 = -\partial^2/\partial \theta^2$ and $\phi(t)$ being a periodic sequence of $\delta$-pulses [1], as well as in an actual experimental realization of the KQR – trapped ultracold atoms in the field of a modulated laser standing wave [2]. The mapping of the KQR to the quasi-random 1d Anderson model has been done in Ref. [3], a similar analogy was exploited in Ref. [4] to demonstrate the DL in a mesoscopic disordered ring threaded by a magnetic flux growing linearly in time. In
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Ref. [5] an analogy between the KQR and band random matrices was pointed out, the latter have been reduced to a 1d nonlinear $\sigma$ model [6]. In Ref. [7] the direct correspondence between the KQR and a 1d nonlinear $\sigma$ model was demonstrated.

The physical origin of dynamic localization in the energy space of driven systems is essentially the same as for its real space counterpart – the Anderson localization – in stationary systems. This is the quantum nature of absorption and emission which does not reduce merely to a quantized step $\hbar \omega$ of the random walk. According to basic principles of quantum mechanics one should consider paths in the energy space between an initial and a final point each of them consisting of many such steps. The transition probability is given by a square modulus of the sum of corresponding amplitudes where all interference terms have to be taken into account. It is these interference terms that makes the picture of independent absorption and emission events incomplete and eventually may lead to dynamic localization. In particular the weak dynamic localization (WDL) exhibits itself as a time-dependent negative correction $\delta D(t)$ to the energy diffusion coefficient which magnitude is controlled by a small parameter $\delta/\Gamma \ll 1$, where $\Gamma = \langle V^2 \rangle / \delta$ is the typical radiation width of energy levels.

The present study consists of three parts. In the first part we develop an analytical theory of weak dynamic localization in a closed system of non-interacting electrons driven by a time-dependent perturbation with an arbitrary function $\phi(t)$. To accomplish this goal we use a variant of the diffusion-cooperon diagrammatic technique [8, 9] in the time domain that has been developed on the basis of the Keldysh formalism for non-equilibrium systems. We focus on the case – most relevant for the quantum dot application – where both the unperturbed system $\hat{H}_0$ and the perturbation operator $\hat{V}$ are described by the Gaussian ensemble of random matrices or by an equivalent random field Hamiltonian considered in the zero-mode approximation. Such description has proved [see [8] and references therein] to be valid for low-energy domain of diffusive quantum dots and is believed to be also accurate for ballistic quantum dots with irregular boundary. We establish an intimate relation between WDL and dephasing by time-dependent perturbation using the notion of no-dephasing points first introduced in Ref. [10]. We show that the existence and character of weak dynamic localization depends crucially on the spectral and symmetry properties of $\phi(t)$. Furthermore we show that by merely changing $\phi(t)$ one can obtain the dependence of $\delta D(t)$ that reproduces the dependence of WL correction to electric conductivity $\delta \sigma(t_\phi)$ on the dephasing time $t_\phi$ in quasi-one dimensional wires, two- and three-dimensional systems.

In the second part we give a brief description of the effect of electron-electron interaction on dynamic localization in closed systems assuming that in the absence of interaction the strong dynamic localization occurs at $t > t_s$.
with the effective temperature at saturation $T^*_s = \sqrt{D_0 t^*_s}$. We show that in contrast to the Anderson localization where the hopping conductance is finite but exponentially small at low temperatures, the strong dynamic localization is destroyed by electron-electron interaction even in the case where the characteristic time of inelastic electron-electron scattering $t_{ee} = t_{ee}(T^*_s)$ is much larger than the localization time $t^*_s$. What is left of DL in closed systems with electron interaction is a suppression of the energy absorption rate $W(t)/W_0 \sim t^*_s/t_{ee}$ at intermediate times, $t^*_s < t < t_{ee}$. For times longer than $t_{ee} \ln(t_{ee}/t^*_s)$ the classical absorption rate $W_0$ is again recovered.

Finally, in the third part we address a realistic case of a quantum dot in a steady-state regime under time-dependent perturbation where both electron-electron interaction and electron escape into leads are taken into account. We show that a signature of dynamic localization can be observed in an almost closed quantum dot with the escape rate $\gamma_{esc} \ll \delta$. This is a plateau at the tail of the Coulomb blockade peak in the dc conductance vs. the gate voltage. We obtain an analytic expression for the shape of the Coulomb blockade peak in driven quantum dots and identify a region of parameters where the plateau can occur. The main results of the first two parts are published in Ref.[11, 12] and a detailed study of the Coulomb blockade regime in a driven quantum dot is presented in [22].

2. Weak dynamic localization

The goal of this section is to show how the cooperon-diffuson diagrammatic technique [14] which is the main theoretical tool for describing weak Anderson localization (WAL) and mesoscopic phenomena, can be extended as to include non-equilibrium processes considered in the time-domain. This consideration is based on the Keldysh technique [13] and is described in detail Ref.[8, 9]. Here we present only the main practical hints which allow to derive the WDL correction along the same lines as those used to derive WAL corrections to conductivity [14].

We start by the standard [13] expression for the energy distribution function

$$f(\varepsilon, t) \equiv \frac{1}{2} - \frac{1}{2} \int h \left( t + \frac{\tau}{2}, t - \frac{\tau}{2} \right) e^{-i\varepsilon \tau} d\tau, \quad (1.5)$$

in terms of the Keldysh Green’s function $G^K(t, t')$:

$$G^K(t, t') = -2\pi i \nu h(t, t'). \quad (1.6)$$

Then the time-dependent absorption rate $W(t) = \frac{D(t)}{\delta}$ which is related to the time-dependent energy diffusion coefficient $D(t)$, is given by:

$$D(t) = -\frac{1}{2\nu} \frac{\partial}{\partial t} \lim_{\tau \to 0} \frac{\partial}{\partial \tau} G^K \left( t + \frac{\tau}{2}, t - \frac{\tau}{2} \right). \quad (1.7)$$
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Next we note that the Keldysh Green’s function for the free electron gas coupled with an external time-dependent field by \( \mathcal{H}_{\text{ef}}(t) = \hat{V} \phi(t) \) can be expressed in terms of retarded \( G^R \) and advanced \( G^A \) Green’s functions as follows:

\[
G^K(t,t') = \int dt_1 dt'_1 G^R(t,t_1) G^A(t'_1,t') h_0(t_1 - t'_1) [\mathcal{H}_{\text{ef}}(t'_1) - \mathcal{H}_{\text{ef}}(t_1)] + \int dt_1 h_0(t_1) G^R(t_1,t') - \int dt_1 G^A(t,t_1) h_0(t_1 - t'_1), (1.8)
\]

where \( h_0(t) \) is the Fourier-transform of the equilibrium distribution function \( \tanh(\epsilon/2T) = 1 - 2f_0(\epsilon) \) which is supposed to hold in the absence of perturbation \( \mathcal{H}_{\text{ef}} \). Note also that the functions \( G^R(A)(t,t') \) take account of the external time-dependent field \textit{in all orders} in \( \mathcal{H}_{\text{ef}} \) and therefore depend on two time arguments and not only on their difference.

The perturbation operator in Eq.(1.8) is not necessarily random and without loss of generality it can be assumed to be traceless \( \text{Tr} \mathcal{H}_{\text{ef}} = 0 \). For instance one can consider the perturbation by a time-dependent space-homogeneous electric field \( \mathbf{E}(t) = -\partial_t \mathbf{A} = E_0 \phi(t) \), where \( \mathcal{H}_{\text{ef}} = -e\mathbf{v} \mathbf{A}(t) \) with zero average of the electron velocity \( \mathbf{v} \) over the Fermi surface.

2.1 Diffuson-Cooperon diagrammatic technique in the time domain

Now we assume that the Hamiltonian \( \hat{H}_0 \) of the unperturbed system is that of free electrons in a Gaussian random impurity field \( U(\mathbf{r}) \) which results in the diffusive electron dynamics with a small elastic mean free path \( \ell \ll L \) compared to the system size \( L \). Averaging over disorder realizations \( U(\mathbf{r}) \) can be done using impurity diagrammatic technique [15]. For relatively weak external fields such that

\[
eE_0 \ell \ll \hbar \omega \quad (1.9)
\]

the essentially nonlinear effect of the field is on the two-particle correlation functions –diffusons and Cooperons– while for the \textit{averaged} single-particle Green’s function an expansion in \( \mathcal{H}_{\text{ef}} \) up to the second order is sufficient:

\[
\langle G^R(A)(t,t') \rangle = \delta(t - t') \left[ G^R(A) + G^R(A) \mathcal{H}_{\text{ef}}(t) G^R(A) \right] + G^R(A) \mathcal{H}_{\text{ef}}(t) G^R(A) \mathcal{H}_{\text{ef}}(t) G^R(A) \]. \quad (1.10)

In Eq.(1.10) it is assumed that the elastic scattering time \( \tau_0 = \ell/v_F \) is the smallest relevant time scale, and

\[
G^R(A)(\xi_p) = \frac{1}{\xi_p^2 + i/2\tau_0}, \quad (1.11)
\]
where \( \xi_p = \frac{v^2}{2m} - \varepsilon_F \).

Using Eq. (1.10) one can decouple the time- and momentum- integrations. The latter for a closed loop of \( G^R(A) \) functions can be done as a simple pole integral \( \nu \int d\xi G^R(A) (\xi) G^R(A) (\xi) \), as in the stationary case. In particular, one can repeat the standard derivation of the diffusons and Cooperons as the corresponding ladder series of the impurity diagrammatic technique.

The diffuson

\[
\langle G^R (r, r'; t_+, t'_+) G^A (r', r; t'_-, t_-) \rangle
\]

\[
= 2\pi \nu \delta (\tau - \tau') e^{i\mathbf{K}_d (t, \tau)} D_\tau (t, t'; r, r') e^{-i\mathbf{K}_d (t', \tau)},
\]

where \( t_{\pm} = t \pm \tau / 2, t'_{\pm} = t' \pm \tau' / 2, \mathbf{K}_d (t, \tau) = A (t + \tau / 2) \mp A (t - \tau / 2) \), obeys the equation [8]:

\[
\left\{ \frac{\partial}{\partial t} - D \nabla_r^2 - i\mathbf{rE}_0 \left[ \phi \left( t + \frac{\tau}{2} \right) - \phi \left( t - \frac{\tau}{2} \right) \right] \right\}
\times D_\tau (t, t'; r, r') = \delta (t - t') \delta (r - r')
\]

Equations (1.13,1.15) should be supplemented with the Neumann boundary conditions:

\[
\frac{\partial D_\tau (t, t'; r, r')}{\partial n} = 0, \quad \frac{\partial C_\tau (\tau, \tau'; r, r')}{\partial n} = 0.
\]

In contrast to the initial transverse gauge, in the longitudinal gauge we switched to in Eqs.(1.12–1.15) the boundary conditions do not depend on the time-dependent perturbation.

Without time-dependent perturbation Eqs.(1.13,1.15) are just simple diffusion equations which have a complete set of stationary solutions \( \phi_\mu (r) \) such that \( D \nabla_r^2 \phi_\mu (r) = E_\mu \phi_\mu (r) \) including the zero mode \( \phi_0 = \text{const} \). One can use this set as a convenient basis for solving Eqs.(1.13,1.15) in the presence of time-dependent perturbation [8]. In the limit

\[
e\mathbf{E}_0 L, \hbar \omega \ll E_{Th},
\]
where $E_{Th} = \hbar E_1 \sim \hbar D/L^2$ is the Thouless energy, the zero mode makes the main contribution, other modes should be treated as perturbations. Then the second-order perturbation theory yields for the zero-mode diffuson and Cooperon [8, 9, 16]:

$$D_\tau(t, t') = \theta(t - t') \exp \left\{-\Gamma \int_{t'}^t \left[ \phi \left( \frac{t'' + \tau}{2} \right) - \phi \left( \frac{t'' - \tau}{2} \right) \right]^2 \, dt'' \right\},$$

(1.18)

$$C_\tau(\tau, \tau') = \theta(\tau - \tau') \exp \left\{-\frac{\Gamma}{2} \int_{\tau'}^\tau \left[ \phi \left( \frac{t + \eta}{2} \right) - \phi \left( \frac{t - \eta}{2} \right) \right]^2 \, d\eta \right\},$$

(1.19)

where

$$\Gamma = \sum_{\mu \neq 0} \frac{(e r_0 \mu_0 E_0)^2}{E_\mu} \sim \frac{(e E_0 L)^2}{E_{Th}}$$

(1.20)

and $r_0 \mu = \int dr \phi_0 r \phi_\mu(r)$.

The validity condition for the zero-mode approximation can be obtained using Eqs.(1.9,1.17,1.20):

$$\frac{\ell}{L} \sqrt{\Gamma E_{Th}} \ll \frac{\hbar \omega}{E_{Th}} \ll 1.$$  

(1.21)

It is remarkable that the same expressions for the diffuson and the Cooperon can be obtained [8, 9] if one starts from the Hamiltonian Eq.(1.1) with $\hat{H}_0$ and $\hat{V}$ taken from the Gaussian Orthogonal ensemble of real symmetric $N \times N$ random matrices with the probability distributions:

$$\mathcal{P}_{V_0} \propto \exp \left[ -\frac{\pi^2 \text{tr} H_0^2}{4N\delta^2} \right], \quad \mathcal{P}_V \propto \exp \left[ -\frac{\pi \text{tr} V^2}{4\Gamma \delta} \right].$$

(1.22)

This demonstrates that for small conductors (quantum dots) and relatively weak external fields where Eq.(1.21) satisfies, the non-random time-dependent perturbation $V_0 \phi(t) = e \hat{r} E_0 \phi(t)$ in a random system is equivalent to a random-matrix perturbation $\hat{V}$ with $\langle V^2 \rangle \sim V_0^2 / g$, where $g = E_{Th}/\delta \gg 1$. This statement is believed to hold not only for diffusive quantum dots but also for generic ballistic quantum dots.

However, perhaps the simplest way to generate a correct diffuson-cooperon diagrammatic technique in the zero-dimensional limit Eq.(1.21) is to consider in Eq.(1.1)

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(r) + V(r) \phi(t)$$

(1.23)
where \( U(\mathbf{r}) \) and \( V(\mathbf{r}) \) are independent Gaussian random fields with zero mean values and the following correlation functions:

\[
\langle U(\mathbf{r}) U(\mathbf{r}') \rangle = \frac{1}{2\pi \nu \tau_0} \delta(\mathbf{r} - \mathbf{r}'), \quad \langle V(\mathbf{r}) V(\mathbf{r}') \rangle = \frac{\Gamma}{\pi \nu} \delta(\mathbf{r} - \mathbf{r}'); \quad \Gamma \tau_0 \ll 1.
\]

(1.24)

We will use this representation below because it corresponds to a minimal deformation of the impure-metal Hamiltonian that is a starting point of a conventional impurity diagrammatic technique and because it helps to avoid subtle effects of the boundary conditions Eq.(1.16) which are important for a deterministic global perturbation.

2.2 One loop DL correction

As a first step of averaging over disorder in Eq.(1.8) we single out the ladder series that represent diffusons and Cooperons. As usual [14] the diagrams are classified with respect to the number of diffuson or Cooperon loops. The zero- and one-loop diagrams are shown in Fig.1a and Fig.1c, respectively. The next step is to properly average the remaining products of retarded and advanced single-particle Green’s functions (denoted by \( R \) and \( A \) in Fig.1). So we obtain the Hikami boxes denoted by the shadowed polygons in Fig.1b and Fig.1d. In order to compute the Hikami boxes, one substitutes Eq.(1.10) for the averaged retarded or advanced Green’s functions in Fig.1a,c retaining only terms second order in the perturbation operator \( \mathcal{H}_{\text{eff}} = V(\mathbf{r}) \phi(t) \). Then one performs averaging over the random field \( V(\mathbf{r}) \). It is shown in Fig.2 as the dashed-dotted line connecting two vertices. In addition to that rules of the impurity diagrammatic technique require additional averaging over the impurity field \( U(\mathbf{r}) \) which is shown by the dotted line in Fig.2. The further calculation of the Hikami boxes is standard and reduces to pole integrals of \( G^{R(A)}(\xi) \) functions Eq.(1.11).

One can easily see that the diagrams Fig.2a and Fig.2b have opposite signs. As the result the Hikami box Fig.2a,b in the zero-loop diagram Fig.1b is proportional to \([\phi(t' + \tau/2) - \phi(t' - \tau/2)]^2\), so that the function \( h(t + \tau/2, t - \tau/2) \) in Eq.(1.6) obeys in the zero-loop approximation the following equation:

\[
\left\{ \frac{\partial}{\partial t} + \Gamma \left[ \phi \left( t + \frac{\tau}{2} \right) - \phi \left( t - \frac{\tau}{2} \right) \right]^2 \right\} h \left( t + \frac{\tau}{2}, t - \frac{\tau}{2} \right) = 0.
\]

(1.25)

If one is interested in the energy resolution \( \Delta \varepsilon \gg \omega \), where \( \omega \) is a typical period of a time-dependent field, it suffices to consider \( \omega \tau \ll 1 \). Then from Eq.(1.25,1.5) we obtain diffusion in the energy space:

\[
\left\{ \frac{\partial}{\partial t} - \Gamma (\partial_t \phi(t))^2 \frac{\partial^2}{\partial \varepsilon^2} \right\} f(\varepsilon, t) = 0.
\]

(1.26)
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If in addition we are interested in the time resolution $\Delta t \gg \omega^{-1}$, the time-independent diffusion coefficient can be defined:

$$D_0 = \Gamma \left( \partial_t \phi(t) \right)^2 \sim \Gamma \omega^2; \quad (1.27)$$
where $(\partial_t \phi)^2$ averaged over time interval much larger than the typical period of oscillation is supposed to be well defined and independent of time.

One can see from Eq.(1.26) that the parameter $\Gamma$ defined in Eqs.(1.20,1.22,1.24) has a physical meaning of an inverse time of making a step $\sim \omega$ in a random walk over the energy space.

The correction $\delta D(t)$ to the energy diffusion coefficient is described by the diagrams Fig.1c,d where the Hikami box is given by diagrams Fig.2c-g. One can easily see that in the zero-mode approximation (zero external momentum in any vertex of the rhomb) three diagrams Fig.2c-e cancel each other. The same is valid for the diagrams Fig.2c'-e'. The remaining diagrams Fig.2f,g have opposite signs, so that the entire Hikami box in Fig.1d is proportional to

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{hikami_boxes.png}
\caption{Hikami boxes: solid lines denote retarded or advanced Green’s functions, Eq.(1.11); the dashed-dotted line and the dotted line represent the $\langle V V \rangle$ and the $\langle U U \rangle$ correlator Eq.(1.24), respectively. The black circle denotes the $V \phi(t)$ vertex in Eq.(1.8); the open circle corresponds to the $V \phi(t)$ vertex in Eq.(1.10).}
\end{figure}
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\[( \phi(t' + \tau/2) - \phi(t' - \tau/2) ) (\phi(t' - \xi) - \phi(t' - \xi - \tau)) . \] Then Eq.(1.7) yields \[\delta D(t) = \frac{\Gamma \delta}{\pi} \int_0^t \partial_t \phi(t) \partial_t \phi(t - \xi) C_{t-\xi/2}(\xi, -\xi) \, d\xi, \quad (1.28)\]

where \( C_{t-\xi/2}(\xi, -\xi) \) is the zero-mode Cooperon given by Eq.(1.19) and \( t \) is time passed since the onset of the time-dependent perturbation \( \phi(t) \). Note that the correction \( \delta D(t) \) is a purely quantum mesoscopic effect that vanishes if the mean level spacing \( \delta \) between energy levels of the non-perturbed system is equal to zero.

Equation (1.28) that describes the weak dynamic localization is the main result of this section. Below we establish some of its implications.

2.3 No-dephasing points and dynamic localization

Let us consider the simplest example of a harmonic perturbation \( \phi(t) = \cos \omega t \). Then for \( \omega \xi \gg 1 \) one can write:

\[C_{t-\xi/2}(\xi, -\xi) = \exp \left\{ -2\xi \Gamma \sin^2 \left( \omega t - \frac{\omega \xi}{2} \right) \right\}. \quad (1.29)\]

A remarkable phenomenon is that the negative exponential that describes dephasing by a harmonic perturbation vanishes on a certain set of no-dephasing points:

\[\omega \xi_k = 2\omega t - 2\pi k, \quad k = 0, \pm 1, \pm 2, ... \quad (1.30)\]

At large \( \Gamma \xi \) only a close vicinity of these points contributes to the integral Eq.(1.28). Expanding \( \sin^2 \) around no-dephasing points and performing the Gaussian integration we arrive at:

\[\frac{\delta D(t)}{D_0} = -\frac{\delta}{\omega} \sum_{\xi_k} \sqrt{\frac{2}{\pi \Gamma \xi_k}} \approx -\sqrt{\frac{t}{t_*}}, \quad (1.31)\]

where

\[t_* = \frac{\pi^3 \Gamma}{2 \delta^2}. \quad (1.32)\]

This result is valid at \( t \ll t_* \) and \( \xi \sim t \gg 1/\Gamma \) which is only compatible if

\[\Gamma \gg \delta. \quad (1.33)\]

Remarkably, the WDL correction Eq.(1.31) increases with time. Equation (1.31) sets a time scale \( t_* \) where the negative WDL correction is of the order of the classical energy diffusion coefficient and one may expect the strong dynamic localization.
Note that it is only because of the no-dephasing points which are zeros of the dephasing function
\[ \Gamma_c(t) = \frac{\Gamma}{2} \left[ \phi(t + \frac{\eta}{2}) - \phi(t - \frac{\eta}{2}) \right]^2, \] (1.34)
that the correction \( \delta D(t)/D_0 \) grows with time and may become of order one despite a small parameter \( \delta/\Gamma \). For instance, the white-noise perturbation \( \phi(t) \) such that the time average \( \phi(t)\phi(t') \propto \delta(t-t') \), results in a constant dephasing function \( \Gamma_c = \Gamma \phi^2 \). Then the WDL correction Eq.(1.28) is finite \( \delta D(t) \sim D_0 \delta/\Gamma \) at \( t \to \infty \) and small compared to \( D_0 \).

In order to understand the physical meaning of no-dephasing condition we consider two electron trajectories in real space with loops traversed in opposite directions (see Fig.3), the traversing time being \( t \). Interference between such trajectories is known to be a cause of the weak Anderson localization. Let us assume that a time-dependent vector-potential \( A(t) \) is present. Then the phase difference between trajectories 1 and 2 in Fig.3 becomes random:
\[ \delta \phi = \int_0^t A(t') \left[ v_1(t') - v_2(t') \right] dt'. \] (1.35)
Using the condition \( v_1(t') = -v_2(t-t') \) we obtain
\[ \delta \phi = \int_{-t/2}^{t/2} \left[ A(t/2 + t') + A(t/2 - t') \right] v_1(t' + t/2) dt'. \] (1.36)
If the typical period of oscillations in \( A(t) \) is large compared to the velocity correlation time, one obtains after averaging over directions of velocity \( v_1 \):
\[ \langle (\delta \phi)^2 \rangle \propto \int_{-t/2}^{t/2} \left[ A(t/2 + t') + A(t/2 - t') \right]^2 dt'. \] (1.37)
For a harmonic \( A(t) = \sin \omega t \) it is easily seen that the phase difference is zero for trajectories of any shape provided that the traversing time \( t \) is synchronized with the period of the field:
\[ t = \frac{2\pi k}{\omega}, \quad k = 1, 2, .. \] (1.38)
that is, the dephasing is absent for trajectories which traversing time is equal to a multiple of the period of perturbation.

2.4 No-dephasing points and generalized time-reversal symmetry

Now let us consider a generic periodic perturbation:
\[ \phi(t) = \sum_{m=1}^{\infty} A_n \cos(m\omega t + \varphi_m). \] (1.39)
First of all we require the diffusion coefficient Eq.(1.27) to be finite:

\[ D_0 = \frac{\Gamma}{2} \sum_{m=1}^{\infty} m^2 A_m^2 < \infty. \]  (1.40)

This is only so when \( A_m^2 \) decreases faster than \( 1/m^3 \). In particular, it is not the case when \( \phi(t) \) is the periodic \( \delta \)-function as in the KQR model [1, 3]. In similar cases the sum Eq.(1.40) should be cut off at \( m \sim E_{Th}/\omega \), so that the energy diffusion coefficient \( D_0 \) grows with increasing \( E_{Th} \) or the size of matrix \( N \sim E_{Th}/\delta \) in the equivalent random matrix model Eq.(1.22).

The cooperon dephasing function Eq.(1.34) that corresponds to Eq.(1.39) takes the form:

\[ \Gamma_c(t) = \Gamma \sum_{m=1}^{\infty} A_m^2 \sin^2(m\omega t + \varphi_m). \]  (1.41)

The no-dephasing points \( t_k \) determined by \( \Gamma_c(t_k) = 0 \) exist only provided that phases of all harmonics are synchronized: \( \varphi_m = m \varphi \) (see Fig.4a). This condition implies that the perturbation \( \phi(t) \) can be made a symmetric function by a shift of the origin \( \tilde{\phi}(t) \equiv \phi(t - \varphi/\omega) \):

\[ \tilde{\phi}(t) = \phi(-t). \]  (1.42)
Figure 1.4. The Cooperon dephasing function $\Gamma_c(t)$ for (a) periodic $\phi(t)$ obeying Eq. (1.42): a regular array of zeros; (b) generic periodic $\phi(t)$: a gap; (c) quasi-periodic $\phi(t)$ with two incommensurate frequencies: a pseudo-gap.

Apparently, this is a generalization of the time-reversal condition for the case of time-dependent perturbation and for quantities that survive averaging over time intervals much larger than the period of oscillations in $\phi(t)$. It has been first established in Ref.[10].

If this condition is violated (see Fig.4b) and $\min \Gamma_c(t) = \gamma > 0$, the Cooperon $C_t(\xi, -\xi)$ acquires an exponentially decreasing factor $e^{-2\gamma \xi}$ that limits the growth of WDL correction for $t\gamma \gg 1$. For a generic case $\gamma \sim \Gamma$ we have $\delta D(t)/D_0 \sim (\delta/\Gamma) \ll 1$ and the one-loop WDL correction given by Eq.(1.28) can be neglected.

The analogous situation arises in the weak Anderson localization when the time-reversal symmetry is violated by a constant magnetic field or magnetic impurities. In this case the one-loop WAL correction also gradually vanishes leading to the anomalous magneto-resistance. The difference is that in the case of WDL the Hamiltonian remains real, represented e.g. by the orthogonal ensemble of random matrices. Yet the time-reversal symmetry can be broken just by time-dependence of perturbation. From this perspective the harmonic perturbation is an important exception rather than a paradigm of a periodic perturbation.
The analogy with the WAL suggests that the main WDL correction in the case where the condition Eq.(1.42) is violated is given by the two-loop diagrams containing only diffusons. Indeed, let us consider the diffuson dephasing function:
\[
\Gamma_d(\tau) = \Gamma \left[ \phi \left( t'' + \frac{\tau}{2} \right) - \phi \left( t'' - \frac{\tau}{2} \right) \right]^2 .
\] (1.43)

For \( \phi(t) \) given by Eq.(1.39) it reduces to:
\[
\Gamma_d(t) = 2\Gamma \sum_{m=1}^{\infty} A_m^2 \sin^2 \left( m \frac{\omega \tau}{2} \right) .
\] (1.44)

As was expected, \( \Gamma_d(t) \) is independent of phases \( \varphi_m \) and has a set of no-dephasing points \( \tau_k = \frac{2\pi k}{\omega} \) for any periodic perturbation Eq.(1.39). As the result, the two-loop WDL correction is a growing function of time [17]:
\[
\frac{\delta^{(2)} D(t)}{D_0} = -\frac{\pi}{24 \ t_*} ,
\] (1.45)

where
\[
t_* = \frac{\pi^3 \Gamma m^2}{2 \delta^2}
\] (1.46)
and \( m^2 = \sum_{m=1}^{N} m^2 A_m^2 / \sum_{m=1}^{N} A_m^2 \).

### 2.5 Incommensurability of frequencies, avoided no-dephasing points and weak Anderson localization in higher dimensions.

Now let us consider the perturbation \( \phi(t) \) as a sum of several incommensurate harmonics:
\[
\phi(t) = \sum_{m=1}^{d} A_m \cos(\omega_m t + \varphi_m) .
\] (1.47)

In this case the Cooperon dephasing function \( \Gamma_c(t) \) is given by:
\[
\Gamma_c(t) = \Gamma \sum_{m=1}^{d} A_m^2 \sin^2(\omega_m t + \varphi_m) ,
\] (1.48)

where all \( \omega_m \) do not have a common multiplier. In this case one has a distribution of avoided no-dephasing points as in Fig.4c.

In order to compute the one-loop WDL correction Eq.(1.28) we expand each of the exponentials [sf. Eq.(1.29)] \( \exp \left\{ -2\xi \Gamma A_m^2 \sin^2 \left( \omega_m t - \frac{\omega_m \xi}{2} + \varphi_m \right) \right\} \)
in a Fourier series using:

\[ e^z \cos(2\omega_m t + 2\varphi_m) = \sum_{n_m = -\infty}^{+\infty} I_{n_m}(z) e^{i n_m (2\omega_m t + 2\varphi_m)}, \]  

where \( I_{n_m}(z) \) is the Bessel function.

Then Eq.(1.28) reduces to:

\[ \delta D(t) = -\frac{\Gamma \delta}{2\pi} \int_0^t d\xi \sum_{n_1 = -\infty}^{+\infty} \ldots \sum_{n_d = -\infty}^{+\infty} \sum_{m=1}^{d} \omega_m^2 A_m^2 (\ln I_{n_m}(z_m))' \times \prod_{m=1}^{d} I_{n_m}(z_m) e^{-z_m} e^{i n_m (2\omega_m t - \omega_m \xi + 2\varphi_m)}, \]

where \( z_m = \xi \Gamma A_m^2 \), and the prime denotes the derivative \( d/dz \).

In order to obtain the behavior of \( \delta D(t) \) averaged over time intervals much larger than the typical period of oscillations one should take into account only \( n_m \) obeying the constraint:

\[ \sum_{m=1}^{d} n_m \omega_m = 0. \]

It is exactly the condition of incommensurability of frequencies \( \omega_m \) that the constraint Eq.(1.51) can be satisfied only if all \( n_m = 0 \). One can see that at such a condition \( \delta D(t) \) does not depend on the phases \( \varphi_m \) and is given by:

\[ \delta D(t) = -\frac{\Gamma \delta}{2\pi} \int_0^t d\xi \sum_{m=1}^{d} \omega_m^2 A_m^2 (\ln I_0(z_m))' \prod_{m=1}^{d} I_0(z_m) e^{-z_m}. \]

Further simplification is possible if we consider \( A_m = 1 \) and \( \Gamma t \gg 1 \). In this limit \( (\ln I_0(z_m))' = 1 \), \( I_0(z_m) \approx e^{z_m}/\sqrt{2\pi z_m} \) and one arrives at [11]:

\[ \frac{\delta D(t)}{D_0} = -\frac{\delta}{\pi \Gamma} \int_{\Gamma}^{t} \frac{dz}{(2\pi z)^{d/2}}. \]

Remarkably, the dependence on \( t \) in Eq.(1.53) coincides with the dependence of WAL corrections to conductivity of a \( d \)-dimensional conductor on the de-phasing time \( \tau_{\varphi} \). In the case of two incommensurate harmonics the WDL correction grows with time logarithmically:

\[ \frac{\delta D(t)}{D_0} = -\frac{\delta}{2\pi^2 \Gamma} \ln(t\Gamma). \]

For three and more incommensurate harmonics \( \delta D(t)/D_0 \sim \delta/\Gamma \) is small. It is yet to be studied whether or not a dynamic localization transition is possible for \( d \geq 3 \) at some \( \Gamma_{crit} \sim \delta \) as it is claimed to be the case for the KQR [18].
3. Role of electron interaction on dynamic localization in closed quantum dots.

So far we considered energy absorption in a closed system of non-interacting fermions under time-dependent perturbation. Although the many-body character of the system was important for making the diffusion in the energy space a mechanism of energy absorption due to the presence of the filled Fermi sea, inelastic processes of interaction were completely disregarded. We have demonstrated how the weak dynamic localization arises in such systems and how much does it resemble the Anderson localization (AL) corrections to conductivity. This analogy makes us believe that WDL corrections for periodic perturbations evolve with time to produce the strong DL in the same way as WAL corrections lead to a vanishing conductance of low-dimensional systems of sufficiently large size in the absence of dephasing mechanisms. So we assume that in a quantum dot without electron interaction at times \( t \gg t^* \) the energy diffusion coefficient becomes exponentially small. By this time the electron temperature rises from a low fridge temperature to the effective temperature \( T^* \sim \omega \Gamma / \delta \).

The question is what happens if electron interaction is present. More precisely, what happens if the dephasing rate \( \gamma_\varphi = 1/t_{ee} \) related with interaction is small but finite:

\[
0 < \gamma_\varphi(T^*) \ll \frac{1}{t^*}.
\]

In the problem of strong AL the conductance \( \sigma \) remains exponentially small provided that the phase-breaking length is much greater than the localization length \( L_\varphi > L_{\text{loc}} \) and the temperature is much smaller than the mean level spacing in the localization volume \( \delta_{\text{loc}} = (\nu L_{\text{loc}}^d)^{-1} \). The first condition ensures localization of eigenstates. The second one makes hopping over preformed localized states an exponentially rare event, so that \( \sigma \sim e^{-(\delta_{\text{loc}}/T)^\alpha} \) with some positive exponent \( \alpha \).

The point of a crucial difference between the strong AL and the strong DL is that in the latter it is the inverse localization time \( t_{\text{loc}}^{-1} \sim \delta^2/\Gamma \) that plays a role of \( \delta_{\text{loc}} \). In the case when \( \delta \) is the smallest energy scale we have

\[
T^* \sim \frac{\omega \Gamma}{\delta} \gg \frac{1}{t^*}.
\]

As a result, no exponentially small factor arises in the hopping over the Floquet states in contrast to the hopping over localized states in a real space.

The qualitative picture that follows from the consideration [12] based on the Fermi Golden Rule is the following. A given electron absorbs energy from the external field during the time \( t^* \) until the strong DL develops itself. At \( t > t^* \) the dynamically localized state with the energy \( T_{\text{eff}} \sim T^* \) above the Fermi sea is formed which is characterized by the exponentially small absorption rate.
So it continues until after a time $t_{ee}$ a collision with another electron breaks the phase-tuning necessary for DL. The "efficiency" of each collision in the destruction of DL is close to 100% because the typical energy transfer is large $T_* \gg 1/t_*$. Then the absorption rate jumps to a classical value and it takes another $t_*$ to reach DL for the second time, but now with energy $2T_*$. So, for a given electron relatively short time intervals $\sim t_*$ of absorption are followed by larger periods $\sim t_{ee}$ of DL, the energy increasing by $T_*$ after each circle of absorption. The averaged absorption rate $W_{in}$ is given by:

$$W_{in} = W_0 \frac{t_*}{t_* + t_{ee}}. \quad (1.57)$$

Were electron-electron collision time $t_{ee}$ energy independent, the averaged absorption rate would stay constant at all times $t > t_*$. However, $1/t_{ee}$ typically increases with increasing the energy $T_{eff}$, so does also the averaged absorption rate. Then we obtain from Eq(1.3):

$$\frac{dT_{eff}^2}{dt} = D_0 \frac{t_*}{t_* + t_{ee}(T_{eff})}. \quad (1.58)$$

The time $t_{ee}$ that corresponds to inelastic collisions of electrons in a quantum dot with the energy transfer $\sim T_{eff} \ll E_{Th}$ has been calculated in Ref.[19]:

$$\frac{1}{t_{ee}(T_{eff})} = \delta \left( \frac{T_{eff}}{E_{Th}} \right)^2. \quad (1.59)$$

For $t_* \ll t_{ee}(T_{eff})$ Eqs.(1.58, 1.59) suggest that the effective electron temperature increases exponentially:

$$T_{eff} = T_* \exp \left[ \frac{t}{2t_{ee}(T_*)} \right] \quad (1.60)$$

rather than remaining a constant $\sim T_*$ as it does for non-interacting electrons. The corresponding absorption rate $W_{in}(t)$ takes the form:

$$W_{in}(t) = W_0 \left( t_* \right) \exp \left[ \frac{t}{2t_{ee}(T_*)} \right], \quad t > t_. \quad (1.61)$$

The exponential growth of effective temperature continues until $t_{ee}(T_{eff})$ decreases down to $t_*$. At larger times $t > t_{ee}(T_*) \ln(t_{ee}(T_*)/t_*)$ the classical absorption rate is restored: $W_{in} = W_0$. The sketch of the time-dependence of absorption rate is shown in Fig.5. The main conclusion of the qualitative picture of Ref.[12] described above is that the dynamic localization is a transient phenomenon that is destroyed at long enough times by an arbitrary small electron interaction.
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Figure 1.5. Absorption rate vs time in a closed system with electron-electron interaction.

Note, however, that the consideration of Ref.[12] is based on the Fermi Golden Rule. The applicability of this description is known [20] to be restricted in quantum dots by the region of relatively large electron energies $T_{\text{eff}} \gg \sqrt{E_{Th} \delta}$. At smaller energies the localization in the Fock space should take place with the effect of vanishing $1/t_{ee}$. Therefore it is likely that at $T_{\ast} \sim \omega \Gamma / \delta < \sqrt{E_{Th} \delta}$ the dynamic localization survives electron-electron interaction.

4. Dynamic localization in an open quantum dot and the shape of the Coulomb blockade peak.

The transient character of DL and difficulty to realize a closed system in condensed matter physics because of the phonon heat transport leaves only one option for possible observation of DL: the steady state in an open system under periodic pumping. In an open system Eq.(1.57) should be generalized to include the dephasing due to electron escape into leads along with dephasing by inelastic electron collisions considered in the previous section:

$$W_{\text{in}} = W_0 \frac{\gamma_{\varphi} t_{\ast}}{1 + \gamma_{\varphi} t_{\ast}},$$

(1.62)

where the energy-dependent dephasing rate is given by:

$$\gamma_{\varphi} = \gamma_{\text{esc}} + \frac{1}{t_{ee}(T_{\text{eff}})}.$$

(1.63)

From Eq.(1.62) it follows that in order to observe any deviation from the classical absorption rate $W_0$ the escape rate $\gamma_{\text{esc}}$ must be much smaller than the inverse localization time $1/t_{\ast} \sim \delta^2 / \Gamma$. Then we obtain the necessary condi-
tion for DL:

\[ \frac{\gamma_{\text{esc}}}{\delta} \ll \frac{\delta}{\Gamma} \ll 1. \] (1.64)

The last inequality Eq.(1.33) is the basic approximation adopted in this study of DL.

Eq.(1.64) shows that the DL effects considered could be observed only provided that the quantum dot is almost closed, i.e. \( \frac{\gamma_{\text{esc}}}{\delta} \ll 1 \). This is exactly the condition where the Coulomb blockade effects [21] are relevant. Under this condition the low-temperature linear dc conductance of the dot is strongly peaked near the value of the gate voltage \( V_g = V_c \) where the states with \( N \) and \( N + 1 \) electrons on the dot are nearly degenerate. At equilibrium conditions where the electron energy distribution inside the dot is Fermi-Dirac with the same temperature as in the leads, the width of the peak is proportional to the temperature and thus can be used as a thermometer. In the case where the dot is subject to time-dependent perturbation, the electron energy distribution inside the dot can be drastically different from that in the leads and a non-trivial question arises which of these two distributions affect the linear dc conductance. The answer for an open dot where the Coulomb blockade effects play no role is given in Refs.[8, 9]. It appears that in this case the linear dc conductance is sensitive only to the electron energy distribution in the leads and thus cannot be used to probe energy dynamics in the dot. However, with the Coulomb interaction inside the dot taken into account and for the case \( T_{\text{eff}} \gg \delta \) the linear dc conductance becomes sensitive to the effective electron temperature \( T_{\text{eff}} \) inside the dot. Below we consider only the extreme case of the Coulomb blockade. We also assume that the inelastic electron interaction is sufficiently strong to produce the Fermi-Dirac form of the electron energy distribution inside the dot, though with the temperature \( T_{\text{eff}} \) much larger than that in the leads. We concentrate on the qualitative changes in the Coulomb blockade peak shape characteristic of the dynamic localization in the approximation of sequential tunneling [21]. More detailed consideration including the role of inelastic co-tunneling and cooling by phonons is presented in Ref.[22].

The steady state of a dot under ac perturbation is determined by the global energy balance \( W_{\text{in}} = W_{\text{out}} \), where the pumping of energy by the ac perturbation is governed by Eq.(1.62) and \( W_{\text{out}} \) is the cooling rate. The most interesting case corresponds to a situation where the main cooling mechanism is due to electron escape into cold leads. Denoting by \( U = V_g - V_c \) the detuning of the gate voltage from the peak center and introducing the parameter \( x = U/(2T_{\text{eff}}) \) we obtain [22] for the case of leads maintained at a temperature \( T_0 \ll T_{\text{eff}} \):

\[ \frac{W_{\text{out}}(U)}{(\gamma/\delta) T_{\text{eff}}^2} = \frac{\pi^2}{12} - x^2 + \frac{2x}{\ln(2 \cosh x)} \int_0^x \xi \tanh \xi \, d\xi, \] (1.65)
where $\gamma = 2\gamma_{esc}(U = 0)$. In the same approximation we obtain for the linear

dc conductance of the dot:

$$
\frac{G(U)}{G_0} = 1 - \frac{x \tanh x}{\ln(2 \cosh x)}. 
$$

(1.66)

Remarkably, the r.h.s. of Eqs.(1.65,1.66) depends only on the single variable $x$, so that one can establish a relationship similar to the Wiedemann-Franz law:

$$
W_{out} = \left(\frac{\gamma}{\delta}\right) T_{\text{eff}}^2 W(G/G_0).
$$

(1.67)

Now, let us assume that

$$
\gamma_{\phi} t_* \ll 1,
$$

(1.68)

and

$$
\frac{\gamma_{esc}(U)}{\delta} \ll \frac{T_{\text{eff}}^2}{E_{T\text{h}}},
$$

(1.69)

where the escape rate is given by

$$
\frac{\gamma_{esc}}{\gamma} = \frac{1}{2} - \frac{|x|}{2 \ln(2 \cosh x)}.
$$

(1.70)
Then we find from Eqs.(1.59,1.62):

\[ W_{in} = T_{eff}^2 \left( \frac{T_*}{E_{Th}} \right)^2. \]  

(1.71)

One can see that due to the fact that both the cooling rate \( W_{out} \) given by Eq.(1.67) and the energy pumping rate \( W_{in} \) given by Eq.(1.71) are proportional to \( T_{eff}^2 \), the effective electron temperature drops out of the global energy balance \( W_{in} = W_{out} \). As the result, the dc conductance of the dot is remarkably independent of the gate voltage:

\[ \frac{G}{G_0} \sim \left( \frac{T_*}{E_{Th}} \right)^2 \frac{\delta}{\gamma}. \]  

(1.72)
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Note that since $G/G_0 < 1$ such a plateau solution exists only when

$$\frac{\gamma}{\delta} > \left(\frac{T_s}{E_{Th}}\right)^2. \quad (1.73)$$

At the same time for large enough detuning $U > U_{\text{min}} \sim T_s$ the condition Eq.(1.69) is fulfilled because the escape rate $\gamma_{\text{esc}}(U)$ decreases and the effective temperature increases with increasing $U$. However, with $U$ and $T_{\text{eff}}$ further increasing the DL regime Eq.(1.68) is destroyed and $G(U)/G_0$ starts to decrease. This happens at $U > U_{\text{max}} \sim E_{Th} \sqrt{\frac{\delta}{\Gamma}}$. The sketch of the the $G(U)$ dependence is shown in Fig.7.

5. Summary

In conclusion, we have considered the manifestation of dynamic localization in a solid-state system – a quantum dot under time-dependent perturbation. We have shown that in a closed dot without electron interaction the dynamic localization is possible and we developed the theory of weak dynamic localization for an arbitrary time-dependent perturbation. Then we considered a realistic case of a dot with electron-electron interaction weakly connected to leads. We have demonstrated that if the dot is subject to a periodic perturbation the dynamic localization can result in a plateau at the tail of the Coulomb blockade peak in the linear dc conductance vs the gate voltage and we established the conditions under which such a plateau arises.

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