Technical note

Hydraulic scale modelling and thermodynamics of mass oscillations in closed surge tanks

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ABSTRACT
The design and results from a hydraulic scale model of mass oscillations in a hydropower plant with a closed surge tank constructed as an underground rock cavern are presented. The results from the model test of an existing hydropower plant at scale 1:65 are compared with field measurements. The main contributions of this work include (1) an assessment of whether hydraulic models may be applied to evaluate hydropower tunnels with closed surge tanks, (2) a novel approach to scale atmospheric air pressure, and (3) an evaluation of the thermodynamic behaviour in the model and prototype. The hydraulic model is shown to provide an accurate representation of the maximum (first) amplitude, with a relative error of less than 4%. An estimate of the period of the oscillations has a relative error of less than 1%. The model has higher dampening compared with the prototype, resulting in the 20% relative error of the second amplitude. Both the model and prototype reveal approximately adiabatic behaviour of the closed surge tank.

Keywords: Closed surge tanks; field studies; hydraulic models; mass oscillations; thermodynamics

1 Introduction

Mass oscillations in hydropower tunnels with closed surge tanks cause large pressure amplitudes, and need to be understood by engineers for control of the hydraulic pressure in the power plants. Mass oscillations are hydraulic transients caused by a change of turbine flow and the inertia of the water mass in the tunnel. In the case of a load rejection in a hydropower plant, the turbine flow is reduced and the water in the headrace tunnel is forced to flow into the surge tank. The water inflow initiates mass oscillations in the tunnel between the surge tank and the upstream reservoir.

The closed surge tank was first introduced by Michaud (1878) as a means to mitigate water hammer in pipes. Johnson (1908) further shows that the closed surge tank may be applied to obtain regulation stability in hydropower plants. Closed surge tanks for hydropower plants may be constructed as rock caverns or steel tanks filled with pressurized air, and are applied where the topography or other factors render them more feasible than open surge tanks connected to atmospheric air.

The thermodynamic behaviour of the closed surge tank is described with the perfect gas law as shown in Eq. (1):

$$pV = mRT$$

where $p$ is air pressure, $V$ is air volume, $T$ is air temperature, $m$ is air mass, $R$ is the specific gas constant for air. However, the exact behaviour is difficult to calculate analytically due to the three unknowns (pressure, volume, and temperature) and the influence of heat transfer. From a literature review, it is seen that several different theories have been applied, varying from isothermal to adiabatic. The earliest researchers on closed surge tanks (De Sparre, 1911; Johnson, 1908; Michaud,
1878) assume isothermal behaviour. Camichel (1918) presented several experiments with some showing adiabatic behaviour and others revealing isothermal behaviour. Camichel (1918) noted that the behaviour depends on the period of the oscillations and the construction material of the closed surge tank. Graze (1968) reported new experiments observing that the heat transfer to the surrounding environment influences the thermodynamic behaviour appearing between of adiabatic and isothermal regimes. Graze (1968) furthermore showed that the polytrophic behaviour.

\[ pTv^n = k \] (2)

is not accurate for closed surge tanks, unless the thermodynamic behaviour is either isothermal or adiabatic \((n) is the polytrophic constant ranging from 1.0 to 1.4 for isothermal and adiabatic conditions, respectively, and \(k is a constant). However, in engineering applications the thermodynamics are usually simplified, and the polytrophic equation is assumed to be valid (Thorley, 2004; Wylie & Streeter, 1993). The thermodynamic behaviour of air in hydraulic systems is still an important research topic, with recent results presented in Zhou, Liu, Karney, & Wang (2013) and Vereide, Tekle, & Nielsen (2015).

The present work investigates whether hydraulic scale models may be applied for evaluation of mass oscillations in hydropower plants with closed surge tanks constructed as underground rock caverns. To the best of our knowledge, no previous attempts at hydraulic scale modelling of such systems are reported in the literature. One of the main challenges is scaling of the atmospheric air pressure, and a novel approach in addressing this issue is proposed in this Note. Furthermore, measurements from both a full-scale existing closed surge tank and the hydraulic scale model are used to evaluate the thermodynamic behaviour.

The hydraulic scale model is constructed in the hydraulic laboratory of the Norwegian University of Science and Technology in Trondheim. The prototype is the 150 MW Torpa hydropower plant in southern Norway. Field measurements are conducted in the closed surge tank during an emergency shutdown from full load. The surge tank is constructed as an underground rock cavern with 13,000 m³ of air at pressure 4.1 MPa.

2 Theoretical background

Transient water flow in closed conduits is described with the continuity and momentum equations (Wylie & Streeter, 1993):

\[ \frac{\partial p}{\partial x} + \frac{\partial p}{\partial t} + \rho a^2 \frac{\partial v}{\partial x} = 0 \] (3)

\[ \frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} + g \sin \varphi + f \frac{v|v|}{2D} = 0 \] (4)

where \(v\) is the water velocity, \(p\) is the water pressure, \(x\) is the longitudinal coordinate, \(t\) is the time, \(\varphi\) is the conduit angle, \(\rho\) is the water density, \(a\) is the wave celerity, \(f\) is the Darcy-Weisbach friction factor, and \(D\) is the conduit internal diameter.

For a hydropower headrace system, the boundary conditions for these differential equations are the upper reservoir, the closed surge tank, and the turbine. The upper reservoir is represented by a fixed water level providing a constant pressure. During an emergency shutdown from the full load, the turbine in the currently investigated system closes in ten seconds with a linear closing law. The flow through the turbine is described as:

\[ Q = \alpha A (2gH)^{1/2} \] (5)

where \(Q\) is the water discharge, \(\alpha\) is the percentage opening, \(A\) is the water flow cross section during full opening, \(g\) is gravity acceleration, and \(H = p/(\rho g)\) is the piezometric head.

For calculation of the thermodynamic behaviour during mass oscillations in the specific type of closed surge tanks presently considered, Goodall, Kjørholt, Tekle, & Broch (1988) have shown that the assumption of adiabatic conditions yields accurate results. Based on these studies, the working hypothesis has been that the thermodynamic behaviour of the closed surge tank is adiabatic and may be described with Eq. (2) and the adiabatic exponent equal to 1.4.

3 Dimensional analysis

A dimensional analysis of the hydropower system is conducted with Buckingham’s (1914) \(\pi\)-theorem to determine the scaling of the hydraulic model. The physical behaviour of the system then depends on 12 parameters: pressure \((p)\), density of water \((\rho)\), water velocity \((v)\), tunnel diameter \((D)\), dynamic viscosity of water \((\mu)\), tunnel length \((L)\), tunnel friction \((f)\), gravity \((g)\), air volume \((V)\), the adiabatic constant for air \((\kappa)\), the wave celerity in water \((a)\), and the tunnel slope \((\sin \varphi)\). By selecting \(\rho\), \(v\) and \(D\) as independent units one derives the \(\pi\)-terms shown in Table 1. It is seen from the dimensional analysis that the system is characterized by the Euler, Reynolds, Froude, and Mach numbers. To scale the mass oscillations of the system correctly, the Euler scaling law is selected to preserve the effects of the pressure and inertial forces. For the present hydraulic system, the Reynolds and Mach numbers cannot be scaled correctly at the same time as the Euler number, due to physical restrictions of the laboratory environment.

Scaling effects due to the different Reynolds numbers of the model and prototype are known to be limited if the flow is turbulent (Hughes, 1993). The scaling factors are therefore selected to ensure that the flow is in the turbulent regime in the hydraulic scale model. The Mach number characterizes the compressibility and the water hammer effects in fluid flow. However, the selection of the Euler scaling law causes the water hammer effects in the hydraulic scale model to reduce compared with the prototype. For most hydropower schemes, the water hammer effects do not significantly influence the mass oscillations.
The presence of such influence in the current hydraulic scale model and the prototype is evaluated in the later discussion.

The Euler model law results in the scaling factors \( L_r \) for length, \( v_r = L_r^{1/2} \) for velocity, \( t_r = L_r/v_r = L_r^{1/2} \) for time, and \( p_r = v_r^2 = L_r \) for pressure. Water density, gravity, viscosity, adiabatic constant, friction factor, and temperature are the same for the model and prototype. The wave celerity is smaller in the scale model compared with the prototype owing to the higher elasticity of the conduit, which is beneficial for obtaining a better scaling of the Mach number. The difference, however, is small and the improvement is limited. The tunnel slope is different in the model and prototype to allow for scaling of the atmospheric air pressure, as described in the next section. As the slope terms in Eqs (3) and (4) are commonly disregarded for practical purposes, their effect is assumed negligible.

### 4 Hydraulic scale model design and operation

The prototype for the hydraulic model is the headrace tunnel of the 150 MW Torpa power plant in southern Norway. The power plant was commissioned in 1989 and is owned by Eidsiva Vannkraft AS. The nominal head of the power plant is 445 m, and the nominal discharge is 35 m³ s⁻¹. The headrace length is 9.6 km, the diameter is 6.56 m, and the tunnel is inclined directly between the reservoir and the power station without a pressure shaft. Figure 1 shows a principle diagram of the power plant.

The closed surge tank of the power plant is constructed as an unlined rock cavern, with a total volume of 17,000 m³. During normal operation, the water depth in the surge tank is 2 m, giving 13,000 m³ of air at pressure of 4.1 MPa. The surge tank is located 300 m upstream of the turbine. The scale of 1:65 is selected based on available laboratory space, and the resulting dimensions of the hydraulic scale model are shown in Table 2.

Some practical considerations are necessary to account for atmospheric air pressure. For most hydraulic scale models only the relative air pressure needs to be considered, as there are no thermodynamic processes. However, for scaling of closed surge tanks the absolute air pressure must be scaled to obtain the correct thermodynamics. In the present work, a novel method has been developed. The method involves placing the closed surge tank at a higher elevation in the model compared with the prototype, to obtain the correct relation between the volume and pressure in the surge tank, while allowing free atmospheric pressure at the inlet reservoir and the outlet. An alternative is to use pressure tanks to scale the atmospheric pressure at the inlet reservoir and the outlet, but this approach was regarded as unfeasible due to higher cost and complexity. The main error introduced by the proposed technique is the need for a longer shaft between the surge tank and the turbine in the model, due to the required height difference between the closed surge tank and the outlet. This will, however, influence a water hammer effect only, as the water mass in the shaft does not oscillate between the surge tank and the upper reservoir.

Another practical consideration relates to the diameter of the pipes close to the turbine. In the prototype, the last 130 m of the headrace has a reduced diameter, leading to a bifurcation pipe that splits the water flow for the two turbines. This detail is not included in the hydraulic scale model due to the added complexity. The main error introduced by this manipulation is the reduced water hammer effect and water velocity close to the...
turbine, which is considered to have a negligible effect on the mass oscillations and thermodynamics of the closed surge tank.

The resulting layout of the hydraulic model is presented in Fig. 2, and pictures of the headrace pipes, the closed surge tank, and the closing valve are shown in Fig. 3. As can be seen from Fig. 2, the novel approach to account for the atmospheric air pressure requires the closed surge tank to be placed 3.83 m above the water level in the upper reservoir to gain the correct relation between the pressure and the water level.

The model is constructed with welded AISI304 stainless steel pipes with internal diameter of 100 mm, and wall thickness of 3 mm. The model rig is fixed by clamping in the flow direction and perpendicular to the flow direction. The closed surge tank is constructed as an acrylic glass box with an air volume of 64.5 dm$^3$ and wall thickness of 10 mm. A vacuum pump is used to set the initial air pressure. The inlet reservoir is a wooden box with a volume of 0.48 m$^3$, with an overflow weir to control the water level. Water inflow is supplied from the water mains. The lumped frictional and singular losses in the pipes are controlled by butterfly valves situated between the inlet reservoir and the closed surge tank, and between the closed surge tank and the outlet. Water flow and shutdown in the model is controlled with a pneumatic-controlled needle valve, which closes from full opening in one second.

The model is equipped with two absolute pressure transducers (GE Druck PTX-1400) with an error of less than 0.15% of full scale 0.6 MPa. One transducer is placed in the surge tank, and one is placed immediately upstream of the outlet needle valve. Water discharge in the pipes is measured with an electromagnetic flow sensor (SITRANS F M 5100 W) with an error of less than 0.4% of full scale 10 m s$^{-1}$.

Field measurements have been collected from Torpa power plant during an emergency shutdown in ten seconds from the full load. Pressure measurements are collected in front of the turbine with a PARO scientific DIQ 73 K sensor with an error of less than 0.04% of full scale of 20 MPa, and from the air pocket in the closed surge with a PARO scientific 8DP000-S sensor with an error of less than 0.01% of full scale of 6 MPa.

The authors collected the measurements upstream of the turbine, while the power plant owner collected the measurements from the closed surge tank. The measurements upstream of the turbine are sampled at 1 Hz. The measurements from the closed surge tank are sampled with lower and unstable frequency, as the measurement system collects samples based on thresholds of water level movement. Table 3 presents the initial ($t = 0$ s) and the end ($t = 3,600$ s) steady state conditions of the mass oscillations measured in the prototype. The headrace velocity is calculated based on produced power (MW), and efficiency curves provided by the power plant operator. The resulting time-series of the measured shutdown are presented in the next section.

5 Results and discussion

The measured shutdown situation at Torpa power plant is recreated in the hydraulic scale model for comparison. The initial parameters are set according to Table 3, and a comparison of the prototype and the model turbine pressure at up-scaled values is presented in Fig. 4a. The comparison of measured and modelled air pressure in the closed surge tank is presented in Fig. 4b. Note that the initial pressure upstream of the turbine is different in the model and the prototype due to different velocity heads in front of the turbines, as described in the previous section. The sum of the pressure head and velocity head is, however, equal in the model and the prototype. The Nash-Sutcliffe efficiency coefficient is 0.90 for the presented turbine pressure time-series, and 0.84 for the closed surge tank air pressure. The relative error of the first amplitude (design pressure) of the turbine pressure is 4%, while the relative error in the second amplitude is 20%. The relative error in the oscillation period is 1%. The repeatability of the experiments is high: the standard deviation of the maximum

| Parameter                  | Initial | End  |
|----------------------------|---------|------|
| Produced power (MW)        | 142     | 0    |
| Upper reservoir level (m)  | 451     | 451  |
| Surge tank water level (m) | 36.2    | 36.3 |
| Air pressure (kPa)         | 4,110   | 4,168|
| Turbine inlet pressure (kPa)| 4,296 | 4,429|
| Turbine velocity head (kPa)| 54     | 0    |
| Headrace velocity (m s$^{-1}$) | 1.0   | 0.0  |
| Headrace headloss (m)      | 7.4     | 0.0  |
| Shaft headloss (m)         | 1.8     | 0.0  |
amplitude from four experiments conducted over two separate days is 7.3 kPa, while the standard deviation of the period is 1.2 s.

The results presented above show that the hydraulic scale modelling of hydropower tunnels with closed surge tanks is possible within a reasonable error. The main error is the dampening of the oscillations, which may be caused by unscaled roughness (head loss is adjusted with valves), air bubbles in the flow, insufficient fixation of the model rig, limited overflow capacity in the upper reservoir, and minor heat transfer occurring over time. It has also been confirmed that the novel approach to account for atmospheric air pressure is fairly suitable. However, this approach raises the elevation of the pipes in parts of the model, which may have a minor effect on the results.

The water hammer effect is not scalable if the focus is on mass oscillations in hydraulic scale models. In addition, Amara, Achour, & Berreksi (2013) showed that in specific cases the water hammer and mass oscillation may occur with harmonic frequencies, and thus may affect each other. In this study, one can see in Fig. 4a at \( t = 100 \) s that water hammer occurs immediately after shutdown in both prototype and model, but the influence on the mass oscillations is in this case seen to be limited for practical purposes.

The field measurements reveal that the thermodynamic behaviour of this specific closed surge tank is approximately adiabatic during the mass oscillations. The hydraulic scale model also exhibits adiabatic behaviour despite the different construction material and size of the surge tank. By comparing the present results with previous studies, it is seen that different thermodynamic behaviour should be expected for different types of surge tanks depending on size, construction material, and period of the mass oscillations. Future studies involving both laboratory and field experiments are necessary to gain better understanding of the thermodynamics in different types of closed surge tanks.

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**Disclosure statement**

No potential conflict of interest was reported by the authors.

**Notation**

- \( \alpha \) = percentage turbine opening (–)
- \( \kappa \) = adiabatic constant (–)
- \( \rho \) = mass density (kg m\(^{-3}\))
- \( D \) = tunnel diameter (m)
- \( g \) = acceleration by gravity (m s\(^{-2}\))
- \( H \) = piezometric head (m)
- \( k \) = constant (–)
- \( L \) = tunnel length (m)
- \( m \) = air mass (kg)
- \( n \) = polytrophic exponent (–)
- \( p \) = pressure (kg m\(^{-1}\) s\(^{-2}\))
- \( Q \) = turbine water flow (m\(^3\) s\(^{-1}\))
- \( R \) = specific gas constant (J kg\(^{-1}\) K\(^{-1}\))
- \( t \) = time (s)
- \( T \) = temperature (K)
- \( v \) = velocity (m s\(^{-1}\))
- \( V \) = volume (m\(^3\))

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