The one-way unlocalizable quantum discord

Zhengjun Xi,1 Heng Fan,2 and Yongming Li1

1College of Computer Science, Shaanxi Normal University, Xi'an, 710062, China
2Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

(Dated: December 21, 2013)

In this paper, we present the concept of the one-way unlocalizable quantum discord and investigate its properties. We provide a polygamy inequality for it in tripartite pure quantum system of arbitrary dimension. Several tradeoff relations between the one-way unlocalizable quantum discord and other correlations are given. If the von Neumann measurement is on a part of the system, we give two expressions of the one-way unlocalizable quantum discord in terms of partial distillable entanglement and quantum disturbance. Finally, we also provide a lower bound for bipartite shareability of quantum correlation beyond entanglement in a tripartite system.

PACS numbers: 03.67.-a, 03.65.Ta

I. INTRODUCTION

Entanglement plays a key role in quantum computation and quantum information, which is a kind of quantum correlation not available in classical world [1]. On the other hand, there exists some other quantum correlations different from entanglement, for example, quantum discord is also a quantum correlation not present in classical system [2, 3]. It is supposed that quantum discord might be responsible for the advantage of a quantum algorithm, the deterministic quantum computation with one qubit [4], which surpasses any corresponding classical algorithms while with vanishing entanglement but non-zero quantum discord. It is also noticed that, while different from entanglement, quantum discord is closely related with entanglement and may have interesting operational interpretations in quantum information processing [3, 12], and also see the reviews [13, 14].

We know that entanglement is an invaluable resource for quantum information processing, and thus it cannot be shared freely among multipartities. This property is described by monogamy inequality, which is one of the most fundamental properties of entanglement [1, 15]. That is, if two qubits A and B are maximally entangled they cannot be correlated at all with third qubit C. In general, there is tradeoff between the amount of entanglement between qubits A and B and the same qubit A and qubit C. This property is purely quantum: in classical world if A and B bits are perfectly correlated, then there is no constraints on correlations between bits A and C [11]. For three qubits the tradeoff is described by Coffman-Kundu-Wootters (CKW) monogamy inequality in terms of the concurrence [12]. The CKW inequality can be extended to n-qubits, it has been proven true only recently [10]. More discussions on monogamy of different quantum correlations measures can be found in [17, 22].

Since quantum discord quantifies the quantum correlation in a bipartite state [2, 3] and might also be a resource in quantum information processing, it is interesting to study whether it also respects monogamy relation. Recently, monogamy relation for quantum discord has already been studied in [23–27]. It is found that monogamy of quantum discord is due to the relationships between the entanglement of formation and the one-way classical correlation, see the inequality [43]. In general, this inequality (43) does not hold. Thus, the monogamy of quantum discord is not universal for any tripartite pure state. As pointed out in [23], polygamy relation for quantum discord can hold for some states.

For the case of entanglement, it is showed that distributed entanglement is by nature polygamy by establishing the concept of the one-way unlocalizable entanglement [29], which is defined by entanglement of assistance [30, 31]. Since quantum discord is an important supplementary to entanglement, it motivates us to propose a similar concept for quantum discord. Thus we can systematically investigate the polygamy of distributed quantum correlations in terms of quantum discord in tripartite quantum system. Based on the duality between the one-way unlocalizable entanglement and the one-way classical correlation [3], in this paper, we present the concept of the one-way unlocalizable quantum discord and investigate its basic properties, and give a polygamy relation. We give the lower and upper bound for the one-way unlocalizable quantum discord. We also derive several tradeoff relations between the one-way unlocalizable quantum discord and other correlations, and provide a polygamy relation of distributed quantum correlations in terms of the one-way unlocalizable quantum discord. In case the von Neumann measurement is on a part of the system, we give two expressions of the one-way unlocalizable quantum discord in terms of partial distillable entanglement [3] and quantum disturbance [32]. Finally, we also provide a lower bound for bipartite shareability of quantum correlations (beyond entanglement) in a tripartite system.

The paper is organized as follows. In Sec III we provide the definition of the one-way unlocalizable quantum discord and investigate its basic properties, and give a polygamy relation of distributed quantum correlations.
In Sec III we study the relations between the one-way unlocalizable quantum discord and entanglement. In Sec IV we discuss monogamy relation for quantum discord and we summarize our results in Sec. V.

II. ONE-WAY UNLOCALIZABLE QUANTUM DISCORD

A. Definition

We now consider a tripartite pure state $|\psi\rangle_{ABC}$ shared between three parties referred to as Alice, Bob and Charlie. The entanglement supplier, Bob performs a measurement on him share of the tripartite state which yields a know bipartite entangled state for Alice and Charlie. Tracing over Bob’s system yields the bipartite mixed state

$$\rho_{AC} = \text{Tr}_B(|\psi\rangle_{ABC}\langle\psi|)$$

(1)

where the maximum is taken over all possible pure state decompositions of $\rho_{AC}$, satisfying $\rho_{AC} = \sum_x p_x |\phi_x\rangle_{AC}\langle\phi_x|$ and $\rho^A_{AC} = \text{Tr}_C(|\phi_x\rangle_{AC}\langle\phi_x|)$. Here, $S(\rho) := -\text{Tr} \log \rho$ is von Neumann entropy. Since $E_u(\rho_{AC})$ measures the maximum average entanglement that can be localized on the subsystem $AC$ with the assistance of $B$, and $E_u(\rho_{AC}) \leq S(\rho_A)$, then the difference between two is the so-called one-way unlocalizable entanglement of a bipartite state $\rho_{AB}$, namely,

$$E_u(\rho_{AC}) = \max_{\{p_x,|\phi_x\rangle\}_{AC}} \sum_x p_x S(\rho^A_{AC})$$

(2)

where $\rho_{AC} = \langle\psi|_{ABC}\langle\psi|\rangle_{ABC}$

(3)

where $\rho_{AB} = \text{Tr}_C(|\psi\rangle_{ABC}\langle\psi|)$. For tripartite pure state $|\psi\rangle_{ABC}$, all possible pure state decompositions of $\rho_{AC}$ can be realized by rank-1 measurements of subsystem $B$, conversely, any rank-1 measurement can be induced from a pure state decomposition of $\rho_{AC}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$. Therefore, the right-hand side of Eq. (2) is the maximum average entropy over all possible rank-1 measurements $\{M_k^B\}$ applied on subsystem $B$. Then, Eq. (2) can be equivalently characterized by

$$E_u(\rho_{AB}) = \min_{\{M_k^B\}} \left[ S(\rho_A) - \sum_k p_k S(\rho^k_A) \right]$$

(4)

where $S(\rho_{AB})$ is the probability of the outcome $k$, and $\rho^k_A = \text{Tr}_B(\rho_{AC})/p_k$ is the state of system $A$ when the outcome is $k$.

At the same time, the one-way classical correlation $\mathcal{J}$ is defined as

$$\mathcal{J}^+(\rho_{AB}) := \max_{\{M_k^B\}} S(\rho_A) - \sum_k p_k S(\rho^k_A)$$

(5)

The relation between the one-way unlocalizable entanglement and the one-way classical correlation is like the relation between the entanglement of assistance and the entanglement of formation.

The quantum mutual information is a measure of the total amount of correlations in the bipartite quantum state, and it was divided into quantum correlation and classical correlation. Then, the one-way unlocalizable quantum discord of the state $\rho_{AB}$ under the measurement elements $\{M_k^B\}$ is defined as the difference between the mutual information and the one-way unlocalizable entanglement, namely,

$$\delta_u(\rho_{AB}) := I(\rho_{AB}) - E_u(\rho_{AB})$$

(6)

$$= I(\rho_{AB}) - E_u(\rho_{AB})$$

(7)

B. Tradeoff relation

To understand the one-way unlocalizable quantum discord, we firstly give an operational interpretation. All bipartite and single-party states are obtained by taking the appropriate partial traces of tripartite pure state. Then, without loss of generally, we suppose that $|\psi\rangle_{ABC}$ is the purification of $\rho_{AB}$. In this case, Eq. (5) can be rewritten as

$$S(\rho^A) = E_u(\rho_{AB}) + E_u(\rho_{AC})$$

(8a)

where $S(\rho^A)$ quantifies the entanglement of the pure state $|\psi\rangle_{ABC}$ with respect to the bipartite cut $A-BC$. This equality is called the Buscemi-Gour-Kim equality. Fig. I graphically illustrates this separation, we follow this figure from the [29]. In fact, the Buscemi-Gour-Kim equality is universal for any tripartite pure state, namely,

$$S(\rho^X) = E_u(\rho^{XY}) + E_u(\rho^{XZ})$$

(8b)

where $X, Y, Z \in \{A, B, C\}$. For the tripartite pure state $|\psi\rangle_{ABC}$, using the Buscemi-Gour-Kim equality, we have

$$\delta_u(\rho_{AB}) + S(A|B) = E_u(\rho_{AC})$$

(9)

where $S(A|B) := S(\rho_{AB}) - S(\rho^B)$ is the von Neumann conditional entropy. Since $S(\rho^C) = S(\rho_{AB})$, using
Eq. (19), we obtain the tradeoff relation between the one-way unlocalizable quantum discord for A and B and the one-way unlocalizable entanglement for C and B, namely,

$$S(\rho^B) = \delta_u^+(\rho^{AB}) + E_u^-(\rho^{CB}).$$  \hspace{1cm} (10)

Here, $S(\rho^B)$ quantifies the entanglement of the pure state $|\psi\rangle^{B(AC)}$ with respect to the bipartite cut $B - AC$. This equation says that it is equal to the sum between the one-way unlocalizable quantum discord for A and B with measurements on B and the one-way unlocalizable entanglement for C and B with measurements on B. We graphically illustrate this separation in Fig 2.

We now list some basic properties of the one-way unlocalizable quantum discord.

1. For any bipartite state $\rho^{AB}$,

$$\frac{I(\rho^{AB})}{2} \leq \delta_u^+(\rho^{AB}) \leq S(\rho^B).$$  \hspace{1cm} (11)

In particular, for any pure state $|\psi\rangle^{AB}$, we have

$$\delta_u^+(\rho^{AB}) = E_u^+(\rho^{AB}) = S(\rho^B).$$  \hspace{1cm} (12)

2. For all bipartite states $\rho^{AB}$ and $\rho^{A'B'}$,

$$\delta_u^+(\rho^{AB}) + \delta_u^+(\rho^{A'B'}) \leq \delta_u^+(\rho^{AB} \otimes \rho^{A'B'}).$$  \hspace{1cm} (13)

3. For a three-qubit pure state $|\psi\rangle^{ABC}$,

$$S(\rho^A) \leq \delta_u^+(\rho^{AB}) + \delta_u^+(\rho^{AC}).$$  \hspace{1cm} (14)

**Proof.**

1. For any bipartite state $\rho^{AB}$, form the result [29], one obtain

$$E_u^-(\rho^{AB}) \leq \frac{I(\rho^{AB})}{2}.$$  \hspace{1cm} (15)

Substituting this equation into Eq. (10), we obtain a lower bound

$$\frac{I(\rho^{AB})}{2} \leq \delta_u^+(\rho^{AB}).$$  \hspace{1cm} (16)

Since $E_u^-(\rho^{AB}) \geq 0$, from Eq. (10), we give the upper bound

$$\delta_u^+(\rho^{AB}) \leq S(\rho^B).$$  \hspace{1cm} (17)

For pure state case, since every post-measurement state $\rho_u^A$ is also pure state, then Eq. (12) can be directly verified.

2. For all bipartite states $\rho^{AB}$ and $\rho^{A'B'}$, we have

$$\delta_u^+(\rho^{AB} \otimes \rho^{A'B'}) = I(\rho^{AB} \otimes \rho^{A'B'}) - E_u^+(\rho^{AB} \otimes \rho^{A'B'})$$

$$\geq I(\rho^{AB}) + I(\rho^{A'B'}) - E_u^+(\rho^{AB}) - E_u^+(\rho^{A'B'})$$

$$= \delta_u^+(\rho^{AB}) + \delta_u^+(\rho^{A'B'}).$$  \hspace{1cm} (18)

where the inequality is due to the additivity of quantum mutual information [37] and the subadditivity of the one-way unlocalizable entanglement [29].

3. Following the results [29], for a three-qubit pure state $|\psi\rangle^{ABC}$,

$$S(\rho^A) - E_a(\rho^{AC}) \leq E_f(\rho^{AB}).$$  \hspace{1cm} (19)

It is actually equivalent to

$$E_u^+(\rho^{AB}) \leq E_f(\rho^{AB}).$$  \hspace{1cm} (20)

Using the Koashi-Winter equality [35], we have

$$E_u^+(\rho^{AB}) \leq S(\rho^A) - J^-(\rho^{AC}).$$  \hspace{1cm} (21)

After some manipulation we obtain

$$S(\rho^A) \leq \delta_u^+(\rho^{AB}) + \delta_u^+(\rho^{AC}).$$  \hspace{1cm} (22)

\[\square\]

### C. Polygamy inequality

Since the Buscemi-Gour-Kim equality [38] is universal for any tripartite pure state, one checks that

$$\delta_u(\rho^{AC}) + S(A|C) = E_a(\rho^{AB}).$$  \hspace{1cm} (23)

Combined Eq. (22), we obtain the conservation relation between the one-way unlocalizable quantum discord and the entanglement of assistance as follows

$$E_a(\rho^{AC}) + E_a(\rho^{AB}) = \delta_u^+(\rho^{AB}) + \delta_u^+(\rho^{AC}).$$  \hspace{1cm} (24)
From the results in [29], there exists a polygamous relation of entanglement of assistance, namely,

$$E_a(\rho^{A(BC)}) \leq E_a(\rho^{AB}) + E_a(\rho^{AC}).$$  \hspace{1cm} (25)$$

The one-way unlocalizable quantum discord is equal to the entanglement of assistance for the pure state $|\psi\rangle^{ABC}$ with respect to the bipartite cut $A - BC$, we then have

$$\delta^-(\rho^{A(BC)}) = E_a(\rho^{A(BC)}).$$  \hspace{1cm} (26)$$

Substituting Eqs. (26) and (27) into Eq. (24), we get the polygamy of the one-way unlocalizable quantum discord

$$\delta^-(\rho^{A(BC)}) \leq \delta^-(\rho^{AB}) + \delta^+(\rho^{AC}).$$  \hspace{1cm} (27)$$

This inequality shows that there exists a polygamy relation of quantum correlations in terms of the one-way unlocalizable quantum discord for a tripartite pure state of arbitrary dimension.

### III. LINKING QUANTUM CORRELATION TO ENTANGLEMENT

First we introduce the measures of information gain and disturbance of a quantum measurement constructed in [32]. Form the result [42], it states that whatever quantum measurement $\{M^E_x\}$ can be modeled as an indirect measurement, in which the input system first interacts with an apparatus $Q$, initialized in a fixed pure state $|0\rangle^Q$, through a suitable unitary interaction $I^A \otimes U^B Q : \mathcal{H}_{ABQ} \rightarrow \mathcal{H}_{ABQ'}$, and

$$(I^A \otimes U^B Q)\left(\rho^{AB} \otimes |0\rangle\langle 0|\right)\left(I^A \otimes U^B Q\right) = \rho^{AB'Q'}.$$  \hspace{1cm} (28)$$

Subsequently, a particular measurement $M^Q$, depending also on $U^B Q$, is performed on the apparatus. In addition, by introducing a reference system $R$ purifying the input state as $|\Psi\rangle^{RAB}$, $\text{Tr}_{R}(|\Psi\rangle^{RAB} |\Psi\rangle^{RAB}) = \rho^{AB}$, we are in the situation schematically represented as in Fig. 3. After the unitary interaction $U^B Q$, the global state is

$$|\Psi\rangle^{RAB'Q'} = (I^A \otimes U^B Q)|\Psi\rangle^{RAB} \otimes |0\rangle^Q,$$

and the measurement on the apparatus can be chosen such that

$$(I^{RAB'Q'} \otimes M^Q) |\Psi\rangle^{RAB'Q'} = \sum_x p_x |v_x^{RAB'Q'} \otimes |x\rangle^X \langle x|,$$

where $v_x^{RAB'Q'}$ are pure states such that

$$\text{Tr}_{Q'}(v_x^{RAB'Q'}) = (I^A \otimes M^B_x) |\Psi\rangle^{RAB}/|\Psi\rangle^{RAB}/p_x = \rho^{AB},$$

and

$$\text{Tr}_{Q'} \rho^{RAB'} = \rho^{AB'} = (I^A \otimes E_x^B)^{\rho^{AB}}(I^A \otimes E_x^B),$$

where $|x\rangle^X$ are the classical register states, $M^B_x := E_x^B E_x^B$.

We note that the one-way unlocalizable quantum discord is defined by considering rank-1 POVMs. The rank-1 POVMs can be replaced by a minimization over orthogonal projectors of rank-1 on an extended Hilbert space [37]. Here, the generalized measurement is actually restricted to complete von Neumann measurements.

Based on the result in [5], we establishes a connection between the partial entanglement and the one-way unlocalizable quantum discord. The one-way unlocalizable quantum discord of a bipartite state $\rho^{AB}$ is equal to the maximal partial distillable entanglement in a von Neumann measurement on $B$,

$$\delta^-(\rho^{AB}) = \max_U \left[ E_D^{AB} |\rho^{AB} |\rho^{M_B} |\rho^{B'Q'} - E_D^{B'} |\rho^{B'Q'} - \rho^{B'Q'} \right],$$  \hspace{1cm} (29a)$$

where the maximum is taken over unitaries $U$ which realize a von Neumann measurement on $B$. Here, $E_D$ is distillable entanglement [39, 40]. For arbitrary measurement of entanglement $E$, one denotes the partial entanglement

$$P_E := E^{AB'} |\rho^{AB'} |\rho^{B'Q'} - E^{B'} |\rho^{B'Q'} - \rho^{B'Q'}.$$  \hspace{1cm} (29b)$$

It quantifies the part of entanglement which is lost when the subsystem $B$ is ignored [3].

Under the measurement $M^B$ on $B$, the quantum disturbance introduced $\delta(\rho^B, M^B)$ and $\delta(\rho^{AB}, M^B)$ are given, respectively,

$$\delta(\rho^B, M^B) := S(\rho^B) - T^{R_{A-B'}X} (\rho^{R_{A}B'X})$$  \hspace{1cm} (29a)$$

and

$$\delta(\rho^{AB}, M^B) := S(\rho^{AB}) - T^{R_{A-B'}X} (\rho^{R_{A}B'X})$$  \hspace{1cm} (29b)$$

where $T^{Y_{-Z}} := S(\rho^Z) - S(\rho^Y)$ is the coherent information and $R$ is a purifying system for which $|\Psi\rangle^{RAB}$ is the purified state of density operator $\rho^{AB}$. Here, $\rho^{R_{A}X}$ is the measurement $R_{A-B'}X$.
and $\rho^{RX}$ are the states reduced from the post-measurement state $\rho^{RAB}X$.

After some manipulation one obtain the following form

$$
\delta(\rho^B, M^B) - \delta(\rho^{AB}, M^B)
=S(\rho^B) - S(\rho^{AB}) + \sum_x p_x S(\rho^B_x).
$$

(30)

Therefore, we obtain the second expressions for the one-way unlocalizable quantum discord based on quantum disturbance, namely,

$$
\delta_u(\rho^{AB}) = \max_{M^B} \left[ \delta(\rho^B, M^B) - \delta(\rho^{AB}, M^B) \right].
$$

(31)

At the same time, the information gain $\iota(\rho^B, M^B)$ and $\iota(\rho^{AB}, M^B)$ are given, respectively,

$$
\iota(\rho^B, M^B) := \mathcal{I}^{RA:X}(\rho^{RX}),
$$

(32a)

$$
\iota(\rho^{AB}, M^B) := \mathcal{I}^{R:X}(\rho^{RX}).
$$

(32b)

After some manipulation one obtain the following form

$$
\iota(\rho^B, M^B) - \iota(\rho^{AB}, M^B)
=S(\rho^B) - S(\rho^{AB}) + \sum_x p_x \left[ S(\rho^B_x) - S(\rho^{RA}_x) \right]
=S(\rho^B) - S(\rho^{AB}) + \sum_x p_x \left[ S(\rho^{AB'}Q') - S(\rho^{B'}Q') \right]
\leq S(\rho^B) - S(\rho^{AB}) + \sum_x p_x S(\rho^A_x).
$$

(33)

The second equality is due to the $\nu^{RAB'}Q'$ is pure state for every $x$. The inequality come form the sub-additivity of the von Neumann entropy. Therefore, we arrive another lower bound for the one-way unlocalizable quantum discord based on information gain, namely,

$$
\delta_u(\rho^{AB}) \geq \max_{M^B} \left[ \iota(\rho^B, M^B) - \iota(\rho^{AB}, M^B) \right]
$$

(34)

with equality if and only if $S(\rho^{RA}_x) = S(\rho^B_x) - S(\rho^A_x)$ for every $x$.

IV. MONOGAMY OF QUANTUM DISCORD

For the tripartite pure state $|\psi\rangle^{ABC}$ with $\rho^A = \text{Tr}_{BC}(|\psi\rangle^{ABC}\langle\psi|)$, $\rho^{AB} = \text{Tr}_C(|\psi\rangle^{ABC}\langle\psi|)$ and $\rho^{AC} = \text{Tr}_B(|\psi\rangle^{ABC}\langle\psi|)$, the Koashi-Winter equality is given as

$$
E_f(\rho^{AB}) + J^{\rightarrow}(\rho^{AC}) = S(\rho^A).
$$

(35)

Here $J^{\rightarrow}(\rho^{AC})$ is the one-way classical correlation between $A$ and $C$, and the entanglement of formation $E_f$ is defined as

$$
E_f(\rho^{AB}) = \min_{\{p_i, |\psi_i\rangle^{AB}\}} \sum_i p_i S(\rho^A_i).
$$

(36)

where the minimum is taken over all pure ensembles $\{p_i, |\psi_i\rangle^{AB}\}$ satisfying $\rho^{AB} = \sum_i p_i |\psi_i\rangle^{AB}\langle\psi_i|$, and $\rho^A_i = \text{Tr}_B(|\psi_i\rangle^{ABC}\langle\psi_i|)$ [32, 33].

For the tripartite pure state $|\psi\rangle^{ABC}$, the monogamic distribution of entanglement of formation and quantum discord can be given as [11]

$$
\delta^{\rightarrow}(\rho^{AB}) + \delta^{\rightarrow}(\rho^{AC}) = E_f(\rho^{AB}) + E_f(\rho^{AC}).
$$

(37)

Here, $\delta^{\rightarrow}(\rho^{XY})$ is the quantum discord of $\rho^{XY}$.

If the entanglement of formation would be smaller than the one-way classical correlation, namely,

$$
E_f(\rho^{XY}) \leq J^{\rightarrow}(\rho^{XY}).
$$

(38)

Substituting this inequality into Eq. (35), there exists a monogamy relation of entanglement of entanglement for the tripartite pure states. Along with this way, combining Eq. (37), we give the monogamy of quantum discord as following

$$
\delta^{\rightarrow}(\rho^{AB}) + \delta^{\rightarrow}(\rho^{AC}) \leq \delta^{\rightarrow}(\rho^{A(BC)}),
$$

(39)

where $\delta^{\rightarrow}(\rho^{A(BC)}) = E_f(\rho^{A(BC)}) = S(\rho^A)$, and $S(\rho^A)$ quantifies the entanglement of the pure state $|\psi\rangle^{ABC}$ with respect to the bipartite cut $A - BC$.

But the inequality (38) does not hold for some states [3, 28]. Thus, the monogamy of quantum discord is not universal for tripartite any pure state (see [3, 24, 26]).

Combined Eq. (38) with Eq. (35), we obtain the conservation relation

$$
E_f(\rho^{AB}) + J^{\rightarrow}(\rho^{AC}) = E_a(\rho^{AC}) + E_u^{\rightarrow}(\rho^{AB}).
$$

(40)

It is easy to verify that the the one-way unlocalizable quantum discord is equal to quantum discord for the pure state $|\psi\rangle^{ABC}$ with respect to the bipartite cut $A - BC$. By the definitions of entanglement of formation and entanglement of assistance, we have

$$
E_f(\rho^{XY}) \leq E_u^{\rightarrow}(\rho^{XY}).
$$

(41)

Using Eq. (37), we obtain an interesting inequality as follows

$$
\delta^{\rightarrow}(\rho^{AB}) + \delta^{\rightarrow}(\rho^{AC}) = E_f(\rho^{AB}) + E_f(\rho^{AC})
\leq E^{\rightarrow}_u(\rho^{AB}) + E^{\rightarrow}_u(\rho^{AC}) = \delta_u(\rho^{AB}) + \delta_u(\rho^{AC}).
$$

(42)

V. CONCLUSION

In this work, we propose the concept of the one-way unlocalizable quantum discord and provided an operational interpretation. We have investigated systematically some foundational properties of the one-way unlocalizable quantum discord, for example, the lower and the upper bound. By extending a bipartite system to a tripartite system, we have established several tradeoffs.
between the one-way unlocalizable quantum discord and other correlations. We have also provided a polygamy relation of distributed quantum correlations in terms of the one-way unlocalizable quantum discord in tripartite quantum systems, and have derived a lower bound for bipartite shareability of quantum correlations in a tripartite system. If the von Neumann measurement is on a part of the system, we have provided two expressions in terms of partial distillable entanglement and quantum disturbance for the one-way unlocalizable quantum discord.

As an important supplementary to quantum entanglement, quantum discord is proved to be also important and necessary in quantum information processing. With recent developments for quantum correlations beyond entanglement such as quantum discord both in experiment and in theory, it will be interesting to explore the new proposed one-way unlocalizable quantum discord further experimentally and theoretically.

Acknowledgments

Y.M. Li is supported by NSFC with grant No.60873119 and the Higher School Doctoral Subject Foundation of Ministry of Education of China with grant No.200807180005. Z.J. Xi is supported by the Superior Dissertation Foundation of Shaanxi Normal University (S2009YB03). H. Fan is supported by NSFC with grant No.10974247 and No.11175248, and ‘973’ program (2010CB922904).

[1] R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Rev. Mod. Phys. 81, 865 (2009).
[2] H. Ollivier, W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001).
[3] L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001).
[4] E. Knill and R. Laflamme, Phys. Rev. Lett. 81, 5672 (1998).
[5] A. Streltsov, H. Kampermann and D. Bruß, Phys. Rev. Lett. 106, 160401 (2011).
[6] M. Piani, S. Ghiribian, G. Adesso, J. Calsamiglia, P. Horodecki and A. Winter, Phys. Rev. Lett. 106, 220403 (2011).
[7] L. Mazzola and M. Paternostro, Nature Scientific Reports, 1, 199 (2011).
[8] D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, and A. Winter, Phys. Rev. A 81, 032324 (2011).
[9] V. Madhok and A. Datta, Phys. Rev. A 81, 032323 (2011).
[10] A. Streltsov, H. Kampermann and D. Bruß, Phys. Rev. Lett. 107, 170502 (2011).
[11] F. Ciccarello, V. Giovannetti, arXiv:1105.5551 (2011).
[12] S. Campbell, T. J. G. Apollaro, C. Di Franco, L. Banchi, A. Cuccoli, R. Vaia, F. Plastina, and M. Paternostro, Phys. Rev. A, 84, 052316 (2011).
[13] K. Modi, A. Brodutch, H. Cable, T. Paterek, V. Vedral, arXiv:1112.6238.
[14] L. C. Céleri, J. Maziero, R. M. Serra, Int. J. Quant. Inf., 9, 1837 (2011).
[15] V. Coffman, J. Kundu and W. K. Wootters, Phys. Rev. A 61, 052306 (2000).
[16] T. J. Osborne and F. Verstraete, Phys. Rev. Lett. 96, 220503 (2006).
[17] M. Seevinck, Quantum Inf. Process., 9, 273 (2010).
[18] A. Kay, D. Kaszlikowski, and R. Ramanathan, Phys. Rev. Lett. 103, 050501 (2009).
[19] M. Pawłowski and C.Brukner, Phys. Rev. Lett. 102, 030403 (2009).
[20] P. Kurzyński, T. Paterek, R. Ramanathan, W. Laskowski, and D. Kaszlikowski, Phys. Rev. Lett. 106, 180402 (2011).
[21] Y. C. Ou and H. Fan, Phys. Rev. A 75, 062308 (2007).
[22] Y. C. Ou, H. Fan, and S. M. Fei, Phys. Rev. A 78, 012311 (2008).
[23] R. Prabhu, A. K. Pati, A. Sen(De), and U. Sen, arXiv:1108.5168 (2011).
[24] G. L. Giorgi, Phys. Rev. A, 84 054301 (2011).  
[25] Sudha, A. R. Usha Devi, A. K. Rajagopal, arXiv:1110:3026.
[26] Xi-Jun Ren, H. Fan, arXiv: 1111:5163.
[27] A. Streltsov, G. Adesso, M. Piani, D. Bruß, arXiv:1112:3167.
[28] N. Li and S. Luo, Phys. Rev. A 76 032327 (2007).
[29] F. Buscemi, G. Gour and J. San. Kim, Phys. Rev. A 80, 012324 (2009).
[30] O. Cohen, Phys. Rev. Lett., 80, 2493 (1998).
[31] D.P. Divincenzo et al., Lect. Notes Comput. Sci., 1509, 247 (1999).
[32] F. Buscemi, M. Hayashi, and M. Horodecki, Phys. Rev. Lett. 100, 210504 (2008).
[33] L. P. Hughston, R. Jozsa, W. K. Wootters, Physics Letters A, 183, 14 (1993).
[34] G. Gour, D. A. Meyer, and B. C. Sanders, Phys. Rev. A 72, 042329 (2005).
[35] W. K. Wootters, Phys. Rev. Lett. 80 2245 (1998).
[36] W. K. Wootters, Quantum Inf. Comput. 1 27 (2001).
[37] M. A. Nielsen and I. L. Chung Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, England, 2000).
[38] J. Preskill, Lecture Notes for Physics 219: Quantum Information and Computation (Caltech, Pasadena, CA, 1999), Chap. 3, http://www.theory.caltech.edu/people/preskill/ph229/
[39] C.H. Bennett, D. P. DiVincenzo, J. A. Smolin and W. K. Wootters, Phys. Rev. A 54, 3824 (1996).
[40] M. B. Plenio and S. Virmani, Quantum Inf. Comput. 7, 1 (2007).
[41] A. Peres, Quantum Theory: Concepts and Methods (Springer, New York, 1995).
[42] M. Ozawa, J. Math. Phys. (N.Y.) 25, 79 (1984).
[43] M. Koashi and A. Winter, Phys. Rev. A 69, 022309 (2004).
[44] F. F. Fanchini, et al, Phys. Rev. A 84 012313 (2011).
[45] J. S. Xu, X. Y. Xu, C. F. Li, C. J. Zhang, X. B. Zou, and G. C. Guo, Nature Comm. 1, 1005 (2010).