PQCD approach to exclusive $B$ decays$^1$

Hsiang-nan Li

Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China

Abstract

I review the recent progress on the perturbative QCD approach to exclusive $B$ meson decays, discussing the comparison of collinear and $k_T$ factorizations, the CP asymmetry in the $B^0 \to \pi^+\pi^-$ decay, penguin enhancement, the branching ratio of the $B^0 \to D^0\pi^0$ decay, and three-body nonleptonic decays.

1 Introduction

Both collinear and $k_T$ factorizations are the fundamental tools of perturbative QCD (PQCD), where $k_T$ denotes parton transverse momenta. For inclusive processes, such as deeply inelastic scattering (DIS) of a hadron, collinear ($k_T$) factorization applies when the process is measured at a large (small) Bjorken variable $x_B$ [1] (2, 3, 4). For exclusive processes, collinear factorization was developed in [5, 6, 7, 8], in which a physical quantity is written as the convolution of a hard amplitude with hadron distribution amplitudes in parton momentum fractions $x$. In the inclusive case the range of a momentum fraction $x \geq x_B$ is experimentally controllable. In the exclusive case the range of a momentum fraction is not controllable, and must be integrated over between 0 and 1. Hence, one must deal with the end-point region with a small $x$. If a hard amplitude does not develop an end-point singularity, collinear factorization works. If such a singularity occurs, collinear factorization breaks down, and $k_T$ factorization [1, 4] should be employed.

Based on the concepts of collinear and $k_T$ factorizations, the PQCD [11] and QCD factorization (QCDF) [12] approaches to exclusive $B$ meson decays have been developed, respectively. The soft-collinear effective theory is a sys-

---

$^1$ based on talks presented at Flavor Physics and CP Violation, Philadelphia, USA, May 2002, at the 3rd Workshop on Higher Luminosity B Factories, Kanagawa, Japan, Aug. 2002, and at Summer Institute 2002, Yamanashi, Japan, Aug. 2002
tematic framework for the study of collinear factorization [13]. As applying collinear factorization to the semileptonic decay $B \to \pi \ell \bar{\nu}$ at large recoil, an end-point singularity from $x \to 0$ was observed [14]. According to the above explanation, we conclude that exclusive $B$ meson decays demand $k_T$ factorization [15, 16, 17]. As shown in [18], predictions for exclusive processes derived from $k_T$ factorization are gauge-invariant. In the PQCD approach the $B \to h_1 h_2$ decay amplitude is written as the convolution [19, 20, 21, 22],

$$A = \phi_B \otimes H^{(6)} \otimes \phi_{h_1} \otimes \phi_{h_2} \otimes S,$$  

(1)

where the six-quark amplitude $H^{(6)}$ corresponds to the diagrams with a hard gluon emitted from the spectator quark [23], and $S$ denotes the Sudakov factor.

Once parton transverse momenta $k_T$ are included [13], the end-point singularities from small momentum fractions in exclusive $B$ meson decays are smeared. The resummation of the resultant double logarithms $\alpha_s \ln^2 (Pb)$, where $P$ denotes the dominant light-cone component of a meson momentum, and $b$ is the variable conjugate to $k_T$, leads to a Sudakov form factor $\exp[-s(P,b)]$. This factor suppresses the long-distance contributions from the large $b$ region with $b \sim 1/\Lambda$, where $\Lambda \equiv M_B - m_b$, $M_B$ ($m_b$) being the $B$ meson ($b$ quark) mass, represents a soft scale. The suppression renders $k_T^2$ flowing into the hard amplitudes of $O(\Lambda M_B)$. The off-shellness of internal particles then remain of $O(\Lambda M_B)$ even in the end-point region, and the singularities are removed. Since the end-point singularities do not exist [13, 24, 25, 26], the arbitrary cutoffs introduced in QCDF [12, 27] are not necessary. Therefore, factorizable, nonfactorizable and annihilation amplitudes can be estimated in a more consistent way in PQCD than in QCDF.

2 CP Asymmetries

Phenomenological consequences for two-body nonleptonic $B$ meson decays derived from collinear and $k_T$ factorizations are quite different. Here we mention only the predictions for the CP asymmetry in the $B_d^0 \to \pi^+ \pi^-$ decay. According to the QCDF power counting rules [12, 27] based on collinear factorization, the factorizable emission diagram gives the leading contribution of $O(\alpha_s^0)$, since the $B \to \pi$ form factor is not calculable. Because the leading contribution is real, the strong phase arises from the factorizable annihilation diagram, being of $O(\alpha_s m_q/M_B)$, and from the vertex correction to the leading
diagram, being of $O(\alpha_s)$. For $m_0/M_B$ slightly smaller than unity, the vertex correction is the leading source of strong phases. In $k_T$ factorization the power counting rules change \[28\]. The factorizable emission diagram is calculable and of $O(\alpha_s)$. The factorizable annihilation diagram has the same power counting as in QCDF. The vertex correction becomes of $O(\alpha_s^2)$. Therefore, the annihilation diagram contributes the leading strong phase. This is the reason the strong phases derived from PQCD and from QCDF are opposite in sign, and the former has a large magnitude. The detailed reason is referred to \[28\]. As a consequence of the different power counting rules, QCDF prefers a small and positive CP asymmetry $C_{\pi\pi}$ \[29\], while PQCD prefers a large and negative $C_{\pi\pi} \sim -30\%$ \[30, 31, 32, 33, 34\]. In the near future the two approaches to exclusive $B$ meson decays, collinear and $k_T$ factorizations, could be distinguished by experiments \[35, 36\]. Significant CP asymmetries are also expected in the $B \to K\pi$ \[11\], $B \to KK$ \[37\] and $B \to \rho K$, $\omega K$ \[38\] decays. The last two modes are especially sensitive to the annihilation contributions. It has been pointed out \[39\] that contribution from intrinsic charms, one of the higher $B$ meson Fock states, reduces the magnitude but does not flip the sign of the CP asymmetries in the $B \to K\pi$ decays. It implies that PQCD has caught the correct leading picture of exclusive $B$ meson decays.

### 3 Penguin Enhancement

The leading factorizable contributions involve four-quark hard amplitudes in QCDF, but six-quark hard amplitudes in PQCD. This distinction also implies different characteristic scales in the two approaches: the former is characterized by $m_b$, while the latter is characterized by the virtuality of internal particles of order $\sqrt{\Lambda M_B} \sim 1.5$ GeV \[11, 30, 31\]. It has been known that to accommodate the $B \to K\pi$ and $\pi\pi$ data, penguin contributions must be large enough. In QCDF one relies on the chiral enhancement by increasing the chiral symmetry breaking scale to a large value $m_0 \sim 3-4$ GeV \[40\]. Because of the renormalization-group evolution effect of the Wilson coefficients, the lower hard scale leads to the dynamical penguin enhancement in PQCD.

Whether the dynamical enhancement or the chiral enhancement is responsible for the large $B \to K\pi$ branching ratios can be tested by measuring the $B \to \phi K$ modes \[28, 41\]. In these modes penguin contributions domi-
nate, such that their branching ratios are insensitive to the variation of the unitarity angle $\phi_3$. Because the $\phi$ meson is a vector meson, the mass $m_0$ is replaced by the physical mass $M_\phi \sim 1$ GeV, and the chiral enhancement does not exist. If the branching ratios of the $B \to \phi K$ decays are around $4 \times 10^{-6}$ [12, 13], the chiral enhancement may be essential for the penguin-dominated decay modes. If the branching ratios are around $10 \times 10^{-6}$ as predicted in PQCD [28, 44], the dynamical enhancement may be essential. However, it should be mentioned that the infrared cutoffs in QCDF have been assumed to be different in the $B \to PP$ and $VP$ decay amplitudes [14]. Introducing two independent sets of free parameters for the $B \to PP$ and $VP$ modes, the $B \to \phi K$ branching ratios can be fit without increasing the $B \to K\pi$ branching ratios. Nevertheless, the $B \to \phi K$ branching ratios are enhanced by large annihilation contributions [45], which seem to violate the QCDF power counting rules.

4 $B \to D^0\pi^0$

The PQCD formalism for $B \to D^{(*)}$ transitions has been developed [16], which holds under the hierarchy,

$$M_B \gg M_{D^{(*)}} \gg \bar{\Lambda},$$

(2)

with $M_{D^{(*)}}$ being the $D^{(*)}$ meson mass. The relation $M_B \gg M_{D^{(*)}}$ justifies perturbative evaluation of the $B \to D^{(*)}$ form factors at large recoil and the definition of light-cone $D^{(*)}$ meson wave functions. The relation $M_{D^{(*)}} \gg \bar{\Lambda}$ justifies the power expansion in the parameter $\bar{\Lambda}/M_{D^{(*)}}$. We have calculated the $B \to D^{(*)}$ form factors as double expansions in $M_{D^{(*)}}/M_B$ and in $\bar{\Lambda}/M_{D^{(*)}}$, and found that the leading PQCD factorization formulas respect the heavy-quark symmetry.

Based on the power counting rules constructed in [16], it can be shown that the relative importance of the different topologies of diagrams for the $B \to D\pi$ decays is given by

$$\text{emission : nonfactorizable} = 1 : \frac{M_D}{M_B},$$

(3)

which approaches $1 : \bar{\Lambda}/M_B$ as the $D$ meson mass $M_D$ reduces to the pion mass of $O(\bar{\Lambda})$. Since the factorizable and nonfactorizable diagrams contribute
to the parameters $a_1$ and $a_2$ in PQCD, respectively, the ratio $|a_2|/a_1 \sim 0.5$ is obtained. Moreover, the imaginary nonfactorizable amplitudes determine the relative phase of the factorizable and nonfactorizable contributions, which is about $-57^\circ$. The PQCD predictions for the $B \to D\pi$ branching ratios [17],

\[ B(B^- \to D^0\pi^-) \sim 5.5 \times 10^{-3}, \]
\[ B(\bar{B}^0 \to D^+\pi^-) \sim 2.8 \times 10^{-3}, \]
\[ B(\bar{B}^0 \to D^0\pi^0) \sim 2.6 \times 10^{-4}, \]

are consistent with the experimental data, including those recently observed for the $\bar{B}_d \to D^{(*)0}\pi^0$ decay [18, 49, 50]. Hence, we are not convinced by the conclusion drawn from the factorization assumption that the $B \to D^0\pi^0$ data hint large final-state interaction [51, 52, 53, 54] (see also [55]).

5 Three-body Decays

Three-body nonleptonic $B$ meson decays have been observed recently [56, 57]. Viewing the experimental progress, it is urgent to construct a reliable framework for these modes. Motivated by its theoretical self-consistency and phenomenological success, we have generalized PQCD to three-body nonleptonic $B$ meson decays [58]. A direct evaluation of the hard amplitudes, which contain two virtual gluons at lowest order, is, on one hand, not practical due to the enormous number of diagrams. On the other hand, the region with the two gluons being hard simultaneously is power-suppressed and not important. Therefore, a new input is necessary in order to catch dominant contributions to three-body decays in a simple manner. The idea is to introduce two-meson distribution amplitudes [59], by means of which the analysis is simplified into the one for two-body decays (number of diagrams is greatly reduced). Both nonresonant contributions and resonant contributions through two-body channels can be included. Moreover, the application of this formalism to three-body baryonic decays [60] is straightforward.

Here we pick up the leading term in the complete Gegenbauer expansion of the two-pion distribution amplitudes $\Phi(z, \zeta, w^2)$ [59]:

\[ \Phi_v(z, \zeta, w^2) = \frac{3F_\pi(w^2)}{\sqrt{2N_c}} z(1 - z)(2\zeta - 1), \]
\[ \Phi_s,t(z, \zeta, w^2) = \frac{3F_{s,t}(w^2)}{\sqrt{2N_c}} z(1 - z), \]  (5)
where the subscripts \( v \), \( s \) and \( t \) stand for the vector, scalar and tensor components, respectively. The variable \( z (\zeta) \) is the parton (pion) momentum fraction, and \( F_{\pi,s,t}(w^2) \) the time-like pion electromagnetic, scalar and tensor form factors with \( F_{\pi,s,t}(0) = 1 \). That is, the two-pion distribution amplitudes are normalized to the time-like form factors.

To calculate the nonresonant contribution, we propose the parametrization,

\[
F^{(nr)}_{\pi}(w^2) = \frac{m^2}{w^2 + m^2}, \quad F^{(nr)}_{s,t}(w^2) = \frac{m_0m^2}{w^2 + m_0m^2},
\]

where the parameter \( m = 1 \) GeV is determined by the fit to the experimental data \( M_{J/\psi}^2 |F_\pi(M_{J/\psi}^2)|^2 \sim 0.9 \) GeV\(^2\) \[61\], \( M_{J/\psi} \) being the \( J/\psi \) meson mass. To calculate the resonant contribution, we parametrize it into the time-like form factors,

\[
F^{(r)}_{\pi,s,t}(w^2) = \frac{M_V^2}{\sqrt{(w^2 - M_V^2)^2 + \Gamma_V^2w^2}} - \frac{M_V^2}{w^2 + M_V^2},
\]

with \( M_V (\Gamma_V) \) being the mass (width) of the resonance meson \( V \). The subtraction term renders Eq. (7) exhibit the features of resonant contributions: the normalization \( F^{(r)}(0) = 0 \) and the asymptotic behavior \( F^{(r)}(w^2) \sim 1/w^4 \), which decreases at large \( w \) faster than the nonresonant parametrization in Eq. (6). Equation (7) is motivated by the pion time-like form factor measured at the \( \rho \) resonance \[62\].

For the \( B^+ \rightarrow \rho^0(770)K^+ \) and \( B^+ \rightarrow f_0(980)K^+ \) channels, we choose \( \Gamma_{\rho} = 150 \) MeV and \( \Gamma_{f_0} = 50 \) MeV \[63\]. The nonresonant contribution \( 0.61 \times 10^{-6} \) to the \( B^+ \rightarrow K^+\pi^+\pi^- \) branching ratio is obtained. Our results \( 1.8 \times 10^{-6} \) and \( 13.2 \times 10^{-6} \) are consistent with the measured three-body decay branching ratios through the \( B^+ \rightarrow \rho(770)K^+ \) and \( B^+ \rightarrow f_0(980)K^+ \) channels, \( < 12 \times 10^{-6} \) and \( (9.6^{+2.5+1.5+3.4}_{-2.3-1.5-0.8}) \times 10^{-6} \) \[54\], respectively. Since the \( f_0 \) width has a large uncertainty, we also consider \( \Gamma_{f_0} = 60 \) MeV, and the branching ratio reduces to \( 10.5 \times 10^{-6} \). The resonant contributions from the other channels can be analyzed in a similar way. For example, the \( K^*(892) \) resonance can be included into the \( K^-\pi \) form factors by choosing the width \( \Gamma_{K^{*+}} = 50 \) MeV.
6 Conclusion

I have briefly reviewed the PQCD approach to two-body nonleptonic $B$ meson decays. This approach is based on the rigorous $k_T$ factorization theorem, in which both meson wave functions and hard amplitudes can be constructed in a gauge-invariant way. The predictions for the branching ratios and the CP asymmetries of various modes are in agreement with the experimental data [34]. The $B \to K\eta'$ data are an exception, which may indicate a significant gluon content of the $\eta'$ meson [64, 65, 66]. The charm penguin contribution could be significant, and requires more investigation [67, 68]. The generalization to three-body nonleptonic decays seem to be successful.

In the future we shall work out the next-to-leading-order and next-to-leading-power corrections, which are expected to be more essential for the $B \to \pi\pi$ decays.

I thank members in the PQCD working group for useful discussions. The work was supported in part by the National Science Council of R.O.C. under the Grant No. NSC-91-2112-M-001-053, by the National Center for Theoretical Science of R.O.C., and by Theory Group of KEK, Japan.

References

[1] G. Sterman, *An Introduction to Quantum Field Theory*, Cambridge, 1993.

[2] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B 242, 97 (1990); Nucl. Phys. B366, 135 (1991).

[3] J.C. Collins and R.K. Ellis, Nucl. Phys. B360, 3 (1991).

[4] E.M. Levin, M.G. Ryskin, Yu.M. Shabelskii, and A.G. Shuvaev, Sov. J. Nucl. Phys. 53, 657 (1991).

[5] G.P. Lepage and S.J. Brodsky, Phys. Lett. B 87, 359 (1979); Phys. Rev. D 22, 2157 (1980).

[6] A.V. Efremov and A.V. Radyushkin, Phys. Lett. B 94, 245 (1980).

[7] V.L. Chernyak, A.R. Zhitnitsky, and V.G. Serbo, JETP Lett. 26, 594 (1977).
[8] V.L. Chernyak and A.R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 544 (1980); Phys. Rept. **112**, 173 (1984).

[9] J. Botts and G. Sterman, Nucl. Phys. **B225**, 62 (1989).

[10] H-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).

[11] Y.Y. Keum, H-n. Li and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).

[12] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).

[13] C.W. Bauer et al., Phys. Rev. D **63**, 014006 (2001); Phys. Rev. D **63**, 114020 (2001); Phys. Lett B **516**, 134 (2001); I.W. Stewart, hep-ph/0207253; hep-ph/0209159.

[14] A. Szczepaniak, E.M. Henley, and S. Brodsky, Phys. Lett. B **243**, 287 (1990).

[15] H-n. Li and H.L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); Phys. Rev. D **53**, 2480 (1996).

[16] M. Dahm, R. Jakob, and P. Kroll, Z. Phys. C **68**, 595 (1995).

[17] B. Melic, Phys. Rev. D **59**, 074005 (1999).

[18] H-n. Li, Phys. Rev. D **64**, 014019 (2001); M. Nagashima and H-n. Li, hep-ph/0202127; hep-ph/0210173.

[19] C.H. Chang and H-n. Li, Phys. Rev. D **55**, 5577 (1997).

[20] T.W. Yeh and H-n. Li, Phys. Rev. D **56**, 1615 (1997).

[21] H.Y. Cheng, H-n. Li, and K.C. Yang, Phys. Rev. D **60**, 094005 (1999).

[22] H-n. Li, hep-ph/0110363.

[23] C.Y. Wu, T.W. Yeh, and H-n. Li, Phys. Rev. D **53**, 4982 (1996); Phys. Rev. D **55**, 237 (1997).

[24] T. Kurimoto, H-n. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007 (2002).
[25] H-n. Li, hep-ph/0102013.

[26] Z.T. Wei and M.Z. Yang, hep-ph/0202018.

[27] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. B606, 245 (2001).

[28] C.H. Chen, Y.Y. Keum, and H-n. Li, Phys. Rev. D 64, 112002 (2001); Phys. Rev. D 66, 054013 (2002).

[29] M. Beneke, hep-ph/0207228.

[30] Y.Y. Keum and H-n. Li, Phys. Rev. D 63, 074006 (2001).

[31] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D 63, 074009 (2001).

[32] C.D. Lü and M.Z. Yang, Eur. Phys. J. C 23, 275 (2002).

[33] K. Ukai and A.I. Sanda, Prog. Theor. Phys 107, 421 (2002).

[34] Y.Y. Keum, H-n. Li, and A.I. Sanda, hep-ph/0201103; Y.Y. Keum, hep-ph/0209002; hep-ph/0209208; Y.Y. Keum and A.I. Sanda, hep-ph/0209014.

[35] Y. Nir, hep-ph/0208080.

[36] J. Rosner, hep-ph/0208248.

[37] C.H. Chen and H-n. Li, Phys. Rev. D 63, 014003 (2001).

[38] C.H. Chen, Phys. Lett. B 525, 56 (2002).

[39] S. Brodsky and S. Gardner, Phys. Rev. D 65, 054016 (2002); S. Gardner, hep-ph/0209080.

[40] N.G. Deshpande, X.G. He, W.S. Hou and, S. Pakvasa, Phys. Rev. Lett. 82, 2240 (1999); W.S. Hou, J.G. Smith, and F. Würthwein, hep-ex/9910014.

[41] H-n. Li, hep-ph/0103305.

[42] X.G. He, J.P. Ma, and C.Y. Wu, Phys. Rev. D 63, 094004 (2001).

[43] H.Y. Cheng and K.C. Yang, Phys. Rev. D 64, 074004 (2001).
[44] S. Mishima, Phys. Lett. B 521, 252 (2002).

[45] D.S. Du et al., Phys. Rev. D 65, 074001 (2002); Phys. Rev. D 65, 094025 (2002); hep-ph/0209233.

[46] T. Kurimoto, H-n. Li, and A.I. Sanda, in preparation.

[47] Y.Y. Keum, T. Kurimoto, H-n. Li, C.D. Lu, and A.I. Sanda, in preparation.

[48] BELLE Coll., K. Abe et al., hep-ex/0109021.

[49] CLEO Coll., T.E. Coan, hep-ex/0110053.

[50] BABAR Coll., B. Aubert et al., hep-ex/0207092.

[51] M. Neubert and A.A. Petrov, Phys. Lett. B 519, 50 (2001).

[52] Z.Z. Xing, hep-ph/0107257.

[53] H.Y. Cheng, Phys. Rev. D 65, 094012 (2002).

[54] C.K. Chua, W.S. Hou and K.C. Yang, Phys. Rev. D 65, 096007 (2002).

[55] J.P. Lee, hep-ph/0109101.

[56] BELLE Coll., A. Garmash et al., Phys. Rev. D 65, 092005 (2002).

[57] BABAR Coll., B. Aubert et al., hep-ex/0206004.

[58] C.H. Chen and H-n. Li, hep-ph/0209043.

[59] D. Muller et al., Fortschr. Physik. 42, 101 (1994); M. Diehl, T. Gousset, B. Pire, and O. Teryaev, Phys. Rev. Lett. 81, 1782 (1998); M.V. Polyakov, Nucl. Phys. B555, 231 (1999).

[60] H.Y. Cheng and K.C. Yang, Phys. Rev. D 66, 014020 (2002); C.K. Chua, W.S. Hou, and S.Y. Tsai, Phys. Rev. D 66 054004 (2002).

[61] Particle Data Group, K. Hikasa et al., Phys. Rev. D 45, S1 (1992), p.I.1.

[62] CMD-2 Coll., R.R. Akhmetshin et al., hep-ex/9904027.
[63] Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).

[64] E. Kou and A.I. Sanda, Phys. Lett. B 525, 240 (2002).

[65] A. Ali and A. Ya. Parkhomenko, Phys. Rev. D 65, 074020 (2002).

[66] M. Beneke and M. Neubert, hep-ph/0210085.

[67] M. Ciuchini et al., Phys. Lett. B 515, 33 (2001); AIP Cpnf. Proc. 602, 180 (2001); hep-ph/0208048. T.N. Pham, hep-ph/0210102.

[68] S. Mishima and A.I. Sanda, in preparation.