I would like to start my tenure as editor of the Logic Column by thanking Jon Riecke, who has edited this column since 1998. The Logic Column serves as a showcase of the many connections between logic and computer science. Logic has been connected with computer science since the early days of Turing. In the past few decades, logical methods have had a considerable impact. To get a sense of the range of applications, consider the 2001 NSF/CISE Workshop on The Unusual Effectiveness of Logic in Computer Science (see http://www.cs.rice.edu/~vardi/logic/). An article derived from the workshop appeared in the Bulletin of Symbolic Logic [Halpern et al. 2001], and it is an exceedingly good read. If you get a copy of that issue of the Bulletin, make sure to also have a look at the article by Buss et al. [2001], which discusses the current state of mathematical logic.

If you have any suggestion concerning the content of the Logic Column, or even better, if you would like to contribute by writing a survey or tutorial on your own work or topic related to your area of interest, feel free to get in touch with me. Topic of interest include, but are not limited to:

- recent results on logic in general, and in applications to computer science in particular;
- reviews of research monographs and edited volumes;
- conference reports;
- relevant results and connections with other fields that make use of logical methods, such as mathematics, artificial intelligence, linguistics, and philosophy;
- surveys of interesting uses of logical methods in computer science.

And while we are on the topic of logical methods in computer science, let me take this opportunity to advertise a new online journal, aptly called Logical Methods in Computer Science. See http://www.lmcs-online.org/ for more details.

**Modeling Confidentiality**

First-time novelists write transparently autobiographical novels; first-time columnists write about what they do. Therefore, this article will be about logical methods applied to security, a topic I...
have been involved with for the past few years. The goal is to illustrate a feature of logical methods: to unify under the umbrella of a formal language seemingly distinct notions that share a common intuition. Some of the results I report reflect ongoing work in collaboration with Sabina Petride, of Cornell University.

There are a number of important concepts in security: confidentiality (keeping data secret), authentication (proving identity or origination), integrity (preventing data modification), and others. In this article, I will focus on confidentiality, arguably one of the core concepts. There are many views on confidentiality in the literature, with many corresponding definitions, in many different guises. My goal here is to show that these definitions can essentially be understood as follow: they capture the fact that unauthorized agents do not know anything about confidential data. The variations between definitions concern the kind of data being protected, and the properties of the data that are meant to remain unknown. The setting where the various definitions will be interpreted is a setting where we can make sense of such knowledge, in a pleasantly abstract way.

The first step is to specify what we mean by an unauthorized agent. Generally, security is studied in an adversarial setting, that is, in the presence of an adversary. Given our focus on confidentiality, we assume that the adversary seeks to circumvent confidentiality, and obtain information about the confidential data. To simplify our problem slightly, we assume that confidentiality is meant to be enforced against such an adversary, and thus that ensuring confidentiality means that the adversary does not know anything about a confidential piece of data.

The second step is therefore to capture the knowledge of the adversary in some general way. A particularly successful formalization of knowledge is due to Hintikka [1962], and has been applied to many fields of computer science; see Fagin et al. [1995] for a survey. The formalization relies on the notion of possible worlds: a possible world is, roughly speaking, a possible way in which the world could be. To drive the intuition, consider the following situation. Suppose I witness Alice murdering Bob in the library, and suppose that it is a matter of fact that Alice used a fire poker, but for whatever reason, I did not notice the murder weapon that Alice used. Thus, there are (at least) two worlds that I consider as possible alternatives to the actual world: the actual world itself, where Alice used a fire poker, and a world where Alice used, say, a brick. I cannot be said to know that Alice murdered Bob using a poker, but for whatever reason, I did not notice the murder weapon that Alice used. Thus, there are (at least) two worlds that I consider as possible alternatives to the actual world: the actual world itself, where Alice used a fire poker, and a world where Alice used, say, a brick. I cannot be said to know that Alice murdered Bob using a poker, since from my point of view, it is possible that Alice did not: there is a world I consider possible where Alice used a brick. On the other hand, I can be said to know that Alice is a murderer, since it will be the case at every world I consider possible. Thus, I can be said to know a fact at a world if that fact is true in all the worlds I consider possible at that world.

To reason about possible worlds and the knowledge of an agent with respect to these worlds, we use epistemic frames. An epistemic frame is a tuple \( F = (W, K) \), where \( W \) is a set of possible worlds (or states) and \( K \) is a relation on \( W \) that represents the worlds that the agent considers as possible alternatives to other worlds; \((w, w') \in K \) if the agent consider \( w' \) as a possible world at world \( w \). We often use the notation \( K(w) \) for \( \{w' \mid (w, w') \in K\} \). We identify a fact with the set of worlds where that fact holds. Thus, a fact is a subset \( S \) of \( W \). Following the intuition above, we say a fact \( S \) is known at a world \( w \) if \( S \subseteq K(w) \), that is, if at every world that the agent considers possible at world \( w \), the fact \( S \) holds at that world. To model the situation in the previous paragraph, consider a simple epistemic frame with three worlds \( W = \{w_1, w_2, w_3\} \), where \( w_1 \) is the world where Alice murdered Bob in the library with a poker, \( w_2 \) is the world where Alice murdered Bob in the library with a fire poker, and \( w_3 \) is the world where Alice murdered Bob in the library with a brick. The situation is modeled by the epistemic frame \( F = (W, K) \), where \( K(w) \) is the set of worlds that the agent considers possible at world \( w \).

1This is under the assumption that I am not subject to illusions, or hallucinations, of course. Philosophers are fond of such counterexamples, which reveal implicit assumptions about the world that may affect our reasoning. When applying these ideas to computer science, we shall assume that our models take into account everything relevant to establish knowledge, including such implicit assumptions.
with a brick, and \( w_3 \) is the world where Alice did not murder Bob. Thus, the fact \( G_1 = \{ \text{Alice murdered Bob in the library} \} \) is represented by the set \( \{ w_1, w_2 \} \), and the fact \( G_2 = \{ \text{Alice murdered Bob with a poker} \} \) is represented by the set \( \{ w_1 \} \). By assumption, the worlds I consider possible at \( w_1 \) are \( \{ w_1, w_2 \} \), and thus \( \mathcal{K}(w_1) = \{ w_1, w_2 \} \). Since \( \mathcal{K}(w_1) = \{ w_1, w_2 \} \subseteq G_1 \), I know the fact \( G_1 \), but since \( \mathcal{K}(w_1) = \{ w_1, w_2 \} \not\subseteq \{ w_1 \} \), I do not know \( G_2 \).

The framework can be easily extended to reason about the knowledge of multiple agents. It suffices to provide a relation \( \mathcal{K}_i \) to every agent \( i \). In this article, since we shall focus on confidentiality with respect to only a single adversary, we only need to reason about the adversary’s knowledge. This has the advantage of simplifying the framework and the notation. Of course, our discussion can be expanded to deal with multiple agents. In fact, we will assume multiple agents, but only model the knowledge of the adversary.

Epistemic frames describe the structure of the model that we want to reason about. They have been quite successful in fields such as economics, where they are used to reason about the knowledge of economic agents [Aumann 1999]. While a lot can be done purely at the level of the model, one big advantage in casting a situation in epistemic frames is that we can define a formal language to let us do the reasoning without having to explicitly manipulate the worlds of the model. The language of epistemic logic starts with a set of primitive propositions \( \Phi_0 \) (describing the basic facts we are interested in reasoning about), and forming more general formulas using conjunction \( \varphi \land \psi \), negation \( \neg \varphi \), and knowledge formulas of the form \( K \varphi \), read “the agent knows \( \varphi \)”. In order to interpret this language in an epistemic frame, that is, to say when a formula of the language is true at a world of the frame, we need to add an interpretation \( \pi \) stating which primitive propositions are true at which worlds. An epistemic structure (also known as a Kripke structure) is a tuple \( M = (\mathcal{W}, \mathcal{K}, \pi) \), where \( (\mathcal{W}, \mathcal{K}) \) is an epistemic frame, and \( \pi \) is an interpretation. The truth of a formula \( \varphi \) at a world \( w \) of structure \( M \), written \( (M, w) \models \varphi \), is established by induction on the structure of \( \varphi \):

\[
(M, w) \models p \text{ if } p \in \pi(w)
\]
\[
(M, w) \models \neg \varphi \text{ if } (M, w) \not\models \varphi
\]
\[
(M, w) \models \varphi \land \psi \text{ if } (M, w) \models \varphi \text{ and } (M, w) \models \psi
\]
\[
(M, w) \models K \varphi \text{ if for all } w' \in \mathcal{K}(w), (M, w') \models \varphi.
\]

The semantics for primitive propositions shows the role of the interpretation. The semantics of negation and conjunction are the obvious ones. The semantics of knowledge formulas follows the intuition outlined above: a formula is known at a world \( w \) if it is true at all the worlds the agent considers possible at \( w \). We write \( M \models \varphi \) if \( (M, w) \models \varphi \) for all worlds \( w \).

We therefore have two tools to reason about the knowledge of agents: a way to model the system with a notion of possible worlds, and a language to express properties of the system. These two tools come together when applying the framework to capture various notions of confidentiality in the literature.

Rather than using epistemic structures in their full generality, we focus on a particular class of structures, inspired by the multiagent systems often used to model distributed systems. We assume a number of agents (named 1 to \( n \), for simplicity), including an adversary, named \( \text{adv} \). We assume that every agent (including the adversary) is in some local state at any global state of the system. We take as the worlds of our model the global states of the system. We furthermore assume that the environment acts like an agent, and has its own local state, to account for the information that needs to be maintained but is not kept in the local state of any agent. Thus, a global state is a
tuple \((s_e, s_{adv}, s_1, \ldots, s_n)\), where \(s_e\) is the local state of the environment, \(s_{adv}\) is the local state of the adversary, and \(s_i\) is the local state of agent \(i \in \{1, \ldots, n\}\). Intuitively, the local state of an agent represents the part of the global state that he can observe. Thus, if an adversary has the same local state in two global states \(s, s'\), then at state \(s\), he should consider state \(s'\) as a possible global state, since he can observe exactly the same local state in both cases. In other words, the basic possible worlds relation for the adversary (called an *indistinguishability relation* since it is based on the idea of distinguishing local states) we consider is the state-identity relation \(K^{local}\), which holds between two states if the adversary has the same local state in both states. Formally, \((s, s') \in K^{local}\) if only only if \(s_{adv} = s'_{adv}\). While we will find it useful to customize the indistinguishability relation of the adversary to control what he can observe, especially when dealing with cryptography, we shall always assume that the adversary cannot distinguish two states where he has the same local state. Thus, if \(K\) is an indistinguishability relation for the adversary, we have \((s, s') \in K\) whenever \(s_{adv} = s'_{adv}\), that is, \(K^{local} \subseteq K\). Putting this all together, we define an *adversarial frame* as a tuple \(F = (S, K)\), where \(S\) is a set of global states, and \(K\) is an indistinguishability relation for the adversary with \(K^{local} \subseteq K\). Similarly, an *adversarial structure* is a tuple \(M = (S, K, \pi)\) where \((S, K)\) is an adversarial frame, and \(\pi\) is an interpretation. Since adversarial structures are just epistemic structures, we can interpret an epistemic language over adversarial structures, where the knowledge operator captures the knowledge of the adversary.

We now explore how we can use this framework to explicate many definitions of confidentiality used in the security literature. As we shall see, all the definitions will be captured semantically, that is, by describing appropriate conditions on adversarial structures, as well as describing appropriate indistinguishability relations for the adversary. Moreover, we will give an interpretation to these semantic conditions in terms of formulas of an epistemic logic. Roughly speaking, this interpretation means the adversary never knows a particular class of formulas; these formulas represent properties of the data defined to be confidential.

### Confidentiality and Information Flow

A particular form of security is to ensure the confidentiality of information among users at different security levels. (This is often called *multilevel security*.) An example is the stereotypical classification of users (and data) in military systems, where security levels include “unclassified”, “classified”, and “top-secret”; users at a given level can access information marked at that level, and at lower levels. The model of the world is that these users share the same system, and the goal is to prevent the system from leaking information about the high-level secrets to lower levels. Generally, these security levels form a hierarchy [Denning 1976]. Consider the following example. A company operates a large computer network. Alice, the CEO, has access to all the data in the company. Bob, a consultant, uses the same system, but has restricted access. The company would like to ensure that Bob cannot gain any information about some of the high-level data that Alice enters in the system. That is, the company would like to prevent any sort of flow of information from high-level data to low-level users. In this setting, a confidentiality property specifies which flows of information are allowed, and which are forbidden. The most general form of confidentiality is to forbid any kind of information to flow from the high-level users to lower-level users. In the discussion that follows, I will suppose that there are only two classes of security levels, *high* and *low*, with the adversary being a low user; however, the ideas readily generalize to multiple security levels.

The general approach goes back to the notion of nondeducibility introduced by Sutherland [1986]. Halpern and O’Neill [2002] have formalized this notion (and others) using possible worlds
and epistemic logic. I describe one of their results in this section. To a first approximation, the intuition is that the low agent should not be able to rule out possibilities as far as the interesting part of the local state of the high agents is concerned. To capture the interesting part of the local state of the high agents, define an information function for agent $i$ to be a function $f$ on global states that depends only on agent $i$’s local state, that is, $f(s) = f(s')$ if $s_i = s'_i$.

If an information function for agent $i$ describes the interesting aspects of the local state of agent $i$ that he seeks to keep confidential, then we can define $f$-secrecy with respect to the adversary: agent $i$ maintains $f$-secrecy in an adversarial frame $F$ if for all global states $s$ and all values $v$ in the image of $f$,

$$ \mathcal{K}(s) \cap f^{-1}(v) \neq \emptyset. $$

In other words, the adversary considers all values of $f$ possible, at every global state.

This fairly simple semantic characterization can be captured syntactically, using the epistemic logic described earlier, in a way that relates to the adversary’s knowledge about the high-level state. Let $\Phi_0$ be an arbitrary set of primitive propositions. If $f$ is an information function for agent $i$, a formula $\phi$ in $M$ is said to be $f$-local if it depends only on the value of $f$, that is, whenever $f(s) = f(s')$, then $(M, s) \models \phi$ if and only if $(M, s') \models \phi$. Thus, in some sense, $\phi$ is a proposition that captures a property of the value of $f$. Of course, we have to account for the possibility that $\phi$ is completely trivial. Say $\phi$ is nontrivial in $M$ if there exists $s, s'$ such that $(M, s) \models \phi$ and $(M, s') \models \neg \phi$. The following result is proved by Halpern and O’Neill [2002].

**Theorem 1.** Let $F = (S, \mathcal{K})$ be an adversarial frame. Agent $i$ maintains $f$-secrecy in $F$ if and only if, for every interpretation $\pi$, if $\phi$ is $f$-local and nontrivial in $M = (S, \mathcal{K}, \pi)$ then $M \models \neg \mathcal{K}\phi$.

Of course, the characterization of confidentiality above is extremely strong. A more realistic form of confidentiality should allow some form of information to leak. Timing information, for example, might fit in this category. In general, given any state, the adversary should be able to rule out states for the high-level agents that are in the distant past, or the distant future. However, formalizing the fact that this kind of information flow is allowed is difficult in practice. It is not obvious how to distinguish allowed timing information flow from attacks that rely on timing channels [Wray 1991].

Similarly, there are cases where we must permit the declassification of some data in order for the system to be able to perform useful work. The classical example used in the literature is a password-checking program: a program that prompts the user for a password, and logs him or her in if the password is correct. Assume the password is a high-level piece of data. If an adversary tries to login with password $p$ and fails, he has gained information about the true password, namely, that it is not $p$. Most work in information flow in the past few years has aimed at understanding this notion of declassification of data [Pottier and Conchon 2000; Zdancewic and Myers 2001; Chong and Myers 2004].

Finally, there are many ways in which the above definitions are insufficiently precise. For one, they do not take the likelihood of states into account. Assume that the adversary initially believes that all the states of the agents are equally likely. If after some interaction with the system, the adversary still believes that all the high-level states are possible, but one is overwhelmingly more likely than the others, then one could easily argue that there has been information leakage, although the above definitions do not capture it. Handling these kind of flows requires more

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2Strictly speaking, $f$-secrecy is defined with respect to any agent, but we already established that we care only about the adversary in this article.
quantitative forms of information flow properties [Gray and Syverson 1998; Halpern and O’Neill 2002].

Confidentiality and Symbolic Cryptographic Protocol Analysis

One thing that the framework of the last section did not take into account is the use of *cryptography* to hide data from the adversary. Defining confidentiality in the presence of cryptography is more challenging. This form of confidentiality is sometimes studied in the literature when considering cryptographic protocols, that is, protocols between agents that aim at exchanging messages with some security guarantees, such as ensuring that the messages remain confidential, or that the origin of the messages is authenticated. In the information-flow setting, we were interested in reasoning about what the adversary could infer about the local state of the high agents. In this section, however, the adversary is allowed to intercept messages, as well as forward and inject new messages into the system. We are interested in reasoning about what the adversary can infer about the messages he has intercepted, despite them being perhaps encrypted. This will be reflected in the language used to capture the confidentiality specification: to capture information flow, the formulas involved in the specification are those whose truth depends on the local state of the other agents; for cryptographic protocol analysis, as we shall see, the formulas involved in the specification are those whose truth depends on the messages intercepted by the adversary.

There are a number of notions of confidentiality that have been studied in the cryptographic protocol analysis literature. A common one is based on the intuition that the adversary is not able to distinguish between states where the agents exchange message $m$ and states where the agents exchange some other message $m'$, for all messages $m$ and $m'$. This is the approach taken, for instance, in the spi calculus of Abadi and Gordon [1999]. Phrasing it this way brings us halfway to the framework of the last section; however, we need to take into account that the adversary should not be able to distinguish encrypted messages for which he does not have the corresponding decryption key. This requires a formalization of what the adversary can do to messages. The view we take in this section is that an adversary can do anything short of attempting to crack encrypted messages. Thus, we treat the particular encryption scheme used by the agents as perfect. (We weaken this assumption in the next section.) Such an adversary was first formalized by Dolev and Yao [1983]. Roughly speaking, a Dolev-Yao adversary can compose messages, replay them, or decrypt them if he knows the right keys. We first define a symbolic representation for messages, where we write $(m_1, m_2)$ for the pairing (or concatenation) of $m_1$ and $m_2$, and $\{m\}_k$ for the encryption of $m$ with $k$. We write $k^{-1}$ for the inverse key of $k$, that is, the key used to decrypt messages encrypted with $k$. We then define a relation $\vdash$, where $H \vdash m$ is interpreted as saying that the adversary can infer message $m$ from a set $H$ of messages. (Intuitively, $H$ is the set of messages he has intercepted). This relation is defined using the following inference rules:

$$
\frac{m \in H}{H \vdash m} \quad \frac{H \vdash \{m\}_k \quad H \vdash k^{-1}}{H \vdash m} \quad \frac{H \vdash (m_1, m_2) \quad H \vdash m_1}{H \vdash m_2} \quad \frac{H \vdash (m_1, m_2)}{H \vdash m_2}. 
$$

Thus, for instance, if an adversary intercepts the messages $\{m\}_{k_1}, \{k_1^{-1}\}_{k_2}$, and $k_2^{-1}$, he can derive $m$ using these inference rules, since

$$
\{\{m\}_{k_1}, \{k_1^{-1}\}_{k_2}, k_2^{-1}\} \vdash m.
$$

(The use of a symbolic representation for messages is the source of the name “symbolic cryptographic protocol analysis” given to this style of protocol analysis.)
We can define an indistinguishability relation by following the formalization of Abadi and Rogaway [2002]. (Similar ideas appear in Abadi and Tuttle [1991] and Syverson and van Oorschot [1994].) Intuitively, the adversary cannot distinguish two states if his local state is the same at both states, except that we identify encrypted messages for which he does not have the key, to capture the intuition that he cannot distinguish encrypted messages. For simplicity, assume that the local states $s_{adv}$ of the adversary simply consist of sets of messages (intuitively, the messages he has intercepted, along with any initial messages, such as public keys.)

Given a message $m$ and a set of keys $K$, let $\lfloor m \rfloor_K$ be the result of replacing every indecipherable message in $m$ by $\Box$. Formally, define

\[
\lfloor p \rfloor_K = p \\
\lfloor k \rfloor_K = k \\
\lfloor (m_1, m_2) \rfloor_K = (\lfloor m_1 \rfloor_K, \lfloor m_2 \rfloor_K) \\
\lfloor \{m\} \rfloor_K = \begin{cases} 
\lfloor m \rfloor_K & \text{if } k^{-1} \in K \\
\Box & \text{otherwise}
\end{cases}
\]

It is easy to check that $m'$ is a submessage of $\lfloor m \rfloor_K$ if and only if $K \cup \{m\} \vdash m'$. We extend $\lfloor - \rfloor$ to sets of messages $H$ by taking $\lfloor H \rfloor_K = \{ \lfloor m \rfloor_K \mid m \in H \}$. Define $\text{Keys}(H) = \{ k \mid H \vdash k \}$. Finally, define the indistinguishability relation $\mathcal{K}^{dy}$ by taking $(s, s') \in \mathcal{K}^{dy}$ if and only if $\lfloor s_{adv} \rfloor_K = \lfloor s'_{adv} \rfloor_{K'}$, where $K = \text{Keys}(s_{adv})$ and $K' = \text{Keys}(s'_{adv})$. In other words, the adversary cannot distinguish two states in which he has intercepted different messages, where the only difference between these messages occurs in the content of encrypted messages for which he does not have the decryption key.

We restrict our attention to message-transmission protocols, protocols in which the goal is for agent 1 to send a message to agent 2 in a confidential way. We assume that the adversary can intercept messages from the network, and can also forward and inject messages into the network. We can associate with a protocol $P$ a set $S_P$ of global states corresponding to the states that the protocol goes through upon execution (including states that result from the adversary intercepting, forwarding, or injecting messages). We assume that the global states in $S_P$ include states for all the possible messages that could be sent. If $\mathcal{M}$ is the set of all messages that could be sent, and $m \in \mathcal{M}$, let $G(m) \subseteq S_P$ be the set of global states where agent 1 sends message $m$ to agent 2. We say a message-transmission protocol $P$ preserves message secrecy if for all global states $s \in S_P$ and all messages $m \in \mathcal{M}$,

\[
\mathcal{K}^{dy}(s) \cap G(m) \neq \emptyset.
\]

In other words, every local state of the adversary is compatible with agent 1 having sent any possible message $m$.

Can we capture this syntactically? Let $\Phi_0$ be a primitive vocabulary. Say $\varphi$ depends only on the message exchanged by the protocol if $(M, s) \models \varphi$ if and only if $(M, s') \models \varphi$, whenever the same message is exchanged in both states $s$ and $s'$. The following result can be proved using techniques similar to those used to prove Theorem 1.

**Theorem 2.** A message-transmission protocol $P$ preserves message secrecy if and only if, for every interpretation $\pi$, if $\varphi$ depends only on the message exchanged by $P$ and is nontrivial in $M = (S_P, \mathcal{K}^{dy}, \pi)$ then $M \models \neg K \varphi$. 
An alternative approach, sometimes used in the literature, leads to a specification which is easier to enforce. This approach uses the $\vdash$ relation directly in the specification. This specification essentially says that the adversary cannot derive the content of the message being exchanged. (This is the approach taken, for instance, by Casper [Lowe 1998], a protocol analysis tool based on the CSP language [Hoare 1985].) Say a message-transmission protocol $P$ preserves message DY-secrecy if, at every state $s \in S_P$ where the message exchanged is $m$,

$$s_{adv} \not\vdash m.$$ 

This specification does not require an indistinguishability relation for the adversary, and this suggests that it can be captured by a specification that does not use knowledge. Indeed, the specification can be captured rather simply if we use the right language. As opposed to the way we have been specifying things until now, this time, we fix a particular vocabulary and a particular interpretation $\pi_0$. Let $\text{has}(m)$ be a fixed class of primitive propositions, one per message $m$, with $\text{has}(m) \in \pi_0(s)$ if and only if $s_{adv} \vdash m$. Let $\text{exchanged}(m)$ be a fixed class of primitive propositions, one per message $m$, with $\text{exchanged}(m) \in \pi_0(s)$ if and only if $m$ is the message exchanged by the protocol at state $s$. The following result follows immediately from the definition of message DY-secrecy.

**Theorem 3.** A message-transmission protocol $P$ preserves message DY-secrecy if and only if, for the model $M_0 = (S_P, K, \pi_0)$ and all messages $m$, $M_0 \models \text{exchanged}(m) \Rightarrow \neg \text{has}(m)$.

This specification does not use knowledge, and uses a particular model with a fixed interpretation. In fact, it can be seen as a form of safety property, following the classification of properties due to Alpern and Schneider [1985]. Roughly speaking, a safety property can be checked independently at all the points of the system; the truth or falsehood of a formula at a point does not depend on the other points of the system. This generally leads to efficient procedures for checking the specification. It is possible to refine the approach by considering more general ways for the adversary to derive messages, and to formally relate the results to specifications based on knowledge [Halpern and Pucella 2002].

**Confidentiality and Cryptography**

In the last section, the framework let us capture confidentiality in cryptographic protocols, under the assumption that the encryption was perfect; we did not allow the adversary to extract any information from an encrypted message for which he did not have the decryption key. Of course, in reality, encryption schemes are not perfect, and they can possibly leak information about the message being encrypted. In this section, we examine how we can capture the confidentiality of encryption schemes.

Cryptography studies, among others, the properties of encryption schemes. Modern cryptography is motivated by two basic tenets. First, encryption schemes are concrete mathematical systems that act on strings (often taken to be bit strings). This view leads naturally to finer confidentiality properties than simply showing that the adversary cannot recover the message being encrypted. Rather, confidentiality should mean that the adversary cannot derive any information about the message being encrypted, including, say, that the first bit of the message is a 1. The second tenet is that we do not impose any restriction on the computations that the adversary can perform on encrypted messages, aside from the fact that they must be feasible computations. Generally, the class of feasible computations is the class of probabilistic polynomial time algorithms [Motwani and
The definition of a probabilistic polynomial time algorithm is asymptotic: the running time of the algorithm is polynomial in the length of the input. Working with such a definition of feasibility is simplified by taking the encryption scheme itself to be defined asymptotically, where the parameterization is given by a security parameter. Intuitively, the larger the security parameter, the harder it is for an adversary to get information about encrypted messages.

The basic definition of confidentiality for an encryption scheme is that the adversary learns nothing about the content of an encrypted message (except possibly, for technical reasons, information about the length of the plaintext). The definitions we use are essentially due to Goldwasser and Micali [1984], but simplified following Goldreich [1998]. In particular, we assume a encryption scheme where the same key is used to encrypt and decrypt messages, and where the encryption is probabilistic: encryption with a given key yields a probability distribution over encrypted messages. We take \( E(x) \) to be the distribution of encryptions of \( x \), when the key is selected at random. Moreover, we assume that for a security parameter \( \eta \), the keys have length \( \eta \), and the scheme is used to encrypt messages of length \( \eta^2 \). Thus, we can simply take the security parameter to be the length of the keys. These restrictions, and the following definitions, are fairly technical, and I will refer to Goldreich [1998, 2001] for intuitions and more in-depth discussions.

The definition we use is that of indistinguishability of encryptions, which says that an adversary cannot distinguish, based on probabilistic polynomial time tests, whether two messages encrypted with a random key are the same message or not, even when provided with essentially arbitrary a priori information. Formally, let \( A \) be a feasible algorithm (which we assume returns 0 or 1). We say a sequence \( (x_\eta, y_\eta, z_\eta)_\eta \), where \( |x_\eta| = |y_\eta| = \eta^2 \) and \( |z_\eta| \) is polynomial in \( \eta \), is \( A \)-indistinguishable if

\[
1 - \Pr[A(E(x_\eta), z_\eta) = A(E(y_\eta), z_\eta)]
\]

is a negligible function of \( \eta \), where \( f(\eta) \) is negligible in \( \eta \) if for all polynomials \( p, f(\eta) \leq 1/p(\eta) \) for all \( \eta \). In other words, two sequences are \( A \)-indistinguishable if the adversary cannot really distinguish, based on the output of \( A \), whether an encrypted message is an encryption of \( x_\eta \) or of \( y_\eta \), even when provided with arbitrary information \( z_\eta \). (For instance, \( z_\eta \) could be the pair \( (x_\eta, y_\eta) \), meaning that even when the adversary knows that the encrypted message is the encryption of either \( x_\eta \) or \( y_\eta \), he cannot tell which is the actual message that was encrypted.) Note that we do not require the probabilities to be equal, but that the difference should not be noticeable by a polynomially-bounded observer.

An encryption scheme is semantically secure if, for all feasible algorithms \( A \), all sequences \( (x_\eta, y_\eta, z_\eta)_\eta \), where \( |x_\eta| = |y_\eta| = \eta^2 \) and \( |z_\eta| \) is polynomial in \( \eta \), are \( A \)-indistinguishable. One of the many achievements of modern cryptography is to show that there are encryption schemes that are semantically secure, assuming the existence of mathematical entities such as one-way functions [Goldreich 2001].

We can translate semantic security of an encryption scheme \( C \) into properties in an adversarial frame, where the states of the adversary are sequences of messages (indexed by the security parameter), along with some initial information. Formally, a local state for the adversary is a pair \( ((x_\eta)_\eta, (z_\eta)_\eta) \), where \( (x_\eta)_\eta \) is the sequence of messages to be encrypted, and \( (z_\eta)_\eta \) is the sequence of a priori information. Let \( S_C \) be the set of all states where the adversary has such a local state. Note that \( S_C \) does not directly model a particular protocol between agents; we are interested in modeling properties of an encryption scheme, not a protocol. To get an adversarial frame, we define an indistinguishability relation \( K^{\text{crypt}} \) as follows: take \( (s, s') \in K^{\text{crypt}} \) if and only if

\[ \text{strictly speaking, this is the definition of an encryption scheme having indistinguishability of encryptions, which can be shown to be equivalent to the traditional definition of semantic security.} \]
(x_\eta, y_\eta, z_\eta)_\eta is A-indistinguishable for every feasible algorithm A, where s_{adv} = ((x_\eta)_\eta, (z_\eta)_\eta) and s'_{adv} = ((y_\eta)_\eta, (z_\eta)_\eta).

We can, up to a point, capture semantic security of the encryption scheme using a knowledge specification. Let \Phi_0 be a primitive vocabulary. Say \varphi depends only on messages but not their length in M when the following properties hold:

1. if s_{adv} = s'_{adv}, then (M, s) \models \varphi if and only if (M, s') \models \varphi;
2. there exists s, s' with s_{adv} = ((x_\eta)_\eta, (z_\eta)_\eta), s'_{adv} = ((y_\eta)_\eta, (z'_\eta)_\eta), and |x_\eta| = |y_\eta| for all \eta, such that (M, s) \models \varphi and (M, s') \models \neg \varphi.

The following result follows almost immediately from the definition of semantic security.

**Theorem 4.** If an encryption scheme C is semantically secure, then, for every interpretations \pi, if \varphi depends only on messages but not on their length and is nontrivial in M = (S_C, K_crypt, \pi) then M \models \neg K \varphi.

This formalizes one intuition behind semantic security, namely that it ensures the adversary cannot derive any (nontrivial) knowledge about the content of encrypted messages, except perhaps their length. It is not clear how to get the other direction of the implication without making stronger assumptions on the language or the models.

This result is unsatisfying compared to the results of the previous section as far as it concerns reasoning about protocols. In particular, the states of the models are more “artificial”, and do not correspond directly to states that arise during the execution of a protocol. A more interesting result would be to characterize the knowledge of an adversary in the context of message-transmission protocols implemented using an encryption scheme with a property such as semantic security. This is an active research area. Some results have been obtained using techniques from programming languages [Lincoln, Mitchell, Mitchell, and Seedorf 1998; Abadi and Rogaway 2002], and logical techniques have been brought to bear on the question [Impagliazzo and Kapron 2003], but no connection to knowledge has yet been established, as far as I know.

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