Collider Prospects for Muon $g - 2$ in General Two Higgs Doublet Model

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Recent progress on muon $g - 2$ measurement prompts one to take it even more seriously. In the general two Higgs doublet model that allows extra Yukawa couplings, we take a simplified approach of single enhanced coupling. We fix the charged lepton flavor violating coupling, $\rho_{\tau\mu} = \rho_{\mu\tau}$, via the one-loop mechanism, for illustrative masses of the heavy scalar $H$ and pseudoscalar $A$, where we assume $m_A \approx m_{H^+}$. Since extra top Yukawa couplings are plausibly the largest, we turn on $\rho_{tt}$ and find that LHC search for $gg \to H, A \to \tau\mu$ gives more stringent bound than from $\tau \to \mu\gamma$ with two-loop mechanism. Turning on a second extra top Yukawa coupling, $\rho_{tc}$, can loosen the bound on $\rho_{tt}$, but LHC constraints can again be more stringent than from $B \to D_{\mu\nu}$ vs $D_{\tau\nu}$ universality. This means that evidence for $H, A \to \tau\mu$ may yet emerge with full LHC Run 2 data, while direct search for $\tau^\pm\mu^\pm bW^+$ or $tcbW^+$ (plus conjugate) may also bear fruit.

I. INTRODUCTION

After extended, meticulous efforts, the Fermilab Muon $g - 2$ experiment announced recently their first measurement [1], $a_\mu$(FNAL) = 116592040(54) × 10$^{-11}$ (0.46 ppm). This confirms the previous result [2] at Brookhaven National Laboratory, combining to give [1]

$$a_\mu(\text{Exp}) = 116592061(41) \times 10^{-11} (0.35 \text{ ppm}) .$$

(1)

Comparing this with the “consensus” theory prediction [3] for the Standard Model (SM), namely $a_\mu(\text{SM}) = 116591810(43) \times 10^{-11} (0.37 \text{ ppm})$, the difference $a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$,

(2)

is at 4.2σ. Eq. (1), however, is consistent with a new lattice result [4] based on staggered fermions. Thus, the issue of the true SM value remains. We shall take Eq. (2) as is and seek 1σ solution with New Physics.

The persistence of the “muon $g - 2$ anomaly” means there is a truly vast theory literature, hence we refer to a very recent comprehensive account [5] for more complete references. Ref. [5] stresses the need for chiral enhancement in solving the muon $g - 2$ anomaly, whereas the two Higgs doublet model (2HDM) is the only possibility without introducing new vector bosons or leptoquarks’. We shall follow the 2HDM that is “not flavor-aligned” [5], which possesses extra Yukawa couplings such as charged lepton flavor violating (CLFV) $\tau - \mu$ couplings, namely Ref. [6] (which descends from Refs. [7, 12]). We expand on the impact at the Large Hadron Collider (LHC) by considering extra top Yukawa couplings [13].

The well-known 2HDM Model I and II invoke a $Z_2$ symmetry to implement the Natural Flavor Conservation (NFC) condition of Glashow and Weinberg [14], i.e. just one Yukawa matrix per quark charge (and for charged leptons as well). But this is “special”, if not ad hoc, so in the general 2HDM (g2HDM) one drops the $Z_2$ symmetry and let Nature reveal her flavor design. First called Model III [15], and following the footsteps of the Cheng-Sher ansatz [16], indeed the emergent fermion mass mixing hierarchies can be exploited to ease the worries [14] of flavor changing neutral couplings (FCNC): extra Yukawa matrices should trickle off when going off-diagonal. The recent emergent alignment phenomenon, that the observed $h$ boson at 125 GeV resembles very closely the SM Higgs boson, brought in a flavor-independent surprise: alignment suppresses [18] FCNC involving the $h$ boson. Nature’s designs for flavor seem intricate.

The alignment control of FCNC is illustrated by $h \to \tau\mu$ search. The CMS experiment found initially [19] an intriguing $2\sigma$ hint, which subsequently disappeared [20]. The full Run 2 data at 13 TeV gives [21],

$$B(h \to \tau\mu) < 0.15\%.$$ (CMS2021)

But since the FCNC $\rho_{\tau\mu}$ arises from the heavy exotic doublet $\Phi^* (\langle \Phi \rangle = 0)$ rather than the mass-giving doublet $\Phi$ (sole source of vacuum expectation value), the $h\tau\mu$ couplings as $\rho_{\tau\mu}c_\gamma$, where $c_\gamma \equiv \cos \gamma$ is the $h - H$ mixing angle between the two CP-even scalar bosons. Thus, alignment, that $c_\gamma$ or $h - H$ mixing is small, can account for Eq. (3) without requiring $\rho_{\tau\mu}$ to be small, which is analogous to another FCNC process, $t \to ch$ (with coupling $\rho_{tc}c_\gamma$). This is the starting point for a one-loop mechanism (see Fig. 1 for $\tau \to\mu\gamma$).
Invoking the one-loop mechanism of Fig. 1 left to account for muon $g - 2$ for $m_A, \mu$ at the weak scale implies rather large $\rho_{\tau\mu}$, at several times $\lambda_\tau \approx 0.01$, the tau Yukawa coupling in SM. Eq. (3) then demands $|\epsilon_{\tau}| \ll 1$, i.e. near the alignment limit. On one hand this calls for a symmetry, which we do not get into. On the other hand, one should turn to $H, A \rightarrow \tau \mu$ (and $H^+ \rightarrow \tau^+ \nu_{(\mu)}$, $\mu^+ \nu_{(\tau)}$) search [23], as it is not hampered by small $c_\gamma$ but at full strength of $s_\gamma (\equiv \sin \gamma) \rightarrow -1$. This needs finite $\rho_{\mu}$ for gluon-gluon fusion production, so let us articulate our approach.

With one Higgs doublet of the SM already fully affirmed, adding a second doublet is the most conservative and simple extension. But, while simple, without a $Z_2$ symmetry to enforce NFC, the g2HDM possesses many new parameters. We therefore take a simplified approach of one large extra Yukawa coupling at a time, especially for muon $g - 2$ anomaly, which are more stringent than from Belle. We shall take $\rho_{\mu}$ and $\rho_{\tau\mu}$, where the Belle experiment has recently updated [24] with full data. This constrains $\rho_{\mu}$ to be considerably smaller than the $\rho_{\mu}$ needed for muon $g - 2$. We will show that a recent search [24] for $gg \rightarrow H \rightarrow \tau \mu$ with 36 fb$^{-1}$ data at 13 TeV, when interpreted in g2HDM, would place bounds on $\rho_{\mu}$ that are more stringent than from Belle.

To enlarge the allowed range for $\rho_{\mu}$, we turn on a second extra top Yukawa coupling, $\rho_{t\mu}$, which can dilute the branching ratio $B(H, A \rightarrow \tau \mu)$, thereby extend the allowed range for $\rho_{\mu}$. However, the product of $\rho_{t\mu}$ and $\rho_{\mu}$ can induce $B \rightarrow D_{\mu\tau}$ rate (as the $\tau$ flavor cannot be detected), thereby break universality with $B \rightarrow D_{e\tau}$. Depending on the $H^+$ mass, $\rho_{t\mu}$ can be in a range such as $\tau^\pm \mu^\pm b^{\pm} \nu^\pm$ or $\tau bW^+$ at the LHC should be of interest.

The purpose of this paper is therefore threefold. First, g2HDM can [6, 11] account for muon $g - 2$ anomaly, which is not new. Second, interesting signatures may emerge soon, not just at Belle II, but also at the LHC, if large $\rho_{\tau\mu}$ is behind the muon $g - 2$ anomaly. Our two LHC contact points above imply that $gg \rightarrow H, A \rightarrow \tau \mu$ may suddenly emerge, perhaps even with full Run 2 data, or detailed work may discover novel signatures such as $\tau^\pm \mu^\pm b^{\pm} \nu^\pm$ or $\tau bW^+$ at the LHC. Third, this illustrates how limited our knowledge of g2HDM really is.

In Sec. II we discuss the one-loop mechanism and find the bound on $\rho_{\tau\mu}$, then discuss flavor constraints on $\rho_{t\mu}$ and $\rho_{\mu}$, that follow, as well as various flavor concerns; in Sec. III we compare $\tau \rightarrow \mu \gamma$ constraint on $\rho_{t\mu}$ with direct search for $gg \rightarrow H, A \rightarrow \tau \mu$, and $B \rightarrow D_{\ell\nu}$ universality ($\ell = e, \mu$) constraint on $\rho_{\mu}$ with $\tau^\pm \mu^\pm b^{\pm} \nu^\pm$ or $\tau bW^+$ search at the LHC; after some discussion in Sec. IV, we offer our summary.

**II. ONE-LOOP MECHANISM FOR MUON $g - 2$**

The Yukawa couplings in g2HDM are [23, 27]
\[
- \bar{\nu}_i \rho_{ij} R f_j H^+ - \frac{1}{\sqrt{2}} \sum_{f=e,\mu} \bar{f}_i \left[ (\lambda^f_i \delta_{ij} c_\gamma + \rho_{ij}^f s_\gamma) H - i \text{sgn}(Q_f) \rho_{ij}^f A \right] \left( \lambda^f_i \delta_{ij} s_\gamma - \rho_{ij}^f c_\gamma \right) R f_j + h.c.,
\]
where $i, j$ are summed over generations, $L, R = (1 \mp s_\gamma)/2$ are projection operators, $V$ is the CKM matrix, with lepton matrix taken as unity due to vanishing neutrino masses. One can therefore read off the $\rho_{\mu\tau}$, $\rho_{\tau\mu}$ and $\rho_{t\mu}$ couplings indicated in Fig. 1.

We do not write down the Higgs potential $V(\Phi, \Phi')$ (except assuming it is CP conserving), as it can be found in many papers traced to Ref. [26], and illustrate with $\rho_{\mu\tau}$ values, that follow, as well as various flavor concerns; in Sec. III we compare $\tau \rightarrow \mu \gamma$ constraint on $\rho_{t\mu}$ with direct search for $gg \rightarrow H, A \rightarrow \tau \mu$, and $B \rightarrow D_{\ell\nu}$ universality ($\ell = e, \mu$) constraint on $\rho_{\mu}$ with $\tau^\pm \mu^\pm b^{\pm} \nu^\pm$ or $\tau bW^+$ search at the LHC; after some discussion in Sec. IV, we offer our summary.
accommodate flavor constraints, especially in the quark $\rho$-gent bound on $H$, $A$

\begin{align}
|\rho_{\nu\tau}| &\lesssim 0.024, 0.037, 0.053 \quad \text{for } m_H = 300 \text{ GeV and } m_{H^\pm} = m_A = 340, 420, 500 \text{ GeV, giving} \\
|\rho_{\tau\tau}| &\lesssim 0.08, 0.19, 0.32.
\end{align}

The decreasing value of $\rho_{\tau\mu}$ with increasing $m_A$, viz. Eq. (6), implies the opposite for $\rho_{\tau\nu}$, which for $m_A \gtrsim 400 \text{ GeV can surpass the } \rho_{\tau\mu} \text{ strength needed for } 1\sigma \text{ solution of muon } g-2.$

In fact, a good part of the dilution effect for $B(H, A \rightarrow \tau \mu)$ is driven by the opening of $A \rightarrow HZ$ decay for $m_A \gtrsim 400 \text{ GeV}, \text{ which is one reason why we increased } \Delta m = m_A - m_H > m_Z.$ With $m_A = m_{H^\pm}$, this means $H^+ \rightarrow HW^+$ also opens up. As we shall see in the next section, a sizable $\rho_{\tau\tau}$ can give rise to $c\gamma \rightarrow bH^+$ production (again, the $c\bar{b}H^+$ coupling is not CKM-suppressed), as well as $c\gamma \rightarrow tH, tA,$ with $c\gamma \rightarrow tA$ suppressed for higher $m_A$. $H^+ \rightarrow HW^+$ decay with $H \rightarrow t\bar{e}$ would lead to additional signatures at the LHC.

Let us comment on a few other flavor concerns. With $\rho_{\tau\mu}$ sizable, it can induce $\tau \rightarrow 3\mu$ decay at tree-level with $\rho_{\mu\mu} \neq 0$. Thus, the bound by Belle $B(\tau \rightarrow 3\mu) \lesssim 2.1 \times 10^{-8},$ puts a constraint on $\rho_{\mu\mu}$. Using formulas from Refs. [33, 38], we find $|\rho_{\mu\nu}| \lesssim (240-320)\lambda_\tau\lambda_\nu$ for $m_H = 300 \text{ GeV and } m_{H^\pm} - m_H = 10-200 \text{ GeV. This implies } |\rho_{\mu\nu}| \lesssim 10\lambda_\mu < 0.01, \text{ which is still rather small.}$

One evades $h \rightarrow \mu\mu$ search in alignment limit, $c_{\tau\tau} = 0$, while one also evades the recent $H, A \rightarrow \mu\mu$ search by CMS, after the multi-mode dilution effects discussed in the next section are taken into account.

The coupling $\rho_{\mu\tau}$ ($\rho_{\tau\mu}$) enters the $H^+\bar{\nu}_\tau (H^+\bar{\nu}_{\mu})$ coupling directly, and can generate $\tau \rightarrow \mu\nu\bar{\nu}_\tau$, where the neutrino flavors are swapped compared with $W$ boson exchange, which the experiment cannot distinguish. Compared with $\tau \rightarrow e\nu\bar{\nu}_e$, this constitutes another test of lepton universality violation at the per mille level, which has been recorded by HFLAV [40]. We have checked that $|\rho_{\mu\tau}| \lesssim 0.9$ is still allowed for our benchmark masses, which is more accommodating than muon $g-2$.

Finally, the $\rho_{\tau\mu} = \rho_{\mu\tau}$ coupling affects $Z \rightarrow \tau\tau, \mu\mu$ decays, which have been precisely measured [20], again at the per mille level. We find the bound to be weaker than the bounds from $\tau$ decays discussed here.
as one should always take the more stringent bound out of $H$ vs $A$. For the cases of $m_A = 340, 420$ GeV, the larger production cross sections for $A$ imply more stringent limit than from $H$, but for $m_A = 500$ GeV, parton densities have dropped too low, and the bound from $H$ is more stringent than $A$, but still more stringent than from $\tau \rightarrow \mu \gamma$. We remark that the case of $m_H, m_A = 300, 340$ GeV, i.e. Fig. 4(left), does provide a 1σ solution to muon $g-2$. But if all other extra Yukawa couplings, $\rho'_{ij}$s ($f = u, d, \ell$), are not stronger than $\lambda_{\text{max}(i,j)}$, the Yukawa strength in SM, then there are little other consequences, except in the $gg \rightarrow A \rightarrow \tau \mu$ probe of $\rho_{tt}$ itself. The reinterpretation of CMS bound of $\rho_{tt} \lesssim 0.02$ from Fig. 4(left) is below $\rho_{tt} \lesssim 0.05$ allowed by $\tau \rightarrow \mu \gamma$, which is really small compared with $\lambda_i \equiv 1$. But there is still discovery potential with full Run 2 data. The allowed range for $\rho_{tt}$ from CMS gets partially restored for heavier $m_A$, where $gg \rightarrow H, A \rightarrow \tau \mu$ production becomes predominantly $H \rightarrow \tau \mu$. However, as $m_H \sim 1$ GeV, while $\Gamma_A \lesssim 2$ GeV for $m_A < 400$ GeV, and remains below 10 GeV for $m_A \sim 500$ GeV, hence satisfy the narrow width approximation. We do not combine the two separate states.

We see that, with only a subset of Run 2 data, the CMS bound is more stringent than the bound from $\tau \rightarrow \mu \gamma$, as one should always take the more stringent bound out.
tion weakens progressively. It is less significant for $A$ production at $m_A = 500$ GeV in Fig. 4(right), because $A \to HZ$ is much stronger. But the corresponding drop in $gg \to H \to \tau\mu$ is rather significant. This is because the $\rho_{tc} \simeq 0.32$ value is about twice as large as the $\rho_{tm}$ value of $\simeq 0.167$. The large $\rho_{tc}$ brings about interest, thereby possible constraints, from production processes at the LHC.

Before discussing $\rho_{tc}$-induced processes, we remark that Fig. 5 where we also give $H^+$ decay branching fractions, is not just for our three benchmark values of $m_A = 340, 420, 500$ GeV. The plot is made by the same approach: for $m_H = 300$ GeV and a given $m_A$, we find the $1\sigma$ muon $g-2$ solution value for $\rho_{tm}$ analogous to Eq. (6), then find the allowed upper bounds for $\rho_{tt}$ and $\rho_{tc}$, analogous to Eqs. (2) and (9), respectively. The plot therefore scans through $m_A$, with $m_H$ fixed at 300 GeV.

Having $\rho_{tc}$ alone with $\rho_{tt}$ small, it can generate $cg \to tH, tA \to tt\bar{c}$ or same-sign top plus jet final state, which can feed the $t\bar{t}W$ control region (CRW) of 4t search by CMS (by different selection cuts, ATLAS is less stringent), where there is now a full Run 2 data study. More significant is a study of $cg \to bH^+ \to bAW^+$ production followed by $A \to t\bar{c}$ and both the top and the $W^+$ decay (semi-)leptonically, which can also feed the CRW of CMS 4t search. In this study, $H^+ \to AW^+$ decay was treated with $m_{H^+} = m_H$, i.e. swapping $H \leftrightarrow A$ from our present case. The $bH^+$ cross section is sizable due to a light $b$ quark and no CKM suppression for the $cbH^+$ coupling. It was found that $\rho_{tc} \simeq 0.15$ is barely allowed for $m_{H^+} \sim 500$ GeV. This may seem to make the large values of $\rho_{tc}$ for our dilution effect untenable.

However, the dilution effect may solve itself. We note that, assuming just two associated extra Yukawa couplings, i.e. $\rho_{tm}$ and $\rho_{tc}$, the two branching ratios are $B(H \to t\bar{c} + tc) : B(H \to \tau\mu) = 5% : 95%, 40% : 60%, 71% : 29%$, for $m_A = 340, 420, 500$ GeV and $(\rho_{tm}, \rho_{tc}) = (0.289, 0.08), (0.192, 0.19), (0.167, 0.32)$, respectively. For the $m_A = m_H = 340$ GeV benchmark, $\rho_{tc}$ is too small to be of concern. Likewise, for $m_A = m_{H^+} = 420$ GeV benchmark, from Fig. 4 of Ref. 13, which is for $m_H = 300$ GeV and $m_A = 350$ GeV, CRW of CMS 4t search would push $\rho_{tc}$ to below 0.4. However, $m_A$ at the much heavier 420 GeV reduces $cg \to tA \to t\bar{t}c$ events that feed CRW, so one is not yet sensitive to $\rho_{tc} \simeq 0.2$, while there is some dilution from $A \to HZ$. For the more efficient $cg \to bH^+$ production, we see from Fig. 5(right) that $H^+ \to cb$ at $\simeq 32\%$ is pure dilution, and even $\tau\nu(\mu) + \mu\nu(\tau)$ at $\simeq 22\%$ together with the $b$ would likely be overwhelmed by single-top background. The dominant ($\simeq 41\%$) $bHW^+$ final state would be $20\% b\tilde{t}W^+$ and $60\% b\tau^+\mu^-W^+$. The latter would be an interesting signature in itself, but would not feed CRW of CMS 4t search.

It is the $m_A = m_{H^+} = 500$ GeV benchmark that CRW of CMS 4t may put a limit on $\rho_{tc}$: $H^+ \to cb$ is slightly below 30%, while $\tau\nu(\mu) + \mu\nu(\tau)$ is no longer a dilution factor. This is due to the dominance of $H^+ \to HW^+$, now at 70%. With $H \to t\bar{c}$ at $\simeq 35\%$, it would feed 4t CRW abundantly, as pointed out in Ref. 28, for the analogous twisted custodial case for $bAW^+$ production. If we take the number there as a guessimate, then $\rho_{tc} \sim 0.15$ would be borderline. This just means that $\rho_{tc}$ should be considerably less than 0.32 from Eq. (8), which means that one cannot reach $\rho_{tt} \simeq 0.1$ as allowed by $\tau \to \mu\gamma$.

We note that, from Fig. 2 and for $m_H \sim 300$ GeV, $m_A = m_{H^+}$ at 500 GeV is already at the “border” of the scan space. Thus, we conclude that for large splitting of $m_A = m_{H^+}$ from $m_H$, the solution to muon $g-2$ would not allow a $\rho_{tc}$ value larger than $\rho_{tt}$ for the one-loop solution. As one moves $m_H$ higher, the discussion should be similar for higher $m_A = m_{H^+}$, requiring larger $\rho_{tt}$ values to solve muon $g-2$.

IV. DISCUSSION AND CONCLUSION

Referring to the 4.2$\sigma$ “white space of disagreement” between theory and experiment, the question “What monsters may be lurking there?” from the April 7 announcement by the Muon $g-2$ experiment, became the quote of the day. We stress that $\rho_{tt}/\lambda \sim 20$, while large, is not “monstrous”, and is up to Nature to choose (and the counterbalancing $\rho_{tt}/\lambda \lesssim 1/10$ as well). This can be compared, for example, with $\tan\beta = \mathcal{O}(10^3)$ in
the muon-specific 2HDM \cite{44}, tailor-made for the muon $g - 2$ by some $Z_4$ symmetry. In a different $Z_4$ arrangement, the $\mu$-phobic extra scalar doublet \cite{45} was introduced with only the $\rho_{\tau\mu} = \rho_{\mu\tau}$ coupling, with further variations of $m_A - m_H < 50$ GeV by considering the $\sim 2\sigma$ deviation from SM in $\tau$ decays, or pushing $m_H$ to be very light \cite{47}. We hope we have illustrated that such elaborations are not necessary, that the g2HDM is versatile enough with its arsenal of extra Yukawa couplings, to provide solution to muon $g - 2$ while hiding from view so far, but with promising LHC implications.

With enhanced $\rho_{\tau\mu}$ implied by muon $g - 2$, it has been suggested \cite{48,49} that, by the natural complexity of extra Yukawa couplings, they could possibly drive electroweak baryogenesis (EWBG), i.e. be the CP-violating source of the baryon asymmetry of the Universe. In fact, part of our motivation for considering $\rho_{\tau\mu}$ and $\rho_{\mu\tau}$ as our primary motivators is that the g2HDM is versatile enough with its arsenal of extra Yukawa couplings, that $\rho_{\mu\tau}$ values needed for muon $g - 2$, dileptonic decays of $H, A, H^{\pm}$ may prevail, which allow exquisite reconstruction \cite{6} of exotic extra Higgs boson masses, hence rather attractive. However, the electroweak production cross section, at the fb level, is rather small. Take the case of $m_H = 300$ GeV, $m_A = m_{H^+} = 340$ GeV for example. If we allow the lowest $\rho_{\tau\mu}$ value of 0.08 in Eq. (9), nominally from $B \to D\ell\nu$ universality constraint, we find that the $c\bar{c} \to bH^+$ cross section, being a strong production process involving a gluon (with not too significant a suppression from charm parton distributions) and only one heavy particle in final state, is almost two orders of magnitude higher than the electroweak production processes. This effect persists for the other two benchmarks where $A + H^+$ are heavier, which allow for larger $\rho_{\mu\tau}$ values. Furthermore, as we have illustrated, multiple decay modes of the heavier $A$ and $H^+$ (see Fig. 5), as well as $H \to \tau^+\tau^-; t\bar{t}(c\bar{c})$ would dilute one another, which works also for the decays of the extra scalar bosons in electroweak pair production. Thus, neither the search for the electroweak production of extra Higgs boson pairs, nor the reconstruction of the extra boson masses, may be as rosy as argued in Ref. \cite{6}.

In summary, we employ large CLFV Yukawa coupling $\rho_{\tau\mu} = \rho_{\mu\tau} \sim 0.2$ in the general 2HDM to account for muon $g - 2$ anomaly through the known one-loop mechanism. The extra Higgs bosons have mass at the weak scale, but one is close to the alignment limit of very small $h - H$ mixing to evade $h \to \tau\mu$. This motivates the check with $gg \to H, A \to \tau\mu$ search, where a subset of LHC Run 2 data already puts more stringent bound on the extra top Yukawa coupling $\rho_{\mu\tau}$ than from $\tau \to \mu\gamma$ through the two-loop mechanism. The stringent constraint on $\rho_{\mu\tau}$ can be eased by allowing a second extra top Yukawa coupling $\rho_{\mu t}$, which motivates the search for $c\bar{c} \to bH^+ \to \tau^+\tau^-; t\bar{t}(c\bar{c})$ plus two $b$-jets and additional jet, with missing $p_T$) at the LHC, on top of possible same-sign top plus jet signatures from $c\bar{c} \to tH, tA \to t\bar{t}c$. Whether these extra Yukawa couplings can drive electroweak baryogenesis should be further studied.

Acknowledgments The work of WSH is supported by MOST 109-2112-M-002-015-MY3 of Taiwan and NTU 110L104019 and 110L92101, and the work of RF and GK are supported by MOST 109-2811-M-002-516 and 109-2811-M-002-540, respectively. The research of CK was supported in part by the U.S. Department of Energy and the University of Oklahoma, and TM by a postdoctoral research fellowship from the Alexander von Humboldt Foundation.
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