One of the approaches to the concrete strength mathematical model

V M Nikonorov¹* and V V Nikonorov²

¹Higher School of Management and Business, Peter the Great St.Petersburg Polytechnic University, Polytechnicheskaya St., 29, St. Petersburg, 195251, Russia
²Department of Roads, Bridges and Tunnels, St. Petersburg State University of Architecture and Construction, 2 nd Krasnoarmeyskaya St., 4, St. Petersburg, 190005, Russia

E-mail: nikanorv@mail.ru

Abstract. The importance of concrete as a structural material is undeniable. The key indicator of concrete as a construction material is strength. Creating and solving a concrete strength mathematical model will allow optimizing raw material costs, increasing the financial result of concrete production, and managing the concrete strength at the stage of its production. The authors reviewed existing concrete strength mathematical models. In the considered models, the concrete strength was influenced by various factors – water-cement ratio, C3S hardness, etc. The authors used the kinetic theory of solid destruction as the basis for their mathematical model of concrete strength. The process of destruction of the sample is considered without deformation. Applying the concept of strained and broken bonds, the authors constructed and solved a concrete strength mathematical model. The main parameters of the model are the speed formation coefficient of the stressed bonds and the speed formation coefficient of destroyed bonds. The model allows estimating the time required for sample destruction at a given load. This is a non-trivial conclusion.

1. Introduction

Relevance. Concrete is a key material in construction. Reasons: significant reserves of raw materials, relatively low production costs; the ability to give the desired shape to the finished product; coupling with metal. The significance of concrete (reinforced concrete) for construction is not in doubt. The most important indicator of a structural material is strength. Accordingly, the concrete strength mathematical model will allow highlighting the factors that affect the strength and the degree of influence of each factor separately. This, in turn, will lead to more precise management of the required concrete strength and possible savings in the cost of concrete production.

The research object is concrete.

The research subject is the concrete strength.

The research aim is to construct and solve a concrete strength mathematical model.

Studies of concrete strength are presented in [1,2,3,4,5].

“Concrete: an artificial stone-like building material obtained by forming and hardening a rationally selected and compacted concrete mixture” [6].

The formula for the compressive strength of concrete [7].

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There are concrete strength mathematical models in the form of a multiple nonlinear regression equation (a 2nd degree polynomial).

\[
S = \alpha \frac{F}{A} K_w
\]  

(1)

- \(S\) – axial compressive strength of concrete, MPa;
- \(F\) – breaking load, N;
- \(A\) – sample area, mm\(^2\);
- \(\alpha\) – scale coefficient;
- \(K_w\) – correction coefficient (for cellular concrete).

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In these models, strength is a function of the volume fraction of the cement stone, the true water-cement ratio (total water-cement ratio), and so on.

The classical Abrams model is well known [8].

\[
S = k \frac{V_v}{A^x} = \frac{V_v}{V_t} \alpha
\]  

(3)

- \(k\) – strength coefficient (cement activity);
- \(A\) – constant;
- \(V_v\) – volume of the shutting water, m\(^3\);
- \(V_t\) – volume of cement \((\rho=1500\text{kg/m}^3)\), m\(^3\);
- \(x\) – water-cement ratio.

Later, after a series of transformations under the hands of researchers, this formula was transformed into the M. Bolomey’s formula. [9]

\[
S = K \left(\frac{1}{x} - \frac{1}{2}\right)
\]  

(4)

- \(K\) – constant.

In [10] the concrete strength is a polynomial of the 2nd degree of \(x\) (water-cement ratio)

\[
S = 50.525 - 2.915x - 0.975x^2.
\]  

(5)

The model of the dependence of concrete strength on porosity \(P\) is known [11].

\[
S = S_0^{-4.84P}.
\]  

(6)

In [12], the time dependence is proposed for predicting the concrete strength without waiting for 28 days

\[
S = \alpha \ln t + \beta
\]  

(7)

- \(\alpha, \beta\) – constants.

The origins of formula (7) can be seen in Sh. Popovich’s work [13].

\[
S = c_1 \log t + c_2
\]  

(8)

- \(c_1, c_2\) – constants that depend on the type of cement, concrete curing, testing, etc.

Also Sh. Popovich gives the formula for the hardness of alite C3S (the main component of concrete) (9). Alite is a tricalcium silicate 3CaO x SiO2. In the first 28 days of hardening, alit makes the main contribution to the concrete strength.
\[
\frac{d^2 S_1}{dt^2} = a_1^2 (s_0 - s_1)
\]

\(S_1\) – hardness C3S in cement paste;
\(S_0\) – hardness C3S at the initial time;
\(a_1^2\) constant, depending on the fineness of the grinding and the cement composition, etc.

[14] provides an interpretation of formula (9), which is extended to the concrete strength as such:

\[
\frac{dS}{dt} = K (S_{lim} + S_0 - S)
\]

\(K = a_1 \exp\left(-\frac{a_2}{t}\right)\)

\(S\) – concrete strength, MPa;
\(S_{lim}\) – limit value \(S\);
\(S_0\) – initial value \(S\);
\(K\) – depends on the product temperature.

2. Methods
The main hypothesis of the research: to build a concrete strength mathematical model, the kinetic theory of solid destruction is applied. We will consider the process of destruction without taking into account the deformation. We start from the strained and broken bonds in the concrete sample. Strained bonds occur in a concrete sample when a destructive load \(F\) is applied. Because of the concrete heterogeneity (filler, cement stone), local overvoltage occur, bonds are torn. We assume that in the process of destruction, strained connections turn into broken ones. Let us apply mathematical modeling methods: we create a concrete strength mathematical model.

Model assumptions.

\[
\frac{dN_t}{dt} = \gamma N_t
\]

\[
\frac{dN_b}{dt} = \phi (N_t - N_b)
\]

\(N_t = \chi F\)

\(N_b (t = 0) = 0; N_b (t = T) = \lambda V\)

where \(N_t\) – the number of strained bonds;
\(N_b\) – number of broken bonds;
\(\gamma\) – speed formation coefficient of strained bonds;
\(\phi\) – speed formation coefficient of broken bonds;
\(\chi\) – the proportionality coefficient between the applied load \(F\) and the number of strained bonds;
\(\lambda\) – the constant that defines the number of bonds per volume unit.
\(V\) – sample volume.

(12) – the rate of increase in the number of strained bonds is proportional to the number of strained bonds. Their number increases in time with the action \(F\).

(13) – the rate of increase in the number of broken bonds is proportional to the difference between the strained and broken bonds.

(14) – the number of strained bonds is proportional to \(F\).

(15) – in the initial moment of time the number of broken bonds is equal to zero. At finite time \(T\), the number of broken bonds is proportional to the sample volume, and all the bonds become broken. It is assumed that there was a complete destruction of the sample.
(12)-(15) – a concrete strength mathematical model on the basis of kinetic theory. The type of mathematical model indicates that (12) and (13) can be reduced to a linear homogeneous 2nd order differential equation with constant coefficients (hereinafter – HDE). The method of solving such an equation becomes clear – calculating the roots of the characteristic equation.

3. Results
We introduce the symbols:
\[ N_t = x; N_b = y \] \hspace{1cm} (16)

Then (12) and (13) we write in the form
\[ \frac{dx}{dt} = \gamma x \] \hspace{1cm} (12’)
\[ \frac{dy}{dt} = \varphi(x - y) \] \hspace{1cm} (13’)

After transformations, the system (11)-(12) is reduced to the form
\[ y'' + y(\varphi - \gamma) - \gamma \varphi y = 0 \] \hspace{1cm} (17)

We obtained a linear homogeneous differential equation with constant coefficients. As the initial conditions are set, we have the Cauchy problem. Accordingly, the type (17) and the presence of initial conditions indicate that the solution is unique. Roots of the characteristic equation
\[ A = \frac{\gamma - \varphi}{2}; B = \frac{\gamma - \varphi}{2} \left( 1 - \sqrt{1 + \frac{4 \gamma \varphi}{(\varphi - \gamma)^2}} \right) \]
\[ \alpha = \frac{\gamma - \varphi}{2}; \beta = \sqrt{1 + \frac{4 \gamma \varphi}{(\varphi - \gamma)^2}} \]

Solution of the HDE we write in the form
\[ N_b = C_1 e^{\alpha t} + C_2 e^{\beta t} \] \hspace{1cm} (18)

From (15)
\[ N_b(t=0) = C_1 + C_2 = 0 \]
\[ N_b(t=T) = C_1 e^{\alpha T} + C_2 e^{\beta T} = \lambda V \]
\[ C_1 + C_2 = 0; C_1 = -C_2 = C \] \hspace{1cm} (19)
\[ \lambda V = \frac{e^{\alpha T} - e^{\beta T}}{e^{\alpha T} - e^{\beta T}} \] \hspace{1cm} (20)
\[ N_b = \frac{\lambda V}{\alpha - \beta} \left( e^{\alpha T} - e^{\beta T} \right) \] \hspace{1cm} (21)

According to the authors, (21) is a solution to the concrete strength mathematical model (12)-(15) based on kinetic theory. It follows from (21) that if \( \gamma \) and \( \varphi \) (\( \alpha = 0 \)) are equal, \( N_b \) tends to infinity, which contradicts (15). To avoid this contradiction, it should be assumed that destruction occurs when a part of the strained bonds becomes broken, and not all the strained bonds become broken. In this case, destruction means that the sample broke into fragments, the original shape is lost. But inside the fragments, there are unbroken bonds. This assumption also removes the “idem per idem” problem in (21).
4. Summary
(21) allows estimating the time required for sample destruction. As the number of broken bonds Nb in a concrete sample can be calculated using its volume. The model can be complicated by assuming that it is enough to break part of all the bonds to destroy it. If we compare the moment of destruction of the sample with the percentage of broken bonds from the number of all bonds (the number of all bonds depends on V), we can estimate at what time point the destruction of the sample will occur.

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