Exclusive $B \rightarrow M \ell^+\ell^-$ ($M = \pi$, $K$, $\rho$, $K^*$) Decays and Determinations of $|V_{ts}|$ (and $|V_{td}/V_{ts}|$)

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Abstract

We examine the possibility for precise determination of $|V_{ts}|$ (and $|V_{td}/V_{ts}|$) from the exclusive decays, $B \rightarrow K\ell^+\ell^-$, $B \rightarrow K^*\ell^+\ell^-$ (and $B \rightarrow \pi\ell^+\ell^-$, $B \rightarrow \rho\ell^+\ell^-$). We show that the ratio $|V_{ts}|$ can be extracted experimentally with a small theoretical uncertainty from hadronic form–factors, if we appropriately constrain kinematical regions of $q^2$. We also give detailed analytical and numerical results on the differential decay width $d\Gamma(B \rightarrow K^*\ell^+\ell^-)/dq^2$, and the ratios of integrated branching fractions, $B(B \rightarrow \rho\ell^+\ell^-)/B(B \rightarrow K^*\ell^+\ell^-)$. We estimate that one can determine the ratio $|V_{ts}|$ from those decays within theoretical accuracy of $\sim 10\%$.
1 Introduction

The determination of the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix is one of the most important issues of quark flavor physics. The precise determination of $V_{td}$ and $V_{ub}$ elements is of special significance, since they are closely related to the origin of CP violation in the Standard Model (SM). Furthermore, the accurate knowledge of these matrix elements can be useful in relating them to the fermion masses and also in searches for hints of new physics beyond the SM. For these reasons many strategies for the accurate determination of $V_{td}$ and $V_{ub}$ are under intensive investigation [1]. The main reason we are interested in $B$ physics is that this area is very likely to yield information about new physics beyond the SM. We expect that new physics will influence experimentally measurable quantities in different ways. For example, most of us expect that $\Delta B = 2$ transitions are more sensitive to new physics than decay rates. New physics may couple differently to $K$ mesons compared to $B$ mesons. Therefore, it is essential to determine the CKM matrix elements in as many different methods as possible.

In the existing literature, we can find several proposals of different methods for precise determination of $V_{td}$ and/or $|V_{td}/V_{ts}|$ [1, 2, 3, 4]:

- $|V_{td}|$ can be extracted indirectly through $B_d - \overline{B}_d$ mixing. However, in $B_d - \overline{B}_d$ mixing the large uncertainty of the hadronic matrix elements prevents us from extracting CKM elements with good accuracy.

- A better extraction of $|V_{td}/V_{ts}|$ can be made if $B_s - \overline{B}_s$ is measured as well, because the ratio $f_{B_d}^2 B_{B_d}/f_{B_s}^2 B_{B_s}$ can be predicted much more reliably.

- The determination of $|V_{td}/V_{ts}|$ from the ratios of rates of several hadronic two–body $B$ decays, such as $\Gamma(B^0 \rightarrow K^{*0}K^0)/\Gamma(B^0 \rightarrow \phi K^0)$, $\Gamma(B^0 \rightarrow K^{*0}K^{*0})/\Gamma(B^0 \rightarrow \phi K^{*0})$, $\Gamma(B^+ \rightarrow K^{*0}K^+)/\Gamma(B^+ \rightarrow \phi K^+)$, and $\Gamma(B^+ \rightarrow K^{*0}K^{*+})/\Gamma(B^+ \rightarrow \phi K^{*+})$ has also been proposed in [2].

- $V_{td}$ can be determined from $K \rightarrow \pi \nu \overline{\nu}$, $B \rightarrow \pi \nu \overline{\nu}$ and $B \rightarrow \rho \nu \overline{\nu}$ decays with small theoretical uncertainty [1, 3].

- In [4] a new method was proposed for the determination of $|V_{td}/V_{ts}|$ from the ratio of the inclusive decay distributions

$$\frac{d\mathcal{B}}{ds}(B \rightarrow X_d \ell^+ \ell^-)/\frac{d\mathcal{B}}{ds}(B \rightarrow X_s \ell^+ \ell^-),$$

where $s$ is the dilepton invariant mass.
In this work, in order to determine $|V_{ts}|$ (and the ratio $|V_{td}/V_{ts}|$) we carefully examine another well known method from an analysis of exclusive decay distributions

$$\frac{d\mathcal{B}}{ds}(B \to \pi \ell^+ \ell^-), \quad \frac{d\mathcal{B}}{ds}(B \to K \ell^+ \ell^-), \quad \frac{d\mathcal{B}}{ds}(B \to \rho \ell^+ \ell^-) \quad \text{and} \quad \frac{d\mathcal{B}}{ds}(B \to K^* \ell^+ \ell^-).$$

It is well known that the experimental investigation and detection of exclusive decays are much easier than those of inclusive ones, although the theoretical understanding of exclusive decays is complicated considerably by nonperturbative hadronic form factors. Exclusive $B \to M \ell^+ \ell^-$ decays have been previously studied in [3, 4] in the framework of the heavy quark effective theory [7]. Later these decays were also examined for new physics effect [8]. As is well known, the investigation of the rare decays $B \to \ell^+ \ell^-$ in future $B$ factories, KEK-B, SLAC-B, B-TeV and LHC-B, would provide us of one of the best way to determine $|V_{ts}|$ (and $|V_{td}/V_{td}|$).

This paper is organized as follows. In Section 2 we present analytic expressions for $d\Gamma/ds(B \to M \ell^+ \ell^-)$, $M = \pi, \rho, K, K^*$, and the ratios of branching fractions $\mathcal{B}(B \to \rho \ell^+ \ell^-)/\mathcal{B}(B \to K^* \ell^+ \ell^-)$ and $\mathcal{B}(B \to \pi \ell^+ \ell^-)/\mathcal{B}(B \to K \ell^+ \ell^-)$.

### 2 Theory of $B \to M \ell^+ \ell^-$ $(M = \pi, K, \rho, K^*)$ Decays

In the Standard Model the process $B \to M \ell^+ \ell^-$ $(M = \pi, K, \rho, K^*)$ is described at quark level by $b \to q \ell^+ \ell^-$ ($q = s, d$) transitions and receives contributions from $Z$ and $\gamma$ mediated penguins and box diagrams. The QCD corrected Hamiltonian for $b \to q \ell^+ \ell^-$ decay can be written as

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}^*V_{ts} \sum_{i=1}^{10} C_i \mathcal{O}_i + \frac{4G_F}{\sqrt{2}}V_{ud}^*V_{ub} \left[ C_1 (\mathcal{O}_1^n - \mathcal{O}_1) + C_2 (\mathcal{O}_2^n - \mathcal{O}_2) \right],$$

where $V_{ij}$ are the CKM matrix elements. The operators are given as

$$\mathcal{O}_1 = (\bar{q}_L \gamma_\mu b_L)^i (\bar{c}_{L,\beta} \gamma^\mu c_{L,\beta})^i,$$

$$\mathcal{O}_2 = (\bar{q}_L \gamma_\mu b_L)^i (\bar{c}_{L,\beta} \gamma^\mu c_{L,\beta})^i,$$

$$\mathcal{O}_3 = (\bar{q}_L \gamma_\mu b_L)^i \sum_{q' = u,d,s,c,b} (\bar{q}_{L,\beta}' \gamma^\mu q_{L,\beta})^i,$$

$$\mathcal{O}_4 = (\bar{q}_L \gamma_\mu b_L)^i \sum_{q' = u,d,s,c,b} (\bar{q}_{R,\beta}' \gamma^\mu q_{R,\beta})^i,$$

$$\mathcal{O}_5 = (\bar{q}_L \gamma_\mu b_L)^i \sum_{q' = u,d,s,c,b} (\bar{q}_{R,\beta}' \gamma^\mu q_{R,\beta})^i.$$
\[ \mathcal{O}_6 = (\bar{q}_L \gamma_{\mu} b_{L,\beta}) \sum_{q' = u, d, s, c, b} (\bar{q}_{R,\gamma}^{q'} \gamma^{\mu} q_{R,\alpha}) , \]

\[ \mathcal{O}_7 = \frac{e}{16\pi^2} \bar{q}_a \sigma_{\mu \nu} (m_b R + m_q L) b_{\alpha} F_{\mu \nu} , \]

\[ \mathcal{O}_8 = \frac{g}{16\pi^2} \bar{q}_a T_{\alpha \beta}^{a} \sigma_{\mu \nu} (m_b R + m_q L) b_{\beta} G_{\mu \nu} , \]

\[ \mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{q}_a \gamma^\mu L b_{\alpha}) (\bar{\ell} \gamma_{\mu} \ell) , \]

\[ \mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{q}_a \gamma^\mu L b_{\alpha}) (\bar{\ell} \gamma_{\mu} \gamma_5 \ell) , \]

\[ \mathcal{O}_1 = (\bar{q}_L \gamma_{\mu} b_{L,\beta}) (\bar{u}_{L,\beta} \gamma^\mu u_{L,\beta}) , \]

\[ \mathcal{O}_2 = (\bar{q}_L \gamma_{\mu} b_{L,\beta}) (\bar{u}_{L,\beta} \gamma^\mu u_{L,\alpha}) , \]

where \( L(R) = \frac{1}{2} (1 \pm \gamma_5) \) are the chiral projection operators.

Using the effective Hamiltonian in Eq. (1), the resulting QCD corrected matrix element for the decays \( b \to q \ell^+ \ell^- \) \((q = d, s)\) can be written as

\[ \mathcal{M} = \frac{G_F \alpha}{2\sqrt{2} \pi} \bar{V}_q V_b \left\{ C_{q_0}^{eff} \bar{q} \gamma_{\mu} (1 - \gamma_5) b \bar{\ell} \gamma_{\mu} \ell + C_{10}^{eff} \bar{q} \gamma_{\mu} (1 - \gamma_5) b \bar{\ell} \gamma_{\mu} \gamma_5 \ell \right\} - 2 C_{q_0}^{eff} \bar{q} i \sigma_{\mu \nu} \frac{q^{\nu}}{q^2} [m_q (1 - \gamma_5) + m_b (1 + \gamma_5)] b \bar{\ell} \gamma_{\mu} \ell , \] (2)

where

\[ C_{q_0}^{eff} = \mathcal{C}_9 + Y_{LD}^q (\hat{s}) . \] (3)

In Eq. (2), \( q' \) is the four momentum transfer to dileptons, \( \hat{s} = q^2/m_b^2 \) and

\[ \mathcal{C}_9 = C_9 \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \omega (\hat{s}) \right\} + Y_{SD}^q (\hat{s}) . \] (4)

The function \( Y_{SD}^q (\hat{s}) \) is the one–loop matrix element of \( \mathcal{O}_9 \) and \( Y_{LD}^q (\hat{s}) \) describes the long distance contributions due to the vector \( J/\psi, \psi' \), \( \cdots \) resonances. The function \( \omega (\hat{s}) \) represents the one gluon correction to the matrix element of the operators \( \mathcal{O}_9 \). Its explicit form can be found in [3, 12]. The explicit forms of the two functions \( Y_{SD}^q (\hat{s}) \) and \( Y_{LD}^q (\hat{s}) \) are given [4] as

\[ Y_{SD}^q (\hat{s}) = g (\hat{m}_c, \hat{s}) [3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6] \]

\[ - \frac{1}{2} g (1, \hat{s}) [4C_3 + 4C_4 + 3C_5 + C_6] \]

\[ - \frac{1}{2} g (0, \hat{s}) [C_3 + 3C_4] + \frac{2}{9} [3C_3 + C_4 + 3C_5 + C_6] \]

\[ - \frac{V_{ub}^* V_{ub}}{V_{tb}^* V_{tb}} [3C_1 + C_2] [g (0, \hat{s}) - g (\hat{m}_c, \hat{s})] , \]
\[ Y_{LD}^q (\hat{s}) = \frac{3}{\alpha^2} \kappa \left\{ - \frac{V_{cd}^* V_{tb}}{V_{tb}^* V_{tb}} C^{(0)} - \frac{V_{us}^* V_{ub}}{V_{tb}^* V_{tb}} [3C_3 + C_4 + 3C_5 + C_6] \right\} \]
\[ \times \sum_{V_i=\psi(1s),..,\psi(6s)} \frac{\pi \Gamma (V_i \to \ell^+ \ell^-) M_{V_i}}{(M_{V_i}^2 - \hat{s} m_b^2 - i M_{V_i} \Gamma_{V_i})}. \]

The function \( g(\hat{m}_q, \hat{s}) \) arises from the one loop contributions of the four quark operators \( \mathcal{O}_1-\mathcal{O}_6 \), \textit{i.e.},
\[ g(\hat{m}_q, \hat{s}) = -\frac{8}{9} \ln \hat{m}_q + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9} (2 + y_q) \sqrt{|1 - y_q|} \]
\[ \times \left\{ \Theta(1 - y_q) \left( \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} - i \pi \right) + \Theta(y_q - 1) 2 \arctan \frac{1}{\sqrt{y_q - 1}} \right\}, \]
where \( y_q = 4 \hat{m}_q^2/\hat{s} \). In Eq. (5), \( C^{(0)} \equiv 3C_1 + 2C_2 + 3C_3 + 4C_4 + 3C_5 + C_6 \). Using \( m_t = 175 \text{ GeV} \), \( m_b = 4.8 \text{ GeV} \), \( m_c = 1.4 \text{ GeV} \), \( \alpha_s (m_W) = 0.12 \) and \( \alpha_s (m_b) = 0.22 \), the numerical values of the Wilson coefficients, which we will use in our further numerical analysis, are
\[ C_1 = -0.26, \quad C_2 = 1.11, \quad C_3 = 0.01, \]
\[ C_4 = -0.03, \quad C_5 = -0.03, \quad C_6 = -0.03, \]
\[ C_7 = -0.32, \quad C_9 = 4.26, \quad \text{and} \quad C_{10} = -4.62. \]

The values of \( C_9 \) and \( C_{10} \) are very sensitive to \( m_t \). We will neglect the second term in \( Y_{LD}^q (\hat{s}) \), since \( 3C_3 + 4C_4 + 3C_5 + C_6 < C_6 \). Masses, widths and leptonic branching ratios of the \( J^P = 1^- \) \( c\bar{c} \) resonances are presented in [13]. Note that as far as short distance effects are considered, the \( u \)-loop matrix element contribution to the \( b \to s \ell^+ \ell^- \) process is negligible due to the smallness of \( V_{us}^* V_{ub} \) compared to \( V_{cb}^* V_{cs} \simeq -V_{ts}^* V_{tb} \), while in the \( b \to d \ell^+ \ell^- \) case, the term proportional to \( V_{ud}^* V_{ub} \) is kept. The factor \( \kappa \) is chosen to have the value \( \kappa = 2.3 \) [14] to reproduce the rate of decay \( B \to X_s J/\psi \to X_s \ell^+ \ell^- \). The phase of \( \kappa \) is fixed, since recent experimental data have determined the sign of the ratio of factorization approach parameters \( a_2/a_1 \) and the phase of \( a_1 \) is expected to be near its perturbative value [13].

At this point, there arises the problem of computing the matrix elements of Eq. (2) between the mesons \( B \) and \( M \) (\( M = \pi, K, \rho, K^* \)) states. The matrix element \( \langle M | \mathcal{M} | B \rangle \) has been investigated in the framework of different approaches, such as chiral perturbation theory [16], three point QCD sum rules [17], relativistic quark model [18], effective heavy quark theory [5], and light cone QCD sum rules [19]–[22].

The matrix elements for \( B \to P \ell^+ \ell^- \ (P = \pi, K) \) decays can be written in terms of the form–factors
\[ \langle P(p_2) | q \gamma_\mu (1 - \gamma_5) b | B(p_1) \rangle = f_+(q^2)(p_1 + p_2)_\mu + f_-(q^2)q_\mu, \]
\[ \langle P(p_2) | \bar{q} i \sigma_{\mu\nu} q' b | B(p_1) \rangle = \left[ (p_1 + p_2)_{\mu} q^2 - (m_B^2 - m_p^2) q_{\mu} \right] \frac{f_T(q^2)}{m_B + m_p}, \] (7)

where \( q = p_1 - p_2 \).

The matrix elements for \( B \to V \ell^+ \ell^- \) (\( V = \rho, \ K^* \)) decays are defined as follows

\[ \langle V(p_2, \epsilon) | \bar{q} \gamma_{\mu}(1 - \gamma_5)b | B(p_1) \rangle = \]
\[ - \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu p_2^\alpha q^\beta \frac{2V(q^2)}{m_B + m_V} - i\epsilon_{\mu}^*(m_B + m_V)A_1(q^2) + i(p_1 + p_2)_{\mu}(\epsilon^* q) \frac{A_2(q^2)}{m_B + m_V} \]
\[ + iq_{\mu}(\epsilon^* q)\frac{2m_V}{q^2} \left[ A_3(q^2) - A_0(q^2) \right], \] (8)

\[ \langle V(p_2, \epsilon) | \bar{q} i \sigma_{\mu\nu} q''(1 + \gamma_5)b | B(p_1) \rangle = \]
\[ 4\epsilon_{\mu\nu\alpha\beta} \epsilon^\nu p_2^\alpha q^\beta T_1(q^2) + 2i \left[ \epsilon^*_{\mu}(m_B^2 - m_V^2) - (p_1 + p_2)_{\mu}(\epsilon^* q) \right] T_2(q^2) + \]
\[ + 2i(\epsilon^* q) \left[ q_{\mu} - (p_1 + p_2)_{\mu} \frac{q^2}{m_B^2 - m_V^2} \right] T_3(q^2), \] (9)

where \( \epsilon \) is the 4–polarization vector of the \( V \)–meson. Using the equation of motion, the form–factor \( A_3(q^2) \) can be written as a linear combination of the form–factors \( A_1(q^2) \) and \( A_2(q^2) \) (see [17]).

\[ A_3(q^2) = \frac{m_B + m_V}{2m_V} A_1(q^2) - \frac{m_B - m_V}{2m_V} A_2(q^2), \]

with the condition \( A_3(q^2 = 0) = A_0(q^2 = 0) \).

Using Eqs. (2), (7), (8), and (9) and summing over the final lepton polarization for \( B \to P \ell^+ \ell^- \) and \( B \to V \ell^+ \ell^- \) decay widths, we get

\[ \frac{d\Gamma}{ds}(B \to P \ell^+ \ell^-) = \frac{G^2 \alpha^2 m_B^5}{2^{8/3} \pi^5} |V_{tb} V_{tb}^\ast|^2 \lambda^{3/2} \]
\[ \times \left\{ \left[ 2m_b C_7 \left( -\frac{f_T(q^2)}{m_B + m_p} \right) + C_9^{\text{eff}} f_+(q^2) \right]^2 + |C_{10} f_+(q^2)|^2 \right\}, \] (10)

\[ \frac{d\Gamma}{ds}(B \to V \ell^+ \ell^-) = \frac{G^2 \alpha^2 m_B^3}{2^{12/3} \pi^5} |V_{tb} V_{tb}^\ast|^2 s \lambda^{1/2} \]
\[ \times \left\{ 16\lambda m_B^4 \left[ |A|^2 + |C|^2 \right] + 2 \left[ |B_1|^2 + |D_1|^2 \right] \frac{\lambda + 12rs}{rs} + 2 \left[ |B_2|^2 + |D_2|^2 \right] \frac{m_B^4 \lambda^2}{rs} \right\} \]
\[ - 4 \left[ \text{Re}(B_1 B_2^\ast) + \text{Re}(D_1 D_2^\ast) \right] \frac{m_B^2 \lambda}{rs} \}, \] (11)

where \( \lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs, r = m_M^2/m_B^2, s = q^2/m_B^2 \). In Eq. (11) \( A, B_1, B_2, C, D_1 \) and \( D_2 \) are defined as

\[ A = C_9^{\text{eff}} \frac{V(q^2)}{m_B + m_V} + 4C_7 \frac{m_b}{q^2} T_1(q^2), \]
\[ B_1 = C^\text{eff}_9 (m_B + m_V) A_1(q^2) + 4 C_7 \frac{m_b}{q^2} (m_B^2 - m_V^2) T_2(q^2), \]

\[ B_2 = C^\text{eff}_9 \frac{A_2(q^2)}{m_B + m_V} + 4 C_7 \frac{m_b}{q^2} \left( T_2(q^2) + \frac{q^2}{m_B^2 - m_V^2} T_3(q^2) \right), \]

\[ C = C_{10} \frac{V(q^2)}{m_B + m_V}, \]

\[ D_1 = C_{10} (m_B + m_V) A_1(q^2), \]

\[ D_2 = C_{10} \frac{A_2(q^2)}{m_B + m_V}. \]

3 Numerical analysis and discussions

Now we consider the differential decay widths, \( d\Gamma/dq^2(B \to (\pi, \rho, K, K^*) + \ell^+ + \ell^-) \). The hadronic formfactors in the framework of light-cone QCD sum rules have been calculated in \[19\]–\[22\]. For those values of the formfactors, we have used the results of \[19, 20\], where the radiative corrections to the leading twist contribution and \( SU(3) \) breaking effects are also taken into account. The \( q^2 \) dependence of the formfactors can be represented in terms of three parameters as

\[ F(q^2) = \frac{F(0)}{1 - a_F \frac{q^2}{m_b^2} + b_F \left( \frac{q^2}{m_b^2} \right)^2}, \]

where the values of parameters \( F(0) \), \( a_F \) and \( b_F \) for the relevant decays, \( B \to \pi \), \( B \to K \), \( B \to \rho \) and \( B \to K^* \), are listed in Table 1. We note that light-cone QCD sum rules method is applicable for those decays in the region of \( m_b^2 - q^2 = \text{few GeV}^2 \), and we found that sum rules works very well up to \( q^2 = 20 \text{ GeV}^2 \). In order to extend our investigation to the full physical phase space, we use the above mentioned parametrization in such a way that up to \( q^2 = 20 \text{ GeV}^2 \) it successfully reproduces the light-cone QCD sum rules predictions.

A few words about error analysis in the differential decay rates and their ratios are in order. In both cases the errors which come from different form–factors are added quadratically since they are theoretically independent of each other. Note also that all errors, which come from the uncertainties of the \( b \) quark mass, the Borel parameter variation, wave functions, non–inclusion of higher twists and radiative corrections, are added in quadrature. The uncertainty in the ratio is estimated as the half distance between the maximum and minimum
In regard to the determination of $|V_{ts}|$, we first consider $B \rightarrow K^* \ell^+ \ell^-$. Since the expression for $B \rightarrow V \ell^+ \ell^-$ decays contains many form–factors (see Eq. (11)), each with its own uncertainty, the error in the differential decay width may be substantial. To reduce these uncertainties, it would be better to choose a kinematical region, in which the contributions from most of these form–factors will be practically negligible. Therefore, we propose to consider the end–point region, $16.5 \text{ GeV}^2 < q^2 < 19.25 \text{ GeV}^2$. In this region the contributions from the terms that are proportional to $1/\lambda$ are much smaller than those from terms proportional to $\sim C_9$ or $\sim C_{10}$. This is due to the fact that the terms which are proportional to $\sim C_7$ contain $1/\lambda^2$. In this region the decay width takes the following form

$$
\frac{d\Gamma}{ds} = \frac{G^2 \alpha^2 m_B^5}{2^{11/3} \pi^5} |V_{tb} V_{*tb}|^2 s \lambda^{1/2} \times \left[ |\tilde{C}_9|^2 + |C_{10}|^2 \right] \left( 1 + \sqrt{\lambda} \right)^2 |A_1|^2 \frac{\lambda + 12 r s}{r (1 + \delta)(1 + \Delta)(1 + d)} ,
$$

(12)

where $\delta$ represents the contributions from the form–factors $V$ and $A_2$. $\Delta$ takes into account the magnetic momentum operator contributions ($\sim \mathcal{O}_7$), and $d(q^2)$ parametrizes the long distance effects. In the above–mentioned region, the long distance contributions are taken into account in the following way:

$$
|C_{eff}^6|^2 + |C_{10}|^2 \simeq \left[ |\tilde{C}_9|^2 + |C_{10}|^2 \right] \left[ 1 + d(q^2) \right] .
$$

| \text{Parameter} | $A_1^{B\rightarrow \rho}$ | $A_2^{B\rightarrow \rho}$ | $V^{B\rightarrow \rho}$ | $T_1^{B\rightarrow \rho}$ | $T_2^{B\rightarrow \rho}$ | $T_3^{B\rightarrow \rho}$ | $f_+^{B\rightarrow \pi}$ | $f_-^{B\rightarrow \pi}$ |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $F(0)$           | 0.26 ± 0.04     | 0.29 ± 0.04     | 0.34 ± 0.05     | 0.15 ± 0.02     | 0.15 ± 0.02     | 0.10 ± 0.02     | 0.30 ± 0.04     | −0.30 ± 0.04    |
| $a_F$            | 0.29            | 0.93            | 1.37            | 1.41            | 0.28            | 0.19            | 1.35            | 1.34            |
| $b_F$            | −0.415          | −0.092          | 0.315           | 0.361           | −0.500          | −0.076          | 0.270           | 0.260           |
| $F(0)$           | 0.34 ± 0.05     | 1.18 ± 0.14     | 0.19 ± 0.03     | 0.19 ± 0.03     | 0.19 ± 0.03     | 0.13 ± 0.02     | 0.35 ± 0.05     | −0.39 ± 0.05    |
| $a_F$            | 0.60            | 0.97            | 1.59            | 1.59            | 0.49            | 1.20            | 1.37            | 1.37            |
| $b_F$            | −0.023          | 0.55 ± 0.29     | 0.615           | 0.615           | −0.241          | 0.098           | 0.350           | 0.370           |

Table 1: $B$ meson decay form factors in a three-parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account (This Table is taken from [19, 20]).
In order to estimate the uncertainties which arise from $d(q^2)$, $\Delta(q^2)$ and $\delta(q^2)$, we present their $q^2$ dependences in Figs. 1, 2 and 3, respectively. In Fig. 1 we take into account all six $J^{PC} = 1^{--}$ $c\bar{c}$ resonance states. From this figure we observe that the long distance contribution is about $\pm 0.18$ of the short distance contribution. As is obvious from Fig. 2, the contribution of the magnetic dipole operator is about $\Delta \simeq -0.13 \pm 0.01$, whose behavior is observed to be almost independent of $q^2$. The same contribution calculated in the framework of the HQET gives numbers quite close to our result, in the range $-0.18 < \Delta < -0.14$ [13], in the same region. In Fig. 3, the dependence of $\delta(q^2)$ on $q^2$ is depicted. As is clear from this figure, the contribution of $\delta$ to the differential decay width is substantial at lower values of $q^2$. However, the error due to the uncertainties in the form–factors only causes about $\sim 1\%$ deviation from the case where central values of all form–factors are used. In Fig. 4 we present the dependence of $d\Gamma(B \to K^*\ell^+\ell^-)/dq^2$ on $q^2$ and indeed, with the observed oscillatory behavior, the uncertainty that $d(q^2)$ brings to the $B \to K^*\ell^+\ell^-$ differential decay rate is about $\sim 8\%$, although its leading uncertainty to the perturbative contribution is about $\pm 0.18$.

We note that the extraction of $|V_{td}/V_{ts}|$ from the decay widths $B \to \rho \ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ in the low invariant mass region, for example $2 \text{ GeV}^2 < q^2 < m_{J/\psi}^2$, becomes more problematic. In this region the contribution of the magnetic dipole operator $O_7$ is large. Therefore the theoretical predictions of the decay widths of the processes $B \to \rho \ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ have significant uncertainties in this region, since the form–factors $T_1$, $T_2$ and $T_3$ play an essential role. In our opinion, it is better to investigate the $B \to \pi \ell^+\ell^-$ and $B \to K \ell^+\ell^-$ decays in this low momentum region for the determination of the ratio $|V_{td}/V_{ts}|$, since both decays are described only by the two form–factors $f_+$ and $f_T$ (when lepton masses are neglected). Various theoretical schemes predict that $f_+(0) \simeq -f_T(0)$ (see [17]–[22]). In this region of $q^2$, that is far from $q^2_{\text{max}}$, the $B^*$ pole contribution [14] is unlikely to violate the SU(3) symmetry predictions.

We now show the possibility for extracting $|V_{td}/V_{ts}|$ from the ratio

$$
\frac{dR}{dq^2} \equiv \frac{d}{dq^2} \left[ \mathcal{B}(B \to \rho \ell^+\ell^-)/\mathcal{B}(B \to K^*\ell^+\ell^-) \right],
$$

(13)

and the ratio

$$
\frac{dR_1}{dq^2} \equiv \frac{d}{dq^2} \left[ \mathcal{B}(B \to \pi \ell^+\ell^-)/\mathcal{B}(B \to K \ell^+\ell^-) \right] \simeq \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{f^{B\to\pi}(q^2)}{f^{B\to K}(q^2)}^2.
$$

(14)

Performing the integrations over $q^2$, from $16.5 \text{ GeV}^2$ to $19.25 \text{ GeV}^2$ in Eq. (13) and from 2
GeV$^2$ to 9 GeV$^2$ in Eq. (14), i.e.

$$
R(16.5 < q^2 < 19.25 \text{ GeV}^2) = \frac{\mathcal{B}(B \to \rho \ell^+ \ell^-)}{\mathcal{B}(B \to K^* \ell^+ \ell^-)},
$$

$$
R_1(2 < q^2 < 9 \text{ GeV}^2) = \frac{\mathcal{B}(B \to \pi \ell^+ \ell^-)}{\mathcal{B}(B \to K \ell^+ \ell^-)},
$$

we get the numerical results

$$
R = 0.85 \pm 0.06,
$$

$$
R_1 = 0.77 \pm 0.06,
$$

(normalized to $|V_{td}/V_{ts}|^2$).

At this point we estimate the number of expected events of the types $B \to V \ell^+ \ell^-$ and $B \to P \ell^+ \ell^-$ in experiments at future $B$ factories. Future symmetric and asymmetric $B$ factories of electronic and hadronic colliders should produce much more than $10^9$ $B$–$\bar{B}$ mesons per year by the year 2010. Assuming $10^9$ $B$ mesons effectively reconstructed, the number of expected events in the corresponding kinematical regions ($16.5 \text{ GeV}^2 < q^2 < 19.25 \text{ GeV}^2$ for $B \to V \ell^+ \ell^-$ decay and $2 \text{ GeV}^2 < q^2 < 9 \text{ GeV}^2$ for $B \to P \ell^+ \ell^-$ decay) are calculated to be

$$
N(B \to K^* \ell^+ \ell^-) = \mathcal{B}(B \to K^* \ell^+ \ell^-) \times 10^9 \sim 140,
$$

$$
N(B \to K \ell^+ \ell^-) = \mathcal{B}(B \to K \ell^+ \ell^-) \times 10^9 \sim 735,
$$

$$
N(B \to \rho \ell^+ \ell^-) = \mathcal{B}(B \to \rho \ell^+ \ell^-) \times 10^9 \sim 5,
$$

$$
N(B \to \pi \ell^+ \ell^-) = \mathcal{B}(B \to \pi \ell^+ \ell^-) \times 10^9 \sim 28.
$$

As can be seen easily, for determination of $|V_{td}|$ or $|V_{td}/V_{ts}|$ we need much more than $10^9$ $B$ mesons produced, and it will be only possible at future hadronic $B$ factories, such as B-TeV and LHC-B, where the double lepton triggering helps high reconstruction efficiencies with more than $10^{11}$ $B$ mesons per year produced. From the above results we conclude that these decays have a good chance to be detected at future $B$ factories.

The problem of accurate determination of CKM matrix elements with different methods receives special attention in connection with the fact that the next generation of $B$ meson decay experiments will be a test of the flavor sector of the SM at high precision as well as allowing the determination of $V_{td}$ and $V_{ub}$ with very high accuracy. Simultaneous determinations of CKM angles and phases $[1, 23]$ would be extremely important to check the consistence within the SM and to search for the hints of new physics beyond the SM. It is well known that the experimental investigation and detection of exclusive decays is much
easier than those of inclusive ones, while the theoretical understanding of exclusive decays is much more complicated due to nonperturbative hadronic form factors. We have found that for the determinations of $|V_{ts}|$ (and $|V_{td}/V_{ts}|$), due to the character of the relevant hadronic form factors, the kinematic region of small $q^2$ is more useful for the $B \to P\ell^+\ell^-$ decays, while the kinematical region of $q^2$ near the end–point is more suitable for the $B \to V\ell^+\ell^-$ decays. And we estimated numerically $|V_{ts}|$ and the ratio $|V_{td}/V_{ts}|$ from the ratios of those decays with a theoretical uncertainty of $\sim 10\%$.

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**Figure Captions**

1. Dependence of $d$ on $q^2$ for $B \rightarrow K^*\ell^+\ell^-$ decay. Dash–dotted $d(q^2) = 0$ line corresponds to the short distance contributions.

2. Dependence of $\Delta$ on $q^2$ for $B \rightarrow K^*\ell^+\ell^-$ decay. Dotted and dash–dotted curves correspond to the cases when the uncertainty is added and subtracted from the central values of all form–factors, respectively.

3. Dependence of $\delta$ on $q^2$ for $B \rightarrow K^*\ell^+\ell^-$ decay. Dotted and dash–dotted curves correspond to the cases when the uncertainty is added and subtracted from the central values of all form–factors, respectively.

4. Dependence of $d\Gamma(B \rightarrow K^*\ell^+\ell^-)/dq^2$ on $q^2$ in units of

$$
\frac{G^2\alpha^2 m_B^3}{2^{11}3\pi^5} |V_{ts}V_{tb}^*|^2 \left[ |\tilde{C}_9|^2 + |C_{10}|^2 \right] \left( 1 + \sqrt{r} \right)^2 .
$$

In this figure the wavy curve numbered as 2 represents the case that takes into account all $1^{--}$ states of $c\bar{c}$ resonances for the central values of all form–factors, while the wavy curves numbered as 1 and 3, correspond to the cases when the uncertainty is added and subtracted from the central values of all form–factors, respectively. The dash–dotted line represents the contribution of only the three lightest resonances. The dotted curve, on the other hand, is for the perturbative result, i.e., $d(q^2) = 0$. 

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References

[1] For example, for the relation between $V_{td}$ and $V_{ub}$, see T. M. Aliev and C. S. Kim, *Phys. Rev.* D58 (1998) 013003.

[2] M. Gronau and J. L. Rosner, *Phys. Lett.* B376 (1996) 205.

[3] G. Buchalla and A. Buras, *Phys. Rev.* D54 (1996) 6782.

[4] C. S. Kim, T. Morozumi and A. I. Sanda, *Phys. Rev.* D56 (1997) 7240.

[5] W. Roberts, *Phys. Rev.* D54 (1996) 863.

[6] W. Roberts and F. Ledroit, *Phys. Rev.* D53 (1996) 3643.

[7] C. Greub, A. Ioannissian and D. Wyler, *Phys. Lett.* B346 (1995) 149.

[8] N. Isgur, M. B. Wise, *Phys. Rev.* D41 (1990) 151;
   M. B. Wise, *Phys. Rev.* D45 (1992) 2188;
   G. Burdman and J. F. Donoghue, *Phys. Lett.* B280 (1992) 287;
   L. Wolfenstein, *Phys. Lett.* B291 (1992) 177;
   T. M. Yan et. al. *Phys. Rev.* D46 (1992) 1148;
   G. Burdman, Z. Ligeti, M. Neubert and Y. Nir, *Phys. Rev.* D49 (1994) 2331.

[9] B. Grinstein, M. J. Savage and M. B. Wise, *Nucl. Phys.* B319 (1989) 271.

[10] M. Misiak, *Nucl. Phys.* B398 (1993) 23; Erratum, *ibid.* B439 (1995) 461.

[11] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini,
   Phys. Lett. B 316 (1993) 127;
   M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Nucl. Phys. B 415 (1994) 403;
   G. Cella, G. Curci, R. Ricciardi and A. Vicere, Nucl. Phys. B 421 (1994) 41;
   *ibid.* Phys. Lett. B 325 (1994) 227.

[12] A. J. Buras and M. Münz, Phys. Rev. D 52 (1995) 186.

[13] Particle Data Group, *Phys. Rev.* D54 (1996) 1.

[14] Z. Ligeti and M. Wise, *Phys. Rev.* D53 (1996) 4937.

[15] T. E. Browder, K. Honscheid and D. Pedrini,
   *Ann. Rev. Nucl. Part. Sci.* 46 (1996) 395.
[16] R. Casalbuoni et. al., Phys. Rep. 281 (1997) 145.

[17] P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, 
Phys. Rev. D53 (1996) 3672; Phys. Lett. B395 (1997) 339.

[18] W. Jaus and D. Wyler, Phys. Rev. D41 (1990) 3405; 
D. Melikhov, N. Nikitin and S. Simula, hep-ph/9704268 (April 1997).

[19] P. Ball, hep-ph/9802394 (February 1998).

[20] P. Ball, V. M. Braun, hep-ph/9805422 (May 1998).

[21] T. M. Aliev, A. Özpıneci and M. Savcı and H. Koru, Phys. Lett. B400 (1997) 194.

[22] T. M. Aliev, A. Özpıneci and M. Savcı, Phys. Rev. D56 (1997) 4260.

[23] A. S. Dighe, M. Gronau and J. L. Rosner, Phys. Rev. D54 (1996) 3309; C. S. Kim, D. London and T. Yoshikawa, Phys. Rev. D57 (1998) 4010.
Figure 1:

Figure 2:
Figure 3:

\[ \delta(B \rightarrow K^+\ell^+\ell^-) \]

\[ q^2 \ (GeV^2) \]

Figure 4:

\[ \frac{d\Gamma}{dq^2}(B \rightarrow K^{*+}\ell^+\ell^-) \]

\[ q^2 \ (GeV^2) \]