A DYNAMICAL ANALYSIS OF THE 47 URSAE MAJORIS PLANETARY SYSTEM

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ABSTRACT

Thirteen years of Doppler velocity measurements have revealed the presence of two planets orbiting the star 47 Ursae Majoris on low-eccentricity orbits. A two-Keplerian fit to the radial velocity data suggests that the inner planet has a period $P_b = 1089.0 \pm 2.9$ days and a nominal $\sin i = 1$ mass $M \sin i = 2.54 \, M_{\text{Jup}}$, while the outer planet has a period $P_c = 2594 \pm 90$ days and a mass $M \sin i = 0.76 \, M_{\text{Jup}}$. These mass and period ratios suggest a possible kinship to the Jupiter-Saturn pair in our own solar system. We explore the current dynamical state of this system with numerical integrations, and compare the results with analytic secular theory. We find that the planets in the system are likely participating in a secular resonance in which the difference in the longitudes of pericenter librates around zero. Alternately, it is possible that the system is in the $7:3$ mean motion resonance (in which case apsidal alignment does not occur). Using a self-consistent fitting procedure in conjunction with numerical integrations, we show that stability considerations restrict the mutual inclination between the two planets to $\sim 40^\circ$ or less, and that this result is relatively insensitive to the total mass of the two planets. We present hydrodynamical simulations which measure the torques exerted on the planets by a hypothesized external protoplanetary disk. We show that planetary migration in response to torques from the disk may have led to capture of the system into a $7:3$ mean-motion resonance, although it is unclear how the eccentricities of the planets would have been damped after capture occurred. We show that Earth-mass planets can survive for long periods in some regions of the habitable zone of the nominal coplanar system. A set of planetary accretion calculations, however, shows that it is unlikely that large terrestrial planets can form in the 47 UMa habitable zone.

Subject headings: methods: numerical — planetary systems — stars: individual (47 Ursae Majoris)

1. INTRODUCTION

Planetary systems with multiple gas giant planets in long-period circular orbits would mark the long-sought true analogs of our solar system. The first extrasolar multiple planet system was discovered around $
u$ And (Butler et al. 1999) and since then, pairs of planets have been detected around HD 168443 (Marcy et al. 2001b; Udry, Mayor, & Queloz 2001), GJ 876 (Marcy et al. 2001a), and most recently, 47 UMa (Fischer et al. 2002). Radial velocity trends among the known planet-bearing stars suggest that additional planetary companions may ultimately be found in more than half of the stars with one detected planet (Fischer et al. 2001).

When first announced in 1996, 30 radial velocity measurements suggested a single 2.5 $M_{\text{Jup}}$ planet in a circular 2.2 AU orbit around 47 UMa (Butler & Marcy 1996). As described in Fischer et al. (2002), 61 subsequent radial velocity measurements (for a total of 91 velocities) suggest that a second, less massive planet also exists. The nominal mass of this second planet is 0.76 $M_{\text{Jup}}$, its eccentricity is small, and its semimajor axis is roughly 3.7 AU. The current best-fit properties of the system (as reported in Fischer et al. 2002) are shown in Table 1.

Of all the multiple systems found to date, 47 UMa is by far the most reminiscent of our own solar system. The star 47 UMa is in mass, age, and rotational velocity to the Sun, with $M = 1.03 \, M_{\odot}$, $\tau = 7$ Gyr, and $P_{\text{rot}} = 24$ days (Noyes et al. 1984; Baliunas et al. 1995). The accumulated radial velocity measurements suggest that 47 UMa bears a startling resemblance to the Jupiter-Saturn pair in our own solar system. The orbital period ratios for both systems are quite similar (2.38:1 for 47 UMa, 2.49:1 for Jupiter-Saturn), and to within the radial velocity measurement errors, the mass ratios are identical (3.34:1 for coplanar configurations of 47 UMa versus 3.34:1 for Jupiter-Saturn). The major difference between the two systems is one of overall scale. The orbital period of 47 UMa b is less than one-fourth that of Jupiter, and the nominal mass is $\sim 2.5$ times greater. We thus expect that the planets in the 47 UMa system will experience more significant mutual perturbations than do Jupiter and Saturn, and on a shorter timescale. A dynamical analysis of the properties of the 47 UMa system constitutes the main goal of this study. In particular, we would like to know how dynamical arguments can constrain the parameter space allowed to the system by the radial velocity observations, and we would like to gain a better idea of whether the apparent kinship between the solar system and 47 UMa goes beyond a superficial resemblance.

2. NUMERICAL INTEGRATIONS OF THE 47 UMa SYSTEM

Because the outer planet in the 47 UMa system has made only two full revolutions since the beginning of the radial velocity observations, the eccentricity, $e_c$, of the outer planet is not well determined. In fact, $e_c = 0.3$ provides almost as good a fit to the radial velocity data as does $e_c = 0.005$. Furthermore, the inclinations of the orbits to the plane of the

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sky are unknown, and there are therefore many different configurations of the system that are consistent with the current radial velocity data set.

One can get an idea of which configurations are allowed for the 47 UMa planets by performing numerical integrations of the star–two-planet system, and in particular, we can determine which ranges of mass factors, mutual inclinations, and outer planet eccentricities are both consistent with the radial velocity data set and also dynamically stable. This approach has met with considerable success in constraining the properties of the multiple planet system orbiting v And (see, e.g., Laughlin & Adams 1999; Rivera & Lissauer 2000; Lissauer & Rivera 2001; Barnes & Quinn 2001; or Gozdziwski et al. 2001).

In a preliminary dynamical analysis performed by Fischer et al. (2002), it was found that during the course of an individual trial integration, the periodic maxima in the osculating eccentricity of the outer planet provide a good running diagnostic of the distance of the system from instability. Once the eccentricity of the outer planet ventures above \( e_c = 0.2 \), the destruction of the system (via the near-parabolic ejection of the outer planet) is generally imminent.\(^4\) Table 2 shows the outcome of a grid of calculations done for various coplanar configurations of the fit to the system listed in Table 1 [with the coplanar “mass factor” given by \( \sin(i) \), where \( i \) is the inclination of the system with respect to the plane of the sky], and with various values for the \( e_c \). All simulations were started at epoch JD 2,451,363.5, the moment of periastron passage for planet c. The orbital elements are considered to be astrocentric. In each case, the simulation was run for either 100 Myr, or until instability occurred (listed as “unst”). For stable configurations, we list the largest eccentricity obtained by the outer planet during the course of the simulation. It is clear that for stability \( e_c < 0.2 \), but that for small values of \( e_c \), the system planets can be up to 6 times heavier than their nominal masses, and yet still be dynamically stable. We note that astrometric limits from the Hipparcos satellite rule out mass factors greater than \( 3 \) (Fischer et al. 2002).

| Parameter                     | 47 UMa b | 47 UMa c |
|-------------------------------|----------|----------|
| \( P (\text{days}) \)       | 1089.0 (2.9) | 2594 (90) |
| \( T_P (\text{JD}) \)       | 2,453,622.9 (33.6) | 2,451,363.5 (495.3) |
| \( e \)                      | 0.061 (0.014) | 0.005 (0.115) |
| \( \omega (\deg) \)        | 171.8 (15.2) | 127.0 (55.8) |
| \( K_c (\text{m s}^{-1}) \) | 49.3 (1.2) | 11.1 (1.1) |
| \( a (\text{AU}) \)         | 2.09 | 3.73 |
| \( a_i \sin(i) (\text{AU}) \) | 4.94 \( \times 10^{-3} \) | 2.64 \( \times 10^{-3} \) |
| \( f(i) (\text{AU}) \)       | 1.35 \( \times 10^{-4} \) | 3.67 \( \times 10^{-10} \) |
| \( M_b \sin(i) (M_{\text{Jup}}) \) | 2.54 | 0.76 |
| \( N_{\text{hit}} \)       | 90 | 90 |
| \( \text{rms} (\text{m s}^{-1}) \) | 7.4 | 7.4 |
| \( x^2 \)                    | 1.06 | 1.06 |

We can compare the results of the integrations with an analytic stability criterion. In the three-body problem, certain values of the energy \( E \) and angular momentum \( L \) make it impossible for close encounters to occur between the outer two bodies in the system. Hence, in the 47 UMa system, close encounters between the two planets can only occur for certain ranges of the initial orbital elements. In particular, collisions between planets are prevented if

\[
- \frac{2m_{\text{tot}}}{G^2 m_{\text{pair}}} L^2 E > S_{\text{crit}},
\]

(1)

where \( m_{\text{tot}} = m_\star + m_b + m_c \) is the total mass of the system, \( m_\star \) is the mass of the star, \( m_{\text{pair}} = m_b m_c + m_b m_e + m_c m_e \), and

\[
S_{\text{crit}} = 1 + \frac{3^{4/3} m_b m_c}{m_\star (m_b + m_c)^{4/3}} - \frac{m_b m_c (11 m_b + 7 m_c)}{3 m_\star (m_b + m_c)^2} + \cdots
\]

(2)

(Marchal & Bozis 1982; Gladman 1993).

When the planets are not in a close encounter, we can express \( E \) and \( L \) approximately in terms of the osculating orbital elements of their Keplerian orbits about 47 UMa. Equation (1) now becomes

\[
\frac{(1 + \alpha \mu)}{(1 + \mu)^3} \left[ \left( 1 - \frac{e_c^2}{2} \right) + \mu \alpha^{-1/2} \left( 1 - \frac{e_c^2}{2} \right) \right]^2 > S_{\text{crit}},
\]

(3)

for the values of \( \alpha \) appropriate to the 47 UMa system given in Table 1. Note that the left-hand side of equation (3) is independent of \( i \) at this level of approximation.

We have applied the analytic stability criterion to the numerical integrations shown in Table 2. The simulations above the ragged line are prevented from undergoing planetary close encounters (that is, they are “Hill stable”) by conservation of energy and angular momentum. The simulations below the line are not prevented from having close encounters according to the analytic criterion, although other mechanisms may maintain stability in practice.

The results of the integrations show some interesting disagreements with the analytic criterion. The threshold for instability tends to occur at lower values of \( e_c \) for increasing masses of the planets (decreasing \( \sin(i) \)). For low values of \( e_c \), some of the high-mass systems are stable despite the fact that they are not constrained to be so by the analytic criterion. It is possible that other protection mechanisms are occurring in these cases, but it is also possible that these systems are unstable on very long timescales. As a point of curiosity, we note that the system represented by the table entry having \( \sin(i) = 1.75 \), \( e_c = 0.1 \) develops instability within several million years. This point in the phase space of initial conditions is part of a narrow gully of instability that eats into the table when \( \sin(i) \approx 1.75 \).

The systems with high \( e_c \) are also intriguing. Some of these systems are unstable despite the fact that the stabil-
It is also worthwhile to point out that the integrations in Table 2 do not represent the last word on the stability of the system. The orbital elements in Table 1 all have associated uncertainties, and if alternate initial conditions are drawn from the 1 σ distributions associated with the parameters, then the stability boundary is subject to change, sometimes with extraordinary sensitivity. Furthermore, the development of instability is a statistical rather than a deterministic process. If stability is observed over a particular interval for a particular simulation, it does not prove that the configuration is unconditionally stable. Numerical experiments of the sort shown in Table 2 must therefore be viewed from a qualitative rather than quantitative standpoint.

Figure 1 shows three sets of numerical integrations. Two are of nominal coplanar configurations of the 47 UMa system with $e_c = 0.005$ and 0.15, respectively. The third integration is of the Jupiter-Saturn pair in our own solar system. The time evolution of the orbital eccentricities and relative longitude of periastron $\Delta \varpi$ are indicated by the red curves in the figure. In each system, the eccentricities of the planets oscillate in antiphase, as they exchange orbital angular momentum. For 47 UMa in the low-$e_c$ case, the planets lie in a secular resonance in which $\Delta \varpi = \varpi_2 - \varpi_1$ librates about zero. This is a stable configuration in which the major axes of each orbit remain roughly aligned, preventing close approaches between the planets. For Jupiter and Saturn, and for 47 UMa in the high-$e_c$ case, $\Delta \varpi$ circulates through 360°. This periodically brings the major axes of the orbits into antialignment, at which point the perturbations between the planets are strongest. In these cases shown in Figure 1, the system remains stable, but larger initial values of $e_c$ lead to instability (see Table 2).

The black curves in Figure 1 show the results of applying classical Laplace-Lagrange secular theory to the same systems. In this theory, the mutual planetary perturbations are expanded as a Fourier series in $e$. Short-period terms, mean-motion resonances, and terms $O(e^4)$ and higher are all neglected, leaving a set of coupled linear differential equations. Following Murray & Dermott (1999), the solution to these equations can be expressed as

$$
e_1 \sin \varpi_1 = e_{11} \sin \psi_1 + e_{12} \sin \psi_2,$$

$$
e_1 \cos \varpi_1 = e_{11} \cos \psi_1 + e_{12} \cos \psi_2,$$

$$
e_2 \sin \varpi_2 = e_{21} \sin \psi_1 + e_{22} \sin \psi_2,$$

$$
e_2 \cos \varpi_2 = e_{21} \cos \psi_1 + e_{22} \cos \psi_2,$$

with $\psi_i = g_i t + \beta_i$, where $t$ is the time and $e_{ij}$, $g_i$, and $\beta_i$ are constants determined by the masses and initial orbital elements of the planets. The subscripts 1 and 2 refer to the inner and outer planets, respectively.

The linear secular theory qualitatively reproduces the form of the evolution found in the numerical integrations. It also predicts the amplitude of the oscillations in $e$, and correctly determines whether or not the planets are librating about the secular resonance. However, the secular periods predicted by the linear theory are wrong. In the case of Jupiter and Saturn, this difference is largely

$$
\text{Throughout this paper we use the term secular resonance to mean a resonance in which the secular argument } \varpi_2 - \varpi_1 \text{ is librating. This should not be confused with the secular resonance in which two of the eigen-frequencies of the Laplace-Lagrange secular Hamiltonian coincide (Kinoshita & Nakai 2001).}
$$
due to the 5:2 near mean-motion resonance between these planets, which is neglected in the secular theory (Murray & Dermott 1999). The discovery of this near-resonance allowed Laplace (1785; see also Laskar 1996) to identify an additional 900 yr periodicity in the orbital elements of Jupiter and Saturn, which is visible in Figure 1 as a high-frequency modulation in Saturn’s eccentricity. Conversely, for the specific set of osculating orbital elements given in Table 1, the planets in the 47 UMa system are not close to any strong resonances (the nearest is the 7:3), and the shortcomings of the secular theory are probably caused by neglect of higher order terms in the masses and eccentricities.

Despite its limitations, we can use the Laplace-Lagrange theory to understand why the 47 UMa planets are in a secular resonance for small $e_c$ but not at larger values, and why Jupiter and Saturn do not lie in the same resonance. Rearranging equation (5), we get

\begin{equation}
0 = e_1 e_2 \cos \Delta \omega (e_{11} e_{21} + e_{12} e_{22}) + (e_{11} e_{22} + e_{12} e_{21}) \cos (\psi_1 - \psi_2). \tag{6}
\end{equation}

Since $\psi_1 - \psi_2$ increases linearly with time, the behavior of $\Delta \omega$ will depend on a quantity $S$, given by

\[ S = \left| \frac{e_{11} e_{22} + e_{12} e_{21}}{e_{11} e_{21} + e_{12} e_{22}} \right|. \tag{7} \]

When $S > 1$, $\Delta \omega$ can take any value and the planets lie outside the secular resonance with $\Delta \omega$ circulating. When $S < 1$, $\Delta \omega$ is restricted to a particular range of values, and the planets lie in the resonance. In the latter case, librations can take place about 0° (major axes aligned) or 180° (major axes antialigned).

Figure 2 shows the extent of the libration regions for the planets in 47 UMa, and for Jupiter and Saturn, as a function of the initial values of $\Delta \omega$ and $e$ of the outer planet. In the case of 47 UMa with low $e_c$, the planets lie deep inside the secular resonance, with $\Delta \omega$ librating about 0°. However, when $e_c$ is larger (for the same initial value of $\Delta \omega$), the system lies outside the resonance. Jupiter and Saturn also lie outside the resonance, but relatively small changes in Saturn’s eccentricity or $\Delta \omega$ would bring the planets into resonance. (The proximity to the secular resonance is apparent...}
from the form of the $\Delta \omega$ curve for Jupiter and Saturn in Fig. 1.)

Of the multiple planet systems known to date, two others (GJ 876 and ν And) are also likely to be involved in the secular resonance. In both of these systems, the libration amplitudes of the outer planets appear to be small (Laughlin & Chambers 2001; Chiang, Trembachnik, & Tremaine 2001), hinting at some sort of dissipative evolution following resonance capture. By contrast, for the particular parameter values listed in Table 1, 47 UMa is undergoing very large librations around the resonance, and Jupiter and Saturn are out of the resonance. Given the current orbital uncertainties for 47 UMa, it is not clear whether this difference is significant, or whether the existence of the possible secular resonance in 47 UMa, and the circulation in the Jupiter-Saturn system, is simply a matter of chance.

3. MUTUALLY INCLINED CONFIGURATIONS

The Doppler radial velocity technique measures only the line-of-sight velocity of the star, and is thus unable to determine the inclinations and nodes of the planetary orbits if the orbits are assumed to be Keplerian. In systems such as GJ 876 (Marcy et al. 2001a), in which the planets are strongly interacting, and where the orbital periods are short enough so that the system can be observed over many cycles, fits to the full N-body motion can potentially determine all of the orbital parameters of the system (Laughlin & Chambers 2001; Rivera & Lissauer 2001). In the 47 UMa system, however, the orbit of the outer planet has been followed with 3–5 m s$^{-1}$ precision for only a single orbital period. It is at present impossible to obtain the inclinations and nodes from the radial velocity curve itself. Numerical integrations of allowed system configurations can, however, place limits on the system.

At a given epoch, the orbit of a planet in a multiplanet system is parameterized by six orbital elements ($a$, $e$, $i$, $\omega$, $\Omega$, and $I$), along with a mass, $m$. Here, the inclination angle $i$ is understood to be with respect to the plane of the sky. For a system model containing two planets, the overall invariance of the radial velocity curve with respect to system rotations around the line of sight can be accounted for by specifying a parameter $\Omega_{12}$ corresponding to the relative difference between the lines of nodes for the two planets, yielding, for two planet systems, a total of 13 system parameters. When fitting to a radial velocity curve, a final parameter corresponding to the velocity zero point is also required.

We generate mutually inclined fits to the 47 UMa radial velocity data by randomly selecting fixed values for the inclinations, $i_1$ and $i_2$, and the angular separation of the lines of nodes, $\Omega_{12}$. The remaining 11 parameters are then obtained by using a Levenberg-Marquardt minimization scheme driving a three-body integrator to obtain a low $\chi^2$ fit to the radial velocities (as described in Laughlin & Chambers 2001). This fitting procedure automatically accounts for the dependence of $\omega_i$ on viewing geometry (see Chiang et al. 2001). Each allowed configuration is then integrated for $10^6$ yr to check stability. In Fischer et al. (2002), and in § 2 above, we found that configurations of the 47 UMa system in which the osculating eccentricity of the outer planet exceeds $e_2 = 0.2$ are generally dynamically unstable on 100 Myr timescales, whereas systems in which $e_2 < 0.2$ are generally stable.

In Figure 3, we plot the results of the orbital integrations of 150 mutually inclined fits to the radial velocity data. The three-dimensional parameter space spanned by $i_1$, $i_2$, and $\Omega_{12}$ is projected onto a two-dimensional space spanned by the mutual inclination, $\cos \Omega_{12} = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos \Omega_{12}$, and the total planetary mass, $m_1 + m_2$. This telescopes the three parameters down to two for easier interpretation. If a system reaches $e_{2,\text{max}} > 0.4$, it is plotted as a filled circle. Systems that reach $0.2 < e_{2,\text{max}} < 0.4$ are plotted as stars, and stable systems are plotted as open circles. The figure shows an interesting result that mutual inclination must be less than 45° for the system to be stable, and this result is surprisingly independent of the total mass in the system.
4. HYDRODYNAMICAL SIMULATIONS

The mass and period ratios of the 47 UMa planets are reminiscent of the Jupiter-Saturn pair in our own solar system. Nevertheless, the 47 UMa system packs at least 3.5 $M_{Jup}$ masses into a region that was within the so-called “snow line” of 47 UMa’s protoplanetary nebula, where temperatures were below 150 K, and where water ice grains would have survived evaporation at the time when planetesimals were forming. It thus seems reasonable to imagine that the 47 UMa planets formed in a colder environment, where icy materials were available to build protoplanetary cores, and then migrated inward.

The phenomenon of planet-disk interaction and migration is now becoming better understood. In early phases, when the mass of a planet is small, an orbiting planet induces a linear spiral response in its parent disk. The spiral pattern in the disk material lying inside the planet’s orbit exerts a positive torque on the planet, which causes outward migration of the planet. The spirals are now becoming better understood. In early phases, the disk material would be present interior to the orbit of the inner planet. The positive torque exerted by this inner disk is likely to cause outward migration of the inner planet, while the outer disk exerts a negative torque on the planet and causes inward migration.

If the mass of the planet, $M_{pl}$, becomes large enough so that the Roche radius,

$$R_{R} = \left(\frac{M_{pl}}{3M_{*}}\right)^{1/3} a_{pl},$$

is larger than the local scale height $h$ of the disk, then the migrating planet can open up a gap if the disk has a sufficiently low viscosity:

$$\alpha_{disk} \leq \alpha_{max} = \frac{1}{40} \frac{M_{pl}}{M_{*}} \left(\frac{a_{pl}}{h}\right)^{2}.$$

In a system where two planets have a semimajor axis ratio of $a_{2}/a_{1} \sim 2$, work by Bryden et al. (2000) and Kley (2000) shows that the ring of gas between the two planets is removed within several hundred orbital periods. Initially, disk material would be present interior to the orbit of the inner planet. The positive torque exerted by this inner disk is likely to cause outward migration of the inner planet, while the outer disk itself is gradually depleted by accretion onto the central star. This means that the nascent 47 UMa system may have experienced a phase in which the two planets were present with effectively their present masses, and were surrounded by an external disk with little disk mass interior to the inner planet. We can therefore ask: would the planets then experience interactions which would lead to migration, resonance capture, or eccentricity pumping? Can the present state of the system put interesting limits on the amount of migration that might or might not have occurred?

We have modeled a hypothetically forerunner to the 47 UMa system using a two-dimensional hydrodynamical code described by Laughlin (1994). In the simulation, two planets, with masses equal to the nominal masses in Table 1, are placed in circular orbits having an initial semimajor axis ratio, $a_{2}/a_{1} = 1.8$. The planets are allowed to interact gravitationally with an external circumstellar disk having a total mass of 1 $M_{Jup}$. Most of the mass of the circumstellar disk lies beyond the orbit of the second planet; the initial surface density profile of the disk has a split-Gaussian profile with

$$\sigma(r) = \sigma_{0} e^{-\left(\frac{r-R_{0}}{w_{i}}\right)^{2}},$$

for $r < R_{0}$, and

$$\sigma(r) = \sigma_{0} e^{-\left(\frac{r-R_{0}}{w_{o}}\right)^{2}},$$

for $r > R_{0}$, with $w_{i} = 1.0$ AU, $w_{o} = 2$ AU, and $R_{0} = 6$ AU.

The disk is modeled with a locally isothermal equation of state, in which the scale height of the disk, $h/r = 0.03$, is assumed constant. The inner and outer radial boundaries of the hydrodynamical simulation are located at $R_{in} = 1$ AU and $R_{out} = 10$ AU. The initial planetary orbital radii lie at $r = 2.0$ and $3.6$ AU. Although both planets are immersed in the disk, they are not allowed to accrete material which passes within their Roche lobes; the initial disk profile assumes that the outer planet has nearly completed the gap-opening process. The planets are given a gravitational softening length $r_{s} = 0.30$ AU in order to minimize erratic local forces arising from individual grid cells. Because of this large softening length, and because we are modeling a phase following gap clearing, material within the Roche lobes of the planets is not given special treatment. The planet and the star are integrated using a 4th order Runge-Kutta method with a time step set at 1/20 of the CFL condition, which limits the hydrodynamic time step. A kinematic viscosity of the order of $\nu = 8 \times 10^{-5}$ AU$^{2}$ yr$^{-1}$ is used. Mass that flows across the inner boundary of the hydrodynamical domain is added to the central star.

The simulation employs 256 logarithmically spaced radial zones and 256 evenly spaced azimuthal zones, and was run for 250 orbits of the inner planet. Figure 4 shows an image of the system at the end of the simulation. The system has settled down to a quasi-steady-state configuration in which the outer planet has set up a trailing spiral response in the disk. The outer Lindblad resonance of the inner planet lies in a region of low surface density, there is little coupling between the disk material and the inner planet.

The top panel in Figure 5 shows the azimuthal component of the accelerations of the two planets due to the disk material. After an initial period of readjustment, which is associated with the transient response of the equilibrium disk to the introduction of the two planets, the disk is seen to exert a constant torque on the outer planet, causing it to experience an azimuthal acceleration of $-1.2 \times 10^{-5}$ AU$^{2}$ yr$^{-1}$. This causes the outer planet to spiral in on a timescale $a_{1}/|\dot{a}| \propto 10^{5}$ yr (as shown in the second panel from the top of Fig. 5). This timescale is consistent with the analytic result that the migration timescale is the viscous timescale of the disk, and this agreement has been previously noted in the simulations of, e.g., Bryden et al. (2000) and Kley (2000).

The net azimuthal torque on the inner planet averages to zero. The inner planet thus shows very little change in semimajor axis (second panel from the bottom of Fig. 5). During this independent phase of migration, the eccentricities of both planets remain near zero. The kinematic viscosity $\nu = 8 \times 10^{-5}$ AU$^{2}$ yr$^{-1}$ used in the simulation corresponds to an “alpha” coefficient (Shakura & Sunyaev 1973) $\alpha = v\Omega/c_{s}^{2} = 0.011$ at the radial location of the outer planet. For the disk under consideration, $\alpha$ must be less than $\alpha_{max} = 0.02$ for the outer planet to be able to maintain a gap. The surface density distribution of the disk (Fig. 4) shows that the outer planet is indeed maintaining a gap in the disk, although some material is lingering in the vicinity of the stable Lagrange points L4 and L5. The larger mass of the inner planet allows it to easily maintain its gap. A low-mass ring of material between the two planets is replenished.
by gas flowing through the gap maintained by the outer planet.

The result of the hydrodynamical simulation suggests that migration of the outer planet occurs at a steady rate, and that the disk torque does not directly couple to the inner planet. We can therefore mimic the effects of the full gas-dynamical calculation by performing much less time-consuming three-body integrations in which a steady azimuthal acceleration $a_\phi = -1.2 \times 10^{-5}$ AU$^2$ yr$^{-1}$ of magnitude suggested by the simulations is applied to the outer planet. This approach has been applied by Lee & Peale (2002) to study the evolution of the GJ 876 system, which is now participating in the 2:1 resonance. Additional simulations of the GJ 876 configuration by Snellgrove, Papaloizou, & Nelson (2001) have studied resonant capture into the 2:1 resonance using both torqued three-body and hydrodynamical approaches.

Figures 6 and 7 show the result of one such simulation. Figure 6 is a surface of section in which the ratio of the planets’ semimajor axes, $a_c/a_b$, is plotted against the orbital phase of the inner planet each time the outer planet reaches aphelion. Mean-motion resonances are visible in this diagram as discrete islands of points. In the initial phases of the evolution, the outer planet migrates inward toward the inner planet. Once the planets are locked in resonance, the quantity $a_c/a_b$ stops decreasing. In the case shown, the two planets skirt the 5:2 resonance, but are captured into the 7:3 resonance. Once the planets are locked in resonance, they begin to migrate inward together so as to maintain the 7:3 commensurability. However, once resonance locking occurs, the amplitude of the secular eccentricity oscillations between the two planets begins to increase, and the eccentricity of the smaller outer planet reaches a value of $\sim 0.2$ within 30,000 yr. In a simple $N$-body calculation, which does not include the back-reaction of the exterior gas disk, the outer planet eccentricity grows to a point where the system is destabilized. Lee & Peale (2002) have addressed this problem by introducing parameterized eccentricity damping, but fully self-consistent hydrodynamical calculations of the disk response to the resonant migration of the planets need to be done.

The best fit to the radial velocity data for 47 UMa indicates that the system may indeed be in the 7:3 resonance, in which case eccentricity damping is likely to have occurred unless the nebula dissipated immediately after the system was captured into resonance. A longer time baseline of observations will nail down the period of the outer planet more accurately, and further simulations will be done to better elucidate the process of eccentricity damping for planets caught in resonance.

In order to gain a better understanding of whether capture into the 7:3 resonance is likely, we have performed a large number of additional three-body integrations in which a constant azimuthal acceleration is applied to the outer planet. For systems in which the planets have initial osculating elements corresponding to those in Table 1, but with
3000 days, capture into the $3:1$ resonance occurred in every case we examined. We have also computed 10,000 simulations in which the outer planet starts with $P_c = 2800$ days (i.e., interior to the $3:1$ resonance), and the initial osculating eccentricity of planet c is chosen randomly to fall between 0.0 and 0.15. The results of these simulations are shown in Figure 8.

This figure indicates that capture into the $7:3$ resonance is possible, but is much less likely than capture into the $5:2$ resonance, which occurs in about 50% of the cases. (Note that many of the symbols in Fig. 8 overlie one another at the $5:2$ resonance.) In the simulations in which the planets avoid being caught in either the $5:2$ or the $7:3$, capture into the lower order $2:1$ commensurability appears to be certain. It is clear that evolution into the locked $5:2$ or $7:3$ commensurabilities depends sensitively on the initial conditions for the integration. When the eccentricity of the outer planet starts with a small value (which would be expected from a standard gap-opening scenario), capture into the $7:3$ is observed in 3% of the simulations in which $e_c < 0.02$. For large values of the initial eccentricity for the outer planet, capture into higher order resonances (e.g., $9:4, 12:5$, etc.) is

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**Fig. 5.**—Results of a hydrodynamical simulation of two planets (with the nominal masses of 47 UMa b and c) embedded in a protoplanetary disk. The two planets start out beyond the $7:3$ resonance, with a semimajor axis ratio $a_2/a_1 = 1.8$. The y-axes of the second and third panels from the top are in units of 2 AU. There is initially $1 M_{\text{Jup}}$ in the external gaseous disk. The top panel of the figure shows that the disk exerts a steady negative torque on the outer planet, but has little coupling to the inner planet. The torque on the outer planet causes the orbit of the outer planet to migrate steadily inward (second from top), whereas the inner planet stays nearly fixed. In this uncoupled migration phase, the eccentricities do not increase (bottom panel). [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 6.**—Surface of section showing the results of a three-body integration in which the torque measured in the hydrodynamical simulation shown in Figs. 4–5 is applied to the outer planet. The orbit of the outer planet decays, skirts the $5:2$ resonance, and lands in the $7:3$ resonance. This simulation shows that capture in the $7:3$ resonance is possible.
occasionally observed. Note that capture into the $7:3$ resonance for small values of $e_c$ requires $e_b \sim 0.05$. When $e_b \sim 0.01$, capture into the $2:1$ resonance is certain for $e_c < 0.04$. All of the simulations resulted in some form of resonance capture, which suggests that pairs of resonant planets may be quite common in the galaxy.

Although no planetary systems have yet been observed in the $5:2$ resonance, this configuration may be particularly ubiquitous. Indeed, Lee & Peale (2002) have suggested that Jupiter and Saturn may once have been participating in a $5:2$ resonance, which was subsequently disrupted, perhaps as a result of interactions with a remnant planetesimal or gas disk in the early solar system.

There is still some uncertainty in the period ratio of the 47 UMa planets, but the system is clearly interior to the $3:1$ resonance. Because capture into $3:1$ resonance is virtually certain within our migration scenario, it appears that the planets must have started interior to the $3:1$ commensurability.

5. ACCRETION OF EARTHLIKE PLANETS

In the solar system, terrestrial planets formed and survived in a large stable zone between 0 and 2 AU from the Sun, which happens to include the Sun’s habitable zone (Kasting, Whitmire, & Reynolds 1993). The size and width of this stable zone is largely determined by the orbits and masses of the giant planets. Most orbits with semimajor axes between 4 and 36 AU are unstable as a result of close encounters with Jupiter, Saturn, Uranus, or Neptune (e.g., Gladman & Duncan 1990). Many orbits in the asteroid belts (roughly 2–4 AU and >36 AU) are stable, but these regions contain a number of unstable orbital resonances associated with the giant planets, and it appears that these resonances prevented the formation and/or survival of terrestrial planets in these parts of the solar system (e.g., Franklin & Lecar 2000; Nagasawa, Tanaka, & Ida 2000; Chambers & Wetherill 2001).

The 47 UMa system contains two giant planets with mass and period ratios similar to Jupiter and Saturn, and orbits
with low eccentricities—a characteristic also shared by the giant planets of the solar system. Although the absolute masses and relative inclination of the planets in the 47 UMa system are not known at present, it is plausible that these giants could have sculpted a system of asteroid belts and terrestrial planets similar to the solar system, but shifted to smaller semimajor axes reflecting the shorter orbital periods of the giants.

Jones, Sleep, & Chambers (2001) have examined the stability of Earth-mass planets in the presence of the inner giant (i.e., before the existence of the outer giant had been established). These authors found that for $\sin i = 1$, orbits inside $\sim 1.3$ AU are stable apart from a narrow unstable zone at the 3:1 resonance at 1.0 AU. For $\sin i = 0.5$, orbits are stable inside $\sim 1.15$ AU, and the unstable zone at the 3:1 is wider. In both cases, stable orbits can exist in a part of the star’s habitable zone. Chambers (2000) made a single simulation of late-stage terrestrial-planet accretion in the presence of the inner giant planet. In this simulation, several small terrestrial planets formed, including a Mars-size body moving on a stable orbit at $a = 0.92$, at the inner edge of the habitable zone.

The presence of the outer giant complicates the dynamics of terrestrial planets. In particular, a strong secular resonance, analogous to the $\nu_6$ resonance in the solar system, is present at $a \sim 0.85$ AU, and any terrestrial planet forming in this region will have an unstable orbit. (In the solar system, the $\nu_6$ resonance marks the inner boundary of the main asteroid belt.) We have examined the stability of test particles in the presence of the two giant planets using $N$-body integrations. The top panel of Figure 9 shows the fates of 280 particles with low-$e$, low-$i$ orbits and semimajor axes $0.4 < a < 2.0$ AU, integrated for 20 Myr using a hybrid-symplectic algorithm (Chambers 1999). In the figure, blue vertical bars indicate the initial semimajor axes of particles that were ejected from the system by close encounters with one of the giant planets. Yellow bars indicate particles that came within 0.1 AU of the star, and we assume that these

![Figure 9](image-url)
particles would ultimately be removed by colliding with the star. Red bars indicate particles that survived for the length of the integration.

Most of the test particles with $a > 1.3$ AU are removed, mainly by ejection onto hyperbolic orbits. This is similar to the result of Jones et al. (2001) for the case with just the inner giant planet. Many of the particles with $a < 1.3$ AU survived, although there are a number of narrow unstable regions associated with resonances with the giant planets. The secular resonance at $a \sim 0.85$ AU removed several particles by inducing large orbital eccentricities so that the particles fell into the star. Gaps at the $3:1$ and $5:2$ mean-motion resonances are also apparent. Two particles survived in the $3:2$ resonance at $a \sim 1.6$ AU, although it is unclear whether they would survive on longer timescales. In many ways the distribution of stable and unstable regions mimics that in the main asteroid belt of the solar system, with Kirkwood gaps at the $\nu_6$, $3:1$, $5:2$, and $2:1$ resonances, and the Hilda asteroids in a stable region at the $3:2$ resonance.

Most of the test particles with $a < 1.3$ AU survived in the simulation. However, this does not provide a robust indication of whether terrestrial planets can form or survive here. Terrestrial planets have mass and so they modify the strength and location of resonances associated with the giant planets, as Earth and Venus modify the $\nu_5$ resonance in the solar system (Namouni & Murray 1999). During the late stages of planetary accretion, many planetary embryos with appreciable mass will be present. These will modify another’s orbits during close encounters as well as causing secular orbital evolution due to more distant perturbations. In particular, embryos can gravitationally scatter one another into unstable resonance regions, and this greatly enhances the ability of resonances to remove material from a protoplanetary disk (Wetherill 1992). Chambers & Wetherill (2001) have shown that if planetary embryos formed in the asteroid belt of the solar system, there is a two-thirds probability that a combination of gravitational scattering and giant-planet resonances would have removed all embryos from the belt. Something similar may have occurred in the region beyond 0.85 AU in the 47 UMa system.

To examine this possibility, we have made four $N$-body simulations of terrestrial-planet accretion in the 47 UMa system. We assume that planetary embryos were able to form rapidly by runaway accretion (Wetherill & Stewart 1993) interior to the orbits of the giant planets. In addition, since the formation/migration timescales are not known for the giant planets, we make the simplifying assumption that the giants had attained their current orbits and masses at the start of the final stage of accretion of the terrestrial planets. To calculate the orbital and accretional evolution of the embryos, we used a hybrid-symplectic $N$-body integrator (Chambers 1999).

Simulation 1 began with 280 lunar-mass bodies ($m \sim 3 \times 10^{-8} M_\oplus$) with $0.3 < a < 2.0$ AU and roughly circular, coplanar orbits. The total initial mass of the embryos is about $2.5 M_\oplus$ (Earth masses), which in the solar system is generally sufficient to form a system of 3 or 4 planets with total mass similar to the terrestrial planets (Chambers 2001). In this simulation, the giant-planet masses are assumed to have their minimum possible values (i.e., we take $\sin i = 1$). Figure 10 shows six snapshots of the evolution in the simulation. Each panel shows the instantaneous values of $a$ and $e$ for surviving embryos, with symbol radius proportional to the radius of the body. Within $10^7$ yr, the inner giant planet has gravitationally ejected most of the embryos with $a > 1.4$ AU, and a gap has developed at the secular resonance at $a = 0.85$ AU. After 0.5 Myr, the secular resonance is almost empty and embryos are also being dynamically excited and removed at the $3:1$ resonance near 1 AU.

By 2 Myr, a large fraction of the initial population of embryos has been lost, while little accretion has taken place. Clearly, the timescale to remove material from the unstable parts of the disk is short compared to the accretion timescale. It is also apparent that depletion of material is not confined to the resonance locations, but extends across most of the disk beyond about 0.7 AU. As a result, appreciable accretion is confined to the innermost part of the disk, which represents a large contiguous region where resonances are absent. At the end of the simulation, this region contains three small terrestrial planets, the largest of which has $m \sim 0.2 M_\oplus$. All of these planets lie inside the inner edge of the habitable zone of 47 UMa. A single unaccreted (lunar mass) body survives in the habitable zone (HZ) at $a = 1.2$ AU. The mass of this object simply reflects the initial mass of the embryos used in the calculation, and as such the size of this “planet” is not very meaningful.

The middle panel of Figure 9 shows the fates of the 280 embryos at the end of simulation 1, using the same coloring as the top panel. In cases where two or more embryos merged, each of the embryos is assigned the same fate as the composite body. Note that the number of surviving objects is much smaller than in the test-particle integration. In particular, embryos are lost in a wide region with $a > 0.5$ AU, and not merely at resonance locations.

If planetary embryos did form in the HZ of 47 UMa, it is unlikely that they would have had a uniform mass. In fact, runaway accretion of planetesimals, which is thought to lead to the formation of embryos, generally produces a few large objects and many smaller ones (Kokubo & Ida 1998). With this in mind, we made a second accretion simulation which began with 14 Mars-sized bodies ($m \sim 3 \times 10^{-7} M_\oplus$) and 140 lunar-mass objects. The total mass and orbital distributions were the same as in simulation 1. Planetary accretion simulations are notoriously stochastic, so we were somewhat surprised that the orbital and accretional evolution in simulation 2 was very similar to the first. The simulation ended with three small planets with $a < 0.7$ AU, with masses $m < 0.2 M_\oplus$, in addition to a single Mars-sized embryo with $a \sim 1.2$ AU which had undergone no accretion.

In simulations 1 and 2, the total mass of embryos in the region 0–2 AU is similar to that which would have existed in a solar minimum-mass nebula (Weidenschilling 1977). However, the giant planets of 47 UMa are more massive than Jupiter and Saturn. In addition, the giants probably migrated inward after they formed, which suggests that 47 UMa had a more massive protoplanetary disk than the Sun. For this reason, we reran simulations 1 and 2 enhancing the initial masses of the embryos by a factor of 5 (the giant-planet masses were unchanged). In each of these simulations, the dynamical evolution progressed through the same stages seen in simulations 1 and 2. However, the embryos were typically removed somewhat more quickly, since their mutual gravitational perturbations were larger, and they were scattered into unstable resonances more easily. Accre-
tion was again confined to the region $a < 0.7$ AU, and in these simulations the larger embryos produced fewer and more massive final planets. Simulation 3 (uniform-mass embryos) ended with two approximately Earth-mass planets with $a \sim 0.4$ and $0.7$ AU. The individual fates of the initial bodies in simulation 3 are shown in the bottom panel of Figure 9. Again, objects are removed from a much wider range of orbits than the test-particle case.

Simulation 4 ended with a large, $2 M_{\oplus}$ planet with $a \sim 0.5$ AU, and a second planet with $m \sim 0.5 M_{\oplus}$ and $a = 1.1$ AU. In common with simulations 1 and 2, simulation 4 ended with a single body in the HZ that had undergone no accretion, so that the final mass of this planet reflects the initial masses of the embryos. However, we note that if large embryos did form during the runaway accretion stage, a large unaccreted embryo could exist in the HZ of 47 UMa, and this body may be large enough to support plate tectonics (as Earth does, but Mars does not) and be considered a habitable planet.

6. DISCUSSION

The discovery of a second planet orbiting 47 UMa raises a series of very interesting dynamical questions, and can serve to provide important constraints on the formation history of the system.

The 47 UMa system is different from the other known multiple-planet extrasolar systems in two important respects. First, both planets lie at considerable distance from the star. The period of the inner planet, 47 UMa b, is 5 times longer than the inner planet period of any of the other known multiple systems (currently GJ 876, v And, HD 168443, HD 83443, and HD 82943), and 47 UMa c has the longest period of any known extrasolar planet. Second, the orbits of both planets are nearly circular, again in stark contrast to the other known multiple-planet systems. Indeed, 47 UMa is currently the best extrasolar analog of our own system.

47 UMa b and c are engaged in a secular exchange of eccentricities with a period of roughly 6000 yr. A first-order
theory does a good job of describing this motion, and verifies that the two planets are stabilized by a secular resonance which maintains the periapses of the planets in libration around an aligned configuration. This secular resonance is also observed in the outer two planets of v And, and is likely a very common state of affairs in the Galaxy. Jupiter and Saturn currently lie in a different region of secular phase space and do not participate in the secular resonance, although these planets would lie in the resonance for smaller values of $e_{\text{Sat}}$ or $\Delta \omega$.

The current time baseline of radial velocity data covers only two full periods of 47 UMa c, and non-Keplerian interactions between the two planets cannot be resolved with 3–5 m s$^{-1}$ precision. The individual and mutual inclinations are therefore unconstrained by the radial velocity measurements. Dynamical calculations show that the system can tolerate mass factors of $\sin(i) = 0.2$ for coplanar configurations. However, if the system is non-coplanar, stability requires that the mutual inclination be 40° or less, and that this value is quite insensitive to the masses of the two planets.

The small eccentricities of the two planets in the 47 UMa system put interesting constraints on the theory of planet-nebula migration. Our results show that when two-planet systems undergo interaction with an exterior circumstellar disk, the outer planet will migrate inward until it becomes trapped in a low-order mean motion resonance with the interior planet. Generally, this occurs in the 2:1 resonance, as is the case with GJ 876, but it appears that other resonances, including the 3:1, 7:3, and 5:2 can also elicit capture. Once two planets are captured in resonance, they migrate inward together, and their eccentricities increase. It is not clear how eccentricity damping can occur to the degree that both planets remain on nearly circular orbits.

It seems quite possible that the class of eccentric giant planets discovered to date may be the end result of resonant migration with eccentricity pumping. Once planetary orbits begin to cross, the smaller planet is generally ejected from the system, leaving behind the larger survivor on an eccentric orbit. This may explain the fact that eccentric giant planets have a larger average mass than the short-period 51 Peg type planets, which presumably were able to migrate all the way in without being affected by a second body.

A possible scenario for 47 UMa might run as follows. The inner planet formed in the outer part of the 47 UMa protoplanetary nebula, possibly beyond the ice line, and then migrated inward toward its present location at 2 AU. During this migration, a second planet may have formed as a result of density enhancements or vortices beyond the inner edge of the exterior disk. The growth of this second planet would have led to further gap opening, and an isolation of the inner planet and a halt to its migration. The remnant disk was then unable to force much further migration of 47 UMa c, accounting for its present circular orbit. Had it migrated further, 47 UMa c would have been locked in the 2:1 resonance, would have had its eccentricity pumped, and would have escaped, leaving 47 UMa b as just another eccentric giant.

Earth-mass planets can survive in the habitable zone of 47 UMa, but accretion simulations show that it is unlikely that they could have formed there unless very massive planetary embryos accreted in the habitable zone prior to the formation of the giant planets.

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