Overview of the rheological models for rocks

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Abstract. This paper introduces the models of rock rheological element model, including classical models and some new models proposed based on the basic models. The classical models are Maxwell model, Kelvin model, Burgers model, and Xi-yuan model. The improvements in the new models mainly focus on using the nonlinear Newtonian body to replace the linear Newtonian body in viscoplastic body. The establishment method and constitutive equations of the new models are also presented. These new models can represent the properties of the corresponding type of rock and soil, and the feasibility of the models has been verified by experiments and modelling. All these researches provide more models in representing the rock rheology behaviour of soil, which is of vital importance in the study of soil and rock mechanics.

1. Instruction
The model for rock creep is one of the major focuses in rock mechanics, with many research achievements made in this topic till now [1]. With the introduction of the 14th five-year plan in China, many large-scale rock projects (including deep-buried tunnels, large landslide prevention and control, etc.) in southwest China have been proposed, such as the construction of the expressway from Xichang to Zhaotong in Yunan province. These large-scale rock projects bring opportunities for the study of rock mechanics along with the new problems and challenges. Among them rock rheology is one of the problems that must be considered in rock mass engineering.

Large-scale slope projects take a large portion in the rock engineering cases. The stability analysis of the large-scale slope is the main concern in such a project. In most researches, the structural plane is taken as the main research object in the rock stability analysis. However, the influence of the rock property on the slope stability is rarely considered. Among many rock properties, the rheological characteristics of the rock are important for the long-term stability of the rock. Therefore, characterizing the rock rheology is meaningful for the research of slope stability, and establishing the rock rheological model is very important. Several classical models have been put forward, and a variety of improved models were proposed. This paper presents the classical models and new models about rock rheology.

2. The classical rheological models
Being the main focus of many domestic and foreign researches, rheological element model is one of the most important models of rock rheology models. The first step of the rheological element combination model is to set some basic elements according to the elastic, plastic and viscous properties of the rock. Then, the elements were combined into a constitutive model that can reflect the rheological properties of a specific type of rock, so as to simulate the stress-strain relationship of the rock. Among many rheological element models, some models including Kelvin model, Burgers model and Xi-yuan model are widely-used.
Making property assumption for the basic elements in the model is an important step in the model establishment. According to the researches in recent years, the following basic elements have been set: rigid body element, elastomer element, viscous body element and plastic body element [1].

Maxwell model and Kelvin model are the most basic ones among the rheological models mentioned above. Most of the models proposed later by domestic and foreign scholars were based on those models, and their improvements were adding new components or recombining the two models according to the basic properties of the rock and soil mass studied. The two basic models are introduced briefly below.

Maxwell model (as shown in Fig. 1) is composed by an elastic element \( a \) and a flow element \( b \) in series, so it can describe materials with both elasticity and flow properties.

![Figure 1. Maxwell model](image)

The constitutive relation for the Maxwell model is \( \varepsilon = \frac{\sigma}{E} + \frac{\eta}{E} \varepsilon \). When \( t=0 \), there is instantaneous strain; the strain increases linearly with time \( t \).

This model can represent the elasticity of the rock, but the elastic limit of the rock cannot be reflected by this model, as if any small stress can generate enough strain, and the rock does not seem to have basic strength [2]. Therefore, Maxwell model is only a basic model. When representing a certain type of rock and soil, multiple components should be combined into this model.

Kelvin model (as shown in Fig. 2) connects elastic element \( a \) and flow element \( b \) in parallel to describe the viscosity and elasticity relationship of the material.

![Figure 2. Kelvin model](image)

The constitutive relation for Kelvin model is \( \sigma = E \varepsilon + \eta \dot{\varepsilon} \). It can be concluded that when \( t=0 \), there is no instantaneous strain, while when \( t \to \infty \), the final maximum strain value is only equal to the instantaneous strain of the elastomer model, which is equivalent to a delay of elastic strain. The final total deformation of the elastomer model is reached gradually as \( t \) goes to infinity, therefore Kelvin model is also called the hysteresis model.

As long as the stress remains stable, although the strain increases slowly with time, the increment will decrease with time and won’t exceed a fixed value, a strain limit for each stress. This value corresponds to the maximum elastic strain of the elastic element under this stress condition. It indicates that the material described has strength, and it is not always flowing. The elastic strain value of rock is generally limited. Therefore, Kelvin model is more suitable for small rock deformation conditions, such as many phenomena related to earthquakes [3].

As mentioned above, Maxwell model and Kelvin model are two basic models of rheology. New rheology models were proposed by adding new rheological elements in series or parallel on these two basic models, trying to increase their elastic and fluidity properties to fit the mechanical properties of rock and soil studied.

Among the new models developed based on the two basic models, Burgers model (Fig. 3) and Xi-yuan model (Fig. 4) are the most representative models. Comparative studies on these two models were conducted by researchers.
The Burgers model, also called the M-K body, is a series of Maxwell models and a Kelvin model.

The Xi-yuan model is a series of Bingham models and a Kelvin model, which is recorded as B-K.

Tao Bo [4] tested the grey-yellow mylonite on the F3 section of the south wall of the Fengshui Road, the 12 coal seam of the Zhagzezhuang Coal Mine of Kailuan Group, by using the triaxial rheological tester developed by the Institute of Geology and Geophysics of the Chinese Academy of Sciences. The rheological properties were studied based on the test results. The creep curves of the experimental results were fitted with the Burgers model and the Xi-yuan model. Finally, a better fit were achieved on the rock model with the ternary model. Therefore, as the stress will enter a viscoplastic state after the viscoelasticity flows to a certain extent, the Xi-yuan model can better reflect this property in geotechnical engineering.

3. Nonlinear rheological model

However, despite many models described above, none of them can represent the accelerated creep stage of rock. To solve this problem, many scholars are devoted to the study of nonlinear rheological element models. This paper mainly focuses on the method of using nonlinear Newtonian bodies to replace linearity in viscoplastic bodies to form nonlinear viscoplastic body. The nonlinear viscoplastic body was combined with a linear viscoelastic rheological model to derive a new type of nonlinear viscoelastic-plastic rheological model.

Deng Ronggui [5] changed the linear viscous body in Bingham body to nonlinear viscous body. The stress of nonlinear viscous body is proportional to the creep acceleration. The one-dimensional creep equation of the improved Bingham body is:

$$\varepsilon(t) = \frac{\sigma_0 - \sigma}{2\eta} t^2$$

Then the improved Bingham body and village mountain body model were combined to obtain a nonlinear viscoelastic-plastic rheological element model which can reflect the accelerated creep. The model is used to analyze the data of fault rock creep test of a large hydropower station in the Yabi River. The results show that the theoretical values are in good agreement with the experimental values.

Cao [6] believed that the crack of the rock would close and elastic deformation would occur when force were applied. As the stress in the rock increased, new cracks were generated, and then increased and expanded, leading to the eventual destruction the rock mass. Therefore, the viscosity coefficient of the rock can be seen as gradually increased at the beginning stage, and the maximum value would be reached quickly. When the cracks expand, the viscosity coefficient would decrease. However, as the
whole process is short, the change of the viscosity coefficient is also fast. Based on the above theory, the viscous model was improved to obtain a new viscous model. The constitutive equation in one-dimensional state is:

\[
\begin{align*}
\sigma &= \frac{A\eta_0}{A+bt+c}\dot{e} \\
\tau &= \frac{A\eta_0}{A+bt+c}\ddot{\gamma}
\end{align*}
\]  

(2)

Song [7] carried out immersion uniaxial tests and triaxial compression creep tests on a certain gypsum breccia. The tests show that the stress level of nonlinear creep was less than that of accelerated creep. Based on the test results, two types of nonlinear Newtonian viscous components, namely SN and SP component, were proposed by adopting the method of stress and time phase dispersion. The SN component is used to describe the nonlinear creep of rock, while the SP component is used to describe the accelerated creep. When \(\sigma = \sigma_0\), the creep equations of the SN element and the SP element can be described as Eqn. (3) and Eqn. (4) respectively:

\[
\begin{align*}
\dot{e} &= \frac{\sigma_0}{\gamma_0} \dot{\gamma} + c_k \\
\dot{\gamma} &= \frac{\eta_0(\sigma_0 - \beta\dot{e})}{\gamma_0(\sigma_0 - \beta\dot{e})}
\end{align*}
\]  

(3)

(4)

Yang [8] et al believe that the causes of creep damage for hard brittle rocks are cracking, expansion and slip of the internal cracks. When the stress in the rock exceeds the long-term strength, the rheological acceleration phase would occur, on which the rheological damage accumulate and the mesoscopic main crack expand rapidly. During this process, the viscous coefficient of the rock decreases gradually, and the failure would occur when the strain of the rock reaches the critical value. Moreover, the above process would also be affected by the stress level, leading to different durations before damage for different stress levels. The higher the stress level and the lower the rock viscous coefficient, the earlier the damage of the rock would occur. In the accelerated creep phase, the creep rate increases nonlinearly and the rock viscous coefficient decreases gradually with time. The stick component could be decomposed into a linear portion and a non-linear portion. In the attenuation and steady-state creep stage, the viscous coefficient \(\eta^M\) remains stable. In the accelerated creep stage, nonlinearization was carried out for \(\eta^M\) to obtain the constitutive equation for nonlinear viscoplastic element, given by:

\[
\dot{\varepsilon}_3 = \frac{\sigma_3 - \sigma_0}{\eta^M} \left[ 1 + A\sigma^m \frac{\left(\frac{\dot{\varepsilon}_3}{\dot{\varepsilon}_3^{n-1}}\right)}{A+bt} \right]
\]  

(5)

Chen [9] conducted limited uniaxial compression rheological tests on the coarse-grained soil of a certain red sandstone. The results show that under the large stress state (axial pressure \(\sigma > 0.8\) MPa), there are three rheological phases for the red sandstone coarse grain: attenuating rheology, steady-state rheology, and accelerating rheology. In order to describe the nonlinear viscoplastic deformation characteristics of such soil, a hyperbolic-based nonlinear viscous element was proposed and combined with a plastic element to form a new nonlinear viscoplastic rheological body (VPB). The rheological equation of the nonlinear elastoplastic rheological model (VPB) under the constant stress \(\sigma_0\) condition could be given by Eqn. (6):

\[
\varepsilon(t) = \frac{H(\sigma_0 - \sigma_d)}{\eta_3} \frac{t}{a+bt}
\]  

(6)

Song [10] studied the viscosity coefficient \(\eta\) based on the research of S.S.Vyalov. Based on the results of sandstone uniaxial compression creep tests, mudstone uniaxial compression creep tests and direct shear creep tests, the Bingham creep equation was transformed into a two-variable linear regression equation, and the parameters for the creep equation could be determined. According to the experimental results, the variation of the viscosity coefficient of the material with time dependent on the stress value applied. Huang [11] proposed an unsteady Newtonian body based on Bingham body, which considered the influence of stress state. The improved Bingham body and Bingham model were combined to obtain the unsteady viscoelastic-plastic rheological model which could consider the influence of stress state. The viscosity coefficient is for this model could be given by Eqn. (7):

\[
\eta^B = \frac{\eta \bar{\tau}_d}{1+3H(3)a\bar{\varepsilon}^2}
\]  

(7)
4. Conclusions and suggestion
This paper presents a detailed review of the classical models and discussions of some latest nonlinear rheological models. The conclusions and suggestion could be drawn as follows:

(1) Great progress was made in the development of rheological models of the geotechnical components over these years. Most of the new models were constructed by adding or improving components based on the classic models. However, the constitutive equations of some new models are extremely complicated, which is inconvenient to use for real engineering. Therefore, it is necessary to establish relatively simple constitutive equations to satisfy the need for engineering application in the future.

(2) Despite the development of rheological tests, uniaxial and triaxial tests in particular, there are many influencing factors to be investigated. The influence of factors such as temperature, size effect, water content and expansion needs to be studied.

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