On the weighting field of irradiated silicon detectors

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Abstract

The understanding of the weighting field of irradiated silicon sensors is essential for calculating the response of silicon detectors in the radiation environment at accelerators like at the CERN LHC. Using 1-D calculations of non-irradiated pad sensors and 1-D TCAD (Technology Computer-Aided Design) simulations of pad sensors before and after irradiation, it is shown that the time-dependence of the weighting field is related to the resistivity of low field regions with ohmic behaviour in the sensor. A simple formula is derived, which relates the time constant of the time-dependent weighting field, \( \tau \), with the resistivity and the extension of the low-field region for pad detectors. As the resistivity of irradiated silicon increases with fluence and finally reaches the intrinsic resistivity, \( \tau \) becomes much larger than the charge-collection time and the weighting field becomes essentially independent of time. The TCAD simulations show that the transition from a time-dependent to a time-independent weighting field occurs at a neutron-equivalent fluence of \( \approx 5 \times 10^{12} \text{ cm}^{-2} \) for a 200 \( \mu \text{m} \) thick pad diode operated at 40 V and \( -20^\circ \text{C} \). It is therefore concluded that the use of a time-independent weighting field calculated with the same method as for a fully-depleted non-irradiated sensor is also appropriate for the simulation of highly irradiated silicon sensors.

Keywords: Silicon detectors, radiation damage, time-dependent weighting field.

1. Introduction

The weighting field describes the electromagnetic coupling of a charge to an arrangement of conducting electrodes. It is used to calculate the signal currents induced in the readout electrodes by charges moving in a detector. The weighting field has first been introduced by Shockley and Ramo [1, 2] to describe the signal generation in vacuum tubes. They also presented an elegant method of calculating weighting fields for different electrode arrangements. In Ref. [3] the method was extended to the presence of fixed space charges. In Ref. [4] the time-dependent weighting vector has been introduced, to describe the situation when the electrodes are not kept at a fixed potential, but connected by linear impedance elements to ground or the power supply, and methods of its calculation were presented. The extension to non-linear media was presented in Ref. [5], and to media with arbitrary conductivity and permittivity in Ref. [6]. A recent overview, which includes the calculation and discussion of the signal shape in partially depleted silicon pad detectors is given in Ref. [7]. It is found that a time-dependent weighting vector is required to describe this situation, and the time constant is given by \( \tau = \rho \cdot \varepsilon_{Si} \) multiplied by a geometrical factor, which is bigger than one and is given by the depletion depth relative to the sensor thickness. The resistivity of the silicon is denoted \( \rho \), and \( \varepsilon_{Si} \) is the dielectric constant of silicon. A typical value of the resistivity of silicon before irradiation is 5 k\( \Omega \)-cm, which gives \( \tau = 5 \text{ ns} \) times the geometric factor.

This paper addresses the question how radiation damage by energetic particles influences the weighting field and thus the signal generation. The electric field in a highly irradiated silicon pad sensor exhibits a so-called double junction: High-field regions at the two electrodes and a resistive low-field region in-between [8]. Thus the field distribution is not too different from the distribution in a partially depleted non-irradiated pad diode with implantations at both electrodes. For partially depleted non-irradiated pad

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diodes analytic results are available [7]. They show that a time-dependent weighting field is required to describe the signal shapes. Thus, they can be considered a good test bed for understanding the detailed and complex TCAD (Technology Computer-Aided Design) simulations, and give some confidence that they also give precise results when applied to radiation-damaged sensors.

In the following section, experimental data from capacitance-voltage (C–V) measurements of pad diodes for different frequencies after irradiation to 1 MeV neutron equivalent fluences, $\Phi_{eq}$, by 24 GeV/c protons are shown. At low $\Phi_{eq}$-values a strong frequency dependence of the capacitance is observed, whereas at high $\Phi_{eq}$ it is independent of voltage and frequency and equal to the geometrical capacitance of the fully depleted pad sensor before irradiation. A similar behaviour is obtained from a model calculation of a partially-depleted non irradiated pad diode. This suggests that the frequency dependence of the capacitance of irradiated sensors is influenced by the presence of a resistive low-field region. However, the frequency at which the capacitance changes for non-irradiated sensors is beyond the 2 MHz maximum frequency of most capacitance bridges, and has not been observed experimentally.

Next, the time-dependent weighting field from the TCAD simulation is compared to the results of the analytical calculation for the non-irradiated diode. Apart from some expected minor differences, the agreement is good. In addition, the analytical model is used to derive a relation between the time constant of the weighting field and the depletion width and resistivity of the non-depleted region.

Finally, the Hamburg Penta-Trap Model, HPTM [10] is introduced, which is used for the TCAD simulations of the electric field and the weighting field of pad diodes irradiated to different fluences. The fluence dependence of the electric field and the weighting field is presented, and used to extract the resistivity of the low-field region as a function of the fluence with the help of the formula derived from the analytic calculation of the non-irradiated pad diode. The results confirm that the time dependence of the weighting field is mainly determined by the resistivity and extension of the low field region of the irradiated sensor.

2. C–V–f results: Data and simulations

Before discussing in detail the weighting field of non-irradiated and irradiated sensors, capacitance-voltage (C–V) results are presented for a pad sensor of an area $A = 0.5 \times 0.5 \text{cm}^2$ with the parameters given in Table 1. In the analysis the parallel capacitance, $C_p$, is used. From the admittance, $Y$, calculated for an electrical model, $C_p$ is obtained using $Y = 1/R_p + i \omega C_p$, with the parallel resistance $R_p$ and the angular frequency $\omega = 2\pi f$. For the measurements the capacitance meter applies the voltage $V + V_{AC} \cdot \sin(\omega t)$ to the device under test, measures the amplitude of the current and the phase difference between current and voltage, and calculates the corresponding values of $1/R_p(V, f)$ and $C_p(V, f)$.

| $d$ [µm] | $N_p$ [10$^{12}$ cm$^{-3}$] | $N_{p^+}$ [10$^{19}$ cm$^{-3}$] | $d_{p^+}$ [µm] | $N_{p^+}$ [10$^{19}$ cm$^{-3}$] | $d_{p^+}$ [µm] |
|----------|------------------|------------------|----------------|------------------|----------------|
| 200      | 3.85             | 1.2              | 2.4            | 1.0              | 2.4            |

Table 1: Pad-diode parameters used for the TCAD simulations: $d$ is the mechanical Si-thickness of the pad diode, $N_p$, the boron density of the silicon bulk, $d_{p^+}$ and $N_{p^+}$ the depth and dopant density of the phosphorus implant, and $d_{p^+}$ and $N_{p^+}$ the depth and dopant density of the backside boron implant.

Before irradiation, the measurements agree with the expectations: Below the full depletion voltage, $V_{fd} \approx 120$ V, $1/C_p^2$ depends linearly on voltage, with the parallel capacitance $C_p = (\varepsilon_{Si} \cdot A)/w(V)$ and the depletion depth $w(V) = \sqrt{2\varepsilon_{Si} \cdot (V + V_{bi})/(q_0 \cdot N_p)}$. Above $V_{fd}$, $C_p \approx (\varepsilon_{Si} \cdot A)/d$. For the frequency range of the measurements, $f = 100$ Hz – 2 MHz, the results do not depend on $f$. The dielectric constant of silicon is denoted $\varepsilon_{Si}$, the elementary charge $q_0$, and the built-in voltage $V_{bi} \approx 0.8$ V.

Next, the $C_p(V, f)$-dependence is calculated using a simple model. The left side of Fig. 1 shows a schematic cross-section of the sensor. The $n^+ p$ diode is at $y = 0$, and the backside $p^+$ implant at $y = d$. The dashed line at $y = w$ indicates the boundary between the depleted and the non-depleted regions of a non-irradiated pad diode. The right side of the figure shows the electrical model used to simulate the expected $C–V–f$ dependence. The capacitance of the depletion region $C_d$, and the capacitance $C_{nd}$ and the
resistance $R_{nd}$ of the non-depleted region are:

$$C_d = (\varepsilon_{Si} \cdot A)/w(V), \quad C_{nd} = (\varepsilon_{Si} \cdot A)/(d - w(V)), \quad R_{nd} = \rho \cdot (d - w(V))/A,$$

with the resistivity of the non-depleted silicon, $\rho = 1/(q_0 \cdot \mu_h \cdot N_p)$.

$$C_p(V, \omega) = C_d \cdot \frac{1 + \omega^2 \cdot \tau_{nd} \cdot (\tau_{nd} + \tau_d)}{1 + \omega^2 \cdot (\tau_{nd} + \tau_d)^2},$$

with $\tau_{nd}(V) = C_{nd}(V) \cdot R_{nd}(V)$ and $\tau_d(V) = C_d(V) \cdot R_{nd}(V)$. The calculated voltage dependence of $C_p$ for different frequencies $f$ is shown in Fig. 2a. Up to $f \approx 3$ MHz, the voltage dependence is: $C_p(V) = (\varepsilon_{Si} \cdot A)/w(V)$. For $f = 3 - 50$ MHz the value of $C_p$ decreases at low voltages, and finally reaches the constant value $C_p = (\varepsilon_{Si} \cdot A)/d$ for $f \gtrsim 50$ MHz. At these high frequencies the entire AC-current of the capacitance meter flows through $C_{nd}$ and $C_d$, and the capacitance is $C_p = (1/C_{nd} + 1/C_d)^{-1} = (\varepsilon_{Si} \cdot A)/d$, the geometrical capacitance of the pad diode.

Fig. 2b shows the measured voltage dependence $C_p(V)$ at different frequencies for the pad sensor irradiated by 24 GeV/c protons to a neutron-equivalent fluence $\Phi_{eq} = 6 \times 10^{15}$ cm$^{-2}$. The measurements were performed at $-30^\circ$C.

Frequently the observation of a frequency-dependent $C_p$ is assumed to be caused by the emission-capture times of the radiation-induced states in the silicon band gap, which can change their charge states only up to
3. Time-dependent weighting vector and weighting field

The time-dependent weighting vector $\vec{W}(\vec{r}, t)$ has been introduced in Ref. [4] to describe the signal in detectors when the electrodes are not grounded. In Ref. [6] it is shown that the same formalism can be used if the material between the electrodes is conducting. The current induced in an electrode at constant potential by a charge $Q_0$ created at $t = 0$ moving on the trajectory $\vec{r}(t)$ with the velocity $\vec{v}(t)$ is

$$I(t) = Q_0 \cdot \int_0^t \vec{W}(\vec{r}(t'), t - t') \cdot \vec{v}(t') \, dt'.$$

The time-dependent weighting vector is related to the time-dependent weighting field $E_w(\vec{r}, t)$ by

$$\vec{W}(\vec{r}, t) = \frac{\partial}{\partial t} \vec{E}_w(\vec{r}, t).$$

The weighting field can be calculated in the following way: First the electrical field $\vec{E}(\vec{r})$ for a sensor biased with the voltage $V$ is calculated. Then, a voltage step $V_0 \cdot \Theta(t)$ with a small $V_0$ value is applied to the readout electrode, and the time-dependent field $E_w(\vec{r}, t)$ is calculated, to obtain

$$E_w(\vec{r}, t) = \frac{1}{V_0} \cdot \left( \vec{E}(\vec{r}, t) - \vec{E}(\vec{r}) \right).$$

$\Theta(t)$ is the Heaviside step function. For numerical calculations $V_0 \cdot \Theta(t)$ has to be replaced by a fast voltage ramp from 0 to $V_0$. Note that the electric field has units voltage/length, the weighting field 1/length, and the weighting vector 1/(length-time).

$E_w(\vec{r}, t)$ has two terms: A quasi-instantaneous step, and a time-dependent term, which accounts for the flow of charges in the conductive material of the sensor and/or the charge flow through the connections of the electrodes to the ground and the power supply. $E_w(\vec{r}, 0^+)$ is equal to the geometric weighting field, $E_{geom}(\vec{r})$, which is obtained by just considering the electrodes and ignoring the effects of the conductive material in the sensor. The time constant of the quasi-instantaneous term is short, but finite, as electric fields propagate with the speed of light. For a sensor of a thickness of 200 µm it is of the order of 10 fs, can be considered instantaneous and thus ignored. When calculating $W(\vec{r}, t)$ using Eq. 4, the instantaneous term $E_{geom}(\vec{r}) \cdot \delta(t)$ gives the instantaneous contribution $E_{geom}(\vec{r}) \cdot \delta(t)$. From Eq. 5 it follows, that, in the

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1 $0^+$ denotes the time just before $t = 0$, and $0^+$ just after $t = 0$. 

absence of an additional time-dependent term, the induced current in the readout electrode from a charge \( Q_0 \) moving with the velocity \( \vec{v}(\vec{r}(t)) \) at time \( t \) is

\[
I(t) = Q_0 \cdot \vec{v}(\vec{r}, t) \cdot \vec{E}_{\text{geom}}(\vec{r}) \cdot \Theta(t),
\]

which is the formula usually used to calculate the signals in detectors.

From Eq. [3] it also follows that \( E_w(\vec{r}, t) \cdot d\vec{r} \) describes the time dependence of the charge induced in the readout electrode by a unit charge which has moved from \( \vec{r} \) to \( \vec{r} + d\vec{r} \) at \( t = 0 \). The corresponding current is \( \vec{W}(\vec{r}, t) \).

### 4. Weighting field of the non-irradiated pad diode

In this section 1-D TCAD simulations are compared to the analytical calculation of an \( n^+p \) pad diode. The schematic layout of the diode and the coordinate system used are shown in Fig. 1 and the parameters are given in Table 1. The silicon bulk is doped with boron and does not contain additional levels in the silicon band gap due to irradiation or impurities. The electrode at \( y = d \) is connected to ground and the electrode at \( y = 0 \) is biased to the voltage \( V \). Given that a 1-D problem is analysed, only the \( y \)-components of the vectors introduced in Sect. 3 are relevant, and vector signs are omitted in the following.

#### 4.1. Analytical calculation

In Ref. [7] the Laplace-transform technique is used to calculate the weighting vector, \( W(V, y, t) \), and the signal current \( I(t) \) for holes and electrons produced at \( t = 0 \) with different \( y \)-distributions. In the following calculation the electrical model shown in Fig. 1 is used to derive an analytical expression for the time-dependent weighting field \( E_w(V, y, t) \) and the weighting vector \( W(V, y, t) \). For the calculation, at \( t = 0 \) a voltage step of height \( V_0 \) is added to the bias voltage \( V \) at \( y = 0 \).

For voltages above the full-depletion voltage, \( V \geq V_{fd} \), the geometric weighting field, \( E_{\text{geom}} = 1/d \). As there are no free charges in the depleted sensor, the resistivity is infinite, and \( E_w(V, y, t) = 1/d \) for \( t > 0 \) independent of \( V \), \( y \) and \( t \). To simplify the formulae, \( d \) is used for the sensor thickness, instead of \( d' = d - (d_n + d_p) \), which takes into account the field-free regions of the highly doped regions. For the figures, in which the analytical results are compared to the TCAD simulations, this small difference is taken into account.

For \( V < V_{fd} \) the weighting field immediately after the voltage step, \( E_w(V, y, 0^+) = E_{\text{geom}} = 1/d \), and the voltage at the depletion depth \( w(V) \) is \( V_w(t = 0^+) = V_0 \cdot C_{nd}/(C_{nd} + C_d) \). The differential equation for \( V_w(t \geq 0^+) \) is obtained from current conservation: The current in the depleted region (right-hand side of the equation) is equal to the current in the non-depleted region (left-hand side)

\[
\frac{V_w(t)}{R_{nd}} + C_{nd} \frac{dV_w(t)}{dt} = C_d \frac{d(V_0 - V_w(t))}{dt},
\]

and the differential equation is

\[
\frac{dV_w(t)}{dt} + \frac{V_w(t)}{R_{nd} \cdot (C_{nd} + C_d)} = 0.
\]

For the given initial conditions the solution is:

\[
\frac{V_w(t)}{V_0} = \frac{C_d}{C_{nd} + C_d} \cdot e^{-t/(R_{nd} \cdot (C_{nd} + C_d))} = \frac{d-w}{d} \cdot e^{-t/\tau}.
\]

With the relations given in Eq. [1] one finds

\[
\tau(V) = e_{Si} \cdot \rho \cdot d/w(V) = \tau_r \cdot d/w(V),
\]

with the dielectric relaxation time \( \tau_r = e_{Si} \cdot \rho \). For the parameters given in Table 1 \( \tau_r = 3.5 \) ns at 20 °C. For \( V < V_{fd} \) and \( t > 0 \) the time-dependent weighting field is obtained by dividing \( (V_0 - V_w)/V_0 \) by \( w \) for \( y < w \), and \( V_w/V_0 \) by \( d - w \) for \( y \geq w \).
\[ E_w(V, y, t) = \begin{cases} \frac{1}{w(V)} \left( 1 - \frac{d-w(V)}{d} e^{-t/\tau(V)} \right) \cdot \Theta(t) & \text{for } 0 < y < w(V) \\ \frac{1}{d} e^{-t/\tau(V)} \cdot \Theta(t) & \text{for } w(V) \leq y < d. \end{cases} \]

Using Eq. 4 gives
\[ W(V, y, t) = \begin{cases} \frac{1}{d} \left( \delta(t) + \frac{d-w(V)}{w(V)\tau(V)} e^{-t/\tau(V)} \right) & \text{for } 0 < y < w(V) \\ \frac{1}{d} \left( \delta(t) - \frac{1}{\tau(V)} e^{-t/\tau(V)} \right) & \text{for } w(V) \leq y < d. \end{cases} \]

\( W \) is zero for \( t < 0 \). The result agrees with the result of Ref. [7], which uses the Laplace transform.

Figs. 3 and 4 show, in addition to the results of the TCAD simulation discussed in Sect. 4.2, the calculated \( E_w(V, y, t) \) for \( V = 20 \) V and \( V = 80 \) V. Both voltages are below the full-depletion voltage \( V_{fd} \approx 120 \) V. For \( t = 0^+ \), \( E_w \approx 1/d = 50 \) cm\(^{-1}\), independent of \( y \). The figures show that \( E_w \) increases as a function of \( t \) to \( 1/w \) in the depletion region \( 0 < y < w \), and decreases to zero in the non-depleted region \( w < y < d \).

Figure 3: Comparison of the analytical calculation (symbols) and the TCAD simulation (lines) of the time-dependent weighting field, \( E_w(V, y, t) \), for the non-irradiated pad diode. (a) Results for \( V = 20 \) V, and (b) results for \( V = 80 \) V.

Figure 4: Comparison of the analytical calculation (lines) and the TCAD simulation (symbols) of the time-dependent weighting field for the non-irradiated pad diode at \( y = 20 \) µm (depleted region) and at \( y = 180 \) µm (non-depleted region). (a) Results for \( V = 20 \) V, and (b) results for \( V = 80 \) V.
4.2. TCAD simulation

With the pad-diode parameters of Table 1, SYNOPSYS TCAD [9] is used to simulate the electric field, \( E(V, y) \), and the weighting field \( E_w(V, y, t) \). Fig. 5 shows \( E(y) \) for \( V = 20, 40, \) and \( 80 \) V. Note the drop of \( E(y) \) due to the high \( n^+\)-doping at \( y = 0 \), and the value of \( \approx 2 \) kV/cm at \( y = 200 \) µm due to the diffusion of electrons from the \( p^+\)-doping into the \( p^+\)-doped Si-bulk. These effects are not taken into account in the analytical calculation. In addition, there are diffusion effects at the transition from the depleted to the non-depleted region, which however can only be seen in a figure with logarithmic \( E \)-scale.

\[
E(y) \approx 2 \text{ kV/cm} \quad \text{at} \quad y = 200 \mu\text{m}
\]

![Figure 5: Electric field of the non-irradiated pad diode for \( V = 20, 40, 80 \) V simulated using SYNOPSYS TCAD at \(-20 \) °C. Note the effects of the \( n^+\)-implant at \( y = 0 \), and of the \( p^+\)-implant at \( y = 200 \mu\text{m} \).

To calculate \( E_w \), the method discussed in Sect. [5] and Eq. [5] with \( V_0 = 1 \) V is used. The voltage at \( y = 0 \) is ramped linearly in time in 50 ps from \( V \) to \( V + 1 \) V, and the electrode at \( y = d \) is kept on ground potential. The \( y \)-dependencies of \( E_w(V, y, t) \) in 1 ns time steps at \( V = 20 \) and \( 80 \) V are shown as lines in Fig. 3 and the \( t \)-dependencies in the depleted region \((y = 20 \mu\text{m})\) and in the non-depleted region \((y = 180 \mu\text{m})\) as symbols in Fig. 6. Overall, the agreement between the TCAD simulation and the analytical calculation is very good. As expected, differences occur at \( y = 0 \), at \( y = w \) and close to \( y = 200 \mu\text{m} \).

It is concluded that the analytical formula, Eq. [11] can be used to calculate the time-dependent weighting field in non-irradiated pad diodes, and that the proposed method using TCAD simulations to calculate time-dependent weighting fields gives reliable results.

5. Weighting field of the irradiated pad diode

For the simulation of the irradiated pad diode the Hamburg Penta Trap Model HPTM [10] is used. The HPTM assumes 5 traps, 2 acceptors and 3 donors, with energies in the band gap taken from microscopic measurements. The introduction rates and the cross-sections for holes and electrons have been obtained by minimising the sum of the squares of the relative differences of simulations and measurements from pad diodes irradiated with 24 GeV/c protons to \( \Phi_{eq} = 0.3, 1.0, 3.0, 6.0, 8.0 \) and \( 13 \times 10^{15} \) cm\(^{-2}\) and annealed at 60 °C for 80 minutes. The experimental data used for the optimisation are \( I–V, \ C–V \)- and charge-collection-measurements with near-infrared light at \(-30\) and \(-20 \) °C. The data are well described by the simulations.

The TCAD simulations are performed for \( \Phi_{eq} = 0, 0.1, 0.25, 0.5, 0.75, 1, 10, \) and \( 100 \times 10^{13} \) cm\(^{-2}\). Fig. 6 shows the electric field \( E(y) \) at \( V = 40 \) V for selected \( \Phi_{eq} \)-values. Compared to the situation before irradiation, \( E(y) \) hardly changes in the depletion region up to \( \Phi_{eq} = 10^{12} \) cm\(^{-2}\), whereas in the non-depleted region a small field appears. It is the result of the ohmic voltage drop given by the product of dark current and silicon resistance. The resistance increases with \( \Phi_{eq} \) because of the increase in generation rate, which results in a decrease of the density of majority charge carriers, in order to satisfy the equilibrium condition \( n_e \cdot n_h = n_i^2 \). The electron, hole and intrinsic densities are denoted \( n_e, n_h \) and \( n_i \), respectively. For higher \( \Phi_{eq} \)-values, \( E(y) \) increases significantly at low \( y \)-values because of the filling of radiation-induced donor traps with electrons from the dark current. At \( \Phi_{eq} = 10^{15} \) cm\(^{-2}\) there is also an evidence for the increase of \( E(y) \) close to \( y = d \), known as double junction in the literature [8], due to the filling of radiation-induced
acceptor traps with holes. At the voltage of 40 V the $E(y)$-field is low in a significant fraction of the pad diode.

The $y$- and $t$-dependencies of $E_w(y, t)$ from the TCAD simulation of the pad diode at 40 V for selected $\Phi_{eq}$-values are shown in Figs. 7 and 8. Up to $\Phi_{eq} = 10^{12}$ cm$^{-2}$, the same $y$- and $t$-dependence is found as for the non-irradiated pad diode. Between $\Phi_{eq} = 10^{12}$ and $10^{13}$ cm$^{-2}$ the situation changes: The time constant
of the change of $E_w(y, t)$ increases, and above $10^{13}$ cm$^{-2}$, $E_w(y, t)$ reaches the constant value of $1/d$, which is the geometric weighting field. It has been verified by TCAD simulations that $E_w(y, t) = 1/d$ for $t > 0$ is also valid at higher voltages if $\Phi_{eq} > 10^{13}$ cm$^{-2}$. This is the justification for using the geometrical weighting field for the calculation of the current signals in highly-irradiated segmented sensors. It is noted that the fluence at which $E_w(y, t)$ becomes a constant is about one order of magnitude lower than the fluence at which the $y$-dependence of the electric field changes, indicating that these effects have different causes.

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**Figure 8**: Time dependence of the simulated weighting field as a function of $\Phi_{eq}$ at $V = 40$ V at $-20$ °C. The upper curves are for $y = 10$ µm, the lower ones for $y = 180$ µm. Left: linear, and right logarithmic scale.

Fig. shows the $t$-dependence of $E_w$ at $y = 10$ µm (depleted region) and at $y = 180$ µm (non-depleted region) for different values of $\Phi_{eq}$. For all $\Phi_{eq}$-values exponential dependencies are observed, with time constants $\tau(V, \Phi_{eq})$, which depend on both $V$ and $\Phi_{eq}$. Using Eq. 10 and $w(40$ V) of the non-irradiated diode, an estimate of the resistivity $\rho(\Phi_{eq})$ of the low field region $y > w$ can be obtained from $\tau(\Phi_{eq})$. As shown in Fig. 9, $\rho(\Phi_{eq})$ increases by about four orders of magnitude from 2.3 kΩ-cm to a value of a few $10^4$ kΩ-cm.

The value expected for the non-irradiated silicon is given by the boron-doping density and the mobility of holes: $\rho = (q_0 \cdot \mu_h \cdot N_d)^{-1} = 2.3$ kΩ-cm. If the density of free charge carriers is dominated by generation-recombination

$$\rho = (2 \cdot q_0 \cdot n_i \cdot \sqrt{\mu_e \cdot \mu_h})^{-1},$$

(13)

with the value of $\rho = 2.1 \times 10^4$ kΩ-cm for silicon at $-20$ °C. Fig. shows that the analysis of the simulated data is in agreement with these expectations. The mobilities of holes and electrons are $\mu_h$ and $\mu_e$, respectively, and $n_i$, $n_h$ and $n_e$ are the intrinsic charge-carrier density, and the densities of holes and electrons. Eq. 13 follows from the relation $\rho = (q_0 \cdot (n_h \cdot \mu_h + n_e \cdot \mu_e))^{-1}$. Equal electron-hole generation gives $\mu_h \cdot n_h = \mu_e \cdot n_e$, the equilibrium condition is $n_h \cdot n_e = n_i^2$, from which follows $\mu_h \cdot n_h + \mu_e \cdot n_e = 2 \cdot n_i \cdot \sqrt{\mu_e \cdot \mu_h}$.

To summarise this section: The TCAD simulations show that for $\Phi_{eq}$ above $10^{13}$ cm$^{-2}$ the weighting field is independent of time and voltage, and equal to the geometric weighting field, i.e. the time-independent weighting field of the fully depleted non-irradiated sensor. At lower voltages and fluences, the time dependence of $E_w$ is related to the resistivity of the low-field region. Its resistivity increases with $\Phi_{eq}$ and reaches approximately the intrinsic resistivity.
Figure 9: Resistivity, $\rho$, in the non-depleted region for $V = 40$ V calculated from the logarithmic slope of the time-dependence of $E_w$ shown in Fig. 8 as a function of $\Phi_{eq}$. Open triangles: Values of $\rho$ obtained from the TCAD simulation. For the calculation of $\rho$ a constant depletion width $w(40$ V$)$ is assumed. The value before irradiation is displayed at $\Phi_{eq} = 10^{11}$ cm$^{-2}$. The uncertainty of $\rho$ determined from the time dependence of $E_w$ at $\Phi_{eq} = 10^{14}$ cm$^{-2}$ is about 100%. The $E_w$-decay time is about 50 µs and the TCAD simulation is only made up to 50 ns.

6. Conclusions

In this paper the weighting field, $E_w$, is investigated, which is needed to simulate the response of radiation-damaged silicon detectors. Usually it is assumed that $E_w$ for irradiated sensors does not depend on time and can be calculated as the difference of the electric field in the biased sensor and 1 V added to the readout electrode minus the electric field in the biased sensor. The paper shows that this assumption is valid, and Eq. 6 can be used to calculate the induced current.

As pointed out in Ref. [7], a time-dependent $E_w$ is required to simulate the response of partially depleted, non-irradiated silicon sensors. Using TCAD simulations of a pad diode with radiation damage described by the Hamburg Penta Trap Model [10], it is shown that for a partially depleted sensor and neutron equivalent fluences $\Phi_{eq} \lesssim 10^{13}$ cm$^{-2}$ a time-dependent $E_w$ is required, whereas at higher fluences $E_w$ is time independent. The reason for the transition is the increase of the resistivity in the low-field region of the sensor due to the increase of the charge-carrier generation rate by radiation damage. At $T = -20$ °C, the temperature of the study, the resistivity of the non-depleted silicon increases by about four orders of magnitude as a function of $\Phi_{eq}$.

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