A ranking-based differential evolution algorithm for hybrid flow shop sustainable scheduling

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A ranking-based differential evolution algorithm for hybrid flow shop sustainable scheduling

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Abstract

With the increasing of environmental pressure, the sustainable production for hybrid flow shops (HFS) has attracted more attention due to its broad industrial applications. In implementing the sustainable production of HFS, the selection of parallel machines for various jobs is a vital step. In light of this, a multi-objective mathematical model for minimization of makespan and energy consumption of HFSP was formulated. The sustainability of parallel machines were evaluated and ranked according to fuzzy TOPSIS method. To solve the multi-objective model of HFSP, an improved differential evolution algorithm was presented to assign the job with the ranked parallel machines, which can narrow the search scope and accelerate the convergence speed. Finally, a case study was presented to evaluate the effectiveness of the proposed ranking-based algorithm and to prove the feasibility of the model. The results showed that the proposed improved algorithm outperforms NSGA-II and PSO in searching for non-dominated solutions, which can effectively solve the sustainable production of HFSP.

Keywords: hybrid flow shop scheduling, sustainable production; improved differential evolution algorithm

Introduction

Hybrid flow shop scheduling problem (HFSP) indicated a combination of the classic flow-shop scheduling problem and the parallel machine scheduling problem, which was characterized by adjusting of jobs processing sequence and allocating the parallel equipment reasonably (Li et al. 2018; Tian et al. 2018; Kong et al. 2020; Xu et al. 2013). With the pressure of environmental problems, sustainable production for HFSP have aroused extensive concerns nowadays. Sustainable production for HFSP can be evaluated with three criterions, i.e., environmental impact, technical performance and
production cost. Most previous research for HFSP reported that the arrangement of parallel machines for various jobs quite affect the sustainability in HFSP.

According to the characteristic of parallel machines, the classic HFSP can be divided into three types, i.e., the HFSP with identical parallel machines (HFSP-IPM), the HFSP with uniform machines (HFSP-UM), and the HFSP with unrelated parallel machines (HFSP-UPM). For HFSP-IPM, a job at a certain stage have the same speed, which referring identical processing time on every machine; For HFSP-UM, the speed of parallel machines is various for each job at a stage, and the processing time is inversely proportional to the speed. HFSP-UPM is the most complex and closed to real industrial manufacturing scheduling problems, which has a wide engineering application (Meng et al. 2019). For HFSP-UPM, the processing time of a job on any parallel machines are independent and the production efficiency quite depends on the matching degree between the job and the machines (Kong et al. 2020).

Hence, to implement sustainable production, reasonable evaluation of sustainability for the parallel machines become an urgent demand for scheduling.

The sustainability of parallel machines can be evaluated from three aspects, i.e., the environmental impact, the technical performance and the production cost. Energy consumption of the parallel machines was supposed to the main source of environmental impact, which has been under extensive research. Drake et al. (Drake et al. 2006) showed that there were significant amounts of energy associated with machine start-up and idling. In light of this, Mouzon (Mouzon et al. 2007) proposed a greedy randomized adaptive search procedure integrated with GA to solve the minimization of total energy consumption of HFSP. The result showed that a significant amount of energy can be saved when non-bottleneck are turned on during a long idle time. Liu (Liu et al. 2019) presented an ultra-low idle state of machines by turning off some auxiliary parts in idle state. To calculate the energy consumption of machine tools with respect to various cutting parameters, the specific energy consumption (SEC) was generally recognized as an essential indicator. Bedsides, subsidiary materials, such as cutting fluid are also have important influence on environmental impact. For production cost of parallel machines, the machining cost, the human labor cost, etc., were generally investigated for optimizing the job scheduling of the workshop (Behnamian et al. 2014). For technical performance, accuracy and reliability in terms of the basic requirements of production have been focused (Dimitrov et al. 2014). In addition, axes configurations, in particular at a high speed spindle rotations and high feedrates, has a significant variation of the dynamic properties for the machines (Brecher et al. 2016). To sum up, a
For the optimization of HFSP, the genetic algorithm (Han et al. 2018; Xu et al. 2017), grey wolf algorithm (Ni et al. 2020), backtracking search algorithm (Lu et al. 2019), ant colony optimization (Kang et al. 2014; Feng et al. 2019), particle swarm optimization (Fang et al. 2020), differential evolution algorithm (Tian et al. 2016; Miao et al. 2015) were developed. It is noted that the selection of machines for various jobs in HFSP was random for those algorithms in the iterations, which result in a lower efficiency. To overcome this problem, local search method with a forward decoding method considering idle time (Wang et al. 2012; Tian et al. 2019), energy-saving capability (Ding et al. 2016), the total weighted tardiness (Ding et al. 2016), etc., were proposed. Nevertheless, the methods mostly neglected the sustainability of unrelated parallel machines, which is not applicable to sustainable production.

To this end, this work presented a sustainable model of HFSP for minimization of makespan and energy consumption. In addition, an improved differential evolution algorithm integrated with the ranking sustainability of parallel machines was developed to support the HSFP sustainable scheduling.

The innovations of the approach are summarized below:

- A multi-objective mathematical model for minimization of makespan and energy consumption of HFSP was formulated.
- The sustainability of parallel machines were evaluated and ranked according to fuzzy TOPSIS method.
- To solve the multi-objective model of HFSP, an improved differential evolution algorithm was presented to assign the job with the ranked parallel machines, which can narrow the search scope and accelerate the convergence speed.

The rest of this paper is organized as: Section II describes a multi-objective sustainable model of HFSP. Section III elaborates a ranking-based differential evolution algorithm considering the sustainability of machines. Section IV presents the solutions to several cases. Finally, a conclusion and some future research issues was drawn in section V.

**Problem formulation**

The hybrid flow shop scheduling problem (Fig.1) is commonly described as follows: there are a
set of $n$ jobs has to be processed at $w$ stages in series. Each job $i$ consists of a pre-determined sequence of operations. Each operation requires one machine selected from a set of available parallel machines, which is denoted as $M_{ij}$. The formal mathematical definition of the problem will be described in detail in the following sections.

Hybrid flow shop scheduling has been extensively examined and the main objective has been to improve production efficiency. However, limited attention has been paid to the consideration of energy consumption with the advent of green manufacturing. In order to reduce resource and energy consumption and achieve sustainable production, this paper set the assignment of machines and the sequence of operations on all the machines as variables in HFSP to minimize the makespan $T$ and energy consumption $E$ to realize sustainable production.

Hypotheses considered in this paper are summarized as follows:

1. Jobs are independent, and have equal priority.
2. After a job is processed on a machine, it is transported to the next machine immediately.
3. All jobs and machines are available at time zero, turning off the idle machines is not allowed and machine failure is not considered.
4. The machine cannot be turned off completely until it has finished all operations assigned to it.
5. The order of operations for each job is predefined and cannot be modified.
6. Pre-emption is not allowed, that is, no task can be interrupted before the completion of its current operation.

The notations used throughout the study are listed in Table 1.

| Notation | Description |
|----------|-------------|
| $S_{ijk}$ | start time for the $i$th job of the $j$th process at machine $k$ |
| $F_{ijk}$ | completion time for the $i$th job of the $j$th process at machine $k$ |
In this work, the process of production can be divided into three stages: (1) processing stage; (2) waiting stage; (3) transportation stage. Each stage corresponding to the consumption of time and energy. The processing time $P_{ijk}$, processing energy consumption per unit time $PE_{ijk}$, waiting energy consumption per unit time $WE_{ijk}$ of each operation on $m$ machine, transportation distance of machine tools in different stages $d_{jk}$ and transportation energy consumption per unit time $TE_{ijk}$ are deterministic and known in advance. The waiting time $W_{ijk}$ was shown in Eq. (1), which was associated with the schedule scenario. It was determined by the completion time $F_{ijk}$ of the work piece and the start time of the next work piece $S_{i(j+k+1)}$ on the same machine. Clearly, the magnitude of transportation time $T_{ijk}$ is determined by the distance between two consecutive machines, which was formulated in Eq. (2). The transport speed $V$ is assumed to be fixed for the convenience of calculation. The makespan means the maximum completed time of all the jobs. It can be described in detail as shown in Eq. (3).

$$W_{ijk} = S_{i(j+k+1)} - F_{ijk}$$  \hspace{1cm} (1)

$$T_{ijk} = \frac{d_{jk}}{V}$$  \hspace{1cm} (2)

$$T = \max \{ F_{ijk} \}$$  \hspace{1cm} (3)

The total energy consumption was composed of the energy consumption for the processing stage $PE_{total}$, the waiting stage $WE_{total}$ and the transportation stage $TE_{total}$. The processing energy is determined by processing time and the processing power per unit time of machine tools. The waiting
energy of machine tools is the energy consumed by machine tools when they are not machining, that is, waiting to process next jobs. The transportation energy consumption $TE_{total}$ is determined by the transportation process of the work-pieces between different stages, which can be calculated as Eq. (5).

Therefore, the second objective of this problem is simplified below.

$$E = PE_{total} + WE_{total}$$ (4)$$TE_{total} = \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} T_{ij,k} \cdot TE_{ijk} \cdot X_{ijk}$$ (5)$$PE_{total} = \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} P_{ij,k} \cdot PE_{ijk} \cdot X_{ijk}$$ (6)$$WE_{total} = \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \left( S_{(i,k+1)} - F_{(i,k)} \right) \cdot W_{ijk} \cdot X_{ijk}$$ (7)

Mathematically, an integer linear programming model of the HFSP was formulated as the following, which will be used throughout the paper.

$$\min_{i,j,k} \{ F_{ijk} \}$$ (8)$$\min_{i,j,k} \{ P_{ijk} \cdot PE_{ijk} + \left( S_{(i,k+1)} - F_{(i,k)} \right) \cdot W_{ijk} \} \cdot X_{ijk}$$ (9)

Subject to:

$$\sum_{k=1}^{\infty} X_{ijk} = 1, i \in \{1,2,\ldots,n\}; j \in \{1,2,\ldots,w\}; k \in \{1,2,\ldots,m\}$$ (10)$$S_{ij} + \sum_{k=1}^{\infty} P_{ijk} \cdot X_{ijk} \leq S_{(i,j+1)} \cdot i \in \{1,2,\ldots,n\}; j \in \{1,2,\ldots,w\}; k \in \{1,2,\ldots,m\}$$ (11)$$F_{(i,j+1)k} - F_{ijk} \geq P_{(i,j+1)k}, \quad \forall i \in n, j \in w, k \in m$$ (12)$$S_{(i,j+1)k} - F_{ijk} \geq 0, \quad \forall i \in n, j \in w, k \in m$$ (13)$$F_{ijk} - F_{(i,j)k} \geq P_{(i,j)k}, \quad \forall i \in n, j \in w, k \in m$$ (14)$$X_{ijk} = \begin{cases} 1, & \text{if machine } k \text{ is selected for operation} \\ 0, & \text{otherwise} \end{cases}, \quad \forall i \in n, j \in w, k \in m$$ (15)$$S_{ij,k} - F_{ijk} \geq 0, \quad \forall i \in n, j \in w, k \in m$$ (16)$$S_{ijk} \cdot F_{ijk} \leq m \cdot X_{ijk}, \quad \forall i \in n, j \in w, k \in m$$ (17)$$0 \leq F_{ijk} \leq T_{ijk}, \quad \forall i \in n, j \in w, k \in m$$ (18)$$T_{max} \leq F_{ijk}, \quad \forall i \in n, j \in w, k \in m$$ (19)$$E_{max} \leq E_{ij,k}, \quad \forall i \in n, j \in w, k \in m$$ (20)

where, Eq.(8) and Eq.(9) is the objective function. Eq.(10) ensures that each operation is assigned to only one machine from its candidate machine set. Eq.(11) means that the operations belonging to the
same job satisfy the precedence constraints Eq.(12) and Eq.(13) ensures that work pieces are processed according to sequence constraints. Eq.(14) governs that at one time a machine can execute one operation and it becomes available for other operations only if the previous operation is completed. Eq.(15) defines the assignment of jobs and the sequence of machines. Eq.(16) represents the precedence relationship among various operations of a job. Eq.(17) ensures that the starting time of each operation in the unselected process plan is zero. Besides, the starting time of each operation should be non-negative. Eq.(18) specifies that the completion time of each operation transported from one machine to another machine is greater than the completion time of the corresponding operation, which should be greater than zero. Eq.(19) implies that the makespan is equal to or greater than the completion time of a schedule that includes the completion time of all jobs. Eq.(20) guarantees that the production processed meet the constrains of makespan and energy consumption.

**Ranking for sustainability of parallel machines**

In HFSP, the selection of machine tool from parallel machines is an important decision-making process. Generally, the selection processes is randomly assigned and then evaluated in the following iteration processes of HFSP, which is time-consuming. To overcome this problem, a heuristic method is proposed to rank and select the sustainability of parallel machines based on the fuzzy TOPSIS method (Tian et al. 2019).

*Fig. 2 The Framework of sustainable evaluation for machine tools*

**Sustainable evaluation system of machines**

To rank the sustainability of parallel machines, the hierarchical structure of this research decision problem is shown in Fig. 2. The sustainability of machines was evaluated by three aspects described as following.
Technical performance: the technical performance refers to the maximum machining quality and efficiency that the machines can achieve, which were indicated by the maximum spindle speed (\(N_1\)), feedrate (\(N_2\)), the accuracy (\(N_3\)) and reliability (\(N_4\)).

Economic performance: the economic performance of machines is indicated by the cost to processing the corresponding jobs. Generally, the cost can be calculated by the summary of the machining cost (\(N_5\)), human cost (\(N_6\)) and maintenance cost (\(N_7\)).

Environmental performance: the most environmental impact was owing to the energy consumption, which can be characterized by the power of machines (working power, \(N_8\) and waiting power, \(N_9\)) and specific energy consumption (SEC, \(N_{10}\)). Besides, the cutting fluid (\(N_{11}\)) is also an important factors for the environment impact.

### Table 2 Membership function of linguistic scale

| Linguistic evaluation of parallel machines | Linguistic evaluation of the weight of criteria | Scale of fuzzy number |
|------------------------------------------|-----------------------------------------------|----------------------|
| Very low (VL)                            | Of little importance (VL)                      | (1,1,3)              |
| Low (L)                                  | Moderately important (MI)                      | (1,2,5)              |
| Good (G)                                 | Important (I)                                 | (3,5,7)              |
| High (H)                                 | Very important (VI)                           | (5,7,8)              |
| Excellent (Ex)                           | Absolutely important (AI)                     | (7,9,9)              |

Based on the construction of the hierarchy, the priority weights of each criteria and the attributes of parallel machines can be calculated with the fuzzy TOPSIS method. Here, the linguistic variables are defined and the corresponding membership function by triangular fuzzy number listed in Table 2. The fuzzy TOPSIS method of calculating priority weights and ranking the parallel machines is described shown in Fig.3.
Step 1: Data preparation

- Isotropy decision matrix
  - Construct the fuzzy performance matrix and choose the appropriate linguistic variables for the parallel machines with respect to criteria.

- Normalization of the fuzzy-decision matrix
  - The normalized fuzzy-decision matrix denoted by $R$ is calculated as follows:
  $$R = \frac{a_{ij}}{a_{ij}^{\max}}, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n$$
  - $a_{ij}^{\max}$ is the maximum value in the $i$th row.

For the elements in $\hat{\Lambda}$, it can be obtained by the rating of experts.

Step 2: Data processing

- Determination of the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS)
  - We can define the FPIS $A^+$ (aspiration levels) and FNIS $A^-$ (the worst levels) as follows:
  $$A^+ = \left\{ \hat{v}_1, \hat{v}_2, ..., \hat{v}_n \right\}$$
  $$A^- = \left\{ \hat{v}'_1, \hat{v}'_2, ..., \hat{v}'_n \right\}$$
  - $\hat{v}_i = (1,1) @ u_i$, $\hat{v}'_i = (0,0) @ u_i$, $i = 1, 2, ..., n$

- Calculate the distance of each parallel machine from FPIS and FNIS
  - The distances $(d^+_{ij} \text{ and } d^-_{ij})$ of each parallel machines from $A^+$ and $A^-$ can be currently calculated by the following equation.
  $$d^+_{ij} = \sum_{k=1}^{n} (\hat{v}_k - \hat{v}'_k) @ u_j, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n$$
  $$d^-_{ij} = \sum_{k=1}^{n} (\hat{v}_k - \hat{v}'_k) @ u_j, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n$$

Where $u_j = v_j \otimes w_j$.

Step 3: Machines ranking

- Determine the fuzzy gap degree
  - $$CC_i = \frac{d^+_{ij}}{d^+_{ij} + d^-_{ij}}, \quad i = 1, 2, ..., m$$
- Machines sustainability ranking based on the fuzzy gap degree
  - (M2 M4 M3 M1—Ascending order)

Fig. 3 Procedure of ranking the parallel machines based on fuzzy TOPSIS method

An improved differential evolution algorithm

Differential Evolution (DE) is a stochastic direct search and global optimization algorithm include genetic operation, i.e., differential mutation, crossover and selection. It is a parallel search evolution strategy that is fairly fast and reasonably robust, which make differential evolution the versatile tool today. Inspired by No Free Lunch Theorem, the ranking of machines’ sustainability introduced in above section was integrated with the DE by the heuristic decoding rule. In this work, a ranking-based differential evolution algorithm (RBDE) was proposed to solve the hybrid flow-shop problems based on the heuristic decoding rule. In this approach, after the genetic operation, active decoding method was carried out and integrated into iterations of the algorithm. The selection pressure to the proper direction could be obtained so as to enhance its performance by accelerating the convergence speed in the iteration of DE. The procedures are described in detail as below.
**Ranking-based Decoding**

Decoding method is the key factor to decode the iterative chromosome sequence into a reasonable production scheduling scheme, which has a great impact on the efficiency of the solution. The heuristic rule proposed in this paper mainly refers to the directional selection of machine tools according to the sustainability of the parallel machines. The parallel machines with the highest sustainability are more suitable for the jobs. In this work, the jobs is encoded by the number and the sequence as shown in Fig.4. In the decoding processes, the jobs would be assigned to the machines with the highest sustainability via the above ranking method.

![Fig.4 heuristic decoding rule](image)

**Calculation procedure**

Based on the above decoding method, the ranking-based integration with the heuristic decoding rule was developed for HFSP sustainable scheduling. The flowchart of the RBDE was illustrated in Fig.5. It can realize directional selection of the most sustainbale machines to accelerate the convergence speed. The scheduling process was presented as follows:

**Initializing:** The population $P_0$ were generated with uniform distribution in search space, and the control parameters $F$ and $Cr$ was provided based on the experience.

**Mutation:** NP mutants were generated with the following mutation strategy in Eq.(21).

$$Mu(NP) = x_{r_1} | P_0 + F \cdot (x_{r_2} - x_{r_1}) | P_0$$  \hspace{1cm} (21)

**Crossover:** Population $P_0$ and its mutant intermediates $NP$ was implemented as Eq. (22).

$$C_r (P_0 + 1) = \begin{cases} x_{r_1}(NP), & \text{if } rand(0,1) \leq cp \\ x_{r_1}(P_0), & \text{otherwise} \end{cases}$$  \hspace{1cm} (22)
where, \( \text{rand}(0,1) \) is the random numbers which are uniform distribution between 0 and 1. \( x_r() \) represents the selection of gene of the \( r \)th chromosome for the population.

**Evacuation & Selection:** Based on the ranking-based decoding method, the parallel machines were selected for various jobs. Then, the energy consumption and makespan could be calculated with the developed HFSP model. In this step, only non-dominated individuals were carried on to the next generations.

**Stopping criteria:** The iteration repeated until it achieved the maximum number of generations.

**Report:** The best individuals’ outcome as the optimal solution.

![Flowchart of the RBDE](image)

**Table 3 The parameters in HFSP**

| processes | machines | job1 | job2 | job3 | job4 | processing power(kW) | waiting power(kW) |
|-----------|----------|------|------|------|------|-----------------------|-------------------|
| process 1 | M1       | 6    | 4.2  | 6.1  | 5.8  | 20                    | 15                |
|           | M2       | 2.3  | 2.6  | 4.1  | 3.4  | 30                    | 25                |
|           | M3       | 4.2  | 3.2  | 5.6  | 4.3  | 25                    | 20                |
|           | M4       | 8.1  | 4.2  | 7.4  | 6.9  | 10                    | 5                 |
|           | M5       | 1.5  | 2.6  | 3.6  | 2.8  | 35                    | 30                |
| process 2 | M6       | 2.3  | 2.4  | 2.2  | 2.9  | 30                    | 25                |
|           | M7       | 3.5  | 3.6  | 3.2  | 2.6  | 23                    | 18                |
|           | M8       | 4.3  | 4.3  | 4.8  | 3.6  | 18                    | 13                |
|           | M9       | 5.2  | 4.5  | 4.8  | 4.8  | 12                    | 7                 |
Case study

A case was presented to testify the feasibility and effectiveness of the proposed methodology for minimizing makespan and energy consumption of HFSP. In this case, four jobs were processed on 25 machines and each job requires 5 steps operations. There were 5 parallel machines in each process. The related data including job number, processing time, process power and wait power were provided in Table 3.

In addition, the sustainability of each machine were evaluated and ranked with the fuzzy TOPSIS method listed in Table 4. The higher rank would be selected in the decoding processing of the RBDE algorithm. Also, the proposed method was implemented in Matlab 2014 and runs on an Intel Core i5 CPU (2.53Ghz/8.00G RAM) PC with a Windows 10 operation system. All the parameters and their selected values are summarized in Table 5.

### Table 4 Ranking value of the machines

| processes | Machines | (value/rank) | job1 | job2 | job3 | job4 |
|-----------|----------|--------------|------|------|------|------|
| process1  | M1       | 0.59/0.40/0.55/0.55/5 | 0.40/0.31/0.48/0.41/2 | 0.55/0.60/0.48/0.35/1 | 0.55/0.60/0.48/0.35/1 |
|           | M2       | 0.30/0.31/0.48/0.41/2 | 0.31/0.48/0.41/2 | 0.48/0.48/0.48/0.48/4 | 0.48/0.48/0.48/0.48/4 |
|           | M3       | 0.50/0.36/0.60/0.48/4 | 0.36/0.60/0.48/4 | 0.60/0.60/0.48/0.35/1 | 0.60/0.60/0.48/0.35/1 |
|           | M4       | 0.58/0.31/0.48/0.48/3 | 0.31/0.48/0.48/3 | 0.48/0.48/0.48/0.48/3 | 0.48/0.48/0.48/0.48/3 |
|           | M5       | 0.19/0.32/0.44/0.35/1 | 0.32/0.44/0.35/1 | 0.44/0.44/0.35/0.28/1 | 0.44/0.44/0.35/0.28/1 |
| process2  | M6       | 0.27/0.28/0.28/0.38/3 | 0.28/0.28/0.38/3 | 0.28/0.28/0.38/3 | 0.28/0.28/0.38/3 |
|           | M7       | 0.41/0.40/0.38/0.28/1 | 0.40/0.38/0.28/1 | 0.38/0.38/0.28/1 | 0.38/0.38/0.28/1 |

Note: P_T means the processing time for each jobs with the corresponding machines.
Table 5 Parameters of the RBDE

| Parameters                        | constraint |
|-----------------------------------|------------|
| initial population size           | 100        |
| range of crossover probability    | [0.1, 1]   |
| range of mutation probability     | [0.01, 0.2]|
| scale factor                      | 0.5        |
| stopping condition                | Maximum number of iterations 200 or convergence condition \( |V| < 10^{-3} \) |

Table 6 Optimization results of RBDE

| No. instances | extreme point1 | extreme point2 | runninng time (min) |
|---------------|----------------|----------------|---------------------|
|               | Makespan (min) | energy consumption (kW.h) | Makespan (min) | energy consumption (kW.h) |                      |
| 1             | 4*5*25         | 21              | 22.9              | 1949              | 34.252               |
| 2             | 8*5*25         | 26.5             | 33.8              | 3089              | 61.710               |
| 3             | 12*5*25        | 30.8             | 38.2              | 4253              | 90.262               |
| 4             | 16*5*25        | 36.6             | 40.3              | 5587              | 117.240              |
| 5             | 20*5*25        | 41.8             | 50.2              | 6791              | 143.815              |
To visualize the performance of RBDE, we selected the pareto front solutions of No.3 and No.9 instance. It can be seen from Fig. 6 that the results are distributed evenly and widely, which denoted the effectiveness of the proposed methods.

To further assess the versatility of the proposed algorithm, the extreme solution points A, A’, B, B’ were selected from the obtained Pareto front solutions in Fig. 6. Gantt charts of the four test instances are showed in Fig. 7.

The optimal solutions corresponding to the four instance are \{8,10,11,4,2,7,12,6,5,3,1,9\}, \{4,6,2,3,10,5,12,8,9,1,11,7\}, \{10,2,12,11,3,6,4,5,7,8,1,9\} and \{7,1,11,9,6,5,10,3,2,12,4,8\} respectively. Different ranking value of parallel machine tools for each process and jobs can be obtained according to section 2, as shown in Table 6. It can be seen that the utilization rates of M1, M8, M9, M10, M13, M11, M18, M21 and M23 in parallel machines is lower from Gantt chart (a) and (b). Besides, it also indicated that the data of parallel machine tool in (c), (d) is double in (a), (b), which is consistent with the ranking of parallel machines.
Table 7 Number of non-dominated solutions found by each algorithm

| NO | instance     | RBDE max | RBDE min | RBDE avg | HMPSO max | HMPSO min | HMPSO avg | HMNSGA-II max | HMNSGA-II min | HMNSGA-II avg |
|----|--------------|----------|----------|----------|-----------|-----------|----------|---------------|--------------|---------------|
| 1  | 4*5*25       | 6        | 4        | 5.2      | 4         | 3         | 3.1      | 5             | 3            | 4.6           |
| 2  | 8*5*25       | 18       | 10       | 14.5     | 15        | 9         | 11.3     | 17            | 10           | 13.2          |
| 3  | 12*5*25      | 21       | 12       | 14.2     | 16        | 11        | 12.5     | 17            | 12           | 13.4          |
| 4  | 16*5*25      | 28       | 10       | 21.7     | 13        | 8         | 10.3     | 23            | 10           | 18.5          |
| 5  | 20*5*25      | 22       | 15       | 19.4     | 18        | 12        | 15.6     | 20            | 13           | 18.9          |
| 6  | 25*5*25      | 28       | 13       | 20.1     | 18        | 12        | 17.4     | 24            | 12           | 18.3          |
| 7  | 4*10*50      | 5        | 3        | 4.5      | 3         | 2         | 2.6      | 4             | 3            | 3.2           |
| 8  | 8*10*50      | 12       | 9        | 11.3     | 11        | 5         | 7.9      | 13            | 7            | 9.8           |
| 9  | 12*10*50     | 24       | 13       | 22.4     | 18        | 11        | 16.3     | 21            | 14           | 18.1          |
| 10 | 16*10*50     | 23       | 15       | 19.5     | 17        | 10        | 12.6     | 20            | 12           | 16.4          |
| 11 | 20*10*50     | 17       | 10       | 12.3     | 14        | 8         | 10.5     | 15            | 9            | 11.2          |
| 12 | 25*10*50     | 15       | 7        | 9.6      | 12        | 6         | 8.3      | 13            | 6            | 8.9           |
Comparisons with other algorithms

In order to test the performance of the proposed algorithms, the heuristic rule was implanted in the NSGA-II and PSO named as HMNSGA-II and HMPSO. The ‘Max.’, ‘Avg.’, and ‘Min.’ column represent the maximum, average, and minimum number of non-dominated solutions, respectively. It can be clearly found from Table 7 that the overall AVG, MAX and MIN yielded by RBDE were better than those generated by HMNSGA-II and HMPSO algorithms in the same computation time. The reason can be explained that the proposed algorithm can take full of the non-dominated solutions to generate excellent offspring and the heuristic rule for iteration operators to disturb old individuals.

Besides, in this paper, we utilize the five performance metrics to evaluate the improved methods, i.e., convergence metric, uniformity performance, diversity metric, hyper-volume and running time.

(1) Convergence metric $γ$, it means the average value of the minimum distance between each reference point in the set $p$ and the reference set $p^*$.

$$
γ = \frac{\sum_{y \in p} \min_{x \in p^*} \text{dis}(x, y)}{|p|}
$$

(23)

Where, $p$ is the solution set obtained by the algorithm $\text{dis}(x,y)$ represents the Euclidean distance between point $y$ in reference set $p^*$ and point $x$ in reference set $p$.

(2) Uniformity performance: Spacing Metric $sp$, it was used to measure the standard deviation of the minimum distance from each solution to others.

$$
sp = \sqrt{\frac{1}{|p|} \sum_{i=1}^{N} (\overline{d} - d_i)^2}
$$

(24)

where, $d_i$ is the minimum distance from $t$ point $i$ on the pareto frontier of the algorithm to the other points, $\overline{d}$ is the average value of all distance $d_i$.

(3) Diversity metric $Δ$, measures the extent of spread achieved among the obtained solutions.

$$
Δ = \frac{|df + dl + \sum_{i=1}^{N-1}(d_i - \overline{d})|}{df + dl + (N-1)d}
$$

(25)

where, $df$ and $dl$ are the Euclidean distances between the extreme solutions and the boundary solutions.
of the obtained non-dominated set. Assuming that there are $N$ solutions on the best non-dominated front.

(4) Hyper-volume, $HV$: The volume of the region in the target space enclosed by the non-dominant solution set and the reference point obtained by the algorithm.

$$HV = \delta \left( \bigcup_{i=1}^{\left| S \right|} v_i \right)$$  \hspace{1cm} \text{(26)}$$

where, $\delta$ is the Lebesgue measure, which was used to measure volume. $\left| S \right|$ represents the number of non-dominated solution sets. $v_i$ means the Super volume composed of the reference point and the $i$th solution in the solution set.

(5) Running time describes the execution time of an algorithm to reflect the efficiency.

For all three algorithms, the population size is set as 100, the number of iterations is 200. For each instance, all the tested algorithms are run 20 times and the performance metrics averaged are collected, as shown in Table 8.

Table 8 shows the statistical performance metrics obtained by each of the three algorithms. It is revealed that RBDE outperforms HMNSAG-II and HMPSO for almost all the metrics, particularly for
$\gamma$, $sp$ and running time, which means that RBDE can achieve the most proximity to the reference front. In terms of convergence $\gamma$, RBDE and HMPSO can quickly converge to a better value than HMNSAG-II. However, HMPSO is easy to fall into local convergence, and the universality and diversity of RBDE and NSGA-II ($\Delta$ and HV) is significantly superior to the HMPSO. From the uniformity performance perspective (i.e, $sp$), it can be observed that DE and PSO outperforms NSGA-II when the problem size is small. However, such advantage will no longer exist if the problem size is getting larger. In large scale problems, RBDE and HMNSGA-II can find more non-dominated solutions, and it performs especially better in solving those problems with less number of stages and machines. In addition, RBDE is the fastest in solving multi-objective HFSP problems, which means the proposed algorithms have a better searching ability than the others for the problem studied in this work.

To further visualize the performance of these algorithms, the pareto fronts are selected to provide a graphical representation for different scale of workpiece instance, i.e, No.2, No.3, No.4 and different scale instance No.3, No.6, No.12, which are selected as typical examples of the small, medium and large scale instances, respectively. Results are demonstrated in Fig. 8.

Fig. 8 shows the optimal pareto frontier as the function of iteration number. As shown, the curves of all the three schemes converge toward the optimal pareto frontier. These figures give an intuitive illustration for some results derived from the performance metrics analysis. As shown in the figure, the RBDE algorithm performs the best among the three algorithms in solution quality and diversity. Besides, it is noted that the RBDE algorithm provides solutions that are relatively closer to the Pareto fronts, whereas the RBDE and HMNSGA-II algorithms provide solutions that are more diversified. This result well explains the above-mentioned performance anomaly between the three algorithms.

Overall, the proposed RBDE algorithm is capable of providing better solutions than HMNSGA-II and HMPSO in terms of quality and distribution. The heuristic rule for modified DE algorithm has a positive effect on the behavior of the proposed algorithm.
Fig. 8 Scatter of RBDE, HMNSGA-II and HMPSO algorithms (● - RBDE, ◆ - HMNSGA-II, ✥ - HMPSO)

Conclusions

This work investigates a multi-objective hybrid flow shop sustainable scheduling model based on a modified differential evolution algorithm. The main contributions of the proposed algorithm are concluded as follows:

1) A sustainable scheduling mathematical model considering makespan and energy consumption model was established for the hybrid flow shop.

2) The sustainability of parallel machines were defined by a hierarchy criteria, which was ranked by fuzzy TOPSIS method.

3) A ranking-based DE was developed to produce feasible scheduling sequences, in which realized active selection of parallel machines with high sustainability. The effectiveness of the proposed methods was verified.

Although the efficiency of the proposed method has been verified, this paper has some limitations. 1) It does not to use actual jobs-shop data to validate this method to provide the best sustainable scheduling and 2) It just focuses on the energy-consumption and makespan problem. The future research should make use of multiobjective optimization tools to achieve the comprehensive
optimization by considering large numbers of variables and constraints, e.g., set-up time of machines, limited resources, and requirements of customer order.

**Author contributions** L.W. and L.K. conceptualised the mathematical model. X.L. made the program. L.W. and F.L. analysed the data and made figures. L.W. wrote the paper. All the authors read and contributed to the submitted version of the manuscript. L.W. and F.L. acquired the funding and were responsible for resources.

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Not applicable.

**Consent to Participate**
All authors agree to participate in the editing of the paper.

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The authors declare that they have no conflict of interest.

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