A model for the emergence of geopolitical division

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In this work, we present a model based on a competitive dynamics that intends to imitate the processes leading to some characteristics of the geopolitical division. The model departs from very simple principles of geopolitical theory and geometrical considerations, but succeeds to explain general features related to the actual process. At the same time, we will propose an evolutionary explanation to the fact that most capitals (in Eurasia) are located far from the borders or coasts and, in many cases, close to the barycenter of the respective countries.

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INTRODUCTION

The combination of both a suitable theory and empirical data can render macro historical predictions possible. This is one of the statement enounced by R. Collins in [1], where he accounts for a successful prediction of the breakup of the Soviet Union. The suitable theory that frames his model [2] is a geopolitical theory for the power of the states. The history of geopolitical theory started in Germany at the beginning of the 20th century with the works by Weber [3] on the development of the state. But it was not until the 70’s, after many years of declining interest on the subject, that the studies on this area of knowledge flourished [4]. Among all the processes studied by geopolitical history, the formation of the modern states system emerges as one of the most relevant, involving feedback processes between the economy, the society and the politics.

The geopolitical configuration of the continents and its evolution was a determinant factor on the economic success of several areas of the world. At the same time it was critical in sealing the future political regimes of the nations. As complex as the spatial evolution of states might be, Collins [2] proposed a simple theoretical model to deal with the main aspects involved in the process of expansion and contraction of the territorial power of states. The model is based on five principles that will be summarized in the following paragraph. These principles describe the factors that promote the expansion or the collapse of a nation. On one side, the expansion of states is favored by the geographical size and availability of resources and by gepositional advantages, e.g. countries with fewer enemies expand at the expenses of other countries with more enemies on the borders. However, on the other side, a strain of resources may arise as a result of overextension, leading to the disintegration of the state. In the same model Collins assumes that states in the middle of a geographical region tend to fragment into smaller unities over time. Finally, the model affirms that some cumulative processes give place to long term simplification, with massive wars between a few contenders.

We can give an example of the processes foreseen by these principles. During historical periods previous to the technological explosion derived from the industrial revolution the states increased their power through conquest and expansion. But this process entailed an increment in the costs of administration and military defense. Therefore, whereas the seek for power promoted the expansion, the increment in size and in administrative duties imposed critical constrains. At the initial stages of expansion the increments in the wealth of a successful state exceeded the incremental costs of defending the additional new territory. However, when borders reached a critical distance from the center of power those costs rose faster than the benefits. As the survival of a polity depended on its military power to neutralize the constant threat posed by contiguous states, the protection of the borders was crucial [5].

In [6] the author analyzes the interplay between the evolution of the geopolitical structure of Europe towards a system of competing states and the social development. This competing structure promoted the creation of a pluralistic environment that succeeded to constrain the power of the ruling classes, as well as the power of the states on the individuals, helping to promote institutions and legal systems that ultimately provided greater freedom than did monolithic empires. Jones affirms that the ultimate configuration of the state system in Europe can not respond to mere geometric aspects and funds his affirmation on the analysis of a collection of complex and interconnected facts that should have influenced on the whole geopolitical process. These conclusions seem to contradict the ideas of Collins, whose model is mainly based on geometrical considerations. The goal of this work is to show that although geometry can not account for the whole geopolitical process, it does impose some constrains with important effects on the geopolitical division process. When the geometry of a given polity is closely related to other indicators such as the military power and the availability of resources some basic aspects of the process of political division can be derived from a model solely based on topological principles.
This task was partially accounted by Artzrouni and Komlos \cite{artzrouni}, where they analyzed the spatial evolution of the European state system throughout a period extending roughly from A.D. 500 to A.D. 1800. Following this previous work, we generalize the model to include some additional aspects, such as the location of the center of power controlling each of the states and the analysis not only the final configuration but of the temporal evolution towards the steady state.

**THE MODEL**

The model presented here is associated to a previous work by Artzrouni and Komlos \cite{artzrouni}, where the authors introduced a spatial predatory model to trace the evolution of the political borders in Europe, from 500 A.D. to 1800 A.D. The dynamics of the model is based on the fact that any political unit has a natural tendency to expand by conquering neighboring areas. A first simplistic assumption is that its perspective of success or failure in combat will depend on its military power. Following the principles stated in \cite{artzrouni} the present model considers that the military power increases with the size of a state but decreases as a result of the over-extension of the border-line and the increment of the distance between the center of power and the borders. Though the present model represents an oversimplification of the real historical process still captures the most essential features. Among the neglected features we can mention the evolution of military technology, as well as the inhomogeneities in the distribution of the population and other economic indicators. In this version of the model we will consider the expansion or collapse by annexation but we will neglect the segregation of big polities into smaller units. Each country struggles for its survival by combating its neighbouring rivals. During the initial stages of the evolution this process leads the successful countries to expand and the defeated states to shrink and disappear. The gains in wealth and power associated to the expansion exceed the incremental costs of defending the annexed territory. Some defeated countries eventually disappear as a continuous loss of territories undermines their military power and can not withstand further attacks from their opponents. At a later stage the length of the expanding borders increase as well as their distances to the center increase, making the costs to protect them go beyond the benefits of expansion. At this point, the states face the dilemma of maintaining or stopping an expansive process involving costs that become increasingly difficult to overcome.

Based on the previous considerations, Artzrouni and Komlos \cite{artzrouni} define the military power of a state, \( P \), as a quantity depending on the geometry of each state. It is expressed as a function of two variables: the area \( A \) and perimeter \( P \) of a country. The states are located on top of a square lattice. The area \( A \) is simply the number of nodes of the grid occupied by the country, whereas the perimeter is the number of bordering nodes. In turn, \( P(A, M) \) is defined as

\[
P(A, M) = \frac{A}{\delta + \exp(\gamma F + \beta)},
\]

where \( \delta, \beta, \gamma \) are positive parameters.

One of the modifications introduced in the present work is the addition of fixed centers of power or capitals with a crucial role on the dynamics of the system. The military power \( P \) will depend on the area \( A \) and on a quantity \( M \) that measures the distance between the frontier points and the center of power. Physically, this quantity is associated to a power of the moment of inertia of the border of a country, relative to its capital.

\[
M_k = \sum_{b \in \Omega_k} [ (i_b - i_{c_k})^2 + (j_b - j_{c_k})^2 ]^\alpha,
\]

where \( \Omega_k \) is the border of the country \( k \), \( i_b \) and \( j_b \) are the coordinates of a border node \( b \) and \( i_{c_k} \) and \( j_{c_k} \) the coordinates of the capital. In the previous model two different geomorphological aspects have been considered: The first one is the existence of natural barriers due to topological aspects of the terrain, that only inhibit the interaction between adjacent nodes. The second one is the existence of costs, generally more easy to defend. We will account only for the effect of the coastal borders on \( M \) through a weighting factor \( \kappa < 1 \). Thus, Eq. (2) results in

\[
M_k = \sum_{b \in \Omega_k^1} [ (i_b - i_{c_k})^2 + (j_b - j_{c_k})^2 ]^\alpha + \sum_{b \in \Omega_k^2} \{ \kappa [ (i_b - i_{c_k})^2 + (j_b - j_{c_k})^2 ] \}^\alpha,
\]

where \( \Omega^1 \) takes into account the inland border and \( \Omega^2 \) the coastal border nodes. \( P(A, M) \) is defined as

\[
P(A, M) = \frac{A}{\exp(\gamma M)}. \tag{4}
\]

According to the previous discussion, we want the military power to decide the outcome of a combat, though we are interested in preserving a certain degree of stochasticity as well. Therefore, we follow the prescription proposed in \cite{artzrouni}. After measuring the \( P \) values of the two confronting countries we call \( P_h \) and \( P_l \) the higher and lower \( P \) values respectively. This identification is irrelevant in case of equality and can be randomly done as shown later. Next we assign the victory to the country with the higher military power with a probability

\[
\phi_h = 1 - 0.5 \exp(-k(P_h/P_l - 1)),
\]

where \( k \) is the parameter that tunes the deterministic character of the dynamics. The higher \( k \) is the higher the
As an example, we will consider a state represented by \( n \) can be a taken into account in the dependence of power by effects of the state’s geometry. This fact is a bigger area can inhibit the efficient exertion of that represents a potentially higher power. On the other hand, creasing function of \( P \) is a monotonic increasing function of \( A \), indicating that a greater area represents a potentially higher power. On the other hand, a bigger area can inhibit the efficient exertion of that power by effects of the state’s geometry. This fact is taken into account in the dependence of \( P \) on \( F \) or \( M \). As an example, we will consider a state represented by \( n^2 \) territorial unities. In two extreme situations the state can be a \( n \times n \) square or a \( n^2 \times 1 \) rectangle. Though both state have the same area, the rate perimeter - surface is lower for the square state, consequently, according to Eq. (1) its military power is higher. A similar conclusion can be derived when considering Eq. (4). According to Steiner theorem, the moment of inertia of a plane figure is minimum when it is calculated about an axis located at the center of mass. Therefore, among all the states with identical shape, the one with its capital being the closest to the center of mass will present the lowest value of \( M \). A possible observation to the situation described above is the fact that borders closer to the center of power should be easier to protect that those far away. At the same time, a wiser military strategy will tend to focus the defense of the country on the borders but without neglecting the capital. To consider this facts we introduce a further modification of Eq. (4), by including a local term that accounts for the relative distance between the capital and the border points involved in a specific conflict, i.e. the points at the border with the eventual opponent country. The military power presents then a global term plus a local factor associated with the conflictive border. We redefine \( P \) as

\[
P(A, M) = \frac{A}{\exp(\gamma M)}.
\]

The calculation of \( M \) is analogous to that of \( M \) but restricted to the border under conflict, i.e. \( \Omega^1 \) and \( \Omega^2 \) in Eq. (4) refer only to the partial border.

As we will show later, the dynamics of this system can be associated to a cluster growth process. Although there is a vast literature on this area \([5, 12]\) and some of these works might show some common features with the one described in this work, the present model includes some unique features closely related to the particular problem here analyzed. In the following section we discuss the results obtained in each case.

**RESULTS**

Throughout all the simulations we have maintained the same initial condition. At the beginning the system is composed by 400 states, each one being a square comprising \( 3 \times 3 \) surface unities with the capital located at the barycenter. The whole system is a square lattice of \( 60 \times 60 \) surface unities. During the first stages of the simulation, the dynamics of the system is purely random, until some differences among the shapes of the states arise, playing a defining role in the further evolution of the system. In order to get a correct interpretation of our results, we analyze first the behavior of a system driven by a random dynamics throughout the whole simulation. This can be very easily achieved by considering that the outcome of a fight is uncorrelated to the military power of the confronting countries. In this case the winner is randomly chosen. Figure 1 summarizes the main findings: the dynamics of the model drives the system to the conformation of a unique big country having conquered all the available territory. The collapse of smallest countries produces an exponential decay in the number of survival countries \( N_s \), with a slowing down when the number of remaining countries is around 10. At this point the disappearance of countries takes much longer. Finally, the system converges to its only steady state.

This trivial stochastic evolution turns much more interesting when, as proposed in the model, it is the value \( P \) what defines the victorious country. According to what was discussed before, we expect the states displaying values of \( P \) monotonically related to their sizes at the beginning. After a transient letting to the disappearing of
the majority of the countries and the conformation of a bunch of powerful states, the effect of the critical size must be apparent. At this point the increasing value of \( F \) or \( M \) dominates the evolution of the states, making their \( P \) values to decrease in case of overextension. The previous affirmations can be supported by analytical results. Considering Eqs. \( 1 \) and \( 4 \), we can state that the optimal shape adopted by any state is the circular one. In our analysis, and only for simplicity, we will consider the octagonal shape. Furthermore, if the location of the capital is taken into account, \( P \) is maximized when the former is located on the center of mass of the corresponding state. We define \( r \) as the radius of the circle circumscribing the octagon. As discussed before, an expanding state will increase its military power until a critical value of \( r \) is attained. At this point the effects of the overextension will start to be noticeable. We can estimate the maximum attainable value of \( P \) at this critical radius \( r_c \).

If we analyze the Eq. \( 1 \) considering that a state expands preserving an octagonal shape and a centered capital, we can find an approximate value of \( r_c \), the point when the maximum military power is achieved. We note that the quantities \( A, F \) and \( P \) can be written now in terms of \( r \).

\[
A(r) = 2(2\sin(\pi/8)r)^2(1 + \sqrt{2}) \\
F(r) = 16\sin(\pi/8)r \\
P(r) = \frac{A(r)}{(\alpha + \exp(\gamma P(r) + \beta))} 
\]

Figure 2 shows the value of \( P(r) \) according to the expression in Eq. \( 6 \) in the \((r, \gamma)\) plane. The existence of critical radius \( r_c \) is evident. Deriving the last of Eqs. \( 6 \) with respect to \( r \) let us find \( r_c \), getting the following expression

\[
e^{2\gamma M r \sin(\pi/8)}(8r_c \gamma \sin(\pi/8) - 1) - \alpha = 0 \tag{7} \]

with a solution for \( r_c \) in terms of Lambert’s \( W \) function,

\[
r_c = \frac{2 + W[2\alpha \exp(-2 - \beta)]}{16\sin(\pi/8)\gamma} \tag{8} \]

The former expression shows us that the critical radius is inversely proportional to \( \gamma \). If we consider that the mean size of the countries is inversely proportional to the square of the radius, we get the following relation: \( N_s \propto \gamma^2 \). This is precisely what we get when plotting the numerical data on the \((N_s, \gamma)\) plane and we fit them with a parabola, as shown in Fig. 3. When we take the location of the capital into account we get the following expressions:

\[
A(r) = 2(2\sin(\pi/8)r)^2(1 + \sqrt{2}) \\
M(r) = 2(2 + \sqrt{2})\sin(\pi/8)r^{2\alpha + 1} \tag{9} \\
B(r) = \frac{A(r)}{\exp(\gamma M(r))} 
\]

In this case the critical radius is inversely proportional to \( \gamma^{1/(2\alpha + 1)} \), therefore \( N_s \propto \gamma^{1/(\alpha + 1)} \). Fig. 4 shows an example of the fitting of numerical data when \( \alpha = 1 \). The fitting shows us that \( N_s \propto \gamma^{1/2} \).

This analysis is not applicable to Eq. \( 5 \) because it involves not only the geometry of the analyzed state, but also that of its neighbours as well as local considerations. All the data on Figs. 6 and 10 correspond to averages over 1000 realizations for each point. The number of surviving states \( N_s \) reaches a stationary value, maintained
over at least 50000 time steps, ten times longer than the time it takes the system to reach the steady value. Although the number of surviving states remains constant, the system is not frozen. We can observe to changes in the shapes and sizes of these surviving countries due to persisting combats. Nevertheless, these fights do not change significatively the conformed map. These results let us confirm that the system evolves, with some countries expanding at cost of the collapse of others, until a metastable situation is achieved.

At this point the analysis of the model described by Eq. (1) is almost through except for the study of the shape and size distribution of the surviving countries. To statistically analyze the geometry of the surviving countries we measure how far from a circle or octagon their shapes are. We consider a quantity $\rho$ herewith defined. First we measure the distance $d_{i,0}$ between each point $i$ on the borderline of the country and its center of mass. Then we calculate $\rho$ as the ratio between the variance of this distances and the area of the country, $A$. If a country is circular, this quantity is equal to zero, while it is $\approx 0.00112$ for an octagon. The expression for $\rho$ is then

$$\rho = \frac{1}{A} \left( \frac{1}{N_{\Omega}} \sum_{i \in \Omega} (d_{i,0})^2 - \left( \frac{1}{N_{\Omega}} \sum_{i \in \Omega} d_{i,0} \right)^2 \right) \tag{10}$$

In Fig. 5 we plot $\rho$ versus the number of surviving states, $N_s$. The filled dots correspond to the present case. We observe the sharp transition as the number of surviving countries grows from 1 to 3. The lower value, $\rho \approx 4 \times 10^3$, corresponds to a unique squared country limited by the shape and size of the lattice. The value of $\rho$ then stabilizes around $5 \times 10^2$. As a reference, a rectangular, almost 1-dimensional country of the same size when $\gamma = 0.1$ would have $\rho \approx 10$. The variation in size, whose mean value is associated to the mean number of surviving states depicted in Fig. 5 is around 50% when the number of surviving countries is over 5, then it decreases abruptly to zero. The introduction of fixed capitals let us add to the previous analysis the study of their locations and of the centrality of the respective countries. One of the aspects we wanted to analyze by means of the modification introduced to the original model was precisely where the capitals of the surviving countries are located. Figure 5 also shows the behaviour of $\rho$ in this case, plotted with empty dots. The most evident fact is that the values are lower than in the previous case, indicating that the countries adopt a more symmetric and circular shape. The results do not differ too much if we consider either Eq. (1) or Eq. (5).

When considering the location of the capital, not only we can measure $\rho$ but we can calculate the distance between the center of mass and the capital. We choose the mean radius of the country to compare the previous distance with a characteristic length. We define $\delta_{c,0} = d_{c,0}/r_m$, where $d_{c,0}$ is the distance between the capital and the center of mass and $r_m$ is the mean radius.

Figures 6 and 7 show the distribution of $\delta_{c,0}$ for different values of $\alpha$ and $\gamma$ for the cases described by Eqs. (1) and (5) respectively. In this case there is an evident difference in the behaviour of the system. The values of $\alpha$ and $\gamma$ were adjusted to get a final configuration with the same number of surviving countries in each case. We can observe that if we consider local aspect related to the military power and the cost of defending the borders we get capitals located closer to the center of mass of the countries.
FIG. 6: Distribution of $\delta_{c0}$ for different pairs of values $\alpha$ and $\gamma$ with $<N_s>=10$. This is the case associated to Eq.(4).

FIG. 7: Distribution of $\delta_{c0}$ for different pairs of values $\alpha$ and $\gamma$ with $<N_s>=10$. This is the case associated to Eq.(5).

states.

To provide a more graphical example of the results we present the outcome of three single realizations in Fig 8 together with a representation of the time evolution of the sizes of the surviving countries. Each realization corresponds to a different choice for $P$. The group (a) corresponds to the model that does not take into account the location of capitals, Eq. (1), while (b) and (c) differ in that the last one reflects the state of a system where $P$ is affected by local contributions, Eq. (5) and (b) corresponds to Eq. (4). When we observe the temporal evolution of the size of the surviving countries the differences are rather evident. The curves related to model associated to Eq. (4) present fluctuations of great amplitude. This behaviour reflects in the distribution of sizes of the states than can be observed in the map showing an instantaneous picture of the system. At a given moment some countries can be six times bigger than others. If fluctuations are strong enough they can lead the system to a trivial state. On the other hand, groups (b) and (c) show a more steady behaviour. After a transient, the size of the countries stabilizes suffering only very small fluctuations. At the same time the distribution of sizes is more even. Groups (b) and (c) differ en that the effect of local considerations is the centralization of the capitals, displayed with stars on the map. This effect was already discussed in Fig. 7.

CONCLUSIONS

Supported by the fact that the historical evolution of geopolitics is a complex process involving a huge variety of causes and effect, the conception of simple models explaining general aspects has been neglected. This fact is the main motivation behind the present work. It is not the intention of this model to provide an accurate and complete description of the process that shaped the present political division of the world. On the contrary, its more modest goal is to show that some general features can be explained departing from very simple assumptions. Based on the ideas sketched by Collins in (Collins 1995, Collins 1978) we have developed a model that succeeded to reproduce a series of geopolitical phenomena, namely the fact that most of the capitals are located in a central position, the fact that most of the surviving countries have cost, with only a few being internal, the possibility of stabilizing a political division with many countries of similar size and overall shape, but also describing the situation when the rise of an empire is possible. Still the model needs some adaptation to be
able to describe the sort of processes that have occurred in The Americas and Africa, and include the possibility of state segregation. These aspects will be taken into account in a future work.

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