Interactions of a Single Goldstino

Taekoon Lee\textsuperscript{\(a,b\)} and Guo-Hong Wu\textsuperscript{\(a\)}

\textit{Department of Physics, Purdue University, West Lafayette, IN 47907\textsuperscript{(a)}}

\textit{Department of Physics, Seoul National University, Seoul, 151-742, Korea\textsuperscript{(b)}}

\textsuperscript{\(\dagger\)}Present address.

Abstract

The single goldstino interaction is given by the goldstino derivative coupling to the supercurrent. In an alternative description, the goldstino couples nonderivatively. We give a simple method to establish the equivalence of the two approaches, valid to all orders in perturbation theory, and for any scattering process involving an arbitrary number of particles, but with a single external goldstino. In the meantime, we find in the nonderivative form of the goldstino interaction a new quartic vertex that has been overlooked, and terms that were included incorrectly in the literature. The phenomenological implication of this new quartic operator is discussed.
Light gravitino is common in some models of supersymmetric extension of the standard model, as in gauge mediated SUSY breaking models [1] and no scale supergravity models [2]. Phenomenologically a light gravitino is interesting because it could be produced in systems with relatively low energies compared to the SUSY breaking scale. An observation of a gravitino emitting process in an accelerator experiment, for example, could determine a very important parameter, the SUSY breaking scale. It is thus important to understand the gravitino interaction with other fields, for example, those in the minimal supersymmetric standard model (MSSM).

The interaction of the helicity $\frac{1}{2}$ longitudinal component of the gravitino with matter becomes stronger as the gravitino mass gets smaller. In the small mass limit, which is valid if the energy in consideration is much bigger than the gravitino mass, one can replace using the SUSY version of the equivalence theorem [3, 4] the gravitino with the goldstino which was eaten by it. Then because the goldstino is the Goldstone fermion of spontaneous SUSY breaking, its coupling with other fields is determined by the well-known derivative coupling of goldstino to the supercurrent, much like the derivative coupling of pions in spontaneous chiral symmetry breaking.

On the other hand, if one works out in a given linearly realized SUSY model, one usually gets goldstino interactions in nonderivative form. In this formalism, the triple vertices of goldstino-boson-fermion are fixed by the goldstino Goldberg-Treiman relation [3, 5] in which the couplings are proportional to the mass splittings of the boson-fermion pairs.

The two different forms of goldstino coupling are expected to give identical amplitude in scatterings with a single external goldstino because the derivative coupling is part of the nonlinearly realized SUSY effective lagrangian [3, 4, 5, 6], which can, in principle, be obtained from the corresponding linearly realized SUSY model by field redefinition [6]. In practice, however, finding the field transformations can be quite involved [6], and to the best of our knowledge, there is no explicit proof of the equivalence of the two formulations of the goldstino interaction. In this letter, we give a simple method for establishing the equivalence of the derivative and nonderivative couplings of the goldstino, without using any
field redefinitions. Our method applies to any scattering process with an arbitrary number of external particles but with one external goldstino, and to all orders in perturbation. By using this method, one can identify the complete set of operators for the nonderivative goldstino couplings in any given model. As a result of this exercise, we find a quartic vertex that has not been discussed previously, and terms that were included incorrectly in the nonderivative formalism.

For simplicity, we apply our method to SUSY QED. However, the method can be straightforwardly used for more complex models, and later we will comment on the goldstino interaction with the MSSM fields. The SUSY QED we consider is comprised of a photon $A_\mu$, a photino $\chi$, with mass $m_\chi$, a complex scalar $\phi$ with mass $m_\phi$, and a massless Weyl fermion $\psi$. The interaction lagrangian is given by:

$$L_{\text{int}} = -eA_\mu \psi \sigma^\mu \bar{\psi} + ieA_\mu (\phi^* \partial^\mu \phi - \partial^\mu \phi^* \phi)$$

$$-\sqrt{2}e(\phi^* \psi \lambda + \phi \bar{\psi} \bar{\lambda}) - \frac{e^2}{2}(\phi^* \phi)^2 - e^2 A_\mu A^\mu \phi^* \phi$$ (1)

where $e$ is the $U(1)$ gauge coupling and $\psi$ and $\phi$ carry a unit charge. Throughout this paper we follow the convention for spinors and metric given in [10], except that our gaugino $\lambda$ is related to the gaugino $\lambda_{WB}$ of [10] by $\lambda = -i\lambda_{WB}$. Though we have chosen, for simplicity, a model with only one Weyl spinor, which is not anomaly free, our method applies to Dirac fermions as well. We will come back to this later.

The derivative coupling of the goldstino to the supercurrent is given by

$$L_D = \frac{1}{F} \partial_\mu \chi^\alpha J^{\mu}_\alpha + h.c.$$ (2)

where $\chi$ denotes the goldstino, $F$ is the goldstino decay constant, and $J^{\mu}_\alpha$ is the supercurrent,

$$J^{\mu}_\alpha = \sigma^\nu \sigma^\rho \psi D_\nu \phi^* - \frac{1}{2\sqrt{2}} \sigma^\nu \sigma^\rho \sigma^\mu \lambda F_{\nu\rho},$$ (3)

with

$$D_\mu \phi = (\partial_\mu + ieA_\mu)\phi, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$ (4)
In the nonderivative form of the goldstino interaction, though the triple vertices are fixed by the Goldberg-Treiman relation, the quartic vertex (or vertices) has not been thoroughly explored. However, our proof of the equivalence of the derivative and nonderivative descriptions of the single goldstino interaction gives the following nonderivative lagrangian,

$$L_{ND} = \frac{m^2}{F} \chi \psi \phi^* + \frac{im\lambda}{\sqrt{2}F} \chi \sigma^{\mu\nu} \lambda F_{\mu\nu} - \frac{em\lambda}{\sqrt{2}F} \phi^* \phi \chi \lambda + h.c. ,$$  \hspace{1cm} (5)

where the first two are the standard terms, while the quartic term will be justified later. We now prove that the lagrangians \((2)\) and \((3)\) give identical amplitude to an arbitrary order in gauge coupling for any scattering process involving a single external goldstino. For this purpose, let us consider the difference between the two lagrangians,

$$\delta L = L_D - L_{ND} = \overline{\mathcal{L}}_1 + \delta V_4 + \mathcal{L}_{\text{gauge}} + \text{(total derivative)} ,$$  \hspace{1cm} (6)

where

$$\overline{\mathcal{L}}_1 = \frac{1}{F}(\partial^2 - m^2_\phi)\phi^* \chi \psi - \frac{1}{F} \partial^2 \psi \chi \phi^*$$

$$- \frac{1}{2\sqrt{2}F}(\partial_\mu \lambda \bar{\sigma}^\mu + im\lambda)\sigma^\rho \bar{\sigma}^\nu \chi F_{\rho\nu} + \frac{1}{\sqrt{2}F}(\partial^2 g_{\mu\nu} - (1 - \frac{1}{\xi})\partial_\mu \partial_\nu)A^\nu \chi \sigma^\mu \lambda + h.c. ,$$  \hspace{1cm} (7)

and

$$\delta V_4 = -\frac{ie}{F} \partial_\mu \chi \sigma^\nu \bar{\sigma}^\mu \psi \phi^* A_\nu + \frac{em\lambda}{\sqrt{2}F} \phi^* \phi \chi \lambda + h.c. ,$$  \hspace{1cm} (8)

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{\sqrt{2}\xi F} \chi \sigma^\mu \bar{\lambda} \partial_\mu \partial_\nu A^\nu + h.c. .$$  \hspace{1cm} (9)

Here \(\xi\) is the gauge fixing parameter, and the corresponding photon propertor is given by

$$D_{\mu\nu}(q) = -\frac{i}{q^2} \left( g_{\mu\nu} - (1 - \xi)\frac{q_\mu q_\nu}{q^2} \right) .$$  \hspace{1cm} (10)

Note that each term in \(\overline{\mathcal{L}}_1\) is proportional to the free field equation, and thus vanishes on-shell, and \(\mathcal{L}_{\text{gauge}}\) is proportional to the divergence \(\partial_\mu A^\mu\).
To prove the equivalence, we need to show the following matrix element between arbitrary initial and final states vanishes,

\[ \delta S_{fi} = \langle f | T e^{i \int dx \mathcal{L}_{\text{int}}(x)} \int dx \delta \mathcal{L}(x) | i \rangle > \]

\[ = \langle f | T e^{i \int dx \mathcal{L}_{\text{int}}(x)} \left[ \int dx \mathcal{L}_1(x) + \int dx \delta V_4(x) \right] | i \rangle > \]

\[ = 0, \]  \hspace{1cm} (11) \]

where \( T \) denotes the time-ordered product. Note that here we ignored \( \mathcal{L}_{\text{gauge}} \) term because its contribution to \( \delta S_{fi} \) vanishes due to the Ward identity. Now because it vanishes when a field in \( \mathcal{L}_1 \), proportional to the free field equation, contracts with an external particle, nonvanishing term can arise only when the field contracts to become an internal line in a Feynman diagram. And because of the free field equation, when such a field contracts, it generates a local operator. This implies that the difference in amplitude between the two formalism is given by local operator insertions. We will show that the sum of such local operator insertions generated by \( \mathcal{L}_1 \) vanishes when combined with \( \delta V_4 \).

Consider

\[ \langle f | T e^{i \int dx \mathcal{L}_{\text{int}}(x)} \int dx \mathcal{L}_1(x') | i \rangle > \]

\[ = \langle f | T \sum_{n=0}^{\infty} \frac{[i \int dx \mathcal{L}_{\text{int}}(x)]^n}{n!} \int dx \mathcal{L}_1(x') | i \rangle > \]

\[ = \langle f | T \sum_{n=1}^{\infty} \frac{[i \int dx \mathcal{L}_{\text{int}}(x)]^{n-1}}{(n-1)!} \int dx dx' \ll i \mathcal{L}_{\text{int}}(x), \mathcal{L}_1(x') \gg | i \rangle > \]

\[ = \langle f | T e^{i \int dx \mathcal{L}_{\text{int}}(x)} \int dx dx' \ll i \mathcal{L}_{\text{int}}(x), \mathcal{L}_1(x') \gg | i \rangle >, \]  \hspace{1cm} (12) \]

where \( \ll \mathcal{O}_1(x), \mathcal{O}_2(x') \gg \) is a local operator obtained by contracting the field in \( \mathcal{O}_2 \) that is proportional to the free field equation with the corresponding one in \( \mathcal{O}_1 \). For example, if

\[ \mathcal{O}_1(x) = \bar{\psi} \psi(x) \quad \mathcal{O}_2(x') = (\partial^2 - m_\phi^2)\phi^* \chi \psi(x'), \]  \hspace{1cm} (13) \]

then

\[ \ll \mathcal{O}_1(x), \mathcal{O}_2(x') \gg = \bar{\lambda} \psi(x) \phi(x)(\partial^2 - m_\phi^2)\phi^* (x') \chi \psi(x') \]

\[ = i \bar{\lambda} \psi \chi (x) \delta(x - x'). \]  \hspace{1cm} (14)
Using (1) and (7), we get (see Fig.1)

$$\int dx dx' \ll i\mathcal{L}_{\text{int}}(x), \mathcal{L}_1(x') \gg = \int dx (\mathcal{L}_2(x) - \delta V_4(x) + \delta V_5(x)) \ , \quad (15)$$

where

$$\delta V_5 = \frac{\epsilon^2}{F} \phi^* \phi \sigma^\mu \lambda + \sqrt{2}\epsilon^2 F A_\mu \phi^* \phi \chi \sigma^\mu \lambda + \frac{\epsilon^2}{F} A_\mu A^\mu \phi^* \phi \chi + \text{h.c.} \ , \quad (16)$$

and

$$\mathcal{L}_2 = \frac{i\epsilon}{\sqrt{2}F} \phi^* \phi \chi (\sigma^\mu \partial_\mu \lambda - \text{im}_\lambda \lambda) + \frac{i\epsilon \phi}{F} \partial_\lambda \psi \sigma^\nu \partial^\mu \chi \phi^* A_\mu + \text{h.c.} \ . \quad (17)$$

As with $\mathcal{L}_1$, every term in $\mathcal{L}_2$ is proportional to a free field equation, and thus vanishes on-shell. Using (11), (12) and (15),

$$\delta S_{f_i} = \langle f | T e^{i} \int dx \mathcal{L}_{\text{int}} \left[ \int dx \mathcal{L}_2(x) + \int dx \delta V_5(x) \right] | i \rangle \ . \quad (18)$$
Figure 2: Local operators generated by the contraction $\ll i\mathcal{L}_{\text{int}}(x),\overline{\mathcal{L}}_2(x') \gg$ (see Eq. (20)). The solid, dashed, and wavy lines denote fermions, scalars, and gauge bosons respectively. Double lines denote fields proportional to their respective free field equations.

Now repeating the same manipulation with $\overline{\mathcal{L}}_2$ we have (see Fig.2)

$$
\delta S_{fi} = <f|Te^i\int dx\mathcal{L}_{\text{int}} \left[ \int dx' \ll i\mathcal{L}_{\text{int}}(x),\overline{\mathcal{L}}_2(x') \gg + \int dx\delta V_5(x) \right]|i> = 0 , \tag{19}
$$

because

$$
\ll i\mathcal{L}_{\text{int}}(x),\overline{\mathcal{L}}_2(x') \gg = -\delta V_5(x)\delta(x-x') . \tag{20}
$$

This completes the proof. We have thus shown that the difference in amplitude is either given by contractions of a field that is proportional to its free field equation with an external particle, or insertions of null operators, or both. In any case, it vanishes. Note that the proof is not only true at tree level, but also in any order of loop expansion because the steps we have taken through (11)–(20) are applicable with a given regularization of loop divergences.

Thus far we have only considered a model with a Weyl fermion. For SUSY QED with a Dirac fermion with mass $m_\psi$, the derivative coupling of the goldstino to the supercurrent can be easily obtained from (2) and (3) with $\psi \rightarrow \psi_i$ and $\phi \rightarrow \phi_i$, where $i = +, -$ denotes the
charges of the fields. Using the above method, the non-derivative coupling of the goldstino is found to be,

\[ \mathcal{L}_{\text{ND}} = \frac{m_{\phi}^2 - m_{\psi}^2}{F} \chi \psi_i \phi_i^* + \frac{im_{\lambda}}{\sqrt{2}F} \chi \sigma^{\mu\nu} \lambda F_{\mu\nu} - \frac{em_{\lambda}}{\sqrt{2}F} \chi \lambda (\phi_+^* \phi_- - \phi_-^* \phi_+) + \text{h.c.} , \]  

(21)

where summation over \( i \) is implied and \( \phi_+ \) and \( \phi_- \) are taken to be mass eigenstates.

The above analysis can be straightforwardly generalized to nonabelian gauge theories. The nonderivative goldstino coupling for SUSY QCD with one quark flavor is given by,

\[ \mathcal{L}_{\text{QCD ND}}^{\text{QCD}} = \frac{m_{\phi}^2 - m_{\psi}^2}{F} \chi \psi \phi_i^* + \frac{im_{\lambda}}{\sqrt{2}F} \chi \sigma^{\mu\nu} \lambda_a F_{\mu\nu}^a - \frac{gm_{\lambda}}{\sqrt{2}F} \chi \lambda_a \phi_i^* T^a_{ij} \phi_j + \text{h.c.} , \]  

(22)

where \( g \) is the gauge coupling and \( T^a \) are the generators of the gauge group. Note that the presence of the third term is again required.

Now a comment is in order on the nonderivative form of the goldstino interactions (3), (21), and (22). Without the quartic vertex, the equivalence would have failed, and so it must be included in the nonderivative form. The nonabelian version

\[ - \frac{gm_{\lambda}}{\sqrt{2}F} \chi \lambda_a \phi_i^* T^a_{ij} \phi_j , \]  

(23)

should be present in the nonderivative form of goldstino interaction with the MSSM fields, but was missing in Ref. 13, and was overlooked in Ref. 11. Since this term must exist model-independently, it is convincing to know that the model discussed in Ref. 14 does have the quartic vertex with the right coefficient. Also note the absence of \( A_\mu \chi \psi \phi^* \) vertex in the nonderivative form, while it exists in the derivative form. Terms of this type in the derivative form in MSSM were mistakenly included in the nonderivative goldstino coupling in Ref. 13 ( (e),(f),(e’), and (f’) of Table 1 in the reference). We also note that existence of this term in the nonderivative form of goldstino interaction would violate the gauge symmetry, and thus should not be allowed. The goldstino phenomenology studied in Ref. 13 is, however, not dependent on the quartic term (23).

The operator (23) contributes directly to the goldstino emission rate through \( \phi_i + \phi_j^* \to \chi + \lambda^a \) and \( \phi_i + \lambda^a \to \chi + \phi_j \), where \( \phi_i \) and \( \lambda^a \) stand for a squark and a gluino respectively.
These two are part of the goldstino production processes considered in Ref. [1] to derive the constraint on the light gravitino mass from cosmology. To see the effect of this quartic operator, we have computed the cross sections for these two processes in the limit when the center of mass energy is much bigger than the squark and gluino masses, as considered in Ref. [1]. In this limit, only the dimension-5 operators in (22) need be considered.

For $\phi_i + \phi_j^* \to \chi + \lambda^a$, we find that the $s$-channel gluon exchange contribution to the cross section is $(g^2 m_\lambda^2 / 48 \pi F^2) T_{ji}^a T_{ji}^{a*}$, in agreement with the result of Ref. [1]. The quartic term contributes $(g^2 m_\lambda^2 / 16 \pi F^2) T_{ji}^a T_{ji}^{a*}$, which is three times as large. The interference term vanishes identically, and the total cross section is simply given by

$$\sigma(\phi_i + \phi_j^* \to \chi + \lambda^a) = \frac{g^2 m_\lambda^2}{12 \pi F^2} T_{ji}^a T_{ji}^{a*},$$

which is 4 times of that given in Ref. [1].

For the process $\phi_i + \lambda^a \to \chi + \phi_j$, the $t$-channel gluon exchange contribution to the cross section depends on the cut imposed on the scattering angle, as obtained in Ref. [1]. We find that the quartic term contribution is given by $(g^2 m_\lambda^2 / 64 \pi F^2) T_{ji}^a T_{ji}^{a*}$, which is small compared to the gluon exchange contribution for the angular cut used in Ref. [1].

The effect of the quartic operator on the total cross section for goldstino production after summing over all processes is expected to be at the few per cent level, depending on the angular cut used. The correction to the cosmological bound on the light gravitino mass is correspondingly small.

Before we conclude, we would like to remark that although both the derivative and the nonderivative forms of goldstino coupling have been used for over two decades, an explicit proof of the equivalence of the two formulations has not been available to the best of our knowledge. In fact, it seems that some discrepancies exist regarding the complete set of operators in the nonderivative form[11, 13]. In this letter, we have given the complete set of operators associated with the single goldstino nonderivative coupling in SUSY QED and QCD. In particular, we point out one quartic operator that has been missing in many discussions of goldstino phenomenology. More generally, it is important to have a simple and
straightforward method available to identify the complete set of operators for the nonderivative goldstino interaction in any given model, and we hope that our method fits into this category.

We thank T. Clark and S. Love for useful conversations. This work was supported in part by the U.S. Department of Energy under grant DE-FG02-91ER40681 (task B). Also one of the authors (T. L.) is supported in part by the KOSEF brain pool program.
References

[1] M. Dine and A.E. Nelson, Phys. Rev. D 48 (1993) 1277; M. Dine, A.E. Nelson, and Y. Shirman, ibid. 51 (1995) 1362; M. Dine, A.E. Nelson, and Y. Shirman, ibid. 53 (1996) 2658.

[2] J. Ellis, K. Enqvist, and D.V. Nanopoulos, Phys. Lett. B 147 (1984) 99.

[3] P. Fayet, Phys. Lett. B 70 (1977) 461; Phys. Lett. B 86 (1979) 272.

[4] R. Casabuoni, S. De Curtis, D. Dominici, F. Feruglio, and R. Gatto, Phys. Lett. B 215 (1988) 313.

[5] T.E. Clark and S.T. Love, Phys. Rev. D 54 (1998) 5723.

[6] E.A. Ivanov and A.A. Kapustnikov, J. Phys. A 11 (1978) 2375; J. Phys. G 8 (1982) 167.

[7] S. Samuel and J. Wess, Nucl. Phys. B 221 (1983) 153.

[8] T.E. Clark, T. Lee, S.T. Love, and G.-H. Wu, hep-ph/9712353, Phys. Rev. D 57 (1998) 5912.

[9] M.A. Luty and E. Ponton, Phys. Rev. D 57 (1998) 4167.

[10] J. Wess and J. Bagger, Supersymmetry and Supergavity, 2nd ed. (Princeton University Press, Princeton, NJ, 1992).

[11] T. Moroi, H. Murayama, and M. Yamaguchi, Phys. Lett. B 303 (1993) 289; T. Moroi, Ph.D. thesis, Effects of the gravitino on the inflationary universe, hep-ph/9503210.

[12] A. Brignole, F. Feruglio, and F. Zwirner, Nucl. Phys. B 501 (1997) 332.

[13] J. Kim, J.L. Lopez, D.V. Nanopoulos, R. Rangarajan and A. Zichichi, Phys. Rev. D 57 (1998) 373.