Fermi spin polaron and dissipative Fermi-polaron Rabi dynamics

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We consider a spin impurity with multiple energy levels moving in a non-interacting Fermi sea, and theoretically solve this Fermi spin polaron problem at nonzero temperature by using a non-self-consistent many-body $T$-matrix theory. We focus on the simplest case with spin half, where the two energy states of the impurity are coupled by a Rabi flip term. At small Rabi coupling, the impurity exhibits damped Rabi oscillations, where the decoherence is caused by the interaction with the Fermi sea, as recently reported in Fermi polaron experiments with ultracold atoms. We investigate the dependence of Rabi oscillations on the Rabi coupling strength and examine the additional nonlinear damping due to large Rabi coupling. At finite temperature and at nonzero impurity concentration, the impurity can acquire a pronounced momentum distribution. We show that the momentum/thermal average can sizably reduce the visibility of Rabi oscillations. We compare our theoretical predictions to the recent experimental data and find a good agreement without any adjustable parameter.

I. INTRODUCTION

Quantum impurity interacting with a many-body environment is a long-lasting research topic in the modern physics [1]. The earliest study can be traced back to the seminal work by Lev Landau in 1933 [2], which led to the fundamental concept of quasiparticles. Over the last fifteen years, this research topic has received renewed interest, due to rapid advances in ultracold atomic physics [3-5]. In particular, the dynamics of a quantum impurity immersed in a non-interacting Fermi sea, namely, Fermi polaron, has been systematically explored both experimentally and theoretically [6-10]. A convenient experimental setup is the use of a highly imbalanced two-component Fermi-Fermi mixture, where minority atoms in a hyperfine state can be well treated as isolated, uncorrelated impurities. For such a system, quasiparticle properties of Fermi polarons, including the ground-state attractive polaron and the excited branch of repulsive polaron, have been well characterized experimentally by the radio-frequency (rf) spectroscopy [11, 16], Ramsey interferometry [14], Rabi oscillation [13, 15, 18], and most recently by Raman spectroscopy [19]. Theoretically, an exactly solvable polaron model with a heavy impurity and a Bardeen-Cooper-Schrieffer (BCS) superfluid environment has also been constructed [20, 22], clarifying several salient features of Fermi polarons in a rigorous way.

In principle, quantum impurity can have internal degrees of freedom and can occupy multiple energy levels. For example, molecule impurity can be trapped inside a nanodroplet of superfluid helium, forming the so-called angulon quasiparticle [24]. The rotational degree of freedom of the molecule can be affected by the many-body environment of helium droplet, as evidenced by a larger effective moment of inertia. This effect is similar to the renormalization of the effective mass for impurity observed in Fermi polarons [6]. In highly imbalanced Fermi-Fermi mixtures, it is also feasible to coherently transfer minority atoms to another hyperfine state, by using an always-on rf field [24]. Thus, impurity atoms can occupy two different hyperfine states and acquire a pseudo-spin degree of freedom. Indeed, in recent Rabi dynamics experiments for Fermi polarons [13, 14, 18], Rabi oscillation between the two hyperfine states is driven by the rf field with reasonably small coupling strength in the linear response regime, where polaron properties are assumed to be unchanged by Rabi coupling.

In this work, we investigate in detail the Fermi spin polaron with a mobile spinor impurity, with the purpose of better understanding the Rabi dynamics of Fermi polarons. We are specifically interested in the dependence of quasiparticle properties of Fermi spin polarons on the Rabi coupling strength, which is less considered in earlier theoretical analyses on dissipative Rabi dynamics [24, 25, 27] (For an exception, see Ref. [26], where a state-of-the-art simulation of Rabi oscillations is presented). This dependence is crucial to examine the small Rabi coupling assumption adopted in recent experimental measurements [13, 15, 18].

Our theoretical investigation is based on a non-self-consistent many-body $T$-matrix theory of Fermi polarons [28, 30], extended to the case of a spinor impurity. In the spinless case of a structureless impurity, such a many-body $T$-matrix approach is fully equivalent to Chevy’s variational ansatz [37, 40], including its finite temperature extension [41]. This approach is particularly useful for a mobile impurity, whose recoil energy suppresses multiple particle-hole excitations near the Fermi surface of the many-body environment. Our results are therefore complement to the two earlier studies [24, 27], which considered the heavy impurity limit using either a spin-model with an Ohmic bath [24] or the functional determinant approach [27].

Our many-body $T$-matrix theory is also convenient to investigate the finite-momentum effect of polarons, which arises due to the nonzero temperature and the finite impurity concentration. This effect is not em-
phasized in a recent Rabi dynamics study based on the finite-temperature variation approach \[26\], but is found to be important for understanding the measured rf spectroscopy \[23\]. We find that the visibility of Rabi oscillations can be sizably reduced by the momentum average due to the thermal momentum distribution of polarons.

The rest of the paper is organized as follows. In the next section (Sec. II), we present the non-self-consistent many-body $T$-matrix theory for Fermi spin polarons at finite temperature. In Sec. III, we discuss in detail the quasiparticle properties of spin polarons, such as self-energy, spectral function and polaron energies, as a function of the Rabi coupling strength. We emphasize the nonlinear effect arising from large Rabi coupling. In Sec. VI, we first compare our theoretical predictions with the experimental data on Rabi oscillation and show that there is a good agreement, without any free fitting parameters. We then examine the effect of the momentum average and the nonlinear dependence of Rabi oscillations on large Rabi coupling strength. Finally, we give a brief summary and outlook in Sec. V.

II. THE NON-SELF-CONSISTENT MANY-BODY $T$-MATRIX THEORY

A. The model Hamiltonian

According to the recent experiments on dissipative Rabi dynamics \[13\], \[14\], \[18\], we consider a spin-1/2 impurity of mass $m_I$ that has two hyperfine energy levels (i.e., $\sigma = \uparrow, \downarrow$), described by the single-particle model Hamiltonian \[24\] \[27\],

$$H_I = \sum_{\mathbf{p}\sigma} \epsilon_{\mathbf{p}\sigma} d_{\mathbf{p}\sigma}^\dagger d_{\mathbf{p}\sigma} + \frac{\Omega}{2} \sum_{\mathbf{p}} \left( d_{\mathbf{p}\uparrow}^\dagger d_{\mathbf{p}\downarrow} + d_{\mathbf{p}\downarrow}^\dagger d_{\mathbf{p}\uparrow} \right), \quad (1)$$

where $d_{\mathbf{p}\sigma}^\dagger$ and $d_{\mathbf{p}\sigma}$ are the creation and annihilation field operators for the impurity with momentum $\mathbf{p}$ in the spin-up ($\sigma = \uparrow$) and spin-down ($\sigma = \downarrow$) states that have the dispersion relations $\epsilon_{\mathbf{p}\sigma} = \epsilon_{\mathbf{p}\uparrow} = \hbar^2 \mathbf{p}^2/(2m_I)$ and $\epsilon_{\mathbf{p}\sigma} = \epsilon_{\mathbf{p}\downarrow} + \Delta$, respectively, $\Delta$ is the detuning, and $\Omega$ is the Rabi coupling strength.

For a spin-1/2 impurity, its non-interacting thermal Green function is a 2 by 2 matrix $G_0(\mathcal{P})$,

$$G_{11}^{(0)} G_{12}^{(0)} = \begin{pmatrix} i\omega_p - \epsilon_{\mathbf{p}\uparrow} - \Omega/2 & -\Omega/2 \\ \Omega/2 & i\omega_p - \epsilon_{\mathbf{p}\downarrow} - \Delta \end{pmatrix}^{-1}, \quad (2)$$

where we have used the short-hand notation $\mathcal{P} = (\mathbf{p}, i\omega_p)$ with fermionic Matsubara frequency $\omega_p = (2p + 1)\pi k_B T$ at temperature $T$ and integer $p = 0, \pm 1, \pm 2, \ldots$. By diagonalizing the matrix, we find two energy levels, $E_{\mathbf{p}}^{(\pm)} = (\epsilon_{\mathbf{p}\sigma}^{(l)} + \Delta)/2 \pm \sqrt{\Delta^2 + \Omega^2}/2$. The associated amplitudes (i.e., wavefunctions) are given by,

$$u^2 = \frac{1}{2} \left[ 1 + \frac{\Delta}{\sqrt{\Delta^2 + \Omega^2}} \right], \quad (3)$$

$$v^2 = \frac{1}{2} \left[ 1 - \frac{\Delta}{\sqrt{\Delta^2 + \Omega^2}} \right], \quad (4)$$

$$uv = \frac{\Omega}{2\sqrt{\Delta^2 + \Omega^2}} \quad (5)$$

The non-interacting impurity Green function can then be conveniently written as,

$$\begin{pmatrix} G_{11}^{(0)} & G_{12}^{(0)} \\ G_{21}^{(0)} & G_{22}^{(0)} \end{pmatrix} = \begin{pmatrix} v^2 uv & u^2 - uv \i\omega_p - E_{\mathbf{p}\uparrow} \i\omega_p - E_{\mathbf{p}\downarrow} \end{pmatrix}, \quad (6)$$

The impurity is moving in and interacting with an ideal Fermi sea of fermionic atoms of mass $m$ described by $\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow}$, where $c_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}}$ are the creation and annihilation field operators for fermionic atoms with momentum $\mathbf{k}$ and single-particle dispersion relation $\epsilon_{\mathbf{k}} = \hbar^2 k^2/(2m)$. The total model Hamiltonian then takes the form,

$$H = H_I + \sum_{\sigma} \frac{g_\sigma}{V} \sum_{\mathbf{k}\mathbf{p}\mathbf{q}} c_{\mathbf{k}\uparrow} d_{\mathbf{q}\uparrow \rightarrow \mathbf{k}\sigma} d_{\mathbf{q}\downarrow} c_{\mathbf{p}\sigma} + \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow}, \quad (7)$$

where $V$ is the system volume and the middle term describes the s-wave contact interactions between the impurity and the Fermi bath with bare interaction strengths $g_\sigma$, which are to be regularized via the relation,

$$\frac{1}{g_\sigma} = \frac{m_r}{2\pi\hbar^2 a_\sigma} - \frac{1}{V} \sum_{\mathbf{k}} \frac{2m_r}{\hbar^2 k^2}. \quad (8)$$

Here $a_\sigma$ ($\sigma = \uparrow, \downarrow$ or interchangeably $\sigma = 1, 2$) is the s-wave impurity-bath scattering length, $m_r \equiv mm_I/(m + m_I)$ is the reduced mass. Throughout the work, we always take $m_I = m$, so $m_r = m/2$. The density ($n$) or the total number ($N = nV$) of fermionic atoms in the Fermi sea can be tuned by adjusting the temperature-dependent chemical potential $\mu(T)$. We often measure the single-particle energy of atoms from the chemical potential and therefore define $\xi_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}} - \mu$. Hereafter, for clarity we will suppress the volume $V$ in expressions, so the summation over the momentum $\sum_{\mathbf{k}}$ in the later equations should be understood as $\sum_{\mathbf{k}} = (1/V) \sum_{\mathbf{k}} = \int d\mathbf{k}/(2\pi)^3$.

B. The diagrammatic theory

We use the non-self-consistent many-body $T$-matrix theory \[28\] \[32\] to solve the Fermi spin polaron problem, within which the motion of the impurity can be described by a series of ladder diagrams that take into account the successive forward scatterings between the impurity and
the atoms in the Fermi bath. By summing up the infinitely many ladder diagrams, as detailed in Appendix A, we find the two-particle vertex function, which takes the following 2 by 2 matrix form,

\[
\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} \\
\Gamma_{21} & \Gamma_{22}
\end{bmatrix} = \begin{bmatrix}
1/g_1 + \tilde{\chi}_{11}(Q) & \tilde{\chi}_{12}(Q) \\
\chi_{21}(Q) & 1/g_2 + \chi_{22}(Q)
\end{bmatrix}^{-1},
\]  

where \(Q \equiv (q, i\nu_q)\) is the short-hand notation of the four dimensional momentum with bosonic Matsubara frequency \(\nu_q = 2q\pi k_B T\) and integer \(q = 0, \pm 1, \pm 2, \ldots\), and the various pair propagators \(\tilde{\chi}_{ij}\) \((i, j = 1, 2)\) are given by,

\[
\tilde{\chi}_{ij}(Q) = \sum_k k_B T \sum_{i\omega_k} G(K) G_{ij}^{(0)}(Q-K). \tag{10}
\]

Here we have introduced \(K \equiv (K, i\omega_K)\) with fermionic Matsubara frequency \(\omega_K = (2k + 1)\pi k_B T\) and integer \(k = 0, \pm 1, \pm 2, \ldots\), and

\[
G(K) = \frac{1}{i\omega_k - \xi_k} = \frac{1}{i\omega_k - \epsilon_k + \mu} \tag{11}
\]

is the thermal Green function for non-interacting fermionic atoms in the Fermi bath. The summation over the fermionic Matsubara frequency in Eq. (10) is easy to carry out. In the single impurity limit, we find that \[35],

\[
\tilde{\chi}_{11} = \sum_k \frac{v^2 (f(\xi_k) - 1)}{iv_q - E_{q-k} - \xi_k} + \frac{u^2 (f(\xi_k) - 1)}{iv_q - E_{q-k}^+ - \xi_k} \tag{12}
\]

\[
\tilde{\chi}_{12} = \sum_k \frac{uv (f(\xi_k) - 1)}{iv_q - E_{q-k} - \xi_k} - \frac{uv (f(\xi_k) - 1)}{iv_q - E_{q-k}^+ - \xi_k} \tag{13}
\]

\[\tilde{\chi}_{21} = \tilde{\chi}_{12}, \text{ and } \tilde{\chi}_{22} \text{ can be obtained from } \tilde{\chi}_{11} \text{ by exchanging the factor } u^2 \text{ with } v^2 \text{ in the square bracket. The function } f(x) = 1/[e^{x/(k_B T)} + 1] \text{ is the Fermi-Dirac distribution at temperature } T. \text{ It is readily seen that the integral in both } \tilde{\chi}_{11} \text{ and } \tilde{\chi}_{22} \text{ has an ultraviolet divergence at large momentum. This divergence is due to the use of the s-wave contact interactions and can be exactly compensated by the counter term in the regularization relation Eq. (35), i.e., } \sum_k 2m_r/(\hbar^2 k^2), \text{ therefore, it is convenient to introduce } \chi_{11} \equiv 1/g_1 + \chi_{11} \text{ and } \chi_{22} \equiv 1/g_2 + \chi_{22}, \text{ and rewrite } \chi_{12} \equiv \chi_{12} \text{ and } \chi_{21} \equiv \chi_{21}. \text{ We will still refer to } \chi_{ij}(Q) \text{ as the pair propagators, without any confusion.}

The integrals in \(\chi_{ij}(Q)\) can be categorized into two types \[35\]. The first is the two-body part, which can be analytically evaluated by using,

\[
\sum_k \left[ \frac{1}{\Omega - \epsilon_q^{(1)} - \xi_k} + \frac{2m_r}{\hbar^2 k^2} \right] = -\frac{i (2m_r)^\frac{3}{2}}{4\pi \hbar^3} \sqrt{\Omega - \zeta_q} \tag{14}
\]

for any complex frequency \(\Omega\). Here \(\zeta_q \equiv \hbar^2 q^2/[2(m + m_f)] - \mu\) is the center-of-mass kinetic energy measured from the chemical potential. Another is the many-body part, which takes the form,

\[
\chi_{\text{eff}}(q, \Omega) = \sum_k \frac{f(\xi_k)}{\Omega - \epsilon_q^{(1)} - \xi_k} \tag{15}
\]

and can be numerically calculated in a very efficient way, as discussed in detail in our recent work (see, i.e., Appendix A of Ref. \[35\]). By defining two constants \(\gamma_{\pm} = (\Delta \pm \sqrt{\Delta^2 + \Omega^2})/2\) and rewriting \(E_{q}^{(\pm)} = \epsilon_q^{(1)} + \gamma_{\pm}\), it is then easy to check that \((Q \equiv (q, i\nu_q) \equiv (q, \Omega))\),

\[
\chi_{11}(Q) = \frac{m_r}{2\pi \hbar^2 a_1} + \frac{im_r^{3/2}}{\sqrt{2\pi \hbar^3}} \left[ v^2 \sqrt{\Omega - \gamma_+ - \zeta_q} + u^2 \sqrt{\Omega - \gamma_- - \zeta_q} \right] + v^2 \chi_{\text{eff}}(q, \Omega - \gamma_+) + u^2 \chi_{\text{eff}}(q, \Omega - \gamma_-) \tag{16}
\]

\[
\chi_{12}(Q) = \frac{im_r^{3/2}}{\sqrt{2\pi \hbar^3}} uv \left[ \sqrt{\Omega - \gamma_+ - \zeta_q} - \sqrt{\Omega - \gamma_- - \zeta_q} \right] + uv \left[ \chi_{\text{eff}}(q, \Omega - \gamma_+) - \chi_{\text{eff}}(q, \Omega - \gamma_-) \right], \tag{17}
\]

\[
\chi_{22}(Q) = \frac{m_r}{2\pi \hbar^2 a_2} + \frac{im_r^{3/2}}{\sqrt{2\pi \hbar^3}} \left[ u^2 \sqrt{\Omega - \gamma_+ - \zeta_q} + v^2 \sqrt{\Omega - \gamma_- - \zeta_q} \right] + u^2 \chi_{\text{eff}}(q, \Omega - \gamma_+) + v^2 \chi_{\text{eff}}(q, \Omega - \gamma_-) \tag{18}
\]

Once the pair propagators \(\chi_{ij}(Q)\) are obtained, we take the matrix inverse to find the 2 by 2 matrix function \(\Gamma(Q) = [\chi(Q)]^{-1}\). The 2 by 2 self-energy of the impurity \(\Sigma_{ij}(\mathcal{P})\) can be obtained by winding back the
out-going leg of the fermionic field operator in the vertex function \( \Gamma(Q)_{ij} \) and by connecting it with the in-coming leg of the fermionic field operator \([28, 33]\). Physically, this describes the single particle-hole excitation across the Fermi surface of the bath \([28, 31, 37]\). Thus, we have

\[
\Sigma_{ij}(P) = \sum_q k_B T \sum_{i \omega_q} \Gamma_{ij}(Q) \frac{1}{i \nu_q - i \omega_P - \xi_{q-p}}. \tag{19}
\]

The consequent matrix inverse leads to the retarded two-to-setting \( 2 \) \( T \)-matrix result Eq. (21) with another approximated imp.-matrix result Eq. (21) with another approximated imp. matrix inverse, only the 11-component of the matrices, such as the two-particle vertex function \( \Gamma_{11}(Q) = \chi_{11}^{-1}(Q) \) and the self-energy \( \Sigma_{11}(p, \omega) \), is nonzero. We note that, the final-state effect in the rf-spectroscopy \([11, 16]\) or Raman spectroscopy \([13]\) and the Fermi bath \((n \downarrow p, \omega)\) at zero momentum \((p = 0)\) in the unitary limit \( 1/a = 0 \). The self-energy is in units of of \( \varepsilon_F \), where \( \varepsilon_F \equiv \hbar^2 k_F^2/(2m) \) and \( k_F = (6\pi^2 n)^{1/3} \) are the Fermi energy and Fermi wavevector, respectively. The temperature \( T \) is equal to the attractive polaron energy (at zero Rabi coupling), \( \Delta = E_{\text{att}} \). We have consider three characteristic Rabi couplings \( \Omega = 0.1\varepsilon_F \) (black solid line), \( 1.0\varepsilon_F \) (red dashed line), and \( 2.0\varepsilon_F \) (blue dot-dashed line). The green dotted line in (a) shows the curve \( y = \omega \). The arrow in (b) points to the attractive polaron energy \( E_{\text{att}} \approx -0.64\varepsilon_F \).

\[
\begin{align*}
\text{C. Analytic continuation and numerical calculations}
\end{align*}
\]

We are interested in the retarded impurity Green functions given by the Dyson equation,

\[
G(p, \omega) = [G_0^{-1}(p, \omega) - \Sigma(p, \omega)]^{-1}, \tag{21}
\]

and the related impurity spectral functions \( A(p, \omega) \),

\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} = -\frac{1}{\pi} \text{Im} \left[ 
\begin{bmatrix}
G_{11}(p, \omega) & G_{12}(p, \omega) \\
G_{21}(p, \omega) & G_{22}(p, \omega)
\end{bmatrix} \right]. \tag{22}
\]

Here, \( G_0(p, \omega) \) and \( \Sigma(p, \omega) \) are given by Eq. (3) and Eq. (23), respectively, with the analytic continuation (i.e., \( \omega_p \rightarrow \omega + i0^+ \)) explicitly performed. As mentioned earlier, this analytic continuation can be trivially done by replacing \( \Omega \) in Eqs. (10)-(18) with \( \Omega + i0^+ \) for the calculations of the retarded pair propagators \( \chi_{ij}(q, \Omega) \). The consequent matrix inverse leads to the retarded two-particle vertex functions \( \Gamma_{ij}(q, \Omega) \), which is to be used in Eq. (20).

In our numerical calculations, we take the Fermi wave-vector \( k_F \equiv (6\pi^2 n)^{1/3} \) and Fermi energy \( \varepsilon_F \equiv \hbar^2 k_F^2/(2m) \) as the units of the momentum (or wavevector) and energy, respectively. The temperature \( T \) is accordingly measured in units of the Fermi temperature \( T_F = \varepsilon_F/k_B \). The choice of this natural units amounts to setting \( 2m = \hbar = k_B = 1 \). As we take the equal mass for the impurity and background fermionic atoms \((m = m_1)\), the reduced mass \( m_r = 1/4 \). In Eqs. (10)-(18), we find that \( m_r/(2\pi \hbar^2 a_i) = 1/(8\pi a_i) \) \((i = 1, 2)\), \( m_r^{-1/2}/(\sqrt{2}\pi) = 1/(8\sqrt{2}\pi) \) and \( \xi_0 = q^2/2 - \mu \). The dimensionless expression of \( \chi_{\text{eff}}(q, \Omega + i0^+) \) can be found in Appendix A of Ref. [33].

### III. Quasiparticle Properties of Fermi Spin Polarons

In the experiments on dissipative Rabi dynamics \([13, 15, 18]\), the interaction between the spin-down impurity

and the Fermi bath \((a_1 \text{ or } a_2)\) is typically small. For convenience, we simply set \( a_2 = 0^- \) and denote \( a_2 = a_1 = a \). Therefore, in Eq. (18) \( \chi_{22}(Q) \rightarrow \infty \). By taking the matrix inverse, only the 11-component of the matrices, such as the two-particle vertex function \( \Gamma_{11}(Q) = \chi_{11}^{-1}(Q) \) and the self-energy \( \Sigma_{11}(p, \omega) \), is nonzero. We note that, the case with a small but positive scattering length \( a_2 > 0 \) might be useful to understand the residual final-state effect in the rf-spectroscopy \([13, 15]\) or Raman spectroscopy \([19]\), and could be addressed in future studies.

#### A. Self-energy

For the case of \( a_2 = 0 \), it is useful to contrast our \( T \)-matrix result Eq. (21) with another approximated im-
Typically a reasonably large Rabi coupling \( \Omega \sim \varepsilon \) like (\( \Omega \) Rabi coupling to the self-energy \( \Sigma(0) \)) always the impurity is larger pioneering investigations of the dissipative Fermi polaron physics without Rabi coupling, which is determined by the self-energy \( \Sigma(0)(p, \omega) \) at \( \Omega = 0 \). In more detail, through strongly driven Rabi oscillations we would instead measure \( \Omega \)-dependent attractive and repulsive polaron energies, and their \( \Omega \)-dependent decay rates. The decay rate is roughly proportional to \(-\text{Im}\Sigma(1)(0, \omega)\). As indicated by the arrows in Fig. 1(b) and Fig. 2(b), we find that the imaginary part of the self-energy \(-\text{Im}\Sigma(1)(0, \omega)\) increases with increasing Rabi coupling strength. The change in \(-\text{Im}\Sigma(1)(0, \omega \sim E_{\text{att}})\) of the attractive polaron branch is particularly significant at large Rabi coupling: it increases from a negligible value \(0.006\varepsilon_F\) to a considerable value \(0.097\varepsilon_F\). As we shall see, this will bring an additional damping to Fermi-polaron Rabi oscillations.

**B. Single-particle spectral function**

Using the self-energy \( \Sigma(1)(p, \omega) \) in Eq. (21), we calculate directly the impurity Green functions \( G_{1}(p, \omega) \) and \( G_{22}(p, \omega) \), and the associated single-particle spectral functions \( A_{11}(p, \omega) \) and \( A_{22}(p, \omega) \). Two example cases of zero-momentum spectral function are shown in Fig. 8 and Fig. 4 for the interaction strengths \(1/(k_F a) = 0\) and \(1/(k_F a) = 0.5\), respectively. In each case, we consider the resonant detuning for attractive or repulsive polarons.

It is readily seen that there are several peaks in the spectral functions \( A_{11}(0, \omega) \) and \( A_{22}(0, \omega) \). Each peak corresponds to a well-define quasiparticle, with its life-
Eq. (21) at zero momentum is useful to approximate the impurity Green function in the blue dot-dashed line on the left. 

\[ \omega \text{ attractive polaron} \] at \( p \) purity spectral functions \( A \) and \( \omega \) blue dot-dashed line on the right indicates the peak position is \( T \). The temperature is \( T = 0.2T_F \) and the Rabi coupling is \( \Omega = 0.5\varepsilon_F \). The blue dot-dashed line indicates the peak position \( \omega_p = E_{\text{att}} \approx -0.64\varepsilon_F \) of the impurity spectral function without Rabi coupling \( \Omega = 0 \).

\[ F = \frac{\omega}{\varepsilon_F} \]

\[ A_{11}(p,0) \] and \( A_{22}(p,0) \) at the interaction strength \( F \) and the Rabi coupling is \( \Omega = 2\varepsilon_F \). The temperature is \( T = 0.2T_F \).

\[ G(0,\omega) = \left[ Z^{-1}(\omega - \varepsilon_P + i\Gamma/2) -\Omega/2 \right]^{-1} \] at zero momentum \( p = 0 \) as \[ 18 \],

\[ \frac{1}{\omega - \Sigma_{11}(0,\omega)} \approx \frac{1}{(\omega - \varepsilon_P) \left( 1 - \frac{\partial \Sigma_{11}(\omega)}{\partial \omega} \right) - i\text{Im}\Sigma_{11}} \] by Taylor-expanding \( \Sigma_{11}(0,\omega) \) near \( \omega = \varepsilon_P \). Following the standard way \[ 22, 35 \] to introduce the residue \( Z = [1 - \partial \Sigma_{11}(\omega)/\partial \omega]^{-1} \) and decay rate \( \Gamma = -2\text{Im}\Sigma_{11} \), we then arrive at Eq. \[ 24 \]. This approximate form of the impurity Green function is very useful to understand the Rabi dynamics of the Fermi polaron in the spin-up state, as suggested in Ref. \[ 18 \]. For example, at the resonant detuning \( \Delta = \varepsilon_P \), it is easy to find that the poles of Eq.
[24] satisfy the equation,

$$\left( E - \varepsilon_{p} + i \frac{\Gamma}{2} \right) \left( E - \varepsilon_{p} \right) - \frac{Z \Omega^{2}}{4} = 0, \quad (26)$$

and are given by $$(\Gamma_{R} \equiv \Gamma/2) \quad [18, 26],$$

$$E_{\pm} = \left( \varepsilon_{p} \pm \frac{1}{2} \sqrt{Z \Omega^{2} - \Gamma_{R}^{2}} \right) - i \frac{\Gamma_{R}}{2}. \quad (27)$$

The form of the quasiparticle energies in the above equation clearly indicates Rabi oscillations with a modified effective Rabi coupling strength $\Omega_{\text{eff}} = \sqrt{Z \Omega}$ and with a damping rate $\Gamma_{R} = \Gamma/2$. Therefore, if one neglects the $\Omega$-dependence of the residue $Z$ and of the decay rate $\Gamma$ (which seems justified from Fig. 4 and Fig. 2) for $\Omega < \varepsilon_{F}$, as we have discussed in the last subsection, one can directly extract both residue and decay rate of Fermi polarons from Rabi oscillations $[13, 15, 18]$. In the strong driving regime, $\Omega \gg \varepsilon_{F}$, instead we anticipate that the effective Rabi coupling strength $\Omega_{\text{eff}}$ will deviate from $\sqrt{Z \Omega}$. More discussions on this point will be provided in the next section.

As shown in Fig. 3 for the unitary impurity-bath interaction, where only attractive polaron exists, we find two peaks with position well described by Eq. (27). The situation becomes a little complicated at the interaction strength $1/(k_{F} a) = 0$ (a) and $1/(k_{F} a) = 0.5$ (b). In the unitary limit $1/(k_{F} a) = 0$, we take the detuning $\Delta = E_{\text{att}} \simeq -0.64 \varepsilon_{F}$, while at the interaction strength $1/(k_{F} a) = 0.5$, we set $\Delta = E_{\text{rep}} \simeq 0.80 \varepsilon_{F}$. These two detunings are indicated by the red dashed lines. The green solid lines show the anticipated energies $E_{\text{n}} = \Delta \pm \sqrt{Z} \Omega/2$, where $Z_{\text{att}} \simeq 0.73$ in (a) and $Z_{\text{rep}} \simeq 0.47$ in (b). The temperature is $T = 0.2T_{F}$.
rity Green function Eq. (24), but it could be obtained by numerically solving the poles of the full impurity Green function Eq. (21).

C. Energies of Fermi spin polarons

We have numerically determined the poles of the full impurity Green function in Eq. (21), by neglecting the imaginary part of the self-energy \( \text{Im} \Sigma_{11} \). The results in the unitary limit and at \( 1/(k_F a) = 0.5 \) as a function of the detuning are shown in Fig. 5 and Fig. 6, respectively, at two Rabi coupling strengths \( \Omega = 0.5 \varepsilon_F \) (see the upper panels of the figures) and \( \Omega = 2 \varepsilon_F \) (lower panels).

In the unitary limit (Fig. 5), we find two energies of the Fermi spin polaron, which basically follow,

\[
E_{\pm} = \frac{1}{2} \left[ \varepsilon_P + \Delta \pm \sqrt{\varepsilon^2 P - \Delta^2} \right],
\]

(28)

if we neglect the small decay rate (i.e., \( \Gamma \ll \varepsilon_F \)). This is particularly evident at the small Rabi coupling (see Fig. 5(a)), where both energies follow \( \varepsilon_P \) and \( \Delta \) far off the resonance and exhibit a well-defined avoided crossing at the resonance \( \Delta = \varepsilon_P \). At the large Rabi coupling (Fig. 5(b)), however, we find that the upper branch of the energies seems to develop an additional structure around zero detuning \( \Delta = 0 \). We attribute it to the strongly modified self-energy \( \text{Im} \Sigma_{11} \) at large Rabi coupling.

At the interaction strength \( 1/(k_F a) = 0.5 \) (Fig. 5), we typically find four poles in the impurity Green function Eq. (21). The pole closest to \( E = 0 \) is a false solution, since we do not include \( \text{Im} \Sigma_{11} \) in finding the poles of Eq. (21). In general, \( \text{Im} \Sigma_{11} \) takes a very large value near zero energy (see, i.e., Fig. 2(b)). Thus, it is meaningless to treat the near-zero-energy solution as a well-defined quasiparticle. We find two avoided crossings located at the resonances \( \Delta = \varepsilon_{\text{att}} \) and \( \Delta = \varepsilon_{\text{rep}} \). Far off the resonances, the other three poles basically follow the trace of \( \varepsilon_P = \varepsilon_{\text{att}} \), \( \varepsilon_P = \varepsilon_{\text{rep}} \) (see the dotted red lines) and \( \Delta \) (thin green line). At large Rabi coupling, the middle pole may disappear at the detuning \( \Delta \approx 0 \), as shown in Fig. 5(b). This is again due to the large value of \( \text{Im} \Sigma_{11} \) near zero energy.

In Fig. 7 we report the energy splitting at the resonant detuning, as a function of the Rabi coupling, as predicted by Eq. (27). By ignoring the small decay rate \( \Gamma_R \), the energy splitting is given by \( \delta E = \sqrt{Z} \Omega \), where \( Z \) is the residue of either attractive polaron or repulsive polaron. For the Rabi coupling \( \Omega \leq \varepsilon_F \), we find that Eq. (27) provides an excellent fit to the numerically extracted quasiparticle energies. At larger Rabi coupling, nonlinear deviation from Eq. (27) becomes sizable, indicating the breakdown of the approximate impurity Green function in Eq. (24).

IV. DISSIPATIVE RABI DYNAMICS OF FERMI POLARONS

Let us now try to better understand the recent experiments on Fermi polaron Rabi oscillations [13, 15, 18], by examining more closely the role played by a reasonably large Rabi coupling strength \( \Omega \sim \varepsilon_F \). Experimentally, the impurity is initially prepared in the non-interacting (or weakly-interacting) spin-down state. At time zero, a simple square pulse is added to transfer the impurity to the spin-up state that is in strongly interaction with the Fermi bath. The frequency of the pulse is suitably chosen, so either the attractive polaron branch or the repulsive polaron branch of the spin-up state is selected to be on resonance (i.e., \( \Delta = \varepsilon_{\text{att}} \) or \( \Delta = \varepsilon_{\text{rep}} \)). After a variable holding time \( t \), the relative population of the impurity in the spin-down state is then determined.

A. Theory of the dissipative Rabi oscillation

Physically, for a single impurity this procedure measures the spin-down occupation,

\[
\begin{aligned}
n_{\downarrow} (t) &= \left\langle \psi (t) \left| \sum_p d_{p \downarrow}^\dagger d_{p \downarrow} \right| \psi (t) \rightangle, \\
&= \left\langle \psi (0) \left| \sum_p d_{p \downarrow}^\dagger (t) d_{p \downarrow} (t) \right| \psi (0) \rightangle
\end{aligned}
\]

(29)

where the time-dependent many-body wavefunction is \( |\psi (t)\rangle = e^{-i\varepsilon t / \hbar} |\psi (0)\rangle \), time-dependent field operator \( d_{p \downarrow} (t) = e^{i\varepsilon t / \hbar} d_{p \downarrow} e^{-i\varepsilon t / \hbar} \), and the initial wavefunction at time \( t = 0 \) is given by,

\[
|\psi (0)\rangle = |\downarrow\rangle_T \otimes |\text{FS}\rangle.
\]

(30)

Here, since the initial spin-down impurity does not interact with the thermal Fermi bath, we have taken \( |\psi (0)\rangle \) as a direct product of a thermal impurity state \( |\downarrow\rangle_T \) and a thermal Fermi sea \( |\text{FS}\rangle \). In the Fermi sea, at finite temperature \( T \) fermionic atoms are occupied into single-particle states according to the Fermi-Dirac distribution. For a single impurity, the thermal probability of the spin-down impurity would be given by a suitable distribution function \( f_k \) that is determined by the type or statistics of the impurity, if it has a momentum \( k \). By substituting Eq. (30) into Eq. (29), we find that,

\[
n_{\downarrow} (t) = \sum_{pk} f_k \left( \text{FS} \left| d_{k \downarrow}^\dagger (t) d_{p \downarrow} (t) d_{k \downarrow} \right| \text{FS} \right).
\]

(31)

It is difficult to exactly evaluate \( n_{\downarrow} (t) \) for long time, which involves a product of four field operators. This is because the effects due to strong correlations will gradually accumulate during the time evolution. For the time scale in the Rabi dynamics experiments (i.e., for a few Rabi oscillations), however, it might be instructive to
consider the following first-order, mean-field type decoupling,

\[ n_\downarrow(t) \approx \sum_p f_p \left\langle d_{p\downarrow}^\dagger(t) d_{p\downarrow}(t) \right\rangle d_{p\downarrow}^\dagger(t), \]

\[ = \sum_p f_p S_{\downarrow\downarrow}(p, t), \]  

(32)
as inspired by the well-known Wick theorem in the diagrammatic theory. Here, in the second line we have introduced for \( t > 0 \),

\[ S_{\downarrow\downarrow}(p, t) \equiv \left\langle \text{FS} \left| d_{p\downarrow}(t) d_{p\downarrow}^\dagger \right| \text{FS} \right\rangle \equiv \left\langle d_{p\downarrow}(t) d_{p\downarrow}^\dagger \right\rangle. \]  

(33)

It is easy to recognize that \( S_{\downarrow\downarrow}(p, t) = iG_{22}(p, t) \) is exactly the impurity Green function in the spin-down channel in the time domain. Therefore, we can determine it directly from the single-particle spectral functions, i.e.,

\[ S_{\downarrow\downarrow}(p, t) = \int_{-\infty}^{+\infty} d\omega A_{22}(p, \omega) e^{-i\omega t}. \]  

(34)

and consequently, we are able to calculate

\[ n_\downarrow(t) \approx \sum_p f_p |S_{\downarrow\downarrow}(p, t)|^2. \]  

(35)

An expression similar to Eq. (35) but without the thermal average has been advised by Adlong et al. (see Eq. (4.16) in Ref. [26]), based on the variational Chevy’s ansatz. To show the usefulness of Eq. (35), let us derive an analytic expression of \( n_\downarrow(t) \) following Ref. [26], by using the approximate impurity Green function in Eq. (24) at zero momentum. It is readily seen that, the spin-down impurity Green function then takes the approximate form,

\[ G_{22}(0, \omega) = \frac{A}{\omega - E_+ + i\Gamma_R/2} + \frac{1 - A}{\omega - E_- + i\Gamma_R/2}, \]  

(36)

where the pole energies \( E_+ \) and \( E_- \) are given by Eq. (28), and \( A \equiv 1/2 - (\mathcal{F}_p - \Delta)/(2\Omega_{\text{eff}}) \) with \( \Omega_{\text{eff}} = \sqrt{Z\Omega^2 + (\mathcal{F}_p - \Delta)^2} \). After some straightforward algebra, we find that,

\[ |S_{\downarrow\downarrow}|^2 \approx e^{-\Gamma_R t} \left[ \cos^2 \frac{\Omega_{\text{eff}} t}{2} + \frac{(\mathcal{F}_p - \Delta)^2}{\Omega_{\text{eff}}^2} \sin^2 \frac{\Omega_{\text{eff}} t}{2} \right], \]  

(37)

which clearly exhibits an oscillation with periodicity \( 2\pi/\Omega_{\text{eff}} \) and damping rate \( \Gamma_R \).

The above equation and Eq. (35) are not applicable for long evolution time. This is partly reflected in the exponential decay of \( |S_{\downarrow\downarrow}|^2 \), which implies that \( n_\downarrow(t \to \infty) = 0 \) for any detuning \( \Delta \). However, at the resonant detuning \( \Delta = \mathcal{F}_p \), the effective bias for the impurity spin would be zero [24]. Therefore, we should anticipate a zero steady-state magnetization, or, \( n_\uparrow(t \to \infty) = n_\downarrow(t \to \infty) = 1/2 \), A possible reason why Eq. (35) can not give a zero steady-state magnetization is that the single impurity condition, i.e., \( \sum_p [d_{p\uparrow}^\dagger(t) d_{p\uparrow}(t) + d_{p\downarrow}^\dagger(t) d_{p\downarrow}(t)] = 1 \), is not strictly satisfied by our approximated mean-field type decoupling.

To rectify this weakness, it is useful to consider the following operator for the spin-down occupation,

\[ \hat{n}_\downarrow(t) = \frac{1}{2} - \frac{1}{2} \sum_p \left[ d_{p\downarrow}^\dagger(t) d_{p\downarrow}(t) - d_{p\downarrow}^\dagger(t) d_{p\downarrow}(t) \right], \]  

(38)

and then calculate

\[ n_\downarrow(t) = \sum_k f_k \left\langle \text{FS} \left| d_{k\downarrow} \hat{n}_\downarrow(t) d_{k\downarrow}^\dagger \right| \text{FS} \right\rangle. \]  

(39)

By using the mean-field decoupling and repeating the steps that lead to Eq. (35), it is easy to derive that,

\[ n_\downarrow(t) \approx \frac{1}{2} + \frac{1}{2} \sum_p \left[ |S_{\downarrow\downarrow}(p, t)|^2 - |S_{\uparrow\downarrow}(p, t)|^2 \right], \]  

(40)

where \( S_{\uparrow\downarrow}(p, t) \) takes the form,

\[ S_{\uparrow\downarrow}(p, t) = \int_{-\infty}^{+\infty} d\omega A_{12}(p, \omega) e^{-i\omega t}. \]  

(41)

The use of Eq. (40) is still restricted to the short-time evolution of a few Rabi oscillations. However, we anticipate that it may provide a more accurate prediction than Eq. (35) at the resonant detuning, the case that we will focus on.

B. Comparison between the theory and experiments

We consider the recent Rabi oscillation experiment carried out at the European Laboratory for Non-linear Spectroscopy (LENS) [17]. There, impurities are the minority fermionic \(^6\text{Li}\) atoms, initially in the weakly interacting hyperfine state \((\downarrow\downarrow)\) (i.e., the second lowest-energy Zeeman state). The impurity concentration is about \( n_{\text{imp}} \approx 0.15n \), where \( n \) is the density of the majority \(^6\text{Li}\) atoms in the hyperfine state \((\uparrow\uparrow)\). The temperature is about \( T \approx 0.13T_F \), where the Fermi energy \( T_F \) is determined by the density \( n \). Therefore, in the Rabi measurement, initially the impurities would follow a Fermi-Dirac distribution, and then calculate

\[ \sum_p f_p = \sum_p \exp \left[ \frac{E_p - \mu_\text{imp}}{k_B T} \right] = n_{\text{imp}}. \]  

(42)

Theoretically, we have solved the impurity spectral functions \( A_{12}(p, \omega) \) and \( A_{22}(p, \omega) \) at the given experimental
For the repulsive polaron, the agreement also justifies the experimental procedure of extracting the residue of the polaron $Z$ from the oscillation periodicity and the effective Rabi coupling strength (i.e., $Z \sim \Omega_{\text{eff}}^2 / \Omega^2$) and of measuring the polaron decay rate $\Gamma$ from the damping of Rabi oscillations (i.e., $\Gamma = 2 \Gamma_R$). However, it should be emphasized that, strictly speaking, the obtained polaron residue and decay rate are not for the zero-momentum polaron at nonzero Rabi coupling, as assumed in the recent theoretical analysis [26] (see, nevertheless, more discussions on the role played by the finite Rabi coupling in Sec. IVD). They are contributed from polarons with different momenta thermally distributed according to $f_p$. This is clearly evidenced by the difference between the dashed line and solid line, as shown in Fig. 8(b). Although the difference due to finite momentum is small, it can lead to a quantitative modification to, for example, the theoretically predicted damping for Rabi oscillations.

For the attractive polaron in Fig. S(b), the difference between the dashed line and solid line is even larger. In this case, it is worth noting that the damping rate of Rabi oscillations does not correspond to the decay rate of Fermi polarons. Even at zero momentum, the damping rate exhibited by the red dashed line is much larger than the decay rate of the attractive polaron. The latter is actually negligible at $T = 0.13 T_F$ [35]. This inequivalence comes from the fact that the imaginary part of the self-energy $\text{Im} \Sigma_{11}$ changes dramatically near the attractive polaron energy, as indicated by the arrow in Fig. S(b). As a result, although the Taylor expansion of $\text{Re} \Sigma_{11}$ is still meaningful, the expansion of $\text{Im} \Sigma_{11}$ near the attractive polaron energy becomes problematic for large Rabi coupling. The use of the approximate impurity Green function Eq. (24) then will strongly underestimate the decay rate at the experimental Rabi coupling strength $\Omega \approx 0.7 \epsilon_F$. In sharp contrast, $\text{Im} \Sigma_{11}$ has a very weak energy-dependence near the repulsive polaron energy, as seen from Fig. 2(b). The approximate impurity Green function Eq. (24) is an excellent approximation at $\Omega \approx 0.7 \epsilon_F$ for repulsive Fermi polarons.

Rabi coupling strength $\Omega \approx 0.7 \epsilon_F$ and at different interaction parameters $1/(k_F a)$, and have consequently calculated $S_{\uparrow \downarrow}(p, t)$ and $\delta S\downarrow\downarrow(p, t)$. By integrating over the momentum with the distribution function $f_p$, we then determine the time-dependence of the spin-down occupation $n_\downarrow(t)$ in Eq. (10).

In Fig. S we compare our theoretical predictions (lines) with the experimental data (circles) for the repulsive polaron at $1/(k_F a) = 1.27$ (a) and for the attractive polaron near the unitary limit $1/(k_F a) = 0.07$ (b) [13]. The red dashed line indicates the result for the zero-momentum polaron, without taking into account the thermal average over the momentum distribution, while the black solid line includes the momentum average at finite temperature. We find a good agreement between theory and experiment [13], without any free adjustable parameters. In particular, for the attractive polaron in Fig. S(b), most of the experimental data locate on the solid line within the experimental error bar. The good agreement partly justifies the approximated mean-field decoupling used to derive Eq. (10) for the short-time evolution of $n_\downarrow(t)$.

FIG. 8. The comparison of the theory (lines) with the experimental data from the LENS group (symbols) [13], for the Rabi oscillations of a repulsive polaron at $1/(k_F a) = 1.27$ (a) and of an attractive polaron near the unitary limit $1/(k_F a) \approx 0$ (b). The data in (a) are extracted from Fig. 2(e) of Ref. [26] and the data in (b) are extracted from Fig. S4(b) of Ref. [13]. For the red dashed lines, we consider the Rabi oscillations of the impurity with zero-momentum. For the black solid lines, we include the momentum average, arising from the thermal distribution of the momentum at finite temperature. In the theoretical calculations, we always take the detuning $\Delta$ that is resonant with either the repulsive polaron energy $E_{\text{rep}}$ or attractive polaron energy $E_{\text{att}}$ at zero Rabi coupling. The impurity density is taken as $n_{\text{imp}}/n = 0.15$, the Rabi coupling is $\Omega = 0.7 \epsilon_F$ and the temperature is $T = 0.13 T_F$, following the experimental condition [13]. In the comparison, we do not include any adjustable free parameters.
C. Importance of the momentum average

Let us now examine more carefully the effect of the momentum average for Rabi oscillations of the attractive polaron. In Fig. 9 we show the oscillations in $n_\downarrow(t)$ contributed from the momentum $k = 0$ (black solid line), $0.5k_F$ (red dashed line), and $k_F$ (blue dot-dashed line). In comparison to the zero-momentum oscillation, a finite momentum gradually increases the periodicity of Rabi oscillations, in addition to causing more damping. In particular, at large momentum (i.e., the $k = k_F$ curve), the oscillation becomes overdamped. After taking into account the thermal distribution function $f_\mathbf{k}$, the final theoretical prediction with momentum average roughly follow the curve at $0.5k_F$ at the given low temperature $T = 0.1T_F$.

D. Dependence on the Rabi coupling

Here we examine the dependence of Rabi oscillations on the Rabi coupling strength. In Fig. 10 and Fig. 11 we respectively report the Rabi oscillations of the repulsive and attractive Fermi polarons at three Rabi coupling strengths: $\Omega = 0.5\varepsilon_F$ (black solid line), $0.7\varepsilon_F$ (red dashed line), and $\varepsilon_F$ (blue dot-dashed line). For the repulsive polaron at $1/(k_F a) = 1$, the visibility or amplitude of oscillations increases with increasing Rabi coupling, indicating a smaller damping rate. This counterintuitive tendency cannot be simply understood from the picture of a zero-momentum Fermi polaron, whose $-\text{Im}\Sigma_{11}$ near the repulsive polaron energy would increase with increasing Rabi coupling, as shown in Fig. 2(b). Therefore, the momentum average tends to decrease the damping rate of Rabi oscillations at large Rabi coupling. On the other hand, a finite momentum increases the damping rate at a fixed Rabi coupling, as we already discussed. It is then readily seen that the Rabi coupling and finite momentum have opposite effects on the damping of Rabi oscillations. Although these two effects may not completely cancel, it seems reasonable to interpret the observed damping of Rabi oscillation (at

\[ \frac{1}{(k_F a)} = 0.0 \text{ and } \Delta = \varepsilon_\text{att} \]
finite Rabi coupling with momentum average) as the decay rate of zero-momentum Fermi polaron (at zero Rabi coupling). This understanding therefore supports the observation found in the LENS experiment [15] that, the damping rate of Rabi oscillations quantitatively matches the predicted quasiparticle peak spectral width $\Gamma$ of repulsive Fermi polarons at zero momentum. For the attractive polaron in the unitary limit, in contrast, the visibility of Rabi oscillations decreases with increasing Rabi coupling. This enhanced damping can be understood from the rapidly changing $-\text{Im} \Sigma_{11}$ near the attractive polaron energy, as shown in Fig. 1(b). If we consider the approximated impurity Green function Eq. (24), the effective polaron decay rate is actually given by $-\text{Im} \Sigma_{11}$ at $\omega = \varepsilon_p + \sqrt{\Omega}/2$ (see, i.e., Eq. (27)), which should increase with increasing Rabi coupling. As a result, the additive effects of the momentum average and Rabi coupling on enhancing the damping rate of Rabi oscillations, we conclude that for attractive Fermi polarons, the damping rate of Rabi oscillations can not be simply interpreted as the zero-momentum quasiparticle decay rate $\Gamma$. We finally note that the periodicity of Rabi oscillations is also slightly affected by a finite Rabi coupling. Large Rabi coupling tends to decrease and increase the periodicity for the repulsive polaron and attractive polaron, respectively. It seems to have the same effect as the momentum average, as shown in Fig. 3. As a result, for repulsive polarons, the combined additive effect of a finite Rabi coupling and momentum average may lead to a smaller periodicity of Rabi oscillations and hence a larger effective Rabi coupling $\Omega_{\text{eff}}$, compared with the expectation from a zero-momentum Fermi polaron, i.e., $\sqrt{\Omega}$.

V. CONCLUSIONS AND OUTLOOKS

In summary, based on the non-self-consistent many-body $T$-matrix approximation, we have presented a general theoretical framework of Fermi spin polarons for a spinor impurity immersed in a Fermi bath. We have focused on the spin-1/2 case with a Rabi coupling $\Omega$ between the two spin states, and have addressed the dependence of quasiparticle properties on the Rabi coupling strength. This turns out to be crucial to understand the recent cold-atom experiments on the dissipative Rabi dynamics of Fermi polarons [12, 15, 18]. In particular, we have confirmed that for the Rabi coupling less than the Fermi energy of the Fermi bath, an approximate impurity Green function provides a reasonable good description of Fermi spin polarons, near the resonant detuning for the repulsive branch. We have then developed an approximate theory for calculating the time-evolution of the spin-down occupation, which is measured in the experiments [12, 15, 18]. This approximate theory relies on a first order, mean-field type decoupling of a correlation function that involves four field operators, which could be accurate for the short-time evolution. We have compared our theoretical predictions on Rabi oscillations with the experimental data [12] and find a good agreement without any adjustable free parameters. We have analyzed in detail the role played by the momentum average on Rabi oscillations, due to the initial thermal distribution of the impurity at finite temperature. We have also addressed the consequence of a finite Rabi coupling at the order of the Fermi energy ($\Omega \sim \varepsilon_p$), which could be significant in real experiments. The effects of both factors (i.e., the thermal momentum average and the finite Rabi coupling) are less considered in previous analyses of the dissipative Rabi dynamics [12, 15, 18].

We have found that, for repulsive polarons, the momentum average and the finite Rabi coupling have opposite effects on the damping of Rabi oscillations. As a result, to a good approximation, we may directly extract the decay rate $\Gamma$ of a zero-momentum repulsive Fermi polaron at zero Rabi coupling from the damping of Rabi oscillations, a procedure that has already been experimentally adopted [12, 15, 18]. For the periodicity of Rabi oscillations, however, the two factors have the same effects: both of them tend to decrease the periodicity and hence lead to a slightly larger effective Rabi coupling strength than the naive theoretical expectation of $\sqrt{\Omega}$. For attractive polarons, on the other hand, the situation turns out to be more complicated. We have emphasized that at low temperature, the zero-momentum decay rate of attractive Fermi polarons is not related to the damping of Rabi oscillations. Both the thermal momentum average and the finite Rabi coupling should be carefully taken into account in analyzing the dissipative Rabi dynamics of attractive Fermi polarons.

We finally comment on the theoretical calculation of Rabi oscillations. To go beyond the approximation of the mean-field decoupling, in Eq. (31) we may consider inserting the unity identity between the field operators $d_{p\downarrow}^\dagger(t)$ and $d_{p\uparrow}(t),$

$$1 = |FS\rangle\langle FS| + \sum_{p,h} c_p^hc_h |FS\rangle\langle FS| c_h^\dagger c_p + \cdots , \quad (43)$$

where the second term stands for the many-body state of the Fermi bath with one-particle-hole excitations, and the terms in the “$\cdots$” denote the many-body states with multiple particle-hole excitations. It is easy to see that the first term in the unity identity $|FS\rangle\langle FS|$ gives rise to the mean-field decoupling. The second term generates the contributions that involve a correlation function,

$$(FS|d_{k\downarrow}d_{p\downarrow}^\dagger(t)c_{q\uparrow}^\dagger c_{p\downarrow}q-k|FS) . \quad (44)$$

We will consider the calculation of this correlation function in future works, with which we may recover the variational results presented in Ref. [20].
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Note added. — After the submission of this manuscript, we were informed by Tomasz Wasak their interesting related theoretical work [42], in which the decoherence and momentum relaxation in Fermi-polaron Rabi dynamics have been analyzed by a kinetic equation approach. An excellent agreement between their theoretical predictions and the LENS experimental data is also demonstrated, without any free parameters. The connection and comparison between our work and their theoretical analysis will be addressed in future studies.

We note also that, most recently a strongly driven Fermi spin polarons beyond the many-body T-matrix approximation and a better description of Rabi dynamics than the current mean-field decoupling approach.

Appendix A: The two-particle vertex function within ladder approximation

The diagrammatic representation of the two-particle vertex functions is shown in Fig. 12. For $\Gamma_{11}(Q)$ in Fig. 12(a) and $\Gamma_{21}(Q)$ in Fig. 12(c), we can write down,

$$\Gamma_{11}(Q) = g_1 - g_1 \tilde{\chi}_{11}(Q) \Gamma_{11} - g_1 \tilde{\chi}_{12}(Q) \Gamma_{21}, \quad (A1)$$

$$\Gamma_{21}(Q) = -g_2 \tilde{\chi}_{21}(Q) \Gamma_{11} - g_2 \tilde{\chi}_{22}(Q) \Gamma_{21}, \quad (A2)$$

where the pair propagators $\tilde{\chi}_{ij}$ ($i,j = 1, 2$) are defined in Eq. (10). By solving these two equations, we find that,

$$\Gamma_{11}(Q) = \frac{(1/g_2 + \tilde{\chi}_{22})}{(1/g_1 + \tilde{\chi}_{11})(1/g_2 + \tilde{\chi}_{22}) - \tilde{\chi}_{12} \tilde{\chi}_{21}} \chi_{11}, \quad (A3)$$

$$\Gamma_{21}(Q) = \frac{-\tilde{\chi}_{21}}{(1/g_1 + \tilde{\chi}_{11})(1/g_2 + \tilde{\chi}_{22}) - \tilde{\chi}_{12} \tilde{\chi}_{21}} \chi_{22}. \quad (A4)$$

We similarly solve $\Gamma_{12}(Q)$ and $\Gamma_{22}(Q)$ by using the diagrams in Fig. 12(b) and Fig. 12(d). It is readily seen that the final expressions for the various two-particle vertex functions can be written in a compact form, as given by Eq. (A9).

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FIG. 12. Diagrammatic representation of the various two-particle vertex functions $\Gamma_{ij}(Q)$. Here, the upper orange line is the Green function of fermionic atoms in the bath, and the bottom purple line is the impurity Green function. The dotted line represents the contact interactions with strengths $g_i$ ($i = 1, 2$). We do not label explicitly the two hyperfine states of the impurity.

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