Trapped Quintessential Inflation

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Abstract

Quintessential inflation is studied using a string modulus as the inflaton - quintessence field. The modulus begins its evolution at the steep part of its scalar potential, which is due to non-perturbative effects (e.g. gaugino condensation). It is assumed that the modulus crosses an enhanced symmetry point (ESP) in field space. Particle production at the ESP temporarily traps the modulus resulting in a brief period of inflation. More inflation follows, due to the flatness of the potential, since the ESP generates either an extremum (maximum or minimum) or a flat inflection point in the scalar potential. Eventually, the potential becomes steep again and inflation is terminated. After reheating the modulus freezes due to cosmological friction at a large value, such that its scalar potential is dominated by contributions due to fluxes in the extra dimensions or other effects. The modulus remains frozen until the present, when it can become quintessence and account for the dark energy necessary to explain the observed accelerated expansion.
To everyone’s surprise recent observations of high-redshift supernovae have suggested that the Universe at present is engaging into a phase of accelerated expansion [1]. This result has been confirmed by a number of other independent observations such as the distribution of large scale structure [2] and also from combinations of the precise CMB anisotropy data with the estimates of the Hubble constant from the Hubble space telescope key project [3]. In fact, many cosmologists now accept the so-called concordance model of cosmology, according to which more than 70% of the Universe content at present is attributed to the so-called Dark Energy. Dark Energy is an elusive substance, with pressure negative enough to cause the observed acceleration [4] (for a recent review see [5]). The simplest type of Dark Energy is a positive cosmological constant $\Lambda$, which however, needs to be fine-tuned to an incredibly small value in order to explain the observations [6]. Hence, despite the fact that considering a non-zero $\Lambda$ is the simplest and by far the easiest to use option, theorists have looked for alternative explanations, which could explain the observations while setting $\Lambda = 0$, as was originally assumed. A promising idea along this direction is to consider that the Universe at present is entering a late-time inflationary period [7]. The credibility of this option has been enhanced by the fact that the generic predictions of the inflationary paradigm in the Early Universe are very much in agreement with the observations. The scalar field responsible for this late-inflation period is called quintessence because it is the fifth element after baryons, photons, CDM and neutrinos [8].

Since they are based on the same idea, it was natural to attempt to unify early Universe inflation with quintessence. Quintessential inflation was thus born [9, 10, 11, 12]. The advantages of this effort are many. The obvious one has to do with the fact that quintessential inflation models allow the treatment of both inflation and quintessence within a single theoretical framework, with hopefully fewer and more tightly constraint parameters. Another practical gain is the fact that quintessential inflation dispenses with a tuning problem of quintessence models; that of the initial conditions for the quintessence field. It is true that attractor - tracker quintessence alleviates this tuning but the problem never goes away completely. In contrast, in quintessential inflation the initial conditions for the late time accelerated expansion are fixed in a deterministic manner by the end of inflation. Finally, a further advantage of unified models for inflation and quintessence is the economy of avoiding to introduce yet again another unobserved scalar field to explain the late accelerated expansion.

For quintessential inflation to work one needs a scalar field with a runaway potential, such that the minimum has not been reached until today and, therefore, there is residual potential density, which, if dominant, can cause the observed accelerated expansion. String moduli fields appear suitable for this role because they are typically characterised by such runaway potentials. The problem with such fields, however, is how to stabilise them temporarily, in order to use them as inflatons in the early Universe. In this letter (see also Ref. [13]) we achieve this by considering that, during its early evolution our modulus crosses an enhanced symmetry point (ESP) in field space. As shown in Ref. [14], when this occurs the modulus may be trapped at the ESP for some time. This can lead to a period of inflation, typically comprised by many sub-periods of different types of inflation such as trapped, eternal, old, slow-roll, fast-roll and so on. After the end of inflation the modulus picks up speed again in field space resulting into a period of kinetic density domination, called kination [15]. Kination is terminated when the thermal bath of the hot big bang (HBB) takes over. During the HBB, due to cosmological friction [16], the modulus freezes asymptotically at some large value and remains there until the present, when its potential density becomes dominant and drives the late-time accelerated expansion [12].

It is evident that, in order for the scalar field to play the role of quintessence, it should not decay after the end of inflation. Reheating, therefore should be achieved by other means. One option is gravitational particle production as discussed in Ref. [17]. However, because such reheating is typically quite inefficient, in this paper we assume that the thermal bath of the HBB is due to the decay of some curvaton field [18, 19] as suggested in Refs. [12, 20]. Note that the curvaton can be a realistic field, already present in simple extensions of the standard model (for example it can be a right-handed sneutrino [21], a flat direction of the (N)MSSM [22, 23] or a pseudo Nambu-Goldstone boson [24, 25] possibly associated with the Peccei-Quinn symmetry [26] etc.). Thus, by considering a curvaton we do not necessarily add an ad hoc degree of freedom. The importance
of the curvaton lies also in the fact that the energy scale of inflation can be much lower than the grand unified scale \[27\]. In fact, in certain curvaton models, the Hubble scale during inflation can be as low as the electroweak scale \[25, 28\].

String theories contain a number of flat directions which are parametrised by the so-called moduli fields. Many of such flat directions are lifted by non-perturbative effects, such as gaugino condensation or D-brane instantons \[29\]. The superpotential, then, is of the form

\[ W = W_0 + W_{np} \quad \text{with} \quad W_{np} = Ae^{-cT}, \]  

where \( W_0 \) is the tree level contribution from fluxes (which can be taken approximately constant), \( A \) and \( c \) are constants, whose magnitude and physical interpretation depends on the origin of the non-perturbative term (in the case of gaugino condensation \( c \ll 1 \)), and \( T \) is a Kähler modulus in units of \( m_P \), for which

\[ T = \sigma + i\alpha \quad \text{with} \quad \sigma, \alpha \in \mathbb{R}. \]  

Hence, the non-perturbative superpotential \( W_{np} \) results in a runaway scalar potential characteristic of string compactifications. For example, in type IIB compactifications with a single Kähler modulus, \( \sigma \) is the so-called volume modulus, which parametrises the volume of the compactified space. Consequently, in this case, the runaway behaviour leads to decompactification of the internal manifold.

The tree level Kähler potential for a modulus, written in units of \( m_P^2 \), is

\[ K = -3 \ln (T + \bar{T}), \]  

and the corresponding supergravity potential is

\[ V_{np}(\sigma) = \frac{cAe^{-c\sigma}}{2\sigma^2 m_P^2} \left[ \left(1 + \frac{\sigma}{3}\right) Ae^{-c\sigma} + W_0 \cos(c\alpha) \right]. \]  

To secure the validity of the supergravity approximation we consider \( \sigma > 1 \). Then, for values of \( c \ll 1 \), we have \( \sigma c > 1 \), and we approximate the potential as

\[ V_{np}(\sigma) \simeq \frac{cAe^{-c\sigma}}{2\sigma^2 m_P^2} \left( \frac{\sigma}{3} Ae^{-c\sigma} - W_0 \right). \]  

In order to study the cosmology, we turn to the canonically normalised field \( \phi \) associated to \( \sigma \), which due to Eq. (3) is given by

\[ \sigma(\phi) = \exp \left( \lambda \phi / m_P \right) \quad \text{with} \quad \lambda = \sqrt{2/3}. \]  

Let us assume that the Universe is initially dominated by the above modulus. The non-perturbative scalar potential for the modulus, shown in Eq. (5), is very steep (exponential of an exponential), which means that the field will soon become dominated by its kinetic density. Once this is the case the particular form of the scalar potential ceases to be of importance.

Now, suppose that while rolling towards large values the modulus crosses an ESP. In string theory compactifications there are distinguished points in moduli space at which there is enhancement of the gauge symmetries of the theory \[30\]. This often results in some massive states of the theory becoming massless at these points. Even though from the classical point of view an ESP is not a special point, as the modulus approaches it certain states in the string spectrum become massless \[31\]. In turn, these massless modes create an interaction potential that may drive the field back to the symmetry point. In that way a modulus can become trapped at an ESP \[14\]. The strength of the symmetry point varies depending on the degree of enhancement of the symmetry at the particular point; the higher the symmetry the stronger the force driving the modulus back to the symmetry point. Such moduli trapping can lead to a period of so-called ‘trapped inflation’

\[ \text{The imaginary part } \alpha \text{ of the } T \text{ modulus can be taken such that the ESP lies in a minimum in the direction of } \alpha, \text{ namely } \cos(c\alpha) = -1. \]
[14], when the trapping is strong enough to make the kinetic density of the modulus fall below the potential density at the ESP. However, it turns out that the number of e-foldings of trapped inflation cannot be very large. Therefore, with respect to cosmology, the main virtue of the ESPs relies on their ability to trap the field and hold it there, at least temporarily, thereby setting the initial conditions by confining the modulus to a precise location in field space. In this sense, quantum effects make the ESP a preferred location.

Let us briefly study the trapping of the modulus at the ESP. Following Ref. [14] we assume that around the ESP there is a contribution to the scalar potential due to the enhanced interaction between the modulus $\phi$ and another field $\chi$, which we take to be also a scalar field for convenience. The interaction potential is

$$V_{\text{int}}(\phi, \chi) = \frac{1}{2} g^2 \chi^2 \tilde{\phi}^2,$$

where $\tilde{\phi} \equiv \phi - \phi_0$ with $\phi_0$ denoting the value of the modulus at the ESP and $g$ being a dimensionless coupling constant. We see that at the ESP the $\chi$ particles become massless. The time dependence of the effective mass-squared of the $\chi$ field results in the creation of particles with typical momentum [14] $k_0 \sim (g\phi_0)^{1/2}$, where $\dot{g}\phi_0^2$ is the kinetic density of the modulus when crossing the ESP (and the dot denotes derivative with respect to the cosmic time $t$). The production takes place when the field is within the production window $|\phi| < \Delta \phi$, where

$$\Delta \phi \sim (\phi_0/g)^{1/2} \sim k_0/g.$$

The effective scalar potential, $V_{\text{eff}}(\phi) = V(\phi) + V_{\text{int}}(\phi)$, near the ESP is

$$V_{\text{eff}}(\phi) \approx V_0 + \frac{1}{2} g^2 \langle \chi^2 \rangle \tilde{\phi}^2$$

where $V_0 \equiv V(\phi_0)$ with $V(\phi)$ being the ‘background’ scalar potential. Following Ref. [14] we have $\langle \chi^2 \rangle \simeq n_\chi/|\phi|$, where $n_\chi$ denotes the number density of $\chi$ particles produced after the crossing of the ESP. This means that $V_{\text{eff}}(\phi) \sim V_0 + g n_\chi |\phi|$ and the field climbs a linear potential since $n_\chi$ is constant outside the production window. The field reaches an amplitude $\Phi_1$ given by

$$\Phi_1/m_P \sim \frac{\phi_0^{1/2}}{g^{5/2} m_P} \lesssim 1,$$

which is determined by its initial kinetic density. The upper bound in the above is due to the requirement that the modulus does not overshoot the ESP without being trapped, since for larger values the coupling softens and the field, instead of falling back to the symmetry point, would keep rolling down its potential [32].

After reaching $\Phi_1$ the field reverses direction and crosses the production window again, generating more $\chi$ particles and, therefore, increasing $n_\chi$. Thus, it now has to climb a steeper potential reaching an amplitude $\Phi_2 < \Phi_1$. The process continues until the ever decreasing amplitude becomes comparable to the production window shown in Eq. (8). At this moment particle production stops. The final value of $n_\chi$ is estimated as $n_\chi \sim g^{-1/2} \phi_0^{3/2}$. After the end of particle production, $\langle \chi^2 \rangle$ remains roughly constant during an oscillation and the modulus continues oscillating in the quadratic interaction potential [cf. Eq. (9)]. Studying this oscillation in Ref. [13] we found that, due to the Universe expansion, the amplitude and frequency decrease as

$$\Phi(t) \sim \frac{\Delta \phi}{a} \quad \text{and} \quad \langle \chi^2(t) \rangle \sim \left( \frac{k_0}{a} \right)^2 = k^2(t),$$

where $k(t)$ is the physical momentum and we have normalised the scale factor $a(t)$ to unity at the end of particle production. When the initial kinetic density of the modulus is redshifted down to the level of $V_0$, trapped inflation begins.

It is easy to check that the amplitude of the oscillations at the onset of trapped inflation is

$$\Phi_1 \sim \frac{\sqrt{m_0 m_P}}{g^{1/2}}.$$
Using this, it can be easily shown that the number of e-foldings of trapped inflation, while the modulus oscillates, is [13]

\[ N_{osc} \sim \ln \left( g^{3/2} \frac{m_P}{m_0} \right)^{1/2}, \]  

(13)

where \( m_0 \) is roughly the Hubble scale during inflation, given by \( m_0 \equiv \sqrt{V_0}/m_P \approx \sqrt{3}H_0 \). Note that \( N_{osc} \) is independent of \( \dot{\phi} \) because the excess of kinetic energy must be redshifted before trapped inflation can begin. Therefore, once trapped inflation begins the only relevant energy scale is \( m_0 \).

The redshifting of the frequency of oscillations means that the modulus becomes lighter. At some point oscillations cease and the field begins to slow-roll down the interaction potential. The number of slow-roll e-foldings is found to be [13]

\[ N_{sr} \sim \frac{4}{3} \ln \frac{1}{g}. \]  

(14)

The field value during this phase of slow-roll trapped inflation is roughly \( |\dot{\phi}| \sim \Phi_{min} \), where

\[ \Phi_{min} \sim m_0/g^2 \]  

(15)

is the minimum amplitude of the oscillations.

Finally, because the interaction potential becomes increasingly depleted, the phase of slow-roll of the trapped modulus comes to an end when the field becomes dominated by its quantum fluctuations. After this moment the field drives a period of eternal inflation, which is oblivious to the scalar potential. During this phase the field value performs a one dimensional random walk in field space, of step determined by the Hawking temperature \( H_s/2\pi \) corresponding to an e-folding of inflation. Hence, after \( N \) e-foldings we have \( \sqrt{\langle \dot{\phi}^2 \rangle} \sim \frac{H_s}{2\pi}\sqrt{N} \). The field obeys

\[ |\dot{\phi}(N)| \sim m_0/g^2 + m_0\sqrt{N}. \]  

(16)

The above results are true under the assumption that trapped inflation continues uninterrupted. However, since the interaction potential becomes gradually depleted, there will be a moment when \( V_{int}(\phi) \lesssim |V'(\phi)| \), in which the dynamics of the modulus begins to be determined by the ‘background’ scalar potential \( V(\phi) \) rather than the interaction potential (where the prime denotes derivative with respect to \( \phi \)). Thus, the stages of trapped inflation can be summarised as follows [13]:

- **Trapping:** After crossing the ESP, the field starts to oscillate around it if \( |V'(\Phi_1)| < gn_\chi \), where \( n_\chi \sim g^{3/2}\dot{\phi}_0^{3/2} \) is the number density of \( \chi \) particles after the first crossing.
- **Trapped inflation:** Trapped inflation occurs whenever \( |V'(\Phi_1)| \lesssim g^{5/2}(m_0m_P)^{3/2} \).
- **Slow-roll:** The oscillations of the field give way to slow-roll motion if \( |V'(\Phi_{min})| \lesssim m_0^3/g^2 \).
- **Eternal inflation:** A phase of eternal inflation occurs when \( |V'(\Phi_{min})| \lesssim g^2m_0^3 \). This phase lasts until the field is sufficiently far away from the ESP so that the scalar potential \( V(\phi) \) becomes the driving force again.

Because ESPs are fixed points of the symmetries, by definition, the scalar potential is flat at these points. Hence, we have

\[ V'_0 \equiv V'(\phi_0) = 0. \]  

(17)

The above means that the ESP is located either at a local extremum (maximum or minimum) or at a flat inflection point of the scalar potential, where \( V'_0 = V''_0 = 0 \) with \( V''_0 \equiv V''(\phi_0) \). This means that the presence of an ESP is expected to deform the non-perturbative scalar potential. To model the ESP we, therefore, introduce a phenomenological contribution \( V_{ph} \) to the non-perturbative scalar potential such that \( V(\phi) = V_{np}(\phi) + V_{ph}(\phi) \) (see Fig. 1). We require that \( V_{ph} \) becomes negligible far away from the ESP so that the steepness of \( V_{np} \) is recovered. To be able to perform analytic calculations we have chosen the following form for \( V_{ph} \):

\[ V_{ph} = \frac{1}{2} \left( \frac{m_P}{\dot{\phi}_0} \right)^2 \dot{\phi}_0^2 \]  

(18)
Figure 1: The deformation of $V_{np}(\sigma)$ at the ESP as modelled through the use of our choice for $V_{ph}(\sigma)$. Case (a) corresponds to a flat inflection point, while Case (b) corresponds to a local minimum.

\[ V_{ph}(\sigma) = -\frac{M}{\cosh^q(\sigma - \Sigma)} , \]  

where $M$ is some density scale and $\Sigma, q$ are positive constants. It can be shown that the functional form of $V_{ph}$ is not important [13]. The results, in fact, depend only on the following parameters

\[ m_0^2 \equiv V''(\phi_0) \quad \text{and} \quad V_0^{(3)} \equiv V'''(\phi_0) . \]  

Combining Eqs. (5) and (18) one can show that $V_0^{(3)}$ can be parametrised as follows

\[ |V_0^{(3)}| \approx 2(\lambda\mu)^3(c\sigma_0)^3\xi^2 m_0^2 m_P , \]  

where $\sigma_0 \equiv \sigma(\phi_0)$, $\mu = 2, 1$ if Eq. (5) is dominated either by the first or second term, respectively. The parameter $\xi$, defined by

\[ \xi \equiv \frac{\sqrt{q}}{c\mu} > 1 , \]  

accounts for the strength of the symmetry point: the smaller the value of $\xi$ is, the larger the inflationary plateau generated by the ESP becomes, and therefore, the stronger the deformation due to the ESP is. The lower bound on $\xi$ is due to the requirement that $V_{ph}$ (i.e. the deforming effect of the ESP) becomes negligible far away from the ESP.

Before moving on let us discuss briefly the particular case where our Kähler modulus is the volume modulus of type IIB compactifications. In this case the ESP must be located within the non-perturbative region of string theory. In this region, the volume of the compact space (measured in string units) is $V_{st} \ll 1$. In terms of the volume modulus $T$ and the string coupling $g_s$ we have $V_{st} \sim g_s^{3/2}(\text{Re} T)^{3/2}$, where we take $\text{Re} T \equiv \sigma$. Therefore, to locate the ESP within the non-perturbative region we require $\sigma_0 < g_s^{-1}$. In this case $V_0^{(3)}$ may be parametrised as [13]

\[ V_0^{(3)} = -2(2\lambda)^3\zeta^2 \frac{m_0^2}{m_P} , \]  

where $\zeta$ is defined as

\[ \zeta \equiv \frac{\sigma_0\sqrt{q}}{2} > 1 . \]
The physical meaning of $\zeta$ is identical to that of $\xi$ in Eq. (21).

Let us study first the case where the ESP creates a flat inflection point such that $V_0' = V_0'' = 0$. In this case the inflationary dynamics, arising from the scalar potential $V(\phi)$ in the neighbourhood of the ESP, can be accounted for by approximating the scalar potential as

$$V(\phi) \approx V_0 + \frac{1}{3!} V_0^{(3)} \phi^3.$$  \hspace{1cm} (24)

To quantify our parameter space we consider that $m_0 \sim 1$ TeV. For a generic Khähler modulus we take $(c\sigma_0) \sim 10$, $c \lesssim 1$ and $g \simeq 0.1$. Then the following cases are possible [13]:

- If $g^2 m_0 < |V_0^{(3)}|$ then the field is unable to drive a long-lasting period of inflation because it escapes trapping before the end of the oscillations.
- If $g^2 m_0 \lesssim |V_0^{(3)}| < g^2 m_0$ then the slow-roll after the end of oscillations is continued with another phase of slow-roll over the scalar potential $V(\phi)$. The number of e-foldings attainable is $N_{sr1} \sim g^2 m_0/|V_0^{(3)}| \lesssim g^{-4}$. For our choice of parameter values we find that this case corresponds to $10^3 < \xi^2 < 10^6$. Then the maximum number of e-foldings in this case is $N_{sr1} \lesssim 10^4$.
- If $|V_0^{(3)}| < g^2 m_0$ then the slow-roll phase after the end of oscillations is continued with a phase of eternal inflation, which, typically, lasts for $N_{et} \gtrsim m_0/|V_0^{(3)}| > g^{-6}$ e-foldings. The end of eternal inflation is followed by another phase of slow-roll over the scalar potential $V(\phi)$ with number of e-foldings $N_{sr2} \sim \sqrt{m_0/|V_0^{(3)}|} \gtrsim g^{-3}$. For our choice of parameter values we find that this case corresponds to $1 < \xi^2 < 10^6$. The maximum number of eternal inflation e-foldings is $N_{et} \sim 10^{10-11}$. Similarly, the maximum number of e-foldings corresponding to the second slow-roll period is $N_{sr2} \sim 10^5$.

In order not to overshoot the ESP, the interaction coupling has to satisfy the bound $g \gtrsim 10^{-3}$. In the interval $10^{-3} \lesssim g \lesssim 10^{-2}$ the resulting slow-roll phase needs not to be preceded by eternal inflation, while in the interval $10^{-2} < g \lesssim 1$ eternal inflation has to occur.

Another important bound on the parameters stems from the requirement that the density perturbations due to the inflaton modulus are not excessive compared to the observations. This requirement translates into the bound [13]

$$\xi^2 \lesssim \frac{10^{-4}}{N^2(c\sigma_0)^2} \frac{m_P}{m_0} \sim 10^4,$$ \hspace{1cm} (25)

where we take $N \sim 10^2$. Fortunately, eternal inflation is followed by enough slow-roll inflation to encompass the cosmological scales. In the case where $c$ is the volume modulus one can show that, the requirement that the inflaton’s perturbations are not excessive translates into [13]

$$\xi^2 \lesssim \frac{10^{-4} m_P}{N_{sr1}^2 m_0},$$ \hspace{1cm} (26)

which is quite easily satisfied.

We move on now to considering the case when the ESP results in an extremum in the scalar potential. This time we have $V_0'' = m_0^2 \neq 0$ and the scalar potential can be approximated as

$$V(\phi) \simeq V_0 + \frac{1}{2} m_0^2 \phi^2 - \frac{1}{2} m_0^2 \phi_t^2,$$ \hspace{1cm} (27)

where $\phi_t$ is defined by $V(\phi_t) = V(0)$, i.e. it is the value of the modulus where the scalar potential has the same height as at the ESP. It is evident that in the case where the ESP corresponds to a local minimum {maximum} $\phi_t$ is positive {negative}. The value of $\phi_t$ depends on the magnitude of $V_0^{(3)}$. Using Eq. (20) we obtain

$$\phi_t \sim \frac{\eta \xi^{-2}}{(c\sigma_0)^2} m_P,$$ \hspace{1cm} (28)
where \( \eta = (m_\phi/m_0)^2 \) is the second slow-roll parameter at the ESP.

Suppose at first that the ESP lies at a local minimum. In this case after trapping the field has to tunnel out, as in old inflation. We assume that the field emerges at \( \approx \phi_c \) after tunnelling. Then, the number of e-foldings of slow-roll after bubble nucleation is \( N_{sr} \sim (m_0/m_\phi)^2 \gtrsim N_H \), where \( N_H \gtrsim 60 \) is the number of e-foldings necessary to produce a flat Universe at scales of at least \( 10^2 H^{-1} \) so that the flatness and horizon problems be solved. Therefore, the mass of the field at the ESP must be \( m_\phi \lesssim \frac{1}{10} m_0 \). Note that, only if there is enough slow-roll inflation after bubble nucleation can we hope to overcome the graceful exit problem of old inflation.

Another bound is due to the requirement that the modulus does not generate excessive density perturbations. This results in the bound [13]

\[
\xi^2 \lesssim \frac{10^{-4}}{N_{sr}^2 (\sigma_0)^3} \frac{m_P}{m_0} \sim 10^4,
\]

with a maximum number of e-foldings \( N_{sr} \sim 10^3 \).

Now suppose that the ESP lies at a local maximum. If the tachyonic mass of the field is \( |m_\phi| \lesssim m_0 \) the field may drive a period of inflation with total number of e-foldings given by [13]

\[
N_{tot} \sim \frac{1}{F_\phi} \ln \left( \frac{|\eta| g^2}{(\sigma_0)^3 \xi^2} \frac{m_P}{m_0} \right),
\]

where \( F_\phi \equiv \frac{1}{2} \left( \sqrt{1 + \frac{4}{3} |\eta|} - 1 \right) \). When \( |m_\phi| \ll m_0 \) inflation is slow-roll and \( F_\phi \approx |\eta| \). When \( |m_\phi| \sim m_0 \) inflation is fast-roll and \( F_\phi = O(1) \). The requirement that the inflaton modulus does not produce excessive density perturbations results in the bound [13]

\[
\xi^2 \lesssim \frac{|\eta|}{10^{14} (\sigma_0)^3} \frac{m_P}{m_0}.
\]

For the above to be consistent with the parameter space for \( \xi \) we need \( 10^{-4} \lesssim |\eta| \lesssim 0.25 \), which is a reasonable range.

After the end of inflation the field rolls away from the ESP. Soon the influence of the ESP on the scalar potential diminishes and \( V(\phi) \approx V_{np}(\phi) \) (i.e. \( V_{ph} \) becomes negligible). The steepness of \( V_{np} \) results in the kinetic domination of the modulus density. As a result a period of kination takes place, during which the field equation is \( \ddot{\phi} + 3 H \dot{\phi} \approx 0 \), which suggests that the density of the Universe scales as \( \rho \approx \frac{1}{2} \phi^2 \propto a^{-6} \) [15]. During kination the scalar field is oblivious of the particular form of the scalar potential as long as the latter does not inhibit kination. Kination is terminated when the density of an oscillating curvaton field, or of radiation due to the curvaton’s decay overtakes the kinetic density of the modulus [20]. For the purposes of this work both of these possibilities are roughly equivalent. Therefore, we assume that the curvaton decays before it dominates the Universe. In this case the end of kination is also the onset of the HBB, i.e. it corresponds to the reheating of the Universe, with reheating temperature \( T_{reh} \sim g_*^{-1/4} \sqrt{H_{reh} m_P} \), with \( g_* \sim 10^2 \) being the number of relativistic degrees of freedom and \( H_{reh} \) being the Hubble parameter at reheating.\(^2\)

As shown in Ref. [12] (see also [16]) after the onset of the HBB the rolling scalar field is subject to cosmological friction which asymptotically freezes the field at a value \( \phi_F \) corresponding to

\[
\sigma_F \sim \left( \frac{m_0 m_P}{g_* T_{reh}^2} \right)^{2/3}.
\]

Because \( \sigma_F > 1 \) the scalar potential is no longer dominated by \( V_{np} \) shown in Eq. (5). Instead it is dominated by a contribution of the form

\[
V(\sigma) \approx \frac{C_n}{\sigma^n} \Rightarrow V(\phi) \approx C_n e^{-b\phi/m_P},
\]

\(^2\)In quintessential inflation, typically, reheating occurs late, which requires the curvaton to decay in advance, in order to account for baryogenesis.
with \( b = n\lambda = \sqrt{\frac{2}{3}} n \). The modulus remains frozen at the value shown in Eq. (32) until the present. This guarantees that there is no dangerous variation of fundamental constants despite the fact that the modulus is not stabilised at a local minimum of the scalar potential.

At present, if the modulus is to account for the required dark energy, its potential density has to satisfy the coincidence requirement \( V(\sigma F) \simeq \Omega_\Lambda \rho_0 \), where \( \Omega_\Lambda \simeq 0.73 \) is the density parameter of dark energy and \( \rho_0 \) is the critical density at present. Using Eqs. (32) and (33) one finds that the coincidence requirement demands

\[
T_{\text{reh}} \sim \sqrt{m_0 m_P \left( \frac{\rho_0}{C_n} \right)^{\sqrt{3}/8n^2}}.
\]

(34)

Because reheating has to occur before nucleosynthesis takes place, the above results in the bound

\[
C_n \lesssim \rho_0 \left( \frac{m_0 m_P}{T_{\text{BBN}}} \right)^{2n\sqrt{2/3}},
\]

(35)

where \( T_{\text{BBN}} \sim 1 \text{ MeV} \) is the temperature at nucleosynthesis. This is an important constraint on the scalar potential in Eq. (33).

Another important constraint is due to the possibility of excessive gravitational wave generation. The spectrum of gravitational waves in models of quintessential inflation features a spike at high frequencies due to the stiff equation of state during kination [12, 33]. In order for these gravitons not to disturb nucleosynthesis one has to impose the bound [13]

\[
m_0 \lesssim (4 \times 10^2)^{3/8} (m_P T_{\text{BBN}})^{1/2} \sim 10^5 \text{ TeV}.
\]

(36)

Thus, we see that inflation has to take place at lower energies than the grand unified scale. This supports the use of a curvaton field for the generation of the density perturbations because low-scale inflation is easier to attain [25, 27, 28].

The scalar potential considered in Eq. (33) may have a multitude of origins. For example, if we use the volume modulus, we may consider a number of \( D^3 \)-branes located at the tip of a Klebanov-Strassler throat. The contribution \( \delta V \), proportional to the warp factor, is [34]

\[
\delta V(\sigma) \sim e^{-8\pi K/3M g_s} \frac{m_1^3}{\sigma^2} \equiv C_2/\sigma^2,
\]

(37)

where \( M \) and \( K \) quantify the units of RR and NS three-form fluxes. To satisfy Eq. (35) we must have \( C_2^{1/2} \lesssim 10^{-20} m_P \). This can be obtained with a choice of fluxes \( K/M g_s \gtrsim 22 \). Taking \( g_s = 0.1 \), only approximately twice as many units of \( K \) flux as those of \( M \) flux are needed.

It is also possible to consider fluxes of gauge fields on \( D7 \)-branes [35]. In this case, the scalar potential obtains a contribution

\[
\delta V \sim \frac{2\pi E^2}{\sigma^3} \equiv C_3/\sigma^3,
\]

(38)

where \( E \) depends on the strength of the gauge fields considered. The constraint in Eq. (35) requires now \( C_3^{1/4} \lesssim 10^{-15} m_P \sim 1 \text{ TeV} \).

Another kind of corrections introducing an uplifting term in the scalar potential \( V(\phi) \), changing its structure at large volume and breaking the no-scale structure, are \( \alpha' \) corrections [36]. The uplifting term is due to a corrected Khähler potential [37]

\[
K = K_0 - 2\ln \left( 1 + \frac{\hat{\xi}}{2(2\Re T)^{3/2}} \right),
\]

(39)

where \( K_0 \) is the tree-level Khähler potential shown in Eq. (3), and \( \hat{\xi} = -\frac{1}{3}\xi(3)\chi e^{-3\phi/2} \) where \( \chi \) is the Euler number of the internal manifold, and \( e^\phi = g_s \) is the string coupling. In this case the uplifting term \( \delta V \) is

\[
\delta V \sim \frac{\hat{\xi} W_0^2}{\sigma^{9/2}} \equiv C_{9/2}/\sigma^{9/2}.
\]

(40)
Thus, the bound in Eq. (35) with \( m_0 \sim 1 \text{ TeV} \) requires \( C_{9/2}^{1/4} \lesssim 10^{-7} m_P \), i.e. it corresponds to the intermediate scale \( C_{9/2}^{1/4} \lesssim 10^{11} \text{ GeV} \).

In closed string theory, in addition to the usual translational modes, vibrational modes of closed strings wrapping around the compact manifold are also present. The risk is that the presence of these modes may disturb nucleosynthesis. The mass of these excitations follows an inverse relation to the radius of the compactification [38] \( m_{KK} \sim m_s/R \sim g_s m_P/\sqrt{\nu_{st}^2/3} \), where \( R \) is the radius of the compactified space defined by the volume of the Calabi-Yau manifold \( \nu_{st} \sim R^6 \). These modes decay at the temperature \( T_{KK} \sim (g_s/\sigma_F)^{3/2} m_P \). Demanding \( T_{KK} > T_{\text{bbn}} \) constrains \( \sigma_F \). Using Eq. (32) we obtain a lower bound on the reheating temperature

\[
T_{\text{reh}} > (g_s g_s)^{−1/4} \sqrt{T_{\text{bbn}} m_0} . \tag{41}
\]

For \( m_0 \sim 1 \text{ TeV} \), this bound results in \( T_{\text{reh}} > 1 \text{ GeV} \). Combining the above with Eq. (34) we get a stronger bound on \( C_{9/2} \), which reads \( C_{9/2}^{1/4} \lesssim 10^{-12} m_P \sim 10^6 \text{ GeV} \).

The future of the modulus after unfreezing depends on the steepness of the potential, or equivalently the value of \( b \) in Eq. (33).

- For \( b \leq \sqrt{2} \), the modulus dominates the Universe for ever, leading to eternal acceleration. This results in future horizons, which pose a problem for the formulation of the S-matrix in string theory [39].
- For \( \sqrt{2} < b \leq \sqrt{3} \), the modulus dominates the Universe but results only in a brief accelerated expansion period. Such is the fate of the \( n = 2 \) case.
- For \( \sqrt{3} < b \leq \sqrt{6} \) the modulus does not dominate the Universe, albeit causing a brief period of accelerated expansion. After this period the modulus density remains at a constant ratio with the background matter density. This is the fate of the \( n = 3 \) case.
- For \( \sqrt{6} < b \leq 2\sqrt{6} \), the modulus also does not dominate the Universe, but still causes a brief period of accelerated expansion. Afterwards the modulus rolls fast down the quintessential tail of the scalar potential with its density approaching asymptotically kinetic domination (and subsequently freezing at a larger value than \( \sigma_F \)). This is the fate of the \( n = 9/2 \) case.
- For \( b > 2\sqrt{6} \) the modulus after unfreezing does not cause any accelerated expansion and so cannot be used as quintessence. As in the previous case the modulus rolls fast down the quintessential tail of the scalar potential with its density approaching asymptotically kinetic domination (and subsequently freezing at a larger value than \( \sigma_F \)). This case corresponds to \( n > 6 \).

The brief acceleration period caused by the unfreezing modulus is due to the fact that the modulus oscillates around an attractor solution [40]. This solution does not result to acceleration, but the oscillations of the system around it have been found to cause brief periods of accelerated expansion, especially just before unfreezing [11]. As shown in Ref. [41] brief acceleration occurs if \( \sqrt{2} < b \leq 2\sqrt{6} \), which corresponds to the range \( \sqrt{3} < n \leq 6 \). In all cases the modulus eventually approaches \( +\infty \) which corresponds to a supersymmetric ground state. If \( \sigma \) is the volume modulus then this final state leads to decompactification of the extra dimensions.

In summary, we have studied quintessential inflation using a string modulus as the inflaton - quintessence field. In our model the periods of accelerated expansion both in the early Universe and at present are due to the dynamical evolution of the compactified extra dimensions, which, in the intermediate epoch, remain frozen due to cosmological friction. Our inflaton modulus is initially kinetic density dominated as it rolls down its steep non-perturbative potential, due to to gaugino condensation or D-brane instantons. We assumed that the modulus crosses an enhanced symmetry point (ESP), which results in substantial particle production. The produced particles generate a contribution to the scalar potential which can trap the modulus at the vicinity of the ESP, giving rise to a limited period of trapped inflation. The latter can be followed by a longer period of inflation after the modulus is released from the trapping. This period is due to the fact that the scalar potential is deformed around the ESP, so that the ESP is located at
either an extremum of a flat inflection point. We have studied carefully all possible cases and showed that a variety of inflationary phases may take place, such as eternal, old, slow-roll and fast-roll inflation. We then investigated the parameter space which produces enough inflation to solve the horizon and flatness problems and found that this is indeed possible for reasonable values of the model parameters, mostly determined by the extend of the effect of the ESP on the form of the scalar potential. After the end of inflation the modulus rolls away from the ESP and it becomes kinetically dominated again. Thus, inflation is followed by a period of kination which is interrupted by the domination of the decay products of a curvaton field. A curvaton has been employed not only to generate the required density perturbations but also because its decay products are able to reheat the Universe. After reheating the evolution of the modulus is dominated by cosmological friction due to the thermal bath of the hot big bang. Consequently, the modulus approaches asymptotically a fixed value, where it becomes stabilised (frozen) despite the fact that it does not lie in a local minimum of the potential. The freezing value of the field is now such that the scalar potential for the canonically normalised modulus is dominated by an exponential contribution, which can be due to a number of string induced effects, such as fluxes or D-branes in the extra dimensions or α’-corrections to the Kähler potential. The modulus remains frozen until the present, when its potential density becomes comparable to the density of the Universe once more. At this moment the modulus is expected to unfreeze and play the role of quintessence. We have shown that the exponential potential can satisfy the coincidence requirement for reasonable values of the parameters. In this model quintessence is about to unfreeze at the present time. Therefore, in the past our model does not offer predictions different from ΛCDM. However, it may conceivably be linked to the intriguing possibility that some of the fundamental constants, such as the fine-structure constant, may begin to vary at present. This issue lies beyond the scope of this work but deserves future consideration.

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