One-loop corrections for $WW$ to $HH$ in HEFT with the electroweak chiral Lagrangian

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Abstract. In this work we present the computation of the one-loop electroweak radiative corrections to the scattering process $WW \rightarrow HH$ within the context of the Higgs Effective Field Theory (HEFT). We assume that the fermionic interactions are like in the Standard Model, whereas the Beyond Standard Model interactions in the bosonic sector are given by the Electroweak Chiral Lagrangian. The computation of the one-loop amplitude and the renormalization program is performed in terms of the involved one-particle-irreducible functions (1PI) and using $R_\xi$ covariant gauges. The renormalization of 1PI functions at arbitrary external momenta is a more ambitious program than just renormalizing the amplitude with on-shell external legs and it has the advantage that they can be used in several scattering amplitudes. In fact, we use here some of the 1PI functions already computed in our previous work (devoted to $WZ \rightarrow WZ$). We will complement them here with the computation of the new 1PI functions required for $WW \rightarrow HH$. From this renormalization procedure we will also derive the full set of renormalized coefficients of the EChL that are relevant for this scattering process. In the last part, we will present the numerical results for the EChL predictions of the one-loop level cross section, $\sigma(WW \rightarrow HH)_{\text{1-loop}}$, as a function of the center of mass energy, showing the relative size of the one-loop radiative corrections respect to the tree level prediction in terms of the EChL coefficients. The results of the one-loop corrections to $WW \rightarrow HH$ for the SM case will be also presented, for comparison with the EChL case, following the same computational method, i.e., by means of the renormalization of 1PI functions.
1 Introduction

The use of Effective Field Theories (EFTs) to study the phenomenological implications of anomalous Higgs couplings beyond the Standard Model (SM) of Particle Physics is nowadays a very common strategy, widely employed, to test at colliders the new Higgs physics implied by those anomalous couplings in a model independent way, namely, without assuming a particular Beyond Standard Model (BSM). The information of the anomalous Higgs couplings is encoded in a set of effective operators, built with the SM fields and with the unique requirement of being invariant under the SM gauge symmetry, $SU(3) \times SU(2) \times U(1)$. The coefficients in front of these operators (usually called Wilson coefficients) are generically unknown and encode the information of the particular underlying fundamental theory that generates such EFT at low energies when the new heavy modes of this theory are integrated out. Depending on the kind of dynamics involved in the fundamental theory, it is more appropriate the use of one EFT or another. Usually, the so-called SMEFT (Standard Model Effective Theory) is more appropriate to describe the low energy behaviour of weakly interacting dynamics, whereas the so-called HEFT (Higgs Effective Field Theory) is more appropriate to describe strongly interacting underlying dynamics (for reviews, see for instance, [1, 2]). Here we choose this second case, the HEFT, and focus on the bosonic sector which is described in terms of the so-called Electroweak Chiral Lagrangian (EChL). The fermionic sector will be assumed to be as in the SM, so that no BSM interactions nor effective operators are considered in the fermionic sector of the HEFT.

Our main goal here is to determine, within this EChL context, the size of the one-loop electroweak (EW) radiative corrections for the subprocess at colliders, $W^+W^- \rightarrow HH$, where two Higgs bosons are produced from the scattering of two $W$ gauge bosons which are radiated from the initial colliding particles (also called $WW$ fusion in the literature). From now on, for brevity, we omit the explicit charges of the $W$ bosons. This $WW$ scattering subprocess is known to be relevant for both types of colliders, $e^+e^-$ and $pp$, with energies at the TeV domain. We also wish to
compare in this work these EW radiative corrections in the EChL context with the corresponding ones of $W W \rightarrow HH$ within the SM context, thus we do here the two computations in parallel. Our calculations of the amplitudes and corresponding cross sections in both cases, the EChL and the SM, are full bosonic one-loop computations, including all kind of diagrams in the loops, and are valid for physical $W$ and $H$ particles in the external legs, with all possible polarizations for the $W$ gauge bosons, longitudinal and transverse. That means that we do not make any approximation for the external legs, and do not use the Equivalence Theorem which replaces the external $W_L$’s by the corresponding Goldstone bosons (GBs) and is valid only at high energies, $\sqrt{s} \gg m_W$. Our computation of the radiative corrections within the HEFT is therefore valid at all energies, from the low energies just above the two Higgs boson threshold production, $2m_H \sim 250$ GeV, up to the typical EFT scale which, in the EChL framework, is set by $4\pi v \sim 3$ TeV with $v = 246$ GeV.

Regarding the technicalities involved in the present computation we follow closely our previous work in Ref. [3] which was addressed to the case of $WZ \rightarrow WZ$ scattering. Concretely, we follow the standard Feynman diagrammatic approach and describe the full renormalization program also in terms of one-loop Feynman diagrams. We organize this computation in terms of the involved one-particle-irreducible (1PI) Green functions, with two, three and four external legs, and perform the renormalization program of these Green functions in covariant gauges. As we showed in our previous work [3], the renormalization of the EChL coefficients must be gauge invariant and therefore independent of the $\xi$ parameter of the $R_\xi$ covariant gauges. This is an important point of doing the analytical computation of the amplitude in covariant gauges. Regarding the numerical evaluations for the $WW \rightarrow HH$ scattering we will choose here in particular the Feynman ‘t Hooft gauge with $\xi = 1$. The renormalization conditions are also fixed here as in Ref. [3], using the on-shell scheme for the EW parameters, like boson masses, $m_W$, $m_Z$, $m_H$, and gauge couplings, and the $\overline{MS}$ scheme for the EChL coefficients. Our work presented here of the full one-loop corrections for $WW \rightarrow HH$ in the EChL is the most complete one in the literature, and improves the previous related works in the literature in various aspects. The first computation of $WW \rightarrow HH$ in [4] was done just for the case with external longitudinal $W$ bosons, replaced by external GBs by using the ET, and include only scalar particles both in the external legs and in the loops, working always with massless GBs. A more recent computation of the one-loop radiative corrections for $WW \rightarrow HH$ in the EChL context, in [5], also refers to the case of longitudinal $W$’s and also uses the ET that replace the external $W_L$ by the GBs which are taken massless. They consider all kinds of loops for the GBs scattering and compute them in the Landau gauge. They make the additional approximation of taking equal the $W$ and $Z$ boson masses (called isospin limit in the literature). Our best improvement respect to these works is that we do not use the ET, i.e. we work with external gauge bosons, instead of GBs, we do not take equal masses for $W$ and $Z$, and we do not take massless GBs, since we work in the Feynman ‘t Hooft gauge. Consequently, the set of Feynman 1-loop diagrams considered here and in [4, 5] are also different. Another important aspect, that we cover in a different way than in those references is the renormalization program, that we implement here in terms of general renormalized Green functions, with generic external momenta, in contrast to Refs. [4, 5] that apply the renormalization program directly to the on-shell scattering amplitude. The advantage of doing renormalization at the more general off-shell Green functions level, is that these same renormalized functions can be used as well for the computation of radiative corrections in another observables. For instance, we have used the same renormalized vertex function $WWH$ here for $WW \rightarrow HH$ than in our previous computation in Ref. [3] for $WZ \rightarrow WZ$. The difference is just in the particular setting of the external legs momenta of the vertex function which must be done properly for each case. On the other hand, the renormalization program using 1PI Green functions instead of just on-shell amplitudes requires the renormalization of a larger set of EChL coefficients. It is, therefore, also more complete in this sense.
relevance of this latter issue, we will devote some part of the present work to the comparison of our results on the renormalization of the EChL coefficients with some related previous results [4–7].

The paper is organized as follows. In Section 2, we briefly describe the main features of the EChL with $R_\xi$ gauge-fixing and set the relevant operators for the $WW \to HH$ scattering process. The diagrammatic computation by means of the 1PI functions is presented in Section 3. The Section 4 is devoted to the renormalization program, including the prescriptions for regularization and renormalization assumed and the summary of all the divergent counterterms. The numerical predictions for this observable within the EChL and the SM are presented and discussed in Section 5. Finally, we conclude in Section 6.

2 Relevant part of the electroweak chiral Lagrangian

In this section we introduce the part of the bosonic EChL that is needed for the present computation of the EW radiative corrections of the $WW \to HH$ scattering, and provide some necessary notation. In the EChL context, the active fields are the EW gauge bosons, $B_\mu$ and $W_\mu^a$ ($a = 1, 2, 3$), their corresponding GBs $\pi^a$ ($a = 1, 2, 3$), and the Higgs boson $H$. The unique requirement for the building of the EChL is the invariance under the EW gauge, $SU(2)_L \times U(1)_Y$, transformations. On the other hand, the scalar sector of the EChL has an additional invariance under the EW chiral $SU(2)_L \times SU(2)_R$ transformation. Under this EW chiral transformation the GBs transform non-linearly. This peculiarity implies multiple GBs interactions among themselves and also with the other fields. The Higgs boson field, in contrast, is invariant under all transformations. Usually the GBs are introduced in a non-linear representation via the exponential parametrization, by means of the matrix $U$, which transforms linearly under the EW chiral transformations:

$$U(\pi^a) = e^{i\pi^a \tau^a / v} , \quad (2.1)$$

where, $\tau^a$, $a = 1, 2, 3$, are the Pauli matrices and $v = 246$ GeV. On the other hand, the Higgs field is a singlet of the EW chiral symmetry and the EW gauge symmetry. Hence the interactions of $H$ are introduced via generic polynomials since there are not limitations from symmetry arguments on the implementation of this field and its interactions with itself and with the other fields, in contrast to linear EFTs as the SMEFT. Finally, the EW gauge bosons are introduced by the gauge invariance principle. Thus, they appear in the following pieces of the EChL:

$$\begin{align*}
\tilde{B}_\mu &= gB_\mu \tau^3 / 2 , \\
\tilde{B}_{\mu\nu} &= \partial_\mu \tilde{B}_\nu - \partial_\nu \tilde{B}_\mu , \\
\tilde{W}_\mu &= gW_\mu^a \tau^a / 2 , \\
\tilde{W}_{\mu\nu} &= \partial_\mu \tilde{W}_\nu - \partial_\nu \tilde{W}_\mu + i[\tilde{W}_\mu, \tilde{W}_\nu] , \\
D_\mu U &= \partial_\mu U + i\tilde{W}_\mu U - iU \tilde{B}_\mu , \\
V_\mu &= (D_\mu U)^\dagger , \\
D_\mu O &= \partial_\mu O + i[\tilde{W}_\mu, O] .
\end{align*} \quad (2.2)$$

As it is usual, the chiral counting arrange the effective operators in the EChL into terms with increasing chiral dimension. The most relevant ones are the leading order Lagrangian, with chiral dimension two, $\mathcal{L}_2$, and the next to leading order one with chiral dimension four, $\mathcal{L}_4$. The relevant EChL for the present computation can then be summarized as follows:

$$\mathcal{L}_{\text{EChL}} = \mathcal{L}_2 + \mathcal{L}_4 , \quad (2.3)$$

In this chiral dimension counting, it is important to keep in mind that all derivatives and masses count as momentum, namely, $\partial_\mu$, $m_W$, $m_Z$, $m_H$, $g v$, $g' v$, $\lambda v \sim O(p)$.

Firstly, the leading order Lagrangian, $\mathcal{L}_2$ is given by,

$$\mathcal{L}_2 = \frac{v^2}{4} \left(1 + 2a \frac{H}{v} + b \left(\frac{H}{v} \right)^2 + \ldots \right) \text{Tr} \left[D_\mu U^\dagger D^\mu U \right] + \frac{1}{2} \partial_\mu H \partial^\mu H - V(H)$$

$$- 3 -$$
Here \( V(H) \) is the Higgs potential, \( \mathcal{L}_{GF} \) and \( \mathcal{L}_{FP} \), are the gauge-fixing Lagrangian and Fadeev-Popov Lagrangian, respectively. From now on, the dots in the presentation of the relevant pieces in the EChL stand for terms that do not enter in our processes of interest, \( WW \to HH \), neither at tree level nor at one-loop level and then we omit them. The Higgs potential in \( \mathcal{L}_2 \) is given by:

\[
V(H) = (-\mu^2 + \lambda v^2)vH + \frac{1}{2}(-\mu^2 + 3\lambda v^2)H^2 + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4 .
\]  

(2.5)

For the posterior discussion on renormalization in this EChL context, it is convenient to define \( m_H^2 = -\mu^2 + 3\lambda v^2 \), then we can eliminate the \( \mu^2 \) parameter in terms of \( m_H^2 \). In this case, the linear term (Higgs tadpole) can be simply written as,

\[
T = (m_H^2 - 2\lambda v^2)v ,
\]

(2.6)

and the minimum of the potential, corresponding to vanishing tadpole, sets \( m_H^2 = 2\lambda v^2 \).

Here, we quantize the EChL as in our previous work [3], i.e., using the linear covariant \( R_\xi \) gauges [8] with the gauge-fixing Lagrangian given by,

\[
\mathcal{L}_{GF} = -F_+F_--\frac{1}{2}F_Z^2 - \frac{1}{2}F_A^2 ,
\]

(2.7)

and the gauge-fixing functions given by:

\[
F_\pm = \frac{1}{\sqrt{\xi}}(\partial^\mu W_\mu^\pm - \xi m_W \pi^\pm) , \quad F_Z = \frac{1}{\sqrt{\xi}}(\partial^\mu Z_\mu - \xi m_Z \pi^3) , \quad F_A = \frac{1}{\sqrt{\xi}}(\partial^\mu A_\mu) .
\]

(2.8)

Here \( \xi \) is the generic gauge-fixing parameter of the \( R_\xi \) gauges. Some comments about the \( \xi \) dependence are worth to be added here. Notice that in our renormalization program, we demand the renormalization of all the 1PI functions involved at arbitrary momentum for the external legs, and not just the finiteness of the one-loop scattering amplitude. Thus, in order to demonstrate explicitly the gauge invariance of the renormalized EChL coefficients, i.e., to check that these are \( \xi \) independent, the computation of the loop diagrams involved in the 1PI functions should be done for arbitrary \( \xi \) parameter, as it was done in our previous work [3] devoted to \( WZ \to WZ \) scattering. All the final scattering amplitudes with external on-shell particles are of course finite and gauge invariant, but the involved 1PI functions are \( \xi \) dependent, so the cancellation of the \( \xi \) dependence in the final one-loop amplitude is an excellent check of the computation. In the present paper, for all the numerical estimates of \( WW \to HH \) we will choose in particular the Feynman ’t Hooft gauge and, accordingly, for definiteness, we set \( \xi = 1 \) in the presentation of all our results.

From this previous \( R_\xi \) gauge-fixing Lagrangian, one derives as usual, the corresponding Fadeev-Popov Lagrangian [9], given by:

\[
\mathcal{L}_{FP} = \sum_{i,j,+,-,Z,A} \bar{c}^i \frac{\delta F_i}{\delta \alpha_j} c^j ,
\]

(2.9)

where \( c^j \) are the ghost fields and \( \alpha_j \) \((j = +, -, Z, A)\) are the corresponding gauge transformation parameters under the local transformations \( SU(2)_L \times U(1)_Y \) given by \( L = e^{ig_2^T \Delta L(x)/2} \) and \( R = e^{ig^A \tau_A \alpha_Y(x)/2} \).

Formally, the expressions in Eqs. (2.7)-(2.9) and the gauge bosons field transformations are the same as in the SM. However, the scalar transformations in this non-linear EFT differ from.
the corresponding ones in the SM. This particularity yields to the absence of interactions among
the Higgs boson and ghost fields, and the presence of new interactions with multiple GBs and two
ghost fields.

Secondly, the relevant chiral dimension four Lagrangian for the computation of all the one-loop
1PI functions involved in the $WW \rightarrow HH$ scattering amplitude is given by:

$$\mathcal{L}_4 = -a_{ddVV1} \frac{\partial\mu H \partial\nu H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] - a_{ddVV2} \frac{\partial\mu H \partial\nu H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu]$$

$$+ \left( a_{11} + a_{H11} \frac{H}{v} + a_{H11} \frac{H^2}{v^2} \right) \text{Tr}[\mathcal{D}_\mu \mathcal{V}_\mu \mathcal{D}_\nu \mathcal{V}_\nu]$$

$$- \frac{m_H^2}{4} \left( 2a_{HHVV} \frac{H}{v} + a_{HHVV} \frac{H^2}{v^2} \right) \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu]$$

$$- \left( a_{HHWW} \frac{H}{v} + a_{HHWW} \frac{H^2}{v^2} \right) \text{Tr}[\hat{W}_{\mu\nu} \mathcal{V}_\mu \mathcal{V}_\nu + i \left( a_{d2} + a_{Hd2} \frac{H}{v} \right) \frac{\partial\nu H}{v} \text{Tr}[\hat{W}_{\mu\nu} \mathcal{V}_\mu]$$

$$+ \left( a_{\Box \nu\nu} + a_{H\Box \nu\nu} \frac{H}{v} \right) \frac{\Box H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] + \left( a_{d3} + a_{Hd3} \frac{H}{v} \right) \frac{\partial\nu H}{v} \text{Tr}[\mathcal{V}_\mu \mathcal{D}_\nu \mathcal{V}_\mu]$$

$$+ \left( a_{dd} \frac{m_H^2}{v^2} + a_{ddV} \frac{m_H^2}{v^2} + a_{ddV} \frac{m_H^2}{v^2} \right) \frac{H}{v} \mathcal{V}_\mu \partial\mu H. \quad (2.10)$$

These relevant effective operators are taken from the full HEFT Lagrangian in Ref. [6, 10], but we
use here a different notation for the EChL coefficients in $\mathcal{L}_4$, that are referred here generically
as $a_i$’s. The correspondence among the two set of coefficients, $a_i$’ here and the coefficients in Ref.
[6, 10] can be summarized, in short, by: $a_{ddVV1} \leftrightarrow c_8, a_{ddVV2} \leftrightarrow c_20, a_{11} \leftrightarrow c_9, a_{HHWW} \leftrightarrow a_W, a_{HHWW} \leftrightarrow b_W, a_{d2} \leftrightarrow c_5, a_{Hd2} \leftrightarrow a_5, a_{\Box \nu\nu} \leftrightarrow c_7, a_{H\Box \nu\nu} \leftrightarrow a_7, a_{d3} \leftrightarrow c_{10}, a_{Hd3} \leftrightarrow a_{10}, a_{\Box \Box} \leftrightarrow c_{10}, a_{H\Box \Box} \leftrightarrow a_{\Box \Box}, a_{ddV} \leftrightarrow c_{\Delta H}, a_{HHVV} \leftrightarrow a_C$ and $a_{HHVV} \leftrightarrow a_C$.

It is worth noticing that, in contrast to our one-loop computation here, for a tree level com-
putation of the scattering amplitude (see for instance [11]) the previous set of operators in $\mathcal{L}_4$
can be reduced to a smaller set by the use of the equations of motion (EOMs). Concretely, one may
use the following EOMs involving the pieces $\Box H$ and $\mathcal{D}_\mu \mathcal{V}_\mu$:

$$\Box H = -\frac{\delta V(H)}{\delta H} - \frac{v^2}{4} \frac{\partial F(H)}{\partial H} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] \Rightarrow$$

$$\Rightarrow \Box H = -m_H^2 \frac{H}{v^2} - \frac{2\kappa m_H^2}{v^2} \frac{H^2}{v^2}$$

$$\text{Tr}[\mathcal{D}_\mu \mathcal{V}_\mu F(H)] = -\text{Tr}[\tau^j \mathcal{D}_\mu \mathcal{V}_\mu \mathcal{D}_\mu F(H)]$$

$$\Rightarrow \text{Tr}[\tau^j \mathcal{D}_\mu \mathcal{V}_\mu] = -\text{Tr}[\tau^j \mathcal{V}_\mu] \frac{2a}{v} \partial\mu H. \quad (2.11)$$

Then, one may eliminate the terms in $\mathcal{L}_4$ involving these two pieces, by re-writing them in terms
of the other effective operators. This reduces in practice the number of effective operators in $\mathcal{L}_4$
as follows:

$$\mathcal{L}_4^{\text{EOMs}} = -(a_{ddVV1} - 4a_{ddVV1}^2 a_{11} + 2a_{ddd}) \frac{\partial\mu H \partial\nu H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] - (a_{ddVV2} + \frac{a}{2} a_{ddd}) \frac{\partial\mu H \partial\nu H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu]$$

$$- \frac{m_H^2}{2} \left( a_{HHVV} - 2a_{\Box \nu\nu} + 2a_{\Box \Box} \right) \frac{H}{v} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu]$$
\[-\frac{m_{H}^{2}}{4} (a_{HVV} - 6\kappa_{3}a_{\Box}VV - 4a_{H\Box}VV + 4ba_{\Box \Box} + 6\kappa_{3}aa_{\Box \Box} + 4aa_{H\Box \Box}) \frac{H^{2}}{v^{2}} \text{Tr} \left[ \mathcal{V}^{\mu} \mathcal{V}_{\mu} \right] \]
\[+ (a_{Hdd} - a_{d\Box}) \frac{m_{H}^{2}}{v^{2}} \frac{H}{v} \partial^{\mu} H \partial_{\mu} H + \left( a_{dW} \frac{m_{W}^{2}}{v^{2}} + a_{ddZ} \frac{m_{Z}^{2}}{v^{2}} \right) \frac{H}{v} \partial^{\mu} H \partial_{\mu} H \]
\[- \left( a_{HWW} \frac{H}{v} + a_{HHWW} \frac{H^{2}}{v^{2}} \right) \text{Tr} \left[ \hat{W}_{\mu \nu} \hat{W}^{\mu \nu} \right] + i \left( a_{d2} + a_{dHd} - \frac{H}{v} \right) \frac{\partial^{\nu} H}{v} \text{Tr} \left[ \hat{W}_{\mu \nu} \mathcal{V}^{\mu} \right] \]

In this reduced Lagrangian, \(a_{d\Box}VV\), \(a_{11}\) and \(a_{d3}\) have been grouped together in the first operator, \(a_{d\Box}VV\) and \(a_{d\Box}A\) in the second operator, etc. We then redefine these combinations of coefficients as follows:

\[
\eta = a_{d\Box}VV_{1} \equiv a_{d\Box}VV_{1} - 4a_{11}^{2} + 2aa_{d3}
\]
\[
\delta = a_{d\Box}VV_{2} \equiv a_{d\Box}VV_{2} + \frac{\alpha}{2}a_{d\Box}A
\]
\[
\tilde{a}_{HVV} \equiv a_{HVV} - 2a_{\Box}VV + 2aa_{\Box}
\]
\[
\tilde{a}_{HVV} \equiv a_{HVV} - 6\kappa_{3}a_{\Box}VV - 4a_{H\Box}VV + 4ba_{\Box \Box} + 6\kappa_{3}aa_{\Box \Box} + 4aa_{H\Box \Box}
\]
\[
\tilde{a}_{Hdd} \equiv a_{Hdd} - a_{d\Box}A
\]

Notice, that in the two first coefficients we have also used the alternative notation given by \(\eta\) and \(\delta\), which is the one frequently used in some related literature. In particular, the contributions to the scattering amplitude \(WW \rightarrow HH\) of these two first operators in the above list with \(\eta\) and \(\delta\) coefficients, were studied firstly in [4, 5]. We will compare our results with these and more references in the next sections. Notice also that some coefficients, like \(a_{H11}, a_{H1H11}, a_{Hd3}\), have disappeared in Eq. (2.12) since, after the use of the EOMs, they contribute to effective operators with at least three Higgs bosons which do not enter in the process of our interest here.

The main consequence of using the EOMs when computing the scattering amplitude is, that these combinations of the EChL coefficients are the ones appearing precisely in the on-shell scattering amplitudes. On the other hand, this means that only these combinations of coefficients are the ones that are really testable at colliders via this particular \(WW \rightarrow HH\) scattering. In particular, only \(\eta\) and \(\delta\) in Eq. 2.13 and not the separate coefficients, \(a_{d\Box}VV_{1}, a_{11}, a_{d3}, a_{d\Box}VV_{2}\), and \(a_{d\Box}A\) are the appropriate parameters for a phenomenological analysis of this scattering \(WW \rightarrow HH\) process. Similarly for the other combinations appearing in Eq. (2.12). However, for our most ambitious computation and renormalization program, where the finite renormalized one-loop scattering amplitude is obtained in terms of finite renormalized one-loop 1PI functions, this reduced Lagrangian is not sufficient and we must use the full Lagrangian in Eq. 2.10. As we will see in the following sections, this full Lagrangian provides not only a finite one-loop amplitude with on-shell external particles but also finite one-loop 1PI functions with arbitrary external momenta (generically off-shell). This renormalization program in terms of one-loop 1PI functions is also relevant for the check of the gauge invariance of the final one-loop amplitude, and to demonstrate the gauge invariance of the renormalized EChL coefficients. The great advantage of using this procedure by means of the 1PI functions to compute the radiative corrections in scattering amplitudes is that the same 1PI one-loop renormalized functions, once computed at arbitrary external momenta, can be used for several processes, by just adjusting the external momenta to the ones of that particular process, including the proper on-shell setting for the external legs when needed. For instance, the one-loop 1PI function \(\tilde{\Gamma}_{HHW}\) can be used for both \(WW \rightarrow HH\) and \(WZ \rightarrow WZ\), the one-loop 1PI function \(\tilde{\Gamma}_{HHH}\) can be used for both \(WW \rightarrow HH\) and \(HH \rightarrow HH\), and similarly with other processes. Therefore, our renormalization program based on 1PI functions is more powerful than just to renormalize concrete scattering amplitudes.
Finally, to end this section we remind that in order to reach the SM tree level vertices from the above presented EChL, one has to set the EFT coefficients to the following reference values: 1) the coefficients in $L_2$, $a$, $b$, $\kappa_3$ and $\kappa_4$ should be set to 1, and 2) all the coefficients $a_i$'s in $L_4$ should be set to zero. Accordingly, the new BSM physics encoded in the EChL is parametrized in terms of the departures from these reference parameter values. The corresponding derived Feynman Rules (FRs) from this EChL that are needed for the present computation, together with the corresponding FRs within the SM, were provided in our previous work, concretely in Appendix A of [3], so we do not repeat them here.

3 Diagrammatic computation using 1PI Green functions

In this section we present our procedure for the computation of the radiative corrections to the amplitude $A(WW \rightarrow HH)$ by means of the 1PI Green functions. We apply this procedure to both cases, the EChL and the SM.

Within the EChL formalism, the full one-loop scattering amplitude can be splitted into two parts corresponding to the leading order (LO), $O(p^2)$, and the next-to-leading order contributions (NLO), $O(p^4)$, which are denoted as $A^{(0)}$ and $A^{(1)}$ respectively, yielding to

$$A^{\text{EChL Full}} \equiv A_{WW \rightarrow HH}^{\text{EChL}} = A^{(0)}(WW \rightarrow HH) + A^{(1)}(WW \rightarrow HH).$$

(3.1)

In this EChL context, the LO amplitude comes from $L_2$ at the tree level, and the NLO one receives typically two contributions. One contribution comes from $L_4$ at the tree level and the other one comes from the loops computed with $L_2$. Thus, these LO and NLO contributions are written generically as,

$$A^{(0)}(WW \rightarrow HH) \equiv A^{\text{EChL Tree}^{(2)}},$$

(3.2)

$$A^{(1)}(WW \rightarrow HH) \equiv A^{\text{EChL Tree}^{(4)}} + A^{\text{EChL Loop}}.$$

(3.3)

The one-loop amplitude in the EChL can also be written in an alternative way, accounting for the quantum corrections expansion, i.e., in powers of $\hbar$. Then, the full one-loop amplitude is written as:

$$A^{\text{EChL Full}} = A^{\text{EChL Tree}^{(2+4)}} + A^{\text{EChL Loop}},$$

(3.4)

where, the tree level amplitude, $O(\hbar^0)$, has contributions from both $L_2$ and $L_4$, generically written as $A^{\text{EChL Tree}^{(2+4)}} = A^{\text{EChL Tree}^{(2)}} + A^{\text{EChL Tree}^{(4)}}$, whereas the one-loop correction, $O(\hbar^1)$, is obtained by computing loops with just $L_2$, generically written as, $A^{\text{EChL Loop}}$. Remember that within the EChL framework, the $a_i$ coefficients in $L_4$, have a double role and will act as well as counterterms of the extra divergences generated by these loops which can not be absorbed by just the renormalization of the parameters in $L_2$.

On the other hand, we wish to compare the EChL predictions with the SM ones using the same procedure of 1PI functions. Thus, we will present in parallel the two predictions for the EChL and SM cases. To our knowledge, our SM computation is the first full bosonic one-loop computation of $WW \rightarrow HH$ scattering using the $R_\xi$ gauges in the literature. This SM amplitude is defined as the sum of the LO contribution, which in this case, is the tree level contribution of $O(\hbar^0)$, and the NLO contribution, which is of $O(\hbar^1)$:

$$A^{\text{SM Full}} = A^{\text{SM Tree}} + A^{\text{SM Loop}}.$$

(3.5)
For the technical description of the one-loop radiative corrections we then organize the one-loop full amplitude in terms of the 1PI Green functions as follows:

\[ A^{\text{ChLfull}} = A^{\text{LO}} + A_{1-\text{legs}} + A_{2-\text{legs}} + A_{3-\text{legs}} + A_{4-\text{legs}} + A_{\text{res}}, \]  

where \( A^{\text{LO}} \) is the result from the LO Lagrangian, i.e., \( A^{\text{LO}} = A^{(0)} \), and \( A_{n-\text{legs}} \) means the contributions to the amplitude from the \( n \)-legs 1PI functions. The LO contribution is that from \( L_2 \) to tree level and therefore it is \( O(h^0) \). The \( A_{n-\text{legs}} \) terms contain the NLO contributions, including the contributions from the \( a_i \) coefficients and also the \( O(h^1) \) contributions from the loop diagrams in the corresponding 1PI functions. Notice also, that we have separated explicitly the extra contribution to the amplitude from the finite residues of the external particles \( A_{\text{res}} \).
the product of several black balls containing each one both LO and NLO contributions, and one has to perform this product accordingly to extract the final result for the amplitude containing all the terms of both orders, $O(\hbar^1)$ and $O(\hbar^1)$.

The tensor amplitude in Eq. (3.8) is obtained by adding the $s$, $t$, $u$ and contact $c$ channel contributions as represented in Fig. 1. These contributions by channels can be written in terms of the full Green functions as follows:

$$iA_{\mu}^{\nu} = i\hat{\Gamma}_{\mu\nu}^{WW} i\hat{\Delta}^{HH} i\hat{\Delta}^{HHH} + i\Gamma_{\mu\nu}^{WA} (-i)\Delta_{\mu \nu}^{AA} i\Gamma_{\mu\nu}^{AH} + i\Gamma_{\mu\nu}^{WWZ} (-i)\Delta_{\mu \nu}^{ZZ} i\Gamma_{\mu\nu}^{ZHH}$$

$$iA_{t}^{\mu} = i\hat{\Gamma}_{\mu\nu}^{WW} (-i)\hat{\Delta}_{\mu \nu}^{WW} + i\Gamma_{\mu\nu}^{WW} \hat{\Delta}_{\mu \nu}^{WW} i\Gamma_{\mu\nu}^{WW} + i\Gamma_{\mu\nu}^{WW} \hat{\Delta}_{\mu \nu}^{WW} i\Gamma_{\mu\nu}^{WW} + i\Gamma_{\mu\nu}^{WW} \hat{\Delta}_{\mu \nu}^{WW} i\Gamma_{\mu\nu}^{WW}$$

$$iA_{u}^{\mu} = i\hat{\Gamma}_{\mu\nu}^{WWZ} (-i)\hat{\Delta}_{\mu \nu}^{WW} + i\Gamma_{\mu\nu}^{WWZ} \hat{\Delta}_{\mu \nu}^{WW} i\Gamma_{\mu\nu}^{WWZ} + i\Gamma_{\mu\nu}^{WWZ} \hat{\Delta}_{\mu \nu}^{WW} i\Gamma_{\mu\nu}^{WWZ} + i\Gamma_{\mu\nu}^{WWZ} \hat{\Delta}_{\mu \nu}^{WW} i\Gamma_{\mu\nu}^{WWZ}$$

$$iA_{c}^{\mu} = i\hat{\Gamma}_{\mu\nu}^{WWHH}$$

At one-loop level, it is convenient to write the full propagators in terms of the self-energies. Following our procedure and conventions defined in [3], we get the following expressions for the full propagators in terms of the LO propagators and the full self-energies:

$$i\hat{\Delta}_{\mu \nu}^{HH}(q^2) = i\Delta_{\mu \nu}^{HH} + i\Delta_{\mu \nu}^{HH} (-i)\hat{\Sigma}_{HH} i\Delta_{\mu \nu}^{HH},$$

$$i\hat{\Delta}_{\mu \nu}^{HH}(q^2) = i\Delta_{\mu \nu}^{HH} + i\Delta_{\mu \nu}^{HH} (-i)\hat{\Sigma}_{HH} i\Delta_{\mu \nu}^{HH},$$

$$-i\Delta_{\mu \nu}^{WW}(q^2) = -i\Delta_{\mu \nu}^{WW} - i\Delta_{\mu \nu}^{WW} i\hat{\Sigma}_{WW} (-i)\Delta_{\mu \nu}^{WW},$$

$$-i\Delta_{\mu \nu}^{WW}(q^2) = -i\Delta_{\mu \nu}^{WW} - i\Delta_{\mu \nu}^{WW} i\hat{\Sigma}_{WW} (-i)\Delta_{\mu \nu}^{WW},$$

$$\hat{\Delta}_{\mu \nu}^{WW}(q^2) = \Delta_{\mu \nu}^{WW} - i\Delta_{\mu \nu}^{WW} \hat{\Sigma}_{WW} i\Delta_{\mu \nu}^{WW}.$$

where all functions on the right hand side are functions of $q^2$ and the LO propagators in the $R_\xi$ gauges are summarized by:

$$i\Delta_{\mu \nu}^{HH} = \frac{i}{q^2 - m_H^2}, \quad -i\Delta_{\mu \nu}^{WW} = \frac{-i}{q^2 - m_W^2},$$

$$-i\Delta_{\mu \nu}^{WW} = \frac{-i\xi}{q^2 - \xi m_W^2}, \quad i\Delta_{\mu \nu}^{WW} = \frac{i}{q^2 - \xi m_W^2}, \quad \Delta_{\mu \nu}^{WW} = 0.$$}

As commented previously, only $\Delta_{\mu \nu}^{HH}, \Delta_{\mu \nu}^{WW}$ and $\Delta_{\mu \nu}^{WW}$ are involved in the LO contribution to the amplitude in Eq. (3.1). On the other hand, the relevant vertex functions at LO are:

$$i\Gamma_{\mu\nu}^{WW} = igm_{WW} g_{\mu\nu}, \quad i\Gamma_{\mu\nu}^{HHHH} = -3i\kappa_3 m_H^2 / v,$$

$$i\Gamma_{\mu\nu}^{WW} = ag_{\mu\nu}, \quad i\Gamma_{\mu\nu}^{WW} = ibg_{\mu\nu} / 2.$$
The result for the LO amplitude in the $R_\xi$ gauge, corresponding to the tree level diagrams in Fig. 2 is given by:

$$A^{(0)} = A_s^{(0)} + A_t^{(0)} + A_u^{(0)} + A_c^{(0)}$$  \hspace{1cm} (3.13)

where the contributions by $s$, $t$, $u$ and contact channels are given, respectively, by:

$$A_s^{(0)} = \frac{g^2}{2} 3a\kappa_3 \frac{m_H^2}{S - m_H^2} \epsilon_+ \cdot \epsilon_-$$

$$A_t^{(0)} = g^2 a^2 \frac{m_W^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2}{T - m_W^2}$$

$$A_u^{(0)} = g^2 a^2 \frac{m_W^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_2 \epsilon_- \cdot k_1}{U - m_W^2}$$

$$A_c^{(0)} = \frac{g^2}{2} b \epsilon_+ \cdot \epsilon_-$$  \hspace{1cm} (3.14)

From this equation, notice that the corresponding result of the SM amplitude at LO is simply obtained from this same formula by setting $a = b = \kappa_3 = 1$. We have checked explicitly the gauge invariance of our LO result above, namely, that the dependence on $\xi$ disappears in the final amplitude as expected. The cancellation of the $\xi$-dependent terms is achieved once the external gauge bosons are on-shell, i.e., by contracting the tensorial amplitude with their corresponding polarization vectors in $A^{(0)}$. Concretely, the cancellation of the $\xi$-dependent terms occurs separately in the two channels $t$ and $u$, and it happens between the contribution of the longitudinal part of the $W$ propagator and the GB propagator in Eq. (3.9).

Finally, we present the result for the complete amplitude to tree level, i.e.,

$$A^{\text{EChL}_{\text{Tree}}^{(2+4)}} = A^{\text{EChL}_{\text{Tree}}^{(2)}} + A^{\text{EChL}_{\text{Tree}}^{(4)}}$$  \hspace{1cm} (3.15)

where $A^{\text{EChL}_{\text{Tree}}^{(2)}} = A^{(0)}$ is given in Eqs. (3.13)-(3.14) and $A^{\text{EChL}_{\text{Tree}}^{(4)}}$ is computed from $\mathcal{L}_4$ and contains the $a_i$ coefficients. As we have explained in the previous section, this can be written in two ways, depending if one uses the EOMs to reduce the list of operators or not. We provide here the short version, i.e., using $\mathcal{L}_4^{\text{+EOMs}}$ in Eq. (2.12).

$$A^{\text{EChL}_{\text{Tree}}^{(4)}} = A^{\text{EChL}_{\text{Tree}}^{(4)}|_s} + A^{\text{EChL}_{\text{Tree}}^{(4)}|_t} + A^{\text{EChL}_{\text{Tree}}^{(4)}|_u} + A^{\text{EChL}_{\text{Tree}}^{(4)}|_c}$$  \hspace{1cm} (3.16)
where the contributions by channels are:

\[ A^{EChL_{tree}}_{u} = \frac{g^2}{2w^2} \left( \frac{1}{T - m_W^2} \right) (3\kappa_3 a_{d2} m_H^2 (S\epsilon_+ \cdot \epsilon_- - 2\epsilon_+ \cdot p_- - \epsilon_- \cdot p_+) + 6\kappa_3 a_{HWW} m_W^2 (S - 2m_W^2) \epsilon_+ \cdot \epsilon_- - 2\epsilon_+ \cdot p_- - \epsilon_- \cdot p_+) - (3\kappa_3 a_{HVH} m_H^4 + a_{d1} \delta_{d1} m_H^2 + a_{dd} m_W^2 + a_{ddz} m_Z^2) (S + 2m_W^2) \epsilon_+ \cdot \epsilon_- ) \]

\[ A^{EChL_{tree}}_{t} = \frac{g^2}{2w^2} \left( \frac{a}{T - m_W^2} \right) (a_{d2} (4m_W^2 m_H^2 \epsilon_+ \cdot \epsilon_- + 2(T - 3m_W^2 - m_H^2) \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2 - 4m_W^2 (\epsilon_+ \cdot k_1 \epsilon_- \cdot p_+ + \epsilon_+ \cdot p_- \epsilon_- \cdot k_2)) - 8a_{HWW} m_W^4 ((T + m_W^2 - m_H^2) \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot p_+ + \epsilon_+ \cdot p_- \epsilon_- \cdot k_2) - 4a_{HVH} m_H^2 (m_W^2 \epsilon_+ \cdot \epsilon_- + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2)) \]

\[ A^{EChL_{tree}}_{c} = \frac{g^2}{2w^2} \left( - \frac{2a_{ddVV1}}{T - m_W^2} \right) (\epsilon_+ \cdot k_2 \epsilon_- \cdot k_1 + \epsilon_+ \cdot k_1 \epsilon_- \cdot k_2) + (-2a_{ddVV2} (S - 2m_W^2) + a_{HHWW} (S - 2m_W^2) + a_{Hd2} S - a_{HVH} m_H^2 \epsilon_+ \cdot \epsilon_- - 2(a_{Hd2} + 4a_{HHWW} \epsilon_+ \cdot p_- \epsilon_- \cdot p_+)) \]

\[ (3.17) \]

Notice that we have used the new coefficients defined in Eq. (2.13). Notice also that the above results are given in terms of the polarization vectors of the initial W gauge bosons. Therefore, our results above apply to all the possible polarized channels, \( W_X W_Y \rightarrow HH \), with \( XY = LL, TT, LT, TL \) by just inserting the proper polarization vectors \( \epsilon_+ \) and \( \epsilon_- \).

We next comment shortly on the comparison of our analytical results in this section for the tree level amplitude within the EChL with the previous literature. First of all, the LO amplitude in Eqs. (3.13)-(3.14) is in full agreement with [12]. Secondly, regarding \( A^{EChL_{tree}} \) we have compared our results with those in [5]. We have checked the full agreement in the contributions from the coefficients, \( \tilde{a}_{d1V1} (= \eta) \), \( \tilde{a}_{d1V2} (= \delta) \), \( a_{d2} (= b_1 \chi) \) and \( a_{Hd2} (= 2b_2 \chi) \) with their results. The other coefficients in our result of Eq. (3.17) were not considered in [5]. On the other hand, the results of [4] in terms of \( \delta \) and \( \eta \) where provided using the Equivalence Theorem, so they can only be compared for the longitudinal modes and in the high energy regime \( \sqrt{s} \gg m_W, m_H \). By an exploration of our amplitudes for the case of the longitudinal modes in that high energy regime, we have also checked agreement of the \( \eta \) and \( \delta \) contributions with that reference. The other parameters were not studied either in that reference.

Finally, it is important to keep in mind the existent hierarchy among the various polarization channels and among the relevance of the various coefficients for each polarization channel. Firstly, it is well known the dominance in the total cross section for this \( WW \rightarrow HH \) process of the longitudinal polarized modes over the transverse modes. Namely, \( \sigma(WW \rightarrow HH) \) is fully dominated by \( \sigma(W_W W_L \rightarrow HH) \). The other polarization channels with initial \( W_T W_T \) or \( W_L W_T \) are highly subdominant at the center of mass energies in the TeV domain. Therefore, by studying the longitudinal polarized case one can approximate quite well the total cross section. This dominance of the \( \sigma(W_W W_L \rightarrow HH) \) over the other polarized channels also happens in the EChL case, in the tree level estimates of the cross section, at both orders the LO and the NLO ones. A recent phenomenological study of the corresponding BSM effects in Ref. [11] for all the polarized channels and considering all the EChL coefficients in Eq. (3.17), has shown that the most relevant coefficients of the EChL, for the \( LL \) modes and at the tree-level NLO, are indeed \( \eta \) and \( \delta \). Here, by ‘the most relevant coefficients’ we mean those EChL coefficients in \( L_4 \) that lead to the largest cross sections in this \( WW \rightarrow HH \) scattering process at the TeV energy domain. For definiteness here, and to
Figure 3. Tree level cross section predictions for $W^+W^- \to HH$ within the EChL setting $a = b = \kappa_3 = \kappa_4 = 1$. All EChL coefficients in the NLO Lagrangian are set to zero except for $\eta$ and $\delta$. Plots in the left column are for non-vanishing $\eta$ and plots in the right column are for non-vanishing $\delta$. The predictions for the total unpolarized case are displayed in the plots of the first row, and the ones for the polarized $LL$ case in the second row. The SM predictions are displayed in all the plots, for comparison.

summarize this $LL$ dominance in the tree level-NLO prediction from the EChL, we show in Fig. 3 our predictions, as a function of the center-of-mass-energy $\sqrt{s}$, of the cross sections: 1) for the total unpolarized case (the two plots on the first row), and 2) for the $LL$ polarized case (the two plots on the second row). We display in this figure the BSM departures respect to the SM predictions from the separate effects of the two most relevant coefficients, assuming different numerical values for those coefficients ($\pm 0.1, 0.01, 0.001, 0.0001$): 1) the effect from $\eta$ is displayed in the plots on the first column; 2) the effect from $\delta$ is displayed in the plots on the second column. We can clearly see in these plots that the cross section for the $LL$ case fully dominates the total (unpolarized) cross section for all the studied cases. Indeed, the two lines for $LL$ and for ‘total’ practically coincide in the studied TeV domain (up to the obvious reducing 1/9 factor in the unpolarized result due to the average over the possible initial helicities). The other evident conclusion from this figure is that large values of the cross sections and large departures from the SM predictions can be reached at the TeV energies for the cases with the larger input coefficients $\eta$ and $\delta$. For a more devoted study of the phenomenological consequences of these tree level predictions within the EChL at NLO we
address the reader to Ref. [11]. In particular, the relevance of these predictions for the di-Higgs production at future $e^+e^-$ colliders via $WW$ fusion has also been explored in that reference. In the following part of the present work, we do not go further in these phenomenological issues and focus instead in our main purpose here: the computation of the EW radiative corrections for the $WW \rightarrow HH$ scattering process.

4 Renormalization procedure

4.1 Generalities

In this section we present our renormalization program to compute the renormalized 1PI functions within the EChL in covariant $R_\xi$ gauges using a diagrammatic approach. These renormalized 1PI functions, denoted here generically by $\hat{\Gamma}$, receive contributions from the tree level Lagrangian $\mathcal{L}_2 + \mathcal{L}_4$, $\Gamma_{\text{Tree}}$; from the one-loop diagrams using the interaction vertices of $\mathcal{L}_2$ only, $\Gamma_{\text{Loop}}$; and from all the counterterms of $\mathcal{L}_2 + \mathcal{L}_4$, $\Gamma_{\text{CT}}$:

$$\hat{\Gamma}_{n-\text{legs}} = \Gamma_{n-\text{legs}}^{\text{Tree}} + \Gamma_{n-\text{legs}}^{\text{Loop}} + \Gamma_{n-\text{legs}}^{\text{CT}}. \quad (4.1)$$

Notice again the double role of $\mathcal{L}_4$ in the chiral Lagrangian approach: on the one hand, it contributes to a tree level scattering amplitude, and on the other hand it also acts as source of new counterterms in order to remove the extra divergences emerging from the loops computed with $\mathcal{L}_2$, which are not removable by a simple redefinition of the parameters in this part of the Lagrangian.

Our analytical computation here is performed with the various softwares associated to Wolfram Mathematica [13] and starts by implementing our model in FeynRules [14], generating and drawing the Feynman diagrams with FeynArts [15] and performing the main calculations with FormCalc and LoopTools [16]. Some extra checks of the involved one-loop divergences were made using FeynCalc [17] and Package-X [18]. The SM results were obtained following the same steps.

The renormalization program followed in this work is similar to the one we already presented in [3] in the EChL context for Vector Boson Scattering (VBS) processes, like $WZ \rightarrow WZ$, etc. Next we briefly summarize the main aspects of the regularization and multiplicative renormalization prescriptions, the renormalization conditions and then we present the new one-loop diagrams, the new divergences, and the solutions for all the counterterms relevant for $WW \rightarrow HH$ scattering.

4.2 Regularization and Renormalization prescriptions

As it is usual, our regularization procedure of the loop contributions is performed with dimensional regularization [19, 20] in $D = 4 - \epsilon$ dimensions. This method preserves all the relevant symmetries in the bosonic sector of the theory, including chiral invariance (Dirac $\gamma_5$ is not involved in this work since we do not consider the fermionic contributions). Consequently, the scale of dimensional regularization is set to $\mu$ and all the one-loop divergences are expressed in terms of:

$$\Delta_\epsilon = \frac{2}{\epsilon} - \gamma_E + \log(4\pi). \quad (4.2)$$

Concerning the renormalization procedure, we generate the counterterms of all the parameters and fields appearing in the tree level Lagrangian, $\mathcal{L}_2 + \mathcal{L}_4$, by the usual multiplicative renormalization prescription that relates the bare quantities (here denoted by a specific sub- or super-script with a label 0) and the renormalized ones (here with no specific sub- or super-script labels). We have the following relations:

$$H_0 = \sqrt{Z_H}H, \quad B_{0\mu} = \sqrt{Z_B}B_\mu, \quad W_{1,2,3}^{0,1,2,3} = \sqrt{Z_W}W_{1,2,3}^{1,2,3}, \quad \pi_0^{1,2,3} = \sqrt{Z_\pi}\pi^{1,2,3},$$

$$\pi_0^{1,2,3} = \sqrt{Z_\pi}\pi^{1,2,3},$$
\[ v_0 = \sqrt{Z_\pi} (v + \delta v), \quad \lambda_0 = Z_H^{-2} (\lambda + \delta \lambda), \]
\[ g'_0 = Z_B^{-1/2} (g' + \delta g'), \quad g_0 = Z_W^{-1/2} (g + \delta g), \quad \xi^0_{1,2} = \xi (1 + \delta \xi_{1,2}), \]
\[ a^0 = a + \delta a, \quad b^0 = b + \delta b, \quad \kappa^0_{3,4} = \kappa_{3,4} + \delta \kappa_{3,4}, \quad a^0_i = a_i + \delta a_i, \quad (4.3) \]

where the \( Z_i = 1 + \delta Z_i \) are the usual renormalization multiplicative constants and we use the generic notation \( \delta p (\delta a_i) \) for the counterterm of each involved EW parameter \( p \) (effective coefficient \( a_i \)).

With these definitions, our final results for both the renormalized 1PI functions and the \( WW \to HH \) scattering amplitude are expressed in terms of the renormalized quantities, \( m_W, m_Z, m_H, g, g', v, a, b, \kappa_3, \lambda \) and the \( a_i \)'s. Notice that \( \kappa_4 \) and the ghost counterterms do not enter in the present computation and we omit to show them for shortness. On the other hand, the renormalization of the covariant gauge fixing parameters have set to a common renormalized \( \xi \) parameter for all the involved EW gauge bosons. For more details on the technicalities of our renormalization method, see Ref. [3].

Next, we summarize the renormalization conditions. As in Ref. [3] we adopt here a hybrid prescription in which we choose the on-shell (OS) scheme for the EW parameters in the lowest order Lagrangian \( L_2 \) and the \( \overline{MS} \) scheme for all the EChL coefficients. The list of conditions are as follows:

- Vanishing (Higgs) tadpole:
  \[ \hat{T} = 0. \quad (4.4) \]

- The pole of the renormalized propagator of the Higgs boson lies at \( m_H^2 \) and the corresponding residue is equal to 1:
  \[
  \text{Re} \left[ \hat{\Sigma}_{HH}(m_H^2) \right] = 0, \quad \text{Re} \left[ \frac{d\hat{\Sigma}_{HH}}{dq^2}(m_H^2) \right] = 0. \quad (4.5)
  \]

- Properties of the photon: residue equal one; no \( A - Z \) mixing propagators; and the electric charge defined like in QED, since there is a remnant \( U(1)_{\text{em}} \) electromagnetic gauge symmetry:
  \[
  \text{Re} \left[ \frac{d\hat{\Sigma}_{AA}}{dq^2}(0) \right] = 0, \quad \hat{\Sigma}_{ZA}(0) = 0, \quad \hat{\Gamma}_\mu^{\gamma\gamma}|_{\text{OS}} = ie\gamma^\mu. \quad (4.6)
  \]

- The poles of the transverse renormalized propagators of the \( W \) and \( Z \) bosons lie at \( q^2 = m_W^2 \) and \( q^2 = m_Z^2 \), respectively:
  \[
  \text{Re} \left[ \hat{\Sigma}_{WW}(m_W^2) \right] = 0, \quad \text{Re} \left[ \hat{\Sigma}_{ZZ}(m_Z^2) \right] = 0. \quad (4.7)
  \]

- The poles of the renormalized propagators in the unphysical charged sector \( \{W^\pm, \pi^\pm\} \) lie at \( q^2 = \xi m_W^2 \). Therefore:
  \[
  \text{Re} \left[ \hat{\Sigma}^{\xi}_{WW}(\xi m_W^2) \right] = 0, \quad \text{Re} \left[ \hat{\Sigma}_{\pi\pi}(\xi m_W^2) \right] = 0. \quad (4.8)
  \]

- \( \overline{MS} \) scheme for all the involved EChL coefficients.
In particular for \( a, b, \kappa_3, \kappa_4 \) in Eq. (2.4) and the \( a_i \)'s in Eq. (2.10)
The above renormalization conditions on all the EChL parameters determine both the divergent and finite parts involved in all the 1PI functions, and therefore also in the one-loop scattering amplitudes. Notice that the residue for the Higgs and photon fields are set to one in the previous conditions, but the resulting residues \( Z_{W(Z)} \) of the gauge bosons \( W(Z) \), are different to one. Since each external \( W \) provides a factor \( Z_{W}^{1/2} \) to the observable \( S \) matrix, then the corresponding contribution from the residues ( \( A_{\text{res}} \) in Eq. (3.6)) of the two external \( W \)'s in the \( WW \rightarrow HH \) scattering is given by:

\[
A_{\text{res}} = \Re \left[ \frac{d\Sigma_{WW}^{T}}{dq^{2}}(m_{W}^{2}) \right] A^{(0)},
\]

(4.9)

In addition, the Higgs tadpole enters in many parts of the different diagrams contributing to the amplitude. However, with the renormalization condition of Eq. (4.4), the \( A_{1-\text{leg}} \) in Eq. (3.6) vanishes.

4.3 Summary of contributions to the renormalized 1PI functions

We emphasize again that our renormalization program in the \( R_{\xi} \) gauges makes finite all the relevant 1PI Green functions for arbitrary momentum of the external legs (hence, generically, off-shell) and no transversality condition for the EW gauge bosons, \( p_{i} \cdot \epsilon(p_{i}) = 0 \), is applied, for those 1PI results. This means, that our renormalization program is more demanding than the usual renormalization program which gets just finite results for the scattering amplitudes with on-shell external legs. Notice also that in this later case the trasversality conditions for the external gauge bosons are usually used as well.

In the following of this section, we collect the various contributions to the renormalized 1PI functions, already mentioned in Section 3, that enter in \( WW \rightarrow HH \) scattering and that were not involved in our previous computation [3] which was addressed to the \( WZ \rightarrow WZ \) case. In particular, we exhibit now the results for the Green functions involving Higgs bosons in the external legs, corresponding to the vertices \( HHH, HWW, \pi WH, AHH, ZHH \) and \( WWHH \). And, for completeness, we also include in Appendix A a short summary of the other renormalized 1PI functions derived in [3] that also enter here, for the \( WW \rightarrow HH \) scattering. For definiteness, all the explicit analytical results presented in the present paper (and in the appendices) are provided in the Feynman ’t Hooft gauge with \( \xi = 1 \).

The results of the 3-legs functions corresponding to \( H(p_{1})H(p_{2})H(p_{3}), H(q)W^{\mu}(k_{1})W^{\nu}(k_{2}), \pi(q)W^{\mu}(p_{W})H(p_{H}) \) and \( V^{\mu}(q)H(p_{1})H(p_{2}) \) (with \( V = A, Z \), are given by the sum of the LO part (if any), loop contributions, EFT coefficient’s contributions and CT contributions, as follows:

\[
i \hat{\Gamma}_{HHH} = -3i\kappa_{3}m_{H}^{2}v + i\Gamma_{HHH}^{\text{Loop}} - 3i\kappa_{3}m_{H}^{2}v \left( \frac{\delta k_{3}}{k_{3}} + \frac{\delta m_{H}^{2}}{m_{H}^{2}} - \frac{\delta Z_{\pi}}{2} - \frac{\delta v}{v} + \frac{3\delta Z_{H}}{2} \right)
\]

\[
+ \frac{i}{v^{3}} (a_{d}a_{d}(p_{4}^{2} + p_{4}^{2} + p_{4}^{2}) + 2(a_{H} - a_{Hd})(p_{1}^{2} + p_{2}^{2} + p_{3}^{2}))
\]

\[
i \hat{\Gamma}_{WW-W+} = ia^{2}v^{2}g^{\mu\nu} + i\Gamma_{WW-W+}^{\text{Loop}} + ia^{2}v^{2} \left( \frac{\delta a}{a} + \frac{2\delta g}{g} + \frac{\delta v}{v} + \frac{\delta Z_{H}}{2} + \frac{\delta Z_{\pi}}{2} \right) g^{\mu\nu}
\]

\[-i\frac{g^{2}}{2v} \left( (-2a_{HWW} + a_{d} + a_{d}a_{d}a_{d})q^{2} + 2a_{HWW}(k_{1}^{2} + k_{2}^{2}) + a_{HV}m_{H}^{2} \right) g^{\mu\nu}
\]

\[
+ (a_{d} + a_{d}) (k_{1}^{2}k_{2}^{2} + k_{2}^{2}k_{3}^{2} + 2(a_{d} - a_{d}) k_{1}^{2}k_{2}^{2} + 2(a_{d} + a_{d}) k_{1}^{2}k_{2}^{2}),
\]

\[
i \hat{\Gamma}_{W-H} = -iag(p_{W} + p_{H})^{\mu} + i\Gamma_{W-H}^{\text{Loop}} - iag \left( \frac{\delta a}{a} + \frac{\delta g}{g} + \delta Z_{H}/2 + \delta Z_{\pi}/2 \right) (p_{W} + p_{H})^{\mu}
\]
\[ \hat{\Gamma}_{W',H}^{\mu
u} = \hat{\Gamma}_{W,H}^{\mu
u} \cdot \]

Similarly, the result of the 4-legs function corresponding to \( W^\mu(p_1) W^\nu(p_2) H(p_3) H(p_4) \) is given by the sum of the LO part, the loop contributions, the EFT coefficients contributions and the CT contributions as follows,

\[
i\hat{\Gamma}_{W',W-HH}^{\mu
u} = i \frac{b g^2}{2} \frac{1}{2v^2} (g^{\mu\nu} (p_2^2 + p_3^2) (-2a_{dV} + 2a_{HWW} + 2a_{H\nu\nu}) + (p_1 + p_2)^2 (-2a_{dV} - a_{Hd})
- 4((p_1 + p_3)^2 + (p_1 + p_4)^2)a_{HWW} + a_{H\nu\nu}m_H^2)
- 4((a_{Hd} - a_{Hd} + 2a_{HWW})(p_2^2 + p_3^2))

\]

In all the previous expressions above, in Eqs. (4.10)-(4.11), the explicit \( a_i \) coefficients entering are the bare \( a_i \) coefficients, but for shortness we have dropped the superindex 0. Therefore, the \( a_i \)'s included in these equations must be all understood rather as \( (a_i + \delta a_i) \), with these \( a_i \)'s being the renormalized coefficients in the \( \overline{MS} \) and \( \delta a_i \) the corresponding divergent CT needed to cancel the new divergences from the loops of the 1PI functions. As we have said, the computation of the loop contributions to all these 1PI functions are performed with the help of FormCalc and LoopTools. For illustrative purposes, we show in Figs. 6-7, all the generic one-loop diagrams entering in the computation of the previous \( \Gamma^{\text{loop}} \) functions. Notice, that since we are working with covariant \( R_\xi \) gauges we have considered all the possible particles propagating in the loops, namely, GBs, Higgs, EW gauge bosons and ghosts.

Next we provide our results for the divergent (singular) parts of these Loop contributions for the relevant 1PI functions in Eqs. (4.10)-(4.11). All these divergent contributions will set the values of the \( \mathcal{O}(\Delta_v) \) counterterms, both for the EW parameters and the \( a_i \) coefficients, that are relevant for our computation. We get the following results:

\[
i\Gamma^{\text{Loop}}_{HHH}^{\mu\nu}|_{\text{div}} = i \frac{\Delta_v}{16\pi^2} \frac{3}{2v^3} \left( 9\kappa_3 \kappa_4 m_H^4 + 12ab(2m_W^4 + m_Z^4) - 2a^3(p_1^2 p_2^2 + p_2^2 p_3^2 + p_3^2 p_1^2) - 2a(b^2 - b)(p_1^2 + p_2^4 + p_3^4) - 2(2m_W^2 + m_Z^4)(p_2^2 + p_2 + p_3^2) \right)
\]

\[
i\Gamma^{\text{Loop}}_{HHW}^{\mu\nu}|_{\text{div}} = i \frac{\Delta_v}{16\pi^2} \frac{g^2}{12v^2} \left( (3a(2 + a^2)q^2 + a(b^2 - b))(k_1^2 + k_2^2) - 3(a - b)(2a - 3a_3)m_H^2 - 18abm_W^2 + 78am_W^2 - 18am_Z^2 \right) g^{\mu\nu}
\]

\[
i\Gamma^{\text{Loop}}_{WW}^{\mu\nu}|_{\text{div}} = i \frac{\Delta_v}{16\pi^2} \frac{g^2}{6v^2} \left( (p_2^2 + p_3^2)(-9(a^2 - b)\kappa_3 m_H^2 - a^3(3p_1^2 + p_2^2 - 6m_H^2 + 3q^2) + a(6p_2^2 - 34m_W^2 + 14m_Z^2 + b(p_2^2 - 6m_H^2 + 18m_W^2 - 3q^2)) - a^2(2 + a^2 + 2b)p_2^2 - a^2(p_2^2 - 11q^2) + 2(m_W^2 + m_Z^2 - b(p_2^2 + q^2)) \right)
\]

\[
i\Gamma^{\text{Loop}}_{\text{HHH}}^{\mu\nu}|_{\text{div}} = i \Gamma^{\text{Loop}}_{\overline{ZHH}}^{\mu\nu}|_{\text{div}} = 0
\]
\[ i \Gamma^{\text{Loop}}_{WWHH|_{\text{div}}} = -i \frac{\Delta_e}{16 \pi^2} \frac{g^2}{\nu^2} (g^{\mu \nu} (3(-8a^4 + 12a^3 \kappa_3 - 12abk_3 + a^2(10b - 3\kappa_4) - 2b^2 + 3b\kappa_4))m_H^2 \]
\[ + 3b((6b - 26)m_W^2 + 6m_Z^2) \]
\[ (p_1^2 + p_2^2)a^2(6 + 6a^2 - 3b) \]
\[ + 6(p_1 + p_2)^2(1 + a^2)(a^2 - b) + ((p_1 + p_3)^2 + (p_1 + p_4)^2)(4a^4 - 5a^2b + b^2)) \]
\[ - 8(4a^4 - 5a^2b + b^2)p_5^{\mu \nu}p_5^{\mu \nu} + 2(4a^4 + a^2b - 2b^2)(p_5^{\mu \nu}p_5^{\nu \mu} + p_5^{\mu \nu}p_5^{\mu \nu}) \]
\[ - 2(20a^4 - 19a^2b + 2b^2)(p_5^{\mu \nu}p_5^{\nu \mu} + 2(4a^4 + a^2b - 2b^2)(p_5^{\mu \nu}p_5^{\nu \mu} + p_5^{\mu \nu}p_5^{\mu \nu}) \]
\[ + 6a^2(2a^2 + b)(p_5^{\mu \nu}p_5^{\nu \mu} + p_5^{\mu \nu}p_5^{\mu \nu}) \]  

(4.12)

Finally, we present the corresponding results in the SM for the Green functions that are involved in the WW → HH computation and were not given in [3]. We use the ‘bar’ notation for all the 1PI functions in the SM, not to be confused with the previous ones of the HEFT. Notice that, contrary to the HEFT, in the SM case, the multiplicative renormalization constant for the EW parameters entering in \( L \) is solved sequentially. The CTs corresponding to \( \Delta \) results finite. This procedure leads to a system of equations by demanding the cancellation of \( O(\Delta_e) \) contributions for each involved Lorentz structure and in each term in the momentum powers expansion of the Green functions, that it is solved sequentially. The CTs corresponding to

\[ 4.4 \text{ Renormalization of the EFT parameters} \]

In this section we present the results for the renormalization of the EFT parameters. These include the EW parameters entering in \( L_2 \), like \( g, g' \), etc., and the EChL coefficients, namely, \( a, b, \kappa_3 \) entering in \( L_3 \), and the \( a_i \) coefficients entering in \( L_4 \).

First, we determine the divergent parts (called in short \( \delta_r \)) of all the counterterms requiring that all the renormalized 1PI functions at arbitrary values of the external leg momenta (generically off-shell) results finite. This procedure leads to a system of equations by demanding the cancellation of \( O(\Delta_e) \) contributions for each involved Lorentz structure and in each term in the momentum powers expansion of the Green functions, that it is solved sequentially. The CTs corresponding to
the $L_2$ parameters in Eq. (2.4), except for $b$, $\kappa_3$ and $\lambda$, and some of the $a_i$'s coefficients in Eq. (2.10) were already derived in our previous work [3]. Respect to this reference, we add now the Green functions with Higgs bosons corresponding to the vertices $HHH$, $HW$, $\pi W$ and $WW^HH$ (notice that the corresponding ones to $AHH$ and $ZHH$ are finite and do not have new EChL coefficients). In particular, we derive $\delta \lambda$ from the tadpole's counterterm; $\hat{\Gamma}_{HHH}$ sets $\delta_0 \kappa_3$, $\delta_0 a_{dd\Box}$, $\delta_0 a_{dd\Box}$ and $\delta_0 a_{dd\Box}$; $\hat{\Gamma}_{HHW}$ sets $\delta_0 a_{HHW}$, $\delta_0 a_{d2}$, $\delta_0 a_{d2}$, $\delta_0 a_{d2}$, $\delta_0 a_{H11}$; $\hat{\Gamma}_{WWW}$ sets $\delta_0 a_{d2}$, $\delta_0 a_{d2}$, $\delta_0 a_{d2}$, $\delta_0 a_{H11}$, $\delta_0 a_{d2}$, $\delta_0 a_{d2}$, $\delta_0 a_{H11}$; and with the singular parts of all the CTs, we check that $\hat{\Gamma}_{HW}$ gives a finite contribution to the scattering amplitude.

Second, these divergent parts of the CTs can also be determined by using the renormalization conditions of Eqs. (4.4)-(4.8). They allow us to write the counterterms as functions of the undressed $1PI$ functions. Then we have used this second procedure as a check of our results that we obtain solving the system described in the previous paragraph. Also, with this second procedure we can determine the finite contributions to the counterterms (if any) and we use them in the final numerical computation of the one-loop cross section in the next section. Therefore, we postpone the estimates of the finite contributions for the next section and focus here in the derivation of the singular parts of the EChL counterterms. For completeness, we also provide in Eq. (A.5) the divergent counterterms for the EW parameters derived in our previous work together with $\delta \lambda$ (that enters now in the $s$ channel) in Eq. (A.5). The corresponding SM results, obtained from the 1-leg and 2-legs Green functions, were presented and compared with the EChL in [3] and we do not repeat them here.

Our results for the divergent parts of the full set of EChL coefficients are then summarized as follows:

\[
\begin{align*}
\delta_e a & = \frac{3 \lambda}{16 \pi^2 2 d t^2} \bigg( (a^2 - b)(a - \kappa_3) m_H^2 + a \bigg( (1 - 3 a^2 + 2b) m_W^2 + (1 - a^2) m_Z^2 \bigg) \bigg), \\
\delta_e b & = -\frac{\Delta_e}{16 \pi^2 2 d t^2} \bigg( (a^2 - b)(8 a^2 - 2 b - 12 a \kappa_3 + 3 \kappa_4) m_H^2 \\
& \quad + 6 a^2 b (2 m_W^2 + m_Z^2) - 6 b (m_W^2 + m_Z^2) - 6 b^2 m_W^2) \\
\delta_e \kappa_3 & = -\frac{\Delta_e}{16 \pi^2 2 m_H^2 t^2} \bigg( \kappa_3 (a^2 - b + 9 a^2 - 6 \kappa_3) m_H^2 - 3 (1 - a^2) \kappa_3 m_H^2 (m_W^2 + m_Z^2) \\
& \quad + 6 (-2 a b + 2 a^2 \kappa_3 + b \kappa_4) (2 m_W^2 + m_Z^2)) \bigg), \\
\delta_e a_{dd\Box} & = -\frac{\Delta_e}{16 \pi^2} \frac{a^4 + a^2 b + b^2}{6}, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{a^2 (a^2 - b)}{6}, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{a (a^2 - b)^2}{6}, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{a (2 a^2 + a^2)}{6}, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{a^2 (a^2 + b)}{6}, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{3 a^2}{4}, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{3 a (a^2 - b)}{2}, \\
\delta_e a_{d2} & = 0, \\
\delta_e a_{d2} & = -\frac{\Delta_e}{16 \pi^2} \frac{3 a (a^2 - b)}{2}.
\end{align*}
\]
where we have used the bold notation for the new EChL coefficients in this computation respect to [3]. As it is expected from the $SU(2)_L \times U(1)_Y$ gauge invariant construction of $\mathcal{L}_4$, we have found no $\xi$-dependence in any of the CTs of the EChL coefficients (in contrast to the results for the CTs of the EW parameters, like $\delta g$ etc that are in general $\xi$ dependent, see [3]). We also see in these results that some of these CTs vanish for the choice $a = b = \kappa_3 = \kappa_4 = 1$, and some others do not, like $a_{d\bar{d}V1}$, $a_{d\bar{d}V2}$, $a_{11}$, $a_{H\bar{H}VV}$, $a_{d3}$, $a_{Hd3}$, $a_{\Box}$ and $a_{H\Box}$.

Some comments about the previous results in Eq. (4.14) are in order. First, we wish to notice that these results, to our knowledge, are the only ones within the EChL that apply to the most general and complete renormalization program of off-shell one-loop 1PI functions and including all types of bosonic loop diagrams in the $R_\xi$ gauges. However, it is pertinent to compare our results with some previous results of the EChL one-loop divergences and counterterms in the literature. We will summarize this comparison in the following. Firstly, we compare with previous works computing the one-loop scattering amplitude. The renormalization of the $W_L W_L \to HH$ process was studied to one-loop within the EChL previously in [4]. It was done by means of the ET, i.e., replacing the external $W_L$'s by the $w$ GB's and studying the corresponding $ww \to HH$ scattering with just chiral loops (meaning loops with only GBs and Higgs in the internal propagators), and assuming massless GBs (as in Landau gauge, i.e., for $\xi = 0$). More recently in [5], the loop contributions to the $W_L W_L \to HH$ scattering amplitude were computed as well by means of the ET, i.e also for $ww \to HH$ scattering, but improving the previous computation of [4], by considering all kind of bosonic one-loop diagrams in this scattering of GBs. They also used the Landau gauge, i.e. with massless GBs, and simplify the computation by assuming the so-called isospin limit with $m_W = m_Z$. We have further improved those two computations, in several aspects. We do not use the ET, i.e, we consider gauge bosons in the external legs, we work in generic $R_\xi$ gauges (i.e. with massive GBs) and we do not work in the isospin limit, i.e. for us $m_W$ and $m_Z$ are different, as corresponds to the physical on-shell gauge boson masses. Furthermore, we consider the full set of 1PI functions involved in the amplitude and include all kind of diagrams in those functions. The full set of one-loop diagrams computed here are in consequence different than in [5]. However, we can make contact with some of the results in this reference, by specifying our results for the particular assumptions and approximations of that reference. For instance, taking into account the differences in the conventions, and setting $m_Z = m_W$, we find agreement for the CTs of $a$, $b$, $\lambda$, $\kappa_3$, and $a_{d3}$. On the other hand, to compare with this reference, it is convenient to use the reduced set of NLO coefficients that, as explained in the previous sections, can be obtained by the use of the equations of motion. Concretely, the EChL NLO coefficients appearing in the scattering amplitude are those presented in Eq. (3.17) and appear within the particular combinations of coefficients given in Eq. (2.13). Therefore these are the ones that should be compared with [5]. From our results in Eq. (4.14), our prediction for the divergences of these combinations are:

$$\delta_\eta = \delta_\lambda = \frac{16\pi^2}{3}(a^2 - b)^2,$$

$$\delta_\delta = \frac{16\pi^2}{12}(a^2 - b)(7a^2 - b - 6),$$

$$\delta_\xi = \frac{16\pi^2}{a(1 - a^2)},$$

$$\delta_\xi(a_{HVV} - 2a_{\Box} + 2aa_{\Box}) = \frac{16\pi^2}{3}(3\kappa_3 a(1 - a^2) + 2b - 2a^2(2 + 3b) + 8a^4)$$
The two first lines in the above equation are in agreement with the result for $\eta$ and $\delta$ in [4, 5], where $a_{11}$, $a_{d3}$ and $a_{dE}$ were not considered. It is interesting to remark that the combinations in Eq. (4.15) indeed vanish for $a = b = \kappa_3 = 1$ as expected in the comparison with the SM.

Secondly, we compare our results with Ref. [6]. In this work, the renormalization of one-loop 1PI functions was performed for off-shell external legs but they considered the pure scalar theory, i.e., only the Higgs and GBs sector of the EChL and worked with massless GBs (as in Landau gauge, with $\xi = 0$). No gauge or ghost fields were included and, therefore, no gauge-fixing. We find agreement in the divergences found for the subset of $a_i$’s involving in the scalar sector (the coefficients in the notation of [6] are specified inside the parentheses). Concretely, we agree in: $a (ac), b (bc), \kappa_3 (\mu_3), a_{d4V1} (c_8), a_{d4V2} (c_20), a_{11} (c_9), a_{H11} (a_9), a_{HH11} (b_9), a_{dE} (c_\Delta H), a_{dV} (c_7), a_{HdV} (a_7), a_{d3} (c_10), a_{Hd3} (a_{10}), a_{\Box} (c_{\Box H})$ and $a_{H\Box} (a_{\Box H})$.

Thirdly, we compare our results with others that do not study scattering amplitudes but are devoted to the renormalization of the Lagrangian. In particular, the renormalization of the EChL was studied in the path integral formalism, using the background field method, in [7, 21, 22]. The most complete comparison of our results should be done with the bosonic loop results of [7, 22] since they also included all loops of scalar and gauge particles. However, the comparison with the path integral results is tricky since they use the equations of motion to reduce the number of operators they also included all loops of scalar and gauge particles. Therefore, some off-shell divergences do not appear in their results and some others are redefined by the use of the equations of motion. They also use redefinitions of the fields (in particular the Higgs field) to reach the canonical kinetic term in the Lagrangian. On the other hand, the parametrization used in [7, 22] is also very different than here and not straightforward to compare with. For example, the divergence canceled by ours $a_{ddV}, a_{ddZ}$ and $a_{Hdd}$ in the $HH$ Green function is absorbed via the Higgs field redefinition in their context.

Finally, we summarize in the following the main results regarding the renormalization group running equations (RGEs) for the NLO EChL coefficients, which complement those given in our previous work [3]. These RGEs can be easily derived from the previous results in Eq. (4.14) and taking into account the relation between the renormalized and bare coefficients given by $a_i^0 = a_i + \delta a_i$. In the $\overline{MS}$ scheme (with $\mu$ being the scale of dimensional regularization in $D = 4 - \epsilon$ dimensions), the running $a_i(\mu)$ can be written as follows:

$$a_i(\mu) = a_i^0 - \delta a_i(\mu), \quad \delta a_i(\mu) = \delta, \quad a_i = \frac{\Delta \gamma_a}{16\pi^2} \mu^2, \quad \delta, \quad a_i = \frac{\Delta \gamma_a}{16\pi^2} \mu^2, \quad (4.16)$$

where the divergent $\delta, \gamma_a$ is written in terms of the anomalous dimension $\gamma_a$ of the corresponding effective operator. The running and renormalized $a_i$’s can then be related, in practice, by:

$$a_i(\mu) = a_i + \frac{\gamma_a}{16\pi^2} \mu^2$$

The set of RGEs for all the $a_i$’s then immediately follow:

$$a_i(\mu) = a_i(\mu') + \frac{1}{16\pi^2} \gamma_a \log \left( \frac{\mu^2}{\mu'^2} \right), \quad (4.18)$$

where the specific value of $\gamma_a$ for each coefficient can be read from Eq. (4.14). For instance, in the case of the two most relevant NLO-EChL coefficients for the present $WW \to HH$ scattering, $\eta$ and $\delta$, we get the following RGE’s:

$$\eta(\mu) = \eta(\mu') - \frac{1}{16\pi^2} \frac{1}{3} (a^2 - b)^2 \log \left( \frac{\mu^2}{\mu'^2} \right),$$
\[ \delta(\mu) = \delta(\mu') + \frac{1}{16\pi^2} \frac{1}{12} (a^2 - b)(7a^2 - 6b - 6) \log \left( \frac{\mu^2}{\mu'^2} \right), \]  

which are in agreement with the RGEs given in [4]. Notice that, in particular, for \( a = b = 1 \) these two EChL coefficients \( \eta \) and \( \delta \) do not run, therefore they are RGEs invariants.

5 Numerical results for \( W^+_L W^-_L \to HH \)

In this section we study the numerical predictions from the EChL for the cross section of the scattering process \( WW \to HH \) and compare the tree level rates with the one-loop rates. We also compare these rates with the SM case which have been computed independently here following the same procedure as for the EChL case. It is also interesting the comparison of this SM case with previous SM results in the literature [23]. Since, as we have already said, the dominant contribution to this scattering process at the TeV domain is that coming from the longitudinally polarized gauge boson modes, we will focus in this section in this most relevant cross section, i.e in \( \sigma(W^+_L W^-_L \to HH) \). In addition, this numerical study of the radiative corrections will be devoted to the most relevant coefficients of the NLO-EChL, that as already said are the parameters \( \eta \) and \( \delta \). For simplicity, the LO-EChL parameters will be set here to the SM default values, i.e. in the following we set \( a = b = \kappa_3 = \kappa_4 = 1 \). All the numerical computations presented here have been performed with the help of FormCalc and LoopTools and, for definiteness, we choose the Feynman ‘t Hooft gauge, i.e we fix \( \xi = 1 \).

First of all, it is worth mentioning that we have done a numerical check of the finiteness of the predicted one-loop cross section in both cases, the EChL and the SM. This is done indirectly, by checking numerically the renormalization \( \mu \) scale independence of the result. This is not a trivial check at all, since the computation of the one-loop amplitude from the 1PI functions amounts to the evaluation of more than 500 one-loop diagrams where each one depends on this \( \mu \) scale. Thus, the cancellation of the \( \mu \) dependence among the various diagrams found in the final result is a quite convincing check. Notice that for the studied case here of \( a = b = 1 \) the two parameters, \( \delta \) and \( \eta \), as already said, do not run, therefore they have equal value at any assumed \( \mu \) scale.

We next summarize our numerical results for \( \sigma(W^+_L W^-_L \to HH) \) as a function of the center-of-mass energy \( \sqrt{s} \) in the two figures 4 and 5. In Fig. 4 we study the effect of \( \eta \), and in Fig. 5 we study the effect of \( \delta \). In both cases, we have explored the following values for those coefficients, \( \pm 0.01 \) and \( \pm 0.001 \), which are allowed by present experimental data. In both plots we have included, for comparison, the following rates: 1) the tree level predictions for the EChL, EChL\(^{(2+4)}\), 2) the full one-loop predictions for the EChL, EChL\(^{\text{Full}}\), 3) the tree level predictions for the SM, SM\(^{\text{Tree}}\) (which coincide with the LO result in the EChL, EChL\(^{(2)}\)), and 4) the full one-loop predictions for the SM. In the lower part of these plots we display the predictions for the relative size of the one-loop correction respect to the tree level prediction, by means of \( \delta_{1\text{-loop}} \) that is defined by,

\[ \delta_{1\text{-loop}} = \frac{\sigma_{\text{Full}} - \sigma_{\text{Tree}}}{\sigma_{\text{Tree}}} \]  

(5.1)

The main features learnt from these two figures are the following:

⋆ We get a one-loop correction in the SM case that is negative and increases in size with energy. The size of \( \delta_{1\text{-loop}} \) can be up to \( \sim -20\% \) at the maximum energy studied of \( \sqrt{s} = 3 \) TeV and it is in accordance with [23].

⋆ The predictions from the EChL, both at tree level and one-loop level, show a clear departure from the corresponding SM prediction. The largest deviations occur for the largest \( |\delta| \) and/or \( |\eta| \) considered values.
One–loop EChL versus SM: $\eta$ effect

Figure 4. Cross section prediction for $W^+_L W^-_L \to HH$ as a function of the energy $\sqrt{s}$ within the EChL at one-loop level (solid lines) and comparison with the tree level prediction (dashed lines). The effect of the NLO parameter $\eta$ is displayed, assuming values for this parameter of $\pm 10^{-2}$ and $\pm 10^{-3}$. The LO parameters are set to $a = b = \kappa_3 = \kappa_4 = 1$. The other NLO-parameters are set to zero. The SM predictions at tree level (pink) and 1-loop level (red) are also included. The relative size of the one-loop prediction respect to the tree level one, defined by means of $\delta_{1\text{-loop}}$ in Eq. (5.1), is displayed at the bottom of this figure. The color code is: red (SM), orange (EChL, $\eta = 10^{-3}$), brown (EChL, $\eta = -10^{-3}$), bright green (EChL, $\eta = 10^{-2}$), green (EChL, $\eta = -10^{-2}$).
One-loop EChL versus SM: $\delta$ effect

![Graph](image)

Figure 5. Cross section prediction for $W^+_L W^-_L \rightarrow HH$ as a function of the energy $\sqrt{s}$ within the EChL at one-loop level (solid lines) and comparison with the tree level prediction (dashed lines). The effect of the NLO parameter $\delta$ is displayed, assuming values for this parameter of $\pm10^{-2}$ and $\pm10^{-3}$. The LO parameters are set to $a = b = \kappa_3 = \kappa_4 = 1$. The other NLO-parameters are set to zero. The SM predictions at tree level (pink) and 1-loop level (red) are also included. The relative size of the one-loop prediction respect to the tree level one, defined by means of $\delta_{1\text{-loop}}$ in Eq. (5.1), is displayed at the bottom of this figure. The color code is: red (SM), orange (EChL, $\delta = 10^{-3}$), brown (EChL, $\delta = -10^{-3}$), bright green (EChL, $\delta = 10^{-2}$), green (EChL, $\delta = -10^{-2}$).

We get a one-loop correction in the EChL case that depending on the value of the coefficient and the value of the energy can be either negative or positive. For $\eta$ we find it negative.
for $\pm 10^{-3}$ and $+10^{-2}$, at all studied energies. But it is positive for $-10^{-2}$ in the interval $1.3\text{TeV} < \sqrt{s} < 3\text{TeV}$. For $\delta$ we find it negative for $+10^{-2}$ and $+10^{-3}$ at all the studied energies. But it is positive for $-10^{-2}$ in the interval $0.9\text{TeV} < \sqrt{s} < 3\text{TeV}$ and for $-10^{-3}$ in the interval $2.5\text{TeV} < \sqrt{s} < 3\text{TeV}$.

- Overall we see that the maximum size of the radiative one-loop correction found in the EChL is about $-15\%$ in both $\eta$ and $\delta$ cases. This is a bit lower than in the SM case.

- Finally, notice that the values of the coefficients $\eta$ and $\delta$ specified in these plots refer to the renormalized parameter values. However, since we have taken in these plots, $a = b = 1$, they do not depend on the $\mu$ scale. This, together with the previously mentioned $\mu$ independence of the sum of all the contributing one-loop diagrams, complements the check of $\mu$ scale invariance of the total cross section result.

6 Conclusions

In this work we have computed the one-loop electroweak radiative corrections to the scattering process $WW \rightarrow HH$ within the context of the Higgs Effective Field Theory, considering that the new Higgs Physics beyond the SM enters only in the bosonic sector and it is given by the Electroweak Chiral Lagrangian. We consider this EChL with all the relevant effective operators of chiral dimensions two and four and present the computation in terms of the involved 1PI Green functions in covariant $R_\xi$ gauges. An ambitious renormalization program for all these one-loop 1PI functions involved is developed, considering the most general case with arbitrary momenta for the external particle legs. This renormalization procedure is more demanding than just requiring a finite result for the one-loop amplitude with external on-shell particles, and it has the advantage of being applicable to several processes sharing some of those 1PI functions with the amplitude under study here. We have applied this same procedure for both cases, the EChL and the SM. In particular, we have used here for $WW \rightarrow HH$ scattering some of the previous renormalized 1PI functions computed in our previous work devoted to $WZ \rightarrow WZ$ scattering. We have used those functions also here and then we have complemented them with the new one-loop 1PI functions for the new vertices involving the Higgs particle, $HHH$, $HWW$, $\pi WH$, $AHH$, $ZHH$ and $WWHH$, whose results are presented here.

One of the most important results contained in this work, are the full set of divergent counterterms derived for the EChL coefficients, summarized in Eqs. (4.14)-(4.15). These set of divergences do also set the corresponding set of RGEs for the involved HEFT coefficients, according to Eqs. (4.16)-(4.19). A small subset of these results have been cross-checked with previous results in the literature which were done following a very different approach to ours and we have found agreement. A discussion on this comparison has also been included in the present work.

The final part of this paper has been devoted to the numerical computation of the one-loop radiative corrections to the cross section of the $W_LW_L \rightarrow HH$ scattering process. Again we have done in parallel both the computation for the EChL and for the SM. In the case of the SM we have found agreement with the previous result in [23]. Our estimate of the one-loop correction respect the tree level cross section in the SM gives a negative value whose maximum size is reached at the largest energy studied of $\sqrt{s} = 3$ TeV and is about $\delta_{1-\text{loop}} \sim -20\%$. In the EChL case, where we have considered the effects from the two most relevant parameters $\eta$ and $\delta$, we find also important one-loop corrections, with a maximum of about $\delta_{1-\text{loop}} \sim -15\%$, a bit lower than in the SM case. The size of this correction depends on the energy and the particular values of the EChL coefficients. The largest departures of the HEFT respect to the SM prediction are found for the largest studied values of $\delta$ and/or $\eta$. There are also some input values for these parameters and energy ranges that
provide a positive one-loop correction although small, being below 5%. All these numerical results are summarized in Figs. 4 and 5.

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Appendices

A Summary of complementary 1PI functions

For completeness, we summarize in this appendix the renormalized 1PI functions already derived in [3] that also enter in the present $WW \to HH$ scattering. We have taken these analytical results from that previous reference, but we have displayed them here by setting $\xi = 1$, as in the main results of this paper. The definition of the EChL coefficients, parameters and functions entering in these complementary functions can be found in [3].

Starting with the EChL, the 1-leg function (Higgs tadpole) is

$$i\tilde{T} = iT^{\text{Loop}} - i\delta T, \quad \delta T = (\delta m_H^2 - m_H^2(-2\delta Z_H + \delta\lambda/\lambda + Z_\pi + 2\delta v/v))v \quad (A.1)$$

Notice that now we are fixing a typo in this counterterm respect to our previous publication.

The 2-legs functions are

$$-i\Sigma_{HH}(q^2) = -i\Sigma_{HH}^{\text{Loop}}(q^2) + i\left(\delta Z_H(q^2 - m_H^2) - \delta m_H^2\right) + i\frac{2a_0}{v^2}q^4,$$

$$i\Sigma_{WW}(q^2) = i\Sigma_{WW}^{\text{Loop}}(q^2) - i\left(\delta Z_W (q^2 - m_W^2) - \delta m_W^2\right),$$

$$i\Sigma_{W\pi}(q^2) = i\Sigma_{W\pi}^{\text{Loop}}(q^2) + i\left(\frac{2\xi_2 - \xi_1}{2}m_W^2 + q^2g^2a_{11}\right),$$

$$-i\Sigma_{\pi\pi}(q^2) = -i\Sigma_{\pi\pi}^{\text{Loop}}(q^2) + i\left((q^2 - m_W^2)\delta Z_\pi - \delta m_W^2 - m_W^2\delta_2\right) - i\frac{g^2}{m_W^2}q^4a_{11}. \quad (A.2)$$

In these formulas above, the $a_i$ coefficients must be understood again as $a_i + \delta a_i$.

On the other hand, the 3-legs functions corresponding to $W^\mu(k_1)W^\nu(k_2)V^\rho(q)$ (with $V = A, Z$) enter in the present work just at the LO, therefore, they take the usual tree level expression:

$$i\Gamma_{W^+W^-A}^{\mu\nu\rho} = -ig_{sw}(g^\mu\nu(k_1 - k_2)^\rho + g^\rho\nu(k_2 - q)^\mu + g^\rho\mu(q - k_1)^\nu),$$

$$i\Gamma_{W^+W^-Z}^{\mu\nu\rho} = -ig_{cw}(g^\mu\nu(k_1 - k_2)^\rho + g^\rho\nu(k_2 - q)^\mu + g^\rho\mu(q - k_1)^\nu). \quad (A.3)$$

In contrast, the $AHH$ and $ZHH$ 1PI functions in the second diagram of Fig. 1 vanish at LO and they get only NLO contributions that are finite.

Next we summarize the loops divergences of all the above 1PI functions. These are:

$$iT^{\text{Loop}}|_{\text{div}} = i\frac{\Delta_1}{16\pi^2} \frac{3}{2v}\left(\kappa_3 m_H^4 + 2a(2m_W^4 + m_Z^4)\right),$$

$$-i\Sigma_{HH}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_1}{16\pi^2} \frac{3}{2v^2}\left(a^2q^4 - 2a^2(2m_W^2 + m_Z^2)q^2 + (3\kappa_3 + \kappa_4)m_H^4 + (4a^2 + 2b)(2m_W^4 + m_Z^4)\right),$$

$$i\Sigma_{WW}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_1}{16\pi^2} \frac{g^2}{12}\left((39 - a^2)q^2 + 3(a^2 - b)m_H^2 + 3(13 - 3a^2)m_W^2 - 9m_Z^2\right),$$

$$i\Sigma_{W\pi}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_1}{16\pi^2} \frac{g^2}{4}\left(a^2q^2 + (a^2 - b)m_H^2 + (13 - 3a^2)m_W^2 - 3m_Z^2\right),$$

$$i\Sigma_{\pi\pi}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_1}{16\pi^2} \frac{g^2}{4}\left(-a^2q^2 - (a^2 - b)m_H^2 - (17/3 - 3a^2)m_W^2 + (7/3)m_Z^2\right),$$

$$-i\Sigma_{\pi\pi}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_1}{16\pi^2} \frac{g^2}{4}\left(a^2q^2 + \frac{q^2}{v^2}(a^2 - b)m_H^2 - (5/3 + 3a^2)m_W^2 - (5/3)m_Z^2\right) \quad (A.4)$$
The resulting divergent part of the EChL counterterms for the EW parameters were also derived in [3]. We include those results here, including now explicitly $\delta, \lambda$, setting $\xi = 1$:

$$\delta Z_H = \frac{\Delta_e}{16\pi^2} \frac{3(a^2-b)}{v^2} (2m_W^2 + m_Z^2), \quad \delta T = \frac{\Delta_e}{16\pi^2} \frac{3}{2v} (\kappa_3 m_H^4 + 2a (2m_W^4 + m_Z^4)), \quad \delta m_H^2 = \frac{\Delta_e}{16\pi^2} \frac{3}{2v^2} ((3\kappa_3^2 + \kappa_4) m_H^4 - 2a^2 m_W^2 (2m_W^2 + m_Z^2) + (4a^2 + 2b)(2m_W^4 + m_Z^4)), \quad \delta m_Z^2 = \frac{\Delta_e}{16\pi^2} \frac{3}{2v} (3(a^2-b)m_H^2 + (78-10a^2)m_W^2 - 9m_Z^2), \quad \delta g' / g' = 0, \quad \delta g / g = -\frac{\Delta_e}{16\pi^2} 2g^2;$$

$$\delta, \xi_1 = \frac{\Delta_e}{16\pi^2} \frac{g^2}{12} (39-a^2), \quad \delta, \xi_2 = \frac{\Delta_e}{16\pi^2} \frac{1}{3v^2} (6(a^2-b)m_H^2 + (73-19a^2)m_W^2 - 14m_Z^2), \quad \delta, \xi_3 = \frac{\Delta_e}{16\pi^2} \frac{1}{v^2} ((a^2-b)m_H^2 - (5/3 + 3a^2)m_W^2 - (5/3)m_Z^2), \quad \delta, \xi_4 = \frac{\Delta_e}{16\pi^2} \frac{2m_H^2 + m_Z^2}{3v^2};$$

$$\delta, \lambda = \frac{\Delta_e}{16\pi^2} \frac{1}{4v^4} (2a^2 (m_H^4 + m_Z^4 + m_W^4 + m_Z^2) + 6 (2m_W^4 + m_Z^2)) - 6a (2m_W^4 + m_Z^2) - 2b (m_H^4 - 3 (2m_W^4 + m_Z^2)) + 3(3\kappa_3^2 - \kappa_3 + \kappa_4)m_H^4 - 6m_H^2 (m_W^2 + m_Z^2).$$

Finally the corresponding results in the SM with $\xi = 1$ are:

$$i \hat{T} = i \hat{T}_{\text{SM}}^{\text{Loop}} - i \delta \hat{T}, \quad \delta \hat{T} = (\delta m_H^2 - m_H^2(-\delta Z_\phi + \delta \lambda / \lambda + 2\delta v / v))v, \quad -i \hat{\Sigma}_{HH} (q^2) = -i \hat{\Sigma}_{HH}^{\text{Loop}} (q^2) + i (\delta Z_\phi (q^2 - m_H^2) - \delta m_H^2),$$

$$i \hat{\Sigma}_{WW} (q^2) = i \hat{\Sigma}_{WW}^{\text{Loop}} (q^2) - i (\delta Z_W (q^2 - m_W^2) - \delta m_W^2);$$

$$i \hat{\Sigma}_{W} (q^2) = \hat{\Sigma}_{W}^{\text{Loop}} (q^2) + i ((q^2 - m_W^2) \delta Z_W + \delta m_W^2 + q^2 \delta \xi_1), \quad \hat{\Sigma}_{W}^{\text{Loop}} (q^2) = \frac{\delta \xi_2 - \delta \xi_4}{2} m_W^2;$$

$$-i \hat{\Sigma}_{\phi} (q^2) = -i \hat{\Sigma}_{\phi}^{\text{Loop}} (q^2) + i ((q^2 - m_W^2) \delta Z_\phi - \delta m_W^2 - m_W^2 \delta \xi_2 - \delta \hat{T} / v), \quad \hat{\Sigma}_{\phi}^{\text{Loop}} (q^2) = \frac{\delta \xi_2 - \delta \xi_4}{2} m_W^2,$$

and again the WWA and the WWZ vertices enter only at the tree level in this amplitude, therefore:

$$i \hat{T}^{\text{Tree}_{WW-A}} = -ig s_w (g^{\mu\nu} (k_1 - k_2)^\rho + g^{\mu\rho} (k_2 - q)\nu + g^{\rho\nu} (q - k_1)^\mu), \quad i \hat{T}^{\text{Tree}_{WW-Z}} = -ig c_w (g^{\mu\nu} (k_1 - k_2)^\rho + g^{\mu\rho} (k_2 - q)\nu + g^{\rho\nu} (q - k_1)^\mu),$$

whereas the $AHH$ and $ZHH$ vertices vanish at the tree level and these 1PI functions only get 1-loop corrections that are finite.

The loop divergences of the above 1PI functions in the SM are:

$$i T_{\text{Loop}}^{\text{div}} = i \frac{\Delta_e}{16\pi^2} \frac{1}{2v} (3m_H^4 + 6 (2m_W^4 + m_Z^4) + m_H^2 (2m_W^2 + m_Z^2))$$
dependence here. shortness. conclusions for the diagrams in the second column corresponding to $\Gamma_{\pi W H}$ dependence due to the behaviour of the scalar loop diagrams in the EChL and the SM. The same a the results in the EChL depend on are present in the SM computation. 

In this Appendix we present the relevant one-loop diagrams entering in the computation of the 1PI B Relevant one-loop diagrams computations. These diagrams were generated with FeynArts [15] and we collect them by the new Green functions, $\Gamma_{\pi W H}$ functions for W W functions for $\pi W H$ Higgs boson or Goldstone bosons and wavy lines refer to all possible EW gauge bosons. Notice the computation in [3].


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In this Appendix we present the relevant one-loop diagrams entering in the computation of the 1PI B Relevant one-loop diagrams computations. These diagrams were generated with FeynArts [15] and we collect them by the new Green functions, $\Gamma_{\pi W H}$ functions for W W functions for $\pi W H$ Higgs boson or Goldstone bosons and wavy lines refer to all possible EW gauge bosons. Notice the computation in [3].

\begin{equation}
-i\Sigma_{HH}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_\epsilon}{16\pi^2} \frac{1}{v^2} (-4(2m_W^2 + m_Z^2)q^2 + 15m_H^4 + 18(2m_W^2 + m_Z^2) + m_H^2(2m_W^2 + m_Z^2))
\end{equation}

\begin{equation}
-i\Sigma_{W+}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_\epsilon}{16\pi^2} g^2 \frac{2}{6} (19q^2 + 6(2m_W^2 - m_Z^2))
\end{equation}

\begin{equation}
-i\Sigma_{W-}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_\epsilon}{16\pi^2} g^2 (2m_W^2 - m_Z^2)
\end{equation}

\begin{equation}
-i\Sigma_{W\pi}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_\epsilon}{16\pi^2} g^2 (-2m_W^2 + 3m_Z^2)
\end{equation}

\begin{equation}
-i\Sigma_{\pi\pi}^{\text{Loop}}(q^2)|_{\text{div}} = i\frac{\Delta_\epsilon}{16\pi^2} g^2 \frac{2}{2} (-4(2m_W^2 + m_Z^2)q^2 + 3m_H^4 + 12m_W^4 + 6m_Z^4 + m_H^2(2m_W^2 + m_Z^2))
\end{equation}

and the resulting divergences of the counterterms are:

\begin{equation}
\delta_\epsilon ZH = \frac{\Delta_\epsilon}{16\pi^2} \frac{1}{v^2} (2m_W^2 + m_Z^2), \quad \delta_\epsilon T = \frac{\Delta_\epsilon}{16\pi^2} \frac{1}{2v} (3m_H^4 + 6(2m_W^2 + m_Z^2) + m_H^2(2m_W^2 + m_Z^2)),
\end{equation}

\begin{equation}
\delta_\epsilon m_H^2 = \frac{\Delta_\epsilon}{16\pi^2} \frac{3}{2v^2} (5m_H^4 - m_H^2(2m_W^2 + m_Z^2) + 6(2m_W^4 + m_Z^4))
\end{equation}

\begin{equation}
\delta_\epsilon ZB = -\frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{6}, \quad \delta_\epsilon ZW = \frac{\Delta_\epsilon}{16\pi^2} \frac{19g^2}{2},
\end{equation}

\begin{equation}
\delta_\epsilon m_Z^2 = -\frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{6} (31m_W^2 - 6m_Z^2)
\end{equation}

\begin{equation}
\delta_\epsilon m_H^2 = \frac{\Delta_\epsilon}{16\pi^2} \frac{g^2}{6v_w} ((10 - 4c_w^2)m_W^2 + 7m_Z^2)
\end{equation}

\begin{equation}
\delta_\epsilon g'/g' = 0, \quad \delta_\epsilon g/g = -\frac{\Delta_\epsilon}{16\pi^2} \frac{2g^2}{2},
\end{equation}

\begin{equation}
\delta_\epsilon \xi_1 = \frac{\Delta_\epsilon}{16\pi^2} \frac{19g^2}{6}, \quad \delta_\epsilon \xi_2 = \frac{\Delta_\epsilon}{16\pi^2} \frac{2}{3v^2} (25m_W^2 - 9m_Z^2),
\end{equation}

\begin{equation}
\delta_\epsilon v/v = \frac{\Delta_\epsilon}{16\pi^2} \frac{2m_W^2 + m_Z^2}{v^2},
\end{equation}

\begin{equation}
\delta_\epsilon \lambda = \frac{\Delta_\epsilon}{16\pi^2} \frac{1}{v^4} (3m_H^4 - m_H^2(2m_W^2 + m_Z^2) + 3(2m_W^4 + m_Z^4))
\end{equation}

B Relevant one-loop diagrams

In this Appendix we present the relevant one-loop diagrams entering in the computation of the 1PI functions for $WW \to HH$ scattering within the EChL. In particular, the corresponding ones to the new Green functions, $\Gamma_{HHH}, \Gamma_{W+H}, \Gamma_{AHH}, \Gamma_{ZH}H$ and $\Gamma_{WWHH}$, which respect to our previous computation in [3]. These diagrams were generated with FeynArts [15] and we collect them by different topologies using a generic notation for the internal propagators: dashed lines refer to both Higgs boson or Goldstone bosons and wavy lines refer to all possible EW gauge bosons. Notice the absence of ghost fields since the Higgs boson does not interact with them in the EChL, but they are present in the SM computation.

The loop diagrams of $\Gamma_{HHH}$ are shown in the first column of Fig. 6. Different from the SM, the results in the EChL depend on $a, b, \kappa_3$ and $\kappa_4$ and there is a different (non trivial) momentum dependence due to the behaviour of the scalar loop diagrams in the EChL and the SM. The same conclusions for the diagrams in the second column corresponding to $\Gamma_{\pi WH}$, but there is no $\kappa_4$ dependence here.

Regarding the $AHH$ and $ZHH$ Green functions, they have the same generic topologies than $\Gamma_{\pi WH}$ but they result finite in both the EChL and SM. We omit the corresponding diagrams for shortness.
Finally the one-loop diagrams for the $WWHH$ 1PI Green function are presented in Fig. 7. Also, the results in the EChL depend on $a$, $b$, $\kappa_3$ and $\kappa_4$ and again there is a different (non trivial) momentum dependence.

Figure 6. Generic loop diagrams for the $HHH$ and $\pi WH$ Green functions in the EChL. The topologies for $AHH$ and $ZHH$ are the same as for $\pi WH$. 
Figure 7. Generic loop diagrams for the $WWHH$ Green functions in the EChL.
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