Self-Organized Bottleneck
and Coexistence of Incongruous States
in a Microwave Phonon Laser (Phaser)

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Abstract

Phenomena of emergence of regular and chaotic fine structure (FS) in stimulated emission (SE) power spectra of an autonomous microwave phonon laser (phaser) have been revealed and investigated experimentally in pink ruby at liquid helium temperatures. The phenomenon of a self-organized bottleneck in evolution of the microwave acoustic FS lines has been observed by means of narrow-range phonon SE spectral analysis. The large-scale phenomenon of coexistence of incongruous (stationary, periodic and chaotic) states in the whole spin-phonon phaser system is revealed in experiments with panoramic power spectra of phonon SE. Phaser active medium is characterized by a very small ratio $R = T_f/T_a << 1$ (see D. N. Makovetskii, arXiv:cond-mat/0402640v1), where $T_f$ is the lifetime of an emitted field quasiparticle (namely, acoustic branch phonon), and $T_a$ is the time of spin-lattice relaxation of an individual active center (AC). Such the medium may be considered as an excitable system by analogy with class-B optical lasers (see C. O. Weiss e.a., Phys. Rev. A, vol.47, p.R1616, 1993), if one takes into account local (dipole-dipole) interactions between ACs. We propose a possible direction for modeling of both the observed phenomena on the basis of three-level cellular automata (S. D. Makovetskiy and D. N. Makovetskii, arXiv:cond-mat/0410460v2; S. D. Makovetskiy, arXiv:cond-mat/0602345v1) which emulate evolution of a bounded phaser-like excitable system with locally interacting ACs in the limit $R \to 0$. 
Introduction

The problem of studying the nonlinear processes occurring in open dissipative systems which possess a complex phase-space structure is challenging for a number of branches of modern physics [1]. In particular, the laser-based and other nonlinear optical systems are studied intensively in this field of science nowadays [2, 3], and a number of important fundamental and applied results has already been obtained. The ideas of quantum electronics have also deeply penetrated into adjacent branches of modern physics, especially into quantum acoustics [4–13]. A series of new nonlinear phenomena, which arise under the quantum amplification or generation of tera-, hyper- or ultrasonic oscillations in nonequilibrium (inverted) acoustic and nanomechanical dissipative systems of various origins, have been predicted theoretically and discovered experimentally during last years [14–20]. The researches concerning the conditions, under which the formation of inverted states and the purposeful control over nonlinear processes in such systems (the acoustic analogs of lasers) become possible, are of significant independent interest.

As has already been emphasized in works [14, 15, 21, 22], if one confines the consideration to the quantum amplification and generation of hypersound, i.e. the acoustic waves within the microwave range of frequencies, the following circumstances invoke the greatest interest. A microwave phonon laser (phaser) emits acoustic radiation with a wavelength of 1–3 µm, which corresponds to the wavelength of electromagnetic radiation in the near-infrared range, where many ordinary lasers generate as well. At the same time, the velocity of hypersound in the phaser active paramagnetic media is 5 orders of magnitude lower than the velocity of electromagnetic waves. Accordingly, the frequency of stimulated emission (SE) of acoustic waves in a paramagnetic phaser amounts to $\omega = 3 \times 10 \text{ GHz}$ only, which is about the same 5 orders of magnitude lower than the frequency of the electromagnetic SE in optical lasers. Therefore, the relative intensity of the spontaneous component $J_{\text{spont}}$ of emission in a microwave phaser is about 15 orders of magnitude lower than that in an ordinary optical (electromagnetic) laser (because $J_{\text{spont}} \propto \omega^3$), whereas the lengths of corresponding acoustic and electromagnetic waves coincide.

The low level of the quantum noise in phasers opens new opportunities for studying the nonlinear phenomena in nonequilibrium systems. This conclusion has already been confirmed experimentally for a paramagnetic phaser with a pink ruby crystal (Cr$^{3+}$: Al$_2$O$_3$). In works [14, 15, 21, 22], a new nonlinear resonance (λ-resonance), which is accompanied by a number of interesting dynamical phenomena, has been revealed experimentally in a non-autonomous ruby phaser and studied in details at liquid-helium temperatures and electromagnetic pumping with carrier frequency about 23 GHz. This λ-resonance arises at modulation of pumping (or Zeeman magnetic field) in the range of ultralow frequencies $\omega_m \approx \omega_\lambda$, where $\omega_\lambda/2\pi \approx 10 \text{ Hz}$ [14, 15, 21, 22]. It is considerably lower than the typical frequencies $\omega_\nu$ of the usual relaxational resonance ($\nu$-resonance) in the ruby phaser ($\omega_\nu/2\pi \approx 70–300 \text{ Hz}$), which was observed earlier in works [23, 24] (note that $\nu$-resonance in the ruby phaser is an acoustic analog
of the well-known laser relaxational resonance).

In works [14, 15, 21, 22], it has been discovered experimentally that a plenty of coexisting states of the spin-phonon system emerges in the domain of the λ-resonance and the very slow regular transitions, which remind autowave motions, occur between them. The period of the full cycle of such transitions reaches giant values and exceeds $10^3 - 10^4$ s at $\omega_m \approx \omega_\lambda$ [14,15,21,22]. This is several orders of magnitude longer than the period $\tau_m = 2\pi/\omega_m$ of an external force that modulates phaser pumping (or Zeeman magnetic field).

It should be mentioned that a kind of slowing-down of transient processes in paramagnetic crystals, which accompanies the saturation of spin transitions, has already been studied earlier [25–27] under conditions of the so-called phonon bottleneck caused by non-coherent (thermal) effects in a resonant spin-phonon system which interacts with a thermostat. However, in the 1970-1980s, it has been proved experimentally [5, 7, 9, 12, 28, 29] that, at least in the microwave range, both the quantum amplification and self-excitation of phonons in phasers arise just in the absence of the phonon bottleneck effect, which could only inhibit the normal functioning of a phaser as a quantum oscillator.

Anyway, the phonon bottleneck effect in pink ruby in the microwave range and at temperatures of liquid helium is absent both in our system and in analogous paramagnetic systems used by other authors in numerous investigations of quantum paramagnetic amplifiers. In particular, our experimental data for pink ruby crystals [9,12,28,29] have demonstrated that at temperatures $\theta = 1,7–4,2$ K the maximal lifetime of resonant phonons with a frequency of about 10 GHz is about $10^4$ times shorter than the time of longitudinal spin relaxation $\tau_1 \approx 0,1–1$ s of chromium ions. This is the direct manifestation of nonoccurrence of phonon bottleneck in the active medium of our phaser.

Thus, the emergence of slow processes, which were observed in paramagnetic non-autonomous phaser [14,15,21,22], cannot evidently be a result of influence of incoherent phonon bottleneck. The slowing-down of the energy exchange in the active medium of non-autonomous paramagnetic phaser, as was shown in [14,15,21,22], reflects the peculiarities of the self-organization of dissipative structures in the spin-phonon system of such a phaser, provided a strong external perturbation on the inversion states of paramagnetic centers. Does such or similar slowing-down exists in an autonomous phaser, where the self-organization emerges in absence of external forces at all? The main goal of this work was experimental search for the answer to this question. The results of theoretical researches and computer studies [30–37], which have been published recently, served as an additional motivation for this work to be fulfilled.

1 The Phaser System

The experimental part of the work was carried out making use of a microwave phaser [14,15,21,22]. As it was already pointed out in works [14,15], a microwave phaser is an acoustic analog of the class-B optical lasers [3]. In particular, both in class-B lasers and in a phaser the slow processes of the population inversion
of active centers (ACs) are the leading ones, while the fast-relaxing system of the field excitations follows the AC-system evolution.

A phaser differs from a class-B laser only by the nature of the stimulated emission (SE). Namely, in a class-B laser the SE is purely electromagnetic, but in a phaser it is mainly acoustic. The pointed difference is very essential, because velocity of acoustic waves is typically 5 orders lower than velocity of light. We will discuss this principal difference later.

The main element of the phaser, which was already described in [14, 15, 21, 22], is a ruby acoustic Fabry–Perot resonator (AFPR) with two flat and parallel acoustic mirrors. The AC-system of ruby phaser consists of trivalent chromium ($\text{Cr}^{3+}$) paramagnetic ions in non-magnetic corundum crystal matrix. The inversion of spin-levels populations of AC-system in ruby phaser is formed by continuous stationary pumping. The pumping field is a microwave electromagnetic field excited from outside in an electromagnetic pump cavity (EPC) with the AFPR inside.

At one of the acoustic mirrors, the thin-film piezoelectric transducer (composed of a textured ZnO film and an Al interlayer) is mounted. This transducer admits to detect an acoustic SE (laser-like phonon emission) that arises in the ARFP when the corresponding ($\text{Cr}^{3+}$) spin transitions are being pumped. In the simplest case the acoustic microwave power spectrum of this SE consists of very narrow lines located at eigenfrequencies of ARFP $\omega_n/2\pi \approx 9 \text{ GHz}$ ($\Delta_n/2\pi = (\omega_n - \omega_{n-1})/2\pi = 310 \text{ kHz}$).

It is important that the thin-film piezoelectric transducer is bi-directional. So, making use of a microwave external generator (that excites microwave acoustic vibrations of the ZnO film), one can inject a phonon flux in the phaser active system independently from the laser-like phonon emission generated by ACs. The frequency of this injected acoustic signal $\omega_{\text{inj}}$ is equal to the frequency of the microwave external generator and generally is not equal to any of eigenfrequencies of AFPR.

Other details concerning the thin-film piezoelectric transducer, the AFPR, the EPC, and the control parameters (the static magnetic field $\vec{H}$, the pump frequency $\omega_{\text{pump}}$, the pump power $P$, etc.), can be found in works [21, 22]. However, in contrast to works [21, 22], all the measurements of microwave phonon SE were carried out provided that any modulation of the control parameters was absent. Therefore, all the SE regimes observed in this work concern exclusively the autonomous phaser.

2 Absorption, Amplification and Self-Excitation of Hypersound in the Phaser System

2.1 Absorption of injected hypersound in the absence of pumping

Provided that the pumping is absent (the pump power $P = 0$), and the amplitude and direction of $\vec{H}$ are far from the lines of the acoustic paramagnetic
resonance (APR) of the system $\text{Al}_2\text{O}_3: \text{Cr}^{3+}$, the absorption of injected hypersound with the frequency $\omega_{\text{inj}}$, is governed by the two following mechanisms:

(M1) Non-resonant attenuation of hypersound in the $\text{Al}_2\text{O}_3: \text{Cr}^{3+}$ crystal. This attenuation is characterized by the “volume” decrement $\eta_{\text{vol}}$ of hypersound in the crystal medium itself. The value $\eta_{\text{vol}}$ depends, first of all, on the crystal matrix perfection. In addition, this value includes also hypersound energy leakage through the lateral (non-mirror) surface of the AFPR;

(M2) Hypersound losses at the acoustic mirrors (one of which is additively loaded by the thin-film piezoelectric transducer). This attenuation is characterized by the effective “mirror” decrement $\eta_{\text{mirr}}$ of hypersound. The value $\eta_{\text{mirr}}$ takes into account leakage of the hypersound energy through imperfect mirror surfaces of the AFPR. So it depends strongly on the mirrors’ quality and the transducer’s loading.

Both mechanisms (M1) and (M2) lead to decreasing of intensity of the injected hypersound in AFPR, because $\eta_{\text{vol}}$ and $\eta_{\text{mirr}}$ are always positive.

If the injected hypersound frequency $\omega_{\text{inj}}$ is scanned, the acoustic microwave resonances are observed in the AFPR at $\omega_{\text{inj}} \approx \omega_n$. These acoustic resonances are similar to ordinary electromagnetic ones in an optical Fabry-Perot resonator. It is obvious that the pointed acoustic resonances one can observe even in pure corundum ($\text{Al}_2\text{O}_3$) AFPR.

At $P = 0$ and $\omega_{\text{inj}} = \text{const}$, the value and direction of $\vec{H}$ can be tuned in such a manner that the APR arises in the $\text{Al}_2\text{O}_3: \text{Cr}^{3+}$ paramagnetic system [7, 38]. The APR is realized when the splitting for a certain pair of energy levels $\mathcal{E}_m(\vec{H})$ and $\mathcal{E}_n(\vec{H})$ in the $\text{Al}_2\text{O}_3: \text{Cr}^{3+}$ system approximately coincides with $\hbar \omega_{\text{inj}}$ and the quantum transition $\mathcal{E}_m \leftrightarrow \mathcal{E}_n$ is allowed for interaction with hypersound [7,38,39]. Additively, the intensity of injected hypersound $J_{\text{inj}}$ must not be excessively high, namely $J_{\text{inj}} < J_{\text{sat}}$, where $J_{\text{sat}}$ is saturation intensity for the quantum transition $\mathcal{E}_m \leftrightarrow \mathcal{E}_n$ in $\text{Al}_2\text{O}_3: \text{Cr}^{3+}$ paramagnetic system.

Under these conditions, the additional losses of hypersound take place at $\vec{H} \approx \vec{H}_{\text{inj}}^{(\text{vert})}$, where $\vec{H}_{\text{inj}}^{(\text{vert})}$ is the vertex of the magnetic-field-line of the APR at the frequency $\omega_{\text{inj}}$. So, we have the third mechanism of the absorption of injected hypersound in our system, namely:

(M3) Resonant paramagnetic attenuation of hypersound, which depends strongly on $\omega_{\text{inj}}$, $\vec{H}$ and $J_{\text{inj}}$, as well as on the hypersound type (longitudinal, fast transverse, slow transverse) [7, 38, 39].

One can not observe this resonant paramagnetic attenuation in pure corundum ($\text{Al}_2\text{O}_3$) or in any other non-magnetic medium. The (M3) kind of hypersound attenuation is caused by spin-phonon interaction in paramagnetic medium (the ruby crystal $\text{Al}_2\text{O}_3 : \text{Cr}^{3+}$ in our case).

Phenomenologically this attenuation is characterized by the effective “magnetic” decrement $\eta_{\text{magn}}^{(\text{null})}$ of hypersound. The superscript (null) indicates that pumping is absent ($P = 0$). Quantitative description of the APR phenomenon and rigorous formulae for hypersound attenuation one can find in [7, 38, 39].

It is obvious, that $\eta_{\text{magn}}^{(\text{null})} > 0$ (as far as $\eta_{\text{vol}}$ and $\eta_{\text{mirr}}$), and no amplification of injected hypersound in AFPR takes place in absence of pumping.
2.2 Quantum amplification of injected acoustic signal and self-excitation of hypersound in ruby phaser

Let the electromagnetic pumping is switched on $(P > 0)$ at a frequency $\omega_{\text{pump}} = \omega_{\text{cav}}$, where $\omega_{\text{cav}}$ is an eigenfrequency of the EPC. The vertex $\vec{H}_{\text{pump}}^{(\text{vert})}$ of the magnetic-field-line of the electron spin resonance (ESR) at the frequency $\omega_{\text{pump}}$ generally does not coincide with that of the APR line $\vec{H}_{\text{inj}}^{(\text{vert})}$ at the frequency $\omega_{\text{inj}}$. But if the condition $\vec{H}_{\text{pump}}^{(\text{vert})} = \vec{H}_{\text{inj}}^{(\text{vert})}$ is satisfied (e. g. after an appropriate tuning of the frequency $\omega_{\text{inj}}$), and the ESR line is saturated, the APR line becomes inverted [7,10]. In other words, the paramagnetic absorption of injected hypersound is changed by its phaser amplification. As for the specific conditions for the inversion of the APR line at the ESR saturation, see [21,22].

If all the non-magnetic losses of hypersound ($\eta_{\text{vol}}$ and $\eta_{\text{mirr}}$) in the AFPR are totally compensated owing to the phaser amplification, the self-excitation of hypersound arises. This self-excitation is possible even in absence of an injected signal: exponential growth of small noisy (thermal) acoustic seed is the cause of the acoustic instability leading to the hypersound self-excitation [5,9,12,28,29].

Conditions of self-excitation of acoustic waves in a phaser are much more complicated than conditions of self-excitation of electromagnetic waves in lasers. Two main causes of the pointed complexity are as follows: (i) the coexistence of longitudinal, transverse and mixed types of acoustic waves at an arbitrary direction of hypersound propagation and (ii) very high anisotropy of spin-phonon interaction in active paramagnetic crystals.

In this work, the geometrical axis of the AFPR $\vec{O}_C$ coincides with the ruby’s crystallographic axis of the third order $\vec{O}_3$. In this case no mixed types of acoustic waves exist along the AFPR axis. Moreover, it is known that, in the case of transverse acoustic waves with the wave vector $\vec{k}_T \parallel \vec{O}_3$, the so-called conical refraction takes place [40]. The last phenomenon is a consequence of degeneration of fast and slow transverse acoustic modes propagating along $\vec{O}_3$ [40]. Hence, only pure longitudinal hypersound modes with the wave vector $\vec{k}_L \parallel \vec{O}_C \parallel \vec{O}_3$ posses a high Q-factor in such an AFPR at its microwave eigenfrequencies $\omega_n$.

When the pump power in our system is slowly increased beginning from $P = 0$, the considered compensation of non-magnetic losses of hypersound achieves (step by step) for different longitudinal AFPR modes $\omega_n$. First of all, self-excitation of hypersound arises at the longitudinal AFPR mode $\omega_1$ which is located most closely to the center of the inverted APR line $\omega_0$. For this mode, the condition

$$Q^{(1)}_{\text{eff}}(P, \vec{H}, \omega_1) < 0,$$

holds true ahead of all other modes $\omega_n$, as $P$ increases. Here, $Q^{(1)}_{\text{eff}}$ is the effective acoustic Q-factor of the phaser system for the mode $\omega_1$ under pumping. This effective acoustic Q-factor is determined by the relation:

$$\frac{1}{Q^{(1)}_{\text{eff}}} = \frac{1}{Q^{(1)}_{\text{vol}}} + \frac{1}{Q^{(1)}_{\text{mirr}}} + \frac{1}{Q^{(1)}_{\text{magn}}} = \frac{1}{Q^{(1)}_{C}} + \frac{1}{Q^{(1)}_{\text{magn}}},$$

(2)
where $Q_{\text{vol}}^{(1)} = k_L/\eta_{\text{vol}}$; $Q_{\text{mirr}}^{(1)} = k_L/\eta_{\text{mirr}}$; $k_L = |\vec{k}_L| = \omega_1/V_L$; $V_L$ is the phase velocity of hypersound; $Q_C^{(1)}$ the Q-factor of the AFPR at the $\omega_1$ mode in the absence of pumping, and $Q_{\text{magn}}^{(1)}$ is the magnetic Q-factor of this mode under pumping.

In contrast to $Q_C^{(1)}$, the magnetic $Q_{\text{magn}}^{(1)}$ can be negative. It looks like

$$Q_{\text{magn}}^{(1)} = -k_L/\alpha_1(P, \vec{H}, \omega_1) \equiv -k_L \left[ K(P, \vec{H}) \sigma(\vec{H}, \omega_1) \right]^{-1},$$

where $\alpha_1$ is the increment of quantum amplification of hypersound for the mode under consideration; $K$ is the inversion ratio for the APR line (in the case of inversion, $K > 0$); $\sigma \approx \eta_{\text{null}}$ is the effective “magnetic” decrement of hypersound with frequency $\omega_1 \approx \omega_0$ in the absence of pumping.

Following works [21, 22], we write down the expression for $\sigma$ at the spin transition $E_m \leftrightarrow E_n$, in the form:

$$\sigma_{mn} = \frac{2\pi^2 C_a \nu^2 g(\nu)|\Phi_{mn}|^2}{(2S + 1)\rho'V_L^2 k_B T},$$

where $C_a$ is the concentration of paramagnetic centers; $\nu = \omega/2\pi$; $g(\nu)$ is the form-factor of the APR line; $\rho'$ is the crystal density; $k_B$ is the Boltzmann constant; $|\Phi_{mn}|$ is the coupling parameter of the spin transition $E_m \leftrightarrow E_n$ with hypersound.

The matrix element $\Phi_{mn}$ for a longitudinal hypersound wave propagating along the ruby’s axis of the third order $\vec{O}_3$ is determined as follows (the coordinate axis $z$ is parallel to $\vec{O}_3$):

$$\Phi_{mn} = \frac{\partial}{\partial \varepsilon_{zz}} \langle \psi_m | \hat{\mathcal{H}} | \psi_n \rangle = \frac{G_{33}}{2} \left( 3 \langle \psi_m | \hat{S}_z^2 | \psi_n \rangle - S(S + 1) \langle \psi_m | \psi_n \rangle \right).$$

Here, $\varepsilon_{zz}$ is the component of the elastic strain tensor; $\hat{\mathcal{H}}$ is the Hamiltonian of spin-phonon interaction [7, 38, 39]; $|\psi_m\rangle$ and $|\psi_n\rangle$ are the wave functions of paramagnetic ion in the crystalline field (these wave functions belong to the ion’s energy levels $E_m$ and $E_n$ respectively); $G_{33}$ is the component of the spin-phonon interaction tensor [7, 38, 39]; and $\hat{S}_z$ is the projection of the vectorial spin operator onto the $z$-axis.

In order to estimate $\Phi_{mn}$, let us use the experimentally found value $G_{33} = 5.8 \text{ cm}^{-1} = 1.16 \times 10^{-15} \text{ erg}$ [38], as well as the wave functions $|\psi_3\rangle$ and $|\psi_2\rangle$ which belong to the energy levels $E_3$ and $E_2$ of the Cr$^{3+}$ ion in the trigonal crystalline field of ruby (in this work, similarly to works [21, 22], $m = 3, n = 2$). From Eq. (5), for the magnetic field $H = 3.92 \text{ kOe}$ directed at an angle $\vartheta = \vartheta_{\text{symm}}$ with respect to the $z$-axis, where $\vartheta_{\text{symm}} \equiv \arccos(1/\sqrt{3}) = 54^\circ 44'$, we find $\Phi_{32} \approx 10^{-15} \text{ erg}$. The choice of $\vartheta = \vartheta_{\text{symm}}$ was dictated by the requirements of the so-called symmetric (or push-pull) pumping regime which was also used.
by us earlier to enhance the inversion ratio in a phaser \cite{21,22}. As the result, at $\nu = 9.1$ GHz, $g(\nu) = 10^{-8}$ s, $C_a = 1.3 \cdot 10^{19}$ cm$^{-3}$, $\rho' = 4$ g/cm$^3$, $V_L = 1.1 \cdot 10^6$ cm/s, $T = 1.8$ K, we find from Eq. (4) that $\sigma_{mn} = \sigma_{32} \approx 0.04$ cm$^{-1}$.

The acoustic Q-factor $Q_C^{(1)}$ for our ruby AFPR (loaded by the piezoelectric film) was measured making use of the pulse-echo method at the frequency $\omega = 9.12$ GHz. The value of $Q_C^{(1)}$ was found to be $5.2 \pm 0.4) \cdot 10^5$ at $H = 0$ and $P = 0$. Whence, $\eta = \eta_{vol} + \eta_{mirr} = \omega/Q_C^{(1)}V_L \approx 0.1$ cm$^{-1}$.

For the case of an autonomous phaser, a simple relation takes place between $\sigma_{32}$, $\eta$ and the threshold value $\alpha_g^{(1)}$, at which the generation of the first mode begins. This relation reads as follows:

$$\alpha_g^{(1)} = \eta = K_g \sigma_{32},$$

where $K_g$ is the critical value of the inversion ratio $K$ at the transition $E_3 \leftrightarrow E_2$. Substituting $\sigma_{32} \approx 0.04$ cm$^{-1}$ and $\eta \approx 0.1$ cm$^{-1}$ into Eq. (6), we find that $K_g \approx 2.5$, which can be ensured liberally by the push-pull pumping scheme.

The autogeneration of longitudinal hypersound in the autonomous ruby phaser was detected provided that external perturbations were absent both in the signal and pump channels. The microwave electromagnetic signal, excited in the hypersound transducer by the microwave phonon SE, was supplied to a heterodyne spectrum analyzer, and the spectra obtained were registered from its screen by a photocamera. All these experiments were fulfilled at temperatures below the superfluid critical point of liquid helium $\theta < 2.1$ K, which allowed us to avoid the difficulties related to the boiling of this cryogenic liquid.

Now, let us consider the observed microwave power spectra of the phonon SE in the autonomous phaser generator in detail.

3 Coarse Structure of the Spectra of Stationary Phaser Autogeneration

Since the frequency width $\Gamma_s$ of the APR line for the spin transition $E_3 \leftrightarrow E_2$ amounts to approximately 100 MHz, and the distance between the AFPR modes is only about 300 kHz, the primary single-mode SE easily transforms to a multimode one, even if the excess over the pumping threshold is comparatively small. In other words, there emerges a coarse structure (CS) of the phonon generation (see Fig. 1). Provided $\omega_{pump} = \omega_{cp} = 23.0$ GHz, and $H = H_0 = 3.92$ kOe, the multimode phonon generation is observed even at $P \geq 50 \mu$W.

Figure 1 near here

Fig. 1: Microwave power spectrum of longitudinal phonon SE at $\Delta_P = \Delta_H = 0$ in the ruby phaser. The frequency interval between neighboring CS modes of phonon SE amounts to 310 kHz. The cryostat temperature is $\theta = 1.7$ K.

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Under the condition of the magnetic-field detuning $\Delta_H \equiv H - H_0 \neq 0$, it is natural that a considerably more intense pumping is needed for the condition $K > K_g$ to be satisfied. On the other hand, the increasing of a pumping level is also necessary if there is some frequency detuning $\Delta_P \neq 0$, where $\Delta_P \equiv \omega_{\text{pump}} - \omega_{cp}^{(0)}$.

If the pumping is switched on abruptly, the transient process, which takes place in the course of establishment of the stationary integral intensity of SE $J_\Sigma$, demonstrates an oscillatory behavior. The frequency $\omega_\nu$ of these damped oscillations (the so called relaxation frequency) for our system has, as it was already mentioned, a value of the order $\omega_\nu/2\pi \approx 10^2$ Hz [23, 24].

The lifetime of this transient process $\tau_{\text{tran}}$ is shorter than 0.3 s at $P > 0.1$ mW and provided that the magnetic-field and pump-frequency detunings are absent ($\Delta_H = 0, \Delta_P = 0$). Therefore, if the phaser system is finely tuned and the pumping is powerful enough, the transient process is rather fast ($\tau_{\text{tran}} \approx \tau_1$). This experimental fact agrees with theoretical estimations [23, 24] of $\tau_{\text{tran}}$ made in the framework of the elementary (balance) model of SE [41, 42]: $\tau_{\text{tran}} \approx \tau_1/A_P$, where $A_P$ is the pumping parameter (for our system, $A_P \approx 2$ [23, 24]). In other words, under the conditions specified above, the process of formation of the stationary CS of the phaser power spectra terminates approximately within the same time interval, over which the stationary resonant absorption in a passive (non-inverted) paramagnetic system becomes settled.

4 Regular Fine Structure and Slow Transient Processes in the Microwave Power Spectra

4.1 Conditions of emergence of a regular fine structure

At $P \geq 4$ mW, $\Delta_P = 0$, and the small magnetic-field detunings $|\Delta_H| \leq 2-3$ Oe, the integral intensity $J_\Sigma$ of multimode SE of an autonomous phaser, as it was already pointed out in works [21, 22], practically does not depend on time. Provided that the pumping is powerful enough and its frequency is tuned finely, the measured magnitude of $J_\Sigma$ weakly (in the limits of several percent) oscillates only if $|\Delta_H|$ is increased to about $\approx 30$ Oe and more [21, 22].

In the corresponding non-stationary microwave phonon power spectra, some CS modes of autonomous phaser become splitted at $|\Delta_H| \geq 30$ Oe. One of the typical cases of such the splitting for a phaser SE mode is shown in work [43]. Thus, the fine structure (FS) spontaneously arises in microwave phonon power spectrum. We stress that no external forcing is needed for emergence of FS, in contrary to other nonlinear phenomena investigated by us at the CS level [14, 15, 21, 22, 43]. The intensities of the FS components $J_F^{(i)}$ observed in [43], were very low, even at $i_{\text{max}} = 2 - 3$. This is several orders of magnitude lower than the intensities of the non-splitted SE components of stationary generation which are depicted at Fig. 1.

However, if the phaser system becomes appreciably detuned by the pump frequency ($\Delta_P >> \Delta_n \approx 0.3$ MHz), the emergence of the FS in the microwave
power spectra of phaser generation is observed even at \( \Delta_H = 1 - 2 \) Oe. In this case, \( J_F^{(i)} \) are of the same order of magnitude as the unsplitted CS components. In Fig. 2, the splitting of the CS mode into two and three intense FS components is demonstrated, as \( \Delta_H \) increases from 0 up to 3.5 Oe. One can also notice very weak additional FS components against the noise background.

![Figure 2 near here](image)

**Fig. 2:** Emergence of a regular FS in the power spectra of an autonomous phaser at \( P = 4 \) mW, \( \Delta_P = 8.8 \) MHz, \( \theta = 1.8 \) K, and for various \( \Delta_H \). Left oscillogram: \( \Delta_H = 0 \) (the FS is absent). Middle oscillogram: \( \Delta_H = 1.5 \) Oe (the two-component FS). Right oscillogram: \( \Delta_H = 3.5 \) Oe (the three-component FS). The range of the frequency sweeping along the horizontal axis is about 6 kHz for each oscillogram.

### 4.2 Mechanism of formation of a regular fine structure

The appearance of the regular FS with a few (2 to 3) components of the splitted CS mode of phonon SE can be explained in the framework of the Casperson–Yariv mechanism [44]. The normalized dispersion \( \Delta V_L / V_L \) of the phase velocity for hypersound with the frequency \( \omega \) in a paramagnetic AFPR possessing a certain eigenfrequency of the CS mode \( \omega_{cs} \approx \omega \) looks like

\[
\frac{\Delta V_L}{V_L} = \frac{\alpha \xi_r}{2k_L(1 + \xi_r^2)} = \frac{q_M \xi_r}{(1 + \xi_r^2)},
\]

(7)

where \( \xi_r \equiv (\gamma_s H_0 - \omega)/\Gamma_s \), \( \gamma_s = h^{-1} \partial (\mathcal{E}_3 - \mathcal{E}_2) / \partial H \), \( H_0 = \omega_{cs}/\gamma_s = \pi n V_L/\gamma_s L \), and \( q_M = \alpha/2k_L \).

Now, let us introduce the additional detuning of the system by changing the static magnetic field \( H \) with respect to its resonant value \( H_0 \). Using the dimensionless magnetic-field detuning \( h = \gamma_s \Delta_H \Gamma_s^{-1} = (\gamma_s H - \omega_{cs})/\Gamma_s \), we have the following formula for the dependence of the dispersion of the hypersound phase velocity on the magnetic field:

\[
\frac{\Delta V_L(h)}{V_L} = \frac{q_M \delta}{(1 + \delta^2)},
\]

(8)

where

\[
\delta \equiv \xi_r + h = \frac{\gamma_s H - \omega}{\Gamma_s}.
\]

(9)

Now, we apply the well-known Casperson–Yariv relation [44] which couples the refraction index \( \eta \) (in our case, this is the acoustic refraction index \( \eta = \eta_{acoust} \)) with the mode frequency \( \omega \) in the resonator (in our AFPR, this is a hypersonic mode, for which Eqs. (8)–(9) hold true):

\[
(\eta_{acoust} - 1) L_A \omega/L = \omega_{cs} - \omega,
\]

(10)
where $L_A$ is the active medium length. In the used phaser system, $L_A = L$; therefore, making use of the relation $\Delta V_L / V_L = 1 - \eta_{\text{acoust}}$ and formulae (8) and (9), we obtain the implicit expression for calculating the dependence $\delta(h)$:

$$h - \delta(h) = \frac{q_M \varpi(h)}{\Gamma_s} \cdot \frac{\delta(h)}{1 + \delta^2(h)}.$$  

(11)

On the right-hand side of Eq. (11), we may put $\varpi = \omega_{cs}$. Then, from Eq. (11 and using the equality $\xi_r = \delta - h$, we find the FS spectrum for the AFPR mode as a function of the magnetic field:

$$\varpi^{(j)}(h) = \omega_{cs} + \left[h - \delta^{(j)}(h)\right] \Gamma_s.$$  

(12)

Here, $\delta^{(j)}$ are the roots of the Bonifacio–Lugiato equation [45]:

$$h = \left(1 + \frac{2M}{1 + \delta^2}\right) \delta,$$  

(13)

and the control parameter $M$ is of the form

$$M = \frac{q_M \omega_{cs}}{2 \Gamma_s} \propto \frac{K C_s k_B^2 |\Phi_{32}|^2}{\Gamma_s^2 k_B \theta}.$$  

(14)

At high magnetic-field detuning $h$, we have $\varpi \rightarrow \omega_{cs}$; while at $h \rightarrow 0$ we see that $|\varpi - \omega_{cs}| \propto h$. Accordingly, the function $\delta(h)$ is single-valued for these extreme cases. In the intermediate range of magnetic-field detunings $h$, the function $\delta(h)$ may be multi-valued. Bifurcation points which determine conditions of this branching of $\delta(h)$ (i.e. splitting of a CS mode into two or three FS components) can be easily calculated by standard methods of the catastrophe theory [46]. We find two bifurcations of codimension 2 (at $M = M^{(0)} = 4$ and $h = h^{(0)}_\pm = \pm 3\sqrt{3}$) and four bifurcations of codimension 1, which take place at $M > M^{(0)}$, $h = h^{(0)}_\pm \approx \pm \sqrt{(M/2)[a(M) \pm b(M)]}$. Here signs in subscript correspond to signs at curly brackets, signs in superscript correspond to signs within square brackets, $a(M) = a + 10 - 2a^{-1}$ and $b(M) = [(a - 4)^3/a]^{1/2}$. So, the function $\delta(h)$ is two-valued at $M = M^{(0)}$ or at $M > M^{(0)}$ and $h = h^{(0)}_\pm$. Accordingly, the function $\delta(h)$ is three-valued if $M > M^{(0)}$ and $h^{(-)}_+ < h < h^{(+)}_+$ or $h^{(-)}_+ < h < h^{(-)}_$. This three-component splitting is typical due to it’s non-critical nature. The two-component splitting is, strictly speaking, critically dependent on variations of $h$, but in real experiment there exist a small but finite range of $h$ in the vicinity of bifurcation point making it possible to observe unhesitatingly the two-component FS (see Fig. 2, middle oscillogram).

Thereby, the “static” Casperson-Yariv model considered above demonstrates a possible mechanism of spontaneous (not connected with external factors) splitting of a CS mode into two or three components, depending on magnetic field detuning. This model gives a satisfactory interpretation of such the splitting FS. Moreover, the Casperson-Yariv model can be improved considerably by making allowance for the effects of hole burning in the amplification line etc. [44].
a series of other results on microwave phonon SE in the autonomous phaser needs alternative approaches for at least qualitative explanation of phaser dynamics observed in our experiments. Very interesting, in particular, are the issues concerning transient processes of the FS emergence, especially in the case where the characteristic time of a transient process $\tau_{\text{tran}}$ is much longer than the spin-lattice relaxation time $\tau_1$ (and a control parameter set is far from the critical one). Processes of self-induced non-critical slowing of evolution have been studied intensively in recent years in nonlinear autonomous systems of various nature, and important results have already been obtained by means of computer modeling [30–37]. In this work, we focus our attention first on two specific issues, namely, whether such slow processes may proceed in an autonomous phaser and which is the nature of coexistence of incongruous states arising during the pointed transient processes in the phaser active medium. The relation between obtained experimental results and those of computer modeling [30–37] will be discussed in details after exposition of experimental data.

4.3 Slow transient processes in phaser during formation of the regular fine structure

Our observations of the spectra of phonon SE in the autonomous phaser showed that, provided $\Delta H \neq 0$ and $\Delta P \neq 0$, the characteristic time of transient processes, which accompany the emergence of the spectral FS, considerably exceeds the time of longitudinal relaxation of Cr$^{3+}$ ions in ruby at the same temperature $\theta$: $\tau_{\text{tran}}(\theta) \gg \tau_1(\theta)$. For example, for the FS depicted in Fig. 2, $\tau_{\text{tran}}(\theta_0) \approx 30$ s (here, $\theta_0 = 1.8$ K). Not only is this by two orders of magnitude longer than $\tau_1(\theta_0)$, but also is much longer than the time of establishing of the stationary CS at $\Delta H = 0$ and $\Delta P = 0$. Moreover, after the process having terminated, the structure of the power spectra is, as a rule, nonstationary. Instead, there appear periodic oscillations of the intensities of the FS components or even the periodic motions of these components along the frequency axis within the limits of several kHz. These oscillations and motions, as well as the preceding transient process, run slowly in comparison to the relaxation time of active Cr$^{3+}$ centers.

At first glance, the anomalously great values of $\tau_{\text{tran}}$ which were observed in the generating phaser system under conditions of detunings $\Delta H \neq 0$, $\Delta P \neq 0$ can be explained as the result of influence of the slowly relaxing subsystem of magnetic nuclei $^{27}\text{Al}$ of aluminium ions Al$^{3+}$ which belong to the corundum crystalline matrix. Such an influence of $^{27}\text{Al}$ nuclei on electron paramagnetic system of active Cr$^{3+}$ centers has been revealed experimentally earlier [12,28,29] at qualitatively different conditions, namely in experiments on phaser amplification of hypersound in ruby. Studying of phaser amplification in [12,28,29] were fulfilled, naturally, below the self-excitation threshold of phaser generation, i.e. at inversion ratios $K < K_g$. In this case, the dominant role of electron-nuclear interactions in slowing-down of transient processes of non-generating phaser was established in [12,28,29] and confirmed in alternative experiments on phaser amplification at the mentioned condition $K < K_g$ (under condition of self-focusing of hypersound) [47]. However, our further researches have demonstrated that
under the phaser generation, when $K > K_g$, this electron-nuclear mechanism of transient processes slowing-down does not dominate.

Really, the slow motions, which were observed in the electron-nuclear magnetic system of ruby phaser amplifier [12, 28, 29], are characterized by the well-known restriction on the maximal time of the transient process $\tau_{\text{tran}}$ in this system — the relaxation time $\tau_{\text{nucl}}^{(z)}$ of the Zeeman reservoir for the subsystem of magnetic nuclei [48, 49]. Under our experimental conditions, $\tau_{\text{nucl}}^{(c)}$ does not exceed 10 s, which is three times shorter than the values of $\tau_{\text{tran}}$ observed at the FS emergence in the power spectra of the phaser generation.

More detailed researches of the power spectra of an autonomous phaser demonstrated that, under definite conditions, the FS formation can occur following much more complicated scenarios than those in the experiments described above. In this case, the ultimate state of the SE line turns out qualitatively different from what is shown in Fig. 2, and the time of the corresponding transient process grows substantially ($\tau_{\text{tran}} > 10^2$ s). Such the situation takes place, e.g., if, provided $\Delta P > \Delta n$, the parameter $\Delta H$ is increased several times with respect to those $\Delta H$ values, for which the FS shown in Fig. 2 was observed. Let us consider these experiments in detail.

5 Super-slow Transient Processes and Selective Chaotization of Fine Structure in Microwave Phonon Power Spectra

5.1 Experimental observation of selective chaotization of fine structure

Provided the same values of pump power and pump frequency detuning as in the previous experiments (i.e. $P = 4$ mW, $\Delta P = 8.8$ MHz), but for the enlarged magnetic-field detuning up to $\Delta H = 15$ Oe, the regular FS (which is shown in Fig. 2) was found to be gradually destroyed. In approximately 10 min after the indicated detuning having been introduced, the form of some FS lines became apparently chaotic. In this work, we use the term “chaotic FS”, not specifying the kind of the disorder observed, because it is not still clear, whether this is a multi-dimensional deterministic chaos or some form of a small-scale spin-phonon turbulence. The experimental data presented below allow us to assert only that the dimension of the phase space, which is necessary to describe the observed chaotic FS, should be much higher than that in the case of the ordinary low-dimension deterministic chaos which was studied in phasers with periodic pump modulation [23, 24, 41, 42].

A typical view of the chaotic FS in an autonomous phaser is shown in Fig. 3. Unlike the regular FS which can be either static or periodically pulsing, the chaotic FS is distinguished for fast, irregular, and mutually non-synchronized pulsations of the amplitudes of its numerous components. Moreover, these chaotic pulsations of amplitudes are accompanied by similar irregular and mu-
tually non-synchronized motions of the FS components along the frequency axis (back and forth in narrow frequency windows).

**Figure 3 near here**

Fig. 3: Chaotic FS of the phonon generation at $\theta = 1.8$ K, $P = 4$ mW, $\Delta P = 8.8$ MHz, and $\Delta H = 15$ Oe. The range of the frequency sweeping along the horizontal axis is about 18 kHz.

From Fig. 3, one can see that the spectral width of such a chaotic FS line of the phonon SE is several times greater than the width of the line with a regular FS. At the same temperature $\theta = 1.8$ K, holding both $\Delta P$ and $\Delta H$ constant (namely, $\Delta P = 8.8$ MHz, $\Delta H = 15$ Oe), but reducing the pump power $P$ by a factor of four, a diminishing of the peak intensity of the chaotic FS components was observed, as well as some narrowing of the FS line as a whole (Fig. 4).

**Figure 4 near here**

Fig. 4: The same as in Fig. 3, but at reduced pump power $P = 0.9$ mW.

The further reduction of $P$ results in disruption of the phonon generation at the chaotic SE mode. Just before this mode extinguishing (at $P \approx 0.1$ mW), its structure becomes somewhat ordered as compared with cases shown at Fig. 3 and Fig. 4.

### 5.2 Superslow transient processes at fine structure chaotization

The disruption of the phonon generation at a chaotic SE mode may happen, as it was observed in our experiments, at fixed $P$ as well, but when varying $\Delta P$ or $\Delta H$. In so doing, contrary to the case of reducing $P$, the chaotization of the nearest neighboring unsplitted CS mode was often observed.

It is essential that the described phenomena of chaotization of FS lines are very selective with regard to detunings of the phaser on the pump frequency and the static magnetic field. Chaotization take place only in the rather narrow windows of $\Delta P$ and $\Delta H$. Outside of these windows, the strongly marked chaotization of the SE lines was not achieved, but some intermediate states of the FS were observed outside such windows as well. For example, a FS line may contain only two or three robust components, which pulse smoothly and move asynchronously along the frequency axis (within an interval of about 20 kHz). A kind of such an intermediate FS state (emerged in the same ruby phaser, but at $\Delta H \approx 30$ Oe) was already exemplified by us in [21, 22].

The described above internally caused chaotic pulsations of the FS components in our autonomous phaser are fast for most fixed sets of the control
parameters used in our experiments. Typical times of the FS intramode motions are less than 1 s. On the other hand, the externally excited transitions from one chaotic state to another (initiated by a step-type changing of at least one of the control parameters) proceed even more slowly than in the case of transitions between various states of a regular FS. Typical values of $\tau_{\text{tran}}$ for transitions between states of a chaotic FS (e.g., between states shown at Fig. 3 and Fig. 4) amount to $\tau_{\text{tran}} \gtrsim 10^3$ s. No external forcing of the system takes place here, so we obviously deal with self-organized bottleneck in transients between selective chaotic states in phaser medium.

Giant times of transient processes are comparable with the characteristic times of superslow motions observed earlier in the non-autonomous (periodically-modulated) ruby phaser [21, 22]. However, the similarity between superslow processes in non-autonomous (studied in [21, 22]) and autonomous (studied in the present paper) phasers at this point comes to end. For the non-autonomous phaser [21, 22], the emergence of superslow motions is a characteristic feature of the whole power spectrum (the typical width of the phonon SE power spectrum amounts to several MHz). In this case, the “firing” of microwave phonon CS modes on one side of the spectrum is accompanied by the “extinguishing” of approximately the same number of CS modes on its opposite side. This phenomenon of alternation of microwave phonon CE modes was observed in [21, 22] under deep modulation of phaser pump at frequencies of nonlinear resonance (about 9.8 Hz in the investigated system) and its even harmonics.

Of the autonomous phaser, typical are the scenarios, when the majority of microwave phonon CS modes are almost stationary, and their frequencies are very close to frequencies of normal modes of the AFPR. Only the small quantity of the CS modes (usually no more than two), as it was described above, are splitted, shifted in frequency and even chaotized at certain combinations of the control parameters. The definite modification of such a combination of the control parameters is needed in order to translocate this selective phonon mode “fever” to the nearest neighboring CS modes. But times of the mode structure reconfiguration are much more greater than times of the individual AC’s relaxaton time. Generally speaking, there are two unusual and interrelated phenomena observed in our autonomous system: coexistence of incongruous states (neighboring of dominant normal and selectively chaotized CS modes) and the self-organized bottleneck in transients between these states in phaser medium.
6 A Possible Direction for Numerical Modeling of Self-Organized Bottleneck and Coexistence of Incongruous States in Phaser and Other Excitable Systems

6.1 Macro- and microscopic approaches to simulating nonlinear phenomena in systems of the phaser type

A theoretical model of the observed highly nonlinear phenomena requires a separate studies which must include spatio-temporal microscopic description of the local interactions between individual ACs. These interactions were considered by us earlier [12] only at the level of nonequilibrium thermodynamics, namely, by using spin-temperature approach [48, 49], which by definition neglects individuality of the ACs.

Some preliminary numerical results towards this direction have been obtained in [35–37, 50]. Below, the spatial phenomena and the transient processes, which were observed in computer experiments on excitable systems [35–37, 50], as well as their relation to the described above results of real experiments (including self-organized bottleneck and coexistence of incongruous states in the autonomous phaser), will be discussed. But first, it is necessary to stop at some conceptual moments dealing with phaser models.

Slow transient processes at the phaser amplification of hypersound (below the threshold of the phaser self-excitation) were observed in 1970–1980 [12, 28, 29]. The characteristic time of those processes did not exceed, as it was said above, the relaxation time of $^{27}$Al nuclei which form an additional energy reservoir in the phaser active medium. This reservoir interacts with the electron spin subsystem of the active chromium centers in the crystalline matrix of Al$_2$O$_3$ owing to a thermal contact between the system of $^{27}$Al nuclei and the electron dipole-dipole ($d$–$d$) reservoir made up by locally interacting Cr$^{3+}$ [48, 49].

Such a contact is possible due to the proximity of the nuclear magnetic resonance (NMR) frequencies of $^{27}$Al nuclei to the characteristic frequencies of electronic $d$–$d$ interactions in the system of Cr$^{3+}$ ions in ruby (all these frequencies are of the order of 10 MHz). As a result, the slow evolution of the joint electron-nuclear system of the ruby phaser amplifier can be described adequately [12, 28, 29] in the framework of the nonequilibrium spin thermodynamics concept [48, 49].

It is known that those concepts are based on the hypothesis about the so-called spin temperatures [48, 49], i.e. some macroscopic characteristics of the energy reservoirs of a nonequilibrium quantum system, the saturation of which is typically ensured by an external coherent field with a frequency close to one of the system’s resonances (NMR, ESR, or APR). We emphasize that it is the fact of saturating the system by an external resonant field that enables us to analyze the distribution of spin level populations in the coordinate system which rotates synchronously with the frequency of the applied field. Under rather general
conditions, such a distribution of spin-level populations is exponential, which allows us to speak about such a macroscopic parameter as the spin temperature.

A different is the situation, when various modes of phaser generation emerge, evolve, and reach their dynamically stable states. Really, even provided that the FS is absent, the spin system of a phaser generator becomes saturated not only due to the pump field, but also due to the fields of microwave phonon SE at several CS frequencies (see Fig. 1), because the single-mode phaser generation is unstable \cite{9,12,28,29}. In this case, individual spin temperatures for all the individual spin reservoirs (which generate the corresponding CS microwave modes) have to be considered. The quantity of simultaneously self-saturating microwave CS modes with various eigenfrequencies reaches about 10 – 20 and more \cite{9,12,28,29}. So the thermodynamical approach is here not so clear as in the case of single-frequency saturation studied in \cite{48,49}.

Again, the occurrence of a regular FS (Fig. 2) not only increases the number of macroscopic parameters, which are necessary for description of the active system behavior, but also calls into question the adequacy of such a thermodynamical approach in general. Concerning the chaotic FS of a phaser generator (Fig. 3), the macroscopic thermodynamical model of saturated spin system \cite{48,49}, which yielded the satisfactory results for a phaser amplifier \cite{12,28,29}, becomes obviously unacceptable now.

In order to understand the mechanisms of formation of the chaotic FS (including such issues as the slowing-down of transient processes, the coexistence of regular and chaotic CS modes, and so on) we have to deal with the simulation of inversion states dynamics at the microscopic level.

Carrying out the direct simulation of the evolution of an active multiparticle system on the basis of the Maxwell–Bloch coupled equation \cite{51} becomes inexpedient even at the number of ACs \( N \gtrsim 10^3 \), because too huge computational resources would be required for this purpose. Alternative is the way to apply discrete models of cellular automata (CA) type \cite{52}. Such models use algorithmic schemes with local information processing (K- or Kolmogorov algorithms \cite{53} and allow the corresponding CA-programs to run effectively on ordinary personal computers even if \( N \gtrsim 10^6 \). It is important that K-algorithms used in most CA-models belong to the \( P \)-time class of complexity \cite{54}, which allows \( N \) to be increased by several orders for middle-class computers.

### 6.2 Possibility of the formation of a self-organized bottleneck in active systems of the phaser type

In works \cite{35–37,50}, a series of computer experiments on a three-level CA (TLCA) model of a class-B phaser-like system was carried out. The prototype model (Zykov-Mikhailov model) \cite{55} of excitable system includes mechanisms of global activation (GA), global inhibition (GI) and local activation (LA) of ACs. The LA mechanism is a deterministic analog of the one-channel (1C) diffusion of excitations. The TLCA model proposed in \cite{35–37,50} takes into consideration local inhibition (LI) too. In the framework of the TLCA model \cite{35–37,50}, the combined LA/LI interaction between ACs is a deterministic analog of the
two-channel (2C) diffusion of excitations. It is well known that in dilute paramagnets the main mechanism of local interactions between ACs is namely LA & LI one. More concrete, this is dipole-dipole magnetic interactions between impurity paramagnetic ACs [26, 27]. Mechanisms of LA and LI are equal partners in dilute paramagnets, in contrary to negligible role of LI in chemical reactions of Belousov-Zhabotinskii type.

The results of computer experiments showed [35–37, 50], that a typical scenario of the evolution of an autonomous class-B system with weak diffusion of excitations is the self-organized formation of vortex-like coherent dissipative structures of the rotating spiral wave (RSW) type. In the case of 2C-diffusion, transient times for RSW dissipative structures stabilization reach giant values (of about $10^4 - 10^6$ times more than individual AC’s relaxation time). This self-organized coherent bottleneck differs qualitatively from the well-known incoherent phonon bottleneck, which is absent in dilute paramagnets used as phaser active media.

In contrast to the phonon bottleneck or any other incoherent bottleneck, the coherent bottleneck observed in computer experiments [35–37, 50] is the result of very slow emergence of dissipative spatio-temporal structures. In some aspects, slowing-down phenomena observed in [35–37, 50] are similar to self-induced slowing-down of transient processes discovered computationally in both a system of interacting oscillators [30–32] and a system of reaction-diffusion type [33, 34]. The last system is very similar to the one modeled in [35–37].

It is interesting, that under conditions of strong competition of LA and LI mechanisms of AC-AC interaction (2C-diffusion) the unexpected phenomenon of RSW “reflections” was discovered in three-level CA model of excitable phaser-like system [35, 36]. In essence, each “reflection” of RSW is, as it was observed in [35, 36], the result of nonlinear transformation of the RSW’s core in surface layer. Such the phenomenon of RSW regeneration at the boundaries of an active medium leads to further increasing of transient time in TLCA. A close phenomenon of optical vortex “self-repairing” was observed earlier in real physical experiments [56].

Moreover, computer experiments [35, 36] demonstrated replications of RSWs in surface layer of three-level active system with 2C-diffusion of excitations. In this case, each collision of RSW with boundary generate several new RSWs, which move from boundary into inner area of the system. Self-induced replications of RSWs may lead to emergence of transient chaos, when attractor becomes effectively unattainable by any digital computer with limited resources.

Hence, plausible looks the assumption that phenomenon of a self-organized bottleneck is the cause of slowing-down of the transient processes in a phaser. The additional evidence for this assumption is the result of computer experiment [36, 37] demonstrating coexistence of regular and irregular spatio-temporal structures in TLCA (transient analog of chimera states [57–59]). This phenomenon correlates with the coexistence between the regular and chaotic FS modes of phaser generation described above. Certainly, the possibility for dissipative structures of the RSW type to exist in a real phaser system still demands additional researches, also spiral structures have already be observed in dissipa-
tive nonlinear optical systems [2, 3, 60–62].

A particular emphasis should be made that, as it was already pointed out in [30–32], the self-organized bottleneck has the origin qualitatively different from that of well-known phenomenon of self-organized criticality (SOC) [63, 64]. The SOC phenomenon arises only in non-autonomous dissipative systems (as a nontrivial response to an external destabilizing force), whereas the self-organized bottleneck [30–37, 50] is a result of internal processes running in the autonomous dissipative systems.

In this aspect, the processes of self-organization observed experimentally in non-autonomous (our previous works [14, 15, 21, 22]) and autonomous (this work) phasers may also be examined from qualitatively different points of view. Really, slow self-organized motions in the spin-phonon system of a non-autonomous phaser [14, 15, 21, 22] arise just under the influence of an external resonant destabilizing force and disappear after switching the latter off. A different picture was observed in this work for autonomous phasers, because here the slowing-down of the transient processes is self-induced and does not require any external perturbations. On the other hand, in the case of a non-autonomous system similar to that described in works [14, 15, 21, 22], the slow motions proceed as long as the corresponding destabilizing force is active, while the transient process in an autonomous system has, of course, a finite (though, may be, very big) characteristic time. As we have shown in this work, the asymptotic state of an autonomous phaser need not necessarily be regular, because the FS may possess a very complicated, turbulent-like structure.

7 Conclusions

The structure and evolution of power spectra in a ruby-based (Cr$^{3+}$ : Al$_2$O$_3$) autonomous microwave phonon laser (phaser) have been studied experimentally at liquid helium temperatures.

The self-organized bottleneck and coexistence of incongruous states (regular and irregular phonon modes) were revealed during investigation of FS of microwave phonon power spectra at simultaneous detunings of both the pump frequency and the magnetic field.

Some possible mechanisms of a selective emergence of the irregular FS in the microwave phonon power spectra, the coexistence of regular and irregular phaser spatial subsystems and the nature of superslow transient processes (self-organized slowing-down) in an autonomous phaser have been discussed.

The analogous phenomena of self-organized slowing-down have been revealed recently by S. D. Makovetskiy in computer experiments with a discrete autonomous excitable systems (DAES) [35, 36], which can be regarded as simplified models of phaser media.

One of the typical scenarios of DAES evolution is the emergence and very slow (Zeno-like) development of complicated coherent autostructures (e. g., rotating spiral waves) which may spatially coexist with irregular ones [37]. The numerically revealed phenomenon of the bottlenecked evolution of DAES may
have the direct relation to the slowing-down of transient processes that were observed experimentally in this work for an autonomous phaser.

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FIGURE CAPTIONS

to the paper of D. N. Makovetskii

“Self-Organized Bottleneck and Coexistence of Incongruous States in a Microwave Phonon Laser (Phaser)”

(figures see as separate PNG- and JPG-files)

Fig. 1: Microwave power spectrum of longitudinal phonon SE at $\Delta P = \Delta H = 0$ in the ruby phaser. The frequency interval between neighboring CS modes of phonon SE amounts to 310 kHz. The cryostat temperature is $\theta = 1.7$ K.

Fig. 2: Emergence of a regular FS in the power spectra of an autonomous phaser at $P = 4$ mW, $\Delta P = 8.8$ MHz, $\theta = 1.8$ K, and for various $\Delta H$. Left oscillogram: $\Delta H = 0$ (the FS is absent). Middle oscillogram: $\Delta H = 1.5$ Oe (the two-component FS). Right oscillogram: $\Delta H = 3.5$ Oe (the three-component FS). The range of the frequency sweeping along the horizontal axis is about 6 kHz for each oscillogram.

Fig. 3: Chaotic FS of the phonon generation at $\theta = 1.8$ K, $P = 4$ mW, $\Delta P = 8.8$ MHz, and $\Delta H = 15$ Oe. The range of the frequency sweeping along the horizontal axis is about 18 kHz.

Fig. 4: The same as in Fig. 3, but at reduced pump power $P = 0.9$ mW.
This figure "Figure1.png" is available in "png" format from:

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