Parity-breaking phases of spin-orbit-coupled metals with gyrotropic, ferroelectric and multipolar orders

Liang Fu

We study Fermi liquid instabilities in spin-orbit-coupled metals with inversion symmetry. By introducing a canonical basis for the doubly degenerate Bloch bands in momentum space, we derive the general form of interaction functions. A variety of time-reversal-invariant, parity-breaking phases is found, whose Fermi surface is spontaneously deformed and spin-split. In terms of symmetry, these phases possess gyrotropic, ferroelectric and multipolar orders. The ferroelectric and multipolar phases are accompanied by structural distortions, from which the electronic orders can be identified. The gyrotropic phase exhibits a unique nonlinear optical property. Based on recent experiments, we identify several interesting quantum materials including pyrochlore oxides, which show evidence of these parity breaking orders.

PACS numbers: 71.10.Ay, 71.10.Hf, 75.70.Tj

Novel physics from strong spin-orbit coupling in quantum materials is currently attracting widespread interest across many disciplines in condensed matter physics. In particular, there is now an intensive investigation of the interplay between spin-orbit coupling and electron correlation in d-orbital and f-orbital systems\textsuperscript{[1–3]}. The majority of studies have been focused on correlated band or Mott insulators, whereas spin-orbit coupling in correlated metals has received less attention. It is well-known that spin-orbit coupling in metals without inversion symmetry generates spin-split energy bands and spin-polarized Fermi surfaces\textsuperscript{[4]}. This has interesting consequences in the presence of electron-electron interactions\textsuperscript{[5–10]}. In contrast, in metals with inversion symmetry, spin-orbit coupling does not generate any band splitting, but instead changes Bloch wavefunctions from being fully spin-polarized to a superposition of different spin-polarizations on different atomic orbitals, i.e., spin-orbit entangled\textsuperscript{[11, 12]}. Because of this hidden nature, the role of spin-orbit coupling in inversion-symmetric materials is often overlooked.

In this Letter, we explore the consequences of the combination of strong spin-orbit coupling and electron interaction in metals with inversion symmetry. By generalizing Landau’s Fermi liquid theory to include spin-orbit coupling, we theoretically predict novel phases of metals arising from Fermi surface instabilities of Pomeranchuk type\textsuperscript{[13, 14]}, which spontaneously break inversion symmetry. The onset of parity-breaking orders splits the original spin-degenerate Fermi surface into two deformed Fermi surfaces that carry opposite, momentum-dependent spin polarizations. We focus on three such phases, having the symmetry of a ferroelectric, a multipolar and an isotropic gyrotropic liquid respectively. The ferroelectric and multipolar phases are generally accompanied by a structural change, from which the primary electronic orders can be revealed. In contrast, the isotropic gyrotropic order does not induce any structural transition for symmetry reasons. Nonetheless, due to its breaking of inversion and mirror symmetries, we find this gyrotropic order can be detected by a novel type of nonlinear optical spectroscopy. Finally, we identify several quantum materials of current interest that show evidence of these parity-breaking orders.

The parity-breaking phases found in this work can be regarded as new examples of electronic nematic liquid crystals\textsuperscript{[14]}, broadly defined as translationally invariant electronic phases that break the point group symmetry of the lattice. We emphasize that these new phases owe their existences to spin-orbit coupling, and will be compared with their counterparts in spin-rotationally-invariant systems\textsuperscript{[13, 17]}. Following the spirit of Landau’s Fermi liquid theory, we focus on Bloch states on the Fermi surface throughout this work. In the presence of spin-orbit coupling, these states are not spin eigenstates. Nonetheless, in systems with both time-reversal ($T$) and inversion ($P$) symmetry, Bloch states remain doubly degenerate at every $k\textsuperscript{[11]}$. To develop Fermi liquid theory of such spin-orbit-coupled systems, we must first choose a basis $\{|\psi_{k,1}\rangle, |\psi_{k,2}\rangle\}$ for the degenerate bands at every point on the Fermi surface. As observed by Blount long ago\textsuperscript{[18]}, due to the absence of any conserved spin quantum number, it has been unknown whether there is a natural and canonical choice of basis: an arbitrary $U(2)$ rotation on the doublet at every $k$ produces a new basis that appears to be as good as the old one.

Here we introduce a canonical basis that we call “manifestly covariant Bloch basis” (MCBB). This is a universal basis defined uniquely by the following gauge fixing condition imposed on the Bloch wavefunction at $r = 0$, a two-component spinor:

\begin{align*}
\psi_{k,1}(r = 0) &= u_k |\uparrow\rangle, \\
\psi_{k,2}(r = 0) &= v_k |\downarrow\rangle.
\end{align*}

where $u_k$ and $v_k$ are real and positive; $\uparrow, \downarrow$ labels electron’s spin. Importantly, the origin of real space coordinate $r = 0$ is chosen to be the center of point group symmetries of the crystal. According to Eq. (1), the wavefunctions of MCBB at $r = 0$ are fully spin-polarized.
We now explicitly construct MCBB by starting from an arbitrary basis \(\{|\phi_{k,1}\}, \{|\phi_{k,2}\}\}\). Because of time-reversal \((T)\) and inversion \((P)\) symmetry, the two members are related to each other \([13]\): \(\phi_{k,2}(r = 0) = PT\phi_{k,1}(r = 0)\), where we have absorbed a \((U(1)\) phase factor by a redefinition. Therefore, the corresponding spinors defined by Bloch wavefunctions at the inversion center form a Kramers doublet: \(\delta E_{T}\phi_{k,2}(r = 0) = \delta E_{PT}\phi_{k,1}(r = 0)\), and thus are orthogonal. This orthogonality condition guarantees one can perform a \((U(2)\) transformation on \(\{|\phi_{k,1}\}, \{|\phi_{k,2}\}\\) to obtain a new basis satisfying \([11]\), or equivalently MCBB.

The advantage of MCBB is its remarkably simple transformation property under point group symmetries, which act on both the electron’s spatial coordinate and spin. For a generic choice of basis, a symmetry action \(G\) will map Bloch states at \(k\) into those at \(Gk\) (or the star of \(k\)) up to a complicated, \(k\)-dependent \((U(2)\) basis transformation \([18]\). In contrast, the defining property \([11]\) guarantees that \(G\) maps the MCBB \(|\psi_{k,\alpha}\rangle\) at \(k\) directly to its partner at \(Gk\),

\[
G : |\psi_{k,\alpha}\rangle \rightarrow U_{\alpha\beta}(G)|\psi_{Gk,\beta}\rangle
\]

(2)

where \(U(G)\) is the \(SU(2)\) matrix representation of \(G\). Furthermore, the MCBB at \(\pm k\) are related by time-reversal symmetry in the same way as spin eigenstates:

\[
T|\psi_{k,\alpha}\rangle = \epsilon_{\alpha\beta}|\psi_{-k,\beta}\rangle.
\]

(3)

Eq.\(2\) and \(3\) show that the two members of the MCBB \(\alpha = 1, 2\) transform identically as spin up and down under symmetry operations. Therefore, for the convenience of presentation, we will refer to the \(\alpha\) index of MCBB as spin, while keeping in mind that the Bloch states \(|\psi_{k,\alpha}\rangle\) are not spin eigenstates. MCBB provides the starting point for our Fermi liquid theory below, and more generally, is expected to have wide applications in studying spin-orbit-coupled systems.

Fermi liquid theory relates the change of energy \(\delta E\) to the change in the quasi-particle distribution function up to second order. The distribution function is a \(2 \times 2\) Hermitian matrix in spin space, which we write as \(n_{\alpha\beta}(k)\) in MCBB, i.e., \(n_{\alpha\beta}(k) = \langle c_{k,\alpha}^\dagger c_{k,\beta}\rangle\). \(\delta E\) is then a quadratic functional of \(n_{\alpha\beta}(k)\), involving Bloch states near the Fermi surface. The form of the energy functional is constrained by symmetry considerations. In dealing with spin-orbit-coupled systems, we find it is convenient to decompose \(n_{\alpha\beta}(k)\) in terms of the density and spin distribution function:

\[
n_{\alpha\beta}(k) \equiv n(k)\delta_{\alpha\beta} + s(k) \cdot \delta_{\alpha\beta}.
\]

(4)

We now express \(\delta E\) in terms of the change in the density and spin distribution function. First, note the transformation property of \(n(k)\) and \(s(k)\) under time reversal and inversion,

\[
T : n(k) \rightarrow n(-k), \quad s(k) \rightarrow -s(-k)
\]

\[
P : n(k) \rightarrow n(-k), \quad s(k) \rightarrow s(-k).
\]

(5)

It follows that when both symmetries are present, \(\delta E\) takes the form

\[
\delta E = \sum_k \epsilon_k \delta n(k) + \sum_{k,k'} F^n(k,k')\delta n(k)\delta n(k')
\]

\[
+ \sum_{k,k'} F^s_{ij}(k,k')s_i(k)s_j(k'),
\]

(6)

where \(F^n\) denotes density-density interaction, and \(F^s\) denotes spin-spin interaction.

The invariance of \(\delta E\) under symmetry transformations imposes constraints on the momentum dependence of \(F^n(k,k')\) and \(F^s(k,k')\). In spin-rotationally-invariant systems, the spin interaction is isotropic in spin space, i.e., \(F^s_{ij} \propto \delta_{ij}\). This is not the case for spin-orbit-coupled systems. Instead, both \(F^n(k,k')\) and \(F^s_{ij}(k,k')\) are constrained by crystal symmetries acting on electron’s coordinate and spin in combination. It can be seen from the symmetry property of MCBB \([2]\) that under a crystal symmetry operation \(G\), \(n(k)\) and \(s(k)\) transform as a scalar and a vector field respectively

\[
G : n(k) \rightarrow n(Gk)
\]

\[
s_i(k) \rightarrow G_{ij}s_j(Gk).
\]

(7)

where \(G_{ij}\) is the \(SO(3)\) matrix representation of \(G\). It then follows that the functions \(F^n(k,k')\) and \(F^s_{ij}(k,k')\) transform as a scalar field and a rank-two tensor field respectively:

\[
G : F^n(k,k') \rightarrow F^n(Gk,Gk')
\]

\[
F^s_{ij}(k,k') \rightarrow G_{ii'}G_{jj'}F^s_{ij'}(Gk,Gk').
\]

(8)

Eq.\(7\) and \(8\) determine the form of the energy functional of Fermi liquids in spin-orbit-coupled systems. The key difference from spin-rotationally-invariant systems lies in the spin-dependent interaction, which is anisotropic in spin space and spin-momentum locked. This motivates us to explore consequences of spin-dependent interactions in spin-orbit-coupled Fermi liquids below.

To proceed, we write the spin-dependent interaction in a separable form given by products of basis functions of \(k, s(k)\) and of \(k', s(k')\):

\[
\delta E_{\text{spin}} = \sum_{k,k'} F^s_{ij}(k,k')s_i(k)s_j(k')
\]

\[
= \sum_{\eta} \sum_{k,k'} F^s_{\eta}(k,s(k))\phi_{\eta}(k')\phi_{\eta}(s(k')).
\]

(9)

Naturally, the basis functions \(\phi_{\eta}(k)\) are grouped into different representations of crystal symmetry group that acts on \(k\) and \(s(k)\) in combination. As a first step, it is instructive to consider isotropic spin-orbit-coupled systems invariant under any combined rotation of space and spin. This full rotational symmetry can emerge in
low-energy theories of crystals with a high point group symmetry. In this case, the basis functions are labeled by three quantum numbers $\eta = (L, J, J_z)$: $L$ is the orbital angular momentum, $J$ is the total angular momentum $J = L + S$ with $S = 1$, and $J_z$ is the $z$-component $J$. The $2J + 1$ basis functions with the same $(L, J)$ and $J_z = -J, -J + 1, \ldots J$ form a multiplet. Associated with each $(L, J)$-multiplet is an interaction parameter $F_{L,J}^s$, which specifies the energy cost for the corresponding channel is a superposition of different (classified by different point group representations. Each channel as in (10). However, these channels now are distinct phases, as we will show below.

Before proceeding, we discuss the effect of crystalline anisotropy that reduces the full rotational symmetry to anisotropic and spin-momentum locked. The presence of three $L = 1$ (i.e., $p$-wave) interaction channels parameterized by $F_1^s, F_2^s$ and $F_3^s$ gives rise to three distinct phases, as we will show below.

Eq. (10) is a key result of this work, showing that the spin-dependent interaction in spin-orbit-coupled Fermi liquids is anisotropic and spin-momentum locked. The form of $\eta$ is given by:

$$\delta E_{\text{spin}} = \sum_{\mathbf{k}, \mathbf{k}'} F_0^s \mathbf{s}(\mathbf{k}) \cdot \mathbf{s}(\mathbf{k}') + F_1^s \left(\mathbf{k} \cdot \mathbf{s}(\mathbf{k})\right) \left(\mathbf{k}' \cdot \mathbf{s}(\mathbf{k}')\right) + F_2^s \left(\mathbf{k} \times \mathbf{s}(\mathbf{k})\right) \cdot \left(\mathbf{k}' \times \mathbf{s}(\mathbf{k}')\right) + F_3^s Q_{ij}(\mathbf{k}) Q_{ij}(\mathbf{k}'),$$  \hspace{1cm} (10)

where $Q_{ij} = Q_{ji}$ is a second-rank tensor constructed from $\mathbf{k}$ and $\mathbf{s}(\mathbf{k})$:

$$Q_{ij}(\mathbf{k}) = \frac{1}{2} \left(\hat{k}_i s_j(\mathbf{k}) + \hat{k}_j s_i(\mathbf{k})\right) - \frac{1}{3} \hat{k} \cdot \mathbf{s}(\mathbf{k}) \delta_{ij}. \hspace{1cm} (11)$$

Eq. (10) is a key result of this work, showing that the spin-dependent interaction in spin-orbit-coupled Fermi liquids is anisotropic and spin-momentum locked. The presence of three $L = 1$ (i.e., $p$-wave) interaction channels parameterized by $F_1^s, F_2^s$ and $F_3^s$ gives rise to three distinct phases, as we will show below.

Before proceeding, we discuss the effect of crystalline anisotropy that reduces the full rotational symmetry to its subgroup—point group of crystals. In this case, Fermi liquid interactions should still be decomposed into different channels as in (10). However, these channels now are classified by different point group representations. Each channel is a superposition of different $(L, J)$-multiplets that belong to the same representation, and involves more interaction parameters. Despite these changes, for many crystal structures such as cubic, tetragonal, trigonal and hexagonal, the four channels parameterized by $F_0^s, \ldots, F_3^s$ in (10) belong to different point group representations, and hence remain orthogonal.

When one or more interaction parameters in (10) are negative and of sufficiently large magnitude, Fermi surface instability takes place in the spin channel, resulting in a transition to a symmetry-breaking phase with a spin-split Fermi surface. We now study these ordered phases, paying particular attention to novel features due to spin-orbit coupling. First, the instability associated with $F_0^s$ in the $s$-wave channel splits the two Fermi surfaces of opposite spin polarizations. One spin-split Fermi surface grows in size and the other shrinks, both keeping their shapes unchanged. This gives rise to ferromagnetism, similar to the case of spin-rotationally-invariant Fermi liquids.

The main focus of this work is on Fermi liquid instabilities in the three $p$-wave interaction channels (i.e., $L = 1$) in spin-orbit-coupled metals. We start from the instability associated with $F_1^s$ in the $(L = 1; J = 0)$ channel. According to (10), this instability generates an Ising order parameter:

$$\eta = \sum_{\mathbf{k}} \mathbf{k} \cdot \mathbf{s}(\mathbf{k}). \hspace{1cm} (12)$$

$\eta$ is a pseudo-scalar, because it is invariant under time-reversal and all reflections, but breaks inversion and all reflections. Therefore the ordered phase with $\eta \neq 0$ is an isotropic gyroscopic liquid. This gyroscopic order parameter splits the original spin-degenerate Fermi surfaces into two with unequal volumes, with opposite spin polarizations. Unlike the case of ferromagnetism, here the spin quantization axis defined in terms of MCBB is not uniform but parallel to the momentum: $\mathbf{s}(\mathbf{k}) \propto \eta \mathbf{k}$, which leads to a hedgehog spin texture over the Fermi surface.

Next, consider the instability associated with $F_2^s$ in the $(L = 1; J = 1)$ channel. According to Eq. (11), this instability generates a vector order parameter

$$\mathbf{P} = \sum_{\mathbf{k}} \mathbf{k} \times \mathbf{s}(\mathbf{k}). \hspace{1cm} (13)$$

We observe that $\mathbf{P}$ has the same symmetry as the ferroelectric polarization: it is odd under inversion and invariant under time-reversal, and transforms as a vector under rotation. Therefore we identify the ordered phase with $\mathbf{P} \neq 0$ as a “ferroelectric” metal in the sense of having a polar axis, despite that a spontaneous polarization is screened by free carriers. The ordered phase has deformed and spin-split Fermi surfaces. The shape deformation and spin splitting is determined by a $\mathbf{k}$-dependent spin-orbit field $\mathbf{h}(\mathbf{k})$ acting on the original spin-degenerate Fermi surface. $\mathbf{h}(\mathbf{k})$ is proportional to the spin-polarization field $\mathbf{s}(\mathbf{k})$ generated by $\mathbf{P}$: $\mathbf{h}(\mathbf{k}) \propto \mathbf{s}(\mathbf{k}) \times \mathbf{P} \times \mathbf{k}$, which has the same form as the Rashba spin-splitting created by internal electric fields in noncentrosymmetric materials. In our case, an interaction-driven instability to Rashba spin splitting of the Fermi
surface generates a polar order parameter, which can be regarded as “inverse Rashba effect”.

Lastly, we consider the instability associated with \( F^* \) in the \((L = 1, J = 2)\) channel. According to Eq. (10), the corresponding order parameter is a traceless symmetric tensor given by

\[
Q_{ij} = \sum_k Q_{ij}(k),
\]

where \( Q_{ij}(k) \) is defined in (11). As a product of spin and momentum, \( Q_{ij} \) is a time-reversal-invariant multipolar order parameter having the \( d \)-wave symmetry under a combined rotation. Furthermore, \( Q_{ij} \) breaks inversion, hence is a second-rank pseudo-tensor. In the sense of having the same symmetry, the multipolar phase with \( Q_{ij} \neq 0 \) is an electronic analog of the chiral nematic liquid crystals\[20\]. Depending on the microscopic interaction, the matrix \( Q_{ij} \) may have two degenerate eigenvalues, in which case the ordered phase is uniaxial. Otherwise, the ordered phase is biaxial. The ordered phase has deformed and spin-split Fermi surfaces determined by the spin-orbit field \( h_{s}(k) \propto s_{ij}(k) \propto Q_{ij} k_{j} \).

To summarize, spin-orbit-coupled Fermi liquids are found to host a variety of parity-breaking phases. In particular, we find three phases due to spin-dependent interaction in the \( p \)-wave channel, with gyrotropic, ferroelectric and multipolar order parameters respectively. Fermi surfaces of these phases are spontaneously deformed and split. The magnitude and direction of the spin-splitting are momentum-dependent, leading to novel spin-textures on the Fermi surface, which can be detected by angle- and spin-resolved photoemission spectroscopy.

We emphasize that the above parity-breaking phases can only exist because of spin-orbit coupling. To see this, we make a comparison with spin-rotationally-invariant Fermi liquids, which have an isotropic spin-dependent interaction of the form \( \sum_{kk'} f^*_P(\mathbf{k} \cdot \mathbf{k'}) s(\mathbf{k}) \cdot s(\mathbf{k'}) \) in the \( p \)-wave channel. The corresponding instability was found to generate two types of ordered phases \[13\] \[14\]. The first type has two spin-split Fermi surfaces that are shifted relative to each other in \( \mathbf{k} \) space. The spin polarization on each Fermi surface is uniform, different from the phases we found. The second type has a mean-field order parameter of the form \( \mathbf{k} \cdot \mathbf{O} s(\mathbf{k}) \), where \( \mathbf{O} \) can be any arbitrary \( SO(3) \) matrix. This large mean-field degeneracy results from the spontaneous breaking of the spin rotational symmetry. Furthermore, it was found that order parameter fluctuations unavoidably destroy the uniform ordered phase, leading to a spatial modulated state \[17\]. In our study, spin-orbit coupling selects and stabilizes an Ising-type gyrotropic order \[12\], which corresponds to \( O = I \). Besides spin-orbit coupling, dipolar interactions in ultracold Fermi gases also lock spin and momentum, hence can generate ordered phases with similar features \[21\].

For the ferroelectric and multipolar phases, the electronic order parameters break rotational symmetry of the crystal, and therefore can couple directly to with lattice distortions that do not enlarge the unit cell. As a result, the transition driven by Fermi liquid instability is accompanied by a structural transition, from which the electronic order can be inferred (as we will show below). In contrast, the time-reversal-invariant gyrotropic order \[12\] is highly symmetric and typically cannot generate any lattice distortion that is associated with atomic displacements. This leads to a hidden order that could be difficult to detect by conventional probes \[22\] \[23\].

We now identify from recent experiments several materials that show evidence of the above parity-breaking orders. First, a recently synthesized material LiOsO\(_3\) was found to undergo a second-order ferroelectric structural transition at low temperature\[24\]. The high-temperature structure is \( D_{3d} \), which is inversion symmetric. A polar axis in \( c \) direction appears in the low temperature phase, reducing the crystal symmetry to \( C_{3v} \). Based on the observation of unusually large residual resistivity and Curie-Weiss behavior of spin susceptibility, it has been suggested that electron correlation plays an important role and possibly drives the structural transition\[26\]. Therefore the low-temperature phase of LiOsO\(_3\) may be an electronic-driven ferroelectric metal.

We further suggest pyrochlore oxides \( \text{A}_2\text{B}_2\text{O}_7 \) as promising candidates for the multipolar phase. The pyrochlore crystal structure has the \( O_h \) point group symmetry. Due to this crystal anisotropy, the five-component multipolar order parameter \( Q_{ij} \) defined in (11) splits into a two-dimensional \( E_u \) representation \( (Q_{xx} - Q_{yy}, 2Q_{zz} - Q_{xx} - Q_{yy}) \) and a three-dimensional \( T_{2u} \) representation \( (Q_{xy}, Q_{yz}, Q_{zx}) \). Recently, the pyrochlore oxide Cd\(_2\)Re\(_2\)O\(_7\) was found to undergo a second-order structural transition from cubic to tetragonal at \( T_c = 200 K \), with an order parameter of the \( E_u \) symmetry\[22\] \[30\]. Remarkably, the lattice change across the transition is extremely small, whereas electrical properties change drastically. In addition, a large mass enhancement above the transition temperature was inferred from transport and optical measurements\[31\] \[32\]. Therefore, the structural transition in Cd\(_2\)Re\(_2\)O\(_7\) may be induced by an electronic transition to the multipolar phase.

In the above examples of ferroelectric and multipolar phases, the appearance of electronic order is inferred, by symmetry consideration, from the structural distortion it couples to. Obviously, more experiments are needed to answer the question whether the driving force for the transition is electronic or structural. To address this issue, it may be useful to study the electronic susceptibility and the phonon frequency, and see which one becomes critical upon approaching the transition temperature from above\[34\]. On the other hand, the isotropic gyrotropic order is much more hidden. Take the example of pyrochlore crystals: the isotropic chiral order belongs
A nonzero $\mathbf{P}$ implies the presence of gyrotropic order.

I thank Tim Hsieh and Vlad Kozii for interesting discussions. This work is supported by David and Lucile Packard Foundation.

[1] W. Witzczak-Krempa, G. Chen, Y. B. Kim and L. Balents, Ann. Rev. Cond. Mat. Phys. 5, 57 (2014).
[2] M. Dzero, K. Sun, V. Galitski and P. Coleman, Phys. Rev. Lett. 104, 106408 (2010).
[3] C. Wang, A. C. Potter and T. Senthil, Science 343, 6171 (2014).
[4] R. Shindou and L. Balents, Phys. Rev. Lett. 97, 216601 (2006).
[5] A. Alexandradinata and J. E. Hirsch, Phys. Rev. B 82, 195131 (2010).
[6] E. Berg, M. S. Rudner and S. A. Kivelson, Phys. Rev. B 85, 035116 (2011).
[7] A. Ashrafi and D. L. Maslov, Phys. Rev. Lett. 109, 227201 (2012).
[8] P. G. Silvestrov and O. Entin-Wohlman, Phys. Rev. B 89, 155103 (2014).
[9] J. Ruhman and E. Berg, Phys. Rev. B 90, 235119 (2014).
[10] Y. Bahri and A. C. Potter, arXiv:1408.6826.
[11] L. Fu and C. L. Kane, Phys. Rev. B 76, 045302 (2007).
[12] X. Zhang, Q. Liu, J. W. Luo, A. J. Freeman and A. Zunger, Nat. Phys. 10, 387 (2014).
[13] I. I. Pomeranchuk, Sov. Phys. JETP 8 361 (1959).
[14] E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein and A. P. Mackenzie, Annu. Rev. Condens. Matter Phys. 1 153 (2010).
[15] C. Wu and S. C. Zhang, Phys. Rev. Lett. 93, 036403 (2004).
[16] C. M. Varma and L. Zhu, Phys. Rev. Lett. 96, 036405 (2006).
[17] C. Wu, K. Sun, E. Fradkin and S. C. Zhang, Phys. Rev. B 75, 115103 (2007).
[18] E. I. Blount, Phys. Rev. B, 32, 2935 (1985).
[19] P. W. Anderson and E. I. Blount, Phys. Rev. Lett. 14, 217 (1965).
[20] T. Lubensky and L. Radzihovsky, Phys. Rev. E 66, 031704 (2002).
[21] B. M. Fregoso and E. Fradkin, Phys. Rev. B 81, 21443 (2010).
[22] P. Hosur, A. Kapitulnik, S. A. Kivelson, J. Orenstein, and S. Raghu, Phys. Rev. B 87, 115116 (2013); P. Hosur, A. Kapitulnik, S. A. Kivelson, J. Orenstein, S. Raghu, W. Cho, and A. Fried, Phys. Rev. B 91, 039908 (2015).
[23] J. Orenstein and J. E. Moore, Phys. Rev. B 87, 165110 (2013).
[24] S. Chakravarty, Phys. Rev. B 89, 087101 (2014).
[25] N. P. Armitage, Phys. Rev. B 90, 035135 (2014).
[26] Y. Shi et al, Nat. Mat. 12, 1024 (2013).
[27] I. A. Sergienko and S. H. Curnoe, J. Phys. Soc. Jpn 72, 1607 (2003).
[28] J. P. Castellan et al, Phys. Rev. B 66, 134528 (2002).
[29] J. Yamaura and Z. Hiroi, J. Phys. Soc. Jpn. 71, 2598 (2002).
[30] I. A. Sergienko et al, Phys. Rev. Lett. 92, 065501 (2004).
[31] Z. Hiroi et al, J. Phys. Soc. Jpn. 71, 1553 (2002).
[32] Z. Hiroi, M. Hanawa, Y. Muraoka and H. Harima, J. Phys. Soc. Jpn. 72, 21 (2003).
[33] N. L. Wang, J. J. McGuire, T. Timusk, R. Jin, J. He, and D. Mandrus, Phys. Rev. B 66, 045304 (2002).
[34] R. M. Fernandes, A. V. Chubukov, J. Schmalian, Nat. Phys. 10, 97 (2014).
[35] I. A. Sergienko and S. H. Curnoe, J. Phys. Soc. Jpn. 72, 1607 (2003).
[36] J. C. Petersen et al, Nat. Phys. 2, 605 (2006).
[37] M. A. Belknap, S. H. Han, X. Wei, and Y. R. Shen, Phys. Rev. Lett. 87, 113001 (2001).