The working group performance modeled by a bi-layer cellular automaton

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The problem ‘human and work’ in a model working group is investigated by means of cellular automata technique. Attitude of members of a group towards work is measured by an indicator of loyalty to the group (the number of agents who carry out their tasks), and lack of loyalty (the number of agents, who give their tasks to other agents). Initially, all agents realize their strategies one-by-one. Agents with the number of scheduled tasks larger than a given threshold change their strategy to unloyal one and start avoiding completing tasks by passing them to their colleagues. Optionally, in some conditions, we allow agents to return to loyal state; hence the rule is hysteretic. Results are presented on an influence of i) the density of tasks, ii) the threshold number of tasks assigned to the agents’ forcing him/her for strategy change on the system efficiency. We show that a ‘black’ scenario of the system stacking in a ‘jammed phase’ (with all agents preferring unloyal strategy and having plenty tasks scheduled for realization) may be avoided when return to loyalty is allowed and either i) the number of agents chosen for task realization, or ii) the number of assigned tasks, or iii) the threshold value of assigned tasks, which force the agent to conversion from loyal strategy to unloyal one, or iv) the threshold value of tasks assigned to unloyal agent, which force him/her to task redistribution among his/her neighbors, are smartly chosen.

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I. INTRODUCTION

In modeling social phenomena, most important obstacle is the complexity of human mind. Individual decisions depend not only on the present status of the environment, but also on the whole history of a given person. For a descriptive theory of individual decisions see [1]. A predictive theory needs causal relations, what undermines free will of human beings. Yet, from the point of view of a modeler, the eternal discussion of the free will is somewhat vain: even in a fully deterministic world, to identify the boundary between ‘yes’ and ‘no’ in a complex multidimensional space of arguments, constructed in our memory, is a hopeless task. (For a recent and provocative formulation of the problem of free will see Ref. [2].) When collective effects are concerned, social modelers pin their hopes in the law of large numbers, where an individual can be reduced to a black box. How deep reduction is legitimated, depends heavily on the modeler’s purpose; social libraries are filled up with self-defending proclamations. For an outstandingly realistic approach we refer to writings of Bruce Edmonds [3]. Also, a set of articles in Ref. [4] can be recommended. Our position here is that it is worthwhile to try to imagine consequences of our memory for collective effects, even if no grounded scientific strategy justifies a concomitant set of assumptions.

Once again, to keep the task simple, at least computationally, cellular automata (CA) are invaluable.

In CA, memory can be introduced directly by an enrichment of the rule, as it was done in the construction of a reversible automaton [5]. There, the cell state at time \( t \) depends not only on the environment state at time \( t−1 \) but also on the state at time \( t−2 \). Another approach is to make the rule dependent of the mean state variable, calculated over the whole system history [6]. Here we refer to switch to a new rule, when a given condition is fulfilled by the cell state. In this way the system refers to its memory in a dynamical way, which cannot be predicted before the simulation is performed; this characteristics of the problem is known as the computational irreducibility [7]. Further, the system can be switched back to the previous rule, if another condition is met. This kind of hysteretic rule has been applied in man-machine systems, where different sets of rules have been switched on by different procedures [8, 9]. Here we apply it as to refer to the human ability to modify the cognitive context as to enable decisions [10]. It remains in a general accordance with the psychological concept of scripts [11], which is activated once; further behavior of an individual is executed according to this script.

Many works that have appeared in the last decade, have shown the usefulness of CA in the field of modeling various aspects of organization’s management systems. Human (group of people) and work (workflow) are key elements of the management system, thus, the analysis of these elements and their relationships is a major subject of research in this area. Robbins et al. [11] formulate

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the three ‘paths of research’ in business management. They are: workflow optimization (work), human group behavior (human), and human-things interaction (human and work).

The study of group behavior using the model based on CA is fairly common, because it is a tool tailored to the nature and dynamics of social processes. Broad discussion of this subject was submitted by *inter alia* Hegelmann and Flache [12]. What provide the universality of CA are emerging attempts to build models within the two other perspectives of research in the field of management. For example, Hassan and Tucker [13] have demonstrated the use of CA to optimize locations of objects (facility layout problem). Optimizing of rearranging—for example, machine in the production floor—leads to minimization of transport costs (time) and thus it is a typical task in the field of workflow optimization. Thirumaran et al. [14] proposed a simulation model which allows to support the analysis of changes in the business processes of customer service web portal. In both approaches, the individual agents (the automaton cells) represent the technical facilities and business rules and do not have a direct relationship with employees.

In this work we are interested in prospect of ‘human and work’ research. In this area, the main goal of researchers is to build CA models, which are tools allowing to explain the global behavior of the analyzed system based on local interactions within groups of employees, and between employees and broadly understood environment. Agents in such models represent the people (employees) and the parameters (attributes). The CA rules represent procedures, which shall be adopted on the basis of more or less simplified assumptions and/or sociopsychological theories and knowledge of the organization and management theory.

The theoretical model in this perspective was proposed by Bin and Zhang [15]. This model allows analyzing the impact of managerial decisions on the behavior of members of a group towards work—measured by specific indicator of loyalty to the group (in accordance with the general sociological message that a group loyal to each other is effective in achieving the objectives). The authors assume, in accordance with the theory of ‘social exchange’, that people are motivated by a desire for social status and respect as much as they are motivated by gains that are material and/or monetary. They introduce, by splitting the group members into ‘economic beings’ and ‘social beings’, the possibility to analyze management policy, which consists of incentives of economic and/or social nature. The direction of movement depends on the assumed policy and the type of agent (a social agent is attracted by social incentives, and economic agent by material ones). The simulation finishes with a state of equilibrium, and the evaluation of the applied policy consists of determining indicator loyalty value for a group for this state.

Another, more complex approach is presented in the work of Shengping and Bin [16]. The proposed model takes into account the types of work performed by a group of employees. The authors introduced the characteristics of the tasks on a scale from ‘hard’ to ‘soft’ work. Hard work must be completed chiefly with technical ability and soft work must be done with social communication ability. Employees were also categorized into groups on the scale of a continuum between ‘working hard’ and ‘social’. An interaction between neighboring agents consisting of ‘reconciling’ the way they cooperate (on a scale hard/social) of each agent with ambient agents (on the principle of the adoption pattern of majority) has also been introduced. Simulated at each step, the overall ‘state behavior’ of the group, is therefore a function of many factors including his/her behavior in last time step, his/her neighbors’ behavior, his/her properties, characteristic and state of the work. Although the precise interpretation of such a complex indicator is difficult, in general, the level of ‘state behavior’ reflects the (average) degree of positive attitude towards work and in a broad sense can be associated with the level of loyalty, which was proposed in Ref. [15].

Management policy analysis in the perspective of the work initiated in Ref. [15] provides a model for the proposed work of Saravacos and Sirakoulis [17]. The authors, in a simple implementation of CA technique, have proposed the characteristics of workers behavior in the seven-point scale from extremely negative to absolutely positive. As the second dimension of agent features indicator depicting each employee’s insistence, his ability to remain ‘uninfluenced’ by his coworkers was adopted. Insistence takes values from one to five, where one denotes an employee who is highly influenced by his neighborhood and thus his behavior is determined by his coworkers; five is the total resistance to the influence of neighbors. For intermediate sizes weights of impact were adopted. Introduced behavioral rules allow to simulate (in each step) attitude change of each agent according to its attitude in the previous step, and the characteristics of its neighbors. Additionally, in the model it was assumed that insistence is an adaptable trait and depends on the extent the employee is conformed to the organizational norms. Organizational norms are determined by the company policy. The organization’s policy is represented by the ‘reward’ coefficient (for positive behavior for agents with low insistence) and ‘punishment’ coefficient (for negative behaviors and a large insistence). Overall assessment of the behavior of group in a given environment is the average ‘loyalty factor’. The main use of the model is to analyze the behavior of the tested group of workers (mainly by a loyalty factor) in the conditions of use of different combinations of reward and punishment levels.

The general idea of the analysis of key factors and relationships in the term ‘human and work’ and the ability to simulate specific managerial decision is also used in the proposed model in this article. Loyalty is measured by the number of loyal agents who carry out their tasks, and the lack of loyalty is expressed by the number of unloyal agents, who give their tasks to other agents.
The latter activity is a channel of interaction between neighbors, what enables collective states. The current strategy of a worker depends on the actual number of his/her awaiting tasks, what makes the changes dynamical and presumably complex. Shifted tasks make the neighboring workers overburdened, what enhances the probability that they will also shift their tasks; this positive feedback allows to expect sharp transitions between collective states.

A related problem in real life is the phenomenon of work stress induced burnout, which can lead to a selfish strategy to shifting duties to colleagues. The problem is related to a number of professional groups, as nurses [18] and police officers [19]. The unloyal state takes the form of absenteeism and/or passivity. Basically, the transition to this state is irreversible; therefore the return to the loyal state, considered below, could be interpreted as the staff turnover.

At the group level, the opposition between the loyal and the unloyal phase is an example of a social dilemma, as defined in Ref. [20]: a dichotomous choice of strategy, and the choice which is individually profitable, but makes worse when universally adopted. The concept of social dilemma is a generalization of the famous prisoner’s dilemma, and it is used to discuss it in the frames of the game theory; for a simple introduction we recommend Ref. [21]. Our assumption here is that the transition of an individual from the loyal to the unloyal phase is triggered by the large number of tasks. However, the unloyal behavior can be seen also as a reasonable strategy of self-preference, when the number of tasks exceeds some threshold. Thus, the frames of game theory enable yet another interpretation of the transition from the unloyal back to the loyal state.

In subsequent sections we define the cellular automaton (Sec. II) and we present main results Sec. IV (including model verification Sec. III) obtained by means of computer simulations. The Sec. V is devoted to the discussion of the results and conclusions.

II. MODEL

The model working group contains \( L \times L \) agents occupying nodes of a square lattice. Each agent may follow one of two strategies (loyal or unloyal to the group). In the latter case an agent distributes his/her task among his/her nearest neighbors. The loyal agents do not bother their neighbors with their own task and during each time step they complete one of their tasks. We start our simulations with group of loyal agents without any tasks assigned to them. Agent loyal to the group having more than some threshold number of tasks assigned to them changes his/her strategy to unloyal one. Unloyal agents having assigned more than assumed threshold number of task shift part of their task to their adjacent neighbors. These neighbors unconditionally take these additional tasks independently on the current number of tasks assigned to them. The number of tasks awaiting for realization by a single agent cannot exceed the agent’s maximal task capacity. Incoming tasks are randomly distributed among different agents. Finally, but optionally, we allow unloyal agents to convert to loyal ones. This step may be realized either as soon as unloyal agent shifts out all the tasks assigned to him/her or when unloyal agent has not more than some threshold value of tasks.

To implement the model described above the CA technique has been chosen.

A cellular automaton [7, 22–24] consists of a regular grid of cells, each in one of a finite number of states. At each time step, a new generation of the cells’ states is created, according to some fixed rule that determines the new state of each cell in terms of the current state of the cell and the states of the cells in its neighborhood. Here, von Neumann neighborhood on \( L \times L \) square bilayer lattice with lateral periodic boundary conditions is assumed, i.e. the site \((i, j)\) has exactly \( z = 4 \) neighbors at sites \((i \pm 1, j), (i, j \pm 1)\). The first lattice layer indicates agents strategy (loyal \( X(i, j) = 0 \) or unloyal \( X(i, j) = 1 \)). The second layer carries information on current number of tasks \( k(i, j) = 0, \cdots, M \) assigned to \((i, j)\)-th agent. We start simulation with \( L^2 \) loyal agents awaiting for their first tasks, i.e.

\[
X(i, j) = 0 \text{ and } k(i, j) = 0 \text{ for } 1 \leq i, j \leq L.
\]

The automata rule is as follow:

1. \( K \) different sites \((i, j)\) are selected randomly. The number of tasks assigned to agents occupying these sites is incremented by \( Z \):

\[
k_{t+1/6}(i, j) \equiv k_t(i, j) + Z, \quad (1a)
\]

\[
X_{t+1/6}(i, j) \equiv X_t(i, j). \quad (1b)
\]

2. Each loyal agent \((X_{t+1/6}(i, j) = 0)\) with more than \( R \) tasks assigned to him/her \((k_{t+1/6}(i, j) > R)\) changes his/her strategy to unloyal one:

\[
X_{t+2/6}(i, j) \equiv 1, \quad (2a)
\]

\[
k_{t+2/6}(i, j) \equiv k_{t+1/6}(i, j). \quad (2b)
\]

3. Each loyal agent \((X_{t+2/6}(i, j) = 0)\) realizes one of his/her task:

\[
k_{t+3/6}(i, j) \equiv \max\{k_{t+2/6}(i, j) - 1, 0\}, \quad (3a)
\]

\[
X_{t+3/6}(i, j) \equiv X_{t+2/6}(i, j). \quad (3b)
\]

4. Each unloyal agent \((X_{t+3/6}(i, j) = 1)\) with more than \( T \) [and \((z - 1) \leq T \leq R\)] tasks assigned to him/her \((k_{t+3/6}(i, j) > T)\) redistributes his/her
own tasks among his/her \((z = 4)\) nearest neighbors

\[
k_{t+4/6}(i,j) ≡ k_{t+3/6}(i,j) - z, \tag{4a}
\]

\[
X_{t+4/6}(i,j) ≡ X_{t+3/6}(i,j), \tag{4b}
\]

and

\[
k_{t+4/6}(i - 1,j) ≡ k_{t+3/6}(i - 1,j) + 1, \tag{4c}
\]

\[
k_{t+4/6}(i + 1,j) ≡ k_{t+3/6}(i + 1,j) + 1, \tag{4d}
\]

\[
k_{t+4/6}(i,j - 1) ≡ k_{t+3/6}(i,j - 1) + 1, \tag{4e}
\]

\[
k_{t+4/6}(i,j + 1) ≡ k_{t+3/6}(i,j + 1) + 1. \tag{4f}
\]

5. The number of tasks scheduled to a single agent cannot exceed maximal agent’s capacity \(M\):

\[
k_{t+5/6} ≡ \min\{k_{t+4/6}(i,j), M\}, \tag{5a}
\]

\[
X_{t+5/6} ≡ X_{t+4/6}(i,j). \tag{5b}
\]

6. The conversion of unloyal agent \((X_{t+5/6}(i,j) = 1)\) to loyal member of the group is possible

\[
X_{t+6/6}(i,j) ≡ 0 \tag{6a}
\]

\[
k_{t+6/6}(i,j) ≡ k_{t+5/6}(i,j) \tag{6b}
\]

(A) after shifting out all of his/her tasks \((k_{t+5/6}(i,j) = 0)\).

(B) Alternatively, we allow for unloyal agents relaxation to loyal ones when they have no more than \(T\) tasks \((k_{t+5/6}(i,j) ≤ \tau)\).

The steps 6(A) or 6(B) are realized only optionally. We will show that realization of these steps is crucial for avoiding ‘black scenario’ of working group total ‘jamming’.

If steps 6(A) and 6(B) are omitted then simply

\[
X_{t+6/6}(i,j) ≡ X_{t+5/6}(i,j), k_{t+6/6}(i,j) ≡ k_{t+5/6}(i,j).
\]

The steps described above are consecutively and synchronously applied to all sites. The results for applying steps 1–5, 1–5+6(A) and 1–5+6(B) are collected in Secs. IV A, IV B 1 and IV B 2, respectively.

III. MODEL VERIFICATION

The model verification for computer models means the model evaluation or ‘process of reaching to sufficient confidence that the model is ready for use in particular case’ [27]. In order to verify [4] our model we inspect single agent temporal evolution of

(A) the number \(\ell\) of his/her unloyal nearest neighbors,

(B) his/her strategy \(X\),

(C) and the number \(k\) of tasks currently scheduled for her/him.

These dependencies are shown in A, B, C panels of Fig. 1. Four sub-figures of Fig. 1 correspond to possibility of agents back conversion to loyalty (1c, 1d) and reasonably (1a, 1c, \(K = 5\)) or enhanced (1b, 1d, \(K = 8\)) tasks delivering. For these set of parameters \((L = 10, M = 16, R = 8, Z = 4, T = 3)\) the system becomes totally ‘jammed’: for long enough times of simulation all agents become overburdened and have to complete \(M\) tasks. The relaxation to loyalty is realized according to the rule 6(A). As expected the integer number of agents’ unloyal nearest neighbors vary between 0 and \(z\) \((\ell \in \{0, 1, 2, 3, 4 = z\})\), panels A of Fig. 1 and the number of tasks assigned to single agent cannot exceed assumed maximal number \(M = 16\) \((k \in \{0, 1, \cdots, M - 1, M\})\), panels C of Fig. 1. For irreversible transition to the unloyal strategy only single transition of state \(X_t = 0 \rightarrow X_{t+1} = 1\) is expected as presented in panels B of Figs. 1a and 1b. On the contrary, for the rule 6(A) multiple transitions \(X_t = 0 \rightarrow X_{t+1} = 1\) and \(X_t = 1 \rightarrow X_{t+1} = 0\) may be observed [panels B of Figs. 1c and 1d].

IV. RESULTS

A. Irreversible transition to the unloyalty

In Fig. 2 the time evolution of spatial distribution of unloyal agents is presented. The back conversions of unloyal agents to loyal ones are excluded, \(i.e.\) we do not realize the optional 6-th step from our algorithm presented in Sec. II. The periodic boundary conditions are assumed. After \(t > 20\) unloyal agents appear more likely at the border of existing unloyal agents’ cluster.

The time evolution of the average fraction of unloyal agents \(|\langle ρ\rangle|\) (percent) and the average number of tasks per agent \(|\langle k\rangle|\) are presented in Fig. 3. Here, \(|\cdot|\) denotes a spatial average over all \(L^2\) agents and \(|\cdots|\) stands for an average over \(N\) different simulations. Obviously, increasing number of sites \(K\) where new tasks are delivered must lead to decreasing time \((τ_u)\) after which all agents become unloyal and to decreasing time \((τ_l)\) after which all agents have to complete maximally allowed number \(M\) of tasks. These times \((τ_u, τ_l)\) dependencies on number \(K\) of agents chosen for new \(Z\) tasks realization are presented in Fig. 4.
FIG. 1: An example of the time evolution of (A) the number of the unloyal agents in agent’s neighborhood $\ell$, (B) the agent state $X$ and (C) the number $k$ of tasks assigned for a single agent. In simulations presented in panels (a) and (b) the back conversions of unloyal agents to loyal ones were excluded. In cases (c) and (d) the step 6(A) is realized. $L = 10$, $M = 16$, $Z = 4$, $R = 8$, $T = 3$.

Please note, that after time $t > \tau$ the newly incoming tasks are lost, as all both unloyal and overburdened agents ignore them. This situation is caused by the assumed numerical technique (CA), in which single lattice cell may stay in finite number of states (here two, for the first automaton layer, and $M$ for the second one). This however, does not influence our results qualitatively, as increasing of maximal agents capacity $M$ results only in a delay in reaching overburden state of all agents (see Fig. 5a); the time of conversion all agents to unloyal group members ($\tau$) remains unchanged. Time $\tau$ grows with $M$ roughly linearly (see Fig. 5b).

This transition can be verified by a comparison with a simplified picture, obtained with a mean field model of a square lattice of agents. The equation is composed in the spirit of Refs. [25, 26], where the set of states is reduced to unloyal (with probability $\rho$) and loyal (with probability $1 - \rho$);

$$\dot{\rho} = f(\rho) = \sum_{i=0}^{2} \left( \frac{4}{i} \right) \left( [1 - \rho]^{4+i} - \rho^{4-i} \right) + (1 - \rho) h,$$

(7)

where the terms on the r.h.s. of Eq. (7) for $i = 0$ are related to a creation/annihilation of unloyal agent in the neighborhood of four unloyal/loyal ones, the terms for $i = 1$ describe a creation/annihilation of unloyal agent in
FIG. 2: The time evolution of spatial distribution of unloyal agents. The back conversions of unloyal agents to loyal ones are excluded. $L = 10$, $M = 16$, $Z = 4$, $R = 8$, $K = 10$, $T = 3$. 
FIG. 3: The time evolution of the average fraction of unloyal agents \( \langle \rho \rangle \) (upper panel) and the average number of tasks per agent \( \langle k \rangle \) (down panel). The back conversions of unloyal agents to loyal ones are excluded. \( L = 10, M = 16, Z = 4, R = 8, T = 3 \). The results are averaged over \( N = 100 \) simulations.

the neighborhood of three unloyal/loyal ones, etc. The last term (field \( h > 0 \)) is a creation of unloyal agent because of an external flow of tasks [25].

For \( h = 0 \) we get three fixed points: \( \rho_0^* = 1, \rho_1^* = 0 \) (both stable) and \( \rho_2^* = \frac{1}{2} \) (unstable). When \( h \) increases to \( h_b \approx 0.06493966 \cdots \), the roots \( \rho_1^* \) and \( \rho_2^* \) merge at the saddle-node bifurcation (see Fig. 6) and the unloyal state \( \rho_0^* = 1 \) remains as the only solution. The same situation appears for the chain of agents and for the Bethe lattice with three neighbors, but with \( h_b \approx \frac{1}{\pi} \). Decrease of \( h_b \) with increasing lattice coordination number seems to be natural, as for larger number of neighbors the transition to unloyal state should appear earlier. Although much details are lost in this description, the basic result—the destabilization of the loyal state with an increasing input of tasks—is reproduced.

B. Hysteretic rules

When back conversion of unloyal agents to loyal ones is excluded it is just a matter of time when the system will be totally overburden. Without agents relaxation to loyalty (\( X = 0 \)) the system inescapably tends to the situation where all agents are unloyal and always have to complete yet \( M \) tasks.

In order to avoid this ‘black scenario’ unloyal agents must have a chance for relaxation to the state of loyalty. However, even when back conversion is allowed for some set of parameters the system ‘jamming’ may occur.

When unloyal agents are allowed to relax to the loyalty the ultimate system fate depends on assumed values of parameters (\( K, T, R \) and \( Z \)). The phase space of parameters \( (K,T,R,Z) \) may be divided into two regions for which system

- either tends to the ‘jamming’ state
  \[ \lim_{t \to \infty} \langle k \rangle = M, \quad \lim_{t \to \infty} \langle \rho \rangle = 1, \]

- or the average number of task awaiting for realization and the average fraction of unloyal agents do not reach their maximal values
  \[ \lim_{t \to \infty} \langle k \rangle < M, \quad \lim_{t \to \infty} \langle \rho \rangle < 1. \]

In terms of statistical physics we can talk about ‘organizational’ phase transition between ‘jamming’ and ‘making-it’ phases. The efficiency of tasks realization in ‘making-it’ phase depends quantitatively on applied relaxation rule [6(A) or 6(B)]. The results of simulations for these two rules will be presented in subsequent two subsections IV B 1 and IV B 2, respectively.
FIG. 5: (Color online) (a) Time evolution of the total number \( L^2 \cdot \langle k \rangle \) of tasks scheduled for realization for various values of maximal agents’ capacity \( M \). (b) The time necessary to make all agents totally overburdened \( \tau \) as a function of parameter \( M \). \( L = 10, Z = 4, R = 4, K = 8, T = 3 \). The results are averaged over \( N = 100 \) independent simulations.

1. Returning to loyalty after completing all tasks \((k = 0)\)

The time evolution of the average fraction \( \langle \rho \rangle \) of loyal agents and the average number \( \langle k \rangle \) of tasks per agent are presented in Figs. 7 and 8, respectively. The results were obtained with the rule 6(A) applied in simulation, i.e., when relaxation to loyalty is possible for an agent who shifted out all of the tasks assigned to her/him. The fraction of unloyal agents varies between \( 14\% < \langle \rho \rangle < 18\% \) for \( Z = 4, R = 8, T = 3 \) in ‘making-it’ phase \((K \leq 7)\) [see Fig. 7a]. For \( K > 7 \) the system reaches a ‘jamming’ phase. Using again statistical physics terminology, we can say that on hyper-plane \((Z = 4, R = 8, T = 3)\) the critical value of parameter \( K_C = 7 \) separates two phases. Similarly, the critical values of \( T_C = 3, R_C = 8 \) and \( Z_C = 4 \) on hyper-planes \((Z = 4, R = 8, K = 7), (Z = 4, T = 3, K = 7) \) and \((R = 8, T = 3, K = 7)\) may be deduced from Figs. 7b, 7c and 7d, respectively. The same critical values of \( K_C, T_C, R_C \) and \( Z_C \) on mentioned above hyper-planes may be observed in Figs. 8a, 8b, 8c and 8d presenting time evolution of the average number \( \langle k \rangle \) of tasks assigned to agents. The system reaches ‘jamming’ phase for \( K > K_C, T > T_C, Z > Z_C \) and \( R < R_C \). Similarly to the case of irreversible transition to the unloyalty (Sec. IV A) the role of maximal capacity parameter \( M \) is qualitatively the same only in this ‘jamming’ phase. For set of model controls parameters corresponding to ‘making-it’ phase the changes of \( M \) parameter do not influence the results of simulations (till \( M \) is larger than thresholds \( R \) and \( T \)).

2. Relaxation to loyal strategy for sufficiently low number of awaiting tasks \((k \leq T)\)

When agents strategy change from unloyal to loyal one occurs as soon as they have less than \( T \) tasks the qualitative picture remains the same, i.e., we observe two regimes in tasks realization in our model working group. The results presented in Figs. 9 and 10 correspond to the step 6(B) instead of 6(A) in our algorithm. The time evolution of the average fraction \( \langle \rho \rangle \) of unloyal working group members and the average number \( \langle k \rangle \) of tasks per agent are presented in Figs. 9 and 10, respectively. The only qualitative difference appears in system response to changing \( R \) and \( T \) parameters: ‘jamming’ phase occurs for small enough values of \( R < R_C \) and large values of \( T > T_C \) when rule 6(A) is applied and it is absent for rule 6(B). The common difference between results of applying rule 6(A) or 6(B) is the average level of loyalty/unloyalty in working group and the average number of tasks awaiting for realization. Selecting for inspection the same hyper-planes as in Sec. IV B 1 we see definitely lower ranges of \( \langle \rho \rangle \) and \( \langle k \rangle \) in ‘making-it’ phase. These differences are presented in Tab. 1. Also the change of
FIG. 7: (Color online). Time evolution of the average fraction $\langle \rho \rangle$ of unloyal agents obtained for 6(A) rule. $L = 10$. The results are averaged over $N = 100$ simulations.
FIG. 8: (Color online). Time evolution of the average number $\langle k \rangle$ of tasks per agent obtained for 6(A) rule. $L = 10$. The results are averaged over $N = 100$ simulations.
The ‘black scenario’ of the system stacking in a ‘jammed’ loyalty. In case of applying hysteretic rules [6(A) or 6(B)] for the parameter \( R \) when rule 6(A) is applied, the transition to the ‘jammed’ state occurs for values of \( R < R_c \). Yet, several results deserve more attention.

First one is related to the role of parameter \( T \), which is the threshold; if a unloyal agent has more than \( T \) tasks, he/she shifts his tasks to his/her neighbors. When the rule 6(B) is applied the larger value of \( T \), the earlier unloyal agents return to loyalty what should result in smoother tasks realization; then it seems that the collective transition to the unloyalty phase should be less likely when \( T \) increases. Yet, the numerical results presented in Fig. 10b indicates that the trend is slightly opposite. On the other hand, we well understand this trend for the rule 6(A) [see Figs. 7b and 8b]. The point is that a unloyal agent does not perform her/his tasks even if their number is less than \( T \). In this super-selfish state agent neither do his/her tasks nor send them to his/her neighbors. He or she is just waiting till having again at least \( T \) tasks and after receiving them above the threshold \( T \) he/she redistributes them among his/her neighbors. Please note however, once tasks are shifted, there is a chance that some of them will be performed if a neighbor is still a loyal agent. This hypothesis may be particularly attractive if we recall clustering of unloyal agents presented in Fig. 2. The tasks shifted by unloyal agents may leave the cluster of unloyal agents as soon as they reach its border.

The second issue is more complex: some numerical results seem to be stable, i.e. they remain constant in a certain range of at least some parameters. An example is the percentage of unloyal agents, shown in Fig. 7a, as dependent on the parameter \( K \). Recalling that \( K \) is a measure of the amount of incoming tasks, we should expect that the amount of unloyal agents increases monotonously with \( K \). More exactly, the stream of incoming tasks is the product \( K \cdot Z \), where \( Z \) is the number of incoming tasks assigned to an agent. We could expect that the amount of unloyal agents should be a function of this product. Indeed, what we observe is that the amount of unloyal agents should be a function of this product. Instead, what we observe is that the amount of unloyal agents increases with \( Z \) but remains stable with \( K \), as shown in Fig. 7a. This puzzle remains to be solved. A plausible hypothesis is that some amount of unloyal agents is functional as they help to transport tasks throughout the system, making the spatial distribution of tasks more homogeneous. Four tasks can be performed by four loyal neighbors in one time step, not in four steps. Within the range of parameters where the stability is observed, the problem of overload is maybe solved locally.

### Table I

| hyper-plane | rule | \(|\rho|\) (%) | \(|k|\) |
|-------------|------|--------------|--------|
| \((Z = 4, R = 8, T = 3)\) | 1–5+6(A) | 14 | 16 | 0.4 | 0.9 |
| | 1–5+6(B) | 0 | 0.5 | 0.5 | 2.5 |
| \((Z = 4, R = 8, K = 7)\) | 1–5+6(A) | 14 | 14 | 0.9 | 0.9 |
| | 1–5+6(B) | 0 | 0.4 | 0.55 | 0.59 |
| \((Z = 4, T = 3, K = 7)\) | 1–5+6(A) | 10 | 16 | 0.75 | 0.85 |
| | 1–5+6(B) | 0 | 0.5 | 0.45 | 0.65 |
| \((R = 8, T = 3, K = 7)\) | 1–5+6(A) | 0 | 14 | 0.1 | 0.9 |
| | 1–5+6(B) | 0 | 2.5 | 0.1 | 0.9 |

### Table II

| \(K_C\) | \(T_C\) | \(R_C\) | \(Z_C\) |
|--------|--------|--------|--------|
| rules 1–5+6(A): | 7 | 3 | 8<sup>a</sup> | 4 |
| rules 1–5+6(B): | 12<sup>b</sup> | | 6 |

<sup>a</sup> For the rule 6(A) the transition to the ‘jammed’ state occurs for \( R < R_C \).

<sup>b</sup> For the rule 6(B) on the hyper-plane \((Z = 4, R = 8, K = 7)\) only ‘making-it’ phase is observed.

<sup>c</sup> For the rule 6(B) on the hyper-plane \((Z = 4, T = 3, K = 7)\) only ‘making-it’ phase is observed.

Critical parameters \( K_C, T_C, R_C \) and \( Z_C \) may be easily observed (see Tab. II).

### V. DISCUSSION AND CONCLUSIONS

In our model working group the crucial point for avoiding system ‘jamming’ is allowing agents for relaxing to loyalty. In case of applying hysteretic rules [6(A) or 6(B)] the ‘black scenario’ of the system stacking in a ‘jammed’ phase may be avoided when

- either the number of agents chosen for task realization \( K \)
- or the number of assigned tasks \( Z \)
- or the threshold value \( R \) of assigned tasks, which force the agent to conversion from loyalty to unloyalty
- or the threshold value \( T \) of tasks assigned to unloyal agents, which force him/her to task redistribution among his/her neighbors

are smartly chosen. The term ‘smartly chosen’ means

- small enough for \( K, T, Z \) parameters and large enough for the parameter \( R \) for the parameter \( R \) when rule 6(A) is applied
- and small enough for \( K \) and \( Z \) parameters for the rule 6(B).
FIG. 9: (Color online). Time evolution of the average fraction $\langle \rho \rangle$ of unloyal agents obtained for 6(B) rule. $L = 10$. The results are averaged over $N = 100$ simulations.
FIG. 10: (Color online). Time evolution of the average number $\langle k \rangle$ of tasks per agent obtained for 6(B) rule. $L = 10$. The results are averaged over $N = 100$ simulations.

(a) $M = 16$, $Z = 4$, $R = 8$, $T = 3$ and various values of $K$

(b) $M = 16$, $Z = 4$, $R = 8$, $K = 7$ and various values of $T$

(c) $M = 16$, $Z = 4$, $T = 3$, $K = 7$ and various values of $R$

(d) $M = 16$, $R = 8$, $T = 3$, $K = 7$ and various values of $Z$
Yet, this hypothesis waits for a confirmation with dedicated numerical tools, as the local correlation functions of the task density.

The third issue is connected with ambiguous role of parameter $R$. It seems reasonable to expect scenario presented in Figs. 7c and 8c, i.e. when increasing of the threshold value $R$ after which loyal agents become unloyal group members helps system in staying in ‘making-it’ phase. However, this natural system response to increasing $R$ is absent when rule 6(B) is applied. Yet, for the variant 6(B), the overall amount of unloyal agents is much smaller. Perhaps a weak increase of $\langle \rho \rangle$ with $R$ is related to a longer lifetime of an unloyal agent, who has to shift more tasks $(R - T)$ to be converted to an loyal agent again. Anyway, this increase seems to be a second-order effect. On the other hand for this rule when scanning $R$ parameter we do not observe a ‘jammed’ phase on $(Z = 4, T = 3, K = 7)$ hyper-plane [see Figs. 9c, 10c and Tab. II].

From the perspective of social simulation, the model and results reported above fall into the category of YA-WOTAS (‘Yet Another Way Of Thinking About Stuff’), i.e. of analogical models rather than explanatory or predictive ones [28]. Yet, this category is shared with most applications based on the technique of CA. According to classic textbooks on social sciences [29], our research can be classified as an exploration; here it deals with mathematical aspects of performance of groups of workers. Our results suggest that the regime where their work is efficient can be stabilized by a tuning of the system parameters. Also we hope that the memory effect captured by hysteretic rules can be inspiring when looking solutions to other problems, even so elusive as those met in social sciences. Yet we should add that our mechanism of switching to the unloyal state by passing duties from a neighbor is more specific than social contagion in general. Therefore, here we do not need to discuss the conditions of an efficient social contagion, formulated in the literature [30, 31].

The model setting and the results can be treated as a social realization of the concept of self-organized criticality [32, 33]. When applied to a large system, these frames suggest a research on the number of ‘topplings’, i.e. events when a given task is passed along a chain of coworkers. In our model, tasks are irreversible and it is only their number which matters. Yet, the unloyal strategy, when applied by the majority of agents, is equivalent to the self-organized critical state, when no task is performed; they are only passed from one agent to another. We note that with the rule 6(A), this collectively unloyal state is absorbing, analogously to the very idea of self-organization. On the other hand, the scale-free distribution of the length of the above remarked chains has been discussed in Ref. [34] as a result of putting tasks with lower priority off. These analogies promisingly link theory of organizations to current problems of statistical mechanics.

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