Complex spectrum of spin models for finite-density QCD

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Talk@Lattice2016
27 July 2016

<HN, M. Ogilvie, and K. Pangeni, PRD93, 094501>
Outline

• Introduction
  - Spin models for QCD at finite $\mu$
  - $CK$ symmetry

• Main Results
  - Complex mass spectrum
  - Sinusoidal oscillation for correlation functions

• Conclusions
Introduction
Yang-Mills in (1+1) dimensions

- Integrate out the spatial links using the character expansion

\[ \beta = 1/T \]

\[ L \]

\[ \text{SU(3) LGT in 2D} \]

\[ \text{SU(3) spin model in 1D} \]

- Construct the transfer matrix with the heat kernel action

\[ T_0 = \langle P_{i+1} | e^{-aH_0} | P_i \rangle \quad \text{where} \quad H_0 = \frac{g^2 \beta}{2} C \]

\[ \langle r' | e^{-aH_0} | r \rangle = \text{drag}(1, e^{-4a/3}, e^{-4a/3}, e^{-3a} \ldots) \]

\[ \text{<P. Menotti and E. Onofri, 1981> etc} \]
Static quarks

- Inserting static quarks in the transfer matrix

\[ T = \langle r' | e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger) e^{-aH_0/2} | r \rangle \]

where \( z_1 = e^{(\mu - M)/T} \) and \( z_2 = e^{(-\mu - M)/T} \)

- Raising and lowering operators

\[ \det(1 + z_1 P) = 1 + z_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + z_1^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + z_1^3 \]

- Transfer matrix is non-Hermitian: A manifestation of the sign problem.

Pure SU(3)

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{1}{e^{4/3}} & 0 & 0 \\
0 & 0 & \frac{1}{e^{4/3}} & 0 \\
0 & 0 & 0 & \frac{1}{e^3}
\end{pmatrix}
\]

With quarks \((z_2 = 0)\)

\[
\begin{pmatrix}
1 + z_1^3 & \frac{z_1}{e^{2/3}} & \frac{z_1^2}{e^{2/3}} & 0 \\
\frac{z_1^2}{e^{2/3}} & 1 + z_1^3 & \frac{z_1}{e^{4/3}} & \frac{z_1^2}{e^{13/6}} \\
\frac{z_1}{e^{2/3}} & \frac{z_1^2}{e^{4/3}} & 1 + z_1^3 & \frac{z_1}{e^{13/6}} \\
0 & \frac{z_1}{e^{13/6}} & \frac{z_1^2}{e^{13/6}} & \frac{1 + z_1^3}{e^3}
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{1}{e^{4/3}} & 0 & 0 \\
0 & 0 & \frac{1}{e^{4/3}} & 0 \\
0 & 0 & 0 & \frac{1}{e^3}
\end{pmatrix}
\]
**CK-symmetric systems**

- Eigenvalues are real or form a complex conjugate pair due to CK-symmetry

\[ T = \text{diag}(e^{-m_0 a}, e^{-m_1 a}, \ldots) \]

- Charge conjugation (C)
- Complex conjugation (K)

\[ \rightarrow \det M(\mu) = [\det M(-\mu)]^* \]

\[ \det M(\mu) = \begin{vmatrix} m_0 & m_1 & m_2 & m_3 \\ m_3 & m_0 & m_1 & m_2 \\ m_2 & m_1 & m_0 & m_3 \\ m_1 & m_2 & m_3 & m_0 \end{vmatrix} \]

- Correlation function: sinusoidal exponential decay if \( m_2 = m_1^* \)

\[ \langle \text{tr}_F P^\dagger(x) \text{tr}_F P(0) \rangle_C \sim \exp(-\text{Re}[m_1 - m_0]x) \cos(\text{Im}[m_1 - m_0]x) \quad (x, L \rightarrow \infty) \]

\[ <\text{HN, M. Ogilvie, and K. Pangeni, 2014}> \]
In (1+1)-dim with static quarks, the results of mass spectrum are exact.

They are also the results for higher dimensions at leading order in strong coupling.

The leading diagrams for $\langle \phi(\vec{x})\phi(0) \rangle$ are the shortest possible paths.

\[<J. Kogut, D. Sinclair, R. Pearson, J. Richardson, and J. Shigemitsu 1981>\]
Results
1. Static quark: \[ T = \langle r' | e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger) e^{-aH_0/2} | r \rangle \]

where \( z_1 = e^{(\mu - M)/T} \) and \( z_2 = e^{(-\mu - M)/T} \)

- **Particle-Antiparticle (C):** \((z_1, z_2) \rightarrow (z_2, z_1)\)

- **Particle-Hole (K):** \((z_1, z_2) \rightarrow (1/z_1, 1/z_2)\)

\[ \text{PH: } \det(1 + z_1 P) \rightarrow \det(1 + P/z_1) \]
\[ \text{K: } \det(1 + z_1 P) \rightarrow \det(1 + z_1 P^\dagger) \]
\[ = z_1^N \det(1 + P/z_1) \]

- **Combined transformation (CK):** \((z_1, z_2) \rightarrow (1/z_2, 1/z_1)\)
1. Static quark: \[ T = \langle r' | e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger) e^{-aH_0/2} | r \rangle \]

- Invariant under \( z_1 \rightarrow 1/z_1 \)
- Hermitian point at \( z_1 = 1 \).
- The Polyakov loop goes to zero for \( \mu >> M \)

Also observed in other methods:

- Strong-coupling
  \(<J. Langelage, M. Neuman, and O. Philipsen, 2014>\>
  \(<T. Rindlisbacher and P. de Forcrand, 2015>\>
  and more

- Complex Langevin
  \(<G. Aarts, E. Seiler, D. Sexty, and I. Stamatescu, 2014>\>
  \(<G. Aarts, F. Attanasio, B. Jäger, and D. Sexty, 2016>\>
  and more
1. Static quark: \[ T = \langle r' \mid e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger) e^{-aH_0/2} \mid r \rangle \]

- Invariant under \( z_1 \rightarrow 1/z_1 \)
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  - [G. Aarts, E. Seiler, D. Sexty, and I. Stamatescu, 2014] and more
1. Static quark: \( T = \langle r' | e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger) e^{-aH_0/2} | r \rangle \)

- The number density saturates \( (n_q \rightarrow 3) \) for \( \mu >> M \)

- Particle-Hole symmetry if \( \mu/T = M/T \gg 1 \rightarrow \text{Half-filling} \)

See also <T. Rindlisbacher and P. de Forcrand, 2015>
1. Static quark: \[ T = \langle r' | e^{-aH_0/2} \det(1 + z_1 P) \det(1 + z_2 P^\dagger) e^{-aH_0/2} | r \rangle \]

- Drops below zero.

- Sinusoidal modulation.

- Could check with other methods:
  Complex Langevin and Reweighting?

See also:
\[ <P. Meisinger and M. Ogilvie, 2014> \]
\[ <O. Akerlund, P. de Forcrand, and T. Rindlisbacher, 2016> \]
2. Heavy quark: \[ T = \langle r' | e^{-aH_0/2} \exp [z_1 \text{tr}_F P] \exp [z_2 \text{tr}_F P^\dagger] e^{-aH_0/2} | r \rangle \]

- Spin model with “complex” magnetic field

\[ H \sim -J \sum_{\langle i,j \rangle} \text{Re} \left[ \text{tr} P_i \text{tr} P_j^\dagger \right] - h_R \sum_i \text{Re} \left[ \text{tr} P_i \right] - i h_I \sum_i \text{Im} \left[ \text{tr} P_i \right] \]

- Region plots where \( m_1 = m_2^* \in \mathbb{C} \) for SU(3)
Conclusions

• We constructed the transfer matrix of strong-coupling lattice QCD with static quarks and heavy quarks.

• The transfer matrix is not Hermitian, but $CK$-symmetric.

• Mass spectrum of spin models for finite-density QCD becomes complex when chemical potential is non-zero.

• Oscillation of correlation functions of the Polyakov loops.