Quantum Purity at a Small Price: Easing a Black Hole Paradox

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ABSTRACT

Following Hawking, it is usual to mimic the effect of collapse space-time geometry on quantum fields in a semi-classical approximation by imposing suitable boundary conditions at the origin of coordinates, which effectively becomes a moving mirror. Suitable mirror trajectories induces a close analogue to the radiance of black holes, including a flux of outgoing radiation that appears accurately thermal. If the acceleration of the mirror eventually ceases the complete state of the radiation field is a pure quantum state, even though it is indistinguishable from an accurately thermal state for an arbitrarily long period of time and in a precise sense differs little from “pure thermal” closely followed by “vacuum”. Suspicions that the semiclassical calculation of black hole radiance gives evidence for the evolution of pure into mixed states are criticized on this basis. Possible extensions of the model to mimic black holes more accurately (including the effects of back reaction and partial transparency), while remaining within the realm of tractable models, are suggested.
(The talk as given reported on work that has now been published [1] [2] and I see no point in duplicating this material. Instead I am taking the opportunity to discuss some joint work contained in my student Christoph Holzhey’s thesis [3]. The discussion here is qualitative. More quantitative discussions including comparison between coarse and fine-grained entropy for particular cases of radiance are contained in the thesis, and we plan to write them up fully in due course.)

Semi-classical calculations of the radiation [4, 5] from black holes indicate that their emission is the same as one might expect from an ideal gray body. It is commonly believed that these calculations are very accurate for black holes having masses much larger than the Planck mass (and away from any extremal limit). This raises a conceptual problem that has been much discussed, as follows. One can certainly imagine forming a black hole from matter in a pure quantum state. One then finds, in an approximation which appears accurate, that it radiates to produce a mixed state. Yet the evolution of a pure into a mixed state would violate the basic principles of quantum mechanics as they are currently understood. Inspired by this conflict, Hawking [6] boldly proposed to jettison some of the usual assumptions of quantum mechanics. However, naturally many physicists are reluctant to tinker with the foundations of our most practically successful physical theory in response to such an esoteric thought experiment, and so far this approach has not been very fruitful.

Indeed there are many things we do not understand about quantum gravity, and one’s first reaction might be to hope that this conceptual problem will resolve itself once we obtain a proper understanding of the ultraviolet properties of gravity,
in particular its behavior under conditions of extreme curvature. According to the classical theory of black holes, such regions – singularities – do inevitably occur. They are repositories for our ignorance, and appear to provide promising scapegoats for any conceptual problems we encounter with black holes.

After a little further reflection, however, it may come to seem less reasonable to hope that any possible behavior near the singularities could help resolve the particular puzzles posed by black hole radiance. For the bulk of the predicted radiation from a massive black hole can be demonstrated to arise in an analysis where large curvatures never appear, and quantum corrections due to fluctuations of the gravitational field ought to be negligible. (Of course because of ultraviolet problems the graviton loop corrections, not only to this but to all physical processes, diverge. However any “reasonable” cutoff procedure, such as supposing that the Planck mass sets the scale for the magnitude of the coefficients of higher-dimensional operators in the effective Lagrangian, leads one to expect a small result.) In other words, the semi-classical approach to the radiance, leading to apparently thermal radiation, governs the bulk of the history of the black hole evaporation process, as measured for example by the energy or the entropy of the emitted radiation. Thus one might be tempted to say – and it has often, in the literature, been stated – that by the time the black hole gets to the Planck mass and quantum gravity effects plausibly come into play it is too late. For at that point there is no longer enough material (energy) left to restore the missing correlations, which are necessary to render a mixed state with arbitrarily gigantic entropy pure.

Here it is demonstrated, in the context a simple soluble model that can mimic many of the features of black hole radiance, that this argument is too quick. The model in question is the imposition of boundary conditions on a moving surface. In
the simplest case one imposes reflection boundary conditions, thus arriving at the moving mirror model. The global structure of the Cauchy problem in this model is quite transparent, and one can address the question of quantum purity on the basis of general principles. On the other hand the moving mirror boundary conditions generally induce radiation, that for suitable trajectories can closely mimic the Hawking radiation from black holes. We shall see that quantum purity is enforced in subtle ways at little cost in energy, even after an arbitrarily long period of apparently thermal radiation. Very roughly, the result of these considerations is to put the paradox back where it belongs, at the Planck scale – and also, as we shall see, to suggest plausible model pictures of how it might be resolved.

Of course the moving mirror model is not new (see the references in [5]). Indeed it was used with a philosophy similar to the one adopted here in two very ingenious papers by Carlitz and Willey [7], which appear not to have received the attention they deserve.

1. Causal Structure of the Mirror Problem

Consider the evolution of a massless scalar field in 1+1 dimensions subject to the boundary condition

$$\phi(z(t), t) = 0 \quad (1.1)$$

along the mirror trajectory $z = z(t)$. The scalar field is defined to vanish on the left-hand side of the mirror. The effect of the boundary condition is of course that rays incident on the mirror reflect off it.

As will be discussed below, the mirror plays the role of the origin $r = 0$ in space – the center of the hole – in the black hole problem. Thus reflection off the mirror
mimics the propagation of an ingoing wave to the center and its emergence as an outgoing wave. The distortion of space-time – essentially the lengthening of space (and shortening of time) near the surface of the hole – in during collapse has a dynamical effect similar to the effect of a rapidly receding mirror in 1+1 dimensional flat space. Indeed the fundamental effect is that rays reflected off a rapidly receding mirror are severely red-shifted – as are the rays, crucial to Hawking’s analysis, which barely avoid being trapped behind the incipient horizon.

Three types of mirror trajectories are illustrated in Figures 1-3.

The first trajectory type describes a mirror that accelerates away from rest at $t = 0$ and approaches the speed of light asymptotically. Let the asymptote light-ray be denoted $A$ as in Figure 1. Since we are dealing, for simplicity, with a massless field we may consider only left-moving modes. Let us define points 1, 2, 4, 5 as in the Figure, and use the same labels to distinguish the rays emanating from these points at $t = 0$.

We see that rays such as 1 and 2, which begin to the left of $A$, intersect the mirror and propagate out to spatial infinity at the right, denoted in deference to the black hole interpretation as $\mathcal{I}^+$. On the other hand rays such as 4 and 5, which begin to the right of $A$, never intersect the mirror. They propagate to the left infinity, denoted as $\mathcal{H}^+$. (This infinity may seem a little funny from the point of view of the metaphorical interpretation of $z$ as the effective position of the black-hole origin. The point is that the effective radial distance from the point of view of wave propagation is most appropriately measured in intervals of the tortoise coordinate $r_*$, which diverges to $-\infty$ at the black hole horizon. By the way, these rays leave the Figure in finite affine time.)

Now consider the problem of the the evolution of a quantum state defined
for $z > 0$ at $t = 0$ into the distant future. Naturally one should consider first the 
ground state, defined by the absence of positive-frequency modes. It is evident that 
the time interval between the arrival of 1 and 2 at a given point in space before they 
reflect is much dilated after they reflect. Thus the frequency of waves is altered, 
and negative-frequency waves can acquire positive-frequency components. This 
would be interpreted as the creation of an excited state on $\mathcal{I}_+$. For an appropriate 
mirror trajectory, as we shall see, the state on $\mathcal{I}_+$ will be a thermal state, with its 
temperature related to the rate of acceleration of the mirror. Clearly all information 
concerning the state of the field $\phi$ in region to the left of A at $t = 0$ is propagated 
to $\mathcal{I}_+$. 

Rays such as 4 and 5 beginning to the right of A propagate undisturbed to $\mathcal{H}_+$. 
Clearly all information concerning the state of the field $\phi$ in region to the right of 
A at $t = 0$ is propagated to $\mathcal{H}_+$. If we start with the ground state on $t = 0$, an 
observer making measurements on $\mathcal{H}_+$ also sees his natural ground state. 

Now according to basic principles of quantum mechanics, which of course are 
certainly not contradicted by anything in the simple model problem under consid-
eration, a pure state localized to the left of A would propagate into a pure state 
on $\mathcal{I}_+$ and a pure state localized to the right of A would propagate into a pure 
state on $\mathcal{H}_+$. However, the ground state at $t = 0$ is not pure when restricted either 
eto either segment. The positive-frequency condition forces consideration of modes 
which extend over both intervals, and introduces correlations between these inter-
vals. Indeed, the two-point function $\langle \phi(1)\phi(4) \rangle$ at $t = 0$, for example, certainly 
does not vanish. Furthermore, this correlation will propagate in a simple way into 
the future, introducing correlations between $\mathcal{I}_+$ and $\mathcal{H}_+$. Thus we should not be 
shocked to find a mixed state if we consider $\mathcal{I}_+$ by itself, without regard to (trac-
ing over) the state on $\mathcal{H}_+$. And this indeed is what we do find: the correlation functions on $\mathcal{I}_+$, for the appropriate trajectory of this type, are precisely thermal, and therefore certainly must be described by a mixed state on $\mathcal{I}_+$.

The phenomenon that may be a shock to one’s intuition is that it is correlations between the rich thermal state on $\mathcal{I}_+$ and the apparently barren desert on $\mathcal{H}_+$ which insure purity of the whole. Thus for example the expectation value of the energy-momentum tensor vanishes, and its multi-point correlators are vacuous (i.e. indistinguishable from the vacuum), when restricted to $\mathcal{H}_+$ – but its cross-correlators between $\mathcal{H}_+$ and $\mathcal{I}_+$ do not vanish. This peculiar phenomenon, whose existence and nature is made quite transparent by the foregoing extremely elementary observations, was noted and emphasized by Carlitz and Willey. (However they somewhat obscured the issue by claiming in effect that particle creation on $\mathcal{I}_+$ is uniquely and locally related to particle creation on $\mathcal{H}_+$, which is not the case.) It shows in as dramatic fashion as one could desire that the purity of a big complicated state with gigantic entropy (in any sense) can be restored at little – here actually at zero – cost in energy.

Now let us consider the mirror trajectory depicted in Figure 2, which is the same as the one discussed for a long interval of time, but such that the mirror eventually stops accelerating. Then all rays eventually intersect the mirror, and get reflected to $\mathcal{I}_+$. Thus we obtain on $\mathcal{I}_+$ a pure state which looks thermal for an arbitrarily long time. Of course once the mirror stops accelerating there is no longer any radiation emitted. The transition to zero acceleration can be done smoothly, so that only a small burst (whose magnitude is essentially independent of the length of the interval over which thermal radiation has occurred) accompanies it. Thus altogether one finds, similar to the previous case, that quantum purity
comes at a small price.

The situation of Figure 2 is not yet a model for complete black hole evaporation. For although positive frequencies at late times are indeed reflected into positive frequencies, and there is no particle production, yet the frequency is highly redshifted. Thus real particles at late times will sense (in the interpretation of the mirror as the locus of the origin) a remnant that delays them for a long time and saps their energy. It is left for the reader to invent witty names for such a remnant.

Finally in Figure 3 we have the situation where the mirror returns to rest. Real particles emitted at late times, which intersect the mirror during its second period of rest, behave as if passing through the origin of empty space in the analogue problem. Thus this provides a model for a black hole that evaporates completely. From the nature of the construction, any pure initial state evolves into a pure final state.

2. Connection to Collapse Geometry and Radiance

Now I would like to indicate, following Unruh [8] a precise connection between the mirror model and collapse geometry, and summarize the resulting correlation functions and radiation flux.

Consider for simplicity a spherically symmetric collapsing shell of matter. We have vacuum inside and outside the shell, while the shell carries a given amount of mass (and possibly other quantum numbers). Thanks to Birkhoff’s theorem, we know the metric in both regions:

\[
ds^2 = \begin{cases} 
  dr^2 - d\tau^2 - r^2 d\Omega^2, & \text{for } \tau + r \leq V_s; \\
  \lambda^2 dt^2 - \lambda^{-2} dr^2 - r^2 d\Omega^2, & \text{for } t + r \geq v_s.
\end{cases}
\]  

(2.1)

Note that in order to exhibit the metric in each region in its familiar (static) form,
two different sets of coordinates had to be used. It is convenient to introduce light-
cone coordinates in each region. In the interior region we use simply $U = \tau - r$
and $V = \tau + r$, whereas in the outer region we first define the tortoise-coordinate
$r_*$ through
\[
\frac{dr_*}{dr} = \frac{1}{\lambda^2},
\]
and then take $u = t - r_*$ and $v = t + r_*$ as light-cone coordinates. The space-time
is described by the metric:
\[
ds^2 = \begin{cases} 
dUdV - r^2d\Omega^2, & \text{for } V \leq V_s; \\
\lambda^2dudv - r^2d\Omega^2, & \text{for } v \geq v_s,
\end{cases}
\]
where $r$ is determined through the relations
\[
V - U = 2r, \quad \text{for } V \leq V_s; \\
v - u = 2r_*(r), \quad \text{for } v \geq v_s. 
\]

When we paste together the two coordinate systems for the interior and exterior
region to form a global coordinate-system, we can choose to coincide with (2.3)
either in the exterior or in the interior region. The first choice is natural from
the point of view of a distant observer, while the second is more convenient to
implement the boundary condition at the origin and to display the complete space-
time structure.

Let us consider first the former choice, that is using $u$-$v$-coordinates in both
regions and looking for a satisfactory coordinate-transformation $U(u)$ and $V(v)$.
In the infinite past the space-time is flat and there is no difference between the
two coordinate systems. This implies that we can choose $V(v) = v$. We find
the function $U(u)$ by demanding that along the worldline $v = v_s$ of the shell the
coordinate $r$ should agree in both systems, because it has a gauge-invariant meaning (it determines the area of a two-sphere at constant radius and time). Applying (2.4) along $v = v_s$ we obtain the implicit relation:

$$r_s \left( r = \frac{v_s - U(u)}{2} \right) = \frac{v_s - u}{2}. \quad (2.5)$$

Differentiating this equation along the worldline of the shell we find, with the help of the defining equation (2.2) for $r_s$,

$$\frac{dU}{du} = \lambda^2(u, v_s), \quad (2.6)$$

so that the metric becomes:

$$ds^2 = \begin{cases} \lambda^2(u, v_s)du dv - r^2 d\Omega^2, & \text{for } v < v_s; \\ \lambda^2(u, v)du dv - r^2 d\Omega^2, & \text{for } v > v_s, \end{cases} \quad (2.7)$$

which is continuous along $v_s$. The metric is, of course, only valid for non-negative values of $r$, i.e. for $v \geq U(u)$. The world-line of the origin is therefore described by

$$v_o(u) = U(u). \quad (2.8)$$

Since nothing can go beyond the regular origin, i.e. to negative $r$, it acts like a perfectly reflecting mirror.

In the $u-v$-frame the shell never crosses the horizon since $r_s$ and $t = v_s - r_s$ diverge as the horizon is approached. On the other hand we know that the shell reaches the origin in finite proper time. In order to describe the whole space-time, including the interior of the black hole it is convenient to use the $U-V$-coordinates,
which provide a complete cover since they contain the origin until the shell reaches it. The space-time is then described by

\[
    ds^2 = \begin{cases} 
    dUdV - r^2d\Omega^2, & \text{for } v \leq v_s; \\
    \lambda^2(u,v)\lambda^{-2}(u,v_s)dUdV - r^2d\Omega^2, & \text{for } v \geq v_s,
    \end{cases}
\]

(2.9)

In spite of its appearance, the metric is regular on the horizon where \( \lambda^2 = 0 \). The origin is stationary at \( V = U \) until the shell reaches it.

For a shell of mass \( M \) one has explicitly for the tortoise coordinate

\[
    r_s(r) = r + 2M \ln |r - 2M| + c,
\]

(2.10)

and thus from (2.5)

\[
    u = U - 4M \ln |(-4M - U + v_s)/2| - 2c.
\]

(2.11)

c is here an arbitrary integration constant. As \( U \) approaches \( U_h = v_s - 4M \), \( u \) diverges, which identifies the line \( U = U_h \) with the future horizon. Alternatively, the finite range of \( U \) implies according to (2.8) that the origin approaches the light-like asymptote \( v = U_h \) at late times as viewed in the \( u-v \)-frame. At very early times, the origin is at rest because as \( u \rightarrow -\infty \), \( U \approx u \). At late times we can invert (2.11) by neglecting the linear term. We find that \( U(u) \) is of the general form

\[
    U(u) = c_1 + c_2 e^{-\kappa u},
\]

(2.12)

where \( \kappa = 1/4M \) is the surface gravity. (This relation is nothing but the familiar transformation [9] between Eddington-Finkelstein and Kruskal-coordinates:

\[
    U_K = -4Me^{-u/4M}.
\]

(2.13)

At late times our coordinate \( U \) therefore agrees with Kruskal \( U_K \), while \( V \) equals \( v \) is always of the Eddington-Finkelstein type.)
The upshot of all this is simply to justify partially but precisely the idea expressed earlier, that the collapse geometry may be modeled by a moving mirror problem. In this model the mirror arises at the origin of coordinates (not the horizon); its “motion” is an effective representation of the distortion of space-time in the collapse. An important feature left out of the model in its simplest form is the non-trivial spatial curvature outside the shell.

For simple forms of matter, e.g. a free massless scalar field, the moving mirror problem is eminently tractable. Any quantity of interest may be calculated explicitly. For example one has for the correlation function of the fields

\[ G_{\text{vac}}(1, 2) \equiv \langle 0| \phi(1)\phi(2) |0 \rangle = \]

\[ = \frac{1}{4\pi} \ln \frac{(U_2 - U_1 + i\delta)(V_2 - V_1 + i\delta)}{(U_2 - V_1 + i\delta)(V_2 - U_1 + i\delta)}. \]  

Indeed this function manifestly satisfies the wave equation with the correct singularity and the moving mirror boundary condition, and reduces to the correct vacuum value before the mirror motion begins. For the energy-momentum tensor describing the emitted radiation one finds (see, for example, [5])

\[ \langle 0| T_{\mu\nu} |0 \rangle = \frac{\delta_{\mu\nu}\delta_{\rho\sigma}}{12\pi} \sqrt{U'} \frac{d^2}{du^2} \sqrt{1/U'}. \]  

All energy \( n \)-point functions are determined by \( \langle 0| T_{\mu\nu} |0 \rangle \) and \( G_{\text{vac}}(1, 2) \). For example the energy two-point function

\[ C_{\mu\nu,\alpha\beta}(1, 2) \equiv G_{\alpha\beta,\mu\nu}^E(1, 2) - G_{\alpha\beta}^E(1)G_{\mu\nu}^E(2). \]  

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is evaluated to be

\[
C_{uu, uu}(1, 2) = \frac{1}{8\pi} \frac{U'(1)^2 U'(2)^2}{(U(2) - U(1))^4},
\]
\[
C_{vv, vv}(1, 2) = \frac{1}{8\pi} \frac{1}{(v(2) - v(1))^4},
\]
\[
C_{uu, vv}(1, 2) = \frac{1}{8\pi} \frac{U'(1)^2}{(v(2) - U(1))^4}.
\]

(2.17)

Not unexpectedly, the correlations diverge for two points connected by a light-like line in the direction of the energy flux in question. Note that there are correlations between leftward and rightward flux, as anticipated of this chapter. The correlations are, however, by no means sharply localized \(^\#1\).

One may compare these expressions to the thermal correlation function (populating only right-movers) which is easily found to be

\[
G_{th}(1, 2) = -\frac{T}{4} (|u_1 - u_2| + |v_1 - v_2|) + \frac{1}{4\pi} \ln \left[ \left( 1 - e^{-2\pi T|u_1 - u_2|} \right) \left( 1 - e^{-2\pi T|v_1 - v_2|} \right) \right],
\]

(2.18)

and to the two-point correlation of outward flux:

\[
C_{uu, uu}(1, 2) = \frac{\kappa^4}{8\pi^2} \frac{e^{2\kappa|u_1 - u_2|}}{(e^{\kappa|u_1 - u_2|} - 1)^4}.
\]

(2.19)

There is perfect agreement if we substitute for \(U\) the particular trajectory \(U \propto -e^{-\kappa u}\). Moreover, with that choice,

\[
\frac{\partial}{\partial u_1} \frac{\partial}{\partial u_2} G_{th}(1, 2) = \frac{\partial}{\partial u_1} \frac{\partial}{\partial u_2} G_{vac}(1, 2),
\]

(2.20)

so that all correlations of outward energy flux will be thermal. Correlations involving \(T_{vv}\) are, of course, not thermal. In fact, we see from (2.17) that the two-point

\(^\#1\) Carlitz and Willey [7] claimed to have found a unique local correlation between quanta on the two sides. Adapting their argument to our notation, they computed (2.17) in the \(u-v\)-frame in the case where a horizon forms and analytically continued to \(\tilde{v}\) defined beyond the horizon. The correlation is local in these analytically continued variables, but not in the physical ones.
correlation \( C_{\nu_1,\nu_2}(1,2) \) for the mirror is the ordinary correlation expected for a vacuum state.

In view of our previous qualitative discussion it is appropriate to note three properties of the radiation:

1. For the mirror trajectory of the specified form, it is precisely thermal, in the sense that all its correlation functions are precisely those one would find for radiation from a black body of temperature \( T \).

2. It is determined locally from the vacuum state in the past, in the sense that the correlation functions in an interval at late times at spatial infinity can be determined by propagating a finite interval of vacuum at \( t = 0 \) forward, bouncing once off the mirror. In particular, the correlations in this interval are not affected by what the mirror does much later.

3. Nevertheless if, as in Figures 2 and 3, the mirror does not asymptote to the speed of light, then the final quantum state is pure. The apparently unlimited entropy of associated with a long period of thermal radiation is thus, from a microscopic point of view, illusory. This radiation is not truly thermal; it is correlated with subsequent “vacuum” fluctuations.

Evidently it is dangerous to think of microscopic, fine-grained entropy as a substance which can be measured locally and once created is never destroyed. Why one can get away with this metaphor for coarse-grained entropy in thermodynamics is another story ...
3. Limitations and Extensions

I hope this discussion has illustrated the use of the moving mirror as a conceptually transparent model for some aspects of black hole radiance. Its strength is that it is easy to think about and is a more-or-less normal quantum mechanical system against which one can check alleged black hole conundrums. Its obvious weaknesses are that the mirror is treated as a given rather than being dynamically determined by the matter, and that the effects of spatial curvature are ignored. I would like to close by mentioning a few directions in which the model invites extension.

As I mentioned before, to simulate complete evaporation of a black hole one would like to bring the mirror back to rest. An important limitation to this possibility, however, is that the mirror may radiate as it is decelerated. Indeed if we define
\[ \sqrt{U'} = e^{-g}, \]
then we obtain from (2.15) the total energy flux radiated after the thermal period in the form:
\[ E = \frac{1}{12\pi} \int_{u_e}^{u_r} (g'^2 - g'') \, du. \]

At \( u_e \), the end of the thermal period, we have \( g \approx \kappa u_e / 2 \) and \( g' \approx \kappa / 2 \). If we demand that the mirror be at rest after \( u = u_r \) (so that \( g = g' = 0 \)) and minimize the integral (3.2), the \( g'' \)-term leads to a constant boundary-term in the variational procedure and a linearly decreasing \( g \) is optimal. The trajectory is therefore of the thermal form (2.12) (with negative \( \kappa \)). The integrated flux decreases with increasing available time. If we suppose that deceleration sets in only when the
hole has reached the Planck mass, then the available energy is quite small and one must stretch out the deceleration process in order to minimize the radiation. Carlitz and Willey thus deduced that the time interval over which the mirror gets back to rest, and space-time returns to normal, would have be much longer than the lifetime of the black hole.

While this sort of slowly cooling remnant appears to be a logically consistent possibility, in the absence of a specific mechanism it seems a sufficiently strange outcome that one is open to alternatives. I find it appealing to imagine that an appropriate analogue of spatial curvature outside the hole, which is surely there but is ignored in the simple mirror model, could play an essential role in mitigating the problem of radiation accompanying mirror deceleration. The analogue would be a potential barrier to the right of the mirror, whose height depends on the mass of the acceleration of the mirror (i.e. mass of the analogue hole) and becomes very large as the hole mass approaches the Planck mass. It is plausible that a mirror hidden behind a sufficiently high potential barrier could decelerate to rest in a limited time without catastrophic emission of radiation. If the barrier becomes truly infinite, we have a mirror analogue of the black hole pinching off into another universe. In this case the wave function on $\mathcal{I}_+$ is insufficient to reconstruct the original state. In a sense information is lost, but of course in the mirror analogue we know exactly where to find it.

Another possible extension is to promote the mirror to a quantum mechanical variable, instead of a fixed source. Its state will then be correlated in a non-trivial way with the emitted radiation. The most primitive effect is that it recoils, and thus its position is described by a wave function correlated with the wave function of the emitted radiation. With a heavy mirror treated semi-classically, this would
presumably provide a model for the intrinsic black hole entropy which appears in classic black hole thermodynamics.

REFERENCES

1. S. Coleman, J. Preskill, and F. Wilczek, *Nucl. Phys.* B370, 577 (1992).
2. C. Holzhey and F. Wilczek, *Nucl. Phys.* B380, 447 (1992).
3. C. Holzhey, Princeton University Thesis, (1992), unpublished.
4. S. Hawking *Comm. Math. Phys.* 43, 199, (1975).
5. N. D. Birrell and P. C. W. Davie, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge (1982).
6. S. Hawking, *Phys. Rev.* D14, 2460, (1976).
7. R. Carlitz and R. Willey *Phys. Rev.* D36, 2327, 2336, (1987).
8. W. Unruh, *Phys. Rev* D14, 870, (1976).
9. C. Misner, K. Thorne and J. Wheeler, *Gravitation*, Freeman, NY (1973).