A note on cosmology in a brane model

Mikhail Z. Iofa
Skobeltsyn Institute of Nuclear Physics
Moscow State University
Moscow 119992, Russia

Abstract

We study some aspects of cosmology in a five-dimensional model with matter, radiation and cosmological constant on the four-dimensional brane(s) and without matter in the bulk. The action of the model does not contain explicit curvature terms on the brane(s). We obtain solution of the generalized Friedman equation as a function of dimensionless ratio of the scales $b^2 = \frac{\mu M^2}{M_{pl}^4}$ ( $\mu$ is the scale in the warp factor in the 5D metric which is taken of order $10^{3\div4} GeV$, $M \geq \mu$ is the 5D fundamental scale ). We assume that there is a hierarchy between 4D and 5D scales. For $b^2 = O(1)$ the age of the Universe is found comparable, but below the current experimental value, for $b^2 \gg 1$ it is obtained much smaller than the experimental bound. Because time dependence of temperature of the Universe in the 5D model is different from that in the standard cosmology, the abundance of $^4$He produced in the primordial nucleosynthesis is obtained about three times more than in the standard cosmology.

1e-mail: iofa@theory.sinp.msu.ru
Recently there has been much activity in the study of the higher-dimensional models as in connection with possible detection of hypothetical large extra dimensions at the future colliders, so as with a possibility of solution of the hierarchy problem [1, 2, 3, 4]. If considered as models of the physical world, these models pose a question of possible cosmologies. In this note we consider a five-dimensional model with the visible world located on a 3-brane embedded in the $AdS_5$ space-time.

We study some aspects of cosmology in a model with matter and cosmological constant(s) on the brane(s) and with cosmological constant, but without matter in the bulk. In 5D models the form of the Friedman equation on 3-branes is different from that in the standard cosmology [6, 7, 8] and contains terms quadratic in components of the energy-momentum tensor on a brane.

The energy-momentum tensor on the visible brane is taken in the form

$$T^B_A = \delta(y)\sqrt{-g^{(4)}} \left[-\text{diag}(\dot{\rho}, \dot{\rho}, \dot{\rho}, \dot{\rho}, 0) + \text{diag}(-\dot{\rho}, \dot{p}, \dot{p}, \dot{p}, 0)\right], \quad (1)$$

where $\dot{\rho}(t)$ and $\dot{p}(t)$ are the sum of densities and pressures of cold matter and radiation and $\dot{\sigma}$ is the cosmological constant on the brane. The visible brane is located at the fixed position at $y = 0$.

We consider a class of solutions to the Einstein equations in 5D model with the metrics of the form

$$ds^2 = g_{AB}dx^A dx^B = -n^2(t, y)dt^2 + a^2(t, y)\gamma_{ij} dx^i dx^j + dy^2. \quad (2)$$

Solution of the Einstein equations is assumed to be invariant under the symmetry $y \to -y$.

The fundamental 4D and 5D scales are $M_{pl}$ and $M$, the gravitational constants are $\kappa_4^2 = 8\pi/M_{pl}^2$ and $\kappa^2 = 8\pi/M^3$. The characteristic scale of the warp factor of the metric in the bulk is $\mu = (\kappa^2 \Lambda/6)^{1/2}$, where $\Lambda$ is the cosmological constant in the bulk.

Possible cosmologies are strongly dependent on whether the curvature terms on 3-brane(s) are introduced explicitly in the 5D action or not. Cosmologies in the models with curvature terms on the branes were investigated in papers [9, 10, 11, 12, 13]. In this note we consider a model without explicit 4D curvature terms in the 5D action.

We study cosmologies by solving the generalized Friedman equation. The age of the Universe is calculated as a function of dimensionless ratio of the scales $b^2 \equiv \mu M_{pl}^2/M^3$. We assume that there is a hierarchy between the Planck and 5D scales $M_{pl}$ and $M$.

Solving Friedman equation we find time dependence of the scale factor and of temperature of the Universe. In the 5D model this dependence is different from that in the standard 4D cosmology. This leads, in particular, to different rates of primordial nucleosynthesis in both cases. We calculate the abundance of $^4$He and compare it with that in the standard cosmology.
Let us introduce the normalized expressions for energy density, pressure and cosmological constant on the visible brane which all have the same dimensionality $GeV$
\[
\rho = \frac{\kappa^2 \hat{\rho}}{6}, \quad \sigma = \frac{\kappa^2 \hat{\sigma}}{6}, \quad \hat{p} = \frac{\kappa^2 \hat{p}}{6}.
\] (3)

From the Einstein equations and the junction conditions for the functions $a(y, t)$ and $n(y, t)$ on the branes is derived the generalized Friedman equation. Using the freedom in the choice of parametrization of space-time variables, the function $n(y, t)$ on the visible brane is made constant on the visible brane
\[n(0, t) = 1.\]

We study cosmology by solving the generalized Friedman equation on the visible brane in the form with the second-order time derivatives of the scale factor $[6, 8]$
\[
\ddot{a}(0, t) + \dot{a}^2(0, t) = \frac{2}{a^2(0, t)} \left\{2(\sigma^2 - \mu^2) - |\sigma| \sum (\rho_i - 3p_i) - \sum \rho_i \sum (\rho_j + 3p_j)\right\},
\] (4)

where $(\rho_i, p_i) = (\rho_m, p_m) (\rho_r, p_r)$ are densities and pressures of the total cold matter and radiation.

In terms of cosmological variable
\[z = \frac{a(0, t_0)}{a(0, t)} - 1,
\]
where $a(0, t_0)$ is the current-time scale factor, $z$-dependencies of cold matter and radiation densities have the following form
\[
\dot{\hat{\rho}}_m = \dot{\hat{\rho}}_{m0}(1 + z)^3, \quad \dot{\hat{\rho}}_r = \dot{\hat{\rho}}_{r0}(1 + z)^4.
\] (5)

As the input we use the current-time values of the Hubble parameter $H_0 \sim 10^{-42} GeV$ and cold matter and radiation densities, or $\Omega_m = \hat{\rho}_{m0}/\hat{\rho}_C \sim 0.2 \div 0.4$ and $\Omega_r = \hat{\rho}_{r0}/\hat{\rho}_C \sim 10^{-4}$, where $\hat{\rho}_C$ is the critical density $\hat{\rho}_C \equiv 3H_0^2M_{pl}^2/8\pi [14, 15, 16]$. The normalized matter density can be written as
\[
\rho_{m0} = \frac{b^2 H_0^2 \Omega_m}{2\mu}.
\]

We assume that $\mu \sim 10^{3+4} GeV$ and $M \geq \mu$ (cf. [1]). First, let us consider the case $1 \leq b^2 < 10^{32}$, the upper limit corresponding to $M \sim \mu \sim 10^{3+4} GeV$. Rearranging the terms in the rhs of (4), we rewrite it as
\[
\frac{\ddot{a}(0, t)}{a(0, t)} + \frac{\dot{a}^2(0, t)}{a^2(0, t)} = 2(\sigma^2 - \mu^2) - |\sigma| \sum (\rho_i - 3p_i) - \sum \rho_i \sum (\rho_j + 3p_j).
\] (6)

\[\text{Having in view a model of the RSI type [17, 18, 19] with matter, below we take a negative cosmological constant on the visible brane. The "effective" cosmological constant } \mu - |\sigma| \text{ is small and positive.}\]
Introducing new variable
\[ u(t) = (z + 1)^{-2} = (a(0, t)/a(0, t_0))^2, \]
we obtain the first integral of Eq.(6) in the form
\[ (\dot{u})^2 = 4 \left[ (\sigma^2 - \mu^2)u^2 - 2|\sigma_1|\rho_{m0}u^{1/2} + \rho_{m0}^2u^{-1} + 2\rho_{m0}\rho_{r0}u^{-3/2} + \rho_{r0}^2u^{-2} + C \right], \quad (7) \]
where \( C \) is an integration constant.

Setting in (7) \( t = t_0 \) and substituting \( \dot{u}/u|_{t=t_0} = 2H_0 \) we obtain
\[ H_0^2 = \sigma^2 - \mu^2 - 2|\sigma_1|\rho_{m0} + (\rho_{m0} + \rho_{r0})^2 + C. \quad (8) \]
The second relation is obtained by taking (6) at \( t = t_0 \) and writing \( \ddot{a}(t_0)/a(t_0) = -q_0H_0^2 \), where \( |q_0| = O(1) \) is the deceleration parameter,
\[ (1 - q_0)H_0^2 = 2(\sigma^2 - \mu^2) - |\sigma|\rho_{m0} - (\rho_{m0} + \rho_{r0})\rho_{m0}. \quad (9) \]

Let us introduce the ratio
\[ r^2 = \frac{H_0^2}{|\sigma|\rho_{m0}} = \frac{2}{b^2\Omega_m |\sigma|} \mu. \quad (10) \]
From Eqs. (8) and (9), neglecting the term quadratic in densities, we obtain
\[ C = |\sigma|\rho_{m0} \left( \frac{3}{2} + \frac{1 + q_0}{2} r^2 \right), \quad \sigma^2 - \mu^2 = |\sigma|\rho_{m0} \left( \frac{1}{2} + \frac{1 - q_0}{2} r^2 \right). \quad (11) \]
Solving the second equation (11) with respect to \( |\sigma| \), we have
\[ |\sigma| \simeq \mu + \frac{\rho_{m0}}{4} + \frac{1 - q_0}{4\mu} H_0^2. \]
Thus, \( |\sigma| \simeq \mu \) and \( r^2 \simeq 2/b^2\Omega_m \).

Substituting in Eq.(7) the equality \( |\sigma|\rho_{m0} = H_0^2/r^2 \) which follows from the definition (10) and setting \( |\sigma| \simeq \mu \), we rewrite Eq.(7) in a form
\[ (\dot{u})^2 = 4 \frac{H_0^2}{r^2} \left[ \frac{1 + (1 - q_0)r^2}{2}u^2 - 2u^{1/2} + \frac{H_0^2}{r^2\mu^2}u^{-1} \left( \frac{1 + \rho_{r0}/\rho_{m0}}{u^{1/2}} \right)^2 + \frac{3 + (1 + q_0)r^2}{2} \right]. \quad (12) \]
For \( u \ll 1 \) the term quadratic in densities, estimated with \( \mu \sim 10^{3+4}GeV, \rho_{r0}/\rho_{m0} \sim 2 \cdot 10^{-4} \) [16] and \( H_0 \sim 10^{-42}GeV \), is of order the last constant term in (12) for \( u < 10^{-(49+50)}r^{-1} \), or \( z \geq 10^{25}b^{1/2} \).
To obtain a qualitative picture and to estimate the age of the Universe, we consider the following regions of $u$ (or $z + 1 = u^{-1/2}$).

(i) Cold matter-dominated region $1 > u > (\rho_{r0}/\rho_{m0})^2 \sim 10^{-8}$ or $1 \leq z < 10^4$, where Eq.(12) can be approximated as

$$u^2 \simeq 4 H_0^2 \frac{r^2}{r^2} \left[ \left( \frac{1}{2} + \frac{(1 - q_0)r^2}{2} \right) u^2 - 2u^{1/2} + \left( \frac{3}{2} + \frac{(1 + q_0)r^2}{2} \right) \right],$$

(13)

(ii) Radiation-dominated region where all $u$-dependent terms are small as compared to the constant term $3 \cdot 10^{-8} > u$ or $10^{-8} > u > 10^{-49/50}/r^2$. In this region Eq.(12) can be written as

$$u^2 \simeq H_0^2 \frac{r^2}{r^2} \left[ \frac{3 + (1 + q_0)r^2}{2} \right],$$

(14)

(iii) Radiation-dominated high-energy region $u < 10^{-50}b$, or $z > 10^{25}r^{1/2}$, where Eq.(12) can be approximated as

$$\dot{u}^2 \simeq 4\rho_{r0}u^{-2}.$$ 

(15)

In both regions (ii) and (iii) we use the approximate equation

$$\dot{u}^2 \simeq 4 \frac{3 + (1 + q_0)r^2}{2} \mu \rho_{m0} + \rho_{r0}^2 u^{-2}.$$ 

(16)

To estimate the age of the Universe we can consider only the matter-dominated period. Integrating Eq.(13), we obtain

$$H_0(t_0 - t) = \frac{r}{\sqrt{2}} \int_{u(t)}^{1} \frac{du}{[(1 + (1 - q_0)r^2)u^2 - 4u^{1/2} + 3 + (1 + q_0)r^2]^{1/2}}.$$ 

(17)

For an estimate we can take $t = 0$ and $u(t) = 0$.

First, let us suppose that both scales $\mu$ and $M$ are of order $10^{3-4} GeV$. In this case $b^2 \sim 10^{32}$ and $r^2 \sim 10^{-30-32}$. For small $r^2$ the integral (17) can be written as

$$H_0t_0 \simeq r \sqrt{2} \int_{0}^{1} \frac{vdv}{[v^4 - 4v + 3 + r^2((1 - q_0)v^4 + 1 + q_0)]^{1/2}} \simeq r \int_{0}^{1} \frac{vdv}{[3(v - 1)^2 + r^2]^{1/2}}$$

(18)

$^3$At the point of transition from the matter-dominated to the radiation-dominated phase $z \sim 10^4$ the terms quadratic in densities are of the same order.
Here we have set \( v^4 - 4v + 3 = (v - 1)^2(v^2 + 2v + 3) \simeq 6(v - 1)^2 \) and \((1 - q_0)v^4 + 1 + q_0 \simeq 2\).

For \( r^2 \ll 1 \) the age of the Universe

\[
t_0 \simeq \frac{r}{\sqrt{3}H_0} \left[ \ln \frac{2\sqrt{3}}{r} - 1 \right]
\]

(19)
is much smaller than that in the standard cosmology.

Next, we take \( \mu \sim 10^{3\div 4} GeV \), keeping the 5D scale \( M \) or, equivalently, \( r^2 \) as free parameters. The age of the Universe \( t_0 \) calculated with \( q_0 = 0, \pm 1/2 \) and \( r^2 = O(1) \) is

| \( r^2 = 2/b^2 \) | \( \Omega_m = \frac{2}{b^2} \)
|----------------|-----------------|
| \( q_0 = +1/2, \ t_0 H_0 = \) | 0.07 | 0.16 | 0.26 | 0.37 | 0.42 | 0.46 | 0.49 | 0.50 | 0.52 | 0.54
| \( q_0 = 0, \ t_0 H_0 = \) | 0.07 | 0.16 | 0.27 | 0.38 | 0.44 | 0.49 | 0.51 | 0.53 | 0.55 | 0.58 | 0.60
| \( q_0 = -1/2, \ t_0 H_0 = \) | 0.07 | 0.16 | 0.27 | 0.40 | 0.47 | 0.54 | 0.56 | 0.59 | 0.62 | 0.67 | 0.72

For \( r^2 \sim 1 \div 10 \) the age of the Universe is comparable, but smaller than that in the standard cosmology in the \( \Lambda \)CDM model (\( t_0 \sim 0.65 H_0^{-1} \) in matter-dominated model with \( \Omega_m = 1 \) and \( \Lambda = 0 \), and \( t_0 \simeq H_0^{-1} \) in the \( \Lambda \)CDM model with \( \Omega_m = 0.28 \) and \( \Omega_m + \Omega_\Lambda = 1 \) [14, 16, 20]).

With \( b^2 = O(1) \) and \( \mu \sim 10^{3\div 4} GeV \) the 5D scale \( M \) is of order \((\mu M_{pl}^2)^{1/3} \sim 10^{14} GeV\).

At times \( t \sim t_0 \) such that \( H_0(t_0 - t) \ll 1 \) solution of (17) is the same as in the standard cosmology

\[
z \simeq 1 + H_0(t_0 - t).
\]

(20)

In the region (ii) where \( y \ll 1 \), from (14) we obtain

\[
u(t) \simeq z^{-2} \simeq (H_0 t) \left( \frac{6 + 2(1 + q_0)r^2}{r^2} \right)^{1/2}. \tag{21}\]

In regions (ii) and (iii) solution of Eq. (16) is

\[
u^2(t) = \frac{2(3 + (1 + q_0)r^2)}{r^2} (H_0 t)^2 + \frac{4\rho_{r0}}{r^2 \mu \rho_{m0}} H_0^2 t. \tag{22}\]

The transition time \( \bar{t} \) from the law \( z \sim (H_0 t)^{-1/4} \) to \( z \sim H_0(t)^{-1/2} \) is

\( \bar{t} \sim 10^{-(7 \div 8)} (3 + (1 + q_0)r^2)^{-1} GeV^{-1} \). This time is considerably smaller than the characteristic times of nucleosynthesis \( 1 \div 10^3 \) s, or \( 10^{24 \div 26} GeV^{-1} \). Thus, at the time of nucleosynthesis we can neglect second term in (22).

Let us compare primordial nucleosynthesis in the standard and non-standard cosmologies.

First, let us find relation between time and temperature of the Universe in the radiation-dominated phase in the non-standard cosmology. At \( z > z_{cr} \) the Universe becomes radiation-dominated. Combining expressions for the radiation density

\[
\rho_r(z) \simeq \rho_{r0}z^4
\]
valid for large $z$ and the expression for the radiation density as a function of temperature $\tilde{T}$ of the Universe [14, 15]

$$\hat{\rho}_r(\tilde{T}) = \frac{\pi^2}{30} g_*(\tilde{T}) \tilde{T}^4$$

and substituting $z^4$ from (22) we have

$$\frac{\pi^2}{30} g_*(\tilde{T}) \tilde{T}^4 \simeq \frac{\hat{\rho}_{m0}}{4(H_0t)^2} \frac{2r^2}{3 + (1 + q_0)r^2} \frac{\hat{\rho}_{r0}}{\hat{\rho}_{m0}}. \quad (23)$$

In the standard cosmology, the Friedman equation in the radiation-dominated period

$$\left( \frac{a(t)}{a(t_0)} \right)^2 = \frac{\kappa_4^2 \hat{\rho}_{r0}}{3} \left( \frac{a(t)}{a(t_0)} \right)^2$$

is solved as

$$z^{-2} = 2t \left( \frac{\kappa_4^2 \hat{\rho}_{r0}}{3} \right)^{1/2} ,$$

where we substituted $a(t_0)/a(t) \simeq z$. Substituting the relation

$$\frac{\kappa_4^2 \hat{\rho}_{r0}}{3} = \frac{\kappa_4^2 \hat{\rho}_{m0} \hat{\rho}_{r0}}{\hat{\rho}_{m0}} = H_0^2 \hat{\rho}_{r0} \Omega_m$$

in the expression for radiation density $\hat{\rho}_r(z) \simeq \hat{\rho}_{r0} z^4$, we obtain

$$\hat{\rho}_r(z) \simeq \frac{\hat{\rho}_{m0}/\Omega_m}{4(H_0t)^2}.$$

The relation connecting time and temperature takes the form

$$\frac{\pi^2}{30} g_*(T) T^4 \simeq \frac{\hat{\rho}_{m0}/\Omega_m}{4(H_0t)^2}. \quad (24)$$

Comparing expressions (23) and (24) we notice the appearance of the large extra factor $\hat{\rho}_{m0}/\hat{\rho}_{r0} \simeq 0.5 \cdot 10^4$ [16] in the non-standard cosmology. The effective numbers of massless degrees of freedom $g_*(\tilde{T})$ and $g_*(T)$ are temperature-dependent and are different in the standard and non-standard cosmologies.

Time dependence of the Hubble parameter in both the standard and non-standard cosmologies is the same $H(t) = 1/2t$. The Hubble parameter expressed as a function of temperature in the non-standard cosmology is

$$H(\tilde{T}) = \tilde{T}^2 \left( \frac{\pi^2 g_*(\tilde{T}) H_0^2}{30} \frac{[3 + (1 + q_0)r^2]}{2r^2} \frac{\hat{\rho}_{m0}}{\hat{\rho}_{r0}} \right)^{1/2}.$$
The freezing temperature $\tilde{T}_F$ of the reaction $n \leftrightarrow p$ is estimated as the temperature at which the Hubble parameter $H(\tilde{T})$ is of order of the reaction rate $\sim G_F^2 \tilde{T}^5$ [15, 14]. Comparing the freezing temperatures $T_F$ and $\tilde{T}_F$ in the standard and non-standard cosmologies we obtain

$$\frac{\tilde{T}_F}{T_F} = \left( \frac{g_*(\tilde{T}_F)}{g_*(T_F)} \frac{[3 + (1 + q_0) r^2] \rho_{m0}}{\rho_{r0}} \right)^{1/6}. \tag{25}$$

At the freezing temperature $\tilde{T}_F$ the effective number of degrees of freedom is $g_*(\tilde{T}_F) = 10.75$. At temperatures below 1 MeV neutrinos decouple from massless particles in equilibrium and the effective number of degrees of freedom is $g_*(T_F) \sim 5$ yielding the factor $\left( \frac{g_*(\tilde{T}_F)}{g_*(T_F)} \right)^{1/6} \sim 1.1$. Taking for an estimate $r^2 = 1; 10; 2 \cdot 10^4$ and $q_0 = 0$, we obtain

$$\frac{\tilde{T}_F}{T_F} \simeq 7.9; 6.5; 5.6.$$

Substituting $T_F \simeq 1 MeV$, we obtain the equilibrium ratio

$$(n/p)(\tilde{T}_F) = \exp \left( - \frac{m_n - m_p}{T_F} \right) \simeq 0.80; 0.76; 0.72$$

as compared with $(n/p)(T_F) \simeq 0.17$ in the standard cosmology.

The mass fraction of $^4$He produced in the nucleosynthesis $X_4 = 4n_{He}/n_N$ starts rapidly approaching the final value at the temperature $T_f \sim 0.1 - 0.3 MeV$ at the time $t_f$. The times of the freeze-out in the standard and non-standard cosmologies are $t_F \simeq 0.75s$ and $\tilde{t}_F \sim 10^{-4}s$. In the standard cosmology the time between the freezing point and $t_f$ is about 2 minutes during which the ratio $n/p$ decreases because of the neutron decay [15, 14]. In non-standard cosmology the time $\tilde{t}_f$ is of order a second and for the estimate of $X_4$ the neutron decay between the times $\tilde{t}_F$ and $t_f$ can be neglected.

As a result, in the non-standard cosmology the mass fraction of $^4$He calculated with the above values of $(n/p)$ is

$$X_4 = \frac{2(n/p)}{(n/p) + 1} \simeq 0.89; 0.86; 0.84$$

which is to be compared with $X_4 \simeq 0.25$ in the standard cosmology [14, 16].

Assuming that there is a large hierarchy of scales, above we considered the case $b^2 \geq 1$. However, because of the large value of the ratio $M_{pl}/\mu$, there can be a hierarchy between $M$ and $M_{pl}$ with $b^2 < 1$, supposing that $b^2$ is not too small.

Let us estimate the age of the Universe and the freezing temperature of the reaction $n \leftrightarrow p$ assuming that $b^2 < 1$ or $r^2 > 1$. We will distinguish two cases (i) $q_0 \neq -1$, and (ii) $q_0 \simeq -1$. In the first case, taking $|q_0| = O(1)$ [16, 15], with large $r^2 \gg 1$ we obtain that $2r^2\Omega_m/(3 + (1 + q_0)r^2) = O(1)$. As it was discussed above, in (25) there is a big number
\[ \hat{\rho}_{m0}/\hat{\rho}_{r0} \sim 2 \cdot 10^4 \text{ which for large } r^2 \text{ yielded } X_4 \sim 0.8. \] We see that \( X_4 \) weakly depends on \( r^2 \), and its value is about three times larger than that in the standard model.

In the case (ii) the ratio of freezing temperatures (25) contains the factor \( 2r^2 \Omega_m/3 \) which for \( r^2 \sim 10^4 \) is of order unity, i.e. \( \tilde{T}_F \sim T_F \). The estimate of the age of the Universe is

\[ H_0 t_0 = \frac{r}{\sqrt{2}} \int_0^1 \frac{du}{\left[ (2r^2 + 1)u^2 + 3 - 4u^{1/2} \right]^{1/2}} \sim \frac{1}{2} \int_1^{1/\sqrt{3/2r^2}} \frac{du}{u}. \]

With \( r^2 \sim 10^4 \) the age of the Universe is obtained considerably above the experimental bound.

In the model with 5D action containing explicit curvature terms on the branes (on the visible brane with the Newton constant) situation is rather different. In this model in the range of times at which \( H^2(t)/\mu^2 \ll 1 \) solution of the generalized Friedman equation is close to solution of the Friedman equation in the standard cosmology with matter (radiation) and cosmological constant of the form \( H^2/H_0^2 = \Omega(z + 1)^n + (1 - \Omega) \), where \( \Omega = \Omega_m(\Omega_r) \), \( n = 3 \) (4) in the cold matter (radiation) dominated Universe (cf. [11, 12, 13]). With the input of the current values of the Hubble parameter and matter density, in terms of the cosmological parameter \( z \), the above range is estimated as \( z < 10^{22} \).

References

[1] V.A. Rubakov, Phys. Usp. 44, 871 (2001), [hep-ph/0104152].

[2] C. Csaki, TASI lectures, [hep-ph/0404096].

[3] T.G. Rizzo, [hep-ph/0409309].

[4] R. Sundrum, TASI lectures, [hep-th/0508134].

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), [hep-ph/9905221].

[6] P. Binétruy, C. Deffayet, U. Ellwanger, D. Langlois, Phys. Lett. B477, 285 (2000), [hep-th/9910219].

[7] P. Binétruy, C. Deffayet, and D. Langlois, Nucl. Phys. B565, 269 (2000), [hep-th/9905012].

[8] T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D62, 024012 (2001), [hep-th/9910076].

[9] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B485, 208 (2000) [hep-th/0005016].

[10] Yu. V. Shtanov, Phys. Lett. B541, 177 (2002) [hep-th/0108153].
[11] V. Sahni and Yu. V. Shtanov, JCAP 0311, 014 (2003), [astro-ph/0202346].

[12] V. Sahni, Yu. V. Shtanov, A. Viznyuk, JCAP 0512, 005 (2005), [astro-ph/0505004].

[13] C. Deffayet, Phys. Lett. B502, 199 (2001), [hep-th/0010186].

[14] E. W. Kolb and M. S. Turner, The Early Universe, 1993.

[15] S. Weinberg, Gravitation and Cosmology, 1972.

[16] Particle Data Group, Astrophysics and Cosmology.

[17] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), [hep-ph/9905221].

[18] C. Csaki, M. Graesser, C. Kolda, J. Terning, Phys. Lett. B462, 34 (1999), [hep-ph/9906513].

[19] C. Csaki, M. Graesser, L. Randall, J. Terning, Phys. Rev. D62, 045015 (2000), [hep-ph/9911406].

[20] V. Sahni, Lect. Notes Phys. 653, 141 (2004) [astro-ph 0403324 ].