Holographic Ricci dark energy and generalized second law

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Abstract

We explore the validity of the generalized second law (GSL) of thermodynamics in flat FRW universe with apparent horizon and event horizon as the boundary. We found that in a universe with holographic Ricci dark energy and dark matter, interacting with each other, the GSL is satisfied at the apparent horizon and partially satisfied at the event horizon under thermal equilibrium conditions. We also analyses the GSL under non-equilibrium conditions and shows that the fulfillment of GSL at the apparent horizon implies that the temperature of the dark energy is greater than that of the horizon. Thus there occurs a flow of dark energy towards the apparent horizon. As a result the entropy of the dark energy decreases and that of horizon increases. This is verified by finding the evolution of the dark energy entropy and horizon entropy in a dark energy dominated universe under non-equilibrium conditions.

Keywords: Dark energy, generalized second law, FRW universe.

1 Introduction

Observations on type Ia supernova have shown that the present universe is dominated by dark energy with negative pressure which provides a mechanism for the accelerating expansion of the universe [1, 2, 3]. Cosmic Microwave Background anisotropies also supporting the cosmic acceleration [4]. Various theoretical models of dark energy are proposed and are mainly concentrating on predicting the strength of this cosmic acceleration. Most of these models are based on scalar or multi scalar fields, e.g. quintessence model [5, 6, 7], K-essence model [8], Chaplygin gas model [9, 10], phantom model [11], quintom model [12]. An alternative approach in understanding the dark energy arises from the holographic principle which states that the number of degrees of freedom related directly to entropy which scales with the enclosing area of the system. On the basis of the cosmological state of the holographic principle proposed by Fischler and Susskind [13, 14], holographic model of dark energy has been proposed and studied widely in the literature [15, 16, 17, 18, 19, 20, 21]. The different dark energy models were mainly varied in the choice of the length scale $L$ appearing the dark energy density, $\rho_{de} = 3M_{pl}L^{-2}$. One of the recent models of dark energy is holographic Ricci dark energy model, where the IR length scale $L$ is taken as the average radius of Ricci scalar
curvature, $R^{-1/2}$ \[15, 19, 22\]. This model has the following advantages. First, the fine tuning problem can be avoided in this model. Secondly, the presence of event horizon is not presumed in this model, so the causality problem be avoided. Moreover the coincidence problem can also be solved effectively in this model. Recently a form of modified holographic Ricci dark energy model (MHRDE) was proposed \[23\] in connection with interaction between dark matter and dark energy. Detailed analysis of this model were found in the references \[24, 25, 26\]. Our study in this letter is with this model of modified holographic Ricci dark energy interacting with the dark mater in the flat FRW universe. Interaction models where the dark energy weakly interacts with dark matter will throw light on how the energy flow between the dark sectors takes place as the universe evolves. It is often assumed that the decay rate from one sector to the other is proportional to the Hubble parameter for a good fit to the expansion history of the universe as determined by the supernova and CMB data \[27\].

Motivated by the discovery of the black hole thermodynamics in the seventies, physicist have tried to analyses the thermodynamics of the cosmological models in an accelerated expanding universe \[28, 29, 30\]. It was Bekenstein and Hawking who proved that the entropy of the black hole is proportional to the area of it’s event horizon \[31, 32\]. By considering the entropy decrease of a black hole during it’s evaporation, the second law of thermodynamics was modified in such a way that the entropy of the black hole together with the entropy exterior to the blackhole is always increasing, and is known as the generalized second law (GSL). The thermodynamic properties of horizons were shown to exist in a more basic level by recasting the Einstein’s field equation in the form of the fist law of thermodynamics \[33, 34\]. Bakenstein’s idea was extended to cosmology by Gibbons and Hawking and have shown that cosmological event horizon also does possess entropy proportional to the horizon area \[35\]. They have shown it for de Sitter universe for which both apparent horizon and event horizon were coincide with each other. Since then many were studied about the validity of GSL in both apparent and event horizons \[36, 37, 38\]. In the cosmological scenario the GSL means that the rate of change of entropy of the horizon together with that of the fluid within the horizon will always greater than equal to zero.

\[
\frac{dS_{\text{horizon}}}{dt} + \frac{dS_{\text{fluid}}}{dt} \geq 0
\] (1)

For interacting holographic dark energy model, Setare \[39\] have derived the constraint on the deceleration parameter, for the fullfilment of the GSL in the non-flat universe enclosed by the event horizon. The first law of thermodynamics to the apparent horizon in FRW universe has been studied in \[40, 41\]. In references \[18, 30\] the validity of the GSL has been studied on the event horizon of a universe driven dark energy. Using a specific model of dark energy, Wang et. al \[42\] have proved that the GSL is obeyed at the apparent horizon of the universe but not on the event horizon. Validity of the GSL was studied for new holographic dark energy in reference \[43\] and have shown that, it is valid on the apparent horizon of the universe but partially valid on the event horizon of the universe. In reference \[44\], it has been argued that, in contrast to the apparent horizon, the GSL of thermodynamics breakdown if one consider the universe is to be enveloped by the event horizon. These studies favors the apparent horizon as the physical boundary of the universe as far as the thermodynamics is concerned. The GSL of thermodynamics has also been studied in the framework of Branworld \[45, 46\] and more general Levelock gravity \[47\].
At present there exist no unique theory which explaining the dark energy fully and since the modified holographic Ricci dark energy (MHRDE) is explaining the current acceleration of the universe without the assistance of fine tuning also explaining the coincidence problem, it is fair to analyze the thermodynamics of the universe with MHRDE. In this work we are examining the GSL of thermodynamics in a universe driven by MHRDE in interaction with dark matter and trying to answer the question whether Interacting MHRDE model favors apparent horizon or/and event horizon as the thermodynamic boundary. The paper is organized as follows. In section 2, we are presenting the relevant characteristics of the interacting MHRDE model. In section 3, we are presenting our analysis on the GSL under thermal equilibrium conditions both on the apparent horizon and event horizon of the universe. In section 4, we are briefly analyzing the conditions imposed by the validity of the GSL at the apparent horizon under non-equilibrium conditions. In section 5 we are presenting the conclusions.

2 Interacting MHRDE model

We consider a flat FRW universe described by the equation,

\[ H^2 = \frac{1}{3} (\rho_{de} + \rho_m) \]  

(2)

where \( H \) is the Hubble parameter, \( \rho_{de} \) is the dark energy density and \( \rho_m \) is dark matter density. The dark energy \( \rho_{de} \) is taken as the modified holographic Ricci dark energy (where the Ricci scalar is taken as the IR cut-off) \[23, 24\],

\[ \rho_{de} = \frac{2}{(\alpha - \beta)} \left( \frac{\dot{H}}{2} + \frac{3\alpha}{2} H^2 \right) \]  

(3)

where \( \dot{H} \) is the derivative of the \( H \) with respect to cosmic time, \( \alpha \) and \( \beta \) are model parameters. The interaction between the dark energy and dark matter can be included through the continuity equations,

\[ \dot{\rho}_{de} + 3H (\rho_{de} + p_{de}) = -Q \]  

(4)

\[ \dot{\rho}_m + 3H \rho_m = Q \]  

(5)

Where \( p_{de} \) is the pressure density of dark energy, dark matter is assumed to be pressure less, \( Q \) is the interaction term and over-dot representing derivative with respect to time. Since there is no conclusive theory for the microscopic origin of the interaction between the dark sectors, one has to assume the form of \( Q \) phenomenologically. The interaction term is chosen as, \( Q = 3bH\rho_m \), where \( b \) is the coupling constant and \( b > 0 \) means energy is transferring from dark energy to cold dark matter. From Friedmann equation the second order differential equation for the weighted Hubble parameter \( h = H/H_0 \) (where \( H_0 \) is present value of the Hubble parameter) can be obtained,

\[ \frac{d^2h^2}{dx^2} + 3 (\beta - b + 1) \frac{dh^2}{dx} + 9\beta (1 - b) h^2 = 0 \]  

(6)
where \( x = \ln a \) with \( a \) as the scale factor of the universe. The solution for this can be obtained as,

\[
h^2 = k_1 e^{-3\beta x} + k_2 e^{-3(1-b)x} \tag{7}
\]

where the constants \( k_1 \) and \( k_2 \) are evaluated using the initial conditions,

\[
h^2|_{x=0} = 1, \quad \frac{dh^2}{dx}|_{x=0} = 3\Omega_{de0}(\alpha - \beta) - 3\alpha \tag{8}
\]

where \( \Omega_{de0} \) is the value of present dark energy density and is related to present matter density \( \Omega_{m0} \) as \( \Omega_{de0} = 1 - \Omega_{m0} \) for the flat universe. The constants \( k_1 \) and \( k_2 \) are evaluated as

\[
k_1 = \frac{\Omega_{de0}(\alpha - \beta) - \alpha - b + 1}{1 - \beta - b}, \quad k_2 = 1 - k_1 \tag{9}
\]

Again using Friedmann equation the dark energy density parameter can be obtained as,

\[
\Omega_{de} = k_1 e^{-3\beta x} + k_2 e^{3(b-1)x} - \Omega_{m0} e^{-3x} \tag{10}
\]

From this the dark energy equation of state parameter can be calculated as,

\[
\omega_{de} = -1 + \left[ \frac{k_1 \beta e^{-3\beta x} + k_2 (1-b) e^{-3(1-b)x} - \Omega_{m0} e^{-3x}}{k_1 e^{-3\beta x} + k_2 e^{-3(1-b)x} - \Omega_{m0} e^{-3x}} \right] \tag{11}
\]

In the non-interaction case, with \( b = 0 \) and with \( \Omega_{de0} = 1 \), the coefficients, \( k_1 = 1 \) and \( k_2 = 0 \) and the equation of state reduces to the standard form, \( \omega_{de} = -1 + \beta \), confirming the earlier observations [26] in this regard. The value of the interaction coupling constant, is chosen as \( b = 0.003 \) in the present case, at which the model explaining the coincidence problem very well [48]. A sample of the evolution of the equation of state parameter with redshift \( z \) (note that \( (1+z) = e^{-x} \)) is shown figure in [1] for the model parameters [23, 24, 48] \((\alpha, \beta) = (1.01, -0.01), (1.2, 0.1)\). The figure shows

![Figure 1: Evolution of the equation of state of interacting MHRDE with redshift z for interaction coupling term b=0.003 and parameter(\alpha, \beta)=(1.01,-0.01)-thick line, (1.2,0.1)-thin line.](image)

that, for the parameters \((1.01,-0.01)\) the equation of state parameter is varying from zero at very large values of redshift and evolves to \( \omega_{de} \to -1 \) as \( z \to -1 \). The present value of \( \omega_{de} \) is found to
around -0.97 from the figure, which is very close to the WMAP value -0.93 \[52\]. as \( z \to -1 \), while for (1.2,0.1) the equation of state parameter \( \omega_{de} > -1 \) as \( z \to -1 \). The deceleration parameter \( q \) is found to be of the form

\[
q = -1 + \frac{3}{2} \left( \frac{k_1 \beta e^{-\beta x} - k_2 (b - 1) e^{-3(1-b)x}}{k_1 e^{-\beta x} + k_2 e^{-3(1-b)x}} \right)
\]  

(12)

The evolution of the \( q \) parameter with redshift is shown in figure 2 for the same set of model parameters. It is clear from the plot that the \( q \) parameter starts with value around 0.5 at large redshift and evolves to negative values, implying that the accelerated expansion commences in the recent past. For the parameters (1.01,-0.01) the transition to the accelerating phase at which \( q \) parameter become negative, is occurred at redshift \( z_T = 0.68 \), which very well in the observationally deducted range, \( Z_T = 0.45 - 0.70 \) \[53\]. The present value of the deceleration parameters from the plot is \( q_0 = -0.57 \) is very much near to the observational prediction -0.60 \[52\].

\[ \text{Figure 2: Evolution of the } q \text{-parameter of interacting MHRDE with redshift. The plots for } (\alpha,\beta) = (1.01,-0.01)-\text{thick line, (1.2,0.1)-thin line with coupling constant } b = 0.003. \]

3 **GSL with thermal equilibrium conditions**

Here we will analyze the validity of the generalized second law of thermodynamics both at the apparent horizon and event horizon as the boundary of the universe. We consider a flat FRW universe, filled with dark energy and dark matter with mutual interaction. According to GSL the entropy of the fluid within the horizon and entropy of the horizon together do not decrease with time \[49, 50\]. In analyzing the validity of GSL we assume that both the dark sectors are in equilibrium with the horizon, hence the temperature of both dark energy and dark matter are same as the temperature of the horizon. Such a thermal equilibrium is inevitable between the dark sectors because of the interaction between them. The dark sectors will soon come to equilibrium with the horizon. As a result the temperatures of all the components of the universe, will finally become equal to that of the the horizon temperature, \( T_h = 1/2\pi R_h \) \[35, 51\], where \( R_h \) is radius of the horizon. Further we will also assume that the first law of thermodynamics given by

\[
TdS = dE + PdV
\]  

(13)
is satisfied at the horizon (both apparent and event horizons) where $T, S, E, P, V$ are the temperature, entropy, internal energy, pressure and volume of the system.

We will analyze first the validity of the GSL at the apparent horizon, with horizon radius $R_A = 1/H$ which is also same as the Hubble horizon for the flat universe. From the above equation the rate of change of entropy of the dark energy and dark matter with respect to $x = \ln a$ are

$$S'_{de} = \frac{E'_{de} + P_{de}V'}{T}, \quad S'_m = \frac{E'_m}{T} \quad (14)$$

here the pressure due to dark matter is taken as zero and $T = T_h$ under equilibrium conditions. The volume and temperature are given as

$$V = \frac{4\pi^3}{3H^3}, \quad T_h = \frac{H}{2\pi} \quad (15)$$

The internal energies of the dark energy and dark matter are

$$E_{de} = \frac{4\pi \rho_{de}}{3H^3}, \quad E_m = \frac{4\pi \rho_m}{3H^3} \quad (16)$$

The entropy of the apparent horizon is $S_A = \pi/H^2$, so the derivative with respect to $x$ is

$$S'_A = \frac{-2\pi H'}{H^3} \quad (17)$$

From equations (14,17), the rate of change in total entropy with respect to $x$ is

$$S' = S'_{de} + S'_m + S'_A = \frac{2\pi}{H^2} + \frac{2\pi}{H^2} (1 + 12\pi (1 + \omega_{de}\Omega_{de})) (0.5 + 1.5\omega_{de}\Omega_{de}) \quad (18)$$

where we used the relation for the $q$–parameter for the flat universe,

$$q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} (1 + 3\omega_{de}\Omega_{de}) \quad (19)$$

where the dot represents the derivative with respect to time. In figure 3, we have plotted the evolution of the rate of total entropy, $S'$ with $x$ using equations (7),(10) and (11) for the model parameters $(\alpha, \beta) = (1.01, -0.01)$. This figure shows that the variation of the total entropy is always negative which implies that the GSL of thermodynamics is not satisfied at the apparent horizon for the model parameters $(1.01,-0.01)$. Next we will check the status of GSL for the model parameters $(1.2, 0.1), (1.2, 0.3), (1.3, 0.3), (1.4, 0.3)$, and the respective rate of the total entropy with $x$ is given in figure 4. The figure shows that the rate of total entropy is positive throughout the evolution of the universe, hence the GSL is satisfied always at apparent horizon for these parameters with MHRDE interacting with matter within the apparent horizon. This shows that the apparent horizon can be a good boundary for studying the cosmology and thermodynamics of the universe with interacting MHRDE. For parameter $(1.01,-0.01)$, the equation of state $\omega_{de} \to -1$, while for other parameters the $\omega_{de} > -1$ throughout the expansion of the universe. This indicates that apparent horizon of
the universe with interacting MHRDE satisfies the GSL of thermodynamics as far as $\omega_{de} > -1$. This limit on the equation of state parameter otherwise indicating that the universe is not entering the phantom phase of expansion at any stage of its evolution as far as $\omega_{de} > -1$. So it can be concluded that the universe consisting of interacting MHRDE satisfy the GSL at the apparent horizon for model parameters for which the universe doesn’t go into the phantom phase.

Next we consider the validity of GSL at the event horizon of the universe with interacting MHRDE. The rate of change of the entropy can be calculated using the relations [14]. For which the volume and temperature are,

\[ V = \frac{4\pi}{3} R_E^3, \quad T_E = \frac{1}{2\pi R_E}, \]

(20)

where $R_E$ is the event horizon radius of the universe. The event horizon distance of a universe can be obtained using the relation [43]

\[ R_E = a(t) \int_t^\infty \frac{dt'}{a(t')} = -\frac{1}{(1+z)H_0} \int_z^{-1} \frac{dz'}{h}. \]  

(21)
where $z$ is the redshift. With the equation (7), the event horizon radius is become

$$R_E = \left( \frac{\gamma}{H_0} \right) \frac{1}{\sqrt{k_2(1+z)^3(1-b)}} \times$$

$$2F_1[0.5, -\frac{0.5}{3\beta - 3(1-b)}, 1 - \frac{0.5}{3\beta - 3(1-b)}, -\frac{k_1}{k_2}(1+z)^{3\beta-3(1-b)}]$$

(22)

where $\gamma = 2.0182$, a constant and $2F_1$ is the hypergeometric function. Using equation (14), the sum of the entropies of dark energy and dark matter can be obtained as,

$$S_{de}' + S_m' = -24\pi^2 (1 + \omega_{de}\Omega_{de}) H R_E^3$$

(23)

Adding the entropy of the event horizon and using the standard relation $\dot{H} = HR_E - 1$ [51], the rate of the total entropy can be obtained as,

$$S' = S_{de}' + S_m' + S_E' = -24\pi^2 (1 + \omega_{de}\Omega_{de}) H R_E^3 + 2\pi R_E^2 - 2\pi H^{-1} R_E$$

(24)

which shows that the GSL is satisfied, i.e. $S' \geq 0$ if

$$\omega_{de}\Omega_{de} \leq -1 + \frac{1}{12\pi H R_E} \frac{d\log R_E}{dx}$$

(25)

This shows that as far as $R_E' \geq 0$, the equation of state $\omega_{de}$ is always less than -1 in a dark energy dominated universe, but on the other hand if $R_E' \leq 0 \omega_{de} < -1$ always. In figure 5 we plot $S'$ for the parameter set $(\alpha, \beta)=(1.01,-0.01)$, and it is showing that the GSL is satisfied at the event horizon only partially. The GSL is satisfied only in the remote past and far future in the evolution.

For other model parameters parameters it is clear form figure [6] that the GSL is not satisfied at

![Figure 5: The plot of $S'$ versus x for the model parameters $(\alpha, \beta)=(1.01,-0.01)$ at the event horizon of the universe with interaction coupling term $b=0.003$.](image)

the event horizon at large redshifts. But when the redshift is less than 2, the GSL is found satisfy with limiting condition that $S' = 0$. So it can be concluded that the GSL is not satisfied through out the evolution of the universe. These results shows that the event horizon of the universe with interacting MHRDE cannot be taken as physical boundary for the thermodynamics of the universe.
Here will analyze the validity of the GSL by assuming that, the different components are not in thermal equilibrium. So the dark energy and dark matter were evolved with different temperature and the horizon have a still different temperature. This is strictly possible when there is no interaction between the components of the universe. So we will take \( Q = 0 \) in the conservation equations \[4\] and \[5\]. The rate of change of the total entropy of the universe enclosed by the apparent horizon can be then obtained as

\[
S' = \frac{12\pi \Omega_{de}}{H} (1 + \omega_{de}) q + \frac{12\pi \Omega_{m}}{H} \frac{q}{T_{m}} = \frac{2\pi H'}{H^3} \tag{26}
\]

where \( T_{de} \) and \( T_{m} \) are the temperatures of the dark energy and dark matter respectively. The validity of the GSL requires that \( S' > 0 \), which gives a constraint on the temperature of the dark energy in a universe dominated with dark energy as

\[
T_{de} \geq -\left(\frac{6q}{1+q}\right) \left(1 + \omega_{de}\right)H \tag{27}
\]

By taking the observational value of \( q_0 \) and \( \omega_{de0} \) from WMAP results, -0.60 and -0.93 respectively, then it can be seen that \( T_{de0} > H_0 \). This inevitably shows that the temperature of the dark energy is greater than that of the event horizon (taking event horizon temperature as \( T_h \sim H \)). As a result of this, there occurs a flow of dark energy towards the apparent horizon. In literature there are examples for the anticipation of the flow of the fluid from within the horizon towards the horizon \[55\] \[50\]. This flow towards the apparent horizon would possibly lead to the crossing of the dark energy across the horizon. Hence the entropy of the dark energy will decrease as the universe evolves. But the validity of the GSL in turn requires that, the decrease in the dark energy entropy is balanced by the increase in the horizon entropy such that the total entropy will never decrease. Consider a universe dominated by dark energy. The entropy of the dark energy can be obtained from simple considerations as \[56\],

\[
S_{de} = \left(\frac{\rho_{de} + p_{de}}{T_{de}}\right) V = \frac{8\pi^2(1 + \omega_{de})\Omega_{de}}{cH^2} \tag{28}
\]
where we have taken the temperature of the dark energy is proportional to the horizon temperature as, \( T_{de} = c \frac{H}{2\pi} \) \([30]\), \( c \) is a numerical constant greater than one to ensure that the dark energy temperature is greater than that of the horizon. The entropy of the apparent horizon is,

\[
S_H = \frac{\pi}{H^2}
\] (29)

Using the expressions (7), (10) and (11), we have plotted \( S_{de} \) and \( S_H \& S_{total} \) for \( c = 1.25 \) and are shown in figures 7 and 8, where \( S_{total} = S_H + S_{de}. \) It is evident the figures that the entropy of the dark energy is decreasing, the entropy of the horizon is increasing and the total entropy comprising the entropies of the dark energy and horizon is also increasing as result the GSL is satisfied.

### 5 Conclusions

In this work we have considered the flat FRW universe filled with holographic Ricci dark energy and dark matter and analyzed the validity of the GSL of thermodynamics at the apparent and
event horizons of the universe. We have done the analysis for both thermal equilibrium and non-equilibrium conditions.

We have first analyzed the validity of the GSL under thermal equilibrium conditions at which the temperature of the dark energy and dark matter were equal to the temperature of the horizon. The thermal equilibrium is achieved mainly through the interaction between the dark energy and dark matter. We have found that, with modified holographic Ricci dark energy, the GSL is satisfied with apparent horizon as the boundary of the universe for all model parameters except $(1.01,-0.01)$. For the parameters $(1.01,-0.01)$, there exist a possibility for the universe to go into the phantom phase of expansion which seems to causes the partial violation of the GSL. Hence the apparent horizon can be taken as the physical boundary for thermodynamic considerations as far as the universe does not entering the phantom phase of expansion or equivalently $\omega_{de} < -1$. This conclusion is in agreement with many other works [36, 37, 38, 18, 17, 43, 44] with different forms of dark energies. In the case of the universe with event horizon as the boundary, we have found that the GSL is partially satisfied for all the parameters including $(1.01,-0.01)$. For the parameters $(1.01,-0.01)$ the GSL is satisfied at the recent past and recent future and violated in other regions. But for other parameters, it is clear from figure 5 that, GSL is not satisfied in the past corresponding to redshift $z > 2$. For $z < 2$, GSL is satisfied for all the model parameters with the condition $dS/dt = 0$. Because of this partial fulfillment of the GSL at the event horizon, it cannot be taken as a physical boundary of the universe. This is also in tune with conclusion with other models of dark energy as noted in the reference cited above.

Under non-equilibrium condition we have obtained the constraints imposed by the validity of the GSL at the apparent horizon. In this case the dark energy and dark matter are non-interacting, hence their temperatures are not equal and are different from the horizon temperature. We have found that for the GSL is to be satisfied the temperature of the dark energy must be greater than that of the apparent horizon. This implies that the dark energy can flow towards the horizon and even cross the horizon. As a result the entropy of the dark energy must decrease. On the other hand the entropy of the horizon and also the total entropy must increase which ensures the validity of the GSL. Assuming the temperature of dark energy, $T_{de} = c(H/2\pi)$, with $c > 1$, we have calculated the entropies of dark energy and horizon. The evolution of these entropies, figures 7 and 8, shows that the entropy of the dark is deceasing but the entropy of the horizon is increasing. The increase in the horizon entropy is compensating the decrease in the entropy of the dark energy, so that the total entropy is always increasing.

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