Entropy of the FRW universe based on the generalized uncertainty principle

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Abstract

The statistical entropy of the FRW universe described by time-dependent metric is newly calculated using the brick wall method based on the general uncertainty principle with the minimal length. We can determine the minimal length with the Plank scale to obtain the entropy proportional to the area of the cosmological apparent horizon.

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I. INTRODUCTION

There has been much interest in the holographic principle for gravity [1–5]. It relates the quantum theory of gravity to quantum field theory without gravity on the boundary with lower dimensions. As for black holes, the entropy of a black hole is proportional to the area of the horizon [6–8]. On the other hand, it has been claimed that the holographic property on the entropy also appears in the cosmological context with a particle horizon [9]. Concerning the holography of the FRW universe, the covariant entropy bound has been described in Ref. [10] and it has been shown that the $k = 1$ FRW universe filled with CFT (radiation-matter) indicates the holographic nature in terms of temperature and entropy [11]. The boundary in the cosmology can be chosen as the cosmological apparent horizon instead of the event horizon of a black hole. It can be shown that the thermodynamic first law can be satisfied and the entropy is $S = A/4G$ with the temperature $T = |\kappa|/2\pi$, where $G$, $A$, and $\kappa$ are the gravitational constant, the area of the apparent horizon, and the surface gravity on the horizon, respectively [12–16].

The brick wall method suggested by 't Hooft [17] can be used for calculating the statistical entropy of a black hole, and the cutoff parameter is introduced to avoid the divergence near the event horizon. The method can help us to understand the origin of entropy in various black holes [18–26]. Since degrees of freedom of a field are dominant near horizon, the brick wall model has often been replaced by a thin-layer model, which makes the calculation of entropy simple [27–29]. Recently, it has been shown that the (thin-layered) brick wall model can be applied to a time-dependent black hole with an assumption of local equilibrium near horizon [30]. It can be also applied to calculation of the entropy for the Friedmann-Robertson-Walker (FRW) universe [31, 32], which is described with time-dependent metric. On the other hand, the generalized uncertainty principle (GUP) modifying the usual Heisenberg’s uncertainty principle has been used in calculating the entropy of various black holes [33–43], where the cutoff parameter is naturally connected with the minimal length.

In this paper, we would like to study the entropy of the FRW universe based on the brick wall method considering the GUP, and find the minimal length giving the entropy proportional to the area. In the section III we review the FRW cosmology briefly. The state of the universe will be assumed to be in the locally thermodynamic equilibrium. Then, the degrees of freedom of a field are dominant near horizon like as the case of the black hole.
so that the thin layer will be considered as an equilibrium system near the horizon. We will calculate the entropy of the FRW universe and determine the minimal length in terms of the Plank scale and the total density parameter in the section III. As a result, we will show that the entropy becomes \( S = A/4G \) when the cosmological constant is dominant in compared with the other matters. Some conclusion and comment are given in section IV.

II. BACKGROUND GEOMETRY OF THE FRW UNIVERSE

The standard metric of the FRW universe is given by

\[
\frac{ds^2}{-dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]},
\]

where \( a(t), k = 0, \pm 1, \) and \( d\Omega^2 = d\theta^2 + \sin^2 \theta \, d\phi^2 \) are the scale factor, the normalized spatial curvature, and the line element on the unit two-sphere, respectively. Before we take the brick-wall method to calculate the entropy of the FRW universe, we need to transform the radial coordinate to the similar form to the metric of black holes as follows:

\[
\frac{ds^2}{1/v^2 \left( -f \, dt^2 - 2HR \, dt \, dR + dR^2 \right) + R^2 d\Omega^2},
\]

where we have defined the new radial coordinate as \( R = ar \) and the Hubble parameter as \( H(t) = \dot{a}/a \). The functions \( v \) and \( f \) are defined by \( v(t, R) = \sqrt{1 - kR^2/a^2} \) and \( f(t, R) = 1 - R^2/R_A^2 \) with the apparent horizon of \( R_A(t) = 1/\sqrt{H^2 + k/a^2} \). The dot denotes the derivative with respect to the time coordinate \( t \). The temperature on the apparent horizon is defined by \( T = \beta^{-1} = |\kappa| \pi \) and the surface gravity on the horizon is given by \( \kappa = \frac{1}{2\sqrt{-h}} \, \partial_a (\sqrt{-h} h^{ab} \partial_b R)|_{R=R_A} \) [44, 45], where the metric \( h_{ab} \) is defined by \( ds^2 = h_{ab}(x) dx^a dx^b + R^2(x) d\Omega^2 \). Then we can write the explicit form of the temperature as

\[
T = \frac{H^2 R_A}{2\pi} \left| 1 + \frac{1}{2H^2 \left( \dot{H} + \frac{k}{a^2} \right)} \right|.
\]

Now, we will review the FRW cosmology briefly. From the metric (11), the Einstein equation is given by

\[
H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho + \frac{1}{3} \Lambda,
\]

\[
\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p),
\]
where $\rho$, $p$, and $\Lambda$ are the energy density, the pressure, and the cosmological constant, respectively. The equation-of-state parameter $\gamma = p/\rho$ is defined by

$$\gamma = \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \frac{\Omega_{\text{rad}}/3 - \Omega_{\Lambda}}{\Omega_{\text{tot}}},$$

(6)

where $\rho_{\text{tot}} = \rho + \rho_{\Lambda} = \rho_m + \rho_{\text{rad}} + \rho_{\Lambda}$ and $p_{\text{tot}} = p + p_{\Lambda} = p_m + p_{\text{rad}} + p_{\Lambda}$ with $\rho_{\Lambda} = \Lambda/8\pi G$ and $\rho_k = -3k/8\pi Ga^2$ [46]. In particular, $\gamma$ becomes $1/3$ for the radiation-dominated, zero for the matter($m$)-dominated, $-1/3$ for the spatial curvature($k$)-dominated, and $-1$ for the vacuum energy($\Lambda$)-dominated universe. In general, the equation-of-state parameter is given by $\gamma = n/3 - 1$ when an energy density satisfies the power law, $\rho \sim a^{-n}$. The subscript “$m$” and “rad” mean the matter and the radiation-dominant, respectively. The density parameter for the type of energy $\rho_i$ ($i = m, \text{rad}, \Lambda, \text{etc}$.) has been defined by $\Omega_i = \rho_i/\rho_c$, where $\rho_c = 3H^2/8\pi G = \rho_{\text{tot}} + \rho_k$ is the critical density. The equations of motion (4) and (5) can be written as

$$H^2 = \frac{8\pi G}{3} (\rho_{\text{tot}} + \rho_k),$$

(7)

$$\dot{H} + H^2 = \frac{4\pi G}{3} (1 + 3\gamma)\rho_{\text{tot}}.$$

(8)

Using Eqs. (7) and (8), the temperature (3) is calculated as

$$T = \beta^{-1} = \frac{1}{2\pi R_A} \left| \frac{1 - 3\gamma}{4} \right|.$$

(9)

The apparent horizon is also given by

$$R_A = \frac{1}{H \sqrt{\Omega_{\text{tot}}}},$$

(10)

where $\Omega_{\text{tot}} = 1 - \Omega_k = 1 + k/(H^2 a^2)$.

III. QUANTUM STATISTICAL ENTROPY OF THE FRW UNIVERSE

Now, let us consider a quantum gas of scalar particles confined within a thin layer near the apparent horizon of the FRW universe and introduce an infinitesimal cut-off parameter $\epsilon$. It is assumed that the scalar field satisfies the Klein-Gordon equation

$$(\Box - \mu^2)\Phi(t, R, \theta, \phi) = 0,$$

(11)
with the boundary condition $\Phi(R_A - \epsilon) = \Phi(R_A)$, where $R_A - \epsilon$ and $R_A$ represent the inner and outer walls of the layer, respectively, and $\mu$ is the mass of the field. From the metric (2), Eq. (11) can be rewritten as

$$-v \partial_t \left[ \frac{1}{v} (\partial_t \Phi + HR \partial_R \Phi) \right] + \frac{v}{R^2} \partial_R \left[ \frac{R^2}{v} (-HR \partial_t \Phi + f \partial_R \Phi) \right] + \left( \frac{1}{R^2} \nabla^2_{\Omega} - \mu^2 \right) \Phi = 0, \quad (12)$$

where $\nabla^2_{\Omega} \equiv \csc \theta \partial_\theta (\sin \theta \partial_\theta) + \csc^2 \theta \partial^2_\phi$ means the Laplacian on the unit two sphere. Then, by using the WKB approximation with $\Phi \sim \exp[i \sigma(t, R, \theta, \phi)]$, we can obtain the relation as

$$f p_R^2 + 2HR \omega p_R + \frac{p_\theta^2}{R^2} + \frac{p^2_\phi}{R^2 \sin^2 \theta} = \omega^2 - \mu^2, \quad (13)$$

where the energy and the momenta of the scalar field are defined by $\omega = -\frac{\partial \sigma}{\partial t}$, $p_R = \frac{\partial \sigma}{\partial R}$, $p_\theta = \frac{\partial \sigma}{\partial \theta}$, and $p_\phi = \frac{\partial \sigma}{\partial \phi}$. Note that Eq. (13) can also be obtained from the relation $p_\mu p^\mu = -\mu^2$ with $p_\mu = (-\omega, p_R, p_\theta, p_\phi)$. Moreover, we obtain $p^2_\mu = g^{\mu \nu} p_\nu p_\mu = f p_R^2 + HR \omega p_R + \frac{p_\theta^2}{R^2} + \frac{p^2_\phi}{R^2 \sin^2 \theta} = \omega^2 - \mu^2 - 2HR \omega p_R - f p_R^2$, from which we have to read the condition for $p_R$ as $p_R^- \leq p_R \leq p_R^+$, where $p_R^\pm$ are defined by $p_R^\pm = [-HR \pm \sqrt{(f + H^2 R^2)\omega^2 - \mu^2 f}]/f$.

From now on, we will assume such a locally equilibrium system that the temperature of thermal radiation is slowly varying near the horizon. The temperature is approximately proportional to the apparent horizon, $T \sim R_A^{-1}$, which will be shown later, and then, in the locally equilibrium system it requires that $\delta T/T \sim \delta R_A/R_A \sim \delta a/a \ll 1$ for $H \ll 1$, where $\delta$ denotes fluctuation of each quantity.

On the other hand, many efforts have been recently devoted to the generalized uncertainty relation [47–52], which is given by

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[ 1 + \lambda \left( \frac{\Delta \mu}{\hbar} \right)^2 \right]. \quad (14)$$

Here $\lambda$ is the GUP parameter, which is related to the proper length, $2\sqrt{\lambda}$. It will effectively play a role of the brick wall cutoff. We simply take the units $\hbar = k_B = c \equiv 1$. Then, one can easily get $\Delta x \geq \sqrt{\lambda}$, which gives the lowest bound of the the minimal length near horizon. Furthermore, based on the generalized uncertainty relation, the 3-dimensional volume of a phase cell is changed from $(2\pi)^3$ into $(2\pi)^3(1 + \lambda p^3)^3$ [47, 49].

Then, the number of quantum states with the energy less than $\omega$ under the GUP is calculated as

$$n(\omega) = \frac{1}{(2\pi)^3} \int dR d\theta d\phi \frac{dp_R dp_\theta dp_\phi}{(1 + \lambda p^3)^3}.$$
\[ S = \beta^2 \frac{\partial F}{\partial \beta} = \beta^2 \int d\omega \frac{\omega n(\omega)}{4 \sinh^2 \frac{1}{2} \beta \omega}. \] (16)

For convenience, we replaced \( \omega \) by \( x \equiv \frac{1}{2} \beta \omega \). Note that \( f \) and \( x_0 = \frac{1}{2} \beta \omega_0 = \frac{1}{2} \beta \mu \sqrt{f} / \sqrt{f + H^2 R^2} \) vanish while \( x_0^2 \) does not for massive scalar fields in the near horizon limit of i.e., \( R \to R_A \). Since \( f/\lambda \) can be taken as a finite value near the horizon, the logarithmic term in Eq. (15) can be neglected in the leading order approximation. Then, the entropy (16) becomes

\[ S \approx \frac{\beta^3}{8 \pi \lambda^3 H} \int_0^\infty dx \frac{I(x)}{\sinh^2 x}, \] (17)

with

\[ I(x) \equiv \int_{R_A - \epsilon}^{R_A} dR \frac{Rx^2 + \frac{\beta^2 f}{4 \lambda H^2 R}}{x^4 - \frac{1}{4} \beta^2 \mu^2 x^2 + \beta^2 / (2 \lambda)}. \] (18)

In the leading order of \( \epsilon \), Eq. (18) is calculated as

\[ I(x) = \frac{R_A x^2 \epsilon}{x^4 - \frac{1}{4} \beta^2 \mu^2 x^2 + \beta^2 / (2 \lambda)} + O(\epsilon^2). \] (19)

Substituting Eq. (19) into Eq. (17), we obtain

\[ S \approx \frac{\beta^3 R_A \epsilon}{8 \pi \lambda^3 H} \int_0^\infty dx h(x), \] (20)

where

\[ h(x) = \frac{x^2}{\sinh^2 x \left( x^4 - \frac{1}{4} \beta^2 \mu^2 x^2 + \frac{\beta^2}{2 \lambda} \right) }. \] (21)
The entropy (20) can be regarded as the complex function
\[ h(z) = z^2 / [\sinh^2 z (z^2 - \alpha_+^2)(z^2 - \alpha_-^2)] \]
with \( \alpha_\pm^2 = \frac{1}{8} \beta^2 \mu^2 \pm i \Gamma^2 \) and \( \Gamma = \sqrt{\beta/\sqrt{2\lambda (1 - \beta^2 \mu^4/32)}}^{1/4} \). If we assume that \( \mu^2 \beta < 4\sqrt{2/\lambda} \), then \( \Gamma \) is real and positive. The function \( h(z) \) has poles at \( z = \pm \alpha_+ \), \( z = \pm \alpha_- \), and \( z = \pm \pi \) where \( n \)'s are non-vanishing integers. The residues of \( h(z) \) are \( 2\pi n (n^2 \pi^4 - \alpha_+^2 \alpha_-^2) / [i(n^2 \pi^2 + \alpha_+^2)(n^2 \pi^2 + \alpha_-^2)] \) at \( z = \pm \alpha_\pm \). At low temperature, the residues of \( h(z) \) at \( z^2 = \alpha_\pm^2 \) decrease exponentially when \( \beta \) goes to infinity so that the leading term of the integral in Eq. (20) comes from the poles at \( z = \pm \pi \).

Using the residue theorem, the entropy can be obtained as
\[ S \approx \frac{1}{4} A \cdot \frac{\beta \epsilon}{24 \lambda^2 H R_A}, \tag{22} \]
where \( A = 4\pi R_A^2 \) is the area of the apparent horizon.

Defining a diagonalized metric, \( \tilde{g}_{\mu \nu} \) from Eq. (2), the proper length is calculated as
\[ 2\sqrt{\lambda} \equiv \int_{R_{A-\epsilon}}^{R_A} dR \sqrt{\tilde{g}_{RR}} = \int_{R_{A-\epsilon}}^{R_A} dR \approx \sqrt{2R_A \epsilon}, \tag{23} \]
which leads to \( \epsilon = 2\lambda / R_A \). Then, we can obtain the entropy as follows:
\[ S \approx \frac{1}{4} A \cdot \frac{\beta}{12\lambda H R_A^2} = \frac{A}{4} \cdot \frac{\pi}{6\lambda H^3} \left( H^2 + \frac{k}{a^2} \right)^{3/2} \left| 1 + \frac{1}{2H^2} \left( \dot{H} + \frac{k}{a^2} \right) \right|^{-1}, \tag{24} \]
where the temperature (3) has been used.

Now, one can rewrite the above entropy formula in terms of matter contents using the equations of motion for the FRW cosmology. Inserting Eqs. (9) and (10) into Eq. (24) and recovering the dimension, the entropy becomes
\[ S \approx \frac{A}{4G} \cdot \frac{2\pi \ell_p^2 \sqrt{\Omega_{\text{tot}}}}{3\lambda |1 - 3\gamma|}, \tag{25} \]
where \( \ell_p = \sqrt{Gh/c^3} \) is the Plank length. Setting the minimal length as \( 2\sqrt{\lambda} = \ell_p \Omega_{\text{tot}}^{1/4} \sqrt{2\pi/3} \approx 1.4472 \times \Omega_{\text{tot}}^{1/4} \ell_p \), the entropy is written as
\[ S \approx \frac{4}{|1 - 3\gamma|} \frac{A}{4G}. \tag{26} \]
According to the recent observational data [53], the density parameter is given by \( \Omega_{\text{tot}} \approx 1 \) for our present universe. Therefore, the minimal length is approximately obtained as \( 2\sqrt{\lambda} \approx 1.4472 \times \ell_p \). Note that the minimal length is exact constant for \( k = 0 \), while it is time-dependent for \( k = \pm 1 \). It is interesting to note that the vacuum energy(Λ)-dominated universe of \( \gamma = -1 \) gives the one quarter area law.
IV. CONCLUSIONS

In conclusion, we have newly studied the cosmological entropy applying the brick wall method based on the GUP to the time-dependent FRW universe. In fact, there is another way to define the cutoff in that we could absorb the state parameter, $\gamma$, into the cutoff by

$2\sqrt{\lambda} = \ell_p \Omega_\text{tot}^{1/4} \sqrt{2\pi/3}/\sqrt{|1 - 3\gamma|/4} \ (\gamma \neq 1/3)$,

which yields the one quarter area law. In that case, we can not avoid the era-dependent cutoff. We do not deal with the holographic property of the FRW universe in this paper, but we hope that we will study it in the next work.

The final comment is that for the radiation dominant era of $\gamma = 1/3$, the entropy is divergent. In the present formulation, the temperature is zero as seen in Eq. [9], which is reminiscent of the extremal black hole so that it is not easy to formulate the free energy and its entropy. We hope this issue will be discussed in elsewhere.

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