No time for isolated clocks

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Abstract. The Wheeler-DeWitt equation provides a model of the Universe as a timeless, isolated system. Page and Wootters developed the conditional probability interpretation (CPI) to account for the experience of time within this timeless model by identifying part of the Universe as the clock. In resolving criticisms of the CPI, some investigators have since concluded that the clock must be completely isolated from the remainder of the Universe. This isolation has also been used to reinforce the conclusion that time must be an illusion. However, some interactions must exist between all subsystems of the Universe as gravitational effects, however weak, cannot be shielded. Here, we present the results of an investigation which used a toy model to consider the implications of including interactions. Although counter intuitive, their inclusion is seen to directly correlate with the accuracy of measurements of the clock state. The case is then made that, if interactions are indeed a necessary feature of the CPI, the viability of treating time as an illusion needs to be reconsidered. We conclude that the experience of time can be interpreted as a real phenomenon, which naturally incorporates an arrow of time, once interactions are taken seriously in the CPI paradigm.

1. Introduction

Time is often taken for granted but, yet, can be a sensitive subject in some fields of physics. Although the concept of time is used within the mathematical framework of physical theories, its description in different theories does not always agree. The most prominent case in point is the disagreement between quantum mechanics and general relativity. Both of these versions of time could be expected to emerge from a single description of time which is contained in a more fundamental theory of quantum gravity. If so, the more fundamental theory requires a single description that is apparently consistent with (at least) two contradictory versions of emergent time. This issue encapsulates one aspect of the infamous ‘problem of time’. For a detailed overview of both this aspect of the problem and various others, see, for example, [1].

A mathematical framework that nicely incorporates both quantum mechanics and general relativity, albeit superficially, is provided by the Wheeler–DeWitt equation [2]. The equation, which is meant to model the quantum nature of the Universe, is expressible as

$$\hat{H}|\Psi\rangle = 0,$$

where $|\Psi\rangle$ can be regarded as the wavefunction of the Universe and the Hamiltonian operator $\hat{H}$ is a suitably quantized version of the Hamiltonian constraint from general relativity. The right-hand side of equation (1) implies that no energy is being exchanged with systems that are (somehow) external to the Universe; in other words, the Universe is, at least effectively, an isolated system.
Irrespective of the legitimacy of the Wheeler–DeWitt equation, an isolated model for the Universe has important ramifications for the interpretation of time. Equation (1) does, however, help to clarify the point by showing that any physical state is annihilated by the Hamiltonian, implying that the Universe as a whole cannot be functionally dependent on time. The conclusion is one of ‘timelessness’: The Universe cannot experience any change in its quantum state, even in the Schrödinger picture.

This timelessness is problematic: If the Universe as a whole includes no notion of time evolution, then why should any smaller subsystem? A solution, which is often referred to as the conditional probability interpretation (CPI) of time, was offered quite a while ago by Page and Wootters [3]. Given that the Universe can be divided into a pair of maximally entangled, weakly interacting subsystems, then one of these – which is typically identified as the clock \( C \) — will necessarily provide a time parameter for the remaining subsystem, which is normally referred to as \( R \). This emergent notion of time circumvents the need for the ‘global time’ parameter \( t \) that is commonly adopted in standard quantum mechanics, and so a means for describing the evolution of subsystems can be recovered.

The CPI has been confronted with a variety of criticisms. The most prominent of these was the concern that, while one pair of states for \( C \) and \( R \) could be described at some given time, this description could not be extended in a meaningful way so as to connect a pair of states at one time to a second pair at a later time [4]. This matter has since been resolved by some different approaches; see, for instance, [5] and [6].

Another concern is the so-called clock-ambiguity problem [7]. To explain, the CPI dynamics will in general be arbitrary, as one can expect a multitude of different ways to partition the Universe into a suitable pair of subsystems and that each such partition would provide its own dynamical description. A proposed resolution to alleviate this concern is to restrict the choice of partitions to subsystems which are perfectly isolated from one another [8]. This is a stronger constraint than that prescribed by the CPI, which stipulates that \( C \) and \( R \) be weakly interacting systems but need not be cut off entirely. The proposal works because, given that \( C \) and \( R \) are indeed isolated, any other partition is related to the first by a local unitary transformation if and only if the resultant subsystems are similarly isolated. There is, however, a cost: An isolated clock leads to the interpretation that any experience of time within the Universe is an illusion. We elaborate on this reasoning in Section 3.

It is our contention that interactions between \( C \) and \( R \) have to be present and then duly accounted for. Doing so, one finds the (perhaps) counter-intuitive outcome that the inclusion of interactions actually improves the accuracy of the clock. Moreover, and as just alluded to, this inclusion is necessary if one is to recover a non-illusionary description of the experience of time. But, irrespective of these perks, the choice of non-interacting subsystems is not even feasible as a strict matter of principle. This is because there is no way to shield them against the effects of gravity.

In Section 2, we present the results of an investigation into the effectiveness of interacting clock systems. This is followed in Section 3 by an assessment of the interpretive issues. We conclude in Section 4 with a brief summary and comment on the outlook.

2. A realistic and interacting clock

A refined version of the CPI [5] (also see [9]) can be applied to physically realistic clocks and their respective environments [10]. In [11], we employed this refined CPI to study just such a clock system, while adopting the viewpoint that the clock and its environment are toy models of a partitioned form of the isolated Universe. This analysis revealed that the idealized case of

\[ \text{With regard to the application, an essential refinement is the replacement of the abstract CPI time parameter — the eigenvalue of the conjugate to the clock’s reduced Hamiltonian — with a more tangible parameter such as the position of the clock’s center of mass.} \]
an isolated clock does not make for a particularly accurate timepiece. The rest of this section recalls this investigation and our conclusions thereof.

For purposes of simplicity, we adopted the assumption that the clock $C$ was much smaller than the environment (or remaining system) $R$. As such, the effects of the interactions — which are regarded as weak as mandated by the CPI — would have no bearing on $R$ and could be directly incorporated into the dynamics of $C$.

As a clock, we selected a coherent-state description of a (weakly) damped harmonic oscillator [13, 14], with the position of the oscillator playing the role of the time. This would provide a time parameter for $R$ in an analogous fashion to how the position of the hands on a clock indicates the time.

We were able to find a Gaussian expression for the probability of finding the clock in a particular position $x$ at some value of abstract clock time $n$ (see below). The associated uncertainty, which could be identified as the width of the Gaussian and regarded as a measure of the inaccuracy of the clock, was found to be of the form

$$\delta = e^{-rn/2} \sqrt{\frac{\hbar}{2m\omega}},$$

where $r$ represents the strength of the damping (i.e., the oscillations are damped by a factor of $e^{-rn}$), $m$ is the mass and $\omega$ is the undamped frequency.

As for the abstract clock time $n$, it initially appears as the external time parameter (i.e., the $t$ in standard quantum mechanics) but ultimately plays the role of an integration variable in the refined CPI formalism. As far as $R$ is concerned, the time parameter is the clock position $x$, as $n$ never appears in any of the final (conditional) probability expressions and so cannot be viewed as a measurable time. Rather, $n$ acts as a ‘synchronization’ tool, whose purpose is to keep track of how $C$ and $R$ are evolving in relation to one another.

In terms of this damped harmonic oscillator, $n$ is also relevant to the condition of weak damping because of the oscillator’s damping factor of $e^{-rn}$. Hence, $n$ must be small but, on the other hand, one would like $n$ to be as large as possible so as to ensure a lengthy ‘run time’ for the clock. (This does not refer to a physical run time but rather to a breakdown of the formalism as the clock progresses through more and more states.) These two conflicting considerations leads one to an operational range of $0 < rn < 1$, where the right-hand side really means a number on the order of unity.

From an inspection of equation (2), it becomes clear that the uncertainty is minimized when the damping parameter $r$ approaches infinity. However, the previous constraint restricts the size of $r$ with respect to $n$. Meaning that, in order for the clock to function effectively for as long as possible, one arrives at the choice of $r = \frac{1}{n}$. This is a essentially a sweet spot for which the operation of the clock is optimized.

What is evident is that the optimal setting for $r$ is not zero, as this would have maximized the uncertainty in equation (2). In short, a completely isolated clock may be ideal (to some) but it is hardly optimal. It appears that the condition of a weakly interacting clock must be taken more literally: the interactions must be weak but never removed completely. This conclusion is further supported by an investigation in [15], which produced a very similar result for the ‘more quantum’ case of an atomic clock. This leaves us with the task of reassessing the role of interactions in the original (Universal) version of the CPI.

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2 Investigations into the inclusion of interactions can be found elsewhere; e.g., [12], where the interactions have been formally inserted into $H$. 
3. Implications of interactions

The discussion to follow is based on parts of [11, 15], where more thorough treatments can be found. We once again regard \( C \) and \( R \) as the maximally entangled subsystems of a partitioned form of the isolated Universe.

Let us suppose that \( C \) and \( R \) are completely isolated as is apparently needed to resolve the clock-ambiguity problem as in [8]. The basic problem now is that this isolated clock must be as timeless as the isolated Universe. This is because of the elimination of any energy transfer to or from the clock, ensuring that its state cannot change, even in the Schrödinger picture, and so the clock can have no inherent sense of time. Nor can it gain a sense of time from its surroundings inasmuch as there are no interactions between \( C \) and \( R \).

Let us clarify a somewhat subtle point: In standard quantum mechanics, a time-independent system, even if completely isolated, is presumed to exist within a larger system that does indeed experience time. In this manner, the state of the smaller system can ‘inherit’ time, allowing for features such as a time-dependent phase. The CPI prohibits this feature out of hand, as the larger system, the isolated Universe, cannot experience time, leaving no sense of time for the clock to inherit.

There is then no inherent ‘flow of time’ for the isolated clock, nor for its similarly isolated complement \( R \). Rather than each subsystem tracking the other as they run through a succession of synchronized pairs of states, the interpretation now becomes that every state of \( C \) exists both ‘simultaneously’ and ‘eternally’ as does every state of \( R \).\(^3\) This is much the same as a series of pages making up a book because every such page must still exist alongside all of the others. A momentum state of \( C \) forms a part of the evolution of \( C \) but, as each other state exists alongside of it, the notion of change must be an illusion: No system ever actually ‘moves’ from one state to another. One is thus left to conclude that the flow of time, as it emerges in the isolated Universe with isolated subsystems, must be an illusionary effect.

Given that real change is indeed a mirage, why is it that the experience of time proceeds in such an orderly fashion, whereby the states of any given system follow a particular order along a distinct arrow towards the future? An argument in favor of the timeless interpretation that addresses this question can be found in [8], where an ad hoc memory system is introduced in order to explain the ordered series of moments that one experiences. With reference to the book analogy, the memory system essentially provides a way to gather up loose pages and arrange them into a story. However, since every possible page is always represented, what one really has is an infinite pile of loose pages rather than a single bound book. A given path through the pile would represent the ‘experience’ of a single book.

Various concerns with this perspective have been raised by the current authors in [17]. Our focus at present will be on the difficulties that non-trivial or irreversible interactions pose when time is regarded as illusional. Fortuitously, the inclusion of such interactions can resolve the clock-ambiguity problem by singling out thermodynamic time and, in this way, altogether nullify what is (in our opinion) the main motivation for an illusionary interpretation of time. This alternative resolution of the clock-ambiguity problem is somewhat involved; see [17] for a detailed account.

Let us suppose that irreversible interactions are now included and consider the state of some system just before and after an interaction takes place. Can this inclusion be incorporated into the above interpretation, written onto the respective pages of the book as it were? The answer is not necessarily. When interpreted as an intervention of one system on another, an interaction requires an associated process of change. This change can be thought of as one system ‘learning’ about another by transferring information about their respective states through an appropriate conduit. The time-is-an-illusion viewpoint, on the other hand, implies that each state (or page)

\(^3\) This picture is essentially describing a quantum version of the Block Universe. For further discussion, see, e.g., [16].
of each system (or book) is locked into an ‘eternal moment’ and, as such, each state is itself essentially an independent system. If one state were somehow removed or changed, there is nothing that is built into this picture to suggest that the remaining states would be affected. In other words, two systems cannot interact in a meaningful way because they are already locked into particular states at whatever ‘moment’ is meant to represent that just after the interaction.

Irreversible interactions also imply an arrow of time. As would clearly be the case for the damped clock in the previous section, there is a continual increase in entropy as a system interacts with its environment. This is another feature which appears to contradict the notion that time is an illusion, as the lack of real change is also suggestive of complete reversibility. One is supposed to get around this contradiction through yet another ad hoc assumption; namely that each state of the system contains the information, or ‘memory’, which would enable it to create the illusion of an arrow of time. This picture necessarily requires each state to ‘know’ enough about the other states so as to produce the illusion of a consistent arrow of time. It remains an open question as to how such a convoluted picture could come to fruition or, indeed, why is it that Nature would so diligently construct such an illusion.

Rather than trying to square these circles, we are inclined to take irreversible interactions seriously and arrive at a different interpretation of the CPI: The experience of time as an emergent feature of the isolated Universe is a real phenomenon. Given that one system has the opportunity to learn about others through a process of change, then each state of the system is directly responsible for the state which follows it. There is then a contiguity in time such that one can identify the final state of a system as having evolved from the initial one.

It is the arrow of time, which is now aimed in the direction of increasing entropy, enables one to differentiate the past from the future. If the experience of time is real, this distinction between past and future can be expected to have physical meaning. Systems would be ‘transforming’ from their present states into future states, while their memories (cf footnote 4) would be referring to states from the past. What is implicit in this interpretation is a postulate of causality. Given this, the intervention of one system on another would be the cause and the resulting transformation would be the effect. In spite of its fundamental nature, causality has fallen out of favor in some areas of physics but is currently experiencing something of a revival. See, for example, [18].

There are still many issues to resolve under the interpretation that time is real. To name just a few, a definitive description of how change occurs, why there should be a low entropy point in the past to evolve from and to clarify the role of causality. Nonetheless, these are not insurmountable challenges; certainly no more so than those facing the interpretation that time is an illusion.

4. Discussion

The presence of interactions is required in principle when dealing with gravitational systems. Our quantitative results in Section 2 reinforce the notion that interacting clocks are a crucial aspect of the CPI and should not be dismissed from the outset. Qualitative arguments, such as those in Section 3, lend further credence to this sentiment.

Even if the experience of time is regarded as real, as would be necessary to accommodate the inclusion of interactions, there are still many interpretive issues to be resolved. For example, causality plays a prominent role in the experience of time but its origin is not really understood. We discuss this sensitive topic in more detail in [17]. In essence, whatever it is that places the future events before us and the past ones behind us must either be put in by hand or else deferred to a more fundamental treatment. But, regardless of how causality does fit into the grand scheme of things, our conclusion remains that interactions are a key ingredient if emergent time is to be regarded as a real phenomenon in an isolated model of the Universe.

4 Memory does not have to imply consciousness in this context. For instance, a sea-side rock can carry the memory of the tide via its pattern of erosion.
Acknowledgments
The research of AJMM received support from an NRF Incentive Funding Grant 85353 and NRF Competitive Programme Grant 93595. KLHB is supported by an NRF bursary through Competitive Programme Grant 93595 and a Henderson Scholarship from Rhodes University. This work is based on the research also supported in part by the National Research Foundation of South Africa (Grant Numbers: 111616).

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