CORRELATION BETWEEN THE HALO CONCENTRATION $c$ AND THE VIRIAL MASS $M_{\text{vir}}$ DETERMINED FROM X-RAY CLUSTERS

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ABSTRACT

Numerical simulations of structure formation have suggested that there exists a good correlation between the halo concentration $c$ (or the characteristic density $\delta_c$) and the virial mass $M_{\text{vir}}$ for any virialized dark halo described by the Navarro, Frenk, & White profile. In this Letter, we present an observational determination of the $c$-$M_{\text{vir}}$ (or $\delta_c$-$M_{\text{vir}}$) relation in the mass range of $\sim 10^{11} M_\odot < M_{\text{vir}} < 10^{16} M_\odot$ using a sample of 63 X-ray luminous clusters. The best-fit power-law relation, which is roughly independent of the values of $\Omega_m$ and $\Omega_\Lambda$, is $c \propto M_{\text{vir}}^{-0.5}$ or $\delta_c \propto M_{\text{vir}}^{-1.2}$, indicating $n \approx -0.7$ for a scale-free power spectrum of the primordial density fluctuations. We discuss the possible reasons for the conflict with the predictions by typical cold dark matter models.

Subject headings: cosmology: theory — galaxies: clusters: general — X-rays: galaxies

1. INTRODUCTION

High-resolution simulations of structure formation have suggested that the virialized dark matter halos with masses spanning several orders of magnitude should follow a universal density profile (Navarro, Frenk, & White 1995, hereafter NFW):

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^{3}}$$,

where $\rho_s$ and $r_s$ are the characteristic density and length, respectively. The former is also related to the characteristic density $\delta_c$ and the critical density $\rho_c = (3H_0^2/8\pi G)\Omega_m(z)$ of the universe at redshift $z$ through $\rho_s = \delta_c \rho_c$, in which $\Omega_m(z) = (1 + z)^3(\Omega_m/\Omega_m(z))$, where $\Omega_m$ is the cosmic density parameter. Although the NFW profile was first obtained in the standard cold dark matter (SCDM) model, subsequent numerical studies have shown that this density profile is independent of mass, the initial density fluctuation, and cosmology (e.g., Cole & Lacey 1996; Navarro, Frenk, & White 1997; Eke, Navarro, & Frenk 1998; Jing 1999). The two free parameters in the NFW profile can be determined from the halo concentration $c = r_s/\Delta$ and the virial mass $M_{\text{vir}}$ if the overdensity of the dark matter with respect to the average value is $\Delta_c$:

$$\delta_c = \frac{\Delta_c}{3} \left( 1 + c \right) - c l(1 + c)^{-1}$$

$$r_s = \frac{1.626 \times 10^{-3} (M_{\text{vir}}/M_\odot)^{1/3} (\Delta_c/200)^{-1/3}}{c} \times \left( \frac{1}{\Omega_m} \right)^{1/3} h^{-2/3} \text{Mpc}$$.

Based on numerical simulations, it has been well established that a good correlation exists between the halo concentration $c$ (or the characteristic density $\delta_c$) and the virial mass $M_{\text{vir}}$ for any particular cosmology (e.g., Navarro, Frenk, & White 1996, 1997; Salvador-Solé, Solanes, & Manrique 1998). For example, given $\Delta_c = 200$, in the mass range of $3 \times 10^{11} M_\odot < M_{\text{vir}} \leq 3 \times 10^{15} M_\odot$, the $c$-$M_{\text{vir}}$ relation can be well fitted by a power-law function: $c = a(M_{\text{vir}}/M_\odot)^b$, with $(a, b) = (891, -0.14)$ and $(186, -0.10)$ for the SCDM model ($\Omega_m = 1.0$, $\Omega_\Lambda = 0$, $h = 0.5$, and $\sigma_8 = 0.65$) and the ΩCDM model ($\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$, $h = 0.75$, and $\sigma_8 = 1.3$), respectively. While the NFW profile and the $c$-$M_{\text{vir}}$ correlation are determined only from simulations for halo masses as small as $M_{\text{vir}} = 10^{11} M_\odot$ or as large as $M_{\text{vir}} = 10^{15} M_\odot$, there is no reason to believe that these results would not be valid for halos that are 1 order of magnitude lower or higher in mass (Burkert & Silk 1999).

On the other hand, observationally we can only measure the distribution of baryonic matter rather than the distribution of the dark halo. It is crucial to link numerical predictions with astronomical observations. In a fully virialized system, the distribution and motion of baryonic matter can be determined by the underlying gravitational potential of the dark halo, if the velocity dispersion or temperature profile can be well measured. It is thus possible to work out the dynamical properties of the dark matter halo, such as $M_{\text{vir}}$, $r_s$, and $c$ (or $\delta_c$), from the observed distribution of luminous matter in a gravitationally bound system. This will eventually allow us to examine observationally whether there is a correlation between $c$ (or $\delta_c$) and $M_{\text{vir}}$. Finally, a comparison of the observationally determined and numerically simulated $c$-$M_{\text{vir}}$ (or $\delta_c$-$M_{\text{vir}}$) correlations may provide a useful way to distinguish different cosmological models. In this Letter, we will make an attempt for the first time to derive the $c$-$M_{\text{vir}}$ and $\delta_c$-$M_{\text{vir}}$ correlations on cluster scales by fitting the observed X-ray surface brightness profiles of clusters to those predicted by the NFW profile as the cluster dark halos under the assumption of isothermality. Note that there is a striking similarity between the distribution of intracluster gas tracing the dark halo of the NFW potential and the conventional $\beta$-model. Makino, Sasaki, & Suto (1998) have explicitly shown that the NFW profile via isothermal hydrostatic equilibrium results in an analytic form of gas distribution:

$$n_{\text{gas}}(r) = n_{\text{gas}}(0)e^{-\alpha(1 + r/r_s)^{\mu(r/r_s)}}$$,

in which $\alpha = 4\pi G\mu m_p R^2/kT$, and $\mu = 0.585$ denotes the mean molecular weight. Briefly, our task is to determine the best-fit parameters of $\alpha$ and $r_s$ for an ensemble of X-ray clusters in terms of equation (4) and then to derive the relevant parameters.
clusters have high X-ray luminosity, \( L_X > 10^{45} \) ergs s\(^{-1} \), and a large fraction of them also have relatively high redshift, \( z \approx 0.1-0.3 \). The best-fit values of \( \alpha \) and \( r \) for all 36 EF clusters have been given by EF for a cosmological model of \( H_0 = 50 \) km s\(^{-1} \) Mpc\(^{-1} \) and \( \Omega_m = 1 \). A conversion of \( r \) into the values in the different cosmological models should be properly made, if needed. The MME clusters are taken from an X-ray flux-limited sample, and thereby they are located at relatively small redshift. We perform the \( \chi^2 \) fit of the observed X-ray surface brightness profiles of the MME clusters to the theoretical prediction \( L_X \propto \int n_e^2 dl \) according to thermal bremsstrahlung, where the integral is made along the line of sight. A cross identification reveals that 16 clusters are listed in both samples, for which our fitted values of \( \alpha \) and \( r \) are fairly consistent with those obtained by EF. We further require that the X-ray temperature should be observationally available, and we take the temperature data from Wu, Xue, & Fang (1999, and references therein). The final merged EF and MME sample that is to be used in our following analysis contains 63 clusters.

3. The \( c-M_{\text{vir}} \) and \( \delta-M_{\text{vir}} \) Correlations

We confine ourselves to a flat cosmological model with \( \Omega_m + \Omega_{\Lambda} = 1 \). In this case, the density contrast depends on the value of \( \Omega_m \) and can be approximated by \( \Delta = 178\Omega_m(z)^{0.45} \). The best-fit values of \( \chi^2 \) for the EF clusters, \( \Omega_m(z) = (1 + z)^2/[(1 + z)^2 - 1]\Omega_m \), and \( \Omega_m(z) = \Omega_m(1 + z)^2/\Omega_m \) (Eke et al. 1998). For a given cosmological model \( \Omega_m, \Omega_{\Lambda} \), we convert our best-fit values of \( \alpha \) and \( r \) into \( \delta \), and then obtain the concentration parameter \( c \) and the virial mass \( M_{\text{vir}} \) by numerically solving equations (2) and (3). In Figure 1, we display an example of the resultant \( c \) and \( \delta \), versus \( M_{\text{vir}} \) for \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \). It is apparent that there exists a strong correlation between \( c \) (or \( \delta \)) and \( M_{\text{vir}} \). We employ the Monte Carlo simulations and the \( \chi^2 \)-fitting method to obtain the best-fit \( c-M_{\text{vir}} \) and \( \delta-M_{\text{vir}} \) relations that are assumed to be power laws. This enables us to include the measurement uncertainties in both axes. Note that for the EF clusters, we have not accounted for the uncertainties arising from the fitted parameters of \( \alpha \) and \( r \), since EF provided no information about their error estimates. The results for a set of cosmological models are listed in Table 1. Additionally, we have tried the orthogonal distance regression technique ODRPACK (e.g., Feigelson & Babu 1992) and reached a steeper power index. For example, in the case of \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \), we find that \( c = 10^{15.47} \pm 0.03 \Omega_m(M_{\text{vir}}/M_\odot)^{-0.52} \), and \( \delta = 10^{16.15} \pm 0.28 \Omega_m(M_{\text{vir}}/M_\odot)^{-0.77} \) (see Fig. 1). Although ODRPACK can simultaneously account for the scatters around \( M_{\text{vir}} \), and \( c \) and \( \delta \), the goodness of the fit has not been improved for our problem in the sense that the reduced \( \chi^2 \) for both the \( c-M_{\text{vir}} \) and \( \delta-M_{\text{vir}} \) relation are significantly larger than those obtained using the \( \chi^2 \)-fitting method along with Monte Carlo simulations. In Table 1, we have also given the power index for a scale-free power spectrum of initial density fluctuations implied by our best-fit \( \delta-M_{\text{vir}} \) relation: \( n \propto \delta^{-3/2} \) (NFW), which yields \( n \approx -0.7 \).

| \( \Omega_m \) | \( \Omega_{\Lambda} \) | \( c-M_{\text{vir}} \) Correlation | \( \delta-M_{\text{vir}} \) Correlation | \( n \) |
|----------|----------|----------------|----------------|------|
| 0.15     | 0.85     | \( c = 10^{17.0(0.04)}(M_{\text{vir}}/M_\odot)^{-0.2} \) | \( \delta = 10^{20.77(0.14)}(M_{\text{vir}}/M_\odot)^{-1.19} \) | -0.72 \( \pm 0.20 \) |
| 0.30     | 0.70     | \( c = 10^{18.2(0.06)}(M_{\text{vir}}/M_\odot)^{-0.3} \) | \( \delta = 10^{21.83(0.14)}(M_{\text{vir}}/M_\odot)^{-1.15} \) | -0.70 \( \pm 0.22 \) |
| 0.50     | 0.50     | \( c = 10^{19.6(0.04)}(M_{\text{vir}}/M_\odot)^{-0.25} \) | \( \delta = 10^{20.99(0.12)}(M_{\text{vir}}/M_\odot)^{-1.16} \) | -0.68 \( \pm 0.24 \) |
| 0.70     | 0.30     | \( c = 10^{20.3(0.02)}(M_{\text{vir}}/M_\odot)^{-0.25} \) | \( \delta = 10^{21.92(0.10)}(M_{\text{vir}}/M_\odot)^{-1.16} \) | -0.68 \( \pm 0.26 \) |
| 1.00     | 0.00     | \( c = 10^{21.3(0.06)}(M_{\text{vir}}/M_\odot)^{-0.31} \) | \( \delta = 10^{22.01(0.14)}(M_{\text{vir}}/M_\odot)^{-1.17} \) | -0.66 \( \pm 0.28 \) |
4. DISCUSSION AND CONCLUSIONS

In a virialized system, the distribution and motion of galaxies and intracluster gas (if their self-gravity is negligible) are determined by the underlying gravitational potential of the dark matter halo, provided that the velocity dispersion and temperature profiles are well measured. Therefore, one can probe the dynamical properties of the dark matter halo, even though it is invisible, by using optical/X-ray observations coupled with the hydrostatic equilibrium hypothesis. In this Letter, we have made an attempt to derive the halo concentration $c$, the characteristic density $\delta_c$, and the virial mass $M_{\text{vir}}$ for galaxy clusters characterized by the NFW profile from the observed distribution and temperature of X-ray-emitting gas. Although the correlation between $c$ (or $\delta_c$) and $M_{\text{vir}}$ has been well predicted from numerical simulations, this is the first time that the $c$-$M_{\text{vir}}$ and $\delta_c$-$M_{\text{vir}}$ correlations have been determined by making use of the real data from astronomical observations.

The correlation between $c$ (or $\delta_c$) and $M_{\text{vir}}$ established in this Letter is applicable to massive halos in the mass range of $10^{14} M_\odot < M_{\text{vir}} < 10^{16} M_\odot$. However, we notice that the resultant slope ($\approx -0.5$) of the $c$-$\log M_{\text{vir}}$ relationship is significantly steeper, while the spectrum ($n \approx -0.7$ $\pm$ 0.3) of the primordial density fluctuations is much flatter than the values predicted by typical CDM spectra for the mass halos with $3 \times 10^{11} M_\odot < M_{\text{vir}} < 3 \times 10^{15} M_\odot$ (Burkert & Silk 1999), especially on cluster scales in which $n \approx -2$ (e.g., Henry & Arnaud 1991; Mathiesen & Evrard 1998; Donahue & Voit 1999; Mahdavi 1999). If our results are not a statistical fluke (note the large dispersion of the X-ray data points in Fig. 1), the above conflict may imply that we should abandon at least one of our working hypotheses: (1) intracluster gas is in hydrostatic equilibrium; (2) intracluster gas has an isothermal temperature profile; (3) the self-gravity of intracluster gas is considerably small when compared with the contribution of the dark matter halo; and (4) the NFW profile provides a precise description of the dark matter distribution. Yet further investigations should be made toward obtaining a robust constraint on the $c$-$M_{\text{vir}}$ and $\delta_c$-$M_{\text{vir}}$ relations before we come up with a detailed study of the possible reasons for the reported discrepancy.

We note from Table 1 that our best-fit $c$-$M_{\text{vir}}$ and $\delta_c$-$M_{\text{vir}}$ relations and the constraints on $n$ are roughly independent of the cosmological models ($\Omega_m$ and $\Omega_\Lambda$). These properties will be significant for distinguishing between different cosmological models when combined with high-resolution numerical simulations.

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