The small axial charge of the $N(1535)$ resonance

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Abstract

There is a natural cancellation between the contributions of the $qqq$ and $qqqqq\bar{q}$ components to
the axial charge of the $N(1535)$ resonance. While the probability of the former is larger than that
of the latter, its coefficient in the axial charge expression is exceptionally small. The magnitude of
two of the corresponding coefficients of the $qqqqq\bar{q}$ components are in contrast large and have the
opposite sign. This result provides a phenomenological illustration of the recent unquenched lattice
calculation result that the axial charge of the $N(1535)$ resonance is very small, if not vanishing [1].
The result sets an upper limit on the magnitude of the probability of $qqqqq\bar{q}$ components as well.

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A number of phenomenological failures of the constituent quark model for the baryons may be repaired by extending the model space beyond that of the basic three quark configurations \( qqq \). The question of key interest is then that of the relative magnitude of the sea-quark configurations, and in particular of the most obvious \( qqqq\bar{q} \) configurations. For most electromagnetic and strong decay observables, this is difficult to estimate, because of the very strong contribution from the transition matrix elements between the \( qqq \) and \( qqqq\bar{q} \) components. The axial current operator of the baryon resonances is an exception, as for this the transition matrix elements are suppressed - i.e. they involve the small components of the spinors - with respect to the diagonal matrix elements, so that the axial charges, to a good approximation, may be expressed as a sum of the diagonal matrix elements of all possible configurations, which takes the form of numerical coefficients \( A_n \) times the corresponding probabilities \( P_n \):

\[
g^*_A \simeq \sum A_n P_n.
\]

The (diagonal) axial charges of baryon resonances are however not accessible experimentally. It is in this regard that the recent result, obtained numerically by an unquenched QCD lattice calculation, that the axial charge of the \( N(1535) \) actually may vanish in the two-flavor case, is so interesting. As that result appears to be insensitive to the quark mass (the magnitude of the value extrapolated to 0 is less than 0.2), it may be taken as a substitute for an experimental value. While the statistical error margins of the calculated values of the axial charge of the \( N(1535) \) are not yet sufficiently narrow to exclude the small value -1/9 given by the conventional constituent quark model with only \( qqq \) configurations, it is interesting to explore the phenomenological consequences of a vanishing axial charge.

If the axial charge of the \( N(1535) \) vanishes, it implies that the sea-quark configurations shall have to cancel the (small) contribution of the \( qqq \) configuration. This makes it possible to put constraints on the sea-quark configurations in the \( N(1535) \). To illustrate this possibility we consider the contributions from all the \( qqqq\bar{q} \) components, which may exist in the \( N(1535) \). These have been enumerated in ref. \[7\]. As all 5 constituents in a \( qqqq\bar{q} \) configuration in the negative parity \( N(1535) \) may be in the ground state, the orbital state of the 4 quarks may be assumed to be completely symmetric. Then either the spin-flavor state has to have the mixed flavor-spin symmetry \([31]_{FS}\) or alternatively the color-spin state has to have one of the mixed flavor symmetries \([31]_{CS}, [22]_{CS}\) or \([211]_{CS}\). There are 5 different
TABLE I: The $qqqq\bar{q}$ configurations in the $N(1535)$ and the corresponding axial charge coefficient $A_n$.

| configuration | flavor-spin | $C_{FS}$ | color-spin | $C_{CS}$ | $A_n$ |
|---------------|-------------|----------|------------|----------|-------|
| 1             | $[31]_{FS}[211]_{F}[22]_{S}$ | $-16$    | $[31]_{CS}[211]_{C}[22]_{S}$ | $-16$    | 0     |
| 2             | $[31]_{FS}[211]_{F}[31]_{S}$ | $-40/3$  | $[31]_{CS}[211]_{C}[31]_{S}$ | $-40/3$  | $+5/6$|
| 3             | $[31]_{FS}[22]_{F}[31]_{S}$  | $-28/3$  | $[22]_{CS}[211]_{C}[31]_{S}$ | $-16/3$  | $-1/9$|
| 4             | $[31]_{FS}[31]_{F}[22]_{S}$  | $-8$     | $[211]_{CS}[211]_{C}[22]_{S}$ | 0        | $-4/15$|
| 5             | $[31]_{FS}[31]_{F}[31]_{S}$  | $-16/3$  | $[211]_{CS}[211]_{C}[31]_{S}$ | $+8/3$   | $+17/18$|

$qqqq\bar{q}$ configurations in the $N(1535)$ that have an appropriate symmetry structure and spin and isospin $1/2$. These are listed in Table I.

The numbering of these configurations are in order of increasing energy, if the hyperfine interaction between the quarks is assumed to depend either on flavor and spin or on color and spin. In the table the matrix elements of the schematic hyperfine splitting operator

$$C_{kS} = -\sum_{i,j} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$$

are listed for both the cases where the operators $\vec{\lambda}$ represent either the generators of the color $SU(3)$ ($k=C$) or the flavor $SU(3)$ group ($k=F$), respectively. (Here the spatial structure of the interaction has been neglected, as all the constituents are in the same orbital ground state). Note that because of their mixed flavor symmetry [211]$_F$, both the configurations (1) and (2) in Table I have to contain a strange quark-antiquark pair. This is as expected on the basis of the observed large $N\eta$ decay branch of the $N(1535)$.

Because the states 1 in Table I has zero total spin, and the antiquark is a strange quark, it contributes no matrix element to the axial charge operator $\sum_i \sigma_z(i) \tau_z(i)$ (calculated here as the matrix element of the third component of the axial vector current). In the table the matrix element of the axial charge of these configurations, combined with the wave function of the antiquark are also listed. The general expression in the flavor-spin coupling scheme for these 5 quark wave functions is:

$$\psi^{(i)}_{t,s} = \sum_{a,b,c} \sum_{Y,T_z,t_z} \sum_{S_z,s_z} C^{[14]}_{[31]_a[211]_a} C^{[31]_a}_{[F^{(i)}]_b[S^{(i)}]_c} [F^{(i)}]_{b,y,T_z[S^{(i)}]_c,S_z} [211; C]_a$$

$$(Y,T,T_z,y,t_z1,1/2,t)(S,S_z,1/2,s_z1/2,s)\bar{\chi}_{y,t_z} \bar{\xi}_{s_z} \varphi[s] .$$

Here $i$ is the number of the $qqqq\bar{q}$ configuration in Table I. $\bar{\chi}_{y,t_z}$ and $\bar{\xi}_{s_z}$ represent the
isospinor and the spinor of the antiquark respectively, and $\varphi$ represents the completely symmetrical orbital wave function. The first summation involves the symbols $C_{[\ldots]}^{[\ldots]}$, which are $S_4$ Clebsch-Gordan coefficients for the indicated color $([211])$, flavor-spin $([31])$ and flavor $([F])$ and spin $([S])$ wave functions of the $qqqq$ system. The second summation runs over the flavor indices in the $SU(3)$ Clebsch-Gordan coefficient (with 9 symbols) and the third over the spin indices in the standard $SU(2)$ Clebsch-Gordan coefficient. In the case of the spin configuration $[22]$ the total spin of the $qqqq$ system vanishes, so that $S = S_z = 0$. These wave functions are given in explicit form in Ref. \cite{8}.

With the results in Table I the explicit expression for the axial charge of the $N(1535)$ takes the form

$$g_A(N(1535)) = -\frac{1}{9}P_3 + \frac{5}{6}P_5^{(2)} - \frac{1}{9}P_5^{(3)} - \frac{4}{15}P_5^{(4)} + \frac{17}{18}P_5^{(5)}.$$ (4)

Here $P_3$ is the probability for the conventional $qqq$ configuration, while $P_5^{(i)}$ represents the probabilities of the $qqqq\bar{q}$ configurations in Table II. Note that the energetically most favorable $qqqq\bar{q}$ configuration (1) does not contribute to the axial charge at all.

The fact that the two $qqqq\bar{q}$ contributions in (4), which are positive, have large coefficients $\sim 1$, while the coefficient of the $qqq$ contribution is small and negative ($-1/9$) immediately suggests the possibility for a considerable cancellation between the $qqq$ valence and the $qqqq\bar{q}$ sea-quark contributions, as the probability of the latter is likely to be considerably smaller than that of the former. If only the first two terms in the expression (4) are taken into account $g_A(N(1535))$ would vanish if $P_5^{(2)} = 2/15P_{qqq}$, which may be a fairly reasonable assumption. The last two remaining $qqqq\bar{q}$ configurations are in expected to have very small probability, as they are energetically unfavorable (Table II).

In ref. \cite{8} it was in fact found that the quark model prediction for the helicity amplitude $A_{1/2}$ for $N(1535) \rightarrow N\gamma$ could be brought qualitatively into line with the empirical values if $P_{qqq} \simeq 0.55$ and $P_{5}^{(3)} \simeq 0.45$. Since the $qqqq\bar{q}$ configurations (1) and (2) in Table II are similar in that both involve an $s\bar{s}$ pair, but the latter is energetically disfavored by the matrix elements of the hyperfine interaction \cite{2}, the helicity amplitude should be similar if the probability $P_{5}^{(2)}$ for the configuration (2) in Table II which has a large axial charge coefficient \cite{4}, falls in the range $(0.25-0.3)P_{5}^{(1)}$. With these numbers $g_A(N(1535))$ comes out to lie in the range 0.03-0.06. If on the other hand one considers both the configurations (2) and (3) in Table II as equally probable: $P_{5}^{(2)} = P_{5}^{(3)}$ and the probabilities of to fall within the
range (0.12-0.15) $P_5^{(1)}$, the numerical value for $g_A(N(1535))$ falls in the range -0.02 to -0.05. This shows that the likely range of values for $g_A(N(1535))$ in the extended quark model, which includes explicit $qqq\bar{q}$ components $-0.05 .. +0.06$, brackets 0. This range would bracket 0 also in the case where the relative $qqq$ probability where increased to $P_3 = 0.7$ and $P_5^{(2)} = 0.3$. It does in any case not appear possible to reach the value 0 for $g_A(N(1535))$, with an overall $qqq\bar{q}$ probability that is larger than 0.45.

The conclusion is therefore that the very small or possibly vanishing axial charge of the $N(1535)$ already at the present level of accuracy constrains the magnitude of the probability of the sea-quark components in the $N(1535)$ to be less than 45%. A more general observation is that the axial charges of the baryon resonances may be useful for setting limits on the probabilities of their sea-quark configurations.

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