Study of Unbound States of $^{15}$Be using Supersymmetric Quantum Mechanics

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The theoretical procedure of supersymmetric quantum mechanics (SQM) is adopted to study the resonance states of the unbound nucleus $^{15}$Be. We have been able to reproduce the unbound state energies without any modification of our constructed density dependent M3Y (DDM3Y) microscopic potential. Our procedure confirmed the existence of $5/2^+$ state and also reproduced the experimentally predicted unbound resonance energy of 1.8 MeV.

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Introduction

Recently there has been a surge in interest in the neutron-unbound nucleus $^{15}$Be. First observation of $^{15}$Be was published in 2013 [1], where a resonance at 1.8 MeV ($5/2^+$) was observed. Subsequently Kuchera et al. [2] confirmed the existence of the $5/2^+$ state. It is very difficult to tackle theoretically the unbound states by conventional methods. The study of resonance state of $^{15}$Be is a challenging problem as it has no ground state while its $5/2^+$ state two-body potential is a shallow well followed by a very low wide barrier. The low barrier does temporarily trap the system leading to a broad resonance. This inadvertently hinders accurate calculation of resonance energy masked by the broad resonance width.

In the present work, we resort to a very effective technique used earlier by us for study of weakly bound nuclear systems [3]. The theoretical procedure of supersymmetric quantum mechanics (SQM) is adopted to tackle the resonance states of unbound nuclei. We have been able to reproduce the unbound state energy without any modification in our constructed microscopic potential. Our procedure confirmed the existence of $5/2^+$ state and also reproduced the experimentally predicted unbound resonance energy [2].

Theory

The SQM was earlier applied to detect low-lying broad resonances of weakly bound nuclei [3]. Its success has prompted us to apply it effectively for unbound nuclei like $^{15}$Be. Success of our theoretical procedure is due to its ability to circumvent the numerical challenges posed by the shallow potential of such nuclear systems. In our present work, we treat the $^{15}$Be nucleus in the framework of a two-body model consisting of an inert core of $^{14}$Be and a single valence neutron.

The two-body potential $v(r)$ is generated microscopically in a single folding model using the density dependent M3Y (DDM3Y) effective interaction [3].

From this potential, SQM generates a family of isospectral potentials (IP), which has a normalizable positive energy solution at a selected energy. This is a lesser known result of SQM, namely a bound state in the continuum (BIC). Now this IP has desirable properties which can be utilized to extract information about unbound resonance states. The microscopic potential constructed from single folding calculation is in general a shallow well followed by a low and wide barrier. For a finite barrier height, in principle, a
system may be temporarily trapped inside the shallow well when its energy is close to the resonance energy. In reality, there is a very high probability for tunnelling through the barrier which gives rise to broad resonance widths. Our technique bypasses this problem and obtains precise resonance energies.

An isospectral partner potential could be constructed by following the ideas extended by Pappademos et al. to scattering states with positive energy in the continuum. Wave functions in the continuum are non-normalizable but following Pappademos et al one can construct normalizable wave functions at a selected energy, which represents a BIC. The BIC represents a solution of the equation with an isospectral potential \( \hat{v}(r, \lambda) \), where \( \lambda \) is a parameter which affects the strength of IP. It follows from theory as well as in practice that resonance energy does not depend on the choice of \( \lambda \). So a suitable choice of \( \lambda \) ensures the stability of the resonance state. It preserves the spectrum of the original potential while adding a discrete BIC at a selected energy.

We solve the two-body Schrödinger equation for a positive energy \( E \) subject to the boundary condition \( \psi_E(0) = 0 \) and normalized to a constant (fixed) amplitude of oscillation in the asymptotic region. The obtained \( \psi_E(r) \) is used to construct the isospectral potential \( \hat{v}(r; \lambda) \). The \( \hat{v} \) approaches \( v \) for \( \lambda \to +\infty \) and develops a deep and narrow well followed by a high barrier near the origin for \( \lambda \to 0^+ \). The deep well and high barrier combination effectively traps the system giving rise to a quasibound state. We calculate the probability of the system to be trapped within this enlarged well-barrier combination as,

\[
C(E) = \int_0^B [\hat{\psi}_E(r')]^2 dr'.
\]

Our method is advantageous when it comes to highly accurate calculation of resonance energy along with numerical ease. In our procedure, the probability \( (C(E)) \) of the system to be trapped in well-barrier combination of \( \hat{v}(r; \lambda) \) shows a sharp peak at the resonance energy for appropriate choice of \( \lambda \). Resonant state in the original potential \( v(r) \) is slightly modified within the well while it gets enhanced by a very large amount in \( \hat{v}(r; \lambda) \) giving rise to a sharp peak in probability \( C(E) \).

Results and Conclusions

Considering \(^{15}\text{Be}\) to be a two-body system \((^{14}\text{Be} + n)\), we investigated the \( \frac{5}{2}^+ \) resonant state. A plot of \( C(E) \) as a function of \( E \) for various \( \lambda \) values shows how the trapping effect of \( \hat{v}(r; \lambda) \) increases as \( \lambda \) decreases. For appropriate choice of \( \lambda \), deep-welled isospectral potentials are constructed. Probability \( C(E) \) of the system for \( \frac{5}{2}^+ \) state is plotted in Fig. 1. It is evident from the plot that there is a resonant state at energy \( E = 1.8 \text{ MeV} \) for \(^{15}\text{Be}\) state. The accuracy in location of resonance energy \( E_R \) could be increased by the choice of an optimum value of \( \lambda \), although in general \( E_R \) is independent of \( \lambda \). This optimized \( \lambda \) value could be used in all further calculations cutting down numerical computational time. This procedure adopted to study resonances
in binary systems can also be used in calculating differential cross sections as a function of energy. Wave functions for resonant states are readily available in our procedure which could be advantageous for further extended calculations. The resonance width ($\Gamma = 470$ keV) obtained semiclassically slightly deviates from the experimental finding ($\Gamma_{exp} = 575 \pm 200$ keV). We also found that $\Gamma$ is independent of $\lambda$. Since the present framework appears very promising we are working on further improvements. The fact that our calculated results agree best with the experimental results for $^{15}$Be can be credited to the construction of the two-body ($^{14}$Be + n) folded potential which accurately reproduced the $E_R$. In conclusion, we note that since the potential $\tilde{v}(r; \lambda)$ is strictly isospectral with the original potential $v(r)$ - although they have widely different shapes - the arbitrary parameter $\lambda$ can be suitably chosen so that $\tilde{v}(r; \lambda)$ has desirable properties. This fact could be used to calculate other observables of $^{15}$Be as we have successfully reproduced the $5^+$ resonant state energy with our constructed folded potential interaction.

[1] J. Snyder et al, Phys. Rev. C88, 031303(R) (2013)
[2] A. N. Kuchera et al, Phys. Rev. C91, 017304 (2015)
[3] S. K. Dutta, D. Gupta, D. Das, Swapan K Saha, Jour. Phys. G: Nucl. Part. Phys. 41, 095104 (2014); see references therein
[4] J. Pappademos, U. Sukhatme and A. Pagnamenta, Phys. Rev. A 48, 3525 (1993).