Modeling and Forecasting Periodic Time Series data with Fourier Autoregressive Model

Abass I. Taiwo*, Timothy O. Olatayo, Adedayo F. Adedotun, Kazeem K. Adesanya
Department of Mathematical Sciences, Faculty of Science, Olabisi Onabanjo University, Ago-Iwoye, Nigeria

Abstract
Most frequently used models for modeling and forecasting periodic climatic time series do not have the capability of handling periodic variability that characterizes it. In this paper, the Fourier Autoregressive model with abilities to analyze periodic variability is implemented. From the results, FAR(1), FAR(2) and FAR(2) models were chosen based on Periodic Autocorrelation function (PeACF) and Periodic Partial Autocorrelation function (PePACF). The coefficients of the tentative model were estimated using a Discrete Fourier transform estimation method. FAR(1) models were chosen as the optimal model based on the smallest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC). The residual of the fitted models was diagnosed to be white noise. The in-sample forecast showed a close reflection of the original rainfall series while the out-sample forecast exhibited a continuous periodic forecast from January 2019 to December 2020 with relatively small values of Periodic Root Mean Square Error (PRMSE), Periodic Mean Absolute Error (PMAE) and Periodic Mean Absolute Percentage Error (PMAPE). The comparison of FAR(1) model forecast with AR(3), ARMA(2,1), ARIMA(2,1,1) and SARIMA(1,1,1)(1,1,1)_{12} model forecast indicated that FAR(1) outperformed the other models as it exhibited a continuous periodic forecast. The continuous monthly periodic rainfall forecast indicated that there will be rapid climate change in Nigeria in the coming yearly and Nigerian Government needs to put in place plans to curtail its effects.

Keywords: Fourier Autoregressive model, Climate change, Periodic time series, Forecasting, Rainfall Series

Introduction
Understanding the variability in climatic time series data of a region over a long period gives one an idea about the climate change of such region [1]. The change in the measures of climatic variables has been attributed to natural and man-made reasons [2].

This change has been termed climate change and [3] defined climate change as a change in the state of the climate that can be identified by changes in the mean and/or the variability of its properties, and that persists for an extended period, typically decades or longer.

Based on the growing consensus among several scientific kinds of literature that in the coming decades, there will be a rapid increase in climatic time series variability level worldwide. This will be unfavourable for crop growth and yields in many regions and countries [4-6]. Patterns of precipitation and storm events are also likely to change and intensity of rainfall events is likely to increase on the average as well [7, 8]. Nigeria like other countries in sub-Saharan Africa is highly vulnerable to the impacts of climate change [9].

The historical climatic record of Nigeria has shown considerable signals of a changing climate [10]. In hence, climate change in Nigeria can be a challenge to sustainable human development and this may lead to more frequent and more severe climate-related impacts that may deter efforts to achieve the country’s development objectives, including the targets of the Nigeria Vision 20:2020 and the

*Email: taiwo.abass@oouagoiwoye.edu.ng
Millennium Development Goals (MDGs) [11, 12]. The challenges will be multifaceted (social, economic, environmental), and its impact on infrastructure will be significant [11]. It is also expected to negatively affect the already limited electrical power supply through impacts on the existing hydroelectric and thermal generation; service interruption [12].

This research article aims at modeling and forecasting Nigerian monthly rainfall series from January 1996 to December 2018 obtained from [13] since the variability changes in rainfall series is significant to climate change [14]. Several statistical methods like Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) had been used to model and forecast Nigerian Monthly Rainfall series but it seems inappropriate because these methods only take into consideration stationarity of the series and determining the presence of seasonal variation [15]. Hence, a Fourier Autoregressive (FAR) model proposed by [15] and has the ability to capture periodic changes from one period in a year to another year will be used to model and forecast Nigerian rainfall series.

Materials and Methods

Fourier autoregressive model

Fourier autoregressive (FAR) model is given as

$$y_{k\omega+v} = \varphi_0 + \sum_{i=1}^{p(v)} \left( \varphi_i(v) \cos \frac{2\pi k}{\omega} + \varphi_i^*(v) \sin \frac{2\pi k}{\omega} \right) y_{k\omega+v-i} + \varepsilon_{k\omega+v}$$

where $v$ is the period index ($v = 1, 2, ..., \omega$), $k$ is the year index ($k = 0, \pm 1, \pm 2, ...$), $\varphi_i(v)$ is the periodic autoregressive coefficient, $\omega$ is the number of seasons and $\varepsilon_{k\omega+v}$ is a white noise with mean zero (0) and periodic variance, $\sigma^2(v)$.

Model building in Fourier autoregressive model

The four basic steps in Fourier autoregressive model building will be carried out as follows:

Model identification for Fourier autoregressive (FAR) model

The identification of the FAR model will be determined using sample periodic Autocorrelation function (PeACF) given as

$$\Gamma_i(v) = \hat{\gamma}_i(v) \sqrt{\hat{\gamma}_0(v) \hat{\gamma}_0(v-\lambda)}$$

where $\hat{\gamma}_i(v)$ is the sample periodic autocorrelation function and sample periodic Partial Autocorrelation function (PePACF) given as

$$\hat{\phi}_{k\omega+v} = \frac{\hat{\gamma}_i(v) - \sum_{j=1}^{k} \hat{\phi}_{k\omega+i-j} \hat{\gamma}_{i-l}}{1 - \sum_{j=1}^{k} \hat{\phi}_{k\omega+i-j} \hat{\gamma}_{i-l}}$$

where

$$\hat{\phi}_{k\omega+i} = \hat{\phi}_{k\omega+i} - \hat{\phi}_{k\omega+i,k\omega+i} \hat{\phi}_{k\omega+i+1}$$

Model estimation of Fourier autoregressive model

The Fourier autoregressive coefficients will be estimated using the discrete Fourier transform estimation method where the discrete Fourier transform is assumed to be a periodic stationary process and this is expressed in matrix form as

$$
\begin{pmatrix}
\gamma_{t1} \\
\gamma_{t2} \\
\vdots \\
\gamma_{t\omega}
\end{pmatrix} = \begin{pmatrix}
1 & \frac{\cos 2\pi}{12} & \frac{\sin 2\pi}{12} \\
1 & \frac{\cos 2\pi(2)}{12} & \frac{\sin 2\pi(2)}{12} \\
\vdots & \vdots & \vdots \\
1 & \frac{\cos 2\pi(\omega)}{12} & \frac{\sin 2\pi(\omega)}{12}
\end{pmatrix}
\begin{pmatrix}
\gamma_{t1-1} \\
\gamma_{t2-1} \\
\vdots \\
\gamma_{t\omega-1}
\end{pmatrix} + \begin{pmatrix}
\cos 2\pi k \\
\cos 2\pi(2) k \\
\vdots \\
\cos 2\pi(\omega) k
\end{pmatrix}
\begin{pmatrix}
\gamma_{t1-k} \\
\gamma_{t2-k} \\
\vdots \\
\gamma_{t\omega-k}
\end{pmatrix}$$

where $\varphi_k = k\omega + v, \text{for } v = 0,1,2, ..., \omega$ and $\omega = 12$.

After model estimation, the most appropriate model will be chosen based on the lowest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC) given as

$$\text{PAIC} = n \ln \hat{\sigma}^2(v) + 2P(v)$$

and

$$\text{PBIC} = n \ln \hat{\sigma}^2(v) + \frac{\ln N}{N} P(v)$$
where $\hat{\sigma}^2_0(v)$ is the periodic estimator of $\sigma^2_0(v)$ and $P(v)$ is the number of periodic autoregressive coefficients in the season respectively.

**Diagnostic Checking in Fourier Autoregressive Model**

After parameter estimation, the assessment of the model adequacy will be done by checking whether residuals $\{e_t\}$ are white noise. Hence a careful analysis of the estimated residuals will be carried out by checking whether all the residuals are white noise and this will be done by computing the sample PEACF and PEPACF of the residuals to determine whether they do not form any pattern and were found to be statistically significant within two standard deviations with $\alpha = 0.05$.

**Forecasting in Fourier autoregressive model (FAR) model**

Given a FAR(1) model as

\[
y_t = \mu + \Phi_1 \cos z (y_{t-1}) - \Phi_1^* \sin z (y_{t-1}) + \epsilon_t \\
= (1 - \Phi_1 \cos z - \Phi_1^* \sin z)(y_{t-1} - \mu) + \epsilon_t
\]

where $z = \frac{2\pi k}{\omega}$ and $\mu$ is a constant.

The general form of the forecast equation is given as

\[
y_{t1}(l) = \mu + [(\Phi_1 \cos z y_{t1}(l-1)) - \mu] + (\Phi_1^* \sin z y_{t1}(l-1) - \mu) \quad l \geq 1
\]

**Forecasting Evaluations**

The forecast evaluations that will be used to measure the accuracy of FAR(1) model are Periodic Root Mean Square Error (PRMSE), Periodic Mean Absolute Error (PMAE) and Periodic Mean Absolute Percentage Error (PMAPE). The forecast evaluations are given as

\[
PRMSE = \sqrt{\frac{1}{tv+1} \sum_{tv=1}^{p-1} (\hat{y}_{tv} - y_{tv})^2}
\]

\[
PMAE = \sum_{tv=1}^{p-1} \frac{\hat{y}_{tv} - y_{tv}}{\hat{y}_{tv}}
\]

\[
PMAPE = \frac{1}{tv+1} \sum_{tv=1}^{p-1} \frac{(\hat{y}_{tv} - y_{tv})^2}{\hat{y}_{tv}}
\]

where $tv = 1, 2, \ldots, p-1$ (Taiwo, 2017). The actual and predicted values for corresponding $tv$ values are denoted by $\hat{y}_{tv}$ and $y_{tv}$ respectively. The smaller the values of PRMSE, PMAE and PMAPE, the better the forecasting performance of the model.

**Results and Discussion**

In order to ascertain the efficiency of Fourier autoregressive models, Nigerian monthly rainfall series from January 1996 to December 2018 collected from [13] was analyzed. The results obtained from Autoregressive (AR), Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA) and Seasonal Autoregressive Integrated Moving Average (SARIMA) models will be compared with FAR model results to ascertain the reasons while FAR is the better model for Nigerian rainfall series.

The rainfall series exhibited a cyclical or periodic variation and this informs the use of Fourier Autoregressive Model. A critical look at the PeACF and PePACF for January to December series showed that the PeACF was stable and PePACF cut off at lag 2, so tentative FAR(1), FAR(2) and FAR(3) were chosen for January to December rainfall series. Discrete Fourier transform estimation method was used to obtain the coefficients of FAR(1), FAR(2) and FAR(3). FAR(1) models estimation is given in equation (12) were chosen as the optimal models based on the smallest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC) given in table 1.
\[
y_{\text{Jan}} = 3.612765 - 0.339286 \cos(t) y_{t-1} - 0.871348 \sin(t) y_{t-1} \\
y_{\text{Feb}} = 8.376009 + 2.040730 \cos(t) y_{t-1} + 0.697592 \sin(t) y_{t-1} \\
y_{\text{Mar}} = 26.85521 - 2.915318 \cos(t) y_{t-1} + 1.072262 \sin(t) y_{t-1} \\
y_{\text{Apr}} = 64.12574 + 7.189011 \cos(t) y_{t-1} - 7.315425 \sin(t) y_{t-1} \\
y_{\text{May}} = 115.6598 + 5.108940 \cos(t) y_{t-1} + 0.519816 \sin(t) y_{t-1} \\
y_{\text{Jun}} = 157.4872 - 6.767992 \cos(t) y_{t-1} - 9.866734 \sin(t) y_{t-1} \\
y_{\text{Jul}} = 212.0417 - 11.81382 \cos(t) y_{t-1} - 11.87484 \sin(t) y_{t-1} \\
y_{\text{Aug}} = 228.8587 - 3.177316 \cos(t) y_{t-1} - 6.206400 \sin(t) y_{t-1} \\
y_{\text{Sep}} = 201.2373 + 18.82645 \cos(t) y_{t-1} + 3.634644 \sin(t) y_{t-1} \\
y_{\text{Oct}} = 107.7105 - 0.157149 \cos(t) y_{t-1} - 0.629027 \sin(t) y_{t-1} \\
y_{\text{Nov}} = 109.7640 - 0.656149 \cos(t) y_{t-1} - 0.529527 \sin(t) y_{t-1} \\
y_{\text{Dec}} = 3.594377 + 0.717928 \cos(t) y_{t-1} + 1.261494 \sin(t) y_{t-1}
\]

where \( t = \frac{2 \pi k}{\omega} \).

The residual of the fitted FAR(1) models was diagnosed using periodic residual autocorrelation. The periodic residual autocorrelation for FAR(1) models showed the residual was white noise, hence the models can be used to forecast Nigerian monthly rainfall series. The in-sample and out-sample forecast were obtained for Nigerian monthly rainfall series based on the FAR(1) models from January to December. The in-sample forecast for FAR(1) model from January to December showed a close look to the original series and out-sample forecast values for FAR(1) in Table-2 and Figure-1 showed a continuous periodic and close reflection of the original series from January 2019 to December 2020.

**Table 1**: Information Criteria for January to December Series

| Month(s) | Inf. Criteria | FAR(1) | FAR(2) | FAR(3) | Month(s) | Inf. Criteria | FAR(1) | FAR(2) | FAR(3) |
|----------|---------------|--------|--------|--------|----------|---------------|--------|--------|--------|
| January  | paic          | 3.430102* | 3.455248 | 3.541566 | July     | aic          | 10.21054* | 10.34706 | 10.52046 |
|          | bic           | 3.578210* | 3.702095 | 3.887152 |          | bic          | 10.35865* | 10.59391 | 10.86005 |
| February | paic          | 5.621061* | 5.684433 | 5.839269 | August   | aic          | 9.501169* | 9.638035 | 9.67562 |
|          | bic           | 5.769169* | 5.931279 | 6.184854 |          | bic          | 9.649277* | 9.884881 | 10.02211 |
| March    | paic          | 7.344116* | 7.479084 | 7.503036 | September | aic          | 9.421283* | 9.584354 | 9.701643 |
|          | bic           | 7.492224* | 7.725931 | 7.848621 |          | bic          | 9.569391* | 9.83120  | 10.04723 |
| April    | paic          | 8.621772* | 8.7137  | 8.721801 | October   | aic          | 9.011679* | 9.250466 | 9.054205 |
|          | bic           | 8.769880* | 8.960547 | 9.067386 |          | bic          | 9.27657*  | 9.497312 | 9.39979 |
| May      | paic          | 8.962050* | 9.014371 | 9.144934 | November  | aic          | 9.030679* | 9.250466 | 9.054205 |
|          | bic           | 9.110158* | 9.261218 | 9.490519 |          | bic          | 9.278877* | 9.497312 | 9.39979 |
| June     | aic           | 8.862177* | 8.929468 | 9.097053 | December  | aic          | 4.626287* | 4.75582  | 4.907723 |
|          | bic           | 9.010285* | 9.176315 | 9.442639 |          | bic          | 4.774395* | 5.002667 | 5.253309 |

**Table 2**: Forecast of Nigerian Rainfall series from January 2019 to December 2020

| Month | AR | ARMA | ARIMA | SARIMA | FAR |
|-------|----|------|-------|--------|-----|
| Jan-19| 37.38354 | 32.35587 | 37.2978 | 2.94521 | 3.8648 |
| Feb-19| 76.19937 | 72.97246 | 74.81843 | 7.97545 | 13.5153 |
| Mar-19| 105.9529 | 116.4021 | 103.7225 | 23.1458 | 132.2314 |
| Apr-19| 120.5798 | 150.8834 | 117.7292 | 93.6134 | 34.0749 |
| May-19| 121.2371 | 167.9942 | 118.1123 | 121.723 | 649.8104 |
| Jun-19| 113.2972 | 164.6139 | 110.2251 | 125.624 | 689.5175 |
| Jul-19| 102.8612 | 143.3183 | 100.0514 | 165.923 | 474.2523 |
| Aug-19| 94.42364 | 111.2287 | 91.93392 | 218.061 | 388.8888 |
| Sep-19| 90.01039 | 77.7493 | 87.7716 | 245.435 | 821.4758 |
| Oct-19| 89.48844 | 51.88299 | 87.36449 | 98.9342 | 105.5695 |
| Nov-19| 91.47476 | 39.87402 | 89.32715 | 9.9567 | 95.0692 |
| Dec-19| 94.29991 | 43.77599 | 92.03366 | 2.43258 | 2.9225 |
| Jan-20| 96.68187 | 61.24982 | 94.26269 | 1.62353 | 5.7203 |
| Feb-20| 98.0003 | 86.54453 | 95.44619 | 10.6348 | 20.9143 |
| Mar-20| 98.2394 | 112.3036 | 95.5976 | 50.7355 | 133.261 |
Table 3 - Forecast Evaluation for Nigerian Rainfall Series FAR(1)

| Month(s) | Mean Absolute Percent Error of FAR(1) | Root Mean Square Error of FAR(1) | Mean absolute Error of FAR(1) |
|----------|--------------------------------------|---------------------------------|-----------------------------|
| January  | 8.24128                              | 1.180193                        | 0.885314                    |
| February | 35.60173                             | 2.484298                        | 1.95046                     |
| March    | 37.45815                             | 8.353536                        | 6.794034                    |
| April    | 20.06252                             | 15.82376                        | 12.14949                    |
| May      | 13.29114                             | 18.75859                        | 15.12987                    |
| June     | 8.275527                             | 17.84486                        | 12.49376                    |
| July     | 12.02743                             | 35.01917                        | 26.36026                    |
| August   | 8.782356                             | 24.56226                        | 19.63751                    |
| September| 9.291498                             | 23.60051                        | 17.65652                    |
| October  | 16.2637                              | 20.4088                         | 16.2637                     |

Figure 1: time plot of Nigerian monthly rainfall from January, 2019 to December, 2020.

In order to ascertain the reasons while Fourier Autoregressive model is the better model for forecasting Nigeria rainfall, the rainfall forecasted values from AR(3), ARMA(2,1), ARIMA(2,1,1) and SARIMA(1,1,1)(1,1,1)_{12} given in Table-2 and Figure-1 is compared with that of FAR(1) model. Based on the comparison, AR(3), ARMA(2,1) and ARIMA(2,1,1) models are not appropriate for forecasting Nigerian rainfall since the forecast values from these models did not reflect the seasonality and periodicity that is usually present in rainfall series. While SARIMA(1,1,1)(1,1,1)_{12} model captured and exhibited the seasonality in Nigerian rainfall series but the periodicities in the series were not resolved. But, FAR(1) model forecast in Table-2 and Figure-1 captured and exhibited the seasonality and periodicity that is presented in the rainfall series. As well, FAR(1) model showed a continuous periodic movement and close reflection to the original series from January 2019 to December 2020. The values of the forecast evaluation for FAR(1) model in Table-3 showed as well the consistent of the forecast since their values were relatively low. Hence, the Fourier Autoregressive model is adequate and suitable for modeling and forecasting Nigerian rainfall series.

Conclusion

The Fourier autoregressive model was used to analyze Nigerian monthly rainfall series collected from [13] from January 1996 to December 2018. FAR(1), FAR(2) and FAR(2) models were chosen
based on \( PeACF \) and \( PeACF \). The coefficients of the tentative model were estimated using a Discrete Fourier transform estimation method. The FAR(1) models were chosen as the optimal model based on the smallest values of Periodic Akaike (PAIC) and Bayesian Information criteria (PBIC). The residual of the fitted models was diagnosed to be white noise. The in-sample forecast showed a close reflection of the original rainfall series while the out-sample forecast exhibited a continuous periodic forecast from January 2019 to December 2020 with relatively small values of PRMSE, PMAE, and PMAPE. The comparison of FAR(1) model forecast with AR(3), ARMA(2,1), ARIMA(2,1,1) and SARIMA\((1,1,1)(1,1,1)_1_2\) showed that AR(3), ARMA(2,1), ARIMA(2,1,1) models are not appropriate for forecasting Nigerian rainfall since the forecast values from these models did not reflect the seasonality and periodicity that is usually present in rainfall series. While SARIMA\((1,1,1)(1,1,1)_1_2\) model captured and exhibited the seasonality in Nigerian rainfall series but the periodicities in the series were not resolved. But, FAR(1) model captured and exhibited the seasonality and periodicity that is presented in the rainfall series. FAR(1) model showed a continuous periodic movement and close reflection to the original series from January 2019 to December 2020. As well, the values of the forecast evaluation for FAR(1) model showed consistent of the forecast since their values were relatively low. Hence, Fourier Autoregressive model is adequate and suitable for modeling and forecasting Nigerian rainfall series. In conclusion, the continuous monthly periodic forecast indicated that there will be rapid climate change in Nigeria in the coming yearly and Nigerian Government needs to put in place plans to curtail its effects.

References
1. Adepitan, J., Falayi E. and Ogunsanwo, F. 2017. Confirmation of Climate change in Southwestern Nigeria through Analysis of Rainfall and Temperature variations over the region. Covenant Journal of Physical and Life Sciences, 5: 37-51.
2. Novotny E. and Stefan, H. 2007. Streamflow in Minnesota: indicator of climate change. Journal of Hydrology, 334: 319-333.
3. Intergovernmental Panel on Climate Change (IPCC). 2007. Climate change: climate change impacts, adaptation and vulnerability. Working Group II; contribution to the Intergovernmental Panel on Climate Change Fourth Assessment Report. Summary for Policymakers, 23.
4. Food and Agriculture Organization, 2008. FAO, Report of the FAO Expert Workshop on Climate Change Implications for Fisheries and Aquaculture. FAO Fisheries Report no.870, Food and Agricultural Organization of the United Nations, Rome, Italy.
5. Yesuf, M., Difale, S., Deressa, T., Ringler C. and Kohlin G. 2008. The Impact of Climate Change and Adaptation on Food Production in Low- Income Countries: Evidence from the Nile Basin, Ethiopia. International Food Policy Research Institute Discussion (IFPRI Paper No. 00828. Environment and Production Technology Division, IFPRL, Washington. D.C.
6. Ibrahim, K., Shamsudin, M., Yacob R. and Radam, A. 2014. Economic impact of climate change on Maize production in Northern Nigeria. Trend in Applied Sciences Research, 9: 522-533.
7. Meehl, G. A., Stocker, T. F., Collins, W. D., Friedlingstein P. and others, 2007. Global climate projections. In: Solomon S, Qin D, Manning M, Chen Z and others (eds) Climate change: the physical science basis. Contribution of Working Group I to the Fourth Assessment Report of the Inter-Governmental Panel on Climate Change. Cambridge University Press, Cambridge, pp: 49-84.
8. Ogunrayi, O., Akinseye, F., Goldbelg V. and Bernhofer, C. 2016. Descriptive Analysis of Rainfall and Temperature trends over Akure, Nigeria. Journal of Geography and Regional Planning, 9: 195- 202.
9. Daramola, M., Eresanya E. and Erhabor, S. 2017. Analysis of Rainfall and Temperature over Climatic zone in Nigeria. Journal of Geography, Environment and Earth Sciences International, 11:1-14.
10. Adiku, S., Dilys, S., Ibrahima, H., Madina, D., Bright, S., Freduah, A., Joseph, P., Seydou, K., Eric, A., Alhassane, L., Jon, F., Dougbedji, A., Myriam, T., Lodoun, D., Daouda and Ousmane, N. 2014. Change Impacts on West African Agriculture: An Integrated Regional Assessment (CIWARA).

11. NEST and Tegler, B. 2011. Climate Change Adaptation Strategy Technical Reports – Nigeria, (CCASTR). Building Nigeria’s Response to Climate Change. Nigerian Environmental Study/Action Team (NEST), Ibadan, Nigeria. National Adaptation Strategy and Plan of Action on Climate Change for Nigeria (NASPA-CCN), 2011. Federal Ministry of Environment Special Climate Change Unit. Building Nigeria’s Response to Climate Change. Ibadan, Nigeria.

12. Nigerian Meteorological Agency, 2018. Lagos Office, Nigeria.

13. Tara, A. C. and Mark B. 2017. Seasonal time-series modeling and forecasting of monthly mean temperature for decision making in the Kurdistan Region of Iraq, Journal of Statistical Theory and Practice, 11(4):604-633.

14. Taiwo, A., I. 2017. Spectral and Fourier Parameter Estimation of Periodic Autocorrelated Time Series Data, Department of Mathematical Sciences, Ph.D. Thesis, Olabisi Onabanjo University, Ago-Iwoye, Nigeria.

15. Adesanya K. K., Taiwo A. I., Adedodun A. F. and Olatayo T. O. 2018. Modeling Continuous Non-Linear Data with Lagged Fractional Polynomial Regression, Asian Journal of Applied Sciences, 6(5): 315-320