Multi-criteria optimization of chassis parameters of Nissan 200 SX for drifting competitions

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Abstract. The work objective is to increase performance of Nissan 200sx S13 prepared for a quasi-static state of drifting on a circular path with given constant radius (R=15 m) and tyre-road friction coefficient (µ=0.9). First, a high fidelity “miMA” multibody model of the vehicle is formulated. Then, a multicriteria optimization problem is solved with one of the goals to maximize a stable drift angle (β) of the vehicle. The decision variables contain 11 parameters of the vehicle chassis (describing the wheel suspension stiffness and geometry) and 2 parameters responsible for a driver steering and accelerator actions, that control this extreme closed-loop manoeuvre. The optimized chassis setup results in the drift angle increase by 14% from 35 to 40 deg.

1. Introduction
The paper deals with chassis parameters optimization of Nissan 200sx S13 (figure 1) with Rear Wheel Drive and 300hp engine prepared for a drifting competition [1], where not a lap time, like in most of motorsport disciplines, but the biggest drift (side slip) angles are evaluated.

Figure 1. Nissan 200sx S13 while drifting on R-const test.
An oversteering without a suitable driver intervention is unstable mode of a vehicle motion. There are a few great papers [2, 3, 4, 5] describing dynamic behaviour of drifting vehicles, but focused on a control theory standpoint and with rather basic vehicle-driver models utilized. This kind of extreme motion states are not covered in classical vehicle dynamics [6, 7, 8].

Possibilities of effective initiation, sustaining and decay of extreme drift angles depend on a drift-vehicle chassis setup. Typically, this setup is selected based on many track tests with trial and error methods. There are, to the author’s knowledge, no published papers dealing with a computer aided selection method of the best drift vehicle parameters.

In order to improve this expensive process a model (called “miMA”) of driver-vehicle-road system adapted for multi-criteria optimization problems has been formulated by the author in Matlab environment (figure 2).

“miMA” software is characterized by:

- multibody spatial model of a vehicle with hundreds of discrete parameters specialized in motorsport, like off-road [9], racing [10, 11], rally [12], drifting [13] or karting;
- effective code yielding proper balance between model accuracy and computation time;
- combined optimization of driver actions for closed-loop manoeuvres, vehicle chassis and motion trajectory;
- implemented genetic algorithm which enable global search of nonlinear and multi-criteria tasks;
- substitution of actual driver actions by additional optimization variables, what emulates driver adaptation without using oversimplified driver models.

Figure 2. „miMA” model of driver-vehicle-road system used for optimisation tasks in motorsport.
In this work the given drift-vehicle (figure 1) has to follow R-const (R=15m) curve (figure 3), in a quasi-static fashion, with the biggest possible drift angle ($\beta_{\text{max}}$) on even and isotropic road with tyre-road friction coefficient $\mu=0.9$. The best setup of the drift-vehicle chassis is sought, together with the driver actions needed for stabilization of unstable oversteering motion. This problem is solved by using an innovative definition of driver-vehicle system optimization (miMA).

2. Definition of “miMA” model
The model of driver-vehicle-road system (figure 3) adapted for optimisation problems is formulated by the author in Matlab environment. “miMA” is characterized by deep specialization in motorsport applications, e.g. rally [8], race [13], drifting [11] and off-road [12]. The vehicle model relates its design parameters ($p$) and driver actions ($\delta$) with the vehicle dynamic characteristics, which can be derived based on its motion states ($\zeta$). A multibody dynamic model with discrete parameters is described by system of nonlinear ordinary differential equations (ODE). Special formulation of closed form solutions of the wheel suspensions makes it possible to avoid time consuming differential algebraic equations (DAE).

The virtual racing driver (figure 3) has to guide a vehicle through a desired path, stabilize it and seek a maximum performance from a vehicle on a specific road surface. The driver actions are described by:

$$\delta = [\delta_h, \delta_b, \delta_a, \delta_c, \delta_g, \delta_e]^T$$

where: $\delta_h$ – steering wheel angle [deg]; $\delta_b$ – normalized position of brake pedal ($\delta_b \in (0,1)$); $\delta_a$ – norm. position of accelerator pedal ($\delta_a \in (0,1)$); $\delta_c$ – norm. position of clutch ($\delta_c \in (0,1)$); $\delta_g$ – gear shift position ($\delta_g \in \{-1,0,1,2,\ldots\}$); $\delta_e$ – normalized position of e-brake ($\delta_e \in (0,1)$).

The driver actions (1) are based on observations (figure 3) of selected vehicle motion states ($\zeta$) and visual perception. The road-environment model includes descriptions of road profiles ($h$), friction potential ($\mu$), wind velocity ($v_w$) and ambient air temperature ($T_a$) and road surface temperature ($T_r$).
3. “miMA” model of Nissan 200 sx S13

The numerical example considers Nissan 200sx S13 (figure 1) with: Rear Wheel Drive, 300 horsepower engine, 1250 kg curb weight (50/50%), limited slip differential and performance tires [1].

The nonlinear multibody vehicle model is presented in figure 4. Main components of “miMA” model are defined in Table 1. The model, described by 26 generalized coordinates \((q)\) and 440 discrete parameters, is suitable for dynamic analyses in a frequency range up to 20 Hz.

The wheel suspension mechanisms, i.e. a McPherson strut with trailing rack&pinion steering system in the front axle and a multi-link in the rear axle, are described by spatial kineto-static models [4]. Magic Formula with the first order lag dynamics [5] is implemented as tyre model.

| Model parts    | Generalized coordinates \((q)\) | Description                                      |
|----------------|----------------------------------|--------------------------------------------------|
| Car body       | \(6 (x, y, z, \phi, \theta, \psi)\) | Position and orientation of rigid body           |
| Wheels         | \(4 (\phi_1, \phi_2, \phi_3, \phi_4)\) | Rotation about wheel bearings                     |
| Suspensions    | \(4 (z_1, \ldots, z_4)\)          | Bounce motion                                    |
| Tyres          | \(4+4 (F_{x1}, \ldots, F_{x4}, \ldots, F_{y1}, \ldots, F_{y4})\) | First-order dynamics of tire horizontal forces |
| Steering sys.  | \(1 (\phi_s)\)                   | Compliance of steering shaft                      |
| Powertrain     | \(1+1+1 (M_r, \phi_m, \phi_w)\)  | Engine torque + differential + shafts compliance |
| SUM            | 26                               |                                                   |
In order to limit the paper scope, it is assumed that: the road is even and isotropic \( (h = 0, \mu = \text{const}) \), temperature effects of tyres, dampers and brakes are neglected. More on tyre temperature influence on a race car performance one may find in [10].

Most of “miMA” model parameters are estimated on the basis of in&outdoor experiments by utilisation of the Institute test rigs and measuring apparatus. Characteristics of the wheels suspension springs (coil springs, bumpers, anti-roll bars) and dampers (figure 4) are determined on the basis of test rig measurements and described as passive elements with nonlinear characteristics.

Major dimensions of the actual suspension mechanisms in Nissan 200sx S13 are measured directly. Those difficult to measure are identified by comparison of kinematic characteristics obtained from measurements and simulations [14]. The test rig utilising a wire based platform [15] for determination of wheel suspensions kinematic characteristics is presented in figure 5.

The considered vehicle was instrumented (figure 6) in order to verify the simulation model on the basis of the measured motion states.

**Figure 5.** Front axle of Nissan 200sx S13 on test rig for kinematic characteristics determination by using wire based platform.

**Figure 6.** Acquisition apparatus in the vehicle cockpit and optical measurement of the vehicle horizontal velocity.
Sample results of measurements and simulations for the considered R-const (figure 3, R =15 m, \( \mu = 0.9 \)) test are presented on a handling diagram in figure 7. An expert drift driver starts in an understeer state, where the steering wheel angle (\( \delta_h \)) increases in proportion to the central acceleration (\( a_c \)), until the front axle saturation at \( a_c = 9 \text{ m/s}^2 \). Next, an oversteer state is generated by excessive application of the engine power. The drift angle (\( \beta \)) increases to 35 deg, what requires a countersteer of \( \delta_h = -550 \text{ deg} \) to continue R-const trajectory as semi-steady state (figure 3), with slight modulations on the steering wheel and the accelerator (not shown). High adequacy of the formulated “miMa” model may be confirmed by comparing the motion states from the road measurements and the simulations. The positively verified model may be further incorporated in an optimization algorithm.

The following 11 chassis parameters (\( p \)) of Nissan “miMA” model (figure 4) are selected as important for an optimization. Its definitions, initial values and ranges are given in Table 2.

**Table 2.** Definitions of Nissan chassis parameters (\( p_i \)) and driver actions (\( \delta_i \)).

| No | Definition                          | Bounds   | Initial | After optimization |
|----|------------------------------------|----------|---------|--------------------|
| p1 | FR steering axis inclination angle [deg] | 10÷16 | 13      | 15                 |
| p2 | FR steering axis caster angle [deg]  | 3÷9     | 6       | 4                  |
| p3 | Steering system ratio [-]           | 12÷20   | 17      | 12                 |
| p4 | Rate of steering Ackerman [-]       | -1÷1    | 0.5     | 0                  |
| p5 | RR wheel camber angle [deg]         | -5÷-1   | -3      | 4                  |
| p6 | FR antiroll bar (linear) stiffness [N/mm] | 5÷50 | 25       | 20                 |
| p7 | RR antiroll bar (linear) stiffness [N/mm] | 0÷20 | 10      | 0                  |
| p8 | Preload of limited slip differential [Nm] | 0÷200 | 100     | 100                |
| p9 | Rate of 2-way limited slip differential [-] | 0÷1 | 1        | 1                  |
| p10| Maximal engine torque [Nm]          | 355÷400 | 355     | 0                  |
| p11| Mass distribution to FR axle [%]    | 45÷55   | 50      | 48                 |
| \( \delta_h \) | Steering wheel angle [deg]         | -700÷0  | -550    | -560               |
| \( \delta_a \) | Accelerator position [-]           | 0÷1     | 0.5     | 0.7                |
4. Vehicle chassis optimization for R-const drifting
The numerical example is based on the following assumptions:

- the vehicle (figure 4) has to: (I) generate the biggest possible drift angle ($\beta$), (II) in a quasi-static fashion (when $\dot{\beta} \to 0$), (III) moving on R-const (R=15m, $\mu=0.9$) curve with a minimal deviation ($\Delta R$), and at (IV) minimal cost of the vehicle setup changes;
- this closed-loop manoeuvre is controlled be an virtual driver by changing only the steering wheel angle ($\delta_h$) and the accelerator position ($\delta_a$) on given ($\delta_g=2$) gear;
- the manoeuvre takes 8 seconds and starts with a drift angle defined by initial conditions;
- steady-state drifting, i.e. state of motion when drift angle velocity vanishes ($\dot{\beta} = 0$), has to be obtained during the last 4 s.

In that case, an optimization algorithm of the driver actions and the vehicle chassis parameters is defined as follows:

- goal functions:
  \[ w = [\max(\beta) \min(\dot{\beta}) \min(\Delta R) \min(c)]_{1 \times 4} \] (2)
- decision variables:
  \[ d = [\delta \ p]_{1 \times 13} \] (3)
- under constraints:
  \[ d_{\min} < d < d_{\max}, \] (4)

where:
- $\delta = [\delta_h \ \delta_a]_{1 \times 2}$ driver actions,
- $p = [p_1 \ldots p_{11}]_{1 \times 11}$ vehicle chassis parameters,
- \(\Delta R\) deviation from R-const trajectory,
- \(c = \sum_{i=1}^{11} p_i\) vehicle reassembly cost.

The problem is solved by using multircriteria Genetic Algorithm after 30k iterations (ca 8hrs on PC).

5. Results of Nissan 300sx chassis optimization
The obtained optimization results for Nissan 200sx (R = 15 m, $\mu = 0.9$) are presented on a drift phase plane (figure 8) defined by the first two ($w_1$ and $w_2$) criteria (2). The obtained V-like profile of each solutions family has the following interpretation. The best solution is described by a maximal drift angle ($\beta$) and a minimal drift angle rate ($\dot{\beta}$), that approaches zero. This state of the drifting vehicle motion fulfills a stability condition from a technical point of view.

In case of the base chassis setup, the vehicle can be driven (figure 8, left) with a steady-state powerslide at $\beta = 35.3$ deg drift angle (figure 3). Other solutions are unstable under the defined work assumptions. In case of the optimized chassis setup (figure 8, right), the steady-state drift angle is increased by 14% to ca. 40 deg. The optimized chassis parameters are given in Table 2.

![Figure 8. Optimization results of Nissan 200sx (R = 15 m, $\mu = 0.9$) on drift phase plane ($\dot{\beta} \ vs \ \beta$).](image-url)
6. Concluding remarks and further steps

In the paper an innovative method of RWD vehicle chassis optimization for quasi-static drifting on defined path with given constant radius and tyre-road grip, is presented.

The high fidelity vehicle model, called “miMA”, with 26 state variables and more than 400 parameters is formulated for the optimisation purposes. ‘miMA’ enables to describe details of a vehicle chassis components (relevant in motorsport) at a reasonable computational time (3 times shorter than real time). Similar results are rarely achievable with commercial general purpose software. The developed model of Nissan 200sx S13 was successfully verified on the basis of track tests with an expert drift-driver. Control problem of oversteering mode stabilization was reduced to a static optimization problem with virtual driver actions included as decision variables, in addition to chassis parameters.

The considered Nissan 200sx S13 with optimized chassis setup (for R = 15 m, µ=0.9) can stably sustain a 14% bigger drift angle, i.e. $\beta = 40$ deg instead 35 deg. It was mainly achieved by changing the front axle inclination angle and anti-roll bar stiffness.

The subsequent step of the drift vehicle preparation usually considers an optimization in cases of dynamic manoeuvres, like a slalom or a drift initiation.

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