Electrostatic screening in nanostructures with multicomponent electron plasma

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Abstract. Screening of the Coulomb interaction accounting for the Friedel oscillations in the structures with multicomponent low-dimensional electron plasma is considered. Calculations are made for nanotubes, double quantum wells (DQW) and superlattices. The binding energy of a donor in DQW is found as a function of the subbands occupation numbers.

1. Introduction
Availability of the mobile electrons in a system results in renormalization of the electron - electron interaction, i.e., screening. The specific form of the screened e-e interaction is determined by the effective dimension and by the energy spectrum of the electron gas. As is known in 3D plasma the bare Coulomb interaction is replaced (due to screening) by the Yukawa law plus the Friedel oscillations \( \cos(2p_F r)/r^3 \) if the electron gas is degenerate; here \( p_F \) is the Fermi momentum, \( r \) is the distance from a point charge, and \( \hbar = 1 \). Similar results in the 2D case read: the regular part of the screened potential at \( \rho \to \infty \) is \( Qa_B^2/\rho^3 \) [1], where \( Q \) is the initial point charge, \( a_B \) is the effective Bohr radius, whereas the oscillating contribution is proportional to \( \sin(2p_F \rho)/\rho^2 \).

The problem of electrostatic screening was considered in a great variety of papers. 2D case was studied e.g. in [2, 3]; in [4] the binding energy of 2D excitons was calculated with allowing for the finite width of the quantum well. However in all those papers the Coulomb interaction was screened by a single-component plasma. Even in the case of quantum well of finite width the authors assumed that only the lowest level of the transverse quantization was populated.

The aim of the present work is to study the screening of the Coulomb interaction in nanostructures containing a few component of the 2D or 1D electron plasma. These components are labeled by the discrete quantum number of transversal quantization (the number of subband) or the number of layer in multilayer structures when tunneling is ignored. We use the theory of linear response and, correspondingly, obtain our results for linear screening when the energy of an electrostatic perturbation is small as compared with the electron Fermi energy. We will consider 2D electron gas in a hollow cylinder (nanotube), double quantum well and multilayer structure.
2. Nanotube

Consider a nanotube with the semiconductor type of the single particle energy spectrum:

\[ \varepsilon_{p,l} = \frac{p^2}{2m} + Bl^2; \quad B \equiv \frac{1}{2ma^2}, \quad l = 0, \pm 1, \pm 2, \ldots \]  

(1)

Here \( p \) is the electron momentum parallel to the tube axis, \( l \) is the number of the subband, \( m \) is the effective mass and \( a \) is the nanotube radius. Thus, the problem in question differs from the similar one for a planar 2D system by only quantization of one of the component of momentum: \( p_x = l/a \). The considered system is infinite and uniform in \( z \)-direction (the nanotube axis) and periodic and uniform in the azimuthal direction \( \varphi \). Hence, the electron Green function \( G \) depends on only differences \( z - z', \varphi - \varphi' \). In the Fourier representation \( G \) is diagonal: \( G(p, p'; l, l') = \delta_{pp'} \delta_{ll'} G(p, l) \).

In what follows we will apply the standard diagram technic to find the \( e - e \) interaction renormalized by screening and we will use the linear theory of screening. We are aware that in strictly 1D system with linear single-particle dispersion law the Luttinger liquid model is valid. The linear screening theory relates to only long-wave length limit in which its results coincide with the ones of the Luttinger model (see e.g. [5]).

However we actually deal with 2D object and we use the parabolic dispersion law for electrons. For such a situation the Luttinger model is not applicable and we do not have any other instrument except RPA to investigate the problem.

The main difference with the strictly 1D situation is dependence of our effective (screened) potential not only on \( z \) (coordinate along the tube axis) but also on \( \varphi \) - azimuthal separation between two electrons on the surface of a hollow cylinder. We discovered that average potential remains finite at \( x/d \to 0 \) rather than 1/\(|x| \ln |x/d| \), otherwise \( \int U_{eff}(x)dx \) diverges while the Fourier component \( U_{eff}(q) \) remains finite at \( q \to 0 \).

\[ V(k, n) = \frac{V^{(0)}(k, n)}{1 + V^{(0)}(k, n) \Pi(\omega; k, n)}, \]

(2)

where \( V^{(0)} \) is the bare Coulomb interaction:

\[ V^{(0)}(k, n) = e^2 \int_{-\infty}^{\infty} \int_{0}^{2\pi} \frac{e^{-ikz - in\varphi} dz d\varphi}{\sqrt{z^2 + 4a^2 \sin^2(\varphi/2)}} = 4\pi e^2 I_n(|k| a) K_n(|k| a), \]

(3)

Here \( e^2 = e^2/\chi \), \( \chi \) is the background dielectric constant, \( I_n, K_n \) are the modified Bessel functions of the 1-th and the 3-rd type. The polarization operator (electron loop) has a form

\[ \Pi(\omega; k, n) = \frac{1}{2\pi^2} \sum_{l-\infty}^{\infty} \int_{-\infty}^{\infty} dp \frac{f_{p-k,l-n} - f_{p,l}}{\varepsilon_{p,l} - \varepsilon_{p-k,l-n} - \omega - i\delta} \]

(4)

\( \delta = +0 \),

\[ 1 \] There is a misprint in the eq.(8) of ref. [5]. Asymptotic behavior of the screened potential \( U_{eff} \) must be 1/|x| ln |x/d| rather than 1/|x| ln |x/d|, otherwise \( \int U_{eff}(x)dx \) diverges while the Fourier component \( U_{eff}(q) \) remains finite at \( q \to 0 \).
where \( f_{\nu \ell} \equiv f(\varepsilon_{\nu \ell}) \) are the Fermi occupation numbers.

We are interested here in the static screening and we put from now on \( \omega = 0 \). By transforming \( V(k, n) \) from eq. (2) back to \( z \)-space we obtain the expansion of the screened interaction in cylindrical harmonics. As \( V^{(0)}(k, n) \) and \( \Pi(0; k, n) \) are even functions of \( n \) we come to the series

\[
V(z, \varphi) = \int_{-\infty}^{\infty} \frac{dk}{(2\pi)^2} \exp(ikz)(V(k, 0) + 2 \sum_{n=1}^{\infty} V(k, n) \cos(n\varphi)).
\]

(5)

The slowest decreasing term at \( |z| \to \infty \) is given by the zero harmonic and reads

\[
V_0(z) = \int_{0}^{\infty} \frac{dk}{\pi} \frac{4\pi e^2 I_0(ka)K_0(ka)}{1 + 4\pi e^2 \Pi_0(k)I_0(ka)K_0(ka)},
\]

(6)

where

\[
\Pi_0(k) \equiv \Pi(0; k, 0) = \frac{m}{\pi^2 k} \sum_{-L}^{L} \ln \left| \frac{k + 2p_l}{k - 2p_l} \right|
\]

(7)

\( L \) is the number of the highest occupied subband at zero temperature, \( p_l = \sqrt{2m(E_F - B l^2)} \) is the Fermi momentum of the \( \ell \)-th subband.

To calculate \( V_0(z) \) in the regime \( |z| >> a \) we write in the eq. (6) \( \cos(kz) = \Re(\exp(ikz)) \), and turn the path of integration to the upper imaginary semiaxis. At large \( |z| \) the monotonous part of the screened potential \( V_0(z) \) is given by a small vicinity of the point \( k = 0 \) in the integral (6), and we get the expansion in inverse powers of the value \( \Lambda \equiv \ln(2|z|/a) \sim C \) (\( C \) is the Euler constant)

\[
V_0(z) \sim \frac{e^2}{z} \left( \frac{maB}{4\pi \kappa_0 \Lambda} \right)^2 \left( 1 - \frac{e^2}{\pi^2 \kappa_0^2} + \frac{4C}{2\Lambda} + \cdots \right),
\]

(8)

where \( \kappa_0 \equiv \Pi_0(k \to 0) = m[1/p_0 + 2 \sum_{l=1}^{L} (1/p_l)]/\pi^2 \); \( \pi \kappa_0 \) is the sum of the partial density of states in occupied subbands. Thus, the Coulomb interaction in nanotubes is screened rather weakly, \( V_0(z) \sim e^2/(z \ln^2(z/a)) \).

Qualitatively different results are obtained for non-zero harmonics. The factor \( I_n(ka)K_n(ka) \) for \( n \neq 0 \) tends to the constant \( 1/2n \) for \( k \to 0 \) and \( \Pi(k, n) \) at \( k = 0 \) remains finite either. Thus, all non-zero harmonics of the Coulomb interaction undergo the screening of the dielectric type, that is, their dependence on the distance \( z \) coincides with that of the bare Coulomb law. Indeed, from eq. (3) we have for the \( n \)-th harmonic of the bare interaction

\[
V_{n}^{(0)}(z) = \frac{e^2}{\pi a} Q_{n-1/2}(1 + \frac{z^2}{2a^2});
\]

\[
V_{n}^{(0)}(z \gg a) \sim \frac{\Gamma(n + 1/2)}{\sqrt{\pi n!}} \frac{e^2}{z} \left( \frac{a}{z} \right)^{2n},
\]

(9)

where \( Q_{n} \) is the spherical function of the second type. The regular part of the screened interaction stems again from the region of small \( k \).

For the \( n \)-th harmonic we get

\[
V_n(z) \sim \frac{\Gamma(n + 1/2)}{\sqrt{\pi n!}} \frac{e^2}{z} \left( \frac{a}{z} \right)^{2n} \left( 1 + \frac{\pi \kappa_n}{maBn} \right)^{-2}, \kappa_n = \Pi(\omega = 0; k = 0; n).
\]

(10)
The role of the effective dielectric constant for the n-th harmonic is played by the quantity

$$\epsilon_n = (1 + \frac{\pi \kappa_n}{ma_B n})^2$$

(11)

that tends to 1 with increasing n. The dielectric type of screening of harmonics with n ≥ 1 can be understood in terms of the classical electrostatics (suggested by M.V.Entin). Each term of the series in eq. (5) corresponds to one of the expansion in multipoles: n = 1 gives dipole-dipole contribution, n = 2 - quadrupole-quadrupole and so on. At very large |z| the fields of all multipoles on the nanotube surface are practically normal to the axis, hence, they cause only the displacements of electrons in the azimuthal direction \(\varphi\). That is why the \(z\)-dependence of the e-e interaction does not change; the system simply is polarized in accord with the dielectric mechanism when only the bound charges are available.

3. Basic relation for a nonuniform system

In the self-consisted field approach (called, also random phase approximation (RPA)) the screened potential is expressed via the Lindchard polarization operator. In case of multicomponent plasma this operator is, obviously, a matrix \(\Pi_{ij}(\omega, \mathbf{q})\), where indices \((i, j)\) label the components, \(\omega, \mathbf{q}\) are the frequency and the wave vector. In what follows we put \(\omega = 0\) (static screening). In a layered structure submersed in a medium with the background dielectric constant \(\varepsilon\) the induced part of the total (i.e. screened) potential \(U_{ind}\) obeys the Poisson equation. For the \(\mathbf{q}\)-th Fourier component we have

$$\left(\frac{d^2}{dz^2} - q^2\right) U_{ind}(\mathbf{q}, z) = -\frac{4\pi e^2}{\varepsilon} \sum_{nm} \Pi_{nm} \varphi_n(z) \varphi_m(z) U_{nm}(\mathbf{q})$$

(12)

where \(\varphi_n(z)\) are the eigenfunction of the transverse motion, \(U_{nm}\) - matrix element of the total potential, while \(\Pi_{nm}\) takes the standard form

$$\Pi_{nm}(\mathbf{q}) = -\sum_k f_n(k) - f_m(k + \mathbf{q} + \mathbf{k})$$

(13)

\(E_n(k)\) is the electron energy in the \(n\)-th subband, \(f_n\) - the Fermi occupation numbers. With the Green function of the eq.(12) \(G(z, z') = \exp(-q|z - z'|)/2q\) we can find its formal solution, then we add the bare (unscreened) potential \(U^0\) to \(U_{ind}\) and take matrix element over the functions \(\varphi_n\). Thus, we arrive to the the closed system of equations for the values \(U_{nm}(\mathbf{q})\):

$$U_{ij}(q) + \frac{2\pi e^2}{q} \sum_{nm} I_{ij, nm}(q) \Pi_{nm}(q) U_{nm} = U_{nm}^0(q), \quad \hat{e}^2 = e^2/\varepsilon.$$  

(14)

Here we introduce the form-factors

$$I_{ij, nm}(q) = \int \varphi_i(z) \varphi_j(z) e^{q|z - z'|} \varphi_n(z') \varphi_m(z') dz dz', \quad (15)$$

which (together with \(\Pi_{nm}(q)\)) define the matrix dielectric function:

$$\varepsilon_{ijnm} = \delta_{in}\delta_{jm} + \frac{2\pi e^2}{q} I_{ij, nm}(q) \Pi_{nm}(q)$$

(16)

As it is seen from the above given equations, in the multicomponent system all the components of the plasma contribute to the screening even with accounting for the finite number \(n_0\) of the occupied subbands. But the contribution of the empty subbands decreases with increasing \(n\) both due to denominators of the polarization operator \(\Pi_{nm}\), and because of decreasing the form-factors \(I_{ij, nm}(q)\) owing to fast oscillations of the functions \(\varphi_n(z)\) at large \(n\). This allows us to confine ourselves to some finite number of subbands for analysis of the screening in multicomponent structures. Below we consider some examples of such structures.
4. Double quantum well

Taking into account only the lowest subband in each of two wells we come to the two-level system for which four equations are given by the relations (14). As the functions \( \varphi_n(z) \) are real we have \( U_{12} = U_{21} \) and, thus, three equations determine the elements \( U_{11}, U_{22} \) and \( U_{12} \). Suppose now that the structure is symmetric relatively its middle plane. Then the functions \( \varphi_n(z) \) are characterized by parity and the two states under consideration \( \varphi_1(z) \) and \( \varphi_2(z) \) are of different parity. In such a case the "oblique" form-factors \( I_{11,12} \) and \( I_{22,21} \) equal zero and the equation for \( U_{12} \) is split off:

\[
U_{12}(q) = \frac{U_{12}^0(q, z_0)}{1 + \gamma_q[\Pi_{12}(q) + \Pi_{21}(q)]I_4(q)}
\]  

(17)

where \( \gamma_q = 2\pi^2/q \), \( I_4(q) = I_{12,12}(q) \). \( U_{12}^0 \) depends on the position \( z_0 \) of the charge creating the bare potential \( U^0(\rho, z) \).

In a symmetric quantum well the functions \( \varphi_1 \) and \( \varphi_2 \) can be taken in the form:

\[
\varphi_1(z) = \frac{\psi_1(z) + \psi_2(z)}{\sqrt{2}}, \quad \varphi_2(z) = \frac{\psi_1(z) - \psi_2(z)}{\sqrt{2}},
\]

where \( \psi_1 \) and \( \psi_2 \) correspond to the states localized either in the first or in the second well. As the simplest approximation one can put \( \psi_1^2(z) = \delta(z-H/2), \psi_2^2(z) = \delta(z+H/2) \), where \( H \) is the separation between the quantum wells. Note, that such a simplified model still allows for tunneling between the wells while the subband eigenfunctions \( \varphi_1(z) \) and \( \varphi_2(z) \) describe the states of electrons collectivized over two wells. For the matrix elements of the bare potential \( U^0(q, z) = \gamma_q e^{-q|z-z_0|} \) we obtain:

\[
U_{11}^0(q) = U_{22}^0(q) = \gamma_q \frac{1}{2} \left( e^{-q|z-H/2|} + e^{-q|z+H/2|} \right), \\
U_{12}^0(q) = U_{21}^0(q) = \gamma_q \frac{1}{2} \left( e^{-q|z-H/2|} - e^{-q|z+H/2|} \right)
\]

(18)

If the external charge is placed in the first well \( (z_0 = H/2) \) the screened potential is defined by its matrix elements:

\[
U_{12}(q) = \frac{\gamma_q I_4(q)}{1 + \gamma_q[\Pi_{12}(q) + \Pi_{21}(q)]I_4(q)}, \\
U_{11}(q) = U_{22}(q) = \frac{\gamma_q I_1(q)}{1 + \gamma_q[\Pi_{11}(q) + \Pi_{22}(q)]I_1(q)}, \\
I_1(q) = I_2(q) = I_3(q) = \frac{1}{2}(1 + e^{-qH}), \quad I_4(q) = \frac{1}{2}(1 - e^{-qH})
\]

(19)

The elements of the polarization operator \( \Pi_{ij}(q) \) are calculated by the standard method (see [1]) and we use their values for zero temperature.

The potential 'seen' by electrons in the first \( (z = H/2) \) or in the second \( (z = -H/2) \) well is the result of averaging \( U(z) \) with \( \psi_1^2(z) \) or \( \psi_2^2(z) \), correspondingly, and can be expressed via \( U_{ij} \) from the eq. (19):

\[
\langle U \rangle_{1,2} = \frac{U_{11} + U_{22}}{2} \pm U_{12},
\]

(20)

We are interesting in the asymptotic behavior of \( \langle U(\rho) \rangle_{1,2} \) for large \( \rho \). There are monotonous part (contribution of small \( q \)) and the Friedel oscillations. The monotonous part can be found if
we put \( q = 0 \) in \( \Pi_1(q) \) and in \( I_1(q) \) but \( I_4(q) \) should be expanded up to the terms of the order of \( q^2 \). In this limit \( \Pi_1 = \Pi_2 = m^*/\pi, \Pi_1 + \Pi_2 = 2(N_1 - N_2)/\Omega = \Pi_0 \), \( N_1, N_2 \) are the occupation numbers of the subbands, \( \Omega = E_2 - E_1 \) is the energy separation between the subbands. We see that asymptotes of the diagonal elements coincide with the case of a single electron 'sheet' but the screening radius is exactly two times smaller

\[
U_{11}(\rho) = U_{22}(\rho) \sim \frac{\tilde{e}^2}{(2q_s)^2 \rho^3}, \quad q_s = 2/a_B
\]  

(21)

For the off-diagonal element we found the same asymptote \( C/\rho^3 \) but now the coefficient \( C \) depends on the subband occupation numbers and on the interwell separation \( H \):

\[
U_{12}(\rho) \sim \frac{\tilde{e}^2 H^2}{2(1 + \pi \tilde{e}^2 H \Pi_0)^2 \rho^3}.
\]

(22)

In the state of thermodynamic equilibrium \( \Pi_0 \) increases with increasing the electron concentration until the second subband starts to fill with electrons. After that \( \Pi_0 \) takes the value \( \Pi_0 = 2(N_1 - N_2)/\Omega = 2m^*/\pi \) and remains constant. The total potential in the plane free from the external charge \( z = -H/2 \) equals \( U_{11} - U_{12} \) and changes its sign for large \( \rho \) and at \( H > a_B/2 \). Physical reason of such overscreening can be explained as follows: if the external charge is placed at \( \rho = 0 \) in the plane \( z = H/2 \) then 'under' it, in the plane \( z = -H/2 \) a charge of the opposite sign is induced and the field of the induced charge in a distant point of this plane exceeds the field of the initial charge for sufficiently large \( H \).

5. Friedel oscillations

It is known that oscillatory contribution to the screened potential at large distances stems from singularities of dielectric function owing to the jump of the Fermi distribution at \( T = 0 \). In our situation these are branching points of radicals which enter into \( \Pi_{11}, \Pi_{22}, \Pi_{12} \). The oscillatory contribution of diagonal elements originates from the points \( q = 2p_1, 2p_2 \):

\[
\tilde{U}_{11}(\rho) \sim -\tilde{e}^2 q_s \left[ \frac{4p_1^2 I_1^2(2p_1)}{[2p_1 + q_s I_1(2p_1)][2 - \sqrt{1 - p_1^2/p_1^2}]^2} \frac{\sin(2p_1\rho)}{(2p_1\rho)^2} \right. - \left. \frac{4p_2^2 I_1^2(2p_2)}{[2p_2 + q_s I_1(2p_2)][2 - \sqrt{1 - p_2^2/p_2^2}]^2} \frac{\sin(2p_2\rho)}{(2p_2\rho)^2} \right]
\]

(23)

and similarly for \( \tilde{U}_{22}(\rho) \). In comparison with result [1] population of the second subband changes the coefficient at \( \sin(2p_1\rho)/(2p_1\rho)^2 \) and adds the term oscillating with the period \( \pi/p_2 \).

More important modifications appear due to off-diagonal element \( U_{12} \). If in the thermodynamic equilibrium the second subband is not occupied, \( U_{12} \) has no singular points at real \( q \). But at \( p_1^2 > 2m^*\Omega \) electrons appear in the second subband and their maximal momentum is found from the relation \( p_2^2 = p_1^2 - 2m^*\Omega \). It is easy to see that in this case one obtains

\[
\Pi_{12}(q) + \Pi_{21}(q) = \frac{2m^*}{\pi} \left[ 1 - \theta(q^2 - 2m^*\Omega) \sqrt{\left( 1 + \frac{2m^*\Omega}{q^2} \right)^2 - \frac{4p_1^2}{q^2}} \right].
\]

(24)

The radical in (24) vanishes in two points \( q^2_+ = (p_1 + p_2)^2 \) and \( q^2_- = (p_1 - p_2)^2 \) but due to \( \theta \)-function contribution of the point \( q_- \) vanishes. Thus, the off-diagonal element gives the term oscillating with combination frequency:

\[
\langle \tilde{U}(\rho) \rangle_{1,2} \sim \tilde{U}_{11}(\rho) \pm B \frac{\sin(p_1 + p_2)\rho}{(p_1 + p_2)^2 \rho^2}
\]

(25)
Coefficient in (25) has been calculated in the framework of $\delta$-model for eigenfunctions $\varphi_1, \varphi_2$ (18). But the structure of the polarization operator matrix $\Pi_{mn}$ depends only on electron spectrum and is independent of the form of the wave functions. It is clear that the presence of the oscillation contribution with the period $2\pi/q_+$ in the asymptotic of the potential is the general property of the two-component systems.

6. Donor in DQW

Consider now a donor localized in the upper ($z_0 = H/2$) plane of DQW. In this case Fourier transform of the potential is given by eq.(19) taken with opposite sign (attraction). The wave function of the ground state of an electron located on this center can be found by variation method. As a trial function we choose

$$\Psi(\rho, z) = \frac{q_s}{\sqrt{2\pi}} e^{-\lambda\rho/a_B} (\varphi_1(z) \cos \theta + \varphi_2(z) \sin \theta),$$

(26)

Where $\lambda, \theta$ are the variation parameters. Calculating energy of the ground state and taking derivatives $\partial/\partial \lambda, \partial/\partial \theta$ we get a system of equations for $\lambda$ and $\theta$. This system was solved numerically. The results of numerical calculations are presented in Fig.1. It is seen that increasing of the electron concentration in the system leads to increasing screening and, as a result, binding energy decreases. A jump in density of states under occupation of the second subband results in a break and faster decreasing of the donor binding energy versus electron concentration.

**Figure 1.** Dependence of the donor ground state energy on the Fermi level position. Arrow shows the beginning of population of the second subband. All energies are given in effective Rydberg. For numerical calculations we assumed $E_1 = 0, E_2 = 1R_Y$. The curves are given for three separations between the quantum wells $\beta = H/a_B$.

**Figure 2.** The binding energy versus donor position inside the barrier between quantum wells. The separation between quantum wells is $\beta = 1$. The values of the Fermi level are given in the figure.

Fig.2 shows the dependence of ground state energy versus donor position inside the barrier between quantum wells. It can be seen that the minimum of the energy is reached in the middle
plane of the structure where (due to symmetry) the off-diagonal matrix element $U_{12}(q)$ vanishes. It should be pointed out that the last circumstance occurs for any symmetric structure and does not depend on the form of the wave functions.

7. Multilayer superlattice

Let us consider an infinite periodic system of the parallel layers in each of them there is 2D electron plasma. We neglect the tunneling and finite width of the layer, in other words, for wave functions we write $\varphi_n(z) = \sqrt{\delta(z - n\Delta)}$, where $\Delta$ is a period of the structure. Just in such a model the problem of screening in superlattice was considered in [7]. A method that was applied in [7] is a very complicate and authors presented numerical calculations only for several particular cases. Below we show that this problem can be solved analytically in the asymptotic case of large distances from external charge and for arbitrary direction relative to the superlattice axis. In the applied model only form-factors $I_{nn,mm} = \exp(-q|n - m|)$ do not vanish. System of equations (14) reads:

$$U_{nn} + \gamma_q \sum_m \Pi_{mm} e^{-q\Delta|n - m|} U_{mm} = \gamma_q e^{-q\Delta|n_0 - n|} \tag{27}$$

where $n_0$ is a number of plane where external charge is located and we can put $n_0 = 0$. Let us introduce Fourier-transform of the $U_{nn}(q)$ over index $n$:

$$U(k, q) = \sum_n U_{nn}(q) e^{-ik\Delta n}, \quad -\frac{\pi}{\Delta} < k < \frac{\pi}{\Delta}$$

and take into account that in a periodic system $\Pi_{mm}$ does not depend on $m$. Multiplying both sides of the eq.(27) by $e^{-ik\Delta n}$ and summing up over $n$ one can easily find:

$$U(k, q) = \frac{\gamma_q Q(k, q)}{1 + \gamma_q \Pi(q) Q(k, q)} \tag{28},$$

where

$$Q(k, q) = \frac{\sinh(q\Delta)}{\cosh(q\Delta) - \cos(k\Delta)} \tag{29}.$$

In the long-wavelength limit $k\Delta, q\Delta \ll 1$ and $q \ll p_F$ one can take $\Pi = m^*/\pi$ and from (28), (29) we obtain:

$$U(k, q) = \frac{4\pi e^2}{\Delta(k^2 + q^2) + 2q_s}. \tag{30}$$

Transforming (30) in r-space we get the potential in $n$-th plane at the distance $\rho$:

$$U(\rho, n) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{q}\rho} d\mathbf{q} \int_{-\pi/\Delta}^{\pi/\Delta} U(k, q) e^{ik\Delta n} d(\Delta k) \tag{31}$$

Integral over $dk$ can be reduced to the limits from $-\infty$ up to $\infty$. As a result we obtain

$$U(\rho, n) = \frac{e^2}{r_n \kappa} \exp(-r_n\kappa), \tag{32}$$

where $\kappa^2 = 2q_s/\Delta$, $r_n^2 = \rho^2 + (n\Delta)^2$. Thus, we can conclude that in this approximation, when $\rho >> \Delta, n >> 1$, screening becomes three-dimensional and isotropic and $\sqrt{\Delta a_B}/2$ stands for the Tomas-Fermi radius.
An anisotropy of the system occurs when one of the above given inequalities is broken, i.e. in the vicinity of the axis \( z \) \( (n >> 1, \rho \sim \Delta) \) and in the directions parallel to the planes \( (\rho \to \infty, n \sim 1) \). We present here the asymptotics at \( \rho = 0, n >> 1 \) and \( n = 0, \rho >> \Delta \):

\[
U(\rho = 0, n >> 1) = \left( 1 + \frac{q_s \Delta}{3} \right)^{-1} \frac{\tilde{e}^2}{|z|} \exp(-\kappa |z|),
\]

(33)

\[
U(\rho >> \Delta, n = 0) = \left( 1 + \frac{q_s \Delta}{2} \right)^{-1/2} \frac{\tilde{e}^2}{\rho} \exp(-\kappa \rho).
\]

(34)

We see that the three-dimensional character of the screening remains (Yukawa law) but there is an anisotropic renormalization (decreasing) of the charge. Knowing screened potential \( U(\rho, n) \) the induced charge can be easily found at the point \( \rho \) of the \( n \)-th plane. This result was obtained in [8] for hydrodynamic model of the 2D plasma (long-wavelength limit). In asymptotic regimes \( \rho >> \Delta, n \sim 1 \) and \( \rho \sim \Delta, n >> 1 \) only small momenta are essential, and results of [8] gives Yukawa law, like our formulas (33), (34) obtained in the framework of RPA.

At last we consider Friedel oscillations of screening in superlattice. A singular contribution into the Fourier-transform \( U(k, q) \) at large distance from the charge along the planes is

\[
U(k, \rho) = \frac{1}{(2\pi)^2} \int \frac{\gamma_q Q(k, q)e^{iq\rho}d\mathbf{q}}{1 + \gamma_n \frac{m_e}{m_\pi} \left[ 1 - \theta \left( 1 - 4p_F^2/q^2 \right) \sqrt{1 - 4p_F^2/q^2} \right] Q(k, q)} \sim \frac{\gamma_q Q(k, q)e^{iq\rho}d\mathbf{q}}{1 + \gamma_n \frac{m_e}{m_\pi} \left[ 1 - \theta \left( 1 - 4p_F^2/q^2 \right) \sqrt{1 - 4p_F^2/q^2} \right] Q(k, q)}
\]

\[
\sim -\tilde{e}^2 q_s \frac{Q(k, 2p_F)\sin(2p_F\rho)}{[2p_F + q_s Q(k, 2p_F)]^2\rho^2}.
\]

Further one has to transform \( U(k, \rho) \) according to

\[
\frac{\Delta}{2\pi} \int_{-\pi/\Delta}^{\pi/\Delta} U(k, \rho)e^{i\kappa \Delta}dk
\]

in order to find the potential \( U_n(\rho) \) in the \( n \)-th layer. The integral of the type

\[
\int_{-\pi}^{\pi} \frac{e^{i\varphi}}{(A - \cos \varphi)^2}d\varphi
\]

can be calculated easily, and we come to the result:

\[
U_n(\rho) = -\tilde{e}^2 q_s \frac{\sinh^2(2p_F\Delta)}{\sinh^2(2p_F\tilde{\Delta})} \coth(2p_F\tilde{\Delta})e^{-2p_F\tilde{\Delta}|n|}\frac{\sin(2p_F\rho)}{(2p_F\rho)^2},
\]

(37)

where \( \tilde{\Delta} \) can be found from the equation

\[
\cosh(2p_F\tilde{\Delta}) = \cosh(2p_F\Delta) + \frac{q_s}{2p_F} \sinh(2p_F\Delta).
\]

(38)

Thus, the coordinate dependence of the oscillating part of the potential (in contrast to monotonic one) remains two-dimensional, i.e. proportional to \( \rho^{-2} \). The amplitude of the oscillating contribution decreases exponentially far away from the plane where external charge is located with decrement \( 2p_F\tilde{\Delta} \). As \( \tilde{\Delta} \neq \Delta \) in accord with (38), the decay length along the \( z \) axis differs from \( (2p_F)^{-1} \cdot \) the period of oscillations in the \( (x, y) \) plane but these two values become equal at \( q_s << p_F \) (2D plasma of high density).
8. Conclusion
It is known that in a single 2D layer the radius of screening of the long-wavelength component of an electrostatic perturbation \( q < 2p_F \) does not depend on the electron concentration and equals \( q_s^{-1} = a_B/2 \). Screened potential of a point charge decreases at infinity as \( 1/\rho^3 \) (monotonic part) and oscillates as \( \sin(2p_F\rho)/\rho^2 \) \([1]\). We have shown in the present work that:

1) Zero azimuth harmonic of the Coulomb potential in nanotubes is screened rather weakly as \( 1/(z \ln^2 z) \).
2) All \( n \)-th \((n \neq 0)\) harmonics are screened in accord with dielectric mechanism and the effective dielectric constant depends on \( n \).
3) In DQW radius of screening depends on difference of the population of subbands because of contribution of the intersubband transitions (off-diagonal element); in the equilibrium case this radius becomes constant as soon as the second subband starts to be populated.
4) Friedel oscillations include contribution with combination period if both subbands of DQW are populated.
5) In infinite periodic system of 2D layers screening of the Coulomb potential becomes three-dimensional (Yukawa law); the role of the radius of screening plays a quantity independent of the electron concentration. Anisotropy of the system manifests itself in the dependence of the preexponential factor on the direction. Amplitude of the Friedel oscillations in the \( n \)-th plane of the superlattice exponentially decreases with increasing \( n \).

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