Geometric-dissipative control of exothermic continuous reactors

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Abstract: The problem of robustly stabilizing through output-feedback control an open-loop unstable exothermic continuous reactor with temperature measurement is addressed. The combination of advanced geometric control and classical mechanics methods yields: (i) solvability of the detailed model-based nonlinear output-feedback problem in terms of passivity, observability and dissipativity, (ii) open-loop energy and closed-loop Lyapunov functions in analytic form, and (iii) the tradeoff between response speed, robustness, and control effort. On the basis of simplified model tailored according to the passivity-observability-dissipativity structure, the advanced geometric observer-based nonlinear controller is realized with an industrial-like PID controller. The methodology is illustrated with numerical simulations.

Keywords: Chemical reactor, output-feedback control, PID control, Lyapunov function, geometric control, passive control, dissipative control, mechanical control.

1. INTRODUCTION

Amundson and Aris (AA) studied the open loop (OL) dynamics and the closed-loop (CL) behavior with linear P, PI and PID control of a two-state continuous exothermic jacketed single-component chemical reactor (Aris and Amundson; 1959a-c), by combining reactor engineering and global dynamics insight, local stability analysis with analytic formula, and global dynamics with analog simulation. The reactor has a rather reach, simple and complex, dynamical nonlinear behavior over its parameter space (Van Heerden, 1953; Uppal et al, 1974), and has become a benchmark for advanced and conventional control development. The related literature is abundant and disperse (in few reactor per se studies and in many ones with the reactor as application example of general-purpose control designs), and here it suffices to mention that, in spite of significant advances with valuable insight, there are still open problems, among them are two addressed in the present study: (i) the first principle-based construction of control Lyapunov functions, and (ii) the rigorous connection between advanced nonlinear and industrial linear control approaches.

Here, the problem of stabilizing with temperature-driven output-feedback (OF) control AA’s reactor at an unstable nonunique steady-state (SS) is addressed, with emphasis on: (i) the tradeoff between response speed, robustness, and control effort, and (ii) the formal connection between advanced nonlinear (passive and dissipative) and industrial (PID) control.

The points of departure are: (i) the nonlocal saturated robust nonlinear state-feedback (NLSF) stabilizing control (Alvarez et al, 1991; El-Farra and Christofides,2003), (ii) the observer-based (Alvarez and Fernández, 2009) geometric passive NLOF geometric control and its connection with linear PI control (Alvarez Ramirez et al, 2002; Gonzalez and Alvarez, 2005; Schaum et al., 2015), (iii) global motion observability (Diaz-Salgado et al. 2012, Moreno and Alvarez, 2015), (iv) classic mechanics (Corben and Stehle, 1960), and (v) thermodynamics-based constructions of Lyapunov functions (Ydstie, 2002; Favache et al, 2011) for control design via dissipation for abstract (Solis-Down, 2013) and electro-mechanical systems (Ortega et al, 2002).

First, the geometric-passive NLSF control problem is solved, with CL globally-robustly stable dynamics accompanied by a Lyapunov function in analytic form. Then, classic dynamics and nonlinear observer theory are applied to solve the nonlocal robust OF stabilization problem with nonlinear and linear PID control schemes, including: (i) solvability, (ii) systematic construction with reduced model dependency, and (iii) simple tuning.

2. CONTROL PROBLEM

Consider a (possibly OL unstable) single-reactant exothermic continuous reactor, where a reactant is converted into product via a first-order reaction rate (ca) with Arrhenius temperature dependency (α), according to the dimensionless dynamic mass and heat balances (Aris, 1969)

\[ \dot{c} = \theta c_a - \lambda_c(\tau, \theta) c, \quad c(0) = c_0 \]  \hspace{1cm} (1a)
\[ \dot{\tau} = \alpha_a(\tau) - \eta(\tau, \theta, \tau_e, \tau_c), \quad \tau(0) = \tau_o \]  \hspace{1cm} (1b)
\[ \tau = m e + r, \quad y = r. \]  \hspace{1cm} (1c)

where \( \alpha_a(\tau) = \exp(a_a - \epsilon_a / \tau), \lambda_c(\tau, \theta) = \theta + \alpha(\tau) \eta(\tau, \theta, \tau_e, \tau_c) = \lambda_c(\theta) \tau - (\theta \tau_e + v \tau_c), \lambda_c(\theta) = \theta + v \)

\( c \) (or \( \tau \)) is the concentration (or temperature) state, \( v \) is the heat transfer number, \( \theta \) is the measured volumetric flow rate input, \( a_a \) is frequency factor-Damkohler number product, \( \epsilon_a \)
is the activation energy, \( y \) is the measured output temperature, \( \tau_r \) (or \( \tau_a \)) is the measured (or unmeasured) feed temperature (or concentration), the coolant temperature \( \tau_c \) is the control input, and \( z \) is the regulated output. In compact vector notation, reactor (1) is written as

\[
\dot{x} = f(x, y, d, u), \quad x(0) = x_0, \quad y = c_y x, \quad z = c_z x
\]  

(2)

where \( x = (c, r)^T \), \( w = c_w \), \( d = (\theta, \tau_c)^T \)

\[
u = \tau_c, \quad c_y = (0 \, 1)^T, \quad c_z = (m \, 1)^T
\]

\( x \) is the state, \( d \) (or \( w \)) is the measured (or unmeasured) input, and \( y \) (or \( z \)) is the measured (or regulated) output, and the constant \( m \) (1c) is an adjustable parameter for control design. For nominal input \( (\tilde{w}, \tilde{d}, \tilde{u}) \), the reactor statics, with \( n_e \) Ss \( \tilde{x} \), are

\[
f(\tilde{x}, \tilde{w}, \tilde{d}, \tilde{u}) = 0, \quad \tilde{y}_i = c_y \tilde{x}_i, \quad \tilde{z}_i = c_z \tilde{x}_i, \quad i = 1, \ldots, n_e \geq 1
\]

Depending on its parameters, the OL reactor (2) has simple or complex nonlinear dynamics (Uppal et al., 1974). To the mass-heat reactor balance (2) we will refer to as Cartesian dynamics.

The problem is to design a dynamic OF stabilizing controller

\[
\dot{\xi} = g(\xi, d, y), \quad \xi(0) = \xi_0, \quad u = \mu_y(\xi, d, y)
\]  

(3)

so that the CL reactor is robustly stable about its (possibly OL unstable) nominal SS \( \bar{x} \), with: (i) solvability conditions, and (ii) simple construction (with model dependency, nonlinearity and coupling as small as possible) and tuning. We are interested in identifying the trade-off between response speed, robustness and control effort, and rigorously connecting advanced nonlinear (passive and dissipative) and industrial (PID) control. AA’s reactor (Aris, 1969) is chosen as case example, with a stable focus \( \bar{x}_f \) (or node \( \bar{x}_n \)), and an unstable SS (saddle \( \bar{x} \)):

\[
\tilde{\theta} = \tilde{\tau}_s = v = 1, \quad \tilde{\tau}_c = \tilde{c}_e = 7/4, \quad a_a = 25, \quad \varepsilon_a = 50
\]  

(4)

\[
\bar{x_f} \approx (0.0892, 2.206)^T, \quad \bar{x_n} \approx (0.964, 1.768)^T
\]  

(5)

3. OPEN-LOOP DYNAMICS

Here the global NL OL reactor dynamics are characterized with notions and tools from nonlinear dynamics and classical mechanics.

3.1 Cartesian dynamics

The reactor Ss are in the line set \( L \), which is inscribed in the trapezoidal invariant set \( X \) (Alvarez et al., 1991; Alvarez et al., 2015), i.e.,

\[
\bar{x}_{1,1-n_e} \in L = \{x \in \mathbb{R}^2 | 0 \leq x_1 \leq c_x, x_2 = \tau_f - m_l x_1 \} \quad (6a)
\]

\[
L \subset X = \{x \in \mathbb{R}^2 | 0 \leq x_1 \leq c_x, \tau_i^c \leq \tau \leq \tau^t(x_i) \} \subset \mathbb{R}^2 \quad (6b)
\]

where

\[
m_l = 1/(1 + v), \quad \tau_c^e = (\tau_e + v \tau_c)/(1 + v),
\]

\[
\tau_i^c = \tau_c^e + m_l \tau_c, \quad \tau^t(x_i) = \tau_i^c + m_l (c_x - x_1)
\]

\( X \) is an invariant set in the sense that any state motion \( x(t) \) born in \( X \) stays in \( X \) (Hubbard and West, 1995).

The reactor example (4) has two invariant sets contained in \( X \) (6b) (see Fig. 1): (i) the separatrix curve \( S_0 \) that contains the saddle \( \bar{x} \) and divides the basins of attraction \( X_f^p \) and \( X_f^n \) of the stable focus \( \bar{x}_f \) and node \( \bar{x}_n \), respectively, and (ii) the curve \( S_a \) that connects the three Ss. The curve \( S_a \) (or \( S_b \)) is an attractive (or repulsive set) with respect to \( X \). The geometry of the OL dynamics of the reactor example (4) is presented in Fig. 1, including: (i) the rhabdoid (yellow) set where the invariants curves \( S_1 \) (or \( S_0 \)) are reasonably close to their tangent line sets \( L_1 \) (or \( L_0 \)) at the unstable saddle SS \( \bar{x} \), and (ii) the rhabdoid (with slashed boundary) where the reactor is expected to operate. These observations suggest that the \( m \)-parametric lines about \( L_1 \) as set of regulated outputs (1c).

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Fig. 1. Geometry of the OL Cartesian reactor dynamics.

Due to Bendixon-Poincaré’s theorem (Hubbard and West, 1995), each motion \( x(t) \) reaches asymptotically (with characteristic time \( \tau_k \)) either a SS point \( \bar{x} \) or a closed orbit \( \bar{x}(t) \) (Alvarez et al., 1991), i.e.,

\[
x_a \in X \Rightarrow x(t) \in X, \quad x(t) \rightarrow \bar{x} \text{ or } \bar{x}(t) \in X
\]  

(7)

By virtue of Lyapunov’s converse and La Salle’s invariance theorems (La Salle and Lefschetz, 1960), there exists an “abstract” energy function \( e(x) \) with dissipation function \( \Delta(x) \):

\[
e = e(x) \geq 0, \quad \dot{e} = \Delta(x) \leq 0 \forall x \in X, \Delta(x) = 0 \forall x \in N \quad (8)
\]

\[
\Delta(x) = d_f(x, \bar{x}, \bar{f}, \bar{u}) e(x), \quad N = \{x | \Delta(x) = 0\} \supset \bar{L}
\]

where \( d_f \) is the directional derivative of \( e \) along \( f \), and the union of limit sets \( L \) is the largest invariant set contained in the null dissipation set \( N \).

When reactor (2) (Alvarez et al., 2015) is monostable with global attractor \( \bar{x} \), \( e = e(x) \) is a single-well (Lyapunov) function with global minimum at \( \bar{x} \). When reactor (2) is bistable, as in example (5), \( e = e(x) \) is a two-well surface with local minima at \( \bar{x}_f \) and \( \bar{x}_n \), a saddle (in between) at \( \bar{x} \), and global minimum at the strongest attractor \( (\bar{x}_f \text{ or } \bar{x}_n) \). When reactor (2) has a limit cycle with unstable focus \( \bar{x}_f \) and orbit curve \( \bar{c}_1 \), \( e = e(x) \) is a sombrero surface with global minimum set (or maximum point) at \( \bar{c}_1 \) (or \( \bar{x}_f \)).

3.2 Newtonian dynamics

Denote by

\[
p = \tau, \quad v = \dot{\tau}, \quad a = \ddot{\tau} = \dddot{\tau}
\]  

(9)

the reactor temperature “position” \( p \), “velocity” \( v \) and “acceleration” \( a \), and apply the coordinate change

\[
p = \tau = \sigma_p(\bar{\tau}), \quad v = \sigma_v(c, \tau, d, u)
\]  

(10a)
\[ \sigma_v(c, \tau, d, u) := c \alpha(\tau) - \eta(p, \theta, \tau, u) \]  
(10b)
to write the reactor dynamics (1) in Newtonian form  
\[ \dot{p} = v, \quad p(0) = p_0, \quad y = p, \quad c = \sigma_v(p, v, d, u) \]  
(11a)  
\[ \dot{v} = -\phi(p) - v(p, v)v + v[\dot{u} + \lambda_u(p, v, \theta)\bar{u}] \]  
(11b)  
\[ + \dot{\phi}(p, v, \dot{d}; \bar{d}, \omega), \quad v(0) = v_0 \]  
(11c)
\[ \bar{u} = u - \bar{u}, \quad \bar{w} = w - \omega, \quad \bar{d} = d - d \]  
\[ \dot{\phi}(p, v, \dot{d}; \bar{d}, \omega) \]  
In Fig. 3 is plotted the set \( \mathcal{H}_1 \) (in yellow) of control-induced robustly attractive (or non-attractive) line sets.

The enforcement of the CL output regulation dynamics  
\[ \dot{z} = -k(z - \bar{z}), \quad z(0) = z_0, \quad k > 0 \]  
(12a)
on the reactor (2) yields the NLSF geometric controller  
\[ \mu(c, \tau, d, w) = \lambda_1(\theta)\tau - \theta \tau - \lambda_u(\theta) \]  
(16)
\[ -k(m \tau + z - \bar{z}) - m[\theta \omega - \lambda(\theta, \tau)c] / v \]  
By construction, the \( m \)-parametric invariant line set  
\[ \bar{L} = \{ x \in \mathbb{R}^2 | m \tau + z \leq \bar{z}, 0 \leq \alpha \} \subset X, \quad \bar{X} \in \mathcal{L} \]  
(17)
contains the OL unstable SS \( \bar{x} \) (18a), is invariant (18b) with restricted stable ZD (13) (18c), and is an asymptotic attractor (18d) for the trapezoidal set \( X \) (6b), i.e.,

\[ \bar{x} \in \mathcal{L} \subset X, \quad x_o \in \mathcal{L} \Rightarrow x(t) \in \mathcal{L} \]  
(18a-b)
\[ \bar{x} \in \mathcal{L}, \quad x_o \neq \bar{x} \Rightarrow x(t) \rightarrow \bar{x} \]  
(18c)
\[ x_o < \bar{x}, \quad x_o \in \mathcal{L} \Rightarrow x(t) \in X, \quad x(t) \rightarrow \bar{x} \]  
(18d)
The application of control (16) to reactor (2) yields the CL system  
\[ \dot{x} = f(x, w, d, \mu(x, d, w)), \quad x(0) = x_o, \quad y = \gamma(x, z = c, x) (19) \]  
By virtue from Seibert's reduction principle (El-Hawwary, 2010), this system is globally robustly stable with respect to \( \bar{x} \) provided the controller is adequately saturated (Alvarez et al, 1991).

\[ \sigma_\alpha(c, \tau, \alpha, d, u) := c \alpha(\tau) - \eta(p, \theta, \tau, u) \]  
(10b)  

4.1 State-feedback control

Since \( v > 0 \), reactor (2) has relative degree one with respect to the input-output pair \( (x, z) \) for each parameter \( m \in \mathbb{R} \) of the \( z \)-output map (4c). The corresponding zero-dynamics (ZD) are (Alvarez et al, 1991)

\[ \dot{c} = -\lambda_z(c, m), \quad c(0) = c_0, \quad \tau = \bar{z} - mc \]  
(13)
where \( \lambda_z(c, m) = \lambda_z(\bar{z} - mc, \theta) \).

The set of output maps with respect to which the reactor is globally \( (u, z) \)-passive is given by (Alvarez et al, 1991)

\[ \mathcal{H}_f = \{ z = mc + \tau \mid \partial_z \lambda_z(c, m) > 0 \forall c \in [0, c_e] \} \]  
(14)
The local version (about the unstable saddle \( \bar{x} \)) of this set is

\[ \mathcal{H}_f = \{ z = mc + \tau \mid -\infty < m < m^* \} \subset \mathcal{H}_f, m_m < m^* < m_m \]  
(15)
with \( m^* \) being the slope of the line set \( \mathcal{L}^* \) where the local ZD (13) about \( \bar{x} \) undergoes transcritical bifurcation over \( m \).

Thus, locally (about \( \bar{x} \)) the ZD (13): (i) are stable (or unstable) for \( m < \) (or >) \( m^* \), and (ii) marginally stable at \( m = m^* \). The bifurcation line \( \mathcal{L}^* \) is below (or above) the eigensubspaces \( L_m \) (or \( L_m \)) with slope \( -m_u \) (or \( -m_s \)) at \( \bar{x} \).
motion $x(t)$ is instantaneously $(x, w)$-observable (Alvarez and Fernandez, 2009; Moreno and Alvarez, 2015).

The related robust convergent NL observer (22a-b) (Alvarez and Fernandez, 2009), and its combination with the NLSF controller (16) yields the NLOF geometric controller

$$
\dot{\delta}_i = \Gamma \delta_i + \kappa_d (d_i - c_d \delta_i), \quad \delta_i(0) = \delta_{i,0}, \quad i = \theta, \tau_e \tag{22a}
$$

$$
x_a = f_a(x_a, d, u) + g(x, d, u, \theta)(y - \hat{x}_2), \quad \hat{x}_a(0) = \tilde{x}_{a,0} \tag{22b}
$$

$$
u = \mu(\hat{x}_a, d, w) \tag{22c}
$$

with $g$, $k_d$ and $\Gamma$, defined in Appendix A. The CL robust nonlocal stability follows from the small-gain theorem (Gonzalez and Alvarez, 2005). This controller with $m = 0$ yields better functioning that its EKF implementation (Diaz-Salgado et al., 2012). The solvability of the NLOF geometric control (22) problem is due to the global $(u, z)$-passivity (14) and $(x, w)$-observability (20) properties.

The NLOF controller (22) is too complex for practical implementation: (i) 7 ODEs (4 linear and three nonlinear ones), (ii) dependency on the reactor model (2), and (iii) feed rate and temperature measurements are needed. The removal of this obstacle is the subject of the next section.

5. NEWTONIAN CONTROL

Here the robust stabilizing NLNSF geometric controller (22) is redesigned with a simplified model built according to passivity and observability properties.

5.1 Prescribed nominal closed-loop dynamics

Recall the nominal CL stable dynamics (19), and apply the coordinate change (10) to obtain the prescribed nominal CL stable Newtonian dynamics

$$
\dot{p} = v, p(0) = p_a; \quad \dot{v} = -\phi_g(p) - v_g(p, v)u, \quad v(0) = v_a \tag{23}
$$

with restoration-dissipation mechanism

$$
e = e_g(p, v) \geq 0, \quad \dot{e} = -v_g(p, v)v^2 \leq 0 \tag{24a-b}
$$

$$
\dot{e}_g(p, v) = v^2/2 + \pi_g(p), \quad \pi_g(p) = \int_0^p \phi_g(r)dr \tag{24a-b}
$$

$$
\phi_g(p) = k[p - p_\lambda \lambda_s p_\lambda] - m[p - \lambda_s] + \phi_\lambda(p) > 0 \tag{24a-b}
$$

$$
v_g(p, v) = (k + \lambda_s p_\lambda)(v - \epsilon_g p_\lambda + \phi_\lambda(p, v, \theta, u, m)) > 0 \tag{24a-b}
$$

$$
[k - \lambda_s p_\lambda \lambda_s p_\lambda] + \phi_\lambda(p, v, \theta, u, m) = \frac{1}{2}(v + k(p - \tilde{v})) \tag{24a-b}
$$

The corresponding CL single-well energy (Lyapunov) function for the example (4) is presented in Fig. 2b, for $(k, m) = (5, m_s = 0.2)$.

5.2 State-feedback control

The enforcement of the prescribed CL nominal dynamics (23) on the OL perturbed ones (11) yields the stabilizing dynamic NLNSF Newtonian controller

$$
\dot{\delta}_i = -\lambda_u (p, v, \theta)u + \mu_d (p, v, \hat{d}, \hat{w}), \quad \dot{\delta}_i(0) = \delta_{i,0} \tag{25a}
$$

$$
u = \ddot{u} + \tilde{u}, \quad \lambda_u (p, v, \theta) > 0 \tag{25b}
$$

which is a 1st-order linear lag driven by a nonlinear PI controller $\mu_n$ with FF $(\mu_{ff})$ and FB $(\mu_{fb})$ stabilizing components:

$$
\mu_n(p, v, \hat{d}, \hat{w}) = \mu_{ff}(p, v, \hat{d}, \hat{w}) + \mu_{fb}(p, v) \tag{26a}
$$

$$
\mu_{ff}(p, v, \hat{d}, \hat{w}) = -\hat{\phi}(p, v, \hat{d}, \hat{w})/v \tag{26b}
$$

$$
\mu_{fb}(p, v, \hat{d}, \hat{w}) = k_p (p) - k_d (p, v, v), \quad \mu_{fb}(p, v) = 0 \tag{26c}
$$

$$
k_p (p) = \phi_g(p) - \phi_\lambda(p), \quad k_p (p, v, v) = 0 \tag{26d}
$$

$$
k_d (p, v, v) = v_g(p, v) - v(p, v)/v \tag{26e}
$$

with $k_p$ (or $k_d$) being the proportional (or differential) gain function.

5.3 Simplified model

Following (Alvarez-Ramirez et al., 2002; Gonzalez and Alvarez; 2005), express the OL dynamics (11) in $t$-parametric form

$$
p = v, \quad \dot{p} = v, \quad \dot{v} = -\ddot{v} - \hat{\phi}'\hat{p} + v(\hat{u} + \tilde{u}) + \tilde{v} \tag{27a}
$$

$$
v(0) = v_a, \quad y = p; \quad i = \tilde{f}(p, v, d, \hat{d}, \hat{w}, \tilde{u}) \tag{27b-c}
$$

where $f_\tilde{t}$ is defined in Appendix A. The unique solution for $(v, i)$ of (27a-b) followed by the substitution of $(v, i) = (\hat{y}, \tilde{y})$ yields

$$
v = \hat{y}, \quad \dot{i} = \tilde{y} + \ddot{y} - \hat{\phi}'\hat{p} - v(u + \tilde{u}) \tag{28}
$$

meaning that the unmeasured state-input pair $(v, i)$ is instantaneously observable from the measured-known signal $(y, u)$. Consequently, $(v, i)$ can be quickly reconstructed (up to measurement error) with an observer driven by $(y, u)$.

Recall the (actual) $t$-parametric dynamics (27), drop the nonlinear state component (27c), enforce the slow-varying assumption ($\lambda_u$ is the inverse of the reactor characteristic time, and $\lambda_{w}$ is the convergence rate of an observer to be designed)

$$
i \approx \tilde{v}, \quad |i| = \lambda_i \ll \lambda_s \ll \lambda_{w}, \quad \lambda_o \approx n_w \lambda_x \tag{29}
$$

for the input $i$, and obtain the linear, second-order, $(v, l)$-observable model for OF control design

$$
p = v, \quad \dot{p} = v, \quad \dot{v} = -\ddot{v} - \hat{\phi}'\hat{p} + v(\hat{u} + \tilde{u}) + \tilde{v} \tag{30a-b}
$$

$$
v(0) = v_a, \quad y = p; \quad i \approx \tilde{v}, \quad \tilde{v} \approx 0 \tag{30c-d}
$$

In terms of the reconstructible load input $\tilde{u}$, the exact model-based NLNSF Newtonian controller (25) is written as follows

$$
\dot{u} = -\tilde{u} - \tilde{u} + \tilde{u} + \tilde{u} \tag{31a-c}
$$

This controller consists of a first-order linear lag driven by the reconstructible output $\tilde{u}$ and a nonlinear PD controller with proportional (or differential) gain function $k_p$ (or $k_d$):

$$
\mu_{pd}(p, v) = -k_p (p) - k_d (p, v), \quad \mu_{pd}(p, v) = 0 \tag{32a}
$$

$$
k_p (p) = \phi_g(p) - \phi_\lambda(p), \quad k_p (p) = 0 \tag{32b}
$$

$$
k_d (p, v) = v_g(p, v) - \ddot{v}/v, \quad k_d (p, v) = \tilde{y}_g > 0 \tag{32c}
$$

5.4 Nonlinear output feedback controller

On the basis of the linear $(v, i)$-observable model (28), set the (improper) observer

$$
\dot{\hat{v}} = -\ddot{v} - \hat{\phi}'\hat{p} + v(\hat{u} + \tilde{u}) + \ddot{v} \tag{33a}
$$

and apply the coordinate change

$$
\chi_v = \hat{v} - k_v \hat{v}, \quad \chi_i = \dot{\hat{v}} \tag{33b}
$$

5.5 Output feedback control

$$
\chi_v = \hat{v} - k_v \hat{v}, \quad \chi_i = \dot{\hat{v}} \tag{33c}
$$

5.6 Disturbance observer

$$
\chi_v = \hat{v} - k_v \hat{v}, \quad \chi_i = \dot{\hat{v}} \tag{33d}
$$

5.7 Robustness of controller

$$
\chi_v = \hat{v} - k_v \hat{v}, \quad \chi_i = \dot{\hat{v}} \tag{33e}
$$
to obtain the proper \((v,\tau,\omega)\)-observer \((33a-c)\). The combination of the NLSF controller \((31)\) with the velocity-load observer \((33a-c)\) yields the *dynamic OF controller*

\[
\begin{align*}
\dot{x}_v &= -\lambda_x x_v + x_t - b_y v - b_v u, \\
\dot{x}_o &= -k_x x_v - b_y \dot{y} - b_u \dot{u}, \\
\dot{y} &= x_o + k_y \dot{y} + u, \\
\dot{u} &= -\dot{\lambda}_u \dot{u} - \dot{\tau}/\omega + \mu_{pd} (y + \ddot{y}, \dot{v}), \\
\dot{\omega} &= \dot{v} + k_x x_v - k_o \dot{u} > 0, \\
\dot{b}_v &= \dot{v} + k_y \dot{v} > 0, \\
\dot{b}_u &= \dot{v} (\dot{\lambda}_v - \dot{\lambda}_u) > 0.
\end{align*}
\]

\(\mu_{pd}\) is a nonlinear PD controller, \(P\) is the set of approximations of the parameters of the Taylor linearizations of the OL \((11)\) and ZD \((14)\) dynamics, and \(K\) is a four-parameter tuning set. The observer \((33a-c)\) has only two linear filters \((33a-c)\). The first-order lag driven by the FB non-\(\mu_{pd}\) (33a-c) ensures CL stability with efficient control effort. That the FF-FB combination is the best way to control difficult processes is a well-known fact in industrial practice (Shinskey, 1988).

5.6 Linear output feedback controller

Choose the linearization of the nonlinear Newtonian dynamics \((24)\) as prescribed dynamics:

\[
\begin{align*}
\dot{p} &= v, \quad p(0) = p_o, \\
\dot{v} &= -\ddot{v}_g v - \dddot{v}_g \dot{p}, \quad v(0) = v_o \quad (35a) \\
\dddot{v}_g &> 0, \quad \dddot{v}_g > 0 
\end{align*}
\]

to obtain the linear OF controller \((33)\) with linear PD FB controller

\[
\mu_{pd}(p, v) = -k_p \dot{y} - k_d v, \quad k_p = \dddot{v}_g - \dddot{v}_o, \quad k_d = \dot{v}_g - \dddot{v}_o 
\]

Controller \((33,38)\) is an upgraded version of an industrial linear PID (primary) temperature controller for exothermic reactors, with the upgrade being: (i) guarantee of nonlocal CL robust stability with enhanced disturbance rejection capability, and (ii) systematic construction and simple tuning.

6. OUTPUT-FEEDBACK CONTROL FUNCTIONING

6.1 Nonlinear control
7. CONCLUSIONS
The output-feedback robust stabilization problem for an open-loop unstable reactor has been resolved by combining geometric control and classic mechanics. Passivity and observability solvability conditions were identified and exploited to tailor models for simplified OF control design.

Geometric control provided the abstract detailed model-solution within a global-nonlinear robust stability framework. Classic mechanics enabled: (i) the derivation of PID control realization, and (ii) the characterization of the underlying restoration-dissipation mechanism in terms of analytic first principle-based open-loop energy and close-loop Lyapunov functions.

APPENDIX A: FUNCTIONS AND PARAMETERS

A.1 Newtonian Dynamics (11)
\[ \phi(p) = \varphi_n(p, d, w, \bar{u}), \quad v(p, v) = v_n(p, v, d, \bar{u}) \]
\[ \phi(\bar{r}_i) = 0, \quad i = 1, \ldots, n_\alpha \]
\[ \varphi_n(p, d, u, w) = \lambda_p(p, \theta) + \lambda_u(p, w) + (\epsilon/p^2) \eta(p, \theta, r, u) - \omega(p) \]
\[ \lambda_u(p, w, \theta) = \lambda_w(p, \theta) - \epsilon/v/p^2 > 0 \]
\[ \phi(p, v, \bar{d}, \bar{w}) = -\lambda(p, v, \bar{d}) - \varphi_n(p, d, w) \]
\[ + \theta r_e - (p - r_e) \lambda \]
\[ \bar{v}_n(p, v, \bar{d}) = v_n(p, v, d, u) - v(p, v) \]
\[ \bar{\phi}_n(p, d, w) = \varphi_n(p, d, u, w) - \varphi(p) \]

A.2 Geometric Observer (22)
\[ g(x, d, \dot{\theta}) = [\partial_x \sigma(x, d, \dot{d})]^{-1} k_o, \quad k_o = (d\zeta \omega, d\zeta \omega^2, \omega^3) \]
\[ d = 1 + 2\zeta, \quad e_d = (1, 0), \quad \Gamma = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad k_d = (2\zeta \omega a_d, \omega^2 a_d)^T \]

A.3 t-parametric dynamics (27)
\[ f_p(p, v, \bar{d}, \bar{w}, \bar{u}) = \dot{\phi}(p, v, \bar{d}, \bar{w}) + \phi(p) + v(p, v) \]
\[ + v\lambda(p, v, \theta) \bar{u}, \quad \dot{\phi}(p) = \varphi(p) - \phi(p) \]
\[ \bar{v}(p, v) = v(p, v) - \bar{v}, \quad \bar{\lambda}_u(p, v, \theta) = \lambda_u(p, v, \theta) - \lambda_u \]

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