Simulation of Fatigue Crack Growth in Weld Joint of Thin Structures Using Characteristic Tensor

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Abstract. It is known that fatigue life of welded structures, such as automotive and ship structures, is greatly influenced by welding residual stress. In case of real structures, the orientation of the stresses produced by static load and that produced by repeated load and that of welding residual stress are generally different from each other and they form a highly complex multi axial stress states. On the other hand, the fatigue behaviour has been mostly studied for uniaxial stress state without residual stress. The crack growth rate is related to the stress intensity factor range $\Delta K$ such as in the Paris Law [1]. To predict the fatigue behaviour of real welded structures, the three dimensional stresses produced by static load, repeated load and welding must be rationally correlated with a scalar value $\Delta K$. The author proposes a Characteristic Tensor which characterizes the singular stress field at crack tip and its invariant parameters, such as principal values or invariants which has similar form as strain energy density, are used to compute $\Delta K$ in a consistent manner. Its potential usefulness is demonstrated through simple examples.

1. Introduction
Fatigue strength of weld joint is much lower than that of the base metal because of stress concentration and welding residual stress. Deterioration of material due to the welding thermal cycle is also a reason. Due to the dramatic progress in numerical simulation and computer technologies, both the stress concentration and the welding residual stress can be computed even for a large component of welded structures [2]. The purpose of present research is to develop a simple method to simulate crack growth in structural component under cyclic load by utilizing these information obtained by Finite Element Method. One of the problems we face is how to estimate the Range of stress intensity factor $\Delta K$ for a crack with arbitrary shape under multiaxial stress state. Most of the existing methods are proposed to obtain $\Delta K$ with high accuracy but their applications are limited to simple geometry and loading conditions. By sacrificing the accuracy to some extent, a simple method to compute $\Delta K$ of complex welded component can be developed. For this, Characteristic tensor $\chi$ which characterizes the singular stress field around crack tip is introduced. Further, invariants (scalar which is independent on coordinate system) of the Characteristic tensor are chosen as Generalized equivalent stress range $\Delta \sigma^*$ which is proportional to the Range of stress intensity factor $\Delta K$. Once $\Delta K$ is computed from $\Delta \sigma^*$, fatigue crack growth under cyclic load can be computed. After presenting the framework of the proposed method using the analogy to the theory of plasticity, how to determine the proportional constant between $\Delta \sigma^*$ and $\Delta K$ and how to determine the direction of crack growth are discussed. Potential capability of the proposed Characteristic Tensor Method (CTM) is demonstrated through examples.
2. Characteristic Tensor Method

2.1. Analogy to Theory of Plasticity

The problem we face when we attempt to simulate fatigue crack growth in welded structures is that there are three stresses we need to take into account, namely stress due to dead load, that varies with applied load and the welding residual stress. This means that the crack growth under fatigue load is the result of very complex multiaxial stress state which involves 18 stress components. On the other hand, the information available from experiment is limited mostly to uniaxial stress state with varying stress ratio [3]. To overcome this gap, concept of Characteristic tensor is proposed based on the analogy between the Theory of Plasticity and the crack growth problem as shown in Table 1. In case of practical elastic-plastic problems, the stress state is multiaxial with 6 components. Since the plastic phenomenon is not strongly influenced by mean stress, deviatoric stress is extracted as a dominant part of the stress tensor. Since the phenomenon is independent on the coordinate system, invariants such as principal value of the deviatoric stress tensor or energy is used to define the equivalent stress as in the case of Tresca’s yield criterion and Mises’ yield criterion, respectively. The equivalent stress which is independent on coordinate system connects the multiaxial stress state in real structures and uniaxial stress state in experiments. Based on the same idea, fatigue tests conducted under uniaxial stress state can be generalized to multiaxial stress state in real structures. In case of crack growth problem, stress field at crack tip is singular when material is elastic and the singularity of the stress state can be extracted as the Characteristic tensor $\chi_{ij}$.

Using average stress $\mu_{ij}$ taken over the volume $V$ around the crack tip, i.e.

$$\mu_{ij} = \int (\sigma_{ij} + \sigma_{R_{ij}} + \Delta \sigma_{ij}) dV / V$$

the Characteristic tensor $\chi_{ij}$ is defined as

$$\chi_{ij} = \mu_{ij} \sqrt{R} \lim R \to 0$$

where, $R$ is the radius of the volume $V$. According to its definition, $\chi_{ij}$ have finite values and characterize the singular stress field around the crack tip. Theoretically, the components of Characteristic tensor $\chi_{ij}$ have the same dimension as the Stress intensity factors $K_1, K_2, K_3$ and linearly related to them.
In case of numerical analysis such as the Finite Element Method, radius $R$ with finite value is used. In this sense, we can calculate only approximate values of the Characteristic tensor $\chi_i$.

Assuming that the Stress intensity factor $K$ is linearly related to Characteristic tensor $\chi_i$, i.e.

$$K = \alpha_i \chi_i$$

(3)

The Range of stress intensity factor $\Delta K$ when stress changes from $(\sigma_{0ij} + \sigma_{Rij})$ to $(\sigma_{0ij} + \sigma_{Rij} + \Delta \sigma_{ij})$ can be defined as,

$$\Delta K = \alpha_i \sqrt{R} \int \left[ (\sigma_{0ij} + \sigma_{Rij} + \Delta \sigma_{ij}) - (\sigma_{0ij} + \sigma_{Rij}) \right] dV / V = \alpha_i \sqrt{R} \int \Delta \sigma_{ij} dV / V$$

(4)

where $\sigma_{0ij}$, $\Delta \sigma_{ij}$ and $\sigma_{Rij}$ are stresses produced by dead load and repeated applied load and residual stress, respectively. As it is readily seen from Eq. (4), the effect of initial stress $(\sigma_{0ij} + \sigma_{Rij})$, or the effect of stress ratio, is not contained in $\Delta K$ when linear relation between $K$ and $\chi_i$ is assumed. One way to solve this problem is to define $\Delta K$ through strain energy release rate $G$.

Let’s assume that Strain energy release rate $G$ and Stress intensity factor $K$ satisfy the following relation.

$$E G = \frac{K^2}{2}$$

(5)

When initial stress $(\sigma_0 + \sigma_r)$ is zero, $\Delta K$ can be conceptually defined as,

$$\Delta K^2 = E \Delta G = E(\text{Energy/Area}) \propto E \left( \frac{1}{2} \sigma^2 V / A \right) \propto \frac{1}{2} \Delta \sigma^2 L = \frac{1}{2} E \Delta \sigma^* \Delta \varepsilon^* = EW^*$$

(6)

where, $\Delta \sigma^* = \Delta \sigma \sqrt{L}$, $\Delta \varepsilon^* = (\Delta \sigma \sqrt{L}) / E$, $W^* = \frac{1}{2} \Delta \sigma^* \Delta \varepsilon^*$

(7)

Since $\Delta \sigma^*$ and $\Delta \varepsilon^*$ have the dimension $<\text{stress}/\text{length}>$ and $<\sqrt{\text{length}>}$, they are named as Generalized stress and strain, respectively. In the same way $W^*$ is called as Generalized strain energy density. It can be shown as the area of the triangle in Fig. 1.

When the initial stress $(\sigma_0 + \sigma_r)$ is not zero, the following relation can be derived.

$$\Delta K^2 \propto \frac{1}{2} \left[ (\sigma_0 + \sigma_r + \Delta \sigma)^2 - (\sigma_0 + \sigma_r)^2 \right] L = \frac{1}{2} \left( \Delta \sigma \right)^2 + 2(\sigma_0 + \sigma_r) \Delta \sigma L$$

$$= \frac{1}{2} E \left( 2 \sigma^* + 2 \sigma_r + \Delta \sigma^* \right) \Delta \varepsilon^* = EW^*$$

(8)

In this case, the Generalized strain energy density $W^*$ becomes the area of the trapezoid shown in Fig. 2. In this way, the Range of stress intensity factor $\Delta K$ can be explicitly related to the initial stress $(\sigma_0 + \sigma_r)$ and the repeated applied stress $\Delta \sigma$.

2.2. Generalization to Multiaxial Stress State

The discussion up to now is for the uniaxial stress state. The idea is generalized to multiaxial stress state through Generalized equivalent stress range $\Delta \bar{\sigma}$ which has the same dimension as $\Delta K$, i.e.

$$\Delta K^2 \propto \frac{1}{2} \left( \Delta \sigma^* \right)^2 + 2(\sigma_0^* + \sigma_r^*) \Delta \sigma^* = \frac{1}{2} (\Delta \bar{\sigma})^2$$

(9)

or,
Further, it can be extended to multiaxial stress state as follows.

\[
\Delta \sigma^* = \left\{ 2 \int_0^1 \phi(\xi) \left( \sigma_{ij}^* + \sigma_{ij}^* \xi \Delta \sigma_{ij}^* \right) d\xi \right\}^{1/2}
\]

where, \( C_{ijkl} \) and \( \sigma_m^* \) are elastic compliance and mean stress. The factor \( \phi(\xi) \) is introduced to take it into account that the energy does not contribute as the driving force for the crack growth when the mean stress is compressive.

2.3. Proportional Constant
As it is discussed, the Range of stress intensity factor \( \Delta K_1 \) and Generalized equivalent stress range \( \Delta \sigma^* \) are linearly related, i.e.

\[
\Delta K_1 = \alpha \Delta \sigma^*
\]

To determine the proportional constant \( \alpha \), simple problem such as a thin plate with a through thickness centre crack can be used.

2.4. Direction of Crack Growth
To simulate the growth of fatigue crack in multiaxial stress state, the direction of crack growth must be determined. In the proposed Characteristic Tensor Method, growth of the crack is simulated by removing element. Although the Range of stress intensity factor \( \Delta K_1 \) can be defined only at the crack tip, it is computed for all elements which are facing the crack surface assuming that the crack tip locates at that element. Using the computed \( \Delta K_1 \) and the crack growth property of the material, the crack growth rate \( da / dN \) and the crack length in the element are computed assuming Paris’ law [1]. If the crack length in the element reaches the size of the element, the element is removed. In this way, the crack grows in the most preferable direction can be achieved. Figure 3 shows growth of a crack in a square plate uniformly loaded on its boundaries. Uniaxial loading is assumed in Case (a). In Case (b) and (c), combination of axial and shear loads are applied. The broken line in the figure represents the principal direction of the applied stress. As it is observed from Fig. 3, the direction of the crack growth in simulation agree fairly well with that of the principal stress.
3. Application to lap joint of thin plates

Example is the fatigue crack growth in lap weld joint as shown in Fig.4. Two sheets of steel plate with 2 mm thickness is assumed to be joined by welding and initial crack with 0.2 mm depth is placed at the weld toe in the middle of the width. Fatigue crack growth under cyclic load with average stress of 63 MPa.

(a) axial=100 MPa                (b) axial=100 MPa, shear=100 MPa   (c) axial=50 MPa, shear=100 MPa

Fig. 3 Fatigue crack growth in square plate under uniaxial and multiaxial stress state.

Fig. 4 Lap weld joint model
Fig. 5 Distribution of stress
Fig. 6 Distribution of welding Residual stress.

Fig. 7 Crack growth with beach mark.
Fig. 8 Influence of welding residual stress on fatigue crack growth.
MPa is computed. Fig. 5 shows the distribution of the stress in the loading direction produced by the load on the surface and the middle cross-section. In the same way, distribution of the welding residual stress is shown in Fig. 6. To trace the crack growth from small size to fairly large size, fine and coarse FE meshes are used. The crack planes with beach mark are shown for both cases with and without welding residual stress in Fig. 7. The growth of crack in the width direction is compared between cases with and without residual stress. As it is seen from Fig. 8, the speed of crack growth becomes four times faster when welding residual stress exists.

4. Conclusions
To simulate the crack growth in components of welded structures such as automotive components under cyclic load and to predict their fatigue life, a simple finite element method called Characteristic Tensor Method (CTM) is developed. For this, a concepts of Characteristic tensor and Generalized equivalent stress range are proposed. Characteristic tensor is defined based on the average of the stress tensor around the crack tip and it is linearly related to the Stress intensity factors. Generalized equivalent stress range is introduced to connect multiaxial stress state in real structures to uniaxial stress state in experiments. The advantages of the proposed method are,

(1) Its simplicity.
(2) Coarse FE mesh can be used.
(3) The effect of initial stress including residual stress is in cooperated in computation of the Range of stress intensity factor in a consistent manner.
(4) The direction of crack growth can be determined in a straightforward manner.

Potential capability of the proposed Characteristic Tensor Method and the influence of residual stress on growth of fatigue crack are demonstrated through examples.

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