Low-energy electron-electron bound states in planar QED

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Abstract

In this talk, we present a parity-preserving QED\textsubscript{3} model with spontaneous breaking of a local $U(1)$-symmetry. The breaking is accomplished by a potential of the $\varphi^6$-type. It is shown that a net attractive interaction appears in the Møller scattering ($s$- and $p$-wave scatterings between two electrons) as mediated by the gauge field and a Higgs scalar. We show, by solving numerically the Schrödinger equation for both the scattering potentials ($s$- and $p$-wave), that in the weak-coupling regime only $s$-wave bound states appear, whereas in the strong-coupling regime $s$- and $p$-wave bound states show up. Also, we discuss possible applications of the model to the phenomenology of high-$T_c$ superconductors \footnote{The author dedicates this work to his daughter Vittoria who attended the talk.} and to the re-entrant superconductivity effect \footnote{Talk given at the XXI Encontro Nacional de Física de Partículas e Campos, October 2000, São Lourenço - MG - Brazil.}.
1 The planar QED and the Higgs mechanism

The action for the parity-preserving QED with spontaneous symmetry breaking of a local $U(1)$-symmetry is given by [3]:

$$S_{\text{inv QED}} = \int d^3x \left\{ -\frac{1}{4} F_{mn} F^{mn} + i \overline{\psi}_+ D \psi_+ + i \overline{\psi}_- D \psi_- - m_e (\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) + - \lambda_{ep} (\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) \varphi^* \varphi + D^{m} \varphi^* D_m \varphi - V(\varphi^* \varphi) \right\}, \quad (1)$$

with the potential

$$V(\varphi^* \varphi) = \mu^2 \varphi^* \varphi + \frac{\zeta}{2} (\varphi^* \varphi)^2 + \frac{\lambda}{3} (\varphi^* \varphi)^3, \quad (2)$$

where the mass dimensions of the parameters, $\mu$, $\zeta$, $\lambda$ and $\lambda_{ep}$ are respectively 1, 1, 0 and 0.

The covariant derivatives are defined as follows:

$$D \psi_{\pm} \equiv (\partial + i \frac{e}{\sqrt{\lambda_c}} A) \psi_{\pm} \quad \text{and} \quad D_m \varphi \equiv (\partial_m + i \frac{e}{\sqrt{\lambda_c}} A_m) \varphi, \quad (3)$$

where $\sqrt{\lambda_c}$ is a coupling constant with dimension of (mass) $^{1/2}$ - the electron charge $e$ is dimensionless. In the action [1], $F_{mn}$ is the usual field strength for $A_m$, $\psi_+$ and $\psi_-$ are two kinds of fermions (the $\pm$ subscripts refer to their spin sign [1]) and $\varphi$ is a complex scalar.

The QED$_3$-action is invariant under the parity symmetry, $P$, whose action is fixed below:

$$x_m \xrightarrow{P} x'_m = (x_0, -x_1, x_2), \quad \psi_\pm \xrightarrow{P} \psi'_\pm = -i \gamma^1 \psi_{\mp}, \quad \overline{\psi}_\pm \xrightarrow{P} \overline{\psi}'_\pm = i \overline{\psi}_{\mp} \gamma^1, \quad A_m \xrightarrow{P} A'_m = (A_0, -A_1, A_2), \quad \varphi \xrightarrow{P} \varphi' = \varphi. \quad (4)$$

The sixth-power potential, $V(\varphi^* \varphi)$, is the responsible for breaking the electromagnetic $U(1)$-symmetry. It is the most general renormalizable potential in three dimensions.

Analyzing the potential $V(\varphi^* \varphi)$, and imposing that it is bounded from below and yields only stable vacua (metastability is ruled out), the following conditions on the parameters $\mu$, $\zeta$, $\lambda$ must be set:

$$\lambda > 0, \quad \zeta < 0 \quad \text{and} \quad \mu^2 \leq \frac{3 \zeta^2}{16 \lambda}. \quad (5)$$

1 The metric is $\eta_{mn} = (+, -, -)$; $m, n = (0, 1, 2)$, and the $\gamma$-matrices are taken as $\gamma^n = (\sigma_z, i\sigma_y, -i\sigma_y)$. 

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We denote $\langle \phi \rangle = v$ and the vacuum expectation value for the $\phi^*\phi$-product, $v^2$, is chosen as

$$\langle \phi^*\phi \rangle = v^2 = -\frac{\zeta}{2\lambda} + \left[ \frac{\zeta}{2\lambda} \right]^2 - \frac{\mu^2}{\lambda},$$

(6)

the condition for the minimum leading as $\mu^2 + \zeta v^2 + \lambda v^4 = 0$.

The complex scalar, $\phi$, is parametrized by $\phi = v + H + i\theta$, where $\theta$ is the would-be Goldstone boson and $H$ is the Higgs scalar, both with vanishing vacuum expectation values.

In order to preserve the manifest renormalizability of the model, the 't Hooft gauge is adopted:

$$\hat{S}_{gf} = \int d^3x \left\{ -\frac{1}{2\xi} \left( \partial^m A_m - \sqrt{2\xi} M_A \theta \right)^2 \right\},$$

(7)

where $\xi$ is a dimensionless gauge parameter.

Replacing the field parametrization for $\phi$ into the action (1), and adding up the 't Hooft gauge, yields the following complete parity-preserving action:

$$S_{\text{QED}} = \int d^3x \left\{ -\frac{1}{4} F_{mn} F^{mn} + \frac{1}{2} M_A^2 A_m A_m - \frac{1}{2\xi} (\partial^m A_m)^2 + i\overline{\psi}_+ D^+ \psi_+ + i\overline{\psi}_- D^- \psi_- - m_e (\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) + \partial^m H \partial_m H + \partial^m \theta \partial_m \theta - \xi M_A^2 \theta^2 + \lambda_{ep} (\overline{\psi}_+ \psi_+ - \overline{\psi}_- \psi_-) ((v + H)^2 + \theta^2) + 2 \frac{e}{\sqrt{\lambda_c}} A^m (H \partial_m \theta - \theta \partial_m H) + e^2 \frac{M_A^2}{\lambda_c} A^m (2vH + H^2 + \theta^2) - \mu^2 ((v + H)^2 + \theta^2) + \frac{\zeta}{2} ((v + H)^2 + \theta^2)^2 - \frac{\lambda}{3} ((v + H)^2 + \theta^2)^3 \right\},$$

(8)

where the physical mass parameters $M_A^2$, $m$ and $M_H^2$ are given by

$$M_A^2 = 2v^2 \frac{e^2}{\lambda_c}, \quad m = m_e + \lambda_{ep} v^2 \quad \text{and} \quad M_H^2 = 2v^2 (\zeta + 2\lambda v^2).$$

(9)

The Møller scattering to be contemplated will include the scatterings mediated by the gauge field and the Higgs ($A_m$ and $H$). The scattered electrons may exhibit either opposite spin polarizations ($e^-(\pm)$ and $e^-(\mp)$) or the same spin polarizations ($e^-(\pm)$ and $e^-(\pm)$). To compute the scattering potentials for the interaction between electrons with opposite spin polarizations ($s$-wave) and with the same spin polarizations ($p$-wave), we refer to the works of [5], where the concept of potential in quantum field theory and in scattering processes is discussed in detail.

The calculation of scattering potentials will be performed in the center-of-mass frame, for in this frame the scattered electrons are correlated in momentum space.
The \(s\)- and \(p\)-wave scattering potentials turn out to be:

\[
V_s(r) = -\frac{1}{2\pi} \left[ 2\lambda_p v^2 K_0(M_Hr) + \frac{e^2}{\lambda_c} K_0(M_Ar) \right],
\]
\[
V_p(r) = -\frac{1}{2\pi} \left[ 2\lambda_p v^2 K_0(M_Hr) - \frac{e^2}{\lambda_c} K_0(M_Ar) \right].
\]

(10)

Now, a particular condition on the parameters is set:

\[
\frac{e^2}{\lambda_c} = \zeta + 2\lambda v^2 \quad (M_H = M_A).
\]

(11)

It should be noticed that this is not the only possible choice, incidentally, it is the simplest one, since our proposal here is not to find the whole parameters range that ensures a net attractive potential in both cases (\(s\)- and \(p\)-wave), but only to verify that such a possibility may indeed be implemented.

2 Low-energy electron-electron bound states

In Section 1, we showed that, in the model presented here, electrons may attract each other in three dimensions through scattering processes where a massive gauge boson and a Higgs are involved. This electronic attraction might favour a bound state.

The radial Schrödinger equation associated to both the electron-electron pairing states, \(s\)- and \(p\)-wave, reads

\[
\frac{d^2g_{s,p}(r)}{dr^2} + \frac{l^2}{r^2} g_{s,p}(r) + 2m_b[E_{s,p}^2 - V_{s,p}(r)]g_{s,p}(r) = 0,
\]

where \(l\) is the orbital angular momentum and \(m_b\) is the reduced effective mass, given by

\[
m_b = \frac{m}{2} = \frac{1}{2}(m_e + \lambda_c v^2).
\]

(13)

By recalling the scattering potentials \(V_s\) and \(V_p\), it can be concluded that for weak-coupling electrons, where \(2\lambda_p v^2 \ll \frac{e^2}{\lambda_c}\), only \(V_s\) is attractive, therefore, only \(s\)-type bound states are available. In the case of strong-coupling electrons, for which \(2\lambda_p v^2 \geq \frac{e^2}{\lambda_c}\), in addition to the possible \(s\)-type bound states (since \(V_p\) is also attractive), potential \(p\)-type bound states could appear.

2.1 Some very preliminary results

In the Table 1, we collect some results we have sucessfully applied (they are assigned an OK) to the case of four planar strong-coupling high-\(T_c\) superconductors. The first two cases displayed in the Table 1 are weak-coupling BCS superconductors. Due to the fact that BCS superconductivity effect is not a quasi-planar phenomenon, the model proposed here does not fit (one assigns an X), as it should be expected, the experimental results at all.
|          | $T_c$ (K) | $\lambda_c$ (Å) | $\lambda_{ep}$ | $2\Delta(0)$ (meV) | $E_b$ (meV) | $v^2$ (meV) |
|----------|-----------|-----------------|----------------|-------------------|-------------|-------------|
| TaS$_2$  | 0.6       | 48000           | 0.41           | 0.182             | -0.384 [X]  | $\approx$ 10 |
| NbSe$_2$ | 7.1       | 48000           | 0.74           | 2.15              | -4.68 [X]   | $\approx$ 1  |
| YBa$_2$Cu$_3$O$_7$ | 87       | 1700            | 2.5            | 30.0              | -30.0 [OK]  | 1.05        |
| Tl$_2$Ba$_2$Ca$_2$Cu$_3$O$_{10}$ | 105   | 48000           | 2.0            | 28.0              | -28.0 [OK]  | 2.10        |
| Bi$_2$Sr$_2$CaCu$_2$O$_8$ | 109   | 5000            | 2.6            | 53.4              | -53.4 [OK]  | 2.73        |
| HgBa$_2$Ca$_2$Cu$_3$O$_8$ | 131   | 1980            | 2.5            | 48.0              | -48.0 [OK]  | 2.20        |

Table 1: Data we try to fit.

3 Conclusions

Four high-$T_c$ and two BCS superconductors have been analyzed. In the high-$T_c$ cases, the model proposed here fits quite well their respective gaps ($2\Delta(0)$). Contrary, for the BCS cases, the model failed, as expected, due to the fact that BCS superconductivity is not a quasi-planar phenomenon. It is now under consideration the most general case, $M_H \neq M_A$, and its possible application to the re-entrant superconductivity effect [2]. Also, we are searching for bound states in the Maxwell-Chern-Simons model coupled to QED$_3$ [6] with spontaneous breaking of the $U(1)$-symmetry. The issue of the thermodynamical and statistical properties (phase transitions, specific heats...) of a planar electron gas (for the ideal case, see [7]) shall be investigated further.

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