Muon and Electron $g - 2$
and the Origin of Fermion Mass Hierarchy

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Abstract

We present a model that explains the electron and the muon $g - 2$ discrepancies simultaneously, and further gives a natural explanation to the charged lepton mass hierarchy. In the model, we introduce lepton-flavor-dependent $U(1)_F$ symmetry and three additional Higgs doublets with $U(1)_F$ charges, to realize that each generation of charged leptons couples to one of the three additional Higgs doublets. The $U(1)_F$ symmetry is softly broken by +1 charges, and the smallness of the soft breaking naturally gives rise to the hierarchy of the Higgs VEVs, which then accounts for the charged lepton mass hierarchy. Since electron and muon couple to different scalar particles, it is possible to explain the electron and the muon $g - 2$ discrepancies simultaneously, with an appropriate choice of the relations of the charged and CP-even neutral scalar particle masses. Here the $U(1)_F$ symmetry suppresses charged lepton flavor violation.
1 Introduction

The Standard Model (SM) explains almost all experimental data, but there are several anomalies in low-energy observables. Among the anomalies, the experimental and theoretical studies on the muon $g - 2$ has been extensively carried out. Comparing the experimental value to its SM prediction, the current discrepancy of the muon $g - 2$ is reported as $[1, 2, 3, 4]$

$$\delta a_\mu = a_\mu^{\text{obs}} - a_\mu^{\text{SM}} = (27.4 \pm 7.3) \times 10^{-10}. \quad (1)$$

Recently, the discrepancy of the electron $g - 2$ is also reported as $[5, 6, 7, 8, 9]$

$$\delta a_e = a_e^{\text{obs}} - a_e^{\text{SM}} = (-8.7 \pm 3.6) \times 10^{-13}. \quad (2)$$

Although the electron $g - 2$ discrepancy is less than $3\sigma$, both discrepancies may be signs of physics beyond the SM.

In the effective theoretical approach, the contribution to the lepton $g - 2$ is described by the chirality-breaking operator, $\sim \ell_L^\sigma \mu_R \ell_R F^{\mu\nu}$. If the chirality-breaking in a new physics model is proportional simply to the lepton mass, as in Minimal Supersymmetric Standard Model (MSSM) with Minimal Flavor Violation (MFV), the contribution to the lepton $g - 2$ is proportional to its mass, and there is a simple relation between the electron and muon $g - 2$

$$\frac{\delta a_e}{\delta a_\mu} \approx \frac{m_e^2}{m_\mu^2} = 2.3 \times 10^{-5}. \quad (3)$$

This relation does not hold experimentally in Eq. (1) and Eq. (2), especially the sign is opposite. Hence, any new physics model that explains both discrepancies must include flavor violation beyond MFV in the interactions of muon and electron. Various sources of flavor violation beyond MFV have been considered to address the discrepancies $[10, 11, 12, 13, 14, 15, 16, 17, 18]$.

A stringent restriction $[12]$ on the model building attempts comes from the absence of the $\mu \to e\gamma$ decay. In the effective theory point of view, this indicates that two operators $\mu_L^\sigma \mu_R F^{\mu\nu}$ and $\mu_L^\sigma e_R F^{\mu\nu}$ appear in a way that breaks MFV, but the operators $\mu_L^\sigma \mu_R F^{\mu\nu}$ and $\mu_L^\sigma e_R F^{\mu\nu}$ are forbidden. The above situation is realized naturally by assuming a muon-specific and/or electron-specific $U(1)$ symmetry. One possibility is that this $U(1)$ symmetry is the same as the accidental muon-number and electron-number symmetries of the SM, namely, the new physics sector respects the muon-number and electron-number symmetries so that $\mu_L^\sigma \mu_R F^{\mu\nu}$ and $\mu_L^\sigma e_R F^{\mu\nu}$ are generated with arbitrary strengths, but $\mu_L^\sigma \mu_R F^{\mu\nu}$ and $\mu_L^\sigma e_R F^{\mu\nu}$ are not generated. Another possibility is that only $\mu_R$ or $e_R$ is charged under a new (anomalous) $U(1)$ symmetry, and there exists a new Higgs field charged under it that couples exclusively to $\mu_R$ or $e_R$. 

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In this paper, we construct a model along the second possibility to explain the electron and muon $g-2$ discrepancies without invoking large charged lepton flavor violation. We go one step further and connect the new $U(1)$ symmetry to the origin of the fermion mass hierarchy.

In our model (which we name ‘lepton-flavored Higgs model’), we introduce a new $U(1)_F$ symmetry under which $\tau_R, \mu_R, e_R$ are charged by $-1, -2, -3$, respectively, and introduce three additional Higgs doublets with $U(1)_F$ charges $+1, +2, +3$. Due to the $U(1)_F$ symmetry, each generation of charged leptons couples to one of the three Higgs doublets. We assume that the $U(1)_F$ symmetry is softly broken with a small amount by $+1$ charges, which naturally generates a hierarchy among the vacuum expectation values (VEVs) of the Higgs doublets. We consider that this hierarchy of VEVs accounts for the charged lepton mass hierarchy and that the charged lepton Yukawa couplings are all $O(1)$. A notable feature of the above setup is that since the charged and heavy neutral scalars in the Higgs sector couple differently to electron and muon, there is little correlation between the new scalar contributions to the electron and muon $g-2$. We survey the space of parameters of the Higgs sector and find that there are sets of parameters that explain the discrepancies simultaneously.

This paper is organized as follows. In Section 2, we explain our lepton-flavored Higgs model. In Section 3, we conduct a numerical search of parameters of the Higgs sector and show that there are sets of parameters that explain both electron and muon $g-2$ discrepancies. Section 4 summarizes the paper.

2 Lepton-Flavored Higgs Model

The lepton-flavored Higgs model includes four Higgs doublets, $H_1, H_2, H_3, H_0$, in addition to the SM leptons. The fields are charged under the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group and a new anomalous $U(1)_F$ symmetry as Table I. Note that since only the right-handed charged leptons have $U(1)_F$ charges, the model does not restrict the Weinberg operator for the tiny neutrino mass.
Table 1: The fields and their charge assignments. $\alpha$ labels the three generations.

| Field      | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | anomalous $U(1)_F$ |
|------------|-----------|-----------|-----------|-------------------|
| $H_1$      | 1         | 2         | +1/2      | +3                |
| $H_2$      | 1         | 2         | +1/2      | +2                |
| $H_3$      | 1         | 2         | +1/2      | +1                |
| $F_H$      | 1         | 2         | +1/2      | 0                 |
| $\ell^\alpha_L$ ($\alpha = 1, 2, 3$) | 1 | 2 | −1/2 | 0 |
| $e_R$      | 1         | 1         | −1        | −3                |
| $\mu_R$    | 1         | 1         | −1        | −2                |
| $\tau_R$   | 1         | 1         | −1        | −1                |

The $U(1)_F$ symmetry is assumed to be softly broken by +1 charges. The soft breaking by +1 charges can be realized by introducing a SM gauge-singlet scalar $S$ with $U(1)_F$ charge 1/2 and demanding that renormalizable interactions preserve the $U(1)_F$. Then one introduces a VEV of $S$ to break the $U(1)_F$. The resulting Nambu-Goldstone boson gains mass from non-renormalizable terms that explicitly violate the $U(1)_F$.

The Yukawa couplings for the SM leptons, which respect the $U(1)_F$ symmetry, are given by

$$-\mathcal{L}_{\text{Yukawa}} = y_1 \bar{\ell}_L^1 H_1 e_R + y_2 \bar{\ell}_L^2 H_2 \mu_R + y_3 \bar{\ell}_L^3 H_3 \tau_R + \text{H.c.}$$

The Higgs potential, where the $U(1)_F$ symmetry is softly broken by +1 charges, is given by

$$-\mathcal{L}_{\text{Higgs}} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 H_3^\dagger H_3 + m_0^2 H_0^\dagger H_0$$

$$- \mu_{12}^2 (H_1^\dagger H_2 + H_2^\dagger H_1) - \mu_{23}^2 (H_2^\dagger H_3 + H_3^\dagger H_2) - \mu_{30}^2 (H_3^\dagger H_0 + H_0^\dagger H_3)$$

$$+ \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_3^\dagger H_3)^2 + \lambda_0 (H_0^\dagger H_0)^2$$

$$+ \sum_{i,j=0,1,2,3; i > j} \lambda_{ij} (H_i^\dagger H_j)(H_j^\dagger H_i) + \sum_{i,j=0,1,2,3; i > j} \rho_{ij} (H_i^\dagger H_j)(H_j^\dagger H_i)$$

Here the scalar quartic couplings are all real. Also, $\mu_{12}, \mu_{23}, \mu_{03}$ are taken real positive through a phase redefinition of $H_1, H_2, H_3, H_0$. It follows that the model does not contain any source of CP violation.

Now we make a crucial assumption on the Higgs potential. We assume

$$m_0^2 < 0, \quad m_1^2 > 0, \quad m_2^2 > 0, \quad m_3^2 > 0.$$

$H_0$ develops a VEV, $\langle H_0 \rangle = v/\sqrt{2}$, which is estimated to be

$$v \simeq \sqrt{-m_0^2/\lambda_0}.$$
The VEV of $H_0$ induces a VEV of $H_2$ through the term $\mu^2_{03}(H_0^1 H_3 + H_3^1 H_0)$. The latter induces a VEV of $H_2$ through the term $\mu^2_{23}(H_2^1 H_3 + H_3^1 H_2)$, which then induces a VEV of $H_1$ through the term $\mu^2_{12}(H_1^1 H_2 + H_2^1 H_1)$. Consequently, writing the VEVs as $\langle H_3 \rangle = v_3/\sqrt{2}$, $\langle H_2 \rangle = v_2/\sqrt{2}$, $\langle H_1 \rangle = v_1/\sqrt{2}$, we get

$$v_3 \simeq \frac{\mu^2_{03}}{m^2_3} v, \quad v_2 \simeq \frac{\mu^2_{23}}{m^2_2} v_3, \quad v_1 \simeq \frac{\mu^2_{12}}{m^2_1} v_2. \quad (8)$$

We arrange the masses such that ($m_t$ denotes the top quark mass)

$$\frac{\mu^2_{03}}{m^2_3} \sim \frac{m_\tau}{m_t} \ll 1, \quad \frac{\mu^2_{23}}{m^2_2} \sim \frac{m_\mu}{m_\tau} \ll 1, \quad \frac{\mu^2_{12}}{m^2_1} \sim \frac{m_\mu}{m_\tau} \ll 1. \quad (9)$$

The arrangement of Eq. (9) is natural because $\mu^2_{03}, \mu^2_{23}, \mu^2_{12}$ break the $U(1)_F$ symmetry while $m^2_3, m^2_2, m^2_1$ preserve it. It follows from Eq. (9) that the lepton Yukawa couplings are all $O(1)$,

$$y_1 \sim y_2 \sim y_3 \sim 1. \quad (10)$$

We thus naturally explain the hierarchy of the charged lepton masses in terms of the small soft breaking of the $U(1)_F$ symmetry.

One can implement a similar structure in the quark sector, where we have five more Higgs doublets whose VEVs are on the order of $(m_b/m_t)v$, $(m_s/m_b)v$, $(m_d/m_s)v$, $(m_c/m_t)v$, $(m_u/m_c)v$, respectively, and which couple exclusively to the right-handed bottom, strange, down, charm and up quarks, respectively. Also, $H_0$ couples only to the right-handed top quark. The resulting model is basically the same as the progressive $U(1)$ model of Ref. [19].

We move to phenomenological aspects of the model. After electroweak symmetry breaking, there appear four CP-even scalar particles, three CP-odd scalar particles, and three charged scalar particles. Since $v_1 \ll v_2 \ll v_3 \ll v$, we can make an approximation that each CP-odd scalar particle comes exclusively from $H_1$, $H_2$ or $H_3$, each CP-even scalar particle comes exclusively from $H_1$, $H_2$, $H_3$ or $H_0$, and each charged scalar particle comes exclusively from $H_1$, $H_2$ or $H_3$. Under the above approximation, the Yukawa couplings in Eq. (1) are rewritten in terms of physical particles as

$$L_{\text{Yukawa}} \simeq \frac{y_1 v_1}{\sqrt{2}} \bar{e} e + \frac{y_2 v_2}{\sqrt{2}} \bar{\mu} \mu + \frac{y_3 v_3}{\sqrt{2}} \bar{\tau} \tau + \frac{y_1 v_1}{\sqrt{2}} h \bar{e} e + \frac{y_2 v_2}{\sqrt{2}} h \bar{\mu} \mu + \frac{y_3 v_3}{\sqrt{2}} h \bar{\tau} \tau + \frac{y_1}{\sqrt{2}} H^0_1 \bar{e} e + \frac{y_2}{\sqrt{2}} H^0_2 \bar{\mu} \mu + \frac{y_3}{\sqrt{2}} H^0_3 \bar{\tau} \tau + \frac{y_1}{\sqrt{2}} A_1 \bar{e} i\gamma_5 e + \frac{y_2}{\sqrt{2}} A_2 \bar{\mu} i\gamma_5 \mu + \frac{y_3}{\sqrt{2}} A_3 \bar{\tau} i\gamma_5 \tau + y_1 H^+_1 \bar{\nu}_e e_R + y_2 H^+_2 \bar{\nu}_\mu \mu_R + y_3 H^+_3 \bar{\nu}_\tau \tau_R + \text{H.c.} \quad (11)$$
where $A_1, A_2, A_3$ denote CP-odd scalar particles, $h, H_1^0, H_2^0, H_3^0$ CP-even scalar particles, and $H_1^+, H_2^+, H_3^+$ charged scalar particles. $h$ has SM-like Yukawa couplings and can be identified with the observed 125 GeV scalar particle.

We concentrate on the contribution of $H_0^1, A_1, H_1^+$ to the electron $g-2$ and that of $H_0^2, A_2, H_2^+$ to the muon $g-2$. They are given by \[ \delta a_e = \frac{1}{16\pi^2} y_1^2 \left( \frac{m_e^2}{m_{H_0^1}^2} \log \frac{m_{H_0^1}^2}{m_e^2} - \frac{7}{6} \right) - \frac{m_e^2}{m_{A_1}^2} \left( \log \frac{m_{A_1}^2}{m_e^2} - \frac{11}{6} \right) - \frac{m_e^2}{6m_{H_1^+}^2} \right), \]

\[ \delta a_\mu = \frac{1}{16\pi^2} y_2^2 \left( \frac{m_\mu^2}{m_{H_0^2}^2} \log \frac{m_{H_0^2}^2}{m_\mu^2} - \frac{7}{6} \right) - \frac{m_\mu^2}{m_{A_2}^2} \left( \log \frac{m_{A_2}^2}{m_\mu^2} - \frac{11}{6} \right) - \frac{m_\mu^2}{6m_{H_2^+}^2} \right). \]

\[ (12) \]

\[ (13) \]

It is important to note that different sets of scalar masses enter the formulas for the electron and muon $g-2$. This allows us to simultaneously explain the negative deviation of the electron $g-2$ and the positive deviation of the muon $g-2$. We comment that the two-loop Barr-Zee diagrams are suppressed by the electron mass or muon mass and hence are negligible.

We derive the masses of scalar particles $H_0^1, A_1, H_1^+, H_0^2, A_2, H_2^+$ from the scalar potential Eq. (5), and show that there exists a parameter region where the discrepancies of electron and muon $g-2$ can be explained, with $O(1)$ values for Yukawa couplings $y_1, y_2$ and without conflicting experimental bounds on the masses of CP-even, CP-odd and charged scalar particles. Expanding the Higgs potential around the VEVs in Eq. (8), the tadpole parameters $t_{H_i}$ for the CP-even scalars should vanish.

\[ t_{H_1} \simeq m_1^2 v_1 - \mu_{12}^2 v_2 + \frac{\lambda_{10} + \rho_{10}}{2} v_1^2 v_1 = 0, \]

\[ t_{H_2} \simeq m_2^2 v_2 - \mu_{23}^2 v_3 + \frac{\lambda_{20} + \rho_{20}}{2} v_2^2 v_2 = 0, \]

\[ t_{H_3} \simeq m_3^2 v_3 - \mu_{30}^2 v_0 + \frac{\lambda_{30} + \rho_{30}}{2} v_3^2 v_3 = 0. \]

Here the hierarchy of the VEVs, $v_1 \ll v_2 \ll v_3 \ll v$, is assumed. The physical Higgs mass spectrum is given by

\[ m_{H_1^0}^2 = m_{A_1}^2 \simeq m_i^2 + \frac{\lambda_{i0} + \rho_{i0}}{2} v_i^2, \]

\[ m_{H_1^+}^2 \simeq m_i^2 + \frac{\lambda_{i0}}{2} v_i^2. \]

Notice that the CP-even and the CP-odd scalars have different masses due to the quartic coupling constants $\rho_{i0}$.
3 Numerical Results

First, let us estimate the contributions of the new scalar particles to $\delta a_e$ and $\delta a_\mu$. From Eq. (12) and Eq. (13),

\[
\delta a_e = 2.5 \times 10^{-13} \times \left( \frac{y_1}{3} \right)^2 \left( \frac{100 \text{ GeV}}{m_{H_1^0}} \right)^2 \left( 4 - \left( \frac{m_{H_1^0}}{m_{H_1^+}} \right)^2 \right) \text{ e cm},
\]

\[
\delta a_\mu = 1.2 \times 10^{-9} \times \left( \frac{y_2}{1} \right)^2 \left( \frac{100 \text{ GeV}}{m_{H_2^0}} \right)^2 \left( 4 - \left( \frac{m_{H_2^0}}{m_{H_2^+}} \right)^2 \right) \text{ e cm},
\]

where $m_{A_i} = m_{H_i^0}$ is assumed. In order to explain the magnitude of the current discrepancies in Eq. (2) and Eq. (1), the Yukawa coupling constants must be $O(1)$ and the new scalar masses must be $O(100)$ GeV. Since the current deviation of the electron $g - 2$ is negative, there is a condition that

\[
m_{H_1^0} > 2m_{H_1^+}.
\]

From Eq. (17) and Eq. (18), this inequality can be satisfied if $\rho_{10}$ is $O(1)$. On the other hand, the current deviation of the muon $g - 2$ is positive, so that

\[
m_{H_2^0} < 2m_{H_2^+}.
\]

This inequality can be satisfied if $\rho_{20}$ is sufficiently small.

In the numerical analysis, we take the VEVs of the four Higgs doublets as follows.

\[
v_3 = \frac{m_\tau}{m_t} v, \quad v_2 = \frac{m_\mu}{m_\tau} v_3, \quad v_1 = \frac{m_e}{m_\mu} v_2,
\]

with $v = 246$ GeV. For the other Higgs sector parameters, we scan them randomly in the following regions.

\[
0 < m_1, m_2, m_3 < 50 \text{ GeV},
\]

\[
0 < \lambda_i, \lambda_{ij} < 1,
\]

\[
0 < \rho_{ij} < 6.
\]

For each parameter set, we impose the vanishing tadpole conditions in Eqs. (14)-(16) and calculate the physical Higgs spectrum. We cannot find the LHC constraint on the lepton-specific Higgs. If the Yukawa couplings are $O(1)$, the charged Higgs mainly decay into the charged lepton and neutrino. In this case, the charged Higgs search is almost the same as the
left-handed slepton search. The current LHC bound on the left-handed slepton is $\sim 300$ GeV for the combination of selectron and smuon [21, 22]. There is no separate mass bound and we impose the following flavor dependent constraints.

$$m_{H_1}^+ > 200 \text{ GeV}, \quad m_{H_{2,3}}^+ > 400 \text{ GeV}. \quad (27)$$

Fig[1] shows the contributions of the new scalar particles to $\delta a_e$ and $\delta a_\mu$. Here we fix the Yukawa coupling constants as $y_e = 8$ and $y_\mu = 4$. Unfortunately, we cannot find any parameter set that gives $\delta a_e$ in the $1\sigma$ region. However, we observe that there are parameter sets that give $\delta a_e$ in the $2\sigma$ region and $\delta a_\mu$ in the $1\sigma$ region. In these parameter sets, the charged scalar masses are around the current experimental bound $\sim 100$ GeV and $\rho_{10}$ is $O(1)$.

Figure 1: Scatter plot of $\delta a_e$ and $\delta a_\mu$. The Yukawa coupling constants are fixed $y_e = 8$ and $y_\mu = 4$ and other input parameters are explained in the text. The green and red lines correspond to the experimental $1\sigma$ and $2\sigma$ bounds, respectively.

4 Summary

We have studied the lepton-flavored Higgs model to explain the electron and the muon $g - 2$ discrepancies simultaneously, and further give a natural explanation to the charged lepton mass hierarchy. In the model, we introduce the $U(1)_F$ symmetry under which $e_R, \mu_R, \tau_R$ are charged.
with $-3, -2, -1$, respectively, and introduce three additional Higgs doublets with $U(1)_F$ charges +3, +2, +1, so that each generation of charged leptons couples to one of the three additional Higgs doublets due to the $U(1)_F$ symmetry. We assume that the $U(1)_F$ symmetry is softly broken by +1 charges, and take the soft breaking to be sufficiently small that the hierarchy of the charged lepton masses are originated from the hierarchy of the Higgs VEVs while the Yukawa couplings are $O(1)$. Note that it is natural to take the soft breaking small. Since electron and muon couple to different scalar particles with $O(1)$ Yukawa couplings, the electron and the muon $g - 2$ discrepancies can be explained simultaneously. Specifically, the negative deviation of $a_e$ is explained when the almost-electron-specific charged scalar particle is much lighter than the CP-even scalar particle. On the other hand, the positive deviation of $a_\mu$ is explained when the mass of the almost-muon-specific charged scalar particle is similar to that of the CP-even scalar particle. We have searched the space of the Higgs sector parameters and found sets that give $a_e$ in 2$\sigma$ region and $a_\mu$ in 1$\sigma$ region.

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