Constraints on Majoron Models, Neutrino Masses and Baryogenesis

James M. Cline
Kimmo Kainulainen
and
Keith A. Olive

School of Physics and Astronomy, University of Minnesota
Minneapolis, MN 55455, USA

Abstract

We derive strong contraints on the Yukawa couplings and the vacuum expectation value in the singlet majoron model, taking into account the possibility of a small gravitationally induced mass for the majoron. If the present baryon asymmetry was created earlier than at the electroweak phase transition, then to preserve it from the combined effects of lepton number violating interactions and electroweak sphalerons, we find that stringent constraints on the Yukawa couplings, $h < 10^{-7}$, and the majoron scale, $v_s < v_{EW}$, must be satisfied. We also carefully rederive baryogenesis bounds on neutrino masses, finding that in general they...
apply not to the masses themselves, but only to related parameters, and they are numerically somewhat less stringent than has previously been claimed.
1 Introduction

There is an ever increasing interest in the possibility for neutrino masses. Aside from the intrinsic interest in new and perhaps tangible physics, neutrino masses have been shown to be quite useful. A neutrino with a small mass, $O(10 \text{ eV})$, could make a significant contribution to the overall energy density of the Universe. Other potential benefits from neutrino masses include MSW neutrino mixing relating to the solar neutrino problem [1], and the possibility of exploiting lepton number violating interactions in conjunction with baryon and lepton number violation in the electroweak sector [2] to generate a baryon asymmetry of the Universe [3].

The very small ratio of expected neutrino masses, relative to other particle masses, made the idea of the seesaw mechanism quite attractive [4]. Numerous models incorporate a neutrino mass seesaw, including unification via SO(10) [5], or simply adding a single complex singlet to the standard model which breaks a global $U(1)_{B-L}$ symmetry, i.e., the singlet majoron model [6]. In both of these models lepton number is spontaneously broken. In a GUT, $(B-L)$-violating effects will generally be small because the breaking scale is generically $M_{\text{GUT}}$. In the majoron model, there are a priori few restrictions on the breaking scale, which might be close to the electroweak scale.

However the possibility of lepton number violation at or above the electroweak scale is potentially dangerous, as was first discussed by Fukugita and Yanagida [7] in the context of a simple model with explicit lepton number violation. The danger occurs when $L$-violating interactions are in equilibrium in the early Universe at the same time that $(B+L)$-violating nonperturbative electroweak interactions, due to sphalerons, are in equilibrium. In this case the baryon asymmetry of the Universe is destroyed independently of the initial $B-L$ asymmetry. This realization has led to a series of contraints on models with additional sources of baryon or lepton number violation [7]-[13]. These bounds are of course evaded if the baryon asymmetry is ultimately produced during or after the electroweak phase transition.

The baryon wash-out bounds also directly apply to the majoron model if the lepton symmetry breaking occurs above the electroweak scale. They take on an especially interesting twist however, when combined with the “expectation” of gravitational corrections to the
singlet sector. Recalling the well-known adage that Nature abhors global symmetries, it is quite reasonable to expect nonrenormalizable contributions to the low energy Lagrangian due to gravitational corrections that violate nongauged symmetries [14]. Although such terms explicitly break global symmetries, their effects are strongly felt only above some cutoff, in this case the Planck scale. Such effects were previously considered in the triplet majoron model [15] and for axions [16]. Nonrenormalizable gravitationally-induced interactions which explicitly break the global $B - L$ symmetry were recently studied in the context of the singlet majoron model [17, 18]. In [17] it was shown that these effects generate a mass $\mathcal{O}(1-2)$ keV or larger for the majoron and that the cosmological consequences of such a massive majoron imply a stringent upper bound on the vacuum expectation value of the singlet field in the broken phase.

In this paper we derive some of the baryon wash-out bounds in detail. In particular we extend the bound coming from decays and inverse decays [11] of heavy neutrinos to the case of spontaneously broken symmetry and find an explicit expression for this bound in terms of the light neutrino seesaw masses, the neutrino mixing matrix and the initial lepton asymmetries. We will then show how the bound on the singlet vacuum expectation value may be further strengthened when combined with the $(B + L)$-violating nonperturbative electroweak effects.

2 The Model

The singlet majoron model extends the Standard Model by merely adding a single complex scalar field, $\sigma$, and a two-component fermion field $\nu_R$, both SU(2)×U(1) singlets. The new interaction Lagrangian terms may be written as

$$hH\bar{L}\nu_R + h_2\sigma\bar{\nu}_R\nu^c_R + h.c. + V(\sigma, H); \quad (1)$$

$$V(\sigma, H) = \lambda_s \left(|\sigma|^2 - v_s^2/2\right)^2 + \lambda_H \left(|H|^2 - v^2/2\right)^2 + \lambda_{sh}(|\sigma|^2 - v_s^2/2)(|H|^2 - v^2/2), \quad (2)$$

where we allow for self-interactions of the scalar singlet as well as interactions with the Standard Model Higgs doublet, $H$. $L$ is the usual left-handed lepton doublet, with generation
indices suppressed. When $\sigma$ acquires a vacuum expectation value we can write

$$\sigma = \frac{1}{\sqrt{2}}(v_s + \rho + i\chi). \quad (3)$$

Upon symmetry breaking, $\nu_R$ acquires a Majorana mass, $M = \sqrt{2}h_2v_s$ and a Dirac mass with $\nu_L$, $m = hv/\sqrt{2}$ where $v/\sqrt{2} = 174$ GeV is the VEV of $H$. For $M \gg m$ the low energy mass eigenstates are determined by the seesaw relations $[4]$ $N = \nu_R + \nu_R^c + (m/M)(\nu_L^c + \nu_L)$ and $\nu = \nu_L + \nu_L^c - (m/M)(\nu_R^c + \nu_R)$ with $m_N \simeq M$ and $m_\nu \simeq m^2/M$. The field $\rho$ has mass $m_\rho \sim v_s$ for self-couplings of order 1, as is often assumed, and the massless majoron is $\chi$.

Because this model introduces only new singlet fields, it is only weakly constrained. If any of the light neutrinos are heavier than $\sim 100$ eV, they must decay fast enough so that neither they nor their decay products overclose the Universe $[19]$. Furthermore, they must decay fast enough to allow for sufficiently long late-time matter domination, so that primordial density fluctuations can grow to form galaxies $[20]$. The observed neutrino pulse from SN 1987A leads to a constraint on the VEV, $v_s$ for neutrinos in the mass range from 100 eV to $\sim 30$ MeV $[21]$. A massless majoron would also conflict with bounds on light particle degrees of freedom $[22]$ unless it decoupled early enough $[23]$. For the very small neutrino masses we will argue for below, the majoron was always sufficiently decoupled.

### 3 The Gravitationally Induced Bound

The gravitationally-induced interactions for the singlet majoron model, considered in $[17]$, were assumed to have the generic form

$$\lambda \frac{\sigma^5}{M_P} \quad (4)$$

with the consequence that for $v_s > v$, a majoron mass

$$m_\chi \simeq \sqrt{20\lambda} \left(\frac{v_s}{v}\right)^{3/2} \text{keV} \quad (5)$$

is generated. Similarly, although we will not be interested in this case, when $v_s < v$, a majoron mass of at least $O(\text{keV})$ will also be produced. We anticipate that $\lambda \sim 1$ (there is
no reason to assume otherwise), but will nevertheless keep all \( \lambda \)-dependence explicit in our results.

An immediate consequence of majorons with a mass in the keV range is that they would overclose the universe, if they were stable. To see this, one must estimate their relic abundance, \( Y = n_\chi / n_\gamma \), given by the ratio of particle species in equilibrium today to the number at the time they decoupled. Above the phase transition where \( \sigma \) gets its VEV, all the particles are massless, so that majorons are in equilibrium at least until the critical temperature \( T_c \), which is of order \( v_s \). If this occurs above the weak scale, but below the threshold for new SUSY or GUT particles, then the spectrum of particles in equilibrium will just be all those of the Standard Model, resulting in the dilution of the majoron number density by a factor 

\[ Y \simeq \left( \frac{1}{27} \right) [22]. \]

This dilution is sufficient to satisfy the constraints from big bang nucleosynthesis [24] since the the majoron at the time of nucleosynthesis would only contribute the equivalent of \( \sim 0.03 \) neutrinos [22, 23], well below the bound 0.3. However, the present density of majorons becomes

\[ \rho_\chi = \frac{1}{2} m_\chi Y n_\gamma \]  \hspace{1cm} (6)

which leads to the constraint

\[ m_\chi < 340 \, \text{eV} \left( \frac{1}{27Y} \right) \left( \frac{\Omega h^2}{0.25} \right) \]  \hspace{1cm} (7)

and which for \( \lambda \gtrsim 0.01 (v/v_s)^3 \) is incompatible with the estimate in (3) [17]. If the majorons are kept in equilibrium to lower temperatures, for example through the processes \( N \leftrightarrow \chi \nu \) or \( \nu \nu \leftrightarrow \chi \chi \), \( Y \) will be larger and the conflict will only be exacerbated. Note that the bound cannot be evaded by assuming the majoron decoupled at a higher temperature (resulting in a smaller value of \( Y \)), since the majoron mass grows faster as a function of \( v_s \sim T_c \) than does \( Y^{-1} \). Thus majorons must decay. But not only must they avoid overclosing the universe, they must decay early enough to allow for the growth of density fluctuations [20].

The recent measurements by COBE [25] of the quadrupole moment in the spectrum of primordial density fluctuations, which is consistent with a flat spectrum and an amplitude

\[ \frac{\delta \rho}{\rho} \simeq 5 \times 10^{-6}, \]

indicates the need for a substantial period of matter domination to allow for the growth of these perturbations in order to form galaxies. Because of the majoron mass
scale, $m_\chi Y \gtrsim 0.1$ keV, galaxy size perturbations will go nonlinear only if the last epoch of matter domination is sufficiently long \[20, 26\]. With such a small value of $\frac{\delta \rho}{\rho}$ it is necessary to assume that majorons never dominate the overall density. This implies that \[26\]

$$\frac{m_\chi Y}{T_D} \lesssim 9.0$$

(8)

where $T_D$ is the majoron decay temperature. Relating $T_D$ to the majoron lifetime gives the limit

$$\left(\frac{m_\chi Y}{1 \text{ keV}}\right) \left(\frac{\tau_D}{1 \text{ sec}}\right)^{1/2} \lesssim 1 \times 10^4.$$  

(9)

Then using

$$\tau_D^{-1} = \Gamma_D = \frac{m_\chi}{16\pi} \left(\frac{m_\nu}{v_s}\right)^2$$

(10)

one can obtain the bound \[17\] on $v_s$,

$$v_s \lesssim 20 \text{ TeV} \left(\frac{m_\nu}{25 \text{ eV}}\right)^{4/7} \left(\frac{1}{27Y}\right)^{4/7} \lambda^{-1/7}$$

(11)

which is interesting in and of itself (note the weak dependence on $\lambda$), as long as $m_\nu \lesssim 25$ eV. In fact the latter condition must be satisfied, because $m_\nu$ here refers to the heaviest neutrino which can result from the decay $\chi \rightarrow \nu \nu$. Such a neutrino, if heavier than the cosmological bound of 25 eV, would itself have to decay into three lighter neutrinos, $\nu \rightarrow 3 \nu'$, with a majoron in the intermediate state. However the $\nu \nu' \chi$ coupling is highly suppressed \[27\]; at low energies the Yukawa interactions of the singlet fields have the form

$$\frac{h_2}{\sqrt{2}} \chi \left(\bar{N} X N - 2 \frac{m}{M} \bar{N} X \nu + \frac{m^2}{M^2} \bar{\nu} X \nu + O \left(\frac{m^4}{M^4}\right) \bar{\nu} X \nu'\right),$$

(12)

where $X = \rho + i \gamma_5 \chi$. Because the rate is suppressed by the tiny factor $(m/M)^{12} = (m_\nu/M)^6$, this decay mode is far too slow to vitiate the 25 eV bound on $m_\nu$.

4 Sphaleron Induced Bounds

We now turn to the baryon wash-out constraints. Any model with lepton number violation faces the danger of baryon number wash-out as a result of the combination of its own
lepton number violating interactions and the \((B + L)\)-violating sphaleron interactions in the Standard Model. This was first noticed by Fukugita and Yanagida \[7\], who showed that the presence of a lepton (or baryon) number violating operator such as \(\frac{h}{M}\nu\nu HH\) will erase any primordial baryon asymmetry if these interactions are in equilibrium simultaneously with sphaleron interactions. The condition that these interactions be out of equilibrium translates into a bound on the neutrino mass \[7, 8, 9\]

\[
m_\nu \lesssim 50 \text{ keV} \left(\frac{100 \text{ GeV}}{T_{BL}}\right)^{1/2} \tag{13}
\]

where \(T_{BL}\) is the temperature at which a \(B - L\) asymmetry was produced or \(\sim 10^{12} \text{ GeV}\) (the maximum temperature for which sphaleron interactions are in equilibrium), whichever is smaller. This type of bound has been extended to general baryon and lepton number violating operators in \[10\] where some general constraints on the majoron model were also considered.

Another, complementary bound on neutrino masses can be obtained considering the lepton number violating decays and inverse decays \(N \leftrightarrow HL\), that inevitably accompany any nonrenormalizable operator such as indicated above. The requirement that these processes are out of equilibrium has been shown to imply \[11\]

\[
m_\nu \lesssim 10^{-3} \text{ eV} \tag{14}
\]

if the \(B - L\) asymmetry is generated above \(T \simeq M\). Remember that both (13) and (14) can be evaded if the baryon asymmetry is ultimately produced during or after the electroweak transition.

Without the bound (11) on the singlet majoron model, it would be natural to assume that \(v_s\) and \(M\) are large. For \(T_{BL} < M \sim v_s\), the lepton-violating decays and inverse decays would have gone out of equilibrium before baryogenesis, evading the bound (14). Then one would be left with only the weaker constraint coming from the dimension-5 operator (13). Until a nonvanishing neutrino mass is measured, this puts no restriction on the scale of baryogenesis. But with the restriction (11) the story changes dramatically, and the window of opportunity for producing the \(B - L\) asymmetry becomes very narrow. With (14) the
bound from decays and inverse decays is the only one available as the dimension-5 operators go out of equilibrium at $T \sim 10$ TeV, for $m_{\nu} \simeq 25$ eV, and cannot be used at $T > v_s$.

The original derivation of the bound (14) was incomplete in the sense that the authors did not solve, nor even formulate the Boltzmann equations for the asymmetries. Because we are studying a theory with spontaneous symmetry breaking, where the lepton-violating decays leading to (14) only exist below the critical temperature $T_c$ at which $v_s$ starts to develop, and since we are finding that $v_s$ must be near the weak scale, it behooves us to reexamine in greater detail under what conditions the validity of (14) is generally assured.

Our basic assumption is that all the other processes proceed fast compared to those that violate lepton number, so that at each instant of time an equilibrium between various chemical potentials is attained. The evolution of all asymmetries is then controlled by the evolution of the individual lepton asymmetries due to the decay processes $N \leftrightarrow HL$.

It is convenient to choose the mass eigenstates of the right-handed sector as the two helicity states of the previously defined Majorana field $N = \nu_R + \nu_R^c$, which does not mix with the light states until temperatures below the weak scale. Above the critical temperature of the phase transition in the singlet sector these are precisely the eigenstates of the conserved right-handed neutrino lepton number. Moreover it is convenient to work in the basis in which the Majorana mass matrix is diagonal. Since we are interested in temperatures above the weak breaking scale, all the other fermions remain massless. As the weak symmetry eventually gets broken, the light neutrinos gain a seesaw mass matrix whose diagonalized form is

$$m_{\nu} = U^T m^T mU,$$  \hspace{1cm} (15)

where $M$ is the heavy Majorana mass matrix, $m$ is the Dirac mass matrix

$$m \equiv h v / \sqrt{2},$$  \hspace{1cm} (16)

and $U$ is the neutrino mixing matrix that relates the flavor and mass eigenstates.

The Boltzmann equation for the evolution of the $\nu_{\ell}$ number density, due to the lepton number violating decays is given by
\[
(\partial_t - 3H)n_{\nu} = \int \prod_{i=1}^{3} \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4(p_1 - p_2 - p_3) \sum_j |M|_j^2 
\times f_h(p_2, \mu_h)f_\nu(p_3, \mu_\ell)e^{\beta E_i} 
\times \left\{ f_{j+}(p_1, \mu_j)(e^{-\beta \mu_j} - e^{\beta(-\mu_h - \mu_h)}) 
+ f_{j-}(p_1, -\mu_j)(e^{\beta \mu_j} - e^{\beta(-\mu_h - \mu_h)}) \right\}
\]

where \( \ell \) is the generation index. The first term in the curly brackets in (18) comes from \( N_{j+} \) and the second from \( N_{j-} \) decays, with the index \( j \) running over all heavy neutrino species and \( H \) is the Hubble expansion rate of the Universe. The maximal lepton number nonconservation below \( T_c \) in the decays \( N_\pm \leftrightarrow H\nu_L, H^*\nu_L^c \) is reflected in the fact that the helicity states \( N_\pm \) decay with equal total rate to both allowed final states with opposite lepton number. Indeed, the matrix element for the decay of \( N_{j\lambda} \) is given by

\[
|M|_j^2 = |h_{j\ell}|^2 M_j(p_j \cdot p_\nu \pm M_j s_{j\lambda} \cdot p_\nu),
\]

where \( s_{j\lambda} \) is the helicity four-vector of \( N_j \) and \((-+)\) refers to the \( H\nu_L^c (H^*\nu_L) \) final state. The various decay channels differ by the spin dependent part in (18), but it is easy to see that this contribution identically vanishes upon the phase space integration.

Subtracting the Boltzmann equation for the number density of \( \nu_L^c \)'s from that of \( \nu_L \)'s and linearizing in the chemical potentials, one finds

\[
(\partial_t - 3H)(n_{\nu_L} - n_{\nu_L^c}) = -4 \frac{\mu_L + \mu_h}{T} \sum_i h_i^2 \frac{M_i(M_i^2 + m_{\nu_L}^2 - m_{\nu_L^c}^2)}{64\pi^3} h_{i\ell} I_{i\ell}.
\]

The function \( I_{i\ell} \) can be evaluated exactly to yield

\[
I_{i\ell} = \int_1^{\infty} du \frac{e^{ux}}{(1 + e^{ux})^2} \ln \left( \frac{\cosh(\alpha_{i\ell} u + \gamma \sqrt{u^2 - 1})}{\cosh(\alpha_h u - \gamma \sqrt{u^2 - 1})} \right) 
\times \frac{\sinh(\alpha_{i\ell} u + \gamma \sqrt{u^2 - 1})}{\sinh(\alpha_h u - \gamma \sqrt{u^2 - 1})},
\]

where \( \alpha_{i\ell} \equiv (M_i^2 + m_{\nu_L}^2 - m_{\nu_L^c}^2)/4M_i T, \alpha_h \equiv (M_i^2 + m_h^2 - m_{\nu_L}^2)/4M_i T \) and \( \gamma \equiv \lambda^{1/2}(M_i^2, m_{\nu_L}, m_{\nu_L^c})/4M_i T \), with \( \lambda(x, y, z) \equiv (x - y - z)^2 - 4yz \). This is a generalization of the inverse-decay
rate used in [29]. Note that the $N_i$ chemical potentials do not appear in the equation (19). This is again due to maximal lepton number violation in the decays; should there be a difference in the decay rates, the $N_i$ asymmetries would be nontrivially coupled to $\nu_\ell$’s. In the present case the evolution of the $N_i$ asymmetry decouples and the $N_i$ asymmetries are driven to zero independently of (and faster than) the asymmetries of the $\nu_\ell$’s. A subtlety in the finite temperature calculation of the decay rate that deserves to be mentioned is that the temperature dependent masses enter only through the kinematics and the dispersion relation. In particular, the spinor functions must be taken identical to the vacuum spinors, but with a modified dispersion relation [28]. In all our calculations we have used the approximate dispersion relation $E = \sqrt{p^2 + m^2(T)}$ where $m(T)$ is the appropriate temperature dependent mass.

Equation (20) can be greatly simplified if one substitutes the Maxwell-Boltzmann approximation for the phase space distributions. In general this is not legitimate, because then, for small $M_i/T$, the enhancement factor due to the singularity in the boson distribution function at $E_h/T \approx 0$ would be ignored. In practice however, this singularity is shielded by the temperature dependent mass of the final state particles and the MB approximation becomes effectively very accurate. We then find the approximate expression

$$I_{i\ell} \simeq \frac{\lambda^{1/2}(M_i^2, m_\ell^2, m_h^2)}{M_i^2} \cdot K_1 \left( \frac{M_i}{T} \right),$$

where $K_1(x)$ is the modified Bessel function. This result coincides with (20) to within few percent over the whole range of interest, and becomes exact for large $M_i/T$.

The Higgs chemical potential $\mu_h$ can be solved in terms of the leptonic chemical potentials from the equilibrium conditions imposed by the fast weak processes. Following [8], and using $\mu_\ell = \mu_{\nu_\ell}$ due to fast weak interactions, we find

$$\mu_h = \frac{4}{21} \sum_{\ell=1}^3 \mu_\ell.$$  \hspace{1cm} (22)

Inserting this result into the equation (19) and expressing the chemical potentials in terms of the asymmetries in the comoving volume, we obtain

$$L_{\nu_\ell} \equiv \frac{n_{\nu_\ell} - n_{\nu_\ell}^c}{s} \simeq \frac{15}{4\pi^2 g_\ast} \frac{\mu_\ell}{T},$$ \hspace{1cm} (23)
where \( s \) is the entropy density and \( g_* \) is the number of relativistic degrees of freedom. We find the following coupled set of equations for the lepton asymmetries

\[
\frac{dL_\ell}{dt} = -(L_\ell + \frac{4}{21} \sum_\nu L_\nu) \Gamma_\ell, \tag{24}
\]

with

\[
\Gamma_\ell \equiv \frac{24}{T^3} \sum_i |h_{\ell}\rangle^2 T \frac{(M_i^2 + m_\ell^2 - m_h^2)}{64\pi^3 M_i} \chi^{1/2}(M_i^2, m_\ell^2, m_h^2) K_1 \left( \frac{M_i}{T} \right). \tag{25}
\]

Eq. (24) is valid as such for both neutrino and left handed charged lepton asymmetries, which is why we replaced the subscript \( \nu \) with \( \ell \) in (24) and (25). One should note that (24) allows for a nontrivial equilibrium solution (some of the \( L_\ell \)'s nonzero) only if at least one of the rates \( \Gamma_\ell \) vanishes, or is small enough [13]. In the opposite case all \( L_\ell \)'s and hence all the other asymmetries as well are driven to zero with a rate controlled by (24). The baryon asymmetry is related to the sum of the individual neutrino or left handed charged lepton asymmetries by

\[
B(t) = -\frac{4}{3} \sum L_\ell(t). \tag{26}
\]

Given the initial asymmetries and the various couplings that appear in the expressions for \( \Gamma_\ell \), one could compute numerically the baryon asymmetry as a function of time from equation (24). For the purpose of obtaining a bound on the neutrino mass, we can make the following, most conservative, simplifying assumption about the parameters involved in (24): “the initial asymmetry corresponding to the smallest of rates \( \Gamma_\ell \) is the largest.” One should bear in mind that the fast electroweak processes enforce the initial conditions for the slow processes such that all initial asymmetries are of the order as \( L_\ell \)'s. The faster decaying asymmetries in eq. (24) rapidly reach their fixed point values and it can be integrated exactly for the slowest one to yield:

\[
L_\ell(t) = L_\ell(t_0) \exp \left( -\frac{33}{29} \int_{t_0}^t dt \Gamma_\ell(t) \right). \tag{27}
\]

The bound on the neutrino mass arises from the requirement that the present baryon asymmetry exceeds the known lower limit of \( B_{min} \simeq 4 \times 10^{-11} \), which in terms of the lepton asymmetries becomes

\[
\left| \sum_\ell L_\ell(t_{EW}) \right| > \frac{4}{3} B_{min}, \tag{28}
\]

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where the time $t_{EW}$ refers to the time of electroweak phase transition. With our conservative assumption we then obtain the bound

$$
\left( \int_{t_0}^{t_{EW}} dt \Gamma_\ell \right)_{\text{min}} \lesssim \frac{29}{33} \ln \left( \frac{L_0^0}{4 \cdot 10^{-11}} \right) \simeq 17,
$$

(29)

using in the last step a reasonable maximum value of $L_{\ell_{\text{max}}}^0 \sim \epsilon n_\gamma/s \lesssim 10^{-2}$ for the initial asymmetry, where $\epsilon$ is some generic CP-asymmetry parameter \[29\]. Up to now our discussion has been quite general and eq. (29) is valid for either explicit or spontaneous symmetry breaking. Assuming that the temperature dependent mass of the $N_i$ is given by

$$
M_i^2(T) = M_0^2(1 - (T/T_c)^2),
$$

(30)

we can write the integral appearing in (29) in the spontaneous symmetry breaking case as

$$
\int_{t_c}^{t_{EW}} dt \Gamma_\ell \simeq 284 \sum_i m_{\ell i}^* \frac{1}{M_i} m_{i\ell} \times I
$$

(31)

where

$$
I \equiv \int_{z_{EW}}^{z_{c}} dz \frac{z^4}{\sqrt{z^2 + (M_{\text{max}}/T_c)^2}} f(z) K_1(z),
$$

(32)

with $z \equiv M_i(T)/T$, $f(z) = (1 + r^2 - r_h^2)\lambda_{1/2}(1, r_1^2, r_h^2)\theta(1 - r_\ell - r_h)$ and $r_\alpha \equiv m_\alpha(T)/M_i(T)$.

The explicit symmetry breaking case can be obtained by taking the limit $t_c \rightarrow 0$, $T_c \rightarrow \infty$.

In the singlet majoron model the various temperature dependent masses that we use in our calculations are given by

$$
M_i^2(T) \equiv 2h_{2i}^2 v_s^2(T) ; \quad v_s^2(T) = v_s^2(1 - T^2/T_c^2)
$$

$$
m_{\ell i}^2(T) = \frac{1}{32}(3g^2 + g') T^2
$$

$$
m_h^2(T) = \frac{1}{16}(3g^2 + g'^2 + 4h_t^2 + 8\lambda_H + \frac{4}{3}\lambda_{sH}) T^2 - \lambda_H v^2 - \frac{1}{2}\lambda_{sH}(v_s^2 - v_{\ell}^2(T)),
$$

(33)

where $g$ and $g'$ are the usual weak and $U(1)_Y$ couplings, $h_\ell$ is the top quark Yukawa coupling, $\lambda_H$, $\lambda_s$, $\lambda_{sH}$, $v$ and $v_s$ are the quartic scalar couplings and VEV’s defined in the equation (2) and $T_c$ is the $(B - L)$-symmetry breaking temperature given by

$$
T_c = v_s \left( \frac{6\lambda_s + 3\lambda_{sH}(v/v_s)^2}{2\lambda_s + \lambda_{sH} + h_t^2} \right)^{1/2},
$$

(34)

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where $\vec{h}^2 \equiv \sum_j h_{2j}^2$. The expression for $m_h(T)$ in (34) assumes that for temperatures $T_c > T > T_{EW}$, the field is located in the temperature dependent asymmetric minimum in the singlet direction with the electroweak symmetry still unbroken, $\langle |\sigma| \rangle = v_s(T)/2$ and $\langle |H| \rangle = 0$. We have computed the integral $I$ for a wide range of parameters and find that it is essentially constant $I \simeq 4$, for $M_i > m_h$. The constant behaviour arises because the integrand $\sim z^3 K_1(z)$ peaks for rather large values of $z \simeq 3$ so that the effects of final state masses and the $M_0/T_c$ dependence are strongly suppressed. Thus the final bound arising from (29) is essentially the same for either spontaneous or explicit breaking. With a constant $I$ the LHS of the equation (29) becomes proportional to the quantity $(m^\dagger M^{-1} m)_{\ell \ell}$.

Our main result now comes from combining (28) and (29), using the constant value of $I$ just determined:

$$
\min_{\ell} (m^\dagger M^{-1} m)_{\ell \ell} < 8 \times 10^{-4} \ln \left( \frac{L_0}{4 \cdot 10^{-11}} \right) \text{ eV} \simeq 10^{-2} \text{ eV},
$$

where again, the maximum value for $L_0$ was used in the last step. Notice that the careful treatment leading to eq. (35) has yielded a somewhat less restrictive bound on the scale of the seesaw neutrino masses, $m^2/M$, than previous investigations. But another interesting feature of (35) is that, unlike previous derivations, it does not give any direct bound on the masses of the neutrinos, because $m^\dagger M^{-1} m$ is different from the mass matrix $m^T M^{-1} m$. For example, consider just two generations and assume that $M^{-1/2}$ is proportional to the Pauli matrices $\sigma_1 + \sigma_2$. Then $m^\dagger M^{-1} m$ is diagonal and nonzero, but the mass matrix vanishes identically, and thus the bound (35) is not directly related to the masses. Such a situation could arise either due to fine-tuning of the mass matrices or (approximate) lepton flavor symmetries. Otherwise one generically expects $m^\dagger M^{-1} m$ to be related to the masses and mixings via

$$
(m^\dagger M^{-1} m)_{\ell \ell} \sim \sum_{i=1}^3 m_{\nu_i} |U_{i\ell}|^2,
$$

as can be seen by writing $M^{-1/2} = V^\dagger D U$, where $D$ is diagonal, and assuming that $V$ is either close to the identity, or has off-diagonal elements of the same order as those of $U$. Then $m^\dagger M^{-1} m = U^\dagger |D|^2 U$, whereas the mass matrix is $U^T D V^T V D U$. If $V^T V = 1 + \epsilon$, then (35) is correct to lowest order in $\epsilon$. If $\epsilon$ is large but the off-diagonal elements of $U$ are
also large, then (36) will again be approximately true, since it is dominated by the largest eigenvalue of $D$, that is, the heaviest neutrino mass, regardless of the generation index $\ell$.

If either of the reasonable assumptions leading to eq. (36) holds, we can combine the baryon-preservation bound (35) with the majoron decay bound (11) to obtain what is potentially a much more stringent limit on the majoron scale than was given previously in eq. (11). Eq. (35) tells us that the mass of the heaviest neutrino $\nu_h$ into which the majoron can decay should satisfy

$$m_{\nu_h} < 10^{-2} U_{h\ell}^{-2} \text{eV},$$

where $\ell$ is the neutrino flavor whose conservation is most weakly violated. Then (11) becomes

$$v_s \lesssim \max \left(200 \text{GeV}(27Y)^{-4/7} U_{h\ell}^{-8/7} \lambda^{-1/7}, v \right).$$

(37)

Note that we cannot obtain a bound for $v_s < v$, since sphaleron interactions will have decoupled by then.

A further application of our bound is a constraint on the Yukawa couplings of the right-handed neutrinos to the Higgs doublet, about which no information has hitherto been available. Rewriting the mass matrices in terms of Yukawa coupling matrices and using the constraint (37) on $v_s$ we obtain

$$\min_\ell \sum_i \frac{|h_{i\ell}|^2}{h_{2i}} < 6 \cdot 10^{-14} \max \left(0.8(27Y)^{-4/7} U_{h\ell}^{-8/7} \lambda^{-1/7}, 1 \right).$$

(38)

Even if the gravitationally induced effects (parametrized by $\lambda$) or the mixing $U_{h\ell}$ are very small, this is quite a strong result, for it says that either all the Yukawa couplings $h_{i\ell}$ are even smaller than that of the electron or there exist large hierarchies within $h_{i\ell}$ with some of the elements being extremely small; $h_{i\ell} \lesssim 10^{-7}$.

The dramatic results (37,38) apply in the case of generic mass matrices for the neutrinos, but in model building it is often interesting to consider the nongeneric case where some combination of flavors $\tilde{L}$ is approximately conserved, for example $\tilde{L} = L_e - L_\mu + L_\tau$. If $\tilde{L}$ is only weakly violated, then the decay vertices we have considered have only very slow $\tilde{L}$-violating channels, since the heavy decaying neutrinos $N$ have pseudo-Dirac masses of order $M$ (which almost conserve $\tilde{L}$) rather than Majorana masses that would violate $\tilde{L}$ maximally. The decays are therefore predominantly of the form $N \to H\nu$ and $\overline{N} \to H^*\bar{\nu}$. The fastest $\tilde{L}$
violating process are $N$-$\overline{N}$ oscillations coming from the small Majorana mass terms $\mu$ [30], followed by the $\bar{L}$-conserving decays. These are faster than any oscillations occurring in the light neutrino sector because the mass splittings, hence the oscillation rate, are much larger for the heavy neutrinos. To be exact, we would have to write the Boltzmann equation also for the asymmetry in $N$ and combine it with that for $\nu$ in order to track the asymmetry in $\bar{L}$. But roughly, we can account for this effect by multiplying the previously computed decay probability by the probability of an oscillation, given by

$$P = \sin^2(2\theta) \sin^2 \left( \frac{\mu M}{2ET} \right)$$

(39)

where for pseudo-Dirac neutrinos the mixing is maximal ($\sin^2(2\theta) = 1$), and $\Gamma$ is the rate of interactions of $N$ which will damp the oscillations if they are much faster than the oscillation rate $\delta M^2/2E = \mu M/E$. $\Gamma$ is given by the sum of rates $\sum_{\ell} \Gamma_{\ell}$ which we computed previously, eq. (25). For very fast oscillations the second $\sin^2()$ factor averages to 1/2, but if $\mu$ is small then our bound (35) starts to disappear. The condition for (35) to be valid in the case of an approximate symmetry is therefore that $\mu M \gtrsim T\Gamma$ at temperatures of order $M$, or in terms of the dimensionless parameter $\mu/M$ that quantifies the small departures from $\bar{L}$ symmetry,

$$\frac{\mu}{M} \gtrsim \frac{1}{\pi^3} \sum_{i\ell} |h_{i\ell}|^2 \sim \frac{\sum_i m_{\nu_i}}{\pi^3(v^2/M)}.$$  

(40)

Therefore even in the extreme case of $M = 100$ GeV and $m_{\nu_3} = 10$ MeV, a very small symmetry breaking $\mu/M \sim 10^{-6}$ would be sufficient for our results to still apply.

5 Caveats

Finally, we discuss some special situations, where our bounds (37) and (38) could be avoided. One such situation is the case in which the baryon asymmetry is actually generated below $T_c$. Since the wash-out due to the decays is not effective for $T \ll M_i(T)$, the asymmetry generated at $T_{BL}$ is safe if $T_{BL} \ll T_c$. However, due to the bound (11) on $v_s$ with $T_c \sim v_s$ this should be seen as an extremely tight constraint on the scale $T_{BL}$!

Another possibility is that the baryon asymmetry is generated at some high scale, and the electroweak symmetry breaks before the $B - L$ symmetry even though $v_s > v$. If this
was the case, then there never was a period where the sphaleron interactions and the lepton number violation were in equilibrium together and the bound \((40)\) would not be valid. To see how generic this is, we must compare the critical temperature for the electroweak breaking (at the field origin) with the \((B-L)\)-symmetry breaking scale \(T_c\) \((34)\). We find

\[
T_{EW} = v \left( \frac{12\lambda + 6\lambda sH (v_s/v)^2}{6\lambda + \lambda sH + 3r} \right)^{1/2},
\]

(41)

where \(r = (2M_W^2 + M_Z^2 + 2m_t^2)/v^2\). Note that if the \(B-L\) symmetry is indeed broken first, the temperature \((41)\) is not the true electroweak breaking temperature, since the field shifts from the origin after \((B-L)\)-breaking. The parameters of the quartic couplings in the scalar potential must always satisfy the relation \(\lambda s < 4\lambda sH\) in order for \(both\) symmetries be broken; otherwise there would be no light neutrino masses at low scales. Moreover, the requirement that the one-loop corrections do not render our vacuum unstable introduces a bound on the Yukawa couplings \(h_{2i}\). This bound can be naively estimated by computing the coefficient of the \(|\sigma|^4 \ln |\sigma|\) term and requiring that this coefficient be nonnegative: 

\[
\frac{5}{4}\lambda s^2 + \frac{1}{16}\lambda sH - \sum_j h_{2j}^4 > 0.
\]

One can then show that the aforementioned constraints with \(v < v_s\) and \(T_{EW} > T_c\) would be satisfied only in a small region of parameter space, where \(\lambda s \ll \lambda sH \ll \lambda \sim O(1)\).

There are two more subtle cases where the bound \((35)\) could be weakened, related to the computation of the integral \(I\) in \((32)\). First, it was assumed that the parameter \(M_{0i}/T_c\) is not excessively large. Using the vacuum stability bound and the bound on the quartic couplings, one can show that \(M_{0i}/T_c \gtrsim \) some number \(\alpha\) would require \(\sqrt{3\alpha} (1 + (\frac{\lambda s}{\lambda})^{1/2})^{-1/2} \gtrsim h_2 \gtrsim 2\lambda s\). This inequality has solutions for \(\alpha \gtrsim 1\), i.e., only for nonperturbative values of couplings \(\lambda, \lambda s, h_2 \gtrsim O(1)\). A more severe problem arises if \(M_{0i}/T_c\) is excessively small. As noted earlier, the integral in \((32)\) gets its largest contribution from the region \(z \equiv M_{0i}(T)/T \approx 3\). For small \(M_{0i}/T_c\) this peaking occurs at temperature \(T_p \approx \sqrt{3}\). It is always possible to make \(M_{0i}\) much less than \(T_c\) merely by letting \(h_2\) become small. When \(M_{0i} < m_h\) our bound scales roughly as \(h_2^{-1}\). However, when \(v_s\) is small, a very small value of \(h_2\) is rather unattractive anyway, since the mass \(M_{0i}\) becomes very small, rendering the seesaw mechanism less natural.
6 Conclusions

While it is clear that small neutrino masses are an interesting and potentially necessary extension to the standard model, the origin of these masses remains unclear. Many of the possibilities involve lepton number violating Majorana masses, as is the case in the singlet majoron model discussed here. The combination of lepton number violation in conjunction with the $B + L$ violation in the standard model leads to the grave possibility of washing out any primordial baryon asymmetry. Thus, the survival of the baryon asymmetry enables one to constrain models such as the singlet majoron model (unless, of course, the baryon asymmetry is produced very late). We were thus able to strengthen the bounds of ref. [17].

We have shown under general circumstances that at least one neutrino mass is constrained to have a value $m_\nu \lesssim 10^{-2}$ eV and the neutrino Yukawa coupling is constrained to $h \lesssim \text{few} \times 10^{-7}$. The singlet VEV is correspondingly constrained by $v_s \lesssim v$. While the majoron model is still viable with a low value of $v_s$, models with a large VEV must be carefully tuned so as to avoid the constraints discussed here as well as in [17].

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