Research on a 3-DOF Planar Cable-Driven Parallel Robot Based on Fuzzy Adaptive PD Computed Torque Control

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\textbf{Abstract.} Cable-driven parallel robots (CDPR) with spatial configuration are widely used while ones with planar configuration are rarely applied. Therefore, this paper studies the modeling and control characteristics of a three degree-of-freedom cable-driven planar parallel robot with great application prospects. Aiming at the shortcomings of the existing control methods towards CDPR, a new method based on fuzzy adaptive PD computing torque control is proposed which contains a reasonable optimal distribution of cable forces. To validate the method, the tracking control simulation research and analysis are carried out for the trajectory of high-order interpolation planning, the results of which show that the control method has favorable dynamic tracking accuracy and mechanical properties. Moreover, compared with the fixed-gain computed torque control, the proposed control method can overcome the influence of model inaccuracy more effectively.

\section{Introduction}
Cable-driven parallel robot (CDPR) is widely used due to its large working space, light inertia, fast moving speed, strong carrying capacity, and reconfigurability. It is used in rehabilitation institutions, radio telescopes, wind tunnel support systems, cargo handling, virtual game systems and other fields \cite{1}. However, most of the application examples are spatial cable parallel configuration, and there are few engineering application examples of planar flexible cable parallel. In fact, in such fields as building exterior cleaning, water cooling tube wall inspection of power plant boiler, which need to drag heavy objects in the facade operation, the parallel robot driven by flexible cable has a great application prospect. Therefore, this paper takes the cable-driven parallel robots with 4 cables and 3 degrees of freedom (3DOF-PCDPR) that can carry heavy objects in a plane as the research object to study its modeling and control characteristics.

The planar flexible parallel robot has the characteristics of multiple input and multiple output, nonlinearity, strong coupling, redundancy constraint and hysteresis \cite{1}. The classical PID control cannot meet its dynamic requirements, so there are a lot of nonlinear control methods for its characteristics, mainly including computational torque control \cite{2}, fuzzy control \cite{3}\cite{4}, neural network \cite{5}, sliding mode variable structure control methods\cite{6}. Computational torque control is the simplest, the most effective and the most classical control method. It greatly improves the control performance through linear decoupling control law, but the control effect depends on the accuracy of the model. Neural network controllers have certain challenges in stability, convergence, and real-time performance. Sliding mode variable structure control methods have chattering problems, while flexible cable parallel robots
are more sensitive to vibration and need to be combined with fuzzy control methods to reduce chattering, which increases the complexity of the algorithm. Although the design method of the fuzzy controller is more dependent on experience, it is highly achievable. It can be called by offline interpolation method in actual application, and the performance is excellent. At present, there have been fuzzy control researches applied to flexible cable parallel robots[3][4], but it is mainly a decentralized control strategy based on cable length space without considering the effect of joint coupling. In addition, the redundantly constrained flexible cable parallel robot also needs to consider the problem of cable force distribution. Typical cable force optimization methods include linear programming, quadratic programming, p-norm optimization and other methods, but these methods do not consider how to reasonably stay away from the cable force boundary constraints. In fact, when the flexible cable has a large span and is at the boundary of the cable force, it is easy to lose stability. Under comprehensive consideration, the method of cable force optimization in this paper is a linear programming based on the real-time update of the cable force boundary conditions, so that the lower limit of the cable force can be appropriately changed with the change of the cable length, and at the same time, a relatively small cable force is used to control the movement of the moving platform. For the control strategy, the cable force optimal distribution model is combined with the PD-type computed torque control strategy to reduce the influence of joint coupling. On this basis, the fuzzy control algorithm is introduced to adjust the PD parameters in real time, and to track the given trajectory, which is of great significance to overcome the shortcomings of the computed torque control effect dependent on the accuracy of the model.

2. Mechanism Description and Modeling of the 3DOF-PCDPR

2.1. Mechanism description and kinematics modeling

For 3DOF_PCDPR, the model diagram is shown in Figure 1. The moving platform can move in the facade under the action of four flexible cables. Each cable is driven by a winch system composed of a motor, a reducer and a roller (the details are not shown in the figure).

![Figure 1 Model sketch](image)

Establish a dynamic coordinate system \( \{P\} \) fixedly connected to the center of mass of the moving platform and a base coordinate system \( \{O\} \) fixedly connected to the earth. Assuming that the flexible cable maintains a straight line and does not occur elastic deformation, according to the principle of vector closed chain and coordinate transformation, the expression of the rope length vector \( \mathbf{l}_i \) and rope length \( l_i \) in the base coordinate system is as equation (1):

\[
\begin{align*}
\mathbf{l}_i &= \mathbf{a}_i - \mathbf{Rb}_i - \mathbf{p} \\
l_i &= \sqrt{\left(\mathbf{a}_i - \mathbf{Rb}_i - \mathbf{p}\right)^T \left(\mathbf{a}_i - \mathbf{Rb}_i - \mathbf{p}\right)}
\end{align*}
\]

where \( i=1, 2, 3, 4 \), \( \mathbf{p}=[x \ y]^T \) is the vector diameter of the center of mass of the moving platform in \( \{O\} \), \( \mathbf{a}_i \) is the vector path in \( \{O\} \) where the \( i-th \) winch rope exits, \( \mathbf{b}_i \) is the vector path in \( \{P\} \) of the
connection point between the \(i\)-th flexible cable and the moving platform, and \(R\) is the rotation matrix of \(\{P\}\) relative to \(\{O\}\), its expression is:

\[
R = \begin{bmatrix}
\cos \beta & -\sin \beta \\
\sin \beta & \cos \beta
\end{bmatrix}
\]  

(2)

\(a_i\) and \(b_i\) are vectors composed of the size parameters of the mechanism, and the movement process remains unchanged, then \(l_i\) can be regarded as a function of \(x\), \(y\), and \(\beta\). Let \(L = [l_1 l_2 l_3 l_4]^T\), and \(X = [x\ y\ \beta]^T\), then the relationship between the speed of rope length change and the generalized speed of the moving platform can be derived:

\[
\dot{L} = -J^\top \dot{X}
\]  

(3)

where

\[
J = \begin{bmatrix}
\frac{\partial l_1}{\partial x} & \frac{\partial l_2}{\partial x} & \frac{\partial l_3}{\partial x} & \frac{\partial l_4}{\partial x} \\
\frac{\partial l_1}{\partial y} & \frac{\partial l_2}{\partial y} & \frac{\partial l_3}{\partial y} & \frac{\partial l_4}{\partial y} \\
\frac{\partial l_1}{\partial \beta} & \frac{\partial l_2}{\partial \beta} & \frac{\partial l_3}{\partial \beta} & \frac{\partial l_4}{\partial \beta}
\end{bmatrix}
\]  

(4)

### 2.2. Kinetic modeling

#### 2.2.1. moving platform

Assuming that the pulling force of the flexible cable on the moving platform is in the same direction as the rope length vector, Then According to D'Alembert's principle and virtual work principle, the dynamic equations of the moving platform can be derived as following:

\[
JT = M\ddot{X} + G
\]  

(5)

where \(T = [T_1 T_2 T_3 T_4]^T\) represents the vector composed of the tension values; \(M = \text{diag}(m\ m\ I)\), and \(m\) and \(I\) are the mass and inertia of the moving platform; \(G = [0\ mg\ 0]^T\).

#### 2.2.2. Cable force optimization

Equation (5) is redundant. For a moving platform in a certain state of motion, the cable tension has infinite solutions. The general solution is:

\[
T = J^* (M\ddot{X} + G) + \lambda N(J)
\]  

(6)

where \(N(J) \in \mathbb{R}^{4 \times 1}\) is the one-dimensional basis vector of the null space of \(J\), which is an arbitrary real number, and \(J^*\) is the Moore-Penrose inverse of \(J\), and the expression is as in formula (7).

\[
J^* = J^\top (JJ^\top)^{-1}
\]  

(7)

For the solution of cable force in a certain state, the following cable force optimization model can be designed:

**Variable:** \(\lambda\)

**Objective function:** \(f(\lambda) = \|T\|_{\text{min}}\)
Restrictions:

\[
\begin{align*}
T &= J^\top \left( M\ddot{X} + G \right) + \lambda N(J) \\
T_i &\geq \varepsilon l_i^2, \varepsilon \geq 0, i = 1, 2, 3, 4
\end{align*}
\]

where \( \varepsilon \) is a constant close to 0, and the cable force optimization model can be solved by linear optimization in the MATLAB optimization toolbox, then \( \lambda \) is determined for specific state of \( J \) and \( \dot{X} \).

2.2.3. Winch system

For the four winch systems, each part is composed of a roller, a reducer and a motor. The roller is to retract and release the rope, the reducer is to reduce the torque, and the motor drives the overall model to move. The dynamic equation of the winch system can be expressed for:

\[
\tau = A_0\dot{\theta} + B_0\dot{\theta} + rT
\]

where \( A_0 = \text{diag}(a_{01}, a_{02}, a_{03}, a_{04}), B_0 = \text{diag}(b_{01}, b_{02}, b_{03}, b_{04}), r = \text{diag}(r_1, r_2, r_3, r_4) \) are the diagonal matrix composed of the equivalent inertia, equivalent viscous friction coefficient and equivalent radius of each winch system. \( r_i = r_{ni} / j, r_{ni} \) is the radius of the \( i \)-th roller, \( j \) is the reduction ratio, \( \tau = [\tau_1 \ \tau_2 \ \tau_3 \ \tau_4]^T \), \( \theta = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4]^T \) are the vectors composed of the torque and rotation angle of each motor.

2.2.4. Overall dynamics model

Assuming that the initial value of the length of each flexible cable constitutes the vector \( L_0 \), it is stipulated that the motor rotates in the positive direction when the flexible cable is contracted, and the relationship among \( \theta, L, X \) is:

\[
\begin{align*}
\theta &= r^{-1}(L_0 - L) \\
\dot{\theta} &= -r^{-1}\dot{L} = r^{-1}J^\top \dot{X} \\
\ddot{\theta} &= -r^{-3}\ddot{L} = r^{-3}J^\top \ddot{X} + r^{-3}J^\top \dot{X}
\end{align*}
\]

Combining (5), (9), (10) can get the final system dynamics model:

\[
J \tau = A\ddot{X} + B\dot{X} + C
\]

where:

\[
\begin{align*}
A &= r^3J^\top A_0J^\top \\
B &= r^3J^\top A_0J^\top + r^3J^\top B_0J^\top \\
C &= rG
\end{align*}
\]

3. Trajectory Tracking Control Strategy of the 3DOF-PCDPR

3.1. Design of overall control law

According to the dynamic equation of the controlled object is equation (11), the ideal tracking trajectory is set as \( X_d \), and the tracking error is defined as \( e = X - X_d = [e_x \ e_y \ e_\beta]^T = [e_1 \ e_2 \ e_3]^T \) then the control law of the PD-type computed torque structure can be expressed as:

\[
\tau = J^\top (A\ddot{X} + Pe + D\dot{e} + B\dot{X} + C) + \lambda N(J)
\]

where \( P = \text{diag}(P_1, P_2, P_3) \) and \( D = \text{diag}(D_1, D_2, D_3) \) represent the proportional gain matrix and the differential gain matrix respectively; \( P_i, D_i \geq 0, i = 1, 2, 3; \lambda \) is determined by the cable force optimization algorithm in (8). Its function is to adjust the redundant internal tension between the flexible cables to ensure that the flexible cables are in tensioned state at all times.
Simultaneous formulas (11) and (13), combined with the characteristics of the full rank of $A$ matrix, obtain the closed-loop system error equation as:

$$
\dot{e} + Pe + D\dot{e} = 0
$$

(14)

Therefore, from the time-invariant positive definiteness of $P$ and $D$, we know that $(e, \dot{e}) = (0, 0)$ is the equilibrium point of global asymptotic stability. However, in the actual control process, the trajectory tracking error cannot be completely eliminated due to the deviation of the model estimates $\hat{A}$, $\hat{B}$ and $\hat{C}$ from the real $A$, $B$ and $C$. Therefore, in order to improve the control performance, a fuzzy logic control method is introduced to adjust the PD gain matrix in real time. The control scheme is shown in Figure 2:

3.2. Fuzzy controller design

In the general fuzzy control process, the precise input is transformed into precise output through fuzzification, fuzzy reasoning based on fuzzy rules, and defuzzification [7].

In terms of the control object in the article, for the $i$-th feedback loop ($i=1, 2, 3$), $P_i$ and $D_i$ are constants; variables $e_i$ and $\dot{e}_i$ are taken as input, $\Delta P_i$ and $\Delta D_i$ are used as output of the fuzzy logic controller.

The basic domains of $e_i$, $\dot{e}_i$, $\Delta P_i$ and $\Delta D_i$ are transformed into the domain {-3, -2, -1, 0, 1, 2, 3} on the fuzzy set through their respective quantization factors, and the corresponding fuzzy subset is {NB, NM, NS, ZO, PS, PM, PB}, each element in the subset means "negative large, negative medium, negative small, zero, positive small, positive middle, positive large". The membership functions are shown in Figure 3. In order to control the stability of the system, it is necessary to make a reasonable selection of the basic domains of $\Delta P_i$ and $\Delta D_i$ under the non-negative bounded constraints of $P_i + \Delta P_i$ and $D_i + \Delta D_i$. 

Then the overall control law can be described as:

$$
\tau = J^T(\hat{A}(\dot{X}_d) + (P + \Delta P)e + (D + \Delta D)e + \dot{\hat{B}}X + \dot{\hat{C}}) + \lambda N(J)
$$

(15)

Figure 2 Control block diagram
Fuzzy reasoning adopts the Mamdani rule. The fuzzy rules are determined based on expert experience and multiple debugging results as shown in Table 1, and the MIN-MAX gravity-center method is used for defuzzification.

| $\Delta P_i, \Delta D_i$ | $\dot{e}_i$ |
|--------------------------|-----------|
| NB | PM,ZO | PS,NS |
| NM | PM,PS | ZO,ZO |
| NS | PM,PM | PS,NS |
| ZO | PB,PB | PM,PM |
| PS | PM,ZO | PS,NS |
| PM | PM,NS | NS,ZO |

4. Simulation Research

4.1. Simulation parameter setting

In order to verify the performance of the control algorithm, a simulation study based on Matlab-Simulink is carried out on the robot, and the necessary parameters are given based on the schematic diagram of the actual physical prototype (size parameters unit:m): $a_1=[7.5, 4.5]^T$, $a_2=[7.5, -4.5]^T$, $a_3=[-7.5, 4.5]^T$, $a_4=[-7.5, -4.5]^T$, $b_1=[0.1, 0.1]^T$, $b_2=[0.1, -0.1]^T$, $b_3=[0.1, -0.1]^T$, $b_4=[0.1, 0.1]^T$. The mass of the moving platform is $m=30$ kg, and the same parameters are set for each drive system: $a_{in}=4.75\times10^{-4}$ kg·m2, $b_{in}=0.0125$N·m·s, $r_{in}=0.15$m, $j=10$. For the control parameters, the position and velocity error gain matrix is selected as $P=\text{diag}(80, 80, 60)$ and $D=\text{diag}(16, 16, 12)$ through multiple experiments. Cable force lower bound regulator $\varepsilon=0.4$. The physical theory domains of $\Delta P_i$ are all $[-40,40]$, and the physical theory domains of $\Delta D_i$ are all $[-7,7]$. The quantization factors of $x_e, \dot{x}_e, y_e, \dot{y}_e, e_\beta, \dot{e}_\beta, e_\beta, \dot{e}_\beta$ are 600, 150, 60, and 60, which can be transformed into the fuzzy domain of the previous chapters through the quantization factor. For the model error, take $\hat{A}=0.9A, \hat{B}=0.9B$ and $\hat{C}=0.9C$. For the motion parameters $x$ and $y$, the expected pose trajectory based on high-order interpolation planning is:

\[
\begin{align*}
    x &= -5 + 0.8t^3 - 0.24t^4 - 0.0192t^5 \\
    y &= -3 + 0.48t^3 - 0.144t^4 + 0.0115t^5
\end{align*}
\]
This trajectory represents that the center of mass of the robot starts from coordinates [-5,-3] within 5s and moves to [5,3] according to a straight path, and the center of mass velocity and acceleration are zero at the beginning and end. The desired posture $\beta_s$ setting remains unchanged at 0 rad.

### 4.2. Simulation results and analysis

The simulation results of the control method adopted in this paper are shown in Figure 4. It can be seen that the $x$ and $y$ outputs basically coincide with the inputs, which shows that the control algorithm has good performance in terms of the path.

![Desired and actual trajectories of fuzzy PD computed torque control](image)

Figure 4 Desired and actual trajectories of fuzzy PD computed torque control

The change of cable force and motor torque is shown in Figure 5. Each flexible cable is far away from the boundary of the flexible cable, and there is a dynamic change at the boundary, indicating that the cable force optimization algorithm is reasonable. It can be seen that when the cable force is relatively large, the change of the motor torque is similar to that of the cable force. When the cable force is relatively small, the change of the motor torque shows other trends more obvious. This is because when the cable force is relatively small and the moving platform is moving at high speed, the motor torque overcomes the inertial force and frictional resistance of the drive system.
Figure 5  Variation curves of cable force and motor torque in fuzzy PD computed torque control process

The real-time change curve of $\Delta P$ and $\Delta D$ based on fuzzy rules is shown in Figure 6. It can be seen that the change of $\Delta P$ is basically in the positive half plane, that is, the pose error gain is always greater than the set initial value $P$, which is beneficial to improve the control dynamic response and tracking accuracy. But at the same time, it is easy to cause overshoot. Therefore, it can be seen that $\Delta D$ repeatedly changes in the positive and negative range. In addition to speed tracking, it will also suppress the possible overshoot caused by the larger pose loop gain.

Figure 6  Variation curves of $\Delta P$ and $\Delta D$ in fuzzy PD computed torque control process

Under the premise that the $P, D$ parameters and other fixed parameters are the same, the control law and the gain fixed computed torque control algorithm proposed in this paper are compared and analyzed based on the trajectory tracking error and its rate of change. As shown in Figure 7 to 9, no matter what kind of control algorithm, the trajectory tracking error changes in a small range. It is worth noting that the magnitude of $e_x$ and $e_\beta$ is $10^{-2}$, which is one magnitude larger than $e_y$. For $e_y$, this is because the
main load of the mechanism is the gravity of the moving platform, and the direction is consistent with. Therefore, $e_y$ is more sensitive to the error of the gravity related term, which indicates that the precise identification of the gravity term should be paid attention to in actual control. For the attitude error $e_\beta$, it is because the size of the moving platform has a very small variation range relative to the rope length, and the attitude of the platform is more sensitive to the rope length error. This indicates that the actual control should pay attention to the precise measurement and feedback of the attitude or rope length. For $e_x$, the x-direction rope length error caused by the model error can compensate each other, so the x-direction error is relatively small, which is a major feature of parallel robots. Further comparing the difference between the two control algorithms, it can be found that the maximum absolute value of the trajectory tracking error and its rate of change under the fuzzy PD control algorithm is smaller than that of the computed torque control, and the maximum reduction is 33%. It shows that fuzzy control shows better performance than computed torque control in this dynamic trajectory tracking.

Figure 7 Variation curves of $e_x$ (left) and $\dot{e}_x$ (right) of the two control algorithms

Figure 8 Variation curves of $e_y$ (left) and $\dot{e}_y$ (right) of the two control algorithms
5. Conclusion

In this paper, for the four-cable three-degree-of-freedom planar cable parallel robot for vertical work, a dynamic equation is established based on the D’Alembert principle and the principle of virtual work. Aiming at the model error, in order to improve the control performance, a control law combining computational torque control, cable force optimization, and fuzzy control methods is proposed. The tracking control simulation research and analysis are carried out for the trajectory of high-order interpolation planning. The results show that the adopted control method has good dynamic tracking accuracy and mechanical performance, and it has greater advantages when compared with the PD-type computed torque control. PD parameters can be adjusted in real time to obtain higher control accuracy. The simulation results verify the rationality of the control method, and have outstanding reference significance for actual control. The main deficiencies in current research are the complicated design of fuzzy control algorithms. Because 3DOF-PCDPR is a multiple-input multiple-output system, and the change of each variable is different, there are many parameters that need to be set, and the membership function and domain of discourse need to be changed accordingly for different situations. Therefore, designing a simpler and more intelligent fuzzy controller based on the work of this paper is the content that should be studied in the future.

Acknowledgments

This article is one of the phased achievements of National major scientific instrument project (2014YQ24044504).

References

[1] Tang X Q. (2014) An overview of the development for cable-driven parallel manipulator [J]. Advances in Mechanical Engineering, (823028): 1-9.
[2] Robert I I, Gallina P. (2003) Translational Planar Cable-Direct-Driven Robots[J]. Journal of Intelligent & Robotic Systems, 37(1):69-96.
[3] Zi B, Qian S. (2017) Design, Analysis and Control of Cable-suspended Parallel Robots and Its Applications[M]. Springer Singapore.
[4] Shao Zhufeng, Tang Xiaojian*, Wang Liping, You Zheng. (2013) A Fuzzy PID Approach for the Vibration Control of the FSPM, International Journal of Advanced Robotic Systems, 10(59):1-8(SCI, EI)
[5] Asl H J, Janabi-Sharifi F. (2017) Adaptive neural network control of cable-driven parallel robots with input saturation[J]. Engineering Applications of Artificial Intelligence, 65:252-260.

[6] Babaghasabha R, Khosravi M A, Taghirad H D. (2015) Adaptive robust control of fully-constrained cable driven parallel robots[J]. Mechatronics, 25:27-36.

[7] PASSINO K M, YURKOVICH S. (1998) Fuzzy Control[M]. Menlo Park, Calif.: Addison-Wesley.