A New Tool to Search for Physics Beyond the Standard Model in $\bar{B} \to D^{*+}\ell^−\bar{\nu}$

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Recent experimental results in $B$ physics from Belle, BaBar and LHCb suggest new physics (NP) in the weak $b \to c$ charged-current and the $b \to s$ neutral-current processes. Here we focus on the charged-current case and specifically on the decay modes $B \to D^{*+}\ell^−\bar{\nu}$ with $\ell = e, \mu$, and $\tau$. The world averages of the ratios $R_D$ and $R_{D_s}$ currently differ from the Standard Model (SM) by $3.8\sigma$ while $\Delta A_{FB} = A_{FB}(B \to D^*\mu\nu) - A_{FB}(B \to D^*e\nu)$ is found to be $4.1\sigma$ away from the SM prediction in an analysis of 2019 Belle data. These intriguing results suggest an urgent need for improved simulation and analysis techniques in $B \to D^*\ell^−\bar{\nu}$ decays. Here we describe a Monte Carlo Event-generator tool based on EVTGEN developed to allow simulation of the NP signatures in $B \to D^*\ell^−\nu$, which arise due to the interference between the SM and NP amplitudes. As a demonstration of the proposed approach, we exhibit some examples of NP couplings that are consistent with current data and could explain the $\Delta A_{FB}$ anomaly in $B \to D^*\ell^−\nu$ while remaining consistent with other constraints. We show that the $\Delta$-type observables such as $\Delta A_{FB}$ and $\Delta S_{5}$ eliminate most QCD uncertainties from form factors and allow for clean measurements of NP. We introduce correlated observables that improve the sensitivity to NP. We discuss prospects for improved observables sensitive to NP couplings with the expected 50 ab$^{-1}$ of Belle II data, which seems to be ideally suited for this class of measurements.

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I. EXECUTIVE SUMMARY

The objective of this paper is to explore the effects of NP couplings in the charged-current semi-leptonic decays of $B$ mesons, rather than the direct production of new particles at the Large Hadron Collider. These decays originate from the underlying quark-level transitions $b \to c\ell^−\bar{\nu}_\ell$, where $\ell = e, \mu$, or $\tau$. At the hadron level they manifest as decays such as $B \to D^{(*)}\ell^−\bar{\nu}_\ell$. In the coming years, the B factories, such as Belle II and hadron $B$ experiments, may conclusively confirm the presence of NP in some of these semi-leptonic $B$ decays.

The charged-current decays $B \to D^{(*)}\tau\bar{\nu}_\tau$ have been measured by the BaBar, Belle and LHCb experiments. Discrepancies with SM predictions of $R_{D^{(*)}} \equiv B(B \to D^{(*)}\tau^−\bar{\nu}_\tau)/B(B \to D^{(*)}\ell^−\bar{\nu}_\ell)\ (\ell = e, \mu)$ [1–10] and $R_{J/\psi} \equiv B(B^+_c \to J/\psi\tau^+\bar{\nu}_\tau)/B(B^+_c \to J/\psi\ell^+\bar{\nu}_\ell)$ [11] have been observed thus far. The SM predictions and the corresponding World-Averaged experimental results from the Heavy Flavor Averaging Group (HFALAV) [12] are shown in Table I. The deviation from the SM in $R_{D}^\ell$ and $R_{D_s}^\ell$ (combined) has a significance of $3.4\sigma$ while that in $R_{J/\psi}$ is $1.7\sigma$ [13]. These measurements suggest the presence of NP that is lepton-flavor universality violating (LFUV) in $b \to c\tau\nu_\tau$ decays.
TABLE I. Measured values of observables that suggest NP in $b \to c \tau \nu$. Measurements presented in this table refer to World Averages (WA). Note that in [15], the most recent lattice data from [16] on $B \to D^* \ell \nu$ form factors were used to obtain the SM prediction for $R_{D^*}^{\ell \nu}$, 0.2586 ± 0.0030.

| Observable | SM Prediction | Measurement (WA) |
|------------|---------------|------------------|
| $R_{D^*}^{\ell \nu}$ | 0.258 ± 0.005 [12] | 0.295 ± 0.011 ± 0.008 [12] |
| $R_{D^*}^{\ell \ell}$ | 0.299 ± 0.003 [12] | 0.340 ± 0.027 ± 0.013 [12] |
| $R_{D^*}^{\mu \nu}$ | 0.283 ± 0.048 [13] | 0.71 ± 0.17 ± 0.18 [11] |
| $R_{D^*}^{\ell \ell}$ | ~1.0 | 1.04 ± 0.05 ± 0.01 [14] |

We will focus on the decay $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ as a laboratory to explore NP effects in $b \to c \ell^- \bar{\nu}_\ell$ transitions. At leading order, the $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ transitions proceed via the SM. However, new interactions can affect these decays at subleading orders. In virtual, the underlying transition is $b \to c \ell X$ where the invisible state $X$ can be a left-handed (LH) neutrino (part of the SM LH doublet of leptons) or a light right-handed (RH) singlet neutrino. Here we will focus on NP scenarios that produce only LH neutrinos in the final state.

Although the theoretical work on NP has concentrated on the semi-leptonic $\tau$ modes, where experimental statistics are limited, attention is now also being paid to the semi-leptonic muon and electron modes where data is plentiful. For example, scaling the Belle results in [17] to Belle II at 50 ab$^{-1}$ we expect a yield of $8 \times 10^6$ events in each of the muon and electron modes. Similarly, scaling the BaBar results in [18] on $B \to D^* \ell \nu$ with a fully reconstructed hadronic tag, we expect $3 \times 10^5$ events with no background.

An additional advantage is that the missing neutrino momentum can be calculated from kinematic constraints of $e^+e^-$ production at the $\Upsilon(4S)$ and the angular distributions can be fully reconstructed. Unlike the $\tau$, which is detected through its decay products, the muon and electron are directly detected in experiment. In contrast, for the semi-leptonic $\bar{B}$ decays to the $\tau$ lepton, the final state contains one or more additional neutrinos from the $\tau$ decay, which complicates the situation. Examining NP in the muon mode is further motivated by the anomalous $(g-2)_\mu$ measurements [19] as well as by the neutral-current LFUV $B$ anomalies in the $b \to s \mu^+ \mu^-$ decays (see for example, Ref. [20]). At first glance, when studying the $B$ anomalies within the framework of an Effective Field Theory (EFT), these anomalies may appear unrelated. However, within an SMEFT framework NP in the $b \to s \mu^+ \mu^-$ transition could imply NP in the $b \to c \mu^- \bar{\nu}_\mu$ decay [21]. In this article, therefore, we will focus on the muon and electron modes, assuming that the electron decay mode is well described by the SM, but NP contributions are allowed in the muon mode.

Although hints for NP have appeared in the ratio of rates such as $R_{D^*\ell\nu}$, establishing NP and diagnosing the type of NP will require examination of deviations from the SM in other observables as well. Several observables can be constructed from a complete differential distribution of events using helicity angles. Fig. 1 shows a schematic definition of the three helicity angles in $\bar{B} \to D^* \to D\pi \ell^- \bar{\nu}_\ell$.

Angular observables are even more interesting as these may provide one or more unambiguous signals for NP. One such angular observable is the forward-backward asymmetry of the charged lepton, $A_{FB}$, which can be reconstructed as the difference between the number of leptons with the lepton’s helicity angle, $\theta_l$ (see Fig. 1), greater and less than $\pi/2$. In Ref. [22], it was shown that NP in the $\mu$ modes can also be detected in the CP-violating triple-product terms in the angular distribution [23, 24].

A non-zero $A_{FB}$ is present in both the muon and electron channels in the SM due to interference between different helicity amplitudes of the virtual $W$ Boson. However, in a $\Delta$-type observable, $\Delta A_{FB} = A_{FB}^\mu - A_{FB}^e$, where one considers the difference between the muon and electron channels, the SM contributions approximately cancel, except for a small residual effect due to the dependence on the muon mass close to its threshold. Furthermore, we find that the observable $\Delta A_{FB}$ has reduced sensitivity to hadronic uncertainties in form factors. Therefore, any deviation from the SM prediction for $\Delta A_{FB}$ is likely due to NP effects. Recently, using the tables of Belle data from Ref. [17], an anomaly in $\Delta A_{FB}$ was reported in Ref. [25]. This could indicate a new signature of LFUV NP.

LFUV NP in the electron and muon sectors is tightly constrained by the measurement of the ratio of rates $R_{D^*\ell\nu}^{\mu\nu} \equiv B(\bar{B} \to D^{(*)} \mu^- \bar{\nu}_\mu)/B(\bar{B} \to D^{(*)} e^- \bar{\nu}_e)$ which is about 5% from unity. We restrict ourselves to NP Scenarios in which a deviation of up to 3% from unity is allowed, which could be tested in the future. Even if the effects of LFUV NP are small in the ratios of decay rates, larger effects may be visible in the angular distributions as functions of $q^2$.

In this paper, we describe a Monte Carlo (MC) Event-generator tool to allow simulations of the NP signatures in $B \to D^* \ell^- \nu$ arising due to the interference between SM and NP amplitudes. We employ our MC tool primarily to study the semi-leptonic decays with a muon and electron in the final state. We assume that the electron decay mode is well described by the SM, but allow for NP contributions in the muon mode. Using this MC tool we generate results for three distinct scenarios with different NP couplings that are consistent with current data and can explain the $\Delta A_{FB}$ anomaly, while remaining consistent with other constraints. Furthermore, using MC simulations we demonstrate that
FIG. 1. Schematic diagram defining various angles in $\bar{B} \to D^* (\to D\pi) \ell^- \bar{\nu}$ decay [22]. We have aligned the coordinate axes so that the decaying $\bar{B}$ meson is at rest at the origin and in this frame the momentum of the $D^*$ meson is oriented along the $z$-axis. Subsequent decays are shown in the rest frames of the corresponding object that is decaying -- $D^* \to D\pi$ is in the rest frame of the $D^*$ and a virtual particle decays into $\ell^- \bar{\nu}$. The polar angles, $\theta^*$ and $\theta_{\ell}$, are respectively defined in these subsequent rest frames, while the azimuthal angle, $\chi$, is defined in the rest frame of the $\bar{B}$ meson.

$\Delta$-type observables, such as $\Delta A_{FB}$ and $\Delta S_5$, eliminate most QCD uncertainties from form factors and allow for clean measurements of NP. We introduce correlated observables that improve the sensitivity to NP. We also discuss prospects for improved observables sensitive to NP couplings with the expected 50 ab$^{-1}$ of Belle II data, which seems to be ideally suited for this class of measurements.

The layout of the remainder of this article is as follows. In Section II, we discuss the theoretical basis of the full angular distribution for $\bar{B} \to D^* (\to D\pi) \ell^- \bar{\nu}$. In Sections III, IV, and V, we present the implementation of our NP MC tool, the signatures of and sensitivity to NP respectively. Section VI describes improvements to be implemented in the future and we conclude in Section VII.

II. THEORY

In the study of NP in charged-current semi-leptonic $B$ decays it is useful to adopt an EFT framework. In an EFT description of the $b \to c\ell^- \bar{\nu}$ decays, one writes down all possible dimension-six four-quark operators at the scale of the $b$-quark mass. The effective Hamiltonian that describes SM and NP effects can be expressed as,

$$H_{\text{eff}} = \frac{G_F V_{cb}}{\sqrt{2}} \left\{ [(1 + g_L) \bar{c} \gamma_\mu (1 - \gamma_5) b + g_R \bar{c} \gamma_\mu (1 + \gamma_5) b] \bar{\mu} \gamma^\alpha (1 - \gamma_5) \nu_\mu \right. $$

$$+ \left[ g_S \bar{c} b + g_P \bar{c} \gamma_5 b \right] \bar{\mu} (1 - \gamma_5) \nu_\mu + g_T \bar{c} \sigma^{\alpha\beta} (1 - \gamma_5) b \bar{\mu} \sigma_{\alpha\beta} (1 - \gamma_5) \nu_\mu \right\} + \text{h.c.} ,$$

(1)

where the factors $g_X$, $X = L, R, S, P$, and $T$, are coupling constants that describe NP effects. As indicated earlier, we have only included LH neutrinos in this EFT, however, we have allowed for both LH and RH NP couplings.

Based on the effective Hamiltonian of Eq. (1), one can express the decay amplitude for the process $\bar{B} \to D^* (\to D\pi) \ell\bar{\nu}$ as [22, 26],

$$\mathcal{M} = \frac{4 G_F V_{cb}}{\sqrt{2}} \left\{ \langle D\pi | \bar{c} \gamma^\mu [(1 + g_L) P_L + g_R P_R] b | \bar{B} \rangle \langle \bar{\ell} \gamma_\mu P_L \nu \rangle 
$$

$$+ \langle D\pi | \bar{c} (g_S P_L + g_R P_R) b | \bar{B} \rangle \langle \bar{\ell} P_L \nu \rangle + g_T \langle D\pi | \bar{c} \sigma^{\alpha\beta} b | \bar{B} \rangle \langle \bar{\ell} \sigma_{\alpha\beta} P_L \nu \rangle \right\},$$

(2)

where $P_{R,L} = (1 \pm \gamma_5)/2$. This decay amplitude contains several hadronic matrix elements that describe the $\bar{B} \to D^* \to D\pi$ transitions through LH and RH scalar and vector currents, as well as a tensor current. The $D^* \to D\pi$
decay is mediated solely by the strong force, so that

$$
(D\pi|D^*(k,\epsilon)) = \epsilon \cdot (p_D - p_\pi),
$$

where $p_{D(\pi)}$ is the four-momentum of the $D(\pi)$, $k = p_D + p_\pi$ is the four-momentum of the $D^*$ and $\epsilon$ is its polarization. Note that these satisfy the on-shell condition $k \cdot \epsilon = 0$.

The remaining parts of the hadronic matrix elements that appear in Eq. (2) are [27]:

$$
\langle D^*(k,\epsilon) | \bar{c} \gamma_\mu b | B(p) \rangle = -i\varepsilon_{\mu\nu\rho\sigma} \epsilon^{\nu} p^{\rho} k^{\sigma} \frac{2V(q^2)}{m_B + m_{D^*}},
$$

$$
\langle D^*(k,\epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(p) \rangle = \epsilon^\mu (m_B + m_{D^*}) A_1(q^2) - (p + k)_{\mu} (\epsilon^\nu q) \frac{A_2(q^2)}{m_B + m_{D^*}} - q_{\mu} (\epsilon^\nu q) \frac{2m_{D^*}}{q^2} [A_3(q^2) - A_0(q^2)],
$$

$$
\langle D^*(k,\epsilon) | \bar{c} \gamma_5 b | B(p) \rangle = -(\epsilon^\nu q) \frac{2m_{D^*}}{m_B + m_e} A_0(q^2),
$$

$$
\langle D^*(k,\epsilon) | \bar{c} \gamma_\mu \gamma_5 b | B(p) \rangle = \varepsilon_{\mu\nu\rho\sigma} \left\{ -\epsilon^{\nu} (p + k)^\rho T_1(q^2) + \epsilon^{\nu} q^\rho \frac{m_B^2 - m_{D^*}^2}{q^2} [T_1(q^2) - T_2(q^2)] + 2 \frac{\epsilon^\nu q^\rho}{q^2} k^\sigma \left[ T_1(q^2) - T_2(q^2) - \frac{q^2}{m_B^2 - m_{D^*}^2} T_3(q^2) \right] \right\}
$$

where $p$ is the four-momentum of the $B$ meson, $q$ represents the four-momentum of the lepton-neutrino pair, while $m_{B(D^*)}$ represents the mass of the $B(D^*)$ meson. Here, $V, A_0, A_1, A_2, A_3, T_1, T_2$ and $T_3$ are the relevant form factors for a $B \to V$ transition. For the Levi-Civita tensor, $\varepsilon_{\mu\nu\rho\sigma}$, we use the convention $\varepsilon_{0123} = +1$.

For easy comparison with similar literature in the field, below we present an alternative notation and its connection to the notation used in this article. Following the presentation in Ref. [25], the effective Lagrangian that describes $b \to c\ell^- \bar{\nu}$ transitions can be written as

$$
\mathcal{L} = -\frac{4G_F}{\sqrt{2}} \sum_i C_i \mathcal{O}_i + h.c.,
$$

where $i = V_L, V_R, S_L, S_R$, and $T$, and $C_i$ represents the Wilson Coefficient (WC) corresponding to the operator $\mathcal{O}_i$. Note the negative sign added to this Lagrangian in order to obtain the correct sign for the SM term (see for example Eq. (20.90) in [28] with errata in [29]). The WC’s can be easily converted into the NP coupling constants that appear in Eq. (1) as follows.

$$
C_{V_L} = 1 + g_L, \quad C_{V_R} = g_R, \quad C_{S_L} = g_S - g_P, \quad C_{S_R} = g_S + g_P, \quad C_T = g_T.
$$

Note that, only $C_{V_L}$ has both SM and NP parts while all other WC’s are NP only. Furthermore, for a $B \to V$ transition, where $V$ denotes a vector meson, the scalar matrix element $\langle V|\bar{q}b|B \rangle = 0$. A consequence of this is that the following condition must be imposed,

$$
C_{S_R} + C_{S_L} = 2 g_S = 0.
$$

Thus, there are only four independent NP parameters that can be used to describe the decay $B \to D^* \ell^- \bar{\nu}$ process, namely $g_L, g_R, g_P$, and $g_T$. We will use these to label the result plots presented in this article.

One can now express the differential decay distribution for $B \to D^* \to D(\to D\pi)\ell^- \bar{\nu}$ as a function of four kinematic variables $-q^2$ and three helicity angles $\theta^*, \theta_\ell$, and $\chi$ (see Fig. 1 for a schematic diagram defining these angles) – in the following form.

$$
\frac{d^4\Gamma}{dq^2 \, d\cos\theta^* \, d\cos\theta_\ell \, d\chi} = \frac{9}{32\pi} \left[ (I_1^* \sin^2\theta^* + I_1^* \cos^2\theta^*) \cos 2\theta_\ell + I_2^* \sin^2\theta^* \cos 2\theta_\ell + I_3^* \sin^2\theta^* \cos 2\theta_\ell + I_4^* \sin^2\theta^* \cos 2\theta_\ell + I_5^* \sin^2\theta^* \cos 2\theta_\ell + I_6^* \sin^2\theta^* \cos 2\theta_\ell + I_7^* \sin^2\theta^* \cos 2\theta_\ell + I_8^* \sin^2\theta^* \cos 2\theta_\ell \right],
$$

where the 12 coefficients $I_i^{(s,c)}(q^2)$ ($i = 1, \ldots, 9$) can be expressed in terms of eight helicity amplitudes that in turn depend on the NP parameters $g_L, g_R, g_P$, and $g_T$. For brevity, the exact dependence of the coefficient functions, $I_i^{(s,c)}$
is given in Appendix A. The distribution for the CP-conjugate process is obtained with the following transformation, $\theta_1 \rightarrow \pi - \theta_1$ and $\chi \rightarrow \pi + \chi$. The various helicity amplitudes transform as $A_{SP} \rightarrow - A_{SP}, A_0 \rightarrow A_0, A_t \rightarrow - A_t, A_{||} \rightarrow A_{||}, A_{\perp} \rightarrow - A_{\perp} (A_{\pm} \rightarrow A_{\mp})$. Note that if one writes, $A = |A|e^{i\phi + i\delta}$ then $\bar{A} = |A|e^{-i\phi + i\delta}$, where $\phi$ is the CP violating weak phase and $\delta$ is the CP conserving strong phase.

The full phase space for the $B \rightarrow D^* \ell^- \bar{\nu}$ decay is obtained by varying the kinematic variables over their allowed ranges which are as follows: $m_f^2 \leq q^2 \leq m_B^2 - m_D^2, 0 \leq \theta_{D^*,t} \leq \pi$, and $0 \leq \chi \leq 2\pi$. One can now construct several observables by integrating the distribution of Eq. (11) over one or more of these kinematic variables. The first of these is the differential decay distribution as a function of $q^2$, constructed by integrating over the full range of allowed values for all three helicity angles.

$$\frac{d\Gamma}{dq^2} = \frac{1}{4} [3 I_1^c - \frac{1}{2} + 2 (3 I_1^c - I_2^c)].$$

Next, one can construct double-differential decay distributions as functions of $q^2$ and one other angle variable at a time, obtained by integrating over the other two angles.

$$\frac{d^2\Gamma}{dq^2 dq^2 \cos \theta^*} = \frac{3}{4} \frac{d\Gamma}{dq^2} \left[ 2 F_L^{D^*} (q^2) \cos^2 \theta^* + F_T^{D^*} (q^2) \sin^2 \theta^* \right],$$

$$\frac{d^2\Gamma}{dq^2 dq^2 \cos \theta_t} = \frac{d\Gamma}{dq^2} \left[ \frac{1}{2} + A_{FB} \cos \theta_t + \frac{1 - 3 \tilde{F}_L^t}{4} \frac{3 \cos^2 \theta_t - 1}{2} \right],$$

$$\frac{d^\Gamma}{dq^2 dq^2 \cos \chi} = \frac{1}{2\pi} \frac{d\Gamma}{dq^2} (1 + S_3 \cos 2\chi + S_9 \sin 2\chi),$$

where $F_L^{D^*} (q^2)$ is the longitudinal (transverse) polarization of the $D^*$, $A_{FB}$ is the charged-lepton forward-backward asymmetry, and $S_9$ is a triple-product asymmetry. The coefficient functions that appear in Eq. (15) can be expressed in terms of the angular coefficients, $I_i^{(s,c)}$, as follows.

$$F_L^{D^*} (q^2) = 1 - F_T^{D^*} (q^2) = \frac{3 I_1^c - I_2^c}{3 I_1^c - I_2^c + 2 (3 I_1^c - I_2^c)},$$

$$A_{FB}(q^2) = \frac{3}{2} \frac{2 I_1^c + I_2^c}{3 I_1^c - I_2^c + 2 (3 I_1^c - I_2^c)},$$

$$\tilde{F}_L^t (q^2) = \frac{I_1^c - 3 I_2^c + 2 (I_1^c - 3 I_2^c)}{3 I_1^c - I_2^c + 2 (3 I_1^c - I_2^c)},$$

$$S_3(q^2) = \frac{4 I_3}{3 I_1^c - I_2^c + 2 (3 I_1^c - I_2^c)},$$

$$S_9(q^2) = \frac{4 I_9}{3 I_1^c - I_2^c + 2 (3 I_1^c - I_2^c)}.$$

Note that there are additional observables that can be extracted from data by performing asymmetric integrals over more than one angles. We discuss some such observables in Section IV.

### III. NEW-PHYSICS IMPLEMENTATION IN EVTGEN

We implement the preceding discussion in the EvtGen MC simulation framework as the BTODSTARLNUNP decay model. This NP generator, BTODSTARLNUNP, can run either in a standalone mode or be integrated into a software framework of a $B$-physics experiment. The model includes SM contributions, various NP parameters as well as their interference. The model takes the NP parameters $\delta V_{t\ell} \equiv g_L, C_{Vt\ell}, C_{StL}, C_{StR},$ and $C_T$ as inputs. The user specifies the NP parameters keeping in mind that the scalar coefficients ($C_{StL}, C_{StR}$) are related to each other by Eq. (10). Each of these parameters can take complex values as inputs and are entered in the user decay file. The default value for each parameter has been set to zero so that when no value is specified for these parameters the code returns SM results. Below we present an example of a user decay file to illustrate the usage of the NP MC generator.

```plaintext
# first argument is cartesian(0) or polar(1) representation of NP coefficients which
# are three consecutive numbers {id, Re(C), Im(C)} or {coeff id, |C|, Arg(C)}
# id==0 \delta V_{VL} -- left-handed vector coefficient change from SM
# id==1 C_{VR} -- right-handed vector coefficient
```
To generate NP the user inputs several arguments in the user decay file. The first of these specifies whether the remaining arguments are to be entered in Cartesian (0) or polar (1) coordinate system. Next, the user enters sets of three values. The first specifies the type of NP coupling ($\delta C_{V_2}, C_{R}, C_{S_L}, C_{S_R}$, and $C_T$), while the second and third represent the real and imaginary parts in Cartesian coordinates, or magnitude and complex phase in polar coordinates. In the above example we have shown how the user can generate events for the SM as well as for a specific NP scenario which in our case is NP scenario 2. A complete version of the NP MC tool with an implementation of the BTODSTARLNUNP decay model can be found in Ref. [30].

IV. SIGNATURES OF NEW PHYSICS

The ratios of branching fractions as well as the differential $q^2$ distributions have limited sensitivity to NP for $b \to c \ell \nu$, $\ell = e, \mu$, which receive tree-level contributions in the SM and are hence unsuppressed. In contrast, angular observables have much better sensitivity to the interference between SM and NP. The optimal sensitivity to NP can be obtained by studying these angular observables as functions of $q^2$. We will examine four angular asymmetries as functions of $q^2$ to make predictions for our NP scenarios, $A_{FB}, S_3, S_5$, and $S_7$. $A_{FB}$ and $S_3$ are previously defined in Section II, while $S_5$ and $S_7$ are the coefficients of $\sin \theta_\ell \sin 2\theta^* \cos \chi$ and $\sin \theta_\ell \sin 2\theta^* \sin \chi$, respectively. These asymmetries can be constructed from the full angular distribution of Eq. (11) through asymmetric integrals shown below.

$$A_{FB}(q^2) = \left( \frac{d\Gamma}{dq^2} \right)^{-1} \int_0^1 \int_{-1}^0 d\cos \theta_\ell \frac{d^2\Gamma}{d\cos \theta_\ell dq^2},$$

(21)

$$S_3(q^2) = \left( \frac{d\Gamma}{dq^2} \right)^{-1} \int_0^{\pi/4} - \int_{\pi/4}^{\pi/2} + \int_{3\pi/4}^{\pi} + \int_{5\pi/4}^{3\pi/2} - \int_{3\pi/2}^{5\pi/4} + \int_{7\pi/4}^{\pi/2} + \int_{7\pi/4}^{\pi/2} d\chi \frac{d^2\Gamma}{dq^2 d\chi},$$

(22)

$$S_5(q^2) = \left( \frac{d\Gamma}{dq^2} \right)^{-1} \int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{3\pi/2}^{2\pi} d\chi \int_0^1 \int_{-1}^0 d\cos \theta^* \frac{d^3\Gamma}{dq^2 d\cos \theta^* d\chi},$$

(23)

$$S_7(q^2) = \left( \frac{d\Gamma}{dq^2} \right)^{-1} \int_0^{\pi/2} - \int_{\pi/2}^{\pi} d\chi \int_0^1 \int_{-1}^0 d\cos \theta^* \frac{d^3\Gamma}{dq^2 d\cos \theta^* d\chi}. $$

(24)

To extract these asymmetries from data, we calculate the integrals in Eqs. (21-24) from binned distributions of the appropriate angular variables. For example, consider $S_5$. This distribution involves asymmetric integrals over both $\cos \theta^*$ and $\chi$. For a given bin of $q^2$, we first divide the events into $\chi$ bins of size $\pi/2$. In each of these bins, we then divide the events into $\cos \theta^*$ bins of size 1. This gives us 8 bins corresponding to the various terms of Eq. (23), which we will label $N_i$ with $i = 1, 2, ..., 8$. To find the value of $S_5$ for a given $q^2$ bin, we then combine the $N_i$'s in the same way as the integrals in Eq. (23), normalized by $\sum_{i=1}^{8} N_i$. 

TABLE II. Angular functions corresponding to angular observables $A_{FB}$, $S_3$, $S_5$, and $S_7$ alongside NP parameters that contribute to each. The dependence on NP parameters has been separated into different orders of $m_l/\sqrt{q^2}$.

| Observable | Angular Function | NP Dependence | $m_l$ suppression order |
|------------|-----------------|---------------|-------------------------|
| $A_{FB}$  | $\cos \theta_\ell$ | $\text{Re}[g_T g_R^*]$ | $O(1)$ |
|           |                  | $\text{Re}[(1+g_L-g_R)(1+g_L+g_R)^*]$ | $O(1)$ |
|           |                  | $\text{Re}[(1+g_L-g_R)g_T^*]$ | $O(m_l/\sqrt{q^2})$ |
|           |                  | $\text{Re}[g_T(1+g_L-g_R)^*]$ | $O(m_L^2/q^2)$ |
|           |                  | $\text{Re}[g_T(1+g_L+g_R)^*]$ | $O(m_L^2/q^2)$ |
| $S_3$     | $\sin^2 \theta^* \sin^2 \theta_\ell \cos 2\chi$ | $|1+g_L+g_R|^2$ | $O(1)$, $O(m_L^2/q^2)$ |
|           |                  | $|g_T|^2$ | $O(1)$ |
| $S_5$     | $\sin 2\theta^* \sin \theta_\ell \cos \chi$ | $\text{Re}[g_T g_R^*]$ | $O(1)$ |
|           |                  | $|1+g_L-g_R|^2$ | $O(1)$, $O(m_L^2/q^2)$ |
|           |                  | $\text{Re}[(1+g_L-g_R)g_T^*]$ | $O(m_l/\sqrt{q^2})$ |
|           |                  | $\text{Re}[g_T(1+g_L-g_R)^*]$ | $O(m_L^2/q^2)$ |
|           |                  | $\text{Re}[g_T(1+g_L+g_R)^*]$ | $O(m_L^2/q^2)$ |
| $S_7$     | $\sin 2\theta^* \sin \theta_\ell \sin \chi$ | $\text{Im}[(1+g_L+g_R)g_T^*]$ | $O(m_l/\sqrt{q^2})$ |
|           |                  | $\text{Im}[(1+g_L-g_R)g_T^*]$ | $O(m_L^2/q^2)$ |
|           |                  | $\text{Im}[(1+g_L-g_R)(1+g_L+g_R)^*]$ | $O(m_L^2/q^2)$ |

When generating our predictions, we used $\Delta A_{FB} = A_{FB}(B \to D^* \mu \nu) - A_{FB}(B \to D^* e \nu)$, $\Delta S_3 = S_3(B \to D^* \mu \nu) - S_3(B \to D^* e \nu)$, and $\Delta S_5 = S_5(B \to D^* \mu \nu) - S_5(B \to D^* e \nu)$, where the electron mode has been generated with the SM only while the muon mode contains both SM and NP contributions. These are $\Delta$-type observables as defined above, which eliminate most of the QCD uncertainties in the form factors, allowing for a clean measurement of LFUV NP. The asymmetry $S_7$ is always zero in the SM, and therefore was not recast into the form of a $\Delta$ observable. The NP dependences of $A_{FB}$, $S_3$, $S_5$, and $S_7$ are given in Table II. Note that these dependencies have different weights, which are dependent on $q^2$. For all theory plots presented in this article, we have only used uncorrelated central values of the FF parameters as listed in Tables V and VI. Unless otherwise stated, we use the CLN parameterization of the hadronic form factors in our predictions.

V. NEW-PHYSICS SENSITIVITY AND RESULTS

The $q^2$ distribution alone has little sensitivity to NP, as shown in Fig. 2. On the other hand, angular asymmetries as functions of $q^2$ are sensitive to NP couplings. In particular, the angular asymmetries in the angle $\theta_\ell$ and $\chi$ can be promising probes of NP as shown in Fig. 2. However, the angular asymmetries remain quite sensitive to form-factor uncertainties. As an example, in Fig. 4, the uncertainty in the predictions for $A_{FB}$ in the SM and with NP with two different form-factor parameterizations is shown. To address this issue we consider differences between angular asymmetries in the muon and electron channels using $\Delta$ observables. As one observes in Fig. 4, using $\Delta A_{FB}$ as an example, the predictions of the $\Delta$ observables are robust against form-factor uncertainties. In the SM the form-factor uncertainties cancel effectively in the $\Delta$ observables while with NP the cancellation is slightly less effective as the NP violates lepton universality.

From our initial scan, we cannot reproduce the experimental-$\Delta A_{FB}$ anomaly with a single NP coupling. Instead, we consider scenarios with several NP couplings. In order to match $\Delta A_{FB}$ from Ref. [25], we require a $g_R$ NP coupling. In order to maintain the LFU BR constraint we also need to add a $g_L$ NP coupling that is comparable to $g_R$. In addition, it is also possible to include a $g_P$ contribution, but in order to satisfy the constraints it must be imaginary. In this section we provide results corresponding to the three distinct NP Scenarios indicated in Table III chosen with the above considerations in mind.

Using the benchmark scenarios above, we show in Fig. 3 the predictions for the $\Delta$ observables. As discussed earlier these observables are sensitive to NP couplings and have much reduced dependence on form-factor uncertainties. In
it is important to measure the ∆ observables as functions of $q_A$ over the full $\theta$ ranges, is divided into twice as many bins. The angular bins are chosen to be sufficiently fine to compare MC data to the theory. The $\cos \theta$ ranges are divided into 15 equal bins, and the $\chi$ range, being twice as large as the $\theta$ ranges, is divided into twice as many bins.

| $g_L$  | $g_R$  | $g_P$ |
|-------|-------|-------|
| Scenario 1: 0.06 | 0.075 | 0.2 i |
| Scenario 2: 0.08 | 0.090 | 0.6 i |
| Scenario 3: 0.07 | 0.075 | 0 |

TABLE III. Values of NP coefficients for three distinct NP scenarios considered in this paper and used for generating the results presented in this section.

the figure, the SM expectations for these quantities are shown using solid black curves. Note that due to lepton mass and helicity effects, $\Delta A_{FB}$ is negative in the low $q^2$ region even in the SM. We find that the SM predictions for $\Delta A_{FB}$ over the full $q^2$ range is $-5.4 \times 10^{-3}$, and with a low-$q^2$ cutoff of 1.14 GeV$^2$ it is $-2.5 \times 10^{-3}$. To optimize sensitivity, it is important to measure the $\Delta$ observables as functions of $q^2$. Furthermore, from Fig. 3 we see that NP couplings produce correlated signatures of deviations from the SM in multiple $\Delta$ observables, such as $\Delta A_{FB}$ and $\Delta S_3$.

In addition to the two $\Delta$ observables, $\Delta A_{FB}$ and $\Delta S_3$, Fig. 3 also shows the $q^2$ dependence of the observable $\Delta S_3$ and $S_7$. $S_7$ represents an angular asymmetry in $\sin \chi$, where $\chi$ is the azimuthal angle between the decay planes. This is a CP-odd triple-product asymmetry, which is predicted to be identically zero in the SM for any $q^2$. We find that NP scenarios with an imaginary $g_P$ are able to produce a small non-zero signal in the $q^2$ distribution of $S_7$ as shown in Fig. 3. The observable $S_3$ is the coefficient of $\cos 2 \chi$ term in the angular distribution and can be extracted using the asymmetric integral defined in Eq. (22). Although $\Delta S_3$ is close to zero in the SM, NP can produce a non-zero $\Delta S_3$ in the $q^2$ range as shown in the lower left plot of Fig. 3.

We note that for the benchmark scenarios described above, we have also checked the constraints from the longitudinal polarization fraction of the $D^*$ meson, $F_L$, and another angular observable $\bar{F}_L$, which are proportional to the coefficients of the $\cos^2 \theta^*$ and $\cos \theta_L$ terms in the angular distribution. These quantities were extracted for the first time by [25] using the binned CP-averaged differential decay distribution data provided by Belle [10]. They obtain a CP-averaged SM prediction for the integrated $\langle \Delta F_L \rangle$ and $\langle \Delta \bar{F}_L \rangle$ to be $(5.43 \pm 0.36) \times 10^{-4}$ and $(-5.20 \pm 0.30) \times 10^{-3}$ respectively. By fitting the data, they also report $\langle \Delta F_L \rangle^{exp} = -0.0065 \pm 0.0059$ and $\langle \Delta \bar{F}_L \rangle^{exp} = -0.0107 \pm 0.0142$. 

FIG. 2. SM predictions for distribution of $B \rightarrow D^* \ell^+ \nu$ events as functions of (clockwise from top left) $q^2$, $\cos \theta^*$, $\chi$, and $\cos \theta_L$ theory shown for the SM (solid black curve) and for NP Scenario 2 (dashed red curve). EvtGen data shown for NP Scenario 2 (solid red histogram). Each plot is fully integrated over three of the four kinematic variables. The $q^2$ range is divided into 23 equal bins, to reflect the expected resolution of experimental measurements. The angular bins are chosen to be sufficiently fine to compare MC data to the theory. The $\cos \theta$ ranges are divided into 15 equal bins, and the $\chi$ range, being twice as large as the $\theta$ ranges, is divided into twice as many bins.
We have verified that our benchmark values satisfy these experimental bounds within a $1\sigma$ confidence interval.

**VI. FUTURE WORK**

Below we present a list of studies that we plan to carry out in the near future.

- Detailed experimental sensitivity studies using full Belle II detector MC simulation, backgrounds, detector efficiencies, etc. will be carried out in the near future.

- The original analysis of Ref. [25] used a single $\Delta A_{FB}$ computed over a large range of $q^2$ (corresponding to $1.0 \leq w \leq 1.5$). Here we consider $\Delta A_{FB}$ and $\Delta S_3$ as functions of $q^2$. In the future we will obtain much improved sensitivity using a 4-dimensional unbinned likelihood fit directly with NP parameters $g_p$, $g_L$, $g_R$, and $g_T$. We will also generate $\Delta$ observables for one-dimensional distributions for comparison.

- Machine learning algorithms, such as neural networks (NNs), can be used in methods to constrain NP parameters in the absence of a known likelihood [31]. Procedures have been recently developed toward this end [31–35]. We will utilize our NP generator to produce MC samples from which $\Delta A_{FB}$ and $\Delta S_3$ as a function of $q^2$ can be extracted. Using this information it is possible to train a NN to distinguish between SM and NP scenarios. These NNs can be built using frameworks such as Keras [36]. The output layer of the NN can then be used in conjunction with methods such as binned template fitting, to help constrain $g_p$, $g_L$, $g_R$, and $g_T$. Comparable methods for constraining NP at the LHC are given in Refs. [37] and [38].

- So far we have investigated New Physics couplings of the pseudoscalar type ($g_p$), and $g_L$, and $g_R$. In the future we will expand our scope to include the tensor coupling $g_T$. 
It is anticipated that the strongest signals of new physics in the charged-current semi-leptonic processes will appear in the $B \rightarrow D^* \tau \nu$ channel. However, this mode is experimentally challenging as the $\tau$ reconstruction is complicated by the presence of multiple neutrinos in the final state. In the future we will investigate signatures of NP in $B \rightarrow D^* \tau \nu$ using this new MC tool to search for most sensitive signatures and $\tau$ decay modes (for example using $\tau \rightarrow \pi \nu$ as discussed in Ref. [26]).
VII. CONCLUSIONS

We have developed a new Monte Carlo New Physics (NP) generator tool for $B \to D^*[\ell\bar{\nu}_\ell]$ with $\ell = \mu, e$ in the EvtGen framework. We used this tool to examine signatures of NP, which are consistent with current data and with the hints of NP in $B \to D^*\mu\nu_\mu$. We assume that the decay $B \to D^*\ell\nu_\ell$ is described by the SM. We found that the angular asymmetries, $A_{FB}, S_5, S_3, S_7$ that can be extracted from the fully reconstructed angular distribution, are sensitive to new physics.

We introduce the $\Delta$ observables, which are obtained by taking the differences between the observables for the muon and the electron modes, to deal with theory uncertainties due to form factors, which might obscure signals of NP. We identify $\Delta_{FB}$, $\Delta_{S_5}$, $\Delta_{S_3}$, and $\Delta_{S_7}$ which are sensitive to new physics.

We also observe that correlated signatures of NP in multiple observables such as $\Delta_{A_{FB}}$ and $\Delta_{S_5}$ are required to confirm the presence of NP (see Fig. 3.)

In this work we also discuss several improvements, both theoretical and experimental, to the MC NP generator, which will be implemented in the future. The NP signatures described here are ideally suited for Belle II at 50 ab$^{-1}$.

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Appendix A: Angular coefficients

The angular distribution of $\bar{B} \to D^* \ell^+ \bar{\nu}_\ell$ presented in Eq. (11) contains 12 coefficients labeled $I_i^{(s,c)}$ with $i = 1, \ldots, 9$. The full list of angular coefficients are presented below as functions of eight helicity amplitudes, $A_{SF}, A_1, A_0, A_{||}, A_\perp, A_{OT}, A_{||,T},$ and $A_{\perp,T}$. These helicity amplitudes depend on hadronic form factors as well as NP coefficients. The form of the eight helicity amplitudes are given in Appendix B.

\begin{align}
I_i^{(s,c)} &= 
\frac{G_F^2}{192\pi^3 m_D^2 q^2} V_{cb}^* |V_{cb}|^2 \left( q^2 - m_D^2 \right) |p_D| B(D^* \to D\pi_i)|\tilde{I}_i^{(s,c)}|, \\
I_1^c &= 4 \left( |A_{SF}|^2 + \frac{m_\ell^2}{q^2} |A_t|^2 \right) + 2 \left( 1 + \frac{m_\ell^2}{q^2} \right) \left( |A_0|^2 + 16 |A_{0,T}|^2 \right) + 8 \frac{m_\ell}{\sqrt{q^2}} \left\{ \text{Re} [A_t A_{SF}^*] - 4 \text{Re} [A_0 A_{0,T}^*] \right\}, \\
I_1^s &= \left\{ \frac{3}{2} (|A_{||}|^2 + |A_\perp|^2) + 8 (|A_{||,T}|^2 + |A_{\perp,T}|^2) \right\} - 16 \frac{m_\ell}{\sqrt{q^2}} \left\{ \text{Re} [A_{||} A_{||,T}^*] + \text{Re} [A_{\perp} A_{\perp,T}^*] \right\} + \frac{m_\ell^2}{q^2} \left\{ \frac{1}{2} (|A_{||}|^2 + |A_\perp|^2) - 24 (|A_{||,T}|^2 + |A_{\perp,T}|^2) \right\}, \\
I_2^c &= -2 \left( 1 - \frac{m_\ell^2}{q^2} \right) \left\{ |A_0|^2 - 16 |A_{0,T}|^2 \right\}, \\
I_2^s &= \frac{1}{2} \left( 1 - \frac{m_\ell^2}{q^2} \right) \left\{ (|A_{||}|^2 + |A_\perp|^2) - 16 (|A_{||,T}|^2 + |A_{\perp,T}|^2) \right\}, \\
I_3 &= - \left( 1 - \frac{m_\ell^2}{q^2} \right) \left\{ (|A_{||}|^2 - |A_\perp|^2) - 16 (|A_{||,T}|^2 - |A_{\perp,T}|^2) \right\}, \\
I_4 &= \sqrt{2} \left( 1 - \frac{m_\ell^2}{q^2} \right) \left\{ 16 \text{Re} [A_{0,T} A_{||,T}^*] - \text{Re} [A_0 A_{||}^*] \right\},
\end{align}
\[ \tilde{I}_5 = 2\sqrt{2} \left\{ (\text{Re} [A_0 A_{\perp}^0] + 4 \text{Re} [A_{||, T} A_{SP}^*]) + \frac{m^2}{q^2} (16 \text{Re} [A_{0, T} A_{\perp, T}^*-] - \text{Re} [A_{||} A_{\perp}^0]) \right. \\
+ \frac{m_t}{\sqrt{q^2}} \left( 4 \text{Re} [A_{||, T} A_{\perp}^0] - 4 \text{Re} [A_0 A_{\perp, T}^0] - 4 \text{Re} [A_{0, T} A_{\perp}^0] - \text{Re} [A_{||} A_{SP}^*] \right) \right\}, \quad (A8) \]
\[ \tilde{I}_6^c = 32 \text{Re} [A_{0, T} A_{SP}^*] + \frac{m_t}{\sqrt{q^2}} \left\{ 32 \text{Re} [A_{0, T} A_{\perp}^0] - 8 \text{Re} [A_0 A_{SP}^*] \right\} - \frac{8 m_t^2}{q^2} \text{Re} [A_0 A_{\perp}^0], \quad (A9) \]
\[ \tilde{I}_6^s = -4 \text{Re} [A_{||} A_{\perp}^0] + 16 \frac{m_t}{\sqrt{q^2}} \left\{ \text{Re} [A_{||} A_{\perp}^0] + \text{Re} [A_{||, T} A_{\perp}^0] \right\} - 64 \frac{m_t^2}{q^2} \text{Re} [A_{||, T} A_{\perp, T}^0], \quad (A10) \]
\[ \tilde{I}_7 = -8\sqrt{2} \text{Im} [A_{SP} A_{\perp, T}^0] - 2\sqrt{2} \text{Im} [A_0 A_{||}^0] + 2\sqrt{2} \frac{m_t^2}{q^2} \text{Im} [A_0 A_{\perp}^0] \\
+ 2\sqrt{2} \frac{m_t}{\sqrt{q^2}} \left\{ 4 \text{Im} [A_{||, T} A_{\perp}^0] - 4 \text{Im} [A_{||} A_{\perp, T}^0] - 4 \text{Im} [A_{||} A_{\perp}^0] - \text{Im} [A_{||} A_{SP}^*] \right\}, \quad (A11) \]
\[ \tilde{I}_8 = -\sqrt{2} \left( 1 - \frac{m_t^2}{q^2} \right) \text{Im} [A_{||} A_{\perp}^0], \quad (A12) \]
\[ \tilde{I}_9 = 2 \left( 1 - \frac{m_t^2}{q^2} \right) \text{Im} [A_{||} A_{\perp}^0], \quad (A13) \]
where \(|p_{D^*}| = \sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)/(2m_B)}\) represents the magnitude of the \(D^*\) 3-momentum, and \(\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca\).

Appendix B: Helicity Amplitudes and Form Factors

The 12 angular coefficients needed to construct the full angular distribution of Eq. (11) were presented in Appendix A. These angular coefficients depend on eight helicity amplitudes that can be further expressed in terms of NP coefficients \((g_P, g_L, g_R,\) and \(g_t)\) and hadronic form factors. We list the helicity amplitudes below [27, 39].

\[ A_{SP} = -g_P \frac{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)}}{m_b + m_c} A_0(q^2), \quad (B1) \]
\[ A_0 = -\frac{(1 + g_L - g_R)(m_B + m_{D^*})}{2m_{D^*}\sqrt{q^2}} \left( (m_B^2 - m_{D^*}^2, -q^2) A_1(q^2) - \frac{\lambda(m_B^2, m_{D^*}^2, q^2)}{(m_B + m_{D^*})^2} A_2(q^2) \right), \quad (B2) \]
\[ A_t = -(1 + g_L - g_R) \frac{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)}}{\sqrt{q^2}} A_0(q^2), \quad (B3) \]
\[ A_{\pm} = (1 + g_L - g_R)(m_B + m_{D^*}) A_1(q^2) \mp (1 + g_L + g_R) \frac{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)}}{m_B + m_{D^*}} V(q^2), \quad (B4) \]
\[ A_{0, T} = \frac{g_T}{2m_{D^*}(m_B^2 - m_{D^*}^2)} \left( (m_B^2 - m_{D^*}^2)(m_B^2 + 3m_{D^*}^2 - q^2) T_2(q^2) - \lambda(m_B^2, m_{D^*}^2, q^2) T_3(q^2) \right), \quad (B5) \]
\[ A_{\pm, T} = g_T \frac{\sqrt{\lambda(m_B^2, m_{D^*}^2, q^2)}}{\sqrt{q^2}} T_1(q^2) \pm (m_B^2 - m_{D^*}^2) T_2(q^2), \quad (B6) \]

The angular coefficients requiring vector and/or tensor type contributions may also require the amplitudes to be expressed in the transversity basis as follows.

\[ A_{||, T} = (A_{+(T)} + A_{-(T)}) / \sqrt{2}, \quad (B7) \]
\[ A_{\perp, T} = (A_{+(T)} - A_{-(T)}) / \sqrt{2}. \quad (B8) \]

The above helicity amplitudes depend on the seven hadronic form factors listed below.

\[ V(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} \sqrt{\lambda} h_V(w(q^2)), \quad (B9) \]
\[ A_1(q^2) = \frac{(m_B + m_{D^*})^2 - q^2}{2\sqrt{m_B m_{D^*}^2}} h_{A_1}(w(q^2)). \quad (B10) \]
\[ A_2(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} \left[ h_{A_2}(w(q^2)) + \frac{m_{D^*}}{m_B} h_{A_3}(w(q^2)) \right], \]  
\[ A_0(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left[ \left( m_B + m_{D^*} \right)^2 - q^2 \right] \frac{1}{2m_{D^*}} h_{A_1}(w(q^2)) \right. 
\left. \quad \quad \quad \quad \quad - \frac{m_B^2 - m_{D^*}^2 + q^2}{2m_B} h_{A_2}(w(q^2)) - \frac{m_B^2 - m_{D^*}^2 - q^2}{2m_B} h_{A_3}(w(q^2)) \right], \]  
\[ T_1(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left( \left( m_B + m_{D^*} \right) h_{T_1}(w(q^2)) - \left( m_B - m_{D^*} \right) h_{T_2}(w(q^2)) \right), \]  
\[ T_2(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left( \left( m_B + m_{D^*} \right)^2 - q^2 \right) \left( \frac{1}{m_B + m_{D^*}} h_{T_1}(w(q^2)) \right. 
\left. \quad \quad \quad \quad \quad - \left( \frac{m_B - m_{D^*}}{m_B} \right)^2 - q^2 \right) \left( \frac{1}{m_B - m_{D^*}} h_{T_2}(w(q^2)) \right), \]  
\[ T_3(q^2) = \frac{1}{2\sqrt{m_B m_{D^*}}} \left( \left( m_B - m_{D^*} \right) h_{T_1}(w(q^2)) - \left( m_B + m_{D^*} \right) h_{T_2}(w(q^2)) \right) 
\left. \quad \quad \quad \quad \quad - 2 \frac{m_B^2 - m_{D^*}^2}{m_B} h_{T_3}(w(q^2)) \right], \]

where the recoil angle, \( w(q^2) \) can be expressed as \( w(q^2) = (m_B^2 + m_{D^*}^2 - q^2)/2m_B m_{D^*} \). The above expressions depend on several lepton and meson masses that are used as input parameters. In our calculations we use the values of meson and lepton masses given in Table IV. We have also used the following values for the quark masses, \( m_b = 5279.63(20) \) GeV and \( m_c = 105.6583745(24) \) GeV.

Note that the above form factors still depend on several additional functions of \( q^2 \), namely \( h_V, h_{A_1}, h_{A_2}, h_{A_3}, h_{T_1}, h_{T_2}, h_{T_3}, R_1, R_2, \) and \( R_3 \). There are several ways of parameterizing these functions using Heavy Quark Effective Theory (HQET). Two such parameterizations are presented in Appendix C.

### Appendix C: Parameterizations of the hadronic form factors

The hadronic form factors described in Appendix B depend on several form factors that appear as functions of \( q^2 \) in HQET. At present there are several ways of parameterizing these functions. Although each parameterization gives a slightly different value for the underlying function, a conclusive identification of the best way to parameterize these functions still eludes us. This problem adds to the theoretical uncertainties associated with the determinations of some of the NP observables discussed in this article.

A commonly used parameterization for the HQET form factors, first presented by Caprini, Lellouch, and Neubert (CLN) in Ref. [41] is given below.

\[ h_V(w) = R_1(w) h_{A_1}(w), \]  
\[ h_{A_2}(w) = \frac{R_2(w) - R_3(w)}{2r_D^*} h_{A_1}(w), \]  
\[ h_{A_3}(w) = \frac{R_2(w) + R_3(w)}{2} h_{A_1}(w), \]  
\[ h_{T_1}(w) = \frac{1}{2(1 + r_D^2 - 2r_D^* w)} \left[ \frac{m_b - m_c}{m_B - m_{D^*}} (1 - r_D^*)^2 (w + 1) h_{A_1}(w) \right. 
\left. \quad \quad \quad \quad \quad - \frac{m_b + m_c}{m_B + m_{D^*}} (1 + r_D^*)^2 (w - 1) h_V(w) \right], \]  

### Table IV

| Masses     | Value (MeV)       |
|------------|-------------------|
| \( m_{D^0} \) | 2010.26(05)       |
| \( m_{D^{*+}} \) | 0.5109989461(31)  |
| \( m_c \)   | 105.6583745(24)   |

TABLE IV. Input values used for meson and lepton masses taken from the Particle Data Group [40]. Numbers in parentheses represent the errors in the last two digits.
$$h_T(w) = \frac{(1-r_D^2)(w+1)}{2(1+r_D^2-2r_D w)} \left[ \frac{m_b-m_c}{m_B-m_D^*} h_{A_1}(w) - \frac{m_b+m_c}{m_B+m_D^*} h_V(w) \right],$$  

(C5)

$$h_{T_2}(w) = -\frac{1}{2(1+r_D^2)(1+r_D^2-2r_D w)} \left[ 2 \frac{m_b-m_c}{m_B-m_D^*} r_D(w+1) h_{A_1}(w) 
+ \frac{m_b-m_c}{m_B-m_D^*} (1+r_D^2) h_{A_2}(w) 
- \frac{m_b+m_c}{m_B+m_D^*} (1+r_D^2) h_V(w) \right],$$  

(C6)

where $r_D = m_D^*/m_B$ and the $w$-dependencies are expressed as

$$h_{A_1}(w) = h_{A_1}(1) \left[ 1 - 8\rho_D^2 z + (53\rho_D^2 - 15)z^2 - (231\rho_D^2 - 91)z^3 \right]$$

(C7)

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2,$$  

(C8)

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2,$$  

(C9)

$$R_3(w) = 1.22 - 0.052(w-1) + 0.026(w-1)^2.$$  

(C10)

The parameter $z$ is related to the recoil angle $w$ through $z(w) = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$. The form factor dependences on $q^2$ are shown in Fig. 5.

| Parameter   | Value               |
|-------------|---------------------|
| $h_{A_1}(1)$ | $0.908 \pm 0.017$   |
| $\rho_D^*  $ | $1.207 \pm 0.026$   |
| $R_1(1)     $ | $1.403 \pm 0.033$   |
| $R_2(1)     $ | $0.854 \pm 0.020$   |

TABLE V. Input values of parameters needed for the CLN parameterization of form factors used here were taken from [27].

Yet another way of parameterizing the HQET form factors is to express them in terms of the leading Isgur-Wise (IW) function $\xi(w)$ [42] and sub-leading IW terms, which represents higher order power corrections to the leading IW function as

$$h_X(w) = \xi(w) \hat{h}_X(w), \quad (X = V, A_1, A_2, A_3, T_1, T_2, T_3)$$  

(C11)

where

$$\hat{h}_X(w) = \hat{h}_{X,0} + \varepsilon_a \delta \hat{h}_{X,\alpha_s} + \varepsilon_b \delta \hat{h}_{X,m_b} + \varepsilon_c \delta \hat{h}_{X,m_c} + \varepsilon_c^2 \delta \hat{h}_{X,m_c^2}.$$  

(C12)

Here, $\varepsilon_a, \varepsilon_b, \varepsilon_c$ denote the expansion coefficients corresponding to the higher order corrections in $\alpha_s$ and $1/m_{b,c}$ respectively which were worked out by [41, 43] using heavy quark symmetry.

The leading term in C12 is

$$\hat{h}_{X,0} = \begin{cases} 1 & \text{for } X = A_1, A_3, T_1, \\ 0 & \text{for } X = A_2, T_2, T_3. \end{cases}$$  

(C13)

The $\alpha_s$ corrections are given as

$$\delta \hat{h}_{V,\alpha_s} = \frac{1}{6z_{cb}(w-w_{cb})} \left[ 4z_{cb}(w-w_{cb})\Omega_w(w) + 2(w+1)((3w-1)z_{cb} - z_{cb}^2 - 1)r_w(w) 
- 12z_{cb}(w-w_{cb}) - (z_{cb}^2-1) \log z_{cb} \right] + V(\mu),$$  

(C14)

$$\delta \hat{h}_{A_1,\alpha_s} = \frac{1}{6z_{cb}(w-w_{cb})} \left[ 4z_{cb}(w-w_{cb})\Omega_w(w) + 2(w-1)((3w+1)z_{cb} - z_{cb}^2 - 1)r_w(w) 
- 12z_{cb}(w-w_{cb}) - (z_{cb}^2-1) \log z_{cb} \right] + V(\mu),$$  

(C15)

$$\delta \hat{h}_{A_2,\alpha_s} = \frac{-1}{6z_{cb}^2(w-w_{cb})^2} \left[ (2 + (2w^2 - 5w - 1)z_{cb} + 2w(2w-1)z_{cb}^2 + (1-w)z_{cb}^3) r_w(w) 
- 2z_{cb}(z_{cb} + 1)(w-w_{cb}) + (z_{cb}^2 - (4w+2)z_{cb} + 3 + 2w)z_{cb} \log z_{cb} \right],$$  

(C16)

$$\delta \hat{h}_{A_3,\alpha_s} = \delta \hat{h}_{A_1,\alpha_s} + \frac{1}{6z_{cb}(w-w_{cb})^2} \left[ 2z_{cb}(z_{cb} + 1)(w_{cb} - w) + (2z_{cb}^3 + 2z_{cb}(2w^2 - 5w - 1) 
- 4w^2 z_{cb}^2 + (4w - 1)z_{cb}^3) \right].$$
\[\delta T_{1,\alpha_0} = \frac{1}{3 z_{cb}(w - w_{cb})} \left[ 2 z_{cb}(w - w_{cb}) \Omega_w(w) + 2 z_{cb}(w^2 - 1) r_w(w) - 6 z_{cb}(w - w_{cb}) \right] \]
\[+ (1 - 2 z_{cb}) \log z_{cb} \right] + T(\mu), \quad (C18)\]
\[\delta \hat{h}_{T_2,\alpha_0} = \frac{w + 1}{3 z_{cb}(w - w_{cb})} \left[ (1 - 2 z_{cb}) r_w(w) + 2 z_{cb} \log z_{cb} \right], \quad (C19)\]
\[\delta \hat{h}_{T_3,\alpha_0} = \frac{1}{3 z_{cb}(w - w_{cb})} \left[ (z_{cb} w - 1) r_w(w) - z_{cb} \log z_{cb} \right], \quad (C20)\]

where

\[z_{cb} = \frac{m_c}{m_b}, \quad w_{cb} = \frac{1}{2} \left( z_{cb} + z_{cb}^{-1} \right) , \quad w_{\pm}(w) = w \pm \sqrt{w^2 - 1}, \quad (C21)\]
\[r_w(w) = \frac{\log w_{\pm}(w)}{\sqrt{w^2 - 1}}, \quad (C22)\]
\[\Omega_w(w) = \frac{w}{2 \sqrt{w^2 - 1}} \left[ 2 \text{Li}_2(1 - w_{-}(w) z_{cb}) - 2 \text{Li}_2(1 - w_{+}(w) z_{cb}) \right. \]
\[+ \text{Li}_2(1 - w_{+}^2(w)) - \text{Li}_2(1 - w_{-}^2(w)) \bigg] - w r_w(w) \log z_{cb} + 1. \quad (C23)\]

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & HQET (3/2/1) & HQET (2/1/0) \\
\hline
$\xi^{(0)}$ & 1 & 1 \\
$\xi^{(1)}$ & $-0.93 \pm 0.10$ & $-1.10 \pm 0.04$ \\
$\xi^{(2)}$ & $+1.35 \pm 0.26$ & $+1.57 \pm 0.10$ \\
$\xi^{(3)}$ & $-2.67 \pm 0.75$ & $-$ \\
\hline
$\lambda^{(0)}_2$ & $-0.05 \pm 0.02$ & $-0.06 \pm 0.02$ \\
$\lambda^{(0)}_2$ & $+0.01 \pm 0.02$ & $-0.06 \pm 0.02$ \\
$\lambda^{(0)}_2$ & $-0.01 \pm 0.02$ & $-$ \\
$\lambda^{(0)}_3$ & 0 & 0 \\
$\lambda^{(0)}_3$ & $-0.05 \pm 0.02$ & $-0.03 \pm 0.01$ \\
$\lambda^{(0)}_3$ & $-0.03 \pm 0.03$ & $-$ \\
\hline
$\eta^{(0)}$ & $+0.74 \pm 0.11$ & $+0.38 \pm 0.06$ \\
$\eta^{(1)}$ & $+0.05 \pm 0.03$ & $+0.08 \pm 0.03$ \\
$\eta^{(2)}$ & $-0.05 \pm 0.05$ & $-$ \\
\hline
& $+0.09 \pm 0.18$ & $+0.50 \pm 0.16$ \\
$\bar{\ell}^{(0)}_1$ & $+1.20 \pm 2.09$ & $-$ \\
$\bar{\ell}^{(1)}_1$ & $-2.29 \pm 0.33$ & $-2.16 \pm 0.29$ \\
$\bar{\ell}^{(0)}_2$ & $-3.66 \pm 1.56$ & $-$ \\
$\bar{\ell}^{(1)}_2$ & $-1.90 \pm 12.4$ & $-1.14 \pm 2.34$ \\
$\bar{\ell}^{(0)}_3$ & $+3.91 \pm 4.35$ & $-$ \\
$\bar{\ell}^{(1)}_3$ & $-2.56 \pm 0.94$ & $+0.82 \pm 0.47$ \\
$\bar{\ell}^{(0)}_4$ & $+1.78 \pm 0.93$ & $-$ \\
$\bar{\ell}^{(1)}_4$ & $+3.96 \pm 1.17$ & $+1.39 \pm 0.43$ \\
$\bar{\ell}^{(0)}_5$ & $+2.10 \pm 1.47$ & $-$ \\
$\bar{\ell}^{(1)}_5$ & $+4.96 \pm 5.76$ & $+0.17 \pm 1.15$ \\
$\bar{\ell}^{(1)}_6$ & $+5.08 \pm 2.97$ & $-$ \\
\hline
\end{tabular}
\caption{Values of input parameters needed for the HQET (3/2/1) and HQET (2/1/0) parameterizations of the hadronic form factors taken from [44].}
\end{table}
FIG. 5. Form factor dependence on $q^2$ for two different FF parametrizations. The shaded band show the region with the 1σ upper and lower limits of the form factor parameters listed in Tables V and VI are considered without any correlation. For the HQET form factors, we show only the 2/1/0 scenario following the analysis presented in Ref. [44].
Here \( \text{Li}_2(x) = \int_0^x dt \log(1-t)/t \) is the dilogarithm function and \( V(\mu), T(\mu) \) are scale factors given as

\[
V(\mu) = -\frac{2}{3} \left( \frac{m_b m_c}{\mu^2} \right) ,
\]

\[
T(\mu) = -\frac{1}{3} \left( \frac{2 m_b m_c}{\mu^2} \right) .
\]

In our calculations we choose the scale \( \mu = 4.2 \text{ GeV} \). The \( 1/m_{b,c} \) corrections in eq. C12 are given as

\[
\delta \hat{h}_{V,m_b} = \hat{L}_1(w) - \hat{L}_4(w) ,
\]

\[
\delta \hat{h}_{V,m_c} = \hat{L}_2(w) - \hat{L}_5(w) ,
\]

\[
\delta \hat{h}_{A_1,m_b} = \hat{L}_1(w) - \frac{w}{w+1} \hat{L}_4(w) ,
\]

\[
\delta \hat{h}_{A_1,m_c} = \hat{L}_2(w) - \frac{w}{w+1} \hat{L}_5(w) ,
\]

\[
\delta \hat{h}_{A_2,m_b} = 0 ,
\]

\[
\delta \hat{h}_{A_2,m_c} = \hat{L}_3(w) + \hat{L}_6(w) ,
\]

\[
\delta \hat{h}_{A_3,m_b} = \hat{L}_1(w) - \hat{L}_4(w) ,
\]

\[
\delta \hat{h}_{A_3,m_c} = \hat{L}_2(w) - \hat{L}_3(w) + \hat{L}_6(w) - \hat{L}_5(w) ,
\]

\[
\delta \hat{h}_{T_1,m_b} = \hat{L}_1(w) ,
\]

\[
\delta \hat{h}_{T_1,m_c} = \hat{L}_2(w) ,
\]

\[
\delta \hat{h}_{T_2,m_b} = -\hat{L}_4(w) ,
\]

\[
\delta \hat{h}_{T_2,m_c} = \hat{L}_5(w) ,
\]

\[
\delta \hat{h}_{T_3,m_b} = 0 ,
\]

\[
\delta \hat{h}_{T_3,m_c} = \frac{1}{2} \left( \hat{L}_6(w) - \hat{L}_3(w) \right) .
\]

where the \( \hat{L}(w) \) functions read

\[
\hat{L}_1(w) = -4(w-1) \hat{\chi}_2(w) + 12 \hat{\chi}_3(w) ,
\]

\[
\hat{L}_2(w) = -4 \hat{\chi}_3(w) ,
\]

\[
\hat{L}_3(w) = 4 \hat{\chi}_2(w) ,
\]

\[
\hat{L}_4(w) = 2 \eta(w) - 1 ,
\]

\[
\hat{L}_5(w) = -1 ,
\]

\[
\hat{L}_6(w) = -2 \left( 1 + \eta(w) \right) \frac{1}{w+1} .
\]

The corrections of order \( 1/m_z^2 \) are included via the subsub-leading reduced IW functions \( \hat{i}_{1-6}(w) \) as [45]

\[
\delta \hat{h}_{V,m_z^2} = \hat{\ell}_2(w) - \hat{\ell}_5(w) ,
\]

\[
\delta \hat{h}_{A_1,m_z^2} = \hat{\ell}_2(w) - \frac{w-1}{w+1} \hat{\ell}_5(w) ,
\]

\[
\delta \hat{h}_{A_2,m_z^2} = \hat{\ell}_3(w) + \hat{\ell}_6(w) ,
\]

\[
\delta \hat{h}_{A_3,m_z^2} = \hat{\ell}_2(w) - \hat{\ell}_3(w) - \hat{\ell}_5(w) + \hat{\ell}_6(w) ,
\]

\[
\delta \hat{h}_{T_1,m_z^2} = \hat{\ell}_2(w) ,
\]

\[
\delta \hat{h}_{T_2,m_z^2} = \hat{\ell}_5(w) ,
\]

\[
\delta \hat{h}_{T_3,m_z^2} = \frac{1}{2} \left( \hat{\ell}_3(w) - \hat{\ell}_6(w) \right) .
\]
The IW functions are expressed, in general, as expansions about \( w = 1 \) as

\[
f(w) = \sum_{n=0} \frac{f(n)}{n!} (w-1)^n
\]

with \( f = \xi, \eta, \chi_2, \chi_3 \) and \( \hat{\ell}_i \). One can further relate the kinematic variable \( w \) with the expansion variable \( z \) as

\[
w(z) = 2 \left( \frac{1+z}{1-z} \right)^2 - 1.
\]

One can then expand the IW functions up to any order in \( z \) as

\[
f(w) = f(0) + 8f(1)z + 16(f(1) + 2f(2))z^2 + \frac{8}{3}(9f(1) + 48f(2) + 32f(3))z^3 + \ldots (\text{higher orders}).
\]

The fitted value of the parameters for the above two scenarios from [44] are given in Table VI.
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