A new analytical model for wind farm power prediction

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Abstract. In this study, a new analytical approach is presented and validated to predict wind farm power production. The new model is an extension of the recently proposed by Bastankhah and Porté-Agel for a single wake. It assumes a self-similar Gaussian shape of the velocity deficit and satisfies conservation of mass and momentum. To estimate the velocity deficit in the wake, this model needs the local wake growth rate parameter which is calculated based on the local turbulence intensity in the wind farm. The interaction of the wakes is modeled by use of the velocity deficit superposition principle. Finally, the power curve is used to estimate the power production from the wind turbines. The wind farm model is compared to large-eddy simulation (LES) data and measurments of Horns Rev wind farm for a wide range of wind directions. Reasonable agreement between the proposed analytical model, LES data and measurements is obtained. This prediction is also found to be substantially better than the one obtained with a commonly used wind farm wake model.

1. Introduction
The energy field nowadays has reached a critical situation. Due to the limited fossil-based fuel resources and their negative impact on the environment and the climate, there is a growing urge to find alternative sources of energy which are clean, sustainable and cost-effective. Renewable energies play an important role in achieving these objectives. With the increasing global interest in renewable energies, wind energy is witnessing a continuous growth at an average annual rate of approximately 25%, as stated by the Global Wind Energy Council [1].

In wind farms, turbine wake effects are responsible for significant power losses, which can be as large as 25% of the total power output [2]. Accurate wind farm power prediction is of vital importance in order to minimize the impact of wakes on the downwind turbines. Various experimental, numerical and analytical studies have been carried out to investigate turbine wake effects. Accurate wake prediction and understanding could be achieved by using numerical methods such as large-eddy simulation (LES) and cutting edge experimental techniques. Although both experimental and numerical approaches provide accurate results, analytical models are preferred to use for wind farm optimization purposes due to their simplicity and low computational cost [3].

Several wind turbine wake models have been proposed to predict velocity deficit inside wind farms [4-8]. One of the most common wake models, which has been extensively used in the literature (e.g., [9]) as well as in commercial softwares (e.g., [10-14]), is the one proposed by Jensen [5]. It considers a top-hat shape for the normalized velocity deficit and states:
where \( U_\infty \) is the undisturbed velocity, \( U_w \) the wake velocity, \( C_T \) the thrust coefficient of the turbine, \( k_{wake} \) the wake spreading parameter, \( d_0 \) the wind turbine diameter and \( x \) the distance behind the turbine. It should be mentioned that this model was derived using only mass conservation \[8\].

Frandsen et al. \[7\] later on proposed the following equation considering a top-hat shape for the velocity deficit and applying the conservation of mass and momentum to a control volume around the turbine:

\[
\frac{\Delta U}{U_\infty} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{C_T}{A_0} \frac{A_w}{A_0}} \right),
\]

(2)

where \( A_0 \) is the area swept by the wind turbine blades and \( A_w \) is the cross-sectional area of the wake. Although these models are commonly used in the literature and commercial softwares, they tend to overestimate power prediction in the full-wake conditions and underestimate it in the partial-wake conditions due to the top-hat velocity deficit assumption.

Recently Bastankhah and Porté-Agel showed that a Gaussian wake model can provide substantially better results in the full-wake and partial-wake conditions \[8\]. They applied mass and momentum conservation to a control volume around one turbine, assuming a self-similar Gaussian profile for the velocity deficit, and derived the following expression:

\[
\frac{\Delta U}{U_\infty} = \left( 1 - \sqrt{\frac{C_T}{8(k^*x/d_0 + \varepsilon)^2}} \right) \times \exp \left( -\frac{1}{2(k^*x/d_0 + \varepsilon)^2} \left( \frac{(z - z_h)^2}{d_0^2} + \frac{(y/d_0)^2}{d_0^2} \right) \right),
\]

(3)

where \( x, y, \) and \( z \) are streamwise, spanwise and vertical coordinates, respectively, and \( z_h \) is the hub height level. \( k^* \) denotes to the wake growth rate which is a function of thrust coefficient and local turbulence intensity \[8\]. Bastankhah and Porté-Agel also proposed the following expression for \( \varepsilon \):

\[
\varepsilon = 0.2 \sqrt{\beta},
\]

(4)

where \( \beta \) is a function of \( C_T \) which is defined as:

\[
\beta = \frac{1}{2} \left( 1 + \sqrt{1 - C_T} \right).
\]

(5)

Inside a wind farm, depending on the direction of the wind, several wind turbines operate in the wakes of many up-wind turbines. To this effect, wind farm models predict wake effects using single wake models and apply superposition principles to deal with multiple wakes. Lissaman \[4\] used an analogy between the expansion of wind turbine wakes and the dispersion of point-source pollutants (e.g., from smoke stacks) in the atmospheric boundary layer to propose a model for the cumulative velocity deficit based on linear superposition of velocity deficits. The model states:

\[
U_i = U_\infty - \sum_k (U_\infty - U_{ki}),
\]

(6)
where $U_i$ is the velocity at turbine $i$ and $U_{ki}$ is the wake velocity of turbine $k$ at turbine $i$ considering only those turbines whose wakes interact with turbine $i$. Later, Katic et al. [5] used the superposition of energy deficits instead of velocity deficits, to take into account the interaction of multiple wakes as follows:

$$U_i = U_{\infty} - \sqrt{\sum_k (U_{\infty} - U_{ki})^2}, \quad (7)$$

they assumed that the kinetic energy deficit of a multiple wake is equivalent to the sum of the energy deficits for individual wake at the down-wind position. Voutsinas et al. [6] followed the same approach as Katic et al. [5], but in order to estimate the energy deficit of each wake, they considered the difference between the inflow velocity at the turbine and the wake velocity as follows:

$$U_i = U_{\infty} - \sqrt{\sum_k (U_k - U_{ki})^2}. \quad (8)$$

Several studies (e.g., [15, 16]) have shown that an enhancement of turbulence intensity occurs inside wind farms with respect to the turbulence level of the incoming wind. Moreover, Bastankhah and Porté-Agel [8] showed that wake growth rate increases as turbulence intensity increases. To these effects, we expect that the assumption of constant wake growth rate, which has been extensively used in the common analytical wind farm models, is not realistic and instead in this paper, we propose an empirical equation for the local wake growth rate which is based on the local turbulence intensity. There have been a few research studies to model added turbulence intensity inside the wind farm [17-19]. These models usually rely on the thrust coefficient of wind turbines and ambient turbulence intensity to calculate added turbulence intensity at the wind turbine hub height. Added turbulence intensity is defined as:

$$I_+ = \sqrt{I_{wake}^2 - I_0^2}, \quad (9)$$

where $I_{wake}$ is the turbulence intensity in the wake and $I_0$ the ambient turbulence intensity. Quarton and Ainslie [17] proposed the following empirical expression to predict the added turbulence intensity generated by a wind turbine:

$$I_+ = 4.8C_T^{0.7}I_0^{0.68}(x/x_n)^{-0.57}, \quad (10)$$

where $x_n$ is the length of the near-wake region which is defined as [20]:

$$x_n = \frac{\sqrt{0.214 + 0.144m(1 - \sqrt{0.134 + 0.124m})} \cdot r_0}{(1 - \sqrt{0.214 + 0.144m})\sqrt{0.134 + 0.124m}(dr/dx)} \quad (11)$$

where $m = \frac{1}{\sqrt{1-C_T}}$, $r_0 = d_0\sqrt{\frac{m+1}{2}}$ and $dr/dx$ is defined by the following expression:

$$dr/dx = \sqrt{\left(\frac{dr}{dx}\right)_a^2 + \left(\frac{dr}{dx}\right)_m^2 + \left(\frac{dr}{dx}\right)_\lambda^2}, \quad (12)$$
where \( \frac{dr}{dx} = 2.5I_0 + 0.005 \), \( \frac{dr}{dx} = \frac{(1-m)\sqrt{1.49+m}}{9.76(1+m)} \), and \( \frac{dr}{dx} = 0.012B\lambda \). \( B \) is the number of blades and \( \lambda \) is the tip speed ratio. Later, Hassan [18] suggested the following expression for the added turbulence intensity:

\[
I_+ = 5.7C_T^{0.7}I_0^{0.68}(x/x_n)^{-0.96}.
\]

Crespo and Hernandez [19], based on their numerical data, suggested the following empirical equation for the ranges of parameters, \( 5 < x/d_0 < 15 \), \( 0.07 < I_u < 0.14 \) and \( 0.1 < a < 0.4 \) where \( a \) is the induction factor:

\[
I_+ = 0.73a^{0.8325}I_0^{0.0325}(x/d)^{-0.32}.
\]

In section 2, the analytical wind farm model is presented. Then, a description of case study (Horns Rev wind farm) is presented in section 3. The results are discussed and compared with LES, observed data and a commonly used analytical wind farm wake model. in section 4. The conclusion and future research are presented in section 5.

2. Description of the new analytical wind farm model

The new wind farm model applies the superposition of the velocity deficit to the recently proposed Gaussian wake model by Bastankhah and Porté-Agel [8]. Next, the details of the new wind farm model and its implementation are given.

2.1. Analytical Model for the velocity deficit

For each turbine in the farm, we apply the Gaussian wake model recently developed by Bastankhah and Porté-Agel [8] (equations (3), (4) and (5)). For single wakes, this model assumes a self-similar Gaussian distribution for the velocity deficit, which conserves mass and momentum. Since in this paper wind turbines operate in a range that thrust coefficient is approximately constant (Figure 3), we assume that \( k^* \) is only dependent to local turbulence intensity. Figure 1 shows the wake growth rate behind a V-80 turbine simulated with LES for a wide range of turbulence intensities of the incoming boundary layer flow at hub height [8]. Based on aforementioned numerical data we propose the following empirical equation for the range of conditions considered in this study \( (0.065 < I_u < 0.15) \) to calculate the wake growth rate:

\[
k^* = 0.3837I + 0.003678,
\]

where \( I \) is the local turbulence intensity at the rotor hub height.

![Figure 1. Wake growth rate for V-80 turbine in boundary layer flow with different turbulence intensities at hub height](image-url)
The problem of multiple wake interactions is solved by applying a new approach of velocity deficit superposition. Previously Lissaman [4] used the velocity deficit superposition. He considered the difference between the undisturbed velocity and the wake velocity (equation (6)) which resulted in an overestimation of the velocity deficit [3]. Here, to calculate the velocity deficit, we instead propose to use the difference between the inflow velocity at the turbine and the wake velocity which is defined as:

\[
U_i = U_\infty - \sum_k (U_k - U_{ki}).
\] (16)

As mentioned before, the single wake model conserves the mass and momentum. It is of vital importance that the velocity deficit superposition conserves mass and momentum. Lissaman [4] justified the linear superposition of the wakes using an analogy between turbine wakes and pollution plumes, whose Gaussian concentration distribution can be superimposed due to the linearity of the process. Since pollutant mass is conserved by applying superposition, in the same way superposition of velocity deficit conserves the linearized momentum deficit.

2.2. Turbulence intensity model
For the local turbulence intensity, we use a top-hat distribution with the wake diameter of \(4\sigma\), which has been derived empirically based on LES data [15]. \(\sigma\) is the standard deviation of the Gaussian-like velocity deficit and it is defined [8] as:

\[
\sigma/d_0 = k^*x/d_0 + \varepsilon.
\] (17)

The enhancement of turbulence intensity for individual turbines is calculated from equation (14). Then, using equation (9) the local turbulence intensity can be found.

Extensive numerical and experimental studies have revealed that the turbulence intensity inside a wind farm increases and quickly reaches an equilibrium after 2-3 rows [15, 21]. A previous study by Frandsen and Thøgersen [22] has also shown that only the effect of neighboring turbines is important to predict the turbulence intensity in the wake of given turbine. To this effect, for every turbine, we consider only the added turbulence intensity resulting from the closeset upstream turbine whose wake has the most significant impact. It is defined as:

\[
I_{+j} = \max \left( \frac{A_w4}{\pi d_0^2} I_{+kj} \right),
\] (18)

where \(I_{+j}\) is the added turbulence intensity at the turbine \(j\), \(A_w\) the intersection between the wake (using equation (17)) and the rotor area, and \(I_{+kj}\) the added turbulence intensity induced by the turbine \(k\) at the turbine \(j\).

2.3. Power prediction
Power generated by each turbine is predicted using the power curve, which gives the power production as a function of wind speed. Here we use the data available for Vestas V-80 wind turbines and we fit a fifth degree polynomial.

3. Case description
The Horns Rev offshore wind farm is selected as a case study, since LES predictions [15, 23] and measurements [10, 24] are available to evaluate the performance of the new analytical wind farm model. The wind farm is located in the North Sea around 15 km off the westernmost point of Denmark. It has 160 MW total rated power capacity and consists of eighty Vestas V-80 wind turbines.
within an area of about 20 km². Each turbine has a rotor diameter of \( d = 80 \text{ m} \) and a hub height of \( H_{\text{hub}} = 70 \text{ m} \) (above sea level). Figure 2 shows a schematic of the Horns Rev wind farm layout where the wind turbines are positioned in a rhomboid shape with a minimum spacing of 7 rotor diameter between two consecutive turbines within 8 columns and ten rows (approximately 7° turned counterclockwise from North-South).

![Figure 2. Layout of Horns Rev wind farm. Distances are normalized by the rotor diameter \( d = 80 \).](image)

The analytical wind farm model needs wind turbine characteristics as an input (power curve and thrust coefficient) to predict the wind farm power. Here, we use the data that was used to validate LES by Wu and Porté-Agel [23]. Figure 3 shows the previously mentioned power curve and thrust coefficient for different wind speeds. The surface roughness for the farm is considered as \( z_0 = 0.05 \text{ m} \). In this paper, the inflow wind condition is defined by a turbulence intensity of 7.7% at hub height and an average velocity of 8 m s\(^{-1}\) at the same height.

![Figure 3. Measured and simulated power curve and thrust coefficient curves with respect to different range of wind speeds for the Vestas V-80 2 MW turbine, (Source: Wu and Porté-Agel, 2014)](image)
4. Results and discussion

In this section, the new analytical wind farm model is used to simulate the turbine wakes and associated power losses in Horns Rev. The results are compared with available power measurements [10, 24] and LES data [15, 23], as well as predictions from an analytical top-hat wake model. The top-hat wake model is based on the model proposed by Katic et al [5]. In this top-hat wake model (e.g., WASP), the wake growth rate is set to 0.04, which is the recommended value for offshore applications [25].

Figure 4 shows the simulated total normalized power output from the Horns Rev wind farm obtained from the new proposed model, LES [15] and the top-hat model [5] for a wide range of wind directions (from 173° to 353°). This allows the evaluation of the model over a variety of full-wake and partial-wake conditions. In this paper, the simulated power is normalized by the power of an equivalent number of stand-alone wind turbines operating in the same incoming wind condition. As it can be seen in Figure 4, while the top-hat model under predicts the normalized power, the proposed analytical model shows a good agreement with LES. A significant power loss of approximately 30% is seen when the wind farm is exposed to wind direction angles (173°, 270° and 353°) corresponding to full-wake conditions with small streamwise distances between consecutive wind turbines. Also, several local maxima (e.g., 185° and 340°) can be distinguished which correspond to large streamwise distances between turbines.

In order to evaluate the performance of different multiple wake superposition approaches, next we compare the normalized power output simulated with LES with the one obtained with the new analytical model using both velocity deficit superposition and energy deficit superposition. Figure 5 shows the normalized power output as a function of turbine row (averaged over columns 2, 3 and 4) in the wind farm. Here, for the calculation of velocity and energy deficits, we use the difference between the inflow velocity at the turbine and the wake velocity. In the case of using the difference between the undisturbed boundary-layer velocity and the wake velocity, due to the large number of turbine rows, it leads to negative velocities. As it can be seen in the Figure 5, velocity deficit superposition shows a good agreement with LES, while energy deficit superposition overestimates the normalized power by about 15%.

Figure 4. Distribution of the normalized Horns Rev wind farm power output obtained with the new analytical model and LES for different wind directions. Red circles correspond to LES, blue circles represent new analytical model and black crosses show top-hat model.

Figure 6 presents the normalized power output for three wind sectors (i.e., 270 ±5°, ±10° and ±15°) predicted by the new analytical model and WASP, as well as the measurements [24]. From this figure it is clear that WASP tends to overestimate the velocity deficit while the proposed model shows a very good agreement with the measurements. This result highlights the fact that assuming a top-hat shape for velocity deficit can lead to significant error in both full-wake and partial-wake conditions.

As mentioned before, the wake growth rate is a function of the local turbulence intensity. Therefore, in the analytical wind farm model we should predict the local turbulence intensity in the farm. Moreover, it is of vital importance to investigate the level of turbulence intensity in order to determine the impact of fatigue loading on the turbines. Figure 7 presents the level of turbulence...
intensity at hub height in the farm simulated using the models of Quarton [17], Hassan [18] and Crespo [19] as well as the LES predictions. From this figure it is clear that Quarton and Hassan models predict turbulence intensities that are much larger than both Crespo model and LES. As shown in the figure, Crespo model, which is used in the new analytical wind farm model, shows an acceptable agreement with LES.

**Figure 5.** Comparison of the simulated power output for $\theta_{\text{wind}} = 270^\circ$ by use of energy and velocity deficit superposition with LES data

**Figure 6.** Comparison of the simulated and observed power output for three wind sectors of (a) $270 \pm 5^\circ$, (b) $270 \pm 10^\circ$ and (c) $270 \pm 15^\circ$
5. Conclusion
A new analytical model is proposed to estimate the power output of a wind farm. Individual wakes are modeled using a self-similar Gaussian distribution for the velocity deficit, which conserves mass and momentum. Wake velocity distribution is predicted by specifying local wake growth rate. In this study, we propose an empirical expression to calculate the wake growth rate based on the local turbulence intensity. The local turbulence intensity is predicted inside the farm by use of empirical models. A velocity deficit superposition is used to consider the effect of multiple wakes. The power curve is used to predict the power production from the wind turbines.

We select the Horns Rev wind farm as a case study since LES predictions and measurements are available for a wide range of inflow conditions. The comparison with the LES and observed data reveals that the obtained power prediction with the new analytical model is in reasonable agreement with the LES and experimental data. Also, it is observed that the power prediction of the new proposed model is substantially better than the one obtained by WAsP.

Future research will focus on developing an expression to calculate wake growth rate for a wider range of inflow conditions. Furthermore, the new analytical model can be used to design alternative layouts so as to maximize wind farm power output while minimizing the fatigue accumulation on the turbines.

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