Implications of axionic hair on shadow of M87∗

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Abstract

Detection of axion field can unfold intriguing facets of our universe in several astrophysical and cosmological scenarios. In four dimensions, such a field owes its origin to the completely anti-symmetric Kalb-Ramond field strength tensor. Its invisibility in the solar system based tests compels one to look for its signatures in the strong field regime. The recent observation of the shadow of the supermassive black hole in the galaxy M87 ushers in a new opportunity to test for the footprints of axion in the near horizon region of black holes, where the gravity is expected to be strong. In this paper, we explore the impact of axion on the black hole shadow and compare the result with the available image of M87∗. Our analysis indicates that axion which violates the energy condition seems to be favored by observations. The implications are discussed.

1 Introduction

Axions are pseudo-scalar fields which appear as closed string excitations in the heterotic string spectrum [1, 2]. In four dimensions, the derivative of such a field is associated with the Hodge dual of the Kalb-Ramond field strength $H_{αµν}$, which plays a significant role in explaining several astrophysical and cosmological observations. The field strength tensor $H_{αµν}$ transforms like a third rank completely anti-symmetric tensor field and is associated with a massless, second rank anti-symmetric tensor $B_{µν}$, the so called Kalb-Ramond field. In higher-dimensional theories such a field is necessary to unify gravity and electromagnetism [3,4].

Apart from the emergence of such 3-forms in the effective low energy action of a type IIB string theory [1,2], they play consequential roles in understanding leptogenesis [5, 6], in explaining the cosmic microwave background anisotropy [7, 8] and in engendering topological defects which are instrumental in imparting intrinsic angular momentum to galaxies [9,10].

The emergence of superstring theory [1, 2] provided a further incentive to investigate the nature and consequences of the Kalb-Ramond field. Its compelling resemblance with space-time torsion [7,10–18] is noteworthy. In general relativity, the third rank torsion tensor $T_{αµν}$ is associated with the anti-symmetric part of the affine connection, i.e., $T_{αµν} = Γ_{αµν} - Γ_{ανµ}$ and is primarily anti-symmetric in two indices. Its association with the Kalb-Ramond field strength $H_{αµν}$ becomes evident only when we consider a special sub-class of the torsion tensor antisymmetrized in all the three indices [11,14–16]. In such a scenario, Einstein gravity with the Kalb-Ramond field in the matter sector is equivalent to a modified theory of

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gravity incorporating the completely anti-symmetric space-time torsion. Due to this remarkable analogy between spacetime torsion and Kalb-Ramond field, gravity theories based on twistors necessitates Kalb-Ramond field [19] and one can show that such a field can successfully generate optical activity in spacetime exhibiting birefringence [20, 21].

Moreover, the inefficacy of general relativity in adequately addressing the dark sector [22–25] indicates the need for either some additional matter fields or some alteration in the gravity sector or both. In such a scenario, inclusion of axions in the matter sector is often considered [26]. Although working with Kalb-Ramond field or completely antisymmetric spacetime torsion corresponds to the same physical scenario, in this work we will concentrate primarily on modification in the matter sector due to the addition of the Kalb-Ramond field. Given the theoretical significance of such a field, it is instructive to search for the signatures of Kalb-Ramond field or axions in the available astrophysical and cosmological observations. The attempts to detect the presence of axion in solar system based tests, e.g. bending of light, perihilion precession of Mercury etc., reveal that such fields cause minuscule changes compared to general relativity and hence cannot be detected by the present level of precision in the solar system based tests [27]. A quest for such a field in the spectrum of quasars have surprisingly revealed that axions which violate the energy condition seems to be favored by astrophysical observations related to black hole accretion [28]. Incidentally, the observed spectrum of the same quasars seem to favor certain classes of alternate gravity theories, e.g., extra dimensions, Einstein Gauss-Bonnet gravity in higher dimensions, etc. [29–31].

The recent observation of the shadow of the supermassive black hole M87* by the Event Horizon Telescope collaboration [32–37] has facilitated direct observations of the near horizon regime of a black hole. This has opened up a new and independent window to test the nature of strong gravity. The aim of this paper is therefore to examine the implications and consequences of axions/Kalb-Ramond field from the observed shadow of M87* which will enable us to understand whether the silhouette of M87* favors the presence of such a field.

The paper is broadly classified into five sections. In Section 2, we study the Einstein field equations with Kalb-Ramond field as the source and revisit the static, spherically symmetric and asymptotically flat black hole solution of such equations. Section 3 is dedicated in investigating the nature of the black hole shadow first in a general spherically symmetric background in Section 3.1 and subsequently in Section 3.2 we specialize to the spacetime with axionic hairs presented in Section 2. In Section 4 we investigate the consequences of the Kalb-Ramond field on the recent observation of the shadow of M87*, the supermassive black hole located at the centre of the galaxy M87 and finally we conclude with a summary of our findings and implications of our results in Section 5.

Throughout the paper, the gravitational constant $G$ and the speed of light $c$ are taken to be unity. The metric convention adopted is $(-,+,+,+,+)$.

## 2 Static spherically symmetric black hole solution in presence of Kalb-Ramond field

In this section we discuss the nature of static, spherically symmetric black hole solution in presence of Kalb-Ramond field minimally coupled to gravity [27, 38]. The Kalb-Ramond field $B_{\mu \nu}$, which transforms like a second rank skew-symmetric tensor field can be considered to be a generalization of the electromagnetic four potential $A_{\mu}$ [1, 11]. The associated field strength tensor $H_{\alpha \mu \nu}$ is given by,

$$H_{\alpha \mu \nu} = \partial_{[\alpha} B_{\mu \nu]} = \frac{1}{3} [\nabla_{\alpha} B_{\mu \nu} + \nabla_{\mu} B_{\nu \alpha} + \nabla_{\nu} B_{\alpha \mu}] = \frac{1}{3} [\partial_{\alpha} B_{\mu \nu} + \partial_{\mu} B_{\nu \alpha} + \partial_{\nu} B_{\alpha \mu}] \tag{1}$$
The action associated with the Kalb-Ramond field in four dimensional Einstein gravity is given by

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{1}{12} H_{\alpha\mu\nu} H^{\alpha\mu\nu} \right]
\]  

(2)

where, \( g \) is the determinant of the metric tensor, \( R \) is the Ricci Scalar and \( \kappa = \sqrt{8\pi G} \), is related to the four dimensional gravitational constant \( G \). The factor of \(-1/12\) has been introduced in the Lagrangian so that one can have the canonical kinetic term as \( \frac{1}{2} (\partial_\mu B_{\lambda\nu})^2 \) in the local inertial frame. Due to the anti-symmetry property, Kalb-Ramond field should possess six independent components in four dimensions. But the propagating degrees of freedom of this field is reduced to three as only the spatial components of the field are dynamical. A gauge symmetry \( B_{\mu\nu} \rightarrow B_{\mu\nu} + \nabla_{[\mu} \chi_{\nu]} \) further reduces the degrees of freedom to zero as the gauge field \( \chi_\mu \) has three spatial components. However, the gauge field \( \chi_\mu \) exhibits a further invariance \( \chi_\mu \rightarrow \chi_\mu + \partial_\mu \psi \), where \( \psi \) is a scalar field and in fact this is the scalar propagating degree of freedom for Kalb-Ramond field in four dimensions. Field equations for Kalb-Ramond field can be derived by varying the action Eq. (2) with respect to the field \( B_{\mu\nu} \) which yields \( \nabla_\mu H^{\mu\nu\rho} = 0 \) as the equations of motion. Additionally, it can be shown that the Kalb-Ramond field satisfies the Bianchi identity \( \nabla_{[\mu} H_{\alpha\beta\gamma]} = 0 \).

The variation of the action Eq. (2) with respect to the metric \( g_{\mu\nu} \) leads to the gravitational field equations

\[
G_{\mu\nu} = 8\pi G T^{(KR)}_{\mu\nu}
\]  

(3)

where, \( G_{\mu\nu} \) is the Einstein tensor and \( T^{(KR)}_{\mu\nu} \) is the energy-momentum tensor for the Kalb-Ramond field given by

\[
T^{(KR)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\tilde{L})}{\delta g^{\mu\nu}}
\]

\[
= \frac{1}{6} \left[ 3H_{\rho\sigma\mu} H^{\rho\sigma}_{\nu} - \frac{1}{2} g_{\mu\nu} (H_{\rho\sigma\delta} H^{\rho\sigma\delta}) \right]
\]  

(4)

such that \( \tilde{L} \) is the Lagrangian for the Kalb-Ramond field.

Since the Kalb-Ramond field has a single propagating degree of freedom in four dimensions one can express it in terms of the Hodge dual of the derivative of a pseudo-scalar field \( \Phi \), known as the axion, where

\[
H^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma} \partial_\sigma \Phi
\]  

(5)

Eq. (5) enables us to establish the connection between the Kalb-Ramond field with the axion and throughout the paper the terms axion and Kalb-Ramond field will be synonymously used.

Since our goal in this paper is to explore the impact of axions on the shadow of the black hole, we first need to derive the static, spherically symmetric and asymptotically flat black hole solution of the Einstien’s equations given in Eq. (3). This enables us to consider a line element of the form

\[
ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega^2
\]  

(6)

such that

\[
e^{\nu(r)} = 1 - \frac{2}{r} + \frac{b}{r^3} + O\left(\frac{1}{r^4}\right)
\]  

(6a)
where, $\nu(r)$ and $\lambda(r)$ are two arbitrary functions of $r$ satisfying Eq. (3). With Kalb-Ramond field as the source the above solution has been worked out previously in [27, 38]. For brevity we do not repeat the derivation here but simply mention the results. Here, the distance $r$ is expressed in units of $M$ and the parameter $b$ which represents the axionic hair for black hole solution has units of $M^2$. Note that in order for Eq. (6) to represent a black hole solution, there must be an event horizon. The radius of the horizon $r_h$ is given by solving for $g^{rr} = e^{-\lambda(r)} = 0$, which yields,

$$r_h = 1 \pm \sqrt{1 - 3b}$$  \hspace{1cm} (7)

where the event horizon $r_{eh}$ is given by the positive root of Eq. (7).

In the next section we will consider the geodesic motion of the photons in the background given by Eq. (6) which will enable us to derive the shape and size of the black hole shadow. It is important to note that the observation of the shadow directly probes the near horizon regime of black holes where $r$ is small. Therefore, although the leading order term with the axion appears as $1/r^3$ correction to the Schwarzschild scenario its impact on the observed shadow is expected to be significant.

### 3 Geodesic motion of photons and the shadow of a black hole

The shadow of a black hole refers to the set of directions in the local sky from where electromagnetic radiation just escapes the black hole event horizon and reaches the observer on Earth [39–43]. When light from a distant astrophysical object or the accretion disk surrounding the black hole arrives in the vicinity of the event horizon, a part of it gets trapped inside the horizon while another part escapes to infinity. This results in a lack of radiation in the observer’s sky leading to a dark patch in the image of the black hole, known as the black hole shadow. The outline of the shadow testifies the signatures of strong gravitational lensing of nearby radiation and hence the shape and size of the shadow can reveal valuable information regarding the nature of strong gravity near the black hole [41,44–47]. Consequently, the image of a black hole can be used as a potential probe to estimate the deviation from general relativity.

While the shape of the shadow bears imprints of the background geometry, the size of the shadow scales directly with its mass, reduces with increase in distance and also exhibits dependence on the background spacetime. For example a non-spinning black hole always casts a circular shadow [39, 40]. In this case the size of the shadow can be used to investigate the deviation from the Schwarzschild scenario in general relativity [39, 40]. Introducing spin to black holes incurs deviation from the circular shape and this has been studied extensively in the past both in the context of general relativity and alternative gravity models [41,44–48]. However, it is important to note that the deviation from circularity becomes apparent only when the angle of inclination of the observer with respect to the rotation axis of the black hole becomes appreciable, i.e., an observer viewing a black hole with zero inclination angle will always see a circular shadow [39, 40].

In the next section we will derive the contour of the black hole shadow in the presence of a general static, spherically symmetric and asymptotically flat metric given by Eq. (6) and subsequently we will consider the special case with axionic hairs where the metric components are given by Eq. (6a) and Eq. (6b).
3.1 Structure of black hole shadow in a general spherically symmetric metric

In this section we will work out the structure of the black hole shadow in a general static, spherically symmetric background given by Eq. (6). For this purpose we will study geodesic motion of photons in this spacetime. We consider a geodesic with an affine parameter $\lambda$ such that the tangent vector is $u^\mu = \frac{dx^\mu}{d\lambda}$. The Lagrangian $\mathcal{L}$ corresponding to the motion of test particles assumes the form,

$$\mathcal{L}(x^\mu, \dot{x}^\mu) = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

such that the action $S$ representing the motion of test particles satisfies the Hamilton-Jacobi equation given by,

$$\mathcal{H}(x^\mu, p_\mu) + \frac{\partial S}{\partial \lambda} = 0$$

where,

$$\mathcal{H} = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu = \frac{k}{2}$$

is the Hamiltonian, $k$ is a constant representing the rest mass of the test particles (which is zero for photons) and $p_\mu$ is the conjugate momentum corresponding to the coordinate $x^\mu$ and is given by,

$$p_\mu = \frac{\partial S}{\partial x^\mu} = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = g_{\mu\nu} \dot{x}^\nu$$

Since the metric does not depend explicitly on $t$ and $\phi$, the energy $E$ and the angular momentum $L$ of the photons are conserved. These constants of motion are given by,

$$E = -g_{tt} u^t = -p_t \quad \text{and} \quad L = g_{\phi\phi} u^\phi = p_\phi$$

respectively. The action $S$ in Eq. (11) can therefore be integrated with the help of Eq. (12a) and Eq. (12b) such that

$$S = -Et + L\phi + \bar{S}(r, \theta)$$

where in the case of a static, spherically symmetrically metric like Eq. (6), $\bar{S}(r, \theta)$ turns out to be separable in $r$ and $\theta$ with, $\bar{S}(r, \theta) = S'(r) + S'\theta(\theta)$. We also note that with the help of Eq. (11), Eq. (10) can be written as,

$$-e^{-\nu(r)} r^2 E^2 + e^{-\lambda(r)} r^2 \left( \frac{dS'}{dr} \right)^2 + L^2 = -\left( \frac{dS_\theta}{d\theta} \right)^2 - L^2 \cot^2(\theta) = -C$$

where the separation constant $C$, known as the Carter constant represents a third constant of motion [49]. Therefore the geodesic equations for $r$ and $\theta$ are given by,

$$\left( \frac{dS'}{dr} \right) = \sqrt{e^{\nu(r)} \left( -\frac{C}{r^2} - \frac{L^2}{r^2} + e^{-\nu(r)} E^2 \right)} = E \sqrt{V(r)} = g_{rr} \hat{r} \quad \text{and} \quad (15)$$
respectively, where

\[
V(r) = -\frac{e^{\lambda(r)}\chi}{r^2} - \frac{e^{\lambda(r)}l^2}{r^2} + e^{\lambda(r)-\nu(r)}
\]

represents the effective potential in which the photon moves, while

\[
\Theta(\theta) = \chi - l^2 \cot^2 \theta
\]

such that \( \chi = C/E^2 \) and \( l = L/E \). The radius of the photon sphere \( r_{ph} \) corresponds to the condition where \( \dot{r} \) vanishes and the effective potential \( V(r) \) has an extrema. This is generally a maxima, which corresponds to an unstable equilibrium of the photon. Given a slight perturbation the photon either falls into the horizon or escapes to infinity. Due to this reason the photon sphere plays a crucial role in determining the boundary of the black hole shadow.

Therefore, \( r_{ph} \) is obtained by solving \( V(r) = V'(r) = 0 \), such that the above conditions yield

\[
\chi + l^2 = r_{ph}^2 e^{-\nu(r_{ph})} \quad \text{and}
\]

\[
\chi + l^2 = \frac{1}{2} r_{ph}^3 e^{-\nu(r_{ph})} \nu'
\]

respectively. The photon sphere in an arbitrary spherically symmetric metric is therefore obtained by solving for \( r \) in the following equation,

\[
r_{ph} \nu'(r_{ph}) = 2
\]

In order to derive the contour of the black hole shadow in the observer’s sky one considers the projection of the photon sphere in the image plane [50]. Note that the largest positive radius obtained by solving Eq. (21) is relevant for the computation of the shadow outline [39,40]. The locus of the shadow boundary is denoted in terms of two celestial coordinates \( \alpha \) and \( \beta \) which are related to \( l \) and \( \chi \) [40,50]. This can be understood by expressing the metric in terms of the tetrads which for a spherically symmetric background assumes the form,

\[\begin{align*}
g_{\mu\nu} &= e^{(a)}_{\mu} e^{(b)}_{\nu} \eta_{ab} \quad \text{where} \\
e^{(t)}_{\mu} &= (e^{\nu/2}, 0, 0, 0) \\
e^{(r)}_{\mu} &= (0, e^{\lambda/2}, 0, 0) \\
e^{(\theta)}_{\mu} &= (0, 0, r, 0) \\
e^{(\phi)}_{\mu} &= (0, 0, r \sin \theta)
\end{align*}\]
respectively. An observer located at a distance \( r_0 \) with an inclination angle \( \theta_0 \) will perceive that the celestial coordinates are given by,

\[
\beta = \lim_{r_0 \to \infty} r_0 v_{(\theta)}(r_0, \theta_0) = \mp \sqrt{\Theta(\theta_0)}
\]

and

\[
\alpha = \lim_{r_0 \to \infty} r_0 v_{(\phi)}(r_0, \theta_0) = -\frac{l}{\sin \theta_0}
\]

Note that \( r_0 \) do not appear in the expression for \( \alpha \) and \( \beta \) since the metric is assumed to be asymptotically flat. Using Eq. (18) it can be shown that

\[
\alpha^2 + \beta^2 = \chi + l^2 = r_{sh}^2
\]

which shows that the shadow is circular in shape where the dependence of its radius \( r_{sh} \) on \( r_{ph} \) is given by Eq. (19). The above discussion clearly elucidates that for any general static, spherically symmetric and asymptotically flat metric the shadow is circular in shape. Further, the radius of the shadow depends only on the \( g_{tt} \) component of the metric and is independent of the distance \( r_0 \) and the inclination angle \( \theta_0 \) of the observer.

### 3.2 Structure of black hole shadow in presence of Kalb-Ramond field

In this section we will compute the contour of the black hole shadow by considering specifically the spherically symmetric metric with axionic hairs (Eq. (6) with Eq. (6a) and Eq. (6b) as metric elements). As discussed in the previous section, the radius of the photon sphere is obtained by solving Eq. (21) which for our specific case leads to

\[
2r^3 - 6r^2 + 5b = 0
\]

Depending on the value of the axion parameter \( b \), Eq. (29) can have three distinct real roots, one distinct and one coincident real root or a single distinct real root. The conditions for the above are enlisted below:

\[
\begin{align*}
\text{Three distinct real roots} & \quad 0 < b < 1.6 \\
\text{Only one real root} & \quad b < 0; \quad b > 1.6 \\
\text{One real and one coincident root} & \quad b = 0; \quad b = 1.6
\end{align*}
\]

The roots of this equation can be obtained analytically by using Cardano’s method [51].
When $0 < b < 1.6$, the three real roots are given by,

\begin{align}
  r_1 &= 1 + 2 \cos \left( \frac{1}{3} \cos^{-1} B \right) \\
  r_2 &= 1 - \cos \left( \frac{1}{3} \cos^{-1} B \right) + \sqrt{3} \sin \left( \frac{1}{3} \cos^{-1} B \right) \\
  r_3 &= 1 - \cos \left( \frac{1}{3} \cos^{-1} B \right) - \sqrt{3} \sin \left( \frac{1}{3} \cos^{-1} B \right)
\end{align}

with $|B| < 1$ where $B = -1 + 5b/4$. In the event, $b = 0$ or $b = 1.6$, $r_2$ coincides with $r_3$, while $r_1$ corresponds to another distinct real root. When $b < 0$ or $b > 1.6$, which is identical to the situation with $|B| > 1$, there is only one real root which is given by,

\[ r_0 = 1 + \left\{ |B| + \sqrt{B^2 - 1} \right\}^{1/3} + \left\{ |B| + \sqrt{B^2 - 1} \right\}^{-1/3} \]

The radius of the photon sphere is depicted clearly in Fig. 1a. In the figure, the blue solid line corresponds to the condition $|B| > 1$ while the red curves constitutes the situation with $|B| < 1$. Since the latter consists of three distinct radii, $r_1$, $r_2$ and $r_3$ are marked with solid, dot-dashed and dotted red lines respectively. Among these the greatest positive root is taken for the computation of the shadow radius. It is important to note that for $b > 1.6$ only one real root exists but the root is always negative (which is unphysical since the photon sphere cannot have a negative radius) and hence we conclude that $b$ can never exceed this limit. The maximum value of $b$ is further reduced from the consideration that for the metric to represent a black hole the event horizon has to be real, which from Eq. (7) implies $b < 1/3 = b_{\text{max}}$. Henceforth, we will consider $b_{\text{max}}$ to be the upper limit of $b$.

For the region $b < 0$, Eq. (29) has only one positive real root $r_0$ which increases as $b$ decreases. It is important to note that $b$ cannot assume arbitrarily large negative values since when $b \lesssim -1.48 = b_{\text{min}}$
the radius of the event horizon \( r_{eh} \) exceeds the photon sphere \( r_{ph} \). Consequently, when \( b \) is lower than \( b_{\text{min}} \) photon circular orbits do not exist. This is an interesting feature the spacetime inherits due to the presence of the Kalb-Ramond background.

Therefore, in the remaining discussion we will limit ourselves in the range \(-1.48 \lesssim b \lesssim 1/3\). We emphasize once again that since we are probing the near horizon regime, \( 1/r^3 \) correction to the metric is expected to exhibit significant effect on the black hole shadow. Moreover, in our regime of interest \( |b|/r^2 \ll 1 \) still holds which enables us to truncate the metric in Eq. (6a) and Eq. (6b) up to the leading order term. However, we verify this approximation explicitly by considering terms up to \( 1/r^4 \) in both \( g_{tt} \) and the \( g_{tr} \) components of the metric and find that this has negligible effect on our results.

Once the dependence of the photon sphere on \( b \) is understood, we can compute the radius of the shadow \( r_{sh} \) in terms of the axionic parameter \( b \) using Eq. (19) and Eq. (28). The variation of \( r_{sh} \) with \( b \) is plotted in Fig. 1b where we have shaded the theoretically allowed region of \( b \). We also note that the shadow expands with negative values of \( b \). This result will have interesting consequences from the observed shadow of M87* which we discuss in the next section.

### 4 Observed shadow of M87* and implications on axionic hair

Using the techniques of VLBI (Very Large Baseline Interferometry), the Event Horizon Telescope (EHT) Collaboration has recently released the image of the supermassive black hole M87* at the centre of the galaxy M87, thereby opening a new window to test gravity in the strong field regime [32–37]. Their analysis reveals that the angular diameter of the shadow of M87* is \((42 \pm 3) \mu\text{as}\) exhibiting a deviation from circularity \( \Delta C < 10\% \) and the axis ratio \( \Delta A < 4/3 \) [32]. This implies that the observed shadow is nearly circular which is further supported from the fact that the jet axis makes an angle of 17° to the line of sight, which is taken to be the inclination angle of the the black hole [32,36,37]. We have already mentioned in the last section that non-circular shadows are only possible if a black hole is observed at high inclination angle. This therefore, justifies our choice in considering the spherically symmetric metric given by Eq. (6), as a first approximation. Hence, the only relevant observable in our context is the angular diameter of the shadow, \( \Delta A \) and \( \Delta C \) being trivially equal to one and zero respectively, satisfying the observed constraint.

The angular diameter of the shadow depends not only on the background metric but also on the mass \( M \) of the black hole and its distance \( D \) from the observer. This has been illustrated in Fig. 2 which shows that

\[
\tan \alpha \approx \alpha = \frac{r_{sh}}{D}
\]

where \( 2\alpha \) is the angular diameter. Since the distance between the black hole and the observer is much much greater than the radius of the shadow \( (r_{sh}) \), \( \alpha \) is very small which justifies the approximation in Eq. (32). We have already expressed the radius of the shadow in terms of the background metric in Eq. (6) (e.g. Fig. 1b). Since the radius is in units of \( GM/c^2 \) the angular diameter scales directly with the black hole mass. Further, black holes at larger distances will cast smaller shadows.

In the previous section we have computed the dependence of the axion parameter \( b \) on the radius of the photon sphere and the shadow. Therefore from the magnitude of the observed angular diameter we can comment on the observationally favored values of \( b \). We however, require independent measurements of the mass and distance of M87*. Based on stellar dynamics and gas dynamics measurements the mass of M87* is reported to be \( M \sim 6.2^{+1.1}_{-0.5} \times 10^9 M_\odot \) [52] and \( M \sim 3.5^{+0.9}_{-0.3} \times 10^9 M_\odot \) [53] respectively while the
distance of the source is reported to be $D = (16.8 \pm 0.8) \text{ Mpc}$ [54–56] from stellar population measurements. Moreover, the mass of the object reported by the EHT Collaboration is $M = (6.5 \pm 0.7) \times 10^9 M_\odot$ [32,36,37]. Further, the observed emission ring is actually expected to be $\sim 10\%$ larger than the true shadow size which is supported by multiple simulations of the accretion flow around M87* [37].

With the above data we plot the angular diameter of M87* with $b$ assuming the mass estimated from the stellar dynamics measurements ($M \sim 6.2 \times 10^9 M_\odot$) in Fig. 3 and from the shadow measurements ($M \sim 6.5 \times 10^9 M_\odot$) in Fig. 4. The distance of the source is taken to be $D \sim 16.8$ Mpc in both the aforesaid figures for the computation of the angular diameter. Also, the theoretically allowed values of $b$ ($-1.48 \lesssim b \lesssim 1/3$) are shaded in both the figures. In Fig. 3a and Fig. 4a the observed angular diameter of $42 \mu\text{as}$ is marked with solid blue line while the error of $\pm 3 \mu\text{as}$ about the centroid value are depicted with blue dashed lines. Figures Fig. 3b and Fig. 4b are same as Fig. 3a and Fig. 4a respectively, except that the observed angular diameter with 10% offset are marked with blue lines.

From Fig. 3a it is clear that if the mass of M87* is assumed to be $M \sim 6.2 \times 10^9 M_\odot$, the observed angular diameter can be reproduced only for negative values of $b$ in the range, $-5.5 \lesssim b \lesssim -0.75$. However, since $b$ cannot assume values lesser than $b_{\text{min}} \sim -1.48$, the theoretical angular diameter can be only as large as $40 \mu\text{as}$ which is still within the observed error bar. The Schwarzschild scenario seems to be allowed only when one considers the 10% offset in the observed angular diameter of the shadow (Fig. 3b) although a negative $b$ still explains the observation better.

In Fig. 4 the angular diameter of M87* is plotted assuming $M \sim 6.5 \times 10^9 M_\odot$. Fig. 4a elucidates that $-3.5 \lesssim b \lesssim 0.2$ can reproduce the observed angular diameter whereas the theoretically allowed range of $b$ ($-1.48 \lesssim b \lesssim 1/3$) restricts the theoretical angular diameter within $38.5 \mu\text{as} \lesssim 2\alpha \lesssim 42.5 \mu\text{as}$. Therefore, assuming $M \sim 6.5 \times 10^9 M_\odot$ the observations can be explained even with $b = 0$ (which falls within the error bar), although a negative axionic parameter seems to be more favored. Moreover, if we consider that the true shadow is 10% smaller than the observed angular diameter, $-0.5 \lesssim b \lesssim 1/3$ can address the observation within the error bars as shown in Fig. 4b.
The above discussion clearly shows that within the domain of the allowed values of $b$ a negative axionic parameter explains the observation better. Such an axion violates the energy condition and has several interesting consequences. It is important to note that $M \sim 3.5 \times 10^9 M_\odot$ (which is based on gas dynamics observations) can reproduce the observed angular diameter only if $b$ assumes a high negative value which is outside the theoretically allowed range. Therefore, similar to the general relativistic scenario, even after incorporating the axionic charge the mass estimations based on gas dynamics are not in agreement with the shadow observations [37]. Interestingly, our present results are in concordance with a previous finding where we estimated the observationally favored signature of $b$ based on the spectral data of quasars.
By comparing the observed spectrum of a set of eighty quasars with the theoretical spectrum from the surrounding accretion disk, we reported that the Kalb-Ramond field violating the energy condition (which is equivalent to a negative axion parameter $b$) seems to be favored by observations [28].

5 Summary

In this paper we aim to investigate the signatures of the Kalb Ramond field or its dual axion from the recent observations of the shadow of the supermassive black hole in the centre of the galaxy M87. This is important, since the weak field tests of gravity lack the necessary precision to discern the presence of such a field while the strong field tests e.g. electromagnetic spectrum emitted from the black hole accretion disk have reported that axion violating the energy condition is observationally favored. Therefore, it is instructive to subject this finding to further tests and the observation of the black hole shadow provides the appropriate opportunity.

In order to accomplish our goal, we compute the contour of the black hole shadow first in a general spherically symmetric background and note that the radius of the shadow depends only on the $g_{tt}$ component of the metric. Subsequently we consider the spherically symmetric solution of Einstein’s equations solved in the Kalb-Ramond background. Such a metric exhibits a perturbation over the Schwarzschild scenario through the axion parameter $b$. Since the axion primarily appears as a $1/r^3$ correction to the Schwarzschild metric, its effect on the black hole image which probes the vicinity of the horizon, is expected to be significant. Yet we can safely ignore the corrections to the metric with large inverse powers of $r$ due to the theoretical restriction on the axion parameter $-1.48 \leq b \leq 1/3$, such that in our regime of interest $b/r^2$ continues to be less than unity. The fact that the magnitude of $b$ is very small from a theoretical consideration is further supported from the observations related to perihelion precession of mercury and bending of light [27]. The theoretical lower bound on negative value of $b$ arises from the absence of any photon circular orbit outside the event horizon. This is an intriguing feature the spacetime exhibits due to the presence of the Kalb-Ramond field.

The choice of the spherically symmetric metric is justified from the fact that the source has a very low inclination angle $\sim 17^\circ$ such that the observed image is nearly circular with axis ratio $\Delta A < 4/3$ and deviation from circularity $\Delta C < 10\%$. Therefore, $\Delta A$ and $\Delta C$ cannot be used to constrain the spin of M87*. We have verified this explicitly in a previous work [47] where the tidal charge parameter of axisymmetric braneworld black holes could be constrained from the observed angular diameter, but nothing could be concluded about its spin from the observational constraint on $\Delta C$ and $\Delta A$.

We evaluate the dependence of the shadow radius on the axion parameter $b$ and find that the radius of the shadow decreases with increase in $b$, or alternatively Schwarzschild metric perturbed with a negative axion parameter casts a larger shadow. Consequently, such an axion can potentially reproduce the observed angular diameter of M87* with both the mass estimations of the object $M \sim 6.2 \times 10^9 M_\odot$ and $M \sim 6.5 \times 10^9 M_\odot$. This is better than the general relativistic scenario which becomes viable only when we consider $M \sim 6.5 \times 10^9 M_\odot$ or allow a 10% offset in the observed image. These considerations however do not exclude a negative axionic charge. This result continues to hold true even when we consider the sub leading corrections to the metric with larger inverse powers of $r$.

The axion with a negative charge parameter has several interesting astrophysical and cosmological implications. It violates the energy condition and such a scenario is often invoked for removal of singularity in geodesic congruences [57], gains ground in bouncing cosmology to prevent the big bang singularity [58], plays a crucial role in altering the Buchdahl’s limit for star formation [13] and can potentially generate a non-zero cosmological constant in four dimensions whose origin is attributed to bulk Kalb-Ramond field in
a higher dimensional scenario [59]. Moreover, the suppression of Kalb-Ramond field has been discussed in several physical scenarios, e.g. in the context of warped brane-world models [60] with bulk Kalb-Ramond fields [61, 62] and the related stabilization of the modulus [63], in the context of higher curvature gravity where the associated scalar degrees of freedom diminishes the coupling of such a field with the Standard Model fermions [64, 65], and in the inflationary era induced by higher curvature gravity [66, 67] and higher dimensions [68].

As a final remark we mention that in the electromagntical domain there is no dearth of spectral data of supermassive black holes while there is only a single observation of black hole shadow on which the present result is based. The real challenge of discerning the nature of strong gravity from the black hole spectrum lies in appropriately modelling the spectrum which depends not only on the background spacetime but also on the nature of the accretion flow. Disentangling the impact of the metric from the spectrum therefore becomes quite non-trivial. The image of the black hole on the other hand provides a much cleaner environment to explore the strong gravity regime. This scope will further enhance as more and more data on black hole images become available in the near future.

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