Lorentz violation and the speed of gravitational waves in brane-worlds

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Abstract

Lorentz violation in a brane-world scenario is presented and used to obtain a relationship between the speed of gravitational waves in the bulk and that on the brane. Lorentz violating effects would manifest themselves in gravitational waves travelling with a greater speed in the bulk than on the brane and this effect is independent of the signature of the extra dimension.

1 Introduction

According to Einstein’s principle of relativity the maximal velocity in our universe, \( c \), is equal to the fundamental speed in the Minkowski space-time. This equality is deeply embedded in 4\( D \) Einstein field equations \cite{1} and is confirmed experimentally \cite{2}. Recent discussions on the alternative claims that the constant entering space-time intervals, or the speed of gravity, is different from \( c \) can be found in \cite{3, 4}.

General relativity however, cannot describe gravity at high enough energies and must be replaced by a quantum gravity theory. The physics responsible for making a sensible quantum theory of gravity is revealed only at the Planck scale. This cut-off scale indicates the point where our old notion of nature breaks down. It is therefore not inconceivable that one of the victims of this break down is Lorentz invariance. It is thus interesting to test the robustness of this symmetry at the highest energy scales \cite{5, 6, 7}. As usual in high energy physics, if the scale characterizing new physics is too high then it cannot be reached directly in collider experiments. In this case cosmology is the only place where the effects of new physics can be indirectly observed. Brane-world models offer a phenomenological way to test some of the novel predictions and corrections to general relativity that are implied by M-theory. Such models usually assume that \( c \) is a universal constant. For alternative approaches where the speed of gravity can be different from \( c \) in a brane-world context, see \cite{8, 9, 10}. It should be emphasized that the assumption that the maximal velocity in the bulk coincides with the speed of light on the brane must not be taken for granted. In this regard, theories with two metric tensors have been suggested with the associated two sets of “null cones,” in the bulk and on the brane \cite{11}. This is the manifestation of violation of the bulk Lorentz invariance by the brane solution. In some brane-world scenarios, the space-time globally violates 4\( D \) Lorentz invariance, leading to apparent violations of Lorentz invariance from the brane observer’s point of view due to bulk gravity effects. These effects are restricted to the gravity sector of the effective theory while the well measured Lorentz invariance of particle physics remains unaffected in these scenarios \cite{12, 13}. In a similar vein, Lorentz invariance violation has been employed to shed some light on the possibility of

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signals travelling along the extra dimension outside our visible universe [14]. In a different approach a brane-world toy model has been introduced [15] in an inflating 5D brane-world setup with violation of 4D Lorentz invariance at an energy scale $k$.

In a previous paper [16], we studied Lorentz violation in a brane-world scenario by introducing a vector field normal to the brane along the extra dimension. This, in fact, was a generalization of the theory suggested by Jacobson and Mattingly [17, 18] who had investigated Lorentz violation in a 4D space-time. In this work, we study local Lorentz violation in the same setting and address the question of the speed of propagation of gravitational waves in the bulk as well as on the brane. We find a relation between the maximal velocity in the bulk and the speed of light on the brane. As it turns out, we find that Lorentz violating effects would manifest themselves in gravitational waves travelling with a speed different from light, providing a possible detection mechanism in gravitational wave experiments. Another interesting outcome is that the speed of light turns out to be greater in the bulk than on the brane irrespective of the signature of the extra dimension. This is interesting since in brane theories without Lorentz violation, the only way the speed of gravitational waves in the bulk could become greater than that on the brane is when the extra dimension is taken to be space-like, which is the usual assumption in brane theories.

2 Field equations

In the usual brane-world scenarios the space-time is identified with a singular hypersurface (or 3-brane) embedded in a five-dimensional bulk. Suppose now that $N^A$ is a given vector field along the extra dimension, effectively making the associated frame a preferred one. The theory we consider consists of the vector field $N^A$ minimally coupled to gravity with an action of the form\(^1\):

$$S = \int d^5x \sqrt{\eta} \left[ \frac{1}{2k_5^2} \left( (5) R + \mathcal{L}_N \right) + \mathcal{L}_m \right],$$

where $k_5^2$ is a constant introduced for dimensional considerations, $\mathcal{L}_N$ is the vector field Lagrangian density while $\mathcal{L}_m$ denotes the Lagrangian density for all the other matter fields. In order to preserve general covariance, $N^A$ is taken to be a dynamical field. The Lagrangian density for the vector field is written as

$$\mathcal{L}_N = K^{ABCD} \nabla_A N^C \nabla_B N^D + \lambda (N^A N_A - \epsilon),$$

where

$$K^{ABCD} = -\beta_1 g^{AB} g_{CD} - \beta_2 \delta^A_C \delta^B_D - \beta_3 \delta^A_D \delta^B_C.$$

Here, $\beta_i$ are dimensionless parameters, $\epsilon = -1$ or $\epsilon = 1$ depending on whether the extra dimension is space-like or time-like respectively and $\lambda$ is a Lagrange multiplier. This is a slight simplification of the theory introduced by Jacobson and Mattingly [17], where we have neglected a quartic self-interacting term of the form $(N^A \nabla_A N^B)(N^C \nabla_C N_B)$, as has been done in [19]. We also define a current tensor $J^A_C$ via

$$J^A_C \equiv K^{ABCD} \nabla_B N^D.$$

Note that the symmetry of $K^{ABCD}$ means that $J^B_D = K^{ABCD} \nabla_A N^C$. With these definitions the equation of motion obtained by varying the action with respect to $N^A$ is

$$\nabla_A J^{AB} = \lambda N^B.$$  

The equation of motion for $\lambda$ enforces the fixed norm constraint

$$g_{AB} N^A N^B = \epsilon, \quad \epsilon^2 = 1.$$  

\(^1\)The upper case Latin indices take the values 0, 1, 2, 3 and 5 while the Greek indices run from 0 to 3.
The choice $\epsilon = 1$ ensures that the vector will be time-like. Multiplying both sides of (5) by $N_B$ and using (6), we find

$$\lambda = \epsilon N_B \nabla_A J^{AB}. \tag{7}$$

One may also project into a subspace orthogonal to $N^A$ by acting the projection tensor $P^C{}_B = -\epsilon N^C N_B + \delta^C_B$ on equation (5) to obtain

$$\nabla_A J^{AC} - \epsilon N^C N_B \nabla_A J^{AB} = 0. \tag{8}$$

This equation determines the dynamics of $N^A$, subject to the fixed-norm constraint.

In taking the variation, it is important to distinguish the variables that are independent. Our dynamical degrees of freedom are the inverse metric $g^{AB}$ and the contravariant vector field $N^A$. Hence, the Einstein equations in the presence of both the matter and vector fields in bulk space are [20]

$$(5) G_{AB} = (5) R_{AB} - \frac{1}{2} g_{AB} (5) R = k^2_{5} (5) T_{AB}, \tag{9}$$

where

$$(5) T_{AB} = (5) T^{(m)}_{AB} + \frac{1}{k^2_{5}} T^{(N)}_{AB}. \tag{10}$$

Here, $T^{(m)}_{AB}$ is the five-dimensional energy-momentum tensor and the stress-energy $T^{(N)}_{AB}$ is considered to have the following form [19, 21]

$$T^{(N)}_{AB} = 2\beta_1 \left( \nabla_A N^C \nabla_B N_C - \nabla^C N_A \nabla_C N_B \right) - 2 \left[ \nabla_C \left( N_A J^{CB} \right) + \nabla_C \left( N^C J_{AB} \right) \right] - \nabla_c \left( N_A J^C_{Y_B} \right) + 2\epsilon N_D \nabla_C J^{CD} N_A N_B + g_{AB} \mathcal{L}_N. \tag{11}$$

### 3 Geometrical setup

Let us now assume that the background manifold $\bar{v}_4$ is isometrically embedded in a pseudo-Riemannian manifold $v_5$ by the map $\mathcal{Y} : \bar{v}_4 \rightarrow v_5$ such that

$$\mathcal{Y}^A_{\mu} \mathcal{Y}^B_{\nu} g_{AB} = \bar{g}_{\mu\nu}, \quad \mathcal{Y}^A_{\mu} N^B g_{AB} = 0, \quad N^A N^B g_{AB} = \epsilon. \tag{12}$$

where $g_{AB}(\bar{g}_{\mu\nu})$ is the metric of the bulk (brane) space $v_5(\bar{v}_4)$ in arbitrary coordinate, $\mathcal{Y}^A(\chi^\mu)$ is the basis of the bulk (brane) and $N^A$ is normal unite vector, orthogonal to the brane. Since $N^A$ is a vector field along the extra dimension, we may write

$$N^A = \frac{\delta^A_5}{\phi}, \quad N_A = (0, 0, 0, \epsilon, \phi), \tag{13}$$

with $\phi$ being a scalar field. The perturbation of $\bar{v}_4$ with respect to a small positive parameter $y$ along the normal unit vector $N^A$ is given by

$$Z^A(x^\alpha, y) = \mathcal{Y}^A + y\phi N^A, \tag{14}$$

where we have chosen $N^A$ to be orthogonal to the brane, thus ensuring gauge independency and having the field $\phi$ dependent on the local coordinates $x^\alpha$ only [22]. The integrability conditions for the perturbed geometry are the Gauss and Codazzi equations. To find the perturbed metric, $g_{\mu\nu}$, we follow the same definitions as in the geometry of surfaces. Consider the embedding equations of the perturbed geometry written in the particular Gaussian frame defined by the embedded geometry and the normal unit vector

$$Z^A_{\mu} Z^B_{\nu} g_{AB} = g_{\mu\nu}, \quad Z^A_{\mu} N^B g_{AB} = 0, \quad N^A N^B g_{AB} = \epsilon. \tag{15}$$
Using equations (14) and (15), we may express the perturbed metric in the Gaussian frame defined by the embedding as

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + 2\phi(x^\alpha)\bar{K}_{\mu\nu} + y^2\phi^2(x^\alpha)\bar{g}^{\alpha\beta}\bar{K}_{\mu\alpha}\bar{K}_{\nu\beta}, \]

where \( \bar{K}_{\mu\nu} \) is the extrinsic curvature of the original brane and the metric of our space-time is obtained by setting \( y = 0 \) (\( g_{\mu\nu} = \bar{g}_{\mu\nu} \)). Using equations (13), (15) and (16), the metric of the bulk is written as

\[ ds^2 = g_{\mu\nu}(x^\alpha,y)dx^\mu dx^\nu + \epsilon\phi^2(x^\alpha)dy^2, \]

where we have used signature \((+ - - - \epsilon)\) everywhere. The Einstein equations (9) contain the first and second derivatives of the metric with respect to the extra coordinate. These can be expressed in terms of geometrical tensors in 4D. The first partial derivatives can be written in terms of the extrinsic curvature

\[ K_{\mu\nu} = \frac{1}{2} \bar{\mathcal{L}}_N g_{\mu\nu} = \frac{1}{2\phi} \frac{\partial g_{\mu\nu}}{\partial y}, \quad K_{A5} = 0. \]

The second derivatives can be expressed in terms of the projection \((5)C_{\mu5\nu5}\) of the bulk Weyl tensor to 5D

\[ (5)C_{ABCD} = (5)R_{ABCD} - \frac{2}{3} \left( (5)R_A[CGD]_B - (5)R_B[CGD]_A \right) + \frac{1}{6} \left( (5)Rg_A[CGD]_B \right). \]

In the absence of off-diagonal terms \((g_{5\mu} = 0)\) the dimensional reduction of the five-dimensional equations is particularly simple \([23], [24]\). Thus, the field equations (9) can be split up into three parts \([16]\)

\begin{align*}
(4)G_{\mu\nu} & = \frac{2}{3} k_5^2 \left[ (5)T_{\mu\nu} + \left( (5)T_{5} - \frac{1}{4} (5)T \right) g_{\mu\nu} \right] - \epsilon (K_{\mu\alpha}K_{\nu}^\alpha - KK_{\mu\nu}) + \frac{\epsilon}{2} g_{\mu\nu} \left( K_{\alpha\beta}K^{\alpha\beta} - K^2 \right) - \epsilon E_{\mu\nu}, \quad (20) \\
\phi_{;\mu}^\mu & = -\epsilon \frac{\partial K}{\partial y} - \phi \left( \epsilon K_{\alpha\beta}K^{\alpha\beta} + (5)\mathcal{R}_5 \right), \quad (21) \\
D_\mu (K_{\mu\nu} - \delta_{\mu\nu} K) & = k_5^2 \frac{(5)T_{5\nu}}{\phi}. \quad (22)
\end{align*}

In the above expressions, \( E_{\mu\nu} \) is the electric part of the Weyl tensor and the covariant derivatives are calculated with respect to \( g_{\mu\nu} \), i.e. \( Dg_{\mu\nu} = 0 \).

### 4 Brane world considerations

To progress any further we need the Einstein field equations on the brane. We therefore concentrate on deriving these equations in this section by presenting a quick and brief review on how this can be done. The reader may consult reference \([16]\) for a detailed discussion.

With the brane-world scenario in mind, it is assumed that the five-dimensional energy-momentum tensor has the form

\[ (5)T_{AB}^{(m)} = \Lambda_5 g_{AB}, \]

where \( \Lambda_5 \) is the cosmological constant in the bulk. Now, using equation (11) we may calculate \((5)T_{\mu\nu}, (5)T_5\) and \((5)T\), obtaining

\begin{align*}
(5)T_{\mu\nu} & = \frac{1}{k_5^2} \left[ -4(\beta_1 + \beta_3)K_{\mu\tau}K_{\nu}^\tau + 2(\beta_1 + \beta_3)KK_{\mu\nu} + \beta_2 g_{\mu\nu}K^2 + \frac{2(\beta_1 + \beta_3)}{\phi} K_{\mu\nu,5} \right. \\
& \quad + \left. \frac{2\beta_2}{\phi} g_{\mu\nu}K_{,5} - (\beta_1 + \beta_3)g_{\mu\nu}K_{\alpha\beta}K^{\alpha\beta} + 2\epsilon \beta_1 \frac{\phi_{,\mu}\phi_{,\nu}}{\phi^2} - \epsilon \beta_1 g_{\mu\nu} \frac{\phi_{,\alpha}\phi_{,\alpha}}{\phi^2} \right] + \Lambda_5 g_{\mu\nu}, \quad (24) \\
(5)T_5 & = \frac{1}{k_5^2} \left[ (\beta_1 + \beta_3)K_{\alpha\beta}K^{\alpha\beta} + \beta_2 K^2 + 2\epsilon \beta_1 g_{\mu\nu} \frac{\phi_{,\mu\nu}}{\phi} - \epsilon \beta_1 \frac{\phi_{,\alpha}\phi_{,\alpha}}{\phi^2} \right] + \Lambda_5, \quad (25)
\end{align*}
\( T = \frac{1}{k_5^2} \left[ -3(\beta_1 + \beta_3)K_{\alpha\beta}K^{\alpha\beta} + 2(\beta_1 + \beta_3)K^2 + 5\beta_2 K^2 + \frac{2}{\phi}(\beta_1 + \beta_3)K_{55} + \frac{8}{\phi}\beta_2 K_{55} \right] - 3\epsilon\beta_1 \frac{\phi_{\alpha}\phi^{\alpha}}{\phi^2} + 2\epsilon\beta_1 g^{\mu\nu} \frac{\phi_{\mu\nu}}{\phi} + 5\Lambda_5. \)  

(26)

Now, upon defining the following new set of parameters

\[
\begin{align*}
\alpha_1 &= 2(\beta_1 + \beta_3), \\
\alpha_2 &= \frac{2\epsilon(\beta_1 + \beta_2 + \beta_3)}{3 - 2\epsilon(\beta_1 + 4\beta_2 + \beta_3)}, \\
\alpha_3 &= \frac{\alpha_1(3 + \epsilon - 2\beta_2)}{6} - \beta_2, \\
\alpha_4 &= \frac{\alpha_1(6 + \epsilon + \alpha_1)}{6}, \\
\alpha_5 &= \epsilon\beta_1,
\end{align*}
\]

(27)

and using equations (24), (25) and (26), equation (20) may be written as

\[
\begin{align*}
(4)G_{\mu\nu} &= \frac{k_5^2}{2} g_{\mu\nu}\Lambda_5 - \frac{3(\epsilon + \alpha_1)}{3 + \alpha_1} (K_{\mu\gamma}K_{\nu}^{\gamma} - KK_{\mu\nu}) - \frac{3(\epsilon + \alpha_3)}{2(3 + \alpha_1)} g_{\mu\nu}K^2 \\
&+ \frac{3(\epsilon + \alpha_4)}{2(3 + \alpha_1)} g_{\mu\nu}K_{\alpha\beta}K^{\alpha\beta} + \left[ \frac{\alpha_1(\alpha_5 + \frac{1}{3}) + 3\alpha_5}{3 + \alpha_1} \right] g_{\mu\nu} \frac{\phi_{\alpha}}{\phi} - \frac{\alpha_5(5 + \alpha_1)}{2(3 + \alpha_1)} g_{\mu\nu} \frac{\phi_{\alpha}\phi^{\alpha}}{\phi^2} \\
&- \frac{2\alpha_1}{(3 + \alpha_1)} \frac{\phi_{\mu\nu}}{\phi} + \left[ \frac{4\alpha_5}{(3 + \alpha_1)} \right] \frac{\phi_{\mu}\phi_{\nu}}{\phi^2} - \frac{3(\epsilon + \alpha_1)}{3 + \alpha_1} E_{\mu\nu}.
\end{align*}
\]

(28)

Note that \((3 + \alpha_1)\) is the coefficient of the four-dimensional Einstein tensor as one may multiply both sides of the above equation by this factor. It therefore provides a relation among the extrinsic curvature, the electric part of the Weyl tensor and scalar field \(\phi\) when \(\alpha_1 = -3\). We take \(\alpha_1 \neq -3\) thereafter. In the spirit of the brane world scenario, we assume \(Z_2\) symmetry about our brane, considered to be a hypersurface \(\Sigma\) at \(y = 0\). Using \(Z_2\) symmetry, the Israel junction conditions are obtained as

\[
K_{\mu\nu}|_{\Sigma^+} = -K_{\mu\nu}|_{\Sigma^-} = -\frac{\epsilon k_5^2}{2(1 + \epsilon\alpha_1)} \left[ \tau_{\mu\nu} - \frac{1}{3} g_{\mu\nu}(1 + \alpha_2) \right].
\]

(29)

To avoid unreal singularities in equations (28) and (29), it would be convenient to take \(\alpha_1 < -3\) [16]. The energy-momentum tensor \(\tau_{\mu\nu}\) represents the total vacuum plus matter energy-momentum. It is usually separated in two parts,

\[
\tau_{\mu\nu} = \sigma g_{\mu\nu} + T_{\mu\nu},
\]

(30)

where \(\sigma\) is the tension of the brane in 5D, which is interpreted as the vacuum energy of the brane world and \(T_{\mu\nu}\) represents the energy-momentum tensor of ordinary matter in 4D. Using equations (29) and (30) and defining the following set of parameters

\[
\begin{align*}
\alpha_6 &= \frac{\alpha_1(1 - 2\alpha_2) - 2\epsilon\alpha_2}{3}, \\
\alpha_7 &= \frac{(\epsilon + \alpha_1)(\alpha_2 + \alpha_2^2)}{9} + \frac{(\alpha_4 - 3\epsilon - 4\alpha_3)(\alpha_2 + 2\alpha_2^2)}{18},
\end{align*}
\]

(31)

we obtain the Einstein field equations with an effective energy-momentum tensor in 4D as

\[
\begin{align*}
(4)G_{\mu\nu} &= \Lambda_4 g_{\mu\nu} + 8\pi GT_{\mu\nu} + k_5^2 \Pi_{\mu\nu} - \frac{3(\epsilon + \alpha_1)}{3 + \alpha_1} E_{\mu\nu} + \left[ \frac{\alpha_1(\alpha_5 + \frac{1}{3}) + 3\alpha_5}{3 + \alpha_1} \right] g_{\mu\nu} \frac{\phi_{\alpha}}{\phi} \\
&- \frac{\alpha_5(5 + \alpha_1)}{2(3 + \alpha_1)} g_{\mu\nu} \frac{\phi_{\alpha}\phi^{\alpha}}{\phi^2} - \frac{2\alpha_1}{(3 + \alpha_1)} \frac{\phi_{\mu\nu}}{\phi} + \left[ \frac{4\alpha_5}{(3 + \alpha_1)} \right] \frac{\phi_{\mu}\phi_{\nu}}{\phi^2},
\end{align*}
\]

(32)

where

\[
\Lambda_4 = \frac{k_5^2}{2} \Lambda_5 + \left[ \frac{-\epsilon k_5^3 + 3k_5^2(-\alpha_1 + 4\alpha_6 + 16\alpha_7 + 2\alpha_4)}{4(3 + \alpha_1)(1 + \epsilon\alpha_1)^2} \right] \sigma^2.
\]

(33)
\[ 8\pi G = \left[ -\frac{2\epsilon k_5^4 + 3k_3^5(-2\alpha_1 + 4\alpha_6)}{4(3 + \alpha_1)(1 + \epsilon \alpha_1)^2} \right] \sigma, \quad (34) \]

and

\[
\Pi_{\mu\nu} = \frac{3}{4(3 + \alpha_1)(1 + \epsilon \alpha_1)^2} \left[ -(\epsilon + \alpha_1)T_{\mu\gamma}T^\gamma_{\nu} + (\frac{\epsilon}{3} + \alpha_6)TT_{\mu\nu} \right] - \left( \frac{\epsilon}{6} - \alpha_7 \right) g_{\mu\nu}T^2 + \left( \frac{\epsilon + \alpha_4}{2} \right) g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} + \left[ \frac{3(\alpha_6 + 8\alpha_7 + \alpha_4)}{4(3 + \alpha_1)(1 + \epsilon \alpha_1)^2} \right] g_{\mu\nu}\sigma T. \quad (35) \]

All these 4D quantities have to be evaluated in the limit \( y \to 0^+ \). They give a working definition of the fundamental quantities \( \Lambda_4 \) and \( G \) and contain higher-dimensional modifications to general relativity. As expected, switching off the effects of Lorentz violation \( (\alpha_i = 0) \) in these equations results in expressions one usually obtains in the brane-worlds models. In the next section, we use the solutions of equation (32) to obtain a relation between the speed of light in the bulk and on the brane.

### 5 Speed of gravitational waves

Let us start by assuming a perfect fluid configuration on the brane. The energy-momentum tensor is therefore written as

\[ T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \quad (36) \]

where \( u, \rho \) and \( p \) are the unit velocity, energy density and pressure of the matter fluid respectively. We will also assume a linear isothermal equation of state for the fluid

\[ p = \gamma \rho, \quad 0 \leq \gamma \leq 1. \quad (37) \]

The weak energy condition [25] imposes the restriction \( \rho \geq 0 \). In this paper we deal with non-tilted homogeneous cosmological models on the brane, \( i.e. \) we are assuming that the fluid velocity is orthogonal to the hypersurfaces of homogeneity. In the standard cosmological models, We can also consider \( \phi(x^\alpha) = \phi(t) > 0 \) [23].

Next, we consider the metric for our 4D universe as

\[ g_{\mu\nu} = \text{diag}(\epsilon_t^2, -a(t)^2 \Upsilon_{ij}), \quad (38) \]

with coordinates \( (t, x^i) \) and the 3-metric \( \Upsilon_{ij} \) on the spatial slices of constant time. Now, using the Israel junction conditions, we have

\[
\bar{K}_{00} = -\frac{\epsilon k_5^5 \bar{g}_{00}}{2(1 + \epsilon \alpha_1)} \left[ \sigma + \rho - \frac{1}{3}(1 + \alpha_2)(4\sigma + (1 - 3\gamma)\rho) \right], \\
\bar{K}_{ii} = -\frac{\epsilon k_5^5 \bar{g}_{ii}}{2(1 + \epsilon \alpha_1)} \left[ \sigma - \gamma \rho - \frac{1}{3}(1 + \alpha_2)(4\sigma + (1 - 3\gamma)\rho) \right]. \quad (39) \]

Now, by substituting the above equations in equations (16), the different 4D sections of the bulk in the vicinity of the original brane have the metric

\[ g_{\mu\nu} = \Omega^2 \text{diag}(Dc_b^2, -a(t)^2 \Upsilon_{ij}), \quad (40) \]

where

\[ \Omega^2 = (1 + y\phi B)^2, \quad D = \left[ \frac{1 + y\phi A}{1 + y\phi B} \right]^2, \quad (41) \]
with

\[
A = \frac{-\epsilon k^2}{2(1 + \epsilon \alpha_1)} \left[ \sigma + \rho - \frac{1}{3} (1 + \alpha_2) (4\sigma + (1 - 3\gamma)\rho) \right],
\]

(42)

\[
B = \frac{-\epsilon k^2}{2(1 + \epsilon \alpha_1)} \left[ \sigma - \gamma \rho - \frac{1}{3} (1 + \alpha_2) (4\sigma + (1 - 3\gamma)\rho) \right].
\]

(43)

From (38), we see that the constant \(c_0\) represents the speed of light on the original brane, whereas from (40) the speed of propagation of gravitational waves in this model is \(Dc_0^2\). Also, within the context of this model, we have \(\frac{-\epsilon k^2}{2(1 + \epsilon \alpha_1)} > 0\), so that irrespective of the signature of the extra dimension, \(A\) is always greater than \(B\) and consequently \(D > 1\). This leads to apparent violations of Lorentz invariance from the brane observer’s point of view due to the bulk gravity effects. Also, this result confirms the existence of a preferred frame at a point in space-time. Now, if the effects of Lorentz violations are ignored \((\alpha_1 = 0)\), the maximal velocity in the bulk becomes more than the speed of light on the brane only when the extra dimension is space-like. It is worth noting that if the energy density of the matter fluid on the brane is zero, we obtain \(A = B\), implying that the maximal velocity in the bulk will be the speed of light on the brane.

There is an interesting analogy between the behavior of gravitational waves wandering into the bulk from the brane and electromagnetic waves crossing one medium into another with different indexes of refraction. This is a reflection of Fermat’s principle when studying gravitational wave propagation, that is, when such waves take advantage of their greater speed in the bulk and travel from one point on the brane to another by taking a path through the bulk space, thus achieving a shorter travel time than an electromagnetic wave traversing the same points. Therefore, gravitational waves travelling faster than light would be a possibility. These faster than light signals, however, do not violate causality since the apparent violation of causality from the brane observer’s point of view is due to the fact that the region of causal contact is actually bigger than the region one would naively expect from the ordinary propagation of light in an expanding universe. Indeed, there is no closed timelike curves in the 5D spacetime that would make the theory inconsistent. The importance of these models is that even extremely small Lorentz violating effects may be measured in gravity wave experiments. For example, an astrophysical event such as a distant supernova might generate gravitational waves which would reach future gravity wave detectors before we actually “see” the event. It is therefore probable that future gravitational wave experiments like LIGO, VIRGO or LISA might discover this unique signature of the existence of extra dimensions. If found, such evidence may strongly influence future developments in elementary particle physics, cosmology and astrophysics [26].

6 Conclusions

In this paper we have studied a brane-world scenario where the idea of Lorentz violation was considered by specifying a preferred frame through the introduction of a dynamical vector field normal to our brane. Such a normal vector was, however, assumed to be decoupled from the matter fields since such fields were assumed to be confined to the brane. The Einstein field equations were obtained on the brane using the SMS formalism [27], modified by additional terms emanating from the presence of the vector field. As we have stressed, our model breaks the 4D Lorentz invariance in the gravitational sector. Particle physics will not feel these effects, but gravitational waves are free to propagate into the bulk and they will necessarily feel the effects of the variation of the speed of light along the extra dimension. We have also shown that within the framework of our model, irrespective of the signature of the extra dimension, the speed of the propagation of gravitational waves is always greater in the bulk than on the brane.
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