Abstract. We consider triangulated orbit categories, with the motivating example of cluster categories, in their usual context of algebraic triangulated categories, then present them from another perspective in the framework of topological triangulated categories.

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1. Introduction

Cluster algebras were introduced and studied by Berenstein, Fomin, and Zelevinsky [8], [9], [10], [1]. It was the discovery of Marsh, Reineke, and Zelevinsky that they are closely connected to quiver representations [15]. This connection is reminiscent of one between quantum groups and quiver representations discovered by Ringel [16] and investigated by many others. The link between cluster algebras and quiver representations becomes especially beautiful if, instead of categories of quiver representations, one considers certain triangulated categories deduced from them. These triangulated categories are called cluster categories.

Cluster categories were introduced by Buan, Marsh, Reineke, Reiten, and Todorov in [6] and, for Dynkin quivers of type $A_n$, in the paper of Caldero, Chapoton, and Schiffler [7]. If $k$ is a field and $Q$ a quiver without oriented cycles, the associated cluster category $C_Q$ is the “largest” 2-Calabi-Yau category under the derived category of representations of $Q$ over $k$. This category fully determines the combinatorics of the cluster algebra associated with $Q$ and, simultaneously, carries...
considerably more information which was used to prove significant new results on
cluster algebras.

The goal of this paper is to introduce topological triangulated orbit categories, and in particular, the motivating example of topological cluster categories. In doing so, we hope to explain the fundamental ideas of triangulated orbit categories to readers from a more homotopy-theoretic, rather than algebraic, background. Our goal, as topologists, is to work the theory of cluster categories backward, by understanding triangulated categories that have similar properties but which arise from purely topological origins. In particular, in [3] we provide sufficient conditions on a stable model category (or more general cofibration category) $\mathcal{C}$, equipped with a self equivalence $F: \mathcal{C} \to \mathcal{C}$, so that the orbit category $\mathcal{C}/F$ admits a triangulated structure.

We begin by presenting the definition of triangulated orbit categories in Section 2. In Section 3, we elaborate on the notion of algebraic triangulated category and discuss the enhanced version of orbit categories in differential graded categories. We conclude that section with a brief introduction to cluster categories, the primary example of interest. In Section 4, we introduce topological triangulated categories and give definitions of topological triangulated orbit categories and the corresponding example of cluster categories.

2. TRIANGULATED ORBIT CATEGORIES

Definition 2.1. Let $\mathcal{T}$ be an additive category and $F: \mathcal{T} \to \mathcal{T}$ a self-equivalence of $\mathcal{T}$. The orbit category of $\mathcal{T}$ by $F$ is the category $\mathcal{T}/F$ with objects those of $\mathcal{T}$ and morphisms defined by

$$\text{Hom}_{\mathcal{T}/F}(X, Y) = \bigoplus_{n \in \mathbb{Z}} \text{Hom}_{\mathcal{T}}(X, F^nY).$$

The composite of a morphism $f: X \to F^nY$ with a morphism $g: Y \to F^pZ$ is given by $(F^ng) \circ f$.

Although the orbit category in fact has many more morphisms than the original category $\mathcal{T}$, we regard it as a kind of quotient; in particular, it comes equipped with a “projection” functor $\pi: \mathcal{T} \to \mathcal{T}/F$, together with an equivalence of functors $\pi \circ F \to F$ which is universal with respect to all such functors.

If we merely require $\mathcal{T}$ to be an additive category, it is not hard to see that the orbit category $\mathcal{T}/F$ is again an additive category, and the projection $\pi: \mathcal{T} \to \mathcal{T}/F$ is an additive functor. However, we are most interested in the case where $\mathcal{T}$ is in fact a triangulated category. The question of whether $\mathcal{T}/F$ still has a natural triangulated structure is much more difficult.

Most basically, we would like to complete any morphism $X \to Y$ in $\mathcal{T}/F$ to a distinguished triangle. If it comes from a morphism $X \to Y$ in $\mathcal{T}$, then there is no problem. However, in general, it is of the form

$$X \to \bigoplus_{i=1}^{N} F^{n_i}Y$$

in terms of maps in $Y$. It is not clear how to complete such a morphism to a triangle in the orbit category.

In [11], Keller gives conditions under which the orbit category associated to some algebraic triangulated categories still possess a natural triangulated structure. He
constructs a triangulated category into which the orbit category embeds, called the *triangulated hull*, then shows under which hypotheses this triangulated hull is in fact equivalent to the orbit category. While his conditions are fairly restrictive, he shows that they hold in several important applications. Most significantly, they hold for the construction of the cluster category.

3. Algebraic triangulated categories

A triangulated category is *algebraic* if it admits a differential graded model, sometimes referred to as an *enhanced* algebraic triangulated category.

We deviate from algebraists’ standard conventions in two minor points. First, in line with grading conventions in topology, we grade complexes homologically (as opposed to cohomologically), so that differentials decrease the degree by 1. Second, we use covariant (as opposed to contravariant) representable functors; the resulting dg categories we obtain are hence the opposite of those obtained dually.

Let \( k \) be a field. A *differential graded category*, or simply *dg category*, is a category \( \mathcal{C} \) enriched in chain complexes of \( k \)-modules. In other words, a dg category consists of a class of objects together with a complex \( \text{Hom}_\mathcal{C}(X, Y) \) of morphisms for every pair of objects \( X, Y \) in \( \mathcal{C} \). Composition is given by the tensor product of chain complexes, i.e.,

\[
\circ : \text{Hom}_\mathcal{C}(Y, Z) \otimes_k \text{Hom}_\mathcal{C}(X, Y) \to \text{Hom}_\mathcal{C}(X, Z)
\]

for all \( X, Y, Z \) in \( \mathcal{C} \) which is associative and admits two-sided units \( 1_X \in \text{Hom}_\mathcal{C}(X, X)_0 \) such that \( d(1_X) = 0 \).

The category of \( \mathbb{Z} \)-graded chain complexes is naturally a dg category. A *dg \( \mathcal{C} \)-module* is a dg enriched functor from \( \mathcal{C} \) to the category of chain complexes. This is precisely saying that a \( \mathcal{C} \)-module \( M \) is the assignment of a chain complex \( M(Z) \) to each object \( Z \) of \( \mathcal{C} \) together with a \( \mathcal{C} \) action

\[
\circ : \text{Hom}_\mathcal{C}(Y, Z) \otimes M(Y) \to M(Z)
\]

which is associative and unital with respect to the composition in \( \mathcal{C} \).

An important class of dg \( \mathcal{C} \) modules are the *free of representable* modules. We say that a \( \mathcal{C} \)-module \( M \) is *free* or *representable* if there exists a pair \( (Y, u) \) which consists of an object \( Y \) in \( \mathcal{C} \) and a universal 0-cycle \( u \in M(Y)_0 \) such that

\[
\text{Hom}_\mathcal{C}(Y, Z) \to M(Z)
\]

is an isomorphism of chain complexes for all \( Z \).

**Example 3.1.** Consider a \( k \) algebra \( R \) as a dg category with one object \( X \), i.e. \( \text{Hom}(X, X) = R \) and composition

\[
\circ : \text{Hom}(X, X) \otimes_k \text{Hom}(X, X) \to \text{Hom}(X, X)
\]

is just the multiplicative structure of \( R \), \( R \otimes_k R \to R \). Then the category of dg \( R \) modules is the category of chain complexes in \( R \).

The motivation for calling a dg category an “enhancement” of a triangulated category stems from the following definition.

**Definition 3.2.** A dg category \( \mathcal{C} \) is *pretriangulated* if it has a zero object, \(*\), such that the following properties hold.
(1) (Closure under shifts) For an \( X \) in \( \mathcal{C} \) and an \( n \) in \( \mathbb{Z} \) the dg \( \mathcal{C} \)-module 
\[
\Sigma^n \text{Hom}_\mathcal{C}(X, -) \text{ given by }
\]
\[
(\Sigma^n \text{Hom}_\mathcal{C}(X, Z))_{n+k} = \text{Hom}_\mathcal{C}(X, Z)_k
\]
with differential 
\[
d(\Sigma^n f) = (-1)^n \cdot \Sigma^n(df)
\]
is representable.

(2) (Closure under cones) Given a 0-cycle in \( \text{Hom}_\mathcal{C}(X, Y) \) the dg \( \mathcal{C} \)-module \( M \) given by 
\[
M(Z)_k = \text{Hom}_\mathcal{C}(Y, Z)_k \oplus \text{Hom}_\mathcal{C}(X, Z)_{k+1}
\]
with differential 
\[
d(a, b) = (d(a), af - d(b))
\]
is representable.

Underlying any dg category \( \mathcal{C} \) is a preadditive category \( Z(\mathcal{C}) \) called the cycle category. The category \( Z(\mathcal{C}) \) has the same objects as \( \mathcal{C} \), but morphisms are now given by 
\[
\text{Hom}_{Z(\mathcal{C})}(X, Y) = \ker(d: \text{Hom}_\mathcal{C}(X, Y)_0 \to \text{Hom}_\mathcal{C}(X, Y)_{-1}),
\]
i.e., the morphisms are the 0-cycles of the complex of morphisms. The homology category \( H(\mathcal{C}) \) of a dg category \( \mathcal{C} \) is a quotient of \( Z(\mathcal{C}) \). In particular, \( H(\mathcal{C}) \) again has the same objects as \( \mathcal{C} \), but morphisms are given by \( \text{Hom}_{H(\mathcal{C})}(X, Y) = H_0(\text{Hom}_\mathcal{C}(X, Y)) \), i.e. morphisms are given by the 0-th homology groups of the homomorphism complexes. It is the case that if \( \mathcal{C} \) is a pretriangulated dg category, the associated homology category \( H(\mathcal{C}) \) has a natural triangulated structure. A proof of this fact can be found [4, §3], but we describe the shifts and distinguished triangles here for completeness.

Let us assume that \( \mathcal{C} \) is a pretriangulated dg category. A shift of an object \( X \) in \( \mathcal{C} \) is an object \( \Sigma X \) which represents the dg module \( \Sigma^{-1} \text{Hom}_\mathcal{C}(X, -) \) described above. One can take all of the shifts of all objects \( X \in \mathcal{C} \) and canonically assemble them into an invertible shift functor \( X \mapsto \Sigma X \) on \( \mathcal{C} \). This functor induces a shift functor on the homology category \( H(\mathcal{C}) \) (see, “closure under cones” in definition 3.2).

The distinguished triangles of \( H(\mathcal{C}) \) are triangles that come from mapping cone sequences in \( \mathcal{C} \). More explicitly, a triangle in \( H_0(\mathcal{C}) \) is distinguished if it is isomorphic to the image of a triangle of the form 
\[
\begin{array}{ccc}
X & \xrightarrow{f} & Y \\
\downarrow & & \downarrow \\
Cf & \xrightarrow{\Sigma f} & \Sigma X
\end{array}
\]
for some \( f: X \to Y \) in \( \mathcal{C} \) (see “closure under cones” in definition 3.2).

Example 3.3. Many examples of pretriangulated dg categories come from additive categories, including the pretriangulated hulls of Keller [11]. In particular, we may want to consider the category of modules over a hereditary \( k \) algebra \( R \). Let \( \mathcal{A} = R - \text{Mod} \). Then to the additive category \( \mathcal{A} \) we can associate a category of complexes \( C(\mathcal{A}) \). \( C(\mathcal{A}) \) is a category with objects the \( \mathbb{Z} \)-graded chain complexes of objects in \( \mathcal{A} \) and morphisms the chain maps which are homogeneous degree 0. This category can be made into a dg category \( C(\mathcal{A}) \) as follows. Given any two chain complexes \( X \) and \( Y \) the chain complex of morphisms \( \text{Hom}_{C(\mathcal{A})}(X, Y) \) is given by 
\[
\text{Hom}_{C(\mathcal{A})}(X, Y)_n = \prod_{k \in \mathbb{Z}} \text{Hom}_{\mathcal{A}}(X_k, Y_{k+n}),
\]
which is the abelian group of graded homogeneous morphisms of degree $n$. The differential on $\text{Hom}_{C(A)}(X, Y)$ is given by

$$df = d_Y \circ f - (-1)^n f \circ d_X$$

where $f \in \text{Hom}_{C(A)}(X, Y)_n$. Composition works as expected.

In this case, the cycle category $Z(C(A))$ is equivalent to the category $C(A)$. The homology category $H(C(A))$ is what is typically called the homotopy category $K(A)$, which is the category of complexes modulo chain homotopies. We claim that $C(A)$ a cofibration category. In this case we let the class of chain homotopy equivalences be the class weak equivalences and let the chain maps which are dimension-wise split monomorphisms be our class of cofibrations.

3.1. The dg orbit category. There is no reason to assume that a triangulated structure on the orbit category, when it exists, is unique. However, when it is the triangulated category associated to a dg category, namely, the dg orbit category, it can be regarded as the solution of a universal problem. Thus, there is a canonical triangulated structure on the orbit category, arising from a dg structure which is unique up to quasi-equivalence.

With this motivation in mind, we give the definition of the dg orbit category.

**Definition 3.4.** [12] Let $A$ be a dg category and $F: A \to A$ a dg functor such that $H_0(F)$ is an equivalence. The dg orbit category $C$ has the same objects as $A$ and morphism complexes defined by

$$\text{Hom}_C(X, Y) = \colim_p \bigoplus_{n \geq 0} \text{Hom}_A(F^n X, F^n Y).$$

Composition can be defined similarly to the ordinary orbit category, and analogously there is a canonical projection functor $\pi: A \to C$. In particular, as categories $H(C) \cong H(A)/F$.

Under some conditions, the dg orbit category $C$ is equivalent to the triangulated hull for the orbit category of a dg category $A$ under an equivalence $F: A \to A$.

3.2. Cluster categories. The primary example of an orbit category is that of the cluster category, first defined by Buan, Marsh, Reineke, Reiten, and Todorov [6] as a generalization of a cluster algebra. Although it can be defined more generally, we consider the specific case of the cluster category associated to an algebra arising from a quiver.

A quiver $Q$ is an oriented graph. We consider here only quivers whose underlying unoriented graph is a Dynkin diagram of type $A$, $D$, or $E$. (Such graphs have no cycles and are of particular importance in the study of Lie algebras.) A representation of $Q$ over a field $k$ associates to every vertex of $Q$ a $k$-vector space and to every arrow in $Q$ a $k$-linear map. The category of representations of $Q$ over $k$ forms an abelian category $\text{rep}(Q)$. In homotopy-theoretic language, the bounded derived category $\mathcal{D}^b(Q)$ is the homotopy category of the model category bounded chain complexes in $\text{rep}(Q)$. The restrictions we have made on the quiver $Q$ assure that both $\text{rep}(Q)$ and $\mathcal{D}^b(Q)$ are well-behaved.

**Theorem 3.5.** [13] The bounded derived category $\mathcal{D}^b(Q)$ admits a self-equivalence $\nu: \mathcal{D}^b(Q) \to \mathcal{D}^b(Q)$.
such that, for every object $X$, there is an isomorphism of functors

$$D \text{Hom}(X, -) \rightarrow \text{Hom}(-, \nu X),$$

where $D = \text{Hom}_k(-, k)$.

Such a self-equivalence is called a Serre functor or Nakayama functor.

Because $D^b(Q)$ is a triangulated category, it has an associated shift functor $\Sigma$.

**Definition 3.6.** The cluster category $C_Q$ associated to a quiver $Q$ is the orbit category of $D^b(Q)$ by the self-equivalence $\nu^{-1} \circ \Sigma^2$.

In fact, the construction of the cluster category can be placed in to a much more general framework.

**Definition 3.7.** Let $d$ be an integer. A sufficiently finitary triangulated category $\mathcal{T}$ is $d$-Calabi-Yau if there exists a Serre functor $\nu$ together with a triangulated equivalence $\nu \rightarrow \Sigma^d$.

From this perspective, we have the following reformulation of the cluster category.

**Proposition 3.8.** [11] The cluster category $C_Q$ is the universal $2$-Calabi-Yau category under the bounded derived category $D^b(Q)$.

4. Topological triangulated categories

4.1. Cofibration categories. Topological triangulated categories are defined in terms of cofibration categories. All cofibrantly generated stable model categories satisfy the conditions for a cofibration category.

**Definition 4.1.** [17] A cofibration category is a category $\mathcal{C}$ equipped with two classes of morphisms, called cofibrations and weak equivalences which satisfy axioms (C1)–(C4).

(C1) All isomorphisms are cofibrations and weak equivalences. Cofibrations are stable under composition. The category $\mathcal{C}$ has an initial object and every morphism from an initial object is a cofibration.

(C2) Given two composable morphisms $f$ and $g$ in $\mathcal{C}$, such that two of the three morphisms $f, g$ and $gf$ are weak equivalences, then so is the third.

(C3) Given a cofibration $i: A \rightarrow B$ and any morphism $f: A \rightarrow C$, there exists a pushout square

\[
\begin{array}{ccc}
A & \rightarrow & C \\
\downarrow^i & & \downarrow^j \\
B & \rightarrow & P
\end{array}
\]

in $\mathcal{C}$ and the morphism $j$ is a cofibration. If additionally $i$ is a weak equivalence, then so is $j$.

(C4) Every morphism in $\mathcal{C}$ can be factored as the composite of a cofibration followed by a weak equivalence.

We use the term acyclic cofibration to denote a morphism that belongs to the class of cofibrations and to the class of weak equivalences. We also note that in a cofibration category a coproduct $B \vee C$ of any two objects in $\mathcal{C}$ exists. The canonical morphisms from $B$ and $C$ to $B \vee C$ are cofibrations. The homotopy category of
a cofibration category is a localization at the class of weak equivalences, i.e., a functor $\gamma : C \to \text{Ho}(C)$ that takes all weak equivalences to isomorphisms which is initial among such functors.

The notion of cofibration category is due to K. S. Brown [5]. The axioms (C1)–(C4) given in [17] are equivalent to the dual of his axioms (A)–(E) [5, 1.1]. If one prefers to work in model categories, one can obtain a cofibration category by restricting to the full subcategory of cofibrant objects and forgetting the fibrations. For the purposes of this article and [3] we restrict to examples that come from model categories whenever a model category structure exists.

Like in the case with dg categories, cofibration categories which satisfy some extra conditions are enhancements of triangulated categories. A cofibration category will be called pointed if every initial object is also terminal. We denote this zero object by $\ast$. In a pointed cofibration category, the axiom (C4) provides a cone for every object $A$, i.e., a cofibration $i_A : A \to CA$ whose target is weakly equivalent to $\ast$.

Given a pointed cofibration category $C$, the suspension $\Sigma A$ of an object $A$ in $C$ is the quotient of the cone inclusion. This is equivalent to a pushout

$$
\begin{array}{ccc}
A & \xrightarrow{i_A} & CA \\
\downarrow & \downarrow & \downarrow \\
\ast & \rightarrow & \Sigma A.
\end{array}
$$

As with pretriangulated dg categories, one can assemble the suspension construction into a functor $\Sigma : \text{Ho}(C) \to \text{Ho}(C)$ on the level of homotopy categories [14].

The class of cofibrations in $C$ allow us to define distinguished triangles in $\text{Ho}(C)$. In particular, each cofibration $j : A \to B$ in a pointed cofibration category $C$ gives rise to a natural connecting morphism $\delta(j) : B/A \to \Sigma A$ in $\text{Ho}(C)$. The elementary distinguished triangle induced by the cofibration $j$ is the triangle

$$
\begin{array}{ccc}
A & \xrightarrow{j} & B \\
\downarrow & \downarrow q & \downarrow \delta(j) \\
B/A & \rightarrow & \Sigma A
\end{array}
$$

where $q : B \to B/A$ is a quotient morphism. A distinguished triangle is any triangle that is isomorphic to the elementary distinguished triangle of a cofibration in the homotopy category.

A pointed cofibration category is stable if the suspension functor $\Sigma : \text{Ho}(C) \to \text{Ho}(C)$ is a self-equivalence. The suspension functor and the class of distinguished triangles make the homotopy category $\text{Ho}(C)$ into a triangulated category.

**Definition 4.2.** A triangulated category is topological if it is equivalent, as a triangulated category, to the homotopy category of a stable cofibration category.

The adjective “topological” does not imply that the category or its hom-sets have a topology, but rather that these examples are constructed by methods in the spirit of abstract homotopy theory.

4.2. **Topological triangulated categories arising from algebraic ones.** One can demonstrate that the cycle category $Z(B)$ of a pretriangulated dg category $B$ is a cofibration category. A closed morphism is a weak equivalence if it becomes an isomorphism in the homology category. A closed morphism $i : A \to B$ is a cofibration if:
the induced chain morphism $\text{Hom}_B(i, Z)$ is surjective for every object $Z$ of $B$

the kernel $B$-module $Z \mapsto \ker \left[ \text{Hom}_B(i, Z) : \text{Hom}_B(B, Z) \to \text{Hom}_B(A, Z) \right]$ is representable.

Notice that given that the module $(C, u)$ represents the kernel of $B(i, -)$, then, by definition, there exists a universal 0-cycle $u : B \to C$ such that for every $Z$ of $B$ the sequence of cycle groups

$$
\begin{array}{cccc}
0 & \to & \text{Hom}_{Z(B)}(C, Z) & \xrightarrow{u^*} \text{Hom}_{Z(B)}(B, Z) & \xrightarrow{i^*} \text{Hom}_{Z(B)}(A, Z) \\
\end{array}
$$

is exact. In particular, $u : B \to C$ is a cokernel of $i : A \to B$ in the category $Z(B)$.

The following proposition is due to Schwede.

**Proposition 4.3.** [17] Let $B$ be a pretriangulated dg category. Then the cofibrations and weak equivalences make the cycle category $Z(B)$ into a stable cofibration category in which every object is fibrant. Moreover, the homotopy category $\text{Ho}(Z(B))$ is equivalent, as a triangulated category, to the homology category $\text{Ho}(B)$. In particular, every algebraic triangulated category is a topological triangulated category.

**4.3. Topological orbit categories.** We now define a topological orbit category as a generalization of a dg orbit category. As a consequence of proposition 4.3 this definition includes all of the previously known algebraic examples. Let $\mathcal{T}$ be a stable cofibration category and $F : \mathcal{T} \to \mathcal{T}$ a standard equivalence of cofibration categories, i.e., a functor inducing a triangulated equivalence on the homotopy category. In the case where $\mathcal{T}$ is a stable model category, then we ask that $F$ be one of the adjoint maps in a derived Morita equivalence (see [18]).

**Definition 4.4.** The topological orbit category $\mathcal{T}/F$ is the homotopy coequalizer of the diagram

$$
\begin{array}{cccc}
\mathcal{T} & \xrightarrow{id} & \mathcal{T} & \xrightarrow{F} \mathcal{T} \\
\end{array}
$$

The definition of a homotopy colimit of (stable) model categories is given in [2], but the definition holds for more general (stable) cofibration categories. In fact, the homotopy colimit of a diagram of model categories is not generally still a model category, but a more general homotopy theory. One can take it to be a cofibration category. The question, then, as to whether an orbit category is triangulated can now be understood as a question of whether or not a homotopy coequalizer of cofibration categories is a stable cofibration category.

**4.4. Topological cluster categories.** We can now consider our motivating example of cluster categories from the point of view of triangulated orbit categories. For simplicity, we work in the context of stable model categories.

**Definition 4.5.** A Serre functor on a stable model category $\mathcal{T}$ is a Quillen functor $\nu : \mathcal{T} \to \mathcal{T}$ inducing a Serre functor of triangulated categories on $\text{Ho}(\mathcal{T})$.

**Definition 4.6.** Let $\mathcal{T}$ be a stable model category which admits a Serre functor. The cluster category of $\mathcal{T}$ is the topological orbit category of $\mathcal{T}$ by the self-equivalence $\nu \circ \Sigma^2$. 
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