Alternative viscous-plastic rheologies for the representation of fracture lines in high-resolution sea ice models

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A little thought experiment...
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A little thought experiment...
...which we can observe on the field.
Motivation?

The overarching motivation

September Arctic Sea Ice Volume (PIOMAS)
Motivation?

We call these deformation lines *Linear Kinematic Features* or *LKFs*.

**LKFs influence**
- Exchange of Energy and Moisture
- Creation of new ice → in leads
- Creation of thick ice → in ridges
- **Influence the mass balance**

**We**
- Observe the LKFs intersection angles in deformation patterns
- Want to reproduce these patterns in sea ice dynamical models

**Figure:** Shear Deformation — From Rampal et al. (2019) — under CC-BY license.
The sea ice Viscous-Plastic (VP) rheological model

The most widely used sea ice model
- Viscous for small deformations $\rightarrow$ Plastic for large deformations (Hibler, 1977)
- Two main components:
  - A yield curve
  - A flow rule

A yield curve
- Transition between Viscous and Plastic in the stress space
- Viscous deformation are slow ($t_{def} \simeq 35$ y)
  - Almost a purely plastic model

A flow rule
- Post-failure deformation
- i.e. the ratio of shear and divergence or convergence
- Can be normal or non-normal to the yield curve

We call rheology the coupling of a yield curve shape and a flow rule.

VP was designed for resolution of $O(100 \text{ km})$
and is now used at resolution of $O(1 \text{ km})$
Models and observation disagree on LKFs intersection angles

**Figure**: PDFs of LKFs half-intersection angles — Derived from Hutter and Losch (2020) – under CC-BY license.

See the work of Nils Hutter on comparing sea ice rheological model here at vEGU21: EGU21-9739
Theory of fracture angles in granular matter

- **Coulomb Angle** $\theta_C$ (Coulomb, 1773):
  The fracture angle depends on the slope of the yield curve, i.e., the stress ratio $\phi$ along the shear line.

- **Roscoe Angle** $\theta_R$ (Roscoe, 1970):
  The fracture angle depends on the orientation of the flow rule, i.e., the strain-rate ratio $\delta$ along the shear line.

- **Arthur Angle** $\theta_A$ (Arthur et al., 1977):
  The fracture angle is the average of $\theta_C$ and $\theta_R$.

  $\implies$ with a normal flow rule, then $\theta_C = \theta_R = \theta_A$
Experimental setup: Uni-axial compression

- Prescribed Strain
- Sea ice
- Open boundaries
- Open water
- Shoreline

Diagram:

- Y-axis: $L_y$
- X-axis: $L_x$
- $2\theta$
Recent results with the same setup

**Ringeisen et al. (2019)**
- Elliptical yield curve with normal flow rule (Hibler, 1979)
- Fracture angles depend on the yield curve slope with a normal flow rule
- Cannot create angles smaller than 30° in uni-axial compression

**Ringeisen et al. (2020)**
- Designed a elliptical yield curve with non-normal flow rule.
- The direction of the flow rule sets the fracture angle → Roscoe angle
- Able to create angles smaller than 30° in uni-axial compression

Here we
- Investigate yield curves that do not have an elliptical shape.
  - Especially Mohr–Coulomb yield curve, known for the modelling granular materials.
  - Insist on good numerical convergence to explore the precise effects of the rheology.
- Idealized compression experiment
- with the MITgcm sea ice package (Losch et al., 2010).
New yield curves: Mohr–Coulomb & Teardrop

Mohr–Coulomb yield curve (MCE)

*non-normal flow rule*

derived from Ip et al. (1991)

\[
\sigma_I - P \dot{\varepsilon}_I, \quad \sigma_{II} - P \dot{\varepsilon}_{II} = \mu \left( P + T \right)
\]

Teardrop yield curve (TD)

*normal flow rule*

modified from Zhang and Rothrock (2005)

\[
\sigma_I - T \dot{\varepsilon}_I, \quad \sigma_{II} - T \dot{\varepsilon}_{II} = \mu (P + T)
\]
Results: Mohr–Coulomb yield curve

- Creates defined shear lines, unlike the formulation of Ip et al. (1991).
Results: Mohr–Coulomb yield curve

- Creates defined shear lines, unlike the formulation of Ip et al. (1991).
- The fracture angles correspond to the Arthur angles.

![Graph showing Mohr–Coulomb yield curve with fracture angles and slope parameter.]
Results: Mohr–Coulomb yield curve

- Creates defined shear lines, unlike the formulation of Ip et al. (1991).
- The fracture angles correspond to the Arthur angles.
Results: Teardrop yield curve

- Creates defined shear lines with small angles.
Results: Teardrop yield curve

- Creates defined shear lines with small angles.
- Modeled angles fit exactly the theoretical angles with normal flow rule.
Conclusions and Outlook

Mohr–Coulomb

- Surprisingly: Shear lines with Arthur angles
  - Contradicts our previous work (Ringeisen et al., 2020).
  - Unknown reason yet.
- Allows to decrease the fracture angles.

Teardrop

Very good agreement with theory. Clean fracture pattern, with issues fixed. Also allows to decrease the angles. Good candidate to reduce the fracture angles overall.

Conclusions

Essential to test our rheological models. We can reduce the fracture angles with non-elliptical yield curves. With a yield curve for granular properties.

Outlook

Yield curves implemented in the MITgcm sea ice package. Currently testing their effect in high-resolution pan-Arctic simulations.
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### Outlook
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Deformation lines in sea ice
- Intersection angles are larger in models than observed.
- Linked to the Viscous-Plastic rheology

Two modified rheologies
- Mohr–Coulomb yield curve — non-normal
- Teardrop yield curve — normal flow rule

Idealized numerical experiment
- Both rheologies allow for smaller angles
- MCE creates fractures with Arthur angles

Investigating rheologies is necessary
- Available in MITgcm now
- Next step: test in pan-arctic setups
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