Combined experimental and numerical analysis of a bubbly liquid metal flow

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Abstract. The paper proposes a combined experimental and numerical procedure for the investigation of bubbly liquid-metal flows. It describes the application to a model configuration consisting of a recirculating GaInSn flow driven by an argon bubble chain. The experimental methods involve X-ray measurements to detect the bubbles and UDV measurements to gain velocity information about the liquid metal. The chosen numerical method is an immersed boundary method extended to deformable bubbles. The model configuration includes typical phenomena occurring in industrial applications and allows insight into the physics of bubbly liquid-metal flows. It constitutes an attractive test case for assessing further experimental and numerical methods.

1. Introduction

Bubbly liquid metal flows occur in different flow regimes for a wide range of gas flow rates: single bubbles, bubble chains, swarms and plumes. In many industrial processes, gas bubbles are injected into liquid metals to promote the mixing of the melt, to control chemical reactions or to remove undesired inclusions. The injection of a large number of gas bubbles leads to complex flow conditions, the prediction of which is extremely challenging. In addition to the interaction with the surrounding liquid, the bubbles generally interact with each other or with container walls. They deform distinctly due to the local pressure field and induce a flow in the surrounding liquid metal, which, in turn, acts upon the bubble. Substantial work, experimental as well as numerical, has been carried out to improve the understanding of these flows in their great diversity [1–7]. The physical and numerical models, however, still lack sophistication as they need reliable experiments for validation. The detection of flow properties in opaque fluids requires more advanced measurement techniques such as neutron- or X-ray radiography, ultrasound techniques or inductive methods. This is also the major reason for the fact that to date there are only a few systematic experimental studies on the behavior of gas bubbles in liquid metals. Driven by this unsatisfactory situation, extensive research was carried out within the last ten years at the HZDR and the last five years in the LIMTECH alliance for the development and application of specific measurement techniques in liquid metals. The present paper focused on work performed within project A5 of this alliance which aimed to develop and qualify powerful and efficient numerical methods for an appropriate description of dispersed liquid metal bubbly flows. In parallel, suitable model experiments were realized to provide an experimental data base.
for code validation. The capabilities of both approaches will be demonstrated focusing on the specific case of a bubble chain rising in an open, narrow container filled with liquid metal. This configuration includes the most important phenomena occurring in complex bubbly flows being relevant for industrial applications. Moreover, the problem is simplified enough to facilitate experimental measurements and direct numerical simulations and in-depth physical analysis.

The paper is organized as follows: The experimental and numerical methods are introduced in section 2. Results for the specific case of a bubble chain in a flat container are presented and discussed in section 3. A summary and conclusions will be given in section 4.

2. Methods for investigating bubbly liquid metal flows

2.1. A setup for investigating single bubbles, bubble chains and small swarms

This paper focuses on argon bubbles in GaInSn in a rectangular, narrow container. Such a flat geometry fulfills two crucial requirements: On one hand, the numerical model resolving very small scales of the flow has to face the limited power of current high-performance computers. A narrow container simply facilitates a higher resolution of the calculation domain. On the other hand, the X-ray radioscopy was chosen as measuring technique which provides an in-situ visualization of the gas bubbles rising in the opaque liquid metal. The high attenuation of X-ray radiation by liquid metals provides a good contrast but restricts the thickness of the fluid container along the direction of the X-ray beam.

GaInSn is liquid at room temperature and, hence, highly suitable for low-cost experiments without heating. A compilation of thermophysical data of this alloy can be found in [8]. In the case considered here, argon gas was injected through a vertically oriented injection nozzle placed at the center of the bottom plate at a height, see figure 1. The vessel employed was made of acrylic glass with extensions $144 \times 200 \times 12 \text{ mm}^3$ and filled with GaInSn up to a height of $144 \text{ mm}$. Acrylic glass was chosen for the wall material because it is transparent and facilitates the X-ray and ultrasonic measurements due to its low X-ray absorption and a sound velocity comparable to the one of the liquid metal. The container walls have a thickness of $10 \text{ mm}$, except at the positions of the ultrasonic sensors where the wall thickness was reduced to $2 \text{ mm}$. The nozzle was a non-wetting medical injection needle (Sterican, Gauge 19) made of stainless steel with an outer diameter $D_o = 1.09 \text{ mm}$ and an inner diameter $D_i = 0.785 \text{ mm}$ with a bevel orifice facing in the x-direction placed at a height of about $5 \text{ mm}$. This simple arrangement allows the investigation of single bubbles, chains and small swarms.
2.2. Experimental methods

The applied experimental procedure combines the X-ray radioscopy to visualize the bubbles with the ultrasound Doppler velocimetry (UDV) to measure velocity profiles of the liquid metal flow. The following measurement equipment has shown to be suitable for investigating bubbly liquid metal flow.

For X-ray measurements, a high-power source was applied (ISOVOLT 450M1/25-55, GE Sensing & Inspection Technologies GmbH) operating with a maximum voltage of 320 kV and a current of 14 mA. A scintillation screen (SecureX HB, Applied Scintillation Technologies) was installed directly behind the vessel containing the liquid metal to convert the proportion of X-rays transmitted through the sample into visible light. The imaging system is completed by a custom-made lens system (Thalheim Optik) and a video camera. Two different types of video cameras were used. The first one was the CCD camera Pixelfly QE (PCO) operated with a frame rate of 23 fps. The second camera was a Pco.edge AIR (PCO) based on an sCMOS-sensor achieving about 100 fps. The images were captured by the Pixelfly camera with a resolution of 696 × 512 pixels, corresponding to a field of view of 90 mm × 150 mm and was used to capture the overall situation. The Pco.edge camera provided a resolution of 1728 × 1022 pixels, corresponding to a field of view of 60 mm × 100 mm and was used for the quantitative analysis. The exposure times (3 ms for Pixelfly and 5 ms for Pco.edge) were adjusted to achieve an acceptable signal-to-noise ratio and a sufficient contrast. Installing lead shields around the observation area was necessary to reduce the background noise arising from scattered radiation originating from regions outside the liquid metal volume and provided significantly increased image quality. Additional lead shielding was installed around the video camera to avoid radiation damage of the video sensors.

The quantitative analysis of the bubble sizes, positions and velocities was performed by off-line data processing using Matlab scripts for image analysis. After subtracting the mean dark current signal, a shading correction was applied using a mean reference image recorded for a situation without gas injection. Gaussian low-pass filtering was applied to reduce the image noise. Then, the intensity gradient was computed. The final image for analysis was obtained by summing up the smoothed image and the gradient image. Separation of individual bubbles and bubble clusters from the background was achieved by using a threshold value. All pixels with a brightness above the threshold value were added to the bubble area. The total image was further segmented to divide bubble clusters into individual bubbles using the binary image resulting from the previous step. This operation includes a watershed segmentation algorithm to distinguish between bubbles whose projections are close to another or even overlap. For each bubble the center of mass was calculated from its brightness distribution. The projected area of a gas bubble was obtained by counting pixels in the segmented zone. The equivalent bubble diameter was calculated from the projected bubble area. Finally, the derived quantities were converted from pixel units to metric units using the scaling of the image. This setup is similar to that applied in [9].

The UDV technique was applied to measure the velocity field in the continuous phase. This is restricted to zones without bubbles because of their blocking effect. A detailed description of the measuring principle can be found in [10, 11]. For the present study, specially manufactured sensor arrays (Richter Sensor & Transducer Technology) were used operating at an emission frequency of 8 MHz. The sensor array consists of 25 single piezo elements and has a total length of about 70 mm. Two adjacent elements were always activated simultaneously. Eight of these pairs of piezo elements were operated for recording vertical profiles of the vertical velocity with a time resolution of about 32 ms. The instrument DOP3010 (Signal Processing SA) was used for multiplexed excitation of the piezo elements and data acquisition. The overall time resolution for all eight measuring lines was about 4 frames per second, and a spatial resolution of about 2.4 mm was achieved in y-direction. The angle of divergence of the acoustic beam was approximately 5°, the sensor array was attached to the bottom wall of the fluid container and the ultrasonic
energy was transmitted through the acrylic glass wall using a water-soluble coupling gel. The following control parameters of the DOP3010 were applied: pulse repetition frequency of 220 ms, 64 emissions per profile and an internal amplification of 0 to 40 dB (slope). A value of 2730 m/s was assumed for the speed of sound within the liquid metal [12].

A gas flow control system (MKS Instruments Deutschland GmbH) was used to cover a wide range of gas flow rates from 10 to 680 sccm (standard cubic centimeters per minute, valid for nitrogen, 1 bar, 0°C). In the case of argon at room temperature 1 sccm corresponds to 1.07 cm³/min or 17.8 mm³/s.

2.3. Numerical method
The numerical simulations were performed using the multiphase code PRIME [13] with the extensions presented in [14] and [15]. It solves the incompressible Navier-Stokes equations on a staggered Cartesian grid by a finite-volume method and employs an immersed boundary method (IBM) for the coupling between fluids and bubbles. The bubbles are treated as deformable objects of axisymmetric shape. Their motion is decomposed into three parts: translation, rotation and change of shape. Translation and rotation are determined by the corresponding ordinary differential equations for linear and angular momentum, respectively, relating these to the force and torque exerted by the fluid onto the bubble.

The instantaneous shape of each individual bubble is represented by a finite series of spherical harmonics. The surface is described in spherical coordinates by the surface points $x(\vartheta, \varphi) = x_p + r(\vartheta, \varphi)(\cos \varphi \sin \vartheta, \sin \varphi \sin \vartheta, \cos \vartheta)^T$ with $\vartheta \in [0, \pi]$, $\varphi \in [0, 2\pi)$, $x_p$ the center point, and the radial approximation

$$r(\vartheta, \varphi) \approx \sum_{n=0}^{N} \sum_{m=-n}^{n} c_{nm} Y_n^m(\vartheta, \varphi). \quad (1)$$

Here, $Y_n^m$ identifies the spherical harmonic function of degree $n$ and order $m$, $c_{nm}$ the associated coefficients, and $N$ the order of the approximation. In the present case, the series is restricted to an axisymmetric shape by reducing the second sum to the contribution $m = 0$. The representation equation (1) allows very accurate determination of geometrical properties such as surface curvature, etc., via appropriate analytical formulae for the spherical harmonic functions [16]. Furthermore, the bubble volume can be controlled exactly by controlling the coefficients in equation (1). The coefficients $c_{nm}$ are computed by balancing the forces on the bubble surface minimizing the potential displacement energy [17]

$$W = \int_S \left[ (p_i - p_o) - 2\sigma H \right] \delta n \, dA \rightarrow \min. \quad (2)$$

This equation describes the balance of the forces by the inner pressure $p_i$, the outer pressure $p_o$ and the surface tension effects proportional to the mean curvature $H$.

In equation (2), viscous stresses are neglected. On the bubble surface a no-slip condition is imposed, since the bubbles in the present liquid metal case have typically an oxide layer on the phase boundary. These assumptions are further discussed in [18] and [19].

Due to the narrow shape of the container considered in the present study, collisions of bubbles with walls, collisions between bubbles and bubble coalescence have to be taken into account. In the present approach, collisions are represented by the model of [20] and coalescence of bubbles is accounted for by the model proposed in [21]. These events, however, are relatively rare in the simulation reported below.
3. Application to a bubble chain

3.1. A bubble chain model problem

The configuration of a bubble chain was chosen for investigation within this study. This choice avoids dealing with the complex bubble swarms but allows investigating essential bubble-bubble interactions, like collisions, coalescence and the effect of bubble wakes. Bubble chains were already addressed in [22] covering low Galilei numbers. A bubble chain in a liquid metal relevant regime was studied in [21], but close validation was not possible due to a lack of detailed experimental data. The present work aims at filling this gap and providing such a validation and, beyond that, aims at providing new quantitative data on the physical phenomena. As mentioned above, the size of the container is $L_x$ in the horizontal x-direction, $L_y$ in the vertical y-direction, and a depth of $L_z$ in the z-direction (figure 1). Bubbles of mean equivalent diameter $d_{ref}$ are injected at the center of the bottom plane at height $y_0$ and rise upwards where they finally pierce through the free surface. The bubbles are introduced at a constant frequency $f_b$ and are supposed to exhibit a Gaussian distribution of their diameter around the mean value $d_{ref}$ with a standard derivation of $\sigma_d$. The present case features a rectangular container of size $(L_x, L_y, L_z) = (1, 1, 1/12)L$ with $L = 144$ mm (figure 1). The bubbles are released with a frequency of $f_b = 30$ Hz. The mean bubble diameter is $d_{ref} = 6$ mm with the standard deviation $\sigma_d = 0.08 d_{ref}$. The liquid metal is GaInSn with $\rho = 6440$ kg/m$^3$ and $\nu = 3.46 \cdot 10^{-7}$ m$^2$/s [8] and the gas bubbles consist of argon with $\rho_b = 1.780$ kg/m$^3$ which is much smaller than the fluid density. The surface tension at the gas liquid interface is $\sigma = 0.553$ N/m. Hence, each bubble starts at $y_0 = 0.8 d_{ref}$. The container size corresponds to $(L_x, L_y, L_z) = (24, 24, 2) d_{ref}$. 

Starting point is the container described in section 2.1 filled with GaInSn (figure 1). If one chooses the mean diameter $d_{ref}$ as the reference length, the setting is then fully described by the container size ratios $L_x/d_{ref}, L_y/d_{ref}$ and $L_z/d_{ref}$, the non-dimensional initial bubble position $y_0/d_{ref}$ and five more non-dimensional parameters related to gravitational driven flow.

![Figure 2](image1.png)

**Figure 2.** Measurement zones with snapshots of the respective measurement data: vertical velocity component obtained by UDV (left, right) and differences in density by X-ray (center).

![Figure 3](image2.png)

**Figure 3.** Snapshot of the simulation showing the bubble chain and the magnitude of the velocity field in the center plane.
and bubble injection:

\[ \pi_{\Delta \rho} = \frac{\rho - \rho_b}{\rho} \]  

(3)

\[ Ga = \frac{g d_{ref}^3}{\nu^2} \]  

(4)

\[ Eo = \frac{\Delta \rho g d_{ref}^2}{\sigma} \]  

(5)

\[ \pi_{\sigma_d} = \frac{\sigma_d}{d_{ref}} \]  

(6)

\[ \pi_l = \sqrt{\frac{\pi_{\Delta \rho} g}{d_{ref} f_b}} \]  

(7)

These parameters result from the Buckingham \( \pi \)-theorem applied to the governing physical variables, namely the gravitational acceleration \( g \), the fluid density \( \rho \), the fluid kinematic viscosity \( \nu \), the surface tension \( \sigma \), the bubble density \( \rho_b \), the mean bubble diameter \( d_{ref} \), its standard derivation \( \sigma_d \) and the bubble release frequency \( f_b \). The relative density difference \( \pi_{\Delta \rho} \), the Galilei number \( Ga \), the Eötvös number \( Eo \) and the relative standard deviation of the bubble diameter \( \pi_{\sigma_d} \) are well-known parameters. The value of \( \sqrt{Ga} \) is a good \emph{a priori} approximation for the bubble Reynolds number for single bubbles rising in stagnant fluids. The parameter \( \pi_l \) given in equation (7) is newly proposed here to complete the characterization of the physical situation. If \( l \) denotes a reference value of the center-center distance of two consecutive bubbles, the ratio \( \pi_l = l/d_{ref} \) characterizes the interaction. Here, \( l = u_c/f_b \) is derived from the characteristic buoyancy velocity

\[ u_c = \sqrt{\pi_{\Delta \rho} g d_{ref}} \]  

(8)

and the characteristic time scale \( 1/f_b \). Small values for \( \pi_l \) reflect high bubble frequencies with small bubble-bubble distances, with the result that interactions are more likely to occur. The limit \( \pi_l = 1 \) describes a bubble release with vanishing distance of the bubble surfaces, so that coalescence should have a sizable impact. In the limit of very large values of \( \pi_l \), the bubbles rise without any bubble-bubble interaction. Note that the Morton number \( Mo \) can be computed from

\[ Mo = Eo^3 Ga^{-2} \pi_{\Delta \rho}^{-2} \]  

and is therefore no independent parameter.

Thus, beyond geometrical parameters, the present case is defined by the following values of the dimensionless parameters defined by equation (3) to equation (7): \( \pi_{\Delta \rho} = 1 \) neglecting the bubble density, \( \sqrt{Ga} = 4200 \) as a first rough bubble Reynolds number approximation, \( Eo = 4.3 \) indicating bubble deformation, \( \pi_{\sigma_d} = 0.08 \) corresponding to a realistic bubble size distribution and \( \pi_l = 1.35 \) which indicates the possibility of bubble-bubble interactions.

### 3.2. Details for the experimental and numerical procedures

The methods described in section 2 were applied to the bubble chain model problem of section 3.1 in the following manner. Each experiment was performed in well-defined steps employing the methods described in section 2.2. First, an initial gas flow rate of 10 sccm was applied, resulting in a detachment of single bubbles at a frequency of about 1.5 bubbles per second. Then, data acquisition was started and the gas flow was increased up to the final value which was 200 sccm. This gas flow rate results in a bubble detachment frequency of approximately 27 Hz and equivalent bubble diameters of about 6 mm, which is close to the desired values. All experiments were conducted at room temperature and atmospheric pressure. Both measurement technologies were not be conducted simultaneously but were carried out subsequently under the same conditions. The analysis of the flow field in the continuous liquid phase by UDV focuses
on regions on both sides of the bubble chain. The comparison with the numerical simulation relies on eight experiments conducted at a gas flow rate of 200 sccm, each averaging the velocity of the continuous phase over 5.3 seconds. Figure 2 contains exemplary results from both the X-ray and UDV measurements.

Employing the method of section 2.3, the simulations were conducted in dimensionless form on an equidistant grid with \((N_x, N_y, N_z) = (960, 960, 80)\) grid points, adding up to roughly \(74 \cdot 10^6\) grid points in total. It resolves the mean bubble diameter \(d_{ref}\) with about 40 points. This ensures an acceptable resolution. A no-slip condition was imposed at the bottom wall and the side walls, the free surface was represented by a free-slip condition. Due to the complexity of the bubble detachment process, the nozzle itself was not resolved by the numerical model. Instead, a simplified detachment model was devised [21]. The bubbles are inserted at the bottom center at height \(y_0 = 4.8\) mm assuming spherical shape, with the diameter selected according to a Gaussian distribution defined by the mean value \(d_{ref}\) and the variance \(\sigma_d\) and a frequency according to \(\pi_t\). The initial velocity of the bubbles is set to the fluid velocity at this position which is justified due to the negligible bubble inertia. At the free surface the bubbles are removed from the simulation by just switching off the corresponding IBM force [21]. This is a very smooth way and an advantage of the IBM, as it avoids the representation of bubbles piercing through the surface which would be numerically very cumbersome. The time step was adjusted instantaneously during the simulation to maintain a CFL number equal to 0.8. Averaging was started after obtaining a constant value for the bubble mean velocity. The initial phase was excluded from averaging. The whole simulation time corresponds to 17.2 seconds in the experiment, the averaging period to 5.3 seconds. It is known that bubble chains and columns tend to form an s-shape which can suddenly shift to the other side \([4, 23–25]\). However, such a behavior was not observed in the present case.

![Figure 4](image)

**Figure 4.** Averaged vertical fluid velocity and streamlines with instantaneous bubble constellation.
3.3. Results for the disperse phase

The center positions of the bubbles are tracked at discrete time steps. The resulting ensemble-averaged trajectory obtained from experiment and simulation are plotted in figure 5 and figure 7, scaled by the container height. Here, $\bar{\psi}$ denotes the average of a bubble property $\psi$ related to $L_0$.

**Figure 5.** Experimentally determined bubble center positions and average trajectory.

**Figure 6.** Experimentally determined bubble center positions, average and standard deviation. Same dataset as in figure 5 but scaled with $d_{ref}$ instead of $L$, horizontally stretched.

**Figure 7.** Numerically determined bubble center positions and average trajectory. Depiction covers the whole domain.

**Figure 8.** Numerically determined bubble center positions, average and standard deviation. Same dataset as in figure 7 but scaled with $d_{ref}$ instead of $L$, horizontally stretched.
a certain elevation. It is obtained by subdividing the container vertically in equal parts and by computing the temporal average for each part independently. The same data were used to create more detailed plots, figure 6 and figure 8, horizontally stretched and scaled by the average bubble size. The standard deviation \( \sigma_{\psi} \) of a bubble property \( \psi \) with respect to the elevation average \( \bar{\psi} \) is then computed by 
\[
\sigma_{\psi}^2 = \psi^2 - \bar{\psi}^2.
\]
Note that there is a small horizontal shift between simulation and experimental data due to the angled injection nozzle. This shift was less than 2 mm and the position of the experimental trajectories were shifted accordingly for comparison.

The bubble centers stay within a band of \( 4d_{\text{ref}} \), whereas most of them occur within a band of \( 2d_{\text{ref}} \). The widening of the trajectories can be quantified by an undulation amplitude of about \( d_{\text{ref}}/4 \) and an increasing deviation \( \sigma \) (figure 6 and figure 8). The simulation results show the three-dimensional character of the bubble chain (figure 8). The slight s-shape of the chain is visible in both the experiment and in the simulation. Undulation and standard deviation are slightly larger in the simulation, but in general the experimental measurements confirm the numerical results.

Each bubble increases its velocity until a rising velocity of approximately \( 2u_c \). The scattering of the bubble velocity increases simultaneously with the scattering of the bubble position. Right after the injection, it is quite low. At a certain height however, located roughly at \( y = 3d_{\text{ref}} = \frac{1}{8}L \), the variance of the rise velocity increases noticeably towards a constant value within the center region. Due to the low initial velocity, the void fraction close to the injection needle is about twice the void fraction in the developed center region (figure 11, figure 12). The experimental and numerical results coincide qualitatively, but the experimentally determined gas fraction is slightly higher. This could be caused by inaccuracies in the computation of the equivalent bubble diameter from the X-ray images. Such an error distorts the void fraction with third power. The void fraction distribution suggests, again, the existence of a center region bounded by a lower injection region and an upper close-to-surface region.

Such regions can be found also by analyzing bubble-bubble distances. Dominating distances between particles or bubbles can be identified by the discrete radial pair correlation function \( g_r \) [26, 27]. To account for the dimensions of the container, the analysis is done here in a two-dimensional manner. For a given distance value \( r \), the value \( g_r(r) \) is then evaluated by determining the number of bubble-bubble distances \( r_{ij} = (|x_i - x_j|^2 + |y_i - y_j|^2)^{1/2} \) lying in the

![Figure 9.](image1.png)  
**Figure 9.** Experimentally determined absolute vertical bubble velocity against container height.

![Figure 10.](image2.png)  
**Figure 10.** Numerically determined absolute vertical bubble velocity against container height.
shell \((r - \frac{\Delta r}{2}, r + \frac{\Delta r}{2})\) supplemented with a shell-specific scaling according to

\[
g_r(r) = \frac{1}{N(N-1)V_{\text{shell}}/V} \sum_{i=1}^{N} \sum_{j=1}^{N} D \left( r_{ij} \in \left( r - \frac{\Delta r}{2}, r + \frac{\Delta r}{2} \right) \right).
\]

(9)

Here, \(N\) is the total number of bubbles at a given instant in time and \(V\) the volume of the container. The operator \(D(x)\) is a discrete version of the Dirac distribution. It is equal to one if the statement \(x\) is true and zero otherwise.

The volume \(V_{\text{shell}}(r, \Delta r)\) is the annulus volume \(2\pi r \Delta r L_z\), regardless whether the considered distance is close to the boundary. The result for the radial pair correlation function is given in figure 13 and leads to the following conclusions. The correlation for the whole container shows a dominant peak at approximately \(2d_{\text{ref}}\) which is equivalent to a surface-to-surface distance of \(1d_{\text{ref}}\). Of course, it is much more revealing to search for subregions associated with characteristic correlations to detect regions of similar physics. Such regions can be found iteratively by checking the sensitivity of the associated correlation function to changes of the region size. Three distinct regions with different characteristic correlations can be found (figure 15). The distance of the bubbles in the injection zone is clearly dominated by one single peak resulting from the injection frequency. This distance is equal to the length scale \(l\) discussed above together with \(\pi_l\). Hence, it corresponds exactly to the dimensionless length scale associated with the injection procedure:

\[
\frac{r_{\text{inj}}}{d_{\text{ref}}} = \frac{l}{d_{\text{ref}}} = \pi_l = 1.35
\]

(10)

Hence, the radial pair correlation function is able to recover the value of the \textit{a priori} known dimensionless injection bubble distance \(\pi_l\). In the large center region, the core region, the dominant peak is at \(2.3d_{\text{ref}}\) with a smaller one at \(5d_{\text{ref}}\) probably caused by pairing scenarios. In the upper part, the close-to-surface region, the bubbles come closer to the free surface and are finally switched off in the simulation which leads to an unspecific correlation pattern.

Dominating directions of the connecting line between two bubbles can be identified by the discrete angular pair correlation function \(g_{\varphi}\). If all particle distances are finite, it can be computed similarly to the radial pair correlation function, reading

\[
g_{\varphi}(\varphi) = \frac{1}{N(N-1)V_{\text{sector}}/V} \sum_{i=1}^{N} \sum_{j=1}^{N} D \left( \varphi_{ij} \in \left( \varphi - \frac{\Delta \varphi}{2}, \varphi + \frac{\Delta \varphi}{2} \right) \right).
\]

(11)

**Figure 11.** Horizontal distribution of time-averaged volume fraction \(\langle \phi \rangle\).

**Figure 12.** Vertical distribution of time-averaged volume fraction \(\langle \phi \rangle\). The upper part of the container is not covered by the X-ray measurements.
Here, \( V_{\text{sector}}(\phi) \) is the volume of the angular sector bounded by \( \phi - \frac{\Delta \phi}{2}, \phi + \frac{\Delta \phi}{2} \) and an outer radius \( r_{\text{max}} \), arbitrarily chosen as \( L \). As before, it is computed in a two-dimensional manner as \( V_{\text{sector}}(\Delta \phi) = \frac{1}{2} r_{\text{max}}^2 \Delta \phi L_z \). As expected, the bubbles are more or less aligned (figure 14).

3.4. Results for the continuous phase
The rising bubbles induce a recirculation flow of the liquid metal upstream in the center and downstream remote from the chain near the sidewalls. The velocity of the recirculating liquid metal was measured by UDV, as illustrated in figure 2. The simulated, time averaged magnitude of the fluid velocity is shown in figure 4, overlapped with a snapshot of the rising bubbles. Here, \( \langle \psi \rangle \) denotes the temporal average. In figure 16, computational and experimental results for the vertical velocity of the liquid metal are compared at five equidistant container heights, i.e. at \((2, 3, 4, 5, 6) \frac{1}{8} L \) which corresponds to \((36, 54, 72, 90, 108) \text{ mm} \). The inner upstream flow generated

**Figure 13.** Numerically determined radial pair correlation function. The distribution for the injection region is scaled by 0.25.

**Figure 14.** Numerically determined angular pair correlation function. The distribution for the whole container is scaled by 0.25.

**Figure 15.** Regions with similar characteristics. The regions differ in terms of the pair correlation function, specific values for volume fraction and the deviation horizontal displacement and rise velocity.
by the rising bubbles is visible, as well as the outer downstream flow. The range of the velocity axis was limited to allow suitable comparison between the UDV data, recorded in the outer flow region, and the simulation results. The velocity gradient between the bubble chain and the walls is approximately constant $|\Delta v_f/\Delta x| \approx 4.7 \, s^{-1} = 0.12 u_c/d_{ref}$. Apparently, the mean flow is not fully symmetric with respect to the $y$-axis for simulation and experiment. This is not an error but a consequence of the persisting s-shape of the bubble chain discussed above. The differences between experimental and numerical velocities are small for the right side and tolerable for the left side of the container. In the experiment, the recirculation reaches slightly higher velocity values compared to the simulation which suggests that the free-slip assumption might not be the optimal choice for free liquid metal surfaces. Apart from that, the numerical and the experimental velocity data are in good agreement. To understand the influence of the rising bubbles on the surrounding liquid metal, the turbulent kinetic energy provides further insight. An example is shown in figure 17 at half the container height. The bubbles produce large turbulent kinetic energy in the surrounding fluid. This energy is transported upwards close to the surface before it is carried downwards by the recirculating fluid near the side walls. Between these regions of increased turbulence, there are zones of low turbulent kinetic energy.

![Figure 16. Profiles of vertical fluid velocity at five equidistant elevations derived from simulation and UDV measurement.](image)

3.5. Results related to the interaction of disperse and continuous phase
The comparison between experimental and numerical results proves the reliability of the numerical approach. This allows a closer look at quantities that are inaccessible with present measurement technology. The three-dimensionality of the bubble shapes cannot be captured by high-speed X-ray measurements and the velocity of the liquid close to the bubble chain cannot be determined in a sufficient manner by UDV. The numerical approach, on the other
hand, includes a three-dimensional representation of each bubble with its individual shape and the entire velocity field. To derive a relative bubble velocity one has to determine a fluid velocity. Here, the fluid velocity $v_f$ close to each bubble is determined by averaging the vertical fluid velocity component over a horizontal range around the bubble, $(\Delta x, \Delta z) = (2, 1) d_{ref}$, excluding the bubble itself. The notation $\tilde{v}$ is used here for such a local average combined with averaging over bubbles at a given height. This definition allows to compute the relative velocity $\bar{v}_{rel} = \bar{v}_b - \tilde{v}_f$ and the bubble Reynolds number $Re_b = \bar{v}_{rel} d_{ref}/\nu$. The result is shown in figure 18. After a very short distance, the relative velocity $\bar{v}_{rel}$ exhibits a fairly constant value in the injection region. It increases in the subsequent center region and saturates at around $3200 d_{ref}/\nu$. Further up, both bubble and fluid velocity decay yielding a constant relative velocity. Obviously, the relation $Re_b \approx \sqrt{Ga}$, known to hold for rising single bubbles in quiescent fluid, is not met here as the present flow is substantially more complex. The evolution of the relative velocity finally approaches $\bar{v}_{rel} = 0.64 \tilde{v}_f$ which corresponds to an absolute bubble velocity of $\bar{v}_b = 1.64 \tilde{v}_f$.

Deformed bubbles can behave significantly different than spherical ones. The deformation can cause a separated wake behind the bubble which increases the drag noticeably [28]. It is
also known, that the near-wall behavior of spherical and non-spherical bubbles differs, so that deformable bubbles tend to drift to the center of channels [29, 30]. Therefore, deformation is an important quantity of interest. The three-dimensional deformation can be measured by the non-sphericity

\[ N_3 = \frac{A - A_{\text{veq}}}{A}, \]

where \( A \) denotes the surface area of the bubble and \( A_{\text{veq}} \) the surface area of the volume-equivalent sphere. The experimental technique only provides two-dimensional projections of the shape which can be assessed with the non-circularity

\[ N_2 = \frac{P - P_{\text{aeq}}}{P}, \]

where \( P \) denotes the perimeter of the projected bubble and \( P_{\text{aeq}} \) the perimeter of the area-equivalent circle. Both \( N_2 \) and \( N_3 \) are zero in case of a spherical bubble and larger than zero in case of deformation. For example, an oblate ellipsoid with an axial ratio of two exhibits a non-sphericity of \( N_3 = 0.09 \). The definitions in equation (12) and equation (13) where derived from definitions of sphericity and circularity [31]. It is obvious, that \( N_2 \) does not approximate \( N_3 \) in case of strong deformation in the direction of projection. Nevertheless, the comparison of both measures can be justified in the case of a symmetry axis orthogonal to the direction of projection and the assumption of minor deformations. For such cases, it can be shown mathematically, that the error \((N_2 - N_3)/N_3\) for an oblate ellipsoid with a deformation according to an axial ratio lower than two is below 5%. Therefore, the non-circularity \( N_2 \) is a good approximation to the non-sphericity \( N_3 \) in the present case.

The deformation of a bubble is closely connected to the ratio of the square of the relative velocity and surface tension. This ratio is expressed by the Weber number \( We = \rho f i_{\text{rel}}^2 d_{\text{rel}}/\sigma \). Large Weber numbers relate to large deformations in case of single bubbles [28]. As expected, this does not hold in the present case as visible in figure 19 together with figure 20. Moreover, the present simulation shows different results for the lower and the upper part of the container. It can be concluded from the comparison of figure 19 with figure 20, that the bubble acceleration is important here: The approach of the final relative velocity allows the bubbles to return to a more spherical shape. This change starts approximately at an elevation of \( y/L = 0.4 \). The final deformation of \( N = 0.03 \pm 0.01 \) in the core region is confirmed by the X-ray measurements.
Nevertheless, the re-deformation in the simulation occurs later than in the experiments. The final deformation, however, is predicted quite well.

4. Conclusions
The paper demonstrates the capability of the experimental and numerical methods for the investigation of bubbly liquid metal flows with deformable bubbles. A model configuration is proposed which is complex enough to contain industrially relevant phenomena and which is simple enough to allow detailed experimental and numerical investigation. Experimental and numerical results were found to be in very good agreement both for the disperse gas phase and for the continuous liquid metal phase. The physical phenomena are presented in detail, such as the connection between deformation and Weber number. In the future, the long-term behavior of the meandering and the influence of the distinct parameters could be studied. Apart from this, more complex configurations like swarms in wide containers or the influence of magnetic fields on coalescence could be investigated.

Along with the application of the method described so far and the physical evaluation of the results reported above further methodological achievements were made recently. An inconvenience of classical immersed boundary methods is the fact that vanishing mass of bubbles or particles can only be handled with difficulties [13]. The method in [14] provides a solution but does not allow a density exactly equal to zero and has comparatively severe stability limits. The problem was recently solved by devising a new formulation of the IBM introducing a better coupling to the fluid. It was presented in [32] for spherical particles and generalized recently. The second achievement is a new hybrid method for phase-resolving DNS of bubbly flows combining the advantages of front tracking and the above IBM. It bases on an equation for the coefficients $c_{nm}$ in equation (1) directly, which guarantees a smooth and stable propagation of the bubble surface even in case of complex shapes, provided the surface point distribution is determined appropriately [33]. These recent developments will be used in future bubbly flow simulations.

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