Weak- to strong pinning crossover

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Material defects in hard type II superconductors pin the flux lines and thus establish the dissipation-free current transport in the presence of a finite magnetic field. Depending on the density and pinning force of the defects and the vortex density, pinning is either weak-collective or strong. We analyze the weak- to strong pinning crossover of vortex matter in disordered superconductors and discuss the peak effect appearing naturally in this context.

Pinning of vortices by material defects is crucial in establishing the defining property of a superconductor, its ability to transport electrical current without dissipation. Collective pinning theory [1], describing the concerted action of many weak pins on the vortex system, is playing a central role in our understanding of this complex statistical mechanics problem [2]. On the other hand, first attempts describing flux pinning go back to Labusch [3], who described the interaction between vortices and strong pinning centers which introduce large (plastic) deformations in the vortex system. In this letter, we describe how these two theories relate to one another; given the density \(n_p\) and force \(f_p\) of pinning centers, as well as the vortex density \(n_v = 1/a^2\), we identify the regimes where individual vortex lines and the bulk vortex lattice are pinned by the collective action of many weak pins or by the independent action of strong pins, see Fig. 1. We naturally recover the peak effect [4] described in the work of Larkin and Ovchinnikov [1] and establish its formal relation to the Landau theory of phase transitions.

In a type II superconductor, the field (\(B\)) induced vortices subject to a current flow \(j\) experience the Lorentz force density \(\mathbf{F}_L = j \times \mathbf{B}/c\) and the resulting vortex motion leads to dissipation. The superconducting response is resurrected through material inhomogeneities pinning the vortices at energetically favorable locations. The pinning force density \(\mathbf{F}_{\text{pin}}\) defines a critical current density \(j_c = c\mathbf{F}_{\text{pin}}/B\) below which the current can flow free of dissipation. Usually, this critical current density is considerably reduced with respect to the depairing current density \(j_0 \sim cH_c/4\pi \lambda \sim c\varepsilon_0/\Phi_0\xi\); here, \(H_c = \Phi_0/2\sqrt{2}\pi \lambda\xi\) is the critical magnetic field, \(\lambda\) and \(\xi\) denote the penetration depth and the coherence length, respectively, \(\Phi_0 = hc/2e\) is the flux unit, and \(\varepsilon_0 = (\Phi_0/4\pi\lambda)^2\) is the (line) energy scale. Below, we focus on the most generic situation of isotropic superconductors and ignore effects due to thermal fluctuations.

When pinning is strong [1, 3, 5] defects act individually and the pinning force density \(\mathbf{F}_{\text{pin}}\) is linear in the density \(n_p\) and average pinning force \(\langle f_{\text{pin}} \rangle\) of defects. The classic arguments characterizing strong pinning go back to Labusch [3], see also [1, 5]: A strong pinning defect induces plastic deformations in the vortex lattice [3, 6, 7] and the energy landscape \(e_{\text{pin}}(\mathbf{R})\) becomes multi-valued in the vortex position \(\mathbf{R}\), see Fig. 2. The averaging over defect locations then has to account for the preparation of the system. We concentrate on the critical current density and thus search for the force against drag; the vortex position then is parametrized through the two-component drag parameter \(\mathbf{R}_d\), fixing the position of the unperturbed lattice with respect to the defect. Dragging the system along the \(x\)-direction, we express the drag force \(-\partial_x e_{\text{pin}}(x, y)\) integrated along \(x\) through the jump \(\Delta e_{\text{pin}}(y) > 0\) in the pinning energy and average over ‘impact parameters’ \(y\),

\[
\langle f_{\text{pin}} \rangle = -\int_0^{L_x} \int_0^{L_y} \frac{\partial_x e_{\text{pin}}(x, y)}{L_x L_y} \, dx \, dy = -\int_0^{a_0} dy \frac{\Delta e_{\text{pin}}(y)}{a_0 a_d(y)},
\]

where \(a_d\) denotes the distance between periodic jumps [4]. For moderately strong pins with deformations not exceeding the lattice constant we have \(a_d \approx a_0\) and assuming a maximal transverse trapping distance \(t_\perp\) along the \(y\)-axis we obtain the mean pinning force

\[
\langle f_{\text{pin}} \rangle \approx -\frac{t_\perp}{a_0} \Delta e_{\text{pin}}(0) \approx -\frac{t_\perp t_\parallel}{a_0} f_p \approx -\frac{S_{\text{trap}}}{a_0^2} f_p, \tag{1}
\]
with the jump $\Delta e_p(0) \approx t_0 f_p$ expressed via the typical impurity force $f_p$ and the bistability range $t_0$ of $e_p(x,0)$; the product $t_0 t_\parallel$ defines the trapping area $S_{\text{trap}}$ associated with the strong pinning center and classified as a weak or strong one.

In order to derive a quantitative criterion for the appearance of strong pinning, we consider a single defect at the origin with a pinning potential $e_p(r)$. Such a defect acts on the vortex system to produce a pinning energy density $E_p(r,u) = \sum e_p(r)\partial^2 \left( R - R_v - u(R_v,z) \right)$, where vortices positioned at $R_v + u(R_v,z)$, $R_v$ the equilibrium positions and $u$ the displacement field. The latter follows from the solution of the implicit equation $(r_v = (R_v,z))$

$$u_\alpha(r_v) = \int d^3r' G_{\alpha\beta}(r_v - r') [-\partial_{u_\beta} E_p](r',u')$$

$$= \sum v_\beta \int dz' G_{\alpha\beta}(r_v - r'_\beta) \phi_p(\beta, R_v + u(R_v,z), z')$$

$$= G_{\alpha\beta}(R_v - R_\alpha, 0) \phi_p(\beta, R_\alpha + u(R_\alpha, 0), 0), \quad (3)$$

with $G_{\alpha\beta}(r)$ the elastic Green's function and $\phi_p = -\nabla u e_p(u)$ the pinning force of the defect. In the last equation we have assumed a moderately strong pinning potential (pinning one vortex at most) of range much smaller than the lattice constant $a_0$ and have chosen $R_\alpha$ as the distance to the vortex closest to the defect. Evaluating (3) for $R_v = (R_\alpha, 0)$, we arrive at the self-consistency equation (note that $G_{\alpha\beta}(r = 0)$ is diagonal)

$$u(R, 0) \approx \overline{C}^{-1} f_p(R + u(R, 0), 0), \quad (4)$$

with the effective elastic constant $\overline{C}^{-1} = \int d^3k/(2\pi)^3 G_{\alpha\beta}(k)$. For weak pinning the displacement $u$ is small and the solution $u(R, 0) \approx f_p(R)/\overline{C}$ is unique. Strong pinning, however, produces multi-valued functions $u(R, 0)$ and $e_p(R)$, cf. Fig. 2. The solution of (4) turns multi-valued as the displacement collapses when $\partial u/R \to \infty$. Assuming a defect symmetric in the plane, $e_p(R, z) = e_p(R, -z)$, and dragging the lattice through the defect center along the $z$-axis, we find $u' = f_p'(x + u) [\overline{C} - f_p'(x + u)]^{-1}$ (note that $x > 0$ implies $u < 0$) and arrive at the (Labusch) criterion in the form

$$\partial_x f_p = -\partial_x^2 e_p = C; \quad (5)$$

hence, in order to produce strong pinning the (negative) curvature of the pinning energy $e_p$ has to overcompensate the lattice elasticity (the Labusch criterion involves the maximal negative curvature above the inflection point). Note that the Labusch criterion tests an individual pinning center and classifies it as a weak or strong one.

When pinning is weak, the elastic forces dominate over the pinning forces and the defects compete; we then are faced with the problem of the statistical summation of individual pinning forces. For weak pins the average $f_p$ vanishes and pinning is due to fluctuations in the pinning density: the forces of the competing pins (with pinning force $f_p$, density $n_p$, and extension $r_p \sim \xi$) add up randomly and produce the pinning energy

$$\langle E_p^{2}(V) \rangle^{1/2} \approx \left[ f_p^2 n_p (\xi/a_0)^2 V \right]^{1/2} \xi; \quad (6)$$

only vortex cores are pinned by the disorder, hence the factor $(\xi/a_0)^2$. Within weak collective pinning theory the sublinear growth of $\langle E_p^{2}(V) \rangle^{1/2}$ with volume turns linear when the displacement $u$ increases beyond the scale $\xi$ of the pinning potential, thus defining the collective pinning volume $V_c$. Each volume of size $V_c$ is pinned independently with a pinning energy $U_c = \langle E_p^{2}(V_c) \rangle^{1/2}$ and we obtain a proper pinning force density

$$F_{\text{pin}} \sim U_c/V_c r_p \sim \left( f_p^2 n_p (\xi/a_0)^2 V \right)^{1/2}; \quad (7)$$

balancing this pinning force density against the Lorentz force density $jB/c$ we find a finite critical current density $j_c \sim cF_{\text{pin}}/B$. The remaining task is the determination of the collective pinning volume $V_c$; its calculation is complicated by the dispersion and anisotropy of the vortex lattice, see below and Ref. [2] for a detailed discussion.

It is instructive to compare the weak- and strong pinning schemes and their dependence on dimensionality, particularly in the limit of a small defect density $n_p$ (in the following, we assume pinning sites characterized by their force $f_p$ and extension $\xi$). An isolated vortex line (1D) is always subject to strong pinning forces as the effective elastic coefficient $\overline{C}$ vanishes due to the diverging integral. At the same time, the deformation of the line due to the pins is large and we cannot ignore their mutual competition. Comparing the elastic energy $\varepsilon_0 \xi^2/L_c$
and the pinning energy $U_c = (f_p^2 n_p L_c \xi^2)^{1/2} \xi$, we find the collective pinning length $L_c \sim (\zeta_0^3/\xi^2)^{1/3}$ and a critical current density

$$j_c \sim j_0 (n_p c^3 f_p^2 / \zeta_0^3)^{1/3}.$$  \hspace{1cm} (8)

This result is valid as long as many pins compete within the volume $\xi^2 L_c$: the condition $n_p \xi^2 L_c > 1$ defines the lower limit $\tilde{n}_{1D} \sim f_p / \zeta_0^3 < n_p$, where the critical current density assumes the value $j_c \sim j_0 (f_p / \zeta_0^3)^{3/2}$.

For small densities $n_p < \tilde{n}_{1D}$ the pins act individually and we determine $j_c$ from the force balance ($\Phi_0/c) j_c \sim \Delta \varepsilon_{\text{pin}} \sim f_p u$, with $u \sim t_1$ the displacement directed along the force. The displacement $u$ and the length $l$ between two subsequent pins fixing the vortex derives from an analysis of the pinned vortex geometry, see Fig. 4 inset: integrating the force equation $\varepsilon_0 \partial_u^2 u = f(z)$ (with $f(z)$ the force per unit length acting on the line) over one pinning center, we find the distortion angle $\theta = \partial_u u \sim u/l \sim f_p / \zeta_0$. A vortex segment of length $l$ deformed by the angle $\theta$ in the direction of the driving force encounters $\theta^2 \xi n_p$ defects (with the trapping length $t_1 \sim \xi$). At the distance $l$, this number is unity, hence $l \sim \sqrt{\zeta_0 / f_p} n_p \xi$ and we obtain the critical current density

$$j_c \sim j_0 (n_p c^3 f_p^2 / \zeta_0^3)^{1/2}.$$  \hspace{1cm} (9)

At the crossover density $\tilde{n}_{1D} \sim f_p / \zeta_0^3$ the critical current density matches up with the weak pinning result; also, the displacement $u \sim l f_p / \zeta_0$ is of order $\xi$ at the crossover density $\tilde{n}_{1D}$ and hence matches the displacement field relevant in the collective pinning scenario. Note that collective pinning dominates over the strong pinning at large densities $n_p > \tilde{n}_{1D}$.

For the vortex lattice (3D bulk pinning; we assume $a_0 < \lambda$) the Labusch criterion offers a distinction between weak and strong pinning centers; using the Green’s function for the vortex lattice (see, e.g., [2]) we find $C \sim \varepsilon_0 / a_0$. According to [3] a pinning center changes from weak to strong at $f_p \sim f_{\text{Lab}} \equiv \varepsilon_0 n_p^2 / a_0$. We first review the weak pinning situation with $f_p < f_{\text{Lab}}$ (where necessary, we encode quantities in this regime with a superscript ‘−’). The determination of the anisotropic collective pinning volume $V_c = R^2 L^3$ has to account for the dispersion in the tilt modulus at intermediate scales $a_0 < R_c < \lambda$, see Ref. [2], and produces the results

$$j_c \sim j_0 \frac{\xi^2}{a_0} \frac{\lambda}{L_c} \left( \frac{a_0}{L_c} \right)^\nu \sim e^{-\xi / (L_c / a_0)^3}, \hspace{1cm} R_c < \lambda;$$  \hspace{1cm} (10)

$$j_c \sim j_0 \frac{\xi^2}{a_0} \frac{\lambda}{L_c} \left( \frac{a_0}{L_c} \right)^6, \hspace{1cm} R_c > \lambda.$$  \hspace{1cm} (11)

The bulk strong pinning result (12) smoothly transforms into the 1D expression (9) at $\tilde{n}_c^w$, where $l \sim a_0$. On the contrary, the strong pinning expression (12) does not match up with the weak bulk collective pinning results (10) and (11) at $f_p = f_{\text{Lab}}$ (we concentrate on low impurity densities with $n_p \xi^2 < 1$, cf. Fig. 1). However, we have to keep in mind that our rough derivation of the strong pinning result (12) breaks down on approaching the critical force $f_{\text{Lab}}$. Indeed, since the displacement field $\mathbf{u}(\mathbf{r})$ turns single valued below $f_{\text{Lab}}$, strong pinning vanishes altogether (with a power $|f_p - f_{\text{Lab}}|^2$, see [15]) and pinning survives only in the form of weak collective pinning due to fluctuations in the impurity density. Within the approximative scheme adopted here the sharp rise of the critical current density at $f_p > f_{\text{Lab}}$ is encoded in a jump $j_{c|_{\text{wp}}} \sim (\lambda^2 / a_0^3) / n_p a_0^2 \xi^2 > 1$ for $n_p < \bar{n}_\lambda \sim 3$.

The crossover from strong to weak pinning at the Labusch condition can be analyzed within a Landau type expansion: We define the free energy functional $\varepsilon_{\text{pin}}(\mathbf{u}, \mathbf{R}_d) = C u^2 / 2 + e_p (\mathbf{R}_d + \mathbf{u})$ from which the self-consistency equation (13) follows by variation. Note that the derivative $-\partial_x \varepsilon_{\text{pin}} = f_p x (\mathbf{R}_d + \mathbf{u})$ provides the force along $x$ acting on a vortex separated from the defect by $\mathbf{R}_d$ and deformed by $\mathbf{u}$, c.f. Fig. 1; it is this force which has to be averaged over in the definition of $\varepsilon(\text{pin})$.

We first concentrate on the trajectory $\mathbf{R}_d = (x, 0)$ with $\mathbf{u} = (u, 0)$. The curvature $c_p(u)$ relevant in (5) assumes a maximal negative value; we denote the corresponding location and value by $u_s$ and $-\kappa$, respectively. Next, we expand the curvature around $u_s$: $c_p(u) \approx -\kappa + \alpha(u - \kappa)$.
The transition between these states is discontinuous and within the (anisotropic) volume $V$, the critical Labusch force. Quantitatively, we compare the collective pinning scenario in terms of the strong pinning force overcompensates the elastic force; this length agrees with the 3D collective pinning length in the non-dispersive regime. The resulting bistable solutions are the signature of the alternative pinning valleys which the collective pinning volume can select beyond the scale $R_c$.

The above discussion sheds light on the general concept of pinning. Pinning is absent in the rigid limit. A finite but large elasticity (with $f_{\text{Lab}} > f_p$) allows only for weak deformations and individual pins cannot hold the lattice as the averaging over individual pinning forces produces a null result. Hence, pinning is due only to fluctuations in the pinning forces and thus collective. Reducing the elasticity, strong pinning defects appear when $f_{\text{Lab}}$ drops below $f_p$; they pin the lattice individually and strong pinning, linear in the defect density $n_p$, outperforms collective pinning. The important role played by the curvature $\varepsilon'' < 0$ in the pinning potential is an interesting topic for numerical studies. The crossover between weak collective and strong pinning can be realized in experiments: increasing the magnetic field towards its critical value $H_{c2}$ leads to a marked softening of the elastic moduli. The reduction in the elastic moduli entails a decrease of the Labusch force $f_{\text{Lab}}$ and triggers the crossover from weak- to strong pinning, producing the well known peak effect in the critical current density $I_c$. We acknowledge discussions with Anatoly Larkin and financial support from the Swiss National Foundation.

Defining the individual force of (equal) pinning centers via $f_p = \max_u \phi_u f_p$, we can translate the expression for the critical current density $j_c$ into an expression for the average pinning force $\langle f_{\text{pin}} \rangle$ is shown in Fig. 2. The above results for the jump in pinning energy and the transverse trapping distance we find the averaged pinning force (c.f. (13)):

$$\langle f_{\text{pin}} \rangle \approx 18(u_c + \nu/C)(C - \kappa)^2/\alpha a_0^2.$$  (14)

Defining the individual force of (equal) pinning centers via $f_p = \max_u \phi_u f_p$, we can translate (13) into an expression for the critical current density $j_c$, extending the strong pinning result (12) to the vicinity of the Labusch point,

$$j_c \sim j_0 (\xi^2/a_0^2) n_p a_0 \xi^2 |f_p/f_{\text{Lab}} - 1|^2.$$  (15)

Comparing with the weak pinning result (11), we note a sharp rise in the critical current density $j_c$ once the strong pinning force overcomes the weak pinning result $I_c$. With the small parameter $\delta = (a_0/\lambda)\sqrt{n_p a_0 \xi^2} < 1$, this crossover appears above but still close to the Labusch point as $f_{\text{Lab}} \propto C$ decreases below $f_p/(1 + \delta)$.

Another remarkable result is the interpretation of the collective pinning scenario in terms of the strong pinning picture; indeed, summing over competing pins within the collective pinning volume $V_c$ produces the corresponding critical Labusch force. Quantitatively, we compare the force gradient $f' \sim |n_p (\xi^2/a_0^2)| V^{1/2} f_p/\xi$ accumulated within the (anisotropic) volume $V = LR^3 = (\lambda/a_0) R^3$ with the elastic parameter $C(R) = \varepsilon_0 \lambda R/a_0^3$ for smooth distortions on the scale $R > \lambda$ (non-dispersive regime) and apply the Labusch criterion (14). We then find the scale $R_c = \lambda L/L_{\text{Lab}} f_p^2 n_p a_0 \xi^2$, where the accumulated pinning force overcompensates the elastic force; this length agrees with the 3D collective pinning length in the non-dispersive regime. The resulting bistable solutions are the signature of the alternative pinning valleys which the collective pinning volume can select beyond the scale $R_c$.

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[10] Alternatively, we find $C_\xi = 1/\xi_0 k_\xi^2$ and cut the integral on the scale $\tilde{l}$, $C_\xi \sim \xi_0/\tilde{l}$; the relation $u \sim f_p/C$ provides the angle $\theta \sim u/\tilde{l} \sim f_p/\xi_0$.
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