Analytical Method for Magnetic Field Calculation of Induction Heater for Heat Supply Systems

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Abstract. This paper presents an analysis of the need and technical feasibility of using electric heaters for heat supply systems. The use of induction heating units in the design of individual heating systems is proposed. Electromagnetic systems for the conversion of electrical energy into heat are considered. The heater is made in the form of coaxial cylinders using ferromagnetic cores. The analysis is based on magnetic equivalent circuits. The ways of optimization of the considered devices used for heating liquids in multi-purpose water heaters are formulated.

1. Introduction
The general trend in the national economy, which is forecasted in the new economic conditions, is a significant reduction in electricity consumption in industrial sectors. However, it is necessary to note the growth of electricity consumption in the domestic sector, agriculture and passenger transport. Further development of the world economy implies careful and rational use of not only electricity, but also other fuel and energy resources. One can expect a change in the entire fuel and energy balance from resource extraction to resource saving and the introduction of new energy saving technologies. From this point of view, the creation of modern electro-technological water-heating equipment for use not only in the traditional fields of industry, agriculture, but also in the thermal power industry to solve the problems of decentralized and reservation of heat supply systems becomes urgent. Therefore, this paper provides an applied analysis of the needs and technical feasibility of using electric heaters, primarily in the application to the heating supply. It is necessary to correctly substantiate the technical requirements for new water heating systems, which can be used for electric heating of premises and to compare these requirements with the technical parameters of existing water heating equipment.

In the analysis of this problem it is necessary to take into account the development of modern decentralized systems of local heat supply using electricity, natural gas and fuel oil. However, in the current years and in the near future, centralized heat supply systems will provide most of the consumers of large cities of Siberia. This objective reality, the ignorance of which can lead to fundamental errors in the assessment of new life support systems under development. Economic calculations based only on the use of centralized heat sources from existing city heating systems show the efficiency of traditional heat sources. But they do not take into account the changes in pricing policy, environmental, comfort and resource-saving factors, which can be technically implemented in the creation of individual heating systems.

As an example, it is possible to consider the option of using electricity to produce heat for the communal needs of the rural population. This is especially important for settlements with significant distance from fuel sources, from each other, from heat supply centers, in mountainous areas and permafrost regions. The heat demand of these consumers is met by small boilers (10-15%), furnaces
and apartment heat generators (85-90%). About 80% of small boiler-houses with heat output up to 3 Gcal/h have obsolete equipment and low efficiency of burning fossil fuel with efficiency not exceeding 10–12%. It leads to the consumption of coals in 2-2.5 times more than for the production of the same amount of heat by thermal power plants and large boiler-houses. Application of electric heating in these cases will allow saving up to 50% of the mass of energy resources.

As it follows from the analysis of the current state of induction heating installations in recent years, new electrotechnological devices of induction type have been created, which made it possible to carry out heating liquids and gases with great efficiency [1]. In spite of different design versions of the devices under consideration, from the electromagnetic point of view these devices are generalized by a single design scheme and, consequently, by a magnetic equivalent scheme. It should be noted that the generalized constructive scheme and the generalized electromagnetic equivalent circuit of induction systems with coaxial cylinders include three-element induction systems "load – inductor – electromagnetic shield" [2]. Multi-element systems of induction heating, having much in common differ principles of optimization: at creation of systems with shielding elements it is necessary to minimize the energy which is allocated in screens. When considering new induction systems with coaxial cylinders, it is necessary to optimize the power allocation, as this factor is the target in this case. It is necessary to analyze the energy characteristics of induction systems as a whole by operational efficiency and power factor.

2. Analysis of electromagnetic relationships in an induction heating installation

The creation of methods for calculating the new induction heating system with coaxial cylinders is based on the theory of induction heating and is its development. Structurally, the new devices are similar to transformers [3]. The inductor is the primary winding and is placed on the magnetic circuit. Secondary windings are made of short-circuited cylinders that cover the inductor from the outside. From the electrical point of view, the secondary windings in the form of coaxial cylinders are connected in parallel and form a layered system, which plays the role of heaters for non-conductive materials such as liquids and gases. The interaction of electromagnetic, thermal and hydrodynamic parameters determines the operational efficiency of the structure as a whole. The possibility of performing a secondary winding in the form of two or more short circuits in the form of coaxial cylinders does not allow the full use of known methods of analysis and calculation from the theory of induction heating. The objectives of this paper include the analysis of a new electromagnetic system and the creation of an affordable analytical method for calculating the induction systems that can significantly reduce the specific surface heating power. It will allow to carry out interconnected optimization of electromagnetic and thermechanical characteristics of heaters.

3. Magnetic equivalent circuit of induction system with coaxial cylinders

The possible variety of technical solutions for flow induction heaters with coaxial cylinders is generalized by a magnetic system including magnetic cores and electrically conductive coaxial cylinders. The design scheme of the generalized system is shown in figure 1 [4].

In the theory of induction heating, there are two approaches to the mathematical description of such systems [5]. The first is based on the known physical laws-Faraday's law (the law of electromagnetic induction) and Joule-Lenz's law (the conversion of electrical energy into heat), which have the form: in differential form

\[ e = -\frac{d\Phi}{d\tau}, \]

\[ q_v = \frac{E^2}{\rho}, \]

in integral form

\[ U = 4,44 \cdot f \cdot B \cdot S, \]

\[ P = U^2 / R. \]

The following notations are used in these equations: \( e \) – electromotive force; \( \Phi \) – magnetic flux, \( \tau \) – time; \( q_v \) – specific volume power; \( E \) – electric field intensity; \( \rho \) – specific electrical resistance; \( U \) –
voltage; \( f \) – frequency; \( B \) – magnetic induction; \( S \) – surface area through which the magnetic flux passes; \( P \) – power; \( R \) – an active conductor resistance.

\[
d_{1}^{M} \quad d_{12}^{n} \quad d_{21}^{n} \quad d_{22}^{n}
\]

\[h_{1}\]

**Figure 1.** Generalized structural scheme of the heating system: 1 – magnetic core; 2 – inductor; 3 – coaxial cylinders.

This approach will be used to analyze the electromagnetic connections between the individual elements of the system: the magnetic core, the inductor and the coaxial cylinders. On the basis of the theory of electric circuits with the use of equivalent circuits, the electric parameters of the heating system with coaxial cylinders are estimated taking into account the edge effect [6, 7].

The second approach is based on Maxwell's equations, which characterize the local parameters of the electromagnetic field:

\[
\text{rot}\vec{H} = \dot{E} / \rho + \frac{\partial \vec{D}}{\partial t} ;
\]

\[
\dot{B} = \mu_{0} \vec{H} ;
\]

\[
\text{div}\vec{B} = 0 ;
\]

\[
\text{rot}\dot{\vec{E}} = -\frac{\partial \vec{B}}{\partial t} ;
\]

\[
\vec{D} = \varepsilon \varepsilon_{0} \vec{E} ;
\]

\[
\text{div}\dot{\vec{E}} = \sigma ,
\]

where \( \vec{H} \) – magnetic field intensity; \( \vec{D} \) – electric induction; \( \mu \) – magnetic permeability; \( \varepsilon \) – dielectric permeability; \( \mu_{0} \) and \( \varepsilon_{0} \) – magnetic and electric constants; \( \sigma \) – density of electric charge.

This approach allows the analysis of processes inside bodies that are in an electromagnetic field, and will be used by us to determine the own resistances of the elements making the induction system.

For the preliminary analysis it was accepted that the considered system has a large length in height and the model can be consider the ratio of parameters per unit of its length.

The magnetic equivalent circuit of the heating system is provided on figure 2.

According to the law of electromagnetic induction in each coaxial cylinder, forming an independent closed contour, electromotive force is induced \( e_{n} \). Its value is determined by the rate of change in the flux linkage of a particular contour \( \psi_{n} \), i.e.

\[
e_{n} = -d\psi_{n} / d\tau ,
\]

where \( n \) – number of magnetic flux distribution paths.

From the analysis of figure 2 it follows that the flux linkage outside of the inductor \( \psi_{0} \) is divided into

\[
\psi_{1} = \Phi_{1}, \psi_{2} = \Phi_{2}, ..., \psi_{n} = \Phi_{n}
\]

therefore

\[
\psi_{0} = \Phi_{1} + \Phi_{2} + ... + \Phi_{n} = \sum_{i=1}^{n} \Phi_{i}
\]

It was taken into account that the flux linkage of each cylinder is created not only by the magnetic flux in the specific gap \( \Phi_{0}, \Phi_{12}, ..., \Phi_{(n-1)n} \), but also by the own magnetic flux of each cylinder \( \Phi_{nc} \) at the course of the electric current \( I_{n} \), i.e. \( \Phi_{n} = \Phi_{(n-1)n} + \Phi_{nc} \), where \( \Phi_{nc} = L_{nc} I_{n} \), E.M.F. of a self-induction
\[ e_a = -L_{ac} \cdot \frac{dI_a}{d\tau} \], \[ L_{ac} \] – inductance of each cylinder. The components of the magnetic flux are created by the inductor so that

\[ \Phi_0 = \sum_{i=1}^{n} \Phi_i = \sum_{i=1}^{n} \frac{I_{w,i,0}}{Z_i^n} = \frac{U_1}{4.44 \cdot f \cdot w_{l,0}}, \] (4)

where \( I_{w,i,0} \) – number of ampere-turns per unit length; \( Z_i^n \) – complex magnetic resistance of each of the magnetic flux paths; \( U_1 \) – voltage on the inductor; \( w_{l,0} \) – number of turns per unit length of the inductor; \( Z_i \) – electrical resistance of each coaxial cylinder.

**Figure 2.** The magnetic equivalent circuit of the heating system.

In the gaps between the inductor and the nearest cylinder to it and between the cylinders, magnetic fluxes can be calculated by the following expressions:

\[ \Phi_{01} = \frac{\dot{H}_{01}}{Z_{01}^m} = \frac{\dot{H}_{01} x_{01}}{\omega} = \mu_0 \dot{H}_{01} \frac{(d_{11}^2 - d_{22}^2)}{4} \], (5)

\[ \Phi_{12} = \frac{\dot{H}_{12}}{Z_{12}^m} = \frac{\dot{H}_{12} x_{12}}{\omega} = \mu_0 \dot{H}_{12} \frac{(d_{21}^2 - d_{12}^2)}{4} \]. (6)

The presence in the equivalent circuit, shown in figure 2, of the magnetic flux component \( \Phi_3 \) (outside the last in the cylinder heating system) is determined by the final value of the system length, i.e. height \( h_2 \) of this cylinder. From the theory of induction heating it is known that the magnetic flux \( \Phi_3 \) can be taken equal to zero for cylinders with a ratio of height to diameter \( h/d \gg 2 \). Such systems are called long. For short systems with a ratio \( h/d < 2 \), it is necessary to consider the ways of the reverse closure of the magnetic flux of the last cylinder, which are characterized by magnetic resistance

\[ Z_{sc}^m = j\omega X_3 \]. (7)
where $X_3$ – reactive resistance of the reverse closure. This resistance is calculated as follows. At $h_2 = h_1$ the value of $X_3$ is determined by the expression

$$X_3 = X_2^{i,0} \frac{K_2}{1 - K_2},$$  
(8)

where $X_2^{i,0}$ – cylinder resistance excluding edge effects; $X_2^{i,0} = \omega \cdot \mu_0 \frac{S}{h_2}$; $S = \pi D_2^2$ – cross-sectional area of the internal cavity of the cylinder; $K_2 = 2.3(\frac{d_2}{h_2} + 2.3)^{1}$ – the correction factor taking into account the final height of the real cylinder (Nagaoka’s coefficient).

When $h_2 < h_1$ the sought resistance of the reverse closure is determined by

$$X_3 = X_2^{i,0} \frac{K_2 \cdot h_1}{h_1 - K_2 \cdot h_2}.$$  
(9)

The magnetic fluxes passing in coaxial cylinders are respectively equal to:

$$\Phi_{1w} = \frac{H_{01}}{Z_{1c}} = \frac{\dot{H}_{01}}{j \omega} \dot{Z}_{1c} = \dot{H}_{01} \left( \frac{x_{1c}}{\omega} - j \frac{r_{1c}}{\omega} \right),$$  
(10)

$$\Phi_{2w} = \frac{H_{12}}{Z_{2c}} = \frac{\dot{H}_{12}}{j \omega} \dot{Z}_{2c} = \dot{H}_{12} \left( \frac{x_{2c}}{\omega} - j \frac{r_{2c}}{\omega} \right)$$  
(11)

where $\dot{Z}_{1c}$, $\dot{Z}_{2c}$ – own electric resistance cylinders; $r_{1c}$, $x_{1c}$ and $r_{2c}$, $x_{2c}$ – active and inductive resistances of the cylinders layers on which the magnetic flux passes.

The subsequent expression allows to carry out preliminary analysis of a new heating system. After the transformations all resistances $Z_i$ are given to equivalent resistance $Z_{EQ}$. Therefore, it turns out that the expression for the number of turns of the inductor $w_1 = \frac{U_1}{U_2}$, where $U_2$ – equivalent electromotive force induced in coaxial cylinders. It turns out that $Z_{EQ}$ at $U_2 = const$ and $P_Z = const$ the total number of turns of the inductor $w_1$ increases due to a decrease in $U_2$. Consequently, the magnetic flux in the magneto-conductor $\Phi_0 = U_1(4.44 \cdot w_1 \cdot f)$ decreases and at constancy of magnetic induction $B = \Phi_0 / S_m$, the cross-section of the magneto-conductor $S_m$, its mass and losses in the magneto-conductor decreases. This allows to make the heating device more compact and less material-intensive. The increase of $w_1$ raises the consumption of copper or aluminum while maintaining the current density in the inductor. Therefore, for each design of the heating device there is an optimal mass ratio of electrotechnical steel and copper (aluminum) used for the inductor.

The analysis of internal interconnections of the heating system parameters showed that the ratio of equivalent sections of cylinders $(l, a_i)$ and their height $(h_i)$, magnetic permeability $(\mu_i)$, specific electrical resistance of the cylinder material $(\rho_i)$ and the distance between them $(b_i)$ is optimization parameters, as it affects the cost of the device and performance indicators.

The optimization task of creating a new system conversion of electrical energy into heat based on a group of coaxial cylinders includes the following feature: the need to increase the total power in the heating system is in conflict with the existing limitation of the specific surface power on a specific cylinder, acting as a liquid or gas heater. The integral temperature of the heated cylinder is determined by the technological, structural and regime requirements and should not lead to boiling of water (100 °C) and overheating of air (120 °C) to prevent sublimates of the dust fraction. These two conflicting requirements are interconnected with two other features: introduction to the system of high power coaxial cylinders while maintaining a relatively low specific surface power on a specific cylinder is possible by installing an additional coaxial cylinder. However, this leads to a significant
decrease $Z_r^{eq}$ and, therefore, an increase in the number of turns of the inductor $w_1$, which complicates the construct of a multilayer inductor and increases the consumption of copper (aluminum).

4. Conclusion
The considered formulation of the optimization problem should be used in the designing of each projected heater, since the optimal design can be achieved in various ways. These include the use of materials for cylinders with different $\rho$, $\mu$, geometric dimensions $a$, $b$, $h$ and quantity of cylinders $n$.

From the technical options of heating electric devices are most suitable for the purpose of reserving heat supply are those that do not require relatively complex and expensive auxiliary systems for their operation. Without going into a detailed analysis of all variety of possible devices to use, it should be concluded that, according to this condition, the devices of a reserve heat supply with a water heat carrier are preferable. They are the easiest to combine with district heating systems. Flat heating elements are considered as additional or independent elements of autonomous heating when it is necessary to maintain comfortable conditions. They are easily mounted in protecting surfaces of the room, induction systems for low-temperature heating of water and air, and some other devices.

Reasonable application of variants of autonomous heat source in heat supply systems is possible as a result of a complex consideration of a specific energy situation, taking into account all the components: electricity, heat and fuel supply.

5. References

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