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**Hodge theory of Kloosterman connections.** (English) Zbl 07536938

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Summary: We construct motives over the rational numbers associated with symmetric power moments of Kloosterman sums, and prove that their $L$-functions extend meromorphically to the complex plane and satisfy a functional equation conjectured by Broadhurst and Roberts. Although the motives in question turn out to be “classical,” we compute their Hodge numbers by means of the irregular Hodge filtration on their realizations as exponential mixed Hodge structures. We show that all Hodge numbers are either zero or one, which implies potential automorphy thanks to recent results of Patrikis and Taylor.

**MSC:**

11G40 $L$-functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture  
11F80 Galois representations  
11L05 Gauss and Kloosterman sums; generalizations  
14F40 de Rham cohomology and algebraic geometry  
32S35 Mixed Hodge theory of singular varieties (complex-analytic aspects)  
32S40 Monodromy; relations with differential equations and $D$-modules (complex-analytic aspects)

**Keywords:** 
connections with irregular singularities; $D$-modules; exponential motives; Fourier transform; Galois representations; irregular Hodge filtration; Kloosterman sums; $L$-adic sheaves; $L$-functions; mixed Hodge modules; potential automorphy

**Full Text:** DOI Link

**References:**

[1] A. ADOLPHSON and S. SPERBER, “On twisted de Rham cohomology,” Nagoya Math. J. 146 (1997), 55-81. · Zbl 0915.14012 · doi:10.1017/S0027763000006218

[2] T. Barnet-Lamb, T. Gee, D. Geraghty, and R. Taylor, “Potential automorphy and change of weight,” Ann. of Math. (2) 179 (2014), no. 2, 501-699. · Zbl 1310.11060 · doi:10.4007/annals.2014.179.2.3

[3] A. BEILINSON, “On the crystalline period map,” Camb. J. Math. 1 (2013), no. 1, 1-51. · Zbl 1351.14011 · doi:10.4310/CJM.2013.v1.n1.a1

[4] D. BROADHURST, “Feynman integrals, $L$-series and Kloosterman moments,” Commun. Number Theory Phys. 10 (2016), no. 3, 527-569. · Zbl 1392.81072 · doi:10.4310/CNTP.2016.v10.n3.a3

[5] D. BROADHURST, “Critical $L$-values for products of up to 20 Bessel functions,” preprint, 2017, https://www.matrix-inst.org.au/wp_Matrix2016/wp-content/uploads/2016/04/Broadhurst-2.pdf.

[6] R. CREW, “On Kloosterman sums and monodromy of a $p$-adic hypergeometric equation,” Compos. Math. 91 (1994), no. 1, 1-36. · Zbl 0806.14018

[7] P. DELIGNE, “Les constantes des équations fonctionnelles des fonctions $L(1/2)$” in “Modular Functions of One Variable, II (Annapolis, 1977),” Lecture Notes in Math. 349, Springer, Berlin, 1973, 501-597.

[8] P. DELIGNE, “Applications de la formule des traces aux sommes trigonométriques” in “ Cohomologie étale,” Lecture Notes in Math. 569, Springer, Berlin, 1977, 168-232.

[9] P. DELIGNE, “Valeurs de fonctions $L$ et périodes d’intégrales” in “Automorphic Forms, Representations and $L$-Functions (Corvallis, 1977), Part 2,” with appendix “Algebraicity of some products of values of the $(\tau,\eta)$-function” by N. Koblitz and A. Ogus, Proc. Sympos. Pure Math. 33, Amer. Math. Soc., Providence, 1979, 313-346.

[10] P. DELIGNE, “La conjecture de Weil, II,” Publ. Math. Inst. Hautes Études Sci. 52 (1980), no. 1, 137-252.

[11] P. DELIGNE and N. KATZ, eds., “Groupes de monodromie en géométrie algébrique, II,” Séminaire de Géométrie Algébriques du Bois-Marie 1966-1969 (SGA7), Lecture Notes in Math. 589, Springer, Berlin, 1977.

[12] P. DELIGNE, B. MALGRANGE, and J.-P. RAMIS, “Singularités irrégulières,” Correspondance et documents, Doc. Math. (Paris) 5, Soc. Math. France, Paris, 2007. · Zbl 1130.14001

[13] J. Denef and F. Loeser, “Weights of exponential sums, intersection cohomology, and Newton polyhedra,” Invent. Math. 106 (1991), no. 2, 275-294. · Zbl 0763.14025 · doi:10.1007/BF01243914

[14] M. DETTWEILER and C. SABBAB, “Hodge theory of the middle convolution,” Publ. Res. Inst. Math. Sci. 49 (2013),
