The pseudo-SU(3) model has been extensively used to study normal parity bands in even-even and odd-mass heavy deformed nuclei. The use of a realistic Hamiltonian that mixes many SU(3) irreps has allowed for a successful description of energy spectra and electromagnetic transition strengths. While this model is powerful, there are situations in which the intruder states must be taken into account explicitly. The quasi-SU(3) symmetry is expected to complement the model, allowing for a description of nucleons occupying normal and intruder parity orbitals using a unified formalism.

1. Introduction
The SU(3) shell model\(^1\) has been successfully applied to a description of the properties of light nuclei, where a harmonic oscillator mean field and a residual quadrupole-quadrupole interaction can be used to describe dominant features of the nuclear spectra. However, the strong spin-orbit interaction renders the SU(3) truncation scheme useless in heavier nuclei, while at the
same time pseudo-spin emerges as a good symmetry\(^2\).

The pseudo-SU(3) model\(^2,3\) has been used to describe normal parity bands in heavy deformed nuclei. The scheme takes full advantage of the existence of pseudo-spin symmetry, which refers to the fact that single-particle orbitals with \(j = l - 1/2\) and \(j = (l - 2) + 1/2\) in the \(\eta\) shell lie close in energy and can therefore be labeled as pseudo-spin doublets with quantum numbers \(\tilde{j} = j\), \(\tilde{\eta} = \eta - 1\) and \(\tilde{l} = l - 1\). The origin of this symmetry has been traced back to the relativistic Dirac equation\(^4\).

A fully microscopic description of low-energy bands in even-even and odd-A nuclei has been developed using the pseudo-SU(3) model\(^5\). The first applications used pseudo-SU(3) as a dynamical symmetry, with a single irreducible representation (irrep) of SU(3) describing the yrast band up to the backbending region\(^5\). On the computational side, the development of a computer code to calculate reduced matrix elements of physical operators between different SU(3) irreps\(^6\) represented a breakthrough in the development of the pseudo-SU(3) model. With this code in place it was possible to include symmetry breaking terms in the interaction.

Once a basic understanding of the pseudo-SU(3) model was achieved and computer codes enabling its application developed, a powerful shell-model theory for a description of normal parity states in heavy deformed nuclei emerged. For example, the low-energy spectra and B(E2) and B(M1) electromagnetic transition strengths have been described in the even-even rare earth isotopes \(^{154}\)Sm, \(^{156,158,160}\)Gd, \(^{160,162,164}\)Dy and \(^{164,166,168}\)Er\(^7,8,9,10\) and in the odd-mass \(^{157}\)Gd, \(^{159,161}\)Tb, \(^{159,163}\)Dy, \(^{159,165}\)Eu, \(^{161,169}\)Tm, and \(^{165,167}\)Er nuclei\(^11,12,13\).

In the present contribution we review recent results obtained using a modern version of the pseudo-SU(3) formalism, which employs a realistic Hamiltonian with single-particle energies plus quadrupole-quadrupole and monopole pairing interactions with strengths taken from known systematics\(^11,12\). Its eigenstates are linear combinations of the coupled pseudo-SU(3) states. The quasi SU(3) approach for intruder states is also discussed, together with its implications regarding a unified description of a system with nucleons occupying normal and intruder parity orbitals.

### 2. The Pseudo SU(3) Basis and Hamiltonian

Many-particle states of \(n_\alpha\) active nucleons in a given normal parity shell \(\eta_\alpha\), \(\alpha = \nu\) (neutrons) or \(\pi\) (protons), can be classified by the group chain \(U(\Omega^N_\nu) \supset U(\Omega^{N/2}_\nu) \times U(2) \supset SU(3) \times SU(2) \supset SO(3) \times SU(2) \supset SU_J(2)\),
where each group in the chain has associated with it quantum numbers that characterize its irreps.

The most important configurations are those with highest spatial symmetry. This implies that \( \tilde{S}_{\pi,\nu} = 0 \) or \( 1/2 \), that is, the configurations with pseudo-spin zero for an even number of nucleons or \( 1/2 \) for an odd number are dominant. In some cases, particularly for odd-mass nuclei, states with \( \tilde{S}_{\pi} = 1 \) and \( \tilde{S}_{\nu} = 3/2 \) must also be taken into account, allowing for coupled proton-neutron states with total pseudo-spin \( \tilde{S} = 1/2, 3/2 \) or \( 5/2 \). Since pseudo-spin symmetry is close to an exact symmetry in the normal parity sector of the space, a strong truncation of the Hilbert space can be invoked. However, the pseudo spin-orbit partners are not exactly degenerate and this introduces a small pseudo-spin mixing in the nuclear wave function.

The Hamiltonian,

\[
H = H_{sp,\pi} + H_{sp,\nu} - \frac{1}{2} \chi \cdot \tilde{Q} - G_{\pi} H_{pair,\pi} - G_{\nu} H_{pair,\nu} + a K_{\pi}^2 + b J^2 + A_{sym} \tilde{C}_2, \tag{1}
\]

includes spherical Nilsson single-particle energies for \( \pi \) and \( \nu \) as well as the pairing and quadrupole-quadrupole interactions, with their strengths taken from systematics\(^{1,3,15}\). Only the parameters \( a \), \( b \) and \( A_{sym} \) are used fit to the data. A detailed description of each term in the Hamiltonian (1) can be found in Ref.\(^2\).

The electric quadrupole operator is expressed as\(^5\)

\[
Q_\mu = e_\pi Q_\pi + e_\nu Q_\nu \approx e_\pi \frac{\eta_\pi + 1}{\eta_\pi} \tilde{Q}_\pi + e_\nu \frac{\eta_\nu + 1}{\eta_\nu} \tilde{Q}_\nu, \tag{2}
\]

with effective charges \( e_\pi = 2.3 \) and \( e_\nu = 1.357 \). The magnetic dipole operator is

\[
T_\mu^1 = \sqrt{\frac{3}{4\pi}} \mu_N \{ g^{S\pi}_\pi L_\mu^\pi + g^{S\nu}_\pi L_\mu^\nu + g^{S\pi}_\nu L_\mu^\nu + g^{S\nu}_\nu L_\mu^\nu \} \tag{3}
\]

where the ‘quenched’ \( g \) factors for \( \pi \) and \( \nu \) are used. To evaluate the M1 transition operator between eigenstates of the Hamiltonian (1), the pseudo SU(3) tensorial expansion of the T1 operator (3)\(^{16}\) was employed.

3. Some representative results

The experimental and theoretical ground-, beta- and gamma-bands in \(^{166}\)Er are shown in Fig. 1. Having a close connection with the rotor Hamiltonian,
the pseudo SU(3) model is particularly well suited to describe these bands. The term proportional to $K^2$ allows the position of the gamma ($K_f = 2$) band-head to be fit, a particularly difficult task in many fermionic models.

Experimental and theoretical B(E2) transition strengths in $^{166}$Er are shown in Table 1. Effective charges used are $1.25e$ and $2.25e$. Transitions between states in the ground-state band are of the order of $e^2b^2$, while those from the $\gamma$- to ground-state bands are far smaller. The agreement with the experimental information is remarkable.

Table 1. Experimental and theoretical B(E2) transition strengths for $^{166}$Er.

| $J_i \rightarrow J_f$ | B(E2)$|e^2b^2 \times 10^{-2}|$ | Exp. | Th. |
|-----------------------|-----------------|------|------|
| $0_{gs} \rightarrow 2_{gs}$ | $580 \pm 27$ | $580$ |
| $2_{gs} \rightarrow 4_{gs}$ | $303 \pm 20$ | $299$ |
| $4_{gs} \rightarrow 6_{gs}$ | $273 \pm 35$ | $265$ |
| $6_{gs} \rightarrow 8_{gs}$ | $258 \pm 35$ | $251$ |
| $2_\gamma \rightarrow 4_{gs}$ | $0.363 \pm 0.027$ | $1.485$ |
| $2_\gamma \rightarrow 2_{gs}$ | $4.915 \pm 0.638$ | $10.310$ |
| $0_{gs} \rightarrow 2_\gamma$ | $15 \pm 1$ | $17$ |
| $4_{gs} \rightarrow 5_\gamma$ | $\geq 0.27$ | $3.22$ |
Fig. 2 shows the yrast and six excited normal parity bands in $^{163}\text{Dy}$. The integer numbers denote twice the angular momentum of each state. Experimental energies are plotted on the left-hand-side of each column, while their theoretical values are shown in the right-hand-side. These results should be compared with the three bands described in an earlier study\textsuperscript{13}, where the same Hamiltonian and parametrization was employed but the Hilbert space was restricted to $\tilde{S}_\pi = 0$ and $\tilde{S}_\nu = \frac{1}{2}$ states. The present description reproduces almost all the data reported for normal-parity bands in this nucleus.

Table 2 shows the B(E2) intra- and inter-band transition strengths for $^{167}\text{Er}$, in units of $e^2b^2 \times 10^{-2}$. Effective charges were 1.3e and 2.3e. Experimental data are shown with error bars in the figure and in parenthesis in the table. As usual, the intra-band transitions are in general two orders of magnitude larger than the inter-band transition strengths. In both cases the agreement with experiment is very good.
Table 2. Theoretical B(E2) transition strengths for $^{167}$Er.

| $J_i^- \rightarrow J_f^-$ | B(E2) | $J_i^- \rightarrow J_f^-$ | B(E2) |
|-------------------------|-------|-------------------------|-------|
| $1/2_i^- \rightarrow 3/2_i^-$ | 275   | $5/2^-_i \rightarrow 7/2^-_i$ | 310   |
| $3/2_i^- \rightarrow 5/2_i^-$ | 59    | $7/2^-_i \rightarrow 9/2^-_i$ | 252   |
| $5/2_i^- \rightarrow 7/2_i^-$ | 25    | $9/2^-_i \rightarrow 11/2^-_i$ | 186   |
| $7/2^-_i \rightarrow 9/2^-_i$ | 15    | $5/2^-_i \rightarrow 9/2^-_i$ | 100   |
| $9/2^-_i \rightarrow 11/2^-_i$ | 10    | $7/2^-_i \rightarrow 11/2^-_i$ | 151   |
| $11/2^-_i \rightarrow 13/2^-_i$ | 10    |                           |       |
| $1/2_i^- \rightarrow 5/2_i^-$ | 415   |                           |       |
| $3/2_i^- \rightarrow 7/2_i^-$ | 353   |                           |       |
| $5/2^-_i \rightarrow 9/2^-_i$ | 328   |                           |       |
| $7/2^-_i \rightarrow 11/2^-_i$ | 308   |                           |       |
| $9/2^-_i \rightarrow 13/2^-_i$ | 301   |                           |       |

The results shown above faithfully display the usefulness of the pseudo SU(3) model in the description of normal parity bands in heavy deformed nuclei. However, as already mentioned, the role of nucleons in intruder parity orbitals cannot be underestimated. The quasi SU(3) symmetry offers the possibility to describe them in similar terms as those occupying normal parity orbitals.

4. Quasi SU(3) symmetry

The “quasi SU(3)” symmetry, uncovered in realistic shell-model calculations in the pf-shell, describes the fact that in the case of well-deformed nuclei the quadrupole-quadrupole and spin-orbit interactions play a dominant role and pairing can be included as a perturbation. In terms of a SU(3) basis, it is shown that the ground-state band is built from the $S = 0$ leading irrep which couples strongly to the leading $S = 1$ irreps in the proton and neutron subspaces. Furthermore, the quadrupole-quadrupole interaction was found to give dominant weights to the so-called “stretched” coupled representations, which supports a strong SU(3)-dictated truncation of the model space.

The interplay between the quadrupole-quadrupole and the spin-orbit interaction has been studied in extensive shell-model calculations\textsuperscript{18} as well as in the SU(3) basis\textsuperscript{14}. In the former case\textsuperscript{15} the authors studied systems with four protons and four neutrons in the pf and sdg shells, and compared the mainly rotational spectra obtained in a full space diagonalization of the realistic KLS interaction with those obtained in a truncated space and a Hamiltonian containing only quadrupole-quadrupole and spin-orbit inter-
actions. They found that for realistic values of the parameter strengths
the overlap between the states obtained in the two calculations is always
better than $(0.95)^2$. They also found that while additional terms in the in-
teraction change the spectrum, the wave functions remain nearly the same,
suggesting that the differences can be accounted for in a perturbative way.

Following these ideas, a truncation scheme suitable for calculations in
a SU(3) basis was worked out. In contrast with what was done in, systems with both protons and neutrons were analyzed, and the interplay
of the quadrupole-quadrupole and spin-orbit interactions was emphasized,
while the pairing interaction was not included in the considerations for
building the Hilbert space.

The SU(3) strong-coupled proton-neutron basis span the complete shell-
model space, and represents an alternative way of enumerating it. In order
to define a definite truncation scheme that is meaningful for deformed nu-
clei, in we investigated the Hamiltonian

$$H = -\frac{\chi}{2} Q \cdot Q - C \sum_i \vec{l}_i \cdot \vec{s}_i$$  \hspace{1cm} (4)

where

$$Q = Q^\pi + Q^\nu; \quad Q^{\pi (\nu)} = \sqrt{\frac{16\pi}{5}} \sum_i r_i^2 Y_{2\mu}(\theta_i, \phi_i)$$  \hspace{1cm} (5)

is the total mass quadrupole operator, which is just the sum of the proton
(\pi) and neutron (\nu) mass quadrupole terms, restricted to work within one
oscillator shell, and \(\vec{l}_i, \vec{s}_i\) are the orbital angular momentum and spin of the
\(i\)-th nucleon, respectively. An attractive quadrupole-quadrupole Hamiltonian
classifies these basis states according to their \(C_2\) values, the larger the
\(C_2\) the lower the energy. The spin-orbit operator can be written as

$$\sum_i \vec{l}_i \cdot \vec{s}_i = -\frac{1}{2} \left[ \frac{(\eta + 3)!}{2(\eta - 1)!} \right]^{1/2} \left[ a_{(\eta,0)1/2}^\dagger \vec{a}_{(0,\eta)1/2} \right]_{(1,1) L=1, J=0}$$  \hspace{1cm} (6)

Results for \(^{22}\text{Ne}\) are presented in Table 3. Modern shell-model
calculations exhibit more mixing of SU(3) irreps than previous ones.
The ground-state band, often described as a pure (8,2) state, has impor-
tant mixing with the spin 1 (9,0) irrep. The \(J = 1\) state with dominant
(6,3) \(L = 1, S = 0\) components mixes strongly with (7,1) \(S = 0\) and others,
in agreement with the shell-model results.

Extensive calculation of the energy spectra and electromagnetic transi-
tions in many even-even, even-odd and odd-odd nuclei along the sd-shell


confirm that the quasi S(3) symmetry can be used as a useful truncation scheme even when the spin-orbit splitting is large.

5. Summary and Conclusions

A quantitative microscopic description of normal parity bands and their B(E2) intra- and inter-band strengths in many even-even and odd-mass heavy deformed nuclei has been obtained using a realistic Hamiltonian and a strongly truncated pseudo SU(3) Hilbert space, including in some cases pseudo-spin 1 states.

In light deformed nuclei the interplay between the quadrupole-quadrupole and spin-orbit interactions can be described in a Hilbert space built up with the leading S=0 and 1 proton and neutron irreps, in the stretched SU(3) coupling. In heavy deformed nuclei this quasi SU(3) truncation scheme will allow the description of nucleons occupying intruder single-particle orbits.

Using the pseudo + quasi SU(3) approach it should be possible to perform realistic shell-model calculations for deformed nuclei throughout the periodic table.

References

1. J. P. Elliott, Proc. Roy. Soc. London Ser. A 245, 128 (1958); 245, 562 (1958).
2. K. T. Hecht and A. Adler, Nucl. Phys. A 137, 129 (1969); A. Arima, et al. Phys. Lett. B 30, 517 (1969).
3. R. D. Ratna Raju, J. P. Draayer, and K. T. Hecht, Nucl. Phys. A 202, 433 (1973).
4. A. L. Blokhin, et. al., Phys. Rev. Lett. 74, 4149 (1995); J. N. Ginocchio, Phys. Rev. Lett. 78, 436 (1997); J. Meng, et. al. Phys. Rev. C 58, R632 (1998).
5. J. P. Draayer, et. al., Nucl. Phys. A 381, 1 (1982).
6. C. Bahri and J. P. Draayer, Comput. Phys. Commun. 83, 59 (1994).
7. T. Beuschel, J. P. Draayer, D. Rompf, and J. G. Hirsch, Phys. Rev. C 57, 1233 (1998); J. P. Draayer, T. Beuschel, D. Rompf, J. G. Hirsch, Rev. Mex. Fis. 44 Supl. 2, 70 (1998); ibid Phys. At. Nuclei 61, 1631 (1998); J. P. Draayer, T. Beuschel, and J. G. Hirsch, Jour. Phys. G - Nucl. Part. Phys. 25, 605 (1999).
8. T. Beuschel, J. G. Hirsch, and J. P. Draayer, Phys. Rev. C 61, 54307 (2000).
9. G. Popa, J. G. Hirsch, and J. P. Draayer, Phys. Rev. C 62, 064313 (2000).
10. J. P. Draayer, G. Popa, and J. G. Hirsch, Acta Phys. Pol. B 32, 2697 (2001); J. G. Hirsch, G. Popa, C. E. Vargas, and J. P. Draayer, Heavy Ion Physics 16, 291 (2002).
11. C. Vargas, J. G. Hirsch, T. Beuschel, and J. P. Draayer, Phys. Rev. C 61, 31301 (2000); J. G. Hirsch, C. E. Vargas, and J. P. Draayer, Rev. Mex. Fis. 46 Supl. 1, 54 (2000).
12. C. E. Vargas, J. G. Hirsch, and J. P. Draayer, Nucl. Phys. A 673, 219 (2000).
13. C. Vargas, J. G. Hirsch, and J. P. Draayer, Phys. Rev. C 66, 064309 (2002); ibid, Phys. Lett. B 551, 98 (2003).
14. C. Vargas, J. G. Hirsch, P. O. Hess, and J. P. Draayer, Phys. Rev. C 58, 1488 (1998).
15. P. Ring and P. Schuck. The Nuclear Many-Body Problem Springer, Berlin (1979); M. Dufour and A. P. Zuker, Phys. Rev. C 54, 1641 (1996).
16. O. Castanos, et. al., Ann. of Phys. 329, 290 (1987).
17. National Nuclear Data Center, http://bnlind2.dne.bnl.gov
18. A. P. Zuker, J. Retamosa, A. Poves, and E. Caurier, Phys. Rev. C 52, R1741 (1995).
19. J. Escher, C. Bahri, D. Troltenier, and J. P. Draayer, Nucl. Phys. A 633, 662 (1998).
20. J. Retamosa, J. M. Udías, A. Poves, and E. Moya de Guerra, Nucl. Phys A 511, 221 (1990).
21. Y. Akiyama, A. Arima, and T. Sebe, Nucl. Phys. A 138, 273 (1969).
22. C. E. Vargas, J. G. Hirsch, and J. P. Draayer, Nucl. Phys. A 690, 409 (2001); ibid. Nucl. Phys. A 697, 655 (2002).