OPTIMAL RETURN AND REBATE MECHANISM BASED ON DEMAND SENSITIVITY TO REFERENCE PRICE

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ABSTRACT. The acceleration of electronic products’ upgrade affects consumers’ purchase behaviour. How to encourage consumers to return old products in order to upgrade to new products and how to optimize such the closed-loop supply chain are important managerial topics. According to the theory of reference price, the closed-loop supply chain model with fixed rebate and variable rebate is established. The analysis results imply that, when consumers’ willingness to return second-hand products depends on manufacturers’ rebates and prices of new products, the profit of closed-loop supply chain decreases. In addition, when consumers are sensitive to price difference, enterprises can adopt low profit margin methods to increase new product demand. Furthermore, the profit of the manufacturer is closely related to whether consumers are loss-seeking or loss-averse. Finally, our analysis provides the insights of the relationship between the optimal return and rebate mechanism and the use time of the previous generation of products.

1. Introduction. The rapid development of electronic products provides more innovative products to consumers as well as more outdated products. Apple Inc. launched one product per year from 2010 to 2012 (iPhone 4; iPhone 4s; and iPhone 5), two products annually (iPhone 5s, iPhone 5c; iPhone 6, iPhone 6 plus; iPhone 6s, and iPhone 6s plus) from 2013 to 2015, and three products per year (iPhone 7, iPhone 7 plus, iPhone SE; iPhone 8, iPhone 8 plus, iPhone X; iPhone XS, iPhone XR, iPhone XS Max; iPhone 11, iPhone 11 Pro and iPhone 11 Pro Max) from 2016 to 2019. The constant replacement of old products has caused a great pressure on our environment. In the field of waste management, EPR (Extended Producer Responsibility) is the environmental protection strategy that aims to reduce the total impact of products on the environment. The measures taken are to make

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the producers of the products responsible for the whole life-cycle of the products, especially including the take-back, recycling and final disposal.

In recent years, the development of reverse logistics has led to positive impacts on enterprises in the treatment of waste products. New value can be created through a series of steps such as treatment, detection, renovation or reconstruction and resale after the recycling of waste products. Savaskan et al. [39] study different return channels and finds that active recycling not only creates the positive green image for enterprises but also creates profits for them. Therefore, in the highly competitive market, enterprises actively recycle their products through different means. The research on closed loop supply chains has garnered great attention [18].

Due to the development of e-commerce, many returning methods are realized online; that is, manufacturers directly collect products from consumers through certain incentive returning methods. Apple Inc. uses the Apple GiveBack rewards program to motivate consumers to trade in their used iPhones and replace with new ones. This program conducts the online evaluation of consumers’ used products and provides them with quotations or discount coupons. If consumers are satisfied with the quotation, the company will send professional staff to inspect the products and pick them up. From the consumer’s perspective, consumers generally do not wait for the product to become unusable before replacing it, especially electronic products. When new products are launched, some consumers will hesitate to replace their old ones, because they consider not only the rebate provided by the manufacturer for used products but also the price of new products [14]. As for the depreciation of second-hand products, most previous studies are conducted on the basis of the fixed rebate [39, 49]. Depending on the situation, some enterprises encourage consumers to return second-hand products in the form of fixed rebates. For example, H&M encourages consumers to return certain amount of used clothes, and gives consumers a coupon. Dell Reconnect gives fixed per-ton tax discount to consumers. While some companies offer different rebates based on the condition of consumers’ second-hand products, such as Apple Inc.’s online testing of used electronic products. Of course, the value of the used product varies due to variance of its conditions. Although Apple Inc. recycles every generation of its products, products like iPhone 4, iPhone 5 and even iPhone 6 have been discontinued. Because the recyclable parts of these early products are no longer well used to make new products, the longer the phone is used, the less value it has left. Therefore, the rebate that the consumer receive is also different. The size of the rebate is determined according to the quality condition of the used products. Generally, the residual value of the used product will decrease with the increase of its use time [8, 37].

Furthermore, changes in consumer demand will also have an impact on the closed-loop supply chain. Regarding consumer buying behavior, consumers usually consciously or unconsciously compare new products’ prices either with what they previously bought or directly with similar products currently on sale. For example, since the launch of the iPhone X series in 2018, the price of the iPhone 8 has dropped to different degrees. For price sensitive consumers, the price performance ratio of iPhone 8 is higher, which will have certain impacts on the demand of iPhone X series. The price comparison psychology of consumers in purchasing products is bound to affect the demand for new products. According to Apple’s 2018 financial statements, iPhone sales in the second and third quarters fell by 32% and 21% respectively (http://www.apple.com/investor). One of the reasons for the decline is its increasingly high pricing. Rosch [38] defines the reference price
that consumers refer to when evaluating the actual price of the product. When purchasing new products, consumers tend to refer to the prices of similar products in the past or the prices of current alternative products. Moon et al. [32] find that nearly 91% of consumers would decide their purchasing behavior based on the reference prices. Therefore, when setting prices for new products, companies need to consider the prices of the last previous generation and even the earlier generation. In reality, consumers will use the latest price of the previous purchase of the product as the reference price [35]; this will affect the market demand. Therefore, this paper applies the reference price to the market demand of closed-loop supply chain and assumes that consumers use the past new price of the now used products as the reference price.

Based on the above realistic background, in this paper, we address the following questions:

(1) How does the change of consumers’ return behavior affect the closed-loop supply chain, in which the manufacturer offering rebates on used products?

(2) How does the change in consumers’ sensitivity to the price difference of two generations of products affect the profits of the closed-loop supply chain?

(3) What is the impact of the speed of upgrading of products on the profitability of the closed-loop supply chain when manufacturers offer variable rebates to consumers?

(4) If the previous generation products have different usage time, which return and rebate mechanism is optimal?

To study these problems, this paper considers a single-stage closed-loop supply chain, including only one manufacturer and one retailer, and proposes two rebate mechanisms: variable rebate and fixed rebate. Combining with demand sensitivity to reference price, we determine the optimal rebate mechanism and returning strategy that enterprises should adopt to recycle used products. Comparing to the study by Genc and De Giovanni [14], the difference is that the impact of demand sensitivity to reference price is considered in this paper. To the best of our knowledge, this is the first study to examine the recycling and rebate mechanisms of closed-loop supply chain considering consumers’ purchase behaviors affected by the price of products purchased in the past.

The main contributions of this paper are three-fold. First, we establish the closed-loop supply chain model based on the fixed rebate and the variable rebate; the variable rebate depends on the current condition of the used product. Second, by using the reference price theory, we develop the demand model that is affected by the price difference between the two generations of products. Finally, by comparing the closed-loop supply chain for two different rebates, we reveal the optimal pricing and returning strategies under the fixed and the variable rebate mechanism. The remainder of this paper is organized as follows. Section 2 reviews the literature and presents the difference between our and other researches. Section 3 describes the model and assumptions. Sections 4 and 5 provide Stackelberg equilibrium outcomes with fixed and variable rebates. Section 6 compares the solutions under the two rebate mechanisms. Section 7 provides a numerical example. Section 8 concludes the paper and suggests follow-up research directions.

2. Literature review. This paper relates to three main streams in the literature on the second-hand product recyclers, the rebate mechanisms and the impact of the reference price on consumer purchasing behavior in closed-loop supply chain.
The research application range of closed-loop supply chain is very wide. For example, in the research of vehicle routing optimization, closed-loop supply chain can reduce the no-load rate, thereby greatly reducing the cost incurred in the transportation process [15, 42]. In addition, the economic benefits of the closed-loop supply chain in recycling waste electronic products [20] also motivate manufacturers take different measures to recycle used products. It is well known that new product’s sales are usually through retailers, third parties or manufacturers, as is the recycling of used products. The research on second-hand product recycling is initially expanded based on the comparison of three different recycling channels by Savaskan et al. [39], and the results show that retailers are closer to consumers and have better recycling effect. As a result, researches have subsequently emerged about manufacturers recycling by themselves [7, 50] or incentivizing retailers [3, 30] or third parties [9, 36] to recycle used products. The recycling channels for second-hand products have changed from the single [31, 46] to the coexistence of multiple recycling channels [13, 47, 48], which also includes competition between different recycling channels or recyclers [5, 20, 45]. These studies are all conducted from the perspective of different recyclers without considering consumers. However, consumers are end-users of products, and their willingness to return products directly affects the recycling rate of waste products, thus affecting the profit of the whole closed-loop supply chain. A small number of studies have considered the impact of consumers on product recycling, but these studies are carried out from the perspective of increasing consumer environmental awareness [11, 12] or from the perspective of different incentives offered by retailers to encourage consumers to return used products [6, 10]. With the development of the Internet, we can not only buy goods online, but also directly recycle them online, which is convenient and fast, realizing the mode that manufacturers directly recycle used products from consumers. Therefore, this paper attempts to consider manufacturers recycling from the perspective of consumers, based on the fact that Apple Inc. has the direct online recycling of used products from consumers.

In terms of rebate mechanism, in closed-loop supply chain, initial studies are mostly about manufacturers offering fixed rebates to encourage recyclers to recycle used products, including manufacturers providing recyclers with the fixed rebate per unit of returned products [10, 39, 53] or providing recyclers with the fixed monetary rebate [11, 27]. Although subsequent studies have added online recycling channels to traditional recycling channels, manufacturers or retailers also provide recyclers with the fixed reward per unit of recycled products [40]. And the used product returning rate is usually an exogenous variable, which is not affected by other factors [1, 2, 52]. However, combined with the reality, different second-hand products have different usage conditions and salvage values, so the discount size varies. Regarding the research on variable rebate, Ray et al. [37] study the different rebate strategies when customers first-time buy the durable product and when customers who currently owned the product buy it again. De Giovanni [10] considers the amount of incentive depended on the returning rate and the amount of effort that retailers put into green activity planning. Moreover, different rebate sizes have certain impacts on consumers’ willingness to return second-hand products, thus affecting the return rate of used products. Therefore, the research on the return rate of second-hand products also expands from exogenous variables to endogenous variables. For example, when Govindan and Popiuc [17] use the revenue sharing contract to study the coordination problem of the closed-loop supply chain, they define the customer's
willingness to return the abandoned unit as the functional discount to the retailer in exchange for recycling equipment and remanufacturing value. This article is to consider both variable rebates and the endogenous variables of consumers’ willingness to return products impact on the closed-loop supply chain. Although this situation is also considered in Genc and De Giovanni’s [14] research, the residual value of second-hand products decreases with the use of the products [21, 23, 37], not depend on the price of the products purchased in the past [14]. Therefore, the variable rebate of the product in this study is determined by the existing quality condition of the used product, which is one of the differences between this paper and the existing research.

In general, the market demand of products is the function of its price, and it decreases marginally as the price increases. For example, Guide et al. [19] consider that the remanufacturing profitability is affected by the quantity and quality of commodity returns and the demand for remanufacturing, and they believe that demand is affected by different prices. Although studies have also considered that demand is influenced not only by the current price of the product but also by the quality of the product [29], however, many studies on consumer behavior, such as that by Uhl and Brown [44], call the final payment price “price impression”, arguing that it affects how consumers respond to prices. Kalwani et al. [26] integrate the psychological theory of price perception into the empirical model of consumer buying behavior. As a result, a series of studies on reference prices have emerged. The original research believes that recent prices or observed information have greater reference effects than earlier prices [4], and there is minimal difference between using the last payment or charge of the brand and the weighted average of the last payment or charge of five prices as the reference price [26]. So, some research use past final payments as the basis for measuring reference prices [34]. All the above literature has shown that consumers’ purchasing behavior is influenced by the reference price, especially the price of the last purchase of the similar product, while the effect of this behavior on the closed-loop supply chain has not been studied. This is also one of the inspirations of this paper.

As far as we know, we are the first to combine product recycling from the perspective of consumers, rebate mechanisms, and demand sensitivity to reference price in closed-loop supply chain. In addition, the uniqueness of this research is that the size of the variable rebate depends on the current condition of the product rather than the price of buying the product in the past. Moreover, the returning rate of second-hand products is no longer an exogenous variable but is affected by the price and rebate of products purchased in the past. Finally, this paper applies the reference price to the market demand of the closed-loop supply chain and assumes the price of the used products are updated by consumers in the past as the reference price.

3. Model description and assumptions. We consider a single-stage closed-loop supply chain, and we assume that some of the products sold in previous periods can be recycled. There is only one manufacturer and one retailer in the closed-loop supply chain. The manufacturer is responsible for the production of new products and the recycling of used products directly from the consumers. The retailer is responsible for selling products. Table 1 displays the notations we employ.

**Assumption 1.** Each unit of returned product can become one unit of new product after reengineering.
We assume that all returned products can be remanufactured; therefore, the remanufacturing rate is 1. Previous studies have made similar assumptions \cite{24, 51}.

**Assumption 2.** The remanufactured product is no different from the new one in the market.

The remanufactured product and the new product are not perceived by consumers, and there is no quality difference between them \cite{39}. Both manufactured and remanufactured products are used to satisfy demands \cite{25, 41}.

**Assumption 3.** Both manufacturers and retailers are risk neutral and symmetrical to each other.

With this assumption, we can properly control the inefficiency and risk-sharing caused by information asymmetry.

**Table 1.** The description of the symbols

| Notations | Description |
|-----------|-------------|
| $p_0$     | The price at which the previous generation of products are sold |
| $p$       | Sales price of new generation products |
| $\omega$  | The unit wholesale price of the new product |
| $c$       | Unit cost of new product |
| $T$       | Product life cycle |
| $t$       | Product usage time |
| $a$       | Market demand scale |
| $b$       | Price sensitivity coefficient |
| $\alpha$  | The quantity of used products returned by consumers independent of price and rebate |
| $\beta$   | Consumers’ willingness to return used products |
| $\delta$  | Discount factor according to product quality |
| $k$       | Price difference (price difference between the new product and the previous one) sensitivity coefficient ( $k \geq 0$) |
| $g$       | Cost of the unit of remanufactured product (including remanufacturing, transportation and detection costs) |
| $l$       | Fixed rebate |
| $d$       | The demand for new products |
| $r_i$     | The quantity of previous generation products returned |
| $\pi_{M_i}$ | Manufacturer’s profit |
| $\pi_{R_i}$ | Retailer’s profit |
| $\pi_i$   | Profits of closed loop supply chains |
| $i = 1, 2$ | Represent the corresponding parameters under fixed rebate and variable rebate respectively |

When new products are launched, the price of new products is the important factor that influences whether consumers are willing to return used products and consider buying new products. Genc and De Giovanni \cite{14} use the simple linear regression of the number of returned mobile phones in France from Statista.com and the relevant average price index for 2012-2015. The results show that there is a linear, negative correlation between returns and pricing. In addition, when consumers receive appropriate rebates, they are willing to return used products.

When the rebate offered by the manufacturer to the consumer is the fixed rebate $l$, the return function of the product is: $r_1(p) = \alpha - \beta(p - l)$ \cite{14, 16}. The
term $\beta(p - l)$ indicates that consumers’ willingness to return used products dependent on the price of new products and the rebates of used products. If $\beta = 0$, consumers’ willingness to return products is independent of product prices and rebates. At this point, the return of products in the closed-loop supply chain is based on the voluntary return of consumers [33].

Ray et al. [37] note in their study that the residual value of the product is $s(T - t)$, where $s$ is the rate of change of the residual value with the residual effective life. Chien [8] believes that the residual value is $v(T - t)$, and $v$ is the residual value of the residual life per unit. Genc and De Giovanni [14] note that the price of the product represents its quality. Therefore, based on the existing research, this paper assumes that $p_0 T$ is the product value of unit life in this paper, and $(T - t)p_0 T$ is the residual value of the used product when it has been used for t time and returned. At this point, when rebate is the variable rebate, which depends on the existing quality of the used products and the price of the new products, the return function of the product is $r_2(p_0, p) = \alpha - \beta[p - \delta(T - t)p_0 T]$.

The influence of reference price on demand depends on the difference between the reference price and the observed price (proposed by Kopalle et al. [28]). As such, we choose the price of the previous generation $p_0$ and the price of new product $p$ as the reference price points that influence market demand for new products. The reference price may be higher than the current selling price. For example, the price of iPhone 7 is higher than that of iPhone 8 and the price of iPhone XS is higher than that of iPhone 11; the reference price may also be lower than the current selling price, such as HUAWEI Mate series and P series. For consumers, reference prices have different effects on demand. When the reference price $p_0$ is higher than the current price $p$, consumers will perceive the gain and result in the increase in demand. At this point, the impact of reference price on demand is $k_1(p - p_0)$; When the reference price is lower than the current price, the consumer will feel the loss and they would be less inclined to make the purchase. In this case, the impact of reference price on demand is $k_2(p - p_0)$. The parameters $k_1$ and $k_2$ are interpreted as the gain and loss effects respectively, since they are associated with perceived gains and losses. If $k_1 > k_2$, consumers are called loss-seeking; If $k_1 < k_2$, they are loss-averse; If $k_1 = k_2$, consumers are loss-neutral [22, 28]. When consumer’s demand elasticity is small, consumer’s demand for the product will be more urgent and relatively rigid. At this time, the price sensitivity of the product is relatively low, and the corresponding consumer’s sensitivity to the reference price difference will also be reduced. This suggests that price difference sensitivity and price sensitivity are positively correlated; Therefore, $kb(p - p_0)$ is the demand component influenced by the difference between the current price and the previous generation’s price. The demand function is $d = D(p, p_0) = a - bp - kb(p - p_0)$, and if $p_0 > p$, $k = k_1$; otherwise, $k = k_2$.

When the rebate is the fixed rebate,

The retailer’s profit function:

$$\max_p \pi_{R_1} = (p - \omega)d$$

(1)

The manufacturer’s profit function:

$$\max_\omega \pi_{M_1} = d(\omega - c) + r_1(c - g - l)$$

(2)

When the rebate is the variable rebate,
The retailer’s profit function:

\[
\max_p \pi_{R_2} = (p - \omega)d \tag{3}
\]

The manufacturer’s profit function:

\[
\max_\omega \pi_{M_2} = d(\omega - c) + r_2[c - g - \delta(T - t)\frac{p_0}{T}] \tag{4}
\]

Assuming that \(T = 1\), \(\delta = 1\); therefore, \(t \in [0,1]\). \(t = 0\) means that the new product is recycled at full price, which can be understood as the situation of new product “seven days no reason to return (Unsupported after activation)”, and this phenomenon is more common on e-commerce platforms. At this time the manufacturer’s profit function:

\[
\max_\omega \pi_{M_2} = d(\omega - c) + r_2[c - g - (1 - t)p_0] \tag{4}
\]

The surplus value of second-hand products (especially electronic products) is related to the degree of a product’s usage. If the recycler considers that the residual value of all second-hand products is the same when recycling used products (such as De Giovanni et al. [12]; Kaya, [27] and savaskcan et al. [39]), it is not only unsuitable to the actual situation, but the recycling mode of such recycling will affect the interests of recyclers and consumers. Based on the real practice, this paper considers two rebate mechanisms, fixed rebate \((l)\) and variable rebate \(( (T - t)\frac{p_0}{T})\). Although this is also reflected in Genc and De Giovanni’s [14] research, the size of variable rebate in Genc and De Giovanni’s [14] work is related to the price \(p_0\) at the time of product purchased. It is worth noting that the residual value of a second-hand product, especially the electronic product, is time-sensitive, and its residual value is closely related to the length of time \(t\). Therefore, the quantification of variable rebates \(( (T - t)\frac{p_0}{T})\) in this paper is more suitable for the real situation. In addition, the consumer’s psychology cannot be ignored. Consumers will consciously or unconsciously compare prices when shopping. The result of the price comparison will affect their purchasing behavior and thus the demand for products. However, most studies did not consider the impact of consumer price comparison psychology on market demand [14, 19]. In this paper, the market demand \((d = D(p, p_0) = a - bp - kb(p - p_0))\) is modeled by using the reference price theory, filling the gaps in previous studies.

In the reference price theory, the difference between the reference price and the price of the new product describes whether a consumer is loss-seeking \((k_1 > k_2)\), loss-averse \((k_1 < k_2)\) or loss-neutral \((k_1 = k_2)\). There are also differences in the impact of different types of consumers on the demand model \((d = D(p, p_0) = a - bp - k_i(b(p - p_0)), i = 1 \text{ or } 2)\); furthermore, combined with the characteristics of the time value sensitivity of electronic products, there is a certain endogenousness between the variable rebate in the model and the usage time of products (variable rebate \( = (T - t)\frac{p_0}{T}\)). This increases the complexity of the model to varying degrees during calculation and quantification.

4. Stackelberg equilibrium with fixed rebate. This part discusses the equilibrium of the closed-loop supply chain when the manufacturer offers the fixed rebate to consumers to incentive them to recycle their used products. Examples of scenario that offer the fixed rebate directly to recyclers are Dell and H&M. In the game between manufacturer and retailer, the manufacturer is the leader and decides the
wholesale price of the product, and the retailer decides the market retail price of the product according to the wholesale price determined by the manufacturer.

For retailer, we can obtain \( \frac{\partial^2 \pi_{R_1}}{\partial p^2} = -2(k+1)b < 0 \) from formula (1). In other words, the retailer’s objective function is concave and has a maximum value. The first derivative of (1) with respect to \( p \) can be obtained \( p_1^* = \frac{a+kbp_0+b(k+1)c}{2b(k+1)} \).

For manufacturer, plugging \( p_1^* = \frac{a+kbp_0+b(k+1)c}{2b(k+1)} \) into formula (2), we can obtain \( \frac{\partial^2 \pi_{M_1}(p_1^*)}{\partial c^2} = -b(k+1) < 0 \), which means that the Manufacturer’s objective function is concave and has a maximum value. With the fixed rebate, we can obtain the Stackelberg equilibrium results as follows.

\[
\omega_1^* = \frac{a + kbp_0 + \beta(g + l) - [\beta - b(k + 1)]c}{2b(k + 1)} \tag{5}
\]

\[
p_1^* = \frac{3(a + kbp_0) + \beta(g + l) - [\beta - b(k + 1)]c}{4b(k + 1)} \tag{6}
\]

\[
\pi_{M_1}^* = \frac{(a + kbp_0)^2 - \beta^2(g + l)^2 - 2bc(a + kbp_0)(k + 1)}{8b(k + 1)}
+ \frac{2\beta^2c(g + l) - c^2[\beta^2 - b^2(k + 1)^2]}{8b(k + 1)}
+ \frac{2(c - g - l)[4b(k + 1)(a + \beta l) - 3\beta(a + kbp_0)]}{8b(k + 1)}
- \frac{2(c - g - l) \{\beta^2(g + l) - \beta c[\beta - b(k + 1)]\}}{8b(k + 1)} \tag{7}
\]

\[
\pi_{R_1}^* = \frac{(a + kbp_0 - \beta(g + l) + [\beta - b(k + 1)]c)^2}{16b(k + 1)} \tag{8}
\]

**Proposition 1.** Higher consumers’ willingness relying on fixed rebate offered by manufacturers to return used products leads to lower pricing strategies, higher retailers’ profits and lower manufacturers’ profits.

**Proof.** See Appendix A. \( \square \)

\( \frac{\partial \omega_1^*}{\partial g} \leq 0, \frac{\partial \pi_{R_1}^*}{\partial g} \leq 0, \frac{\partial \pi_{M_1}^*}{\partial g} \geq 0, \frac{\partial \pi_{M_1}^*}{\partial c} \leq 0 \). The results show that although consumers are more willing to return their products, consumers’ willingness to return products relying on new product price and fixed rebates offered by manufacturers. The main reason for the decline of manufacturers’ profits may be that with the increase of consumers’ willingness to return second-hand products, the products on the market basically come from the remanufactured products of recycled second-hand products rather than the new products made from raw materials, which will reduce the market share of products and the profits of manufacturers. Manufacturers are profitable but their profits are decreasing, which is in line with the fact that some enterprises have not tried their best to recycle used products.

To illustrate how changes in consumers’ willingness to return affect the entire closed-loop supply chain, we offer the following Figure 1. We assume that \( \alpha = 0.8, a = 1, b = 0.7, l = 0.03, q = 0.02 \). If customers are loss-seeking \( c=0.1, p_0=1 \); otherwise customers are loss-averse, \( c=0.6, p_0=0.5 \). And we use these parameter values throughout the paper, which meet all the assumptions and conditions in this article.
Figure 1. $\pi_1^*$ when $\beta$ varies

From Figure 1, we find that, whether consumers are loss-seeking or loss-averse, for different $k$, increasing in $\beta$ makes the closed-loop supply chain’s profits decrease.

**Proposition 2.** Under certain conditions, whether the demand is loss-seeking or loss-averse, the wholesale price and retail price of the new generation products are inversely proportional to the price difference sensitivity coefficient of consumers. As the price difference sensitivity coefficient increases, the profit of manufacturers and retailers increases when the demand is loss-seeking; however, when the demand is loss-averse, retailer’s profit decreases, and manufacturer’s profit increases, but the increase slows down.

**Proof.** See Appendix B.

If $c < p_0 < a - \beta(c - g - l)$, then $k_1 > k_2$, which means that consumers are more responsive to perceived gains than equally sized losses, and the customer is loss-seeking. If $p_0 < a - \beta(c - g - l)$ and $p_0 < c$, we can get $k_1 < k_2$, meaning that the customer is loss-averse. Because $p_0 \neq c$, so there is no situation where the customer is loss-neutral. However, whether the customer is loss-seeking or loss-averse, we can always obtain $\frac{\partial \pi_1^*}{\partial k} < 0, \frac{\partial \pi_1^*}{\partial k} < 0$. This result indicates that, when the price of the previous generation of products is within a certain range, the higher the sensitivity coefficient of consumers to price difference, the lower the price of the new generation of products will be.

From formula (7) and (8), we can obtain
\[
\frac{\partial \pi_{M1}}{\partial k} = \frac{(a + kb_{p0})(b_{p0}(k + 2) - a) - b^2(c - 2p_{0})(k + 1)^2 + \beta(c - g - l)\beta(a - b_{p0}) - \beta(c - g - l)}{8b(k + 1)^2(b_{p0} - c)k - a - \beta(c - g - l) + b(2p_{0} - 3c)}\]
\[
\frac{\partial \pi_{R1}}{\partial k} = \frac{16b(k + 1)^2}{10b(k + 1)^2(g + l)}\]
respectively. To illustrate how changes in the sensitivity coefficient of consumers to price difference affect the profits of the retailer and the manufacturer, we offer the following Figures 2 - Figure 5. Figure 2 and Figure 3 respectively represent the relationship between manufacturer and retailer profits and $k$ when consumers are loss-seeking; Figure 4 and Figure 5 respectively represent the relationship between manufacturer and retailer profit and $k$ when consumers are loss-averse; Figure 6 represents the relationship between the total profit of the supply chain and $k$ when consumers are loss-averse.

If consumers are loss-seeking, we can obtain the following Figure 2 and Figure 3.

If consumers are loss-averse, we can obtain the following Figure 4 - Figure 6.

When the market demand is loss-seeking, in this case $p_1^* < p_0$. Figure 2 and Figure 3 illustrate the relationship between retailer’s and manufacturer’s profit and the parameter: reference price difference sensitivity coefficient $k$. The higher the
sensitivity coefficient of consumers to the price difference, the higher the demand for products, and the higher the profits of both manufacturers and retailers will be. Clearly, the total profits of the closed-loop supply chain increase.

When the market demand is loss-averse, in this case $p^*_1 \geq p_0$. Figure 4 - Figure 6 illustrate the relationship between retailer’s profit, manufacturer’s profit and the total supply chain profit and the parameter $k$. With the increase of the price difference sensitivity coefficient $k$, the market demand is gradually decreasing, and the retailer’s profit is decreasing; the manufacturer’s profit is increased but the increase is small; And the trend of the total profit of closed-loop supply chain is the same as that the manufacturer.
Therefore, when enterprises are faced with price-sensitive or loss-averse consumers, they should price the new generation of products reasonably and not excessively, otherwise it will affect market demand and then market profits. For example, almost every new generation of iPhone launched in China will see its price rise. In the past two years, Apple Inc.’s iPhone sales volume in China has decreased sharply. On one hand, the price of other brands of mobile phones, such as HUAWEI, is also rising, but the increase is much lower than that of Apple’s; On the other hand, consumers are relatively sensitive to product prices.

5. Stackelberg equilibrium with variable rebate. In this section, we consider the equilibrium of the closed-loop supply chain when the manufacturer provides variable rebates to consumers to encourage them to return second-hand products. Similar to the fixed rebate, in the Stackelberg game, the manufacturer is the leader, and the retailer is the follower.

For the retailer, we can obtain \( \frac{\partial^2 \pi_{R_2}}{\partial p^2} = -2(k + 1)b < 0 \) from formula (3). In other words, the retailer’s objective function is concave and has a maximum value. The first derivative of (3) with respect to \( p \) can be obtained as \( p^*_2 = \frac{a + kbp_0 + b(k + 1)\omega}{2b(k + 1)} \).

For the manufacturer, plugging \( p^*_2 \) into formula (4), we can obtain \( \frac{\partial^2 \pi_{M_2}(p^*_2)}{\partial p^2} = -b(k + 1) < 0 \), which means that the manufacturer’s objective function is concave and has a maximum value. With the variable rebate, we can obtain the Stackelberg equilibrium results as follows.

\[
\omega^*_2 = \frac{a + kbp_0 + \beta[g + (1 - t)p_0] - [\beta - b(k + 1)]c}{2b(k + 1)} \quad (9)
\]

\[
p^*_2 = \frac{3(a + kbp_0) + \beta[g + (1 - t)p_0] - [\beta - b(k + 1)]c}{4b(k + 1)} \quad (10)
\]

\[
\pi^*_{M_2} = \frac{(a + kbp_0)^2 - \beta^2g + (1 - t)p_0)^2 - 2bc(a + kbp_0)(k + 1)}{8b(k + 1)}
+ \frac{2\beta^2c[g + (1 - t)p_0] - c^2[\beta^2 - b^2(k + 1)^2]}{8b(k + 1)}
+ \frac{2[c - g - (1 - t)p_0]{4b(k + 1)[\alpha + \beta(1 - t)p_0] - 3\beta(a + kbp_0)}}{8b(k + 1)}
- \frac{2[c - g - (1 - t)p_0]{\beta^2[g + (1 - t)p_0] - \beta c[\beta - b(k + 1)]}}{8b(k + 1)}
\]

\[
\pi^*_{R_2} = \frac{(a + kbp_0 - \beta[g + (1 - t)p_0] + [\beta - b(k + 1)]c)^2}{16b(k + 1)} \quad (11)
\]

**Proposition 3.** A higher consumers’ willingness relying on variable rebate offered by manufacturers to return second-hand products leads to lower pricing, higher retailers’ profits and lower manufacturers’ profits.

**Proof.** See Appendix C. \( \square \)
be that with the increase of consumers’ willingness to return, the products in the market are basically recycled from second-hand products without the input of new products produced by raw materials, which will affect the market share of products and thus the manufacturers’ profits. Manufacturers are profitable but their profits are decreasing, which is in line with the fact that some enterprises have not tried their best to recycle used products.

To illustrate how changes in consumers’ willingness to return affect the entire closed-loop supply chain, we offer the following Figure 7.

From Figure 7, we find that, whether consumers are loss-seeking or loss-averse, for different \( k \), an increase in \( \beta \) makes the closed-loop supply chain’s profits decrease.

**Proposition 4.** Under certain contains, no matter the demand is loss-seeking or loss-averse, the wholesale price and the retail price of the new generation products are inversely proportional to the price difference sensitivity coefficient of consumers. However, with the increase of price difference sensitivity coefficient, if the market demand is loss-seeking, the profit of manufacturers and retailers increases; if the market demand is loss-averse, retailer’s profit decreases and manufacturer’s profit increases slowly.

**Proof.** See Appendix D.

If \( t > 1 - \frac{b}{\beta} \), \( c < p_0 < \frac{a-\beta(c-g)}{b-\beta(1-t)} \), then \( k_1 > k_2 \), which suggests that customers are loss-seeking; If \( t < 1 - \frac{b}{\beta} \), \( p_0 > \frac{a-\beta(c-g)}{b-\beta(1-t)} \) and \( p_0 < c \), then \( k_1 < k_2 \), meaning that consumers are loss-averse. Because \( p_0 \neq c \), so there is no situation where market demand is loss-neutral. In both cases, we can obtain \( \frac{\partial \pi_2^M}{\partial k} < 0 \), \( \frac{\partial p_2^*}{\partial k} < 0 \), which indicates that, when the time and price of the previous generation of products are within a certain range, the smaller the sensitivity coefficient of consumers is to the price difference, the higher the price of the new generation of products will be.

From formula (11) and (12), we can obtain

\[
\frac{\partial \pi_2^M}{\partial k} = \frac{\beta[c-g-(1-t)p_0]([6(a-bp_0)-\beta[c-g-(1-t)p_0])]}{8k(k+1)^2} + \frac{k^2c(k+1)^2(c-2p_0)+(a+kbp_0)[bp_0(k+2)-a]}{8k(k+1)^2},
\]

\[
\frac{\partial p_2^*}{\partial k} = \frac{\{a+kbp_0-\beta[g+(1-t)p_0]+[\beta-b(k+1)c]\}(2b(p_0-c)-(a+kbp_0)+\beta[g+(1-t)p_0]-[\beta-b(k+1)c])}{16k(k+1)^2}.
\]

respectively. To illustrate how changes in the sensitivity coefficient of consumers to the price difference affect the profits of the retailer and the manufacturer, we offer the following Figure 8 - Figure 12.

If consumers are loss-seeking, we can obtain the following Figure 8 and Figure 9.

If consumers are loss-seeking, we can obtain the following Figure 10 - Figure 12.
When the market demand is loss-seeking, in this case $p^*_2 < p_0$. Figure 8 and Figure 9 depict that the profits of both manufacturers and retailers increase with the increase of the price difference sensitivity coefficient, $\frac{\partial \pi^*_R}{\partial k} > 0$, $\frac{\partial \pi^*_M}{\partial k} > 0$. This finding indicates that higher sensitivity coefficient of consumers to the price difference increases the demand for products and the profits of both manufacturers and retailers. Obviously, the total profits of the closed-loop supply chain increase.

When the market demand is loss-averse, in this case $p^*_2 \geq p_0$. Figure 10 - Figure 12 illustrate the relationship between retailer’s profit, manufacturer’s profit and the total supply chain profit and the parameter $k$. With the increase of the price difference sensitivity coefficient $k$, the market demand is gradually decreasing, and
the retailer’s profit is decreasing; the manufacturer’s profit is increased but the increase is small; and the trend of the total profit of closed-loop supply chain is the same as that of the manufacturer.

**Proposition 5.** Longer use of the previous generation product by consumers leads to lower pricing and higher retailers’ profits. If consumers are loss-seeking, in the case of manufacturers providing the variable rebate to consumers, it is profitable when the product is used for a certain amount of time; otherwise, if consumers are loss-averse, it is always profitable for manufacturers to provide consumers with variable rebates without considering the usage time of second-hand products.

**Proof.** See Appendix E.

\[ \frac{\partial \omega^*_2}{\partial t} < 0, \quad \frac{\partial p^*_2}{\partial t} < 0, \] which indicates that the longer consumers use the products of the previous generation, the lower the pricing of the new products will be. \[ \frac{\partial \pi^*_R}{\partial t} \geq 0, \] which implies that retailers’ profits are positively correlated with the product’s usage time.

From formula (11), we can obtain

\[
\frac{\partial \pi^*_M}{\partial t} = \frac{b p_0 (k+1)(4a+4\beta(1-t)p_0-\beta c) - 3\beta p_0 (a+k p_0) - \beta p_0 (c-g-(1-t)p_0)}{4b(k+1)}. \]

To illustrate how changes in the usage time of previous generation products affect the profit of the retailer and the manufacturer, we offer the following Figure 13 - Figure 15.

**Figure 13.** \( \pi^*_R \) when \( t \) varies

**Figure 14.** \( \pi^*_M \) when \( t \) varies

**Figure 15.** \( \pi^*_2 \) when \( t \) varies

Figure 13 and Figure 14 depict the relationship between the retailer’s and the manufacturer’s profit and the length of time the product was used. When consumers
are loss-seeking, as shown by the black line in the figure, for different $\beta$ and $k$, the increase in $t$ makes the retailer’s and the manufacturer’s profits increase; from Figure 14, we can obtain $\frac{\partial \pi_{M_2}^*}{\partial t} > 0$. Although manufacturers’ profits are positively correlated with the product’s usage time, manufacturers’ profits are negative when $t$ is less than a certain value. For $\pi_{M_2}^* = 0$, $t_0 = \frac{-4ab^{-3a\beta-2\sqrt{2(X+Y+Z)}}}{\beta p_0(8b-\beta+8bK)}$ (where $X = b^2(k+1)^2[(2\alpha - 2\beta g + \beta c)\alpha - \beta g + \beta c - 2\beta c(a + kbp_0)], Y = b\beta(k+1)[(a + kbp_0)^2 + (a + kbp_0)^2 + (a + kbp_0)(3\beta g - 3\alpha - 2\beta c + 2b^2(k+1)^2) + \beta^2(a + kbp_0)^2], Z = \beta^2(c - g - p_0) + 5b\beta(kp_0 - c - ck) + 4b\beta(g + 2p_0 + ck) + 2k\beta p_0)$). Therefore, when the product is used for a period of time $t_0 < t \leq 1$, it is profitable for the manufacturer to offer consumers the variable rebate incentive to return the used product. Correspondingly, the total profit of the closed-loop supply chain is positive only when the use time of the previous generation product exceeds a certain range, as is shown in Figure 15.

If consumers are loss-averse, as shown by the blue line in the figure, for different $\beta$ and $k$, as $t$ increases, the retailer’s profit increases, but it is far below its increase when consumers are loss-seeking; however, from Figure 14, we can see that the manufacturer’s profit has increased but the increase is not large, and even with the increase in product use time $t$, the profit has the tendency to decline. But overall, manufactures’ profits are positive. Therefore, for consumers who are loss-averse, it is always profitable for the manufacturer to offer consumers the variable rebate incentive to return the used product. Similarly, the change in the total profit of the closed-loop supply chain is the same as that of the manufacturer, as is shown by the blue lines in Figure 15.

6. Comparison of fixed and variable rebate mechanisms. In this section, we analyze and compare the optimal price and the profit of the closed-loop supply chain under the two rebate mechanisms to investigate the optimal rebate mechanism and the pricing strategy according to the usage time of the previous generation products.

We separately calculate the difference between the equilibrium results of the two rebate mechanisms and obtain the following results.

$$\omega_1^* - \omega_2^* = \frac{\beta[l - (1-t)p_0]}{2b(k + 1)}$$  (13)

$$p_1^* - p_2^* = \frac{\beta[l - (1-t)p_0]}{4b(k + 1)}$$  (14)

$$\pi_{M_1}^* - \pi_{M_2}^* = \frac{l - (1-t)p_0}{8b(k + 1)} \left\{ \beta^2[2g + l + (1-t)p_0] + 8b\beta(c - g)(k + 1) \right\} - \frac{l - (1-t)p_0}{8b(k + 1)} \left\{ \alpha + \beta l + \beta(1-t)p_0 \right\} + \frac{l - (1-t)p_0}{8b(k + 1)} \left\{ 6\beta(a + kbp_0) - 2\beta c[\beta - b(k + 1)] \right\}$$  (15)

$$\pi_{R_1}^* - \pi_{R_2}^* = \frac{\beta[l - (1-t)p_0 - l]}{16b(k + 1)} \left\{ 2(a + kbp_0) - \beta[2g + l + (1-t)p_0] + 2c[\beta - b(k + 1)] \right\}$$  (16)

By comparing the sizes of the equilibrium results under the two rebate mechanisms, the following propositions can be obtained.
Proposition 6. If $t \in [1 - \frac{1}{\rho_0}, 1]$, whether consumers are loss-seeking or loss-averse, under certain conditions, the closed-loop supply chain under the variable rebate mechanism is better than the closed-loop supply chain under the fixed rebate mechanism.

Proof. See Appendix F. \qed

When $1 \geq t \geq 1 - \frac{1}{\rho_0}$, $\omega_1 - \omega^*_2 \geq 0$ and $p^*_1 - p^*_2 \geq 0$. For retailers, $\pi^*_R_1 - \pi^*_R_2 \leq 0$. For manufacturers, when $\beta < 8b(k+1)$, $p_0 \leq \frac{8b(k+1)[a - \beta(c - g - t)] + \beta(2c[\beta - b(k+1)] - 2(2g + t) - 6a)\beta}{6b^2}$, or $\beta \geq 8b(k+1)$, $p_0 \leq \frac{8b(k+1)[a - \beta(c - g - 2t)] + \beta(2c[\beta - b(k+1)] - 2(2g + t) - 6a)\beta}{6b^2}$. Therefore, under certain conditions, the closed-loop supply chain is optimal under the variable rebate mechanism.

When second-hand products are used for a long time, the rebates that recycling companies provide to consumers are best determined based on the quality of the second-hand products, rather than fixed rebates, so that companies can profit when recycling second-hand products.

Proposition 7. If $t \in [0, 1 - \frac{1}{\rho_0})$, whether consumers are loss-seeking or loss-averse, under certain conditions, the closed-loop supply chain under the fixed rebate mechanism is better than the closed-loop supply chain under the variable rebate mechanism.

Proof. See Appendix G. \qed

When $0 \leq t < 1 - \frac{1}{\rho_0}$, $\omega_1 - \omega^*_2 < 0$ and $p^*_1 - p^*_2 < 0$. For retailers, $\pi^*_R_1 - \pi^*_R_2 \geq 0$. For manufacturers, when $\beta < 8b(k+1)$, $p_0 \leq \frac{8b(k+1)[a - \beta(c - g - t)] + \beta(2c[\beta - b(k+1)] - 2(2g + t) - 6a)\beta}{6b^2}$. Therefore, under certain conditions, the closed-loop supply chain is optimal under the fixed rebate mechanism.

Figure 16 depicts the relationship between the difference in profits between the two retailers and the usage time of the previous generation of products. Figure 17 depicts the relationship between the difference in profits between the two manufacturers and the usage time of the previous generation of products. Accordingly, Figure 18 depicts the relationship between the difference in profits between the two closed loop supply chains and the usage time of the previous generation of products.

Whether consumers are loss-seeking or loss-averse, we find that the closed-loop supply chain is optimal under the fixed rebate mechanism when the product is used for a short time, and the closed-loop supply chain is optimal under the variable rebate mechanism when the product is used for a long time.

Under the constraints of consumers' willingness to return and the price of the previous generation of products, when companies recycle second-hand products that have been used for a short period of time, in order to obtain higher profits, companies should provide consumers with the fixed rebate to encourage them to return second-hand products.

7. Numerical example. In order to more intuitively describe and compare the relationship between the various variables in the model, this chapter uses numerical analyses to analyze the influence of several parameters on the equilibrium results of the two rebate mechanisms. First of all, the setting of $a = 1$ is a benchmark, which
means that the original market demand scale is 1. In addition, there is a maximum value α for the consumer’s return amount, independent of price and rebate. That is, when β = 0, the largest value of consumers’ returned products is no more than 1, in line with the above assumption that some products sold in the previous period can be recycled. At this time, the closed-loop supply chain adopts a passive return strategy [14, 33], so we set α = 0.8 in this paper. The setting of other parameter values is not only based on existing research [14], but also on the basis of satisfying the various assumptions and conditions we introduced earlier. The main purpose is to better pay attention to the impact of consumers’ return willingness β, price difference sensitivity coefficient k and second-hand product usage time t in the closed-loop supply chain. Therefore, α = 0.8, a = 1, b = 0.7, l = 0.03, g = 0.02; if customers are loss-seeking, c = 0.1, p = 1; if customers are loss-averse, c = 0.6, p = 0.5.

For the value setting of β, k and t, as long as the assumptions and different conditions are met, the combination of other values of the parameter β, k and t will not affect our conclusions. When setting the parameter combination value of β, k and t in the paper, it meets the different conditions and restrictions in each proposition at the same time, which not only makes the results more concise and clear, but also easy to analyze and compare, and saves a lot of unnecessary charts. The equilibrium solutions and the impact of different parameters on equilibrium solutions are shown in the following Tables.
7.1. Equilibrium solutions and sensitivity analysis with $\beta$. When $k = 0.5, t = 0.93$

Table 2. If consumers are loss-seeking

| $\beta$ | with fixed rebate mechanism | with variable rebate mechanism |
|--------|-----------------------------|-------------------------------|
|        | $p_1^*$ $\omega_1^*$ $\pi_{M1}^*$ $\pi_{R1}^*$ $\pi_1^*$ | $p_2^*$ $\omega_2^*$ $\pi_{M2}^*$ $\pi_{R2}^*$ $\pi_2^*$ |
| 0      | 0.989 0.693 0.225 0.092 0.317 | 0.989 0.693 0.193 0.092 0.285 |
| 0.2    | 0.987 0.688 0.215 0.094 0.309 | 0.989 0.692 0.191 0.093 0.284 |
| 0.4    | 0.986 0.683 0.205 0.095 0.301 | 0.988 0.691 0.189 0.093 0.282 |
| 0.6    | 0.982 0.679 0.196 0.097 0.293 | 0.988 0.690 0.187 0.093 0.280 |
| 0.8    | 0.980 0.674 0.186 0.098 0.285 | 0.987 0.689 0.185 0.094 0.279 |
| 1      | 0.977 0.669 0.177 0.100 0.277 | 0.987 0.688 0.183 0.094 0.277 |

Table 3. If consumers are loss-averse

| $\beta$ | with fixed rebate mechanism | with variable rebate mechanism |
|--------|-----------------------------|-------------------------------|
|        | $p_1^*$ $\omega_1^*$ $\pi_{M1}^*$ $\pi_{R1}^*$ $\pi_1^*$ | $p_2^*$ $\omega_2^*$ $\pi_{M2}^*$ $\pi_{R2}^*$ $\pi_2^*$ |
| 0      | 0.989 0.860 0.445 0.018 0.463 | 0.989 0.860 0.471 0.018 0.489 |
| 0.2    | 0.963 0.807 0.341 0.026 0.367 | 0.963 0.808 0.369 0.026 0.395 |
| 0.4    | 0.937 0.755 0.205 0.035 0.275 | 0.937 0.756 0.269 0.035 0.304 |
| 0.6    | 0.911 0.702 0.141 0.046 0.287 | 0.911 0.704 0.172 0.045 0.217 |
| 0.8    | 0.885 0.650 0.046 0.058 0.104 | 0.886 0.652 0.078 0.057 0.135 |
| 1      | 0.858 0.598 -0.047 0.071 0.024 | 0.860 0.600 -0.013 0.071 0.058 |

From Table 2 and Table 3, we can obtain the equilibrium results under the two rebate mechanisms when $k = 0.5, t = 0.93$. Furthermore, whether the consumer is loss-seeking or loss-averse, and either in the fixed or variable rebate mechanism, we can observe that $\pi_1^* > 0, \pi_2^* > 0$ with different $\beta$. However, as consumers’ willingness to return the products of the previous generation $\beta$ increases, $p_1^*, \omega_1^*, p_2^*, \omega_2^*$ decrease. Furthermore, $\pi_{R1}^*, \pi_{R2}^*$ increase, $\pi_{M1}^*, \pi_{M2}^*, \pi_1^*, \pi_2^*$ decrease.

This finding explains that, although some enterprises know that recycling second-hand products is profitable, when consumers’ willingness to return second-hand products increases, the profit of the closed-loop supply chain decreases; this will lead to other enterprises not actively recycling second-hand products. Under such circumstances, it is necessary for the government to implement certain measures to encourage enterprises to recycle second-hand products, such as providing certain subsidies, using financial incentives and even making laws and regulations to restrain the recycling behavior of enterprises.

7.2. Equilibrium solutions and sensitivity analysis with $k$. When $\beta = 0.5, t = 0.93$

Table 4. If consumers are loss-seeking

| $k$ | with fixed rebate mechanism | with variable rebate mechanism |
|-----|-----------------------------|-------------------------------|
|     | $p_1^*$ $\omega_1^*$ $\pi_{M1}^*$ $\pi_{R1}^*$ $\pi_1^*$ | $p_2^*$ $\omega_2^*$ $\pi_{M2}^*$ $\pi_{R2}^*$ $\pi_2^*$ |
| 0   | 1.088 0.746 0.168 0.081 0.249 | 1.095 0.761 0.157 0.078 0.235 |
| 0.2 | 1.035 0.714 0.181 0.087 0.268 | 1.041 0.726 0.169 0.084 0.253 |
| 0.4 | 0.998 0.690 0.194 0.093 0.287 | 1.003 0.701 0.182 0.090 0.272 |
| 0.6 | 0.970 0.673 0.207 0.099 0.307 | 0.975 0.682 0.194 0.096 0.290 |
| 0.8 | 0.949 0.659 0.221 0.106 0.327 | 0.953 0.667 0.208 0.103 0.311 |
| 1   | 0.931 0.648 0.235 0.112 0.347 | 0.935 0.656 0.221 0.109 0.330 |
Table 5. If consumers are loss-averse

| $k$ | $p_1^*$ | $\omega_1^*$ | $\pi_{M_1}^*$ | $\pi_{R_1}^*$ | $p_2^*$ | $\omega_2^*$ | $\pi_{M_2}^*$ | $\pi_{R_2}^*$ | $\pi_1^*$ | $\pi_2^*$ |
|-----|---------|-------------|---------------|-------------|---------|-------------|---------------|-------------|---------|---------|
| 0   | 1.123   | 0.818       | 0.155         | 0.065       | 0.220   | 1.124       | 0.820         | 0.186       | 0.065   | 0.251   |
| 0.2 | 1.024   | 0.773       | 0.173         | 0.053       | 0.226   | 1.024       | 0.775         | 0.203       | 0.052   | 0.255   |
| 0.4 | 0.952   | 0.741       | 0.185         | 0.044       | 0.229   | 0.953       | 0.743         | 0.215       | 0.043   | 0.259   |
| 0.6 | 0.899   | 0.717       | 0.194         | 0.037       | 0.231   | 0.899       | 0.719         | 0.225       | 0.037   | 0.262   |
| 0.8 | 0.857   | 0.699       | 0.202         | 0.032       | 0.234   | 0.858       | 0.700         | 0.232       | 0.032   | 0.263   |
| 1   | 0.824   | 0.684       | 0.208         | 0.028       | 0.236   | 0.825       | 0.685         | 0.237       | 0.027   | 0.265   |

From Table 4 and Table 5, we can obtain the equilibrium results under the two rebate mechanisms when $\beta = 0.5$, $t = 0.93$. Under either the variable or the fixed rebate, we can observe that, as consumers’ sensitivity coefficient to price difference $k$ increases, $p_1^*, \omega_1^*, p_2^*, \omega_2^*$ decrease. However, if consumers are loss-seeking, $\pi_{R_1}^*, \pi_{M_1}^*$, $\pi_{R_2}^*$, $\pi_{M_2}^*$ increase, and the total profits of the closed-loop supply chain $\pi_1^*$, $\pi_2^*$ increase, as is shown in Table 4; if customers are loss-averse, $\pi_{R_1}^*$, $\pi_{R_2}^*$ decrease, and $\pi_{M_1}^*$, $\pi_{M_2}^*$, $\pi_1^*$, $\pi_2^*$ increase, but the increase is significantly lower than that when consumers are loss-seeking, which is shown in Table 5.

In fact, most consumers are sensitive to the fluctuations of product prices, which affects their behavior of buying new products to a large extent. In cases of high price volatility, consumers will either choose other products with similar brands but lower prices or wait until the new products’ prices decrease, which will affect the market demand for new products. Therefore, enterprises can appropriately lower the price when pricing, stimulate consumers’ purchase demand and realize more sales with small profits.

7.3. Equilibrium solutions and sensitivity analysis with $t$ under the variable rebate mechanism. When $k = \beta = 0.5$

Table 6. If consumers are loss-seeking

| $t$ | $p_2^*$ | $\omega_2^*$ | $\pi_{M_2}^*$ | $\pi_{R_2}^*$ | $\pi_2^*$ |
|-----|---------|-------------|---------------|-------------|---------|
| 0   | 1.099   | 0.912       | -0.531        | 0.037       | -0.494  |
| 0.2 | 1.075   | 0.864       | -0.308        | 0.047       | -0.261  |
| 0.4 | 1.051   | 0.817       | -0.122        | 0.058       | -0.064  |
| 0.6 | 1.027   | 0.769       | 0.026         | 0.070       | 0.096   |
| 0.8 | 1.004   | 0.721       | 0.136         | 0.084       | 0.220   |
| 1   | 0.980   | 0.674       | 0.209         | 0.098       | 0.307   |

Table 7. If consumers are loss-averse

| $t$ | $p_2^*$ | $\omega_2^*$ | $\pi_{M_2}^*$ | $\pi_{R_2}^*$ | $\pi_2^*$ |
|-----|---------|-------------|---------------|-------------|---------|
| 0   | 1.098   | 0.924       | 0.080         | 0.020       | 0.100   |
| 0.2 | 1.098   | 0.817       | 0.127         | 0.024       | 0.151   |
| 0.4 | 0.956   | 0.793       | 0.165         | 0.028       | 0.193   |
| 0.6 | 0.944   | 0.769       | 0.194         | 0.032       | 0.226   |
| 0.8 | 0.932   | 0.745       | 0.213         | 0.037       | 0.250   |
| 1   | 0.920   | 0.721       | 0.223         | 0.042       | 0.265   |

The equilibrium results under variable rebate mechanisms when $k = \beta = 0.5$ can be observed in Table 6 and Table 7. In addition, under the variable rebate mechanism, the pricing of new products $p_2^*, \omega_2^*$ decrease with the increase of the
use time $t$ of the previous generation of products, and the retailer’s profit $\pi^*_R$ and the manufacturer’s profit $\pi^*_M$ increase. However, if the market demand is loss-seeking, only when the use time $t$ exceeds the certain period of time $\pi^*_M > 0$. Accordingly, $\pi^*_M > 0$ only when the use time $t$ exceeds the certain period of time, as is shown in Table 6; if the market demand is loss-averse, manufacturer’s profit has always been positive, and so has the total profit of the closed-loop supply chain, as is shown in Table 7.

It can be observed from the numerical example analysis that, if the market demand is loss-seeking, when the use time of the previous generation product is shorter than a certain range, the manufacturer and the closed-loop supply chain have negative profit. From this perspective, enterprises occupy the market and speed up the updating of products; however, they cannot rush to pursue consumers to quickly update new products, otherwise it will cause the loss. Therefore, it is important for enterprises to decide when to encourage consumers to return the products and upgrade the new products; if the market demand is loss-averse, companies will not have concerns in this regard.

7.4. Comparison of the two rebate mechanisms for different $t$.

When $k = \beta = 0.5$

**Table 8. If consumers are loss-seeking**

| $t$        | $p^*_1 - p^*_2$ | $\omega^*_1 - \omega^*_2$ | $\pi^*_M - \pi^*_M$ | $\pi^*_R - \pi^*_R$ | $\pi^*_1 - \pi^*_2$ |
|------------|-----------------|--------------------------|---------------------|---------------------|---------------------|
| $t \in [0, 1 - \frac{1}{p})$ | 0.1 | -0.104 | -0.207 | 0.615 | 0.055 | 0.670 |
| 0.3 | -0.080 | -0.160 | 0.410 | 0.044 | 0.454 |
| 0.5 | -0.060 | -0.112 | 0.243 | 0.032 | 0.275 |
| $t \in [1 - \frac{1}{p}, 1]$ | 0.98 | 0.001 | 0.002 | -0.004 | -0.001 | -0.005 |
| 0.99 | 0.002 | 0.005 | -0.006 | -0.002 | -0.008 |
| 1 | 0.004 | 0.007 | -0.009 | -0.002 | -0.011 |

**Table 9. If consumers are loss-averse**

| $t$        | $p^*_1 - p^*_2$ | $\omega^*_1 - \omega^*_2$ | $\pi^*_M - \pi^*_M$ | $\pi^*_R - \pi^*_R$ | $\pi^*_1 - \pi^*_2$ |
|------------|-----------------|--------------------------|---------------------|---------------------|---------------------|
| $t \in [0, 1 - \frac{1}{p})$ | 0.1 | -0.050 | -0.100 | 0.085 | 0.018 | 0.103 |
| 0.3 | -0.038 | -0.076 | 0.043 | 0.014 | 0.057 |
| 0.5 | -0.026 | -0.052 | 0.009 | 0.010 | 0.019 |
| $t \in [1 - \frac{1}{p}, 1]$ | 0.98 | 0.002 | 0.005 | -0.032 | -0.001 | -0.033 |
| 0.99 | 0.003 | 0.006 | -0.032 | -0.001 | -0.033 |
| 1 | 0.004 | 0.007 | -0.033 | -0.002 | -0.035 |

From Table 8 and Table 9, the following conclusions can be derived when $k = \beta = 0.5$. Whether consumers are loss-seeking or loss-averse, if $t \in [0, 1 - \frac{1}{p})$, $p^*_1 - p^*_2 < 0$, $\omega^*_1 - \omega^*_2 < 0$; While $\pi^*_M - \pi^*_M > 0$, $\pi^*_R - \pi^*_R > 0$ and $\pi^*_1 - \pi^*_2 > 0$. As $t$ increases, the price difference and profit margin are reduced in equilibrium. Therefore, in this case, the fixed rebate mechanism is optimal. If $t \in [1 - \frac{1}{p}, 1]$, $p^*_1 - p^*_2 > 0$, $\omega^*_1 - \omega^*_2 > 0$. While $\pi^*_M - \pi^*_M < 0$, $\pi^*_R - \pi^*_R < 0$ and $\pi^*_1 - \pi^*_2 < 0$. As $t$ increases, the price difference and the profit margin are increased in equilibrium. Therefore, in this case, the variable rebate mechanism is optimal.

Therefore, when enterprises encourage consumers to return second-hand products, it is not that enterprises blindly provide consumers with the fixed or variable
rebate. In practice, when recycling products, enterprises need to consider the use time of the returned products, and then decide whether to offer consumers the fixed or variable rebate to optimize the profit of the closed-loop supply chain.

8. Conclusion. In this paper, regarding consumer returning behavior, we consider consumers’ return behavior to be affected by product price and rebate; in addition, using the reference price theory, we consider demand as influenced by the last generation of product price. Then, we establish the fixed rebate and the variable rebate mechanism model of the closed-loop supply chain. Under two different rebate mechanisms we discuss the optimal pricing and returning decision; through comparative analysis, we derived the following basic conclusions.

First, higher consumers’ willingness relying on the rebate offered by manufacturers to return second-hand products leads to lower pricing, higher retailers’ profits and lower manufacturers’ profits. This finding is consistent with the observation that some enterprises do not actively recycle second-hand products. Although recycling can be profitable, the increase of consumers’ willingness to return will lead to the decrease of profits in the supply chain as they rely on the rebate offered by enterprises. In this case, the government can take certain measures, such as providing corresponding subsidies to encourage enterprises to take green actions or set corresponding recycling standards to restrain enterprises’ recycling behavior.

Second, the wholesale and retail prices of new generation products are inversely proportional to the price difference sensitivity coefficient of consumers. When consumers are loss-seeking, the profit of manufacturers and retailers increases with the increase of the price difference sensitivity coefficient, and the total profit of the closed-loop supply chain increases; however, if consumers are loss-averse, the retailer’s profit is inversely proportional to the price difference sensitivity coefficient, while the manufacturer’s profit and the total profit of the closed-loop supply chain are directly proportional to it; although the profit of the manufacturer and the total profit of the closed-loop supply chain increase with the increase of the price difference sensitivity coefficient, the increase is smaller than that of the manufacturer and the total profit when the consumer is loss-seeking. So enterprises need to consider the price difference between two generations of products when releasing new products, and they can adopt low price method to increase product demand and sell more goods at small profits when customers are loss-averse and are sensitive to price difference.

Third, the longer use of the previous generation product by consumers leads to lower pricing and higher retailers’ profits; if customers are loss-seeking, this is only profitable when the manufacturer offers the variable rebate to the consumer when the product is used for a certain amount of time; however, if customers are loss-averse, manufacturers do not need to consider the length of use of second-hand products, and it is always best for manufacturers to provide consumers with variable rebates. Therefore, before the launch of new products, it is especially important that the enterprise provides consumers with the incentive of changing the rebate to return the second-hand products and update the new products. It is not recommended that consumers should upgrade their products as quickly as possible. If the enterprise adopts the strategy of the variable rebate mechanism too early, it will lead to the situation that the product returning rate increases and the profit of the enterprise is negative when consumers are loss-seeking.
Finally, when companies recycle used products, the length of time second-hand products have been used is the important factor for enterprises to consider. This basis is important for companies to adopt the variable rebate mechanism or the fixed rebate mechanism. If second-hand products are used for a short period of time, it is better for the enterprise to offer consumers the fixed rebate to encourage them to return the second-hand products. If the second-hand products are used for a long time, it is better for the enterprise to offer consumers the variable rebate to encourage them to return the second-hand products.

This article only considers a single-level closed-loop supply chain, and the situation of the two-level closed-loop supply chain can be considered in future research. In addition, the existence of market competition, such as the reliability and sustainability of two or more suppliers and the suppliers has not been covered in this paper. What’s more, the research by Mardani et al. [30] provides us with more thoughts and views on green and sustainable supply chains, which makes us see this paper. What’s more, the research by Mardani et al. [30] provides us with more thoughts and views on green and sustainable supply chains, which makes us see this article only discusses the limitations of the economic benefits generated by closed-loop supply chains, while environmental benefits, social responsibility and research methods are worthy of further discussion in future research.

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Appendix. The proof of each proposition is as follows.

Appendix A

Proof. The profit of reverse supply chain is nonnegative, otherwise it is meaningless to conduct product recycling; therefore \( r_1(c - g - l) \geq 0 \), that is \( g + l \leq c \). We can obtain \( \frac{\partial \omega_1^*}{\partial \beta} = \frac{g + l - c}{2b(k + 1)} \), \( \frac{\partial \pi_1^*}{\partial \beta} = \frac{g + l - c}{2b(k + 1)} \) separately from formula (5) and (6), and \( \frac{\partial \pi_1^*}{\partial k} = \{c - g - l)(a + kp_0 - \beta(g + l) + c[\beta - b(k + 1)]\} = \{c - g - l)(l - p_1^*) \} separately from formula (8) and (7). Because \( c - g - l \geq 0 \) and \( p_1^* > c > l \), therefore \( \frac{\partial \omega_1^*}{\partial \beta} \leq 0, \frac{\partial \pi_1^*}{\partial \beta} \leq 0, \frac{\partial \pi_1^*}{\partial k} \geq 0, \frac{\partial \pi_1^*}{\partial k} \leq 0 \).

Appendix B

Proof. When \( p_0 = p_1^* \), namely \( p_0 = \frac{3(a + kp_0) + \beta(g + l) - b(g + l + c)}{b(k + 1)} \) (obviously \( p_0 \neq c \)). In the case of \( p_0 > c \), when \( p_0 > p_1^* \), then \( k_1 > \frac{3a + \beta(g + l - c) + b(c - 4p_0)}{6p_0 - c} \). When \( p_0 \leq p_1^* \), then \( k_2 \leq \frac{3a + \beta(g + l - c) + b(c - 4p_0)}{6p_0 - c} \). Therefore, \( k_1 > k_2 \). If \( p_0 < c \), correspondingly, we can know \( k_1 < k_2 \).

From formula (5) and (6) we can obtain \( \frac{\partial \omega_1^*}{\partial k} = \frac{\beta(c - g - l) - (a - bp_0)}{2b(k + 1)^2} \), \( \frac{\partial \pi_1^*}{\partial k} = \frac{\beta(c - g - l)}{2b(k + 1)^2} \). If \( \beta(c - g - l) < (a - bp_0) \) and \( p_0 > c \), namely \( c < p_0 < a - \beta(c - g - l) \), then \( \frac{\partial \omega_1^*}{\partial k} < 0, \frac{\partial \pi_1^*}{\partial k} < 0 \); If \( \beta(c - g - l) < (a - bp_0) \) and \( p_0 < c \), namely \( p_0 < a - \beta(c - g - l) \), and \( p_0 < c \), we can get \( \frac{\partial \omega_1^*}{\partial k} < 0, \frac{\partial \pi_1^*}{\partial k} < 0 \). So whatever demand is loss-seeking or loss-averse, we can obtain \( \frac{\partial \omega_1^*}{\partial k} < 0, \frac{\partial \pi_1^*}{\partial k} < 0 \). 


Appendix C

Proof. Because the profit of the reverse supply chain is nonnegative, it is not rational to recycle the product otherwise; therefore, \( r_2[c - q - (1 - t)p_0] \geq 0 \), namely, \( g + (1 - t)p_0 \leq c \). We can obtain \( \frac{\partial \omega^*_2}{\partial \beta} = \frac{g + (1 - t)p_0 - c}{2b(k + 1)} \), \( \frac{\partial \pi^*_2}{\partial \beta} = \frac{g + (1 - t)p_0 - c}{4b(k + 1)} \) separately from formula (9) and (10), and \( \frac{\partial \pi^*_2}{\partial \beta} = \frac{[c - g - (1 - t)p_0][4b(k + 1)(1 - \beta)p_0 - 3(a + b)p_0 - c(\beta - b(k + 1))]}{8b(k + 1)} \) = \( \left[ c - g - (1 - t)p_0 \right] \left[ (1 - \beta)p_0 - p_2^* \right] \) separately from formula (12) and (11). Because \( c - g - (1 - t)p_0 > 0 \) and \( p_2^* > c > (1 - t)p_0 \), therefore \( \frac{\partial \omega^*_2}{\partial \beta} \leq 0, \frac{\partial p_2^*}{\partial \beta} \leq 0, \frac{\partial \pi^*_2}{\partial \beta} \geq 0 \), \( \frac{\partial \pi^*_2}{\partial \beta} \leq 0 \). □

Appendix D

Proof. When \( p_0 = p_2^* \), namely \( p_0 = \frac{3(a + b)p_0 + \beta g + (1 - t)p_0 - \beta b(k + 1))}{4b(k + 1)} \) (obviously \( p_0 \neq c \). (1) In the case of \( p_0 > c \), when \( p_0 > p_2^* \), then \( k_1 > \frac{\beta c - g - (1 - t)p_0 - (a - b)p_0)}{b(p_0 - c)} \). When \( p_0 \leq p_2^* \), then \( k_2 \leq \frac{3a + \beta g + (1 - t)p_0 - c}{b(p_0 - c)} \). Therefore, \( k_1 > k_2 \); If \( p_0 < c \), correspondingly, \( k_1 < k_2 \).

We can obtain \( \frac{\partial \omega^*_2}{\partial k} = \frac{\beta [c - g - (1 - t)p_0] - (a - b)p_0}{2b(k + 1)} \), \( \frac{\partial \pi^*_2}{\partial k} = \frac{\beta [c - g - (1 - t)p_0] - 3(a - b)p_0}{4b(k + 1)} \) separately from formula (9) and (10). If \( \beta [c - g - (1 - t)p_0] < (a - b)p_0 \) and \( p_0 > c \), namely, \( t > 1 - \frac{b}{\beta} \) and \( c < p_0 < \frac{a - \beta (c - g)}{b(1 - t)} \), then \( \frac{\partial \omega^*_2}{\partial k} < 0, \frac{\partial \pi^*_2}{\partial k} < 0 \); If \( t < 1 - \frac{b}{\beta} \), \( p_0 > \frac{a - \beta (c - g)}{b(1 - t)} \) and \( p_0 < c \), we can get \( \frac{\partial \omega^*_2}{\partial k} < 0, \frac{\partial \pi^*_2}{\partial k} < 0 \). Thus, whatever demand is loss-seeking or loss-averse, then \( \frac{\partial \omega^*_2}{\partial k} < 0, \frac{\partial \pi^*_2}{\partial k} < 0 \). □

Appendix E

Proof. We can obtain \( \frac{\partial \omega^*_2}{\partial t} = \frac{-\beta p_0}{2b(k + 1)}, \frac{\partial \pi^*_2}{\partial t} = \frac{-\beta p_0}{4b(k + 1)} \) separately from formula (9) and (10); therefore, \( \frac{\partial \omega^*_2}{\partial t} < 0, \frac{\partial \pi^*_2}{\partial t} < 0 \). From formula (12), we can obtain \( \frac{\partial \pi^*_2}{\partial t} = \frac{\partial p_0}{2b(k + 1)}d_2^* \geq 0 \). □

Appendix F

Proof. When \( l - (1 - t)p_0 \geq 0 \), that is \( 1 \geq t \geq 1 - \frac{1}{\beta}, \omega^*_1 - \omega^*_2 \geq 0; p_1^* - p_2^* \geq 0 \).\]

For retailers, with fixed rebate, \( d_1^* = \frac{a + bkp_0 - \beta g + (1 - t)p_0 + c[\beta - b(k + 1)]}{4} \geq 0 \), with variable rebate, \( d_2^* = \frac{a + bkp_0 - \beta g + (1 - t)p_0 + c[\beta - b(k + 1)]}{4} \geq 0 \); therefore, \( \pi^*_{R_1} - \pi^*_{R_2} = \frac{\beta [(1 - t)p_0 - l(d_1^* + d_2^*)]}{4b(k + 1)} \leq 0 \).

Now we prove that when \( l - (1 - t)p_0 \geq 0 \), \( \pi^*_{M_1} - \pi^*_{M_2} \leq 0 \). To simplify the expression, We set \( f(t) = \frac{\beta^2[2g + (1 - t)p_0] + 8b\beta(c - g)(k + 1) - 8b(k + 1)(a + \beta t + \beta(1 - t)p_0)}{8b(k + 1)} + \frac{6\beta(a + bkp_0) - 2b[\beta - b(k + 1)]}{8b(k + 1)} \), then \( \pi^*_{M_1} - \pi^*_{M_2} = [l - (1 - t)p_0]f(t) \).

If \( l - (1 - t)p_0 \geq 0 \), that is \( t \in [1 - \frac{L}{p_0}, 1] \). Take the first derivative of \( f(t) \) with respect to \( t \), and we can get \( \frac{df}{dt} = \frac{\beta \omega^*}{8b(k + 1)} \).

(1) When \( \beta < 8b(k + 1), \frac{df}{dt} > 0 \), \( f(t) \) is increasing monotonically. At this time, \( \pi^*_{M_1} - \pi^*_{M_2} = \frac{\beta [(1 - t)p_0 - l(d_1^* + d_2^*)]}{4b(k + 1)} \leq 0 \). Only if \( f(1) \leq 0 \), that is \( p_0 \leq \frac{8b(k + 1)(a - \beta - c - g + \beta(2b + 1) - 6a)}{6b(k + 1)^2} \), then \( f(t) \leq 0 \). Because \( l - (1 - t)p_0 \geq 0 \), therefore \( \pi^*_{M_1} - \pi^*_{M_2} = [l - (1 - t)p_0]f(t) \leq 0 \).
(2) When $\beta \geq 8b(k+1)$, $\frac{df(t)}{dt} < 0$, $f(t)$ is increasing monotonically. At this time, $f_{max}(t) = f(1 - \frac{t}{p_o}) = \frac{\beta(6(a+kbp_o)+2b(g+1)-2c[\beta-b(k+1)])}{8b(k+1)} + \beta(c-g) + (a+2\beta)l$.

Only if $f(1 - \frac{t}{p_o}) \leq 0$, that is $p_o \geq \frac{8b(k+1)[\alpha-\beta(c-g-2l)]+\beta(2c[\beta-b(k+1)]-2b(g+1)-6a)}{6kb^2}$, then $f(t) \leq 0$. Because $l-(1-t)p_o \geq 0$, therefore $\pi_{M_1}^* - \pi_{M_2}^* = [l-(1-t)p_o]f(t) \leq 0$.

Overall, when $l-(1-t)p_o \geq 0$, $\pi_{M_1}^* - \pi_{M_2}^* \leq 0$.

□

Appendix G

Proof. When $l-(1-t)p_o < 0$, that is $0 \leq t < 1 - \frac{1}{p_o}$, $\omega_1^* - \omega_2^* < 0$; $p_1^* - p_2^* < 0$. For retailers, with fixed rebate, $d_1^* = \frac{a+kbp_o-\beta(g+1)+c[\beta-b(k+1)]}{4}$, and with variable rebate, $d_2^* = \frac{a+kbp_o-\beta[g+(1-t)p_o]+c[\beta-b(k+1)]}{4} \geq 0, d_1^* + d_2^* = \frac{2(a+kbp_o)-\beta[2g+f(1-t)p_o]+\frac{2c[\beta-b(k+1)]}{4}}{4} \geq 0$; therefore, $\pi_{R_1}^* - \pi_{R_2}^* = \frac{\beta(l-\frac{l}{p_0}-(d_1^*+d_2^*))}{4b(k+1)} \geq 0$.

Now we prove that when $l-(1-t)p_o < 0$, $\pi_{M_1}^* - \pi_{M_2}^* \geq 0$. To simplify the expression. We set $f(t) = \frac{\beta^2(2g+f(1-t)p_o)+8b\beta(c-g)(k+1)-8b(k+1)[\alpha+\betaf(1-t)p_o]+\frac{6b\beta(a+kbp_o)+2b\beta[\beta-b(k+1)]}{8b(k+1)}[\beta]}{8b(k+1)}$, then $\pi_{M_1}^* - \pi_{M_2}^* = [l-(1-t)p_o]f(t)$.

If $l-(1-t)p_o < 0$, that is $t \in [0, 1 - \frac{1}{p_o})$. Take the first derivative of $f(t)$ with respect to $t$, and we can get $\frac{df(t)}{dt} = \frac{\beta(6(a+kbp_o)+2b(g+1)-2c[\beta-b(k+1)])}{8b(k+1)} + \beta(c-g) + (a+2\beta)l$.

(1) When $\beta < 8b(k+1)$, $\frac{df(t)}{dt} > 0$, $f(t)$ is increasing monotonically. At this time, $f_{max}(t) = f(1 - \frac{t}{p_o}) = \frac{\beta(6(a+kbp_o)+2b(g+1)-2c[\beta-b(k+1)])}{8b(k+1)} + \beta(c-g) + (a+2\beta)l$.

Only if $f(1 - \frac{t}{p_o}) \leq 0$, that is $p_o \leq \frac{8b(k+1)[\alpha-\beta(c-g-2l)]+\beta(2c[\beta-b(k+1)]-2b(g+1)-6a)}{6kb^2}$, then $f(t) \leq 0$. Because $l-(1-t)p_o \geq 0$, therefore $\pi_{M_1}^* - \pi_{M_2}^* = [l-(1-t)p_o]f(t) \geq 0$.

(2) When $\beta \geq 8b(k+1)$, $\frac{df(t)}{dt} < 0$, $f(t)$ is increasing monotonically. At this time, $f_{max}(t) = f(0) = \frac{\beta(6(a+kbp_o)+2b(g+1)+p_o)-2c[\beta-b(k+1)]}{8b(k+1)} + \beta(c-g) + (a+2\beta+\beta p_o)$.

Only if $f(0) \leq 0$, that is $p_o \leq \frac{8b(k+1)[\alpha-\beta(c-g-l)]+\beta(2c[\beta-b(k+1)]-2b(g+1)-6a)}{2b[\beta-2kb-8b]}$, then $f(t) \leq 0$. Because $l-(1-t)p_o \geq 0$, therefore $\pi_{M_1}^* - \pi_{M_2}^* = [l-(1-t)p_o]f(t) \geq 0$.

Overall, when $l-(1-t)p_o < 0$, $\pi_{M_1}^* - \pi_{M_2}^* \geq 0$.

References

[1] A. Atasu and G. C. Souza, How does product recovery affect quality choice?, Production and Operations Management, 22 (2013), 991–1010.

[2] A. Atasu, L. N. Van Wassenhove and M. Sarvary, Efficient take-back legislation, Production and Operations Management, 18 (2009), 243–258.

[3] R. Batarfi, M. Y. Jaber and S. M. Aljazzar, A profit maximization for a reverse logistics dual-channel supply chain with a return policy, Computer & Industrial Engineering, 106 (2017), 58–82.

[4] R. A. Briesch, L. Krishnamurthi, T. Mazumdar and S. P. Raj, A comparative analysis of reference price models, Journal of Consumer Research, 24 (1997), 202–214.

[5] K. Cao, Q. Bo and Y. He, Optimal trade-in and third-party collection authorization strategies under trade-in subsidy policy, Kybernetes, 47 (2018), 854–872.

[6] J. Chen and B. Chen, Competing with customer returns policies, International Journal of Production Research, 54 (2016), 2093–2107.

[7] X. Chen, K. Li, F. Wang and X. Li, Optimal production, pricing and government subsidy policies for a closed loop supply chain with uncertain returns, J. Ind. Manag. Optim., 16 (2020), 1389–1414.

[8] Y. Chien, The effect of a pro-rata rebate warranty on the age replacement policy with salvage value consideration, IEEE Transactions on Reliability, 59 (2010), 383–392.

[9] X. Chu, Q. Zhong and X. Li, Reverse channel selection decisions with a joint third-party recycler, International Journal of Production Research, 56 (2018), 5969–5981.
[10] P. De Giovanni, State- and control-dependent incentives in a closed-loop supply chain with dynamic returns, *Dyn. Games Appl.*, 6 (2016), 20–54.

[11] P. De Giovanni, Closed-loop supply chain coordination through incentives with asymmetric information, *Ann. Oper. Res.*, 253 (2017), 133–167.

[12] P. De Giovanni, P. V. Reddy and G. Zaccour, Incentive strategies for an optimal recovery program in a closed-loop supply chain, *European J. Oper. Res.*, 249 (2016), 605–617.

[13] P. De Giovanni and G. Zaccour, A two-period game of a closed-loop supply chain, *European J. Oper. Res.*, 232 (2014), 22–40.

[14] T. S. Genc and P. De Giovanni, Optimal return and rebate mechanism in a closed-loop supply chain game, *European J. Oper. Res.*, 269 (2018), 661–681.

[15] A. Goli, E. B. Tirkolaee and G.-W. Weber, A perishable product sustainable supply chain network design problem with lead time and customer satisfaction using a Hybrid Whale-Genetic algorithm, in Golinska-Dawson P (eds. Logistics Operations and Management for Recycling and Reuse). EcoProduction (Environmental Issues in Logistics and Manufacturing), Springer, Berlin, Heidelberg, (2020), 99–124.

[16] J. Gönisch, Buying used products for remanufacturing: negotiating or posted pricing, *Journal of Business Economics*, 84 (2014), 715–747.

[17] K. Govindan and M. N. Popiuc, Reverse supply chain coordination by revenue sharing contract: A case for the personal computers industry, *European J. Oper. Res.*, 233 (2014), 326–336.

[18] K. Govindan, H. Soleimani and D. Kannan, Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future, *European J. Oper. Res.*, 240 (2015), 603–626.

[19] V. D. R. Guide, R. H. Teunter and L. N. V. Wassenhove, Matching demand and supply to maximize profits from remanufacturing, *Manufacturing & Service Operations Management*, 5 (2003), 303–316.

[20] Q. He, N. Wang, Z. Yang, Z. He and B. Jiang, Competitive collection under channel inconvenience in closed-loop supply chain, *International Journal of Production Economics*, 183 (2017), 259–272.

[21] A. Mardani, D. Kannan, R. E. Hooker, S. Ozkul, M. Alrasheedi and E. B. Tirkolaee, Evaluation of green and sustainable supply chain management using structural equation modelling: A systematic review of the state of the art literature and recommendations for future research, *Journal of Cleaner Production*, 249 (2020), 119383.
[34] D. S. Putler, Incorporating reference price effects into a theory of consumer choice, *Marketing Science*, 11 (1992), 287–309.

[35] K. N. Rajendran and G. J. Tellis, Contextual and temporal components of reference price, *Journal of Marketing*, 58 (1994), 22–34.

[36] V. Ramani and P. De Giovanni, A two-period model of product cannibalization in an atypical closed-loop supply chain with endogenous returns: The case of DellReconnect, *European J. Oper. Res.*, 262 (2017), 1009–1027.

[37] S. Ray, T. Boyaci and N. Aras, Optimal prices and trade-in rebates for durable, remanufacturable products, *Manufacturing & Service Operations Management*, 7 (2005), 208–228.

[38] E. Rosch, Cognitive reference points, *Cognitive Psychology*, 7 (1975), 532–547.

[39] R. C. Savaskan, S. Bhattacharya and L. N. V. Wassenhove, Closed-loop supply chain models with product remanufacturing, *Management Science*, 50 (2004), 239–252.

[40] A. A. Taleizadeh and R. Sadeghi, Pricing strategies in the competitive reverse supply chains with traditional and e-channels: a game theoretic approach, *International Journal of Production Economics*, 215 (2018), 48–60.

[41] R. H. Teunter, Economic order quantities for stochastic discounted cost inventory systems with remanufacturing, *International Journal of Logistics Research & Applications*, 5 (2002), 161–175.

[42] E. B. Tirkolaee, P. Abbasion and G.-W. Weber, Sustainable fuzzy multi-trip location-routing problem for medical waste management during the COVID-19 outbreak, *Science of the Total Environment*, 756 (2021), 143607.

[43] E. B. Tirkolaee, A. Mardani, Z. Dashtian, M. Soltani and G.-W. Weber, A novel hybrid method using fuzzy decision making and multi-objective programming for sustainable-reliable supplier selection in two-echelon supply chain design, *Journal of Cleaner Production*, 250 (2020), 119517.

[44] J. N. Uhl and H. L. Brown, Consumer perception of experimental retail food price changes, *Journal of Consumer Affairs*, 5 (1971), 174–185.

[45] N. Wang, Q. He and B. Jiang, Hybrid closed-loop supply chains with competition in recycling and product markets, *International Journal of Production Economics*, 217 (2019), 246–258.

[46] W. Wang, P. Zhang, J. Ding, J. Li, H. Sun and L. He, Closed-loop supply chain network equilibrium model with retailer-collection under legislation, *J. Ind. Manag. Optim.*, 15 (2019), 199–219.

[47] J. Wei, Y. Wang, J. Zhao and E. D. R. Santibanez Gonzalez, Analyzing the performance of a two-period remanufacturing supply chain with dual collecting channels, *Computer & Industrial Engineering*, 135 (2019), 1188–1202.

[48] Z. Wu, X. Qian, M. Huang, W.-K. Ching, H. Kuang and X. Wang, Channel leadership and recycling channel in closed-loop supply chain: The case of recycling price by the recycling party, *Journal of Industrial and Management Optimization*, (2020).

[49] X. Wu and Y. Zhou, The optimal reverse channel choice under supply chain competition, *European J. Oper. Res.*, 259 (2017), 63–66.

[50] Y. Xiao, Choosing the right exchange-old-for-new programs for durable goods with a rollover, *European J. Oper. Res.*, 259 (2017), 512–526.

[51] C.-T. Zhang and M.-L. Ren, Closed-loop supply chain coordination strategy for the remanufacture of patented products under competitive demand, *Appl. Math. Model.*, 40 (2016), 6243–6255.

[52] S. Zhao and Q. Zhu, Remanufacturing supply chain coordination under the stochastic remanufacturability rate and the random demand, *Ann. Oper. Res.*, 257 (2017), 661–695.

[53] B. Zheng, C. Yang, J. Yang and M. Zhang, Dual-channel closed loop supply chains: Forward channel competition, power structures and coordination, *International Journal of Production Research*, 55 (2017), 3510–3527.

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