THE JOINT LARGE-SCALE FOREGROUND–CMB POSTERIORS OF THE 3 YEAR WMAP DATA

H. K. Eriksen,1,2 C. Dickinson,3 J. B. Jewell,3 A. J. Banday,4 K. M. Górska,5,5 and C. R. Lawrence5

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ABSTRACT

Using a Gibbs sampling algorithm for joint CMB estimation and component separation, we compute the large-scale CMB and foreground posteriors of the 3 yr WMAP temperature data. Our parametric data model includes the cosmological CMB signal and instrumental noise, a single power law foreground component with free amplitude and spectral index for each pixel, a thermal dust template with a single free overall amplitude, and free monopoles and dipoles at each frequency. This simple model yields a good fit to the data over the full frequency range from 23 to 94 GHz. We obtain a new estimate of the CMB sky signal and power spectrum, and a new foreground model, including a measurement of the effective spectral index over the high-latitude sky. A noteworthy result in this respect is the detection of a common spurious offset in all frequency bands of $\sim 13$ $\mu K$, as well as a dipole in the V-band data. Correcting for these is essential when determining the effective spectral index of the foregrounds. Fortunately, the CMB power spectrum is not significantly affected by these issues, as our new spectrum is in excellent agreement with that published by the WMAP team. The corresponding cosmological parameters are also virtually unchanged.

Subject headings: cosmic microwave background — cosmology: observations — methods: numerical

1. INTRODUCTION

A major challenge in CMB research is component separation, which can be summarized in two questions. First, how can we separate reliably the valuable cosmological signal from confusing foreground emission? Second, how can we propagate accurately the errors induced by this process through to the final analysis products, such as the CMB power spectrum and cosmological parameters?

During the last few years, a new analysis framework capable of addressing these issues in a statistically consistent approach has been developed. This framework is Bayesian in nature, and depends critically on the Gibbs sampling algorithm as its main computational engine. The pioneering ideas were described by Jewell et al. (2004) and Wandelt et al. (2004) and later implemented for modern CMB data sets for temperature and polarization by Eriksen et al. (2004) and Larson et al. (2007), respectively. Applications to the 1 yr and 3 yr WMAP data (Bennett et al. 2003a; Hinshaw et al. 2007b) were described by O’Dwyer et al. (2004) and Eriksen et al. (2006, 2007a). These papers mainly focused on the cosmological CMB signal, and adopted the foreground corrected data provided by the WMAP team.

Recently this algorithm was extended to include internal component separation capabilities by Eriksen et al. (2007b). Using rather general parameterizations of the foreground components, this method produces the full joint foreground-CMB posterior, and therefore allows us both to estimate each component separately through marginalized statistics and to propagate the foreground uncertainties through to the final CMB products. The implementation of this algorithm used in this paper is called “Commander,” and is a direct descendant of the code presented by Eriksen et al. (2007a).

In this Letter, we apply the method to the 3 yr WMAP temperature observations (Hinshaw et al. 2007). This data set, with five frequency bands, allows only very limited foreground models; however, the analysis provides an instructive demonstration of the capabilities of the method. For a comprehensive analysis of a controlled simulation with identical properties to this data set, see Eriksen et al. (2007b).

2. DATA

We consider the 3 yr WMAP temperature data, provided on LAMBDA in the form of sky maps from 10 “differencing assemblies” covering the frequency range between 23 and 94 GHz. Since our current implementation of the Gibbs foreground sampler can only handle sky maps with identical beam response (Eriksen et al. 2007b), we downgrade each of these maps to a common resolution of 3 FWHM and repixelize at a HEALPix resolution of $N_{\text{side}}=64$, corresponding to a pixel size of 55’.

These 10 maps are then co-added by frequency into five single frequency band maps at 23, 33, 41, 64, and 94 GHz (K, Ka, Q, V, and W bands, respectively). The power from the instrumental noise is less than 1% of the CMB signal at $l=50$ in the V and W bands, and less than 2% at $l=100$ (Eriksen et al. 2007b). To regularize the noise covariance matrix at high spatial frequencies, we added 2 $\mu K$ per $N_{\text{side}}=64$ pixel of uniform white noise. This noise is insignificant at low multipoles, but dominates the signal near the spherical harmonic truncation limit of $l_{\text{max}}=150$. We then have five frequency maps at a common resolution of 3 FWHM, with signal-to-noise ratio of unity at $l \sim 120$, and strongly bandwidth limited at $l_{\text{max}}=150$.

We choose to include such high $l$-values in the analysis for two reasons. First, our main goal is an accurate approximation of the CMB likelihood at $l \leq 50$. In order to ensure that the degradation process (i.e., smoothing and noise addition) does not significantly affect these multipoles, it is necessary to go well beyond $l \sim 80$–100. Second, significant information on the spatial distribution of foregrounds is obtained by going to higher resolution.

The cost of this treatment of the noise is high $\chi^2$ values in the analysis; however, the analysis provides an instructive demonstration of the capabilities of the method. For a comprehensive analysis of a controlled simulation with identical properties to this data set, see Eriksen et al. (2007b).
The frequency specifications of the WMAP observations and, eventually, high-sensitivity Planck maps. See also Eriksen et al. (2007b) for a comprehensive analysis of a simulation with identical instrumental properties, for which similar behavior is observed.

We impose the base WMAP Kp2 sky cut (Bennett et al. 2003b) on the data, but not the point-source cuts. The base mask is downgraded from its native \( N_{\text{side}} = 512 \) resolution to \( N_{\text{side}} = 64 \) by excluding all low-resolution pixels for which any one of its subpixels is excluded by the high-resolution mask. A total of 42,081 pixels are included, or 85.6% of the sky.

The frequency bandpass of each map is modeled as a top-hat function, and implemented in terms of effective frequency as a function of spectral index as described by Eriksen et al. (2007b). We refer the interested reader to that paper for full details of the algorithm, and for a comprehensive analysis of a realistic simulation corresponding to the same data and model used in this Letter.

Finally, we estimate a new set of cosmo 3. MODEL AND METHODS parameters within the standard Λ CDM model. For this analysis, we follow the approach of Eriksen et al. (2007a) and replace the low-1 part of the WMAP likelihood with a new Blackwell-Rao Gibbs-based estimator (Chu et al. 2005). No ancillary data sets beyond the 3 yr WMAP temperature and polarization data are included in the analysis. The CosmoMC code (Lewis & Bridle 2002) is used as the main MCMC engine.

4. RESULTS

Figure 1 shows the marginal posterior mean and rms maps for the CMB sky signal, the foreground amplitude, and the foreground spectral index. Table 1 gives the corresponding results for the monopole and dipole coefficients for each frequency band. The FDS dust template amplitude relative to 94 GHz and an assumed spectral index of \( \beta = 1.7 \) is \( \hat{a}(\nu) = 0.917 \pm 0.003 \). The CMB power spectrum is discussed separately below. The full set of results, both individual Gibbs
TABLE 1

| Band | Monopole (μK) | Dipole X (μK) | Dipole Y (μK) | Dipole Z (μK) |
|------|---------------|---------------|---------------|---------------|
| K    | -11.8 ± 0.5   | 1.7 ± 1.2     | -2.2 ± 0.8    | 2.2 ± 0.1     |
| Ka   | -16.6 ± 0.5   | 0.9 ± 1.2     | 0.9 ± 0.8     | -1.3 ± 0.1    |
| Q    | -12.8 ± 0.5   | 1.9 ± 1.2     | -0.9 ± 0.8    | 0.4 ± 0.1     |
| V    | -11.1 ± 0.5   | 1.6 ± 1.2     | -3.9 ± 0.8    | 4.0 ± 0.1     |
| W    | -12.6 ± 0.5   | 1.7 ± 1.2     | -0.9 ± 0.8    | 1.0 ± 0.1     |

Note. — Means and standard deviations of the marginal monopole and dipole posteriors.

As seen in the bottom right panel of Figure 1, the impact of the Gaussian prior of $β = -3 ± 0.3$ on the foreground spectral index varies over the sky: At low latitudes, the posterior rms is $Δβ = 0.006$, which means that the likelihood dominates over the prior by a factor of $∼50$. On the other hand, at high latitudes the rms peaks at $Δβ = 0.229$, which implies that the prior is in fact stronger than the likelihood in these regions.

The single most surprising aspects of the solution presented above are the monopole and, possibly, the V-band dipole coefficients, listed in Table 1. Most notably, there is a strong detection of a roughly 13 μK offset common to all frequency bands. Formally speaking, these offset values are only optimal within the current model; however, this type of signal is not degenerate with any other component in the model. Further, we have attempted to fit several other models assuming no offsets at one or more bands. These all result in strong, visible residuals in the CMB map, and considerably higher $χ^2$ values overall.

We have also performed a simple analysis to cross-check the monopole coefficients: First we corrected the raw downgraded sky maps described in § 2 using the template fits presented by Hinshaw et al. (2007) for the Q, V, and W bands. If the foreground model is accurate, then these maps are supposed to consist of CMB and noise only. We then imposed the conservative $Kp0$ sky cut to these maps (Bennett et al. 2003b), and computed the high-latitude averages. The values obtained from this exercise were $m_Q = 8.2$ μK, $m_V = -6.8$ μK, and $m_W = -14.4$ μK. Adopting a parametric frequency spectrum for these monopoles on the form $m_ν = m_0 + m_1(ν/23$ GHz)$^3$, we find a best-fit model of $m_ν = [-15.3 + 135(ν/23$ GHz)$^3]$ μK, in excellent agreement with the results presented above. Similar results were obtained for the full-resolution $N_{side} = 512$ differing assembly maps.

To demonstrate the quality of the signal model presented in this Letter, we show in the right column of Figure 2 the difference maps between the raw WMAP data and the Commander model for all five frequency bands. The data fidelity of the Commander model is striking, with residuals typically smaller than $∼3–5$ μK, comparable to the noise level.

The marginal maximum posterior CMB power spectrum is shown in Figure 3, together with the maximum-likelihood/pseudo-$C_l$ hybrid spectrum computed by the WMAP team. Perhaps the most notable difference is in the $l = 21$ multipole, which looks anomalous in the WMAP spectrum, as noted by other authors.

The cosmological parameters corresponding to the Commander

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*See http://www.astro.uio.no/~hke.*

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![Fig. 2.—Comparison of raw data and signal models for the five WMAP frequency bands. All maps are shown at an angular resolution of 3° FWHM. Left column: Raw downgraded sky maps. Middle column: Total signal of one single joint Gibbs sample. (Other samples look similar up to instrumental noise.) Right column: The difference between the raw maps and the Commander signal.](image-url)
spectrum for a standard six-parameter ΛCDM model are $\Omega_b h^2 = 0.0222 \pm 0.0007$, $\Omega_c = 0.243 \pm 0.036$, $\log(10^{10} A_s) = 3.027 \pm 0.068$, $h = 0.730 \pm 0.032$, and $\tau = 0.089 \pm 0.030$. Corresponding values for the unmodified WMAP likelihood are $\Omega_b h^2 = 0.0221 \pm 0.0007$, $\Omega_c = 0.242 \pm 0.035$, $\log(10^{10} A_s) = 3.030 \pm 0.068$, $h = 0.730 \pm 0.032$, and $\tau = 0.091 \pm 0.030$. These values refer to marginal means and standard deviations.

Clearly, the agreement between the two sets of results is excellent, and this provides a strong confirmation of the WMAP results: At the level of precision of the WMAP experiment, details in the foreground model used for foreground correction on large angular scales appear to have only a minor impact on the CMB temperature power spectrum.

5. CONCLUSIONS

We have presented the first exact Bayesian joint foreground-CMB analysis of the 3 yr WMAP temperature data. We have established a new estimate of both the CMB sky signal and the power spectrum, a detailed foreground model consisting of a foreground amplitude and spectral index map and a dust template amplitude, and also provided new estimates of the residual monopole and dipole coefficients in the WMAP data.

The detection of significant nonzero offsets in the WMAP data is the new result of the greatest immediate importance for the CMB community. These new monopole and dipole estimates could have a significant impact on several previously published results, especially those concerning the foreground composition in the WMAP data.

Taking a longer perspective, the most important aspect of this analysis is a demonstration of feasibility of joint Bayesian foreground-CMB analysis. This will be essential for Planck, whose high sensitivity and angular resolution demand more accurate foreground separation than WMAP. Considering the flexibility, power, and accuracy of the method employed in this Letter, together with its unique capabilities for propagating uncertainties accurately all the way from the postulated foreground model to cosmological parameters, we believe that this should be the baseline analysis strategy for Planck on large angular scales, say $l \leq 200$.

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