Experimental study on the ultrasonic nonlinear damage characteristics of expansive soil during constant amplitude dry-wet cycles

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- Wave velocity;
- Low-stress shear strength.

**Abstract.** Various control parameters of the dry-wet cycles can affect the development of expansive soil damage. It is important to study the fatigue damage characteristics of expansive soil under dry-wet cycles of constant amplitude. This paper considers expansive soil from Baise in Guangxi, China as the research object. Based on tests of the P-wave velocity and low-stress shear strength of expansive soil under 0-6 constant amplitude dry-wet cycles, the attenuation laws for the P-wave velocity were analyzed, the damage variable of expansive soil was characterized by P-wave velocity, and the rationality of this damage variable was verified by measuring the low-stress shear strength values of expansive soil specimens. Based on the experimental P-wave velocity results, a nonlinear empirical model of fatigue damage for expansive soil was constructed. As a result, the P-wave velocity of an expansive soil sample decreases nonlinearly with an increasing number of dry-wet cycles and that the damage degree increases nonlinearly with an increasing number of cycles. The cohesive force and damage degree of expansive soil show an attenuation relationship that is well fit by a logarithmic function; in general, the damage characteristics of expansive soil are closely related to the dry-wet cyclic loading path.

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1. Introduction

Expansive soil is a special type of unsaturated soil whose mineral composition mainly consists of strongly hydrophilic clay minerals (Montmorillonite and Illite). Because of the distinctive engineering properties of expansive soil such as fissuring, expansion and shrinkage, and supersolidity, its applications remain significantly limited [1]. These properties are sensitive to natural climatic conditions, which cause expansive soil to repeatedly expand, shrink, and deform. Because the soil is constrained by its surrounding, repetitive tensile stress is applied in the interior of the soil [2,3]. Through the functionality of this tensile stress, many micro-crack cores will form in the expansive soil itself and its interior structure, and the micro-cracks will gradually expand with an increasing number of stress cycles and ultimately form macro-cracks, resulting in fatigue damage to the soil and attenuation of the soil strength [4]. Therefore, studying the damage characteristics of expansive soil is vitally important to understand the nature of fissure development and to guide design and treatment practices in expansive soil engineering.
At present, the question of how to measure the degree of development of material fatigue damage under cyclic loading has received considerable attention from researchers. To theoretically describe the development of fatigue damage to materials, it is necessary to establish damage variables and cumulative fatigue damage models. Since the emergence of Miner’s theory of linear cumulative fatigue damage, various theories and models of fatigue damage have continued to emerge and evolve [5–7]. Nevertheless, there have been few studies on fatigue damage to expansive soil under the action of dry-wet cycles and related theories. Some scholars have performed related research. For example, Gens and Alonso [8] proposed an elastic-plastic conceptual model to describe the expansion and shrinkage characteristics of reshaped expansive soil, known as the G-A model of expansive soil. Lu et al. [9] described the behavior of the non-damaged part of unsaturated soil using a nonlinear constitutive model. The properties of the damaged part were described using a damage evolution equation and two yield surfaces (corresponding to loading yield and shear yield). Furthermore, a consolidation model for unsaturated undisturbed expansive soil was established. Shi et al. [10] considered the influences of the water loss rate and dry-wet cycles on the depth of crack development and established a crack development depth calculation model that considers the water loss rate and cumulative damage. Tang et al. [11] adopted the method of multi-scale analysis to study the microstructure and mechanical properties of expansive soil under freeze-thaw cycles and established a unified expression considering the resulting microdamage to the stress-strain characteristics of the soil. Although these studies are highly valuable, the damage characteristics of and models for expansive soil materials still need to be further studied and explored.

In practice, due to the irregular variations of rainfall and groundwater, the distribution of soil moisture content shows spatial irregularities even at the same depth. In addition, the effect of the atmosphere on the underground moisture content generally decreases with increasing depth, causing the distribution of soil moisture content to gradually vary with the depth of the soil layer. Therefore, it is essential to consider the laws governing the effects of various dry-wet cycle parameters (initial moisture content, dry-wet cycle times, cycle amplitude, etc.) on fracture and damage development in expansive soil.

The mechanical characteristics of expansive soil are closely related to the dry-wet cyclic loading path, and the changes in the mechanical properties of the soil are bound to exhibit significant differences with different loading paths. Therefore, it is necessary to study the attenuation laws for the mechanical properties of expansive soil with respect to the cycle amplitude. This study considers the case of dry-wet cycles of constant amplitude, which is helpful for providing an important research foundation for further experimental study on the effects of dry-wet cycles on expansive soil.

The main indicators used for the definition of damage variables include stress and strain [12], CT number [13], and ultrasonic wave velocity [14]. As a non-destructive, fast and economical method, ultrasonic testing technology has been widely used in rock mass engineering, and the extent of the use of this technology in laboratory-based experimental research is showing an upward trend [15,16]. The characteristics of sound waves in soil (wave velocity, waveform, amplitude, frequency, etc.) are related to the acoustic characteristics of the rock mass, and there are great differences among different rock masses because of their different structures, mechanical properties, initial stresses, and so on. Moreover, the research results concerning acoustic waves in rock masses can serve as an important reference for acoustic behavior of soil materials, and various parameters, mechanical models, and theories related to rock engineering can be applied to soil materials [14,17].

This study used a non-metallic ultrasonic instrument to test the P-wave velocities of expansive soil samples with different initial moisture contents after different numbers of dry-wet cycles under different constant amplitude conditions, analyzed the attenuation laws for the P-wave velocity of expansive soil subjected to constant amplitude dry-wet cycles, defined a damage variable on the basis of the P-wave velocity, and explored the fatigue damage to expansive soil after constant amplitude dry-wet cycles. The rationality of the damage variable defined on the basis of the P-wave velocity was analyzed based on the experimental results for the low-stress shear strength of expansive soil. Finally, based on the results of ultrasonic tests, a nonlinear fatigue damage model for expansive soil was established and the experimental damage results were estimated. The findings of this study will deepen existing understanding of the physical and mechanical properties and damage behavior of expansive soil under the action of dry-wet cycles and promote the development of damage theory and engineering applications for unsaturated soil.

2. Testing programme

2.1. Expansive soil samples

Because of the influences of structure, inhomogeneity, stress history, and so on, undisturbed expansive soil specimens show poor comparability in terms of internal fracture development and wave velocity variations; even under the same testing conditions, and it is difficult to collect uniform undisturbed soil samples in the field. Therefore, the expansive soil samples used in this study were reshaped. The samples were taken from
Table 1. Basic physical indexes of the expansive soil samples.

| Density (g.cm⁻³) | Liquid limit (%) | Plasticity index | Optimal water content (%) | Maximum dry density (g.cm⁻³) | Particle composition (mm/%) | Free swelling ratio (%) |
|------------------|------------------|------------------|---------------------------|-----------------------------|----------------------------|------------------------|
| 2.002            | 56               | 35               | 17.46                     | 1.80                        | > 0.075                   |                       |

Baise in Guangxi Province, China from a soil depth of 2.0 m away from direct atmospheric influence. The physical properties of the expansive soil samples are listed in Table 1.

2.2. Definition of the damage variable
The changes to the microstructure and some macroscopic physical properties of a material caused by material damage correspond to the degree of damage. Therefore, first, it is necessary to select an appropriate damage variable to describe the degree of damage to a material.

The ultrasonic wave velocity is usually measured using the ultrasonic pulse transmission method, which is practical, convenient, and fast. In this study, a non-metallic ultrasonic meter was used to measure the wave velocities of expansive soil subjected to constant amplitude dry-wet cycles. The P-wave (longitudinal wave) velocity was adopted as a basis for a variable defined to quantify the damage degree of an expansive soil sample. The formula is as follows [18,19]:

\[
D = 1 - \left( \frac{V_p}{V_0} \right)^2 \quad (0 < D < 1),
\]

where \( D \) denotes the damage degree of the expansive soil sample, \( V_p \) denotes the P-wave velocity of the expansive soil after some number of dry-wet cycles, and \( V_0 \) denotes the initial P-wave velocity of the uncycled expansive soil sample.

The damage variable \( D \) defined in Eq. (1) is a relative value; any specimen will be damaged in the process of its creation and no completely damage-free material exists. Therefore, \( D = 0 \) simply corresponds to the value of the damage variable for a specimen during its 0th dry-wet cycle.

2.3. Testing process
2.3.1. Establishment of the initial moisture content
The damage to the mechanical properties of expansive soil is closely related to the dry-wet cyclic loading path [20]. Therefore, it is essential to study the laws governing the damage development of expansive soil with different initial moisture contents. In practical engineering applications, due to the action of atmospheric forces, the moisture content in a body of expansive soil will exhibit a water content gradient that decreases or increases with increasing depth. To consider the effect of the initial moisture content, controlled moisture contents of \( w = 15\%, 17\%, 19\%, \text{and} 21\% \) were initially established.

2.3.2. Sample preparation
The recovered expansive soil samples were air dried, crushed, and sifted through a 2 mm mesh. Soil samples were prepared with initial moisture contents of 15\%, 17\%, 19\%, and 21\%, and the samples were sealed for 24 hours to ensure that the initial moisture content was evenly distributed throughout the soil. Then, the static pressure method [21] was applied to compact the soil samples into specimens with a dry density of 1.7 g.cm⁻³ (61.8 mm in diameter and 20 mm in height). With this method, the dry density of the specimens could be accurately controlled to within an error of ±0.02 g.cm⁻³. Compared with the ASTM standard compaction method, the static pressure method has the following advantages:

1. Soil saving;
2. Saving the time of making sample; for example, it takes about 30 minutes to complete an ASTM standard compaction process, while the static pressure method can produce 6-8 samples in 30 minutes;
3. High accuracy; the static pressure method controls the density of the soil sample through the weight of the soil.

2.3.3. Constant amplitude dry-wet cycles
Characteristic seasonal climate variations lead to alternating cycles of dry and wet conditions. In this study, cycle amplitudes \( \Delta w \) of ±10\%, ±7.5\%, ±5.0\%, and ±2.5\% were selected for investigation. Taking an initial moisture content of 17\% and a cycle scheme with a cycle amplitude of ±10\% as an example, the control process is given in Figure 1.

The specimens were divided into 94 groups with 4 specimens in each group, for a total of 376 specimens. The controlled wetting process was conducted as in the following process. A filter paper and a piece of permeable stone with a diameter of approximately 79.8 mm were placed above and below each specimen to ensure that the soil could uniformly absorb water and to prevent the expansion and disintegration of the
soil. Each specimen was placed in coverless glassware with a certain amount of water in the bottom so that the bottom permeable stone could freely absorb water, and the top of the specimen was exposed to a micro-spray sprinkler to simulate atmospheric precipitation. The specified cycle amplitude was used to calculate the desired moisture content of the specimen. After the desired moisture content was reached (within ±0.1%), the specimen was completely sealed with cling film and tape and placed in a moisture retention box for 24 hours to ensure that the moisture content inside and outside of the specimen was evenly distributed. The controlled drying process was conducted as follows. Since the temperature of expansive soil in Guangxi can reach approximately 40°C in summer, to simulate the de-wetting process, the temperature of the test oven was set to 40°C to dry the specimen. When the specimen had dried to a sufficient extent, corresponding to the specified cycle amplitude (within ±0.1% of the desired change in mass), drying was stopped and the specimen was completely sealed with fresh film and placed in the moisturizing cylinder. These wetting and drying processes were repeated 6 times, as shown in Figure 2.

2.3.4. Ultrasonic testing system
In this study, a TH204 non-metallic ultrasonic testing system was used, as shown in Figure 2. This instrument uses the Windows XP operating system and mature sound wave processing software, with which a wealth of waveform parameters can be obtained.

The whole system is light, compact, and easy to operate and offers powerful processing functions with high precision. The main technical specifications of the TH204 non-metallic ultrasonic testing system are as follows:
(a) The display is a 10.4-inch LCD;
(b) It has two operation modes: touch and mouse;
(c) As a power supply, it has a built-in 12 V rechargeable lithium battery, which is able to support 6–8 hours of continuous operation. It can also be connected to a large-capacity 12 V battery or 220 V AC power;
(d) It has 2 USB ports;
(e) It has 4 channels, which can be operated one at a time, two at a time, or all at once;
(f) The sampling rate is 20 MHz;
(g) It supports sampling intervals of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, and 10 µs as well as 0.02, 0.05, 0.1, 0.2, 0.5, 1, and 2 ms;
(h) The tested time ranges 0.05 µs to 163.84 s;
(i) With its memory function, it can store 2 GB of measurement data;
(j) In terms of amplifier bandwidth, it supports upper side band cut-off frequencies of 1 MHz, 100 kHz, 10 kHz, 1 kHz, and none as well as lower side band cut-off frequencies of 100 Hz, 1 kHz, 10 kHz, 100 kHz, and none;
(k) The pulse voltage is 160 V/1000 V;
(l) The transmission pulse width is continuously adjustable from 2 to 100 µs;
(m) The acoustic system is equipped with a variety of ultrasonic probes (Figure 2), which are made
of coated wafer materials. The focus part of the probe uses impedance matching, backing, piezoelectric elements, etc., to achieve good performance in terms of time and temperature stability, lack of depolarization, frequency error, pass band conditions, and simple harmonic vibration frequencies between the sensor and the acoustic transmission equipment.

During ultrasonic testing, a coupling agent was applied to the surfaces of the soil samples once they had reached the desired water content initially and after each of the 6 cycles. The pulse reflection method was employed to test the acoustic wave characteristics of the samples. The wave velocity test results of each specimen after dry wet cycle varied. In order to reduce the effects of errors on the results, the limit error method was used to process the collected acoustic data, that is, the probability that the error of a single measurement does not exceed the limit error (that is, the error equals to 3 times of standard deviation.). When the wave velocity exceeds the range of the probability values, it will be considered negligible or wrong. The average of other values within the limit probability range is taken as the statistical result.

2.3.5. Direct shear strength test under low-stress conditions

Most failures of expansive soil slopes are shallow slumps. In this case, the shape of the sliding surface is similar to a cylindrical surface and the general depth range is no greater than 5 m [22]. In addition, for normal geological structures, the instability of expansive soil slopes mostly manifests as shallow landslides, and for many of these slopes, the depth of the sliding surface is no greater than 2 m. Therefore, the actual vertical pressure on expansive soil is often less than 50 kPa. However, in conventional direct shear tests, vertical pressures of 50, 100, 200, and 300 kPa or 100, 200, 300, and 400 kPa are usually considered, which are not consistent with the realistic situation and consequently, the results are not suitable for guiding construction and engineering design [23]. Therefore, to obtain the true shear strength of the expansive soil specimens, direct shear strength tests were carried out with vertical pressures of 5, 15, 30, and 50 kPa. In this study, vertical pressures of 5, 10, and 30 kPa are replaced by the same weight of dry sand.

Soil samples that had undergone the desired number of cycles were saturated using vacuum saturation method. Specifically, the samples were placed into a saturation instrument for vacuum saturation, with an air extraction time of 4 hours and a soaking time of 24 hours. Then, the saturated soil samples were removed and an intelligent electric quadruple direct shear apparatus was employed to perform a fast shear test (Figure 2), with the shear rate set to 0.8 mm/min.

3. Test results and analysis

3.1. Attenuation laws for the P-wave velocity

3.1.1. Number of dry-wet cycles

Curves showing the variations in the P-wave velocities of expansive soil samples with the same initial moisture contents subjected to different numbers of constant amplitude dry-wet cycles are presented in Figure 3.

According to Figure 3, the P-wave velocity of an expansive soil sample decreases nonlinearly with an increasing number of cycles for a given initial moisture content and cycle amplitude. Overall, the attenuation curves can be divided into two stages. The first stage corresponds to the 1st and 2nd cycles; in this stage, the slope of the curve is large and the wave velocity decays rapidly. The second stage corresponds to the 3rd–6th cycles; in this stage, the wave velocity tends to stabilize with an increasing number of cycles. Crack initiation in the compacted expansive soil occurs mainly in the 1st and 2nd cycles, and the crack development tends to stabilize with an increasing number of cycles. The attenuation of acoustic energy occurs mainly through the scattering of acoustic waves. The reason for this scattering attenuation is the loss of acoustic energy caused by the scattering of acoustic waves by voids and micro-cracks in the material. Accordingly, the attenuation of the P-wave velocity will increase as the degree of the development of the micro-cracks in the expansive soil increases and, therefore, a higher degree of crack development will result in stronger scattering when sound waves propagate through the soil. Similarly, as the number of micro-cracks tends to stabilize with an increasing number of dry-wet cycles, the corresponding wave velocity will tend to stabilize.

3.1.2. Dry-wet cycle amplitude

Curves showing the variations in the average P-wave velocities of expansive soil samples with the same initial moisture contents subjected to the same number of dry-wet cycles with different cycle amplitudes are presented in Figure 4.

According to Figure 4, the average P-wave velocities of expansive soil samples subjected to the same number of dry-wet cycles decrease as the amplitude of the dry-wet cycles increases, given the same initial moisture content. For a given initial moisture content, the higher the moisture content of the sample is after the wetting process, the greater the reduction in the moisture content will be during the drying process. The shrinkage force acting on expansive soil during drying is directly proportional to the variation in moisture content, also called the cycle amplitude. The larger the cycle amplitude is, the greater the tensile stress and the easier the cracks (damage) can be generated in the soil. In addition, at a given drying temperature,
the larger the cycle amplitude is, the longer the drying process will be; consequently, the ageing effect of the tensile stress will act over a longer period of time and the crack development time will also be longer, leading to greater extensive crack development. Therefore, the P-wave velocity will suffer greater attenuation.

3.2. Damage laws for expansive soil

3.2.1. Number of dry-wet cycles

The observed damage degrees of expansive soil specimens with different moisture contents under dry-wet cycles of constant amplitude were fitted via nonlinear regression and an empirical equation for nonlinear fatigue damage evolution was established as follows:

\[ D = b \ln N - a \quad (N \geq 1) \]

(2)

where \( D \) is the damage degree of the expansive soil, \( N \) is the number of dry-wet cycles, and \( a \) and \( b \) are coefficients related to the initial moisture content and cycle amplitude.

Figure 5 shows the fitted nonlinear curves for the damage degree (quantified on the basis of the P-wave velocity) versus the number of dry-wet cycles. As the number of cycles increases, the damage degrees of expansive soil samples with different moisture contents increase nonlinearly. In addition, the slopes of the fitted curves gradually decrease with an increasing number of cycles, that is, the damage growth tends to become gentler as the number of cycles increases. The higher the moisture content is, the smaller the value of the damage variable. The larger the cycle amplitude, the greater the degree of damage. For a given cycle amplitude, under the action of dry-wet cycles, tensile and compressive stresses are cyclically produced in the sample, leading to the formation of fatigue cracks in the soil structure. These fatigue cracks gradually expand as the number of cycles increases. When a propagating wave encounters a crack, reflection, diffraction, and other phenomena will occur, attenuating the wave velocity. The greater the attenuation, the greater the corresponding degree of damage. The larger the cycle amplitude is, the larger the tensile and compressive stresses will be, the more cracks will form, and ultimately, the greater the fatigue damage will be. Considering that expansive soil swells in the presence of water, when the original moisture content is restored, the higher the moisture content is, the higher the degree of fracture healing in the sample will be, resulting in less attenuation of the wave velocity.
3.2.2. Dry-wet cycle amplitude
Curves showing the cumulative damage degree versus the cycle amplitude are presented in Figure 6. In the first cycle, the damage degree decreases with decreasing cycle amplitude. Between cycle amplitudes of 3% and 2.5%, the slope of the damage curve is steeper than that at higher cycle amplitudes. The concept of the cycle amplitude is interpreted from the mechanical point of view, that is, the stress amplitude (the amplitude of the change in the matrix suction), which is mainly due to the periodic variations in soil moisture occurring during the dry-wet cycles, resulting in periodic variations in the matrix suction. The larger the cycle amplitude is, the larger the variations in matrix suction and the larger the degrees of soil expansion and contraction will be, which will accelerate the expansion and interconnection of cracks.

As the number of cycles increases, the more closely clustered the damage degree curves become, indicating that the damage degree tends to stabilize with an increasing number of cycles. However, it should be noted that the damage degrees in cycles 2–6 do not monotonically increase as the cycle amplitude increases, indicating that the 1st dry-wet cycle effect plays a critical role in crack generation.

4. Rationality analysis of the damage variable defined on the basis of the P-wave velocity
The cyclic action of dry-wet conditions will lead to a decrease in the shear strength of expansive soil, a change that is closely related to fatigue damage [24]. To verify the rationality of using the P-wave velocity as a basis for defining a measure of the damage degree for expansive soil, it is necessary to establish the inherent relationship between the shear strength of expansive soil and its degree of damage.

For an initial moisture content of 17%, the test results for the variations in the internal friction angles of expansive soil samples with the number of dry-wet cycles are shown in Figure 7. It appears that the internal friction angle is not strongly affected by the number of dry-wet cycles and it is difficult to establish a proper functional relationship with the damage degree. Therefore, this study instead focuses on the corresponding changes in the cohesion and damage degree of expansive soil with different initial moisture contents under the action of dry-wet cycles. Based on the experimental results for the cohesion and damage degree of expansive soil specimens, the cohesion-damage relationships for expansive soil with
different initial moisture contents were nonlinearly fitted. The results are shown in Figure 8 and the fitting formula is as follows:

\[ c = a + \beta D + \gamma D^2 \quad (N \geq 1), \]  

where \( c \) is the cohesion of the expansive soil, \( D \) is the damage degree of the expansive soil, and \( a, \beta, \) and \( \gamma \) are the coefficients related to the cycle parameters.

It can be seen from Figure 8 that there is a clear nonlinear attenuation relationship between the cohesive force of expansive soil and the damage degree defined in terms of P-wave velocity. For a given initial moisture content, the larger the cycle amplitude is, the steeper the slope of the curve is, that is, the cohesion is increasingly attenuated as the degree of damage increases. This relationship is not difficult to explain. Under the action of dry-wet cycles, the periodic changes in the water content of the expansive soil cause periodic variations in the matrix suction, leading to internal fatigue damage to the soil and the formation of micro-cracks. With the accumulated effect of constant amplitude cyclic action, the micro-cracks gradually expand, the average length of the acoustic propagation path increases, and the measured acoustic time becomes longer than that of the intact expansive soil when calculating the damage degree. The more cracks there are, the greater the damage degree is; at the same time, the more highly developed the crack network in the expansive soil becomes, the worse the integrity of the soil structure and the lower the strength of the soil will be. In addition, by using the functional relationship expressed in Eq. (3), the degree of damage to expansive soil can be indirectly measured to calculate the soil cohesion.

5. Empirical damage model

5.1. Nonlinear fatigue damage model based on P-wave velocity

According to the experimental results presented in Section 3.1, the P-wave velocity of expansive soil decreases with an increasing number of dry-wet cycles and
Figure 6. Curves of damage degree versus cycle amplitude.

Table 2. Fitted coefficients and correlation coefficients.

| Initial moisture content (%) | Constants | Correlation coefficient $R^2$ |
|------------------------------|-----------|-------------------------------|
|                              | $a$       | $b$  | $d$  | $e$  | $f$  | $z_0$ |                                  |
| 15%                          | -17.692   | -32.311 | -0.356 | 2.850 | -1.736 | 873.383 | 0.9702 |
| 17%                          | -15.939   | -39.464 | -0.083 | 2.640 | 0.315  | 968.547 | 0.9720 |
| 19%                          | -26.421   | -33.017 | 1.458  | 2.403 | -0.121 | 933.130 | 0.9733 |
| 21%                          | -6.837    | -15.183 | 0.576  | 1.700 | -1.774 | 990.514 | 0.9930 |

an increasing cycle amplitude; thus, the attenuation of the P-wave velocity is related to both of these parameters. The P-wave velocities of expansive soil specimens with the same initial water contents were fitted to a quadratic polynomial to characterize the attenuation relationship with respect to cycle number and cycle amplitude, and the results are shown in Figure 9. The fitting formula is as follows:

$$V_p = z_0 + a \Delta w + bN + d(\Delta w)^2 + eN^2 + fN(\Delta w),$$ \hspace{1cm} (4)

where $N$ is the number of cycles, $\Delta w$ is the cycle amplitude, and $a$, $b$, $d$, $e$, $f$, and $z_0$ are coefficients related to the initial moisture content. The specific values of the fitted coefficients and the correlation coefficients $R^2$ obtained for the polynomial fits are shown in Table 2.

Substituting Eq. (4) into Eq. (1) yields the following result:

$$D(N) = 1 - \left[ \frac{z_0 + a \Delta w + bN + d(\Delta w)^2 + eN^2 + fN(\Delta w)}{V_0} \right]^2.$$ \hspace{1cm} (5)

Thus, the final nonlinear fatigue damage model developed...
which reflects that the damage degree of expansive soil presents nonlinear characteristics with the change of dry-wet cycles. The nonlinear fatigue damage empirical model can be used to estimate the damage degree of expansive soil. According to the damage degree, the model can be used to estimate the strength parameters of expansive soil.

5.2. Damage estimation for expansive soil

It can be seen from Eq. (5) that the damage degree of expansive soil with a given initial moisture content is related to the number of dry-wet cycles and the cycle amplitude. Eq. (5) can be used to calculate the damage degree of expansive soil with a certain initial moisture content subjected to a particular number of dry-wet cycles with a given cycle amplitude. The results for the damage degree as calculated with Eqs. (1) and (5) under different initial moisture conditions are compared in Figure 10. The measured values are represented by Eq. (1) and the calculated values are represented by Eq. (5). In Figure 10, it can be seen that the damage variable values calculated through
Eqs. (1) and (5) are in good agreement. Thus, under the condition of a known initial moisture content, the damage degrees of expansive soil subjected to different numbers of dry-wet cycles of constant amplitude can be satisfactorily estimated using Eq. (5). Although the fitting degree of some results is not high, it reflects the inherent variability of soil and the accumulation of test process error. Therefore, further research is required in this field.

6. Conclusions

The P-wave velocities and low-stress shear strengths of expansive soil subjected to 0-6 dry-wet cycles of constant amplitude were tested in this study. According to the research results of existing literature, the damage variable of expansive soil was characterized by P-wave velocity and its rationality was verified based on the value of the low-stress shear strength. Based on experimental results for the P-wave velocity, a nonlinear empirical model for the fatigue damage of expansive soil was established. The main conclusions are as follows.

For a given initial moisture content and a given cycle amplitude, the P-wave velocity of an expansive soil sample decreases nonlinearly with an increasing number of cycles and the overall variation curve can be divided into two stages. For a given initial moisture content, the average P-wave velocity of expansive soil subjected to a given number of cycles decreases with the increasing cycle amplitude.

The proposed damage variable for expansive soil increases nonlinearly with an increasing number of cycles and the rate of increase tends to be gentle. The higher the moisture content, the smaller the value of the damage variable. The larger the cycle amplitude is, the larger the value of the damage variable is.

There is a clear nonlinear attenuation relationship
between the cohesion of expansive soil and its degree of damage. The P-wave velocity can be successfully used to reflect a representative damage variable for expansive soil. For a given initial moisture content, the larger the cycle amplitude is, the greater the cohesion is attenuated with an increasing degree of damage. The attenuation function relating to the degree of fatigue damage and the low-stress strength of expansive soil can be used to predict the direct shear strength.

The damage characteristics of expansive soil are closely related to the dry-wet cyclic loading path, and the development of the damage characteristics shows significant differences under different cyclic loading methods. The development of damage to expansive soil is a very complex process that is controlled by many factors affecting the cyclic process, not just a single factor. In the future, it is necessary to further explore the factors and rules governing the damage behavior of expansive soil.

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References

1. Mehta, B. and Sachan, A. “Effect of mineralogical properties of expansive soil on its mechanical behavior”, Geotechnical and Geological Engineering, 35(6), pp. 2923–2934 (2017). DOI: 10.1007/s10706-017-0289-6

2. Tang, C.S., Cui, Y.J., Shi, B., Tang, A.M., and Liu, C. “Desiccation and cracking behaviour of clay layer from slurry state under wetting-drying cycles”, Geoderma, 166(1), pp. 111–118 (2011). DOI: 10.1016/j.geoderma.2011.07.018

3. Wei, B.X. and Huang, Z. “Assessment of change in acoustic wave velocity of compacted expansive soil through experiments”, Scientia Iranica, 24(1), pp. 130–142 (2017). DOI: 10.24200/sci.2017.2383

4. Noormy, I. and Schweing, C. “Later extension of compacted-fill slopes in expansive soils”, Journal of Geotechnical and Geoenvironmental Engineering, 141(1), p. 04014083 (2015). DOI: 10.1061/(ASCE)GT.1943-5606.0001190
5. Miner, M.A. “Cumulative damage in fatigue”, *ASME Journal of Applied Mechanics, 67*(9), pp. A159-A164 (1945).

6. Jardin, A., Leblond, J., Berghenaz, D., and Portigliatti, M. “Definition and experimental validation of a new model for the fatigue of elastomers incorporating deviations from Miners linear law of cumulative damage”, *Fatigue and Fracture of Engineering Materials and Structures, 32*(6), pp. 439-448 (2009).

7. Lynn, A.K., and Duquesnay, D.L. “Computer simulation of variable amplitude fatigue crack initiation behaviour using a new strain-based fatigue damage model”, *International Journal of Fatigue, 24*(9), pp. 977-986 (2002). DOI: 10.1016/S0142-1123(02)00067-5

8. Gens, A. and Alonso, E.E. “A framework for the behaviour of unsaturated expansive clays”, *Canadian Geotechnical Journal, 29*(6), pp. 1013-1023 (1992). DOI: 10.1139/t81-062 (1984).

9. Lu, Z., Chen, Z., Fang, X., Guo, J., and Zhou, H. “Structure damage model of unsaturated expansive soil and its application in multi-field couple analysis on expansive soil slope”, *Applied Mathematics and Mechanics-English Edition, 27*(7), pp. 891-900 (2006). DOI: 10.1007/s10488-006-0704-1

10. Shi, B., Chen, S., Han, H., and Zheng, C. “Expansive soil crack depth under cumulative damage”, *The Scientific World Journal*, p. 498437 (2014). DOI: 10.1155/2014/498437

11. Tang, L., Cong, S., Ling, X., Xing, W., and Nie, Z. “A unified formulation of stress-strain relations considering micro-damage for expansive soils exposed to freeze-thaw cycles”, *Cold Regions Science and Technology, 153*, pp. 164-171 (2018). DOI: 10.1016/j.coldregions.2018.05.006

12. Xue, X.H., Yang, X.G., Zhang, W.H., and Dai, F. “A soil damage model expressed by a double scalar and its applications”, *ACTA Mechanica, 225*(9), pp. 2667-2683 (2014). DOI: 10.1007/s10488-014-0971-1

13. Xu, J., Li, Y., Lan, W., and Wang, S. “Shear strength and damage mechanism of saline intact loess after freeze-thaw cycling”, *Cold Regions Science and Technology, 164*, pp. 102779 (2019). DOI: 10.1016/j.coldregions.2019.05.005

14. Chen, J., Wang, H., and Yao, Y. “Experimental study of nonlinear ultrasonic behavior of soil materials during the compaction”, *Ultrasonics, 69*, pp. 19-24 (2016). DOI: 10.1016/j.ultras.2016.03.001

15. Wang, P., Xu, J., Fang, X., Wang, P., Zheng, G., and Wen, M. “Ultrasonic time-frequency method to evaluate the deterioration properties of rock suffered from freeze-thaw weathering”, *Cold Regions Science and Technology, 143*, pp. 13-22 (2017). DOI: 10.1016/j.coldregions.2017.07.002

16. Saroglou, C. and Kallimogiannis, V. “Fracturing process and effect of fracturing degree on wave velocity of a crystalline rock”, *Journal of Rock Mechanics and Geotechnical Engineering, 9*(5), pp. 797-806 (2017). DOI: 10.1016/j.jrmge.2017.03.012

17. Xu, Y., Li, J., Duan, J., Song, S., Jiang, R., and Yang, Z. “Soil water content detection based on acoustic method and improved Brutsaert’s model”, *Geoderma, 359*, p. 114003 (2020). DOI: 10.1016/j.geoderma.2019.114003

18. Zhao, M.J. and Xu, R. “The rock damage and strength study based on ultrasonic velocity”, *Chinese Journal of Geotechnical Engineering, 22*(6), pp. 720-722 (2000).

19. Jin, J.F., Zheng, H.B., Wu, Y., Guo, Z.Q., and Zhou, X.J. “Method selection for defining damage variable of rock subjected to static loadings and cyclic impacts”, *Nonferrous Metals Science and Engineering, 4*(1), pp. 85-90 (2013).

20. Thiyagaraj, T. and Rao, S.M. “Influence of osmotic suction on the soil-water characteristic curves of compacted expansive clay”, *Journal of Geotechnical and Geoenvironmental Engineering, 136*(12), pp. 1697-1702 (2010). DOI: 10.1061/(ASCE)GT.1943-5606.0000389

21. Huang, Z., Wei, B.X., Zhang, L.J., Chen, W., and Peng, Z.M. “Surface crack development rules and shear strength of compacted expansive soil due to dry-wet cycles”, *Geotechnical and Geoenvironmental Engineering, 37*(4), pp. 2947-2957 (2019). DOI: 10.1007/s10706-018-0781-y

22. Xiao, J., Yang, H.P., Zhang, J.H., and Tang, X.Y. “Surface failure of expansive soil cutting slope and its flexible support treatment technology”, *Advances in Civil Engineering, p. 1099608* (2018). DOI: 10.1155/2018/1099608

23. Yang, H.P., Zheng, J.J., and Zhang, R. “Addressing expansive soils”, *Civil Engineering, 77*(3), pp. 62-69 (2007). DOI: 10.1061/cieseg.000112

24. Dong, J.G., Xu, G.Y., Lv, H.B., and Yang, J.Y. “An instrument for wetting-drying cycle of expansive soil under simulated loads and experimental research”, *Journal of Engineering Research, 7*(3), pp. 1-12 (2019).

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