We recently found that Gamma–Ray Burst energies and luminosities, in their comoving frame, are remarkably similar. This, coupled with the clustering of energetics once corrected for the collimation factor, suggests the possibility that all bursts, in their comoving frame, have the same peak energy $E'_p$ (of the order of a few keV) and the same energetics of the prompt emission $E'_\gamma$ (of the order of $2 \times 10^{48}$ erg). The large diversity of bursts energies is then due to the different bulk Lorentz factor $\Gamma_0$ and jet aperture angle $\theta_{\text{jet}}$. We investigated, through a population synthesis code, what are the distributions of $\Gamma_0$ and $\theta_{\text{jet}}$ compatible with the observations. Both quantities must have preferred values, with log–normal best fitting distributions and $\langle \Gamma_0 \rangle \sim 275$ and $\langle \theta_{\text{jet}} \rangle \sim 8.7^\circ$. Moreover, the peak values of the $\Gamma_0$ and $\theta_{\text{jet}}$ distributions must be related – $\theta_{\text{jet}}^2 \Gamma_0 = \text{const}$: the narrower the jet angle, the larger the bulk Lorentz factor. We predict that $\sim 6\%$ of the bursts that point to us should not show any jet break in their afterglow light curve since they have $\sin \theta_{\text{jet}} < 1/\Gamma_0$. Finally, we estimate that the local rate of GRBs is $\sim 0.3\%$ of all local SNIb/c and $\sim 2.5\%$ of local hypernovae, i.e. SNIb/c with broad absorption lines.
1. Introduction

The spectral energy correlations in GRBs are still matter of hot debate. The isotropic equivalent energy $E_{\text{iso}}$ of the prompt phase of long GRBs correlates with the rest frame peak $E_p$ of the $\nu F_\nu$ spectrum \[1\], \[3\]: $E_p \propto E_{\text{iso}}^{0.5}$. A similar correlation (obeyed also by short events – \[11\]) exists between the isotropic equivalent luminosity $L_{\text{iso}}$ and $E_p$ \[24\]: $E_p \propto L_{\text{iso}}^{0.5}$.

If GRBs emit their radiation within a jet of opening angle $\theta_{\text{jet}}$, the true energy $E_\gamma\sim E_{\text{iso}}\theta_{\text{jet}}^2$ can be estimated \[7\]. For $\sim 30$ GRBs with known $\theta_{\text{jet}}$, $E_\gamma$ is tightly correlated with $E_p$ \[3\], \[8\].

The presence of outliers of the $E_p - E_{\text{iso}}$ correlation \[3\], \[20\], \[23\] \[5\] and the presence of possible instrumental biases \[3\], \[19\], caution about the use of these correlations either for deepening into the physics of GRBs or for cosmological purposes. However, even if instrumental selection effects are present, it seems that they cannot produce the correlations we see \[3\], \[20\], \[23\].

Moreover, a correlation between $E_p$ and $L_{\text{iso}}$ is present within individual GRBs as a function of time \[3\], \[12\], \[3\] \[14\].

A new piece of information recently added to the puzzle is that the energetics in the comoving frame, emit a sample of \[22\]. The large black dot corresponds to our main assumptions, i.e. all bursts, in the comoving frame, emit $E_\gamma\sim 2 \times 10^{48}$ erg at $E_p\sim 1.5$ keV independent of their $\Gamma_0$. $E_p$ is smaller (2$\sigma$) than the mean value derived in \[15\], in order to be able to reproduce GRBs lying quite close to the $E_p \propto E_{\text{iso}}^{1/3}$ line. GRBs with different $\Gamma_0$ would lie on the $E_p\sim E_\gamma$ line, giving rise to the $E_p - E_\gamma$ relation. Then, by assuming a given aperture angle $\theta_{\text{jet}}$ we can calculate $E_{\text{iso}}$. The GRB will move to the right by the quantity \[1/(1-\cos \theta_{\text{jet}})\] if $\theta_{\text{jet}}>1/\Gamma_0$, and by the quantity $2\Gamma_0^2$ otherwise. In the latter case, the relation between $E_p$ and $E_{\text{iso}}$ becomes $E_p \propto E_{\text{iso}}^{1/3}$. This implies that region (III) of Fig. 1 is forbidden. The other forbidden regions are region (II) because this would correspond to $\theta_{\text{jet}}> 90^\circ$, and region (I) because we assume $1<\Gamma_0<8000$. All our simulated bursts will then lie on the white part of the plane. The distribution of the simulated bursts in this plane depends on the chosen distributions of $\Gamma_0$ and $\theta_{\text{jet}}$. We thus have a tool to find what are the best fitting distributions.

2. Simulation set up

Fig. 1 shows the $E_p - E_{\text{iso}}$ plane. The black points are GRBs belonging to the complete Swift sample of \[22\]. The large black dot corresponds to our main assumptions, i.e. all bursts, in the comoving frame, emit $E_\gamma\sim 2 \times 10^{48}$ erg at $E_p\sim 1.5$ keV independent of their $\Gamma_0$. $E_p$ is smaller (2$\sigma$) than the mean value derived in \[15\], in order to be able to reproduce GRBs lying quite close to the $E_p \propto E_{\text{iso}}^{1/3}$ line. GRBs with different $\Gamma_0$ would lie on the $E_p\sim E_\gamma$ line, giving rise to the $E_p - E_\gamma$ relation. Then, by assuming a given aperture angle $\theta_{\text{jet}}$ we can calculate $E_{\text{iso}}$. The GRB will move to the right by the quantity \[1/(1-\cos \theta_{\text{jet}})\] if $\theta_{\text{jet}}>1/\Gamma_0$, and by the quantity $2\Gamma_0^2$ otherwise. In the latter case, the relation between $E_p$ and $E_{\text{iso}}$ becomes $E_p \propto E_{\text{iso}}^{1/3}$. This implies that region (III) of Fig. 1 is forbidden. The other forbidden regions are region (II) because this would correspond to $\theta_{\text{jet}}> 90^\circ$, and region (I) because we assume $1<\Gamma_0<8000$. All our simulated bursts will then lie on the white part of the plane. The distribution of the simulated bursts in this plane depends on the chosen distributions of $\Gamma_0$ and $\theta_{\text{jet}}$. We thus have a tool to find what are the best fitting distributions.
The steps are: i) select a redshift from the assumed redshift distribution (that is taken from [22], which includes an evolutionary term); ii) select a $\Gamma_0$ and calculate $E_p$ and $E_\gamma$; iii) select a $\theta_{\text{jet}}$ and calculate $E_{\text{iso}}$; iv) choose a viewing angle and decide if it is pointing at us or not; v) calculate the peak flux in the appropriate band (assuming a typical Band spectrum) and decide if the burst belongs to the complete Swift sample [22] or not. Bursts in this sample have a peak flux larger than 2.6 ph cm$^{-2}$ s$^{-1}$, and almost 90% of them have a measured redshift. The steps are repeated until the number of simulated Swift bursts matches the real ones. Finally, we repeat 1,000 times each simulation to see how many times we can get a reasonable agreement with several observational constraints. First, we compare the simulated points of the complete Swift sample with the real ones in the $E_p - E_{\text{iso}}$ plane. Then we compare them also in the observed planes $E_p^{\text{obs}}$–Fluence and $E_p^{\text{obs}}$–Peak Flux (irrespective if the redshift is known or not). Finally, we compare the distribution of simulated vs real flux and fluences of the BATSE and GBM bursts (down to limiting values that are not affected by incompleteness).

### 2.1 Results

We performed several simulations considering first that both $\Gamma_0$ and $\theta_{\text{jet}}$ have no preferred values, i.e. assuming that they are distributed as power–laws, changing the corresponding slopes. None of these cases is in agreement with the data. Then we assumed a broken power law either for $\Gamma_0$ or for $\theta_{\text{jet}}$, or for both. For the latter case we do find some agreement, but the distribution of the simulated points in the $E_p - E_{\text{iso}}$ plane describes a linear correlation, instead of the observed...
$\theta_{\text{jet}}$ and $\Gamma_0$ of GRBs

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Figure 2: Simulations assuming log–normal distributions of $\theta_{\text{jet}}$ and $\Gamma_0$ and the relation $\theta_{\text{jet}}^{5/2}\Gamma_0=$constant. Left panel: simulated points and real data (black) in the $E_p–E_{\text{iso}}$ plane. Yellow points are all simulated bursts, blue points are those pointing at us, red contours are the distribution of simulated bursts (1 and $\sigma$) brighter than the peak flux limit of the Swift complete sample (i.e. 2.6 ph cm$^{-2}$ s$^{-1}$). Right panel: Simulated (contours) and real points (black squares) are compared in the $E_{p,\text{obs}}$–Fluence observational plane.

| Distrib. | sample | $\sigma$ | $\mu$ | Mode | Mean | Median |
|----------|--------|---------|-------|------|------|--------|
| $\theta_{\text{jet}}$ | ALL    | 0.916±0.001 | 1.742±0.002 | 2.5° | 8.7° | 5.7° |
|         | PO     | 0.874±0.010 | 3.308±0.013 | 12.7° | 40.0° | 27.3° |
|         | PO Swift | 0.527±0.032 | 1.410±0.043 | 3.1° | 4.7° | 4.1° |
| $\Gamma_0$ | ALL    | 1.475±0.002 | 4.525±0.002 | 11   | 274  | 92    |
|         | PO     | 1.452±0.020 | 2.837±0.025 | 2    | 49   | 17    |
|         | PO Swift | 0.975±0.060 | 5.398±0.083 | 85   | 355  | 221   |

Table 1: Parameter values ($\mu$ and $\sigma$) obtained by fitting a log–normal function to the distributions of $\Gamma_0$ and $\theta_{\text{jet}}$ (Fig. 3), for all the simulated bursts (ALL), for those pointing to us (PO) and for those pointing to us and with a peak flux larger than 2.6 ph cm$^{-2}$ s$^{-1}$ (the flux limit of the complete Swift sample) (PO Swift). For each distribution are reported the three moments: the mode, the mean and the median.

$E_p \propto E_{\text{iso}}^{0.6}$. We then tried log–normal distributions both for $\Gamma_0$ and $\theta_{\text{jet}}$. In addition we assumed that there is a relation between the average values of the two distributions. The best results are obtained with $\theta_{\text{jet}}^{5/2}\Gamma_0=$constant (Fig. 4). Note that the slope of the $E_p–E_{\text{iso}}$ correlation of bright bursts is harder than for faint ones (see the blue points in Fig. 4). But, curiously, these bright GRBs sample the distribution of the whole ensemble of bursts (yellow points) better than the fainter ones. This is because, if we improve our detector sensitivity, we preferentially see GRBs with larger opening angles. This makes them less energetic and enhances their probability to point at us. Fig. 5 shows (left panel) the distribution of $\Gamma_0$ of all simulated bursts (black), those pointing at us (blue) and those (red) that are pointing at us and have a peak flux larger than 2.6 ph cm$^{-2}$ s$^{-1}$ (i.e. the flux limit of the complete Swift sample). The green points correspond to the few GRBs of measured $\Gamma_0$ (left) or $\theta_{\text{jet}}$ (right). Tab. 4 reports the parameters of the best fitting log–normal distributions values of $\Gamma_0$ and $\theta_{\text{jet}}$ for all bursts (ALL), for those pointing at us (PO) and for those pointing at us with peak flux larger than 2.6 ph cm$^{-2}$ s$^{-1}$ (the flux limit of the complete Swift sample) (PO Swift).
\( \theta_{\text{jet}} \) and \( \Gamma_0 \) of GRBs

3. Conclusions

The crucial assumption of this study is that all bursts have the same \( E'_p = 1.5 \text{ keV} \) and \( E'_\gamma \sim 2 \times 10^{48} \text{ erg} \). Although there could be a dispersion of these values, our results still hold if the width of this dispersion is not larger than the dispersion of the observed quantities. The fact that these values are independent of \( \Gamma_0 \) suggests that the dissipation mechanism giving rise to the prompt emission is not the transformation of bulk kinetic into random energy. If our assumption is true, then the \( E_p - E_\gamma \) relation is produced by the distribution of \( \Gamma_0 \) values, and must be linear (both \( E_p \) and \( E_\gamma \) are proportional to \( \Gamma_0 \)). In turn, the \( E_p - E_{\text{iso}} \) relation results from a distribution of jet aperture angles, with the caveat that, for small values of \( \Gamma_0 \), the radiation collimation angle is \( 1/\Gamma_0 \), not \( \theta_{\text{jet}} \). These bursts will never have a jet–break in the light curve of their afterglow, and could be mistaken as outliers. In our simulations we find that these should be about 6% of the GRBs pointing at us. Another important outcome of our study is that we can calculate the fraction of all GRBs (whether aligned or misaligned) with respect to SN Ibc, as a function of redshift. Taking the recent estimates of the SN Ibc of [18], we find that, locally (i.e. up to \( z \sim 1 \)), GRBs are 0.3% of all SN Ibc.

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