Analysis of unsteady flow in compressor cascade based on modal POD methods

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Abstract: There are complex secondary flow structures in the compressor cascade passage, which seriously suppresses the performance of the compressor. The flow structures are highly unstable, especially for highly loaded compressor cascades. The reduced-order model based on the modal decomposition method can capture the main characteristics of the flow field and offer a reference for the application of unsteady flow control methods. In this study, the POD methods are used to analyze the flow in a high-speed compressor cascade. The methods can extract the mode that reflects the main energy of the flow field. Both the design and off-design conditions are analyzed. The decomposition methods can obtain the global energy, time coefficient and spatial characteristics of the flow field. The result showed that the flow field of the cascade has high steadiness and regularity. The zeroth POD mode under design conditions occupies over 99% of the energy of the flow field. The fluctuation velocity of off-design conditions is larger. The first five POD modes can contain over 90% of the energy of the fluctuation velocity field.

1. Introduction
The increase of compressor load makes it easier to generate large-scale flow separation in the passage, which inhibits the performance of the compressor. The flow field in the compressor is usually unsteadiness. There are also differences in the separation structure under different working conditions. Although the flow field changes with time, the internal flow structure is still not very clear. So how to capture the orderly structure in the flow process has important academic significance and engineering application value for understanding the mechanism of flow separation and flow control.

High-precision numerical and experimental methods are gradually applied to the field of fluid mechanics. A large amount of data improves the accuracy of calculation, but it brings a new problem for the analysis and understanding of the flow field. The evolution of the internal flow structure of the turbomachinery in time and space is usually orderly. So it is necessary to use flow field post-processing methods to extract the main characteristics of the complex flow. It is easier to understand the physical properties from a simple structure.

In order to obtain the main characteristics of the flow field, an important method is to construct the flow field reduced-order model. The POD method is one of the earliest mode decomposition method [1]. This method decomposes the flow field into several spatial orthogonal modes and sorts the modes according to the energy (eigenvalue). The POD method loses the phase information of the system, a certain mode obtained by decomposition contains flow field structures with different frequencies. Initially, it was mainly used to analyze the flow of flat plate and cylindrical flows. Recently, this method

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has been gradually introduced into the unsteady flow of airfoil, and turbomachinery. Rochuon et al. [2] extract the dominant modes of a rotor-stator interface of a centrifugal compressor by the use of POD. Wang et al. [3] studied the unsteady flow in compressor cascade using POD. Clark [4] use the POD method to reconstruct and predict the CFD result in a two-dimensional compressor cascade. Yao et al. [5] obtained the mode component of tip flow with POD and expected to understand the mechanism of the rotating instability and rotor-stator interaction phenomenon. Zambonini et al. [6] used the POD to decompose the flow in a compressor cascade and to clarify the underlying cause-effect relations, which predominate the dynamics of the present corner separation flow. Huang [7-9] introduced the Lyapunov exponent to evaluate the overall control effect. The POD method was used to analyze the separated flow with and without pulsed excitation. The result showed that when the jet frequency is consistent with the main frequency of separation flow in the two-dimension compressor cascade, the control effect is best. The spatial distribution of local vortex is more orderly using flow control. Xu et al. [10] use the POD method to analysis the unsteady separation flow in a two-dimension compressor cascade with boundary layer suction. The conclusion is the same as Huang et al [7-9]. Shi et al. [11]used the POD method to analysis the leakage flow in a compressor cascade.

Many researchers have used POD to study the unsteady flow of the two-dimensional compressor cascade. More attention is paid to the boundary layer separation of the suction surface. In this study, the numerical method is used to solve the unsteady flow field of a three-dimensional compressor cascade. At the same time, the main flow field structure in the cascade is analyzed by the POD method. Both the design and the off-design condition is analyzed. It is expected to be helpful for the application of flow control.

2. POD method

POD method is using an orthogonal transformation to convert the flow field into a linear combination of the temporal coefficients $c_n^k$ and basis function (mode, $n\phi$). The decomposition methods include classical and snapshot methods. The snapshot approach is used in this study. We take the $V$ velocity field as an example.

$$N \sum_{n=1}^{N} c_n \phi_n$$

(1)

$N$ is the total number of the snapshot.

In fact, solving the mode is equivalent to solving the following maximum problem.

$$\max \left\{ \frac{1}{N} \sum_{n=1}^{N} V_n \phi_n \right\}$$

(2)

subject to $\| \phi_n \|^2 = 1$

Using the variational method, the maximization is transformed into solving the eigenvalues of the correlation matrix $B$.

$$B \beta_n = \lambda_n \beta_n$$

(3)

Where $\lambda_n$ is the eigenvalue of the correlation matrix, $\beta_n$ is eigenvectors.

The correlation matrix $B$ can be defined as

$$B = \frac{1}{N} (VV^T)$$

(4)

The spatial mode $\phi_n$ can be obtained by

$$\phi_n = \beta_n V_n$$

(5)

The eigenvalues $\lambda_n$ have a clear physical meaning. The total energy of the dynamic system can be expressed as the sum of the eigenvalues $E = \sum_{j=1}^{N} \lambda_j$. The percentage of energy in each mode is defined as
The dimension of the original flow field can be reduced by selecting the first $M$ modes that contain most of the energy. The reconstructed flow field represented by $M$ POD basis functions is

$$V^{\text{POD}} = \sum_{n=1}^{M} c_n \varphi_n$$  \hspace{1cm} (6)

3. Numerical method and validation

3.1. Compressor cascade

The cascade profile is NACA65-K48 which is design by DLR[12]. Qin et al[13]’s result showed that the fluctuating velocity is small. We get the same conclusions in this study. The chord length ($C$) of the blade is 60mm. The span height ($h$) is 100mm. The inlet angle $\beta_1$ is $132^\circ$ at the design condition. The outlet angle $\beta_2$ is $90^\circ$. The pitch length is 33mm. The Stagger angle is $112.05^\circ$. Fig. 1 gives the sketch of the blade profile.

A numerical study by means of unsteady Reynolds-averaged Navier–Stokes is performed by a commercial soft ANSYS CFX 14.0. SST k-ω model is used for turbulence treatment. The transition model is the $\gamma$-0 model. The computational grids were generated by Gambit. And the number of passage mesh is about 1.5 million. Fig. 2 shows the mesh of the compressor cascade. The inlet of the cascade gives the total pressure, total temperature and flow angle. The inlet Mach number is 0.7. The outlet gives the average static pressure. The wall is set with the adiabatic and non-slip condition. Translational periodic boundary conditions are applied on two sides of the cascade. The time step is $1 \times 10^{-6}$. Maximum inner iterations were set to 15 which produced sufficiently low residual. Fig. 3 shows the monitor points and the sections used in this study. We mainly studied 9 flow planes along the streamwise and 6 planes along spanwise.

The time-averaged results are compared with the experimental results[14] in order to evaluate the feasibility of the numerical method. Fig. 4 shows the mass-averaged total pressure loss coefficient with the different incident angles at the outlet. The change of numerical results is close to the experiment, and the overall error is less than 3%. The error at -2°is maximum. The main reason is the same inlet velocity distribution as the design conditions. Fig. 5 (a) shows the pitchwise-averaged total pressure loss coefficient at the design point. The variation of the numerical results is consistent with the experiment. The error near the endwall is large because the measured point is less. Fig. 5 (b) and (c) shows the oil flow results and the numerical limiting streamlines on the wall. The numerical results capture the main structure of the flow field, and the area of the separation is close to the experiment. The cross flow of the end wall is also very clear. In addition, Qin et al. [13] used the same numerical method to study the flow in this cascade. Therefore, the numerical method used in the paper is reasonable.
4. Results and discussion

4.1. Spectrum analysis of unsteady flow

Fig. 6 shows the spectrum results of the total pressure of different monitor points. When the incident angle is -2, there are three characteristic frequencies, which are 2200 Hz, 4400 Hz, and 6600 Hz, with dominant amplitudes. The characteristic frequencies are multipliers and have dominant amplitudes. Therefore, it can be considered that they correspond to the same vortex. The frequency of 2200 Hz has the highest amplitude, so 2200 Hz is used as the reference frequency. On the other hand, it shows that the flow field in the baseline cascade is uniform, and the formation and development of the vortex structure is periodic. The amplitude at 10% height of span is highest which indicating that the turbulent pulsation near the end wall is stronger. The amplitude of 4400 Hz is higher than the frequency of 2200 Hz at 40% height of span, but both have very low amplitudes. It indicates that the intensity of the vortex of 4400 Hz and 2000Hz are similar near the midspan. For the case of 0 degrees, the amplitude of 3100 Hz is more obvious, and the amplitudes at 1500 Hz and 6200 Hz are lower. So 3100 Hz is used as the reference frequency. The amplitudes at different heights are close because the fluctuating pressure is more low under the design conditions. For the case of 4 degrees, it can be found that its amplitude is significantly higher than the first two cases indicating that the flow is more complicated. The amplitudes of different heights are close because the separation is more serious at the off-design condition. 300 snapshots including 6 periodic flow fields were selected in this study for all cases.
Fig. 6 The spectrum of total pressure at the outlet

Fig. 7 Instantaneous velocity components at monitor point 1

Fig. 7 shows the instantaneous velocity component at monitor point 1. The velocity components of the three cases are very uniform and periodically distributed indicating that the flow field is very orderly. The value changes very little, which also suggests that although there are well-organized shedding vortex structures at cascade outlet, the fluctuating velocity is low. So the energy carried by the fluctuating flow field is lower than that of the time-average flow field. The change of velocity is less than 0.05m/s at the design condition. The change of velocity at off-design conditions is high, but still low compared to the time-average velocity.

4.2. POD analysis

4.2.1. Energy characteristic

We first use the POD method to decompose the original flow field in order to quantitatively explain the steady flow is domination. Fig. 8 shows the energy ratio of the zero-order mode at different spanwise. For the design condition, the zeroth-order mode (time-averaged flow field) contains more than 99.8% of the energy of the flow field. The flow of the 4-degree case is the most complex, so its zeroth order mode contains the lowest energy. As the span height increases, the proportion of the zeroth-order energy gradually decreases, because the secondary flow near the end wall is stronger. Fig. 9 shows the energy ratio of zero-order POD modes at different streamwise sections. The zeroth-order energy ratio still exceeds 99.8% at the design condition. The lowest point appears at 40% chord length for the case of 4 degrees. The energy ratio of zero-order modes is continuously reduced after 100% of the chord length for the case of -2 degrees.
Fig. 10 shows the mode energy ratio at different spanwise sections. The energy of the first-order mode gradually decreases with the increase of span height for all cases. The convergence speed of the mode energy ratio also gradually reduced. The first three modes below 30%H contain more than 90% of the energy of the fluctuating velocity for the case of 0 degrees. As the span height increases, the energy contained in the first three modes gradually decreases. It suggests that the first few modes can reflect the flow characteristics of the separation vortex. The separation near the end wall is more obvious and the large scale separation structure contains more energy. The separation flow in the middle of the cascade is weak, so the energy of the dominant mode is no longer significant. Similar conclusions can also be drawn under off-design conditions. Therefore, the main flow characteristics and the large-scale flow structure of the fluctuating velocity field can be grasped by analyzing the first three modes. Fig. 11 shows the mode energy ratio at the different streamwise planes. The first five modes contain more than 90% of the energy of the fluctuating velocity. The energy distribution before 100% C varies greatly. The energy distribution is close after 100% C due to the small change in the flow field.

4.2.2. Temporal characteristics
We mainly analyze the time coefficients at 5%H and 40%H according to the distribution of energy coefficients. Fig. 12 shows the variation of the time characteristic at 5%H. It can be seen that the first four POD modes of different cases exhibit periodic changes. The first two modes have a higher amplitude. Every mode has a dominant frequency after Fourier transformation. The first two modes have higher energy and the same frequency. The frequency is 2200Hz, 3000Hz and 1200 Hz, respectively. The third-order and fourth-order modes reflecting the high-frequency structure near the end-wall have higher frequencies and lower amplitude. The amplitude of the time coefficient is the largest under 4 degrees, which shows fluctuating velocity is the most intense.
The amplitude of the temporal coefficients at 40%H still decreases with the increase of mode order for all cases. The amplitude changes less and is lower than that of the endwall. Because there are the separations of the boundary layer and corner near the endwall. The secondary flow near the end wall is more intense for the low aspect ratio cascades used in this study and is the main source of loss. The distribution of the energy coefficients shown in Fig. 10 is close suggests that the amplitude of the time coefficient can express the energy. The boundary layer separation of -2 degrees is weakest, and the amplitude is the lowest. It can be seen from the result of the spectrum that the higher-order modes still represent high-frequency structures. The dominant frequency is consistent with the reference frequency.

Fig. 12 The temporal coefficients and the spectrum of temporal coefficients at 5%H

Fig. 13 The temporal coefficients and spectrum of temporal coefficients at 50%H

Fig. 14 shows the time coefficients and spectrum of temporal coefficients at 60% C. The first four modes of different cases exhibit periodic changes. The two adjacent modes have the same dominant frequency but there is a certain phase shift. The first-order mode frequency is 1500 Hz for the case of 0 degrees, which corresponds to a new flow field structure. Fig. 15 shows the time coefficients and spectrum of temporal coefficients at 120% C. The dominant frequency of the first two modes is still consistent with the reference frequency. And the trailing edge vortex shedding is the main structure of
the flow field. The spectrum of temporal coefficients indicates that there are two main shedding vortices in the trailing edge.

Fig. 14 The temporal coefficients and the spectrum of temporal coefficients at 60%C

Fig. 15 The temporal coefficients and the spectrum of temporal coefficients at 120%C

5. Conclusion
This study introduces POD methods to analyze the unsteady flow in a high-speed compressor cascade. Both the design condition and off-design conditions are studied. The main conclusions are as follows.

Fourier transform shows there is a dominant frequency for every incidence angle. The velocity field at the outlet is also well-organized. The time-averaged flow field in the cascade used in this study occupies most of the energy of the flow field. As incidence angles increase, the energy of fluctuating velocity gradually increases. The energy of the time-averaged flow field is still more than 98% for the case of 4 degrees. The fluctuation near the endwall is the most intense.

The first five POD modes of the fluctuating velocity field contain more than 90% of the energy. The convergence speed of the mode energy ratio is slowest at midspan.

The time coefficients obtained by the POD methods exhibit periodic changes. The peak value of the time coefficients can reflect the mode energy. The fast Fourier transform of the POD coefficients show
the first four modes near the end wall have a single dominant frequency.

The spatial characteristics obtained by POD indicates that the mode mainly shows the fluctuation caused by the suction surface separation and the trailing edge vortex shedding. The separation of the suction surface is more intense for the case of 4 degrees. The trailing edge shedding structure is more intense for the angle of 2 degrees.

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