SO(10)-GUT Coherent Baryogenesis

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A model for GUT baryogenesis, coherent baryogenesis within the framework of supersymmetric SO(10), is considered. In particular, we discuss the Barr-Raby model, where at the end of hybrid inflation charge asymmetries can be created through the time-dependent higgsino-gaugino mixing mass matrix. These asymmetries are processed to Standard Model matter through decays via nonrenormalizable \((B-L)\)-violating operators. We find that a baryon asymmetry in accordance with observation can be generated. An appendix is devoted to provide useful formulas and concrete examples for calculations within SO(10).

1. INTRODUCTION

Grand Unified Theories (GUTs) generically predict a scalar potential and thereby a large amount of vacuum energy when the scalar fields are displaced from the minimum which breaks the symmetry down to the Standard Model. It is hence often argued that cosmic inflation may be implemented by the scalar field dynamics of a GUT \(1\). A well-known paradigm is supersymmetric (SUSY) hybrid inflation \(2, 3\), and it has led to various models using different grand unified gauge groups, see \(e.g.\) \(4, 5, 6\).

Another feature of GUTs, which is of possible relevance for cosmology, is obviously the violation of baryon minus lepton number, \(B-L\), due to the unification and mixing of baryons and leptons, since this can lead to mechanisms for generating the observed baryon asymmetry of the Universe (BAU). While leptogenesis is strictly speaking not necessarily implemented into a GUT – it can be operative within the Standard Model minimally extended by right handed neutrinos with Majorana mass terms – we suggested a scenario relying on baryon-lepton unification, coherent baryogenesis \(7, 8\), and implemented it within a Pati-Salam supersymmetric hybrid inflationary model. We give

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a brief review of this mechanism and our calculational formalism inspired by kinetic theory in
section 2.

Being a product group, the Pati-Salam gauge group \([9]\) gives, strictly speaking, not rise to a GUT, and it is therefore desirable to devise models based on the proper GUT SO(10). A way of breaking the SO(10)-symmetry, which is particularly suitable for SUSY hybrid inflation, is the mechanism proposed by Barr and Raby. Kyae and Shafi extended this model by global symmetries which restrict the superpotential to contain only couplings consistent with hybrid inflation \([6]\). This model has however a rather complicated Higgs sector.

The purpose of this paper is to show that coherent baryogenesis naturally occurs within a SUSY-SO(10)-framework. Therefore, we want to keep the discussion as simple as possible and use the minimal superpotential suggested by Barr and Raby, leaving aside the issues which Kyae and Shafi focus on. In turn, we also point out that we do not extend the minimal model by \textit{ad hoc} terms just in order to make our mechanism viable.

In section 3 we put together the features of the Barr-Raby model which are important for our baryogenesis scenario and present in some detail the derivation of the higgsino-gaugino mixing mass matrix. We also devote an appendix to the conventions and techniques we apply for calculations within SO(10), with the intention to make this article self-contained and easily comprehensible to the reader who is not familiar with SO(10)-model building, and furthermore we want to provide a useful help for accessing the papers mentioned above.

Putting our considerations into work, we present a numerical study of SO(10)-coherent baryogenesis in section 4. There, we also discuss the decay processes of the higgsinos \textit{via} nonrenormalizable, \((B - L)\)-violating couplings. The result is an estimate of the produced baryon asymmetry for a particular set of parameters.

2. COHERENT BARYOGENESIS

Coherent baryogenesis relies on the production of particles due to a time dependent mass term, a phenomenon which we refer to as preheating. Preheating has been extensively studied for scalars \([10, 11, 12, 13]\) and for fermions \([14, 15, 16]\), using the technique of Bogolyubov transformations. A special feature of fermionic preheating is that fermions can be amply produced when their mass term crosses zero. When one considers instead of a mass term a mixing mass matrix, charge \(C\) and parity \(P\) may be violated and an asymmetry can be stored within different fermionic species. A nonsymmetric mass matrix however leads to the violation of the orthogonality of par-
particle and antiparticle modes. One therefore needs a formalism which is independent of a basis in terms of particle and antiparticle creation and annihilation operators and thereby generalizes the Bogolyubov transformation approach. This is developed in Ref. [17] and shall be briefly reviewed in the following.

We consider several fermionic flavours, mixing through a mass matrix $M(\eta)$, which is a function of the conformal time $\eta$. In a spatially flat Friedmann-Lemaître-Robertson-Walker Universe, described by the metric $g_{\mu\nu} = a^2(\eta) \times \text{diag}(1, -1, -1, -1)$, we rescale the fields such that for the mass terms, there is the replacement $M(\eta) \rightarrow a(\eta) M(\eta)$.

Our goal is to compute the charge density, which is a bilinear form in the fermionic fields. We therefore introduce the Wigner function

$$i S^<_ij(k, x) = - \int d^4r \text{e}^{ik\cdot r} \langle \bar{\psi}_j(x-r/2) \psi_i(x+r/2) \rangle,$$

where $i, j$ are flavour indices and $(i\gamma^0 S^<_j)^\dagger = i\gamma^0 S^<_i$ is hermitean. The Wigner transform is the Fourier transformation of the two point function w.r.t. its relative coordinate while keeping the center of mass coordinate fixed. One can hence consider it as an analogue to a classical phase space density defined in quantum theory. Since we assume here spatial homogeneous conditions, one can ignore the center of mass coordinate in the following and consider $i S^<_j$ as a Fourier transform. Our formalism is applicable for a 2-point function with general density matrix, but in view of our applications in inflation we prefer to write it with respect to the vacuum from the outset.

When decomposing the mass matrix $M$ into its hermitean and antihermitean parts,

$$M_H = \frac{1}{2} \left( M + M^\dagger \right), \quad M_A = \frac{1}{2i} \left( M - M^\dagger \right),$$

we find that $i S^<_j$ obeys the Wigner space Dirac equation

$$\left( \# + \frac{i}{2} \gamma^5 \partial_\eta - (M_H + i\gamma^5 M_A) \text{e}^{-\frac{i}{2} \gamma^5 \partial_\eta} \right)_{ij} i S^<_j = 0.$$  

The mass matrix $M$ emerges generically from Yukawa couplings to scalar field condensates, $\mathcal{L}_{\text{Yu}} = -y\phi \bar{\psi}_R \psi_L + \text{h.c.}$. In the model we consider here, $M$ is the higgsino-gaugino mixing mass matrix.

A crucial point is the time-dependence of $M$, which is not only the source of particle production. The matrices $M$ and $dM/d\eta$ both contribute to $\text{CP}$-violating phases, which – provided $M$ and $dM/d\eta$ are linearly independent – can not be removed by time-independent redefinitions of the fermionic fields.
In order to simplify the Wigner-Dirac equation \([2]\), which is, besides the flavour indices, also endowed with a \(4 \times 4\) spinor structure, we make use of the fact that for spatially homogeneous \(i \gamma_0 S_h^<\), the helicity operator \(\hat{h} = \hat{k} \cdot \gamma^0 \gamma^5\) commutes with the Dirac operator in \([2]\) and decompose the Wigner function as \([18, 19]\)

\[-i \gamma_0 S_h^< = \frac{1}{4} (1 + \hat{k} \cdot \sigma) \otimes \rho^\mu g_{\mu h},\]  

where we have omitted the flavour indices, \(\hat{k} = k/|k|\) and \(\sigma^\mu, \rho^\mu (\mu = 0, 1, 2, 3)\) are the Pauli matrices, and \(h = \pm 1\) are the eigenvalues of \(\hat{h}\). We multiply \([2]\) by \(\rho^\mu\), take the Dirac trace and integrate the hermitean part over \(k_0\). Introducing the 0th momenta of \(g_{\mu h}\), \(f_{\mu h} = \int (dk_0/2\pi) g_{\mu h}\), we note that the functions \(f_{ij}^{\mu h}\) explicitly read

\[
\begin{align*}
  f_{0h}^{ij}(x, k) &= -\int \frac{dk_0}{2\pi} \int d^4r e^{ik \cdot r} \langle \bar{\psi}_{hj}(x-r/2) \gamma^0 \psi_{hi}(x+r/2) \rangle, \\
  f_{1h}^{ij}(x, k) &= -\int \frac{dk_0}{2\pi} \int d^4r e^{ik \cdot r} \langle \bar{\psi}_{hj}(x-r/2) \gamma^5 \psi_{hi}(x+r/2) \rangle, \\
  f_{2h}^{ij}(x, k) &= -\int \frac{dk_0}{2\pi} \int d^4r e^{ik \cdot r} \langle \bar{\psi}_{hj}(x-r/2) (-i \gamma^5) \psi_{hi}(x+r/2) \rangle, \\
  f_{3h}^{ij}(x, k) &= -\int \frac{dk_0}{2\pi} \int d^4r e^{ik \cdot r} \langle \bar{\psi}_{hj}(x-r/2) \gamma^0 \gamma^5 \psi_{hi}(x+r/2) \rangle.
\end{align*}
\]

Therefore, the \(f_{\mu h}(x, k)\) can be interpreted as follows: \(f_{0h}\) is the charge density, \(f_{3h}\) is the axial charge density, and \(f_{1h}\) and \(f_{2h}\) correspond to the scalar and pseudoscalar density, respectively.

From the Wigner-Dirac equation \([2]\), we can now derive the following system of equations:

\[
\begin{align*}
  f_0^{0h} + i [M_H, f_1^{1h}] + i [M_A, f_2^{2h}] &= 0, \\
  f_1^{1h} + 2h |k| f_{2h} + i [M_H, f_0^{0h}] - \{M_A, f_3^{3h}\} &= 0, \\
  f_2^{2h} - 2h |k| f_{1h} + \{M_H, f_3^{3h}\} + i [M_A, f_0^{0h}] &= 0, \\
  f_3^{3h} - \{M_H, f_2^{2h}\} + \{M_A, f_1^{1h}\} &= 0,
\end{align*}
\]

where the prime denotes a derivative with respect to \(\eta\). It is understood that \(M\) and the \(f_{\mu h}\) are flavour matrices. Note that the commutators in \([5]\), which mix particle flavours, are essential for the production of the charges \(f_{0h}\), and thus for our scenario. Moreover, one can infer, that a necessary condition for \(f_0^{0h} \neq 0\) is a nonsymmetric \(M\). We already anticipated this when noting that for such a mass matrix the orthogonality of particle and antiparticle modes is violated. Note that the tree level dynamics given by Eqns. \([5]\) closes for \(f_{\mu h}\). When rescatterings, as described through nonlocal quantum loop corrections, are included, off-shell effects may become important, and one would have to solve for the full dynamics of the \(g_{\mu h}\). In the nonrelativistic regime and
close to equilibrium however, in which off-shell effects are suppressed, it is possible to include rescatterings into Eqns. 5 and still retain closure for the equations for the $f_{\mu h}$.

Now we fix the initial conditions for a Universe, which is void of fermions at the end of inflation. For an initially diagonal slowly evolving mass matrix, the Wigner functions for a zero particle state zero particles are (cf. Ref. [17]):

\[
\begin{align*}
    f_{0h}^{ab} &= L_h^{ab} L_h^b + R_h^{ab} R_h^b, \\
    f_{1h}^{ab} &= -2\Re(L_h^a R_h^b), \\
    f_{2h}^{ab} &= 2\Im(L_h^a R_h^b), \\
    f_{3h}^{ab} &= L_h^{ab} L_h^b - R_h^{ab} R_h^b,
\end{align*}
\]  

(6)

with

\[
L_h^{ab} = \delta_{ab} \sqrt{\frac{\omega_a + \hbar k}{2\omega_a}}, \\
R_h^{ab} = \delta_{ab} \frac{M_h^{*a}}{\sqrt{2\omega_a(\omega_a + \hbar k)}},
\]

where $\omega_a = \sqrt{k^2 + |M_{aa}|^2}$. Note, that for the case of real $M_{aa}$, this reduces just to the usual choice of the components of the basis spinors in chiral representation. For a nondiagonal, but hermitean, $M$, one obtains the initial conditions by an appropriate unitary transformation. If additionally $M_A \neq 0$, as is the case for the SO(10) example discussed in the following, a biunitary transformation is necessary for diagonalization.

Since $f_{0h}$ is the zeroth component of the vector current, the charge of the species $a$ carried by the mode with momentum $k$ and helicity $h$ is simply $q_{ah}(k) = f_{0h}^{aa} - 1$. Note also, that the Lagrangean

\[
\mathcal{L} = \bar{\psi}_a \gamma_\mu \psi_a - \bar{\psi}_b (\mathcal{M}_H + i\gamma^5 M_A)_{ba} \psi_a
\]

is $U(1)$ symmetric, and thus $\sum_a q_{ah}(k)$ is conserved, as we shall verify explicitly for the SO(10) example discussed here.

The scenario for coherent baryogenesis is as follows: initially, there are zero fermions described by appropriate initial conditions for the $f_{\mu h}$, and $M$ is approximately constant in time. Then a phase transition occurs during which $M$ changes rapidly, which leads to fermion production. Eventually, $M$ stops evolving and the produced number of fermions as well as the charges $f_{0h}$ stored within the different species are frozen in. We emphasise that $f_{0h}^{ii}$ should not be confused with the number of produced particle pairs at preheating, which in our language can be expressed in terms of the $f_{ih} (i = 1, 2, 3)$ as given in Ref. [17].
3. THE BARR-RABY MODEL

One possibility to break SO(10) down to the Standard Model is to use a Higgs multiplet $A$ in the adjoint representation $45$ and another pair of Higgses $C$ and $\bar{C}$ in the spinor representations $16$ and $\overline{16}$. The apparently most simple implementation of this pattern of symmetry breaking, which is in accordance with particle physics observations, has been suggested by Barr and Raby \[20\]. In the following, we review the features of this model as far as they are relevant for our baryogenesis scenario, in particular we present in some detail the derivation of the higgsino-gaugino mass matrix. In the appendix, we give account of the conventions we use, in particular how the charges under the Standard Model group

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

are assigned to the various multiplets of SO(10).

We consider the superpotential

$$W \supset \kappa S (CC - \mu^2) + \frac{\alpha}{4M_S} \text{tr} A^4 + \frac{1}{2} M_A \text{tr} A^2 + T_1 A T_2 + M_T T_2^2$$

$$+ \bar{C}' \left( \xi P A \frac{M_S}{M_C} + \zeta Z_1 \right) C + \bar{C}' \left( \xi P A \frac{M_S}{M_C} + \xi Z_2 \right) C' + M_{C'} C' \bar{C}' ,$$

where the additional fields $S, P, Z_1, Z_2$ are singlets, $T_1$ and $T_2$ 10-plets of SO(10). Furthermore, there are the spinor $C'$ and the conjugate spinor $\bar{C}'$.

Let us first discuss the purely adjoint sector. The potential is at its minimum, when the condition

$$- F_A^* = \frac{\partial W}{\partial A} = 0$$

is met. When $\langle A \rangle = \text{diag}(a_1, a_2, a_3, a_4, a_5) \otimes i\sigma_2$, it follows

$$\frac{\alpha}{M_S} a_i^3 - M_A a_i = 0 .$$

This can be solved by either $a_i = 0$, or $a_i = a$, where

$$a = \pm \sqrt{\frac{M_A M_S}{\alpha}} .$$

In order to step towards the Standard Model, it is possible to break SO(10) down to the left-right symmetric group

$$G_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
by choosing for \( \langle A \rangle \) the Dimopoulos-Wilczek (DW) form

\[
\langle A \rangle = \begin{pmatrix}
a & a \\
a & a \\
0 & 0
\end{pmatrix} \otimes i \sigma_2. \tag{13}
\]

Note, that \( \langle A \rangle \) being of DW form is proportional to the \((B - L)\) operator given in Eqn. \( \text{A26} \). In the appendix, we give account of the explicit construction of the tensor- and spinor representations of SO(10) and the conventions we use.

The two Higgs doublets of the MSSM are contained within \( T_1 \) and are identified with the four components which remain massless by the superpotential \( \mathbf{5} \) when using the DW-form for \( \langle A \rangle \). The additional six degrees of freedom of \( T_1 \), two colour triplets, become heavy and hence invisible at low energies, such that there is doublet-triplet splitting. The second \( \mathbf{10} \)-plet \( T_2 \) becomes necessary since a direct mass-term for the triplet components of \( T_1 \) would lead to a disastrous rapid higgsino-mediated proton decay.

The Higgs fields \( C \) and \( \bar{C} \) reduce the SO(10) symmetry to SU(5). When minimizing the scalar potential, the absolute values of their VEVs are \( \langle C \rangle = \langle \bar{C} \rangle = \mu \), and they point in the SU(5)-singlet direction with the quantum numbers of a right-handed neutrino.

Both sectors, the spinorial and the adjoint, in combination reduce the SO(10)-symmetry to the Standard Model group \( G_{SM} \). However, they need to be linked together in order to get a congruency of the assignment of Standard Model quantum numbers and to remove all pseudo-Goldstone modes from the particle spectrum. The obvious candidate term to add to the superpotential, \( \bar{C}AC \), however destabilizes the DW form \( \text{13} \) by altering the expression for the \( F \)-term \( \text{10} \) when the spinors get a nonzero VEV. Barr and Raby therefore suggested to add the additional spinors \( C' \) and \( \bar{C}' \), which get a zero VEV. The conditions for potential minimization now become

\[
- F_{C'}^* = \left( \xi \frac{PA}{MS} + \zeta \frac{Z_1}{Z_2} \right) C + M_{C'} C' = 0, \tag{14}
\]

\[
- F_{\bar{C}'}^* = \bar{C} \left( \xi \frac{PA}{MS} + \zeta \frac{Z_1}{Z_2} \right) + M_{C'} \bar{C}' = 0. \tag{15}
\]

When comparing with Eqn. \( \text{A26} \), we note that in the DW-form \( \text{13} \) we can identify \( \langle A \rangle \equiv \frac{3}{2} a (B - L) \). If we assume that the VEV of \( P \) is fixed, then \( Z_1 \) and \( Z_2 \) settle to

\[
Z_1 = -\frac{3}{2} \frac{\zeta / \zeta Z}{MS} \langle P \rangle a, \tag{16}
\]

\[
Z_2 = -\frac{3}{2} \frac{\xi / \xi Z}{MS} \langle P \rangle a. \tag{17}
\]
since \( C \) and \( \bar{C} \) point into the right-handed neutrino direction, where \( B - L = 1 \). We have hence achieved a link between the spinorial and adjoint sector without changing the form of \(-F_A^*\).

In our model, \( CP \)-violation arises from the phase between \( \zeta \) and \( \xi \) and therefore from couplings of the adjoint to the spinor multiplets. Let us label the multiplets of the Standard Model group \( \mathbf{16} \) by \( K \). The representations \( \mathbf{16} \) and \( \mathbf{45} \) harbour as multiplets with common \( G_{SM} \) quantum numbers \( K = (3, 2, \frac{1}{6}), \ K = (\bar{3}, 1, -\frac{2}{3}) \) and \( K = (1, 1, 1) \). These multiplets therefore mix through the higgsino mass matrix. The corresponding conjugate multiplets in \( \mathbf{\overline{16}} \) and \( \mathbf{45} \) are labeled by \( \bar{K} \). Furthermore, all these representations contain the singlet \( (1, 1, 0) \).

The spinor pair with 32 complex degrees of freedom breaks the 45-dimensional \( SO(10) \) down to the 24-dimensional \( SU(5) \). The 21 Goldstone modes come from the multiplets \( K = (3, 2, \frac{1}{6}), \ K = (\bar{3}, 1, -\frac{2}{3}), \ K = (1, 1, 1) \) plus one linear combination of the singlets \( K = (1, 1, 0) \) within \( \mathbf{16} \) and \( \mathbf{\overline{16}} \). The 45-dimensional adjoint reduces the \( SO(10) \)-symmetry to the 15-dimensional \( G_{LR} \). Because of the DW VEV being proportional to the \( (B - L) \) operator, the 30 Goldstone modes can be identified with the multiplets for which \( B - L \neq 0 \), that are all colour triplets.

Hence, by the supersymmetric Higgs-mechanism, there is a mixing of the higgsino modes with the gaugino sector, through the Lagrangean terms

\[
\sqrt{2} g \varphi^* T^a \psi \lambda^a + \text{h.c.,} \quad (18)
\]

where \( T^a \) is a generator of \( SO(10) \), normalized as \( \text{tr} (T^a)^2 = 1 \), \( \lambda^a \) a gaugino and \( \varphi \) the scalar superpartner of the \( \psi \)-fermion. This will induce higgsino-gaugino mixing mass terms for both multiplets, \( A \) and \( C \) when \( K = (3, 2, \frac{1}{6}) \) or \( K = (\bar{3}, 1, -\frac{2}{3}) \), and only for \( C \), when \( K = (1, 1, 1) \).

Let us consider possible mass terms involving only the adjoint Higgs. If we denote the components of either \( (3, 2, \frac{1}{6}) \) or \( (\bar{3}, 1, -\frac{2}{3}) \) by \( b_K \), we have (cf. appendix A)

\[
\text{tr} A^2 = -6a^2 - 2b_K b_{\bar{K}} , \quad (19)
\]

\[
\text{tr} A^4 = 6a^4 + 4a^2 b_K b_{\bar{K}} + b_K^2 b_{\bar{K}}^2 . \quad (20)
\]

Hence, the portion \( \frac{\alpha}{4M_S} \text{tr} A^4 + \frac{1}{2} M_A \text{tr} A^2 \) of the superpotential \( \mathbf{S} \) gives for these modes a zero mass term

\[
m_K = \frac{\alpha a^2}{M_S} - M_A = 0 \quad \text{for} \ K = (3, 2, \frac{1}{6}) \text{ and } K = (\bar{3}, 1, -\frac{2}{3}) , \quad (21)
\]

where we have used the VEV \( \mathbf{11} \) for \( a \). This result is expected, since the multiplets in question are Goldstone. In contrast, we find

\[
m_K = \frac{\alpha a^2}{M_S} \quad \text{for} \ K = (1, 1, 1), (1, 3, 0) , \quad (22)
\]
\[ m_K = -2\frac{\alpha a^2}{M_S} \quad \text{for } K = (8, 1, 0). \] (23)

Let us now discuss the mixing of the adjoints and spinors. \( \psi_{A_K} \) and \( \psi_{C'_K} \) get a mixing mass term through
\[
\frac{\delta^2 W}{\delta A_K \delta C'_K} = \xi \langle \bar{C} \rangle (P) \sqrt{2} M_S. \] (24)

The derivation of this term is instructive and works as follows: In the block-diagonal basis, we have
\[
A_{10}^{\text{BL}} = \begin{pmatrix} 0 & 0 \\ \overline{10} & 0 \end{pmatrix}, \] (25)
which transforms according to \( (A30) \) to the off-diagonal basis as
\[
A_{10} = U^{-1}_{\text{BLOCK}} A_{10}^{\text{BL}} U_{\text{BLOCK}} = \frac{1}{2} \begin{pmatrix} \overline{10} & i\overline{10} \\ i\overline{10} & -\overline{10} \end{pmatrix}. \] (26)

The single degrees of freedom \( \overline{10} \) are represented by the ten antisymmetric \( 5 \times 5 \) matrices with two nonvanishing entries of the value \( 1/\sqrt{2} \). Without loss of generality, we pick the matrix with \( -1/\sqrt{2} \) in the first row, fourth column, corresponding to one degree of freedom of the \( \bar{K} = (\bar{3}, 2, -1/6) \)-multiplet. In order to let the tensor \( A \) act on a spinor, we make use of Eqn. (A24) and represent it in terms of the \( \Gamma \)-operators as
\[
A = -\frac{\sqrt{2}}{16} ([\Gamma_1, \Gamma_4] + i[\Gamma_1, \Gamma_9] + i[\Gamma_6, \Gamma_4] - [\Gamma_6, \Gamma_9]). \] (27)

When paired with the spinor
\[
\Psi = \frac{1}{2} \left[ \chi_2 \chi_3 \chi_5^\dagger - \chi_3^\dagger \chi_2 \chi_5 \right] |0\rangle, \] (28)
a \( G_{SM} \) singlet is formed, and after anticommuting the \( \chi_i \)-operators, we find
\[
\langle 0 | \chi_1 \chi_2 \chi_3 \chi_4 \chi_5 A \Psi = \frac{1}{\sqrt{2}}, \] (29)
from which we immediately obtain Eqn. (24). The higgsino-gaugino mixing terms can be derived from (18) in a very similar way.

Finally, for the mixing among the spinors we have
\[
\frac{\delta^2 W}{\delta C_K \delta C'_K} = \xi a_K \frac{a(P)}{M_S}, \] (30)
where \( \alpha_K = \frac{3}{2} \Im (B - L)_K - 1 \) or explicitly,

\[
\alpha_K = \begin{cases} 
-1 & \text{for } K = (3, 2, \frac{1}{3}) \\
-2 & \text{for } K = (\bar{3}, 1, -\frac{2}{3}) \\
0 & \text{for } K = (1, 1, 1) 
\end{cases}
\]  

(31)

We have used here the VEVs \([16, 17]\) and again the proportionality of \( \langle A \rangle \) to the \( (B - L) \) operator.

We are now in the position to write down the higgsino-gaugino mass matrix:

\[
\left( \begin{array}{cccc}
\psi_{\lambda_K} & \psi_{A_K} & \psi_{C_K} & \psi_{C'_K} \\
0 & -i\sqrt{2}\gamma_K g a & g(C) & 0 \\
i\sqrt{2}\gamma_K g a & m_K & 0 & \xi \langle C \rangle \langle P \rangle M_S \\
G(C) & 0 & \kappa \langle S \rangle & i\alpha_K \zeta \langle P \rangle M_S \\
0 & \xi \langle C \rangle \langle P \rangle M_S & i\alpha_K \zeta \langle P \rangle M_S & M_{C'} \\
\end{array} \right)
\times
\left( \begin{array}{c}
\psi_{\lambda_K} \\
\psi_{A_K} \\
\psi_{C_K} \\
\psi_{C'_K} \\
\end{array} \right) + \text{h.c.},
\]

(32)

where

\[
\gamma_K = \begin{cases} 
\frac{1}{2} & \text{for } K = (3, 2, \frac{1}{3}) \\
1 & \text{for } K = (\bar{3}, 1, -\frac{2}{3}) \\
0 & \text{for } K = (1, 1, 1) 
\end{cases}
\]  

(33)

The mass matrix is nonsymmetric, therefore being endowed with the necessary prerequisites for coherent baryogenesis.

4. SIMULATION OF COHERENT BARYOGENESIS

The superpotential \([8]\) is of the type suitable for hybrid inflation. We assume that symmetry breaking by the adjoint sector has already taken place before or during inflation and is preserved throughout the subsequent history of the Universe, such that possible monopoles are diluted. We therefore do not consider the dynamics of the field \( A \). For a discussion of the role of cosmic strings formed at the transition \( G_{LR} \rightarrow G_{SM} \) after inflation, we refer to Ref. \([21]\).

The part of the scalar potential relevant for hybrid inflation reads

\[
V = \kappa^2 \left| C^2 - \mu^2 \right|^2 + 2\kappa^2 |SC|^2,
\]

(34)

where we have used \( C = \bar{C}^* \) due to the vanishing of the \( D \)-terms and have written \( C \equiv \bar{C} \).

During inflation, the VEVs of \( C \) and \( \bar{C} \) are sitting at a minimum located at zero, and \( S \) rolls down a logarithmic slope until reaching the critical value \( S_{cr} = \mu \), such that the value zero for \( C \)
and $\bar{C}$ becomes a maximum. The waterfall regime begins, at the end of which the scalar fields settle down to the supersymmetric ($V = 0$) minimum $S = 0, |C| = |\bar{C}| = \mu$. This is a rapid phase transition which brings coherent baryogenesis along. We simulate this scenario for the parameter $\kappa = 0.05$, a damping rate $\Gamma = 0.02\mu$ and also take account of the expansion of the Universe, cf. FIG. I.

Damping partly comes into play because of the perturbative decay of the inflaton. More important at the beginning of the waterfall regime is however the phenomenon of tachyonic preheating: Since the fields $C$ and $\bar{C}$ attain a negative mass square term, modes with momenta less than this mass get produced exponentially fast. Here we mimic this effect by introducing the damping rate $\Gamma$. For numerical studies of this process, see Refs. [22, 23, 24, 25, 26, 27, 28]. Damping will also receive a contribution from fermionic preheating. A proper treatment of fermionic preheating, which includes rescatterings, would require techniques used in Ref. [29], which have so far not been applied to the question of inflaton thermalisation through decay into fermions.

In order to keep the discussion simple, we do not take the dynamics of the singlet fields $Z_1$ and $Z_2$ into account here. In principle, their VEVs only get fixed when $C$ and $\bar{C}$ acquire nonzero VEVs. A possible way to fix $Z_1$ and $Z_2$ already during inflation is for example to shift the spinors away from the zero VEV, as proposed in Ref. [5] and is also applicable to SO(10)-models [6].

For the remaining parameters, we choose $\mu = 0.5 \times 10^{16}$GeV, $M_S = 550\mu$, $M_{C'} = 0.02\mu$, $g = 0.2$, $\zeta = -0.02$, $\xi = 0.05i$, $a = 25\mu$ and $\langle P \rangle = 24\mu$. 

![FIG. 1: Epoch of phase transition in the SO(10)-model](image-url)
The charge numbers which are plotted in figures 2 and 3 refer to the mass matrix, which is diagonal after the phase transition is completed. Hence, there is mixing among $\psi_{\lambda K}$, $\psi_{A K}$, $\psi_{C K}$ and $\psi_{C K}'$. The mass matrix is diagonalized via a biunitary transformation. It turns out that the higgsino-gaugino mass matrix in the supersymmetric vacuum has two heavy and two light
eigenvalues. We only display the charge production corresponding to the light mass eigenvalues which we label by $q_1$ and $q_2$. There is no substantial charge asymmetry stored within the heavy flavours, $|q_3|, |q_4| \ll |q_1|, |q_2|$. The apparent symmetry $\sum_{i=1}^{4} q_i = 0$ results from the overall $U(1)$ symmetry of the fermionic fields and is a useful check for the numerical results. We shall assume here that the resonant decay into gauge bosons and scalar Higgs particles is not important. This can be justified by noting that the necessary conditions for a resonant decay into bosons are not met for our choice of parameters.

The $q_i$ are charges stored within the mass eigenstates which are Dirac fermions of the generic mixing form

$$
\Psi_i = \begin{pmatrix}
  i\alpha^L_{\lambda K} \lambda_K + i\alpha^L_{A K} \psi_{A K} + i\alpha^L_{C K} \psi_{C K} + i\alpha^L_{C' K} \psi_{C' K} \\
i\beta^R_{\lambda K} \bar{\lambda}_{\bar{K}} + i\beta^R_{A K} \bar{\psi}_{A K} + i\beta^R_{C K} \bar{\psi}_{C K} + i\beta^R_{C' K} \bar{\psi}_{C' K}
\end{pmatrix}, \quad i = 1, 2, 3, 4,
$$

(35)

where the coefficients $i\alpha^L_X$ and $i\beta^R_X$ are determined by the biunitary transformation diagonalizing the higgsino-gaugino mass-matrix. For the example we discuss here, the transformation matrices and the coefficients are determined numerically.

Now, we discuss how the $q_i$ charges get transformed to $B-L$ charge stored within fermionic matter. Decay into SUSY Standard Model particles can take place through the gaugino components $\lambda_K, \bar{\lambda}_{\bar{K}}$ and through the Higgsinos $\psi_{C K}, \bar{\psi}_{C K}$. The relevant operator for gaugino decay from the gauge supermultiplet Lagrangean is

$$
\sqrt{2}g \{\lambda FF + h.c.\}
$$

(36)

while the higgsinos decay through the dimension five couplings

$$
i\gamma_1 \frac{\bar{C}F\bar{C}F}{M_S},
$$

$$
i\gamma_2 \frac{C^a F C^a F}{M_S},
$$

$$
i\gamma_3 \frac{C^a \Gamma^b F C^a \Gamma^b F}{M_S},
$$

(37)\(38)\(39)

which are added to the superpotential $\mathbf{S}$, and where $F$ are the Standard Model matter fields and the right-handed neutrino, contained in $\mathbf{16}$, and $\Gamma$ denotes the operators defined in $\mathbf{A6}$ and $\mathbf{A7}$. The index $i = 1, 2, 3$ denotes the matter generation.

While the coupling $\mathbf{36}$ is universal for all three generations of Standard Model matter, $i\gamma_1$, $i\gamma_2$ and $i\gamma_3$ may be different for the three generations. We assume that $3\gamma_2, 3 \gg 2\gamma_2, 3 \gg 1\gamma_2, 3$, such that only the $3\gamma_2, 3$ are of relevance for the decays. In contrast, we require that the decays through
and $\gamma_1$ are not possible since the corresponding right-handed neutrinos are heavier than the decaying particle, such that only $\gamma_1$ is relevant. We now argue of which order these couplings should be for realistic scenarios.

For nonthermal leptogenesis, one usually assumes that two of the three Majorana neutrinos from the different generations of matter fermions are heavier than half of the mass of the inflaton fields, which are the $\nu^c$-like components of $C$ and $\bar{C}$ and the singlet $S$, such that their decay into right handed neutrinos is kinematically forbidden. Through the coupling the right handed neutrinos acquire Majorana masses

$$i m_{\nu^c} = i \gamma_1 \langle C \rangle^2 / M_S ,$$

such that the requirement $2, 3 m_{\nu^c} > m_I / 2$ reads

$$2, 3 \gamma_1 > \frac{1}{\sqrt{2}} \frac{M_S}{\mu} ,$$

where we have used $\langle C \rangle = \mu$. It appears to be reasonable that also the couplings $2, 3 \gamma_2, 3$ are of the same order, as we shall assume.

For the scenario we discuss, the lightest right-handed neutrino is important for the reheating process. The coupling also allows for the decay of the inflaton fields at the rate

$$\Gamma_\nu = \frac{1}{8\pi} m_I \left( \frac{1 \gamma_1 \langle C \rangle^2}{M_S} \right)^2 .$$

The Universe becomes radiation dominated and entropy production stops, when $\Gamma_\nu = H$, where $H$ denotes the Hubble expansion rate. The reheat temperature at this time is

$$T_R = 0.55 \sqrt{\frac{1}{g_*}} \Gamma_\nu m_{\text{Pl}} ,$$

and we take the estimate $g_* = 220$, the number of relativistic degrees of freedom after reheating. The value for the Planck mass is $m_{\text{Pl}} = 1.2 \times 10^{19}\text{GeV}$. The mass of the lightest right-handed neutrino is therefore proportional to the reheat temperature:

$$m_{\nu^c} = 7.7 \times \frac{1}{\sqrt{\kappa}} \frac{\mu}{m_{\text{Pl}}} T_R .$$

Taking for the highest reheat temperature allowed by the gravitino bound $T_R = 10^{11}\text{GeV}$ and for the parameters we use, we find $m_{\nu^c} < 5 \times 10^9\text{GeV}$ while $2, 3 m_{\nu^c} > 2 \times 10^{14}\text{GeV}$. Therefore,
a fortuitous hierarchy of five orders of magnitude for the Majorana masses is required. This is usually assumed for scenarios of nonthermal leptogenesis.

For the coherent baryogenesis model we consider here however, we can also allow for all Majorana masses to be larger than $m_1/2$. Under these circumstances, $S$ and the $\nu^c$-like components of $C$ and $\bar{C}$ cannot decay through the term (37) into two right-handed neutrinos but decay to three particles via the operators (38) and (39). Since these processes involve no Majorana particles, leptogenesis is absent for this scenario.

We also have to deal with the fact, that for $\alpha_K = 0$, namely for $K = (1, 1, 1)$, $\psi_{C_K}$ and $\psi_{C'_K}$ do not mix, cf. the mass matrix (32). Therefore we assume that also the fields $C'$ and $\bar{C}'$ may decay through couplings of the above type, suppressed however by additional powers of $\langle R \rangle / M_S$, where $R$ is some singlet with a VEV.

Note however, that for $K = (\bar{1}, 1, 1)$ the mass matrix (32) is block-diagonal, such that only the pairs $\lambda_K - \psi_{C_K}$ and $\psi_{A_K} - \psi_{C'_K}$ are mixed. Only for the second pair, the $CP$-violating parameters $\xi$ and $\zeta$ are relevant and an asymmetry is generated, which vanishes however after the decay into matter.

The charges $q_i$ gets processed differently to $B-L$ when $\Psi_i$ decays through its various components. Let us denote the $B-L$ number resulting from the decay of a component $X$ of a $\Psi_i$ quantum by $T_X$. By the couplings (36), the reactions

$$\lambda_K \rightarrow F'^*_{K} + \nu^c$$

$$\lambda_{\bar{K}} \rightarrow F''_{K} + \nu^c$$

are induced, where one of the particles on the right hand side is a scalar, the other one a fermion. Due to its Majorana mass term coming from the operator (37), the right handed neutrino $\nu^c$ is its own antiparticle and therefore carries effectively $B-L = 0$ at tree-level. The resulting $B-L$-charge is therefore the one stored within $F'^*_{K}$ and $F''_{K}$ and we find

$$T_{\lambda_K} = \frac{1}{3}, \quad K = (3, 2, \frac{1}{6}),$$

$$T_{\lambda_{\bar{K}}} = \frac{1}{3}, \quad K = (\bar{3}, 2, \frac{1}{6}),$$

$$T_{\bar{\lambda}_K} = 1, \quad K = (1, 1, 1),$$

$$T_{\bar{\lambda}_{\bar{K}}} = -1, \quad \bar{K} = (1, 1, -1).$$
Similarly, the coupling (37) allows for the decay reaction
\[ \psi \bar{C} \rightarrow F_K^* + \nu^c, \]  
(54)
Hence, the charges get transformed to
\[ T_{\psi \bar{C} K} = -\frac{1}{3}, \quad \bar{K} = (\bar{3}, 2, -\frac{1}{6}), \]  
(55)
\[ T_{\psi \bar{C} K} = \frac{1}{3}, \quad \bar{K} = (3, 1, \frac{2}{3}), \]  
(56)
\[ T_{\psi \bar{C} K} = -1, \quad \bar{K} = (1, 1, -1). \]  
(57)

We can calculate the term (38) \( \propto \gamma_2 \) using the techniques explained in the appendix A. It is however easier to note that (cf. Ref. [30])
\[ (3, 2, \frac{1}{6}) \otimes (\bar{3}, 1, \frac{1}{3}) \supset (1, 2, \frac{1}{2}) \subset 10, \]  
(58)
as well as
\[ (1, 1, 0) \otimes (1, 2, -\frac{1}{2}) = (1, 2, -\frac{1}{2}) \subset 10. \]  
(59)
The components of \( \psi_{C K} \) for \( K = (3, 2, \frac{1}{6}) \) therefore decay to
\[ \psi_{C_d} \rightarrow d^c + e^*, \]  
(60)
\[ \psi_{C_u} \rightarrow d^c + \nu^*, \]  
(61)
where \( \psi_{C_d} \) denotes the \( d \)-quark like higgsino, \( \psi_{C_u} \) the \( u \)-quark like one. The charges hence get transformed as
\[ T_{C_K} = \frac{4}{3}, \quad \text{for} \quad K = (32, \frac{1}{6}). \]  
(62)
The \( u^c \)-quark like higgsino with \( K = (\bar{3}, 1, -\frac{2}{3}) \) decays through the \( \gamma_3 \)-coupling (39). We note (cf. Ref. [30])
\[ (\bar{3}, 1, -\frac{2}{3}) \otimes (\bar{3}, 1, \frac{1}{3}) = (3, 1, -\frac{1}{3}) \oplus (6, 1, -\frac{1}{3}) \subset 120, \]  
(63)
\[ (1, 1, 0) \otimes (\bar{3}, 1, \frac{1}{3}) = (3, 1, \frac{1}{3}) \subset 120, \]  
(64)
and therefore have the reaction
\[ \psi_{C_{u^c}} \rightarrow d^c + d^c, \]  
(65)
and the charge conversion
\[ T_{C_K} = \frac{2}{3}, \quad \text{for} \quad K = (3, 1, -\frac{2}{3}). \]  
(66)
Finally, the $e^c$ like higgsino $K = (1, 1, 1)$ turns into matter via the $\gamma_2$-coupling \(38\), as can be seen by

\[
(1, 1, 1) \otimes (1, 2, -\frac{1}{2}) = (1, 2, \frac{1}{2}) \subset 10, \quad (67)
\]

\[
(1, 1, 0) \otimes (1, 2, -\frac{1}{2}) = (1, 2, -\frac{1}{2}) \subset 10. \quad (68)
\]

Consequently, the decay reaction is

\[
\psi_{C^e} \rightarrow e^* + \nu^*, \quad (69)
\]

and the resulting asymmetry

\[
T_{C_K} = 2, \text{ for } K = (1, 1, 1). \quad (70)
\]

The procedure to obtain the produced $B-L$-density is as follows: We first integrate the produced charge numbers $q_i$ over momentum space in order to obtain charge densities $Q_i$. From the biunitary diagonalization of $M$ in the supersymmetric minimum the contributions of $\lambda_K, \bar{\lambda}_K, \psi_{C_K}$ and $\psi_{\bar{C}_K}$ to the $\Psi_i$ are determined, which gives the branching ratios and therefore the respective contributions for the decays of the Dirac fermions to Standard Model matter. As a formula, this reads

\[
n^0_{B-L} = \sum_{i=1}^{4} Q_i \frac{3|\alpha_{\lambda_K}^L|^2 2g^2 T_{\lambda_K} + |i\alpha_{C_K}^L|^2 \left(\frac{3\gamma_{i}(C)}{M_S}\right)^2 T_{C_K} - 3|\beta_{\lambda_K}^R|^2 2g^2 T_{\lambda_K} - |i\beta_{C_K}^R|^2 \left(\frac{1}{M_S}\right)^2 T_{C_K},}{3|\alpha_{\lambda_K}^L|^2 2g^2 + |i\alpha_{C_K}^L|^2 \left(\frac{3\gamma_{i}(C)}{M_S}\right)^2 + 3|\beta_{\lambda_K}^R|^2 2g^2 + |i\beta_{C_K}^R|^2 \left(\frac{1}{M_S}\right)^2} \quad (71)
\]

where the factors of three come from the presence of three generations of matter and

\[
\frac{2}{3} \text{ for } K = \left(3, 2, \frac{1}{6}\right), \quad (72)
\]

\[
3 \text{ for } K = \left(\bar{3}, 1, -\frac{2}{3}\right), \quad (72)
\]

\[
2 \text{ for } K = (1, 1, 1). \quad (72)
\]

The total $(B-L)$ number density produced at the phase transition is taking account of the multiplicity of colour and flavour given by

\[
n^0_{B-L} = 6n^0_{(3, 2, \frac{1}{6})} + 3n^0_{(3, 1, -\frac{2}{3})}. \quad (73)
\]

A study of the parametric dependence of the produced asymmetry is beyond the scope of this paper, which is to show that coherent baryogenesis is viable with the gauge group $\text{SO}(10)$, and shall be discussed elsewhere. Therefore, we content ourselves with presenting just one typical numerical example here. The parameters yet to be specified are the $^3\gamma_{2,3}$ and $^1\gamma_1$. We can set effectively $^1\gamma_1 = 0$ because it is either very small due to the restrictions given by the reheat temperature \(14\).
and the gravitino bound or, in the case of $1^m_{\nu c} > m_f/2$ decays through the coupling \[37\] do not take place. In accordance with the relation \[42\] we choose $3\gamma_2 = 3\gamma_1 = 0.05M_S/\mu$. Then, we find

$$n_{B-L}^0 = 2.5 \times 10^{-7} \mu^3.$$  

(74)

In order to estimate the baryon to entropy ratio, we express the entropy density $s$ through the reheat temperature $T_R$ as

$$s = 2\pi^2 g_* T_R^3 / 45,$$  

(75)

and the Hubble expansion rate is given by

$$H = 1.66 \sqrt{g_*} \frac{T_R^2}{m_{Pl}},$$  

(76)

where $m_{Pl} = 1.22 \times 10^{19}$GeV is the Planck mass.

During the epoch of coherent oscillations, that is between the end of inflation and the onset of radiation era, the Universe is matter dominated and expands by a factor

$$\frac{a}{a_0} = \left( \frac{H_0}{H} \right)^{2/3},$$  

(77)

where $H_0$ is the expansion rate at the end of inflation, given by

$$H_0 = \sqrt{\frac{8\pi V}{3 m_{Pl}^2}}.$$  

(78)

Putting everything together, we find

$$\frac{n_B}{s} \approx \frac{1}{3} \frac{n_{B-L}^0}{s} \left( \frac{a_0}{a} \right)^3 \approx \frac{1}{4} \frac{n_{B-L}^0}{V_0} T_R,$$  

(79)

where we have taken account of a division by three for sphaleron transitions promoting $(B-L)$ to $B$ asymmetry.

The value for the vacuum energy at the end of inflation is $V_0 = \kappa^2 \mu^4$, and by Eqn. \[79\], we find

$$\frac{n_B}{s} = 1.0 \times 10^{-10},$$  

(80)

where we have chosen $T_R = 2 \times 10^{10}$GeV. Hence, it appears that in order to get a BAU in accordance with observation, there has to be a reheat temperature of order of the upper bound allowed by the requirement that gravitinos shall not be overproduced. However, our estimate of entropy production is rather crude. It is conceivable that initially the decay of $C$, $\bar{C}$ and $S$ is enhanced due to tachyonic and also fermionic preheating. This could lead to an initial radiation-like equation of state \[27\] or shorten the matter-dominated era and would therefore lead to less dilution of the initial asymmetry.
For the case $m_{\nu C} < m_f/2$, it is of interest to compare the result \((80)\) with the baryon asymmetry resulting from nonthermal leptogenesis, which is given by \((31)\)

\[
\frac{n_B}{s} = 0.5 \frac{\epsilon_1}{m_f} T_R ,
\]

where

\[
\epsilon_1 = 2 \times 10^{-10} \left( \frac{1 m_{\nu C}}{10^6 \text{GeV}} \right) \left( \frac{m_{\nu 3}}{50 \text{meV}} \right)
\]

is the maximal $CP$-violation which may arise from the decay of the lightest right-handed neutrino \((32)\), and $m_{\nu 3}$ denotes the heaviest mass eigenvalue of the light neutrino mass matrix. When we assume $m_{\nu 3} = 50 \text{meV}$ and use the same parameters as for the coherent baryogenesis example and Eqn. \((45)\) for $m_{\nu C}$, we find $n_B/s = 6 \times 10^{-12}$. Therefore our example corresponds to a point in parameter space where coherent baryogenesis dominates over leptogenesis. However, we expect that also the opposite case may occur for a different set of parameters.

5. CONCLUSIONS

In this paper, we show that during the phase transition terminating SUSY SO(10) hybrid inflation, a charge asymmetry within the Higgsino sector may be produced through the mechanism of coherent baryogenesis and subsequently turned into baryons. $CP$-violation is provided by the couplings of the spinorial to the adjoint representations and occurs at tree-level. This is very different from leptogenesis, a one loop effect, where $CP$-violation is sourced by the matrix of Yukawa couplings between the neutrinos and the Standard Model Higgs field. Since the spinor-adjoint couplings are an indispensable part of the Barr-Raby model, coherent baryogenesis naturally occurs at the end of SUSY-SO(10) hybrid inflation. Together with similar results which we found for the Pati-Salam group \((7)\), this indicates that effects from fermionic preheating are generically of importance for the generation of the BAU in hybrid-inflationary scenarios.

Coherent baryogenesis however has been neglected in the standard picture of baryogenesis in SUSY-GUT hybrid inflation so far, which is as follows \((33)\): The inflaton decays into right-handed neutrinos which then decay out-of-equilibrium into Standard Model matter, leaving behind a $B-L$ asymmetry \textit{via} the leptogenesis mechanism \((34)\). Since the decaying Majorana neutrinos are not produced by the thermal background, this scenario is often called nonthermal leptogenesis. However, it imposes strong constraints on the hierarchy of the masses of right-handed neutrinos, as we discuss in section \((4)\). We also emphasize that coherent baryogenesis relaxes this constraint
and allows for all Majorana masses to be larger than the inflaton mass, since the mechanism
does not rely on leptogenesis and the decay of Majorana particles. In conclusion, the relations of
the parameters of hybrid inflationary models to the BAU as suggested e.g. in Refs. [5, 6, 33] by
considering leptogenesis, should be altered, since coherent baryogenesis turns out to be an additional
source of the BAU, which may dominate in some regions of parameter space.

We emphasize that nonthermal leptogenesis and coherent baryogenesis should not be confused
with the often discussed thermal leptogenesis mechanism [34, 35, 36, 37, 38], an appealing feature
of which is that the BAU is generated from a Universe which is – within horizon scale – in thermal
equilibrium. Leaving aside primordial density fluctuations, all cosmological observables including
the BAU would then be predictable from an effective theory valid up to the Majorana mass scale
of the neutrinos.

Grand Unified Theories open up many possible paths for the generation of the BAU and it is yet
not known where the actual asymmetry originates from. The various mechanisms allow to establish
relations to the parameters of the underlying models. While leptogenesis renders constraints on the
neutrino sector, coherent baryogenesis is of interest since it is a scenario of GUT-baryogenesis and
thereby related to the dynamics of symmetry breaking.

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APPENDIX A: SO(10)

Besides by tensors, orthogonal groups may also be represented by spinors, which satisfy a Clif-
ford algebra. In order to construct group-transformation invariants, both types of representations
need to be linked together via Dirac gamma matrices. For the familiar case of the Poincaré group
SO(3,1), it is often convenient to use a specific representation for these matrices. In contrast,
one better circumvents the tedious task of explicitly constructing ten $32 \times 32$ gamma matrices for
SO(10). Mohapatra and Sakita [39] have therefore devised a very useful technique for performing
calculations involving spinors and tensors, employing just abstract commutation and anticommu-
tation relations.

On the other hand, when it comes to symmetry breaking, one has to choose a certain convention,
that is a certain basis, how the particles of the Standard Model are assigned to the representation \textbf{16} of SO(10). This assignment fixes in turn the definition of the charge operators and hence the quantum numbers of certain entries in vectors and tensors of SO(10).

While the paper by Mohapatra and Sakita \cite{39} does not provide much details of tensor representations and symmetry breaking, such a discussion can be found in the comprehensive work by Fukuyama \textit{et al.} \cite{30}, where in turn spinors are neglected. The coupling of spinors to tensors is explained for SO(10) by Nath and Syed \cite{40}. In the paper by Barr and Raby \cite{20}, which contains the model we consider here, a basis where tensors nicely decompose into blocks of SU(5)-representations is chosen. Unfortunately, the choice of basis and normalizations is not explicitly given, but has to be inferred by the reader.

In the following, we give some detailed account of the construction of SO(10)-singlets, following the conventions of Barr and Raby. Explicit expressions for the charge operators acting on spinors and tensors as well as for the accommodation of the Standard Model particles and the right-handed neutrino in the representation \textbf{16} are given, which shall ensure an easier and faster comprehensibility of the Barr and Raby analysis as well as of our calculations.

\textbf{Charge Assignments}

We denote by \(Q\) the electric charge, by \(Y\) the weak hypercharge and by \(I_L^3\) the weak isospin. The charges which are not gauge symmetries of the Standard Model are baryon minus lepton number \(B - L\) as well as the SU(2)\(_R\)-isospin \(I_R^3\) and the less known charge \(X\). There are linear dependencies among these charges, which are given by

\begin{align*}
Q &= I_L^3 + Y, \\
B - L &= 2(Y - I_R^3), \\
B - L &= \frac{1}{5}(4Y - X).
\end{align*}

Note that, when comparing to the conventions by Fukuyama \textit{et. al.} \cite{30}, we have twice as large values for \((B - L)\), such that for a single lepton, we have \((B - L) = -1\).

In table \textbf{I} we give the charge numbers of the Standard Model particles and of the right-handed neutrino.
TABLE I: Quantum numbers of matter

|   | Q   | $I_L^3$ | $I_R^3$ | Y  | B - L | X  |
|---|-----|---------|---------|-----|-------|----|
| $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ | \(\frac{2}{3}\) | \(\frac{1}{2}\) | 0 | \(\frac{1}{6}\) | \(\frac{1}{3}\) | -1 |
| \(\frac{-1}{3}\) | \(\frac{-1}{2}\) | 0 | \(\frac{1}{6}\) | \(\frac{1}{3}\) | -1 |
| $u^c$ | -2/3 | 0 | -1/2 | -2/3 | -1/3 | -1 |
| $d^c$ | 1/3 | 0 | 1/2 | 1/3 | -1/3 | 3 |
| $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$ | 0 | 1/2 | 0 | -1/2 | -1 | 3 |
| \(-1\) | -1/2 | 0 | -1/2 | -1 | 3 |
| $\nu^c$ | 0 | 0 | -1/2 | 0 | 1 | -5 |
| $e^c$ | 1 | 0 | 1/2 | 1 | 1 | -1 |

SO(2N) in an SU(N) Basis

This section contains a review of the paper by Mohapatra and Sakita [39], but adopts the basis conventions of Barr and Raby [20].

Let us introduce $N$ operators $\chi_i$ ($i = 1, \ldots, N$), acting on an antisymmetric Fock space, which obey the following anticommutation relations:

\[
\{\chi_i, \chi_j^\dagger\} = \delta_{ij}, \quad (A2)
\]

\[
\{\chi_i, \chi_j\} = 0. \quad (A3)
\]

The operators defined as

\[
T^i_j = \chi_j^\dagger \chi_j \quad (A4)
\]
satisfy the SU(N) algebra:

\[
[T^i_j, T^k_l] = \delta^i_l T^j_k - \delta^i_k T^j_l. \quad (A5)
\]

We now introduce the $2N$ operators

\[
\Gamma_j = -i(\chi_j - \chi_j^\dagger), \quad j = 1, \ldots, N, \quad (A6)
\]

\[
\Gamma_{N+j} = \chi_j + \chi_j^\dagger, \quad (A7)
\]

which obey by Eqns. [A2, A3] the Clifford algebra

\[
\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}, \quad i, j = 1, \ldots, 2N. \quad (A8)
\]
and hence, the algebra of generators of $SO(2N)$ is given by

$$\Sigma_{ij} = \frac{1}{2i} [\Gamma_i, \Gamma_j].$$

(A9)

Since the dimension of the spinor representation of $SO(2N)$ is $2^N$, a concrete representation could be constructed for $SO(10)$ in terms of $32 \times 32$-matrices, which however shall not be done here.

The spinor states can be constructed by letting the $N$ creation operators $\chi_i^\dagger$ act on the “vacuum” $|0\rangle$, such that the spinor representation is $2^N$-dimensional, as it should.

It is well known, that the spinor representation of $SO(2N)$ is reducible. We therefore define

$$\Gamma_0 = i^N \prod_{i=1}^{2N} \Gamma_i = \prod_{j=1}^{N} (1 - 2n_j),$$

(A10)

where we have introduced the number operators

$$n_j = \chi_j^\dagger \chi_j.$$  

(A11)

The chiral projectors $\frac{1}{2}(1 \pm \Gamma_0)$ give therefore rise to the two irreducible $2^{N-1}$-dimensional representations containing only even (case “+”) or only odd (case “−”) numbers of creation operators.

Now let $\Psi$ be an $SO(2N)$ spinor state. We are interested in calculating products of the form

$$\Psi^T B \Gamma_{i_1}...\Gamma_{i_M} \Psi,$$

(A12)

involving a certain number of $\Gamma$ matrices. The matrix $B$ is necessary since $\Psi^T$ does not transform as a conjugate spinor when acted upon with an infinitesimal $SO(10)$-transformation $\epsilon_{ij}$:

$$\delta \Psi = i \epsilon_{ij} \Sigma_{ij} \Psi,$$

(A13)

$$\delta \Psi^\dagger = -i \epsilon_{ij} \Psi^\dagger \Sigma_{ij},$$

$$\delta \Psi^T = i \Psi^T \epsilon_{ij} \Sigma_{ij}^T.$$

We require from $B$ the property

$$B^{-1} \Sigma_{ij}^T B = -\Sigma_{ij},$$

(A14)

such that

$$\delta (\Psi^T B) = i \epsilon_{ij} \Psi^T B B^{-1} \Sigma_{ij}^T B = -i \epsilon_{ij} (\Psi^T B) \Sigma_{ij},$$

(A15)

i.e. $\Psi^T B$ transforms as a conjugate spinor. The condition [A14] can be met if

$$B^{-1} \Gamma_i^T B = \pm \Gamma_i.$$ (A16)
By choosing the minus-sign in the latter equation, we find

\[ B = \prod_{i=1}^{N} \Gamma_i, \quad (A17) \]

because for \( i = 1, \ldots, N \) the \( \Gamma_i \) are represented by antisymmetric matrices, while for \( i = N+1, \ldots, 2N \) by symmetric ones.

For \( N = 5 \), we can arrange the Standard Model particles in the spin-16 representation, which is projected out of the 32-dimensional spinor \( \Psi \) by \( \frac{1}{2} (1 - \Gamma_0) \Psi \). Defining

\[
\begin{align*}
    u_i &= \frac{1}{2} \varepsilon^{ijkl} \chi_i \chi_j \chi_k \chi_l |0\rangle, \\
    d_i &= \frac{1}{2} \varepsilon^{ijkl} \chi_i \chi_j \chi_k \chi_l |0\rangle, \\
    u^c_i &= \chi_i \chi_j \chi_k \chi_l |0\rangle, \\
    d^c_i &= \chi_i |0\rangle, \\
    \nu &= \chi_5 |0\rangle, \\
    e &= \chi_4 |0\rangle, \\
    \nu^c &= \chi_1 \chi_2 \chi_3 \chi_5 |0\rangle, \\
    e^c &= \chi_1 \chi_2 |0\rangle,
\end{align*}
\]

where \( i, k, l = 1, 2, 3 \). Cf. also Ref [41], where the doublet and triplet blocks are interchanged.

The next task is to construct the charge operators. For example, the ladder operators associated with the left isospin take \( u \leftrightarrow d \) and \( \nu \leftrightarrow e \). They are therefore given by

\[
\begin{align*}
    I^+_L &= \chi_5 \chi_4, \\
    I^-_L &= \chi_4 \chi_5.
\end{align*}
\]

The weak isospin operator is hence

\[
I^3_L = \frac{1}{2} [I^+_L, I^-_L] = \frac{1}{2} (n_5 - n_4). \quad (A20)
\]

By comparison with the charge numbers in table \( \text{II} \) we can identify

\[
Y = \frac{1}{3} \sum_{i=1}^{3} n_i - \frac{1}{2} \sum_{j=4}^{5} n_j = \frac{1}{12i} (|[\Gamma_1, \Gamma_6] + [\Gamma_2, \Gamma_7] + [\Gamma_3, \Gamma_8]) - \frac{1}{8i} (|[\Gamma_4, \Gamma_9] + [\Gamma_5, \Gamma_{10}]), \quad (A21)
\]

where we have used

\[
[\Gamma_{5+j}, \Gamma_j] = -4in_j + 2i. \quad (A22)
\]
When identifying the indices of the $\Gamma$ operators with matrix rows and columns as implied by Eqn. \[A9\], we can explicitly write down the suitably normalized $Y$ in tensor representation:

$$Y = \text{diag} \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right) \otimes \sigma_2.$$ \[A23\]

Generally, we use as rule for conversion of the operator to the tensor representation

$$\left[\frac{1}{4}[\Gamma_i, \Gamma_j]\right]_{ab} = \delta_{ia} \delta_{jb} - \delta_{ja} \delta_{ib},$$ \[A24\]

which reads for the special case of the charge operators

$$\frac{1}{4} [\Gamma_{5+i}, \Gamma_i] = n_i - \frac{1}{2} = \text{diag} \left(\delta_{1i}, \delta_{2i}, \delta_{3i}, \delta_{4i}, \delta_{5i}\right) \otimes \sigma_2.$$ \[A25\]

Now, we easily find the other charge operators. Putting everything together, we have in spinor and in tensor representation

$$Q = \frac{1}{3} \sum_{i=1}^{3} n_i - n_4 = \text{diag} \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1/2, -1/2\right) \otimes \sigma_2,$$ \[A26\]

$$I_L^3 = \frac{1}{2} (n_5 - n_4) = \text{diag} \left(0, 0, 0, -1/2, 1/2\right) \otimes \sigma_2,$$

$$I_R^3 = \frac{1}{2} (1 - n_4 - n_5) = \text{diag} \left(0, 0, 0, -1/2, -1/2\right) \otimes \sigma_2,$$

$$B - L = \frac{2}{3} \sum_{i=1}^{3} n_i - 1 = \text{diag} \left(2/3, 2/3, 2/3, 0, 0\right) \otimes \sigma_2,$$

$$Y = \frac{1}{3} \sum_{i=1}^{3} n_i - \frac{1}{2} \sum_{j=4}^{5} n_j = \text{diag} \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1/2, -1/2\right) \otimes \sigma_2,$$

$$X = -2 \sum_{i=1}^{5} n_i + 5 = \text{diag} \left(-2, -2, -2, -2, -2\right) \otimes \sigma_2,$$

where we have used the normalization convention \[A1\].

The operator representation for the charge operators $Q$ is suitable for finding the charge eigenvalues $q$ of the spinors through $Q\Psi = q\Psi$.

Tensors can be constructed from the fundamental 10-dimensional vector $\Phi_{10}$ by taking antisymmetric products, such that a rank $n$ tensor is of dimension $10 \cdot 9 \cdot \ldots \cdot (10 - n + 1)/n!$. Explicitly, for the vector and the rank two tensor, the charges implied by the gauge-covariant derivatives are given by the eigenvalue equations

$$Q_q \Phi_{10} = q \Phi_{10},$$ \[A27\]

$$[Q_q, \Phi_{45}] = q \Phi_{45},$$

where $Q$ is acting here by matrix multiplication.

---

1 This is of course strictly speaking no equality but an assignment of an operator acting in Fock space to an operator acting in tensor space.
The Tensor Representations

In order to perform calculations such as $16.45.16$, $tr45^4$ and $16.10.16$, we need to identify the Standard Model multiplets within $10$ and $45$, just as we did for the $16$ in (A18). We first note, that under SU(5), the fundamental representation of SO(10) decomposes as $10 = 5 \oplus \bar{5}$. Let us denote an element of $5$ in the representation $10$ of SO(10) by $\Phi^5_{10}$, an element of $\bar{5}$ by $\bar{\Phi}^5_{10}$. Since they obey

$$X \Phi^5_{10} = 2 \Phi^5_{10} \quad \text{and} \quad X \bar{\Phi}^5_{10} = -2 \bar{\Phi}^5_{10},$$

they are of the form

$$\Phi^5_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} a_1 \\ \vdots \\ a_5 \\ -ia_1 \\ \vdots \\ -ia_5 \end{pmatrix} \quad \text{and} \quad \bar{\Phi}^5_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_5 \\ i\bar{a}_1 \\ \vdots \\ i\bar{a}_5 \end{pmatrix},$$

with $\sum_{i=1}^{5} |a_i|^2 = 1$ and $\sum_{i=1}^{5} |\bar{a}_i|^2 = 1$. To remove this inconvenient mixing of the upper and lower five-blocks, we introduce the unitary transformation

$$U_{\text{BLOCK}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1_5 & i1_5 \\ 1_5 & -i1_5 \end{pmatrix}, \quad U_{\text{BLOCK}}^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1_5 & 1_5 \\ -i1_5 & i1_5 \end{pmatrix},$$

such that

$$U_{\text{BLOCK}} \Phi^5_{10} = \begin{pmatrix} a_1 \\ \vdots \\ a_5 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{and} \quad U_{\text{BLOCK}} \bar{\Phi}^5_{10} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \bar{a}_1 \\ \vdots \\ \bar{a}_5 \end{pmatrix}.$$

By this change of basis, the charge operators become diagonal, for example

$$U_{\text{BLOCK}} X U_{\text{BLOCK}}^{-1} = 2 \begin{pmatrix} 1_5 & 0 \\ 0 & -1_5 \end{pmatrix}.$$
We can therefore immediately see how the entries of 45 transform under SU(5), namely

\[
U_{\text{BLOCK}} \Phi_{45} U_{\text{BLOCK}}^{-1} = \begin{pmatrix}
\begin{array}{c|c|c|c}
24 & 1 & 0 & 10 \\
\hline
10 & 24 & 1 & 0
\end{array}
\end{pmatrix},
\]

(A33)

where the single entries represent $5 \times 5$-blocks and the blocks in the upper left and the lower right are to be related to each other by the factor of minus one. The SU(5)-singlet 1 has here the form of the matrix $1/\sqrt{5} \mathbb{1}_5$. The arrangement of the $G_{SM}$-multiplets contained in 24 can be schematically written as

\[
\begin{array}{cccc|c|c|c}
1 & 2 & 3 & 4 & 5 \\
\hline
1 & \mathbb{1} & \mathbb{1} & \mathbb{1} & \mathbb{1} \\
2 & (8, 1, 0) & (1, 1, 0) & (3, 2, -\frac{5}{6}) & : \\
3 & \mathbb{1} & \mathbb{1} & \mathbb{1} & \mathbb{1} \\
4 & (\bar{3}, 2, \frac{5}{6}) & (3, 2, 0) & \mathbb{1} & \mathbb{1} \\
5 & \mathbb{1} & \mathbb{1} & \mathbb{1} & \mathbb{1}
\end{array}
\]

(A34)

and finally 10 of SU(5) decomposes into

\[
\begin{array}{cccc|c|c|c}
1 & 2 & 3 & 4 & 5 \\
\hline
1 & \mathbb{1} & \mathbb{1} & \mathbb{1} & \mathbb{1} \\
2 & (\bar{3}, 1, -\frac{2}{3}) & (3, 2, \frac{1}{6}) & : \\
3 & \mathbb{1} & \mathbb{1} & \mathbb{1} & \mathbb{1} \\
4 & -(3, 2, \frac{1}{6}) & (1, 1, 1) & \mathbb{1} & \mathbb{1} \\
5 & \mathbb{1} & \mathbb{1} & \mathbb{1} & \mathbb{1}
\end{array}
\]

(A35)

where the matrix is imposed to be antisymmetric, since it is identified with the antisymmetric part of $5 \otimes 5$ of SU(5).

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