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Thermal Roots of Correlation-Based Complexity

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Thermal Roots of Correlation-Based Complexity

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Bayesian maxent lets one integrate thermal physics and information theory points of view in the quantitative study of complex systems. Since net surprisal (a free energy analog for measuring “departures from expected”) allows one to place second law constraints on mutual information (a multimoment measure of correlations), it makes a quantitative case for the role of reversible thermalization in the natural history of invention, and suggests multiscale strategies to monitor standing crop as well. It prompts one to track evolved complexity starting from live astrophysically observed processes, rather than only from evidence of past events. Various gradients and boundaries that play a role in availability flow, ranging from the edge of a wave-packet to the boundary between idea-pools, allow one to frame wide-ranging correlations (including that between a phenomenon and its explanation) as delocalized physical structures.

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Key Words: correlations; niche networks; information engines; maximum entropy

1. INTRODUCTION

Chaisson in this journal recently discussed the evolution of complex systems and its empirically observed correlation with free-energy density [1]. This paper is partly about the need he cites for putting such energy flows into an agreeable information-theory context. However, “perched on the dawn” is not the whole story, since humans also face a decline in one of their major sources of free energy [2]. Thus as we (for reasons that have nothing to do with fossil fuels [3]) move past the prime of earth’s “age of plants and animals,” a quantitative look at what we want to protect about evolved complexity is timely as well.

Historically, information theory since the days of Shannon and Weaver [4] and Jaynes [5, 6] saw entropy and ensembles as tools for applying gambling theory

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interdisciplinary umbrella for relating versa. you learn about A by knowing B, and vice versa. Mutual information is defined as what which energy and mass might flow, etc. regions or control volumes in and out of not be occupied with particles), or as individual particles, as collections of particles, which may or may not be occupied with particles), or as regions or control volumes in and out of which energy and mass might flow, etc. Mutual information is defined as what you learn about A by knowing B, and vice versa.

In Mathematical terms, one can say for subsystems A and B that mutual information

$$M[A:B] \equiv S_A + S_B - S_{AB}, \quad (1)$$

where $S_A$ and $S_B$ are uncertainties associated with each system taken alone, while $S_{AB}$ is uncertainty about the combination given all available information, including that associated with correlations. It is relatively easy [22] to prove that $M[A:B] \geq 0$. In a sense, therefore, $M[A:B]$ is a quantitative measure of how systems A and B taken together may be more than the sum of their parts.

For example, imagine that you have two drawers (A and B) for your socks. Suppose you know that each drawer contains $N$ socks, and that the socks are identical except they are either black or white with equal probability, the amount of uncertainty (average surprisal) about the socks in A and B given this information is $2N$ bits, i.e. $k \ln(2)$ or one bit for each sock. However, if you also know that socks were put into the drawers by breaking up matched pairs, one into each drawer, knowledge of the content of one drawer will also tell you what is in the other. Hence, the “two drawer” (whole system) uncertainty is reduced to $N$ bits. The added knowledge about how the drawers were filled therefore provides $2N$ minus $N$, or $N$ bits of mutual information.

2. Augmenting the Sum of Parts
Let us begin with correlation itself, i.e. with quantitative ways of seeing the whole as more than the sum of its parts. Statistics courses often focus on second-moment pair, or variance-based, measures like correlation-coefficient and covariance. The focus here instead is on logarithmic (e.g. bitwise) measures of correlation, like mutual information, which a priori at least operate on all scales.

In the context of general system theory [21], begin by defining two subsystems A and B, for example, as individual particles, as collections of particles, as individual states (which may or may not be occupied with particles), or as regions or control volumes in and out of which energy and mass might flow, etc. Mutual information is defined as what you learn about A by knowing B, and vice versa.

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from the process by which the drawers were filled, or from a peek at their contents. Entropy increases in deterministic model physical systems in fact depend on the tossing out of correlations between subsystems too small to investigate with “a peek.” Thus, entropy increases in physical systems can be seen as a loss of mutual information between those systems and their environment.

For example consider a single-atom bipartitioned cell like the $N$-atom cell in Figure 1. Using one of the pistons to capture that one ideal gas atom on either side of the removable partition decreases the entropy of the gas by $k \ln(2)$ or one bit. If an observer is taking notes, it can also increase the mutual information between that gas and its environment by one bit since they can afterwards answer the true–false question correctly: In which of the two partitions does the atom reside?

To take a more general view, begin with system A having $N$ accessible states so that a priori uncertainty about A is $S_A = k \ln N$. Then consider an observer B, with sufficient added information about A to limit the number of accessible states to $\Omega < N$. Observer B therefore has uncertainty about A of $S_{A/B} = k \ln \Omega$. What we can learn about A by knowing B also is then the mutual information between B and A, i.e. $M[A:B] = S_A - S_{A/B}$. The second law assertion that $S_{A/B}$ can increase but not decrease with time if systems A and B are isolated, thus also says quite generally that the mutual information between isolated systems A and B can decrease but not increase with time.

Even though delocalized mutual information and physical entropy are connected in this way, the latter is historically treated as an extensive property “distributed locally” throughout the system. As shown by Eq. (1), uncertainty may be treated as extensive i.e. as a sum of parts, as long as subsystem correlations within that system (mutual information) may be ignored on the
This is the case with many thermodynamic probabilities by defining for each probability type, the ‘surprisal’ as

\[ S = k \ln \Omega \]

where \( \Omega \) is the effective number of accessible states. The relationships described here translate seamlessly into quantum mechanical terms [6].

3. MAXENT AND NET SURPRISAL

A robust tool for estimating uncertainties in the face of added information might be called “the maxent best-guess machine.” The added information is normally written as “expected averages,” although the approach in principle can accommodate a wide range of added information types.

One first writes entropy in terms of probabilities by defining for each probability \( p_i \) a “surprisal” \( s_i = k \ln[1/p_i] \), in units determined by the value of \( k \). The average value of this surprisal reduces to \( S = k \ln \Omega \) when the \( p_i \) are all equal, making it simply a logarithmic measure of the effective number of accessible states \( \Omega \equiv e^{S/k} \). The relationships described here translate seamlessly into quantum mechanical terms [6].

3.1. Availability Minimization

The standard problem and its solution has been reiterated myriad times since its statement by Jaynes. The bottom line is that the problem of entropy maximization under expectation-value constraints can be recast through Lagrange’s method of undetermined multipliers as a constraint-free minimization of generalized availability in information units. This minimized availability (defined without reference to any specific physical system or conservation laws), for example [23], becomes the appropriate free energy for any given thermodynamic ensemble when divided by energy’s Lagrange multiplier \( 1/kT \).

3.2. Departures and Net Surprisal

Although availability (hence free energy) minimization is quite useful for locating the most likely states, finite departures from expected are better measured with whole system changes in uncertainty relative to a reference state [24, 25], or net surprisals. Net surprisal (also called the Kullback–Leibler divergence [26] or relative entropy [27]) is defined as

\[ I_{\text{net}} = -k \sum_{i=1}^{n} p_i \ln \left( \frac{p_{\text{ref}}}{p_i} \right) \geq 0, \]

where \( p_{\text{ref}} \) is the state probability based only on ambient state information, while \( p_i \) takes into account all that is known. From the solution to the standard problem mentioned earlier, one can then show under fairly general conditions near ambient [23, 25] that derivatives of availability under ensemble conditions are also derivatives of net surprisal.

For example, systems in thermal contact with an ambient temperature bath may be treated as canonical ensemble systems with constrained average energy. Thus, a temperature deviation from ambient \( T_o \) for a monatomic ideal gas gives for that system

\[ \frac{dW}{dV} = \frac{2N}{k} \frac{dT}{T}, \]

where \( \bar{x} = x - 1 - \ln x \geq 0 \). If that system is also in contact with an ambient pressure bath (i.e. able to randomly share volume and energy), volume deviations add \( N \bar{x} \). For grand canonical systems whose molecule types might change (e.g. via chemical reaction), one instead adds \( N \bar{x} \Xi \bar{x} \) for each molecule type \( j \) whose concentration varies from ambient [23, 25, 28].

Even more specifically, a problem offered to intro physics students at the University of Illinois asked how cool the room must be for an otherwise unpowered device to take boiling water in at the top, and return it as ice water at the bottom. Since the 2nd Law allows conversion of one form of net surprisal reversibly into another (famously without providing any clues how to pull it off), one can simply...
set net surprisals equal for the initial and final states, and then solve for $T_{room}$.

Of course, the net surprisal measure is not only relevant to inference about systems for which physically conserved energy is of interest. For example, it meets the requirements for an information measure proposed by Gel-Mann and Lloyd [29], and includes the Shannon information measure discussed there as a special case. It can also be useful in applied statistics, as in the assessment of student responses to multiple-choice test questions [23].

3.3. A Special Case

More importantly, the mutual information measure defined earlier is a special case. To see this, again consider two subsystems. The joint probability that system $I$ is in state $i$ and system $J$ is in state $j$ might be written $p_{ij}$ ≥ 0. This obeys $\sum_i \sum_j p_{ij} = 1$, where the $i$ indices are run over all possible states for subsystem $I$, while the $j$ indices are run over all possible states for system $J$. From the joint probabilities one can calculate marginal probabilities like $p_i(\equiv \sum_j p_{ij})$, which ignore the state of other subsystems. From these probabilities, the values for joint entropy $S_{IJ}/k \equiv (-\ln p_{ij})$, and marginal entropies like $S_{I}/k \equiv (-\ln p_i)$, follow immediately.

Mutual or correlation information between systems $I$ and $J$, denoted here as $M_{IJ}$ and defined by Eq. (1) as the sum of marginal entropies $S_{I} + S_{J}$ minus the joint entropy $S_{IJ}$ thus becomes

$$M_{IJ} = -k \sum_i \sum_j p_{ij} \ln \frac{p_{ij}p_i}{p_j} \geq 0.$$  \hfill (3)

From Eq. (2), it is easy to see that mutual information is the net surprisal that follows on learning that two systems (here $I$ and $J$) are not independent.

One interesting feature of such correlation measures is that they refer to the relationship between system $A$ and system $B$, and thus may be quite independent of models for system $A$ or $B$ per se. One can also express Bar-Yam's multiscale complexity measures [30, 31] by combining subsystem mutual information terms, without reference to uncertainty about the state of individual subsystems taken separately. For instance with three binary variables (e.g. Ising model spins $I, J$, and $K$), the amount of “intermediate scale complexity” $C(2)$ associated with two or more spins can be written as

$$M_{IJK} + M_{JKI} + M_{IKJ} - M_{IIK}, \hfill (4)$$

where $M_{IJK}$ is the three-system (all-scale) mutual information. Similarly the “large scale complexity” $C(3)$ can be written as

$$2M_{IJK} - M_{IIK} - M_{IKJ} - M_{JKI}, \hfill (5)$$

These relationships generalize nicely for $N$ spins, via the fact that the sum over size-scales from 2 to $N$ is the $N$-system mutual information. Such mutual information expressions also, as discussed earlier, connect such multiscale measures to constraints provided by the second law.

Hence the maxent formalism allows us to connect the isolated-system 2nd law to observations like “If Jimmy and Alice did not talk to each other, there is no way Jimmy could have known what Alice was planning to do.” This assertion is about correlations between subsystems (Jimmy and Alice) that are quite independent of one's thermodynamic models for Jimmy and Alice, as are the multiscale complexity measures $C_2$ and $C_3$ given earlier. Thus armed with statistical inference tools that underpin traditional thermodynamic applications, but which require no physical assumptions a priori short of some state invariants, we now take a look at some of the more complex system areas where applications (already underway in many fields) will likely continue to develop.

Examples of correlated subsystem pairs include photon or electron pairs with opposite but unknown spins, a single strand of messenger RNA and the sequence of nucleotides in the gene from which it was copied, a manuscript and a copy of that manuscript created with a xerox machine (or a video camera), your understanding of a subject before being given a test and the answer key used by the teacher to grade that test (hopefully), enzymes and coenzymes with site specificity, tissue sets treated as friendly by your immune system, metazoans who developed from the same genetic blueprints (e.g. identical twins), families that share similar values, and cities which occupy similar niches in different cultures (e.g. sister cities). However, does this relationship between such complex systems and the homogeneous systems of physical thermodynamics have any consequence in practice?

4. Correlation Physics

The amount of entropy associated with the flow of thermal energy often dwarfs that associated with the flow of information, per se. For example, the 2nd law dumps 1/40 eV per nat of erased memory into a room temperature ambient, but this is negligible compared to other sources of heat in present day computers. Thus thermodynamics is seldom today a direct hindrance to information flow.

On the other hand, more light on a subject (literally free energy to work with) rarely hinders discovery. Perhaps the most striking evidence of this practical connection between thermal physics and complex systems comes from the correlation illustrated by Chaisson, between evolved complexity and free-energy flux density [1]. The latter is a measure of the rate at which ordered energy is being thermalized (for the most part irreversibly). The former is evidence, on the contrary, that correlations are being effectively created and preserved.

Mutual information (e.g. that two spins are correlated, or that two gases have not been well mixed) also plays a well-known role in physical systems.
That the only Zurek [40] among others suggests generalizing the Maxwell’s demon problem of the parents’ socks in another.

Changes in energy, such as the job of considering engines whose primary function is simply the ambient dynamic cost, in energy per nat of correlation information, is simply the ambient temperature, and around 60 bits eV of energy if operating near the freezing point of water. The equations for these engines are compared graphically to those of heat engines and heat pumps in Figure 2.

Cameras, tape recorders, and copying machines may be considered such information engines, as are forms of life that take in chemical energy available for work from plant biomass, and thermalize that energy at ambient temperature while creating correlations between objects in their environment and their survival needs, or in the form of thermalized work at room temperature. In binary information units, this amounts to producing about 55 bits of information per electron volt of thermalized work at room temperature, and around 60 bits eV of energy if operating near the freezing point of water. The equations for these engines are compared graphically to those of heat engines and heat pumps in Figure 2.

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complementary to the survival of groups of individuals.

6. MONITORING CORRELATIONS
6.1. Inventories of Standing Crop
Perhaps the simplest thing to do is count. Doing this objectively in practice, of course, is far from trivial. Further, more detailed consideration (e.g., enumeration of alternatives) may be necessary to put such measures into 2nd law terms. Nonetheless, one might sketch the outline of correlation measures in the following way across three quite different levels of complex system organization: the state of a molecule, a metazoan, and a community of individuals.

- $N_{\text{correlated states/fermion}} \times N_{\text{fermions/molecule}}$
- $N_{\text{assignments/molecule}} \times N_{\text{molecules/metazoan}}$
- $N_{\text{niches/metazoan}} \times N_{\text{metazoans/community}}$

Presumably the first term in each case has an upper limit. For example, assume that individual elements each occupy no more than one correlated state directed inward and/or outward from each of the physical boundary types that comprises the level of which they are part.

Thus with a community of individuals we might conceptualize niches as focussed inward and outward with respect to the boundaries of self (physical skin), family (gene-pool), and culture (meme-pool). Although the latter two boundaries between groups of correlated codes are geometrically complicated (to say the least), they are physical boundaries nonetheless. Roles taking care of self (skin IN), friends (skin OUT), family (gene-pool IN), hierarchy (gene-pool OUT), culture (meme-pool IN), and profession (meme-pool OUT) have thus developed, as have (respectively) the related lore and participant/leadership obligations of patient/doctor, colleague/mentor, sibling/parent, citizen/leader, dancer/priest, and professional/scholar. In other words, this inventory for the case of a community simply asks: “In how many of these six areas are individuals, on average, fortunate enough to be able to make a name for themselves?” If this is decreasing, things are perhaps getting worse for the community, independent of what other indicators have to say.

On the level of metazoan, we might similarly consider molecule assignments pointing inward and outward with respect to molecule surface, cell-membrane, and organ. For example, hormone molecules required to convey signals from one organ system to
another might be seen as charged with an intertissue (organ OUT) assignment.

On the level of an individual molecule, the relevant boundaries for correlated fermion states might be the fermion wave-packet, the appropriate nuclear or electronic shell, and the atom comprising the molecule. Electrons involved in covalent bonds might in this sense be involved in correlations directed outward from the atom to which they were initially assigned. Internal to atoms, counting correlations in second-law (mutual information) terms may be easier still. For example, the mutual information between upspin and downspin electrons comprising a He atom’s K-shell is simply one bit.

In the expressions mentioned earlier, the number of states/assignments/niches per agent is an average, so that each total can also be determined by a sum of all the states/assignments/niches in the larger unit. Each of the three bullets given earlier, in sequence, attempts to estimate the correlation-information associated with order on a larger size scale. Thus each assignment of a metazoan molecule builds on a certain number of correlated fermion-states within that molecule, just as each niche for a community metazoan will build on a certain number of correlated molecule-assignments within that metazoan. Nonetheless each assignment or niche, as an emergent phenomenon, is quite distinct from (hence something more than) the sum of its constituent states or assignments, respectively.

### 6.2. Process Indicators

Ultimately, the goal is to look at the rates at which the aforementioned correlations are created, minus the rates at which they are lost. One strategy, of
7. SYNTHESE AND CONCLUSIONS

Here we have discussed net surprisal tools that can be applied to mutual information between parts of more complex systems, independent of the existence of models for individual subsystems per se. Such applications were conspicuously launched by Shannon in the late 1940s. Although their application in everyday and cutting-edge applications is expanding today at an even faster rate, awareness of a crossdisciplinary foundation for applying them remains incomplete.

We also discuss the utility of net surprisals for treating reversible equilibration problems (like a device for reversibly converting hot water into cold), and suggest that physical boundaries in complex systems (like code-pool boundaries) provide a platform for the quantitative monitoring of subsystem correlations in systems as complex as human communities today. The hard work of putting these observations to use is ahead.

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REFERENCES

1. Chaisson, E.J. Complexity: An energetics agenda. Complexity 2004, 9, 14.
2. Musser, G. The climax of humanity. Sci Am 2005, 293, 44.
3. Ward, P.D.; Brownlee, D. The Life and Death of Planet Earth; Copernicus: New York, 2000.
4. Shannon, C.E.; Weaver, W. The Mathematical Theory of Communication; University of Illinois Press: Urbana, IL, 1949.
5. Jaynes, E.T. Information theory and statistical mechanics. Phys Rev 1957, 106, 620.
6. Jaynes, E.T. Information theory and statistical mechanics II. Phys Rev 1957, 108, 171.
7. Kuhn, T.S. The Structure of Scientific Revolutions, 2nd ed.; University of Chicago Press: Chicago, IL, 1970.
8. Grandy, J.W.T. Foundations of Statistical Mechanics, Vols. I and II; D. Reidel Publishing: Dordrecht, 1987.
9. Plischke, M.; Bergersen, B. Equilibrium Statistical Physics; Prentice Hall: New York, 1989.
10. Garrod, C. Statistical Mechanics and Thermodynamics; Oxford University Press: New York, 1995.
11. Reif, F. Fundamentals of Statistical and Thermal Physics; McGraw-Hill: New York, 1965.
12. Katz, A. Principles of Statistical Mechanics: The Information Theory Approach; W. H. Freeman: San Francisco, 1967.
13. Giffin, B.A. Statistical Physics of Materials; Wiley: New York, 1973.
14. Kittel, C.; Kroemer, H. Thermal Physics; W. H. Freeman: New York, 1980; p 39.
15. Stowe, K. Introduction to Statistical Mechanics and Thermodynamics; Wiley: New York, 1984; p 116.
16. Baierlein, R. Thermal Physics: Cambridge University Press: New York, 1999.
17. Schroeder, D.V. An Introduction to Thermal Physics; Addison-Wesley: San Francisco, 2000.
18. Marijuan, P.C. First conference on foundations of information science. Biosystems 1996, 38, 87.
19. Castle, J.; Emmenish, W.; Henkes, R.; Miller, R.; Rayne, J. Science by Degrees: Temperature from Zero to Zero (Westinghouse Search Book Series); Walker and Company: New York, 1965.
20. Moore, T.A. Six Ideas that Shaped Physics: Unit T; McGraw-Hill: New York, 1998.
21. von Bertalanffy, L. General Systems Theory: Foundations, Development, Applications; G. Braziller: New York, 1968.
22. Lloyd, S. Use of mutual information to decrease entropy. Phys Rev A 1989, 39, 5378.
23. Fraundorf, P. Information Physics: From Energy to Codes, 2003; arXiv: physics/0611022.
24. Evans, R.B. PhD Dissertation; Dartmouth College: Hanover, NH, 1969.
25. Tribus, M.; McIrvine, E.C. Energy and information. Sci Am 1971, 224, 179.
26. Kullback, S.; Leibler, R.A. On information and sufficiency. Ann Math Stat 1951, 22, 79.
27. Badil, R.; Politi, A. Complexity: Hierarchical structures and Scaling in Physics; Cambridge University Press: Cambridge, UK, 1997.
28. Fraundorf, P. In: Proceedings 5th International Conference on Complex Systems; Bar-Yam, Y, Eds.; Perseus/InterJournal: Reading, MA, 2004.
29. Gell-Mann, M.; Lloyd, S. Information measures, effective complexity, and total information. Complexity 1996, 2, 44.
30. Bar-Yam, Y. Multiscale complexity/entropy. Adv Complex Syst 2004, 7, 47.
31. Gheorghiu-Svirschevska, S.; Bar-Yam, Y. Multiscale analysis of information correlations in an infinite-range, ferromagnetic Ising system. Phys Rev E 2004, 70, 066115.
32. Brillouin, L. Science and Information Theory, 2nd ed.; Academic Press: New York, 1962.
33. Scully, M.O.; Zubairy, M.S.; Agarwal, G.S.; Walther, H. Extracting work from a single heat bath via vanishing quantum coherence. Science 2003, 299, 862.
34. Schneider, T.D. Theory of molecular machines. II. Energy dissipation from molecular machines. J Theor Biol 1991, 148, 125.
35. Schneider, T.D. Evolution of biological information. Nucleic Acids Res 2000, 28, 2794. http://www-lmmb.ncifcrf.gov/toms/.
36. Wootters, W.K.; Zurek, W.H. A single quantum cannot be cloned. Nature 1982, 299, 802.
37. Gottesman, D.; Lo, H.-K. From quantum cheating to quantum security. Phys Today 2000, 53, 22, quant-ph/0111100.
38. Grosse, I.; Herzel, H.; Buldyrev, S.V.; Stanley, H.E. Species independence of mutual information in coding and noncoding DNA. Phys Rev E 2000, 61, 5624.
39. Bennett, C.H. Demons, engines and the second law. Sci Am 1987, 257, 108.
40. Zurek, W.H. In: 1989 Lectures in Complex Systems, Jen, E., Ed.; Addison-Wesley: Redwood CA, 1989. No. II in Lectures in the Sciences of Complexity; pp. 49–65.
41. Penrose, O. Foundations of Statistical Mechanics; Pergamon Press: Oxford, 1970; pp. 220–230.
42. Odum, E.P. Fundamentals of Ecology 3rd ed.; W. B. Saunders: Philadelphia, 1971.
43. Chaisson, E.J. The cosmic environment for the growth of complexity. Biosystems 1998, 46, 13.