Abstract. In an era where accumulating data is easy and storing it inexpensive, feature selection plays a central role in helping to reduce the high-dimensionality of huge amounts of otherwise meaningless data. This report overviews concepts and algorithms of feature selection, surveys existing feature selection algorithms for classification with a categorizing framework based on the complexity: filter, embedded, and wrappers methods. Some real-world applications are included to demonstrate the use of feature selection in data mining. As a result, the report proposes extensive tests on diverse datasets. We conclude this work by identifying trends and challenges of feature selection research and development while providing a toolbox of 10 methods selected from recent literature.

Keywords: Feature selection, variable ranking, variable selection, feature selection, space dimensionality reduction, pattern discovery, filters, wrappers

1 Introduction

Feature selection for classification is a well-researched problem, aimed at reducing the dimensionality and noise in data sets. Sometimes in many learning domains, a human operator defines the potentially useful features. However, not all of these features may be relevant and some of them may be redundant. In such a case, automatic feature selection can be employed for removing irrelevant, redundant, and noisy information from the data, often leading to better performance in learning and classification tasks. For supervised learning, feature selection algorithms maximize some function of predictive accuracy. Because we are given class labels, it is natural that we want to keep only the features that are related to or lead to these classes. Generally, feature selection for supervised machine learning tasks can be accomplished on the basis of the following underlying hypothesis: “a good feature subset is one that contains features highly correlated with (predictive of) the class, yet uncorrelated with (not predictive of) each other [7]”. We may define a feature which is highly correlated with the class as “relevant”, whereas a feature which is uncorrelated with the others as not “redundant”. Redundant features are those which provide no more information than the currently selected features, and irrelevant features provide no useful information in any context. Figure 1 shows an example of feature redundancy. Note that the data can be grouped in the same way using only either feature x or feature y. Therefore, we consider features x and y to be redundant. Figure 2 shows an example of an irrelevant feature. Observe that feature y does not contribute to class discrimination. Used by itself, feature y leads to a single class structure which is uninteresting. Note that irrelevant features can misguide classification results, especially when there are more irrelevant features than relevant ones. In unsupervised learning, we are not given class labels. Unsupervised learning
Fig. 1. In this example, features $x$ and $y$ are redundant, because feature $x$ provides the same information as feature $y$ with regard to discriminating the two clusters.

Fig. 2. In this example, we consider feature $y$ to be irrelevant, because if we omit $x$, we have only one class, which is uninteresting.
is a difficult problem. It is more difficult when we have to simultaneously find the relevant features as well. Feature selection is a widely recognized important task in machine learning, data mining and recently successfully applied in Social Signal Processing domain [16,4,20,12,17,3] and analysis of personality [18,19]. In high-dimensional data-sets feature selection improves algorithms performance and classification accuracy since the chance of overfitting increases with the number of features.

Feature selection techniques can be partitioned into three classes [10]: wrappers, which use classifiers to score a given subset of features; embedded methods, which inject the selection process into the learning of the classifier; filter methods, which analyze intrinsic properties of data, ignoring the classifier. Most of these methods can perform two operations, ranking and subset selection: in the former, the importance of each individual feature is evaluated, usually by neglecting potential interactions among the elements of the joint set [5]; in the latter, the final subset of features to be selected is provided. In some cases, these two operations are performed sequentially (ranking and selection) [12,18,24,15,23]; in other cases, only the selection is carried out [9]. Generally, the subset selection is always supervised, while in the ranking case, methods can be supervised or not.

Generally, feature selection is \( NP\text{-}hard \) [10]; if there are \( n \) features in total, the goal is to select the optimal subset of \( m \ll n \), to evaluate \( \binom{n}{m} \) combinations; therefore, sub-optimal search strategies are considered. With the filters, features are first considered individually, ranked, and then a subset is extracted, some examples are MutInf [24], Relief-F [15], and SW Relief-F [23]. Conversely, with wrapper and embedded methods, subsets of features are sampled, evaluated, and finally kept as the final output, for instance, FSV [18], SVM-RFE [12], Ens.SVM-RFE [23], and SW SVM-RFE [23].

The paper is organized as follows. A brief overview of the existing methods is given in Section 2. Section 3 contains the experimental evaluations and results. Finally, conclusions are provided in Section 4.

2 Existing Methods

Among the most used feature selection strategies, Relief-F [15] is an iterative, randomized, and supervised approach that estimates the quality of the features according to how well their values differentiate data samples that are near to each other; it does not discriminate among redundant features, and performance decreases with few data. Similar problems affect SVM-RFE (RFE) [12], which is an embedded method that selects features in a sequential, backward elimination manner, ranking high a feature if it strongly separates the samples by means of a linear SVM.

An effective yet fast filter method is the Fisher method [9], it computes a score for a feature as the ratio of interclass separation and intraclass variance, where features are evaluated independently, and the final feature selection occurs by aggregating the \( m \) top ranked ones. Other widely used filters are based on mutual information, dubbed MI here [24], which considers as a selection criterion the mutual information between the distribution of the values of a given feature and the membership to a particular class; Even in the last case, features are evaluated independently, and the final feature selection occurs by aggregating the \( m \) top ranked ones.
Selecting features in unsupervised learning scenarios is a much harder problem, due to the absence of class labels that would guide the search for relevant information. In this scenario, we compare our approach against the recent unsupervised graph-based filter dubbed Inf-FS [21]. In the Inf-FS formulation, each feature is a node in the graph, a path is a selection of features, and the higher the centrality score, the most important (or most different) the feature. It assigns a score of importance to each feature by taking into account all the possible feature subsets as paths on a graph. Another unsupervised method is the Laplacian Score (LS) [13], where the importance of a feature is evaluated by its power of locality preserving. In order to model the local geometric structure, this method constructs a nearest neighbor graph. LS algorithm seeks those features that respect this graph structure.

Finally, for the wrapper method, we include the feature selection via concave minimization (FSV) [1], where the feature selection process is injected into the training of an SVM by a linear programming technique.

2.1 Approaches

Table 1 reports their type, that is, $f$ = filters, $w$ = wrappers, $e$ = embedded methods, and their class, that is, $s$ = supervised or $u$ = unsupervised (using or not using the labels associated with the training samples in the ranking operation). Additionally, we report their computational complexity (if it is documented in the literature); Table 1 lists the

| Acronym   | Type | Cl. | Compl.             |
|-----------|------|-----|--------------------|
| SVM-RFE [12] | e    | s   | $O(T^2n\log_2n)$  |
| Inf-FS [21] | f    | u   | $O(n^{2.37}(1 + T))$ |
| Relief-F [15] | f    | s   | $O(iTnC)$         |
| FSV [18]   | w    | s   | N/A                |
| MutInf [24] | f    | s   | $\sim O(n^2T^2)$  |
| mRMR [16]  | f    | s   | $O(n^3T^2)$       |
| Fisher [9]  | f    | s   | $O(Tn)$           |
| LaplacianScore [13] | f | u   | N/A                |
| MCFS [2]   | f    | u   | N/A                |
| L0[12]     | w    | s   | N/A                |

Table 1. List of the feature selection approaches considered in the experiments, specified according to their type, class (Cl.), and complexity (Compl.). As for the complexity, $T$ is the number of samples, $n$ is the number of initial features, $i$ is the number of iterations in the case of iterative algorithms, and $C$ is the number of classes. The complexity of FSV cannot be specified since it is a wrapper (it depends on the chosen classifier).
3 Experimental Evaluation

This section reports preliminary results obtained on two different scenarios.

3.1 Datasets

Table 2 shows the diverse scenarios we selected for the experimental section. We consider the datasets dealing with few training samples and many features (samples in the table), classes that severely overlap (Complexity), or whose samples are noisy or redundant (Redundancy) due to: i) complex scenes where the object to be classified is located (as in the VOC series) or ii) many outliers (as in DEXTER prepared for the NIPS 2003 variable and feature selection challenge).

3.2 Features Extracted with a Deep Convolutional Neural Network Architecture

We use the PASCAL VOC-2007 [6] dataset as reference benchmark. For this reason, we report results of seven state-of-the-art feature selection methods selected from Table 1. This experiment considers as features the cues extracted with a deep convolutional neural network architecture (CNN). We selected the recent pre-trained model called GoogLeNet [22], which achieves the new state of the art for classification and detection in the ImageNet Large-Scale Visual Recognition Challenge 2015 (ILSVRC15).

We use the 1,024-dimension activations of the last five layers as image descriptors (5,120 features in total). The VOC-2007 edition contains about 10,000 images split into train, validation, and test sets, and labeled with twenty object classes. A one-vs-rest SVM classifier for each class is learned and evaluated independently and the performance is measured as mean Average Precision (mAP) across all classes. Classification results over the entire set of deep features is 81.6%, while varying the cardinality of

| Name    | #Samples | # Classes | #Features | Complexity | Redundancy |
|---------|----------|-----------|-----------|------------|------------|
| VOC-07  | 10K      | 20        | n.s.      | X          | X          |
| DEXTER  | 2.6K     | 2         | 20K       | -          | X          |

Table 2. Benchmarks used in the classification tasks. The abbreviation n.s. stands for not specified

| Object Recognition - Ranking CNN Features | First 128F Selected | First 256F Selected |
|-------------------------------------------|---------------------|---------------------|
| Fisher | ReliefF | MutInf | FSV | SVM RFE | mRMR | Inf-FS | Fisher | ReliefF | MutInf | FSV | SVM RFE | mRMR | Inf-FS |
| AP     | 76.29   | 51.43  | 71.48 | 75.09 | 74.60 | 71.96 | 67.28 | 78.51   | 66.35  | 77.97 | 77.07 | 77.33 | 78.01 | 74.62 |

Table 3. VOC 2007 classification results achieved in terms of average precision (AP). The total amount of features is 5,120 CNN GoogLeNet [22].
the selected features according to our method, we improved the mAP by 0.2% while removing up to 30% of irrelevant features.

Table 3 shows how well important features are ranked high by several feature selection algorithms. As visible, Fisher method achieved the best performance in terms of mean average precision (mAP) followed by the wrapper method FSV and the embedded SVM-RFE. Notably, in those classes where Fisher does not perform the best result (i.e. person, cars, birds, etc.), the deviation from the best score achieved by any of the other methods is on the average up to 1.2%.

3.3 Large-Scale Feature Ranking and Selection

This experiment shows how relevant and/or un-redundant features are ranked high by our feature selection algorithm when dealing with a big amount of features. As reference benchmark we use the DEXTER [11] from the 2003 NIPS feature selection challenge, which consists of 20,000 features (see Table 2 for details). Moreover, depending on the outcomes of feature selection techniques under the different splits training/testing, we also want to investigate how stabled are the partial ranked list produced, that is, how often the same subsets of features are selected with the same ordering. For this reason, we employ the Kuncheva’s stability measure [14].

The testing protocol adopted in this experiment consists in splitting the dataset up to 60% for training and 30% for testing. The feature ranking has been calculated using only the training samples, and then applied to the testing samples. Classification is performed using a linear SVM. For setting the best parameters we used a 10-fold cross validation on the training data. Figure 3(a) shows the results in terms of average precision (AP). The experiment starts with a pool of 20,000 features characterizing the training data. These features are ranked and selected, generating different subsets of different cardinality (ranging from 10 to 1,000 where values are logarithmically spaced).
Results show again that Fisher outperforms all the other methods, showing a curve which maintains an upward movement without taking falls. It can happen whenever a set of irrelevant features is added to the selected set. As for SVM-RFE and ReliefF where the curves bounce back. It is noteworthy that this behavior has a correspondent low stability index as shown in Figure 3(b). The stability measure represents the similarity between the set of rankings generated over the different splits of the dataset. The similarity between sequences of size \( k \) can be seen as the number of elements \( r \) they have in common. As for the Kuncheva index, the higher its value, the larger the number of commonly selected features in both rankings. In the very first positions, Fisher shows a high stability whereas the highest performance is achieved. The average time spent overall the applications of feature selections are also shown in the legend of Figure 3. Moreover, Fisher needs only 3.7 seconds to rank the 20K features of the DEXTER dataset.

4 Concluding Remarks

The task of a feature selection algorithm is to provide with a computational solution to the feature selection problem motivated by a certain definition of relevance. This algorithm should be reliable and efficient. The methods proposed in this report are based on quite different principles (as the evaluation measure used, the precise way to explore the search space, etc) and loosely follow different definitions of relevance. In this work a way to evaluate the methods was proposed in order to understand their general behavior on the particularities of relevance, irrelevance, redundancy and sample size of synthetic data sets. To achieve this goal, a set of controlled experiments using different datasets were used and carried out and the methods compared in terms of accuracy, precision and ranking stability. Finally, we integrated the proposed set of feature selection methods in our code library (DOWNLOAD the last version from MATLAB Central home of File Exchange) with uniform input and output formats to facilitate large scale performance evaluation and application.

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