Flux qubits on semiconducting quantum ring

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Abstract. The ability to control the quantum state of a single electron in a quantum ring made of a semiconductor is at the heart of recent developments towards a scalable quantum computer. A peculiar dispersion relation of quantum rings allows to steer the ground state properties by the magnetic flux and offers spin and orbital degrees of freedom for quantum manipulations. We show that such ring can be effectively reduced to the two-state system forming a qubit on orbital or spin degrees of freedom.

1. Introduction
Quantum information and computation are one of the fastest expanding areas of modern physics. The basic units of quantum computation are the two state systems, usually called qubits, which are the superpositions of quantum states [1]. These include microscopic objects such as ions in the electromagnetic traps, atoms in cavities, nuclear spins in molecules [2] and charge and flux states in mesoscopic solid state systems [3, 4, 5]. The latter are easier to integrate using standard circuit technology but the large number of degrees of freedom makes it more difficult to maintain the coherence.

To built the qubit a necessary condition which has to be fulfilled is that the system under consideration has to be treated in a good approximation as a two state quantum system. Thus the energy of next lower and higher lying states has to be at least five qubit splittings away to avoid leakage of quantum information [4, 6]. An attractive candidate for a solid state qubit is based on semiconducting quantum dot, which allow controlled coupling of one or more electrons using rapidly switchable voltages applied to electrostatic gates. Recently a flux qubit built on semiconducting quantum ring has been proposed [7].

The size of a flux qubit (similar to a big molecule) places it on the border line between the two kinds of quantum bits discussed above. The small number of degrees of freedom together with the small size should help to decouple it from the environment but simultaneously it can be easily coupled to other qubits. The system is an arbitrary doubly connected conductor or semiconductor. The phase coherence length of an electron in the system must be longer then its circumference. The two opposite persistent current (PC) states with a well defined orbital magnetic moments are the basis states. Quantum tunneling via the controllable potential barrier close to one of the degeneracy point leads to a formation of a qubit states beeing the quantum superposition of two persistent current states [7]. In that model we considered spinless electrons which was a reasonable assumption for metallic rings but an oversimplification for semiconductor ones.

In the present paper we include spin into considerations. In such a system apart from orbital, there is a spin degree of freedom resulting in a forthfold deenercy in full symmetry systems.
To make a qubit i.e. a two state system we have to break this symmetry. It can be done by the application of the magnetic field $B_c$ parallel to the axis of the ring and/or by breaking spatial symmetry of the system. Because we consider orbital as well as spin magnetic moments of an electron, the appropriate qubits will be called orbital and spin qubits respectively. The general state is of the form

$$|Q_{orb,spin}⟩ = |n⟩ \otimes |s⟩.$$  

(1)

where $n$ is the orbital angular momentum quantum number yielding at a given $B_c$ persistent currents corresponding to orbital momentum parallel $l \equiv \uparrow$ and antiparallel $l \equiv \downarrow$ to external field and $s = \pm \frac{1}{2}$ is the spin quantum number inducing the spin magnetic moment $\mu_s = g_s \mu_B s$, where $g_s$ is the Lande factor. In principle it is feasible to make a two state system, well separated from the other states, based on the general form (1). It would require however, the mixing potential acting simultaneously on the spin and orbit degrees of freedom. To make it easier to fabricate, we consider in the present paper two kinds of systems, having fixed one of the degrees of freedom. The first is the system which is in a superposition of states with fixed $s$ and well defined orbital angular quantum numbers $n$ and $m$, degenerate at $B_c$, corresponding to opposite PC $l \equiv \uparrow, \downarrow$; we call it an orbital qubit

$$|Q_{orb}^\pm⟩ = \alpha |n, +⟩ \pm \beta |m, s⟩,$$  

(2)

where $|\alpha|^2 + |\beta|^2 = 1$. The second type of qubit is a spin qubit, which is formed by the states of the same orbital number $n$ but opposite, well defined, spin quantum numbers

$$|Q_{spin}^\pm⟩ = |n, \frac{1}{2}⟩ \pm |n, -\frac{1}{2}⟩.$$  

(3)

In Chapter 2 we discuss the formation of orbital and spin qubits. In Chapter 3 we present coherent manipulations on qubits, the conclusions are given in Chapter 4.

2. Orbital and spin qubits on semiconductor quantum rings.

Let us consider a defect-free mesoscopic semiconducting quasi 1D ring of radius $R$ ($2\pi R < L_\phi$) in the presence of static magnetic flux $\phi$, $\phi = B_c \pi R^2$, $B_c$ is the applied magnetic field perpendicular to the plane of the ring. The quantized energy spectrum of electrons is numbered by the orbital quantum number $n = 0, \pm 1, ...$ and spin quantum number $s = \pm \frac{1}{2}$

$$E_{n,s}(\phi') = \frac{\hbar^2}{2m^* R^2} \left[ (n - \phi')^2 + 2g_s \frac{m^*}{m_e} s \phi' \right],$$  

(4)

where $\phi' = \frac{\phi}{\pi 0}$, $\phi_0 = \frac{\hbar}{e}$, and $m^*/m_e$ is the ratio of the effective mass to the normal mass of an electron. We assume $T \ll \Delta_{\text{min}}$ where $\Delta_{\text{min}}$ is the smallest energy gap in the quantized energy spectrum, in order to neglect thermal excitations.

At $\phi = 0$ the energy levels with $n \neq 0$ are fourfold degenerate due to orbital (clockwise and anticlockwise persistent current) and spin degrees of freedom. In order to build a qubit one has to remove the fourfold degeneracy. This can be done by the application of $B_c$ which couples independently to spin and orbital moments, changing the energy of the states so that they are no longer equivalent. Let us take as an example GaAs quantum rings of the radius $R = 40nm$ where $g_s = -0.4$ and $m^*/m_e = 0.067$ [8]. The energy spectrum (4) is shown in Fig.1.

At first we develop a qubit on the spin degrees of freedom. For the formation of qubit, to make it relatively insensitive to the fluctuations of the flux, one should choose a nondegenerate orbital state in a flat part of the orbital energy spectrum at integer $\phi'$. The energy gap between the ground $n$ and the first excited $n' = n \pm 1$ states of the same spin is independent on $n$ and
Figure 1. The energy spectrum (4) of an electron in a 1D quantum ring of the radius \( R = 40 \text{nm} \) made of GaAs with spin \( s = -\frac{1}{2} \) (solid line) and \( s = \frac{1}{2} \) (dashed line). The spin qubit A has spin splitting gap \( \hbar \omega_{Q,s} \) much smaller than the gap to the first excited state \( \Delta_{\text{orb}} \). The orbital qubit B has energy splitting \( \hbar \omega_{Q,\text{orb}} \) 5 times smaller than the spin splitting \( \Delta_s \). The orbital qubits can be also build at integral \( \phi / \phi_0 \) (e.g. point C). The energy unit is \( \hbar^2 / 2m^* R^2 \simeq 3.4K \).

equal to \( \Delta_{\text{orb}} = \frac{\hbar^2}{2m^* R^2} \simeq 3.4K \). On the other hand the spin splitting \( \hbar \omega_{Q,s} \) is linear in \( \phi' \) but in small fields e.g. at \( \phi' = 1 \), where the ground state correspond to \( n = 1 \) (see point A in Fig.1) it is
\[
\hbar \omega_{Q,s} = E_{1, \frac{1}{2}} - E_{1, -\frac{1}{2}} = 0.18K,
\]
i.e. much smaller than the orbital gap. If the ring is occupied by a single electron (\( N_e = 1 \)) the spin qubit is formed at A. Such a two level system formed by electron spin-up and down states is one of the most promising candidates for a qubit. It is separated from the higher excited states by the energy gap yielding the ratio \( \Delta_{\text{orb}} / \hbar \omega_{Q,s} \gg 5 \) and therefore is robust against fluctuations of the electric and magnetic field [4, 6].

For the formation of an orbital qubit one should choose a pair of degenerate orbital states well separated from the higher states. As the energy spectrum of the orbital motion is periodic and spin splitting linear in \( \phi' \), to obtain a considerable spin splitting would require the application of stronger magnetic field \( B_e \). If we introduce to the ring a controllable potential barrier [7], the tunneling of electron close to the degeneracy points will lead to the superposition of states with opposite persistent current but with the same spin direction leading to a formation of the orbital qubit. For example let us assume \( \phi' = 3.5 \) and the number of electrons \( N_e = 1 \). Then \( E_{3,s} = E_{4,s} \) and the point B in Fig.1 acquires a spin splitting
\[
\Delta_s = E_{3(4), \frac{1}{2}} - E_{3(4), -\frac{1}{2}} = 0.63K
\]
In the presence of the energy barrier [7] of finite length \( a \) and height \( V_0 \) quantum tunneling (which conserves the spin direction) should thus lead to an orbital qubit i.e. a quantum superposition of the two opposed current states with fixed spin \( s = -\frac{1}{2} \). Assuming resonable length \( a \) and height \( V_0 \) of the barrier qubit energy splitting \( \hbar \omega_{Q,\text{orb}} \sim 0.12K \) is five times smaller than \( \Delta_s \) and the unwanted excitations to higher states will be strongly reduced [4, 6]. To increase
the $\Delta_z/\hbar\omega_{Q,\text{orb}}$ ratio even further one can take higher fields, e.g. half integral $\phi' \approx 10$, which are still feasible to obtain. In a similar way an orbital qubit with fixed opposite spin $s = 1/2$ can be built for a ring with $N_e = 3$. With this number of electrons one can also build orbital qubits at integral $\phi'$ (e.g. point $C$ in Fig.1).

The above discussion was based on the theoretical spectrum of the single channel ring. To support our idea one should consider a more realistic model which takes into account a few radial channels. The high-quality quantum rings have been recently fabricated on AlGaAs-GaAs [9] heterostructures and InGaAs [10] and the energy spectrum has been investigated both theoretically and experimentally.

The idea to implement a qubit in that type of structures is the construction of quantum superposition states of two degenerate (or nearly degenerate) basis states. As we have shown the basis states could be of different origin and depending on them the coherent coupling would be obtained by the tunneling via the potential barrier or by the transverse magnetic field.

3. Coherent manipulations of qubits

In order to use quantum rings in quantum computation it is necessary to establish a way to perform single qubit operations and to implement efficient quantum logic gates on pairs of qubits.

In a pseudospin notation [7, 11] the qubit hamiltonian can be written as

$$H_Q = -\frac{1}{2}B_z \hat{\sigma}_z - \frac{1}{2}B_x \hat{\sigma}_x.$$  \hspace{1cm} (7)

where $\hat{\sigma}_z, \hat{\sigma}_x$ denote Pauli spin matrices. For the orbital qubit the term $B_z$ being an effective magnetic field can be tuned by varying the magnetic flux threading the ring

$$B_z = E_\uparrow - E_\downarrow = \begin{cases} \Delta \left[ 1 - \frac{2\phi}{\phi_0} \right] & \text{for odd } N_e \\ -\Delta \cdot \frac{2\phi}{\phi_0} & \text{for even } N_e \end{cases}.$$  \hspace{1cm} (8)

The effective magnetic field $B_z$ describes the tunneling amplitude $\hbar\omega_{\uparrow\downarrow}$ between the two potential wells and can be tuned by changing the barrier parameters. With these two external control parameters the elementary single-bit operations i.e. $z$ and $x$ rotations [11] can be performed. The measurement of the orbital qubit states can be done by SQUID magnetometer inductively coupled to it [7]. The two orbital qubits can be coupled magnetically by the flux the circulating currents generate in a similar way as the superconducting flux qubits [4, 11, 12]. Such direct inductive coupling is always switched on. A controllable interqubit coupling which can be switched on and off can be achieved when it is mediated by an $LC$ circuit [13]. Two qubit operations and entanglement can be also mediated by the non-classical electromagnetic field of the high-$Q$ microcavity [14, 15, 16].

For spin qubit $B_z$ and $B_x$ are real magnetic fields which can be used to perform all single qubit operations on a Bloch sphere. Cirquits of nano rings are scalable and they can be initialized to the many qubit ground state by relaxation at low temperature. Reading out the spin states can be done in a similar way as in $QD$ using spin dependent tunneling or by spin-charge conversion [19]. The single qubit rotations can be achieved if the magnetic field could be pulsed exclusively onto one ring e.g. by a scanning probe tip.

By connecting such rings in a series one can achieve all qubit gates. The two spins can be coupled to each other by an effective exchange interaction via tunneling barriers with the strength which can be controlled by gate voltages [17]. Both types of qubits can be driven by microwave pulses resonant with the level separation.
The performance of qubits is limited by the interaction with noisy environment. Relaxation and decoherence of orbital qubits has been discussed in [7, 18]. It was shown there that the relaxation and decoherence times are of the order of one to several microsec and that the quality factor giving the number of quantum logic operations is of the order of $10^4$.

At present the general mechanisms of relaxation and decoherence of spin qubits in QDs are known, for a review see e.g. [19]. For QRs, that is the ring-shaped QDs, these mechanisms should be similar (although some caution is necessary because of the additional orbital degree of freedom) giving the relaxation and decoherence times of the order of milisec.

4. Conclusions
Quantum computing is based on the controlled evolution of quantum mechanical state. Contrary to classical bits qubits can be in a quantum superposition of different logical values and coherently coupled into entangled states. It is unclear whether atomic physics implementations could ever be scaled up to large-scale quantum computation. Interest in solid state nanosystems in quantum information processing is motivated by the prospect of scalable device fabrication. In this paper we discussed the formation of spin and orbital qubits on semiconductor quantum rings and coherent manipulations performed on them. Because the basic states are the magnetic moment states the qubits will be sensitive to the flux noise, but relatively insensitive to the charge noise. As semiconductor quantum rings are now experimentally accessible these results demonstrate the feasibility of operating single or few electrons in quantum rings as quantum bits. Thus nanometre-scale devices in the form of rings are, besides quantum dots, promising physical realizations of systems to be used for quantum state engineering.

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