Inflation in Kaluza-Klein Theory: Relation between the Fine-Structure Constant and the Cosmological Constant

Li-Xin Li and J. Richard Gott, III
Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544
(April 28, 1998)

In this paper we investigate a model of an inflationary universe in Kaluza-Klein theory, which is a four-dimensional de Sitter space plus a one-dimensional compactified internal space. We find that the energy scale for inflation can be predicted from the fine-structure constant in a self-consistent solution of the semi-classical Einstein equations including the Casimir effect. From the observed value of the fine-structure constant, we obtain an energy scale for inflation of \( \epsilon = 1.84 \times 10^{16} g_*^{1/4} \) GeV, where \( g_* \) is a dimensionless number depending on the spin and number of matter fields existing in the universe. This value is consistent with the values often discussed for inflation and grand unification. The wave function for this model predicts a high probability for forming such universes, independent of the value of the cosmological constant. The tunneling probability favors the creation of inflationary universes with a compactified dimension, over those with all macroscopic dimensions.

PACS number(s): 98.80.Cq, 04.62.+v, 04.50.+h.

Kaluza-Klein theory is a five-dimensional theory of gravity and electrodynamics, which is a combination of Einstein’s theory of gravity and Maxwell’s theory of electrodynamics [1]. In this theory the electric charge is quantized and the fine-structure constant \( \alpha = e^2 \) is determined by the circumference \( b \) of the one-dimensional internal space via \( \alpha = 64\pi^2 G/b^2 \) where \( G \) is the four-dimensional Newton’s gravitational constant which is related to the five-dimensional gravitational constant \( G_5 \) via \( G_5 = G b \). (Throughout we use units where \( c = \hbar = 1 \). If instead of the electron charge \( e \), considering quarks, we use the electric charge \( e/3 \) as the fundamental charge, the value of \( b \) will be three times larger than the value we will quote.) In this theory, the five-dimensional gravitational constant \( G_5 \) is the unique fundamental coupling constant, the four-dimensional gravitational constant and the fine-structure constant are determined by the value of \( G_5 \) and the circumference of the internal space. To keep the four-dimensional Newton’s gravitational constant and the fine-structure constant invariant with time, the circumference of the internal space must not change with time. Modern accelerators have probed matter at scales as small as \( 10^{-16} \) cm without finding any evidence for internal dimensions [3]. By contrast, our four-dimensional spacetime has a spatial scale as large as (or greater than) \( 10^{28} \) cm. If both the external and internal (if it exists) scales originate from the Planck scale (~ \( 10^{-33} \) cm) as modern cosmology suggests, the rate of change in the internal scale must be sufficiently small compared with the expansion of the four-dimensional observed universe. Observations limiting the variation of coupling constant with time also place strict restriction on the rate of change in internal dimensions. Primordial nucleosynthesis in Kaluza-Klein theory implies that \( 1.01 \geq b_N/b_0 \geq 0.99 \) where \( b_N \) is the circumference of internal space at the epoch of primordial nucleosynthesis and \( b_0 \) is the circumference of the internal space today [3]. (By contrast, for the expand scale factor \( a \) of the observed four-dimensional universe, in the Big Bang theory we have \( a_N/a_0 \sim 10^{-10} \).) Thus, naturally, in cosmology with extra dimensions people try to find solutions with the external dimensions expanding while the internal dimensions remain static. But at present no mechanism for keeping the internal spatial scale static has been found. In this paper, we give a model of an inflationary universe in Kaluza-Klein theory, which is static in the one-dimensional internal dimension while expanding in the external dimensions. We find that this model can solve the semi-classical Einstein equations with the Casimir effect [3] or vacuum polarization considered, and the fine-structure constant \( \alpha \) is related to the energy scale for inflation, \( \epsilon \), via \( \epsilon/\epsilon_p \simeq 0.0176 g_*^{1/4} \alpha^{1/2} \), where \( \epsilon_p = G^{-1/2} \) is the Planck energy, and \( g_* \) is a dimensionless number determined by the spin and number of matter fields in the early universe \( (g_* \sim 100) \). The idea that the energy-momentum tensor required to produce the geometry of spacetime with internal dimensions (via Einstein equations) is provided by the Casimir effect plus a cosmological constant has been investigated by Weinberg [3] and Candelas and Weinberg [3] for the case of four-dimensional flat Minkowski spacetime plus extra dimensions. They have found that in order to produce a reasonable value of the gauge coupling constant, an enormous number (greater than 1000) of matter fields are needed which is not supported by observations. Our model is a four-dimensional de Sitter space plus a compactified one-dimensional flat internal space. For this model, we find that in order to produce the correct value of the fine-structure constant \( \alpha = 1/137.036 \) it is required that the energy scale for inflation is \( \epsilon = 1.84 \times 10^{16} g_*^{1/4} \) GeV which for any reasonable value of \( g_* \sim 1 - 100 \) is consistent with values often discussed for inflation and grand unification [3].

Our model is \( dS^4 \times S^1 \) where \( dS^4 \) is a four-dimensional de Sitter Space and \( S^1 \) is a compactified one-dimensional flat Euclidean space with circumference \( b \). The metric is
\[ ds^2 = -d\tau^2 + r_0^2 \cos^2 \frac{\tau}{r_0} \left[ d\psi^2 + \sin^2 \psi \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right] + dq^2, \]  

(1)

where \((\tau, \psi, \theta, \phi)\) are the usual global coordinates on \(dS^4\), \(r_0\) is the radius of \(dS^4\), \(q\) is the Cartesian coordinate on \(S^1\), and \((\tau, \psi, \theta, \phi, q)\) is identified with \((\tau, \psi, \theta, \phi, q + nb)\) where \(n = 0, \pm 1, \pm 2, \ldots\). Extending to the Euclidean section by setting \(\tau \rightarrow -ir_0(\chi - \pi/2)\), \(dS^4 \times S^1\) is extended to \(S^4 \times S^1\) where \(S^4\) is a four-sphere with radius \(r_0\) embedded in a five-dimensional flat Euclidean space. The corresponding metric extended from Eq. (1) is

\[ ds^2 = r_0^2 \left\{ d\chi^2 + \sin^2 \chi \left[ d\psi^2 + \sin^2 \psi \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right] \right\} + dq^2, \]  

(2)

where \((\chi, \psi, \theta, \phi)\) are spherical coordinates on \(S^4\) with \(0 < \chi < \pi\), \(0 < \psi < \pi\), \(0 < \theta < \pi\), and \(0 \leq \phi < 2\pi\). Again, \((\chi, \psi, \theta, \phi, q)\) is identified with \((\chi, \psi, \theta, \phi, q + nb)\). A prominent feature of this model is that, in the Lorentzian section \(dS^4 \times S^1\), the four-dimensional internal space (the de Sitter space) is exponentially expanding while the one-dimensional compactified internal space \(S^1\) is static; in the Euclidean section \(S^4 \times S^1\), both the external space \(S^4\) and the internal space \(S^1\) are static and can have comparable scales.

Various Lorentzian spacetimes can be obtained from different continuations of the Euclidean space \(S^4 \times S^1\): 1) if we make the continuation \(\chi \rightarrow \pi/2 + i\tau/r_0\), \(S^4\) is extended to \(dS^4\) with the global coordinates having closed spatial sections, which can be used to describe a closed universe created from nothing \([8, 10]\); in our Kaluza Klein model, \(S^4 \times S^1\) is extended to \(dS^4 \times S^1\), which is a four-dimensional closed de Sitter universe with a compactified one-dimensional internal space and can be used to describe the creation of a closed five-dimensional inflationary universe with one dimension compactified and static; 2) if we first make the continuation \(\psi \rightarrow \pi/2 + i\tau\) and then make the continuation \(\chi \rightarrow it\) and \(\tau \rightarrow i\tau/2 + \psi\) \([11]\), \(S^4\) is extended to \(dS^4\) with an open spatial section describing an open inflation \([12, 13]\), which can be used to describe the creation of an open inflationary universe created from nothing \([11]\); in this case, in our Kaluza-Klein model, \(S^4 \times S^1\) is extended to an open de Sitter universe with a compactified one-dimensional internal space, i.e., creation of an open inflationary universe with a static compactified one-dimensional internal space (see \([10]\) for a non-static internal dimension case); 3) if we make the continuation \(\phi \rightarrow it\), \(S^4\) is extended to a de Sitter space with closed timelike curves (CTCs) which describes a universe created from itself \([7]\); in this case in our Kaluza-Klein model \(S^4 \times S^1\) is extended to a de Sitter space with CTCs and a static and compactified one-dimensional internal space; 4) if we make the continuation \(q \rightarrow it\), \(S^4 \times S^1\) is extended to a five-dimensional Einstein static universe with CTCs (or without CTCs if \(S^1\) is unfolded). Thus, we make our calculations in the Euclidean section \(S^4 \times S^1\) and then continue the results to the Lorentzian spacetimes we are interested in (except for the response function of a particle detector, which has to be calculated in the appropriate Lorentzian spacetime).

The space \(S^4 \times S^1\), with the metric in Eq. (2), is conformally flat. With the coordinate transformation

\[ r = \frac{\sin \chi}{2[\cos \chi + \cosh (q/r_0)]}, \quad p = \frac{\sinh (q/r_0)}{2[\cos \chi + \cosh (q/r_0)]}, \]  

(3)

the metric in Eq. (2) can be written as \(ds^2 = \Omega^2 ds^2\), where \(\Omega = 2r_0 \cos \chi + \cosh (q/r_0)\) and \(ds^2\) is the metric of the five-dimensional flat Euclidean space

\[ ds^2 = dr^2 + r^2 \left[ d\psi^2 + \sin^2 \psi \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right] + dp^2. \]  

(4)

Thus \(S^4 \times S^1\) is conformally flat. Consider a massless and conformally coupled scalar field. The Hadamard function for the Minkowski vacuum in five-dimensional flat Euclidean space is

\[ G^{(1)}_M (X, X') = \frac{1}{4\pi^2} \frac{1}{\sigma^3} = \frac{1}{4\pi^2} \left[ \frac{1}{r^2 + r'^2 - 2rr' \cos \Theta_3 + (p - p')^2} \right]^{3/2}, \]  

(5)

where \(X = (r, \psi, \theta, \phi, p)\), \(X' = (r', \psi', \theta', \phi', p')\), \(\sigma\) is the Euclidean distance between \(X\) and \(X'\), and \(\cos \Theta_3 = \cos \psi \cos \psi' + \sin \psi \sin \psi' \cos \Theta_2\) with \(\cos \Theta_2 = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi - \phi')\). The corresponding Hadamard function for the conformal Minkowski vacuum in \(S^4 \times R^1\) with the metric in Eq. (2) is given by \(G^{(1)} (X, X') = \Omega^{-3/2} (X) \tilde{G}^{(1)} (X, X') \Omega^{-3/2} (X') \[3\], which is

\[ G^{(1)}_{CM} (X, X') = \frac{1}{2^{7/2} \pi^2 r_0^3} \frac{1}{\{\cosh [(q - q')/r_0] - \cos \Theta_4\}^{3/2}}, \]  

(6)

where \(X = (\chi, \psi, \theta, \phi, q)\), \(X' = (\chi', \psi', \theta', \phi', q')\), and \(\cos \Theta_4 = \cos \chi \cos \chi' + \sin \chi \sin \chi' \cos \Theta_3\) (\(\Theta_4\) is the angular distance between \(X\) and \(X'\) on \(S^4\)). The Hadamard function for the massless conformally coupled scalar field in the
adapted conformal Minkowski vacuum (i.e. the conformal Minkowski vacuum with multiple images) in the Euclidean space $S^4 \times S^1$ is given by the summation of Eq. $(7)$ with multiple images

$$G^{(1)}(X, X') = \frac{1}{27/2 \pi^2 r_0^3} \sum_{n=-\infty}^{\infty} \frac{1}{\cosh ([q - q' + nb]/r_0) - \cos \Theta_4}^{3/2}. \tag{7}$$

The expectation value of the energy-momentum tensor is obtained from the Hadamard function by

$$\langle T_{ab} \rangle = \frac{1}{2} \lim_{X' \to X} D_{ab} G^{(1)}(X, X'), \tag{8}$$

where for a massless conformally coupled scalar field in a five-dimensional spacetime

$$D_{ab} = \frac{5}{8} \nabla_a \nabla_b - \frac{3}{8} \nabla_a \nabla_b + \frac{3}{8} g_{ab} \left( g^{cd} \nabla_c \nabla_d - \frac{1}{3} g^{cd} \nabla_c \nabla_d \right) + \frac{3}{16} \left( R_{ab} - \frac{1}{2} R g_{ab} \right), \tag{9}$$

where $R_{ab}$ is the Ricci tensor and $R = g^{ab} R_{ab}$. Inserting Eq. $(11)$ into Eq. $(8)$, the $n = 0$ term diverges as $X' \to X$, thus renormalization must be taken. However, here the renormalization is simplified by noting that the $n = 0$ term in Eq. $(8)$ is just the Hadamard function in $S^4 \times R^1$ given in Eq. $(10)$. For the massless conformally coupled scalar field in odd dimensional conformally related spaces, there is no trace anomaly and the renormalized energy-momentum tensors are related by

$$\langle T^b_a \rangle_{\text{ren}} = \Omega^{-4} \langle T^b_a \rangle_{\text{ren}}. \tag{10}$$

For the Minkowski vacuum in $R^5$ with metric in Eq. $(4)$ we have $\langle T^a_b \rangle_{\text{ren}} = 0$. Thus for the conformal Minkowski vacuum in $S^4 \times R^1$ with metric in Eq. $(4)$ we also have $\langle T^b_a \rangle_{\text{ren}} = 0$, by Eq. $(10)$. The $n = 0$ term’s contribution to the renormalized energy-momentum tensor of the adapted conformal Minkowski vacuum in $S^4 \times S^1$ happens to be the renormalized energy-momentum tensor in $S^4 \times R^1$, which is thus zero as discussed above. Therefore, all contributions to the renormalized energy-momentum tensor in $S^4 \times S^1$ come from the $n \neq 0$ terms in the expansion of the Hadamard function in Eq. $(8)$, a purely Casimir effect. Inserting all $n \neq 0$ terms in Eq. $(8)$ into Eq. $(10)$, we obtain the renormalized energy-momentum tensor of the adapted conformal Minkowski vacuum in $S^4 \times S^1$

$$\langle T^\mu_\nu \rangle_{\text{ren}} = \frac{3A}{8\pi r_0^3} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix}, \tag{11}$$

where $A = \frac{1}{b^2} \sum_{n=1}^{\infty} \frac{3 \sinh^2 n \beta + 4}{n^2 \sinh^2 \beta}$ and $\beta = b/(2r_0)$, the coordinates are $(\chi, \psi, \theta, \phi, q)$. The renormalized energy-momentum tensor in Eq. $(11)$ has the form of radiation if we regard $q$ as Euclidean time.

The Ricci tensor of $S^4 \times S^1$ with metric in Eq. $(4)$ is $R_{ab} = (3/r_0^2) (g_{ab} - dq_adq_b)$. The Ricci scalar is $R = g^{ab} R_{ab} = 12/r_0^2$. The five-dimensional semi-classical Einstein equations are

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi G_5 \langle T^b_a \rangle_{\text{ren}}, \tag{12}$$

where $\Lambda$ is the cosmological constant and $G_5$ is the five-dimensional gravitational constant. Inserting Eq. $(11)$ into Eq. $(12)$, we find that the semi-classical Einstein equations are solved if

$$r_0^3 = 5G_5 A, \tag{13}$$

and $\Lambda = 18G_5 A/r_0^5 = 18/5r_0^2$. The asymptotic solution of Eq. $(13)$ as $r_0 \to \infty$ is $b = (10G_5/n)^{1/5} r_0^{2/5}$. Numerical calculation shows that this solves Eq. $(13)$ very accurately for any $r_0 > l_p$ where $l_p = G_5^{1/3}$ is the five-dimensional Planck length. For example, for $r_0 = l_p$, the asymptotic solution gives $b = 1.2606 l_p$, while the numerical solution is $b = 1.2594 l_p$, the relative error is only 0.1%. Thus the asymptotic solution solves Eq. $(13)$ very accurately for $r_0 > l_p$. If there are many conformally coupled matter fields instead of only one scalar field, the renormalized energy-momentum tensor is given by Eq. $(11)$ multiplied by a factor $g_*$ determined by the number of species and spins of the matter fields. Then the asymptotic solution is
Using the four-dimensional Planck length $l_p = G^{1/2}$ and $G = G_5/b$, Eq. (14) can also be written as

$$\frac{b}{l_p} = \left( \frac{10g_s}{\pi} \right)^{1/4} \left( \frac{r_0}{l_p} \right)^{1/2} \simeq 1.36g_s^{1/4} \left( \frac{r_0}{l_p} \right)^{1/2}.$$  

(15)

The fine-structure constant is $\alpha = 64\pi^3G/b^2 = 64\pi^3(l_p/b)^2$. For an observer in $dS^4 \times S^1$, which is a Lorentzian extension of $S^4 \times S^1$, the effective cosmological constant is $\Lambda_{\text{eff}} = 3/r_0^2 = (5/6)\Lambda$. Thus Eq. (17) gives a correlation between $\Lambda_{\text{eff}}, \alpha$ and $G$

$$G\Lambda_{\text{eff}} = \frac{15g_s}{2048\pi^2} \alpha^2.$$  

(16)

In inflation theory people usually use an inflation potential instead of the effective cosmological constant. The inflation potential is related to the effective cosmological constant via $\Lambda_{\text{eff}} = 8\pi GV$. If we write the inflation potential as $V = \epsilon^4$ where $\epsilon$ is the energy scale for inflation, Eq. (14) thus gives a correlation between the energy scale of inflation and the fine-structure constant

$$\frac{\epsilon}{\epsilon_p} = \left( \frac{15^{1/4}g_s^{1/4}}{27/2\pi^2} \right) \alpha^{1/2} \simeq 0.0176g_s^{1/4} \alpha^{1/2},$$  

(17)

where $\epsilon_p = G^{-1/2}$ is the Planck energy. If at the epoch of inflation the fine-structure constant and the four-dimensional gravitational constant have the same values as today (as argued in the beginning of the paper), we have $\epsilon \simeq 1.8 \times 10^{16}g_s^{1/4} \text{ GeV}$ and $b = 52l_p = 8.4 \times 10^{-31}$ cm (or $\epsilon \simeq 0.61 \times 10^{16}g_s^{1/4}$ Gev and $b = 1.56 \times 10^4l_p = 2.5 \times 10^{-30}$ cm if charge is quantized in units of $e/3$). The value of $\epsilon$ is not very sensitive to $g_s$ which is expected to be of order $10^2$ and an energy scale often discussed for inflation and grand unification 

$S^4 \times S^1$ is a compact five-dimensional Euclidean space. The Euclidean action of a compact manifold plays an important role in Hartle and Hawking’s no-boundary proposal for quantum cosmology, which gives the probability for a universe created from nothing [1]. With the Wick rotation $t \to -i\tau$, the Euclidean Hilbert-Einstein action is $I_g = -\int d^5x\sqrt{g}(R - 2\Lambda)$. For $S^4 \times S^1$, we have $R = 12/r_0^2$ and thus $I_g = -\int d^5x\sqrt{g}(12/r_0^2 - 2\Lambda) V_5$ where $V_5 = 8\pi^2r_0^5/5$ is the volume of $S^4 \times S^1$. For the self-consistent (i.e., the Einstein equations are solved) case, we have $\Lambda = 18/5r_0^2$ and thus $I_g = -4\pi^2r_0^2/5$ for $S^4 \times S^1$. The vacuum polarization of conformal fields (which is pure Casimir effect for the case of $S^4 \times S^1$) gives rise in the Euclidean regime to a radiation-like energy-momentum tensor [Eq. (11)], whose Euclidean action is $I_m = \int d^5x\sqrt{g}\rho$ where $\rho$ is the “energy” density (in this case it is minus the $q - q$ component of $\langle T_{\mu\nu}\rangle_{\text{ren}}$). From Eq. (11) we have $\rho = 3\Lambda/2\pi^2r_0^6 = 3/(10\pi G r_0^2)$ where Eq. (13) has been used, thus $I_m = \rho V_5 = 4\pi^2r_0^2/5$. Notice that $I_m$ and $I_g$ have the same magnitude but opposite sign, thus the total Euclidean action is $I = I_g + I_m = 0$ (this result is independent of $g_s$), and thus the probability for the creation of the universe from nothing is $P = 1$ in the Hartle and Hawking formulation. This means that the creation of universes with different radius $r_0$ (and thus different effective cosmological constant $\Lambda_{\text{eff}} = 3/r_0^2$) has equal probability.

For a five-sphere $S^5$ with metric $ds^2 = r_0^2 (d\Omega_6^2 + \sin^2 \alpha d\Theta_6^2)$ where $r_0$ is the radius of $S^5$, we have $R = 20/r_0^2$ and $V_5 = \pi^3 r_0^5$. Such an $S^5$ is a Euclidean solution of the five-dimensional vacuum Einstein equations with a cosmological constant $\Lambda = 6/r_0^2$. (The renormalized energy-momentum tensor is zero for a conformally coupled scalar field in the conformal Minkowski vacuum in $S^5$ due to the fact $S^5$ is conformally flat and there is no trace anomaly in odd-dimensional spaces.) Thus for $S^5$ we have $I = I_g = -(1/16\pi G_5) \int d^5x\sqrt{g}(R - 2\Lambda) = - (\pi^2/2G_5) r_0^4 = - (\pi^2/2G_5) (6/\Lambda)^{3/2}$. The probability for the creation of a universe from an instanton is given by $P = e^{-I}$. (For the necessity of the absolute value of $I$ in the expression for $P$, see [13]. This is obviously true for quantum tunneling processes in ordinary quantum mechanics in flat spacetime.) Thus the probability for the creation of a universe from $S^5$ is $P = \exp \left[-(\pi^2/2) (r_0/l_p)^3\right] = \exp \left[-(\pi^2/2G_5) (6/\Lambda)^{3/2}\right]$, which is always less than unity (for $r_0 > l_p$, we have $P < 0.7\%$). Therefore the creation of a universe from $S^4 \times S^1$ (which has a probability equal to unity) is more probable than from $S^5$, which could explain why we are not living in a universe with five macroscopic dimensions.

Suppose there is a point-like particle detector moving along a geodesic in the spacetime of $dS^4 \times S^1$, with $q = $ constant. Since the vacuum state is de Sitter invariant, we can always choose global coordinates in de Sitter space
such that $\psi = \text{constant}$, $\theta = \text{constant}$, and $\phi = \text{constant}$ long the geodesic. The response function of the particle detector is

$$\mathcal{F}(\Delta E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-i\Delta E(\tau-\tau')} G^+(X(\tau), X(\tau')),$$

where $G^+$ is the Wightman function of the detected field, $\tau$ is the proper time of the detector’s worldline, $\Delta E$ is the energy difference between an excited state of the detector and its ground state. The Hadamard function in $dS^4 \times S^1$, in the global de Sitter coordinates, is obtained from Eq. [9] by extending $\chi \to \pi /2 + i\tau /r_0$. The Wightman function is equal to one half of the corresponding Hadamard function with $\tau$ replaced by $\tau - i\epsilon /2$ and $\tau'$ replaced by $\tau' + i\epsilon /2$ where $\epsilon$ is a positive infinitesimal real number [22,17]. On the worldline of the particle detector, the Wightman function so obtained is

$$G^+(\Delta \tau) = \sum_{n=-\infty}^{\infty} \frac{1}{\cosh(n b/r_0) - \cosh[(\Delta \tau - i\epsilon)/r_0]^{3/2}},$$

where $\Delta \tau = \tau - \tau'$. Inserting Eq. [13] into Eq. [18], the integral of the $n = 0$ term can be worked out with the residue theorem by choosing a contour closed in the lower-half plane of complex $\Delta \tau$. The result is a Fermi-Dirac-like distribution with temperature $T = 1/2\pi r_0$ (the Boltzmann constant is taken to be unity)

$$\frac{d\mathcal{F}}{d\tau} = \frac{1}{8\pi r_0} \frac{(\Delta E r_0)^2 + 1/4}{e^{2\pi \Delta E r_0} + 1},$$

where $\tau = (\tau + \tau') / 2$. (Similar conclusions for an accelerated particle detector in odd dimensional Minkowski spaces can be found in [23].) The terms with $n \neq 0$ cannot be worked out with the residue theorem because a closed contour on which the integrand is analytic does not exist. The integrals of the $n \neq 0$ terms can only be worked out numerically. However, it can be estimated that the contributions of the $n \neq 0$ terms are negligible compared to the $n = 0$ term’s contribution, because, for large $\Delta E r_0$, the oscillation of the integrand causes the contributions from different terms with different $n$ to cancel each other; for small $\Delta E r_0$, the contribution of the $n \neq 0$ terms is of the order of $(\Delta E r_0)^2$. Thus the $n = 0$ term dominates the contribution to the response function.

Thus, consideration of Kaluza-Klein theory in terms of inflation offers some promising features. Given the observed value of the fine-structure constant a self-consistent solution including the Casimir effect for an early inflationary state suggests an effective value for the cosmological constant which is quite reasonable (corresponding to an energy scale of $1.84 \times 10^{16} g_0^{1/4}$ Gev where $g_0 \sim 100$). Interestingly, when the renormalized energy-momentum tensor due to the Casimir effect is included in the Euclidean action it makes tunneling to create an inflationary universe with a compactified dimension quite probable.

This work is supported by NSF grant AST95-29120 and NASA grant NAG5-2759.

[1] T. Kaluza, Preus. Acad. Wiss. K 1, 966 (1921); O. Klein, Zeit. Phys. 37, 895 (1926).
[2] E. W. Kolb and M. S. Turner, The Early Universe (Addison-Wesley Publishing Company, Redwood City, 1990).
[3] E. W. Kolb, M. J. Perry, and T. P. Walker, Phys. Rev. D 33, 869 (1986).
[4] H. B. G. Casimir, Proc. Kon. Ned. Akad. Wet. 51, 793 (1948).
[5] S. Weinberg, Phys. Lett. B 125, 265 (1983).
[6] P. Candelas and S. Weinberg, Nucl. Phys. B 237, 397 (1984).
[7] A. R. Liddle, Phys. Rev. D 49, 739 (1994); E. F. Bunn, A. R. Liddle, and M. White, Phys. Rev. D 54, R5917 (1996).
[8] A. Vilenkin, Phys. Lett. B 117, 25 (1982).
[9] J. B. Hartle and S. W. Hawking, Phys. Rev. D 28, 2960 (1983).
[10] A. D. Linde, JETP 60, 211 (1984); and Lett. Nuovo Cim. 39, 401 (1984).
[11] S. W. Hawking and N. Turok, hep-th/9802038 (1998).
[12] J. R. Gott, Nature 295, 304 (1982); J. R. Gott, in Inner Space /Outer Space, edited by E. W. Kolb et al (University of Chicago Press, Chicago,1986).
[13] M. Bucher, A. S. Goldhaber, and N. Turok, Phys. Rev. D 52, 3314(1995).
[14] B. Ratra and P. J. E. Peebles, Phys. Rev. D 52, 1837 (1995).
[15] A. D. Linde, Phys. Lett. B 351, 99 (1995); A. D. Linde and A. Mezhlumian, Phys. Rev. D 52, 6789 (1995).
[16] J. Garriga, hep-th/9804106 (1998).
[17] J. R. Gott and L. -X. Li, astro-ph/9712342 (1997), to appear in Phys. Rev. D.
[18] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge University Press, Cambridge, 1982).
[19] B. S. DeWitt, in General Relativity: An Einstein Centenary Survey, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979).
[20] E. Fahri, A. H. Guth, and J. Guven, Nucl. Phys. B 339, 417 (1990).
[21] A. Linde, gr-qc/9802038 (1998).
[22] L. -X. Li and J. R. Gott, Phys. Rev. Lett. 80, 2980 (1998).
[23] S. Takagi, Prog. Theo. Phys. 74, 142 (1985).