The foundations of quantum theory  
and its possible generalizations*

V. A. Franke†

Saint Petersburg State University, Saint Petersburg, Russia

Abstract

Possible generalizations of quantum theory permitting to describe in a unique way the development of the quantum system and the measurement process are discussed. The approach to the problem based on the Lindblad’s equation for the statistical operator is reviewed. The Tomonaga-Schwinger like equation of this type is introduced to establish Lorentz invariance. The application of tachyonic field to overcome divergences arising in this equation is analyzed. Other approaches to the problem are shortly discussed.

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†E-mail: valentin.alf.franke@gmail.com
Since the discovery of quantum mechanics Albert Einstein and some other physicists were not satisfied with its standard (Copenhagen) interpretation. The interest in this field rises today because (a) the theorists have approached in their constructions the Planck scale \( l_{pl} \approx 10^{-33} \) cm; \( m_{pl} \approx 10^{-5} \) gr, where a new physics may be found; (b) the astronomical observations carried out by man-made satellites permit to verify cosmological models describing the very beginning of the Universe history, when the observable part of the Universe was much smaller than an atom; hence, to construct such models one has to apply quantum theory to the Universe as a whole; this requires a generalization of conventional interpretation of quantum theory; (c) intensive investigations of the domain, intermediate between microscopic and macroscopic world, are now carried out; this is related to the interpretation of quantum theory.

In the period of eighty years some physicists have formulated the following requirements which are desirable to be fulfilled and which the quantum theory does not satisfy or does not exactly satisfy: (1) the theory should describe in a unified way the existing reality and not only the relations between observations (as J. S. Bell said, “the theory should be not about observables but about beables”); (2) being formulated in terms of probabilities the quantum theory should permit a statistical derivation; an unknown today subquantum world should be found, where the corresponding statistics takes place; (3) some conditions of locality and causality should be fulfilled in the mentioned subquantum world [1].

We shall concentrate ourselves mainly on the first point because it is a necessary prerequisite to the investigations of the second and third points and because it is very little known about the second point. But beforehand we shall describe the important result of J. S. Bell concerning causality in the hypothetical subquantum world.

Let two spin 1\(^2\) particles are prepared in a common state of spin 0 in the spacetime region \( \sigma \) at the time \( t_0 \). Let the particles move to the regions A and B correspondingly, where two observers measure simultaneously at the time \( t_1 > t_0 \) the spin projections of these particles onto the directions \( \vec{A} \) and \( \vec{B} \). Let the spin-orbital interaction be absent. Then the quantum mechanics teaches us and the experiment confirms that the probability to find both spin projections to be positive is equal to \( w(\vec{A}, \vec{B}) = \sin^2 \theta \), where \( \theta \) is the angle between the directions \( \vec{A} \) and \( \vec{B} \).

Trying to explain in an objective way the correlation between the measurements in the regions A and B one introduces hidden variables existing in the region \( \sigma \). Let us denote all these variables by one symbol \( \alpha \). Then one assumes that there exists a probability density \( \rho(\alpha) \) such that

\[
\rho(\alpha) \geq 0, \quad \int d\alpha \rho(\alpha) = 1. \quad (1)
\]

Furthermore, one assumes that for a definite value \( \alpha \) of hidden variables the conditional probabilities \( P(\alpha, \vec{A}) \) and \( P(\alpha, \vec{B}) \) exist to get positive results when the spin projections in the directions \( \vec{A} \) and \( \vec{B} \) are measured. These probabilities fulfill the conditions

\[
P(\alpha, \vec{A}) \geq 0, \quad P(\alpha, \vec{A}) + P(\alpha, -\vec{A}) = 1, \quad (2)
\]

\[
P(\alpha, \vec{B}) \geq 0, \quad P(\alpha, \vec{B}) + P(\alpha, -\vec{B}) = 1, \quad (3)
\]

for all \( \vec{A} \) and \( \vec{B} \). From the conventional probability theory it follows that

\[
w(\vec{A}, \vec{B}) = \int d\alpha \rho(\alpha) P(\alpha, \vec{A}) P(\alpha, \vec{B}). \quad (4)
\]

J. S. Bell has shown that no functions exist fulfilling the conditions [1], (2), (3) and the equality \( w(\vec{A}, \vec{B}) = \sin^2 \frac{\theta}{2} \) even approximately (the discrepancy turns out to be about 30\%). Here it is important that \( P(\alpha, \vec{A}) \) does not depend on \( \vec{B} \) and \( P(\alpha, \vec{B}) \) does not depend on \( \vec{A} \).
due to relativistic causality which one assumes to hold in the subquantum world of hidden variables.

It follows that no theory of subquantum world where the conventional relativistic causality holds can be built. In every theory of subquantum world the relativistic causality must arise at higher level of quantum and macroscopic world.

Returning to the problem of unified description of reality it is natural to assume the following principle of macroscopic definiteness: there exists really only one four-dimensional macroscopic picture of the world where all macroscopic quantities are defined with macroscopic precision.

The most straightforward way to implement this principle and to ensure simultaneously the unified description of reality is related with the generalization of quantum theory. One has to change the equation describing the time evolution of quantum system in such a way that each superposition of macroscopically different quantum states transforms in time automatically into one of these states with conventional probabilities. The description of microscopic systems should not change significantly. The generalized theory should be irreversible in time. So one has to modify the Schrödinger equation which is reversible. The simple possibility is to use the equation for statistical operator (quantum density matrix) ρ. This permits to describe irreversibility. We consider firstly such a possibility in formal manner and afterwards discuss some physical problems and other approaches.

If it is known that an object possesses a quantum state vector (a wave function) ψₙ with the probability wₙ, we attribute to it a ρ-matrix

\[ ρ = \sum_n w_n |ψ_n⟩⟨ψ_n|. \] (5)

When ρ = |ψ⟩⟨ψ| one calls the state of the object pure and otherwise mixed. The average value of any hermitian operator A in the state ρ is equal to

\[ tr(Aρ) = \sum_n w_n ⟨ψ_n|ρ|ψ_n⟩. \]

The representation (5) of a mixed state is not unique. It becomes unique if one requires that ⟨ψₙ|ψₘ⟩ = δₙₘ and if all the wₙ are different. The ρ-matrix characterizes the mixed state completely. No more information can be extracted from such state by experiment than it is contained in the ρ-matrix.

In the conventional quantum theory the ρ-matrix fulfills the equation

\[ \frac{dρ}{dt} = -i[H, ρ], \] (6)

which is equivalent to Schrödinger equation for the wave functions ψₙ. [ , ] is a commutator. The ρ-matrix is hermitian (ρ = ρ†), normalized (Tr ρ = 1) and positive definite (⟨ψ|ρ|ψ⟩ ≥ 0 ∀ ψ). The equation (6) preserves these properties and is reversible in time.

The solution of the equation (6) has the form

\[ ρ(t) = U(t)ρ(0)U†(t), \] (7)

where the unitary operator U(t) is equal to U(t) = exp(−iHt). The transformation (7) is linear and preserves the hermicity, normalization and positive definiteness of ρ. It describes a process reversible in time.

To introduce the irreversibility in time one has to generalize the transformation (7). It must stay linear to conserve the probabilities wₙ in the eq-n (5). These probabilities describe our knowledge of the system and cannot change in time. Furthermore the hermicity, normalization and positive definiteness of ρ should be preserved as before to avoid complex and negative probabilities and to conserve the probability.
It is known for a long time \[2\] that a most general linear transformation of the \(\rho\)-matrix preserving the hermicity and normalization can be written in the form

\[
\rho(t) = \sum_n \lambda_n A_n \rho(0) A_n^\dagger, \tag{8}
\]
where \(\lambda_n\) are real numbers and \(A_n\) are operators such that \(\sum_n \lambda_n A_n^\dagger A_n = I\).

If one takes

\[
\lambda_n > 0 \ \forall \ n, \tag{9}
\]
the positive definiteness is also preserved, because it follows from the equality

\[
\rho(0) = \sum_m w_m \langle \psi_m | \rho(0) | \psi_m \rangle, \quad w_m > 0,
\]
that in this case

\[
\langle \psi | \rho(t) | \psi \rangle = \sum_{n,m} w_m \lambda_n \langle \psi | A_n | \psi_m \rangle \langle \psi_m | A_n^\dagger | \psi \rangle \geq 0 \ \forall \ \psi.
\]

But under the condition (9) the relation (8) is not the most general transformation preserving the positivity condition. For example a transformation

\[
\rho(t) = \sum_n \lambda_n A_n \rho(0) A_n^\dagger + \sum_m \hat{\lambda}_m (B_m \rho(0) B_m^\dagger)^*, \tag{10}
\]
where

\[
\sum_n \lambda_n A_n^\dagger A_n + \sum_m \hat{\lambda}_m B_m^\dagger B_m = I,
\]
\(\lambda_n > 0, \hat{\lambda}_m > 0 \ \forall \ n, m\) and \(^*\) designates complex conjugation, preserves positivity, hermicity and normalization, but cannot in general be written in the form (8) with \(\lambda_n > 0\).

One calls each linear transformation of \(\rho\)-matrices preserving hermicity, normalization and positivity conditions a positive dynamical transformation, and one calls the transformation of the form (8) with \(\lambda_n > 0\) completely positive dynamical transformations.

The role of completely positive transformations is seen from the following example. Consider a system consisting of two objects which do not interact before the time moment \(t = 0\) possess at this moment the \(\rho\)-matrices \(\rho^{(1)}_{AB}(0)\) and \(\rho^{(2)}_{ik}(0)\). Common \(\rho\)-matrix at \(t = 0\) is the product

\[
\rho_{Ai,Bk}(0) = \rho^{(1)}_{AB}(0)\rho^{(2)}_{ik}(0).
\]
Let these objects interact in the time interval \((0, t_1)\) and let the evolution of the system be described in this interval by a unitary operator \(U_{Ai,Bk}\). Then at the moment \(t_1\) we have the \(\rho\)-matrix

\[
\rho_{Di,Em} = \sum_{A,i,B,k} U_{Di,Ai} \rho^{(1)}_{AB}(0) \rho^{(2)}_{ik}(0) U_{Bk,Em}^\dagger.
\]
If we make observations at the time \(t_1\) only on the first object and are not interested in the second, we can get the \(\rho\)-matrix of the first object putting

\[
\rho^{(1)}_{DE}(t_1) = \sum_l \rho_{Di,El}(t_1) = \sum_{l,i,k} \sum_{A,B} U_{Di,Ai} \rho^{(1)}_{AB}(0) \rho^{(2)}_{ik}(0) U_{Bk,El}^\dagger.
\]
Because

\[
\rho^{(2)}_{ik}(0) = \sum_p \lambda_p |\psi_p(0)\rangle_i \langle \psi(0)|_k, \quad \lambda_p > 0,
\]

\[4\]
one gets

$$\rho^{(1)}_{DE}(t_1) = \sum_{p,l} \lambda_p \left( \sum_i U_{DL,Ai} |\psi_p(0)\rangle_i \right) \times \rho^{(1)}_{AB}(0) \left( \sum_k k |\psi_p U_{Bk,Ei}^\dagger \right).$$  \hspace{1cm} (11)

This is a completely positive transformation of the $\rho$-matrix $\rho^{(1)}$. It is irreversible even when the initial states $\rho^{(1)}(0)$ and $\rho^{(2)}(0)$ are pure because of the summation over the index $l$ in the eqn (11). Let us stress that the irreversibility arises not because the two systems interact but because the information about the second system is lost completely. And this is a common rule: the development of a state in quantum theory becomes irreversible if some information about the considered system disappears.

The described situation is typical for irreversible physical processes. That’s why one assumes commonly that only completely positive transformations are of physical interest. But this is not exactly the case. In quantum field theory one has often to do with indefinite metric. In this case the operators $U_{DL,Ai}$ are only pseudounitary and the transformation (11) may not be completely positive. Nevertheless under appropriate conditions it can be positive. This is a special way to get rid of the indefinite metric by introducing irreversibility. That’s why the positive but not completely positive dynamical transformations are worth of some attention. But it is very hard unsolved mathematical problem to find the general form of positive dynamical transformations.

Today it is known that the equation (10) defines a most general positive transformation only for a $\rho$-matrices of dimension $2 \times 2$ \cite{3} but not in other cases. The problem of describing general positive transformations is connected with Artin’s theorem which solves the 5-th Hilbert problem. Artin has proven that each rational function which is positive everywhere can be represented as a sum of squares of rational functions.

Let us write the dynamical transformation in the form

$$\rho_{ik}(t) = \sum_{lm} L_{ik,lm} \rho_{lm}(0),$$

where $L_{ik,lm} = L_{ki,ml}^\ast$ to ensure hermicity. It is enough to preserve positivity for pure initial states $\rho_{lm} = |\psi_i \rangle |\psi_m \rangle^\ast$. The positivity condition in this case looks like

$$\sum_{i,k,l,m} \psi_{ik}^\ast \psi_{li}^\ast L_{ik,lm} \psi_{lj} \psi_{mi}^\ast \geq 0 \\forall \psi_{ik}^\ast, \psi_{lj}.$$ 

According to Artin’s theorem there exists a representation

$$\sum_{i,k,l,m} \psi_{ik}^\ast \psi_{li}^\ast L_{ik,lm} \psi_{lj} \psi_{mi}^\ast = \sum_{\nu} \left( P_{\nu}^\ast P_{\nu}^\ast Q_{\nu}^\ast Q_{\nu} \right),$$  \hspace{1cm} (12)

where $P_{\nu}, Q_{\nu}$ are polynomials with respect to $\psi_i, \psi_k^\ast, \psi_i^\ast, \psi_m^\ast$. Here the extension of Artin’s theorem to complex region is used. After the right side of the equation (12) is reduced to a common denominator this denominator should cancel with the nominator. How to fulfill this condition in general is unknown because the proof of Artin’s theorem is based on Zorn’s lemma and that’s why it is unconstructive. From here on we consider only completely positive dynamical transformations.

After the differentiation of the relation

$$\rho(t) = \sum_n \lambda_n(t) A_n(t) \rho(0) A_n^\dagger$$

with respect to time and introduction of some new denotations one gets the equation

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_n \alpha_n \left( 2B_n \rho B_n^\dagger - B_n^\dagger B_n \rho - \rho B_n^\dagger B_n \right), \\alpha_n > 0,$$  \hspace{1cm} (13)
where \( B_n \) are some operators. It is easy to verify directly that this equation preserves hermicity, normalization and positivity. The equation (13) is called the Lindblad’s equation [4].

Via this equation one can describe the decay of the macroscopically indefinite state into macroscopically definite ones. Let us describe an example. Consider the simplest case when only one operator \( B \) is present in (13) and \( H = 0 \). Then
\[
\frac{d\rho}{dt} = \alpha \left( 2B\rho B^\dagger - B^\dagger B\rho - \rho B^\dagger B \right).
\]
Let the operator \( B \) be hermitian, so that \( B = B^\dagger \). Consider the frame where \( B \) is diagonal. Let \( b_1, b_2, \ldots \) are the eigenvalues of \( B \). Then
\[
\frac{d(b_1|\rho|b_2)}{dt} = \alpha \left( 2b_1(b_1|\rho|b_2)b_2 - b_1^2(b_1|\rho|b_2) - (b_1|\rho|b_2)b_2^2 \right).
\]
or
\[
\frac{d(b_1|\rho|b_2)}{dt} = -\alpha(b_1 - b_2)^2(b_1|\rho|b_2).
\]
The solution is
\[
(b_1|\rho(t)|b_2) = \exp(-\alpha(b_1 - b_1)^2t) \times (b_1|\rho(0)|b_2).
\]
So all the nondiagonal matrix elements of \( \rho \) disappear when the time goes and the matrix \( \rho \) becomes a mixture of eigenstates of the operator \( B \).

The constant \( \alpha \) should be very small because otherwise the conventional quantum mechanics of microscopic objects will be destroyed. But when \( B \) is a macroscopic operator, the quantity \( \alpha(b_1 - b_2)^2 \) may be large, and the initial state may decay into eigenstates of this operator.

Now let us return to the general equation (13). Let all the operators \( B_n \) be macroscopic and all the numbers \( \alpha_n \) very small. The operators \( B_n \) may not commute exactly with the Hamiltonian \( H \) and with each other. But the average values of these commutators are much smaller than the average values of the operators \( B_n \). So one should expect that the superposition of the eigenstates of operators \( B_n \) belonging to macroscopically different eigenvalues will be destroyed.

We see that via replacing the Schrödinger equation by the equation (13) with appropriate operators \( B_n \) and numbers \( \alpha_n \) we can formally fulfill the principle of macroscopic definiteness in the case of nonrelativistic physics.

The other proposed approach to the problem is based on a stochastic differential equation for the quantum state vector \( \psi \) [5]. One assumes that \( \psi \) is a stochastic quantity and writes for it an equation similar to Schrödinger’s but with some noise. This approach is equivalent to considering the probability distribution \( W(\psi) \) over the Hilbert space of all quantum state vectors \( \psi \).

Clearly
\[
W(\psi) \geq 0
\]
and
\[
\int d\mu(\psi)W(\psi) = 1,
\]
where \( d\mu(\psi) \) is some measure on the Hilbert space. For this distribution one may write an evolutionary equation
\[
\frac{dW(\psi)}{dt} = L(W(\psi))
\]
where \( L \) is some linear operator preserving the conditions (14), (15). This approach has the advantage that one can directly restrict the functional \( W(\psi) \) to be zero on macroscopically
indefinite states. But the following additional restriction must be fulfilled. If two distributions 
\( W_1(\psi) \) and \( W_2(\psi) \) correspond to one and the same \( \rho \)-matrix at the initial time, i.e.

\[
\rho(t_1) = \int d\mu(\psi) W_1(t_1, \psi) |\psi\rangle \langle \psi| \\
= \int d\mu(\psi) W_2(t_1, \psi) |\psi\rangle \langle \psi| ,
\]

then this equality should persist with time. To find the general condition under which such requirement takes place is equivalent to discover the general form of positive dynamical transformation. It is extremely difficult.

That’s why it seems easier to write firstly the equation (13) for the \( \rho \)-matrix and then to verify whether it can be represented in the form (16) with appropriate \( W(\psi) \). If not, one has to look for other equation of the type (13). Let us remark that one and the same equation (13) can be represented in the form (16) in many ways, because the representation of the \( \rho \)-matrix (1) is not unique.

One further approach, popular today, is called "the method of decoherent histories". One assumes that the Universe behaves such as if somebody measures periodically or continuously some set of macroscopic quantities. Let these "measurements" take place at the time moments \( t_1, t_2, \ldots \) At each moment \( t_i \) one defines a set of projectors \( p_{in} \) on subspaces of the Hilbert space, such that

\[
p_{in}p_{im} = p_{in}p_{im} = 0 \quad \forall \ m \neq n, \forall \ i \\
\sum_n p_{in} = I \quad \forall \ i.
\]

Each subspace corresponds to definite values of macroscopic quantities fixed with macroscopic precision but is large enough to contain the superpositions of microscopic states. One assumes that the probability that the macroscopic quantities have corresponding values is equal to

\[
\langle \psi | p_{1n_1}p_{2n_2} \cdots p_{N-1,nN-1}p_{NN}p_{N-1,nN-1} \cdots p_{2n_2}p_{1n_1} | \psi \rangle ,
\]

where \( \psi \) is the initial state of the Universe. We have used the Heisenberg representation. This approach gives for each quantity \( \langle \psi | p_{in} | \psi \rangle \) the exact conventional quantum value only if all projectors \( p_{in} \) commute in the Heisenberg representation (for all \( i \) and \( n \)). This is very difficult to achieve because to do this one has to solve the Heisenberg equations of motion. But one may hope that, if the projectors \( p_{in} \) are macroscopic and do not exactly commute, the deviation of the values of \( \langle \psi | p_{in} | \psi \rangle \) from the conventional quantum values will be very small.

This approach is connected with the method based on the Lindblad’s equation. If one appropriately defines the \( \rho \)-matrix at each moment \( t_i \), introduces the Schrödinger representation and goes to the limit \( t_{i+1} - t_i \to 0 \), one gets the Lindblad’s equation (13).

Let us now take into account the requirement of the Lorentz invariance assuming that the space-time is flat. The simplest way to do this consists in going to interaction representation assuming that the field operators fulfill the conventional equations for free fields. We consider the \( \rho \)-matrix \( \rho(\sigma) \) which depend on the spacelike hypersurface \( \sigma \) and write down an equation similar to one of Tomonaga-Schwinger:

\[
\frac{\delta \rho(\sigma)}{\delta \sigma(x)} = -i[H_{int}(x), \rho] +
+ \alpha \sum_n \left\{ 2A_n(x)\rho(\sigma)A_n^\dagger(x) - A_n^\dagger(x)A_n(x)\rho(\sigma) - \rho(\sigma)A_n^\dagger(x)A_n(x) \right\} .
\]
To fulfill the Bloch integrability condition one has to assume that all operators $H_{int}(x)$, $A_n(x)$, $A_n^\dagger(x)$ commute when they are taken at different points of the hypersurface $\sigma$. This is not possible if $A_n$ are macroscopical operators as we have assumed in the nonrelativistic case. Nevertheless it is worth to investigate the case when the $A_n$ are local operators commuting at different points on $\sigma$.

For simplicity let us consider the case, when only one free complex scalar field is present, and write the equation (17) as follows

\[
\frac{\partial \rho(\sigma)}{\partial \sigma(x)} = \alpha (2\phi(x)\rho(\sigma)\phi^\dagger(x) - \phi^\dagger(x)\phi(x)\rho - \rho\phi^\dagger(x)\phi(x)).
\]  

Let $\rho$ be a vacuum state

\[
\rho = |\Omega\rangle \langle \Omega |.
\]

Taking into account that

\[
\phi(x) = \int \frac{d^3p}{\sqrt{2p_0}} (b^\dagger(\vec{p})e^{ipx} + a(\vec{p})e^{-ipx}),
\]

where $b^\dagger, a$ are creation and annihilation operators, one sees that the term $\phi(x)\rho\phi^\dagger(x)$ renders the vacuum state $|\Omega\rangle \langle \Omega |$ into all possible one particle states with comparable probabilities. This leads to a strong divergence: the vacuum state disappears immediately. Because of positivity of all corresponding quantities this divergence cannot be renormalized. The difficulty remains when one goes to more complicated theories because according to axiomatic field theory no local operators can annihilate the vacuum state if all conventional requirements are fulfilled. But without the positive energy condition this result cannot be proved. That’s why there exist local tachyonic fields annihilating the vacuum state. Such fields can be substituted into equation (18) for the $\phi(x)$ without creating divergences.\footnote{To use tachyonic fields in this context has proposed P. R. Pearle [6]} The tachyonic field makes the vacuum unstable by conventional tool, but if this field interacts with other fields very weakly such instability may be unobservable.

Let us look for a spin zero field $\phi(x)$ satisfying the causality conditions

\[
[\phi(x), \phi(y)] = 0 \quad \forall \, x, y : (x - y)^2 < 0,
\]

\[
[\phi(x), \phi^\dagger(y)] = 0 \quad \forall \, x, y : (x - y)^2 < 0
\]

and annihilating the vacuum state:

\[
\phi(x)|\Omega\rangle = 0.
\]

Let us put

\[
F(x) = i\langle \Omega |[\phi(x), \phi^\dagger(0)]|\Omega\rangle,
\]

\[
F_r(x) = \theta(x^0)F(x),
\]

\[
F_a(x) = \theta(-x^0)F(x),
\]

where $\theta(x^0) = 1$ if $x^0 > 0$, $\theta(x^0) = 0$ if $x^0 < 0$, and define the Green function

\[
G(x) = i\langle \Omega |T(\phi(x)\phi^\dagger(0))|\Omega\rangle,
\]

Due to the equality (19) one gets $G(x) = F_r(x)$. Let us define Fourier transform of the $F(x)$:

\[
\tilde{F}(k) = \frac{1}{(2\pi)^2} \int d^4xe^{ikx}F(x)
\]
and similarly $\tilde{F}_r(k) = \tilde{G}(k)$ and $\tilde{F}_a(k)$.

In analogy with conventional derivation of Lehmann representation one gets

$$
\tilde{G}(k) = \tilde{F}_r(k) = f(k^2 + i\epsilon \text{sgn } k^0),
$$

$$
\tilde{F}_a(k) = f^*(k^2 + i\epsilon \text{sgn } k^0),
$$

$$
\tilde{F}(k) = \tilde{F}_r(k) - \tilde{F}_a(k) = 2i \text{Im } f(k^2 + i\epsilon \text{sgn } k^0),
$$

(22)

where the function $f(s)$ is analytic in all the complex $\mathbb{C}$-plane without, perhaps, the positive part of real axis. From the eqns (20) and (21) it follows that

$$
\text{Re } \tilde{F}(k) = 0, \quad \text{Im } \tilde{F}(k) \geq 0,
$$

so that for each real $s \equiv k^2$

$$
\text{Im } f(s \pm i\epsilon) \geq 0.
$$

To get the general representation for the $f(s)$ let us put

$$
\xi_+(s) = \frac{1}{2}(f(s) + f^*(s^*)),
$$

$$
\xi_-(s) = -\frac{i}{2\sqrt{-s}}(f(s) - f^*(s^*)),
$$

where the $\sqrt{-s}$ is positive if $s < 0$ and has a cut at $0 \leq s < +\infty$. The functions $\xi_{\pm}(s)$ have the same analytic properties as $f(s)$. Let us assume that the functions $\xi_{\pm}(s)$ decrease at $|s| \to \infty$ faster than $|s|^{-\eta}$ with some positive $\eta$. Then the dispersion relations hold

$$
\xi_\pm = \frac{1}{\pi} \int_{s'=0}^\infty ds' \frac{\text{Im } \xi_{\pm}(s' + i\epsilon)}{s' - s}.
$$

Consequently it takes place the relation

$$
f(s) = \frac{1}{2\pi} \left( \int_0^\infty ds' \frac{\text{Im } f(s' + i\epsilon) - \text{Im } f(s' - i\epsilon)}{s' - s} + 
+ i\sqrt{-s} \int_0^\infty ds' \frac{\text{Im } f(s' + i\epsilon) + \text{Im } f(s' - i\epsilon)}{\sqrt{-s'}} \right). 
$$

(23)

Defining at $0 \leq s \leq +\infty$ arbitrary nonnegative decreasing fast enough functions $\text{Im } f(s' + i\epsilon)$ and $\text{Im } f(s' - i\epsilon)$, one gets from the eqn (23) the function $f(s)$ fulfilling all requirements.

One sees from the eqns (22) and (23) that the function $\tilde{F}(k)$ differs from zero at all $k^2 < 0$ if it is not zero everywhere. That’s why by definitions (20) and (21) the tachyonic spectrum is continuous and contain all the negative part of the $k^2$-axis.

We can now put

$$
\phi(x) = \frac{1}{(2\pi)^2} \int d^4k \ a(k)e^{-ikx},
$$

where the $a(k)$ are annihilation operators fulfilling the relation
\[ [a(k), a(k')] = \eta(k)\delta^4(k - k'), \]
\[ [a(k), a(k')] = 0, \]
\[ a(k)|\Omega\rangle = 0, \]
where \( \eta(k) = 2(2\pi)^2 \text{Im} f(k^2 + i\epsilon \text{sgn} k^0). \) Inserting these operators \( \phi(x) \) into the equation (18) one sees that no divergence appears because the right side vanishes if the \( \rho \) is a vacuum state. All other purely tachyonic states transform into the vacuum when the time goes.

If such a tachyonic fields interact with all other nongravitational fields very weakly and with the gravitation in a conventional way, they can exist without a contradiction with the experiment. They can even be a part of the hidden matter in the Universe. It is worth to investigate the possibility of existing of such fields in cosmology.

But we have to take into account that the operator \( \phi(x) \) of a tachyonic field is not a macroscopic operator. That’s why it is not guaranteed that the equation of the type (17) with such a field will lead to the required decay of macroscopically indefinite states. This point is not investigated today and it is worth to do this.

Until now we have tried to describe the decay of macroscopically indefinite states only formally. But one can ask what from the additional terms in the Lindblad’s equation can be derived. Much work has been done to get such terms from the interaction of the object with the environment (the thermal bath). The considerations are like our simple example on the page 4. But the second system is now the environment. Such considerations permit to get quantitative results. Especially, the time of the decay of macroscopically indefinite states can be calculated. In this way it was shown that the macroscopically definite states are stable against decay and the macroscopically indefinite are not, what is an interesting result. It is to stress here that the needed irreversibility arises not because the environment acts on the system, but because some information about the system disappears in the environment.

However these considerations do not solve the problem completely, because the environment can be included into the quantum system (at least in principle). So one has to consider a new larger environment and so on. Finally all the Universe is included into the quantum system, and there is no environment more. One may try to solve the problem asking oneself how some information about the quantum system can disappear completely, i.e. in such a way that it cannot return to the system. One may imagine three ways leading to this disappearance.

Firstly the system may radiate electromagnetic, gravitational and other waves that disappear at the infinity. This is possible in flat as well as in curved space-time. Secondly in rapidly expanding Universe the connection between two parts of a system may become impossible because the appropriate light cones do not intersect. Thirdly if our space-time is a surface in a space of higher dimensions, the information can flow into the additional dimensions.

At least in the first and second case a very long time is needed before the information disappears completely. To fulfill the principle of macroscopic definiteness one has to require that the macroscopically indefinite states decay much more rapidly. So one is led to consider a possibility that the future circumstances influence the present state of affairs leading to the decay of such states. This is in accordance with the Bell’s proof that the subquantum world cannot be causal in conventional relativistic sense. If the only result of the action of the future on the present is the decay of macroscopically indefinite states then this action cannot contradict the conventional macroscopic causality.

Clearly this is only a very preliminary discussion of the problem. But one has to bear it in mind.

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