Remote transfer of Gaussian quantum discord

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Abstract: Quantum discord quantifies quantum correlation between quantum systems, which has potential application in quantum information processing. In this paper, we propose a scheme realizing the remote transfer of Gaussian quantum discord, in which another quantum discordant state or an Einstein-Podolsky-Rosen entangled state serves as ancillary state. The calculation shows that two independent optical modes that without direct interaction become quantum correlated after the transfer. The output Gaussian quantum discord can be higher than the initial Gaussian quantum discord when optimal gain of the classical channel and the ancillary state are chosen. The physical reason for this result comes from the fact that the quantum discord of an asymmetric Gaussian quantum discordant state can be higher than that of a symmetric one. The presented scheme has potential application in quantum information network.

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1. Introduction

Quantum correlation, which can be quantified by quantum discord [1–3], is a fundamental resource for quantum information processing. It has been shown that quantum correlation (no quantum entanglement) can be used to complete some quantum computation tasks [4–6]. Quantum discord can also be utilized as a resource for entanglement distribution [7], quantum state merging [8, 9], remote state preparation [10], local broadcasting [11, 12] and quantum key distribution [13, 14]. Recent research shows that quantum discord bounds the amount of distributed entanglement [15], cannot be shared [16], and can be frozen in the non-Markovian dynamics [17]. Presently, quantum discord has been extended to the region of continuous variables for Gaussian states [18, 19] and certain non-Gaussian states [20]. Gaussian quantum discord has been experimentally demonstrated [21–24].

Entanglement swapping [25–28], which makes two independent quantum states without direct interaction become entangled, is important to build quantum information network [29]. In fact, it represents the quantum teleportation of entangled state [27]. Entanglement swapping has been experimentally demonstrated in both discrete and continuous variables region [30, 31].

In this paper, we propose a scheme to realize the remote and unconditional transfer of Gaussian quantum discord based on applying the technique of entanglement swapping. The Gaussian quantum discord is transferred remotely and unconditionally by using an other quantum discordant state or an Einstein-Podolsky-Rosen (EPR) entangled state as ancillary state. Two quantum states that never have direct interaction become quantum correlated after transfer of Gaussian quantum discord. A more interesting result is that the output Gaussian quantum discord can be higher than the initial quantum discord at some given conditions, which will never happen...
in entanglement swapping. The maximum output quantum discord is obtained when optimal gain in classical channel and squeezing parameter of the EPR entangled state are chosen. The simplest way to transfer the Gaussian quantum discord is to use a coherent state as ancillary state, where the output quantum discord can also be higher than the initial quantum discord when optimal gain is chosen. This feature is useful for constructing a quantum information network. The physical reason for this result is that the quantum discord of an asymmetric Gaussian quantum discordant state can be higher than that of a symmetric one, which is also proved in the paper. Here, the asymmetric (symmetric) Gaussian quantum discordant state means that the correlated noise variances on two quantum correlated beams are different (same).

2. The remote transfer scheme

2.1. Using a Gaussian discordant state as ancillary state

Fig. 1 shows the schematic for the remote transfer of Gaussian quantum discord. Alice and Bob, who represent two nodes in a quantum information network, own two independent Gaussian discordant states \((\hat{a}, \hat{b})\) and \((\hat{c}, \hat{d})\), respectively. In order to transfer the quantum correlation to remote station, Alice and Bob send one of each states (\(\hat{b}\) and \(\hat{c}\)) to the middle station owned by Claire. Claire interferes quantum modes \(\hat{b}\) and \(\hat{c}\) on a 50% beam-splitter (BS) and measures the amplitude and phase quadratures of the output modes \(\hat{e}\) and \(\hat{f}\) by two homodyne detection (HD) systems, respectively. The measurement results are fedforward to Bob through the classical channel. Bob performs phase space displacement on quantum mode \(\hat{d}\) with amplitude and phase modulators (EOMx and EOMp), respectively. Finally, the quantum discord between quantum modes \(\hat{a}\) and \(\hat{d}\) are measured by an verifier.

The amplitude and phase quadratures of an optical mode \(\hat{\sigma}\) are defined as \(\hat{X}_\sigma = \hat{\sigma} + \hat{\sigma}^\dagger\) and \(\hat{Y}_\sigma = (\hat{\sigma} - \hat{\sigma}^\dagger)/i\), respectively. The variances of amplitude and phase quadratures for a vacuum (coherent) state are \(V(\hat{X}_\sigma) = V(\hat{Y}_\sigma) = 1\). The Gaussian quantum discordant state can be obtained by correlated (anti-correlated) displacement of two coherent states in the amplitude (phase) quadrature with a discording noise \(V\frac{21}{21}\), which is a separable state. The two modulated coherent states are \(\hat{a} = \hat{c}_1 + \hat{s}_1\) and \(\hat{b} = \hat{c}_2 + \hat{s}_2\), respectively, where \(\hat{c}_1\) and \(\hat{c}_2\) represent two independent coherent state at same frequency, \(\hat{s}_1 = \hat{X}_s + i\hat{Y}_s\) and \(\hat{s}_2 = \hat{X}_s - i\hat{Y}_s\) stand for the discording signal with a variance \(V(\hat{X}_s) = V(\hat{Y}_s) = V\). The covariance matrix of the prepared
Gaussian discordant state is given by

\[ \sigma = \begin{pmatrix} A & C \\ C & B \end{pmatrix}, \]

where

\[ A = B = \begin{pmatrix} 1 + V & 0 \\ 0 & 1 + V \end{pmatrix}, \quad C = \begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix}. \]

For Alice and Bob’s Gaussian discordant state, we have \( V = V_A \) and \( V = V_B \) in the covariance matrix, respectively.

Quantum discord is defined as the difference between two quantum analogues of classically equivalent expression of the mutual information. For a bipartite quantum system \( \rho_{AB} \), the total classical and quantum correlations is given by mutual information \( I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \), where \( S(\rho) \) is the von Neumann entropy. Another expression of mutual information based on conditional entropy is \( J^\rho(\rho_{AB}) = S(\rho_A) - \inf_{(\rho_B)} S(\rho_B)(A|B) \), which is known as one-way classical correlation. The infimum describes all possible measurements on subsystem \( B \).

The difference between these two expressions of mutual information is defined as quantum discord \( D_{AB} = I(\rho_{AB}) - J^\rho(\rho_{AB}) \). An explicit expression of Gaussian quantum discord for a two-mode Gaussian state is given by \[ D_{AB} = f(\sqrt{I_2}) - f(\sqrt{V_+} + f(\sqrt{V_-}), \]

where \( f(x) = (\frac{x+1}{2}) \log_2(\frac{x+1}{2}) - (\frac{x-1}{2}) \log_2(\frac{x-1}{2}), \)

\( V_\pm = \sqrt{\Delta \pm \sqrt{\Delta^2 - 4\det(\sigma)}} \),

are the symplectic eigenvalues of a two-mode covariance matrix \( \sigma \) with \( \det(\sigma) \) as the determinant of covariance matrix and \( \Delta = \det A + \det B + 2\det C \), and

\[ E^{\min} = \begin{cases} 2I_2^2 + (I_2^2 - I_4 + I_4^2 - 2I_4)(I_4 - I_2) & a) \\ I_2^2 - 2I_4^2 - 4I_4^2 & b) \end{cases} \]

where a) applies if \( (I_4 - I_2I_2)^2 \leq I_2^2(I_4 + 1)(I_4 + 1) \) and b) applies otherwise. \( I_4 = \det A, I_2 = \det B, I_3 = \det C, I_4 = \det \sigma \) are the symplectic invariants. A two-mode Gaussian state is quantum correlated when Gaussian quantum discord \( D_{AB} > 0 \).

In Fig. 1, the output modes from the 50% beam-splitter are \( \hat{c} = (\hat{b} + \hat{\epsilon})/\sqrt{2} \) and \( \hat{f} = (\hat{b} - \hat{\epsilon})/\sqrt{2} \). The measured photocurrents of two homodyne detection systems are \( \hat{i}_1 = (\hat{X}_b - \hat{X}_e)/\sqrt{2} \) and \( \hat{i}_2 = (\hat{Y}_b + \hat{Y}_e)/\sqrt{2} \), respectively. They are sent to Bob through the classical channel, respectively. The output beam is

\[ \hat{d}' = d + \sqrt{2}g\hat{i}_1 + i\sqrt{2}g\hat{i}_2, \]

where \( g \) describes Bob’s (suitably normalized) amplitude and phase gain factor in the classical channels, where we have assumed that the gain in two channels is identical. The covariance matrix of the output states \( \hat{d}' \) and \( \hat{d} \) is given by

\[ \sigma_{out} = \begin{pmatrix} A' & C' \\ C' & B' \end{pmatrix}, \]

where

\[ A' = \begin{pmatrix} 1 + V_A & 0 \\ 0 & 1 + V_A \end{pmatrix}, \quad B' = \begin{pmatrix} V_d & 0 \\ 0 & V_d \end{pmatrix}, \quad C' = \begin{pmatrix} gV_A & 0 \\ 0 & -gV_A \end{pmatrix}, \]

with \( V_d = 1 + 2g^2 + g^2V_A + (1 - g)^2V_B \). Finally, the quantum discord of the output states can be verified by Eq. (3).
2.2. Using an EPR entangled state as ancillary state

For the case of transferring Gaussian quantum discord with an EPR entangled state as ancillary state, the quantum discordant state at Bob’s station is replaced by an EPR entangled state, whose covariance matrix is given by

$$\sigma_E = \begin{pmatrix} A_E & C_E \\ C_E & B_E \end{pmatrix}, \quad (8)$$

where

$$A_E = B_E = \begin{pmatrix} V_E & 0 \\ 0 & V_E \end{pmatrix}, \quad C_E = \begin{pmatrix} \sqrt{V_E^2 - 1} & 0 \\ 0 & -\sqrt{V_E^2 - 1} \end{pmatrix}, \quad (9)$$

in which $V_E = \cosh 2r$ is the variance of the EPR entangled state, where $r$ is the squeezing parameter.

After the transfer of the quantum discordant state, the covariance matrix of the output state becomes

$$\sigma_{E_{out}} = \begin{pmatrix} A' & C' \\ C' & B'_E \end{pmatrix}, \quad (10)$$

where $A'$ and $C'$ are same with that in Eq. (7), and

$$B'_E = \begin{pmatrix} V_{E'd} & 0 \\ 0 & V_{E'd} \end{pmatrix}, \quad (11)$$

with $V_{E'd} = (g^2 + 1)V_E - 2g\sqrt{V_E^2 - 1} + g^2(1 + V_A)$.

2.3. The effect of imperfect detection efficiency

We also consider the effect of the imperfect detection efficiency on the proposed transfer schemes. The detection efficiency can be looked as the loss of the detected optical mode, which is usually modeled by a beam splitter with transmission efficiency of $\eta$. In this way, the detected optical mode $\hat{\nu}$ turns into $\sqrt{\eta}\hat{\nu} + \sqrt{1-\eta}\hat{\nu}$, where $\hat{\nu}$ represent the vacuum noise induced by imperfect detection efficiency. For analyzing the effect of imperfect detection efficiency on the proposed scheme, we consider the detection efficiency on the optical modes $\hat{a}$, $\hat{e}$, $\hat{f}$, and $\hat{d}'$, respectively.

3. Results and analysis

Figure 2(a) shows the dependence of output quantum discord on the Alice’s discording noise $V_A$ with a Gaussian quantum discordant state with discording noise $V_B$ as ancillary state. The output quantum discord with $V_B = 20$, $g = 1$ (red dashed line) is lower than the Alice’s initial quantum discord (black solid line). From the expression of $V_{d'}$, we can see that the output quantum discord is independent on Bob’s discording noise when a unit gain factor is chosen. When the optimal gain factor is chosen, the output quantum discord can be improved (blue dotted line with $V_B = 20$, $g = 0.88$). The most important result is that the output quantum discord can be higher than the initial quantum discord when optimal gain factor and Bob’s discording noise are chosen. For example, the green dash-dotted line in Fig. 2(a) corresponds to $V_B = 0$, $g = 0.26$, which means that two independent coherent states are used as ancillary state. It is clear that the output quantum discord is higher than the initial quantum discord when the discording noise of initial state $V_A > 18$. Therefore we can complete the transfer of Gaussian quantum discord using coherent state as ancillary state, which is the simplest way for the practical application.

Figure 2(b) shows the dependence of output quantum discord on the discording noise with an EPR entangled state as ancillary state. Comparing the red dashed ($r = 0.46$, $g = 1$) and
Fig. 2. Dependence of quantum discord of output state on the discording noise with a quantum discordant state (a) and an EPR entangled state (b) as ancillary state.

blue dotted lines \((r = 1.15, g = 1)\), we find that the higher entanglement the higher output quantum discord can be obtained when unit gain factor is chosen. When the optimal squeezing and gain factor are chosen, the output quantum discord (green dash-dotted line with \(r = 0.33, g = 0.32\)) can also be higher than the initial quantum discord (black solid line) when the initial discording noise \(V_A > 7.6\). Comparing the green dash-dotted lines in Fig. 2(a) and 2(b), we find that the maximum output Gaussian quantum discord is obtained when an EPR entangled state is used as ancillary state with optimal parameters. The purple dash-dotted-dotted lines in Fig. 2 correspond to the case of imperfect detection efficiency with \(\eta = 0.9\) and optimal gain factors. Comparing the green dash-dotted and purple dash-dotted-dotted lines, it is obvious that the quantum discord is decreased due to the imperfect detection efficiency.

Figure 3(a) shows the dependence of output quantum discord on Bob’s discording noise and gain factor for the case with a quantum discordant state as ancillary state, where Alice’s discording noise is chosen to be 50. It is obvious that the maximum output quantum discord is obtained at \(V_B = 0\), i.e. the ancillary states are two independent coherent states. This confirms...
that using coherent state as ancillary state is the simplest way to transfer the Gaussian quantum discord. Figure 3(b) shows the dependence of output quantum discord on squeezing parameter and gain factor when an EPR entangled state is used as an ancillary state, in which Alice’s discording noise is fixed to 50. The maximum output quantum discord is obtained with optimal squeezing parameter and gain factor. Comparing the maximum output quantum discord in Fig. 3(a) and 3(b), we find that entanglement is helpful to obtain maximum output quantum discord.

The relation between quantum discord of the output states and the gain factor when Alice’s discording noise $V_A = 50$ is shown in Fig. 4. In Fig. 4(a), the black solid, red dashed and blue dotted lines correspond to Bob’s discording noise of 0, 20 and 50, respectively. The optimal gain factor increases with the increasing of Bob’s discording noise. The maximum output quantum discord is obtained when coherent state is used as ancillary state. In Fig. 4(b), the black solid, red dashed and blue dotted lines correspond to squeezing parameter $r = 0, 0.33$ and 1.15, respectively. The optimal gain factor is close to unit with the increasing of squeezing parameter. There is an optimal squeezing that maximize the output quantum discord, which is $r = 0.33$. In this case, the maximum output Gaussian quantum discord of 0.92 is obtained.

4. Quantum discord of an asymmetric Gaussian discordant state

We now analyze the physical reason for why output quantum discord can be higher than initial quantum discord. Comparing the covariance matrixes of the initial and output Gaussian discordant states, we find that the initial state is a symmetric quantum discordant state, which has same noise variances on its two quantum correlated beams, while the output state is an asymmetric quantum discordant state, which has different noise variances on its two quantum correlated beams. This implies that the quantum discord of an asymmetric Gaussian quantum discordant state may be higher than that of a symmetric one. For proving this conclusion, we assume the discording noise modulated on two coherent states are different in the preparation of the quantum discordant state. For example, assuming the two modulated coherent states are $\hat{a} = \hat{c}_1 + \hat{s}_1$ and $\hat{b} = \hat{c}_2 + T\hat{s}_2$, respectively, where $T$ is the attenuation of the discording signal. In this case, the elements in the covariance matrix of the prepared Gaussian quantum discordant
state [Eq. (1)] becomes
\[
B = \begin{pmatrix}
1 + T^2 V & 0 \\
0 & 1 + T^2 V
\end{pmatrix},
\]
\[
C = \begin{pmatrix}
TV & 0 \\
0 & -TV
\end{pmatrix},
\]
(12)

When \( T \neq 1 \), the prepared state becomes an asymmetric Gaussian quantum discordant state.

Figure 5(a) shows the Gaussian quantum discord for the prepared state. It is obvious that the quantum discord with attenuation (red dashed line, \( T = 0.5 \)) is higher than that without attenuation (black solid line, \( T = 1 \)) when discording noise \( V > 4 \). Fig. 5(b) shows the dependence of quantum discord on the attenuation \( T \) at different discording noises. When \( V = 50 \) (red dashed line), it is clear that there is an optimal attenuation \( T \) that maximize the quantum discord. When \( 0.1 < T < 1 \), the quantum discord is higher than the symmetric Gaussian quantum discordant state. However, when \( V = 1 \) (black solid line), the quantum discord of the asymmetric Gaussian quantum discordant state is not higher than that of the symmetric one.

A symmetric Gaussian discordant state can also be turned into an asymmetric state by attenuate one or both optical modes. Here, we attenuate the optical modes \( \hat{a} \) and \( \hat{b} \) by two beam splitters with transmission efficiencies \( T_1 \) and \( T_2 \), respectively. The attenuated optical modes become
\[
\hat{a} = \sqrt{T_1} \hat{a} + \sqrt{1 - T_1} \hat{v}_1 = \sqrt{T_1} (\hat{c}_1 + \hat{s}_1) + \sqrt{1 - T_1} \hat{v}_1 \quad \text{and} \quad \hat{b} = \sqrt{T_2} \hat{b} + \sqrt{1 - T_2} \hat{v}_2 = \sqrt{T_2} (\hat{c}_2 + \hat{s}_2) + \sqrt{1 - T_2} \hat{v}_2,
\]
respectively.

Figure 6 shows the dependence of quantum discord on the attenuations \( T_1 \) and \( T_2 \) with discording noise \( V = 50 \). When \( T_1 = 1 \) and \( 0 < T_2 < 1 \), the Gaussian discordant state is an asymmetric state. It is obvious that the quantum discord with the optimal attenuation \( T_2 \) is higher than the discord of a symmetric Gaussian discordant state, which corresponds to the quantum discord at \( T_1 = T_2 = 1 \). This is similar to what we obtained in Fig. 5(b), which confirms that the quantum discord of an asymmetric Gaussian discordant state can be higher than a symmetric one with optimal attenuation. When the attenuation on the mode \( \hat{a} \) is also considered, the quantum discord is decreased along with the decrease of \( T_1 \). This is because the asymmetry between two optical modes is reduced by attenuating both modes.

So we draw a conclusion that the quantum discord of an asymmetric Gaussian quantum discordant state can be higher than that of a symmetric one prepared in this way when optimal
Fig. 6. Dependence of quantum discord on the attenuations $T_1$ and $T_2$ with discording noise $V = 50$.

attenuation and discording noise are chosen. This comes from the fact that the one-way classi-
cal correlation $J^{\rightarrow} (\rho_{AB})$ is measurement dependent. The attenuation of one mode (asymmetry) lead to the decrease of total and classical correlations, while the difference between them (quan-
tum discord) can be increased. This result is compatible with what obtained by Madsen et al in [23], where the attenuation of one mode of a Gaussian quantum discordant state lead to in-
creasing of quantum discord. In [23], transmission of one mode of a Gaussian discordant state in a lossy channel is mimicked by a beam-splitter with variable transmission efficiency. They observed the increase of quantum discord along with the increasing of the attenuation, which lead to an asymmetric Gaussian quantum discordant state. This property of Gaussian quantum discord will have potential application in quantum information tasks. For example, in the pre-
sented remote transfer scheme, the output quantum discord can be higher than the input discord. Since the Gaussian quantum discordant state is resilient to loss, it may be used to extend the transmission distance of quantum communication.

5. Conclusion

In conclusion, a remote transfer scheme of Gaussian quantum discord is proposed. The quantum correlation emerges between two independent quantum modes that without direct interaction after the remote transfer. In the remote transfer scheme with a Gaussian discordant state as ancillary state, the output quantum discord is independent on discording noise of the ancil-
ary state when unit gain is chosen. The maximum output quantum discord is obtained with two independent coherent states as ancillary state in the case of optimal gain. In the remote transfer scheme with an EPR entangled state as ancillary state, the maximum output quantum discord is obtained when optimal squeezing and gain are chosen. Comparing schemes with a Gaussian quantum discordant state and an EPR entangled state as ancillary state, we find that entanglement in the ancillary state is helpful to improve the output quantum discord.

Notably, the output Gaussian quantum discord can be higher than the initial quantum discord. The physical reason for this phenomenon is that the quantum discord of an asymmetric Gaussian quantum discordant state can be higher than that of a symmetric one. The remote transfer of Gaussian quantum discord has potential application in the quantum information network. Especially, the coherent states can serve as the ancillary states, which might save the quantum resources in the practical application.
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