Lifetime Estimations and a Non-Monotonic Initial Energy Density in Heavy Ion Collisions at RHIC and LHC

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Abstract—We highlight some connections between the final state hadronic observables and the initial conditions using a recently found new exact family of solutions of relativistic hydrodynamics. These relations provide explicit examples of the scaling behaviour in relativistic hydrodynamics and provide an advanced estimate of the lifetime and the initial energy density in , , and  GeV Au + Au collisions at RHIC and  TeV Pb + Pb and  TeV Xe + Xe as well as ,  and  TeV \( p + p \) collisions at LHC energies. A surprising result is that these advanced estimates yield a non-monotonic increase of the initial energy density with increasing collision energy at the RHIC energy range.

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INTRODUCTION

Recently, we have published a series of manuscripts that presents a new family of exact solutions of relativistic hydrodynamics \cite{1}, and its applications to the evaluation of the pseudorapidity distributions \cite{2}, the longitudinal HBT radii \cite{3}, the estimation of the initial energy densities \cite{4} as well as the application of these results to the analysis of experimental data at RHIC and LHC energies \cite{5}. In this conference contributions we highlight some of the most beautiful results, that are given in full detailed in refs. \cite{1–5}.

NEW, EXACT SOLUTIONS OF RELATIVISTIC, PERFECT FLUID HYDRODYNAMICS

The equations of relativistic perfect fluid hydrodynamics are given in terms of the entropy density, denoted by \( s \), the four velocity \( u^\mu \), normalized as \( u^\mu u_\mu = 1 \), and the energy-momentum four tensor \( T^{\mu\nu} \), as follows. These fields depend on \( x = x^\mu = (t, r_x, r_y, r_z) \).

\begin{align}
\partial_\mu (su^\mu) &= 0, \\
\partial_\nu T^{\mu\nu} &= 0.
\end{align}

For perfect fluids, the energy-momentum four tensor is

\[ T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}, \]

where \( \varepsilon \equiv \varepsilon(x) \) is the energy density and \( p \equiv p(x) \) is the pressure, and \( g^{\mu\nu} = \text{diag}(1,−1,−1,−1) \) stands for the Minkowskian metric. The above set of equations is closed by the following equation of state:

\[ p = \tau^2 \varepsilon = \varepsilon/\kappa, \]

where \( \tau = 1/\sqrt{\kappa} \) is a temperature independent, average value of the speed of sound, that was measured by the PHENIX collaboration in \cite{7}: \( \tau = 0.35 \pm 0.05 \), corresponding to \( \kappa = 0.13 \). An exact and analytic, finite and accelerating, 1+1 dimensional solution of relativistic perfect fluid hydrodynamics was recently found by Csörgö, Kasza, Csanád and Jiang (CKCJ) \cite{1} as a family of parametric curves, obtained using the Rindler-coordinates: \( \tau, \eta_x = \left( \sqrt{r_z^2 - r_x^2}, 0.5 \ln \left[ \frac{t + r_z}{t - r_z} \right] \right) \), where \( \tau \) stands for the longitudinal proper time and \( \eta_x \) is the space-time rapidity. The four-velocity is chosen as \( u^\mu = (\cosh(\Omega), \sinh(\Omega)) \) and it was assumed that the fluid rapidity \( \Omega = \Omega(\eta_x) \) is independent of the proper time. The new class of CKCJ solutions was pre-
sent in [1] and the corresponding pseudorapidity distributions were obtained as parametric curves in [2].

We present this formulae, as well as an advanced estimation of the initial energy densities [4], and the results on the longitudinal HBT radii of [3], as derived from the CKCJ exact solution of relativistic hydrodynamics. This solution is an explicit function of the longitudinal proper-time \( \tau \), while its the dependence on the space-time rapidity \( \eta \), is given in terms of parametric curves, parameterized in terms of \( H = \Omega - \eta \). These solutions are constrained to a cone inside the normal coordinate, as indicated on Fig. 1. If \( \Delta \eta = \tau \Delta \eta_{\text{ss}} \), the finite mid-rapidity density, given by \( \langle N \rangle / (2\pi\Delta^2 y)^{1/2} \).

The CKCJ solution was shown to describe the pseudorapidity density distribution of \( p + p \) collisions at \( \sqrt{s} = 7 \) and 8 TeV in [1]. The description of the pseudorapidity distribution of 40–50% Pb + Pb collisions at \( \sqrt{s_{NN}} = 5 \) TeV was given in [2], while fits to Au + Au collision data at \( \sqrt{s_{NN}} = 200 \) GeV were shown in [3]. Several other plots were shown in conference presentations that indicate that the CKCJ solution of relativistic hydrodynamics describes well the pseudorapidity distributions from \( p + p \) through Xe + Xe and Au + Au to Pb + Pb collisions, from the colliding energies of \( \sqrt{s_{NN}} = 20 \) GeV to the currently largest LHC energy of \( \sqrt{s} = 13 \) TeV. Most recently, we have described the CMS measurement of the pseudorapidity density in Xe + Xe collisions at \( \sqrt{s_{NN}} = 5.44 \) TeV [12] in the 0–80% centrality class, as indicated on Fig. 3.

**LIFE-TIME AND INITIAL ENERGY DENSITY ESTIMATION AT RHIC ENERGIES**

In this section we present a new method, which is developed to determine the initial energy density of the expanding fireballs, as a function of only one parameter, the initial proper time. This method can be summarized as follows: (i) Evaluate the effective temperature \( T_{\text{eff}} \) from fits to the invariant momentum distribution of identified hadrons, as a function of \( m_T - m \). (ii) Determine the acceleration parameter \( \lambda \) from a fit with Eq. (5), by using Eqs. (6), (7) from fits to measured pseudorapidity density data. See refs. [1, 5] for details. (iii) Fit the \( m_T \) dependence of the longitudinal HBT-radii \( R_{\text{long}} \), to evaluate the life-time parameter of the medium. This HBT radius parameter was derived from the CKCJ solution in [4] as follows:

\[
R_{\text{long}} = \tau_f \Delta \eta_{\text{ss}} = \frac{\tau_f}{\sqrt{\lambda(2\lambda - 1)}} \sqrt{T_f} \frac{m_T}{m_T - m},
\]

where \( \tau_f \) is the life-time parameter, and \( T_f \) stands for the kinetic freeze-out temperature. (iv) Finally, one can use the fitted parameters to evaluate the initial energy density by our new formula that was calculated exactly from the CKCJ solution. It corrects the Bjorken estimation by taking into account the non boost-invariant expansion and the finite width of the pseudo-rapidity distribution, as represented by \( \lambda \neq 1 \).
Fig. 1. The top left panel indicates the result of fits with \( A \exp[-(m_T - m)/T_{\text{eff}}] \) to the invariant momentum distribution of \( \pi^\pm \), \( K^\pm \), \( p \) and \( \bar{p} \) to \( \sqrt{s_{NN}} = 62.4 \) GeV \( \text{Au} + \text{Au} \) collisions in the 0–5% centrality class. The top right panel shows the same in the 30–40% centrality class. Bottom left panel shows the excitation function of the effective temperature of positively and negatively charged pions in the 0–5% centrality class. Bottom right panel shows the same in the 30–40% centrality class. The bottom panels are indicating a non-monotonic behaviour of the pion slope parameters with increasing energy, in both centrality classes.

Fig. 2. Description of PHOBOS results [6] on the pseudorapidity distributions of \( \pi^\pm \) GeV, 0–30% \( \text{Au} + \text{Au} \) collisions with the CKCJ solutions of relativistic hydrodynamics, using Eqs. (5)–(7). Details of the fitting method are described in [5].
As well as the work, done by the pressure during the fireball evolution [3]:

\[ \varepsilon_{Bj} = \varepsilon_{Bj}^0 (2\lambda - 1) \left( \frac{\tau_f}{\tau_0} \right)^{\frac{1}{1+\frac{1}{\varepsilon}} - 1} \]  

(9)

where \( \varepsilon_{Bj}^0 \) is Bjørken’s estimate and it can be expressed as [10]:

\[ \varepsilon_{Bj}^0 = \left( \frac{\langle E_T \rangle}{S_1 T_0} \right) \frac{dN}{d\eta_{\parallel} d\eta_{\perp}} \mid_{\eta_\parallel = 0}. \]  

(10)

In this Bjørken estimate, \( \langle E_T \rangle \) is the average thermalized, transverse energy and \( S_1 \) stands for the overlap area of the colliding nuclei.

We have gone through on these steps, so we estimated the initial energy density of RHIC \( \text{Au + Au} \) collisions at 3 different colliding energies, namely: \( \sqrt{s_{NN}} = 62.4, 130, \) and \( 200 \) GeV. Here, we can only highlight one of the surprising results, which indicate an unexpected feature of the strongly interacting quark-gluon plasma (sQGP) created in \( \text{Au + Au} \) collisions at RHIC energies. Our advanced initial energy density estimate of \( \sqrt{s_{NN}} = 130 \) and \( 200 \) GeV collisions is detailed in [5].

Figure 5 indicates the initial energy densities as a function of the colliding energies in the 0–30% centrality class. This figure immediately highlights the surprising feature we have just mentioned. Although Bjorken’s estimate suggests a monotonic increase of the initial energy density with increasing energy, our advanced initial energy density estimate indicates a non-monotonic behavior.

This result is to be considered as preliminary and treated carefully, due to the limitations of the CKCJ solution: this solution does not include transverse dynamics, and the temperature dependence of the speed of sound is replaced by an average value. However, we have cross-checked in [5], that our analytical results on the initial energy density yields similar time evolution for the energy density in the center of the fireball to that of an 1 + 3 dimensional numerical solution of the equations of relativistic hydrodynamics using the lattice QCD equation of state [11]. This comparison provided a surprisingly good agreement, suggesting that the non-monotonic behavior of the initial energy density with increasing energy of...
the RHIC beam energy scan may be a robust feature of the data. Of course, it is conceivable that the match is a coincidence, since [11] did not make explicit the centrality class of the calculation. In addition, of course, the question may arise about the initial energy density of LHC energies predicted by the CKCJ solution. We have also estimated with our new method the lower limit of the initial energy density of Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, in the 10–20% centrality class. The result satisfied our expectation, namely the lower limit of the initial energy density at such a high colliding energy is higher than our initial energy density estimate at $\sqrt{s_{NN}} = 200$ GeV, the top RHIC energy for Au + Au collisions.

In summary, we highlighted in this work two of our remarkable theoretical results from [5]:

(i) A simple and beautiful formula was found to describe the pseudorapidity density distribution from the CKCJ solution of relativistic hydrodynamics. This formula is given by Eqs. (5)–(7), and it describes not only $p + p$ and heavy ion data at RHIC and LHC energies, but it also describes excellently the recent CMS data on Xe + Xe collisions exceedingly well.

(ii) We outlined our new method to extract the initial energy density of high energy proton–proton and heavy ion collisions, that corrects Bjorken’s oversimplified initial energy density estimate for realistic pseudorapidity density distributions and for taking into account the work done by the non-vanishing pressure during the expansion.

The excitation function of the initial energy density is summarized in Fig. 5, indicating a non-monotonic behaviour. These advanced estimates of the initial energy density may thus become a new tool to search for the critical point of on the QCD phase diagram: in the vicinity of the QCD critical point, several quantities may behave in a non-monotonic manner, including life-time related observables, such as the estimated initial energy density. Indeed, a non-monotonic behaviour of the HBT-radii has been observed in the RHIC beam energy scan, pointing to a QCD critical point near $\mu_B \approx 95$ MeV, corresponding to $\sqrt{s_{NN}} \approx 47.5$ GeV [13]. Our data analysis related to the estimations of the initial energy density of Au + Au collisions at RHIC supports independently the possibility of this kind of a scenario. This scenario is also supported by the rather robust nature of Fig. 1, that indicates a non-monotonic dependence of the slope parameters of the single-particle transverse mass spectra on $\sqrt{s_{NN}}$ in a similar energy range.

Further, more detailed studies are necessary to investigate possible shock-wave effects at lower colliding energies, together with the effects arising from a possible proper-time dependence of the acceleration parameter $\lambda$ and from using a more realistic, 1 + 3 dimensional expansion, the temperature dependence of the speed of sound and using a lattice QCD equation of state.

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