Constraint on the radius of five-dimensional dS spacetime with GW170817 and GRB 170817A

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The recent detections of the gravitational wave (GW) event GW170817 and its electromagnetic counterpart GRB 170817A produced by a binary neutron star (NS) merger is a new milestone of multi-messenger astronomy. The time interval between these two signals has attracted widespread attention from physicists. In the braneworld scenario, GWs could propagate through the bulk while electromagnetic waves (EMWs) are bounded on the brane, i.e., our Universe. Therefore, the trajectories of GWs and EMWs may follow different paths. If GWs and EMWs are originated simultaneously from the same source on the brane, they are expected to arrive at the observer successively. Consequently, the time delay between GW170817 and GRB 170817A may carry the information of the extra dimension. In this paper, we try to investigate the phenomenon in the context of a five-dimensional dS (dS\textsubscript{5}) spacetime. We assume two special models for our Universe, i.e., the de Sitter model and the Einstein-de Sitter model, and compare the gravitation horizon radius and photon horizon radius in each case. Our results show that the dS\textsubscript{5} radius will contribute to the time delay in the latter case. With the data of the observation, we constrain the dS\textsubscript{5} radius to \( \ell \gtrsim 7.5 \times 10^{7}\) Tpc. After considering the uncertainty in the source redshift and the time-lags given by different astrophysical processes of the binary NS merger, we find that our constraint is not sensitive to the redshift in the range of \((0.005, 0.01)\) and the time-lag in the range of \((-100\text{s}, 1.734\text{s})\).

I. INTRODUCTION

The nature of gravitational waves (GWs) is the perturbations of the spacetime. But such perturbations are so weak that it took humans decades to detect them. Since GWs were first detected by the LIGO and Virgo collaborations in 2015\cite{1}, in just two or three years, they have detected more than ten GW events through gravitational observation. These events involve the binary black hole mergers as well as the binary neutron star (NS) merger\cite{1, 2, 3, 4, 5, 6}. The latter is a special case, because the binary NS is widely expected to radiate both short gamma ray bursts (SGRBs) and GWs during its merger\cite{7, 8, 9, 10, 11, 12, 13, 14, 15, 16}. In the event GW170817, the collaboration of the LIGO-Virgo detectors, the \textit{Fermi} Gamma-ray Burst Monitor (GBM), and the SPectrometer on board INTEGRAL Anti-Coincidence Shield (SPI-ACS) found that the gravitational signal GW170817 originated from the merger of a binary NS is followed by the electromagnetic (EM) signal GRB 170817A\cite{6, 17, 18, 19}. Since the source of GRB 170817A is very close to the source of GW170817 and the time interval between the two signals is only 1.7 seconds, most people believe that GRB 170817A is an EM counterpart of GW170817\cite{6, 17, 18, 19, 20}.

Although it is now widely accepted that a binary NS merger could emit GWs and SGRBs, the time interval between the events GW170817 and GRB 170817A is still nerve-racking. So far, there are plenty of researches attempting to explain the time delay. From the perspective of the gravity itself, the linearized Einstein equation has various forms in different modified gravities, so the propagation speed of the introduced extra GW polarization modes could deviate from the speed of light\cite{21, 22, 23, 24, 25, 26}. On the other hand, the binary NS could undergo some exotic astrophysical processes during its merger. In these processes, the generation of the GWs and SGRBs may not be simultaneous\cite{11, 12, 13, 14, 15, 16}.

In higher-dimensional theories, the time delay between the GWs and SGRBs is also reasonable\cite{27, 28, 29, 30, 31, 32}. After one hundred years of development of the extra dimensional theories from Kaluza-Klein (KK) theory\cite{33, 34, 35}, most famous higher-dimensional theories agree with that our Universe is a four-dimensional hypersurface (called brane) embedded in a higher-dimensional spacetime. In this scenario, the elementary particles and interactions in the Standard Model are confined on the hypersurface, while the gravity could propagate through the bulk. It indicates the possibility that the trajectory of five-dimensional null geodesics might deviate from light, which hence may result in the time delay\cite{29, 31, 32}.

Apart from the time delay, extra dimensions have another important effect on GWs, which may reveal the
number of spacetime dimensions. In higher-dimensional theories, since GWs could propagate through the bulk, they will leak into extra dimensions (called gravitational leakage) when propagating in our Universe. Therefore, for a given higher-dimensional theory, the amplitude of GWs will decay faster than the expectation in the four-dimensional theory. It is then expected that the gravitational leakage will reduce the amplitude of the observed GWs and make the four-dimensional observer to misjudge the travel distance of the GWs. Generally speaking, the more extra dimensions in the spacetime, the faster the amplitude of GWs will decay during their propagation. According to this and the data of GW170817, the modified amplitude of the GWs in a specific higher-dimensional theory was used to constraint the number of spacetime dimensions [36–39]. The results shown that, the number of spacetime dimensions could be larger than four.

On the other hand, it is well known that some extra dimensional theories could unify gravity and electromagnetism [33–35] and solve the huge hierarchy between the fundamental scales of gravity and electromagnetism [40–43]. All of these features indicate that it is important to investigate the structure of extra dimensions and it is worth paying close attention to revealing the information of it from the time delay and its other effects.

Recently, a five-dimensional theory with static spherically symmetric anti-de Sitter (AdS) spacetime was studied in Refs. [29, 31, 32]. The brane is embedded as a curved hypersurface in this model and the null trajectories of GWs and electromagnetic waves (EMWs) are concerned. Similar to most extra-dimensional theories, EMWs are confined on the brane while GWs could propagate through the bulk. It was found that the gravitational horizon radius on the brane is effected by the structure of the bulk spacetime. Then the discrepancy between the gravitational horizon radius and photon horizon radius, which may eventually result in the time delay, could be used to reveal the feature of the AdS5 radius. In Ref. [31], the authors analysed such model with the assumption that our Universe is closed. The cases that our Universe is dominated by either the dark energy or the non-relativistic matter were discussed. In both cases, the present-day spatial curvature density is constrained to the scale of $10^{-10}$ which is much smaller than the result obtained by the Planck collaboration [44, 45]. Later, it was found that the time delay also occurs even if our Universe is flat [32]. In this case, the AdS5 radius is required to be smaller than 0.535 Mpc at 68% confidence level.

In this work, we focus on the braneworld embedded in a five-dimensional de Sitter (dS5) spacetime, which is on account of the following motivations. The cosmological observations indicate that the phase of our very early Universe is a dS phase and that the Universe may ultimately dominated by the dark energy (which is also a dS phase). So it is worth generalizing the bulk spacetime into a dS5 case where the braneworld could be realized by a quantum creation from the holographic dS/CFT correspondence [46, 47]. On the other hand, it was already found that the dS4 brane with a modified Friedmann-Lemaître-Robertson-Walker (FLRW) equation could be constructed in the dS5 bulk [48–51]. The mass, entropy, holography, and other properties of the five-dimensional Schwarzschild-dS black hole were also discussed in Refs. [51–55]. All these breakthroughs indicate that studying the structure of the dS5 spacetime has a guiding significance for us to understand our Universe.

Inspired by Refs. [29–32], we then try to constrain the dS5 radius with the observed time delay between the detections of GW170817 and GRB 170817A. For a dS5 bulk, we would like to embed our Universe inside the cosmological horizon so that the scale factor of our Universe could increase from $a(t_0 = 0) = 0$. Indeed, it might require a large dS5 radius. On the other hand, it has been proved that the standard cosmology will be recovered on the brane as long as the dS5 radius is extremely large [48–51]. Therefore, if such braneworld is true, the constraint from the observed time delay should allow the existence of the large dS5 radius. We use two specific models of our Universe to calculate the discrepancy between the gravitational horizon radius and photon horizon radius. With the event GW170817/GRB 170817A, we obtain the lower bound on the dS5 radius, $\ell \geq 7.5 \times 10^2$ Tpc, which is consistent with the above analysis.

The following context is arranged as follows. In Sec. II, we construct the brane model in a five-dimensional static spherically symmetric dS spacetime, where our Universe is embedded. We then give the abstract forms of the gravitational horizon radius and photon horizon radius in Sec. III. In order to express the gravitational horizon radius in a practical form, we convert the unknown quantity into the observable quantities in Sec. IV. Combined with the data of GW170817 and GRB 170817A, we give the constraint on the dS5 radius in Sec. V. Finally, we make a short conclusion in Sec. VI.

### II. EMBEDDED UNIVERSE

We start from a five-dimensional spherically symmetric spacetime with the metric:

$$ds^2 = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2d\Sigma_k^2,$$  \hspace{1cm} (1)

where $T$ is the coordinate used to denote the sequence of events, $d\Sigma_k^2$ is a metric on a locally homogeneous three-dimensional surface of constant curvature $k$:

$$d\Sigma_k^2 = \frac{1}{1 - kr^2}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2,$$ \hspace{1cm} (2)

and $R$ is the spatial coordinate denoting the radial coordinate distance of the hypersurface $\Sigma_k$ from the coordinate origin. Here and after, we assume that the geometry of the bulk is dominated by a bulk cosmological constant. Then we can get a series of Schwarzschild-like solutions.
of the metric by solving the Einstein equation with the bulk cosmological constant:

\[
  f(R) = \begin{cases} 
    k (k \neq 0) & \text{for Minkovskian bulk} \\
    k - \frac{R^2}{\ell^2} - \frac{\mu}{R^2} & \text{for dS bulk} \\
    k + \frac{R^2}{\ell^2} - \frac{\mu}{R^2} & \text{for AdS bulk}
  \end{cases}
\]  

(3)

where the parameter \( \ell \) is the dS\(_5\) (AdS\(_5\)) radius and \( \mu \) is the Schwarzschild-like mass. For a dS\(_5\) spacetime, there are a cosmological horizon at

\[
  R_{\text{ch}} = \sqrt{(k\ell^2 + \sqrt{k^2\ell^2 - 4\mu\ell})}/2
\]

(4)

and a black hole horizon at

\[
  R_{\text{bh}} = \sqrt{(k\ell^2 - \sqrt{k^2\ell^2 - 4\mu\ell})}/2
\]

(5)

for a positive \( k \). In this paper we will set \( \mu = 0 \) and \( k > 0 \) for convenience. Therefore, the black hole horizon vanishes and the cosmological horizon reduces to \( R_{\text{ch}} = k\ell \). Note that an observer with velocity \( V^\mu = (1, 0, 0, 0, 0) \) follows a time-like geodesic inside the cosmological horizon.

To embed our Universe in the dS\(_5\) spacetime, we introduce the following constraint

\[
  -f(R)dt^2 + f(R)^{-1}dR^2 = -dt^2,
\]

(6)

such that the induced metric of the four-dimensional submanifold (three-dimensional brane and one-dimensional time) coincides with the FLRW metric of our Universe:

\[
  ds_4^2 = -dt^2 + R^2 \left( \frac{1}{1 - kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)
\]

(7)

with

\[
  R = a(t)
\]

(8)

on the brane. Here \( t \) and \( a(t) \) are the cosmological time and the scale factor of our Universe, respectively. The constraint condition (6) implies that the brane is a four-dimensional hypersurface embedded in the dS\(_5\) spacetime. The diagram of our Universe with a time interval is shown in Fig. 1. It is obvious that our Universe at each moment corresponds to a three-dimensional sphere \( S^3 \) with a constant radial distance \( R \) on the hypersurface.

In the braneworld scenario, the particles and interactions in the Standard Model are confined on the brane while gravity propagates through the bulk. Consequently, EMWs and GWs may follow different null curves. Since GWs follow the five-dimensional null geodesics, their trajectory (called shortcut) is expected to be the shortest path from the source to the observer. Assuming that GWs and EMWs are emitted simultaneously, the difference between their trajectories could lead to a time interval between the detections of them and make the viewers on the brane misjudge the speed of GWs. In other words, the four-dimensional viewers may find that the speed of GWs is not equal to the speed of light. In the next section, to figure out this question, we will calculate the gravitational horizon radius and photon horizon radius.

III. HORIZON RADIUS

For convenience, we would like to assume that the projections of the events \( A, B, B' \), and \( C \) on the 3-sphere (see Fig. 1) are lined along the radial direction \( r \). Then the four-dimensional null geodesic connecting events \( A, B' \), and \( C \) and the five-dimensional null geodesic connecting events \( A \) and \( B \) have the fixed angular variables, \( \theta \) and \( \phi \). And their trajectories follow

\[
  ds_4^2 = -dt^2 + a^2 dr^2,
\]

(9)

\[
  ds_5^2 = -f(R)dt^2 + f(R)^{-1}dR^2 + R^2 dr^2,
\]

(10)

respectively, where we have introduced a coordinate transformation, i.e., \( dr^2 = \frac{1}{1 - kr^2} dr^2 \), for the sake of simplification. From the metric (10), one finds two killing vectors, \( \left( \frac{\partial}{\partial \tau} \right)^M \) and \( \left( \frac{\partial}{\partial \nu} \right)^M \), and defines the following
quantities:

\[
\kappa_T = g_{MN} U^M \left( \frac{\partial}{\partial T} \right)^N = -f \frac{dT}{d\lambda}, \tag{11}
\]

\[
\kappa_\varphi = g_{MN} U^M \left( \frac{\partial}{\partial \varphi} \right)^N = R^2 \frac{d\varphi}{d\lambda}, \tag{12}
\]

where \( U^M = \frac{dx^M}{d\lambda} \) is a unit space-like vector tangent to the geodesic with \( \lambda \) the affine parameter of the five-dimensional null geodesic. Obviously, the quantities \( \kappa_T \) and \( \kappa_\varphi \) are conserved along the five-dimensional null geodesic. For this five-dimensional null geodesic, the combination of Eqs. (10), (11), and (12) gives

\[
-\frac{\kappa_T^2}{f} + \frac{1}{f} \left( \frac{dR}{d\lambda} \right)^2 + \frac{\kappa_\varphi^2}{R^2} = 0, \tag{13}
\]

\[
\frac{dT}{d\lambda} + \kappa_T = 0, \tag{14}
\]

\[
\frac{d\varphi}{d\lambda} - \kappa_\varphi = 0. \tag{15}
\]

and further

\[
dR^2 = \frac{R^2}{\kappa_T^2} \left( R^2 \kappa_T^2 - f \kappa_\varphi^2 \right) d\varphi^2, \tag{16}
\]

\[
dR^2 = \frac{f^2}{R^2 \kappa_T^2} \left( R^2 \kappa_T^2 - f \kappa_\varphi^2 \right) d\tilde{T}^2. \tag{17}
\]

It is obvious that the comoving distance between events \( A \) and \( B \) on the brane can be obtained by integrating Eq. (16):

\[
\begin{align*}
\tilde{r}_g &= \int_{\tilde{r}_A}^{\tilde{r}_B} \frac{1}{\sqrt{1 - \kappa_\varphi^2}} d\tilde{r} = \int_{\tilde{r}_A}^{\tilde{r}_B} d\tilde{\varphi} \\
&= \int_{R_A}^{R_B} \left( \frac{R^4}{s - R^2 f} \right)^{-\frac{1}{2}} dR,
\end{align*}
\tag{18}
\]

where we have used the definition

\[
s \equiv \kappa_\varphi^2 / \kappa_T^2. \tag{19}
\]

Note that, since the GWs originated from a binary system are finally detected on the brane, the radial distances, \( R_A \) and \( R_B \), could be converted into the redshifts of the source and the observer, respectively. Here and after, we rescale the present-day scale factor \( a_B \) as the unit and use tildes to denote the rescaled quantities. Accordingly, the rescaled gravitational horizon radius could be expressed as

\[
\tilde{r}_g \equiv a_B r_g = \int_{\tilde{r}_A}^{\tilde{r}_B} \tilde{R}^{-2} \left( \frac{1}{s} + \frac{1}{f^2} - \tilde{k} \tilde{R}^{-2} \right)^{-\frac{1}{2}} d\tilde{R}, \tag{20}
\]

where \( \tilde{k} \equiv k/a_B^2 \) and \( \tilde{R} \equiv R/a_B \) are the rescaled spatial curvature of a 3-sphere and the rescaled radial coordinate, respectively. The observer redshift has been set to zero and the source redshift is marked as \( z_A \).

Another crucial quantity is the rescaled photon horizon radius:

\[
\tilde{r}_\gamma \equiv a_B r_\gamma = \int_{t_A}^{t_B} \frac{1}{\tilde{a}} dt, \tag{21}
\]

where \( \tilde{a} \equiv a/a_B \) is the rescaled scale factor. Note that the cosmological times, \( t_A \) and \( t_B \), correspond to the moments the GWs are emitted and detected, respectively. As we have mentioned, from the viewpoint of any five-dimensional observer, events \( A \) and \( B \) are causally connected by a five-dimensional null geodesic. Assuming that the GWs and EMWs are emitted from the binary star merger at the same cosmological time \( t_A \), the discrepancy between the gravitational horizon radius and photon horizon radius may result in a time delay \( \Delta t \) between the detections of the GWs and EMWs. Accordingly, in the case of \( \tilde{r}_g > \tilde{r}_\gamma \), one has an approximate formula

\[
\tilde{r}_g - \tilde{r}_\gamma \approx c \Delta t \tag{22}
\]

under the low-redshift case \( z \ll 0.1 \).

Here we should note that if a discrepancy between \( \tilde{r}_g \) and \( \tilde{r}_\gamma \) in (22) appears, it will give us an opportunity to explain the detected time delay. In this case, the time delay is expected to carry the information of the extra dimension (see Eqs. (20) and (22)) and one could read out the feature of the dS5 radius. Mathematically, if the integral of Eq. (20) deviates from Eq. (21), one can use the observational data of some specific GWs events to constrain the structure of the extra dimension through Eq. (22). However, one finds that the quantity \( s \) in Eq. (20) is still unknown. Therefore, one could not obtain the gravitational horizon radius directly from the observed data with Eq. (20). To solve this problem, we would use the method introduced in Refs. [31, 32] in the next section.

**IV. PARAMETER TRANSFORMATION**

In the previous section, based on Eq. (16) we have got the gravitational horizon radius on the brane. However, the unobservable quantity \( s \) therein makes Eq. (20) not so practical. To eliminate the quantity \( s \), one should recall Eq. (17), with which the following equation is obtained:

\[
\int_{T_A}^{T_B} \frac{d\tilde{T}}{\tilde{a}_B} = \left[ \frac{R^2}{R^2 - f \tilde{s} j} \right]_{R_B}^{R_A} d\tilde{R}, \tag{23}
\]

where \( \tilde{T} \equiv a_B T \) is the rescaled coordinate time, \( \tilde{f} \equiv f/a_B^2 = \tilde{k} - R^2/\tilde{f}^2 \) is the rescaled function, and \( \tilde{T}_A \equiv a_B T_A \) and \( \tilde{T}_B \equiv a_B T_B \) are the rescaled coordinate times of the emission and detection of GWs, respectively. In principle, the quantity \( s \) could be solved from this equation directly as long as the coordinate time interval, \( T_{AB} \equiv \tilde{T}_B - \tilde{T}_A \), is known. However, for any four-dimensional observer, the detected time interval is the
cosmological time interval, $t_{AB} \equiv t_B - t_A$, which is related to the source redshift. So one should use Eqs. (6) and (8) to express the rescaled coordinate $\tilde{T}$ in terms of the cosmological time $t$. Accordingly, one finds that the coordinate time interval $\tilde{T}_{AB}$ obeys
\[
\int_{\tilde{T}_A}^{\tilde{T}_B} d\tilde{T} = \int_{t_A}^{t_B} \frac{\sqrt{F_1 + H^2\tilde{u}^2}}{F_1} dt,
\] 
(24)
where the definition $F_1(t) \equiv \tilde{k} + \tilde{a}^2/\ell^2$ has been employed and the quantity $H(t)$ is the Hubble constant. Based on Eqs. (23) and (24), the expression of the quantity $s$ in terms of the observable quantities could be obtained by solving the following parameter equation:
\[
\int_{\tilde{R}_A}^{\tilde{R}_B} \sqrt{\frac{\tilde{R}^2}{R^2} - \frac{1}{f s}} d\tilde{R} = \int_0^{z_A} \frac{\sqrt{F_2 + H^2}}{F_2 H} dz,
\]
(25)
where we have utilized the definition $F_2(z) \equiv \tilde{k}(1+z)^2 + \ell^{-2}$. Finally, with Eq. (20), one could express the gravitational horizon radius in terms of the dS$_5$ radius and a series of the observable quantities, i.e., the source redshift $z_A$, the present-day Hubble constant $H_B$, and the present-day spatial curvature density $\Omega_k$. Recalling Eq. (22), the dS$_5$ radius could be constrained under the low-redshift case for some specific GWs events. In the next section, we will use the data of the event GW170817/GRB 170817A [6, 17, 18] and the present-day spatial curvature density [44, 45] to constrain the dS$_5$ radius.

V. CONSTRAINT

In 2017, the LIGO and Virgo detectors detected the gravitational signal GW170817 originated from the merger of a binary system, which is located at the relatively close distance of $40^{+8}_{-3}$ Mpc from Earth [6, 19]. Then, 1.7 s later, GBM and SPI-ACS detected an EM signal, i.e., GRB 170817A, originated from the same place [17, 18], which is believed to be an EM counterpart of the event GW170817. The time delay between GW170817 and GRB 170817A opens a wide range of researches on the exotic physics, and could be explained by: (1) the astrophysical process of a binary NS merger [11–16]; (2) the modified propagation speed of the GWs [21–26]; (3) the shortcut of the five-dimensional null geodesic [27–29, 31, 32].

It was found that the hot torus orbiting around a rapidly spinning black hole will be formed within 100 ns after the binary NS merger and could result in SGRBs [11, 12, 14]. However, the exact process of the binary NS merger is still unknown. So the time-lag between the emissions of SGRBs and GWs is highly model-dependent. In other words, different astrophysical processes of the binary NS merger predicted in exotic astrophysical models will lead to different time-lags. In Refs. [15, 16], the authors pointed out that, if the long-lived binary-merger product forms after the binary NS merger, a black hole-torus system emitting SGRBs will eventually appear through the collapse of the binary-merger product. In such process, SGRBs are expected to be produced later than the peak magnitude of the emission of GWs and the time-lag could exceed $10^3$ s easily [15, 16]. The time-lag could be reversed in a different consideration. If the crust-core model is applied to the binary NS merger, then the crust will crack with the emission of GRBs seconds before the merger [13]. As a conclusion, these models with different astrophysical processes give a window of the time-lag, i.e., $(-100$ s, $1000$ s) [20]. Therefore, the astrophysical processes could be used to explain the time delay detected on the observations.

The present four-dimensional gravitational theories could be divided into two categories by Weyl criterion [56]. In the first class of theory, the propagation speed of GWs equals to the speed of light, while in the second class, the speed is not consistent with the speed of light. It is obvious that the time delay and the window of the time-lag of the events GW170817 and GRB 170817A could give a constraint on the second class of theory. According to Ref. [20], by ignoring the contribution of the intergalactic medium dispersion and considering the window of the time-lag $(-100$ s, $1000$ s), the speed of the GWs, $c_g$, is constrained to
\[
-2.4 \times 10^{-13} \leq \frac{c_g - c_s}{c_s} \leq 2.5 \times 10^{-14}.
\]
(26)
Note that, in Ref. [20], the travel distances of the SGRBs and GWs are chosen as 26 Mpc and the window of the time-lag is $(0$ s, $10$ s), so the speed of the GWs, $c_g$, is constrained to $-3 \times 10^{-15} \leq \frac{c_g - c_s}{c_s} \leq 7 \times 10^{-16}$. It implies that the modification on the speed of GWs is extremely small.

Note that this constraint is based on the four-dimensional theories. In higher-dimensional theories, EMWs are confined on the brane while GWs could propagate through the bulk. So the trajectory of the GWs could be the shortest path (called shortcut) in the bulk. Therefore, even if the GWs and SGRBs are emitted from the same source simultaneously and follow the null curves, they are expected to arrive at the observer successively and result in the time delay [29, 31, 32]. In this case, the gravitational horizon radius on the brane is affected by the structure of the extra dimension and the four-dimensional observer may misjudge the propagation speed of the GWs. In conclusion, for a given higher-dimensional gravitational theory which allows a shortcut, the constraint on the propagation speed of the GWs (26) will be changed. To show this, we will qualitatively analyse the modified constraint here. As shown in Fig. 1, for the event GW170817/GRB 170817A, when the GWs are detected, the SGRBs is arriving at the point $B'$. Then the timelike curve $BC$ denotes the time delay between the detections of GW170817 and GRB 170817A.
For a given higher-dimensional theory and under the low-redshift, the propagation speed of the GWs, $c_g'$, viewed by the five-dimensional observer obeys:

$$\frac{L_q}{c_g'} = \frac{L_q}{c_g} = \delta t_1 + \Delta t \equiv \Delta t_1, \quad (27)$$

$$\frac{L_q}{c_g'} = \frac{L_q}{c_g} = \delta t_2 - \Delta t \equiv \Delta t_2, \quad (28)$$

where $L_\gamma$ is the travel distance of the SGRBs from A to C (see Fig. 1), $L_q \equiv L_\gamma - \Delta L$ is the travel distance of the GWs from A to B along the shortcut, $\Delta t = 1.734$s is the time delay for the event GW170817/GRB 170817A, $c_\gamma$ is the propagation speed of the SGRBs on the brane, $\delta t_1 = 100\,\text{s}$ and $\delta t_2 = 1000\,\text{s}$ correspond to the different time-lags between the emissions of the SGRBs and GWs, and $c_{g1}'$ and $c_{g2}'$ are respectively the upper and lower bounds on $c_g'$. According to Eqs. (27) and (28), the bounds could be expressed as

$$c_{g1}' = \frac{L_q c_\gamma}{L_\gamma - c_\gamma \Delta t_1} = c_g - \frac{\Delta L}{L_\gamma - c_\gamma \Delta t_1}, \quad (29)$$

$$c_{g2}' = \frac{L_q c_\gamma}{L_\gamma + c_\gamma \Delta t_2} = c_g - \frac{\Delta L}{L_\gamma + c_\gamma \Delta t_2}, \quad (30)$$

or

$$\frac{c_{g1}' - c_\gamma}{c_\gamma} = \frac{c_{g2}' - c_\gamma}{c_\gamma} = -\delta_1, \quad (31)$$

$$\frac{c_{g2}' - c_\gamma}{c_\gamma} = \frac{c_{g2}' - c_\gamma}{c_\gamma} = -\delta_2, \quad (32)$$

where $\delta_1 \equiv \frac{\Delta L}{L_\gamma - c_\gamma \Delta t_1}$ and $\delta_2 \equiv \frac{\Delta L}{L_\gamma + c_\gamma \Delta t_2}$ are the correction terms determined by the shortcut in the higher-dimensional theory, and $c_{g1} = \frac{c_\gamma L_\gamma}{L_\gamma - c_\gamma \Delta t_1}$ and $c_{g2} = \frac{c_\gamma L_\gamma}{L_\gamma + c_\gamma \Delta t_2}$ are respectively the upper and lower bounds on $c_g$ in the four-dimensional theory. One finds that, for a given higher-dimensional theory allowing a shortcut, the propagation speed of the GWs is modified as follows:

$$-(2.4 \times 10^{-13} + \delta_2) \leq \frac{c_{g2}' - c_\gamma}{c_\gamma} \leq 2.5 \times 10^{-14} - \delta_1, \quad (33)$$

which shows that both the upper and lower bounds are lower.

In the next context, we will focus on the brane model constructed in Sec. II that may allow the existence of the shortcut and assume that GWs and SGRBs are originated from the same source simultaneously with the same speed. Further, we will use the observed time delay in the event GW170817/GRB 170817A to constrain the size of the dS$_5$ radius.

### A. Approximation

Before we calculate the constraint on the dS$_5$ radius, we should consider some approximate conditions which are useful to simplify our calculations in the following context. In Sec. II, we have set $k > 0$ and $\mu = 0$ so that the black hole horizon vanishes. As shown in Fig. 1, our Universe is a four-dimensional hypersurface embedded in the dS$_5$ spacetime. At a constant cosmological time $t_1$, it is a 3-sphere with a constant radial distance $R_1 = a(t_1)$. For Big Bang theory, our Universe (the brane) arises from a singularity, at which we have $a(t_0) = 0$. With this consideration, our Universe is to be embedded inside the cosmological horizon. On the other hand, one finds that, if events on the brane are causally connected by four-dimensional null or time-like geodesics, they are still causally connected by five-dimensional ones. In other words, the causal structure inside the cosmological horizon is not broken. Note that, as our Universe is inside the cosmological horizon, the present-day radius $R_B$ of our Universe should be smaller than $\sqrt{\ell^2}$, i.e., $k \ell^2 > 1$. Moreover, since the four-dimensional cosmological model will be recovered on the brane in the case of a large dS$_5$ radius [48–51], we further expect that $R_B^2 \ll k \ell^2$. To coincide with the assumption that our Universe mainly contains the dark energy, the dark matter, and the ordinary matter, the dynamics of the present-day Universe should follow the Friedmann equation introduced in the $\Lambda$CDM model. Then, in order to show the contribution of each component in our Universe, one has to keep the higher-order terms in Eq. (25). However, these higher-order terms will eventually make it hard to convert the quantity $s$ into the observable quantities. Therefore, we would like to discuss the constraint in two special cases, i.e., the Universe dominated by the dark energy (de Sitter model) and the dark matter (Einstein-de Sitter model), respectively.

### B. de Sitter model

The four-dimensional Friedmann equation in the de Sitter model is given by

$$H^2 = H_B^2 \left( \Omega_\Lambda \left( \frac{\Omega_\mu}{a^2} \right) \right), \quad (34)$$

where $\Omega_\Lambda$ is the present-day effective cosmological constant density and $H_B$ is the present-day Hubble constant. Replacing the Hubble constant in Eq. (24) with Eq. (34), one finds that the rescaled coordinate time interval $\tilde{T}_{AB}$ between the emission and detection of the GWs obeys the following equation:
tanh\left(\frac{T_{AB} H_B \sqrt{-\Omega_k}}{\ell}\right) = \frac{\sqrt{-\Omega_k} \sqrt{1 + \ell^2 H_B^2(1 - \Omega_k)(1 + z_A - \sqrt{1 + z_A(2 + z_A)\Omega_k})}}{(1 + z_A)\Omega_k (1 - \ell^2 H_B^2(1 - \Omega_k)) - \sqrt{1 + z_A(2 + z_A)\Omega_k}}.

(35)

The integral of Eq. (23) has a similar form:

\begin{align*}
\tanh\left(\frac{T_{AB} H_B \sqrt{-\Omega_k}}{\ell}\right) &= \frac{\sqrt{-\Omega_k} \sqrt{1 + Q\ell^2 \left(1 + z_A\right) \sqrt{\frac{Q}{H_B^2} + \Omega_k} - \sqrt{\frac{Q}{H_B^2} + (1 + z_A)^2 \Omega_k}}}{(1 + z_A)\Omega_k (1 - Q\ell^2) - \sqrt{\frac{Q}{H_B^2} + \Omega_k} \sqrt{\frac{Q}{H_B^2} + (1 + z_A)^2 \Omega_k}},
\end{align*}

(36)

where \( Q \equiv \frac{1}{s} + \frac{1}{\ell^2} \) is a new parameter introduced for convenience. It is obvious that Eqs. (35) and (36) are equivalent as long as \( s = \ell^2 [H_B^2(1 - \Omega_k)\ell^2 - 1]^{-1} \). So it is the solution of Eq. (25). Finally, one obtains the gravitational horizon radius in terms of the observable quantities as follows:

\begin{align*}
\hat{r}_g &= \frac{\arctan\left(\frac{\sqrt{-\Omega_k}}{\sqrt{\Omega_k + \frac{1}{\ell^2 H_B^2(1 - \Omega_k)}}}\right) - \arctan(\sqrt{-\Omega_k})}{H_B \sqrt{-\Omega_k}},
\end{align*}

(37)

which indeed equals to the photon horizon radius (21). It implies that, in our model, if the Universe is dominated by the dark energy, the extra dimension may not cause the discrepancy between the gravitational horizon radius and photon horizon radius. Therefore, there is no time delay between the GWs and EMWs emitted simultaneously.

One should note that Eq. (24) highly depends on cosmological models. So one may get a different solution of the quantity \( s \) in other cosmological models. In the next subsection, we will give the gravitational and photon horizon radii in the Einstein-de Sitter model.

### C. Einstein-de Sitter model

Now, we assume that our Universe is dominated by the dark matter. In this case Eq. (35) and hence the solution of \( s \) will change. Since the calculation is extremely tedious, we do not show all results here. Instead, for the event GW170817/GRB 170817A, we give the gravitational horizon radius and photon horizon radius as follows:

\begin{align*}
\hat{r}_g &\approx \frac{\arctan\left(\frac{H_B \sqrt{-\Omega_k}}{\sqrt{C_2 + H_B^2 \Omega_k}}\right) - \arctan\left(\frac{H_B \sqrt{-\Omega_k}}{\sqrt{C_2 + H_B^2 \Omega_k}}\right)}{H_B \sqrt{-\Omega_k}}.
\end{align*}

(38)

\begin{align*}
\hat{r}_\gamma &= \frac{2 \left[\arctan(\sqrt{-\Omega_k}) - \arctan\left(\frac{\sqrt{-\Omega_k}}{\sqrt{1 + z_A(2 + z_A)\Omega_k}}\right)\right]}{H_B \sqrt{-\Omega_k}}.
\end{align*}

(39)

Here \( C_1 \) and \( C_2 \) are very complex parameter functions of the dS\(_5\) radius, \( z_A, H_B, \) and \( \Omega_k \). We also do not show them here. Generally speaking, the two horizon radii (38) and (39) are different and the discrepancy between them will result in the time delay. Taking advantage of the events GW170817 and GRB 170817A, we can obtain the constraint on the dS\(_5\) radius.

From the data of the LIGO-Virgo detectors, the source redshift of GW170817 is about \( z_A = 0.008^{+0.002}_{-0.003} \) [6, 19]. The collaboration of the LIGO-Virgo detectors, GBM, and SPI-ACS shows a time delay about \( \Delta t = 1.74^{+0.05}_{-0.05} \) s between GW170817 and its EM counterpart GRB 170817A [6, 17–20]. Concerning the uncertainty, we choose three sets of the redshifts (0.01, 0.008, 0.005). In addition, we first assume that the GWs and EMWs are emitted simultaneously. Then, with respect to the present-day spatial curvature density, the dS\(_5\) radius is shown in Fig. 2(a). Based on the Planck data [44, 45], the present-day Hubble constant is chosen as \( H_B = 67.36 \) km s\(^{-1}\) Mpc.

From Fig. 2(a), one finds that the dS\(_5\) radius increases with the decrease of \( |\Omega_k| \). The uncertainty in the measured source redshift of GW170817 could result in a small correction to the constraint on the dS\(_5\) radius. It means that the dS\(_5\) radius is not sensitive to the redshift ranging from 0.005 to 0.01. The analysis of the cosmic microwave background by Planck collaboration has given a constraint on the present-day spatial curvature density \( |\Omega_k| \lesssim 10^{-3} \) (see Table 5 in Ref. [44] and Table 4 in Ref. [45]). According to it, we constrain the dS\(_5\) radius...
to $\ell \gtrsim 7.5 \times 10^2$ Tpc. As shown in Fig. 2(a), the bound also denotes a constraint on the present-day spatial curvature density $|\Omega_k| \lesssim 3.6 \times 10^{-11}$. Therefore, if the constraint on the spatial curvature density obtained by the future Planck collaboration contradicts to this result, the five-dimensional model considered in this paper should be ruled out then.

The previous context mainly bases on the assumption that the GWs and SGRBs are emitted simultaneously. Here we will consider the contribution of different astrophysical processes of the binary NS merger on the dS$_5$ radius. Since we have assumed that the GWs have the same speed as the SGRBs, the shortcut will naturally result in a larger gravitational horizon radius than the photon horizon radius. Therefore, we choose the window ranging from $-100 \text{s}$ to $1.734 \text{s}$ and investigate the effect of them on the dS$_5$ radius. As shown in Fig. 2(b), an interesting result is that such time-lag introduced in the present alternative astrophysical models has little effect on the constraint. It could be explained by the fact that the dimensionless discrepancy $\Delta r \equiv (\tilde{r}_g - \tilde{r}_\gamma)/L$ with $L = 1 \text{ Mpc}$ between the two horizon radii decreases extremely fast with the increase of the dS$_5$ radius (see Fig. 2(c)).

**VI. CONCLUSION**

The recent detection of GW170817 by the LIGO-Virgo detectors and the following detection of GRB 170817A by GBM and SPI-ACS imply a binary NS merger near the NGC 4399 [6, 17, 18] and prove the link between SGRBs and binary NS mergers [19]. It therefore opens a wide range of astrophysical researches on the NS (e.g. Refs. [57–63]). On the other hand, the time delay between the GWs and SGRBs may reveal the existence of exotic physics. In higher-dimensional theories, GWs could pass through the bulk while EMWs are confined on the brane (see Fig. 1). Assuming that both the GWs and SGRBs follow null curves and are originated from the same source simultaneously, a discrepancy between the gravitational horizon radius and photon horizon radius may appear and result in the time delay observed by the detectors. Since the gravitational horizon radius is affected by the structure of the extra dimension, it is then expected that the time delay will carry some information of the extra dimension. Therefore, the time delay is a key to investigating the structure of the extra dimension [29, 31, 32].

We considered a static spherically symmetric dS$_5$ spacetime, where our Universe (viewed as a brane) is embedded. The brane is located inside the cosmological horizon so that the scale factor could increase from $a(t = 0) = 0$ to $a(t = t_B)$. We calculated the gravitational horizon radius and photon horizon radius under the assumption that our Universe is dominated by the dark energy/the dark matter. We found that, in the for-
mer case, the discrepancy between the two horizon radii vanishes, while in the latter case, it appears and the observed time delay may result from it. Accordingly, we used the time delay to constrain the $dS_5$ radius with respect to different present-day spatial curvature densities (see Fig. 2). We found that the constraint on $\Omega_k$ given by the Planck collaboration \cite{118, 119} leads to a constraint on the $dS_5$ radius $\ell \gtrsim 27.1 \text{ Mpc}$. Moreover, the extra condition, $k \ell^2 \gtrsim 1$, used to embed our Universe excludes out some parameter regions and gives a stronger constraint on the $dS_5$ radius, e.g., $\ell \gtrsim 7.5 \times 10^2 \text{ Tpc}$ for $z_A = 0.01$ and $\ell \gtrsim 1.1 \times 10^3 \text{ Tpc}$ for $z_A = 0.005$. Consequently, we gave a lower bound on the $dS_5$ radius as $\ell \gtrsim 7.5 \times 10^2 \text{ Tpc}$. We also found that this bound gives a constraint on the present-day spatial curvature density $|\Omega_k| \lesssim 3.63 \times 10^{-11}$, which is useful to judge the validity of the five-dimensional model considered in this paper.

We also simply analysed the sensitivity of our constraint to the parameters. We investigated the effect of the uncertainty in the source redshift on our constraint. The result shows that, for the event GW170817, the uncertainty gives little effect on the constraint. Note that this constraint was obtained by assuming the simultaneous emissions of the GWs and SGRBs. However, for different astrophysical processes of the binary NS merger, there is a time-lag between the emissions of the GWs and SGRBs. Therefore, we chose some non-vanishing time-lags to show the effect of different astrophysical processes on the constraint. Since the GWs follow the shortcut which is a null curve, we considered a window of the time-lag ranging from $-100 \text{ s}$ to $1.734 \text{ s}$. We finally found that, the constraint is not sensitive to the time-lag in this range.

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