Application of Hybrid Conjugate Gradient Algorithms in Inverse Problems of Electromagnetic Tomography

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Abstract. Based on the existing conjugate gradient algorithm, a new MN conjugate gradient method for inverse problems of electromagnetic tomography is proposed. The composition of the electromagnetic tomography system and the characteristics of the positive and inverse problems are described, and the solutions of the inverse problems are given. The existing typical conjugate gradient algorithms and the hybrid conjugate gradient methods using convex combinations are summarized. Based on four typical models of EMT system, PRP, HCG and new MN methods are compared in terms of image reconstruction accuracy, iteration times and iteration time. It is concluded that all three conjugate gradient methods can satisfy the termination conditions. The MN method can reduce the number of iterations and the calculation time, and has excellent numerical performance in practical calculation.

1. Introduction

Electromagnetic tomography (EMT) is a kind of electrical tomography technology based on electromagnetic induction principle. EMT technology has the advantages of non-intrusion and non-contact\cite{1}. It is more suitable for the detection of the internal structure of space in the unavailable or inaccessible environment. It is widely used in medical field and industrial non-destructive testing\cite{2,3}. In view of physical structure, EMT system is generally composed of excitation system, detection system and PC mechanism\cite{2}. The excitation system consists of an excitation signal source circuit and excitation coils. The excitation system mainly forms the excitation magnetic field in the object field space. When the exciting magnetic field encounters a conductive or magnetic medium in the field, it changes and forms a changed object field space. The detection system consists of a detection coil and a detection circuit to obtain the detection values in all directions of the object space for image reconstruction. The function of PC is to form spatial sensitivity coefficient matrix of object field, preprocess the detected value, and design reasonable reconstruction algorithm to realize image reconstruction. In the technical point of view, the research of electromagnetic tomography technology includes two parts\cite{4}: forward and inverse problems. The positive problem is mainly to establish the physical model and sensitivity model of the system, and to study the influence of the change of system parameters, etc. The inverse problem is to reconstruct and display images from measured data.
2. Forward and inverse problems

In the sine wave excitation, the electromagnetic field of EMT system with the Maxwell equation group of harmonic forms are as follows:

\[
\begin{align*}
\nabla \times \vec{H} &= \vec{J} + j\omega \vec{D} \\
\nabla \times \vec{E} &= -j\omega \vec{B} \\
\n\nabla \cdot \vec{B} &= 0 \\
\n\nabla \cdot \vec{D} &= 0
\end{align*}
\]

Define the magnetic vector position, which satisfies

\[
\nabla \times \vec{A} = \vec{B}
\]

According to the distribution of excitation coils in EMT system, the corresponding Neumann boundary conditions are determined:

\[
\mu^{-1} \left( \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} \right) = j\omega \delta \vec{A}
\]

\[
\frac{\partial \vec{A}}{\partial n} \mid_{x^2+y^2=R} = \mu_0 \hat{I}
\]

The complete mathematical description of EMT system is obtained, where \(R\) is the radius of the circumference of the excitation coil, \(\mu_0\) is the permeability of air, and \(\hat{I}\) is the current density of the excitation coil. In the forward problem, the solution of sensitivity matrix is the key, and it is also an important index to solve the inverse problem. Sensitivity is defined as:

\[
s_{ij}(k) = \frac{A_{ij}(k) - A_{ij}(\mu_1)}{A_{ij}(\mu_2) - A_{ij}(\mu_1)} \cdot \frac{1}{\mu_2 - \mu_1} \cdot \omega(k)
\]

where \(e_0, \Gamma\) is the number of subdividing units and coils in the field object. The inverse problem of EMT system can be simplified as follows:

\[
s x = b
\]

Conjugate gradient algorithm is recognized as the most effective method to solve large-scale unconstrained linear equations. It has the characteristics of fast convergence and simple algorithm. Formula (5) is optimized into unconstrained continuous differentiable equations.

\[
\min f(x) = \min (s x - b)
\]

Assuming that \(f(x)\) is continuously differentiable, \(d_k\) is search direction, the iterative formula of conjugate gradient algorithm is

\[
x_{k+1} = x_k + \alpha_k d_k
\]

\[
d_k = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k d_{k-1}, & k \geq 1 \end{cases}
\]
where \( g(x) = \nabla f(x) \).

For different conjugate coefficients \( \beta_k \), various conjugate gradient methods with different numerical experimental results and convergence properties have been developed. The more famous conjugate coefficients \( \beta_k \) are:

\[
\beta_{k}^{FR} = \frac{\| g_k \|^2}{\| g_{k-1} \|^2}, \quad \beta_{k}^{PRP} = \frac{\langle g_k, g_k - g_{k-1} \rangle}{\| g_{k-1} \|^2}, \quad \beta_{k}^{CD} = \frac{\| g_k \|^2}{d_{k-1}^T g_{k-1}}
\]

\[
\beta_{k}^{HS} = \frac{g_k^T (g_k - g_{k-1})}{d_{k-1}^T y_{k-1}}, \quad \beta_{k}^{LS} = \frac{g_k^T (g_k - g_{k-1})}{d_{k-1}^T g_{k-1}}, \quad \beta_{k}^{DY} = \frac{\| g_k \|^2}{d_{k-1}^T y_{k-1}}.
\]

FR method is easy to produce small step size. Under the precise line search, the convergence is different according to the different objective functions. CD method has good convergence effect, but the numerical results are not satisfactory. PRP method and HS method can not achieve both global convergence and experimental data. In order to construct a new algorithm with good convergence property and excellent numerical performance, the conjugate parameter \( \beta_k \) above is directly improved.

3. Hybrid Conjugate Gradient Algorithms

In order to give full play to the advantages of each conjugate method, the conjugate gradient algorithm with different properties is fused.

In the reference\[7\], PRP algorithm and FR algorithm are combined to form conjugate coefficients by convex combination:

\[
\beta_{k}^{HCG} = \theta_k \beta_{k}^{FR} + (1-\theta_k) \beta_{k}^{PRP}
\]

where \( \theta_k \in [0,1] \). The iteration direction generated by this algorithm under standard Wolfe condition is descending.

In the reference\[8\], LS algorithm and DY algorithm are combined to form conjugate coefficients by convex combination:

\[
\beta_{k} = \theta_k \beta_{k}^{DY} + (1-\theta_k) \beta_{k}^{LS}
\]

where \( \theta_k \in [0,1] \). The advantage of this algorithm is that the iteration direction not only satisfies the famous D-L condition, but also conforms to Newton direction, and this property does not depend on any line search method.

For EMT system, different conjugate gradient algorithms are mixed to extract the maximum parameters, and a new conjugate coefficient is defined as:

\[
\beta_{k}^{MN} = \frac{\| g_k \|^2 - \max \{0, g_k^T g_{k-1}\}}{\max \{\| g_{k-1} \|^2, d_{k-1}^T y_{k-1}\}}
\]

The formula (12) integrates the characteristics of the conjugate gradient coefficient mentioned above. It can not only improve the convergence of the algorithm, increase the step size, but also reduce the number of iterations and shorten the calculation time. The concrete steps are as follows:

Set accuracy \( \varepsilon \leq 10^{-7} \).

Step 1: Selection of initial points \( x_1 \in \mathbb{R}^n \), set \( k = 1 \), and calculate \( g_1 = g(x_1) \);

Step 2: If \( \| g_k \| \leq \varepsilon \), then iteration stop. Otherwise, step size \( \alpha_k > 0 \) is determined by line search conditions:

Step 3: Set \( x_{k+1} = x_k + \alpha_k d_k \) and \( g_{k+1} = g(x_{k+1}) \).

Step 4: Set \( d_k = -g_k + \beta_k^{MN} d_{k-1} \), \( \beta_k^{MN} \) calculated by formula (12).
Step 5: Set \( k = k + 1 \), jump to Step 2.

The experiment adopts Wolfe line search condition and is realized by MATLAB programming. The termination condition of the algorithm is \( \| g_k \| \leq 10^{-7} \), or the iteration time of the algorithm exceeds 3600s. The MN algorithm is compared with the basic PRP algorithm and the fusion HCG method. Four typical imaging examples of the object space of EMT system are shown in Table 1. In Table 1, M is the name of the conjugate gradient algorithm; I is the number of iterations of the algorithm to solve a test problem; T is the calculation time of the algorithm to solve a test problem. S is the termination condition of the algorithm, so that \( S = 1 \), otherwise, \( S = 0 \).

As can be seen from Table 1, (1) Either conjugate gradient algorithm can satisfy the termination condition within 600 seconds, and can successfully solve the image reconstruction problem of EMT system; (2) For four typical models, MN method is superior to PRP and HCG methods in terms of iteration times and computation time; (3) When the object is at the edge of the object field, the three conjugate gradient algorithms have the fastest imaging speed, which is related to the high sensitivity around the object field.

| original image | M | I  | T/s | S |
|---------------|---|----|-----|---|
| PRP           | 78| 64.2134 | 1 |
| HCG           | 45| 51.4252 | 1 |
| MN            | 34| 40.1254 | 1 |
| PRP           | 64| 65.7658 | 1 |
| HCG           | 47| 56.1543 | 1 |
| MN            | 36| 39.7853 | 1 |
| PRP           | 88| 71.0346 | 1 |
| HCG           | 78| 58.5655 | 1 |
| MN            | 67| 49.9987 | 1 |
| PRP           | 80| 65.7789 | 1 |
| HCG           | 79| 70.3475 | 1 |
| MN            | 45| 37.9801 | 1 |

4. Conclusion

Based on conjugate gradient method, a new hybrid conjugate gradient algorithm MN method for EMT system is constructed, which has sufficient descent under any line search condition. The algorithm has excellent numerical performance in practical calculation.

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