Astrophysical Implications of the Superstring-Inspired $E_6$ Unification and Shadow Theta-Particles

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Abstract. We have developed a concept of parallel existence of the ordinary (O) and mirror (M), or shadow (Sh) worlds. $E_6$ unification, inspired by superstring theory, restores the broken mirror parity at the scale $\sim 10^{18}$ GeV. With the aim to explain the tiny cosmological constant, we consider the breakings: $E_6 \rightarrow SO(10) \times U(1)_Z$ — in the O-world, and $E_6 \rightarrow SU(6)' \times SU(2)_Y'$ in the Sh-world. We assume the existence of shadow $\theta$-particles and the low energy symmetry group $SU(3)'_C \times SU(2)'_L \times SU(2)'_Y \times U(1)'_Y$ in the shadow world, instead of the Standard Model. The additional non-Abelian $SU(2)_\theta$ group with massless gauge fields, “thetons”, has a macroscopic confinement radius $1/\Lambda_{\theta}$. The assumption that $\Lambda_{\theta} \approx 2.3 \cdot 10^{-3}$ eV explains the tiny cosmological constant given by recent astrophysical measurements. Searching for the Dark Matter (DM), it is possible to observe and study various signals of theta-particles.

Keywords: mirror world, shadow world, shadow axion, theta particle, dark energy, dark matter, unification, $e6$

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INTRODUCTION

The present talk is devoted to the problem of cosmological constant. Our model is based on the following assumptions:

• Grand Unified Theory (GUT) is inspired by Superstring theory [1], which predicts $E_6$ unification in the 4-dimensional space, occurring at the high energy scale $\sim 10^{18}$ GeV.

• There exists a Mirror World (MW) [2, 3], which is a mirror duplication of our Ordinary World (OW), or Shadow World (ShW) (hidden sector) [4, 5], which is not identical with the O-world, having different symmetry groups. The mirror (M), or shadow (Sh) matter interacts with ordinary matter only via gravity, or other very weak interactions.

• The Shadow world is responsible for the dark energy (DE) and dark matter (DM).

• We assume that $E_6$ unification had a place in the O- and M-worlds at the early stage of our Universe. This means that at very high energy scale $\sim 10^{18}$ GeV the mirror world exists and the group of symmetry of the universe is $E_6 \times E_6'$ [6, 7] (where the superscript ‘prime’ denotes the M- or Sh-world).

MIRROR WORLD WITH BROKEN MIRROR PARITY

At low energies we can describe the ordinary and mirror worlds by a minimal symmetry $G_{SM} \times G_{SM}'$, where $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ stands for the Standard Model (SM) of observable particles: three generations of quarks and leptons and the Higgs boson. Then $G_{SM}' = SU(3)'_C \times SU(2)'_L \times U(1)'_Y$ is its mirror gauge counterpart having three generations of mirror quarks and leptons and the mirror Higgs boson. The M-particles are singlets of $G_{SM}'$ and the O-particles are singlets of $G_{SM}$. If the ordinary and mirror worlds are identical, then O- and M-particles should have the same cosmological densities. But this is in the immediate conflict with recent astrophysical measurements. Mirror parity (MP) is not conserved [8]. In the case of the broken MP the VEVs of the Higgs doublets $\Phi$ and $\Phi'$: $\langle \Phi \rangle = v$, $\langle \Phi' \rangle = v'$ are not equal: $v \neq v'$. We have introduced the parameter characterizing the violation of MP: $\zeta = v'/v \gg 1$. Then the masses of fermions and massive bosons in the mirror world are scaled up by the factor $\zeta$ with respect to
the masses of their counterparts in the ordinary world: 

\[ m_{\nu_a} = \zeta m_{\nu_a}, M_{\nu_{\mu} \nu_{\tau}} = \zeta M_{\nu_{\mu} \nu_{\tau}}, \]

while photons and gluons remain massless in both worlds.

In the language of neutrino physics, the O-neutrinos \( \nu_\ell \) are active neutrinos, while the M-neutrinos \( \nu'_\ell \) are sterile neutrinos. If MP is conserved (\( \zeta = 1 \)), then the neutrinos of the two sectors are strongly mixed. But it seems that the situation with the present experimental and cosmological limits on the active-sterile neutrino mixing do not confirm this result. MP is spontaneously broken, \( \zeta \gg 1 \), and active-sterile mixing angles should be small: \( \theta_{\nu\nu} \sim \frac{1}{\zeta} \).

Then we have the following relation between the masses of the light left-handed neutrinos: 

\[ m_{\nu_a} \approx \zeta^2 m_\nu. \]

Also the seesaw mechanism described by Refs. [8] predicts that so called right-handed neutrinos \( N_a \) with large Majorana mass terms have equal masses in the O- and M(Sh)-worlds: \( M_{\nu_a} = M_{\nu_a}' \). They are created at seesaw scale \( M_R \) (or \( M_R' \)) in the O- (or M(Sh)-)world. And even in the model with broken MP, we have the same seesaw scales in both worlds: \( M''_R = M_R \).

**SUPERSTRING THEORY AND E6 UNIFICATION**

The ‘heterotic’ superstring theory \( E_8 \times E_8' \) was suggested as a more realistic model for unification of all gauge interactions with gravity [1]. This ten-dimensional Yang-Mills theory can undergo spontaneous compactification. The integration over six compactified dimensions of the \( E_8 \) superstring theory leads to the effective theory with the \( E_6 \) unification in the four-dimensional space.

In the present investigation at the scale \( \sim 10^{18} \) GeV we adopt for the O-world the breaking \( E_6 \rightarrow SO(10) \times U(1) \), while for the Sh-world we consider the breaking \( E_6' \rightarrow SU(6)' \times SU(2)' \), thus being able to explain the small value of the cosmological constant \( CC \), due to the additional \( SU(2)' \) gauge symmetry group appearing in the Sh-world, which has a large confinement radius.

We assume that in the ordinary world, from the SM up to the \( E_6 \) unification, there exists the following chain of symmetry groups:

\[
SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow [SU(3)_c \times SU(2)_L \times U(1)_Y]_{\text{SUSY}} \\
\rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z \\
\rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_Z \rightarrow SO(10) \times U(1)_Z \rightarrow E_6.
\]

We consider the following chain of possible symmetries in the Sh-world:

\[
SU(3)'_c \times SU(2)'_L \times SU(2)'_R \times U(1)'_Y \rightarrow [SU(3)'_c \times SU(2)'_L \times SU(2)'_R \times U(1)'_Y]_{\text{SUSY}} \\
\rightarrow SU(3)'_c \times SU(2)'_L \times SU(2)'_R \times U(1)'_X \times U(1)'_Z \\
\rightarrow SU(4)'_c \times SU(2)'_L \times SU(2)'_R \times U(1)'_Z \rightarrow SU(6)' \times SU(2)'_R \rightarrow E'_6.
\]

Now we are confronted with the question: What group of symmetry \( SU(2)' \), unknown in the O-world, exists in the Sh-world, ensuring the \( E'_6 \) unification?

**NEW SHADOW GAUGE GROUP SU(2)' AND THETA-PARTICLES**

In the present paper we consider the idea of the existence of theta-particles, developed by L.B. Okun [9]. In those works it was suggested that in Nature there exists the symmetry group \( SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_Y \), i.e. with an additional non-Abelian \( SU(2)_R \) group whose gauge fields are neutral, massless vector particles – thetons, having a macroscopic confinement radius \( 1/A_\theta \).

We assume that the group of symmetry \( G_\theta' = SU(3)'_c \times SU(2)'_L \times SU(2)'_R \times U(1)'_Y \) exists in the Shadow World at low energies instead of the SM'. By analogy with theory [9], we have shadow thetons \( \Theta_{\mu \nu}' \), which belong to the adjoint representation of \( SU(2)'_R \), three generations of shadow theta-quarks \( q_\theta' \), shadow leptons \( l_\theta' \), and two thetascalars \( \phi_\theta' \) as doublets of \( SU(2)'_R \). Shadow thetons have a confinement radius \( 1/A_\theta \), and \( A_\theta \sim 10^{-3} \) eV provides the tiny cosmological constant. We also consider a complex scalar field \( \phi_\theta \), which is a singlet under the symmetry group \( G_\theta' \). This singlet scalar field has its origin from 27-plet of the \( E'_6 \) unification.
THE RUNNING OF COUPLING CONSTANTS IN THE O- AND SH-WORLDS

In this work we consider the running of all the gauge coupling constants in the SM and its extensions which is well described by the one-loop approximation of the renormalization group equations (RGEs), since from the Electroweak (EW) scale up to the Planck scale \( M_{Pl} \) all the non-Abelian gauge theories with rank \( r \geq 2 \) appearing in our model are chosen to be asymptotically free. With this aim we consider only the Higgs bosons belonging to the \( N + \bar{N} \) representations for \( SO(N) \) or \( SU(N) \) symmetry breaking (see [7]).

The running of the inverse coupling constants are given by the following expressions:

\[
\alpha_i^{(t)-1}(\mu) = \frac{b_i^{(t)}}{2\pi} \ln \frac{\mu}{\Lambda_i^{(t)}},
\]

where \( \mu \) is the energy scale. For compactness of notation, we denote by \( \alpha_i^{(t)-1} \) the inverse of various coupling constants and by \( X^{(t)} \) the various scales and values belonging to either OW (the non-primed symbols) or ShW (the primed symbols). In Eq. (3) \( \alpha_i^{(t)} = (g_i^{(t)})^2/4\pi \) and \( g_i^{(t)} \) is the gauge coupling constant of the gauge group \( G_i^{(t)} \). Here \( i = 1, 2, 3 \) correspond to \( U(1), SU(2) \) and \( SU(3) \) groups of the SM\(^{(t)} \). A big difference between the EW scales \( v \) and \( v' \) will not cause the same difference between the gauge scales \( \Lambda_i \) and \( \Lambda_i' \): \( \Lambda_i' = \xi \Lambda_i \) with \( \xi \approx 1.5 \) for \( \xi = 30 \).

For the energy scale \( \mu \geq M_{\text{ren}}^{(t)} \), where \( M_{\text{ren}}^{(t)} \) is the renormalization scale, we have the following evolution for the inverse coupling constants given by RGE in the one-loop approximation:

\[
\alpha_i^{(t)-1}(\mu) = \alpha_i^{(t)-1}(M_{\text{ren}}^{(t)}) + \frac{b_i^{(t)}}{2\pi} \mu^{(t)},
\]

where \( \mu^{(t)} = \ln \left( \mu/M_{\text{ren}}^{(t)} \right) \) is the evolution parameter.

As an example of the evolutions (1) and (2) we have used the following parameters: supersymmetric breaking scale in the O-world \( M_{SUSY} = 10 \text{ TeV}, \zeta = 30 \), i.e. supersymmetric breaking scale in the Sh-world \( M_{SUSY}' = 300 \text{ TeV} \), seasea scale \( M_R = M_R' = 2.5 \cdot 10^{14} \text{ GeV} \).

The running of the inverse coupling constants as functions of \( x = \log_{10} \mu \) is presented for O-world in Fig. (a) and for Sh-world in Fig. (b). We start in Fig. (a) with \( G_{SM} \) and \( M_{\text{ren}} = M_t \), where top-quark mass is \( M_t = 174 \text{ GeV} \). Fig. (a) starts with \( G_\theta \) and \( M_{\text{ren}} = M_t' = \zeta M_t = 5.22 \text{ TeV} \). In these pictures Figs. (b) show the running of the gauge coupling constants near the scale of the \( E_6 \) unification (for \( x \geq 15 \)). The coefficients (slopes) \( b_i \), describing the running of the coupling constants with our choice of gauge groups and particle content, are given in Tables (1, 2).

FIGURE 1. Figure (a) presents the running of the inverse coupling constants \( \alpha_i^{-1}(x) \) in the ordinary world from the Standard Model up to the \( E_6 \) unification for SUSY breaking scale \( M_{SUSY} = 10 \text{ TeV} \) and seasea scale \( M_R = 2.5 \cdot 10^{14} \text{ GeV} \). This case gives: \( M_{SUSY} = M_{E_6} = 6.98 \cdot 10^{17} \text{ GeV} \) and \( \alpha_{E_6}^{-1} = 27.64 \). Figure (b) is the same as (a), but zoomed in the scale region from \( 10^{15} \text{ GeV} \) up to the \( E_6 \) unification to show the details.
The boson $\phi$ in the present paper we give a very simple explanation for the smallness of the cosmological constant. Figure (a) presents the running of the inverse coupling constants $\alpha_i^{-1}(x)$ in the shadow world from the Standard Model up to the $E_6$ unification for shadow SUSY breaking scale $M'_{\text{SUSY}} = 300$ TeV and shadow seesaw scale $M_R' = 2.5 \cdot 10^{14}$ GeV; $\zeta = 30$. This case gives: $M'_{\text{SUSY}} = M_{E_6} = 6.98 \cdot 10^{17}$ GeV and $\alpha_6^{-1} = 27.64$. Figure (b) is the same as (a), but zoomed in the scale region from $10^{15}$ GeV up to the $E_6$ unification to show the details.

**TABLE 1.** The coefficients $b_i$ in the O-world with the breaking $E_6 \rightarrow SO(10) \times U(1)_Z$.

| NonSUSY groups: $b_i$: | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|------------------------|-----------|-----------|-----------|
| $SU(3)_C$              | 3         | 19/6      | -41/10    |
| $SU(2)_L$              | -1        | $SU(2)_L \times SU(2)_R$ | $b_{22} = -2$ |
| $U(1)_Y$               | -33/5     | $U(1)_Z$  | $SO(10)$  |
| $b_{10} = 1$           |           |           |           |

**TABLE 2.** The coefficients $b_i$ in the Shadow World.

| NonSUSY groups: $b_i$: | $SU(3)_C'$ | $SU(2)_L'$ | $SU(2)_R'$ |
|------------------------|------------|------------|------------|
| $SU(3)_C'$             | 3          | 19/6       | 3          |
| $SU(2)_L'$             | -1         | $SU(2)_L' \times SU(2)_R'$ | $U(1)_{Y'}$ |
| $SU(2)_R'$             | -2         | -33/5      | $U(1)_{Y'}$ |
| $U(1)_{Y'}$            | -9         | -33/5      | $SU''(6)$  |
| $b_{10} = 11$          |           |           |           |

The running of $\alpha_2^{-1}(\mu)$ with slopes $b_{2\theta} = 3$ and $b_{2\theta}^{SU} = -2$, given by Fig. 2a,b, shows that it is easy to obtain the value $\Lambda'_6 \sim 10^{-5}$ eV.

**SHADOW AXION, COSMOLOGICAL CONSTANT AND DARK ENERGY**

In the present paper we give a very simple explanation for the smallness of the cosmological constant. There exists an axial $U(1)_A$ global symmetry in our theory with a current having $SU(2)'_A$ anomaly, which is spontaneously broken at the scale $f_\theta$ by a singlet complex scalar field $\phi_\theta$, with a VEV $\langle \phi \rangle = f_\theta$, i.e.

$$\phi = (f_\theta + \sigma) \exp(i \alpha_\theta / f_\theta).$$

(5)

The boson $\alpha_\theta$ (imaginary part of the singlet scalar field $\phi_\theta$) is an axion and could be identified with a massless Nambu-Goldstone (NG) boson if the $U(1)_A$ symmetry is not spontaneously broken. However, the spontaneous breaking of the
global $U(1)_A$ by $SU(2)_{\theta}'$ instantons inverts $a_\theta$ into a pseudo Nambu-Goldstone (PNG) boson.

A singlet complex scalar field $\phi_\theta$ reproduces a Peccei-Quinn (PQ) model \[10\]. In the shadow world with shadow $\theta$-particles the vacuum energy density is: $\rho_{\text{vac}} = (\Lambda_\theta')^4$. Near the vacuum, a PNG mode $a_\theta$ emerges the following PQ axion potential:

$$V_{\text{PQ}}(a_\theta) \approx (\Lambda_\theta')^4 (1 - \cos(a_\theta/f_\theta)).$$

This axion potential exhibits minima at

$$\cos(a_\theta/f_\theta) = 1, \quad (a_\theta)_{\text{min}} = a_n = 2\pi n f_\theta, \quad n = 0, 1, \ldots$$

For small fields $a_\theta$ we expand the effective potential near the minimum:

$$V_{\text{eff}} \approx (\Lambda_\theta')^4 \left( 1 + \frac{1}{2} (a_\theta/f_\theta)^2 + \ldots \right) = (\Lambda_\theta')^4 + \frac{1}{2} m^2 a_\theta^2 + \ldots,$$

and hence the PNG axion mass squared is given by:

$$m^2 \sim \Lambda_\theta'^4 / f^2_\theta.$$ 

Let us assume that at the cosmological epoch when $U(1)_A$ was spontaneously broken, the value of the axion field $a_\theta$ was deviated from zero, and it was $a_{\theta,\text{in}} \sim f_\theta$. The value of the scale $f_\theta \sim 10^{18}$ GeV (near the $E_6$ unification breaking scale) makes it natural that the $U(1)_A$ symmetry was broken before inflation, and the initial value $a_{\theta,\text{in}}$ was inflated above the present horizon. So after the inflation breaking scale, and in particular in the present universe, the field $a_\theta$ is spatially homogeneous (constant), and the initial energy density corresponding to $a_{\theta,\text{in}}$ is also spatially homogeneous:

$$\rho_{\text{in}} = V(a_{\theta,\text{in}}) \simeq \Lambda_\theta'^4 (1 - \cos(a_{\theta,\text{in}}/f_\theta)).$$

For the expanding universe the equation of motion (EOM) of the classical field $a_\theta$ is:

$$\frac{d^2 a_\theta}{dt^2} + 3H \frac{da_\theta}{dt} + V'(a_\theta) = 0,$$

where $H$ is the Hubble parameter \[11\]: $H = 1.5 \times 10^{-42}$ GeV. For small $a_\theta$ we have: $V'(a_\theta) = m^2 a_\theta$. If $\Lambda_\theta' \sim 10^{-3}$ eV and $f_\theta \sim 10^{18}$ GeV, then from Eq. (9) we obtain a value of the axion mass:

$$m \sim \Lambda_\theta'^2 / f_\theta \sim 10^{-42} \text{ GeV}.$$ 

Now, it is natural to assume that the initial velocity $a_{\theta,\text{in}}$ was small: $a_{\theta,\text{in}} \sim H f_\theta$. Then, for $3H^2 \gg m^2$ the potential curvature $V''(a_\theta)$ in the above EOM can be neglected, and we have a solution with $a_\theta$ remaining the constant in time.

For the present epoch the critical density of the universe is:

$$\rho_c = 3H^2/8\pi G = (2.5 \times 10^{-12} \text{ GeV})^4.$$ 

According to the Particle Data Group \[11\], the fraction of the dark energy corresponds to

$$\rho_{\text{DE}} \approx 0.75 \rho_c \approx (2.3 \times 10^{-3} \text{ eV})^4.$$ 

Now, having $m^2 < 3H^2$, we see that the classical PNG field $a_\theta$ does not start the oscillation and in the present epoch its energy density remains constant (does not scale with the time) and saturates the dark energy fraction of the universe:

$$\rho_{\text{DE}} = \rho_{\text{vac}} = \min V_{\text{eff}} \simeq \Lambda_\theta'^4,$$

which means that $\Lambda_\theta' \approx 2.3 \times 10^{-3}$ eV.

In this case, for the present epoch, the energy of the PNG field $a_\theta$ can imitate dark energy, providing the equation of the state $p = w \rho$ with $w \approx -1$, but not exactly equal to $-1$, as a quintessence. Of course, to claim that this can explain the present amount of the dark energy, one must assume that the major constant contributions to the cosmological term are canceled by some means, i.e. true cosmological constant is almost zero by some (yet unknown) symmetry, or by dynamical reasons. Also the gravity itself can be modified so that it does not feel the truly constant terms in the
energy. In this case one can ascribe the present acceleration of the universe by such a PNG quintessence field, with implication that the acceleration will not be forever, but it will finish as soon as $m^2 \sim 3H^2$ will be achieved. After that the PQ classical energy will behave as a dark matter component and not as a dark energy.

Here we have suggested a model when our universe was trapped in the vacuum $[15]$, and exists there at the present time with a tiny cosmological constant $CC$:

$$CC = \rho_{\text{vac}} \simeq (\Lambda_{\theta}^4)^{4} \simeq (2.3 \times 10^{-3} \text{ eV})^{4}. \quad (16)$$

Such properties of the present axion lead to the 'LCDM' model of the accelerating expansion of our universe $[11]$. By this reason, the axion $a_\theta$ could be called an ‘acceleron’, and the field $\sigma$ given by Eq. $(5)$ is an ‘inflaton’.

**DARK MATTER**

The existence of dark matter in the universe, which is non-luminous and non-absorbing matter, is now well established by astrophysics.

For the ratios of densities $\Omega_{X} = \rho_{X}/\rho_{\text{crit}}$, cosmological measurements give the following density ratios of the total universe $[11]$: $\Omega_0 = \Omega_r + \Omega_M + \Omega_{\Lambda} = 1$. Here $\Omega_r$ is a relativistic (radiation) density ratio, and $\Omega_\Lambda = \Omega_{\text{DE}}$. The measurements give: $\Omega_{\text{DE}} \sim 75\%$ - for the mysterious dark energy, $\Omega_M \approx \Omega_\Lambda + \Omega_{DM} \sim 25\%$, $\Omega_\Lambda \approx 5\%$ - for (visible) baryons, $\Omega_{DM} \approx 21\%$ - for dark matter. Here we propose that a plausible candidate for DM is a shadow world with its shadow quarks, leptons, bosons and super-partners, and the shadow baryons are dominant: $\Omega_{DM} \approx \Omega_\Lambda$. Then we see that $\Omega_{\theta} \approx 5\Omega_\Lambda$, what means that the shadow baryon density is larger than the ordinary baryon density.

The new gauge group $SU(2)\theta$ gives the running of $(\alpha')^{-1}_{\theta}(\mu)$. Near the scale $\Lambda_{\theta} \sim 10^{-3}$ eV, the coupling constant $g'_{\theta}$ grows infinitely. But at higher energies Fig. $2$ gives that this coupling constant is comparable with the electromagnetic one. Here we would like to emphasize that shadow quarks $q'_{\theta}$ of the first generation are stable, and can participate in the formation of shadow “hadrons”, which can be considered as good candidates for the Cold Dark Matter (CDM). So we have the two types of shadow baryons: baryons $b'$ constructed from shadow quarks $q'$ which are singlets of $SU(2)\theta$, and baryons $b''_{\theta}$ constructed from the quark $q'$ and two shadow $\theta$-quarks $q'_{\theta}$, in order to preserve $\theta$-charge conservation. Then, $\Omega_{b'} = \Omega_{b''_{\theta}} \approx 5\Omega_\Lambda$. We shall study in detail the DM in a forthcoming communication.

The present work opens the possibility to specify a grand unification group, such as $E_6$, from Cosmology.

**REFERENCES**

1. M. B. Green, J. H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge, 1988.
2. T. D. Lee and C. N. Yang, *Phys. Rev.* 104, 254 (1956).
3. I. Yu. Kobzarev, L. B. Okun and I. Ya. Pomeranchuk, *Yad. Fiz.* 3, 1154 (1966) [Sov. J. Nucl. Phys. 3, 837 (1966)].
4. K. Nishijima and M. H. Saffouri, *Phys. Rev. Lett.* 14, 205 (1965).
5. E. W. Kolb, D. Seckel, M. S. Turner, *Nature* 314, 415 (1985).
6. C. R. Das, L. V. Lapershvili, *Int. J. Mod. Phys. A* 23, 1863 (2008); *Phys. Atom. Nucl.* 72, 377 (2009).
7. C. R. Das, L. V. Lapershvili, A. Tureanu, arXiv: 0902.4874 [hep-ph].
8. Z. Berezhiani, A. Dolgov and R. N. Mohapatra, *Phys. Lett. B* 375, 26 (1996); Z. Berezhiani, in *Ian Kogan Memorial Collection “From Fields to Strings: Circumnavigating Theoretical Physics”*, edited by M. Shifman et. al., World Scientific, Singapore, Vol. 3, 2005, pp. 2147-2195; *Eur. Phys. J. ST* 163, 271 (2008).
9. L. B. Okun, *JETP Lett.* 31, 144 (1980); *Pisma Zh. Eksp. Teor. Fiz.* 31, 156 (1979); *Nucl. Phys. B* 173, 1 (1980).
10. R. D. Peccei and H. R. Quinn, *Phys. Rev. Lett.* 38, 1440 (1977).
11. Particle Data Group, C. Amster et. al., *Phys. Lett. B* 667, (2008).