New Representation for the 2-to-4 Gluon Vertex in High Energy QCD *

Carlo Ewerz

Cavendish Laboratory, Cambridge University
Madingley Road, Cambridge CB3 0HE, UK

and

DAMTP, Cambridge University
Silver Street, Cambridge CB3 9EW, UK

email: carlo@hep.phy.cam.ac.uk

Abstract

A new representation for the two-to-four gluon vertex arising in the context of unitarity corrections is derived which involves only BFKL kernels. We discuss possible implications of this representation, including the possibility of finding the NLO corrections to the vertex.

*Work supported in part by the EU Fourth Framework Programme ‘Training and Mobility of Researchers’, Network ‘Quantum Chromodynamics and the Deep Structure of Elementary Particles’, contract FMRX-CT98-0194 (DG 12 - MIHT), and by the German Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie.
1 Introduction

The scattering of small color dipoles is an important process which can be studied in perturbative QCD. At large energy, the leading logarithms of the energy can be resummed resulting in the BFKL equation \[1\]. It describes the \(t\)-channel exchange of two gluons which are interacting via the so-called BFKL kernel. With increasing energy the parton densities become larger and parton recombination effects become important. In this situation subleading corrections to the BFKL equation have to be taken into account. These so-called unitarity corrections have recently been studied in the perturbative framework \[2, 3, 4\]. They are characterized by exchanges with more than two reggeized gluons in the \(t\)-channel. Consequently, the quantities of interest in this context are amplitudes describing the production of \(n\) gluons in the \(t\)-channel. Those \(n\)-gluon amplitudes have been found to exhibit interesting properties.

The two most important properties of the \(n\)-gluon amplitudes are the field theory structure in these amplitudes and their conformal invariance in impact parameter space. The amplitudes with up to four gluons have been investigated in \[2\]. The three–gluon amplitude was shown to be a superposition of two–gluon (BFKL) amplitudes. The same is true for a part of the four–gluon amplitude, the so–called reggeizing part. The \(t\)-channel evolution of the other part of the four–gluon amplitude starts with a two–gluon state coupled to external particles. At some point during the \(t\)-channel evolution this two–gluon state undergoes a transition to a four–gluon state. The coupling of these two states is mediated by the two–to–four gluon vertex \(V_{2\to4}\). The vertex turns the set of quantum mechanical \(n\)-gluon states into a field theory. This field theory structure was shown to be present also in the five– and six–gluon amplitudes \[4\], indicating that the whole set of unitarity corrections can be formulated as an effective field theory. Also in those amplitudes the two–to–four vertex plays a prominent role. Further, the vertex \(V_{2\to4}\) was shown to be conformally invariant in two–dimensional impact parameter space \[3\]. Together with the conformal invariance of the \(n\)-gluon states \[5\] this suggests that the effective field theory will be a conformal field theory. Recently, also the five– and six–gluon amplitudes have been shown to be conformally invariant \[3\].

The two–to–four vertex is the simplest number–changing element and thus a key element for the understanding of the effective field theory. It is therefore important to study its properties and to gain deeper insight into its structure. In this letter we give a new representation of the vertex which we expect to be helpful in this respect. Surprisingly, it is possible to write the two–to–four vertex in terms of full BFKL kernels only. In other words, it can be expressed through simpler and well–understood elements. We are thus able to establish a relation between the vertex and the BFKL kernel. Although the new representation is rather long, it might be quite useful for the analysis of the vertex and its properties. We hope that it is also helpful for the comparison with other approaches to the
unitarization problem like the dipole picture [7] or the operator expansion [8]. As we will discuss, it might also allow one to find the NLO corrections to the two–to–four gluon vertex.

The paper is organized as follows. In section 2 we recall some facts about the BFKL kernel and the two–to–four gluon vertex. In section 3 we display and explain the new representation for this vertex. We also give a similar representation for another useful function arising in the analysis of unitarity corrections. Section 4 contains a discussion of the potential uses of the new representation and we close with a short summary.

2 The BFKL kernel and the two-to-four gluon vertex

The perturbative Pomeron is given by a two–gluon amplitude $D_2$ solving the BFKL equation

$$\omega D_2(\bm{k}_1, \bm{k}_2) = D_{(2;0)}(\bm{k}_1, \bm{k}_2) + \int \frac{d^2l}{(2\pi)^2} \frac{1}{l^2(q-l)^2} K_{\text{BFKL}}(l, q-l; \bm{k}_1, \bm{k}_2) D_2(l, q-l),$$

where all momenta are two–dimensional transverse momenta, $q = \bm{k}_1 + \bm{k}_2$ is the total momentum transfered in the $t$-channel, and $\omega$ is the complex angular momentum. $D_{(2;0)}(\bm{k}_1, \bm{k}_2)$ is an inhomogeneous term describing the coupling of the BFKL Pomeron to external particles, for example to virtual photons via a quark loop. The integral kernel in (1) is the BFKL kernel

$$K_{\text{BFKL}}(l, q-l; \bm{k}, q-k) = -N_c g^2 \left[ q^2 - \frac{k^2(q-l)^2}{(k-l)^2} - \frac{(q-k)^2l^2}{(k-l)^2} \right]$$

$$+ (2\pi)^3 k^2(q-k)^2 \left[ \beta(k) + \beta(q-k) \right] \delta(k-l).$$

The coupling constant $\alpha_s = g^2/(4\pi)$ is kept fixed, and the function $\beta$ is given by

$$\beta(k^2) = \frac{N_c}{2} g^2 \int \frac{d^2l}{(2\pi)^2} \frac{k^2}{l^2(k-l)^2}.$$

The amplitudes $D_n$ describing the production of $n$ gluons in the $t$-channel are described by a set of coupled integral equations (for a review see [9]). The four–gluon amplitude for example satisfies

$$\left( \omega - \sum_{i=1}^{4} \beta(k_i) \right) D_4 = D_{(4;0)} + K_{2\to4} \otimes D_2 + \sum K_{2\to3} \otimes D_3 + \sum K_{2\to2} \otimes D_4$$

where we have for simplicity suppressed all color indices. The convolution involves an integration like the one in (1), and the kernel $K_{2\to2}$ is essentially given by the
first square bracket in (4). The kernels $K_{2-m}$ have been derived in [3]. For a
detailed description of the integral equation and the kernels we refer the reader to [3]. The three–gluon amplitude $D_3$ can be shown to reggeize, i.e. to be a
superposition of two–gluon amplitudes. Then starting from eq. (4), the four–
gluon amplitude can be shown to consist of two parts, $D_4 = D_4^R + D_4^I$, the first of
which (the reggeizing part) is a superposition of two–gluon amplitudes $D_2$. The
second part, $D_4^I$, has the structure $D_4^I = G_4 \cdot V_{2 \rightarrow 4} \cdot D_2$, where $G_4$ is the Green’s
function of the four–gluon state. The vertex $V_{2 \rightarrow 4}$ thus couples the four–gluon to
the two–gluon state. Its color structure is

$$V_{2 \rightarrow 4}^{a_1a_2a_3a_4}(k_1, k_2, k_3, k_4) = \delta_{a_1a_2} \delta_{a_3a_4} V(k_1, k_2; k_3, k_4) + \delta_{a_1a_3} \delta_{a_2a_4} V(k_1, k_3; k_2, k_4) + \delta_{a_1a_4} \delta_{a_2a_3} V(k_1, k_4; k_2, k_3).$$ (5)

$V$ should be understood as an integral operator in momentum space, and when
acting on $D_2$ its detailed form as derived in [2] is

$$(VD_2)(k_1, k_2; k_3, k_4) = \frac{g^4}{4} \times$$

$$\times \left\{ 2 \left[ c(1234) - b(124, 3) - b(134, 2) - b(234, 1) + b(12, 34) + b(34, 12) + a(13, 2, 4) + a(14, 2, 3) + a(23, 1, 4) + a(24, 1, 3) - a(1, 2, 34) - a(2, 1, 34) - a(3, 12, 4) - a(4, 12, 3) \right]$$

$$+ [t(123, 4) + t(124, 3) + t(134, 2) + t(234, 1) - t(12, 34) - t(34, 12) - s(13, 2, 4) - s(13, 4, 2) - s(14, 2, 3) - s(14, 3, 2) - s(23, 1, 4) - s(23, 4, 1) - s(24, 1, 3) - s(24, 3, 1) + s(1, 2, 34) + s(1, 34, 2) + s(2, 1, 34) + s(2, 34, 1) + s(3, 12, 4) + s(3, 4, 12) + s(4, 12, 3) + s(4, 2, 12) \right\},$$ (6)

where the numbers in the arguments stand for the indices of the corresponding
momenta $k_i$ and a string of numbers corresponds to the sum of the momenta.
The functions in this expression are

$$a(k_1, k_2, k_3) = \int \frac{d^3l}{(2\pi)^3} \frac{k_1^2}{[1 - k_2^2][1 - (k_1 + k_2)^2]} D_2 \left( 1, \sum_{j=1}^{3} k_j - 1 \right),$$ (7)

$$b(k_1, k_2) = a(k_1, k_2, k_3 = 0),$$ (8)

$$c(k_1) = b(k_1, k_2 = 0),$$ (9)

$$s(k_1, k_2, k_3) = \frac{2}{N_c g^2} \beta(k_1) D_2(k_1 + k_2, k_3),$$ (10)

$$t(k_1, k_2) = s(k_1, k_3 = 0, k_2)$$ (11)

The first three of these correspond to real gluon emission, the last two describe
virtual corrections.
In \cite{10} it was pointed out that the function $G$ introduced in \cite{2},

\[
G(k_1, k_2, k_3) = \frac{g^2}{2} \left[ 2c(123) - 2b(12, 3) - 2b(23, 1) + 2a(2, 1, 3) \\
+ t(12, 3) + t(23, 1) - s(2, 1, 3) - s(2, 3, 1) \right],
\]

is a very useful tool for studying the vertex $V_{2\rightarrow4}$ because the vertex function $V$ can be represented as a superposition of $G$-functions,

\[
(VD_2)(k_1, k_2; k_3, k_4) = \frac{g^2}{2} \left[ G(1, 23, 4) + G(2, 13, 4) + G(1, 24, 3) + G(2, 14, 3) \\
- G(12, 3, 4) - G(12, 4, 3) - G(1, 2, 34) - G(2, 1, 34) \\
+ G(12, -, 34) \right].
\]

Already the simpler function $G$ is conformally invariant and infrared finite by itself. However, it does not vanish when its second argument vanishes, whereas the vertex function $V$ does have this property for all of its arguments. The function $G$ does not occur as an isolated object in the analysis of the integral equations, and only combinations similar to (13) are found \cite{3}. We therefore consider the function $G$ to be of less fundamental significance than the full vertex function $V$. Nevertheless, it is a convenient object for computational purposes, for example for discussing the conformal invariance of the unitarity corrections \cite{10, 11, 6}.

**3 New representation for the vertex $V_{2\rightarrow4}$**

We now give a representation of the vertex function $V$ — and thus of the full vertex $V_{2\rightarrow4}$ — which involves only BFKL kernels and free propagators $(1/k^2)$.

Let us outline how the new representation can be derived by reconsidering the usual derivation of $V$ (see \cite{2, 4}). The basic idea is to express the kernels $K_{2\rightarrow m}$ with $m = 3, 4$ in the integral equation (4) in terms of two–to–two kernels $K_{2\rightarrow2}$. (This was done for the two–to–three kernel $K_{2\rightarrow3}$ also in \cite{11, 12}.) Then the color tensors in the integral equation can be shown to arrange in such a way that the kernel $K_{2\rightarrow2}$ can be replaced in these expressions by the full BFKL kernel by adding and subtracting appropriate trajectory functions $\beta$.

To display the formula, we first define $\mathcal{K}$ to be the product of a full BFKL kernel with the two propagators entering from above,

\[
\mathcal{K}(q_1, q_2; k_1, k_2) = \frac{1}{N_c g^2 q_1^2 q_2^2} K_{\text{BFKL}}(q_1, q_2; k_1, k_2),
\]

\footnote{In those references also the two–to–four kernel $K_{2\rightarrow4}$ was expressed in terms of the kernel $K_{2\rightarrow2}$. The identity given there is not suited for deriving a new representation of the full vertex $V_{2\rightarrow4}$ in terms of full BFKL kernels as we give it here. We have found a different way to express $K_{2\rightarrow4}$ in terms of $K_{2\rightarrow2}$ which can be seen in the last square brackets of equation (15) below.}
where (cf. (3)) the kernel $K_{\text{BFKL}}$ includes the trajectory functions $\beta$. The vertex function $V$ (see (3)) can then be written as

$$
(V D_2)(k_1, k_2; k_3, k_4) = -\frac{g^4}{4(2\pi)^3} \int \left( \prod_{i=1}^{4} d^2 l_i \right) \delta \left( \sum_{j=1}^{4} l_j - \sum_{j=1}^{4} k_j \right) \times
$$

$$
\times \left\{ -[D_2(l_1 + l_2 + l_3, l_4) + D_2(l_1 + l_2 + l_3 + l_4) + D_2(l_1 + l_3 + l_4)] 
+ D_2(l_1, l_2 + l_3 + l_4) - D_2(l_1 + l_2, l_3 + l_4) - D_2(l_1 + l_3, l_2 + l_4) 
- D_2(l_1 + l_2, l_2 + l_3 + l_4)] 
\times [K(l_1, l_2; k_1, k_2)\delta(l_3 - k_3)\delta(l_4 - k_4) 
+ K(l_1, l_3; k_1, k_3)\delta(l_2 - k_4)\delta(l_1 - k_2) 
+ K(l_2, l_3; k_2, k_3)\delta(l_1 - k_1)\delta(l_2 - k_3) 
+ K(l_2, l_4; k_2, k_4)\delta(l_1 - k_1)\delta(l_3 - k_4)] 
+ D_2(l_1 + l_2 + l_4, l_3) - D_2(l_1 + l_2, l_3 + l_4) - D_2(l_1 + l_3, l_2 + l_4) 
- D_2(l_1 + l_3, l_2 + l_4)] 
\times [K(l_1, l_4; k_1, k_4)\delta(l_2 - k_2)\delta(l_3 - k_3) 
+ K(l_2, l_3; k_2, k_3)\delta(l_1 - k_1)\delta(l_4 - k_4) \right\}
$$

$$
+ \frac{g^4}{4(2\pi)^3} \int \left( \prod_{i=1}^{3} d^2 l_i \right) \delta \left( \sum_{j=1}^{3} l_j - \sum_{j=1}^{4} k_j \right) \times
$$

$$
\times [D_2(l_1 + l_2, l_3) - D_2(l_1 + l_3, l_2) + D_2(l_1, l_2 + l_3)] 
\times \left\{ [K(l_1, l_2; k_1 + k_2, k_3) - K(l_1 - k_1, l_2; k_2, k_3)]\delta(l_3 - k_4) 
- [K(l_1, l_3; k_1 + k_2, k_4) - K(l_1 - k_1, l_3; k_2, k_4)]\delta(l_2 - k_3) 
- [K(l_1, l_3; k_1, k_3 + k_4) - K(l_1 - k_1, l_3 - k_3; k_2, k_3)]\delta(l_2 - k_2) 
+ [K(l_2, l_3; k_2, k_3 + k_4) - K(l_2, l_3 - k_4; k_2, k_3)]\delta(l_1 - k_1) \right\}
$$

$$
+ \frac{g^4}{2(2\pi)^3} \int \left( \prod_{i=1}^{2} d^2 l_i \right) \delta \left( \sum_{j=1}^{2} l_j - \sum_{j=1}^{4} k_j \right) D_2(l_1, l_2) \times
$$

$$
\times [K(l_1, l_2; k_1 + k_2, k_3 + k_4) - K(l_1 - k_1, l_2; k_2, k_3 + k_4) 
- K(l_1, l_2 - k_4; k_1 + k_2, k_3) + K(l_1 - k_1, l_2 - k_4; k_2, k_3)]
$$

(15)

By introducing further $\delta$-functions for the arguments of the $D_2$'s one can easily isolate the vertex $V$ as an integral operator acting on $D_2$. The above representation can be shown to be equivalent to eq. (3) by a somewhat tedious but straightforward computation.

Since eq. (15) is rather complicated we try to make it more transparent by using a diagrammatic notation. We define a diagram for $K$, the BFKL kernel
including the propagators for the gluons entering from above,

\[ \mathcal{K}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{k}_1, \mathbf{k}_2) = \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{\mathbf{k}_1 \cdot \mathbf{k}_2}. \]  

Let us further introduce a pictorial notation for the momentum arguments of the BFKL amplitude \( D_2 \). We write

\[ D_2(l_1 + l_2, l_3 + l_4) = D_2(\; \; \; \; \; \; \;), \]

and the generalization of the notation to other combinations of the four momenta \( l_i \) is obvious. Now equation (15) can be rewritten as

\[ (V D_2)(\mathbf{k}_1, \mathbf{k}_2; \mathbf{k}_3, \mathbf{k}_4) = \]

\[ - \frac{g_4^4}{4} \left\{ - \left[ D_2(\; \; \; \; \; \; \;\; \; \; ) + D_2(\; \; \; \; \; \; \;\; \; \; ) + D_2(\; \; \; \; \; \; \;\; \; \; ) + D_2(\; \; \; \; \; \; \;\; \; \; ) \right] 
- D_2(\; \; \; \; \; \; \;\; \; \; ) - D_2(\; \; \; \; \; \; \;\; \; \; ) - D_2(\; \; \; \; \; \; \;\; \; \; ) \right] \times (||H|| + ||H||) \]

\[ + \left[ D_2(\; \; \; \; \; \; \;\; \; \; ) + D_2(\; \; \; \; \; \; \;\; \; \; ) - D_2(\; \; \; \; \; \; \;\; \; \; ) \right] \times (||H|| + ||H||) \]

\[ + \left[ D_2(\; \; \; \; \; \; \;\; \; \; ) + D_2(\; \; \; \; \; \; \;\; \; \; ) - D_2(\; \; \; \; \; \; \;\; \; \; ) - D_2(\; \; \; \; \; \; \;\; \; \; ) \right] \times 
\times (||H|| + ||H||) \} \]

\[ + \frac{g_4^4}{4} \left\{ [D_2(\; \; \; \; \; \; \;\; \; \; ) - D_2(\; \; \; \; \; \; \;\; \; \; )] \times 
\times (||H|| - ||H|| - ||H|| + ||H||) \right. 
\left. - ||H|| + ||H|| + ||H|| - ||H|| \right\} \]

\[ + \frac{g_4^4}{2} D_2(\; \; \; \; \; \; \;\; \; \; ) \times \left( ||H|| - ||H|| \right. - ||H|| + ||H|| \right) \]. \] \hspace{1cm} (18)

Here an integration with the weight \( 1/(2\pi)^3 \) over the loop momentum is implied as are the \( \delta \)-functions according to the gluon lines that are not involved in the interactions.

The different sums of \( D_2 \)'s in the new representation (18) are familiar objects: The combinations of the first integral, consisting of three and four \( D_2 \)'s which occur together with their sum (seven \( D_2 \)'s), are the momentum space factors corresponding to two different color structures of the reggeizing part \( D_4^R \) in the four–gluon amplitude. The combination of three \( D_2 \)'s in the second integral is the momentum part of the three–gluon amplitude \( D_3 \). Together with the above discussion of the kernels \( K_{2 \rightarrow m} \) this emphasizes that the new representation is closely related to the elements entering the original integral equation (14).
Interestingly, also the function $G$ (see eq. (12)) can be expressed through BFKL kernels,

$$G(k_1, k_2, k_3) = g_3 D_2 \left( l_1, l_2 \right) D_2 \left( l_1 + l_2, l_3 \right) \times$$

$$\times \left[ \mathcal{K}(l_1, l_2; k_1 + k_3, k_2) - \mathcal{K}(l_1, l_2; k_1, k_2 + k_3) - \mathcal{K}(l_1, l_2 - k_3; k_1, k_2) \right]$$

$$+ \frac{g_3^3}{2(2\pi)^3} \int \left( \prod_{i=1}^{3} d^2 l_i \right) \delta \left( \sum_{j=1}^{3} l_j - \sum_{j=1}^{3} k_j \right) \times$$

$$\times \left[ D_2(l_1 + l_2, l_3) - D_2(l_1 + l_2 + l_3) \right] \times$$

$$\times \left[ \mathcal{K}(l_1, l_2; k_1, k_2)\delta(l_3 - k_3) + \mathcal{K}(l_1, l_3; k_1, k_3)\delta(l_2 - k_2) + \mathcal{K}(l_2, l_3; k_2, k_3)\delta(l_1 - k_1) \right]. \quad (19)$$

To make this easier to read, we again write it using the graphical notation introduced above:

$$G(k_1, k_2, k_3) = g^3 D_2 \left( \mid \mid \right) \times \left( \mid \mid \mid \mid \right)$$

$$+ \frac{g^3}{2} \left[ D_2 \left( \mid \mid \right) - D_2 \left( \mid \mid \mid \right) + D_2 \left( \mid \mid \mid \right) \right] \times$$

$$\times \left( \mid \mid \mid + \mid \mid \mid + \mid \mid \mid \right). \quad (20)$$

Here we again find the specific combination of three $D_2$ amplitudes which is the momentum part of the three-gluon amplitude.

Concerning the relation of the two new representations for $V$ and $G$ we should mention that it is not possible to derive the representation (15) from (19) and (13) without decomposing the BFKL kernel according to (2).

4 Possible applications of the new representation

In the preceding section we have found a relation between the two–to–four vertex and the much simpler BFKL kernel. Now we turn to possible applications of this relation.

Firstly, the new representation is useful for the further investigation of the vertex itself. The emergence of conformal invariance in the complicated vertex seems much more natural in view of the fact that the vertex is build from conformally invariant BFKL kernels and free propagators. The new representation might also be useful for studying the interesting but difficult question how the vertex behaves under crossing, i.e. what its properties are under the exchange of one of the two incoming and one of the four outgoing gluons.
Unitarity corrections have also been studied in Mueller’s dipole picture \[7\]. Its relation to the field theory structure arising in the \(t\)-channel approach has not yet been clarified beyond the one–ladder (BFKL) approximation. We hope that a structure similar to our new representation of the vertex can also be identified in the dipole picture.

Recently, the NLO corrections to the BFKL kernel have been calculated \[13, 14\]. A natural question is whether it is possible to find also the NLO corrections to the other elements of the effective field theory of unitarity corrections, especially the corrections to the two–to–four vertex. On first sight this appears to be extremely difficult, since it would require to derive and solve in NLO the full integral equations describing the \(n\)-gluon amplitudes. On the other hand, our new representation shows that the vertex is closely related to the BFKL kernel. If this relation could be shown to hold in NLO as well, then it would be possible to obtain the NLO two–to–four gluon vertex simply from eq. (15) by replacing the LO BFKL kernel by its NLO modification

\[
\mathcal{K} \rightarrow \mathcal{K}^{\text{NLO}},
\]

as calculated in \[13\]. The crucial point is that it is most probably easier to prove the relation between the vertex and the BFKL kernel in NLO than to use the full machinery of the integral equations to compute the NLO vertex. We expect that the origin of the new representation lies in some kind of bootstrap relation for the kernels \(K_{2 \rightarrow n}\) in the integral equations together with the symmetry of the amplitudes in color space. Further, it would be necessary to prove the regularization of certain parts of the \(n\)-gluon amplitudes (i. e. their being superpositions of BFKL amplitudes) in NLO. These properties are very general features of QCD in the high energy limit and should presumably also hold in NLO, although a proof is still missing. In the absence of a full computation from first principles, the representation (13) together with the replacement (21) can at least be used as an educated guess for the NLO two–to–four gluon vertex. This conjectured form can then be tested in view of more general properties expected for transition vertices in the effective theory of unitarity corrections. The conjecture can also be applied to other object of interest in high energy QCD. The triple–Pomeron vertex, for example, can be obtained from the two–to–four vertex after projection onto three BFKL Pomeron states \[15, 16\]. Its NLO corrections can thus also be obtained from the NLO vertex.

Finally, we expect that a representation in terms of BFKL kernels can also be constructed for higher transition vertices like the two–to–six transition found in \[4\]. The BFKL kernel thus turns out to play an important role not only in the two–gluon state but in the whole set of unitarity corrections.
5 Summary

We have shown that the two–to–four gluon vertex can be expressed in terms of BFKL kernels, thus establishing the relation between these two elements of the effective field theory of unitarity corrections. The underlying principles of the effective field theory leading to this relation are still unknown. We hope that the new representation can help in finding and understanding them. We have indicated possible applications of the new representation which deserve further study, among them the possibility of finding the NLO corrections to the two–to–four gluon vertex.

Acknowledgements

I would like to thank Jochen Bartels, Peter Landshoff, and Bryan Webber for helpful discussions.

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