Local time on the exceptional set of dynamical percolation and the incipient infinite cluster.

Summary: In dynamical critical site percolation on the triangular lattice or bond percolation on \( \mathbb{Z}^2 \), we define and study a local time measure on the exceptional times at which the origin is in an infinite cluster. We show that at a typical time with respect to this measure, the percolation configuration has the law of Kesten’s incipient infinite cluster. In the most technical result of this paper, we show that, on the other hand, at the first exceptional time the law of the configuration is different. We believe that the two laws are mutually singular, but do not show this. We also study the collapse of the infinite cluster near typical exceptional times and establish a relation between static and dynamic exponents, analogous to Kesten’s near-critical relation.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
60J55 Local time and additive functionals
82B43 Percolation
82B27 Critical phenomena in equilibrium statistical mechanics
60J67 Stochastic (Schramm-)Loewner evolution (SLE)
60D05 Geometric probability and stochastic geometry

Keywords:
dynamical percolation; local time; critical phenomena; Palm measure

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