Incremental Variational Inference for Latent Dirichlet Allocation

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Abstract

We introduce incremental variational inference and apply it to latent Dirichlet allocation (LDA). Incremental variational inference is inspired by incremental EM and provides an alternative to stochastic variational inference. Incremental LDA can process massive document collections, does not require to set a learning rate, converges faster to a local optimum of the variational bound and enjoys the attractive property of monotonically increasing it. We study the performance of incremental LDA on large benchmark data sets. We further introduce a stochastic approximation of incremental variational inference which extends to the asynchronous distributed setting. The resulting distributed algorithm achieves comparable performance as single host incremental variational inference, but with a significant speed-up.

1. Introduction

Approximate Bayesian inference has become mainstream in machine learning (Bishop et al., 2006; Murphy, 2012) and enjoyed a (re)gained interest in the statistics community (Wang & Titterington, 2006; Armagan & Dunson, 2011). It constitutes an appealing alternative to Markov Chain Monte Carlo when one is interested in probabilistic data modelling. Approximate inference techniques are pragmatic, postulating an approximate model family and trying to find the best model within this family by optimizing a surrogate objective (Wainwright & Jordan, 2008). They are also practical, as the code implementing these inference algorithms is relatively easy to de-bug. For example, variational inference monotonically increases the variational objective. Hence, the bound provides a sanity check for correctness and can be used to monitor convergence.

The amount of data being generated and collected today is tremendous. For example, at the time of writing, there are almost 5 million articles in Wikipedia. Amazon S3 holds trillions of objects and over 6 billion hours of video are watched each month on YouTube. In 2012, the number of active Facebook users had surpassed 1 billion. The trend of “big data growth” presents enormous challenges for industry and creates a need to invent new algorithms capable of ingesting and processing massive data sets.

Stochastic variational inference (Hoffman et al., 2013) was a first step in this direction in the context of approximate inference. It relies on stochastic optimization (Robbins & Monro, 1951) and was designed to handle very large data sets by processing the data sequentially. The drawback of stochastic variational inference requires to adjust additional parameters like the learning rate and the mini-batch size. Moreover, it does not share the attractive property of batch variational inference of monotonically increasing the bound while inferring the model parameters.

The increasing availability of distributed architectures, such as multi-processor and grid-computing hardware, provides an opportunity to devise distributed inference algorithms able to take advantage of the infrastructure and perform well at scale. Recent attempts in this direction include the work by Smola & Narayanamurthy (2010); Newman et al. (2009); Asuncion et al. (2009). However, stochastic variational inference cannot easily be adapted to the distributed optimization setting.

To address the shortcomings of stochastic variational inference, we introduce incremental variational inference, which generalizes incremental EM proposed by Neal & Hinton (1998). Like stochastic variational inference, incremental variational inference processes the data sequentially. However, it does not require to adjust the learning rate. By maintaining a set of local statistics, it also preserves the property of monotonically increasing the variational objective at each iteration. We further propose a stochastic modification of incremental variational inference...
that can be executed in a distributed environment. We report significant horizontal speed-up, while sacrificing very little predictive performance.

In this paper, we focus on Latent Dirichlet Allocation (LDA) [Griffiths & Steyvers, 2004; Blei et al., 2003], a popular generative model for documents. However, it should be noted that the approximate inference scheme we introduce is general and it is applicable to any latent variable model with a set of local and global variables.

Topic Models like LDA make the simplifying assumption that documents can be represented as bag-of-words. This means they ignore the sequential structure of the text. More specifically, LDA postulates the existence of a collection of $K$ topics, each of which is defined as a categorical distribution over a vocabulary of size $V$. It further assumes that each document in a corpus of $D$ documents is generated according to a document-specific categorical distribution over these topics.

Let us denote word $n$ in document $d$ by $x_{nd}$ and its topic assignment by $z_{nd}$. The generative model is defined as follows:

$$z_{nd} \mid \theta_d \sim \text{Categorical} (\theta_d),$$

$$x_{nd} \mid z_{nd}, \{ \phi_k \}_{k=1}^K \sim \text{Categorical}(\phi_{z_{nd}}),$$

where $\theta_d \sim \text{Dirichlet}(\alpha_0 \mathbf{1}_K)$ and $\phi_k \sim \text{Dirichlet}(\beta_0 \mathbf{1}_V)$. The parameters $\alpha_0$ and $\beta_0$ are non-negative reals.

The paper is organized as follows. In Section 2 we review batch and stochastic variational inference for LDA. In Section 3 we introduce incremental variational inference and its stochastic counterpart. The asynchronous distributed inference algorithm for LDA is described in Section 4. After discussing related work in Section 5 we present results on several large benchmark data sets in Section 6.

2. Variational Inference for LDA

Bayesian inference is often difficult in practice as it requires the computation of analytically intractable integrals. One can circumvent this problem by resorting to Markov Chain Monte Carlo (MCMC) to simulate samples from the posterior. For example, collapsed Gibbs sampling has proven to be very successful for inference in LDA [Griffiths & Steyvers, 2004]. However, convergence of MCMC is notoriously difficult to verify. A more pragmatic approach is to consider deterministic approximations like variational inference [Bishop et al., 2006] or expectation propagation [Minka & Lafferty, 2002]. These methods turn the inference problem into an optimization problem, which is often more easy to tackle and to monitor convergence.

Variational inference maximizes a lower bound to the log marginal likelihood of the data by approximating the true posterior by postulating a simpler distribution, which is parametrized by a set of free parameters. In the case of LDA, the variational bound is given by

$$\ln p(X) \geq \ln p(X, Z, \Theta, \Phi) + H[q(Z, \Theta, \Phi)]$$

$$= \ln p(X) - KL[q(Z, \Theta, \Phi)||p(Z, \Theta, \Phi|X)],$$

where $X = \{x_{nd}\}_{n,d}$, $Z = \{z_{nd}\}_{n,d}$, $\Theta = \{\theta_d\}_d$ and $\Phi = \{\phi_k\}_k$. The notation $\langle \cdot \rangle$ denotes an expectation wrt $q(Z, \Theta, \Phi)$, $H[p]$ is the differential entropy and $KL[q||p]$ is the Kullback-Leibler divergence wrt $q$. Maximizing this bound is equivalent to minimising the Kullback-Leibler divergence between the true posterior $p(Z, \Theta, \Phi|X)$ and the approximate posterior $q(Z, \Theta, \Phi)$. In general, this minimization problem is still problematic, unless we further restrict the form of $q(Z, \Theta, \Phi)$.

Mean field variational inference (MVI) assumes the latent variables and the parameters are independent when conditioning on the data, that is,

$$q(Z, \Theta, \Phi) = \prod_{n,d} q(z_{nd}) \times \prod_q q(\theta_d) \times \prod_k q(\phi_k).$$

It is easy to show that in this case the lower bound is maximised when the factors are defined as follows [Blei et al., 2003]:

$$q(z_{nd}) = \text{Categorical}(\pi_{nd}), \ \ \ \pi_{knd} \propto e^{\langle \ln \theta_{kd} \rangle + \langle \ln \phi_{n,k} \rangle},$$

$$q(\theta_d) = \text{Dirichlet}(\alpha_d), \ \ \ \alpha_{kd} = \alpha_0 + \langle m_{kd} \rangle,$n,d$$

$$q(\phi_k) = \text{Dirichlet}(\beta_k), \ \ \ \beta_{vk} = \beta_0 + \langle m_{vk} \rangle,$$ (2)

where $m_{kd}$ is the (unobserved) number of times topic $k$ appeared in document $d$ and $m_{vk}$ the (unobserved) number of times word token $v$ was assigned to topic $k$ in the corpus. Hence, the special quantities $\langle m_{kd} \rangle$ and $\langle m_{vk} \rangle$ are expected counts under the variational approximation. They are respectively given by $\sum_{n} \pi_{knd}$ and $\sum_{n,d} \delta_v(x_{nd}) \pi_{knd}$. The function $\delta_v(\cdot)$ is Dirac’s delta centred at $v$. The expectations $\langle \ln \theta_{kd} \rangle$ and $\langle \ln \phi_{vk} \rangle$ are respectively given by $\psi(\alpha_{kd}) - \psi(\sum_k \alpha_{kd})$ and $\psi(\beta_{vk}) - \psi(\sum_k \beta_{vk})$.

MVI is a coordinate ascent method that converges to a local maximum of the variational bound [Beal, 2003]. Cycling through the updates for variational parameters in (2) ensures a monotonic increase of this bound. MVI is a batch inference approach: every update of the variational parameter $\beta_{vk}$ requires updating all word-specific proportions $\pi_{nd}$ beforehand, which is costly when the corpus is large. Stochastic variational inference (SVI) was recently proposed in the context of LDA to address this problem [Hoffman et al., 2010; 2013]. The goal was to speed up inference and to scale up LDA to very large data sets.

SVI optimizes the lower bound by stochastic optimization [Robbins & Monro, 1951]. It maintains a set of local and global parameters, which characterize the variational posteriors. Local variables are the indicator variables $Z$ and the document-topic proportions $\Theta$, which are respectively characterized by the local parameters $\{\pi_{nd}\}_{n,d}$ and...
The global variables are topic-word proportions $\Phi$, which are characterized by the global parameters $\{\beta_k\}_k$. SVI considers a noisy, but unbiased estimate of the gradients of the variational parameters associated to the global variables.

This leads to the following updates (document $d$ being picked at random) \cite{Hoffman2013}:

$$
\beta_k^{(t)} = (1 - \rho_t)\beta_k^{(t-1)} + \rho_t \hat{\beta}_k,
$$

$$
\hat{\beta}_{vk} = \beta_0 + D \sum_{n=1}^{N_d} \delta_v(x_{nd}) \pi_{kn,d},
$$

where $\sum_t \rho_t = \infty$ and $\sum_t \rho_t^2 < \infty$. Throughout this work, we will use the learning rate $\rho_t = (t + \tau)^{-\kappa}$, where $\kappa \in (0.5, 1]$ and $\tau \geq 0$.

Intuitively, the second term on the right hand side of (3) is a noisy, but unbiased estimate of the expected number of counts appearing in (2), namely $\{m_{vk}\}$. The variational parameters associated to the local variables (that is, $\pi_{kn,d}$ and $\alpha_d$) can be computed as in MVI. Typically, mini-batches are used to stabilize the gradients. An interesting property of SVI is that it corresponds to natural gradients with respect to the variational distribution \cite{Hoffman2010}.

The intrinsic noise of the stochastic gradients can impede the convergence of SVI. Variance reduction techniques have been proposed to address this issue \cite{Wang2013, Paisley2012}. SVI is also sensitive to the learning rate decay schedule and choice of mini-batch size \cite{Ranganath2013}. Next, we derive incremental variational inference for LDA, which does not require to choose and adjust the learning rate. Importantly, it ensures a monotonic increase of the bound and convergence to a local maximum of the log marginal likelihood like MVI.

### 3. Incremental Variational Inference for LDA

Incremental variational inference (IVI) computes updates in a similar fashion as incremental EM \cite{Neal1998}. Each iteration performs a partial variational E-step before performing a variational M-step. This amounts to maintaining a set of global statistics associated to the global variables, which are updated incrementally in the variational E-step by first subtracting the old statistics associated to a data point (or a mini-batch) and adding back the corresponding new one. The updated global statistics are then used in the variational M-step. This is to be contrasted with SVI. Indeed, SVI uses a noisy estimate of the global statistics, which is based exclusively on the mini-batch that is considered in the current iteration. In the case of LDA, IVI leads to the following incremental update:

$$
\beta_{vk}^{(t)} = \beta_0 + \langle m_{vk} \rangle + \sum_{n=1}^{N_d} \delta_v(x_{nd}) (\pi_{kn,d}^{(t)} - \pi_{kn,d}^{(t-1)}),
$$

while the updates for $\pi_{kn,d}$ and $\alpha_d$ are the same as in MVI as they are associated to the local variables. The main advantage of IVI is that it ensures a monotonic increase of the bound and does not require to have seen all the data points to make progress. The price we have to pay is that we have to store the previous set of proportions $\pi_{kn,d}$, which can be costly when the number of topics $K$ is large as the additional memory requirements scale as a constant factor times the number of words in the corpus. IVI for LDA is summarized in Algorithm 1.

### 4. Distributed Variational Inference for LDA

To speed up inference in the context of large data sets, SVI and IVI process document sequentially. In this section, we further scale up IVI by extending it to the distributed setting. We introduce asynchronous distributed incremental
Algorithm 2 Distributed IVI (D-IVI)

1: Initialize \( \beta^{(0)} \) randomly; set \( \alpha_{kd} = \alpha_0 \).
2: Set the step-size schedule \( \rho_t \).
3: Split documents into \( P \) disjoint subsets \( \{D_1, \ldots, D_P\} \).
4: for \( t = 1, 2, \ldots, \infty \) do
5: for each processor \( p \in \{1, \ldots, P\} \) in parallel do
6: Sample a document \( d \) uniformly from \( D_p \).
7: repeat
8: \( \pi_{knd}^{(t)} \propto e^{(\ln \theta_{kd} + \ln \phi_{x_{nd}})} \)
9: \( \alpha_{kd} = \alpha_0 + \sum_{n=1}^{N_d} \pi_{knd} \)
10: until \( \alpha_{kd} \) and \( \pi_{knd} \) converge.
11: end for
12: \( \hat{\beta}_{vk} = \beta_0 + (m_{vk} + \sum_{n=1}^{N_d} \delta_v(x_{nd}) (\pi_{knd}^{(t)} - \pi_{knd}^{(t-1)}) \)
13: \( \beta_k(t) = (1 - \rho_t) \beta_k(t-1) + \rho_t \hat{\beta}_k \)
14: end for

5. Related Work

Collapsed variational inference for LDA \cite{Teh2006} is the de facto standard for learning topic models on corpora of moderate size. Recently, SVI was introduced to scale up inference and make it possible to handle massive corpora \cite{Hoffman2010, Hoffman2013, Foulds2013}. Zhai et al. (2013) take this work one step further by developing stochastic collapsed variational inference. Modifications of SVI, such as subsampling from data non-uniformly \cite{Gopalan2013} or using control variates \cite{Wang2013}, have been proposed to reduce the variance in the noisy gradient and further speed up convergence. Along similar lines, Paisley et al. (2012) develop an algorithm that allows for direct optimization of the variational lower bound for variance reduction in stochastic gradient and Mandt & Blei (2014) propose a variance reduction scheme tailored to SVI by averaging successively over the sufficient statistics of the local variational parameters. All these methods, however, require to tune at least the learning rate and the minibatch size. By contrast, IVI uses an incremental method to reduce the variance of the noisy natural gradient and has no learning rate. Our work is most closely related to the work by Hughes & Sudderth (2013). They generalize previous incremental variants of the EM algorithm and develop the memoized online variational inference algorithm which is analogous to IVI, but they do not consider the stochastic and distributed extensions of IVI.

Various implementations and improvements have been explored for developing distributed algorithms for LDA to improve scalability in terms of memory and computation. Most works consider parallel algorithms that are synchronous. Besides, these studies parallelize batch variational inference. For example, Nallapati et al. (2007) describe distributed mean-field variational EM for LDA. Like in the case of D-IVI it relies on the fact that the expensive variational E-step can easily be parallelized because the local variable are conditionally independent. However, the master node waits until each of the workers completes its job to perform the M-step. Wolfe et al. (2008) investigate the parallelization of both the E- and M-step of variational EM for LDA. Each node computes partial statistics in a local E-Step, sends these to a central node, and receives back completed statistics relevant for completing its local M-Step. This distributed version of LDA produces identical results to the sequential version of the algorithm but it requires a global synchronization step. Zhai et al. (2012) proposed a distributed variational inference algorithm using the MapReduce framework, where the E-step is done in the MapReduce and the M-step in the Reducer. Another set of works attempt to distribute MCMC algorithms (Smola & Narayanamurthy, 2010; Newman et al., 2009; Nallapati et al., 2007; Thiesson et al., 2004; Wolfe et al., 2008), where workers concurrently run several Gibbs sam-
plers and perform a global update of the topic counts after the synchronization. Up to our knowledge, only Asuncion et al. (2009) propose an asynchronous approach for LDA, which is based on Gibbs sampling unlike D-IVI.

6. Experiments and Results

We carry out two types of experiments. First, we study the performance of IVI and S-IVI for LDA. We also benchmark IVI against MVI and SVI on large document collections. Second, we measure speed-ups that are obtained with our distributed algorithm (D-IVI).

Hardware: Experiments were all run on a 32-core machine with 3.6 GHz Intel Core i7-3820 processors and a total of 128GB of RAM.

Data: We benchmark IVI on four corpora: Associated Press articles, Newsgroup documents, Wikipedia articles and the scientific abstracts from Arxiv repository (Mandt & Blei, 2014). Besides, we used two additional large corpora to evaluate D-IVI: reviews from Amazon website and New York Times articles (Mandt & Blei, 2014). The characteristics of the datasets are reported in Table 1.

Experimental Setup: To quantitatively evaluate the model, we estimate the predictive probability over the vocabulary (Blei et al., 2003). We wish to achieve high average per-word likelihood on held-out test documents. Under this metric, a higher score is better, as a better model will assign a higher probability to the held-out words. We learn the topics on the training corpus. We use half of each test document to estimate its topics proportions and use the remainder to compute the predictive distribution over the vocabulary. In all the experiments, we set the number of topics $K$ to 100, the Dirichlet hyperparameters $\alpha_0$ to 0.5 and $\beta_0$ to 0.05. For stochastic methods, we set the forgetting constant $\kappa$ to 0.9 and the delay $\tau$ to 1.

6.1. IVI Prediction Results

In the first set of experiments, we compare the different inference algorithms for LDA, using our own implementation of MVI, SVI and IVI. Figure 1 shows that IVI converges to a solution which is comparable or better than MVI, SVI and S-IVI and IVI converges faster than the other algorithms. We first compare the performances of IVI and MVI at the point where MVI converges to a solution. IVI yields the same result after processing half (Newsgroup) to tenth (Arxiv) of the documents that MVI has processed. Besides, we observe that IVI gives consistently better predictive performance than MVI when both of them converges to a solution.

In Section 3 we mentioned that S-IVI does not maintain strictly accurate sufficient statistics, but it uses statistics computed as decaying average of recently visited data. Hence, it requires less memory than IVI and improves SVI in terms of accuracy and speed. Figure 1 provides the experimental support for these claims.

In the second set of experiments, we evaluate IVI with various mini-batch sizes by computing the average predictive log likelihood on the test set.

Next, we turn our attention to Figure 2. Fixing the hyperparameters and the number of topics, we explored the effect of varying mini-batch sizes on all four corpora. IVI converges faster to a good solution for smaller ones. However, larger mini-batches lead to better final performance.

6.2. D-IVI Convergence and Speed-up Results

The purpose of these experiments is to measure speed-ups obtained with D-IVI. We report the performance of D-IVI on a single processor which corresponds to S-IVI for reference; and compare it to the performance of D-IVI for a varying number of processors. We are interested in two aspects of performance: the quality of the model learned and the time taken to learn the model. We record wall clock time and the log predictive probability on Customer Review, New York Times and Arxiv corpus. In the experiments, computations were done on $P$ processors for D-IVI where $P = \{1, 2, 4, 8, 16, 32\}$. These results are averaged over 5 runs with random initializations. The results in Table 2 and Figure 3 show that the log predictive proba-
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Table 1. Characteristics of data sets used in experiments.

|                      | AP       | Newsgroup | Wikipedia | Arxiv    | Customer Review | NYT     |
|----------------------|----------|-----------|-----------|----------|----------------|---------|
| Number of documents in training set | 1246     | 13888     | 39565     | 782385   | 452944         | 290000  |
| Number of documents in test set     | 1000     | 5000      | 10000     | 100000   | 100000         | 10000   |
| Average number of words per document| 198      | 249       | 260       | 116      | 151            | 232     |
| Number of words in vocabulary       | 10473    | 27059     | 42419     | 141927   | 120043         | 102660  |

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Figure 1. Per-word predictive probability for LDA as a function of the number of processed documents. We compare results for the Associated Press, Newsgroup, Wikipedia and Arxiv data sets. Incremental approaches (IVI and S-IVI) converge to a higher value on all datasets. We reported results for 2 mini-batch sizes.

Figure 4. Convergence results (per-word predictive probability for LDA model as a function of number of documents processed so far) for D-IVI on Arxiv, Customer Review and New York Times for varying number of processors. As the number of processors increases, the rate of convergence slows down.

...
These results suggest that asynchronous D-IVI converges to a solution that exhibits a performance close to one obtained with S-IVI.

**Simulated Delays:** Next, we add delays to some workers to explore the robustness of D-IVI. Figure 4 provides results when each processor sleeps with a probability for a small amount of time before sending the latest sufficient statistics to the master. The delay length is chosen randomly from a normal distribution with the mean $\mu$ (in seconds) and $\sigma = \mu/5$. We have specified the upper limit of $\mu$ as twice the average time required to compute the sufficient statistics of a mini-batch.

Here, we report performance by plotting log predictive probability against number of documents seen so far. Figure 4 shows that as the number of processors increases, the rate of convergence slows down, since more iterations are needed for information to propagate to all the processors. However, it is important to note that one iteration in real time of D-IVI is up to number of processors times faster than one iteration of S-IVI, so D-IVI converges much more quickly than S-IVI (see Table 2 for time results).

**Simulated Delays:** Next, we add delays to some workers to explore the robustness of D-IVI. Figure 4 provides results when each processor sleeps with 0.5 probability for a small amount of time before sending the latest sufficient statistics to the master. The delay length is chosen randomly from a normal distribution with the mean $\mu$ (in seconds) and $\sigma = \mu/5$. We have specified the upper limit of $\mu$ as twice the average time required to compute the sufficient statistics of a mini-batch.

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Finally, we test if D-IVI is robust to extremely stale parameters by increasing the delay. Figure 5 shows the results of this case. For CR corpora, the computation time of the sufficient statistics for a mini-batch of 1000 documents is 26 seconds in average. Here, each processor sleeps with 0.25 probability and the average delay is set to twice (50 seconds, $\mu=200$), 5-times and 10-times the computation time for a mini-batch.

We see that the D-IVI algorithm still converges even with considerable delays of 5 and 10 times the processing time for a mini-batch. Despite no formal convergence guarantees, D-IVI algorithm performs well empirically in all experiments we conducted on the three real-world data sets considered.

**Table 2.** Log-prediction-probability (LPP) and runtime (in terms of seconds per iteration) of the D-IVI for different number of mini-batch sizes and number of processors.

| Dataset       | Mini-batch Size | Number of Processors | 1   | 2   | 4   | 8   | 16  | 32  |
|---------------|-----------------|----------------------|-----|-----|-----|-----|-----|-----|
| Customer Review (CR) | 1000  | LPP               | -7.25 | -7.25 | -7.25 | -7.28 | -7.28 | -7.28 |
|                |                | Time               | 13626 | 8015 | 4167 | 3299 | 2428 | 2259 |
| New York Times (NYT) | 1000  | LPP               | -7.26 | -7.26 | -7.26 | -7.26 | -7.28 | -7.28 |
|                |                | Time               | 13162 | 7607 | 4126 | 3082 | 2237 | 2113 |
| Arxiv         | 1000  | LPP               | -7.21 | -7.21 | -7.24 | -7.24 | -7.24 | -7.24 |
|                |                | Time               | 13043 | 7538 | 3875 | 2757 | 1883 | 1659 |
|                | 2000  | LPP               | -7.49 | -7.49 | -7.51 | -7.51 | -7.51 | -7.51 |
|                |                | Time               | 12935 | 6916 | 3902 | 2879 | 1987 | 1728 |
|                | 5000  | LPP               | -7.48 | -7.48 | -7.48 | -7.48 | -7.50 | -7.50 |
|                |                | Time               | 12427 | 6510 | 3407 | 2360 | 1748 | 1428 |
|                | 1000  | LPP               | -7.63 | -7.63 | -7.63 | -7.65 | -7.66 | -7.66 |
|                |                | Time               | 16996 | 9601 | 5087 | 3776 | 2534 | 2185 |
|                | 2000  | LPP               | -7.55 | -7.55 | -7.56 | -7.56 | -7.56 | -7.56 |
|                |                | Time               | 17845 | 9857 | 5100 | 3678 | 2453 | 2158 |
|                | 5000  | LPP               | -7.52 | -7.52 | -7.52 | -7.54 | -7.54 | -7.54 |
|                |                | Time               | 17957 | 10030 | 4760 | 3228 | 2105 | 1835 |
7. Conclusion

We introduced incremental variational inference as an alternative to stochastic variational inference. The algorithm does not require to adjust the learning rate. We showed experimentally that the incremental approach converges faster and often to a better local optimum of the variational objective. Incremental variational inference processes documents sequentially. It scales thus similarly to stochastic variational inference and is suitable when we can afford to incur an additional memory cost (which scales as $O(KN)$).

We further modified incremental variational inference to accommodate a stochastic variant, which can be adapted to distributed environments. This enabled us to further scale variational inference. We showed experimentally that the proposed asynchronous algorithm is robust to noise and outdated parameters, and produces solutions that are very close to the single host solutions. The horizontal speed-up saturates when then number of processors increases as communication cost increases and more passes over the data are necessary to ensure convergence to the same level of accuracy.

We left the convergence analysis of incremental variational inference to future work, as well as its application to other probabilistic models. Indeed, the incremental variational algorithms proposed in the paper are generic. They can be applied to any model with local and global variables and are by no means restricted to their application to LDA.

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