THE TOTAL IRREGULAR VALUE FOR DOUBLE LAYERED NEIGHBOURLY IRREGULAR FUZZY CHEMICAL GRAPHS

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Abstract: In this paper, using Neighbourly Irregular Fuzzy Graph and Neighbourly Irregular Chemical Graph, we introduce Neighbourly Irregular Fuzzy Chemical Graphs (NIFCG). We also discuss about the Total Irregular value for Double Layered Neighbourly Irregular Fuzzy Chemical Graphs (NIFCG) using edge cut method.

Keywords: Edge cut, Neighbourly Irregular Fuzzy Chemical Graphs, Double Layered Fuzzy Chemical Graphs, Total Irregular Fuzzy Chemical Graphs.

1. INTRODUCTION
A.Rosenfeld [1975] [7] A.Kauffman [1973] along with Yeh and Bang introduced the concepts of fuzzy graph. Inspired the papers of A.Nagoorgani [1] , who introduced notations and some basic definitions for fuzzy graphs and fuzzy irregular graphs. J.Arockia Aruldoss [2] who introduced Neighbourly Irregular Chemical graphs inspired by S.Gnana Bhragsam [5] who introduced the NI graphs.

Throughout the paper we considered Neighbourly Irregular fuzzy chemical graph (NIFCG) and we used edge cut method to find The Total Irregular Value For Double Layered Neighbourly Irregular Fuzzy Chemical Graphs.

Section two contains basic definitions and we introduce a new definitions for Neighbourly Irregular and Neighbourly Total Irregular Double Layered Fuzzy Chemical Graphs, in section three presents the theoretical concepts and finally we give conclusion on Total Irregular Value For DLNI FCG.

2. BASIC DEFINITIONS AND NEIGHBOURLY IRREGULAR, NEIGHBOURLY TOTAL IRREGULAR DOUBLE LAYERED FUZZY CHEMICAL GRAPHS DEFINITIONS
A not empty set S has a fuzzy subset of mapping $\sigma$: S tends to [0,1] such that $\sigma(x)$ in [0,1] then $0 \leq \sigma(x) \leq 1$ where $x \in S$ .

2.1 Definition
A Fuzzy graph is a pair of functions $G: (\sigma, \mu)$ where V is a fuzzy subset of $\sigma$, a symmetrical fuzzy relation $\mu$ on $\sigma$. i.e., $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ $\exists \mu(u,v) \leq \sigma(u) \land \sigma(v) \forall u,v \in V$.

2.2 Definition
Let $G = (\sigma, \mu)$ where $\tau (u) \leq \sigma (u)$ for all $u \in V$ and $\rho (u,v) \leq \mu (u,v) \forall u,v \in V$ which is said to be the fuzzy graph $H = (\tau, \rho)$ is called fuzzy Subgraph.
2.3 Definition
Let $G = (\sigma, \mu)$ be a fuzzy graph, if there is some vertices with distinct degrees which is adjacent to the vertex is called *Irregular Graph*.

2.4 Definition
Let $G = (\sigma, \mu)$ be a fuzzy graph, if there is a vertex which is adjacent to vertices with distinct total degrees where $G$ is *Total Irregular*.

2.5 Definition
A *Neighbourly Total Irregular Fuzzy Graph* is said to be $G = (\sigma, \mu)$ if every two adjacent vertices of fuzzy graph have different total degrees.

2.6 Definition
Let $V$ be a non empty set of vertices. A *Neighbourly Irregular Double Layered Fuzzy Graph* is a two identical function $D_L(G) = (\sigma_{DL}, \mu_{DL})[3]$ where fuzzy subset of $V$ is $\sigma_{DL}$, $\mu_{DL}$ is a symmetric Double Layered fuzzy relation of $\sigma_{DL}$. ie, $\sigma_{DL}: V \rightarrow [0,1] \triangleright \mu_{DL}(u,v) \leq \sigma_{DL}(u) \land \sigma_{DL}(v).[7]$

2.6 Example

![Neighbourly Irregular Double Layered Fuzzy Graph](image)
2.7 Definition
Let $D_L(G) = (\sigma_{DL}, \mu_{DL})$ be a Double Layered Fuzzy graph. The total degree of a vertex $u \in V$ is defined by $td_{DL(G)}(u) = d_{DL(G)}(u) + \sigma_{DL(G)}(u)[6]$. If each vertex of $D_L(G)$ has the distinct degrees. This is known as \textit{Neighbourly Total Irregular Double Layered Fuzzy Graphs}.

2.7 Example

![Diagram of a Double Layered Fuzzy Graph]

\textbf{Neighbourly Total Irregular Double Layered Fuzzy Graph}

2.8 Definition
An \textit{edge cut} is a set of edges of $D_LFG$ which is removing on edges to obtain the required graph. It is denoted by $i_e$, $D_L(G) - e_j$, for some $j$’s such that not all $e_j$’s are removed.

2.8 Example

![Diagram of a Double Layered edge cut fuzzy graph]

\textbf{Double Layered edge cut fuzzy graph} $(D_L(G) - e_2, e_3, e_5, e_9)$. 
2.9 Definition

Let \( V \) be a non empty set of atoms of corresponding molecules of \textit{Neighbourly Irregular Chemical graph}. A NIFC graph is a two identical function \( G = (\sigma, \mu) \). ie, \( \sigma : V \rightarrow [0,1] \) and \( \mu : V \times V \rightarrow [0,1] \) \( \mu(u,v) \leq \sigma(u) \land \sigma(v) \ \forall \ u, v \in V. \)

2.10 Definition

Let \( V \) be a non empty set of atoms of corresponding molecules of \textit{Neighbourly Irregular Double Layered Fuzzy Chemical Graph}[8]. A NIDLCG is a pair of functions \( DLFC(G)=(\sigma_{DLFC}, \mu_{DLFC}) \) where \( \sigma_{DLFC} \) is a fuzzy subset of \( V \), \( \mu_{DL} \) is a symmetric Double Layered fuzzy relation of \( \sigma_{DL}. \)

ie, \( \sigma_{DLFC} : V \rightarrow [0,1] \) \( \mu_{DLFC}(u,v) \leq \sigma_{DLFC}(u) \land \sigma_{DLFC}(v). \)
2.10 Example

![Neighbourly Irregular Double Layered Fuzzy Chemical Graph](image)

2.11 Definition

Let $D_L(G) = (\sigma_{DL}, \mu_{DL})$ be a Double Layered fuzzy graph. The total valencies of an atom of corresponding molecules of total NI(DL)FCG $u$ belongs to $V$ is stated as $td_{DLFC}(u) = d_{DLG}(u) + \sigma_{DLFC}(u)$. If every two atoms of $D_{LFC}(G)$ has the distinct total valencies of atoms. Then $D_{LFC}(G)$ is **Neighbourly Total Irregular Double Layered Fuzzy Chemical Graphs**.
2.11 Example

Sicl₄ (silicon tetrachloride) [2]  Neighbourly Total Irregular Double Layered Fuzzy Chemical Graph

2.12 Definition
An edge cut is a set of edges of a Double Layered Chemical Graph which is by removing on edges to obtain the required graph. It is denoted by Dₐ₋ₑ(G) for some j’s such that not all eᵢ’s are removed.

2.12 Example

BrF₃ (bromine trifluoride)[2]  Dₐ₋ₑ(G)
3. SOME RELATED THEOREMS BASED ON NEIGHBOURLY IRREGULAR FUZZY CHEMICAL GRAPHS.

3.1 THEOREM
A Neighbourly Irregular Double Layered Edge Cut Fuzzy Chemical Graph is an Irregular Fuzzy Chemical Graph.

Proof
Define \( D_{LC}(G) = (\sigma_{DLC}, \mu_{DLC}) \), by \( \sigma_{DLC}(u) = 0.6, \sigma_{DLC}(v) = 0.5, \sigma_{DLC}(w) = 0.2, \sigma_{DLC}(x) = 0.7, \sigma_{DLC}(p) = 0.3, \sigma_{DLC}(q) = 0.4, \sigma_{DLC}(r) = 0.8, \sigma_{DLC}(s) = 0.9 \) and \( \mu_{DLC}(u,v) = 0.4, \mu_{DLC}(v,w) = 0.2, \mu_{DLC}(v,x) = 0.5, \mu_{DLC}(u,p) = 0.3, \mu_{DLC}(v,q) = 0.3, \mu_{DLC}(q,r) = 0.3, \mu_{DLC}(q,s) = 0.4 \) are Neighbourly Irregular Fuzzy[1] Chemical Graph. Therefore no two adjacent atoms has the same valencies of an atoms.

3.2 THEOREM
A Neighbourly Total Irregular Double Layered Edge Cut Fuzzy Chemical Graph is an Neighbourly Irregular Fuzzy Chemical Graph.

Proof
Define \( D_{LC}(G) = (\sigma_{DLC}, \mu_{DLC}) \), by \( \sigma_{DLC}(u) = 0.9, \sigma_{DLC}(v) = 0.7, \sigma_{DLC}(w) = 0.5, \sigma_{DLC}(x) = 0.8, \sigma_{DLC}(p) = 0.4, \sigma_{DLC}(z) = 0.2, \sigma_{DLC}(a) = 0.4, \sigma_{DLC}(b) = 0.3, \sigma_{DLC}(c) = 0.2, \sigma_{DLC}(d) = 0.1, \sigma_{DLC}(e) = 0.6, \sigma_{DLC}(f) = 0.8 \) and \( \mu_{DLC}(u,v) = 0.4, \mu_{DLC}(v,w) = 0.2, \mu_{DLC}(w,y) = 0.1, \mu_{DLC}(v,b) = 0.3, \mu_{DLC}(a,b) = 0.3, \mu_{DLC}(c,d) = 0.1, \mu_{DLC}(d,x) = 0.1, \mu_{DLC}(c,e) = 0.2, \mu_{DLC}(c,f) = 0.2, \mu_{DLC}(f,z) = 0.2 \) ie, two adjacent vertices have distinct degrees[6] but G is NIFG.
3.3 THEOREM
Let $D_{LC}(G) = (\sigma_{DLC}, \mu_{DLC})$, be a Double Layered Fuzzy Graph. In some Neighbourly Total Irregular Double Layered Fuzzy Chemical Graphs then there exist $\sigma_{DL}$ is a constant function by using edge cut method, then $D_{LFC}(G)$ is a Double Layered Fuzzy Chemical Graph which is NI.

Proof
Given $D_{LFC}(G) = (\sigma_{DLC}, \mu_{DLC})$
Using edge cut method, To prove $D_{LFC}(G)$ is a Neighbourly Total Irregular Double Layered Fuzzy Chemical Graph.
Wkt, each pair of the two adjacent atoms are distinct in the total valencies of an atom. By using edge cut method there exist two adjacent atoms in Double Layered graph[3] $v_4$ and $v_8$ with valencies of an atoms $\delta_4$ and $\delta_8$.[1]

ie, $d_{DLFC}(v_4) = \delta_4$ and $d_{DLFC}(v_8) = \delta_8$

Also assume that $\sigma_{DLFC}(v_4) = \sigma_{DLFC}(v_8) = k$, a constant where $k \in [0,1]$

To prove: $d_{DLFC}(v_4) \neq d_{DLFC}(v_8)$

ie, $td_{DLFC}(v_4) \neq td_{DLFC}(v_8)$

$\delta_4 + k \neq \delta_8 + k$

$\delta_4 \neq \delta_8$

Ie, valencies of an adjacent atom of $D_{LFC}(G)$ are different. This is verified for two identical adjacent atoms in $D_{LFC}(G)$.
Therefore every Neighbourly Total Irregular Double Layered Edge Cut Fuzzy Chemical Graph has a Neighbourly Irregular Double Layered Fuzzy Chemical Graphs.

3.2 Example

![Diagram](Image)

TeCl₄(Tellurium tetrachloride) [4] Neighbourly Irregular Fuzzy Chemical Graph
3.4 THEOREM

Let $D_{L}(G) = (\sigma_{DL}, \mu_{DL})$, be a Double Layered Edge Cut Fuzzy Chemical Graph. If $D_{L}(G)$ Neighbourly Irregular[5] Fuzzy Chemical Graphs (NIFCG) there exist $\sigma_{DL}$ is a constant function then $D_{LFC}(G)$ is a NTIFCG.

Proof

Let $D_{LFC}(G) = (\sigma_{DLFC}, \mu_{DLFC})$ is a Neighbourly Irregular[5] Double Layered Fuzzy[3] Chemical Graph[8] using edge cut method.

ie, The valencies of each pair of two adjacent atoms are different.

Let us assume that the two adjacent atoms are different in the total valencies of an atom.

By using edge cut method there exist two adjacent atoms in Double Layered graph $v_3$ and $v_5$ with valencies of an atoms $\delta_3$ and $\delta_5$ respectively.

ie, $d_{DLFC}(v_3) = \delta_3$

$\delta_3 + k$ is a constant where $k \in [0,1]$. Therefore,

Assume that $\sigma_{DLFC}(v_3) = \sigma_{DLFC}(v_5) = k$, a constant where $k \in [0,1]$. Therefore,

$$td_{DLFC}(v_3) = d_{DLFC}(v_3) + \sigma_{DLFC}(v_3) = \delta_3 + k$$

$$td_{DLFC}(v_5) = d_{DLFC}(v_5) + \sigma_{DLFC}(v_5) = \delta_5 + k$$

To prove: $td_{DLFC}(v_3) \neq td_{DLFC}(v_5)$

$$\delta_3 + k \neq \delta_5 + k$$

$$\delta_3 - \delta_5 \neq k - k$$

$$\delta_3 - \delta_5 \neq 0$$
\[ \delta_3 \neq \delta_5 \]

ie, \[ \delta_3 \neq \delta_5 \]

Therefore \( t_{DLFC}(v_3) \neq t_{DLFC}(v_3) \)

For any two adjacent atoms with different valencies, its total valencies are also different, then \( \sigma_{DLFC} \) is a constant function. The above result is satisfied for every pair of adjacent atoms in \( DLFC(G) \).

Therefore every Neighbourly Irregular Double Layered Fuzzy Chemical Graph have a Neighbourly Total Irregular Double Layered Fuzzy Chemical Graphs for using edge cut method.

### 3.4 Example

Pcl₃ (phosphorus trichloride) [2] Neighbourly Irregular Fuzzy Chemical Graphs

Neighbourly Irregular Double Layered Edge Cut Fuzzy Chemical Graph
4. Conclusion

In this paper proposes the Total Irregular value for Double Layered Neighbourly Irregular Fuzzy Chemical Graphs using edge cut method. And we define some new definitions on Double Layer Fuzzy Chemical Graphs. Further work can be done by using Vertex cut method in Double Layered Fuzzy graph in NIC Graphs.

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