EMPirical abundance scaling laws and implications for the gamma process in core-collapse supernovae

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Received 2007 December 19; accepted 2008 May 3

ABSTRACT

Analyzing solar system abundances, we have found two empirical abundance scaling laws for \( p \)- and \( s \)-nuclei with the same atomic number. The first scaling law is that \( s/p \) ratios are almost constant over a wide range of atomic numbers where the \( p \)-nuclei are lighter than the \( s \)-nuclei by 2 or 4 neutrons. The second law is that \( p/p \) ratios are almost constant where the \( s \)-nuclei are lighter than the first \( p \)-nuclei by 2 neutrons. These scalings provide evidence that most \( p \)-nuclei are dominantly synthesized by the \( \gamma \)-process in supernova explosions. These scalings lead to a novel concept of the “universality of the \( \gamma \)-process,” in that the \( s/p \) and \( p/p \) ratios of nuclei produced by individual \( \gamma \)-processes are almost constant. We have calculated the ratios produced by the \( \gamma \)-process based on core-collapse supernova explosion models under various astrophysical conditions, and found that the scalings hold for materials produced by individual \( \gamma \)-processes independent of the astrophysical conditions assumed. The universality originates from three mechanisms: the shifting of the \( \gamma \)-process layers in order to keep their peak temperature, the weak \( s \)-process in presupernovae, and the independence of the \( s/p \) ratios from nuclear reactions. The results further suggest an extended universality, that the \( s/p \) ratios in the \( \gamma \)-process layers are not only constant but also centered on a specific value of 3. With this specific value and the first scaling law, we estimate that the ratio of \( s \)-process abundance contributions from AGB stars to massive stars will be almost 6.7 for the \( s \)-nuclei of \( A > 90 \). We find that large enhancements of the \( s/p \) ratios for Ce, Er, and W are evidence that the weak \( s \)-process actually occurred before supernovae.

Subject headings: nuclear reactions, nucleosynthesis, abundances — supernovae: general

1. INTRODUCTION

Light elements such as H and He were mainly synthesized in the big bang, whereas heavier elements were synthesized primarily in many different stellar nucleosynthesis processes after the formation of the first generation of stars in the Galaxy. The solar system was formed from the interstellar medium (ISM), the composition of which was provided by stellar nucleosynthesis. Solar system abundances are therefore an important record of these stellar nucleosynthesis processes and the Galactic chemical evolution (GCE). Solar system abundances also provide evidence that two neutron capture reaction processes happened before the formation of the solar system. The first evidence for this is a pair of two abundance peaks near three neutron magic numbers, \( N = 50, 82 \), and 126. These two peaks correspond to the \( s \)- and \( r \)-processes, which originated from the positions of the two nucleosynthesis flows in the nuclear chart (Burbidge et al. 1957). Further evidence for the \( s \)-process is an empirical relation, \( \sigma N_a \sim \text{constant} \), where \( \sigma \) and \( N_a \) are the neutron capture cross-section and the solar abundance for the pure \( s \)-nuclei, respectively (Seeger et al. 1965; Käppeler et al. 1990; Gallino et al. 1998). This can be understood in terms of a “steady flow.” In this way, the evidence observed in solar abundances correlates with the fundamental mechanisms of nucleosynthesis.

We here focus our attention on the origin of \( p \)-process isotopes (\( p \)-nuclei). These have the following features. First, they cannot be synthesized by neutron-capture reactions, because they are on the neutron-deficient side of the \( \beta \)-stability line in the nuclear chart (see Fig. 1). Second, their isotopic fractions are small (typically 0.1%-1%). About 50 years ago, Burbidge et al. suggested that the \( p \)-nuclei may be synthesized by \( (\gamma, n) \) or \( (p, \gamma) \) reactions (Burbidge et al. 1957). Arnould (1976) proposed the \( p \)-process in presupernova phases, and Woosley & Howard (1978) proposed the \( \gamma \)-process in supernovae (SNe). In these pioneering works, the \( p \)-nuclei are synthesized mainly by photodisintegration reactions from pre-existing nuclei affected by the \( s \)-process in earlier evolutionary states of progenitors. Woosley & Howard (1978) pointed out the anticorrelation between the photodisintegration reaction rates and the solar abundances for the \( p \)-nuclei, which is evidence for the \( \gamma \)-process. After these works, detailed calculations reproduced the relative solar abundances of most \( p \)-nuclei within a factor of 3 (Rayet et al. 1995; Rauscher et al. 2002; Arnold & Goriely 2003).

Nevertheless, their origin has long been discussed, with many possible nuclear reactions suggested over the last 50 years, and their astrophysical sites have not been uniquely identified. The proposed nuclear reactions are rapid proton capture reactions in novae or Type I X-ray bursts in neutron stars (rp-process; Schatz et al. 1998, 2001), proton-induced reactions by Galactic cosmic rays (Audouze 1970), the \( \gamma \)-process in core-collapse SNe (Arnould 1976; Woosley & Howard 1978; Rayet et al. 1995), explosive nucleosynthesis in Type Ia SNe (Howard et al. 1991; Arnould & Goriely 2003; Kusakabe et al. 2005), supernova-driven supercritical accretion disks (Fujimoto et al. 2003), and neutrino-induced reactions in SN explosions (the \( \nu \)-process; Woosley et al. 1990; Hoffman et al. 1996). The origin of the \( p \)-nuclei is crucial to our understanding of how the solar system material formed and
evolved. In a previous paper (Hayakawa et al. 2004), we reported two empirical scaling laws found in the solar system abundances, which provide evidence that the most probable origin of the $p$-nuclei is the SN $\gamma$-process. The empirical laws lead a novel concept of "the universality of the $\gamma$-process," i.e., that each SN $\gamma$-process arising from different conditions in individual SN explosions should reproduce $N(s)/N(p)$ constant over a wide range of atomic numbers (Hayakawa et al. 2004, 2006a). The purpose of this paper is to report a detailed analyses of these scalings. We also present the mechanisms of this universality in core-collapse SN $\gamma$-process model calculations.

2. ANALYSES OF THE SOLAR ABUNDANCES

2.1. Discovery of the First Scaling

The $p$-nuclei are on the neutron-deficient side of the $\beta$ stability line, as shown in Figure 1. Typical $p$-nuclei are even-even isotopes, in which both proton and neutron numbers are even. They are isolated in the nuclear chart, and around them isotopes are unstable. There are 22 pairs of a $p$-nucleus and an $s$-nucleus that is heavier than the $p$-nucleus by 2 neutrons. In most cases, the $s$-nuclei are pure $s$-nuclei that are dominantly synthesized by the $s$-process, since they are shielded by stable isobars against $\beta^-$-decay after freezeout of the $r$-process. Nine elements have two $p$-nuclei. We here define the first and second $p$-nuclei as those that are lighter than the $s$-nucleus by 2 and 4 neutrons, respectively. Figure 1 shows a partial nuclear chart as a typical example. $^{134}$Ba is a pure $s$-nucleus shielded by a stable isobar $^{134}$Xe against $\beta$-decay after the freezeout of the $r$-process. The stable isotopes $^{132}$Ba and $^{130}$Ba are the first and second $p$-nuclei, respectively. It is noted that $^{144}$Sm is the 2nd $p$-nucleus and $^{148}$Sm is the $s$-nucleus, but the first $p$-nucleus $^{146}$Sm is unstable. There are 35 $p$-nuclei: 22 first $p$-nuclei, 10 second $p$-nuclei, and 3 other $p$-nuclei. The 3 other $p$-nuclei are odd-$N$ isotopes of $^{115}$Sn, $^{139}$La, and $^{180}$Ta, and are beyond of the scope of this discussion of solar abundance ratios.

Here we discuss the isotope abundance ratios of a $p$-nucleus and an $s$-nucleus with the same atomic number, taking the abundance ratios of the $s$-nucleus to the $p$-nucleus, $N(s)/N(p)$, where $N(s)$ and $N(p)$ are the solar isotope abundances of the $s$- and $p$-nuclei, respectively. For example, the solar isotope abundances of the $p$-nucleus $^{132}$Ba and the $s$-nucleus $^{134}$Ba are 0.101% and 2.417%, respectively, and thus the isotope abundance ratio of $N(s)/N(p)$ is 23.93. We have previously reported the scaling law that $N(s)/N(p)$ ratios are almost constant over a wide region of atomic numbers (Hayakawa et al. 2004). We also reported another empirical scaling between two $p$-nuclei with the same atomic number, as shown in Figure 2 (Hayakawa et al. 2004).

2.2. Second Scaling

In addition to the first scaling, we also report a second empirical scaling law. In a previous paper (Hayakawa et al. 2004), we reported another empirical scaling between two $p$-nuclei with the same atomic number, as shown in Figure 2 (Hayakawa et al. 2004). Nine elements have two $p$-nuclei. As shown in Figure 3, the $N(s)/(1st p)/N(s)/(2nd p)$ ratios are almost constant over a wide region of atomic numbers, except for a large deviation of Er, the reason for which will be discussed later.
The observed ratios are almost constant within a factor of 2. The ratios in the region of 49 number. The circles and triangles show over a wide region are constrained at a constant value of 23, and the dot-dashed lines are 11.5 and 46. The observed ratios are almost constant within a factor of 2.

The $N_{\odot}(s)/N_{\odot}(1st\ p)$ and $N_{\odot}(1st\ p)/N_{\odot}(2nd\ p)$ ratios show a clear correlation (Hayakawa et al. 2004), but the question of whether the ratios of $N_{\odot}(s)/N_{\odot}(2nd\ p)$ can be described by the first scaling law remains. Here we present the ratios of $N_{\odot}(s)/N_{\odot}(1st\ p)$ and $N_{\odot}(s)/N_{\odot}(2nd\ p)$ in Figure 2. There is a clear correlation between them. The ratios are almost constant over a wide range of atomic numbers, with some deviations, which are discussed later. The ratios in the region of $49 \leq Z \leq 72$, except for Ce and Er, are centered around their average of 23.2. In addition, most ratios over a wide region are constrained at a constant value of $N_{\odot}(s)/N_{\odot}(p) \approx 23$, within a factor of 2. We summarize the isotope abundances and their ratios in Table 1.

The first scaling law shows a strong correlation between $p$- and $s$-nuclei with the same atomic number, which indicates that the origin of the $p$-nuclei is strongly correlated with the $s$-nuclei. This is consistent with the previous theoretical calculations that the $p$-nuclei are produced by the $\gamma$-process ($p$-process) in SN explosions (Arnould 1976; Woosley & Howard 1978; Rayet et al. 1990, 1995; Prantzos et al. 1990; Rauscher et al. 2002; Arnold & Goriely 2003). In the $\gamma$-process models, pre-existing nuclei in massive stars are affected by the weak $s$-process during presupernova evolutionary stages. These pre-existing nuclei originate from early generations of stars through interstellar media. The $p$-nuclei are produced from them by photodisintegration reactions such as ($\gamma$, $n$) reactions in a huge photon bath at extremely high temperatures in SN explosions. Previous calculations indicated that the $p$-nuclei are produced via two nuclear reaction paths. The first is direct ($\gamma$, $n$) reactions from heavy isotopes. The second is the EC/$\beta^+$ decay from the neutron-deficient unstable nuclei after the freezeout of the $\gamma$-process. These neutron-deficient nuclei are first transmuted by successive photodisintegration reactions, such as ($\gamma$, $n$) reactions from heavier isotopes, and subsequently transmuted by downflows of ($\gamma$, $p$) and ($\gamma$, $\alpha$) reactions. The first scaling law suggests that the first of these reaction paths is likely to play a more important role than the latter. The contributions of these two nucleosynthesis paths will be discussed quantitatively later. The charged particle reactions in the rp-process (Schatz et al. 1998, 2001) and proton-induced reactions via cosmic rays (Audouze 1970) change the proton number of the seed nuclei. In the $\nu$-process, the charged current interaction, which has a larger effect than the neutral current interaction, also changes the proton number (Goriely et al. 2001; Heger et al. 2005). Therefore, the scaling does not emerge from the dominant charged particle processes and the $\nu$-process. The first scaling law thus provides evidence that the SN $\gamma$-process is the most probable origin of the $p$-nuclei.

Figure 4 shows the solar abundances of the $p$-nuclei and $s$-nuclei that are members of the first scaling law. The correlation between the $p$- and $s$-nuclei can be observed in this figure. Seeger et al. (1965) pointed out that the empirical relation $\sigma N_s \sim N_p$ for the pure $s$-nuclei. We would like to stress that the abundances of the $s$-nuclei that are the members of the first scaling, $N_s$, is almost constant in the two mass regions of $50 < N < 82$ and $82 < N$, respectively. The abundance pattern of the $p$-nuclei shows a tendency similar to that of the $s$-nuclei. The abundances of the $p$-nuclei are almost constant in the two mass regions of $50 < N < 82$, and $82 < N$, respectively. There are some enhancements of the $p$-nuclei, for example $^{112}$Sn and $^{190}$Hg. Their partner $s$-nuclei, $^{116}$Sn and $^{198}$Hg, also show enhancements. These enhancements show that the abundances of the $p$-nuclei are proportional to those of the $s$-nuclei with the same atomic number. This is consistent with the first scaling law that the $N_{\odot}(s)/N_{\odot}(p)$ ratios are almost constant over a wide range of atomic numbers.

2.2. Reason for the Deviations

Figure 2 shows some deviations from the average value of 23. These deviations can be explained by contributions from other nucleosynthesis processes or of other astrophysical origin.

Ce, Er, and W.—The three large deviations for Ce, Er, and W can be explained by an exceptional contribution from the $r$-process because the heavier isotopes in the pairs, $^{140}$Ce, $^{166}$Er, and $^{182}$W, are not shielded against $\beta^+$-decay after the freezeout of the $r$-process. The $r$-process contributions to $^{166}$Er and $^{182}$W are larger than the $s$-process contributions in theoretical calculations (Arlandini et al. 1999). In addition, the effect of the neutron magic number $N = 82$ should contribute to the deviation for Ce.

Gd.—The small deviation for Gd can be explained by a contribution from a weak branch of the $s$-process. The unstable nucleus $^{153}$Sm is known as a branching point of the $s$-process,
where the neutron capture reaction and $\beta$-decay compete. The $p$-nucleus $^{152}$Gd is produced by the $s$-process branch $^{150}$Sm($n$, $\gamma$)$^{151}$Sm($e^{-}\bar{\nu}$)$^{151}$Eu($n$, $\gamma$)$^{152}$Eu($e^{-}\bar{\nu}$)$^{152}$Gd.

As the result, the $N(s)/N(2nd p)$ ratio decreases from the $N(s)/N(1st p)$ ratio in SNe.

Kr.—The deviation of $N(s)/N(80Kr)/N(s)/N(78Kr)$ may originate from a weak branch of the $s$-process. As shown in Figure 5, $^{79}$Se is a branching point, and $^{80}$Kr is synthesized by a weak branch of the

\[
^{150}\text{Sm}(n, \gamma)^{151}\text{Sm}(e^{-}\bar{\nu})^\text{s}\text{Eu}(n, \gamma)^{152}\text{Eu}(e^{-}\bar{\nu})^{s}\text{Gd},
\]

Table 1

| Element | Z | \(2nd\, p\) | \(1st\, p\) | \(s\) | \(N(2nd\, p)/N(1st\, p)\) | \(N(s)/N(2nd\, p)\) | \(N(s)/N(1st\, p)\) | \(N(2nd\, p)/N(1st\, p)\) |
|---------|---|-------------|-------------|-----|----------------|-----------------|----------------|----------------|
| Se      | 34 | 74 76       | 0.89        | 9.36| 10.5           |
| Kr      | 36 | 78 80       | 0.35        | 2.25| 6.43           |
| Sr      | 38 | 84 86       | 0.56        | 9.86| 17.6           |
| Mo      | 42 | 92 94 96    | 14.84       | 9.25| 16.68          |
| Ru      | 44 | 96 98 100   | 5.52        | 1.88| 12.6           |
| Pd      | 46 | 102 104     | 1.02        | 11.14| 10.9           |
| Cd      | 48 | 106 108 110 | 1.25        | 0.89| 12.49          |
| In      | 49 | 113 115     | 4.3         | 95.7$^b$| 22.3$^b$       |
| Sn      | 50 | 112 114 116 | 0.97        | 0.65| 14.53          |
| Te      | 52 | 120 122     | 0.096       | 9.36| 10.5           |
| Xe      | 54 | 124 126 128 | 0.10        | 0.09| 1.91           |
| Ba      | 56 | 130 132 134 | 0.106       | 0.101| 2.417          |
| La      | 57 | 138         | 0.0902      |     |
| Ce      | 58 | 136 138 140 | 0.19        | 0.25| 88.48$^{ac}$   |
| Sm      | 62 | 144         | 3.1         |     |
| Gd      | 64 | 152 154     | 0.20$^b$    | 2.18| 10.9$^b$       |
| Dy      | 66 | 156 158 160 | 0.06        | 0.10| 2.34           |
| Er      | 68 | 162 164 166 | 0.14        | 1.61$^b$| 33.6$^b$ |
| Yb      | 70 | 168 170     | 0.13        | 3.05| 23.5           |
| Hf      | 72 | 174 176     | 0.162       | 5.206| 32.14         |
| Ta      | 73 | 180         | 0.012       |     |
| W       | 74 | 180 182     | 0.13        | 26.3$^a$| 202$^a$ |
| Os      | 76 | 184 186     | 0.02        | 1.58| 79             |
| Pt      | 78 | 190 192     | 0.01        | 0.79| 79             |
| Hg      | 80 | 196 198     | 0.15        | 9.97| 66.5           |

Note.—The isotope abundances are taken from De Bievre & Taylor (1993).

$^a$ The contribution of the $\beta$-process.

$^b$ The contribution of the $s$-process.

$^c$ The effect of the neutron magic number.

Fig. 4.—Solar abundances of the $p$- and $s$-nuclei, used in the scalings. The dot-dashed lines show the neutron magic numbers \(N = 50\) and \(82\). The dotted lines show constant values around the centers of the $p$- and $s$-nucleus abundances in each region.

**TABLE 1**

**List of the $p$-Process Isotopes**

| Element | \(Z\) | \(2nd\, p\) | \(1st\, p\) | \(s\) | \(N(2nd\, p)/N(1st\, p)\) | \(N(s)/N(2nd\, p)\) | \(N(s)/N(1st\, p)\) | \(N(2nd\, p)/N(1st\, p)\) |
|---------|---|-------------|-------------|-----|----------------|-----------------|----------------|----------------|
| Se      | 34 | 74 76       | 0.89        | 9.36| 10.5           |
| Kr      | 36 | 78 80       | 0.35        | 2.25| 6.43           |
| Sr      | 38 | 84 86       | 0.56        | 9.86| 17.6           |
| Mo      | 42 | 92 94 96    | 14.84       | 9.25| 16.68          |
| Ru      | 44 | 96 98 100   | 5.52        | 1.88| 12.6           |
| Pd      | 46 | 102 104     | 1.02        | 11.14| 10.9           |
| Cd      | 48 | 106 108 110 | 1.25        | 0.89| 12.49          |
| In      | 49 | 113 115     | 4.3         | 95.7$^b$| 22.3$^b$       |
| Sn      | 50 | 112 114 116 | 0.97        | 0.65| 14.53          |
| Te      | 52 | 120 122     | 0.096       | 9.36| 10.5           |
| Xe      | 54 | 124 126 128 | 0.10        | 0.09| 1.91           |
| Ba      | 56 | 130 132 134 | 0.106       | 0.101| 2.417          |
| La      | 57 | 138         | 0.0902      |     |
| Ce      | 58 | 136 138 140 | 0.19        | 0.25| 88.48$^{ac}$   |
| Sm      | 62 | 144         | 3.1         |     |
| Gd      | 64 | 152 154     | 0.20$^b$    | 2.18| 10.9$^b$       |
| Dy      | 66 | 156 158 160 | 0.06        | 0.10| 2.34           |
| Er      | 68 | 162 164 166 | 0.14        | 1.61$^b$| 33.6$^b$ |
| Yb      | 70 | 168 170     | 0.13        | 3.05| 23.5           |
| Hf      | 72 | 174 176     | 0.162       | 5.206| 32.14         |
| Ta      | 73 | 180         | 0.012       |     |
| W       | 74 | 180 182     | 0.13        | 26.3$^a$| 202$^a$ |
| Os      | 76 | 184 186     | 0.02        | 1.58| 79             |
| Pt      | 78 | 190 192     | 0.01        | 0.79| 79             |
| Hg      | 80 | 196 198     | 0.15        | 9.97| 66.5           |

Note.—The isotope abundances are taken from De Bievre & Taylor (1993).

$^a$ The contribution of the $\beta$-process.

$^b$ The contribution of the $s$-process.

$^c$ The effect of the neutron magic number.
$^{82}$Kr is located on the main flow of the $s$-process, and the isotope abundance of $^{82}$Kr is about 4 times larger than that of $^{80}$Kr. If we take the ratio $N_{\odot}(^{82}$Kr)/$N_{\odot}(^{80}$Kr) instead of $N_{\odot}(^{80}$Kr)/$N_{\odot}(^{82}$Kr), it is about 3.3.

Mo and Ru.—Four isotopes of $^{92,94}$Mo and $^{96,98}$Ru located near the neutron magic number $N = 50$ show large deviations. Isotope abundances of typical $p$-nuclei in a mass region around Mo and Ru are $0.3\%-1\%$, whereas the isotope abundances of $^{92,94}$Mo and $^{96,98}$Ru are $14.8\%$, $9.25\%$, $5.54\%$, and $1.87\%$, respectively. Thus, the ratios of Mo and Ru are lower than the average value of 23 by an order of magnitude. Previous $\gamma$-process (or $p$-process) model calculations tried to reproduce the relative abundances of all the $p$-nuclei. However, the calculated results showed an underproduction of Mo and Ru. (Rayet et al. 1995). The large deviations in the $N_{\odot}(s)/N_{\odot}(p)$ ratios are consistent with the previously calculated results, and suggest that their origin may be different from the other $p$-nuclei.

Sm.—The $N_{\odot}(^{140}$Sm)/$N_{\odot}(^{144}$Sm) ratio shows a large deviation, like those of Mo and Ru. The $p$-nucleus $^{144}$Sm is on the neutron magic number $N = 82$. This situation is similar to Mo and Ru, and the large deviation suggests that Mo, Ru, and $^{144}$Sm may have the same origin.

Os and Pt.—The uncertainties for $^{184}$Os and $^{186}$Pt are about 50% and 100%, respectively (De Bievre & Taylor 1993). These deviations may originate from the large uncertainties of the measured abundances; measurements with higher precision would be desirable.

Figure 6 shows the corrected $N_{\odot}(s)/N_{\odot}(p)$ ratios. The open and filled circles are the original $N_{\odot}(s)/N_{\odot}(p)$ ratios and the corrected ratios, respectively. The $s$-process abundances of $^{168}$Er, and $^{182}$W are taken from calculated results in a stellar $s$-process model (Arlandini et al. 1999). The $s$-process abundance for $^{152}$Gd is taken from a calculated result based on neutron capture reaction rates measured at a n_TOF facility (Abbondanno et al. 2004). For Kr, we take $N_{\odot}(^{82}$Kr)/$N_{\odot}(^{78}$Kr) instead of $N_{\odot}(^{80}$Kr)/$N_{\odot}(^{78}$Kr). In order to focus on the $\gamma$-process, we omit the ratios of Mo, Ru, and Sm. Most corrected ratios are centered around 23 within a factor of 2 (see Fig. 6). We note that the ratio of In decreases after the correction of the $r$-process contribution. $^{115}$In is not shielded against $\beta$-decay and is produced by the $r$-process, as Ce, Er, and W, and thus the $N(\text{In})/N(\text{In})$ ratio decreases. This suggests that $^{115}$In may have contamination from other processes, such as the $s$-process.

2.3. Universality of the $\gamma$-Process

The universality of nucleosynthetic processes is an important concept for understanding stellar nucleosynthesis. The solar system was formed from the interstellar medium (ISM) originating from many stellar nucleosynthesis episodes in the Galaxy. The stellar nucleosynthesis environments, such as mass, metallicity, and explosion energy, were different, and hence the abundance distributions of the synthesized nuclides may be different. However, astronomical observations of very metal-deficient stars have reported “universal” abundance distributions for $Z > 56$ (Sneden et al. 1998, 2000; Honda et al. 2004), which are in agreement with the abundance distribution of the $r$-process nuclides in the solar system. These facts suggest a uniform site and/or uniform conditions for the synthesis of the $r$-process nuclei. Otsuki et al. explained the universality of the $r$-process by a neutrino-energized wind model (Otsuki et al. 2003).
The $p$- and $s$-nuclei were produced in different stellar environments. Thus, the mass distribution of the synthesized nuclei may depend on the astrophysical conditions. Nevertheless, the observed $N_s/N_p$ ratios in the solar system are almost constant over a wide range of atomic numbers. This leads to a novel concept, the universality of the $\gamma$-process, according to which the $N(s)/N(p)$ ratios of nuclides produced by individual $\gamma$-processes are constant over a wide region. We would like to stress that the concept of the universality of the $\gamma$-process has been derived only from analysis of solar abundances, independent of nucleosynthesis calculations.

The situation of the $\gamma$-process is similar to that of the $s$-process. It is known that $\sigma N_s$ is roughly constant for the $s$-process over a large range of nuclear masses, where $\sigma$ and $N_s$ are the neutron capture cross-section and the solar abundance, respectively (Seeger et al. 1965; Käppeler et al. 1990; Gallino et al. 1998). In this case, one could infer a "universality of the $s$-process," arising from the conditions being such that a steady neutron-capture flow was achieved. The difference in $\sigma N_s$ for $A < 90$ and $A > 90$ then points to two different $s$-process components. Smaller deviations from constant $\sigma N_s$ may signal interesting nuclear phenomena, such as a weak branch. If we could observe abundances of neighboring $s$-only isotopes in stars of various ages, we might expect their abundance ratios to be inversely proportional to the cross section ratio, just as they are in the solar system. The universality of the $\gamma$-process suggests that one might find roughly constant $N(s)/N(p)$ ratios with observation of abundances of $p$- and $s$-nuclei in stars.

The universality of the $\gamma$-process suggests three possible mechanisms for the scaling relations apparent in the solar system. First, a single supernova could have occurred near the solar system before its formation and strongly affected the material from which it formed. Second, if the $\gamma$-process occurs only under a specific uniform astrophysical condition in the Galaxy, the mass distribution of materials produced by individual $\gamma$-processes will be almost same. Third, the $\gamma$-processes could occur under various astrophysical conditions, but $N(s)/N(p)$ ratios remain almost constant over a wide region of atomic numbers, independent of the astrophysical conditions. In the last case, the question of why the scaling relations hold for materials produced by individual $\gamma$-processes independent of the astrophysical conditions remains. This universality is thus essential for understanding the nature of the $\gamma$-process and for constraining the nucleosynthesis site of the $\gamma$-process.

3. SUPERNova MODEL Calculations

3.1. $\gamma$-Process in Core-Collapse Supernova Models

Solar abundances show two empirical scalings, which provide evidence that the most probable origin of the $p$-nuclei is the $\gamma$-process in SNe. One question we have to ask here is whether a standard $\gamma$-process model in a core-collapse SN can reproduce these scalings. In previous studies (Rayet et al. 1990, 1995), calculated results were presented in the form of $F(X)/F_0(X)$, where $F(X)$ is the ratio of the calculated abundance to the solar abundance for an isotope $X$, and $F_0$ is the average of $F(X)$. However, these results have not been presented in the form of $N(s)/N(p)$.

We present $N(s)/N(p)$ ratios calculated under various astrophysical conditions (Iwamoto et al. 2004). A massive star evolves from the main sequence to the core-collapse stage and explodes (Nomoto et al. 2004). Solar abundances are adopted as the initial composition of the massive star, which is affected by the weak $s$-process in various evolutionary states. The mass distribution of the seed nuclei affected by the weak $s$-process are used as the initial composition at the SN explosion. Most of the reaction rates are taken from a common astrophysical nuclear data library of REACLIB (Thielemann 2000).

![Figure 7: Calculated ratios, $N(s)/N(p)$, for models with $M = 25 M_\odot$, $Z = Z_\odot$, and $E = 10^{51}$ erg (filled circles); $M = 40 M_\odot$, $Z = Z_\odot$, and $E = 10^{51}$ erg (filled triangles); $M = 25 M_\odot$, $Z = Z_\odot$, and $E = 20 \times 10^{51}$ erg (open triangles); and $M = 25 M_\odot$, $Z = 0.05 Z_\odot$, and $E = 10^{51}$ erg (open squares). The dashed line marks $N(s)/N(p) = 3$, and the dot-dashed lines 1 and 9. The ratios in all the models are centered around 3, within a factor of 3, over a wide range of atomic number for $Z > 40$.](image)

We have suggested that there are three possibilities to account for the universality of the $\gamma$-process. The calculated results show that even if the $\gamma$-process should occur under the various astrophysical conditions, the $N(s)/N(p)$ ratios in individual nucleosynthesis episodes result in an almost constant value over a wide range independent of the SN conditions assumed.

3.2. $\gamma$-Process Layers Correlated with Astrophysical Conditions

Our calculated results have verified the universality of the $\gamma$-process. However, why the scaling laws hold for the different nucleosynthesis episodes remains an open question. Here we discuss the mechanism of the universality of the $\gamma$-process in detailed calculations. The supernova explosion is a dynamical phenomenon, and physical conditions in individual layers of the progenitor, for example temperature and density, are different. Thus, the physical conditions of each $\gamma$-process layer are key to understanding the universality of the $\gamma$-process. It is known that the mass regions of the $\gamma$-process layers depend on the astrophysical conditions and are shifted to the outer layers as the explosion energy or the mass of the progenitor increases (Arnould 1976; Woosley & Howard 1978; Rayet et al. 1990, 1995).

In the present calculations, the mass regions in the $\gamma$-process models are $1.97–2.8 M_\odot$ for $10^{51}$ ergs and $2.87–4.63 M_\odot$ for $20 \times 10^{51}$ ergs, respectively. The density and temperature of the layers at the peak of the explosion are $(1.7–3.5) \times 10^9$ K and $(0.13–0.95) \times 10^6$ g cm$^{-3}$, respectively, for model 1, $(1.7–3.5) \times 10^9$ K and $(0.12–0.90) \times 10^6$ g cm$^{-3}$ for model 2, $(1.6–3.5) \times 10^9$ K
and (0.14–2.1) × 10^6 g cm^{-3} for model 3, and (1.7–3.5) × 10^7 K and (0.03–0.21) × 10^6 g cm^{-3} for model 4. Although the mass regions are different, the temperature ranges are approximately equal. This shift is also confirmed in our calculations and contributes to the universality.

3.3. Proportionality between the p- and s-Nuclei in the Solar System

The p-nuclei are predominantly produced by SNe, whereas the s-nuclei are continuously produced by the main s-process in asymptotic giant branch (AGB) stars (Gallino et al. 1998; Busso et al. 1999), as shown in Figure 8. The question we have to ask here is why the abundances of the p-nuclei is proportional to the s-nuclei in the solar system. A weak s-process in different evolutionary stages of massive stars gives a hint to the answer. It is well known that the weak s-process produces s-nuclei of A < 90 from iron seeds. However, we focus our attention on the weak s-process effect on the heavy nuclei of A > 90. The pre-existing nuclei in the mass region of A > 90 are irritated by successive neutrons in the weak s-process, and thereby their abundance pattern is changed. We calculate the mass distribution created by the weak s-process in different evolutionary stages of a massive star with M = 25 M_{⊙}, Z = Z_{⊙}. The open and filled circles in Figure 9 show the solar abundances adopted as the initial abundances in the massive star, and the abundances of the s-processed seed nuclei. The abundance distribution of the mass region of A > 90 is locally changed to that of the main s-process in the AGB stars. For example, there are two r-process abundance peaks around A = 130 and 195, and an r-process hill around A = 150 (open circles in Fig. 9), but these peaks and hill clearly disappear after the weak s-process (filled circles in Fig. 9). To clarify the effect of the neutron irradiation, we present the abundance ratios of the s-processed...
seeds to the solar abundances, $N(s)/N(p)$, for the 22 $s$-nuclei in Figure 10. These $s$-nuclei are the members of the first scaling relation. The ratios are almost constant over the wide region of $A > 90$; thus, the mass distribution of the $s$-processed nuclei of $A > 90$ is approximately equal to the AGB $s$-process one. The abundances of the $p$-nuclei are proportional to those of the seed nuclei at the SN explosions. Therefore, the abundance change created by the weak $s$-process is the reason for the proportionality between the $p$- and $s$-nuclei.

The $N(s)/N(p)$ ratios in the mass region of $A < 90$ are lower than the average value of 23 (see Fig. 2). This can be understood by taking into account the enhancement of the $s$-processed seed in the mass region of $A < 90$. The abundance ratios of $N(s)/N(p)$ at $A < 90$ are larger than those at $A > 90$, as shown in Figure 10, and the abundances of the $p$-nuclei are proportional to those of the massive star $s$-nuclei. Therefore, the $N(s)/N(p)$ ratios for $A < 90$ are lower than those for the higher mass region.

We note that there are three deviations in Figure 10, for Ce, Er, and W. The solar abundances of $^{140}\text{Ce}$, $^{166}\text{Er}$, and $^{182}\text{W}$ are generated by both the $s$- and $r$-processes, since they are not shielded against $\beta^-$-decay. The partial abundances originating from the $r$-process disappear after the weak $s$-process, and hence the $N(s)/N(p)$ ratios decrease.

3.4. Contributions of Two Nuclear Reaction Paths

Here we study effects of two nucleosynthesis paths in the $\gamma$-process (Rayet et al. 1990; Rapp et al. 2006; Rauscher 2006). The first path consists of direct ($\gamma, n$), ($\gamma, p$), ($\gamma, \alpha$) reactions from heavier isotopes. The ($\gamma, n$) reactions are dominant near the $\beta$ stability line. The second path is $\beta^+$ decay after the freezeout of the $\gamma$-process. The seed nuclei are transmuted to light isotopes by successive photodisintegration reactions. This reaction flow reaches to the neutron-deficient region where the ($\gamma, \alpha$) or ($\gamma, p$) reaction rate is larger than the ($\gamma, n$) reaction rate. The neutron-deficient nuclei are subsequently transmuted to lighter elements by the downflow of the ($\gamma, \alpha$) and ($\gamma, p$) reactions. After the freezeout of the $\gamma$-process, the nuclei on the neutron-deficient side decay to stable isotopes.

Woosley & Howard (1978) reported an anticorrelation between the photodisintegration reaction rates and the solar abundances of the $p$-nuclei, and the ($\gamma, n$) reactions play an essential role in the synthesis of the $p$-nuclei. The first scaling law suggests that the contribution of the first path is dominant. However, $\beta^+$ decay from the neutron-deficient nuclei may break the scaling law, since the mass distribution on the neutron-deficient side of the flow depends on nuclear properties such as particle separation energies.

Here we study an effect of the second path on the final $N(s)/N(p)$ ratios. Figure 11a shows the $N(s)/N(p)$ ratios before and after $\beta^-$ decay of the neutron-deficient nuclei on the second path. The ratio before the $\beta^-$ decay corresponds to that contributed from only the first path. Figure 11b shows percentages of the $p$-nuclei populated from only the first path in relation to the final abundance due to both the paths. These $N(s)/N(p)$ ratios and percentages are obtained using a model of $Z = Z_s$, $M = 25 M_\odot$, and $E = 10^{51}$ erg.

In the mass region of $58 < Z < 82$, corresponding to $82 < N < 82$, the second path flow is located on the neutron-deficient side of the $\beta$ stability line, typical by mass unit $\Delta A \approx 2–6$. Thus, the $p$-nuclei in this mass region are produced by the two reaction paths. The calculated results show that the $N(s)/N(p)$ ratios before $\beta^+$ decay are almost constant in this mass region (filled circles in Fig. 11a). However, the percentage contributed by the first path scatters over a wide range of 5%–100% (see Fig. 11b). This indicates that the contributions of the second path to the individual $p$-nuclei are different. In fact, the variance of the final $N(s)/N(p)$ ratios after $\beta$-decay (Fig. 11a, open circles) is larger than that before the $\beta$-decay (filled circles). However, the final $N(s)/N(p)$ ratios are still almost constant, because both the $p$- and $s$-nuclei are populated by $\beta^+$-decay. Therefore, the second path does not drastically change the ratios in this mass region.

In the mass region $40 < Z < 58$, corresponding to $50 < N < 82$, the nucleosynthesis flow mostly proceeds along the $\beta$ stability line. A similar result was previously reported (Rayet et al. 1990). The ratio of ($\gamma, n$) to ($n, \gamma$) reaction rates is important for the position of the second path in the nuclear chart. Since the neutron separation energy increases with decreasing the proton number, the ($\gamma, n$) reaction rates in lower mass regions are lower than those in higher mass regions. Therefore, the position of the second path in the lower mass regions is close to the $\beta$-stability line. The second path proceeds mostly on the $\beta$-stability line in the mass region $N < 82$. The $N(s)/N(p)$ ratios before and after $\beta$-decay (Fig. 11a, filled circles) are lower than those after $\beta$-decay (open circles). This is because the second path is located almost on the $\beta$-stability line, but some neutron-deficient isotopes...
are still produced. The $\beta$-decay from the neutron-deficient isotopes populates the $s$-nuclei, whereas the $p$-nuclei are predominantly produced via the first path (see Fig. 11b). Therefore the $\beta$-decay increases the $N(s)/N(p)$ ratios.

In the mass region $32 < Z < 40$, there are only three elements, Se, Kr, and Sr. The ratios of these three elements scatter. The ratio before $\beta$-decay is approximately equal to that after $\beta$-decay, since the second path is located almost on the $\beta$-stability line and the $p$-nuclei are predominantly produced via the first path.

In this section, we have discussed the contributions of the two nucleosynthesis paths. In most cases, in the mass region $40 < Z < 58$, the $\beta$-decay of the second path nuclei populates only the $s$-nuclei in a pair, and hence the $N(s)/N(p)$ ratios increase after the $\beta$-decay. We would like to stress that the final ratios in the mass regions $40 < Z < 58$ and $58 < Z < 82$ are almost constant. The second path thus shows that the ratios are almost constant over a wide range of atomic numbers. Therefore, the contributions of both the first and second paths have an essential role in the first scaling relation. It should be noted that ($\gamma, n$) reactions have dominant role in the two mass regions $N < 50$ and $50 < N < 82$, and experimental measurements of ($\gamma, n$) reaction rates on neutron-deficient isotopes (e.g., Mohr et al. 2000) will be of importance for the quantitative calculation of the $\gamma$-process and the scaling relations.

3.5. Effects of ($\gamma, \alpha$) Reactions

The calculated $N(s)/N(p)$ ratios in Figure 7 show some deviations. We here discuss the reason for two deviations for Dy and Er in view of the nuclear data input for the nucleosynthesis flow. Figure 12 shows a typical second path flow at the freezeout of the $\gamma$-process. In this mass region, the neutron-deficient nuclei are mainly transmuted to lighter elements by ($\gamma, \alpha$) reactions. After the freezeout, the nuclei on the second path decay to $p$- and/or $s$-nuclei. In most cases, both the $p$- and $s$-nuclei have contributions from $\beta$-decay, and the final $N(s)/N(p)$ ratios are not drastically changed from the ratios before $\beta$-decay, as in the case of Yb. The $N(s)/N(p)$ ratios for Dy calculated by the various stellar models are systematically larger than those of other $p$-nuclei (see Fig. 7). This can be understood by considering the effects of $\beta$-decay from the second path. The $s$-nucleus $^{158}\text{Dy}$ is strongly populated by the $\beta$-decay, but the $p$-nucleus $^{158}\text{Dy}$ is predominantly produced by the first path (see Fig. 11b). As a result, the $N(^{158}\text{Dy})/N(^{156}\text{Dy})$ ratio increases after $\beta$-decay. In contrast, the calculated ratios of Er are systematically lower than the average value. The $p$-nucleus $^{164}\text{Er}$ is populated by the $\beta$-decay, but the $s$-nucleus $^{166}\text{Er}$ is not populated (see Fig. 11b). As the result, the $N(^{164}\text{Er})/N(^{166}\text{Er})$ ratio decreases after $\beta$-decay. The abundances populated by the $\beta$-decay are determined by the position and width of the second path, which depends strongly on the ($\gamma, \alpha$) reaction rates. Thus, the final $N(s)/N(p)$ ratios are sensitive to the ($\gamma, \alpha$) reaction rates for neutron-deficient isotopes.

Rauscher (2006) calculated photodisintegration reaction rates with various nuclear models and presented branching points where the ($\gamma, \alpha$) or ($\gamma, p$) reaction rate competes against the ($\gamma, n$) reaction rate. The results showed that the branching points depend on the nuclear model. The discussion above shows that the actual ($\gamma, \alpha$) reaction rates of Hf and Yb isotopes may be lower than the reaction rates used in the present calculations, and the second path may be located far from the $\beta$-stability line. If so, both the $p$- and $s$-nuclei of Dy and Er are populated by $\beta$-decay, as in the case of Yb, and the final $N(s)/N(p)$ ratios are not largely changed from those before $\beta$-decay. Thus, reaction rates based on experimental

![Diagram](https://via.placeholder.com/150)

**Fig. 12.**—Partial nuclear chart of Dy, Er, and Yb isotopes and a schematic view of nucleosynthesis flows. The $p$-nuclei are synthesized by two nuclear reaction paths: (1) direct photodisintegration reactions such as ($\gamma, n$) from heavier isotopes, and (2) a flow located in neutron-deficient side and subsequent $\beta$-decay. Arrows show a typical second path flow.
data with high accuracy are desired. The \((\gamma,\alpha)\) reaction rates have been systematically studied using \(\alpha\) particle induced reactions (Gyurký et al. 2006; Kiss et al. 2006; Özkân et al. 2007). A new generation of \(\gamma\)-ray sources from laser Compton scattering (LCS) \(\gamma\)-rays in an energy range of MeV has been developed (Litvinenko et al. 1997; Ohgaki et al. 1991; Aoki et al. 2004) and widely used for studies of nuclear astrophysics (Utsunomiya et al. 2003; Mohr et al. 2004; Shizuma et al. 2005; Hayakawa et al. 2006b). The LCS sources are powerful tools for measuring nuclear properties such as photo-induced reaction rates, because of the sharp edge in their energy spectra, and the tunable energy. The measurements of \((\gamma,\alpha)\) reaction rates for the \(p\)-nuclei using the LCS \(\gamma\)-rays should contribute to the improvement of \(\gamma\)-process models (Utsunomiya et al. 2006).

### 3.6. Extended Universality

The first scaling relation leads to constant \(N(s)/N(p)\) ratios across a range of atomic numbers in individual \(\gamma\)-processes, but their absolute values are considered to depend strongly on astrophysical conditions. However, most \(N(s)/N(p)\) ratios calculated in the four different SN models are not only constant but also centered on a specific value of 3, within a factor of 3 (see Fig. 7). These results suggest an extended universality, that the \(N(s)/N(p)\) ratios produced by individual SN \(\gamma\)-processes are centered on the specific value of 3.

### 3.7. Frequency of the \(s\)-Process in AGB Stars and Massive Stars in the Galaxy

The universality of the first scaling relation is important for studying the frequency of \(s\)-process episodes in AGB and massive stars. Figure 8 shows a schematic chart for the Galactic chemical evolution (GCE). The \(s\)-nuclei in the solar system originate mainly from the main \(s\)-process in the AGB stars (Gallino et al. 1998; Busso et al. 1999). The abundance, \(N_s(s)\), is proportional to the abundance synthesized by the individual main \(s\)-processes and the frequency of the formation of AGB stars, whereas \(N_s(p)\) is proportional to the \(\gamma\)-process abundance and the frequency of SNe. Using the estimated frequency of SNe and the calculated abundances of the \(p\) and \(s\)-nuclei in single nucleosynthesis, we can estimate the frequency of the AGB \(s\)-process relative to the \(\gamma\)-process.

The extended universality gives a ratio of \(s\)-nucleus abundance originating from AGB stars to that from massive stars. The extended universality shows that the \(N_{SN}(s)/N_{SN}(p)\) ratio produced by a single SN is almost equal to 3 in the mass region of \(A > 90\). The \(s\)-process abundances in the solar system are generated by both the AGB and massive stars associated with SNe; namely, \(N_s(s) = N_{AGB}(s) + N_{SN}(s)\). The \(p\)-nuclei originate predominantly from SNe; namely, \(N_s(p) = N_{SN}(p)\). Using the first scaling law of \(N_s(s)/N_s(p) = 3\) and the extended universality of \(N_{SN}(s)/N_{SN}(p) = 3\), we obtain the result that the ratio of \(N_{AGB}(s)/N_{SN}(s)\) in the solar system should be about 6.7 in the mass region of \(A > 90\).

### 3.8. Proposal for Astronomical Observations of Indium to Verify the Extended Universality

The verification of the extended universality of the \(\gamma\)-process is essential for the discussion given above. Recent progress in spectroscopic studies of metal-poor stars has enabled isotope separation of several heavy elements, such as Eu, by measurements of hyperfine splitting and isotope shifts (Snedden et al. 2002; Lambert & Prieto 2002; Aoki et al. 2003). The observation of \(^{113}\text{In}\) is the most promising among all the \(p\)-nuclei in extrasolar objects, for two reasons: (1) it has only two odd-\(N\) stable isotopes, a \(^{113}\text{In}\) \(p\)-nucleus and a \(^{115}\text{In}\) \(s\)-nucleus; and (2) the \(^{115}\text{In}\) \(p\)-nucleus has an isotopic fraction as large as 4.29% in the solar system. In contrast, most \(p\)-isotopes are even-even nuclei and have many isotopes. There are three other odd-\(Z\) or odd-\(N\) \(p\)-nuclei, \(^{115}\text{Sn}\), \(^{138}\text{La}\), and \(^{185}\text{Ta}\), but their isotopic fractions are small (0.012%–0.34%). The absorption line of \(^{115}\text{In}\) (4511.31 Å) has been detected in the solar photosphere (Goldberg et al. 1960; Lambert et al. 1969). Recently, the \(\text{In}\) line at 4511 Å was also detected in Sun-like stars (Gonzalez 2006). If the indium isotopic fractions were observed in metal-deficient \(r\)-process enhanced stars, it would be valuable evidence for the universality of the \(\gamma\)-process. The calculated \(N(\text{\text{^{115}In}})/N(\text{\text{^{113}In}})\) ratios in the four models are 3.1–8.5, corresponding to the \(^{113}\text{In}\) fraction of 11%–24%. The fraction of \(p\)-isotopes in metal strongly affected by a single SN should be enhanced, although the enhancement may be small because of the mixing of materials from other layers.

### 4. EVIDENCE FOR THE WEAK \(s\)-PROCESS

We have suggested that the weak \(s\)-process before SN explosions is important in setting the proportionality between the \(p\)- and \(s\)-nuclei in solar abundances. The three large deviations of \(N_s(s)/N_s(p)\) for Ce, Er, and W give a hint to understanding the weak \(s\)-process. The ratios of the three elements are larger by an order of magnitude than those of the other elements (see Fig. 2). These deviation originate from enhancements of heavier isotopes, \(^{140}\text{Ce}, \text{^{166}}\text{Er}, \text{^{182}}\text{W}\), in their pairs. These three isotopes are not shielded against \(\beta\)-decay after the freezeout of the \(r\)-process, and the contributions of the \(r\)-process for \(^{166}\text{Er}\) and \(^{182}\text{W}\) are larger than those of the \(s\)-process (Arlandini et al. 1999).

The solar abundances can be expressed by

\[
N_s(Z, N) = N_p(Z, N) + N_s(Z, N) + N_s(Z, N).
\]

The abundances of typical \(s\)-nuclei are expressed by \(N_s(Z, N) = N_s(Z, N)\), but the abundances of the three heavier isotopes are expressed by \(N_s(Z, N) = N_s(Z, N) + N_s(Z, N)\). The first scaling law indicates that the abundance of the \(p\)-nucleus, \(N_p(Z, N)\), is proportional to the heavier isotope abundance of \(N_s(Z, N) + N_s(Z, N)\) at SN explosions; namely, \(N_p(Z, N - 2) = \alpha[N_s(Z, N) + N_s(Z, N)]\), where \(\alpha\) is about 1/23 at the solar system. The abundances of the \(p\)-nuclei can be expressed by \(N_p(Z, N - 2) = N_p(Z, N - 2) + N_p(Z, N - 2)\), where \(N_p(Z, N - 2) = N_s(Z, N)\), and \(N_p(Z, N - 2) = N_s(Z, N)\).

The weak \(s\)-process occurs in massive stars before SN, and the mass distribution of pre-existing heavy elements is changed to that of the \(s\)-process (see Fig. 10). The abundance \(N_s(Z, N) + N_s(Z, N)\) is changed to \(N_s(Z, N)\), and the \(p\)-nucleus abundance \(N_p(Z, N - 2)\) originating from \(N_s(Z, N)\) vanishes; for example, \(N_p(\text{^{164}}\text{Er}) = (1/23)N_s(\text{^{166}}\text{Er})\). Because the solar abundances of \(^{140}\text{Ce}, \text{^{166}}\text{Er}, \text{^{182}}\text{W}\) are produced by both the \(r\)- and \(s\)-processes, the \(N_s(s)/N_s(p)\) ratios of these three elements can be expressed by \(N_s(Z, N) + N_s(Z, N)/N_s(Z, N - 2)\), which is larger than the \(N_s(Z, N)/N_p(Z, N - 2)\) of neighboring \(p\)-nuclei by a factor of \(N_s(Z, N)/N_s(Z, N)\). In this way, three large enhancements in \(N(s)/N(p)\) ratios are observed in the solar abundances. In contrast, if the weak \(s\)-process does not occur before the \(\gamma\)-process, the abundances of \(p\)-nuclei in these three elements should also be proportional to those of heavier isotopes. Therefore, the three deviations for Ce, Er, and W are evidence that the weak \(s\)-process in massive stars occurs before the \(\gamma\)-process.
1957). Takahashi & Yokoi (1983) suggested that a stable isotope in contrast, the first temperature environments of $^{163}\text{Dy}$ becomes to unstable to $^{164}\text{Er}$ becomes to unstable by atomic effects and decays to $^{163}\text{Ho}$. $^{166}\text{Er}$ is synthesized by the $s$- and $r$-processes, since it is not shielded against $\beta$-decay after the $r$-process.

4.1. Thermometer for the $s$-Process

The question of why the abundance of $^{164}\text{Er}$ is an order of magnitude larger than those of neighboring $p$-nuclei, as shown in Figure 4, was already being asked fifty years ago (Burbidge et al. 1957). Takahashi & Yokoi (1983) suggested that a stable isotope $^{163}\text{Dy}$ becomes to unstable to $\beta$-decay at typical $s$-process temperatures because of the atomic effect of ionized ions, and $^{165}\text{Er}$ may be synthesized by a weak branch of the $s$-process,

$$^{165}\text{Dy}(e^-\nu)^{165}\text{Ho}(n,\gamma)^{166}\text{Ho}(e^-\nu)^{164}\text{Er}.$$  

The $\beta$-decay rate of $^{163}\text{Dy}$ depends on the thermodynamic conditions of the $s$-process, and hence the abundance ratio of $N_s(^{160}\text{Er})/N_r(^{160}\text{Er})$ is sensitive to the temperature. However, the evaluation of $N_s(^{160}\text{Er})/N_r(^{160}\text{Er})$ seems to be difficult, because $^{160}\text{Er}$ is produced by the $s$- and $\gamma$-processes, and $^{166}\text{Er}$ is produced by the $s$- and $r$-processes, as shown in Figure 13. We here propose a new method to evaluate the $s$-process abundances of $^{164,166}\text{Er}$ using the empirical scalings, $N(s)/N(p) = 23$ and $N(1st\,p)/N(2nd\,p) = 1$. Because the second $p$-nucleus $^{162}\text{Er}$ is dominantly synthesized by the $\gamma$-process, we can evaluate the $p$-process abundance of $^{164}\text{Er}$ and the seed $s$-nucleus abundance of $^{160}\text{Er}$ using the scaling laws. The resulting ratio, $N_s(^{160}\text{Er})/N_r(^{160}\text{Er})$, is 2.2 at the solar system. Arlandini et al. (1999) calculated the component of all $s$-nuclei using a stellar model for low-mass AGB stars with $^{13}$C burning. The calculated ratio $N_s(^{160}\text{Er})/N_r(^{160}\text{Er})$ is 3.8, which is consistent with the present evaluated ratio of 2.2 within a factor of 2. Therefore, the empirical scalings support the $s$-process calculations by Arlandini et al.

5. OTHER ORIGINS

Our discussion shows that the two scalings laws are evidence that $27$ $p$-nuclei are predominantly produced by the SN $\gamma$-process. Another model discussed in the literature (Howard et al. 1991) is the $\gamma$-process in the outer layers of exploding white dwarf stars. Those outer layers should have been enriched in $s$-process nuclei as the ashes of He shell burning settled on them in the star’s AGB phase. If the burning front passing through them during the explosion tends to give the right set of conditions, a $\gamma$-process might occur. Although the $p$-isotope yields from exploding white dwarf stars are highly uncertain, they can be enormous, and may be significant contributors to the solar system’s supply of the $p$-isotopes. The question of whether the $\gamma$-process in white dwarf stars can satisfy the scaling laws still remains.

There are $35$ $p$-nuclei altogether; the origin of the other $8$ $p$-nuclei, $^{92,94}\text{Mo}$, $^{96,98}\text{Ru}$, $^{144}\text{Sm}$, $^{115}\text{Sn}$, $^{138}\text{La}$, and $^{180}\text{Ta}$, also remains an open question.

5.1. Even-Even Nuclei

The first scaling relation in solar abundances shows a large deviation for Mo and Ru isotopes (see Fig. 2). The isotope abundances of these nuclei are about $10$ times larger than those of typical $p$-nuclei. It is well known that $\gamma$-process calculations underestimate the abundances of these isotopes, $^{92,94}\text{Mo}$ and $^{96,98}\text{Ru}$. Recently, nucleosynthesis in the early neutrino wind in core-collapse SNe ($\nu$-$p$-process) has been studied as the origin of these isotopes (Pruet et al. 2006; Fröhlich et al. 2006; Wanajo 2006).

Burbidge et al. (1957) pointed out that the mass distribution of the $p$-nuclei shows two peaks near the two neutron magic numbers $N = 50$ and $82$, and suggested that the $p$-nuclei may be synthesized by $(p,\gamma)$ and $(\gamma, n)$ reactions. The abundance pattern around the Mo region is similar to that of Sm. As shown in Figure 4, the abundances of $^{92,94}\text{Mo}$ are larger than those of lighter $p$-isotopes, $^{84}\text{Sr}$ and $^{78}\text{Kr}$, and the abundance of $^{144}\text{Sm}$ is also larger than those of lighter $p$-nuclei. In contrast, the abundances of the seed $s$-nuclei, $^{96}\text{Mo}$, $^{100}\text{Ru}$, and $^{148}\text{Sm}$, are consistent with those of the same mass region; namely, their abundances are lower than those of $s$-nuclei in the lighter mass regions beyond the neutron magic numbers. This fact suggests the possibility that these five $p$-nuclei, $^{92,94}\text{Mo}$, $^{96,98}\text{Ru}$, and $^{144}\text{Sm}$, might be synthesized mainly from $s$-nuclei located in lighter mass regions beyond the neutron magic numbers by particle-induced reactions such as $(p,\gamma)$, $(\alpha,\gamma)$, or $(n,\gamma)$. Recently, proton-capture experiments have been carried out up to $A \sim 120$ (Özkan et al. 2002; Spyrou et al. 2007). However, the reaction rates in a heavy mass region of $A \sim 140$ have not been measured. These rates are of importance for understanding the origin of the $p$-nuclei near the neutron magic numbers $N = 50$ and $82$.

Recent progress in meteorite science has provided crucial hints about the origin of Mo and Sm. Yin et al. (2002) measured isotopic fractions of Mo in primitive meteorites with high accuracy, and isotopic fractions of the $p$-nucleus $^{94}\text{Mo}$ and the $r$-nucleus $^{100}\text{Mo}$ show different anomalies in comparison with the solar abundances. Yin et al. concluded that there are at least three patterns of isotopic anomalies of the Mo isotopes. This result suggests that the nucleosynthesis site of $^{94}\text{Mo}$ is not correlated with the $r$-process site. Isotopic anomalies of Sm in primitive meteorites were also measured, and the results show that nucleosynthesis sites supplying the $p$- and $r$-isotopes are disconnected or only weakly connected (Andreassen & Sharma 2006). It would be useful to measure the isotope abundances of both Sm and Mo in individual meteorite samples in order to answer the question of whether the nucleosynthesis sites of Sm and Mo are same or not.

5.2. Odd-A Nuclei

The $\gamma$, $s$, and $r$-processes have all been proposed as the origin of $^{115}\text{Sn}$. $^{115}\text{Sn}$ can be synthesized through a $\beta$ unstable isomer in $^{113}\text{In}$ after the freezeout of the $r$-process, or by a weak branch of the $s$-process by $^{113}\text{Cd}(e^-\nu)^{113}\text{In}(n,\gamma)^{114}\text{In}(e^-\nu)^{114}\text{Sn}(n,\gamma)^{115}\text{In}$.

However, previous studies reported that $^{115}\text{Sn}$ cannot be produced in sufficient quantity by the $s$-process (Németh et al. 1994). Németh et al. suggested that the population of the isomer in $^{113}\text{Cd}$ should
follow by thermal-equilibrium conditions in the s-process environment $kT = 12-30$ keV. In this case, the population ratio of the isomer depends only on the temperature of the environment. Here we point out that recent studies of the s-process have suggested the possibility that the s-process may occur at low temperatures $kT < 10$ keV. If this is the case, the population of the excited states in $^{113}$Cd does not follow thermal-equilibrium conditions, and thus the $^{115}$Sn abundance depends on a partial neutron-capture reaction cross-section to the isomer of $^{113}$Cd. The neutron-capture reaction cross-section for the $^{113}$Cd isomer has never measured with high accuracy at any energy; this measurement would be useful.

5.3. Odd-Odd Nuclei

Two isotopes, $^{138}$La and $^{180}$Ta, are classified in a group of odd-odd nuclei. These two isotopes have the following unique features. First, their isotope abundances are small, 0.090% (for $^{138}$La) and 0.012% ($^{180}$Ta). Second, they are shielded against both $\beta^+$ and $\beta^-$ decay by stable isobars, and thus they are produced only by direct nuclear reactions, for example ($\gamma, n$) reactions from heavy isotopes or ($\gamma, e^- e^+$) reactions from isobars.

The cosmic-ray process (Audouze 1970) and the $\nu$-process (Woosley et al. 1999) have been proposed as the origin of $^{138}$La. Heger et al. (2005) suggested that about 90% in the solar abundance of $^{138}$La can be explained by the $\nu$-process, and Byelikov et al. (2007) quantitatively evaluated the contribution of a key charged current reaction to $^{138}$La. A study of primitive meteorites provides a hint as to the origin of $^{138}$La. Shen & Lee (2003) reported isotope abundance anomalies of several elements, including Ti and La, in Ca-Al-rich inclusions, which are believed to originated from the early solar nebula. The abundance ratios of $^{138}$La/$^{139}$La correlate with those of $^{50}$Ti/$^{48}$Ti (Shen & Lee 2003). However, just why $^{138}$La correlates with $^{50}$Ti remains an open question.

The origin of $^{180}$Ta is a hot topic in nuclear astrophysics, since $^{180}$Ta has the unique feature that the metastable state of $^{180}$Ta is the only isomer (half-life of $\geq 10^{15}$ yr) existing in the solar system, while the ground state is unstable against $\beta^-$ decay (half-life of 8 hr). As the origin of $^{180}$Ta, the s-process (Yokoi & Takahashi 1983; Belic et al. 1999), the $\gamma$-process (Woosley & Howard 1978), the $\nu$-process (Woosley et al. 1999), and the cosmic-ray process (Audouze 1970) have been suggested. A recent $\nu$-process calculation overproduces the solar abundance of $^{180}$Ta relative to $^{16}$O, which is based on a new experimental result using a $^{180}$Hf($^3$He,$t$)$^{180}$Ta reaction to evaluate the charged current reaction rate (Byelikov et al. 2007).

In explosive nucleosynthesis such as the $\gamma$- and $\nu$-processes, the population ratio of the metastable isomer to the $\beta^-$-unstable ground state at freezeout is an important factor. The transition probability from the isomer to the ground state was measured using real photons (Belic et al. 1999), and the population ratio from ($\gamma, n$) reactions was also measured (Goko et al. 2006). To estimate exactly the residual abundance of $^{180}$Ta, a time-dependent calculation is required. However, for estimating the isomer population, a thermal equilibrium approximation (Rauscher et al. 2002) is widely used, by which the population ratio of the isomer to the ground state can be calculated from the Maxwell-Boltzmann population at critical temperature $T_{\text{crit}}$, which is the lowest temperature of the thermal equilibrium region.

It should be noted that exited states located above the isomer are also populated in high-temperature environments, and may decay to the isomer. On other words, the excited states above the isomer also contribute to the residual abundance of $^{180}$Ta. Mohr et al. (2007) calculated the temperature $T_{\text{crit}}$ based on a measured transition probability (Belic et al. 1999), and concluded that $T_{\text{crit}}$ is 40.4 keV, which leads to $P_{\text{m}} / P_{\text{total}}$ of 0.35.

Here we discuss the population of the isomer under the thermal equilibrium approximation. The population of an excited state is followed to the Maxwell-Boltzmann population presented by

$$P_i / P_{\text{gs}} = \frac{2J_i + 1}{2J_{\text{gs}} + 1} \exp\left(\frac{E_i}{kT}\right).$$

(1)

In a temperature range of $T' = 0.1-1.0$, all excited states lower than a few hundred keV energy are populated (see Fig. 14). After the freezeout, each excited state decays to the ground state or the isomer. The final population ratio of the isomer to the total is given by

$$P_{\text{m}} / P_{\text{total}} = \frac{\sum (P_i / P_{\text{gs}})}{\sum (P_i / P_{\text{gs}}) + \sum (P_i / P_{\text{gs}})} ,$$

(2)

where $i$ and $j$ indicate levels decaying to the isomer and the ground state, respectively. We calculate the contribution of the excited levels presented in Figure 14, since the contribution of the higher levels is lower than 0.1%. In previous calculations, the residual ratios of 0.3-0.5 have been used (Rauscher et al. 2002). However, the present calculated ratios are 0.06, 0.13, 0.20, and 0.26 for the assumed critical temperature of $T'_{\text{crit}} = 0.20, 0.25, 0.30,$ and 0.35, respectively. This result shows that the residual abundance of $^{180}$Ta should decrease from previous estimates if the thermal equilibrium is a good approximation.

6. CONCLUSION

We present two empirical scaling laws for $p$-s-nuclei with the same atomic number in solar system abundances. The first and the second $p$-nuclei are lighter than the $s$-nucleus by 2 and 4 neutrons, respectively. The first scaling law is the correlation between the $p$-s-nuclei. The corrected $N(p)/N_s$ (1st $p$) and $N_p/N_s$ (2nd $p$) ratios are almost constant over a wide region of atomic numbers, and are mostly centered around a value of 23, within a factor of 2. These scaling laws are evidence that the $\gamma$-process is the most probable origin of the 27 $p$-nuclei.

The scaling relations in the solar system lead to a novel concept, the universality of the $\gamma$-process, according to which the abundance ratios $N(s)/N(p)$ produced by individual SN $\gamma$-processes are almost constant over a wide region of atomic numbers. The $\gamma$-process calculations for core-collapse SNe under various astrophysical conditions support the universality of the $\gamma$-process. Three mechanisms contribute to the universality. The first mechanism is the weak $s$-process. It is well known that light elements of $A < 90$ are produced by successive neutron capture reactions from iron seeds in the weak $s$-process. However, pre-existing heavy elements of $A > 90$ are also irradiated by neutrons, and their mass distribution is changed locally to that of the main $s$-process in asymptotic giant branch stars. Therefore, the mass distribution patterns of the seed nuclei in supernova explosions are almost identical. The second mechanism is the shifting of the mass regions of the $\gamma$-process layers to keep their peak temperature. The $\gamma$-process layers are shifted to outer layers as the explosion energy or mass increases. Inside the $\gamma$-process layers, the seed nuclei are completely destroyed by photodisintegration reactions, whereas the $p$-nuclei outside the $\gamma$-process layers are not synthesized, because of their low temperature. Therefore, the physical conditions of the $\gamma$-process layers are almost identical. The third mechanism is independence from two nucleosynthesis flows. The $p$-nuclei are produced by two flows: direct ($\gamma, n$)
reactions from heavier isotopes, and $\beta$-decay after downflow by \((\gamma, p)\) and \((\gamma, \alpha)\) reactions from heavier elements, which may break the scaling laws. We calculated the contributions of both the flows for each p-nucleus. The \((\gamma, n)\) reactions are dominant in the light mass region of \(N < 82\). The \(^{12}C\)-decay after the downflow contributes to the p-nucleus abundances in the heavy mass region of \(N > 82\), but does not drastically change the abundance ratios, because both the p- and s-nuclei are populated by \(^{12}C\)-decay in most cases. In this way, the scaling laws hold for individual \(\gamma\)-processes.

In addition, we find that large deviations of \(N_{SN}(s)/N_{SN}(p)\) for Ce, Er, and W in the solar system are evidence that the weak s-process actually occurs before SNe. The present calculated results suggest an extended universality, that the \(N_{SN}(s)/N_{SN}(p)\) ratios produced in individual SNe may be constant around a specific value of 3, independent of the stellar conditions. With the extended universality of \(N_{SN}(s)/N_{SN}(p) = 3\) and the first scaling law of \(N_{SN}(s)/N_{SN}(p) = 23\), we estimate that the ratio of the s-process abundance contributions from AGB stars to that from massive stars associated with SNe is about 6.7 for the s-nuclei of \(A > 90\) in the solar system. Astronomical observations of indium isotope separation would be useful for verifying the extended universality. This observation could provide crucial input for identifying the \(\gamma\)-process site and GCE for p- and s-nuclei.

This work has been supported in part by Grants-in-Aid for Scientific Research in Japan (18340071).

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