Universality in few-body systems*

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Abstract Low-energy universality in atomic few-body systems as a result of a large two-body scattering length has gained a lot of attention recently. Here, I discuss recent progress in describing the three-body recombination of cold atoms in terms of a finite set of universal scaling functions and review results for the recombination length of $^{133}$Cs atoms obtained with these functions. Furthermore, I will consider the inclusion of effective range corrections and the relevance for further calculations in atomic and nuclear physics.

1 Introduction

Non-relativistic three-body systems of identical bosons with a large two-body scattering length display a rich set of universal features. In particular, if $a = \pm \infty$ the system has infinitely many three-body bound states (trimers) with an accumulation point at the scattering threshold. In the zero-range limit the binding energies of these trimers are given by

$$B_3^{(n)} = (\exp[-2\pi/s_0])^{n-n_*} \kappa_2^2/m,$$

(1)

where $\kappa_2$ is the binding wavenumber of the branch of Efimov states labeled by $n_*$ and $s_0 \approx 1.00624$ in the case of identical bosons.

Efimov who found these results pointed out that these universal features are also valid in the case of large but finite scattering length as long as it remains large compared to the range $R$ of the underlying interaction [1]. The implications of these discoveries are manifold, in particular, as they are independent of the details

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of the interaction and therefore apply to systems of nucleons but also to cold atoms which have an unnaturally large scattering length. These findings have become even more important over the last years since Efimov's results have been rephrased in terms of an effective field theory (EFT). It turns out that this EFT is the appropriate low-energy theory for non-relativistic particles with short-range interactions: It allows a model-independent calculation of observables, the inclusion of external currents is straightforward and corrections due to the finite range of the interaction can be included systematically. This framework should therefore be ideally suited to compute electroweak few-nucleon observables which are, for example, of interest in nuclear astrophysics (pd → γ3He).

The leading-order (LO) implications of this EFT, also known as Efimov physics, are interesting by itself. In nuclear physics these consequences can be seen in correlations between different observables. If, for example, the binding energy of the triton is computed with different nucleon-nucleon potentials which reproduce the same two-body observables but give different values for the neutron-deuteron scattering length, a plot of these atom-dimer scattering lengths versus the corresponding results for the triton binding energy will give an approximately linear correlation which is known as the Phillips line.

The first experimental evidence for Efimov physics has recently been presented by the Innsbruck group [2]. They carried out experiments with ultracold 133Cs atoms, using a magnetic field to control their scattering length. They observed a resonant enhancement in the three-body recombination rate at $a \approx -850 a_0$ that can be attributed to an Efimov trimer close to the three-atom threshold. The scattering-length-dependent loss rate at 10 nK can be fit well by the universal formula for zero temperature derived in [3].

In the following, I will discuss several applications of the EFT for short-range interactions relevant to the themes mentioned above. I will briefly review the theoretical basis on which this EFT is built in Sect. 2. Then I will discuss recent work on universal functions which allow to parameterize the three-body recombination of identical bosons. In Sect. 4, I will consider how to account for finite-range corrections within this framework and will give results for observables which have been calculated up to next-to-next-to-leading order (NNLO) in the EFT expansion. I will end with a short conclusion.

2 An EFT for short-range interactions

The EFT for short-range interactions is formulated in terms of the minimal set of degrees of freedom, i.e., heavy boson or fermion fields only, and is valid if the underlying potential is short-ranged and the involved momentum is smaller than the inverse range of the potential. The most general Lagrangian describing non-relativistic bosons interacting through contact interactions only is given by

$$\mathcal{L} = \psi^\dagger \left[ i \partial_t + \frac{\nabla^2}{2M} \right] \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 - \frac{D_0}{6} (\psi^\dagger \psi)^3 + \cdots,$$

where the ellipses denote interactions with more derivatives and/or more fields. With the corresponding power counting this EFT is an expansion in $R/a$, where $R$