Well-Covered Graphs Without Cycles of Lengths 4, 5 and 6

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Abstract

A graph \( G \) is well-covered if all its maximal independent sets are of the same cardinality. Assume that a weight function \( w \) is defined on its vertices. Then \( G \) is \( w \)-well-covered if all maximal independent sets are of the same weight. For every graph \( G \), the set of weight functions \( w \) such that \( G \) is \( w \)-well-covered is a vector space. Given an input graph \( G \) without cycles of length 4, 5, and 6, we characterize polynomially the vector space of weight functions \( w \) for which \( G \) is \( w \)-well-covered.

Let \( B \) be an induced complete bipartite subgraph of \( G \) on vertex sets of bipartition \( B_X \) and \( B_Y \). Assume that there exists an independent set \( S \) such that each of \( S \cup B_X \) and \( S \cup B_Y \) is a maximal independent set of \( G \). Then \( B \) is a generating subgraph of \( G \), and it produces the restriction \( w(B_X) = w(B_Y) \). It is known that for every weight function \( w \), if \( G \) is \( w \)-well-covered, then the above restriction is satisfied.

In the special case, where \( B_X = \{x\} \) and \( B_Y = \{y\} \), we say that \( xy \) is a relating edge. Recognizing relating edges and generating subgraphs is an \( \text{NP} \)-complete problem. However, we provide a polynomial algorithm for recognizing generating subgraphs of an input graph without cycles of length 5, 6 and 7. We also present a polynomial algorithm for recognizing relating edges in an input graph without cycles of length 5 and 6.

1 Introduction

Throughout this paper \( G = (V, E) \) is a simple (i.e., a finite, undirected, loopless and without multiple edges) graph with vertex set \( V = V(G) \) and edge set \( E = E(G) \).

Cycles of \( k \) vertices are denoted by \( C_k \). When we say that \( G \) does not contain \( C_k \) for some \( k \geq 3 \), we mean that \( G \) does not admit subgraphs isomorphic to \( C_k \). It is important to mention that these subgraphs are not necessarily induced. Let \( G(\hat{C}_{i_1}, \ldots, \hat{C}_{i_k}) \) the family of all graphs which do not contain \( C_{i_1}, \ldots, C_{i_k} \).

Let \( u \) and \( v \) be two vertices in \( G \). The distance between \( u \) and \( v \), denoted \( d(u, v) \), is the length of a shortest path between \( u \) and \( v \), where the length of
There exists a polynomial time algorithm, which solves the following problem:
Input: A graph \( G = (V, E) \) ∈ \( G(\hat{C}_4, \hat{C}_5, \hat{C}_6, \hat{C}_7) \).

Question: Find \( WCW(G) \).
In order to prove Theorem 1, the following notion has been introduced in [10]. Let $B$ be an induced complete bipartite subgraph of $G$ on vertex sets of bipartition $B_X$ and $B_Y$. Assume that there exists an independent set $S$ such that each of $S \cup B_X$ and $S \cup B_Y$ is a maximal independent set of $G$. Then $B$ is a generating subgraph of $G$, and it produces the restriction: $w(B_X) = w(B_Y)$. Every weight function $w$ such that $G$ is $w$-well-covered must satisfy the restriction $w(B_X) = w(B_Y)$. The set $S$ is a witness that $B$ is generating. In the restricted case that the generating subgraph $B$ is isomorphic to $K_{1,1}$, call its vertices $x$ and $y$. In that case $xy$ is a relating edge, and $w(x) = w(y)$ for every weight function $w$ such that $G$ is $w$-well-covered. The decision problem whether an edge in an input graph is relating is NP-complete [3]. Therefore, recognizing generating subgraphs is NP-complete as well. However, recognizing relating edges can be done polynomially if the input graph is restricted to $G(\hat{C}_4, \hat{C}_6)$ [9], and recognizing generating subgraphs is a polynomial problem when the input graph is restricted to $G(\hat{C}_4, \hat{C}_6, \hat{C}_7)$ [10].

In Section 2 we consider some general properties of $WCW(G)$. In Section 3 we analyze the structure of $WCW(G)$ for graphs without cycles of length 5. In Section 4 we characterize polynomially relating edges in graphs without cycles of length 5 and 6. In Section 5 we characterize polynomially generating subgraphs in graphs without cycles of length 5, 6 and 7. In Section 6 we improve on Theorem 1 by presenting a polynomial algorithm which solves the following problem:

**Input:** A graph $G = (V, E) \in G(\hat{C}_4, \hat{C}_5, \hat{C}_6)$.

**Question:** Find $WCW(G)$.

# 2 The Vector Space $WCW(G)$

## 2.1 A Subspace of $WCW(G)$

In this subsection we describe a procedure, which receives as its input a graph $G = (V, E)$, and returns a vector space of weight functions $w : V \to \mathbb{R}$ such that $G$ is $w$-well-covered. This space is a subspace of $WCW(G)$.

Recall that a vertex $v \in V$ is simplicial if $N[v]$ is a complete graph.

**Theorem 2** Let $S$ be the set of all simplicial vertices in $G = (V, E)$, and $A = \{a_1, ..., a_k\}$ be a maximal independent set of $G[S]$. Define a weight function $w : V \to \mathbb{R}$ as follows:

- for every $1 \leq i \leq k$ choose an arbitrary value for $w(a_i)$;
- for every $v \in V \setminus A$ define $w(v) = w(N(v) \cap A)$.

Then $G$ is $w$-well-covered.

**Proof.** Let $X$ be a maximal independent set of $G$. Since $a_i$ is simplicial, $|N[a_i] \cap X| = 1$ for every $1 \leq i \leq k$. Therefore, $w(X) = \sum_{1 \leq i \leq k} w(a_i) = w(A)$.
Consequently, the collection of all weight functions obtained in accordance with Theorem 2 is a subspace of $W CW(G)$.

### 2.2 The Dimension of $W CW(G)$

In this subsection we present for each integer $k$, a family of graphs $\{C_{m,k,r} \mid m \geq k \geq 1, r \geq 1\}$ with the following properties:

1. $wcdim(C_{m,k,r}) = k$ for each $1 \leq k \leq m$ and for each $r \geq 1$.
2. The size of $C_{m,k,r}$ limits to infinity when $m$ goes to infinity.

Denote the vertices of the cycle $C_m$ by $v_1, ..., v_m$. The graph $C_{m,k,r}$ is obtained from $C_m$ by adding new $k$ disjoint cliques, $A_1, ..., A_k$, each of them is of size $r$. All vertices of $A_i$ are adjacent to $v_i$, for each $1 \leq i \leq k$.

For every $1 \leq i \leq k$, all vertices of $A_i$ are simplicial. A weight function $w$ defined on the vertices of $C_{m,k,r}$ belongs to $W CW(C_{m,k,r})$ if and only if it satisfies the following conditions.

1. $w(x) = w(y)$ for each $x, y \in \{v_i\} \cup A_i$ for each $1 \leq i \leq k$.
2. $w(v_i) = 0$ for each $k + 1 \leq i \leq m$.

If $w \in W CW(C_{m,k,r})$ then the weight of every maximal independent set in the graph is $\Sigma_{1 \leq i \leq k} w(v_i)$, and $wcdim(C_{m,k,r}) = k$.

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**Figure 1:** The graph $C_{6,3,4}$. 

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4
3 Graphs Without Cycles of Length 5

Let \( G = (V, E) \in G(\hat{C}_5) \), and let \( w : V \rightarrow \mathbb{R} \). In this section we find a necessary condition that \( G \) is \( w \)-well-covered.

Define \( L(G) \) to be the set of all vertices \( v \in V \) such that one of the following holds:

1. \( d(v) = 1 \).
2. \( d(v) = 2 \) and \( v \) is on a triangle.

For every \( v \in V \) define \( D(v) = N(v) \setminus N(N_2(v)) \), and let \( M(v) \) be a maximal independent set of \( D(v) \).

The fact that \( G \in G(\hat{C}_5) \) implies that for every \( v \in V \), the subgraph induced by \( D(v) \) can not contain a path of length 3. Therefore, every connected component of \( D(v) \) is either a \( K_3 \) or a star. (\( K_1 \) and \( K_2 \) are restricted cases of a star.)

**Lemma 3** Let \( G = (V, E) \in G(\hat{C}_5) \), and let \( v \in V \). Then every maximal independent set of \( N_2(v) \) dominates \( N(v) \cap N(N_2(v)) \).

**Proof.** Let \( v \in V \), and let \( T \) be a maximal independent set of \( N_2(v) \). Assume on the contrary that \( T \) does not dominate \( N(v) \cap N(N_2(v)) \). Let \( u \in (N(v) \cap N(N_2(v))) \setminus N(T) \), and let \( u' \in N(u) \cap N_2(v) \). Clearly, \( u' \not\in T \) but \( u' \) is adjacent to a vertex \( t \in T \). The fact that \( t \in N_2(v) \) implies that there exists a vertex \( x \in N(t) \cap N(v) \). Hence, \( (v, u, u', t, x) \) is a cycle of length 5, which is a contradiction. Therefore, every maximal independent set of \( N_2(v) \) dominates \( N(v) \cap N(N_2(v)) \).

**Corollary 4** Let \( G = (V, E) \in G(\hat{C}_5) \), and let \( v \in V \). If \( D(v) = \emptyset \) then every maximal independent set of \( N_2(v) \) dominates \( N(v) \).

**Proof.** If \( D(v) = \emptyset \), then \( N(v) \cap N(N_2(v)) = N(v) \), and, consequently, by Lemma 3 every maximal independent set of \( N_2(v) \) dominates \( N(v) \).

**Theorem 5** Assume that \( G = (V, E) \in G(\hat{C}_5) \) is \( w \)-well-covered for some weight function \( w : V \rightarrow \mathbb{R} \). Then \( D(v) \neq \emptyset \) implies \( w(v) = w(M(v)) \) for every \( v \in V \setminus L(G) \).

**Proof.** Let \( v \in V \setminus L(G) \), let \( T \) be a maximal independent set of \( N_2(v) \), and let \( S \) be a maximal independent set of \( G \setminus N[v] \), which contains \( T \). Then \( S \cup \{v\} \) and \( S \cup M(v) \) are two maximal independent sets of \( G \). The fact that \( G \) is \( w \)-well-covered implies that \( w(S \cup \{v\}) = w(S \cup M(v)) \). Therefore, \( w(v) = w(M(v)) \).
4 Relating Edges in Graphs Without Cycles of Length 5 and 6

In this section we prove that recognizing relating edges in an input graph, which does not contain cycles of length 5 and 6, is a polynomial problem.

Theorem 6 Let $G = (V, E) \in G(\tilde{C}_5, \tilde{C}_6)$ and let $xy \in E$. Then $xy$ is relating if and only if $N_2(\{x, y\})$ dominates $N(x) \triangle N(y)$.

Proof. Assume that $N_2(\{x, y\})$ dominates $N(x) \triangle N(y)$. The following algorithm returns a witness that $xy$ is relating.

Construct a set $S_x$ as follows. For every vertex $x' \in N(x)$ add to $S_x$ a vertex $x'' \in N(x') \cap N_2(x)$. The set $S_x$ is independent, because if $x''_1$ and $x''_2$ were two adjacent vertices in $S_x$, then there existed two distinct vertices $x'_1 \in N(x) \cap N(x''_1)$ and $x'_2 \in N(x) \cap N(x''_2)$. Hence, $(x, x'_1, x''_1, x'_2, x''_2)$ was a cycle of length 5.

Construct similarly an independent set $S_y$ by choosing a vertex $y'' \in N(y') \cap N_2(y)$ for every $y' \in N(y)$. The fact that $G$ does not contain cycles of length 6 implies that $S_x \cup S_y$ is independent too. Let $S$ be a maximal independent set of $G \setminus N[\{x, y\}]$ which contains $S_x \cup S_y$. Then $S \cup \{x\}$ and $S \cup \{y\}$ are maximal independent sets of $G$. Therefore, $xy$ is related, and $S$ is the witness.

Assume $xy$ is relating. Let $S$ be a witness that $xy$ is relating. Then $S \cap N_2(\{x, y\})$ dominates $N(x) \triangle N(y)$. □

5 Generating Subgraphs in Graphs Without Cycles of Lengths 5, 6 and 7

In this section we prove that recognizing generating subgraphs in an input graph, which does not contain cycles of lengths 5, 6 and 7, is a polynomial problem.

Theorem 7 Let $G \in G(\tilde{C}_5, \tilde{C}_6, \tilde{C}_7)$, and let $B$ be an induced complete bipartite subgraph of $G$ on vertex sets of bipartition $B_X$ and $B_Y$. Then $B$ is generating if and only if $N_2(B)$ dominates $N(B_X) \triangle N(B_Y)$.

Proof. Assume that $B$ is generating. Let $S$ be a witness of $B$. Then $S \cap N_2(B)$ dominates $N(B_X) \triangle N(B_Y)$, therefore $N_2(B)$ dominates $N(B_X) \triangle N(B_Y)$.

Suppose that $N_2(B)$ dominates $N(B_X) \triangle N(B_Y)$. Let $S_X$ be a maximal independent set of $N_2(B_X) \cap N_3(B_Y)$, and let $S_Y$ be a maximal independent set of $N_2(B_Y) \cap N_3(B_X)$. The fact that $G$ does not contain cycles of length 6 implies that $S = S_X \cup S_Y$ is independent. The fact that $G$ does not contain cycles of length 5 implies that there are no edges between $S_X$ and $N(B_Y) \cap N_2(B_X)$. Similarly, there are no edges between $S_Y$ and $N(B_X) \cap N_2(B_Y)$.

Assume on the contrary that there exists a vertex $x' \in N(B_X) \cap N_2(B_Y)$ which is not dominated by $S$. Clearly, $x'$ is adjacent to a vertex $x'' \in N_2(B_X) \cap N_2(B_Y)$. Hence, $x''$ is a neighbor of a vertex $v \in S_X$. Clearly, $v$ is adjacent to a vertex $w \in N(B_X) \cap N_2(B_Y)$.
Let $x_1$ be a neighbor of $x'$ in $B_X$, and let $x_2$ be a neighbor of $w$ in $B_X$. If $x_1 = x_2$ then $(x_1, x', x'', v, w)$ is a cycle of length 5. Otherwise, let $y$ be any vertex of $B_Y$. Then $(x_1, x', x'', v, w, x_2, y)$ is a cycle of length 7. In both cases we obtained a contradiction. Therefore $S$ dominates $N(B_X) \cap N_2(B_Y)$. Similarly, $S$ dominates $N(B_Y) \cap N_2(B_X)$. Let $S^*$ be any maximal independent set of $G \setminus N[B]$ which contains $S$. Then $S^*$ is a witness that $B$ is generating.

6 The Vector Space of Well-Covered Graphs Without Cycles of Lengths 4, 5, and 6

In this section $G = (V, E) \in G(\hat{C}_4, \hat{C}_5, \hat{C}_6)$. Therefore, $L(G)$ is the set of all simplicial vertices of $G$. For each $v \in V$ every connected component of $N(v)$ is either a $K_1$ or a $K_2$. Also, every connected component of $D(v)$ is either a $K_1$ or a $K_2$.

6.1 A polynomial characterization of $WCW(G)$.

Theorem 8 Let $G \in G(\hat{C}_4, \hat{C}_5, \hat{C}_6)$. There exists a polynomial time algorithm which finds $WCW(G)$.

The proof of Theorem\ref{thm:poly} is based on the polynomial characterization of well-covered graphs without cycles of length 4 and 5, found by Finbow, Hartnell and Nowakowski.

Theorem 9 \ref{thm:poly} Let $H = (V, E) \in G(\hat{C}_4, \hat{C}_5)$. Then $H$ is well-covered if and only if one of the following conditions holds.

1. There exists a set $\{v_1, ..., v_k\} \subseteq V$ of simplicial vertices such that $|N[v_i]| \leq 3$ for every $1 \leq i \leq k$, and $\{N[v_i]|1 \leq i \leq k\}$ is a partition of $V$.

2. $H$ is isomorphic to $C_7$ or to $T_{10}$.

The following lemmas together with Theorem\ref{thm:poly} imply Theorem\ref{thm:poly}.

Lemma 10 Let $w : V \rightarrow \mathbb{R}$ be a weight function defined on $G$, and assume that $G$ is $w$-well-covered. Let $v \in V \setminus L(G)$. If there exists a vertex $u \in N[v]$ such that $D(u) \neq \emptyset$ then $w(v) = w(M(v))$.

Proof. If $D(v) \neq \emptyset$ then according to Theorem\ref{thm:poly} $w(v) = w(M(v))$, and the lemma holds.

Suppose $D(v) = \emptyset$ and there exists a vertex $u \in N(v)$ such that $D(u) \neq \emptyset$. Let $T$ be a maximal independent set of $(N_2(v) \cap N_3(u)) \cup (N_2(u) \cap N_3(v))$, let $S_1$ be a maximal independent set of $G$ such that $S_1 \supseteq T \cup \{u\}$, and let $S_2 = (S_1 \setminus \{u\}) \cup M(u) \cup \{v\}$. Clearly, $w(S_1) = w(S_2)$, therefore $w(u) = w(M(u) \cup \{v\})$. However, by Theorem\ref{thm:poly} $w(u) = w(M(u))$. Hence $w(v) = 0 = w(M(v))$. \hfill \blacksquare
Lemma 11 Assume that \( L(G) \neq \emptyset \). Let \( w : V \rightarrow \mathbb{R} \) be a weight function defined on the vertices of \( G \). Then \( G \) is \( w \)-well-covered if and only if \( w(v) = w(M(v)) \) for every \( v \in V \setminus L(G) \).

**Proof.** If part: Assume that \( G \) is \( w \)-well-covered. Let \( v \in V \setminus L(G) \). It is enough to prove that \( w(v) = w(M(v)) \).

If there exists a vertex \( u \in N_v \) such that \( D(u) \neq \emptyset \) then \( w(v) = w(M(v)) \) by Lemma 10.

Suppose that \( D(u) = \emptyset \) for every \( u \in N_v \). Let \( H \) be the subgraph of \( G \) induced by \( \{ v \in V : D(v) = \emptyset \} \). The fact that \( L(G) \neq \emptyset \) implies that \( H \neq G \). Let \( C \) be the connected component of \( H \) which contains \( v \). By Theorem 6 all edges of \( C \) are relating. Therefore, all vertices of \( C \) are of the same weight. There exists a vertex \( x \in C \) which is adjacent to a vertex \( y \notin C \). Clearly, \( D(y) \neq \emptyset \). Lemma 10 implies that \( w(x) = w(M(x)) = 0 \). Hence, \( w(z) = 0 = w(M(z)) \) for every vertex \( z \in C \).

Only if part: Assume that \( w(v) = w(M(v)) \) for every \( v \in V \setminus L(G) \), and \( L(G) \neq \emptyset \). Since \( L(G) \) is the set of simplicial vertices in the graph, Theorem 2 implies that \( w \in VS(G) \).

Corollary 12 Assume that \( L(G) \neq \emptyset \). Then \( G \) is well-covered if and only if \( D(v) \) is a copy of \( K_1 \) or \( K_2 \), for every \( v \in V \setminus L(G) \).

Lemma 13 If \( L(G) = \emptyset \) then all edges of \( G \) are relating.

**Proof.** \( L(G) = \emptyset \) implies \( D(v) = \emptyset \) for every \( v \in V \). Therefore, \( N_2(\{x, y\}) \) dominates \( N(x) \triangle N(y) \) for every edge \( xy \). Hence, by Theorem 6 all edges of \( G \) are relating.

Lemma 14 Assume that \( L(G) = \emptyset \). Then the following holds.

1. If \( G \) is isomorphic to either \( C_7 \) or \( T_{10} \) Then \( w \in WCW(G) \) if and only if there exists \( k \in \mathbb{R} \) such that \( w \equiv k \).
Otherwise, $WCW(G)$ contains only the zero function.

**Proof.** By Lemma 13, all edges of $G$ are relating. Therefore, if $w \in WCW(G)$, then all vertices of $G$ are of the same weight. Hence, it should be decided which of the following two cases holds.

1. $G$ is well-covered. Hence, $w \in WCW(G)$ if and only if there exists $k \in \mathbb{R}$ such that $w \equiv k$. In this case $wcdim(G) = 1$.

2. $G$ is not well-covered. Hence, $w \in WCW(G)$ if and only if $w \equiv 0$. In this case $wcdim(G) = 0$.

Since $L(G) = \emptyset$, there are no simplicial vertices in $G$. Consequently, the first condition of Theorem 9 does not hold. By Theorem 9 the graph is well-covered if and only if it is isomorphic to $C_7$ or to $T_{10}$. 

An example of the above is the graph $D_{12}$. Clearly, $L(D_{12}) = \emptyset$, and the graph does not contain simplicial vertices. All edges in the graph are relating. The graph is not well-covered because $\{v_3, v_6, v_9, v_{12}\}$ and $\{v_3, v_5, v_7, v_9, v_{12}\}$ are two maximal independent sets of with different cardinalities. Therefore $WCW(D_{12})$ contains only the zero function.

### 6.2 The Algorithm and its Complexity

The following algorithm receives as its input a graph $G = (V, E) \in G(\hat{C}_4, \hat{C}_5, \hat{C}_6)$, and finds $WCW(G)$. All elements of $WCW(G)$ can be obtained by this algorithm.

**Algorithm 15 Vector Space**

1. Find $L(G)$.

2. **If** $G$ is isomorphic to $C_7$ or to $T_{10}$
   
   (a) Assign an arbitrary value for $k$.
   
   (b) For each $v \in V$ denote $w(v) = k$

3. **Else**
   
   (a) Find a maximal independent set $S$ of $L(G)$, and assign arbitrary weights to the elements of $S$.
   
   (b) For each vertex $l \in L(G) \setminus S$ denote $w(l) = w(N(l) \cap S)$.
   
   (c) For each $v \in V \setminus L(G)$
      
      i. Find $D(v)$.
      
      ii. Construct a maximal independent set $M(v)$ of $D(v)$.
      
      iii. Denote $w(v) = w(M(v))$
Correctness of the algorithm. If the condition of Step 2 holds, then by Theorem 9 and Lemma 13 the algorithm returns \( WCW(G) \).

Assume that the condition of Step 2 does not hold. Denote the elements of the set \( S \) found in Step 3a by \( S = \{s_1, ..., s_{|S|}\} \). Then \( wcdim(G) = |S| \), and \( w(s_1), ..., w(s_{|S|}) \) are the free variables of the vector space. Every connected component of \( L(G) \) contains at most 2 vertices. According to Step 3b, if \( l_1 \) and \( l_2 \) are two vertices of the same connected component of \( L(G) \), then \( w(l_1) = w(l_2) \). Therefore, if \( x \in V \setminus L(G) \), and \( T_1, T_2 \) are two maximal independent sets of \( G[N(x) \cap L(G)] \), then \( w(T_1) = w(T_2) \).

In Step 3c, \( D(v) \subseteq L(G) \), for every vertex \( v \in V \setminus L(G) \). The set \( M(v) \) can be constructed in more than one possible way. However, \( w(M(v)) \) is uniquely defined by Step 3b. If \( L(G) \neq \emptyset \), by Lemma 14 the algorithm returns \( WCW(G) \). If \( L(G) = \emptyset \), by Lemma 14 the algorithm returns \( WCW(G) \).

Complexity analysis. Steps 1 and 2 run in \( O(|V|) \) time. Steps 3a and 3b can be implemented in \( O(|E|) \) time. Step 3c is a loop with \( |V| \) iterations. Each iteration can be implemented in \( O(|E|) \) time. Therefore, the total complexity of Step 3c is \( O(|V| |E|) \), which is the total complexity of the whole algorithm as well.

7 Open Question

In this paper we presented a polynomial algorithm whose input is a graph in \( G(\hat{C}_4, \hat{C}_5, \hat{C}_6) \), and its output is \( WCW(G) \). On the other hand, there is a polynomial characterization of well-covered graphs without cycles of lengths 4 and 5 [8]. Thus it is a natural step in learning \( w \)-well-covered graphs to ask whether the following problem is polynomially solvable.

Input: A graph \( G = (V, E) \in G(\hat{C}_4, \hat{C}_5) \).

Output: \( WCW(G) \).

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