The estimation of phase-coupled channels of EEG signals by patients with traumatic brain injury during cognitive and motor tests

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Abstract. Identical inter-channel phase coherency of electroencephalogram (EEG) signals is determined for healthy subjects during cognitive and motor tests. EEG signal phase is evaluated at the points of it wavelet spectrogram ridge. Areas of interest of the cortex at cognitive and motor tests for group of healthy subjects are determined. Inter-channel EEG phase coherency for patients after a traumatic brain injury are represented.

1. Introduction

Under the new approach, phase-coupled pairs of EEG signals are considered for the group of healthy subjects and for the group of people with traumatic brain injury during cognitive and motor tests [1]. The coherence is used to estimate the phase locking of signals in two EEG channels. Normally, the coherence of two signals is estimated with the aid of normalized complex mutual correlation that is calculated as a product of components of Fourier signals [2-4].

The coherence of two EEG channels is determined as a linear dependence of two signals at a certain frequency [3]. Let \( x_i(f) \) and \( x_j(f) \) be the complex Fourier transforms of time series \( x_i(t) \) and \( x_j(t) \) of channels \( i \) and \( j \), respectively. Then, the cross spectrum is defined as

\[
S_{ij}(f) = \langle x_i(f)x_j^*(f) \rangle
\]

(1)

where * denotes complex conjugation and <> denotes expected mean value. The expected mean value is calculated as a mean value for a relatively large number of time intervals. The coherence is determined as the normalized cross spectrum:

\[
C_{ij}(f) = \frac{S_{ij}(f)}{(S_{ii}(f)S_{jj}(f))^{1/2}}
\]

(2)

and connectivity is determined as the absolute value of the coherence:

\[
Co_{ij}(f) = |C_{ij}(f)|
\]

(3)

The phase coupling is calculated using phases of signals \( i \) and \( j \). When the Fourier transforms of the signals are represented as \( x_i = r_i \exp(i\Phi_i) \) and \( x_j = r_j \exp(i\Phi_j) \), the cross spectrum is calculated as

\[
S_{ij}(f) = \langle r_i r_j \exp(i\Delta\Phi) \rangle
\]

(4)
where $\Delta \Phi = \Phi_i - \Phi_j$ is the phase difference of the signals in channels $i$ and $j$ at a certain frequency.

For calculation of the phase coupling, the cross spectrum is normalized by amplitudes $\langle r_i^2 \rangle^{1/2}$ and $\langle r_j^2 \rangle^{1/2}$. If the signals in two channels are independent, phase difference $\Delta \Phi_{i,j}$ is a random quantity and the coupling is absent. The phase coupling or phase locking is determined as unweighted mean value

$$P = \langle \exp(i\Delta \Phi) \rangle$$ (5)

Then, the phase difference is averaged in a certain frequency interval that is determined using neurophysiological data. Normally, such intervals correspond to the delta (2–4 Hz), theta (4–8 Hz), and alpha (8–12 Hz) EEG rhythms. Based on the calculation of the averaged phase difference for pairs of channels and choosing a cutoff threshold, we obtain the phase-coupled pairs of brain regions.

The analysis of coherence or phase locking involves averaging of coherence or phase difference over different time intervals in a certain frequency range that is determined using practical neurophysiological results. Such an approach and the presence of the threshold level of coherence that is used to select the phase-coupled pairs of signals are disadvantages of the coherent analysis that lead to instability in the determination of the inter-channel phase coupling of substantially nonstationary electrophysiological signals [5]. In this regard, it is expedient to develop a method for determination of the phase-coupled pairs of signals using the results of single tests and more stable determination of the phase-locking threshold.

We proposed an approach to the estimation of the inter-channel phase connectivity of the EEG [1] is based on the calculation and comparison of the phases of the signals at the points of the ridges of their wavelet spectrograms having the property of stationarity of the phase, which is not in any of the above approaches. In the article the determination of the phase-coupled pairs of EEG signals for the group of the healthy subjects (8 people) and for the group of patients with traumatic brain injury (5 people) is represented. Under the new approach to estimation of the inter-channel phase connectivity of EEG signals, phase-coupled pairs of EEG signals are determined during cognitive and motor tests [1]. Two channels are phase-coupled if the following condition is satisfied:

$$|\varphi_{i,j}(t)| \leq \text{const}$$ (6)

where $\varphi_{i,j}(t) = n\phi_i(t) - m\phi_j(t)$, where $\phi$ is phase of EEG signal, $n$ and $m$ are the integers. The definition of phase-coupled pairs of EEG signals can be useful for the monitoring the treatment of people with traumatic brain injury. The recovery of phase-coupled EEG pairs can be used as indicator of proper treatment.

2. Methods

In the well-known book by Mallat [6], the ridges of the wavelet spectrograms are defined as the points of the stationary phase, that is in them the derivative of the phase with respect to time is equal to the frequency. In [7], devoted to computer modeling of smooth music, it was shown that at the points of time- and frequency-asymptotic ridges of wavelet spectra the phase is stationary in the sense that at these points $d\phi/dt \approx \omega$.

The method for the estimation of the inter-channel phase coupling of EEG is based on the calculation and comparison of the phase characteristics of the EEG signals in different channels at the ridge points of the wavelet spectrograms (stationary-phase points).

The approach to estimating the inter-channel phase connectivity of the EEG at the points of the ridges of their wavelet-spectrograms with the stationary phase is considered as an inverse task to the problem of modeling ridges. In our approach is shown that for the amplitude and phase of the amplitude-modulated signal $x(t) = A(t)e^{i\phi(t)}$ is true [1]:

$$A(t) \approx |W(t,f_r)| \cap \phi(t) \approx \arctan\left(\frac{\text{Im}W(t,f_r)}{\text{Re}W(t,f_r)}\right)$$ (7)

Based on this, first we find the ridge with the maximum value $|W(t,f_r)|$ at each reference point $\tau_i$ of the Morlet wavelet:

$$W(\tau, f) = \sqrt{f} \int x(t)\psi((t-\tau)f)dt$$ (8)

$$\psi(\eta) = \frac{1}{\sqrt{2\pif_b}}e^{2\pi if_c\eta}e^{-\frac{\eta^2}{f_b}}$$ (9)
where $T = 1/f$ is the oscillation period and $F_b = F_c = 1$ [8].

Then, we calculate the phase difference of two signals $x_i(t)$ and $x_j(t)$ at the points of the ridges of their wavelet spectra (6) and select const to estimate their phase coherence.

In this case, the points of the wavelet spectrograms that lie outside the ridge are not used in the estimation of the phase characteristics of signals. When the ridge points do not satisfy the condition for asymptotic ridges, we obtain errors in the calculations of the phase difference. However, we assume that such an error is significantly less than the errors related to the averaging of the phase difference in a certain interval of the ridge frequencies. Also, an applying of the Morlet wavelet in other biomedical fields is used [9].

3. Results

Under the new approach by estimation of the inter-channel phase connectivity results of phase coherence during two cognitive (CT1 and CT2) and motor (MT1) tests for the group healthy subjects (8 people) and for the group patients with traumatic brain injury are considered [10].

During the calculation–logical cognitive test (CT1), doctor randomly called some items that belong to the category "clothes" or "food" to the subject. During the test, the subject counts in the mind the quantity of items belonging to one of these categories. At the end of the test, he declares the result.

When performing the spatial-pattern cognitive test (CT2), the doctor randomly called the time. The subject must imagine in the mind the dial of the clock and the position of the clock hands on it in accordance with the time mentioned. If both clock hands are in the same half of the dial, he says "yes," and if they are in different halves, he keeps silent.

When performing the motor test (MT1), the subject stands on a stabilographic platform. The position of his center of gravity is displayed on the screen. He must hold itself inside a circle of a certain diameter. The duration of every test was 60 seconds.

Figure 1 shows an example of Morlet wavelet spectrograms of the signal for a pair of the EEG channels with the selected ridge line $R_i$.

![Figure 1. Morlet wavelet spectrograms of the signal for a pair of the EEG channels with the selected ridge: a) first channel; b) second channel.](image)

Figure 2 represent the histograms of the values of portions $\rho_{i,j} = n_{i,j}/N$, where $n_{i,j}$ is number of reference points of ridges with $|\Delta \varphi_{i,j}| < 0.05\pi$, where $N$ is a summary number of EEG signal reference points during the test. The first pair of channels (Figure 2a) can be referred to a phase-coupled pair. Another pair (Figure 2b) can be referred to a phase-unconnected pair. Figure 2a shows, that $\rho_{i,j} < 0.1$. 

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can be considered as a background. We consider the threshold \( \rho_{i,j}^{thr} = 0.1 \) and we will assume that above this value of the points portions the ridge correspond to the phase-coupled pairs of channels.

![Histograms](image1)

**Figure 2.** Histograms of portions \( \rho_{i,j} \) for the phase difference at ridge points of the wavelet spectrograms for two EEG channels: a) phase-coupled pair of EEG channels; b) uncoupled pair of EEG channels.

Distribution graphs of the portions of the reference points for pairs of EEG leads, based on the histograms obtained, were built and they were sorted in order of increasing \( \rho_{i,j} \) with EEG records without tests, with cognitive tests and with a motor test. These distributions are shown in Figure 3.

![Distribution Graphs](image2)

**Figure 3.** Distribution of the fraction of sampling points \( \rho_{i,j} \) over pairs of EEG leads arranged in the ascending order with numbers \( n_{i,j} \) of pairs of the EEG channels. Blue line is EEG in the absence of testing. Red line is EEG for the CT1 test. Purple line is EEG for the CT2 test. Green line is EEG for the MT1 test.

The records of 19-channel EEG were analyzed, therefore the number of pairs of channels is 171. Figure 3 represent that for a certain value on the abscissa axis for each test corresponds, possibly, a different pair of EEG channels. We consider the threshold \( \rho_{i,j}^{thr} = 0.1 \) and we will assume that above this value of the points portions the ridge correspond to the phase-coupled pairs of channels.

Based on the obtained pairs of EEG leads, it is possible to find phase-coupled pairs of the EEG channels that appear in most people in a group of 8 healthy subjects during cognitive tests (CT1 and CT2) and motor test (Figure 4).

Figure 4 shows, the prefrontal regions of the left hemisphere are predominantly activated in the calculation–logical cognitive test (CT1). The prefrontal regions of the right hemisphere are predominantly activated in the spatial-pattern cognitive test (CT2). A larger number of phase-coupled pairs reflects the different efforts required to perform each of the tests, which, according to the complexity of performing from easy to difficult, can be arranged in the following sequence: MT1 – CT1 – CT2. The records of the 19-channel EEG were analyzed for the group healthy subjects (8 people) and for the group patients with traumatic brain injury (5 people). In accordance with published works, the prefrontal regions of the left and right hemispheres are predominantly activated in the
calculation–logical and spatial-pattern tests, respectively [11-12]. Our method confirms this. There are presence phase-coupled pairs of EEG signals in the left hemisphere in the calculation–logical test and in the right hemisphere in spatial-pattern test. Phase-coupled pairs that appear in most people in a group of 8 healthy subjects are represented in Figure 4 (red line: phase-coupled pairs that are present in most healthy subjects during CT1, CT2 and MT1 tests).

Figure 5 shows phase-coupled pairs of channels for the group of patients with traumatic brain injury (5 people). These patients have significant communication connectivity. Phase-coupled pairs of EEG channels for each patient are represented by special color.

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![Figure 4](image1.png)

**Figure 4.** Phase-coupled pairs that appear in most people in the group of 8 healthy subjects during cognitive tests (CT1 and CT2) and motor test (MT1). a) MT1; b) CT1; c) CT2.

![Figure 5](image2.png)

**Figure 5.** Phase-coupled pairs that appear for the group patients with traumatic brain injury during cognitive tests (CT1 and CT2) and motor test (MT1). a) MT1; b) CT1; c) CT2. Phase-coupled pairs of EEG channels for each patient are represented by special color.
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Figure 5 represents that phase-coupled pairs that appear in most people in the group of patients with traumatic brain injury practically are absent. Coincident phase-coupled pairs are represented in only two patients during the test CT2: Fp1 – Fp2 and coincident phase-coupled pairs are represented in three patients during test CT2: Fp1-Fp2, Fp1-F8, Fp2-F7, F3- F8 и Fp1-F4. These are EEG channels of the frontal region of the brain. The activation of the left hemisphere during the performance of the CT1 test is absent by patients with traumatic brain injury. Also, the activation of the right hemisphere during the performance of the CT2 test is absent by patients with traumatic brain injury.

4. Conclusions
The new approach to estimation of the inter-channel phase connectivity of EEG signals based on the calculation of the phase in points of ridges in the wavelet spectrogram with stationarity is proposed. Under the new approach, phase-coupled pairs of EEG signals are determined for healthy subjects during two cognitive (calculation–logical and the spatial-pattern) and motor tests. The coincident phase-coupled pairs of EEG signals for the group of healthy subjects (8 subjects) and the non-coincident phase-coupled pairs of EEG signals for the group of people with traumatic brain injury (5 subjects) are represented. Areas of interest of the cortex at cognitive and motor tests for group of healthy subjects are determined. The prefrontal regions of the left hemisphere are predominantly activated in the calculation–logical cognitive test. The prefrontal regions of the right hemisphere are predominantly activated in the spatial-pattern cognitive test. A larger number of phase-coupled pairs reflects the different efforts required to perform each of the tests, which, according to the complexity of performing from easy to difficult, can be arranged in the following sequence: motor test – calculation–logical cognitive test – spatial-pattern cognitive test. Our method confirms this. The definition of phase-coupled pairs of EEG signals can be useful for the monitoring the treatment of people with traumatic brain injury. The recovery of phase-coupled EEG pairs can be used as indicator of proper treatment.

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