Regular R–R and NS–NS BPS black holes

Matteo Bertolini\textsuperscript{a,1} and Mario Trigiante\textsuperscript{b}

\textsuperscript{a}International School for Advanced Studies ISAS-SISSA and INFN
Sezione di Trieste, Via Beirut 2-4, 34013 Trieste, Italy

\textsuperscript{b}Department of Physics, University of Wales Swansea, Singleton Park
Swansea SA2 8PP, United Kingdom

Abstract

We show in a precise group theoretical fashion how the generating solution of regular BPS black holes of $N = 8$ supergravity, which is known to be a solution also of a simpler $N = 2$ STU model truncation, can be characterized as purely NS–NS or R–R charged according to the way the corresponding STU model is embedded in the original $N = 8$ theory. Of particular interest is the class of embeddings which yield regular BPS black hole solutions carrying only R–R charge and whose microscopic description can possibly be given in terms of bound states of D–branes only. The microscopic interpretation of the bosonic fields in this class of STU models relies on the solvable Lie algebra (SLA) method. In the present article we improve this mathematical technique in order to provide two distinct descriptions for type IIA and type IIB theories and an algebraic characterization of $S \times T$–dual embeddings within the $N = 8, d = 4$ theory. This analysis will be applied to the particular example of a four parameter (dilatonic) solution of which both the full macroscopic and microscopic descriptions will be worked out.

e-mail: teobert@sissa.it, m.trigiante@swansea.ac.uk

\textsuperscript{1}Address after November 1st: NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark.
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1 Introduction

After the characterization of $D$–branes as R–R charged non–perturbative states of closed superstring theory [1], there have been successful microscopic computations of the entropy of some extremal and non–extremal black hole configurations which reproduced, at the microscopic level, the expected Beckenstein–Hawking behavior [2]–[6]. However, despite these encouraging results, an open problem, nowadays, is still to find a general recipe to give this correspondence based on first principles other than specific computations. Actually, while gravity seems to describe the quantum properties of all black holes in a unified but incomplete way (since it provides only a macroscopic description), string theory seems to give nice answers but losing the unified character of the properties of different black holes. It would be necessary to find a microscopic but still unified way of describing black hole physics in the context of string theory. In the last two years there have been in fact various attempts, especially within the AdS/CFT correspondence [7], to give an answer to this question relying on some general principles but a definite answer has not been found yet. For recent progress in this direction see for example [8]–[14].

A complementary strategy, which could be helpful in this respect, is to take advantage of the $U$–duality properties of BPS black holes (like for instance the invariance of the entropy under $U$–duality transformations) and use them to infer the common underlined structure of very different black holes sharing the same entropy. In particular, if one is able to give a precise correspondence between at least one macroscopic solution (ultimately the 5 parameters generating one) and its microscopic description, and is really able to act on it via duality transformations, one could derive the microscopic stringy description of any macroscopic solution. Even of those solutions (as the pure NS–NS ones) for which a microscopic entropy counting has not been achieved yet\textsuperscript{2}. The possibility of having a control, both at macroscopic and microscopic level, on all regular black holes with a given entropy could shed further light on the very conceptual basis of the microscopic entropy within string theory. This is the spirit this paper relies on.

Some time ago it has been shown that the generating solution of regular $N = 8$ black holes can be characterized as a solution within the $STU$ model [16]. Such a model is a $N = 2$ truncation of the $N = 8$ original theory [17]. Any regular BPS black hole solution within this latter model preserving 1/8 of the original supersymmetries (as any other regular BPS black hole solution should be: indeed the 1/2 and 1/4 solutions of $N = 8$, $D = 4$ supergravity have vanishing entropy, [18]) is, modulo $U$–duality transformations, a 1/2 BPS soliton of the $N = 2$ theory. This important result enables one to concentrate on the

\textsuperscript{2}See [15] for earlier qualitative results on the microscopic interpretation of NS–NS black hole entropy.
simpler structure of the \textit{STU} model, and then to generate more general (and complicated) solutions by \textit{U–duality} transformations. Without specification of the proper embedding in the mother \( N = 8 \) theory these \textit{STU} model solutions can be NS–NS, R–R or of a mixed nature. This distinction, from the 4 dimensional point of view, relies on the identification of the relevant (dimensionally reduced) 10 dimensional fields which enter dynamically in the solution. Of particular interest will be the solutions which carry only charge with respect to vector potentials deriving from the dimensional reduction of 10 dimensional R–R forms. In principle these BPS black holes can be described microscopically by means of a suitable system of D–branes compactified on an internal manifold (in the weak coupling limit). The choice of a particular point in the moduli space of the theory defining the values of the scalar fields of the solution at radial infinity, although uninfluential as far as the macroscopic properties of the black hole are concerned (\textit{no–hair} theorem), determines the particular D–brane configuration corresponding to the given solution in the opposite regime of the string coupling constant. We shall consider in what follows two main embeddings of the \textit{STU} model in the \( N = 8 \) theory: one in which the vector fields have a 10–dimensional interpretation in terms of NS–NS fields as opposed to the other in which the vector fields have a R–R origin. These two categories of embeddings are \textit{equivalence classes} with respect to the action of \( S \times T \) duality (to be defined more rigorously in the sequel and in particular in the appendix). Within the equivalence class of the R–R charged solutions, two particular representative embeddings will be considered: one in which the six real scalars of the \textit{STU} model (three dilatons and three axions) come only from the compactified components of the metric \((G_{ij})\) and an other in which the three axions derive from compactified components of the anti–symmetric tensor in 10 dimensions \((B_{ij})\). As we shall see, the two embeddings are related by \( T–duality \) along three orthogonal directions of the internal torus. The former will be described in the framework of type IIB theory and the latter in the framework of type IIA theory. Black hole solutions in the above type IIB embedding could be possibly interpreted microscopically in terms of a system of D3–branes at angles, while those in the type IIA embedding may be interpreted in terms of D0 and D4–branes with magnetic fluxes on the world volume of the latter (since the fields \( B_{ij} \) are coupled to the D4–branes in a gauge invariant combination with the flux density \( F_{ij} \)). The embeddings of the \textit{STU} model within the \( N = 8 \) theory and the consequent interpretation of its fields (scalar and vector) in terms of dimensionally reduced 10 dimensional fields are achieved using the powerful tool of solvable Lie algebras (SLA). This technique will be briefly reviewed at the beginning of section 2. In the subsections 2.1 and 2.2 the embeddings yielding NS–NS and R–R charged black holes will be defined, together with the action of \( S \times T \) duality on the fields. The mathematical details will
be postponed to the appendix. In section 3 we shall consider, as an explicit example, a four parameter solution of the STU model, which can be easily characterized, at the microscopic level, in terms of a bound state of D4 and D0–branes. The relation between macroscopic and microscopic parameters will be particularly simple and immediate.

2 The microscopic “nature” of the STU model

The 10 dimensional interpretation of the fields characterizing a solution depends on the embedding of the STU model inside the $N = 8$ theory. An efficient technique for a detailed study of these embeddings is based on the so–called Solvable Lie Algebra (SLA) approach. In the following we shall summarize the main features of this formalism while we refer to [19] for a complete review on the subject.

The solvable Lie algebra technique consists in defining a one to one correspondence between the scalar fields spanning a Riemannian homogeneous (symmetric) scalar manifold of the form $\mathcal{M} = G/H$ ($G$ being a non–compact semisimple Lie group and $H$ its maximal compact subgroup) and the generators of the solvable subalgebra $Solv$ of the isometry algebra $G$ defined by the well known Iwasawa decomposition:

$$G = \mathcal{H} \oplus Solv$$

(2.1)

where $\mathcal{H}$ is the compact algebra generating $H$. A Lie algebra $G_s$ is solvable if for some integer $n \geq 1$, its $n^{th}$ order derived algebra vanishes:

$$D^{(n)}G_s = 0 \quad \text{where} \quad D^{(k)}G_s \equiv \left[D^{(k–1)}G_s, D^{(k–1)}G_s\right]$$

Since the 70–dimensional scalar manifold $\mathcal{M}_{scal}$ of $N = 8$ supergravity has the above coset structure with $G = E_{7(7)}$ and $H = SU(8)$, it can be globally described as the group manifold generated by a solvable Lie algebra $Solv_7$, whose parameters are the scalar fields $\phi_i$:

$$Solv_7 = \{T_i\} \quad \phi_i \leftrightarrow T_i \quad i = 1, \ldots, 70$$

Indeed the solvable group generated by $Solv_7$ acts transitively on $\mathcal{M}_{scal}$. Considering the $N = 8, d = 4$ theory as the dimensional reduction on a torus $T^6$ of type IIA or IIB supergravity theories in $d = 10$, the solvable characterization of the NS–NS and R–R scalars in the four dimensional theory was worked out in [20, 21] and is achieved by decomposing

\footnote{In those papers the correspondence generators–scalars referred just to one of the two maximal theories. In the present paper we wish to give a precise geometrical characterization of the bosonic fields of the two theories and to find the relation between them.}
the solvable algebra $\text{Solv}_T$ with respect to the solvable algebra $\text{Solv}_T + \text{Solv}_S$, where $\text{Solv}_T$ generates the moduli space of the torus $\mathcal{M}_T = \text{SO}(6,6)/\text{SO}(6) \times \text{SO}(6)$ ($T = \text{SO}(6,6)$ being the classical $T$–duality group), and $\text{Solv}_S$ generates the two dimensional manifold $\text{SL}(2,\mathbb{R})/\text{SO}(2)$ spanned by the dilaton $\phi$ and the axion $B_{\mu\nu}$ ($S = \text{SL}(2,\mathbb{R})$ being the $S$–duality group of the classical theory). Since in the formalism outlined above $\text{Solv}_T$ is naturally parameterized by the moduli scalars $G_{ij}$, $B_{ij}$ ($i, j$ denoting the directions inside the torus), and $\text{Solv}_S$ by $\phi$ and $B_{\mu\nu}$, the complement of $\text{Solv}_T + \text{Solv}_S$ inside $\text{Solv}_7$ is a 32–dimensional subspace of nilpotent generators parameterized by the 32 R–R scalars $[\cdot]$. The general structure of the solvable algebra defined by the decomposition (2.1) is the direct sum of a subspace of the Cartan subalgebra CSA and the nilpotent space spanned by the shift operators corresponding to roots whose restriction to this Cartan subspace is positive:

$$\text{Solv} = \mathcal{C}_K \oplus \sum_{\alpha \in \Delta^+} \{E_{\alpha}\}$$

(2.2)

$\mathcal{C}_K$ is the non–compact part of the CSA and $\Delta^+$ is the space of those roots which are positive (non vanishing) with respect to $\mathcal{C}_K$.

In the case of the $N = 8$ theory in $d = 4$, $\text{Solv}$ is generated by the generators of the whole Cartan subalgebra of $\mathcal{E}_{7(7)}$ (which denotes the algebra generating the group $E_{7(7)}$, whose Cartan generators are non–compact) and all the shift operators corresponding to the positive roots of the same algebra. A suitable basis of Cartan generators is parametrized by the radii of the internal torus $\rho_i$ (i.e. of the cycles along orthogonal compact directions $x^i$) plus the dilaton $\phi$; the shift operators corresponding to the positive roots of $\text{Solv}_T$ are parametrized by the remaining $T^6$ moduli; finally, the shift operators corresponding to the positive spinorial roots of $\text{Solv}_T$ are naturally parameterized by the R–R scalars. The precise correspondence between the positive roots of $E_{7(7)}$ and type IIA and type IIB fields is summarized in the appendix. Although this correspondence is fixed by the geometry, in what follows we shall define algebraically two different classes of embeddings of the $\text{STU}$ model within the $N = 8$ theory which describe NS–NS or R–R charged solutions respectively. The embeddings within each class are related by an $S \times T$ transformation which preserves, as a general property, the NS–NS and R–R nature of the fields. This transformation is implemented on the scalar and vector fields through the action of the automorphisms $(\text{Aut}(S \times T))$ of the $S \times T$ duality group $\text{SL}(2,\mathbb{R}) \times \text{SO}(6,6)$ on the corresponding generators and weights, respectively. As far as the action on the

\[\text{Depending on whether the 32 representation of \text{SO}(6,6) has positive or negative chirality (i.e. 32}^+ \text{ and 32}^-\), described within two different constructions of the \mathcal{E}_{7(7)}\text{ algebra), the corresponding theory will be type IIA or type IIB, as we shall see.\]
generators is concerned, the inner automorphisms will amount to a “rotation” of the Cartan generators (separately acting on the radii and the dilaton) and a redefinition of the NS–NS scalars $G_{ij}$ ($i \neq j$) and $B_{ij}$ and of the R–R fields within the $32^+$ or $32^-$ (type IIA or type IIB respectively). The action of the outer automorphisms of $D_6$ (the Dynkin diagram of $SO(6,6)$) differs from the one described above in the fact that it exchanges the $32^+$ with the $32^-$ weights of $SO(6,6)$ and thus the type IIA theory with type IIB theory. This is consistent with the characterization of $T$–duality along a compact direction which we shall give in the sequel.

Let us now recall the main concepts on how to define the embedding of the $STU$ model from the reduction of the central charge matrix $Z_{AB}$ of the $N = 8$ theory to its skew–diagonal form (normal form), $Z^N$:

$$Z^N = \begin{pmatrix} Z_1 \epsilon & 0 & 0 & 0 \\ 0 & Z_2 \epsilon & 0 & 0 \\ 0 & 0 & Z_3 \epsilon & 0 \\ 0 & 0 & 0 & Z_4 \epsilon \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (2.3)$$

The $U$–duality invariant properties of an $N = 8$ BPS black hole solution are represented by the $SU(8)$ invariants which may be built out of $Z_{AB}$. There are 5 invariants intrinsically associated with the central charge matrix: the four norms of its skew–eigenvalues $Z_\alpha$ and the overall phase of the latter [30]. They represent the five duality–invariant quantities characterizing a 1/8 BPS black hole (i.e. its essential macroscopic degrees of freedom). The skew–eigenvalues of the central charge matrix do not depend on 16 of the 70 scalar fields (which are associated with the centralizer of $Z^N$ [14], to be defined in the sequel) while they depend on the 56 quantized charges $\vec{Q} = (p,q)$ and the remaining 54 scalars through the entries $Z_{AB}$. Since we are interested in the generating solution, we may start looking for a minimal consistent truncation of the $N = 8$ theory on which the four complex (eight real parameters) skew–eigenvalues of $Z_{AB}$ are independent parameters. This may be achieved by means of an $SU(8)$ gauge fixing which amounts to setting to zero all the central charge entries except the skew–diagonal ones. The above gauge fixing corresponds to a 48 parameter $U$–duality transformation on the 56 quantized charges and the 54 scalars in the expression of the central charge skew–eigenvalues, which makes $Z^N$ depend only on eight quantized charges $\vec{Q}^N = (p^N, q^N)$ (the normal form of the quantized charges) and on 6 scalar fields defining in turn the vector and the scalar content of an $STU$ model. The generating solution will be a solution within this $STU$ model, depending only on five of the eight quantized charges (resulting from fixing a residual $SO(2)^3$ gauge transformations acting on $Z^N$). After the $SU(8)$ gauge fixing
the entries of $Z^N$ will coincide with the skew–diagonal entries of $Z_{AB}$ and therefore the result of this procedure will depend on the initial basis in which the central charge matrix is written.

The embedding of the $STU$ model, resulting from the above gauge fixing, within the $N = 8$ theory can be characterized geometrically as follows [10], [18]. We define the centralizer of $\vec{Q}^N$ as its little group $G_C = SO(4, 4)$ contained inside $E_{7(7)}$ (i.e. $G_C \cdot \vec{Q}^N = \vec{Q}^N$). The maximal compact subgroup $H_C = SO(4)^2$ of $G_C$ is the centralizer of $Z^N$ and the homogeneous manifold $G_C/H_C$ is spanned by the aforementioned 16 scalar fields which the skew–eigenvalues of the central charge do not depend on. We define the normalizer of $\vec{Q}^N$ as the maximal subgroup $G_N = \left[ \text{SL}(2, \mathbb{R}) \right]^3$ of $G_N$ is the normalizer of $Z^N$ and the six dimensional homogeneous coset $G_N/H_N$ defines the scalar manifold of the $STU$ model we are interested in:

$$M_{STU} = \left[ \frac{\text{SL}(2, \mathbb{R})}{\text{SO}(2)} \right]^3$$

(2.4)

The scalar content of this model, in terms of the $N = 8$ scalars, is defined by embedding $\text{Solv}(M_{STU})$ into $\text{Solv}_7$, $\vec{Q}^N$ defines the quantized charges of the model, while as usual the real and imaginary parts of the skew eigenvalues $Z_\alpha$ of the central charge define the physical dressed electric and magnetic charges of the interacting $N = 2$ model.

As previously stressed, the above defined $SU(8)$ gauge fixing procedure, when applied to $Z_{AB}$ in different bases, yields $STU$ models embedded differently inside the original theory (the embedding of the algebras $G_N$ and $G_C$, generating $G_N$ and $G_C$, inside $E_{7(7)}$ would in general depend on the original basis of $Z_{AB}$). In the following subsections we shall define two relevant classes of embeddings of the $STU$ model in the $N = 8$ theory, in which the vector fields have a NS–NS or a R–R ten dimensional origin respectively, once the latter theory is interpreted as the low energy supergravity of a type II string on $T^6$.

2.1 The NS–NS $STU$ model

Let us consider the central charge matrix in a basis $Z_{\hat{A}\hat{B}}$ in which the index $\hat{A}$ of the 8 of $SU(8)$ splits in the following way: $\hat{A} = (a = 1, \ldots, 4; a' = 1', \ldots, 4')$, where $a$ and $a'$ index the $(4, 1)$ and $(1, 4')$ in the decomposition of the 8 with respect to $SU(4) \times SU(4)' = SU(8) \cap SO(6, 6)$ (this is the basis considered by Cvetic and Hull in defining their NS–NS 5–parameter solution, [22]). The group $SU(4) \times SU(4)'$ is the maximal compact subgroup of the classical $T$–duality group and decomposing with respect to it the 28 of $SU(8)$ will
define which of the entries of $Z_{\alpha\beta}$ correspond to R–R and which to NS–NS vectors (the former will transform in the spinorial of $SU(4)^2 \equiv SO(6)^2$):

$$28 \rightarrow (6, 1') + (1, 6') + (4, 4') \quad (2.5)$$

the $(6, 1') + (1, 6')$ part consists of the two diagonal blocks $Z_{ab}$ and $Z_{a'b'}$ and define the 12 NS–NS (complex) charges, while the spinorial $(4, 4')$ correspond to the off–diagonal block $Z_{aa'}$ and define the 16 (complex) R–R charges. The skew–diagonal elements which will define $Z_{NS}^N$ correspond then to NS–NS charges $(Z_{12}, Z_{34}, Z_{1'2'}, Z_{3'4'})$ and therefore the corresponding STU model will contain 4 NS–NS vector fields. Let us work out the embedding of $G_N$ and $G_C$ within $E_{7(7)}$. Let the simple roots of $E_{7(7)}$ be $\alpha_n$ whose expression with respect to an orthonormal basis $\epsilon_n$ is the following:

$$\begin{align*}
\alpha_1 &= \epsilon_1 - \epsilon_2 ; \\
\alpha_2 &= \epsilon_2 - \epsilon_3 ; \\
\alpha_3 &= \epsilon_3 - \epsilon_4 \\
\alpha_4 &= \epsilon_4 - \epsilon_5 ; \\
\alpha_5 &= \epsilon_5 - \epsilon_6 ; \\
\alpha_6 &= \epsilon_5 + \epsilon_6 \\
\alpha_7^\pm &= -\frac{1}{2} (\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5 \mp \epsilon_6) + \frac{\sqrt{2}}{2} \epsilon_7
\end{align*} \quad (2.6)$$

As previously stated, in the SLA formalism the Cartan subalgebra is parametrized by the six radii of the torus and the dilaton, in particular the orthonormal elements $H_i$, are multiplied by the scalars $\ln(\rho_{i+3}) \ (i = 1, \ldots, 6)$, see (A.4).

The group $H_C = SO(4)^2 \subset SO(6)^2 \subset SO(6, 6)$ consists of four $SU(2)$ factors acting separately on the blocks $(1, 2), (3, 4), (1'2'), (3'4')$ of the central charge matrix. The centralizer at the level of quantized charges $G_C$ on the other hand is the group $SO(4, 4)$ regularly embedded in $SO(6, 6)$. If the latter is described by the simple roots $\alpha_1, \ldots, \alpha_6$, a simple choice, modulo $S \times T$–duality transformations, for the Dynkin diagram of $G_C$ would be $\alpha_3, \alpha_4, \alpha_5, \alpha_6$. The solvable subalgebra of $G_C$ consists of NS–NS generators only. The algebra $G_N$, being characterized as the largest subalgebra of $Solv_7$ which commutes with $G_C$, is immediately defined, modulo isomorphisms, to be the $[SL(2, \mathbb{R})]^3$ algebra corresponding to the roots $\beta_1 = \sqrt{2} \epsilon_7, \beta_2 = \epsilon_1 - \epsilon_2$ and $\beta_3 = \epsilon_1 + \epsilon_2$. The scalar manifold of the corresponding STU model has the form:

$$M_{STU} = \frac{G_N}{H_N} = \frac{SU(1, 1)}{U(1)}(\beta_1) \times \frac{SO(2, 2)}{SO(2) \times SO(2)}(\beta_2, \beta_3) \quad (2.7)$$

\[5\] Let us recall that the Dynkin diagram of $E_{7(7)}$ is constructed by adding to the $D_6$ Dynkin diagram consisting of the simple roots $(\alpha_i)_{i=1,\ldots,6}$ the highest weight $\alpha_7^\pm$ of one of the spinorial representations $32^\pm$ of $SO(12)$. We call $E_{7(7)}^\pm$ the algebra obtained by attaching $\alpha_7^\pm$ to $\alpha_5$; $E_{7(7)}^\pm$ the algebra obtained by attaching $\alpha_7^-$ to $\alpha_6$. As previously anticipated the scalars of type IIA or type IIB theories parameterize respectively $Solv(E_{7(7)}^+)$ or $Solv(E_{7(7)}^-)$. 

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The reason why the above expression has been written in a factorized form is to stress the different meaning of the two factors from the string point of view: the group \( SU(1, 1)(\beta_1) \) represents the classical \( S \)-duality group of the theory and the corresponding factor of the manifold is parameterized by the dilaton \( \phi \) and the axion \( B_{\mu\nu} \). In the same way it can be shown that the second factor is parameterized by the scalars \( G_{44}, G_{55}, G_{45} \) and \( B_{45} \) and its isometry group acts as a classical \( T \)-duality, i.e. its restriction to the integers is the perturbative \( T \)-duality of string theory. This non–symmetric version of the \( STU \) model is the same as the one obtained as a consistent truncation of the toroidally compactified heterotic theory and therefore describes the generating solution also for this theory (the string interpretation of the 4 scalars spanning the second factor in \( M_{STU} \) is in general non generalizable to the heterotic theory). Therefore its corresponding microscopic structure should be given in terms of NS states (fundamental string and NS5–brane states).

2.2 The R–R \( STU \) model

Let us start with the central charge matrix \( Z_{AB} \) obtained from \( Z_{\hat{A}\hat{B}} \) through an orthogonal conjugation, such that the new index \( A \) of the 8 of \( SU(8) \) assumes the values \( A = 1, 1', 2, 2', \ldots, 4, 4' \), the unprimed and primed indices spanning the 4 of the two \( SU(4) \) subgroups previously defined. Let us now consider the decomposition of \( SU(8) \) with respect to its subgroup \( U(1) \times SU(2) \times SU(6) \) (which is the decomposition suggested by the Killing spinor analysis of the 1/8 BPS black holes) such that the 8 decomposes into a \( (1, 2, 1) \) labeled by \( i = 4, 4' \) and a \( (1, 1, \tilde{6}) \) labeled by \( \tilde{i} = 1, 1', \ldots, 3, 3' \). The 28 decomposes with respect to \( U(1) \times SU(2) \times SU(6) \) in the following way:

\[
28 \rightarrow (1, 1, 1) + (1, 1, 15) + (1, 2, 6) \quad (2.8)
\]

where the singlet represents the diagonal block \( Z_{ij} \), the \( (1, 1, 15) \) the diagonal block \( Z_{i\tilde{j}} \) and the \( (1, 2, 6) \) is spanned by the off diagonal entries \( Z_{i\tilde{j}} \). The skew–diagonal entries which survive the previously defined gauge fixing procedure and which thus enter the new normal form of the central charge \( Z_{RR}^N \) are now \( Z_1 = Z_{1,1'}, Z_2 = Z_{2,2'}, Z_3 = Z_{3,3'} \) and \( Z_4 = Z_{4,4'} \), which are R–R charges.

It is interesting to notice that these four (complex) charges are part of the set of 10 R–R (complex) charges entering the diagonal blocks \( (1, 1, 1) + (1, 1, 15) \). These charges can be immediately worked out either by directly counting the entries with mixed primed and unprimed indices \( (Z_{ab}) \) contained in these two blocks or, in a group theoretical fashion, by decomposing the \( (1, 1, 1) + (1, 1, 15) \) in \( (2.8) \) and the \( (4, 4') \) in \( (2.6) \) with respect to a common subgroup \( U(1) \times SU(3) \times SU(3)' \equiv [U(1) \times SU(6)] \cap [SU(4) \times SU(4)'] \). Both
the decompositions contain a common representation \((1, 1, 1') + (1, 3, 3')\) describing 10 R–R central charges. The 3 and 3′ are spanned by the values 1, 2, 3 and 1′, 2′, 3′ of the indices \(a\) and \(a'\) of the 4 and 4′ respectively. These charges correspond to the 1 + 9 vectors of an \(N = 2\) truncation of the \(N = 8\) theory with scalar manifold \(SU(3, 3)/U(3) \times SU(3)\). A truncation of this theory yields in turn the \(STU\) model corresponding to the normalizer of \(Z_{RR}^N\). The 4 complex charges in \(Z_{RR}^N\) will indeed depend on the 8 R–R quantized magnetic and electric charges \(\vec{Q}_{RR}\) and the 6 scalar fields of the new \(STU\) model. Therefore, differently to the previous defined class, in this case all gauge fields (and hence the corresponding charges) come from R–R 10 dimensional forms. The centralizer \(SO(4, 4)\) of \(\vec{Q}_{RR}^N\) is now no more contained inside \(SO(6, 6)\) and therefore its solvable algebra contains R–R generators as well. As a common feature of the truncations belonging to this class, the scalars entering each quaternionic multiplet split into 2 NS–NS and 2 R–R. Indeed the centralizer \(SO(4, 4)\) is now the isometry group of the manifold \(SO(4, 4)/SO(4) \times SO(4)\) describing 16 hyperscalars and therefore its solvable algebra has 8 R–R and 8 NS–NS generators.

In order to specify a particular truncation within the class one should define the simple roots of \(SO(4, 4)\) and of the isometry group \([SL(2, \mathbb{R})]^3\) of the \(STU\) model, which in turn determines \(Solv(M_{STU})\) and thus the scalar content of the model. An interesting possibility is the one where the system of simple roots for \(SO(4, 4)\) is chosen to be:

\[
\gamma_1 = \epsilon_1 + \epsilon_2; \quad \gamma_3 = \epsilon_3 + \epsilon_4; \quad \gamma_4 = \epsilon_5 + \epsilon_6 \\
\gamma_2 = \alpha^7
\]

(2.9)

Here the root \(\beta_1 = \sqrt{2}\epsilon_7 = \sum_{i=1}^4 \gamma_i + 2\gamma_2\) belongs to the \(SO(4, 4)\) root space. In the solvable language, since the Cartan generator and the shift operator corresponding to this root are parameterized by \(\phi\) and \(B_{\mu\nu}\) respectively, these two scalars are now part of a hypermultiplet, known as the universal sector. The isometry group of the \(STU\) model which commutes with the above defined \(SO(4, 4)\) centralizer is generated by a \([SL(2, \mathbb{R})]^3\) algebra which is regularly embedded in the isometry group \(GL(6, \mathbb{R})\) of the classical moduli space of \(T^6\) and defined by the following roots:

\[
\beta_1 = \epsilon_1 - \epsilon_2; \quad \beta_2 = \epsilon_3 - \epsilon_4; \quad \beta_3 = \epsilon_5 - \epsilon_6
\]

(2.10)

The scalar manifold of this \(STU\) model is now symmetric with respect to \(S, T, U\) since it is contained in the moduli space of \(T^6\) (its scalars are all NS–NS but there is no \(\phi\) and

\footnote{Notice that any solution within a Calabi–Yau compactification of type II string lies in this class. Indeed all the vector fields surviving the compactification come from R–R forms, both for type IIA and type IIB theories.}
\[ B_{\mu\nu} : \]

\[ \mathcal{M}_{STU} = \frac{G_N}{H_N} = \frac{SU(1,1)}{U(1)}(\beta_1) \times \frac{SU(1,1)}{U(1)}(\beta_2) \times \frac{SU(1,1)}{U(1)}(\beta_3) \]  

(2.11)

From table 2 we can read out the scalar content of this model: \( G_{45}, G_{67}, G_{89} \) and 3 radii (the latter being Cartan generators). The interesting feature of the above embedding is that all the excited scalar fields come from the metric tensor \( G \) rather than from the Kalb–Ramond field \( B \) (on the contrary, and this is a common feature of all embeddings falling in this class, all charges are R–R). Suppose we are working in the framework of type IIB theory, then this particular embedding could possibly be described in the weak string coupling regime by a system of D3–branes at angles.

Let us denote the compact directions of the torus are \( x^4, x^6, x^7, x^9 \) and the non–compact space–time coordinates are \( x^{0,1,2,3} \). It is interesting to consider an other embedding which is obtained from the one above by acting on it by means of a \( T \)–duality along the directions \( x^5, x^7, x^9 \) of the internal torus. In the SLA language this operation is characterized by three transformations \( \psi_{\tau_1}, \psi_{\tau_2}, \psi_{\tau_3} \) in \( Aut(S \times T) \) (see the appendix), where \( \tau_i \) are rotations on the root space obtained by multiplying the outer automorphism of \( D_6 (\alpha_5 \leftrightarrow \alpha_6) \) with suitable Weyl transformations on \( D_6 \). The automorphisms \( \psi_{\tau_i} \) act on the Cartan subalgebra by sending \( H_{\epsilon_4, \epsilon_5} \to -H_{\epsilon_4, \epsilon_5} \) and therefore the corresponding fields transform in the following way: \( \rho_k \to \rho_k^{-1}, \ (k = 5, 7, 9) \). The action of the automorphism \( \psi_{\tau} = \psi_{\tau_1 \tau_2 \tau_3} \) may be extended from the Cartan subalgebra to the shift operators \( E_\alpha \), and therefore on the corresponding axions, using the recipe (A.11): \( \psi_{\tau}(E_\alpha) \propto E_{\tau(\alpha)} \). Since

\[ \tau(\epsilon_1 - \epsilon_2) = \epsilon_1 + \epsilon_2 \]
\[ \tau(\epsilon_3 - \epsilon_4) = \epsilon_3 + \epsilon_4 \]
\[ \tau(\epsilon_5 - \epsilon_6) = \epsilon_5 + \epsilon_6 \]

the corresponding axions will transform as follows (see appendix):

\[ \psi_{\tau} : \begin{cases} 
G_{45} \to B_{45} \\
G_{67} \to B_{67} \\
G_{89} \to B_{89} 
\end{cases} \]  

(2.12)

A \( T \)-duality along an odd number of internal directions, which in our characterization amounts to inverting the sign to an odd number of \( \epsilon_i \), maps \( 32^- \) into \( 32^+ \) (see the appendix) that is type IIB theory into type IIA and this is consistent with known properties of \( T \)-duality. This new embedding \( SL(2, \mathbb{R})^3 \subset SU(3,3) \subset E^+_{7(7)} \), obtained by acting with the \( T \)-duality \( \tau \) in on the embedding of \( SL(2, \mathbb{R})^3 \) in \( E^-_{7(7)} \) defined by eqs. (2.10) and
is a type IIA embedding in which the axions come only from internal components of the antisymmetric tensor $B_{ij}$. It is natural to interpret black hole solutions within this embedding, for instance, in terms of systems of D0 and D4 branes with magnetic flux on the world volume of the latter, as anticipated in the introduction. The mathematical details associated with the two R–R embeddings described above (i.e. the one in type IIB and the $T$–dual in type IIA theory) will be dealt with in the appendix. These two embeddings provide a convenient framework in which to look for the generating solution and to give it a suitable D–brane interpretation in which a microscopic entropy counting is affordable. In the next section we shall fix on the type IIA R–R embedding and consider, just as an example, a known four parameter solution, which is a pure dilatonic one, and work out both its macroscopic and microscopic description, consistently with the 10 dimensional interpretation of its scalar and vector fields given in the appendix.

### 3 Example: a pure R–R solution and its microscopic description

Let us now consider a specific example, namely a four parameter solution within the $STU$ model. From the macroscopic point of view this solution is analogous to the one described in [23]. Other macroscopic solutions of the $STU$ model have been obtained, for instance, in [24, 25, 26, 27].

Let us very briefly remind the structure of the $STU$ model while a complete treatment has been carried out in [26, 27]. The $STU$ model is characterized by a $N = 2$ supergravity theory coupled to 3 vector multiplets whose scalars spans the manifold $M_{STU}$, eq.(2.4). The total number of scalar fields in the game is 6 ($z_i = a_i + ib_i$, $i = 1, 2, 3$) while the number of charges ($p^A, q_A$) is 8 (4 electric and 4 magnetic). In the framework of the $STU$ model, the local realization on moduli space $M_{STU}$ of the $N = 2$ supersymmetry algebra central charge $Z$ and of the 3 matter central charges $Z^i$ associated with the 3 matter vector fields are related to the $N = 8$ central charge eigenvalues in the following way:

$$Z = i Z_4, \quad Z^i = h^{ij} \nabla_j Z = \Pi^i_{\hat{i}} Z^\hat{i}$$

(3.1)

where $\Pi^i_{\hat{i}} = 2b_i(r)$ is the vielbein transforming rigid indices $\hat{i}$ (the one characterizing the eigenvalues of the $N = 8$ central charge in its normal form, eq.(2.3)) to curved indices $i$ (see [26], section 3, for details).

The killing spinor equations characterizing the BPS black hole solution translate into first order differential equations for the relevant bosonic fields once suitable ansätze are adopted. As a standard procedure, the vanishing of the gravitino transformation rule along killing spinor directions implies a condition for the metric while the vanishing of the
dilatino transformation rules translates into equations for the scalar fields. The ansätze for the metric $G_{\mu\nu}$ and the complex scalars $z^i$ are the following:

$$ds^2 = e^{2U(r)} dt^2 - e^{-2U(r)} d\vec{x}^2 \quad \left(r^2 = \vec{x}^2\right)$$

$$z^i \equiv z^i(r)$$

(3.2)

After some algebra one can see that the structure of the first order BPS equations turns out to be the following:

$$\frac{dz^i}{dr} = \mp 2 \left(\frac{e^{U(r)}}{r^2}\right) h^{ij*} \partial_j |Z(z, \bar{z}, p, q)|$$

$$\frac{dU}{dr} = \mp \left(\frac{e^{U(r)}}{r^2}\right) |Z(z, \bar{z}, p, q)|$$

(3.3)

which is a system of first order differential equations. Working out the geometric structure of the $STU$ model, the above system of equations can be made explicit in terms of the scalar fields and the quantized charges $(p^\Lambda, q_\Lambda)$ characterizing the model. This has been explicitly achieved in [26, 27] (where the same conventions and notations have been used), see in particular the appendices for explicit formulæ.

In this section we shall focus a particular (4–parameter) regular solution for which the microscopic description turns out to be particularly nice. On this solution the central charge eigenvalues $\{Z_\alpha\} \equiv \{Z_i, Z_4\}$ are pure imaginary. This condition fixes not only the $[SO(2)]^3$ symmetry of the model but also the overall phase $\phi$ of the four central charge eigenvalues ($\phi = 0 \, \text{mod} \, 2\pi$), yielding just four independent invariants $|Z_\alpha|$.

Since the central charge $Z_4$ is set to be imaginary (i.e. $Z$ real), the system of eqs. (3.3) may be rewritten in the simpler form:

$$\frac{dz^i}{dr} = \mp \left(\frac{e^{U(r)}}{r^2}\right) h^{ij*} \nabla_j \bar{Z}(z, \bar{z}, p, q) = \mp \left(\frac{e^{U(r)}}{r^2}\right) Z^i(z, \bar{z}, p, q)$$

$$\frac{dU}{dr} = \mp \left(\frac{e^{U(r)}}{r^2}\right) Z(z, \bar{z}, p, q)$$

(3.4)

It is possible moreover to show that the reality of $Z$ is consistent with the regularity of the solution provided we set $p^0 = 0$. The conditions $Z_i = -\bar{Z}_i$ (and therefore $Z^i = -\bar{Z}^i$, see eq.(3.1)) imply that the three axions are double–fixed: $a_{1,2,3}(r) \equiv a_{1,2,3}^f$. They require also that three electric quantized charges vanish, namely: $q_1 = q_2 = q_3 = 0$.

Hence the quantized charges left are $(q_0, p_1^1, p_2^3, p_3^3)$ and the system of first order differential equations our solution has to fulfill reduces considerably. Indeed the equations for
the dilatons and for \( \mathcal{U} \) decouple from the axions and may be solved independently:

\[
\begin{align*}
\frac{db_1}{dr} &= \pm \left( \frac{e^\mathcal{U}}{r^2} \right) \sqrt{-\frac{b_1}{2b_2b_3}} (p^1b_2b_3 - p^2b_1b_3 - p^3b_1b_2 + q_0) \\
\frac{db_2}{dr} &= \pm \left( \frac{e^\mathcal{U}}{r^2} \right) \sqrt{-\frac{b_2}{2b_1b_3}} (-p^1b_2b_3 + p^2b_1b_3 - p^3b_1b_2 + q_0) \\
\frac{db_3}{dr} &= \pm \left( \frac{e^\mathcal{U}}{r^2} \right) \sqrt{-\frac{b_3}{2b_1b_2}} (-p^1b_2b_3 - p^2b_1b_3 + p^3b_1b_2 + q_0) \\
\frac{d\mathcal{U}}{dr} &= \pm \left( \frac{e^\mathcal{U}}{r^2} \right) \frac{1}{2\sqrt{2}b_1b_2b_3} (p^1b_2b_3 + p^2b_1b_3 + p^3b_1b_2 + q_0)
\end{align*}
\]

The fixed values for the scalar fields (namely the values the scalars get at the horizon, \( \text{fix} \)) are:

\[
\begin{align*}
\frac{da_1}{dr} &= 0 = -(a_3b_2 + a_2b_3) p^1 + (a_3b_1 - a_1b_3) p^2 + (a_2b_1 - a_1b_2) p^3 \\
\frac{da_2}{dr} &= 0 = -(a_3b_1 + a_1b_3) p^2 + (a_3b_2 - a_2b_3) p^1 + (a_1b_2 - a_2b_1) p^3 \\
\frac{da_3}{dr} &= 0 = -(a_1b_2 + a_2b_1) p^3 + (a_1b_3 - a_3b_1) p^2 + (a_2b_3 - a_3b_2) p^1 \\
\text{Im} Z &= 0 = (a_3b_2 + a_2b_3) p^1 + (a_3b_1 + a_1b_3) p^2 + (a_2b_1 + a_1b_2) p^3
\end{align*}
\]

The fixed values for the scalar functions (namely the values the scalars get at the horizon, \( \text{fix} \)) are:

\[
\begin{align*}
b_1^{fix} &= -\sqrt{\frac{q_0p^1}{p^2p^3}} , & b_2^{fix} &= -\sqrt{\frac{q_0p^2}{p^1p^3}} , & b_3^{fix} &= -\sqrt{\frac{q_0p^3}{p^1p^2}} \\
a_1^{fix} &= 0 , & a_2^{fix} &= 0 , & a_3^{fix} &= 0
\end{align*}
\]

Introducing four harmonic functions as follows:

\[
H_I(r) = A_I + k_I/r \quad (I = 0, 1, 2, 3) \\
k_0 = \sqrt{2} q_0 , \quad k_i = \sqrt{2} p^i
\]

it is easy to see that the following ansätz for the \( b_i \) and the scalar function \( \mathcal{U} \):

\[
\begin{align*}
b_1 &= -\sqrt{\frac{H_0H_1}{H_2H_3}} , & b_2 &= -\sqrt{\frac{H_0H_2}{H_1H_3}} , & b_3 &= -\sqrt{\frac{H_0H_3}{H_1H_2}} \\
\mathcal{U} &= -\frac{1}{4} \ln (H_0H_1H_2H_3)
\end{align*}
\]

satisfies both the first and second order differential equations. Choosing the metric to be asymptotically flat and standard values for the dilatons at infinity, the four constants
$A_i$ are set to be all equal to 1. The solution, consisting of the three $b_i$, the double–fixed $a_i$ and $U$ is expressed in terms of 4 independent charges (and four harmonic functions): $q_0, p^1, p^2, p^3$. According to the ansätze (3.2) the metric has the following form:

$$ds^2 = (H_0H_1H_2H_3)^{-1/2} dt^2 - (H_0H_1H_2H_3)^{1/2} d\vec{x}^2$$  \hspace{1cm} (3.10)

and the macroscopic entropy, according to Beckenstein–Hawking formula, reads:

$$S_{macro} = 2\pi \sqrt{q_0 p^1 p^2 p^3}$$  \hspace{1cm} (3.11)

As far as the vector fields are concerned, their form in terms of the harmonic functions introduced above is analogous to the one in the solution of [23] and we shall not give it here. Let us now move to the microscopic description of the above solution. This will be easily obtained starting from the explicit expression of the $N = 8$ central charge eigenvalues. Comparing eq.s (3.3) and (3.5) one can see that the central charge eigenvalues at spatial infinity, when written in terms of the quantized charges, reads (in rigid indices):

$$Z_1 = \frac{i}{2\sqrt{2}} (q_0 - p^2 - p^3 + p^1)$$

$$Z_2 = \frac{i}{2\sqrt{2}} (q_0 + p^2 - p^3 - p^1)$$

$$Z_3 = \frac{i}{2\sqrt{2}} (q_0 - p^2 + p^3 - p^1)$$

$$Z_4 = \frac{i}{2\sqrt{2}} (q_0 + p^2 + p^3 + p^1)$$  \hspace{1cm} (3.12)

According to the previous discussion, if the $STU$ model is embedded in the full $N = 8$ theory as discussed in subsection 2.2, the microscopic description of the above solution can be given in terms of the intersection of four bunches of parallel D–branes. In particular, if we consider the type IIA embedding defined in the same subsection, the central charge $Z_4$ (which represents the $N = 2$ graviphoton dressed charge) and the matter charges $Z_i$ are related to the gauge fields coming from the 10 dimensional R–R 3–form $A_{MNP}$ (and its Hodge dual) coupled to D2 (and D4)–branes and from the R–R 1–form $A_M$ (and its Hodge dual) coupled to D0 (and D6)–branes. Our solution is hence described, at the microscopic level, as a 1/8 supersymmetry preserving intersection of 4 bunches of these D–branes. The fact that each of the central charge eigenvalues is real or pure imaginary (in our case they are all pure imaginary) implies that the solution is pure electric, that is it is not made of electromagnetic dual objects.

Let us be more precise and try to deduce the microscopic interpretation of our solution from the algebraic framework consistently built in [16],[27] and in the present paper.
(see the appendix). In appendix A of [26] the $Sp(8, \mathbb{R})$ representation of the $SL(2, \mathbb{R})^3$ generators that we have been using for our study of the $STU$ model is given. In particular, the Cartan generators have the form:

\[
\begin{align*}
    h_1 &= \frac{1}{2} \text{diag} (-1,1,-1,-1,1,1,1,1) \\
    h_2 &= \frac{1}{2} \text{diag} (-1,-1,1,1,1,1,1,1) \\
    h_3 &= \frac{1}{2} \text{diag} (-1,-1,1,1,1,1,1,1)
\end{align*}
\]

(3.13)

If we interpret these Cartan generators as corresponding (in the type IIA framework) to the roots $\epsilon_1 + \epsilon_2$, $\epsilon_3 + \epsilon_4$ and $\epsilon_5 + \epsilon_6$, so that we can interpret their diagonal values as the scalar product of these roots with 8 weights $W^{(\lambda)}$, the only set of weights on which all the three generators are non singular as in (3.13) is:

\[
\{W^{(1)} - , W^{(6)} - , W^{(7)} - , W^{(16)} - , W^{(29)} - , W^{(34)} - , W^{(35)} - , W^{(44)} - \}
\]

(3.14)

A possible correspondence is:

\[
\begin{align*}
    h_1 &= -\frac{1}{2} H_{\epsilon_1 + \epsilon_2} \\
    h_2 &= -\frac{1}{2} H_{\epsilon_3 + \epsilon_4} \\
    h_3 &= -\frac{1}{2} H_{\epsilon_5 + \epsilon_6} \\
    \{p^0, p^1, p^2, p^3\} &= \{W^{(29)} - , W^{(7)} - , W^{(16)} - , W^{(6)} - \} \\
    \{q_0, q_1, q_2, q_3\} &= \{W^{(1)} - , W^{(35)} - , W^{(44)} - , W^{(34)} - \}
\end{align*}
\]

(3.15)

Since our solution is charged only with respect to $\{q_0, p^1, p^2, p^3\}$ from table 3 we may read off the corresponding R–R fields: $A_\mu, A_{\mu 6789}, A_{\mu 4589}, A_{\mu 4567}$. The configuration of microscopic objects coupled with these forms consists in 3 bunches of orthogonal D4–branes ($N_1$, $N_2$, $N_3$, respectively) wrapped on the internal torus $T^6$ with $N_0$ D0–branes on top of them where the D4–branes are positioned in the following way:

The above configuration is $1/8$ supersymmetric and adding any number of $D0$ branes the number of preserved supersymmetries does not change, [29]. The previous analysis about the microscopic interpretation of the vector fields suggests a precise relation between $\{q_0, p^1, p^2, p^3\}$ and $\{N_0, N_1, N_2, N_3\}$ in the order. This relation can be easily derived by

\footnote{After a suitable change of basis by means of the matrix $M$ defined in [26].}

\footnote{Which is a general feature of the $Sp(8)_D$ representation of $SL(2, \mathbb{R})^3$.}
Table 1: The position of the $D4$ branes on the compactifying torus: for any given brane the directions labeled with $\times$ are Neumann while those labeled with $\cdot$ are Dirichlet.

|     | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|-----|-------|-------|-------|-------|-------|-------|
| $N_1$ | $\cdot$ | $\cdot$ | $\times$ | $\times$ | $\times$ | $\times$ |
| $N_2$ | $\times$ | $\times$ | $\cdot$ | $\times$ | $\times$ | $\times$ |
| $N_3$ | $\times$ | $\times$ | $\times$ | $\times$ | $\cdot$ | $\cdot$ |

writing the expression of the $E_7(7)$ quartic invariant, $J_4$, as in [30]:

$$J_4 = (|Z_1| + |Z_2| + |Z_3| + |Z_4|) (|Z_1| - |Z_2| - |Z_3| + |Z_4|) (|Z_1| + |Z_2| - |Z_3| + |Z_4|)$$

$$(-|Z_1| - |Z_2| + |Z_3| + |Z_4|) + 8|Z_1||Z_2||Z_3||Z_4| (\cos \phi - 1)$$

(3.16)

where, as well known, the entropy of the solution is $S = \pi \sqrt{J_4}$. In the case at hand $\phi = 0 \mod 2\pi$ and the last term in the above equation drops out (according to the fact that it is a four, rather than a five parameters solution). The above expression reduces to:

$$J_4 = s_0 s_1 s_2 s_3$$

(3.17)

where, using relations (3.12), it follows:

$$s_0 \equiv (|Z_1| + |Z_2| + |Z_3| + |Z_4|) = \sqrt{2} q_0$$

$$s_1 \equiv (|Z_1| - |Z_2| - |Z_3| + |Z_4|) = \sqrt{2} p_1$$

$$s_2 \equiv (-|Z_1| + |Z_2| - |Z_3| + |Z_4|) = \sqrt{2} p_2$$

$$s_3 \equiv (-|Z_1| - |Z_2| + |Z_3| + |Z_4|) = \sqrt{2} p_3$$

(3.18)

As noticed in [30], the charge vector basis we have chosen turns out to be the suitable one for the microscopic identification, as for reading off the values of the integers $N_I$ from the relations (3.12). First notice that the 4 dimensional charge of a wrapped $Dp$–brane is $Q_p = \hat{\mu}_p \cdot V_p / \sqrt{\alpha'}$ where $\hat{\mu}_p = \sqrt{2} \pi (2 \pi \sqrt{\alpha'})^{3-p}$ is the normalized $Dp$–brane charge density in ten dimensions. Provided the asymptotic values of the dilatons, which parameterize the radii of the compactifying torus and which has been taken to be unitary, it turns out that, in units where $\alpha' = 1$, the four dimensional quanta of charge for any kind of (wrapped) $Dp$–brane is equal to $\sqrt{2} \pi$. On the contrary, our quantized charges $(p_A, q_A)$ are integer valued. The entropy formula (3.11) is reproduced microsco pically by the above $D$–branes configuration, table if we have precisely $N_0 = q_0$, $N_1 = p_1$, $N_2 = p_2$, $N_3 = p_3$, 16
consistently with the result of our previous geometrical analysis. Indeed the microscopic entropy counting for this configuration has been performed in \[31\] and gives:

\[
S_{\text{micro}} = 2\pi \sqrt{N_0 N_1 N_2 N_3}
\]

which exactly matches expression (3.11). From the configuration in table II one can obtain, by various dualities, other four parameters solutions. For instance, \(T\)-dualizing on the whole \(T^6\), one obtains a configuration made of \(N_0\) D6–branes and 3 bunches of \((N_1, N_2, N_3)\) D2–branes on the planes \((x^4, x^5), (x^6, x^7), (x^8, x^9)\) respectively.

4 Discussion

The main aim of the present article was to define in a precise mathematical fashion a connection between the \textit{macroscopic} analysis of 1/8 BPS black hole solutions of \(N = 8, d = 4\) supergravity carried out in \([16, 18, 26, 27]\) and the \textit{microscopic} description of the subclass of these solutions carrying R–R charge in terms of D–branes. To this end it was necessary to single out in the \(U\)-duality orbit of 1/8 BPS black holes those charged with respect to R–R vector fields, once the \(N = 8\) theory is interpreted as the low–energy limit of type II superstring on \(T^6\). The first step in this direction was to describe group theoretically the embedding of a class of \(STU\) models yielding the generating solution of R–R charged 1/8 BPS black holes within the \(d = 4\) maximal supergravity. To achieve this we used the SLA techniques developed in \([20, 21]\) in order to characterize geometrically the R–R and the NS–NS ten dimensional origin of the fields in the \(N = 8, d = 4\) theory. We improved them in order to provide also two distinct descriptions of the \(N = 8, d = 4\) theory as deriving from type IIA or type IIB theories in ten dimensions. A SLA characterization of the effect of \(T\)-duality transformations on compact directions is also formulated in terms of the action of automorphisms of the \(S \times T\)-duality group \(SL(2, \mathbb{R}) \times SO(6, 6)\) on the SLA generating the scalar manifold of the theory. Besides characterizing the class of embeddings of the \(STU\) model describing R–R charged generating solutions, we defined a \(U\)-dual class of \(STU\) models describing NS–NS charged solutions in the same mathematical fashion. The \(U\)-duality relation between the two classes of \(STU\) models can be inferred from their embedding in the larger \(N = 8\) theory (this transformation is in the automorphism group of \(U = E_7(7)\) but not in \(Aut(S \times T) = Aut(SL(2, \mathbb{R}) \times SO(6, 6))\), since it does not preserve the R–R and NS–NS identities of the fields, while the models within each class are related by transformations in \(Aut(S \times T)\) )

Eventually we focused on two particular representatives of the R–R class of \(STU\)
models and interpreted their fields in terms of ten dimensional type IIA or type IIB fields. As an example a particular 4–parameter solution of this model was considered and a microscopic interpretation of it was given in terms D–branes (a configuration of $D_4$ and $D_0$–branes, in the framework of type IIA theory).

The utility of the mathematical apparatus constructed in the present article (whose details can be found in the appendix) is more general. Indeed it allows to characterize the bosonic sector of whatever model embedded in the $N = 8, d = 4$ theory in terms of dimensionally reduced fields of type IIA or IIB theories, and therefore to interpret microscopically its solutions. Moreover, a precise prescription is given as to how to act on a particular truncation of the $N = 8, d = 4$ theory by means of $S \times T$ duality (at the classical level). This method can be easily extended to describe also $U$–dual embeddings by considering the action of the whole $Aut(E_{7(7)})$ on the model.

Within the two $T$–dual $R$–$R$ embeddings of the $STU$ model discussed in subsection 2.2 it would be interesting to find the generating solution in a form which is easily interpreted in terms of bound states of D–branes, recovering as an example the two $T$–dual microscopic configurations described in [24]. One of these configurations consists of a system of $D_4$ and $D_0$–branes in type IIA theory analogous to the one represented in table I but with a magnetic flux switched on the world volume of one bunch of parallel $D_4$–branes (which would imply additional effective $D_2$ and $D_0$ charges,[33]). As already pointed out in the introduction, this flux would correspond in the solution to the presence of non–trivial axions coming from NS–NS $B$ field components (which would contribute to a non vanishing real part of the central charges). An embedding where to look for this solution is the type IIA embedding described in subsection 2.2. The type IIB configuration described in [32] consists of 4 sets of $D_3$–branes at angles and is obtained from the one described above by a $T$–duality along the internal directions $x^{5,7,9}$ (these angles being related to the flux in the dual configuration and consistent with the supersymmetry requirement, [29]). The corresponding solution has to be looked for in the type IIB embedding of section 2.2. In the limits in which the magnetic flux is sent to zero or the system of $D_3$–branes is set to be orthogonal [14], one recovers the four parameter solution described in the previous section (or its $T$–dual).

The importance of finding both a macroscopic and microscopic description of the generating solution relies in the fact that it would allow to have a precise control on both the macroscopic and microscopic structures of all regular stringy black holes related by $U$–duality transformations, and hence sharing the same entropy. Starting from a configuration for which a microscopic entropy counting is known (as for instance pure D–branes configurations) one can then have an entropy prediction and a description, both
at macroscopic and microscopic levels, of those configurations for which a microscopic
entropy counting is out of present reach (as for example pure NS–NS state configurations).
This could help in revealing the underlying common properties of very different black
holes sharing the same entropy and hence giving some insights into the basic properties
of stringy oriented microscopic entropy counting.

Although there exist in the literature some macroscopic 5 parameter generating solu-
tions, [35, 36, 27], the interpretation, at the microscopic level, of the parameters entering
these solutions is quite difficult, especially as far as the fifth one is concerned. Hence a sim-
ple and clear description of the generating solution, both at macroscopic and microscopic
level, is still missing. The completion of this program is left to a future work.

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Appendix A: SLA Description of $N = 8$, $d = 4$ from Type IIA and
type IIB Theories and S, T–dualities.

SLA of $\mathcal{E}_{7(7)}^\pm$ and the scalar fields:
As already stated in section 2, in the SLA representation of a theory, the scalar fields
are parameters of the solvable Lie algebra generating the scalar manifold (in most super-
gravities at the classical level the scalar manifold can be described as a solvable group
manifold). The $N = 8$ theory in four dimensions can be interpreted as the low energy
limit of type IIA or type IIB theories on $T^6$. Depending on the two interpretations we
shall give two different SLA descriptions of the scalar manifold which are consistent with
the geometric characterization of $T$–duality to be given in the sequel.

As anticipated in subsection 2.1 one may construct the $\mathcal{E}_{7(7)}$ algebra in two ways,
depending on whether the $D_6$ Dynkin diagram consisting of the roots $(\alpha_i)_{i=1,\ldots,6}$ in (2.6)
is extended by attaching the highest weight of the $32^+$ of $SO(6,6)$ ($\alpha^+_7$) to $\alpha_5$ or the
highest weight of the $32^-$ ($\alpha^-_7$) to $\alpha_6$. Thus we obtain two $\mathcal{E}_{7(7)}$ isomorphic algebras,
namely $\mathcal{E}_{7(7)}^\pm$. As previously stated we parameterize with the scalars of the $N = 8$ theory
deriving from a dimensional reduction of type IIA of type IIB theory the SLA of $\mathcal{E}_{7(7)}^+$ and
\( \mathcal{E}_{\gamma(t)} \) respectively.

The SLA is generated by the non-compact Cartan generators (in the case of maximal supergravities the whole Cartan subalgebra contributes to the SLA) and the shift operators corresponding to roots with positive restriction on the non-compact Cartan generators. As far as the common NS-NS sector is concerned, a suitable basis of non-compact Cartan generators will be parametrized by the radii of the torus and by the ten dimensional dilaton:

\[
\begin{align*}
\mathcal{C}_K(\text{IIB}) &= -\sum_{i=1}^{6} \sigma_i H_{\epsilon_i + \frac{\epsilon_7}{\sqrt{2}}} + \phi H_{\sqrt{2}\epsilon_7} \\
\mathcal{C}_K(\text{IIA}) &= -\sum_{i=1}^{5} \sigma_i H_{\epsilon_i + \frac{\epsilon_7}{\sqrt{2}}} - \sigma_6 H_{-\epsilon_6 + \frac{\epsilon_7}{\sqrt{2}}} + \phi H_{\sqrt{2}\epsilon_7} \\
\sigma_i &= \ln(\rho_{i+3})
\end{align*}
\]

where, as previously stated, \( \rho_k \) \((k = 4, \ldots, 9)\) are the radii of the internal torus along the directions \( x^k \). In the expressions (A.1) the overall coefficient of \( H_{\sqrt{2}\epsilon_7} \) is the four dimensional dilaton:

\[
\phi_4 = \phi - \frac{1}{2} \sum_{i=1}^{6} \sigma_i = \phi - \frac{1}{4} \ln(\det(G_{ij}))
\]

The strange non-orthonormal basis in (A.1) is defined by the decomposition of the \( U \)-duality group in \( d \) dimensions with respect to the \( U \)-duality group in \( d + 1 \) dimensions for maximal supergravities [20]:

\[
\mathcal{E}_{r+1(r+1)} \to O(1,1)_r + \mathcal{E}_{r(r)} \quad (r = 10 - d)
\]

where \( \mathcal{E}_{r(r)} \) is obtained by deleting the extreme root in the Dynkin diagram of \( \mathcal{E}_{r+1(r+1)} \) (on the branch of \( \alpha_1, \alpha_2, \ldots \)) and substituting it with the Cartan generator \( O(1,1)_r \) orthogonal to the rest of the Dynkin diagram. These \( O(1,1)_r \) define the basis in (A.1) and are naturally parametrized by \( -\sigma_{7-r} \).

As far as the the remaining NS-NS fields are concerned they parameterize the positive roots of \( SL(2, \mathbb{R}) \times SO(6,6) \). According to the definition of the ordering relation

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9 The criterion of positivity is defined for instance by fixing an orthonormal basis of non-compact Cartan generators with a certain order and ordering the restrictions of the roots to this basis in a lexicographic way: e.g. with respect to the system \( H_{\epsilon_1}, H_{\epsilon_2} \) the root \( \epsilon_1 - \epsilon_2 \) is positive while with respect to \( H_{\epsilon_2}, H_{\epsilon_1} \) the root \( \epsilon_2 - \epsilon_1 \) is positive.

10 They may be characterized in terms of the diagonal elements of the vielbein matrix \( V_k^i \) of the torus in a basis in which it is triangular and the square of their product gives therefore the determinant of the metric \( G_{ij} \).
among the roots with respect to the orthonormal basis \((H_{\epsilon})\), which determines the roots contributing to the SLA, one can have different equivalent SLAs, usually related by automorphisms of \(D_6\). It is natural to associate always the fields \(G_{ij}\) \((i \neq j)\) with the roots \(\pm (\epsilon_i - \epsilon_j)\) and \(B_{ij}\) with the roots \(\pm (\epsilon_i + \epsilon_j)\), since in the representation \(12\) of \(SO(6,6)\) in which the generators have the form:

\[
(M_{\Lambda \Sigma})_\Delta^\Gamma = \eta_{\Lambda \Delta} \delta_\Sigma^\Gamma - \eta_{\Sigma \Delta} \delta_\Lambda^\Gamma \\
\eta_{\Lambda \Delta} = \text{diag}(++ + + + + - - - -) \\
H_{\epsilon_i} = M_{ii+6}
\]

the shift operators corresponding to the former roots have a symmetric \(6 \times 6\) off–diagonal block, while those corresponding to the latter roots have an antisymmetric off–diagonal block. Using a lexicographic ordering with respect to the basis \((H_{\epsilon})\) the roots contributing to the SLA and the corresponding scalar fields are listed in table 2.

Finally the R–R fields, as already pointed out, parameterize the shift operators corresponding to the roots which are weights of the \(32^\pm\) of \(SO(6,6)\). These roots are:

\[
32^+: \alpha^+ = -\frac{1}{2} (\pm \epsilon_1 \pm \epsilon_2 \pm \epsilon_3 \pm \epsilon_4 \pm \epsilon_5 \pm \epsilon_6) + \sqrt{\frac{2}{2}} \epsilon_7 \\
\text{(odd number of “+” signs within brackets)}
\]

\[
32^-: \alpha^- = -\frac{1}{2} (\pm \epsilon_1 \pm \epsilon_2 \pm \epsilon_3 \pm \epsilon_4 \pm \epsilon_5 \pm \epsilon_6) + \sqrt{\frac{2}{2}} \epsilon_7 \\
\text{(even number of “+” signs within brackets)}
\]

Indeed the chirality operator \(\gamma\) is easily computed in terms of the product of the Cartan generators \((H_{\epsilon_i})_{i=1,\ldots,6}\) in the spinorial representation \((S(H_{\epsilon_i}))_{i=1,\ldots,6}\):

\[
\{\gamma_\Lambda, \gamma_\Sigma\} = 2\eta_{\Lambda \Sigma} \\
S(H_{\epsilon_i}) \equiv \gamma_i \gamma_{i+6}, \quad (i, 1, \ldots, 6) \\
\gamma = \gamma_1 \gamma_2 \cdots \gamma_{12} = -S(H_{\epsilon_1})S(H_{\epsilon_2}) \cdots S(H_{\epsilon_6})
\]

it is easy to check that \(\gamma\) is positive on the \(\alpha^+\) and negative on the \(\alpha^-\). A precise correspondence between the spinorial roots and scalar fields from type IIA and type IIB theories is again given in table 2.

**S, T–duality:**
The aim of the following discussion is to characterize the effect (at the classical level) of \(S\) and \(T\)–duality on the embedding of a theory in the \(N = 8\) one, from the point of view of the scalar fields, as the action of transformations in \(Aut(S \times T)\) on the SLA generating the scalar manifold.
Automorphisms of a semisimple Lie algebra $G$ are isomorphisms of the algebra on itself and can be *inner* if their action can be expressed as a conjugation of the algebra by means of a group element generated by the algebra itself, or *outer* if they do not admit such a representation (see for instance [37]). A generic automorphism may be reduced, through the composition with a suitable (nilpotent) inner automorphism, to an isometric mapping which leaves the Cartan subalgebra stable. Let us focus on the latter kind of transformations, which we shall denote by $\psi_\tau$. It can be shown that the restriction of the group $\{\psi_\tau\}$ to the Cartan subalgebra is isomorphic to the automorphism group of the root space $\Delta$ consisting of the transformations $\tau$ on the weight lattice leaving the Cartan–Killing matrix invariant (*rotations*). It can be shown that inner automorphisms $\psi_\tau$ correspond to $\tau$ in the Weyl group of $G$ while in the case of outer $\psi_\tau$, $\tau$ may be reduced, modulo Weyl transformations, to symmetries of the Dynkin diagram (permutations of the simple roots).

Conversely, given a rotation $\tau$ on $\Delta$, one may associate with it an automorphism $\tilde{\psi}_\tau$ on the whole $G$ whose action on its canonical basis reads:

$$\tilde{\psi}_\tau(H_\beta) = H_{\tau(\beta)} ; \tilde{\psi}_\tau(E_\alpha) \propto E_{\tau(\alpha)}$$

$$\alpha, \beta \quad \text{roots} \quad (A.7)$$

A general $\psi_\tau$ has the form $\psi_\tau = \tilde{\psi}_\tau \cdot \omega$, where $\omega$ is an automorphism leaving the Cartan subalgebra of $G$ pointwise fixed (these automorphisms are all inner [37]).

In the case in which $G = SL(2, \mathbb{R}) \times SO(6, 6)$ the rotation corresponding to an outer automorphism $\psi_\tau$ can be reduced (modulo Weyl transformations) to the only symmetry transformation of $D_6$, i.e. $\alpha_5 \leftrightarrow \alpha_6$, or equivalently $\epsilon_6 \rightarrow -\epsilon_6$. It can be shown in particular that rotations on the root space amounting to a change of sign of an odd number of $\epsilon_i$ ($i = 1, \ldots, 6$) define outer automorphisms. Since the automorphisms preserve algebraic structures, they will map solvable subalgebras into solvable subalgebras. Of course we do not expect all the automorphisms of $S \times T$ to be automorphisms of $E_{7(7)}$ since, for instance, the Dynkin diagram of the latter does not have symmetries. Indeed it is easy to check that outer automorphisms of $SO(6, 6)$ map $E_{\gamma,\epsilon}^{\pm}$ into $E_{\gamma,\epsilon}^{\mp} \subset E_{7(7)}$ (this derives from the fact that changing sign to an odd number of $\epsilon_i$ maps $\alpha^\pm$ into $\alpha^\mp$).

We characterize algebraically a $T$–duality transformation “*(large radius) $\leftrightarrow$ (small radius)*” along a compact direction $x^k$ ($k = 4, \ldots, 9$) as the action of an outer automorphism $\psi_\tau$ corresponding to $\tau : \epsilon_{k-3} \rightarrow -\epsilon_{k-3}$, and an $S$–duality transformation “*(strong coupling) $\leftrightarrow$ (weak coupling)*” as a $\psi_\tau$ such that $\tau : \epsilon_7 \rightarrow -\epsilon_7$.

Let us consider as an example a $T$–duality transformation along the direction $x^9$ and its effects on $\rho_9$ ($\sigma_6$) and the dilaton $\phi$ starting from type IIB fields. According to the
geometrical prescription given above:

\[
\left(\phi - \frac{\sigma_6}{2}\right) H\sqrt{\pi r} - \sigma_6 H e_6 = \left(\phi' - \frac{\sigma'_6}{2}\right) H\sqrt{\pi r} - \sigma'_6 H e_6
\]

\[
\sigma'_6 = -\sigma_6 \Rightarrow \rho'_9 = 1/\rho_9
\]

\[
\phi' = \phi - \sigma_6 = \phi - \ln(\rho_9)
\]  \hspace{1cm} (A.8)

where the primed fields are the corresponding type IIA fields and the last equation is the known transformation rule for the dilaton under \(T\)-duality along a compact direction (in the units \(\alpha' = 1\)). As far as the other fields are concerned, the action of this automorphism is to map the roots \(\epsilon_i \pm \epsilon_6\) into \(\epsilon_i \mp \epsilon_6\). If we extend the rotation \(\tau : \epsilon_6 \rightarrow -\epsilon_6\) to the whole Lie algebra using the simple recipe \([A.7]\) the fields \(G_{i9}\) and \(B_{i9}\) will be mapped (modulo proportionality constants \(c_{1,2}\) to be fixed) into \(G'_{i9}\) and \(B'_{i9}\) respectively:

\[
G_{i9}E_{\epsilon_i-\epsilon_9} + B_{i9}E_{\epsilon_i+\epsilon_9} = G_{i9}E_{\epsilon_i+(-\epsilon_9)} + B_{i9}E_{\epsilon_i-(-\epsilon_9)} = c_1 B'_{i9}E_{\epsilon_i+(-\epsilon_9)} + c_2 G'_{i9}E_{\epsilon_i-(-\epsilon_9)}
\]  \hspace{1cm} (A.9)

The transformation on the R–R fields, applying a similar rationale, can be read off table 2.

**Vector fields:**

As far as the vector fields are concerned, we refer to the conventions of \([16]\) in which the corresponding representation \(Sp(56)_D\) \([\pi]\) of \(E_7(7)\) was described in terms of 56 weights \(W^{(\lambda)}\) \((\lambda = 1, \ldots, 56)\), whose difference from the \textit{highest weight} \(W^{(51)}\) are suitable combinations of the simple roots with positive integer coefficients (the first 28 weights correspond to \textit{magnetic} charges, the last 28 to \textit{electric} charges). In the conventions introduced in this paper, depending on whether we consider \(E_7(7)\) (type IIA/IIB) we have two set of weights \(W^{(\lambda)}\). These weights provide a suitable basis also for the two representations \(28\) and \(\overline{28}\) in which the \(56\) decomposes with respect to \(SU(8)\): the \(28\) is generated by \(W^{(\alpha)}\) with \(\alpha = 1, \ldots, 28\) and the \(\overline{28}\) by \(W^{(\alpha+28)} = -W^{(\alpha)}\). We talk about representation \(56\) when we consider the quantized charges \((p^\alpha, q_\beta)\) and of representation \(28 + \overline{28}\) referring to the \textit{dressed} charges to be defined below. The weights \(W^{(\lambda)}\) can be naturally put in correspondence with the vector fields obtained from the dimensional reduction of the type IIA or IIB theory respectively. Both the first 28 magnetic charges and the last 28 electric charges decompose into a first set of 16 R–R charges (which contribute to a \(32^\pm\) of \(SO(6,6)\)) and a second set of 12 NS–NS charges. Representing these weights (as well as the \[\text{By } Sp(56)_D \text{ we denote the } 56 \text{ symplectic representation of } E_7(7) \text{ in which the Cartan generators are diagonal.}\]
roots for the scalar fields in table 2) in the basis of \((\epsilon_i)_{i=1,...,7}\) the correspondence weights ↔ vectors (or roots ↔ scalars) becomes natural and consistent with our characterization of \(S \times T\) duality. Indeed, as far as the R–R fields are concerned, the natural correspondence is between the inner indices of the dimensionally reduced form (which gives rise either to a scalar or to a vector) and the number and positions of the “+” signs multiplying the \((\epsilon_i)_{i=1,...,6}\) in the corresponding weight. In tables 2 and 3 this correspondence has been “nailed” down for a particular \(S \times T\)–duality gauge (so that the fields (weights) of IIB and IIA are related by a \(T\)–duality along the compact direction \(x^9\) (automorphism \(\epsilon_6 \rightarrow -\epsilon_6\)), making it possible to infer the transformation rules of the fields under a generic \(S \times T\) transformation.

We would like ultimately to be able to use table 3 in order to infer which kind of D–brane charges characterize a particular solution and from it infer the particular supersymmetric brane configuration which reproduces it (just as we have done for the relatively simple example of the four parameter solution). Table 3 has to be applied to the vector in the \(28 + \overline{28}\) of \(SU(8)\) consisting of the following dressed charges (i.e. the charges of the branes coupled to the moduli of the internal torus), which are the effective microscopic physical charges one could measure:

\[
\begin{pmatrix}
y^{\alpha}(\phi) \\
x_{\beta}(\phi)
\end{pmatrix} = \mathcal{C} \mathcal{L}(\phi) \mathcal{C} \begin{pmatrix} p^\alpha \\ q_\beta \end{pmatrix} = -\mathcal{L}^{-1}(\phi) \begin{pmatrix} p^\alpha \\ q_\beta \end{pmatrix} \tag{A.10}
\]

where we have used the notation of \([26,29]\): \(\phi\) is the point of the moduli space at radial infinity, \(\mathcal{L}(\phi)\) is the coset representative of the scalar manifold (solvable group element) in its \(Sp(56)_D\) representation, \(\mathcal{C}\) is the symplectic invariant matrix and \((p^\alpha, q_\beta)\) are the usual quantized charges \((\alpha, \beta = 1, \ldots, 28)\).

If we consider, as we did in the present paper, a theory embedded in the \(N = 8, d = 4\) one (i.e. a consistent truncation), then the embedding can be perturbative or non-perturbative depending on whether the vectors described in the Lagrangian of the truncation derive from the same forms as the 28 vectors appearing in the Lagrangian of the larger theory, or some of them derive from the dimensional reduction of the corresponding Hodge dual forms. From the geometrical point of view these two situations are represented respectively by the case in which the \(y^n\) \((n = 1, \ldots, n_v \ (# \ of \ vectors))\) span a space generated by a subset of the magnetic weights \(W(\alpha)\) only (and therefore the \(x_n\) are expressed in the basis of the electric weights \(W(\alpha + 28)\)), or by the case in which \(y^n\) is also expressed

\[\text{\footnotesize Footnote 12: For example the vector } A_{ijkl} \text{ corresponds to the weight } (1/2)(..+i..+j..+k..+l..).\]
\[\text{\footnotesize Footnote 13: In the general case also to the R–R scalars.}\]
\[\text{\footnotesize Footnote 14: If the vector on the right hand side of (A.10) was projected on the Young basis of the } 28 + \overline{28} \text{ of } SU(8) \text{ we would have obtained on the left hand side the central charge vector } (Z^{AB}, \overline{Z}_{AB}) \text{ [16].}\]

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in terms of part of the electric weights $W^{(\alpha + 28)}$. The correspondence between the electric and magnetic charges $(y^n, x_n)$ and the weights $W^{(\lambda)}$ may be inferred from the embedding of the SLA which generates the scalar manifold of the truncation, $\text{Solv}_{\text{trunc}}$, within the $\text{Solv}(\mathcal{E}_{7(7)}^\perp)$ of the larger theory (which defines which scalar fields are switched off in the process of truncation). Indeed, once the Cartan generators $H_i$ in $\text{Solv}_{\text{trunc}}$ are expressed in the $\text{Sp}(2n_v)_D$ representation in which they are diagonal, interpreting their diagonal elements as the scalar product of the vectors $\gamma_i$ and $2n_v$ of the $W^{(\lambda)}$s it is possible to characterize the embedding consistently in terms of vector fields as well.

This procedure was applied in section 3 for the type IIA embedding of the STU model described in subsection 2.2. From the form of the Cartan generators in the $\text{Sp}(8)_D$ we could give a consistent interpretation of the electric and magnetic charges $(x_n$ and $y^n$) in terms of the weights $W^{(1)}$ (eqs. (3.13)) and thus, applying table 3, we were able to tell which of the ten dimensional forms these charges were associated with. In the dilatonic solution discussed in section 3, the electric charge $x_0 = -q_0$, corresponds to the weight $W^{(1)}$ which, in the original $N = 8$ theory is magnetic and therefore the embedding is non-perturbative. The 3 magnetic charges $y^i$ coincide with $-p^i$ which in turn correspond to magnetic weights of the $N = 8$ theory (the one to one correspondence between the dressed charges $(y, x)$ and the quantized ones $(p, q)$ on our solution is related to the fact that the solution is dilatonic and that we are considering a square torus ($b_i = -1$)). If axions were switched on the correspondence between dressed and quantized charges would be less trivial (since the coset representative would not be diagonal) and table 3 could give the microscopic interpretation of the dressed charges only.

In the literature the vector of dressed charges $(y(\phi), x(\phi))$ in the $N = 8$ theory is also written in the following equivalent complex form:

$$x_{ij}(\phi) + iy_{ij}(\phi) = -\frac{1}{\sqrt{2}} \left(\Gamma^{AB}\right)_{ij} Z_{AB} \quad (A.11)$$

where $\left(\Gamma^{AB}\right)_{ij}$ is an $SO(8)$ rotation and the antisymmetric couple of indices $(ij)$ run in the $28$ of $SO(8)$.

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| IIA | IIB | $\alpha_{m,n}^+ (IIB/IIA)$ | $\epsilon_1$-components | $\alpha_2$-components |
|-----|-----|--------------------------|------------------------|---------------------|
| $A_9$ | $B_9$ | $a_{1,1}$ | $(0, 0, 0, 0, 0, 0, 1, 1, 1, 1)$ | $(0, 0, 0, 0, 0, 0, 1, 1, 1, 1)$ |
| $G_9$ | $G_9$ | $a_{1,2}$ | $(0, 0, 0, 0, 0, 0, 1, 1, 1, 1)$ | $(0, 0, 0, 0, 0, 0, 1, 1, 1, 1)$ |
| $A_8$ | $A_8$ | $a_{2,3}$ | $(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)$ | $(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)$ |
| $B_7$ | $B_7$ | $a_{1,1}$ | $(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)$ | $(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)$ |
| $G_7$ | $G_7$ | $a_{1,2}$ | $(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)$ | $(0, 0, 0, 1, 1, 1, 1, 1, 1, 1)$ |
| $G_9$ | $G_9$ | $a_{3,4}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_{7,8}$ | $A_{7,8}$ | $a_{1,4}$ | $(0, 0, 1, 1, 1, 1, 1, 1, 1, 1)$ | $(0, 0, 1, 1, 1, 1, 1, 1, 1, 1)$ |
| $A_7$ | $A_7$ | $a_{1,6}$ | $(0, 0, 1, 1, 1, 1, 1, 1, 1, 1)$ | $(0, 0, 1, 1, 1, 1, 1, 1, 1, 1)$ |
| $B_6$ | $B_6$ | $a_{1,1}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $G_6$ | $G_6$ | $a_{1,2}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $B_7$ | $B_7$ | $a_{2,3}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $G_7$ | $G_7$ | $a_{2,4}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_6$ | $A_6$ | $a_{1,9}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_{4,5}$ | $A_{4,5}$ | $a_{1,10}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $B_6$ | $B_6$ | $a_{1,1}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $G_6$ | $G_6$ | $a_{1,2}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $B_7$ | $B_7$ | $a_{2,3}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $G_7$ | $G_7$ | $a_{2,4}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_5$ | $A_5$ | $a_{1,12}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_{4,5}$ | $A_{4,5}$ | $a_{1,13}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_{4,5}$ | $A_{4,5}$ | $a_{1,14}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $B_6$ | $B_6$ | $a_{1,1}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $G_6$ | $G_6$ | $a_{1,2}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $B_7$ | $B_7$ | $a_{2,3}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $G_7$ | $G_7$ | $a_{2,4}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_5$ | $A_5$ | $a_{1,12}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_{4,5}$ | $A_{4,5}$ | $a_{1,13}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |
| $A_{4,5}$ | $A_{4,5}$ | $a_{1,14}$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ | $(0, 0, 1, 0, 0, 0, 0, 0, 0, 0)$ |

Table 2: The correspondence between the positive roots $\alpha_{m,n}^+$ of the U-duality algebra $E_{7(7)}^\pm$ and the scalar fields parameterizing the moduli space for either IIA or IIB compactifications on $T^6$. The notation $\alpha_{m,n}$ for the positive roots was introduced in 20.  

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| IIA | IIB | $W^{(\lambda)\pm}$ (IIB/IIA) | $\epsilon_i$-components: IIB/IIA |
|-----|-----|-------------------------------|----------------------------------|
| $A_2$ | $A_{0,2}$ | $W^{(1)}$ | $\uparrow (-1, -1, -1, -1 \pm 1, 0)$ |
| $A_{1,5,678}$ | $A_{0,5,678}$ | $W^{(14)}$ | $\downarrow (-1, 1, 1, 1, 1 \pm 1, 0)$ |
| $A_{4,478}$ | $A_{4,478}$ | $W^{(13)}$ | $\uparrow (1, -1, 1, 1, 1 \pm 1, 0)$ |
| $A_{4,578}$ | $A_{4,578}$ | $W^{(14)}$ | $\downarrow (1, 1, -1, 1, 1 \pm 1, 0)$ |
| $A_{4,456}$ | $A_{4,456}$ | $W^{(15)}$ | $\uparrow (1, 1, 1, -1 \pm 1, 0)$ |
| $A_{4,567}$ | $A_{4,567}$ | $W^{(16)}$ | $\downarrow (1, 1, 1, -1 \pm 1, 0)$ |
| $A_{4,567}$ | $A_{4,567}$ | $W^{(17)}$ | $\uparrow (1, 1, 1, -1 \pm 1, 0)$ |
| $A_{4,567}$ | $A_{4,567}$ | $W^{(18)}$ | $\downarrow (1, 1, 1, -1 \pm 1, 0)$ |
| $A_{4,567}$ | $A_{4,567}$ | $W^{(19)}$ | $\uparrow (1, 1, 1, -1 \pm 1, 0)$ |
| $A_{4,567}$ | $A_{4,567}$ | $W^{(20)}$ | $\downarrow (1, 1, 1, -1 \pm 1, 0)$ |
| $B_{1,4}$ | $B_{1,4}$ | $W^{(17)}$ | $(-1, 0, 0, 0, 0, 0)$ |
| $B_{4,5}$ | $B_{4,5}$ | $W^{(18)}$ | $(0, -1, 0, 0, 0, 0)$ |
| $B_{4,6}$ | $B_{4,6}$ | $W^{(19)}$ | $(0, -1, 0, 0, 0, 0)$ |
| $B_{7,7}$ | $B_{7,7}$ | $W^{(20)}$ | $(0, 0, 0, -1, 0, 0)$ |
| $B_{8,8}$ | $B_{8,8}$ | $W^{(21)}$ | $(0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(22)}$ | $(0, 0, 0, 0, 0, 1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(23)}$ | $(0, 0, 0, 0, 0, 1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(24)}$ | $(0, 0, 0, -1, 0, 0, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(25)}$ | $(0, 0, 0, -1, 0, 0, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(26)}$ | $(0, 0, 0, 0, -1, 0, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(27)}$ | $(0, 0, 0, 0, -1, 0, 0)$ |
| $B_{0,9}$ | $B_{0,9}$ | $W^{(28)}$ | $(0, 0, 0, 0, 0, 1, 0)$ |
| $A_{0,45678}$ | $A_{0,45678}$ | $W^{(29)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,479}$ | $A_{4,479}$ | $W^{(30)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{5,52}$ | $A_{5,52}$ | $W^{(31)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{6,6}$ | $A_{6,6}$ | $W^{(32)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,7}$ | $A_{4,7}$ | $W^{(33)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,459}$ | $A_{4,459}$ | $W^{(34)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,460}$ | $A_{4,460}$ | $W^{(35)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,47}$ | $A_{4,47}$ | $W^{(36)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,45}$ | $A_{4,45}$ | $W^{(37)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,48}$ | $A_{4,48}$ | $W^{(38)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,45}$ | $A_{4,45}$ | $W^{(39)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,46}$ | $A_{4,46}$ | $W^{(40)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $A_{4,47}$ | $A_{4,47}$ | $W^{(41)}$ | $(-1, -1, -1, -1, -1, -1, 0)$ |
| $B_{1,4}$ | $B_{1,4}$ | $W^{(42)}$ | $(1, 0, 0, 0, 0, 0, 0)$ |
| $B_{4,5}$ | $B_{4,5}$ | $W^{(43)}$ | $(0, 1, 0, 0, 0, 0, 0)$ |
| $B_{4,6}$ | $B_{4,6}$ | $W^{(44)}$ | $(0, 1, 0, 0, 0, 0, 0)$ |
| $B_{7,7}$ | $B_{7,7}$ | $W^{(45)}$ | $(0, 0, 1, 0, 0, 0, 0)$ |
| $B_{8,8}$ | $B_{8,8}$ | $W^{(46)}$ | $(0, 0, 0, 1, 0, 0, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(47)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(48)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(49)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(50)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(51)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(52)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(53)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(54)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(55)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |
| $G_{0,9}$ | $G_{0,9}$ | $W^{(56)}$ | $(0, 0, 0, 0, 0, -1, 0)$ |

Table 3: Correspondence between the weights $W^{(\lambda)\pm}$ of the 56 of $\mathcal{E}_7^{\pm}$ and the vectors deriving from the dimensional reduction of type IIA and type IIB fields.