Hints for possible low redshift oscillation around the best fit ΛCDM model in the expansion history of the universe

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We search for possible deviations from the expectations of the concordance ΛCDM model in the expansion history of the Universe by analysing the Pantheon Type Ia Supernovae (SnIa) compilation along with its Monte Carlo simulations using redshift binning. We demonstrate that the redshift binned best fit ΛCDM matter density parameter Ωm0 and the best fit effective absolute magnitude M oscillate about their full dataset best fit values with considerably large amplitudes. Using the full covariance matrix of the data taking into account systematic and statistical errors, we show that at the redshifts below z ≈ 0.5 such oscillations can only occur in 4 to 5% of the Monte Carlo simulations. While statistical fluctuations can be responsible for this apparent oscillation, we might have observed a hint for some behaviour beyond the expectations of the concordance model or a possible additional systematic in the data. If this apparent oscillation is not due to statistical or systematic effects, it could be due to either the presence of coherent inhomogeneities at low z or due to oscillations of a quintessence scalar field.

I. INTRODUCTION

In 1998, two independent groups [1, 2] confirmed that the Universe is undergoing a phase of accelerated expansion, which has been attributed to the cosmological constant [3], thus establishing ΛCDM as the concordance model of modern cosmology. Despite its simplicity and consistency with most cosmological observations for almost two decades [4–10], ΛCDM faces some challenges at the theoretical level [11–14], as well as at the observational one, since recent observations revealed some inconsistencies between the measured values of the basic parameters of ΛCDM [15–22].

The most prominent tension in the context of ΛCDM is the so-called “H0 tension”, which describes the discrepancy between the Planck measurement of the Hubble parameter H0 [10] and the measurement published from Type Ia supernovae (SnIa) data that ranges from 4.4σ [9] to 6σ depending of the subset of SnIa that is used [23]. Moreover, a tension that is currently at a 2–3σ level, is the so-called “growth tension”, which refers to the mismatch between the σ8 (density rms matter fluctuations in spheres of radius of about 8 Mpc) and/or Ωm0 (matter density parameter) measurement of the Planck mission [10] with Weak Lensing (WL) [24–28] and Redshift Space Distortion (RSD) data [29–36].

In order to explain the aforementioned challenges a plethora of theories have been proposed in the literature to solve the theoretical [37–43] and the observational challenges of ΛCDM. In particular, for the observational challenges the mechanisms that have been proposed and can alleviate one or even both of these tensions simultaneously include early [44–48] and late dark energy models [49–55], interacting dark energy models [56–61], metastable dark energy models [18, 62–64], modified gravity theories [65–70] as well as modifications of the basic assumptions of ΛCDM such as non zero spatial curvature [71, 72], and many more [73–76] (see also the reviews [35, 77, 78] and references within).

The measurement of H0 that has been published by the SnIa data leading to the “H0 tension” is based on the assumption that SnIa can be considered as standard candles, thus allowing to probe the Hubble parameter through the apparent magnitude

\[ m(z) = M + 5 \log_{10} \left[ \frac{d_L(z)}{1 \text{Mpc}} \right] + 25, \]  

(1.1)

where \( d_L(z) \) is the luminosity distance, which in a flat Universe can be expressed as

\[ d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')}, \]  

(1.2)

while \( M \) corresponds to the corrected, over stretch and color, absolute magnitude.

Alternatively, the apparent magnitude can be expressed in terms of the dimensionless Hubble-free luminosity distance \( D_L \equiv H_0 d_L/c \) as

\[ m(z) = M + 5 \log_{10} [D_L(z)] + 5 \log_{10} \left( \frac{c/H_0}{1 \text{Mpc}} \right) + 25. \]  

(1.3)

Clearly, from Eq. (1.3) it is evident that the parameters \( H_0 \) and \( M \) are degenerate and since in the context of ΛCDM both of these are assumed to be constant, usually,
TABLE I: The best fit values with the 1σ error of $\mathcal{M}$ and $\Omega_0m$ for the four redshift bins with equal number of datapoints for the real data. Notice that for first three redshift bins the $\sigma$ distance ($\Delta \sigma$) of the best fit from the full dataset best fit is at least 1σ and on the average it is larger than 1.2σ. In the simulated Pantheon data such large simultaneous deviations for the first three bins occurs for about 2% of the datasets.

| Bin     | $z$ Range | $\mathcal{M} \pm 1\sigma$ error | $\Delta \sigma_\mathcal{M}$ | $\Omega_0m \pm 1\sigma$ error | $\Delta \sigma_{\Omega_0m}$ |
|---------|-----------|---------------------------------|------------------------------|-------------------------------|-------------------------------|
| Full Data | 0.01 < $z$ < 2.26 | 23.81 ± 0.01 | - | 0.29 ± 0.02 | - |
| 1st     | 0.01 < $z$ < 0.13 | 23.78 ± 0.03 | 1.14 | 0.07 ± 0.17 | 1.35 |
| 2nd     | 0.13 < $z$ < 0.25 | 23.80 ± 0.06 | 1.48 | 0.56 ± 0.19 | 1.34 |
| 3rd     | 0.25 < $z$ < 0.42 | 23.75 ± 0.06 | 0.99 | 0.18 ± 0.11 | 1.05 |
| 4th     | 0.42 < $z$ < 2.26 | 23.85 ± 0.06 | 0.69 | 0.33 ± 0.06 | 0.50 |

a marginalization process is performed \cite{4, 9, 79} over the degenerate combination

$$\mathcal{M} ≡ M + 5 \log_{10} \left[ \frac{c/H_0}{1\text{Mpc}} \right] + 25$$

$$= M - 5 \log_{10}(h) + 42.38, \quad (1.4)$$

where $h ≡ H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. However, in our analysis we choose to keep $\mathcal{M}$ in order to avoid any loss of crucial information.

The latest (and largest thus far) compilation of SnIa that has been published is the Pantheon dataset \cite{9}, consisting of 1048 SnIa in the redshift range 0.01 < $z$ < 2.3. Using Eqs. (1.1)-(1.4), the corresponding $\chi^2$ function reads

$$\chi^2(\mathcal{M}, \Omega_0m) = V_{\text{SnIa}} C_{ij}^{-1} V_{\text{SnIa}}^T,$$  

where $V_{\text{SnIa}} ≡ m_{\text{obs}}(z_i) - m(z)$ and $C_{ij}^{-1}$ is the inverse covariance matrix. The covariance matrix can be considered as the sum of two matrices: a diagonal matrix that is associated with the statistical uncertainties of the apparent magnitude $m_{\text{obs}}$ of each SnIa and a non-diagonal part that is connected with the systematic uncertainties due to the bias correction method \cite{9}.

In Refs. \cite{70, 80} it was shown that the best fit $\Lambda$CDM parameter values for the best fit parameters $\mathcal{M}$ and $\Omega_0m$ of redshift binned Pantheon data oscillate around the full dataset best fit at a level that is consistently larger than 1σ for the first three out of four redshift bins. If this variation is due to statistical fluctuations, then the same variation is anticipated to be evident in simulated Pantheon-like datasets. In this analysis we will address the following questions:

- How likely is this behaviour of the data in the context of the $\Lambda$CDM model?

- In how many realizations we can see more than the $\sigma$ deviations of the real data ($\sigma^{\text{real}}$) for both $\mathcal{M}$ and $\Omega_0m$ in the first three or any three out of four redshift bins?

- In how many realizations we can see more than the $1\sigma$ deviations for both $\mathcal{M}$ and $\Omega_0m$ in the first three or in any three out of four redshift bins?

The structure of the paper is the following: In Section II we describe the statistical analysis and the comparison of the constructed simulated datasets with the actual Pantheon data searching for abnormalities of the real data in the context of the reported level of Gaussian uncertainties. Finally, in Section III we summarize our results and discuss possible extensions of the present analysis.

II. REAL VERSUS MONTE CARLO DATA

In our Monte Carlo statistical analysis we split the Pantheon dataset \cite{9} into four redshift bins, consisting of equal number of datapoints (262). We then find the best fit parameters $\mathcal{M}$ and $\Omega_0m$ and $\sigma$ uncertainties in the context of a $\Lambda$CDM model for each bin, with $\mathcal{M}$ and $\Omega_0m$ being allowed to vary simultaneously. We also find the corresponding best fit for the full Pantheon dataset and identify the $\sigma$ distance between the best fit parameter values in each bin and the best fit value of the full dataset. The results of the tomography for the real data can be seen in Table I. Clearly, all first three bins of the real data best fits of $\mathcal{M}$ and $\Omega_0m$ differ by at least 1σ from the full dataset best fits.

In order to estimate the likelihood of such a $\sigma$ deviation of best fit values in the first three bins, we construct 1000 simulated Pantheon-like datasets, with random apparent magnitudes $m$ obtained from a multivariate normal distribution with a mean value equal to the best fit $\Lambda$CDM value of the real data using the full covariance matrix of the real data. The corresponding probability distribution is of the form

$$f_m(m_1, \ldots, m_k) = \frac{\exp \left[ -\frac{1}{2} (m - \bar{m})^T C^{-1} (m - \bar{m}) \right]}{\sqrt{(2\pi)^k |C|}},$$  

where $C$ is the full non-diagonal covariance matrix including both statistical and systematic errors, $m$ is the vector $\{m_1, m_2, \ldots, m_k\}$ and $\bar{m}$ corresponds to the mean value of the apparent magnitude vector. Using this multivariate normal distribution we construct the simulated datasets and find the percent fraction of them where all first three redshift bins have best fit $\Lambda$CDM parameter values $\mathcal{M}$ and $\Omega_0m$ that have simultaneously $\sigma$ distance from the real data best fit more than $k \sigma = \sigma_k \sigma$. The variations for the first three bins is anticipated to be evident in simulated Pantheon-like datasets.
These results for the parameters $M$ ($\sigma_k = \sigma_{k,M}$) and $\Omega_{0m}$ ($\sigma_k = \sigma_{k,\Omega}$) are shown in Fig. 1.

According to Fig. 1, the probability that all three first bins differ simultaneously more than $1\sigma$ from the best fit of each simulated full dataset in the context of $\Lambda$CDM is less than 5%. This is an effect approximately at $2\sigma$ level. The statistical level of this effect increases to nearly $3\sigma$ (or about 2.7% of the simulated datasets) when only the statistical part of the covariance matrix is taken into account in the construction of the simulated datasets.

In fact, this probability is even smaller if we consider the exact $\sigma$ differences that are shown in Table I and find the fraction of simulated datasets with simultaneous $\sigma$ differences larger that the exact corresponding $\sigma$ differences of the real data. In particular we find that the probability to have simultaneously $1.14\sigma$ difference (or larger) in the first bin, $1.48\sigma$ difference (or larger) in the second bin and $0.99\sigma$ difference (or larger) in the third bin for $M$, is $1.3 \pm 0.7\%$. Similarly, for $\Omega_{0m}$ we find the same probability to be $1.4 \pm 2\%$. Even though this decrease of probability is interesting to note, it is not generic as it is based on the fine tuned $\sigma$ deviations of the real data bins from the full data best fits ($1.14\sigma$, $1.48\sigma$ and $0.99\sigma$).

Therefore, we adopt the more generic and conservative statistical level of significance of 5% corresponding to the simultaneous deviation of at least $1\sigma$ for all three lowest $z$ bins. Note that a similar oscillating effect was also observed in Refs. [70, 80] even though its statistical significance was not quantified using simulated data as in the present analysis.

Moreover, it is interesting to check if this behaviour is also evident for any three out of four bins. In 1000 Monte Carlo realizations we find that the number of simulated datasets where the derived $\Omega_{0m}$ in any 3 bins is more than $1\sigma$ away from the best fit $\Omega_{0m}$ to the whole (random) data sample is $10.4 \pm 2.2\%$ while the corresponding number of cases for $M$ is $11.1 \pm 2.4\%$ as it is demonstrated in Fig. 2. The probability is smaller if we consider the exact $\sigma$ difference of Table I. In particular, we derive the number of cases where the derived $\Omega_{0m}$ in any 3 bins is more than $\sigma_{\text{real}_i}$ away from the best fit $\Omega_{0m}$ to the whole (random) data sample is $7.5 \pm 1.5\%$, while the corresponding number of cases for $M$ is $7.4 \pm 1.5\%$. A summary of the results can be seen in Table II. These results indicate that the aforementioned oscillating effect is much more prominent at low $z \lesssim 0.5$ where the dark energy density is more prominent than in the fourth bin, which involves higher $z$. This fact favors the possibility that the effect has a physical origin since a systematic effect would probably affect equally all four redshift bins.

III. CONCLUSION - OUTLOOK

We performed a redshift tomography of the Pantheon data dividing them into four redshift bins of equal number of datapoints and searched for hints of abnormal oscillation behaviour for the best fit parameter values of $M$ and $\Omega_{0m}$ in these bins with respect to the corresponding best fits of the full Pantheon dataset.

We constructed 1000 simulated Pantheon-like datasets and found that including both systematic and statistical uncertainties, the percentage of the simulated Pantheon dataset with a similar amplitude oscillating behaviour is $\simeq 5\%$. Considering only statistical uncertainties in the construction of the simulated datasets this probability decreases to about 2.7%.

While the statistical significance of the oscillations reduces when we consider any 3 bins out of 4 bins, we emphasise that the first three bins covering the 75% of the total data points are all at relatively low redshifts ($z < 0.42$) where dark energy is dominant. Hence, concerning the physical origin of the aforementioned effect,
we anticipate that the importance of the first three bins is amplified compared to any other three bin combination. Plausible physical causes for such low \( z \) oscillating behavior of the data include the following

- The presence of large scale inhomogeneities at low \( z \) including voids or superclusters \([81, 82]\).
- Dark energy with oscillating density in redshift. Such oscillations may be induced \( e.g. \) by scalar field potentials with a local minimum \([83, 84]\).

Finally, some interesting extensions of the present analysis include the following

- Further investigation for a similar oscillating behaviour in other data (\( e.g. \) BAO or \( H(z) \) cosmic chronometer data \([78]\)). Clearly, if such oscillations are observed in other cosmological datasets, the overall statistical significance of such an effect would be considerably boosted.
- Construction of physical models that naturally lead to such an oscillating low \( z \) behavior of the data.
- Forecasts with future SNIa compilations, \( e.g. \) by the LSST survey, to ascertain whether this oscillatory effect would be more prominent in upcoming data.

- Making some internal consistency checks such as using “Robustness” criterion \([85]\) or/and looking for redshift evolution in the light curve parameters of the data \([86]\) to determine whether the Pantheon sample is statistically consistent or is contaminated with systematics.

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