Abstract

Using Promise Theory as a calculus, I review how to define agency in a scalable way, for the purpose of understanding semantic spacetimes. By following simple scaling rules, replacing individual agents with ‘super-agents’ (sub-spaces), it is shown how agency can be scaled both dynamically and semantically.

The notion of occupancy and tenancy, or how space is used and filled in different ways, is also defined, showing how spacetime can be shared between independent parties, both by remote association and local encapsulation. I describe how to build up dynamic and semantic continuity, by joining discrete individual atoms and molecules of space into quasi-continuous lattices.

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1 Introduction

This is the second part of a series of papers on semantic spacetimes, exploring the idea of scaling of the structure and tenancy of space, i.e. describing how properties (i.e. information) reside within a space and occupy it. These notes contain technical ideas to show an approach unifying semantics with dynamics, for later refinement.

The laying out of patterns in space is the basis for the encoding of information, as well as of functional behaviours in systems. Semantics add strong constraints to the nature of spacetime that dynamics alone cannot provide in a natural way, e.g. how do we explain intentional inhomogeneity is space? Does topology have meaning? Is there a smallest grain size in spacetime? The role of functional asymmetry is another of the key features of semantically rich systems that requires a simple explanation.

Examples of scale and tenancy in spacetimes with semantics include everything from the mundane, where people occupy housing, offices, or hotels rooms, to computer programs that live in ‘the cloud’, where data live on a network, and how we fill storage resources, or search for items in storage warehouses. The structure of spacetime is key to answering all of these issues.

The theoretical calculus for this paper is promise theory [1,2]. Elements of spacetime are described by autonomous agents that cooperate through the keeping of promises. Promises describe observer semantics and shared dynamics of agents in a unified way. The scope of promise information is local.

2 Properties of spacetime agency

The properties of spacetime refer both to the notions of location, adjacency, and to the possibly inhomogeneous properties exhibited by both. In paper I [1], it was seen that one could construct a model of spacetime in which the elements of space were agents, or units of agency, representing locations, and the topology or connectivity was made through promises, as well as the functional semantics. The purpose of this construction was to attribute a necessary but sufficient degree of capability to space with the aim of understanding and utilizing it operationally\(^1\). What is elegant about this is the small number of concepts required in order to build an entire picture of functional spaces. A semantic spacetime, then, is more or less simply:

- A collection of autonomous agents, usually denoted \(A_i\).
- A set of promises made between these agents, usually denoted \(\pi\) (or as a matrix \(\Pi\)).

This is the starting point.

\(^1\)I am grateful to Susan Bennet for making this crisp observation.
2.1 Agency by attachment, encapsulation, and association

Agency is that property which ascribes intentionality and semantics to an object. In the natural sciences, the idea that space and time could have semantics has been skirted and avoided. Indeed, one normally suppresses this issue altogether, by making spacetime a neutral backdrop to what goes on\textsuperscript{2}. However, once one embraces the notion that space and time can have semantics, it is possible to describe all kinds of structures from cell biology to databases as special cases of a generalized idea of semantic spacetimes, while embracing concepts of adjacency, regularity, symmetry, and relativity, normally associated with their physics.

![Figure 1: Local versus non-local scaling of agency, by encapsulation and by association.](image)

Agency, i.e. intentionality and meaning, may be attributed to any kind of process, smart or dumb, by intelligent, thinking observers, but this does not make its perception less important, or less real. Abstraction informs us that there are two ways by which agency can be wielded:

- Locally, by encapsulation of intentions within a spacetime element (what we call autonomy).
- Non-locally (remotely), by association of a spacetime element with a source of agency which is elsewhere (what we call subordination).

These two modes will be studied in the remainder of these notes in the guise of the more prosaic terminology:

- The scaling of local agency (by encapsulation, absorption, clique and membership, etc).
- The scaling of remote agency (intentionality at a distance, by attachment, correlation, entanglement, etc).

These two notions are complementary, and yet they also become intertwined as we scale up.

2.2 Recap of the propagation model, and observation

The basic model underlying promise theory is one of transmission of influence, through signalling of intent and by measurement, that guides causation. There is a basic asymmetry already encoded into this elementary intentional process: the asymmetry of ‘have’ and ‘have not’. For influence to be transmitted from an agent $A$ to an agent $A'$, it requires a promise to be communicated by $A$, and accepted by $A'$:

$$A \xrightarrow{-b} A'$$

$$A' \xrightarrow{b} A$$

\textsuperscript{2}The exception here is Einstein’s theory of general relativity, which ascribes semantics of gravitational force to spacetime curvature, or vertex functions in the Regge calculus.
How we describe causation within a promise model depends on how we understand the agencies involved. However, Shannon’s basic model of transmitter and receiver has to be at the core of all other considerations [3] (see fig 2). It enshrines the way in which source and observer break the translational symmetries of the system in order to make a scale for measurements.

![Diagram of Shannon's basic source-sink view of information transmission forms the basis of a promise-promisee interaction, and all transmission of influence between locations. This model encapsulates the way in which observation breaks spacetime symmetries.](source-sink-diagram.png)

The recipient of the source’s communicated intent (the semantics), might not be the same as the recipients of the impact of the intent (dynamics). Thus transmitted information has these two aspects, which must be understood together. This is why promise theory is a harder set of constraints to satisfy than a theory in which we take the semantics as trivial. Causation is transferred through vector promises (see section 2.3), especially so-called ‘use-promises’. These, in turn, may be interpreted as special cases of the following general semantics:

1. $A_1$ can influence $A_2$ (change/causation)
2. $A_1$ is connected to $A_2$ (topology)
3. $A_1$ is part of $A_2$ (containment)

**Example 1** Examples include the following quasi-transitive and quasi-causative relations:

- $A_1 \xrightarrow{Instructions} A_2$  
- $A_1 \xrightarrow{Precedes/Follows} A_2$  
- $A_1 \xrightarrow{Affects} A_2$  
- $A_1 \xrightarrow{Is a special case of} A_2$  
- $A_1 \xrightarrow{Generalizes} A_2$  
- $A_1 \xrightarrow{Neighbour} A_2$

These examples lead to what we call quasi-transitivity, which in turn allows the propagation of intent$^3$.

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$^3$This is similar to the way in which the equations of motion of a dynamical variable allow the propagation of state from an initial configuration to a final one, over a ‘free’ interval. One expects to derive other kinds of measurements from this basic evolutionary structure.
2.3 The tensor structure of a promise body

The spatial relationships encoded by promises do not necessarily follow the graph of promiser-promisee bindings. It is useful to have a language by which to discuss these. Tensors are well-known structures that interrelate constellations of points in a space. Promises bind agents (the elements in a semantic space) in two ways:

- Binding together agents that receive information about intent (scope, promisee).
- Binding agents who must cooperate in keeping the promise (promiser and body)

We write a promise from $A$ to $A'$ in the form

$$A \xrightarrow{b(A_1A_2...)} \sigma \rightarrow A', \quad (9)$$

where $b$ is the body of the promise, possibly depending on specific agencies, and $\sigma$ is a collection of agents known as the scope of the promise.

If no other agent is involved in keeping a promise, i.e. no agent appears mentioned in the promise body $b$, then we call it a scalar promise. If a single agent is involved in defining the promise, we call it a vector promise. Promises of higher dependence can also exist, but might be decomposable into multiple vector promises. There is thus a notion of promise rank, in the same way as for tensors (see paper I, section 3.3.4). This distinction between scalar and vector promises becomes increasingly important as we apply promises to tenancy, so let’s begin by examining this in more detail (see fig 3).

**Lemma 1 (Use promises acquire tensor rank 1 implicitly)** A use-promise such as

$$\pi : S \; \xrightarrow{b} \{ R \}$$

also written

$$S \; \xrightarrow{U(b)} \{ R \} \quad (10)$$

is a special case in which there is an implicit reference to the acceptance of a promise in a binding initiated by a remote promiser. Hence the promisees $\{ R \}$ are implicitly involved parties in the promise body. Since all promises are separable into promises between pairs of agents, the rank of a use-promise

$$R_i \; \xrightarrow{±b} \; S,$$  

is always 1 (a vector).

A promise, conditional on a promise from an external agent, is also a special case, in which the promise body refers to a promise quenched by an external agent through the dependency. This does not refer to the agent quenching the promise, however. The conditional promise law (see [2]) implies it will be accompanied by a use-promise, however, which will lead to a vector relationship. Thus conditional promises lead implicitly to vector relationships also.

**Definition 1 (Tensor promises)** Promise rank is determined by the number of agencies related by a promise.

0. A scalar promise refers to no other agency in its body, i.e. expresses a property of just the promiser.

1. A vector promise refers to one other agency in its body, i.e. a property in relation to one other independent agent.

2. A rank $n$ tensor promise, at point $A$, refers to $n$ other independent agencies in its body. In all cases, the number of promisees is undefined.
A scalar promise might be: I promise to brush my teeth every day. A vector promise might be: I promise to read a message from agent X. A 2-tensor promise might be: I promise to relay information from X to Y. An n-tensor promise might be: I promise to select give my money to one of A₁, …, An that has the cheapest goods.

A promise body has the form:
\[ b = b(A₁, \ldots, Aₙ) \]  

A set of vector promises that correlates the promiser with the points Aₙ appearing in the body form a matroid, or a vector basis, for a local region of the space by choosing to make the adjacent agencies A₁, A₂, … (see figure 3). Intuitively, the set of paths radiating out from a point to promisees, describe its adjacencies (or virtual adjacencies), and represents the possible different directions by which information can be passed to neighbouring agents. Those directions, which are local (adjacent) to the promiser, acquire the role of a spanning set of component directions, i.e. promise vectors at the promiser’s location. If the promise body refers to any remote agencies, i.e. which are not a direct neighbour, one may resolve ‘virtual adjacencies’ (such as tunnels or overlaid spatial structures) in terms of the the actual vectors of this local set.

In a general promise graph, each agent location has no knowledge of other agents, i.e. it has only local information. This is analogous to the inhomogeneous constraints that explain most influential fields of force in fundamental physics. We thus rediscover a natural representation of gauge theories in promises, where scalar and vector properties depend on location, due to the inhomogeneous symmetry properties⁴.

2.4 Completeness of promise information

The bindings between promiser and promisee relate to the two aspects of behaviour:

- **Dynamics**: (change and measure) agencies mentioned in the promise body involved in the keeping of the promise.

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⁴In the original paper [4], I suggested that the appearance of other agent identities in promise bodies was undesirable as it let to potential inconsistency. This was a potential issue when dealing with non-scalar promises. However, this was before the role of impositions was properly understood in Promise Theory. The inconsistencies arise when trying to make promises on behalf of non-local agencies.
• **Semantics**: (purpose) the promisees to whom intent is communicated.

Promisees are the intended recipients of the outcome, but there is also the scope of the promise by which agents receive the pure kinematic or dynamic information content about the promiser’s intent. Influence can thus also occur between pairs of agents via indirection.

**Example 3** Intent and execution can be communicated separately, with different consequences. Imagine promising to follow someone without informing them of that promise. Then the promise body would refer to the person, but the promisee would not include the person.

The agent being followed might still observe a correlation with the promiser, without understanding the information of the promise to signal the possible origin of the behaviour, and could even speculate about the existence of a promise, as emergent behaviour, but still has formally incomplete information.

We may define the completeness of information to another agent by saying that

**Definition 2 (Promise with complete information)** In a tensor of rank \( n \), made by agent \( \mathcal{A} \), referring either explicitly or implicitly to \( n \) other agents, the promise provides complete information iff all \( n \) agents referred to in the promise body are in the scope of the promise, i.e. are promisees or informees of \( \mathcal{A} \).

A promise with complete information thus makes other involved agents aware of promises made about them.

### 2.5 Defining promise matrices

It is useful to define two matrices:

**Definition 3 (Promise matrix)** Let \( i, j \) label a collection of \( n \) agents, where \( i = 1, \ldots, n \). The promise matrix \( \Pi_{ij} \) is the matrix of all promises between \( \mathcal{A}_i, \mathcal{A}_j \), stripped of the agent labels, i.e. in which the agents are implicit. Hence, the complete set of promises between the agents may be written:

\[
\bigcup_{i,j=1}^{n} \mathcal{A}_i \pi_{ij} \mathcal{A}_j = \sum_{i,j=1}^{n} \mathcal{A}_i \pi_{ij} \mathcal{A}_j,
\]

(13)

**Note**: this notation defines the meaning of \( \sum \) and \( + \) for the rest of the paper.

**Definition 4 (Promise adjacency matrix)** The directed graph adjacency matrix which records a link if there is a promise of any type between the labelled agents.

\[
\Pi_{ij} = \begin{cases} 1 & \text{iff } \mathcal{A}_i \xrightarrow{b_{ij}} \mathcal{A}_j, \\ 0 & \forall b_{ij} \neq \emptyset \end{cases}
\]

(14)

These may be further decomposed into useful subsets, for instance, the rank decomposition of the promise matrix, into matrices of promises of rank \( r \), will be useful:

\[
\Pi_{ij} = \sum_{r=0}^{\Pi_{ij}^{(r)}},
\]

(15)

The concept of an agent’s interior and exterior promises will also be defined below.
2.6 Differentiating spatial role: agent types and labels

We may benefit from differentiating types of agents, when applying promise theory to functional spaces. A priori, agents have no type: they are homogeneous, structureless, universal elements (analogous to biological ‘stem cells’) that may only be differentiated via the promises they make. A type may thus be defined either by identification of a role [2], or by explicitly making a scalar promise.

Consider a universal (typeless) agent $A_∅$, that makes no initial promises, or empty promises to everyone:

$$A_∅ \rightarrow^\emptyset *.$$

(16)

We may add a scalar promise, with additional scope $σ$:

$$A_∅ \rightarrow^{+bla}^σ *.$$

(17)

Any agent in the scope $\{*, σ\}$ may now identify the former agent as being equivalent to an agent of type $b$, making no promise. In other words, within the scope, the agent effectively has a new name:

$$A_{bla} \rightarrow^{+}^σ *.$$

(18)

Thence

$$bla \rightarrow^{+}^σ *.$$

(19)

i.e., we can drop the promiser’s agent symbol ‘$A$’ and label agents simply by their names. I’ll use this notion from here on to write agent types implicitly, e.g.

$$T_1 \rightarrow^\emptyset A ≡ A_1 \rightarrow^+ T A$$

(20)

$$R_2 \rightarrow^\emptyset A ≡ A_2 \rightarrow^+ R A$$

(21)

$$H_3 \rightarrow^\emptyset A ≡ A_3 \rightarrow^+ H A$$

(22)

and so on. In this way, we can move scalar labels from the promise body to the agent’s identifier at will. This is in keeping with the idea that the agent’s name is the basic promise that identifies it.

2.7 Promise valency and saturation

To develop the formal chemistry of intent, we need to clarify how many agents a promise can support. In other words, how many ‘slots’, ‘binding sites’ or occupiable resources does an agent have, to share between promisees?

Declaring a finite number of such slots, explicitly allows for a simple discussion of resource exclusivity around promises. The concept is basically analogous to the valences (oxidation numbers) of electrons in physical chemistry. Think also of the binding sites for receptors, viruses and major histocompatibility proteins in biology.

Definition 5 (Valence of an agent promise, and overcommitting) A promise which provides $+_b$ to a number of other agents may specify how many agents $n$ for which the promise will be kept exclusively. The valency number of an exclusive promise is a positive integer $n$, written

$$A \rightarrow^{+b#n} \{A_1, \ldots, A_p\}.$$

(23)

A promise body may be called over-promised (or over-committed) if $p > n$. 


**Example 4** A reserved parking area promises 10 spaces, to 20 employees. The parking promise is over-committed, since it cannot keep all of its promises simultaneously.

Over-promising is not a problem unless all of the promisees accept the promise, and promise to use it. Thus a separate concept of saturation arises by using up all of the valence slots:

**Definition 6 (Use-promise saturation)** Suppose we have

\[
A \xrightarrow{+b#n} \{A_1, \ldots, A_p\} \quad (24)
\]

\[
\{A_1, \ldots, A_m\} \xrightarrow{-b#m} A \quad (25)
\]

is saturated if \( m \geq n \).

It is useful to define a function whose value is the net valence of a particular type of promise body.

**Definition 7 (Net valence of a promise graph and utilization)** \( \pm b \), for a collection of agents \( \{A_i\} \):

\[
\text{Valence}(b; \{A_i\}) = \sum_i \text{Valence}(b; A_i) \quad (26)
\]

\[
= n - m \quad (27)
\]

Hence we may assign an integer value to the level of usage, or a rational fraction \( m/n \) for utilization of the resource. If this fraction exceeds unity, or the net valency is negative, the keeping of the promise effectively becomes a queue of length \( |m - n| \), requiring the agent to multiplex its resources in time to keep its promise.

**Example 5** Consider the two agents \( A_1, A_2 \):

\[
A_1 \xrightarrow{+b#2} A_2 \quad (28)
\]

\[
A_2 \xrightarrow{-b#3} A_1 \quad (29)
\]

\( A_1 \) offers two possible slots for its promise of \( +b \), while \( A_2 \) requests three units of it, leaving a net deficit:

\[
\text{Valence}(b; A_1, A_2) = -1 \quad (30)
\]

This notation allows us to simplify the discussion of occupancy and tenancy in later sections.

**Example 6** Consider the following promises made by a network switching device:

\[
\text{switch} \xrightarrow{+(10Gb)#48} \text{client} \quad (31)
\]

\[
\text{client} \xrightarrow{+(1Gb)#1} \text{switch} \quad (32)
\]

The switch makes 48 promises offering 10Gb ‘bandwidth’ to the clients. The client accepts one valency slot (leaving 47 more), and promises to consume only 1Gb of the maximum possible 10Gb.

### 2.8 The language structure of promise bodies

In order to even comprehend one another’s promises, agents need a common language, with which to express body content. The problem of how agents come to develop a mutually acceptable language for information exchange between independent agents has been studied in connection with linguistics both
of the traditional variety and in the biology of the genetic code [5]. It is not a simple problem, and I shall not try to address it here in full; however, there are some simple things we can say about it.

In all cases, what we derive from these studies is that, regardless of whether language is executed continuously or discretely\textsuperscript{5}, the possible intended meanings form a discrete alphabet of symbols, representing capabilities, intentions, and so on. Thus, semantics constrain promise bodies to a set of linguistic atoms (morphemes) which are discrete. In nature, we see this in everything from gene codons, to cells and organisms, to Chinese ideograms\textsuperscript{6}.

The representation of promises in arbitrary natural language is unlikely to be simple, since the metaphoric basis of natural languages do not lend themselves to direct mapping. However, we can imagine formulating a set of restricted Domain Specific Languages, \(\ell(\alpha) = \{\beta_0^{(\alpha)}\}\), consisting of symbols \(\beta_0^{(\alpha)}\), whose purpose is the represent specific intentional behaviours, and which map to a set of concepts that can be represented in a natural language \(N(\alpha)\) of all agencies in a coordinate patch:

\[
\ell(\alpha) \rightarrow N(\alpha).
\]  

**Assumption 1 (Discreteness of promise body encodings)** The language of promise bodies is assumed to be a discrete language pattern, of fixed, but unspecified, alphabet \(\beta\), comprising basis symbols \(\beta_\alpha\).

\[
b(A_i) = \sum_{\alpha=1}^{C} c_\alpha \beta_\alpha,
\]  

\textsuperscript{5}Messages may be sent with or without words, with body postures, or coded by melody (frequency division multiplexing) rather than representations discrete in time (time division multiplexing). These are well known in information theory.

\textsuperscript{6}One can speculate about the reason for the size of discrete patterns used to convey meaning. Dynamical scales will ultimately place limits of the comprehension of an agent. If agents could have infinite resolution, there would be no limit to what information could be conveyed in an arbitrary promise. But a recipient has to be able to parse this information in a finite time, shorter than that which is needed to keep its promise. This suggests that information density must be finite, whatever the nature of the agents. Even a concept like 'happiness' cannot have an infinite number of shades of grey! Protein size limits the size of a gene; variety of length and time scales limit the complexity of a key used by a human or a computer, and even the density of musical notes in the scale is limited to approximately quarter tones by the size of the human ear.
for some coefficients $c_a$, and $C = \dim(\beta(A_i))$ for agent $A_i$.

Based on this decomposition of intent, we may now consider a local observer view on the measurement of agent structure, assuming finiteness of information. Consider the syntax space of alphabetic strings in a set of languages $\ell(\alpha) \in \mathcal{L}$, where $\alpha$ labels different languages, each of which might have its own alphabet.

In order for agencies with different languages to be able to communicate intent, these languages must be mutually comprehensible, i.e. we must be able to map parts of them to one another. The existence of a discrete alphabet of symbolic words allows us to think of the alphabet of intentions as a matroid or spanning set $\beta$, over a vector space. We may assume that the language of agent $A_i$ has $\dim(\beta(A_i))$ dimensions or classes of intention, so that $a = 1, \ldots, \dim(\beta)$ (see figure 4). This alphabet may or may not necessarily be shared between all agents, but needs to be partially translatable for agents to make promises to one another.

Suppose we have two alphabets with symbols $\beta$ and $\beta'$. In order for a promise body to be encodable in either language, we must have:

$$b = \sum_{a=1}^{\dim(\beta)} c_a \beta_a = \sum_{a'=1}^{\dim(\beta')} c_{a'} \beta'_{a'}$$

(35)

i.e. both languages have to be able to span the promise body, or represent it in their own spanning sets or words. We need not require a single common spanning set of body parts to span every message, the language of agents does not have to be a global symmetry across spacetime, but we do require local continuity in the couplings, in order for information to be passed on.

### 2.9 Agent homogeneity, and transmission of intent

A related issue concerns the ability for an observational arbiter to distinguish between different agents. This depends on the ability of the agent to comprehend the language being promised. Suppose an observer suspects that an agency is non-atomic, i.e. it contains internal structure.

- Multiple agencies within a single agent might be identified if:
  - If an agent seems to make independent partial-promises to different promisees, it could be natural to formally resolve it into separate sub-agents for the independent promises (‘disaggregation’).

- A compound promise could be resolved into several simpler promises if:
  - The details of the promise body can be expressed as a number of independent items that can be made (+) or consumed (-) independently.
  - If, by emergent agreement, a set of primitives (like a table of elements) can be seen to form a spanning set for the promises made by an ensemble of one or more indistinguishable agents$^7$.

To know this information for sure, it would have to be promised. For this, we can define the concept of an agent directory (see section 3.8.2).

**Definition 8 (Homogeneity of agent languages, and transmission of intent)** Agents $A_1$ and $A_2$ may be said to have distinguishable promises that can be resolved by a receiver $A_r$ iff the measures of the

$^7$The latter case is like the Millikan experiment for measuring electric charge. If you look for differences, then the smallest difference may be assumed to be an elementary.
promises, which overlap with the receiver, are unequal. Suppose $A_1, A_2$ each make a promise to an observing receiver $A_r$:

$$A_1 \xrightarrow{+b_1} A_r \quad (36)$$

$$A_2 \xrightarrow{+b_2} A_r \quad (37)$$

$A_r$ might judge these two agents identical if $b_1 \cap b_2 = b_r \neq \emptyset$, i.e. if both promises contain a common part (the intersection $b_1 \cap b_2$), which the promisee promises to see:

$$A_r \xrightarrow{-(b_1 \cap b_2)} A_1, A_2. \quad (38)$$

In other words, if the receiver filters its perceptions according to what is common to all agents, then it is unable to distinguish them.

We can break this into two cases: if sources $A_1$ and $A_2$ share common components (e.g. share common genes), i.e. $b_1 \cap b_2 \neq \emptyset$, then the receiver can observe a similarity between the agents. If, further, the receiver only perceives the influence of what is common between them, i.e. it promises to accept $-b_r$, then the agents will perceive an elementary unit of promise equal to:

$$b_1 \cap b_2 \cap b_r \neq \emptyset. \quad (39)$$

**Example 7** In genetics, the body elements correspond to genes. A gene can be passed on (+) from a parent to a child, but whether or not it is ‘expressed’ or activated depends on the proteins use the gene (−) during morphogenesis. Thus, simply passing genes from generation to generation need not result in transmission of attributes (promises kept). Similarly, environmental conditions can play a role in activating or de-activating particular gene promises in different circumstances.

**Example 8** For instance, imagine one agent believes it is promising to deliver a letter to a recipient. The agent receiving what the promiser considers a letter might, in fact, be promising to evidence in an investigation as a DNA sample on the letter. The rest of the letter vehicle has no semantic value. A second agent then promises to deliver a blood sample to the same recipient. This also qualifies as an evidential DNA sample to the recipient. Since the agency of DNA is encapsulated by both the letter and the blood sample: DNA ⊂ Letter and DNA ⊂ Blood, an agent that can only measure DNA would see the letter and the blood sample as being equivalent sources of DNA.

DNA, itself, is a vehicle (agency) for genes that are embedded within it. Exactly the same argument now applies at the level of DNA as a container. The presence or absence of an allele (gene flavour) within a strand of DNA indicates a similarity of intent.

The impact of a promise is defined through its binding strength, or effective coupling constant. In earlier work, I defined the notion of a trajectory for an agent, and the corresponding notion of a generalized force, obeying Newtonian semantics [6, 7]. Intuitively, one expects a force to be something that impacts an agent’s trajectory

$$F : b \rightarrow b + \delta b \quad (40)$$

Though, readers should note that promise trajectories are rarely Newtonian. From this, one may construct a generalized force, which with the help of assessment function $\alpha(\pi)$ takes on a familiar form of a field-charge like interaction:

$$F \simeq \alpha \left( \begin{array}{c} \text{Field} \xrightarrow{+b} R, R \xrightarrow{-b} \text{Charge} \end{array} \right) . \quad (41)$$
See [6, 7] for the details. This gives us a simple notion of a coupling strength by which to define such a measure of impact. The analogies to physics are attractive, but we should beware that the trajectories are ‘rough walks’ not smooth curves, in spite of the analogy to differential notation.

2.10 Continuity and spatial homogeneity of promise semantics

Promises comprise information transmitted between agents. The effective transmission of information requires the existence of a common language [3, 8]. If each agent is an autonomous entity, how may agents learn a common language, or equilibrate different languages, in order to understand one another’s promises?

Consider the existence of a language operation that transforms a body string $b_1$ by agent $A_1$ into a body string $b_2$ for agent $A_2$.

$$
\begin{align*}
    b^{(a)} &= L_{ab}(b^{(b)}) \\
    b^{(b)} &= L_{ba}(b^{(a)}).
\end{align*}
$$

In order for $L(\cdot)$ to be faithful and express transitive properties such as long-range order, we need piecewise reversibility. Substituting (43) into (42)

$$
\begin{align*}
    b^{(b)} &= L_{ba}(L_{ab}(b^{(b)}))
\end{align*}
$$

which implies that

$$
L_{ab}(L_{ba}) = 1
$$

or the inverse relationship is the transpose:

$$
L_{ab} = L_{ba}^{-1}.
$$

In a general matrix representation, this implies that universal representation of the matrices representing $L$ belong to the unitary group over language space $a, b$:

$$
L^\dagger L = I.
$$

The full unitary symmetry (if we take the general solution to this seriously, in the absence of other constraints) allows for general rotations of symbols. Thus so-called entangled states are, in principle, allowed for in this observation.

The set of transformations represented by $L$ does not have to be assumed a global symmetry. The index labels gloss over the piecewise locality of the assumption that $L_{ij}(A_i)$ is a transformation that takes place at the location $A_i$, on its way from $A_j$. Similarly $L_{ij}(A_j)$ takes place at $A_j$ on its way from $A_i$.

The limit of locality is thus the adjacency length between $A_i$ and $A_j$

$$
L_{ab}(A_i^{(b)}) \cdot L_{ba}(A_j^{(a)}) = 1.
$$

If $A_i$ and $A_j$ are nearest neighbours, this is straightforward. However, if we regard the transmission of a promise through intermediate proxy agents, then comprehension and message integrity depend on the existence of non-local correlations, somewhat analogous to entanglement in quantum mechanics.

This symmetry is closely related to the observation that, even with a common language, in any promise relationship between agents:

$$
U(U(+b)) \neq +b
$$

$$
- - b \neq +b
$$

\footnote{There is an analogy here with local gauge symmetries, as imagined by Weyl.}
Both relations imply a kind of long-range cooperation between the agents. These are analogous to the global symmetries of particle physics.

On seeing this familiar symmetry of the physical world, it is tempting to look for a conserved quantity, or a conservation law for the alphabets $\beta$, but it cannot be the case that the alphabets are preserved. A conservation law would make the transfer of alphabetic messages into a zero sum game: what was passed on to a neighbour would be lost by the sender. This is not how evolution works. Instead, the process of equilibration is more like an epidemic duplication$^9$. The transformations of language at a location are more likely to be non-conserved, in general, and depend on the proper time (evolutionary change). One expects transmission of symbols in both time and space, but without conservation. Thus one could imagine dividing the inter-lingual transformations into two parts:

$$L = L(A) + L(A, \tau)$$

The first part could lead to a zero-sum conserved current of symbols from one agent to another, allowing migration without preservation, while the latter part allows symbols to be duplicated and spread. It might be fruitful, in the future, to consider how this process takes place, and compute Kubo relations for the transmission of promises [9].

In order to cooperate, agents interacting at a distance need to have a sufficient level of similarity to local agents in order to be able to make sense of what they are promising to one another (see figure 5). This is true regardless of whether they directly adjacent or not. This must be a semantic equilibrium, mediated by a dynamic exchange process, in order to this cooperative behaviour to emerge. Unlike elementary physics, where locality is more obvious, cooperation between agencies could be long range, as long as there is adjacency over a long range. Semantics tend to follow humans, companies, organizations, races, countries, etc, and humans form multiple outposts with geographic separation.

\[\text{Figure 5: Agents need to have similar structure to make promises across agent boundaries, referring to internals of the other. The promise bodies do not have to be identical as long as each agent recognizes its own version of the other's promise.}\]

$^9$Why, for example, would genes be preserved in number and type across species? If that were the case, all species would eventually equilibrate into one, and what was gained by one species would be lost by another.
2.11 Inter-agency language translations

It is possible, in principle, to construct a linear transformation of one language into another:

$$\beta'_a = L_{a'a}(\beta_a).$$  \hspace{1cm} (52)

Then, from the linearity, we may use the distributive law to say that a body

$$b = \sum_{a=1}^{\dim(\beta)} c_a \beta_a = \sum_{a=1}^{\dim(\beta)} c_a L_{aa'}(\beta'_a)$$  \hspace{1cm} (53)

Thus, as long as the matric $L$ exists, the languages will be translatable. If the dimensions of the languages are not the same, only a subset of meanings will be translatable from one to the other. This might be asymmetric, allowing one agent to understand another, but not vice versa. At the level of atomic intentions, we can introduce coding transformations a transition matrix for mapping

$$\ell(\alpha)(A_i) \in L_{\alpha\beta}(\ell(\beta)(A_j))$$  \hspace{1cm} (54)

for some invertible matrix-set of maps $L_{\alpha\beta}$.

**Example 9** Consider a body language with alphabet $\beta = \{SEND, RECEIVE, SEEK, FORWARD, BACK\}$, and $\beta' = \{PUT, GET, APPEND\}$, then we can translate these:

- $c'_1 \beta'_1 = PUT = c_1 \beta_1 = SEND$  \hspace{1cm} (55)
- $c'_2 \beta'_2 = GET = c_2 \beta_2 = RECEIVE$  \hspace{1cm} (56)
- $c'_3 \beta'_3 = APPEND = c'_2 \beta'_2 + c'_4 \beta'_4 + c'_1 \beta'_1 = SEEK + FORWARD + SEND$  \hspace{1cm} (57)

Hence there is a translation matrix:

$$L_{a'a} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$  \hspace{1cm} (58)

which, in this case, is not invertible. Hence the language is translatable in one direction only.

In principle, however, it should be possible to restrict $\ell(\alpha)$, such that translation may be performed faithfully as a bijection, by postulating a ‘table of elements’ for the chemistry of all promises in a semantic spacetime:

$$\ell(\alpha) \leftrightarrow \ell(\beta) \ \forall \ a, b \in \mathcal{L}.$$  \hspace{1cm} (59)

From here, the requirement for transmission of intent is that there be a piecewise continuity by partial overlap between neighbouring languages (see figure 6) becomes:

$$\ell(\alpha) \cap L_{a,a\pm 1}(\ell(\alpha+1)) \neq \emptyset \ \forall \ a$$  \hspace{1cm} (60)

so that for $a \pm 1$ a neighbouring language of $\ell(\alpha)(A_i)$, i.e. $\Pi^\text{ext}_{ij}(A_i) = 1$, a promise binding, of the following kind, may be mutually and equivalently comprehended:

$$\begin{align*}
\ell(\alpha) &\implies \ell(\beta) \\
A_i^{(\alpha)} + b &\implies A_j^{(\alpha)} \\
A_i^{(\alpha)} &\implies A_i^{(\alpha)}
\end{align*}$$  \hspace{1cm} (61)
This assumes that both $L_{ab}$ and $L_{ba}$ exist over the relevant bodies. The solution of the continuity relation (60) would then take the form:

$$\ell^{(a)} = \{\beta^{(a)}\} \cup \{L_{a,a\pm 1}(\beta^{(a\pm 1)})\} \; \forall \; \alpha$$

i.e. the language of an agent in a coordinate patch $\alpha$ should consist of all interior body symbols, together with native translations of neighbouring languages.

### 3 The scaling of agency

To describe the spatial structure of agents, in which the composition of elements is consistent, we need to describe how agency scales collectively, through aggregation and reduction of elementary agents. The concepts of scaling are familiar to physicists for dynamics, but here we also want to extend them to incorporate semantics. This might be motivated by questions of the following kind:

- How does a team promise something as a unit?
- How does an organization appear as a coherent entity?
- How does a collection of components promise to be a car?

Since we can aggregate promises into a single promise, and aggregate agents into a single agent, the ability to detect or resolve parts within a whole depends on the observer’s capabilities. Similarly, the ability for an agent to perceive a collection of individual agents with a collective identity (i.e. a super-agent) depends on the capabilities of the observer agent. Elementarity and composability of agency thus go together with a hierarchy of observable agency, which needs to be elucidated.

#### 3.1 Subspaces

A partial region of a semantic space, at any scale, may be called subspaces. We may distinguish the boundaries of such a region in any way convenient. A subspace is assumed to be connected, but not necessarily homogeneous of isotopic.

**Definition 9 (Subspace)**  A subspace is a collection of agents, in which every agent is adjacent to at least one other. The agents may be:

1. Identified and associated by an external observer by their (-) promises, or
2. Intentionally labelled and coordinated by its members with a (+) promise.

Subspaces can be defined by partitioning a space, or by constructing a space agent by agent. There is good reason to consider both of these points of view, so let’s describe them below.
3.2 Independence of agents under aggregation

We begin by considering how to identify discrete, elementary components within a system of autonomous agents, making promises. If we assume that sufficient knowledge of agents in available, then an observer can assess the independence of agents by the absence of mutual information, i.e. zero overlap.

Definition 10 (Independent agent) Two agents $A_1$ and $A_2$ are independent iff the following overlap relation holds:

$$A_1 \cap A_2 = \emptyset.$$  \hspace{0.5cm} (63)

An agent that it independent of another agent may be said to be outside or exterior to the agent. An agent that overlaps with another agency may be said to be inside or interior to it.

3.3 Composition of agents (sub- and super-agency)

The treatment of a collection of agents as a single entity is a choice made by any observer. It can be made with or without promises from the composite agents themselves (see figure 7). Agency can be defined recursively to build up hierarchies of component parts. In paper I, I showed how spatial boundaries can be defined by membership to a group or role. We still have to explicate the relationship between the internal members and the structure of the whole, as perceived by an observer.

![Agent structure](image)

Figure 7: Agent structure consists of an element that makes a number of exterior promises, some of which are scalar, some vector, etc. Interior promises are invisible from the outside.

We define a collective super-agent as a spacetime structure that has collective agency, i.e. its intended semantics relate to a collection of agents surrounded by a logical boundary, with collective semantics (see figure 7).

Definition 11 (Bare super-agent) A super-agent of size $S$ is any bounded agency composed of individually separable agencies, partially or completely linked by internal vector promises. The bare super-agent is defined by the closed graph, without any external adjacencies. It is a doublet:

$$A_{\text{super}} = (\{A_i\}, \Pi_{ij}^{\text{int}}), \quad i = 1, 2 \ldots S.$$  \hspace{0.5cm} (64)

where $A_i$ is an internal agent of $A_{\text{super}}$, and $\Pi_{ij}$ is the promise adjacency matrix between the $S$ internal constituents.
**Definition 12 (Dressed super-agent)** A dressed super-agent is the bare super-agent together with its set of exterior promises. It is a triplet:

\[
A_{\text{super}} = (\{A_i\}, \Pi_{ij}^{\text{int}}, \Pi_{\text{ext}}^{\text{ext}}), \quad i = 1, 2 \ldots S. \tag{65}
\]

Super-agency allows promises to exist within and without a super-agent boundary. We call these interior and exterior promises, respectively.

**Definition 13 (Exterior promises)** Exterior promises are made by agencies within the super-agent boundary, to agents outside. They represent inputs and outputs of the super-agent, i.e. how it interacts with an external space.

**Definition 14 (Interior promises)** Internal promises are made by agents inside the super-agent boundary to other agents inside the boundary. They represent the bindings that make the super-agent behave as a single cohesive entity.

In principle an observer could draw a line around any collection of agents and call it a cell or composite super-agent. This is an assessment any agent can make, as part of its definition of an agency scale. However, it might still be of interest to distinguish special criteria by which such an arbitration might occur. In component design, for instance, the choice of boundary has often to do with the a choice interface an agent wants to interact with.

![Figure 8: Three ways of binding collective agency. Another way is to simply make an arbitrary collection.](image)

The alternatives fall into three basic categories (see figure 8):

(a) A membership in a group or associative role, where the central membership authority may be either inside or outside the boundary. In this case, we are identifying a group of symmetrical agents.

\[
A_{\text{host}} \xrightarrow{+\text{membership}} \{A_{\text{tenant}}\} \quad \{A_{\text{tenant}}\} \xrightarrow{-\text{membership}} A_{\text{host}} \tag{66}
\]

(b) A total graph or collaborative role. In this case, we are identifying agents with coordinated behaviours.

\[
A_i \xrightarrow{\pm\text{membership}} A_j \quad \forall A_i, A_j \in A_{\text{super}}. \tag{68}
\]
Let us now define the converse properties:

**Definition 15 (Sub-agent)** A sub-agent is an agent assessed to be a resident of the internal structure forming a composite (super) agent.

**Definition 16 (Residency)** A sub-agent \( A \) is resident at a location \( L \) iff it is defined to be within the boundary of the agent:

\[
A \cap L \neq \emptyset.
\]  

(69)

Since an observer can form their own judgement about super-agent boundaries, we cannot say that residence is the same as a promise of adjacency.

There are two types of adjacency, somewhat spacelike, which may or may not be interchangeable (see figure 9). Normal ‘physical’ adjacency promises, and resident adjacency, which might link agents virtually even though they are not physically adjacent. I’ll return to this topic when discussing tenancy below.

![Figure 9: A resident adjacency forms a super-agent by accreting to a seed agency that represents the location anchor.](image)

Lesser agents can become satellites of other agents. This leads to a hierarchy or ‘planetary’ structure, by accretion into super-agents.

### 3.4 Super-agent surface boundary

Super-agent boundaries may be formed with different structural biases.

- Simple aggregations of agents, related through membership to a single leader (leader may be inside or outside the super-agent), and the connections are made through the leader as a proxy-hub.
- A cluster of agents linked by cooperative vector promises.
- Strongly cooperative agents which are inseparable without breaking an external promise. E.g. an organism made of components that are all different and non-redundant to the functioning of the whole.
Interior promises are those entirely within the surface boundary of a super-agent. We may define interior and exterior promise matrices for any agency, using a matrix analogous to the adjacency matrix:

\[
\Pi_{\text{int}}^{ij} = \begin{cases} 
1 & \text{iff } A_i \xrightarrow{} A_j \\
0 & \text{otherwise}
\end{cases} \\
A_i, A_j \in A_{\text{super}} 
\]  
\( (70) \)

\[
\Pi_{\text{ext}}^{i\epsilon} = \begin{cases} 
1 & \text{iff } A_i \xrightarrow{} A_\epsilon \\
0 & \text{otherwise}
\end{cases} \\
A_i \in A_{\text{super}}, A_\epsilon \not\in A_{\text{super}}
\]  
\( (71) \)

**Definition 17 (Surface of a super-agent)** The exposed surface \( \Sigma \) of the agent is the subset of interior/internal agents that have adjacencies to agencies outside the super-agent.

\[
\Sigma \equiv \{ A_i \} \subset A_{\text{super}} \mid \Pi_{\text{ext}}^{i\epsilon} \neq 0 
\]  
\( (72) \)

A super-agent surface may also make new explicit promises that are not identifiable with a single component agency (see section 3.10).

**Example 10** In the molecular example above, the super-agent makes interior promises

\[
\begin{align*}
A_1 & \xrightarrow{+H} * \\
A_2 & \xrightarrow{+H} * \\
A_3 & \xrightarrow{+O} *
\end{align*}
\]  
\( (73-75) \)

from internal agents, and collectively \( M = \{ A_1, A_2, A_3 \} \)

\[
M \xrightarrow{+H_2O} * 
\]  
\( (76) \)

This promise implies some interior structure, and it does not emanate from any smaller agent. Thus if an external observer is able to resolve the component agents within \( M \), the promise of \( H_2O \) is no longer a promise.

As noted in section 2.9, an observing agent may or may not be able to discern an internal elementary agent within a super-agent, i.e. whether the agent has internal structure, or whether it is atomic. This depends on whether the agent promises transparency across its surface.

**Definition 18 (Super-agent transparency)** All promises whose scope extends beyond the boundary pass transparently through the surface of the super-agent. Scope includes the list of promisees.

### 3.5 Sub-agents as the subject of a promise: emission and absorption

An agent may itself be the subject of a promise body. Agents can conceivably promise to spawn new agencies: cells multiply, particles transmute into clusters, humans give birth, organizations spin off departments into new organizations, etc.

**Assumption 2 (Emission and absorption, parent-child relationships)** It is within the allowed behaviours of an agent to emit (send) or absorb and incorporate (receive) a child sub-agent, which has a formally distinguishable identity to the parent.

The exchange of a sub-agent as an independent promise implies the exchange of the promises made by it too. However, there is no assumption about the promises having been kept before or after the keeping of the promise to emit or absorb the sub-agent. This might be reified later. The promising of an agent is thus somewhat like the passing of a point reference, with possibly late binding.

\( \text{These ionic properties may reach outside the super-agent boundary too in some circumstances, allowing oxidation, etc.} \)
Lemma 2 (Emission and residency) In order to not violate the autonomy of agents, an agent \( A_{\text{parent}} \) could only make a promise to produce an agent \( A_{\text{child}} \) if, at the outset \( A_{\text{child}} \subseteq A_{\text{parent}} \) i.e. resident.

The understanding that agents embody strings of information (with an arbitrary physical realization) makes all of the arguments simple. An agent might spawn another, such as a device \( A_{\text{device}} \) emitting a network packet \( A_{\text{packet}} \):

\[
A_{\text{device}} \xrightarrow{+A_{\text{packet}}} A_{\text{network}}.
\] (77)

The agent referred to in the body can, in turn, promise another agent in its body, e.g. a verifier of the checksum authenticator:

\[
A_{\text{packet}} \xrightarrow{+A_{\text{checksum}}} A_{\text{verifier}}.
\] (78)

In order to be an intermediary, the exchanged package should promise to both sender and receiver:

\[
A_{\text{checksum}} \xrightarrow{+\text{structure}} \{A_{\text{packet}}, A_{\text{verifier}}\}.
\] (79)

This does not necessarily imply the existence of a promise about another promise. A promise of the existence of an agent has to result in an autonomous agent, by the rules of promise theory.

![Diagram](image)

Figure 10: Agents may be emitted if they start as residents.

Definition 19 (Emission of a body part)

\[
A_{\text{super}} \xrightarrow{+A_{\text{sub}}} A_{\text{recipient}}
\] (80)

\[
A_{\text{super}} \xrightarrow{A_{\text{super}} \rightarrow \{A_{\text{super}} \rightarrow A_{\text{sub}}\}} A_{\text{recipient}}
\] (81)

Definition 20 (Absorption of a body part)

\[
A_{\text{recipient}} \xrightarrow{-A_{\text{sub}}} A_{\text{sender}}
\] (82)

\[
A_{\text{recipient}} \xrightarrow{A_{\text{super}} \rightarrow \{A_{\text{super}} + A_{\text{sub}}\}} A_{\text{sender}}
\] (83)

The signs inside the set braces imply union and complement removal, i.e. set difference.

3.6 Is there an empty space? The existence of a ground state.

Is there a discrete lower bound on agency? Can we infer the existence of a state of empty space in any system?

\[
A \xrightarrow{\emptyset} A
\] (84)

Suppose there is a lowest level to the hierarchy of agency at which point no new intention can be inserted. The promises made by the agent are purely names, since a name is a promise that identifies the presence of the agent. A name or label cannot be subdivided without simply resulting in more than one name, i.e. without increasing the number of promises.
**Hypothesis 1 (Lowest level of hierarchy)** There is a level at which names cannot be subdivided without losing the ability to function and be understood (or connected to). The ground state is a gaseous state of maximal symmetry. At this level, the only promise that an object can make is its name (this is a tautology).

Does an agent that makes no promises even exist? As mentioned in paper I, it is completely disconnected from the rest of a space, so we may consider it either to be non-existent relative to an existing spacetime, or be an entirely separate spacetime\(^{11}\). If we do not allow a complete absence of promises, then this suggests that empty spacetime must, at least, promise adjacency. That is, empty space requires at least a promise of adjacency. The promise might not be kept all the time, however, as is the case in a gaseous phase. This issue need further study.

### 3.7 Agency scales

Super-agency is a scaling transformation, in the sense of a dynamical system. It is therefore an important bridge between dynamics and semantics, with consequences for both. Because promises incorporate semantics, which may be arbitrarily applied to a collection of agents, boundaries for super-agency may be defined around any collection of agents.

#### 3.7.1 Definition of semantic agent scales

**Definition 21 (Agency scale)** A named collection of agencies considered to be the irreducible entities of space, i.e. it defines a set of possibly aggregate atomic agencies that are to be considered the set of addressable agents at the scale concerned.

Unlike dynamical scales, there is no \textit{a priori} identifiable measuring stick for semantic scales: they are non-ordered symbolic quantities, and must be promised independently by an observer, by promise types and body constraints. An agency scale is a thing in between a semantic scale and a dynamical scale. The identification of agency is a semantic issue, but the scale is a well-defined dynamical unit.

**Example 11** Suppose we have atomic agents \(A_1, A_2, A_3\), which make exterior promises:

\[
\begin{align*}
A_1 \xrightarrow{+H} & \ast & (85) \\
A_2 \xrightarrow{+H} & \ast & (86) \\
A_3 \xrightarrow{+O} & \ast & (87)
\end{align*}
\]

and interior promises

\[
\begin{align*}
A_1 \xrightarrow{+e} & \ast & (88) \\
A_2 \xrightarrow{+e} & \ast & (89) \\
A_3 \xrightarrow{-e,-e} & \ast \text{ (valency 2)} & (90)
\end{align*}
\]

We may combine these agents into a super-agent by defining a scale:

\[
\text{Molecular} \equiv \{M\},
\]

where \(M \equiv \{A_1, A_2, A_3\}\). \textit{At the molecular scale, we now have a single super-agent A, instead of three resolvable atomic agents.}

\(^{11}\)If one can imagine a disconnected agent that makes no promises at all, then from there, one can imagine postulating the existence of an infinite number of empty spacetimes. These matters become technical, and we have no way of deciding whether they are real or fictional, assuming that there is a distinction.
3.7.2 Explicit schematic example of agent scaling

Example 12 Consider figure 11.

Figure 11: The transformation of promises under a coarse-graining transformation. How promises appear to emanate from the super-agent surface.

At the level of atomic agents, we have exterior promises:

\[
\pi_1 : A_1 \xrightarrow{+b_1} A_5 \\
\pi_2 : A_2 \xrightarrow{+b_1} A_5 \\
\pi_3 : A_4 \xrightarrow{+b_2} A_5 \\
\pi_4 : A_4 \xrightarrow{-b_3} A_6
\]

and interior promises

\[
\pi_5 : A_3 \xrightarrow{+b_4} A_2 \\
\pi_6 : A_2 \xrightarrow{+b_5} A_4 \\
\pi_7 : A_6 \xrightarrow{+b_6} A_7
\]

The super-agents are defined by:

\[
S = \langle\{A_1, A_2, A_3, A_4\}, \{\pi_5, \pi_6\}\rangle \\
R = \langle\{A_5, A_6, A_7\}, \{\pi_7\}\rangle
\]

Scales have to be defined semantically in a promise model, since the interpretation of an aggregate boundary is arbitrary; thus, if we define the following scales:

Atomic \(=\) \(\{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}\) \hspace{1cm} (101)

Hybrid \(=\) \(\{S, A_5, A_6, A_7\}\) \hspace{1cm} (102)

Super \(=\) \(\{S, R\}\) \hspace{1cm} (103)

Then, at hybrid scale, the promises (92-98) above collapse to:

\[
S \xrightarrow{+(b_1 \cup b_2)} A_5 \\
S \xrightarrow{-b_3} A_6
\]

and at super scale:

\[
S \xrightarrow{+(b_1 \cup b_2), -b_3} R
\]
Example 13 The human eye is an example of a coarse grained receptor, i.e. a use-promise to accept radiation in certain frequency range. Light emitted at all frequencies in the range may be promised by emitters; however, only three coarse receptors (for red, green, and blue) cover this range. Thus colour vision in humans is limited to interpretation through these three channels.

3.7.3 Gauss’ law for coarse-grained agencies and their external promises

The divergence theorem (Gauss’ theorem) is a simple universal identity that applies to vector fields enclosed by a boundary. We may apply it to vector promises too. In its well-known form, if may be written:

$$\int_{\sigma} \vec{V} \cdot d\sigma = \int_{V} \nabla \cdot \vec{V} \, dv \tag{107}$$

i.e. the integral of vector flux emanating from a surface $\sigma$ is the result of the divergence of the vector field generated from the volume enclosed by it.

Using definitions for unadorned graph theory, we can show that a coarse graining is a simple application of this result. In other words, the promises that come out of any volume or grain of space are only a result of what agencies are inside it. Consider a grain consisting of a number of agents with a surface boundary, and let

$$\pi_{ij} = \pi^\text{int}_{ij} + \pi^\text{ext}_{ij}. \tag{108}$$

We observe that $\pi_{ij}$ is the $j$-th component of a local basis vector $\hat{e}_j$, surrounding any agent $A_i$. We can define a vector field in this basis by

$$\vec{\pi}(A_i) = \sum_{j} b_{ij} \hat{e}_j = \sum_{j \in \text{grain}} b_{ij} \pi_{ij}, \tag{109}$$

where $b_{ij}$ is the body (semantics) of agent $A_i$’s promise to agent $A_j$, where $i,j$ run over all agents (or just the nearest neighbours of $A_i$) that are picked out by $\pi_{ij}$. The components of the $\vec{\pi}(A_i)$ as a row vector are thus easily constructed:

$$\vec{\pi}(A_i) = (b_{i1}, b_{i2}, \ldots). \tag{110}$$

The values are not really important, as long as they can be defined, since they will cancel out of the sums. The derivative of this vector is anti-symmetric in $i,j$:

$$d_j \vec{\pi}(A_i) = (b_i (i+j) - b_{ij}, \ldots) \tag{111}$$

and thus the divergence is the sum of these where $i = j$. It is thus easy to see that the sum over the volume enclosed by any grain surface cancels everywhere except for those vector components that protrude through the surface:

$$\int d(\text{vol}) \sum_i (d_i \cdot \vec{\pi}(A_i)) = 0 + \text{exterior contributions} \tag{112}$$

$$= \sum_{i \in \text{surface}} (b_{i \epsilon_1}, \ldots) \cdot \hat{\sigma} \tag{113}$$

$$= \sum_{i \in \text{surface}} \vec{\pi}^\text{ext}(A_i) \cdot \hat{\sigma} \tag{114}$$

where $\hat{\sigma}$ is the unit surface vector. This is exactly (107) for $\vec{V} = \vec{\pi}$, summed over the grain. i.e.

$$\sum_{i \in \text{surface}} \vec{\pi}(A_i) = \sum_{\epsilon \in \text{neighbours}} \vec{\pi}^\text{ext}(A_\epsilon) \tag{115}$$

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3.8 Coarse-graining (agent aggregation)

Coarse-graining is what happens when one invokes a change of scale from a high level of detail to a low level of detail, i.e. from a large number of smaller agents to a smaller number of larger agents, by aggregation. This is usually done for systems assumed to have infinite detail, in the so-called continuum limit.

In a discrete system, it is straightforward to define a coarse graining by aggregation of autonomous agents into collections. In physics (dynamically), one does this by defining characteristic lengths (see the discussion for pseudo-continuous information in [10]). In a semantically labelled theory, there are no such easily defined lengths, and we are forced to define granular scales explicitly as sets, see definition 21 (section 3.7).

**Definition 22 (Coarse-graining)** A transformation in which the collection of agencies representing space-time are changed for a smaller set, with a corresponding reduction in detail. Coarse graining involves:

- The summation of bulk properties, and relabelling of composite super-agents by a single collective label.
- The elimination of all interior promises, which are not visible between the agents at the coarser scale.
- Loss of information to exterior from the above.

In a spacetime without boundaries, we identify self-similarity by the idea that a system is functionally and dynamically similar in its promises and behaviours, before and after coarse-graining by the coarse graining function:

**Definition 23 (Coarse-graining function)** A many-to-one function whose domain is the semantic space-time \(\langle\{A_i\}, \pi_{ij}^M\rangle\), composed of a collection of agents \(\{A_i\}\) at scale \(M\), along with its promise matrix \(\pi_{ij}^M\), and whose co-domain is the image doublet \(\langle\{S_k\}, \pi_{kl}^{M'}\rangle\), at scale \(M'\):

\[
\langle\{A_i\}, \pi_{ij}^M\rangle \rightarrow G(\langle\{A_i\}, \pi_{ij}^M\rangle) = \langle\{S_k\}, \pi_{kl}^{M'}\rangle
\]

(116)

where \(i, j = 1\ldots \text{dim}M\), and \(k, l = \text{dim}M'\), where \(\text{dim}M' \leq \text{dim}M\).

The function \(G()\) is a dimensionless renormalization transformation. Scale-free behaviour can not usually apply to a bounded space, as boundaries pin the system at a fixed scale. It might still be possible to compute effective equations for dynamical systems however [11,12]. Two spacetimes may be considered equivalent iff:

**Definition 24 (Spacetime equivalence function \(\simeq\))** The equivalence of two semantic spacetimes designated by \(\langle\{A_i\}, \pi_{ij}^M\rangle\), and \(\langle\{S_k\}, \pi_{kl}^{M'}\rangle\) may be written

\[
\langle\{A_i\}, \pi_{ij}^M\rangle \simeq \langle\{S_k\}, \pi_{kl}^{M'}\rangle.
\]

(117)

where there exists a permutation group relabelling of \(i, j \rightarrow k, l\) \(\pi_{ij}^M \equiv \pi_{kl}^{M'}\) and \(\text{dim}M = \text{dim}M'\).

The transmission of influence by promise exchange is a dynamical property that must also be affected by coarse graining. This is the essence of the Renormalization Group in scaling systems [11]. When a coarser grain size influences a smaller grain from the top down, it takes the form of effective boundary conditions on the behaviour of the smaller scale. This tends to introduce non-linearity, as it modifies the propagation law for behavioural trajectories. If a smaller scale interacts with a larger scale, from the bottom up, it takes the form of a perturbation, which probes the stability of the larger grains.

There is no a priori connection between scale and influence; this depends, as usual, on what exterior promises are made by the agencies regardless of grain-size. Multi-grain-size models are perfectly possible. Indeed, they are the natural state of materials like steel and plastics.
Figure 12: Agency coarse graining. If there is a mismatch in scale between promiser and receptor, the promise might not make sense. In some cases, it might be possible to build a large scale solution from smaller grains, but not vice versa.

3.8.1 Coarse graining of interior and exterior promises

Which promises are lost through coarse graining? Effective external promises, made by a super-agent, have their origin in the internal constituent agents, but they do not preserve all of the internal details, and they might add new ones (see section 3.10). In order to scale agency, we therefore need to understand the scaling of both interior and exterior promises, and how they preserve the semantics of a system during a scaling transformation.

Example 14 The chemistry promised by an atom does not depend strongly on the nature of the isotope of a chemical element, only on the electron structure. Thus electron shells made exterior promises from the surface of the agent.

3.8.2 Promiser coarse graining, and coarse-graining directories

Promises that remain exterior to a given grain, under a coarse graining process, may be combined under the idempotency rule for promises, and the combinatorics between common end-points. Hence promises of the same type will merge into a single maximal promise, defined below.

It remains possible for a single agent (coarse or fine grained) to make different promises to multiple recipients. These cannot be combined. However, promising from different sub-agents to the same recipient becomes an instance for the idempotence rule once the difference between the promisers is coarse-grained away. Promises with uniform promisees are idempotent, or cumulative by virtue of the set union. The general scaling rule for promisers is thus:

Definition 25 (Scaling rule for promiser) Let \( b_i \) be a set of promise bodies of the same type, i.e. \( \tau(b_i) = \tau \), \( \forall b_i \), originating from a set of distinct agents \( A_i \). Then we may define the coarse graining of the set of agents, over constant \( \tau, \varepsilon \), by:

\[
G_n \left( A_1 \xrightarrow{b_1} A_c, \ldots, A_n \xrightarrow{b_n} A_c \right) = G_n \left( A_1, \ldots, A_n \right) \frac{\text{max}(\{b_i\})}{\text{A_super}} A_c
\]

(118)

where \( \text{max}(x_j) \) represents the union of the sets pertaining to set \( x_j \) intended for agent \( A_j \):

\[
\text{max}(x_j) \equiv \bigcup_i x_i.
\]

(120)
Not this rule does not depend on the sign of the promise body $b_i$.

In other words, the coarse-graining of a set of promises is equivalent to a possibly smaller set of aggregate agents, which promise the composite effect of the individual promises. The total number of promises is reduced from $n$ to 1 in this coarse graining.

A coarse-graining procedure is not a bijection, i.e. it is not an invertible map or function. When a coarse graining of promisers is undertaken, information about the specific fine-grain promises is lost. In order to reverse a coarse-graining process, we would have to preserve the information lost.

**Definition 26 (Coarse-graining directory)** The information lost during coarse-graining in equation (119), takes the form

$$
\pi_{\text{directory}}(\tau) = \left\{ A_1 \xrightarrow{b_1\in\beta_1} A_\varepsilon, \ldots, A_n \xrightarrow{b_n\in\beta_n} A_\varepsilon \right\}
$$

for promises of type $\tau$, expressed in language $\beta$. Hence, whenever we group agents under a common umbrella, by coarse-graining, there will be associated map, of the form,

$$
\langle \tau, \pi_{\text{directory}}(\tau), \beta \rangle, \quad \forall \tau,
$$

that preserves the lost information, and could restore the detailed view of which sub-agent is able to keep a promise perceived at the level of the whole. This is the information that becomes unavailable at the coarser scale$^{12}$. We may call such a map the super-agent directory or index map, and write its promise:

$$
A \xrightarrow{\text{directory}(A)} A'
$$

Coarse-graining replaces $\pi_{\text{directory}}(\tau)$ with a single promise of type $\tau$, and an effective promise body. So, for each external promise of type $\tau(b_i) = \tau$, $\forall i$, coarse-graining leads to a surjective map, between the collection of promises made by sub-agents and the single effective promise made by the super-agent. This map cannot be inverted, however it can be resolved to direct the external agent to an appropriate microscopic part of the super-agent.

**Lemma 3 (Resolvability of super-agent detail)** Let $A_{\text{super}}$ be a super-agent at scale $M_1$, formed by coarse graining promises at scale $M_0$. From in equation (119), we see that the information lost during coarse-graining $M_0 \rightarrow M_1$, takes the form of a collection of promises called a directory or index

$$
\pi_{\text{directory}}(\tau)
$$

By preserving, this directory we may expose and resolve the scale $M_0$ under $M_1$:

$$
\pi_{M_1} + \pi_{\text{directory}}(\tau) \geq \pi_{M_0}.
$$

How an external agent resolves a microscopic inspection of a super-agent will be discussed in section 4.1.1.

### 3.8.3 Promisee coarse graining

For scaling of the promisees, the scaling rule is simpler than for the promiser. It basically amounts only to a redefinition of scope. Information about the scope within the promisee is lost to coarse-graining, but can be restored by using the coarse-graining directory (section 3.8.2), through scale transduction (section 4.1.4).

$^{12}$Note, this information is related to the entropy of the system, but it is not related to disorder, merely a loss of information about order.
Definition 27 (Scaling rule for promisee)

\[ G_n \left( A_1 \xrightarrow{b_1} A_{\varepsilon_1}, \ldots, A_n \xrightarrow{b_n} A_{\varepsilon_n} \right) = A_1 \xrightarrow{b_1} G_n \left( A_{\varepsilon_1}, \ldots, A_{\varepsilon_n} \right), \ldots \]
\[ \ldots, A_n \xrightarrow{b_n} G_n \left( A_{\varepsilon_1}, \ldots, A_{\varepsilon_n} \right), \]
\[ = A_1 \xrightarrow{b_1} A_{\text{super}}, \ldots, A_n \xrightarrow{b_n} A_{\text{super}}, \]  \hspace{1cm} (125)

and \( G_n(\sigma) \) is obtained by replacing any \( A_{\varepsilon_1} \ldots A_{\varepsilon_n} \) with \( A_{\text{super}} \).

In other words, the coarse graining of a set of promises, in which only the promisees are aggregated, leads to the same set of promises (because promises originate from the promisers), made to the smaller set of aggregate agents, with only loss of detail and possibly redefined scope. This is now a question of definition, as the information about specific promisees and scope membership is lost as agents are aggregated.

3.9 Intra-agent language uniformity

The assumption of the previous sections has been that every agent shares a single uniform language. As super-agency forms by aggregation of parts, there is nothing to constrain the language inside the super-agent boundary. Hence, we must expect linguistic diversity inside a super-agent, limiting the interactions on the interior.

Definition 28 (Language of a super-agent) For the purpose of all exterior promises, the effective language of a super-agent \( S \) must be considered the union of all sub-languages:

\[ \beta_{\text{super}} = \bigcup_i \beta_i, \]  \hspace{1cm} (126)

where \( i = 1, \ldots, \dim S \). The promised language will only be understood with a reduced probability however, since the coverage by sub-agents is non-uniform.

This information about linguistic diversity is also lost under coarse-graining, hence it must become part of the coarse-graining directory.

3.10 Irreducible promises at scale \( M \), and collective behaviour

In addition to the exterior promises, which emanate from composite super-agencies, as the remnants of microscopic promises belonging to component sub-agencies, we may also observe that completely new promises are possible at each scale, which do not belong to any specific agent inside the surface.

Definition 29 (Irreducible super-agent promises) Let \( M \) be an agency scale, and \( A_s \) be a super-agent formed by an aggregation of agents. A promise with body \( b_s \) made by \( A_s \)

\[ \pi_M : A_s \xrightarrow{b_s} A_\gamma, \]  \hspace{1cm} (127)
\[ A_s = \{ A_i, \ldots \} \]  \hspace{1cm} (128)

may be called irreducible iff there is no set of sub-agents \( A_i \in A_s \), for which

\[ b_s \subset \bigcup_a b_a, \quad a = 1, \ldots, \text{all promises to } A_\gamma \]  \hspace{1cm} (129)

where \( b_a \) are existing bodies promised to \( A_\gamma \) by \( A_i \), and \( A_i \neq A_s \).
In other words, if there exists a combination of promises made by one or more sub-agents (and we assume that the sub-agent is not the same as the super-agent), then that is semantically equivalent to the full promise made by the composite agent, when one could say that the super-agent promise could be reduced to the promise of one of its components. As long as no single agent, working alone, can make such a promise, it makes sense to talk about the collective super-agency making a new promise that is not explicit in the capabilities of its sub-agencies. Thus, irreducible promises at scale $M$ take into account emergent effects, and collective effects of agent interactions.

**Figure 13:** A radio is an example of a composite ‘super’-agent, with interior agencies and promises of circuit connectivity, and exterior promises to play music. The radio and listener define one agency scale, but the components and a repair engineer could form another. Scales of interaction are not necessarily subject to uniform coarse-graining in a world of semantics.

**Example 15** A radio is a composite agent that makes the exterior promise of playing radio signals on a loudspeaker (see fig 13). Interior promises are made by many component agents like resistors, capacitors, transistors, etc. The radio has a semantic surface which interacts with people as promisees. Does this mean that the components inside the radio have to interact with the components inside a person (cells, organs etc)? Not really. Implicitly, this might be partially true by the distributive rule, but clearly that is not a requirement. Any agency can interact, semantically, with any other agency at any agency scale, provided there is a physical channel for the communication to work.

Irreducibility is thus a result of collective phenomena in the underlying semantics and dynamics. The latter answers classic objections against the naive reductionism of systems. Yes, we can decompose systems into a sum of parts, but one must not throw away the promises when doing do, else it will not be possible to reconstruct both semantics and dynamics. This is a consequence of boundary information or systemic topology.

The form or collective identity of a super-agent can be enough to signal a promise, by association. This is an emergent promise in the sense of [2], i.e. one inferred by a potential promisee rather than given explicitly by an agency. Nevertheless, some scaled objects do make promises: tables, chairs

**Law 1 (Reduction law)** When reconstructing a system from components from finer-grained scale $M'$ to a scale $M$, all super-agencies at scales below $M$, and their component promises must be retained in order to reconstruct the system as the sum of its parts. This is achieved if each super-agent promises its coarse-graining directory, providing dynamic transparency.

---

13In fact, in the case of a radio, one could argue that it is the outer casing which makes the promise of being a radio, and that the other components are tenants of the outer casing agent. I’ll return to the issue of tenancy in the latter part of these notes.
Thus increase of detail downwards may not be at the expense of loss of upward irreducible information. If we try to view a compound agent as a collection of parts, with component promises (on the interior of the super-agent), the component promises revealed do not replace the high level exterior promise: they are simply prerequisites for it. The only reason to disregard irreducible promises belonging to a coarse grain is because one is focusing on the internals of an agent in isolation (what one calls a closed system in physics).

If a super-agent has no agency of its own, how can it make a promise? Clearly, a collection of agents has agency through its members, and these may or may not have the ability to keep promises related to scale $M$. So where do promises come from in a super-agent?

- **+** promises: the collective appearance of a super-agent, at a certain scale, must provide the information to signal a promise. The declaration of the promise may or may not come from a single agent. The keeping of an irreducible promise does not come from a single agent, by definition. New irreducible promises are dependent on the individual agents, only through their cooperative behaviours.

**Example 16** A troop unit promises to surround a house (+ promise). The troop leader can make the promise on behalf of the group, but no single agent can keep this promise, but collectively they can. In this case, the promise would often be given by a team commander, with a centralized source of intent, and subordinate agents. However, a team can also arrive at this promise by cooperative consensus.

- **−** promises: a promise to accept another promise, made by an external agent, might only apply to certain sub-agents with the appropriate capabilities. It must be provided as an exterior promise based on the by cooperative agreement amongst the agents.

**Example 17** The Very Large Array of radio telescopes in Mexico has 27 receivers. Each receiver (− promise) is coordinated with the others so that they act as a single super-agent. No single agent can see what the full array can see working together, up to diffraction. Hence the combined array can make promises that individual agents can’t.

A super-agent’s agency is thus conditional on the existence of its sub-agencies, even though its promises are not locatable in any single agent. In both cases above, the promise made by a super-agent could not be made by a single agency (this is what we mean by a ‘host’), or by distributed consensus. Irreducible promises are thus conditional, not only on the uniform cooperation of the sub-agents about a single promise, but on their making all the promises that indirectly lead to the irreducible property.

**Lemma 4 (Irreducible promises are conditional promises, for all scales greater than $M_0$)** Irreducibility is not expressible directly as a sum of component promises of the same type as the irreducible promise, but it is second-order expressible in terms of sub-agent promises, by building on the existence of these contributing promises.

With no promises to build on, a collective agent cannot make any kind of promise, since nothing can be communicated, and it has no independent agency. Crudely, a super-agent is indeed the sum of its parts, as far as agency and promise-keeping are concerned; but, it is not merely a direct sum as new promises are possible through cooperation.

If the scope of exterior promises extends to the interior of a super-agent boundary, that scope becomes ambiguous under coarse-graining. External observers can no longer see which agents are make or receive the interior promises, nor is there any way to refer to them independently after coarse-graining. Observers can only assume that the scope of a promise includes all sub-agents, but this might not be the case, and might result in erroneous expectations. Indeed, there is no reason why the sub-agents would even all use the same body language. We explore this more in section 4.1.
3.11 Scaling of promise impact, and generalized force

As we scale agency to deal with larger entities, it seems unreasonable to expect the impact of a single promise from a sub-agent to have the same impact on the super-agent as on the microscopic parts before coarse-graining. When might we be able to disregard a promise? If system is stable, i.e. small influence leads to small effect. Without going into too many details, I’ll sketch how this can be dealt with.

Example 18 In dynamics one has the notion of change of momentum (force) and pressure (force per unit area). It seems natural to try to construct such a notion of force density or pressure for promises too. However, we would note that even though momentum transfer alters in magnitude, the semantics of momentum transfer do not change, i.e. Newton’s law of momentum conservation does not change.

An effective force (see section 2.9) is one possible measure of impact. It’s attraction is that it works for semantics and dynamics, though it is clearly modelled after physical dynamics. If we tried to scale this relation, we would scale the promises first, but then we also need to scale the assessment function. I’ll leave that exercise for another time, or as an exercise to the reader.

The coupling of one scale to another has some coupling strength which depends on or describes the transmission effect of information between agencies. In physics we have the law of conservation of momentum as the currency of influence, and energy as a ‘stored wealth’. For semantics, we need an impact if intent where intent is preserved, but not necessarily outcome. This is more analogous to the conversation of charge in physics.

There is thus a reasonable algorithmic procedure for progressively disregarding the impacts of certain promises relative to the scale of certain agencies, as we coarse-grain a system.

The characterization of weaknesses, cracks, and defects, in spatial promise structures, is an obvious follow-up question to this notion of semantic impact. If two super-agencies or subspaces come into contact, could there be catastrophic outcomes by which one region might not be able to withstand the influence of the other? This introduces concepts like dynamical stability and material failure. Here, instability transduces the smallest dynamical effect into the largest semantic importance.

4 Agent hierarchies

A full range of agent dynamics should be able to mimic all the processes of the natural world, as well as artificial behaviours based on computation. Promise theory’s goal is then to explicate the semantics of these processes. As in the scaling and renormalization of physics, this leads to hierarchical ideas. However, the implications for semantics go further than those dynamical ideas, as function is often tied specifically to a fixed scale.

In the foregoing sections, I’ve shown explicitly how the sub-agencies of one (super-) agent can be contained within its boundary, and even promised to others as a resource, by emission. Agency thus exists and interacts in coarse, bounded ‘packages’, much like Milner’s notion of bigraphs (see paper I), and all the time on top of a substrate of basic adjacency promises that we call fundamental spacetime.

4.1 Resolving interior details of super-agent structure during coupling

Every time an observer zooms out by coarse-graining, the detail wiped out by the formation of a grain can be captured as a map called a directory or index (see discussion in section 3.8.2). Preserving this map can help an external agent to resolve and interact with the sub-agencies inside a super-agent’s boundary.

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14 Agents can be exchanged, by emission from one agent and absorption by another, as the actual information of the promise body. This is how one models the exchange of forces in quantum theory, via gauge bosons like the photon, or gluons, etc.

15 In information technology, these maps are called variously directory services, name services, indirection tables, or data indices.
This is paradoxical: in order for an agent to exchange promises with a super-agent at scale \( M \) (which has no physical boundary or form other than its constituent parts), an external agent perceiving the effective promise at scale \( M \) would surely have to be directed to an available sub-agent to provide the directory is available to the external agent.

**Example 19** A bank super-agent might promise to give you cash, but the promise still has to be given meaning and carried out by an actual bank teller. The bank itself has no interface to bind to without its sub-agents.

As a fictitious boundary, a super-agent could simply be imagined by the assessments of an observer, with no coordinated intent of its own. However, a promise made to or by a super-agent has to be a promise made to or by its components somehow. If a promise binding is only conceptual, this might not be an issue, but if an actual transfer of information is implied, there must always be a real source and a real receiver.

**Definition 30 (Transparency)** An agent may be called transparent if it promises an index or directory of all its internal sub-agents and their promises.

In the remainder of this section, we address how such a coupling of agency scales, given what we know about irreducible promises (see section 3.10), from the dual perspectives of dynamics and semantics.

### 4.1.1 Distribution or dispatch of promises at super-agent boundaries

So, what happens at boundaries when a promise is made to a super-agency? What happens to the information? Let’s try to answer the question dynamically first, since everything is dependent on what is dynamically possible.

When we make a new promise at scale \( M \) to a super-agent, we need to understand what this means for the component sub-agencies at a finer-grained scale within it. Two possibilities present themselves:

- **Distribution/flooding (broadcast):** Promise bindings made to a super-agent are broadcast or diffused throughout the sub-agents that comprise it, spanning multiple agent locations, like the behaviour of a gas or fluid flooding into contact with an interface.

- **Direction/dispatch (switched):** Promises are routed to a subset of sub-agents, or representative binding sites, in a solid state, making an exterior use-promise on the surface is responsible for accepting the promise. The routing can be direct from the promise to the interior sub-agent (if the super-agent exposes its directory), or it can be made via a proxy routing agent inside the super-agent (if it exposes only a gateway).

Why and how these possibilities should happen at all merits some further discussion. The details almost certainly depend on the scale and context. The generality of the questions (and the occurrence of examples in the natural and technological worlds) is what makes them most intriguing.

**Example 20** Consider an example of an extended super-agent \( \{a, b, c, d\} \) bound by some cooperative promises, which we neglect to mention here. These may occupy a space of similar extent \( \{A, B, C, D\} \), as in figure 14. This scenario is a realization of many possible scenarios, e.g. a journey in many legs (plane, train, network routing), in which the promise of multi-tenant sharing of several sequential host resources forms a journey in which several hosts have to cooperate as a super-agency (an inter-network cloud) (see figure 14). A traveller (a tenant of the journey) has to be authorized for passage by each stage of the journey. This requires promises to authenticate credentials to be distributed throughout the path, and the collaboration of the hosting agencies in trusting the credentials.
Figure 14: Path tenancy. The circled agents form a single super-agent that occupies the corresponding space in the line above. The circles region is simply a super-agent with exterior promises at its end-points. An agent binding to each site in transit can only do so at the scale of the sub-agents but the tenancy binding can be made as a distributive promise to the super-agent.

**Example 21** Directing or dispatching promises, through a specialized agent, is like using a reception desk, service portal, in an office or hotel. Routing of information requires the underlying adjacency infrastructure to be able to direct messages to particular addresses.

We may now state the two methods formally, for clarity:

**Definition 31 (Distributive promise, at scale $M$ (flooding))** A promise made to a super-agent $A_s$

$$A \xrightarrow{+b} A_s$$

(130)

is assumed made to all agents within $A_s$: $A \xrightarrow{+b} A_i$, $\forall A_i \in A_s$.

(131)

The agents $A_i$ voluntarily accept the promise, if they are suitable recipients, hence selecting by brute force rather than intentional labelling.

**Example 22** In information technology, flooding is used to make a ‘bus architecture’. Ethernet and wireless transmission are examples.

**Definition 32 (Directed promise, at scale $M$ (dispatch))** A promise of type $\tau$, made to the super-agent, is assumed directed to a named subset of (one or more) members, on behalf of the entire super-agent.

$$A \xrightarrow{+b} A_i, \quad A_i \subset A_s.$$  

(132)

The subset $A_i$ voluntarily accept the promise, if they are suitable recipients, which we may assume is likely, given the intentional direction.

**Example 23** In information technology, dispatch to a directed address is used in queue managers, like load balancers, or memory and storage devices, to route data to a labelled destination.

Stating these methods does not imply that they are possible in all cases. To understand whether diffusion of promise information is realizable we need to understand the small scale adjacency structure of spacetime, and its effect on promise scope.
4.1.2 Adjacency between external agents and super-agents

Can promises, made by an exterior agent, reach all the internal sub-agencies in a super-agent, then be comprehended and accepted? A promise made to a super-agent has to be transmitted along the network of underlying adjacencies.

Both dispatch and distribution approaches to dissemination and binding assume that promises can be made directly between the sub-agents of neighboring super-agents. The communication needed to make and keep such promises depends greatly on the network substrate of adjacency made at the lowest spacetime level. So the question becomes one about how spacetime adjacency is wired (see figure 15).

Example 24 To visualize promises made at coarse-grained scale, imagine a water authority that promises electricity to a town. Both these agencies are super-agents composed of many sub-agents. Where (which agent) does the promise come from, and who receives it? What adjacency allows the promise to be transmitted?

A generic promise made in the name of the company, depending on its legal department might make the promise. Every resident in the town is a potential recipient, as long as they can receive the information directly or indirectly, i.e. as long as they are in scope. The adjacency might be by postal communication and by water pipe.

In figure 15a, the external agent promising to a super-agent is directly adjacent to every sub-agent inside it. In figure 15b, the external agent only connects to a binding site. How these adjacencies come about, in practice, depends on the phase of the agents. There are two possibilities:

- Agents in a disordered gaseous state (no long range order), agents have no prior knowledge about one another without random walk meetings, and binding to one another. Thus discovery is a kind of Monte Carlo search\(^\text{16}\), and communication is like broadcasting or flooding with messenger agents.

- Agents in an ordered phase, can assign fixed coordinate locations which can be indexed and used to access agents by design. This requires a mapping in the index between promises and locations, and adjacency between the index and each agent inside the super-agent boundary. Thus an index or directory service must act as a switch, routing promises to intended destination.

This, in turn, can be done in two ways:

- by exchange of agent contents at the boundary, and exterior lookup

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\(^{16}\)This is somewhat like the way the immune system binds to cell sites.
– by encapsulation of agent contents and routing with interior lookup

Figure 16: A schematic ‘virtual’ coarse-grained view of promising, on top of the real adjacencies between sub-agents. If adjacency is mediated through intermediate adjacencies, the entire structure has to support the dispatch and/or flooding of messages to keep promises through the substrate of true adjacencies (whether gas or solid).

4.1.3 Semantics of promise scope for super-agency

The semantics of scope need to be clarified during scaling, since scope represents the boundary of information about a promise’s intent, and the ability to distribute a promise depends on the underlying adjacencies of spacetime. If two agents are not adjacent, they might not be able to occupy the same scope.

**Lemma 5 (The scope of a promise to a super-agent)** Consider a promise made to a super-agent \( S_M \) at scale \( M \):

\[
\pi : A \xrightarrow{b} S_M. \tag{133}
\]

Without the coarse-graining directory \( \pi_{\text{directory}}(S_M) \) for super-agent \( S_M \), the scope of \( \pi \) is only defined to the boundary of \( S_M \). It is not possible to say which sub-agents of \( S_M \) are in scope of the promise. If access to the directory is promised to the promiser:

\[
S_M \xrightarrow{+\text{directory}} A \tag{134}
\]

\[
\sigma_M \rightarrow \sigma_M + \sigma_{\text{directory}} \tag{135}
\]

If agent \( S_M \) promises access to its coarse-graining directory, an external agent \( A \) can infer the scope of a promise in terms of the sub-agents of \( S_M \).

The promise could be visible to all the sub-agents \( A_i \in S_M \), which are adjacent to \( A \), as well as the extra scope \( \sigma \). However, a promiser can say which agents are reachable by a promise message if and only if the directory \( \pi_{\text{directory}}(S_M) \) is available.

The implication of this is that promises do not scale automatically by replacing an agent with a super-agent: the scope of a promise made to a super-agent is not necessarily distributive, because of the loss of information in coarse-graining.

36
4.1.4 Scale transduction: coupling to a super-agent boundary (gateways and advertisements)

If all the specific actionable agencies are concealed by a super-agent boundary, how can any promise message reach them from outside? The sub-agents are the agencies that must ultimately act to keep any promises of the super-agent. This becomes a question about information transmission. Loss of transparency, i.e. loss of information both about adjacency and agent type hinders promise resolution.

As seen in section 4.1.2, the underlying adjacency of spacetime remains even under the coarse-graining of super-agency, though it might not be visible inside a boundary. A promise could thus reach the sub-agents by flooding, or by direct dispatch, following the adjacencies within the boundary, provided sufficient adjacency exists. Dispatch can be directed by the promiser (if transparency is granted to an external agent by access to the coarse-graining directory) or by an intermediary agency (acting as a relay gateway) within the super-agent. It is assumed that there are no changes to agent semantics simply by aggregation:

Assumption 3 (Promisee autonomy is preserved in super-agency) Once a promise reaches a sub-agent, it is up to the sub-agent to accept the promise or not, and behave accordingly, unless it has voluntarily promised to subordinate itself to another agent. Even if the promise is transmitted as an imposition, autonomy can never be violated.

Thus the scaling of cooperation remains does not change the rules of autonomy; the ‘voluntary cooperation’ assumption of autonomous agents persists. It is up to sub-agents to use any promise made to them. However, this does assume that they are in the scope of such an external promise. Hence to ensure the coupling of an external agent to a super-agent, transparency has to be restored or relayed.

![Figure 17: Scale transduction for incoming promises to a super-agent: (a) the coarse-graining directory is internal to the super-agent and the external agent makes its promise to a gateway ‘receptor’; (b) the coarse graining directory is exposed and makes the super-agent transparent to the external promiser so that it can promise directly to the sub-agents; (c) there is no directory, and internal information is lost. Promises are flooded to all sub-agents, and may or may not be picked up. The efficiency of flooding depends on the solid or gaseous state of the super-agent.](image)

Under coarse-graining, a super-agent is seemingly replaced by only its boundary, possibly with a promise to access its index/directory of interior information.

Promises from external agents outside the boundary can only refer to the super-agent as promisee or body-tensor coordinate. How then would the internal sub-agents know what to do with the promise? How do they find one another?

In order for one scale to couple to another scale, we can thus introduce the idea of a scale transducer: a combination of necessary and sufficient conditions for promises made to a super-agent boundary, to be resolved without mentioning any sub-agencies.

Example 25 What part of a radio makes the exterior promise of being a radio, rather than a collection of electronic components? Whether the device is switched on or off, its function is not clear without a
promise. The agency that explains this is usually the packaging of the radio, i.e. the casing. It might further be packaged in a box, but that is not a part of the radio itself. In this case, the enveloping a casing becomes an agent with a promise that tracks the super-agent boundary\textsuperscript{17}.

**Definition 33 (Scale transducer)** Let \( A_s \) be a super agent at scale \( M = \{ A_s \} \), and let \( \pi_s \) be a promise made \( A_s \), by any agency \( A \) (see figure 17). Recalling that the scope of a super-agent includes all agents inside it, and coarse graining limits scope (see section 3.8.3), we define a scale transducer by:

1. A number of promises addressed to the super-agent boundary, along exterior adjacencies.
2. One or more sub-agents that make exterior use-promises, within the super-agent, to accept the promise made to the collective:
   \[
   \exists A_i \in A_s : A_i \xrightarrow{U(\pi_s)} A. \quad (136)
   \]
3. An index for directory \( \pi_{\text{directory}} \) is promised externally to agents outside the super-agent boundary. This provides them with information about how to address their promises.
4. If there is more than one agent adjacent to the super-agent, i.e. more than one possible agent that can make promises with the exterior, these agents can be located through the directory.
5. One or more sub-agents in \( A_i \) play the role of gateway and dispatcher, to conditionally forward messages to interior agents with matching use-promises. This is a standard entry point for the agent, e.g. it is the agent adjacent to all exterior agencies, thus forming a skin or boundary between exterior and interior agencies.
   \[
   \exists A_i, A_j \in A_s : A_i \xrightarrow{U(\pi_s)} A \quad \xrightarrow{+\pi_s, \pi_s} A_j \quad i \neq j
   \]  
   (137)  
   (138)

In this case, the super-agent must advertise the location of one or more gatekeepers, entry-points or directory agencies that promise to relay information.

(a) The sub-agents inside a super-agent boundary of a given type \( \tau \) are advertised in the index/directory \( \pi_{\text{directory}}(\tau) \). They are symmetrical with respect to what they promise (promise type \( \tau \)), but they might not necessarily be equivalent in how much they will promise (their promise bodies might differ in all details except the type). This means that the directory must advertise any promises to this effect also.

(b) If a gateway is used as a proxy relay (see figure 17a), it must additionally make promises that select promisees from the sub-agents inside \( A_s \), e.g. policies might be distributed (by flooding) or directed by dispatch.

(c) Gateway agents might also need to translate between the language \( \beta_s \) assumed for the super-agent, and the language(s) of recipient sub-agents \( \beta_i \), as part of transducing through the opaque boundary.

We may note that messages from an external agent might be forcibly constrained to a particular route by spacetime structure, i.e. by a limited adjacency (a bottleneck, as in section 4.1.2). Also, a gatekeeper need not be a single agent, or even a localized cluster in the role of gatekeeper. It could, itself be a fully distributed collective agency, embedded within a specialized set of sub-agents, and coordinated by mutual

\[\text{\textsuperscript{17}I've usually drawn agents schematically as atomic dots, but the shape of an agent is not defined. Nothing prevents it from appearing as a shell.}\]
cooperation. e.g. like a cellular skin. This idea of exterior agents binding to specialized ‘docking sites’ leads us naturally to consider the idea of tenancy in semantic spacetime structures. Indeed, I’ll return to this later in these notes.

From the list of requirements for transducing promises between scales above that the following corollary applies:

**Lemma 6 (Dynamical requirements for coupling between external agent and super-agent)** Any agent $A$ can probe scale information in a super-agent, at scales finer than its boundary, provided the super-agent promises its coarse-graining directory:

$$
\pi_{\text{directory}}(S_M) : S_M \xrightarrow{+\text{directory}(S_M)} A \quad (139)
$$

$$
\pi_{\text{directory}}(S_M) : A \xrightarrow{-\text{directory}(S_M)} S_M \quad (140)
$$

Alternatively, we can say that a coarse-grained agent $S_M$ may be made transparent by promising its coarse-graining directory.

**Example 26** To couple to a single interface in an electronic device, there has to be an exterior promise to bind to (e.g. USB), and an address of the agent inside.

**Example 27** In order to affect a nucleus within an atom (e.g. NMR), a field basically floods its promise to all sub-agencies blindly, hoping to excite the resonance (receptor use-promise).

### 4.1.5 Addendum on scaling of scale transduction itself: queue dispatch

Since scale transduction is itself a dynamical process, dependent on underlying spacetime, its efficiency is also subject to scaling issues. Consider first the coupling issue from a semantic perspective, of interfacing instead. When a promise is made to a super-agent boundary (as in equation (133)), the promise information has to go to an agent that is listening. A super-agent is not such a real agent, it is only an abstraction. This suggests that, in the case of super-agents, the boundary itself might be represented by an explicit agency that can perform routing and forwarding of messages between the super-agent’s fictitious boundary and its sub-agencies.

While this direct dispatched routing of promises makes sense semantically, dynamically, the idea seems contrary to the notion of scaling: to replace a scaled mass of agents by a single gatekeeper or router creates an obvious bottleneck and fragile dependence. This is the cost of coarse-graining, especially in a discrete spacetime, and it suggests that the grain size should never become too large, else a super-agent becomes hindered by interfacing issues.

**Example 28** In queueing theory, a dispatcher is an agent that processes a queue of incoming messages and routes them to a service agent [13, 14]. Load balancers introduced into networks as ‘middle boxes’ are single-agent dispatchers to multiple sub-agents within an super-agent of servers. These middle boxes break the equivalence of the system under re-scaling, by forcing all adjacency through a single route.

**Example 29** A reception desk at a company accepts promises and information on behalf of the collective organization of sub-agents. An agent works at this desk to receive messages, and dispatches, routes or forwards these messages to other relevant agents using an internal directory for responsibility.

An alternative is for the super-agent boundary to promise flooding contact and allow the surface agents to coordinate internally, as redundant gatekeepers, each deciding how to resolve what happens if multiple agents receive the same message. If promises are not, by their nature, exclusive to a single gateway, then coordinating exclusivity adds $N^2$ complexity of promise coordination.
**Example 30** Using the coarse-graining directory to give transparency, any external agent could perform its own dispatching / load sharing without loss of scalability. For example, a directory service used by software could replace ‘middle box’ load balancers in software, with the help of a software interface used to select a specific sub-agent server within the super-agency of all servers.

Let’s summarize the implications of loss of scope from the past previous section.

- Routing of messages to internal agents through a single gateway breaks scale invariance.
- A scale transducer may be introduced, for mediating interactions directly by granting transparency, using a directory.
- Directory promises allow external agents to look up lists of sub-agents encapsulated within enabling scale transduction, i.e. a kind of microscope for crossing the semantic scale boundary.

### 4.2 Addressability in solid structures, and the tenancy connection

To see how we can route messages back to the specific sub-agents, we need to understand addressing. The ability to give every agent a predictable address, and then be able to have messages forwarded to it uniquely, using that address, depends on a number of promises being kept. To illustrate this, let’s construct a semi-lattice by iterating a simple asymmetric message pattern.

![Figure 18: Two promise bindings, leading from a source agent S to two different recipient agents R₁ and R₂. This forms the basis of a routing structure for address decomposition, where part of an address can lead to selection or rejection of a particular route.](image)

Let \( S \) and \( R_i \) be two types of roles for a set of agents \( A_i \), and consider bindings between two kinds of promise:

- A vector promise to dispatch messages to a recipient agent \( R \):
  \[
  S \xrightarrow{\text{dispatch message to } R} R \tag{141}
  \]

- A use-promise to accept messages from a source \( S \), only if its address is compatible with the agent’s conditional expression for forwarding:
  \[
  R \xrightarrow{\text{message \| addressed to me}} S \tag{142}
  \]

Building on these two promises, we may construct uni-directional adjacency-like binding for use as a template to build larger structures.

In the figure 18, we see a node with two such promise bindings sporting different conditionals in different directions. Notice how the choice to forward a message from \( S \) to \( R \) is a voluntary act by \( S \). It
can send in different directions to purposely separate messages, or it can send along different paths for traffic management or load balancing. Note also that the difference between a flooding promise (sending messages to all recipients) is simply a scalar version of the dispatch promise, in which we take away a target from the promise body, i.e. without exclusivity to promisee/body vector. The result is the same, but the efficiency is compromised; efficient routing is assisted by long-range cooperation, and ultimately by long-range order.

The receiver \( R \), has the last word in accepting a message. So no message will arrive at the wrong location no matter whether it was forwarded by targeted dispatch or broadcasted to all agents. Agents have to promise their unique identities, both so that they may be recognized by neighbours, and so that they can recognize messages directed to them.

The consequence of creating an ordered tree from these promises is to create a dumb filter, which routes messages along a unique path depending on their address.

**Example 31** Coin sorting machines create unique pathways the sort and select different sized coins, allowing the to roll only one way through a maze of pathways. This is the basic principle of a semantic sorting process. The pathways for a treelike structure, and the end points of the tree all have a unique address. By placing a coin of a particular kind into the process at the root, it is like placing a message with an address (the type of coin), and having it sorted until it reaches its destination. Coins with the same address will end up at the same location.

Armed with this tool for spatial sorting, we may iterate these promise patterns to generate semi-lattices of greater size. Having a coordinate system within a super-agent boundary, for example, would allow agents to be located in a targeted manner, assuming only that they are connected. To iterate the pattern, we simply make each receiver into a source for the next iteration, and so on.

**Example 32** Figure 19 shows to iterated patterns formed from branching source-receiver iteration. The first (a) is a simple tree structure. Every leaf node of the tree has a unique address, and can be reached from the root by a unique path. This is the property of trees (and spanning trees).

![Figure 19](image)

The second case (b) is also a tree formed from three-way branchings \( S \rightarrow (R_1, R_2, R_3) \), iterated homogeneously and isotropically. The agents then fall into a three dimensional, Cartesian arrangement, which we may call the Cartesian semi-lattice. By filling in some redundant promises from
each point, one can arrange multiple routes from any node to any other, but still only in one direction (radially outwards from the origin).

By making the promises in each of the three dimensions sort forwarding of messages according to a different component in a vector tuple, forwarding can be encoded as a purely local operation\textsuperscript{18}, e.g. forward only if the tuple value is greater than the current tuple address of the agent for the current lattice location. Furthermore, by completing the reverse direction, as a mirror image, with opposite semantics, addresses can also be navigated in the opposite direction, completing the lattice.

4.3 Conditions for a uniform coordinate covering of an ensemble of agents

What are the conditions for being able to address agents using contiguous coordinates without loss of locatability? This is a slightly different question to the one about naming and promise body continuity, because it requires us to preserve the partial ordering of agents in a lattice. It is helpful to explain addressability by introducing two concepts that cover the semantic and dynamic aspects of location:

Definition 34 (Semantic addressing) An agent is said to have a semantic address if it is labelled only by a tuple of names that do not form part of an ordered pattern. Semantic addresses contain no relevant kinematic or dynamical information about an agent’s location in a space.

Semantic addresses act only as sign-posts, and a directory is needed to map which adjacency will eventually lead to the named agents. This is the approach used in Internet routing.

Example 33 Internet addresses (aka IP addresses) are semantic addresses, despite being composed of numbers, because they have no requirements of spatial order. Any agent can assign itself an address with any number, and these numbers do not imply information about where agents can be found. In order to locate IP addresses, a directory called a routing table is needed, which maps the random numbers of the addresses to physical adjacencies of the cabling. This is the function of a router or switch. The advertisement of these local directories is performed by services, which are called routing protocols (BGP, OSPF, RIP, etc).

Definition 35 (Numeric (metric) addressing) An agent is said to have a numeric or metric address if it is labelled by a tuple of values in which each value map to a unique integer. Numeric addresses represent kinematic or dynamic information about a space.

Agents with numeric addresses are partially ordered in a multi-dimensional lattice. The addresses form a coordinate system in the usual sense of mathematics.

In lieu of setting up a proof, I’ll hypothesize this informally for now:

Lemma 7 (Hypothesis: Conditions for a uniform coordinate covering of an ensemble of agents) placement of fixed and ordered address labels on an ensemble of agents, in a voluntary cooperation structure

- Fixed locations.
- Addresses ordered by location.
- Promises to sort and relay messages to destination.
- Long range order in address promises, i.e. cooperation in behaving uniform sorting/routing to addresses.
- Overlapping regions of $\beta$-language relevant to address

\textsuperscript{18}This is essentially how routers and switches forward datagrams in information infrastructure.
This does not refer to any particular topology, so it can be solved by multiple adjacency patterns. A suitable address-sorting process can be satisfied in a number of different ways, reflecting the encoding of the address in relation to the structure of spacetime. Network structures are typically tree-like, and IP addresses are prefix-based with distributed routing tables based on tree branching assumptions. Toroidal structure and Cartesian lattices using tuples are based on a pre-ordered layout, as in a warehouse, for instance.

**Example 34** In networking, regions of a network (like the Internet) can become cut off from the rest by a loss of routing information. Because the Internet is a gas, with semantic rather than numerical (metric) addressing, each super-agent boundary must contain a routing table that points to the next signpost to the destination. To prevent this from getting out of hand, a ‘default route’ is normally used as a wildcard (go this way to find any unspecified agent $A_2$), allowing regions to compress information about how to reach non-local agencies by handing off to centralized routing hubs.

There is an interesting suggestion here, that semantic naming tends to favour the formation of a hierarchy in order to scale. Such a hierarchy is unnecessary for metric naming. This warrants further study.

### 4.4 Efficiency of addressing in a semantic space

Consider a network of agents, with unique names and which are all interconnected by a sufficient number of adjacencies to allow full percolation. Suppose these agents promise to cooperate in relaying messages to one another, by passing the message along one of their adjacencies until it reaches the unique name (i.e. address). How much information has to be available to each agent in order to know how to forward messages to every other agent?

The maximum size of directory information may be computed as a sum over every location, which keeps a table of every other named agent in the space, paired with the ‘next hop’ neighbouring agent that brings the message closer to its destination. This applies for every agent in the space.

- If every agent is independent, then every agent needs a list of all $N - 1$ agents, with

  \[(\text{Agent name, direction of agent})\]  

  and a direction in which to forward. In total memory required for this information is of order $N(N - 1)$ in the number of agents. If there are few agents, this is easy. If there are many, the search cost rises linearly, and the distribution of information by flooding brings high cost.

**Example 35** This is exactly like routing in the Internet, imagining there are no network CIDR summarization prefixes, which would correspond to super-agent boundaries.

- If one can replace atomic agents with super-agents, which can handle their own internal forwarding, then the amount of information one needs to exchange is less.

  \[(\text{Super-agent of every node, direction of super-agent})\]  

  Aggregation of clusters leads to a cost of order $(N - 1) \log N$ memory. Scaling of addresses now depends on the ability to delegate responsibility to agents ‘further down the line’ by using super-agent container names to route messages, as in the coin sorting machine or lattice. This leads naturally to hierarchical naming and routing.
Example 36 Postal addresses refer first to town, then street, then building, and so on. By referring to larger container boundaries first, one can delegate the detail of finding the final destination to agencies within the boundary of the super-agency, e.g. the town. This assumed encapsulation comes at a price, however. It introduces inter-dependency into the end-to-end communication.

Today, postal addressing also now uses metric post codes, which are non-hierarchical. Given modern computational resources, a simple brute force approach can be used to look up these codes from directory information.

Example 37 This is like IP routing with CIDR prefixes. Internet (IP) addresses were originally designed to reduce routing cost by aggregated along certain prefixes, originally of fixed length (called class A, B, C networks). By grouping addresses under a smaller number of prefix patterns, and assuming that all such addresses were contained in the same super-agent boundary, routing tables could be kept small. Later, as these limited prefixes became consumed, they were subdivided into more, causing routing table growth.

• If there is a regular lattice, e.g. \((x, y, z)\), with long range order, and tuple addresses, then the amount of information is now of order 1. It is like asking which way is ‘up’? Irregularities (like holes) can be routed locally at no extra cost.

Example 38 For a Cartesian lattice, one knows left or right, forwards or backwards for each address because of the ordering of the integers.

Delegation, or deferred evaluation, is an attractive idea for scaling linearizable searches, however we must note that, an agent cannot ask another agent for help without already being able to know how to reach it; so, with no basic pattern to compress by, there is no way of centralizing this routing information in the manner of a coarse-graining directory.

Consider the following worst-case scenario in which every agent has a random name, i.e. a spacetime addressed by random numbers. Then, every location has to have a complete map of every other location, with zero possible compression. The need to flood all that information to all parts of spacetime adds a significant cost to promise keeping, and might exceed the capabilities of any or all agents.

Example 39 Instead of handing out metric addresses to visiting mobile devices (as a parking lot, or hotel, would do to its visitors), the internet hands out local semantic addresses (by DHCP), and tries to map them into its routing infrastructure. This makes sense for ephemeral gas-phase devices, but is quite inefficient for the repurposing of solid phase agents, like virtual machine slots, or process containers.

In practice, we see, from the stages of address scaling above, that the information is compressible only if each agent can replace a collection of addresses with a single promise. Hence to coarse-grain addresses into a hierarchy of containers, without loss of information, we need to restore the information lost using a directory at each super-agent boundary\(^{19}\). If we want to keep directory information small, we need long range order in the structural addressing promises (and presumably the adjacencies too) to enable logarithmic aggregate summarizability. Asymmetric tree structures can be adequate, but bi-directional lattices, like a Cartesian lattice, are better still\(^{20}\).

\(^{19}\)This corresponds roughly to the design of the Border Gateway routing tables, for IP addressing, as known from the Border Gateway Protocol BGP.

\(^{20}\)One can see the historical reasons why the Internet was not designed as a Cartesian lattice, but modern datacentre fabrics still have the opportunity to repair this choice.
4.5 Summary of agency properties in semantic spaces

The scaling behaviour described thus far allows us to ‘inflate’ (or scale-up) any functional arrangement of promises by substituting an arbitrary agent with a super-agent composed of sub-agents making similar promises. Then, one may define the exterior promises in such a way as to integrate the sub-agent members seamlessly to agents on the outside of the super-agent boundary. There is a progression:

$$\text{agent} \rightarrow \text{super-agent} \rightarrow \text{role} \rightarrow \text{subspace}$$

(145)

In other words, as an algorithm to scale given a single agent, we replace it with a black-box super-agent. Then we proceed to fill it with multiple sub-agents that are connected to the outside agents by exterior promises. These similar sub-agents are symmetrical with respect to the outside, so they form a role by association. Eventually, as we scale each of the original agencies and connect them to scale the promises, what remains is a set of non-overlapping sub-spaces, one for each agent, embedded in a larger semantic spacetime.

Example 40 Name-spaces, walled/gated communities, zones of privilege, service providers, etc are examples of agencies which scale from a single agent to collections bounded by some kind of contact surface.

Semantic spacetime (agents) have a number of scalable properties:

- They have discrete languages of intentions, easily translatable, in order for promises to be effectively communicated.
- They can be observed and interpreted at a multitude of scales, at the behest of an observer.
- They can effectively cluster their own promises into coarse-grains, through cooperation.
- The number of agencies can grow or shrink, i.e. spacetime itself grows or shrinks, as new points are added or retired.
- There are simple rules for transforming from one agency scale to another, analogous to renormalization transformations.
- Promises behave like tensors in general, with directionality.
- Causal influence is passed by vector promises, and principally through use-promises, by the principle of autonomy.
- Biological organisms offer a useful measuring stick for spacetimes with strong semantics.

So far I’ve focused on preserving symmetries and semantics, while piecing together the underlying connectivity of space. The asymmetry in these promises for routing to fixed addresses has a general utility, and it can be associated with the idea of tenancy, which is the subject of the latter part of this paper. Functionally, this asymmetry is the most important tool for making anything happen in space or time, and is worth exploring in more depth.

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21 This is essentially the method of top-down decomposition used in procedural programming, but with constraints that allow parallelized scaling of execution. One could imagine programming and representation languages that support this kind of model in many different walks of life. This is probably how we should be teaching programming, and service management, instead of the linear imperative models of today. In paper III, we’ll see why the linear storyline approach has its own naturality.
5 Occupancy and tenancy of space

Let’s now turn to a different topic: how to fill the space we’ve built up. A semantic space is richer in structure than its underlying connective graph so it contains information that goes beyond pure adjacency. In particular, as we add autonomous observers with their own agency, we quickly arrive at the need for agents to extend their realm of autonomous control through occupancy and ownership of resources. The question of occupancy and tenancy are thus about how we draw the boundaries of agency on a background of spatial adjacency. So far we’ve focused on symmetry and scale in discussing agency, however strong functional semantics are a result of asymmetry, hence we must now pursue the effects of broken symmetry.

5.1 Definitions of occupancy and tenancy

Tenancy goes beyond simple aggregate membership in a cluster. A tenant is understood to be an agent that ‘occupies’ or utilizes a resource or service, provided by a host, often in a temporary manner, and for mutual benefit (symbiosis). Tenants have separate identities. When we think of tenancy in everyday affairs, we do not usually imagine a tenant as merging with its host, and becoming a part of it (though merger and acquisition is certainly a process one can discuss, as absorption). Tenancy is rather an association between two separate agency roles (host and tenant), each of which retains its autonomy.

To relate this to our spacetime discussion, consider the following question (which, at first glance, might seem purely facetious): does a suit occupy space when no one is wearing it, or does space occupy the suit? The space inside a suit is simply empty before someone climbs into it. Try replacing ‘suit’ with ‘car’ and ‘wear’ with ‘sit inside’.

This peculiar question is closely related to the considerations surrounding the kinds of motion described in paper I, section 5.12. Since we are modelling space as a resource, this is not only a meaningful question, it is essential to understand what kind of volume a suit occupies. Does the presence of suit matter replace space, occupy it, attach to it, or overlap with it? These have different semantics.

Recall, from paper I, that motion of the second and third kinds distinguish between the idea that space and its occupants are either: ii) a visitation by a separate entity, or iii) a change in the state of the same entity. In other words, does space get filled by matter or does matter transform the nature of space?

To address some of these issues, we need to formulate definitions using promises, building up the distinctions in a rational way. Let’s begin with occupancy. Its semantics are different from mere presence, as there is an assumption of valency. Within the scope of promise theory, we can define the following:

**Definition 36 (Occupancy)** An asymmetric association of one agent (the occupier) with another representing a host (the location) at agency scale $M$, in which the valence of a promise made by the host is reduced by its binding to the occupier. The resource $R$ may be any scalar, vector or tensor type:

\[
\text{Host}_M \xrightarrow{+R} A_I \quad \text{(146)}
\]

\[
\text{Occupier}_M \xrightarrow{-R} \text{Host}_M \quad \text{(147)}
\]

In other words, a host makes a finite promise $+R$ to a number of agents in scope, and each occupier reduces the valency by making a use-promise $-R$

\[
\text{Valence}(R, \text{Host}) = \text{Valence}(R, \text{Host}, \text{Occupier}) + 1 \quad \text{(148)}
\]

**Example 41** Our understanding of the semantics of occupancy has many possible interpretations. Here are some examples:

- Occupation of a territory without necessarily being there. e.g. a table reservation.
Occupation of space.

Occupy a car, a suit, a dress.

Occupy a time slot in a calendar.

Filling a space with something.

In physics, bosons can occupy the same space, like voices in a song, but Fermions have exclusion, like the bodies in the choir themselves they occupy space.

From here, we may state a basic template for tenancy for application to a variety of special cases:

**Definition 37 (Tenancy)** Tenancy refers to the conditional occupancy of a location, by an agent, together with the provision of one or more services by the host, which may be considered a function \( f(R) \) of resource \( R \). These services are provided conditionally on a promise \( C \) from the tenant:

\[
\begin{align*}
\text{Host}_M & \xrightarrow{+R\#n\mid C} A_T \\
\text{Host}_M & \xrightarrow{-C} \text{Tenant}_M \\
\text{Tenant}_M & \xrightarrow{+C} \text{Host}_M \\
\text{Tenant}_M & \xrightarrow{-R\#1} \text{Host}_M \\
\text{Host}_M & \xrightarrow{f(C,R)|-R} \text{Tenant}
\end{align*}
\]

(149)

This is the basic template for tenancy, which may be extended by additional promises.

**Example 42** A landlord promises a rentable space for a single occupant \(+R\#1\), conditionally on the signing a contract of terms (i.e. the promise to abide by terms and conditions) \(+C\).

\[
L \xrightarrow{+R\#1\cdot f(C,R)|C} A_T
\]

(150)

A tenant quenches this exclusive resource, by signing up and promising the terms:

\[
T \xrightarrow{+C,-R} L.
\]

(151)

The terms and conditions contain a composite promise body, detailing the services \( f(C,R) \) offered as part of the promise:

\[
C = \{+\text{payment}, \text{termination date}, \ldots\}
\]

(152)

and

\[
f(C,R) = \{+\text{power}, +\text{heating}, \ldots\}
\]

(153)

Tenancy is a service-like relationship between a host and a tenant. This may be contrasted with the notion of residency at a location, which is related to definition of boundaries within an observer’s realm. Tenancy is a also relative concept (relative to promise semantics).
5.2 Laws of tenancy semantics

It is basic to promise theory that we distinguish between a promise made by an agent, and the agency itself. Hence, we begin by noting that:

**Assumption 4** Promises are neither occupants nor tenants of the promisers or promisees, since they have no independent agency.

- Tenancy and occupancy requires two agencies to become associated.
- Agents can be promised, but promises are not agents (they do not possess independent agency).

In a sense, a promise emanating from an agent seems to be attached at the location represented by the agent. However, we do not call this tenancy. A promise is a property of an agent, but it has no independent agency, thus it cannot be a tenant.

**Example 43** An agent $A$ can be the subject of a promise, e.g.

$$A_1 \xrightarrow{+A} A_2, \quad (154)$$

but it is not the promise itself, which belongs to $A_1$.

The semantics of the promises in (149) select an inherent directionality for the provision and use of a resource.

**Lemma 8 (Tenancy flows in the direction of the resource being used)** Tenancy flows towards the host, i.e. towards source of the hosted resource.

It is important to bear in mind the semantics when looking at host and tenant. Consider the following case in figure 20. From the perspective of a renter going directly to a hosting apartment block, the tenant

![Diagram](image_url)

Figure 20: Identifying tenants and hosts correctly requires us to follow the tenancy law carefully. In each case, the arrows point towards to host resource sought by the tenant.

1. A renter may be a tenant of either an office block (providing multiple offices to multiple renters),
2. A renter may be a tenant of a broker (providing multiple client offices to multiple renters),
3. An office block or single office may be a tenant of a broker (offering multiple renters as a resource) to multiple office blocks or office.
Definition 38 (The host:tenant binding is 1:N) A host can have any number tenants, at any one time, keeping full promises, up to and including the valency of the host resource promise.

There is an exclusivity between a tenant and a resource, which is a question of definition. Tenancy with a super-agent scales like any other promise (see section 4.1.1). When we speak of a tenancy, it refers to a single relationship, even though an agent might be engaged in multiple similar tenancies.

Example 44 A rider on a horse is a tenant of the horse. A rider cannot ride a herd of horses, at the same time. Moreover, the rider and horse are not joined by encapsulation, forming the embodiment of a super-agent. A driver in a car however, is a tenant of the car, and is encapsulated by it. The car is a tenant of the driver’s direction. Hence, while the two have independent agency, they seem to form an encapsulated super-agent.

Example 45 In the OSI network model, the layers from L1-L7 form a tower of dependence, in which network resources (at the bottom) are shared out between different applications and users which are tenants of the basic service. These layers farther up the stack depend on the lower layers, hence the arrow of tenancy points down to the L1 physical layer. L2 is a tenant of L1, L3 is a tenant of L2 and so on.

Network encryption is a tenant of L3, and computing applications are tenants of the encrypted stream.

When these layers are implemented as encapsulations, the tenancy increases into the core of the encapsulation, and the host is the outside part. This seems to be the opposite of the way we are taught to think about networking, from a software engineering perspective.

Lemma 9 (Causation is partially ordered by pre-requisite dependency) Promises and intentions may be partially ordered by conditional dependencies, from the conditional promise law. This leads to a hierarchy of directional intent, for fixed semantics.

We can distinguish tenancy from simple scaled agency by this partial ordering of tenants to hosts in the direction of a named resource. However, in most cases, the law of complementarity of promises allows us to transform one tenancy into the reverse relationship interpreted as a different promise. In either case, the orientability of tenancy gives agents topological ‘hair’ which can be combed in a certain direction, as a vector field.

5.3 Forms of tenancy

Let’s look at some familiar exemplars to see how this general pattern is realized in different scenarios.

• Club membership, or passenger with ticket

The issue of club membership is one where an agent associates itself as one of a group of typed agents: a vector promise directed to a specific host. The host offers the tenant a membership, and the tenant accepts the membership lease.

\[
\begin{align*}
C & \rightarrow \text{membership fee} \\
R & \rightarrow \text{membership credentials} \\
\text{f}(C,R) & \rightarrow \text{benefits and services}
\end{align*}
\]

Membership in a club is a label, i.e. a property of an agent. However, in the case that a separate agency validates this label as evidence of an association, we can view the members as guests of the hosting club. The condition \( C \) is typically some kind of subscription, the membership itself is promised with a badge or access credentials, and the additional services that accompany membership require showing of the credentials.
If a club is exclusive, then the promise of $+R$ has finite valency, else it has infinite or unlimited valency.

- **Employment** An immediate corollary of membership is employment at an organization.

$$
C \rightarrow \text{work performed} \quad (158)
$$

$$
R \rightarrow \text{employee status/badge} \quad (159)
$$

$$
f(C, R) \rightarrow \text{benefits and wages} \quad (160)
$$

In this case, an employee is a tenant of the hosting company that pays for membership with his/her daily work. Tenancy is fulfilled by access or credentials (the company badge), and benefits include wages, lunch, travel costs, etc. Tenancy is always symbiotic, by nevertheless asymmetrical. The relative values of $C$ and $f(C, R)$ are in the eyes of the beholders. When trading promises, what is valuable to one party is usually not valuable to the other, else they would not be motivated to trade.

- **Privileged access (territorial access)**

A further corollary is the use of credentials to gain access to territories, e.g. foreign visas, password entry, identity cards, etc.

$$
+ C \rightarrow \text{identity credentials} \quad (161)
$$

$$
- C \rightarrow \text{authentication/access control} \quad (162)
$$

$$
R \rightarrow \text{access passport/visa} \quad (163)
$$

$$
f(R) \rightarrow \text{territorial access/resources} \quad (164)
$$

- **Shared exclusive resource usage (multi-tenancy)**

Now consider the case where we add a finite valency to a limited resource, as well as a condition of fair sharing. A fair sharing promise, up to a maximum valency of $n$, becomes an additional constraint on the host, of the form:

$$
+ R_j \# n \mid \sum_{i}^n R_i \leq R, \quad (165)
$$

for each qualifying tenant $\text{Tenant}_j$, paying its tenancy cost $C_j$.

See figure 21

So the total promise set becomes:

$$
\begin{align*}
\text{Host} & \xrightarrow{+R_j \# n \mid C_j \cdot (\sum_{i}^n R_i \leq R)} \text{Tenant}_j \\
\text{Tenant}_j & \xrightarrow{-R_j \# m} \text{Host} \\
\text{Tenant}_j & \xrightarrow{+C_j} \text{Host} \\
\text{Host} & \xrightarrow{-C_j} \text{Tenant}_j
\end{align*}
$$

(166) (167) (168) (169)

where $m < n$ by necessity due to the valencies. Services $f(C, R)$, based upon $R$ might be subject to additional constraints, but they are also naturally limited by the constraint (165).
• **Representation by proxy** (spokesperson)

In some cases a hosting agency’s purpose is to be a proxy or representative for a client. This is the case for modelling agencies, writers’ agents, sales representatives, public facing spokespersons, and even accountancy firms. Examples include ‘Intel inside’, goods on a shelf in the shop that represent their brands.

In this case, the value added service to signing up is the representation of the tenant itself:

\[ + f(C, R) \rightarrow \text{Tenant representation} \]  

Representation or brokering for the tenant does not necessarily imply constraints on the tenant’s autonomy (this depends on other promises). This is not exchange of the tenant, like sending a letter, or transporting a passenger. Notice, furthermore, that nothing promised here can prevent the tenant or host from acting as separate entities in other ways.

• **Catalysis** (special semantic environments)

In a chemical process, some tenants need the help of a tailored environment to make a transition to a new state. A host plays the role of catalyst

A pit-stop for tyre change, or a port/dock for loading and offloading, or repair of transport vessels.

In the human realm, start-up labs and incubators are catalysts for companies and biological processes. The womb is a host for infant morphogenesis.

In each case \( f(C,R) \)

**Example 46** Users are tenants of multi-user software, logging into walled communities with login credentials. Processes are tenants of operating systems. Operating systems are tenants of computer hardware. Computers are tenants of networks and datacentres.

### 5.4 Tenancy and conditional promises

It should already be apparent from the definition of tenancy, in section 5.1, that there is a likeness between the pre-condition for tenancy (denoted \( C \)) and the resource relationship (denoted \( R \)). From the conditional promise law [2], a conditional binding to provide service \( S \) takes the form

\[
A_T \xrightarrow{+b} A_1, \quad \begin{array}{c} A_1 \xrightarrow{S|b} A_2 \\ A_1 \xrightarrow{-b} A_2 \end{array} \quad \simeq A_1 \xrightarrow{S} A_2
\]

(171)
Notice how the exchange of the condition has the same structure as the tenancy relationship. This is because both are examples of a generic client-server relationship, based on vector promises.

This can be formalized this further to show that a tenancy is really a conditional promise (see figure 22).

\[
H \xrightarrow{+R} A_7 \quad \text{vs} \quad A \xrightarrow{-c} D
\]
\[
T \xrightarrow{-R} H \quad \text{vs} \quad D \xrightarrow{+c}
\]
\[
H \xrightarrow{+f(C,R)|-R} T \quad \text{vs} \quad A \xrightarrow{+b|c} A_7
\]

Thus the tenant is the assumed recipient of functional promises derived from the tenancy relationship, whereas in a general conditional promise this is unspecified.

Note, we shouldn’t worry too much that the sign of the \( +R \) maps to a \( -c \), as the complementarity rule (see [2], section 6.2.2) allows us to re-interpret the signs. For example, \( +R \) could represent the active garbage collection of resources, while \( -R \) represents quenching with resources. Similarly, \( +R \) could represent employment, while \( -R \) is work done to fulfill the employment moniker. In both these cases, the \( + \) promise takes on the character of a receipt of service, often associated with \( - \) promises.

![Figure 22: The likeness between tenancy and a conditional promise involving a third party.](image)

The tenancy relationship is just an extended version of the basic client-server relationship, with the special focus on identity.

### 5.5 Remote tenancy

If we consider the case in which tenancy is not between agents that are actually adjacent to one another, then the promises are delivered by proxy, in the sense of a delivery chain (see [2], section 11.3, and figure 23).

When carried out via proxy, every adjacent node in a connective path through the adjacencies of the carrying spacetime becomes a possible point of failure or loss of integrity, and the cost of promising explicit integrity increases as the square of the number of agents along the path taken via adjacent agents.

### 5.6 Asymmetric tenancy

The semantics of tenancy are always asymmetrical, by definition. Adjacency is usually symmetric and mutual, at least when locations are equally weighted. However, if a superior location is next to an inferior location, according to some weighted importance ranking, then the symmetry is broken, e.g. pilot fish

---

22The description of services in [2] treats clients and essentially faceless, generic entities.
surrounding a whale, shops surrounding a mall. Unifying locations like malls, hubs, planets are natural host-roles to shops, spokes, and satellites, regardless of their relative size, because they connect agencies into an accessible nexus. Their ‘size’ may be thought of in terms of their network centrality [15], for example, which gives them semantic importance.

Lemma 10 (Adjacency is a form of tenancy, or tenancy is ‘rich adjacency’) By symmetrizing over the 
host-tenant promises, and directing unspecified promisees to mutual neighbours, we reproduce adjacency.

\[
H \xrightarrow{+R} (A_T = T), \quad T \xrightarrow{+R} (A_T = H)
\]

(175)

\[
T \xrightarrow{-R} H, \quad H \xrightarrow{-R} T
\]

(176)

\[
H \xrightarrow{f(C,R)} H, \quad T \xrightarrow{f(C,R)} H
\]

(177)

which reduces to

\[
H \xrightarrow{+R, f(C,R)} T
\]

(178)

\[
T \xrightarrow{+R, f(C,R)} H
\]

(179)

Thus \(R\) plays the role of adjacency, and identifying \(R \rightarrow \text{adj} \) and \(f(C, R) \rightarrow 0\), we see that adjacency is equivalent to mutual tenancy in its weakest form.

5.7 Scaling of occupancy and tenancy

The ability to use space and time in a functional and operational way is the key to building organisms and organized processes. When we speak of scaling these semantic forms, we implicitly expect to preserve symmetries, asymmetries, and functional relationships, while inflating the overall size of a semantic space by introducing more agents. Coarse-graining should then allow us to see the functional equivalence of the larger and the smaller system.

The asymmetry inherent in the ideas of occupancy and tenancy suggests that we are not generally going to see scale-free phenomena. What characterizes tenancy and occupancy is the retention of a differentiated cooperative relationship between agencies. Specific agents are bound together with intentional directionality. This contrasts with the idea of absorbing new agencies into a singular agency.
Example 47 In a business partnership, or symbiosis, businesses or organisms retain their separate identities and work together for mutually beneficial returns. In a merger or acquisition, one company or organism subsumes the other, hoping to control it without worrying about explicit cooperation.

The homogeneity of host-tenant semantics often play a role in the coordinated, functional usage of space. Long range order helps us to utilize space in a regular way. Without it, many aspects of space and time are simply opportunistic.

Example 48 In a parking lot, the spaces need to be homogeneous in size else you might not be able to park your car in just any space. The same applies to the width or refrigerators, washing machines and kitchen appliances, block and sector sizes on disks.

We need to account for both strong and weak couplings, homogeneity and inhomogeneity, to understand the wealth of possibilities in the world around us.

5.7.1 Extending tenancy with structural memory

Homogenization is a forgetting process, while inhomogeneous differentiation encodes a memory into spacetime. In a semantic spacetime, memory is encoded through promises, and structure might refer to any one of the aggregation, residency, occupancy and tenancy candidates.

Let’s contrast the ideas of scaling by absorption and tenancy more carefully. Consider the two scenarios in figure 24, which contrasts a symmetrical form of cooperation (a) with an asymmetric tenancy configuration (b). The solid circles represent agency scales, forming various levels of super-agency.

Figure 24: The scaling of membership can retain a memory of its process. The introduction of an agency scale can introduce artificial asymmetry. In (b), a promise binding is made to the super-agent, but this has yet to be realized by a physical agency within this virtual/logical boundary.

In the first case (a), the cooperative arrangement is completely symmetric, and no information about the order in which the agents came together is retained in the structure. If we assume that the tenancy binding is based on a promise with some body \( X \), then we may characterize this arrangement by:

\[
A_i \xrightarrow{\pm X} A_j, \quad \forall i, j = 1, 2, 3, 4. \tag{180}
\]

\[
A_i \xrightarrow{\pm C(X)} A_j \tag{181}
\]

We see exchange promises for \( \pm X \), and symmetrizing coordination promises \( C(X) \). In the second case, an intermediate step is apparent which singles out a distinction between the set \( A_s = \{ A_1, A_2, A_3 \} \) and \( A_4 \):

\[
A_i \xrightarrow{\pm X} A_j, \quad \forall i, j = 1, 2, 3. \tag{182}
\]

\[
A_i \xrightarrow{\pm C(X)} A_j, \quad \forall i, j = 1, 2, 3. \tag{183}
\]

\[
A_s \xrightarrow{\pm X} A_4 \tag{184}
\]

54
In this case, we see a memory of the structure which sees \( A_4 \) joining an already established agency. As yet, the agent \( A_4 \) is not completely symmetrical with the other three. The asymmetry of intent remains, although an observer watching how these promises are kept might not be able to tell the difference between scenarios (a) and (b). This depends on how we interpret the promise between \( A_4 \) and the super-agent \( A_s \). There are two possibilities:

- \( A_4 \) binds distributively, assuming that each of the agencies \( A_1, A_2, A_3 \in A_s \) makes individual promises so as to behave symmetrically (by virtue of (183)), allowing:

\[
A_4 \xrightarrow{\pm X} A_j \quad \forall i, j = 1, 2, 3
\]  

The addition of these promises completes the symmetry that turns scenario (b) into (a), eliminating the memory of their inequivalence to an observer, and absorbing \( A_4 \) effectively into a cooperative entity. Regardless of how a partial observer might draw its super-agency boundaries, it is now able to identify a symmetrical role by cooperation (see [2]) between all four agents. This exercise gives us a clue about what absorption means, in a formal sense.

- \( A_4 \) binds only to a single representative of the collective \( A_s \). This remains asymmetric, and we would consider \( A_4 \) to be a non-resident occupant or tenant of \( A_s \). The memory of the inequivalence is coded into the intentional behaviour, by promises.

These figures illustrate how the promise configurations document the history by which an arrangement of agents was constructed, and allow an entity formed by multi-layered cooperation to retain a memory of past states.

What we see, in figure 24, is that adding promises can effectively remove asymmetry between host and tenant, meaning that tenancy can be eliminated by ‘acquiring’ an agency\(^{23}\). However, it is important to also consider scaling in which the tenant never becomes a part of the host, i.e. we maintain the strict asymmetry, as this implies no loss of autonomy between the tenants.

### 5.7.2 Scaling of the tenancy law

Now, let’s see how the extended scaling of internal structure in either host or tenant affects the promises made in a tenancy relationship. This scaling applies to all promises between coarse-grains, not just tenancy promises.

As the number of sub-agents in internal structure grows, it becomes natural to consider them both as embedded subspaces of the surrounding semantic space. Such a subspace may be either a solid lattice, with long range order, a gaseous state, or forest-like (molecular).

Consider the full tenancy relationship:

\[
\begin{align*}
A_{\text{super}} & \xrightarrow{+X \# n \mid C} A_{\text{tenant}} \\
A_{\text{super}} & \xrightarrow{\pm C} A_{\text{tenant}} \\
A_{\text{super}} & \xrightarrow{f(C,X) - X} A_{\text{tenant}}, A^? \\
A_{\text{tenant}} & \xrightarrow{-X \# m} A_{\text{super}}
\end{align*}
\]  

Suppose we try to add agents in the manner of a scale perturbation. We preserve the implied roles (the tenant \( A_{\text{tenant}} \), and host \( A_{\text{super}} \)), and the structure of the binding between them. What features would

\(^{23}\)This happens frequently in merger and acquisition of companies of course.
change, if we now attempted to scale this relationship by adding new internal structure to either host or
tenant? With the further addition of $A_{\text{pert}}$, assuming this adds to the valency, this becomes:

$$A_{\text{super}} + A_{\text{pert}} \xrightarrow{+X \#(n+1) \pm C} A_{\text{tenant}}$$ (190)

$$A_{\text{super}} + A_{\text{pert}} \xrightarrow{\pm C} A_{\text{tenant}}$$ (191)

$$A_{\text{pert}} \xrightarrow{\pm C} A_{\text{super}}$$ (192)

$$A_{\text{super}} + A_{\text{pert}} \xrightarrow{f(C,X) \pm X} A_{\text{tenant}}, A?$$ (193)

$$A_{\text{tenant}} \xrightarrow{-X \#m} A_{\text{super}} + A_{\text{pert}}$$ (194)

Hence, the symmetry and internal structure of the super-agent has a material effect on the binding prop-
ties in a tenancy arrangement. This is a formal scaling, but it does not really explain what happens to
communicate intent across a coarse-grain, as discussed in earlier.

The exterior promises of a host need to scale to provide valency $n$ binding sites. The first issue to
satisfy is the valency binding constraints, we seem to need $m \leq n$ in the tenancy promises. There are
several ways this might be solved:

- One sub-agent is allocated to one tenant.
- Sub-agents host multiple tenants each.
- Several sub-agents working together service a single tenant.

Similarly, we have to answer the question: how does a tenant know with whom it should interact?
Will tenant and host sub-agent be able to locate one another (see figure 15)? Referring to figure 15, and
the previous discussion of promising at super-agent boundaries in section 4.1.1, we ask: should external
tenants bind (a) directly to independent sub-agents, or should they (b) go through brokers and interfaces?

Example 49 A tenant in figure 25 might represent a person or a population looking for an unoccupied
apartment through a fronting organization; or a car looking for a parking space in a collection of parking
lots, a cubicle in an office space, and so on. While the super-agencies like apartment blocks and parking
lots formally promise space (valency), the potential tenant needs to locate the empty slots in order to bind
to them.

5.7.3 Distributive scaling of tenancy relationships

In order for an outside agent to form an adjacency or a tenancy binding to sub-agent, its has to know of the
other’s existence. In all cases, tenant and host need to be able to locate one another, or be introduced,
in order to communicate, both form a promise binding and to keep the promise. There are only two
possibilities here:

- Host and tenant are directly adjacent (see figure 15a).
- Host and tenant can communicate with the aid of intermediaries (see figure 15b). This requires
  cooperation between the sub-agents.

The super-agent coarse-graining directory plays the key role here in making these details transparent.

---

24 This is actually a hierarchy problem. At the lowest fundamental level of agency, there is no intermediate solution to this: either
agents are adjacent or they aren’t. We have to assume that they can sense and discover one another somehow (see section 4.1.1).

56
Figure 25: Tenants, en masse, can contact a front organization like a website to search for housing, but still need to connect with the agency’s internal components (individual landlords) that can provide actual rather than logical service. e.g. a person needs to make direct contact with the apartment block that offers apartments to rent, not merely the super-agency of ‘real estate’ which has no physical reality. This if one tenant is asking for more service than a single host can offer, an issue becomes whether the service can be delivered in practice, in spite of the apparent size of the super-agency.

Example 50 Viewing virtual or switched private networks as tenants of a series of hosts that cooperate as super-agency: this requires tenancy at each independent host, and coordination between them. In addition, the agencies are connected by adjacency promises between the chained carrier hosts (see figure 26), or virtual adjacencies via intermediaries or proxies (see [2], section 11.3). The scaled tenancy relationship allows tenant spaces to appear contiguous, even though they might be distributed. Structures like overlays and tunnels act as virtual adjacencies, which rely on a substrate which we coarse-grain away.

Figure 26: Shared network requires the distribution of the entire tenancy binding. The two hosts cooperate on handing over their resources to make a collaborative channel, while maintaining the integrity of the segmentation of tenants.

To scale tenancy on the host side, there are some basic requirements:

Lemma 11 (Distributed tenancy law) In order to distribute a tenancy to a collection of autonomous hosts sub-agents, there must exist:

- A cooperative agreement between all sub-agents acting as hosts to share its local resources according to the collective multi-tenancy agreement.
- An index for adjacency coordinates of the host sub-agent, if not directly adjacent.
Example 51 A customer of a bank can visit any branch to access their account, by mutual cooperation between the branches, else the customer has to deal directly with their home branch. (Remarkably, this is still an issue even in 2015, in spite of the prowess of information technology.)

Example 52 Binding sites on cells, immune cell responses, MHC etc, allows a chance encounter for antigen to locate a cellular tenancy.

5.7.4 The significance of functional asymmetry

The economics of tenancy (regardless of the currency used, e.g. energy, money, prestige, etc) brings tenants and hosts together, leading to an asymmetric relationship (see figure 27). Functionality is associated with broken symmetry: the default state of maximal symmetry has very little going for it to attach semantics to. As seen in earlier sections, pure scaling of agency, in the absence of constraints, does nothing to break symmetries, and hence leads to a maximal radial (spherical) symmetry, like a cell; however, boundary conditions from the environment break symmetry allowing functional behaviour. There are many examples of this in the world of biological organisms.

Example 53 Reproductive organisms begin as a single egg, which grows and differentiates. Initially it grows symmetrically, dividing into a ball of cells, then boundary conditions from the environment during morphogenesis create preferred directions with chemical signals. This leads to dorsal-ventral asymmetry, etc. Selective apoptosis leads to further local asymmetries, with functional consequences, such a fingers. Cephalization (formation of a head) is associated with the appearance of brains in the nervous systems of organisms, and the brain is at the front where the organism meets and senses its environment.

Example 54 The shift from a spherical symmetry to axial and bilateral symmetries has happened prolifically in biological evolution. Axial symmetry is typically associated with orientation of an organism with respect to a flow, e.g. a jelly fish. In computational terms organisms orient along their input-output axes.

Example 55 A bottle (box, container) is usually a round axially symmetric structure, but this is irrelevant to its function. It does not matter whether the axial structure is symmetric or asymmetric, round, square, oblong, ellipsoid, etc. The functional shape of a bottle only depends on one end being open to be able to receive the substance it will contain. This asymmetry makes it bind with specificity with functional consequences.

Figure 27 shows organisms that can be modelled as tenancy relationships with various levels of symmetry and asymmetry. The more decentralized organisms are, the more symmetrical they tend to be, and the less ‘smart’, i.e.’ with functionality that is more uncoordinated. In biology, brains are associated with ‘cephalization’, or the evolution of asymmetry to deal with an axis of input-output. Segmentation along that axis (e.g. figure 27d,e,f), or the formation of cells or functional compartments can form through non-linear transmission of boundary information in the manner of Bénard cells, from the presence of a preferred length scale, and aids in semantic differentiation. The segments then often work together in the manner of a host-tenant relationship, binding through specific functional promises. Thus scaling of tenancy (a strong coupling regime) looks very different to the scaling of simple aggregate residency, such as how herds, swarms, and societies scale (weak-coupling regime).

Example 56 Mitochondria and cells co-exist, with mitochondria residents of their super-agent cells. Herds and flocks scale in the manner of residency, but remain loosely coupled without direction intentional ‘encoded’ functional cooperation.

The scaling of intentionality tends to promote asymmetry, through specialization. This places much greater emphasis on strongly coupled, and hence fragile, semantics. It is no longer sufficient to have
Figure 27: Functional behaviour is more associated with symmetry breaking, than residual symmetry. Maximal symmetry or disorder may be regarded as the default state of structure, with minimal cooperation. Cooperation is a force for order, and symmetry breaking.

safety in numbers, as in a herd or tissue design. Functional uniqueness, like the components in a radio, bring fragility.

5.8 Tenancy as a state of order

The structural memory of tenancy leads to asymmetry, but the long range effects of that asymmetry depend on the phase in which spacetime elements find themselves. The phase becomes a part of the strong or weak coupling constraints.

5.8.1 Tenancy in gaseous state

If agents that are seeking to bind in some kind of tenancy relationship are in a gaseous state, it is easier for them to rearrange and attach directly to a host binding site. Conversely, this makes their meeting more haphazard, since no agent is in a known location. Agents are free to move, so it is harder to trust their identity. On the other hand, they can move around and form adjacencies directly without the need to hand off to intermediaries.

Any spacetime location that is not fixed is in a gaseous state, as a free agent. In technology, this applies to humans, mobile phones, vehicles, satellites, etc. There should be sufficient similarity between agents that they can cooperate? Else they will remain inert noble gases.

Example 57 The Internet, for example, appears superficially solid, with all of its cables and boxes, but its lack of a robust adjacency relationships makes it just a slow liquid (like the amorphous solids or pitch)

25It seems likely that survival instincts and intelligence would evolve to be strongest in those species that cannot find obvious safety in numbers.
Moreover, the lack of a fixed spatial coordinate system means that it has a cellular structure on a large scale: it has small regions of brittle coordinatization, loosely floating in a structureless soup. Names and addresses have no global significance within the scope of a symmetrical pattern, and hence have no long range predictive value.

In contrast to simple absorption, any cooperative tenancy configuration must bring additional stability to the combined system of agents in order to persist: a symbiosis which confers a positive advantage to being a solo agent. This suggests an instability leading to the condensation of cooperation, first into amorphous liquids and swarms, and then into more rigid crystalline structures. The binding of a tenant and a host into an $H - T$ molecule is locally solid, but might be globally a gas. One could imagine a phase transition in which a bi-partite crystal was formed when scaled, with the structure of an alloy, formed from donor and a recipient.

As a general principle, flexibility and agility require fluidity; but, in order to stabilize semantics, time or change need to be eliminated. The more time plays a role, the less significant the information is. This points us towards solid structures.

### 5.8.2 Tenancy in a solid state

The solid state is familiar to us through regular spacetime structures including hotels, parking lots, warehouses, hard-disks, and computer memory, to cite a few examples. In a solid spacetime, agents are not free to alter their adjacencies over ‘time’ (see paper I, section 5.12), so we trust their identities and relative promises more easily.

At the lowest atomic levels of agency, tenancy in a solid state must lead to asymmetry of space itself, as we’ll see in addressable structures. When agents are locked into a solid structure, tenancy often leads to a large scale asymmetry, such as that seen in biological organisms. This seems to have implications for being able to pinpoint functional locations. Hence the loss of maximal, random spatial symmetry (or the rise of so-called long range order) is associated with functional behaviours, and hence functional promises. There are numerous ways in which functional asymmetry can manifest and materialize:

- Tenants and hosts are sufficient in number to be able to symmetrize on a larger scale, say in $n$-dimensions, and give the appearance of minimal loss of symmetry, e.g. a metallic cubic crystal, such as steel, with tenant impurities.
- Back to back, tessellating structures mixing various orientations are a possible solution to long range order however.
- Hierarchical structures which fill partial spaces, with self-similarity (fractals).

At coarser scales, where ‘virtual’ agencies can form tenancy bindings on top of an underlying solid adjacency substrate. The fixed locations cannot easily change and be replaced, so they are easy for other agents to trust local neighbours, however being locked into a fixed number of neighbours now mean that long range promise relationships have to be prosecuted through intermediaries (the so-called end-to-end problem discussed in [2]). The presence of intermediaries means that information integrity is now in peril, and trust in non-local relationships is not automatic.

The tighter coupling of agents and reliance on intermediaries to transmit information leads to the possibility of topological defects in the structure, such as crack propagation, and sudden catastrophes. The homogeneity of a solid crystal is important, as mentioned earlier. Re-usability of space suggests quantization of resources in a ‘first normal form’. The relational data normalization rule of first normal form is about the re-usability of space [16, 17].

**Example 58** In a parking lot, the spaces need to be the same size else you might not be able to park your car in just any space. The same applies to the width or refrigerators, washing machines and kitchen appliances, block and sector sizes on disks.
6 Multi-tenancy, and co-existent ‘worlds’

Multi-tenancy in information systems is one of the key challenges of operational effectiveness. As the custodian of the shared resource, the host has to be able to keep promises on an individual basis, while still being constrained by the behaviours of all its tenants. This means that there will be contention at the host. Moreover, the host is often the seat of mediation between the tenants and the outside world, acting as a gate-keeper, and sometimes as a barrier or firewall between them.

How the host isolates these resources from one another is one of the key issues in a functional space. One tenant should not be able to bring down another through its misbehaviour, either directly or through the host as proxy. This is the thinking behind insulated private rooms, isolated electrical circuits, multi-user systems and even virtual machines in computing. In these scenarios, the special semantic role of the host makes it vulnerable: a ‘single point of failure’.

6.1 Defining multi-tenancy

What are the promises that make a spacetime multi-tenant? See the example figure 26.

Definition 39 (Multi-tenancy at scale $M$) Consider a agency scale $M = \{H, T_1, T_2, \ldots, T_m\}$ for some $m > 1$. An agent $H$ is said to exhibit multi-tenancy if a number of autonomous agencies $\{T_1, T_2, \ldots, T_m\}$, called the tenants (making no a priori promises to one another) independently form a tenancy binding to $H$, in the role of host, for a share of a single resource $R$ that $H$ promises.

The issues for multi-tenancy include all those for general tenancy, and also segregation, mutual isolation, addressability of tenants, scaling of naming, sharing of resources between tenants, and the possibility of tenants sharing amongst themselves, without involving the host.

6.2 Branching processes: subroutines, worlds, and asymmetry with hierarchy

A brief digression on understanding the dynamics of aggregation of tenants around a host, introduces the notion of a branching process (and its inverse, the merge, confluence or aggregation process). Branching is what happens as the possible states or locations of a system fan out and increase in number from $n$ to $n'$, where $n' > n$, selecting a preferred direction, either over space or in time.

A branching process is an evolutionary sequence of changes in which the level segregation or multiple agency increases either in space (over distance) or time (spacelike hypersurfaces) (see figure 28). The branch points are associated with instabilities of the system both dynamically (bifurcations) and semantically (if-then-else reasoning).

Branching has an arrow of directionality from the root, along each local vector to nearest neighbour, which marks a gradient from fewer to more states. Thus as the process unravels (in time or space), the number of outcomes increases, favouring an increase in system entropy, and decreasing the likelihood of arriving at any chosen one outcome. The structure is not necessarily linear in total: it could be radial.

\[
\begin{align*}
H & \xrightarrow{+R\#(C)} A_1, \\
H & \xrightarrow{-C} T_i, \\
T_i & \xrightarrow{+C} H, \\
T_i & \xrightarrow{-R\#i} H, \\
H & \xrightarrow{+f(C,R)\rightarrow R} T_i \\
\forall i = 1 \ldots m
\end{align*}
\]
As a companion to branching, we also have a notion of hierarchy, which is sometimes confused with it. Similar to branching, hierarchy comes about by ranking of states. Thus, if one assigns greater importance to having more or less states, one could call a branching process a hierarchy, as the branching asymmetry implies an order from head to tail.

**Definition 40 (Promise hierarchy)** An iterated process, over a series of promises $\pi_i$, in which the promises $\pi_i$ exhibit asymmetric semantics, with respect to the promiser and promisee. Hence the process generates a chain or sequence of ranked or ordered elements (agents), in the manner of a semi-lattice.

**Example 59** For example, a promise to use (dependency, requirement, etc) is asymmetric. Promises of mutual adjacency, on the other hand, have symmetry, hence there is no preferred direction for semantics. A branching process over mutual (symmetric) adjacency does not form a hierarchy, only a tree or forest structure.

**Definition 41 (Branching hierarchy)** A branching process in which there is directionality of both dynamical branching and semantic intent.

**Example 60** Dependency trees, and taxonomies form branching classification hierarchies.

Because the branching of a single host into multiple tenants has the same asymmetry, clearly branching is a key process in interpreting multi-tenancy. It is the inverse of the accretion of multiple tenants to the single host. The converse of the tenancy law is that tenancy is a branching process of host resource usage, branching from one host into multiple tenants, and thenceforth. Semantically, branching may come about in a number of ways:

- Through a breakdown of cooperation between existing agents, leading to a fine-graining\textsuperscript{26} or fragmentation of roles.

- As a spawning of new agents, increasing in number.

\textsuperscript{26}The vernacular ‘dis-aggregation’ is common.
Dynamically, the branching can be either

- Due to a change in the receiver, e.g. a (-) use-promise no longer aggregates agents.
- Due to a change at the source, i.e. a (+) promise now differentiates agents.

Branching processes lead to proliferation of ‘parallel worlds’ (disconnected sub-spaces), and possibly a change of dynamical scaling. Keeping track of these worlds becomes a divergent problem of knowledge management, without a counter-process. If there is exponential growth of agency, this may be intractable, in the sense of computational complexity.

Branching processes may be examined through the lenses of semantics and dynamics:

- **Semantic**: Disambiguation, tenancy, security, etc.
- **Dynamic**: Isolation, bifurcation, cell division, renormalization, etc

Figure 29 contrasts a few of the scale issues mentioned thus far. Notice how, at what might be perceived in one way, at one scale, could be perceived another way from the perspective of a different scale.

| Semantics | Dynamics |
|-----------|----------|
| Single-agent | Multi-agent |
| Super-agent | Sub-agent fragmentation |
| Absorption | Occupancy (tenancy) |
| Member | Associate/symbiote |
| Boundary | Cooperative |

Figure 29: Dynamical and semantic aspects of multi-tenancy eventually merge through the semantics of scaling. This illustrates the importance of scale in the total semantic content of an observer’s realm of assessment.

### 6.3 Tenant world segregation, and resource multiplexing, in privately hosted spaces

A host shares its resources into slots, which can be occupied by a number of tenants in a number of ways:

- Segmentation or partitioning of adjacencies in an underlying graph, or spacetime.
- Labelling of regions by promises, marking gradients (stigmergic typing of agents).
- Growing and spawning new agents.

**Example 61** *Division multiplexing is a common strategy in hosting:*

- A parking lot (host) divides up its space into slots that can be used by cars (tenants).
- A hard disk (host) divides its surface platter into sectors and blocks, for user-data (tenants).
- A time-sharing computer operating kernel (host) divides up its computational resources into processes that can be used by jobs (tenants).
Memory address spaces (host) divide up addresses into pages, for occupation by process data (tenants).

Isolation of a subspace at the host can come about by two means:

- The absence of adjacency between the tenants.
- The absence of reachability: i.e. an agent cannot be reached because it has:
  - No adjacency path/route, with or without the help of intermediaries (vector promises).
  - No name or (unique) identity, or address, to locate it by (scalar promises).

Thus, both scalar promises (e.g. names) and vector promises (e.g. adjacency) play roles in the segmentation of shared spaces.

6.4 Mouth formation at host boundary, and gatekeeping credentials for tenants access

As remarked in paper I, the picture of spacetime described in Milner’s bigraphs represents a containment view of the world; such a representation is not an efficient way of addressing objects for ease of locating them. On the other hand, it is a useful way to describe the semantics of interfaces.

One of the values of the concept of multi-tenancy is the ability to have the host act as a broker for mediating contact with the tenants. We see this in many everyday scenarios:

Example 62 Interface scenarios:

- A security checkpoint at the mouth of a host building, or secure area.
- Passport control at the airport interface of a host country.
- Gated access via host to locked tenant storage units.
- The eyes and mouth of a host organism mediate contact to the tenant organs: brain, stomach, etc.

A tenancy leads to a form of super-agency seeded on (and mediated by) a host. The host acts as a kind of moderator or proxy for certain communications with the outside world, though it might not be the only source of adjacency, if the scale of the tenancy relation is based on promises that are made over a fabric of lower-level adjacency.

6.5 Tenancy formation and privacy as an additional promise

There are two questions concerning tenant segmentation and containment in a hosted super-agent:

1. Who gets to decide whether a tenant or member can join a hosting collective? This might simply be part of the evolution of the design, or it might be a decision made by the host, or indeed all of the tenants in concert.

2. How are tenants kept disjoint from one another, and how is access to the tenants moderated? The connectivity involved in a tenancy with a host favours a radial symmetry between host and tenants. This can be folded (see figure 30) leading to the functional asymmetry.

The default assumption is that there is no cooperation between tenants:
Assumption 5 (Default null adjacency promise for tenant segregation) The default adjacency state, at scale $M$, between tenants is no adjacency. This is most easily accomplished in a spacetime gas phase. If tenancy is built on a connected lattice in the first instance, then this isolation might require additional promises to block adjacency. However, the latter is a losing strategy: the amount of information needed to ‘lock down’ every agency is too large. You need to compress the pattern into a list of exceptions.

Independent semantics of tenants might well go beyond this simple dynamical observation however. Segregation of assets might be viewed as being an important requirement of tenancy. Naturally, this promise can only be kept by a single agent, hence only host-mediated resources can be segregated as a promise to tenants.

![Figure 30](image)

Figure 30: How the preferred host role leads to an axial symmetry. From spherical symmetry (a), a external flow selected a direction (b), which eventually organizes along an axis. Segmentation along the axis then marks out different levels of hierarchy in multi-stage tenancy.

If resources are mediated by the host, as is natural for this reason, it can also act as a moderator, gatekeeper, or security monitor. The point of a gatekeeper is to limit adjacency to a narrow bottleneck, or checkpoint (see figure 31). While this does forces a fixed scale limitation on the choke point, it simplifies the semantics of authentication. The host has to be able to verify the identity of tenants to keep its promises. Having a gatekeeper interface at the host is a simple way to do this. This helps to turn the branching process of the tenancy into a hierarchy, in which the host is the gatekeeper to upper levels, mediating contact to the hosts below.

**Example 63** Many of the classic security blunders have been due to relying on the lack of addressability, in the belief that an item that cannot be named would not be accessed. This is a form of ‘security through obscurity’. Systems that base isolation on prevention are much harder to police than

As an exercise to the reader, consider what promises lead to ‘secure semantics’? How are keys or addresses for accessing tenants assessed by the host, and by users? What agency plays the role of gatekeeper, if not the host? Implement a ‘secure’ system using conditional promises to segregate tenants.

The functional utility of the asymmetric tenancy structure thus seems to lie the following observation:

**Law 2 (Hosting of input and output leads to axial symmetry)** The functional arrangement of input/output mediated by the host leads to a natural head-tail asymmetry, in which the head is favoured in a hierarchy of longitudinal stages. This is known as cephalization.
6.6 Spacetime sharing by tenants: serial time and parallel space

When tenants subscribe to a resource in parallel, they share the resources of the host. This is called multiplexing in the resource domain. When a host-tenant network has net valency less than zero, tenants can multiplexed in the time domain. Over-subscription of a resource could lead to the need for time-sharing. Time division multiplexing is the way this is done in a serial queue27.

Example 64 Time-division multiplexing (queue-processing) of an oversubscribed service:

- Time-sharing of apartments.
- Car rental, or recycling of vehicles.
- Aircraft/bus passenger seating is only fixed for the duration of a journey.

Finite-duration promises are the key to all queueing systems.

6.7 Multi-stage multi-tenancy, and solid spacetime fabrics

This section is a continuation of the iterated solid state tenancy structures, used for locating agent addresses, described in section 4.2. I return to the topic to emphasize that iterated multi-tenancy leads to important scaling properties, both in the information technology realm and in the biological realm. Network fabrics include toroidal networks, Clos networks, and Batcher-Banyan networks [18]. This example is about the popular 2x2 Clos networks.

Example 65 The example of a 2-valent Clos network is instructive because it is a dynamically simple case that is increasingly used in datacentres. Semantically, it is more complicated, because it combines multi-tenancy with multi-homing (multiple hosts) in order to create a cooperative self-organizing structure. To make addressing work as a repeated, tessellating pattern, each agent needs to be both a host and a tenant for different promises.

Today, Clos networks are intimately connected with a particular implementation involving the Border Gateway Protocol (BGP) [19, 20]. BGP is a network route advertisement service that implements a set of promises for promising route information between super-agencies known as Autonomous Systems (ASN)28.

27 In data communications, so-called frequency division multiplexing is what corresponds to normal parallel resource sharing of R.
28 The similarity to promise terminology should not be a surprise: the two ideas are very closely related. BGP predates Promise Theory by many years, but through Promise Theory it gains a special clarity that cannot be seen when focusing on its irrelevant protocol.
Figure 32 shows the basic tenancy and dual homing. Two adjacencies upwards, from each agent in the fabric (shown in bold), connect each tenant in a lower tier to two redundant hosts in the tier above (for resilience and load sharing). Multiple adjacencies downwards in a tier connect each agent, now in the role of host, to its sub-tenants. Thus there are two tenancies back to back promising resources:

The characteristic of the tree structure in a Clos network is that each branch terminates at a definite ‘leaf location’ with a definite and unique address. This means that every agent in the pattern knows that ‘down’ means a specific location, and everywhere else is ‘up’. Hence, each agent engages in two cooperative relationships, framed as tenancies:

- $R^\uparrow$: tenants forward messages that don’t belong below me upwards for the host to aggregate and deal with. This could mean routing to one of the other parallel tenants, or it could mean sending the message out into the wider world beyond the host’s boundary.

- $R^\downarrow$: hosts forward messages that belong belong me downwards to the tenant they belong to. They know which direction to send the message, because the tenancy requires this information to be paid up as part of the condition $C$.

The tenancy boundaries thus lead to progressive layers of concentric nested agency. The agents (which are all network switches) play the dual roles of host and tenant with respect to these different services, in different layers. The tessellating pattern of all of these woven into a fabric allows it to scale to number of independent addresses that are greater than the fixed valency $v$ of any one host$^{29}$.

When forwarding upwards and out, what was a host for the downward routing is now a tenant of each of the lower layer agents that provide addressing data. So the semantics of the roles are reversed depending on the process we consider.

![Diagram](image)

**Figure 32:** Each agent promises to form tenancy agreements with two hosts above it.

The promises all rely on the correct positioning of the nodes to work. In practice, the Border Gateway Protocol is typically used to equilibrate that information and make sure the promises can be defined relatively, with self-organizing consistency.

Keeping a simple notation for representing a pattern this complex is a challenge; however we can illustrate the intent, along with the essential concepts of vector promises, and tenancies. I denote the $i$th agent $A_i^{(n)}$ in tier $n$, either by $T_i^{(n)}$ or $H_i^{(n)}$, depending on its role, where the indices $i$ simply run from 1 to 2 for the dual homed hosts, and $j$ runs from 1 to the downward valency $v$. Let’s formalize the pattern using the multi-tenancy promises. We can label the tiers by superfix $(n)$, and the tenants/hosts within a

---

$^{29}$At the time of writing, the construction of a standard switch has valency of 48 possible tenants downwards, with fixed channel capacity, and a valency of two hosts upwards, each with greater capacity than the downward channels, for allow for aggregation.
Figure 33: The structure can be replicated across datacentre super-agents ($D_n$). Notice that each tenant always has two adjacencies upwards, connecting it to redundant hosts. Eventually, as a message goes up, it will reach the boundary of the Clos super-agent and then the rules for regular (solid state) addressing must change.

tier by subfix $i$. Without taking into account the vertical edge conditions, we can write the tessellating mutual tenancies by the pattern:

\[
T(n) + R_i(n) (H_{2i-1}^{(n+1)}) \# 2C_i \rightarrow H_{2i-1}^{(n+1)} \quad \forall i = 1, 2 \quad (196)
\]

\[
T(n) + R_i(n) (H_{2i}^{(n+1)}) \# 2C_i \rightarrow H_{2i}^{(n+1)} \quad (197)
\]

\[
H(n) \rightarrow R_j(n) (T_j^{(n-1)}) \# vC_j \rightarrow T_j^{(n-1)} \quad \forall j = 1 \ldots v \quad (198)
\]

\[
R_i(n) A_i^{n+1} \rightarrow +\text{forward up to } A_i^{n+1} \text{ if it represents the best path (route)}
\]

\[
R_i(n) (A_i^{n-1}) \# v \rightarrow +\text{forward to best path (route)} \quad \forall i = 1 \ldots v
\]

\[
C_i \rightarrow -R_i^{(n-1)} \text{ i.e. accept the upward forwarding from below as a trade (ACL)}
\]

\[
C_i \rightarrow -R_i^{(n+1)} \text{ i.e. accept the downward forwarding from above as a trade (ACL)}
\]

\[
f_i(C_i) \rightarrow \text{Inform about known addresses from below (BGP)}
\]

The summary of the parts may be written in terms of these vector promises:

\[
R_i(n) A_i^{n+1} = +\text{forward up to } A_i^{n+1} \text{ if it represents the best path (route)}
\]

\[
R_i(n) (A_i^{n-1}) \# v = +\text{forward to best path (route)} \quad \forall i = 1 \ldots v
\]

\[
C_i = -R_i^{(n-1)} \text{ i.e. accept the upward forwarding from below as a trade (ACL)}
\]

\[
C_i = -R_i^{(n+1)} \text{ i.e. accept the downward forwarding from above as a trade (ACL)}
\]

\[
f_i(C_i) = \text{Inform about known addresses from below (BGP)}
\]
Each $v$-valent agent is a host to the $v$ agents below it, and a tenant of the agents above it. The upward valency is 2. As long as we are far away from the lower edge of this pattern, each host also knows that it has two possible routes downward to its tenants also, because of the interwoven tenancy agreements. However, at the bottom edge, there is only a single adjacency to the final address.

Throughout this patterns, the (-) use-promises play the role of access control lists (ACL) for accepting data. I have suppressed most of this to avoid overwhelming with detail; however, those details are important the security and autonomy of the fabric. Without individual tenant control over its choices, the fabric becomes a homogeneous and isotropic solid state space, with long range order. Certainly, this is a good illustration of how such order is valuable, both semantically and dynamically, but it doesn’t really explain how it comes about in practice between uncooperative agents.

When the Internet Protocol was designed, the routing of messages in this way was not conceived in such dense and regular spaces. The Internet was assumed to be a sparse network with clusters at the edges. Gradually, however, as the density increased, and super-agent boundaries were drawn around organizations, a cooperative agreement protocol (BGP) was introduced for mutual benefit. The sole effect of this protocol is to propagate homogeneity along point-to-point adjacencies. Thus, today, a Clos fabric is implemented as a BGP multi-tenant array, just like a tenancy between two utterly independent organizations who want to cooperate in order to forward messages within a shared address space.

Although we think of the Internet as having a single global address space, there is no reason why this should be the case. What makes it true in practice is the need to cooperate between ‘peers’, i.e. between private super-agents who can work symbiotically.

![Diagram of agent spacetime](image)

**Figure 34:** The large scale structure of agent spacetime could easily have different coordinate patches, falling into independent namespaces (called BGP autonomous systems (AS)). Each super-agent (AS) can choose whether to cooperate with the community of addressing or go it alone. This has become common due to the poor scaling of the IPv4 address space, with Network Address Translation for partial splicing at the edges.

As discussed earlier, it is the breaking of symmetries that leads to functional differentiation in a structure. As a final comment on this example of a Clos regular spacetime, we can note that there is very little differentiation. So let’s focus a moment on the natural residual symmetry of the structure. The functional asymmetry of the Clos network is a compelling example of how multi-tenancy orients a structure, but it has a less obvious cephalization. The structure has no obvious brain that forms a master host for the entire structure, but the head is clearly at the top or mouth of the outside world. The scaling is a symmetrical interior scaling of the super-agent boundary: more of a starfish than a cephalopod.

Given such a level of long range order, and insignificant anisotropy along the up-down left-right axes, it makes sense to study the residual symmetry. On paper, we draw these networks as trees with promise adjacencies that cross over one another (see figure 35), as if they were in a two dimensional...
Cartesian lattice. However, we should not be fooled by the clumsiness of a paper drawing. The fact that the adjacencies cross one another should be a sign that this is wrong. Tree-structures have a radial symmetry, and hence the host-tenant decomposition lends itself naturally to polar coordinates, centred on the host.

If we allow the structure to untangle itself, by going to three dimensions (see figure 36), then its true structure begins to make more sense. First the dual hosting can be symmetrized axially, going up and down instead of just up. The then twisted pairs near the bottom of figure 35 can be untwisted to form rings. What one is left with is a hollow tube forming a toroidal geometry, with symmetrical up-down mouths at the top and the bottom. Thus, we have eliminated the quasi-cephalization of the structure and revealed its natural form, which has no asymmetry. It can work top to bottom or bottom to top interchangeably. The final form is shown in figure 37. This tendency for us to orient structures into hierarchies is a common habit in human affairs. However, it often leads to scaling issues and bottlenecking of promises.

It is interesting to think about how Software Defined Networking has tried to recentralize network controls with brain-like controllers. The annals of science would suggest this would be associated with a natural asymmetry. But where is the asymmetry in a datacentre network fabric? Where should the brain be located in order to fulfill its roles as a rapid correlator of sensor input, and coordinator of reaction?
6.8 Collapsing private worlds (part 1: cross-cooperation)

Tenants are assumed to be initially isolated from one another by default. However, we need to ask at what scale is this isolation true? The assumption might not be compatible with the underlying adjacencies of spacetime, since the tenancy promises could easily be made on top of an adjacency substrate.

Example 66 Two students sitting an exam occupy desks next to one another, with line of sight. They make no promises to communicate, but they are physically adjacent, not isolated. Similarly, two rival companies share offices and computing resources in a hosting unit. They make no promises to interact, they might even promise not to interact, but they have an underlying adjacency making the assumption ambiguous.

Segmentation, and non-interference rest on the default assumption about tenants that they make no promises to one another, i.e. that their only notable promises are with the host only. Sometimes separate tenancies may need to be combined. This process can be carried out following the scaling rules in earlier sections.

Example 67 Several military units under a common command are combined to carry out a mission, e.g. different NATO country members collaborate.

Example 68 The section of this document that you are reading now, and the next, are separate tenants of the total host document. However, they would better be written as a single section, and their isolation can be worked around by making promises to cooperation (either through the adjacency of the host, or otherwise).

If one tenant promises to sub-let part of its space to another, by making a promise transverse to the longitudinal axis, this is ok, as it does not affect the sharing promises of the host. If the intermediary funnels resources from one tenant to the other, this could undermine the host’s intentions, but it can never cause the host to break its promise, thanks to the locality of promises (see figure 38).

However, the host might want to prevent this. It’s channel for this is in setting the conditions it is willing to accept from the tenants (conditional $-C$). Thus, at best, the host can try to protect its own interests locally, and ask for the tenants to promise good behaviour. It can punish bad behaviour (tit for tat) but it cannot prevent it.
Figure 38: Tenants might promise to cooperate and break their isolation. They are free to do this, unless the host agreement’s conditions \( C \) forbid it, in which case the host might cease to keep its own promises.

**Example 69** Choice of agency semantics are a design decision in a functional world. For commodity items like radios, television sets, watches, and so on, we design agencies to be easy to grasp by asking the question: what semantics do we wish to expose to an observer?. Consider the illustration in figure 39, showing two choices of agency boundaries for a common appliance. The listener of the radio only needs to see the outer surface of the agency, not its internal components. The battery is a component that needs to be changed frequently compared to the lifetime of the device, so from the perspective it might seem to be a separable piece. However, few users of a radio would want to have the battery alongside the radio as a separate entity. Thus, an agent in the role of radio listener (appliance user), a single agency offers the preferred semantics of scale. However, another agent in the role of radio-maintainer could prefer to see the agent at a different scale, where replaceable components are exposed as separable entities.

6.9 Collapsing private worlds (part 2: failures of scalability)

... is a cooperative promise. It is not uncommon for mergers and acquisitions of tenants to take place, in any realm of tenancy, so knowing that this can be done without violating the isolation of any other tenants is important.

Following on from the subject of scaling agents in section 5.7, we can also consider how segregated tenants and hosts behave when they promise to act as collective entities. Does this change the tenancy
relationship? From the viewpoint of the host, the addition of a cooperative agreement changes nothing. Any resources are still provided as \( +R_i \) and \( +R_j \) to tenants \( i \) and \( j \) respectively; thus sharing has to be re-routed by the tenants to one another. This places them in the host of mutual host for one-another, which might not be optimal in terms of semantics (it is an unwanted complication of the design). The solution for the long-term would be to renegotiate a new host-tenant relationship for the new composite entity.

The asymmetric resource tenancy shown in figure 38 is another example of how a semantic space remembers the historical process of its genesis, (recall figure 24). The resulting network is a non-optimal promise bindings that are not simple scalings of the intended relationships can come about by ‘hot-wiring’ the promises to work around the lack of scalability. Such ad hoc cooperation can solve a temporary need, but creates a new scale pinning, through the asymmetry of the promise, which will further stifle scaling should further super-agency

6.10 Namespaces and hierarchies as multi-tenancy in identity space

A namespace is a cooperative tenancy in which members are isolated from their surroundings by a gateway host. The host promises to make names of its tenants unique, usually by extending the names hierarchically under the common umbrella of its own name. This is now sometimes called ‘disambiguation’ in software. It is essentially the same as adding more lines to place addresses so bound location by containment within. It is a different strategy opt uniqueness than tuple-coordinatization.

Example 70 The same street names appear in many of the local towns, often based on names of historical figures, but this is not a problem, because each exemplar can make a unique address by adding the name of the local town to disambiguate from the others.

Similarly, the ‘High Street’ is a common fixture in British towns. Since most towns have a High Street, the name of the town and district can be added for uniqueness.

The term ‘namespace’ is a popular concept in computing, but the concept of namespaces are clearly in common and widespread use.

Definition 42 (Namespace) A namespace \( N \) is an isolated subspace of a semantic space, formed from a collection of agents. Inside a namespace, all agent coordinates and names are unique by mutual cooperation.

\[
A_i \xrightarrow{+name_i, #name_i} A_j \quad \forall i, j \in N \quad (202)
\]

\[
A_i \xrightarrow{\pm C(name)} A_j \quad (203)
\]

Outside the namespace, names can be made unique by transforming the names according to some bijective function, e.g. either extending the name with a boundary prefix identifier, or performing a name translation.

\[
A_i \xrightarrow{f(name_i, N)} A_k \quad \forall i \in N, k \not\in N \quad (204)
\]

\[
A_k \xrightarrow{-f(name_i, N)} A_i \quad (205)
\]

The hierarchy implicit in nesting namespaces allows us to view these as tenancy relationships, where a host is appointed as the gatekeeper mediating adjacency between the namespace and the outside:

Example 71 Namespace examples:

- A family surname labels a namespace in which Christian names can be made unique relative to other families. Today, this is not a very successful scheme, as it has not scaled well to modern populations.
• Filesystem trees, like the Unix or Windows hierarchical directories name sub-directories and files tenants of their parents, in a forest graph structure.

• Recursion with local variables in functions and subroutines uses the functional closure as the host agent.

• Subnets and networks form a two or more level hierarchy of attachment. A layer 2 broadcast domain is a namespace, in which an IP router is the hosting gateway prefixing addresses with their subnet prefix.

• Taxonomy and classification hierarchies use subject categories as hosting agencies which contain sub-categories.

• Programming class hierarchies, class member functions etc are hosted within named objects, much like a file-tree.

### 6.11 Topology and the indexing of coordinate-spaces in solid-state

As we saw in section 4.2, addressing and tenancy are related through the notion of routing between autonomous agents. The basic asymmetry of tenancy is what allows message sorting by address to be implemented by cooperation, and the ability to scale this is therefore connected to how we scale tenancy.

Branching hierarchies are the most common form of classification sorting (‘disambiguation’) in information technology. They follow in the Aristotelian tradition of taxonomy, widely embraced during the 19th century. It is not uncommon to shoehorn models into a tree structure out of habit. This should not be necessary, however. The challenge of any coordinate system, for address encoding, is to mimic the structure of spacetime in the naming conventions of the points. This enables predictability and emergent routing from local autonomy. The Cartesian tuple-based coordinates discussed in paper I are flexible, and are not tied to any particular origin.

Since sub-agent names are interior (local) to a given super-agency, they can be organized as that agency sees fit. If the size of the namespace is not too large, almost any naming scheme is workable. However, as the number of internal sub-agents grows, the efficacy of tuples depends on the internal connectivity. This need not be a hindrance to using tuples for the following reason: tuples can always be fitted to a spacetime as a covering, with a possibly complex boundary. Put simply, even if the tuple space is not fully populated with points in a Cartesian lattice, one can use the points that are available and ignore the others (see figure 40).

A namespace can be covered by an arbitrary set of coordinate tuples of the right size so as to span all agents in a spacetime uniquely, giving each agent its own unique ID. For example, a tree reference

/tenant/container/sub-component

is trivially written as a tuple:

(tenant, container, sub-component)

The principal different between these two forms is that the order of the tuple components does not imply any particular ordering of dominance or containment. An agent can approach the identification of points by iterating over the tuple members in any order to parse the space according to the desired semantics.

### 7 Applications of multi-tenancy

With a number of tools, principles, and concepts for multi-tenancy under our belts, we are now approaching a vision for how one could design and operationalize environments, both as isolated ‘organisms’, and as fully connected ecosystems of autonomous agencies.
Let’s consider some examples to illustrate these points, focusing particularly on the world of information infrastructure, or what is now called ‘cloud computing’. These cases have immediate utility. We can summarize the basic principles covered so far as a number of points to cover for each analysis:

1. Determine language of promise bodies.
2. Determine the conceptual head of the organism, from input/output flow.
3. Describe segmentation or tenancy/sharing relationships.
4. Explain the role of spacetime phase (solid, gas, hybrid, etc).

In these examples, I will sketch out some outlines only, leaving the details as an exercise to the reader. Completing a full analysis of a comprehensive system would be a significant undertaking.

7.1 Example: Processing element in IT infrastructure

Example 72 Modern IT infrastructure (what is now termed cloud computing) is built from arrays of processing nodes, storage devices, and network switching devices. There is a plethora of terminology which I will try to avoid. In this example, I want to focus only on the processing nodes. A processing node (sometimes called a ‘compute node’ or ‘machine’) generally consists of a physical computer (also called a ‘server’ for historical reasons). It acts as a host which promises an operating system (+f(C,R)) providing tenancy to processes. These sometimes consist of virtual machines or ‘containers’ (+R).

The purpose of hosting these machines and containers is to support the running of software applications within the tenants. For now, I’ll disregard the details of the applications and simply assume that there are isolated processes (see figure 41).

An array of computing machines can be thought of as a semantic spacetime, with each host representing a location. Taking the view that the hosting service is a ‘front’ for the internal tenants, each host may be modelled as a super-agency, with internal structure consisting of the tenants. Each host has a name and an address, and the processes inside them have names and addresses too.

Let’s consider the four points:

I. The language of promise bodies.

We always start by expressing promises that we know we can keep. Promising something vague or undefinable is an act of self-sabotage. Simple promises, close to the capabilities of the agents are
Figure 41: A tenant-oriented infrastructure. Notice the limited self-similarity between the process container scale and the physical container scale.

preferred. Here, the alphabet of promises may include promises to execute programs at a certain rate (based on the CPU clock speed), transmit data, isolate processes from one another, etc.

2. Determine the conceptual head of the organism, from input/output flow.

The head of a processing node is where the interfaces to the outside world enter (see figure 41). These are connected directly to the kernel, and the kernel (represented by the dotted line) acts as a kind of brain for the hosting of the tenants. The tenants are the processes, which have virtual interfaces connecting them to the actual network interfaces, via the kernel.

3. Describe segmentation or tenancy/sharing relationships.

Segmentation of the host’s body is in terms of the process containers. The isolation is maintained by two-mode operation at the hardware level, and thereafter mediated by the kernel.

4. Explain the role of spacetime phase (solid, gas, hybrid, etc).

The phase of the spacetime is undefined here. Inside a super-agent, processes come and go, addresses typically change, looking like a gas. Outside the host, the Internet at large has the structure of a gas, for much the same reason. However, inside a datacentre, fabric design with regular symmetrical arrays (e.g. in leaf-spine Clos networks) and fixed networking has the structure of a solid. This looks not unlike a biological cell, with structure floating around inside and outside a boundary.

How does this scale up? Scaling can be done in one of two ways: either on the interior of the super-agent boundary of the host, or on the exterior:

1. Interior agents can be scaled by parallelizing each sub-agency of the super-agent into a larger sub-agent. The number of sub-agents grows linearly and the super-agents remains constant. This is how we build a mainframe, or NUMA-scaled cluster. Each component is made stronger by adding
parallel numbers, but the structure of the design is constant. It is low level redundant scaling, which is known to lead to best effort reliability, according to the reliability folk theorem [21, 22].

2. Exterior agents can be scaled by parallelizing entire hosts. This increases the total number of super-agents with constant number of interior sub-agents. This is how we build a server farm, like a cloud environment.

The main difference between these two is the adjacency structure, or how communication between the agents is wired.

What can we say about the host-tenant asymmetries? Within each super-agent, there are actually many host-tenancy relationships working together. Referring to figure 41, we see a hierarchy of agency at a typical host. The stippled circle illustrates the super-agent boundary of the host (running the system kernel), and the solid ellipses demark various sub-agents associated with it. The sub-agents form a variety of multi-tenant bindings, with different semantics. These are summarized in this table:

| Role of Host                  | Role of Tenant                 |
|------------------------------|--------------------------------|
| Process container            | Process interface              |
| Physical container           | Process container              |
| Physical container           | Physical interface             |
| Physical container interface | Process container interface    |
| Physical network             | Process network                |

A process container houses several process interface tenants, each of which pay by mediating a connection between the internals of the application and the process via logical network channels. The process network channel interfaces are themselves tenants of the physical network in the manner of figure 26.

In a similar way, but at a coarser scale, the physical interfaces are tenants of the physical host super-agent, mediating physical network connections by direct analogy. Ironically, the process interfaces bind as tenants of the physical interfaces, drawing their agency from the network resource provided by the physical interface host. Such reversals are completely unproblematic, as they are simply semantics pointing towards a resource in a particular viewpoint of the collaborative ecosystem.

It is habitual in IT modelling to simply impose a hierarchy on these resource agencies, from a single viewpoint; however, the imposition of a hierarchy, without reference to the promises they make, leads to viewpoints that masquerade as authoritative, but are not. What one learns from Promise Theory is that, for every asymmetric relation, there is a complementarity transformation which reverses the preferred interpretation and the natural ordering.

Let’s focus on only one issue: addressability. Can we assign a coordinate system of names in a way that is faithful to the structure, and conducive to locating resources from any viewpoint? The possible varieties of naming of agencies are extensive, though not all are used in practice:

| Agency                  | Promised identifier(s)                          |
|-------------------------|------------------------------------------------|
| Physical interface      | { MAC-addresses }, { IP-addresses }             |
| Process interface       | { IP-addresses }                                |
| Namespace               | Namespace umbrella name                        |
| Physical container      | Hostname                                       |
| Physical IP-address     | { DNS translation names }                       |
| Process IP-address      | { Namespace translation names }                |
| Process container       | Process container name                         |

Following industry practice, it is normal to coarse grain away several of the distinctions between these promises. This leads to problems in tracing the origins of promises made by some of the coarse-grained agencies.
Well-known problems associated with design of the Domain Name Service (DNS) (especially reverse lookup) can be cited as an example of how coordinatization based on perceived containers, rather than general tuples leads to difficulty in tracing inter-agency collaborative processes. I’ll leave it as an exercise to the reader to design an improved DNS service which acts as an invertible, based on the resolvability lemma (lemma 3).

7.2 Example: tenant-oriented hosting infrastructure

In the previous example, the low-level resource container (hardware) was considered to be a super-agent boundary for processes running inside. However, we have great freedom to change the nature of a hierarchy in Promise Theory.

Functional thinking is invariably top-down. Computer science teaches top-down decomposition of problems through a logical branching process. Often this leads to trouble at lower levels due to inconsistency. The bottom-up aggregation process avoids inconsistency, and permits logical scaling, however we don’t interface with systems from the bottom up. The ‘cephalization’ of agency around physical connectivity tends to fix attention on the host and its role as a kind of manager or brain. However, the applications (the minds of the system) are run inside the tenants, and there are more tenants than hosts. From a human perspective, then, the tenants see their concerns as a focal point, since they pay rental fees to support the host’s existence as their service provider. The customer service viewpoint thus weighs in semantically, where as the engineering viewpoint of the previous example was attractive dynamically.

Example 73 In a computing platform, design for hosting software systems, multiple applications run as tenants of an infrastructure provided for them. This separation allows delegation with cooperation, sometimes called DevOps: tenants deal with content development, while the host deals with operational delivery.

By analogy with the radio example 69 above, the agencies, which a user of the application would like to see, are shown in figure 42. The agent, in the role of user, does not want to see the internals or even the ‘battery’ that makes the thing work, it only cares about the surface boundary of the application, and its exterior promises. The illustration in figure 42 shows how tenants would like to conceptualize spacetime, placing their concerns as the top-most or outer-most interface. Once again, this is no limitation, since Promise Theory tells us that we have significant freedom to turn hierarchies and viewpoints upside down for convenience. With this viewpoint, the application tenant is in focus, and, from this user perspective, the application subsumes its infrastructure within the outer boundary. It wears user semantics on the outside.

The four concerns are:

1. The language of promise bodies

   This will include the exterior promises made by the application to its users, and the interior promises made between the layers of hosts and tenants, described below.

   At each scale of agency, one can imagine creating a ‘compiler’ from a domain specific body language to a more explicit pattern-generated explication of meaning for the lower level components.

   $$\beta_M \to \beta_{M'} + \beta_M$$  \hspace{1cm} (206)

2. Determine the conceptual head of the organism, from input/output flow

   The head of the organism is now the application-user interface (see figure 43).

---

An analogy might be the following: do we consider the mind to be a tenant of the body, or the body to be a tenant of the mind? Dynamically (physically) the former makes sense, but semantically the latter is a highly convenient viewpoint.
3. Describe segmentation or tenancy/sharing relationships

There are two layers of tenancy:

(a) Application users are tenants of the application platform.
(b) The application is a tenant of the infrastructure platform.

The tenancies are based on approximately the following trades. For the user-application tenancy:

\[ R = \text{application account login} \]
\[ C = \text{user identification credentials (money?)} \]
\[ f(C, R) = \text{application functionality} \] (207)

and for the application-platform tenancy,

\[ R = \text{platform login by application} \]
\[ C = \text{application identification credentials (money?)} \]
\[ f(C, R) = \text{pay by use computer, storage, network resources} \] (208)

Maintainers of the application, and maintainers of the infrastructure on which the application resides will make different choices, depending on what semantics they wish to expose.

At the level of computers, network, and storage, the picture might look something like figure 45. The infrastructure is composed of three types of agent: machine, storage, and network, which make promises accordingly. Applications are thus tenants of the platform infrastructure, and the main resources are represented the infrastructure scale in figure 45:
Figure 43: The application-platform interface exposes the resource knobs needed by application.

4. Explain the role of spacetime phase (solid, gas, hybrid, etc)

From the application perspective, this is a single atomic agency, ready to bind to a user in a gaseous state. Inside the application, the state of the interior depends on the underlying infrastructure that powers the application.

The agency scales of interest are:

\[
\begin{align*}
\text{User} : A &= \{\text{application, user}\} \\
\text{Application} : T &= \{\text{computation, storage, network, namespace, application, user}\} \\
\text{Infrastructure} : M &= \{\text{process container, disk, router, application, box, computation, storage, network, namespace, interface, names/addresses, application, user}\}
\end{align*}
\]

Notice how, when increasing levels of detail, the higher levels are not replaced by the lower levels, since they still contain unique information (see section 3.10). Also, here I have introduced a semantic category scale ‘box’ to symmetrize over the resource providers that are interconnected by network channels (see figure 44).

\[
\begin{align*}
\text{Box} : A &= \{\text{computer, storage, router, interface}\}
\end{align*}
\]
In addition to these 'physical' agencies, there are other more abstractly defined agencies at play within an information system.

- Names and addresses of machines and storage (IP/DNS addresses), for use by tenant name-services. These are private per application.
- Data from machines and storage, for use by tenant networks.
- Transport of data, for use by computer machines and storage (also represented as super-agency 'box').
- Directory service lookups, for use by computers and storage, in order to locate others in their private network.

![Diagram of network bindings to resource containers](image)

Figure 44: The network bindings to resource containers

The network binding to a resource agent has the form of a tenancy also (see figure 44), as a process container might be able to bind to multiple networks, in principle. An application might choose to branch its logic into multiple private channels, just as it chooses to branch sub-functions into private branches. In this reverse viewpoint, like in the previous example, applications can be considered to form private tenancies on top of the common spacetime, by using the intermediate agencies of the spacetime as proxies.

IT applications share a common infrastructure, provided for all applications by mutual cooperation. This forms a semantic spacetime, which is entirely interior to what we may call Application Space. Each application creates its own private world, as a tenant of the application itself.

Applications may thus be viewed as 'atomic' super-agents, either locally or globally; thus, from a promise perspective, it is irrelevant whether the architecture is monolithic (implemented inside a closed space) or distributed (without fixed boundary). The promise abstractions are identical in both cases; the only difference is one of scale.

8 Epilogue: abstract agents, and knowledge modelling

So far, most of the agencies we’ve discussed have been real entities, in a physical spacetime. It is also possible, even common, to construct entirely virtual 'knowledge spaces' of a more abstract kind, whose very structure is a representation of its semantics. This section is a bridge between the notion of tenancy and the treatment knowledge spaces in paper III.
8.1 Classification, categorization, and disambiguation of tenants

The branching of names into taxonomies is closely allied to the way narratives and storylines branch in logical reasoning. This notion is extensive, and will be explored in paper III.

Example 74 The subsections in this section about tenancy form a hierarchy under the namespace of multi-tenancy. Trying to untwine and separate the concepts in to identify what is common and what is independent is a tricky challenge, which can quickly descend into a exercise of listing every related concept one can imagine. Partially ordering these to account for dependencies is a further challenge. This is one of the issues we struggle with when organizing information and representing knowledge.

Classification does not only apply to idea, but also ideas we have about physical entities. Hence the power of a semantic space lies in making the distinction between pure information and information about a separate entity moot: in both cases the information has to be encoded by something physical, whether in the thing itself or in a proxy thing elsewhere.

Example 75 Exclusive clubs are one thing, but categorization of tenants leads to explosions of new agency:

- Premium customers
- Schools for separate sex, race etc
• Cabin classes on airlines (economy, business, first class, etc)
• Car/truck parking
• Taxonomies of species or subject categories.
• Periodic table of elements.
• You can share a public bathroom, or you can have a private one

Multi-tenancy is a bridge between the understanding of resources and the differential categorization of knowledge into concepts.

8.2 The economics of scale

Consider the example in figure 46, logical reasoning creates a natural branching process that subdivides agencies into categories, often from a reductionist standpoint. Conversely, category labels act as logical hosts that unify similar contributors under a common label. A book, such as an anthology of articles, is an aggregation of chapters, where the space $R$ is rented out to chapters for text $C$. The book provides a vehicle for the chapters to be marketed under a common brand. In the same way, authors come together into categories of fiction, poetry, biography, etc, and the physical exemplars of the books share the same category, each contributing something to occupy the hosting of the category as a ‘market brand’. The brand category develops relationships with an audience at a different level or scale than a single book or author can, hence there is a value to the hosting. This is sometimes called the economics of scale.

Figure 46: Orientation of host-tenancy roles. With host-ness to the left and tenancy increasingly to the right, we see that it is not always possible to think of tenants as being simple isolated ‘things’, especially when the promises they represent are more abstract. A tenant shares some space in a host, but what kind of space? A branching process might result in new tenants (+), or new hosts (-).
9 Summary and comments

Continuing on from paper I, we’ve seen that it is possible to define semantic spacetimes with cooperative structures that scale in both semantics and dynamics. The importance of scale to both semantics and dynamics is frequently underestimated in system behavioural analysis, especially when it comes to semantics. The goal of Promise Theory is to place these two aspects (semantics and dynamics) on the same kind of footing, for a wide range of spacelike scenarios, to make a physics of information systems. Even in this deceptively simple topic, and this cursory sketch of a full treatment, there is a lot to swallow. Perhaps this is why computer science has yet to answer some of these basic questions about scaling.

Multi-agent distributed systems delocalize semantics, and potentially make systems harder to understand. This is the value in coarse graining and going to a higher level. As intuited by Milner [23], the design of a system is largely about what faces does one wish to expose to whom? But lacking a proper understanding of spacetime prevents one from implementing the interfaces and moving between the abstract and physical viewpoints.

One thing that comes more clearly out of the approach discussed here is the often-noted misunderstanding about purely reductionist thinking in cooperative phenomena. One cannot take a system apart and expect to see all phenomena in the isolated parts. At each new scale, there are new promises that cannot be reduced to the promises at lower levels. Thus we need rules for preserving information during scaling and coarse-graining. However, we also find that this information lost to coarse-graining can be captured as a directory (or index) of the internal details, so that the complete picture can be represented for probing downwards. Such services are well known as naming or directory services in the world of information technology, but here we see that this is a fundamental requirement for semantic scaling.

Some obvious conclusions drop out of the formalism nicely: central hosted authority is not scale invariant, for instance. Separation of agency scales is a convenience, but one must add to a simple dynamical picture the answer to the semantic question: why would you ignore or trust the promises from a finer-grained substrate of agents? The ability to rely on the integrity of the substrate implies a long range order, and data integrity, but also uniform semantics (see [2] section 11.3).

Lastly, the natural point of departure from this paper II is the issue of abstract agency, naming and categorization. This is a bridge to understanding the more abstract issues of relationships and encoded memory in a functional space. The ultimate goal of this work is to reach a unified description of semantic spacetimes, where the layout of space itself is a realization of knowledge relationships and the comprehension of mind-brain-like structures. I’ll return to these issues in paper III.

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