Dynamics of the Electromagnetic Ion Cyclotron Nonlinear Solitary Structures in the Inner Magnetosphere

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Abstract. The nonlinear interaction of the electromagnetic ion cyclotron (EMIC) frequency waves with plasma particles in the inner magnetosphere is studied. The emission is considered to be circularly polarized electromagnetic waves propagating along the almost constant dipole geomagnetic field in the equatorial region of the inner magnetosphere. Under the action of the ion cyclotron ponderomotive force excitation of the magnetosonic waves through the amplitude modulation of the EMIC waves is investigated. Two dimensional nonlinear Schrodinger equation for the EMIC waves is derived. In the stationary case two solutions of the nonlinear Schrodinger equation with distinct natures are found. The generation of both vortices and of a quasistatic magnetic field across the geomagnetic field lines is discussed.

1. Introduction

Electromagnetic ion cyclotron waves, commonly called EMICs, appear to play a multifaceted role in the Earth’s inner magnetosphere. These waves can be excited by spontaneous plasma instability when plasma is injected via substorms from the geomagnetic tail into the inner magnetosphere (see e.g. [1]). This plasma injection conserves the magnetic momentum of the hot ions in the plasma sheet, producing an energetic ion plasma component with energy up to 30 keV near L-shell values of 4 or 5. Due to its anisotropy in velocity space, this population of energetic ions can drive the EMIC waves through Landau cyclotron resonances. Another common source of electromagnetic ion cyclotron waves is the pulsation of the magnetopause due to variations in the solar wind dynamic pressure [2]. In 2005, the LAZIO-SIRAD detector was installed on the International Space Station with the objective of clarifying the correlation between such electromagnetic waves and earthquakes by measuring changes in the number of charged particles and the strength of magnetic fields at low L-shells.

Electromagnetic ion cyclotron waves have been investigated for a long time with both ground-based instruments and satellite experiments. Initial observations by satellite were made by Russell et al. [3], Gurnett [4], and Kintner and Gurnett [5]. EMIC waves have been seen in association with intense electron fluxes on the auroral field lines with the S3-3 satellite [6, 7], the Freja satellite [8], the Fast satellite [9], and the Polar satellite [10]. EMIC waves have also been observed in the equatorial plane at various L values by the GEOS satellites [11], the Akebono satellite [12, 13, 14], the Equator-S satellite [15], and the CRESS satellite [16, 17]. In this equatorial region, EMIC waves are generated by the ion cyclotron instability driven by the anisotropic distribution of ring-current energetic ions during magnetic storms (see [15,18] and references therein). They are characterized by harmonic
structures, with the electric and magnetic field power between the first multiples of the proton cyclotron frequency. EMIC waves are known to take part in the precipitation of electrons (see e.g.,\cite{17-20}). Parrot et al. \cite{21} reported the observation of electromagnetic harmonic emissions with the microsatellite DEMETER. These emissions were detected in the ELF range (500 Hz to 2 kHz) in the upper ionosphere during large magnetic storms, and it was shown that they could be related to the electromagnetic ion cyclotron waves.

In the present work, we assume that electromagnetic ion cyclotron waves are present with sufficiently large amplitude so that their nonlinear structure and the conditions for parametric instabilities become important. After displaying the linear dispersion relation with a kinetic-theory response for the ions, we develop a nonlinear two-component fluid description of the electromagnetic ion cyclotron waves and explore their properties with respect to propagation speed versus amplitude.

2. Basic equations for electromagnetic ion cyclotron waves

We consider the nonlinear dynamics of electromagnetic fields in the inner magnetosphere. Our investigation is also applicable to the upper regions of the Earth’s ionosphere. We consider the propagation of circularly polarized nonlinear electromagnetic waves propagating along the dipole geomagnetic field in the equatorial region of the inner magnetosphere. We assume that the electromagnetic emission frequency $\omega_0$ is close to the proton gyro-frequency $\omega_{cp}$, i.e., $\omega_0 \approx \omega_{cp} = 2.98 L^3$ kHz, where $L = r_{eq}/R_E$ is the shell parameter of the dipole geomagnetic field, $r_{eq}$ is the radial distance to the magnetic field line in the equatorial plane, and $R_E$ is the Earth’s radius. Further, we consider regions around $L = 2$. The circularly polarized electric field is given by $E_i = E_i(x,y,t)\exp[i(k_0 y - \omega_0 t)]$. Here $k_0$ is the $y$-component of the wave number of the electromagnetic emission. Thus, the pumping electromagnetic waves are assumed to propagate almost along the homogeneous and constant dipole geomagnetic field $B_0$ oriented along the $y$-axis. The amplitude $E(x,y,t)$ becomes a slowly varying function of spatial coordinates and time due to the nonlinear interaction of the electromagnetic waves with plasma. The electromagnetic wave fluctuations are in the $x$ and $z$ directions, transverse to the geomagnetic field ($y$-direction).

Taking the inner magnetosphere plasma to be collisionless and non-degenerate EMIC waves are described by the following dispersion equation \cite{22},

$$N_0^2 = \frac{k_0^2 c^2}{\omega_0^2} = 1 - \frac{\omega_{pe}^2}{\omega_0^2} - \frac{\omega_{pp}^2}{\omega_0^2} (\omega_0 - \omega_{cp} I_s(k_0 v_p)), \quad (1)$$

Where $\omega_{pe}$ and $\omega_{pp}$ are the electron and proton Langmuir frequencies, respectively; $v_p$ is the proton thermal velocity; $c$ is the speed of light in a vacuum and $I_s(z)$ is the appropriate plasma dispersion function. It follows from the dispersion equation (1) that for the range of frequencies $\omega_0 \gg |\omega_0 - \omega_{cp}| >> k_0 v_p$, the refractive index $N$ is complex, i.e., $N = N_0 + i\eta$, where

$$N_0^2 = \frac{k_0^2 c^2}{\omega_0^2} = 1 - \frac{\omega_{pe}^2}{\omega_0\omega_{pe}} - \frac{\omega_{pp}^2}{\omega_0(\omega_0 - \omega_{cp})} I_s, \quad \eta = \frac{\pi}{8 N_0 k_0\omega_0 V_p} \exp\left( -\frac{c^2(\omega_{cp} - \omega_0)^2}{2\omega_0^2 N_0^2 V_p^2} \right), \quad (2)$$

Outside the resonance region $|\omega_0 - \omega_{cp}| >> k_0 v_p$, the absorption of proton cyclotron waves is exponentially small. From Eq. (2), by maximizing $\eta$ with respect to $k_0$, we find that the limit to the resonance occurs at $k_0 = |\omega_{cp} - \omega_0|/\sqrt{2V_p}$. By using this expression for the frequency difference in the expression for $N_0^2$ of Eq. (2), which dominates near resonance ($\omega_0 \approx \omega_{cp}$), we obtain the maximum value for the refraction index as $N_{0\text{max}} = (c\omega_{cp}^2/\sqrt{2V_p}\omega_{cp}^2)^{1/3}$. The EMIC waves are left-hand circularly polarized and resonate with energetic ions. The formation of coherent structures including vortices and solitons, requires the balancing of the dispersion in the $\omega(k)$ waves with nonlinear terms in the wave dynamics.

3. Nonlinear two-component fluid description of EMICs

Here we will discuss the basic equations that describe the nonlinear modulation of electromagnetic
ion cyclotron waves propagating along the magnetic field $B_{0y}$. We assume that the modulation frequency is much smaller than the ion gyro-frequency. Then the slow motion of the plasma can be described by the following single-fluid MHD equations for the plasma density $\rho$ and velocity $\mathbf{v}$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla \rho}{\rho} - \frac{1}{4\pi \rho} \mathbf{B} \times \nabla \times \mathbf{B} + \frac{\mathbf{f}}{m_p},$$

(3)

Here, $\mathbf{B}$ is the total magnetic field, $\mathbf{f}$ is the ponderomotive force incorporating the motion of the protons, and $v_s = \sqrt{T_e / m_p}$ is the proton sound velocity with $T_e$ the electron temperature.

In our consideration, the ponderomotive force $\mathbf{f}$ has two components [23]:

$$f_s = -\frac{m_p \omega_p}{\omega_0 - \omega_{cp}} \frac{\partial}{\partial x} \frac{e^2 |E|^2}{m_p^2 \omega_0^2}, \quad f_p = -\frac{m_p \omega_p}{\omega_0 - \omega_{cp}} \frac{\partial}{\partial y} \frac{e^2 |E|^2}{m_p^2 \omega_0^2} + \frac{m_p \omega_p}{\omega_0 - \omega_{cp}} \frac{\partial e^2 |E|^2}{\partial t},$$

(4)

where $|E| = (e_0 / c) |A|$, with the electric field $\mathbf{E}$ being related to the vector potential $\mathbf{A}$ as $\mathbf{E} = -\partial \mathbf{A} / \partial t$.

3.1 Nonlinear Schrodinger equation. Now we will derive the nonlinear Schrodinger equation, using the Maxwell equation for the ion cyclotron waves:

$$\nabla^2 E_x - \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2} = \frac{4\pi n_e c^2}{m_p} \frac{\partial P_x}{\partial t}.$$  

(5)

For our problem, the contribution of the electron current density is smaller than that of the protons. Here $P_x = P_0 - iP$ is the ion momentum due to a rapidly varying electromagnetic field. A simple calculation of $P_x$ follows from the equation of motion (3):

$$P_x = \left[ \frac{ieA}{\omega - \omega_p} + \frac{e\omega_p}{\omega_p - \omega_0} \frac{1}{\omega_0} \frac{\partial A}{\partial t} \right] e^{i(kz - \omega_p t)}.$$  

(6)

Recall that according to our assumption all quantities have both fast and slow temporal-spatial scales. Substituting Eq.(6) into Eq.(5), we obtain for the ion cyclotron waves a nonlinear Schrodinger equation of the following form,

$$2i \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial y} \frac{\partial}{\partial y} \right) A + \frac{c^2}{\omega_0} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) A - \frac{\omega_p \Delta \omega}{\Gamma} A - \frac{\omega_p^2 \omega_0}{\Gamma (\omega_0 - \omega_{cp})} \frac{\partial n_p}{\partial y} A = 0,$$

(7)

where the group velocity is $v_g = 2(\omega_p - \omega_0) / k_0$ and the expressions for $\Gamma$ and $\Delta \omega$ are given in [24].

3.2 Parametric Decay instability of large-amplitude plane waves. In order to consider modulational instabilities (leading to the excitation of magnetosonic waves), we linearize Equations (3), (4) and (7) and search for plane wave solutions of the form $\exp[i(qr - \Omega t)]$. We obtain the following dispersion equation:

$$\left[ (\Omega - q_y v_g)^2 - \frac{c^2 q_y^2}{2\Gamma} \right] \left( \Omega^2 - \Omega_s^2 \right) = B \left( \frac{eA}{m_p c^2} \right)^2,$$

(8)

where the expression for $B$ is given in [24]. Here $V_s$ is the Alfvén speed, and $q^2 = q_x^2 + q_y^2$. In Eq. (8), $\Omega_s$ are the fast and slow magnetosonic frequencies [24]. Equation (8) has several types of complex solutions. For example, when $\Omega \approx q_y v_g \approx \Omega_s$ or $\Omega \approx q_y v_g \approx \Omega_s$ we have the excitation of fast or slow magnetosonic waves by the electromagnetic waves. These instabilities also lead, in general, to the existence of two different solutions.

4. Coherent nonlinear EMIC structures

In the previous section, we mentioned that the amplitude modulation of the electromagnetic waves leads to an excitation of magnetosonic waves whose amplitude grows exponentially. Eventually, the
wave amplitude stops growing due to the influence of nonlinear terms that were ignored in the linear analysis. Now we take into account nonlinear terms due to the ponderomotive force, which redistributes the protons and changes the density of the plasma. The convective derivative term \((v_p \cdot \nabla)v_p\) continues to be ignored, at least as long as the wave does not steepen too much.

4.1 Envelope solitons. Here we show that there are soliton solutions for the nonlinear Schrödinger equation in Equation (7). We look for solutions that propagate as \(\xi = y - v_gt\) and assume that any dependence on this variable is faster than that on the spatial coordinate \(x\) or the time \(t\). Hence, for the ponderomotive force in Equation (4), we assume that \(|f_x| \gg |f_y|\). Thus we can simplify that and the expressions for ponderomotive force \(f_y\) and density perturbation may be written as

\[
f_y = -m_p c^2 \frac{\omega_p}{\omega_p - \omega_b} \frac{\partial}{\partial \xi} e^2 |E|^2, \quad \frac{\delta n}{n_0} = \frac{1}{(\omega_p - \omega_b)} \frac{\omega_p}{m_p c^2} \frac{e^2 A^2}{v_{g}^2 - v_s^2}, \quad (9)
\]

Substituting the expressions of Equation (9) into the nonlinear Schrödinger equation (7), we obtain

\[
\left( \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial x^2} \right) A - \frac{\omega_p A}{c^2} A + \frac{\omega_p^2}{(v_{g}^2 - v_s^2)} \frac{1}{m_p c^2} \frac{e^2 A^3}{v_{g}^2 - v_s^2} = 0. \quad (10)
\]

Introducing the new dimensionless variables \(\xi'\) and \(x'\) and the appropriate dimensionless function \(U\) we can obtain the following nonlinear Schrödinger equation describing two-dimensional symmetrical solitons (see [24])

\[
\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} U \right) - U + U^3 = 0, \quad (11)
\]

which is written in terms of the cylindrical coordinate \(\rho = \sqrt{\xi'^2 + x'^2}\). The solutions of this equation have been described by Gurevich and Shvartsburg [25], with the boundary conditions \(U \to 0\) for \(\rho \to \infty\) and either \(U = \text{const}\) or \(dU/d\rho = 0\) at \(\rho = 0\). The boundary condition \(dU/d\rho = 0\) at \(\rho = 0\) leads to \(U\) having discrete values \(U_n\) at \(\rho = 0\). The fundamental mode has \(U_1 = 2.2\); this is a soliton solution, peaked at \(\rho = 0\) and falling off as \(e^{\rho^2} / \rho^n\) as \(\rho \to \infty\). The higher-order modes for \(n > 1\) are damped oscillatory wave functions with \((n - 1)\) zero crossings and have initial values \(U_{2} = 3.3, U_{3} = 4.1, U_{4} = 4.6\), etc. Hence equation (11) has a soliton solution only for moderate initial values of \(U\). For comparatively large initial values of \(U\), this equation describes nonlinear stationary waves propagating with a group velocity \(v_g\). Numerical calculations of Eq. (11) are shown in Fig. 1.

![Figure 1. Numerical calculations of Eq. (11)](image-url)
4.2 Generation of vortices and magnetic field. In this subsection we show that in a homogeneous plasma, electromagnetic ion cyclotron waves can lead to the generation of vortices and a quasi-static magnetic field. Previously, these problems have been considered in laser plasma physics by Tsintsadze et al. [26,27].

To derive the equation for vortices, we use the momentum equation for the proton species,

\[ \frac{\partial}{\partial t} \left( m_p \mathbf{v} + \frac{e}{c} \mathbf{A} \right) = -e \nabla \phi + \frac{e}{c} \mathbf{v} \times \mathbf{B} + \mathbf{v} \times (\nabla \times m_p \mathbf{v}) - \frac{1}{n_p} \nabla p v - \nabla \left( \frac{m_p v^2}{2} \right) + \mathbf{f}. \]  

(12)

The expression for \( f \) is given in Equation (4), and \( \mathbf{B} = B_0 e_y + \nabla \times \mathbf{A} \). Taking the curl of both sides of Equation (12), we obtain

\[ \frac{\partial}{\partial t} \mathbf{\Omega} = \nabla \times (\mathbf{v} \times \mathbf{\Omega}) + \nabla \times \mathbf{f}, \]  

(13)

where \( \mathbf{\Omega} = \nabla \times (m_p \mathbf{v} + \frac{e}{c} \mathbf{A}) \) is the vorticity of the canonical ion momentum.

If the characteristic spatial scale length and the characteristic time scale satisfy the inequality \( l > v t \), we may then neglect the first term on the right-hand side in Equation (13) compared to the term on the left-hand side and obtain the following simple relationship between the vorticity and the source:

\[ \mathbf{\Omega} = \frac{\alpha_p}{(\omega_p - \omega_b)^2} \mathbf{k}_y \times \nabla \frac{e^2 A^2}{m_p c^2}. \]  

(14)

We now introduce the canonical momentum circulation \( G \), defined as

\[ G = \oint (\mathbf{P} + \frac{e}{c} \mathbf{A}) \cdot d\mathbf{l} = \int \mathbf{\Omega} d\mathbf{S}. \]  

(15)

Due to the existence of the last term in \( f_y \) of Equation (4), the flux of \( \mathbf{\Omega} \) through a surface bounded by any closed ion fluid contour, as well as the canonical momentum circulation, is not conserved, namely,

\[ \frac{dG}{dt} = \frac{d}{dt} \oint (\mathbf{P} + \frac{e}{c} \mathbf{A}) \cdot d\mathbf{l} = \frac{\alpha_p}{(\omega_p - \omega_b)^2} \frac{\partial}{\partial t} \oint \frac{e^2 A^2}{m_p c^2} \cdot \mathbf{k}_y \cdot d\mathbf{r}. \]  

(16)

Hence, if there are initially no vortices, they will be generated by the electromagnetic ion cyclotron waves over time.

Note that since the wave number \( k_y \) is directed along the \( y \)-axis and the ponderomotive force is a function of \((x, y)\), then the vorticity has only one component, \( \Omega_y \). Thus we can rewrite Equation (13) as

\[ \frac{d}{dt} \ln \Omega = \frac{k_y \alpha_p m_p}{(\omega_p - \omega_b)^2} 1 \frac{\partial}{\partial t} \frac{e^2 A^2}{\Omega z c^2} \frac{\partial}{\partial x} \frac{e^2 A^2}{m_p c^2}. \]  

(17)

Equation (17) is a generalization of the Hasegawa-Mima equation, and it clearly breaks the frozen-in condition [23], given that \( d(\ln(\Omega) / n)/dt = 0 \).

A simple expression for the magnetic field that is generated by the non-stationary ponderomotive force can be obtained from Equation (14) if we assume that \( \nabla \times m_p \mathbf{v} \ll eB_z / c \). We then obtain

\[ \frac{eB_z}{m_p c} = \frac{\alpha_p e^2}{(\omega_p - \omega_b)^2} k_y \frac{\partial}{\partial x} \left( \frac{eA}{m_p c^2} \right). \]  

(18)

Let us estimate this quantity for the following numerical values: \( \omega_p = 0.37 \times 10^7 \text{Hz}, \omega_0 - \omega_p \approx k_0 v_p / \omega_p, \partial / \partial x 1 / L_{oc} \approx 1.6 \times 10^{-9} \text{cm}^{-1}, k_0 \approx 10^{-7} \text{cm}^{-1}, eA / m_p c^2 \approx 10^{-6}, v_p \approx 10^5 \text{cm/sec}. \) Substituting these parameters into Equation (22), we obtain the following estimate for the
normalized magnetic field perturbation \( eB_z / m_e c \approx 3.7 \text{ Hz} \), or, in unnormalized form,
\[ B_z \approx 0.5 \times 10^{-4} G. \]

5. Conclusions
In this paper, we have studied the nonlinear interaction of the electromagnetic ion cyclotron frequency waves with plasma particles in the inner magnetosphere. The emission is considered to be circularly polarized electromagnetic waves propagating along the almost constant dipole geomagnetic field in the equatorial region of the inner magnetosphere. We studied excitation of the magnetoionic waves through the amplitude modulation of the electromagnetic ion cyclotron waves, and obtained a two-dimensional nonlinear Schrödinger equation for the EMIC waves. In the stationary case, we found two solutions of the nonlinear Schrödinger equation with distinct natures. For sufficiently small amplitudes of the EMIC field, there exists a two-dimensional bright soliton, whereas for larger amplitudes the solution is oscillating. The generation of both vortices and of a quasi-static magnetic field across the geomagnetic field lines was discussed.

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