Distributed manipulation of two-qubit entanglement with coupled continuous variables

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We study the dynamics of two qubits separately sent through two coupled resonators, each initially containing a coherent state field. We present analytical arguments and numerical calculations for the qubit-field system under different two-qubit initial states, photon hopping strengths, and detunings. In far off-resonant regime, the maximal entanglement of two qubits can be generated with the initial qubit state in which one qubit is in the excited state and the other is in the ground state, and the initially maximal two-qubit entanglement can be frozen and fully revived even for large mean photon number. When the qubits are both initially in their excited states or ground states, the qubit-qubit entanglement birth and death apparently appear in the regime where the photon hopping strength is close to qubit-field detuning, and its peaks do not decrease monotonically as the interaction time increases. It is interesting to observe that when there is photon hopping strength between two fields, the field-field entanglement can be larger than one and increases as the initial amplitude of the coherent state grows. By postselecting the fields both in their coherent states, the entanglement of two initially unentangled qubits can be largely improved. Our present setup is fundamental for the distributed quantum information processing and applicable to different physical qubit-resonator systems.

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I. INTRODUCTION

Manipulation of the entanglement dynamics between the qubit and light field has become a significantly important issue for both the fundamental quantum theories and experiments [1–8], where reliable quantum information processing (QIP) and computation rely on coherent manipulation of physically realizable systems in which information is stored and by means of which information is processed or transmitted. However, only the dynamics in a qubit-field system where the field evolves in low-dimensional subspace is analytically easy to handle.

Since the continuous-variable (CV) physical system contains an infinite-dimensional spectrum of eigenstates and can be efficiently generated by a classical monochromatic current [9], the CV system, as a kind of excellent quantum resource that most resembles a classical electromagnetic field [10, 11], has attracted much attention in many fields of QIP recently, such as quantum transport [12, 13], quantum storage [14, 15], quantum memory [16, 17], quantum computation [18], quantum transport [19], and quantum memory [20, 21].

Based on resonant Jaynes-Cummings (JC) interaction, Lee et al. [16] demonstrated that an ebit could reciprocate between two non-local qubits and two separate coherent states via postselection, in which the CV systems are able to reliably accumulate more than one ebit when a series of qubit pairs interact with the CV systems; Yönac and Eberly [18] reported the collapse and revival behaviors of entanglement of two separate qubits each interacting with a CV system; Guo et al. have shown that coherent-state control and entanglement transfer between two non-local qubits and two spatially separated CV systems are possible [19].

In general experiment associated with entanglement manipulation, there may exist two common modulations, i.e., the hopping strength between the CV systems and the detuning in the JC interaction. Previous efforts typically concentrate on the entanglement dynamics between a pair of non-local qubits and the spatially separate CV systems, where the dynamics is analytically solvable based on separate JC models. The coupled-cavity system involving two non-local qubits and the coupled thermal fields has been investigated under hopping and detuning modulations, which can exhibit interesting features, such as maximal qubit-qubit entanglement generation and freezing [22]. However, the dynamics involving two non-local qubits and the coupled CV systems have not been extensively investigated due to their infinite-dimension Hilbert space and complicated mutual-interaction processes when considering the hopping between the CV systems. To our knowledge, a convincingly analytical treatment of the dynamics between two non-local qubits and the coupled CV systems is still absent, which is relevant with the recent progresses in the arrays of interacting micro-cavities and their coupling to qubits [20–25].

In this paper, we present the numerically exact solution to the dynamics between two qubits and two coupled coherent-state fields. Our setup, differing further from the previous setups, where two sites each involving a qubit and a CV field evolve independently based on the resonant Jaynes-Cummings interaction [16, 18], focuses on the modulation induced by the hopping between the CV fields and the detuning between the qubit and the local CV field. The entanglement dynamics between two qubits depends on initial two-qubit states, photon hopping strengths, and qubit-field detunings. We present analytical arguments and numerical calculations for the qubit-field system within far off-resonant regime. In far off-resonant regime, the maximal entanglement of two qubits can be generated with the initial qubit state in which one qubit is in the excited state and the other is in the ground state, while the initially maximal two-qubit entanglement can be frozen. Particularly, the maximal qubit-qubit entanglement can be fully revived by tuning the qubit-field detuning even when the mean photon number of the coherent state field.

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is large, which is obviously different from the resonant case where the maximal entanglement can not be fully revival in Ref. [18]. When the qubits are both initially in their excited states or ground states, the qubit-qubit entanglement apparently appears in the regime where the strength of photon hopping is close to qubit-field detuning. It is interesting to observe that when there is hopping between two CV systems, the field-field entanglement can be larger than one and increases as the initial amplitude of the coherent state grows. By postselecting the fields both in their coherent states, the entanglement of two initially unentangled qubits can be largely improved. Even under the far-off-resonant condition, the initially unentangled qubits can become the maximally entangled by measuring the non-maximally entangled fields both in their coherent states.

The present idea can be generalized to other coupled CV systems, including the squeezed coherent state and displaced coherent state, and has the potential application in the distributed QIP, such as the quantum transfer and entanglement protection. Our present results are fundamental and promising for the distributed QIP and applicable to different physical qubit-resonator systems.

II. SYSTEM HAMILTONIAN

FIG. 1: (Color online) Schematic of our setup for two non-local qubits sent through two coupled resonators, each interacting with a CV system.

To investigate the entanglement dynamics between qubits and the coupled continuous variables, let us first consider that two identical qubits 1 and 2 are respectively sent through two coupled resonators, each initially containing a coherent state field $|\alpha\rangle$ ($i = 1, 2$). For convenience, we assume $\alpha$ is a real number throughout this paper and $|\alpha\rangle = \sum_{n=0}^{\infty} A_n |n\rangle$ where $A_n = \alpha^n e^{-\alpha^2/2}/\sqrt{n!}$. In the picture rotating at local field frequency, the interaction Hamiltonian under the rotating-wave approximation is ($\hbar = 1$):

$$H_I = \sum_{i=1}^{2} \left[ \Delta_i a_i^\dagger a_i + g(S_i^+ a_i + S_i^- a_i^\dagger) + J(a_i^\dagger a_2 + a_2 a_i) \right]$$

where $S_i^+ = |e_i\rangle \langle g_i|$ and $S_i^- = |g_i\rangle \langle e_i|$ with $|e_i\rangle$ and $|g_i\rangle$ being the excited state and ground state of the $i$th qubit. $a_i^\dagger$ and $a_i$ are respectively the creation and annihilation operators for the $i$th field mode, $g$ describes the coupling strength between the qubit and field mode, and $\Delta$ is the detuning between the qubit’s transition and field mode. $J$ represents the coherent photon hopping strength between two resonators. In the limit of zero hopping ($J = 0$) and zero detuning ($\Delta = 0$), the present system reduces to a pair of noninteracting atom-cavity systems[16], each described by the resonant JC interaction.

We assume the initial state of two qubits is a pure state $|\psi_d(0)\rangle$, then the evolution of the qubit-field system is:

$$|\Psi(t)\rangle = e^{-iH_I t} |\psi_d(0)\rangle |\alpha\rangle_1 |\alpha\rangle_2.$$  \hspace{1cm} (2)

Since the total excitation number operator $M = \sum_{i=1}^{2} (|e_i\rangle \langle e_i| + a_i^\dagger a_i)$ commutes with $H_I$, the excitation number of the qubit-field system is conserved during its evolution. For specific $\alpha$, we truncate the total excitation number at $M$ in the Hilbert space, therefore the system evolution is:

$$|\Psi(t)\rangle = \sum_{N=0}^{M} \sum_{N_1=0}^{N} U^N_{gg_m_n}(t) |e_1 g_2\rangle |n_1\rangle_1 |n_2\rangle_2$$

$$+ \sum_{N=0}^{M} \sum_{N_1=0}^{N} U^N_{gen_m_n}(t) |g_1 e_2\rangle |n_1\rangle_1 |n_2\rangle_2$$

$$+ \sum_{N=0}^{M} \sum_{N_1=0}^{N} U^N_{gen_{m_1}n_2}(t) |e_1 e_2\rangle |n_1\rangle_1 |n_2\rangle_2,$$ \hspace{1cm} (3)

where $U^N_{gg_m_n}(t), U^N_{gen_m_n}(t), U^N_{gen_{m_1}n_2}(t)$ and $U^N_{gen_{m_2}n_1}(t)$ are the coefficients of the corresponding state components $|e_1 g_2\rangle |n_1\rangle_1 |n_2\rangle_2$, $|g_1 e_2\rangle |n_1\rangle_1 |n_2\rangle_2$, $|g_1 g_2\rangle |n_1\rangle_1 |n_2\rangle_2$ and $|e_1 e_2\rangle |n_1\rangle_1 |n_2\rangle_2$, respectively.

III. QUBIT-QUBIT ENTANGLEMENT

We assume each field is initially prepared in the coherent state with mean photon number $\bar{n} = \alpha^2$, and discuss the entanglement dynamics of two qubits with different initial states based on numerical truncation. For example, when $\bar{n} = 1$, the Hilbert space is safely cut off at $M = 15$; when $\bar{n} = 100$, the Hilbert space is safely cut off at $M = 210$. When $\alpha \gg 1$, the width of the photon number distribution obeys $1 \ll \Delta n \ll \alpha^2$. Therefore it is safely to truncate the Fock state basis to $M = 10 + 2\alpha^2$ for special $\alpha$ value.

In this section, we use the Wootters’s concurrence $C$ as the entanglement measure for two qubits expressed in the standard qubit basis $|e_1 e_2\rangle$, $|e_1 g_2\rangle$, $|g_1 e_2\rangle$, $|g_1 g_2\rangle$, which is defined as[20]:

$$C = \max \left\{ 0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right\}, \hspace{1cm} (4)$$

where $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$ are the eigenvalues arranged in decreasing order of the following matrix:

$$\xi = \rho (\sigma_y \otimes \sigma_y)^* \rho (\sigma_y \otimes \sigma_y), \hspace{1cm} (5)$$

where $\sigma_x$ is the corresponding Pauli matrix, and $\rho$ is the two-qubit reduced density matrix. $C = 0$ is for two unentangled qubits, and $C = 1$ stands for the maximal entanglement of two qubits. Thus, the reduced density matrix $\rho$ for the qubits
is calculated by tracing out the fields, and the elements of \( \rho \) are straightforwardly solved by the numerical simulation.

![FIG. 2: (Color online) The concurrence \( C \) for two qubits depends on the interaction time \( gt/\pi \) and: (a) the hopping strength \( J/g \) \( \Delta = 5g \); (b) the detuning \( \Delta/g \) with \( J = 5g \), where \( \alpha = 1 \) and qubits are initially in the state \( |\epsilon_1g_2\rangle \).](image)

![FIG. 3: (Color online) The concurrence \( C \) for two qubits depends on the interaction time \( gt/\pi \) and: (a) the hopping strength \( J/g \) \( \Delta = 0 \); (b) the detuning \( \Delta/g \) with \( J = 0 \), where \( \alpha = 1 \) and the qubits are initially in the maximally entangled state \( (|\epsilon_1g_2\rangle + |g_1\epsilon_2\rangle)/\sqrt{2} \).

We plot the concurrence of two qubits with initial qubit state \( |\psi_0(0)\rangle = |\epsilon_1g_2\rangle \) against the interaction time \( gt \) and the hopping strength \( J \) (the detuning \( \Delta \)) in Fig. 2(a) (Fig. 2(b)). The appearance of \( C = 1 \) is clearly visible in Fig. 2 (a) and (b). This is because when large detuning condition is satisfied, the probability for energy exchange between the qubits and field modes is close to zero, and the two qubits couple with each other by exchanging the virtual excitation of field modes. This can be understood by the effective Hamiltonian

\[
H_{\text{eff}} = -\frac{3}{2}\sum_{i=1}^{2}\left[\frac{g^2}{2\Delta_1}\hat{b}_i^\dagger\hat{b}_i + \frac{g^2}{2\Delta_2}\hat{b}_i^\dagger\hat{b}_i\right]|e_i\rangle\langle e_i| - |g_i\rangle\langle g_i| + \frac{g^2}{2\Delta_1} |e_i\rangle\langle e_i| + \lambda(S_i^+S_i^- + S_i^-S_i^+), \tag{6}
\]

where \( b_1 = (a_1 + a_2)/\sqrt{2}, b_2 = (a_1 - a_2)/\sqrt{2}, \Delta_1' = \Delta + J, \Delta_2' = \Delta - J, \) and \( \lambda = \frac{\Delta}{2\Delta_1} - \frac{\Delta}{2\Delta_2} \). Under the large detuning condition \( \Delta_1', \Delta_2' \gg \sqrt{\bar{n}} + 1g/\sqrt{2} \), the evolution of the two-qubit system is \( \Psi(t) = e^{-i\lambda t}\left[\cos(\lambda t)|\epsilon_1g_2\rangle - i\sin(\lambda t)|g_1\epsilon_2\rangle\right] \), which is independent of the field states and accounts for the periodic oscillation behavior in Fig. 2. Therefore, for large hopping strength or local qubit-field detuning, the coupled CV systems can generate the maximal qubit-qubit entanglement.

We now consider the case that are initially prepared in the maximally entangled state \( |\psi_0(0)\rangle = (|\epsilon_1g_2\rangle + |g_1\epsilon_2\rangle)/\sqrt{2} \). For \( J \gg g \) in Fig. 3(a) or \( \Delta \gg g \) in Fig. 3(b), the maximal qubit-qubit entanglement can be frozen. This is because that the large detuning condition \( \Delta_1', \Delta_2' \gg \sqrt{\bar{n}} + 1g/\sqrt{2} \) is satisfied in this case, therefore, the effective Hamiltonian of Eq. (6) becomes valid in the system evolution, in which the maximally entangled state for two qubits becomes an eigenstate of this effective Hamiltonian. We also plot the two-qubit concurrence against the interaction time for different amplitudes of initial coherent state in Fig. 4. The result shows that the concurrence \( C \) becomes less stable as the amplitude \( \alpha \) increases for fixed \( J \) and \( \Delta \) values. This is due to the fact that the probability that the qubits exchange energy with the fields increases with the mean photon numbers \( \bar{n} \). When the mean photon numbers are large enough, the large detuning condition \( \Delta_1', \Delta_2' \gg \sqrt{\bar{n}} + 1g/\sqrt{2} \) is not satisfied, therefore the effective Hamiltonian of Eq. (6) loses its validness in the system evolution. Unlike the resonant situation in Ref. [18], the effects of the detuning \( \Delta \) on the two-qubit concurrence \( C \) are considered in Fig. 5. We observe that the period of the collapse and revival of qubit-qubit entanglement is delayed when

![FIG. 4: (Color online) The concurrence \( C \) for two qubits versus the interaction time \( gt/\pi \) for different amplitudes \( \alpha \) of the initial coherent state with \( \Delta = 0 \) and \( J = 10g \). The qubits are initially in the state: (a) \( |\epsilon_1g_2\rangle \); (b) \( (|\epsilon_1g_2\rangle + |g_1\epsilon_2\rangle)/\sqrt{2} \).

![FIG. 5: (Color online) The concurrence \( C \) for two qubits versus the interaction time \( gt/\pi \) with \( \alpha = 10 \) and \( J = 0 \). The qubits are initially in the state \( (|\epsilon_1g_2\rangle + |g_1\epsilon_2\rangle)/\sqrt{2} \).](image)
fields conditioned on two qubits leaving their resonators both in the ground states. The fields after this postselection are in a pure state:

\[ |\Psi_f(t)\rangle = N_f \sum_{l=0}^{M} \sum_{m=0}^{M} U_{l,m}(t) |l\rangle_1 |m\rangle_2, \]

where \(|l\rangle_1\) and \(|m\rangle_2\) represent the Fock state basis for the field 1 and 2, respectively. \(U_{l,m}(t)\) is the time-dependent coefficient of state component \(|l\rangle_1 |m\rangle_2\), and \(N_f\) is the normalization constant.

We take von Neumann entropy to measure field-field entanglement. Taking a partial trace over the field 2, we can obtain the reduced density matrix for the field 1:

\[
\rho_{f1} = Tr_2(|\Psi_f(t)\rangle\langle\Psi_f(t)|)
= \sum_{m=0}^{M} \sum_{l=0}^{M} N_f^2 U_{l,m}(t) U_{l,m}^*(t) |l\rangle_1 \langle l|, \]

and the von Neumann entropy of the field 1 is explicitly calculated by \(\epsilon = -Tr(\rho_{f1} \log_2 \rho_{f1})\).

A. Photon hopping modulation

With different photon hopping strengths between the resonators, the evolution dynamics for the qubit-field system in the present paper is very different from that in Ref. [16], and four subsystems involving the Jaynes-Cummings interaction and the field-field interaction constitute the whole system.

We plot the entanglement \(\epsilon\) of the field against the amplitude \(\alpha\) and the interaction time \(gt\) in Fig. 7 [(a) - (c)]. For \(\alpha = 0\), the qubit-field system is simplified to the model with two coupled vacuum fields in Ref. [20], where \(\epsilon\) can be 1 for sure. Fig. 7 [(d) - (f)] show that the probability \(P\) of two qubits leaving the resonators in their ground states, where \(P = \sum_{l=0}^{M} \sum_{m=0}^{M} |U_{l,m}(t)|^2\). It is interesting to see that when \(\alpha < 1\), \(\epsilon\) can be larger than 1 and \(\epsilon\) becomes larger as \(\alpha\) increases, which is very different from the result with the maximal field-field entanglement \(\epsilon = 1\) in Ref. [16]. This is because that the entanglement between the two Jaynes-Cummings subsystems is not conserved due to their interaction, which can help to improve the entanglement between the fields. However, it should be noted that the photon-hopping itself can not lead to the field-field entanglement for the initial coherent states. The Jaynes-Cummings interaction makes each field deviate from the coherent state so that the photon hopping can enhance the field-field entanglement. When \(\alpha > 0\), the oscillation behaviors of the probability \(P\) in Fig. 7 (d) and (e) are similar to that in Ref. [16]. When the large detuning condition is satisfied, as the case with \(J \gg \sqrt{\alpha^2 + 1/g} \sqrt{2}\) plotted in Fig. 7 (c) and (f), the probability for energy exchange between the qubits and field modes is close to zero, which makes the probability \(P\) in Fig. 7 (f) experience no oscillation behaviors for small \(\alpha\). It is interesting to observe that when \(\alpha > 1\), the field-field entanglement \(\epsilon\) keeps larger than 1 whenever the qubits leave
FIG. 7: (Color online) The degree of entanglement \( \varepsilon \) for the field 1 depends on the amplitude \( \alpha \) of the initial coherent state and the interaction time \( gt \) (in units of \( \pi \)) when \( \Delta = 0 \): (a) \( J = 0.1g \); (b) \( J = g \); (c) \( J = 10g \). Probability \( P \) for the qubits leaving the resonators in their ground states when \( \Delta = 0 \): (d) \( J = 0.1g \); (e) \( J = g \); (f) \( J = 10g \).

the resonators in their ground states after the first moments of oscillation.

To see how a pair of unentangled qubits interact with the highly-entangled coupled CV fields, we send the second pair of qubits with the initial state \( |g_1g_2\rangle \) into the respective resonator, containing the rest entangled field state \( |\Psi_f(t)\rangle \). Applying the Hamiltonian in Eq. (1) again to obtain the system evolution after an interaction time \( t' \):

\[
|\Psi_{a_f}(t')\rangle = e^{-iHt'}|g_1g_2\rangle|\Psi_f(t)\rangle.
\] (10)

To investigate the entanglement dynamics of the second pair of qubits, we trace \( |\Psi_{a_f}(t')\rangle \) over the field variables and obtain the reduced density matrix \( \rho' \) for two qubits:

\[
\rho' = \sum_{l=0}^{M} \sum_{m=0}^{M} \langle l_2|m_2|\Psi_{a_f}(t')\rangle\langle \Psi_{a_f}(t')|l_1\rangle\langle m_1|l_2\rangle.
\] (11)

Taking the definition of Eq. (4), we plot the concurrence \( C \) against the amplitude \( \alpha \) and the interaction time \( gt \) in Fig. 8 [(a) - (c)]. We find that if the photon hopping strength is small enough, as shown in Fig. 8 (a), two qubits are able to become maximally entangled when interacting with highly-entangled field states. However, two qubits are not able to become maximally entangled as the photon hopping strength increases even for \( \alpha = 0 \), as shown in Fig. 8 (b) and (c), due to the entanglement loss induced by the photon hopping.

In order to improve the degree of entanglement for the second pair of qubits, we measure the fields with the projection onto the coherent state of amplitude \( \alpha \) similar to Ref. [16]:

\[
|\Psi_a(t')\rangle = z\langle \alpha|\langle \alpha|\otimes |\Psi_{a_f}(t')\rangle.
\] (12)

The concurrence of the qubits after measuring the fields is plotted in Fig. 8 [(d) - (f)]. After postselecting the cavity fields both in their coherent states, the entanglement of two qubits exhibits sharp oscillating behaviors and can be 1.

B. Detuning modulation

Assume the photon-hopping becomes so weak that can be ignored, we directly generalize the entanglement reciprocation with resonant Jaynes-Cummings interaction in Ref. [16] to the situation with detune Jaynes-Cummings interaction.

Based on the processes from Eq. (7) to Eq. (9), we plot the entanglement \( \varepsilon \) of the field against the amplitude \( \alpha \) and the interaction time \( gt \) in Fig. 9 [(a) - (c)], and the probability \( P \) for two qubits leaving the resonators in their ground states in Fig. 9 [(d) - (f)] under different qubit-field detunings. For \( \alpha = 0 \), \( \varepsilon \) can be 1 for sure, meaning the complete ebit in two qubits can be transferred to the fields under different detunings. Even when the detuning increases to a certain degree, such as \( \Delta = g \) plotted in Fig. 9 (b) and (e), the fields can be with one complete ebit whenever the qubits leave their resonators, which implies the entanglement reciprocation [16] tolerates the qubit-field detuning within a wide range. However, for large detuning case \( \Delta \gg \sqrt{g^2 + 1g^2} \) in Fig. 9 (c) and (f), \( \varepsilon \) is not able to keep in 1, which is very different from that in the resonant situation [16]. This is because under the large detuning regime, the maximally entangled state for two qubits is an eigenstate of the effective Hamiltonian in Eq. (6), therefore an ebit for the qubits can not be completely trans-
onators, which are in the state of qubits with the initial state \(q_1, q_2\). The interaction time \(\alpha\) is large enough. The concurrence for the second pair of qubits after postselecting the fields both in their coherent states when \(\Delta = 0\): (a) \(\Delta = 0.1g\); (b) \(\Delta = g\); (c) \(\Delta = 10g\). The concurrence for the second pair of qubits after postselecting the fields both in their coherent states when \(\Delta = 0\): (a) \(\Delta = 0.1g\); (b) \(\Delta = g\); (c) \(\Delta = 10g\). The degree of entanglement \(e\) for the field 1 depends on the amplitude \(\alpha\) of the initial coherent state and the interaction time \(gt\) (in units of \(\pi\)) when \(J = 0\): (a) \(\Delta = 0.1g\); (b) \(\Delta = g\); (c) \(\Delta = 10g\). Probability \(P\) for the qubits leaving the resonators in their ground states when \(J = 0\): (d) \(\Delta = 0.1g\); (e) \(\Delta = g\); (f) \(\Delta = 10g\).

To conclude, we have numerically study the exact dynamics for two qubits based on the coupled coherent state fields, and the modulations on the entanglement reciprocation induced by photon hopping between two CV systems and the qubit-field detuning. In far off-resonant regime, the maximal entanglement of two qubits can be generated with the initial qubit state in which one qubit is in the excited state and the other is in the ground state, while the initially maximal two-qubit entanglement can be frozen and fully revival even for large mean photon number. For example, the maximal entanglement is fully revival even for large mean photon number \(\bar{n} = 100\) by choosing \(\Delta = 100g\), and the period of the entanglement collapse and revival is delayed as the detuning increases further. When the qubits are both initially in their excited states or ground states, the qubit-qubit entanglement concentrates to appear at \(\Delta \sim J\), and its peaks do not decrease monotonically as the interaction time increases. When there is photon hopping strength between two CV systems, the field-field entanglement can be larger than 1 and increases as the initial amplitude of the coherent state grows. By postselecting the fields in their coherent states, the entanglement of two initially unentangled qubits can be largely improved.

V. CONCLUSION

In order to see whether it is possible for two unentangled qubits to retrieve the ebit from rest entangled CV systems under different qubit-field detunings. We send the second pair of qubits with the initial state \(|g_{1g2}\rangle\) into their respective resonators, which are in the state \(|\Psi_f(t)\rangle\). Based on the equations from Eq. (10) to Eq. (12), we plot the two-qubit concurrence against the amplitude of the initial coherent state and the interaction time \(gt\) in Fig. 10 [(a) - (c)], and the two-qubit concurrence after measuring the fields with the projection onto their coherent states of amplitude \(\alpha\) in Fig. 10 [(d) - (f)] under different qubit-field detunings. We find that if the detuning is small enough, as shown in Fig. 10 (a), two qubits are able to retrieve an ebit from the entangled CV systems for \(\alpha = 0\). As the detuning increases, the entanglement that the qubits can retrieve from the entangled CV systems decreases, especially for the large detuning situation in Fig. 10 (c), the entanglement retrieved by the qubits is close to zero. This is because the probability for energy exchange between the qubits and CV systems is close to zero under the condition \(\Delta \gg \sqrt{\alpha^2 + 1g2}\). After measuring the fields, it is interesting to observe that two qubits can become the maximally entangled state, as plotted in Fig. 10 [(d) - (f)]. Even for the large detuning situation plotted in Fig. 10 (f), the qubits can become the maximally entangled when \(\alpha\) is large enough.

Therefore, the entanglement dynamics between the qubits and the coupled CV systems here becomes different from that in the system involving two uncoupled CV systems. When there is photon hopping strength between two CV systems, the field-field entanglement can be larger than 1 and increases as the initial amplitude of the coherent state grows. By postselecting the fields in their coherent states, the entanglement of two initially unentangled qubits can be largely improved.
vides the fundamental setup for manipulating the qubit-qubit entanglement with coupled CV systems and is applicable for different physical systems.

VI. ACKNOWLEDGEMENT

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