An Optimization Framework for Task Sequencing in Curriculum Learning

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Abstract—Curriculum learning is gaining popularity in (deep) reinforcement learning. It can alleviate the burden on data collection and provide better exploration policies through transfer and generalization from less complex tasks. Current methods for automatic task sequencing for curriculum learning in reinforcement learning provided initial heuristic solutions, with little to no guarantee on their quality. We introduce an optimization framework for task sequencing composed of the problem definition, several candidate performance metrics for optimization, and three benchmark algorithms. We experimentally show that the two most commonly used baselines (learning with no curriculum, and with a random curriculum) perform worse than a simple greedy algorithm. Furthermore, we show theoretically and demonstrate experimentally that the three proposed algorithms provide increasing solution quality at moderately increasing computational complexity, and show that they constitute better baselines for curriculum learning in reinforcement learning.

Index Terms—Curriculum Learning, Transfer Learning, Reinforcement Learning

I. INTRODUCTION

Reinforcement Learning (RL) has recently been successfully applied to a number of tasks whose complexity would have appeared overwhelming only a few years ago [1], [2]. In such large and complex environments, classical exploration strategies designed for Markov Decision Processes (MDPs), aiming at visiting every state the most efficiently, are inadequate. One approach actively investigated is the use of transfer learning [3] to generalize from previous similar tasks, and more recently the application of transfer learning to sequences of tasks of increasing complexity forming a curriculum. Curriculum Learning is often employed in (Deep) RL [4], [5] to let the agent progress more quickly towards better behaviors, but curricula are mostly designed by hand. Curriculum learning has the potential to greatly increase the quality of the behavior discovered by the agent. However, at the moment, creating an appropriate curriculum requires significant human intuition.

Initial attempts have been made towards automating curriculum generation. Two challenges have been defined within curriculum learning: task generation [6], [7], and task sequencing [7]–[9]. Task generation is concerned with creating a set of intermediate tasks for the agent to train on, forming the curriculum. The tasks have to be strongly related to a final task, so that transferring knowledge from one task to the next through the curriculum makes the agent more competent at the final task of interest. Task sequencing, on the other hand, is the problem of selecting and sorting previously generated tasks optimally, finding the order that makes the agent learn the final task fastest, or achieve a better performance. We consider the sequencing problem.

Sequencing methods introduced in the literature until now aim at minimizing the total training time. Curriculum generation is, however, a costly process in terms of samples taken in the different tasks, to establish their optimal order. Unless curricula can be efficiently reused, this limitation conflicts with the objective of using curriculum to learn faster. We argue that curriculum learning should not be considered just as a way to achieve the optimal policy more quickly, but also as a way to improve the quality of the discovered behaviors.

In domains complex enough that the optimal policy cannot be learned in any feasible amount of time, the knowledge originated in the curriculum can shape the exploration and contribute to discovering policies of higher value. Furthermore, when training time is a secondary concern with respect to adapting online to a critical task, the role of the curriculum should be to equip the agent with reusable knowledge that can make the exploration in the final task more limited and efficient. We propose three different metrics in addition to total training time, from whose optimization these alternative uses of curriculum learning naturally emerge.

In this paper, we propose an optimization framework for task sequencing in curriculum learning, composed of the problem definition, several candidate performance metrics for optimization, and three benchmark algorithms. The algorithms are guaranteed to return an optimal curriculum within a search space that we study and characterize. Furthermore, we evaluate the algorithms on a number of experiments ranking their solutions with respect to the global optimum, and the most popular baselines: learning with no curriculum and with a random curriculum.

II. RELATED WORK

Curriculum Learning in reinforcement learning is an increasingly popular field, with successful examples of applications in first-person shooter games [5], [10], real-time strategy games [4], lifelong tasks [11], and real-world robotics.
Curriculum Learning has initially been introduced in supervised learning [13]. Closely related fields are multi-task reinforcement learning [14], and lifelong learning [15], where the agent attempts to maximize its performance over the entire set of tasks, which may not all be related to one another. Conversely, in curriculum learning, the intermediate tasks are generated specifically to be part of the curriculum, and the curriculum is optimized for a single final task.

Task Selection has been studied for general transfer learning, and presents common aspects with the task selection that is part of sequencing in curriculum learning. Several approaches consider learning a mapping from source tasks to target tasks, and estimating the benefit of transferring between the tasks [16]–[18]. Nonetheless, transfer learning is usually performed between two tasks, a source and a target, and task selection methods have never been leveraged to achieve longer sequences.

Curriculum learning has been used to guide the exploration by generating a sequence of goals or initial states within the same environment [19], [20]. On the other hand, we consider the problem of sequencing entirely different tasks.

The automatic generation of curricula has been divided into two sub-problems: task generation [9], [21], that is the problem of creating a set of tasks such that transferring from them is most likely beneficial for the final task; and task sequencing [8], [9], [21], whereby previously generated tasks are optimally selected and ordered.

Current methods for task sequencing attempt to determine the optimal order of tasks either with [8], [22] or without [9], [21] executing the tasks. However, none of the existing methods offers any guarantee on the quality of their solution. Task sequencing methods are the closest to our work, so we discuss them here in further detail. Svetlik et al. [9] propose a method to create a curriculum graph without executing any task, therefore independently of the agent. The method is based on a manually specified task descriptor, and a heuristic measure of task similarity. The curriculum is meant to minimize time-to-threshold. Da Silva and Costa [21] proposed an object-oriented extension of that method, and introduced a task generation and transfer learning procedure, also based on the object-oriented representation. Both methods do not explore the space of curricula, but directly build a single curriculum, and therefore return a solution with no quality guarantee. The experimental evaluation is, in both cases, conducted against learning with no curriculum. In our framework, on the other hand, the computed solution is guaranteed to be at least better than no curriculum, and to be the optimal one within a precisely defined set. Furthermore, our algorithms evaluate the curriculum on the agent, and return the best curriculum tailored to the particular agent and metric of choice. Narvekar et al. [8] frame the curriculum learning problem as a higher-level MDP, where the state space is formed by all the of possible policies, and the actions select the next task to learn. Such an MDP, however, is too complex to be solved directly, and the authors propose a greedy algorithm, guided by a measure of policy change. In its greedy selection, the algorithm prefers tasks that modify the policy the most. However, while the aim of the method is to minimize time-to-threshold, the greedy selection is not driven by it. Therefore, there is no guarantee that the task that causes the largest policy change actually changes the policy in the right way, so as to minimize time-to-threshold. As a consequence, this method has no quality guarantee either. In this case as well, the comparison is performed against learning with no curriculum.

### III. Background

#### A. Reinforcement Learning

We model tasks as episodic Markov Decision Processes. An MDP is a tuple \( \langle S, A, p, r, \gamma \rangle \), where \( S \) is the set of states, \( A \) is the set of actions, \( p : S \times A \times S \rightarrow [0, 1] \) is the transition function, \( r : S \times A \rightarrow \mathbb{R} \) is the reward function and \( \gamma \in [0, 1] \) is the discount factor. If a state is represented as a vector \( s = \langle v_1, \ldots, v_d \rangle \) of \( d \) variables the representation of the state space is said to be factored. Episodic tasks have absorbing states, that are states that can never be left, and from which the agent only receives a reward of 0.

For each time step \( t \), the agent receives an observation of the state and takes an action according to a policy \( \pi : S \times A \rightarrow [0, 1] \). The aim of the agent is to find the optimal policy \( \pi^* \) that maximizes the expected discounted return

\[
G_0 = \sum_{t=0}^{t_M} \gamma^t r(S_t, A_t),
\]

where \( t_M \) is the maximum length of the episode.

Sarsa(\( \lambda \)) is a learning algorithm that takes advantage of an estimate of the value function \( q_{\pi}(s, a) = E_\pi[G_t \mid S_t = s, A_t = a] \). We represent the value function with a linear function approximator, so that the learning algorithm computes an estimate \( \hat{q}(s, a) = \theta^T \phi(s, a) \) of \( q_{\pi}(s, a) \) as a linear combination of features \( \phi \).

#### B. Transfer Learning

Curriculum learning leverages transfer learning to transfer knowledge through the curriculum, in order to benefit a final task. Transfer takes place between pairs of tasks, referred to as the source and the target of the transfer. We use a transfer learning method based on value function transfer [3], which uses the learned source q-values, representing the knowledge acquired in the source task, to initialize the value function of the target task. Several metrics have been designed to evaluate transfer learning [3], which will be our starting point in the curriculum learning context.

### IV. Problem Definition

Let \( \mathcal{M} \) be a set of MDPs constituting the domain, and \( \mathcal{T} \subset \mathcal{M} \) be a finite set of candidate tasks for the curriculum. We define, in the context of this work, a curriculum as a sequence of tasks in \( \mathcal{T} \) without repetitions:

**Definition.** [Curriculum] Given a set of tasks \( \mathcal{T} \), a curriculum over \( \mathcal{T} \) of length \( l \) is a sequence of tasks \( c = \langle m_1, m_2, \ldots, m_l \rangle \) where each \( m_i \in \mathcal{T} \).
Let $m_f \in M$ be a final task, that is, the task the designer wants the agent to learn more efficiently through the curriculum, and $C_i^f$ be the set of all curricula over $T$ of length $i$. In the rest of this paper we will drop the superscript wherever the set of candidate tasks is implicit in the context.

We define $C_{\leq L} := \bigcup_{i=0}^{L} C_i$ as the set of all curricula of length at most $L$. We represent with $C_0$ the set containing the empty curriculum of length 0, denoted with $\emptyset$. The empty curriculum corresponds to learning the final task directly. Given a performance metric $P : C_{\leq L} \times M \to \mathbb{R}$, which evaluates curricula for a specific final task, we consider the problem of finding an optimal curriculum $c^*$, such that:

$$P(c^*, m_f) \geq P(c, m_f) \ \ \forall c \in C_{\leq L}.$$  

V. PERFORMANCE METRICS

Existing curriculum learning methods are tailored to a single performance metric, which is usually the total training time. However, in practice, the number of trials required to automatically generate a curriculum is often greater than the number of trials needed to learn the final task from scratch, making curricula’s utility arguable in terms of training time. It is possible that curricula can be reused, for instance by adjusting them for different agents, but this has not been significantly addressed in the literature yet. Curriculum learning, however, can be beneficial in different scenarios, not only by reducing training time. In the rest of this section we identify two such cases, and propose corresponding performance metrics.

The first scenario is in critical applications, in which the agent has to adapt online so as to explore in the final task as efficiently as possible. The final task cannot be entirely predicted beforehand, but a simulated model of it is available. Examples may range from trading agents, where exploration in the final task may lead to money loss, to smart thermostats, where exploration may result in higher bills, or generally unhappy occupants. Both these domains have simulators available for them, but the real-world task will require online learning to adapt to the particular context. The agent has to be equipped with the best initial knowledge to maximize a given objective. We propose two metrics for such a scenario: jumpstart and regret.

Jumpstart is borrowed from general transfer learning [3]. It evaluates the average reward of the agent within the first $E$ episodes:

$$P_j(c, m_f) := \frac{1}{E} \sum_{i=1}^{E} G_i^f,$$

where $G_i^f$ is the return obtained during episode $i$ in task $m_f$. Jumpstart can be used if it is critical that the agent is deployed in the final task with the highest possible initial performance.

Regret is one of the metrics used to optimally balance exploration and exploitation in single-task learning, whereby the agent minimizing regret attempts to converge to the optimal policy while acting suboptimally as little as possible. The regret metric, with respect to a given performance threshold $g$ (which can be the value of the optimal policy when known), is defined as follows:

$$P_r(c, m_f) := -(Ng - \sum_{i=1}^{N} G_i^f),$$

where $N$ is the number of episodes executed in the final task, and $Ng - \sum_{i=1}^{N} G_i^f$ is the difference between the return the agent would achieve if it obtained a return $g$ at each episode and the return actually achieved. This difference is the regret with respect to a policy achieving $g$, and we intend to minimize it, therefore it is multiplied by $-1$ since $P_r$ is maximized.

Note that the number of episodes to execute in the final task, $N$, does not need to be high enough to achieve convergence to the optimal policy, but it may be much lower. It acts as a planning horizon in the final task, so that the corresponding curriculum will minimize the regret within that limit. We expect that, in practice, designers will use a function balancing jumpstart and regret for learning in critical applications, so that the agent begins acting in the final task with a good performance, and improves as fast as possible from there.

Jumpstart and regret are metrics over the quality of the exploration (how it starts, and how it proceeds respectively) in the final task. Hence, they are evaluated on every episode. In the following two metrics, we focus on the value of the learned policy rather than on exploration. Every $K$ learning episodes, we introduce an evaluation phase, in which the current policy is executed $Q$ times to estimate the expected return achieved by the policy. During this phase, no updates are performed to the value function (or the policy). In the rest of this section, we denote with $E$ the set of the evaluation steps.

The second scenario is the use of curriculum learning to discover policies in complex environments, where the optimal policy is unknown and has never been learned. Curriculum learning has been leveraged with such an intent to learn more sophisticated behaviors in extremely large domains, such as the games of StarCraft [4] and Doom [5]. The curricula, however, are designed by hand, and do not explicitly maximize any measure of quality. We propose a metric, max-return, which will allow the automation of curriculum generation in such cases. With max-return, we are exclusively interested in the value of the policy learned within a given horizon:

$$P_m(c, m_f) := \max_{I \in E} G_I^f,$$

where $G_I^f = \frac{1}{Q} \sum_{i=1}^{Q} G_i^f$ is the average return over the $Q$ episodes in the evaluation step $I$ on the final task. Max-return is variation of asymptotic performance [3] with the difference that the behavior of the agent is not expected to converge by the end of the training time.

Lastly, we consider what is currently the most common use of curriculum learning, that is to reduce total training time. Time-to-threshold evaluates the number of actions executed throughout the curriculum in order to achieve a given threshold performance $g$ during an evaluation step in the final task. Let $a(m_i)$ be the number of actions the agent executed in task $m_i$ before moving on to the next task in the curriculum,
$a_g(m_f)$ be the number of actions the agent executed in the final task until the evaluation step in which the policy achieves an average return of $g$. The time-to-threshold metric is defined as follows:

$$\mathcal{P}_t(c, m_f) := -(a_g(m_f) + \sum_{m_i \in c} a(m_i)),$$

where, similarly to the regret, we intend to minimize the time to threshold, therefore the total time is multiplied by $-1$. Time-to-threshold is the only metric in which each task contributes to the total performance explicitly. In the other metrics the intermediate tasks affect the performance exclusively through transfer learning, and its effect on the behavior in the final task. Time-to-threshold, as defined above, is a metric for strong transfer [3]. This situation is similar to having a reward in an MDP along the way, rather than only in the final state, and as such it is easier to build heuristics for it, which has been done in previous work [8], [21]. Nonetheless, it is in principle possible to define a weak form of time-to-threshold as $\mathcal{P}_t(c, m_f) := -a_g(m_f)$, where the training time during the curriculum is not taken into account.

All the proposed performance metrics represent some form of quality of learning in the final task: at the beginning (jumpstart), during (regret and time to threshold), and by the end (max return). In order to maximize them directly, it is therefore necessary to collect samples in the final task. However, each metric requires the execution of the final task for a fixed number of episodes. This number will depend on the available computation power, and may be (and certainly is in the case of jumpstart) much lower than the number of episodes required to learn the final task to convergence.

VI. ALGORITHMS FOR TASK SEQUENCING

As defined in Section [V] task sequencing is a combinatorial optimization problem, and therefore any general search algorithm could in principle be used to solve it. We considered a number of search methods, and identified three algorithms that perform well across all the evaluated metrics. We consider a purely greedy search algorithm, and propose two variants with increasing solution quality. These algorithms fulfill the design goal of being independent of the metric, the agent (including the transfer learning method), and the domain. We expect that by taking one or more of those elements into account, more efficient but less general methods may be designed.

The first algorithm that we consider is a purely greedy one, shown in Algorithm 1. It takes a set of candidate tasks $T$, a final task $m_f$, and a maximum curriculum length $L \leq |T|$. The algorithm returns a locally optimal curriculum $c^*$. The set of candidate curricula is initially set to all curricula of length 1 in Line 1. All candidate curricula are evaluated in Line 4 and the best candidate is selected in Line 7. If this curriculum does not improve on the current best one the algorithm terminates (hence its greedy nature; Line 9). Otherwise, the next set of candidates is obtained by calling GenerateCandidates with the current best curriculum as input. The function GenerateCandidates is shown in Algorithm 2. The next set of candidates is computed by appending, to each of the seed curricula, all tasks that do not already belong to that curriculum. For example, if $T = \{m_1, m_2, m_3\}$, and GenerateCandidates is invoked on $\{(m_1, m_2)\}$, it returns $\{(m_1, m_2, m_1), (m_1, m_2, m_3), (m_2, m_1), (m_2, m_3)\}$.

In order to characterize the set of curricula that is exhaustively searched by GreedySearch, we define the prefix of a curriculum as follows:

**Definition.** [Curriculum Prefix] A curriculum $c_i = \langle m_1^i, \ldots, m_l^i \rangle$ is a prefix of a curriculum $c_j = \langle m_1^j, \ldots, m_l^j \rangle$ iff $l < p \wedge m_k^i = m_k^j \forall k \in [1, l]$.

Let $C \subseteq C_{l^l}$ be a set of curricula of length $l' < l$, and $C_l(C) \subseteq C_l$ be the set of curricula of length $l$ that have $c \in C$ as a prefix. Furthermore, let $v_l^*(C) := \max_{c_i \in C_l(C)} \mathcal{P}(c_i, m_f)$ be the value of the best curriculum in $C_l(C)$, and $c_l^*(C)$ be an optimal curriculum with value $v_l^*(C)$. In order to simplify the notation of the following theorem, we will set $v_l^* := v_l^*(\{c_{l-1}^*\})$, that is, the value of the optimal curriculum in $C_l(\{c_{l-1}^*\})$.

**Algorithm 1 GreedySearch**

**Input:** $T$, $m_f$, and $L$

**Output:** curriculum $c^*$

1: $P \leftarrow \emptyset$, $v^* \leftarrow \mathcal{P}(\emptyset, m_f)$, $l \leftarrow 1$, $C \leftarrow C_1$
2: while $l \leq L$ do
3:     for $c \in C$ do
4:         $v \leftarrow \mathcal{P}(c, m_f)$ // evaluate curriculum
5:         $P \leftarrow P \cup \{(v, c)\}$
6:     end for
7:     $\langle v', c' \rangle \leftarrow \langle v_i, c_i \rangle \in P \text{ s.t. } v_i \geq v_j \forall j \in [1..|P|]$ if $v' < v^*$ then
8:         $v^* \leftarrow v'$, $c^* \leftarrow c'$
9:     break
10: end if
11: $P \leftarrow \emptyset$
12: $C \leftarrow \text{GenerateCandidates}(\{c^*\}, T)$
13: $l \leftarrow l + 1$
14: end while
15: return $c^*$

**Algorithm 2 GenerateCandidates**

**Input:** seeds, $T$

**Output:** candidate set $C$

1: $C \leftarrow \emptyset$
2: for $c \in \text{seeds}$ do
3:     $E \leftarrow \{m_i \in T | m_i \notin c\}$
4:     for $m \in E$ do
5:         append $m$ to $c$
6:         $C \leftarrow C \cup c$
7:     end for
8: end for
9: return $C$
**Theorem 1.** GreedySearch returns an optimal curriculum in $S_g := C_0 \cup_{i=1..L} C_i(\{c^*_i\})$ where $l$ is the first integer in $[1..L]$ for which $v^*_l < v^*_{l-1}$ if such an integer exists, otherwise $l := L$.

**Proof.** The proof proceeds by showing that the algorithm evaluates all the curricula in $S_g$, while storing the current best one. Since $l \geq 1$, $S_g \supseteq C_0 \cup C_1$, that is $S_g$ contains at least the empty curriculum, and all curricula of length $1$. This is verified by Line 1 where the empty curriculum is evaluated, the set of candidates is set to $C_1(\{c^*_0\}) = C_1$, and evaluated in Line 3. Further sets of candidates $C_i(\{c^*_i\})$ for $l \in [1..L]$ are generated in Line 4 unless the value of the best curriculum of the previous round $v^*_i$ fails to improve on its predecessor $v^*_{i-1}$ (Line 5). Since $S_g$ did not belong to the previous round.

For each length $l$ considered, GreedySearch evaluates $|T| - l + 1$ curricula, therefore its asymptotic computational complexity is $O(L|T|)$.

The greedy search continues to the next curriculum length only if constantly improving on the current best curriculum. However, this is easily prone to stop at a local maximum. The first variant that we propose to improve on GreedySearch is to continue the search up to the maximum length $L$, regardless of the value of evaluated curricula. The resulting algorithm, GreedyDepthSearch, is shown in Algorithm 3.

**Algorithm 3 GreedyDepthSearch**

**Input:** $T$, $m_f$, and $L$  
**Output:** curriculum $c^*$

1: $P \leftarrow \emptyset, v^* \leftarrow \mathcal{P}(\emptyset, m_f), l \leftarrow 1, C \leftarrow C_1$
2: while $l \leq L$ do
3:     for $c \in C$ do
4:         $v \leftarrow \mathcal{P}(c, m_f)$ // evaluate curriculum
5:         $P \leftarrow P \cup \{(v, c)\}$
6:     end for
7:     $(v', c') \leftarrow (v_i, c_i) \in P$ s.t. $v_i \geq v_j \forall j \in [1..|P|]$
8:     if $v' > v^*$ then
9:         $v^* \leftarrow v', c^* \leftarrow c'$
10:    end if
11: $P \leftarrow \emptyset$
12: $C \leftarrow \text{GenerateCandidates}(\{c'\}, T)$
13: $l \leftarrow l + 1$
14: end while
15: return $c^*$

**Theorem 2.** GreedyDepthSearch returns an optimal curriculum in $S_d := C_0 \cup_{i=1..L} C_i(\{c^*_i\})$.

**Proof.** The algorithm generates all the $C_i(\{c^*_i\})$ with $i \in [1..L]$ in Line 12 and evaluates each one of them in Line 4.

Note how, in Line 12, GreedyDepthSearch does not use the optimal curriculum to generate the next set of candidates, since that may not belong to the previous round. Since $S_g \supseteq S_d$, the number of curriculum evaluations of $\text{GenerateCandidates}$ is larger than GreedySearch, but also the curriculum found by GreedyDepthSearch is guaranteed to be at least as good as the curriculum found by GreedySearch, that is:

$$\max_{c \in S_d} \mathcal{P}(c, m_f) \geq \max_{c \in S_g} \mathcal{P}(c, m_f).$$

This improvement is achieved at no asymptotic cost, since the complexity of GreedyDepthSearch is also $O(L|T|)$.

Lastly, we propose a variant of Algorithm 3 aimed at searching a set larger than $S_d$ in order to find a better solution. Instead of only expanding the best curriculum of the previous round, the Algorithm 4 expands the best $M$. ParallelGreedyDepthSearch searches on several branches of curriculum, rather than one as the previous algorithms. The evaluation of the $M$ branches can be parallelized, hence the name.

**Theorem 3.** Let $C^*_M = \{\emptyset\}$ be the set of the best $M$ curriculum of length zero, which contains just the empty curriculum for any $M$. Let $C_i^M \subseteq C_i(C^*_{i-1})$ be the set of the best $M$ curricula of length $i$ obtained by adding one task to the elements of $C^*_M$.

**ParallelGreedyDepthSearch** returns an optimal curriculum in $S_p(M) := C_0 \cup_{i=1..L} C_i(C^*_M)$.

**Proof.** The empty curriculum is immediately evaluated in Line 1 which accounts for $C_0$. Then, for each $l = 1..L$ the set $C_l(C^*_L)$ is generated in Line 6 by adding one task to the best $M$ tasks of the previous round (Line 12). All the curricula generated in Line 6 are evaluated in Line 8.

For $M = 1$, all the sets $C^*_L$ contain a single element $c^*_L$ and we retrieve GreedySearch, that is $S_p(1) \equiv S_d$. For $M > 1$, however, $|S_p(M)| > |S_d|$. Indeed, the asymptotic complexity of ParallelGreedyDepthSearch is $O(ML|T|)$, and we expect a corresponding increase in
solution quality. While increasing $M$ generally does provide better solutions, and indeed with $M$ large enough ParallelGreedyDepthSearch searches exhaustively the whole space of curricula, it is also the case that $S_p(p) \not\supseteq S_p(q)$ with $p > q$. Therefore, increasing $M$ does not guarantee that a better solution will be found, until $M$ is large enough that all solutions are evaluated, which is exponential in the number of tasks. In the experimental section, we discuss the effect of $M$ on the quality of the solution in more detail.

VII. EXPERIMENTAL EVALUATION

We evaluate our algorithms on two domains, BlockDude and Gridworld, implemented within the software library Burlap\(^1\). We used the implementation of Sarsa($\lambda$) available in Burlap, along with Tile Coding as the function approximator.

A. GridWorld

GridWorld is an implementation of an episodic grid-world domain used in the evaluation of existing curriculum learning methods \cite{dulac2016batch,amato2015curriculum}. Each cell can be free, or occupied by a fire, pit, or treasure. An example GridWorld is shown in Figure 1\(a\). The agent can move in the four cardinal directions, and the actions are deterministic. The reward is $-2500$ for entering a pit, $-500$ for entering a fire, $-250$ for entering a cell next to a fire, and 200 for entering a cell with the treasure. The reward is $-1$ in all other cases. The episodes terminate under one of these three conditions: the agent falls into a pit, reaches the treasure, or executes a maximum number of actions (50). The variables fed to tile coding are the distance from the treasure (which is global and fulfills the Markov property), and distance from any pit or fire within a radius of 2 cells from the agent (which are local variables, and allow the agent to learn how to deal with these objects when they are close, and transfer this knowledge).

B. BlockDude

BlockDude is another domain available in Burlap, which has also been used for curriculum learning \cite{dulac2016batch}. It is a puzzle game where the agent has to stack boxes in order to climb over walls and reach the exit. An example task for BlockDude is shown in Figure 1\(b\). The available actions are moving left, right, up, pick up a box and put down a box. The agent receives a reward of $-1$ for each action taken. The variables used as input to tile coding are distance from the exit, distance from each box, distance from each edge of the map, direction of the agent (binary) and whether or not it is holding a box (also binary).

C. Transfer Learning

We used an egocentric representation (using distances with respect to the agent) and local variables to favor transfer, as described separately for the two domains. We also normalized the variables in $[0, 1]$, so that the input is invariant to the scale of the domain. Furthermore, we used a particular value-function transfer \cite{minton1992structured} inspired by Concurrent Layered Learning \cite{Nickel2014}. In Concurrent Layered Learning, an agent learns a complex behavior by incrementally learning sub-behaviors (layers). The more complex behaviors (higher layers) directly depend on the easier ones (lower layers), and during training all the layers are updated simultaneously.

This concept was implemented by carrying over the features of the source task into the target task, along with its parameters. Let $V_i$ be the set of variables defined for the source task, and $V_j$ the variables defined for the target task, in a source-target pair along the curriculum. Let $q_i(s, a) = \theta^T_i \phi_i(s, a)$ be the value function of the source task. The value function for the target task, $q_j$, is defined as:

$$q_j(s, a) := \begin{cases} \theta^T_i \phi_i(s, a) + \theta^T_j \phi_j(s, a), & \text{if } V_i \subseteq V_j, \\ \theta^T_j \phi_j(s, a), & \text{otherwise}. \end{cases}$$

where $\phi_j$ is the feature vector of the target task. Therefore, if the variables of the target are compatible with the variables of the source, so that the features of the source are defined in the target, the features and their parameters are carried over. The new parameters, introduced in the target, are set to 0 so that the imported features initially dominate the behavior. In our experiments we do not remove features, and the number of features and parameters grows with every transfer. However, it is possible to perform feature selection, and remove the features that do not affect the value function significantly.

D. Results

We chose two relatively small domains so that we could perform a thorough evaluation, by computing and analyzing all curricula within the given maximum length. We considered the metrics of jumpstart, time-to-threshold, and regret. We did not use max-return as the agent could always learn the optimal policy on these domains when training without a curriculum.

In our experiments, we perform an evaluation phase each $K = 10$ episodes in order to estimate the quality of the learned policy. As the environment is deterministic, we can perform each evaluation step $Q = 1$ times. Each curriculum has been executed 10 times, and its value for each metrics estimated as the average over those trials.

We ran two sets of experiments per domain, one in which the number of tasks is high and the maximum length is low, and one in which, on the contrary, the number of tasks is low, but the maximum length is high. For GridWorld, the first set of experiments has parameters $n := |\mathcal{T}| = 12$ and $L = 4$,

\(^1\)http://burlap.cs.brown.edu

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**Fig. 1:** The experimental domains.
while the second $n = 7$, and $L = 7$. For BlockDude, the first set of experiments has parameters $n = 18$ and $L = 3$, while the second $n = 9$ and $L = 5$. These parameters were chosen so that the total number of curricula does not exceed 20000. For both domains, the intermediate tasks have been generated manually, by varying the size of the environment, adding and removing elements (pits and fires in GridWorld, and columns and movable blocks in BlockDude). We intentionally created both tasks that provide positive and negative transfer towards the final task, in order to test the ability of the sequencing algorithm to choose the most appropriate ones. All tasks are run for a number of episodes that ensures that the agent has converged to the optimal policy, and were determined at the time of task generation.

We begin by analyzing the performance of ParallelGreedyDepthSearch for different values of $M$ between 1 and 100. Figure 2 shows the ranking of the curricula computed by ParallelGreedyDepthSearch, among all possible curricula, as $M$ increases, where the optimal curriculum is 1st in the ranking. Figure 3 shows the same results while minimizing regret. In both cases, it is possible to see a steep initial improvement for small values of $M$, and a subsequent slower improvement as $M$ increases. It is also possible to see that the function is not monotone, and for some $M$ the computed curriculum is worse than for smaller values, as expected since $S_p(p) \not\subseteq S_p(q)$ with $p > q$. This effect of $M$ is stronger on the experiments with higher maximum curriculum length. Conversely, on time-to-threshold, the algorithm computes the optimal curriculum for all values of $M$, therefore we do not show that graph. Time-to-threshold naturally favors small curricula, and the optimal curriculum is never longer than 1 task within the four experiments, making GreedySearch just as effective as the other methods. Given these results, we chose $M = n$ for the rest of the experiments, because it is enough to overcome the initial large step, and is not too computationally demanding.

In Table I and II we show the results of the experiments on GridWorld and BlockDude respectively. The header row shows the number of candidate tasks, maximum length, and the total number of curricula of each experiment. Each cell contains the ranking and the value of the curriculum computed by each algorithm, for the corresponding metric. We also included the optimal curriculum (first column of each experiment), learning with no curriculum (denoted as $C_0$), and the average value of a random curriculum. Since the random curriculum is just an average of other curricula, it has no ranking of its own. The regret is normalized in $[-1,0]$ with 0 being the value of the policy achieving no regret. Since we are maximizing, for all metrics a bigger value is always better. Lastly, random curricula often do not reach the threshold for the time-to-threshold metric, in which case they are assigned a time of $-\infty$. If this is the case, their average performance on time-to-threshold is omitted.

All three algorithms are consistently better than both learning with no curriculum and a random curriculum, which are the most commonly adopted baselines, often by a large margin. Time-to-threshold, the one used the most in the literature, also appears to be the easiest to optimize. On the other hand, regret appears to be the most difficult metric. ParallelGreedyDepthSearch is always better than GreedyDepthSearch which in turn always improves on GreedySearch. The performance of all the algorithms degrades faster as the maximum length increases, rather than as the number of candidate tasks increases.

Lastly, we provide two examples of the effect on the final task of regret and jumpstart respectively, which have not been previously used explicitly in curriculum generation. In Figure 4 we show the learning behavior of the agent over the final task in the Block Dude domain with $n = 18$ and $L = 3$. The agents explore with an $\epsilon$-greedy policy, with $\epsilon$ decreasing...
TABLE I: Results of all the algorithms on the GridWorld domain. In each cell, the first number is the ranking, while the second is the value of the corresponding curriculum.

|      | Opt | Greedy | Depth | Parallel | C0   | Random | Opt | Greedy | Depth | Parallel | C0   | Random |
|------|-----|--------|-------|----------|------|--------|-----|--------|-------|----------|------|--------|
| GW   | n = 18; L = 3; tot = 5221 | -601,74 | -860,03 | -860,03 | -738,29 | 7669 | - | 135 | -827,68 | -773,71 | 104 | 13620 |
| JS   | -1,1851 | 1743 | -0,2639 | -0,2306 | -223,35 | 11499 | -1,2776 | 1038 | -0,4332 | -0,3887 | 259 | -4535 |
| Reg  | -155,63 | -315,6 | -315,6 | -315,6 | -1351,6 | -1643,7 | 1 | 1 | -1643,7 | -1643,7 | 1 | -2788,1 |

TABLE II: Results of all the algorithms on the BlockDude domain. In each cell, the first number is the ranking, while the second is the value of the corresponding curriculum.

|      | Opt | Greedy | Depth | Parallel | C0   | Random | Opt | Greedy | Depth | Parallel | C0   | Random |
|------|-----|--------|-------|----------|------|--------|-----|--------|-------|----------|------|--------|
| BD   | n = 9; L = 5; tot = 18730 | -44,77 | -49,04 | -49,04 | -47,32 | 7933 | -49 | 8 | -49,4 | -49,4 | 7 | -5221 |
| JS   | -0,3475 | 95 | -0,4507 | -0,4507 | -3977 | 16435 | -0,4244 | 131 | -0,5232 | -0,5232 | 2 | -4871 |
| Reg  | -2749,3 | -2749,3 | -2749,3 | -2749,3 | -2981,7 | 3 | -3828 | 1 | -3828 | -3828 | 1 | 152 |

We introduced a framework for task sequencing in curriculum learning, proposing three metrics in addition to time-to-threshold, which is the one mostly used in the literature. With the exception of regret minimization, which has been used to explore efficiently in single tasks, the other metrics are directly borrowed from, or variations of, general transfer learning metrics. Since transfer learning is a fundamental component of curriculum learning, this is a reasonable first step. However, it is possible that metrics specific to curriculum learning will be designed in future work.

We also introduced three algorithms that are general (they do not depend on the metric, agent, or domain), simple to implement, and perform consistently better than the current popular baselines of learning with no curriculum, and with a random curriculum. All three algorithms improve by design over learning with no curriculum. The algorithms maximize the proposed metrics directly, rather than through proxies [9] or heuristics [8]. This guarantees that the solution is optimal within a well-defined set of curricula, but also requires that...
a number of curricula, linear in the number of tasks, is entirely evaluated, including sampling the final task for a fixed number of episodes (not necessarily until convergence). The sampling requirements directly apply the structure of our framework to scenarios where the performance in the final task is more critical than the time spent preparing for it, for instance when a simulator is available. This is also a consequence of the generality of the proposed algorithms. We expect that by targeting a method to a specific metric and transfer learning technique the sample requirement can be drastically reduced. Nonetheless, we experimentally showed that the proposed algorithms perform well across all metrics, several cases returning a curriculum close to the global optimum, and the data suggest they provide more valid baselines for future research on curriculum learning.

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