Performance Analysis of Receive Antenna Selection in Multi-Antenna Systems under Finite Constellation Size

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Abstract—Antenna selection (AS) is regarded as one of the most prospective technologies to reduce hardware cost but keep relatively high spectral efficiency in multi-antenna systems. By selecting a subset of antennas to transceive messages, AS greatly alleviates the requirement on RF chains. This paper studies receive antenna selection in single-input multiple-output (SIMO) systems, namely the antenna-selection SIMO (AS-SIMO) systems, from the perspective of digital modulation. The receiver, equipped with multiple antennas, selects an optimal antenna subset to receive messages from the single-antenna transmitter. By assuming independent and identical distributed (i.i.d) flat fading Rayleigh channels, we first analyze the input-output mutual information, also referred as symmetric capacity, of AS-SIMO systems when the modulation style is BPSK/QPSK/16QAM. To reduce the computation complexity of the capacity, closed-form approximated expressions of the symmetric capacity based on asymptotic theory are given for the first time to approach the exact results. Compared with the conventional derivations, our approximation holds much lower computation complexity with the guarantee of high precision. Next, this asymptotic approximation technique is extended to estimate the symbol error rate (SER) of antenna-selection SIMO systems and approximated expressions for SER are proposed which indicates much lower complexity. Finally, a special scenario of single-antenna-selection is detailedly investigated and series expressions of the symmetric capacity are formulated for the first time. Beside analytical derivations, simulation results are provided to demonstrate the approximation precision of the derived results. Experiment results show that the asymptotic theory has a remarkable approximation effect.

Index Terms—Antenna selection, Rayleigh fading, symmetric capacity, symbol error rate, asymptotic theory.

I. INTRODUCTION

Multiple-antenna system can drastically improve the spectral efficiency by deploying multiple antennas at the transceivers and will serve as a significant technology in the upcoming 5th generation cellular networks (5G), namely, massive multiple-input multiple-output (MIMO) [11], [2]. To promise communication, each antenna should be connected with a radio-frequency (RF) chain, which results in high hardware cost and energy consumption in multi-antenna systems [3], thus there is an urgent need to address these challenges. In this respect, antenna selection (AS) technology [4] has gained significant attentions in recent years aiming for design of high-efficiency transmission schemes.

Antenna selection is regarded as an alternative to alleviate the requirement on the RF transceivers by selecting a subset of antennas to transceive signals. At the base station (BS), antenna selection has been applied into practical communication systems for both uplink and downlink transmission. To analytically measure the performance of MIMO system with antenna selection (AS-MIMO), Molisch et al. defined the upper capacity bound [4], [5] of AS-MIMO which treated the MIMO system as several independent single-input multiple-output (SIMO) systems, and this defined upper bound serves as an important evaluation criterion for the performance of antenna selection technology. Furthermore, their work was later extended to massive AS-MIMO by Y. Gao et al. in [6] to investigate this selection technology in large-scale systems. Through the work of Molisch, it is clear that investigations in antenna-selection SIMO systems (AS-SIMO) are the foundation for explorations of AS-MIMO systems.

In the past decades, many researches on the AS-SIMO systems were presented, the concerns of which may be divided into two parts, i.e. channel capacity and symbol error rate (SER). The concept of hybrid selection/maximal-ratio combining (H-S/MRC) was first proposed in [7], which means selecting $L$ antennas corresponding to the largest Signal to Noise Ratio (SNR) and doing maximal-ratio combining (MRC). It is apparent that this is definitely the optimal selection strategy in SIMO system and many works focused on the channel capacity of the H-S/MRC systems. To measure the performance of the H-S/MRC systems under Rayleigh fading, the works in [8], [9] derived the analytical expressions of the channel capacity, the basic model of which was further utilized to estimate the outage probability of the single-antenna-selection AS-MIMO systems [10]. Moreover, the calculation of the channel capacity for the AS-SIMO are greatly simplified by the asymptotic theory in [11] and [12] which also discussed the limit behavior of the antenna selection. Nevertheless, it should be noticed that all of these works calculated the channel capacity by assuming that the input signals follow the Gaussian distribution. In practical communication systems, digital modulation is utilized to carry the messages which is constrained by finite constellation size [13], thus the input messages are not continuous, not to mention being Gaussian-distributed. Under this circumstances, the famous Shannon formula $C = \log(1 + SNR)$ can not be directly used and only the definition of the mutual information can be applied to estimate the performance of AS-SIMO, which is also termed as symmetric capacity.

The calculation for symmetric capacity consumes high computation complexity especially for high order modulation,
such as 64QAM [13]. Since there does not exist any closed-form expressions for the mutual information of digital modulation systems, [15] and [16] derived the series expression of symmetric capacity in flat Rayleigh and Nakagami-\(m\) channels when BPSK and QPSK are used, which greatly simplified the calculation of mutual information. In [17], a simple approximation method is put forward to approximate the symmetric capacity of high order modulation. Moreover, [18] proposed some upper and lower bounds for any modulation styles for Nakagami-\(m\) channels. Nevertheless, all of them just considered the simplest single-input single-output (SISO) systems and none of them investigated the symmetric capacity for antenna selection systems. In 2006, Conti et al. investigated the symmetric capacity for \(M\)-QAM in H-S/MRC systems accompanied with adaptive modulation [19]. In addition, [20] discussed the symmetric capacity in AS-SIMO systems in correlated channels. However, they did not propose closed-form expressions or approximated results but directly calculated the symmetric capacity on the basis of the definition of mutual information in [14], the computation complexity of which is prohibitively complex especially for large-scale antenna array and high-order modulation styles.

Besides the rich results on the channel capacity, the theoretical results on the bit error rate (BER) or symbol error rate in AS-SIMO systems were also studied in the literature. [21] and [22] detailedly discussed the SER of H-S/MRC systems for any modulation styles in flat Rayleigh fading channels, and a canonical expression of SER for both \(M\)-PSK and \(M\)-QAM were proposed. The works in [24]–[26], [24] comprehensively investigated the error performance of AS-SIMO under Nakagami-\(m\) fading channels in both correlated and uncorrelated cases and analytical expressions for SER were given. Additionally, the works in [27] and [28] studied the SER over generalized fading channels of AS-SIMO systems. [27] considered an antenna selection protocol which is adaptive with the receive SNR and corresponding analytical expressions for SER were derived. The work in [28] further explored the error performance of AS-SIMO for Rayleigh, Nakagami-\(m\) and Rician fading types in the limit of high SNR and asymptotically theoretical results were exhibited. Even though the analytical expressions of SER for AS-SIMO systems in any types of wireless multi-path channels have been proposed, and the canonical expression formulated in [21], [22] are much simpler than the conventional method shown in [9], it was still hard to use especially when the total amount of receive antennas or selected antennas are large, say large-scale antenna scheme. In this case, there should be a much easier method to calculate or approximate the real SER for the AS-SIMO systems.

**Contribution and Organization**

This paper studies the SIMO system with receive antenna selection under finite constellation size from the point of digital modulation. As stated before, both the calculation for capacity and SER are complex in AS-SIMO systems especially when the receiver is deployed with very large-scale antenna array. To settle these challenges, our study analyzed the symmetric capacity and SER of AS-SIMO systems and much simpler expressions for the capacity and SER are formulated to simplify the calculation under flat Rayleigh fading. Taken together, the key contributions of this paper can be summarized as follows:

- The scenario when multiple antennas are selected is studied in detail and corresponding analytical expressions for the input-output mutual information for BPSK/QPSK/16QAM are derived. To obtain closed-form approximated expressions of the mean symmetric capacity and reduce the computation complexity of the analytical results, asymptotic theory is utilized to estimate the exact values. To the best of our knowledge, this is the first time to make analytical derivations on symmetric capacity in antenna selection system.

- The asymptotic theory is further applied into the analysis of SER in AS-SIMO systems and approximated expressions of SER are formulated which hold much lower complexity than conventional derivations. Simulation results show that the asymptotic theory has a fantastic approximation effect for SER.

- The special case when only one receive antenna is activated is detailedly investigated. Under this simple setup, series expressions for the mean symmetric capacity of BPSK/QPSK are derived for the first time. Then, on the basis of the approximation method introduced in [17], a high-precision approximated series expression for the symmetric capacity of 16QAM is proposed for single-antenna-selection.

The remaining parts of this paper is structured as follows. In section II the system model for the SIMO system with receive antenna selection is formulated. Section III describes the multiple antennas selection in SIMO system and asymptotic approximation is applied to estimate the input-output mutual information. Demonstration about the SER is shown in Section IV. Section V investigates the special case when only one antenna is activated at the receiver and analytical series expressions for the symmetric capacity of BPSK/QPSK/16QAM are derived. Next, simulation results are provided in Section VI to verify the former analysis and derivations. Finally, Section VII concludes the paper.

**Notations:** Throughout this paper, scalars and vectors are denoted by non-bold and bold lower case, respectively. \(C\) stands for the complex plain and \(\mathbb{R}\) stands for the real plain. Hermitian of vector \(h\) is indicated with \(h^\dagger\). The \(\ln(\cdot)\) represent the natural logarithm, \(|\cdot|\) denotes the Euclidean norm and \(E[\cdot]\) is the expectation operator. Additionally, the mutual information between the random variable \(X\) and \(Y\) is denoted by \(I(X;Y)\). Moreover, \(\mathcal{N}(\mu, \sigma^2)\) and \(\mathcal{CN}(\mu, \sigma^2)\) represent the real and complex Gaussian distribution with mean \(\mu\) and variance \(\sigma^2\), respectively.

**II. System Model and Problem Statement**

In this paper, we consider a SIMO system. The transmitter is equipped with one antenna and the receiver is equipped with \(N_t\) antennas. The received signal vector at the receiver reads

\[
y = hx + w,\tag{1}
\]
where $x$ is the transmitted signal with unit power constrained by finite constellation size, such as BPSK: $w \sim \mathcal{N}(0, I_N)$ is the additive Gaussian noise. Considering independent and identically distributed (i.i.d) Rayleigh flat fading channels, the elements in channel matrix $h \in \mathbb{R}^{N_r \times 1}$ have the same probability density function (PDF) as follows:

$$p(a) = \frac{2a}{\gamma} e^{-\frac{a^2}{\gamma}},$$

where $\gamma$ is the average Signal to Noise Ratio (SNR) at each antenna of the receiver.

Now, suppose only $L$ antennas are activated at the receiver and it is clear that the receive antennas corresponding to the strongest $L$ branches are selected. Let $\hat{h}$ and $\tilde{y}$ denote the channel matrix and the received signal after antenna selection. Assume that the channel side information is only available at the receiver, and receive beamforming \cite{29} is used. The result obtained by beamforming is written as

$$\tilde{y} = \frac{\hat{h}^\dagger}{||\hat{h}||} y.$$  

(A. Symmetric Capacity)

On the basis of \cite{29}, the relationship $I(x; \tilde{y}) = I(x; y)$ holds, for the receive beamforming is a lossless operation i.e., $\tilde{y}$ can be generated from $y$ by $\tilde{y} = \frac{\hat{h}^\dagger}{||\hat{h}||} y$. Next, we will use BPSK as an example to demonstrate the calculation of the input-output mutual information, which is also referred as symmetric capacity. The instantaneous symmetric capacity for BPSK over AWGN channels is given by \cite{15}:

$$I(x; y) = C_b(a) = \ln 2 - \int_{-\infty}^{+\infty} e^{-\frac{u^2}{2}} \ln \left(1 + e^{-2\sqrt{\gamma}u - 2a}\right) du,$$

where $a$ is the instantaneous SNR. Next, let we take receive antenna selection into consideration. Define $\Gamma$ as the received SNR after antenna selection and let $p_{\text{snr}}(a)$ indicate its PDF. The mean symmetric capacity, which is also termed as ergodic capacity, can be written as

$$C_{\text{BPSK}} = \mathbb{E}[C_b(a)] = \int_{0}^{+\infty} C_b(a)p_{\text{snr}}(a) da.$$  

(B. Symbol Error Rate)

Let us now turn to the SER of AS-SIMO systems. On the basis of \cite{30, 31}, the SER for $M$-PSK over AWGN channels is given by

$$p_{\text{err}}(\gamma) = \frac{1}{\pi} \int_{0}^{\Theta} \exp \left(-\frac{c \text{MQAM}}{\sin^2 \theta} \gamma\right) d\theta,$$

where $c_{\text{MQAM}} = \sin^2(\pi/M)$, $\Theta = \pi(M-1)/M$ and $\gamma$ denotes the average SNR at each receive antenna. Additionally, the SER for $M$-QAM is written as

$$p_{\text{err}}(\gamma) = \frac{q}{\pi} \int_{0}^{\pi/2} \exp \left(-\frac{c_{\text{MQAM}}}{\sin^2 \theta} \gamma\right) d\theta - \frac{q^2}{4\pi} \int_{0}^{\pi/4} \exp \left(-\frac{c_{\text{MQAM}}}{\sin^2 \theta} \gamma\right) d\theta,$$

where $c_{\text{MQAM}} = 3/(2(M-1))$, $q = 4(1 - 1/\sqrt{M})$ and $\gamma$ denotes the average SNR. We still use BPSK as an example, the mean SER (for BPSK this is also the BER) in wireless multi-path channels is given by

$$p_{\text{err}}(\gamma) = \frac{1}{\pi} \int_{0}^{+\infty} \int_{0}^{\Theta} \exp \left(-\frac{c_{\text{BPSK}}}{\sin^2 \theta} \gamma\right) p_{\text{snr}}(a) d\theta da.$$
III. Analysis of Symmetric Capacity

As previously stated, \(5\) can be used to calculate the ergodic capacity of BPSK in AS-SIMO system. On the basis of \(5\), the mean symmetric capacity for other modulation styles can be derived. Since QPSK can be treated as the superposition of two orthogonal BPSK, its ergodic capacity can be written as

\[
C_{\text{QPSK}} = 2 \int_0^{+\infty} C_b(a)p_{\text{snr}}(a) \, da,
\]

where \(C_b(a)\) is the symmetric capacity for BPSK over AWGN channels shown in \(4\). Next, let us turn to 16QAM. Since 16QAM is the superposition of two orthogonal 4ASK, it is sufficient to discuss the scenario of 4ASK. By \(17\), the instantaneous symmetric capacity of 4ASK is approximated as

\[
C_{\text{4ASK}}(a) \approx C_b(0.2a) + C_b(0.2a(1 + e^{-0.2a}))^2, \tag{13}
\]

where \(a\) is the instantaneous SNR. As a result, the mean symmetric capacity of 16QAM in AS-SIMO systems is estimated as:

\[
C_{\text{16QAM}} \approx 2 \int_0^{+\infty} C_b(0.2a)p_{\text{snr}}(a) \, da + 2 \int_0^{+\infty} C_b(0.2a(1 + e^{-0.2a}))^2 \, p_{\text{snr}}(a) \, da. \tag{14}
\]

Even though the analytical formulas to the ergodic capacity for different modulation styles have been derived, but Fourier transform must be done to acquire \(p_{\text{snr}}(a)\). In the rest part of this section, closed-formed approximated expressions will be given.

When the receiver is equipped with a large-scale antenna array, the calculation for the ergodic capacity can be continuously simplified. By the asymptotic theory of ordered statistics \(32\), \(33\), \(\Gamma\) is termed as a trimmed sum whose distribution converges to be normal as \(N_r\) tending to infinity. Furthermore, \(11\) found that the convergence speed is fast as \(N_r\) increases, thus it makes sense to approximate \(\Gamma\) as a Gaussian random variable \(y \sim N(\mu_g, \sigma_g^2)\). By \(11\), the approximation effect is fantastic even though \(N_r\) is of limited length and its mean and variance are decided by \(11\)

\[
\mu_g = L\gamma \left(1 + \ln \frac{N_r}{L}\right) \tag{15a}
\]

\[
\sigma_g^2 = L\gamma^2 \left(2 - \frac{L}{N_r}\right). \tag{15b}
\]

As a result, \(p_{\text{snr}}(a)\) can be asymptotically approximated as:

\[
p_{\text{snr}}(a) = \frac{1}{\sqrt{2\pi L\gamma^2 \left(2 - \frac{L}{N_r}\right)}} e^{-\frac{(a - L\gamma \left(1 + \ln \frac{N_r}{L}\right))^2}{2L\gamma^2 \left(2 - \frac{L}{N_r}\right)}}. \tag{16}
\]

To verify the fast convergence of \(\Gamma\), Fig. 1 compares the empirical CDF of \(\Gamma\) by Monte-Carlo simulation and the approximated distribution derived from \(16\). As can be seen from this graph, there are scarcely any difference between the dashed and solid lines, which suggests that the approximation effect by asymptotic theory is remarkable. Additionally, the two curves for the empirical and approximated results almost coincide with each other even when \(N_r\) is at a moderate level, such as \(N_r = 32\), which means this theory is efficient and robust in approximation.

Next, using the relatively tight upper bound for BPSK propose in \(34\), an approximated expression for the mean symmetric capacity of BPSK can be obtained, which treats the tight upper bound as an approximation of the instantaneous symmetric capacity. Let \(\tilde{C}_{\text{BPSK}}\) denote the approximation of the ergodic capacity, \(\tilde{C}_{\text{BPSK}}\) can be written as follows:

\[
\tilde{C}_{\text{BPSK}} = \ln 2 - \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi \sigma_g^2}} e^{-\frac{(x - \mu_g)^2}{2\sigma_g^2}} \ln (1 + e^{-x}) \, dx \tag{a}
\]

\[
\approx \ln 2 - \frac{e^{-\frac{\sigma_g^2}{2}}}{\sqrt{2\pi}} \ln (1 + e^{-\mu_g}), \tag{18}
\]

where the step “(a)” is detailedly shown in Appendix A. As explained earlier, QPSK is equivalent to the superposition of two orthogonal BPSK, thus the approximated ergodic capacity for QPSK is

\[
\tilde{C}_{\text{QPSK}} = 2\tilde{C}_{\text{BPSK}} = \ln 4 - 2e^{-\frac{\sigma_g^2}{2}} \ln (1 + e^{-\mu_g}). \tag{19}
\]

As discussed above, \(C_{\text{4ASK}}(a) \approx C_b(0.2a) + C_b(0.2a(1 + e^{-0.2a}))^2\), where the mean of \(C_b(0.2a)\) can be easily calculated on the basis of \(18\). As for the other term \(a_1 = a(1 + e^{-0.2a})^2\), it can be approximated as a Gaussian random vari-
able with the same mean and variance. Since \( a \sim \mathcal{N}(\mu_a, \sigma_a^2) \), the mean and variance of \( a_1 \) are:

\[
\mu_1 = \mathbb{E}[a_1] = \mathbb{E}[a(1 + e^{-0.2a})^2] \\
= -\frac{2}{5} \left( \sigma_a^2 - 5\mu_a \right) e^{-\frac{2\sigma_a^2}{5}} + \mu_a - \left( \frac{2}{5} \sigma_a^2 - \mu_a \right) e^{-\frac{2\sigma_a^2}{5}} - 2\mu_a
\]

\[
\sigma_1^2 = \mathbb{E}[a_1^2] - \mu_1^2 = \mathbb{E}[a^2(1 + e^{-0.2a})^4] - \mu_1^2 \\
= 36\sigma_a^4 + (100 - 120\mu_a) \sigma_a^2 + 100\mu_a^2 e^{-\frac{2\sigma_a^2}{5}} \\
+ \frac{4\sigma_a^4 + (100 - 40\mu_a) \sigma_a^2 + 100\mu_a^2 e^{-\frac{2\sigma_a^2}{5}}}{25} + \sigma_a^2 \\
+ \frac{24\sigma_a^4 + (150 - 120\mu_a) \sigma_a^2 + 150\mu_a^2 e^{-\frac{2\sigma_a^2}{5}}}{25} \\
+ \frac{16\sigma_a^4 + (25 - 40\mu_a) \sigma_a^2 + 25\mu_a^2 e^{-\frac{2\sigma_a^2}{5}}}{25} + \mu_a^2.
\]

(20)

Therefore, the mean symmetric capacity for 16QAM is approximated as follows:

\[
\tilde{C}_{16QAM} = 2 \left( \ln 2 - e^{-\frac{0.2\sigma_a^2}{2}} \ln (1 + e^{-0.2\mu_a}) \right) \\
+ 2 \left( \ln 2 - e^{-\frac{0.2\sigma_a^2}{2}} \ln (1 + e^{-0.2\mu_a}) \right)
\]

\[
= \ln 16 - 2e^{-\frac{0.2\sigma_a^2}{2}} \ln (1 + e^{-0.2\mu_a}) \\
- 2e^{-\frac{0.2\sigma_a^2}{2}} \ln (1 + e^{-0.2\mu_a}).
\]

IV. Analysis of Symbol Error Rate

The previous section has studied the symmetric capacity of AS-SIMO systems and this part moves on to consider the error performance of the AS-SIMO systems. Win et al. derived a simple canonical expression of SER for HS/MRC system [22], which transforms the mean SER of the HS/MRC system into the superposition of SERs of a series of MRC systems. Among their derivations, both the case for \( M \)-QAM and \( M \)-PSK were given, which are listed as follows:

\[
P_{e,\text{MPSK}} = \sum_{k=1}^{N_r-L+1} \sum_{n=2}^{N_r-L+1} \frac{N_r-L+1}{A_{n-1}} P_{e,\text{MPSK}}(k, \gamma) \\
+ \sum_{n=2}^{N_r-L+1} A_{n-1} P_{e,\text{MPSK}}(1, c_n).
\]

(22)

\[
P_{e,\text{MQAM}} = \sum_{k=1}^{N_r-L+1} \sum_{n=2}^{N_r-L+1} \frac{N_r-L+1}{A_{n-1}} P_{e,\text{MQAM}}(k, \gamma) \\
+ \sum_{n=2}^{N_r-L+1} A_{n-1} P_{e,\text{MQAM}}(1, c_n),
\]

where

\[
P_{e,\text{MRC}}(i, j) = \frac{1}{\pi} \int_{0}^{\theta} \left[ \frac{\sin^2 \theta}{\sin^2 \theta + j c_{\text{MQAM}}} \right]^{i} d\theta
\]

(23)

Moreover, the coefficients in (22) satisfies

\[
A_{n,k} = \frac{1}{q_n(k_n-k)} f_n(\mu_n-k),
\]

(24)

where \( f_n(k) \) denotes the \( k \)th derivative of \( f_n(x) = x^k e^{(x-q_n)} \) at \( x = 0 \) and \( f(x) = \prod_{n=1}^{N_r-L+1} (q_n^{\mu_n})^{\mu_n} \). Additionally, \( q_n = 1/(c_{\text{MPSK}}c_n) \) for \( M \)-PSK and \( q_n = 1/(c_{\text{MQAM}}c_n) \) for \( M \)-QAM. In (22), (23) and (24), the expression of \( c_n, \mu_n, c_{\text{MPSK}} \) and \( c_{\text{MQAM}} \) can be found in Section I.

Even though the expressions in (22) can be utilized to calculate the mean SER, it can be further simplified with the asymptotic result. As stated before, the receive SNR in SIMO system with receive antenna selection asymptotically follows Gaussian distribution \( \mathcal{N}(\mu_g, \sigma_g^2) \). Therefore, the mean SER for BPSK (for BPSK, this is also BER) is approximated as follows:

\[
P_{e,\text{BPSK}} \approx \frac{1}{\pi} \int_{0}^{\theta} \left[ 1 - \frac{1}{\sqrt{2\pi\sigma_g^2}} e^{\frac{(x-\mu_g)^2}{2\sigma_g^2}} \right] dx d\theta
\]

(25)

For QPSK, the expression is similar as that of BPSK. Next, we investigate the SER of 16QAM, and the approximation for mean SER of 16QAM is written as:

\[
P_{e,\text{16QAM}} \approx \frac{3}{\pi} \int_{0}^{\theta} \left[ 1 - \frac{1}{\sqrt{2\pi\sigma_g^2}} e^{\frac{(x-\mu_g)^2}{2\sigma_g^2}} \right] dx d\theta
\]

(26)

During the calculation of (22), the total number of integration is proportional to \( N_r \), in other words, the computation complexity of (22) is \( O(N_r) \). Nevertheless, only 1 or 2
integrations is calculated in (25) and (26), thus the computation complexity is \( O(1) \). Under this circumstance, the simplification effect of the asymptotic theory is obvious.

V. SINGLE ANTENNA SELECTION

In this part, a special case will be discussed when only one antenna is activated at the receiver. Since the SER of this special scenario has been investigated in [35], this part will focus on the derivation of symmetric capacity. The case of BPSK will be first investigated, then the derived results will be utilized to analyze other modulation styles, say QPSK and 16QAM.

Suppose that the channel side information (CSI) is only available at the receiver and only one antenna is activated, the ergodic capacity of the antenna selection system is written as

\[
C_{\text{BPSK}} = \ln 2 - \int_0^{+\infty} \int_0^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \ln \left( 1 + e^{-2au - 2aN} \right) \left( 1 - e^{-\frac{y^2}{2}} \right)^{N_r - 1} \frac{2a}{\gamma} e^{-\frac{y^2}{2}} du\]

\[
= \ln 2 - \int_0^{+\infty} \int_0^{+\infty} e^{-\frac{u^2}{2}} \left( 1 - e^{-\frac{y^2}{2}} \right)^{N_r - 1} \frac{2a}{\gamma} e^{-\frac{y^2}{2}} du\]

\[
= \ln 2 - \int_0^{+\infty} \int_0^{+\infty} e^{-\frac{u^2}{2}} (2u + 4a) \left( 1 - e^{-\frac{y^2}{2}} \right)^{N_r} \frac{e^{-2au - 2a^2}}{1 + e^{-2au - 2a^2}} du\]

\[
= \int_0^{+\infty} \frac{4a}{\sqrt{2\pi}} \left( 1 - e^{-\frac{y^2}{2}} \right)^{N_r} \frac{1}{1 + e^{-2au - 2a^2}} du\]

(28)

for the change of variable, \( u \rightarrow u - a \), and the step “(b)” is due to the fact that the first and second integrands are odd and even functions of \( u \). Based on Binominal Formula Expansion (BFE), (28) can be written as

\[
C_{\text{BPSK}} = \ln 2 - \int_0^{+\infty} \int_0^{+\infty} \frac{4a}{\sqrt{2\pi}} \left( 1 - e^{-\frac{u^2}{2}} \right)^{N_r} \frac{e^{-\frac{(u-a)^2}{2}} + e^{-\frac{(u+a)^2}{2}}}{1 + e^{-2au - 2a^2}} du\]

(29)

Based on the Taylor series expansion

\[
\frac{1}{1 + e^{-2au}} = \sum_{k=0}^{+\infty} (-1)^k e^{-2kau},
\]

(30)

\( T_i \) is written as

\[
T_i = \sum_{k=0}^{+\infty} (-1)^k \int_0^{+\infty} e^{-\frac{(u-a)^2}{2} - (2k+1)au - (\frac{a}{2} + \frac{k}{2})u^2} du.
\]

(31)
Moreover, the term \(B(i, k)\) in (31) can be further simplified by repeat applications of the trick of “integration by parts”. After some lines of derivations, \(B(i, k)\) is given by

\[
B(i, k) = \int_0^{+\infty} \int_0^{+\infty} e^{-\frac{x^2}{2} - (2k + 1)xu - \left(\frac{u+1}{\gamma}\right)u^2} \, du \, dx \\
= \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \left(1 - \frac{2k + 1}{\sqrt{1 + 2\frac{1}{\gamma}}}\right) + \frac{(2k + 1)^2}{1 + 2\frac{1}{\gamma}} B(i, k),
\]

(32)

And more detailed derivations for this simplification can be found in Appendix B. Consequently, \(B(i, k)\) can be obtained by solving (32).

\[
B(i, k) = \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \left(1 - \frac{2k + 1}{\sqrt{1 + 2\frac{1}{\gamma}}}\right) + \frac{(2k + 1)^2}{1 + 2\frac{1}{\gamma}} B(i, k).
\]

(33)

As a result,

\[
T_i = \sum_{k=0}^{+\infty} (-1)^k \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \left(1 - \frac{2k + 1}{\sqrt{1 + 2\frac{1}{\gamma}}}\right).
\]

(34)

By substituting (34) into (32), the ergodic capacity can be written as the sum of a series, namely

\[
C_{\text{BPSK}} = \ln 2 - \sum_{i=0}^{N_r} \left( \frac{N_r}{i} \right) (-1)^i \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \sum_{k=0}^{+\infty} \frac{(-1)^k}{\sqrt{1 + 2\frac{1}{\gamma}} + \frac{1}{2} + k}.
\]

(35)

To accelerate the convergence of the series in (35), the following formula of \(\beta\)-function can be utilized (36).

\[
\beta(x) = \sum_{k=0}^{+\infty} \frac{(-1)^k}{x + k} = \sum_{k=0}^{+\infty} \frac{2^{-(k+1)}(-1)^k}{x(x+1)(x+2) \cdots (x+k)}.
\]

(36)

By putting the result back to (36), the ergodic capacity is written as follows:

\[
C_{\text{BPSK}}(\gamma) = \ln 2 - \sum_{i=0}^{N_r} \left( \frac{N_r}{i} \right) \beta(g(i, \gamma)) \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \times \sum_{k=0}^{K} \frac{2^{-(k+1)}(-1)^k}{g(i, \gamma)(g(i, \gamma) + 1)(g(i, \gamma) + 2) \cdots (g(i, \gamma) + k)},
\]

(37)

where \(g(i, \gamma) = \frac{1 + 2\frac{1}{\gamma}}{2} + \frac{1}{2}\) and \(K\) should be set to infinity. However, it should be noticed that the series expansion in (37) converges much faster than the one in (35), thus it is sufficient setting \(K\) to a tiny value such as \(K = 5\) to get a relatively high-precision estimation of the ergodic capacity.

Since QPSK can be treated as the superposition of two orthogonal BPSK, the ergodic capacity for QPSK is given by

\[
C_{\text{QPSK}}(\gamma) = \ln 4 - \sum_{i=0}^{N_r} \left( \frac{N_r}{i} \right) \beta(g(i, \gamma)) \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \times \sum_{k=0}^{K} \frac{2^{-(k+1)}(-1)^k}{g(i, \gamma)(g(i, \gamma) + 1)(g(i, \gamma) + 2) \cdots (g(i, \gamma) + k)}.
\]

(38)

Next, let us turn to 16QAM which is the superposition of two orthogonal 4ASK. As stated before, the instantaneous symmetric capacity of 4ASK is approximated as

\[
C_{\text{4ASK}}(g) \approx C_b(g(0.2g) + C_b(g(1 + e^{-0.2g})^2),
\]

(39)

where \(g\) represents the instantaneous SNR with the PDF \(\frac{1}{\gamma}e^{-\frac{\gamma}{2}}\). For simplicity, the variable \(g_1 = g(1 + e^{-0.2g})^2\) can be approximated as an exponential variable following \(\frac{1}{\gamma}e^{-\frac{\gamma}{2}}\) with the same mean, thus

\[
\gamma_1 = E[g(1 + e^{-0.2g})^2] = 2\gamma(2\gamma e^{4\gamma^3} + 275\gamma^2 e^{100\gamma + 1250})
\]

(40)

Now, consider the single antenna selection, the ergodic capacity for the 4ASK case is given by \(C_{\text{BPSK}}(0.2\gamma) + C_{\text{BPSK}}(0.2\gamma_1)\). As a result, the ergodic capacity for the 16QAM is

\[
C_{\text{16QAM}}(\gamma) = \ln 4 - \sum_{i=0}^{N_r} \left( \frac{N_r}{i} \right) \beta(g(i, 0.2\gamma)) \frac{\sqrt{2\pi}}{2} \frac{1}{1 + 2\frac{1}{\gamma}} \times \sum_{k=0}^{K} \frac{2^{-(k+1)}(-1)^k}{g(i, \gamma)(g(i, \gamma) + 1)(g(i, \gamma) + 2) \cdots (g(i, \gamma) + k)}
\]

(41)

VI. SIMULATION RESULTS

In this part, numerical and simulation results derived in the preceding sections are given. As stated before, there are no closed-form expressions for the symmetric capacity, thus Monte-Carlo simulation is utilized to obtain the exact value, which will be used to examine the feasibility and validity of the former derivations.

Fig. 2 presents the mean symmetric capacity for different antenna deployment styles when multiple antennas are selected. The approximated ergodic capacity for 16QAM is calculated by (14) and the analytical results for BPSK and QPSK are obtained by (5) and (12). Additionally, the simulated values are acquired on the basis of Monte-Carlo simulation. As can be seen from this figure, the analytical results accurately meet with the simulated results, which further supports the derivations in (22). Moreover, the approximation effect of 16QAM by (14) is fantastic since the simulated and approximated curves are nearly coincident with each other, which indicates that the approximation method introduced in (17) is still applicable for multi-antenna systems.

Since Fig. 2 plots the situation when the receiver is deployed with conventional small-scale antenna array, Fig. 3 illustrates the simulated and approximated ergodic capacity versus SNR for BPSK, QPSK and 16QAM when the receiver is equipped with large number of antennas. \(N_r\) ranges between 32 and 64 when \(L\) varies between 10 and 20. Simulated results are obtained by Monte-Carlo simulation and approximated values.
for BPSK, QPSK and 16QAM are calculated by (18), (19) and (21), respectively. From the graph above we can see that the approximation effect for all these three modulation styles are good, which indicates the expressions in (18), (19) and (21) are accurate enough for estimating the capacity. Additionally, the approximation effect is relatively good even though $N_r$ is not too huge, say 32, which is consistent with the former statements.

Fig. 4 compares the simulated and asymptotic approximated BER for BPSK in AS-SIMO system when $L = 5$. Besides, the simulated and approximated SER for 16QAM are plotted in Fig. 5. The simulated result are obtained by Monte-Carlo experiments and the asymptotic approximated SER of BPSK and 16QAM are derived by (25) and (26), respectively. As is shown in these two graphs, when SNR is low, the asymptotic approximation has a good performance even though $N_r$ is small, such as $N_r = 8$. When SNR is large, the approximation effect slightly degrades especially when $N_r$ is small. In fact, BER or SER is small when SNR is very high and the logarithm operation $10 \log(\cdot)$ will enlarge the difference even though the difference is small, which is the reason for the misalignment of the dashed and solid lines when $\gamma$ is large. As suggested before, (25) and (26) posses much lower complexity than (22). Considering the remarkable approximation performance shown in Fig. 4 it makes sense to calculate the mean SER of AS-SIMO systems based upon asymptotic theory. By now, the simulation results for multiple antennas selection has been exhibited and discussed, the next part will concentrate on the special case when only one receive antenna is activated.

The simulated and approximated symmetric capacity versus SNR in a SIMO system under Rayleigh fading channels with single antenna selection is presented in Fig. 6. $N_r$ increases from 1 to 8 and the modulation style is BPSK. Both the empirical value (by Monte-Carlo simulation) and the approximate value (by (27)) are presented for comparison.
More specifically, the approximated results are acquired by summing up the first 8 terms of the series in (37). It can be seen from the curves in Fig. 6 that the series expression in (37) has a remarkable approximation performance and precision even though the summary times are limited, which further verifies the fast convergence of the series in (37). It should be noticed that the situation when $N_r = 1$ is just the single-input single-output (SISO) scenario discussed in [15]. Furthermore, by comparing the situation of $N_r \geq 1$ and $N_r = 1$, the gain originated from the selection diversity is apparent.

Fig. 7 shows the change of the function value in (37) with the increment of $K$, which indicates the convergence of the series in (37). $N_r$ ranges from 1 to 8 and SNR is set to be 0dB. The solid circled lines represented the exact symmetric capacity by Monte-Carlo simulation which are constant. As shown in Fig. 7 it is sufficient to set $K = 5$ when SNR is 0dB. In summary, these results in Fig. 6 and Fig. 7 show that (37) is a promising candidate to decide the symmetric capacity in SIMO system under single-antenna-selection.

Fig. 8 plots the simulated and analytical (or approximated) ergodic capacity of BPSK, QPSK and 16QAM versus SNR in SIMO system when only one antenna is activated at the receiver as $N_r$ increases from 2 to 8. The analytical results for BPSK and QPSK are obtained by (37) and (38). Moreover, the approximated results for 16QAM are based on (41). As can be seen from this figure, the approximation effect gets improved with the increment of $N_r$. In summary, (37), (38) and (41) are effective tools for the calculation of ergodic symmetric capacity in SIMO system with antenna selection.
tailed analyze the performance of SIMO systems under receive antennas. Additionally, we derive the closed-form asymptotic expressions for symmetric capacity of SIMO systems. Next, the asymptotic approximation is further utilized to analyze the SER of SIMO systems with receive antenna selection. Compared with the conventional derivations of the capacity or SER, our results indicate a much lower complexity.

In this paper, for the sake of brevity, only the SIMO system with receive antenna selection under Rayleigh fading is investigated. For other wireless multi-path channel models, e.g., the Nakagami-m fading channels, the analysis of the symmetric capacity for antenna selection technology is still an open problem. Furthermore, the analysis of the symmetric capacity in MIMO system with antenna selection under finite constellation size will be suggested in the future.

VII. CONCLUSION AND FUTURE TRENDS

In this paper, by assuming i.i.d Rayleigh flat fading, we detailed analyze the performance of SIMO systems under receive antenna selection from the view of digital modulation. The analytical or approximated expressions for symmetric capacity of BPSK, QPSK and 16QAM are derived for the first time. QPSK and 16QAM are derived for the first time.

In this paper, for the sake of brevity, only the SIMO system with receive antenna selection under Rayleigh fading is investigated. For other wireless multi-path channel models, e.g., the Nakagami-m fading channels, the analysis of the symmetric capacity for antenna selection technology is still an open problem. Furthermore, the analysis of the symmetric capacity in MIMO system with antenna selection under finite constellation size will be suggested in the future.

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Therefore,

\[
\bar{C}_{\text{BPSK}} = \ln 2 - \sum_{n=1}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2_g}} e^{-\frac{(x-\mu)^2}{2\sigma^2_g}} \left(-1\right)^{n-1} e^{-nx} \frac{1}{n} \, dx
\]

\[
= \ln 2 - \sum_{n=1}^{+\infty} \left(-1\right)^{n-1} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2_g}} e^{-\frac{(x-\mu)^2}{2\sigma^2_g}} \, dx \cdot \frac{1}{n} \, dx
\]

\[
= \ln 2 - \sum_{n=1}^{+\infty} \left(-1\right)^{n-1} e^{-\frac{a^2}{2}} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k}{k!} \, kb
\]

(44)

By (31), the final result is written as

\[
\bar{C}_{\text{BPSK}} = \ln 2 - e^{-\frac{a^2}{2}} \ln \left(1 + e^{-b}\right).
\]  

(45)

### APPENDIX A

**Calculation of the Integration in (26)**

\[
\bar{C}_{\text{BPSK}} = \ln 2 - \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2_g}} e^{-\frac{(x-\mu)^2}{2\sigma^2_g}} \ln \left(1 + e^{-x}\right) \, dx.
\]  

(42)

Based on the Taylor series of \(\ln(1 + x)\),

\[
\ln \left(1 + e^{-x}\right) = \sum_{n=1}^{+\infty} \frac{(-1)^{n-1} e^{-nx}}{n}. \tag{43}
\]

Following the same steps, the results can be expressed in a more general form

\[
\ln 2 - \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2_g}} e^{-\frac{(x-\mu)^2}{2\sigma^2_g}} \ln \left(1 + e^{-kx}\right) \, dx
\]

(46)

\[
= \ln 2 - e^{-\frac{a^2}{2}} \ln \left(1 + e^{-kb}\right).
\]
APPENDIX B
SIMPLIFICATION OF $B(i, k)$

\[ B(i, k) \]
\[ = \int_{0}^{+\infty} \int_{0}^{+\infty} e^{-\frac{u^2}{2}} \left( \frac{e^{-\left(\frac{i}{k} + \frac{1}{2}\right)\alpha^2}}{-2\left(\frac{i}{k} + \frac{1}{2}\right)} \right) d\alpha d\gamma \]
\[ = \int_{0}^{+\infty} \int_{0}^{+\infty} e^{-\frac{u^2}{2}} \left( \frac{e^{-\left(\frac{i}{k} + \frac{1}{2}\right)\alpha^2}}{-2\left(\frac{i}{k} + \frac{1}{2}\right)} \right) |_{0}^{+\infty} d\alpha d\gamma \]
\[ = \int_{0}^{+\infty} e^{-\frac{u^2}{2}} \left( \frac{e^{-\left(\frac{i}{k} + \frac{1}{2}\right)\alpha^2}}{-2\left(\frac{i}{k} + \frac{1}{2}\right)} \right) \left|_{0}^{+\infty} \right. \]
\[ = \frac{\sqrt{2\pi} \left( \frac{2k+1}{1+\frac{2}{7}} \right)}{2 \left( 1 + \frac{2}{7} \right)} - \frac{\sqrt{2\pi} \left( \frac{2k+1}{1+\frac{2}{7}} \right)}{2 \left( 1 + \frac{2}{7} \right)} \left( \frac{2k+1}{1+\frac{2}{7}} \right) (2k+1)^2 B(i, k). \]

(47)