Particle Finite Element Simulation of Fresh Cement Paste – Inspired by Additive Manufacturing Techniques

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The properties of hardened concrete structures, fabricated by novel additive manufacturing techniques with fresh concrete, are significantly influenced by the manufacturing process. A numerical framework for the simulation of extrusion processes in additive manufacturing techniques based on layered extrusion is proposed. The flow behavior of fresh concrete is approximated with a regularized Bingham model and in order to deal with large deformation, governing equations are solved via the Particle Finite Element Method.

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1 Theoretical Framework

Fresh concrete is considered as a homogenized material, covering the domain Ω in the time interval (0, T), whose balance of momentum and mass are given as

$$\rho \dot{v} = \nabla \cdot \sigma + b \quad \text{and} \quad \nabla \cdot v = 0 \quad \text{in} \quad \Omega \times (0, T),$$

where \( \rho \) is the density, \( v \) the velocity, \( \sigma \) is the Cauchy stress tensor and \( b \) denotes the external body force vector. Note that the domain \( \Omega \) is given in the updated configuration. Boundary conditions on the Neumann boundary \( \partial \Omega^\square \) are given as \( \sigma (x, t)n = t^\square (x, t) \) where \( n \) is the normal vector and \( t^\square \) are prescribed tractions. Boundary conditions on the Dirichlet boundary \( \partial \Omega^d \) are defined by prescribing the velocities \( v^d(x, t) \) as \( v(x, t) = v^d(x, t) \). Furthermore, the Cauchy stresses are split into the hydrostatic pressure \( p \) and the deviatoric part \( \tau \) as \( \sigma = \tau - pI \) with the second order identity tensor \( I \). The deviatoric part of the Cauchy stresses is approximated with a regularized Bingham model as

$$\tau = 2 \left( \mu + \frac{\tau_0}{|\gamma|} \left( 1 - e^{-m|\gamma|} \right) \right) \dot{\varepsilon}(v) \quad \text{with:} \quad |\gamma| = \sqrt{2\varepsilon(v) : \dot{\varepsilon}(v)}$$

in which \( \mu \) is the viscosity, \( \tau_0 \) the dynamic yield stress, \( \varepsilon \) the strain rates and \( m \) a regularization parameter.

2 Numerical Framework

The numerical framework is based on the Particle Finite Element Method (PFEM) [1]. In PFEM the governing equations are discretized by means of Finite Elements in an updated Lagrangian framework, by updating nodal positions in each time step. In order to deal with large deformations and distorted elements, the domain is re-meshed from time to time when necessary. The re-meshing algorithm is based on a Delaunay triangulation in association with the \( \alpha \)-shape method to identify connected elements.

The discretized form of (1), by taking into account all boundary conditions, is given as

$$\begin{bmatrix} M_{vv} & 0 & 0 \\ 0 & M_{vp} & 0 \\ 0 & 0 & M_{pp} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{N} \end{bmatrix} = \begin{bmatrix} K_{vv} & -G_{vp} & -F_v \\ -G_{vp} & 0 & \bar{F}_p \\ -F_v & \bar{F}_p & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ N \end{bmatrix} = 0$$

where \( M_{vv} = \int_{\Omega} \bar{N}_v^T \rho \bar{N}_v \, d\Omega \) is the mass matrix, \( K_{vv} = \int_{\partial\Omega^\square} \bar{B}_v^T \bar{C} \bar{B}_v \, d\Omega \) the stiffness matrix, \( G_{vp} = \int_{\partial\Omega^d} \bar{N}_v \, t^d \, d\Omega^n \) the gradient matrix and \( F_v = \int_{\partial\Omega^d} \bar{N}_v \, \rho \, d\Omega^n \) the external force vector with the shape functions of the velocity and pressure field \( \bar{N}_v \) and \( \bar{N}_p \), the discretized differential operator \( \bar{B}_v \) that is compatible with the velocity field and \( \bar{N}_p \) the discretized differential operator \( \bar{B}_p \) that is compatible with the pressure field and \( m = [1, 0, 0]^T \) given for 2D elements. Due to the assumption of incompressibility and equal order elements, the LBB condition is violated and a proper stabilization technique must be introduced to solve the system of equations. Therefore, a stable MINI elements with an additional bubble node in the velocity element are used to solve the system of equations [3]. The influence of the bubble shape function \( N_b \) of this additional node vanishes on the element boundaries and the approximated velocity field is given as

$$v = \sum_{i=1}^{n} N_i \bar{v}_i + N_b \bar{v}_b + \frac{N_b}{n} \sum_{i=1}^{n} \bar{v}_i$$

with:

$$\begin{cases} n = 3, N_b = 27\xi_1\xi_2(1 - \xi_1 \xi_2) & \text{in 2D} \\ n = 4, N_b = 256\xi_1\xi_2\xi_3(1 - \xi_1 \xi_2 \xi_3) & \text{in 3D} \end{cases}$$

Additional degrees of freedom from the bubble nodes are statically condensed and the system of equations is solved monolithically with an iterative correction to deal with the nonlinearity of the Bingham model.

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3 Numerical Application

The well-known slump flow test is taken as a verification example for the flow of fresh concrete. A reference solution for a virtual concrete with material parameters $\mu = 50$ Pa s, $\tau_0 = 50$ Pa and $\rho = 2300$ kg/m$^3$ is given in [4]. The simulation is carried out in 3D for various mesh discretizations with a constant time step $\Delta t = 0.005$ s and $m = 5000$. The capacity $|\tau|/\tau_0$ is depicted in Figure 1a for various time instances. The analytical solution along with the final axisymmetric shape is given in Figure 1b and the diameter over time is shown in Figure 1c. A very good agreement with the analytical solution is observed.

As an application test case, three additively manufactured concrete layers (30 cm length) are analyzed. The geometry of the extrusion nozzle is shown in Figure 2a and the printing speed and inlet velocity are $v_{\text{print}} = v_{\text{inlet}} = 0.1$ m/s. The material properties are chosen as follows: $\mu = 4$ Pa s, $\tau_0 = 200$ Pa and $\rho = 2000$ kg/m$^3$. As can be observed in Figure 2b, the layers are stacked and the material is strong enough to bear its self-weight. Nevertheless, through the inlet flow and a resulting dynamic impact on lower layers, these layers start to yield and unintended displacements might occur (see bottom part of Figure 2b). Furthermore, by analyzing the cross section (see Figure 2c) of the printed structure, it is observed, that the capacity $|\tau|/\tau_0$ is almost reached. For the static case, the yield stress can be thought of as a strength of already printed layers.

4 Conclusions

In this paper we present a numerical model to simulate layered extrusion-based additive manufacturing of fresh concrete with PFEM. In the numerical simulation of a typical additive manufacturing of fresh concrete test scenario it is observed, that yielded regions are small and the material is only flowing in a small region around the extrusion nozzle. Material parameters are inspired by lower bound values in order to demonstrate these effects more clearly. Hence, the material response is governed by the regime before yielding, which is approximated with a large viscosity for the regularized model. In order to assess the stresses of the material at rest more accurately, the physics of the model can be enhanced by an elasto-viscoplastic formulation. A promising approach is based on a classical model, where the viscous behavior is defined by an overstress function.

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