The multi-stream flows and the dynamics of the cosmic web

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Abstract. A new numerical technique to identify the cosmic web is proposed. It is based on locating multi-stream flows, i.e. the places where the velocity field is multi-valued. The method is local in Eulerian space, simple and computationally efficient. This technique uses the velocities of particles and thus takes into account the dynamical information. This is in contrast with the majority of standard methods that use the coordinates of particles only. Two quantities are computed in every mesh cell: the mean and variance of the velocity field. Ideally in the cells where the velocity is single-valued the variance must be equal to zero exactly, therefore the cells with non-zero variance are identified as multi-stream flows. The technique has been tested in the Zel’dovich approximation and in the N-body simulation of the ΛCDM model. The effect of numerical noise is discussed. The web identified by the new method has been compared with the web identified by the standard technique using only the particle coordinates. The comparison has shown overall similarity of two webs as expected, however they by no means are identical. For example, the isocontours of the corresponding fields have significantly different shapes and some density peaks of similar heights exhibit significant differences in the velocity variance and vice versa. This suggests that the density and velocity variance have a significant degree of independence. The shape of the two-dimensional pdf of density and velocity variance confirms this proposition. Thus, we conclude that the dynamical information probed by this technique introduces an additional dimension into analysis of the web.

Keywords: cosmological simulations, cosmic flows, cosmic web

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1 Introduction

The morphology of the cosmic web is an active field of research in modern cosmology. A wide variety of methods have been suggested, see e.g. [1, 2, 7, 8, 18, 21, 28, 30, 36] just to mention a few. However, the majority of methods share one common feature: they study the cosmic web as a property of the density field derived either from the distribution of particles in space in cosmological N-body simulations or from galaxies in the redshift space. Alternatively or may be better to say complementary the web can be viewed as a giant multi-stream flow where the velocity is a multi-valued function. Therefore it can be statistically described by the mean velocity and its dispersion. The statistical relation of the velocity dispersion with the density was studied in small N-body simulations of the ΛCDM model in [11] and in N-body simulations with scale-free initial conditions in [12]. The power spectrum of the velocity dispersion and its relation with the multi-stream flows has been recently studied both analytically and in a group of large N-body simulations of the ΛCDM model in [32]. Our purpose is different from both studies. We describe a simple technique which is capable to identify multi-stream flows. In many respects it is independent of the previous studies. First, the power spectrum carries no information about the geometry of the structure while identifying the structure is the first necessary step for the geometrical studies. It can be also formulated in different terms: the power spectrum deals exclusively with the amplitudes of the Fourier transform of the field while its geometrical information is mainly stored in the
phases of the Fourier transform. Thus, identifying objects is an important step in probing the phase information via geometrical analysis of the fields. Second, the analysis of the geometry poses very different challenges than computing one-point statistics or power spectrum. The statistical information must be split into pieces corresponding to the objects themselves in the process of their identification. Only after it is done it can be cumulated into statistics corresponding to the object classes. The second part cannot be accurate if the classification of individual objects contains a systematic error and this is the challenge.

We begin with a short overview of the origin and evolution of the concept of multi-stream flows in the context of the formation of the large-scale structure in the universe. The notion of three-stream flows was introduced by Zel’dovich [42], in his paper on pancakes. He briefly discussed the formation of three-stream flows and illustrated it by the evolution of Eulerian coordinate as a function of Lagrangian coordinate with time. However, he himself used it mainly as a useful auxiliary theoretical tool helping to describe the nonlinear structures known at present as ”Zel’dovich’s pancakes”. Before 1980 he himself seemed not to think that the multi-stream flows as a physical phenomenon might have relevance to the formation of the structures in the universe. Similar views were common in the west as well. For instance, multi-stream flows were barely mentioned in Peebles’ book [29] published in 1980 and it was only in the context of critical remarks on the pancake model.

Doroshkevich et al. [16] explored the evolution of the multi-stream flows in the one-dimensional N-body simulation a little beyond the three-stream flow stage up to seven streams. This simulation has shown that the multi-stream flow region remains relatively thin in the comoving coordinates in contrast with a simple extrapolation of the Zel’dovich approximation (ZA) where it grows unlimitedly. The quantitative comparison of the thicknesses of the three-stream flow in ZA and in the one-dimensional numerical simulation was presented in the review [37]. The geometry of generic caustics in ZA in two dimensions was discussed in [3], the authors also provided a table of generic singularities occurring in the potential flows in three-dimensional space. A detailed study of the phase space in self-similar gravitational collapse in one, two and three dimensions was presented in [17]. A high resolution two-dimensional distribution of particles obtained in ZA that clearly showed the structure of multi-stream flows was demonstrated in [9], and in the high-resolution two-dimensional N-body simulations in [26].

The multi-stream flows were directly addressed in the model based on the adhesion approximation (AA) suggested in [19] (see also [5, 38]). The AA model was designed to control the runaway growth of the thickness of the pancakes in ZA by introducing artificial viscosity term in the Euler equation. This term has no effect on the motion in voids leaving it as it was in ZA but does not allow the formation of multi-stream flows by annihilating the transverse to pancakes or filaments velocities. Thus, the high density walls and filaments are formed in AA instead of multi-stream flows in ZA. The density profile controlled by the adopted value of the viscosity coefficient becomes smooth and the velocity remains single valued. It is worth stressing that AA modifies the multi-stream flows by transforming them in single-stream flows of a special kind, characterized by high density. An important feature of the model consists in allowing the mass flow within the pancakes and filaments. The model incorporates most of the features of the hierarchical clustering process which is characteristic of the cosmological models dominated by cold dark matter. In addition, the AA model predicted several new features including the continuous flow of mass from walls to filaments to clumps, the multiple merger of clumps, the collapse of some voids, the presence of substructure of hierarchical type in voids, formation of the next generations of the filaments and pancakes. It was quantita-
tively tested against the two-dimensional [23] and three-dimensional [27] N-body simulations and was also used for predicting the structure in the forthcoming SDSS [39, 40]. The AA model has been recently used as a framework for the study of massive galaxy formation [13].

Another significantly simpler modification of ZA was suggested in [10]. Since the ZA model has a serious problem with the runaway growth of the three-stream flow regions the authors proposed to filter out the perturbations with the scales smaller than the scale of nonlinearity i.e. the scale corresponding to the r.m.s density contrast fluctuation being equal to unity. The tests of the model dubbed the truncated Zel’dovich approximation (TZA) against three-dimensional N-body simulations generally confirmed what was expected: the large-scale structure was reproduced quite accurately but the structures on small scales were erased. Two years earlier it had been shown that even replacing the small-scale perturbations with a new statistically independent realization would not change significantly the large-scale structure [24]. The large-scale structure proved to be quite robust. The evolution of the structure with time could be crudely probed by generating a sequence of particle distributions using TZA with the scale of nonlinearity corresponding to each chosen epoch.

A very similar modification of ZA as far as the large-scale dynamics is concerned was suggested by Bond, Kofman and Pogosyan [6] (the BKP model). The only but crucial difference between the BKP and TZA models consists in a different choice of the scale separating the large-scale and small-scale dynamics. The authors of BKP model sedulously insisted that the borderline must be chosen in such a way that the large-scale motion was strictly single-stream flow. The authors also emphasized the coherence of the filtered large-scale density field as well as the strain tensor field on the scales of tens of Mpc between rare high density peaks. The model has been declared to be able to explain the filamentary appearance of the cosmic web by means of the correlation bridges between high rare peaks in the large-scale density field – the idea somewhat similar to the proposition made earlier in [14, 15] and [40]. However, the chosen condition for the scale separating large-scale dynamics from small scale dynamics means that the filaments are the enhancements of density but not multi-stream flows that is contrary to the both the TZA and AA models.

It is probably worth mentioning one more difference between AA on the one hand and TZA and BKP on the other although it is of a rather technical kind. The TZA and BKP models similarly to ZA use the strain tensor as a primary initial field (referred to as the deformation tensor in TZA). The AA model is based on the linear velocity potential defined by the relation \( v = -\nabla \Phi \), which coincides up to a constant factor with the linear gravitational potential in the growing mode. In principle, both the velocity potential and strain fields contain exactly same information since one of them can be easily computed from the other. However, repackaging of the same information may be useful if it helps to identify the most significant variable that determines the structure or serves a particular goal of the model better. Summarizing this short discussion of three theoretical models of the large-scale structure we stress one difference essential for this paper. The multi-stream flows are characteristic for both TZA and AA while the BKP model eludes them in the large-scale dynamics.

The ZA model defines pancakes as the regions between the surfaces on which the density is formally singular i.e. as the multi-stream flows. The three-dimensional N-body simulation by Klypin and Shandarin [22] revealed that the most conspicuous features besides clumps are filaments rather than pancakes. Combining this finding with the results of [3] naturally caused the definition of filaments to become similar to that of pancakes: filaments are the multi-stream flows having very oblong shapes. The similarity is based on dynamics rather than on density contrast, the both pancakes and filaments are nonlinear but unvirialized.
concentrations of mass therefore in the dynamical hierarchy they are in an intermediate position between the density peaks that have not collapsed yet and virialized halos. At the same time another commonly used definition of objects as peaks above certain height in the density field [4] was also broadened to include pancakes and filaments [6]. On the one hand the former is more physical but at the time seemed to be harder to implement especially when the dynamical information about the structure was scarce. On the other the latter seems to be more practical but involves a free parameter, the threshold that determines which peak should be considered an object. This difference between two definitions was probably the reason of the strongest discrepancy between the models declared in [6]: the formation of the objects in BKS is in inverse order than in ZA. It is worth pointing out that despite considerable overlapping between two definitions the objects they describe are not identical.

Despite the inability of current N-body simulations to resolve properly the phase space there is no doubt that the multi-stream flows must be present in the collisionless medium in the nonlinear regime. Therefore even the limited information about multi-stream flows would introduce a new dimension into the analysis of the cosmic web as it does in virialized halos [25]. This must be based on using the velocity field. Velocities bring about dynamical information totally independent of the density and gravitational potential fields in a general case. There are of course special cases like the systems in virial or thermal equilibrium where certain relations of statistical nature between velocities, coordinates of the particles and gravitational potential could be derived.

Recently there have already been made some attempts to incorporate dynamical information into the analysis of cosmic web [18, 21]. Both approaches are based on the analysis of the eigen values of the Hessian of the gravitational potential at the nonlinear stage. Both groups appeal to the analogy with ZA and claim that this statistics provides a dynamical classification of the cosmic web. The only difference between two approaches is in the different choices of the amplitude to be assigned to the eigen values of the Hessian. However, as we have already mentioned neither density nor gravitational potential contains dynamic information in a general case since the both are computed from the particle coordinates only and absolutely independent of their velocities. We have already mentioned a couple of special cases when the gravitational potential and velocities could be related in statistical sense. The growing mode in the linear regime represents another special case. The dynamical evolution in ZA is described by a map

\[ \mathbf{x}(\mathbf{q}, t) = \mathbf{q} - D(t)\nabla \Phi_v(\mathbf{q}), \]  

(1.1)

where \( \Phi_v \) is the velocity potential not the gravitational potential. It is only due to the degeneracy that exists in the growing mode at the linear stage the velocity potential proportional to the gravitational potential \( \Phi_v = \text{constant} \times \Phi_g \). Hence they can be used interchangeably with the proper choice of the constant. For instance, the gradient of the gravitational potential is proportional to the velocity and it can be computed from the gravitational potential in the growing mode in the linear regime. The ZA model is of course an extrapolation and therefore required additional scrutiny before it was accepted as a reasonable approximation for the initial phase of the nonlinear regime (see e.g. [37]). However, neither the gradient of the potential can be used for mapping nor its second derivatives represent the deformation tensor in a general case. If being correct the reasoning presented in [21] and [18] must be valid for arbitrary velocities of the particles since the analysis and thus the conclusions do not depend on the velocities at all. This hardly can be true. Thus, even the suggested statistics
may provide some merits for the study of the density and gravitational potential fields at the nonlinear stages its interpretation as a dynamical characteristics cannot be accepted.

The velocity dispersion was studied in the context of the structure formation in \cite{11, 12}. However, both papers studied only one-point statistics and the scaling properties of the density — kinetic energy relation. Recently the thorough analysis of the generation of velocity dispersion as well as velocity divergence and vorticity by orbit crossing has been reported in \cite{32}. The major goal of the paper was to estimate the power spectra for these fields in cosmological N-body simulations and compare them with analytical predictions. In addition, it has been shown that measuring the velocity divergence and vorticity by means of the Delaunay tessellation technique developed in \cite{41} had considerably better noise properties than the standard interpolation methods (see also \cite{34}). However, the authors also argued that the Delaunay tessellation technique was not adequate for estimating the velocity dispersion tensor. The goal of this paper is quite different from above studies. We introduce a new numerical technique to identify the cosmic web as the multi-stream flows and present the results of the first tests, which are very encouraging. The method is based on a simple statistics derived from the coordinates and velocities of the particles: the mean and variance of velocities in every mesh cell. Here we use a uniform spatial mesh although the homogeneity of the mesh is not vital for the method. We estimate the role of noise caused by numerical effects and compute the two-dimensional pdf of the density and variance of velocity which demonstrates a significant level of independence of these quantities. We also compare the ability of several diagnostics to identify the multi-stream flows in the ZA model, where the multi-stream flows can be ascertained independently. We compare the appearance of the multi-stream flow web with the web derived from the density field as well as with the distribution of particles themselves.

Although there is no agreement between different groups on the exact definition of the filaments and walls/pancakes the most agree that the filaments comprise more mass than any of the rest constituents of the web (i.e. clusters/clumps, walls/pancakes, field/void). For instance, the fraction of mass in filaments and walls in the ΛCDM simulations to be almost 50% of the total mass, considerably more than in clusters \cite{1}. Both filaments and walls are nonlinear but unvirialized objects in contrast to clusters/clumps which are mostly virialized objects. The suggested method allows to quantitatively analyze the dynamics of these objects because they are multi-stream flows. This work is under way and the results will be reported in detail in a separate paper.

The rest of the paper is organized as follows. Section 2 describes the idea of the method, section 3 describes the numerical technique and N-body simulation used in the tests. The noise analysis and comparison with the Zel’dovich approximation is given in section 4. The total volume devoid of multi-stream flows is evaluated in section 5. The appearance of the multi-flow web is compared with the density based web in section 6, and the mean velocity field is shown in section 7. Finally the discussion and summary of the results are given in section 8. In the rest of the paper density is given in the units of the mean dark matter density, velocities in km/s and \(\sigma_v^2\) in \((\text{km/s})^2\).

## 2 The idea of the method

The physical idea of the method can be illustrated by a simple example. Let us consider the formation of a one-dimensional pancake in ZA for a simple sinusoidal perturbation \cite{3, 37, 42}. Using the comoving coordinates one can easily write down explicit expressions for the position, velocity and density as a function of the Lagrangian coordinate \(q\) and the linear
growth factor $D$ that can effectively play the role of time

$$x = q - D\sin q, \quad v \equiv dx/dD = -\sin q, \quad \rho = |1 - D \cos q|^{-1}. \quad (2.1)$$

The solution predicts a shell-crossing singularity in density at $x = 0$ at $D = 1$ and the formation of a three-stream flow afterward. At small $\delta D = D - 1$ it approximately describes the generic phase space structure of the shell crossing regions at their early stages. At later times it evolves into a multi-stream flow with five then seven etc. streams which is beyond the scope of ZA. The density and velocity profiles in the three-stream flow and its vicinity are shown in the top panels in figure 1. The velocity and density are shown for each stream along with the total density in the three-stream flow region. Two bottom panels show the mean velocity, velocity variance $\sigma_v^2(x)$ (solid line) and the density of kinetic energy $\epsilon$ (dashed line) defined bellow. The variance of velocity, $\sigma_v^2(x)$, and the mean kinetic energy $\epsilon(x)$ at every Eulerian point $x$ are defined as follows

$$\sigma_v^2(x) \equiv \frac{\sum_{i=1}^{n_s} \rho_i(x) \Delta v_i^2(x)}{\sum_{i=1}^{n_s} \rho_i(x)}, \quad \epsilon(x) \equiv \frac{1}{2} \frac{\sum_{i=1}^{n_s} \rho_i^2(x) \Delta v_i^2(x)}{\sum_{i=1}^{n_s} \rho_i(x)}, \quad (2.2)$$
where $\rho_i(x)$ and $v_i(x)$ are the density and velocity in each stream and $\Delta v_i \equiv v_i(x) - \bar{v}(x)$, $n_s$ is the number of streams. The mean velocity is defined as

$$\bar{v}(x) \equiv \frac{\sum_{i=1}^{n_s} \rho_i(x)v_i(x)}{\sum_{i=1}^{n_s} \rho_i(x)}.$$  \hspace{1cm} (2.3)

The mean velocity represents the average velocity in the mesh sites where more than one particle make contribution to density and velocity, therefore we will also refer to this velocity as a bulk velocity: $v_b \equiv \bar{v} \equiv \langle v \rangle$. Figure 1 shows all these quantities at the nonlinear stage when the shell crossing has already happened and the three-stream flow has formed. Passing by we note that the mean velocity of the three-stream flow does not coincide with the velocity in one of the streams as may appear in the figure, but they are quite similar in this particular example.

Comparing the shapes of $\sigma^2_v$ and $\epsilon$ curves with the density and velocity curves one may notice remarkable differences. The most important, however not surprising feature of the $\sigma^2_v$ and $\epsilon$ curves is that they are identical zeros beyond the multi-stream flow regions. Less obvious and more surprising feature is that their shapes are so similar to each other and practically inverse to the shape of the total density curve in the shell crossing region. The both features suggest that the $\sigma^2_v$ and $\epsilon$ functions could be complimentary characteristics to the density in the studies of the dark matter cosmic web. The fact that they are equal exactly to zero beyond the multi-stream flow regions may mean that they can be good tracers of multi-stream flows.

The discussed example is too oversimplified since it assumes continuous medium and very high spatial resolution. In reality the noise caused by discreteness undoubtedly violates this idealization. The top right panel of figure 1 provides an illustration of the discreteness effects. The red points show the particles in the phase space that initially were distributed uniformly on the regular grid. The blue points show their position in Euler space and the blue dashed lines mark the grid in Eulerian space. If the bulk velocity and $\sigma^2_v$ are estimated from the particle positions and velocities directly then the shot noise can be rather high. For instance, the cell with two particles would have both $\bar{v}$ and $\sigma^2_v$ too low compared to the values predicted by the idealized model shown in the bottom panels. Considering the cell on the right from the origin with a single particle one finds that $\bar{v}$ is too high while $\sigma^2_v$ is zero in the three-stream flow. Using the CIC method one can expect to reduce the shot noise but if the size of the particles becomes too large some of the multi-stream flow may be erased. It is also easy to envisage that increasing the spacial resolution by reducing the size of the grid cells one can erase $\sigma^2_v$ entirely when the grid cells become too small. The optimization of the grid cell and particle sizes undoubtedly depends on the environment: in very dense clumps it must be completely different from voids. Here we restrict the analysis by considering the simplest possible approach setting the both grid cell and particle sizes to the mesh size. The refinement of the technique is left to the future work.

3 The numerical technique and N-body simulation

3.1 Numerical technique

Since $\sigma^2_v$ and $\epsilon$ are so similar we shall limit the discussion to $\sigma^2_v$ only. The velocity variance depends on density less than $\epsilon$ and therefore it carries more independent information. The physical dimensions of $\sigma^2_v$ suggest that it can be directly related to the gravitational potential. However a more thorough analysis may reveal some additional attractive features of $\epsilon$. We use
the standard CIC (cloud-in-cell) method for the evaluation of $\bar{v}$ and then $\sigma_v^2$. The particles are modeled as constant density cubes of size $l = L/N_p$ where $L = 512$ $h^{-1}$ Mpc is the size of the simulation box and $N_p = 512$ is the number of particles in one dimension. Both $\bar{v}$ and $\sigma_v^2$ are evaluated on a uniform cubic mesh with the cells of the same size as that of particles except the results of appendix A, where the effects of the size of particles are studied. However, it is worth mentioning that the uniformity of the mesh is not a requirement of the method. In general each particle contributes a volume weighted fraction of its velocity to eight neighboring mesh sites. Each velocity fraction equals the fraction of the volume of the overlap of the particle cloud with the mesh cell. It also can be viewed as a fraction of its linear momentum since all the particles have the same masses. The bulk velocity assigned to the mesh site is the sum of the contributions from all the particles overlapping with it divided by the total mass in the cell. The variance $\sigma_v^2$ is evaluated in a similar manner. It is worth stressing that we conduct the further analysis without additional filtering of the density field which is also computed using the CIC scheme.

3.2 The cosmological model and simulation parameters

We applied this method to the pure dark matter N-body simulation in the $512$ $h^{-1}$ Mpc cubic box using PM (particle mesh) code [20, 31]. The number of particles was $512^3$ and the mesh in the gravitational force solver was $1024^3$. The parameters of the $\Lambda$CDM cosmological model were as follows: $h = H_0/(100 \text{ km/s} \cdot \text{Mpc}) = 0.72$, $\Omega_{\text{tot}} = 0.25$, $\Omega_b = 0.043$, $n = 0.97$, $\sigma_8 = 0.8$, the initial redshift $z_{\text{in}} = 200$. The choice of the parameters is related to the main purpose of the study: to test this new technique and illustrate its performance by applying it to a realistic cosmological model.

4 Testing the method

We compare one point statistics of $\sigma_v^2$ field with that of particle velocities ($v_p^2$) and bulk velocities ($\bar{v}^2$) in three cases: initial state of the N-body simulation, the beginning of the non-linear stage in the ZA model, and the final stage in the N-body simulation. We also show the joint one point statistics $f(\sigma_v^2, \rho) = n(\sigma_v^2, \rho)/512^3$. Here $n(\sigma_v^2, \rho)$ is the number of mesh sites per bin of constant size in $\log_{10}(\rho) - \log_{10}(\sigma_v^2)$ space for three cases mentioned above. This statistics is similar to the scatter plot $\log_{10}(\rho) - \log_{10}(\sigma_v^2)$, however it uses all $512^3$ data points which is not feasible to show in the scatter plot. We also compute $f(\sigma_v^2, \bar{v}^2)$ in a similar manner. Then we test the technique in ZA with similar initial conditions and finally compute various statistics for the fields obtained in the N-body simulation.

4.1 Initial linear state

We begin with computing the bulk velocity field $\bar{v}^2$ and $\sigma_v^2$ at the initial state when all the fields are in the linear regime. At the initial amplitude we do not expect to see any multi-stream flows and therefore $\sigma_v^2$ field must be identically equal to zero. However even at the very beginning the small displacements of particles from unperturbed positions on the regular grid result in small overlapping of particles with each other. It is also easy to imagine that more than one particle may overlap with a mesh cell. This causes the generation of small but nonetheless nonzero $\sigma_v^2$. The left panel of figure 2 shows three functions $f(v_p^2) = n(v_p^2)/512^3$ (in blue), $f(\bar{v}^2)$ (in green) and $f(\sigma_v^2)$ (in red), which are the fractions of particles or sites per equally sized logarithmic bins. In the ideal situation one expects that $f(v_p^2) = f(\bar{v}^2)$ and $f(\sigma_v^2) \equiv 0$ as it was described in section 2. The first condition is satisfied with high accuracy.
Figure 2. The fractions of mesh cells per bin: \( f(\sigma_v^2) \) vs \( \sigma_v^2 \) (red), \( f(\bar{v}^2) \) vs \( \bar{v}^2 \) (green), and the fraction of particles \( f(v_p^2) \) vs \( v_p^2 \) (blue). The red peaks at the left boundary show the fractions of the sites with \( \sigma_v^2 < 10^{-3} \) \((\text{km/s})^2\). The vertical dashed line in the middle panel marks the threshold \( \sigma_{vc}^2 = 3080 \) \((\text{km/s})^2\).

but the second is obviously not. However, typically \( \sigma_v^2 \) is about three orders of magnitude smaller than \( v_p^2 \) or \( \bar{v}^2 \) therefore we conclude that this test has been passed without a serious problem. In the multi-stream flows the \( \sigma_v^2 \) is expected to be smaller than \( v_p^2 \) but not by orders of magnitude (see figure 1 for a simple illustration).

An additional question may occur. Does the generation of noise in \( \sigma_v^2 \) depend on the density or bulk velocity of the cell? Naively one may think that \( \sigma_v^2 \) is generated in the regions where particles are crowded and much less in the underdense regions with \( \rho < 1 \) where the particles are running away from each other. The answer to this question happens to be not that simple as the top right hand side panel in figure 3 shows. The figure displays the two-dimensional histogram \( f(\sigma_v^2, \rho) \) for the initial state. The maximum of the function is at \( \rho \approx 1, \log_{10}(\sigma_v^2) \approx 5.5 \). The peak is approximately of a triangle shape, extending to somewhat higher and lower values of \( \rho \). The third direction is toward the lower values of \( \log_{10}(\sigma_v^2) \). This relatively complicated shape of \( f(\sigma_v^2, \rho) \) indicates that the kinematics of particles is not simply contraction for \( \rho > 1 \) and expansion for \( \rho < 1 \) even at this early stage. The overall shape of the histogram shows no strong correlation between \( \sigma_v^2 \) and \( \rho \). Although related to the velocity gradients the true nature of this effect is discreteness of the model and therefore it can be lessened by minimizing discreteness.

The left panel in the top row of figure 3 shows the fraction \( f(\sigma_v^2, \bar{v}^2) \) that illustrates the relation of the velocity variance with the bulk velocity at every mesh site. The peak of the distribution has an elongated shape hinting at a modest correlation between two quantities. The dashed line is \( \bar{v}^2 = \sigma_v^2 \) that separates 'cold' flows above the line where \( \bar{v}^2 > \sigma_v^2 \) from 'hot' flows where \( \bar{v}^2 < \sigma_v^2 \). At the initial stage the fraction of sites with 'hot' flows is negligible as it must be.

4.2 The Zel’dovich approximation

The Zel’dovich approximation represents a natural framework for testing the proposed method of identifying the multi-stream flows using \( \sigma_v^2 \) field. Regardless of its accuracy the Zel’dovich approximation shares an important common feature with N-body simulations which is the formation of multi-stream flows at the non-linear stage. The advantage of the Zel’dovich approximation is due to relative easiness of the exact analytic prediction of multi-stream flows.
Figure 3. Left column: The fractions of mesh sites per bin: \( f(\sigma^2_v, \bar{v}^2) \). Right column: The fractions of sites per bin \( f(\sigma^2_v, \rho) \). The vertical white lines mark the threshold \( \sigma^2_{vc} = 3080 \text{ (km/s)}^2 \) and the horizontal line marks the threshold \( \rho_c = 2.5 \). They optimize the selection of multi-stream flows as excursion sets in \( \sigma^2_v \) or \( \rho \) fields respectively. Diagonal dashed lines are \( \bar{v} = \sigma_v \).

The Zel’dovich approximation in three-dimensional space is given by eq. (1.1). Consider specific volume which is the inverse of density. It is given by the Jacobian

\[
\frac{V(q, t)}{\langle V \rangle} = J = \det \left( \frac{\partial x_i}{\partial q_j} \right).
\]  

The Jacobian can be expressed in terms of the invariants \( (I_1, I_2, I_3) \) or the eigen values
(\lambda_1 > \lambda_2 > \lambda_3) of the deformation tensor \( d_{ij} = -\partial s_i/\partial q_j \)

\[
J(q, t) = 1 - D(t)I_1(q) + D^2(t)I_2(q) - D^3(t)I_3(q) = [1 - D(t)\lambda_1(q)][1 - D(t)\lambda_2(q)][1 - D(t)\lambda_3(q)],
\]

where

\[
I_1 = d_{11} + d_{22} + d_{33}, \quad I_2 = M_{11} + M_{22} + M_{33}, \quad I_3 = \begin{vmatrix}
    d_{11} & d_{12} & d_{13} \\
    d_{12} & d_{22} & d_{23} \\
    d_{13} & d_{23} & d_{33}
\end{vmatrix},
\]

and \( M_{ij} \) are the minors of the corresponding elements of the determinant \( I_3 \). The condition \( 1 - D(t_0)\lambda_1(q) < 0 \) determines the progenitor of the multi-stream flows in Lagrangian space. For the beginning of the non-linear stage studied here it is sufficient to use a little simpler condition \( J(q, t_0) < 0 \), see for instance [35]. The following mapping of the progenitor particles to Eulerian space defines the multi-stream flows in Eulerian space (eq. (1.1)). Thus, the prediction of the multi-stream flows via any indicator including the density or velocity variance threshold can be directly compared with the exact prediction described above. The accuracy of this comparison is entirely determined by the resolution of the numerical realization of the necessary fields and the mapping itself since the dynamics is exact. It is worth stressing that the progenitor determines exactly only the shape (i.e. geometry and topology) of the multi-stream flows but not the density in the multi-stream flows since there are other streams sharing the same volume with the progenitor in Eulerian space. Ideally this comparison should be made by computing the surface encompassing the progenitor particles and comparing it with the surface encompassing the multi-stream flows corresponding to a chosen diagnostic. However, this is not a simple task, therefore here we use a simpler method which consists of the following steps.

The necessary initial random fields were generated on a \( 512^3 \) grid in a \( 512 h^{-1} \) Mpc cubic box with the power spectrum corresponding to the \( \Lambda \)CDM model with the parameters given in section 3.2. The fields were smoothed with a gaussian filter with the comoving radius \( r_g = 1.35 h^{-1} \) Mpc. This choice corresponds to \( \sigma_\delta \approx 1 \) at \( z = 0 \) according to the linear theory. However, after mapping according to eq. (1.1) actually measured \( \sigma_\delta \approx 1.6 \) and \( \rho_{\text{max}} \approx 207 \bar{\rho} \) due to nonlinearity of ZA. The choice of the parameters has a goal to make the ZA model as close as possible to the N-body simulation.

As the first step the progenitor is identified in Lagrangian space as a set of particles (i.e. grid sites) satisfying the above criterion, \( J(q, t_0) < 0 \). Then these particles were mapped to Eulerian space where they form inhomogeneous clouds that can be seen in red in every panel of figure 4 and 5. Figure 4 also shows the isocontours of various fields described bellow. The contours are semi-transparent, therefore the particles screened by the surfaces look darker then in the openings at the slab faces or when they are outside of the regions enclosed by contour surfaces. The progenitor particles are used for generating the density field \( \rho_p(x) \) via the CIC method. The density of the progenitor has no much sense by itself because other streams are present in the same volume, but it can be used for identifying the mask, that is the field

\[
m(x) = \theta(\rho_p(x) - \rho_{pc}),
\]

where \( \rho_{pc} \) is a chosen critical value and \( \theta(x) \) is the Heaviside step function. The maximum of the progenitor density reached \( 110 \bar{\rho} \). The study of the progenitor density field with Mayavi2, which is a general purpose, cross-platform tool for 3-D scientific data visualization [33], revealed that the parts of the progenitor with lowest density had striped appearance in
Figure 4. Each panel shows the progenitor particles mapped to Eulerian space (red dots) and isocontours of six different fields in Eulerian space. The slab is $64 \times 64 \times 20 \, h^{-1} \, \text{Mpc}$. From top left to right to bottom the fields are as follows: top left panel ($P_{11}$) — the progenitor mask in E-space, top right panel ($P_{12}$) — $\sigma^2_v$, (P21) — $\rho$, (P22) — $\text{div}(\langle v \rangle)$, (P31) — $\text{shear}(\langle v \rangle)_{yy}$, and (P32) — $|\text{curl}(\langle v \rangle)|$.

The last five fields are placed in the descending order goodness of identifying the multi-stream flows quantified by the correlation coefficient between the masks of the field and that of the progenitor.

Eulerian space as can be clearly seen in the middle row panels (2,1) and (2,2) in figure 4. This is due to insufficient density of particles or, in other words, mass resolution for the given scale of smoothing. The regions with $\lambda_1 > 1/D(t_0)$ but with both $\lambda_2 < 0$ and $\lambda_3 < 0$ become
compressed only along one direction while expanding along two orthogonal directions. As a consequence they may form multi-stream flows with quite low densities that appear striped revealing the Lagrangian positions of particles on regular grid. Fortunately this impediment is easy to alleviate. Since the progenitor density is used only for identification of the mask of the multi-stream flows, one can artificially double the number of particles in the progenitor. This is readily achieved by adding particles at \((i+1/2, j+1/2, k+1/2)\) positions on the grid. The displacement vectors for these additional particles needed for mapping are obtained by trilinear interpolation from the grid points. This simple shortcut mainly solved the problem as illustrated by panel (1,1) in figure 4 that shows the contour surface of the progenitor along with the progenitor particles. The visual inspection of these and other regions of the simulation cube with Mayavi2 showed that the choice of the threshold at a fiducial value of \(\rho_{pc} \approx \langle \rho_p \rangle\) results in a good correspondence between the distribution of the progenitor particles and the progenitor mask as illustrated by the top left panel \((P_{11})\) in figure 4.

After generating the progenitor mask in Eulerian space six different fields were tested as potential multi-stream flow identifiers. Five of them are illustrated by figure 4: (top right panel \(P_{12}\) — the velocity variance, \(\sigma_v^2\)), \(P_{21}\) — full density (not just the progenitor density), \(\rho\), \(P_{22}\) — divergence of the bulk velocity, \(\nabla \cdot \langle \nu \rangle\), \(P_{31}\) — one of the diagonal shear components of the bulk velocity , and \(P_{32}\) — the magnitude of the curl of the bulk velocity \(|\text{curl}(\nu)|\). The curl of the bulk velocity field may be thought as a particular relevant probe for the multi-stream flows because of Kelvin’s circulation theorem banning generation of the curl everywhere but within the multi-stream flows. As mentioned before the \(\sigma_v^2\) field also exactly equals zero everywhere but within the multi-stream flows. Therefore it is not worse than the curl field in this respect. However, both conclusions are exact only in the ideal case of continuous medium. The discreteness of numerical simulations results in generating numerical noise. Generally the field obtained from another field by numerical differentiation is more susceptible to numerical noise than the parent field. The test shows that it is just the case.

The five fields are placed in the descending order of accuracy of identifying the multi-stream flows quantified by the cross correlation coefficient described below. The sixth studied field was one of the off-diagonal components of the shear tensor. It happened to be the least accurate identifier of the multi-stream flows and therefore is not shown. In the evaluation of the fields themselves all \(512^3\) particles have been used. For each of all tested fields the multi-stream flow mask was generated according to equation \(m_f(i,j,k) = \theta(f(i,j,k) - f_c)\) at a large number of thresholds \(f_c\). At every threshold the cross correlation coefficient with the progenitor mask was evaluated. The cross correlation coefficient of two masks is defined in the standard manner:

\[
C_{12} \equiv \frac{\langle (m_1(i,j,k) - \langle m_1 \rangle)(m_2(i,j,k) - \langle m_2 \rangle) \rangle}{\sigma_{m_1} \sigma_{m_2}}.
\]

Figure 4 displays the contours of the fields at the levels maximizing the cross correlation of the multi-stream flow mask generated by one of the fields with the progenitor mask. The values of the maximum of the cross correlation coefficient are given in the third column of table 1, the volume fractions of the multi-stream flows at the maximum of the cross correlation coefficient are given in the second column. The fourth column provides the values of the cross correlation coefficient when the volume fraction of the multi-stream flows equals the volume fraction of the progenitor mask that is \(\text{VF}_p = 0.073\). Except for the curl there is no significant difference between the maximum of the cross correlation coefficient and its value at \(\text{VF} = \text{VF}_p\). The third digit is provided to show that there is a formal difference between values in the third
| Field               | Volume Fraction (VF) of multi-stream flows at maximum of CCC | Maximum of Cross Correlation Coefficient (CCC) at VF$_p$ = 0.073 |
|---------------------|-------------------------------------------------------------|---------------------------------------------------------------|
| $\sigma^2_v$       | 0.082                                                       | 0.853                                                         | 0.851 |
| Density             | 0.075                                                       | 0.620                                                         | 0.619 |
| Divergence          | 0.097                                                       | 0.473                                                         | 0.465 |
| Shear (diagonal)    | 0.084                                                       | 0.304                                                         | 0.303 |
| Curl (magnitude)    | 0.202                                                       | 0.287                                                         | 0.194 |
| Shear (off diagonal)| 0.048                                                       | 0.168                                                         | 0.162 |

**Table 1.** Performance of various fields as possible indicators of multi-stream flows.

and forth columns rather than emphasizing its statistical significance. It is worth stressing that no other smoothing was applied except the gaussian filtering of the initial displacement field as described before. The visual impression of figure 3 is in a good agreement with the values of the cross correlation coefficients displayed in table 1.

Only two fields $\sigma^2_v$ and $\rho$ generate the masks of the multi-stream flows that correlate with the progenitor mask at the levels greater than 0.5 with the $\sigma^2_v$ field to be the clear winner. Other fields suffer from numerical noise much stronger than density and $\sigma^2_v$. One might probably anticipate this because all of them were obtained by differentiation of the velocity field and numerical differentiation usually boosts noise. The best density and velocity variance masks were determined by the critical values of $\rho_c/\bar{\rho} = 2.5$ and $\sigma^2_v = 3080$ (km/s)$^2$ respectively.

The middle panel of figure 2 shows $f(v^2_p) = n(v^2_p)/512^2$ (in blue), $f(\bar{\nu}^2)$ (in green) and $f(\sigma^2_v)$ (in red). The vertical dashed line marks the threshold $\sigma^2_v = 3080$ (km/s)$^2$ that optimized the prediction of multi-stream flows.

Similarly to the linear case the middle row in figure 3 shows two two-dimensional histograms: fractions of mesh sites with given $\sigma^2_v$ and $\bar{\nu}^2$ on the left and $\sigma^2_v$ and $\rho/\bar{\rho}$ on the right. The vertical white lines in both panels mark $\sigma^2_v = 3080$ (km/s)$^2$ and the horizontal line marks $\rho_c/\bar{\rho} = 2.5$. Two lines in the right hand side panel partitions the $\sigma^2_v - \rho/\bar{\rho}$ plane into four domains. The both indicators predict a mesh site to belong to a multi-stream flow if $\sigma^2_v > \sigma^2_v$ and $\rho_c > \rho_c$ (top right corner) that happens in 4.9% of mesh sites. The both indicators also agree that the mesh site does not belong to a multi-stream flow if $\sigma^2_v < \sigma^2_v$ and $\rho_c < \rho_c$ (bottom left) that happens in 89.2% of sites. However, the indicators make opposite predictions for sites corresponding to the top left and bottom right corners of the diagram. According to the density field additional 2.6% of mesh sites with the parameters corresponding to the top left corner where $\rho_c > \rho_c$ but $\sigma^2_v < \sigma^2_v$ are the members of multi-stream flows while $\sigma^2_v$ field assigns 3.3% of sites from bottom right corner to multi stream flows where $\rho_c > \rho_c$ but $\rho_c < \rho_c$. Thus, the two indicators agree in roughly 5/8 and disagree in 3/8 i.e. nearly 40% of all cases. In order to illustrate the nature of this discrepancy we plot the progenitor particles in Eulerian space together with the contours of the regions corresponding to the top left and to the bottom right corners in two panels of figure 5. The part of the multi-stream flows common to both criterions is excluded. The blue contours in the panel on the right shows the regions ascribed to multi-stream flows by the density criterion only. One can clearly see that many of these regions do not contain red particles, therefore these regions have relatively high densi-
ties ($\rho > 2.5\bar{\rho}$) but have not yet formed multi-stream flows. On the other hand the left hand side panel showing the sites, where $\sigma^2_v > \sigma^2_{vc}$ but $\rho_c < \rho_c$, does not have this problem — this is seen with particular clarity via Mayavi2 allowing to manipulate three-dimensional images.

Thus we conclude that the $\sigma^2_v$ field is considerably better indicator then density field if both are computed on a regular Cartesian grid via the CIC method without additional smoothing of the field. Other tested potential indicators of the multi-stream flows ($\nabla \cdot \mathbf{v}_b$, $\text{curl}(\mathbf{v}_b)$, diagonal, and off-diagonal components of the shear tensor of the bulk velocity) suffer from noise considerably stronger then either density or $\sigma^2_v$ fields.

### 4.3 Final nonlinear state in the N-body simulation

The statistics $f(v^2_p)$, $f(\bar{v}^2)$ and $f(\sigma^2_v)$ are shown in the right hand side panel of figure 2. The statistics of particle velocities $f(v^2_p)$ and bulk velocities $f(\bar{v}^2)$ look very similar bellow the maximum as in other panels. However at the high value end $f(\bar{v}^2)$ drops considerably faster than $f(v^2_p)$ which is in contrast with other panels. The high value end of $f(\sigma^2_v)$ is much closer to $f(\bar{v}^2)$ then in both linear and ZA cases.

As many as 11.76% of cells have $\sigma^2_v = 0$ exactly in excellent agreement with the fraction of completely empty cells with $\rho = 0$ being equal to 11.74%. In addition there are 21.6% of cells with $0 < \sigma^2_v < 1 \times 10^{-9}$. It is shown by the red bar next to the left boundary of the panel. As we will see in figure 6 the cumulative fraction $F(\sigma^2_v < \sigma^2_{v})$ remains practically constant in the range $10^{-9} < \sigma^2_v < 10$ (km/s)$^2$.

The two-dimensional histograms $f(\sigma^2_v, \bar{v}^2)$ and $f(\sigma^2_v, \rho)$ are shown in the bottom panels of figure 3. There is an easially seen correlation between $\sigma^2_v$ and $\rho$ especially at $\rho > 1$. However, the relatively flat peak of the histogram (red) spreads out for about two orders of magnitude along the both axes. The correlation is probably related to the correlation of the gravitational potential depth with density. But the width of the peak suggests that the medium density sites ($1 < \rho < 100$) belong to the systems that are not completely relaxed with $\sigma^2_v$ to be quite independent of $\rho$. 

---

**Figure 5.** The contour in the left panel shows the regions where $\sigma^2_v > \sigma^2_{vc}$ but $\rho < \rho_c$, i.e. included into multi-stream flows by the velocity variance criterion but not by the density criterion. The contour in the right panel shows the regions where $\rho > \rho_c$ but $\sigma^2_v < \sigma^2_{vc}$ i.e. the sites ascribed to multi-stream flows according to the density threshold but not to the velocity variance threshold. The red dots are the progenitor particles mapped to Eulerian space. The contours in the right panel miss the red dots in most of cases.
Figure 6. The cumulative distribution functions (CPF): the probability to find a mesh site with $\sigma^2_v < \sigma^2_{vc}$ (solid) and that with $\rho < \rho_c$ (dashed). Red and blue curves correspond to the Zel’dovich approximation and N-body simulation respectively. The horizontal gives the threshold in (km/s)$^2$ for $\sigma^2_v$ and in dimensionless units for $\rho/\bar{\rho}$. The red dots mark the thresholds at which the multi-stream flows are identified by the density and $\sigma^2_v$ fields: $\rho_c = 2.5$ on the red dashed line and $\sigma^2_v = 3080$ (km/s)$^2$ on the red solid line.

The left hand side panel shows no correlation between bulk velocity $\bar{v}^2$ and variance $\sigma^2_v$. In this respect the ZA and N-body structures are qualitatively similar. The multi-stream flows in both cases are ‘cold’ in the sense that $\sigma^2_v < \bar{v}^2$: this inequality is violated in about 0.5% of sites in the former and in about 0.6% of sites in the latter. Taking into account the resolution of the simulation it is tempting to relate these sites to the halos corresponding to clusters of galaxies. However, it is worth stressing that the comparison was made on scale of $1h^{-1}$ Mpc and therefore cannot be directly related to whole halos in the current work. The question is interesting and is currently under special investigation. The results will be published separately.

5 The total volume in voids

There are 11.74% of the mesh cells completely empty, they obviously have $\sigma^2_v = 0$ exactly as well (except a tiny fraction due to numerical errors). However, additional 21.6% of cells with tiny $\sigma^2_v < 10^{-9}$ (km/s)$^2$ cannot not be considered belonging to multi-stream flow regions by any stretch of the imagination. Figure 6 shows two pairs of cumulative distribution functions (CPF): one is the probability to find a mesh site with $\sigma^2_v < \sigma^2_{vc}$ (solid) and the other the probability with $\rho < \rho_c$ (dashed). The both CPFs are given in the distributions obtained in the ZA model (red) and in the N-body simulation (blue).

The CPF of $\sigma^2_v$ in the n-body simulation grows from 33.24% at $\sigma^2_v = 1 \times 10^{-9}$ to 33.67% at $\sigma^2_v = 1$ (i.e. by 0.43%! ) and to 35.26% at $\sigma^2_v = 10$ (i.e. by 1.59% from $\sigma^2_v = 1$). Then the rate of growth becomes significantly greater and the CPF($\sigma^2_v$) reaches 46.27% and 60.70% at $\sigma^2_v = 100$ and 316 respectively. The long plateau from $\sigma^2_v = 10^{-9}$ to about 10 (ten orders of magnitude!) shows that in about one third of the volume the velocity is a single valued function, i.e. no multi-stream flow regions (that can be resolved with the resolution of the
Figure 7. The left column shows three contours of the velocity variance field: $\sigma_v^2 = 64605$ (top), $\sigma_v^2 = 35800$ (middle) and $\sigma_v^2 = 17593 \text{(km/s)}^2$ (bottom). The right column shows contours of the density field: $\rho = 12.1\bar{\rho}$ (top), $\rho = 6.9\bar{\rho}$ (middle) and $\rho = 3.9\bar{\rho}$ (bottom). The levels in each row correspond to the same fractions of volume encompassed by the contours: which is 0.01, 0.02 and 0.04 in the top, middle and bottom panels respectively. Both fields are obtained in the N-body simulation.

This number can serve as a low physical limit for the total volume in voids for the resolution of this simulation. For a given mass resolution of N-body simulations one can give a physical definition of voids as the regions where the shell-crossing has not yet occurred. Therefore neither halos nor filaments nor sheets could have formed in these regions. The multi-stream flow regions can apparently appear in such places with the growth of the resolution of the simulation, but they will be on smaller scale.
Figure 8. A part of a thin slice of $1 \, h^{-1}$ Mpc thickness through N-body simulation box. The filled contours show the values of $\sigma_v^2$ as indicated by the color column. Heavy red contours show the CIC density field with $\rho = 1$, the white thin contours inside red contours show $\rho = 3, 10$ and 50. The labels show the distances in $h^{-1}$ Mpc.

6 The multi-stream flows versus the density web

In this section the multi-flow web is compared with the standard representation of the web by the isocontours of the dark matter density. The both fields are generated by the CIC algorithm with particle size same as the mesh size, $1h^{-1}$ Mpc. Appendix A briefly discusses how the reduction of the particle size modify some statistics and appearance of the structure.

Figure 7 shows the contours of the $\sigma_v^2$ (on the left) and $\rho$ (on the right) fields obtained in the N-body simulation at three levels each. The levels are chosen in a such manner that the fractions of volumes encompassed by the contours are same in each row, which are 0.01, 0.02 and 0.04 in the top, middle and bottom rows respectively. At the highest thresholds in the top panels the both fields appear basically as isolated peaks. As the thresholds become lower from the top to bottom panels the both fields turn into connected web. Although the structures in two columns resemble each other they are conspicuously different. The multi-stream flows are considerably less fragmented and filaments are more voluminous than in the density web. The voids look emptier however this does not mean that there are no multi-stream flows there. They are present there but at lower levels of $\sigma_v^2$ than large
filaments emerge. The particles at small peaks emerging at the same threshold as large filaments are considerably 'colder' as the values of $\sigma_v^2$ indicate. The latter is not surprising since small isolated density peaks form less deep potential wells. Unfortunately plotting contours in 3D at lower thresholds becomes less practical because of obstruction of distant structures by close contours. Therefore we turn to 2D plots of thin slices.

Figure 8 shows the density contours superimposed on the field of $\sigma_v^2$ in a thin slice (1 $h^{-1}$Mpc) randomly selected from the simulation cube. The filled contours show the regions of the $\sigma_v^2$ field between the levels marked on the color column. The density contours correspond to the following levels: red to $\rho = 1$ and three white contours correspond to $\rho = 3, 10$ and 50. One can see that although the overall structures are very similar, they by no means are identical. We consider the both observations as good news. The former is the evidence of the robustness of the identification of the filaments and other structures while the latter suggests that dynamical information in the $\sigma_v^2$ field brings an additional dimension to the characteristics of the web. A closer inspection show that the highest peaks in two fields do not have identical shapes. It will be better seen in the following figures where we zoom in two regions: one with the lowest mean density (void region) and the other with the highest density in the slice (clumps and filaments).

Figure 9. The same slice as in figure 8. The filled contours show the regions with $\sigma_v^2$ displayed by the color column. Dots are the particles contributed to the density and $\sigma_v^2$ in this slice.
Figure 10. A zoomed low density region in the top right corner of figure 8 and 9. The filled contours show the values of $\sigma_v^2$ as indicated by the color column. The black and white contour lines show a few density peaks above unity in units of the mean density. The solid, dashed and dotted lines correspond to $\rho = 0.5, 0.25$ and $0.1$ respectively. The heavy white line at about $(130, 105)$ shows one small contour with $\rho = 2$.

However, we first show the superposition of particles contributing to the density and $\sigma_v^2$ in this slice on the $\sigma_v^2$ field in figure 9. The color map is slightly different from the previous figure, which allows to see the structures at low values of $\sigma_v^2$ better. In general, we conclude that the correspondence of two fields is very good as it should be apart from numerical noise.

6.1 The slice through a void

The region from the top right corner of the slice is zoomed in figure 10. The color map is changed again in order to adopt better for the low density environment. The levels of $\sigma_v^2$ are shown by the color column while the density contours are shown by black lines. The density in the most of the region is below the mean except a few peaks shown by black and white heavy line. The impression is similar to that of figure 8: the fields have many similarities but also substantial differences, perhaps even greater differences than in figure 8. In particular, the empty regions seem to be significantly emptier in $\sigma_v^2$ than in $\rho$ in a good agreement with
the discussion in section 5. Although the density contours are pushed to such a low values that one may completely disregard them as being subgrid noise the values of the $\sigma_v^2$ contours are also very low and still there is some similarity between both fields in some parts of the figure. Thus, an optimistic conclusion would be that the combination of both fields may allow to probe low density regions more reliably than any of them alone. On the other hand, a pessimistic conclusion would relate it to the correlation of the $\rho$ and $\sigma_v^2$ fields at small $\sigma_v^2$. The further study is obviously required in order to come to a more certain conclusion.

6.2 The slice through clumps and filaments

Figure 11 shows the relatively high density region in the bottom left corner of figure 8 and 9. The figure contains a few density peaks with $\rho > 10$ (white dashed lines) three of which are higher than $\rho = 30$ and two even higher with $\rho > 60$. There are also a few filaments connecting the clumps. Again the both fields are rather similar but there are notable differences as well. Two highest density peaks in the bottom left corner reach $\rho > 60$ but only one exceeds $\sigma_v^2 > 10^5$. However, the peak of $\sigma_v^2$ extends to the density level as low as the mean density.
On the other hand a little lower density peak with \( \rho < 60 \) coincides with a quite substantial region with \( \sigma_v^2 > 10^5 \). Similar observations can be made with the peaks of lower density. Thus, we conclude that the density and \( \sigma_v^2 \) are two parameters that have a substantial degree of independence and therefore may better characterize the clumps and filaments as any of them alone. In addition, the figure shows that the filaments are the multi-stream flows in agreement with both the TZA and AA models and contrary to the prediction of the BKP model that implies that the filaments have not reached the state of multi-stream flows.

7 The bulk velocity

Finally, we present the bulk velocity field defined by eq. (2.3) in figure 12. We superimpose the velocity on a copy of the slice shown in figure 11 where we keep only three highest levels of the density contours for clarity. These levels show the density peaks only. The three-dimensional velocities are projected on the slice plane and shown by the arrows.

The highest density peak sitting at the largest patch of high \( \sigma_v^2 \) is in the bottom left corner approximately at (17, 39). The overall pattern of the mean velocity field clearly indicates that the whole region shown in the figure is falling on this clump. The mass inside three filaments is streaming toward the clump confirming one of the predictions of the AA model [19]. The second largest clump at (28, 54) is falling on the first clump accompanied by the surrounding filaments. The filaments are highly inhomogeneous and the streams are far from being uniform. The velocity field in the vicinity of the first clump looks roughly circular giving the impression of quasi-spherical (actually quasi-circular in this plane because we see only the cross section) accretion but this is misleading if we look at the flux of mass \( \rho \vec{v} \) shown by arrows in figure 13. It is worth stressing that in order to accommodate the arrow lengths in the figure, their lengths are made proportional to the square root of the magnitude instead of the magnitude. Thus the difference between the fluxes from three adjacent filaments compared to that from three surrounding voids is considerably more sizable than it appears in the figure.

8 Summary

We present a new numerical technique that allows to identify the multi-stream flows, i.e. the regions where the velocity field is multi-valued. According to [42] these regions have reached a clear physical verge to be recognized as strongly nonlinear structures. At the same time most of them remain unvirialized. The multi-stream flows form the web which is in general similar by appearance to the web identified via density field. Nonetheless many features are very different both in low and high density environments as seen in figure 10 and 11. These include the difference in shapes of the contours around peaks and the lack of monotonic relation between the heights of peaks in the \( \sigma_v^2 \) and \( \rho \) fields. These differences is particularly easy to quantify for the fields generated by means of the Zel’dovich approximation. Aside from the question of its accuracy ZA provides a good framework for testing the predictions of various indicators potentially capable to identify the multi-stream flows because the multi-stream flows are easy to predict theoretically. The test of six of such indicators (including \( \sigma_v^2 \), \( \rho \), \( \text{div}(\vec{v}) \), \( \text{curl}(\vec{v}) \), one diagonal and one off-diagonal component of the shear tensor derived from the \( \vec{v} \) field) showed that the \( \sigma_v^2 \) an \( \rho \) fields are considerably less noisy then the remaining four indicators mentioned above. This may be not surprising since we have not use any smoothing of the nonlinear fields and those four indicators were obtained by numerical differentiation of the bulk velocity \( \vec{v} \). Comparing the performance of the \( \sigma_v^2 \) an \( \rho \) fields at the
thresholds that maximized the agreement with the theoretical prediction of the multi-stream flows we found that the both indicators agree on the formation of the multi-stream flows in 4.9% of mesh sites. However, the density field predicts that another 2.6% of all sites also belong to the multi-stream flow web while the $\sigma^2_v$ field predicts that 3.3% of sites should be added to the web. The problem is that these additional 2.6% and 3.3% of sites do not overlap with each other, they are completely different sets of the mesh sites. Thus, two indicators agree in roughly 5/8 and disagree in 3/8 of all sites they predict to form the web of multi-stream flows. The $\sigma^2_v$ is in a considerably better agreement with the theoretical prediction than the density field as can be seen in table 1.

Ideally $\sigma^2_v$ is identically equal to zero everywhere where the velocity is single-valued but the simplest numerical implementation based on the ordinary CIC method generates noise. However, it is worth stressing that the present results by no means indicate any difference between $\sigma^2_v$ and $\rho$ fields in this respect. Other schemes like TSC (triangular shape density cloud) or even more sophisticated methods may be worth trying. It is interesting to note that the study did not find any indication of the correlation between the bulk velocity $\bar{v}^2$ and velocity variance $\sigma^2_v$ as the one-point joint statistics $f(\sigma^2_v, \bar{v}^2)$ shows in the left panels of

**Figure 12.** The mean velocity field $\bar{v}$ projected on the slice plane is superimposed on the $\sigma^2_v$ field shown in figure 11. White contours show peaks with density above 10, 30 and 60 in the units of the mean density with dashed, dotted and heavy solid lines respectively.
Figure 13. The mean flux of mass $\rho \vec{v}$ projected on the slice plane is shown by arrows. In order to accommodate the arrows their lengths are made proportional to the square root of the magnitude.

the middle and bottom rows in figure 3. This diagrams also show that the multi-stream flow web represents mostly 'cold' flows in the sense that the velocity variance $\sigma_v^2$ is smaller than the bulk velocity $\bar{v}^2$ in all but 0.5% in the ZA fields and 0.6% in the N-body simulation. The dashed lines in the figures separate these two sectors. Immediate candidates for 'hot' sites are the interiors of the halos corresponding to clusters of galaxies. However, the study of this interesting question is beyond the scope of the current work.

Although one can see a positive correlation between the $\sigma_v^2$ and density that determines the multi-stream flows in figure 3 (right panels in the middle and bottom rows), the spread of points along both axes ($\rho$ and $\sigma_v^2$) is very large suggesting that these parameters retain a considerable level of independence. Therefore $\sigma_v^2$ may serve as a second parameter characterizing the dynamical environment in the nonlinear density peaks before they reached virial equilibrium as well as for filaments and pancakes. This is useful even in the common part of the web where the predictions of the density and $\sigma_v^2$ fields are in agreement.

The field $\sigma_v^2$ naturally defines the physical condition for voids as the regions devoid of multi-stream flows. The identification of the multi-stream flows of course depends on the resolution (in particular the mass resolution) of the simulation, but this is the fundamental limitation for all fields obtained in modern N-body simulations. The $\sigma_v^2$ field by no means
Figure 14. Cumulative distribution functions: density on the left and $\sigma^2_v$ on the right. Dashed lines correspond to the fields obtained with different sizes of the particles: $d = 1$ (red), 0.5 - (green), 0.25 (blue) and 0.1 (magenta) in the Zel’dovich approximation. Here the sizes of particles are given in the mesh units. The solid lines correspond to the fields obtained with $d = 1$ (red), 0.5 (green), 0.25 (blue) in the N-body simulation.

is worse than commonly used density field in this respect. This condition is physically more clear than one based on an essentially arbitrary density threshold. It is worth stressing that about 22% of the total volume has nonzero density but no multi-stream flows. For the resolution of the ΛCDM simulation used here ($\approx 1 \, h^{-1}\text{Mpc}$) we find the volume fraction in voids is at least about one third of the total volume. This is considerably greater than at the percolation transition evaluated for a similar simulation in [36], therefore the volume devoid of multi-stream flows consists predominantly of one connected region.

The issue of voids has of course an additional important aspect if one takes into account the multiscale nature of the cosmic web. The statement that some region is empty always emphasizes or occasionally implies the small scale limit on objects or structures being dealt with. This could be galaxies or halos or filaments that are being resolved in observations or N-body simulations. A simulation with higher resolution is likely to add additional multi-stream flows in voids as well as in filaments which themselves are the multi-stream flows of a greater scale. Thus, one may find a smaller scale filaments within a larger filaments if they have not completely coalesced with other filaments or halos. A word of caution however should be said. Identifying multi-stream flows is not the same as counting the number of streams. The former is easier than the latter. We have not addressed the problem of counting streams here.

The mean velocity field $\bar{v}_i$ defined by eq. (2.3) shows the mass flow along the filaments toward the clumps in agreement with the prediction of the adhesion approximation [19]. Some filaments seem to move predominantly in the transverse direction as a whole (see figure 12). The mean velocity field seems to be quite smooth without significant convergence in these cases. A somewhat similar observation can be made concerning the clumps. In some of them as in one at (17, 39) in figure 12 and 13 the velocity field seems to converge significantly stronger than in the other similar clump at (28, 54) in the same figures where the velocity field looks notably more steady.

The filaments with lengths over 50 $h^{-1}$ are shown to be the multi-stream flows in a qualitative agreement with both the truncated Zel’dovich approximation [10] and the adhesion approximation [19].
Figure 15. The same slice through the simulation box as in figure 8. The top panels shows the density contours and $\sigma_v^2$ field as colored regions for the particles in CIC twice smaller by linear size than the grid cells. The bottom panel shows the fields with particles four times smaller by linear size than the grid cells.
Summarizing, we would like to stress the importance of the study of dynamics in fil-
aments and pancakes, which are highly nonlinear but unviriailized dynamical systems. The
suggested technique may be a useful tool to serve this purpose.

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A Effect of particle sizes

Here we briefly discuss the effects related to the size of particles in the CIC generator of the
density and $\sigma_v^2$ fields from their positions and velocities. The grid cells are kept the same.

Figure 14 shows the cumulative distribution functions of density and $\sigma_v^2$ fields obtained
for a set of the particle sizes in the Zel’dovich approximation (dashed lines) and N-body
simulation (solid lines). The colors of the lines corresponds to the sizes as follows: red —
d=1, green — 0.5, blue — 0.25 and magenta — 0.1. There is no magenta solid line because
the N-body simulation was not analyzed at $d = 0.1$. At high densities and high values of $\sigma_v^2$
the effect is very small while at the lowest densities and lowest $\sigma_v^2$ it reaches the maximum.
The explanation is quite simple: the particles that overlap with two or more neighboring cells
at larger $d$ end up in a single cell at smaller $d$. In particular this increases the number of cells
with the single particles in voids which is clearly seen as a little bump at $\rho = 1$ at smallest
sizes of the particles. The density CPF is affected very little at $\rho > 0.5$ while the differences
in CPFs of $\sigma_v^2$ grow more steadily with the decrease of $\sigma_v^2$.

The horizontal parts of the curves start practically at $\rho = 0$ and $\sigma_v^2 = 0$ and continue
up to $\rho \approx 0.1$ or $\sigma_v^2 \approx 10(\text{km/s})^2$. The differences in the amplitudes of CPFs between
CPF($\rho$) and CPF($\sigma_v^2$) at low values of the argument is mostly due to the cells with a single
particle where $\rho = 1$ and $\sigma_v^2 = 0$.

Figure 15 shows the density and $\sigma_v^2$ fields in the same slice as shown in figure 8 obtained
for $d = 0.5$ and 0.25 in the top and bottom panels respectively. The colors of the lines showing
the density contours and filled contours showing the $\sigma_v^2$ field are kept the same as in figure 8.
The figure demonstrates that reducing the particle sizes keeps the regions with high values
of $\rho$ and $\sigma_v^2$ practically without changes while the voids look emptier in the $\sigma_v^2$ field but get
more junk in the $\rho$ field. The latter is obviously explained by the cells with one or a few
particles. It is hard to determine the critical value of $\sigma_v^2$ separating the physical signal from
numerical noise from the CPFs or contour plots of the both fields.

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