This talk discusses the power behaved corrections to fragmentation functions for the current jet in non-singlet deep inelastic scattering. These corrections are estimated by means of a renormalon model using a dispersive approach. The assumption of a universal infrared finite coupling enables us to provide quantitative estimates.

1 Introduction

The experimental study of final state, single hadron, momentum distributions is now being carried out at HERA. Like the energy distribution in $e^+e^-$, this quantity naturally depends on the parton to hadron fragmentation functions. While it is not possible to calculate fragmentation functions within perturbation theory, one can study their evolution with $Q^2$ (the logarithmic scaling violation) perturbatively and this enables us to extract $\alpha_s$ from the data. However, reliance on a purely perturbative approach still causes problems as it does not account for higher twist pieces. Though these are suppressed by powers of a large scale, their functional dependence on the phase space parameters generally leads to sizable effects, even comparable in some cases to the size of the NLO term of the corresponding perturbative estimate. This would naturally mean that one must account for these pieces before any comparison with data is made.

The most reliable way in which one can model these effects is the renormalon approach. Several papers have been written on these techniques and their applications to structure functions, fragmentation functions in $e^+e^-$, and event shape variables. The phenomenological success has been encouraging enough to continue applying these techniques.

For details of the renormalon technique we can refer to the review talks in these proceedings. For our purposes we provide a very brief introduction to the dispersive approach in section...
2. In section 3 we define the observables considered and explain our notation. The calculations are described in section 4 and finally quantitative results are presented in the form of a figure.

2 The Dispersive Approach

Our estimate of the leading power corrections to the perturbative results are based on the approach of reference 8. Non-perturbative effects at long distances are assumed to give rise to a modification $\delta \alpha_{\text{eff}}(\mu^2)$ in the QCD effective coupling at low values of the scale $\mu^2$. The effect on some observable $F$ is then given by a characteristic function $\mathcal{F}(x, \epsilon)$ as follows:

$$
\delta F(x, Q^2) = \int_0^{\infty} \frac{d\mu^2}{\mu^2} \delta \alpha_{\text{eff}}(\mu^2) \mathcal{F}(x, \epsilon = \mu^2/Q^2) \tag{1}
$$

where

$$
\mathcal{F}(x, \epsilon) = -\epsilon \frac{\partial}{\partial \epsilon} \mathcal{F}(x, \epsilon) \tag{2}
$$

The characteristic function is obtained by computing the relevant one-loop graphs with a non-zero gluon mass $\mu$.

Arbitrary finite modifications of the effective coupling at low scales would generally introduce power corrections of the form $1/k^2_p$ into the ultraviolet behaviour of the running coupling itself. Such a modification would destroy the basis of the operator product expansion. This leads to the constraint that only terms in the small-$\epsilon$ behaviour of the characteristic function that are non-analytic at $\epsilon = 0$ will lead to power behaved non-perturbative contributions. In fact we can choose to express our results only in terms of $\alpha_s$ without the need to explicitly introduce $\alpha_{\text{eff}}$.

Then the results can be expressed in terms of ordinary rather than the logarithmic moment integrals of the coupling. The relevant moment integral in our case (or more generally for any $1/Q^2$ correction) is

$$
A_2 = \frac{C_F}{2\pi} \int_0^{\infty} \frac{d\mu^2}{\mu^2} \mu^2 \delta \alpha_s(\mu^2) \tag{3}
$$

3 Fragmentation in DIS

For the usual reasons (absence of complications due to the remnant jet, similarity with $e^+e^-$) one chooses to work only in the current hemisphere of the Breit frame. For more details regarding the Breit frame the reader is referred to 9 and references therein. Since we wish to include only particles in the current hemisphere, we define the fragmentation function $F^h$ for a given hadron species as a function of the variable $z = 2p_h.q/Q^2$, which measures the fraction of the hadron’s momentum along the current direction and takes values $0 < z < 1$ in the current hemisphere.

The observable we wish to study is then given by

$$
F^h(z, x, Q^2) = \frac{d^3\sigma^h}{dx dQ^2 dz} / \frac{d^2\sigma^h}{dx dQ^2} \tag{4}
$$

where $x = Q^2/(P.q)$ if $P$ is the incoming hadron momentum. The denominator of this expression is the fully inclusive deep-inelastic cross section

$$
\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{Q^4} \left\{ \left[1 + (1 - y)^2\right] F_T(x) + 2(1 - y)F_L(x) \right\} \tag{5}
$$

where $F_T(x) = 2F_1(x)$ and $F_L(x) = F_2(x)/x - 2F_1(x)$ are the usual transverse and longitudinal structure functions. The numerator is given by

$$
\frac{d^3\sigma^h}{dx dQ^2 dz} = \frac{2\pi\alpha^2}{Q^4} \left\{ \left[1 + (1 - y)^2\right] F_T^h(x, z) + 2(1 - y)F_L^h(x, z) \right\} \tag{6}
$$
where $F^h_i(x, z) = 2F^h_i(x, z)$ and $F^h_i(x, z) = F^h_i(x, z)/x - 2F^h_i(x, z)$ are generalized transverse and longitudinal structure functions. The parton model yields $F^h_L(x, z) = F_L(x) = 0$ and

$$F^h_L(x, z) = \sum_q e_q^2[\alpha(x)D^h_q(z) + \bar{q}(x)D^h_q(z)],$$  

while

$$F^h_L(x) = \sum_q e_q^2[\alpha(x) + \bar{q}(x)] = f(x)$$

where $D^h_q$ and $D^h_{\bar{q}}$ are the quark and antiquark fragmentation functions and $q(x), \bar{q}(x)$ are the corresponding parton distribution functions. The $O(\alpha_s)$ result for the numerator in (4) can be expressed in terms of

$$F^h_i(x, z) = \sum_q e_q^2 \int_x^1 d\xi \int_\zeta^1 d\zeta \times \left\{ K_{i,qq}(\xi, \zeta)q(x/\xi)D^h_q(z/\zeta) + K_{i,qg}(\xi, \zeta)q(x/\xi)D^h_g(z/\zeta) + K_{i,gg}(\xi, \zeta)g(x/\xi)D^h_g(z/\zeta) \right\}$$

where the $F^h_i$ denote the longitudinal or transverse pieces that appear in the numerator on the RHS of (4). In the $K_{i,qq}$ for instance, the $i$ stands for the longitudinal or transverse contribution according to what is required and the double suffix denotes the contribution from an incoming as well as a fragmenting quark/anti-quark. Similarly the other terms have corresponding suffixes that represent incoming gluons (photon-gluon fusion) and fragmenting quarks/anti-quarks or incoming quarks/anti-quarks and fragmenting gluons. The variable $\zeta$ denotes the longitudinal momentum of the partons while $\xi = Q^2/(2p\cdot q)$ with $p$ the incoming parton momentum. In terms of the squared matrix elements

$$K_{i,qq}(\xi, \zeta) = \frac{\alpha_s}{2\pi} C_F C_{i,q}(\xi, \xi - \xi \zeta)$$

and

$$K_{i,qg}(\xi, \zeta) = \frac{\alpha_s}{2\pi} T_R C_{i,g}(\xi, \xi - \xi \zeta).$$

while

$$K_{i,gg}(\xi, \zeta) = \frac{\alpha_s}{2\pi} C_F C_{i,q}(\xi, 1 - \xi + \zeta \xi)$$

where the $C_i$ represent the matrix elements squared for scattering off incoming quarks or gluons as denoted by the suffixes ($q$ or $g$). It is emphasised that we do not compute the contribution with incoming gluons here. It is expected that for the kinematic range (small $x$ values) at HERA the above could be an important effect while perhaps it is not too significant for fixed target experiments.

### 4 Power corrections

As mentioned earlier we calculate the power corrections based on the dispersive approach. This means computing the first order correction with a finite gluon mass $\epsilon$. The coefficient of the $\epsilon \log \epsilon$ term then corresponds to the coefficient of the $1/Q^2$ power behaved term. Computing with a massive gluon matrix element and phase space and then taking double moments with respect to $x$ and $z$ we can express the convolution in that equation as a product involving double moments of the $K$ functions and single moments of parton distributions and fragmentation functions. Then keeping only the leading non analytic terms in $\epsilon$ one finds
The $K$ functions differ from the corresponding $K$ functions we had in \cite{2} only by a factor which we choose to include in the definition of our phenomenological parameter $\Delta_2$. Here we have included the virtual contribution calculated in \cite{1}. The expression given above for the gluon fragmentation contribution $\tilde{K}_{T,gg}$ is valid only for $M > 2$. There is an infrared divergence at $M = 0$, because we integrate the real gluon contribution over one hemisphere only, which does not suffice to cancel the divergent virtual contribution at $\zeta = 0$. For $M = 1$ there is a contribution of $8\sqrt{\epsilon}$ instead of an $\epsilon \ln \epsilon$ term, implying a $1/Q^2$-correction to this moment of the gluon fragmentation function, as is the case in $e^+e^-$ annihilation. For $M = 2$ the $1$ becomes $-1$ in the coefficient of $\epsilon \ln \epsilon$. All of these changes represent extra contributions at the point $\zeta = 0$, which we can ignore because the fragmentation function at any finite $z$ depends only on the behaviour at $\zeta > z > 0$.

The $\ln \epsilon$ terms generate the logarithmic scaling violations in the structure and fragmentation functions while the $\epsilon \ln \epsilon$ terms give rise to $1/Q^2$ power corrections, as mentioned earlier. We also need to consider power corrections in the denominator of \cite{2} and the results for this are taken from an earlier publication \cite{1}.

### 5 Results and conclusions

We assume in what follows that the quark fragmentation function is independent of flavour and that $D_q = D_\bar{q}$ which should be reasonable if one confines the discussion to the light quarks and if we sum over the fragmentation into all charged particles. One can combine the leading order results \cite{2} with the power corrections and express the results as

\[
F^h(z; x/Q^2) = D_q(z) + \frac{A_T}{Q^2 f(x)} \int_0^1 d\xi \int_0^1 d\zeta \frac{\xi}{\zeta} F(x/\xi) \{ [H_{T,qq}(\xi, \zeta) - H_{T,gg}(\xi, \zeta)] D_q(z/\zeta) + H_{T,gg}(\xi, \zeta) D_g(z/\zeta) \} ,
\]

where $f(x)$ is the charge-weighted parton distribution and $H_{T,q}(\xi)$ is the higher-twist coefficient function for the transverse structure function \cite{1}

\[
H_{T,q}(\xi) = \frac{4}{(1 - \xi)_+} - 2 - 4\xi + 4\delta(1 - \xi) + \delta'(1 - \xi) .
\]

*Note that the definition here differs from that in Refs.\cite{2} by a factor of $-1/\xi$. 
The negative contribution from the structure function comes about because of our normalization to the structure functions in equation (4). Also the longitudinal contribution is identical between the numerator and denominator of equation (4) which means it does not appear in the final result. We represent our final result as

\[ F^h(z; x, Q^2) = D_q(z) \left( 1 + \frac{A_2}{Q^2} H(z; x) \right) \]  

(18)

We plot the function \( H(z, x) \) as a function of \( z \) for different values of \( x \).

For the plot we use the ALEPH\textsuperscript{10} parametrizations of the light quark and gluon fragmentation functions for charged hadrons at \( Q = 22 \) GeV, and the corresponding MRST (central gluon)\textsuperscript{11} parton distributions. Thus the predictions are at \( Q^2 = 484 \) GeV\(^2\), but \( H(z; x) \) depends only weakly (logarithmically) on \( Q^2 \), and in any case our method is not reliable at the level of logarithmic variations. Results become insensitive to \( x \) below the values shown in Fig.1. Recall, however, that we have not computed the singlet contribution, which may well be important at low \( x \) because of the increase in the gluon distribution there.

The predicted power corrections are qualitatively similar to those for fragmentation functions in \( e^+e^- \) annihilation\textsuperscript{4}, though somewhat larger in magnitude. Part of the increase comes from the negative higher-twist correction to the transverse structure function in the denominator of Eq.(4). The contribution from gluon fragmentation, although subject to further corrections\textsuperscript{9} is estimated to be relatively small for \( z > 0.2 \).

The last point that needs to be made is that inclusion of electroweak effects does not change
the qualitative situation and in fact just means redefining the function $f(x)$ to include electroweak couplings rather than just the quark charges.

Acknowledgments

The work presented here was carried out in collaboration with G.E.Smye and B.R.Webber. I would like to thank Trinity College, the University of Cambridge and the organizing committee of Moriond QCD for financial support.

References

1. ZEUS Collaboration, M.Derrick et al., Z. Phys. C67, 1995, 93; H1 Collaboration, S.Aid et al., Nucl. Phys. B445, 1995, 3. C.Adloff et al., Nucl. Phys. B504, 1997, 3.
2. M.Dasgupta and B.R.Webber, Phys. Lett. B382, 1996, 273.
3. E.Stein, M.Meyer-Hermann, L.Mankiewicz and A.Schafer, Phys. Lett. B376, 1996, 177; M.Meyer-Hermann, M. Maul, L.Mankiewicz, E.Stein and A.Schafer, Phys. Lett. B383, 1996, 463.
4. M.Dasgupta and B.R.Webber, Nucl. Phys. B484, 1997, 247.
5. Yu.L.Dokshitzer, A.Lucenti, G.Marchesini and G.P.Salam, Milan preprint, IFUM-601-FT.
6. M.Dasgupta and B.R.Webber, Eur.Phys.J. C 1, 1998, 539.
7. V.M.Braun, these proceedings; G.Sterman, these proceedings;
8. Yu.L.Dokshitzer, G.Marchesini and B.R.Webber, Nucl. Phys. B469, 1996, 93.
9. M.Dasgupta and B.R Webber, JHEP04, 1998, 017
10. ALEPH Collaboration, D.Buskulic et al., Phys. Lett. B357, 1995, 487; ibid 364, 1995, 247 (E).
11. A.D.Martin, R.G.Roberts, W.J. Stirling and R.S.Thorne, Univ.Durham preprint DTP/98/10 (1998).