The Energy Dependence of Black Hole Horizon in Quantum Gravity Theory

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The energy dependence of deflection angle is a common prediction in some quantum gravity theories. For low energy photons, the deflection angle recovers to the prediction of GR. But it reduced to zero for infinite energy photons. In this paper, we develop an effective approach to calculate the trajectory of photons and other deflection-related quantities semiclassically by replacing $h_{\mu\nu}$ with $h_{\mu\nu} \times f(E)$ to include the correction of quantum gravity. This approach could provide more information for photons traveling in external gravitational field. We compute the Einstein radius of gravitational lensing and the horizon of black hole with this method and find that they are all energy dependent and decrease to zero as energy increases to infinity.

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I. INTRODUCTION

The general relativity (GR) is the most accurate gravity theory nowadays and it had been tested by diferent types of experimentes in the past one hundred years. The perihelion advance of Mercury is one of the earliest evidence. $\Delta \phi$ is about 43'' per century in GR and the current measurement is 42.98 $\pm$ 0.4'' per century $[3]$. Light bending, another important evidence, was firstly observed by Dyson and Eddington in 1919 and then confirmed with much better accuracy by the very long baseline radio interferometry $[2,4]$. The light travel time delay named Shapiro delay, which was firstly introduced by Shapiro $[4]$, can also be used to test GR. The new measurements are consistent with the prediction of GR very well $[1,6]$.

The gravitational lensing, as a result of the bending light, is also an important way to test GR which contains three main types: strong lensing, weak lensing and microlensing. Lots of strong lensing system had already been observed by diferent experiments, such as the Lenses Structure and Dynamics survey $[6,8]$, the Sloan Lens Advanced Camera for Surveys $[10,11]$, the Baryon Oscillation Spectroscopic Survey $[12]$, the Strong Lensing Legacy Survey $[13,15]$ and the Dark Energy Survey $[10]$.

But GR, as one type of classical physics, is conflict with the idea of quantum theory. GR can be quantized directly but it is not a renormalizable theory. Then higher-derivative terms, such as $R^2, R_{\mu\nu}, R_{\mu\nu\rho\sigma}$ and so on, were introduced to solve such difficulties $[17,19]$. Such theories are renormalizable but the annoying massive spin-2 ghost is unavoidable at the same time. The theorists then introduced the infinite derivative terms and built several kinds of nonlocal gravity models $[20,22]$. Such models can kill two birds with one stone which avoid the annoying massive ghost and divergence problem at small distance.

In this paper, we develop an effective approach to compute the trajectory of photons and other deflection-related issues semiclassically in quantum gravity by redefining the perturbed metric $h_{\mu\nu}$. Our approach could provide more information. We adopt with this method to compute the Einstein radius of gravitational lensing and the horizon of black hole in this draft.

This draft is organized as follows: In Sec. II we review the calculation of deflection angle with classical and semiclassical method. In sec. III, we present the new effective approach and our conclusions are summarized in Sec. IV.

II. THE DEFLECTION ANGLE

Light bending is an important prediction of GR and is about 1.75'' for the photons gazing the sun. The classical deflection angle is described by the following formula

$$\theta_{GR} = \frac{\kappa}{2} \int_{-\infty}^{\infty} \partial_y [h_{00} + h_{11}] dx^1. \quad (1)$$

Here we assume that the gravitational source and the trajectory of photons lie in $x - y$ plane. And the perturbed metric is defined as follows

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \quad (2)$$

The gravitational deflection angle can also be calculated semiclassically within the framework of quantum gravity $[23,30]$ which provides us much more information. The Feynman diagram of photons scattering in the external gravity field is shown in Fig. 1 and the corresponding amplitude is

$$\mathcal{M}_{\rho\gamma'} = \frac{1}{2} \kappa h_{\mu\rho}(k) \left[ -\eta_{\mu\nu} \eta_{\lambda\rho} p'_{\nu} + \eta_{\lambda\rho} p'_{\mu} p_{\nu} + 2 \eta_{\mu\nu} \partial_{\lambda} p'_{\rho} \partial_{\gamma'} + \eta_{\mu\nu} \eta_{\lambda\rho} p'_{\rho} - \eta_{\mu\nu} \partial_{\gamma'} p'_{\rho} + \eta_{\mu\nu} \eta_{\lambda\rho} p'_{\rho} \right] c'_{\mu}(p) c'_{\nu}(p'),$$

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where $\epsilon_\mu(p)$ ($\epsilon_\nu(p')$) is the polarization vectors of photons. $h^{\lambda \rho}_{\text{ext}}(k)$ denotes the gravitational field in momentum space. It has different formula for specific quantum gravity theory.

Here we briefly review the result of nonlocal gravity as an example \[31\]. The gravitational action for nonlocal gravity is expressed as

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R + G_{\mu\nu} a(\Box) - \frac{1}{\Box} R^{\mu\nu} \right),$$  \hspace{1cm} (3)

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ and $a(\Box) = e^{-\Box/\Lambda}$. $\Lambda$ is the energy scale non-locality and the current constrain on energy scale $\Lambda$ \[32, 33\] is $\Lambda > 0.01$ eV.  \hspace{1cm} (4)

For nonlocal gravity, $h^{\lambda \rho}_{\text{ext}}(k)$ is presented as

$$h^{(E)\mu\nu}_{\text{ext}}(k) = \kappa M \left( \frac{\eta^{\mu\nu}}{2k^2} - \frac{\eta^{\mu0}\eta^{\nu0}}{k^2} \right) \exp(-\frac{k^2}{\Lambda^2}).$$ \hspace{1cm} (5)

After careful calculations, we finally got the energy dependence of deflection angle \[31\] which is governed by the following equation

$$\frac{1}{\theta_{\text{GR}}^2} = \frac{1}{\theta^2} e^{2\theta^2} \left( -\frac{2\theta^2}{\lambda^2} + \frac{2}{\lambda^2} Ei(-\frac{2\theta^2}{\lambda^2}) \right),$$ \hspace{1cm} (6)

where $\lambda = \frac{\Lambda}{E}$ and $\theta$ is the deflection angle. The exponential integral $Ei(x)$ is defined as follows

$$Ei(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt.$$ \hspace{1cm} (7)

Defining $y = \frac{2\theta^2}{\lambda^2}$ and Eq.(6) becomes

$$\frac{e^{-y}}{y} + Ei(y) - \frac{\lambda^2}{2\theta_{\text{GR}}^2} = 0.$$ \hspace{1cm} (8)

The deflection angle is derived by solving the above equation numerically \[31\] which is energy dependent.

\[\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{The Feynmann diagram of the interaction between external gravitational field and photon.}
\label{fig1}
\end{figure}\]

\[\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{The $f(E)$ in nonlocal gravity theory.}
\label{fig2}
\end{figure}\]

$\Lambda/E \to \infty$ when $\theta \to \theta_{\text{GR}}$. In other words, it recovers the prediction of GR for low energy photons. $\theta \to 0$ when $\Lambda/E \to 0$ and there is no deflection for photons with sufficiently high energy photons. The transition begins at about $10^4\Lambda$.

In \[23, 25\], the authors calculated the gravitational deflection angle in HDG model and found that it decreases to zero at $\log_{10}|\beta| \sim 89$. In Ref. \[34\], the authors systematic studied this issue and found that the energy dependence of deflection angle is a generic conclusion in several quantum gravitational theories.

This phenomenon is a new way to explore the quantum effect of gravity and it can be tested by X-ray \[35–42\] or gamma ray \[43–45\] observations. Only Chandra have good enough angular resolution to test this effect. However such measurement has never been done before.

\section{III. THE EFFECTIVE APPROACH}

Here we develop an effective way to deal with the photon deflection problems for quantum gravity theories. Firstly, we introduce a dimensionless function $f(E)$ which is

$$f(E) = \frac{\theta(E)}{\theta_{\text{GR}}}.$$ \hspace{1cm} (9)

From the above definition, we can see that the form of $f(E)$ depends on the specific quantum gravity theory and it has the following behavior

$$f(E) \to 1 \hspace{0.5cm} \text{for low energy photons},$$ \hspace{1cm} (10)

$$f(E) \to 0 \hspace{0.5cm} \text{for high energy photons}.$$ \hspace{1cm} (11)

The $f(E)$ in nonlocal gravity theory is shown in Fig. \ref{fig2}.

Then we define

$$h'_{\mu\nu} = h_{\mu\nu} \times f(E).$$ \hspace{1cm} (11)
Substituting Eq. (11) into Eq. (2), we can conveniently reproduce the deflection angle of quantum gravity. This semiclassical method could provide much more information and it can be used to deal with much more deflection-related issues in quantum gravity theories, such as the trajectory of photons by solving the geodesic equation with the effective perturbed metric, the gravitational lensing problems and the black hole horizon. Eq. (11) is actually a bridge connecting gravity phenomenon and quantum gravity theory. For weak gravity case, $h_{\mu\nu} \propto GM/r$ and Eq. (11) is equivalent to the replacement $M \to M \times f(E)$.

The image illustration of gravitational lensing is shown in Fig. 3. The mass of the gravitation object is located at L. The observer and light source are located at O and S respectively. But due to the light bending, the source appears at position I. $\kappa$, $\zeta$ and $\xi$ are angles between different direction. In GR,

$$\xi - \zeta = \frac{4GM}{c^2} \frac{dS_{\text{ds}}}{dS_{\text{S}}},$$

(12)

where $\xi_E \propto \sqrt{M}$ is the 'Einstein radius'. From the above discussions, we can see that the Einstein radius is a function of photon energy while including the quantum gravity correction with $M \to M \times f(E)$.

In this draft, we adopt with this method to compute the horizon of black hole. The Schwarzschild metric has the following formula

$$ds^2 = - \left( 1 - \frac{M}{2r'} \right) dt^2 + \left( 1 - \frac{M}{2r'} \right)^{-1} dr^2 + r'^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

(13)

Doing the coordinate transformation as follows:

$$r = r' \left( 1 + \frac{M}{2r'} \right),$$

$$r' = \frac{1}{2} \left( \sqrt{r'^2 - 2Mr} + r - M \right),$$

$$x = r' \sin(\theta) \cos(\phi),$$

$$y = r' \sin(\theta) \sin(\phi),$$

$$z = r' \cos(\theta),$$

(14)

we can get the corresponding cartesian coordinates form of Schwarzschild metric, which is

$$ds^2 = - \frac{1 - M/2r'}{1 + M/2r'^2} dt^2 + (1 + M/2r')^4 [dx^2 + dy^2 + dz^2].$$

(15)

Then the replacement for the space component $h_{xx,yy,zz}$ is $f(E)[4M/2r^2 + 6(M/2r')^2 + 4(M/2r')^3 + (M/2r')^4]$. Then we let

$$1 + f(E)g(r) = 1 + f(E)[4M/2r^2 + 6(M/2r')^2 + 4(M/2r')^3 + (M/2r')^4],$$

(16)

and

$$g(r) = 4M/2(\sqrt{r'^2 - 2Mr} + r - M) + 6[M/2(\sqrt{r'^2 - 2Mr} + r - M)]^2 + 4[M/2(\sqrt{r'^2 - 2Mr} + r - M)]^3 + [M/2(\sqrt{r'^2 - 2Mr} + r - M)]^4.$$

Converting to the original polar coordinate form Eq. (15), the coefficient of $dr^2$ is

$$[1 + f(E)g(r)][1/(2\sqrt{r'^2 - 2Mr} - (r - M)^2)/(2(r'^2 - 2Mr)^{3/2})]].$$

(17)

The quantum-corrected black hole horizon can be deviated by light-like hypersurface condition, which is

$$f(E) \to 0 \text{ as } E \gg 0. \text{ And the above equation becomes}$$

$$[1 - f(E)g(r)][2\sqrt{r'^2 - 2Mr}]/(1 - (r - M)^2) = 0 \Rightarrow r_{\text{horizon}} \to 0.$$

(19)

From the above analysis, it is obviously that the horizon of Schwarzschild black hole is energy dependent and tends to zero for photons with infinitely energy. This is a reasonable result of quantum gravity because there is no deflection for photons with high enough energy. For Kerr and Kerr-Newman black hole, the calculation process much more complicated and similarly result is expected.

**IV. SUMMARY**

In this draft, we developed an effective method to calculate the deflection-related quantities. Replacing $h_{\mu\nu}$ with $h_{\mu\nu} \times f(E)$ is the main idea of our approach in order to include the correction of quantum gravity. For weak gravitation field, it is equivalent to the mass correction as $M \to M \times f(E)$. By this approach, we can easily
calculate the trajectory of photons and other deflection-related quantities, such as the Einstein radius of gravitational lensing and the horizon of black hole, in quantum gravity theory. From our analysis we found that the Einstein radius of gravitational lensing and the black hole horizon are all energy dependent and they decrease to zero as energy increases to infinity.

The energy dependence of black hole horizon is an unexpected result. It maybe conflict with the currently black hole thermodynamics theory because the size of black hole is also energy dependent.

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