A recent analysis of experimental data \cite{1} found that the angular momenta of nuclear fission fragments are uncorrelated. Based on this finding, the authors concluded that the spins are therefore determined only after scission has occurred. We show here that the nucleon-exchange mechanism, as implemented in the well-established event-by-event fission model \texttt{FREYA}, while agitating collective rotational modes in which the two spins are highly correlated, nevertheless leads to fragment spins that are largely uncorrelated. This fact invalidates the reasoning of those authors. Furthermore, it was reported \cite{1} that the mass dependence of the average fragment spin has a sawtooth structure. We demonstrate that such a behavior naturally emerges when shell and deformation effects are included in the moments of inertia of the fragments at scission.

PACS numbers:

I. INTRODUCTION

A recent article by Wilson \textit{et al.} \cite{1} addressed the generation of angular momentum in nuclear fission. In particular, based on the analysis of unique and extensive experimental data taken at the ALTO facility of the IJC Laboratory in Orsay, the authors concluded that there is no significant correlation between the angular momenta (or "spins") of the two fission fragments. Because the authors assume that any spin generated prior to scission must result in strongly correlated fragment spins, they conclude that their observation implies that the fragment spins must be generated after the two fragments are no longer in contact.

Using the fission model \texttt{FREYA} \cite{2,3}, we demonstrate here that a fission treatment based on correlated rotational modes in the dinuclear complex prior to scission may in fact endow the fragments with spins that are nevertheless approximately independent. This fact invalidates the above assumption and hence casts doubt on the central conclusion in Ref. \cite{1} regarding the mechanism for the angular momentum generation.

Furthermore, the data analysis in Ref. \cite{1} revealed that the average magnitude of the fragment spins have a sawtooth-like dependence on the mass number of the fragment. We show that when realistic moments of inertia are employed for the fledging fragments at scission, the calculation naturally yields a sawtooth behavior of the mean spin. This in turn provides an explanation for the observation in Ref. \cite{1} that the light fragment may carry more angular momentum than its heavy partner.

In Sect. II we recall the main relevant features of the theoretical treatment. Then, in Sect. III we discuss the correlations between the fragment spin magnitudes as well as their directions. In Sect. IV we address the mass-number dependence of the mean spin magnitude and we finally present a concluding discussion in Sect. V.

II. MODEL

The fission model \texttt{FREYA} \cite{2,3} employs Monte Carlo techniques to the selection of the mass, charge, and velocity of the primary fragments as well as to their subsequent decays by sequential neutron evaporation and photon radiation, thereby generating large samples of complete fission events. The model is in many respects similar to other fission simulation treatments, such as \texttt{CGMF} \cite{4}, \texttt{FIFRELIN} \cite{5}, and \texttt{GEF} \cite{6}, although these model differ in many details. For example, \texttt{FREYA} is the only one that conserves angular momentum throughout an event.

Of particular relevance is the selection of the fragment spins \cite{7}, which was recently discussed in some detail \cite{8}. The procedure employed in \texttt{FREYA} is motivated by the Nucleon-Exchange Transport model \cite{9,10} which considers the effect of multiple nucleon transfers between the two parts of a dinuclear complex. Because each nucleon carries baryon number (and possibly electric charge) as well as linear and angular momentum, the associated observables exhibit a diffusive evolution as a function of the number of transfers. This mechanism was found to be the dominant cause of dissipation in strongly damped nuclear reactions \cite{11} and it is expected to also play an important role during the late stages of fission, as the system develops a binary character prior to scission \cite{12}.

A detailed study of the effect of nucleon exchange on the dynamical evolution of the fragment spins considered the agitation of the six dinuclear rotational modes (the transverse modes wriggling and bending, which are doubly degenerate, and the coaxial modes twisting and tilting) \cite{13,14}. In particular, expressions were derived for the associated mobility coefficients which determine the time scales. When the relaxation time for a particular mode is short compared with the collective evolution, the associated spin distribution quickly readjusts to the evolving geometry and so it remains in local equilibrium.
Invoking this idealized limit for all six dinuclear modes, Moretto and Schmitt [13] formulated a statistical equilibrium model for the fragment spins in fission and heavy-ion reactions. However, the mobility coefficients for the dinuclear rotational modes differ significantly in magnitude, causing the relaxation time for the wriggling mode to be very fast, whereas the tilting mode is being agitated only quite slowly. As a rough way of taking this complexity into account, FREYA implements the effect of the nucleon-exchange mechanism by assuming full relaxation of the transverse modes (wriggling and bending), while the coaxial modes (twisting and tilting) are not agitated [10].

Generally, the fissioning complex has an overall angular momentum $S_0$, of which the fragments receive their share, $S_f = (I_f/I_{tot})S_0$, where $I_f$ is the moment of inertia of the fragment at scission and $I_{tot} = I_L + I_H + I_R$ is the total moment of inertia, with $I_R$ being the moment of inertia for the relative fragment motion. These rigid-rotation contributions tend to be negligible relative to the contributions from the statistical fluctuations.

The sampling of those is complicated by the fact that the individual fragment spins are not independent, due to the coupling imposed by conservation. There are three angular momentum vectors at scission: the spins of the two dinuclear partners, $S_L$ and $S_H$, and their relative angular momentum, $L$. Because the system is isolated, their sum is conserved, $S_L + S_H + L = S_0$, leaving then six rotational degrees of freedom. These are conveniently represented by the six dinuclear normal modes in terms of which the rotational part of the collective Hamiltonian is diagonal.

The modes considered (all six modes in the statistical treatment [13] and only the four transverse modes in FREYA) are then populated statistically. Thus the amplitude $s_m$ of a given mode $m$ is sampled from $P_m(s_m) \sim \exp(-s_m^2 / 2\Delta T S_m)$, where $\Delta T$ is the moment of inertia for that mode [13]. The wriggling mode of inertia is $I_+ = (I_L + I_H)/(I_L + I_H + I_R)$, while that for bending is $I_- = I_L I_H/(I_L + I_H)$. (The temperature $T$ employed by FREYA takes account of the fact that the distortion of the fragments at scission reduces the available statistical energy somewhat [13].) Once the normal modes have been agitated, yielding $s_+$ for wriggling and $s_-$ for bending, the individual fragment spins then readily follow [13], resulting in

$$S_f = S_f + (I_f/I_{tot})s_+ \pm s_-, \quad (1)$$

and their variances are $\sigma^2_L = 2(1 - I_f/I_{tot})I_f T S_m$.

### III. SPIN CORRELATIONS

As discussed above, the two fragment spins are not independent, due to angular momentum conservation. Indeed, the spin contributions from the two fragments to wriggling are perfectly parallel, while the contributions to bending are anti-parallel. When these dinuclear modes are populated statistically, the resulting correlation coefficient for the individual fragment spins amounts to

$$c(S_L, S_H) \equiv \frac{\langle S_L \cdot S_H \rangle - \langle S_L \rangle \langle S_H \rangle}{\sigma_L \sigma_H} \quad (2)$$

$$= -\left\{I_L I_H / (I_R + I_L) (I_R + I_H)\right\}^{1/2}.$$

This quantity is generally rather small because the moment of inertia for the relative fragment motion, $I_R$, is typically an order of magnitude larger than those of the individual fragments, $I_L$ and $I_H$, $I_R \gg I_f$. Thus, even though the fragment spins are strongly coupled for each of the normal modes, the resulting spins are expected to be relatively independent. This expectation is indeed borne out by actual FREYA simulations [13], as illustrated in Table II and Fig. I.

| Case: | $^{235}$U(n,f) | $^{238}$U(n,f) | $^{239}$Pu(n,f) | $^{252}$Cf(sf) |
|-------|--------------|--------------|---------------|---------------|
| $S_L = \langle S_L \rangle$ | 4.27 | 4.43 | 4.58 | 5.08 |
| $S_H = \langle S_H \rangle$ | 5.66 | 5.80 | 5.93 | 6.33 |
| $c(S_L, S_H)$ (%) | 0.2 | 0.2 | 0.1 | 0.1 |
| $f_1$ (%) | -8.2 | -8.3 | -8.3 | -8.4 |
| $f_2$ (%) | -10.9 | -11.3 | -11.7 | -13.5 |

TABLE I: The average magnitudes of the primary fission fragment spins, $S_L$ and $S_H$, and the associated correlation coefficients $c(S_L, S_H)$ for four fission cases: $^{235}$U(n,f), $^{238}$U(n,f), $^{239}$Pu(n,f), and $^{252}$Cf(sf), as obtained with FREYA using either monotonically increasing moments of inertia or moments of inertia with a refined $A$ dependence (numbers in parentheses). Also shown are the amplitudes $f_2$ in the distribution of the spin opening angles, $P(\phi_{LH}) = 1 + f_1 \cos \phi_{LH}$.

![FIG. 1: (Color online) The distribution of the opening angle $\phi_{LH}$ between the angular momenta of the two fission fragments from $^{238}$U(n,f), as obtained by FREYA using either the standard moment of inertia, $I_f(A_f) = 0.5 I_{rigid}(A_f)$ (dots) or the modified form depending on the shapes of the fragments at scission, $I'_f(A_f)$ (open circles). Also shown are the lowest-order Fourier approximations, $P(\phi_{LH}) = 1 + f_1 \cos \phi_{LH}$.](image)
The magnitudes of the fragment spins, \( S_f = |S_f| \), have rather wide distributions with average values \( \overline{S_f} \equiv \langle S_f \rangle \approx 5 - 6h \). The associated covariance is given by \( \sigma(S_L, S_H) = \langle S_L S_H \rangle - \overline{S_L} \overline{S_H} \), so the spin magnitude correlation coefficient is \( c(S_L, S_H) = \sigma(S_L, S_H)/\sigma(S_L)\sigma(S_H) \). Table I lists the average spin magnitudes and the correlation coefficient for four fission cases of frequent interest. The correlation coefficients are essentially zero, indicating that the primary spin magnitudes are largely uncorrelated, in accordance with the reported experimental finding [1]. (Unfortunately, the mutual independence of the two fragment spins is not quantified in Ref. [1], so it is not possible to make a precise comparison.)

While the experimental data [1] cannot yield information on the fragment spin directions, it is noteworthy that those are also largely uncorrelated in FREYA. The degree of correlation between the fragment spin directions is brought out by the distribution of the opening angle between the two fragment spins, \( \phi_{LH} \), which is given by \( \cos \phi_{LH} = S_L \cdot S_H/|S_L S_H| \). This function is shown in Fig. 1 for \( 238 \text{U}(n,f) \). (The other cases look very similar.) As was shown recently [3], the undulation of the fragment spin opening angle is generally well represented by the first harmonic, \( P(\phi_{LH}) \approx 1 + f_1 \cos \phi_{LH} \). Table I shows the amplitudes \( f_1 \) for the four cases considered. As seen, they are all rather small, being of the order of 10%, showing that the spin directions are also largely uncorrelated.

**IV. MASS DEPENDENCE**

In addition to the qualitative finding that the fragment spins are largely uncorrelated, Ref. [1] presented data for the dependence of the average spin magnitude \( \overline{S_f} \) on the fragment mass number \( A_f \), revealing a sawtooth-like structure of \( \overline{S_f}(A_f) \) (see Fig. 2). We now show that such a behavior emerges naturally due to the variation of the moments of inertia of the fragments at scission.

When the dinuclear rotational modes are sampled from thermal distributions, as in FREYA, then \( \langle S_f^2 \rangle \approx 2I_f T_S \) (see above), so the mean fragment spin scales approximately as the square root of the moment of inertia, \( I_f \). To illustrate the close connection between \( \overline{S_f} \) and \( I_f \), we first note that the standard version of FREYA [3] assumes that the moment of inertia of a fragment is given by \( I_f(A_f) = c_{\text{rot}} I_{\text{rigid}}(A_f) \), with the global reduction factor being \( c_{\text{rot}} = 0.5 \). Thus the \( A_f \) dependence of the moment of inertia is monotonic and the calculated \( \overline{S_f}(A_f) \) is rather featureless. This is illustrated in Fig. 2 where \( \overline{S_f}(A_f) \) obtained with the standard version of FREYA is shown for \( 238 \text{U}(n,f) \) and \( 252 \text{Cf}(sf) \) together with the data reported in Ref. [1]. [The figure also shows that the effect of neutron evaporation on the spin magnitudes is negligible, thus lending support to the assumption made in the data analysis.]

However, as pointed out in Ref. [1], fragments near the doubly magic nucleus \(^{132}\text{Sn}\) have rather rigid spherical shapes and abnormally low level densities, as a consequence of which they cannot accommodate very much angular momentum. Indeed, in order to match the measured \( \overline{S_f} \) values near \(^{132}\text{Sn}\), it is necessary to reduce the rigid moment of inertia by 0.2 rather than 0.5. On the other hand, away from this special region the fragments have deformed equilibrium shapes and are likely significantly further distorted at scission. Thus they have considerably larger moments of inertia and can acquire correspondingly larger angular momenta. (These expectations were indeed borne out by recent microscopic studies of scission configurations [17–13].)

For illustrative purposes, we have roughly incorporated these effects into modified moments of inertia, \( I'_f(A_f) \), using microscopic results on the fragment equilibrium shapes and, as in the standard FREYA, one global scaling parameter. The resulting mean spin magnitudes then ac-
quire a sawtooth-like behavior that roughly matches the experimental data, as illustrated in Fig. 2. Also shown is \( \left[ \mathcal{J}_f^{(s)}(A_f) \right]^{1/2} \) (rescaled for convenience), bringing out the close connection between the moment of inertia and the mean spin magnitude. The remarkably similar mass dependence of those two quantities suggests that the measurements of \( \mathcal{J}_f^{(s)}(A_f) \) in effect provide information on the geometry of the scission configurations.

Finally it should be noted that the modified moments of inertia tend to favor the light fragments (because of their deformations), so these will now have larger spins than their heavy partners, as found experimentally \footnote{This is also consistent with recent microscopic studies \cite{18, 19}.} \footnote{However, importantly, fragment spins remain approximately uncorrelated after the modification of the moments of inertia, as illustrated in Table 1 and Fig. 1.}. This finding demonstrates that the absence of spin correlation can not be taken as evidence for the fragments having acquired their spins independently and it thus invalidates the assumption underlying the data interpretation advanced in Ref. \footnote{In fact, the independent population of rotational states in the fragments after their separation is hard to reconcile with the principle of angular momentum conservation for isolated systems.}. (In fact, the independent population of rotational states in the fragments after their separation is hard to reconcile with the principle of angular momentum conservation for isolated systems.)

We have furthermore demonstrated that the sawtooth-like mass dependence of the average fragment spin can be understood as a reflection of the moments of inertia of the fragments at the time of their formation. Such a behavior would arise also in other simulation codes (such as Refs. \cite{4–6}) if similar moments of inertia were were employed. (It should also be noted that \( \mathcal{J}_f^{(s)}(A_f) \) would still display a sawtooth form even in the extreme scenario where only the wriggling modes were populated, causing the two fragment spins to be perfectly parallel in each event.) The finding, based on the observed sawtooth behavior of \( \mathcal{J}_f^{(s)}(A_f) \), that the light fragment carries more angular momentum than its heavy partner, was anticipated recently on the basis of microscopic studies \cite{18, 19}. Finally we have pointed out that the strong connection between the moments of inertia of the fissioning system at the time of scission, which in turn would be very helpful for the further development of fission theory.

**ACKNOWLEDGMENTS**

This work was supported by the Office of Nuclear Physics in the U.S. Department of Energy under Contracts DE-AC02-05CH11231 (J.R.) and DE-AC52-07NA27344 (R.V.) and was supported by the DOE Office of Nuclear Nonproliferation and the LLNL LDRD Program under Project No. 20-ERD-031 (R.V.).

[1] J. Wilson et al., Angular momentum generation in nuclear fission, *Nature* **590**, 566 (2021).
[2] J. Randrup and R. Vogt, Calculation of fission observables through event-by-event simulation, *Phys. Rev. C* **80**, 024601 (2009).
[3] J.M. Verbeke, J. Randrup, and R. Vogt, Fission Reaction Event Yield Algorithm FREYA 2.0.2, *Comp. Phys. Comm.* **222**, 263 (2018).
[4] P. Talou, I. Stetcu, P. Jaffke, M.E. Rising, A.E. Lovell, and T. Kawano, Fission fragment decay simulations with the CGMF code, *arXiv:2011.10444*.
[5] O. Litaize, O. Serot, and L. Berge, Fission modelling with FIFRELIN, *Eur. Phys. J. A* **51**, 177 (2015).
[6] K.-H. Schmidt, B. Jurado, C. Amoureux, and C. Schmitt, General description of fission observables: GEF model code, *Nucl. Data Sheets* **131**, 107 (2016).
[7] J. Randrup and R. Vogt, Refined treatment of angular momentum in FREYA, *Phys. Rev. C* **89**, 044601 (2014).
[8] R. Vogt and J. Randrup, Angular momentum effects in fission, *Phys. Rev. C* **103**, 014610 (2021).
[9] J. Randrup, Theory of transfer-induced transport in nuclear collisions, *Nucl. Phys. A* **327**, 490 (1979).
[10] J. Randrup, Transport of angular momentum in damped nuclear reactions, *Nucl. Phys. A* **383**, 468 (1983).
[11] W.U. Schröder and J.P. Huizenga, *Treatise on heavy-ion science*, ed. D.A. Bromley (Plenum, New York, 1984) p. 115.
[12] J. Blocki, Y. Boneh, J.R. Nix, J. Randrup, M. Robel, A.J. Sierk, and W.J. Swiatecki, One-body dissipation and the super-viscosity of nuclei, *Ann. Phys. A* **113**, 330 (1978).
[13] T. Døssing and J. Randrup, Dynamical evolution of angular momentum in damped nuclear reactions: (I) Accumulation of angular momentum by nucleon transfer, Nucl. Phys. A 433, 215 (1985).
[14] T. Døssing and J. Randrup, Dynamical evolution of angular momentum in damped nuclear reactions: (II) Observation of angular momentum through sequential decay, Nucl. Phys. A 433, 280 (1985).
[15] L.G. Moretto and R.P. Schmitt, Equilibrium statistical treatment of angular momenta associated with collective modes in fission and heavy-ion reactions, Phys. Rev. C 21, 204 (1980).
[16] R. Vogt and J. Randrup, Event-by-event study of photon observables in spontaneous and thermal fission, Phys. Rev. C 87, 044602 (2013).
[17] M. Albertsson, B.G. Carlsson, T. Døssing, P. Möller, J. Randrup, and S. Åberg, Correlation studies of fission-fragment neutron multiplicities, Phys. Rev. C 103, 014609 (2021).
[18] A. Bulgac, I. Abdurrahman, S. Jin, K. Godbey, N. Schunck, and I. Stetcu, Fission fragments intrinsic spins and their correlations, [arXiv:2012.13422].
[19] P. Marevic, N. Schunck, J. Randrup, and R. Vogt, Angular momentum of fission fragments from microscopic theory, [arXiv:2101.08406].
[20] W.U. Schröder, J.R. Birkelund, J.R. Huizenga, W.W. Wilcke, and J. Randrup, Effect of Pauli blocking on exchange and dissipation mechanisms operating in heavy-ion reactions, Phys. Rev. Lett. 44, 308 (1980).