Research on Path Optimization Problem Based on Satisfaction Degree in Fuzzy Demand Environment

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Abstract. Aiming at the vehicle routing problem with fuzzy demand, this paper takes the fuzzy demand of the customer as the starting point, and first defines the demand satisfaction function; takes the maximum satisfaction and the minimum cost as the goal, constructs the fuzzy demand path optimization multi-objective planning model (MORG-FD), and the characteristics of MORG-FD are analyzed; combined with the actual case of a dairy product distribution for simulation analysis, considering the changes in the distribution plan under different demand parameters, and compared with the commonly used methods, the results show that MORG-FD has good structural characteristics, strong applicability, can provide a way of thinking for solving the problem of fuzzy demand vehicle routing, and can also provide guidance for actual distribution route planning.

1. Introduction
The Vehicle Routing Problem (VRP) is a very important problem in the logistics distribution system. Since Dantzing [1] raised this question in 1959, many scholars have conducted in-depth research on it. It also derives VRP problems with time windows, decomposable requirements, and open-ended issues. In these questions, the demand, distribution center and time window are all deterministic assumptions, but in reality, some information appears to be vague, for example, the demand of a certain customer is about 60 units, and the driving time of the vehicle from point A to point B is about 20~30min, etc. The resulting fuzzy VRP is mainly reflected in fuzzy demand VRP and fuzzy time VRP. Therefore, when the performance of customer information is fuzzy, the common solution is to quantify the fuzzy information and transform it into certain information to participate in the model construction and solution.

Common methods for quantifying fuzzy information include constructing fuzzy chance constraint models based on fuzzy credibility theory [2-5]. There are also Literature [6-8] constructing fuzzy expected value models based on fuzzy credibility theory. The genetic hybrid particle swarm algorithm, genetic tabu search algorithm and tabu particle swarm algorithm are used to solve this problem.

Combining customer satisfaction with VRP with fuzzy time windows is also an effective method to deal with fuzzy VRP [9-10]. Brito [11] and others designed a hybrid ant colony algorithm to solve this problem using fuzzy linear programming. [12-13] respectively proposed how to achieve a win-win solution between cost and satisfaction when facing fuzzy path problems. And to solve this problem were established multi-site and multi-warehouse.

In summary, fuzzy VRP is different from deterministic VRP that has been studied more maturely. Although the literature [2-5] has some research on fuzzy demand VRP, when faced with fuzzy demand, customer demand satisfaction has not been effectively resolved. Therefore, based on the existing VRP,
this article treats customer needs as fuzzy variables, and based on the degree of satisfaction, takes the shortest total driving distance, minimizes the distribution cost and maximizes the average customer satisfaction as the goal, constructs the MORG-FD model, and Simulate analysis based on actual cases.

2. Problem description and model assumptions

2.1. Problem description
This article describes the vehicle routing problem with fuzzy requirements as follows: A distribution Customer location is clear but needs are vague. A vehicle can serve multiple customers, but each customer can only be served once. Delivery vehicles beginning and end points are distribution centers. Find a reasonable vehicle scheduling plan to maximize customer satisfaction and minimize corporate distribution costs.

2.2. Model assumptions and symbol description
Before establishing the model, make the following basic assumptions: 1) All delivery vehicles are of the same model, and the delivery vehicles are fully loaded from the delivery center; 2) In the process of transportation, vehicle accidents, road congestion, weather conditions and other factors are not considered; 3) The capacity of the distribution center and the number of available vehicles can meet customer needs, and multiple vehicles can be dispatched at the same time. Symbols and variable descriptions are shown in Table 1:

| Symbol and variable description                                      |
|---------------------------------------------------------------------|
| $V = \{0,1,2,\ldots,N\}$                                          |
| 0 means distribution, $\{1,2,\ldots,N\}$ means customer point collection |
| $R = \{1,2,\ldots,K\}$                                              |
| Delivery vehicle collection                                         |
| $D_i$                                                               |
| Customer $i$’s needs                                               |
| $Q$                                                                 |
| Maximum load of the vehicle                                        |
| $C$                                                                 |
| Fixed cost per unit of vehicle                                      |
| $a$                                                                 |
| Transport cost per unit distance                                    |
| $d_{ij}$                                                            |
| Distance from node $i$ to $j$                                       |
| $f_i(q_i)$                                                          |
| Demand satisfaction function                                        |
| $\theta$                                                            |
| Demand satisfaction rate                                            |

2.3. Fuzzy demand variables
When the information of customer I is fuzzy, the demand of customer I is expressed by fuzzy number, customer satisfaction is the satisfaction degree of customer demand, which is defined as the demand satisfaction function. If the customer demand is about 60 units, the customer demand is expressed by fuzzy number $[40,55,65,80]$. In $[40,55]$, $[65,80]$ satisfaction decreases gradually, and in $[55,65]$ satisfaction is 1. The satisfiability function and function diagram are shown in equation 1 and figure 1 respectively.

$$f_i(t_i) = \begin{cases} 
q_i - s_i, & s_i < q_i < s_i; \\
1, & 1, s_i \leq q_i \leq e_i; \\
E_i - q_i, & e_i < q_i < E_i; \\
0, & others;
\end{cases} \quad (1)$$

Figure 1 Demand Satisfaction Function Diagram
3. MORG-FD model

Based on the above analysis, the following MORG-FD model is established:

\[
\begin{align*}
\min \ Z_2 &= C \sum_{j=1}^{N} \sum_{i=1}^{N} q_{ij} + \sum_{i=1}^{N} \sum_{j=1}^{K} a_{ij} x_{ij} \\
\max \ Z_2 &= \frac{1}{N} \sum_{i=1}^{N} f_i(t_i) \\
\sum_{j=1}^{N} \sum_{i=1}^{N} d_{ijk} x_{ijk} &= Q, i \neq j, k \in \{1, \ldots, K\} \\
\sum_{k=1}^{K} \sum_{j=1}^{N} x_{ijk} &\leq K \\
\sum_{k=1}^{K} \sum_{j=1}^{N} x_{ijk} &= 1, j \in \{1, \ldots, N\} \\
\sum_{i=1}^{N} \sum_{j=1}^{N} x_{ijk} &= 1, i \in \{1, \ldots, N\} \\
\sum_{k=1}^{K} \sum_{j=1}^{N} x_{ijk} &\leq 1, i, j, k \in \{1, \ldots, N\} \\
\sum_{j=1}^{N} \sum_{i=1}^{N} \sum_{p=j}^{N} x_{ijk} &\leq 1, i, j, k \in \{1, \ldots, N\} \\
\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{K} x_{ijk} &\leq 1, i, j, k \in \{1, \ldots, N\} \\
x_{ijk} &= 0 \text{ or } 1, i \neq j \\
\forall i \in \{0, 1, \ldots, N\}, \forall j \in \{0, 1, \ldots, N\}, \forall k \in \{1, \ldots, K\}
\end{align*}
\]

In the above expression, the objective function equation (2) is the minimum transportation cost; equation (3) represents the maximum average customer satisfaction, and the individual customer satisfaction is solved by equation (1); Equations (4)-(7) are constraints; Equation (4) is the constraint of vehicle loading capacity; Equation (5) shows that the number of delivery vehicles does not exceed \(K\); Equations (6) and (7) represent that each customer has and can only have one vehicle; Equation (8) means that the vehicle must leave the customer point after reaching the customer point; Equation (9) means that only one vehicle for each route departs from the distribution center and returns to the distribution center; Equation (10) represents the minimum service level constraint to ensure that the satisfaction of each customer is not less than \(\theta\); Equation (11) indicates that the demand of a single customer does not exceed the maximum load capacity of the delivery vehicle; Equation (12) indicates that the decision variable is 0-1 variable.

The model in this paper adds the objective function of average satisfaction degree on the basis of literature (9), and combines fuzzy demand with path optimization through satisfaction degree. Therefore, through the relationship between satisfaction degree and demand, the binary function \(f(x, y)\) is constructed. It must satisfy that \(x\) increases with increasing \(f(x, y)\), and \(y\) decreases with increasing \(f(x, y)\). For example, \(f(x, y) = \omega_1 x + \omega_2 y^{-1}\), therefore,

\[
f = \omega_1 \left( \frac{1}{N} \sum_{i=1}^{N} f_i (t_i) \right) + \omega_2 \left( C \sum_{j=1}^{N} \sum_{i=1}^{K} x_{ijk} + \sum_{i=1}^{N} \sum_{j=1}^{K} \sum_{k=1}^{N} a_{ij} x_{ijk} \right)^{-1}
\]
4. Experiment results
Suppose a company uses 10 vehicles to deliver to 10 customers, and each customer demand is represented by a fuzzy number. The maximum capacity of this vehicle is 180 boxes, the fixed cost $c_0$ per vehicle is 150 yuan, and the transportation cost per unit distance $a$ is 3 yuan. The characteristic data of each node is shown in Table 2.

| Node | Demand range |
|------|--------------|
| 0    | (40,55,65,80) |
| 1    | (35,50,60,75) |
| 2    | (60,75,85,100) |
| 3    | (20,35,40,55) |
| 4    | (55,70,80,95) |
| 5    | (95,110,120,135) |
| 6    | (45,60,70,85) |
| 7    | (25,40,50,65) |
| 8    | (20,35,45,60) |
| 9    | (30,45,55,70) |

4.1. Solution based on MORG-FD model
In the delivery process, due to the influence of human subjective factors, each customer has different requirements for delivery satisfaction, and each customer satisfaction is independent of each other. Therefore, this article focuses on constraint (10), in (0,1) Set different values of $\theta$ between. The solution results are shown in Table 3:

| $\theta$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Function value | 0.4360 | 0.4359 | 0.4354 | 0.4353 | 0.4353 | 0.4352 | 0.4314 | 0.4349 | 0.4348 |
| Satisfaction | 0.464 | 0.568 | 0.664 | 0.744 | 0.772 | 0.842 | 0.868 | 0.918 | 0.986 |
| Cost | 499.4 | 525.5 | 695.2 | 786.2 | 796.1 | 806.3 | 889.5 | 1013.2 | 1214.7 |

Solve simulation experiments based on MORG-FD model. In this paper, the value of $\theta$ is 0.7 and $\alpha_1, \alpha_2$ is 0.5. Finally, it is solved that the optimal delivery route is three, the distribution plan of each vehicle is shown in Table 4.

| Tuck | Route | Actual volume | satisfaction | cost |
|------|-------|---------------|--------------|------|
| 1    | 0-7-8-9-1-0 | 0-60-36-35-50-0 | 0-1-0.84-1-0.8-0 | 318.3 |
| 2    | 0-2-4-10-5-0 | 0-46-28-39-68-0 | 0-0.84-0.72-0.76-0.92-0 | 305.4 |
| 3    | 0-3-6-0 | 0-71-109-0 | 0-0.84-0.96-0 | 265.8 |

4.2. Solution based on the method of literature (9)
In literature 9, the method to solve this problem is traditional fuzzy simulation. For example, the customer demand is about 50 boxes, then the actual delivery volume is 50 boxes. The delivery vehicle is fully loaded and departs from the delivery center until the delivery vehicle does not meet the ordering requirements of the next delivery point, then the vehicle returns to the delivery center for replenishment. After obtaining the demand of each customer in this way, the genetic algorithm with the same parameters of the solution method in this paper is adopted, and the simulation experiment is carried out by Matlab. 4 delivery routes are obtained, and a total of 4 delivery vehicles are required. The experiment shows 4 delivery routes the distribution plan is shown in Table 5.
4.3. Scanning method
This method uses polar coordinates to indicate the location of each demand point, with the distribution center as the starting point, the angle is set to zero, and the service area is divided in clockwise or counter clockwise direction, with vehicle capacity as the restriction condition. There are four distribution routes solved by the scanning method. The distribution plan of each vehicle is shown in Table 6.

| Truck | Route        | Actual volume  | Satisfaction | Cost   |
|-------|--------------|----------------|--------------|--------|
| 1     | 0-1-10-8-0   | 0-60-47-45-0   | 0-1-1-1-0     | 322.5  |
| 2     | 0-4-3-0      | 0-32-80-0      | 0-1-1-0       | 280.8  |
| 3     | 0-2-6-0      | 0-55-115-0     | 0-1-1-0       | 284.4  |
| 4     | 0-5-7-9-0    | 0-75-65-38-0   | 0-1-1-1-0     | 302.7  |

4.4. Result analysis
The solution in this paper focuses on combining the degree of satisfaction of customer needs with the cost of enterprise distribution when facing the vague customer needs. First of all, from the aspect of total driving distance, the total driving distance of this proposal is reduced by about 50km compared with the reference (9). Secondly, in terms of the average customer satisfaction rate and vehicle usage, the average customer satisfaction in literature (9) is the best, but the solution in this paper uses one less delivery vehicle than the solution in literature (9). Then, in terms of distribution costs, this article reduces about 300 yuan. Finally, in the literature (9), during the distribution process, it may happen that the distribution vehicle is fully loaded and starts from the distribution center, and the distribution vehicle may pull the remaining goods back to the distribution center, resulting in a waste of capacity. The solution in this paper will comprehensively consider the customer satisfaction and distribution circumstance, avoid this situation from happening. The results of the scheme are shown in Table 7:

|     | value | Vehicles | Delivery distance(km) | Satisfaction | %    |
|-----|-------|----------|-----------------------|--------------|------|
| MORG-FD model | 0.7   | 0.431    | 3                     | 889.5        | 146.4 | 86.80 |
| Literature (9)      | 1     | 0.395    | 4                     | 1190.4       | 196.8 | 100.00 |
| Scanning method     | 0.330 | 4        | 4                     | 1181.7       | 193.9 | 65.99 |

By setting different satisfaction parameters a, observe the changes in demand satisfaction and cost. It can be seen from Table 3 above that for a given lower value of a, the cost of the enterprise has been reduced, and the satisfaction rate of the customer's demand has also been reduced. The satisfaction rate of demand is low for a long time, which is easy to cause the loss of corporate customers. With the continuous increase of a, the satisfaction rate of customers with demand increases, but the cost of the enterprise is also increasing. Blindly increasing the satisfaction rate of customer demand will easily cause the enterprise the high cost may put the company in a state of long-term loss and hinder the long-term development of the company.

5. Conclusion
Aiming at the vehicle routing problem with fuzzy demand, this paper aims at minimizing total driving distance and maximizing average customer satisfaction, and constructs a satisfaction-based MORG-FD model, which fully considers the fuzzy information in reality, and can effectively solve the problem, which can provide a theoretical basis for the optimization of the distribution path of the logistics enterprise. When defining the customer’s delivery volume-demand satisfaction function, the fuzzy
interval is selected by direct definition, and the influence of the fuzzy interval on satisfaction and delivery results is not considered. It can be considered as the next research direction.

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