this page contains a survey of contrast enhancement techniques based on histogram equalization. the authors discuss various methods such as histogram equalization (he), brightness preserving he (bphe), and multi-brightness preserving he (mbphe). they also introduce new techniques such as weighting mean brightness error (wmbhe) and dual sub-image histogram equalization (dsihe). the authors conclude that some researchers have focused on improvement of histogram equalization based contrast enhancement such as mean preserving bi-histogram equalization (bbhe), and minimum mean brightness error bi-histogram equalization (mmbbebh). they also mention that dual sub-image histogram equalization (dsihe) is another improvement of bbhe. the authors conclude that these methods can perform good contrast enhancement, but also cause annoying side effects depending on the variation of gray level distribution in the histogram.
on local minima and assigns specific gray level ranges for each partition before equalizing them separately. These partitions further go through a repartitioning test to ensure the absence of any dominating portions. This method outperforms other present approaches by enhancing the contrast well without introducing severe side effects, such as washed out appearance, checkerboard effects etc., or undesirable artifacts [21]. The brightness preserving dynamic histogram equalization (BPDHE), which is an extension to HE that can produce the output image with the mean intensity almost equal to the mean intensity of the input, thus fulfil the requirement of maintaining the mean brightness of the image [22].

Multilevel Component-Based Histogram Equalization (MCBHE) where we combine the global histogram equalization, BPDHE, multiple graylevel thresholding, and connected component analysis to produce an image with improved global and local contrast and with minimal distortion [22]. Weighting mean-separated sub-histogram equalization (WMSHE) method is to perform the effective contrast enhancement of the digital image [26].

The paper is organised as follows: HE for digital input image is reviewed together with their mathematical formulation and BBHE, DSIHE, MMBBHE, and generalization of BBHE, namely - Recursive Mean Separate Histogram Equalization (RMSHE) in section II. Mean Brightness Preserving Histogram Equalization (MBPHE) will be presented in section III and Dynamic Histogram Equalization (DHE) in section IV. Section V includes the Brightness Preserving Dynamic Histogram Equalization (BPDHE) and Multilevel Component-Based Histogram Equalization (MCBHE) in section VI respectively. In section VII, Weighting Mean-Separated Sub-Histogram Equalization (WMSHE) is discussed. Paper concludes with Section VIII containing discussion of various Histogram Equalization techniques.

II. HISTOGRAM EQUALIZATION (HE)

For a given image $X$, the probability density function $p(X_k)$ is defined as

$$p(X_k) = \frac{n_k}{n} \quad (1)$$

For $k = 0, 1 \ldots L - 1$, where $n_k$ represents the number of times that the level $X_k$ appears in the input image $X$ and $n$ is the total number of samples in the input image. Note that $p(X_k)$ is associated with the histogram of the input image which represents the number of pixels that have a specific intensity $X_k$. In fact, a plot of $n_k$ vs. $X_k$ is known histogram of $X$. Based on the probability density function, the cumulative density function is defined as

$$c(X) = \sum_{j=0}^{k} p(X_j) \quad (2)$$

Where $X_k = x$, for $k = 0, 1 \ldots L - 1$. Note that $c(X_{L-1}) = 1$ by definition. HE is a scheme that maps the input image into the entire dynamic range, $(X_0, X_{L-1})$, by using the cumulative density function as a transform function. Let’s define a transform function $f(x)$ based on the cumulative density function as

$$f(x) = X_0 + (X_{L-1} - X_0)c(x) \quad (3)$$

Then the output image of the HE, $Y = \{ Y (i, j) \}$, can be expressed as

$$Y = f(X) = \{ f(X (i, j)) \mid X (i, j) \in X \} \quad (4)$$

The high performance of the HE in enhancing the contrast of an image as a consequence of the dynamic range expansion. Besides, HE also flattens a histogram. Base on information theory, entropy of message source will get the maximum value when the message has uniform distribution property [4]. As addressed previously, HE can introduce a significant change in brightness of an image, which hesitates the direct application of HE scheme in consumer electronics.

A. Brightness Preserving Bi-Histogram Equalization (BBHE)

This method divides the image histogram into two parts as shown in Fig.1. In this method, the separation intensity $X_T$ is presented by the input mean brightness value, which is the average intensity of all pixels that construct the input image. After this separation process, these two histograms are independently equalized. By doing this, the mean brightness of the resultant image will lie between the input mean and the middle gray level [2].

![Bi-histogram equalization](image_url)

Figure 1. Bi-histogram equalization. The histogram with range from 0 to $L-1$ is divided into two parts, with separating intensity $X_T$. This separation produces two histograms. The first histogram has the range of 0 to $X_T$, while the second histogram has the range of $X_T+1$ to $L-1$.

B. Dualistic Sub-Image Histogram Equalization (DSIHE)

Following the same basic ideas used by the BBHE method of decomposing the original image into two sub-images and then equalize the histograms of the sub-images separately, [4] proposed the so called equal area dualistic sub-image HE (DSIHE) method. Instead of decomposing the image based on its mean gray level, the DSIHE method decomposes the images aiming at the maximization of the Shannon’s entropy [5] of the output image. For such aim, the input image is decomposed into two sub-images, being one dark and one bright, respecting the equal area property (i.e., the sub-images has the same amount of pixels). In [4], it is shown that the brightness of the output image $O$ produced by the DSIHE method is the average of the equal area level of the image $I$ and the middle gray level of the image, i.e., $L/2$. The authors of [4] claim that the brightness of the output image generated by the DSIHE method does not present a significant shift in relation to the brightness of the input image, especially for the large area of the image with the same gray-levels (represented by small areas in histograms with great concentration of gray-levels), e.g., images with small objects regarding to great darker or brighter backgrounds.
C. Minimum Mean Brightness Error Bi-HE Method (MMBEBHE)

Still following the basic principle of the BBHE and DSIHE methods of decomposing an image and then applying the HE method to equalize the resulting sub-images independently, [6] proposed the minimum mean brightness error Bi-HE (MMBEBHE) method. The main difference between the BBHE and DSIHE methods and the MMBEBHE one is that the latter searches for a threshold level \( l_r \) that decomposes the image \( I \) into two sub-images \( I[0, l_r] \) and \( I[l_r+1, L-1] \), such that the minimum brightness difference between the input image and the output image is achieved, whereas the former methods consider only the input image to perform the decomposition.

Once the input image is decomposed by the threshold level \( l_r \), each of the two sub-images \( I[0, l_r] \) and \( I[l_r+1, L-1] \) has its histogram equalized by the classical HE process, generating the output image. Assumptions and manipulations for finding the threshold level \( l_r \) in \( O(L) \) time complexity was made in [6]. Such strategy allows us to obtain the brightness \( l_m \) \( (O[0, l] \cup O[l+1, L-1]) \) of the output image without generating the output image for each candidate threshold level \( l \), and its aim is to produce a method suitable for real-time applications.

D. Recursive Mean-Separate HE Method (RMSHE)

Recall that the extensions of the HE method described so far in this section were characterized by decomposing the original image into two new sub-images. However, an extended version of the BBHE method named recursive mean-separate HE (RMSHE), proposes the following. Instead of decomposing the image only once, the RMSHE method proposes to perform image decomposition recursively, up to a scale \( r \), generating \( 2^r \) sub-images. After, each one of these sub-images \( I'(l_s, l_f) \) is independently enhanced using the CHE method. Note that when \( r = 0 \) (no sub-images are generated) and \( r = 1 \), the RMSHE method is equivalent to the CHE and BBHE methods, respectively. In [7], they mathematically showed that the brightness of the output image is better preserved as \( r \) increases. Note that, computationally speaking, this method presents a drawback: the number of decomposed sub-histograms is a power of two.

III. MEAN BRIGHTNESS PRESERVING HISTOGRAM EQUALIZATION (MBPHE)

The mean brightness preserving histogram equalization (MBPHE) methods basically can be divided into two main groups, which are bisections MBPHE, and multi-sections MBPHE. Bisections MBPHE group is the simplest group of MBPHE. Fundamentally, these methods separate the input histogram into two sections. These two histogram sections are then equalized independently.

However, bisections MBPHE can preserve the mean brightness only to a certain extent. However, some cases do require higher degree of preservation to avoid unpleasant artifacts [7]. Furthermore, bisections MBPHE can only preserve the original mean brightness if and only if the input histogram has a quasi-symmetrical distribution around its separating point [8]. But, most of the input histograms do not have this property. This condition leads to the failure of bisections MBPHE in preserving the mean intensity in real life applications.

Works in [7], [8]-[19] are a few of multi-sections MBPHE. Multi-sections MBPHE group has a better mean brightness preservation as compared with the group of bisections MBPHE. In multi-sections MBPHE, the input histogram is divided into \( R \) sub-histograms, where \( R \) is any positive integer value. Each sub-histogram is then equalized independently. The creation of the sub-histograms can be carried out recursively (e.g. by using the mean or median intensity value), or based on the shape of the input histogram itself (e.g. using the locations of local maximum or local minimum). Yet, in these methods, the detection of the separating points’ process normally requires complicated algorithms, which then associated with relatively high computational time. Furthermore, these methods usually increase the hardware requirement in the implementations for consumer electronic products. In addition, most of these methods put too much constrain on keeping the mean intensity value. As a consequence, not much enhancement could be obtained from most of these methods [20].

IV. DYNAMIC HISTOGRAM EQUALIZATION (DHE)

This Dynamic Histogram Equalization (DHE) technique takes control over the effect of traditional HE so that it performs the enhancement of an image without making any loss of details in it. DHE divides the input histogram into number of sub-histograms until it ensures that no dominating portion is present in any of the newly created sub-histograms. Then a dynamic gray level (GL) range is allocated for each sub-histogram to which its gray levels can be mapped by HE. This is done by distributing total available dynamic range of gray levels among the sub-histograms based on their dynamic range in input image and cumulative distribution (CDF) of histogram values. This allotment of stretching range of contrast prevents small features of the input image from being dominated and washed out, and ensures a moderate contrast enhancement of each portion of the whole image. At last, for each sub-histogram a separate transformation function is calculated based on the traditional HE method and gray levels of input image are mapped to the output image accordingly. The whole technique can be divided in three parts – partitioning the histogram, allocating GL ranges for each sub-histogram and applying HE on each of them [21].

V. BRIGHTNESS PRESERVING DYNAMIC HISTOGRAM EQUALIZATION (BPDHE)

The brightness preserving dynamic histogram equalization (BPDHE), which is an extension to HE that can produce the output image with the mean intensity almost equal to the mean intensity of the input, thus fulfills the requirement of maintaining the mean brightness of the image [22]. This method is actually an extension to both MPHEBP and DHE. Similar to MPHEBP, the method partitions the histogram based on the local maximums of the smoothed histogram. However, before the histogram equalization taking place, the method will map each partition to a new dynamic range, similar to DHE. As the change in the dynamic range will cause the change in mean brightness, the final step of this method
involves the normalization of the output intensity. So, the average intensity of the resultant image will be same as the input. With this criterion, BPDHE will produce better enhancement compared with MPHEBP, and better in preserving the mean brightness compared with DHE [22].

VI. MULTILEVEL COMPONENT BASED HISTOGRAM EQUALIZATION (MCBHE)

The MCBHE algorithm starts just like the BPBHE algorithm by decomposing the input image \( I \) into two

Sub images using the original mean brightness\( \mu_I \). We refer to these two sub images as the background \( I_b \) and foreground \( I_f \) sub images, where

\[
I = I_b \cup I_f
\]

\[
I_b(m,n) = \{I(m,n) \mid I(m,n) < \mu_I, \forall I(m,n) \in I \}
\]

\[
I_f(m,n) = \{I(m,n) \mid I(m,n) \geq \mu_I, \forall I(m,n) \in I \}
\]

(6)

Unlike BPBHE that equalizes each sub image using the conventional histogram equalization, the gray levels of each sub image in MCBHE are processed using multiple gray level thresholding and connected component analysis. This type of analysis is one of the techniques used in image segmentation applications such as tumor detection [23] and handwriting recognition [24]. In this approach the image under consideration is threshold at multiple predetermined gray levels. At each of these threshold levels, sequential labeling is used to identify those connected components that are below or above the current threshold. Depending on the application and the segmentation objective, some features of these components are extracted and saved to be analyzed later along with values obtained for other components at other levels. The benefit of this approach is that it allows for capturing the change and variation of gray levels within the object as the threshold value is changed. Actually, this is the reason for incorporating this approach in MCBHE, as it will help enhancing the local details [25].

VII. WEIGHTING MEAN-SEPARATED SUB-HISTOGRAM EQUALIZATION (WMSHE)

Weighting mean-separated sub-histogram equalization (WMSHE) method is to perform the effective contrast enhancement of the digital image. This method consist the following steps:

A. A. Separation of histogram based on our proposed weighting mean function.

B. B. Achieving contrast enhancement by equalizing sub-histogram respectively in small-scale detail.

1) Histogram Separation

Suppose that an input image \( W \) is composed of \( V \) discrete gray levels and is denoted by \( \{G_0, G_1, ..., G_n, ..., G_{V-1}\} \), the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) for an input image \( W \) is expressed as:

\[
PDF(G_n) = \frac{n^h}{n}
\]

where \( h = 0, 1, ..., V - 1 \).

(7)

Where \( n^h \) denotes the number of pixels that correspond to the value \( h \), and \( n \) is the total number of the pixels in the input image \( W \). Then the weighting mean value \( X_c \) can be calculated by using our proposed weighting mean function, which can be expressed as follows:

\[
X_t = \frac{\sum_{i=a}^{b} l \times CDF(l)}{\sum_{i=a}^{b} CDF(l)}
\]

(8)

Where \([a, b]\) represents the sub-interval of histogram, \( l \) represents the corresponding gray-level, and \( t \) represents the recursion level. Notice that the sub-interval \([a, b]\) is initialized as \([0, 255]\) and \( t \) is initialized as 1. Here we consider that the existing sub-histograms defined over a gray-level range \([X_r, X_{r+1}]\) at the recursion level \( t=1 \) \((0 \leq r \leq t-1)\). In this paper, the recursion level \( t \) is experimentally set at 6.

2) Piecewise Transformed Function

After the ideal weighting mean values are determined, we can directly decide the optimum number of sub-images based on the histogram separation, which can be expressed as follows:

\[
W_k = \{W(x, y) \mid X_k \leq W(x, y) \leq X_{k+1}, \forall W(x, y) \in W \}
\]

(9)

Where \( W_k \) represents each sub-image, and \( k = 0, 1, 2... t-1 \). Then the relationship between gray-level \( G \) and each sub image \( W_k \) is defined as the respective PDF:

\[
PDF_k(G_n) = \frac{n^h}{n_k}
\]

(10)

Where \( k = 0, 1, ..., t-1 \), and \( h = X_k + 1, X_k + 2, ..., X_{k+1} \).

Notice that \( n_k \) denotes the number of pixels that correspond to the value \( h \), and \( n_k \) is the total number of the pixels in \( k \)-th sub image, \( PDF_k(G_n) \) is associated with the histogram of \( k \)-th sub image, which represents the frequency of a specific input gray level \( G_n \). In the next step, the respective CDF \(_k \) is defined for each sub-image based on the respective PDF:

\[
CDF_k(G_i) = \sum_{j=X_{k+1}}^{n_k} PDF_k(G_j)
\]

(11)

Where \( k = 0, 1, ..., t-1 \), and \( h = X_k + 1, X_k + 2, ..., X_{k+1} \).

Finally, the piecewise transformed function is used to map the equalized image. This is characterized by utilizing \( CDF_k \) of sub-image \( W_k \) for \( k \) segments [26].

VIII. DISCUSSION

The comparative study of Histogram Equalization based methods shows that the cases which require higher brightness preservation and not handled well by HE, BBHE and DSIHE, have been properly enhanced by RMSHE. MMBEBHE is the extension of BBHE method that provides maximal brightness preservation. Though these methods can perform good contrast enhancement, they also cause more annoying side
effects depending on the variation of gray level distribution in the histogram [6].

DHE ensures consistency in preserving image details and is free from any severe side effects. BPDHE can preserve the mean brightness better than BBHE, DSIHE, MMMBEBHE, RMSHE, MBPHE, and DHE. MCBHE is simple and heuristic method for contrast enhancement in grayscale images and able to enhance the quality of images such that both global and local contrast is enhanced with minimum distortion in the image appearance [25]. WMSHE achieves the best quality through qualitative visual inspection and quantitative accuracies of Peak Signal-to-Noise Ratio (PSNR) and Absolute Mean Brightness Error (AMBE) compared to other state-of-the-art methods [26].

ACKNOWLEDGMENT

The authors would like to thank Associate Professor Akshay Gridhar for their insightful and valuable suggestions.

REFERENCES

[1] Rafael C. Gonzalez, and Richard E. Woods, “Digital Image Processing”, 2nd edition, Prentice Hall, 2002.
[2] Yeong-Taeg Kim, “Contrast enhancement using brightness preserving Bi-Histogram equalization”, IEEE Trans. Consumer Electronics, vol. 43, no. 1, pp. 1-8, Feb. 1997.
[3] J.-Y. Kim, L.-S. Kim, and S.-H. Hwang, “An advanced contrast enhancement using partially overlapped sub-block histogram equalization,” IEEE Trans. Circuits Syst. Video Technol., vol. 11, no. 4, pp. 475-484, Apr. 2001.
[4] Y. Wang, Q. Chen, and B. Zhang, “Image enhancement based on equal area dualistic sub-image histogram equalization method,” IEEE Trans. on Consumer Electronics, vol. 45, no. 1, pp. 68-75, Feb. 1999.
[5] C. Shannon, “A mathematical theory of communication,” Bell Syst. Tech. J., vol. 27, pp. 379-423, 1948.
[6] S.-D. Chen and A. Ramli, “Minimum mean brightness error Bi-Histogram equalization in contrast enhancement,” IEEE Trans. on Consumer Electronics, vol. 49, no. 4, pp. 1310-1319, Nov. 2003.
[7] S.-D. Chen and A. Ramli, “Contrast enhancement using recursive Mean-Separate histogram equalization for scalable brightness preservation,” IEEE Trans. on Consumer Electronics, vol. 49, no. 4, pp. 1301-1309, Nov. 2003.
[8] Chao Wang and Zhongfu Ye, “Brightness preserving histogram equalization with maximum entropy: a variational perspective”, IEEE Trans. Consumer Electronics, vol. 51, no. 4, pp. 1326-1334, November 2005.
[9] K. Wongsrirong, K. Kittayuansiriwat, F. Cheevaasvit, K. Dejhan, and A. Somboonkaew, “Contrast enhancement using multipeak histogram equalization with brightness preserving”, The 1998 IEEE Asia-Pacific Conference on Circuit and Systems, pp. 24-27, November 1998. [10] M. Abdullah-Al-Wadud, M. H. Kabir, M. A. A. Dewan, and Oksam Chae, “A dynamic histogram equalization for image contrast enhancement”, IEEE Trans. Consumer Electronics, vol. 53, no. 2, pp. 593 - 600, May 2007.
[11] K. S. Sim, C. P. Tso, and Y. Y. Tan, “Recursive sub-image histogram equalization applied to gray scale images”, Pattern Recognition Letters, vol. 28, no. 10, pp. 1209-1221, July 2007.
[12] David Menotti, Laurent Najman, Jacques Facon, and Araujo de A. Araujo, “Multi histogram equalization methods for contrast enhancement and brightness preserving”, IEEE Trans. Consumer Electronics, vol. 53, no. 3, pp. 1186 -1194, August 2007.
[13] Chun-Ming Tsai, and Zong-Mu Yeh, “Contrast enhancement by automatic and parameter-free piecewise linear transformation for color images”, IEEE Trans. Consumer Electronics, vol. 54, no. 2, pp. 213-219, May 2008.
[14] Haidi Ibrahim, and Nicholas Sia Pik Kong, “Brightness preserving dynamic histogram equalization for image contrast enhancement”, IEEE Trans. Consumer Electronics, vol. 53, no. 4, pp. 1752 - 1758, November 2007.
[15] Nicholas Sia Pik Kong, and Haidi Ibrahim, “Improving the visual quality of abdominal magnetic resonance images using histogram equalization”, In Technology and Applications in Biomedicine, 2008. ITAB 2008. International Conference on, pp. 138-139, Shenzhen, China, May 2008.
[16] Nicholas Sia Pik Kong, and Haidi Ibrahim, “Color image enhancement using brightness preserving dynamic histogram equalization”, IEEE Trans. Consumer Electronics, vol. 54, no. 4, pp. 1962 - 1968, November 2008.
[17] Nymikhavgya Sengke, and Heung Kook Choi, “Brightness preserving weight clustering histogram equalization”, IEEE Trans. Consumer Electronics, vol. 54, no. 3, pp. 1329 - 1337, August 2008.
[18] Guy-Hee Park, Hwa-Hyun Cho, and Myung-Ryul Choi, “A contrast enhancement method using dynamic range separate histogram equalization”, IEEE Trans. Consumer Electronics, vol. 54, no. 4, pp. 1981 -1987, November 2008.
[19] Mary Kim and Min Gyo Chung, “Recursively separated and weighted histogram equalization for brightness preservation and contrast enhancement”, IEEE Trans. Consumer Electronics, vol. 54, no. 3, pp. 1389 - 1397, August 2008.
[20] Chen Hee Ooi, Nicholas Sia Pik Kong, and Haidi Ibrahim, “Bi-Histogram Equalization with a Plateau Limit for Digital Image Enhancement”, IEEE Trans. Consumer Electronics, Vol. 55, No. 4, NOVEMBER 2009.
[21] M. Abdullah-Al-Wadud, Md. Hasanal Kabir, M. Ali Akber Dewan, and Oksam Chae, “A dynamic histogram equalization for image contrast enhancement”, IEEE Trans. Consumer Electron., vol. 53, no. 2, pp. 593-600, May 2007.
[22] Haidi Ibrahim, and Nicholas Sia Pik Kong, “Brightness preserving dynamic histogram equalization for image contrast enhancement”, IEEE Trans. Consumer Electronics, vol. 53, no. 4, pp. 1752 - 1758, November 2007.
[23] S. Armaro, M. Giegr, C. Moran, J. Blackburn, K. Doi, H. Macmahon, “Computerized detection of pulmonary nodules in CT Scans,” RadioGraphics 1999: 19:1303-1311.
[24] S. Wu, A. Amin, “Automatic thresholding of gray-level using multistage approach,” In the proceedings of the Seventh International Conference on Document Analysis and Recognition (ICDAR’01), Vol. 1, p.493, 2003.
[25] Iyad Jafar ,and Hao Ying,” Multilevel Component-Based Histogram Equalization for Enhancing the Quality of Grayscale Images”, IEEE EIT, pp. 563-568, 2007.
[26] Pei-Chen Wu, Fan-Chieng Cheng, and Yu-Kung Chen, “A Weighting Mean-Separated-Sub-Histogram Equalization for Contrast Enhancement”, IEEE Trans. Biomedical Engineering and Computer Science, 2010.

AUTHORS PROFILE

Manpreet Kaur was born in July, 26, 1987 in Ludhiana, Punjab (India) and she received the B.Tech degree (with distinction) in Computer Science & Engineering from Guru Nanak Dev Engineering College Ludhiana, Punjab, India, in 2009. She is currently pursuing the Master degree in Computer Science & Engineering from Guru Nanak Dev Engineering College Ludhiana, Punjab, India. Her research interests include image processing.

Jasdeep Kaur received the B.Tech degree (with distinction) in Computer Science & Engineering from Ludhiana Group of Colleges of Engineering and technology Ludhiana, Punjab, India, in 2009. She is currently pursuing the Master degree in Computer Science & Engineering from Guru Nanak Dev Engineering College Ludhiana, Punjab, India. Her research interests include image enhancement.

Jaspreet Kaur was born in Ludhiana, Punjab (India). In 2009, she received the B.Tech degree (with distinction) in Computer Science & Engineering from Guru Nanak Dev Engineering College Ludhiana, Punjab, India. She is currently pursuing the Master degree in Computer Science & Engineering from Guru Nanak Dev Engineering College Ludhiana, Punjab, India. Her research interests include image enhancement and noise reduction.