Wind Turbine Power Curve Modelling Based on Hybrid Relevance Vector Machine

Bo Jing\textsuperscript{1,}\textsuperscript{*}, Zheng Qian\textsuperscript{1}, Anqi Wang\textsuperscript{1}, Tianyang Chen\textsuperscript{1} and Fanghong Zhang\textsuperscript{2}

\textsuperscript{1} School of Instrumentation and Optoelectronic Engineering, Beihang University, Haidian Dist., Beijing, China
\textsuperscript{2} CSIC HaiZhuang Wind power Co., Ltd, Yubei Dist., Chongqing, China

\textsuperscript{*}Email: jingbo@buaa.edu.cn

Abstract. Wind turbine power curve (WTPC) is important for energy assessment, condition monitoring and abnormal detection. In recent years, researchers proposed a number of WTPC modelling approaches to continuously improve the model performance. In this paper, Relevance Vector Machine (RVM) is applied for WTPC modelling for the first time. Combine single-input RVM and multi-input RVM, this paper proposes a hybrid RVM method (HRVM) to further improve the fitting accuracy. Firstly, we analyse the features of model outputs of both single-input RVM and multi-input RVM. According to the analysis, the confidence interval of single-input RVM is used to limit the power output range of multi-input RVM. At last, SCADA data collected from three wind turbines are used to test the model performance. The results show that, compared with typical WTPC model approaches, HRVM achieves a good balance between fitting accuracy and computation cost.

1. Introduction

Wind turbine power curve (WTPC) as a function of wind speed and power generation of wind turbine, which has great significance for energy assessment, condition monitoring and abnormal detection [1, 2]. Recent literatures proposed different types of WTPC modelling approaches, which can be mainly divided into parametric approaches and non-parametric approaches. Polynomial regression (PR) as a simple but effective parametric fitting approach has been wildly used [3]. Besides, Logistic functions are proved to have good fitting accuracy. Compared with four-parameter Logistic function (4PL), five-parameter Logistic function (5PL) has better fitting accuracy [4]. Other parametric approaches such as hyperbolic tangent functions, maximum principle method and power coefficient based model were also applied for power curve fitting [3, 5, 6]. In general, parametric approaches have the advantages of simple structure and fast training speed, but they are sensitive to anomalies within the training data, and has poor robustness. Non-parametric approaches do not impose any prespecified expressions, the modelling process is entirely depends on training data. Among them, Method of Bin (MOB) is the most commonly used approach for wind farm operators [7]. But the fitting accuracy often fails to meet the requirements, especially at the connection point of the straight line. In order to improve the model performance, researchers proposed different interpolation methods instead of MOB, including Fourier series interpolation and spline interpolation [8]. With the expansion of computer science, machines learning techniques are gradually applied for power curve fitting. [9] used Gaussian process (GP) for data preprocessing, then WTPC is created by neural networks. In [10], both Fuzzy Cluster, BP neural network (BPNN) and Monte Carlo were used for WTPC modelling. In addition to wind speed, [11]...
added yaw angle, pitch angle, rotor speed and wind direction as the model input. Then, stochastic gradient boosted regression trees was utilized to build the power curves. The results show that the fitting accuracy increases with the increase of input parameters. According to [10, 12], machine learning based models has strong nonlinear generalization capabilities in power curve modelling. However, these approaches usually have complex modelling process with large calculation cost, especially for multi-input WTPC modelling approaches.

This paper proposes a WTPC modelling approach based on hybrid Relevance Vector Machine (HRVM). The main contributions are: 1) as a Bayesian framework based probabilistic supervised learning method, RVM is applied for WTPC modelling for the first time; 2) we use the confidence interval of single-input RVM to correct the fitted outliers of multi-input RVM. The experimental results show that, compared with typical WTPC modelling approaches, HRVM achieves a good balance between fitting accuracy and training consumption.

2. Methodology

2.1. Relevance vector machine

Relevance vector machine was first proposed by Michael E. Tipping [13]. Based on Bayesian framework, it can obtain sparse solutions for regression and classification. The basic theory of RVM is briefly introduced in this part.

Assume that the targets are samples from the model with additive noise, the probability expression of the target function is defined as

$$p(t_n) = N(t_n | y(x_n; w), \sigma^2)$$

(1)

Where $N(.)$ represents Gaussian distribution density function; $t_n$ is the target value; $w$ is the weight vector; $x_n$ is the input vector. Therefore, the likelihood of the training set is

$$p(t | w, \sigma^2) = (2\pi \sigma^2)^{-\frac{N}{2}} \exp\left(-\frac{||t - \Phi w||^2}{2\sigma^2}\right)$$

(2)

Where $\Phi$ is the matrix of kernel function. In relevance vector regression, Gaussian prior distribution is used over $w$ to avoid the over-fitting problem caused by a variety of model parameters. In this case, equation (2) can be expressed as

$$p(w | \alpha) = \prod_{i=0}^{N} \frac{\alpha_i}{\sqrt{2\pi}} \exp\left(-\frac{\alpha_i w_i^2}{2}\right)$$

(3)

Where $\alpha$ is hyperparameter vector, which is the key feature of RVM. According to Gaussian convolution and a series of simplification calculation, the predictive distribution is expressed as

$$p(t* | t) = \int p(t_n | w, \alpha_{MP}, \sigma^2_{MP}) \cdot p(w | \alpha_{MP}, \sigma^2_{MP}) dw$$

(4)

Where $t*$ is the new test target; $\alpha_{MP}$ and $\sigma^2_{MP}$ is the most-probable values of $\alpha$ and $\sigma^2$, which can be solved by iterative calculation. Based on the fore-mentioned equations, when a new input vector $x*$ is entered, we calculate the predicted mean value $y*$ and variance $\sigma^2*$ by

$$y* = (\sigma^2_{MP} \sum \Phi^T \Phi)^{-1} \cdot \Phi^T \phi(x*)$$

$$\sigma^2* = \sigma^2_{MP} + (\phi(x*)^T \sum \phi(x*))^{-1}$$

$$\sum = (\sigma^2_{MP} \Phi^T \Phi + \text{diag}(\alpha_0, ..., \alpha_N))^{-1}$$

$$\phi(x*) = [1, K(x*, x_1), ..., K(x*, x_N)]^T$$

(5)

2.2. Hybrid relevance vector machine

On the basis of RVM, this paper proposes a hybrid RVM (HRVM) for WTPC modelling. Figure 1 shows the flow chart.

Where the $\mu$ is the mean value of the fitted power output; $\sigma$ is the standard deviation; $\eta$ is the hyperparameter defined in HRVM.
2.2.1. **Data preprocessing.** With the development of SCADA systems, more status parameters are monitored, which can be directly used for data filtering. Accordingly, a simple but effective data preprocessing method is applied based on SCADA parameters. Firstly, we use the operation mode and the wind curtailment mode to filter the stacked outliers, since most of them are caused by operation status and power limitation of wind farm. Then, wind speed and power output are used to filter the unreasonable data points. Table 1 lists the detailed filter conditions for data preprocessing.

| Parameter name                  | Select value                              | Unit     |
|---------------------------------|-------------------------------------------|----------|
| Operation mode                  | 32, (normal working state)                | N/A      |
| Wind curtailment mode           | \(P_{\text{rated}}\), (without wind curtailment) | kW       |
| Wind speed                      | \(V \in [V_{\text{cut-in}}, V_{\text{cut-off}}]\) | m/s      |
| Power output                    | \(P \in [P_{\text{rated}} \times 1\%, P_{\text{rated}} \times 120\%]\) | kW       |

In Table 1, the operation mode describes the working state of wind turbine; wind curtailment mode describes whether the wind turbine is in power limit state. Specific parameter settings depend on the type of wind turbine. Although there are still some spark outliers after data filtering, due to the robustness of RVM, they have little impact on the fitting accuracy of WTPC.

According to [14], power expression of wind turbine can be written as

\[
P = 0.5 \cdot C_p(\lambda, \beta) \rho A v^3
\]

(7)

Where \(\rho\) is air density; \(A\) is impeller sweeping area; \(C_p\) as a function of \(\lambda\) and \(\beta\), is power coefficient; \(\beta\) is pitch angle; \(\lambda = \omega R/v\) (\(\omega\) is rotor speed, \(R\) is radius of impeller). For a specific wind turbine, we can approximate equation (7) to

\[
P = C_p(\beta, \omega, v) \cdot v^3 \cdot C
\]

(8)

Where \(C\) is the constant. According to equation (8), wind speed, rotor speed, and pitch angle are selected as the input parameters of multi-input RVM.

2.2.2. **HRVM.** After data preprocessing, both single-input RVM (SRVM) and multi-input RVM (MRVM) are trained separately. Figure 2 shows the fitting results in one of the test wind turbines.

As shown in Figure 2, WTPC based on SRVM is a deterministic curve, which approximately reflects the performance of the target wind turbine. However, SRVM has inevitable systematic errors, due to there is no deterministic relationship between wind speed and power output [10]. Compared with SRVM, WTPC based on MRVM generally has better fitting accuracy. However, some fitted
outliers occurred during the test process, the main reason is that the complex mapping relationship of multi-input model, which leads to the wrong training.

In order to improve the model performance, we use the confidence interval (CI) of SRVM to limit the power output range of MRVM, expressed as

$$\text{CI} = [\mu_{\text{single}} - \eta \cdot \sigma_{\text{single}}, \mu_{\text{single}} + \eta \cdot \sigma_{\text{single}}]$$

(9)

Where $\mu_{\text{single}}$ is the predicted power output of SRVM; $\sigma_{\text{single}}$ is the predicted standard deviation of SRVM. Once $\eta$ is determined, a unique CI is generated.

![Figure 2](image1.png)  ![Figure 3](image2.png)

**Figure 2.** Fitting results of both single-input RVM and multi-input RVM.

**Figure 3.** Relationship between multi-input RVM and the CI of single-input RVM.

In Figure 3, we define the predicted power outside CI as the fitted outliers, and limit the amplitude of them to the boundary of CI (as shown in Figure 1). Hyperparameter $\eta$ determines the width of CI, thus it has a great impact on model performance. Theoretically, $\eta \in [0, +\infty]$, if $\eta=0$, the predicted power of HRVM is equal to SRVM. On the contrary, if $\eta=+\infty$, the predicted power of HRVM is equal to MRVM. Therefore, we can use one-dimensional optimization method such as Golden Section Search to find the optimal value of $\eta$. The fitting result of HRVM is shown in Figure 4.

![Figure 4](image3.png)

**Figure 4.** Fitting results of Hybrid RVM.

Compared with using MRVM alone (as shown in Figure 2), the edges of predicted power outputs of HRVM are smoother, and the fitted outliers are effectively corrected.

3. Case study

In this study, three wind turbines from different wind farms are selected for testing. Among them, WT1 is located in a plain wind farm, WT2 and WT3 is located in hilly wind farms. All wind turbines are horizontal axis wind turbines with an installed capacity of 2MW. Data collection period is from 6/1/2018 to 8/31/2018 and the first 60 days of data for training, the rest of the data is for testing. According to IEC 61400-12-1 [7], 10-min-average of SCADA data is used for modelling. To evaluate the model performance, we analyse the fitting accuracy and training time of eight WTPC modelling
approaches, including five typical power curving fitting method and three RVM based methods. Table 2 and Table 3 list the mean absolute percentage error (MAPE) and root mean square error (RMSE) of different WTPC approaches. The relationship between training set length and training time of WTPC modelling approaches is shown in Figure 5.

### Table 2. MAPE of different WTPC modelling approaches.

|        | 5PL | 9PR | BPNN | SVM | GP  | SRVM | HVM | HRVM |
|--------|-----|-----|------|-----|-----|------|-----|------|
| WT1    | 7.4[8] | 7.0[5] | 6.5[4] | **5.0[1]** | 7.1[6] | 7.1[6] | 6.3[3] | 6.2[2] |
| WT2    | 7.4[8] | 7.3[7] | 3.5[3] | **2.5[1]** | 4.5[5] | 7.1[6] | 4.2[4] | 3.4[2] |
| WT3    | 9.3[8] | 8.6[5] | 7.1[4] | **6.0[1]** | 8.6[5] | 8.7[7] | 7.0[3] | 6.3[2] |
| Average| 8.0[8] | 7.6[6] | 5.7[3] | **4.5[1]** | 6.7[5] | 7.6[6] | 5.8[4] | 5.3[2] |

[.] is the ranking of fitting accuracy.

### Table 3. RMSE of different WTPC modelling approaches.

|        | 5PL | 9PR | BPNN | SVM | GP  | SRVM | HVM | HRVM |
|--------|-----|-----|------|-----|-----|------|-----|------|
| WT1    | 42.9[5] | 43.5[6] | 39.5[4] | **35.2[1]** | 45.6[8] | 44.8[7] | 36.0[2] | 36.0[2] |
| WT2    | 67.6[8] | 65.6[7] | 29.9[3] | **27.6[2]** | 36.7[5] | 65.5[6] | 30.5[4] | **27.5[1]** |
| WT3    | 71.4[7] | 70.3[6] | 54.7[4] | **48.1[1]** | 52.0[3] | 71.4[7] | 65.4[5] | 49.9[2] |
| Average| 60.6[7] | 59.8[6] | 41.3[3] | **37.0[1]** | 44.8[5] | 60.6[7] | 44.0[4] | 37.8[2] |

[.] is the ranking of fitting accuracy.

In Table 2 and Table 3, 5PL is five-parameter logistic functions; 9PR is 9-order Polynomial regression; SVM is support vector machine and GP is Gaussian process. Among them, 5PL, PR and SRVM are single-input (wind speed only) WTPC modelling approaches and the others are multi-input models (wind speed, rotor speed and pitch angle). The results show that: 1) WTPC models with multi-inputs generally has better fitting accuracy than single-input ones, among them, SVM has the lowest MAPE and RMSE while the fitting accuracy of the proposed HRVM is close to SVM; 2) HRVM can effectively correct the fitting outliers of MRVM, compared with MRVM the average MAPE and RMSE of HRVM decreased by 8.6% and 14.1% respectively.

![Figure 5. The relationship between training set length and training time of WTPC modelling approaches.](image)

In Figure 5, 5PL, 9PR are not shown due to their training time is negligible. Although 5PL and 9PR have small training consumption, the fitting accuracy is lower than multi-input WTPC approaches. As mentioned above, SVM has the best fitting accuracy, but the computation cost is large. The training time of SVM for 3000 data points is 57 seconds, which is over 33 times than that of HRVM. GP has the longest training time for 3000 data points (215 seconds), because the inverse process of covariance matrix consumes a lot of computing resources. In general, HRVM achieves a good balance between fitting accuracy and computation cost, it has the best comprehensive performance.
4. Conclusion
This paper proposes a WTPC modelling approach based in HRVM. Firstly, as a supervised learning method with strong generalization ability and high training speed, RVM is used for power curve fitting for the first time. Secondly, we use the CI of SRVM to limit the output range of MRVM. Compared with using MRVM alone, the average MAPE and RMSE of HRVM decreased by 8.6% and 14.1% respectively. At last, we make a comparative analysis of eight WTPC models by measured data collected from three wind turbines in different wind farms. The results show that WTPC modelling approaches with multi-inputs generally has better fitting accuracy than single-input ones. Among them, HRVM achieves a good balance between fitting accuracy and computation cost, which has the best comprehensive performance.

Acknowledgement
This work is supported by the National Natural Science Foundation of China (No. 61573046) and Program for Changjiang Scholars and Innovative Research Team in University (No. IRT1203).

References
[1] Manwell JF, McGowan JG and Rogers AL 2009 Wind energy expland (United Kingdom: AJohn Wiley and Sons, Ltd.)
[2] Pandit RK and Infield D 2018 SCADA-based wind turbine anomaly detection using Gaussian process models for wind turbine condition monitoring purposes IET Renew. Power Gen. 12 1249-1255
[3] Lydia M, Suresh Kumar S, Immanuel Selvakumar A and Edwin Prem Kumar G 2014 A comprehensive review on wind turbine power curve modeling techniques Renew. Sust. Energy Rev. 30 452-460
[4] Lydia M, Immanuel Selvakumar A, Suresh Kumar S and Edwin Prem Kumar G 2013 Advanced algorithms for wind turbine power curve modeling IEEE Trans. on Sust. Energy 4 827-835
[5] Taslimi-Renani E, Modiri-Delshad M, Elias MFM and Rahim NA 2016 Development of an enhanced parametric model for wind turbine power curve Appl. Energy 177 544-552
[6] Teyabeen AA, Akkari FR and Jwaid AE 2017 Power curve modelling for wind turbines IET Renewable Power Gen. 12 1249-1255
[7] IEC 61400-12-1 2013 Wind turbines-Part 12-1 Power performance measurements of electricity producing wind turbines (Commission IET)
[8] Ciulla G, D’ Amico A, Di Dio V and Lo Brano V 2019 Modelling and analysis of real-world wind turbine power curves: Assessing deviations from nominal curve by neural networks Renew. Energy 140 477-492
[9] Manobel B, Sehnke F, Lazzúis JA, Salfate I, Felder M and Montecinos S 2018 Wind turbine power curve modeling based on Gaussian processes and artificial neural networks Renew. Energy 125 1015-1020
[10] Yan J, Zhang H, Liu Y, Han S and Li L 2019 Uncertainty estimation for wind energy conversion by probabilistic wind turbine power curve modelling Appl. Energy 239 1356-1370
[11] Janssens O, Noppe N, Devriendt C, Walle RVD and Hoecke SV 2016 Data-driven multivariate power curve modeling of offshore wind turbines Engineering App. of Artificial Intel. 55 331-338
[12] Hu Y, Qiao Y, Liu J and Zhu 2019 Adaptive confidence boundary modeling of wind turbine power curve using SCADA data and its application IEEE Trans. on Sust. Energy 10 1330-1341
[13] Tipping ME 2001 Sparse Bayesian learning and the relevance vector machine J. of Machine Learning Research 1 211-244
[14] Dai J, Yang X, Hu W, Wen L and Tan Y 2018 Effect investigation of yaw on wind turbine performance based on SCADA data Energy 149 683-696