An algorithm based on ABNL-MRF model for SAR image segmentation

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Abstract. In this letter, a segmentation method is proposed based on an adaptive Bayesian non-local MRF (ABNL-MRF) model. Aimed at the multiplicative speckle existing in SAR images, a patch-similarity measure based on a ratio of probability is introduced firstly. Then non-local similar pixel-blocks are adopted to guide the segmentation of the noisy image. However, non-local method over-smoothes edge regions and makes them inaccurate though it is robust for speckle. A rectification index is designed according to the homogeneity degree of local window. Based on the value of rectification index, a non-local constraint term is adaptively integrated into the potential function. Thus a suitable prior is defined that guarantees both the smoothness of the denoised image and the preservation of its structure. Experimental results demonstrate the very good performance of the proposed method.

1. Introduction

A crucial problem for SAR image segmentation is how to make the speckle to be suppressed and the detail characteristics preserved effectively. Bayesian segmentation method based on the Markov random fields (MRF) [1] has received great attention for its way to model and process image data, as it seamlessly integrates spatial interaction between pixels into the process of image segmentation. Despite their success, MRF models still leave much room for improvement.

Recently, the NL-means (non-local means) filter [2] was introduced to remove white Gaussian noise while effectively preserving details of images. From that, many variations of the NL-means method have been proposed in the segmentation and denoising area. However, few works have tried to tackle the task of combining NLM method with Bayesian segmentation within MRF framework.

Based on these considerations, a Bayesian method incorporates non-local spatial relationships is proposed, which consists of two steps. At the first step, a novel definition for patch-similarity measure is introduced, which is more suitable for multiplicative speckle. At the second step, ABNL-MRF method is proposed by adding a non-local constraint term into the standard potential function. The smoothness of the denoised image and the preservation of its structure have also been considered. A rectification index is designed according to the homogeneity degree of local window and added into the prior energy function. Experimental results prove the effectiveness of the proposed algorithm.
2. Markov Random Field
The Markov Random field model was presented in 1984 [3]. Let denote a two dimensional image of which size is M×N. It is assumed that Y is the observation of S and X the labeling. Define a neighborhood system for
\[ N = \{ N_i | \forall i \in S \} \]
where \( N_i \) is the set of pixels neighboring \( i \). The neighboring relationship has the following properties:
1. a pixel is not neighboring to itself: \( i \notin N_i \),
2. the neighboring relationship is mutual: \( i \in N_j \iff j \in N_i \).

Suppose that both the a priori probabilities \( P(X = x) \) of configuration \( x \) and the likelihood densities \( P(Y = y | X = x) \) of the observation \( y \) is give. Based on the principle of maximum a posterior (MAP), the best estimate one can get from these random observations is to maximizes the a posteriori probability of labeling field,
\[ \hat{x} = \text{Arg max}(P(X = x | Y = y)) \]

Given the observed image, then the posterior probability, based on the Bayesian theory, is proportional to
\[ P(X = x | Y = y) \propto P(Y = y | X = x)P(X = x) \]

According to the Hammersley-Clifford theorem [3], the density of \( X \) is given by the following Gibbs density
\[ P(X = x) = \frac{1}{Z_s} \exp(-U_s(x | x_{N_i}) / T) \]
where \( Z_s \) is the normalized constant, and \( T \) is the temperature value. \( U_s \) is the energy function which is a sum of clique potentials \( V_s(x) \) over all possible cliques \( C \)
\[ U_s(x) = \sum_{c \in C} V_s(x) \]

The segmentation problem based on MRF method turns to be the optimization of a combinatorial energy function
\[ E(X | Y) = \log(P(Y | X)) - U_s(X) \]

3. Patch-similarity Measure
The patch-similarity measure in NL-means filter is defined as [2],
\[ w(i, j) = \frac{1}{Z_i} \exp\left(-\frac{1}{h^2} \left\| Y_{N_i} - Y_{N_j} \right\|_{L^2,G}^2 \right) \]
where \( Y_{N_i} \) and \( Y_{N_j} \) denote two patches which have the same size and shape centered at pixel \( i \) and \( j \), respectively; \( h \) acts as a filtering parameter; \( \left\| \right\|_{L^2,G}^2 \) is the weighted Euclidean norm using a Gaussian kernel with standard deviation \( G \), and \( Z_i \) is the normalized constant.

However, Equation (1) is only robust to additive noise and invalid for multiplicative speckle of SAR image [4].
Numerous extensions based of the NL-Means proposal have been proposed. Coupe et al. [5] extended the NL-means method to speckled image by defining the Pearson distance to measure the relativity of two image patches. The noise is assumed to be a mixture of additive and multiplicative noise, and the multiplicative noise obeys Gaussian distribution. Deledalle et al. [6] designed a kind of relativity measurement combined of the distance between current restored-image patches and the distance between noisy patches. The relativity is approached using statistical inference in an iterative procedure. Frery et al. [7] present analytic expressions for several kinds of stochastic distances, such
as Kullback-Leibler, Rényi, Bhat- charyya, and Hellinger distances, as well as for the $\chi^2$ distance [8] under the complex Wishart model.

For SAR image, a new patch-similarity measure based on ratio distance is defined and proved robust to multiplicative speckle [4]:

$$w(i,j) = \frac{\| Y_{N_i} \cdot Y_{N_j} \|_{E,L,G}}{\sum_{k=1}^{M} G(k) \cdot p\left( \frac{Z_{N_i}(k)}{Z_{N_j}(k)} \right)}$$  

$$p\left( \frac{Z_{N_i}(k)}{Z_{N_j}(k)} \right) = \frac{2(2L - 1)!}{\Gamma^2(L)} \cdot \frac{(r_{ij}(k))^{2L-1}}{[(r_{ij}(k))^{2} + 1]^{2L}}$$

$$r_{ij}(k) = \frac{Z_{N_i}(k)}{Z_{N_j}(k)}$$

where $L$ is the equivalent number of looks (ENL) of SAR image; $M$ is the number of pixels in the local neighbor window; $Z$ is the observed value of the patch; $\Gamma(\bullet)$ is the gamma function.

In above definition of patch-similarity measure, not only the grey value of a single pixel but also the entire neighbor structure are taken into account. For the multiplicative noise model, the difference between the pixel blocks is transformed to a ratio and is mapped to the probability density function. Thus it has great superiority in suppression for speckle noise. As is shown in Fig.1, the blue marker is the center pixel and its neighborhood. The blocks with red markers are similar to the center one; the green pixel is not similar whereas it has the same value with the pixel labelled blue.

![Figure 1](image.png)

**Figure 1.** The block matching result based on the patch-similarity measure.
(a) The result of region I.
(b) The result of region II.

## 4. Non-local MRF method

A large amount of intrinsic speckle brings trouble for SAR image segmentation. In SAR image, the neighboring pixels of a given pixel are likely to be corrupted by speckle. However, traditional Potts model in MRF method [3] only takes the impact of the neighborhood pixels into account, thus it cannot effectively suppress speckle; meanwhile, the assumption that each pixel in the neighborhood is identical leads to poor description of edge region and inaccurate segmentation.

Therefore, a non-local MRF model for multiplicative speckle noise is designed. The clique function can be defined
\[ V'_{NL}(x_i, x_j) = \begin{cases} 
0, & x_i = x_j \\
\beta \cdot \sum_{k=1}^{M} G(k) \left[ p\left(\frac{Z_{N_i}(k)}{Z_{N_j}(k)}\right) \right], & x_i \neq x_j 
\end{cases} \] \tag{11}

where \( \beta \) is the potential parameter; \( M \) is the pixel number in the local neighborhood. Then the prior energy function is,

\[ U(x_i) = \sum_{j \in \Omega_i} V'_{NL}(x_i, x_j) \]

\[ = \beta \cdot \sum_{j \in \Omega_i} \sum_{k=1}^{M} G(k) \left[ p\left(\frac{Z_{N_i}(k)}{Z_{N_j}(k)}\right) \right] \cdot \left[1 - \delta(x_i, x_j)\right] \] \tag{12}

where \( \Omega_i \) is the search window of pixel \( i \); \( \delta(t) \) is Dirac function whose value is one when \( t = 0 \) or zero otherwise. The local neighborhood \( N_i \) and the search window \( \Omega_i \) are square.

Compared with traditional MRF method with Potts model, the non-local MRF incorporates the non-local ideal. By adding a patch-similarity measure term into the standard potential function, it enables the robustness to speckle noise as shown in (11) and (12). The patch-similarity measure has been introduced in (8).

5. Our proposed ABNL-MRF

However, it should be noted that the non-local method makes the segmentation in edge region over-smooth and inaccurate, nevertheless it is insensitive to speckle. So a rectification should be taken according to the region type. In different types of regions, various value of potential parameter and size of search window should be chosen to suppress speckle noise, so as to keep a balance between excessive smooth and structure preserving.

Inspired by the definition of CV (Coefficient of Variation), the rectification index is designed as

\[ ratio(i) = \frac{\text{var}(Y_{N_i})}{\text{mean}(Y_{N_i})} \] \tag{13}

where \( \text{var}(\bullet) \) and \( \text{mean}(\bullet) \) are the intensity variance and mean of the image patch.

Then the prior potential function defined (11) and (12) are redefined by introducing the variable of \( ratio \) as

\[ V'_{ABNL}(x_i, x_j) = \begin{cases} 
0, & x_i = x_j \\
\frac{\beta}{\text{ratio}(i)} \cdot \sum_{k=1}^{M} G(k) \left[ p\left(\frac{Z_{N_i}(k)}{Z_{N_j}(k)}\right) \right], & x_i \neq x_j 
\end{cases} \] \tag{14}

\[ U(x_i) = \sum_{j \in \Omega_i} V'_{ABNL}(x_i, x_j) = \frac{\beta}{\text{ratio}(i)} \cdot \sum_{j \in \Omega_i} \sum_{k=1}^{M} G(k) \left[ p\left(\frac{Z_{N_i}(k)}{Z_{N_j}(k)}\right) \right] \cdot \left[1 - \delta(x_i, x_j)\right] \] \tag{15}

where the value of \( \text{ratio}(i) \) and the size of search window \( \Omega_i \) adaptively alter according to the
homogeneity degree of the local window [9]. In the smooth region, a larger search window and potential parameter are chose to suppress speckle noise, that is, the value of $\text{ratio}(i)$ should be smaller and $\Omega_i$ be larger; on contrast, in edge region, the search window and potential parameter are reduced so as to keep the image geometry structure and detail characteristic. Thus, the non-local information is adaptively integrated into the prior energy function.

6. Experimental results
In this section, the proposed method is compared with the traditional MRF method [3] and the multi-resolution Markov random field (MRMRF) method [10]. The experiments are performed using Matlab on Windows platform. The computer configuration is Intel(R) Core(TM) i7-2006 CPU with a frequency of 3.47 GHz.

![Figure 2. The segmentation results for a four-look SAR image (733×822).](image)

(a) SAR image 1 of the harbor. (b) MRF method. (c) MRMRF method. (d) Our proposed method.

Segmentation results are given in Fig.2, respectively. The testing SAR image is a harbor in Taiwan acquired by Radarsat-1 satellite (C-band, 4-looks, 10-m resolution). For computational purposes, we have fixed a similarity square neighborhood $N_i$ of 7×7 pixels; the search windows $\Omega_i$ has varied according to the value of $\text{ratio}(i)$. The Standard deviation of the Gaussian kernel function $G$ has been fixed to 10, and $\beta$ has been fixed to 1. As it can be seen, compared with Fig.2 (b) and Fig.2 (c), the regions in Fig.2 (d) are smoother and are described more accurately with speckles cleanly removed and edge structure effectively preserved.

Besides the visual subjective evaluation, two objective criterions [11] are utilized for the evaluation of segmentation: the normalized log measure $|D|$ and the variance $RI_{\text{var}}$ of the ratio image.
The normalized log measure $|D|$ is expressed as

$$|D| = \left| \sum_{k=1}^{m} \frac{n_k}{n} \ln r_k \right|$$  \hspace{1cm} (16)

where $n_k$ is the number of pixels of segment $k$; $n$ is the total number of pixels of the SAR image; $r_k$ denotes the ratio image of segment $k$. The closer the value of $|D|$ approaches 0, the better regional homogeneity the segmentation results have. $RI_{var}$ describes the change of the pixel value in the ratio image. It is given by

$$RI_{var} = \sum_{k=m}^{n} \frac{n_k}{n-1} (r_k - \overline{r}_k^2)$$ \hspace{1cm} (17)

As shown in Table 1, the proposed algorithm outperforms both the MRF and MRMRF algorithm with smaller value of $|D|$ and $RI_{var}$. The better performance of the suppression to speckle noise and the preservation to edge structure comes because the non-local information was utilized to guide the segmentation. Moreover, the prior model is adaptively adjusted to the degree of homogeneity of the local window.

Table 1. Comparison of performance for segmentation results

| Segmentation methods | $|D|$  | $RI_{var}$  |
|----------------------|-------|-------------|
|                      | Region I | Region II | Region I | Region II |
| MRF                  | 0.1283 | 0.1155 | 0.0050 | 0.0040 |
| MRMRF                | 0.1075 | 0.1073 | 0.0047 | 0.0037 |
| Ours                 | 0.1020 | 0.1011 | 0.0041 | 0.0032 |

7. Conclusion
An adaptive Bayesian method for SAR image segmentation is presented. In the method, the non-local information is adopted to guide the noisy image segmentation. According to different region types, the value of rectification index and the size of search window are adaptively adjusted to match the local property of a region. Thus, the segmentation results are more robust to noise while keeping the detail features accurately. Experimental results illustrate a better and higher accuracy performance of the proposed method.

It cannot be claimed that our framework provides an ultimate solution to the problem of accurate segmentation in computer vision. One limitation of this work is how the computational cost can be reduced for real-time processing. Another limitation is that the introduced patch-similarity measure is based on the assumption of homogeneity. It does not always hold in the SAR image segmentation. This is an open question and subject to the future research.

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