Matter-wave interference, Josephson oscillation and its disruption in a Bose-Einstein condensate on an optical lattice

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Using the axially-symmetric time-dependent mean-field Gross-Pitaevskii equation we study the Josephson oscillation in a repulsive Bose-Einstein condensate trapped by a harmonic plus an one-dimensional optical-lattice potential to describe the experiments by Cataliotti et. al. [Science 293 (2001) 843, New J. Phys. 5 (2003) 71.1]. After a study of the formation of matter-wave interference upon releasing the condensate from the optical trap, we directly investigate the alternating atomic superfluid Josephson current upon displacing the harmonic trap along the optical axis. The Josephson current is found to be disrupted upon displacing the harmonic trap through a distance greater than a critical distance signaling a superfluid to a classical insulator transition in the condensate.

1. INTRODUCTION

The experimental loading of a cigar-shaped Bose-Einstein condensate (BEC) in both one- and three-dimensional periodic optical-lattice potentials has allowed to study the quantum phase effects on a macroscopic scale such as interference of matter waves. There have been several theoretical studies on a BEC in a one- and three-dimensional optical-lattice potentials. The phase coherence between different sites of a trapped BEC on an optical lattice has been established in recent experiments through the formation of distinct interference pattern when the traps are removed.

In the experiment on matter-wave interference from two pieces of coherent BEC, an interference pattern comprised of a large number of dark and bright patches is formed. This is similar to the well-known double-slit interference pattern in optics. As the number of slits in the experiment on interference of light is increased, the number of bright patches in the interference pattern is reduced and one has only a few prominent bright patches for interference of light from an optical grating with very large number of narrow slits. A similar phenomenon has emerged in matter-wave interference from a BEC in an optical-lattice trap created by a standing-wave laser field, which can be considered to be a large number of coherent sources of matter wave. When a BEC trapped in an axially-symmetric harmonic and an one-dimensional optical-lattice trap is released from the joint traps a definite interference pattern composed of three peaks is usually formed. With the increase of lattice spacing the interference pattern evolves to \((2N + 1)\) peaks with \(N\) peaks symmetrically located in a straight line on opposite sides of a central peak.

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Cataliotti et al. [2] prepared a BEC on a joint harmonic plus an optical-lattice trap. Upon displacing the harmonic trap along the optical lattice, the BEC was found to execute the Josephson oscillation by quantum tunneling through the optical-lattice barriers. In a later experiment they [3] found that for a larger displacement of the harmonic trap the Josephson oscillation is disrupted. Their measurement of the Josephson oscillation is disrupted by quantum tunneling through the optical-lattice barriers. In the experiments of Cataliotti et al. [2,3] with repulsive $^{87}$Rb atoms, the axial and radial trap frequencies were $\nu_\omega = 2 \pi \times 9$ Hz and $\omega = 2 \pi \times 92$ Hz, respectively. The optical potential created with the standing-wave laser field of wave length $\lambda = 795$ nm is given by $V_{\text{opt}} = V_0 E_R \cos^2(\kappa_L z)$, with $E_R = h^2 \kappa_L^2 / (2 m)$, $\kappa_L = 2 \pi / \lambda$ and $V_0$ (<< 12) the strength.

In terms of the dimensionless laser wave length $\lambda_0 = \sqrt{2} \lambda / l \simeq 1$, $E_R / (\hbar \omega) = 4 \pi^2 / \lambda_0^2$.

2. MEAN-FIELD MODEL AND RESULTS

The time-dependent BEC wave function $\Psi(r; \tau)$ at position $r$ and time $\tau$ is described by the following mean-field nonlinear Gross-Pitaevskii (GP) equation [7,8]

$$
\left[ -i \hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2 \nabla^2}{2m} + V(r) + G N |\Psi(r; \tau)|^2 \right] \Psi(r; \tau) = 0, \quad (1)
$$

where $m$ is the mass and $N$ the number of atoms, $G = 4 \pi \hbar^2 a / m$ the strength of interaction, with $a$ the scattering length. In the presence of the combined traps $V(r) = \frac{1}{2} m \omega^2 (\rho^2 + \nu^2 y^2) + V_{\text{opt}}$ where $\omega$ is the angular frequency of the harmonic trap in the radial direction $\rho$, $\nu \omega$ that in the axial direction $y$, and $V_{\text{opt}}$ is the optical-lattice potential. The axially-symmetric wave function can be written as $\Psi(r, \tau) = \psi(\rho, y, \tau)$, where $0 \leq \rho < \infty$ and $-\infty < y < \infty$. Transforming to dimensionless variables $\hat{\rho} = \sqrt{2} \rho / l$, $\hat{y} = \sqrt{2} y / l$, $t = \tau \omega$, $l \equiv \sqrt{\hbar / (m \omega)}$, and $\varphi(\hat{\rho}, \hat{y}; t) \equiv \hat{\rho} \sqrt{\frac{1}{2} / \sqrt{8}} \psi(\rho, y; \tau)$, Eq. (1) becomes [7,8]

$$
\left[ -i \frac{\partial}{\partial t} - \frac{\partial^2}{\partial \hat{\rho}^2} + \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} - \frac{\partial^2}{\partial \hat{y}^2} + \frac{1}{4} \left( \hat{\rho}^2 + \nu^2 \hat{y}^2 \right) + \frac{V_{\text{opt}}}{\hbar \omega} \right. 
\left. - \frac{1}{\hat{\rho}^2} + 8 \sqrt{2} \pi n \left| \frac{\varphi(\hat{\rho}, \hat{y}; t)}{\hat{\rho}} \right|^2 \right] \varphi(\hat{\rho}, \hat{y}; t) = 0, \quad (2)
$$

where $n = Na / l$. In terms of probability $P(y, t) \equiv 2 \pi \int_0^\infty d\hat{\rho} |\varphi(\hat{\rho}, \hat{y}; t)|^2 / \hat{\rho}$, the normalization of the wave function is $\int_0^\infty d\hat{y} P(y, t) = 1$. The probability $P(y, t)$ is useful in the study of the formation and evolution of the interference pattern and Josephson oscillation.

In the experiments of Cataliotti et al. [2,3] with repulsive $^{87}$Rb atoms, the axial and radial trap frequencies were $\nu \omega = 2 \pi \times 9$ Hz and $\omega = 2 \pi \times 92$ Hz, respectively. The optical potential created with the standing-wave laser field of wave length $\lambda = 795$ nm is given by $V_{\text{opt}} = V_0 E_R \cos^2(\kappa_L z)$, with $E_R = h^2 \kappa_L^2 / (2 m)$, $\kappa_L = 2 \pi / \lambda$ and $V_0$ (<< 12) the strength. In terms of the dimensionless laser wave length $\lambda_0 = \sqrt{2} \lambda / l \simeq 1$, $E_R / (\hbar \omega) = 4 \pi^2 / \lambda_0^2$. 

In the experiments of Cataliotti et al. [2,3] prepared a BEC on a joint harmonic plus an optical-lattice trap.
Figure 1. $P(y,t)$ vs. $y$ and $t$ for the trapped BEC after the removal of combined optical and harmonic traps at $t = 0$ for (a) $c = 1$, (b) $c = 2.4$, (c) $c = 3.5$ and (d) $c = 4.6$. The interference pattern has led to 3, 5, 7 and 9 peaks in these cases.

Hence $V_{\text{opt}}$ of Eq. (2) becomes

$$V_{\text{opt}} = V_0 \frac{4\pi^2}{\lambda_0^2} \left[ \cos^2 \left( \frac{2\pi c \lambda_0}{c} y \right) \right],$$

where the parameter $c$ controls the spacing $c\lambda_0/2$ between the optical-lattice sites. The experimental condition of Cataliotti et al. [2,3] is obtained by taking $c = 1$.

The GP equation (2) is solved by the Crank-Nicolson method [11]. An interference pattern is formed by suddenly removing the combined traps at time $t = 0$ on the ground-state solution. To study the time evolution of the system we plot in Figs. 1 (a), (b), (c) and (d) $P(y,t)$ vs. $y$ and $t$ for $c = 1, 2.4, 3.5$ and 4.6, respectively. The variation of $c$ of Eq. (3) corresponds to a variation of the spacing between successive sites. The increase in $c$ simulates an increase in the distance between the sites and a decrease in the number of occupied sites. In these plots one can clearly see the central condensate and the moving interference peak(s). As the number of occupied optical-lattice sites decreases with the increase of $c$, the interference pattern develops more and more peaks.

We have illustrated in Fig. 1 the formation of interference pattern upon releasing the BEC from the joint optical and harmonic traps. The formation of interference pattern implies phase coherence and superfluidity in the condensate. The Josephson oscillation is a direct manifestation of superfluidity while the condensed atoms freely tunnel through the high optical lattice barriers. The absence of interference pattern after displacing the harmonic trap implies the loss of superfluidity and a disruption of the Josephson oscillation. This phenomenon represents a superfluid to a classical insulator transition and can be studied by a mean-field approach. The superfluid to a quantum Mott insulator
Figure 2. Contour plot of $P(y, t)$ vs. $y$ and $t$ for $V_0 = 2E_R$ after a harmonic trap displacement of 30 µm showing the Josephson oscillation with frequency 8.45 Hz.

Figure 3. Contour plot of $P(y, t)$ vs. $y$ and $t$ for $V_0 = 5E_R$ after a harmonic trap displacement of 70 µm demonstrating the disruption of the Josephson oscillation.

transition as in Ref. [5] can only be understood by a field-theoretic approach beyond mean-field theory. We study the Josephson oscillation using both approaches, e.g., upon releasing the BEC from the joint traps as in the previous study [7] and by following the condensate directly after displacing the harmonic trap. Although a free expansion is the only way to observe the Josephson oscillation experimentally, for numerical purpose the direct approach seems to be more precise and involves less computer memory and time.

In Fig. 2 we exhibit a contour plot of $P(y, t)$ vs. $y$ and $t$ for $V_0 = 2E_R$ after displacing the harmonic trap by a distance 30 µm as in the experiment [2]. The Josephson sinusoidal oscillation around the displaced trap center at $y = -30$ µm is clearly visible in this plot from which the frequency of oscillation can be obtained reasonably accurately. However, when the displacement of the harmonic trap is increased beyond a critical value, the oscillatory motion is disrupted as shown in Fig. 3 for $V_0 = 5E_R$ and a harmonic trap displacement of 70 µm in agreement with experiment [3]. In this case, unlike in Fig. 2, the condensate does not cross the center of the displaced trap at $y = -70$ µm. We performed a direct study of the Josephson oscillation for different $V_0$ for a displacement of the harmonic trap below the critical value for the disruption of superfluidity. In Fig. 4 we plot the Josephson frequencies calculated from this study as well as those calculated by allowing an expansion of the BEC as in Ref. [7]. In this figure we also plot [2] the experimental results as well as those obtained by using the tight-binding approximation. The three-dimensional results obtained after expansion in Ref. [7] and obtained directly here are in agreement with each other as well as with experiment [2]. The present direct results fit a smooth line and hence seem to be more accurate than the results of Ref. [7]. The results for tight-binding approximation [2] are slightly different from the full
Figure 4. Josephson frequency vs. $V_0$: solid circle with error bar - experiment [2]; solid triangle - tight-binding approximation [2]; square - indirect result after expansion [7]; star with full line - present direct result.

Figure 5. Axial width ($R_y$) of central peak by peak separation $d$ vs. time spent in displaced trap: experiment [3] - star - displacement of 30 $\mu$m, triangle - displacement of 120 $\mu$m; present theory - full lines.

three-dimensional results.

Finally, in Fig. 5 we plot the axial width $R_y$ of the central peak normalized to peak separation $d$ for $V_0 = 5E_R$ vs. time spent in the displaced trap for displacements of 30 $\mu$m and 120 $\mu$m and compare with experiment [3]. For the displacement of 30 $\mu$m, $R_y/d$ remains constant, whereas, for 120 $\mu$m, $R_y/d$ increases to unity with time. However, the theoretical increase is much faster than in experiment.

To conclude, we have provided an account of matter-wave interference, Josephson oscillation and its disruption using the three-dimensional mean-field GP equation. The results are in agreement with recent experiments by Cataliotti et al. [2,3]. The present results for Josephson frequencies are slightly different from those of tight-binding approximation. Further studies in three dimension are needed to understand this difference.

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