Shear modulus anomaly of multi-component superconductor: the case of \( M_x\text{Bi}_2\text{Se}_3 \)

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Abstract. A pair of split superconducting transitions in the presence of a symmetry breaking field is a very definitive signature of multi-component superconductivity[1, 2]. We theoretically study the shear modulus anomaly across such pair of split transitions[3]. The talk will be focused on \( M_x\text{Bi}_2\text{Se}_3 \), a nematic superconductor candidate, for which no experimental confirmation of the split transition has been made so far. We propose that the shear modulus \( C_{66} \) must vanish at the lower transition: a very clear signature detectable by experiments. The observation of shear modulus anomalies would be a conclusive test for the nematic superconductivity hosted by the material.

1. Introduction

\( \text{Bi}_2\text{Se}_3 \) is an archetypical topological insulator[4, 5]. By intercalating with electron donors (\( M = \text{Cu}, \text{Sr} \) or \( \text{Nb} \)), the material become superconducting at low temperature[6, 7, 8, 9]. Numerous experiments with an applied magnetic field parallel to the basal plane indicate a two-fold symmetric response emerging in the superconducting phase[10, 11, 12, 13, 14], inconsistent with the trigonal crystal structure. This prompts the suggestion that the superconductivity may be nematic: along with the usual \( U(1) \) symmetry-breaking, it also breaks the lattice rotationally symmetry spontaneously.

Spontaneous symmetry breaking necessitates that the two-fold direction be randomly chosen from three equivalent high-symmetry directions every time a sample turns superconducting in zero field. In contrary, the two-fold direction for a given sample is persistent[15, 16, 17]. A pre-existing, symmetry-breaking pinning field must be added to the theory by hand to account for this effect[18]. This pinning field has been linked to small lattice distortions[15].

With the lattice rotational symmetry already broken in the normal phase by the pinning field, the original symmetry argument for the nematic superconductivity is lost. One is faced with a dilemma: either there is some exotic pairing mechanism[19, 20, 21] that explicitly disfavors the s-wave pairing, or the superconductivity is single-component (i.e. not nematic) yet unusually susceptible to directional perturbations. To settle the issue, we believe a directly test for the multi-component nature of the order parameter in some other way is needed. To this end, the present author and collaborator have examined the extended phase diagram of the material under applied stress[18], and the physics of half vortices in such a nematic superconductor[22]. As an s-wave order parameter is forbidden by symmetry to couple linearly couple with shear strain, the shear modulus anomaly may prove to be the most promising test yet.
2. Ginzburg-Landau theory for nematic superconductor

The order parameter $\vec{\eta}$ for a nematic superconductor can be thought of as a two-dimensional vector in the basal plane of the crystal. To be exact, the two-component quantity transforms in a two-dimensional irreducible representation of the lattice group $D_{3d}$ for $M_xBi_2Se_3$. The Ginzburg-Landau (GL) free energy compatible with the symmetry can be written down:

$$F_0 = a|\eta|^2 + \lambda|\eta|^4 + (\Gamma_1 + \Gamma_2 \cos 6\theta)|\eta|^6 \quad (1)$$

In fact $\vec{\eta}$ behaves as a double-headed vector, since an overall negative sign is the same as a global phase shift of $\pi$, and two opposite directions are equivalent. For concreteness we assume $\Gamma_2 > 0$, leading to the “natural preference” of $\theta = \pm \pi/6$ or $\pi/2$ in equilibrium in the absence of the pinning field.

The pinning field is experimentally reported to always favors either the $a$ or $a^*$ crystallographic direction. It can be modelled by a term in the GL theory:

$$F_\Delta = -\Delta |\eta|^2 \cos 2\theta. \quad (2)$$

The sign of $\Delta$ determines the favored direction and is sample-dependent. A positive (negative) $\Delta$ favors $\theta = 0$ ($\pi/2$).

When $\Gamma_2$ and $\Delta$ are of the same sign, the mismatch in their preferences splits the superconducting transition into two[1, 2]. We stress that both signs of $\Delta$ are naturally found in the samples[16, 17]: without any need of further tuning, some of the crystals must exhibit the physics discussed here. Consider cooling down from the normal phase. At the so-called upper transition, superconductivity sets in, and the order parameter is pinned at $\theta = 0$: see Figure 1. It should be noted that there is no spatial symmetry breaking here: the horizontal direction has already been singled out by the pinning field, and both the normal and the (upper) superconducting phases have only a two-fold spatial symmetry. The physics is controlled by the $O(\eta^2)$ term in the free energy. At a lower temperature, when the $O(\eta^6)$ terms become important, the competition between $\Delta$ and $\Gamma_2$ drives the so-called lower transition. The remaining two-fold symmetry is spontaneously broken, and the order parameter drifts away from $\theta = 0$ toward $\theta = \pm \pi/6$: see Figure 2. The double transitions are unique to a multi-component superconductor, but the observation of the lower transition can be challenging[18]. But we will presently show that the shear modulus anomaly signature is quite dramatic and, we believe, hard to miss in an experiment.

3. The interplay between shear strain and nematic superconductivity

The in-plane shear strain is coupled to the superconducting sector in the usual fashion:

$$\epsilon_1 = \epsilon_{xx} - \epsilon_{yy}, \quad \epsilon_2 = 2\epsilon_{xy}$$

$$F_\epsilon = -g|\eta|^2 \epsilon_1 \cos 2\theta - g|\eta|^2 \epsilon_2 \sin 2\theta + \frac{c_2}{2} (\epsilon_1^2 + \epsilon_2^2). \quad (3)$$

The geometry of $\epsilon_1$ and $\epsilon_2$ are sketched in Figure 3 and 4. We stress that a multi-component order parameter is required to even write down (3): the linear coupling of $\epsilon_1$ and $\epsilon_2$ to an s-wave order parameter is forbidden by symmetry. The $2 \times 2$ shear modulus tensor $c_{i,j}$ can be computed with the standard method; we refer the reader to ref [3] for the full treatment that leads to the plot Figure 5. However, the most important qualitative features can be understood using symmetry arguments alone, and we will attempt to do so in this paper.

Below the upper transition, $|\eta| \neq 0$ nucleated along the $\theta = 0$ direction, as shown in Figure 1. Either by comparing with the geometry of $\epsilon_1$ as depicted in Figure 3, or by consulting the
Figure 1. Upper superconducting phase. The order parameter is pinned at the horizontal by the symmetry-breaking pinning field.

Figure 2. Order parameter in the lower superconducting phase. The two-fold spatial symmetry is broken by the order parameter tilting away from the initial orientation.

Figure 3. In-plane shear strain component $\epsilon_1$.

Figure 4. In-plane shear strain component $\epsilon_2$.

Figure 5. The qualitative behavior of the shear modulus tensor $c_{i,j}$ in the $(\epsilon_1, \epsilon_2)$ basis. In Voigt’s notation, $c_{1,1} = (C_{11} + C_{22} - 2C_{12})/4$, $c_{2,2} = C_{66}$, and $c_{1,2} = (C_{16} - C_{26})/2$.

GL equation $\partial F/\partial \epsilon_1 = 0$, it is easy to convince oneself that $\epsilon_1$ is spontaneously induced by the superconductivity below the upper transition, with a definite sign determined by $\text{sgn}(g)$. A step discontinuity in $c_{1,1}$ is the result.

In contrast, $c_{2,2}$ merely shows a kink: $\epsilon_2$’s coupling to $\vec{\eta}$ implies that the response must change in the superconducting phase, but the two-fold symmetry forbids spontaneous generation of $\epsilon_2$ in equilibrium, resulting in a weaker anomaly. The off-diagonal $c_{1,2}$ is inconsistent with the two-fold symmetry and remains zero in the upper phase.

Below the lower transition, the equilibrium $\theta$ becomes non-zero, spontaneously breaking the remaining two-fold symmetry, as shown in Figure 2. As a result, $\epsilon_2$ (see Figure 4) is induced spontaneously below the transition. Furthermore, at the critical point, a deformation mode must become completely soft, as the crystal is ready to go one way or the opposite at any infinitesimal push, and it is easily seen that $\epsilon_2$ is such mode. This is in contrast with the upper transition, where no spatial symmetry is broken (the broken symmetry is $U(1)$ only), and no soft mode is resulted.

The component $c_{1,2}$ is no longer symmetry-forbidden, and becomes non-zero below the lower
transition. Its asymptotic behavior (the square root of reduced temperature) just below the transition point simply follows from the standard meanfield critical exponent.

4. Conclusion

In summary, we argue that more test for the nematic superconductivity hosted by $\text{M}_x\text{Bi}_2\text{Se}_3$ is needed, beyond the various two-fold anisotropic responses probed in existing experiments, and we believe that the shear modulus anomalies associated with the double superconducting transitions may be the best opportunity yet. Any observation of shear modulus anomaly constitutes strong support for multi-component superconductivity, but the vanishing of the shear modulus component $c_{2,2}$ at the lower transition is an especially dramatic signal that, we believe, is hard to miss in an experiment. We hope our findings help to shed new light on the nature of superconductivity hosted by $\text{M}_x\text{Bi}_2\text{Se}_3$.

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References

[1] Sigrist M, Joynt R and Rice T M 1987 Phys. Rev. B 36 5186–5198
[2] Volovik G E 1988 Pis’ma Zh. Eksp. Teor. Fiz. 48 39–42
[3] How P T and Yip S K 2021 Phys. Rev. B 104 L020506
[4] Zhang H, Liu C X, Qi X L, Dai X, Fang Z and Zhang S C 2009 Nat. Phys. 5 438–442
[5] Liu C X, Qi X L, Zhang H, Dai X, Fang Z and Zhang S C 2010 Phys. Rev. B 82 045122
[6] Hor Y S, Williams A J, Checkelsky J G, Roushan P, See J, Xu Q, Zandbergen H W, Yazdani A, Ong N P and Cava R J 2010 Phys. Rev. Lett. 104 057001
[7] Kriener M, Segawa K, Ren Z, Sasaki S and Ando Y 2011 Phys. Rev. Lett. 106 127004
[8] Shruti, Maurya V K, Neha P, Srivastava P and Patnaik S 2015 Phys. Rev. B 92 020506
[9] Asaba T, Lawson B J, Tinsman C, Chen L, Corbae P, Li G, Qiu Y, Hor Y S, Fu L and Li L 2017 Phys. Rev. X 7 011009
[10] Matano K, Kriener M, Segawa K, Ando Y and Zheng G Q 2016 Nat. Phys. 12 852–854
[11] Pan Y, Nikitin A M, Araogi G K, Huang Y K, Matsushita Y, Naka T and de Visser A 2016 Sci. Rep. 6 28632
[12] Yonezawa S, Tajiri K, Nakata S, Nagai Y, Wang Z, Segawa K, Ando Y and Maeno Y 2017 Nat. Phys. 13 123–126
[13] Smylie M P, Willa K, Claus H, Koshelev A E, Song K W, Kwok W K, Islam Z, Gu G D, Schneeloch J A, Zhong R D and Welp U 2018 Sci. Rep. 8 7666
[14] Sun Y, Kttaka S, Sakakibara T, Machida K, Wang J, Wen J, Xing X, Shi Z and Tamegai T 2019 Phys. Rev. Lett. 123 027002
[15] Kuntsevich A Y, Bryzgalov M A, Prudkoglyad V A, Martovitskii V P, Selivanov Y G and Chizhevskii E G 2018 New J. Phys. 20 103022
[16] Kuntsevich A Y, Bryzgalov M A, Akzhanov R S, Martovitskii V P, Rakhmanov A L and Selivanov Y G 2019 Phys. Rev. B 100 224509
[17] Kawai T, Wang C G, Kandori Y, Honoki Y, Matano K, Kambe T and Zheng G Q 2020 Nat. Commun. 11 235
[18] How P T and Yip S K 2019 Phys. Rev. B 100 134508
[19] Brydon P M R, Das Sarma S, Hui H Y and Sau J D 2014 Phys. Rev. B 90 184512
[20] Wan X and Savrasov S Y 2014 Nat. Commun. 5 4144
[21] Wang J, Ran K, Li S, Ma Z, Bao S, Cai Z, Zhang Y, Nakajima K, Ohira-Kawamura S, Čermák P, Schneidewind A, Savrasov S Y, Wan X and Wen J 2019 Nat. Commun. 10 2802
[22] How P T and Yip S K 2020 Phys. Rev. Research 2 043192