Democratic Seesaw Mass Matrix Model
and New Physics

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Abstract

A seesaw mass matrix model is reviewed as a unification model of quark and lepton mass matrices. The model can understand why top-quark mass $m_t$ is so singularly enhanced compared with other quark masses, especially, why $m_t \gg m_b$ in contrast to $m_u \sim m_d$, and why only top-quark mass is of the order of the electroweak scale $\Lambda_W$, i.e., $m_t \sim O(\Lambda_W)$. The model predicts the fourth up-quark $t'$ with a mass $m_{t'} \sim O(m_{W_R})$, and an abnormal structure of the right-handed up-quark mixing matrix $U_{R^u}$. Possible new physics is discussed.

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1. Why seesaw mass matrix?

The seesaw mechanism

\[ M_f \simeq m_L M_F^{-1} m_R . \quad (1.1) \]

was first proposed [1] in order to answer the question why neutrino masses are so invisibly small. And then, in order to explain why quark masses are so small compared with the electroweak scale \( \Lambda_W \), the seesaw mechanism was applied to the quarks [2]. However, the observation [3] of the top-quark with the large mass \( m_t \sim O(\Lambda_W) \) brought a new situation to the seesaw mass matrix model: Why is the top quark mass \( m_t \) singularly large compared with \( m_b \) in the third family in contrast to \( m_u \sim m_d \) in the first family? Why is the top-quark mass \( m_t \) of the order of \( \Lambda_W \)? It seems that the observation of the large top-quark mass rules out the application of the seesaw mass matrix model to the quarks.

In the present talk, I would like to point out that the largeness of \( m_t \), especially, \( m_t \sim O(\Lambda_W) \), is rather preferable to the seesaw mass matrix model, and as an example, I will review a specific model of a seesaw type mass matrix model, “democratic seesaw mass matrix model” [4,5]. The most of the works were done in the collaboration with H. Fusaoka. I would like to thank him for his energetic collaboration.

The basic idea is as follows. We consider an \( SU(2)_L \times SU(2)_R \times U(1)_Y \) gauge model. We assume vector-like fermions \( F_i \) in addition to the three-family quarks and leptons \( f_i \) (\( f = u, d, \nu, e; i = 1, 2, 3 \)). These fermions and Higgs scalars belong to

\[
\begin{align*}
 f_L &= (2, 1) , \quad F_L = (1, 1) , \quad \phi_L = (2, 1) , \\
 f_R &= (1, 2) , \quad F_R = (1, 1) , \quad \phi_R = (1, 2) ,
\end{align*}
\]

of \( SU(2)_L \times SU(2)_R \). Then, the mass matrix for \( (f, F) \) is given by

\[
 M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z \\ \kappa Z & \lambda O_f \end{pmatrix} . \quad (1.3)
\]

For simplicity, we have taken

\[
 m_L = m_R / \kappa = m_0 Z . \quad (1.4)
\]

We assume that the matrix \( Z \) is universal for \( f = u, d, \nu, e \). Further, we assume that the heavy fermion mass matrix \( M_F \) has a form [(unit matrix) + (rank-one matrix)]:

\[
 M_F = \lambda m_0 O_f = \lambda m_0 (1 + 3 b_f X) , \quad (1.5)
\]
where $b_f$ is an $f$-dependent complex parameter, $1$ is the $3 \times 3$ unit matrix, and $X$ is a rank-one matrix normalized by $\text{Tr} M_F = 0$ at $b_f = -1/3$. Then, for $b_f = -1/3$, we will find [4,5,6] the following mass spectrum,

\begin{align}
    &m_1, m_2 \sim \frac{x}{X} m_0, \\
    &m_3 \simeq \frac{1}{\sqrt{3}} m_0 \sim O(m_L), \\
    &m_4 \simeq \frac{1}{\sqrt{3}} \kappa m_0 \sim O(m_R), \\
    &m_5, m_6 \sim \lambda m_0 \sim O(M_F),
\end{align}

independently of the datails of the matrix $Z (\sim O(1))$. (Also see Fig. 1 later.) Therefore, if we assume that $\text{Tr} M_F = 0$ for up-quark sector, we can naturally understand why only the top quark has a mass of the order of the electroweak scale $\Lambda_W \sim O(m_L)$. This point will also be emphasized by T. Satou [7] in this session from more general study of the seesaw quark mass matrix.

2. Why democratic $M_F$?

So far, we have never assumed that the rank-one matrix $X$ is a democratic type. Next, I would like to talk about why our model is called “democratic” [8].

We know that we can always take the rank-one matrix $X$ as a democratic type

\begin{equation}
    X = \frac{1}{3} \left( \begin{array}{ccc}
        1 & 1 & 1 \\
        1 & 1 & 1 \\
        1 & 1 & 1
    \end{array} \right),
\end{equation}

without losing generality. The naming “democratic” for the model is motivated by the following phenomenological success [4] of taking $M_F$ “democratic”: if we assume that the matrix $Z$ is given by a diagonal matrix

\begin{equation}
    Z = \left( \begin{array}{ccc}
        z_1 & 0 & 0 \\
        0 & z_2 & 0 \\
        0 & 0 & z_3
    \end{array} \right) \propto \left( \begin{array}{ccc}
        \sqrt{m_e} & 0 & 0 \\
        0 & \sqrt{m_\mu} & 0 \\
        0 & 0 & \sqrt{m_\tau}
    \end{array} \right),
\end{equation}

we can obtain reasonable values of the quark masses $m_q$ and Cabibbo-Kobayashi-Maskawa (CKM) [9] matrix $V$. For example, we can obtain the successful relation [10]

\begin{equation}
    \frac{m_u}{m_c} \simeq \frac{3}{4} \frac{m_e}{m_\mu},
\end{equation}
for $b_u \simeq -1/3$. So, hereafter, we call the seesaw mass matrix model with (2.1) and (2.2) the “democratic seesaw mass matrix model”.

Such a structure of the matrix $Z$ was suggested from the following phenomenology: Experimentally well-satisfied charged lepton mass formula [11]

$$m_\tau + m_\mu + m_e = \frac{2}{3} \left( \sqrt{m_\tau} + \sqrt{m_\mu} + \sqrt{m_e} \right)^2 \quad (2.4)$$

can be derived from the bi-liner form

$$M_e \propto Z \cdot 1 \cdot Z , \quad (2.5)$$

where

$$Z = \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{pmatrix} , \quad z_i \equiv x_i + x_0 , \quad x_1 + x_2 + x_3 = 0 , \quad x_0^2 = (x_1^2 + x_2^2 + x_3^2)/3 . \quad (2.6)$$

The form (2.5) suggests a seesaw mass matrix model with a U(3)-family nonet Higgs boson [12]. However, in the present talk, I will skip this topic because I have no time sufficient to discuss it.

3. **Phenomenology of** $m_q^i$ ($q = u, d$) **and** $V$

We take the rank-one matrix $X$ as the democratic form (2.1). Then, the successful results for $m_q^i$ and $V$ are obtained from the following assumptions and inputs.

[Assumption I]: The matrix $Z$ takes a diagonal form $Z = \text{diag}(z_1, z_2, z_3)$, when $X$ is in a democratic basis (2.1).

[Assumption II]: The parameter $b_f$ takes $b_e = 0$, in the charged lepton sector.

The assumption II was put in order to fix the parameters $z_i$ as a trial. Then, the parameters $z_i$ are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} . \quad (3.1)$$

In Fig. 1, we show the behavior of mass spectra $m_q^i$ ($i = 1, 2, \cdots, 6$) versus the parameter $b_f$. As seen in Fig. 1, the third fermion mass $m_3^f$ is sharply enhanced at $b_f = -1/3$. Also, note that the masses $m_2^f$ and $m_3^f$ (masses $m_1^f$ and $m_2^f$) degenerate at $b_f = -1/2$ and $b_f = -1$, and the degeneration disappears for the case of arg$b_f = 0$. 


Fig. 1. Masses $m_i (i = 1, 2, \cdots, 6)$ versus $b_f$ for the case $\kappa = 10$ and $\kappa/\lambda = 0.02$. The solid and broken lines represent the cases $\arg b_f = 0$ and $\arg b_f = 18^\circ$, respectively. The figure was quoted from Ref. [5].

In addition to (2.3), we can obtain many interesting relations [4,5]:

$$\frac{m_c}{m_b} \simeq 4 \frac{m_{\mu}}{m_{\tau}}, \quad \frac{m_d m_s}{m_b^2} \simeq 4 \frac{m_{e} m_{\mu}}{m_{\tau}^2}, \quad \frac{m_u}{m_d} \simeq \frac{3 m_s}{m_c}, \quad (3.2)$$

around $b_u \sim -1/3$, and $b_d \sim -e^{i\beta_d} (1 \ll \beta_d^2 \neq 0)$. Therefore, we put the following assumption.

[Assumption III]: We fix the values of $|b_f|$ for the quark-sector as

$$b_u = -\frac{1}{3}, \quad b_d = -e^{i\beta_d} (1 \ll \beta_d^2 \neq 0). \quad (3.3)$$

The former means the ansatz of “the maximal top-quark-mass enhancement”, but, at present, there is not good naming for the latter.

For phenomenological fitting, we have used the following inputs: $\kappa/\lambda = 0.02$ from the observed ratio $m_c/m_t$; $\beta_d = 18^\circ$ from the observed ratio $m_d/m_s$. Then we obtain reasonable quark mass ratios and CKM matrix parameters [4]:

$$|V_{us}| = 0.220, \quad |V_{cb}| = 0.0598, \quad |V_{ub}| = 0.00330, \quad |V_{td}| = 0.0155. \quad (3.4)$$

(The value of $|V_{cb}|$ is somewhat larger than the observed value [13] $|V_{cb}|_{\text{exp}} = 0.041 \pm$
0.003. For the improvement of the numerical value, see Ref. [5].

4. Application to neutrino mass matrix

As seen in Fig. 1, the choice of \( b_f \approx -\frac{1}{2} \) gives

\[
m_1 \ll m_2 \approx m_3 ,
\]

\[
U_L \approx \left( \begin{array}{ccc}
1 & \frac{1}{\sqrt{2}} \left( \sqrt{m_e} m_\mu - \sqrt{m_e} m_\tau \right) & \frac{1}{\sqrt{2}} \left( \sqrt{m_e} m_\mu + \sqrt{m_e} m_\tau \right) \\
-\sqrt{m_e/m_\mu} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\sqrt{m_e/m_\tau} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array} \right) .
\]

(4.1)

On the other hand, the atmospheric neutrino data (Kamiokande) [14] have suggested a large neutrino mixing \( \sin^2 2\theta_{\mu\tau} \approx 1 \) with \( \Delta m^2_{\tau\mu} \approx 1.6 \times 10^{-2} \text{ eV}^2 \), and the solar neutrino data (with MSW effects) [15] have suggested a neutrino mixing \( \sin^2 2\theta_{e\mu} \approx 0.007 \) with \( \Delta m^2_{\mu e} \approx 6 \times 10^{-6} \text{ eV}^2 \). The results (4.1) and (4.2) are preferable to these data.

In order to make the model more explicit, we put the following assumption: We assume that \( \nu_R \) has a Majorana mass of the order of \( \xi m_0 (\xi \gg \lambda \gg \kappa \gg 1) \) in addition to the heavy neutrino masses \( M_N \sim O(\lambda m_0) \). Then, for example, for \( b_\nu = -0.41 \), we obtain

\[
\sin^2 2\theta_{\mu\tau} \approx 0.58 , \quad \Delta m^2_{\tau\mu} \approx 1.0 \times 10^{-2} \text{ eV}^2 ,
\]

\[
\sin^2 2\theta_{e\mu} \approx 0.0061 , \quad \Delta m^2_{\mu e} \approx 6 \times 10^{-6} \text{ eV}^2 ,
\]

with \( \xi m_0 = 1.9 \times 10^9 \text{ GeV} \). More details have been given in Ref. [16].

5. Abnormal Structure of \( U_R^d \)

In the down-quark sector, where the seesaw expression \( M_f \approx m_L M_F^{-1} m_R \) is valid, the mixing matrices \( U_L^d \) and \( U_R^d \) are given by

\[
U_L^d = \left( \begin{array}{cc}
A_d & \frac{1}{\lambda} C_d \\
\frac{1}{\lambda} C_d' & B_d \end{array} \right) , \quad U_R^d \approx \left( \begin{array}{cc}
A_d^* & \frac{\kappa}{\lambda} C_d^* \\
\frac{\kappa}{\lambda} C_d'^* & B_d \end{array} \right) .
\]

(5.1)

On the contrary, in the up-quark sector, where the seesaw expression is not valid
any longer, the mixing matrices $U^u_L$ and $U^u_R$ are given by

$$U^u_L = \begin{pmatrix} +0.9994 & -0.0349 & -0.0084 & -0.0247 \frac{1}{\lambda} & +6 \times 10^{-5} \frac{1}{\lambda} & +4 \times 10^{-6} \frac{1}{\lambda} \\ +0.0319 & +0.9709 & -0.2373 & -0.2051 \frac{1}{\lambda} & -0.4346 \frac{1}{\lambda} & +0.0259 \frac{1}{\lambda} \\ +0.0165 & +0.2369 & +0.9714 & +0.8990 \frac{1}{\lambda} & +0.8431 \frac{1}{\lambda} & -0.0444 \frac{1}{\lambda} \\ +0.0934 \frac{1}{\lambda} & +0.1114 \frac{1}{\lambda} & -1.0365 \frac{1}{\lambda} & +0.5774 & +0.5774 & +0.5772 \\ -0.0118 \frac{1}{\lambda} & +0.1649 \frac{1}{\lambda} & +0.0209 \frac{1}{\lambda} & -0.7176 & +0.6961 & +0.0215 \\ -0.0064 \frac{1}{\lambda} & -0.1011 \frac{1}{\lambda} & +0.7927 \frac{1}{\lambda} & -0.3894 & -0.4267 & +0.8163 \end{pmatrix} ,$$

(5.2)

$$U^u_R = \begin{pmatrix} +0.9994 & -0.0349 & -0.0084 & -0.0247 \frac{\xi}{\lambda} & +6 \times 10^{-5} \frac{\xi}{\lambda} & +4 \times 10^{-6} \frac{\xi}{\lambda} \\ +0.0319 & +0.9709 & -0.2373 & -0.2051 \frac{\xi}{\lambda} & -0.4346 \frac{\xi}{\lambda} & +0.0259 \frac{\xi}{\lambda} \\ +0.0256 \frac{\xi}{\lambda} & +0.3459 \frac{\xi}{\lambda} & -0.0747 \frac{\xi}{\lambda} & +0.5773 & +0.5773 & +0.5774 \\ +0.0165 & +0.2369 & +0.9713 & +0.3274 \frac{\xi}{\lambda} & +0.2716 \frac{\xi}{\lambda} & -0.6160 \frac{\xi}{\lambda} \\ -0.0118 \frac{\xi}{\lambda} & +0.1649 \frac{\xi}{\lambda} & +0.0209 \frac{\xi}{\lambda} & -0.7176 & +0.6961 & +0.0215 \\ -0.0064 \frac{\xi}{\lambda} & -0.1011 \frac{\xi}{\lambda} & +0.7927 \frac{\xi}{\lambda} & -0.3894 & -0.4267 & +0.8161 \end{pmatrix} .$$

(5.3)

Note that the right-handed up-quark mixing matrix $U^u_R$ has a peculiar structure as if third and fourth rows of $U^u_R$ are exchanged in contrast to $U^u_L$.

Why does such an abnormal structure appear in $U^u_R$? In order to see this, let us change the heavy fermion basis from the “democratic basis” to the “diagonal basis”:

$$M_F \rightarrow \tilde{M}_F \equiv AM_F A^{-1} = m_0 \lambda \begin{pmatrix} 1 + 3b_f & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

(5.4)

Then, the Hermitian matrices $H_L \equiv \tilde{M} \tilde{M}^\dagger$ and $H_R \equiv \tilde{M}^\dagger \tilde{M}$ take the following forms:

$$H_L = \tilde{M} \tilde{M}^\dagger = m_0^2 \begin{pmatrix} \tilde{Z}^T \tilde{Z} & \lambda \tilde{Z}^T \tilde{O}_u \\ \lambda \tilde{O}_u \tilde{Z} & \lambda^2 \tilde{O}_u^2 + \kappa^2 \tilde{Z} \tilde{Z}^T \end{pmatrix}$$

$$H_R = \tilde{M}^\dagger \tilde{M} = m_0^2 \begin{pmatrix} \kappa^2 \tilde{Z}^T \tilde{Z} & \kappa \lambda \tilde{Z}^T \tilde{O}_u \\ \kappa \lambda \tilde{O}_u \tilde{Z} & \lambda^2 \tilde{O}_u^2 + \tilde{Z} \tilde{Z}^T \end{pmatrix}$$
\[
\begin{pmatrix}
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda
\end{pmatrix}
\times
\begin{pmatrix}
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda
\end{pmatrix}
\times
\begin{pmatrix}
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda \\
\lambda & \lambda & \lambda
\end{pmatrix}
\]

where \( \bar{Z} = AZ \) and \(* \sim O(1)\). The result\((5.5)\) means that the top quark \( t \equiv u_3 \) and the fourth up-quark \( t' \equiv u_4 \) consist of the following components

\[
t \equiv u_3 \simeq (u_{3L}, U_{1R}) \,,
\]

\[
t' \equiv u_4 \simeq (U_{1L}, u_{3R}) \,,
\]

although \( u \simeq (u_{1L}, u_{1R}) \), \( c \simeq (u_{2L}, u_{2R}) \), \( u_5 \simeq (U_{2L}, U_{2R}) \) and \( u_6 \simeq (U_{3L}, U_{3R}) \). Therefore, we can expect a single \( t' \) production through the exchange of the right-handed weak boson \( W_R \) as we state later.

6. New physics from DSMM

Since we want to observe new effects from the present model, we take \( \kappa = 10 \) tentatively. Then, we can expect \( m_{t'} \simeq \kappa m_t \sim \) a few TeV. The single \( t' \) production may be observed through the exchange of \( W_R \) as \( d + u \rightarrow t' + d \), with \( |V_{t'd}^{R}| = 0.0206 \) and \( |V_{ud}^{R}| = 0.976 \). For example, we will observe the production \( p + p \rightarrow t' + X \) at LHC.

On the other hand, in the present model, FCNC effects appear proportionally to the factor \([17]\)

\[
\xi^I = U_{fF}U_{fF}^\dagger \,, \text{ where } U = \begin{pmatrix}
U_{ff} & U_{fF} \\
U_{Ff} & U_{FF}
\end{pmatrix} \,.
\]

Note that the FCNC effects appear visibly in the modes related to top-quark, because the large elements are only \( (\xi^u_R)_{tc} = -0.00709 \) and \( (\xi^u_R)_{tu} = -0.000284 \), and the other elements are harmlessly small, e.g., \( (\xi^u_R)_{cu} = \kappa^2(\xi^u_L)_{cu} = 2.01 \times 10^{-6} \), \( |(\xi^d_R)_{ds}| = \kappa^2|(\xi^d_L)_{ds}| = 4.03 \times 10^{-8} \), and so on. For example, we may expect the single top-quark production \( e^- + p \rightarrow e^- + t + X \) at HERA. Unfortunately, the values \( (\xi^u_L)_{tu} = -8.85 \times 10^{-8} \) and \( (\xi^u_R)_{tu} = -2.84 \times 10^{-4} \) lead to an invisibly small value of the cross section \( \sigma(e^- + p \rightarrow e^- + t + X) \sim 10^{-8} \) pb. Only possibility of
the observation will be at a future TeV collider: for example, $e^- + e^+ \rightarrow t + \bar{t}$ at JLC:

\[
\begin{align*}
\sigma &= 6.0 \times 10^{-7} \text{ pb at } \sqrt{s} = 0.2 \text{ TeV} , \\
\sigma &= 3.1 \times 10^{-5} \text{ pb at } \sqrt{s} = 2m_t = 0.36 \text{ TeV} , \\
\sigma &= 1.1 \times 10^{-4} \text{ pb at } \sqrt{s} = 0.5 \text{ TeV} , \\
\sigma &= 7.5 \times 10^{-4} \text{ pb at } \sqrt{s} = 0.7 \text{ TeV} ,
\end{align*}
\]

\[\text{(6.2)}\]

where $\sigma = \sigma(t\bar{t}) + \sigma(c\bar{t})$.

7. Summary

(i) Seesaw Mass Matrix with $M_F = [(\text{unit matrix}) + (\text{rank-one matrix})]$ can naturally understand the observed facts $m_t \gg m_b$ in contrast to $m_u \sim m_d$, and $m_t \sim \Lambda_W$.

(ii) Democratic seesaw mass matrix model with the input $b_e = 0$ can give reasonable quark mass ratios and CKM matrix by taking $b_u = -1/3$ and $b_d = -e^{i18^\circ}$, and a large neutrino mixing $\nu_\mu - \nu_\tau$ by taking $b_\nu \simeq -1/2$.

However, at present, we must take ad hoc parameter values $b_u = -1/3$, $b_\nu \simeq -1/2$, $b_d \simeq -1$, $b_e = 0$. I do not know whether there is some regularity among the values of $b_f$ or not, and what is the meaning of the parameter $b_f$.

(iii) The model will provide new physics in abundance: (a) $m_{t'} \sim$ a few TeV: we may expect a fourth up-quark production. (b) Abnormal structure of $U_R^h$: we may expect a single top-quark production.

However, whether these effects are visible or not in the near future depends on the value of $\kappa$ although we tentatively take $\kappa = 10$ at the present study. If $\kappa \simeq 10$, these effects cannot observe until starting of JLC. Rather, there is a possibility that the effects due to the abnormal structure of $U_R^h$ are sensitive to the $K^0-\bar{K}^0$ mixing which was pointed by T. Kurimoto [18]. However, since our right-handed current structure is different from the conventional $SU(2)_L \times SU(2)_R$ models, more careful study will be required.

(iv) Present model is still a semi-phenomenological model, so that an embedding of the present model into a field-theoretical unification scenario is hoped.

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Fig. 1