Effect of bending moment on the fatigue strength of a bolt in bolt/nut assembly

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Abstract
This study describes an approach to quantify the effect of the bending moment on the fatigue strength of a bolt in bolt/nut assembly. To confirm the validity of the conventional fatigue design methodology based on the nominal stress acting on the specified nominal stress area, both the fatigue tests with various bending stress ratios (nominal bending stress/ nominal total axial stress) and the 3D-FE stress analysis for the corresponding conditions were conducted. The results from fatigue tests using a newly developed fatigue testing fixture clearly show as bending stress ratio increases, the (virtual) fatigue strength expressed by the nominal stress also increases. The results from 3D-FE analysis show that the magnitude of the local bending stress on the thread root is lower than the one estimated from nominal axial stress and the stress concentration factor for purely tensile loading. The results, however, also show that the magnitude of the local stress acting on the bolt thread root is affected by the angular position of nut against the loading axis due to the existence of the incomplete thread of nut at the bearing face side. Finally, it is concluded that the conventional fatigue design methodology is practically acceptable for tensile force combined with bending moment loading, albeit with results that are slightly conservative.

Keywords : Machine element, Bolt/nut assembly, Fatigue design, Stress area, Bending moment, Fatigue testing, 3D-FE stress analysis

1. Introduction

The bolt/nut assemblies in multi-bolted tensile joints are normally subjected to the combined effect of bending moment and tensile force due to the eccentricity of the bolt axis to the loading axis (Bickford, 1995). Fatigue design of the bolt/nut assembly in bolted joints is normally performed using the fatigue limit diagram (Haigh diagram) where the nominal tensile stress on the bolt thread root is used as a load. When bending moment also acts on bolt/nut assembly, nominal (maximum) bending stress ($\sigma_{ben}$) is added to the nominal tensile stress ($\sigma_{ten}$) by tension, with the assumption that the stress area of the thread has a circular cross-section (for example, VDI 2230 Blatt 1, 2014). On the other hand, the fatigue strength diagram of a bolt in bolt/nut assembly represents strictly tension-tension loading obtained by the fatigue test specified in ISO 3800, or by the fatigue strength of the bolt material and the fatigue notch factor. Ishibashi (1969) proposed the hypothesis that the fatigue notch factor can be determined from the local stress acting on the notch at depth $\varepsilon_o$ from the surface. This hypothesis was extended to bolt/nut assemblies under purely tensile loading by Yoshimoto (1983) who included the effect of residual stress, and later modified by Hagiwara et al. (1990). Many studies largely confirmed the validity of Ishibashi's and Yoshimoto’s hypothesis (Hagiwara and Kamiya, 2008; Hagiwara, et al., 2007; Furukawa, et al., 2012; Hagiwara, et al., 2013). The validity of the above-mentioned fatigue design methodology using nominal stress and the fatigue strength data under purely tensile loading was also partially confirmed experimentally (Hagiwara and Yoshimoto, 1987). However, the effect of the bending moment on the fatigue strength itself has not yet been considered because it is very difficult to exclude the effects of other factors such as
dispersion of clamp force in the bolted joint and geometrical variability of the bolt and nut. In addition, the validity of the methodology using the nominal stress area for fatigue design is somewhat flawed because the stress area specified in ISO 898-1 is a strictly theoretical one based on the conversion by formula of the tensile strength of the threaded portion.

This study aims to clarify the effect of the bending moment on the fatigue strength of bolt/nut assemblies and to propose more appropriate method to consider the effect of the bending moment on the fatigue strength. Fatigue tests were conducted for bolt/nut assembly subjected to fatigue loading with various bending stress ratios (nominal bending stress/ nominal total axial stress) using newly developed fatigue testing fixtures by which the bending stress ratio can be controlled accurately. Furthermore, 3D-FE analysis using newly developed 2-step zoomed models was performed to obtain the local (maximum) stress distribution on the bolt thread root with enough accuracy to apply Ishibashi’s hypothesis to the bolt/nut assembly subjected to tensile force and bending moment and to clarify the effect of the bending moment on the fatigue notch factor.

2. Nomenclature

\[ A^* : \text{Cross sectional area of bolt thread} \]
\[ A_{s,nom} : \text{Nominal stress area specified in ISO 898-1} \]
\[ d_2 : \text{Nominal pitch diameter of bolt thread} \]
\[ d_3 : \text{Nominal root diameter of bolt thread} \]
\[ F_b : \text{Tensile force acting on bolt/nut assembly} \]
\[ M_b : \text{Bending moment acting on the first thread of bolt in bolt/nut assembly} \]
\[ P : \text{Pitch of thread} \]
\[ R_b : \text{Bending moment ratio } \left[ = \frac{\sigma_{ben}}{\left( \sigma_{ten} + \sigma_{ben} \right)} \right] \]
\[ s : \text{Offset (eccentricity) of the bolt/nut assembly to the loading axis in fatigue test} \]
\[ Z_{s,nom} : \text{Nominal modulus of section assumed for threaded portion of bolt} \]
\[ Z^* : \text{Modulus of section of bolt thread cross section} \]
\[ \beta : \text{Fatigue notch factor} \]
\[ \varepsilon_o : \text{Depth of the layer from surface of notch} \]
\[ \sigma : \text{Nominal stress on bolt thread (in general)} \]
\[ \sigma^* : \text{Local stress on bolt thread root (in general)} \]
\[ \sigma_{sto} : \text{Fatigue strength expressed by nominal stress} \]
\[ \sigma_{sto}^* : \text{Fatigue strength expressed by local stress} \]
\[ \sigma_b : \text{Stress on bolt shank} \]
\[ \sigma_{ben} : \text{Nominal (maximum) bending stress on bolt thread by } M_b \]
\[ \sigma_{ten} : \text{Nominal tensile stress on bolt thread by } F_b \]
\[ \sigma_T : \text{True fracture stress of bolt material} \]
\[ \sigma_{wo} : \text{Fatigue limit of bolt material by rotating bar bending fatigue test} \]
\[ \sigma_{0.2} : \text{Proof stress or yield strength of bolt material} \]

3. Conventional Fatigue design methodology for bolt/nut assemblies

Figure 1 shows the typical procedure for fatigue design of bolted joints subjected to tensile fluctuating load (Hagiwara and Yoshimoto, 1987). To determine the load acting on bolt/nut assembly in Step I, the tensile force \( F_b \) and the bending moment \( M_b \) (including the effect of the transverse force) acting on the bolt/nut assembly via bearing faces of bolt and nut are obtained. Then the nominal stress acting on the thread root is calculated using the nominal stress area \( A_{s,nom} \) specified and the nominal modulus of section \( Z_{s,nom} \) as,

\[ \sigma = \sigma_{ten} + \sigma_{ben} = \frac{F_b}{A_{s,nom}} + \frac{M_b}{Z_{s,nom}} \]

where

\[ A_{s,nom} = \frac{\pi}{4} \left( \frac{d_2 + d_3}{2} \right)^2 \]

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For the determination of the fatigue strength of a bolt in bolt/nut assembly in Step II, there are two methods available – one by fatigue test and the other from the fatigue notch factor and the fatigue strength of the bolt material. The fatigue testing method is specified in ISO 3800 in which purely tensile (tension-tension) loading with various stress ratios (min/max) or conditions (e.g., $\sigma_{\text{m}} = \text{const.}$ is permitted). The fatigue notch factor of a bolt thread root is obtained using Ishibashi’s hypothesis in the latter case.

Figure 2 shows the concept to determine the fatigue notch factor. Ishibashi (1969) proposed that the fatigue notch factor $\beta$ is corresponding to the local stress at the depth $\varepsilon_0$ from the surface of the notch. Figure 2 (b) shows the fatigue limit diagram (Haigh diagram) drawn based on Ishibashi’s hypothesis expanded by Yoshimoto (1983) and modified by Hagiwara et al. (1990). In the fatigue limit diagram, the fatigue strength of notched specimen such as thread decreases because the diagram is drawn by the nominal stress. The value of $\varepsilon_0$ were estimated in the former studies to be approximately 30 $\mu$m (Hagiwara and Kamiya, 2008, Hagiwara, et al., 2007, Furukawa, et al., 2012). According to this concept, the fatigue strength corresponding to the actual loading in the bolt/nut assembly can be estimated theoretically. However, this analysis has not yet been performed because it requires local stress analysis using 3D-FE bolt/nut model with very fine mesh, a requirement that has not yet been adequately realized. Most previous studies estimate the stress concentration on the thread root using 2D axi-symmetric model due to the constraint of the number of elements for a 3D model. Therefore, in section 5 below, 3D-FE analysis for a bolt/nut assembly is performed using newly developed...
2-step zoomed models that includes the results obtained by fatigue testing performed in section 4.

For the comparison of the load and the strength in Step III in Fig. 1, it is convenient to use the fatigue limit diagram shown in Fig. 2 (b) to consider the effect of the loading condition on the bolt/nut assembly. The clamp force (preload) of a bolt/nut assembly in bolted joints normally gives the minimum stress $\sigma_{\text{min}} = \sigma_m - \sigma_a$.

4. Fatigue tests
4.1 Development of fatigue testing fixture to control bending stress ratio

Figure 3 shows the testing fixtures for the fatigue test of bolt/nut assembly. The newly developed testing fixture shown in Fig. 3 (b) is designed for a conventional 1-axis fatigue testing machine that is not subjected to any harmful effects of reaction force or bending moment on the machine. The bending stress ratio $R_B = \sigma_{\text{ben}} / (\sigma_{\text{ben}} + \sigma_{\text{ten}})$ can be set by selecting the appropriate offset value (location of clearance holes). The load sensor is concentrically placed to control the preload of two of bolt/nut assemblies that are intended to align the upper and lower axes of test frame. The preload in each bolt is minimal ($F = 0.2 \text{ kN}$) making it possible to remove the sensor when the minimum load $2W_{\text{min}}$ is applied during the fatigue test. This type of testing fixture is called “open joint”, and the clamp force does not affect the load (stress) in bolt/nut assembly, rather, it can be controlled to a high degree of precision by the loading condition $(2W)$ of testing machine.

Figure 4 shows the relationship between the tensile force $W$ acting on a bolt and the stress $\sigma$, acting on the bolt shank by the tensile force and the bending moment. In the 3D-FE analysis, SBN (solid bolt/nut) model is used instead of bolt/nut assembly model (Hagiwara and Kawamura, 2018). SBN model has the same axial and flexural rigidities as the actual bolt/nut assembly, and can be made by very small quantity of elements. The result clearly shows that the load on the bolt/nut assembly to be tested can be determined precisely by the 3D-FE analysis with SBN model.

Figure 5 shows the method to control the loading condition during the test. The preload $F$ does not affect the force-stress relationship after the separation at the contact plane [Fig. 5 (a)], and there is a linear relationship between the applied tensile force $W$ and the nominal axial stress $\sigma = \sigma_{\text{ben}} + \sigma_{\text{ten}}$. From Fig. 5 (b), the bending stress ratios are determined to be $R_B = 0.17$ for $s = 51 \text{ mm}$, and $R_B = 0.38$ for $s = 79 \text{ mm}$.

4.2 Testing conditions

The fatigue tests were carried out on M10, PC (property class) 8.8 bolt mated with M10, PC10, style 1 nut. The loading condition of $R_B = W_{\text{min}} / W_{\text{max}} = 0.1$ was selected to avoid the local plastic region [S to P in Fig. 2 (b)] and to...
Fig. 4 Relationship between tensile force \( W \) and the stress \( \sigma_b \) acting on the bolt shank. The maximum stress \( \sigma_{b\text{max}} \) is observed at the inner edge of the bolt shank (loading axis side), and the minimum stress \( \sigma_{b\text{min}} \) on the outer edge due to the bending moment additionally applied on the bolt/nut assembly. The calculation by 3D-FEM with SBN model coincides accurately with measurements using strain gauges.

Fig. 5 Control of the nominal stress on the first thread root of a bolt. The results show that (a) the preload on the load sensor does not affect the \( W-\sigma_b \) relationship after separation (termination of contact) at the bearing face of the sensor, and (b) the tensile force \( W \) for respective nominal stress applied during the test can easily be determined.

exclude the instability of testing at \( W=0 \). The test is in accordance with the staircase method in the combined test method specified in ISO 3800, which is the same method described in JSME 14 S-N fatigue test method.

By using the testing fixtures shown in Fig. 3, we can test the bolt/nut assemblies with the bending stress ratios \( R_B=0 \)
by the conventional one [Fig. 3 (a)], and \( R_B = 0.17 \) and 0.38 by the newly developed one [Fig. 3 (b)], respectively. With this newly developed testing fixture, the angular position of test nut is controlled such that 1/2 \( P \) height of nut thread at the bearing face side is oriented in the direction of the loading axis [see Fig. 9 (b)].

### 4.3 Results and considerations

Figure 6 shows the results of the fatigue tests by the staircase method. The results show that as the bending stress ratio \( R_B \) increases, so does the virtual fatigue strength \( \sigma_{AV} \), expressed by nominal stress using the specified nominal stress area \( A_{s\text{nom}} \). There are two possible causes that explain such differences in the fatigue strength. One is the inadequacy of the assumption for the stress area to calculate the nominal stress, the other is the decrease of the fatigue notch factor or stress concentration factor itself by bending moment loading.

Figure 7 shows the cross section of bolt thread perpendicular to the axis. The actual thread cross section shown in Fig. 7 (a) can be obtained by the formula proposed by Fukuoka and Nomura (2006a). The bigger modulus of section of the actual thread generates a smaller bending stress on the thread root. However, the difference of the fatigue strength

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Fig. 6 Results of fatigue tests conducted. The fatigue strengths were obtained by staircase method with 6 samples in accordance with the combined test method specified in ISO 3800 with cut-off number of cycles \( N_G = 5 \times 10^6 \). The fatigue strength expressed by nominal stress increases with the increase of the bending stress ratio \( R_B \).

Fig. 7 Shape and dimensions of the cross section of bolt thread perpendicular to thread axis; (a) for actual one and (b) for assumed stress area specified in ISO 898-1. The actual thread cross section has bigger area and modulus of section. However, these values would be smaller considering the fundamental deviations of the actual thread with 6g thread tolerance.
cannot fully be explained from the difference of cross section since both the modulus of section and sectional area become smaller in the actual thread considering the fundamental deviations. Furthermore, the effects of the crest side of thread and of the eccentricity of thread axis on the stress distribution are still unclear. Therefore, 3D-FE stress analysis is necessary to clarify the effect of bending moment on the fatigue strength of bolt/nut assembly as shown in Fig. 6.

5. Analytical considerations of the fatigue strength by the local stress

5.1 3D-FE models for zooming method

Fukuoka et al. (2006b) proposed a method to make 3D-FE model for a bolt/nut assembly and conducted FE analysis for a single bolted-joint subjected to external tensile force. They found that the maximum stress acting on the bolt thread root is at the height of 0.5P from the bearing surface of nut (i.e., the bolt thread root mated with the first full thread of nut). However, the countersink or chamfer at the bearing face of nut was not modeled. Furthermore, experiences of axi-symmetric FE analysis suggest that the mesh was too coarse to quantify the stress distribution on the thread root. Therefore, 3D-FE zoomed models are introduced into this study to quantitatively evaluate the stress distribution for individual testing condition.

Figure 8 shows the models for 3D-FE stress analysis by zooming method. As the first step, the bolt/nut assembly model shown in Fig. 8 (a) was made using the method proposed by Fukuoka et al. (2006b) to obtain the macroscopic deformation of bolt threads, in which the external loads $F_b$ and $M_b$ were determined from the analysis using overall model shown in Fig. 4 (a). The coefficients of friction between mating threads and between bearing faces are both assumed to be 0.2. Then thread length corresponding height 2P of the bolt was sectioned in the vicinity of the first thread root, then remeshed using finer elements. The elements along the thread root were further divided into several “truncated pyramid shape” elements. The displacements of corresponding nodes on the sectioned surfaces and the contact points on the mating thread in Fig. 8 (a) are used as the boundary condition of this zoomed model with interpolated displacements for the additional nodes concerned. The forces on the nodes are inadequate to be used as boundary condition because the procedure of redistribution of forces to the additional nodes is very complex.

![3D-FE models for stress analysis on bolt thread root.](image)

Fig. 8 3D-FE models for stress analysis on bolt thread root. The tensile force $F_b$ and the bending moment $M_b$ acting on the bolt/nut assembly model (a) can be obtained by using the model shown in Fig. 4 (a). The zoomed model (b) sectioned from bolt/nut assembly model has a finer mesh, especially in the area along the thread root. The displacements of nodes on the sectioned surfaces and the contact points are used as the boundary condition for the zoomed model.

5.2 Local stress on the thread root and corresponding fatigue strength

Figure 9 shows the stress distributions on the bolt thread root. For purely tensile loading ($R_b = 0$), the maximum local stress was expected to be observed on the first thread root of the bolt mated with nut thread at 1P height from the bearing face. In this case, the location of maximum stress was shifted to 1.125P height due to the existence of a countersink that decreases the rigidity of nut thread. For smaller bending stress ratio ($R_b = 0.17$) the location of maximum stress is further shifted toward the position where maximum nominal bending stress is applied. For larger
bending stress ratio \( (R_B =0.38) \), the maximum stress acting on the bolt thread root is nearest to the loading axis. This finding suggests that the magnitude of the maximum local stress is affected by the nut position against the loading axis (see 5.3).

The maximum local stress induced by the same nominal stress (200 MPa in this case) clearly decreases with the increase of the bending stress ratio \( R_B \). Assuming that \( \epsilon_0 \) in Ishibashi’s hypothesis [see Fig. 2 (a)] is 30 \( \mu \)m, we can estimate the fatigue notch factor from the local stress distribution shown in Fig. 9 (a) and the nominal stress.

Table 1 shows the fatigue strength \( \sigma_{AN^*} \) derived from the fatigue strength \( \sigma_{AN} \) expressed by the nominal stress and the fatigue notch factor \( \beta \) obtained. The fatigue strength \( \sigma_{AN^*} \) exhibits no significant difference by the bending stress ratio \( R_B \). This finding indicates that the virtual increase of the fatigue strength \( \sigma_{AN} \) shown in Fig. 6 can be explained by the decrease of the stress concentration and of the fatigue notch factor for higher bending stress ratio as shown in Fig. 9 (a).

### 5.3 Effect of the location of incomplete thread of nut

As pointed out in 5.2, the results shown in Fig. 9 suggest that the location of nut against the loading axis affects the magnitude of the local stress.

![Fig. 9 Maximum local stress distribution on the bolt thread root in relation to the bending stress ratio \( R_B \). The higher the bending stress ratio, the smaller the maximum local stress on the bolt thread root. The maximum stress is applied in the bolt thread root mated with nut thread(s) of 1.125P height from the bearing face for \( R_B =0 \) (green dot). This location of maximum stress shifts toward the position where nominal maximum bending stress is applied with increasing bending stress ratio \( R_B \) (blue and pink dots).](image)

**Table 1** Comparison of fatigue strength \( \sigma_{AN^*} \) expressed by local stress based on Ishibashi’s hypothesis. The fatigue notch factor \( \beta \) can be obtained from the results shown in Fig. 9 (a) on the assumption the depth \( \epsilon_0 \) is equal to 30 \( \mu \)m.

| \( R_B \) | \( \sigma_{AN} \) [MPa] | \( \beta \) | \( \sigma_{AN^*} = \beta \cdot \sigma_{AN} \) [MPa] |
|----------|-----------------|-------|-----------------|
| 0        | 83.9 (–)        | 3.39  | 284.4 (–)       |
| 0.17     | 96.5 (+15%)     | 2.88  | 277.9 (–2.3%)   |
| 0.38     | 113.3 (+35%)    | 2.67  | 302.5 (+6.4%)   |
Figure 10 shows the maximum local stress at $\varepsilon_o = 30 \, \mu m$ in relation to the nut position. The maximum stresses for position 1/2 $P$ at which the fatigue tests took place for $R_B = 0.17$ and 0.38 represent lowest values. On the other hand, for position 1/4 $P$, the magnitude of the maximum stress becomes highest. The fatigue notch factors of this condition are $\beta = 3.27$ for $R_B = 0.17$ and $\beta = 3.07$ for $R_B = 0.38$. These values are slightly smaller than $\beta = 3.39$ for $R_B = 0$. However, the differences are not significant because there are many factors other than $\beta$ which may influence the fatigue strength of bolt/nut assembly. Therefore, the conventional design methodology using the nominal stress on the nominal stress area $A_{s, \text{nom}}$ is practically acceptable, albeit the results are slightly conservative.

6. Conclusions

The main conclusions obtained in this study are summarized as follows:

1. The fatigue tests for bolt/nut assemblies were conducted with various bending stress ratio using a newly developed testing fixture.
2. With increasing bending stress ratio, the fatigue strength of a bolt expressed by the nominal stress on the imaginary circular cross section having the sectional area $A_{s, \text{nom}}$ also increases.
3. This virtual increase of the fatigue strength can be explained by the decrease of the fatigue notch factor derived from 3-D FE analysis using 2-step zooming models for a bolt/nut assembly under tensile force combined with bending moment loading.
4. The maximum local stress on the bolt thread root is affected by the rigidity of nut thread at the bearing face side which is determined from nut thread height and the specifications for the countersink. Considering the effect of the position of nut against the loading axis on the magnitude of the local stress, the conventional method using the nominal stress based on the specified nominal stress area $A_{s, \text{nom}}$ appears to be acceptable as a fatigue design methodology.
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