Stability of a three-dimensional plate made of a nonlinear viscoelastic material under superimposed finite deformations

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Abstract. In this paper, the stability of a three-dimensional plate made of a nonlinear viscoelastic material under finite perturbations is considered. A fairly broad computational experiment has been performed. The permissible boundaries of the region with respect to the final, initial perturbations are established for the given parameters of loading and structures. Finite sequences of bifurcation points are constructed, which confirm, in contrast to stability under small perturbations, the existence of a sequence of stable equilibrium states. New phenomena and characteristic effects are established.

1. Introduction

In works [1-3], studies were carried out on the stability of the simplest problems from nonlinear elastic and nonlinear viscoelastic material under finite disturbances when the subcritical state is homogeneous. This paper provides a solution to the stability problem for a three-dimensional plate made of a nonlinear viscoelastic material under finite perturbations. The problem of stability of a three-dimensional plate with respect to small perturbations was considered in [4, 5]. Analysis of the main process of plate deformation is reduced to solving a nonlinear boundary value problem with respect to finite perturbations. Solutions for displacement perturbations are chosen in the form of series by eigenfunctions which are solutions to the corresponding linearized problems and satisfy the geometric boundary conditions [1, 6, 7]. After applying the principle of possible displacements, the question of stability of the ground state of a nonlinear problem is reduced to studying the stability of the zero solution of an infinite system of ordinary differential equations with constant coefficients, the number of terms in which is specified by choosing the elastic potential. For the resulting system of equations, a function was constructed that, under certain restrictions on the initial perturbations, is the Lyapunov function [1-3, 6].

2. Materials and methods

In this work, we study the stability of rectangular plates made of a nonlinear viscoelastic material, when its long-term behavior is described by the Murnaghan potential.

Consider a plate made of a viscoelastic isotropic material. In the initial undeformed state the plate has thickness $h$ and is referred to the Cartesian coordinate system $x_1, x_2, x_3$, so that it is bounded by
the \( x_i = \pm \frac{h_i}{2} \) planes. Let the plate be transferred from the undeformed state by a uniform deformation to the state \( V \), which is characterized by coordinates \( x_1, x_2, x_3 \), so that

\[
0 \quad x_1 = \lambda_1 x_1; \quad 0 \quad x_2 = \lambda_2 x_2; \quad 0 \quad x_3 = \lambda_3 x_3,
\]

where \( \lambda_i = \text{const} \).

The thickness of the pre-deformed plate is equal to \( h = \lambda i h_0 \).

Let the initial dimensions of the plate are

\[
0 \leq x_i \leq L; \quad 0 \leq x_2 \leq L_i; \quad -\frac{h_0}{2} \leq x_1 \leq \frac{h_0}{2}.
\]

Assume that the plate is loaded along the \( ox_1 \)-axis. Then

\[
S_{ss}^0 = S_{22}^0 = 0.
\]

Movements will be expressed in terms of coordinates \([2]\).

Deformations and algebraic invariants are expressed in terms of elongations \([1]\).

For isothermal case \( \theta = \text{const} \), the energy equation \( \psi + \eta \dot{\theta} - \eta \theta - tr S + \rho \frac{\dot{h}}{\dot{h}} - r = 0 \) holds identically.

In this case, for a nonlinear viscoelastic body when its behavior in limiting cases is described by the Murnaghan potential \([7]\), for the components of stress tensor \( S_{ii}^\infty (i = 1, 2, 3) \), we obtain

\[
S_{11}^\infty = \lambda(\theta) A_{i0}^0 + a(\theta) A_{i0}^{02} + b(\theta) A_{i0}^0 + \left( \lambda_2 - 1 \right) \left( \mu(\theta) + b(\theta) A_{i0}^0 \right) + 3 \left( \lambda_2 - 1 \right) a(\theta) + \left( \frac{c(\theta)}{3} \right);
\]

\[
S_{22}^\infty = \lambda(\theta) A_{i0}^0 + a(\theta) A_{i0}^{02} + b(\theta) A_{i0}^0 + \left( \lambda_2 - 1 \right) \left( \mu(\theta) + b(\theta) A_{i0}^0 \right) + 3 \left( \lambda_2 - 1 \right) a(\theta) + \left( \frac{c(\theta)}{3} \right) = 0;
\]

\[
S_{33}^\infty = \lambda(\theta) A_{i0}^0 + a(\theta) A_{i0}^{02} + b(\theta) A_{i0}^0 + \left( \lambda_2 - 1 \right) \left( \mu(\theta) + b(\theta) A_{i0}^0 \right) + 3 \left( \lambda_2 - 1 \right) a(\theta) + \left( \frac{c(\theta)}{3} \right) = 0,
\]

where

\[
A_{i0}^0 = \frac{1}{2} \left( \lambda_i - 3 \right); \quad A_{i0}^2 = \frac{1}{4} \left( \lambda_i - 1 \right) \left( \lambda_i - 1 \right); \quad A_{i0}^4 = \frac{1}{2} \left( \lambda_i - 1 \right) \left( \lambda_i - 1 \right) \left( \lambda_i - 1 \right).
\]

From relations \((5)\) follow the dependencies of \( \lambda_2 \) and \( \lambda_3 \) in terms of \( \lambda_1 \) and, consequently, algebraic invariants \( A_{i0}^0 (i = 1, 2, 3) \) \((6)\) of the strain tensor components and, accordingly, the stress tensor component \((5)\) will depend on the elongation of \( \lambda_1 \) only. Introducing dimensionless quantities by formulas

\[
\frac{\lambda}{2(\lambda + \mu)} = \nu; \quad \frac{\mu}{2(\lambda + \mu)} = 0.5 - \nu; \quad \frac{\lambda + 2\mu}{2(\lambda + \mu)} = 1 - \nu;
\]
\[
\frac{a}{2(\lambda + \mu)} = a_i^i; \quad \frac{b}{2(\lambda + \mu)} = b_i^i; \quad \frac{c}{2(\lambda + \mu)} = c_i^i; \\
\frac{E}{2(\lambda + \mu)} = \frac{\lambda(1+\nu)(1-2\nu)}{2\nu(\lambda + \mu)} = E_i^i; \\
\frac{S_{11}^0}{2(\lambda + \mu)} = S_{11}^0; \quad \frac{S_{22}^0}{2(\lambda + \mu)} = S_{22}^0; \quad \frac{S_{33}^0}{2(\lambda + \mu)} = S_{33}^0;
\]

\[
x_1 = \frac{x_1}{L} \quad (0 \leq x_1 \leq 1); \quad x_2 = \frac{x_2}{L} \quad \left(0 \leq x_2 \leq \frac{L_2}{L}\right); \quad x_3 = \frac{x_3}{L} \quad \left(0 \leq x_3 \leq \frac{L_3}{L}\right);
\]

as a result, we write relations (5) in the dimensionless form

\[
\begin{align*}
S_{11}^0 &= \nu(\theta) A_1^0 + a_1(\theta) A_1^0 + b_1(\theta) A_2^0 + \left(\lambda_i^2 - 1\right) \left(0.5 - \nu(\theta) + b_1(\theta) A_1^0\right) + \\
&\quad + 3 \left(\lambda_i^2 - 1\right) \left(a_i(\theta) + \frac{c_i(\theta)}{3}\right); \\
S_{22}^0 &= \nu(\theta) A_1^0 + a_1(\theta) A_1^0 + b_1(\theta) A_2^0 + \left(\lambda_i^2 - 1\right) \left(0.5 - \nu(\theta) + b_1(\theta) A_1^0\right) + \\
&\quad + 3 \left(\lambda_i^2 - 1\right) \left(a_i(\theta) + \frac{c_i(\theta)}{3}\right) = 0; \\
S_{33}^0 &= \nu(\theta) A_1^0 + a_1(\theta) A_1^0 + b_1(\theta) A_2^0 + \left(\lambda_i^2 - 1\right) \left(0.5 - \nu(\theta) + b_1(\theta) A_1^0\right) + \\
&\quad + 3 \left(\lambda_i^2 - 1\right) \left(a_i(\theta) + \frac{c_i(\theta)}{3}\right) = 0.
\end{align*}
\]

The relation for perturbations of the stress tensor [1-3]

\[
S^m = S^{m}_{11} + S^{m}_{22} + \cdots + S^{m}_{nn}
\]

for Murnagahn's potential will be written in the form

\[
S_{ij} = S_{ij}^m(1) + S_{ij}^m(2) + S_{ij}^m(3) + S_{ij}^m(4) + S_{ij}^m,
\]

where

\[
\begin{align*}
S_{ij}^m(1) &= \left[ \delta_{ij} \left( \nu + 2aA_i^0 \right) + 2E_i^0 E_j^0 \right] E_{ss}^0(1) + \delta_{ij} b_i \left( E_{ss}^0(1) E_{ss}(1) + E_{ik}^0 E_{ik}^0(1) E_{ik}^0 \right) + \\
&\quad + 2 \left(0.5 - \nu + b_1 A_i^0\right) E_{ij}^0(1) + c_i E_{jk}^0(1) E_{kk}^0 + c_i E_{kk}^0(1) E_{kk}; \\
S_{ij}^m(2) &= \left[ \delta_{ij} \left( \nu + 2aA_i^0 \right) + 2E_i^0 b_i \right] E_{ss}^0(2) + \delta_{ij} b_i \left( E_{ss}^0(2) E_{ss}^0(2) + E_{ik}^0 E_{ik}^0(2) E_{ik}^0 + E_{ik}^0(2) E_{ik}^0(2) E_{ik}^0 + E_{ik}^0(1) E_{ik}^0(1) \right) + \\
&\quad + 2 \left(0.5 - \nu + b_1 A_i^0\right) E_{ij}^0(2) + c_i E_{jk}^0(2) E_{kk}^0 + c_i E_{kk}^0(2) E_{kk}^0 +
\end{align*}
\]
\[ +c_1 E_{ikj}(1) E_{kij}(1) + 2b_i E_{ij}(1) E_{iij}(1); \]

\[ S_1^-(3) = 2\delta_j a_i E_{ij}(1) E_{ij}(2) + \delta_j b_i \left[ E_{ij}(1) E_{iij}(2) + E_{iij}(1) E_{ij}(2) \right] + \]

\[ +c_1 \left[ E_{ij}(1) E_{ij}(2) + E_{ij}(2) E_{ij}(1) \right] + 2b_i E_{ij}(1) E_{iij}(2) + 2b_i E_{ij}(2) E_{iij}(1); \]

\[ S_1^+(4) = 2\delta_j a_i E_{ij}(1) + \delta_j b_i E_{ij}(2) + c_1 E_{ij}(2) E_{iij}(2) + 2b_i E_{ij}(2) E_{iij}(2); \]

\[ S_{jm} = \int G^{(1)} \left( 0, q - q'; E, \theta \right) \frac{\partial E(\tau')}{\partial \tau'} d\tau. \]

Solving a nonlinear problem

\[ \int \left[ \left( \delta_x + u_{x}^{0} \right) T_{xij} + T_{xij} u_{x,j} + T_{xij} u_{x,k} \right] \delta_{ij} dV + \rho \int u_{ij} \delta_{ij} dV = 0. \]

with respect to finite perturbations we will seek in the form of series [1]

\[ u_i(\xi_i, t) = \sum f_{im}(t) \cdot \phi_{imm}(\xi_i), \quad i = 1, 2, 3; \quad n, m = 1, 2, \ldots, \infty, \]

where functions \( \phi_{imm}(\xi_i) \) satisfy geometric boundary conditions and are chosen as known solutions to the corresponding linearized stability problem of nonlinear viscoelasticity. For the case of hinged support of the plate edges, the boundary conditions are

\[ u_i = 0; \quad M_{x_i} = 0; \quad u_i = 0 \quad \text{at} \quad x_i = 0; \quad x_i = L_i; \]

\[ u_i = 0; \quad M_{x_i} = 0; \quad u_i = 0 \quad \text{at} \quad x_i = 0; \quad x_i = L_i, \]

where \( M_{x_i} \), \( M_{x_i} \) are moments with respect to the corresponding axes.

Basic functions \( \phi_{imm}(\xi_i) \) are chosen in the form [7] where \( (m, n) \) is the number of half-waves along the \( Ox_i \) and \( Oy_x \) axes, respectively

\[ \phi_{imm}(\xi_i) = C_i \sin \left( \frac{m\pi x_i}{L} \right) \sin \left( \frac{n\pi x_i}{L} \right); \]

\[ \phi_{2imm}(\xi_i) = C_2 \cos \left( \frac{m\pi x_i}{L} \right) \sin \left( \frac{n\pi x_i}{L} \right); \]

\[ \phi_{3imm}(\xi_i) = C_3 \sin \left( \frac{m\pi x_i}{L} \right) \cos \left( \frac{n\pi x_i}{L} \right). \]

The choice of basic functions in this form ensures the fulfillment of the geometric boundary conditions (9). For the Murnaghan potential, the system of equations has the form

\[ A f^{(1)} + B f^{(1)} + C f^{(1)} f^{(1)} + D f^{(1)} f^{(1)} + L(\theta^{(1)}) f^{(1)} + K_1 f^{(1)} + K_2 f^{(1)} f^{(2)} + K_3 f^{(1)} f^{(2)} f^{(3)} + \cdots = 0, \]

and

\[ A_{mkl} f_{kl}^{(1)} + B_{mkl}(f_{kl}) + D_{mkl}(\theta_{kl}) + C_{mkl}(f_{kl}) f_{kl} + K_{mkl} f_{kl} + \]

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\[ + B_{mkl}(f_{kl}) + D_{mkl}(\theta_{kl}) + C_{mkl}(f_{kl}) f_{kl} + K_{mkl} f_{kl} + \]
\[ + K_{only\delta} \dot{f}_{kl} \dot{f}_{dy} + K_{only\delta\gamma} \dot{f}_{kl} \dot{f}_{\delta\gamma1} + K_{only\delta\gamma\delta} \dot{f}_{kl} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} + 
\]
\[ + K_{only\delta\gamma\delta\gamma} \dot{f}_{kl} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} = 0, \tag{12} \]

Coefficients of this system are determined by the formulas

\[ A_{nml} = \int_{V} \varphi_{nml} \varphi_{m} dV; \]
\[ B_{nml} = \int_{V} \left[ \left( \delta_{ij} + u_{i,j} \right) E_{z \mu n}^{(0)} \varphi_{\mu k i} \right] dV \int_{-\infty}^{\infty} G^{(1)} \dot{f}_{i} d\tau; \]
\[ D_{nml} = \int_{V} \left[ \left( \delta_{ij} + u_{i,j} \right) \psi_{(1) \varphi_{\mu n j i}} \right] dV \int_{-\infty}^{\infty} G^{(1)} \dot{\varphi}_{j} d\tau; \]
\[ C_{nml} = \int_{V} E_{z \mu n}^{(0)} \varphi_{\mu \pi j} dV \int_{-\infty}^{\infty} G^{(1)} \dot{f}_{i} d\tau; \]
\[ K_{nml} = \int_{V} \left[ \left( \delta_{ij} + u_{i,j} \right) S_{j \mu n} + \dot{S}_{j \mu n} \varphi_{\pi k i} \right] \varphi_{\pi k i} dV; \]
\[ K_{nml\delta} = \int_{V} \left[ \left( \delta_{ij} + u_{i,j} \right) S_{j \mu n l} \varphi_{k \pi i j} + \dot{S}_{j \mu n l} \varphi_{k \pi i j} \right] \varphi_{k \pi i j} dV; \]
\[ K_{nml\delta\gamma} = \int_{V} \left[ \left( \delta_{ij} + u_{i,j} \right) S_{j \mu n l \delta \gamma1} + \dot{S}_{j \mu n l \delta \gamma1} \varphi_{k \pi i j} \right] \varphi_{k \pi i j} dV; \]
\[ K_{nml\delta\gamma\delta} = \int_{V} \left[ S_{j \mu n l \delta \gamma1} \varphi_{k \pi i j} \right] \varphi_{k \pi i j} dV; \]
\[ K_{nml\delta\gamma\delta\gamma} = \int_{V} \left[ S_{j \mu n l \delta \gamma1} \varphi_{k \pi i j} \right] \varphi_{k \pi i j} dV. \tag{13} \]

The stability condition for the zero solution of system (12), as was shown in [1-3, 6], is the positivity of the function

\[ \Pi = \frac{1}{2} A_{nml} \ddot{f}_{kl} + \frac{1}{2} B_{nml} \ddot{f}_{kl} + \frac{1}{3} K_{nml\delta} \ddot{f}_{kl} \dot{f}_{\delta\gamma1} + \frac{1}{4} K_{nml\delta\gamma} \ddot{f}_{kl} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} + 
\]
\[ + \frac{1}{5} K_{nml\delta\gamma\delta} \ddot{f}_{kl} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} + \frac{1}{6} K_{nml\delta\gamma\delta\gamma} \ddot{f}_{kl} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} \dot{f}_{\delta\gamma1} + 
\]
\[ + \int_{-\infty}^{\infty} C_{nml} \dot{f}_{\mu n} d\tau + \int_{-\infty}^{\infty} D_{nml} \dot{f}_{\mu n} d\tau + \int_{-\infty}^{\infty} B_{nml} \dot{f}_{\mu n} d\tau; \tag{14} \]

for perturbations satisfying a system of equations

\[ A_{nml} \dot{f}_{kl} (0) + B_{nml} \left( \dot{f}_{kl} (0) \right) + C_{nml} \left( f_{kl} (0) \right) + D_{nml} \left( \dot{f}_{kl} (0) \right) = 0; \]
\[ K_{nml} \dot{f}_{kl} (0) + K_{nml\delta} \dot{f}_{kl} (0) \dot{f}_{dy} (0) + K_{nml\delta\gamma} \dot{f}_{kl} (0) \dot{f}_{dy} (0) \dot{f}_{\delta\gamma1} (0) + 
\]
In this case, for a viscoelastic material in coefficients (13) of ratios (14), (15), the actual parameters should be replaced by complex ones. Obviously, the defined values of perturbations will also be complex. In this case, the real part will coincide with the corresponding solution to the nonlinear elasticity problem for finite perturbations, and the imaginary part will give the damping coefficient. Since the separation into real and imaginary parts was used to solve the problem of stability of nonlinear elastic bodies under finite perturbations, all the results described in [1-3], will take place here as well.

3. Results and discussion
The results of the computational experiment are shown in figures 1-4 for the given geometric and physical parameters.

Figure 1. The dependence of the permissible initial perturbation $|f|$ on the elongation parameter $\lambda_1$ at $\frac{h}{L} = 10^{-3}$; $a - \pi \frac{L_1}{L} = 1$; $b - \pi \frac{L_1}{L} = 0.5$.

In figure 1, on the phase plane $|f| - \lambda_1$, the regions of permissible perturbations are constructed in which, as in the stability problems of nonlinear elastic bodies, a hierarchy of stable states is observed. Curves 1 – 4 correspond to different potentials. Here and below, in figure 1 – 4 curve 1 corresponds to the Murnaghan potential, curve 2 corresponds to the two-constant potential, curve 3 corresponds to the Mooney potential, curve 4 corresponds to the Treloar potential. They limit the range of permissible perturbations. Figure 2 shows the dependencies of the dimension of the strange attractor [8] of the corresponding dynamical system on the value of the load parameter (elongation $\lambda_1$).
Figure 2. The dependence of the dimension of the strange attractor $\gamma_m$ on the elongation $\lambda_1$ at $\frac{h}{L} = 10^{-3}$; a – $\pi \frac{L_1}{L} = 1$, b – $\pi \frac{L_1}{L} = 0.5$.

Obviously, the results obtained above can be easily generalized to the case of a circular plate of radius $R$ if $L = L_1 = R$, $m = n$ are putted everywhere in them. Here we will limit ourselves to the results of numerical simulations. They are shown in figure 3 – 4.

Figure 3. Dependence of the permissible initial perturbation $|f|$ on the elongation parameter $\lambda_1$; a – $\pi \frac{h}{R} = 0.1$; b – $\pi \frac{h}{R} = 0.2$.

Figure 3 shows the regions of permissible perturbations – regions of stability that are bounded on the $|f| - \lambda_1$ plane by curves that represent the boundary of an infinite sequence of permissible stable equilibrium states. Figure 4 shows the relationship between the dimension of the strange attractor of a dynamical system [9, 10] describing the behavior of a circular plate made of a nonlinear viscoelastic material and the load parameter.
4. Conclusions
Analysis of figures 1 – 4 shows the following features:

- reducing the size ratios of structures also reduces the stability region with respect to initial perturbations and elongations;
- for all the considered cases, the dimension of the strange attractor decreases with increasing load parameter values.

We also emphasize that the obtained dimension of the strange attractor for the considered problem allows us to recommend the number of terms in $u_i(\xi_k, t) = \sum_{n,m} f_{nm}(t) \cdot \varphi_{nm}(\xi_k)$ when the structure operates in different ranges of changes in initial stresses and deformations [1].

Using the stability criterion with respect to finite perturbations [1] makes it possible to obtain a limited sequence of acceptable values of initial perturbations in which the main deformation process will be stable for specific values of the load parameter.

Setting acceptable values of initial perturbations leads to obtaining the area of change in the load parameter in which the main deformation process will be stable.

It should be noted that similar problems were considered in [4] in the framework of the three-dimensional linearized stability theory for nonlinear elastic bodies.

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