An Adaptive Aeromagnetic Compensation Method Based on Local Linear Regression

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Abstract. Aeromagnetic compensation plays an important role in airborne magnetic survey to eliminate the magnetic interference from the aircraft. However, the aeromagnetic compensation methods at present still cannot suppress the interference to the extremely low noise level of a high-sensitivity scalar magnetometer due to the common assumption about the time-invariation and the linearity of the Tolles-Lawson model depicting the aircraft magnetic interference. In this paper, an adaptive method based on local linear regression is proposed to improve the precision of aeromagnetic compensation. Instead of only using the whole historical calibration data all at once to estimate the model coefficients in advance as the traditional approach does, the proposed method calculates the coefficients in real-time by local linear regression during an aeromagnetic survey, using not only the historical calibration data but also the online measuring data, and both of them are required to be similar in a limited time window. The measured data were used to test the proposed method and the experimental results demonstrated its efficiency.

1. Introduction
Aeromagnetic compensation is essential to an airborne high-sensitivity scalar magnetometer to remove the magnetic interferences caused by the aircraft and its motions[1]. Generally, this kind of magnetic interferences cannot be removed by filtering because their bands often overlap with the useful magnetic signals of interest. Hence, in aeromagnetic compensation, the magnetic interferences are represented by a mathematical model with a set of unknown coefficients and once the coefficients are determined, the magnetic interference can be evaluated and eliminated in real-time during an aeromagnetic survey. At present, the Tolles-Lawson equation[2] is the most widely used model and many methods have been proposed to estimate its coefficients. For determining the coefficients, the training data including the scalar and vector magnetic field signals are recorded at first during a so-called calibration flight with the aircraft performing rolls, pitches and yaws at four cardinal headings[3], and then are used to establish a linear equation group to estimate the coefficients of the Tolles-Lawson model[4]. In practice, the estimated coefficients will remain unchanged throughout an actual aeromagnetic survey.

Researchers have been working on improving the accuracy of the estimated coefficients in different aspects, such as dealing with the multicollinearity of the linear equation group for obtaining the coefficients[5-10], designing a better filter to pre-process the signals[11][12][13], and extending the
conventional model to cover more influential factors[14][15], etc. However, the noise level of the aeromagnetic compensated result is still usually much higher than the measurement precision of a high-sensitivity scalar magnetometer[16], which can be explained below.

Given the complexity of the electromagnetic environment around the airborne magnetometer, e.g., various magnetic sources[17][18], non-smooth aircraft motions[3] and hysteresis effects[19], it can be inferred that a single model with a limited set of fixed coefficients is difficult to describe all the signature of the magnetic interference with an extremely high accuracy. However, designing a more complex model for the magnetic interference cannot help to solve the problem. Firstly, the problem of incomplete sampling during the calibration flight has led to the Tolles-Lawson model to be ill-conditioning[3] and it will persist in a more complex model. Secondly, the coefficients are inherently time-varying with the earth’s magnetic field[5]. Thirdly, it is also well known that an overly complex model does not necessarily guarantee a good generalization ability[20], which will limit its application in practice. In addition, since using the whole calibration data all at once to obtain only one set of the model coefficients, the traditional approach can be just taken as an overall optimal solution on average. In other words, some latent local optimal compensated results during an aeromagnetic survey are inevitably missed by the traditional approach.

According to the above analysis, we propose an adaptive aeromagnetic compensation method based on local linear regression in this paper. The coefficients to evaluate the current interference are updated in real-time using a limited training data set which is also built on-line with the signals from both the historical calibration flight and the current carrying-out survey flight. In addition, the criteria for determining how to select the appropriate signals are given. The measured data are used to test the proposed method and the experimental results proved its effectiveness. Finally, we discussed the limitations of the proposed method and the possible direction of improvement.

2. Background

2.1. The Time-Variation of the Tolles-Lawson Model

Most of the traditional methods are based on the Tolles-Lawson model, which takes the aircraft magnetic interference as the vector superposition of the permanent, induced and eddy-current source fields on the aircraft and can be written as below:

\[ \mathbf{H}_f = \mathbf{H}_{\text{per}} + \mathbf{H}_{\text{inc}} + \mathbf{H}_{\text{edd}}. \]

For an airborne scalar magnetometer, the intensity of the measured aircraft magnetic interference is given by the projection of \( \mathbf{H}_f \) on the direction of the total field vector. Given the intensity of the geomagnetic field is much larger than the interference, in practice, the directions of the total field and the geomagnetic field are often assumed to be the same. This means that the observed value of the aircraft magnetic interference is also related to the angle between the geomagnetic field vector and the interference vector. For instance, the intensity of the measured aircraft magnetic interference will take a very small value if the angle is approximately equal to 90°. In fact, the angle is not a constant and varies with the motion of the aircraft in the geomagnetic field. However, it is not reflected explicitly in the current form of the Tolles-Lawson model as below:

\[ H_f = \sum_{i=1}^{3} p_i u_i + \sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij} u_i u_j + \sum_{i=1}^{3} \sum_{j=1}^{3} b_{ij} u_i \hat{u}_j \]

where \( u_i \) and \( \hat{u}_j \) denote the direction cosines and their derivatives of the angles formed by the geomagnetic field vector and the axes of the aircraft, and \( p_i, a_{ij}, b_{ij} \) denote the coefficients to be estimated. Since the angle between the geomagnetic field vector and the interference vector is difficult to be determined, it is necessary and reasonable to take it as one of the factors leading to the time-variation of the coefficients.
Not only does it have a direct effect on the induced and the eddy-current fields\cite{13}, but the varying geomagnetic field causes the time-variation of the model in another way. Generally, the coefficients are estimated by solving the equation:

\[
\mathbf{bpf}(\mathbf{H}_T) = \mathbf{bpf}(\mathbf{A})\eta
\]  

(3)

where \(\mathbf{H}_T\) is the column vector of the measured total field intensity signal, \(\mathbf{A}\) is the matrix with its columns consisting of the direction cosines and their derivatives according to (2), \(\mathbf{bpf}\) denotes a band-pass filter to extract the component related to the aircraft manoeuvres, and \(\eta\) is the corresponding coefficient vector. Besides, since

\[
\mathbf{bpf}(\mathbf{H}_T) = \mathbf{bpf}(\mathbf{H}_T) + \mathbf{bpf}(\mathbf{H}_E)
\]  

(4)

where \(\mathbf{H}_E\) denotes the geomagnetic field signal, it is assumed implicitly in (3) that the band-pass filtered result contains no geomagnetic field signal, i.e.,

\[
\mathbf{bpf}(\mathbf{H}_E) = 0.
\]  

(5)

However, the equation (5) does not always hold in practice due to the geomagnetic gradient. The authors have proposed a method to model \(\mathbf{bpf}(\mathbf{H}_E)\) as a linear function of the altitude, latitude and longitude of the aircraft and extended the traditional Tolles-Lawson model\cite{13}. However, as the geomagnetic gradient changes in different flight areas, the estimated gradient coefficients will become ineffective.

2.2. Analysis of the Problem from the Perspective of Regression

The estimation of the Tolles-Lawson model’s coefficients can be turned into a classical problem of solving over-determined equations, or linear regression. Thus, the principle and basic theory of linear regression can be adopted to have an insight into the procedure of estimating the coefficients. For most methods, the optimization goal is to select a coefficient vector \(\hat{\eta}\) to minimize the Euclidean distance as shown below:

\[
\hat{\eta} = \arg \min_{\eta} \| \mathbf{bpf}(\mathbf{H}_T) - \mathbf{bpf}(\mathbf{A})\eta \|_2.
\]  

(6)

By rewriting \(\mathbf{bpf}(\mathbf{A})\) as \(\mathbf{X}\) and \(\mathbf{bpf}(\mathbf{H}_T)\) as \(\mathbf{Y}\), according to the least-square method, the solution of (6) can be given as

\[
\hat{\eta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}.
\]  

(7)

The corresponding estimation error can be evaluated with the distance between \(\mathbf{Y}\) and its orthogonal projection on the column space of the matrix \(\mathbf{X}\) as below:

\[
\epsilon = \| \mathbf{Y} - \mathbf{X}\hat{\eta} \|_2.
\]  

(8)

However, the rank deficiencies of the matrix \(\mathbf{bpf}(\mathbf{A})\) make it more complex in practice than that as depicted in (6)-(7). The reason is that the data recorded during the calibration flight is incomplete for determining the coefficients unless the aircraft can fly and carry out rolls, pitches and yaws with its back upside-down\cite{3}. Besides, the inherent similarity among the columns of the matrix \(\mathbf{A}\) is another source causing this problem. Many methods have been proposed to deal with this problem but in fact it can hardly be resolved completely. It can be inferred that the actual estimation error will not be less than \(\epsilon\). In general, the data recorded during the calibration flight are used all at once to solve (6) but only one set of coefficients are generated, which means that there is a strong assumption about the calibration data. In light of this, the estimated coefficients can be considered as the overall optimal solution on average. As is shown in Figure 1, the estimation error (or the residual aircraft magnetic interference) curve obtained by the recursive least-square method has fluctuations of different amplitudes throughout the range and many of them have larger amplitudes than others. Therefore, when the coefficients are applied to the calculation of the aircraft interference in an aeromagnetic survey, it can be inferred that good compensation results cannot always be guaranteed due to the deviation of the signatures of the
measured data from those of the calibration data. For instance, the time-variation of the Tolles-Lawson model discussed above will change the signatures of the measured data. Based on the above analysis, we will try to propose an adaptive aeromagnetic compensation method in this paper to obtain potential local optimal results.

![Figure 1](image.png)

Figure 1. The band-pass filtered total field signal (red) and the corresponding estimating error (blue) obtained by solving (6) with the recursive least-square method.

3. Methodology

3.1. The Framework of the proposed method

In order to perform aeromagnetic compensation in an adaptive way, the relationship between the interference in the signal from the high-sensitivity scalar magnetometer and the corresponding reference signals from other sensors such as the three-axis flux magnetometer, the altimeter and the GPS receiver, should be evaluated in real time. In practice, this relationship is often represented as a function between the interference and the reference signals, such as the Tolles-Lawson model. It means that the coefficients of this function should be calculated online during an aeromagnetic survey, and therefore the training data set used to estimate the coefficients should also be determined in real time. After that, the interference at the current sampling point can be evaluated and eliminated. Figure 2 illustrates the basic framework of the proposed method.

The reference signals include the Tolles-Lawson channels, i.e., the columns of the matrix $\mathbf{A}$, as well as the optional latitude, longitude and altitude of the aircraft. The primary signals are the scalar total field signals, which herein have been compensated preliminarily using the coefficients obtained with the traditional method, but probably still contain residual interferences. The proposed method is designed to eliminate the residual interferences in an adaptive manner.

As shown in Figure 2, the current measured data to be processed is specified by a time window with a finite length, which is further divided into two sub-windows, namely the Neighbour Window from $T_{k-L}$ to $T_{k-1}$ and the Host Window from $T_k$ to $T_{k+L}$ respectively. The current sampling point is defined as the first one $T_k$ in the host window. The History Data collected from the historical calibration flight is taken as a sample database for locating the segments of the signals most similar to those in the host window. The Picked Data block consisting of the signals in the Neighbour Window and those selected from the History Data will be used to calculate the coefficients at the current sampling point.
Consequently, the corresponding interferences in the Host Window can be predicted. The compensated results are obtained by subtracting the predicted interferences from the primary signals in the Host Window. The Host Window and the Neighbour Window will be updated with new coming data, and the operations above will execute in a new loop.

Figure 2. The framework of the proposed adaptive aeromagnetic compensation method. The small squares of different colours corresponding to the primary signals and reference signals constitute the larger data block, denoted as History Data, Picked Data and Real-Time Data, respectively. In these data blocks, the squares in the same column are obtained at the same sampling time $T_i (i \in N)$ and the squares in the same row are generated from the same channel $CH_j (j = 0 \cdots m)$. The estimated coefficients are indicated specifically with $C_j (j = 1 \cdots m)$. The processing steps are identified by a set of sequential numbers.

3.2. Picking the Appropriate Data to Estimate the Coefficients
According to the local regression theory, the estimated function is usually fitted by using the observations close to the target sampling point. However, this is not suitable for the current situation. In an actual aeromagnetic survey, not only the aircraft magnetic interference, but also the magnetic anomaly signals of interest to be detected are probably related to the reference signals and will have an impact on the estimation process of the coefficients. As a result, the magnetic anomaly signals will be weakened due to being mistaken as the magnetic interference. This can be demonstrated informally with (7) and (8) by assuming $Y$ contains the magnetic anomaly signals. It means that in adaptive aeromagnetic compensation the data for estimating the coefficients should be treated more prudently.

In this paper, the Picked Data block for calculating the coefficients consists of the signals selected from the history calibration data set and those from the Neighbour Window. Generally, the primary signals in the History Data are believed to contain no magnetic anomaly signals. The similarity between the selected signal segments from the History Data and the signals in the Host Window needs to be minimized and herein the similarity measure is given by the following matrix 2-norm:

$$d_{kp} = \|bpf(W_k) - bpf(W_p)\|_2$$

(9)

where $W_k$ is a $m \times (E + 1)$ matrix with each row containing the whole sample points from the
corresponding reference channels in the Host Window and \( \mathbf{W}_p \) is also a \( m \times (E + 1) \) matrix with each row containing the sample points between \( T_p \) and \( T_{p+E} \) from the corresponding reference channels in the History Data. For \( p \in \{s, s+1, \cdots q - E\} \), if \( d_{kp} \) takes the minimal value then the corresponding signal segments from \( T_p \) to \( T_{p+E} \) will be chosen. After that, The Picked Data will be passed through the band-pass filter \( \mathbf{bpf} \) first and subsequently the recursive least-square method is applied to estimate the coefficients. In practice, the coefficients obtained by the traditional method can be taken as the initial values for the recursive process. Once the current compensation process is finished, the Neighbour Window and the Host Window will move forward by one sampling point.

3.3. Compensating with Different Weights

In general, the magnetic interference at the current sampling point is evaluated using the estimated coefficients according to (2). In this way, the interferences in the Host Window can be given by

\[
\mathbf{\beta}_k = (\mathbf{bpf}(\mathbf{W}_k))^T \mathbf{\tilde{h}}_k
\]  

where \( \mathbf{\tilde{h}}_k \) is the current estimated \( m \times 1 \) coefficient vector, \( \mathbf{\beta}_k \) is the estimated \( (E + 1) \times 1 \) interference vector corresponding to the primary signal in the Host Window. Thus, the compensated result can be given by

\[
\mathbf{r}_k = \mathbf{P}_k - \mathbf{\beta}_k
\]  

where \( \mathbf{P}_k \) is the \( (E + 1) \times 1 \) primary signal vector in the current Host Window and \( \mathbf{r}_k \) is the compensation result vector.

However, since the time spans from each sampling point in the Host Window to the Neighbour Window are different, the credibility of \( \mathbf{\tilde{h}}_k \) with respect to the different sampling points are not identical. Herein, the sampling point corresponding to a narrower time span is assumed to be more inclined to accept \( \mathbf{\tilde{h}}_k \). Therefore, a weight vector with its elements monotonically decreasing is introduced into (11)

\[
\mathbf{r}_k = \mathbf{P}_k - \omega \mathbf{\beta}_k
\]  

where \( \mathbf{I} \) is an \( (E + 1) \times (E + 1) \) identity matrix and \( \omega \) is the introduced \( (E + 1) \times 1 \) weight vector by defining its \( s \)th element \( \omega(s) \) as below:

\[
\omega(s) = e^{-s/E}
\]  

where \( s \in \{1, 2, \cdots E + 1\} \). Obviously, the function \( \omega(s) \) is convex downward in its definition domain, meaning the effectiveness of \( \mathbf{\tilde{h}}_k \) is dropping quickly at the initial stage and then slowly with the time spans becoming wider.

4. Experiments and Analysis

4.1. Environmental Conditions

In practice, it is well known that the satisfactory result of aeromagnetic compensation depends on not only the adopted algorithm itself but also other multiple factors such as sensors, installation, external electromagnetic environment and even the pilot’s skills, etc. Herein the purpose of the experiments is to verify the advantages of the proposed adaptive method over those non-adaptive methods, rather than prove it is the state of the art.

The tests and analysis on the proposed method are based on the empirical data of two calibration flights conducted on the same day. Since the data are offline, the proposed method is executed by replaying the data in order to reproduce the real-time scene. One set of the calibration data is taken as the History Data and provides the initial estimated coefficients, and the other is used to test the proposed method. The sampling rate is 10 Hz. The width of the Neighbour Window and the Host Window is 50. The residual interference is extracted from the compensated total field signal with a Butterworth filter with a passband from 0.06 to 1.0 Hz.
4.2. Compensation Result without Magnetic Anomaly

The proposed method was tested first without considering the magnetic anomaly signals. The non-adaptive method in [14] was adapted. The residual interferences obtained with the adaptive and non-adaptive methods as well as the magnetic interference in the uncompensated signal are plotted in Figure 3. It is obviously that the residual interference in the compensated signal using the proposed adaptive method has a much smaller fluctuation amplitude than the non-adaptive method does. Their standard deviations are 0.025 nT and 0.059 nT, respectively. This experimental result illustrates that the adaptive method can achieve a better performance for aeromagnetic compensation when no magnetic anomaly exists.

4.3. Compensation Result with Magnetic Anomaly

One of the main concerns for the adaptive method is that it would damage the magnetic anomaly signals of interests. A segment of magnetic anomaly signal generated from a simulated magnetic dipole is added to the range of the total field signal from the 8600th to 8999th sample points. The test result is presented in Figure 4. For clarity, the range where the magnetic anomaly exists has been indicated with the green rectangle.
As can be seen in Figure 4, the waveform of the magnetic anomaly in the compensated signal using the non-adaptive method is almost identical to that in the uncompensated signal. However, this is not the case for the adaptive method. The waveform of the magnetic anomaly in the compensated signal using the adaptive method generally maintains its original varying trend at the beginning, but as the adaptive process continues, it gradually deviates from the other two waveforms. Nonetheless, the presence of the magnetic anomaly signals has been monitored.

Generally, the adaptive method should be adjusted when a magnetic anomaly signal is detected. However, in practice it is usually difficult to detect the magnetic anomaly signal before eliminating the interference. The dilemma can hardly be completely avoided in adaptive aeromagnetic compensation. Since the detection methods can be carried out in the Host Window and the corresponding result will be taken as an indicator to adjust the weights, to some extent, the weight vector introduced in (12) can be taken as a trade-off for the dilemma. Specifically, the weight should take smaller values when a magnetic anomaly occurrence is claimed, or vice versa.

5. Conclusion
In this paper, an adaptive aeromagnetic compensation method based on local linear regression is presented. The compensation coefficients are updated in real time with the current measured data as well as part of the history calibration data. The experimental results show that when there are no magnetic anomalies in the signals, the proposed method is able to provide better aeromagnetic compensation results than the non-adaptive methods. However, the waveform of the original magnetic anomaly in the signals can be distorted by the adaptive method. It means that the proposed method at present is more appropriate for those tasks without concerning the magnetic anomaly. Therefore, the future work will focus on improving the proposed adaptive method and making it suitable for magnetic anomaly detection.

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