From a 1D completed scattering and double slit diffraction to the quantum-classical problem: A new approach

N L Chuprikov
Tomsk State Pedagogical University, 634041, Tomsk, Russia

Abstract.
We present a new approach to the quantum-classical problem, which treats it as the problem of modelling the quantum phenomenon described by a coherent superposition of microscopically distinct substates (CSMDS) as a compound one consisting of alternative subprocesses creating unremovable contexts for each other, or as that of reducing a non-Kolmogorovian quantum probability space to underlie a CSMDS to the sum of Kolmogorovian ones. We develop such models for a 1D completed scattering and double slit diffraction. The quantum-classical problem disappears when, in quantum theory with its integral superposition principle, CSMDSs obey the ”either-or” rule to guide alternative random events. There is no observable which could be associated with the whole ensemble of statistical data described by a CSMDS, because such data are incompatible – in the case of a CSMDS, any observable splits into noncommuting observables associated with the substates. To calculate the average value of any observable as well as to introduce characteristic times is meaningful only for the substates of a CSMDS. Ignoring this feature in the conventional description of CSMDSs just leads to paradoxical results (e.g., to the Hartman effect and passing a particle through two slits in the screen simultaneously).

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1. Introduction

A 1D completed scattering and double slit diffraction represent the most simple one-particle scattering problems to arise in quantum mechanics, and their conventional quantum-mechanical models are commonly accepted to give an exhaustive, maximally possible in quantum mechanics, description. The only reason to compel physicists to address these phenomena again and again is an irresistible urge to explain a counterintuitive character of particle’s properties to follow from these models: by them a particle may tunnel with a superluminal velocity through an opaque potential barrier and pass simultaneously through two slits in the screen.

However, despite these paradoxical properties, in the literature nobody calls in question the models themselves. And an important role in this case is played by the fact that they are consistent with Bell’s analysis of the thought Einstein-Podolsky-Rosen-Bohm (EPR-Bohm) experiment, from which it follows the nonexistence of local hidden (objective, predetermined) variables and, on the contrary, the existence of nonzero quantum correlations between events separated by a spatial-like interval. Against this background the above properties of a (micro)particle look as its inherent ones.

So far the EPR-Bohm experiment and these two one-particle phenomena are at the heart of the long-standing debates on the problem of the quantum-to-classical transition (quantum-classical problem) and foundations of quantum mechanics (see. e.g., [1]). Their common feature is that in all the cases we meet pure quantum states to represent coherent superpositions of microscopically distinct substates (CSMDSs) whose conventional description prevents the quantum-to-classical transition when one tries to keep the quantum-mechanical superposition principle as an universal law, both at the micro- and macro-levels (the Schrödinger’s cat paradox).

In fact the conventional treatment of CSMDSs and observed violation of Bell’s inequalities compel investigators to search for solving the quantum-classical problem beyond the idealization of isolated system, by assuming that there is some unremovable, external for the studied system factor to suppress the action of the superposition principle at the macro-level and, thereby, to transform its time-dependent CSMDS into some substate of the CSMDS.

However, by the following reasons this programmm raises objections. Indeed, this programmm dooms quantum theory to be unable to explain such phenomena – within it quantum phenomena described by CSMDSs remain ”unspeakable” (in Bell’s terminology [2]). Besides, within it quantum mechanics, with its superposition principle, looses its status of a universal theory valid both on the micro- and macro-scales. At the same time Bell himself considered that ”The quantum phenomena do not exclude a uniform description of micro and macro worlds...” [2]. In answering the question ”wave or particle?”, he said, together with de Broglie, ”wave and particle” [ibid].

Our aim in this paper is to present an alternative programmm of solving the quantum-classical problem, which renders ”speakable” the micro-world, with keeping the idealization of isolated systems and initial status of quantum mechanics as an universal
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theory. This program is based on the studies of Bell’s inequalities and CSMDSs, from the viewpoint of classical probability theory (see, respectively, review [3] and paper [4]), as well as on the developed in [5, 6, 7, 8] novel quantum-mechanical approach to a 1D completed scattering and a double slit diffraction, which treats both these one-particle processes as compound ones, i.e., it treats them in the spirit of classical physics where a particle can be either transmitted or reflected by the potential barrier and cannot pass through two slits simultaneously.

These approaches show that the conventional treatment of CSMDSs and experimental violation of Bell’s inequalities cannot be considered as a finally established fact. As is shown in [5, 6, 7, 8], the ”either-or” rule to guide mutually exclusive random events, must and can be extended onto the micro-level, i.e., onto CSMDSs. This step allows one both to overcome the interpretational problems surrounding CSMDSs and consistently define characteristic times for the particle’s dynamics described by such states. The latter important, because the conventional description of CSMDSs leads to anomalously short or even negative values of the tunnelling time.

Note that within the conventional description of CSMDSs, where their properties are ”unspeakable”, all interpretational problems are reduced to the question of the unambiguous interpretation of the corresponding experimental data. In this connection, it is relevant to quote from Bohr [9] who wrote, regarding the foundations of quantum mechanics, that ”the unambiguous interpretation of any measurement must be essentially framed in terms of the classical physical theories” or, else, ”it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms”.

As is known, this requirement have led Bohr to the complementarity principle by which, in particular, the wave and corpuscular properties of a particle cannot be observed simultaneously. However, this requirement is only a part of truth, because a consistent interpretation of any experiment must be also based on that theory to describe the phenomenon under study (and the interpretation of the experimental violation of Bell’s inequalities is a representative example). This means that the unambiguous interpretation of statistical experimental data obtained in studying quantum phenomena can be reached if only quantum theory respects classical probability theory.

In particular, the unambiguous interpretation of experiments associated with CSMDSs is realizable if only quantum mechanics (with its integral superposition principle) respects the ”either-or” rule to guide incompatible events. Since the contemporary quantum theory of CSMDSs does not obey this requirement, the complementarity principle, based essentially on this theory, does not reflect the inherent nature of the wave-particle duality. The models [5, 6, 7, 8] show that in the processes under study a particle exhibits simultaneously both corpuscular and wave properties, which respect each other. (We have to stress that these models call in question only the complementarity of the wave and corpuscular properties of a particle; they do not touch the validity of other aspects of the complementarity principle.)

The plan of the paper is as follows. In Sections 2 we show that the interpretational
problems to appear at present in studying the temporal aspects of tunnelling result from the fact that the conventional model of a 1D completed scattering does not allow in principle a consistent resolution of the tunnelling time problem (TTP). A new model of this process and solving the TTP on its basis are presented in Section 3. In Section 4 we revise the model of a double slit diffraction. In Section 5 we discuss solving the quantum-classical problem from the viewpoint of these two models.

2. On the impossibility of a consistent solving of the TTP within the conventional model of a 1D completed scattering

As is known, the quantum-mechanical model of tunnelling a particle through a one-dimensional static potential barrier (hereinafter referred to as "conventional model of tunnelling" (CMT)) has been included in many textbooks on quantum mechanics as the representative of an exhaustive description of quantum phenomena. However, studying the temporal aspects of a 1D completed scattering (see reviews 10 11 12 13 14 15 16 17 and references therein) showed that the simplicity of this process is illusive.

Now it is well known that the CMT predicts the Hartman effect – for a particle tunnelling through a single opaque potential barrier the tunnelling time saturates with increasing the barrier’s width – the (usual) Hartman effect. Moreover, for a particle tunnelling through several successive barriers, the tunnelling time saturates even with increasing the space between the barriers – the generalized Hartman effect. Both these properties of the tunnelling time concept introduced in the CMT say about a superluminal effective velocity of a tunnelling particle. And what is important is that the Hartman effect has been found both in the case of the (nonrelativistic) Schrödinger equation (see, e.g., 18 19 20 21 22 16 23 24) and the (relativistic) Dirac equation (see, e.g., 25 26 27). It is evident that these paradoxical results need a proper explanation.

Deep controversy raised by the Hartman effect has not yet been overcome. All attempts to reconcile this prediction of the CMT with special relativity and, thus, to justify the existing tunnelling time concepts as those to characterize a particle, have not been successful. In this connection, it is worthwhile to point here to the concluding diagnoses made by Winful and Nimtz: by Winful "...the group delay in tunnelling is not a transit time ..." [21]; by Nimtz "...tunnelling modes propagate in zero time. They arise via virtual particles" [28]. Summing up both these statements we conclude that so far there is no consistent definition of the tunnelling (transit) time for real (not virtual) particles.

In [21] Winful points to the possible ways to overcome the interpretational problems associated with the Hartman effect. In particular, he says that "...the duration of the tunnelling event will simply be the temporal extent of the wave packet, assumed propagating at its initial velocity" [21]. However, it is difficult to agree with this statement. Yes, this ad hoc definition of "the duration of the tunnelling event" does not lead to the Hartman effect. However, it is incorrect to equate a particle with the
(spreading) wave packet to describe its state. As well as it is inconsistent to determine the tunnelling time on the basis of the wave packet to describe the whole ensemble of particles, including reflected particles.

This also concerns another, referred in [21], ”luminal” characteristic time for a tunnelling particle – ”the net-flux delay” – which is introduced ”by dividing [the] dwell time by the transmission coefficient”. Again, the Hartman effect disappears in this case. However, the renormalization by hand cannot legalize the time quantity to describe the whole ensemble of particles, to-be-transmitted and to-be-reflected, as that to describe only tunnelling particles.

What is the reason to prevent consistent definition of the tunnelling (or transit) time within the CMT? The answer is that ”... an incoming peak or centroid does not, in any obvious physically causative sense, turn into an outgoing peak or centroid...” (see [29] as well as [11, 21]). That is, the CTM does not provide the knowledge of the time evolution of transmitted particles at all stages of scattering, which is needed for a consistent definition of the tunnelling (transit) time. It is evident that it can be done only within the model to treat a 1D completed scattering as a compound process consisting of two subprocesses – transmission and reflection.

As was shown in [5, 6, 7, 8], the standard Schrödinger equation allows one to develop such a model. All characteristic times introduced in [5, 6, 7, 8] are in a full agreement with special relativity. Let us show this in details.

3. A new quantum-mechanical model of a 1D completed scattering

3.1. Backgrounds

A 1D completed scattering is considered in [5, 6, 7] in the following setting. A particle impinges a symmetrical potential barrier \( V(x) \) \( (V(x - x_c) = V(x_c - x)) \) confined to the finite spatial interval \([a, b]\) \((a > 0)\); \( d = b - a \) is the barrier width, the point \( x_c \) is the midpoint of the barrier region. At the initial instant of time, long before the scattering event, the state of a particle \( \Psi^{(0)}_{\text{full}}(x) \) approaches the in-asymptote

\[
\Psi^{\text{in}}_{\text{full}}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A^{\text{in}}(k) \exp[i(kx - E(k)t/\hbar)] dk,
\]

which is supposed to be a normalized function to belong to the set \( S_\infty \) consisting from infinitely differentiable functions vanishing exponentially in the limit \(|x| \rightarrow \infty\); \( E(k) = \hbar^2k^2/2m \). Without loss of generality, it is also supposed that

\[
< \Psi^{(0)}_{\text{full}}|\hat{x}|\Psi^{(0)}_{\text{full}} > = 0, \quad < \Psi^{(0)}_{\text{full}}|\hat{p}|\Psi^{(0)}_{\text{full}} > = \hbar k_0 > 0, \quad < \Psi^{(0)}_{\text{full}}|\hat{x}^2|\Psi^{(0)}_{\text{full}} > = l_0^2, \quad (1)
\]

where \( l_0 \) and \( k_0 \) are given parameters (\( l_0 << a \)); \( \hat{x} \) and \( \hat{p} \) are the operators of the particle’s position and momentum, respectively. For the Gaussian wave packet \( A^{\text{in}}(k) = (2l_0^2/\pi)^{1/4} \exp[-l_0^2(k - k_0)^2] \). For a completed scattering the average velocity \( \hbar k_0/m \) of incident particles, i.e., the velocity of the centroid of the incident wave packet, is supposed to be much more than the rate of its spreading.
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For any value of $t$ the wave function to describe the particle’s state has the form

$$
\Psi_{full}(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A^{in}(k) \Psi_{full}(x; k) \exp[-iE(k)t/\hbar]dk;
$$

(2)

where $\Psi_{full}(x; k)$, the stationary state of a particle, can be written as follows

$$
\Psi_{full}(x; k) = \begin{cases} 
eq 1 & x \leq a; \\
a_{full} \cdot u(x - x_c; k) + b_{full} \cdot v(x - x_c; k) & a \leq x \leq b; \\
a_{out}(k)e^{ik(x-d)} & x > b; 
\end{cases}
$$

(3)

$u(x - x_c; k)$ and $v(x - x_c; k)$ are such real solutions to the Schrödinger equation that $u(x_c - x; k) = -u(x - x_c; k)$, $v(x_c - x; k) = v(x - x_c; k)$; $\frac{du}{dx}v - \frac{dv}{dx}u = \kappa$ is a constant;

$$
a_{out} = \frac{1}{2} \left( \frac{Q}{Q^* - P} - \frac{P}{Q^*} \right); \quad b_{out} = -\frac{1}{2} \left( \frac{Q}{Q^* + P} \right);
$$

(4)

$$
a_{full} = \frac{1}{\kappa} (P + P^*b_{out}) e^{ika}; \quad b_{full} = \frac{1}{\kappa} (Q + Q^*b_{out}) e^{ika} = \frac{1}{\kappa} Q^* a_{out} e^{ika};
$$

$$
Q = \left( \frac{du(x - x_c)}{dx} + iku(x - x_c) \right) \bigg|_{x=b}; \quad P = \left( \frac{dv(x - x_c)}{dx} + ikv(x - x_c) \right) \bigg|_{x=b}.
$$

Note, in the case of the rectangular barrier of height $V_0$ we have

$$
u = \sinh(\kappa x), \quad v = \cosh(\kappa x), \quad \kappa = \sqrt{2m(V_0 - E)/\hbar} \quad (E < V_0);
$$

$$
u = \sin(\kappa x), \quad v = \cos(\kappa x), \quad \kappa = \sqrt{2m(E - V_0)/\hbar} \quad (E \geq V_0).
$$

(5)

3.2. Searching-for the incoming waves causally connected to the transmitted and reflected ones

Now we can proceed to the crucial step of our approach – to finding the incoming waves connected causally to the outgoing ones both for transmission and reflection. For this purpose we have to formulate the first two physical requirements on the searched-for stationary waves: (1) a causal relationship between the incoming wave and the outgoing wave implies the continuity of the wave function to describe each subprocess, as well as the continuity of the corresponding probability current density; (2) the superposition of the incoming waves for transmission and reflection must give the incoming wave to describe the whole scattering process.

In accordance with these requirements our aim now is to find two solutions to the Schrödinger equation, $\Psi_{tr}(x; k)$ and $\Psi_{ref}(x; k)$, such that the amplitudes of the incoming wave of $\Psi_{tr}(x; k)$ ($\Psi_{ref}(x; k)$) and the transmitted (reflected) wave of $\Psi_{full}(x; k)$ are equal by modulus, and, besides, $\Psi_{tr}(x; k) + \Psi_{ref}(x; k) = \Psi_{full}(x; k)$. Thus, for $x \leq a$

$$
\Psi_{tr}(x; k) = A_{tr}^{in}e^{ikx}, \quad \Psi_{ref}(x; k) = A_{ref}^{in}e^{ikx} + b_{out}e^{ik(2a-x)}
$$

(6)

where $A_{tr}^{in} + A_{ref}^{in} = 1; |A_{tr}^{in}| = |a_{out}|, |A_{ref}^{in}| = |b_{out}|$.

As was shown in [3], there are two pair of solutions to the Schrödinger equation, whose incoming waves obey these requirements. In one pair $\Psi_{ref}(x; k)$ is an even function, relative to the midpoint $x_c$; but in another pair, it is an odd one. As it
will be seen from the following, only the last pair of solutions is associated with the subprocesses. For this pair

\[ A_{ref}^n = b_{out} (b_{out}^* - a_{out}^*) \equiv b_{out}^* (b_{out} + a_{out}) ; \quad A_{tr}^n = a_{out}^* (a_{out} + b_{out}) \equiv a_{out} (a_{out}^* - b_{out}^*) . \]

We have to stress that not only \( A_{tr}^n + A_{ref}^n = 1 \), but also \( |A_{tr}^n|^2 + |A_{ref}^n|^2 = 1 \). In terms of the (real) transmission and reflection coefficients, \( T(k) \) and \( R(k) \), we have

\[ A_{ref}^n = \sqrt{R} (\sqrt{T} \pm i\sqrt{R}) \equiv \sqrt{R} \exp(i\lambda), \quad A_{tr}^n = \sqrt{T} (\sqrt{T} \mp i\sqrt{R}) \equiv \sqrt{T} \exp \left[ i \left( \lambda \pm \text{sign}(\lambda) \frac{\pi}{2} \right) \right] ; \]

\[ \lambda = \pm \arctan(\sqrt{T/R}); \quad T = |a_{out}|^2, \quad R = |b_{out}|^2 . \]  

For these amplitudes

\[
\Psi_{ref}(x; k) = \begin{cases} 
\kappa^{-1} \left( PA_{ref}^n + P^* b_{out} \right) e^{ika} u(x - x_c; k) & (a \leq x \leq b); \\
-b_{out} e^{ik(x-d)} + A_{ref}^n e^{ik(2x_c-x)} & (x \geq b); 
\end{cases} \\
\Psi_{tr}(x; k) = \begin{cases} 
\kappa^{-1} P A_{tr}^n e^{ika} u(x - x_c; k) + b_{out} v(x - x_c; k) & (a \leq x \leq b); \\
(a_{out} + b_{out}) e^{ik(x-d)} - A_{ref}^n e^{ik(2x_c-x)} & (x \geq b). 
\end{cases} \tag{7}
\]

So, the solutions \( \Psi_{tr}(x; k) \) and \( \Psi_{ref}(x; k) \) aimed to describe the subprocesses are determined by Exps. \( \text{6 and 7} \). As is seen, both the solutions contain waves to impinge the barrier from the right and return backward, without crossing the point \( x_c \). So that, unlike \( \Psi_{full}(x; k) \), each of the solutions \( \Psi_{tr}(x; k) \) and \( \Psi_{ref}(x; k) \) describes the scattering problem with two sources of particles. Thus, they themselves cannot describe the subprocesses in the original problem where there is only one source of particles.

### 3.3. Wave functions for transmission and reflection

Note that the "extra" waves in the region \( x > x_c \) disappear in the superposition \( \Psi_{tr}(x; k) + \Psi_{ref}(x; k) \). Moreover, the incident wave \( A_{ref}^n e^{ikx} \) describes particles which do not cross the point \( x_c \), as \( \Psi_{ref}(x; k) = 0 \) for all values of \( k \). Therefore, in the region \( x < x_c \), the only causal counterpart to the transmitted wave is the incident wave \( A_{tr}^n e^{ikx} \).

Thus, for symmetrical barriers, the midpoint of the barrier region is a particular point for the subprocesses. It divides the \( OX \)-axis into two parts where they are described by different solutions of the Schrödinger equation. Transmission and reflection are described, respectively, by the functions \( \psi_{tr}(x; k) \) and \( \psi_{ref}(x; k) \):

\[
\psi_{ref}(x; k) \equiv \Psi_{ref}(x; k), \quad \psi_{tr}(x; k) \equiv \Psi_{tr}(x; k) \quad (x \leq x_c); \\
\psi_{ref}(x; k) \equiv 0, \quad \psi_{tr}(x; k) \equiv \Psi_{full}(x; k) \quad (x \geq x_c). \tag{8}
\]

The main peculiarity of \( \psi_{tr}(x; k) \) and \( \psi_{ref}(x; k) \) is that each of them contains one incoming and one outgoing wave. As is seen from \( \text{8} \), despite the fact that either is presented, in the regions \( x < x_c \) and \( x > x_c \), by different solutions of the Schrödinger equation, these functions as well as the corresponding probability current densities are continuous at the point \( x_c \). Note that the rejected even solution \( \Psi_{ref}(x; k) \) does not lead to continuous wave functions for subprocesses.

So, by this approach, reflected particles never cross the point \( x_c \) in the course of scattering. This result agrees with the well known fact that, for a classical particle to impinge from the left a smooth symmetrical potential barrier, the midpoint of the
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barrier region is the extreme right turning point, irrespective of the particle's mass and the barrier's form and size. By the terminology of [30] this fact means that in the regions $x < x_c$ and $x > x_c$ a particle moves under different experimental contexts. In a sense, this explains why transmitted particles are described by the wave function $\psi_{tr}(x; k)$ represented in these regions by (though properly matched but) different solutions of the Schrödinger equation – different contexts imply different time evolutions of the ensemble.

For narrow in $k$-space wave packets, i.e., in the limit $l_0 \to \infty$, the wave packet $\Psi_{\text{full}}(x, t)$ (see (2)) as well as the ones $\psi_{tr}(x, t)$ and $\psi_{ref}(x, t)$ formed respectively from $\psi_{tr}(x; k)$ and $\psi_{ref}(x; k)$ obey the following relations

$$\Re\langle \psi_{tr}(x, t)|\psi_{ref}(x, t)\rangle = \int_{-\infty}^{x_c} \Re [\Psi_{tr}(x, t)\Psi_{ref}(x, t)] \, dx = 0.$$ 

Therefore, despite the existence of interference between $\psi_{tr}$ and $\psi_{ref}$, for any $t$ we have

$$\langle \Psi_{\text{full}}(x, t)|\Psi_{\text{full}}(x, t)\rangle = T + R = 1;$$
$$T = \langle \psi_{tr}(x, t)|\psi_{tr}(x, t)\rangle = \int_{-\infty}^{x_c} |\Psi_{tr}(x, t)|^2 \, dx + \int_{x_c}^{\infty} |\Psi_{\text{full}}(x, t)|^2 \, dx;$$
$$R = \langle \psi_{ref}(x, t)|\psi_{ref}(x, t)\rangle = \int_{-\infty}^{x_c} |\Psi_{ref}(x, t)|^2 \, dx;$$

constants $T$ and $R$ are the transmission and reflection coefficients, respectively.

Eqs. (2) just support the idea that a 1D completed scattering can be presented as a compound process consisting of two alternative subprocesses – transmission and reflection. These subprocesses are inseparable from each other, because either subprocess creates an unremovable context for its counterpart.

Note, for wave packets of any width, $R$ remains unchanged at all stages of scattering. However $T$ is now constant only at the initial and final stages, i.e., long before and long after the scattering event: $T = \int_{-\infty}^{\infty} |A_{tr}(k)|^2 T(k) \, dk = \int_{-\infty}^{\infty} |a_{out}(k)|^2 T(k) \, dk = 1 - R$. At the very stage of scattering, $dT/dt = I_{tr}(x_c + 0, t) - I_{tr}(x_c - 0, t) \neq 0$: here $I_{tr}$ is a probability current density to correspond to $\psi_{tr}(x, t)$.

Thus, in the general case, for times to correspond to the scattering event, the quantum mechanical formalism does not allow one to entirely exclude the interference terms from $\psi_{tr}(x, t)$, in partitioning the whole scattering process into alternative subprocesses. It should be stressed however that, in our numerical calculations for wave packets whose initial width was comparable with the barrier width, the relative deviation of $T$ from $1 - R$ did not exceed several percentages.

Of course, the alternation of the Schrödinger evolution of the subprocesses at the point $x_c$ leads also to other peculiarities of the subprocesses. Let us consider in detail the case of narrow in $k$-space wave packets when the variation of the norm $T$ is negligible. For example, of interest is the fact that, for such packets, reflected particles are affected at the point $x_c$ by an extra average force to push particles out from the barrier region, backward into the left out-of-barrier region –

$$\frac{d < \hat{p} >_{ref}}{dt} = \left\langle \frac{dV}{dx} \right\rangle_{ref} - \frac{\hbar^2}{2m} \left| \frac{\partial \psi_{ref}}{\partial x} \right|_{x=x_c}^2 ;$$
here angle brackets denote averaging over the corresponding ensemble of particles. For transmitted particles, in the analogous expression
\[
\frac{d \langle \hat{p} \rangle_{tr}}{dt} = \left\langle -\frac{dV}{dx} \right\rangle_{tr} + \frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi_{tr}}{\partial x^2} |_{x=x_c+0} - \frac{\partial^2 \psi_{tr}}{\partial x^2} |_{x=x_c-0} \right),
\]
the second term equals to zero. Indeed, in the limit \( l_0 \to \infty \), we have
\[
\left| \frac{\partial \psi_{tr}}{\partial x} \right|^2 \bigg|_{x=x_c+0} - \left| \frac{\partial \psi_{tr}}{\partial x} \right|^2 \bigg|_{x=x_c-0} = \kappa^2 \left( |a_{full}|^2 - |a_{tr}|^2 \right) = 0,
\]
because the coefficients \( a_{full}(k) \) and \( a_{tr}(k) \) are equal by module (see (11)).

What is important for introducing the group scattering times is that, in this limiting case, the alternation of the Schrödinger evolution of the subprocesses at the point \( x_c \) does not lead to extra terms in the time derivatives for the particle’s position \( x \):
\[
\frac{d < \hat{x} >_{tr}}{dt} = \frac{1}{m} \langle p \rangle_{tr}, \quad \frac{d < \hat{x} >_{ref}}{dt} = \frac{1}{m} \langle p \rangle_{ref}.
\]
Our next step is to present, in the limit \( l_0 \to \infty \), characteristic times for the subprocesses of a 1D completed scattering.

3.4. Characteristic times for transmission and reflection

3.4.1. Exact and asymptotic group times for transmission and reflection

A new approach implies introduction of two different group transmission times – the exact group transmission time \( \tau_{ex}^{tr} \) and the asymptotic group transmission time \( \tau_{as}^{tr} \). By (6), the former is introduced as the difference \( t_2^{tr} - t_1^{tr} \), where \( t_1^{tr} \) and \( t_2^{tr} \) are such instants of time that
\[
\frac{1}{T} \left( < \psi_{tr}(x,t,t_1^{tr})|\hat{x}|\psi_{tr}(x,t,t_1^{tr}) > \right) = a; \quad \frac{1}{T} \left( < \psi_{tr}(x,t,t_2^{tr})|\hat{x}|\psi_{tr}(x,t,t_2^{tr}) > \right) = b;
\]
here \( < \psi_{tr}(x,t)|\hat{x}|\psi_{tr}(x,t) > = \int_{x_c}^{\infty} x |\Psi_{tr}(x,t)|^2 dx + \int_{-\infty}^{x_c} x |\Psi_{full}(x,t)|^2 dx.
\]
Analogously, for reflection the exact group time \( \tau_{ex}^{ref} \) is defined as \( \tau_{ex}^{ref} = t_2^{ref} - t_1^{ref} \), where \( t_1^{ref} \) and \( t_2^{ref} \) are such instants of time that
\[
\frac{1}{R} \left( < \psi_{ref}(x,t_1^{ref})|\hat{x}|\psi_{ref}(x,t_1^{ref}) > \right) = \frac{1}{R} \left( < \psi_{ref}(x,t_2^{ref})|\hat{x}|\psi_{ref}(x,t_2^{ref}) > \right) = a. \quad (10)
\]
As regards \( \tau_{as}^{tr} \), it describes the influence of the potential barrier on a particle within the wide spatial interval \( [a - L_1, b + L_2] \) where \( L_1, L_2 \gg l_0 \gg d \). In this case, instead of the exact wave functions for transmission, one can use the corresponding incoming and outgoing waves,
\[
\psi_{tr}^{in,out}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A_{in}^{tr}(k) f_{tr}^{in,out}(k) \exp[i(kx - E(k)t/\hbar)]; \quad (11)
\]
\[
f_{tr}^{in}(k) = \sqrt{T} \exp \left[ i \left( \lambda + sign(\lambda) \frac{\pi}{2} \right) \right], \quad f_{tr}^{out}(k) = \sqrt{T} \exp[i(J(k) - kd)]; \quad J = \arg(a_{out}).
\]

Long before and long after the scattering event the motion of the centroid of the wave packet \( \psi_{tr}(x,t) \) is described, respectively, by the expressions
\[
< \hat{x} >_{tr}^{in} = \frac{\hbar t}{m} < k >_{tr}^{in} - < \lambda L >_{tr}^{in}, \quad < \hat{x} >_{tr}^{out} = \frac{\hbar t}{m} < k >_{tr}^{out} - < J >_{tr}^{out} + d;
\]
here \( <k>^\text{out}_\text{tr} = <k>^\text{in}_\text{tr} = <k>\) (see (11)); the brackets \( <\ldots>^\text{in, out}_\text{tr} \) denote averaging over the wave packets \( \psi_{^\text{in, out}_\text{tr}}^m(x, t) \); the prime denotes the derivative with respect to \( k \).

The time \( \tau_{\text{tr}}(L_1, L_2) \) spent by the centroid, located at the point \( \hat{x} >_{\text{tr}} \), in the interval \( [a - L_1, b + L_2] \) is

\[
\tau_{\text{tr}}(L_1, L_2) \equiv t^{(2)}_{\text{tr}} - t^{(1)}_{\text{tr}} = \frac{m}{\hbar} \frac{<J^\prime >_{\text{tr}}^\text{out} - <\lambda'>_{\text{tr}}^\text{in} + L_1 + L_2}{<k>_{\text{tr}}}.
\]

The values of \( t^{(2)}_{\text{tr}} \) and \( t^{(1)}_{\text{tr}} \) obey the equations

\[
<\hat{x} >^\text{in}_{\text{tr}} (t^{(1)}_{\text{tr}}) = a - L_1; \quad <\hat{x} >^\text{out}_{\text{tr}} (t^{(2)}_{\text{tr}}) = b + L_2.
\]

The term \( \tau_{\text{as}}^\text{as} (\tau_{\text{as}}^\text{as} = \tau_{\text{as}}(0, 0)) \) is just the asymptotic group transmission time,

\[
\tau_{\text{as}}^\text{as} = \frac{m d_{\text{as}}^\text{eff}}{\hbar} \quad d_{\text{as}}^\text{eff} = <J^\prime >_{\text{tr}}^\text{out} - <\lambda'>_{\text{tr}}^\text{in}.
\]

Analogously, \( \tau_{\text{ref}}(L_1) \) spent by the reflected centroid, located at the point \( \hat{x} >_{\text{ref}} \), in the interval \( [a - L_1, x_c] \) is

\[
\tau_{\text{ref}}(L_1) \equiv t^\text{ref} - t^\text{ref} = \frac{m}{\hbar} \frac{<J' - F' >_{\text{ref}}^\text{out} - <\lambda'>_{\text{ref}}^\text{in} + 2L_1}{<k>^\text{ref}_{\text{in}}};
\]

here the instants of time \( t^\text{ref} \) and \( t^\text{ref} \) obey equations

\[
<\hat{x} >^\text{ref}_{\text{in}} (t^\text{ref}) = a - L_1; \quad <\hat{x} >^\text{ref}_{\text{out}} (t^\text{ref}) = a - L_1.
\]

Then the term \( \tau_{\text{as}}^\text{as} (\tau_{\text{as}}^\text{as} = \tau_{\text{as}}(0)) \) is just the asymptotic group reflection time,

\[
\tau_{\text{as}}^\text{as} = \frac{m d_{\text{as}}^\text{eff}}{\hbar} \quad d_{\text{as}}^\text{eff} = <J' - F' >_{\text{ref}}^\text{out} - <\lambda'>_{\text{ref}}^\text{in}.
\]

Parameters \( d_{\text{tr}}^\text{eff} \) and \( d_{\text{as}}^\text{eff} \) can be interpreted as effective barrier’s widths associated with the transmission and reflection, respectively. These quantities, together with asymptotic group scattering times \( \tau_{\text{as}}^\text{as} \) and \( \tau_{\text{as}}^\text{as} \) can be negative by value, unlike exact group scattering times \( \tau_{\text{tr}}^\text{ex} \) and \( \tau_{\text{ref}}^\text{ex} \).

Note that the average starting points \( x_{\text{tr}}^\text{start} \) and \( x_{\text{ref}}^\text{start} \) for transmission and reflection, respectively, are determined by expressions \( x_{\text{tr}}^\text{start} = -<\lambda'>^\text{in}_{\text{tr}} \) and \( x_{\text{ref}}^\text{start} = -<\lambda'>^\text{ref}_{\text{in}} \). That is, they differ from \( x_{\text{full}}^\text{start} \) to characterize the whole ensemble of particles. This result distinguishes our approach from the CMT based on the implicit assumption that transmitted particles start, on the average, from the point \( x_{\text{full}}^\text{start} \) (in the setting considered, \( x_{\text{full}}^\text{start} = 0 \) (see (11)).

Let us consider in detail tunnelling a particle, with a given energy \( E \), through the rectangular potential barrier of height \( V_0 \) (\( E \leq V_0 \)). Note, firstly, that for symmetric potential barriers \( F'(k) \equiv 0 \). Therefore \( d_{\text{tr}}^\text{eff}(k) = d_{\text{ref}}^\text{eff}(k) \equiv d_{\text{eff}}(k) \) and \( x_{\text{tr}}^\text{start} = x_{\text{ref}}^\text{start} \equiv x_{\text{start}} = -\lambda(k) \). In this case

\[
\tau_{\text{as}}^\text{as}(k) = \tau_{\text{ref}}^\text{as}(k) = \frac{m d_{\text{eff}}(k)}{\hbar k}.
\]

\[
d_{\text{eff}}(k) = \frac{4}{k} \left[ \frac{k^2 + \kappa_0^2 \sinh^2(\kappa d/2)}{4k^2 + \kappa_0^2 \sinh^2(\kappa d)} \right] \left[ \kappa_0^2 \sinh(\kappa d) - k^2 \kappa d \right].
\]
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\[ x_{\text{start}}(k) = -2 \frac{\kappa_0^2}{\kappa} \cdot \frac{(\kappa^2 - k^2) \sinh(\kappa d) + k^2 \kappa \cosh(\kappa d)}{4k^2 \kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}; \]

here \( \kappa_0 = \sqrt{2mV_0/\hbar} \) (see also (5)).

As is seen, like the phase time \( \tau_{\text{ph}} \) defined in the STM for rectangular barrier,

\[ \tau_{\text{ph}}(k) = \frac{m}{\hbar \kappa} \cdot \frac{2k \kappa^2 (\kappa^2 - k^2) + \kappa_0^4 \sinh(2\kappa d)}{4k^2 \kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}. \]

\( \tau_{\text{as}}(k) \) saturates too with increasing the barrier’s width \( d \). However, this fact does not at all mean that the effective velocity of a particle tunnelling through a wide rectangular barrier becomes superluminal. It is demonstrated by Fig. 1 which shows the function \( \langle \dot{x} \rangle_{tr}(t) \) to describe scattering the Gaussian wave packet \( (l_0 = 10\text{nm}, E_0 = \hbar^2 k_0^2/2m = 0.05eV) \) on the rectangular barrier \( (a = 200\text{nm}, b = 215\text{nm}, V_0 = 0.2eV) \). (Note, in this case the deviation of \( T \) from \( 1 - R \) does not exceed five percentages, though the wave-packet’s and barrier’s widths are of the same order.)

![Figure 1](image-url)

Figure 1. The centroid’s positions for \( \psi_{tr}(x,t) \) (circles) and for the corresponding freely moving wave packet (dashed line) as functions of time \( t \).

This figure shows explicitly a qualitative difference between the exact and asymptotic group times – the latter is not an approximation of the former. While the former gives the time spent by the centroid just in the barrier region (in the CMT this role is played by \( \tau_{\text{ph}} \)), the latter describes the influence of the barrier on the centroid in the course of the whole process. More precisely, the quantity \( \tau_{tr}^{\text{as}} - \tau_{\text{free}} \), where \( \tau_{\text{free}} = md/\hbar k_0 \), is the time delay to appear in the motion of the centroid of the transmitted wave packet, as compared with that of the corresponding freely moving packet (it starts from the point \( x_{\text{start}}^{\text{tr}} \), rather than from \( x_{\text{full}}^{\text{tr}} \)), in the course of a 1D completed scattering. In the case considered, \( \tau_{tr}^{\text{ex}} \approx 0.155\text{ps}, \tau_{tr}^{\text{as}} \approx 0.01\text{ps}, \tau_{\text{free}} \approx 0.025\text{ps} \).

As is seen, the influence of an opaque rectangular barrier on the transmitted wave packet has a complicated character. The exact group time says that the centroid’s
velocity inside the barrier region is much smaller than outside. While the asymptotic
group time tells us that the total influence of the barrier on the transmitted wave packet
has an accelerating character: the transmitted packet moves ahead the corresponding
packet moving freely. However, we have to take into account that this effect is related to
the asymptotically large spatial interval, as \( L_1, L_2 \gg l_0 \gg d \). In this case the saturation
of the asymptotic group transmission time, with increasing the barrier’s width, does not
at all mean that the centroid of the transmitted wave packet passes the barrier with a
superluminal velocity.

At the end of this section it is worthwhile to note that the effective width \( d_{\text{eff}} \) for
the \( \delta \)-potential \( V(x) = W\delta(x-a) \) is zero in our approach. That is, the asymptotic group
transmission time, like the exact one, is zero in this case. This result is well expected for
a point-like object to pass through the barrier region of the \( \delta \)-potential, which is zero.
Note that in this case \( x_{\text{start}}(k) = -2mh^2W/(h^4k^2 + m^2W^2) \).

Within the CMT we have an opposite situation. Now \( x_{\text{start}}(k) = x_{\text{full}}(k) = 0 \), but
\( d_{\text{eff}} = 2mh^2W/(h^4k^2 + m^2W^2) \). This result is usually explained by the nonlocality of
tunnelling a particle through \( \delta \)-potential. However, in our opinion, this explanation is
questionable, because it is based implicitly on the illegitimate substitution for a particle
by the wave packet to describe its state (more precisely, the state of the corresponding
ensemble of particles).

3.5. The dwell and Larmor times for transmission and reflection

Note, the concepts of the exact group transmission and reflection times are considered,
in our approach, as auxiliary ones. For example, it in fact unfit for timing reflected
particles in the limiting case \( l_0 \to \infty \). The point is that the centroid of a too wide wave
packet \( \psi_{\text{ref}}(x,t) \) simply does not enter the barrier region (Eqs. (10) have no roots in
this case). The main role in timing a scattering particle in the barrier region is played
here by the dwell and Larmor times, closely connected with each other.

Remind that the dwell times are introduced in [6] for the stationary scattering, i.e.,
for a particle with a given energy \( E \). So, the transmission (\( \tau_{\text{dwell}}^{\text{tr}} \)) and reflection (\( \tau_{\text{dwell}}^{\text{ref}} \))
dwell times read as

\[
\tau_{\text{dwell}}^{\text{tr}}(k) = \frac{1}{I_{\text{tr}}} \int_a^b |\psi_{\text{tr}}(x;k)|^2 dx \equiv \frac{1}{I_{\text{tr}}} \int_a^{x_c} |\Psi_{\text{tr}}(x;k)|^2 dx + \frac{1}{I_{\text{full}}} \int_{x_c}^b |\Psi_{\text{full}}(x;k)|^2 dx \quad (14)
\]

\[
\tau_{\text{dwell}}^{\text{ref}}(k) = \frac{1}{I_{\text{ref}}} \int_a^{x_c} |\psi_{\text{ref}}(x;k)|^2 dx \equiv \frac{1}{I_{\text{ref}}} \int_a^{x_c} |\Psi_{\text{ref}}(x;k)|^2 dx; \quad (15)
\]

where \( I_{\text{tr}} = I_{\text{full}} = T(k)h k/m, I_{\text{ref}} = R(k)h k/m \).

The Larmor times introduced in [6] for the nonstationary case read as

\[
\tau_{\text{tr}}^L = \frac{1}{T} \int_0^\infty \varpi(k) T(k) \tau_{\text{dwell}}^{\text{tr}}(k) dk, \quad \tau_{\text{ref}}^L = \frac{1}{R} \int_0^\infty \varpi(k) R(k) \tau_{\text{dwell}}^{\text{ref}}(k) dk; \quad (16)
\]

\( \varpi(k) = |A^{\text{in}}(k)|^2 - |A^{\text{in}}(-k)|^2 \); for a completed scattering, \( |A^{\text{in}}(k_0)| \gg |A^{\text{in}}(-k_0)| \).

For the rectangular barrier, for \( E < V_0 \), we have

\[
\tau_{\text{dwell}}^{\text{tr}}(k) = \frac{m}{2\hbar k \kappa^3} \left[ (\kappa^2 - k^2) \kappa d + k_0^2 \sinh(\kappa d) \right]; \quad (17)
\]
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\[ \tau_{dwell}^{ref}(k) = \frac{mk}{\hbar \kappa} \cdot \frac{\sinh(\kappa d) - \kappa d}{\kappa^2 + \kappa_0^2 \sinh^2(\kappa d/2)} \].

(18)

In the CMT [19], the dwell time \( \tau_{dwell} \) for rectangular barrier \((E < V_0)\) reads as

\[ \tau_{dwell}(k) = \frac{mk}{\hbar \kappa} \cdot \frac{2k\kappa(k^2 - k_0^2)}{4 \kappa^2 \kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}. \]

(19)

As is seen from (17) and (19), unlike \( \tau_{dwell} \) to appear in the CMT, \( \tau_{dwell}^{tr} \) increases exponentially rather than saturates in the limit \( d \to \infty \). Thus, in our approach, both the exact group time and the dwell time, contrary to the corresponding concepts introduced in the CMT, do not lead to the Hartman effect. By our approach the opaque barrier strongly delays, on the average, the motion of a particle when it enters the barrier region.

Moreover, as is shown in [8] for a particle tunnelling through the system of two identical rectangular barriers of width \( d \) and height \( V_0 \), with the distance \( l \) between them, this approach does not lead to the generalized Hartman effect which was found within the CMT in [31]. In the opaque-barrier limit, when \( V_0 \to \infty \) (or \( k_0 \to \infty \)) and \( d \) is fixed, for the phase time \( \tau_{ph} \) and dwell time \( \tau_{dwell} \) introduced in the CMT we have

\[ \tau_{ph} \approx \frac{2m}{\hbar k k_0}, \quad \tau_{dwell} \approx \frac{2mk}{\hbar k_0^3}. \]

As is seen, both these quantities do not depend on the distance between the barriers and diminish when \( k_0 \to \infty \).

In order to show the behavior of \( \tau_{tr}^{dwell}(k) \) and \( \tau_{ref}^{dwell}(k) \) in this limit, let us firstly note that these quantities possess the property of additivity. They can be written as follows, \( \tau_{dwell}^{tr} = \tau_{tr}^{(1)} + \tau_{gap}^{tr} + \tau_{tr}^{(2)} \) and \( \tau_{dwell}^{ref} = \tau_{ref}^{(1)} + \tau_{gap}^{ref} \); here \( \tau_{tr}^{(1)} \) and \( \tau_{ref}^{(1)} \) describe the first barrier; \( \tau_{gap}^{tr} \) and \( \tau_{gap}^{ref} \) do the gap between the barriers, and \( \tau_{tr}^{(2)} \) describes the second barrier (remind that reflected particles do not cross the midpoint of this symmetric structure). As is shown in [8], in the above limit,

\[ \tau_{ref}^{dwell} \approx \tau_{ref}^{(1)} \approx \tau_{dwell}, \quad \tau_{tr}^{(1)} = \tau_{tr} \approx \frac{m}{4 \hbar k k_0} e^{2k_0 d}, \quad \tau_{tr}^{gap} \approx \frac{mk_0^2}{8 \hbar k^4} (kl - \sin(kl)) e^{2k_0 d}. \]

As is seen, \( \tau_{tr}^{dwell}(k) \) increases exponentially in this limit, and what is also important is that it depends on the distance \( l \) between the barriers. Moreover, in the course of passing a particle through the structure, it spends the most part of time just in the space between the opaque barriers.

Analogous situation arises in another opaque-barrier limit, when \( d \to \infty \) and \( V_0 \) is fixed. Now the explicit expressions for the characteristic times becomes somewhat complicated (see [8]), giving no qualitatively new information, and we omit this case.

4. On a strictly symmetrical setting of the double-slit experiment

As is well known, the main puzzle of a double slit diffraction is that within its conventional quantum model a particle possesses mutually exclusive properties - its wave and corpuscular properties are incompatible with each other. But this result cannot be considered as a finally established fact. Here we present a novel model of a
double slit diffraction where this process is treated, like a 1D completed scattering, as a compound one-particle process. Within this model the wave and corpuscular properties of a particle are in a peaceful coexistence with each other.

Of course, the way of decomposing this one-particle process into alternative subprocesses is different. As is seen from Section 3, the key role in decomposing a 1D completed scattering into subprocesses is played by the fact that at the final stage of the process the particle’s state is a CSMDS in which substates do not interfere with each other, being localized in the non-overlapped spatial regions. Just the final substates to describe transmission and reflection were used for reconstructing their whole evolution.

At first glance, in the case of a double-slit diffraction we meet a more comfortable situation, because here the particle’s state is a CSMDS at all stages of scattering. However, the problem is that the norms of the one-slit substates to enter this superposition do not give unit (the norm of the CSMDS) because of the interference between them. This means that these (initially) distinct one-slit substates to appear in the one-slit experiments associated with the first and second slits, cannot be associated with the alternative subprocesses of the double-slit experiment. Just this fact is usually interpreted as the incompatibility of the wave and corpuscular properties of a particle in the double-slit experiment.

In this connection, our aim is to show, by the example of a strictly symmetrical setting of the double-slit experiment, how to transform the above CSMDS, with the interfering one-slit substates, into that with substates to describe alternative subprocesses. It is assumed that the YZ-plane coincides with the plane of the first screen to have two parallel identical slits centered on the planes \( y = -a \) (first slit) and \( y = a \) (second slit), and the wave function \( \Psi_{\text{two}}(x, y, z, t) \) to describe a particle when both the slits are opened has the form

\[
\Psi_{\text{two}}(x, y, z, t) \equiv \Phi(x, y, z, t); \quad (20)
\]

\[
\Phi(x, y, z, t) = \frac{1}{\sqrt{2}} [\Psi_{\text{one}}(x, y-a, z, t) + \Psi_{\text{one}}(x, y+a, z, t)]
\]

where \( \Psi_{\text{one}}(x, y, z, t) \) is the ”one-slit” wave function; it is such that \( \Psi_{\text{one}}(x, -y, z, t) = \Psi_{\text{one}}(x, y, z, t) \), and \( \Psi_{\text{one}}(0, y, z; k) = 0 \), if \( |y| > d/2 \); \( d \) is the slit’s width; \( a > d/2 \). It is also assumed that a particle impinges the first screen from the left, and the second screen – the particle’ s detector – coincides with the plane \( x = L \) (\( L > 0 \)).

It is evident that in this case

\[
\Psi_{\text{two}}(x, -y, z; k) = \Psi_{\text{two}}(x, y, z; k).
\]

Thus, the \( y \)-th components of the probability current densities associated with the first and second slits balance each other on the plane \( y = 0 \), and the \( y \)-th component of the probability current density associated with their superposition is zero on this plain (see also [32] where this experiment is analyzed within the Bohmian approach).

It is evident that, if to insert along the symmetry plane an infinitesimally thin two-side ”mirror” (we use here this word, bearing in mind the analogue between this
quantum experiment and its counterpart in classical electrodynamics) to elastically scatter particles, the wave function \( \Psi \) and, hence, the interference pattern to appear in this experiment on the second screen should remain the same; inserting the mirror does not disturb the condition (21).

The coincidence of the particle’s states in the original (without the mirror) and modified (with the mirror) settings of the double-slit experiment leads to the following two conclusions: (i) the ensemble of particles freely moving between the first and second screens in the original setting is equivalent to the ensemble of particles in the modified setting where they a priori cannot cross the plane \( y = 0 \) occupied by the mirror; (ii) in the original experiment a particle always passes only through one of two open slits. Figuratively speaking, the above procedure of inserting the mirror can be considered as a non-demolishing ”which-way” measurement.

From (i) it follows that the wave function \( \Psi \) can be rewritten in the form
\[
\Psi_{\text{two}}(x, y, z, t) = \psi_{\text{two}}^{(1)}(x, y, z, t) + \psi_{\text{two}}^{(2)}(x, y, z, t); \tag{22}
\]
where \( \psi_{\text{two}}^{(1)} \) and \( \psi_{\text{two}}^{(2)} \) are piecewise continuous wave functions defined as follows,
\[
\psi_{\text{two}}^{(1)}(x, y, z, t) = \begin{cases} 
0, & y > 0; \\
\Phi(x, y, z, t), & y < 0 
\end{cases}; \quad \psi_{\text{two}}^{(2)}(x, y, z, t) = \begin{cases} 
\Phi(x, y, z, t), & y > 0; \\
0, & y < 0; 
\end{cases}
\]
\[
\psi_{\text{two}}^{(1)}(x, 0, z, t) = \psi_{\text{two}}^{(2)}(x, 0, z, t) = \Phi(x, 0, z, t)/2. \]
We have to stress that these expressions are valid both for \( x > 0 \) and for \( x < 0 \). That is the region of the particle’s source is also divided by the symmetry plane into two parts.

The substates \( \psi_{\text{two}}^{(1)} \) and \( \psi_{\text{two}}^{(2)} \) describe the alternative subprocesses – passing a particle through the first slit (provided that the second one is opened) and passing a particle through the second slit (provided that the first one is opened). These subprocesses are inseparable from each other – either subprocess creates an unremovable context for its counterpart.

Note, in this case, the second screen to play the role of the measurement device can be considered as consisting of two (sub)devices – the left half of the screen \( y < 0 \) to detect particles passing through the first slit, and the right half of the screen \( y > 0 \) associated with the second slit. The same concerns the particle’s source located in the region \( x < 0 \). So that particles to impinge upon the the left (right) half of the second screen are emitted, in fact, from the left (right) half of the particle’s source.

So, though CSMDSs (20) and (22) give the same wave function, the physical meaning of \( \Psi_{\text{one}}(x, y - a, z, t) \) and \( \Psi_{\text{one}}(x, y + a, z, t) \) to enter (20) differs cardinaly from that of \( \psi_{\text{two}}^{(1)}(x, y, z, t) \) and \( \psi_{\text{two}}^{(2)}(x, y, z, t) \) to enter (22). The second pair describes the alternative subprocesses of a double slit diffraction, because \( \| \Psi_{\text{two}} \|^2 = \| \psi_{\text{two}}^{(1)} \|^2 + \| \psi_{\text{two}}^{(2)} \|^2 \); here \( \| \Psi \| \) stands for \( \langle \Psi | \Psi \rangle \). As regards the first pair, there are no alternative subprocesses of a double slit diffraction which could be associated with this pair, because \( \| \Psi_{\text{two}} \| \neq \| \Psi_{\text{one}}(x, y - a, z, t) \| + \| \Psi_{\text{one}}(x, y + a, z, t) \| \).

We have to stress that the following two one-particle processes – passing a particle through the first slit when the second one is opened, and passing a particle through the first slit when the second one is closed – have different experimental contexts (see also
Section 6. As a consequence, they are described by different wave functions (see also [30]). By our approach, \( \psi^{(1)}_\text{two}(x, y, z, t) \) describes the first process, and \( \Psi_\text{one}(x, y + a, z, t) \) describes the second one.

Note, the ”transformation” of the one-slit state \( \Psi_\text{one}(x, y + a, z, t) \) into \( \psi^{(1)}_\text{two}(x, y, z, t) \), after opening the second slit (i.e., after transforming the experimental context), results from the interference of the state \( \Psi_\text{one}(x, y + a, z, t) \) with \( \Psi_\text{one}(x, y - a, z, t) \). Thus, the interference plays a twofold role in this experiment. On the one hand, namely the interference makes it impossible to associate the initial one-slit wave functions with alternative subprocesses of a double slit diffraction. On the other hand, namely the interference makes it possible to decompose a double slit diffraction into the alternative subprocesses described by the inseparable one-slit substates \( \psi^{(1)}_\text{two} \) and \( \psi^{(2)}_\text{two} \).

Note also that the CSMDS to describe a double slit diffraction, being written in the form (22), is similar to a mixed state, in the sense that it obeys the ”either-or” rule. Indeed, from (22) it follows that a particle, despite (and, simultaneously, due to) its wave properties, can pass either through the first slit or through the second one – the whole ensemble of particle consists of two parts. However, unlike pure states to constitute a mixed state, the alternative substates \( \psi^{(1)}_\text{two} \) and \( \psi^{(2)}_\text{two} \) to enter CSMDS (22) are inseparable from each other.

So, the main result of the model is that CSMDS (20) with the initially distinct one-particle substates, hidden due to the interference between them, has been transformed into CSMDS (22) with the inseparable one-slit substates, distinct just due to the interference.

5. The quantum-classical problem as that of reducing a non-Kolmogorovian quantum probability space to underlie a CSMDS to the sum of classical ones

So, from the physical viewpoint the main innovation in a new wave-packet approach is that it treats a 1D completed scattering and double slit diffraction as compound processes, i.e., in the spirit of the classical physics. By this approach, a micro-particle like a macro-particle can pass only through one of two open slits in the screen, as well as it can either be transmitted or reflected by the potential barrier, and so on. Thus, by this approach the abbreviation CSMDS is in fact equally applicable both to micro- and macro-particles.

From the mathematical viewpoint its innovation is the representation of the wave function to describe either of these two one-particle scattering processes as a CSMDS whose unit norm equals to the sum of norms of its substates. That is in fact, this approach represents the probability space associated with either process as the sum of two probability subspaces associated with the subprocesses.

Our next step is to show that the presented here approach to the one-particle scattering phenomena can be taken as the basis for the alternative program of resolving the long-standing quantum-classical problem, with keeping the linear formalism of
quantum mechanics and idealization of isolated systems.

As is known, by the quantum-classical problem is meant the conflict to arise, within the contemporary quantum-mechanical description of CSMDSs, between the superposition principle and the "either-or" rule to guide, in classical probability theory, mutually exclusive random events (the conflict which is inadmissible for theory pretending to the role of a universal one). The most important milestones in developing the modern vision of this problem are the famous Schrödinger’s cat paradox and Bell’s theory of the EPR-Bohm experiment. Both are aimed to demonstrate in the most sharp form the existence of a deep contradiction between the quantum laws of the micro-world and classical laws of the macro-world.

Figuratively speaking, on the road between the micro- and macro-scales, Schrödinger and Bell go in the opposite directions. Schrödinger demonstrates the appearance of the above conflict when one attempts to extend quantum laws onto the macro-scales – Schrödinger calls in question the validity of the superposition principle at the macro-scales. While Bell, by developing the classical-like analysis of the EPR-Bohm experiment, is aimed to show the appearance of this conflict when one attempts to apply classical laws onto the micro-scales – Bell calls in question the universal validity of such fundamental notion of classical physics as "causal external world", i.e., the world whose existence is independent of an observer, and which is guided by the principles of special relativity.

Note that, since quantum mechanics has been developed by its founders as a universal theory, the quantum-classical problem can be also interpreted as the problem of the (in)completeness of quantum mechanics. From this viewpoint, Schrödinger calls in question the completeness of quantum mechanics at the macro-scales, and Bell, on the contrary, calls in question its incompleteness at the micro-scales.

At present the most of scientists to deal with the quantum-classical problem treats it as the measurement (or macro-objectification) problem whose resolution is impossible within the idealization of isolated systems. This viewpoint is based on the widespread interpretation of the experimentally observed violation of Bell’s inequalities. It is considered that their experimental violation falsifies the assumption on the existence of local hidden variables (or other assumptions to concern the (non)locality and/or (non)reality of the micro-world) to underlie Bell’s inequalities.

Within this picture, solving the quantum-classical problem to arise for CSMDSs is impossible without suggesting that there is "everything else" (e.g., the mechanisms of decoherence or localization) to influence (together with the considered potential) the dynamics of the system under study, reducing its original state - a CSMDS - to one of its microscopically distinct substates (the review of the approaches based on this idea, which constitute the decoherence and spontaneous-localization programs of solving the measurement (or macro-objectification) problem, is done in [33]; see also [34, 35]).

Undoubtedly, extending quantum theory onto open micro-systems is of importance, because there are many interesting physical problems when the influence of external factors on a micro-system is essential. It should be stressed also that the deep analysis of
the foundations of quantum mechanics, which was carried out within these programs, is important for searching for the possible ways of resolving the quantum-classical problem. However the ways of resolving this problem, presented in these programs themselves, are unacceptable, as they create a clearly deadlock situation in physics. In fact they condemn quantum mechanics, with its inherent superposition principle, as a weak theory being unable to describe the macro-world and leaving unspeakable the micro-world.

In this connection, of importance are approaches to have tested the validity of Bell’s proof of the nonexistence of local hidden variables, from the mathematical viewpoint. For example an important aspect of this question is pointed out in [36, 37] to argue that in order to judge on (non)locality in the thought EPR and EPR-Bohm experiments, one has at least to introduce a correct space-time structure into their mathematical models. As is shown, this step is sufficient for explaining the (original) EPR experiment.

Other arguments against the nonexistence of local hidden variables have been developed within the approaches to reveal, from the viewpoint of quantum mechanics and classical probability theory, all assumptions to underlie the classical-like derivation of Bell’s inequalities (see the pioneer works by Fine [38], Pitowsky [39] and Accardi [40], as well as the recent review [3] and papers [41, 42, 43, 44]).

As was shown by Fine [38], apart from the explicit assumption on the existence of local hidden variables, the derivation of Bell’s inequality is based also on the implicit assumption that there is a compatible joint distribution to describe experimental data obtained in the EPR-Bohm experiment for different orientations of particle’s detectors. From the viewpoint of quantum mechanics this assumption is improper a priori, because such data are obtained in fact for noncommuting observables and, thus, there is no compatible joint quantum distribution to describe them. That is, it is not surprising that quantum probabilities violate this inequality.

However, this assumption is also at variance with classical probability theory (see [39, 40]) where Bell’s type inequalities have been known yet before Bell’s theorem, and their violation means simply that the probabilities to enter these inequalities are incompatible – they describe statistical (experimental) data which cannot be associated with a common Kolmogorovian probability space. A detailed analysis of Bell’s theorem, from the viewpoint of probability theory, is done in the studies reviewed in [3].

As regards the peculiarities of its experimental testing, as was stated in [43], ”Strictly speaking, there does not exist [Bell’s] inequality such that all the three means involved in it would be spin correlations. It is therefore meaningless to speak of verification of [this] inequality…”. Local hidden variables and probabilities, consistently introduced for the experiments to test Bell’s inequality, obey the inequality to differ from Bell’s one [ibid].

Thus, the experimental violation of Bell’s inequality does not falsify the existence of local hidden variables. Rather it falsifies the existence of ”Bell’s local hidden variables” introduced inconsistently. The main lesson to follow from the critique of Bell’s theorem is that quantum mechanics and classical probability theory respect each other in describing the EPR-Bohm experiment – introducing a common local hidden variable for different
experimental contexts contradicts both these theories.

The above studies to revise the role of Bell’s inequalities are of great importance for solving the quantum-classical problem, because they, in part, rid of obstacles the road from the macro- to the micro-world, for local hidden variables. (We have to stress that it is also the great service of Bell who introduced these inequalities for the analysis of this problem.) However, they leave untouched the contradiction between the superposition principle and ”either-or” rule to guide local hidden variables. As before, the conventional quantum-mechanical description of CSMDSs prevents extending local hidden variables onto the micro-level and, vice versa, extending the superposition principle onto the macro-level – the Schrödinger cat paradox has remained unresolved.

In fact we return in the epoch preceding Bell’s one. Again, due to the conventional description of CSMDSs, the standard quantum mechanics based on the idealization of isolated systems looks as theory complete at the micro-level but incomplete at the macro-level. Thus, the presented in [33] programs of solving the quantum-classical problem, which aimed to make this theory complete at the macro-level, remain relevant as before.

At the same time, the above models show that the conventional description of CSMDSs is not a finally established fact. In studying such states, we have to take into account that a CSMDS, as the EPR-Bohm experiment, deals with incompatible statistical data. Indeed, this pure state implies that the system under study evolves under some complex experimental context consisting of several elementary contexts to imply alternative variants of evolution. For example, in the case of a 1D completed scattering, two elementary contexts are associated with two detectors, for transmitted and reflected particles; in the case of a double slit diffraction, either of two slits creates its own (elementary) experimental context.

Of importance is to stress that the elementary contexts are integral parts of the whole (complex) experimental context to be the ”calling card” of the phenomenon under study. This means that the elementary contexts are inseparable from each other, what, in its turn, results in the inseparability of the corresponding subprocesses.

From the viewpoint of classical probability theory, a CSMDS describes statistical data to belong to a non-Kolmogorovian probability space. This was shown explicitly in [3] by the example of a double slit diffraction. According to classical probability theory, each quantum phenomenon described by a CSMDS should be treated as a compound process to consist of several alternative subprocesses. This implies that the non-Kolmogorovian probability space to underlie a CSMDS should be reduced to the sum of Kolmogorovian ones associated with the subprocesses.

What is important is that quantum mechanics needs the same! Indeed, the above studies of the EPR-Bohm experiment teach us that the non-Kolmogorovness of some model of a quantum phenomenon means that it deals in fact with noncommuting observables. That is, there is no observable which could be associated with incompatible statistical data described by a CSMDS. In particular, introducing any characteristic time as well as calculating the expectation value of any one-particle observable over the whole ensemble of particles described by a CSMDS is meaningless.
So, we arrive at the following two conclusions: (i) the contemporary description of CSMDSs is contradictory; (ii) the quantum-classical problem should be considered as a purely quantum-mechanical problem, namely as that of modelling the quantum phenomenon described by a CSMDS as a compound one consisting of alternative subprocesses, or as that of reducing a non-Kolmogorovian quantum probability space to underlie a CSMDS to the sum of classical ones.

The presented here approach to a 1D completed scattering and double slit diffraction just demonstrates how to resolve this problem on the basis of the standard Schrödinger equation. As was shown, this cannot be done only for wide (in the momentum space) wave packets scattering on a 1D potential barrier, at the very stage of scattering. However, this fact does not prevent the quantum-to-classical transition, because for the initial and final stages of a 1D completed scattering, when the distances between the wave packets and potential barrier are asymptotically large, the subprocesses can be distinguished in the general case too.

So, by our approach the "either-or" rule holds for all coherent superpositions of distinct substates, both for micro- and macro-particles. That is, in the Schrödinger’s cat paradox, the long-suffering cat is either died or alive, independently of an observer, because at any instant of time the radioactive nucleus either has already decayed or has yet non-decayed.

6. Some remarks on the concepts of reality and contextuality in a new wave-packet approach

There is a widespread viewpoint that, within the statistical interpretation of quantum mechanics, the one-particle wave function (to describe a particle in the infinite number of identical experiments) corresponds to nothing in the physical world. A new wave-packet approach justifies and simultaneously falsifies such a viewpoint.

Indeed, on the one hand, any averaging over the CSMDSs to describe a 1D completed scattering and double slit diffraction is indeed meaningless – for either of these phenomena, there is no one-particle observable which could be introduced for the whole ensemble of particles described by the corresponding CSMDS, because it deals with incompatible statistical data. On the other hand, such observables can be introduced for their substates associated with Kolmogorovian probability subspaces.

For these phenomena, every one-particle observable splits into two obserables for the inseparable subprocesses. This means that in the case of a CSMDS every observable must be endowed with the additional index to specify the corresponding subprocess. So, in the case of a 1D completed scattering the particle’s position $x$ and momentum $p$ split, respectively, into the pairs $(x_{tr}, x_{ref})$ and $(p_{tr}, p_{ref})$. Analogously, in the case of a double slit diffraction we have the pairs $(x_{two}^{(1)}, x_{two}^{(2)})$ and $(p_{two}^{(1)}, p_{two}^{(2)})$. Quantum mechanics implies the existence of local hidden variables, and the wave functions of subprocesses give the one-particle distributions (e.g., $x_{tr}$-distribution, $x_{two}^{(1)}$-distribution, $x_{two}^{(2)}$-distribution) to reflect statistically the inherent properties of a (micro)particle, as a local entity.
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We have to stress once more that both the models are non-contextual in the sense that they imply the dependence of all one-particle variables only on the potential under which a particle moves. They do not imply any dependence of the particle’s dynamics on some external factor, environment or an observer. At the same time the subprocesses are contextual. As was said above, each subprocess creates an unremovable context for its counterpart.

Note, in both models the contextuality of subprocesses is associated with symmetry. So, the context to arise for each subprocess of a 1D completed scattering is different in the spatial regions to lie on the different sides of the point $x_c$, because, from the classical viewpoint, reflected particles cannot \textit{a priori} cross the midpoint $x_c$ of a symmetric potential barrier. It is the main reason why each subprocess is described in these regions by different (properly matched at the point $x_c$) solutions of the Schrödinger equations.

Similarly, the symmetry of the considered setting of the double slit experiment leads to the piecewise continuous wave functions $\psi_{two}^{(1)}(x, y, z, t)$ and $\psi_{two}^{(2)}(x, y, z, t)$ to describe the subprocesses evolving on the different sides of the symmetry plane $y = 0$. Inserting the two-side mirror along this plane changes the experimental context. However it keeps the symmetry of the original double slit experiment. As a consequence, the original experiment, where particles move freely between the screens, and the modified ("which-way") experiment, with the mirror inserted between the screens, are described by the same wave function.

7. Conclusion

So, we have developed a new approach to a 1D completed scattering and double slit diffraction, by which these two one-particle processes are modelled as compound ones consisting of two alternative inseparable subprocesses (each subprocess creates an unremovable context for its counterpart). As was shown by Accardi, by the example of a double slit diffraction, the probability spaces to underlie CSMDSs are non-Kolmogorovian. Thus, from the viewpoint of probability theory, our approach reduces a non-Kolmogorovian quantum probability space to the sum of two Kolmogorovian (classical) subspaces.

Since a non-Kolmogorovian probability space describes incompatible statistical data, there are no one-particle observable which could be defined on this space. In the case of a CSMDS consisting of two alternative substates, any observable splits into two noncommuting observables. Calculating the expectation value of any physical observable as well as introducing characteristic times for a particle described by a CSMDS are meaningful only for its substates. Just ignoring this feature, in the current description of such pure states, leads to paradoxical effects (e.g., Hartman effect and simultaneous passage of a particle through two slits in the screen).

Our models give the basis for resolving the quantum-classical problem. By them, a micro-object described by a CSMDS is guided by the "either-or" rule. In this case, the abbreviation CSMDS is equally applicable to micro- and macro-particles. Of course, a
complete resolution of the quantum-classical problem implies revising all current models of quantum phenomena where CSMDSSs appear. All they should be treated as compound phenomena consisting of alternative inseparable subprocesses.

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