Degenerate Thermal Pulses in AGB Stars

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We report on the discovery of a new kind of thermal pulse in intermediate mass AGB stars. Deep dredge-up during normal thermal pulses on the AGB leads to the formation of a long, unburnt tail to the helium profile. Eventually the tail ignites under partially degenerate conditions producing a strong shell flash with very deep subsequent dredge-up. The carbon content of the intershell convective region ($X_C \sim 0.6$) is substantially higher than in a normal thermal pulse ($X_C \sim 0.25$) and about 4 times more carbon is dredged-up than in a normal pulse.
1. Introduction

It is now just over 30 years since thermal pulses were discovered in AGB stars by Schwarzschild and H"arm (1965) and over 20 years since third dredge-up was discovered by Iben (1975), and we are still learning about the consequences of these events for stellar evolution and nucleosynthesis. Although much is known there are still many uncertainties, especially concerning third dredge-up (Frost and Lattanzio (1996)) and mass-loss. We are presently studying the effects of hot bottom burning (HBB) on intermediate mass AGB stars during their thermally pulsing evolution. During these calculations we found a new kind of thermal pulse which we report on in this paper.

2. Degenerate Thermal Pulses

2.1. Setting the Scene: Deep Dredge-Up

We consider the evolution of a $5M_\odot$ star with $Z = 0.004$, as calculated with an updated version of the Mount Stromlo Stellar Structure Program. Opacities are taken from Iglesias and Rogers (1993), the mass-loss rate is from Vassiliadis and Wood (1993)\textsuperscript{1}, and we treat dredge-up in the way described by Frost and Lattanzio (1996).

The star evolves as expected for the first 36 thermal pulses. We find very deep dredge-up, as also found by Vassiliadis and Wood (1993), with the dredge-up parameter $\lambda$ reaching close to unity after the first few pulses. This turns out to be crucial: immediately after a pulse, the deep dredge-up cools the intershell region and does not allow the helium shell to burn outward as much as it does when $\lambda$ is smaller. This is shown in Figure \ref{fig:figure1} where

\textsuperscript{1}Except that we do not include the modification for $M > 2.5M_\odot$, which reduces the mass-loss rate for these stars.
Fig. 1.— The time variation of the helium luminosity and the position of the three points within the helium shell, at $Y = 0.05$, 0.35, and 0.70. The upper panel shows the results for pulse 4 and the lower panel is for pulse 35.
we plot the time variation of the helium luminosity and the position of three representative points within the helium shell. The upper frame of the figure shows the case for the 4th pulse, when $\lambda = 0.78$. The period of extended helium burning following the pulse peak will be referred to as the “subpulse”, and the figure shows clearly that it is primarily during the subpulse that the helium shell advances outward in mass. The lower frame of the figure shows the same situation for the 35$^{th}$ pulse, when $\lambda = 0.93$. Here we see that the deep dredge-up extinguishes helium shell burning very quickly, with the result that the shell moves outward only a small distance in mass. In particular, note that the bottom of the shell, at low helium values, hardly moves at all. This results in the helium shell developing a very long ‘tail’ with $Y \approx 0.1$, as shown in Figure 2. Note that this tail develops only when the dredge-up becomes very deep, typically for $\lambda \gtrsim 0.8$. In summary, during the first few pulses the advance in mass of the helium shell matches that of the hydrogen shell. However, once $\lambda$ approaches unity the bottom of the helium shell starts to fall behind the rest of the shell, thereby widening it in mass. These developments are clearly illustrated in Figure 3.

With successive pulses both the density and degeneracy at the shell base rise. This is shown in Figure 4, which presents conditions at the point where the helium abundance is 0.05. Because of the cooling effect of dredge-up, the temperature experienced by the long tail of the helium shell is relatively low and helium does not burn (during the subpulse) as it normally does in the absence of deep dredge-up. Figure 4 shows that each of the early pulses (with low $\lambda$) experiences a brief time at high temperature, as represented by the temperature spikes. At this time the helium in the shell is usually burned, and the helium shell advances. But later pulses do not experience the brief high-temperature phase, and the helium tail begins to develop. Throughout all these phases, the density continues to rise as the core contracts. After a few tens of pulses, the tail of the helium shell extends down to densities not usually encountered by the helium shell in AGB stars and the bottom of the helium shell is mildly degenerate.
Fig. 2.— Abundance profile of the helium shell after the 34\textsuperscript{th} pulse. Note the long helium tail.
Fig. 3.— The time variation of various important positions (in mass) within the model. From top to bottom these are: the hydrogen shell, the centre of the helium shell (where $Y = 0.35$) and the bottom of the helium shell (where $Y = 0.05$).
Fig. 4.— Radius, temperature, density and degeneracy parameter at the base of the helium shell (where $Y = 0.05$) as functions of time. Note that the earliest pulses (and the first few following a degenerate pulse) show a brief rise in temperature. This is extinguished once
2.2. A Degenerate Pulse

During the 38th thermal pulse, the tail of the helium shell ignites. The peak of the burning is now centered at the base of the shell, rather than near the shell center, which is the normal case. This results in a very deep convective shell which extends from the bottom of the helium shell almost to the hydrogen shell, as clearly illustrated in Figure 5, which shows the convective boundaries during pulses 36–39. The isolated convection zone seen in Figure 5 at the bottom of the normal intershell convection zone in pulse 37 will be discussed below.

Thermal pulse number 38 occurs at higher temperatures and is greater in strength than normal. The abundances of $^4\text{He}$ and $^{12}\text{C}$ left by the intershell convective zone are about 0.34 and 0.58, respectively, compared with 0.74 and 0.24 in the previous pulse. Thus a large quantity of $^{12}\text{C}$ has been produced. Large expansion is driven by the pulse and very deep dredge-up occurs. The extent of dredge-up ($\lambda \approx 1.8$) is such that roughly 80% of the carbon is dredged-up by the envelope. Nearly four times the normal mass of carbon is dredged-up in a single pulse! The surface ratio of carbon to oxygen rises from 0.36 at the beginning of the pulse to 1.04 at the end, creating a carbon star in a model that will undergo significant Hot Bottom Burning during the next interpulse phase.

2.3. A Failed Degenerate Pulse

Another feature seen in Figure 5 is a small convective region at the bottom of the usual intershell convective zone for the 37th pulse (at $M_r \sim 0.890M_\odot$). This is exactly the same phenomenon we have discussed above, but in this case the flash was not strong enough for the convective zone at the base of the shell to grow and engulf the usual intershell convective zone. This event is really a failed degenerate pulse. The amount of helium burned is quite
Fig. 5.— Convective zones versus model number for pulses 36–39 of the model described in the text. Note that the $x$-axis is actually the model number in a post-processing nucleosynthesis code, not the evolution model number.
2.4. Degenerate Sub-Pulses

After the degenerate 38th pulse, the helium shell is reduced in width and is now closer to a normal position found when dredge-up is shallower. Subsequent evolution, shown in Figures 3 and 4, shows that as deep dredge-up continues, the same situation develops once more. The helium shell widens because the low helium tail of the shell remains unburned after each pulse, and a second degenerate pulse occurs 20 pulses later at pulse 58. Following this we again find the helium shell widening, but the next degenerate pulse occurs after only 11 more pulses. This degenerate pulse is different: it occurs during the dredge-up phase of the 69th pulse as shown in Figure 5. The development of this “subpulse” is due to a complex interplay between helium burning during the subpulse and the cooling produced by envelope convection, resulting in a small helium inversion in the tail of the helium shell.

2.5. Other Occurrences of Degenerate Pulses

During the calculation of nine evolutionary sequences ($M = 4, 5$ and $6M_\odot$, each with $Z = 0.004, 0.008$ and 0.02) we found six degenerate pulse events. Three occurred during the evolution of the $5M_\odot Z = 0.004$ case described here. One occurred for the $5M_\odot Z = 0.008$ case, and two during the evolution of the $6M_\odot$ model with $Z = 0.004$. It would certainly be premature of us to claim to have isolated any critical ranges in mass or composition for these events.
Fig. 6.— Convective zones versus model number for pulses 68 and 69.
3. **Physical and Numerical Dependence**

We propose that degenerate pulses and subpulses are caused by extremely deep dredge-up cooling the helium shell, thereby preventing burning of the lower helium shell which normally occurs during the subpulse phase. A long tail (in mass) of low helium abundance ($Y \lesssim 0.1$) forms and grows increasingly denser until finally it ignites under mildly degenerate conditions, either at the beginning of a pulse (a degenerate pulse) or during dredge-up (a degenerate subpulse).

### 3.1. The effect of the entropy of mixing

To test the effects which the numerical treatment of dredge-up and the entropy of mixing have on the occurrence of degenerate pulses we evolved a $5M_\odot Z = 0.004$ star from the main-sequence through the thermally pulsing AGB evolution with Wood’s (1981) entropy treatment removed (hereafter Case E). This reduced $\lambda$ from an average of 0.95 to 0.67. Figure 7 shows the time variation of the mass of the H-exhausted core together with masses at the middle and bottom of the helium shell. Also plotted on the same figure for comparison are the shell positions for our standard $M = 5M_\odot Z = 0.004$ model (discussed above) with deep dredge-up and degenerate pulses. No degenerate pulses occur in Case E. We see in this Figure that the tail of the helium shell burns during each pulse and the shell remains relatively thin in mass. Thus pulses proceed normally.

To test the possibility that the entropy treatment itself may affect the occurrence of degenerate pulses, we evolved another $5M_\odot Z = 0.004$ model with the treatment included but with $\lambda$ restricted to a maximum value of 0.67. $\lambda$ tended to vary from pulse to pulse for this model, though never exceeding 0.67, and the mean value was 0.54. Otherwise the results were very similar to Case E with the helium shell remaining very narrow in mass.
Fig. 7.— Core mass, position of the middle of the helium shell ($Y = 0.35$), and bottom of the helium shell ($Y = 0.05$) for both the standard case (with high $\lambda$) and Case E (with lower $\lambda$).
and no degenerate pulses or subpulses occurring.

### 3.2. The effect of time steps

To test whether time-stepping influences the development of degenerate thermal pulses we ran a case with twice the time resolution of our standard $5M_\odot Z = 0.004$ model, and evolved it through 45 thermal pulses. We saw all the features described for the standard model and a degenerate pulse occurred during the 42nd pulse. Overall, this model is similar to the standard model, though the first degenerate pulse occurs some 4 pulses later. Clearly, the fine details of stellar evolutionary calculations are dependent on numerical treatment, including time-stepping (see also Straniero et al. (1997)). The important result of this test sequence is that reducing the time steps makes only a small difference to the behaviour of the helium shell when deep third dredge-up is present.

### 3.3. The effect of spatial zoning

In Figure 3 we showed the variation with time of the position (in mass) of the hydrogen and helium shells of the standard $M = 5M_\odot Z = 0.004$ model. The reported positions of the helium shell base in Figure 3 are somewhat ragged. This is due to the method used to define the mass interior to $Y = 0.05$ in the model: we select the point “at” $Y = 0.05$ by selecting the shell $j$ so that $Y(j) < 0.05$ and $Y(j + 1) > 0.05$. In addition to the raggedness of the position of the point at $Y = 0.05$, the reported position of the shell bottom apparently moves slowly inward following the two degenerate pulses. This is a non-physical effect due to numerical diffusion as model zoning is changed. Both the above features indicate that the mesh spacing may not be fine enough at the bottom of the helium shell.

To test whether mesh spacing influences the occurrence of degenerate pulses, we
evolved another $5M_{\odot} Z = 0.004$ star with exactly the same conditions as the standard case, except that mesh spacing was halved everywhere. This star experienced a total of 65 complete pulses before convergence difficulties terminated calculations during the 66\textsuperscript{th} pulse. A degenerate pulse occurred during the 40\textsuperscript{th} pulse, and a degenerate subpulse during the 51\textsuperscript{st} pulse.

As a further test, we evolved another case with mesh spacing returned to normal throughout the entire model except at the base of the helium shell. Between $Y = 0$ and 0.15, the mesh spacing was made 10 times finer than normal. The results are shown in Figures 8 and 9. The usual thickening of the helium shell occurred, but once the pulse number exceeded 30, the base of the helium shell began to move outwards again, the temperature rose at the base, and the density and degeneracy fell. The star barely avoided a degenerate pulse at this time, with the base of the helium shell burning gently. However, the easing of conditions at the shell base was short-lived and a degenerate pulse finally occurred at pulse 52. Afterwards, the base of the helium shell remained essentially stationary in mass. Consequently, the helium shell thickened again and a degenerate subpulse was triggered at the end of the 72\textsuperscript{nd} pulse, where the calculations were terminated.

We conclude from the above tests that the details, \textit{but not the occurrence}, of degenerate pulses may be influenced by numerical details. Degenerate pulses inevitably occur if dredge-up is deep enough for long enough.

\textbf{4. Conclusion}

We believe that the development of degenerate thermal pulses is inevitable provided two criteria are met: 1) dredge-up is very deep, providing cooling of the helium shell soon after ignition; and 2) the star lives long enough on the AGB for the helium shell to
Fig. 8.— The time variation of various important positions (in mass) within the model. From top to bottom these are: the hydrogen shell, the centre of the helium shell (where $Y = 0.35$) and the bottom of the helium shell (where $Y = 0.05$). This should be compared
Fig. 9.— Radius, temperature, density and degeneracy parameter at the base of the helium shell (where $Y = 0.05$) as functions of time. This should be compared with the lower resolution calculation in Figure 4.
reach (partially) degenerate conditions. Whether these conditions are realised in real stars remains an open question. The two aspects of AGB evolution which are the most uncertain are the extent of dredge-up and the rate of mass-loss (hence AGB lifetime), precisely the two phenomena which govern the occurrence or not of degenerate thermal pulses.

It seems prudent to look for some observational consequence of degenerate thermal pulses. The nucleosynthetic consequences are currently being investigated, and will be reported elsewhere.
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