Axial charge segregation during a first order phase transition in the presence of hypermagnetic fields

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Abstract.
We study the scattering of fermions off a finite width bubble wall during a first order phase transition in the presence of a background hypermagnetic field. We show that the chiral nature of the fermion coupling with the background field in the symmetric phase generates an axial asymmetry in the scattering processes. We briefly discuss possible implications of such axial charge segregation as a source of CP violation during a process of non-local baryogenesis.

1. Introduction
The problem of baryogenesis is still one of the open questions in cosmology, despite the large amount of work devoted to find a viable explanation. The conditions for developing a baryon asymmetry in an initially symmetric universe were laid down by Sakharov in 1967 \cite{Sakharov:1967dj} and we are still looking for a scenario that effectively encompasses them. These three well-known ingredients are: (1) existence of interactions that violate baryon number, (2) C and CP violation and (3) departure from thermal equilibrium.

1.1. Electroweak baryogenesis
The standard model (SM) of electroweak interactions meets all these requirements, provided that the electroweak phase transition (EWPT) is first order since, at this stage of the universe evolution, it is the only possible source of departure from thermal equilibrium. Nonetheless, it is well known that neither the amount of CP violation within the minimal SM, nor the strength of the EWPT are enough to generate a sizable baryon number \cite{Dine:1982ah, Dine:1983af}. Regarding the existence of processes violating baryon number, the SM provides us with a valuable mechanism, tied to the vacuum structure of a non-Abelian theory. The Sphaleron \cite{Kuzmin:1985mm} is a static and unstable solution of the field equations of the EW model, corresponding to the top of the energy barrier between topologically distinct vacua. Each minimum corresponds to configurations with zero gauge field energy but different baryon number. In such a way, topological transitions between different minima are associated to baryon number \((n_B - \bar{n}_B)\) violation \cite{Kuzmin:1985mm} and can either induce baryogenesis or wash out the previously generated asymmetry. That is why one of the difficult steps along the baryogenesis process is to preserve the baryon asymmetry. This requires that the transition between different topological vacua is supressed in the broken phase, when the universe recovers thermal equilibrium, leading to a rather difficult bound to meet for the parameters involved in the PT \cite{Dine:1983af, Dine:1982ah}.

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1.2. Non-local baryogenesis
Among the different scenarios for baryogenesis proposed in the framework of the EWPT, non-local baryogenesis offers an interesting possibility: it does not require that the three Sakharov conditions occur in a single process at a single space-time point, making them easier to be fulfilled (see [7] for a review). In particular, it takes advantage of $CP$ violation and out-of-equilibrium conditions that can develop during the EWPT and of anomalous baryon violation processes occurring with a large rate in the symmetric phase. In this scenario, the $CP$ violating charge asymmetry generated in the bubble wall can be transported back in the symmetric region by particle reflection off the bubble wall and be converted to baryon number outside the bubble, in the symmetric region. Moreover, in [8, 9], it was shown that the required $CP$ violation can be obtained through the generation of an asymmetry in another quantum number, i.e. in any non conserved charge which is not orthogonal to baryon number. These authors proposed an extension of the SM, where a $CP$ violating phase is introduced, at tree level, in the scalar potential of a two doublets Higgs model, acting as an effective chemical potential for fermionic hypercharge. As a consequence, an axial charge segregation is generated between the two phases, providing a bias for baryon over anti-baryon production in the sphaleron solution.

1.3. Electroweak baryogenesis in the presence of (hyper)magnetic fields
An interesting attempt to link the baryogenesis process to the EWPT results from taking into account the effect of another possible ingredient: primordial magnetic fields. Before the EWPT, these fields would in fact couple through hypercharge and receive the name of hypermagnetic fields, belonging to the $U(1)_Y$ group. Magnetic fields seem to be pervading the entire universe and their generation may be either primordial or associated to the process of structure formation. They have been observed in galaxies, clusters, intracluster medium and high redshift objects [10]. Although there is at present no conclusive evidence about the origin of magnetic fields, their existence prior to the EWPT cannot certainly be ruled out and it is important to investigate their possible influence on different cosmological events and in particular on the baryogenesis process (see [11], for a review). In fact, it has been shown that they act on each of the three Sakharov ingredients.

The first mechanism that was largely explored was the effect of the presence of a large scale magnetic field on the dynamics of the phase transition. This process has been analytically studied classically [12], to one-loop order [13] and up to the contribution of ring diagrams [14, 15], that have proved to be crucial for the description of the long wavelength properties of the theory [16]. It has also been addressed by means of lattice simulations [17]. These calculations all agree that the strength of the PT is enhanced in presence of hypermagnetic fields, although not enough to satisfy the requirements imposed by baryon number preservation. However, other authors [18] reach the conclusion that these fields inhibit the first order phase transition.

Regarding the sphaleron transitions, contrary to the previous encouraging results, it has been found [19] that the presence of the magnetic field is counter-productive for the desired result, since the coupling of the sphaleron dipole moment to the external field lowers the potential barrier, making easier the erasure of the asymmetry.

On another hand, the influence of magnetic fields on the enhancement of $CP$ violation has received much less attention and this is the aspect that we want to examine here. In [20] and [21], it has been shown that the external field is able to produce an axially asymmetric scattering of fermions off first order phase transition bubble walls. This $CP$ violating reflection is due to the chiral nature of the couplings of right- and left-handed modes with the external field in the symmetric phase. This mechanism produces an axial charge induced transitions which can then be transported in the broken symmetry phase where sphaleron induced transitions - taking place with a large rate - can convert it into baryon number generation. In this way, this mechanism is providing an additional source of $CP$ violation within the SM.
The work is organized as follows: in Sect. 2 we describe the Higgs field vacuum expectation value evolution through a first-order PT and the consequent behavior of the masses of the particles involved in the PT. In Sect. 3 we present the movement equations for fermions in the symmetric and the broken symmetry phases in the presence of a background (hyperor)magnetic field. We devote Sect. 4 to the study of the scattering of fermions off the bubble walls: using analytical and numerical solutions of the above equations in both phases, we compute the transmission and reflection coefficients of fermions through the wall and we show that these coefficients differ for distinct helicity modes, leading to an axial asymmetry on both sides of the wall. Finally, in Sect. 5, we discuss the possible implications of such axial charge segregation as a source of \( CP \) violation during a process of non-local baryogenesis.

2. Kink solution in a first order phase transition
A first order PT implies the coexistence of two phases and an order parameter that undergoes a discontinuous change. In the case of the EWPT, the two phases are the symmetric one, built on the false vacuum of the effective potential, and the broken symmetry phase, characterized by the true vacuum; the order parameter is the expectation value of the Higgs field. The conversion from one phase to the other happens through the nucleation and propagation of true vacuum bubbles. The region separating both phases is called the wall and its properties depend on the effective, finite temperature, Higgs potential. Under the assumption that the wall is thin and that the PT happens when the energy densities of both phases are degenerate, it is possible to find a one-dimensional analytical solution for the (dimensionless) Higgs field strength \( \varphi(z) \), called the kink [22]. This is given by

\[
\varphi(z) = 1 + \tanh \left( \frac{z}{\lambda} \right),
\]

where \( z \) is the coordinate along the direction of the phase change and \( \lambda \) is the width of the wall. It can be expressed in terms of the effective potential parameters and the temperature at which the transition takes place. (See for instance Ref. [13] for explicit expressions of the effective potential and the parameters involved).

If we are interested in describing the evolution of particles from the symmetric to the broken phase, one of the aspects that we must take care of is their interaction with the bubble walls. In particular, we are interested in the scattering from the symmetric to the broken phase since particles in the high temperature phase are reflected (or transmitted) off the advancing bubble walls, while most particles in the low temperature phase are unable to catch up with the receding walls. During this processes, since particles acquire mass through the Higgs boson, masses cannot be treated as constant quantities and some assumption must be adopted for their evolution. If we consider the above behavior of the Higgs field vacuum expectation value, the profile for the mass will be \( m(z) = m_0(1 + \tanh(z/\lambda)) \). In terms of the kink solution we can see that \( z = -\infty \) represents the region outside the bubble, that is the region in the symmetric phase, where the particles are massless. Conversely, for \( z = +\infty \), the system is inside the bubble, that is in the broken phase, and the particles have acquired a finite mass \( m_0 \).

The problem can be further simplified by considering the limit when the width of the wall approaches zero. In such a case, the kink solution becomes a step function, \( \Theta(z) \), and consequently the expression for the particles’ mass becomes \( m(z) = m_0 \Theta(z) \).

3. Dirac equation for fermions in the symmetric and broken symmetry phases in a background (hyperor)magnetic field
When scattering is not affected by diffusion, the problem of fermion reflection and transmission through the wall can be casted in terms of solving the Dirac equation, with a position dependent fermion mass proportional to the Higgs field [23].
In the presence of an external magnetic field, we need to consider that fermion modes couple differently to the field in the symmetric and in the broken symmetry phases. In [20] and [21] we have derived the Dirac equation for such fermions and explicitly constructed the solutions in both phases, with a constant magnetic field, requiring that these match at the interface \( z = 0 \).

In [20], working with a step function for the mass profile, we obtained rather simple analytical solutions but we also encountered some problems derived from the discontinuous behavior of the Higgs field value. In [21], we used the more realistic kink solution for the mass evolution and had to resort to numerical methods for obtaining the solution in the symmetric phase. The results obtained with both mass profiles essentially coincide. We will report here the results of this last more realistic case. After exhibiting the Dirac equation in both phases, we only present graphs for the transmission and reflection coefficients for fermions interacting with the wall. These coefficients were built from the spinor wave functions through the incident, reflected and transmitted currents. We refer the interested reader to the original papers for the details on the methods we employed for solving these equations and the matching procedure at the interface.

For definiteness, we have worked with fermions moving in the normal direction to the bubble wall, \( \hat{z} \), i.e. along the \( \hat{z} \) axis, with a constant (hyper)magnetic field in this same direction. In another work [24], this restriction has been lifted, solving the Dirac equation in three spatial dimensions, and obtaining essentially the same result.

### 3.1. Symmetric phase

In the symmetric phase, for \( z \leq 0 \), the coupling of fermions with the hypermagnetic field is chiral. Let

\[
\Psi_R = \frac{1}{2} (1 + \gamma_5) \Psi_L = \frac{1}{2} (1 - \gamma_5) \Psi \tag{2}
\]

represent, as usual, the right and left-handed chirality modes for the spinor \( \Psi \), respectively. Then, the equations of motion for these modes, as derived from the electroweak interaction Lagrangian, are

\[
(i \partial_\mu - \frac{y_L}{2} g' A_\mu) \Psi_L - m(z) \Psi_R = 0
\]

\[
(i \partial_\mu - \frac{y_R}{2} g' A_\mu) \Psi_R - m(z) \Psi_L = 0, \tag{3}
\]

where \( y_{R,L} \) are the right and left-handed hypercharges corresponding to the given fermion, respectively, \( g' \) the \( U(1)_Y \) coupling constant and we take \( A^\mu = (0, A) \) representing a four-vector potential having non-zero components only for its spatial part, in the rest frame of the wall. These equations can be written as a couple of sets of equations, that describe the independent motion of the spin component parallel and antiparallel to the magnetic field respectively. The second is obtained from the first one by changing \( B \) to \( -B \). Both set of equations involve modes coupled with \( y_R \) and \( y_L \) and in the limit when \( y_R = y_L = e \), they decouple as in the case when describing the interaction of fermions with the magnetic field through their electric charge.

### 3.2. Broken symmetry phase

We now look at the corresponding equation in the broken symmetry phase. For \( z \geq 0 \) the coupling of the fermion with the external field is through the electric charge \( e \) and thus, the equation of motion is simply the Dirac equation describing an electrically charged fermion in a background magnetic field, namely,

\[
\left\{ i \partial_\mu - e A_\mu \gamma^\mu - m(z) \right\} \Psi = 0. \tag{4}
\]
4. Fermion scattering off the bubble walls and axial charge segregation

We study here the scattering of left and right-handed fermions off a finite width kink wall during the EWPT in the presence of a background hypermagnetic field. Using analytical and numerical solutions of the above equations in both phases, we compute the transmission and reflection coefficients of fermions through the wall, for the case when they move from the symmetric to the broken symmetry phase. Since these are related to the corresponding coefficients for fermions incident from the broken symmetry phase, it is easy to obtain the total flux of fermions for the two distinct chirality modes in the symmetric phase. We work in particular with top quarks, since all other fermions have small Yukawa couplings and therefore penetrate the bubble wall with little reflection.

Once we have found the solutions for the above equations, for modes with spin components parallel and antiparallel to the magnetic field, prepared as incident waves coupled with $y_L$ or with $y_R$, the problem is completely defined by matching the solutions and their derivatives across the interface.

Since in the broken symmetry phase there should not be a propagating component corresponding to the $Z^0$ field [12, 13], the magnetic field strength $B$ in the symmetric phase is related to $B'$ in the broken one and Weinberg’s angle $\theta_W$ by

$$ B' = \frac{B}{\cos \theta_W}, \quad (5) $$

which in turn implies that the coupling of the fermion with the magnetic field is given by

$$ eB' = g' B. \quad (6) $$

We find that the amplitudes for the distinct axial modes in the symmetric phase are not the same, meaning that there is the possibility of building an axial asymmetry during the scattering of fermions off the wall. To quantify the asymmetry, we need to compute the corresponding reflection and transmission coefficients. These are built from the reflected, transmitted and incident currents of each type. Recall that for a given spinor wave function $\Psi$, the current normal to the wall is given by

$$ J = \Psi^\dagger \gamma^0 \gamma^3 \Psi. \quad (7) $$

The currents need to be computed in the asymptotic regions far away from the wall where the amplitudes represent plane waves with well defined direction of motion.

For a left-handed incident particle, the reflection and transmission coefficients are given as the ratios of the corresponding reflected and transmitted currents, to the incident one, respectively, projected along a unit vector normal to the wall. These are

$$ R_{l \rightarrow r} = -J^r_{\text{ref}}/J^l_{\text{inc}}, \quad T_{l \rightarrow l} = J^l_{\text{tra}}/J^l_{\text{inc}}, \quad (8) $$

where indexes $l$ and $r$ denote helicity modes. The corresponding coefficients for the axially conjugate process are

$$ R_{r \rightarrow l} = -J^l_{\text{ref}}/J^r_{\text{inc}}, \quad T_{r \rightarrow r} = J^r_{\text{tra}}/J^r_{\text{inc}}. \quad (9) $$

For an incident wave coupled with $y_L$ ($y_R$), the fact that the differential equations mix up the solutions means that the reflected wave will also include a component coupled with $y_R$.
Reflection Coefficients

Figure 1. Coefficients $R_{l\rightarrow r}$ and $R_{r\rightarrow l}$ as a function of the magnetic field parameter $b$ for $\xi = 3.5$, $\epsilon = 7.03$, $y_R = 4/3$, $y_L = 1/3$. The value for the $U(1)_Y$ coupling constant is taken as $g' = 0.344$, corresponding to the EWPT epoch. The dots represent the computed values.

$(y_L)$. We have prepared the incident fermion from the symmetric phase in such a way that when coupled with a given chirality, it corresponds to the same helicity. Since in the symmetric phase the fermion mass is asymptotically zero, both the chirality and helicity operators can be simultaneously defined and their eigenvalues coincide. This is no longer the case when the fermion moves in the broken symmetry phase where its mass is different from zero. Nevertheless, since scattering off the wall does not change the direction of the fermion spin, the fermion helicity is preserved during transmission and reversed upon reflection.

Our results are displayed in Figure 1 and Figure 2, where we are working with magnitudes scaled by the width of the domain wall, $\lambda$. $b$ is the parameter representing the magnetic field strength given by $b = \lambda^2 B$; $\epsilon$ is the energy parameter, $\epsilon = \lambda E$, where $E$ is the energy of the incident particle, and $\xi$ represents the heigth of the barrier, being twice the ratio of the fermion mass to the Higgs mass. Figure 1 shows the coefficients $R_{l\rightarrow r}$ and $R_{r\rightarrow l}$ as a function of the magnetic field parameter $b$ for a value of $\xi = 3.5$, an energy parameter $\epsilon = 7.03$, hypercharge values $y_R = 4/3$, $y_L = 1/3$ and for a value of $g' = 0.344$, as appropriate for the EWPT epoch. Notice that when $b \to 0$, these coefficients approach each other and that the difference grows with increasing field strength.

Figure 2 shows the reflection and transmission coefficients as a function of the energy parameter $\epsilon$ scaled by twice the height of the barrier $2\xi$. Figure 2a shows the coefficients $R_{l\rightarrow r}$ and $T_{l\rightarrow l}$ and Fig. 2b the coefficients $R_{r\rightarrow l}$ and $T_{r\rightarrow r}$ for $b = 0.5$.

Since the solutions are computed assuming that the transmitted waves are not exponentially damped, their energies have to be taken larger than the height of the barrier, $\epsilon \geq \sqrt{4\xi^2 - g'b}$, for waves with spin components parallel to the magnetic field, whereas for antiparallel spin components, $\epsilon \geq \sqrt{4\xi^2 + g'b}$. These requirements basically mean that, by energy conservation, the energy of the incident particle must be larger than the mass the particle will acquire while crossing the phase boundary. It can be checked that $R_{r\rightarrow l} + T_{r\rightarrow r} = 1$ and $R_{l\rightarrow r} + T_{l\rightarrow l} = 1$ within the numerical precision of the calculation, which means that the analysis respects unitarity.

5. Discussion and conclusions

We have studied the scattering of fermions off a first order EWPT bubble wall with a finite width in the presence of a background magnetic field directed along the fermion direction of motion. In the symmetric phase, the chiral nature of the coupling of fermions to the hypermagnetic field implies that it is possible to build an axial asymmetry during the scattering of fermions off the wall. We have computed reflection and transmission coefficients showing explicitly that they differ for left and right-handed incident particles from the symmetric phase.
Reflection and transmission coefficients as a function of the energy parameter $\epsilon$ scaled by twice the height of the barrier $2\xi$ for $b = 0.5$ and $\xi = 3.5$, $y_R = 4/3$, $y_L = 1/3$, $g' = 0.344$.

Figure 2a (left panel) shows the coefficients for incident, left-handed helicity modes and Fig. 2b (right panel) for incident, right-handed helicity modes. In both figures, the dots represent the computed values.

The results of this more realistic, albeit numerical calculation where we allow for a finite wall width are in qualitative and quantitative agreement with those previously found in Ref. [20], where the wall was modeled as a step function.

Some remarks about the particular physical situation we have considered are in order here. Let us first look at the situation in which the direction of the magnetic field is reversed with respect to the case studied here, changing $B$ to $-B$ in the two sets of equations we had to solve. The only effect of this switch will be the interchange of one set of equations with the other, leaving intact the coupling. Physically this is also easy to understand since the fermion coupling with the external field is through its spin. Changing the direction of the field exchanges the role of each spin component but since each chirality mode contains both spin orientations, it does not affect the final probabilities. Now suppose that the original direction of motion of the fermion is not parallel to the direction of the magnetic field. The effect of this situation is also negligible since a boost can be done to the frame where the transverse momentum vanishes [9]. The case has been explicitly addressed in [24], obtaining a similar result to the present one.

Finally, under the assumptions of $CPT$ invariance and unitarity, the total axial asymmetry - which includes contributions both from particles and antiparticles - can be quantified in terms of the particle asymmetry. Let $\rho_i$ represent the number density for species $i$. Then the densities in left-handed and right-handed axial charges are obtained by taking the differences $\rho_L - \rho_{\bar{L}}$ and $\rho_R - \rho_{\bar{R}}$, respectively. It is straightforward to show [9] that $CPT$ invariance and unitarity imply that the above net densities are given by

$$
\rho_L - \rho_{\bar{L}} = (f^s - f^b)(R_{r\to l} - R_{l\to r}),
\rho_R - \rho_{\bar{R}} = (f^s - f^b)(R_{l\to r} - R_{r\to l}),
$$

(10)

where $f^s$ and $f^b$ are the statistical distributions for particles or antiparticles (since the chemical potentials are assumed to be zero or small compared to the temperature, these distributions are the same for particles or antiparticles) in the symmetric and the broken symmetry phases, respectively. Thus, the asymmetry in the axial charge density is finally given by

$$
(\rho_L - \rho_{\bar{L}}) - (\rho_R - \rho_{\bar{R}}) = 2(f^s - f^b)(R_{r\to l} - R_{l\to r}).
$$

(11)

This asymmetry, built on either side of the wall, is dissociated from non-conserving baryon number processes and can subsequently be converted to baryon number in the unbroken
symmetry phase where sphaleron induced transitions are taking place with a large rate. It is interesting to notice that, with this mechanism, we are not generating a net excess of one type of particle (left or right-handed) over the other; it is merely a segregation between the two sides of the bubble wall.

The relation of this axial asymmetry to $CP$ violation is to be found in the dynamics of the scattering process, that can be thought of as describing the mixing of two levels: right and left-handed quarks coupled to an external magnetic field. When the two chirality modes interact only with the external field, they evolve separately. It is only the scattering with the bubble wall what allows a finite transition probability for one mode to become the other. Since the modes are coupled differently to the external field, these probabilities are different. $CP$ is violated in the process because, though $C$ is conserved, $P$ is violated and thus is $CP$.

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