Vacuum nonlinear electrodynamic polarization effects in hard emission of pulsars and magnetars

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Abstract. The nonlinear electrodynamics influence of pulsar magnetic field on the electromagnetic pulse polarization is discussed from the point of observation interpretation. The calculations of pulsar magnetic field impact on the electromagnetic pulse polarization are made in such a way to make it easier to interpret these effects in space experiments. The law of hard emission pulse propagation in the pulsar magnetic field according to the vacuum (nonlinear electrodynamics is obtained. It has been shown, that due to the birefringence in the vacuum the front part of any hard emission pulse coming from a pulsar should be linearly polarized and the rest of pulse can have arbitrary polarization. The observational possibilities of vacuum birefringence are discussed. In this paper we give the estimations of detector parameters such as effective area, exposure time and necessity of polarization measurements with high accuracy. The combination of large area and extremely long exposure time gives the good opportunity to search the fine polarization effects like vacuum nonlinear electrodynamics birefringence.

Keywords: X-ray telescopes, gamma ray theory, X-rays

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1 Introduction

Although about a few hundreds of pulsars are discovered in hard X-rays and gamma-rays and their temporal and spectral parameters were studied with good accuracy, there is no sufficiently complete data about polarization properties of these astrophysical objects. A very limited number of space experiments, which were able to study polarization in hard emission, were carried out at all. Mainly, there were observations of solar flares, such as polarization measurements at Soviet and Russian satellites InterCosmos [1] and Coronas-F [2] and RHESSI space observatory, which give the principle possibility of polarization detection of solar flare gamma-rays and cosmic gamma-ray bursts (GRB) [3].

Polarimetry is a powerful diagnostic tool for specific phenomena at work with cosmic sources in the radio-wave and optical energy bands. But very few results are available at high photon energies: the only significant observation in the X-gamma energy range, to date, is the measurement of the linear polarization fraction of \( P = (19 \pm 1)\% \) of the 2.6 keV emission of the Crab nebula by a Bragg polarimeter on board OSO-8 [4]. At higher energies, hard-X-ray and soft-gamma-ray telescopes that have flown into space in the past (for ex. COMPTEL CGRO, [5]) were not optimized for polarimetry, and their sensitivity to polarization was poor. Currently active missions INTEGRAL IBIS [6–8], and SPI [9, 10], have provided some improvement, with, in particular, mildly significant measurements of \( P = (28 \pm 6)\% \) (from 130 to 440 keV), [11] and \( P = (47 \pm 19)\% \) (from 200 to 800 keV), [12] for the Crab Nebula.

A number of Compton polarimeter/telescope projects have been developed, some of which also propose to record photon conversions to \( e^+e^- \) pairs. A variety of technologies have been considered, such as scintillator arrays POGO [13], GRAPE [14], POLAR [15], Si or Ge microstrip detectors MEGA [16], ASTROGAM [17] or combinations of those (Si + LaBr3 for GRIPS [18], Si + CsI(Tl) for TIGRE [19], semiconductor pixel detectors CIPHER [20], liquid xenon LXeGRIT [21] and gas mixture CF4 at 3 atm [22]) time projection chambers (TPC), IXPE [23], XIPE [24], Gas Pixel Detectors (GPD) based on proportional counters [25].

It is supposed that hard emission of astrophysical sources contained strongly magnetized neutron stars should be more-less polarized. It could be caused by geometry of emitting areas, i.e. accretion disk (in the case of neutron star in double system) or magnetic field configuration as well as by highly anisotropy of primary electron beams. The other reason is
connected with influence on the X and gamma rays propagation of specific physical conditions in neutron star magnetosphere.

However, there are the other kind of physical processes, which can lead to polarization effects. It means the influence of very strong magnetic fields on the X and gamma rays polarization. The critical value is so called Schwinger field $B_q = \frac{m^2 c^3}{(e \hbar)} = 4.41 \cdot 10^{13}$ G, above which the nonlinear electrodynamic effects in vacuum become significant. It is impossible to implement such fields in ground experiments, so the astrophysical observations are the only way to test nonlinear electrodynamic models. The strong magnetic field is not so rare in the Universe.

Some gamma-ray pulsars are characterized by magnetic fields near $B_q$ value, but for so called magnetars it could be even higher and reach about $10^{15}$ G. In such objects in the vicinity of strongly magnetized neutron star there are favorable conditions for nonlinear electrodynamic effects, in particular, vacuum birefringence [26–28]. This effect may have different manifestations. One is the dramatically increasing of linear polarization of the thermal radiation in the isolated neutron stars [29], from a level of a few per cent up to even 100 per cent, depending on the viewing geometry and the surface emission mechanism. Currently, this vacuum nonlinear electrodynamics prediction can be tested by measuring the polarization degree of isolated neutron star thermal emission.

The predictions for the high polarization degree of the pulsar radiation, caused by the vacuum birefringence were obtained in several papers. Systematic analysis for the thermal radiation can be found in [30, 31]. Multiwavelength description for the radiation coming from the rotation-powered pulsars was obtained in [32] and showed that the polarization degree of synchrotron radiation emitted from the outside of the light cylinder is less then 20%, while for the curvature radiation it is estimated, up to 40% — 60%. Most of the cited papers use the effective empirical approach which accounts that the electromagnetic radiation with the frequency $\omega$ at the distances $r < l_A = (90 \pi c/\alpha \omega)(B_q/B)^2$ (where $\alpha$ is a fine structure constant) propagates in magnetic field $B$ adiabatically i.e. its polarization instantaneously adapts to the change of the magnetic field strength. For the distances $l_B = B/|n \cdot \nabla B|$ (where $n$ is the unit vector at the propagation direction) the wave polarization can not exactly follow the variation of the magnetic field, and at the outer region $r \gg l_B$ polarization state become frozen. Accounting the region of the space where the wave polarization is changing substantially helps to reduce the calculations significantly. At the same time, such description needs auxiliary assumptions and agreements. In addition to the results of the listed above papers, we propose fully analytical approach, based on isotropic geodesic research in the effective space-time.

First observations [33] of the optical polarization from the thermally emitting isolated neutron star RX J1856.5-3754 have shown that the linear polarization degree degree of thermal emission is $16.43 \pm 5.26$ per cent in that star.

It is argued in [33] that, the no matter how the thermal photons are produced, such high value of the linear polarization in the signal is extremely unlikely to be reproduced by the models in which the QED vacuum polarization effects are not accounted for.

In this paper we present the results of calculations of other vacuum birefringence manifestation in high magnetic field. This calculations could be studied only by polarization measurements of the pulsed hard emission.

The observations only in the hard X and gamma rays are preferable to avoid the mentioned above polarization effects caused by the pulsar magnetosphere plasma influence, which are weaker for high energy photons.
The general covariance equations of nonlinear electrodynamics in post-Maxwellian approximation ($|\mathbf{E}| < B_q$, $|\mathbf{B}| < B_q$) have the form:

$$
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^n} \left\{ \sqrt{-g} Q^{mn} \right\} = -\frac{4\pi}{c} j^m, \quad (1.1)
$$

$$
\partial F_{mn} \partial x^k + \partial F_{nk} \partial x^m + \partial F_{km} \partial x^n = 0,
$$

where:

$$
Q^{mn} = \left[ 1 + \xi (\eta_1 - 2 \eta_2) J_2 \right] F^{mn} + 4\xi \eta_2 F^{mi} F_{ik} F^{kn}
$$

here notations were introduced, such as determinant of the metric tensor $g$ and $\xi = 1/B_{q}^2$.

The magnitude of the dimensionless post-Maxwellian parameters $\eta_1$ and $\eta_2$ depends on the choice of vacuum nonlinear electrodynamics model. In the Heisenberg-Euler nonlinear electrodynamics [34], which is the direct consequence of quantum electrodynamics, the parameters $\eta_1$ and $\eta_2$ have the values $\eta_1 = \alpha/(45\pi) = 5.1 \cdot 10^{-5}$, $\eta_2 = 7\alpha/(180\pi) = 9.0 \cdot 10^{-5}$; in the Born-Infeld theory [35] they are equal $\eta_1 = \eta_2 = a^2 B_q^2/4$, where for the constant $a^2$ only lower estimation is known $a^2 > 1.2 \cdot 10^{-32} \text{G}^{-2}$. The effects of the vacuum polarization in the strong electromagnetic fields were studied in [36], the birefringence and the photon splitting for uniform magnetic field were analyzed in [37].

The birefringence in vacuum and its influence on the emission spectrum and photon propagation in the neutron star magnetosphere were discussed in details in [38], where it has been presented that vacuum effects determine the polarization properties of normal modes of electromagnetic waves propagating in the vicinity of a neutron star. It has also been shown, that the magnetic vacuum effects can change the emission spectral parameters that should lead to the peculiarities in the energy spectra of X and gamma rays pulsars [39, 40].

In general, the description of polarized radiation transfer through the pulsar magnetosphere requires taking into account the different depolarisation effects, mostly caused by the magnetosphere plasma influence. The combination of vacuum and plasma birefringence can dramatically impact on the observed wave polarization, even at the low values of the local magnetic field. Joint consideration of these factors may be found in details in the papers [41–43]. However, as it has been noted at [29], for the radiation with the sufficiently high frequency, vacuum birefringence become more dominant and the plasma only negligibly affects on the wave propagation. The photon energy ranges for which plasma effects are suppressed by vacuum birefringence, depends on the plasma density $n_e$ and can be estimated as:

$$
E_\gamma > 0.035 \text{eV} \left( \frac{B}{10^{12} \text{G}} \right) \left( \frac{P}{1 \text{s}} \right) \left( \frac{n_{\text{GJ}}}{n_e} \right), \quad (1.2)
$$

where $B$ is the field induction, $P$ is rotation period, and $n_{\text{GJ}}$ — Goldreich-Julian value [44]. In the further, we assume the photon energy is sufficient to satisfy the vacuum dominant condition (1.2), so we can bypass the details about of the radiation source and depolarization effects coupled to magnetosphere plasma.

Nonlinear electrodynamics effects caused by the non-uniform and non-stationary magnetic field of a neutron star including beam bending in the dipole magnetic field, electron-positron pair production, photon splitting and modulation by the low-frequency electromagnetic wave scattering in magnetic field of inclined rotator are discussed in [45]. Quantum
electrodynamic effect in the accreting neutron stars, in particular, the one and two photon Compton scattering in the strong magnetic field and its acting on the emitting processes \cite{46–48} and the vacuum polarization effects in the field of charged compact object were also studied \cite{49}.

It should be noted, that pulsar also have a strong gravitational field, which is also can impact on the electromagnetic emission pulses. Because the gravitational field can only bend the trajectories of the electromagnetic beams \cite{50, 51}, and it does not change their polarization, we will neglect gravitational effects in this work.

The calculations and the modeling of nonlinear electrodynamic influence of pulsar magnetic field on the electromagnetic pulse polarization were made repeatedly in many particular cases and with the use of different coordinate systems \cite{28, 52, 53}. However, in these cases the coordinates were chosen from the reasons of convenience and simplicity of the analytical calculations, and often it was not appropriate for observational results interpretation. In this work we avoid this drawback and make the calculations of pulsar magnetic field influence on the electromagnetic pulse polarization in such a way to make it easier to interpret these effects in space experiments.

2 The law of hard emission pulse propagation in the pulsar magnetic field according to the vacuum nonlinear electrodynamics

Let us consider a pulsar with radius $R_N$, which magnetic dipole moment $\mathbf{M}$ rigidly connected with the pulsar “body”. Then, we suppose that the initial point of Cartesian system is placed into the pulsar center. We should direct mutually orthogonal axes $X$, $Y$ and $Z$ in such a way, that $Z$ axis will pass through the center of the Earth, and the directions of $X$, $Y$ will be specified later. Considering, that at some time $t_0$ the short X or gamma-ray burst with arbitrary polarization had occurred in the point $r_s = \{x_s, y_s, z_s\}$ of pulsar magnetosphere. This emission will propagate as pulses along the different rays in the different directions underwent nonlinear electrodynamic impact from pulsar magnetic field. Let us consider that detector on a satellite is placed in the point $r_d = \{x_d, y_d, z_d\}$ in the vicinity of the Earth, that allows to measure the hard emission pulse polarization.

We will calculate here the polarization state of pulse coming to detector. To study this process, which is interesting for us, there is no any necessity to solve the system of nonlinear differential equations in the partial derivatives of the first order (1.1). As it has been shown in \cite{54}, the propagation of the electromagnetic pulse in outer persistent electromagnetic field according to vacuum nonlinear electrodynamics may be described by the equations for isotropic geodesic line in an effective space-time with the metric tensor $G_{pm}^{(1, 2)}$:

\[
\frac{dk^i}{d\sigma} + G_{m}^{(1, 2)} \left[ \frac{\partial G_{mn}^{(1, 2)}}{\partial x^p} - \frac{1}{2} \frac{\partial G_{np}^{(1, 2)}}{\partial x^m} \right] k^n k^p = 0, \tag{2.1}
\]

where $\sigma$ is an affine parameter, $x^n = \{x^0 = ct, x^1 = x, x^2 = y, x^3 = z\}$, $k^n = dx^n/d\sigma$ is isotropic four-dimensional wave vector:

\[
G_{mi}^{(1, 2)} \frac{dx^m}{d\sigma} \frac{dx^i}{d\sigma} = 0. \tag{2.2}
\]

The metric tensor $G_{pm}^{(1, 2)}$ in post Maxwellian approximation of vacuum nonlinear electrodynamics (1.1) depends on the metric tensor $g_{lm}$ of pseudo Riemannian space-time, from the
outer electromagnetic field tensor $F_{ip}$ and from the polarization of considering electromagnetic wave [55]:

$$G^{(1,2)}_{mn} = \gamma^{mi} + 4\epsilon_{1,2}\xi F_{in} F_{n}^m,$$

$$G^{(1,2)}_{mn} = \gamma_{mn} - 4\epsilon_{1,2}\xi F_{nk} F_{-n}^k,$$

where index “1” refers to the first linearly polarized normal mode, and index “2” is refers to the second normal mode, which has the orthogonal polarization relatively to the first one. The indexes of the electromagnetic field tensor $F_{nk}$ in the relation (2.3) can be upped using the metric tensor of pseudo Riemannian space-time $\gamma^{mi}$. The tensor components $G^{(1,2)}_{(1,2)}$ and $G^{(1,2)}_{mn}$ with post Maxwellian accuracy are inversed to each other:

$$G^{(1,2)}_{(1,2)}G^{(1,2)}_{mn} = \delta_n^m + O(\epsilon^2 F_{ip} F_{pm} F_{mk} F_{kn}),$$

where $\delta_n^m$ is the Kronecker tensor.

The existence of two relations $G^{(1)}_{mn}$ and $G^{(2)}_{mn}$ or effective space-time metric tensor means that according to the vacuum nonlinear electrodynamics electromagnetic wave birefringence occurs, i.e. at $\eta_1 \neq \eta_2$ any electromagnetic wave splits onto the two normal modes with mutually orthogonal linear polarization, which pass in the outer field with non-equal velocities. As it has been shown in [53] the occurrence of effective space-time with the metric tensor $G^{(1,2)}_{mn}$, does not mean that Einstein Principe of Equivalence violets in the electrodynamics [56]. The presence of the second terms in the expression (2.3) means that not only gravitational field, described by the metric tensor $g_{nk}$, affects on the electromagnetic wave propagation, but outer electromagnetic field as well.

The equations (2.1)–(2.2) are easier to solve, by choosing as independent variable coordinate $z$, not the affine parameter $\sigma$. In this case we have from the equations (2.1):

$$\frac{d^2 x^0}{dz^2} = \left\{ G^{(1,2)}_{mn} \frac{dx^0}{dz} - G^{(1,2)}_{(1,2)} \right\} \left\{ \frac{\partial G^{(1,2)}_{mn}}{\partial x^p} - \frac{1}{2} \frac{\partial G^{(1,2)}_{mp}}{\partial x^m} \right\} \frac{dx^n}{dz} \frac{dx^p}{dz},$$

$$\frac{d^2 x}{dz^2} = \left\{ G^{(1,2)}_{mn} \frac{dx}{dz} - G^{(1,2)}_{(1,2)} \right\} \left\{ \frac{\partial G^{(1,2)}_{mn}}{\partial x^p} - \frac{1}{2} \frac{\partial G^{(1,2)}_{mp}}{\partial x^m} \right\} \frac{dx^n}{dz} \frac{dx^p}{dz},$$

$$\frac{d^2 y}{dz^2} = \left\{ G^{(1,2)}_{mn} \frac{dy}{dz} - G^{(1,2)}_{(1,2)} \right\} \left\{ \frac{\partial G^{(1,2)}_{mn}}{\partial x^p} - \frac{1}{2} \frac{\partial G^{(1,2)}_{mp}}{\partial x^m} \right\} \frac{dx^n}{dz} \frac{dx^p}{dz}.$$

The equation (2.2) in this case has the form:

$$G^{(1,2)}_{mn} \frac{dx^n}{dz} \frac{dx^m}{dz} = 0.$$

Because the relation (2.5) is the first integral of the equation (2.4), the equation system (2.4)–(2.5) is linearly dependent. Thus, one of the equations of this system can be omitted. It is easier to choose the first equation of system (2.4) as such one. These equations allow us to find the equation for rays $x = x(z)$, $y = y(z)$ and the law of electromagnetic pulse propagation along these rays $x^0 = ct = x^0(z)$. Because for each normal mode we are interested in only one ray along which pulse passes from the point $r = r_s$ to point $r = r_d$, we should to require that the relations

$$x^0(z_s) = ct_0, \quad x(z_s) = x_s,$n

$$y(z_s) = y_s, \quad x(z_d) = x_d, \quad y(z_d) = y_d.$$

$$\frac{d^2 x^0}{dz^2} = \left\{ G^{(1,2)}_{mn} \frac{dx^0}{dz} - G^{(1,2)}_{(1,2)} \right\} \left\{ \frac{\partial G^{(1,2)}_{mn}}{\partial x^p} - \frac{1}{2} \frac{\partial G^{(1,2)}_{mp}}{\partial x^m} \right\} \frac{dx^n}{dz} \frac{dx^p}{dz},$$

$$\frac{d^2 x}{dz^2} = \left\{ G^{(1,2)}_{mn} \frac{dx}{dz} - G^{(1,2)}_{(1,2)} \right\} \left\{ \frac{\partial G^{(1,2)}_{mn}}{\partial x^p} - \frac{1}{2} \frac{\partial G^{(1,2)}_{mp}}{\partial x^m} \right\} \frac{dx^n}{dz} \frac{dx^p}{dz},$$

$$\frac{d^2 y}{dz^2} = \left\{ G^{(1,2)}_{mn} \frac{dy}{dz} - G^{(1,2)}_{(1,2)} \right\} \left\{ \frac{\partial G^{(1,2)}_{mn}}{\partial x^p} - \frac{1}{2} \frac{\partial G^{(1,2)}_{mp}}{\partial x^m} \right\} \frac{dx^n}{dz} \frac{dx^p}{dz}.$$
should valid as the initial conditions to the equations (2.4)–(2.5). To solve the equations (2.4)–(2.5) with post-Maxwellian accuracy it is enough to take in Maxwellian approximation the pulsar dipole magnetic field induction vector $\mathbf{B}$ components:

$$
\mathbf{B} = \frac{3(M \mathbf{r}) \mathbf{r} - M r^2}{r^5}.
$$

It means that non-zero components of pulsar magnetic field tensor in relation (2.3) are:

$$
F_{21} = -F_{12} = B_z, F_{13} = -F_{31} = B_y, F_{32} = -F_{23} = B_x.
$$

(2.7)

The non-zero components of pseudo-Riemannian space-time metric tensor $g_{\mu\nu}$ if neglecting the gravitational field take a form:

$$
g_{00} = 1, g_{11} = g_{22} = g_{33} = -1. \quad (2.8)
$$

By substituting relations (2.7)–(2.8) into expressions (2.3), we find the explicit form of non-zero components of effective space-time metric tensor $G^{(1,2)}_{\mu\nu}$:

$$
G^{(1,2)}_{00} = 1,
$$

$$
G^{(1,2)}_{\alpha\beta} = -\delta_{\alpha\beta} \left( 1 + 4\xi\eta_{1,2} \left[ \frac{3(M \mathbf{r})^2}{r^8} + \frac{M^2}{r^6} \right] + 4\xi\eta_{1,2} \left[ \frac{9(M \mathbf{r})^2}{r^{10}} x_\alpha x_\beta + \frac{M_\alpha M_\beta}{r^6} - \frac{3(M \mathbf{r})}{r^8} (M_\alpha x_\beta + x_\alpha M_\beta) \right] \right),
$$

(2.9)

where $\alpha, \beta \ldots = 1, 2, 3, x_\alpha = \{-x, -y, -z\}, M_\alpha = \{-M_x, -M_y, -M_z\}$.

The detailed solution of the equations (2.4)–(2.5) with an effective space-time metric tensor, which are satisfied the initial conditions (2.6) is presented in appendix A.

Next, we find the propagation law for arbitrary electromagnetic pulse, let us describe the electromagnetic pulse passing $x^0 = x^0(z)$ along the ray connecting the points $\mathbf{r}_s$ and $\mathbf{r}_d$:

$$
t_{1,2}(z) = t_0 + \frac{(z - z_s)}{c} + \eta_{1,2} \xi \left[ \tilde{t}(z) - \tilde{t}(z_s) \right],
$$

where notation is introduced

$$
\tilde{t}(z) = \left[ 25(M_x x_s + M_y y_s)^2 + q^2 (16 M^2 - M_z^2) \right] \left[ \frac{z(3\rho^2 + 2q^2)}{64\rho^4 q^6} + \frac{3}{64q^4} \tan \left( \frac{z}{q} \right) \right] + \frac{1}{\rho^6 q^2} \left[ 5z(M_x x_s + M_y y_s)^2 + 16q^2 M_z (M_x x_s + M_y y_s) + 3M_z^2 q^2 z \right] + \frac{9}{4\rho^8} \left[ z(M_x x_s + M_y y_s)^2 - q^2 M_z (2M_x x_s + 2M_y y_s + M_z) \right].
$$

(2.10)

To study vacuum birefringence manifestation in pulsar magnetic field it is necessary to carry out polarization measurements of pulsed hard emission. We should consider the hard emission pulse of finite duration which occurs in the polar area due to the accretion of matter. In the area with strong magnetic field it splits into two modes polarized in mutually normal planes in such a way that their pulse fronts coincide at the initial time. Due to the vacuum nonlinear electrodynamics birefringence this pulse splits on two pulses with mutually orthogonal linear polarizations. These pulses propagate in the pulsar magnetic field with non-equal velocities. They have fronts, which coincided at the initial time. The front of faster
mode comes to the detector before, than the front of slow mode on a time interval equal \( \Delta t = |t_2(z_d) - t_1(z_d)| \). It means that during the time \( \Delta t \) only fast normal mode of a pulse will pass though the detector and it will detect the linear polarization at this part of the pulse.

After a time \( \Delta t \) the front of other normal pulse mode comes to the detector. Superposition of these normal modes in time produces in the detector emission with elliptic or circular polarization. Hence, according to the vacuum nonlinear electrodynamics the front part of any hard emission pulse with duration \( \Delta t \) coming from a pulsar should be linearly polarized in contrast to the rest of the pulse which in general case will have elliptic polarization.

Analogously, the faster mode pulse rear edge will leave the detector before slow mode pulse rear edge. Thus, polarization of the hard emission pulse rear part with duration \( \Delta t \) will be also linear, but orthogonal to the linear polarization of pulse front. This polarization effect indicates on that neutron star magnetic dipole fields are good objects for studying the vacuum nonlinear electrodynamics and verification of its predictions.

Thus, to test the predictions of vacuum nonlinear electrodynamics, it is necessary to control the polarization state of hard emission pulses coming from magnetars and pulsars on all their duration.

By using the expressions (A.3) and (A.13), we may to calculate the time \( \Delta t = |\Delta \tau| \):

\[
\Delta \tau = t_2(z_d) - t_1(z_d) = (\eta_2 - \eta_1)\xi \left\{ \frac{3\pi}{128q^7} - \frac{z_s(3\rho^2 + 2q^2)}{64r_s^4q^6} - \frac{3}{64q^7}\tan\left(\frac{z_s}{q}\right) \right\} \times (2.11)
\]

\[
\times \left[ 25(M_x x_s + M_y y_s)^2 + q^2(16M^2 - M_z^2) \right] - \frac{1}{8r_s^q q^9} \left[ 5z_s (M_x x_s + M_y y_s)^2 + 16q^2 M_z (M_x x_s + M_y y_s) + 3M_z^2 q^2 z_s \right] -
\]

\[
- \frac{9}{4r_s^2} \left[ z_s (M_x x_s + M_y y_s)^2 - q^2 M_z (2M_x x_s + 2M_y y_s + M_z z_s) \right],
\]

where \( r_s = \sqrt{x_s^2 + y_s^2 + z_s^2} \).

This follows from (2.11), that time interval \( \Delta t \), which characterizes the delay of signals, carried by electromagnetic waves polarized normally to each other, essentially depends on the difference of post-Maxwellian parameters \( \eta_1 - \eta_2 \). Thus, it is different in various models of vacuum nonlinear electrodynamics. In particular, in the Heisenberg-Euler electrodynamics this delay time may be about 1 mcs for typical pulsar, while in the Born-Infeld theory it is strictly equal to zero.

3 Observational possibilities of vacuum nonlinear electrodynamics effect

From the experimental point of view the effect of delaying of electromagnetic signals emitted from the poles of rotating neutron star will be revealed differently in the case of slow varying and burst-like or pulsed emission. In the case of slow varying emission the time dependence of detected intensity on one polarization mode will be shifted relatively to the time dependence of orthogonal mode intensity. This delay time will depend on the angle \( \beta \) between neutron star magnetic momentum vector and the radius-vector of the point of detector place. Lag effect will manifest itself in another way in the case of burst-like or pulsed emission (photon beam) which duration is higher than time \( \Delta t \), defined by expression (2.11).

In this case, if arbitrary polarized pulses are emitted, the detected pulses will have the variable polarization along their length, i.e. at \( \eta_1 > \eta_2 \) the pulse front part of duration \( T = \Delta t \) should be polarized normally to the neutron star magnetic meridian plane and the
other part of a pulse will be polarized randomly in general case. If we assume, that neutron star angular velocity $\Omega$ is sufficiently small, then the linear velocity of points on the neutron star surface will be much less than light velocity in vacuum. For this reason electromagnetic signal propagation time in the area $r \sim 5R_s$ of strong magnetic field, where the nonlinear electrodynamic and gravitational actions on these signals are most significant, will be much less than the star rotation period. Thus, the area of nonlinear electromagnetic action will be in the near field of magneto-dipole emission. It has been shown [57] that the lag time of signals with two main polarizations will be modulated due to the star rotation. It is well-known that time profiles of pulsar X- and gamma rays emission look like a sequence of pulses with period equal to the neutron star rotation period and pulse duration is determined mainly by the emission beam width.

In the case of very narrow beam the pulsar emission time profile can be presented as periodic pulses, which duration much less than period. At $\eta_1 > \eta_2$ it is obvious, that the pulse front will be polarized linearly, which plane is normal to the neutron star magnetic meridian. As it follows from (2.11) the duration of this polarized pulse front will change from zero to maximal value according to (2.11). Because the pulse duration $\tau$ is less than period $2\pi/\Omega$, the periodic change of intensity for time $\tau$ will not be significant, thus for a real analysis it is possible to assume that polarized part of a pulse is constant. Its value will depend on observation line direction relatively to the neutron star meridian plane.

It has been mentioned above that vacuum nonlinear electrodynamics effects, caused by the acting of very strong fields (including birefringence), are the same as nonlinear effects in a matter. In this case, the strong magnetic field might be considered as an effective matter. Thus, the emission parameters, determined by vacuum nonlinear electrodynamics effects, could be masked principally by nonlinear influence of a matter. In particular, due to the Faraday effect, the non-polarized emission, scattered on plasma electrons, becomes partially linearly polarized due to the inhomogeneous distribution of polarization plane rotation angles.

Such an effect has been analyzed in cases of light propagation in the magnetized stellar wind [58], strongly magnetized optically thick accretion disk [59], and magnetized cone shell, which can be considered as the model of relativistic jet [60]. However, nonlinear electrodynamics effects, caused by the matter influence, will dominate on the vacuum effects, mainly for photons from long wave bands of electromagnetic spectrum, i.e. optics and radio. Indeed, the polarization properties of matter are determined significantly by dielectric constant $\varepsilon$, for which the expression [61] is well-known:

$$\varepsilon = 1 - \frac{4\pi N_e \hbar^2 e^2}{m_e E^2}, \quad (3.1)$$

where $E_\gamma$ is the photon energy, $N_e$ is the electron concentration in plasma, $m_e$ is the electron mass, $\hbar$ is the Plank constant. If to substitute in eq. (3.1) the typical $N_e \sim 10^{19} \text{cm}^3$ then we obtain $\varepsilon - 1 = -10^{-11}$ for $E_\gamma = 0.1 \text{MeV}$.

Thus, the matter influence on the polarization properties of X ray and gamma emissions is pitifully. This conclusion is confirmed by digital calculations, which shows that polarization degree of emission after it had passed through magnetic plasma decreases with decreasing of its wave length [62].

The vacuum nonlinear effects also could be connected with some exotic processes, for example, with mutual transformation in magnetic field of photons and light Goldstone bosons (axions). Electromagnetic emission occurred from the axion decay in magnetic field can be highly polarized [63]. However, according to [63] axion input in dielectric constant is inversely
proportional to the square of the photon frequency (or energy) (compare with (3.1)). Thus, such processes can not be significant for the determination of polarization properties of the high energy photons.

To estimate the expected level of polarization degree as for pulsars, so for magnetars it is necessary, of-course, take into account the realistic values of energy spectrum parameters. However, in this paper we emphasize just on the pure vacuum nonlinear electrodynamics effects, which should be the most pure in X and gamma rays, for which polarization effects, caused by photon interaction with matter, could be neglected. Because such vacuum nonlinear electrodynamics effect as birefringence does not depend on the photon energy, we estimate below the possibility of its observation from only expected values of Crab-like source intensity in the different energy ranges without detailed modeling for different types of energy spectrum.

It has been shown above, nonlinear electrodynamics birefringence of X and gamma rays in the strong magnetic field of pulsars and magnetars can be revealed as about 1 mcs lag of two signal modes almost 100% polarized in mutually normal directions. To measure such lag experimentally, it is necessary to realize accurate polarization measurements, allowing to obtain the mean pulsation curve for the linearly polarized and non-polarized pulse components with fine time resolution (about 0.1 mcs). It is also necessary to measure with high accuracy (∼ 0.1 mcs) the arrival time of detected quanta during the all-time of observations.

This effect is valid for the photons propagated in strong magnetic field and its origin is not important. In the case of magnetars it implies the same prediction for quiescent state and for outbursts. However, the typical duration of magnetar outbursts is about a few or dozen milliseconds and corresponds to the emitting area about hundreds kilometres. This means, that the outburst emission, has been generated in the pulsar magnetosphere, where the magnetic field is weaker than near neutron star surface. On the other hand, it is assumed, that a periodic pulsation, observed in the quiescence state, has been caused by the emission from the polar areas, where the matter accretes along the magnetic field lines, i.e. near the surface. Therefore, it could be expected, that vacuum birefringence will be more significant in the case of pulsations in quiescent state and the estimations below were performed only for this case.

Make a final choice in favor of one or another experimental method, let us analyze the main factors, that determine the sensitivity of polarization measurements. For example, considering the Compton scattering technique as the most universal method of the polarization measurements in hard X ray and gamma rays astronomy. Let the total number of signal counts be $I$, and $N$ be the background (noise) counts. Then for expected signal to noise ratio expressed in number of standard deviation $\sigma$, for given exposure time, we have for the polarization measurements an expression:

$$\sigma = \frac{\mu IP}{\sqrt{2(I + N)}},$$

(3.2)

where $P$ is the polarization degree and $\mu$ is the instrumental polarization factor, in which source position on the sky is also taken into account.

According to (3.2), it is possible to obtain the estimation of minimally detectable polarization degree $P_{\text{min}}$. As it follows from (3.2), to increase the sensitivity it is necessary to minimize noise, increase the useful signal and increase the instrumental polarization factor $\mu$. The number of useful signal counts depends on the detected flux intensity $J$, which is
expressed in cm$^{-2}$s$^{-1}$ and equal to
\[ I = JS_{(\text{eff})}\Delta t, \tag{3.3} \]
where $S_{\text{eff}}$ is the effective detector area, $\delta t$ is the time set of signal statistics, which in the case of burst or pulse like signal is equal to the burst (pulse) duration and in the case of slow varying signals is determined by the exposure time.

Thus, it follows from (3.3), that for increasing the useful signal it is necessary maximally increase the detector effective area and the exposure time. Thus, the increasing of the detector area is inevitably linked to increasing of its mass and sizes. It is necessary to estimate the reasonable limits, which are defined of course by the source of observable luminosity.

To estimate the conditions for the detection of 100% polarization, i.e. $P_{\text{min}} = 1$, let us put $\mu = 1$ and neglect the noise. Then, no less than about 25 counts should be detected at each time interval (bin), on which polarization is measured to provide the 3$\sigma$ significance level. The time bin should be also no more than 1 mcs at least, to reveal the lag in about 1 mcs.

Then in the case of continuous signal from pulsar no less than $25T/(2(1\text{ mcs}))$ counts should be accumulated for pulsation period $T$. For simplicity, we assume that the pulsation profile is rectangular with pulse phase duration equal to the pulsation period $T$. If the intensity of the signal pulse component is $J$ (in cm-2s-1), then in the case of detector be with the effective area $S_{\text{eff}}$ for one pulsation period $J \circ T \circ S_{\text{eff}}/2$ counts will be detected. Then, to satisfy the condition of 100% polarization detection with 1 mcs time resolution, the following expression should be valid:
\[ \frac{25T}{(1\text{ mcs})} = JS_{\text{eff}}Tn, \tag{3.4} \]
where $n$ is the number of pulsation periods for a total exposure time.

Equality (3.4) can be rewritten as
\[ \frac{25T}{(1\text{ mcs})} = JS_{\text{eff}}\Delta t, \]
where $\Delta t$ is the total exposure time of a source.

It is well known, that the Crab pulsar is the most intensive in the different energy bands among the other pulsars. Its spectrum could be approximated very good in a wide energy range by the power law with power index $\sim 2$ [64, 65]. Then for the Crab pulsar intensity the following estimations could be obtained for the different energies: $J(E = 20–100\text{ keV}) = 4.6 \times 10^{-2}\text{ phot/cm}^2\text{/s}, J(E = 0.1–1.0\text{ MeV}) = 1.2 \times 10^{-2}\text{ phot/cm}^2\text{/s}$. Taking into account, that the Crab pulsation period is equal 33 ms, from (3.4) we obtain the estimations of factor $S_{\text{eff}}\Delta t$.

For the energy range 20–100 keV the effect can be detected in the case of 1 Crab intensity, if $S_{\text{eff}} = 10^3\text{ cm}^2$, $\Delta t \sim 100\text{ ks}$ and for 1 mCrab intensity, if $S_{\text{eff}} = 10^4\text{ cm}^2$, $\Delta t \sim 3\text{ Ms}$. For the energy range 0.1–1.0 MeV we have respectively if $S_{\text{eff}} = 10^4\text{ cm}^2$, $\Delta t \sim 100\text{ ks}$ for 1 crab and if $S_{\text{eff}} = 10^5\text{ cm}^2$, $\Delta t \sim 10\text{ Ms}$ for 1 mCrab. It is necessary to note, that an effective area is determined not only by the detector geometry area, but also by the efficiency of scattering or any other process, which is using for the polarization measurements. For the well-known polarization techniques, including the Compton polarimeters, the efficiency is no more than 10%, commonly it is about a few percent. Thus, the polarimeter geometry area should be taken at least on order more than the obtained above estimations of its effective area.
To choose the optimal observational method, it is necessary to analyze the main background factors in the different energy ranges. Generally, the detector background counts $N$ could be presented as a sum of parts, caused by natural or Galactic and Meta-galactic diffuse background $N_{\text{diff}}$ and intrinsic background of detector and satellite $N_{\text{int}}$. The possible signal to noise ratio (SNR) could be estimated from the pulsar energy spectrum, which can be typically taken as a Crab-like. The corresponding SNR values are presented in the table 1 in the relative units, in which SNR in the 2–20 keV range is taken equal to 1. These values are obtained for the instrument with FOV $2\pi$ sr, i.e. the intensities of all background components were multiplied on $2\pi$.

As it follows from the table 1, if taking into account only the natural background, the best SNR value is obtained for the ranges of soft (0.1–1.0 MeV) and high energy (0.1–0.5 GeV) gamma rays. However, it is necessary to note, that at the energies less than 0.1 MeV the Meta-Galactic diffuse background is dominating and its input in detector counts is proportional to the instrument FOV. While at energies $>0.1$ MeV the noise counts are determined mainly by the spacecraft and detector intrinsic background and weakly depends on FOV.

The SNR values, in which the contribution of intrinsic background has been taken into account, are also presented in the table 1. In this case, SNR has favorable values for the range of hard X rays and soft gamma rays. It can be explained by the energy spectrum of intrinsic background being extremely hard, (see, for ex. [66]) in presentation $E^{J}$ it can even grow with energy. Of course, the intrinsic background depends strongly on the spacecraft mass, which should be as low as possible, but such energy dependence of the intrinsic background intensity is valid for satellites.

## 4 Conclusions

Thus, we can conclude, that for research of vacuum nonlinear electrodynamics effect the optimal energy range is 0.05–1.0 MeV, which can be extended up to about 10 MeV. As for the factor $S_{\text{eff}} \Delta t$, it should be understood, that increasing of the instrument geometry area can’t be infinite due to the limited resources of space experiments. Obviously, the detector area about $10^{4}$ cm$^{2}$ is near that limit, which can be still implemented in the space experiment. Concerning the exposure time, it could take about all the time of the experiment, which can take years, in the case of constant orientation of the instrument on a source or in the case of monitor observations with wide FOV telescope.

Thus, just the long time monitor observations are necessary for searching and revealing of nonlinear electrodynamics effects. Considering the discussed above features it seems that using the monitor instruments, based on the Compton polarimeters, is the most realistic way to perform the search and observation of the vacuum nonlinear electrodynamics birefringence. The Compton polarimeters have a real advantage in view of SNR optimization, because

| Energy ranges, MeV | SNR  | $\frac{I}{N_{\text{diff}}}$ | $\frac{I}{(N_{\text{diff}}+N_{\text{int}})}$ |
|--------------------|------|-----------------------------|-----------------------------------------------|
| 0.002–0.001       | 0.05–1.0 | 1                           | 5                                            |
| 0.5–1.0           | 100–500    | 150                         | 6                                            |

Table 1. Signal to noise ratio of polarized fraction of pulsar emission in different energy range for different background models.
they are based on the detection of pair coincidences of incident and scattered photons, that allows to eliminate the instrument background effectively. Besides, such instruments give an opportunity to locate the source of detected gamma quanta by revealing of useful signals from the background cause by spacecraft and detector intrinsic noise and the atmosphere gamma rays (in the case of near-Earth observations). By this, the random coincidences are the most dangerous in limits of trigger time $\tau$ of those noise events, which can be detected independently in the diffuse and detector of scattered quanta. As it is well known, if the noise count $N$ is constant and equal in diffuser and detector, the count of random coincidences $M$ is equal $M = N^\tau$. Thus, the value $M$ can be made negligible in the case of very small time window $\tau$, that is in accordance with discussed above time resolution about 0.1 mcs.

Because in the Compton process quanta scatter presumably at 90° relative to the incidence direction and in the case of linear polarization they are scattered normally to the polarization plane, intensity of scattered quanta will be modulated harmonically on the Azimuth angle. The measurements by the Compton polarimeters mean the obtaining of histogram of coincidence pair distribution on the Azimuth angle and its approximation by the harmonic function. Then, this function amplitude can be used as the measure of polarization degree. The main worsening the instrument polarization capabilities factor is the Coulomb scattering, which decreases exponentially the instrument polarization factor and minimal detectable polarization degree respectively. For example, CGRO EGRET instrument, intended for observations in the range of high energy gamma rays scatter factor, decrease effectively the polarization factor in $10^{-4}$ times that made this experiment non-sensitive to polarization measurements.

To increase the efficiency of polarimetry measurements it is necessary to maximize the number of pair coincidences corresponding to the interactions in two neighbor detector pixels. For this, the radius of detector pixel should be about one mass absorption length $\lambda$. In the case of the most popular crystals for gamma quantum detection dense as the semiconductor as CdZnTe for example, or as scintillator LaBr$_3$, CeBr$_3$ and Ce:GAGG, the pixel radius should be chosen about 0.5–1.0 cm. Thus, the Compton polarimeter instrument which is useful for vacuum nonlinear electrodynamics birefringence observations. should be based on large area ($\sim 10^4$ cm$^2$) detector consists from small pixels of about 1 cm size.

One of the most prominent future gamma-ray polarimetry mission has been mentioned above — it is e-ASTROGAM. Its payload consists from silicon tracker, CsI(Tl) pixel calorimeter and the anticoincidence system. The linear polarization of the incident photon fluxes might be measured by both Compton scattering and pair creation. In the energy range 0.2–2 MeV the e-ASTROGAM will be able to achieve a Minimum Detectable Polarization (MDP) at the 99% confidence level as low as 0.7% for a Crab-like source in 1 Ms [67]. It means that for detection of 100% polarization at the 1 mcs pulse front, in the case of Crab-like source (in view of its pulse duration about dozens milliseconds), the exposure time must be about $10^2$ Ms.

It should be taken into account, that in the most Compton telescopes the reconstruction of the incident photon direction provides an uncertainty area which has the shape of a thin cone arc. However, there is an alternative way to use the gas TPC, that is also called Electron Tracking Compton Camera (ETCC), which provides the tracking of the recoil electron from the first Compton interaction with a measurement of the direction of the recoil momentum. It allows to decrease the length of the arc and therefore to improve dramatically the sensitivity of the detector [22], and references therein.

Some of these telescopes are sensitive to the photon energies up to tens of MeV in the Compton mode, but their sensitivity to polarisation above a few MeV is either nonexistent
or undocumented. The GPD technique, based on proportional counters, gives a good opportunity for X-ray polarimetry. For example, IXPE or XIPE missions provide 1% MDP in 1 300 ks for 10 mCrab sources in the 2–8 keV range [68]. It means that the similar exposure time just about 300 ks is needed for detection of 100% polarization at pulse front in the case of Crab-like source. Such exposure time is quite achievable in these missions. However, it necessary to note that in the soft X-ray range nonlinear vacuum electrodynamic polarization effects could be masked by the polarization due to photon scattering in the pulsar magnetosphere matter.

Wide-field gamma ray (0.02–3.0 MeV) telescope Gammascope, which is elaborated now at SINP MSU as part of Russian space program is able to realize the polarization measurements based on the Compton technique. This instrument is the position sensitive detector (PSD) with the coding mask of quasi-spherical (dodecahedron) configuration. It FOV of about $2\pi$ sr provides the continuous observation of a half of the sky during the space experiment.

The instrument should consist from six PSD modules placed on the bottom parts of the dodecahedron frame and from six coding mask panels from Wf or Ta placed on the top parts of the dodecahedron frame, i.e. the each PSD module has an opposite coding mask panel and all system observes half of the sky. Due to such configuration, the exposure time of each source in the instrument FOV will be about the all-time of experiment, i.e. few years. It is supposed, that PSD should consist of large number of cylindrical scintillator ($\text{CeBr}_3$ or Ce:GAGG) pixels of about 0.5 cm diameter and 2 cm height. The dense scintillator like CsI(Tl) or BGO will be used as an active shield. The total area in the compact configuration should be $\sim 10^3 \text{cm}^2$ and $\sim 10^4 \text{cm}^2$ in the optimal case.

The special mode of double coincidences has been foreseen for the polarization measurements. The combination of the large area and extremely long exposure time gives the good opportunity to search the fine polarization effects like vacuum nonlinear electrodynamics birefringence.

Based on the obtained results, we may conclude that the modern observation technique provides the good opportunities for the vacuum nonlinear electrodynamics effects detection, in particular this applies to the vacuum birefringence, in hard emission of pulsars and magnetars. However, it needs highly sophisticated polarization measurements with large area instruments during the years of continuous observations.

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A Detailed calculations

Let us start the solution of the system of equations (2.4)–(2.5) with the initial conditions (2.6). By using the relation (2.9), we obtain for the first integral (2.5):

\[
c^2 \left( \frac{dt}{dz} \right)^2 - \left( \frac{dr}{dz} \right)^2 \left\{ 1 + 4\xi \eta_{1.2} \left[ \frac{3(M \cdot r)^2}{r^8} + \frac{M^2}{r^6} \right] + 4\xi \eta_{1.2} \frac{3(M \cdot r)}{r^6} \left[ r \frac{dr}{dz} - \left( M \frac{dr}{dz} \right)^2 \right] \right\} = 0.
\]
Substituting (2.9) into the last two equations of system (6), we obtain:

\[
\frac{d^2 x}{dz^2} = \eta_{1,2} \left\{ \frac{180}{r^{12}} (M \ r)^2 x^2 z + \frac{12}{r^{10}} (M \ r) [9(M \ r) z - z(14xM_x + 8yM_y) + (3x^2 + 8y^2)M_z] + \frac{12}{r^8} [z(M^2 + M_z^2) - 9(M \ r) M_z + (3xM_x + yM_y)M_z] \right\} \left( \frac{dx}{dz} \right)^3 + \frac{360}{r^{12}} (M \ r)^2 xyz - \frac{24}{r^{10}} (M \ r) [3z(yM_x + xM_y) + 5xyM_z] + \frac{24}{r^8} M_z [xM_y + yM_x] \right\} \left( \frac{dy}{dz} \right)^2 + \frac{180}{r^{12}} (M \ r)^2 y^2 z + \frac{12}{r^{10}} (M \ r) [9(M \ r) z + (8x^2 + 3y^2)M_z - z(8xM_x + 14yM_y)] + \frac{12}{r^8} [z(M^2 + M_z^2) + (xM_x + 3yM_y)M_z - 9(M \ r) M_z] \right\} \left( \frac{dx}{dz} \right)^2 \left( \frac{dy}{dz} \right) + \frac{24}{r^{10}} (M \ r) [15(M \ r) y + 3(2x^2 + y^2)M_y + 8y(xM_x - zM_z)] - \frac{360}{r^{12}} (M \ r)^2 y(2x^2 + y^2) + 2(2x^2 + y^2) + \frac{12}{r^{10}} (M \ r) [21(M \ r) x + 25x^2 M_x + 8x(yM_y - zM_z) + 6y^2 M_x] - \frac{180}{r^{12}} (M \ r)^2 [3x^2 + 8y^2 - \frac{12}{r^8} [5(M \ r) M_x + x(4M_x^2 + M_y^2 - M_z^2) + \frac{12}{r^8} (M \ r)^2 xy^2 + \frac{12}{r^8} [(M \ r) M_x - x(2M_x^2 + M_y^2 + M_z^2) - \frac{12}{r^8} (M \ r)^2 M_y + z(M_x^2 - M_y^2 - 4M_z^2) \right\} \left( \frac{dx}{dz} \right)^2 + \frac{12}{r^{10}} (M \ r) [24z(M \ r) + (25x^2 + 19y^2)M_z + 8z(xM_x - yM_y)] - \frac{180}{r^{10}} (M \ r)^2 [3x^2 + 8y^2] - \frac{12}{r^8} [20(M \ r) M_z + (xM_x - yM_y)M_z + y(M_x^2 - M_y^2) - \frac{360}{r^{12}} (M \ r)^2 xy - \frac{24}{r^8} (yM_x + zM_y)M_z] \left( \frac{dy}{dz} \right) + \frac{180}{r^{12}} (M \ r)^2 x(x^2 + y^2) + \frac{12}{r^{10}} (M \ r) [(3x^2 - 5y^2)M_z - 24(M \ r) x + x(8yM_y + 14zM_z)] + \frac{12}{r^8} [6(M \ r) M_x - x(2M_x^2 + M_y^2 + M_z^2) - (yM_y + 3zM_z)M_z] \right\}.
\]

\[
\frac{d^2 y}{dz^2} = \eta_{1,2} \left\{ \frac{180}{r^{12}} (M \ r)^2 y^2 z + \frac{12}{r^{10}} (M \ r) [9(M \ r) z - z(14yM_y + 8xM_x) + (3y^2 + 8x^2)M_z] + \frac{12}{r^8} [z(M^2 + M_z^2) - 9(M \ r) M_z + (3yM_y + xM_x)M_z] \right\} \left( \frac{dy}{dz} \right)^3 + \frac{360}{r^{12}} (M \ r)^2 xyz - \frac{12}{r^{10}} (M \ r)^2 xy^2 - \frac{12}{r^8} [(M \ r) M_x - x(2M_x^2 + M_y^2 + M_z^2) - (yM_y + 3zM_z)M_z] \right\}.
\]
This, we substitute $x$. Therefore, the successive approximation is convenient method to solve these equations. For solution
\[
\frac{-24}{r^{10}} (\mathbf{M} \mathbf{r}) [3z(xM_y + yM_x) + 5xyM_z] + \frac{24}{r^8} M_z [xM_y + yM_x] \left( \frac{dy}{dz} \right)^2 \left( \frac{dx}{dz} \right) + \\
\frac{12}{r^{10}} (\mathbf{M} \mathbf{r})^2 x^2 z + \frac{12}{r^{10}} (\mathbf{M} \mathbf{r}) [9(\mathbf{M} \mathbf{r}) z + (8y^2 + 3x^2)M_x - z(8yM_y + 14xM_x)] + \\
\frac{12}{r^8} [z(M^2 + M_z^2) + (yM_y + 3xM_x)M_z - 9(\mathbf{M} \mathbf{r}) M_z] \left( \frac{dy}{dz} \right) \left( \frac{dx}{dz} \right)^2 + \\
\frac{24}{r^{10}} (\mathbf{M} \mathbf{r}) [15(\mathbf{M} \mathbf{r}) x + 3(2y^2 + x^2)M_x + 8x(yM_y - zM_z)] - \frac{360}{r^{12}} (\mathbf{M} \mathbf{r})^2 x(2y^2 + \\
x^2) + \frac{24}{r^8} [(zM_z - yM_y)M_x - 3(\mathbf{M} \mathbf{r})M_x + x(M^2 - M_z^2)] \left( \frac{dy}{dz} \right) \left( \frac{dx}{dz} \right) + \\
\frac{12}{r^{10}} (\mathbf{M} \mathbf{r}) [21(\mathbf{M} \mathbf{r}) y + 25y^2 M_y + 8y(xM_x - zM_z) + 6x^2 M_y] - \\
\frac{180}{r^{12}} (\mathbf{M} \mathbf{r})^2 y(3y^2 + 2x^2) - \frac{12}{r^8} [5(\mathbf{M} \mathbf{r}) M_y + y(4M_y^2 + M_x^2 - M_z^2) + \\
(xM_x - zM_z)M_y] \left( \frac{dy}{dz} \right)^2 + \left[ \frac{12}{r^{10}} (\mathbf{M} \mathbf{r}) [(8y^2 + 5x^2)M_y - 9(\mathbf{M} \mathbf{r}) y + y(14xM_x + \\
+ 8zM_z)] - \frac{180}{r^{12}} (\mathbf{M} \mathbf{r})^2 y^2 x^2 + \frac{12}{r^8} [(\mathbf{M} \mathbf{r}) M_y - y(2M_y^2 + M_x^2 + M_z^2) - \\
-(3xM_x + zM_z)M_y] \left( \frac{dx}{dz} \right)^2 + \left[ \frac{12}{r^{10}} (\mathbf{M} \mathbf{r}) [24z(\mathbf{M} \mathbf{r}) + (25y^2 + 19x^2)M_z + \\
+ 8z(yM_y - xM_x)] - \frac{180}{r^{12}} (\mathbf{M} \mathbf{r})^2 z(3y^2 + x^2) - \frac{12}{r^8} [20(\mathbf{M} \mathbf{r}) M_z + (yM_y - xM_x)M_z + \\
+ z(M_y^2 - M_x^2 - 4M_z^2)] \left( \frac{dy}{dz} \right) + \left[ \frac{24}{r^{10}} (\mathbf{M} \mathbf{r}) [3xyM_x + 3y^2 M_x + 5xzM_y - \\
- \frac{360}{r^{12}} (\mathbf{M} \mathbf{r})^2 x y z + \frac{24}{r^8} (xM_z + zM_x)M_y \left( \frac{dx}{dz} \right) + \frac{24}{r^{12}} (\mathbf{M} \mathbf{r})^2 (x^2 + y^2) + \\
+ \frac{12}{r^{10}} (\mathbf{M} \mathbf{r}) [(3y^2 - 5x^2)M_y - 24(\mathbf{M} \mathbf{r}) y + y(8xM_x + 14zM_z)] + \\
+ \frac{12}{r^8} [6(\mathbf{M} \mathbf{r}) M_y - y(2M_y^2 + M_x^2 + M_z^2) - (xM_x + 3zM_z)M_y] \right) \right] .
\]

The equations (A.1) and (A.2) are nonlinear, for which the conventional methods of solution [69, 70] are not applicable. However, they contain the small parameter $\xi \mathbf{M}^2 / r^6$. Therefore, the successive approximation is convenient method to solve these equations. For this, we substitute $x = x_{1,2}(z), y = y_{1,2}(z)$ and $t = t_{1,2}(z)$ in the form of expansions on the small parameter:
\[
t_{1,2}(z) = T(z) + \eta_{1,2} \xi \left[ \bar{t}(z) - \bar{t}(z_s) \right], \\
x_{1,2}(z) = X(z) + \eta_{1,2} \xi \left[ \bar{x}(z) - \bar{x}(z_d) + \frac{(z_d - z)}{(z_d - z_s)} \left[ \bar{x}(z_d) - \bar{x}(z_s) \right] \right], \\
y_{1,2}(z) = Y(z) + \eta_{1,2} \xi \left[ \bar{y}(z) - \bar{y}(z_s) + \frac{(z_d - z)}{(z_d - z_s)} \left[ \bar{y}(z_d) - \bar{y}(z_s) \right] \right],
\]
where $T(z), X(z), Y(z)$ are unknown functions of zero approximation, and $\bar{t}(z), \bar{x}(z), \bar{y}(z)$ are of the first approximation.
It follows from expressions (2.6), that the initial conditions for the functions in these expressions have the form:

\[ T(z) = ct_0, \quad X(z) = x_s, \quad Y(z) = y_s, \quad X(z_d) = x_d, \quad Y(z_d) = y_d. \]  
\[ \text{(A.4)} \]

In the Maxwellian approximation equations (A.1)–(A.2) give:

\[ c^2 \frac{d^2 T(z)}{dz^2} = \frac{d^2 X(z)}{dz^2} = \frac{d^2 Y(z)}{dz^2} = 0, \]
\[ c^2 \left( \frac{dT(z)}{dz} \right)^2 - \left( \frac{dX(z)}{dz} \right)^2 - \left( \frac{dY(z)}{dz} \right)^2 = 1. \]

It is easy to obtain the solution of these equations satisfying the initial conditions (A.4):

\[ T(z) = t_0 + \frac{N}{c} (z - z_s), \quad \mathbf{R} = \mathbf{r}_s + (z - z_s) \mathbf{N}, \]  
\[ \text{(A.5)} \]

where notations

\[ \mathbf{R} = \{X(z), Y(z), z\}, \quad N = |\mathbf{N}|, \]
\[ N_x = \frac{(x_d - x_s)}{(z_d - z_s)}, \quad N_y = \frac{(y_d - y_s)}{(z_d - z_s)}, \quad N_z = 1 \]

are used for the convenience of further calculations. With the use of expressions (A.5) equations (A.2) in the post-Maxwellian approximations have the form:

\[ \frac{d^2 z(x)}{dx^2} = \frac{1}{N_{12}} \left\{ \left[ \frac{180}{\rho_{12}}(\text{MR})^2 X^2 z + \frac{12N^2}{\rho_{10}}(\text{MR})[9(\text{MR})z - z(14XM_x + 8YM_y) + \right. \]
\[ + (3X^2 + 8Y^2)M_z] + \frac{12N^4}{\rho^6}[z(M^2 + M_z^2) - 9(\text{MR})M_z + (3XM_x + YM_y)M_z] \right\} N_z^2 + \frac{360}{\rho_{12}^2} (\text{MR})^2 XY z + \]
\[ + \frac{24N^4}{\rho_{10}} M_z[XM_y + YM_x] - \frac{24N^2}{\rho_{10}^2} (\text{MR})[3YzM_x + 3zM_x + 5XYM_z] \right\} N_y^2 + \frac{180}{\rho_{12}^2} (\text{MR})^2 Y^2 z + \]
\[ + \frac{12N^2}{\rho_{10}^2} (\text{MR})[9(\text{MR})z + (8X^2 + 3Y^2)M_z - z(8XM_x + 14YM_y)] + \frac{12N^4}{\rho^6}[z(M^2 + M_z^2) + \]
\[ + (XM_x + 3YM_y)M_z - 9(\text{MR})M_z \right\} N_x N_y + \frac{24N^2}{\rho_{10}^2} (\text{MR})[15(\text{MR})Y + 3(2X^2 + Y^2)M_y + \]
\[ + 8YM_z - M_y M_z] - \frac{360}{\rho_{12}^2} (\text{MR})^2 Y(2X^2 + Y^2) + \frac{24N^4}{\rho^6}[(zM_z - XM_y)M_y - 3(\text{MR})M_y + \]
\[ + Y(M_z^2 - M_y^2)] \right\} N_x N_y + \frac{12N^2}{\rho_{10}^2} (\text{MR})[21(\text{MR})X + 25XM_x + 14YM_y + 8X(YM_y - zM_z) - \]
\[ - \frac{180}{\rho_{12}^2} (\text{MR})^2 X(3X^2 + 2Y^2) - \frac{12N^4}{\rho^6}[5(\text{MR})M_x + XM_x (4M_x^2 + M_y^2 - M_z^2) + (YM_y - zM_z)M_z] \right\} N_x^2 + \]
\[ + \frac{12N^2}{\rho_{10}^2} (\text{MR})[(8X^2 + 5YM_y)X + X(14YM_y + 8zM_z)] - \frac{180}{\rho_{12}^2} (\text{MR})^2 XY^2 + \]
\[ + \frac{12N^4}{\rho^6} [(\text{MR})M_x - X(2M_x^2 + M_y^2 + M_z^2) - (3YM_y + zM_z)M_z] \right\} N_y^2 + \]
\[ + \frac{12N^2}{\rho_{10}^2} (\text{MR})[24z(\text{MR}) + (25X^2 + \]
\[ + 19Y^2)M_z + 8z(XM_x - YM_y)] - \frac{180}{\rho_{12}^2} (\text{MR})^2 z(3X^2 + Y^2) - \frac{12N^4}{\rho^6} [20(\text{MR})M_z + \]

\[ -16 - \]
\[ + (XM_x - Y M_y) M_z + 3(M_2^2 - M_y^2 - 4M_2^2) \right] N_x + \left[ \frac{24N^2}{\rho^{10}}(MR)[3YM_z + 3XM_y + 5Y zM_x] - \frac{360}{\rho^{12}}(MR)^2 X Y z - \frac{24N^4}{\rho^8}(YM_z + zM_y) M_z \right] N_y + \frac{180}{\rho^{10}}(MR)^2 X (X^2 + Y^2) + \mean{\frac{12N^4}{\rho^8}[(2Y^2 - 5Y^2)] M_x + 24(MR)X + 3(2Y^2 + X^2)M_x + 8X(Y M_y - zM_z) - \frac{360}{\rho^{12}}(MR)^2 X (2Y^2 + X^2) + \frac{24N^4}{\rho^8} (z M_z - Y M_y) M_z - 3(MR) M_x + X (M_2^2 - M_y^2) \right] N_y N_x + \left[ \frac{12N^2}{\rho^{10}}(MR)[21(MR) Y + 25Y^2 M_y + 6X^2 M_y + 8Y (X M_x - 8z M_z)] - \frac{180}{\rho^{12}}(MR)^2 Y (3Y^2 + 2X^2) - \frac{12N^4}{\rho^8} [5(MR) M_y + Y (4M_2^2 + M_y^2 - M_2^2)] + \frac{8z M_z)] - \frac{180}{\rho^{12}}(MR)^2 Y X^2 + \frac{12N^4}{\rho^8} [(MR) M_y - Y (2M_2^2 + M_y^2 + M_2^2) + (3X M_x + z M_z) M_y] N^2 + \left[ \frac{12N^4}{\rho^{10}}(MR)[24z (MR) + (25Y^2 + 19X^2) M_z + 8z (Y M_y - X M_x)] - \frac{180}{\rho^{12}}(MR)^2 z (3Y^2 + X^2) - \frac{12N^4}{\rho^8} [20(MR) M_z + (Y M_y - XM_x) M_z + z (M_2^2 - M_y^2 - 4M_2^2)] \right] N_y + \left[ \frac{24N^2}{\rho^{10}}(MR) \times \right. \left[ 3YM_z + 3Z M_z + M X z] - \frac{360}{\rho^{12}}(MR)^2 X Y z - \frac{24N^4}{\rho^8} (X M_x + z M_x) M_y \right] N_y + \left[ \frac{180}{\rho^{12}}(MR)^2 Y (X^2 + Y^2) + \frac{12N^2}{\rho^{10}}(MR)[3Y^2 + 5X^2] M_y - 24(MR) Y + Y (8X M_x + 14z M_z)] + \left. \frac{12N^4}{\rho^8} [6(MR) M_y - Y (2M_2^2 + M_2^2 + M_z^2) - (X M_x + 3z M_z) M_y] \right) \right\}, \]

where

\[
\rho = \sqrt{(z + p)^2 + q^2}, \quad X = x_s + N_x (z - z_s), \quad Y = y_s + N_y (z - z_s), \quad (A.8)
\]

\[
p = \frac{[(x_d - x_s)(x_s z_d - z_s x_d) + (y_d - y_s)(y_s z_d - z_s y_d)]}{(r_d - r_s)^2},
\]

\[
q^2 = \frac{(z_d - z_s)^2 [(x_d y_d - y_s x_d)^2 + (x_d z_d - z_s x_d)^2 + (y_d z_d - z_s y_d)^2]}{(r_d - r_s)^4}.
\]
The first integral (A.1) in this approximation gives the equation for determination of the function \( \tilde{t}(z) \):

\[
\begin{align*}
\frac{cN}{dz} \left\{ \frac{d\tilde{t}(z)}{dz} \right\} - 2\left[ N_x^2 + N_y^2 + 1 \right] & \left[ \frac{3(M R)^2}{N^8 \rho^8} + \frac{M^2}{N^6 \rho^6} \right] - \\
-N_x \frac{d\tilde{x}(z)}{dz} + N_y \frac{d\tilde{y}(z)}{dz} & + \frac{2}{N^6 \rho^6} \left[ \frac{3(M R)^2}{N^2 \rho^2} (R N) - (M N) \right] \right\}^2 = 0.
\end{align*}
\]

(A.9)

Before the integration of the equations (A.7) and (A.9), we should make some estimations. According to the statement of the problem, the source of the hard emission is on the neutron star surface or in its magnetosphere. Taking the neutron star radius equal to the 10 km, we may assume that \( \sqrt{x_s^2 + y_s^2 + z_s^2} < 10^2 \) km. Detector of hard emission is on the near-Earth satellite. Usually, two types of orbits are appropriate for such observations. First one is the low-altitude circular orbit, which lays mainly under the Earth radiation belts. Its radius no more than approximately 7 \cdot 10^3 \text{km}. The other type of an orbit is high-apogee elliptical one, on which satellite mainly is out of the Earth magnetosphere. The apogee altitude for such orbits (INEGAL space observatory for ex.) can be about 10^5 \text{km}. Therefore, we can use approximations \(|x_d| \leq 10^5 \text{km} \) and \(|y_d| \leq 10^5 \text{km} \) for coordinates \(x_d\) and \(y_d\).

The coordinate \( z_d \) value in the chosen coordinate system is the same as the distance from the pulsar to the Earth. Because the nearest pulsar is on the distance of about a few kps, i.e. \( \sim 10^{17} \text{km} \) from the Earth, we assume that \( z_d \sim 10^{17} \text{km} \). Substituting these values in the expressions (A.6), we obtain the following estimations: \( N_x \sim N_y \sim 10^{-12} \), \( N_z = 1 \). Therefore, we can simplify significantly the equations (A.7) and (A.9), keep there only asymptotically main terms in the expansion on the small parameters \( N_x \) and \( N_y \sim 10^{-12} \).

In this case it follows from the equations (A.5) and (A.8), that \( R = \{x_s, y_s, z\} \), \( X = x_s, Y = y_s, N_x = N_y = 0, N = 1, p = 0, q^2 = x_s^2 + y_s^2 \), \( \rho = \sqrt{x_s^2 + y_s^2} \). Thus, equations (A.7) have the form:

\[
\begin{align*}
\frac{d^2 \tilde{x}(z)}{dz^2} & = \frac{180}{\rho^{12}} (M R)^2 x_s (x_s^2 + y_s^2) + \frac{12}{\rho^{10}} (M R) \left(3x_s^2 - 5y_s^2\right)M_x - 24(M R) x_s + \\
& + x_s (8y_s M_y + 14z M_x) + \frac{12}{\rho^8} \left[ 6(M R) M_x - x_s (2M_x^2 + M_y^2 + M_z^2) - (y_s M_y + 3z M_z) M_x \right],
\end{align*}
\]

(A.10)

\[
\begin{align*}
\frac{d^2 \tilde{y}(z)}{dz^2} & = \frac{180}{\rho^{12}} (M R)^2 y_s (x_s^2 + y_s^2) + \frac{12}{\rho^{10}} (M R) \left(3y_s^2 - 5x_s^2\right)M_y - 24(M R) y_s + \\
& + y_s (8x_s M_x + 14z M_y) + \frac{12}{\rho^8} \left[ 6(M R) M_y - y_s (2M_y^2 + M_x^2 + M_z^2) - (x_s M_x + 3z M_z) M_y \right].
\end{align*}
\]

The first integral (A.9) also becomes more simple in this case

\[
\begin{align*}
\frac{c}{dz} \left\{ \frac{d\tilde{t}(z)}{dz} \right\} - 2\left[ \frac{3(M R)^2}{\rho^8} + \frac{M^2}{\rho^6} \right] & \left[ \frac{3(M R)}{\rho^2} z - M_z \right] \right\}^2 = 0.
\end{align*}
\]

(A.11)

To solve the equations (A.10), we obtain:

\[
\begin{align*}
\tilde{x}(z) & = \frac{3}{64q^3} \sqrt{\frac{z}{q}} \left[ 2M_x M_z q^4 - 175zx_s(M_x x_s + M_y y_s)^2 + \\
& + 10q^2(5M_z x_s - 3z M_x)(M_x x_s + M_y y_s) - 5zx_s q^2(16M_y^2 + 15M_z^2) + \\
& + 80y_s q^2 M_x M_y \right] + \frac{1}{64q^2 q^6} \left[ 30(5x_s z M_z + M_x q^2)(M_x x_s + M_y y_s) + \\
& + 5x_s q^2(16M_y^2 + 15M_z^2) - 175zx_s(M_x x_s + M_y y_s)^2 + \\
& + 10q^2(5M_z x_s - 3z M_x)(M_x x_s + M_y y_s) - 5zx_s q^2(16M_y^2 + 15M_z^2) + \\
& + 80y_s q^2 M_x M_y \right] \left[ \frac{3(M R)}{\rho^2} z - M_z \right]
\end{align*}
\]

(A.12)
\[+2q^2 M_x(3M_z - 40M_y y_s) + 5x_s q^2(16M_y^2 + 15M_x^2) + 175x_s(M_x x_s + M_y y_s) + \sqrt{\tfrac{1}{32\rho^4 q^4}} \left[ x_s q^2(16M_y^2 + 15M_x^2) + (6M_x q^2 + 50x_s z M_z) \right] \times (M_x x_s + M_y y_s) + 2q^2 M_x(M_z - 8M_y y_s) + 35x_s(M_x x_s + M_y y_s)] + \frac{5}{8\rho^2 q^2} \left[ x_s(M_x x_s + M_y y_s)^2 + q^2(2M_z M_x z - M_x^2 z) + 2(M_x q^2 + M_x x_s + M_y y_s) \right] - \frac{9x_s(M \cdot R)^2}{4\rho^8}.

\]

\[\tilde{y}(z) = \frac{3}{64q^6} \tan \left( \frac{z}{q} \right) \left[ 2M_y M_x q^4 - 175z y_s(M_x x_s + M_y y_s)^2 + 10q^2(5M_z y_s - 3M_y z)(M_x x_s + M_y y_s) - 5z y_s q^2(16M_x^2 + 15M_y^2) + 80x_s z q^2 M_x M_y \right] + \frac{1}{64q^4 q^6} \left[ 30(5M_z y_s + M_y q^2)(M_x x_s + M_y y_s) - 2q^2 M_x(3M_z - 40M_y x_s) + 5y_s q^2(16M_x^2 + 15M_y^2) + 175y_s(M_x x_s + M_y y_s) + M_y y_s)] + \frac{5}{8\rho^2 q^2} \left[ y_s(M_x x_s + M_y y_s)^2 + q^2(2M_z M_y y_s - M_x^2 y_s) + 2(M_y q^2 + M_z y_s)(M_x x_s + M_y y_s) \right] - \frac{9y_s(M \cdot R)^2}{4\rho^8}.

By integrating the equation (A.11), we find the explicit dependence \(\tilde{t}(z)\), i.e. the law of a hard emission pulse move along the beams from it common source to the detector:

\[\tilde{t}(z) = \left[ 25(M_x x_s + M_y y_s)^2 + q^2(16M^2 - M_x^2) \right] \left[ \frac{z(3q^2 + 2q^2)}{64q^4 q^6} + \frac{3}{64q^6} \tan \left( \frac{z}{q} \right) \right] + \frac{1}{8\rho^4 q^4} \left[ 5z(M_x x_s + M_y y_s)^2 + 16q^2 M_z(M_x x_s + M_y y_s) + 3M_x q^2 z \right] + \frac{9}{4\rho^8} \left[ z(M_x x_s + M_y y_s)^2 - q^2 M_z(2M_x x_s + 2M_y y_s + M_z) \right].\]

It is necessary to note, that the expressions (A.12) and (A.13) are the partial solutions of non-uniform equations (A.10) and (A.11). From the mathematics rules, they should be added by the general solutions of corresponding uniform equations, i.e. make replacement

\[\tilde{t}(z) \rightarrow \tilde{t}(z) + a_0, \quad \tilde{x}(z) \rightarrow \tilde{x}(z) + a_1 + b_1 z, \quad \tilde{y}(z) \rightarrow \tilde{y}(z) + a_2 + b_2 z,\]

where \(a_0, a_1, a_2, b_1, b_2\) are the arbitrary constants.

However, if to substitute these general solutions (A.10) and (A.11) in the expressions (A.3) then all constants \(a_0, a_1, a_2, b_1, b_2\) exactly cancel. Therefore, further we will use more simple expressions (A.12) and (A.13).
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