Sampling Issues in Bibliometric Analysis

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Abstract

Bibliometricians face several issues when drawing and analyzing samples of citation records for their research. Drawing samples that are too small may make it difficult or impossible for studies to achieve their goals, while drawing samples that are too large may drain resources that could be better used for other purposes. This paper considers three common situations and offers advice for dealing with each. First, an entire population of records is available for an institution. We argue that, even though all records have been collected, the use of inferential statistics and significance testing is both common and desirable. Second, because of limited resources or other factors, a sample of records needs to be drawn. We demonstrate how power analyses can be used to determine in advance how large the sample needs to be to achieve the study’s goals. Third, the sample size may already be determined, either because the data have already been collected or because resources are limited. We show how power analyses can again be used to determine how large effects need to be in order to find effects that are statistically significant. Such information can then help researchers to develop reasonable expectations as to what their analysis can accomplish. While we focus on issues of interest to bibliometricians, our recommendations and procedures can easily be adapted for other fields of study.

Key words

Bibliometrics; Sampling; Population; Power analysis; Percentiles
1 Introduction

Statistical significance tests are frequently used with bibliometric data. For example, Opthof and Leydesdorff (2010) compared leading scientists (professors) at the Academic Medical Center of the University of Amsterdam using the Kruskal-Wallis test. Bornmann (2013a) used binary regression models calculating differences between four universities taking into account two covariates: the length of publications and the number of authors. Statistical significance tests are strongly connected to questions of sampling, since these tests are usually applied to the analysis of samples in order to obtain information about an underlying population (Levy & Lemeshow, 2008). In bibliometrics, several papers have been published which deal with the use of significance tests and effect sizes (e.g. Bornmann & Williams, 2013; Schneider, 2012; Schneider, 2013), but the literature on sampling of populations is scarce. In one of the rare papers, Bornmann and Mutz (2013) argue for clusters in a two-stage sampling design (“cluster sampling”), in which, firstly, one single cluster is randomly selected from a set of clusters (e.g. consecutive publication years, in which an institution have published) and secondly, all the bibliometric data (publications and corresponding citation metrics) is gathered (census) for the selected cluster. Then, this cluster sample can be statistically analyzed.

This paper deals with issues around samples and populations in bibliometrics. In many institutional evaluations, bibliometricians have complete publication and citation records for all the papers of an institution. In other words, they have the bibliometric population data for this institution. However, this may not be true for all bibliometric analyses of institutions, and when it is true it raises issues about what forms of statistical analyses are appropriate. This paper addresses two issues: first, the appropriateness of using inferential statistics when the entire population of records is available (Bornmann, 2013b); and second, the use of sampling when it is impractical to gather information for all institutional citation records (Bornmann &
Mutz, 2013). In particular, how does a bibliometrician go about determining how large a
sample needs to be in order to achieve the goals of the analysis? Conversely, when the sample
size has already been determined, how large do effects need to be in order for them to be
statistically significant? Answering such questions can help the researcher decide how large a
sample is needed; or, if the sample has already been drawn, answering these questions can
help the researcher form reasonable expectations as to what the analysis can accomplish.

2 Methods

2.1 Used data

This study uses percentiles of citations to measure institutional citation impact.

Cross-field and cross-time-period comparisons of citation impact for institutional
evaluation purposes are only possible if the impact is normalized (standardized) (Bornmann &
Marx, 2013; Schubert & Braun, 1986). For its citation impact to be normalized, a paper needs
to have a reference set: all the papers published in the same publication year and subject
category. Percentiles have been proposed as an alternative to normalization on the basis of
central tendency statistics (arithmetic averages of citation counts) (Bornmann, Leydesdorff, &
Mutz, 2013; Bornmann & Mutz, 2011; Bornmann, Mutz, Marx, Schier, & Daniel, 2011;
Schreiber, in press). Percentiles are based on an ordered set of publications in a reference set,
whereby the fraction of papers at or below the citation counts of a paper in question is used as
a standardized value for the relative citation impact of this focal paper. This value can be used
for cross-field and cross-time-period comparisons. If the normalized citation impact for more
than one paper is needed in a research evaluation study (and this is the rule in institutional
evaluations), this percentile calculation is repeated (by using corresponding reference sets for
each one).

Following the practice of Incites (Thomson Reuters,
http://incites.thomsonreuters.com/), we use inverted percentiles in our examples, where low
percentile values mean high citation impact. Hence citation impact above average (in the field and publication year) is defined as percentiles less than 50. It is a trivial matter to use non-inverted percentiles instead if appropriate for the data being analyzed.

2.2 **Using bootstrapping to verify that the statistical methods employed are appropriate for percentile data**

A possible statistical problem in this study is that percentiles have a uniform rather than normal distribution. When variables are normally distributed, cases tend to be clustered near the mean, while extreme values in either direction are less common. With percentile rankings, however, in the population there will be just as many cases in the first percentile as there are in the 50th and the 99th. t tests assume that dependent variables are normally distributed, which raises the question of whether analyses based on t tests (which includes the power analyses presented here) are potentially biased.

To assess such concerns, bootstrapping is often used as an alternative to inference based on parametric assumptions when those assumptions are in doubt (Cameron & Trivedi, 2010). Bootstrapping resamples observations (with replacement) multiple times. Standard errors, confidence intervals and significance tests can then be estimated from the multiple resamples. We used bootstrapping techniques to assess the correctness of the power analysis presented in this paper. Specifically, using real data for the years 2001 and 2002 from three research institutions in German-speaking countries (Williams & Bornmann, in preparation), we employed bootstrapping techniques to determine whether null hypotheses were rejected as often as our power calculations suggested they should be. We found that our power estimates were always within a few percentage points of what we actually found with the data. Additional analyses showed that significance values and standard errors produced by t tests involving percentile data were almost identical to the significance tests and standard errors...
produced by bootstrapping. We therefore feel confident that the statistical techniques we use in this paper are appropriate and that our findings are valid\textsuperscript{1}.

### 2.3 Statistical package

For the calculation of the statistical procedures in this paper, we used Stata (StataCorp, 2013)\textsuperscript{2}. Appendix A contains the code used. However, many/most other statistical packages could also be used for these calculations (e.g. SAS or R).

### 3 Results

We consider three common situations. First, an entire population of records is available for an institution. We argue that, even though all records have been collected, the use of inferential statistics and significance testing is both common and desirable. Second, because of limited resources or other factors, a sample of records needs to be drawn. We demonstrate how power analyses can be used to determine in advance how large the sample needs to be to achieve the study’s goals. Third, the sample size may already be determined, either because the data have already been collected or because resources are limited. We show how power analyses can be used to determine how large effects need to be in order to find effects that are statistically significant. Armed with such information the researcher can form realistic expectations as to what the analyses might find.

#### 3.1 Using inferential statistics to analyse a population

It could be argued that there is no need to compute significance tests or confidence intervals (CIs) given bibliometric population data for an institution. That is, we do not need to estimate parameters or make inferences about the larger population because the information on the entire population of papers is available. For example, do we really need to use CIs to

\textsuperscript{1} Additional details are available upon request.

\textsuperscript{2} In particular, we used the power and sample size routines included with Stata 13. These include such programs as onemeans and twomeans as well as several other types of routines for methods not used in this paper.
estimate a range of plausible values for the mean when we already have all the information to
determine what the population mean is? By way of analogy, a public opinion poll may
estimate, subject to some degree of sampling error, who is leading in an election. But, once
the election has been held we no longer need to estimate the levels of support because we
know who actually got the most votes.

However, in situations similar to institutional evaluations, it is actually quite common
to go ahead and estimate significance tests and CIs anyway. Bielby (2013), for example, notes
that significance tests are widely used in class action employment lawsuits even when all
employee records are available for analysis. Two rationales are typically offered for treating
what appears to be a population as though it were a sample.

First, the current cases might be thought of as being a sample from a larger super
population that includes future cases as well (Gelman, 2009). As researchers from Canada’s
Manitoba Center for Health Policy (2001, p. 1) put it,

[A majority of us] reached the conclusion that even when one has data on the full population,
one only has that data cross-sectionally in time. In a sense, the data can be viewed as a sample
from possible states in the Province as they unfold over time. Therefore, it made sense to us to
try to indicate whether differences which are certainly real across units are statistically
significant when one considers the data to be a one-time sample of the unfolding of the
universe.

A second rationale, and a perhaps more compelling one, is to think of observed cases
as repeated trials that are products of an underlying stochastic process. If we tossed a coin 100
times, we wouldn’t think that we had the entire population of coin tosses; a different set of
tosses is possible and, because of chance factors, would likely yield somewhat different
results. As Gelman (2009, p. 1) puts it,
Another frame is to think of there being an underlying probability model. If you’re trying to understand the factors that predict case outcomes, then the implicit full model includes unobserved factors (related to the notorious “error term”) that contribute to the outcome. If you set up a model including a probability distribution for these unobserved outcomes, standard errors will emerge.

So, for example, in an employment discrimination case, if women are making less than men it might be that chance factors caused some women to be unlucky with their wages (like tossing a coin and getting five tails in a row) even though overall the process by which wages are set is fair.

For bibliometrics, we argue that the observed citation impact of papers (measured by percentiles) allows us to make inferences about the underlying process that generated those impacts and the extent to which citations may have been influenced by random factors. The success of a paper, or of an entire institution, is presumably affected by the quality of the material in the papers, but is also partly determined by chance, e.g. how often a paper or collection of papers gets cited might be affected by how many people chose to read a particular issue of a journal or who happened to learn about a paper because somebody casually mentioned it to them (Bornmann & Daniel, 2008). Put another way, if we could somehow repeat the citation analyses over and over, the citation impact of papers (percentiles of citations) would not be exactly the same for each repetition, just like doing 100 coin tosses over and over would not yield the exact same number of heads each time. Hence, even when all existing citation records for an institution are available, inferential methods can still be used to test whether, say, a high impact score for an institution could just be due to luck, or whether apparent differences in the average percentiles for two institutions are too large to attribute to chance alone.
3.2 Using power analysis when a sample needs to be drawn

While it is desirable to have all institutional records for an evaluation study it is not always practical. Percentile data need to be purchased from other sources (e.g. from InCites provided by Thomson Reuters), and the cost of obtaining percentiles for all records may be prohibitive. In other cases, a bibliometrician may wish to supplement the information contained in the bibliographic records; for example, add information about the authors (e.g. their academic status) or more refined codings of the topic matter. It may be impractical or too expensive to do this for all the records and hence a sample will need to be selected.

However, how does a bibliometrician decide how big a sample needs to be drawn? Samples that are either too small or unnecessarily large both have disadvantages. As StataCorp (2013, p. 1) notes, “A study with too few subjects may have a low chance of detecting an important effect, and a study with too many subjects may offer very little gain and will thus waste time and resources.” To determine optimal sample size, power analyses are often conducted before a sample is collected. A typical use of power analysis is to determine how large the sample must be to detect an effect of a given size. That is, how large does the sample need to be that we can be reasonably confident that we will correctly reject the null hypothesis when the null hypothesis is false?

So, for example, suppose that an institution believes that it is above average in terms of how often its publications get cited. If the papers of the institution really are above average, how much above average does it need to be, and how large does the sample need to be, in order to detect statistically significant differences from the average score in the reference sets (percentile=50)? A power analysis can be used to address such questions (see Table 1 and Figure 1).
A technical explanation of the mathematics behind power analysis is beyond the scope of this paper\(^3\), but we can explain several of the key components behind such an analysis. We interpret the results as follows:

- **28.87** is the population standard deviation (\(\sigma\)) for percentile rankings (Waner & Costenoble, 1996)\(^4\). It is common to assume that the sample standard deviation will be the same, although the bibliometrician could choose some other value if there were reason to believe otherwise.

- **Power** = Pr(rejecting h0| h0 is false). We set power at .8, meaning that we want a sample size that is large enough that we will correctly reject the null 80 percent of the time when it is false. If more power is deemed necessary a larger value can be chosen, but this will also require a larger sample size.

- **Alpha** (\(\alpha\)) = Pr(rejecting h0| h0 is true). \(\alpha = .05\) is a commonly used criterion for rejection; differences between the null and alternative hypotheses must be large enough that we would expect to only reject the null 5 percent of the time when the null is true. More stringent (e.g. .01) or less stringent (.10) values can be chosen, depending on how costly we feel it would be to reject the null when we shouldn’t.

- **\(\mu_0\)** is the value of the mean specified under the null hypothesis. In this case we chose the known population mean of 50, but we could have chosen higher or lower values if we had felt they were more appropriate. For example, a major research institution that considers itself among the world’s elite might want to see whether it exceeds a more demanding value like 25. Conversely, a teaching

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\(^3\) Numerous sources, such as StataCorp (2013) can be consulted by those wishing to see a more technical and mathematical discussion.

\(^4\) More specifically, percentile rankings have a uniform distribution with values ranging from 0 to 100. As Waner and Costenoble (1996) and others note, the formula for the standard deviation of a variable with a uniform distribution is (highest value – lowest value) / square root of 12 = 100/ 3.464 = 28.87.
oriented regional college might feel that a more modest value like 75 is appropriate.

- $\mu_a$ is the hypothesized alternative value for the mean. In this case we specify differences from the mean that are as small as 2.5 percent and as large as 10 percent. The smaller the hypothesized difference, the larger the sample size needs to be in order to be reasonably confident that a false null hypothesis will be rejected.

- Delta ($\Delta$) is a standardized measure of effect size, which equals $(\mu_a - \mu_0)/\sigma$. So, for example, when $\mu_a = 47.5$, $\Delta = (47.5 - 50)/28.87 = -.0866$. The larger the effect size is, the smaller the sample needs to be to produce statistically significant effects. This and other standardized measures can be useful when it is not otherwise clear how substantively significant differences are. If, for example, we knew that students in an experimental teaching program scored one grade level higher than their counterparts in traditional programs, such a difference might have a great deal of intuitive meaning to us. But if instead we knew that they scored 7 points higher on some standardized test, effect size measures could help us to assess how large such a difference really is.

Bibliometricians may have a clear idea of whether or not being 5 points above average is substantively important, but if not measures of effect size can help to guide the analysis and the sample selection.

- N is the sample size that is needed, given the values that have been specified for alpha, power, the null and alternative hypotheses, and the standard deviation. In this case N is estimated while the other values have been specified by the researcher. As shown later it is possible to instead fix the value of N (e.g. set the sample size at 200) and then estimate other quantities, e.g. how much power does the sample have?
The results in Table 1 and Figure 1 tell us that the sample size needs to be 1,049 or greater to be reasonably confident that a real difference from 50 (the population average) of as little as 2.5 points will be found to be statistically significant. A 5 point difference only requires a sample size of 264, and a difference as large as 10 points only requires a sample size of 68. Hence, a researcher who felt that only differences of five points or greater were worth caring about might choose to draw a much smaller sample than a researcher who felt that a difference of as little as 2.5 points was important.

If the institution wants to collect a smaller sample, it could specify a higher value for alpha (e.g. .10) or a lower value for the power. A smaller sample will increase the chances of rejecting the null when we shouldn’t or accepting the null when it is false. Conversely, if we had the resources and wanted more precise and powerful estimates, we could make alpha smaller (e.g. .01) and/or make power higher (e.g. .9). As the results in Table 2 and Figure 2 show, to meet both of these more stringent standards sample sizes would have to be almost twice as large as before.

3.3 Using power analysis when a sample has already been drawn: target means and minimum detectable differences

There may also be situations in which the sample size is already known. Perhaps the data have already been collected; or, available resources only allow the collection of a limited number of records. In such instances, bibliometricians may wish to know what the smallest possible effect and corresponding “target mean” (i.e. the mean of the sample) will have to be in order to detect statistically significant results when the null is false. This is also referred to as the “minimum detectable difference.” So, for example, suppose an institution can only afford to collect percentile data for 200 cases. Table 3 and Figure 3 show how large differences from the average score in the reference sets will have to be in order to achieve statistical significance.
The results in Table 3 and Figure 3 show that, in order for the bibliometricians to be reasonably confident that results would be statistically significant at the .01 level, the true mean for the institution would need to be more than 7 points better (42.96) than the average percentile in the reference set (50). Using the .05 level, the institution would still need to average almost 6 points better (44.25). The less demanding .1 level of significance would require a real difference of slightly over 5 points (44.91). If the institution correctly believed that it was 4 points better than average, a sample size of only 200 would not be large enough to reasonably guarantee that the institution’s mean would be found to be statistically significantly better than the average impact in the reference sets. If this is not considered acceptable the researcher may wish to choose a less demanding value for alpha or, better yet, see if there is some way for additional data to be collected.

The above sorts of calculations can also be useful even when records for all publications have been collected. For example, an institution with relatively few publications can determine how much above average it has to be in order to expect statistically significant results. A power analysis may show that, even if an institution is above average, a statistical analysis is unlikely to yield statistically significant results. Conversely, for a larger institution, a power analysis may reveal that even trivial differences from the average in the reference sets are likely to be statistically significant.

### 3.4 Other possible analyses

Similar calculations can be done for other purposes. We might want to know how large sample sizes need to be to detect differences between two institutions, or how large the sample needs to be to see whether an institution has an exceptionally large number of “excellent” publications, e.g. publication that rank among the top 10 percent of all those cited. With Stata and probably other programs, such calculations are straightforward.
At the same time several factors can make power analyses more complicated. Several analyses involving different variables may be planned, and the optimal sample sizes for each may differ. If subsample analyses are also planned (e.g. papers in certain fields only) that too needs to be taken into account when determining sample size, i.e. sample sizes for each subsample must also be large enough to achieve the study’s goals. Assumptions made in the calculations (e.g. sample standard deviations) may prove to be inaccurate, causing the original calculations of needed sample sizes to be too optimistic or pessimistic. In order to ensure that sample sizes are sufficiently large researchers may wish to choose somewhat more stringent values for power and alpha.

Finally, while we have focused on issues of interest to bibliometricians, similar concerns about samples and sample size occur in many areas of research. Our recommendations and procedures can easily be adapted for other fields of study.

4 Discussion

Bibliometricians will sometimes enjoy the luxury of having complete records for an institution. However, even in such cases the use of inferential statistics is appropriate and helpful. The observed values did not have to come out as they did. Chance factors could have increased the number of citations a paper received or else decreased them. Further, even when all records are available, a power analysis can be useful for determining what the reasonable expectations are for the study. A power analysis can indicate how difficult it is to get statistically significant results even when the citation impact of a small institution really is above average in the reference set; or conversely, how easy it is for the citation impact of a large institution to achieve statistically significant results even if the substantive differences between it and the reference sets are trivial.

In other situations, a sample will need to be drawn. Before drawing the sample, it is important to assess how large the sample needs to be to achieve its goals and provide the best
allocation of resources. If an institution feels that it is about 4 points above average, then even if it is right a sample that is too small may fail to support its beliefs. There is little point in conducting a study if it is likely doomed to failure before it even gets started. But, if an institution spends money collecting far more data than is necessary, it may have to cut back on expenditures in other important areas, e.g. data analysis. The examples and guidelines provided in this paper can help guide researchers when deciding how large their samples ought to be and what they can reasonable expect from their data once they have it.
Appendix A: Stata 13.1 code

* Table 1 and Figure 1
power onemean 50 (47.5(-2.5)40), sd(28.87) table graph(name(Fig1, replace))

* Table 2 and Figure 2
power onemean 50 (47.5(-2.5)40), sd(28.87) alpha(.01) power(.9) ///
    graph(name(Fig2, replace)) table

* Table 3 and Figure 3
power onemean 50, sd(28.87) n(200) power(.8) direction(lower) ///
    alpha (.01 .05 .1) graph(name(Fig3, replace)) table

* Additional editing to the graphs was done in the Stata Graph Editor
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Table 1. Estimated sample size for a one-sample t test

| alpha | power | N   | delta  | m0    | ma   | sd   |
|-------|-------|-----|--------|-------|------|------|
| .05   | .8    | 1049| -.0666 | 50    | 47.5 | 28.87|
| .05   | .8    | 264 | -.1732 | 50    | 45   | 28.87|
| .05   | .8    | 119 | -.2598 | 50    | 42.5 | 28.87|
| .05   | .8    | 68  | -.3464 | 50    | 40   | 28.87|
Table 2. Estimated sample size for a more stringent one-sample t test

| alpha | power | N   | delta | m0   | ma   | sd  |
|-------|-------|-----|-------|------|------|-----|
| .01   | .9    | 1988| -.0866| 50   | 47.5 | 28.87|
| .01   | .9    | 500 | -.1732| 50   | 45   | 28.87|
| .01   | .9    | 224 | -.2598| 50   | 42.5 | 28.87|
| .01   | .9    | 128 | -.3464| 50   | 40   | 28.87|
Table 3. Estimated target mean for a one-sample mean test

| alpha | power | N  | delta | m0    | ma    | sd  |
|-------|-------|----|-------|-------|-------|-----|
| .01   | .8    | 200| -.2437| 50    | 42.96 | 28.87|
| .05   | .8    | 200| -.1991| 50    | 44.25 | 28.87|
| .1    | .8    | 200| -.1764| 50    | 44.91 | 28.87|
Figure 1. Estimated sample size for a one-sample test

$t$ test

$H_0: \mu = \mu_0$ versus $H_a: \mu \neq \mu_0$

Parameters: $\alpha = .05, 1-\beta = .8, \mu_0 = 50, \sigma = 28.87$
Figure 2. Estimated sample size for a more stringent one-sample mean test

\[ t \text{ test} \]

\[ H_0: \mu = \mu_0 \text{ versus } H_a: \mu \neq \mu_0 \]

Parameters: \( \alpha = .01, 1-\beta = .9, \mu_0 = 50, \sigma = 28.87 \)
Figure 3. Estimated target mean for a one-sample mean test

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**Figure 3. Estimated target mean for a one-sample mean test**