Acoustoelectric pumping through a ballistic point contact in the presence of magnetic fields

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The acoustoelectric current, $J$, induced in a ballistic point contact (PC) by a surface acoustic wave is calculated in the presence of a perpendicular magnetic field, $B$. It is found that the dependence of the current on the Fermi energy in the terminals is strongly correlated with that of the PC conductance: $J$ is small at the conductance plateaus, and is large at the steps. Like the conductance, the acoustoelectric current has the same functional behavior as in the absence of the field, but with renormalized energy scales, which depend on the strength of the magnetic field, $|B|$.

I. INTRODUCTION

Acoustoelectric currents induced in a two dimensional electron gas (2DEG) by a surface acoustic wave (SAW) have been observed in point contacts (PC’s) defined in a GaAs/AlGaAs heterostructure by a split gate. This problem has been analyzed theoretically in several papers. In particular, it has been realized that the acoustoelectric current in a ballistic PC does not simply represent the drag of the electrons by the SAW, but constitutes an example of a pumping phenomenon. Pumping in conducting nanostructures, first considered in Ref. 1, is recently a subject of considerable theoretical and experimental interest. The pumping current is excited not by applying a voltage bias to the nanostructure terminals, but rather by periodically and adiabatically varying certain properties of the system, i.e., certain parameters of the system Hamiltonian. In the case of the acoustoelectric current, the periodic perturbation is formed by the moving potential profile created by the acoustic wave, of frequency $\omega_0$ and wave vector $q$,

$$\delta U(r, t) = A \cos(\omega_0 t - qr). \tag{1}$$

As the frequency of the acoustic wave is small, this perturbation can be considered as adiabatic; moreover, the potential $\delta U$ can be presented as $\delta U(r, t) = \lambda_1(t) \cos qr + \lambda_2(t) \sin qr$ with $\lambda_1(t) = A \cos \omega t$ and $\lambda_2(t) = A \sin \omega t$ being the ‘pumping parameters’. This form of the pumping field is very similar to the one suggested by Thoules in his pioneering discussion of quantized charge transfer.

This paper is devoted to the calculation of the acoustoelectric current flowing through a ballistic PC subject to a perpendicular magnetic field due to a SAW propagating along the PC. We employ a model of non-interacting electrons. Our main result is that the acoustoelectric current dependence on the Fermi energy in the leads in the presence of the magnetic field $B$ has the same functional form as for $B = 0$, with the energy scales (or alternatively, the length scales) which characterize the PC, being renormalized and replaced by field dependent quantities. (This feature is known for the quantum conductance of the PC).

The derivation of the acoustoelectric current, in the presence of a magnetic field, requires quite a lengthy calculation. We therefore have organized the paper as follows. The next section describes the formulation of the problem, and gives the final result for the field-dependent acoustoelectric current. We discuss there its dependence on the Fermi energy of the terminals, on the magnetic field and on the wave vector of the SAW. Section III includes the formal derivation of a time-dependent current, induced by a weak time-dependent perturbation, in a nanostructure of a general geometry, subject to a magnetic field, and (a possible) dc bias. The formalism is based on time-dependent scattering states. At the end of that section we obtain the pumped acoustoelectric current, and relate its form to the formula for the pumping current derived by Brouwer. The expression for the acoustoelectric current necessitates the knowledge of the full scattering matrix of the PC, in the presence of the magnetic field. We carry out this calculation for a saddle-point potential in Sec. IV, and derive there the result reported in Sec. II.

II. FORMULATION AND RESULTS

A quite realistic model for the confining potential of a PC in the plane of a 2DEG is the saddle-point potential...
\[ U(x, y) = \frac{1}{2md^2} \left( -\frac{x^2}{L^2} + \frac{y^2}{d^2} \right). \] (2)

Here \( m \) is the electron mass, \( L \) is the length of the PC and \( d \) is its width (\( \hbar = 1 \)). For \( L \gg d \) this potential corresponds to a waveguide in the \( x \)-direction with parabolic walls (at \( |x| \lesssim L \)) adjusted to horns (at \( |x| \gtrsim L \)) with opening angle \( d/L \). These horns represent the left and right terminals at \( x = \mp \infty \). The electronic states in this confining potential are labeled by the energy \( E \) and the transverse channel number \( n \). The transmission amplitude of an electron with energy \( E \) propagating in the \( n \)th channel is

\[ t(\varepsilon_n) = \left( 1 + e^{-2\pi \varepsilon_n} \right)^{-1/2}, \] (3)

where \( \varepsilon_n \) is the (dimensionless) energy for the longitudinal propagation,

\[ \varepsilon_n = (E - E_n)/\delta, \quad E_n = \Delta(n + 1/2), \] (4)

with

\[ \delta = 1/mLd, \quad \Delta = 1/md^2, \] (5)

being the two energy scales of the saddle point potential. The energy \( E_n \) is the threshold for the propagation in the \( n \)th channel: modes having a large positive \( \varepsilon_n \) are propagating (\( t \) is close to 1), while modes with large negative \( \varepsilon_n \) are evanescent (\( t \) is exponentially small).

The motion of an electron through a saddle-point potential, when a constant magnetic field \( B \) is applied perpendicularly to the \( x \) \( y \)-plane, has been considered by Fertig and Halperin [2]. Remarkably enough, they find the same form, Eqs. (3) and (4), for the transmission coefficient, where now the channel index \( n \) refers to the Landau level to which the edge state belongs, and the two energy scales (3) are modified, to be \( \delta_B \) and \( \Delta_B \) [see Eq. (3) below]. For example, at strong magnetic fields such that \( \omega_B^2 = |eB|/mc \gg \Delta \), then \( \Delta_B \approx \omega_B \) while \( \delta_B \approx \Delta/\omega_B \). It therefore follows that, (as in the absence of the field), the conductance (at zero temperature, in units of \( e^2/h \)) is

\[ G = \sum_n t^2(\varepsilon_n), \] with \( E \) [see Eq. (3)] being the Fermi energy in the terminals, \( E_F \). When \( \Delta_B \gg \delta_B \) the conductance as a function of \( E \) is a step-like function, with plateaus of width \( \Delta_B \) and steps of width \( \delta_B \). Increasing the magnetic field makes the intervals between the steps larger and the width of the steps smaller.

In the following sections we calculate the acoustoelectric current, in the presence of a magnetic field normal to the 2DEG plane, and find that it consists of the sum over the contributions from the various channels \( n \),

\[ J = J_0 \sum_n F(\varepsilon_n, p). \] (6)

Here, \( J_0 \) is the nominal current,

\[ J_0 = 2e\omega_0 |A|^2 / \delta_B^2, \] (7)

\( \varepsilon_n \) is defined as in Eq. (4) with \( E = E_F \), and \( p \) gives the SAW wave vector in dimensionless units,

\[ p = aq, \quad a = \left( \frac{1}{2md_B^2} \frac{\Delta^2 + \delta_B^2}{\Delta_B^2 + \delta_B^2} \right)^{1/2}. \] (8)

At zero magnetic field, \( a = (Ld/2)^{1/2} \), while at strong fields it tends to \( \ell_B(L/2d)^{1/2} \), where \( \ell_B = e|eB|/\Delta^2 \) is the magnetic length. The function \( F(\varepsilon, p) \), given in Eq. (3) below, is the same one which appears in the calculations of the zero-field acoustoelectric current. This function is exponentially small well above and below the threshold, where \( \varepsilon \gg 1 \). Near the threshold, where \( \varepsilon \lesssim 1 \), this function for \( p \gg 1 \) displays damped oscillations as function of \( p \) with a period \( \Delta p \approx 1 \), and tends to zero at \( p \to 0 \),

\[ F(\varepsilon, p) = 2\pi e^{-\pi \xi^3(\varepsilon)} c(\varepsilon) \text{erf} \left( \frac{p}{\sqrt{2a}} \right), \quad p \ll 1, \]

\[ F(\varepsilon, p) = 4\pi t^2(\varepsilon) e^{-\alpha p^2} \text{erf} \left( \frac{p^2}{2} - \frac{\pi}{4} - \gamma_c \right), \quad p \gg 1. \] (9)

Here \( \gamma_c = 2\varepsilon \ln p + \arg \Gamma(1/2 - i\varepsilon) \), \( c(\varepsilon) \approx 1 \) and \( \sigma \approx d/L \) is a screening parameter. It is introduced phenomenologically in order to model the screening of the SAW potential in the wide banks of the PC. Since the expression (3) for the acoustoelectric current is the same function as in the absence of the magnetic field, but with modified, field-dependent, arguments, we may discuss its properties borrowing from our previous results.

As in the \( B = 0 \) case, the modes whose thresholds \( E_n \) are far from the Fermi energy \( E_F \) do not contribute to the current. This is expected for the evanescent modes; for the propagating ones this seems to be “counterintuitive”. It means that in a free channel with almost no reflection, the acoustoelectric current is zero. As a result, the current is large only when the Fermi energy is close to one of the thresholds, \( |E_F - E_n| \lesssim \Delta_B \). Hence, as in the \( B = 0 \) case, the dependence of the acoustoelectric current on the Fermi energy closely follows that of the conductance. Since \( \delta_B < \delta \), increasing the magnetic field reduces the domains where the current is strong, the squeezing field at strong fields being \( \omega_B/\Delta \). This is in agreement with the expectation that pumping is quenched by a magnetic field due to the chirality of the electron states. Less expected is that, on the other hand, the magnetic field enhances the magnitude of the nominal current \( J_0 \), Eq. (3), since \( \delta_B < \delta \). This means that while away from the threshold the current is reduced, the field enhances the current at the threshold, the enhancement factor at strong fields being \( (\omega_B/\Delta)^2 \). The field changes also the oscillation period in \( q \), which is of the order \( a^{-1} \) and hence increases with the field. At strong fields \( a/a_B = d/\ell_B \). We also note that the acoustoelectric current in a saddle-point potential PC is invariant
with respect to $B \rightarrow -B$, in agreement with the up-down symmetry discussed in Ref. [4].

The scaling of the characteristic energies with magnetic field is unique to the saddle-point potential and is not a generic feature. However, since the saddle-point potential is a good approximation to the actual confining potential of a PC in a 2DEG, one expects that at least the trends predicted by the scaling will be valid for a realistic confining potential.

III. TIME-DEPENDENT RESPONSE

The acoustoelectric current, driven by a SAW, is an example of a dc current induced by an ac perturbation. Quite generally, such a current can be obtained using the concept of time-dependent scattering states.

In this section we use this formulation to derive the expression for the current flowing in a multi-terminal ballistic nanostructure of a complicated geometry, which may be subject to a dc bias, in the presence of a magnetic field. The result is given in Eq. [13] below, and is valid up to second order in the strength of the ac perturbation. We then specify to the “pumping” conditions, namely, a dc current induced in an unbiased nanostructure, and show that the result is a generalization of Brouwer’s formula for the pumped charge, for the case of spatially distributed pumping parameters.

In order to use the time-dependent scattering state formalism, one writes the current density operator, $j(r, t) = -(ie/2m)\nabla \Psi(r, t)\nabla \Psi(r, t) + h.c.$, with $\nabla = \nabla - (ie/c)A(r, t)$ (where $A$ denotes the vector-potential) representing the electron field operator $\Psi$ with the time-dependent scattering state.

$$\Psi(r, t) = \int \frac{dE}{2\pi} \sum_{\alpha\sigma} a_{\alpha\sigma}(E)\chi_{\alpha\sigma}(E|r, t).$$  
(10)

The state $\chi_{\alpha\sigma}(E|r, t)$ is a solution of the Schrödinger equation with a time-dependent Hamiltonian

$$H(r, t) = H_0(r) + \delta U(r, t),$$  
(11)

excited by an electron coming with energy $E$ from channel $n$ of terminal $\alpha$. The solutions are normalized to unit incoming flux. Although the time-dependent scattering state is labeled with the energy $E$ of the incoming wave, it contains components with energies $E' \neq E$ since the Hamiltonian is time-dependent. In [4], the operator $a_{\alpha\sigma}(E)$ annihilates an electron of energy $E$ in the $n$th channel of lead $\alpha$. The thermal average of these operators is defined, in general, by the temperature and the chemical potential of the terminal connected to the lead, i.e.,

$$\langle a_{\alpha\sigma}^\dagger(E)a_{\alpha'\sigma'}(E')\rangle = 2\pi\delta(E - E')\delta_{\alpha\sigma,\alpha'\sigma'}f_{\alpha}(E),$$  
(12)

where $f_{\alpha}(E) = (e^{(E - \mu_{\alpha})/k_BT} + 1)^{-1}$ is the Fermi distribution with the chemical potential $\mu_{\alpha}$ in terminal $\alpha$. The next step is to find the average of the current density operator. Then, the total current entering terminal $\beta$ is given by

$$J_\beta(t) = \int_{C_\beta} d\mathbf{r}\langle j(\mathbf{r}, t)\rangle_{r \rightarrow \infty},$$  
(13)

where the notation $r \rightarrow \infty$ means that $r$ approaches infinity in lead $\beta$, whose cross section is denoted $C_\beta$. Therefore, it is sufficient to obtain the states $\chi_{\alpha\sigma}$ for $r \rightarrow \infty$.

In the case at hand, $H_0$ is the Hamiltonian of the PC and $\delta U(r, t) = \int d\omega \exp(-i\omega t)\delta U(r, \omega)$ is the potential created by the SAW. A straightforward calculation yields that, to second order in the perturbation $\delta U$, the time-dependent scattering state is

$$\chi_{\alpha\sigma}(E|r, t) = e^{-iEt}\left[\chi_{\alpha\sigma}(E|r) + \int d\omega' d\omega'' G(E'|r, \omega')\delta U(r', \omega')e^{-i\omega' t} \times G(E''|r', \omega'')\chi_{\alpha\sigma}(E|r')\right],$$  
(14)

where we have denoted $E' \equiv E + \omega'$ and $E'' \equiv E + \omega' + \omega''$. Here $G$ is the Green function of the Hamiltonian $H_0$, and $\chi_{\alpha\sigma}(E|r)$ is the scattering state of $H_0$, i.e., in the absence of the time-dependent perturbation. The asymptotic behavior of the latter at $r \rightarrow \infty$ was found in [4].

$$\chi_{\alpha\sigma}(E|r)|_{r \rightarrow \infty, \beta} = \delta_{\alpha\beta}\sum_{\alpha\sigma} w_{\alpha\sigma}^-(E|r)$$  

$$+ \sum_{\beta m} w_{\beta m}^+(E|r)S_{\beta m,\alpha\sigma}(E),$$  
(15)

where $S_{\beta m,\alpha\sigma}(E)$ is the scattering matrix element from $\alpha\sigma$ to $\beta m$ for the Hamiltonian $H_0$, and $w^-, w^+$, are the incoming and the outgoing waves, respectively. The asymptotic behavior of the time-dependent scattering state [4] is derived from this relation and the asymptotic behavior of the Green function [4],

$$G(E|r, r')|_{r \rightarrow \infty} = -\delta \sum_{m} w_{\beta m}^+(E|r)\hat{G}_{\beta m}(E|r')$$  
(16)

where the “hat” indicates that the magnetic field is inverted. One finds

$$\chi_{\alpha\sigma}(E|r, t)|_{r \rightarrow \infty, \beta} = e^{-iEt}\left[\delta_{\alpha\beta}\sum_{\alpha\sigma} w_{\alpha\sigma}^-(E|r)$$  

$$+ \sum_{\beta m} S_{\beta m,\alpha\sigma}(E)w_{\beta m}^+(E|r)$$  

$$+ \int d\omega' d\omega'' \sum_{\beta m} S_{\beta m,\alpha\sigma}(E', E)w_{\beta m}^+(E'|r)$$  

$$\times S_{\beta m,\alpha\sigma}(E'', E', E)w_{\beta m}^+(E''|r)\right],$$  
(17)

where $\delta_{\alpha\beta}$ is the Kronecker delta.
with

\[ S_{\beta m,\alpha n}^{(1)}(E', E) = \]
\[ -i \int dr \chi_{\beta m}(E'|r) \delta u(r, \omega') \chi_{\alpha n}(E|r), \]
\[ S_{\beta m,\alpha n}^{(2)}(E'', E', E) = \]
\[ -i \int dr dr' \chi_{\beta m}(E'|r') \delta u(r', \omega'') \]
\[ \times G(E'|r', r) \delta u(r, \omega') \chi_{\alpha n}(E|r). \]  \hspace{1cm} (18)

Upon inserting this expression into Eq. (13) for the current, using Eq. (12), one finds that the current is given in terms of current density matrix elements for waves \( u_{\beta m}^\pm \). Only diagonal matrix elements contribute, and using the normalization of \( u_{\beta m}^\pm \) to a unit flux, one obtains

\[ J_\beta(t) = -e \int \frac{dE}{2\pi} \left\{ \sum_n f_\beta(E) \right. \]
\[ \left. - \sum_{\alpha m} f_\alpha(E)|S_{\beta m,\alpha n}(E,t)|^2 \right\}, \]  \hspace{1cm} (19)

where a time-dependent scattering matrix, \( \hat{S} \), has been introduced,

\[ \hat{S}_{\beta m,\alpha n}(E,t) = S_{\beta m,\alpha n}(E) \]
\[ + \int d\omega e^{-i\omega t} S_{\beta m,\alpha n}^{(1)}(E', E) \]
\[ + \int d\omega' d\omega'' e^{-i(\omega'+\omega'')t} S_{\beta m,\alpha n}^{(2)}(E'', E', E). \]  \hspace{1cm} (20)

The result (10) gives the current induced in the system by a weak time-dependent perturbation to second-order in the perturbation. (The first order contribution to “internal potential” in terms of the scattering matrix was calculated in Ref. 10.) It hence may serve to obtain a general ac response in the presence of a dc bias. In the following discussion, however, we confine ourselves to the case of a dc response, in the absence of a dc bias.

To obtain the dc, pumped current we now assume that the nanostructure is unbiased, i.e., all terminals have the same chemical potential, and select from the general expression only the time-independent terms. The resulting expression is

\[ J_d^\beta = e \int \frac{dE}{2\pi} \int d\omega \left( f(E) - f(E + \omega) \right) \]
\[ \times \sum_{\alpha m n} |S_{\beta m,\alpha n}^{(1)}(E', E)|^2. \]  \hspace{1cm} (21)

Note that the second order contribution to the time dependent scattering matrix \( S^{(2)} \) does not enter this expression. To derive it, we have used the following properties of the scattering states and the Green function:

\[ \sum_{\alpha n} S_{\beta m,\alpha n}^* \chi_{\alpha n} = \sum_{\alpha n} \hat{S}_{\alpha n,\beta m}^* \chi_{\alpha n} = \hat{\chi}_{\beta m}, \]  \hspace{1cm} (22)

and

\[ G(E|r_2, r_1) - G^*(E|r_1, r_2) \]
\[ = -i \sum_{\alpha n} \chi_{\alpha n}(E|r_2) \chi_{\alpha n}^*(E|r_1). \]  \hspace{1cm} (23)

The form (21) of the pumping current can be presented as a generalization of the adiabatic pumping formula given in Ref. 10. One notes that \( S^{(1)} \), Eq. (18), in the adiabatic approximation is just the correction \( \delta S \) to the scattering matrix due to the small perturbation \( \delta u \) (see also Ref. 24). In terms of scattering states this correction is

\[ \frac{\delta S_{\beta m,\alpha n}(E)}{\delta u(r, \omega)} = -i \hat{\chi}_{\beta m}(E|r) \chi_{\alpha n}(E|r), \]  \hspace{1cm} (24)

and consequently,

\[ J_{dc}^{\beta} = e \int \frac{dE}{2\pi} \int d\omega \omega \left( \frac{\partial f(E)}{\partial E} \right) \]
\[ \times \int dr dr' \delta u(r, -\omega) \delta u(r, \omega) \]
\[ \times \sum_{\alpha m n} \frac{\delta S_{\beta m,\alpha n}(E)}{\delta u(r_2, \omega)} \frac{\delta S_{\beta m,\alpha n}(E)}{\delta u(r_1, -\omega)}. \]  \hspace{1cm} (25)

It follows that the above derivation gives the pumping current for a multi-terminal device, in the presence of a magnetic field, which is pumped by a weak modulation of the “distributed parameters” \( \delta u(r) \).

**IV. THE ACOUSTOELECTRIC CURRENT**

When the time-dependent potential is created by a SAW moving along the x-direction, one has

\[ \delta u(r, \omega) = A e^{iqx} \delta(\omega - \omega_0) + A^* e^{-iqx} \delta(\omega + \omega_0). \]  \hspace{1cm} (26)

Then introducing this into \( S^{(1)} \) given by Eq. (18), one finds that at zero temperature, the current pumped by the SAW is given by

\[ J_d^{dc} = e \omega_0|A|^2 \sum_{\alpha m n} \left( |\tilde{M}_{\beta m,\alpha n}^g|^2 - |\tilde{M}_{\beta m,\alpha n}^\mathfrak{M}|^2 \right). \]  \hspace{1cm} (27)

with

\[ \tilde{M}_{\beta m,\alpha n}^g(E) = \int dr \hat{\chi}_{\beta m}(E|r)e^{iqx} \chi_{\alpha n}(E|r), \]  \hspace{1cm} (28)

where now \( E = E_F \).

The evaluation of the integrals (28) requires the solution of the scattering problem of a saddle-point potential in the presence of a magnetic field. This may be accomplished by choosing the symmetric gauge for the vector potential, \( A = (B/2)[-y, x, 0] \), and changing the variables \( x, p_x \) into \( \xi, s \), as has been done in
Ref. 19. As discussed there, $\xi$ corresponds to the $x$-coordinate of the guiding center, while $s$ describes the cyclotron motion about that center. The advantage of the $\{\xi, s\}$-representation is that the Hamiltonian, which includes the magnetic field, takes again the same form as a saddle-point potential,

$$\mathcal{H}_0 = \delta_B \left( -\frac{\partial^2}{\partial \xi^2} - \frac{1}{4\lambda^2} \right) + \frac{1}{2} \Delta_B \left( -\frac{\partial^2}{\partial s^2} + s^2 \right)$$

(29)

but with magnetic field dependent energy scales:

$$\delta_B = 2\sqrt{\gamma^2 - \lambda^2}, \quad \Delta_B = 2\sqrt{\lambda^2 - \gamma^2}.$$  

(30)

That is, in the $\{\xi, s\}$-representation, the variables are separated. The parameters of this transformation are given by

$$\begin{bmatrix} x \\ p_y \end{bmatrix} = \begin{bmatrix} \ell e^{-\theta_1} \cos \phi & \ell \sqrt{2} e^{\theta_2} \sin \phi \\ \frac{1}{2} e^{-\theta_1} \sin \phi & -\frac{1}{\sqrt{2}} e^{\theta_2} \cos \phi \end{bmatrix} \begin{bmatrix} \xi \\ s \end{bmatrix}.$$  

(31)

Here,

$$\ell = (2m\lambda)^{-1/2}, \quad \tan 2\phi = -\omega_B/4\gamma, \quad \tanh 2\theta_1 = -\lambda_-/\gamma, \quad \tanh 2\theta_2 = \gamma/\lambda_+,$$

(32)

where

$$\lambda^2 = \frac{1}{4\omega_B} + \frac{1}{2}(\Delta^2 - \delta^2), \quad \gamma^2 = \frac{1}{8}(\Delta^2 + \delta^2),$$

$$\lambda_\pm = \frac{1}{2}\lambda \pm \sqrt{\lambda^2 + \left(\frac{1}{4\omega_B}\right)^2}.$$  

(33)

The scattering solutions of Hamiltonian $28$, of energy $E$, are labeled by $\alpha n$, where $\alpha = 1, 2$ correspond to the left and right terminals, and $n$ is the channel index. $\chi_n(E|\xi, s)$ describes an electron entering the PC from $x = -\infty$, and $\chi_n(E|\xi, s)$ corresponds to an electron coming from $x = +\infty$. In the presence of the magnetic field, $\chi_n$ describes an electron entering from the upper left corner (assuming $B > 0$) of the $\{x, y\}$-plane, while $\chi_n$ belongs to an electron coming from the right lower corner. We write the scattering states in the form

$$\chi_{\alpha n}(E|\xi, s) = C\Phi_n(s)\chi_{\alpha n}(\varepsilon_n|\xi), \quad \alpha = 1, 2.$$  

(34)

Here $C$ is the normalization constant, chosen to be real (to be determined below), and $\Phi_n$ are the eigenfunctions of the $s$-part of the Hamiltonian $28$, i.e. harmonic oscillator wave functions, corresponding to eigenergies $E_n = \Delta_B (n + 1/2)$ and normalized to unity, $\int_{-\infty}^{\infty} ds \Phi_n^2(s) = 1$. Then $\chi_{\alpha n}(\varepsilon_n|\xi)$ are the scattering states of the one-dimensional problem, defined by the $\xi$-part of the Hamiltonian $28$, with (dimensionless) energies $\varepsilon_n$,

$$\chi_{1,2}(\varepsilon_n|\xi) = -it(\varepsilon_n)\mathbf{E}(\varepsilon_n, \pm \xi),$$  

(35)

and $\mathbf{E}$ is the complex Weber (parabolic cylinder) function, as defined in Ref. 23.

The integrals $28$ are not matrix elements because the functions $\chi$ and $\bar{\chi}$ belong to different Hamiltonians, one with $B$ and the other with $-B$. As can be appreciated from the above analysis of the saddle-point potential in the presence of a magnetic field, the transformation $21$ turns the scattering problem formally into the one in the absence of the field, but in the mixed representation (momentum, coordinate). It follows that in order to perform the integrals (28), it is convenient to convert them first into matrix elements, which are then straightforwardly transformed in accordance with (31). This can be done by using the relations $22$ to obtain

$$M^q_{\beta m,\alpha n}(E) = \sum_{\beta', m'} S_{\beta m,\beta' m'}(E) M^q_{\beta' m',\alpha n}(E),$$  

(36)

with

$$M^q_{\beta m,\alpha n}(E) = \int d\mathbf{r} \chi^*_{\beta m}(\mathbf{E}|\mathbf{r}) e^{ixE} \chi_{\alpha n}(\mathbf{E}|\mathbf{r}) \equiv \langle \beta m|e^{ixE}|\alpha n\rangle.$$  

(37)

Hence, the calculation of the acoustoelectric current is reduced to (i) obtaining the scattering matrix elements (including their phases, which are not derived in Ref. 19); (ii) normalizing the scattering states (34) to a unit flux; and (iii) evaluating the matrix elements (37).

(i) Because of the variable separation in the Hamiltonian $30$, the scattering matrix does not mix the modes $n$,

$$S_{\alpha \beta}(E) = \delta_{\alpha \beta} S_{\alpha \beta}(\varepsilon_n),$$  

(38)

where $S_{\alpha \beta}(\varepsilon_n)$ is the $2 \times 2$ scattering matrix for the one-dimensional problem defined by the $\xi$-part. The absolute values of these matrix elements have been derived in Ref. 19: $|S_{11}(\varepsilon_n)| = |S_{21}(\varepsilon_n)| = t(\varepsilon_n)$, and $|S_{12}(\varepsilon_n)| = |S_{22}(\varepsilon_n)| = r(\varepsilon_n) \equiv e^{-\pi\varepsilon_n} t(\varepsilon_n)$. To find the phases of the matrix elements $S_{\alpha \beta}$ we use the relation $24$

$$\mathbf{E}(\varepsilon, -\xi) = -ie^{-\pi\xi}\mathbf{E}(\varepsilon, \xi) + it(\varepsilon)^{-1}\mathbf{E}^*(\varepsilon, \xi),$$  

(39)

which enables us to write $\chi_2$ in terms of $\chi_1$ and $\chi_1^*$. A second expression connecting $\chi_1$ with $\chi_2$ is provided by Eq. (22). From these relations, and the fact that $\chi_1$ and $\chi_1^*$ are linearly independent, we obtain

$$S_{11}(\varepsilon_n) = S_{22}(\varepsilon_n) = -r(\varepsilon_n),$$

$$S_{12}(\varepsilon_n) = S_{21}(\varepsilon_n) = -it(\varepsilon_n).$$  

(40)

(ii) In order to deduce the normalization constant $C$ in $24$ we need to obtain the flux transmitted at $x = +\infty$ due to the scattering state $\chi_n(E|\xi, s)$ coming from $x = -\infty$. Utilizing the (bra|ket) notations, such that $(x|E|1En) \equiv \chi_n(E|x, y)$, we write the $x$-component of the flux at the PC cross-section at $x_0$.
$$J_x(x_0) = \mathcal{R}\left( \int dx \, dy \langle 1E_n | x y \rangle \delta(x - x_0) v_x(x y | 1E_n) \right)$$

$$= \mathcal{R}\left( \langle 1E_n | \delta(x - x_0) v_x | 1E_n \rangle \right)$$

$$= C^2 t^2(\varepsilon_n) \mathcal{R}\left( \int d\xi \int ds \Phi_n(s) \mathbf{E}^*(-\varepsilon_n, \xi) \times \delta(x - x_0) v_x \Phi_n(s) \mathbf{E}(-\varepsilon_n, \xi) \right). \quad (41)$$

This calculation requires the velocity operator, $v_x = p_x/(m + (eB/2mc))$ in the ($\xi$, $s$) representation. Following again Ref. 31, we have

$$\begin{bmatrix} p_x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{\ell} e^{\theta_1} \cos \phi & \frac{1}{\sqrt{2\ell}} e^{-\theta_2} \sin \phi \\ -2\ell e^{\theta_1} \sin \phi & \frac{1}{\sqrt{2\ell}} e^{-\theta_2} \cos \phi \end{bmatrix} \begin{bmatrix} -i\partial/\partial \xi \\ -i\partial/\partial s \end{bmatrix}, \quad (42)$$

so that $v_x$ is a linear combination of $\partial/\partial \xi$ and $\partial/\partial s$. Similarly, [see Eq. (31)], the operator $x$ in $\delta(x - x_0)$ is a linear combination of $\xi$ and $s$. However, at $x_0 \to +\infty$, the term linear in $s$ can be neglected. As a result, the terms involving $\partial/\partial s$ in Eq. (11) disappear, leading to

$$J_x(x_0) = C^2 t^2(\varepsilon_n) 2\delta_B \alpha \int d\xi \delta(\alpha - x_0)$$

$$\times \Re \left[ \mathbf{E}^*(-\varepsilon_n, \xi) \left( -i \frac{\partial}{\partial \xi} \right) \mathbf{E}(-\varepsilon_n, \xi) \right], \quad (43)$$

in which $a$, Eq. (31), is the coefficient relating $x$ to $\xi$ in Eq. (11). Using the asymptotic expressions22 of $\mathbf{E}(a, x)$ at $x \to +\infty$, enables one to perform the $\xi$-integration, and with the requirement $J_x(x_0) = |S_{12}|^2$, we find

$$C = \left( \frac{1}{2\delta_B} \right)^{1/2}. \quad (44)$$

(iii) The matrix elements $M_{\alpha, \beta}^\varepsilon$ can be written in the ($\xi$, $s$) representation

$$M_{\beta, \alpha}^\varepsilon(E) = \int_{-\infty}^{+\infty} ds d\xi \chi_{\beta}^*(E|\xi, s) \chi_{\alpha}(E|\xi, s)$$

$$\times \exp[iq(a_1 + bs)]. \quad (45)$$

Here $b$ is the coefficient relating $x$ to $s$ in (31),

$$b = \left( \frac{1}{m\Delta_B} \frac{\Delta_B^2 - \Delta^2}{\Delta_B + \Delta^2} \right)^{1/2}. \quad (46)$$

It is zero at $B=0$, increases with $B$, being of the order of the magnetic length $\ell_B$ when $\omega_B \simeq \Delta$, and saturates at $\ell_B$ when $\omega_B \gg \Delta$. Since $s \approx 1$ and the SAW wavelength $2\pi/q \gg \ell_B$, the second term in the exponent can be neglected. Then the $s$-integration becomes trivial, and the resulting expression is

$$M_{\beta, \alpha}^\varepsilon(E) = \delta_{\alpha, \beta} M_{\beta}(\varepsilon, n, p) \quad (47)$$

with

$$M_{\beta}(\varepsilon, n, p) = \frac{t^2(\varepsilon)}{2\delta_B} \int_{-\infty}^{+\infty} d\xi \exp(ip\xi)$$

$$\times \mathbf{E}^*(-\varepsilon, \xi) \mathbf{E}(-\varepsilon, \xi), \quad \alpha, \beta = 1, 2, \quad (48)$$

where we have used Eq. (13) and the notation $\xi_{1, 2} \equiv \pm \xi$.

Inserting the results (38) and (47) into the expression for the pumped current induced by the SAW, Eq. (23), we find

$$J_{\beta}^\text{dc} = \epsilon \omega |A|^2 \sum_{\alpha \beta} \sum_{\mu \nu} (S_{\beta \mu}(\varepsilon_n) S_{\beta \nu}^*(\varepsilon_n)$$

$$M_{\beta \nu \alpha}(\varepsilon_n, p) - (p \to -p). \quad (49)$$

We parametrize the matrix elements $M_{\alpha, \beta}$ using the representation22

$$\mathbf{E}(-\varepsilon, \xi) = k^{-1/2} W(-\varepsilon, \xi) + ik^{1/2} W(-\varepsilon, -\xi),$$

$$k^{\pm 1} = \sqrt{1 + e^{-2\pi i}} \pm e^{-\pi i},$$

where $W$ are the real Weber functions. We then find

$$M_{12} = \frac{t(\varepsilon)}{2\delta_B} \left( K(\varepsilon, p) \mp ir(\varepsilon) G(\varepsilon, p) \right),$$

$$M_{12} = \frac{t(\varepsilon)}{2\delta_B} \left( 2H(\varepsilon, p) \mp t(\varepsilon) G(\varepsilon, p) \right), \quad (51)$$

where

$$K(\varepsilon, p) = \int_{-\infty}^{+\infty} d\xi \left( W^2(-\varepsilon, \xi) + W^2(-\varepsilon, -\xi) \right) \cos p\xi,$$

$$G(\varepsilon, p) = \int_{-\infty}^{+\infty} d\xi \left( W^2(-\varepsilon, -\xi) - W^2(-\varepsilon, \xi) \right) \sin p\xi,$$

$$H(\varepsilon, p) = \int_{-\infty}^{+\infty} d\xi W(-\varepsilon, \xi) W(-\varepsilon, -\xi) \cos p\xi. \quad (52)$$

Inserting Eqs. (21) and (41) into the current equation, (13), we obtain that $J_{\beta}^\text{dc} = -J_{\beta}^\text{dc}$, where $J$ is given by Eq. (4) above, with

$$F(\varepsilon, p) = t^2(\varepsilon) G(\varepsilon, p) H(\varepsilon, p). \quad (53)$$

The integrals $G$ and $H$ were calculated in Ref. 42.

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