The linear conductance of a tunnel junction in series with an ohmic resistor is determined in the high temperature limit. The tunneling current is treated nonperturbatively by means of path integral techniques. Due to quantum effects the conductance is smaller than the classical series conductance. The reduction factor is found to be nonanalytic in the environmental resistance for vanishing resistance. This behavior is a high temperature manifestation of the Coulomb blockade effect.

PACS numbers: 73.23.Hk, 73.40.Gk, 73.40.Rw

In the last decade or so tremendous progress was made in our understanding and control of Coulomb blockade phenomena. These effects arise in nanostructures with small capacitances when the single electron charging energy $E_C = e^2/2C$ becomes significant. So far work has concentrated on the low-temperature, typically millikelvin region to satisfy the seemingly obvious condition $k_BT \ll E_C$ for the manifestation of charging effects. However, recently Pekola and coworkers have demonstrated that there are remarkable signatures of the Coulomb blockade effect also for temperatures in the few Kelvin region well above the charging energy. In fact, charging phenomena in the high-temperature region can be used for precision thermometry.

Further, in most of the theoretical studies of charging effects in metallic nanostructures it is assumed that the tunneling resistance $R_T$ of the tunnel junctions in the device exceeds the von-Klitzing resistance $R_K = h/e^2 \approx 25.8k\Omega$. Under the condition $R_T \gg R_K$ quantum fluctuations of the charge are strongly suppressed. Hence, in a sense this latter condition ensures that charging effects are not washed out by quantum fluctuations in much the same way as the above-mentioned temperature constraint ensures stability against thermal fluctuations.

It is now interesting to examine whether signatures of the Coulomb blockade effect persist when thermal fluctuations are large and the tunneling resistance may become small. This question will be addressed for the fundamental problem of an ultrasmall tunnel junction biased by a voltage source $V$ via leads of impedance $R$.

We aim at the zero-bias differential conductance $G = \partial I / \partial V |_{V=0}$ where $I$ is the current and $V$ the applied voltage. Our approach is based on the path integral formulation of the tunnel junction and employs the generating functional

$$ Z[\xi] = \text{tr} T_\beta \exp\left\{ -\frac{1}{\hbar} \int_0^{\beta \hbar} d\tau \left[ H - I\xi(\tau) \right] \right\} \quad (1) $$

where $H$ is the Hamiltonian of the system for $V = 0$ and $I$ the microscopic current operator for charge flow in the leads. $T_\beta$ is the time ordering operator for the imaginary-time variable $\tau$. Introducing a phase variable $\varphi$ which is related to the voltage across the tunnel junction via a Josephson-type relation, the generating functional may be written as a path integral.

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FIG. 1. Circuit diagram of a tunnel junction with capacitance $C$ and tunneling resistance $R_T$ biased by a voltage source $V$ via leads of impedance $R$. Specificallly, we consider the circuit depicted in Fig. 1. We aim at the zero-bias differential conductance $G = \partial I / \partial V |_{V=0}$ where $I$ is the current and $V$ the applied voltage. Our approach is based on the path integral formulation of the tunnel junction and employs the generating functional

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\[ Z[\xi] = \int D[\varphi] \exp \left\{ -\frac{1}{\hbar} S[\varphi, \xi] \right\} \]  
\[ \text{with the effective Euclidean action} \]
\[ S[\varphi, \xi] = S_C[\varphi] + S_T[\varphi] + S_R[\varphi, \xi]. \]
\[ \text{where} \]
\[ S_C[\varphi] = \int_0^{\hbar} d\tau \frac{h^2 C}{2e^2} \varphi^2 \]
describes Coulomb charging of the junction and
\[ S_T[\varphi] = 2 \int_0^{\hbar} d\tau \int_0^{\hbar} d\tau' \alpha(\tau - \tau') \sin^2 \left[ \frac{\varphi(\tau) - \varphi(\tau')}{2} \right] \]
\[ \text{quasi-particle tunneling across the junction} \]
The kernel \( \alpha(\tau) \) is determined by the tunneling resistance \( R_T \) and may be written as
\[ \alpha(\tau) = \frac{1}{\hbar \beta} \sum_{n=-\infty}^{+\infty} \tilde{a}(\nu_n) e^{-i\nu_n \tau} \]
where the \( \nu_n = 2\pi n / \hbar \beta \) are Matsubara frequencies and
\[ \tilde{a}(\nu_n) = -\frac{\hbar}{4\pi} \frac{R_K}{R_T} |\nu_n|. \]
The last term in the action (3) reads
\[ S_R[\varphi, \xi] = \frac{1}{2} \int_0^{\hbar} d\tau \int_0^{\hbar} d\tau' k(\tau - \tau') \]
\[ \times \left[ \alpha(\tau) + \frac{e}{\hbar} \xi(\tau) - \varphi(\tau') - \frac{e}{\hbar} \xi(\tau') \right]^2 \]
For \( \xi(\tau) \equiv 0 \) this is the well-known influence functional of an Ohmic lead resistance \( R \) with the kernel
\[ k(\tau) = \frac{1}{\hbar \beta} \sum_{n=-\infty}^{+\infty} \tilde{k}(\nu_n) e^{-i\nu_n \tau} \]
where
\[ \tilde{k}(\nu_n) = -\frac{\hbar}{4\pi} \frac{R_K}{R} |\nu_n|. \]
Note that the influence functional (3) of the lead resistance is at most quadratic in the phase while the tunneling term (4) with a trigonometric phase dependence renders the path integral (3) non-Gaussian. This is due to the discrete charge transfer across the tunnel junction.
The conductance \( G \) can be calculated from the Kubo formula
\[ G = \lim_{\omega \to 0} \frac{1}{i\omega} \text{Im} \left[ \lim_{\nu_n \to -i\omega + i\delta} \int_0^{\hbar} d\tau e^{i\nu_n \tau} \langle I(\tau)I(0) \rangle \right] \]
where the current-current correlator \( \langle I(\tau)I(0) \rangle \) reads in terms of the generating functional (3)
\[ \langle I(\tau)I(0) \rangle = \left. \frac{\hbar^2}{Z(0)} \delta^2 Z[\xi] \right|_{\xi(\tau) \equiv 0} . \]
Now, using the path integral representation (5) and Eq. (6), the correlator may be written as
\[ \langle I(\tau)I(0) \rangle = \frac{1}{Z} \int D[\varphi] \exp \left\{ -\frac{1}{\hbar} S[\varphi(0)] \right\} \]
\[ \times \left( 2 e^2 k(\tau) + I[\varphi, \tau]I[\varphi, 0] \right) \]
where \( S[\varphi] = S[\varphi, \xi \equiv 0] \)
\[ Z = \int D[\varphi] \exp \left\{ -\frac{1}{\hbar} S[\varphi(0)] \right\} \]
is the partition function. Further, the current functional \( I[\varphi, \tau] \) is given by
\[ I[\varphi, \tau] = 2e \int_0^{\hbar} d\tau' k(\tau - \tau') \varphi(\tau'). \]
To calculate the conductance from Eq. (6), we first need to determine the Fourier components
\[ C(\nu_n) = \int_0^{\hbar} d\tau e^{i\nu_n \tau} \langle I(\tau)I(0) \rangle . \]
Using Eqs. (7), (9), and (11), one finds that \( C(\nu_n) = C_1(\nu_n) + C_2(\nu_n) \) where
\[ C_1(\nu_n) = 2e^2 \tilde{k}(\nu_n) \]
and
\[ C_2(\nu_n) = \frac{1}{Z} \int D[\varphi] \exp \left\{ -\frac{1}{\hbar} S[\varphi(0)] \right\} F[\varphi, \nu_n]. \]
Here the functional
\[ F[\varphi, \nu_n] = 4e^2 \beta \tilde{k}(\nu_n) \hat{\varphi}(\nu_n) \sum_{m=-\infty}^{+\infty} \tilde{k}(\nu_m) \hat{\varphi}(\nu_m) \]
is given in terms of the Fourier coefficients \( \hat{\varphi}(\nu_n) \) of the phase variable. Since the action \( S[\varphi] \) is invariant under a global phase shift, we may put \( \hat{\varphi}(0) = 0 \).
According to the decomposition of \( C(\nu_n) \) into (13) and (14), the conductance (6) splits into \( G = G_1 + G_2 \) where
\[ G_1 = \lim_{\omega \to 0} \frac{1}{i\omega} \text{Im} \frac{2e^2}{\hbar} \tilde{k}(-i\omega + \delta) = \frac{1}{R_1}. \]
Since the kernel \( \tilde{k}(\tau) \) has the analytic properties of an equilibrium correlation function, the analytic continuation of \( \tilde{k}(\nu_n) \) is unique (13).
To proceed we first note that in terms of the Fourier coefficients $\tilde{\varphi}(\nu_n)$ the action functional may be written

$$S[\varphi] = S_0[\varphi] + \sum_{k=2}^{\infty} S_{2k}[\varphi]$$

where

$$S_0[\varphi] = \hbar \sum_{n=1}^{\infty} \lambda(\nu_n) |\tilde{\varphi}(\nu_n)|^2$$

(17)

with the eigenvalues

$$\lambda(\nu_n) = \hbar \beta \left( \frac{\hbar \nu_n^2}{2E_c} + \frac{g}{2\pi} \right).$$

(18)

Here $g = R_K \left( R_T^{-1} + R^{-1} \right)$ is the dimensionless parallel conductance of the tunneling and lead resistances. Further,

$$S_{2k}[\varphi] = \left( \frac{-1}{2k} \right) \sum_{l=1}^{2k-1} \sum_{\nu_{n_1}, \ldots, \nu_{n_2k-1}} \tilde{\varphi}(\nu_{n_1}) \cdots \tilde{\varphi}(\nu_{n_{2k-1}}) \cdots \tilde{\varphi}(\nu_{n_{2k-1}})$$

(19)

Now, in the high-temperature limit we may expand about the Gaussian action $S_0[\varphi]$ using

$$\exp(-S[\varphi]/\hbar) = \exp(-S_0[\varphi]/\hbar)[1 - S_4[\varphi]/\hbar - S_6[\varphi]/\hbar - S_4[|\tilde{\varphi}|^2]/2\hbar^2 + \cdots].$$

The ratio of path integrals (14) can then be evaluated by means of

$$\frac{\int D[\varphi] \exp \left\{ -\frac{1}{\hbar} S_0[\varphi] \right\} \tilde{\varphi}(\nu_k) \tilde{\varphi}(\nu_l)}{\int D[\varphi] \exp \left\{ -\frac{1}{\hbar} S_0[\varphi] \right\}} = \frac{\delta_{k,-l}}{\lambda(\nu_k)}$$

and Wick’s theorem for higher order products of the Fourier coefficients $\tilde{\varphi}(\nu_n)$. Since

$$\frac{1}{\lambda(\nu_k)} = \frac{\beta E_C}{2\pi k^2 + g\beta E_C |k|}$$

is of order $\beta E_C$, the expansion (19) gives a high-temperature series in powers of $\beta E_C$.

Proceeding along these lines, one obtains from Eqs. (14) and (19)

$$C_2(\nu_n) = \frac{4e^2 \hbar}{\lambda(\nu_n)} \left( 1 + 2\frac{\beta}{\lambda(\nu_n)} \sum_{m=-\infty}^{\infty} \frac{\tilde{\alpha}(\nu_{n+m} - \nu_n - \tilde{\alpha}(\nu_m)}{\lambda(\nu_m)} \right) + \mathcal{O}(\beta E_C)^3, \quad (20)$$

In view of Eqs. (8) and (13), the high-temperature series for $C_2(\nu_n)$ yields a corresponding expansion of the conductance

$$G_2 = \lim_{\omega \to 0} \frac{1}{\hbar \omega} \ln C_2(-i\omega + \delta). \quad (21)$$

However, the analytic continuation and the limit $\omega \to 0$ involve a subtlety. As is readily seen from Eq. (15), a factor $1/\lambda(\nu_n)$ in Eq. (20) gives in this limit a factor $2\pi i/g\beta \hbar \omega$. While the $1/\omega$-divergences are canceled by $\omega$-factors stemming from the numerators in Eq. (21), as a net result, each $1/\lambda(\nu_n)$ factor reduces the order in $\beta E_C$ of the corresponding term in the high-temperature expansion of $G_2$ by one order. Hence, the quantum correction to $G_2$ of a given order in $\beta E_C$ depends on higher-order terms in the series expansion of $C_2(\nu_n)$.

An analysis of contributions of all orders (19) shows that a term of the series (13) with a product

$$S_{2k_1}[\varphi] S_{2k_2}[\varphi] \cdots S_{2k_l}[\varphi]$$

of quartic or higher-order actions gives quantum corrections to $G_2$ of order $(\beta E_C)^{k_1+k_2+\ldots+k_l-\ell}$ and of higher orders. This proves that the terms of the expansion of $C_2(\nu_n)$ given explicitly in Eq. (20) suffice to calculate the leading order quantum corrections to $G_2$.

Now, combining Eqs. (20) and (21) one finds after some algebra the explicit high-temperature result for $G_2$ that may be added to Eq. (16). This gives for the conductance

$$G = \frac{1}{R_T + R} \left\{ \frac{R_T}{R_T + R} \left[ \frac{\gamma + \psi(1 + u)}{u} + \psi'(1 + u) \right] \beta E_C \frac{\pi}{\nu^2} \right. + \mathcal{O}(\beta E_C)^2 \right\}, \quad (22)$$

where $\gamma$ is Euler’s constant and

$$u = \frac{g \beta E_C}{2\pi^2} = \frac{R_K (R_T + R) \beta E_C}{R_T R} \frac{2\pi^2}{R_K}. \quad (23)$$

Further, $\psi(z)$ and $\psi'(z)$ denote the digamma function and its derivative, respectively.

The predicted quantum corrections to the classical conductance $1/(R_T + R)$ reduce the conductance due to the Coulomb blockade effect. In Fig. 2 the result (22) is depicted for fixed temperature and tunneling resistance $R_T$ as a function of $R/R_K$. Typical lead resistances are in the range of 100Ω so that $R/R_K$ is usually very small. Then, the parameter $u$ defined in (23) is large even for temperatures $T$ well above $E_C/k_B$, and one can use the asymptotic expansion of the digamma function to give for $R \ll R_T, R_K$

$$G = \frac{1}{R_T} \left[ 1 + 2 \frac{R}{R_K} \ln \left( \frac{R}{R_K} \right) + \mathcal{O} \left( \frac{|\ln(\beta E_C)| R}{R_K} \right) \right]. \quad (24)$$
This shows that for small lead resistance the influence of the electromagnetic environment cannot be treated as a weak perturbation since a finite resistance gives rise to a reduction of $G$ that is non-analytic in $R$. This large effect of even a small lead resistance is clearly seen in Fig. 2.

\[ \frac{G}{RT + R} \]

**FIG. 2.** The ratio of the conductance $G$ and the classical conductance $1/(RT + R)$ is shown vs the environmental resistance $R$ in units of $R_K$ for $RT = R_K$ and $\beta E_C = 1/4$. The full line shows the result (22), and the dotted line depicts the approximation (25) for moderate-to-high resistance.

On the other hand, for values of $R/R_K$ of order 1 or larger, $u$ becomes small in the high-temperature limit and the digamma functions in Eq. (22) may be expanded to yield

\[
G = \frac{1}{RT + R} \left\{ 1 - \frac{RT \beta E_C}{3} + \mathcal{O}(\beta E_C^2, u \beta E_C) \right\}.
\]

(25)

We note that in the limit of weak tunneling, $RT \gg R, R_K$, this result may be approximated further to read

\[
G = RT^{-1} \left[ 1 - \beta E_C / 3 + \mathcal{O}(\beta E_C^2) \right],
\]

which is in accordance with the conventional approach based on perturbation theory in the tunneling term. Within this framework higher order terms in $\beta E_C$ can be evaluated explicitly [11]. However, the conventional approach misses the fact that the quantum corrections to Ohm’s law vanish in the limit $R/R_T \to \infty$ according to (22). For large $R$ many charges fluctuate across the tunnel junction on the relevant time scale $RC$ and lead to a suppression of charging effects.

In summary, we have shown that the conductance $G$ of a tunnel junction is affected by Coulomb charging effects even in the high-temperature range where $k_B T$ is large compared to the single electron charging energy $E_C$. In particular, in the experimentally relevant range of small environmental impedance $R$, the corrections to Ohm’s law are significant, since $G(R)$ is nonanalytic at $R = 0$. In view of the fact that the conductance can be measured very accurately, the predicted effects should be readily observable.

The nonclassical behavior of the conductance is also important for the precise determination of junction parameters from data in the “classical” regime. In this context it is important to note that the result (22) remains valid for small tunneling resistance $RT \ll R_K$ as long as $NR_T R_K$ where $N$ is the number of transport channels. Since $N$ is typically above $10^4$ for metallic tunnel junctions, the theory can be used to determine junction parameters also in the region of strong tunneling.

The authors would like to thank Daniel Esteve and Philippe Joyez for valuable discussions. Financial support was provided by the Deutsche Forschungsgemeinschaft (Bonn).

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