Hořava-Lifshitz cosmology with generalized Chaplygin gas

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We investigate cosmological scenarios of generalized Chaplygin gas in a universe governed by Hořava-Lifshitz gravity. We consider both the detailed and non-detailed balance versions of the gravitational background, and we include the baryonic matter and radiation sectors. We use observational data from Type Ia Supernovae (SNIa), Baryon Acoustic Oscillations (BAO), and Cosmic Microwave Background (CMB), along with requirements of Big Bang Nucleosynthesis (BBN), to constrain the parameters of the model, and we provide the corresponding likelihood contours. We deduce that the present scenario is compatible with observations. Additionally, examining the evolution of the total equation-of-state parameter, we find in a unified way the succession of the radiation, matter, and dark energy epochs, consistently with the thermal history of the universe.

I. INTRODUCTION

Recently Hořava proposed a power-counting renormalizable theory with consistent ultra-violet (UV) behavior [1,4]. Although presenting an infrared (IR) fixed point, namely General Relativity, in the UV the theory exhibits an anisotropic, Lifshitz scaling between time and space. Due to these novel features, there has been a large amount of effort in examining and extending the properties of the theory itself [5–30]. Additionally, application of Hořava-Lifshitz gravity as a cosmological framework gives rise to Hořava-Lifshitz cosmology, which proves to lead to interesting behavior [31,32]. In particular, one can examine specific solution subclasses [33–40], the phase-space behavior [41,42], the gravitational wave production [43–49], the perturbation spectrum [50–58], the matter bounce [59,60], the black hole properties [61,62], the dark energy phenomenology [63–70], the observational constraints on the parameters of the theory [71,72], the astrophysical phenomenology [73–80], the thermodynamic properties [81–91] etc. However, despite this extended research, there are still many ambiguities if Hořava-Lifshitz gravity is reliable and capable of a successful description of the gravitational background of our world, as well as of the cosmological behavior of the universe [8,9,11,28,92,95].

Although the foundations and the possible conceptual and phenomenological problems of Hořava-Lifshitz gravity and its associated cosmology are still an open issue, it is worth investigating different cosmological scenarios in this gravitational background. In this regard, it would be interesting to examine cosmological scenarios where apart from the gravitational sector there exists a generalized Chaplygin gas [96–103].

The generalized Chaplygin gas (GCG) is an alternative to the Quintessence model which had attracted great interest in recent times. The scenario can explain the acceleration of the universe via an exotic equation of state, which mimics a pressureless fluid at the early stages of evolution of the Universe, and a cosmological constant at late times. It is therefore interesting to consider the GCG scenario as a unified description for dark matter and dark energy [96–103]. The background evolution fits well the observational data [104,105], however the cosmological behavior is indistinguishable from that of the ΛCDM scenario while fitting with the structure formation data as well as with data from Cosmic Microwave Background radiation [104,105]. Additionally, the scenario is plagued by the presence of instabilities as well as oscillations which are not observed in the matter power spectrum. Although this is a serious drawback of GCG scenario it is not the final verdict for its fate as a model for the unified description of dark matter and dark energy.

As it was shown by Reis et al. in [106], allowing for small entropy perturbations can eliminate instabilities and oscillations in the matter power spectrum, even in the linear regime, for a region of the parameter space where the GCG model behaves quite differently from the ΛCDM one. Furthermore, it was shown by Avelino et al. in [107], in the GCG scenario the transition from dark matter to dark energy behavior is never smooth, and hence the linear theory which was used by [104,105] to rule out GCG as a unified model, may break down late in the matter-dominated era, even on large cosmological scales. Therefore, nonlinear effects should be necessarily taken into account when confronting cosmological observations. Moreover, the addition of baryons in the GCG scenario can also improve the behavior of the matter power spectrum [108]. In summary, the GCG as a unified model of dark matter and dark energy is not completely ruled out and it deserves further investigations. Finally, we
mention that GCG in the presence of cold dark matter as well as baryonic matter (where GCG acts as a normal dark energy candidate) is one of the most well-fit scenarios with cosmological observations, amongst all the exotic models that have been considered so far in cosmology.

In the present work we construct the cosmology of a generalized Chaplygin Gas in a universe governed by Hořava-Lifshitz gravity. Additionally, we use observational data from Type Ia Supernovae (SNIa) [110], Baryon Acoustic Oscillations (BAO) [111] and Cosmic Microwave Background (CMB) [112], together with the Big Bang Nucleosynthesis (BBN) conditions, to constrain the various parameters of the model. Furthermore, in order to be general we perform the analysis both with and without the detailed-balance condition of the gravitational sector.

The manuscript is organized as follows: In section II we present Hořava-Lifshitz cosmology in both its detailed-balance and beyond-detailed-balance version. In section III we construct the scenario of a generalized Chaplygin Gas in Hořava-Lifshitz gravitational background and we extract the cosmological equations. In section IV we present Hořava-Lifshitz cosmology in both its detailed-balance and beyond-detailed-balance version. In section V we use observational data in order to constrain the various parameters of the scenario. In section VI we discuss the cosmological implications, focusing on the evolution of the total equation-of-state parameter of the universe and of the expansion rate. Finally, section VII is devoted to the summary of our results.

II. HOŘAVA-LIFSHITZ COSMOLOGY

In this section we briefly review the scenario where the cosmological evolution is governed by Hořava-Lifshitz gravity [31, 32]. The dynamical variables are the lapse and shift functions, $N$ and $N_i$, respectively, and the spatial metric $g_{ij}$ (roman letters indicate spatial indices). In terms of these fields the full metric is written as:

$$ds^2 = -N^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

where indices are raised and lowered using $g_{ij}$. The scaling transformation of the coordinates reads: $t \to t^3 t$ and $x^i \to lx^i$.

A. Detailed Balance

The gravitational action is decomposed into a kinetic and a potential part as $S_g = \int dt d^3 x \sqrt{|g|}(\mathcal{L}_K + \mathcal{L}_V)$. The assumption of detailed balance [8] reduces the possible terms in the Lagrangian, and it allows for a quantum inheritance principle [1], since the $(D+1)$-dimensional theory acquires the renormalization properties of the $D$-dimensional one. Under the detailed balance condition the full action of Hořava-Lifshitz gravity is given by

$$S_g = \int dt d^3 x \sqrt{|g|}N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2}{2w^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu}{2w^2} \sqrt{g} R_{ij} \nabla_j R_k^l + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} - \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left[ \frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right] \right\},$$

where

$$K_{ij} = \frac{1}{2N} (g_{ij} - \nabla_i N_j - \nabla_j N_i)$$

is the extrinsic curvature and

$$C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k (R^l_i - \frac{1}{4} R \delta^l_i)$$

is the Cotton tensor, and the covariant derivatives are defined with respect to the spatial metric $g_{ij}$. $\epsilon^{ijk}$ is the totally antisymmetric unit tensor, $\lambda$ is a dimensionless constant and the variables $\kappa$, $\mu$ and $\Lambda$ are constants with mass dimensions $-1$, $0$ and $1$, respectively. Finally, we mention that in action (2) we have already performed the usual analytic continuation of the parameters $\mu$ and $\Lambda$ of the original version of Hořava-Lifshitz gravity, since such a procedure is required in order to obtain a realistic cosmology [33, 34, 68, 87] (although it could fatally affect the gravitational theory itself). Therefore, in the present work $\Lambda$ is a positive constant, which as usual is related to the cosmological constant in the IR limit.

In order to add the matter component we follow the hydrodynamical approach (which is most suitable for phenomenological analysis) of adding a cosmological stress-energy tensor to the gravitational field equations, by demanding to recover the usual general relativity formulation in the low-energy limit [9, 41, 72]. Thus, this matter-tensor is a hydrodynamical approximation with $\rho_m$ and $p_m$ (or $\rho_m$ and $w_m$) as parameters. Note that this $\rho_m$ is the total matter energy density, that is it accounts for both the baryonic $\rho_b$ as well as the dark matter $\rho_dm$. Similarly, one can additionally include the standard-model-radiation component (corresponding to photons and neutrinos), with the additional parameters $\rho_r$ and $p_r$ (or $\rho_r$ and $w_r$).

In order to investigate cosmological frameworks, we impose the projectability condition [8] and we use an FRW metric

$$N = 1, \quad g_{ij} = a^2(t) \gamma_{ij}, \quad N^i = 0,$$

with

$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - K r^2} + r^2 d\Omega^2,$$

where $K <,=,> 0$ corresponding to open, flat, and closed universe respectively (we have adopted the convention of taking the scale factor $a(t)$ to be dimensionless...
and the curvature constant $K$ to have mass dimension 2). By varying $N$ and $g_{ij}$, we extract the Friedmann equations:

$$H^2 = \frac{\kappa^2}{6(3\lambda - 1)} (\rho_m + \rho_r) + \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right]$$

$$- \frac{\kappa^4 \mu^2 \Lambda K}{8(3\lambda - 1)^2 a^2}, \quad (7)$$

$$\dot{H} + \frac{3}{2} H^2 = -\frac{\kappa^2}{4(3\lambda - 1)} (w_m \rho_m + w_r \rho_r) - \frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right]$$

$$- \frac{\kappa^4 \mu^2 \Lambda K}{16(3\lambda - 1)^2 a^2}, \quad (8)$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter.

The term proportional to $a^{-4}$ is the usual “dark radiation term”, present in Hořava-Lifshitz cosmology \cite{31,32}, while the constant term is just the explicit cosmological constant. Finally, as usual, $\rho_m$ follows the standard evolution equation

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad (9)$$

while $\rho_r$ (standard-model radiation) follows

$$\dot{\rho}_r + 3H(\rho_r + p_r) = 0. \quad (10)$$

If we require expressions (14) to coincide with the standard Friedmann equations, in units where $c = 1$ we set \cite{31,32},

$$G_{\text{cosmo}} = \frac{\kappa^2}{16\pi(3\lambda - 1)}$$

$$\kappa^4 \mu^2 \Lambda = \frac{1}{8(3\lambda - 1)^2}, \quad (11)$$

where $G_{\text{cosmo}}$ is the “cosmological” Newton’s constant, that is the one that is read from the Friedmann equations. We mention that in theories with Lorentz invariance breaking $G_{\text{cosmo}}$ does not coincide with the “gravitational” Newton’s constant $G_{\text{grav}}$, that is the one that is read from the action, unless Lorentz invariance is restored \cite{113}. For completeness we mention that in our case

$$G_{\text{grav}} = \frac{\kappa^2}{32\pi}, \quad (12)$$

as it can be straightforwardly read from the action \cite{24} (our definitions of $G_{\text{cosmo}}$, $G_{\text{grav}}$ coincide with those of \cite{16,93}). Thus, it becomes obvious that in the IR ($\lambda = 1$), where Lorentz invariance is restored, $G_{\text{cosmo}}$ and $G_{\text{grav}}$ coincide.

In our work the running of $\lambda$ is not a problem, since the whole relevant cosmological history, that is after inflation, is obviously inside the IR limit, that is with $\lambda = 1$. On the other hand, the divergence of $\lambda$ from 1, that is the quantum gravitational features of the theory, become significant at very early times, that is close to the Big Bang or the cosmological bounce. This was additionally shown in \cite{78}, where a detailed analysis based on SNIa, BAO and CMB observations, as well as BBN considerations, constrained $\lambda$ inside a narrow window around its infrared value: $|\lambda - 1| < 0.02$. In summary, since in this work we are interested in post-inflation evolution, we set $\lambda = 1$, and we simplify our notation using $G$ for the (coincided gravitational and cosmological) Newton’s constant.

\subsection{Beyond Detailed Balance}

The aforementioned formulation of Hořava-Lifshitz cosmology has been performed under the imposition of the detailed-balance condition. However, in the literature there is a discussion whether this condition leads to reliable results or if it is able to reveal the full information of Hořava-Lifshitz gravity \cite{31,32}. Therefore, one needs to investigate also the Friedman equations in the case where detailed balance is relaxed. In such a case one can in general write \cite{43,62}:

$$H^2 = \frac{2\sigma_0}{(3\lambda - 1)} (\rho_m + \rho_r) + \frac{2}{(3\lambda - 1)} \left[ \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \right]$$

$$+ \frac{\sigma_2}{3(3\lambda - 1)} \frac{K}{a^2}, \quad (13)$$

$$\dot{H} + \frac{3}{2} H^2 = -\frac{3\sigma_0}{(3\lambda - 1)} (w_m \rho_m + w_r \rho_r) - \frac{3}{(3\lambda - 1)} \left[ \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6} \right]$$

$$+ \frac{\sigma_2}{6(3\lambda - 1)} \frac{K}{a^2}, \quad (14)$$

where $\sigma_0 \equiv \frac{\kappa^2}{12}$, and the constants $\sigma_i$ are arbitrary (with $\sigma_2$ being negative and $\sigma_1$ positive). As we observe, the effect of the detailed-balance relaxation is the decoupling of the coefficients, together with the appearance of a term proportional to $a^{-6}$.

Finally, if we force \cite{13,14} to coincide with the standard Friedmann equations, we obtain:

$$G_{\text{cosmo}} = \frac{6\sigma_0}{8\pi(3\lambda - 1)}$$

$$\sigma_2 = -3(3\lambda - 1), \quad (15)$$

while in this case the “gravitational” Newton’s constant
Thus, (17) provides the pressure of the Chaplygin gas as
\[ p_c = \frac{-A}{\rho_c}, \]  
with \( A \) a positive constant and \( 0 < \beta \leq 1 \). \( \rho_c \) and \( p_c \) denote the energy density and pressure of the Chaplygin gas, and henceforth a subscript \( \text{c} \) will denote quantities pertaining to the Chaplygin gas. The simple Chaplygin gas is the special case of \( \beta = 1 \). Finally, considering the covariant conservation of energy-momentum, \( \rho_c \) satisfies the standard evolution equation
\[ \dot{\rho}_c + 3H(\rho_c + p_c) = 0. \]  
Equations (17) and (18) can be easily solved to yield
\[ \rho_c = \rho_{c0} \left[ A_s + \frac{1}{\alpha^{3(1+\beta)}} \right] \]  
where \( \rho_{c0} \) denotes the present-day density of the Chaplygin gas (in what follows, a subscript 0 will always denote the present-day value of a quantity), and we have introduced the parameter
\[ A_s \equiv A_{\rho_{c0}^{-1(1+\beta)}}. \]  
Thus, (17) provides the pressure of the Chaplygin gas as
\[ p_c = \frac{-A_{\rho_{c0}}A_s}{\alpha^{3(1+\beta)}} \left[ A_s + \frac{1}{\alpha^{3(1+\beta)}} \right] \]  
Clearly, \( A_s \) is therefore just the present value of the equation-of-state parameter of the generalized Chaplygin gas. Furthermore, \( \beta \) is related to the sound velocity of the Chaplygin gas at the present time, which is given by \( \beta A_s \).  

The most important advantage of the Chaplygin gas scenario is that it behaves like dark matter at early times \( (a \ll 1) \) and as a cosmological constant at late times \( (a \gg 1) \), smoothly interpolating between the two phases.

Thus, since in this work we are interested in investigating the generalized Chaplygin gas in Hořava-Lifshitz cosmology, we first separate the total matter energy density \( \rho_m \), that was present in the Friedmann equations of the previous section, into the baryonic \( \rho_b \) and the dark matter \( \rho_{dm} \) parts, and then we use the generalized Chaplygin gas energy density \( \rho_c \) instead of \( \rho_{dm} \). Therefore, using also the definitions \( (11) \), the Friedmann equations (7), (8) of the detailed balance case take the form:
\[ H^2 = \frac{8\pi G}{3} [\rho_b + \rho_c + \rho_r] + \left[ \frac{K^2}{2a^2} + \frac{\Lambda}{2} \right] - \frac{K}{a^2} \]  
\[ \dot{H} + \frac{3}{2} H^2 = -4\pi G \left[ \rho_c + \frac{1}{3} \rho_r \right] - \left[ \frac{K^2}{4a^4} + \frac{3\Lambda}{4} \right] - \frac{K}{2a^2}. \]  

Similarly, for the beyond-detailed-balance case the Friedmann equations (19), (20), using also the identifications (19), become
\[ H^2 = \frac{8\pi G}{3} [\rho_b + \rho_c + \rho_r] \]  
\[ + \frac{2}{(3\lambda - 1)} \left[ \frac{\sigma_1}{6} + \frac{\sigma_2 K^2}{6a^4} + \frac{\sigma_3 K}{6a^6} \right] - \frac{K}{a^2} \]  
\[ \dot{H} + \frac{3}{2} H^2 = -4\pi G \left[ \rho_c + \frac{1}{3} \rho_r \right] \]  
\[ - \frac{3}{(3\lambda - 1)} \left[ - \frac{\sigma_1}{6} + \frac{\sigma_2 K^2}{18a^8} + \frac{\sigma_3 K}{6a^{10}} \right] - \frac{K}{2a^2}. \]  

IV. OBSERVATIONAL CONSTRAINTS

In the conventional gravitational background of general relativity, the cosmological scenarios of generalized Chaplygin gas have been extensively constrained using observational data. In particular, in [103, 104, 107, 114, 115] SNIa observations were used, in [107, 108] data from large scale structure, in [116, 117] CMB data, while in [114] the authors used gravitational lensing observations. Finally, constraints from combined data sources have been obtained in [118] and [119].

In the present work we will consider observations in order to constrain the scenario of a generalized Chaplygin gas in a universe governed by Hořava-Lifshitz gravity. In particular we will use data from SNIa, BAO, CMB and BBN to constrain the Chaplygin gas parameters together with the parameters of Hořava-Lifshitz gravity. For completeness, we perform our analysis separately for the detailed-balance and the beyond-detailed-balance scenario.

A. Constraints on the Detailed-Balance scenario

We first consider the detailed-balance version of the theory. It proves more convenient to use the redshift \( z \)
instead of the scale factor as the independent variable, through the relation $1 + z \equiv a_0/a = 1/a$. Furthermore, we use the dimensionless density parameters

$$\Omega_i \equiv \frac{8\pi G}{3H^2} \rho_i, \quad \Omega_K \equiv -\frac{K}{H^2 a^2},$$

and we introduce the dimensionless parameter

$$\omega \equiv \frac{\Lambda}{2H^2}.$$  

Finally, we use the dimensionless expansion rate

$$E(z) \equiv \frac{H(z)}{H_0}.$$  

Using the above definitions, the Friedman equation becomes:

$$E^2(z) = \Omega_{b0}(1 + z)^3 + \Omega_c(1 + z)^5 + \Omega_r(1 + z)^4 + \Omega_{K0}(1 + z)^2 + \left[\omega + \frac{\Omega_{K0}^2}{4\omega}(1 + z)^4\right],$$  

where

$$F(z) = \left[A_s + (1 - A_s)(1 + z)^{3(1 + \beta)}\right]^{1/\beta}.$$  

Obviously, $E(z = 0) = 1$, which leads to the condition:

$$\Omega_{b0} + \Omega_c + \Omega_{r0} + \Omega_{K0} + \omega + \frac{\Omega_{K0}^2}{4\omega} = 1.$$  

As we have already mentioned above, the term $\frac{\Omega_{K0}^2}{4\omega}$ is the coefficient of the dark radiation term, which is a characteristic feature of the Hořava-Lifshitz gravitational background. Since this dark radiation component has been present also during the time of nucleosynthesis, it is subject to bounds from Big Bang Nucleosynthesis (BBN). As discussed in detail in [77], if the upper limit on the total amount of Hořava-Lifshitz dark radiation and kination-like (a quintessence field dominated by kinetic energy [120, 121]) components allowed during BBN is expressed through the parameter $\Delta N_\nu$ of the effective neutrino species [122, 123], then we obtain the following constraint:

$$\frac{\Omega_{K0}^2}{4\omega} = 0.135\Delta N_\nu\Omega_{r0}.$$  

In this work, in order to ensure consistency with BBN, we adopt an upper limit of $\Delta N_\nu \leq 2.0$ following [124].

In most studies of dark energy models it is usual to ignore curvature (e.g. [123, 133]), especially concerning observational constraints. This consideration is well motivated since most inflationary scenarios predict a high degree of spatial flatness, and moreover the CMB data impose stringent constraints on spatial flatness in the context of constant-$w$ models (for example a combination of WMAP+BAO+SNIa data [138] provides the tight simultaneous constraints $0.0179 \leq \Omega_{K0} \leq 0.0081$ and $-0.12 \leq 1 + w \leq 0.14$, both at 95% confidence).

However, it is important to keep in mind that due to degeneracies in the CMB power spectrum (see [139] and references therein), the limits on curvature depend on assumptions regarding the underlying dark energy scenario. For example, if we use a linear $w$ (that is $w(a) = w_0 + (1 - a) w_a$) instead of a constant one, the error on $\Omega_{K0}$ is of the order of a few percent, that is much [140, 142] (see [143, 145] for the constraints on curvature for different parameterizations). Additionally, in [145] it was shown that for some models of dark energy the constraint on the curvature is at the level of 5% around a flat universe, whereas for others the data are consistent with an open universe with $\Omega_{K0} \sim 0.2$. In [141] it was proposed that geometrical tests, such as the combination of the Hubble parameter $H(z)$ and the angular diameter distance $D_A(z)$, using (future) data up to sufficiently high redshifts $z \sim 2$, might be able to decouple curvature from dark energy evolution, though not in a model-independent way. Furthermore, in [146, 147] the authors highlighted the pitfalls arising from ignoring curvature in studies of dynamical dark energy, and recommended to treat $\Omega_{K0}$ as a free parameter to be fitted along with the other model parameters. Lastly, note that in the present work the spatial curvature plays a very crucial role, since Hořava-Lifshitz cosmology coincides completely with ΛCDM if one ignores curvature [31, 52]. Therefore, and following the discussion above, we prefer to treat $\Omega_{K0}$ as a free parameter.

In summary, the scenario at hand involves the following parameters: $\Omega_{b0}$, $\Omega_{c0}$, $\Omega_{r0}$, $\Omega_{K0}$, $\omega$, $\Delta N_\nu$, $H_0$, $A_s$ and $\beta$. We fix the parameters $\Omega_{m0}(\equiv \Omega_{b0} + \Omega_{c0})$, $\Omega_{b0}$, $H_0$ and $\Omega_{r0}$ at their 7-year WMAP best-fit values [148], namely $\Omega_{m0} = 0.27$, $\Omega_{c0} = 8.14 \times 10^{-5}$ and $H_0 = 71.4$ Km/sec/Mpc. Therefore, there are five remaining parameters, namely $\Omega_{K0}$, $\omega$, $\Delta N_\nu$, $A_s$ and $\beta$, which are subject to the two constraint equations (31) and (32).

We choose to treat $\Delta N_\nu$, $A_s$ and $\beta$ as free parameters and use these constraints to eliminate $\Omega_{K0}$ and $\omega$ for a given choice of curvature, as:

$$\omega(K; \Delta N_\nu, A_s, \beta) = 1 - \Omega_{m0} - (1 - 0.135\Delta N_\nu)\Omega_{r0} - 0.73 \text{ sgn}(K) \sqrt{\Delta N_\nu}\sqrt{\Omega_{r0} - \Omega_{m0}\Omega_{r0} - \Omega_{r0}^2}$$

$$\Omega_{K0}(|\Delta N_\nu, A_s, \beta|) = \frac{0.54 \Delta N_\nu\Omega_{r0} \omega(K; \Delta N_\nu, A_s, \beta).}$$

Note that in order to obtain $\omega$ and $\Omega_{K0}$ from the above equations (33) and (34), the type of curvature $K$ has to be chosen a priori. Therefore, we treat positive ($K > 0$) and negative ($K < 0$) curvature separately. Finally, note that according to (34) if $\Delta N_\nu = 0$ then it forces the spatial curvature to be zero.

Let us now proceed to constrain the three free parameters $\Delta N_\nu$, $A_s$ and $\beta$. We perform a likelihood analysis
using the SNIa, BAO and CMB data (see the appendix of [77] for the details of the procedure). In Fig. 1 we present the likelihood contours of $A_s$ and $\Delta N_{\nu}$ for three different choices of $\beta$, one at its best-fit value, and the other two at the extremes of its allowed values. As we observe, the scenario at hand is in general compatible with observations. Furthermore, note that the contours are smaller for negative curvature, indicating that negative curvature is generally disfavored for these models. Finally, the best-fit values together with the corresponding 1σ bounds arising from the analysis, for the two choices of curvature, are shown in Table I.

### Table I: Best-fit values and 1σ confidence intervals for the free parameters $A_s$, $\Delta N_{\nu}$ and $\beta$ in the detailed balance scenario, arising from a likelihood analysis using SNIa, BAO and CMB data.

|                | $K < 0$                   | $K > 0$                   |
|----------------|---------------------------|---------------------------|
| $A_s$          | $1.96 \times 10^{-13}$; (0, 0.04) | $3.75 \times 10^{-11}$; (0, 0.04) |
| $\Delta N_{\nu}$ | 0; (0, 0.09)            | $2.75 \times 10^{-2}$; (0, 0.09) |
| $\beta$        | $3.21 \times 10^{-5}$; (0, 1)     | $3.89 \times 10^{-3}$; (0, 1)     |

**B. Constraints on the Beyond-Detailed-Balance scenario**

In this subsection we consider the cosmology of a generalized Chaplygin gas in Hořava-Lifshitz gravity where the detailed balance condition has been relaxed. Our cos-
mic fluid consists of the baryonic matter, the standard-model radiation and the Chaplygin gas. Starting with the Friedmann equation (33) and proceeding similarly to the previous subsection, we can write the dimensionless Hubble parameter $E(z)$ as:

$$E^2(z) = \Omega_{b0}(1 + z)^3 + \Omega_c0F(z) + \Omega_{r0}(1 + z)^3 + \Omega_{K0}(1 + z)^2 + \left[\omega_1 + \omega_3(1 + z)^4 + \omega_4(1 + z)^6\right],$$

where as usual

$$F(z) = \left[A_s + (1 - A_s)(1 + z)^{(1+\beta)}\right].$$

In (35) we have introduced the dimensionless parameters $\omega_1$, $\omega_3$ and $\omega_4$, related to the model parameters $\sigma_1$, $\sigma_3$ and $\sigma_4$ through:

$$\omega_1 = \frac{\sigma_1}{6H_0^2},$$

$$\omega_3 = \frac{\sigma_3H_0^2\Omega_{K0}^2}{6},$$

$$\omega_4 = -\frac{\sigma_4\Omega_{K0}}{6}.$$  

Furthermore, we consider the combination $\omega_4$ to be positive, in order to ensure that the Hubble parameter is real for all redshifts (note that $\omega_4 > 0$ is required also for the stability of the gravitational perturbations of the theory [11]). For convenience we moreover assume $\sigma_3 \geq 0$, that is $\omega_3 \geq 0$.

The present scenario involves the following parameters: $H_0$, $\Omega_{b0}$, $\Omega_{c0}$, $\Omega_{r0}$, $\Omega_{K0}$, $\omega_1$, $\omega_3$, $\omega_4$, $A_s$ and $\beta$. Similarly to the detailed-balance case, we fix the parameters $\Omega_{m0}$, $\Omega_{b0}$, $H_0$ and $\Omega_{r0}$ at their 7-year WMAP best-fit values [148]. Thus, the remaining free parameters are $\Omega_{K0}$, $\omega_1$, $\omega_3$ and $\omega_4$, $A_s$ and $\beta$, which are subject to the same two constraints as discussed in the previous subsection. The first one arises from the Friedman equation at $z = 0$, which leads to

$$\Omega_{m0} + \Omega_{r0} + \Omega_{K0} + \omega_1 + \omega_3 + \omega_4 = 1.$$  

We use this constraint to eliminate the parameter $\omega_1$.

The second constraint arises from BBN considerations. The term involving $\omega_3$ represents the usual dark-radiation component. In addition, the $\omega_4$-term represents a kination-like component (a quintessence field dominated by kinetic energy [120, 121]), also called a "stiff fluid". If $\Delta N_\nu$ represents the BBN upper limit on the total energy density of the universe beyond standard model constituents, then as shown in the Appendix of [77], we have the following constraint at the time of BBN ($z = z_{\text{BBN}}$) [123, 125]:

$$\omega_3 + \omega_4(1 + z_{\text{BBN}})^2 = \omega_{3\text{max}} \equiv 0.135\Delta N_\nu\Omega_{r0}. \quad (39)$$

It is clear that BBN imposes an extremely strong constraint on $\omega_4$, since its largest possible value (corresponding to $\omega_3 = 0$) is $\sim 10^{-24}$. This is in agreement with precision constraints on stiff fluid densities at the time of BBN derived in [126]. Finally, $\omega_{3\text{max}}$ denotes the upper limit on $\omega_3$. In the following, we use expression (39) to eliminate $\omega_4$. For convenience, instead of $\omega_3$ we define a new parameter

$$\alpha \equiv \frac{\omega_3}{\omega_{3\text{max}}},$$

which has the interesting physical meaning of denoting the ratio of the energy density of the Hořava-Lifshitz dark radiation to the total energy density of Hořava-Lifshitz dark radiation and kination-like components at the time of BBN.

In summary, in the scenario at hand we have the following five free parameters: $\Delta N_\nu$, $\Omega_{K0}$, $\alpha$, $A_s$ and $\beta$. We use SNIa, BAO and CMB data to perform a likelihood analysis, and we construct the likelihood contours for the parameters $A_s$ and $\Delta N_\nu$ for various fixed choices of the other parameters $\alpha$, $\beta$ and $\Omega_{K0}$.

In Fig. 2 we present the contours of $A_s$ and $\Delta N_\nu$ obtained by varying $\alpha$ around its best-fit value, while keeping $\beta$ and $\Omega_{K0}$ fixed at their best-fit values. We find that as $\alpha$ increases, the contours shrink and rotate counter-clockwise, placing tighter constraints on $\Delta N_\nu$.

In Fig. 3 we present the contours of $A_s$ and $\Delta N_\nu$ for four different choices of spatial curvature, with $\alpha$ and $\beta$ fixed at their best-fit values. We observe that curvature constraints are quite tight. For example the 1-$\sigma$ regions shrink as $\Omega_{K0}$ increases from 0.001 to 0.002 (as shown in the figure) and they almost completely disappear for $\Omega_{K0} \gtrsim 0.003$ (which is not shown).

In Fig. 4 we show the likelihood contours of $A_s$ and $\Delta N_\nu$, with $\alpha$ and $\Omega_{K0}$ fixed to their best-fit values, and two extreme choices of $\beta$. We deduce that the contours expand slightly for the larger value of $\beta$.

Finally, the best-fit values, along with the corresponding 1-$\sigma$ confidence intervals are presented in Table II.

| Parameter | Value  |
|-----------|--------|
| $A_s$     | $2.47 \times 10^{-10}$; (0, 2) |
| $\Delta N_\nu$ | $5.23 \times 10^{-4}$; (0, 2.0) |
| $\alpha$ | $5.17 \times 10^{-7}$; (0, 1) |
| $\Omega_{K0}$ | $1.29 \times 10^{-3}$; (0.003, 0.003) |
| $\beta$ | $4.89 \times 10^{-5}$; (0, 1) |

Table II: Best-fit values and 1-$\sigma$ confidence intervals for the free parameters $A_s$, $\Delta N_\nu$, $\alpha$, $\Omega_{K0}$ and $\beta$, in the beyond-detailed-balance scenario, arising from a likelihood analysis using SNIa, BAO and CMB data.
FIG. 2: (Color online) **Beyond Detailed Balance**: Likelihood contours of \( A_s \) and \( \Delta N_\nu \) for four choices of \( \alpha \), with the other parameters fixed at their best-fit values. Color scheme as in Fig. 1.

FIG. 3: (Color online) **Beyond Detailed Balance**: Likelihood contours of \( A_s \) and \( \Delta N_\nu \) for four choices of \( \Omega_{K0} \), with the other parameters fixed to their best-fit values. Color scheme as in Fig. 1.
FIG. 4: (Color online) **Beyond Detailed Balance:** Likelihood contours of $A_s$ and $\Delta N_\nu$ for two extreme different choices of $\beta$, with the other parameters fixed to their best-fit values. Color scheme as in Fig. 1.

V. COSMOLOGICAL IMPLICATIONS

In the previous sections we constructed cosmological scenarios of a generalized Chaplygin gas in a universe governed by Hořava-Lifshitz, and we used observational data in order to impose constraints on the model parameters. In recent past, people have discussed different cosmological scenarios in Hořava-Lifshitz gravity. Maeda et al. [127] have recently studied the bouncing as well as oscillating universe in Hořava-Lifshitz gravity in presence of both curvature and cosmological constant. One very interesting feature of their study is the possibility of a quantum tunneling from the oscillating spacetime to an inflationary scenario. In another interesting work, Bertolami and Zarro [128] have discussed quantum cosmology via minisuperspace model for projectable Hořava-Lifshitz gravity without detailed balanced condition. They considered the presence of a cosmological constant and when it is positive they recovered the classical GR solution with accelerating expansion. In the present section we discuss some of the cosmological implications in our case where we consider the presence of generalised Chaplygin gas. In particular, we focus on the evolution of the dimensionless expansion rate defined in (28), and on the evolution of the equation-of-state parameter of the total cosmic fluid of the universe, defined as $w = p_{tot}/\rho_{tot}$, with the total pressure and energy density given by

$$p_{tot} = p_c + \frac{1}{3}p_r + \frac{2}{K^2} \left[ \frac{K^2}{\Lambda a^4} - 3\Lambda \right]$$

(41)

$$\rho_{tot} = \rho_c + \rho_b + \rho_r + \frac{2}{K^2} \left[ \frac{3K^2}{\Lambda a^4} + 3\Lambda \right]$$

(42)

for the detailed-balanced scenario, and

$$p_{tot} = p_c + \frac{1}{3}p_r + \left[ \frac{\sigma_1}{6\sigma_0} + \frac{\sigma_3 K^2}{18\sigma_0 a^4} + \frac{\sigma_4 K}{6\sigma_0 a^6} \right]$$

(43)

$$\rho_{tot} = \rho_c + \rho_b + \rho_r + \left[ \frac{\sigma_1}{6\sigma_0} + \frac{\sigma_3 K^2}{6\sigma_0 a^4} + \frac{\sigma_4 K}{6\sigma_0 a^6} \right]$$

(44)

for the beyond-detailed-balance one, as it can be easily extracted from the corresponding Friedmann equations. Therefore, replacing the scale factor by the redshift, and using the density parameters at present, as well as the Hubble parameter as a function of the redshift (from relations and ) for the detailed and beyond detailed-balance case respectively, we straightforwardly acquire the total equation-of-state parameter as a function of the redshift $w(z)$.

A. Detailed Balance

Let us first examine the evolution of the total equation-of-state parameter $w(z)$ of the universe. In order to make our presentation simpler we will use the best-fit value set of Table 1 as our fiducial choice for the various model parameters, denoting by $w_{bf}(z)$ the corresponding $w(z)$. That is, the subscript “bf” for a parameter will imply the best-fit value of the parameter and for a variable will imply that all parameters determining that variable have been fixed to their best-fit values. In Fig. 5 we present the evolution of $w_{bf}(z)$.

As we observe, at very early times the cosmic equation of state is close to $1/3$, since the standard-model radiation and the dark radiation from the Hořava-Lifshitz gravitational background dominate, as expected. At intermediate redshifts, the Chaplygin gas dominates and it behaves like a dust for quite a long time, leading to an equation of state close to zero. Finally at late times, the de Sitter phase of the Chaplygin gas dominates. Interestingly, this evolution is consistent with the thermal history of our universe, with the succession of the radiation, matter, and “dark energy” epochs, and this feature acts as an advantage of the scenario at hand. Furthermore, note that we do not need any additional mechanism to describe the late-time universe acceleration (that is the dark energy), since this is obtained through a unified way by the generalized Chaplygin gas. That is, this crucial property of the Chaplygin gas is still exhibited by the present Hořava-Lifshitz embedded scenario.
We now focus on the evolution of the dimensionless expansion rate $E(z)$ defined in [23]. In order to acquire the basic information, in Fig. 6 we depict the evolution of the fiducial expansion rate $E_{bf}(z)$, that is with all parameters fixed to their best-fit values presented in Table I. As expected, $E_{bf}(z)$ starts from very large values, and it steadily decreases towards its present value 1.

It would be interesting to investigate how the expansion rate is affected by each parameter. In order to quantify this, we define the quantity $Q(z)$, which is the fractional change in the expansion rate $E_{bf}(z)$ as each parameter is varied within its allowed limits, while all the other parameters are kept fixed at their best-fit values, that is

$$Q(z) = \frac{E(z) - E_{bf}(z)}{E_{bf}(z)}.$$  \hspace{1cm} (45)

Note that $A_s$ and $\beta$ have well-defined upper and lower limits, while for $\Delta N_{\nu}$ we use the standard BBN upper limit of $\Delta N_{\nu} < 2.0$ [122]. In Fig. 7 we present $Q(z)$ for all the parameters of the scenario. As we see, both $A_s$ and $\Delta N_{\nu}$ can have a strong impact on the expansion rate, while $\beta$ has a very weak impact. The fact that the effect of $\beta$ is small was expected, since in the contour plots of Fig. 1 we did not obtain a significant change upon varying $\beta$.

Finally, from Fig. 7 we deduce that the impact of both the generalized Chaplygin gas parameters $A_s$ and $\beta$ is realized mainly at low redshifts, while it weakens at high ones. This is explained by the fact that at high redshifts the Chaplygin gas is subdominant and thus the expansion dynamics is driven by the (standard-model and dark) radiation. On the other hand, $\Delta N_{\nu}$, which is directly related to the amount of dark radiation, can have a significant impact on the expansion rate at arbitrarily high redshifts.

### B. Beyond Detailed Balance

Let us now repeat the analysis of the previous subsection, in the beyond-detailed-balance version of the examined scenario.

In Fig. 8 we depict the evolution of $w_{bf}(z)$, that is the total equation-of-state parameter of the universe, with all the model parameters fixed to their best-fit values presented in Table II. As we observe, the behavior of $w_{bf}(z)$ is similar to that of the detailed-balance case, that is we obtain an initial radiation era, a relatively long matter one, and finally an era of accelerated expansion.

Concerning the dimensionless expansion rate of the universe $E(z)$, in Fig. 9 we depict its fiducial value $E_{bf}(z)$ as a function of the redshift. Again we see that $E_{bf}(z)$ starts from very large values and it steadily decreases, compatibly with observations.
FIG. 7: (Color online) **Detailed Balance**: Fractional variation $Q(z)$ of the dimensionless Hubble parameter $E_{bf}(z)$, as each of the parameters $A_s$, $\Delta N_\nu$ and $\beta$ varies within its allowed limits as indicated in the graphs.

FIG. 8: (Color online) **Beyond Detailed Balance**: Evolution of the fiducial total equation-of-state parameter of the cosmic fluid $w_{bf}(z)$, with all parameters fixed to best-fit values presented in Table II.

FIG. 9: (Color online) **Beyond Detailed Balance**: Evolution of the fiducial dimensionless Hubble parameter $E_{bf}(z)$, with all parameters fixed to their best-fit values presented in Table II.
We now proceed to the investigation of the effect that the various model parameters have on the expansion rate. In Figures 10, 11, 12, and 13 we demonstrate the fractional change $Q(z)$ of the expansion rate $E_{bf}(z)$ as each parameter is varied within its allowed limits, as it is indicated in the graphs, while all other parameters are kept fixed at their best-fit values. From Fig. 10 we deduce that $A_s$ can have a strong impact on the expansion rate at low redshifts, while from Fig. 11 we see that $\Delta N_{\nu}$ can affect significantly the expansion rate at large redshifts. On the other hand, form Figures 12, 13 and 14 we observe that $\Omega_{k0}$ and especially the parameters $\alpha$ and $\beta$ have negligible impacts. Similarly to the detailed balance case, the fact that the effect of $\beta$ is small was expected, since in the contour plots of Fig. 4 we did not obtain a significant change upon varying $\beta$.

The explanation of the effect of the various parameters...
on the expansion rate is easily obtained, knowing the behavior of the various terms as a function of the redshift. In particular, at high redshifts the radiation (standard-model and dark one) dominates and drives the expansion, while the Chaplygin gas is subdominant. However $\Delta N_{0}$, which quantifies the Hořava-Lifshitz dark radiation and kination-like components, can have a significant impact on the expansion rate at arbitrarily high redshifts.

VI. CONCLUSIONS

In this work we investigated the cosmological scenario of generalized Chaplygin Gas in a universe governed by Hořava-Lifshitz gravity, both in the detailed-balance as well as in the beyond-detailed balance version of the theory. Furthermore, in order to obtain a realistic cosmology we have included the baryonic matter and standard-model radiation sectors.

After extracting the cosmological equations, we used data from SNIa, BAO and CMB observations, as well as arguments from Big Bang Nucleosynthesis, in order to impose constraints on the various parameters of the scenario. For the detailed-balance case we have three free parameters, namely the exponent $\beta$ of the generalized Chaplygin Gas, its present equation-of-state parameter $A_{s}$, and the effective neutrino parameter $\Delta N_{\nu}$ which quantifies the total amount of Hořava-Lifshitz dark-radiation and kination-like components allowed during BBN. The best fit values of the parameters together with the corresponding $1\sigma$ confidence intervals, arisen from the likelihood analysis for open and closed universe, were shown in Table I while in Fig. 1 we presented the corresponding likelihood contours. We deduced that the scenario at hand is compatible with observations, however the data lead to strong bounds on $A_{s}$ and $\Delta N_{\nu}$ and the spatial curvature $\Omega_{K0}$. Finally, the scenario at hand depends only slightly on $\beta$.

In the beyond-detailed-balance scenario the free parameters are five, namely $\beta$, $A_{s}$, $\Delta N_{\nu}$, the curvature density $\Omega_{K0}$, and the parameter $\alpha$, which is the ratio of the dark-radiation energy density to the sum of dark-radiation and kination-like energy densities at the time of BBN. The best fit values of the parameters together with the corresponding $1\sigma$ confidence intervals, arisen from the likelihood analysis, were shown in Table II. Additionally, in Figures 2-4 we presented the likelihood contours of the model parameters. From this analysis we deduced that the entire (consistently with BBN) range of $0 \leq \Delta N_{\nu} \leq 2.0$ is allowed, for suitable choices of the other parameters. We also found tight constraints on the curvature and on $A_{s}$.

After the observational elaboration, we investigated the cosmological implications of the examined scenarios. In particular, we focused on the evolution of the total equation-of-state parameter of the universe, and of the expansion rate. In the detailed-balance case, we saw that in general at very early-times is radiation that dominates, at intermediate redshifts is the Chaplygin gas that dominates, behaving like matter for a long time, and finally at late times the dominant Chaplygin gas behaves like dark energy, triggering the accelerating expansion (see Fig 5). This evolution is consistent with the thermal history of the universe, and this is an advantage of the present scenario. Note moreover that the present accelerated era is described in a unified way, without the need of any additional mechanism. Concerning the qualitative dependence on the various parameters, we found that the expansion rate has a strong dependence on both $A_{s}$ and $\Delta N_{\nu}$ at low redshifts, while at high redshifts, where the Chaplygin gas is not dominant, this dependence weakens and disappears (see Fig 7). Lastly, the dependence on $\beta$ is very weak at all redshifts.

In the beyond-detailed-balance scenario we found a similar behavior for the total equation-of-state parameter of the universe, that is we obtained successively a radiation, a matter, and a dark energy era (see Fig 8). Qualitatively, $A_{s}$ has a weak impact on the expansion rate at lower redshifts, $\Delta N_{\nu}$ has a significant effect at large redshifts, while $\Omega_{K0}$ and especially $\alpha$ and $\beta$ have a negligible impact on the expansion rate at all redshifts (see Figures 10-14).

In summary, the generalized Chaplygin gas in Hořava-Lifshitz gravitational background is compatible with observations, and can successfully reproduce the expansion history of the universe. However, we should mention that the present analysis does not enlighten the discussion about the possible conceptual problems and instabilities of Hořava-Lifshitz gravity, nor it can address the questions concerning the validity of its theoretical background, which is the subject of interest of other studies. It just analyzes the phenomenological consequences and the cosmological implications of the generalized Chaplygin gas in such a gravitational background, and thus its results can be taken into account only if Hořava-Lifshitz gravity passes successfully the necessary theoretical tests.

In the same lines, the present work does not address the problem of undesirable instabilities and oscillations in the matter power spectra, that GCG scenario faces in standard Einstein gravity. Although, as discussed in the Introduction, there are several approaches which try to solve this problem in conventional gravity, it may be quite interesting to see whether it can be addressed in the context of Hořava-Lifshitz gravity. Let us make some comments in this perspective. As discussed in [98, 102], GCG can be modelled in terms of scalar fields having both canonical as well as non-canonical kinetic energy terms, and thus perturbing the GCG one has to essentially perturb these scalar fields. In the case of Hořava-Lifshitz gravity one expects in general the linear perturbations to be quite different due to higher-order curvature-terms present in the action. These higher-order gradient terms can in principle generate non-adiabatic pressure perturbations, which can help to cure the instabilities or oscillations in the power spectra even in the large scales. The effect can be prominent with scalar fields having
non-canonical kinetic energy terms, for instance of Dirac-Born-Infeld form, which is typical for GCG-like equation of state.

In conclusion, the generalized Chaplygin gas scenario in Hořava-Lifshitz cosmology, can have rich cosmological consequences. The present study is a first step in this direction, where we have considered the background evolution and have confronted the model with a variety of currently available observational data. The next step is to consider the inhomogeneous GCG model in Hořava-Lifshitz gravity, which will be our goal in the near future.

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