Quantum discord induced by white noises

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We discuss the creation of quantum discord between two two-level atoms trapped in an optical cavity in a noisy environment. It is shown that nonzero steady-state quantum discord between atoms can be obtained when the white-noise field is separately imposed on atoms or cavity mode, while the steady-state quantum discord reaches zero if both cavity mode and atoms are driven simultaneously by white-noise fields. In particular, we demonstrate that white-noise field in different cases can play a variously constructive role in the generation of quantum discord.

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1. Introduction

Quantum entanglement, as a valuable physical resource, has been widely applied to most quantum-information processing tasks [1-3]. It is a type of correlation lies in the superposition principle and the tensor product structure of the state space of composite quantum systems. However, entanglement can not distinguish the classical correlation and quantum correlation encoded in a quantum system exactly, since some other kinds of nonclassical correlations could exist even if the entanglement is vanishing. In order to capture the pure quantum correlations in a quantum system, Ollivier and Zurek [4] introduced the so-called quantum discord, which is the discrepancy between two quantum extensions of classical mutual information. Similar idea was suggested by Henderson
and Vedral [5,6] simultaneously and independently. Quantum discord is a more general measure of quantum correlation may include entanglement but is an independent one [7,8], it is more useful than entanglement to describe the quantum correlation involved in a quantum system. Moreover, it is shown that quantum discord can provide a speedup for some certain tasks over their classical counterparts both theoretically [9-12] and experimentally [13], in this sense quantum discord can also be regarded as a physical resource in quantum information theory, so it is desirable to investigate the quantum discord further and more broadly.

On the other hand, a real quantum system will interact with the environmental noise inevitably. The interaction between the system of interest and the noise, which usually models surroundings of the system, leads to a decoherence process [14,15]. As a consequence, the system may end up in a mixed state which is no longer suitable for quantum information processing. In order to prevent or minimize the environmental noise, numerous proposals have been made, such as loop control strategies [16,17], quantum error correction [18,19] and exploiting the decoherence-free subspace [20-22]. Recent studies show that quantum discord is more robust than entanglement against decoherence. For example, under the conditions in which entanglement sudden death (ESD) [23] can occur, the quantum discord will just vanish asymptotically [24-26] or instantaneously at some time points [27]. Moreover, quantum discord can not be destroyed, even can be frozen, under some certain decoherence channels [28,29]. In spite of this, this kind of quantum correlation will still be destroyed due to the destructive effects of the environmental noise at the asymptotic infinite-time limit.

In this paper, we investigate the generation of quantum discord in a noisy environment, by which we find that environmental noise can also play a constructive role in creation of quantum discord. Our system consists of two two-level atoms trapped in a leaky optical cavity. Both the cavity mode and the atoms are driven by external white-noise fields. We focus on the problem of generating quantum discord with only incoherent sources are available. The results show that the quantum discord of the two atoms can indeed be created and even reach a stable value in such a situation. This paper is organized as follows. In Sec.II, we firstly introduce our model and recall the calculations of quantum discord briefly, and then discuss the noise-assisted generation of quantum discord.
detailedly and compared with the generation of entanglement in three different cases: only atoms are driven by white-noise field; only cavity mode are driven by white-noise field; and both cavity mode and atoms are driven by the same white-noise field. The conclusion is drawn finally.

2. Quantum discord induced by white noises

Our system consists of two identical two-level atoms, labeled by 1 and 2, trapped in a leaky optical cavity, see Fig. 1. Each atom has a ground state $|g\rangle$ and an excited state $|e\rangle$. The coupling strength between atoms and cavity mode is $g$. We assume that the distance between the atoms is much larger than an optical wavelength, therefore dipole-dipole interaction can be neglected. The cavity mode and atoms are driven by the external white-noise fields, intensities of which will be characterized in terms of effective photon numbers $m_T$ and $n_T$, respectively. The Hamiltonian [30-32] describing the interaction between atoms and cavity mode reads

$$H = \omega_a a^+ a + \sum_{i=1,2} \omega_i \sigma_i^+ \sigma_i^- + g(a^+ \sigma_i^- + \text{H.c.}), \quad (1)$$

where $\omega_a$ is the transition frequency of atom, $\omega_c$ is the frequency of the cavity mode, $a$ is the annihilation operator of cavity mode, and $\sigma_i^-$ ($i = 1, 2$) is the lower operator of the $i$th atom. To simplify the representation, we turn to the interaction picture with respect to $H_0 = \omega_c a^+ a + \sum_{i=1,2} \omega_i \sigma_i^+ \sigma_i^-$, now the Hamiltonian is given by

$$H_I = \sum_{i=1,2} g(a^+ \sigma_i^- + \text{H.c.}), \quad (2)$$

where we have assumed the atom-cavity coupling is on resonance.

There are two channels that the relaxation of atom-cavity system can take place, one is atom spontaneous emission at rate $2\gamma$ and the other is cavity decay at rate $2\kappa$. The master equation governing the time evolution of the global system is given by (setting $\hbar = 1$)

$$\dot{\rho} = -i[H, \rho] + L(\rho), \quad (3)$$

with the Liouvillian $L(\rho)$ as follow

$$L(\rho) = (n_T + 1)\gamma \sum_{i=1,2} (2\sigma_i^- \rho \sigma_i^+ - \sigma_i^+ \sigma_i^- \rho - \rho \sigma_i^+ \sigma_i^-)$$

$$+ \gamma (m_T + 1) \sum_{i=1,2} (2a^+ a \sigma_i^- - a^+ \sigma_i^- a - \sigma_i^- a^+ a \rho - \rho a^+ \sigma_i^- a).$$
Here we do not specify the white-noises, but their intensities \( m_T \) and \( n_T \) refer to the effective particle numbers.

Before our discussion, it is necessary to recall the quantum discord briefly. For a bipartite quantum system, if \( \rho^{ab} \) denotes the density operator of the joint system, and \( \rho^a \) (\( \rho^b \)) denotes the reduced density operator of subsystem \( a \) (\( b \)), then the quantum discord between subsystems \( a \) and \( b \) can be obtained as follow

\[
Q(\rho) = I(\rho) - C(\rho),
\]

(5)

where \( I(\rho) = S(\rho^a) + S(\rho^b) - S(\rho^{ab}) \) is the quantum mutual information and \( C(\rho) \) is the classical correlation between the two subsystems. As discussed in ref. [5,6], the classical correlation is provided by \( C(\rho) = \max_{B_k} \{ S(\rho^a) - S(\rho^a | B_k) \} \), where \( \{ B_k \} \) is a set of von Neumann measurements performed on subsystem \( b \) locally, \( S(\rho^a | B_k) = \sum_k p_k S(\rho_k) \) is the quantum conditional entropy, \( \rho_k = (\mathbb{1} \otimes B_k)\rho(\mathbb{1} \otimes B_k)/\text{Tr}(\mathbb{1} \otimes B_k)\rho(\mathbb{1} \otimes B_k) \) is the conditional density operator corresponding to the outcome labeled by \( k \), and \( p_k = \text{Tr}(\mathbb{1} \otimes B_k)\rho(\mathbb{1} \otimes B_k) \). Here \( \mathbb{1} \) is the identity operator performed on subsystem \( a \).

Due to the complicated optimization involved, it is usually intractable to obtain the analytical expressions of quantum discord. So far as we know, the analytical expression of quantum discord is obtained only for some specific states [7,8], hence we will calculate the quantum discord of the atoms in a numerical way. Generally, the maximization of the classical correlations is achieved by a positive operator-valued measure (POVM) [5,6,33], however, for two qubits system, which is our case, Hamieh et al. [34] have proved that the projective measurement is the POVM, which maximizes the classical correlations. Therefore, it is sufficient for us to evaluate the discord using the following set of projectors, \( \{|\psi_1\rangle\rangle_1, |\psi_2\rangle\rangle_2| \), in which \( |\psi_1\rangle = \cos \theta |g\rangle + e^{i\phi} \sin \theta |e\rangle \) and \( |\psi_2\rangle = \)}
\[ -\cos \theta |e\rangle + e^{-i\phi} \sin \theta |g\rangle \]. For the purposes for obtaining the maximum of \( S(\rho^o) - S(\rho||B_k)) \), the parameters \( \theta \) and \( \phi \) vary from 0 to \( 2\pi \). We cutoff the intracavity photon numbers at a value of 5 in the simulations. In this paper, we will only focus on the cases of only incoherent sources are available, \( i.e. \), the initial state of the joint system is \( |g\rangle_1|g\rangle_2|0\rangle_c \) where the subscript 1 or 2 denotes the atom 1 or 2 and subscript \( 'c' \) denotes cavity mode. In order to explicitly show the dependence of quantum discord on the noise intensities, atom spontaneous emission rate and cavity leaky rate, we will examine the noise-induced quantum discord in the following three different cases: noise drives only the cavity mode; noises drive the two two-level atoms only; and noises drive both cavity mode and atoms.

Firstly, we will examine the noise-induced quantum discord between atoms in the case of only atoms are driven by noises, thereby we can set \( m_T = 0 \) in Eq. (4). We have plotted the amounts of both quantum discord of the two two-level atoms as a two-variable function of the intensity of the noise \( n_T \) and time \( t \) in Fig. 2. For \( n_T = 0 \), we can see that there is no quantum correlation between two atoms at any time, namely the vacuum field can not induce quantum discord between atoms. For a finite value of \( n_T \), it is shown that steady-state quantum discord between two atoms can be generated. The behavior of the amount of steady-state quantum discord is nonmonotonic with noise intensity, it increases to a maximum value for an optimal intensity of the noise and then decreases. The maximum value of steady-state quantum discord appears at a small noise intensity \( (n_T \approx 0.7) \). The larger \( n_T \) is not conducive to construct steady-state quantum discord. It is especially interesting that for a given noise intensity the quantum discord between atoms evolves from zero to a stationary value monotonically, which is quite different from the case of generation of entanglement where entanglement increases first and then drops [30-32]. That is to say the white-noise always plays a constructive role for quantum discord between atoms during the time evolution.

It is also worthwhile to study the steady-state quantum discord of the atoms as a function of intensity of noise and the cavity leaky rate or the atom spontaneous emission rate, these results are shown in Fig. 3 (a) and (c), respectively. As a comparison, we also show the steady-state entanglement between atoms in Fig.3 (b) and (d). From the upper two plots in Fig. 3, one can find that a proper cavity leaky rate can help increase the steady-state quantum discord of the two
atoms, but the cavity leakage almost has no effects on creation of steady-state entanglement, the steady-state entanglement only rises in a very small region and is extremely tiny. From Fig. 3(c), one can find that the behavior of the steady-state quantum discord between atoms exhibits a double resonance on both atomic spontaneous emission rate and noise intensity at an intermediate value. Contrasting to the steady-state quantum discord, which decreases to zero asymptotically with $n_T$ and $\gamma$ increasing, the steady-state entanglement between atoms vanishes suddenly and drastically, see Fig. 3(d), this phenomenon is reminiscent of the well known ESD [23], where the entanglement of a system will suddenly disappears when interacting with a noisy environment. This comparison between Fig. 3(c) and (d) shows that the quantum discord is more robust than entanglement against the noisy environments.

Now we come to the case that noise is only imposed on the cavity mode, i.e., $n_T = 0$. The time evolutions of quantum discord and between atoms is shown in Fig. 4. Similar to Fig. 2, the quantum discord between atoms evolves to a stationary value for a nonzero $m_T$. The steady-state quantum discord increases to a maximum value for an optimal noise intensity. Contrast to Fig. 2, the steady-state quantum discord holds a relative larger value when the noise intensity is large. That is to say, when noise is imposed on the cavity mode only, it is more easier to obtain the quantum discord between atoms than that the noises are imposed on the atoms. Moreover, we can see from Fig. 4 that the behavior of quantum discord between atoms is nonmonotonic during the time evolution for a given noise intensity, it firstly increases to a maximum value and then decreases to a stationary value. It means that the noise intensity plays both constructive and destructive roles in the generation of quantum discord between atoms. At the beginning the constructive effect is dominate, later the destructive effect is dominate and finally they are balanced where the quantum discord arrives at a stationary value. A weak vibration during the evolution can also be found in a short time especially obviously for $m_T \approx 1$. These are somewhat like the case for entanglement.

The dependence of the steady-state quantum discord and entanglement between atoms on the cavity leaky rate and atom spontaneous emission rate are shown in Fig. 5. From the right two plots in Fig. 5, one can find that, the steady-state entanglement between atoms can not be generated irrespective of the value of $\kappa$ and $\gamma$. From Fig. 5(a) one can see that the behavior of steady-state
quantum discord between atoms shows a resonance on both cavity leaky rate and noise intensity like Fig. 3(c). From Fig. 5(c), it is surprising that the steady-state quantum discord decreases rapidly along with increasing of atom spontaneous emission and then a slight revival occurs with the atomic spontaneous emission rate increasing for a larger $m_T$. This means that an appropriate atomic spontaneous emission is conducive for generating quantum discord between atoms if the environmental noise is strong.

In the following, we will consider that noises are imposed on both cavity mode and atoms. Here we assume the two noises are the same, i.e., $n_T = m_T$. The quantum discord between atoms as a function of noise intensity ($n_T = m_T$) and time are shown in Fig. 6. Again the entanglement between atoms can not be generated. However, different from the previous two cases, when cavity mode and atoms are driven by the same noise, the quantum discord between atoms evolves to zero instead of a stationary nonzero value for any noise intensity. The quantum discord between atoms during the time evolution is nonmonotonic about noise intensity, it increases to a maximal value for an optimal noise intensity and then asymptotically decreases to zero for a large intensity. This can be understood as that the combination of the noises imposed on the cavity modes and atoms strengthens the destructive role in quantum discord, even though they have different effects on the quantum discord separately. The joint effects of the external noises lead to a vanishing quantum discord between atoms.

In an experimental scenario, our atomic level structures can be achieved by alkali metal atoms and the leaky optical cavity can be realized by either the conventional Fabry-Perot cavity or the new type microtoroidal cavity [35-37]. The parameters concerning in this scheme is the coupling constant $g$, cavity leaky rate $\kappa$ and atomic spontaneous emission rate $\gamma$. In the current available experiments [38,39], they take $g \sim 2\pi \times 100$MHz and $g^2 \sim 20\gamma \kappa$. With these parameters, the quantum discord will come to a steady value at about $1\mu s$.

3. Conclusion and discussion

In conclusion, we have discussed the creation of quantum discord between two two-level atoms trapped in a leaky optical cavity in three different cases. It has been shown that there exist quite
differences between the noise-assisted generation of quantum discord and quantum entanglement. One can find that nonzero steady-state quantum discord between atoms can be generated when the white-noise field is separately imposed on atoms or cavity mode, even though the steady-state quantum discord reaches zero when both cavity mode and atoms are driven by two identical white-noise fields. Although quantum discord between atoms in the noisy environment is small but it can be distillable [40], which may provide a broader way to acquire quantum information resource and to process quantum computation. In addition, the effects of cavity leaky rate and atom spontaneous emission rate on the quantum discord between atoms are also discussed. These results may play a guiding part in generating desirable quantum discord between atoms in noisy environment.

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References

1. A. K. Ekert, “Quantum cryptography based on Bells theorem,” Phys. Rev. Lett. 67, 661-663 (1991).
2. C. H. Bennett and S. J. Wiesner, “Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states,” Phys. Rev. Lett. 69, 2881-2884 (1992).
3. C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, “Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels,” Phys. Rev. Lett. 70, 1895-1898 (1993).
4. H. Ollivier and W. H. Zurek, “Quantum Discord: A Measure of the Quantumness of Correlations,” Phys. Rev. Lett. 88, 017901 (2001).
5. L. Henderson and V. Vedral, “Classical, quantum and total correlations,” J. Phys. A 34, 6899-6905 (2001).
6. V. Vedral, “Classical Correlations and Entanglement in Quantum Measurements,” Phys. Rev. Lett. 90, 050401 (2003).
7. S. Luo, “Quantum discord for two-qubit systems,” Phys. Rev. A 77, 042303 (2008).
8. M. Ali, A. R. P. Rau, and G. Alber, “Quantum discord for two-qubit X states,” Phys. Rev. A 81, 042105 (2010).
9. S. L. Braunstein, C. M. Caves, R. Jozsa, N. Linden, S. Popescu, and R. Schack, “Separability of Very Noisy Mixed States and Implications for NMR Quantum Computing,” Phys. Rev. Lett. 83, 1054-1057 (1999).
10. D. A. Meyer, “Sophisticated Quantum Search Without Entanglement,” Phys. Rev. Lett. 85, 2014-2017 (2000).
11. A. Datta, S. T. Flammia, and C. M. Caves, “Entanglement and the power of one qubit,” Phys. Rev. A 72, 042316 (2005).
12. A. Datta, A. Shaji, and C. M. Caves, “Quantum Discord and the Power of One Qubit,” Phys. Rev. Lett. 100, 050502 (2008).
13. B. P. Lanyon, M. Barbieri, M. P. Almeida, and A. G. White, “Experimental Quantum Computing without Entanglement,” Phys. Rev. Lett. 101, 200501 (2008).
14. L. Mazzola, S. Maniscalco, J. Piilo, K.-A. Suominen, and B. M. Garraway, “Sudden death and sudden birth of entanglement in common structured reservoirs,” Phys. Rev. A 79, 042302 (2009).
15. L. Mazzola, S. Maniscalco, K.-A. Suominen, and B. M. Garraway, “Reservoir cross-over in entanglement dynamics,” Quantum Inf. Process 8, 577-585 (2009).
16. H. M. Wiseman and G. J. Milburn, “Quantum theory of optical feedback via homodyne detection,” Phys. Rev. Lett. 70, 548-551 (1993).
17. S. Mancini, D. Vitali, P. Tombesi, and R. Bonifacio, “Stochastic control of quantum coherence,” Europhys. Lett. 60, 498-504 (2002).
18. P. W. Shor, “Scheme for reducing decoherence in quantum computer memory,” Phys. Rev. A 52, R2493-R2496 (1995).
19. A. R. Calderbank and P. W. Shor, “Good quantum error-correcting codes exist,” Phys. Rev. A
20. M. B. Plenio, V. Vedral, and P. L. Knight, “Quantum error correction in the presence of spontaneous emission,” Phys. Rev. A 55, 67-71 (1997).

21. D. A. Lidar, I. L. Chuang, and K. B. Whaley, “Decoherence-Free Subspaces for Quantum Computation,” Phys. Rev. Lett 81, 2594-2597 (1998).

22. A. Beige, D. Braun, B. Tregenna, and P. L. Knight, “Quantum Computing Using Dissipation to Remain in a Decoherence-Free Subspace ,” Phys. Rev. Lett 85, 1762-1765 (2000).

23. T. Yu and J. H. Eberly, “Sudden death of entanglement,” Science 323, 598-601 (2009).

24. T. Werlang, S. Souza, F. F. Fanchini, and C. J. Villas Boas, “Robustness of quantum discord to sudden death,” Phys. Rev. A 80, 024103 (2009).

25. T. Werlang and G. Rigolin, “Thermal and magnetic quantum discord in Heisenberg models,” Phys. Rev. A 81, 044101 (2010).

26. A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acín, “Almost all quantum states have nonclassical correlations,” Phys. Rev. A 81, 052318 (2010).

27. B. Wang, Z. Y. Xu, Z. Q. Chen, and M. Feng, “Non-Markovian effect on the quantum discord ,” Phys. Rev. A 81, 014101 (2010).

28. L. Mazzola, J. Piilo, and S. Maniscalco, “ Sudden transition between classical and quantum decoherence,” Phys. Rev. Lett. 104, 200401 (2010).

29. L. Mazzola, J. Piilo, and S. Maniscalco, “Frozen discord in non-Markovian depolarizing channels,” arxiv: 1006.1805v1.

30. M. B. Plenio and S. F. Huelga, “Entangled Light from White Noise,” Phys. Rev. Lett. 88, 197901 (2002).

31. X. X. Yi, C. S. Yu, L. Zhou, and H. S. Song, “Noise-assisted preparation of entangled atoms,” Phys. Rev. A 68, 052304 (2003).

32. J. B. Xu and S. B. Li, “Control of the entanglement of two atoms in an optical cavity via white noise,” New J. Phys. 7, 72 (2005).

33. P. Giorda and M. G. A. Paris, “Gaussian quantum discord,” Phys. Rev. Lett. 105, 020503 (2010).
34. S. Hamieh, R. Kobes, and H. Zaraket, “Positive-operator-valued measure optimization of classical correlations,” Phys. Rev. A 70, 052325 (2004).

35. T. Aoki, B. Dayan, E. Wilcut, W. P. Bowen, A. S. Parkins, K. J. Vahala, and H. J. Kimble, “Observation of strong coupling between one atom and a monolithic microresonator,” Nature (London) 443, 671-674 (2006).

36. D. W. Vernooy, V. S. Ilchenko, H. Mabuchi, E. W. Streed, and H. J. Kimble, “High-Q measurements of fused-silica microspheres in the near infrared,” Opt. Lett. 23, 247-249 (1998).

37. Jia-sen Jin, Chang-shui Yu, Pei Pei, and He-shan Song, “Positive effect of scattering strength of a microtoroidal cavity on atomic entanglement evolution,” Phys. Rev. A 81, 042309 (2010).

38. C. J. Hood, T. W. Lynn, A. C. Doherty, A. S. Parkins, and H. J. Kimble, “The Atom-Cavity Microscope: Single Atoms Bound in Orbit by Single Photons,” Science 287, 1447-1453 (2000).

39. P. W. H. Pinkse, T. Fischer, P. Maunz, and G. Rempe, “Trapping an atom with single photons,” Nature (London) 404, 365-368 (2000).

40. M. Forster, S. Winkler, and S. Wolf, “Distilling nonlocality,” Phys. Rev. Lett. 102, 120401 (2009).
Fig. 1 Schematic illustration of the system composed of an optical cavity and two two-level atoms. The cavity leaky rate is $2\kappa$ and the cavity mode is driven by the white-noise field with intensity $m_T$. The atom has a ground state $|g\rangle$ and an excited state $|e\rangle$ with atom spontaneous emission rate $2\gamma$ (see the inset), and the atoms are driven by the white-noise field with intensity $n_T$.

Fig. 2 (Color online) Quantum discord of the two atoms between atoms as a function of noise intensity $n_T$ and time $t$ (in units of $1/g$) in the case of $m_T = 0$. The parameters are chosen as $\gamma = 0.1g$ and $\kappa = 1.5g$.

Fig. 3 (Color online) (a) The steady-state quantum discord and (b) the steady-state entanglement between atoms as a function of noise intensity $n_T$ and cavity leaky rate $\kappa$ (in units of $g$) in the case of $m_T = 0$ and $\gamma = 0.1g$. (c) The steady-state quantum discord and (d) the steady-state entanglement between atoms as a function of $n_T$ and atom spontaneous emission rate $\gamma$ (in units of $g$) in the case of $m_T = 0$ and $\kappa = 2g$.

Fig. 4 (Color online) Quantum discord between atoms between atoms as a function of noise intensity $m_T$ and time $t$ (in units of $1/g$) in the case of $n_T = 0$. The parameters are chosen as $\kappa = 0.1g$ and $\gamma = 0.2g$.

Fig. 5 (Color online) (a) The steady-state quantum discord and (b) the steady-state entanglement versus noise intensity $m_T$ and cavity leaky rate $\kappa$ (in units of $g$) in the case of $n_T = 0$ and $\gamma = 0.1g$. (c) The steady-state quantum discord and (d) the steady-state entanglement versus $m_T$ and atom spontaneous emission rate $\gamma$ (in units of $g$) in the case of $n_T = 0$ and $\kappa = 0.1g$.

Fig. 6 (Color online) Quantum discord between atoms between atoms as a function of noise intensity $n_T (= m_T)$ and time $t$ (in units of $1/g$). The parameter are chosen as $\kappa = g$ and $\gamma = 0.1g$. 
Fig. 1.
Quantum discord

Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.