On the resummation of double logarithms in the process $H \to \gamma\gamma$

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Abstract

The decay of the Higgs boson into two photons is discussed. Using Sudakov technique, we study a new type of double logarithms in the case of the light quark loop. First, we examine the origin of such logarithms at the one loop level. Then we perform a two-loop analysis, and finally we present an explicit result of the resummation of double logarithms in all order in QCD. The phenomenological applications are discussed.

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1 Introduction

The neutral scalar Higgs boson $H$ is an important ingredient of the Standard Model (SM) and is the only SM elementary particle which has not been detected so far. The lower limit on $m_H$ of approximately 70 GeV was obtained from direct searches at LEP [1].

The properties of the Higgs particle were studied in details by many authors (see, for example reviews [3, 4, 5, 6, 7, 8, 9] and references therein).

The decay of the Higgs boson into two photons will play an important and unique role. It is of interest for the following reasons:

- for the light Higgs masses this decay mode provides a clear signature for the search at hadron colliders
- the $\gamma\gamma$ width determines the cross section for the Higgs production in $\gamma\gamma$ collisions [10, 11, 12]
- it is well known that the $H \rightarrow \gamma\gamma$ vertex can serve as a counter of the particles with masses larger than the Higgs boson mass.

It is widely accepted that a detailed experimental investigation will be performed at the Next Linear $e^+e^-$ Collider which will also operate in the photon-photon mode [10, 13], as a Photon Linear Collider (see the review of physical applications in [14]). The possibility of studying scalar Higgs boson in the s-channel production, for example in a reaction

$$\gamma + \gamma \rightarrow H \rightarrow q\bar{q}$$

is not so obvious, because of large background from direct quark-antiquark production. Fortunately, it was demonstrated that using the polarizations of initial photons one can suppress background process at high energy despite the QCD corrections to the background process is large [11, 12, 13]. If the task of efficient suppression will be achieved, one can hope that the Higgs will be investigated in detail in $\gamma\gamma$ collisions and the idea of precise determination of the $H\gamma\gamma$ vertex will then become an important phenomenological issue.

Therefore, the $H\gamma\gamma$ vertex should be investigated as thoroughly as possible in the SM as well in the Supersymmetric extension of the SM (SSM).

In the present paper we study QCD corrections to the decay rate in the limit of the small ratio of the quark and Higgs masses, $m_q$ and $m_H$. In this limit, an interesting type of QCD double logarithms (DL) appears and we make an attempt of their resummation. The fact of the emergency of DL in the process $H \rightarrow \gamma\gamma$ was pointed in [16]. The similar phenomena was found and studied in [17] in the another physical process, $\gamma\gamma \rightarrow q\bar{q}$ in the $J_Z = 0$ channel. The authors of [15] have performed a careful analysis of the origin of the DL appearance and studied DL terms up to two-loop accuracy.

There are two reasons why the problem of DL in $H \rightarrow \gamma\gamma$ seems to be worth of being investigated. The first reason is more or less academical. Once we have found a process where a new type of DL (in a sense that it is not a Sudakov-like logarithms)
appear it is interesting to understand their origin and even resum them. From QED and QCD we know a few examples where DL appear and can be resummed in observable quantities (see, for example, reviews \[18, 19\], and also recent paper \[15\] and references therein). We claim now that the $H \rightarrow \gamma\gamma$ decay considered here is the next new example.

The second reason of interest is phenomenological. As we mentioned above the precise measurement of the $H \rightarrow \gamma\gamma$ decay width will be performed at Next Linear Collider. We note at the very beginning that in the SM the contribution of light quark to the amplitude is proportional to the mass of this quark and therefore is small. However with an improvement of precision, the contribution of heaviest from 'light quarks', the b-quark, can become visible in future precise measurements at NLC. In addition we want to stress that in the SSM the coupling of the $b$ quark with the Higgs may be large due to an additional large factor $tg(\beta)$ and that the contribution of the $b$ quark at large $tg(\beta)$ may be as important as the contribution of top quark or $W$ boson.

In present paper we start with the one-loop analysis and demonstrate that one-loop and two-loop contributions contain a new sort of double logarithms

$$\xi = \alpha_s \log^2 \left( \frac{m^2_q}{m^2_H} \right).$$

If the ratio $m_q/m_H$ is quite small the parameter $\xi$ can be large enough and the QCD expansion in $\xi$ is not valid anymore. For example, considering a situation when the Higgs has a mass $m_H = 500$ GeV and decays into two photons through $b$-quark loop and $b$ quark has a mass $m_b = 4.5$ GeV, we see that indeed $\xi \approx 1$ and resummation of double logarithms is necessary. In the present paper we show that this goal becomes feasible when the physical origin of DL is understood and QCD series are properly organized. First, we examine an origin of such log’s by using Sudakov technique and then perform a two-loop calculation in DL approximation. The resummation of DL terms can be done and we present an exact answer in DLA.

The paper is organized as follows. In the section 2 we discuss the one-loop amplitude and the two-loop QCD correction. Section 3 contains the resummation of the DL contributions in all orders the perturbative QCD and a brief discussion.

## 2 One and two loop results

We start with considering the amplitude of the process $H \rightarrow \gamma\gamma$ at leading one-loop level. The amplitude of the Higgs decay into two photons through quark loop can be presented in the following form

$$M^{(\text{quarks})}(H \rightarrow \gamma\gamma) = e^*_1 e^*_2 d^{\mu\nu} \left( G_F \sqrt{2} \right)^{1/2} \frac{\alpha}{4\pi} N_c \sum_q Q^2_q F(x_q),$$

where the tensor $d_{\mu\nu}$ is defined as

$$d^{\mu\nu} = (k_1 k_2) g^{\mu\nu} - k_1^\mu k_1^\nu,$$

$e_1, e_2$ and $k_1, k_2$ are polarization vectors and momenta of the photons, $N_c$ - number of colors, $Q_q$ - the electric charge of the quark $q$ and the variable $x_q$ is defined by the
following relation:

\[ x_q = \frac{m_q^2}{m_H^2}. \]  

(4)

The corresponding decay width was calculated in [20, 21] at the one-loop level. In our notations, the quark formfactor \( F \) defined by Eqs. (2, 3) reads

\[ F(z_q) = \frac{1}{z_q} \left( 1 - \frac{1}{z_q} \right) \left[ \ln \left( \frac{\sqrt{z_q} + \sqrt{z_q - 1}}{\sqrt{z_q} - \sqrt{z_q - 1}} \right) - i\pi \right]^2 - 4 \], \quad z_q = \frac{m_H^2}{4m_q^2} = \frac{1}{4x_q} > 1. \]

Now we demonstrate how to extract the double logarithmic contribution from the amplitude using the Sudakov technique. To understand a basic properties of the amplitude in DLA we consider first a scalar part of amplitude without numerator and then discuss the structure of the numerator separately. The scalar part of the amplitude reads

\[ \tilde{F}(x_q) = \frac{-2i m_H^2}{\pi^2} \int_{-\infty}^{\infty} (k_1 + q)^2 - m_q^2 + i0 \left[ (k_2 - q)^2 - m_q^2 + i0 \right] \left[ q^2 - m_q^2 + i0 \right]. \]  

(5)

This integral is very simple and can be calculated explicitly. However, we are interested in terms containing double logarithms. The most suitable and simplest method of calculation was suggested by Sudakov in [26]. It consists in a direct integration in the momentum space splitting the internal momenta into the parallel and perpendicular components to the plane of the external momenta \( k_1 \) and \( k_2 \). To this end, one introduces new variables \( \alpha \) and \( \beta \) using Sudakov-decomposition for the vector \( q \)

\[ q = \alpha k_1 + \beta k_2 + q_\perp, \quad d^4q = \frac{m_H^2}{2} d\alpha d\beta d^2q_\perp. \]  

(6)

We obtain for \( \tilde{F} \) (see also consideration of the similar integral in DLA in Ref. [15])

\[ \tilde{F}(x_q) = \frac{-i}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\alpha d\beta \int_{0}^{\infty} d\rho \]

\[ \times \frac{1}{\left[ \beta - (x_q + \rho - \alpha\beta) + i0 \right] \left[ \alpha + (x_q + \rho - \alpha\beta) - i0 \right] \left[ (x_q + \rho - \alpha\beta) - i0 \right]} \]  

(7)

where

\[ \rho = \frac{-q_\perp^2}{m_H^2}. \]  

(8)

In Eq. (7) we have performed an integration over the angles in the transverse subspace, using the fact that the integrand does not contain a dependence on these variables. It is then evident from (7), that the double logarithmic contribution at \( x_q \ll 1 \) comes from the integration region where

\[ \rho, x_q \ll |\alpha|, |\beta| \ll 1. \]  

(9)

To carry out the integration over \( \rho \) we may apply the relation

\[ \frac{1}{\left[ (x_q + \rho - \alpha\beta) - i0 \right]} = P \frac{1}{x_q + \rho - \alpha\beta} + i\pi \delta(x_q + \rho - \alpha\beta). \]  

(10)
In the above expression, $P$ means the principal value.

One can check that the first term in Eq. (10) does not contribute to the DL asymptotics of the formfactor and we do not consider it here. We get for the $\tilde{F}$ in DLA

$$\tilde{F}(x_q) = \int \frac{d\alpha d\beta}{\alpha \beta} d\rho \delta(\rho - (\alpha \beta - x_q)) = \int \int \frac{d\alpha d\beta}{\alpha \beta} \delta(\alpha \beta - x_q).$$

(11)

In the last relation, we replace the strong inequality (9) by usual inequality, limiting the integration region in accordance with the logarithmic character of integration.

Let us now consider the numerator of the amplitude. It is clear from presented derivation that only the soft momenta $q$ give contribution to DL. That means that we may neglect the dependence on $q$ in the numerator of the integrand:

$$\frac{4x_q}{2m_H^4 m_q} d_{\mu\nu} Sp \left[ \gamma^\mu (\hat{k}_1 + \hat{q} + m_q)(\hat{k}_2 - \hat{q} - m_q)\gamma^\nu \right] = 4x_q.$$ 

(12)

We see that the numerator is proportional to the second power of the small quark mass. One power of the mass appears because of the Yukawa coupling constant and the second power is due to the trace. It is important here that only the scalar term of the numerator of the $t$-channel quark propagator $(\hat{q} + m_q)/(q^2 - m_q^2)$ contributes to the amplitude. That has a simple physical explanation. The Higgs particle has spin zero and positive parity. Therefore, the initial and final total momenta of the subprocess $\bar{q}q \rightarrow \gamma\gamma$ are zero. And, the amplitude is suppressed by the mass of the light quark $(O(m_q/E_q))$ at high energy due to the nonconservation of quark helicity. We see that the quantum numbers of the external particles force to have a situation when the $t$-channel quark propagator behaves like a scalar boson propagator, which is singular enough to produce DL. Really, the scalar part of the quark propagator is more singular than the vector part in the limit of the soft momenta and this fact guarantees the appearance of the new non-Sudakov DL. The possibility of the emergence of such DL in the mass suppressed amplitudes due to soft quark have been pointed and explained in [15], where helicity nonconserving amplitude $\gamma\gamma \rightarrow q\bar{q}$ was studied in DLA.

Despite the same origin the observed double logarithms in the $H \rightarrow \gamma\gamma$ process demonstrate some new peculiarities. For example, this phenomena 1) occurs not in the differential distribution or in the special helicity amplitude but rather in the decay width, 2) it occurs in the quark loop, but not as a correction to the Born amplitude. Therefore, the discussed phenomena is new in a sense that it appears in a very specific and unique circumstances, and of course, represents a new manifestation of the non-Sudakov type of DL.

The integration in Eq. (11) is trivial. Taking into account expression (12) for the numerator we get the formfactor $F(x_q)$ in the DLA:

$$F(x_q) = 4x_q \ln^2(x_q).$$

(13)

Now we consider a QCD correction to the formfactor $F$ defined by Eq. (2). This QCD correction was calculated in Refs. [22, 16, 24, 23]. In the limit of small quark masses the correction has a simple form

$$F(x_q) = F(x_q)^{\text{one-loop}} \left(1 - \frac{1}{6} \frac{\alpha_s C_F}{4\pi} \ln^2(x_q) + ... \right),$$

(14)
where $C_F$ is given by relation
\[ t^a t^a = C_F I, \] (15)
and $t^a$ is the color SU(N) group generators in the fundamental representation ($C_F = 4/3$ for QCD). There are four different topologies of the two-loop diagrams, which are shown in Fig. 1 (b-e). Here we calculate them in the DL approximation by using Sudakov technique.

Let us start from diagram 1b and consider first a scalar integral. Here and below we always use the Feynman gauge for the gluon fields; the gluon propagator in this gauge is
\[ \frac{-i\delta_{ab} g^{\mu\nu}}{k^2 + i0}. \] (16)
The scalar part of the $F^{(1b)}(x_q)$ reads
\[ \tilde{F}^{(1b)}(x_q) = \left( \frac{-2im_H^2}{\pi^2} \right)^2 \int_{-\infty}^{\infty} \frac{d^4q d^4Q}{\left( (k_1 + q)^2 - m_q^2 + i0 \right) \left( (k_1 + Q)^2 - m_q^2 + i0 \right)} \times \]
\[ \frac{1}{\left( (k_2 - q)^2 - m_q^2 + i0 \right) \left( (k_2 - Q)^2 - m_q^2 + i0 \right) \left( q^2 - m_q^2 + i0 \right) \left( (Q - q)^2 - m_q^2 + i0 \right)}. \] (17)
In the Sudakov variables
\[ q = \alpha k_1 + \beta k_2 + q_\perp \quad \text{and} \quad Q = A k_1 + B k_2 + Q_\perp, \] (18)
we have
\[ \tilde{F}^{(1b)}(x_q) = -\int_{-\infty}^{\infty} \frac{d\alpha d\beta d^2q}{\pi^2} \left[ \frac{1}{\left[ \beta - (q_\perp q^2 - \alpha \beta) + i0 \right] \left[ \alpha + (q_\perp q^2 - \alpha \beta) - i0 \right]} \times \right] \]
\[ \left[ \begin{array}{c} 1 \\ \frac{dAdBd^2Q}{\pi^2} \left[ B - (q_\perp Q^2 - AB) + i0 \right] \\ 1 \\
\end{array} \right] \times \]
\[ \left[ \begin{array}{c} 1 \\ \frac{A + (q_\perp Q^2 - AB) - i0}{\left( (Q - q)^2 - (A - \alpha)(B - \beta) \right) - i0} \end{array} \right], \] (19)
where we have switched to dimensionless Euclidian vectors $\vec{Q}$ and $\vec{q}$, noticing that the transverse momenta are spacelike. Note that the external integral in Eq. (19) coincides with the one loop one. It is then evident that in order to get the DL contribution one should integrate over the following region
\[ q^2, x_q \ll |\alpha|, |\beta| \ll 1, \quad \vec{q}^2 \ll \vec{Q}^2; \]
\[ \vec{Q}^2, |\alpha| \ll |A| \ll 1, \quad \vec{Q}^2, |\beta| \ll |B| \ll 1. \] (20)
Making corresponding simplifications in Eq. (19), performing integration over $\vec{Q}$ and $\vec{q}$ with the help of Eq. (17) and taking into account only the absorptive parts of the corresponding propagators, we get
\[ \tilde{F}^{(1b)}(x_q) = \frac{1}{\alpha} \frac{d\alpha}{\alpha} \frac{1}{\beta} \frac{d\beta}{\beta} \theta(\alpha \beta - x_q) \int_{|\alpha| < |A| < 1} \frac{dA}{A} \int_{|\beta| < |B| < 1} \frac{dB}{B} \theta(AB) = \frac{1}{6} \ln^4(x_q). \] (21)
Figure 1: Feynman diagrams corresponded to the amplitude (2): (a) one-loop diagram corresponded to the zeroth order in $\alpha_s$, (b-e) two-loop diagrams corresponded to the first order in $\alpha_s$. External wavy lines represent photons, internal wavy lines - gluons, internal solid lines - quarks, external solid lines are the Higgs bosons.
Because the essential momenta $q$ and $Q$ are soft in DL approximation we can again neglect the dependence on them in the numerator in DL approximation. The numerator of the integrand then takes the form

$$-\frac{\alpha_s C_F}{4\pi} \frac{4x_q}{4m_q m_H^4} d_{\mu\nu} S \left[ \gamma^\nu \left( \hat{k}_1 + \hat{q} + m_q \right) \gamma^\lambda \left( \hat{k}_1 + \hat{Q} + m_q \right) \left( \hat{k}_2 - \hat{Q} - m_q \right) \gamma^\times \right] = -4x_q \frac{\alpha_s C_F}{4\pi}. \quad (22)$$

Combining Eqs. (21, 22) we obtain

$$F^{(1b)}(x_q) = -\frac{1}{6} \frac{\alpha_s C_F}{4\pi} \ln^2(x_q) F(x_q)^{(\text{one-loop})}. \quad (23)$$

Remaining two-loop diagrams of Fig. 1(b-e) do not contribute to the DL asymptotics of the amplitude because both the electromagnetic vertex convoluted with a polarization vector of real external photon and the quark propagator do not contain double logarithms. These properties can be easily checked at the one loop level. Only one large logarithm may appear in these structures in the Feynman gauge but not the double one.

Thus, we have restored the exact result (14). It is clear from (14) that when the mass of the Higgs boson is large enough so that in spite of smallness of the QCD coupling constant $\alpha_s$

$$\frac{\alpha_s C_F}{4\pi} \ln^2(x_q) \sim 1, \quad (24)$$

the perturbative QCD expansion in $\alpha_s$ is not valid and we should modify two-loop result.

In the next section we obtain the quark formfactors $F$ in all orders of QCD in the following region of Higgs boson masses

$$\frac{\alpha_s C_F}{4\pi} \ll \frac{\alpha_s C_F}{4\pi} \ln(1/x_q) \ll \frac{\alpha_s C_F}{4\pi} \ln^2(1/x_q) \sim 1. \quad (25)$$

In this region we expect to get the correct result in the framework of the DLA.

3 Resummation and Discussion

The resummed form factor can be presented in the form (see Fig. 2)

$$F(x_q) = \left( \frac{8\pi i}{G_F \sqrt{2}} \right)^{1/2} \frac{1}{\alpha N_c Q^2 m_H^4} \int d^4 q \int_{-\infty}^{\infty} d_{\mu\nu} \times tr \left[ S(k_1 + q)V(H \rightarrow q\bar{q})S(q - k_2)R^{\mu\nu}(q\bar{q} \rightarrow \gamma\gamma) \right], \quad (26)$$

where the amplitude $R^{\mu\nu}(q\bar{q} \rightarrow \gamma\gamma)$ is defined in such a way that it contains no two-quark cuts in the $\gamma\gamma$- channel, $V(H \rightarrow q\bar{q})$ is the exact QCD vertex which contains all possible gluon exchanges, $S$ is the exact quark propagator. Double logarithms appear when we have consistent logarithmic integrations. Therefore, only region (9) can yield the DL contribution to the RHS of Eq. (26) after integration over $q$. As it is clear from the above
discussion we should not consider QCD corrections to the amplitude $R^{μν}(q\bar{q} \rightarrow γγ)$ and to the quark propagators $S$ in the DLA, taking them in the Born approximation. One has to know only the expression for the $H \rightarrow q\bar{q}$ vertex $V(p_2, p_1)$ in DLA in the following kinematical region

$$m_q^2 \ll |p_1^2|, |p_2^2| \ll (p_2 - p_1)^2 = m_H^2,$$

$$p_2 = k_1 + q, \quad p_1 = q - k_2.$$  \hfill (27)

Let us remind that in general $F$ is proportional to $m_q^2$. One power of $m_q$ appears from the $Hq\bar{q}$-vertex, the another one - from the trace over Dirac indexes. The physical reasons for that were explained in the previous section. There was also noted that one of the two powers of $m_q$ appears from the quark propagator $(\hat{q} + m_q)/(q^2 - m_q^2)$ from the $R^{μν}(q\bar{q} \rightarrow γγ)$ amplitude. Due to this circumstance we are not forced to consider the $O(m_q)$ corrections to the $Hq\bar{q}$-vertex. Therefore, the exact result for this vertex in DLA is given by multiplication of the Born expression with the well-known Sudakov form factor

$$exp \left[ -\frac{α_s C_F}{2π} \ln \left| \frac{(p_2 - p_1)^2}{p_2^2} \right| \ln \left| \frac{(p_2 - p_1)^2}{p_1^2} \right| \right].$$  \hfill (28)

Therefore, the quark form factor $F$ in the DLA can be found from Eqs. (11) and (12) by multiplying the integrand in the RHS of this equation with the Sudakov form factor (28) and the result reads

$$F(x_q) = 4x_q \int_{-1}^{1} \frac{dα}{α} dβ \theta(αβ - x_q) exp \left[ -(α_s C_F/2π) \ln |α| \ln |β| \right].$$  \hfill (29)

The integration over $β$ can be easily performed and we have

$$F(x_q) = \frac{8x_q \ln^2(x_q)}{Z} \int_{0}^{1} \frac{dy}{y} \left( 1 - exp(-Zy(1 - y)) \right),$$  \hfill (30)

where $Z = α_s C_F \ln^2(x_q)/(2π)$. We have found that following representation is convenient

$$F(x_q) = F(x_q)^{(one-loop)} \sum_{n=0}^{∞} \frac{2Γ(n + 1)}{Γ(2n + 3)} \left( -\frac{α_s C_F}{2π} \ln^2(x_q) \right)^n =$$
\[
F(x_q)^{\text{(one-loop)}} \left[ 1 - \frac{1}{6} \left( \frac{\alpha_s C_F}{4\pi} \ln^2(x_q) \right) + \frac{1}{45} \left( \frac{\alpha_s C_F}{4\pi} \ln^2(x_q) \right)^2 + \ldots \right].
\] (31)

The Eqs. (29,31) are our final results. They correctly describe the quark form factor \( F \) in the region of the Higgs boson masses where the relations (25) are fulfilled.

In Fig.3 we plot relative contributions of two-loop (dashed curve), three-loop (dashed-dotted curve), four-loop (dotted curve) order results (31) and exact result in DLA (30) (solid curve) as a function of \( Z \). All results are normalized on the one-loop result. We see that at small enough \( x_q \), large \( Z \), the perturbation theory does not work and the resummation of DL is mandatory. The middle curve in Fig. 3 corresponds to the exact in DLA result. We see that the later slightly decreases the decay width with increasing \( Z \).

In conclusion, we have discussed a QCD perturbative corrections to the \( H \to \gamma\gamma \) process in the limit of small quark masses in the loops. We have found a new manifestation of the non-Sudakov type of double logarithms and have performed one- and two-loop calculation to find the origin of such double logarithms. Then we have resummed them and have derived the exact result for the quark-loop form factor \( F \) with double logarithmic accuracy (24,31). These results will be important in the situation when the Higgs boson is heavy enough to guarantee smallness of \( m_q/m_H \) and when the mass \( m_q \) itself is not very small to give visible contribution to the amplitude. Such a situation can be realized 1) in the Standard Model with \( M_H \approx 500\text{GeV} \) for the \( b \) quark loop and 2) in the Supersymmetric extension of the Standard Model, where the coupling of \( b \) quark with Higgs can be large. For the first application we need to measure the decay width of the \( H \to \gamma\gamma \) process with very high precision. The feasibility of that depends on experimental side, of course, and should be studied separately. We should understand, of course, that at very high Higgs masses two-loop electroweak corrections became very important due to terms proportional to \( G_F m_H^2 \). Fortunately, they have been calculated \([27]\). The second application seems to be very promising and important phenomenologically. The crucial fact here will be the value of Higgs mass and the value of \( t_g(\beta) \).

The main purpose of this paper was to attract an attention to new non-Sudakov type of double logarithms which appears in the \( H \to \gamma\gamma \) process.

In our opinion, the results presented here may have applications to other cases. One of the most interesting is \( \gamma\gamma \to q\bar{q} \) in the \( J_Z = 0 \) channel, where such non-Sudakov, mass-suppressed DL terms are phenomenologically important, as it was demonstrated in Ref. [15].

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Figure 3: The relative contribution of the two-loop (dashed curve), the three-loop (dashed-dotted curve), the four-loop (dotted curve) and the exact in DLA result for the formfactor (31) (solid curve) as a function of $Z$. All results are normalized on the one-loop result. The one-loop contribution corresponds to one in such a normalization.