ADJUSTMENT OF THE ELECTRIC CHARGE AND CURRENT IN PULSAR MAGNETOSPHERES

YURI LYUBARSKY
Physics Department, Ben-Gurion University, P.O. Box 653, Beer-Sheva 84105, Israel
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ABSTRACT

We present a simple numerical model of the plasma flow within the open field line tube in the pulsar magnetosphere. We study how the plasma screens the rotationally induced electric field and maintains the electric current demanded by the global structure of the magnetosphere. We show that even though bulk of the plasma moves outwards with relativistic velocities, a small fraction of particles is continuously redirected back forming reverse plasma flows. The density and composition (positrons or electrons, or both) of these reverse flows are determined by the distribution of the Goldreich–Julian charge density along the tube and by the global magnetospheric current. These reverse flows could significantly affect the process of the pair plasma production in the polar cap accelerator. Our simulations also show that formation of the reverse flows is accompanied by the generation of long-wavelength plasma oscillations, which could be converted, via the induced scattering on the bulk plasma flow, into the observed radio emission.

Key words: plasmas – pulsars: general

1. INTRODUCTION

The pulsar activity is believed to be associated with the generation of relativistic electron–positron plasma near the magnetic polar caps. This plasma flows along the open field line tube and eventually escapes from the magnetosphere forming a relativistic pulsar wind. Well within the light cylinder, the plasma currents do not distort significantly star’s magnetic field, therefore in the frame corotating with the star, the plasma just moves along the axis of the rotating static dipole. An important point is that this motion could by no means be considered as free streaming because the basic electrolydynamics dictates the charge and current densities at each point of the flow.

First of all, the charge density in the plasma should be equal to the local Goldreich–Julian charge density

$$\rho_{\text{GJ}} = -\frac{\Omega \cdot B}{2\pi e},$$

where $B$ is the local magnetic field and $\Omega$ is the angular velocity of the neutron star. This condition is a generalization of the standard condition of the quasi-neutrality: deviation of the local charge density from $\rho_{\text{GJ}}$ results in a longitudinal electric field, which redistributes the charges to establish $\rho = \rho_{\text{GJ}}$. The necessary field is weak in the sense that the corresponding potential is small as compared with the total rotationally induced potential; this is because the energy of the secondary plasma particles is small as compared with the total potential.

Scharlemann (1974) and Cheng & Ruderman (1977) noted that in the plasma flow along the curved magnetic field lines, some difference should be maintained between the velocities of the electrons and positrons. The condition $\rho = \rho_{\text{GJ}}$ implies $n^+ - n^- \propto B \cos \theta$, where $n^\pm$ are the number density of positrons and electrons, correspondingly, and $\theta$ is the angle between the pulsar rotation axis and the magnetic field. Continuity of the particle flow implies $n^\pm V^\pm \propto B$, where $V^\pm$ are the average velocities of the positrons and electrons, correspondingly. One sees that the average particle velocities should vary along a curved magnetic field line in accordance with the variation of $\theta$. In a highly relativistic flow, $V^\pm$ are close to $c$ therefore even a small variation of $\theta$ implies a significant variation of the Lorentz factor of the positrons and electrons. Since the energy spread of the secondary particles is large, the electric field, which ensures the adjustment of the charge density, easily shifts the low-energy tail of the distribution function to the negative-momentum domain thereby forming a return particle flux (Lyubarskii 1992; Lyubarskii 1993a). Therefore, one can expect counterstreaming plasma flows in the open field line tube.

If the charge density in the flow is adjusted via sending “extra” charges backwards, an electric current appears in the flow. However, the electric current is not a free parameter, which could be adjusted to maintain the necessary distribution of local charges. The current in the open field line tube is dictated by the global structure of the magnetosphere. The magnetic field lines are bent backwards with respect to the rotation direction in order to allow the plasma to stream along them at a velocity smaller than the speed of light even though the rotation velocity becomes superluminal beyond the light cylinder. The current should be distributed such that the necessary magnetic field is maintained (formally, the current along any magnetic field line is determined from the condition of the smooth transition of the flow through the light cylinder; Ingraham 1973; Contopoulos et al. 1999; Timokhin 2006). Generally, the current required by the global structure of the magnetosphere is not matched with the current established in the course of the charge density adjustment. This would result in some induction electric field, which ensues additional redistribution of the charge flows. These considerations show that the self-consistent distribution of charges and currents in the open field line tube could be maintained only in a nontrivial, generally unsteady, multistream state.

As the first step to understanding the electric charge and current adjustment in the pulsar magnetospheres, we study here a simple model of the plasma motion within a narrow tube with a distributed background charge imitating the Goldreich–Julian charge. The plasma production is not addressed here; we just assume that the relativistic electron–positron plasma with large enough density (significantly larger than the Goldreich–Julian density) is injected continuously into the tube. The aim of this study is to keep track of how the plasma screens out the background charge and at the same time maintains the current imposed at the outer edge of the tube. The system evolution is studied numerically by the particle-in-the-cell (PIC) method.
It will be shown that the electric charge and current densities are adjusted in the plasma flow such that the condition $\rho = \rho_{\text{GJ}}$ is fulfilled at any point and at the same time the current imposed by the boundary conditions is maintained. This justifies the magnetohydrodynamics (MHD) approach in studying the global structure of the pulsar magnetosphere. The charge and current adjustment is achieved via formation of relatively weak backward flows. This could have important implications for the models of the plasma production in pulsars because the plasma flow from the magnetosphere onto the polar cap could alter significantly the electric structure of the polar gap accelerator. Moreover, long-wavelength plasma oscillations are excited in the flow in the process of continuous redistribution of charges. The induced scattering of these oscillations by the bulk plasma flow could give rise to the observed pulsar radio emission.

The paper is organized as follows. In the following section, the numerical method is outlined. In Section 3, we present the results of PIC simulations of the relativistic plasma flow within a narrow tube with a distributed background charge. In Section 4, we discuss the plasma oscillations observed in the simulations. In Section 5, we generalize the model adding an arbitrary current at the outer edge of the tube. The conclusions are given in Section 6.

2. QUASI-ONE-DIMENSIONAL ELECTRODYNAMICS

In this section, a simple generalization of the one-dimensional PIC code is described, which pinpoints the basic qualitative properties of the plasma motion within the open field line tube. Since the particles move along the magnetic field lines like beads on a wire, their motion is governed only by the longitudinal electric field,

$$\frac{dp}{dt} = \pm eE_z. \quad (2)$$

Therefore, one can use the particle mover from the electrostatic one-dimensional PIC code (e.g., Birdsall & Langdon 1991). The problem is that in our case the electric field is not one-dimensional because the plasma moves within a narrow tube; therefore, one has to solve Maxwell’s equations within the tube. We take into account the effects of finite transverse dimensions by substituting $1/R$, where $R$ is the tube radius, for the transverse derivatives in Maxwell’s equations. The same substitution is sometimes made in the Poisson equation when studying steady state flows in the open field line tube (Fawley et al. 1977).

Maxwell’s equations in the frame rotating with the star could be written as customary Maxwell’s equations with specific source terms (e.g., Fawley et al. 1977):

$$\nabla \cdot E = 4\pi (\rho - \rho_{\text{GJ}}); \quad (3)$$

$$\nabla \cdot B = 0; \quad (4)$$

$$\nabla \times B = \frac{4\pi}{c} (\mathbf{j} - \mathbf{\bar{j}}) + \frac{\partial E}{\partial t}; \quad (5)$$

$$\nabla \times E = -\frac{\partial B}{\partial t}. \quad (6)$$

Well inside the light cylinder, $\Omega r \ll 1$, one can take $\mathbf{\bar{j}} = \rho_{\text{GJ}}$; $\mathbf{j} = 0$.

Evolution of the fields is governed by Equations (5) and (6); Equations (3) and (4) are satisfied identically provided the initial conditions satisfy these equations and the charge is conserved.

Within the cylindrical tube, the evolution equations are written as

$$\frac{\partial E_z}{\partial t} = \frac{1}{r} \frac{\partial B_{\phi}}{\partial r} - \frac{4\pi}{c} j, \quad (7)$$

$$\frac{\partial E_z}{\partial t} = -\frac{\partial B_{\phi}}{\partial z}, \quad \frac{\partial B_{\phi}}{\partial t} = \frac{\partial E_z}{\partial z} + \frac{\partial E_z}{\partial r}.$$

Here, $B_{\phi}$ is the azimuthal magnetic field created by the currents in the plasma. Well within the light cylinder, this field is very weak as compared with the pulsar magnetic field, therefore it does not affect the particle motion; the particles move in the $z$ direction along the pulsar magnetic field lines. We assume that the walls of the tube are highly conductive so that $\rho = 0$ at the boundary of the tube, $r = R$. Substituting the transverse derivatives by $1/R$, we get a one-dimensional set of equations

$$\frac{\partial E_z}{\partial t} = \frac{B_{\phi}}{R} - \frac{4\pi}{c} j, \quad \frac{\partial E_z}{\partial t} = -\frac{\partial B_{\phi}}{\partial z}, \quad \frac{\partial B_{\phi}}{\partial t} = -\frac{\partial E_z}{\partial z} - \frac{E_z}{R}. \quad (8)$$

which is qualitatively equivalent to the original Equations (7). Here we took into account that $E_z$ goes to zero at the wall so that $\partial E_z/\partial r \Rightarrow -E_z/R$. Equations (8) could be reduced to the one-dimensional, inhomogeneous Klein–Gordon equation. At the scale much less than the radius of the tube, these equations are reduced to the one-dimensional Maxwell equations. At larger scales, they correctly describe the exponential decreasing of the electric field from a localized charge due to the presence of conducting walls. For boundary conditions, one sets $E_z$ at the bottom edge of the tube and $B_{\phi}$ at the outer edge.

We start from the empty tube therefore $B_{\phi}(t = 0) = 0$, whereas the initial electric field is found from the solution to the Gauss Equation (3) with $\rho = 0$. With the same prescription for the transverse derivatives as in Equations (8), one can write

$$E_z(t = 0) = \Phi/R; \quad E_z(t = 0) = -\Phi/3z; \quad \mathbf{\Phi} \text{ is found from the modified Poisson equation}$$

$$\frac{d^2 \Phi}{dz^2} - \frac{\Phi}{R^2} = 4\pi \rho_{\text{GJ}}. \quad (9)$$

With the initial conditions thus found, the evolution Equations (8) are solved by the leap-frog method:

$$E_z(z, t + \Delta t) - E_z(z, t) = \frac{\Delta t}{R} B_{\phi}(z, t + 0.5\Delta t)$$

$$- \frac{4\pi}{c} \frac{\Delta t}{R} j(z, t + 0.5\Delta t); \quad E_z(z + 0.5\Delta z, t + \Delta t) - E_z(z + 0.5\Delta z, t)$$

$$= -\frac{\Delta t}{\Delta z} \left[ B_{\phi}(z + \Delta z, t + 0.5\Delta t) - B_{\phi}(z, t + 0.5\Delta t) \right];$$

$$B_{\phi}(z, t + 0.5\Delta t) - B_{\phi}(z, t - 0.5\Delta t)$$

$$= -\frac{\Delta t}{\Delta z} \left[ E_z(z + 0.5\Delta z, t) - E_z(z - 0.5\Delta z, t) \right]$$

$$- \frac{\Delta t}{R} E_z(z, t).$$
The current density, \( j \), is found in each step by calculating the charge transferred by particles through each grid face. We use the first-order particle weighting, i.e., the particle is assumed to be a cloud of uniform density, the cloud width being equal to the grid size.

Stability of the scheme is analyzed as usual by writing down the dispersion relation for the homogeneous system:

\[
\left( \frac{2R}{\Delta r} \right)^2 \sin^2 \left( \frac{\omega \Delta t}{2} \right) = 1 + \left( \frac{2R}{\Delta z} \right)^2 \sin^2 \left( \frac{k \Delta z}{2} \right). \tag{11}
\]

One sees that under the condition

\[
(\Delta r)^2 \leq \frac{(\Delta z)^2}{1 + (\Delta z/2R)^2} \tag{12}
\]

the frequency \( \omega \) is real, which implies stability. In the simulations, we used \( \Delta t = 0.5 \Delta z \).

3. SCREENING OF THE ROTATIONALLY INDUCED ELECTRIC FIELD

The rotationally induced electric field implies specific source terms in Maxwell’s equations written in the rotating frame of reference. Deep inside the light cylinder these terms are reduced to the Goldreich–Julian charge density of Equation (1), which could be considered as a background charge distributed within the tube. Our main interest here is with the compensation of the background charge in the relativistic plasma flow. In the static system, the particles of both charge species would easily be redistributed to provide total neutrality. The situation is less trivial when the plasma flows through the tube. We investigate the simplest case when the plasma density in the flow, \( n \), is large enough to compensate the background charge, \( e n \gg \rho_{GJ} \).

For the sake of simplicity, we assume that the radius of the tube, \( R \), is constant. In the pulsar magnetosphere, the open field line tube expands with the altitude, which results in decreasing of the particle density as well as of the charge and current densities. However, the Goldreich–Julian charge density (Equation 1) decreases by the same factor because the pulsar magnetic field decreases in the expanding tube. Therefore, only variation of the angle between the magnetic field and the rotation axis, but not the tube expansion, results in the discrepancy between \( \rho \) and \( \rho_{GJ} \). Thus, the plasma flow in a tube of a constant radius with a varying background charge seems to be a reasonable model for studying screening of the Goldreich–Julian charge in the open field line tube. Note also that only variation of the background charge along the tube results in the nontrivial behavior of the plasma. If \( \rho_{GJ} = \text{const.} \), the charge density would be adjusted already at the bottom of the tube and then the acquired charge would be just transferred by the flow further into the tube. As we are interested in a continuous charge redistribution in the plasma flow along the tube, we take the background charge to be varying along the tube. The simplest choice is a linearly growing background charge density

\[
\rho_{GJ} = \rho_0 z/l_0, \tag{13}
\]

where \( l_0 \) is the length of the tube. Then the system is neutral at the bottom of the tube, whereas at higher altitudes charge neutrality may be maintained only via redistribution of charges in order to compensate the growing background charge.

We took the radius of the tube much smaller than the tube length, \( R = l_0/30 \), which means that different parts of the tube are not connected electrically. The initial electric field was found from the solution to the Poisson Equation (9) with \( E_z = 0 \) at both edges of the tube. One can see that far from the edges of the tube, \( z \gg R \) and \( l_0 - z \gg R \), the solution to Equation (9) with the background charge density (Equation 13) is \( \Phi = -4\pi\rho_0 R^2 z/l_0 \). Then the initial longitudinal electric field is constant far from the edges of the tube,

\[
E_z(t = 0) = 4\pi\rho_0 R^2/l_0, \tag{14}
\]

whereas the transverse electric field increases linearly from the bottom to the upper edge of the tube,

\[
E_x(t = 0) = -4\pi\rho_0 Rz/l_0. \tag{15}
\]

We start injecting plasma from the bottom into the empty tube. At the bottom boundary, we use the condition \( E_z(z = 0) = 0 \), which follows from the continuity of the transverse component of the electric field. At the upper boundary, we take \( B_z(z = l_0) = 0 \). Then the plasma moves as within an infinitely long tube until the flow approaches the outer edge of the tube by the distance \( R \), i.e., while the plasma is disconnected electrically from the outer boundary. Electrons and positrons are injected continuously with the same density and distribution function; the last is shown in Figure 1. The pair density in the flow is \( n = 10 \) pairs per cell. We choose \( \rho_0 = 0.1en \) so that the screening of the background charge would require redistribution of a small fraction of the particles. The electron charge, \( e \), was chosen such that \( \omega_p \Delta z/c = 0.3 \), whereas the radius of the tube was \( R = 1000 \Delta z \). Introducing the characteristic plasma frequency, \( \omega_p = \sqrt{8\pi e^2 n/m} \) (recall that \( n \) is the number density of pairs therefore the total particle number density is \( 2n \)), one sees that \( \omega_p R/c = 300 \). With the chosen parameters, the total potential drop in the empty tube is

\[
e\Phi_0 = 4\pi e \rho_0 R^2 = \frac{\rho_0}{2en} m \omega_p^2 R^2 = 4500 mc^2, \tag{16}
\]
which is much larger than the energy of the injected particles. Therefore, the first injected particles are accelerated to high energies. However, within the plasma, the electric field is nearly shorted out. Compensation of the background charge is provided by decelerating and sending back “extra” charges (electrons in our case); therefore, within the plasma flow, a small electric field is maintained sufficient to redirect the slowest particles.

Evolution of the fields and the particle phase space distribution is shown in Figures 2 and 3. One sees that an electron backflow is indeed formed and the plasma efficiently screens the longitudinal electric field. At the bottom of the tube, a relatively large electric field arises with the total potential drop roughly $10mc^2/e$. This is because in the injected plasma, there are no particles with the energy less than $10mc^2$ so that a “double layer” is formed in order to shift the low-energy tail of the electron distribution to the negative-momentum region. In the remainder part of the tube, the field is very low so that the total potential drop is comparable with that near the bottom edge. The distribution of the electric potential along the tube is determined by the distribution of the background charge and by the particle distribution function in the low-energy part. The background charge dictates how many electrons should be removed from the flow and the potential is adjusted such that the necessary amount of electrons is redirected back.

As the plasma moves farther into the tube, the increasing number of electrons is sent back in order to compensate the increasing background charge. The increasing reverse flux of electrons implies the increasing electric current flowing in the plasma and therefore the increasing azimuthal magnetic field. Note that the plasma screens immediately only the longitudinal electric field, whereas the transverse electric field remains nonzero; it only decreases as compared with the initial transverse field. As the transverse field varies along the tube, the curl of the electric field arises, which is balanced, according to the induction law, by the growing magnetic field.

In order to check whether a steady state solution is possible, at least in the average, we extend the tube to $z > l_0$ assuming that the background charge density remains constant at $z > l_0$:

$$\rho_{GJ} = \begin{cases} \rho_0 z / l_0; & z < l_0; \\ \rho_0; & z > l_0. \end{cases}$$  \hspace{1cm} (17)

One can anticipate that in the region with $\rho_{GJ} = \text{const.}$, the reverse plasma flow is not formed therefore this region would not affect the region $z < l_0$ so that a steady state solution would be eventually formed there. The simulations show that this is indeed the case (see Figures 4 and 5). The transverse field gradually decreases to zero and a steady state is formed with...
$E = 0$ and $\rho = \rho_G$. In the course of time, the steady zone extends further into the tube.

One can easily estimate the current, and therefore the azimuthal magnetic field, established in the steady state. In the region $z > l_0$, the background charge density is constant, $\rho_G = \rho_0$; the particle redistribution does not occur there and the reverse particle flux is not formed. Then the current may be presented as $j = ec(n_+ - n_-)$, where $n_+$ and $n_-$ are the number densities of positrons and electrons, correspondingly. Here we assume that all particles move with the speed of light. In the pulsar magnetospheres, the particle distribution is wide, $\Delta \gamma \sim \gamma \sim 100$, and in the presence of the longitudinal electric field, the distribution function is just shifted in the momentum space as a whole so that the fraction of the particles with the modulo velocities significantly less than the speed of light remains negligibly small in any case. Taking into account that charge neutrality implies $e(n_+ - n_-) = \rho_0$, one finds finally $j = c\rho_0$. When the steady state is achieved, the magnetic field approaches

$$B_{\phi,0} = 4\pi R \rho_0. \quad (18)$$

This estimate assumes that the electric field in the tube is screened completely. However, some small field remains in order to decelerate the necessary amount of electrons and send them backwards. Therefore, the charge density in the plasma is a bit less than the background charge density, which implies that the magnetic field remains a bit less than that of Equation (18) (see Figure 5).

4. PLASMA OSCILLATIONS EXCITED IN THE COURSE OF THE CHARGE ADJUSTMENT

Inspection of Figures 4 and 5 shows that strong fluctuations of the longitudinal electric field are present in the region $0 < z < l_0$ where the background charge density varies. In Figure 6, we show the power density spectrum of the fluctuations, $P = (8\pi)^{-1}|E_{z,k}|^2$, in this region as well as in the region $z > l_0$. One sees that in the last region, where the background charge density is constant so that additional redistribution of charges does not occur, only short-wavelength fluctuations are present, which could be attributed to the numerical noise. In the region $0 < z < l_0$, the fluctuation spectrum is extended to the long-wavelength domain, which means that the charge redistribution is accompanied by the generation of long-wavelength plasma oscillations.

The longitudinal fluctuations in the relativistic, one-dimensional plasma are described by the dispersion relation (e.g., Lyubarskii 1996)

$$\omega^2 \int f_+(p) + f_-(p) \gamma^3(\omega - kv)^2 dp = 1, \quad (19)$$

where $f_{\pm}$ are the distribution functions of electrons and positrons, correspondingly. The characteristic frequency of the long-wavelength oscillations,

$$\omega_0 = \omega_p \left( \int \frac{(f_+ + f_-)dp}{\gamma^3} \right)^{1/2}, \quad (20)$$

is determined by the low-energy particles. In Figure 7, we show the numerical solution to the dispersion relation (19) for the distribution functions found in our simulations (and shown in Figure 1); one sees that $\omega_0 \approx 0.06\omega_p$ in this case. According to Figure 6(a), the maximum of the fluctuation energy, $kP$, occurs at $k \sim 0.05\omega_p/c$, which is close to $\omega_0/c$. In order to check this scaling, we performed the same simulations with $\omega_p$ three times larger than in the simulations shown in Figures 2–5. In this case,
Fluctuations presumably arise because we inject equal amount of electrons and positrons, whereas in the true steady state, the injected flux of electrons should be less by \((1/2)\rho_0/e\) than the flux of positrons. This is because an electron redirected at a point \(z = z_1\) contributes twice into the charge density at \(z < z_1\) so that the total amount of injected electrons should be less than that of positrons by the amount of the electrons redirected along the whole tube. If this condition is not satisfied, extra charges (electrons in our case) are expelled from the flow at the scale of the Debye length, which is determined by the frequency \(\omega_0\) for the injected plasma. Therefore, a “double layer” arises at the bottom of the tube; it is clearly seen in Figure 3 and less clearly (because of small scale) in Figures 4 and 5. Such a double layer is unsteady; it oscillates at the timescale of \(1/\omega_0\) thus slightly modulating the injected plasma flow. This could give rise to the electric field fluctuations in the tube because if the plasma flow is somehow modulated, the decelerating electric field should fluctuate in order to keep the number of redirected particles fixed (the last is determined by the distribution of the Goldreich–Julian charge density, which does not vary). On the other hand, one has to stress that the modulation occurs at the frequency \(\omega_0\) for the injected plasma, which is less than the frequency \(\omega_0\) for the plasma in the tube because the lowest energy particles in the injected plasma have energy \(\gamma \sim 10\), whereas in the tube the particle distribution is shifted to \(\gamma \sim 1\). So the presented interpretation is not straightforward.

In real pulsars, the characteristics of the injected plasma depend on the conditions in the polar cap region; therefore, there is no reason to expect that the polar cap accelerator injects the particles in such a way that the difference in the positron and electron fluxes is adjusted to the structure of the magnetosphere in the light cylinder region (recall that \(\rho_0\) is referred to the Goldreich–Julian density far from the polar cap, somewhere at \(\Omega r \sim 1\)). Moreover, the plasma production process is by no means steady, therefore one can expect that the plasma flow in the open field line tube is strongly modulated. Any modulation of the plasma flow would result in fluctuations of the decelerating field along the tube, thus giving rise to long-wavelength plasma oscillations in the flow. Therefore, we believe that such fluctuations are generic in pulsar magnetospheres. Induced scattering of these oscillations on the bulk plasma flow could give rise to high brightness temperature pulsar radio emission (Lyubarskii 1992, 1993b, 1996).
5. ADJUSTMENT OF THE ELECTRIC CURRENT IN THE PLASMA FLOW

One sees that when the system relaxes to the steady, on the average, state, the current (and therefore the azimuthal magnetic field) is established, which is determined by the distribution of the Goldreich–Julian charge density. However, there is no reason to expect that this current is consistent with the current necessary to maintain the global structure of the magnetosphere. As the condition $E \cdot B = 0$ is fulfilled within the plasma flow, the structure of the pulsar magnetosphere satisfies the ideal MHD equations and as such do not admit additional constraints on the current because the current distribution in any MHD system is uniquely determined. Specifically, in the pulsar magnetospheres, the current distribution is determined from the condition of the smooth plasma transition through the light cylinder (Ingraham 1973; Contopoulos et al. 1999; Timokhin 2006). Let us check what happens if the current “demanded” by the global MHD solution differs from the current established in the course of the charge density adjustment, i.e., if the field (18) does not fit the MHD structure of the magnetic field. For this purpose, we simulated plasma flow within a tube of length $l_0$ with a (generally varying) magnetic field imposed at the upper edge of the tube. As an initial condition, we used the distributions of the fields and particles found at the end of the previous simulations when the steady state was achieved in the region $0 < z < l_0$. We imposed the condition of free-particle escape from the upper edge of the tube and no particle inflow through this edge. Conditions at the bottom edge remained the same as in the previous simulations, i.e., the plasma is continuously injected into the tube and $E_z(z = 0) = 0$. When we impose the condition $B_\phi(z = l_0) = B_{\phi,0}$ at the upper edge of the tube, nothing changes and the initial state persists forever. However, when we slowly vary $B_\phi(z = l_0)$ at the upper boundary from $B_{\phi,0}$ to another value, $B_{\phi,1}$, a discrepancy between the current in the plasma and the imposed $B_\phi$ results in the induction electric field, which eventually establishes the necessary current.

In the simulations, the magnetic field at the upper boundary was changed according to the formula

$$B_\phi(z = l_0) = B_{\phi,1} + (B_{\phi,0} - B_{\phi,1}) \exp(-t/\tau). \quad (21)$$

In Figures 8 and 9, we show how the system is relaxed to the state with the azimuthal magnetic field $B_{\phi,1} = 0.5B_{\phi,0}$. One sees that the electric field arises near the upper edge of the tube so that positrons are decelerated until some of them are redirected back thus compensating the initial current. Eventually, a steady state is formed with the new magnetic field and with both electron and positron backflows. The electrons are redirected along the whole tube compensating variation of the background charge density. The positron backflow is formed near the upper edge of the tube.

Relaxation to the state with the current larger than the initial one is achieved via formation of an enhanced electron backflow; see Figure 10. In Figure 11, we show the state with the current having the sign opposite to that of the Goldreich–Julian charge. Such a state is achieved by sending more positrons back. So one concludes that the system could easily maintain any current imposed at the outer edge, the adjustment being achieved via formation of an appropriate particle backflow.

6. CONCLUSIONS

We have demonstrated that the relativistic plasma flow in the open field line tube of the pulsar easily screens the rotationally induced longitudinal (along the magnetic field) electric field and maintains any current demanded by the boundary conditions. This justifies the MHD approach to the structure of the pulsar magnetosphere provided the plasma density in the flow exceeds the Goldreich–Julian density. An important point is that the adjustment of the electric charge and the current is achieved via formation of a particle backflow with the density roughly $\rho_{\text{GJ}}/e$. Depending on the magnetospheric current and on the distribution of the Goldreich–Julian charge density along the open field line tube, the backflow could consist of electrons, positrons, or both.

In these simulations, the pair plasma with the density significantly larger than $\rho_{\text{GJ}}/e$ was continuously injected from the
In the pulsar magnetosphere, the plasma is assumed to be produced at the bottom of the open field line tube where primary particles are accelerated in the potential gap (e.g., Hibschman & Arons 2001). The present study shows that one should expect a particle flux coming from above to the gap. When the plasma fills the gap from above, the electric field is shorted out, the particle acceleration is terminated and the pair production ceases. Presumably this means that the polar accelerator could not operate in the steady state regime. One can expect some kind of spasmodic plasma production such that the required current is maintained only on average. Future models of the polar cap accelerator should take into account the plasma flow onto the gap from the magnetosphere.

Another implication of this study is that the charge and current adjustment is accompanied by the continuous generation of long-wavelength plasma oscillations. As a result of the induced scattering of these oscillations in the bulk plasma flow, these oscillations could be converted into the electromagnetic waves with the frequency \( \omega \sim \omega_p \sqrt{\gamma_{\text{max}}} \), where \( \gamma_{\text{max}} \sim 100 \) is the Lorentz factor corresponding to the maximum in the particle energy spectrum (Lyubarskii 1996). The energy of these waves exceeds the energy of the initial oscillations \( \sim \omega_p \sqrt{\gamma_{\text{max}}}/\omega_0 \) times. One can speculate that this process gives rise to the observed pulsar radio emission (for more details, see Lyubarskii 1996).

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