Estimation Models for the Refractive Index Response Curve of EFBGs

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Abstract—Estimation models for the surrounding refractive index response of EFBGs were developed from computer simulations and tested on experimental data. Numerical simulations were used to investigate the fundamental core mode’s effective refractive index dependence with the fiber radius and the surrounding refractive index. An empirical mathematical model was developed upon the simulation results, and it was used to derive the estimation models. The response curve is estimated from a reference calibration curve using two different approaches: interpolation and extrapolation. The interpolation is performed using the wavelength response of the EFBGs at two different surrounding refractive index values. Then, the response curve is obtained solving a simple pair of linear equations. The extrapolation is performed using the wavelength shift observed during the etching process, which is used to derive a multiplicative factor for the calibration curve. The response curves estimated by the model were compared with the experimental results, showing good agreement within the experimental uncertainties. Considering the surrounding refractive index ranging from 1.333RIU to 1.458RIU, with the sensors’ response ranging from 1544.84nm to 1552.70nm, the maximum deviation between the estimation and the experimental results were 0.26nm and 0.41nm, and the maximum standard deviation were 0.03nm and 0.05nm, respectively for interpolation and extrapolation.

Index Terms—Estimation, etched fiber Bragg grating, modeling, optical fiber sensor, refractive index measurement.

I. INTRODUCTION

Etched fiber Bragg gratings (EFBG) have been applied as refractive index sensors since the first demonstration on FBGs inscribed in thinned, non-uniform thinned and micro-structured fibers [2]–[5]. Recently, Urrutia et al. published a comprehensive review of optical fiber refractometers were they compared the performance of interferometers, grating-based and resonance-based structures [6]. EFBG applications still draw the attention of many researchers [7]–[13]. Bekmurzaeva et al. proposed a biosensor based on an EFBG functionalized with aptamer as a ligand for thrombin detection [7]. Kumar et al. reported an EFBG coated with graphene oxide for ethanol detection in petrol showing a significant detection enhancement when compared with uncoated EFBG [8]. Corotti et al. reported an EFBG coated with diphenylalanine nanotubes capable to detect methanol in methanol-ethanol blend vapors [9]. Korganbayev et al. presented a compact sensor for multi-parametric measurements using a partially etched chirped fiber Bragg grating [10]. Bui et al. proposed a novel method for biochemical sensors using etched fiber Bragg gratings integrated in a fiber ring laser structure, without the need of a spectrometer [11]. For chemical and biochemical detection purposes, higher sensitivities of their tunability through a controlled etching process [1]. In the early years of the 21st century, the potential application of EFBGs as a refractive index sensor has attracted great interest from the optical fiber sensor community. Iadicicco et al.’s seminal work is a remarkable example, in which they presented experimental, numerical and theoretical results in the study of refractive index sensors based on FBGs inscribed in thinned, non-uniform thinned and micro-structured fibers [2]–[5]. Recently, Urrutia et al. published a comprehensive review of optical fiber refractometers were they compared the performance of interferometers, grating-based and resonance-based structures [6]. EFBG applications still draw the attention of many researchers [7]–[13]. Bekmurzaeva et al. proposed a biosensor based on an EFBG functionalized with aptamer as a ligand for thrombin detection [7]. Kumar et al. reported an EFBG coated with graphene oxide for ethanol detection in petrol showing a significant detection enhancement when compared with uncoated EFBG [8]. Corotti et al. reported an EFBG coated with diphenylalanine nanotubes capable to detect methanol in methanol-ethanol blend vapors [9]. Korganbayev et al. presented a compact sensor for multi-parametric measurements using a partially etched chirped fiber Bragg grating [10]. Bui et al. proposed a novel method for biochemical sensors using etched fiber Bragg gratings integrated in a fiber ring laser structure, without the need of a spectrometer [11]. For chemical and biochemical detection purposes, higher sensitivities
are an important feature for obtaining detection limits in the order of ng/mL. Shivananju et al. studied EFBG coated with polyelectrolytes thin films, using computational simulation and experimental measurements [14]. They provided an estimation for the detection limit of the sensor considering the noise levels of the interrogation system. They found that sufficiently narrowed EFBGs can provide a competitive platform for real-time measurements of molecular interactions while simultaneously leveraging the high multiplexing capabilities of fiber optics. In previous works, our group showed that the refractive index sensitivity and the dynamic range of EFBG inscribed in multimode fibers are higher than those verified on single-mode fibers [15]. We have also shown that such gratings can be used in simultaneous measurements of refractive index and temperature [16]. These features make this device very promising for distributed sensing and/or multi-parameter measurements. Tsigaridas et al. presented a theoretical and experimental study on the effect of the etching process over the FBG properties [17]. They proposed a simple analytical expression between the effective refractive index and the radius of the etched FBG. Namiq and Ibsen presented a technique to measure the diameter of an optical fiber during the etching process using an FBG [18].

In this article, we present a model developed from numerical simulations that allows estimating the surrounding refractive index response of an EFBG. Since the EFBG response curve depends on the fiber’s characteristics (dimensions and material properties), the estimation of the response requires at least the determination of a reference curve. The results using the estimation models are compared with experimental measures, and show consistency within the experimental uncertainties. The results presented in this work are complementary to those reported by Tsigaridas et al. [17], and Namiq and Ibsen [18], and can be useful tools in EFBG analysis and design.

II. NUMERICAL SIMULATIONS AND MODELING

In order to understand the EFBG response, a series of computational simulations were performed using the commercially finite element solver COMSOL Multiphysics® with the Wave Optics module.

Fig. 1 presents the effective refractive index ($n_{\text{eff}}$) of the propagating modes as function of the fiber radius $r$, with surrounding refractive index (SRI) set to 1.31RIU (in order to simulate an HF aqueous solution). Over the fiber radius interval from 6 $\mu$m to 15 $\mu$m, the calculated $n_{\text{eff}}$ fits very well to a single exponential function, presented as Equation (1), as proposed by Tsigaridas et al. [17]. The fit results were $\eta = (0.4246 \pm 0.0015) \mu m^{-1}$, $n_{e0} = (1.44616 \pm 0.00001)$RIU, and $n_{e1} = (-0.09792 \pm 0.00091)$RIU. The maximum deviation of the simulated $n_{\text{eff}}$ using this function was less than $3 \times 10^{-5}$RIU. Adopting a grating period $\Lambda$ of 532.45nm, and the Bragg wavelength ($\lambda = 2n_{\text{eff}} \Lambda$), it was possible to calculate the fiber radii corresponding to 1nm wavelength shift intervals. These wavelength shifts values were chosen for the sake of comparison with the experimental results.

\[ n_{\text{eff}}(r) = n_{e0} + n_{e1} \exp(-\eta r) \]  \hspace{1cm} (1)

Based on the results presented in Fig. 1, seven different radii were chosen to calculate $n_{\text{eff}}$ as a function of the SRI. Fig. 2 presents these results, with the SRI ranging from 1.31RIU to 1.445RIU. For each radius, the calculated $n_{\text{eff}}$ fits very well to a non-linear function in the form of (2). In this equation, $n$ is the SRI, while $A$, $B$, $C$ and $k$ are parameters related to the fiber characteristics, such as core and cladding geometry and refractive index. The $k$ values were approximately the same for all radii, with mean value of 0.3128 $\pm$ 0.0026. Through this, Equation (2) was used again to fit the simulated $n_{\text{eff}}$, but with $k$ fixed at 0.3128. These fit results are presented in Fig. 2. The maximum deviation of the calculated $n_{\text{eff}}$ using (2) with this fixed $k$ was less than $1 \times 10^{-3}$RIU. It is worth mentioning that our group has studied a simplified version of (2) with $k = 1$ [16], which is still a good approximation. For the sake of comparison, the maximum deviation of the calculated $n_{\text{eff}}$ using that simpler equation ($k = 1$) was less than $3 \times 10^{-4}$RIU.

\[ n_{\text{eff}}(n) = A + \frac{B}{(C - n)^k} \]  \hspace{1cm} (2)

The dependence of $A$, $B$ and $C$ with the fiber radius is presented in Fig. 3. The $C$ parameter is linear with the fiber
9 parameters is derived, which can be used to describe substituting which was expected after the results presented in Fig. 1. These results were modelled using (3). A surface fit is presented in Fig. 4, while Table I presents as a function of the SRI and the fiber radius, simultaneously.

The dependence on \( r \), while both \( A \) and \( B \) present an exponential behaviour, which was expected after the results presented in Fig. 1.

The dependence on \( r \) of \( A, B \) and \( C \) is presented in (3). By substituting \( A(r), B(r) \) and \( C(r) \) into (2), an equation with 9 parameters is derived, which can be used to describe \( n_{\text{eff}} \) as a function of the SRI and the fiber radius, simultaneously. A surface fit is presented in Fig. 4, while Table I presents the corresponding fit parameters. The maximum deviation of the calculated \( n_{\text{eff}} \) using the surface fit was less than \( 5 \times 10^{-5} \text{RIU} \).

\[
A(r) = a_0 + a_1 \exp(-a_2 r) \\
B(r) = b_0 + b_1 \exp(-b_2 r) \\
C(r) = c_0 + c_1 r \\
n_{\text{eff}}(n, r) = A(r) + \frac{B(r)}{[C(r) - n]^k}
\]  

(3)

Over the SRI range and fiber radii considered, the combined contribution of \( b_0 \) and \( c_1 \) is significantly small (about 2% to the \( n_{\text{eff}} \) at \( n = 1.44 \)), and the exponents \( a_2 \) and \( b_2 \) differ for less than 10%. This means that the surface equation may be simplified by making \( \eta = a_2 = b_2 = 0 \) and \( c_1 = 0 \), and this is presented as (4). The maximum deviation of the calculated \( n_{\text{eff}} \) using (4) was less than \( 2 \times 10^{-4} \text{RIU} \), and is significantly lesser for smaller SRI values. This is consistent with the results presented in Fig. 1, since Equations (1) and (4) share the same functional structure for fixed SRI, whose equivalence is presented in (5). In this equation, \( n_c \) refers to the SRI during the cladding etching (\( n_c = 1.31 \)). In order to compare with the results of (3), the fit results using (4) are also presented in Table I.

\[
n_{\text{eff}}(n, r) = a_0 + \left[ a_1 + \frac{b_1}{(c_0 - n)^k} \right] \exp(-\eta r) \tag{4}
\]

\[
n_{\text{eff}}(n_c) = a_0 \tag{5}
\]

A. Estimating the Sensor’s Response

Equation (4) is useful to estimate the change in the \( n_{\text{eff}} \) for two different \( r \) values at the same \( n \), which is presented as (6).

\[
\Delta_r n_{\text{eff}} = n_{\text{eff}}(n, r) - n_{\text{eff}}(n, r_0) = \left[ a_1 + \frac{b_1}{(c_0 - n)^k} \right] \left[ \exp(-\eta r) - \exp(-\eta r_0) \right] \tag{6}
\]

Therefore, for a pair of EFBG which differ only by their fiber radius, if (4) is a good approximation, then (6) may be used to estimate the relative change in \( \Delta_r n_{\text{eff}} \) as a function of \( n \). For convenience, \( n_c \) will be used as SRI reference, and this is presented at (7). In this equation, the denominator was substituted by \( n_{\text{eff}}(n_c) \), from (5). The remarkable property of (7) is that it does not explicitly depend on \( r \).

\[
\frac{\Delta_r n_{\text{eff}}(n)}{\Delta_r n_{\text{eff}}(n_c)} = \frac{a_1 + \frac{b_1}{(c_0 - n_c)^k}}{a_1 + \frac{b_1}{(c_0 - n_{\text{eff}}(n_c))^k}} = \frac{1}{n_{\text{eff}}(n_c)} \left[ a_1 + \frac{b_1}{(c_0 - n)^k} \right] \tag{7}
\]

Since the Bragg wavelength is proportional to \( n_{\text{eff}} \), \( \Delta_r \lambda \) is also proportional to \( \Delta_r n_{\text{eff}} \). Through this, the wavelength difference of a fiber pair can be mapped in terms of \( \Delta_r \lambda(n_c) \) and \( n \), which is presented in (8). In this, \( \Delta_r \lambda(n_c) \) is simply represented as \( \Delta \lambda_c \).

\[
\Delta_r \lambda(n) = \frac{1}{n_{\text{eff}}(n_c)} \left[ a_1 + \frac{b_1}{(c_0 - n)^k} \right] \Delta \lambda_c \tag{8}
\]
At this approximation, if we compare two fibers which differ only by their radii, at the same SRI, their wavelength difference from the same reference should be proportional. This result is presented in (9).

$$\Delta \lambda_2(n) = \frac{\Delta \lambda_{c2}}{\Delta \lambda_{c1}} \implies \Delta \lambda_2(n) = \frac{\Delta \lambda_{c2}}{\Delta \lambda_{c1}} \Delta \lambda_1(n) \quad (9)$$

Equation (9) suggests that it is possible to estimate the wavelength response of a EFBG (\(\lambda_2\)) by knowing the response of a similar one (\(\lambda_1\)), and the ratio of their wavelength shift during the cladding removal (at \(n = n_c\)). An obvious reference to be used in (9) is \(n_0\) for no cladding removal, which is insensitive to the SRI, that is, \(n_{eff} \rightarrow n_0 \implies \lambda_0 = 2 n_0 \Lambda\). By using this reference, Equation (9) can be rewritten as (10), and \(\Delta \lambda_c\) must be reinterpreted as the wavelength shift during the cladding removal.

$$\lambda_2(n) = \lambda_0 \left(1 - \frac{\Delta \lambda_{c2}}{\Delta \lambda_{c1}}\right) + \frac{\Delta \lambda_{c2}}{\Delta \lambda_{c1}} \lambda_1(n) \quad (10)$$

Equation (10) can be used in interpolation. If one knows the response curve for \(\lambda_1\), and \(\lambda_2\) at two different SRI, then by solving \(\lambda_2 = h_0 + h_1 \lambda_1\) for \(h_0\) and \(h_1\), it is possible to estimate \(\lambda_2\) over the same SRI range. Additionally, we can remove the first term in the right side of (10) by subtracting the wavelength at another SRI reference \((n_0)\), resulting in (11).

$$\lambda_2(n) - \lambda_2(n_0) = \frac{\Delta \lambda_{c2}}{\Delta \lambda_{c1}} [\lambda_1(n) - \lambda_1(n_0)] \quad (11)$$

Equation (11) can be used in extrapolation. By knowing the response curve for \(\lambda_1\), the wavelength shift of \(\lambda_1\) and \(\lambda_2\) during the cladding removal, and \(\lambda_2\) at \(n_0\), then it is possible to estimate \(\lambda_2\) over the same SRI range of \(\lambda_1\).

B. Interpolation Procedure

1) For a first sensor, construct its wavelength response curve (wavelength vs SRI) using the experimental data fitting with (12). This equation relates to (2) by \(A' = 2 \Lambda A\) and \(B' = 2 B \Lambda\):

$$\lambda_1(n) = A' + B' \left(\frac{n}{C - n}\right)^k \quad (12)$$

2) For the first, and a second sensor with same characteristics except the fiber radius after the cladding removal, measure their wavelength response at two different SRI values \(n_a\) and \(n_b\). Let these wavelengths be \(\lambda_{1a}\) and \(\lambda_{1b}\) for the first sensor, and \(\lambda_{2a}\) and \(\lambda_{2b}\) for the second one, respectively;

3) From (10), expressed as \(\lambda_2 = h_0 + h_1 \lambda_1\), solve the following linear system for \(h_0\) and \(h_1\):

$$\begin{bmatrix} \lambda_{2a} = h_0 + h_1 \lambda_{1a} \\ \lambda_{2b} = h_0 + h_1 \lambda_{1b} \end{bmatrix} \quad (13)$$

4) Finally, the second sensor’s wavelength response curve is given by:

$$\lambda_{2e}(n) = h_0 + h_1 \lambda_1(n), \quad (14)$$

where the \(e\) index refers to the interpolation procedure.

C. Extrapolation Procedure

1) For a first sensor, measure its wavelength shift during the cladding removal, \(\Delta \lambda_c\);

2) Follow the same first step of the Interpolation procedure, that is, for the first sensor, construct its wavelength response curve \(\lambda_1(n)\) using the experimental data fitting with (12);

3) For a second sensor, measure its wavelength shift during the cladding removal, \(\Delta \lambda_c\);

4) For the first and second sensor, measure their wavelength responses at some SRI reference \(n_a\). Let these wavelengths be \(\lambda_{1a}\) and \(\lambda_{2b}\), for the first and second sensor, respectively;

5) Finally, the second sensor’s wavelength response curve is given by:

$$\lambda_{2e}(n) = \lambda_{2a} + \frac{\Delta \lambda_{c2}}{\Delta \lambda_{c1}} [\lambda_1(n) - \lambda_{1a}], \quad (15)$$

where the \(e\) index refers to the extrapolation procedure.

D. Testing the Estimation Models

In order to test these estimation models, it was used the same radii of Fig. 2, whose values are presented in Table II. The SRI reference was \(n_0 = n_c = 1.31\). The effective refractive index \(n_{eff}\) was calculated using (3). The wavelength \(\lambda_{c}\) was obtained from the calculated \(n_{eff}\), considering \(\Lambda = 532.45\text{nm}\). The reference for \(\Delta \lambda_c\) was \(\lambda_0 = 2 n_0 \Lambda = (1540.014 \pm 0.008)\text{nm}\), with \(n_0 = (1.44616 \pm 0.00001)\text{RIU}\), obtained from the fit using (1), which was presented in Fig. 1.
TABLE II
PARAMETERS AND RESULTS FOR THE ESTIMATION MODELS APPLIED OVER SEVEN DIFFERENT FIBER RADII. EXTRAPOLATION USES THE WAVELENGTH SHIFT DURING THE CLADDING REMOVAL (Δλc). THE VALUES FOR δmax AND σ WERE CALCULATED WITH THE SRI RANGING FROM 1.310RIU TO 1.445RIU

| i  | r [μm] | n_{eff} [RIU] | λc [nm] | Δλc [nm] | Extrapolation | Interpolation |
|----|--------|---------------|---------|----------|---------------|--------------|
|    |        |               |         |          | δ_{max} [nm] | σ [nm]       | δ_{max} [nm] | σ [nm]       |
| 1  | 11.05  | 1.44528       | 1559.08 | -0.94    | 0.10         | 0.01         | 0.09         | 0.01         |
| 2  | 9.34   | 1.44431       | 1538.05 | -1.97    | 0.20         | 0.03         | 0.18         | 0.03         |
| 3  | 8.36   | 1.44334       | 1537.02 | -3.00    | 0.14         | 0.02         | 0.04         | 0.01         |
| 4  | 7.70   | 1.44245       | 1536.04 | -3.97    |              |              |              |              |
| 5  | 7.17   | 1.44149       | 1535.04 | -4.98    | 0.19         | 0.03         | 0.06         | 0.01         |
| 6  | 6.74   | 1.44055       | 1534.04 | -5.97    | 0.42         | 0.07         | 0.14         | 0.02         |
| 7  | 6.37   | 1.43960       | 1533.03 | -6.98    | 0.69         | 0.12         | 0.29         | 0.04         |

For the interpolation, the second reference for λ_{2} was arbitrarily chosen as n = 1.43. Fig. 5 presents the wavelength shift estimation for the seven radii presented in Table II. The response curve used as reference for both extrapolation and interpolation was r_{4} = 7.70μm with Δλ_{c4} = −3.97nm (fourth row). Table II also presents the estimation results. In general, both maximum deviation (δ_{max}) and standard deviation (σ) for the extrapolation are higher than the interpolation. The higher deviations occur with the lower radii, which present higher sensitivity.

III. MATERIALS AND METHODS

In order to test the estimation models, 7 FBGs were inscribed and the cladding were reduced using different etching times to present different refractive index sensibilities. The fiber gratings were inscribed in a single mode fiber (Draka, G.652) using the phase-mask technique, with an ArF excimer laser (Coherent Xantos 500XS) emitting at 193nm. An Ibsen photonics phase mask with 1073.5nm pitch was used. The exposure time was 4min, with pulse energy of 4mJ at 250Hz repetition rate. The chemical etching of the cladding was performed in aqueous solution of hydrofluoric acid (HF). The reaction was stopped by the fiber immersion in water, and neutralized by its immersion in sodium hydroxide (NaOH) solution. The etching process was performed with 7 fibers, and stopped after observing wavelength shifts from 1nm to 7nm, in 1nm intervals. These 7 gratings were named as FBG_{n}, with n ranging from 1 to 7. After each etching, the fibers were immersed in water at room temperature (n = 1.333) in order to register a wavelength reference and the corresponding wavelength shift due to the cladding removal (Δλ_{c}). All measurements were realized using an Optical Spectrum Analyser—OSA (Yokogawa, AQ6375), with ±5pm of wavelength stability, and an ASE as light source. The refractive indexes of the samples were measured at the same temperature with an Abbe refractometer (Biobrix - 2W AJ) with 2×10^{-4} resolution, operating in sodium D-line at 589nm. The measurements were performed 5 times under repeatability conditions.

IV. RESULTS AND DISCUSSIONS

Chemical etching is an exothermic process that initially causes a red shift in the wavelength (Δλ_{RS}).
However, as the evanescent field penetrates the external medium, the wavelength presents a blue shift. Fig. 6 presents this process for FBG2. After the wavelength shift returned to zero, the acid solution is replaced by a lower concentration to construct seven different sensors with better control of the process. Through this, it was possible to achieve one in order to reduce the etching speed and to have a red shift due to the cladding removal $\Delta \lambda_c$ (using water as reference), and the observed red shift during the etching $\Delta \lambda_{RS}$ for each sensor. Theoretically, $\lambda_0$ and $\Delta \lambda_{RS}$ should be the same for all sensors, since the fiber model, the grating period and experimental procedure were the same. Also, $\Delta \lambda_{RS}$ should be proportional to the temperature change, which depends on the reaction rate, and consequently the acid concentration. For all sensors, the “high concentration” stage of the chemical etching was performed in aqueous solution of HF at 40%. In this case, $\Delta \lambda_{RS}$ varied between 1nm to 7nm, in approximately 1nm intervals. Table III presents the initial wavelength responses $\lambda_0$, the wavelength shift due to the cladding removal $\Delta \lambda_c$ (using water as reference), and the observed red shift during the etching $\Delta \lambda_{RS}$ for each sensor. The wavelength shifts of all four sensors were optimized using Equations (14) and (15), respectively. The mean characteristic exponent was determined as $k = 0.101 \pm 0.008$, and it was fixed for all EFBGs in the next step, when the parameters $A'$, $B'$ and $C$ were optimized again. The asymmetric wavelength shifts between the curves presented in Fig. 7 are consistent with the experimental $\Delta \lambda_c$ values presented in Table III and Fig. 8, and they contrast with the symmetry presented by the simulation results (Fig. 2). The C parameter was approximately constant with $C = (1.4612 \pm 0.0007)$RIU. The results for $A'$ and $B'$ are presented in Fig. 8 as a function of $\Delta \lambda_c$. As (10) suggests, $A'$ and $B'$ have a strict linear dependence with the wavelength shift due to the cladding removal $\Delta \lambda_c$. As $\Delta \lambda_c \rightarrow 0$ it was expected $B' \rightarrow 0$, however $B' = 0$ at $\Delta \lambda_c = (0.73 \pm 0.03)$nm through extrapolation, which coincides with the observed red shift $\Delta \lambda_{RS}$ during the etching. This suggests the etching process does not simply reduces the cladding, but also result in an annealing effect changing the wavelength permanently. Through this, the measured $\Delta \lambda_c$ should be corrected, and our best estimation for that is subtracting the red shifts $\Delta \lambda_{RS}$ observed during the etching.

### A. Estimating the Experimental Results

Fig. 9 presents the estimation of the experimental wavelength response of the EFBGs, through interpolation and extrapolation, using Equations (14) and (15), respectively. The wavelength shift of FBG4 was used as the calibration reference.
TABLE IV
FIT RESULTS FOR EACH EFBG SENSOR USING (12) OVER THE DATA PRESENTED IN FIG. 7

| Sensor | $A'$[nm] | $B'$[nm ⋅ RIU] | $C$[RIU] |
|--------|---------|--------------|---------|
| $FBG_1$ | 1547.67 ± 0.08 | 2.60 ± 0.06 | 1.4613 ± 0.0006 |
| $FBG_2$ | 1545.02 ± 0.26 | 4.16 ± 0.21 | 1.4613 ± 0.0014 |
| $FBG_3$ | 1542.90 ± 0.10 | 5.51 ± 0.08 | 1.4609 ± 0.0003 |
| $FBG_4$ | 1539.48 ± 0.09 | 6.90 ± 0.07 | 1.4607 ± 0.0002 |
| $FBG_5$ | 1536.36 ± 0.23 | 8.67 ± 0.18 | 1.4611 ± 0.0007 |
| $FBG_6$ | 1534.02 ± 0.24 | 10.00 ± 0.19 | 1.4616 ± 0.0005 |
| $FBG_7$ | 1530.29 ± 0.15 | 11.83 ± 0.13 | 1.4609 ± 0.0006 |

Fig. 8. Fit parameters ($A'$ and $B'$) from (12) over the data presented in Fig. 7 as a function of the wavelength shift due to the cladding removal ($Δλ_c$). The red squares represent the extrapolation for $B' → 0$, when the sensor should be insensitive to the SRI. This value correspond to $Δλ_c = (0.73 ± 0.03)$nm and $A' = (1552.59 ± 0.08)$nm.

Fig. 9. Extrapolation (top) and interpolation (bottom) of the wavelength response for seven EFBGs. Solid line represents the experimental data fitting with (12). The wavelength response references used in the interpolation procedure (second step) are represented with circles.

curve. Water was the SRI reference for all wavelength shifts ($n_w = 1.333$RIU), since it was used to clean the fibers after the etching, determining $Δλ_c$ (Fig. 6). Table V presents the effective etching wavelength shift ($Δλ'_e$), which was determined as the wavelength shift due to the cladding removal subtracted by the red shift during the etching, $Δλ'_e = Δλ_e − Δλ_{RS}$. This effective shift $Δλ'_e$ was used in place of $Δλ_c$ in (15). The extrapolation procedure is simpler than the interpolation because it requires only the etching wavelength shifts $Δλ'_e$ to estimate the wavelength response. The interpolation procedure is expected to have a better accuracy, but it requires at least another SRI reference ($n_b$), which demands further measurements. For the interpolation procedure, the second SRI reference was chosen to be $n_b = 1.447$RIU. Using these two SRI references, it was possible to calculate the $h_1$ parameter from (14). As expected, $h_1 ≈ Δλ'_e / Δλ_{ref}$, with maximum relative difference of 5% (for $FBG_2$).

The agreement between the estimated curves and the experimental data fitting was evaluated through maximum deviation ($δ_{max}$) and standard deviation ($σ$), considering the SRI interval from 1.333RIU to 1.458RIU. Table V presents $δ_{max}$ and $σ$ between the estimated and experimental curves responses for each $FBG$. In general, the results for extrapolation and interpolation are similar. The largest deviation occurred for $FBG_6$, which can be visualized in Fig. 9. However, when compared to the simulation (Fig. 5 and Table II), the accuracy of the estimation for
the experimental results is embarrassingly good for both extrapolation and interpolation. These results show that the proposed estimation methods are useful and reliable to estimate the response curve of an EFBG using only a calibration curve of a similar sensor and few additional measurements.

V. CONCLUSION

In this work, an empirical mathematical model was proposed to describe the response curve of the EFBG external refractive index. The model was developed from the results of numerical simulations and it was verified experimentally. From this model, it was possible to construct two estimation approaches, in which the response curve of an EFBG could be predicted based on a reference calibration curve of a similar EFBG with a different etching. The extrapolation approach is simpler than the interpolation, since it only needs one value of surrounding refractive index for the wavelength response reference. However, the interpolation model provided better accuracy when compared to the experimental measurements. The proposed mathematical model and the estimation approaches can successfully predict EFBGs’ response curves, which may be useful for the design of sensors with arbitrary sensitivity.

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TABLE V

RESULTS FOR THE ESTIMATION MODELS APPLIED OVER THE EFBG SENSORS. THE WAVELENGTH SHIFT DUE TO THE CLADDING REMOVAL WAS CORRECTED WITH THE RED SHIFT DURING THE ETCHING (∆λ′ = ∆λ − ∆λRBG). THE VALUES FOR δmax AND σ Were Calculated With the SRI RANGING FROM 1.333RIU TO 1.458RIU

| Sensor | ∆λ′ [nm] | δmax [nm] | σ [nm] | δλc/δλc4 | δmax [nm] | σ [nm] | h1 |
|--------|----------|------------|--------|-----------|------------|--------|-----|
| FBG1   | -1.72    | 0.06       | 0.01   | 0.37      | 0.05       | 0.01   | 0.37 |
| FBG2   | -2.63    | 0.07       | 0.02   | 0.56      | 0.19       | 0.03   | 0.59 |
| FBG3   | -3.67    | 0.03       | 0.00   | 0.79      | 0.02       | 0.01   | 0.78 |
| FBG4   | -4.65    |reference   |reference|          |reference   |         |     |
| FBG5   | -5.79    | 0.17       | 0.02   | 1.24      | 0.07       | 0.02   | 1.22 |
| FBG6   | -6.66    | 0.41       | 0.05   | 1.43      | 0.26       | 0.03   | 1.39 |
| FBG7   | -7.84    | 0.04       | 0.01   | 1.69      | 0.17       | 0.03   | 1.73 |
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