Bose-Einstein correlations of unstable particles *

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Abstract

Within a field theoretical formalism suited to treat inhomogeneous hot quantum systems, we derive the two-particle correlation function for particles having a spectral width $\gamma$ in the region of their emission. We find, that this correlation function measures the radius $R_0$ of the thermal source only in case $\gamma R_0 > 1$.

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In the field of relativistic heavy-ion collisions the analysis of Bose-Einstein correlations has attracted much attention \[1, 2, 3, 4\]. The general hope is to extract information about the size of a source radiating pions by studying their two-particle correlations. These correlations are typical quantum effects, hence quantum field theory is the proper framework to address the problem theoretically. Two types of sources are physically interesting: Classical currents and thermal sources – the physical reality being a mixture of these cases.

The problem of a free quantum field radiating from a classical current is exactly solvable \[5, pp.438\], also at finite temperature \[6\]. Conversely, it is quite difficult to find a consistent theoretical description for bosons radiating thermally from a local ”hot spot”. One reason is, that such a physical situation corresponds to a non-equilibrium state. Another reason for this difficulty is, that in thermal states a breakdown of perturbation theory may occur when it is expressed in terms of stable particles having zero width \[7\].

With the present paper we attempt to study these two aspects, i.e., we address the questions: 1. How are Bose-Einstein correlations affected by gradients in the temperature distribution, and 2. How are they affected by a finite lifetime of the bosons.

In our model, we assume an entirely thermal radiation of pions from a “hot spot”, which acquire a nonzero spectral width in the region of their generation, e.g. by coupling to $\Delta_{33}$-resonances. All thermally generated pions are propagating out of the hot spot.

This picture is grossly simplified as compared to the physical reality: At least a substantial fraction of pions generated in relativistic heavy-ion collisions is not produced thermally, but arises from the decay of the $\Delta_{33}$-resonance outside the “hot spot”. However, on one hand it is legitimate to study only a partial aspect of the full problem. On the other hand, the formalism introduced here is general enough to be extended to the resonances as well.

Quantum field theory for non-equilibrium states comes in two flavors: The Schwinger-Keldysh method \[8\] and thermo field dynamics (TFD) \[9\]. For the purpose of the present paper, we prefer the latter method: Apart from its technical elegance, the problem of an inhomogeneous temperature distribution has been solved explicitly in TFD, up to first order in the temperature gradients \[10, 11\]. This solution includes a nontrivial spectral function of the quantum field under consideration: It employs a perturbative expansion in terms of generalized free fields with continuous mass spectrum \[7\].
described by a statistical operator (or density matrix) $W$, and the measurement of an observable $E$ will yield an average that is calculated as the trace of $EW$ over the Hilbert space of the system. In thermo field dynamics (TFD), this is simplified to the calculation of a matrix element

$$\left\langle E(t, x) \right\rangle = \frac{\langle 1 | EW | 1 \rangle}{\langle 1 | W | 1 \rangle}$$

and the Hilbert space is doubled [9]. The thermal pion field is described by two scalar field operators $\phi_x, \tilde{\phi}_x$ and their adjoints $\phi^*_x, \tilde{\phi}^*_x$, with canonical commutation relations. The field $\phi_x$ is expanded into momentum eigenmodes: $a_{kl}^+(t)$ creates a pion with momentum $k$ and charge $l = \pm 1$, a second set of operators (commuting with $a^+, a$) exists for the tildean field $\tilde{\phi}_x$ [9, 11].

These operators do not excite stable on-shell pions. Rather, they are obtained as an integral over more general operators $\xi, \tilde{\xi}$ with a continuous energy parameter $E$:

$$\left(\begin{array}{c} a_{kl}(t) \\ a^+_{kl}(t) \end{array}\right) = \int_0^\infty \int d^3q \; A^{\infty/\varepsilon}_{\pm}(E, (q + k)/\varepsilon) \left(\begin{array}{c} \xi_{Eql} \\ \tilde{\xi}_{Eql} \end{array}\right)^* e^{-iEt}$$

$$\left(\begin{array}{c} a^+_{kl}(t) \\ -\tilde{a}_{kl}(t) \end{array}\right)^T = \int_0^\infty \int d^3q \; A^{\infty/\varepsilon}_{\pm}(E, (q + k)/\varepsilon) \left(\begin{array}{c} \xi_{Eql} \\ -\tilde{\xi}_{Eql} \end{array}\right)^T \tilde{B}^+(E, q, k) e^{iEt}.$$  

The principles of this expansion have been derived in ref. [7]. $A(E, k)$ is a positive weight function. For equilibrium states, this function is the spectral function of the field $\phi_x$. For non-equilibrium systems, the existence of a spectral decomposition cannot be guaranteed [9]. We may expect however, that close to equilibrium the field properties do not change very much. Thus, with this formalism we study a quantum system under the influence of small gradients in the temperature, with local spectral function $A(E, k)$. Corrections to such a picture only occur in second order of temperature gradients [10, 11].

A thorough discussion of the $2 \times 2$ Bogoliubov matrices was carried out in ref. [12]. For the purpose of the present paper, we simply state their explicit form as

$$\tilde{B}(E, q, k) = \left(\begin{array}{cc} \delta^3(q - k) + N_l(E, q, k) & -N_l(E, q, k) \\ -N_l(E, q, k) & \delta^3(q - k) \end{array}\right),$$

where $N(E, q, k)$ is the Fourier transform of a space-local Bose-Einstein distribution function

$$N_l(E, q, k) = \frac{1}{(2\pi)^3} \int d^3z \; e^{-i(q - k)z} n_l(E, (q + k)/2, z)$$

$$n_l(E, (q + k)/2, \beta) = \frac{1}{e^{\beta E} - 1}.$$  


The $\xi$-operators have commutation relations

$$\left[ \xi_{E Kl}, \xi_{E' K'l'} \right] = \delta_{ll'} \delta(E - E') \delta^3(k - k').$$

(5)

Similar relations hold for the $\tilde{\xi}$ operators, all other commutators vanish, see [7]. They act on the "left" and "right" statistical state according to

$$\xi_{E Kl} | W \rangle = 0, \quad \tilde{\xi}_{E Kl} | W \rangle = 0,$$

(6)

$$\xi_{E Kl} = 0, \quad \tilde{\xi}_{E Kl} = 0 \quad \forall E, k, l = \pm 1.$$

With these rules, all bilinear expectation values can be calculated exactly. Higher correlation functions have a perturbative expansion in the spectral function.

Of these, we are interested in the two-particle correlation function, which is the probability to find in the system a pair of pions with momenta $p$ and $q$:

$$c_{ll'}(p, q) = \frac{\langle \hat{a}_{pl}(t) \hat{a}_{ql'}(t) \hat{a}_{ql'}(t) \hat{a}_{pl}(t) \rangle}{\langle \hat{a}_{pl}(t) \hat{a}_{ql}(t) \rangle \langle \hat{a}_{ql'}(t) \hat{a}_{pl}(t) \rangle} = 1 + \delta_{ll'} F_{\infty}(p, q) F_{\infty}(q, p),$$

(7)

For simplicity, we abbreviate the mean momentum of this pair by $Q = (p + q)/2$. Using the above rules of thermo field dynamics, the functions $F_{\infty}$ and $F_{\in}$ are calculated as

$$F_{\infty}(p) = \int_0^\infty dE \int d^3z \mathcal{A}_{\mathcal{X}}(E, p) |_{z}(E, p, z)$$

$$F_{\in}(p, q) = \int_0^\infty dE \int d^3z \left( \mathcal{A}_{\mathcal{X}}(E, p) \mathcal{A}_{\mathcal{X}}(E, Q) \right) \frac{1}{2} e^{i(p-q)z} n_l(E, Q, z).$$

(8)

Before we use the above expression to obtain numerical results, we perform an expansion of $F_{\in}$ around the mean momentum $Q$. This is consistent with the restriction, that the spectral function acquires corrections in second order of the gradients:

$$F_{\in}(p, q) = F_{\in}(p, q)$$

$$+ \int_0^\infty dE \int d^3z e^{i(p-q)z} \left( \frac{i}{2} \nabla_{Q} \mathcal{A}_{\mathcal{X}}(E, Q) \nabla_{z} |_{z}(E, Q, z) \right) + O(\nabla_{\mathcal{X}}^2)$$

$$F_{\in}(p, q) = \int_0^\infty dE \int d^3z e^{i(q-p)z} \mathcal{A}_{\mathcal{X}}(E, Q) |_{z}(E, Q, z).$$

(9)

The first part of this expansion is, apart from the folding with the spectral function, also obtained in other calculations of the correlator [2, 3, 4]. This standard expression for the correlation function therefore is

$$f_{ll'}(p, q) = 1 + \delta_{ll'} F_{\in}(p, q) F_{\in}(q, p).$$

(10)
and our result for \( c_{ll'}(p, q) \) differs from \( \tau_{ll'}(p, q) \) in first order of gradients in the distribution function. For the purpose of generalizing our result it is worthwhile to note that the gradient term in (9) is just one half of the Poisson bracket of \( \mathcal{A} \) and \( n \).

We have calculated these correlation functions with a simple parameterization of the pion spectral function,

\[
\mathcal{A}_\downarrow(\mathcal{E}, \mathbf{p}) = \frac{\varepsilon \mathcal{E} \gamma}{\pi} \frac{\infty}{\sqrt{\varepsilon - \mathcal{E} \gamma}} + \Delta \mathcal{E} \varepsilon \gamma \varepsilon
\]

where \( \Omega_p = \sqrt{m_{\pi}^2 + \mathbf{p}^2 + \gamma^2} \) and \( m_{\pi} = 140 \text{ MeV} \). This parameterization has been motivated and related to a more serious field theoretical approach in ref. [13]. To gain information about the \textit{maximal} influence exerted by the occurrence of a nonzero spectral width, we studied only the case of an energy and momentum independent \( \gamma \) equal for both charges. The temperature distribution was taken as radially symmetric gaussian

\[
T(z) = T(r) = T_0 \exp \left( -\frac{r^2}{2R_0^2} \right),
\]

with chemical potential \( \mu = 0 \) and \( R_0 = 5 \text{ fm} \). The local equilibrium pion distribution for a given momentum \( k \) is obtained by folding \( n \) with the spectral function. Hence, the mean radius of this particle distribution function acquires a \( \gamma \)-dependence. We define the rms radius \textit{orthogonal} to the direction of \( Q \) as

\[
R_n = \sqrt{\frac{I_2}{I_0}} \quad I_j = \int_0^\infty \int_0^\infty dE \mathcal{A}(\mathcal{E}, k) \left( e^{E/T(\nabla)} - \infty \right)^{-\infty}.
\]

Note, that \( R_n \) is \textit{not} the 3-dimensional rms radius of the distribution function (which would be \( I_4/I_2 \)). Rather, \( R_n \) is half the product of angular diameter and distance between detector and source. A constant temperature over a sphere of radius \( R_0 \) would yield an \( R_n = R_0/\sqrt{3} \), while its 3-D rms radius is \( R_0\sqrt{3/5} \).

The correlation functions, \( c_{ll'}(p, q) \) calculated according to (11) and \( \tau_{ll'}(p, q) \) as defined in eqn. (10) then may be fitted by a gaussian form, i.e.,

\[
c_{ll'}(p, q) \approx 1 + \exp \left( -R^2(p - q)^2 \right)
\]

and similarly for \( \tau_{ll'}(p, q) \) with parameter \( \overline{R} \). We assume for our conclusion, that \( c_{ll'}(p, q) \) is the correlation function measured experimentally.

In the figure, we have plotted the two fit parameters \( R, \overline{R} \) and \( R_n \) as function of \( \gamma \). The principal result of the calculation is, that for \textit{small} values of \( \gamma \) the...
Consequently, the measured correlation radius $R$ is always larger than $\bar{R}$ as expected from the function $\tilde{c}_0(p,q)$. The deviation is such that for small enough $\gamma$, $R \approx \bar{R} + 1/\gamma$. For larger $\gamma$, the small differences between $R$, $\bar{R}$ and $R_n$ may be attributed to our use of a gaussian temperature distribution: $n(T(r))$ is not strictly gaussian, only in the (unphysical) limit $\gamma \to \infty$ one reaches $R = \bar{R} = R_n = R_0/\sqrt{2}$.

Before we interpret this result, we have to admit that our calculation is very crude: Neglecting energy and momentum dependence of the $\gamma$ in the spectral function can be a first step only. Also, in a more realistic calculation the partially coherent production of pions would have to be taken into account, forcing $c_{\nu}(p,p) < 2$.

Within this limitation however we feel safe when stating the following answer to the questions raised in the introduction: A finite lifetime or nonzero spectral width $\gamma$ of the bosons is essential, if one wants to infer the thermal source radius $R_0$ from correlation measurements. To be more precise, only for $\gamma R_0 \geq 1$ the correlation function measures the mean radius of the particle distribution function.

This result is also in agreement with our view of the equilibration process: The
The equilibration rate of a distribution function is, to lowest order, given by the spectral width of the particle [12]. Consequently, a very small $\gamma$ corresponds to a system that does not equilibrate – hence the correlation function approaches the “quantum limit” $c_{ll'}(p,q) \rightarrow 1 + \delta_{ll'} \delta_{pq}$, and the correlation radius obtained by a gaussian fit becomes infinite.

Also, for a given energy, $1/\gamma$ is a measure for the spatial size of the pion “wave packet”, which must be smaller than the object to be resolved. In stars emitting photons having only their thermal width $\gamma \approx \alpha T \approx 0.5/137$ eV, the condition $\gamma R_0 \gg 1$ is always satisfied. For the typical thermal pion sources occurring in relativistic heavy-ion collisions however, this criterion can be violated: Pions of 100 MeV momentum in nuclear matter have an effective $\gamma$ of only a few MeV [13].

We conclude, that in the Hanbury Brown-Twiss analysis of relativistic heavy-ion collisions one may measure correlation radii, i.e., by gaussian fits to the correlation function, which are much larger than the actual radius of the thermal source. Most certainly it is not possible to infer the size of a thermal source by correlation measurements, when the spectral function of the measured bosons in the region of their generation is unknown. As a rule of thumb we suggest, that pion correlation functions should be measured for momenta of $|Q| \approx 350$ MeV, since then the mixing of pions with $\Delta_{33}$-resonance/nucleon-hole excitations is largest.

As a final note we emphasize again, that in common calculations of the correlation function one has to introduce ad-hoc random phases between several classical sources and then obtains the “standard” correlator $\overline{c}_{ll'}(p,q)$. Instead, we relied on a proper field theoretical treatment correct up to first order in gradients of the temperature. We found, that the non-equilibrium character of the system must be taken serious when calculating the correlation function.

References

[1] R.Hanbury Brown and R.Q.Twiss, Nature 178 (1956) 1046
[2] M.Gyulassy, S.K.Kauffmann and L.W.Wilson, Phys.Rev. C 20 (1979) 2267
[3] S.Pratt, Phys.Rev.Lett. 53 (1984) 1219
[4] G.F.Bertsch, P.Danielewicz and M.Herrmann, Phys.Rev. C 49 (1994) 442
[5] C.Itzykson and J.B.Zuber, Quantum Field Theory (McGraw-Hill, New York 1980)
[6] P.A.Henning, Ch.Becker, A.Lang and U.Winkler, Phys.Lett. B\textbf{217} (1989) 211

[7] N.P.Landsman, Ann.Phys. \textbf{186} (1988) 141

[8] N.P.Landsman and Ch.G.van Weert, Phys.Rep. \textbf{145} (1987) 141

[9] H.Umezawa, \textit{Advanced Field Theory: Micro, Macro and Thermal Physics},
(American Institute of Physics, 1993)

[10] P.A.Henning, Nucl.Phys. \textbf{A567} (1994) 844

[11] P.A.Henning, Habilitation thesis (TH Darmstadt 1993)
Physics Report in press

[12] P.A.Henning and H.Umezawa,
Phys.Lett. \textbf{B303} (1993) 209 and Nucl.Phys. \textbf{B417} (1994) 463

[13] P.A.Henning and H.Umezawa, Nucl.Phys. \textbf{A571} (1994) 617
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