Accurate prediction of electroweak observables and impact on the Higgs mass bound

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Abstract

I discuss the importance of the $O(g^4m_t^2/M_W^2)$ corrections to the effective electroweak angle and $M_W$ in the indirect determination of the Higgs mass. I emphasize the rôle of a very precise $M_W$ measurement on the $M_H$ estimate.

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1. Introduction

One of greatest achievement of the program of accurate verification of the Standard Model of electroweak interaction (SM) carried out at LEP and SLC during the last decade, has been the prediction of the top mass. After the experimental discovery of the top quark by the CDF collaboration (with a mass at the right place indicated by the electroweak fits) the challenge of precision physics has moved towards the only remaining unknown particle of the SM, namely the Higgs. However, in this case the game is much harder. The reason is clearly connected to the different behavior of the virtual effects of the two particles in the relevant electroweak corrections: power-like for the top, much milder and just logarithmic for the Higgs. To appreciate how much this logarithmic behavior makes hard the game for the theorists (and the experimentalists also) I consider the effective electroweak mixing angle, $\sin^2 \theta_{\text{lept}}^{\text{eff}} \equiv s_{\text{eff}}^2$, that is the most important quantity in the determination of $M_H$, and write it as

$$s_{\text{eff}}^2 \sim (c_1 + \delta c_1) + (c_2 + \delta c_2) \log y; \quad y \equiv (M_H/100 \ \text{GeV}).$$

In Eq. (1) I identify the l.h.s. with the experimental result that, I assume, carries no error. In the r.h.s. $\delta c_i$ represent the theoretical uncertainty in the corresponding coefficients connected to the fact that we have computed $c_i$ in perturbation theory through certain order in the perturbative series and therefore we do not know their exact values because of higher order contributions. From Eq. (1) one obtains

$$y = y_0 \exp \left[ -\frac{\Delta_{\text{th}}}{c_2} \right] \quad \Delta_{\text{th}} = \delta c_1 + \delta c_2 \log y$$

where $y_0$ is the value corresponding to $\delta c_1 = \delta c_2 = 0$. To see the effect of $\Delta_{\text{th}}$ in extracting $M_H$ I take

$$c_2 \sim \frac{\alpha}{2\pi(c^2 - s^2)} \left( \frac{5}{6} - \frac{3}{4} c^2 \right) \sim 5.5 \times 10^{-4}; \quad \Delta_{\text{th}} \sim \pm 1.4 \times 10^{-4} (3)$$

where $s^2 \sim 0.23$, $c^2 = 1 - s^2$. In Eq. (3) I estimate $c_2$ through the Higgs leading behavior of the correction $\Delta \hat{r}$ relevant for $s_{\text{eff}}^2$ [1, 2] while for $\Delta_{\text{th}}$, I take the value estimated in the 1995 CERN report on ‘Precision calculation for the Z resonance’ [3]. The latter has been obtained comparing the output of five different codes that implement different renormalization schemes and have built in several options for resumming known effects. At the time of the report, the knowledge of the electroweak part of the radiative corrections included, besides the complete one-loop order, the leading logarithms of $O(\alpha^n \log^n M_Z/m_f)$ (here $m_f$ is a generic fermion mass) [4, 5], the
\(O(\alpha^2 \log M_Z/m_f)\) term while for the two-loop top contribution only the leading \(O(g^4 m_t^4/M_W^2)\) correction was known. Therefore the comparison of the various codes was mainly measuring the scheme-dependence error induced by the ignorance of the next term in the two-loop top contribution, namely the \(O(g^4 m_t^2/M_W^2)\) corrections. Inserting the values of Eq. (3) into Eq. (2) yields \(y \sim 1.29 y_0\). We see that a theoretical uncertainty coming from two-loop unknown contributions (that are supposed to be not even the dominant part) makes an error in the indirect determination of the Higgs mass of 29%!

2. Recent advance in higher order calculations

The above example clearly tells us that to extract accurate indirect information on the Higgs one needs not only very precise experiments but also a very good control of the theory side. This brings in the issue of what error we can associate to our theoretical predictions. They are affected by uncertainties coming from two different sources: one that is called parametric and it is connected to the error in the experimental inputs used in our predictions. The second one is called intrinsic and it is related to the fact that our knowledge of the perturbative series is always limited, usually to the first few terms. Concerning parametric uncertainties, \(\alpha(0)\), \(G_\mu\) and \(M_Z\) are very well measured, \(m_t\) and \(\alpha_s\) are not so precisely known while for \(M_H\) there is not at all direct evidence. The scale of the weak interactions is given by the mass of the intermediate vector bosons, so what actually matters in our predictions is not \(\alpha(0)\) but \(\alpha(M_Z)\). The latter contains the hadronic contribution to the photon vacuum polarization, \((\Delta \alpha)_h\), that cannot be evaluated in perturbation theory. Fortunately, one can use a dispersion relation to relate it to the experimental data on the cross section for \(e^+e^-\) annihilation into hadrons. In the recent years there has been a lot of activity on this subject. Several new analyses appeared that differ in the treatment of the experimental data [3, 8] and in the amount of theoretical input used to evaluate them [9, 10]. The situation is not yet settled down (and probably will not be till new experimental data on the \(e^+e^-\) cross section in the low and intermediate energy region are available), so a conservative approach is still to use the value given by the most phenomenological analyses [9], \(\alpha(M_Z)^{-1} = 128.90 \pm 0.09\).

The status of the intrinsic uncertainties has actually improved since the CERN report. A sizeable amount of work on radiative corrections has been completed in the recent past. In this talk, I will discuss only the information that is now available on the \(O(g^4 m_t^2/M_W^2)\) corrections.

The fact that the top is heavier than the other known particles suggests to organize its two-loop contribution to the various radiative parameters
as a series in \( m_t \). The first two terms of this series are enhanced by factors \((m_t^2/M_W^2)^n\) \((n = 1, 2)\) while the remaining ones are at most logarithmic in nature. The leading contribution that scales as \( m_t^4 \) is completely available for arbitrary value of the Higgs mass since few years [6]. The next term, i.e. the \( O(g^4m_t^2/M_W^2) \) correction, has been recently incorporated in the theoretical calculation of \( M_W \) [11], \( s_{eff}^2 \) [12, 13] and the partial widths of the Z into fermions but the \( b \) quark [14]. Indeed in the case of the \( b \) there are specific vertex corrections of the same order not yet computed. To gauge the residual scheme dependence, \( O(g^4) \), this incorporation has been performed in three electroweak resummation approaches and two different ways of implementing the relevant QCD corrections [12]. One of the approaches \((\overline{\text{MS}})\) employs \( \hat{\alpha}(M_Z) \) and \( \sin^2\hat{\theta}_W(M_Z) \equiv \hat{s}^2 \), the \( \overline{\text{MS}} \) QED and electroweak mixing parameters evaluated at the scale \( \mu = M_Z \), while the other two (OSI and OSII) make use of the on-shell parameters \( \alpha \) and \( \sin^2\theta_W \equiv 1 - M^2_W/M^2_Z \). As expected, the dependence on the electroweak scale \( \mu \) cancels through \( O(g^4m_t^2/M_W^2) \). However, because complete \( O(g^4) \) corrections have not been evaluated, the \( \overline{\text{MS}} \) and OSI formulations contain a residual \( O(g^4) \) scale dependence. On the other hand OSII is, by construction, strictly \( \mu \)-independent. In table 1 the predictions for \( s_{eff}^2 \) and \( M_W \) in this three different frameworks are shown. The QCD corrections are implemented on the base of a top pole mass parameterization (for results with QCD corrections implemented in terms of running \( \overline{\text{MS}} \) top mass see Ref.[12]). For each entry of the Higgs mass the first row corresponds to the value obtained including only the \( O(g^4m_t^4/M_W^4) \) contribution while the second one contains also the \( O(g^4m_t^2/M_W^2) \) part. I will not discuss in detail the effect of the \( O(g^4m_t^2/M_W^2) \) corrections in the electroweak fits (see Bob Clare’s talk [15]) but I would like to point out few things that can be easily read from table 1. i) The incorporation of the \( O(g^4m_t^2/M_W^2) \) corrections reduces the scheme dependence to the level of \( 4 \times 10^{-5} \) in \( s_{eff}^2 \) and 2 MeV in \( M_W \). ii) The \( O(g^4m_t^2/M_W^2) \) values for \( s_{eff}^2(M_W) \) are generally higher (lower) than the corresponding \( O(g^4m_t^4/M_W^4) \) results. In the indirect determination of \( M_H \) this fact favors a lighter value of the mass. iii) In general the \( O(g^4m_t^2/M_W^2) \) OSI and \( \overline{\text{MS}} \) results are very close. The OSI resummation is actually the natural generalization to \( O(g^4m_t^2/M_W^2) \) of the one proposed by Consoli-Hollik-Jegerlehner [16] for the reducible \( O(g^4m_t^4/M_W^4) \) term and it is the one presently implemented in ZFITTER [17]. On the other side our \( \overline{\text{MS}} \) approach [4] is quite similar to the one implemented in TOPAZ0 [8]. This explain why in the new version of the famous LEP-EWWG \( \Delta \chi^2 \) vs. \( M_H \) blue-band plot [13] the ZFITTER and TOPAZ0 lines are very close especially for large values of \( M_H \) and the blue band seems to have disappeared. With respect to this a comment is in order. The new
Table 1. Predicted values of $M_W$ and $s_{\text{eff}}^2$ in different frameworks for $m_t = 175$ GeV with QCD corrections based on pole top-mass parameterization. The first row of each $M_H$ entry is obtained including only the $O(g^4 m_t^4/M_W^2)$ corrections. The $O(g^4 m_t^2/M_W^2)$ result is presented in the second row (only the last two different digits are shown).

| $M_H$ | $s_{\text{eff}}^2$ | $M_W$ (GeV) |
|-------|-------------------|--------------|
| OSI   | OSII  | MS   | OSI | OSII | MS   |
| 65    | .23131 | .23111 | .23122 | 80.411 | 80.422 | 80.420 |
| 32    | 34    | 30   | 05  | 04   | 06   |
| 100   | .23153 | .23135 | .23144 | 80.388 | 80.397 | 80.396 |
| 53    | 55    | 52   | 82  | 81   | 83   |
| 300   | .23212 | .23203 | .23203 | 80.312 | 80.316 | 80.319 |
| 10    | 14    | 10   | 08  | 06   | 08   |
| 600   | .23251 | .23249 | .23243 | 80.256 | 80.257 | 80.263 |
| 49    | 52    | 49   | 54  | 52   | 54   |
| 1000  | .23280 | .23282 | .23272 | 80.215 | 80.213 | 80.221 |
| 77    | 79    | 77   | 14  | 13   | 14   |

The precise electroweak measurements allow to constrain significantly the value of the Higgs mass. A global fit to all data gives a strong indication for a light Higgs with an upper limit at 95% C.L. $M_H < 215$ GeV [15]. However, the current estimates of $M_H$ depend crucially on the world average $s_{\text{eff}}^2 = 0.23149 \pm 0.00021$, and this follows from a combination of experimental results that are not always in good harmony. The data presented at the
Table 2. Values in the MS scheme of $b_i$ ($i = 1 - 4$) in Eq. (4) and $d_i$ ($i = 1 - 5$) in Eq. (5) and their ratio.

| $i$ | $b_i$       | $d_i$       | $|b_i/d_i|$ |
|-----|-------------|-------------|------------|
| 1   | $2.26 \times 10^{-3}$ | $-7.2 \times 10^{-4}$ | $\sim 3.1$ |
| 2   | $4.26 \times 10^{-2}$  | $-6.4 \times 10^{-3}$ | $\sim 6.6$ |
| 3   | $-1.20 \times 10^{-2}$ | $6.7 \times 10^{-3}$  | $\sim 1.8$ |
| 4   | $1.94 \times 10^{-3}$  | $-1.1 \times 10^{-3}$ | $\sim 1.8$ |
| 5   | $-1.0 \times 10^{-4}$  | $-1.0 \times 10^{-4}$ | $-1.0 \times 10^{-4}$ |

recent Winter conferences [13] show a better agreement than the previous ones [19] but still the most precise LEP result ($s_{\text{eff}}^2 = 0.23213 \pm 0.00039$ from $A_{th}^k$) and the SLAC data ($s_{\text{eff}}^2 = 0.23084 \pm 0.00035$) are quite far apart. To show how much the low value of SLAC is important for a light $M_H$ determination I consider $s_{\text{eff}}^2$ and use the parameterization [20]

$$
\frac{\sin^2 \theta_{\text{eff}}^{\text{lept.}}}{0.23151} - 1 = b_1 \ln \left( \frac{M_H}{100 \text{ GeV}} \right) + b_2 \left[ \frac{(\Delta \alpha)_h}{0.0280} - 1 \right]
+ b_3 \left[ \left( \frac{m_t}{175 \text{ GeV}} \right)^2 - 1 \right] + b_4 \left[ \frac{\alpha_s(M_Z)}{0.118} - 1 \right] \quad (4)
$$

that in the range $75 \text{ GeV} \leq M_H \leq 350 \text{ GeV}$, with the other parameters within their $1 - \sigma$ errors, approximates the detailed calculations of Ref. [12] with average absolute deviations of $\approx 4 \times 10^{-6}$ and maximum absolute deviations of $(1.1 - 1.3) \times 10^{-5}$ depending on the scheme while outside the above range, the deviations increase reaching $(2.6 - 2.8) \times 10^{-5}$ for $M_H = 600 \text{ GeV}$ (the values of the $b_i$ coefficients for the MS scheme are presented in table 2). Employing in Eq. (4) $m_t = 174.1 \pm 5.4 \text{ GeV}$, $\alpha_s(M_Z) = 0.118 \pm 0.003$, $(\Delta \alpha)_h = 0.0280 \pm 0.0007$ and the LEP average for $s_{\text{eff}}^2$ ($s_{\text{eff}}^2 = 0.23186 \pm 0.00026$) I obtain a 95% C.L. upper bound $M_H < 610 \text{ GeV}$. For the same values of $m_t$, $\alpha_s(M_Z)$ and $(\Delta \alpha)_h$ the use of the SLAC value for $s_{\text{eff}}^2$ in Eq. (4) gives instead a 95% C.L. upper bound $M_H < 110 \text{ GeV}$. Clearly is the SLAC result that mainly pushes the electroweak fit towards a light Higgs mass. Notice that a fit to LEP data alone (excluding the direct determination of $m_t$) gives a light Higgs ($M_H = 56^{+101}_{-31}$) but at the price of a low top ($m_t = 156^{+12}_{-10}$) [3]. There is another observation to be made with respect to $s_{\text{eff}}^2$. This observable is very sensitive to $(\Delta \alpha)_h$. As I said, the accuracy we know this quantity is presently under discussion. The most conservative error [3] ($\delta(\Delta \alpha)_h = 7 \times 10^{-4}$) makes it the bottleneck in
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\[ \ln(M_H/(100 \text{ GeV})) \quad M_W \quad s_{\text{eff}}^2 \]

| \(\delta m_t = 3 \text{ GeV}, \delta M_W = 35 \text{ MeV} \) | \(0^{+0.663}_{-0.815} \) | \(0 \pm 0.647 \) |
| \(\delta (\Delta \alpha)_h = 0.0007 \) | |

| \(\delta m_t = 1 \text{ GeV}, \delta M_W = 20 \text{ MeV} \) | \(0^{+0.404}_{-0.455} \) | \(0 \pm 0.623 \) |
| \(\delta (\Delta \alpha)_h = 0.0007 \) | |

| \(\delta m_t = 1 \text{ GeV}, \delta M_W = 20 \text{ MeV} \) | \(0^{+0.352}_{-0.390} \) | \(0 \pm 0.428 \) |
| \(\delta (\Delta \alpha)_h = 0.0002 \) | |

Table 3. Errors on \(\ln(M_H/(100 \text{ GeV}))\) determined from \(M_W\) (Eq. (5)) and \(s_{\text{eff}}^2\) (Eq. (4)) for \(M_H = 100 \text{ GeV}\), \(\delta s_{\text{eff}}^2 = 0.00021\), \(\delta g(M_Z) = 0.003\) and different values of \(\delta m_t, \delta M_W\) and \(\delta (\Delta \alpha)_h\).

the improvement of the \(M_H\) determination. The recent more theoretically oriented analyses [10] give an error on \((\Delta \alpha)_h\) ranging form \(\delta (\Delta \alpha)_h = 1.6 \times 10^{-4}\) to \(\delta (\Delta \alpha)_h = 4.5 \times 10^{-4}\). Using a smaller error for \((\Delta \alpha)_h\) implies to weight more \(s_{\text{eff}}^2\) in the \(M_H\) fit that means we have to trust more the \(s_{\text{eff}}^2\) results.

This state of affairs strongly suggests the desirability of obtaining constraints on \(M_H\) derived from future precise measurements of \(M_W\). Similarly to Eq. (4) I parameterize the result for \(M_W\) as [20]

\[
\frac{M_W}{80.383} - 1 = d_1 \ln \left( \frac{M_H}{100 \text{ GeV}} \right) + d_2 \left[ \frac{(\Delta \alpha)_h}{0.0280} - 1 \right] + d_3 \ln^2 \left( \frac{M_H}{100 \text{ GeV}} \right) + d_4 \left[ \left( \frac{m_t}{175 \text{ GeV}} \right)^2 - 1 \right] + d_5 \left[ \frac{\alpha_s(M_Z)}{0.118} - 1 \right]
\]

where the \(d_i\) coefficients are shown in table 2 and notice that to obtain an accuracy in the parameterization similar to that of Eq. (4) I need to introduce an extra term proportional to \(\ln^2(M_H/100 \text{ GeV})\). Comparing the coefficients of the Eq. (4) and Eq. (5) we see that at equal level of experimental accuracy (which is, in fact, the current situation) \(s_{\text{eff}}^2\) is more sensitive than \(M_W\) by a factor \(\approx 2.7\) in \(\ln(M_H/100)\) (taking also into account the \(\ln^2(M_H/100)\) term of Eq. (4)). On the other side, \(M_W\) has the welcome characteristic to be not so sensitive to \((\Delta \alpha)_h\). Let us now consider future scenarios where the experimental errors in the various quantities that enter in Eq. (4) and Eq. (5) are somewhat reduced and compare
the indirect determination of $M_H$ from $M_W$ and $s^2_{\text{eff}}$, separately. To make a simple comparison I use central values that give the same Higgs mass, so I choose $s^2_{\text{eff}} = 0.23151$, $M_W = 80.383$ GeV, $(\Delta \alpha)_h = 0.0280$, $m_t = 175$ GeV, $\alpha_s(M_Z) = 0.118$ that correspond to $M_H = 100$ GeV. Table 3 presents 3 possible scenarios in all of which I assume no improvement in the $s^2_{\text{eff}}$ and $\alpha_s(M_Z)$ determination (i.e. $\delta s^2_{\text{eff}} = 0.00021$ and $\delta \alpha_s(M_Z) = 0.003$) while the errors in $m_t$, $M_W$ and in the last case also in $(\Delta \alpha)_h$ get reduced. One sees that a determination of $M_W$ at the level of 35 Mev together with an improvement in $m_t$ to $\delta m_t = 3$ GeV gives an information on $M_H$ competitive with the one that is presently obtained from $s^2_{\text{eff}}$. Such a scenario is consistent with the expectation of Tevatron Run 2. A further reduction in $\delta M_W$ and $\delta m_t$, that can be foreseen at LHC, will make $M_W$ more effective than $s^2_{\text{eff}}$ in determining $M_H$ even in a situation in which the error on $(\Delta \alpha)_h$ will be significantly reduced.

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