Comment on "New modes of halo excitations in the $^6$He nucleus"

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We try to explain the differences in the $^6$He dipole strength function in 1 and 2. We perform the full basis calculation of the strength function with the same renormalized interaction as in 1 and show that the size of the basis, needed for converged calculations of the $^6$He continuum spectrum, is much larger than that for the discrete spectrum. The renormalized interaction of 1 therefore cannot be used for the continuum spectrum calculations with the same basis as for the ground state.

In a recent paper 1 the continuum properties of $^6$He were discussed within a three body $nna$ model. The strength functions were computed using the hyper-spherical harmonics approach. The authors argued that the main features of the continuum spectrum of the $nna$ system can be obtained using only a few hyper-spherical harmonics with the hyper-spherical quantum number $K \leq 6$, if one simultaneously renormalizes the $na$ interaction by introducing additional attraction.

However the dipole strength function in 1 differs significantly from our recent calculation 2, where we used the same three-body model. The purpose of this comment is to explain these differences.

We show that the continuum state calculations of the $nna$ system demand a much larger basis, $K \leq 100$, compared to the bound state. In such case the simple prescription to renormalize the $na$ interaction aiming at the bound state might not be adequate for the continuum.

The method we use is, in principle, similar to that of 1 – the angular part of the wave function is expanded in terms of the hyper-spherical harmonics and then the resulting system of coupled hyper-radial equations is solved. However our approach has the following modifications 3: i) We use Faddeev rather than the Schrödinger equation; ii) Prior to solving the hyper-radial equations we calculate the eigenvalues $\lambda_n$ as functions of the hyper-radius $\rho$, of the angular part of the Faddeev operator. The quantities $\lambda_n(\rho)\rho^{-2}$ then serve as effective potentials in the radial equations; iii) We use analytic long-distance solutions of the Faddeev equations for short range potentials 4 which greatly enhances the method and allows fully converged calculations.

The $na$ interaction, originally fit to the scattering data, was somewhat modified in 1, 2, 3 in order to compensate for the limited basis,

\[
\hat{V}_{na}(r) = \exp\left(-r^2/2.35^2\right) \times 50 \hat{P}_s - 48.3 \hat{P}_p - 23.0 \hat{P}_d - 11.71 \mathbf{s} \cdot \mathbf{s}
\]  

Here $\hat{P}_l$ is a projection operator on the corresponding orbital momentum state $l$ and $\mathbf{s}$ is the neutron spin.

All lengths are in fm and energies in MeV. Due to the over attraction this interaction fails to reproduce the experimental $na$ phase shifts.

For the $nn$ interaction we use a simple Gaussian 1, similar to the one used in 3. The potential reproduces the experimental effective range in s-wave and scattering lengths in s- and p-waves,

\[
\hat{V}_{nn}(r) = \exp\left(-r^2/1.8^2\right) \times (2.92 + 45.22 s_1 s_2 - 12.08 s_1 + s_2 + 26.85 \hat{S}_{12}),
\]

where $s_1, s_2$ are the spins of the neutrons, $\hat{S}_{12}$ is the usual tensor operator.

In Fig. 1 we illustrate the convergence of the lowest angular eigenvalue using, as examples, calculations with $K_{\text{max}} = 9, K_{\text{max}} = 19, K_{\text{max}} = 99$. Obviously the basis limited to $K < 9$, used in 1, is by far not converged. It is only the curve for $K_{\text{max}} \approx 100$ that finally converged towards the large-distance asymptotics. This long tail of the eigenvalue arises, in the present case, due to large scattering length in the $nn$ subsystem 1. The scale of convergence towards the asymptotic behavior is then around 20 fm.

The convergence problems for the lowest effective potential is illustrated in Fig. 2. The converged curve has the minimum value of about 0.15 MeV and the low (0.3 MeV) barrier extending into the asymptotic region beginning at about 25 fm. In contrast the curve for $K_{\text{max}} = 9$ has minimum and maximum values at 0.7 MeV and 1.4 MeV, respectively. A more realistic potential calculated with a correct $na$ interaction 2, is accidently close to the curve for $K_{\text{max}} = 19$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The lowest angular eigenvalue for $^6$He ($J^* = 1^-$) calculated with different $K_{\text{max}}$. Also shown is the eigenvalue obtained from the asymptotic large distance expansion of the Faddeev equations 4.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The lowest angular eigenvalue for $^6$He ($J^* = 1^-$) calculated with different $K_{\text{max}}$. Also shown is the eigenvalue obtained from the asymptotic large distance expansion of the Faddeev equations 4.}
\end{figure}
We have to note, however, that due to the additional attraction introduced in the $n\alpha$ interaction in [1] the converged dipole strength function here is excessively shifted towards lower energies and therefore is not realistic. The realistic dipole strength function [2] is close to the curve $K_{max} = 19$. In this case the limited basis is approximately compensated by the over-attractive potential. Note that for the $0^+$ bound state calculations the corresponding compensatory basis was $K_{max} = 6$ [1]. This shows that the continuum spectrum demands more careful and accurate calculations.

In conclusion, in order to explain the differences between the dipole strength functions from [1] and [2] we have performed a fully converged calculation of the $J^\pi = 1^-$ continuum spectrum of $^6$He using the same renormalized $n\alpha$ interaction as in [1]. We have shown that the number of basis functions needed for converged calculations of the $^6$He continuum spectrum is much larger than that for the discrete spectrum. We conclude that the technique of [1] to renormalize the $n\alpha$ interaction to compensate for the limited basis becomes ambiguous when applied to the continuum spectrum. The reason for the differences is therefore that this renormalized interaction is not appropriate for continuum calculations with the same basis size as for the ground state.

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