ULTRAEFFICIENT INTERNAL SHOCKS

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ABSTRACT

Gamma-ray bursts are believed to originate from internal shocks that arise in an irregular relativistic wind. The process has been thought to be inefficient, converting only a few percent of the kinetic energy into gamma rays. We define ultraefficient internal shocks as those in which the fraction of emitted energy is larger than the fraction of energy given to the radiating electrons at each collision. We show that such a scenario is possible and even plausible. In our model, colliding shells that do not emit all their internal energy are reflected from each other, causing subsequent collisions and thereby allowing more energy to be emitted. As an example, we obtain about 60% overall efficiency even if the fraction of energy that goes to electrons is \( \epsilon_e = 0.1 \), provided that the shells’ Lorentz factor varies between 10 and \( 10^4 \). The numerical temporal profile reflects well the activity of the source that ejects the shells, though numerous collisions take place in this model.

Subject headings: gamma rays: bursts — relativity — shock waves

1. INTRODUCTION

A widely accepted mechanism for producing a cosmological gamma-ray burst (GRB) is the deceleration of relativistically expanding shells. The kinetic energy of the shells is converted into internal energy by relativistic shocks. These shocks can be due to collisions with the ambient medium (external shocks) or to shocks inside the shell itself due to nonuniform velocity (internal shocks). Electrons are heated by the shocks, and the internal energy is radiated via synchrotron and inverse Compton emission, with broken power-law spectra (see, e.g., Sari, Piran, & Narayan 1998).

Most bursts have a highly variable temporal profile with a variability timescale significantly shorter than the overall duration. In the external-shocks scenario this variability is due to irregularity in the surrounding material, but the efficiency is extremely low (Fenimore, Madras, & Nayakshin 1996; Sari & Piran 1997). Thus, GRBs are believed to be produced in the other way: internal shocks. The inner engine itself should be variable in this scenario, because the observed temporal profile follows very closely the operation of the source (Kobayashi, Piran, & Sari 1997, hereafter KPS97). Using a simple model, we have estimated that the hydrodynamic efficiency of this process (transforming kinetic energy to internal energy) is about 10%.

Kumar (1999) argues that the conversion efficiency from bulk motion to gamma radiation is only 1%. His argument is based on three points. (1) The hydrodynamic efficiency is, as mentioned above, typically 10%. (2) It is only electrons that are radiating. Even in equipartition among protons, magnetic field, and electrons, the electrons have only a third of the internal energy. (3) The amount of the radiated energy within the gamma-ray band is about one-third of the total. Combining these three factors gives the low efficiency of \( (1/10)(1/3)(1/3) = 1\% \).

Such a low efficiency results in severe energy demands on the source. Moreover, it is difficult to reconcile with the energy ratio between the GRB and its afterglow. According to the internal-external-shock model, the remaining kinetic energy not converted to radiation by internal shocks is radiated during the afterglow stage, external shock does not suffer from problem (1), and the energy released in the afterglow should be considerably higher than that in the GRB. However, it seems that the energy during the afterglow is only a tenth of that during the GRB, rather than 10 times as large (Frontera et. al. 2000; Kumar & Piran 2000; Freedman & Waxman 2001). Even though the observational constraints are not very good, because most of the energy is released at very early radiative stages when the afterglow is not observed, a factor of 10 more energy in the afterglow seems to be excluded.

A possible solution to these problems is to assume large angular fluctuation in the shells (Kumar & Piran 2000). This model provides clear predictions in the form of afterglow variability whose amplitude decays in time. It also predicts that the afterglow may sometimes be more energetic than the GRB.

In this paper, we suggest a simple alternative solution that overcomes the problems suggested by Kumar. We show that if the distribution of Lorentz factors is not uniform, but instead its logarithm is distributed uniformly, then the typical ratio of Lorentz factors between neighboring shells is considerably larger than predicted by all previous models. As a result, the hydrodynamic efficiency can be close to 100%, even for a reasonable spread of Lorentz factors. (A similar calculation was recently done by Beloborodov 2000) The main point of this paper is the possibility of “ultraefficient” internal shocks. We define ultraefficient internal shocks as those in which the emitted fraction of kinetic energy is larger than \( \epsilon_e \), the fraction of internal energy that goes into electrons (and is then radiated) at each collision. We will show that such a scenario is possible and even reasonable.

2. ULTRAEFFICIENT INTERNAL SHOCKS

Internal shocks can occur within a variable, relativistic wind produced by a highly variable source. We represent the irregular wind as a succession of relativistic shells with a random distribution of Lorentz factors, in a manner similar to that in KPS97. Beloborodov (2000) has shown that the
Internal shocks can convert most of the kinetic energy to internal energy if the fluctuation of the initial Lorentz factors, $A^2 = (\langle \gamma^2 \rangle - \langle \gamma \rangle^2)/\langle \gamma \rangle^2$, is large. Though it is maximally 1/3 if the initial Lorentz factors take random values between $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$, it is not limited if the distribution is uniform in logarithmic space between $\log \gamma_{\text{min}}$ and $\log \gamma_{\text{max}}$.

The efficiency can be estimated by an equation similar to equation (19) of KPS97. In the most efficient case, the masses of the shells are taken to be equal and the efficiency is given by a simple form,

$$\langle \epsilon \rangle \sim 1 - a^{1/2} \log a/(a - 1), \quad (1)$$

where $a = \gamma_{\text{max}}/\gamma_{\text{min}}$. The efficiency is plotted as a function of the fluctuation, $A^2 = -1 + (a + 1) \log a/(a - 1)$, in Figure 1. This analytic estimate fits the result of our numerical simulation and has the asymptotic form $\langle \epsilon \rangle = A^2/2$, which Beloborodov estimated for a small fluctuation. Our expression generalizes Beloborodov's estimate and also gives a reasonable estimate of the efficiency for large fluctuations.

Though we have seen that a large hydrodynamic efficiency is possible if the fluctuation of the initial Lorentz factors is large, it is not reasonable that all the internal energy is emitted after each collision, since electrons do not have most of the internal energy. Defining $\epsilon_\gamma$ as the fraction of energy given to electrons, we expect $\epsilon_\gamma < 1$. Even at equipartition with protons $\epsilon_\gamma = 1/2$. Under these circumstances, the total emitted energy in Beloborodov's model is still limited by $\epsilon_\gamma$.

However, if $\epsilon_\gamma < 1$, the merger produced by a collision is expected to stay hot after the emission. As a result, the merger will spread, transforming the remaining internal energy back to kinetic energy (Kumar 1999). A simplified description of this process is to assume that the two shells reflect with a smaller relative velocity. The difference in the kinetic energy before and after the collision is the emitted internal energy. The reflecting shells will collide into other neighboring shells. Since in this way a large number of collisions are caused, the overall efficiency from kinetic energy to radiation could be larger than $\epsilon_\gamma$. This is the key ingredient of our model. Before going on with this model, we present some hydrodynamic simulations, which show that these simplified assumptions are reasonable.

### 3. HYDRODYNAMIC SIMULATION

To estimate the conversion efficiency of the internal-shocks process, it is important to understand how high-velocity shells interact with slower ones and dissipate kinetic energy. We consider here a collision of two equal-mass shells with very different Lorentz factors, $\gamma/\gamma_s \gg 1$.

The rapid and slower shells are denoted by the subscripts $r$ and $s$, respectively. We assume that the widths of the shells are comparable in the interstellar medium (ISM) rest frame. This is a reasonable assumption, because in this frame the width is given directly by the "inner engine." Even so, the width of the rapid shell $l_r$ is much larger when viewed from the rest frame of the rapid shell $l_r/l_s \sim (\gamma_s/\gamma_r)^2/2$. Then, the dense "slower" shell with Lorentz factor $\gamma_s \sim \gamma_s/2\gamma_r$ collides with the low-density "rapid" shell in this frame, $\rho_s/\rho_r \sim \gamma_s/\gamma_r$. This is a planar analog of the evolution of a relativistic fireball (Sari & Piran 1995; Kobayashi, Piran, & Sari 1999). When the slower shell begins to interact with the rapid-shell material, two shocks are formed: a forward shock propagating into the rapid shell and a reverse shock propagating into the slower one. After the reverse shock crosses the slower shell, the profile of the shocked rapid-shell material approaches that of its fireball analog: a "blast wave" that sweeps and collects the surrounding material. Once the shock wave crosses the rapid shell, the hydrodynamical structure is as follows. There is a shocked rapid shell, the analog of the blast wave, and a shocked slow shell, which has cooled down and is the analog of the adiabatically cooling "fireball ejecta." The structure of such a system in the fireball case was studied by Kobayashi & Sari (2000).

Since $\gamma_s^2 \gg \rho_s/\rho_r$, the slow shell is considerably decelerated by the relativistic reverse shock, down to $\sim (\gamma_s/\gamma_r)^{3/4}$, and heated to a relativistic temperature at the crossing time $t_r \sim (\gamma_s/\gamma_r)^{3/4}l_r/c$. Then it cools adiabatically and follows a planar version of the Blandford & McKee (1976) solution (Sari 2001), in which a given fluid element evolves with a bulk Lorentz factor of $\gamma \propto t^{-1/2}$. Therefore, the motion becomes Newtonian ($\gamma \sim (\gamma_s/\gamma_r)^{3/4}(t_r/t_c)t^{-3/2} \sim 1$) when the forward shock crosses the rapid shell at $t_c = l_r/c$. On the other hand, since the forward shock itself evolves as $\gamma \propto t_c^{-1/2}$, it slows down to $\gamma \sim (\gamma_s/\gamma_r)^{1/2}/2$ at the crossing time $t_c$.

We have developed a relativistic code with an exact Riemann solver to solve relativistic hydrodynamics problems (Kobayashi et al. 1999). Using this code, we numerically study the collision of two equal-mass slabs with $\gamma_s = 10$, $\gamma_r = 10^5$, and the same width in the ISM frame. The initial condition in the rapid-shell comoving frame is as follows:

$$\hat{\gamma}_r = 1, \quad \hat{\gamma}_s \approx 50, \quad \rho_r = 1, \quad \rho_s = 100, \quad \hat{l}_r \approx 5000, \quad \hat{l}_s = 1.$$ 

The mass density outside the slabs and the homogeneous pressure are $\rho = 10^{-8}$ and $p = 10^{-10}$. The mass density and the pressure are measured in the comoving frame of each fluid. Our adiabatic simulations represent a case in which $\epsilon_\gamma$ is very small. In this simulation, we use $c = 1$. 

![Graph showing efficiency vs. A](image-url)
To compare the analytical estimates with the numerical simulation, we define the effective Lorentz factor of each shell as \( \left< \gamma \right> \equiv \int \gamma \rho dl / \int \rho dl \), with effective mass \( m = \gamma[\rho + (3 + \beta^2)\rho] \). The numerical simulation then gives \( \left< \gamma \right>_x \sim 5.3 \) and \( \left< \gamma \right>_y \sim 1.6 \) at the crossing time, values that are in agreement with the analytical estimates. The thin line in Figure 2a shows the numerical density profile at this time. After the forward shock crosses the rapid shell, a rarefaction wave begins to propagate into the rapid shell, transforming the internal energy to kinetic and accelerating the rapid shell to \( \gamma \sim \gamma/\gamma_x \). The center of mass moves with a Lorentz factor of \( \sim (\gamma_x/\gamma_y)^{1/2} \sim 5 \) in the rapid-shell comoving frame. At the end of the simulation, \( t = 10^5 \), about 40\% of the rapid-shell material is slower than the center of mass and goes with the slower shell (see Fig. 2b).

### 4. TWO-SHELL COLLISION

A collision of two shells is the elementary process in our model. A rapid shell catches up to a slower one and the two merge to temporarily form a single shell (denoted by the subscript \( m \)). Using conservation of energy and momentum, the Lorentz factor of the merged shell \( \gamma_m \) and the internal energy \( E_{\text{int}} \) produced by the collision are given by

\[
\gamma_m \sim \sqrt{\frac{m_x \gamma_x + m_y \gamma_y}{m_x \gamma_x + m_y \gamma_y}},
\]

\[
E_{\text{int}} = m_x (\gamma_x - \gamma_m) + m_y (\gamma_y - \gamma_m).
\tag{2}
\]

After a fraction \( \epsilon_c \) of the internal energy is emitted isotropically in the local frame of the merged shell (center-of-mass frame), the shells will spread, transforming the remaining internal energy back to kinetic energy. If the widths are the same in the center-of-mass frame, each mass is conserved before and after the collision. However, as we have seen, some fraction of the rapid-shell material generally goes with the slower shell after the collision. We parameterize the mass splitting as \( m_f = (1 - \delta)m_x \) and \( m_i = m_x + \delta m_y \). We show below that the total efficiency is not sensitive to \( \delta \).

The Lorentz factors of the reflected shells in the center-of-mass frame are given by

\[
\Gamma_r = \left[ M^2 + (m)^2 - (m_f)^2 \right] / 2m_f M,
\]

\[
\Gamma_s = \left[ M^2 + (m)^2 - (m_i)^2 \right] / 2m_i M,
\tag{3}
\]

where \( M = (m_x \gamma_x + m_y \gamma_y - \epsilon_c E_{\text{int}}/c^2)/\gamma_m \). The Lorentz factors in the laboratory frame are

\[
\Gamma_r = \Gamma_r \gamma_m - \sqrt{(\Gamma_r^2 - 1)(\gamma_m^2 - 1)},
\]

\[
\Gamma_s = \Gamma_s \gamma_m + \sqrt{(\Gamma_s^2 - 1)(\gamma_m^2 - 1)}.
\tag{4}
\]

The shells are compressed once by shocks, but they spread when reflecting. For simplicity, we assume that the width of the shells \( l_i \) is constant.

### 5. MULTIPLE-SHELL COLLISION

We consider a wind consisting of \( N \) shells. Each shell is characterized by four variables: \( \gamma_i, m_i, l_i, \) and \( R_i \). We assume that the initial Lorentz factor of each shell is distributed uniformly in logarithmic space between \( \log \gamma_{\text{min}} \) and \( \log \gamma_{\text{max}} \). The masses are assumed to be correlated with the Lorentz factors as \( m \propto \gamma^\eta \). For \( \eta = -1 \) and \( \eta = 0 \) the shells initially have equal energy and equal mass, respectively, and for \( \eta = 1 \) the shells initially have equal density under an assumption of equal shell width. We assume a constant value \( l \) for the initial widths and the initial separations between the shells. Then, the initial position of the shells is \( R_i = 2(i-1)l \).

The evolution of the system in time is basically the same as in KPS97, but equation (4) is used to calculate the Lorentz factor of the reflecting shells for the next time step. We follow the evolution of shells until there are no more collisions, i.e., until the shells are ordered by increasing value of the Lorentz factors.

The conversion efficiency from the kinetic energy of the shells to radiation can be calculated by using the initial and final kinetic energy as \( \left< \epsilon \right> = 1 - \frac{\sum \gamma_i^{f} \gamma_i^{l} / \sum \gamma_i^{l}}{\sum \gamma_i^{f} / \sum \gamma_i^{l}} \), where the superscripts \( f \) and \( l \) represent the initial and final values, respectively. It depends on the model parameters \( \gamma_{\text{max}}, \gamma_{\text{min}}, N, \eta, \epsilon, \) and \( \delta \) and on the specific realization: the set of random Lorentz factors assigned to each shell. For each choice of the parameters of the model, we have evaluated the efficiency for 100 realizations. The mean efficiency and its standard deviation are listed in Table 1 for the conserved-equal-mass case \((\eta = \delta = 0)\).

The efficiency approaches an asymptotic value as the Lorentz factor ratio \( \gamma_{\text{max}}/\gamma_{\text{min}} \) and the number of the shells \( N \) increase. The asymptotic value depends on \( \eta, \epsilon, \) and \( \delta \). The efficiency is plotted in Figure 3 as a function of \( \eta \) for \( \delta = 0, 0.4, \) and \( 0.8 \). In the range \(-1 \leq \eta \leq 1\), it is not very sensitive to \( \delta \) and peaks around \( \eta \sim 0 \), i.e., the most efficient case is the equal-mass case.

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**Fig. 2.** (a) Lorentz factor \( \gamma \) vs. distance \( x \) from the contact surface at \( t = 5000 \) (light line) and \( 6000 \) (heavy line). The Lorentz factor \( \gamma \) and position \( x \) are measured in the rapid shell frame, and the density \( \rho \) is the fluid local frame. (b) Mass fractions for \( \gamma > 5 \) (solid) and \( \gamma < 5 \) (dashed).
TABLE 1

EVALUATION OF EFFICIENCY

| N   | $\gamma_{\text{min}}$ | $\gamma_{\text{max}}$ | Efficiency ($\epsilon_e = 0.1$) (%) | Efficiency ($\epsilon_e = 0.5$) (%) |
|-----|------------------------|------------------------|-------------------------------------|-------------------------------------|
| 30  | $10^2$ | $10^3$ | $9.2 \pm 2.3$ | $16.2 \pm 3.0$ |
| 30  | $10^2$ | $10^4$ | $40.0 \pm 9.2$ | $67.5 \pm 9.3$ |
| $10^2$ | $10^3$ | $10^4$ | $15.1 \pm 1.5$ | $17.7 \pm 1.5$ |

The efficiency is plotted as a function of $\epsilon_e$ for the conserved-equal-mass case ($\eta = \delta = 0$) in Figure 4. It is interesting that the efficiency can be larger than $\epsilon_e$ because the energy released in GRBs can be much larger than that in the afterglow. For instance, in the case of $N = 100$, $\gamma_{\text{min}} = 10^2$, $\gamma_{\text{max}} = 10^4$, $\eta = \delta = 0$, and $\epsilon_e = 0.1$, the internal shocks can convert about 60% of the kinetic energy to radiation. The remainder is converted to thermal energy by external shock. However, only a fraction ($\epsilon_e = 0.1$) of that is emitted as the afterglow, and the efficiency of the external shock is only 4%. The ratio of energy from the GRB to that from the afterglow for these parameters is about 15!

Figure 5 shows the resulting temporal structure in the equal-mass case. It is a superposition of pulses from the elementary two-shell collisions. Though numerous collisions take place during the evolution, the number of peaks in the profile is of order $N$. Since all peaks have widths of the same order of magnitude, the differing amplitude of the peaks originates mainly from the difference in the internal energy produced by the collisions. In Figure 5b, we plot the initial Lorentz factor as a function of the time when shells were emitted by the source. We evaluate $E_{\text{int}}$ for all pairs of shells if the inner shell is faster. We assign to the ejection time of the inner shell (see Fig. 5c). The result resembles well the temporal profile in Fig. 5a. Therefore, despite the complicated nature of the two-shell collisions, the observed burst closely follows the inner-engine temporal profile.

In fact, Sari (1997) has shown that also during the afterglow the fraction of energy that can be emitted may exceed $\epsilon_e$. However, this will be spread over many decades of time.
Neighboring shells collide on a time scale of $2\gamma^2 l/c$. The matter is moving toward the observer, and the resulting observer time scale is $l/c$. On the other hand, the difference in observer time due to the location of a given shell within the wind is of order $Nl/c$. We observe the pulses arising from the collisions according to their positions within the wind. In Figure 5d, we plot the shell’s index against the time when radiation from the shell is observed. Although the light curve in Figure 5a is the superposition of all pulses, in Figure 5d we plot only the pulses that are higher than 1/10 of the highest one. We can see a clear correlation between index and time; the temporal profile reflects the activity of the source.

The deviation from the correlation, i.e., some groups of circles in a line from upper left to lower right, is due to the fact that when a rapid inner shell collides with an outer neighbor, the boosted shell collides in turn with another neighbor farther out. Since the masses of the shells are equal, the Lorentz factors of the inner and outer shells are just switched at each collision, if the radiation is negligible. Then, if the initial Lorentz factor of a shell is peculiarly high, the index of the shell with the high Lorentz factor propagates outward. In this sequence, with the radiation loss the pulse from the collisions damps quickly, and the overall correlation is not destroyed.

The temporal structures for equal-energy cases ($\eta = -1$) are plotted in Figure 6. The merger at each collision is assumed to split into the original masses (Fig. 6a) or into the modified masses ($\delta = 0.4$; Fig. 6c). The initial distributions of $\gamma$ are the same as in Figure 5. For $\eta = -1$, $E_{\text{int}}$ takes almost the same value for most of the collisions (KPS97), so most of the peaks have comparable amplitudes. The profiles still reflect the activity of the source well.

### 6. CONCLUSIONS

We have shown that the conversion efficiency from the kinetic energy of relativistic shells to radiation can be close to 100% if the source produces shells of comparable masses with different Lorentz factors, especially when the logarithms of the Lorentz factors are distributed uniformly (see also Beloborodov 2000). It had been assumed in our previous work that the Lorentz factors themselves were distributed uniformly. With this distribution, the most efficient case is one in which the source produces shells with comparable energy rather than comparable masses, and the highest efficiency is less than 40% (KPS97).

However, this high efficiency is achieved assuming that all the internal energy is emitted at each collision. This is not reasonable, because only electrons radiate effectively, and they do not have all the internal energy. After the electrons radiate, a large amount of the internal energy remains in protons. Using a hydrodynamic simulation, it has been shown that the hot merger produced by a collision spreads, transforming the remaining internal energy back to kinetic. A simplified description of this process is that the shells reflect each other with a smaller relative velocity, after the collision. The difference in the kinetic energy gives the radiated internal energy.

Since the reflecting shell collides into the outer neighbor shell, the index of the shell with a high Lorentz factor propagates outward until the high value decays by the radiation loss. Such a shell might not go through many collisions, but its high “kinetic energy” does. Therefore, the internal shock process is very efficient, even if the fraction of internal energy emitted at each collision is small. Previously, the efficiency in the case of $\varepsilon_e < 1$ had been estimated as smaller by a factor of $\varepsilon_e$ than in the corresponding fully radiative case. Our ultraefficient internal-shocks scenario shows this to be a significant underestimate. Though the efficiency that we have estimated is bolometric efficiency, the efficiency from the kinetic energy of the shells to gamma radiation is also high if the fraction of the energy radiated in the BATSE band is not very small.

Numerous collisions occurred in our ultraefficient internal-shocks model, making the peak width wider than in the previous internal-shocks model. However, the number of main peaks is still almost the same as the number of shells that the source emitted. There is a strong correlation between the time at which we observe a pulse and the emission time of the corresponding shell from the source. This correlation persists even for a small-$\varepsilon_e$ case where a larger number of collisions happen. The temporal structure reproduces the activity of the source.

We have shown that the efficiency of the internal shock process is not limited by $\varepsilon_e$, while that of the external shock is. If a fraction $\zeta$ of the kinetic energy of an explosion is converted to radiation by internal shocks, all the remaining fraction is converted to thermal by external shock in the afterglow stage, and a fraction $\varepsilon_e$ of the thermal is emitted. Thus, we can roughly estimate the ratio between the energy released in afterglow and that in GRB as $\sim \varepsilon_e (1-\zeta)/\zeta$.

Assuming $\varepsilon_e = 0.1$, the ratio is $1/10$ for $\zeta = 0.5$ and...
decreases as $\zeta$ increases. If the efficiency of internal shocks is indeed very large, the luminosities of GRB and the afterglow are expected to be anticorrelated.

As a general argument, in the internal-shock model the luminosity distribution of multipeak bursts would be narrower than that of bursts with only few peaks (Kumar & Piran 2000), because the number of peaks is almost the number of the shells $N$, as we have shown. We have verified that the dispersion of the efficiency is proportional to $1/\sqrt{N}$ in our numerical model.

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