Emergence and expansion of cosmic space as due to M0-branes

Alireza Sepehri 1,2*, Mohammad Reza Setare 3†, Salvatore Capozziello 4,5,6‡

1 Faculty of Physics, Shahid Bahonar University, P.O. Box 76175, Kerman, Iran.
2 Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran.
3 Department of Science, Campus of Bijar, University of Kurdistan, Bijar, Iran.
4 Dipartimento di Fisica, Universita di Napoli Federico II, I-80126 - Napoli, Italy.
5 INFN Sez. di Napoli, Compl. Univ. di Monte S. Angelo, Edificio G, I-80126 - Napoli, Italy,
6 Gran Sasso Science Institute (INFN), Viale F. Crispi, 7, I-67100, LAquila, Italy

Recently, Padmanabhan [arXiv:1206.4916 [hep-th]] discussed that the difference between the number of degrees of freedom on the boundary surface and the number of degrees of freedom in a bulk region causes to the accelerated expansion of the universe. The main question arises as to what is the origin of this inequality between the surface degrees of freedom and the bulk degrees of freedom? We answer this question in M-theory. In our model, first M0-branes are compactified on one circle and N D0-branes are created. Then, N D0-branes join to each other, grow and form one D5-branes. Next, D5-brane is compactified on two circle and our universe-D3-brane, two D1-brane and some extra energies are produced. After that, one of D1-branes which is more close to universe-brane, gives it’s energy into it, leads to an increase in difference between number of degrees of freedom and occurring inflation era. With the disappearance of this D1-brane, the number of degrees of freedom of boundary surface and bulk region become equal and inflation ends. At this stage, extra energies that are produced due to the compactification cause to an expansion of universe and deceleration epoch. Finally, another D1-brane, dissolves in our universe-brane, leads to an inequality between degrees of freedom and happening a new phase of acceleration.

* alireza.sepehri@uk.ac.ir
† rezakord@ipm.ir
‡ capozziello@na.infn.it
I. INTRODUCTION

About three years ago, Padmanabhan suggested that the accelerated expansion of the universe is due to the difference between the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom through a simple equation \( \Delta V = \Delta t (N_{\text{sur}} - N_{\text{bulk}}) \) where \( V \) is the Hubble volume in Planck units and \( t \) is the cosmic time in Planck units [1]. Up to date, many discussions have been done on the Padmanabhan proposal [2-8]. For example, in one paper, with the help of this idea, the Friedmann equations of an \((n + 1)\)-dimensional Friedmann-Robertson-Walker universe corresponding to general relativity, Gauss-Bonnet gravity, and Lovelock gravity have been obtained [2]. In another research, the idea of treating the cosmic space as an emergent process has been applied to brane cosmology, scalar-tensor cosmology, and f(R) gravity and the corresponding cosmological equations in these theories have been derived [3]. In another investigation, using Padmanabhan suggestion, author obtained the Friedmann equations of universe not only in four dimensional space-time and Einstein gravity, but also in higher dimensional space-time and other gravities like Gauss-Bonnet and Lovelock gravity with any spacial curvature [4]. Some other authors, have extended the evolution equation in Padmanabhan idea to give the Friedmann equation in the nonflat universe corresponding to \( k = \pm 1 \) by taking into account the invariant volume surrounded by the apparent horizon [5]. In another scenario, authors showed that applying Padmanabhans conjecture to non-Einstein gravity cases encounters serious difficulties and has to be heavily modified to get the Friedmann equation [6]. In another paper, author applied derived equations of universe in the Padmanabhan model with the corrected entropy-area law that follows from Generalized Uncertainty Principle (GUP) and obtain a modified Friedmann equations due to the GUP [7]. and in more recent research, the Padmanabhan idea has been constructed in Bionic system and shown that all degrees of freedom inside and outside the universe are controlled by the evolutions of BIon in extra dimension and tend to degrees of freedom of black F-string in string theory [8]. The BIon is a configuration of two branes which are connected by a wormhole [9,12].

Now, the main question arises as to what is the origin of difference between the number of degrees of freedom on the boundary surface and the one in a bulk? To answer this question, we use the method in [12]. In that paper, \( k \) fundamental strings decay to \( N \) pairs of M0-branes. Then, these branes glue to each other and form an M3, an anti-M3 and a wormhole
between them. Our universe is located on one of these branes and interact with other brane via the wormhole. Extending this idea, we propose a new model which allows to construct our universe from M0-branes in M-theory. In this proposal, first, we will compactify M0-branes on one circle and obtain the relevant action for N D0-branes. Then, we will show that these D0-branes may join to each other and make a transition to a D5-brane. Next, we will compactify this D5-brane on two circles and derive the relevant actions for one D3-brane, two D1-branes and some extra energies. Our universe is located on this D3-brane and two D1-branes are the main causes of inequality between number of degrees freedom on the surface horizon and in a bulk and occurring inflation and late-time acceleration. Also, extra energies are responsible for happening of deceleration epoch.

The outline of the paper is as follows. In section II, we will construct D0-brane from M0-brane and consider the relation between their algebra. We also show that D0-branes can join each other and form a D5-brane. Then, we will compactify D5-brane on two circles and obtain one D3-brane and two D1-branes. In section III, we will show that the D1-brane which is more close to D3-brane dissolves in it and leads to inflation. Also, in this section, we will argue that another D1-brane is the main cause of second phase of acceleration and inequality between number of degrees of freedom on the holographic surface and one in a bulk. The last section is devoted to summary and conclusion.

II. THE BIRTH OF UNIVERSE IN M-THEORY

In this section, we will show that the origin of universe is M0-branes. Recently, some authors have proposed an action based on Lie 3-algebras to describe M2-branes [13–17]. Some other authors have considered the case of infinite dimensional Lie 3-algebras based on the Nambu-Poisson structure of three dimensional manifolds. They argued that the model contains self-dual 2-form gauge fields in 6 dimensions, and the result may be interpreted as the M5-brane world-volume action [18]. Extending these methods, we propose a new model which allows to construct our universe-brane from M0-branes. To this end, first, we obtain the relevant action for N M0-branes by replacing Nambu-Poisson structure of two dimensional manifolds in D-branes by the structure of three dimensional one. At second stage, we will compactify them on one circle and derive the action for N D0-brane. We show that N D0-branes join to each other, grow and form a D5-brane. Then, this brane is
compactified on two circles and our universe-brane and two D1-branes are created.

First, we introduce the Born-Infeld action for M0-brane by replacing two dimensional Nambu-Poisson bracket [19–24] in the action of Dp-branes by three dimensional Nambu-Poisson bracket [13–18] and applying Lie 3-algebra [12].

\[
S_{M0} = T_{M0} \int dt Tr(\sum_{M,N,L=0}^{10} ([X^M, X^N, X^L], [X^M, X^N, X^L]))
\]

(1)

where \(X^M = X^M_\alpha T^\alpha\) and

\[
\begin{align*}
[T^\alpha, T^\beta, T^\gamma] &= f^{\alpha\beta\gamma}_{\eta} T^\eta \\
\langle T^\alpha, T^\beta \rangle &= h^{\alpha\beta} \\
[X^M, X^N, X^L] &= [X^M_\alpha T^\alpha, X^N_\beta T^\beta, X^L_\gamma T^\gamma] \\
\langle X^M, X^N \rangle &= X^M_\alpha X^N_\beta \langle T^\alpha, T^\beta \rangle
\end{align*}
\]

(2)

where \(X^M (i=1,3,...10)\)’s refer to transverse scalars to M0-brane. One can show that by compactifying M-theory on a circle of radius \(R\), the above action transits to ten dimensional action for D0-brane [22, 24]. To this end, we apply the method in [25] and define \(<X^M> = \frac{R}{l_p^2}\) where \(l_p\) is the Planck length. We obtain [12]:

\[
\begin{align*}
S_{M0} &= -T_{M0} \int dt Tr(\sum_{M,N,L=0}^{10} ([X^M, X^N, X^L], [X^M, X^N, X^L])) = \\
&= -T_{M0} \int dt Tr(\sum_{M,N,L,E,F,G=0}^{10} \varepsilon_{MNLDEFG} X^M X^N X^L X^E X^F X^G) \\
&= -6T_{M0} \int dt Tr(\sum_{M,N,E,F=0}^{9} \varepsilon_{MN10DEFG} X^M X^N X^{10} X^E X^F X^{10}) \\
&= 6T_{M0} \int dt \sum_{M,N,L,E,F,G=0}^{9} \varepsilon_{MNLDEFG} X^M X^N X^L X^E X^F X^G = \\
&= -6T_{M0} \left(\frac{R^2}{l_p^2}\right) \int dt Tr(\sum_{M,N,E,F=0}^{9} \varepsilon_{MN10DEFG} X^M X^N X^E X^F) \\
&= 6T_{M0} \int dt \sum_{M,N,L,E,F,G=0}^{9} \varepsilon_{MNLDEFG} X^M X^N X^L X^E X^F X^G = \\
&= -6T_{M0} (\frac{R^2}{l_p^2}) \int dt Tr(\sum_{M,N=0}^{9} [X^M, X^N]^2) - \\
&= 6T_{M0} \int dt \sum_{M,N,L,E,F,G=0}^{9} \varepsilon_{MNLDEFG} X^M X^N X^L X^E X^F X^G =
\end{align*}
\]
\[ S_{D0} = 6T_{M0} \int dt \sum_{M,N,L,E,F,G=0,\neq 10}^{9} \varepsilon_{MNLDEFG} X^M X^N X^L X^E X^F X^G \]

\[ S_{D0} + V_{Extra,1} \quad (3) \]

where \( V_{Extra,1} = -6T_{M0} \int dt \sum_{M,N,L,E,F,G=0,\neq 10}^{9} \varepsilon_{MNLDEFG} X^M X^N X^L X^E X^F X^G \). Also, \( T_{M0} \) and \( T_{D0} \) denote tensions of \( M0 \) and \( D0 \) respectively and \( T_{D0} = \frac{1}{g_s l_s} \) where \( g_s \) and \( l_s \) are the string coupling and string length respectively. Clearly, the action (3) for compactified \( M0 \)-branes is equal to the sum of relevant action for \( D0 \)-brane \[12, 19–24\].

\[ S_{D0} = -T_{D0} \int dt Tr(\sum_{m=0}^{9} [X^m, X^n]^2) \quad (4) \]

and some extra energies that are produced due to compactification. Now, we can construct other \( Dp \)-branes from \( D0 \)-branes by substituting following rules \[12, 19–24\]:

\[ \Sigma_{a=0}^{p} \Sigma_{m=0}^{9} \rightarrow \frac{1}{(2\pi l_s)^p} \int d^{p+1}\sigma \sum_{m=p+1}^{9} \Sigma_{a=0}^{p} \lambda = 2\pi l_s^2 \]

\[ [X^a, X^i] = i\lambda \partial_a X^i \quad [X^a, X^b] = i\lambda F^{ab} \]

\[ i, j = p + 1, ..., 9 \quad a, b = 0, 1, ...p \quad m, n = 0, 1, ..., 9 \quad (5) \]

in action (4) and doing some mathematical calculations [12]:

\[ S_{Dp} = \sum_{a=0}^{p} S_{D0} = -\sum_{a=0}^{p} T_{D0} \int dt Tr(\sum_{m=0}^{9} [X^m, X^n]^2) = \]

\[ -T_{Dp} \int d^{p+1}\sigma Tr(\sum_{a,b=0}^{p} \sum_{i,j=p+1}^{9} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F^{ab})^2 \}) \quad (6) \]

which is in agreement with results of [19–24] for \( D1, D3 \) and \( D5 \)-branes. Here \( T_{Dp} = \frac{T_{D0}}{(2\pi l_s)^p} \) is the brane tension, \( l_s \) is the string length, \( g_s \) is the string coupling and \( F_{ab} \) is the field strength. To compactify this \( Dp \)-brane on one circle, we need to replace some gauge fields by some scalar fields use of following laws \[12, 18–25\]:

\[ T_{Dp} \int d^{p+1}\sigma Tr(\sum_{a,b=0}^{p} \sum_{i,j=p+1}^{9} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F^{ab})^2 \}) \rightarrow \] 

\[ i\lambda F_{ap} \rightarrow \partial_a X^p \quad i\partial_p X^i \rightarrow \frac{1}{\lambda} [X^p, X^i] \quad (7) \]

Replacing these equations in action (6) and doing some algebra, we obtain:
\[ S_{Dp} = -T_{Dp} \int d^{p+1}\sigma Tr(\Sigma_{a,b=0}^{p} \Sigma_{i,j=p+1}^{9} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \}) = -T_{D(p-1)} \int d^{p}\sigma Tr(\Sigma_{a,b=0}^{p-1} \Sigma_{i,j=p}^{9} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \}) - T_{D1} \int d^{2}\sigma Tr(\Sigma_{a,b=0}^{1} \Sigma_{i,j=2}^{9} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \}) - V_{\text{separation}} \]

\[ S_{Dp} = S_{D(p-1)} + S_{D1} - V_{\text{separation}} \]

\[ S_{Dp} + V_{\text{separation}} = S_{D(p-1)} + S_{D1} \]  

(8)

This equation shows that we need to some extra energies that are applied for separating D1-brane from Dp-brane \( V_{\text{separation}} = -T_{D1} \int d^{2}\sigma Tr \Sigma_{i,j=2}^{9} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \}. \) To supply this energy, we use of extra energies that are produced due to compactification of M0-branes. For this, we compactify \( V_{\text{Extra}} \) on the circle with defining \( \langle X^{p} \rangle = \sqrt{\frac{T_{m}}{12T_{M0}\lambda^2}} \):

\[ V_{\text{Extra,1}} = -6T_{M0} \int dt \Sigma_{M,N,L,E,F,G=0}^{8} \varepsilon_{MNL}^{D} \varepsilon_{EFG}^{D} X^{M} X^{N} X^{L} X^{E} X^{F} X^{G} = -6T_{M0} \int dt Tr( \Sigma_{M,N,E,F=0}^{8} \varepsilon_{MNP}^{D} \varepsilon_{EFG}^{D} X^{M} X^{N} X^{P} X^{E} X^{F} X^{P} - 6T_{M0} \int dt \Sigma_{M,N,L,E,F,G=0,\neq p}^{8} \varepsilon_{MNL}^{D} \varepsilon_{EFG}^{D} X^{M} X^{N} X^{L} X^{E} X^{F} X^{G} = -6T_{M0} \frac{T_{D1}}{12T_{M0}\lambda^2} \int dt Tr( \Sigma_{M,N,E,F=0}^{9} \varepsilon_{MNP}^{D} \varepsilon_{EFG}^{D} X^{M} X^{N} X^{E} X^{F} X^{G} = -6T_{M0} \int dt \Sigma_{M,N,L,E,F,G=0,\neq p}^{8} \varepsilon_{MNL}^{D} \varepsilon_{EFG}^{D} X^{M} X^{N} X^{L} X^{E} X^{F} X^{G} = -6T_{M0} \frac{T_{D1}}{2\lambda^2} \int dt Tr( \Sigma_{M,N=0}^{9} [X^{M}, X^{N}]^2 ) - 6T_{M0} \frac{T_{D1}}{2\lambda^2} \int dt Tr( \Sigma_{M,N=2}^{9} [X^{M}, X^{N}]^2 ) - 6T_{M0} \frac{T_{D1}}{2\lambda^2} \int dt Tr( \Sigma_{M,N=0}^{1} [X^{M}, X^{N}]^2 ) - 6T_{M0} \int dt \Sigma_{M,N,L,E,F,G=0,\neq p}^{8} \varepsilon_{MNL}^{D} \varepsilon_{EFG}^{D} X^{M} X^{N} X^{L} X^{E} X^{F} X^{G} = V_{\text{separation}} + V_{\text{Extra,2}} + V_{\text{Extra,3}} \]  

(9)

where we define:

\[ V_{\text{Extra,2}} = -6T_{M0} \frac{T_{D1}}{2\lambda^2} \int dt Tr( \Sigma_{M,N=0}^{2} [X^{M}, X^{N}]^2 ) \]
\[ V_{\text{Extra},3} = -6T_{M0} \int dt \Sigma_{M,N,L,E,F,G=0}^{N} X^{M} X^{N} X^{L} X^{E} X^{F} X^{G} \] (10)

These extra energies supply required energy for compactifying D(p-1) on another circle. Following equation (6), we can write:

\[
S_{D(p-1)} = S_{D(p-2)} + S_{D1} - V_{\text{separation}} \rightarrow \\
S_{D(p-1)} + V_{\text{separation}} = S_{D(p-2)} + S_{D1} \rightarrow \\
S_{Dp} + 2V_{\text{separation}} = S_{D(p-1)} + S_{D1} + V_{\text{separation}} = S_{D(p-2)} + S_{D1} + S_{D1} \] (11)

For example, by compactifying one D5-brane on two circles, one D3-brane and two D1-branes are created:

\[
S_{D(p-1)} = S_{D(p-2)} + S_{D1} - V_{\text{separation}} \rightarrow \\
S_{D(p-1)} + V_{\text{separation}} = S_{D(p-2)} + S_{D1} \rightarrow \\
S_{D5} + 2V_{\text{separation}} = S_{D(4)} + S_{D1} + V_{\text{separation}} = S_{D(3)} + S_{D1} + S_{D1} \] (12)

This equation shows that the origin of D3-brane is D5-brane. On the hand, this D5-brane is produced by joining and growing D0-branes. Also, D0-branes are created by compactifying M0-branes. If we assume that our universe is located on D3-brane, we can claim that the main cause of the birth of universe is the transition of N M0-branes to a D3-brane via process
\[ 5M0 \rightarrow 5D0 + \text{extra energy} \rightarrow D5 + \text{extra energy} \rightarrow D3 + D1 + D1. \]

III. THE PADMANABHAN IDEA IN D3-D1 SYSTEM

In this section, we will construct the Padmanabhan idea in D3-D1 system and argue that the expansion of universe is controlled by the evolution of branes in extra dimensions. We will show that first D1-brane which is more close to our universe dissolves in it, increase inequality between number of degrees of freedom on the holographic surface and inside a bulk and leads to the inflation. Then, this brane gives its energy to our universe brane, annihilates and inflation ends. After that, extra energies that are produced in compactifications cause an expansion and deceleration epoch. Finally, another D1-brane interacts with our universe and leads to second phase of acceleration.
Using equation (6), we can obtain relevant action for D3 and D1-branes:

\[ S_{D3} = -T_{D3} \int d^4 \sigma Tr(\Sigma^3_{a,b=0} \Sigma^9_{i,j=4} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4}(F_{ab})^2 \}) \] (13)

\[ S_{D1} = -T_{D1} \int d^2 \sigma Tr(\Sigma^1_{a,b=0} \Sigma^9_{i,j=2} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4}(F_{ab})^2 \}) \] (14)

Let us now build the Padmanabhan idea in D3-D1 system. For this, we need to compute the contribution of this system to the degrees of the surface and the bulk. To this end, we write the following relations between these degrees of freedom and the energy of D1 and D3-branes,

\[ N_{\text{sur}} \sim E_{D3} \quad N_{\text{bulk}} \sim E_{D3-D1} = E_{D3} + E_{D1} \]

\[ N_{\text{sur}} - N_{\text{bulk}} \simeq E_{D1} \] (15)

where \( E_{D3} \) and \( E_{D1} \) are energies of D3 and D1-branes respectively. Now, we want to calculate these energies by using the action (13):

\[ H_{D3} = \Sigma^3_{a,b=0} \Sigma^9_{i,j=4} \Pi_i(\partial_t X^i) - L_{D3} \quad \Pi = \frac{\partial L}{\partial(\partial_t X^i)} = -(\partial_t X^i) \]

\[ L_{D3} = Tr(\Sigma^3_{a,b=0} \Sigma^9_{i,j=4} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4}(F_{ab})^2 \}) \]

\[ E_{D3} = -T_{D3} \int d^4 \sigma H = -T_{D3} \int d^4 \sigma Tr(\Sigma^3_{a,b=0} \Sigma^9_{i,j=4} \{ \partial_a X^i \partial_b X^i + \frac{1}{2\lambda^2} [X^i, X^j]^2 - \frac{\lambda^2}{4}(F_{ab})^2 \}) \] (16)

\[ H_{D1} = \Sigma^1_{a,b=0} \Sigma^9_{i,j=2} \Pi_i(\partial_t X^i) - L_{D1} \quad \Pi = \frac{\partial L}{\partial(\partial_t X^i)} = -(\partial_t X^i) \]

\[ L_{D1} = Tr(\Sigma^1_{a,b=0} \Sigma^9_{i,j=2} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4}(F_{ab})^2 \}) \]

\[ E_{D1} = -T_{D1} \int d^2 \sigma H = -T_{D1} \int d^2 \sigma Tr(\Sigma^1_{a,b=0} \Sigma^9_{i,j=2} \{ \partial_a X^i \partial_b X^i + \frac{1}{2\lambda^2} [X^i, X^j]^2 - \frac{\lambda^2}{4}(F_{ab})^2 \}) \] (17)
Minimizing the relevant actions (13) and (14) and also energies (16) and (17) for D3 and D1-branes yields the following condition [20]:

$$\partial_\sigma X^i = \pm \frac{i}{2} \varepsilon^{ijk} [X^j, X^k]$$ (18)

The desired solution is given by:

$$X^i = \pm \frac{\alpha^i}{2\sigma}$$ (19)

where the $\alpha^i$ are an $N \times N$ representation of the SU(2) algebra,

$$[\alpha^i, \alpha^j] = i\varepsilon^{ijk} \alpha^k$$ (20)

Using equation (18), we can obtain the minimum energy of D3 and D1-branes:

$$E_{D3,min} = 7T_{D3} \int d^4\sigma Tr(\Sigma_3^{a,b=0} \left\{ \frac{\lambda^2}{4} (F_{ab})^2 \right\})$$

$$E_{D1,min} = 7T_{D1} \int d^2\sigma Tr(\Sigma_1^{a,b=0} \left\{ \frac{\lambda^2}{4} (F_{ab})^2 \right\})$$ (21)

Following rules in (7), we can obtain the solutions for gauge fields in D3 and D1-branes:

$$\lambda F_{01} \text{ in D1-brane } \rightarrow \partial_t X^1 \text{ in D3-brane}$$

$$F_{ab} \text{ in D3-brane } \rightarrow [X^a, X^b] \text{ in D1-brane } \Rightarrow$$

$$X^i \sim \frac{1}{2\sigma_1}, \quad A^i \sim \frac{1}{2\sigma_3} \quad \text{ in D1-brane}$$

$$X^i \sim \frac{1}{2\sigma_3}, \quad A^i \sim \frac{1}{2\sigma_1} \quad \text{ in D3-brane}$$ (22)

where $\sigma_1$ and $\sigma_3$ are coordinates of D1 and D3-branes respectively. With going time, $\sigma_1$ is decreased and reduced to zero at the end of inflation but $\sigma_3$ is increased. For this reason, we assume $\sigma_1 \sim \frac{1}{t}$ and $\sigma_3 \sim t$. Choosing these approximations needs some further discussion and explanations. According to this model, the universe is located on the D3-brane, thus due to time evolution and universe expansion, the D3-brane expands and $\sigma_3$ which is the
coordinate of D3-brane, grows and has a direct relation with time. On the other hand, the main cause of inflation is dissolving of D1-brane into D3-brane which represent our universe. Therefore, by passing time and universe inflation, $\sigma_1$, which is the coordinate of D1-brane, decreases and thus it is related with the inverse of time. Furthermore, due to the evolution in time and the disappearing of D1-brane, gauge fields which are stick to it, have to vanish. As can be seen from equation (22), the gauge field on D1-brane is related to $\sigma_3$, and thus by evolving time and disappearing D1-brane and the gauge field, $\sigma_3$ increases.

With this assumption and using equations (16), (17) and (22), and also condition in (18), we can calculate energy of D1 and D3-branes and number of degrees of freedom on the holographic surface and one in a bulk:

\[
N_{\text{sur}} - N_{\text{bulk}} \simeq E_{D1} \simeq 14\pi^2 l_s^4 T_{D1} \left[ \frac{t_{\inf} - t}{t_{\inf}} \right]
\]

\[
N_{\text{sur}} \simeq E_{D3} + (E_{D1,\inf} - E_{D1}) \simeq 15\pi^2 l_s^4 T_{D3} \left[ \frac{t^5}{60} + \frac{t_{\inf} t^4}{12} - \frac{t_{\inf}^2 t^3}{12} - \frac{t_{\inf}^3 t^2}{24} + \frac{t_{\inf}^4 t}{2} + 14\pi^2 l_s^4 T_{D1} \left[ \frac{t}{t_{\inf}} \right] \right]
\]

where $(E_{D1,\inf} = 14\pi^2 l_s^4 T_{D1})$ is the energy of D1-brane at the beginning of inflation, $(E_{D1,\inf} - E_{D1})$ is the amount of energy which dissolves in D3-brane and $t = t_{\inf}$ is the time of end of inflation. As can be seen from this equation, difference between number of degrees of freedom on the holographic surface and bulk decreases with time and shrinks to zero at the end of inflation ($t = t_{\inf} \Rightarrow N_{\text{sur}} = N_{\text{bulk}}$). This means that our calculations are consistent with the Padmanabhan idea and thus our model works.

These equations help us to obtain the relation between some of cosmological parameters like deceleration parameter and evolutions of D1 and D3-branes. To this end, first, we calculate Hubble parameter via following equation:

\[
N_{\text{sur}} = \frac{4\pi r_A^2}{l_p^2} \quad r_A = \frac{1}{H} \Rightarrow
\]

\[
N_{\text{sur}} = \frac{4\pi}{(l_pH)^2} \simeq 15\pi^2 l_s^4 T_{D3} \left[ \frac{t^5}{60} + \frac{t_{\inf} t^4}{12} - \frac{t_{\inf}^2 t^3}{9} - \frac{t_{\inf}^3 t^2}{24} + \frac{t_{\inf}^4 t}{2} + 14\pi^2 l_s^4 T_{D1} \left[ \frac{t}{t_{\inf}} \right] \right] \Rightarrow
\]

\[
H = \sqrt{\frac{15\pi^2 l_s^4 T_{D3} \left[ \frac{t^5}{60} + \frac{t_{\inf} t^4}{12} - \frac{t_{\inf}^2 t^3}{9} - \frac{t_{\inf}^3 t^2}{24} + \frac{t_{\inf}^4 t}{2} + 14\pi^2 l_s^4 T_{D1} \left[ \frac{t}{t_{\inf}} \right] \right]}{\left( \frac{t^5}{60} + \frac{t_{\inf} t^4}{12} - \frac{t_{\inf}^2 t^3}{9} - \frac{t_{\inf}^3 t^2}{24} + \frac{t_{\inf}^4 t}{2} + 14\pi^2 l_s^4 T_{D1} \left[ \frac{t}{t_{\inf}} \right] \right)}}
\]

(24)
where $H$ is the Hubble parameter and $l_p$ is the Planck length. Using this equation, we can obtain deceleration parameter during inflation era in terms of time:

$$q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 \simeq \sqrt{15\pi l_p^3 T_{D3} t_{inf}^4 \left[ \frac{t^4}{24t_{inf}^4} + \frac{t^3}{6t_{inf}^3} - \frac{t^2}{6t_{inf}^2} - \frac{t}{24t_{inf}} \right]} - 14\pi^2 l_p^4 T_{D1} t \left[ 1 - \frac{t}{t_{inf}} \right]$$  

(25)

This equation indicates that while the age of universe ($t$) is increased, the deceleration parameter reduces to lower negative values, turns over a minimum, increases and tends to zero at $t = t_{inf}$ (see figure 1). This means that the D1-brane is disappeared at the end of inflation, however the rate of acceleration of universe is increased very fast and tend to large values in this epoch.

![Inflation Era](image)

FIG. 1: The deceleration parameter for inflation era of expansion history as a function of the $t$ where $t$ is the age of universe. In this plot, we choose $t_{inf} = 380000$, $T_{D3} = 10000$, and $l_s = 0.1$.

With the disappearance of D1-brane, inflation ends and deceleration epoch begins. At this stage, extra energies that are produced in previous compactifications of branes cause to expansion of universe. Some of these energies are introduced in equation (10). Applying $V_{Extra,2} = \left( \frac{T_{D1}}{2\pi} \right) \int dt Tr(\sum_{M,N=0}^{2} [X^M, X^N]^2)$ in this equation, assuming $\sigma_{extra} \sim \frac{1}{t}$ and $\sigma_3 \sim t$, and using equations (13), (22) and (18), we can obtain the amount of extra energy and also energy of D3-branes and number of degrees of freedom on the holographic surface and one in the bulk region:
\[ N_{\text{sur}} - N_{\text{bulk}} \simeq E_{\text{extra,in\text{f}f-t}} \simeq -\left(\frac{T_{D1}}{2\lambda^2}\right) \int dt TR(\Sigma^2_{0=M,N=0}[X^M, X^N]^2) \]
\[ \simeq \left(\frac{T_{D1}}{4\pi^2 l_s^4}\right) \left[\frac{t_{\text{dec}} - t_{\text{inf}}}{(t - t_{\text{inf}})^3}\right] \]
\[ N_{\text{sur}} \simeq E_{D3} \simeq E_{D3,\text{inf}} + (E_{\text{extra,inf-dec}} - E_{\text{extra,inf-t}}) + 5T_{D3} \int d^4\sigma TR(\Sigma^3_{a=0}\{\frac{\lambda^2}{4}(\partial_a(\frac{1}{\sigma^3}))^2\}) \simeq \]
\[ E_{D3,\text{inf}} + 15\pi^2 l_s^4 T_{D3}\left[\frac{(t_{\text{dec}} - t)^5}{60} + \frac{(t_{\text{dec}} - t_{\text{inf}})(t_{\text{dec}} - t)^4}{12} - \frac{(t_{\text{dec}} - t_{\text{inf}})(t_{\text{dec}} - t)^3}{9} \right. \]
\[ \left. \frac{(t_{\text{dec}} - t_{\text{inf}})(t_{\text{dec}} - t)^2}{24} \right] + \left(\frac{T_{D1}}{4\pi^2 l_s^4}\right) \left[\frac{(t_{\text{dec}} - t_{\text{inf}})^3}{(t - t_{\text{inf}})^3}\right] \]
\[ \text{(26)} \]

where \( E_{D3,\text{inf}} = \frac{480\pi^2 l_s^4 T_{D3}^2 t_{\text{inf}}^4}{72} \) is the energy of D3-brane at the end of inflation, \( (E_{\text{extra,inf-dec}} - E_{\text{extra,inf-t}}) \) is the amount of energy that dissolves in D3-brane during deceleration era and \( t = t_{\text{dec}} \) is the age of universe at the end of deceleration epoch.

Similar to inflation era, difference between the number of degrees of freedom on the holographic surface and bulk decreases with time and shrinks to zero at the end of deceleration \((t = t_{\text{dec}} \Rightarrow N_{\text{sur}} = N_{\text{bulk}})\).

The Hubble parameter during this era can be calculated as:

\[ N_{\text{sur}} = \frac{4\pi l_s^2}{l_p^2} \Rightarrow r_A = \frac{1}{H} \]
\[ N_{\text{sur}} = \frac{4\pi}{(l_p H)^2} \simeq E_{D3,\text{inf}} + 15\pi^2 l_s^4 T_{D3}\left[\frac{(t_{\text{dec}} - t)^5}{60} + \frac{(t_{\text{dec}} - t_{\text{inf}})(t_{\text{dec}} - t)^4}{12} - \frac{(t_{\text{dec}} - t_{\text{inf}})(t_{\text{dec}} - t)^3}{9} \right. \]
\[ \left. \frac{(t_{\text{dec}} - t_{\text{inf}})(t_{\text{dec}} - t)^2}{24} \right] + \left(\frac{T_{D1}}{4\pi^2 l_s^4}\right) \left[\frac{(t_{\text{dec}} - t_{\text{inf}})^3}{(t - t_{\text{inf}})^3}\right] \Rightarrow \]
\[ H = \frac{4\pi}{l_p \sqrt{N_{\text{sur}}} \frac{1}{H}} \]
\[ \text{(27)} \]

With the help of this equation, we can derive deceleration parameter during deceleration epoch in terms of time:

\[ q = \frac{d}{dt} \left(\frac{1}{H}\right) - 1 \simeq \sqrt{15\pi^2 l_s^4 T_{D3}(t_{\text{dec}} - t_{\text{inf}})^4}\left[\frac{(t_{\text{dec}} - t)^4}{24(t_{\text{dec}} - t_{\text{inf}})^4} - \frac{(t_{\text{dec}} - t)^3}{6(t_{\text{dec}} - t_{\text{inf}})^3} \right. \]
\[ \left. + \frac{(t_{\text{dec}} - t)^2}{6(t_{\text{dec}} - t_{\text{inf}})^2} + \frac{(t_{\text{dec}} - t)}{24(t_{\text{dec}} - t_{\text{inf}})} \right] + \left(\frac{T_{D1}}{4\pi^2 l_s^4}\right) \left[1 - \left(\frac{(t_{\text{dec}} - t_{\text{inf}})^2}{(t_{\text{dec}} - t_{\text{inf}})^2}\right)\right] \]
\[ \text{(28)} \]

In figure 2, we present the deceleration parameter in terms of time. As can be seen from this figure, deceleration parameter increases to higher positive values, turns over a maximum,
decreases and tends to zero at the end of deceleration epoch. Thus our model is consistent with previous predictions for deceleration era.

\[ N_{\text{sur}} - N_{\text{bulk}} \simeq E_{D1,t-t_{\text{dec}}} \simeq 14\pi^2 l_s^4 T_{D1} \left[ \frac{(t_{ac} - t_{\text{dec}}) - (t - t_{\text{dec}})}{(t_{ac} - t_{\text{dec}})} \right] \]

\[ N_{\text{sur}} \simeq E_{D3} \simeq E_{D3,\text{inf}} + E_{D3,\text{dec}} + (E_{D1,t_{ac}-t_{\text{dec}}} - E_{D1,t-t_{\text{dec}}}) + \]

\[ 5T_{D3} \int d^4 \sigma Tr(\Sigma^3_{a=0} \{ \frac{\chi^2}{4} (\sigma_a(\frac{1}{\sigma_1}))^2 \}) \simeq E_{D3,\text{inf}} + E_{D3,\text{dec}} + 15\pi^2 l_s^4 T_{D3} \left[ \frac{(t_{ac} - t)^5}{60} \right] + \]

\[ \frac{(t_{ac} - t_{\text{dec}})(t_{ac} - t)^4}{12} - \frac{(t_{ac} - t_{ac})^2(t_{ac} - t)^3}{9} = \frac{(t_{ac} - t_{\text{dec}})^3(t_{ac} - t)^2}{24} + \]

\[ \frac{(t_{ac} - t_{\text{dec}})^4(t_{ac} - t)}{2} \right] - 14\pi^2 l_s^4 T_{D1} \left[ \frac{(t - t_{\text{dec}})}{(t_{ac} - t_{\text{dec}})} \right] \]

(29)

In above equation, \( E_{D3,\text{dec}} = \frac{480\pi^2 l_s^4 T_{D3}(t_{\text{dec}}-t_{\text{inf}})^5}{72} \) is the amount of energy that D3-brane acquires during deceleration era, \( (E_{D1,t_{ac}-t_{\text{dec}}} - E_{D1,t-t_{\text{dec}}}) \) is the amount of energy of D1-brane that dissolves in D3-brane during late time acceleration and \( t = t_{ac} \) is the age of

FIG. 2: The deceleration parameter for deceleration era of expansion history as a function of the \( t \) where \( t \) is the age of universe. In this plot, we choose \( t_{\text{inf}} = 380000, t_{\text{dec}} = 40000000 \) yr, \( T_{D3} = 10000 \) and \( l_s = 0.1 \).

After a period of time, second D1-brane become close to our universe D3-brane, dissolves in it and leads to present phase of acceleration. Similar to previous epochs, we assume that \( \sigma_1 \sim \frac{1}{t} \) and \( \sigma_3 \sim t \), and use equations (13), (14) and (22) and also condition in (18), to calculate the energy of D1 and D3-branes and number of degrees of freedom on the holographic surface and bulk:
universe at the end of present acceleration epoch. Similar to previous epochs, difference between number of degrees of freedom on the holographic surface and bulk decreases with time and shrinks to zero at the end of late time acceleration ($t = t_{ac} \Rightarrow N_{sur} = N_{bulk}$). The Hubble parameter during this era can be obtained as:

$$H = \frac{4\pi}{l_p\sqrt{N_{sur}}}$$

Using this equation, we can calculate the deceleration parameter during present acceleration epoch in terms of time:

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \approx \sqrt{\frac{15\pi^2 l_s^4 T_{D3}(t_{ac} - t_{dec})^4}{60(t_{ac} - t_{dec})^4}} + \frac{(t_{ac} - t)^3}{6(t_{ac} - t_{dec})^3}$$

$$- \frac{(t_{ac} - t)^2}{6(t_{ac} - t_{dec})^2} - \frac{(t_{ac} - t)}{24(t_{dec} - t_{inf})} - 14\pi^2 l_s^4 T_{D1}\left(\frac{t - t_{dec}}{t_{ac} - t_{dec}}\right)$$

(31)

In figure 3, we show the deceleration parameter during late time acceleration era in terms of time. It is clear that deceleration parameter is negative at present stage which is a signature of acceleration. This result is in agreement with recent experimental data and thus our model works.

IV. SUMMARY AND DISCUSSION

We constructed the Padmanabhan idea in M-theory and argued that the birth and expansion of universe are controlled by the evolution of branes in extra dimensions. To this end, first, we obtained the relevant action for $N$ M0-branes by replacing Nambu-Poisson structure of two dimensional manifolds in D-branes by the structure of three dimensional one. At second stage, we compactified them on one circle and derived the action for $N$ D0-brane. We showed that $N$ D0-branes join to each other, grow and form a D5-brane. Then, we compactified this brane on two circles and obtained the action for our universe-brane,
two D1-branes and some extra energies that are created due to this compactifications. Next, we discussed that one of D1-branes which is more close to our universe-brane, dissolves in it, leads to an increase in difference between number of degrees of freedom on the holographic surface and bulk region and happening inflation era. After a short time, this D1-brane annihilates, the number of degrees of freedom on the boundary surface and bulk region become equal, inflation ends and deceleration epoch begins. During this era, extra energies that are produced due to compactification are the main causes of expansion. Finally, we argued that interaction of another D1-brane with our universe-brane leads to an inequality between degrees of freedom and occurring a new phase of acceleration.

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