Decoupling limits in multi-sector supergravities

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Abstract. Conventional approaches to cosmology in supergravity assume the existence of multiple sectors that only communicate gravitationally. In principle these sectors decouple in the limit $M_{\text{pl}} \to \infty$. In practice such a limit is delicate: for generic supergravities, where sectors are combined by adding their Kähler functions, the separate superpotentials must contain non-vanishing vacuum expectation values supplementing the naïve global superpotential. We show that this requires non-canonical scaling in the naïve supergravity superpotential couplings to recover independent sectors of globally supersymmetric field theory in the decoupling limit $M_{\text{pl}} \to \infty$.

Keywords: cosmology of theories beyond the SM, inflation, particle physics - cosmology connection, physics of the early universe, supersymmetry and cosmology

Sadly, Sjoerd Hardeman passed away only a few days before the completion of this letter. We wish to commemorate him as a splendid physicist and a good friend.
1 Introduction

Multiple sectors are a common feature in supergravity cosmology and phenomenology. These sectors are necessary to either incorporate inflation or supersymmetry breaking or are a consequence of string model-building. In particular to study inflation, it is desirable to separate the dynamics of all fields that do not contribute to the exponential expansion of the Universe from the inflaton fields that do. Since gravity is the weakest possible interaction, the inflationary sector is assumed to only couple gravitationally to an unknown “hidden” sector that may also break supersymmetry by itself. Whereas it is natural for a rigid supersymmetric theory to be separated into several sectors, the restrictive structure of supergravity forces the different sectors to couple not only non-locally through graviton exchange but also directly. For this reason embedding supersymmetric theories as sectors into a supergravity can be notoriously difficult, see e.g. [1–8].

Though multiple sector supergravities are a long studied subject, the context of cosmology has seriously sharpened the question. In supergravity models of inflation it is commonly noted that one seeks a consistent truncation of the scalar sector. This is necessary but not sufficient. Even with a consistent truncation one may have dominating instabilities towards the naively non-dynamical sectors, that can move them away from their supersymmetric critical points. One needs either a symmetry constraint or an energy barrier to constrain the dynamics to the putative inflaton sector.

Moreover, during inflation, supersymmetry is broken. Although it is frugal to consider scenarios where the inflaton sector is also responsible for phenomenological supersymmetry breaking (see e.g. [9–11]), this need not be so. For instance, in a generic gauge-mediation scenario, the mechanism responsible for supersymmetry breaking need not involve the fields that drive inflation. This example immediately shows that the generic cosmological set-up must be able to account for a sector that breaks supersymmetry independently of the inflationary dynamics.

This notion is our starting point. We consider a multiple-sector supergravity that decouples in the strictest sense in the limit $M_{\text{pl}} \rightarrow \infty$. In this limit the action must then be the sum of two independent functions

$$S[\phi, \bar{\phi}, z, \bar{z}] = S[\phi, \bar{\phi}] + S[z, \bar{z}],$$

such that the path integral factorizes. For a globally supersymmetric field theory with a standard kinetic term this can be achieved by demanding that the independent Kähler and superpotentials sum

$$K_{\text{susy}}(\phi, \bar{\phi}, z, \bar{z}) = K^{(1)}(\phi, \bar{\phi}) + K^{(2)}(z, \bar{z}), \quad W_{\text{susy}}(\phi, z) = W^{(1)}(\phi) + W^{(2)}(z).$$

The issue we address here is that in supergravity complete decoupling in the sense of (1.1) appears to be impossible, even in principle. Even with block diagonal kinetic terms from a sum of Kähler potentials, the more complicated form of the supergravity potential

$$V_{\text{sugra}} = e^{K/M_{\text{pl}}^2} \left( K^{ab} D_a W^b \bar{D}_b \bar{W} - \frac{3|W|^2}{M_{\text{pl}}^2} \right), \quad DW_{\text{sugra}} = \partial W_{\text{sugra}} + \partial K_{\text{sugra}} W_{\text{sugra}} M_{\text{pl}}^2,$$

\[ \text{(1.3)} \]

\[ ^1 \text{As example we consider the simplest case, a model with two uncharged scalar supermultiplets } X^a = (\phi, z) \text{ that are singlets under all symmetries. Gauge interactions and global symmetries will not change this general argument provided the two sectors are not mixed by symmetries or coupled by gauge fields. Therefore, we will also ignore } D\text{-terms in the supergravity potential below.} \]
implies that there are many direct couplings between the two sectors. It raises the immediate question: if the low-energy $M_{\text{pl}} \to \infty$ globally supersymmetric model must consist of decoupled sectors, what is the relation between $K_{\text{sugra}}, W_{\text{sugra}}$ and $K_{\text{susy}}, W_{\text{susy}}$, or vice versa given a globally supersymmetric model described by $K_{\text{susy}}, W_{\text{susy}}$, what is the best choice for $K_{\text{sugra}}, W_{\text{sugra}}$ such that the original theory can be recovered in the limit $M_{\text{pl}} \to \infty$?

In this note we shall show that the scaling implied by the explicit factors of $M_{\text{pl}}$ in the supergravity potential (1.3) is an incomplete answer to this question. The direct communication between the sectors, controlled by $M_{\text{pl}}$, has serious consequences for both the ground state structure (solutions to the equation of motion, i.e. the cosmological dynamics) and the interactions between the two sectors. To be explicit, the first guess at how the rigid supersymmetry and supergravity Kähler potentials and superpotentials are related

$$K_{\text{sugra}}(\phi, \bar{\phi}, z, \bar{z}) = K_{\text{susy}}^{(1)}(\phi, \bar{\phi}) + K_{\text{susy}}^{(2)}(z, \bar{z}) + \ldots, \quad W_{\text{sugra}}(\phi, z) = W_{\text{susy}}^{(1)}(\phi) + W_{\text{susy}}^{(2)}(z) + \ldots,$$

with $\ldots$ indicating Planck-suppressed terms and possibly a constant term, suffers from the drawback that the ground states of the full theory are no longer the product of the ground states of the individual sectors, except when both (rather than only one) ground states are supersymmetric [12, 13] (see also [14–16]). This directly follows from considering the extrema of the supergravity potential

$$\nabla_i V = \frac{D_i W}{W} V + e^{K/M_{\text{pl}}^2}|W|^2 \left( \nabla_i \left( \frac{D_j W}{W} \right) \frac{D_j W}{W} + \frac{1}{M_{\text{pl}}^2} \frac{D_i W}{W} \right),$$

$$\nabla_i \nabla_\alpha V = \frac{D_\alpha W}{W} \nabla_i V + \frac{D_i W}{W} \nabla_\alpha V - \frac{D_i W}{W} \frac{D_\alpha W}{W} V + \frac{D_\alpha W}{W} \left( V + \frac{2}{M_{\text{pl}}^2} e^{K/M_{\text{pl}}^2}|W|^2 \right) + e^{K/M_{\text{pl}}^2}|W|^2 \left( \nabla_i \nabla_\alpha \left( \frac{D_j W}{W} \right) \frac{D_j W}{W} + \nabla_\alpha \nabla_i \left( \frac{D_j W}{W} \right) \frac{D_j W}{W} \right).$$

Supersymmetric ground states, for which the covariant derivatives of $W$ vanish on the solution, $D_i W = 0$ and $D_\alpha W = 0$, are still product solutions. But for Kähler and superpotentials that sum (1.4), even if only one sector is in a non-supersymmetric ground state, by which we mean $D_i W = 0$, $D_\alpha W = 0$, we can neither conclude that sector 2, labeled by $i$, is in a minimum, for which $\nabla_i V$ would vanish, nor that the condition for sector 1, labeled by $\alpha$, to be in a local ground state is independent of the sector 2 fields $z^j$, which would mean that $\nabla_i \nabla_\alpha V = 0$. The former is only true when

$$\nabla_i \left( \frac{D_\beta W}{W} \right) \frac{D_\beta W}{W} = 0.$$

The second requires, in addition,

$$\nabla_i \nabla_\alpha \left( \frac{D_\beta W}{W} \right) \frac{D_\beta W}{W} + \nabla_\alpha \nabla_i \left( \frac{D_\beta W}{W} \right) \frac{D_\beta W}{W} = 0.$$

\[\text{To derive (1.6) note that, since $DW/W$ is Kähler invariant and since the Levi-Civita connection $\nabla$ of the field space manifold does not get cross-contributions in a product manifold,}

$$\nabla_i \frac{D_\alpha W}{W} = \partial_i \frac{D_\alpha W}{W} = D_i \frac{D_\alpha W}{W}.$$
and also sharpens the first condition (1.7) to
\[ D_i D_\alpha W = 0. \tag{1.9} \]

Equations (1.7–1.9) are conditions for decoupling which apply not only to the ground state of the full system but also to other critical points of the potential, for instance along an inflationary valley. Generically these conditions are not met on the solution (the second derivative need not vanish at an extremum; recall that \( D_i W \) does not vanish identically but only on the solution). Hence, generically the ground states of hidden sectors mix and this spoils many cosmological supergravity scenarios that truncate the action to one or the other sector (see e.g. [17] and references therein). It is this issue that is particularly relevant for inflationary model building, where a very weak coupling between the inflaton sector and all other sectors has to persist over an entire trajectory in field space where the expectation values of the fields are changing with time (see e.g. [18–22]). At the same time, one is interested in the generic situation in which both sectors may contribute to supersymmetry breaking. 4

2 Natural multi-sector supergravities

There is a well-known natural way to construct supergravity potentials for which the ground states (and critical points) do separate better. This obvious combination of superpotentials automatically satisfies (1.7–1.9) and hence does ensure that if one of the ground states is supersymmetric, the ground state of the other sector is a decoupled field theory ground state whether it breaks supersymmetry or not. This is if we choose a product of superpotentials, keeping the sum of Kähler potentials as before,

\[ K_{\text{sugra}}(\phi, \overline{\phi}, z, \overline{z}) = K_{\text{sugra}}^{(1)}(\phi, \overline{\phi}) + K_{\text{sugra}}^{(2)}(z, \overline{z}), \quad W_{\text{sugra}}(\phi, z) = \frac{1}{M_{\text{pl}}^{3}} W_{\text{sugra}}^{(1)}(\phi) W_{\text{sugra}}^{(2)}(z). \tag{2.1} \]

3 These conditions are merely sufficient not necessary. However, it is clear that the restrictive nature of supergravity enforces conditions on the unknown sectors for the system to be separate. 

4 This situation has to be contrasted to phenomenological models appropriate for studying gravity mediated supersymmetry breaking, such as an ansatz [23]

\[ K(\phi, \overline{\phi}, z, \overline{z}) = K_0(\phi, \overline{\phi}) + z^i \overline{z}^j K_{ij}^{(1)}(\phi, \overline{\phi}), \quad W(\phi, z) = W_0(\phi) + z^i \overline{z}^j W_{ij}(\phi). \]

or equivalently, if \( W \neq 0 \),

\[ G(\phi, \overline{\phi}, z, \overline{z}) = G_0(\phi, \overline{\phi}) + z^i \overline{z}^j G_{ij}^{(1)}(\phi, \overline{\phi}) + z^i \overline{z}^j G_{ij}^{(2)}(\phi, \overline{\phi}) + z^i \overline{z}^j \ldots. \]

In models like these, it is understood that \( \dot{z} = 0 \) and the \( z \)-sector can remain in its supersymmetric critical point throughout the evolution of the supersymmetry breaking fields. For inflation, such an expectation is unrealistic, as the supersymmetry-preserving sector can become unstable during the inflationary dynamics, see e.g. a recent discussion of the case in which the inflaton field \( \phi \) is solely responsible for supersymmetry breaking during inflation ([11] and references therein). In this relatively simple case, and except for very fine-tuned situations, the generic scenario appears to be that one or more of the \( z \)-fields are destabilized somewhere along the inflationary trajectory and they trigger an exit from inflation (in other words, they become “waterfall” fields, and inflation is of the hybrid kind [24]). This implies that the pattern of supersymmetry breaking today is not related to the one during inflation, and also, since the waterfall fields are forced away from their supersymmetric critical points, that supersymmetry is broken by both sectors as the Universe evolves towards the current vacuum.
This is well-known [25–27] and has recently been emphasized in the context of cosmology [14–17, 19, 20, 22, 28, 29]. This ansatz conforms to the more natural description of supergravities in terms of the Kähler invariant function

\[ G(X, X) = \frac{1}{M^2_{\text{pl}}} K_{\text{sugra}}(X, X) + \log \left( \frac{W_{\text{sugra}}(X)}{M^3_{\text{pl}}} \right) + \log \left( \frac{W_{\text{sugra}}(X)}{M^3_{\text{pl}}} \right), \]  

(2.2)

which can be defined if \( W \) is non-zero in the region of interest.\(^5\) This Kähler function in turn underlies a better description of multiple sectors in supergravity where \( G \) is a sum of independent functions

\[ G(\phi, \bar{\phi}, z, \bar{z}) = G^{(1)}(\phi, \bar{\phi}) + G^{(2)}(z, \bar{z}), \]  

(2.3)

such that the two sectors are separately Kähler invariant. The sum implies the product superpotential put forward above. This is the simplest ansatz that still allows some degree of calculational control when both sectors break supersymmetry—as well as optimizing decoupling along the inflationary trajectory. One of the simplest models of hybrid inflation in supergravity, \( F \)-term inflation [32, 33], is in this class.

3 Decoupling

Given that we have just argued that a product of superpotentials is a more natural framework to discuss multiple sector supergravities, the obvious question arises how to recover a decoupled sum of potentials for a globally supersymmetric theory in the limit where gravity decouples, i.e. in which

\[ V_{\text{sugra}} = e^{K/M^2_{\text{pl}}} \left( |DW|^2 - \frac{3|W|^2}{M^2_{\text{pl}}} \right) \rightarrow V_{\text{susy}} = \sum_j |\partial_j W^{(j)}|^2. \]  

(3.1)

For a two-sector supergravity defined by eqs. (2.1) one would not find this answer, if one takes the standard decoupling limit \( M_{\text{pl}} \rightarrow \infty \) with both \( K = K^{(1)} + K^{(2)} \) and \( W = M_{\text{pl}}^{-3} W^{(1)} W^{(2)} \) fixed.\(^6\) Instead, the product structure of the superpotential introduces a cross-coupling between sectors,

\[ V_{\text{eff}} = \frac{1}{M^3_{\text{pl}}} \left( |W^{(2)}|^2 |\partial_\alpha W^{(1)}|^2 + |W^{(1)}|^2 |\partial_\alpha W^{(2)}|^2 \right) \neq V_{\text{susy}}, \]  

(3.2)

whose behavior under the limit \( M_{\text{pl}} \rightarrow \infty \) is best examined at the level of the superpotential.

\(^5\)We expect this condition to hold around a supersymmetry breaking vacuum with almost vanishing cosmological constant. It also holds in many models of supergravity inflation, although a notable exception is [30, 31].

\(^6\)Strictly speaking the decoupling limit sends \( M_{\text{pl}} \rightarrow \infty \) while keeping the fields \( \phi, z \) fixed with \( W^{(i)}/M^3_{\text{pl}} \) a holomorphic function of \( \phi/M_{\text{pl}} \) or \( z/M_{\text{pl}} \) and \( K^{(i)}/M^2_{\text{pl}} \) a real function of \( \phi/M_{\text{pl}}, \bar{\phi}/M_{\text{pl}} \) or \( z/M_{\text{pl}}, \bar{z}/M_{\text{pl}} \). The limit zooms in to the origin so \( K \) must be assumed to be non-singular there. Formally the decoupling limit does not exist otherwise. Physically it means that one is taking the decoupling limit w.r.t. an a priori determined ground state, around which \( K \) and \( W \) are expanded. If \( K \) is non-singular at the origin, the overall factor \( e^{K/M^2_{\text{pl}}} \) yields an overall constant as \( M_{\text{pl}} \rightarrow \infty \), which may be set to unity, i.e. the constant part of \( K \) vanishes. In the decoupling limit, both \( K \) and \( W \) may then be written as polynomials. Letting the coefficients in \( W \) and \( K \) scale as their canonical scaling dimension such that \( W \) has mass dimension three and \( K \) has mass dimension two, then gives the rule of thumb that both \( K \) and \( W \) are held fixed as \( M_{\text{pl}} \rightarrow \infty \).
Supergravity is sensitive to the expectation value \( W_0 = \langle W \rangle \) of \( W \), which relates the scale of supersymmetry breaking to the expectation value of the potential, i.e. the cosmological constant

\[
\Lambda^2 M^2_{pl} = \langle V \rangle \sim \langle DW^2 \rangle - \frac{3}{M^2_{pl}} \langle W^2 \rangle = m^4_{\text{susy}} - 3 \frac{W^2_0}{M^2_{pl}}. \tag{3.3}
\]

The vacuum expectation value cannot vanish in a supersymmetry breaking vacuum with (nearly) zero cosmological constant, such as our Universe. Therefore, in the following we assume \( \langle W \rangle \neq 0 \) in the region of interest. Instead of the usual way to incorporate it, \( W_{\text{sugra}} = W_0 + W_{\text{dyn}} \) with \( W_{\text{dyn}} = W_{\text{susy}} + \ldots \), we include the vacuum expectation value for a two-sector product superpotential by writing

\[
W(\phi, z) = \frac{1}{M^3_{pl}} W^{(1)} W^{(2)} = \frac{1}{M^3_{pl}} \left( W^{(1)}_0 + W^{(1)}_{\text{dyn}}(\phi) \right) \left( W^{(2)}_0 + W^{(2)}_{\text{dyn}}(z) \right) \\
= \frac{1}{M^3_{pl}} \left( W^{(1)}_0 W^{(2)}_0 + W^{(1)}_0 W^{(2)}_{\text{dyn}}(\phi) + W^{(1)}_{\text{dyn}}(\phi) W^{(2)}_{\text{dyn}}(z) \right). \tag{3.4}
\]

This is physically equivalent to a sum of superpotentials except for the last term. Note again, that if one uses the standard scaling, \( \frac{\phi}{M_{pl}} \rightarrow 0; \frac{z}{M_{pl}} \rightarrow 0 \) with all couplings in \( W^{(\text{total})} \) having the canonical scaling dimensions, this last term contains renormalizable couplings involving the scalar partner of the goldstino, and these are not Planck-suppressed: if supersymmetry is broken by the \( \phi \) sector, terms of the form \( \phi z^2 \) are renormalizable and would survive the \( M_{pl} \rightarrow \infty \) limit, leading to a direct coupling between the two sectors.\(^7\) If both sectors break supersymmetry then mass-mixing terms \( \phi z \) also survive. All such (relevant) terms are of course absent if none of the two sectors break supersymmetry, but this is not the case we are interested in. One would have expected that these cross-couplings naturally vanish in the decoupling limit.

The point of this note is simply to remark that the realization that each of the superpotentials \( W^{(j)} = W^{(j)}_0 + W^{(j)}_{\text{dyn}} \) contains a constant term can resolve this conundrum by assuming a non-standard scaling for the constituent parts \( W^{(j)}_0, W^{(j)}_{\text{dyn}} \). To achieve a decoupling we need that the cross term \( W^{(1)}_{\text{dyn}} W^{(2)}_{\text{dyn}} \), which contains the coupling between the two sectors, scales away in the limit \( M_{pl} \rightarrow \infty \). As a result the first term in (3.4) has to diverge, because its product with the cross term should remain finite. In particular we can choose an overall scaling

\[
W = \frac{1}{M^3_{pl}} \left( \frac{W^{(1)}_0 W^{(2)}_0}{\sim M^{3+r}_{pl}} + \frac{W^{(1)}_0 W^{(2)}_{\text{dyn}}}{\sim M^{3}_{pl}} + \frac{W^{(2)}_0 W^{(1)}_{\text{dyn}}}{\sim M^{3}_{pl}} + \frac{W^{(1)}_{\text{dyn}} W^{(2)}_{\text{dyn}}}{\sim M^{3-r}_{pl}} \right), \tag{3.5}
\]

with \( r > 0 \). Let us account for dimensions by introducing an extra scale \( m_A \) such that

\[
W^{(1)}_0 = m_A^{3-r} A M^{3+r}_{pl} + A, \quad W^{(1)}_{\text{dyn}} = M^3_{pl} \frac{W^{(1)}_{\text{susy}}}{W^{(2)}_0}, \quad W^{(1)}_{\text{dyn}} = M^3_{pl} \frac{W^{(2)}_{\text{susy}}}{W^{(1)}_0}, \tag{3.6}
\]

\(^7\)For a product of superpotentials we can always choose a Kähler gauge at every point with \( \langle K \rangle = \langle \partial_\phi K \rangle = 0 \) without mixing the superpotentials. In that case \( F \)-term supersymmetry breaking is given by the linear terms in the expansion of \( W^{(1)} \) and \( W^{(2)} \): \( \langle D_\phi W \rangle \sim \langle \partial_\phi W^{(1)} \rangle, \langle D_\phi W \rangle \sim \langle \partial_\phi W^{(2)} \rangle \).
with $W^{(j)}_{\text{susy}}$ fixed as $M_{\text{pl}} \to \infty$. Formally one can choose an inhomogeneous scaling with $A \neq 0$, but as we shall see it has no real consequences. For any $A$ it is easily seen that with this scaling,

$$D_{\alpha} W = \partial_{\alpha} W^{(1)}_{\text{susy}} + \frac{m r^{-3}}{M_{\text{pl}}^2} W^{(2)}_{\text{susy}} \partial_{\alpha} W^{(1)}_{\text{susy}}$$

$$+ \frac{\partial_{\alpha} K^{(1)}}{M_{\text{pl}}^2} \left( m A^{3-r} M_{\text{pl}}^{3-r} + W^{(1)}_{\text{susy}} + W^{(2)}_{\text{susy}} + \frac{m r^{-3}}{M_{\text{pl}}^2} W^{(1)}_{\text{susy}} W^{(2)}_{\text{susy}} \right) \to \partial_{\alpha} W^{(1)}_{\text{susy}},$$

(3.7)

in the limit $M_{\text{pl}} \to \infty$ if and only if $0 < r < 2$ and thus

$$V_{\text{sugra}} = e^{K/M_{\text{pl}}^2} \left( |D W|^2 - \frac{3 |W|^2}{M_{\text{pl}}^2} \right) \to \sum_j |\partial_j W^{(j)}_{\text{susy}}|^2 - 3 m_{\text{susy}}^2 (3-r) M_{\text{pl}}^{2(r-1)} + \mathcal{O} \left( \frac{1}{M_{\text{pl}}} \right),$$

(3.8)

For $r < 1$ the manifestly constant term in the potential vanishes as well and we recover the strict decoupled field theory result, with the gravitino mass going to zero as $m_{3/2} = \langle W \rangle M_{\text{pl}}^{-2} = m_{\text{susy}}^2 M_{\text{pl}}^{-2} = m_{\text{susy}}^2 \sqrt{3 M_{\text{pl}}}$.\footnote{This also ensures that the mixing between the two sectors in the fermion mass matrix $m_{i\alpha} = e^{G/2} (\nabla G_{\alpha} + G_{\alpha})$ does not interfere with the decoupling, since it is proportional to the gravitino mass and thus vanishes naturally.} We see that the gravitino mass is independent of $r$ in physical scales.

The parameter $r$ should not be larger than unity for the new decoupling limit to be well defined. For the special case $r = 1$ [25], the potential has an additional overall “cosmological” constant. For a generic non-gravitational field theory in which $M_{\text{pl}} \to \infty$ this is just an overall shift of the potential, which we can arbitrarily remove since it does not change the physics. Nevertheless from a formal point of view, we know that absolute ground state energy of a globally supersymmetric theory equals zero, as a result of the supersymmetry algebra $\{Q, Q\} = H$. For this reason it is more natural to restrict the value of $r$ to the range $0 < r < 1$.

4 Concluding remarks

Let us conclude with a comment on the physical meaning behind the scaling (3.6). It may appear that we have changed the canonical RG-scaling of the theory. This is not quite true. For the interacting terms in the potential, it is the coefficients in the product $W^{(2)}_{\text{dyn}} W^{(1)}_{\text{susy}}$ that ought to obey canonical RG-scaling. This precisely corresponds to holding $W^{(j)}_{\text{susy}}$ fixed as $M_{\text{pl}} \to \infty$ (see footnote 6). On the other hand, the scaling of the constant term in the potential has changed from its canonical value. However, this is very natural in a supersymmetric theory. The constant term, $\prod_j W^{(j)}_{0}$, equals the ground state energy. Precisely supersymmetric theories can “naturally” explain non-canonical scaling of the cosmological constant (at the loop level; the scaling of the bare ground state energy can be different in every model). A non-integer power is strange but $r = 1$ is certainly a viable option in a supersymmetry-breaking ground state: it is the natural scaling in theories with higher supersymmetry [34] when combined with a subleading log($M_{\text{pl}}/m_{\text{susy}}$) breaking. Our engineering analysis only focuses on power-law scaling and these can always have subleading logarithms. 
(\(r = 2\) would correspond to the cosmological constant for a spontaneously broken \(\mathcal{N} = 1\) theory due to mass splitting).

Finally, the novel scaling in (3.6) can be readily generalized to an arbitrary number of sectors. For \(s\) sectors, writing \(W^{(j)} = W^{(j)}_0 + W^{(j)}_{\text{dyn}}\),

\[
W = \frac{1}{M_{\text{pl}}^{3(s-1)}} \prod_{j=1}^{s} W^{(j)}
\]

\[
= \frac{1}{M_{\text{pl}}^{2(s-1)}} \left[ \prod_{j=1}^{s} W^{(j)}_0 + \sum_{k=1}^{s} \left( W^{(k)}_{\text{dyn}} \prod_{j \neq k}^{s} W^{(j)}_0 \right) + \sum_{l>k}^{s} \left( W^{(k)}_{\text{dyn}} W^{(l)}_{\text{dyn}} \prod_{j \neq k,l}^{s} W^{(j)}_0 \right) + \ldots \right] .
\]

(4.1)

We want the last and all further terms to scale away as \(M_{\text{pl}}^{-r}\) and higher with \(r > 0\), while the second term(s) should be constant. As a consequence the first term will scale as \(M_{\text{pl}}^r\). Assuming a scaling that is homogeneous across sectors, this implies

\[
W^{(j)}_0 \sim M_{\text{pl}}^{3(s-1)+r}, \quad W^{(j)}_{\text{dyn}} \sim M_{\text{pl}}^{(3-r)(s-1)},
\]

(4.2)

for each of the \(j \in \{1, \ldots, s\}\). With this scaling, a general term consisting of \(l\) dynamical superpotentials and \(s-l\) constant parts, scales as

\[
\frac{W^{(j)}_0}{M_{\text{pl}}^{3(s-1)}} \sim M_{\text{pl}}^{r(1-l)} ,
\]

(4.3)

and as constructed any term containing dynamical interactions between sectors, \(l > 2\), is Planck-suppressed. To ensure a vanishing constant term as in eq. (3.8), \(r\) is again limited to the range \(0 < r < 1\).

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