The Quasi-Infra-Red Fixed Point at Higher Loops

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We calculate the four-loop $\beta$-function for the generalised Wess-Zumino model. We use the result and Padé-Borel summation to discuss the domain of attraction of the quasi-infra-red fixed point of the top-quark Yukawa coupling in the supersymmetric standard model, and argue that the domain is in fact substantial.
In this note we present the four-loop $\beta$-function for a many-field Wess-Zumino model with arbitrary cubic interactions. (This calculation is a straightforward generalisation of the existing one of Avdeev et al\[1\] for the single field and coupling case). We use the result to write down the $O(\lambda_t^9)$ terms in the $\beta$-function for the top-quark Yukawa coupling in the supersymmetric standard model. We then use this result to discuss issues regarding the limitations of perturbation theory in the standard running analysis whereby the low energy supersymmetric standard model is matched onto a much reduced (in terms of the number of free parameters) theory at high energies. In such analyses, putative supersymmetric spectra are often presented as scatter-plots, for ranges of certain of the free parameters, subject to certain cuts. One such cut, frequently applied, is that the Yukawa couplings remain perturbative throughout: in for example Ref. [2] one finds that $\lambda_{t,b,\tau} \leq 3.5$ has been enforced. What is the basis and reliability of such cuts, given the (presumably) asymptotic nature of the perturbation expansion? This question becomes particularly interesting in the context of the possible quasi-infra-red fixed point (QIR FP) behaviour of $\lambda_t$ [3]–[5].

We begin with the generalised Wess-Zumino model, defined by the superpotential:

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k.$$  
(1)

where $\Phi_i$ is a multiplet of chiral superfields. The $\beta$-functions for the Yukawa couplings $\beta^i_{Y}^{jk}$ are given by

$$\beta^i_{Y}^{jk} = Y^p (ij \gamma^k)_{p} = Y^{ijp} \gamma^k_{p} + (k \leftrightarrow i) + (k \leftrightarrow j),$$  
(2)

where $\gamma$ is the anomalous dimension for $\Phi$.

The result for $\gamma$ through three loops is as follows [8]:

$$16\pi^2 \gamma^{(1)} = P$$  
(3a)

$$(16\pi^2)^2 \gamma^{(2)} = -S_4$$  
(3b)

$$(16\pi^2)^3 \gamma^{(3)} = \frac{3}{2} \zeta(3) M + 2 Y^* S_4 Y - \frac{1}{2} S_7 - S_8$$  
(3c)

where our notation follows that of Ref. [7], except that here we have no gauge coupling:

$$P^i_{j} = \frac{1}{2} Y^{ikl} Y_{jkl},$$  
(4a)

$$S^i_{4j} = Y^{imn} P^p_{m} Y_{jpm}$$  
(4b)

$$(Y^* S_4 Y)_{j}^i = Y^{imn} S_4^p_{m} Y_{jpm}.$$  
(4c)

$$S^i_{7j} = Y^{imn} P^p_{m} P^q_{n} Y_{jpq}$$  
(4d)

$$S^i_{8j} = Y^{imn} (P^2)^p_{m} Y_{jpm}$$  
(4e)

$$M^i_{j} = Y^{ikl} Y_{kmn} Y_{lrs} Y^{pmr} Y^{qns} Y_{jpq}.$$  
(4f)
Our result for $\gamma^{(4)}$ is as follows:

\[(16\pi^2)^4\gamma^{(4)} = \frac{5}{6} Y^{ikm} S^l k Y_{jlm} + (\frac{5}{3} - 2\zeta(3)) Y^{ikm} S^l k Y_{jlm} + \frac{4}{3} Y^{ikm} (P S_4 + S_4 P)^l k Y_{jlm} \]

\[-5 Y^{ikq} Y^{kmp} S^l m Y^{iqm} Y^{lkm} Y_{jlm} - (\frac{3}{2}\zeta(3) + \frac{3}{4}\zeta(4)) Y^{ikm} M^l k Y_{jlm} \]

\[+(2\zeta(3) - 1) [Y^{ikm} (P^{3l})^l k Y_{jlm} + Y^{ikm} (P^{2l})^l k P^n m Y_{jln}] \]

\[+ \frac{4}{3} Y^{ikm} P^l k (S_4)^m n Y_{jln} - 10\zeta(5) Y^{ikl} Y^{kln} Y_{jlm} Y^{suv} Y^{rty} Y^{pqr} Y^{vts} Y_{jlp} \]

\[-(6\zeta(3) - 3\zeta(4)) \frac{1}{2} Y^{ikl} P^n t Y^{kln} Y_{jln} Y^{prq} Y^{pqr} Y^{suv} Y_{jlp} \]

\[+ \frac{1}{2} Y^{ikl} Y^{kln} Y_{jln} Y^{pqr} Y^{npt} P^n t Y_{jlp} + Y^{ikl} Y_{kmt} P^n t Y_{jln} Y^{pqr} Y^{suv} Y_{jlp} \].

We have explicitly calculated the requisite Feynman diagrams for the most part. (This was in any case necessary since in Ref. [1] results are given for subsets of, not individual graphs.) The results for several of the Feynman integrals are given in Ref. [7]. In two places, however, we have relied on previous authority: specifically, in Table 1 of Ref. [1], we have used their total for the set of graphs 4.1 to finesse the calculation of one particular graph; and we have used the result of Ref. [8] for the Feynman integral that arises in the calculation of the graph 4.5.

For the supersymmetric standard model superpotential, which is (retaining only $\lambda_t$)

\[W = \lambda_t H_2 Q \tilde{t} \]

where $Q = \begin{pmatrix} t \\ b \end{pmatrix}$, we find, using $\beta_{\lambda_t} = (\gamma_{H_2} + \gamma_Q + \gamma_{\tilde{t}})\lambda_t$ [9]:

\[16\pi^2 \beta_{\lambda_t}^{(1)} = 6\lambda_t^3 \]

\[(16\pi^2)^2 \beta_{\lambda_t}^{(2)} = -22\lambda_t^5 \]

\[(16\pi^2)^3 \beta_{\lambda_t}^{(3)} = [102 + 36\zeta(3)]\lambda_t^7 \]

and finally from Eq. [5]:

\[(16\pi^2)^4 \beta_{\lambda_t}^{(4)} = -[678 + 696\zeta(3) - 216\zeta(4) + 1440\zeta(5)]\lambda_t^9. \]

A specific aspect of the running analysis that has become popular in recent years is the possibility that $\lambda_t$ may exhibit QIRFP behaviour. The main attraction of this philosophy is the idea that a large range of input couplings at high energies may produce the same value of $\lambda_t$ at $M_Z$. It is clearly interesting to inquire as to the extent to which
this range is limited by the requirement of perturbative believability. Let us review the QIRFP paradigm: the one-loop equations governing the evolution of the $\lambda_t$ and the gauge couplings are

$$\frac{dy_t}{dt} = y_t(6y_t - \frac{16}{3} \alpha_3 - 3\alpha_2 - \frac{13}{15} \alpha_1), \quad \text{and} \quad \frac{d\alpha_i}{dt} = b_i \alpha_i^2$$

where $y_t = \frac{1}{4\pi} \lambda_t^2$, $\alpha_i = \alpha_{1,2,3}$, $b_i = \left(\frac{33}{5}, 1, -3\right)$, and $t = \frac{1}{2\pi} \ln \mu$. We have made the approximation that $y_t >> y_\tau, y_b$, which involves the assumption that it is not the case that $\tan \beta >> 1$; it is straightforward to consider the more general case but we omit it here for simplicity. From Eq. (9) we obtain:

$$y_t(M_X) = \frac{y_t(M_Z)}{f(0) - 6F y_t(M_Z)}$$

where

$$F = \int_0^T f(\tau) d\tau, \quad T = \frac{1}{2\pi} \ln \frac{M_X}{M_Z}$$

and

$$f(\tau) = \left[\frac{\alpha_3(T)}{\alpha_3(\tau)}\right]^{\frac{16}{57}} \left[\frac{\alpha_2(T)}{\alpha_2(\tau)}\right]^{\frac{3}{7}} \left[\frac{\alpha_1(T)}{\alpha_1(\tau)}\right]^{\frac{13}{57}}.$$ \hspace{1cm} (11)

We will take $M_X$ to be the unification scale, requiring that $\alpha_2(T) = \alpha_1(T)$. Of course it is a straightforward matter to integrate the differential equations numerically, but the partial analytic solution above is nevertheless useful in order to see what is going on. From Eq. (10) we see that $y_t$ suffers a Landau pole unless

$$y_t(M_Z) < f(0)/(6F).$$ \hspace{1cm} (13)

Another nice way of thinking about this result is as follows. Suppose

$$y_t(M_X) >> 1/(6F).$$ \hspace{1cm} (14)

Then it is easy to see from Eq. (10) that in this limit

$$y_t(M_Z) = \overline{y}_t = f(0)/(6F)$$

which is independent of $y_t(M_X)$! Of course $\overline{y}_t$ is the value of $y_t(M_Z)$ such that the Landau pole occurs at $M_X$, and hence represents an upper limit; but it is more productive to think of $y_t(M_X)$ as the input, as follows. Depending on the value of $F$, there may be a wide range of values of $y_t(M_X)$ which all lead to the same value of $y_t(M_Z)$. The corresponding value,
\( y_t(M_Z) = f(0)/(6F) \), is called a quasi-infra-red fixed point of the evolution equations. The prefix quasi- is occasioned by the fact that \( f/(6F) \) is of course a function of \( T \). This behaviour is illustrated in Fig. 1.

![Fig.1: Plot of \( y_t \) against \( \log_{10}(\mu/(1\text{GeV}) \) for various values of \( y_t(M_X) \)](image)

If we take the limit \( T \to \infty \), then we approach genuine fixed point behaviour \[^3\]. It is easy to see this in the approximation \( \alpha_1 = \alpha_2 = 0 \), for which the equations for \( \frac{dy_t}{dt} \) and \( \frac{d\alpha_3}{dt} \) exhibit an infra-red fixed point such that

\[
\frac{y_t}{\alpha_3} = -3 + \frac{16}{3} = 0.39.
\]

For \( \alpha_3(M_Z) = 0.1 \) this gives \( y_t(M_Z) = 0.039 \). Substituting in Eq. (11) and (12) (with \( \alpha_1(M_Z) = 0.0167, \alpha_2(M_Z) = 0.0320 \) one finds \( f(0) = 9.98 \) and \( F = 17.88 \) so that \( y_t = 0.093 \). Thus QIRFP and IRFP behaviour are quite distinct; it is the former which is relevant for the supersymmetric standard model. (This was first pointed out for the standard model in Ref. \[^4\].)

We now see that since \( 1/(6F) \approx 0.01 \) we have that there is a wide range for \( y_t(M_X) \) such that

\[
1/(6F) << y_t(M_X) < 1
\]

(16)

where the upper limit is a naive constraint for perturbative believability. (Note that the constraint \( \lambda < 3.5 \) used in Ref. \[^2\] would correspond to \( y_t < 0.975 \).) We want to address this latter restriction; does our four-loop calculation afford any insight into it? In the literature, cuts of the type \( \lambda_t \leq 3.5 \) alluded to above are sometimes motivated by requiring
\[ \beta^{(2)} \leq x \beta^{(1)} \], where \( \beta^{(L)} \) is the appropriate \( L \)-loop \( \beta \)-function, and \( x \) is some convincing fraction: \( \frac{1}{4} \) for example. This leads, using Eq. (7) to \( y_t \leq 0.86 \). A naive extension of this approach would obviously suggest a drastic curtailment of the acceptable range of \( y_t(M_X) \), since for instance \( \beta^{(4)} \leq \frac{1}{4} \beta^{(3)} \) gives \( y_t \leq 0.16 \). This is illustrated in Fig. 2, where we plot \( y_t(M_X) \) against \( y_t(M_Z) \) for \( L = 1, \cdots 4 \).

**Fig.2:** Plot of \( y_t(M_X) \) against \( y_t(M_Z) \). The solid, dashed, dotted and dash-dotted lines correspond to one, two, three and four-loop \( \beta \)-functions respectively.

We wish to argue that the (presumably) asymptotic nature of the perturbation series for the \( \beta \)-functions means that in fact the actual domain of attraction of the QIRFP is more accurately represented by the one-loop than by the four-loop approximation.

A striking feature of Eq. (8) is the broad similarity to the corresponding result [12] for \( O(n) \phi^4 \). Most importantly, note the characteristic alternating sign behaviour, suggesting the possibility of Borel summability. (For a review on the problem of resummation of perturbation theory see Ref. [13].) We have not found in the literature any discussion of the large order behaviour of the Wess-Zumino model; but for the supersymmetric anharmonic oscillator, it has been noted [14] that while the supersymmetric case does represent a

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1 Such naive constraints may be even more restrictive in other sectors of the theory: in particular the soft supersymmetry breaking sector. By demanding that \( \frac{\beta^{(2)}}{m_Q^2} \leq \frac{1}{4} \frac{\beta^{(1)}}{m_Q^2} \), we get [10] \( y_t \leq 0.32 \), for any theory with the commonly assumed universal form of the soft supersymmetry breaking parameters (specific models might originate smaller upper bounds; the “\( P = \frac{1}{3}Q \)” class of models [11], for example, gives \( y_t \leq 0.21 \)).
bifurcation point with respect to some behaviour, nevertheless the characteristic factorial divergence $L!$ persists. It is interesting to note, however, that the exact form for the gauge $\beta$-function in an $N = 1$, $d = 4$ theory without chiral fields,\(^{15}\)\(^{16}\)

$$\beta_g = \frac{g^3}{16\pi^2} \left[ \frac{-3C(G)}{1 - 2C(G)g^2(16\pi^2)^{-1}} \right],$$

(17)

clearly has a finite radius of convergence. Now this result may not hold in DRED (dimensional reduction with minimal subtraction); it was shown in Ref.\(^{17}\) that the scheme in which Eq. (17) is valid differs from DRED. In any case, for the Wess-Zumino model, there is certainly no indication from Eq. (15) that the Yukawa $\beta$-function is other than an asymptotic series.\(^3\) Now the existence of renormalon singularities\(^{18}\) implies that in asymptotically un-free theories (such as the Wess-Zumino model) amplitudes are not Borel-summable in general; but specific renormalisation group functions may be so. It seemed to us worthwhile exploring the consequences of naive Padé-Borel (PB) summation of the $y_t$-dependence of $\beta_{y_t}$.

PB summation proceeds as follows (see for example Ref.\(^{19}\)). Given a series

$$f(x) = \sum_{n=0} a_n x^n$$

(18)

one defines

$$B(x) = \sum_{n=0} a_n \frac{x^n}{n!}$$

(19)

and calculates $[N, M]$ Padé approximants to $B(x)$, $B_{N,M}$. Then the PB-summed version of $f(x)$ is given by

$$F_{N,M}(x) = \int_0^\infty e^{-t} B_{N,M}(xt) \, dt.$$  

(20)

Essentially this construction amounts to a guess of the coefficients of powers of $x$ beyond those originally calculated, incorporating (for $x^L$) a factor $L!$.

We have calculated the $[1, 1]$ and the $[2, 2]$ Padés for the $t$-Yukawa $\beta$-function, i.e. the series

$$\frac{dy_t}{dt} = y_t \left\{ 0 + 6y_t - \frac{1}{4\pi} 22y_t^2 + (\frac{1}{4\pi})^2 \left[ 102 + 36\zeta(3) \right] y_t^3 

- (\frac{1}{4\pi})^3 \left[ 678 + 696\zeta(3) - 216\zeta(4) + 1440\zeta(5) \right] y_t^4 + \cdots \right\}$$

(21)

\(^3\) Note that if this was true the same would hold for the exact result of Ref.\(^{16}\) for $\beta_g$, when chiral fields are present.
We have written the series in this form in order that the $O(y_t^L)$ term in the curly brackets should correspond to the $L$th order contribution in perturbation theory. The $L!$ factor produced by the PB process for the $O(y_t^L)$ term in the series then correctly mimics the expected $L!$ growth of the $L$-loop perturbation theory contributions. The $[1,1]$ PB approximation for the series

$$\frac{dy_t}{dt} = y_t(0 + ay_t - by_t^2)$$  \hspace{1cm} (22)

is easily calculated as

$$\frac{dy_t}{dt} = y_t \left[ \frac{2a^2}{b} - \frac{4a^3}{b^2y_t} e^{\frac{2a}{by_t}} E_1 \left( \frac{2a}{by_t} \right) \right],$$  \hspace{1cm} (23)

where $E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$ is the exponential integral function. The $[2,2]$ PB approximation for the series

$$\frac{dy_t}{dt} = y_t(0 + ay_t - by_t^2 + cy_t^3 - dy_t^4)$$  \hspace{1cm} (24)

is found to be

$$\frac{dy_t}{dt} = y_t \left[ \sigma r_1 r_2 + \frac{r_1^2 r_2 (\sigma r_1 - a)}{(r_1 - r_2) y_t} e^{-\frac{r_1}{y_t}} E_1 \left( -\frac{r_1}{y_t} \right) + \frac{r_1 r_2^2 (\sigma r_2 - a)}{(r_2 - r_1) y_t} e^{-\frac{r_2}{y_t}} E_1 \left( -\frac{r_2}{y_t} \right) \right],$$  \hspace{1cm} (25)

where

$$\sigma = \frac{a^2d + 3b^3 - 4abc}{4ac - 6b^2},$$  \hspace{1cm} (26)

and $r_1$ and $r_2$ are the roots of

$$(3bd - 4e^2)r^2 + (6ad - 12bc)r + 12(2ac - 3b^2) = 0.$$  \hspace{1cm} (27)

In each case the PB approximation involves the exponential integral function; this is a generic feature. Note that with $a \ldots d$ as given in Eq. (21), the roots of Eq. (27) are both negative and real. It follows that in Eq. (23), no singularity is encountered in the exponential integrals; the same is true of Eq. (23). In the $[1,2]$ PB, however, the roots of the analogous quadratic have opposite signs. Now such a pole may represent a physical renormalon singularity; but since we do not encounter it in the $[2,2]$ Padé, and because we have no specific knowledge of the asymptotic behaviour for this theory, we choose to ignore this case.

We have investigated the running of $y_t$ and $\alpha_{1,2,3}$ by adding the perturbative contributions involving the gauge couplings to Eq. (7) and evolving the gauge couplings using the perturbative $\beta$-functions. This seems justified on the grounds that $\alpha_{1,2,3}$ remain small over
the range of interest. The results are displayed in Fig. 3, where we plot $y_t(M_X)$ against $y_t(M_Z)$ for our $[1,1]$ PB and $[2,2]$ PB approximations. For purposes of comparison, we also give the results obtained using the one-loop and four-loop $\beta$-functions. (Since we only know explicitly the pure Yukawa contribution at the four-loop level, we must omit the gauge contributions at this level; but in the region of interest we know that the Yukawa contributions are dominant.)

![Graph](image)

*Fig. 3: Plot of $y_t(M_Z)$ against $y_t(M_X)$. The solid and dotted lines correspond to the one and four-loop perturbative $\beta$-functions, and the dot-dashed and dashed lines to the $[1,1]$ and $[2,2]$ PBs respectively.*

For small $y_t$, the $[1,1]$ and $[2,2]$ PB results should approach the two-loop and four-loop perturbative results respectively; this is clearly seen in the $[2,2]$ case. For large $y_t$, the asymptotic nature of the perturbation series implies that lower orders in perturbation theory should be more accurate than higher orders; accordingly, we see that as $y_t$ increases, the PB evolution starts to resemble the one-loop perturbative behaviour. Furthermore, the $[2,2]$ PB is closer to the one-loop result than the $[1,1]$. We have also repeated this exercise for the $[2,1]$ PB; the results are almost indistinguishable from those for the $[2,2]$ Padé and indicate that the successive PBs may be converging quite rapidly in the region of interest.

The one-loop perturbative evolution displays the QIRFP behaviour as explained in our earlier analysis; we see, for instance that values of $y_t(M_X)$ in the range $0.2 < y_t(M_X) < 1$ lead to values of $y_t(M_Z)$ in the range $0.089 < y_t(M_Z) < 0.092$. The upper limit on $y_t(M_X)$, as discussed earlier, is a rough constraint for perturbative believability, and if
one wished to include higher-loop perturbative contributions there is a *prima facie* case for restricting the allowed range of \( y_t(M_X) \) still further. The PB results show the same QIRFP behaviour; however, we believe they are reliable up to a larger value of \( y_t(M_X) \).

Of course, there must still be an upper limit on \( y_t \) beyond which we can no longer trust the PB approximation. Some indication of where this upper limit is can be obtained by a comparison with alternative means of implementing the PB programme; clearly one can no longer trust the PB results in regions where different approaches give qualitatively different behaviour. For instance, one could write

\[
\frac{dy_t}{dt} = 6y_t^2 \left[ 1 - \frac{1}{4\pi} \frac{22}{6} y_t + \left( \frac{1}{4\pi} \right)^2 \frac{1}{6} [102 + 36\zeta(3)] y_t^2 + \ldots \right]
\]

and then perform the PB procedure on the series \( 1 + ay_t + by_t^2 + cy_t^3 + \ldots \). Note that the \( L \)-loop contribution to the perturbation expansion corresponds to the \( O(y_t^{L-1}) \) term in this series. This appears at first to be a drawback, as the PB process will not now exactly match the \( L! \) growth of the \( L \)-loop perturbation theory contribution. However we might well expect behaviour of the \( L \)-loop contribution like \( L!L^a \), and our uncertainty as regards the value of \( a \) means that we cannot really distinguish between growth like \( L! \) or like \( (L - 1)! \).

If one evolves \( y_t \) alone (setting \( \alpha_{1,2,3} = y_b = y_\tau = 0 \)), using the \([1,1] \) or \([2,1] \) PB obtained in this way, one finds a fixed point at large \( y_t \), for \( y_t \sim 100 \) and \( y_t \sim 8 \) respectively, in contrast with the Landau pole behaviour of the previous PBs. However, in the presence of the other couplings, the behaviour is somewhat modified. In fact, the evolution is practically indistinguishable from that for the \([2,2] \) PB displayed in Fig. 3 over the range shown. Moreover, for larger \( y_t(M_X) \), the fixed point is obscured by the development of a Landau pole in \( y_b \) and by the increasing size of \( y_t^3 \alpha_3 \) terms which have not been Borel summed. Nevertheless, the last vestige of a fixed point is discernible in the \([2,1] \) PB evolution at \( y_t \sim 6 \). This may perhaps be taken as a signal that we should not trust any of our PBs beyond this point. Nevertheless we have considerably extended the domain of attraction of the QIRFP as compared to the perturbative case beyond \( y_t(M_X) = 1 \), and perhaps optimistically as far as \( y_t(M_X) \sim 5 \); we now see that values of \( y_t(M_X) \) in the range \( 0.2 < y_t(M_X) < 5 \) lead to values of \( y_t(M_Z) \) in the range \( 0.089 < y_t(M_Z) < 0.094 \). Note that the fixed point for the \([2,2] \) PB is roughly 2% higher than the effective upper limit on \( y_t(M_Z) \) which applies in the one-loop perturbative case; for a fixed value of \( m_t \), this leads to a reduction in \( \tan \beta \) of around 4.5%.
In conclusion, we have demonstrated by means of Padé-Borel summation that the
domain of attraction of the quasi-infra-red fixed point in the supersymmetric standard
model is in fact large. There has been some recent speculation\cite{20} on the possible rôle
of QIRFP behaviour in the theory above $M_X$ in assuring universality of the soft-breaking
parameters. Because the energy range between $M_{\text{Planck}}$ and $M_X$ is much smaller than
that between $M_X$ and $M_Z$, rapid evolution of the couplings is essential if a QIRFP is to
be approached in this region. Since the approach to the QIRFP appears to be quicker for
larger initial couplings, it might be of interest to use PB summation to explore the region
of larger coupling with more confidence.

Acknowledgements

While part of this work was done, two of us (IJ and TJ) enjoyed the hospitality of
the Aspen Center for Physics. We thank Professor Avdeev for correspondence, and David
Barclay for conversations. PF was supported by a scholarship from JNICT.
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