Although entanglement is widely recognized as one of the most fascinating characteristics of quantum mechanics, nonlocality remains to be a big labyrinth. The proof of existence of nonlocality is as yet not much convincing because of its strong reliance on Bell’s theorem where the assumption of realism weakens the proof. We demonstrate that entanglement and quantum nonlocality are two equivalent aspects of the same quantum wholeness for spacelike separated quantum systems. This result implies that quantum mechanics is indeed a nonlocal theory and lays foundation of understanding quantum nonlocality beyond Bell’s theorem.

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Since the classic works of EPR (Einstein, Podolsky and Rosen [1]) and Bell [2, 3], quantum nonlocality and entanglement have been recognized as two crucial notions in modern understanding of quantum phenomena. As entanglement is an essential “resource” in practical applications of quantum information [4], various aspects [4, 5, 6, 7] of it have been extensively studied in recent years. However, our current understanding of nonlocality is largely accepted in literature and serves as the basis of our current understanding of nonlocality. Nonlocality remains to be a big labyrinth. The proof of existence of nonlocality is as yet not much convincing because of its strong reliance on Bell’s theorem. The proof is based on some counterfactual reasonings that assume local realism could be equally interpreted as implying that quantum mechanics is a local non-realistic theory. On the other hand, there are other spacelike separated systems. By assuming local realism weakens the proof. We demonstrate that entanglement and quantum nonlocality are two equivalent aspects of the same quantum wholeness for spacelike separated quantum systems. This result implies that quantum mechanics is indeed a nonlocal theory and lays foundation of understanding quantum nonlocality beyond Bell’s theorem.

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Since Werner’s seminal work [13], many elegant ideas [12, 13, 14, 15, 16, 17, 18] have been proposed to understand nonlocality of mixed states. Yet, the relationship between nonlocality and entanglement for mixed states is still unclear [11, 18] and remains one of the most challenging problems in the field. Here we first give a generic locality condition (i.e., a condition that should be satisfied by any local theory and can thus be regarded as a proper definition of locality) for any multipartite system (with its subsystems being spacelike separated). We then prove that the locality condition, also underlying Bell’s inequalities, is satisfied if and only if the states of any spacelike separated quantum system are non-entangled. As such, any entangled state is quantum mechanically nonlocal, i.e., entanglement and quantum nonlocality are two side of the same attribute of entangled systems. It is anticipated that the result may be a starting point for penetrating the mystery of nonlocality of nature in general and quantum nonlocality in particular.

The concept of entanglement is well justified in the sense that one, at least, has a definition of entangled states [19]. Quantification [5] and manipulation [6] of entanglement are hot topics of fundamental interest in the fields of quantum mechanics and quantum information theory; various separability criteria [7] have been found to classify quantum states into entangled and separable ones. By contrast, quantum nonlocality still remains, to a large extent, an intuitive notion. Roughly spoken, it usually means the impossibility of simulating certain quantum predictions by local realistic theories [8]. As is well known, local realism represents a world view [9] which states that a physical system has local objective properties, independent of any observations on other spacelike separated systems. By assuming locality and realism, the celebrated Bell inequalities (as well as their various generalizations [3, 11, 19, 20, 21]) can be derived and impose an upper bound on correlations of the results of the “Bell experiments”. Astonishingly, the upper bound is violated quantum mechanically by a large class of entangled states [11, 19, 20, 21], as confirmed by many experiments [4, 22, 23]. To Bell [3], the quantum violations of Bell’s inequalities imply that quantum mechanics is a nonlocal theory. This attitude is largely accepted in literature and serves as the basis of our current understanding of nonlocality.

However, the attitude is not unquestionable. Anyway, Bell’s inequalities are derived from two underlying assumptions: locality and realism. Thus, the experimentally confirmed conflict between quantum mechanics and local realism could be equally interpreted as implying that quantum mechanics is a local non-realistic theory. On the one hand, it is natural for some authors (see, e.g., Ref. [24]) to refute realism, rather than locality, in quantum mechanics. On the other hand, there are also attempts [25] to prove the nonlocality of quantum mechanics without explicitly assuming realism. Yet, the proof is based on some counterfactual reasonings that are controversial (see Ref. [26] and references therein). Currently, whether quantum mechanics is indeed a nonlocal theory remains an open question, whose answer can be approached only when we have a solid foundation of nonlocality.

Now it is a well established fact that biparticle [15, 20, 21] and multipartite [27] entangled pure states always lead to certain violation of Bell-like inequalities. This convincingly indicates that there may exist a close and direct relationship between entanglement and nonlocality. Yet, for mixed states the situation is very puzzling [12, 13, 14, 15, 16, 17, 18]. Werner [12] first demonstrated that there are entangled states (the Werner states) that do not violate any Bell inequality. Subsequent works show that the Werner states possess “hidden nonlocality” [14, 17], which, however,
FIG. 1: The spacetime diagram of two spacelike separated particles. The particles A and B are located in two coloured locations which are spacelike separated. W denotes the overlap of the backward light cores of the particles A and B; the remaining parts of the backward light cores of the particles A and B are denoted, respectively, by U and V, which are spacelike separated such that no causal signal can connect them.

can be uncovered only by invoking generalized measurements \[14, 16, 17, 18\] instead of the standard von Neumann measurement. Despite of many elegant efforts \[13, 14, 16, 17, 18\] along the line of the “Bell paradigm”, there are still some open problems \[14, 17, 18\] on the relationship between nonlocality and entanglement. The difficulty encountered in our current understanding of nonlocality might suggest that Bell’s theorem is not sufficient for uncovering nonlocality of quantum mechanics. Then a fundamental problem arises here: In what sense we classify quantum states into local and nonlocal ones?

In this work we provide answers to the above-mentioned open questions. This is achieved by understanding quantum nonlocality at a deeper level going beyond Bell’s theorem. As one might intuitively envision, nonlocality inherent in entanglement is a purely quantum phenomenon. Meanwhile, realism underlying Bell’s theorem is a world view that logically has nothing to do with quantum mechanics. Thus, in order to reveal nonlocality as a fundamental feature of nature, it is necessary to discard the premise of realism; what concerns us first is the condition that any local theory must satisfy.

Local causality (or simply, locality) means that the experimental results obtained from a physical system at one location should be independent of any observations or actions made at any other spacelike separated locations. Unfortunately, the locality condition \[2, 14, 17, 28, 29\] was previously given in company with realism. Since here one needs a locality condition without any pre-assumption other than locality, the condition must be expressed by quantities that are experimentally observable for localists.

In order to obtain the desired locality condition in mathematical terms, let us consider two particles A and B in the spacetime diagram shown in Fig. 1. They are located, respectively, in two spacetime regions denoted by 1 and 2, which are spacelike separated. According to local causality, events occurring in the backward light core of a particle (e.g., particle A or B) may affect the events (e.g., detecting measurement results) occurring on the particle. Thus, events in the overlap W of the backward light cores of the two particles may be “common causes” \[13\] of the events in regions 1 and 2, though events in region 1 should not be causes of events in region 2 (and vice versa) as required again by local causality.

Without loss of generality, one can assume that the observables of A and B have only a finite number of outcomes. Then consider local measurements on the two particles. By the standard rule of probability, the requirement of locality can be mathematically expressed as

\[
P(a_i, b_j | C) = P(a_i | C) P(b_j | C) \tag{1}
\]

for all observed results in the spacelike separated regions 1 and 2 and for any common causes C in W. Here \(P(a_i, b_j | C) (i, j = 1, 2, 3, \ldots)\) is a joint probability of measuring any observables in region 1 (with outcome \(a_i\)) and in region 2 (with outcome \(b_j\)) conditioned on a given element in the set C of all common causes in W: \(P(a_i | C) = P(b_j | C)\) is the local probability of getting the outcome \(a_i\) (\(b_j\)) when measuring the observable in region 1 (region 2) conditioned on the given common cause in C. Thus, equation (1) has a physically apparent interpretation in accord with locality: The given common cause can affect the probabilities with regard to particles A and B; conditioned on the same cause, measurements in region 1 and region 2 must be mutually independent effects. Importantly, the common causes in our analysis represent physically a set of certain individual events whose occurrences can be assigned corresponding probabilities.

Denoting the joint probability of getting outcomes \(a_i\) and \(b_j\) as \(P(a_i, b_j)\) and the probability of common causes in W as \(P(C)\) (see Fig. 1), then the rule of conditional probability gives

\[
P(a_i, b_j) = \sum_C P(a_i, b_j | C) P(C) \tag{2}
\]

where the summation may also mean integration, if necessary. Let us denote the set of common causes in W by \(C = \{\mu | \mu = 1, 2, 3, \ldots\}\), with \(\lambda_\mu \geq 0\) being the probability for the cause \(\mu\) to occur and \(\sum_\mu \lambda_\mu = 1\). Thus, for a given cause \(\mu\), equation (1) becomes \(P(a_i, b_j | \mu) = P(a_i | \mu) P(b_j | \mu)\).

Obviously, equation (2) can be assigned an operational meaning. Namely, one can either measure directly \(P(a_i, b_j)\), or monitor the common causes first. Conditioned on a specific common cause (\(\mu\), say) being detected, \(P(a_i, b_j | C)\) is then measured. Theory of proba-
bility insures that the two procedures are equivalent. Of course, for a localist, equation (4) should be right.

Some further comments in support of the locality condition (11) are noteworthy. First of all, for localists the probabilities in the locality condition are all measurable quantities and can be measured, in principle, with an arbitrary accuracy. Given a statistical ensemble, experimenters can always observe the relative frequencies among all the outcomes; After the number of the observations tends to infinity, the relative frequencies will tend to the true probabilities of the corresponding outcomes. Moreover and more importantly, the locality condition given in a probabilistic terms is presented without resorting to any specific theory (realistic or quantum) as it only involves observable quantities; theory enters the picture when one predicts the probability of each outcome. This feature of the locality condition enables one to test locality versus any specific theory, i.e., to test whether nature is local (30). For instance, when one uses hidden-variable theories to assign the probabilities in equation (4), the locality condition (11) reduces exactly to Bell’s locality condition (2, 3, 11, 28, 29). Then following Bell’s reasoning yields the usual Bell inequalities which must be satisfied by any local realistic theory (2, 3). Bell’s locality condition (2, 3, 11, 28, 29) has been well justified in various aspects in the context of Bell’s inequalities (11, 28, 29) and is now widely accepted.

The generic locality condition (11) allows one to use quantum mechanics, instead of hidden-variable theories, when predicting the local probabilities [e.g., quantum mechanics, instead of hidden-variable theories, the context of Bell’s inequalities (11, 28, 29) and is now satisfied by any local realistic theory (2, 3)]. Bell’s locality condition (2, 3, 11, 28, 29) has been well justified in various aspects in the context of Bell’s inequalities (11, 28, 29) and is now widely accepted.

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The two remarks are in order here. The state $\rho_{AB}$ in equation (4) is called “classically correlated” (13). Apparently, this classical correlations can only originate from the ‘common history’ of the particles A and B, as can be seen from the spacetime diagram (Fig. 1). In the particular case where the observed results in $U$ or $V$ are independent on $W$, then one always has $\rho_{AB} = \rho_{AB}$.

Inverting the above reasoning, it is easy to see that any state given in equation (4) satisfies locality. Thus, for the two spacelike separated particles locality is a necessary and sufficient condition for the states having the form specified in equation (4). Consequently, the state (11) is local by definition. On the contrary, any state that cannot be written as equation (4) is nonlocal.

Importantly, since the above reasoning does not require any specification of the particles, it is general enough that it is valid for any biparticle systems. Furthermore, its generalization to multiparticle systems is straightforward, and the resulting states satisfying locality are still given by a convex sum of direct products of the local density operators, similarly to equation (4).

Surprisingly, the form of $\rho_{AB}$ in equation (4) is just the “mathematical” definition of separable (i.e., non-entangled) two-particle states, as is suggested by Werner (13) and now widely accepted. Meanwhile, $\rho_{AB}$ is local by definition as it comes from the “physical” criterion of locality, which clarifies the physical content, and justifies Werner’s definition, of separable states from another perspective. Consequently, the two basic notions—entanglement of quantum states and nonlocality of measured results—are equivalent for spacelike separated systems; local measurements performed on entangled (separable) states give nonlocal (local) results. This result is a further support of the viability of the locality condition (11).

When the particles A and B are not spacelike separated, the locality condition given in equation (11) can be reasonably called Einstein’s separability condition. In this broader sense, locality is identical to Einstein’s separability when the particles are spacelike separated, and Einstein’s separability is a necessary and sufficient condition for the separability of states for any quantum systems.

\[ \rho_{AB} = \sum_{\mu} \lambda_{\mu} \rho_{A\mu} \rho_{B\mu} \]  

(4)

which represents the state of the two particles allowed by the locality condition (11). The state $\rho_{AB}$ is a convex sum, with weights $\lambda_{\mu}$, of direct products of the local density operators $\rho_{A\mu}$ and $\rho_{B\mu}$. Different set of common causes leads to different convex sum of the local density operators.
This fact may open up an exciting perspective for understanding nonlocality (or more generally, Einstein’s inseparability) and entanglement as a unified concept in quantum mechanics, i.e., quantum wholeness: Entanglement represents the mathematical inseparability of quantum states, while nonlocality physically manifests itself in the correlations of certain measurement results. Thus, one has to accept the existence of a quantum weirdness in nature: Quantum entanglement induces exotic influences for a composite system even when the constituent parts are spacelike separated. Here one is confronted with a situation where an entangled quantum system must be regarded as a holistic entity; any attempt to describe the entangled system locally must fail for certain quantum predictions.

As we proved, locality permits only the classically correlated (i.e., separable) states in quantum mechanics. Or equivalently, whatever the set of common causes is, the locality assumption cannot be fulfilled by any entangled state. However, entanglement is ubiquitous in quantum mechanics as well as in practical quantum information processing as an essential resource. Thus, locality is in conflict with quantum mechanics, namely, quantum mechanics is definitely a nonlocal theory. We believe that we have for the first time proved in simple terms the intrinsic nonlocal feature of quantum mechanics in a clear-cut way, without resorting to the realism assumption or other counterfactual reasonings. Actually, the realism assumption used in deriving Bell’s inequalities is redundant and even detrimental for the purpose of uncovering quantum nonlocality.

One might wonder whether the apparent nonlocality of quantum mechanics could be used for superluminal signaling. If this were the case, then quantum mechanics would violate the relativistic causality which forbids any superluminal causal action. Fortunately, quantum mechanics in its current form is still in a “peaceful coexistence” with relativity in the sense that nonlocality does not lead to superluminal information transmitting. To see this, recall that the locality assumption, according to Shimony, consists of two independent factors: (i) the outcome independence and (ii) the parameter independence, which demand that any measurement outcome of particle A should be independent, respectively, on the outcomes and on the experimental settings of the spacelike separated particle B. Explicit calculation shows that quantum mechanics violates the outcome independence, which nevertheless cannot be used for superluminal information transmitting. However, violation of the parameter independence may imply superluminal communication.

To prove the parameter independence (see, e.g., (18) in the two-particle case (the multiparticle generalization is straightforward), it is sufficient to prove $P(a_i|b) = \sum_j P(a_i, b_j) = P(a_i)$ for any chosen setting $b$. Actually, this is always true as $\sum_j P(a_i, b_j) = \text{Tr}(\rho_{AB} \hat{P}_A \sum_j \hat{P}_B) = \text{Tr}(\rho_A \hat{P}_A) = P(a_i)$. Here we have used the simple fact that $\sum_j \hat{P}_B = \hat{I}_B$ ($\hat{I}_B$ is the unit operator for particles B); $\rho_A = \text{Tr}_B(\rho_{AB})$ is the reduced density operator for particle A. Thus, quantum mechanics respects the parameter independence, implying that quantum nonlocality cannot result in superluminal signaling.

Based on the equivalence between nonlocality/inseparability of measurements and entanglement of states, quantum nonlocality has acquired the same solid basis as quantum entanglement. The two equivalent notions are two distinct aspects of the same quantum wholeness. This fact, being interesting in its own right on fundamental issues of quantum mechanics, might be important in quantum information science, where manipulating entanglement is a vital task for processing information. This practical impulse has greatly enriched our current knowledge on entanglement. We anticipate that future works on entanglement and nonlocality may be mutually promoted to deepen our understanding of the weird quantum wholeness.

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One can prove the following fact: If $\langle \phi | \langle O | \phi \rangle | \psi \rangle = 0$ for any product state $|\phi\rangle |\psi\rangle$ for a two-particle system (generalization to multiparticle cases is straightforward), then the two-particle operator $\hat{O} \equiv 0$. To this end, let $|i\rangle$ be any state of particle A, and $|k\rangle$ and $|l\rangle$ any two orthonormal states of particle B. Then for the product states $|\varphi_1\rangle = |i\rangle \otimes \frac{1}{\sqrt{2}}(|k\rangle + |l\rangle)$ and $|\varphi_2\rangle = |i\rangle \otimes \frac{1}{\sqrt{2}}(|k\rangle + i|l\rangle)$, one necessarily has $\langle \varphi_1 | \hat{O} | \varphi_1 \rangle = \frac{1}{2} \left( \langle ik | \hat{O} | ik \rangle + \langle ik | \hat{O} | id \rangle + \langle il | \hat{O} | ik \rangle + \langle il | \hat{O} | id \rangle \right)$ and $\langle \varphi_2 | \hat{O} | \varphi_2 \rangle = \frac{1}{2} \left( \langle ik | \hat{O} | ik \rangle + i \langle il | \hat{O} | ik \rangle - i \langle il | \hat{O} | id \rangle + \langle il | \hat{O} | id \rangle \right)$.

Since $\langle \varphi_1 | \hat{O} | \varphi_1 \rangle = \langle \varphi_2 | \hat{O} | \varphi_2 \rangle = \langle ik | \hat{O} | ik \rangle = \langle il | \hat{O} | il \rangle = 0$, one obtains from the above equations that $\langle ik | \hat{O} | il \rangle = 0$. Particularly, choosing $|i\rangle$ to be the eigenvectors of the Hermitian operator $\langle k | \hat{O} | l \rangle + \langle k | \hat{O} | l \rangle^\dagger$ for particle A and using $\langle ik | \hat{O} | il \rangle = 0$, it is easy to see that $\langle k | \hat{O} | l \rangle + \langle k | \hat{O} | l \rangle^\dagger = 0$. Similarly, one can prove $\langle k | \hat{O} | l \rangle - \langle k | \hat{O} | l \rangle^\dagger = 0$. Therefore, $\langle k | \hat{O} | l \rangle = 0$, which means that $\hat{O} \equiv 0$. In particular, for $\hat{O} = \rho_{AB} - \sum_\mu \lambda_\mu \rho_{A\mu} \rho_{B\mu}$, $|\phi\rangle = |a_i\rangle$ and $|\psi\rangle = |b_j\rangle$, equation 41 and the result proved in this note then give equation 42.