Magnetic Interaction Between Stars And Accretion Disks

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Abstract.
In this review I consider modern theoretical models of coupled star–disk magnetospheres. I discuss a number of models, both stationary and time-dependent, and examine what physical conditions govern the selection of a preferred model.

1. Introduction

In this paper I review recent theoretical progress in our understanding of magnetic interaction between Young Stellar Objects (YSOs), in particular, Classical T-Tauri Stars (CTTSs), and their accretion disks. That such interaction takes place, we have no doubt, as there is now ample observational evidence of strong (of order $10^3$ G) magnetic fields in these systems (e.g., Johns-Krull et al., 1999; Guenther et al., 1999).

Most of the theoretical work on magnetically linked star–disk systems, both analytical and numerical, has focussed on examining the structure and role of a large-scale axisymmetric magnetic field with (at least initially) dipole-like topology (see Fig. 1). This field presumably arises due to the star’s internal magnetic dipole moment. Studying such a large-scale star–disk magnetosphere will also be the focus of this paper. I will thus ignore the effects of any small-scale intermittent loops that may be generated by the turbulent dynamo action in the disk.

While I am restricting myself to only large spatial scales, I will consider a variety of temporal scales. The shortest relevant time-scale is the rotation period, typically a few days for CTTSs; the longest time-scale is the accretion disk life-time, which can be $\sim 10^6$ years or more (Königl, 1991; Kenyon and Hartmann, 1995). Among the models developed to date, there exist a dichotomy with respect to the system’s behavior on the rotation time-scale. More specifically, in some models a direct magnetic connection between the disk and the star is maintained in a stationary configuration, whereas in other models it is not. In this paper I will review both of these classes of models.

Before I proceed, I would like to list some of the most important questions related to the subject of this review:

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1) What physical parameters determine whether a direct star–disk coupling via a large-scale dipole-like magnetic field can be maintained on the rotation-period time-scale?

2) If a quasi-stationary magnetically-coupled configuration does exist, what is its structure and how does it evolve on the longer (e.g., accretion) time-scale?

3) If the magnetic link is disrupted, then what is the non-steady process? Are there periodic or quasi-periodic openings and closings of the field (due to magnetic reconnection) or there is a transition to a wind-supported permanent stationary open-field configuration without the link?

4) Is it possible that both scenarios are possible under different physical circumstances?

5) In either scenario, what are the effects of turbulent viscosity and magnetic diffusivity? And what is the role of winds and jets?

6) What are the implications for the time-variability of the accretion flow and for the angular momentum and energy exchange? What are the observational consequences that would allow one to discriminate between the models?

Although I will not be able to answer all of these questions in this review, I will use them as the main guiding themes in my discussion.

Figure 1. The general geometry of a magnetically-linked star–disk system.
2. Non-stationary models

The study of magnetic interaction between YSOs and their disks has been pioneered by Königl (1991). He has successfully applied the model of Ghosh and Lamb (1978), developed originally for accreting neutron stars, to explain several important observational features of T-Tauri stars, such as their relatively long rotation periods, UV-excesses, and inverse P-Cygni profiles. In this steady-state model, the stellar magnetic field penetrates the disk over a finite range of radii both inside and outside of the corotation radius \( r_{\text{co}} \equiv (GM^*/\Omega^2)^{1/3} \), where \( M^* \) is the mass of the star and \( \Omega^* \) is its angular velocity. The spin-up magnetic torque due to the field lines connecting to the disk inside \( r_{\text{co}} \) is balanced by the spin-down torque by the lines connecting to the disk outside \( r_{\text{co}} \). Königl has proposed that a magnetic field of \( 10^3 \) G at the stellar surface (a value consistent with observations, see Bouvier et al., 2003) can disrupt the disk at a few stellar radii (but well inside the corotation radius) and channel the accretion flow to higher stellar latitudes. He also has estimated the typical time needed to bring the star into the spin equilibrium with the disk to be \( \sim 10^5 \) years, much shorter than the typical accretion time for CTTSs.

Although this model has been very successful in explaining many spectral and variability features, it has not considered the dynamics of the magnetic field itself. The presence of a strong magnetic field has just been inferred, but no equations governing the magnetosphere structure have been solved. It turns out that there exists a very robust mechanism that leads to the breaking of the magnetic link on the rotation-period time-scale. This presents a serious obstacle for all steady-state models and thus gives us the motivation to consider nonstationary models.

The basic idea can be explained as follows. Both the star and the disk are fairly good conductors and so the magnetic field can generally be considered frozen into them. In addition, they rotate with different angular velocities (except at \( r_{\text{co}} \)). Therefore, the field lines are twisted by the differential rotation and toroidal magnetic flux is generated out of the poloidal flux. As the toroidal field builds up, the corresponding field pressure tends to push the field lines outward and inflate them. At first, the poloidal field structure changes very little, but, after the relative star-disk twist angle exceeds one radian or so, the field lines start to expand faster and faster (at an angle of \( \sim 60^\circ \) with respect to the rotation axis) and tend to open up, thereby destroying the magnetic link between the star and the disk.

This opening process is essentially identical to a similar process that have been studied extensively in solar-corona context (e.g., Aly, 1984, 1995; Low, 1977, 2001; Mikić and Linker, 1994; Uzdensky, 2002b).
Indeed, the opening of coronal magnetic arcades, brought about by displacements of the field-line footpoints on the photosphere, is one of the leading mechanisms for Coronal Mass Ejections (Low, 2001). A significant amount of work on this process has also been done in the accretion-disk context. Thus, in the force-free approximation, it has been shown, using both simple analytical and semi-analytical arguments (Aly and Kuijpers, 1990; van Ballegooijen, 1994; Lynden-Bell and Boily, 1994; Uzdensky et al., 2002a; Uzdensky, 2002a; Lynden-Bell, 2003), and via numerical solutions of the force-free Grad–Shafranov equation (Uzdensky et al., 2002a) that such an opening occurs at a finite twist angle (see Uzdensky, 2002b for a review). In addition, full numerical 2D MHD simulations (without the force-free assumption) have demonstrated the opening process at work as a part of the overall cycle (Hayashi et al., 1996, 2000; Goodson et al., 1997, 1999; Goodson and Winglee, 1999; Matt et al., 2002). Thus, at present there is no doubt that, if stellar dipole magnetic field penetrates a conducting disk over a wide range of radii, the twisting of the field lines will open them, thereby breaking the star–disk connection everywhere, with perhaps the exception of the inner disk region where the magnetic field is strong enough to make the matter corotate with the star. This process naturally results in a non-steady behavior, which has lead to the development of a number of time-dependent models. In addition, however, there exist several alternatives, leading to a small number of distinct stationary models. I shall discuss the non-steady models first.

After the field lines expand and effectively open up, a natural question to ask is: what happens next? Currently, the situation is not entirely clear and there is no unique answer to this question. There are two drastically-different possibilities that are most often discussed.

In the first scenario (Lovelace et al., 1995), developed in the neutron star context, once the field lines open, they stay open indefinitely. A steady state is then achieved, although it is very different from the original one, as the magnetic link between the disk and the star has been severed on most of the field lines (see Fig. 2). One can identify three topologically-distinct regions in the magnetosphere: the stellar wind region (region I), where the field lines extend from the star to infinity, the disk wind region (region II), where the field lines return from infinity to the disk, and the closed field region (region III) — the remnant of the linked magnetosphere, where the field enforces corotation of the disk with the star. Thus, the configuration here is stationary, but the magnetic link extends only over a small part of the disk.

In the second scenario, the situation is really time-dependent, with quasi-periodic cycles of field inflation and opening due to twisting followed by the field closing through reconnection and subsequent con-
traction back to the initial state. This picture has been suggested by van Ballegooijen (1994) and has subsequently been studied in extensive numerical simulations by a number of authors (Goodson et al., 1997, 1999; Goodson and Winglee, 1999; Hayashi et al., 2000; Matt et al., 2002). Some recent observational results also seem to favor this point of view (e.g., Bouvier et al., 2003). Let us consider this most interesting scenario in more detail.

First, notice that in the Lovelace et al. (1995) scenario the poloidal magnetic field reverses across the separatrix between regions I and II. This makes the separatrix an obvious prospective site for reconnection. Indeed, the presence of a rather large anomalous or numerical resistivity has routinely lead to reconnection in the numerical simulations by Hayashi et al. (1996, 2000) and by Goodson et al. (1997, 1999) and Matt et al. (2002).

It is also important to realize that most of the toroidal flux, generated in the twisting process, has now been evacuated radially to infinity.

\footnote{We also have to mention that Uzdensky et al. (2002b) were sceptical about the possibility of reconnection, but this is because they had used the van Ballegooijen (1994) self-similar model describing a uniformly-rotating disk. In that model finite-time field-opening occurs without current-sheet formation along the separatrix. In a more realistic case of non-uniform rotation, Uzdensky (2002a) has argued that there will be finite-time partial field opening accompanied by asymptotic thinning of the separatrix current-concentration region, which can be regarded as current-sheet formation. As the current layer becomes thinner and thinner, a reconnection process may be triggered by anomalous resistivity or the Hall effect.}
(since the toroidal flux on an inflated flux tube is concentrated near the tube’s apex). Therefore, magnetic field in both open-field regions is essentially poloidal (it is exactly poloidal in the force-free framework, but some toroidal field may be present in the MHD-wind regime where matter inertia is important). This means that if magnetic reconnection does occur somewhere along the separatrix, the inner newly-reconnected field lines (connected to both the star and the disk) find themselves out of force-free balance: they have very strong poloidal-field tension that tries to pull them back towards the star but almost no toroidal-field pressure. As a result, in the absence of a powerful outgoing wind (see the discussion below), these inner reconnected field lines contract on the alfvénic time-scale. If both reconnection (exhibited as a flare) and the subsequent contraction and relaxation occur quickly enough, then the resulting closed field lines have very little residual twist, similar to the original dipole-like state. This sets the stage for a new cycle. The continuing differential rotation gradually twists the lines up again and the whole sequence of events repeats, with the natural period of the order of the rotation period. As for the other, outer, newly-reconnected field lines, they, together with the apex regions of the field lines that are still expanding somewhere far away, form a toroidal plasmoid (in a sense, a flying spheromak). The closed magnetic flux surfaces comprising such a plasmoid have the shape of tori nested around a circular line (an O-point in poloidal projection). Each plasmoid is not magnetically connected to either the disk or the star and is out of equilibrium; it then just flies away. If the motion of these plasmoids is collimated towards the axis (i.e., if they are flying mostly vertically), then they can feed the jet, providing an explanation for the observed knotty jet structure (Goodson et al., 1999). The plasmoids will be ejected out with the time intervals equal to the opening/closing period. For CTTSs, however, this period is expected to be too short compared with the observed interval between jet knots (Goodson et al., 1999).

Whereas the effective field-opening time is about a fraction of the rotation period, the time between opening and reconnection is not certain. It depends on the intricate details of the reconnection process and, in particular, is intimately related to the so-called reconnection-trigger, or sudden-onset, problem, very well known in studies of flares in solar physics (Priest, 1984). Here is what it means in the context of our problem. As the field lines start to open, one by one, and the current sheet is formed along the separatrix, how long does one wait before reconnection starts? In other words, how much flux is opened before it is reconnected back? For example, one can imagine that reconnection is triggered as soon as the first few field lines have opened; then one will see the ejection of small islands, separated by the time it takes the
critical amount of flux to open (much shorter than the rotation period). Or, in the opposite extreme, it may be that a large portion or all of the flux opens and only long after that reconnection somehow starts; then one will see finite-size plasmoids ejected with the time interval equal to the sum of the opening time (days) and the uncertain time delay before reconnection onset [for example, in the simulations by Goodson et al. (1999) the total cycle period was about a month].

Also not clear is how much flux is reconnected in each event before the reconnection process shuts off. This question is important because it determines the size of the ejected plasmoids. Indeed, it may be that reconnection proceeds until a large fraction of the flux is reconnected, in which case there will be large-amplitude oscillations in all of the system’s parameters (Hayashi et al., 1996, 2000; Goodson et al., 1999). Alternatively, reconnection may stop very quickly after it has begun, and then one will see very small plasmoids ejected but the larger-scale open-field structure will stay intact, as seen in numerical simulations by Fendt (2000). In fact, the model of Lovelace et al. (1995) can be viewed as an extreme manifestation of this latter scenario. Indeed, in this model it has been assumed, without much discussion or argumentation, that there is no reconnection at all. One can actually bring forth some arguments in favor of this point of view.

In general, there is a competition between field-line closing via reconnection and field-line opening by the wind. Reconnection will be stopped if the wind flowing along the open field lines is so strong that any newly-reconnected field lines are swept open by it (B.C. Low, private communication). More specifically, if there were no flows along the unreconnected field lines (i.e., no wind), then a newly reconnected closed field line on the inner side of the reconnection region would contract rapidly, with the field-line apex moving out of the reconnection region towards the star with the poloidal Alfvén velocity $V_{A,\text{pol}}$. However, if there is a background outflow such as a wind, then one has to add the velocity of this outflow. If the latter is larger than $V_{A,\text{pol}}$, then the resulting apex motion will be directed outward, i.e., the field line will open again. Thus, I suggest the following physical criterion for determining when the re-closing of the open field lines via reconnection will occur. I propose that if the prospective reconnection site is located outside of the Alfvén radius (along the separatrix field line), so that the wind there is super-Alfvénic with respect to the reconnecting poloidal field, then everything will be swept outward by the wind and hence reconnection will not take place and the magnetic link will not be re-established. One then will get a helmet-streamer configuration like that of Lovelace et al. (1995). In the opposite case, reconnection will occur
and will lead to the closing of (a portion of) the field lines, leading to a cyclic behavior.

To summarize, the system’s behavior depends on both the reconnection physics (one needs to know where and when reconnection will be triggered) and on the wind physics: one needs to have a model for the wind to determine everything self-consistently.

3. Steady-state models

I shall now switch gears and discuss the few existing steady-state models in which the magnetic link between the star and the disk remains unbroken. First, to maintain the link, one must find a mechanism that could stop the twisting process. One obvious possibility is the toroidal resistive slippage of the field lines with respect to the plasma in the disk. Let us examine this possibility in more detail and see under what conditions it can work.

The situation depends critically on the disk’s effective magnetic diffusivity (which we shall sometimes call the resistivity). Unfortunately, the value of this diffusivity is not very well known (e.g., Bouvier et al., 2003). Nevertheless, one can set a reasonable upper limit by assuming that it is caused by the same turbulence that facilitates angular momentum transport across the disk. Thus, one can set the magnetic diffusivity \( \eta \) to be equal to the Shakura–Sunyaev kinematic viscosity: \( \eta \sim \nu_{\text{turb}} = \alpha c_s h \), where \( c_s \) is the speed of sound, \( h \) is the disk half-thickness, and \( \alpha \simeq 0.01 - 0.1 \). This range of the \( \alpha \)-values is consistent with the results of numerical MHD simulations of the Magneto-Rotational Instability (e.g., Brandenburg et al., 1996; Stone et al., 1996). It is also consistent with the level of MHD turbulence that is necessary for the ejection of disk winds, as follows from the work of Ferreira (1997) combined with the results of Ferro-Fontan and Gomez de Castro (2003).\(^2\) The effective magnetic diffusivity of this kind leads to the toroidal slippage of field lines with respect to the disk with the relative drift velocity \( \Delta\nu_\phi = (\eta/h)|B_\phi/B_z|d \sim \alpha c_s |B_\phi/B_z|d \), where the subscript \( d \) designates the disk’s surface. For a thin disk, \( c_s/v_K \approx h/r \ll 1 \); thus, the slippage velocity is usually much smaller than the differential rotation velocity \( r\Delta\Omega(r) \equiv r(\Omega_K(r) - \Omega_*) \). There are, however, two special circumstances when this is not so. They are very important as they point us toward the ways to get a steady-state

\(^2\) The conditions for launching MHD winds from accretion disks are found to be close to those necessary for the operation of the Magneto-Rotational Instability (Ferro-Fontan and Gomez de Castro, 2003).
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configuration. I would like to stress, however, that both of these circumstances are somewhat unusual and hence the resulting steady states are not very natural. The first of the two schemes is realized when the field lines under consideration are very close to the corotation radius (so that \( \Delta \Omega \ll \Omega_K \)); it provides the conceptual basis for the model developed by Shu et al. (1994a) (see also Shu et al., 1994b; Najita and Shu, 1994; Ostriker and Shu, 1995). The second scheme requires a very large ratio \( |B_\phi/B_z|d \) and provides the basis for the model developed by Bardou and Heyvaerts (1996) and by Agapitou and Papaloizou (2000).

Let us discuss the first scheme first. Usually, its main idea can be readily dismissed because for a steady state to exist globally, it must exist for all the field lines, and in the majority of models most of the magnetic flux crosses the disk a finite distance away from \( r_{\text{co}} \). However, the model developed by Shu et al. (1994a) solves this problem by assuming that almost all of the magnetic flux is “trapped” and concentrated in the so-called X-region, a very close vicinity of the corotation radius (which the authors of the model call the X-point). This model is one of the most promising, well-developed, and sophisticated models and has gained a lot of popularity and observational support (e.g., Johns-Krull and Gafford, 2002) over the last few years. The disk’s magnetosphere consists of three parts (see Fig. 3). Field lines emanating from the inner part of the X-region connect to the star and form the magnetic funnel that directs the accretion flow. A second portion of the field lines also connects to the star but carries no mass flow; it forms what the authors call the dead zone. Finally, the remaining field lines, those emanating from the outermost part of the X-region, are open and carry the wind that plays a key role in removing the excess angular momentum from the disk. In addition, there are some open stellar field lines that extend from the star to infinity. Thus, the general topology of the poloidal magnetic field is similar to the helmet streamer configuration of Lovelace et al. (1995). An important difference however is that in the Lovelace et al. (1995) model, the poloidal flux is spread smoothly over the entire disk surface and the corotation radius plays no special role, whereas in the Shu model the flux is concentrated close to the X-point, with almost no field at \( r > r_{\text{co}} \). Because of that, the differential rotation in the latter model is weak (\( \Delta \Omega \ll \Omega_* \)), and even a small disk resistivity is sufficient to eliminate the twisting and hence to ensure a steady state. In addition, inside of the corotation radius, the disk density drops rapidly as the plasma is uplifted to form the funnel flow; as a result, the corotation with the star is enforced by the strong magnetic field there.

The inner radius of the disk in the Shu model essentially coincides with the X-point [see, however, Ostriker and Shu (1995), who put it at \( r_{\text{in}} = 0.74r_{\text{co}} \)]. As the accreting matter enters the X-region, some
part of it is loaded onto the open field lines and forms the outgoing X-wind. The rest of the plasma gradually diffuses through the magnetic field in the X-region and is loaded onto the funnel-region field lines and then falls onto the star. At the same time, most of the angular momentum of this falling matter is taken away by the magnetic field and is transported back to the inner portion of the disk, while only a small fraction ends up on the star. This provides an effective control mechanism for the star’s spin and suggests a plausible explanation of the relatively-long rotation periods of CTTSs (Shu et al., 1994a).

This very interesting model is not free of its own problems and inconsistencies, however. Thus, for example, it is highly unlikely that the poloidal field on the inner side of the X-region will not diffuse towards the star. Indeed, these field lines are very strongly bent so that there is a highly concentrated current, essentially an equatorial current sheet, between the X-point and the inner edge of the disk [i.e., the kink-point of Ostriker and Shu (1995)]. Any small amount of resistivity will then cause the field to slip inward through this current layer. As for the plasma flow, it will not be able to counter this diffusion because it is in the same direction. Thus, the resistive-MHD Ohm’s law immediately tells us that this configuration cannot be in a steady state.

The Shu model has also been criticized by Hartmann (1997). He pointed out that their solution requires very fine tuning, so that the inner disk radius (determined by the balance between the stellar magnetic field and the accretion flow) is equal to $r_{\text{co}}$ (determined by $\Omega_*$). Hartmann considers this situation unacceptable, citing an example of DR Tau, where accretion rate has been observed to change on the time scale far too short for $\Omega_*$ to adjust. He seems to favor the van Balle-
 gooijen (1994) viewpoint and concludes that “the entire magnetosphere might be a complicated, time dependent structure”. He also emphasizes the importance of magnetospheric reconnection, noting that it “should lead to substantial heating and flare activity” (Hartmann, 1997).

Now let us consider the second possibility for a steady state. In this scenario, the balance between the twisting due to the differential rotation and the turbulent resistive slippage is made possible by a very large ratio of the toroidal to vertical magnetic field at the disk surface, of the order of $\alpha^{-1} r/h \gg 1$. Such high values are usually considered to be unlikely. Indeed, the angle between the field lines and the disk is determined by the entire solution in the magnetosphere and cannot be arbitrary. The density of matter above the disk is typically so low that magnetic forces completely dominate the dynamics there. In this force-free regime, the toroidal field at the disk surface, $B_{\phi,d}$, increases in proportion to the twist at first, but then reaches a maximum and starts to decline during the rapid-expansion phase; it goes to zero as the field approaches the open state. The maximum value of $B_{\phi,d}$, achievable in a force-free magnetosphere, depends sensitively on the way the poloidal magnetic flux is distributed across the disk, that is on the function $\Psi_d(r)$. Usually, as it turns out, this maximum value is of the same order as the vertical magnetic field and hence the minimum angle between the disk and the projection of the magnetic field vector on the $\theta-\phi$ plane is of order one. In this case, the differential rotation cannot produce the required very large values of the disk toroidal field. The primary physical reason for this is that most of the toroidal magnetic flux, which is being continuously generated by twisting, becomes concentrated near the field-line apex (i.e., the farthest from the star point on a field line). As the field expands, it becomes energetically favorable for the toroidal flux to escape to infinity by opening the poloidal field lines. Coming back to the question of the effects of the disk resistivity, we see that, with the disk toroidal field limited by the opening and flux-escape process, the toroidal resistive slippage, even in a turbulent disk, cannot be fast enough to significantly affect the twisting process.

On the other hand, for a certain class of functions $\Psi_d(r)$, the maximum value $|B_{\phi}/B_z|_{d,max}$ of the ratio of the toroidal to vertical field components at the disk surface, allowed by the force-free solution in the magnetosphere, can be large. In particular, if $\Psi_d(r) \sim r^{-n}$, then $|B_{\phi}/B_z|_{d,max} \sim O(1/n)$ in the limit $n \to 0$ (Lynden-Bell and Boily, 1994; Bardou and Heyvaerts, 1996; Agapitou and Papaloizou, 2000; Uzdensky et al., 2002a).

One can then picture the following evolutionary scenario. Let us start with a non-steady cyclic configuration such as that described by Goodson et al. (1997), (1999). During the first part of the cycle, as
the field expands and approaches the open state, the field lines at the disk surface are inclined away from the star [i.e., \((B_r/B_z)_d > 0\)] and hence diffuse a little bit outward.\(^3\) Then, during the second part of the cycle, as the reconnected field contracts back to the nearly potential state, the field lines may be inclined towards the star at the disk surface [i.e., \((B_r/B_z)_d < 0\)]; they will then diffuse inward. (Note also that, for the field lines inside \(r_{co}\), such an inclination is conducive to loading of matter onto the field lines; thus, accretion can take place during this phase of the cycle.) Then, one can ask what happens on a much longer time scale, when the radial diffusion of magnetic field has to be included. Here, two possibilities immediately come to mind.

It may be, as suggested by van Ballegooijen (1994), that the little diffusive displacements will, over time, redistribute the disk’s magnetic flux so that the net displacement over one cycle will become zero. Then, an averaged steady state will be established, i.e., the cycles of field opening, reconnection, and closing will produce no net secular evolution in the magnetic flux distribution. Since the amount of the outward radial displacement depends on the exact moment of reconnection, it follows that the physics of reconnection again plays a crucial role in determining the long-term magnetic flux distribution.

On the other hand, as I have discussed above, if \(\Psi_d(r) \sim r^{-n}\), then \(|B_\phi/B_z|_{d,\text{max}} \sim O(1/n)\) in the limit \(n \to 0\). Thus, it is in principle possible for the system to achieve exact, not time-averaged, steady state if the disk’s flux redistributes in such a way that the corresponding value of \(n \equiv -d \ln \Psi_d/d \ln r\) becomes very small. Since the value of \(|B_\phi/B_z|_d\), necessary for the balance between the differential rotation and toroidal resistive slippage, is inversely proportional to the disk’s effective resistivity, we see that in this case the disk flux distribution \(\Psi_d(r)\) is essentially determined by the resistivity. In the case of Shakura–Sunyaev turbulent resistivity, one can obtain the following upper limit: \(n < C|B_\phi/B_z|_{d,\text{max}}^{-1} \sim \eta/rh \Delta \Omega = O(\alpha h/r) \ll 1\), where \(C\) is a finite number. Note also that for a steady state to be maintained, one must worry not only about the toroidal direction, but also about the radial direction. This requirement gives not just an upper limit, but in fact determines implicitly the entire function \(n(\eta)\), or, more generally, the dependence \(\Psi_d(r)[\eta(r)]\) (Bardou and Heyvaerts, 1996; Agapitou and Papaloizou, 2000). Such a stationary field configuration, possible in principle, is very different from the dipole field; in particular, it leads to a dramatic decrease in the torque between the star and the disk (Agapitou and Papaloizou, 2000).

\(^3\) with the initial velocity of order the rms turbulent velocity in the disk, \(v_{\text{turb}} \sim \alpha c_s\); hence the characteristic radial footpoint displacement over a rotation period scales as \(\Delta r \sim v_{fp} \Delta \Omega^{-1} \sim \alpha c_s/\Delta \Omega \sim \alpha h \Omega K(r)/\Delta \Omega \ll r\).
4. Summary

In conclusion, I would like to give the following approximate list of the major theoretical approaches to the problem of magnetically-coupled star–disk magnetospheres:

1) Very rich non-stationary scenario (Aly and Kuipers, 1990; van Ballegooijen, 1994; Hayashi et al., 1996, 2000; Goodson et al., 1997, 1999; Goodson and Winglee, 1999; Uzdensky et al., 2002a, 2002b; Matt et al., 2002) with cycles of field inflation, opening, reconnection, contraction, and accretion. Both accretion and outflows occur intermittently, with variability on the differential rotation period (or somewhat longer) time-scale. The amplitude of these oscillations (e.g., how much poloidal flux is opened and then reconnected in each cycle) depends strongly on the physics of reconnection and is not very well constrained. For example, the steady-state model of Lovelace et al. (1995) can be considered a limiting case where no reconnection takes place at all, and thus the oscillation amplitude is zero.

2) The steady-state X-wind model of Shu et al. (1994a). The model of Lovelace et al. (1995) can be considered a bridge model between the Goodson and Shu models.

3) Finally, a steady-state closed magnetosphere with the poloidal vertical magnetic field that threads the disk scaling as $B_z(r) \sim r^{-[2+O(\eta)]}$ — models of Bardou and Heyvaerts (1996) and of Agapitou and Papaloizou (2000). These models take into account the field’s radial diffusion in the disk over a long (compared with $\Omega^{-1}$) time scale.

At present, it is apparently too early to select one of these models as the preferred one based on purely theoretical considerations. More rigorous theoretical work, in conjunction with more sophisticated and thorough numerical simulations and comparison with observations, is needed to sort things out.

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