Particle properties outside of the static limit in cosmology

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Received Day Month Year  
Revised Day Month Year

It is shown that in the rest frame of the observer in expanding Universe states of particles with negative energy exist. The properties of such states are studied. The comparison with the case of negative energies of particles in black holes and rotating coordinates out of the static limit is made.

Keywords: Negative energy; Penrose process; expanding Universe.

PACS numbers: 04.20.-q, 98.80.Jk

1. Introduction

The existence of particles with negative energies and the possibility of observations of consequences of their existence is well known in the black hole physics due to the Penrose process.\textsuperscript{1,2} If one considers the energy of massive particle at rest on space infinity as equal to $mc^2$ ($c$ is the velocity of light) then there is an illusion that the negative energy is possible only in case of the very strong external (for example gravitational) field. Really, for velocity $v \ll c$ the full negative energy $E$ of the particle moving on the distance $r$ from the attracting massive body with the mass $M$

$$E = mc^2 + \frac{m v^2}{2} - G\frac{mM}{r} < 0 \Rightarrow r < \frac{GM}{c^2} = \frac{r_g}{2},$$  \hspace{1cm} (1)$$

where $G$ is gravitational constant, $r_g$ is the gravitational radius. However the definition of the energy depends on the choice of the Killing vector which corresponds to translation in time of the reference frame. This leads to the possibility of existence of negative energy in noninertial reference frame.
In papers \[3,4\] it was shown that the description of particle movement in rotating coordinates is very similar to the description of movement in the field of rotating black hole in Boyer-Lindquist coordinates. In the rotating coordinates also exists a surface called the static limit out of which no body can be at rest in the chosen coordinates. Out of the static limit in rotating coordinate system the processes similar to Penrose process in black holes are possible.

Here we shall discuss the third possibility of existence of particle states with negative energy — that of the expanding Universe.

\section*{2. Negative Energies in Expanding Universe}

Take the interval of the Friedmann homogeneous and isotropic expanding Universe in the Robertson-Walker form

\[ ds^2 = c^2 dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \]  

where \( k = \pm 1, 0 \) for three types of Friedmann models, \( d\Omega^2 = d\theta^2 + \sin^2 \theta \, d\varphi^2 \). In coordinates \( t, r, \theta, \varphi \) the background matter defining the metric of space-time is at rest. For simplicity take the case \( k = 0 \). Then the distance from the observer at rest relative to the background matter at the point \( r = 0 \), for example of the astronomer on the nonrotating Earth, to the object with the coordinate \( r \) will be defined by the product \( ar \) (see Refs. \[6,7\]). Let us use the new coordinates \( t, D, \theta, \varphi \):

\[ D = a(t) \, r, \quad dD = a \, dt + \dot{a} \, dr. \]

Then

\[ dr = \frac{dD}{a} - \frac{D \dot{a}}{a^2} \, dt \]  

and the interval \[2\] becomes

\[ ds^2 = \left( 1 - \frac{D^2 \dot{a}^2}{c^2 a^2} \right) c^2 dt^2 + \frac{2D \dot{a}}{a} dD dt - D^2 dt^2 - D^2 d\Omega^2. \]

Consider particle movement in metric of the general form

\[ ds^2 = g_{00}(dx^0)^2 + 2g_{01}dx^0 dx^1 + g_{11}(dx^1)^2 + g_{0\Omega}d\Omega^2, \]

where \( g_{11} < 0, g_{0\Omega} < 0 \) and \( g_{00}g_{11} - g_{01}^2 < 0 \). From the condition \( ds^2 \geq 0 \) for any possible displacements of particles one obtains the limitation on the velocity component

\[ g_{11} - \sqrt{g_{01}^2 - 2g_{01}g_{11}g_{00} + g_{11}^2 \left( \frac{dx^0}{dx^1} \right)^2} \leq \frac{dx^1}{dx^0} \leq \sqrt{g_{01}^2 - 2g_{01}g_{11}g_{00} + g_{11}^2 \left( \frac{dx^0}{dx^1} \right)^2} - g_{11}. \]
One can see from (7) that the surface $g_{00} = 0$ plays the role of the static limit. In the chosen coordinate system in the region $g_{00} < 0$ no physical body can be at rest. In this region movement will be observed either from the observer if $g_{01} > 0$ corresponding to the expanding Universe (5) with $\dot{a} > 0$ or to the observer for $g_{01} < 0$ in case of contracting Universe (5) with $\dot{a} < 0$.

The particle energy for the free movement in space-time with metric (6) is defined by the canonical 4-momentum

$$ E = p_0 c = mc g_{0k} \frac{dx^k}{d\tau} = mc \frac{dx^0}{d\tau} \left( g_{00} + g_{01} \frac{dx^1}{dx^0} \right). $$

Here $\tau$ is the particle proper time. If the metric does not depend on time then the energy is conserved. The necessary condition for the energy to be negative is

$$ \frac{dx^1}{dx^0} < - \frac{g_{00}}{g_{01}}. $$

Using the limitation (7) one obtains the limitations on possible values of the energy of particle

$$ mc \frac{dx^0}{d\tau} \left( g_{01}^2 - g_{11} g_{00} - |g_{01}| \sqrt{g_{01}^2 - g_{11} g_{00} - g_{11} g_{00} \left( \frac{d\Omega}{dx^0} \right)^2} \right) \leq E \leq mc \frac{dx^0}{d\tau} \left( g_{01}^2 - g_{11} g_{00} + |g_{01}| \sqrt{g_{01}^2 - g_{11} g_{00} - g_{11} g_{00} \left( \frac{d\Omega}{dx^0} \right)^2} \right). $$

From inequalities (10) one has that at the region where $g_{00} > 0$ the particle energy is always positive, on the static limit the minimal energy can be zero, in the region $g_{00} < 0$ (out of the static limit) states with zero and negative energy with any absolute value are possible.

For the metric (5) the static limit is

$$ D_s = \frac{c a}{|\dot{a}|} = \frac{c}{|h(t)|}, $$

where $h(t) = \dot{a}/a$ is the Hubble parameter. The energy of the freely moving particle is

$$ E = E' \left( 1 - \frac{\dot{a}^2 D^2}{a^2 c^2} + \frac{\dot{a} D}{a} \frac{dD}{dt} \right), $$

where $E' = mc^2 dt/dr$. So in these coordinates particles with negative energies are moving in coordinate $D > D_s$ so that the velocity is slower than some definite value

$$ \frac{dD}{dt} < c^2 a \frac{D^2}{D_0} \left( \frac{D_s^2}{D_s^2} - 1 \right). $$

Let us rewrite the inequality (13) in terms of coordinates $t, r, \theta, \phi$. Then we obtain in the case $\dot{a} > 0$

$$ v = a \frac{dr}{dt} < -c \frac{D_s}{D}, \quad D > D_s. $$
This has a meaning similar to that obtained by us for the case of rotating coordinate frame $t, r, \theta, \varphi$: particles with negative energies close to the static limit ($D \to D_s$) must move with velocities close to the light velocity in direction of the observer in expanding Universe.

Now let us discuss a special case of the de Sitter Universe with the scale factor $a = a_0 \exp H t$, where $a_0$ and $H$ are constants. The Hubble constant is constant and the static limit $D_s = c/H$ is constant and it is equal to the cosmological event horizon for the observer at the origin

$$L_H = a(t)c \int_t^\infty \frac{dt'}{a(t')} = \frac{c}{H}.$$  \hfill (15)

So processes (Penrose processes) with particles with negative energy are not seen by the observer. This is analogous to situation of nonrotating black holes where particles with negative energy exist only inside the event horizon.\[^{8}\] However for scale factor $a = a_0 t^\alpha$ ($0 < \alpha < 1$) the cosmological horizon does not exist and some visible consequences of existence of particles with negative energies can be observed.

In this paper we consider particles as classical particles. Surely if they are quantum new features appear.\[^{9,10}\]

Acknowledgments

This research is supported by the Russian Foundation for Basic research (Grant No. 18-02-00461 a). The work of Yu.V.P. was supported by the Russian Government Program of Competitive Growth of Kazan Federal University.

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