Spin 1 fields in Riemann-Cartan space-times via Duffin-Kemmer-Petiau theory

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We consider massive spin 1 fields, in Riemann-Cartan space-times, described by Duffin-Kemmer-Petiau theory. We show that this approach induces a coupling between the spin 1 field and the space-time torsion which breaks the usual equivalence with the Proca theory, but that such equivalence is preserved in the context of the Teleparallel Equivalent of General Relativity.

Dedicated to Professor Gerhard Wilhelm Bund on the occasion of his 70th birthday.

I. INTRODUCTION

The Duffin-Kemmer-Petiau (DKP) equation is a first order relativistic equation, similar to the Dirac’s one, which describes fields of spin 0 and 1 [1–4]. The basic aspects and properties of the DKP equation which are necessary to the comprehension of this work can be found in the references [5,6], where it was adopted the same metric signature. For a historical review covering the theory until the decade of 1970 we refer to reference [7].

Recently there have been a renewed interest in DKP theory. For instance, it has been studied in the context of QCD [8], covariant hamiltonian dynamics [9], in the Causal Approach [10], in the context of five-dimensional galilean covariance [11], in the scattering $K^+$-nucleus [12], and in curved space-times [5,13], among other situations.

One important question concerning the DKP theory is about the equivalence or not of its spin 0 and 1 sectors to the Klein-Gordon (KG) and the Proca theories, respectively. This is an old question for which, nowadays, still lacks a complete answer. Recently, there have been some efforts to give strict proofs of equivalence between the KG equation and the spin 0 sector of the DKP equation in various situations [14–16]. In the same context, some aspects regarding the minimal interaction with the electromagnetic field have been clarified [6,17]. Moreover, the equivalence between the DKP and the KG and the Proca fields for spin 0 and 1 has also been proved in the context of a riemannian space-time [5].

On the other hand, the study of the DKP theory for massive spin 0 fields minimally coupled to Riemann-Cartan (RC) space-time has been carried out in the reference [18], where it was shown that, in the context of Einstein-Cartan theory, the DKP formalism naturally induces an interaction between the spin 0 field and the space-time torsion, breaking the equivalence with the KG equation, which does not present any interaction with torsion. In the same reference it was also discussed the conceptual differences between this kind of interaction and that which appears in the context of the Teleparallel Equivalent of General Relativity (Teleparallelism theory), where the spin 0 sector of DKP field and KG field are completely equivalent.
Our aim in this paper is to complete this analysis by studying massive spin 1 fields in Riemann-Cartan space-times using the DKP theory, both in the context of Einstein-Cartan and Teleparallelism theories, and comparing the results with those obtained in the framework of Proca’s field approach. In the next section we present the DKP theory in Minkowski space-time. In section 3 we introduce minimally coupling to the Riemann-Cartan space-time and select the spin 1 sector in order to compare the results with those obtained through Proca’s field. In section 4 we analyse both the DKP and the Proca fields in the context of the Teleparallelism theory and, in section 5, we present our concluding remarks.

II. DKP FIELD IN MINKOWSKI SPACE-TIMES

The Duffin-Kemmer-Petiau equation in Minkowski space ($M^4$) is given by

$$i\beta^a \partial_a \psi - m \psi = 0,$$

where $a = 0, 1, 2, 3$ are spatiotemporal Minkowski indexes. The matrices $\beta^a$ obey the DKP algebra, given by

$$\beta^a \beta^b \beta^c + \beta^c \beta^b \beta^a = \beta^a \eta^{bc} + \beta^c \eta^{ba};$$

with $\eta^{ab}$ being the metric tensor of Minkowski space-time with signature $(+−−−)$. The DKP equation (1) is very similar to the Dirac’s one, but the algebraic properties of $\beta^a$ matrices, which have no inverses, make it more difficult to deal with. This equation can also be obtained from the Lagrangian density

$$\mathcal{L} = \frac{i}{2} \overline{\psi} \beta^a \partial_a \psi - \frac{i}{2} \left( \partial_a \overline{\psi} \right) \beta^a \psi - m \overline{\psi} \psi,$$

where $\overline{\psi} = \psi^\dagger \eta^0$, $\eta^0 = 2 \left( \beta^0 \right)^2 - 1$ and we choose $\beta^0$ to be hermitian and $\beta^i$ ($i = 1, 2, 3$) anti-hermitian.

It can be shown [2,4] that DKP algebra has only 3 inequivalent irreducible representations, with degrees 1, 5 and 10. The first one is trivial ($\beta^a = 0$), having no physical significance, while the other two represent fields of spin 0 and 1, respectively. Moreover, for any representation, one can define a set of operators (Umezawa’s “projectors”) which selects the scalar and vector sectors of the DKP field [4].

III. THE EINSTEIN-CARTAN THEORY

From now on we follow the definitions and notations from the references [5,18] for the DKP field in curved manifolds. We remember that the covariant derivative $\nabla$ in the Einstein-Cartan theory (which assumes a Riemann-Cartan space-time geometry) has an affine connection $\Gamma_{\mu\nu}^\alpha$, not necessarily symmetric in the lower indexes, whose antisymmetric part $Q_{\mu\nu}^\alpha$ is the Cartan torsion, i.e.,

$$Q_{\mu\nu}^\alpha = \frac{1}{2} \left( \Gamma_{\mu\nu}^\alpha - \Gamma_{\nu\mu}^\alpha \right).$$

Then we can write the affine connection as

$$\Gamma_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha - K_{\mu\nu}^\alpha,$$

where $\Gamma_{\mu\nu}^\alpha$ is the Christoffel symbol (or the riemannian part of the connection) and $K_{\mu\nu}^\alpha$ is the contorsion tensor, defined as

$$K_{\mu\nu}^\alpha = -Q_{\mu\nu}^\alpha - Q_{\nu\mu}^\alpha + Q_{\mu\nu}^\alpha.$$
The covariant derivative of the DKP field is given by \[5\]
\[
\nabla_\mu \psi = D_\mu \psi = \left( \partial_\mu + \frac{1}{2} \omega_{\mu a b} S^{a b} \right) \psi
\]
and is formally similar to that of Dirac’s field \[19\]. In this expression \(\omega_{\mu a b}\) is the spin connection and \(S_{a b} = \beta_a \beta_b - \beta_b \beta_a\). The matrices \(\beta^\mu\) are defined through contraction with the tetrad (or vierbein) fields \(e^\mu_\alpha\), i.e. \(\beta^\mu = e^\mu_a \beta^a\), and they satisfy the generalized DKP algebra
\[
\beta^\mu \beta^\nu \beta^\sigma + \beta^\sigma \beta^\nu \beta^\mu = \beta^\nu g^{\sigma \alpha} + \beta^\alpha g^{\nu \mu},
\]
where \(g^{\nu \mu}\) is the Riemann-Cartan metric tensor. In the Einstein-Cartan theory the spin connection can be written in terms of the affine connection and the tetrad field as \[19\]
\[
\omega_{\mu a b} = \gamma_{\mu a b} - K_{\mu a b},
\]
where \(K_{\mu a b} = -K_{\mu b a} = e_\alpha^a e_\beta^b K_{\mu \alpha \beta}\),
\[
\gamma_{\mu a b} = -\gamma_{\mu b a} = e_\mu^i (C_{\alpha i a} - C_{\alpha i b} - C_{\alpha i b}),
\]
being \(C_{\alpha i a}\) the Ricci rotation coefficients†.
The Lagrangian density for the DKP field minimally coupled \[20,21\] to the Riemann-Cartan manifold is given by \[18\]
\[
L = \sqrt{-g} \left[ i \bar{\psi} \partial_\mu \psi - \frac{i}{2} \left( \nabla_\mu \bar{\psi} \right) \beta^\mu \psi - m \bar{\psi} \psi \right],
\]
from which we get the equation of motion for the massive DKP field in the Einstein-Cartan theory as
\[
i \beta^\mu \nabla_\mu \psi + \frac{i}{2} K_{\sigma \mu} \beta^\mu \psi - m \psi = 0.
\]
We can promptly see that this equation differs from the one that would be obtained from the Minkowskian DKP equation of motion (1) through the minimal coupling procedure as is usual in Einstein-Cartan theory \[22,23\].

A. Spin 1 sector

Now we use the Umezawa’s “projectors” \(R^\mu\) and \(R^{\mu \nu}\) in order to analyse the spin 1 sector of the theory. We remember that each component of \(R^\mu\psi\) is a vector and each one of \(R^{\mu \nu}\psi\) is a second rank antisymmetric tensor \[4–6\]. Applying these operators on the left of equation of motion (14) we get, respectively,
\[
m R^\alpha \psi = i \tilde{D}_\nu \left( R^{\mu \nu} \psi \right),
\]
\[
m \left( R^{\mu \nu} \psi \right) = i \left( \tilde{D}^\nu R^\mu \psi - \tilde{D}^\mu R^\nu \psi \right),
\]
where the derivative operator \(\tilde{D}_\nu\) is defined as
\[
\tilde{D}_\mu = \nabla_\mu + \frac{1}{2} K_{\sigma \mu}^\sigma.
\]
Combining both the equations (15) we get the equation of motion for the massive vector field \(R^\alpha \psi\)
\[
\tilde{D}_\beta \tilde{D}_\alpha T^{\alpha \beta \mu} + m^2 \left( R^\mu \psi \right) = 0,
\]
or, written explicitly,
\[
\left( \nabla_\beta + \frac{1}{2} K_{\sigma \beta}^\sigma \right) \left( \nabla_\alpha + \frac{1}{2} K_{\sigma \alpha}^\sigma \right) T^{\alpha \beta \mu} + m^2 \left( R^\mu \psi \right) = 0,
\]
where we have defined \(T^{\alpha \beta \mu} = g^{\alpha \beta} (R^\mu \psi) - g^{\alpha \mu} (R^\beta \psi)\).

†The brackets in this expression denote antisymmetrization of the enclosed indexes.
1. Proca’s field

In Minkowski space-time the Lagrangian density for Proca’s field is given by
\[ \mathcal{L}_M = -\frac{1}{4} F_{ab} F^{ab} + \frac{1}{2} m^2 A_a A^a, \] (19)
where \( F_{ab} = \partial_a A_b - \partial_b A_a \) is the field strength tensor.

When the procedure of minimal coupling to the Riemann-Cartan manifold is performed on the above Lagrangian we obtain
\[ \mathcal{L}_U = \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + F_{\mu\nu} Q^{\mu\nu} A^\sigma - Q_{\mu\nu} Q^{\mu\nu} A_\rho A^\rho \right), \] (20)
where
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \] (21)

The equations of motion for the field \( A_\mu \), obtained from this Lagrangian, are given by
\[ (\nabla_\nu + K_{\alpha\nu}^\sigma) (\nabla_\sigma A_\mu - \nabla_\mu A_\sigma) + m^2 A_\mu = 0, \] (22)
or
\[ (\nabla_\beta + K_{\alpha\beta}^\sigma) \nabla_\alpha T_{\alpha\beta\mu} + m^2 A_{\mu} = 0, \] (23)
being \( T^{\alpha\beta\mu} = g^{\alpha\beta} A_\mu - g^{\alpha\mu} A^\beta \). These equations show the usual interaction between Proca’s field and torsion, as known in literature, and are very different from equations (17) and (18) given by the DKP approach for the spin 1 field in the Einstein-Cartan theory.

B. Comparison between DKP and Proca fields

In order to compare the results obtained through DKP field with those from Proca’s one we will use the explicit representation of \( \beta \) matrices given in [6]. With this representation we have
\[ R^\mu \psi = \begin{pmatrix} \psi_\mu \\ 0_{3x1} \end{pmatrix}, \quad R^{\mu\nu} \psi = \begin{pmatrix} \psi^{\mu\nu} \\ 0_{3x1} \end{pmatrix}, \quad \mu = 0, 1, 2, 3, \]
where the ten-component DKP field is
\[ \psi = (\psi^0, \psi^1, \psi^2, \psi^3, \psi^4, \psi^5, \psi^6, \psi^7, \psi^8, \psi^9)^T. \]

We have denoted the field components as
\[ \psi^4 = \psi^{23}, \psi^5 = \psi^{31}, \psi^6 = \psi^{12}, \psi^7 = \psi^{10}, \psi^8 = \psi^{20}, \text{ and } \psi^9 = \psi^{30}, \]
so that we get from equation (15)
\[ \psi^{\mu\nu} = \frac{i}{m} (\bar{D}^{\nu} \psi^{\mu} - \bar{D}^{\mu} \psi^{\nu}). \] (24)

Finally, we can define \( \psi^\mu = \sqrt{m^2} A^\mu \), with \( A^\mu \) being a real vector field. Then, using the explicit form of \( \psi \) given above, the DKP Lagrangian (13) can be rewritten in terms of the fields \( A^\mu \) as
\[ \mathcal{L} = \sqrt{-g} \left( -\frac{1}{4} U_{\mu\nu} U^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \right), \] (25)
where
\[ U^{\mu\nu} = \bar{D}^{\mu} A^{\nu} - \bar{D}^{\nu} A^{\mu}. \] (26)

The Lagrangian above can be written explicitly as
\[ \mathcal{L} = \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + F_{\mu\nu} Q^{\mu\nu} A^\sigma + Q_{\mu\nu} Q^{\mu\nu} A_\rho A^\rho - \frac{1}{4} F_{\mu\nu} \Sigma^{\mu\nu} + \frac{1}{2} \Sigma_{\mu\nu} Q^{\mu\nu} A^\sigma - \frac{1}{16} \Sigma_{\mu\nu} \Sigma^{\mu\nu} \right), \] (27)
where the tensor $\Sigma^{\mu\nu}$ is defined as

$$\Sigma^{\mu\nu} = K^{\alpha\beta}_{\mu} A_{\nu} - K^{\alpha\beta}_{\nu} A_{\mu}.$$  \hspace{1cm} (28)

Due to the presence of the last three terms in the above Lagrangian it is not equivalent to the minimally coupled Proca’s Lagrangian \((20)\). In fact, it is straightforward to see from \((26)\) that the minimally coupled DKP Lagrangian (equation \((13)\) or \((27)\)) is equivalent to that obtained from the Minkowskian Proca Lagrangian by means of the following non-minimal substitution\(^\dagger\)

$$\partial_a \rightarrow \tilde{D}_a.$$  \hspace{1cm} (29)

IV. DKP FIELD IN THE TELEPARALLEL THEORY

The analysis of DKP field coupled to grivation in the framework of the Teleparallel Equivalent of General Relativity \([24]\) was developed in reference \([18]\). Specifically, it was considered the spin 0 sector of the theory and the results were compared to those obtained from KG field. Here we will extend such analysis to the spin 1 sector of the DKP theory and compare the results with those obtained from Proca’s field.

We remember that the DKP Lagrangian minimally coupled to the Riemann space-time (which is the space-time of General Relativity and is a special case of the Riemann-Cartan space-time whose torsion vanishes identically) can be equivalently written in terms of the Teleparallel structure, which describes fields in a Weitzenböck space-time (another special case of a Riemann-Cartan space-time whose curvature vanishes identically).

To construct the equations for DKP field in the Teleparallel framework we will start from the corresponding equations for DKP theory in General Relativity, as given in reference \([6]\). Then, we will simply replace the riemannian quantities in these equations by the corresponding teleparallel ones, according to the rules \([18]\)

$$\Gamma_{\alpha\beta}^\mu \rightarrow \Gamma_{\alpha\beta}^\mu + K_{\alpha\beta}^\mu ,$$  \hspace{1cm} (30a)

$$\tilde{\nabla}_\mu \psi \rightarrow \left( \partial_\mu - \frac{1}{2} K_{\mu\alpha\beta} S^{\alpha\beta} \right) \psi ;$$  \hspace{1cm} (30b)

$$\tilde{\nabla}_\mu \tilde{\psi} \rightarrow \tilde{\psi} \left( \tilde{\nabla}_\mu + \frac{1}{2} K_{\mu\alpha\beta} S^{\alpha\beta} \right) .$$  \hspace{1cm} (30c)

Making so, the General Relativity Lagrangian written in terms of the Teleparallel structure is given by \([18]\)

$$L = e \left\{ \frac{i}{2} \bar{\psi}^\beta \left( \partial_\mu - \frac{1}{2} K_{\mu\alpha\beta} S^{\alpha\beta} \right) \psi - \bar{\psi} \left( \tilde{\nabla}_\mu + \frac{1}{2} K_{\mu\alpha\beta} S^{\alpha\beta} \right) \bar{\psi} \psi \right\} - m \bar{\psi} \psi ,$$  \hspace{1cm} (31)

from which we get the equation of motion

$$i \beta^\mu \left( \partial_\mu - \frac{1}{2} K_{\mu\alpha\beta} S^{\alpha\beta} \right) \psi - m \bar{\psi} \psi = 0 .$$  \hspace{1cm} (32)

A. Spin 1 sector

Now we apply the operators $R^\mu$ and $R^{\mu\nu}$ on the equation of motion \((32)\). From the results of \([6]\) and making use of the rules \((30)\) we get

$$R^\mu \psi = \frac{i}{m} D_\nu (R^{\mu\nu} \psi)$$  \hspace{1cm} (33a)

$$R^{\mu\nu} \psi = \frac{i}{m} \left[ D^\nu (R^\mu \psi) - D^\mu (R^\nu \psi) \right] ,$$  \hspace{1cm} (33b)

\(^\dagger\)Besides, of course, the global multiplication by the factor $\sqrt{-g}$. 

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where the covariant derivative $D_\mu$ is defined as
\[ D_\mu = \nabla_\mu + K_\mu, \quad \text{with} \quad \nabla_\mu = \partial_\mu + \Gamma_\mu, \tag{34} \]
In these last expressions $\Gamma_\mu$ is the Cartan connection \[18\]
\[ \Gamma_{\mu\nu}^\alpha = e^\alpha_i \partial_\mu e^i_\nu, \]
which is associated with the Weitzenböck space, and $K$ is the corresponding contorsion tensor, as given by equation (6).

Combining the equations (33a) and (33b) we obtain the equation of motion for the spin 1 DKP field in the Teleparallel framework
\[ D_\nu [D^\nu (R_\mu \psi) - D_\nu (R^\mu \psi)] + m^2 (R^\mu \psi) = 0. \tag{35} \]

**B. Proca’s field**

The General Relativity Lagrangian density of Proca’s field, written in terms of the Teleparallel structure, is given by
\[ L = e \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \right). \tag{36} \]
This expression could be obtained from the Proca’s Lagrangian in Minkowski space by means of the following prescription \[25\] **
\[ \partial_a \rightarrow D_\mu \equiv \partial_\mu + \Gamma_\mu + K_\mu = \nabla_\mu + K_\mu. \]

The stress tensor for the Proca’s field in the Teleparallel structure is given by
\[ F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu. \]

By using the explicit form of $D_\mu$ and the definition of torsion and contorsion tensors, it is an easy task to verify that
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \]
and from the Lagrangian (36) we get the teleparallel version of the Proca’s equation
\[ D_\nu F^{\nu\mu} + m^2 A^\mu = 0. \tag{37} \]

Thus, comparing equations (35) and (37), we conclude that the DKP and the Proca theories for massive spin 1 fields in the context of the Teleparallel description of General Relativity are completely equivalent. This is an expected result because both Proca theory and the spin 1 sector of DKP theory give the same results in the context of General Relativity, of which the Teleparallelism is an equivalent description.

**V. CONCLUDING REMARKS**

In the reference \[18\] it was shown that the spin 0 sector of DKP theory is not equivalent to Klein-Gordon theory in the context of Einstein-Cartan theory with minimal coupling procedure. Differently to what happens with the KG theory, in which does not appear any interaction between the scalar field and the space-time torsion, in the DKP theory this interaction naturally arises. It is interesting to notice that the concept of a scalar field interacting naturally (i.e. through a minimal coupling procedure) with torsion \[26\] is useful in the context of a quantum theory of matter fields.

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\[5\]The derivative $D_\mu$ is sometimes referred to as the *teleparallel version* of General Relativity’s covariant derivative \[25\], because it is nothing more than the General Relativity covariant derivative written in terms of the teleparallel quantities.

\[**\]Besides, we must multiply the whole lagrangian by a factor $e = \det|\epsilon^a_\mu| = \sqrt{-g}$ to make it a scalar density.
in a Riemann-Cartan space-time because it gives the possibility of constructing a renormalizable theory [27,28].

Here we completed this analysis by studying the massive spin 1 sector of DKP theory. We showed that the spin 1 sector of DKP theory and Proca theory are inequivalent in the context of Einstein-Cartan theory with minimal coupling. Although in this context Proca’s formalism allows an interaction between massive spin 1 fields and the space-time torsion (see the Lagrangian (20)), the DKP formalism presents a more general interaction with the torsion, containing all the terms present in the Proca Lagrangian plus three additional terms (see equation (27)).

Still extending the analysis of reference [18], we considered the massive spin 1 sector of DKP theory in the framework of the Teleparallel Equivalent of General Relativity. Nevertheless the fact that in this framework the gravitational field is associated with the space-time torsion and not with curvature, the DKP and the Proca approaches give identical results. This was an expected result since both formalisms are equivalent in the context of General Relativity.

Finally, as it is well known, the application of the minimal coupling procedure to the Maxwell Lagrangian induces a coupling to the space-time torsion which breaks the gauge invariance of the theory, a consequence which is usually avoided by the introduction of non minimal couplings. From the results of reference [18] and those of the present work we saw that, in the context of Einstein-Cartan theory, the DKP field with minimal coupling is equivalent to performing non-minimal couplings in the KG or Proca’s fields. Then, it seems interesting to investigate if the use of DKP field in the study of massless spin 1 fields†† on Einstein-Cartan backgrounds can give further insights on the incompatibility between gauge invariance and the interaction with torsion. This question is presently under our investigation.

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††We observe that DKP formalism for massless spin 1 fields cannot be obtained as a limiting case of the massive theory, as it happens in the case of Maxwell and Proca theories.
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