Quark-meson coupling model with short-range quark-quark interactions

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Short-range quark-quark correlations are introduced into the quark-meson coupling (QMC) model phenomenologically. We study the effect of the correlations on the structure of the nucleon in dense nuclear matter. With the addition of correlations, the saturation curve for symmetric nuclear matter is much improved at high density.

More than a decade ago, Guichon proposed a relativistic quark model for nuclear matter, where it consists of non-overlapping nucleon bags bound by the self-consistent exchange of scalar (σ) and vector (ω) mesons in mean-field approximation (MFA). This model has been developed as the quark-meson coupling (QMC) model, and successfully applied to various phenomena in nuclear physics. Recently, the model has been further extended using relativistic constituent quark models.

So far, the use of the QMC model has been limited to the region of small to moderate densities because it has been assumed that the nucleon bags do not overlap. It is therefore of great interest to explore ways to extend the model to include short-range quark-quark interactions, which may occur when nucleon bags overlap at high density. In this report we will introduce these short-range correlations in a very simple way, and calculate their effect on the quark structure of the nucleon in medium. We refer to this model as the quark-meson coupling model with short-range correlations (QMCs).

Let us consider uniformly distributed (iso-symmetric) nuclear matter with nuclear density $\rho_B$. At finite density the nucleon bags start to overlap with each other, and a quark in one nucleon may interact with quarks in other nucleons in the overlapping region. Since the interaction between the quarks is short range, it may be reasonable to treat it in terms of contact interactions. An additional interaction term of the form, $\mathcal{L}_{\text{int}} \sim \sum_{i \neq j} \bar{\psi}_q(i) \Gamma_\alpha \psi_q(i) \bar{\psi}_q(j) \Gamma_\alpha \psi_q(j)$, is then added to the original QMC lagrangian density. Here $\psi_q(i)$ is a quark field in the $i$-th nucleon and $\Gamma_\alpha$ stands for a combination of $\gamma$ matrices (with or without the isospin and color generators). In this report we shall consider only u and d quarks and simulate the short-range interaction using scalar- and vector-type couplings in MFA: $\Gamma_\alpha = 1$ and $\gamma_0$.

Next we consider the probability for the nucleon bags to overlap, using a simple geometrical approach. To measure the overlap of nucleons we treat them as the
MIT bag with the radius $R$. For any two nucleons the overlapping volume (measured in units of the nucleon volume $V_N(= 4\pi R^3/3)$) is $V_{ov}(y) = 1 - 3y/4 + y^3/16$ (for $y \leq 2$).\textsuperscript{b} and $V_{ov}$ vanishes beyond $y = 2$, where $y = s/R$ with $s$ a distance between the two centers of the nucleon bags.

In a nucleus with $A$ nucleons, any given nucleon may overlap with $(A-1)$ others. If the nucleons are distributed according to some two-particle density function $\rho_2(r_1, r_2)$, then the overlapping volume per nucleon (in units of $V_N$) is

$$V_A = (A-1) \int \int d{r_1}d{r_2}\rho_2(r_1, r_2)V_{ov}(|r_1 - r_2|/R),$$  \hspace{1cm} (1)

where the function $\rho_2$ may be given as the product of single-particle densities $\rho_1(r_1)$ (normalized to unity), $\rho_1(r_2)$ and the two-nucleon correlation function $F(|r_1 - r_2|)$.

In the limit $A \to \infty$, we then find the overlapping volume as $V_\infty = (\rho_B V_N) \times (1 - \xi)$ in the uniform matter, where

$$\xi = \frac{1}{V_N} \int ds H(s)V_{ov}(s/R),$$  \hspace{1cm} (2)

with $H(s) = 1 - F(s)$. Here $\xi$ describes the effect of two-nucleon correlation in the overlapping part. If there were no correlation between two nucleons (namely, $F(s) = 1$ or $\xi = 0$) $V_\infty$ were 0.32 (when $R = 0.8$ fm and $\rho_0 = 0.15$ fm$^{-3}$, the saturation density of nuclear matter).

For nucleons separated less than $\sim 1$ fm, some modification of the two-nucleon density is expected and this is described by the correlation function $F(s)$. In principle, one could calculate the correlation function within QMC self-consistently, as in the relativistic Brueckner-Bethe-Goldstone formalism etc.\textsuperscript{c} It is, however, not easy. In this exploratory study we shall use a phenomenological form of the correlation function.

We use a convenient form of the correlation function proposed by Miller and Spencer\textsuperscript{d} – so-called the Miller-Spencer correlation function. We then find

$$H(s) = 1 - F(s) = 1 - \left(1 - \frac{1}{4}g_p^2(y)\right)(1 + f(s))^2,$$  \hspace{1cm} (3)

where $g_p$ describes the Pauli (exchange) correlations and is given by $g_p(y) = (3/y)j_1(y)$, where $y = sk_F$ with the Fermi momentum $k_F$. The function $f(s)$ describes the strong repulsion in the core part and is parametrized as $f(s) = -(1-\beta s^2)\exp(-\alpha s^2)$, where $\alpha$ and $\beta$ are parameters.\textsuperscript{c}

The Bethe-Goldstone theory places a restriction on the overall size of $f(s)$. The nuclear matter density times integral of the square of $f(s)$ is known as the “wound” integral, $\kappa$:

$$\kappa = \rho_B \int ds f(s)^2.$$  \hspace{1cm} (4)

This quantity, which is the convergence parameter of the hole-line expansion, has a typical value of 0.12 around $\rho_B = \rho_0$.\textsuperscript{c} A next condition on $f(s)$ is obtained by the consistency requirement:

$$\int ds \left(1 - \frac{1}{4}g_p^2(y)\right)(2f(s) + f(s)^2) = 0,$$  \hspace{1cm} (5)
to ensure the normalization on \( \rho_2 \). Another quantity of interest is the correlation length \( \ell_c \) defined by

\[
\ell_c = - \int_0^\infty ds (2f(s) + f(s)^2).
\]

(6)

We shall use the conditions \( \alpha \) and \( \beta \) to determine the parameters \( \alpha \) and \( \beta \) and check the value of “wound” integral \( \kappa \) numerically at the same time.

Now it is natural to choose the probability of overlap to be proportional to

\[
\kappa = \frac{\partial E}{\partial \sigma} \quad \text{for the thermodynamic condition:} \quad \frac{\partial E}{\partial \sigma} = 0.
\]

In Eq.(7), \( \langle \gamma \rangle = \frac{\partial E}{\partial \sigma} \rangle \) are, respectively, the mean-field values of the \( \sigma \) and \( \omega \) mesons and \( g_0^q \) and \( g_1^q \) are, respectively, the \( \sigma \)- and \( \omega \)-quark coupling constants in the usual QMC model. The new coupling constants \( f^q_{s(v)} \) have been introduced for the scalar (vector)-type short-range correlations, and are given by \( f^q_{s(v)} = (f^q_{s(v)}/M^2)V_{\infty} \) (M = 939 MeV, the free nucleon mass). Note that since the coupling constants have dimension of \((\text{energy})^{-1}\), we introduce dimensionless coupling constants \( f^q_{s(v)} \).

In Eq.(7), \( \langle \bar{q}q \rangle \) and \( \langle q \rangle \) are, respectively, the average values of the quark scalar density and quark density with respect to the nuclear ground state, which are approximately given by the values at the center of the nucleon in local density approximation.

Now we can solve the Dirac equation Eq.(3), as in the usual QMC, with the effective quark mass

\[
m_q^* = m_q - (g_0^q \bar{q}q + f^q_{s(v)} \langle \bar{q}q \rangle),
\]

(8)

instead of the bare quark mass. The Lorentz vector interaction shifts the nucleon energy in the medium

\[
\epsilon(k) = \sqrt{M^2 + k^2} + 3(g_0^q \bar{q}q + f^q_{s(v)} \langle \bar{q}q \rangle),
\]

(9)

where \( M^* \) is the effective nucleon mass, which is given by the usual bag energy.

The total energy per nucleon at density \( \rho_B \) is then expressed as

\[
E_{\text{tot}} = \frac{4}{(2\pi)^3\rho_B} \int_{k_F}^k dk \sqrt{M^2 + k^2} + 3(g_0^q \bar{q}q + f^q_{s(v)} \langle \bar{q}q \rangle) + \frac{1}{2\rho_B}(m_\sigma^2 \sigma^2 - m_\omega^2 \omega^2),
\]

(10)

where \( m_\sigma \) and \( m_\omega \) are respectively the \( \sigma \) and \( \omega \) meson masses. The \( \omega \) field created by the uniformly distributed nucleons is determined by baryon number conservation:

\[
\omega = 3g_0^q \rho_B / m_\omega^2 = g_\omega \rho_B / m_\omega^2 \quad \text{where} \quad g_\omega = 3g_0^q.
\]

The \( \sigma \) field is given by the thermodynamic condition: \( \partial E_{\text{tot}} / \partial \sigma = 0 \). This gives the self-consistency condition (SCC) for the \( \sigma \) field

\[
\sigma = -3g_0^q S_N(\sigma) - g_\sigma C_N(\sigma).
\]

(11)

where \( \partial M^*/\partial \sigma \rangle = -3g_0^q S_N(\sigma) = -g_\sigma C_N(\sigma). \) Here \( g_\sigma = 3g_0^q S_N(0) \) and \( C_N(\sigma) = S_N(\sigma) / S_N(0) \), with the quark scalar charge defined by \( S_N(\sigma) = \int d\vec{r} \bar{q}q \).
actual calculations, the quark density $\langle \psi_q^\dagger \psi_q \rangle$ in the total energy may be replaced by $3\rho_B$, and the quark scalar density, contributing to the effective quark mass, is approximately given as $\langle \bar{\psi}_q \psi_q \rangle = (m_{2\sigma}/g_{\sigma})\sigma$ because of the SCC.

Now we are in a position to show our main results. We choose $m_q = 5$ MeV and the bag radius of the nucleon in free space $R_0$ to be 0.8 fm. The bag constant $B$ and usual parameter $z$, which accounts for the center of mass correction and gluon fluctuations, in the bag model are determined to fit the free nucleon mass with $R_0 = 0.8$ fm – we find $B^{1/4} = 170.0$ MeV and $z = 3.295$. The coupling constants $g_{\sigma}$ and $g_{\omega}$ are determined so as to reproduce the average binding energy of symmetric nuclear matter ($-15.7$ MeV) at the saturation density $\rho_0$.

In Fig. 1, we show $\xi$ in Eq. (2). As an example, we set $\bar{f}_q = 40, \bar{f}_q^v = 8$ to fit the (observed) energy per nucleon at high $\rho_B$ (see Fig. 2). If we fix $\ell_c = 0.75 \ (0.7)$ fm, the conditions (5) and (6) give $\alpha = 1.05 \ (1.20)$ fm$^{-2}$, $\beta = 0.617 \ (0.708)$ fm$^{-2}$ and $\kappa = 0.12 \ (0.10)$ at $\rho_0$. We then find that $g_{\sigma}^2 = 50.09 \ (48.11)$, $g_{\omega}^2 = 37.22 \ (33.80)$ and $K = 360 \ (370)$ MeV. As shown in Fig. 1, $\xi$ gradually decreases and it becomes almost constant at high density.

In Fig. 2, we present the saturation curve for symmetric nuclear matter. We can see that QMCs can provide the energy per nucleon, which lies in the empirical region (enclosed with the dotted curves). It may imply the importance of the
short-range correlations at high density. If we choose a large value of $\bar{f}_q^q$ the energy becomes larger at high $\rho_B$ and it also makes the effective nucleon mass large. In QMCs we find $M^*/M \sim 0.84$ at $\rho_0$. We also show the scalar mean-field values at finite density in Fig. 3. In the original QMC the strength of the scalar field goes up to about 500 MeV at $\rho_B \sim 4 \rho_0$, while in QMCs it is much reduced at high $\rho_B$ because of the short-range (repulsive) interaction.

Turning next to the size of the nucleon itself, as measured by the root-mean-square (rms) radius of the quark wave function, we see in Fig. 4 that the short-range correlations give a little enhancement. This effect is, however, not strong within the present parameter set, and the increase of the size around $\rho_0$ lies in the upper-limit value analysed by the electron scattering experiment.

We summarize the role of scalar- and vector-type short-range correlations. The scalar-type correlation modifies the effective quark mass in a nuclear medium as (see Eq. (8))

$$m_q^* = m_q - \left[ 1 + \frac{\bar{f}_q^q}{g_\sigma^2 M^2 V_\infty(\rho_B)} \right] (g_\sigma \sigma).$$

In Eq. (12) the mean-field part ($g_\sigma \sigma$ term) provides a density dependence on the quark mass of order of $O(\rho_B)$ at low density, while the correlation part gives higher order contributions. This leads to the reduction of the scalar mean-field value in matter, which may affect the in-medium nucleon properties.

The vector-type correlation as well as the $\omega$ meson shifts the total energy. The
\(\omega\) and the correlation contribute to the energy as (see Eq.(9))

\[
3\left(g_2^q\omega + f_2^q\langle\psi_q^\dagger\psi_q\rangle\right) = \frac{g_2^q}{m_\omega^2}\left[1 + 9\frac{f_2^q}{g_2^q}\frac{m_\omega^2}{M^2}V_\infty(\rho_B)\right]\rho_B.
\] (13)

Since \(V_\infty\) depends on the density the correlation again gives the contribution of higher order in \(\rho_B\). It thus enhances the energy at high density. If we introduce the effect of higher-configurations like 3-body, 4-body etc., such correlations may have dependences of higher power of \(\rho_B\) and play an important role not only at high density but also near the region of the phase transition to quark-gluon plasma. Those terms may correspond to higher order terms appearing in the chiral effective lagrangian for nuclear matter.

In conclusion, we have studied the effect of short-range quark-quark correlations associated with nucleon overlap in MFA. We have found that a repulsive vector-type correlation makes the saturation curve close to the empirical one at high density. Furthermore, we have shown that the scalar- and vector-type correlations considerably modify the properties of nuclear matter at high density. While our inclusion of the correlations has been based on a quite simple, geometrical consideration, in the future we would hope to formulate the problem in a more sophisticated, dynamical way.

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