Jupiter and Super-Earth embedded in a gaseous disc

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ABSTRACT

In this paper we investigate the evolution of a pair of interacting planets - a Jupiter mass planet and a Super-Earth with the mass $5.5 M_\oplus$ - orbiting a Solar type star and embedded in a gaseous protoplanetary disc. We focus on the effects of type I and II orbital migrations, caused by the planet-disc interaction, leading to the Super-Earth capture in first order mean motion resonances by the Jupiter. The stability of the resulting resonant system in which the Super-Earth is on the internal orbit relatively to the Jupiter has been studied numerically by means of full 2D hydrodynamical simulations. Our main motivation is to determine the Super-Earth behaviour in the presence of the gas giant in the system. It has been found that the Jupiter captures the Super-Earth into the interior 3:2 or 4:3 mean motion resonances and the stability of such configurations depends on the initial planet positions and eccentricity evolution. If the initial separation of planet orbits is larger or close to that required for the exact resonance than the final outcome is the migration of the pair of planets with the rate similar to that of the gas giant at least for time of our simulations. Otherwise we observe a scattering of the Super-Earth from the disc. The evolution of planets immersed in the gaseous disc has been compared with their behaviour in the case of the classical three-body problem when the disc is absent.

Key words: planets formation, numerical simulation, protoplanetary discs, migration, mean motion resonances

1 INTRODUCTION

More than ten years have passed since the discovery of first extrasolar low mass planets (with masses 0.02, 4.3 and $3.9 M_\oplus$) around pulsar PSR B1257+12 (Wolszczan & Frail 1992) and the first hot Jupiter orbiting 51 Pegasi, a Solar-type star (Mayor & Queloz 1995). Since then, more than two hundred gas giants have been found around main sequence stars and only four low (less massive than $10 M_\oplus$) mass planets. The first among these four, with a mass $7.5 M_\oplus$ orbiting the nearby star Gliese 876 has been discovered by the high-precision radial velocity monitoring at the W. M. Keck Observatory in Hawaii (Rivera et al. 2005). The second one, OGLE-2005-BLG-390Lb with a mass of $5.5 M_\oplus$, has been found by using the gravitational microlensing technique by OGLE, PLANETS and MOA collaborations (Beaulieu et al. 2006). Finally the last two, with masses about 5 and $8 M_\oplus$, has been observed around star Gliese 581 (Udry et al. 2007) thanks to HARPS (High Accuracy Radial Velocity for Planetary Searcher), a very precise spectograph located on the ESO telescope at La Silla in Chile. All three stars harboring low mass planets are M dwarfs. The planets in this mass range are often called Super-Earths and we will adopt this name here. These few examples are just a preview of what is to come soon. The microlensing and the high-precision radial-velocity planet-search programmes can bring new discoveries at any time. Even a network of small ground-based telescopes is able to increase the number of known Earth-like planets using the Transit Timing Variation (TTV) method. The French-European mission COROT, which is capable to detect low mass planets, is already operating and collecting data. In 2009 the Kepler satellite will be launched and also will search for Earth-size planets. In other words, it is a perfect time to have a closer look at the possible planet configurations formed as a result of the evolution of young planetary systems in which different mass planets might co-exist with each other. Here we focus on the resonant configurations between two planets - a gas giant (we will call it Jupiter) and a Super-Earth, orbiting a Solar-type star. Our prediction of the occurrence of the commensurabilities in such systems may turn out to be particularly valuable in the case of the TTV observations, because the differences in the time intervals between successive transits, caused by planet-planet interactions, are largest near mean-motion resonance (Agol et al. 2005).

It has been recognized that resonant structures may form as a result of large scale orbital migration in young planetary systems due to tidal interactions between plan-
ets or protoplanets and a gaseous disc, in which the whole system is still embedded. The different mass objects, which we expect to find in forming planetary systems, will migrate with different rates. The final configurations will depend on the intricate interplay among many physical processes including planet-planet, disc-planet and planet-star interactions. One scenario is that the convergent migration brings the giant planets closer to each other and they can become locked in low order commensurability (Bryden et al. 2004, Kley 2000) as it is observed at least in three multiplanet systems (GJ 876, HD 82943 and 55 Cnc). Also, the low mass planets can undergo the convergent migration and form a resonant structure (Papaloizou & Szuszczewicz 2005). The pulsar planets around PSR B1257+12 might be an outcome of such scenario. The differences in the migration rates of low and high mass planets may also lead to the convergent migration (Hahn & Ward 1996). We will consider this type of convergent migration here as a mechanism for capture of a Super-Earth by a Jupiter into mean motion commensurabilities and we will follow the pair of planets further in their evolution to establish the stability of their configuration. It has been found already by Thommes (2003) that for typical protoplanetary disc parameters the low mass planets can be captured by massive planet into exterior resonances. The resonant captures play also an important role in modelling terrestrial planet formation and their survival in the presence of migrating gas giant. The early divergent conclusions on the occurrence of terrestrial planets in hot Jupiter systems (Levison et al. 2001, Armitage 2003, Raymond et al. 2003, Mandell & Sigurdsson 2003) have been clarified by most recent studies (Fogg & Nelson 2005, 2007, Zhou et al. 2007, Raymond et al. 2006, Mandell et al. 2007) which predict that terrestrial planets can grow and be retained in the hot-Jupiter systems both interior and exterior of the gas giant. The most relevant feature of these investigations for the present study is the possibility of the existence of terrestrial planets on the internal orbits relative to Jupiter. Such planets could become captured in the mean motion commensurability which is then maintained during their evolution so an outcome in this case will be the Jupiter and the low-mass planet migrating together towards the star.

Our aim here is to consider in full detail the evolution of the close pair of planets: a gas giant on the external orbit and a Super-Earth on the internal one in the young planetary system, when both planets are embedded in a gaseous disc. We have performed 2D hydrodynamical simulations in order to determine the occurrence of the first order mean motion resonances in such systems and at the same time to examine the possible planet configurations as the outcome of the convergent orbital migration.

Our paper is organized as follows. In Section 2 we summarize shortly few facts about two types of migration, which we will use in the present study. In Section 3 we discuss the Jupiter and Super-Earth system in the context of the restricted three body problem when the disc is absent. Moreover we have performed simple N-body calculations in order to determine the Super-Earth dynamics in a presence of Jupiter and compare it with the classical results of celestial mechanics. This provides us with a well studied and understood framework in which we can present the results of our hydrodynamical simulations of a young planetary system, where planets are still embedded in a gaseous disc. In Section 4 we describe our hydrodynamical simulations and show the planetary orbit evolution together with the changes in the protoplanetary disc structure. The discussion and our conclusions are given in Section 5.

2 SLOW AND FAST MIGRATION

Migration due to planet-disc interaction might play an important role in shaping up planet configurations in the planetary systems as we have already mentioned in the previous section. From the dynamical point of view one of the most important consequences of this process is an occurrence of planets locked in mean-motion resonances and this will be our main concern here.

The migration rates for different planet masses has been estimated by number of authors, see the review by Papaloizou et al. (2006). Their results have been illustrated in Fig. 1 where we plot the migration time of a planet as a function of its mass. There are two mass regimes which are of interest here, namely (0.1 - 30M_{\oplus}) and (150 - 1500M_{\oplus}) for which in a typical protoplanetary disc we can talk about two different types of migration, called simply type I and type II respectively.

The migration time for low mass planets embedded in a gaseous disc (type I migration) has been derived by Tanaka et al. (2002) in the form

\[ \tau_I = (2.7 + 1.1\gamma)^{-1} \frac{M_*}{m_p} \frac{M_*}{2p} \left( \frac{H}{r} \right)^2 \Omega_p^{-1} \]  

(1)

Here \( m_p \) is mass of the planet, \( r_p \) is the distance from the central star \( M_* \), \( \Sigma_p \) is the disc surface density, \( H/r \) and \( \Omega_p \) are the disc aspect ratio and angular velocity respectively. The coefficient \( \gamma \) depends on the disc surface density profile, which is expressed as \( \Sigma(r) \propto r^{-\gamma} \). Assuming \( \gamma = 0 \) for the flat surface density distribution, \( \Sigma_p = 2000 \; \text{kg/m}^2 \), \( H/r = 0.05 \) and \( r_p = 5.2 \; \text{AU} \) we have calculated the migration time as a function of the planet mass and draw it in Fig. 1.

Type II migrators open a gap in the disc and their evolution is determined by the radial velocity drift in the disc

\[ v_r = \frac{3}{2} \frac{\nu}{r_p} \]  

(2)

where \( \nu \) is a kinematic viscosity parameter. The migration time can be estimated as (Lin & Papaloizou 1993)

\[ \tau_{II} = \frac{r_p}{v_r} = \frac{2r_p^2}{3\nu} \]  

(3)

It has been shown in Fig. 1 for \( r_p = 5.2 \; \text{AU} \) and different values of \( \nu (10^{-5}, 2 \cdot 10^{-5}, 3 \cdot 10^{-5}, \ldots, 9 \cdot 10^{-5} \) and \( 10^{-6} \) expressed in the dimensionless units discussed in Section 4.4. The migration time in this formulation does not depend on the planet mass. In drawing lines for a given viscosity parameter we have taken into account the condition for a gap opening in the disc which reads

\[ \frac{m_p}{M_*} > \frac{40\nu}{r_p^2 \Omega_p} \]  

(4)

and from which it is clear that the bigger is viscosity \( \nu \) the bigger is the mass of the planet, which is able to open a gap. Recently, Edgar (2007) has argued that similarly like
in the case of type I migration also here the migration time depends on the planet mass and the disc surface density. His relation gives for our set of parameters somewhat faster migration for the Jupiter in comparison with the equation (3), so it does not change our argument here.

We have chosen the mass of the type I and type II migrators to be 5.5 $M_\oplus$ and 1 Jupiter mass respectively. Their locations have been marked in Fig. 1 accordingly to their mass ratios to the mass of the central star equal $m_1$ and $m_2$, respectively, has to satisfy the following criterion

$$\Delta > 2.4(m_1 + m_2)^{1/3}$$

(5)

in order to be Hill stable. This means that for the planetary orbit separations fulfilling this condition planets will not suffer from the close encounters at any time. The close encounter will result in the scattering of the Super-Earth, which may leave the system, fall onto the star or the Jupiter. In our case, for the Jupiter ($m_1 = m_J$) and Super-Earth ($m_2 = m_{SE}$) considered in this paper this criterion gives

$$\Delta > 0.24,$$

(6)

which is the same as for the Jupiter and a massless body, since $m_{SE}/m_J \ll 1$. We have found how good is this simple analytical prediction in our case by performing direct orbit integration. Employing a simple N-body code, which uses Bulirsch-Stoer integrator and proved itself to be a convenient way to obtain high-accuracy solutions of the differential equations (see e.g. Terquem and Papaloizou 2007), we have calculated the planetary orbits in the region where the interior 3:2, 4:3 and 5:4 Jupiter mean-motion resonances are located. We have put the Super-Earth at the fixed location $a_{SE} = 1$, defining in this way our unit of distance, and the Jupiter initial semi-major axes have been varied in the range from 1.15 till 1.45 in these units. The initial eccentricities of both planets have been set to zero. Our results have been summarized in Table 1 where the number of conjunctions for each initial configuration that the system survives without suffering a close encounter is given in the column 3. When the Jupiter is located at the distance 1.24 or smaller we have observed the orbit crossing and scattering of the Super-Earth. Our result agrees very well with the stability criterion given by Gladman (1993).

Hill stability criterion is useful for predicting close encounters between planets but is not telling us much about other properties of motion as for example whether or not the system is chaotic. Wisdom (1980) basing on the resonance...
overlapping criterion found a boundary between chaotic and stable motions in the form
\[
\Delta > 1.5a_Jm^{2/7}
\]  
(7)
where \( m \) denotes mass ratio of the two biggest bodies in the system. The coefficient in front of the mass in the original paper was 2, but it has been improved later by numerical simulations [Duncan et al., 1989] and we have adopted here the more recent value 1.5. The occurrence of the mean motion resonances is therefore very important for stability of the system. The width of the resonance regions for the circular restricted three-body problem has been derived by Wisdom (1980) (see also Lecar et al. (2001)). In Fig. 2 we have plotted the width of 3:2 and 4:3 interior resonances for zero eccentricity given by Lecar et al. (2001) for the Jupiter (vertical dashed lines). In this Figure the semi-major axis ratio is obtained by dividing the semi-major axis of the Jupiter by that of the small body. Winter & Murray (1997) have discussed the various analytical models that have been used in the study of interior first order resonances. Analyzing simple pendulum model they derived formula describing maximum deviation of semi-major axis from the nominal value given by exact mean motion commensurability. Their results for 3:2 and 4:3 resonances are presented also in Fig. 2 for comparison (solid lines). The theoretical prediction illustrated by the pendulum model (similar results were obtained using Hamiltonian approach) that for higher value of the eccentricity of the small body the first order resonance width is larger, has not been verified by direct integration (Winter & Murray 1997). The size of the resonant libration region derived by Winter & Murray (1997) performing numerical integrations of the full equations of motion is quite similar to that given by Lecar et al. (2001). The plot of maximum libration widths has been supplemented with a few examples of the mean motion commensurability known in our Solar System. The dots denote Hilda and Thule groups of asteroids. This groups are particularly interesting because they are locked in the mean motion resonances with Jupiter. Note that asteroids are located exactly within regions predicted by analysis of the restricted three-body problem.

We have performed an analysis of the chaotic regions for our system (the Super-Earth and Jupiter around the Solar-type star) in the similar way it has been done for asteroid belt. First, let us evaluate the size of the chaotic region using equation (7). Substituting in this equation for \( m \) the mass ratio of Jupiter and Sun we have found that the small body (here the Super-Earth) should be outside the region of large scale chaos if
\[
\Delta > 0.21a_J.
\]  
(8)
According to this criterion the Super-Earth would be in the region of large scale chaos when the Jupiter’s semi-major axis is smaller than 1.27.

Next, we have examined the properties of the planetary orbits in the Hill stable region, which means for the Jupiter semi-major axis having value in the range between 1.24 and 1.45. We have found two families of regular orbits separated from each other by the zone of the chaotic behaviour, which is located between 1.277 and 1.289. It should be pointed out here that the fact of the existence of the chaotic zone outside the region indicated by the resonance overlap criterion is not entirely unexpected. It should be kept in mind that criterion
chaotic zone. This part of our investigations has been also summarized in Table 1.

In the presence of gas giant, the dynamics of the Super-Earth with a mass of 5.5 $M_\oplus$ is similar to that of the massless body in the restricted three-body problem. This gives us an opportunity to determine some of the properties of its motion by means of analytical, well understood methods (Murray & Dermott 1999). The extensive discussion of the classical three-body problem (Solar-type star, Jupiter and Super-Earth) brought us to the stage where we can predict stability and the character of the planetary orbits. At the same time we have prepared the well defined framework for our main study, which aim is to determine how this picture will be modified in a young planetary system in the presence of the gaseous protoplanetary disc in which the planets are embedded.

4 THREE-BODY PROBLEM WITH A GASEOUS DISC

4.1 Description of the performed simulations

The system described in the previous Section has been considered again, but this time the planets are embedded in a gaseous disc. We investigate the orbital evolution of the Super-Earth and Jupiter in the same region as before taking into account the disc-planet interaction. The disc has a constant aspect ratio $\frac{H}{r} = 0.05$. It extends from 0.33 till 4 (the unit of distance is the initial semi-major axis of the Super-Earth as it has been chosen in the previous Section). The kinematic viscosity is constant in space and its value is $\nu = 10^{-5}$ in units of $a_{SE}^3(GM_*/a_{SE}^3)^{1/2}$, where $a_{SE}$ is the initial semi-major axis of the Super-Earth and $G$ is the gravitational constant. This value, with a disc aspect ratio of 0.05 should correspond to $\alpha = 4 \cdot 10^{-3}$ at the distance 1 in our units in the standard $\alpha$-parametrization for the viscosity in the disc. The planets has been placed in the flat part of the surface density profile with $\Sigma = 2 \cdot 10^3 (5.2 \text{au}/a_{SE})^2$ kg m$^{-2}$: the standard value attributed to the minimum mass solar nebula at 5.2 au when $a_{SE} = 5.2$ au. The initial eccentricity of both planets is set to zero. We perform numerical simulations using hydrodynamical code NIRVANA, for details of the numerical scheme and code adopted, see Nelson et al. (2000). We have used the resolution $384 \times 512$ grid-cells in the radial and the azimuthal direction respectively. We are aware of the fact that adopted resolution might be not high enough to simulate properly the evolution of the Super-Earth in general, because the corotation region around planet is covered only by a few grid cells. However, we think that adopted resolution is a good compromise between the accuracy of the simulations and reasonable computational time. The computational domain is a complete ring where the azimuthal coordinate $\phi = 0$ corresponds to $\phi = 2\pi$. We put the open boundary conditions in the radial direction. The potential is softened with the parameter $b = 0.04 r$ in such a way that the $1/r$ law has been changed into $1/\sqrt{r^2 + b^2}$.

As in the case described in previous Section, where the disc was absent we have investigated 5.5$M_\oplus$ and 1$M_J$ planets with the initial positions specified in Table 1. The Super-Earth is located initially at the distance 1 in each simulation.

Figure 3. The structure of phase space trajectories of the Super-Earth in three different planet configuration. From the top to the bottom panels, the Jupiter initial semi-major axis is 1.27, 1.28 and 1.30 correspondingly. The distance from the origin is measured by the Super-Earth eccentricity, $e$ and the polar angle is given by the resonance phase $\phi$. 
Table 1. The initial separation of planets in Super-Earth units (in Jupiter units) and their final configurations. The double line denotes the Hill stability boundary.

| Initial Super-Earth location | Initial Jupiter location | Final outcome of the evolution without disc | Final outcome of the evolution with disc |
|-----------------------------|--------------------------|---------------------------------------------|-----------------------------------------|
| 1.0 (0.870)                 | 1.15 (1.0)               | scattering (before first conjunction)        | scattering                              |
| 1.0 (0.862)                 | 1.16 (1.0)               | scattering (before first conjunction)        | scattering                              |
| 1.0 (0.855)                 | 1.17 (1.0)               | scattering (before first conjunction)        | scattering                              |
| 1.0 (0.847)                 | 1.18 (1.0)               | scattering (after 39 conjunctions)          | scattering                              |
| 1.0 (0.840)                 | 1.19 (1.0)               | scattering (after 20 conjunctions)          | scattering                              |
| 1.0 (0.833)                 | 1.20 (1.0)               | scattering (after 2 conjunctions)           | 4:3                                     |
| 1.0 (0.826)                 | 1.21 (1.0)               | scattering (after 15 conjunctions)          | scattering                              |
| 1.0 (0.820)                 | 1.22 (1.0)               | scattering (after 9 conjunctions)           | 4:3                                     |
| 1.0 (0.813)                 | 1.23 (1.0)               | scattering (after 107 conjunctions)         | 4:3                                     |
| 1.0 (0.806)                 | 1.24 (1.0)               | scattering (after 5 conjunctions)           | 4:3                                     |
| 1.0 (0.800)                 | 1.25 (1.0)               | survival (more than 100,000 conjunctions)  | 4:3                                     |
| 1.0 (0.794)                 | 1.26 (1.0)               | survival (more than 100,000 conjunctions)  | 3:2, scattering                         |
| 1.0 (0.787)                 | 1.27 (1.0)               | survival (more than 100,000 conjunctions)  | 3:2, scattering                         |
| 1.0 (0.781)                 | 1.28 (1.0)               | survival (more than 100,000 conjunctions)  | 3:2                                     |
| 1.0 (0.769)                 | 1.30 (1.0)               | survival (more than 100,000 conjunctions)  | 3:2                                     |
| 1.0 (0.741)                 | 1.35 (1.0)               | survival (more than 100,000 conjunctions)  | 3:2                                     |
| 1.0 (0.690)                 | 1.45 (1.0)               | survival (more than 100,000 conjunctions)  | 3:2                                     |

The Jupiter mass planet is located on the external orbit at the different locations. Such configuration allows to attain convergent migration (Fig. 1).

4.2 Super-Earth in mean motion resonance with Jupiter

In agreement with the theoretical predictions (Fig. 1) for our particular choice of the system parameters, the Jovian mass planet migrates faster than the Super-Earth, and thus planets approach each other and may become eventually captured in a mean motion resonance. In order to investigate how an outcome of such evolution depends on the initial configurations of planets we have performed our simulations with a wide range of the planetary separations (see Table 1). We are particularly interested in the conditions for which the first order mean motion resonances \( p+1 : p \) for \( p \geq 2 \) are attained (here \( p \) is an integer). The 2:1 commensurability was not taken into account because of the computational time constraints. The resonances located closer to the Jupiter like 5:4, 6:5 and with higher \( p \) are not possible, because for such small separations the system become unstable and Super-Earth is scattered from the disc. We have found that two outcomes are possible, namely the Super-Earth is ejected from the disc or planets become locked in 3:2 or 4:3 mean motion resonances.

Our results has been summarized in Table 1 column 4 and on the right hand side of the Fig. 2 where the most likely outcome of the planet evolution is given for every initial semi-major axis ratios. In Fig. 3 we show also two examples of the resonant trapping occurred for the relative planet separation 0.45 (upper curve) and 0.24 (lower curve). In the first case, the differential migration brought planets into 3:2 commensurability and in the second one into 4:3. The semi-major axis ratio of planets librates exactly within regions of the resonance width marked by the solid, horizontal lines in Fig. 4. We plot there the resonance widths for \( (p+1) : p \) commensurabilities with \( 2 \leq p \leq 4 \) in a case of circular orbits in the restricted three body problem where the secondary is a Jovian mass planet (Wisdom 1980; Lecar et al. 2001). The width of 3:2 and 4:3 resonances have already been illustrated in Fig. 2 and discussed in the previous Section. As we can see the 4:3 and 5:4 mean motion resonances partially overlap each other, which might have a significant effect on the stability of the system. Indeed planets located initially in this region were immediately scattered from the disc. Both commensurabilities shown in Fig. 4 are stable for the whole time of our simulations. We have followed their evolution for not longer than 2500 orbits (the unit of time is defined as the orbital period on the initial orbit of the Super-Earth divided by \( 2\pi \)) because after that time the surface density is significantly reduced due to viscous evolution of the disc. However, not all planetary pairs initially locked in the mean motion resonance survived further evolution (always limited to 2500 orbits). In the case of 3:2 commensurability the libration width increases with time. The amplitude of the
Jupiter and Super-Earth embedded in a gaseous disc

Figure 4. The evolution of the ratio of semi-major axes for the Jupiter and Super-Earth embedded in a gaseous disc. In the case of upper curve the initial semi-major axis ratio is 1.45 and planets became locked into 3:2 resonance. For the lower curve, the initial semi-major axis ratio is 1.24 and the attained commensurability is 4:3. The solid horizontal lines are the 3:2, 4:3 and 5:4 resonance widths in the circular three-body problem. Note that 4:3 and 5:4 resonances partially overlap each other.

oscillations is bigger for planets with a separation smaller than the lower boundary of the 3:2 resonance region. So in fact in the case of Jupiter placed at 1.26 and 1.27 the amplitude grows and eventually the Super-Earth is scattered from the disc. We have illustrated it for the Jupiter at 1.26 in Fig. 4. The direct reason of this scattering is the excitation of the Super-Earth eccentricity to the high value, which favours close encounters and ends with the ejection of the planet.

We have also investigated eccentricity evolution of the Super-Earth in few other cases. Fig. 5 shows eccentricity of the smaller planet versus semi-major axis ratio. The dashed, vertical lines denote the resonances width for circular case, the solid lines instead the maximum libration width obtained from the formula given by Winter & Murray (1997) in the pendulum approach in the case of the classical restricted three body problem. We have already introduced these quantities in the previous Section. The eccentricity of the Super-Earth locked in 3 : 2 commensurability increases and at the end of simulation reaches value about 0.5. Eccentricity of planets approaching 3 : 2 mean motion resonance from higher value of semi-major axis exceed value of 0.5. For the planets attaining commensurability from the lower value of the semi-major axis ratio than needed for the exact 3:2 resonance, the oscillations around exact position of the resonance increase and eventually Super-Earth is scattered from the disc (Fig. 5).

The Super-Earth locked in 4:3 mean motion resonance in all simulations approach eccentricity value around 0.3. This is a lower value of eccentricities than in the case of 3:2 commensurability. The amplitude libration around the position of the exact resonance is smaller than for 3:2 and there is no trend in the amplitude to increase.

Till now we have concentrated on the orbital elements evolution occurring due to the tidal planet disc interaction. Another effect of this interaction in the case of the giant planet is the gap opening process, which changes the surface density profile and may affect the Super-Earth migra-
Figure 6. Four examples of the Super Earth eccentricity evolution. For the 4:3 commensurability (left) we are presenting the eccentricity as a function of semi-major axis ratio for the planets with their initial separation 0.24 (gray colour) and 0.22 (black colour) and for the 3:2 commensurability (right) the initial planet separation reads 0.45 (gray colour) and 0.27 (black colour). The resonance widths are the same as in Fig. 2.

Figure 7. The disc profile and planet locations at the time of 3:2 resonant capture. The initial position of Jupiter is 1.30.

Figure 8. The disc profile and planet locations at the time of 4:3 resonant capture. The initial position of Jupiter is 1.24.

and ongoing process of the gap opening in the disc leads to the resonant configurations for a range of the initial planet locations.

5 DISCUSSION AND CONCLUSIONS

In this paper we have investigated the evolution of Super-Earth and Jupiter mass planets embedded in gaseous protoplanetary discs where terrestrial planet is moving on the internal orbit and gas giant on the external one. We have found that according to the simple theoretical prediction different rates of migration lead to the planet capture in the mean motion resonances. For the system considered here the only possible first order commensurabilities are with $p \leq 3$. For higher $p$ the resonance locations will be in the Hill unstable zone and in the regions where the resonances begin to overlap therefore the system will be unstable.

The essential feature of our scenario or differently, the reason for the occurrence of the mean motion resonances in our system, is the convergent migration of planets embedded in the protoplanetary discs. We have mentioned already in Section 2 that for the given masses and disc profile we can control the convergence of the migration by the viscosity in the parent disc, at least for the standard range of parameters, as in the case considered here. In general, the situation may be more complex. First of all, it has been found that the torque acting on the low-mass planets might be considerably smaller than the analytic linear estimate and its value does depend on the viscosity in the parent disc, at least for the standard range of parameters, as in the case considered here. In general, the situation may be more complex. First of all, it has been found that the torque acting on the low-mass planets might be considerably smaller than the analytic linear estimate and its value does depend on the viscosity (see Masset et al. (2006) for the discussion). Moreover, it has been also proposed that even the planets as small as Super-Earths can open a gap in the disc and slow down their orbital migration (Matsumura et al. (2007) and references therein). Finally, combining the equation (4) with the gap opening formula suggested for a disc with a very small viscosity (Lin & Papaloizou 1993, Crida et al. 2006) constructed the generalized criterion which is more restrictive than equation (4). It is then required to consider in detail the disc profile, masses of the planets and their locations.
in the disc in order to determine whether or not we might expect the convergent migration.

Our results for the case of the mature planetary systems when the disc is absent are consistent with those predicted analytically for the classical planetary three-body problem. From the direct orbit integration we have found the Hill stability region which occurs when the Jupiter and Super-Earth orbit relative separation is 0.24 or bigger, which is exactly what we get from the formula \[ \frac{1}{2} \frac{m_2^2}{m_1} \left( \frac{r_1}{r_2} \right)^3 = \frac{1}{2} \frac{m_1^2}{m_2} \left( \frac{r_2}{r_1} \right)^3 \]. The chaotic orbits have been identified in the configurations with the initial semi-major axis ratios in between 1.277 and 1.289. This chaotic zone might be connected with 13:9 resonance, which partially overlap with the 3:2 commensurability.

As has been summarized in Table [1] the planets embedded in the disc show the same behaviour as planets in matured planetary system (i.e. where the disc is absent) for the relative separation between planets less than 0.20. So for these separations the presence of the gas in the system does not change the outcome of the evolution. The planets are unstable in the Hill sense (Gladman 1993) and also they are in the region with the resonance overlapping (Wisdom 1984). The first difference between young and mature planetary systems has been found for the relative separation 0.20. The Super-Earth has been trapped by Jupiter in 4:3 mean-motion resonance very quickly preventing in this way the scattering. This was not the case for 0.21 and the Super-Earth has been ejected from the disc. For the even higher separations from 0.22 till 0.24 which are also Hill unstable we have obtained the 4:3 resonance. The last 4:3 commensurability we have got for the Super-Earth separated from Jupiter by 0.25. Next two configurations with separations 0.26 and 0.27, located inside the chaotic zone of Wisdom (1980) passed through unstable 3:2 resonance. All other investigated by us structures, namely for the separations 0.28-0.45 have been locked in 3:2 commensurability.

The presence of the disc and its interactions with the planets might be a key to the attaining resonant configurations like those observed between Jupiter and groups of asteroids in our system. The fate of the Super Earth in the case of the presence of Jupiter mass planet on the external orbit is determined by separation of the planets and their eccentricity evolution. The resonant structures with 3:2 and 4:3 commensurabilities are easy to attain but their stability requires further studies. Our scenario indicates the possibility that terrestrial planets and giant planets coexist in relatively close proximity. It has an interesting implication for the astrobiological studies. For example, if we consider a system, which contains a giant planet in or near its habitable zone, taking into account a simple stability analysis as in Section 3 we might conclude that there is no dynamical room for terrestrial planets in the habitable zone, close to the Jupiter mass planet. However, as it has been shown here, if we allow for the possibility of mean motion resonance locks among planets, this is no longer necessarily the case. This hypothesis can be verified by the discoveries of Jovian type planets in the habitable zones, similar to the low mass gas planet which has been found recently by Fischer et al. (2007) in 55 Cancri system, so the intensive search for the low mass planets which may be present close to the gas giants is more than justified. The discovery of the resonant configurations would provide constraints also on the migration processes themselves. If the future observations will find primarily smaller planets locked in the interior mean motion resonances and not those locked in the exterior ones as in Thommes (2001), or other way round, than we will get important information on the characteristics of the protoplanetary disc in which these planets form. Finally, the occurrence of mean motion resonances may increase the chance of a detection of the terrestrial planets. The best example is provided by the transit timing measurements, which may detect additional planets (which are as small as the Super-Earth considered here) in the system via their gravitational interaction with the transiting Jupiter mass planet, taking advantage of the resonance induced by migration.

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