Adaptive reasoning and procedural fluency in three-dimensional

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Abstract. This research is motivated by the difficulty of students determining the angle between a line and a plane in the construction of three dimensions. Students sometimes hesitate to identify the algorithm that will be used to solve the distance and angle problems in the three-dimensional material in the lecture of Capita Selecta. Based on this, this study aims to analyze the adaptive reasoning abilities and procedural fluency of students in solving three-dimensional problems. The method used in this study is a qualitative method with a descriptive design. The research was conducted on 20 students. The results of this study indicate students can think logically in choosing the right concepts and situations and are accurate in doing calculations. Students also have shown the correct procedure by making fields through the CF line and \( \perp \) ACH fields. The algorithms used by students are varied so that students are found to have not been precise in determining the projection length of the CF line. It can affect the angle between the CF line and the ACH plane obtained.

1. Introduction

A question will become a problem only if the issue shows a challenge that cannot be solved by a known routine procedure. Many three-dimensional geometry problems in their resolution are related to proof, thus requiring the ability to reason and use a procedure. The ability to reason and use a procedure must be possessed by every student, to support the success of learning mathematics [1]. The use of reasoning and selection of appropriate procedures is part of the form of mathematical proficiency that has five components [1], namely 1) Conceptual Understanding, 2) Procedural Fluency, 3) Strategic Competence, 4) Adaptive Reasoning, 5) Productive Disposition.

Adaptive reasoning is seen as a mathematical skill, while procedural fluency is sawed as a form of mathematical knowledge [2], which in this study was identified through students’ answers in solving three-dimensional problems in Capita Selecta lectures. Adaptive reasoning involves students’ ability to think logically about relationships between mathematical concepts [3], adaptive reasoning also refers to students' skills to think logically, reflect, explain, and justify [1]. Some indicators of adaptive reasoning [4] are 1) logical thinking in choosing the right mathematical concepts and situations; 2) using the right procedure.

Procedural fluency is defined as the ability to carry out procedures flexibly, efficiently, and effectively [5]. Procedural fluency describes the ability of students to carry out mathematical procedures in a flexible, accurate, and efficient manner [1]. Procedural fluency can be said if you know when to use mathematical procedures, know-how mathematical knowledge, where learners' abilities are remembered quickly and perform procedures correctly [6,7]. Procedural Fluency
indicators, according to [4] are 1) able to recognize, choose, and apply the right formula; 2) accurate in calculations; 3) exact algorithm use.

Geometry is one of the most challenging parts of mathematics to learn. Many students face some difficulties in recognizing and understanding geometric objects, especially in three-dimensional geometry objects and their properties. This situation arises when students take part in the capita middle-class lectures in which there are three-dimensional trigonometry and geometry material. Three-dimensional geometry is a part of geometry, which in his study includes the construction of three-dimensional or geometric shapes [8]. For students to better understand the three dimensions, it is necessary to be reminded of angles and distances, which in the learning process require space responsiveness.

In the learning of angles and distances in three-dimensional space there is a prerequisite knowledge that students must have [9] including 1) using special properties that apply in certain flat structures, 2) determining the position relationship between points, lines, and fields, 3) determine the lines parallel to other lines in space, 4) determine the projection of a point on a line and on a plane, 5) determine the projection of a line on a plane, 6) use the terms of the perpendicular plane of the plane and the implications of the perpendicular line of the plane, 7) using the Pythagorean theorem, trigonometric comparison concepts and basic formulas, 8) using distance theorems including formulas in trigonometry. Determining the angle and distance in the three dimensions required student skills in drawing and understanding the three-dimensional space. It is not easy if students do not master how to make a segment that shows the distance between points, lines, and fields and determine the angle in three-dimensional space.

To find out more clearly about the adaptive reasoning and procedural fluency of students, researchers conducted an unstructured interview of students who had received three-dimensional material in the Capita Selecta lecture. From interview results, there are still many students who have difficulty in determining the distance and angle of a three-dimensional shape. One member of the research team had conducted a study related to three-dimensional learning difficulties in students, which showed that abstract three-dimensional material was the biggest reason [10]. Students also do not understand the steps that must be taken to solve the distance and angle problems in the three-dimensional material. Therefore this study aims to analyze the adaptive reasoning abilities and procedural fluency of students in solving three-dimensional problems. The renewal that emerged in this study, where researchers conducted an in-depth analysis of the ability of adaptive reasoning and procedural fluency in the material that must be mastered by students of Mathematics Education. Because the third dimension is a prerequisite material from several mathematics subjects.

2. Method

The research method used in this study is a qualitative descriptive design. The research subjects of students who took the Capita selecta lectures were 20 students, then grouped based on the theory of three high, medium and low cognitive levels [11]. From the grouping, a random sample of 5 students was taken who had similarities in problem-solving based on indicators of procedural fluency and adaptive reasoning. Three cognitive levels are obtained by taking into account the student's final score. Based on the university's determination of the total score of 100, the cognitive level is low with a range of values from 0 to 49, the cognitive level is medium with a range of values from 50 to 69, and 70 to 100 high. To know the description of the strength of procedural fluency and adaptive reasoning, given a test with a three-dimensional geometry problem that includes knowledge of lines, distances, fields, and angles. Data collection techniques in this research are through the study of documentation and interviews. The documentation study was conducted to obtain data on the extent to which the adaptive reasoning ability and procedural fluency of students. The documents obtained are the results of student work when completing three-dimensional geometry problems.

The data analysis technique used in this study is a circular model data analysis [12]. The steps of data analysis in the circular model are data reduction, data display, and concluding. At the stage of data reduction, it is done by examining indicators of adaptive reasoning and mathematical procedural
fluency conducted by students when solving mathematical problems. The data display stage is to present each mathematical fluency indicator that is done by students when solving mathematical problems. The next stage concludes the results of the analysis that has been done and gives meaning descriptively.

3. Result and Discussion

The problem presented in this study is a matter of description aimed at finding angles between lines and fields. In solving this problem, the adaptive reasoning ability and procedural fluency of students are needed in 1) determining the position relationship between points, lines and planes, 2) determining the line projection in a plane, 3) using the conditions of the field's straight line and the implications of the field's straight line, 4) using the Pythagorean theorem, concepts trigonometry comparisons and basic formulas, 5) using distance theorems including formulas in trigonometry. The results of this study reveal that adaptive reasoning and procedural fluency are needed in shaping abstract relational skills to three-dimensional link concepts of real number operations, Pythagorean theorem, high line propositions, angular definitions, angular definitions, and trigonometric principles such as in research conducted by Firmasari and Santi [10]. The following problems are giving by confusing T points on the AD line:

From the cube ABCD.EFGH on the below which has lateral \( \sqrt{3} \) cm, and T on AD with AT = 1 cm.

\[
\text{determine the cosine angle between CF and ACH!}
\]

Note: \( \sqrt{3} = 1.732 \)

![Figure 1. Mathematical problems are given](image1)

To completely, students must understand that the angle is forming by two rays that have the same endpoint. While the angle between the line and the plane is the same as the angle between the line and the line projection of the line on the plane. The steps to resolve the above problems according to indicators of adaptive reasoning and procedural fluency are as follows.

![Figure 2. Angles formed by CF lines and ACH fields](image2)
Table 1. Settlement according to indicators of adaptive reasoning and procedural fluency.

| Indicator                        | Solution                                                                 |
|----------------------------------|--------------------------------------------------------------------------|
| Adaptive reasoning               | • Drawing a CF line and make an ACH field                                |
|                                  | • Make a line projection in the ACH field.                               |
| Using the right procedure        | Make the fields go through the CF line and the H ACH field so that the two fields intersect in the CP line |
| So \(\angle (\text{CF, ACH}) = \angle (\text{CF, CP}) = \angle \text{PCF} = \theta\) |                                                                          |
| Procedural fluency               | • Using the Pythagorean formula                                          |
|                                  | • Using cosine functions and cosine rules                                |
| Accurate in calculations         | \(\text{CF} = \sqrt{BC^2 + BF^2} = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2} = \sqrt{6}\) |
|                                  | \(\text{PF} = \text{CF} = \sqrt{EF^2 + EP^2} = \sqrt{(\sqrt{3})^2 + \left(\frac{1}{2}\sqrt{6}\right)^2} = 3 \sqrt{\frac{1}{2}}\) |
|                                  | \(\cos \theta = \frac{\text{PF}^2 + \text{CF}^2 - \text{PF}^2}{2 \text{PF} \cdot \text{CF}} = \frac{\left(\frac{9}{2}\right)^2 + (\sqrt{3})^2 - \left(\frac{9}{2}\right)^2}{2 \cdot \frac{9}{2} \cdot \sqrt{6}} = \frac{1}{3} \sqrt{3}\) |
| Exact algorithm use              | • At \(\Delta BCF\) Determine the length of the CF line by the Pythagorean formula |
|                                  | • At \(\Delta PEF\) determines the length of the PF line                |
|                                  | • Determine the value of \(\cos \theta\) on \(\angle \text{PCF}\)        |

The results of problem-solving by five students had some differences but still led to an indicator of adaptive reasoning and procedural fluency. In adaptive reasoning with indicators Thinking logically in choosing the right mathematical concepts and situations, students can already draw the CF line and make the ACH field visible in figure 3. The CF line projection in the ACH field is shown in figures 3 (1) and 3 (3), whereas in figure 3 (2), students have not demonstrated any CF line projections in the ACH field.

Figure 3. Adaptive reasoning
Adaptive reasoning with indicators using the right procedure is that students have made a field through the CF line and \( \perp \) the ACH field so that the angle \( \theta \) can be drawn between the CF line and the ACH field, shown in Figure 3 (1) and 3 (3). In figure 3 (2), students do not make a field that goes through the CF line and \( \perp \) the ACH field so that it does not show an angle yet.

In procedural fluency, we will pay attention to the steps of problem-solving by some students. In the first indicator able to remember, choose, and apply the right formula, Figure 4 (1) student A has used the Pythagorean formula and cosine rules appropriately. For the second indicator to be accurate in a calculation, student A does the count to get the OC line length accurately (figure 4 (1)). But in the exact indicator using the algorithm, Student A has a different step, seen in Figure 4 (2) is not yet right in determining the projection length of the CF line. As a result of influencing the results when determining the angle of the CF line and the ACH plane, as shown in figure 4 (3) which shows the angle \( \sqrt{3} \).

The results of problem-solving by student B can be seen in Figure 5. In the first indicator able to remember, choose, and apply the right formula, Figure 5 (1) student B has used the Pythagorean formula and cosine rules appropriately. The algorithm carried out by student B by first determining the intersection point between the AH line and the ED line, and given the name Q point is seen in figure 3 (2). Intended to make the plane go through the CF line and intersect the ACH plane. After that, the application of the Pythagorean formula and calculation is made precisely, as shown in Figure 5 (3), which results in an angle of \( 1/3 \sqrt{3} \).
Figure 5. Procedural fluency of students B

Student C in solving problems looks more straightforward, but even so, it does not mean the answer is not right. C students do the algorithm by making BDF fields that intersect ACH fields. From the intersection of the two areas obtained the intersection point F'. Point F' is located on the diagonal line of FD space, so we get Δ F'CF. Δ F'CF to determine the angle between the CF line and the ACH plane. The angle obtained by student C is 1/3 √3, and the angle value is correct.

Figure 6. Procedural fluency of students C

4. Conclusion
In adaptive reasoning, students can think logically in choosing the right concepts and situations. Then students also have shown the correct procedure by making a plane through the CF line and the H ACH plane so that the two fields intersect at the CP line and form an angle θ. While in procedural fluency, students can choose and apply when to use the Pythagorean formula and to use cosine functions and cosine rules. From some of the results of problem-solving, we can see the variety of algorithms used
by students. Some students have not been precise in determining the length of the CF line projection, so it affects the angle between the CF line and the ACH field obtained. Overall, the indicators are accurate in calculations, and there are no errors seen by students in making calculations.

References
[1] Kilpatrick J, Swafford J, Findell B 2001 *Adding It Up: Helping Children Learn Mathematics* (Washington D C: National Academy Press) p 115-155
[2] Dhlamini Z B and Luneta K 2016 Exploration of the Levels of Mathematical Proficiency Displayed by Grade 12 Learners in Responses to Matric Examinations *International Journal of Educational Sciences* 13 2 p 231-246
[3] Aaron W R and Herbst P G 2015. Teachers’ perceptions of students’ mathematical work while making conjectures: An examination of teacher discussions of an animated geometry classroom scenario. *International Journal of STEM Education, 2 10*: 1-13.
[4] Soebagyo J 2016 Learning profile in accommodating mathematical proficiency *Jurnal Euclid* 3 2 p 474
[5] Sullivan P 2011 *Teaching Mathematics: Using Research-Informed Strategies* (Australia: Australian Council for Educational Research)
[6] Zakaria E and Zaini N 2009 Conceptual and procedural knowledge of rational numbers in trainee teachers *European Journal of Social Sciences* 9 2 p 202-217
[7] Graven M and Stott D 2012 Conceptualizing Procedural Fluency as a Spectrum *Proceedings of the 18th Annual National Congress of the Mathematics Education in South Africa* 1 p 146-156
[8] Iswadji D 2001 *Geometri ruang* (Yogyakarta: Universitas Negeri Yogyakarta)
[9] Krismanto A 2008 *Pembelajaran Sudut dan Jarak dalam Ruang Dimensi Tiga* (Yogyakarta: PPPPTK)
[10] Firmasari S and Santi D P D 2019 Rigorous Mathematical Thinking: Why are Cognitive Levels Important in Three-Dimensional Learning for Pre-service teachers? *J. Phys.: Conf. Ser.* 1280 042040
[11] Firmasari S et al 2019 Rigorous Mathematical Thinking Based On Gender in The Real Analysis Course *J. Phys.: Conf. Ser.* 1157 042106
[12] Satori D and Komariah A 2017 *Qualitative Research Methodology* (Bandung: Alfabeta)