Equivalent material parameters of mechanical joint

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Abstract: The characteristics of the mechanical joint is the significant element influencing the analyzing results of the whole structure. The stiffnesses of the mechanical joint are deduced from the relationship between the displacement and uniform distribution of the joint in this direction, which is obtained by the experiment. The equivalent material parameters of the mechanical joint, including the modulus of elasticity and the poisson ratio, are derived from the stiffness of the joint by adopting the Hertz theory, the contact theory of Hertz–Mindlin and the Greenwood and Williamson contact model. The paper take a structure containing mechanical joint as an example, and the computational model of the structure was set up by the finite element method. A comparison of the computational and testing results is shown to illustrate the feasibility and validity of the equivalent material parameters.

1. Introduction

The machine tool is a complete system, which is made up of multiple components connected with many joints. The characteristics of joints in a machine tool have an important effect on the static behaviors and dynamic behaviors of the machine tool. About 90 percent of the general damping, over 55 percent of the dynamic adaptability, 60–80 percent of the general dynamic stiffness and 85–90 percent of the static deformation of a machine tool originate from the joints[1,2]. The practical importance of mechanical joints has motivated a lot of theoretical and experimental studies.

The performance of mechanical joints had been widely researched in the theory and test for so many years. The original research of Hertz gave a result for the frictionless and non-adhesive contact, which composed of a single spherical convex or concave convex body with a flat surface. Greenwood and Williamson[3] introduced asperity-based models, which assumed the same spherical asperities with Gaussian distribution of the heights applied the Hertz contact theory to each convex independently. Majumdar[4] proposed the fractal parameters to represent the surface roughness of multiscale properties and set up the fractal contact model of the elastic–plastic rough surfaces. Kogut[5] established a static contact model of friction for the elasticplastic rough surfaces, which included the precise analysis of the elastoplastic contact, the adhesion, and the sliding initiation of rough surface of the statistical characterization. On account of the inverse relation between the function of the frequency response and the dynamic stiffness in the multiple-degree-of- freedom system, Mao[6] proposed a parameter identification way of the high precision. On base of the way, the test experimental data of the dynamic structure including the joint, which is used to gain the dynamic characteristic parameters of the joint. Shi[7] measured the mesoscale stiffness of the contact surface and damping of the contact surface on the base of the contact resonance, and the analytic expressions for stiffness of the contact surface and damping of the contact surface of the contact of Hertz and the exponential rough distribution of the
height were derived. Lee\cite{8} conducted experimental studies on the contact behavior of a nanoscale layered surface in micro-devices, and compared their experimental results with those from contact models with different statistically rough surfaces.

Based on the previous research findings, a new method is proposed where the theoretical and test data of the joint can be used directly for the numerical calculation. The equivalent characteristic parameters $E$ and $\mu$ of the joint are deduced. In addition, the effectiveness of the method is verified by selecting a structure including joint as an example. The calculated results are compared with the measured results. The accuracy and feasibility of the equivalent material parameters are evaluated in the study.

2. Characteristic of the mechanical joint

Suppose that a structure is set up of the component I and the component II. In the finite element analysis, the structure is divided into three parts, A, B and an mechanical joint, as shown in Figure 1. The characteristic parameters of the part A are the same as the characteristic parameters of the component I. The characteristic parameters of the part B are the same as the characteristic parameters of the component II. The characteristic parameters of the joint are not only related to the materials of two blocks associated with the joint, but also related to the pressure of the contact region, the status of the media between joint, the method of processing the contact surface, the roughness of the contact surface and so forth\cite{9}.

![Fig.1 A structure of joint](image)

Through the test, the relationship between the normal displacement and the mean compressive stress of the joint is expressed as

$$\lambda_n = c p_n^m$$  \hspace{1cm} (1)

in which $p_n$ is the mean compressive stress of the joint, $\lambda_n$ is the joint’s normal displacement, $c$ and $m$ are the characteristic parameters determined by the component material, the roughness of the contact surface, the medium between the contact surfaces, the method of the processing, and so forth.

Therefore, the joint’s stiffness in the normal direction is

$$k_n = \frac{dp_n}{d\lambda_n} = c^{-1} m^{-1} p_n^{1-m} = a_n p_n^{b_n}$$  \hspace{1cm} (2)

The joint’s stiffness in the tangential direction $k_\tau$ is gained by the similar method

$$k_\tau = a_\tau p_\tau^{b_\tau}$$  \hspace{1cm} (3)

in which $p_\tau$ is the compressive stress of the joint in the tangential direction, $\lambda_\tau$ is the joint’s deformation in the tangential direction. The relationship of $p_\tau - \lambda_\tau$ rests with $p_n$, $a_\tau$ and $b_\tau$. $a_\tau$ and $b_\tau$ are the tangential characteristic parameters of the joint, which gained by the compressive stress, the component material, the lubrication of the joint, the method of the processing, the roughness of the contact surface, and so forth.

3. Equivalent characteristic parameters of the mechanical joint

The rough surface of the joint could be described by the simplified model consisted of one rough surface and one flat plane\cite{3}. In the model, the single asperity of the rough surface is the spherical
bump of equal radius of curvature R, as shown in Figure 2. When beared by the normal load $P$, the spherical bump would have a normal deformation $\delta$.

![Fig.2 Contact of a spherical bump and a plane](image)

Based on the theory of Hertz, the relation between the load $P$ and the displacement $\delta$ in the normal direction is given by

$$P = \frac{4}{3} E_j R^\frac{1}{2} \delta^\frac{3}{2}$$

(4)

in which, $E_j$ is the joint’s modulus of the elasticity.

Thus, the normal stiffness of the single asperity in the normal direction is

$$k_n = \frac{dP}{d\delta} = 2E_j R^\frac{1}{2} \delta^\frac{1}{2}$$

(5)

The formula (7) can also be written as follows

$$E_j = k_n \sqrt{\frac{\pi}{a}}$$

(6)

in which, $a$ is the actual contact area of two parts.

According to the contact theory of Hertz–Mindlin and the Greenwood and Williamson contact model[9], $\mu_j$ is gained by

$$\mu_j = \frac{2k_n}{k_n - A}$$

(7)

In which, $\mu_j$ is the joint’s poisson ratio, $A$ is a constant related to contact conditions.

When the stiffnesses of the joint $k_n$ and $k_\tau$ are obtained by the test, the parameters of the joint $E_j$ and $\mu_j$ could be obtained from the formula (6) and (7).

4. Example

A structure including joint, as shown in Figure 3, consisted with two parts A and B. These parts are the steel component with size of $100 \times 50 \times 18$mm. The contact surfaces of part A and part B are polished, and the contact roughness of the component surface is $0.8\mu$m. The contact surface of the structure is the oil free. The part B is stationary, and the part A is born with the loads $F_t$ and $F_n$. $F_t$ is the tangential load, and $F_n$ is the normal load. The bearing point of the force is located at the left hand of the part A[10].
On base of the finite element method, a computational model of the structure is composed of three parts, as shown in Figure 4. The characteristic parameters of the component I are the same characteristic parameters of part A. The characteristic parameters of the component II are the same characteristic parameters of part B. The joint component is Part III. The characteristic parameters of the joint is gained from the test data of formula (6) and formula (7). Point ‘a’ is the place where the displacement transducer is placed.

A comparison between X direction calculation and test deformation under the tangential load $F_t$ can be shown in Figure 5, where $F_n = 0.5 \times 10^{-2}$ MPa in Fig.5(a) and $F_n = 2 \times 10^{-2}$ MPa in Fig.5(b).

Through the comparisons above, it shованed that the calculated deformations of the test point are very similar to experimental results. The equivalent parameters of the mechanical joint is accurate and effective, which are acquired by the experiment, the contact theory of Hertz–Mindilin and the Greenwood and Williamson contact model.
5. Conclusion
The mechanical joints are widely used in the engineering, the characteristics of joints are usually complex and nonlinear. The stiffness of the joint $k_n$ and $k_\tau$ are acquired by the method of theory and test. The equivalent characteristic parameters of the joint $E_j$ and $\mu_j$ are gained by the contact theory of Hertz–Mindlin and the Greenwood and Williamson contact model.

Numerical analysis and experimental study are carried out for an example of a structure including a joint. The X direction deformations of test points are surveyed and calculated. It shows that the calculated deformations results are very similar to the experimental deformations values by the comparison results, which indicating that the equivalent characteristic parameters of the joint are reliable. The way presented gains a new way of analyzing the performance of the mechanical structure including the joint in this paper.

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