Supply-demand optimization in the product distribution process through the branching method

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Abstract. Interaction and relationship between seller-buyer for the availability of certain products always involve the distribution process from suppliers to a number of customers so that demand at each destination is fulfilled. If the product cannot be fulfilled, the customer will find another supplier to meet the product's shortcomings. Thus the supplier is trying to prevent the supply of other competitors of the same type. Effective logistical arrangements in the distribution of goods need to pay attention to the behavior characteristics of shipping costs. The purpose of this study is to minimize shipping costs with a distribution scheme that can optimize product loads. By using the Balinski linear approach, the problem of sending fixed costs can be solved by the branching method. The branching method gradually produces new problem-solving branches with different objective functions and constraints. Each value branch could be compared so that the optimal solution for the cost is found.

1. Introduction

The product distribution process is very important for a company. In distribution management, distribution managers are responsible for managing company logistics, such as product inventory being delivered on time and economically. Another responsibility is to determine whether distribution services must use private vehicles or must rent. Successful companies realize that there is no cheap distribution to develop their business. Unless distribution is managed effectively and efficiently, so that procurement, manufacturing, and distribution performance can meet expectations.

In the problem of product distribution and delivery, there are fixed costs and variable costs. The problem of fixed costs is modeled as an integer programming problem 0 - 1. Fixed costs arise when investment or rental of transport vehicles. Variable costs in the distribution process include fuel costs and engine maintenance costs. Product distribution and shipping costs will change according to the distance that must be traveled product loads. This distance will be aligned with the cost of fuel, so the further the distance will provide a greater variable cost. The problem of product distribution and delivery will be simplified by linear formulation. Branching method is a very useful method of solving fixed cost distribution problems [1]. Distribution formulation is completed periodically by updating the upper and lower limits so that the optimal solution is obtained. The solution obtained is very useful for solving large-scale fixed cost problems.

Distribution problems in distribution problems can be solved by approximation techniques [2]. The problem of redistribution in the Navy supply system is carried out on small samples. This technique behaves well when fixed costs are higher and few routes are used, at present, no improvement at this lower threshold will come. The approximation technique described is very strong and the general conditions produce an optimal solution to the distribution problem with a fixed cost.

Several studies have discussed the branching method [1, 9], the inventory problems [2, 3, 6], and minimizing cost in logistics [4, 5, 7, 8]. The branching method provides the lower and upper limits for fixed cost distribution problems so that each branch can be considered a solution. In contrast to previous
studies, this study can encourage the linear nature of finding optimal solutions through the Balinski approach. There is at least one optimal point to guarantee that the problem has a solution.

2. Preliminaries

The distribution process is a distribution problem that involves the distribution of products originating from $m$ suppliers to $n$ customers (customers). Inventory at each supplier $i$ is unit $a_i$ and demand for each agent $j$ is unit $b_j$. Suppose $x_{ij}$ is the number of products sent from supplier $i$ to customer $j$ with the shipping cost per unit of the product being $c_{ij}$ and a fixed cost of $f_{ij}$. Distribution problems in this study use fixed costs. Binary integer programming for fixed costs is $y_{ij}$.

In general, a mathematical model for distribution with fixed costs can be formulated as follows:

\[ P : \min Z(P) = \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} x_{ij} + f_{ij} y_{ij}) \] (1)

subject to:

\[ \sum_{i=1}^{m} x_{ij} = a_i \quad \text{for } i = 1, 2, \ldots, m \] (2)

\[ \sum_{j=1}^{n} x_{ij} = b_j \quad \text{for } j = 1, 2, \ldots, n \] (3)

\[ x_{ij} \geq 0 \quad \text{for all } (i,j) \]

\[ y_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \\ 1, & \text{if } x_{ij} > 0 \end{cases} \] (4)

We assume that the inventory owned by source $a_i$ is balanced by the demand owned by the destination $b_j$.

\[ \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad \text{and} \quad a_i, b_j, c_{ij}, f_{ij} \geq 0 \]

Product distributions in connection with demand will be met according to available inventory. At the time of distribution of the product will be considered matters relating to the number of loads and total distribution costs.

3. The product distribution process through the branching method

Development of a linear approach of the fixed cost distribution problem [1] by relaxing the integer constraint on $y_{ij}$ with that property as follows:

\[ y_{ij} = x_{ij}/m_{ij} \] (4)

where

\[ m_{ij} = \min(a_i, b_j) \] (5)

The problem of distribution is relaxed into a fixed cost distribution problem that is transformed into a classic distribution problem without fixed costs. Distribution unit costs are formulated as follows:

\[ c_{ij} = c_{ij} + f_{ij}/m_{ij} \] (6)

This relaxation distribution problem can be called $PB$ and this formulation is used to develop a feasible initial solution for the fixed cost distribution problem. The optimal solution of the $PB$ distribution problem is $(x^B_{ij}, y^B_{ij})$, then a feasible solution is obtained $(x^B_{ij}, y^B_{ij})$ from $P$ with the following conditions as follows:

\[ y^B_{ij} = \begin{cases} 0, & \text{if } x^B_{ij} = 0 \\ 1, & \text{if } x^B_{ij} > 0 \end{cases} \] (7)

The optimal value $Z(PB)$ is the lower limit on the optimal value $Z^*(P)$ of the fixed cost distribution problem. So the solution $(x^B_{ij}, y^B_{ij})$ becomes a feasible solution that gives an upper limit on $Z^*(P)$. Thus $Z(PB) \leq Z^*(P) \leq Z(P)$ can be written as follows:
\[
\sum \sum c_{ij} x_{ij}^B \leq Z^*(P) \leq \sum \sum \left( c_{ij} x_{ij}^B + f_{ij} y_{ij}^B \right).
\] (8)

The solution of a relaxation distribution (PB) problem is integer, the lower limit of inequality in Equation (8) of the optimal (PB) solution can be rounded up to the nearest interval.

4. Supply-demand optimization

Supply and demand allocation is a matter of distribution. Problem-solving uses the formulation of distribution problems with the Balinski linear approach. Certain cells are selected according to certain selection criteria. This method branches off progressively by excluding selected cells to find the optimal solution. We assume that the distribution problem formulation uses fixed costs, with the coefficient of costs \( c_{ij} \), fixed costs \( f_{ij} \) and \( m_{ij} \).

The branching process of fixed cost distribution problems are as follows:

The first step : Formulate Balinski relaxation distribution matrix problem with equation (6)
Second step : Solve it as a classic distribution problem and identify the charge as \( x_{ij}^B \).
Third step : Check the termination conditions for certain branches with the following conditions:
   a. \( \sum \sum c_{ij} x_{ij}^B = \sum \sum \left( c_{ij} x_{ij}^B + f_{ij} y_{ij}^B \right) \) or
   b. \( \sum \sum c_{ij} x_{ij}^B \geq Z \) value, so far stop and end this branch.
Fourth step : Select cells that are partially loaded, i.e. cells with \( 0 < x_{ij}^B < m_{ij} \).
Fifth step : For each cell identified in Step 4, by counting \( \Delta = f_{ij} - \left( f_{ij} / m_{ij} \right) x_{ij}^B = f_{ij} \left( 1 - x_{ij}^B / m_{ij} \right) \).
The value \( \Delta = f_{ij} \left( 1 - x_{ij}^B / m_{ij} \right) \) is the differentiation of fixed costs between the problem of relaxation distribution and the problem of distribution of fixed costs to load \( x_{ij}^B \) in cells \( (i, j) \).
Sixth step : Select the cell \((s, t)\) with the highest \( \Delta \) among those identified in fifth step. Break the bond by choosing \( \Delta \) with the largest \( f_{ij} \). If there is more than one \( \Delta \), then choose one randomly.
In sixth step produces two branches, namely: (a) the \( Y \) branch, the selected cell is given a charge. The associated fixed costs are summarized as \( Z_{y\text{cost}} \) to be combined with the objective function and \( f_{st} = 0 \) in this fixed cost distribution problem; (b) in the selected branch \( N \), is excluded by setting costs related to a very large amount.

Branch \((s, t)\) : contains cells \((s, t)\).
Seventh step : Note \( Z_{y\text{cost}} = f_{st} \), where \( Z_{y\text{cost}} \) represents fixed costs summarized along branch \( Y \) \((s, t)\). Set \( f_{st} = 0 \) in the fixed cost distribution problem.
Eighth step : Repeat Steps 1-6 with the distribution problem of fixed costs that have been adjusted.
Branch \((s, t)\) : except cell \((s, t)\).
Ninth step : Set \( c_{st} = M \) in the fixed cost distribution problem.
Tenth step : Repeat Steps 1-6 with the fixed cost distribution problem that has been adjusted.

The branching method utilizes the relationship presented in inequality in Equation (8), which provides the lower and upper limits for fixed cost distribution problems.
This method sequentially looks for fixed costs \( f_{ij} \) given by distribution problems. This method can encourage the linear nature of finding optimal solutions by sequentially increasing the lower limit obtained from the Balinski formulation. This method guarantees that there is at least one optimal point, identified by the lowest upper limit.
5. Numerical example
We apply the problem of product distribution to the allocation of supply and demand in companies that have three branch factories that operate together. The three branches are called Plant 1, Plant 2, and Plant 3. Product distribution is carried out to meet the market supply based on product demand data provided by Agent 1, Agent 2, and Agent 3 in the maximum and minimum demand periods.

The vehicle used for product distribution is a truck with a load capacity of 30 tons with a fixed cost of IDR 45,000,000 per unit per month. The cost of fuel needed is calculated starting from departing to returning to the warehouse of each plant at a price of IDR 5,500 per liter for a distance of 2 km. The cost distribution matrix is prepared by formulation \((f_{ij}, c_{ij})\) where \(f_{ij}\) is a fixed cost that is the cost vehicle rental, and \(c_{ij}\) are variable costs, i.e. fuel costs.

5.1. Maximum demand period
The distribution cost for the maximum demand period can be seen in Table 1.

|          | Agent 1  | Agent 2  | Agent 3  | Supply (ton) |
|----------|----------|----------|----------|--------------|
| Plant 1  | (720,343)| (720,375)| (630,402)| 14.130       |
| Plant 2  | (720,79)| (720,206)| (630,192)| 14.130       |
| Plant 3  | (720,510)| (720,628)| (630,404)| 14.330       |
| Demand (Ton) | 13.890 | 16.130  | 12.570  | 42.590       |

The relaxation distribution cost \(C_{ij}\) from the Balinski linear approach in Equation (6) when the maximum demand period can be seen in Table 2.

|          | Agent 1  | Agent 2  | Agent 3  | Supply (ton) |
|----------|----------|----------|----------|--------------|
| Plant 1  | 343,052  | 375,051  | 402,05   | 14.130       |
| Plant 2  | 79,052   | 206,051  | 192,05   | 14.130       |
| Plant 3  | 510,052  | 628,05   | 404,05   | 14.330       |
| Demand (Ton) | 13.890 | 16.130  | 12.570  | 42.590       |

Based on Balinski relaxation and branching processes, the optimal solution is found in Table 3. Table 3 shows each plant 14.130/30 tons in one month (30 days). Each plant is sent a load of 16 trucks so that 48 trucks per day is needed on a maximum period.

|          | Agent 1  | Agent 2  | Agent 3  | Supply (ton) |
|----------|----------|----------|----------|--------------|
| Plant 1  | 0        | 14.130   | 0        | 14.130       |
| Plant 2  | 13.890   | 240      | 0        | 14.130       |
| Plant 3  | 0        | 1,760    | 12.570   | 14.330       |
| Demand (Ton) | 13.890 | 16.130  | 12.570  | 42.590       |
5.2. Minimum demand period

The distribution cost for the minimum demand period can be seen in Table 4.

Table 4. The distribution cost \((f_{ij}, c_{ij})\) on minimum demand period (million).

|          | Agent 1 | Agent 2 | Agent 3 | Supply (ton) |
|----------|---------|---------|---------|--------------|
| Plant 1  | (405,193) | (585,305) | (585,373) | 7.650        |
| Plant 2  | (495,55) | (585,167) | (585,178) | 13.703       |
| Plant 3  | (495,350) | (585,511) | (585,375) | 11.603       |
| Demand (Ton) | 9.540 | 11.386 | 12.030 | 32.956       |

The relaxation distribution cost \(C_{ij}\) from the Balinski linear approach in Equation (6) when the minimum demand period can be seen in Table 5.

Table 5. Relaxation distribution cost \(C_{ij}\) on minimum demand period (million).

|          | Agent 1 | Agent 2 | Agent 3 | Supply (ton) |
|----------|---------|---------|---------|--------------|
| Plant 1  | 193,053 | 305,076 | 373,076 | 7.650        |
| Plant 2  | 55,052  | 167,051 | 178,049 | 13.703       |
| Plant 3  | 350,052 | 511,051 | 375,051 | 11.603       |
| Demand (Ton) | 9.540 | 11.386 | 12.030 | 32.956       |

Based on Balinski relaxation and branching processes, the optimal solution is found in Table 6. Table 6 shows Plant 1 for \(\frac{1.7650}{30} = 255\) in one month (30 days). Plant 1 distribution has sent a load of 9 trucks per day. The same way for Plant 2 (16 trucks) and Plant 3 (13 trucks). Total product distribution is done 38 trucks per day is needed on a minimum period.

Table 6. Optimal solutions to problems of distribution fixed costs minimum demand period.

|          | Agent 1 | Agent 2 | Agent 3 | Supply (ton) |
|----------|---------|---------|---------|--------------|
| Plant 1  | 0       | 7.650   | 0       | 7.650        |
| Plant 2  | 9.540   | 3.736   | 427     | 13.703       |
| Plant 3  | 0       | 0       | 11.603  | 11.603       |
| Demand (Ton) | 9.540 | 11.386 | 12.030 | 32.956       |

5.3. Results and Discussion

We use two data scenarios in the numerical example, which are the maximum demand period and the minimum demand period. This is to show the difference between demand in the high season and low season. Two distribution request scenarios are made to look in more detail at resolving the average demand problem.

The fixed costs observed in this study are vehicle rental costs. Fuel costs are variable costs that change according to changes in distribution. Both costs affect the costs the company budgeted for product distribution. The company packages products weighing 40 kg per bag to fulfill supplies according to the agent's request. Product distribution uses trucks with a payload capacity of 30 tons. The shipping amount and weight of the package are integers.

The branching method can solve problems related to integers. This method combined Balinski's formulation and completion by selecting certain cells to be loaded maximally and providing minimal distribution costs.
Numerical simulations during the period of maximum demand obtained minimal results from the distribution of products from 3 plants with 48 trucks. The distribution of each plant for 16 trucks spent Rp.421,244,600,000 per month. While optimization of product distribution in the minimum period consists of 9 trucks for Plant 1, 16 trucks for Plant 2, and 13 trucks for Plant 3. The cost required is Rp.239,924,100,000 per month using 38 units of trucks, the vehicle is carrying 814,900 bags.

6. Conclusion and Future Research
Factors and characteristics of distribution cost are developed as important factors in logistical arrangements. One factor that influences this is the behavior of distribution costs. In this study, the distribution costs are considered fixed costs. The fixed costs include distribution problems that are difficult to solve. The distribution costs are minimized by using the branching method. The numerical example shows the problem of fixed costs on the maximum and minimum period that have different quantities of supply and demand. We use the branching method in the iteration process to get the upper and lower limits that correspond to the constraints. The two limits can be compared to get the optimal solution which is the minimum distribution costs needed to maintain optimal inventory and can meet every customer demand. The next research agenda, fixed costs, and variable costs will be tested for the dominance or importance/weight of both.

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