Control of neuronal excitability by calcium binding proteins: a new mathematical model for striatal fast-spiking interneurons

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INTRODUCTION

Calcium binding proteins, such as parvalbumin (PV), are abundantly expressed in distinctive patterns in the central nervous system but their physiological function remains poorly understood. Notably, at the level of the striatum, where PV is only expressed in the fast-spiking (FS) interneurons. FS interneurons form an inhibitory network modulating the output of the striatum by synchronizing medium-sized spiny neurons (MSN). So far the existing conductance-based computational models for FS neurons did not allow the study of the coupling between PV concentration and electrical activity. In the present paper, we propose a new mathematical model for the striatal FS interneurons that includes apamin-sensitive small conductance \( Ca^{2+} \)-dependent \( K^+ \) channels (SK) and the presence of a calcium buffer. Our results show that a variation in the concentration of PV can modulate substantially the intrinsic excitability of the FS interneurons and therefore may be involved in the information processing at the striatal level.

Keywords: calcium dynamics, parvalbumin, striatal fast-spiking interneurons, excitability, mathematical model

Calcium binding proteins, such as parvalbumin (PV), are abundantly expressed in distinctive patterns in the central nervous system but their physiological function remains poorly understood. Notably, at the level of the striatum, where PV is only expressed in the fast-spiking (FS) interneurons. FS interneurons form an inhibitory network modulating the output of the striatum by synchronizing medium-sized spiny neurons (MSN). So far the existing conductance-based computational models for FS neurons did not allow the study of the coupling between PV concentration and electrical activity. In the present paper, we propose a new mathematical model for the striatal FS interneurons that includes apamin-sensitive small conductance \( Ca^{2+} \)-dependent \( K^+ \) channels (SK) and the presence of a calcium buffer. Our results show that a variation in the concentration of PV can modulate substantially the intrinsic excitability of the FS interneurons and therefore may be involved in the information processing at the striatal level.
In this study, we propose a new conductance-based computational model for striatal FS interneurons that includes the influence of PV and the presence of SK channels providing coupling between excitability and calcium dynamics during the spike generation. This model allows us to investigate the effect of variations in the concentration of PV on striatal FS interneurons activity. We show that excitability of FS neurons depends on PV concentration in the concentration of PV on striatal FS interneurons activity. We generation. This model allows us to investigate the effect of variations between excitability and calcium dynamics during the spike generation.

**MATERIALS AND METHODS**

**FS NEURON COMPUTATIONAL MODEL**

Our computational model is adapted from the conductance-based model of Erisir (Erisir et al., 1999) of a FS neocortical interneuron. The ionic currents of the Erisir model consist of a fast transient Na⁺ current $I_{Na}$, a slow delayed rectifier potassium current of Kv3.1 type $I_{Kv3}$, a slow delayed rectifier potassium current of Kv1.3 type $I_{Kv1}$, and a passive leak current $I_{leak}$. We add to this model a HVA calcium current $I_{Ca}$ (Stocker, 2004) and a SK potassium current, $I_{SK}$. Charge conservation governs the membrane potential dynamics through the following equation:

$$C_m \frac{dV}{dt} = -I_{Na} - I_{Kv1} - I_{Kv3} - I_{Ca} - I_{SK} - I_{leak} + I_{app}$$

(1)

where $C_m$, $V$ are the membrane capacitance and potential of the FS neuron, $I_{app}$ is an external applied current. The ionic currents are given by:

$$I_{Na} = g_{Na}m^3h(V - V_{Na})$$

(2)

$$I_{Kv1} = g_{Kv1}n^4(V - V_{K})$$

(3)

$$I_{Kv3} = g_{Kv3}n^2(V - V_{K})$$

(4)

$$I_{SK} = g_{SK}k^2(V - V_{K})$$

(5)

$$I_{Ca} = g_{Ca}a_{Ca}^2(V - V_{Ca})$$

(6)

$$I_{leak} = g_{leak}(V - V_{leak})$$

(7)

where $m$ and $h$ are respectively the activation and inactivation gating variables of the $I_{Na}$ current, $n_1$, $n_3$, $k$, $a$ are respectively the activation variables of $I_{Kv1}$, $I_{Kv3}$, $I_{SK}$, $I_{Ca}$ currents. The kinetic of the $m$ and $a$ activation variables are considered fast compared to the other gating variables and are set to their steady-state value $m = m_{\infty}(V)$, $a = a_{\infty}(V)$. The membrane capacitance was set to 30 pF, the leak conductance $g_{leak}$ to 2.5 nS and the leak reversal potential to −68 mV, to match the known experimental membrane capacitance (25–30 pF), membrane resistance (400 MΩ), and resting membrane potential (−70 mV). The other reversal potentials are $V_{Na} = 74$ mV, $V_{K} = −90$ mV, $V_{Ca} = 80$ mV, and the maximal ionic conductances are $g_{Na} = 700$ nS, $g_{Kv1} = 2$ nS, $g_{Kv3} = 300$ nS, $g_{Ca} = 30$ nS, $g_{SK} = 2$ nS. The dynamic of the other gating variables (excepted $x$) are governed by:

$$\frac{dx}{dt} = \alpha_x(V)(1 - x) - \beta_x(V)x$$

(8)

$$x_{\infty} = \frac{\alpha_x}{\alpha_x + \beta_x}$$

(9)

$$\tau_x = \frac{1}{\alpha_x + \beta_x}$$

(10)

where $x = h, n_1, n_3$. The kinetics of the $\alpha_x, \beta_x$ are exactly as published in Maniccia et al. (2007). The kinetics of the $a$ variable was adapted from Roussel et al. (2006) and follows:

$$a_{\infty} = \frac{1}{1 + \exp \left(\frac{V - \theta}{\gamma}\right)}$$

(11)

$$\tau_a = \frac{8.0}{1 + \exp(-6.22(V - 30))} + \frac{0.1(V + 8.9)}{\exp(0.2(V + 8.9)) - 1}$$

(12)

The $k$ activation variable for SK channels is $Ca^{2+}$ dependent and voltage independent. The equation for its time evolution was taken from Goldberg et al. (2009):

$$\frac{dk}{dt} = \frac{(k_{\infty}([Ca^{2+}]_i) - k)}{\tau_k}$$

(13)

$$k_{\infty} = \frac{[Ca^{2+}]_i}{K_{SK} + [Ca^{2+}]_i}$$

(14)

$$\tau_k = \frac{1}{K_{SK} + [Ca^{2+}]_i}$$

(15)

where $K_{SK} = k_{off,SK}/k_{on,SK}$. The values of $k_{on,SK} = 0.4 \mu M^{-1} ms^{-1}$ ($Ca^{2+}$-binding rate) and $k_{off,SK} = 0.2 ms^{-1}$ (from Goldberg et al., 2009). In presence of PV, the coupling of the calcium dynamic is done via the following equations:

$$\frac{d[Ca^{2+}]_i}{dt} = -\frac{I_{Ca}}{2FA_D} - \gamma([Ca^{2+}]_i - [Ca^{2+}]_{rest}) - \frac{d[PV_{Ca}]}{dt}$$

(16)

$$\frac{d[PV_{Ca}]}{dt} = k_{on,ca}[Ca^{2+}]_i[PV]_i - k_{off,ca}[PV_{Ca}]_i$$

(17)

$$\frac{d[PV_{Mg}]}{dt} = k_{on,mg}[Mg^{2+}]_i[PV]_i - k_{off,mg}[PV_{Mg}]_i$$

(18)

where $[Ca^{2+}]_i$ and $[PV]_i$ represent respectively the free intracellular $Ca^{2+}$ concentration and the concentration of free PV, $[PV_{Ca}]_i$ and $[PV_{Mg}]_i$ are the concentration of PV bound to $Ca^{2+}$ and Mg²⁺. The total PV concentration $[PV]_T = [PV]_i + [PV_{Ca}]_i + [PV_{Mg}]_i$. We assume that $[Mg^{2+}]_i$ is constant as in (Lee et al., 2000). $[Mg^{2+}]_i$ was set to 500 $\mu M$ in agreement with the values found within neurons (300–600 $\mu M$; Li-Smerin et al., 2001). The association and dissociation constant of PV with $Ca^{2+}$ and $Mg^{2+}$ are $k_{on,ca} = 0.1 \mu M^{-1} ms^{-1}$, $k_{off,ca} = 0.001 ms^{-1}$, and $k_{on,mg} = 0.0008 \mu M^{-1} ms^{-1}$, $k_{off,mg} = 0.025 ms^{-1}$ (Lee et al., 2000). We consider $Ca^{2+}$ fluxes across a shell of thickness $d = 0.2 \mu m$ under cell surface (area $A = 3000 \mu m^2$). The inward flux is $-I_{Ca}/2FA_D$ (F is the Faraday constant). The term $\gamma([Ca^{2+}]_i - [Ca^{2+}]_{rest})$ is the clearance mechanism associated with the $Ca^{2+}$ fluxes across the plasma membrane or storage organelles ($\gamma = 1 ms^{-1}$, $[Ca^{2+}]_{rest} = 0.07 \mu M$). In presence of the slow or fast buffer, the coupling of the calcium dynamic is done...
via the following equations:

\[
\frac{d[Ca^{2+}]}{dt} = -\frac{I_{Ca}}{2FAd} - y \left([Ca^{2+}]_i - [Ca^{2+}]_{rest}\right) - \frac{d[BCa]_i}{dt}
\]

(19)

\[
\frac{d[BCa]_i}{dt} = k_{on}[Ca^{2+}]_i[B]_i - k_{off}[BCa]_i
\]

(20)

where \([B]_i\), \([BCa]_i\) are the concentration of free and bound buffer (slow or fast). The total buffer concentration \([B]_T = [B]_i + [BCa]_i\). The list of model parameters are shown in Table 1. Numerical simulations traces are obtained after an initial integration of 4 s. The equations of the model are numerically solved using a fourth-order Runge–Kutta integration method (Press et al., 1992). The bifurcation diagram was built with the software XPPAUT 6.10 (Free Software Foundation Inc., Cambridge, USA).

RESULTS

EXPERIMENTAL RELEVANCE OF FS NEURON COMPUTATIONAL MODEL

We use a computational model, based on experimental data, to investigate how \(Ca^{2+}\) buffering by PV affects striatal FS interneuron excitability. Striatal FS neurons selectively express the slow calcium buffer PV (Kawaguchi et al., 1995). We have strong evidence that they also possess apamin-sensitive small conductance SK channels. Therefore we propose a FS interneuron model, adapted from Erísir et al. (1999), that includes the presence of SK channels and of PV calcium binding proteins. Since PV possesses mixed \(Ca^{2+}/Mg^{2+}\) binding sites, we take into account in our model the competition between \(Ca^{2+}\) and \(Mg^{2+}\) for PV binding sites (see Materials and Methods). Until recently, parvalbumin was considered as a slow calcium buffer similar to the synthetic chelator EGTA (Schwaller et al., 2009) but new evidence shows that under certain physiological conditions, at high concentration, PV may act as a fast calcium buffer similar to the synthetic chelator BAPTA (Franconville et al., 2011; Eggermann and Jonas, 2012). Therefore we have included three buffering conditions in our study: PV with its mixed \(Ca^{2+}/Mg^{2+}\) binding sites, a slow buffer similar to EGTA, and a fast buffer similar to the metal-free form PV. For these three buffering conditions, we have studied the effect of the calcium buffer on the excitability of FS neurons, for different buffer concentrations. In our model, the slow calcium buffer has a \(K_D = 0.1 \mu M\) and a \(k_{on} = 0.01 \mu M^{-1} \text{ms}^{-1}\) similar to EGTA (Schwaller et al., 2002; Schwaller, 2009). The fast calcium buffer has a \(K_D = 0.01 \mu M\) and a \(k_{on} = 0.1 \mu M^{-1} \text{ms}^{-1}\) similar to those of metal-free form PV (Eberhard and Erne, 1994; Lee et al., 2000).

Figure 1A shows a bifurcation diagram of our FS model, with PV as calcium buffer, where the bifurcation parameter is the applied current (\(I_{app}\)). Our model generates stable oscillations in the physiological range. A supercritical Hopf bifurcation (HB) at \(I_{app} = 44 \text{ pA}\) gives rise to a small window of periodic solutions that lose stability at \(I_{app} = 45 \text{ pA}\). For \(I_{app} > 44 \text{ pA}\) there is a branch of unstable periodic solutions that ends at a saddle-node of limit cycle (SNLC), for \(I_{app} = 68 \text{ pA}\). For \(I_{app} = I_{SNLC}\), a branch of stable

| Table 1 | Model parameters. |
|-----------------|-----------------|-----------------------------|-----------------------------|
| Definition      | Parameters      | Values                      | Reference                    |
| Sodium conductance | \(g_{Na}\)      | 700 nS                      | Jolivet et al. (2004)        |
| Sodium reversal potential | \(V_{Na}\)      | 74 mV                      |                               |
| Kv1 potassium conductance | \(g_{K1}\)      | 2 nS                      |                               |
| Kv3 potassium conductance | \(g_{K3}\)      | 300 nS                      | Goldberg et al. (2009)       |
| SK potassium conductance | \(g_{K}\)       | 30 nS                      | Roussel et al. (2006)        |
| Potassium reversal potential | \(V_{K}\)      | -90 mV                      | Jolivet et al. (2004)        |
| SK affinity for calcium | \(K_{Sc}\)     | 0.5 \mu M                   |                               |
| Calcium conductance | \(g_{Ca}\)      | 30 nS                      |                               |
| Calcium reversal potential | \(V_{Ca}\)      | 80 mV                      |                               |
| Membrane capacitance | \(C_{m}\)       | 30 pF                      |                               |
| Leak conductance | \(g_{leak}\)     | 2.5 nS                      |                               |
| Leak reversal potential | \(V_{leak}\)    | -68 mV                      |                               |
| \(Ca^{2+}\) extrusion rate | \(\gamma\)     | 1 ms\(^{-1}\)               |                               |
| Shell thickness | \(d\)             | 0.2 \mu m                   |                               |
| \(Mg^{2+}\) concentration | \([Mg^{2+}\]j\) | 500 \mu M                   | Li-Smerin et al. (2001)      |
| Resting \(Ca^{2+}\) concentration | \([Ca^{2+}]_{rest}\) | 0.07 \mu M                 |                               |
| Cell surface | \(A\)             | 3000 \mu m\(^2\)           |                               |
| \(Ca^{2+}\) binding rate to PV | \(k_{on,ca}\)   | 0.1 \mu M\(^{-1}\) ms\(^{-1}\) | Lee et al. (2000)            |
| PV affinity for \(Ca^{2+}\) | \(K_{2,ca}\)     | 0.01 \mu M                 | Eberhard and Erne (1994)     |
| \(Mg^{2+}\) unbinding rate from PV | \(k_{off}\)     | 0.025 ms\(^{-1}\)          | Lee et al. (2000)            |
| PV affinity for \(Mg^{2+}\) | \(k_{2,mg}\)     | 31 \mu M                   | Eberhard and Erne (1994)     |
| \(Ca^{2+}\) binding rate (fast buffer) | \(k_{on}\)     | 0.1 \mu M\(^{-1}\) ms\(^{-1}\) | Lee et al. (2000)            |
| Affinity for \(Ca^{2+}\) (fast buffer) | \(K_{D}\)       | 0.01 \mu M                 | Eberhard and Erne (1994)     |
| \(Ca^{2+}\) binding (slow buffer) | \(k_{on}\)     | 0.01 \mu M\(^{-1}\) ms\(^{-1}\) | Schwannekar et al. (2002)    |
| Affinity for \(Ca^{2+}\) (slow buffer) | \(K_{D}\)       | 0.1 \mu M                  | Schwannekar et al. (2002)    |
periodic solutions emerges. From that point the amplitude of the oscillations decreases as the applied current, $I_{\text{app}}$, increases and the repetitive firing disappears at a supercritical Hopf bifurcation (not shown). A stable steady-state time series at the repetitive firing disappears at a supercritical Hopf bifurcation and a stable periodic solution obtained at a Hopf bifurcation point ($I_{\text{app}} = 40 \text{ pA}$), giving rise to a branch of unstable periodic orbits (thick dotted line) and a branch of unstable steady-states at (thin dashed line) $I_{\text{app}} = 40 \text{ pA}$. As the applied current increases further a branch of stable periodic solutions emerges from a saddle-node on a limit cycle (SNLC). Stable periodic solutions exist for $I_{\text{app}} > 69 \text{ pA}$. (B) Time series showing a stable steady-state solution at $I_{\text{app}} = 20 \text{ pA}$ and (C) stable periodic solution at $I_{\text{app}} = 100 \text{ pA}$ (the corresponding current values are displayed as dashed vertical lines in the bifurcation diagram shown in (A)).

**SUMMATION OF [Ca$^{2+}$], TRANSIENTS DURING TRAINS OF ACTION POTENTIAL**

During a train of AP, $[\text{Ca}^{2+}]_i$ increases due to the summation of $[\text{Ca}^{2+}]$ transients. For trains of AP of sufficiently long duration, $[\text{Ca}^{2+}]_i$ will reach a steady-state plateau level and fluctuate between a lower and upper level. At steady-state, calcium influx and clearance mechanisms compensate (Helmchen et al., 1996; Neher, 1998). During train of action potentials, evoked by 5 s depolarizing step current of 100 pA, we have investigated the time course to reach the steady-state using three buffering conditions. In
the first condition, we have simulated PV with mixed Ca\textsuperscript{2+}/Mg\textsuperscript{2+} binding sites. In the second and third conditions, we have simulated respectively a slow buffer similar to EGTA and a fast buffer similar to the metal-free form of PV. Figure 3A shows the summation and lower envelope of [Ca\textsuperscript{2+}], transients, in the presence of PV, during the first 1000 ms of a train of action potentials (I\textsubscript{app} = 100 pA). The time course of the lower envelope of [Ca\textsuperscript{2+}], transients in the presence of PV is shown in Figure 3B. As previously demonstrated (Lee et al., 2000), the time to reach the steady-state is delayed and the plateau level increases as the PV concentration increases. A similar behavior is observed for the slow and fast buffer systems (Figures 3C,D). For the fast buffer the time to reach the steady-state is shorter than the time for the slow buffer (Figure 3C). Moreover, for trains of AP of short duration, the envelope of Ca\textsuperscript{2+} transients follows a similar time course both for PV and the slow buffer (Figure 3D). Whereas for trains of AP of longer duration, once PV is saturated with Ca\textsuperscript{2+}, the lower envelope of Ca\textsuperscript{2+} transients follows a similar time course both for PV and the fast buffer (Figure 3C). In Figures 3E,F, the plots of the buffer occupancy show that PV and the slow buffer partially saturate after a few hundred milliseconds, whereas the fast buffer system is already saturated at this time. The lower envelope of [Ca\textsuperscript{2+}], transients determines the Ca\textsuperscript{2+} available for the activation of SK channels during the interspike intervals and therefore the modulation of the firing frequency. In the following section, we study the effect of different calcium buffer concentrations on the excitability of FS neurons. Using 5 x step current of 100 pA, we only account for Ca\textsuperscript{2+} transients occurring at the steady-state plateau level during the 4th and 5th second of the train. The duration of the step current was chosen to allow Ca\textsuperscript{2+} transients and
spike-frequency adaptation, occurring during trains of AP, to reach their steady-state.

**REGULATION OF FS EXCITABILITY BY FAST AND SLOW CALCIUM BUFFERS**

We have investigated the effect of different calcium buffer concentrations on FS excitability for PV, the slow and fast buffer systems. Previous studies have shown that the somatic PV concentration ranges from 0.8 to 70.6 μM in hippocampal dentate gyrus basket cells and from 55 to 1788 μM in cerebellar basket cells (Eggermann and Jonas, 2012). In the simulations, we have used a 5-s long depolarizing current of 100 pA and buffer concentration ranging between 0 and 1500 μM. For PV, we observe a decrease in excitability as the total buffer concentration PV_T increases from 50 to 1000 μM (Figure 4A). As the buffer concentration increases from 0 to 1500 μM, the mean frequency spiking drops from 39 to 30 Hz (Figure 4B). We have observed a similar behavior for the slow and fast buffer (Figure 4B). This demonstrates that changes in the level of parvalbumin concentration changes the firing rate of the FS interneurons. This regulatory effect occurs in a similar way for the fast and slow buffers.

To understand the change in firing frequencies, we have investigated the effect of the buffer concentration on the activation of the SK current. We have considered the amplitude of the I_{SK} current, the amplitude, and decay time of the Ca^{2+} transients. To avoid the effect of summation of Ca^{2+} transients, only those transients occurring at the steady-state plateau level were taken into consideration. For low PV concentrations, the decay time of the calcium transients is slower than the decay time at high PV concentration. Moreover, at low PV concentration, the amplitude of Ca^{2+} transients are higher than the amplitude at high PV concentration (Figure 5A). Between two AP, the intracellular calcium concentration drops to 0.1 μM (low buffer concentration, PV_T = 50 μM) and 0.3 μM (high buffer concentration, PV_T = 1000 μM). This value of 0.3 μM is sufficient to activate a significant fraction of the SK channels that have a K_{D} for calcium of 300–700 nM (Hirschberg et al., 1998; Xia et al., 1998). The amplitude of the I_{SK} current is relatively constant between two AP at low PV concentration (15 pA) while it slowly increases from 19 to 35 pA at high PV concentration (Figure 5B). This also increases the duration of the AHP and therefore reduces the firing frequency. Figures 5C–F show the results for the slow and fast buffers. As for PV, the residual calcium level between two action potentials is higher at high buffer concentration. It will activate more SK channels, increase the duration of the AHP, and decrease the firing frequency. The time course of the other ionic currents were similar when the buffer concentration was increased in the PV, slow, and fast buffering conditions (not shown).

**DISCUSSION**

In this paper, we present a new conductance-based single compartment computational model for striatal FS interneurons. Our model is adapted from the model of Erisir et al. (1999) for FS neocortical interneurons. It differs from the former model (and its modifications by Golomb et al., 2007; Ermentrout and Wechselberger, 2009) in that it includes the presence of a calcium buffer protein similar to PV. The dynamic of the calcium buffer is included in the FS model of Erisir et al. (1999) by the addition of a HVA calcium current and a SK current. Our model differs also from other FS models (Jolivet et al., 2004; Lewis and Rinzel, 2004; Mancilla et al., 2007) in that it preserves the dynamic of neurons belonging to class 2 excitability, as it is the case experimentally for FS neurons (Tateno et al., 2004).

PV is considered as a slow buffer similar to EGTA, but recent results show that it can behave like BAPTA, under physiological condition, at high concentration (Franconville et al., 2011; Eggermann and Jonas, 2012). Therefore, we have investigated the effect of PV and the effect of a calcium buffer in both cases, slow and fast binding kinetics. Our results show that calcium buffers, through modulation of the level of residual [Ca^{2+}]; and its coupling to SK channels during a train of action potentials, control the excitability of FS interneurons. The SK current activation depends on the level of residual [Ca^{2+}]; between AP. The residual Ca^{2+} concentration increases with buffer concentration, as the calcium buffer can act as a source of calcium ions during that period. Therefore the SK current between AP increases with buffer concentration leading to lower firing frequencies due to prolonged AHP. Our results show that this provides a very robust mechanism for controlling the excitability of FS interneurons. In our simulation, we have used high concentration of PV, such as 1000 μM. Such a high value of buffer concentration is exceptional and so far it has only been found in cerebellar basket cells (Eggermann and Jonas, 2012). Nevertheless our main conclusion does not rely on this specific value of PV concentration. As it is shown in Figure 4B, the firing
frequency decreases as the buffer concentration increases from 0 to 1500 µM. This means that a similar decrease in the frequency of firing will be observed if the buffer concentration is raised from 10 to 50 µM, in the PV concentration range found in hippocampal basket cells (Eggermann and Jonas, 2012). Moreover, the regulation by the calcium buffer concentration appears to produce similar effects for PV, fast, and slow calcium buffers. In addition, it appears not to depend on the type of Ca\(^{2+}\)-activated K\(^+\) conductance providing coupling between excitability and Ca\(^{2+}\) dynamics, as the regulation appears to be same in our FS interneuron model and in cerebellar granule cells where this effect is mediated by BK channels (Gall et al., 2003).

During a train of AP, [Ca\(^{2+}\)]\(_i\) increases due to the summation of Ca\(^{2+}\) transients. For sufficiently long trains of AP, [Ca\(^{2+}\)]\(_i\) will reach a steady-state plateau and fluctuate between a lower and an upper level. During the accumulation phase of Ca\(^{2+}\), we have observed a faster initial decay and higher amplitude of the Ca\(^{2+}\) transients for the slow buffer (data not shown; Markram et al., 1998). The differences in the Ca\(^{2+}\) transients between the slow and fast buffers attenuate at the steady-state plateau level where PV, slow, and fast buffers are already saturated. In agreement with previous published work (Helmchen et al., 1996; Lee et al., 2000) our model predicts a build-up in Ca\(^{2+}\) and different degrees of buffer occupancy in the three conditions (PV, slow, and fast buffers; Figures 3E,F). In our model, the summation of Ca\(^{2+}\) transients is responsible for the spike-frequency adaptation through the progressive activation of SK channels. Due to supra-linear summation, the fast buffer saturates rapidly whereas PV and the slow buffer lead to a slow build-up in Ca\(^{2+}\). This implies that: for trains of AP of short duration, the neuron containing a fast buffer will display spike-frequency adaptation, whereas neurons containing PV or a slow buffer will display little or no spike-frequency adaptation. Those effects will be more pronounced at a higher buffer concentration than at a lower buffer concentration (Figures 3B–D).

The purpose of our model was to propose a basic mechanism for the regulation of excitability of FS neurons by calcium buffering. Despite the use of a single compartment model with a limited set of conductances and currents, the simplicity of our model and the use of buffers with different kinetics validates...
our simulations to other neuronal types. Indeed, the proposed mechanism will remain valid providing that the mechanisms of excitability remain the same, as in FS neurons, and that the conductance of the Ca\textsuperscript{2+}-activated K\textsuperscript{+} channels is sufficient to obtain a strong coupling between excitability and Ca\textsuperscript{2+} dynamics during the spike generation. A possible improvement in our model would be to explore the competitive binding between different calcium binding partners by considering Ca\textsuperscript{2+} diffusion, immobile, and mobile buffers (Markram et al., 1998). Parvalbumin is considered as a slow mobile buffer and SK channels form a complex with calmodulin and act as high affinity, fast Ca\textsuperscript{2+} binding partners (Stocker, 2004). Moreover, it has been shown that in acutely dissociated CA1 hippocampal pyramidal neurons, SK channels are tightly coupled with L-type calcium channels, within a distance of 50–150 nm (Marrion and Tavalin, 1998). This spatial coupling of SK channels and Ca\textsuperscript{2+} sources promotes the formation of nano or microdomains that can modify the efficiency of calcium buffering depending on the mobility and affinity of the calcium buffers. If the Ca\textsuperscript{2+} sensor is within ~20–50 nm of the Ca\textsuperscript{2+} source, a high affinity calcium buffer like BAPTA and not the slow EGTA will be able to interfere with the Ca\textsuperscript{2+} signaling. While if the Ca\textsuperscript{2+} sensor and Ca\textsuperscript{2+} source are located in microdomains (between 50 nm and a few hundred nanometers) both BAPTA and EGTA will interfere with the Ca\textsuperscript{2+} signaling (Neher, 1998; Fakler and Adelman, 2008).

In addition to their already documented role in Ca\textsuperscript{2+} homeostasis, Ca\textsuperscript{2+}-binding proteins appear to play an active role in modulating neuronal intrinsic excitability. Although, information storage is usually believed to be mediated by long-term modifications in the strength of synaptic transmission, activity-dependent changes in the neuronal intrinsic excitability also occur, causing forms of non-synaptic plasticity (Aizenman and Linden, 2000; Armano et al., 2000). Changes in the calcium buffering capacity might have an effect on this regulation. This could be the result of changes in the localization or in the level of expression of Ca\textsuperscript{2+} binding proteins. The mathematical model we present here provides a valuable tool for the investigation of the functional role of parvalbumin in the regulation of the activity of the striatal FS interneurons. This study paves the way for further theoretical work to assess the impact of calcium buffering on the activity of the striatal FS interneurons network.

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