Group Approach to the Quantization of Non-Abelian Stueckelberg Models

V. Aldaya¹, M. Calixto²,¹ and F. F. López-Ruiz¹

¹ Instituto de Astrofísica de Andalucía (IAA-CSIC), Apartado Postal 3004, 18080 Granada, Spain
² Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Paseo Alfonso XIII 56, 30203 Cartagena, Spain
E-mail: valdaya@iaa.es, Manuel.Calixto@upct.es, flopez@iaa.es

Abstract. The quantum field theory of Non-Linear Sigma Models on coadjoint orbits of a semi-simple group G are formulated in the framework of a Group Approach to Quantization. In this scheme, partial-trace Lagrangians are recovered from two-cocycles defined on the infinite-dimensional group of sections of the jet-gauge group \( J^1(G) \). This construction is extended to the entire physical system coupled to Yang-Mills fields, thus constituting an algebraic formulation of the Non-Abelian Stueckelberg formalism devoid of the unitarity/renormalizability obstruction that this theory finds in the standard Lagrangian formalism under canonical quantization.

1. Introduction

Non-Linear \( \sigma \)-fields have attracted a renewed attention as an essential ingredient in generalized, non-Abelian Stueckelberg models [1] formulated in an attempt to replace the Higgs mechanism in non-abelian gauge theories [2, 3, 4, 5]. Unfortunately, those models can accommodate either unitarity or renormalizability but not both [6]. In this paper we propose a non-perturbative description of Non-Linear Sigma Fields minimally coupled to Yang-Mills vector potentials on the framework of a Group Approach to Quantization (GAQ) [7, 8, 9].

Group quantization of non-Abelian gauge groups had been only achieved consistently in \( 1 + 1 \) dimensions by representing the corresponding Kac-Moody group [10]. In fact, the non-trivial cohomology of such groups provided a co-cycle [11] which is easily translated to a WZW Lagrangian. In \( 3 + 1 \) dimensions, however, the Mickelsson two-cocycle is absent and a bit more involved construction is required. One of the new ingredients will be the consideration of the jet-gauge group \( J^1(G) \) of the original rigid symmetry group \( G \) as the target space for the physical fields, so that those fields are mapps from the space-time manifold \( M \) onto \( J^1(G) \), providing the Goldston-like scalar fields \( \varphi^a(x) \) along with the corresponding vector potentials \( A^a_\mu(x) \) [12, 13]. These fields constitute de new local group \( G^1(M) \). The other ingredient refers to the use of a class of two-cocycles that, even though they are coboundaries, and, therefore trivial from some mathematical points of view, they define a central extension endowed with a canonical (left- or right-) invariant form [14] which gives a physical Lagrangian for fields living on a coadjoint orbit of \( G^1(M) \). The corresponding Lagrangian can then be seen as a (covariant) partial trace of the standard \( \sigma \)-model full-trace (chiral) Lagrangian \( Tr(U^{-1} \partial_\mu U U^{-1} \partial^\mu U) \), \( U \in G \) coupled to the vector potentials according to a Minimal Coupling prescription addressed by one of the
two-cocycles involved in the central extension of the local group.

In this paper we review very briefly, in Sec.2, the standard classical approach to the gauge theory of internal interactions, and revisit this classical theory under the equivalent scheme of jet-gauge groups. Then, in Sec.3, we propose the new framework, based entirely on symmetries, to account for a non-canonical group quantization of massive gauge theories without Higgs.

2. Internal gauge interactions: the standard Utiyama theory revisited

In the standard formulation of gauge theories, the well-known Minimal Coupling Principle for internal gauge symmetries establishes that if the action of some matter fields $\psi^\alpha$, $\alpha = 1, ..., N$

$$S = \int \mathcal{L}_{\text{matt}}(\psi^\alpha, \partial_\mu \psi^\alpha) d^4x,$$

is invariant under a rigid internal Lie group $G$, then the modified action

$$\hat{S} = \int \left[ \mathcal{L}_{\text{matt}}(\psi^\alpha, \partial_\mu \psi^\alpha) + \mathcal{L}_0(F_{\mu\nu}^{(a)}) \right] d^4x$$

is invariant under the local group $G(M)$, where $(M)$ is the Minkowski space-time

$$D_\mu \psi^\alpha \equiv \partial_\mu \psi^\alpha - gA^{(a)}_\mu X^\alpha_{(a)\beta} \psi^\beta$$

is usually known as covariant derivative, and

$$F_{\mu\nu}^{(a)} \equiv \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + g \epsilon^{\alpha\beta\gamma} (A^{(b)}_\mu A^{(c)}_\nu - A^{(b)}_\nu A^{(c)}_\mu)$$

is known as curvature of the connection $A^{(a)}_\mu$ (the gauge potentials or Yang-Mills fields).

The constant matrix $X^\alpha_{(a)\beta}$ is the representative of the group generator in the direction, $(a) = 1, ..., \dim G$, acting on the field components, so that, the infinitesimal variation of the matter fields is

$$\delta^{(a)} \psi^\alpha = X^\alpha_{(a)\beta} \psi^\beta$$

Note that in terms of this infinitesimal variation the covariant derivative is written as

$$D_\mu \psi^\alpha \equiv \partial_\mu \psi^\alpha - gA^{(a)}_\mu \delta^{(a)} \psi^\alpha$$

2.1. Including the group parameters in Gauge Theory

We now present a conceptual and structural revision of the standard formulation of gauge theories.

Let $G$ be a Lie group of internal symmetry, the rigid group, and $G(M)$ the gauge group (also named current group), whose elements are the mappings from Minkowski space-time $M$ into $G$:

$$G(M) \equiv \{ M \to G \}.$$

Let us introduce the group of the 1-jets of the gauge group $G(M)$ [12],

$$J^1(G(M)) = \mathbb{R}^2 \times M / \sim^1$$

The formal definition of $J^1(G(M))$ is fully analogous to that of the $(q^4, \dot{q}^2)$-space in Lagrangian Mechanics, or $(\psi^\alpha, \dot{\psi}_\mu^\alpha)$-space in Lagrangian Field Theory, when one desires to vary independently coordinates and velocities (momenta) according to the modified Hamilton principle. That is:

$$J^1(G(M)) \equiv G(M) \times M / \sim^1$$

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where the equivalence relation $\sim^1$ is defined as follows:

$$(\varphi, m) \sim^1 (\varphi', m')$$

for all $(\varphi, m), (\varphi', m')$ belonging to $G(M) \times M$, if and only if

$$m = m'$$

$$\varphi(m) = \varphi'(m)$$

$$\partial_\mu \varphi(m) = \partial_\mu \varphi'(m).$$

This definition may easily be extended to the order $r^{th}$. A natural coordinate system for $J^1(G(M))$ would be

$$\{x^\mu, \varphi^a, \varphi^a_{\mu}\}$$

where $x^\mu$ parametrize locally $M$, and $\varphi^a$ and $\varphi^a_{\mu}$ correspond to $g^a(\varphi(m))$ and $\partial_\mu (g^a(\varphi(m)))$, respectively, under the quotient by the equivalence relation $\sim^1$, for a local coordinate system in $G$, $\{g^a\}$.

Thus, the relevant group in Gauge Theory, extending the standard gauge group $G(M)$, will be that constituted by the mapping from $M$ to $J^1(G(M))$,

$$G^1(M) \equiv \Gamma(J^1(G(M))) \equiv \{M \to J^1(G(M))\},$$

parametrized by fields $\varphi^a(x)$, $\varphi^a_{\mu}(x)$, where $\varphi^a_{\mu}$ is not necessarily the derivative of $\varphi^a$ except for jet-extensions, that is:

$$j^1: G(M) \to G^1(M)$$

$$\varphi \mapsto j^1(\varphi)/\varphi^a_{\mu} = \partial_\mu \varphi^a.$$

In this formalism $\varphi^a_{\mu}$ are essentially the gauge potentials $A^a_{\mu}$:

$$A^a_{\mu} \equiv \varphi^a_{\mu},$$

which are (not) pure gauge if $\varphi^a_{\mu}$ are (not) jet-extensions, i.e. are (not) of the form $\partial_\mu \varphi^a$, where, $\theta^a_{b}(\mu)$ is the (non-constant) invertible matrix defining the (right-) invariant canonical 1-form on the group

$$\theta^R_{\mu}(\alpha) = \theta^a_{b}(\alpha) \partial_\mu \varphi^b$$

(d$UU^{-1}$ for linear groups)

dual to the (right-invariant) generators $X^R_{a(\mu)} = X^b_{(a)} \delta^a_{b(\mu)} = \delta_{(a(\mu))} \varphi^b$, that is:

$$\theta^a_{b(c)} X^c_{(a)} = \delta^a_{b}.$$  

Notice that for pure gauge we have $A^a_{\mu} = \theta^a_{b(\mu)} \partial_\mu \varphi^b \equiv \theta^a_{\mu}$ and, hence, the curvature $F(A_{\mu}) = F(\theta_{\mu}) = 0$, that is, $\theta_{\mu}$ can be seen as a flat connection.
2.2. Lagrangian formalism on jet-gauge groups: gauge parameters $\varphi^a \in G(M)$ as “matter” fields

Standard variational calculus varies Lagrangians depending on the “field configuration” space $E$, with generic field coordinates $\{x^\mu, \psi^a\}$, and the corresponding field derivatives $\{\partial_\mu \psi^a\}$. Our configuration space will be now $J^1(G(M))$, with coordinates $\{x^\mu, \varphi^a, A^{(a)}_\mu\}$. Lagrangians are accordingly functions

$$\mathcal{L} = \mathcal{L}(x^\mu, \varphi^a, A^{(a)}_\mu; \partial_\mu \varphi^a, \partial_\nu A^{(a)}_\mu).$$

We can proceed to formulate some sort of Utiyama’s Theorem on $J^1(G(M))$:

If the action of some “matter” fields $\varphi^a$, $a = 1, \ldots, \dim(G)$

$$S = \int \mathcal{L}_{\text{mat}}(\varphi^a, \partial_\mu \varphi^b) d^4x,$$

is invariant under a rigid internal Lie group $G$, then the modified action

$$\tilde{S} = \int [\mathcal{L}_{\text{mat}}(\varphi^a, D_\mu \varphi^b) + \mathcal{L}_0(F^{(a)}_{\mu\nu})] d^4x$$

is invariant under the local group $G(M) \subset G^1(M)$, where

$$D_\mu \varphi^a \equiv \partial_\mu \varphi^a - g A^{(b)}_\mu X^a_{(b)},$$

the new “covariant” derivative, $X^a_{(b)}$ are the components of the generators of the group $G$ acting on itself and

$$F^{(a)}_{\mu\nu} \equiv \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + \frac{g}{2} \epsilon^{abc} A^{(b)}_\mu A^{(c)}_\nu - A^{(b)}_\mu A^{(c)}_\nu$$

is the curvature of the connection $A^{(a)}_\mu \equiv \theta^{(a)}_b \psi^b_\mu$ (the gauge potentials or the Yang-Mills fields).

We should remark that the Minimal Coupling now occurs in an affine manner. In fact, in the expression

$$\partial_\mu \varphi^a - g A^{(b)}_\mu X^a_{(b)}$$

the matrix $X^a_{(b)}$ is invertible and this way, the minimal coupling above is proportional to

$$\theta^{(a)}_b [\partial_\mu \psi^b_\nu, A^{(c)}_\mu] = \theta^{(a)}_\mu - g A^{(a)}_\mu$$

where $\theta^{(a)}$ are, for concreteness, the components of the right-invariant canonical 1-form on $G$, $\theta^R$, dual to the right-invariant vector fields $X^R_{(a)}$ ($X^R_{(a)} = [\theta^R_{(a)}]^{-1}$). We shall omit the $R$ script if not required.

2.3. Non-linear $\sigma$-Lagrangians for “matter” fields: The non-Abelian Stueckelberg formalism

The new minimal coupling, when written in the form $\theta - A$, strongly suggests the introduction of “matter” of the $\sigma$-model type:

$$\mathcal{L}_\sigma^G = \frac{1}{2} \text{Tr}[\theta_\mu \theta^\mu], \quad \theta_\mu = \partial_\mu UU^{-1}.$$ 

In fact, this Lagrangian is $G$-invariant (left- and right-invariant, indeed; that is, chiral) and the new minimal coupling gives rise to

$$\tilde{\mathcal{L}}_\sigma^G = \frac{1}{2} \text{Tr}[(\theta_\mu - g A_\mu)(\theta^\mu - g A^\mu)]$$

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which is gauge-invariant even though it contains mass terms for the $A_\mu$ fields, with total Lagrangian

$$L_{\text{MYM}}^G = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + m^2 \hat{L}_G$$

This Lagrangian $L_{\text{MYM}}^G$ actually corresponds to the non-Abelian Stueckelberg formalism for massive Yang-Mills fields although, to be precise, the scalar fields $\varphi^a$ are here the parameters of the local group $G(M)$ themselves, whereas in the standard (non-Abelian) Stueckelberg formalism the corresponding scalar fields are external fields behaving under $G$ in exactly the same way as our $\varphi^a$'s.

Unfortunately the $\sigma$-sector Lagrangian

$$L_\sigma^G = \frac{1}{2} \text{Tr}(\theta^R_\mu \theta^L_\mu) = \frac{1}{2} \text{Tr}(\theta^L_\mu \theta^R_\mu) = \frac{1}{2} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) = \frac{1}{2} g_{ab}(\varphi) \partial_\mu \varphi^a \partial^\mu \varphi^b$$

is not properly quantizable due essentially to its non-polynomic character; in fact, the “metric” function $g_{ab}(\varphi)$ may depend on any power of $\varphi$ so that, standard perturbation theory would require, roughly speaking, an infinite number of bare natural constants; that is as many constants as coefficients in the power expansion of $g_{ab}(\varphi)$ [15]. In more accurate terms, we might conclude that physical field theories involving a non-linear sigma sector of scalars can be either renormalizable or unitary but not both [6].

3. A “partial-trace” alternative to the non-Abelian Stueckelberg formalism

A way out to the troubles found in facing the canonical quantization of Lagrangians containing those non-quadratic kinetic terms associated with non-Abelian sigma fields consists in rearranging the theory in such a manner that we shall be able to find an infinite number of symmetries providing as many non-trivial Noether invariants as parameters there are in the solution manifold of the physical problem. With this extended symmetry at hand one may try to formulate enough (generalized) Ward-like identities to reduce the infinities appearing in canonical perturbation theory or, as we shall make here, to resort to a non-standard, symmetry based, Group Approach to Quantization [7, 8, 9].

3.1. The partial-trace Lagrangian

We restrict the whole trace on $G$ to a partial trace on a quotient manifold $G/H$, where $H$ is the isotropy subgroup of a direction $\lambda = \lambda^a T_a$ in the Lie algebra of $G$ under the adjoint action

$$\lambda \to U \lambda U^\dagger$$

where $\lambda^a$ are real numbers subjected to $\text{Tr}(\lambda^2) = 1$.

Defining $\Lambda \equiv U \lambda U^\dagger$ the claimed $G/H - \sigma$ Lagrangian has the expression:

$$L_{\sigma}^{G/H} = \frac{1}{2} \text{Tr}([-i U^\dagger \partial_\mu U, \lambda]^2) = \frac{1}{2} \text{Tr}(\theta^L_\mu, \lambda)^2 = \frac{1}{2} \text{Tr}(\theta_\mu, \Lambda)^2$$

and the minimally coupled version:

$$\hat{L}_{\sigma}^{G/H} = \frac{1}{2} \text{Tr}([-i U^\dagger D_\mu U, \lambda]^2) = \frac{1}{2} \text{Tr}(\theta_\mu - A_\mu, \Lambda)^2$$

is again invariant under the gauge transformations

$$U \to U' U, A_\mu \to U' A_\mu U^\dagger - i \partial_\mu U' U^\dagger$$
The total Lagrangian for the Massive Yang-Mills theory is then

$$\mathcal{L}_{\text{YM}}^G = \mathcal{L}_{\text{YM}}^G + m^2 \mathcal{L}_\sigma^G$$

with ordinary $\mathcal{L}_{\text{YM}}^G \equiv -\frac{1}{4} \text{Tr}(F^{\mu\nu}F_{\mu\nu})$.

Let us remark that under the change of variables

$$\tilde{A}_\mu = U^\dagger (A_\mu - \theta_\mu)U = U^\dagger A_\mu U + iU^\dagger \partial_\mu U$$

(along with analogous change $\phi = U^\dagger \psi$ for eventual fermions) the Lagrangian above may be written as

$$\mathcal{L}_{\text{MYM}}^{G/H} = -\frac{1}{4} \text{Tr}(F^{\mu\nu}(\tilde{A})^2) + \frac{1}{2} m^2 \text{Tr}([\tilde{A}_\mu, \lambda]^2)$$

It should be noted that for the case $G = SU(2)$ with standard spherical basis $T_\pm, T_0$ and $\lambda = \lambda^0 T_0$, that is $H = U(1)$, the mass term would be written

$$\frac{1}{2} m^2 \text{Tr}([\tilde{A}_\mu, \lambda]^2) = m_W^2 \tilde{W}_\mu^+ \tilde{W}_\mu^-$$

where, $\tilde{W}_\mu^{\pm} = \tilde{A}_\mu^{\pm} \pm i\tilde{A}_\mu^3$ and $m_W = m\lambda^0$ remaining $\tilde{W}_\mu^0 = \tilde{A}_\mu^3$ massless (see [16] for more details).

3.2. Quantum symmetries: an attempt to Group Quantization

For linear systems, the canonical quantization rules realize the Lie algebra of the Heisenberg-Weyl group. This means postulating equal-time commutation relations

$$[\phi(x), \pi(y)] = i\delta(x - y).$$

For non-linear systems the Heisenberg-Weyl group must be replaced with a (more involved) symmetry group of the solution manifold, keeping the general idea of considering as basic conjugate operators those group generators giving central terms under commutation.

We should be able to identify such symmetry group by analyzing the symplectic potential (or Liouville 1-form) in the solution manifold (rather than the Lagrangian) [17], which generalizes $p_i dq^i$ from particle mechanics. The symplectic potential can be obtained by integrating the Lagrangian Poincaré-Cartan [18] form on a Cauchy hypersurface $\Sigma$

The symplectic potential for the $\sigma$-sector is given by (see Ref. [16])

$$\Theta_{G/H}^\sigma = \int_\Sigma \text{Tr}([\theta_\mu, \Lambda][-i\deltaUU^\dagger, \Lambda])d\sigma^\mu$$

which is (semi-)invariant under the local “Euclidean group”

$$U \rightarrow U^\dagger U$$

$$\theta_\mu \rightarrow U^\dagger \theta_\mu U^\dagger + \theta_\mu'$$

Even more, the total symplectic potential

$$\Theta_{\text{MYM}}^{G/H} = \int_\Sigma \text{Tr}(F^{\mu\nu}(\tilde{A})\delta \tilde{A}_\nu + m^2 \tilde{A}_\mu^\dagger[-i\deltaUU^\dagger, \lambda])d\sigma_\mu$$

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is invariant under the local “Massive Yang-Mills group” $G(M)^{G/H}_{\text{MYM}}$ with group law given by $(n^\mu$ characterizes the Cauchy surface):

$$
U''(x) = U'(x)U(x),
$$
$$
\theta'_\mu(x)n^\mu = U'(x)\theta_\mu(x)n^\mu U'(x) + \theta'_\mu(x)n^\mu,
$$
$$
A''_\mu(x) = U'(x)A_\mu(x)U'(x) + A'_\mu(x),
$$
$$
F''_{\mu\nu}(x) = U'(x)F_{\mu\nu}(x)U'(x) + F'_{\mu\nu}(x),
$$
$$
\zeta'' = \zeta\zeta \exp \left( i \int \Sigma d\sigma^\mu(x) J_\mu(U', A', F'; U, A, F) \right),
$$
$$
J_\mu = J^\text{YM}_\mu + J^\sigma_\mu,
$$
$$
J^\text{YM}_\mu = \frac{1}{2\sqrt{2}} \text{Tr} \left( (A'^\nu - \theta'^\nu)U'F_{\mu\nu}U' - F'_{\mu\nu}U'(A'^\nu - \theta'^\nu)U' \right),
$$
$$
J^\sigma_\mu = m^2 \text{Tr} \left( \lambda(U'(A_\mu - \theta_\mu)U' - (A_\mu - \theta_\mu)) \right).
$$

In fact, $G^{G/H}_{\text{MYM}}$ is nothing other than the left-invariant canonical 1-form $\theta^{L(C)}$ of this $U(1)$-centrally extended group.

Denoting $E^\mu_a(x)$ the generator of translations in $A^a_j(x)$ and $\hat{A}^a_j(x)$ the generator of translations in $F^\mu_a(x) \equiv E^\mu_a(x)$ (in much the same way $\hat{p}$ is the generator of translations in $q$ in standard Quantum Mechanics) the Lie algebra of $G(M)^{G/H}_{\text{MYM}}$ can be adopted as equal-time $(d\sigma_\mu \rightarrow d\sigma_0)$ basic “canonically conjugate” commutators:

$$
\begin{align*}
[\hat{G}_a(x), \hat{G}_b(y)] &= iC_{ab}^c \hat{G}_c(x) \delta(x - y), \\
[\hat{A}^j_i(x), \hat{E}^k_b(y)] &= i\delta^j_b \delta^k_b \delta(x - y), \\
[\hat{G}_a(x), \hat{A}^j_b(y)] &= iC_{ac}^j \hat{A}^c_j(x) \delta(x - y) - i\delta^j_b \delta^k_b \delta(x - y), \\
[\hat{G}_a(x), \hat{E}^\mu_b(y)] &= iC_{ab}^c \hat{E}^\mu_c(x) \delta(x - y) - im^2 \delta^\mu_b \delta^c_a \delta(x - y)
\end{align*}
$$

The symmetry above requires some comments concerning the structure of the central extension by $U(1)$, which defines, after all, the structure of our proposal for the quantization of the Massive Yang-Mills Theory. On the one hand, the dynamical content of the fields $A^a_i$, $i = 1, 2, 3$ and their canonical momenta $E^\mu_b$, reflected by the second line in (2), is a consequence of the (current) cocycle $J^M_\mu$, and reproduces the ordinary duality coordinate-momentum for Yang-Mills fields; on the other, the dynamical character of the gauge group parameters themselves, $\varphi^a(x)$, as well as the associated momenta $E^\mu_0(x)$ is dictated by the cocycle $J^\sigma_\mu$, responsible for the central term in the last line of (2). This cocycle is trivial from the strict mathematical point of view but highly non-trivial from the physical point of view as it contributes with a non-trivial piece to the symplectic potential and/or Lagrangian at the classical level and to the corresponding quantization form $\Theta^{G/H}_{\text{MYM}} + \frac{2\pi}{\hbar}$. The interest of this sort of coboundaries with non-trivial physical content was first remarked by Saletan [19] and widely used in GAQ and in the representations of semisimple Lie groups as the Virasoro group and Kac-Moody groups (see [14] and references therein).

A unitary, irreducible representation of this (infinite-dimensional) Lie algebra on wave functionals $\Psi(E^\mu)$ in the “electric-field representation” $E^\mu_a (E^0_a \equiv m^2 \text{Tr}(T_aA))$ can be achieved
as:

\[
\hat{E}_a^\mu \Psi(E) = (E_a^\mu - m^2 \delta_0^\mu \lambda_0) \Psi(E),
\]

\[
\hat{A}_a^\mu \Psi(E) = i \frac{\delta}{\delta E_a^\mu} \Psi(E),
\]

\[
\hat{G}_a \Psi(E) = \left( \nabla \cdot \vec{E}_a + i C_{ab} (E_b^\mu - m^2 \delta_0^\mu \lambda_0) \frac{\delta}{\delta E_b^\mu} \right) \Psi(E)
\]

where the last line accounts for the non-Abelian “Gauss law” when the constraint condition \( \hat{G}_a \Psi(E) = 0 \) is required. A more detailed study of the quantization process will be presented elsewhere [20].

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