PHOTOSPERIC EMISSION FROM STRATIFIED JETS

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ABSTRACT

We explore photospheric emissions from stratified two-component jets, wherein a highly relativistic spine outflow is surrounded by a wider and less relativistic sheath outflow. Thermal photons are injected in regions of high optical depth and propagated until the photons escape at the photosphere. Because of the presence of shear in velocity (Lorentz factor) at the boundary of the spine and sheath region, a fraction of the injected photons are accelerated using a Fermi-like acceleration mechanism such that a high-energy power-law tail is formed in the resultant spectrum. We show, in particular, that if a velocity shear with a considerable variance in the bulk Lorentz factor is present, the high-energy part of observed gamma-ray bursts (GRBs) photon spectrum can be explained by this photon acceleration mechanism. We also show that the accelerated photons might also account for the origin of the extra-hard power-law component above the bump of the thermal-like peak seen in some peculiar bursts (e.g., GRB 090510, 090902B, 090926A). We demonstrate that time-integrated spectra can also reproduce the low-energy spectrum of GRBs consistently using a multi-temperature effect when time evolution of the outflow is considered. Last, we show that the empirical $E_p - \tau_p$ relation can be explained by differences in the outflow properties of individual sources.

Key words: gamma-ray burst: general – radiation mechanisms: thermal – radiative transfer – scattering

1. INTRODUCTION

Gamma-ray bursts (GRBs) are the most powerful explosions in the universe. Prompt emission of GRBs are observed mainly in the energy range of 10 keV to MeV and show rapid time variability in their lightcurves. Their spectra are often modeled by an empirical “Band” function (Band et al. 1993; Preece et al. 2000; Kaneko et al. 2006, 2008), which is a smoothly jointed broken power-law whose physical origin is not yet identified. The typical low- and high-energy photon indices are distributed around $\alpha_{ph} \sim -1$ and $\beta_{ph} \sim -2.5$, respectively, while the spectral peak (break) energy is distributed around $E_p \sim$ a few 100 keV.

The most widely discussed model for the prompt emission mechanism is the internal shock model (Rees & Meszaros 1994; Sari & Piran 1997). In this model, it is assumed that relativistically moving shells emanate from the central engine with diverse velocities, and shocks form as a result of collisions of the shells. Shocks are accompanied by particle acceleration, and eventually gamma rays are produced by relativistic electrons via synchrotron radiation. The internal shock model can naturally explain the observed rapid time variability in the lightcurve and non-thermal nature of the spectra. However, it is known that the model suffers from poor radiation efficiency, given that only the kinetic energy associated with the relative motion of shells can be released (Kobayashi et al. 1997; Lazzati et al. 1999; Guetta et al. 2001; Kino et al. 2004). This contradicts with observations that show high efficiencies of a few tens of percent (Fan & Piran 2006; Zhang et al. 2007). Another difficulty is the low-energy spectral index ($\alpha_{ph}$). A non-negligible fraction of observed GRBs show hard spectra at low energies that cannot be explained by synchrotron emission (Crider et al. 1997; Preece et al. 1998; Ghirlanda et al. 2003).

These well-known difficulties in the internal shock model have led researchers to reconsider photospheric emission (e.g., Thompson 1994; Eichler & Levinson 2000; Mészáros & Rees 2000; Rees & Mészáros 2005; Lazzati et al. 2009; Beloborodov 2011; Pe’er & Ryde 2011; Mizuta et al. 2011; Nagakura et al. 2011; Ruffini et al. 2013; Xu et al. 2012; Béguel et al. 2013; Lundman et al. 2013; Lazzati et al. 2013), which is a natural consequence of the original fireball model (Goodman 1986; Paczynski 1986). In this model, prompt gamma rays are released at the photosphere when the fireball becomes optically thin. The peak energy of the spectra is determined by the temperature at the photosphere. This model has an advantage in that high emission efficiencies can be naturally achieved. Strong observational support for this scenario has been provided by the recent detection of quasi-thermal emission by the Fermi Large Area Telescope (LAT), notably GRB 090902B (Abdo et al. 2009; Ryde et al. 2010, 2011; Pe’er et al. 2012).

However, although there are many advantages, the photospheric emission model must overcome several difficulties. The major flaw is reproduction of the observed broad non-thermal spectra, because the photons that originate from regions of very high optical depths are inevitably thermalized. Photospheric emission can, in principle, explain the hard low-energy spectra that cannot be accounted for by synchrotron emission, by considering a superposition of emission components that...
have different temperatures (multi-color blackbody). This is because the Rayleigh–Jeans part of the emission component has a much harder photon index ($\alpha_{\text{ph}} = 1$) than do those inferred in all observed GRBs. We note, however, that the existence of outflow models that can naturally account for the low-energy spectra is still debated. Recent theoretical studies that explore peak energies of photospheric emissions on the basis of spherical (one-dimensional) outflow models (e.g., Levinson 2012; Beloborodov 2013; Vurm et al. 2013) suggest that it is difficult to regulate the temperature of the emissions to be well below the typical observed peak energies $E_P$, particularly when dissipation is present (but see, e.g., Levinson & Globus 2013, for a mechanism that might overcome this difficulty). Although dissipative processes are not considered, an outflow structure in two dimensions suggested by Lundman et al. (2013) might be a key ingredient in resolving this difficulty. On the basis of hydrodynamical simulations of jet propagation (Zhang et al. 2003), Lundman et al. (2013) modeled an outflow with a Lorentz factor gradient in the lateral direction. By solving the transfer of photons within the jet, they showed that the low-energy spectra can be reproduced by the multi-color temperature effects as a result of the superposition of the photons released from different lateral positions that have various Lorentz factors.

In contrast, the high-energy part of the observed spectra is difficult to reproduce by the superposition of thermal emissions, because an unreasonably high temperature must be realized within the outflow. To overcome this difficulty, dissipative processes around the photosphere have been suggested in previous studies. Various mechanisms such as shocks (Pe’er et al. 2005, 2006; Ioka et al. 2007; Lazzati & Begelman 2010), magnetic reconnection (Giannios 2006; Giannios & Spruit 2007; Giannios 2008) and proton–neutron nuclear collisions (Beloborodov 2010; Vurm et al. 2011) have been proposed. All models are accompanied by the generation of relativistic electrons (and positrons) that upscatter the thermal photons to create the non-thermal spectra (Giannios 2012). However, it is uncertain whether dissipative processes can operate efficiently enough to deposit in relativistic electrons (and positrons) the copious amounts of energy needed to reproduce observations.

Although the details are not well studied, Fermi-like acceleration of photons at the shocks formed below the photosphere as a result of bulk Compton scattering can be regarded as an alternative mechanism for producing high-energy non-thermal spectra (Eichler 1994). Presence of relativistic electrons (and positrons) is not required in this scenario because the energy of the background fluid can be directly transferred to the photons as they propagate. Shocks formed below the photosphere are inevitably mediated by radiation (Levinson & Bromberg 2008). The properties of relativistic radiation mediated shocks have been explored in various astrophysical contexts (e.g., shock breakouts) including GRBs. Budnik et al. (2010) performed the most detailed and fully self-consistent analysis of relativistic radiation mediated shocks (see also Katz et al. 2010). Although not as sophisticated as Budnik et al. (2010), Bromberg et al. (2011) explored the properties of photon acceleration in the context of shocks within GRB jets. These studies have shown that the existence of accelerated photons above the thermal peak energy is an inherent feature in these shocks. However, Budnik et al. (2010) and Bromberg et al. (2011) assume a one-dimensional stationary planar shock in their analysis, and the obtained spectra are far from those that are typically observed. Therefore, for a more accurate estimation, a global photon transfer calculation that incorporates the multi-dimensional structure of the jet must be performed. Ioka et al. (2011) explored the properties of shocks that are formed in radiation-dominated jets. They claimed that photons that are accelerated at the shock can generate spectra that are close to the Band function. However, given that their argument is based on a simple analytical framework in which the transfer of photons is not solved, detailed calculations are required to form firmer conclusions.

While the previous studies focused on the shocks, any form of large velocity gradients that are present in the outflow can give rise to photon acceleration. For example, a Fermi-like mechanism can operate also when velocity shear exists in the transverse direction of the outflow, as in the case of the acceleration of the cosmic rays (e.g., Jokipii et al. 1989; Ostrowski 1990, 1998). GRB jets are likely to possess rich internal velocity structures in the transverse directions. For example, numerical studies that explore the launching mechanism of the relativistic jets suggest that the produced jets have transversely stratified structures in which the bulk Lorentz factor increases toward the jet axis (McKinney 2006; Nagataki et al. 2007; Nagataki 2009, 2011; McKinney & Blandford 2009). Because the highly magnetized jets seen in these simulations are cold, subsequent magnetic field dissipation must occur in order to produce a hot fireball which leads to an efficient photospheric emission. Although the flow structure after the dissipative process is highly uncertain, the stratified structure that originated at the central engine might still remain at this stage. Moreover, a rich velocity structure can also develop after the launching phase as the jets drill through the progenitor star envelopes. A large number of hydrodynamic simulations regarding the jet propagation show that large velocity gradients appear within the jet (in radial and transverse directions) as a result of the interaction with the stellar envelope (Zhang et al. 2003; Mizuta et al. 2006, 2011; Morsony et al. 2007; Lazzati et al. 2009; Nagakura et al. 2011). It is worth noting that large velocity gradient in the transverse directions produced by the strong recollimation shocks seen in these studies will likely provide an efficient acceleration site for photons as Levinson (2012) pointed out.

Motivated by this background, we explore the photon acceleration within a jet with velocity structures in the transverse direction and its effect on the resulting spectra of the photospheric emissions by solving the transfer of photons. As mentioned earlier, mainly focusing on the origin of the low-energy spectra, Lundman et al. (2013) carried out a similar study. By performing a photon-transfer calculation, they evaluated the photospheric emission from a relativistic jet that has a continuously decaying velocity profile in lateral direction $\Gamma \propto \theta^{-p}$ at the outer region. However, while they succeeded in reproducing the spectra below the peak energy, the imposed velocity gradient was not large enough to trigger efficient photon acceleration and, therefore, the high-energy part was too soft compared with the Band spectra. We aimed to investigate the cases with efficient photon accelerations, focusing on jets that have a velocity shear in the transverse direction. This can be considered as a limiting case of $p = \infty$ in Lundman et al.’s (2013) work, a parameter region that was not explored. As a first step, this study considers a simple stratified two-component jet structure wherein a...
fluid properties of the spine and sheath regions are described by the standard adiabatic fireball model (e.g., Piran 2004; Mészáros 2006). Here, we briefly review the fireball model and show how we determine the radial evolution of the Lorentz factor (velocity), $\Gamma$, and electron number density, $n_e$, which are necessary background fluid information for evaluating the photon transfer.

The dynamics of the fireball can be characterized by three independent parameters: the initial fireball radius, $r_i$, the kinetic luminosity, $L$, and the dimensionless entropy that characterizes the baryon loading, $\eta \equiv L/Mc^2$, where $M$ and $c$ are the mass outflow rate and the speed of light, respectively. The fireball initially accelerates as it expands by converting its internal energy into kinetic energy (acceleration phase). As a result, the bulk Lorentz factor of the flow increases with radius, $r$, as $\Gamma(r) \simeq (r/r_i)$. If $\eta$ is larger than the critical value given by $\eta_c = (\sigma_T L/8\pi^2 m_p c^3)^{1/4}$ (Piran 2004; Mészáros 2006), the fireball becomes transparent to radiation during the acceleration phase, where $\sigma_T$ and $m_p$ are the Thomson cross section and proton rest mass, respectively. In contrast, if $\eta < \eta_c$, the fireball becomes transparent after the acceleration has ceased and the flow asymptotically approaches a constant Lorentz factor $\Gamma = \eta$ (coasting phase). In this case, the acceleration continues up to the saturation radius, $r_s = \eta r_i$. In the present study, we focus on the latter case and consider the parameter space in which $\eta < \eta_c$ is satisfied in the spine and sheath regions. Hence, the radial evolution of Lorentz factor is given by the following:

$$\Gamma(r) = \begin{cases} \frac{r}{r_i} & \text{for } r \leq r_s, \\ \eta & \text{for } r > r_s. \end{cases} \quad (1)$$

In the relativistically expanding outflow, the electron number density in the comoving frame is given by the following:

$$n_e(r) = \frac{M}{4\pi r^2 m_p \Gamma \beta c} = \frac{L}{4\pi r^2 m_p \eta \Gamma \beta c^3}. \quad (2)$$

where $\beta$ is the velocity of the flow normalized by the speed of light. Here, we assumed that there are no strong dissipative processes in the outflow that might create copious pair plasma. Given that we consider the case of $\eta < \eta_c$, the electron number density decreases with radius as $n_e \propto r^{-3}$ below the saturation radius $(r \leq r_s)$ and as $n_e \propto r^{-2}$ at larger radii $(r > r_s)$.

Given the electron number density and bulk Lorentz factor of the flow, the optical depth to Thomson scatterings for the photons propagating in the radial direction to reach infinity can be evaluated as follows:

$$\tau(r) = \int_r^\infty \sigma_T n_e(r') \Gamma(r')(1 - \beta(r')) dr' \simeq \begin{cases} \frac{\tau_{ph}}{\tau} & \left[ 1 + \frac{1}{2} \left( \frac{\tau}{\tau_{ph}} \right)^2 \right] \right] \quad & \text{for } r \leq r_s, \\ \frac{\tau_{ph}}{\tau} & \quad & \text{for } r > r_s, \end{cases} \quad (3)$$

where we have assumed $\Gamma \gg 1$. Here, $\tau_{ph}$ is the photospheric radius that corresponds to the radius where the optical depth becomes unity ($\tau = 1$).

2.1.2. Spine-sheath Structure

The radial profiles of the fluid quantities ($n_e(r)$ and $\Gamma(r)$) in the spine and sheath regions are different because we impose...
different values on their fireball parameters \((r_1, L_1 \text{ and } \eta_1)\). In the present study, while we assume the same value for \(r_1\) in both regions, the dimensionless entropy of the spine, \(\eta_{0_1}\), is taken to be larger than that of the sheath, \(\eta_{1}\). Hence, the saturation radius in the spine region, \(r_{s0} = \eta_0 r_1\), is larger than that of the sheath, \(r_{s1} = \eta_1 r_1\), and the terminal Lorentz factor of the former (\(\eta_{0_1}\)) is larger than that of the latter (\(\eta_{1}\)). Regarding the evolution of the Lorentz factor in the spine and sheath, they have equal values \((\Gamma_{0}(r) = \Gamma_{1}(r) = r/r_1)\) up to \(r = r_{s1}\). At larger radii \((r > r_{s1})\), a velocity shear begins to develop and the difference in the Lorentz factor increases with radius up to \(r_{s0}\) because the spine is in the acceleration phase while the sheath is in the coasting phase \((\Gamma = \eta_{1})\). Thereafter, the spine also enters the coasting phase \((\Gamma_{0} = \eta_{0})\), and the difference in the Lorentz factor is constant. In determining the kinetic luminosities of the spine, \(L_{0}\), and sheath, \(L_{1}\), we assume that the mass outflow rate is equal in both regions \((L_0/\eta_0 = L_1/\eta_1)\). Therefore, kinetic luminosity of the spine is larger by a factor of \(\eta_0/\eta_1\). Under these assumptions, the photospheric radius in the sheath, \(r_{\text{phi}}\), is larger than that in the spine, \(r_{\text{phi}}\), by a factor of \((\eta_0/\eta_1)^2\) (see Equation (4)). Figure 1 shows a schematic diagram of our model.

Hereafter, the quantities corresponding to the spine and sheath regions are denoted by subscripts 0 and 1, respectively.

### 2.2. Photon Transfer in a Spine-sheath Jet

Having determined the background fluid properties \((\Gamma \text{ and } n_e)\), we evaluate the resultant photospheric emission by solving the propagation of photons that are injected far below the photosphere. The photon transfer is evaluated by performing a three-dimensional test particle Monte Carlo simulation. In GRB jets, opacity of photons is strongly dominated by the scatterings with electrons. Therefore, we neglect the absorption process and consider in our calculations only the scattering process by the electrons. Furthermore, we do not take into account the thermal motion of the electrons in evaluating the scattering for simplicity.

#### 2.2.1. Initial Condition

The photons are initially injected within the jet at the surface of a fixed radius where the velocity shear begins to develop \(r_{\text{inj}} = r_{s1}\). For the cases considered in this study, \(r_{\text{inj}}\) is always located far below the photosphere \((\tau(\theta_{\text{inj}}) \gg 1)\). Therefore, a tight coupling between the photons and matter is expected. For this reason, we can safely assume that the photons have an isotropic distribution with energy distribution given by a Planck distribution in the comoving frame. According to the fireball model, the radial evolution of the comoving temperature is given by the following:

\[
T'_{\text{inj}} = \begin{cases} 
\left(\frac{L}{4\pi r^2 c a}\right)^{1/4} \left(\frac{L}{4\pi r^2 c a}\right)^{-1} \left(\frac{\eta}{\eta_1}\right)^{1/2} & \text{for } r \leq r_s, \\
\left(\frac{L}{4\pi r^2 c a}\right)^{1/4} \left(\frac{L}{4\pi r^2 c a}\right)^{-1} \left(\frac{\eta}{\eta_1}\right)^{-2/3} & \text{for } r > r_s,
\end{cases}
\]

where \(a\) is the radiation constant. Hence, we adopt the temperature at the corresponding radius given by the aforementioned equation \(T'_{\text{inj}} = T'(r_{\text{inj}})\) for the comoving temperature of the injected photons. While the photons are isotropic in the comoving frame, they are strongly beamed in the laboratory frame as a result of the Doppler boosting effect. As a result of this effect, the radiation intensity of the blackbody emission in the laboratory frame is given by the following:

\[
I_{\text{inj}}(\nu) = D(\Gamma_{\text{inj}}, \theta_1^2) B_{\nu}(T'_{\text{inj}}, \nu/D(\Gamma_{\text{inj}}, \theta_1)),
\]

where \(\Gamma_{\text{inj}} = \Gamma(r_{\text{inj}})\) is the bulk Lorentz factor of the flow at \(r = r_{\text{inj}}\) determined from Equation (1). Here, \(B_{\nu}(T', \nu) = 2h\nu^3e^{-h\nu/k_{B}T'}/[\exp(h\nu/k_{B}T') - 1]^{-1}\) is the Planck function, where \(h\) and \(k_B\) are the Planck constant and the Boltzmann constant, respectively, and \(D(\Gamma, \theta_1) = \Gamma(1 - \cos\theta_1)^{-1}\) is the Doppler factor, where \(\theta_1\) is the angle between the photon-propagation direction and the fluid-velocity direction (radial direction). In our calculations, the initial propagation direction and frequency of the injected photons are drawn from a source of photons given by (6).

The results of our calculation are insensitive to the assumed position of the injection radius as long as \(r_{\text{inj}} \leq r_{s1}\) is satisfied. This is because, at a radius far below the photosphere \((\tau(r) \gg 1)\), the photon energy distribution evaluated by solving the photon transfer does not deviate from the Planck distribution if velocity shear is not present (see the next section), and its temperature evolution is well described by Equation (5). The temperature of the injected photons in the spine, \(T'_{\text{inj}}\), is higher than that in the sheath, \(T'_{\text{inj}}\), by a factor of \((\eta_0/\eta_1)^{1/2}\), given that the kinetic luminosity of the former is higher by a factor of \((\eta_0/\eta_1)^2\).

Correspondingly, the luminosity of the injected photons in the laboratory frame \((L_{\text{inj}} \propto r_{\text{inj}}^2 \Gamma_{\text{inj}}^2 T_{\text{inj}}^4)\) in the spine is higher than that of the sheath by a factor \((\eta_0/\eta_1)^2\).

#### 2.2.2. Boundary Conditions

After the photons are injected, we track their path in the jet in three dimensions by performing Monte Carlo simulations (Section 2.2.3) until they reach the outer or inner boundary of the calculation. Using spherical coordinates \((r, \theta, \phi)\), the outer boundary \(r\) is set at a radius, \(r_{\text{out}} = 500r_{\text{phi}}\), where the photons can be safely considered to have escaped given that the optical depth is \(\tau(r_{\text{out}}) = 2 \times 10^{-3} \ll 1\). While there is no boundary in the \(\phi\) direction, the outer boundary in the \(\theta\) direction is set at \(\theta_{\text{obs}} = \theta_{1}\), which corresponds to the edge of the whole jet. As for the inner boundary, we adopt a radius slightly below the injection radius \(r_{\text{inj}} = 0.5r_{\text{inj}}\). For photons that have reached the outer boundaries, we assume that they escape freely to \(r = \infty\) without being scattered or absorbed. In contrast, we assume that the photons are simply absorbed in the inner boundary. We note, however, that the fraction of absorbed photons is negligible because most of the photons in ultra-relativistic outflows are strongly collimated as a result of the relativistic beaming effect and essentially streamed outward (e.g., Pe'er & Ryde 2011; Beloborodov 2011).

The spectra of the emission are evaluated from the photons that have reached the outer boundaries. As a result of the relativistic beaming effect, these photons are highly anisotropic and concentrated mostly within a cone of half-opening angle \(\sim \Gamma^{-1}\) in the direction of the fluid velocity (radial direction) at the last scattering position. Hence, the observed emission spectra depend significantly on the angle between the direction to the observer and the jet axis, \(\theta_{\text{obs}}\). In the present study, we evaluate the spectrum for observers located at direction \(\theta_{\text{obs}}\) by recording all photons that have reached the outer boundary and that are propagating in the direction within a cone of half-opening angle \(\Gamma^{-1}\), which is small enough to regard that the emission is uniform within the cone. From the recorded photon flux, we calculate the isotropic-equivalent luminosity by multiplying the photon.
flux by a factor $4\pi/d\Omega$, where $d\Omega = 2\pi[1 - \cos(1/\Gamma)]$ is the solid angle of the cone.

### 2.2.3. Monte Carlo Simulation for Solving Photon Transfer

Here, we briefly describe the Monte Carlo code used to solve the photon transfer. As noted previously, we neglect the thermal motions of the electrons and take into account only the scattering processes. Hence, the rest-frame of the fluid is equivalent to that of the electrons. Under the aforementioned assumptions, the propagation of the photons is performed by directly tracking the path of the individual photons in the three-dimensional space of the calculation. Each photon has a specified position, propagation direction and frequency, and these quantities are updated by using a uniform random number.

Within the jet, the photons travel along straight paths before they are scattered by the electrons. First, the code determines the distance for the photons to travel before the scattering by drawing the corresponding optical depth $\delta \tau$. The probability for the selected optical depth to be in the range of $[\delta \tau, \delta \tau + d\delta \tau]$ is given as $\exp(-\delta \tau d\tau)$. Then, from the given optical depth $\delta \tau$, the distance $l$ to the scattering event is determined from the integration along the straight path of photons, which can be expressed as follows:

$$\delta \tau = \int_0^l n_e \Gamma (1 - \beta \cos \theta_v) \sigma_{sc} dl, \quad (7)$$

where $\theta_v$ is the angle between the direction of fluid velocity and photon. Here, $\sigma_{sc}$ is the total cross section for the electron scattering and is given as follows:

$$\sigma_{sc} = \begin{cases} 
\sigma_T & \text{for } h\nu_{cmf} \leq 100 \text{ keV}, \\
\sigma_{KN} & \text{for } h\nu_{cmf} > 100 \text{ keV}, 
\end{cases} \quad (8)$$

in our code, where $\sigma_{KN}$ is the total cross section for Compton scattering, and $\nu_{cmf}$ is the frequency of the photon in electron (fluid) comoving frame. (The frequency $\nu_{cmf}$ is evaluated by performing a Lorentz transformation using local fluid velocity.) Given the distance $l$ from the above equation, we update the position of the photons to the scattering position by shifting them from the initial position with the given distance in the initial direction of photon propagation. Unlike the case of Equation (3), the optical depth calculated by Equation (7) is not limited to photons propagating in the radial direction. The path of integration is along the straight path of photons, which can travel in an arbitrary direction. For a given value of $\delta \tau$, the distance $l$ strongly depends on the propagation direction of the photons in the case of a relativistic flow ($\Gamma \gg 1$). As is obvious from the above equation, the mean free path of photons $l_{mfp} = [n_e \Gamma (1 - \beta \cos \theta_v)]^{-1}$ is sensitive to the photon-propagation direction, given that the factor $\Gamma (1 - \beta \cos \theta_v)$ varies largely from $\sim (2\Gamma)^{-1}$ (for $\cos \theta_v = -1$) up to $\sim 2\Gamma$ (for $\cos \theta_v = 1$), depending on the value of $\theta_v$. Hence, a photon tends to travel a larger distance in the fluid-velocity (radial) direction because the mean free path of the photon tends to be larger. Hereafter, quantities measured in the comoving frame of the fluid (electron) are denoted by a tilde.

In evaluating the integration in Equation (7), we use two different methods depending on the frequency and position of the photon. For photons located above the saturation radius $r \geq r_s$ ($\Gamma = \text{const}$) that satisfy $h\nu_{cmf} \leq 100 \text{ keV}$ ($\sigma_{sc} = \sigma_T = \text{const}$), analytical integration can be performed, as shown by Pe’er (2008). Consider a photon path originating from a radius $r_1$ that has an angle $\theta_{v,1}$ with respect to the fluid-velocity (radial) direction at the original position. In this case, the optical depth to reach a radius $r_\Pi$ along the straight photon path can be expressed as follows:

$$\Delta \tau_{r_\Pi} = 2\Gamma^2 r_{ph} \left[ \frac{\theta_{v,\Pi}}{r_\Pi \sin \theta_{v,\Pi}} - \frac{\theta_{v,1}}{r_1 \sin \theta_{v,1}} + \beta \left( \frac{1}{r_1} - \frac{1}{r_\Pi} \right) \right]$$

$$= 2\Gamma^2 r_{ph} \left[ \frac{\theta_{v,\Pi} - \theta_{v,1}}{r_1 \sin \theta_{v,1}} + \beta \left( 1 - \frac{\cos \theta_{v, \Pi}}{\cos \theta_{v,1}} \right) \right], \quad (9)$$

where $\theta_{v,\Pi}$ is the photon angle at the final position ($r = r_\Pi$). In the second equality, we have used the relation $r_1 \sin \theta_{v,1} = r_\Pi \sin \theta_{v,\Pi}$ that holds for an arbitrary straight line. Hence, in this case, we determine the corresponding propagation length $\Delta \tau_{r_\Pi} = r_{ph} \cos \theta_{v,\Pi} - r_1 \cos \theta_{v,1}$ from the drawn optical depth by solving Equation (9). (Equation (9) is solved separately in the spine and sheath region because the values of $r_{ph}$ and $\Gamma (\beta)$ are different in each region.)

In contrast, when the photons have higher energies ($h\nu_{cmf} > 100 \text{ keV}$) or are located below the saturation radius ($r < r_s$), the integration is solved numerically. In this case, we divide the calculation region into a mesh in spherical coordinates ($r, \theta, \phi$). Then, the integration is conducted by assuming that the physical quantities (velocity and number density) within the individual mesh are uniform and have the values corresponding to those at the position of the mesh center. We adopt 500 grid points, which are logarithmically spaced for the mesh in the $r$ coordinate, we adopt 800 uniformly spaced grid points in the range $r_\Pi \leq r \leq r_\Pi$. As for the mesh in the $\theta$ coordinate, we adopt 1000 uniformly spaced grid points for the $\phi$ coordinate ($0 \leq \phi < 2\pi$). The resolution of the grid is sufficiently high to reproduce the result obtained by the analytical solution given by Equation (9) (corresponding to infinite resolution) in cases when $r \geq r_s$ and $h\nu_{cmf} \leq 100 \text{ keV}$ are satisfied.

Given the position for the scattering from the aforementioned procedure, the four-momentum (the energy and propagation direction) of a photon after the scattering is determined based on the differential cross section for Thomson and Compton scattering. In our code, the scattering process is evaluated in the rest-frame of the fluid (electron). First, the four-momentum of the photon before the scattering is Lorentz-transformed into the fluid rest-frame. For the photons that satisfy $h\nu_{cmf} \leq 100 \text{ keV}$, the differential Thompson cross-section is used, while the differential Compton cross-section is used at higher energies ($h\nu_{cmf} > 100 \text{ keV}$). The scattering angle—or, equivalently, the propagation direction of the outgoing photon in the fluid rest-frame—is drawn from the differential cross-sections. Regarding the energy of the outgoing photons, we assume that it is conserved before and after the scattering (elastic scattering) in the case of Thomson scattering ($h\nu_{cmf} \leq 100 \text{ keV}$). In contrast, in the case of Compton scattering ($h\nu_{cmf} > 100 \text{ keV}$), energy loss resulting from the recoil effect is properly taken into account. The code then Lorentz-transforms the outgoing photon four-momentum back into the laboratory frame.

The aforementioned procedure is repeated until all the injected photons reach the boundary of the simulation grids.

### 3. RESULTS

In this section, we show the obtained photon spectra on the basis of the model described in the previous section. We inject $N = 2 \times 10^8$ photon packets in each calculation. In all cases, we use a fixed value of $\theta_1 = 1^\circ$ for the half-opening angle of the jet.
which is smaller than that of the typically observed values. We emphasize, however, that the resulting spectra do not vary for wider jets (larger $\theta_0$) as long as the observer angle stays in the range $\theta_{\text{obs}} \lesssim \theta_0 - \eta_0^{-1} \sim 0.86$, since the fluid properties within the beaming cone $(\theta - \theta_{\text{obs}}) \lesssim 0.14 \eta_0^{-1} \theta_0$ do not change. In the figure, we also show an analytical solution for the expected emission when the photons are in complete thermal equilibrium up to a radius $r$, which can be obtained as follows:

$$L_\nu(\nu) = 8\pi^2 r^2 \int_{\cos \theta_i}^{1} D(\nu_0, \theta) \beta_r(T'(r), \nu/\nu_0, \theta) \cos \theta \, d \cos \theta,$$

(10)

where $\theta$ is the angle between the line of sight and fluid-velocity (radial) direction. The green, blue, and purple solid lines correspond to the solutions for $r = r_{\text{ph}}(\tau = 1)$, $r = r_{\text{ph}}/2 (\tau = 2)$, and $r = r_{\text{ph}}/4 (\tau = 4)$, respectively. Regarding the case of $r = r_{\text{ph}}$, the peak energy, $E_p$, and luminosity, $L_p \sim \nu_p L_\nu(\nu_p)$, where $\nu_p = E_p/c$. The obtained spectrum agree well with the rough estimate given earlier. As for the cases of $r = r_{\text{ph}}/2$ and $r = r_{\text{ph}}/4$, the peak energy and the luminosity are larger by a factor $\sim 2^{3/2} \sim 1.6$ and $\sim 4^{3/2} \sim 2.5$ than the case of $r = r_{\text{ph}}$, respectively, because the temperatures at these radii are larger by the same factor. As shown in the figure, the peak luminosity of the numerical result agrees well with that of the analytical estimate for $r = r_{\text{ph}}$. In contrast, the spectrum extends up to higher energies and the peak energy is close to that for $r = r_{\text{ph}}/2$ because the coupling between the photon and matter is not complete near the photosphere $\tau \sim 1$, as shown in previous studies (Pe'er 2008; Beloborodov 2011; Béguel et al. 2013). As a result, photons that decouple with the matter at moderate optical depth $(\tau \lesssim 5)$ are observed at higher energies.

Regarding the shape of the spectrum, while the emission is dominated by photons that escaped from the on-axis region ($\theta \lesssim \Gamma^{-1}$) at energies near the peak energy and above, the low-energy part ($\nu \ll \nu_\text{p}$) is dominated by those from the off-axis region. The off-axis component becomes prominent at low energies because the Doppler factor is smaller, which leads to a lower peak photon energy $\sim 4D(\nu_0, \theta) \Gamma k_B T'$. Therefore, the low-energy part of the spectrum can be approximated as a superposition of blackbody spectra from the off-axis region that have different peak energies (multi-color blackbody).

As a result, the low-energy slope of the spectra is somewhat softer than that expected from the Rayleigh–Jeans part of a single blackbody ($\nu L_{\nu} \propto \nu^2$) and can be approximated as roughly $\nu L_{\nu} \propto \nu^{2.4}$.

3.1 Uniform (Non-stratified) Jet

Before we look at a stratified jet, we first present results for a one-component uniform jet that does not have structures in the $\theta$ direction ($\theta_0 = \theta_1 = 1'$). In this case, a thermal spectrum is expected, in contrast with a stratified jet, which we discuss later. The isotropic equivalent kinetic luminosity and the dimensionless entropy (terminal Lorentz factor) are set to be $L_0 = 10^{53} \text{ erg s}^{-1}$ and $\eta_0 = 400$, respectively. As described in the previous section, we inject the photons at a radius of $r_{\text{inj}} = 4 \times 10^{10} \eta_0, 400 L_{0,53}^{1/8} \text{ cm}$ with intensity given by a blackbody of temperature $k_B T_{\text{inj}} = 1.7 r_{1,8}^{-1/2} \eta_0^{-1} L_{0,53}^{1/4} \text{ keV}$ (see Section 2 for details), where $\nu_0 = 400/L_{0,53} = 10^{53} \text{ erg s}^{-1}$ and $r_{1,8} = 10^8 \text{ cm}$. The corresponding optical depth at the injection radius is $\tau(r_{\text{inj}}) \sim 23$. The results are insensitive to the value of $r_{\text{inj}}$ as long as $\tau(\alpha) > 1$ is satisfied, as noted in Section 2. The advected photons lose energy adiabatically because of the expansion of the flow during the coupling with matter becomes weak near the photosphere ($r_{\text{ph}} \sim 9.2 \times 10^{11} L_{0,53}^{3/8} \theta_0^{-3/4}$). The expected temperature at the photosphere can be calculated as $k_B T_{\text{ph}}' = 0.38 \nu_0^{1/8} L_{0,53}^{5/8} \eta_0^{3/12} \text{ keV}$.

The observed peak energy of the photospheric emission is expected to be $E_p \sim 8 \eta_0 k_B T_{\text{ph}}' \sim 660 \nu_0^{1/8} L_{0,53}^{5/12} \text{ keV}$ because the photon energy is boosted by factor $D \sim 2 \eta_0$ as a result of the Doppler effect. Also, the luminosity of the emission can be estimated as $L_p \sim L_{\nu}(r_{\text{ph}}/r_\text{ph})^{3/2} \sim 1.2 \times 10^{52} \nu_0^{3/8} L_{0,53}^{1/8} \theta_0^{-1/8} \text{ erg s}^{-1}$.

In Figure 2, a red solid line denotes the numerical result. Here, we assume that the observer is aligned to the jet axis ($\theta_{\text{obs}} = 0'$). We note, however, that the result does not change if the observer angle stays in the range $\theta_{\text{obs}} \lesssim \theta_1 - \eta_0^{-1} \sim 0.86$, since the fluid properties within the beaming cone $(\theta - \theta_{\text{obs}}) \lesssim 0.14 \eta_0^{-1} \theta_0$ do not change. In the figure, we also show an analytical solution for the expected emission when the photons are in complete thermal equilibrium up to a radius $r$, which can be obtained as follows:

$$L_\nu(\nu) = 8\pi^2 r^2 \int_{\cos \theta_i}^{1} D(\nu_0, \theta) \beta_r(T'(r), \nu/\nu_0, \theta) \cos \theta \, d \cos \theta,$$

(10)

where $\theta$ is the angle between the line of sight and fluid-velocity (radial) direction. The green, blue, and purple solid lines correspond to the solutions for $r = r_{\text{ph}}(\tau = 1)$, $r = r_{\text{ph}}/2 (\tau = 2)$, and $r = r_{\text{ph}}/4 (\tau = 4)$, respectively. Regarding the case of $r = r_{\text{ph}}$, the peak energy, $E_p$, and luminosity, $L_p \sim \nu_p L_\nu(\nu_p)$, where $\nu_p = E_p/\hbar$. The obtained spectrum agrees well with the rough estimate given earlier. As for the cases of $r = r_{\text{ph}}/2$ and $r = r_{\text{ph}}/4$, the peak energy and the luminosity are larger by a factor $\sim 2^{3/2} \sim 1.6$ and $\sim 4^{3/2} \sim 2.5$ than the case of $r = r_{\text{ph}}$, respectively, because the temperatures at these radii are larger by the same factor. As shown in the figure, the peak luminosity of the numerical result agrees well with that of the analytical estimate for $r = r_{\text{ph}}$. In contrast, the spectrum extends up to higher energies and the peak energy is close to that for $r = r_{\text{ph}}/2$ because the coupling between the photon and matter is not complete near the photosphere $\tau \sim 1$, as shown in previous studies (Pe'er 2008; Beloborodov 2011; Béguel et al. 2013). As a result, photons that decouple with the matter at moderate optical depth $(\tau \lesssim 5)$ are observed at higher energies.

Regarding the shape of the spectrum, while the emission is dominated by photons that escaped from the on-axis region ($\theta \lesssim \Gamma^{-1}$) at energies near the peak energy and above, the low-energy part ($\nu \ll \nu_\text{p}$) is dominated by those from the off-axis region. The off-axis component becomes prominent at low energies because the Doppler factor is smaller, which leads to a lower peak photon energy $\sim 4D(\nu_0, \theta) \Gamma k_B T'$. Therefore, the low-energy part of the spectrum can be approximated as a superposition of blackbody spectra from the off-axis region that have different peak energies (multi-color blackbody).

As a result, the low-energy slope of the spectra is somewhat softer than that expected from the Rayleigh–Jeans part of a single blackbody ($\nu L_{\nu} \propto \nu^2$) and can be approximated as roughly $\nu L_{\nu} \propto \nu^{2.4}$.

3.2 Stratified Jet

Here, we show the results for a two-component stratified jet. In all cases, the half-opening angle of the spine is fixed at $\theta_0 = 0.5'$.

As mentioned in Section 3.1, when a uniform jet is assumed, spectra tend to be thermal-like with slight modifications from blackbodies originating from a sphere with radius $r = r_{\text{ph}}/5$. In contrast, the appearance of the spectrum can deviate significantly from a thermal one when a strong velocity shear is present in the outflow, given that photons that cross the shear flow multiple times can gain energy through a Fermi-like acceleration mechanism. This can be understood as follows. Under the assumption of elastic scattering, the energy gain of photons in a single scattering event can be expressed as follows:

$$\frac{\nu_{sc}}{\nu_{in}} = \frac{1 - \beta \cos \theta_{in}}{1 - \beta \cos \theta_{sc}},$$

(11)
where \( v_{in} \) (\( \nu_{in} \)) and \( \theta_{in} \) (\( \theta_{\infty} \)) are the frequency and angle between the fluid-velocity and photon-propagation direction before (after) the scattering, respectively. Hence, if \( \theta_{\infty} < \theta_{in} \), photons gain energy and vice versa. Photons that have crossed the boundary layer from the sheath region to the spine region tend to gain energy when they are scattered there (upscatter). This is simply because the photons in the sheath region tend to have larger angle between their propagation direction and fluid velocity than do those in the spine region. In contrast, photons that have crossed the boundary layer from the spine region to the sheath region tend to lose energy (downscatter) for the same reason. Consequently, some fraction of photons that cross the boundary layer multiple times can gain because the energy gain by the upscattering overcomes the downscattering, on average. This mechanism can give rise to a non-thermal spectrum at the high frequencies.

To obtain a rough estimation of the average energy gain and loss rate (\( \nu_{in}/\nu_{in} \)) for each process, we approximate the radially expanding spine and sheath regions as a plane parallel flow. Under the aforementioned consideration, the typical angle between the photon-propagation direction and the fluid direction for the photons in the spine (sheath) region can be estimated as roughly \( \theta_{in} \sim \Gamma_{\infty}^{-1} \) (\( \theta_{\infty} \sim \Gamma_{in}^{-1} \)). Because the angle \( \theta \) is conserved along the photon’s path in the case of a plane parallel flow, the typical energy gain rate by the upscattering in the spine region can be evaluated by substituting \( \theta_{in} = (\theta_{in})_{1} \sim \Gamma_{\infty}^{-1} \) and \( \theta_{\infty} = (\theta_{\infty})_{0} \sim \Gamma_{in}^{-1} \) in Equation (11) and is given as follows:

\[
\frac{\nu_{in}}{\nu_{in}}_{up} \sim \frac{1 - \beta_{1} \cos(\theta_{in})}{1 - \beta_{0} \cos(\theta_{\infty})} \sim \frac{1}{2} \left[ 1 + \left( \frac{\Gamma_{0}}{\Gamma_{1}} \right)^{2} \right]. \tag{12}
\]

Similarly, the typical energy-loss rate by the downscattering in the sheath region is given as follows:

\[
\frac{\nu_{in}}{\nu_{in}}_{down} \sim \frac{1 - \beta_{1} \cos(\theta_{\infty})}{1 - \beta_{0} \cos(\theta_{in})} \sim \frac{1}{2} \left[ 1 + \left( \frac{\Gamma_{1}}{\Gamma_{0}} \right)^{2} \right]. \tag{13}
\]

From Equations (12) and (13), it is clear that the energy gain by the upscattering overcomes the energy loss by the downscattering (\( \nu_{in}/\nu_{in} \)) in all cases, the acceleration per each cycle of crossing (\( \nu_{in}/\nu_{in} \)) is 1. It is also clear that the acceleration per each cycle of crossing (\( \nu_{in}/\nu_{in} \)) is \( \nu_{in}/\nu_{in} \sim (1/4)[2 + (\Gamma_{0}/\Gamma_{1})^{2} + (\Gamma_{1}/\Gamma_{0})^{2}] \approx 1 \). The corresponding optical depth is \( \tau_{inj} \approx 100 \) for the spine and \( \tau_{inj} \approx 1 \) for the sheath. The various lines in the figure show the cases where the observer angle with respect to the jet axis is \( \theta_{obs} = 0^\circ \) (red), \( 0^\circ \) (green), \( 0^\circ \) (blue), \( 0^\circ \) (purple), \( 0^\circ \) (light blue) and \( 0^\circ \) (black). The observed luminosity spectrum in the case of spine-sheath jet in which the spine jet with half-opening angle of \( \theta_{1} = 1^\circ \) and the sheath jet with \( \theta_{0} = 0.5^\circ \) is embedded in a wider sheath outflow with half-opening angle of \( \theta_{1} = 1^\circ \). The values used for dimensionless entropy (terminal Lorentz factor) and kinetic luminosity are chosen as \( \eta_{0} = 400 \) and \( L_{0} = 10^{53} \) erg s\(^{-1} \) for the spine and \( \eta_{1} = 200 \) and \( L_{1} = (\eta_{in}/\eta_{0})L_{0} = 5 \times 10^{52} \) erg s\(^{-1} \) for the sheath, respectively. The initial radius of fireball is chosen as \( r_{i} = 10^{8} \) cm in both regions. The various lines in the figure show the cases where the observer angle with respect to the jet axis is \( \theta_{obs} = 0^\circ \) (red), \( 0.25^\circ \) (green), \( 0.5^\circ \) (blue), \( 0.75^\circ \) (purple) and \( 0.6^\circ \) (light blue) and \( 0.75^\circ \) (black). (A color version of this figure is available in the online journal.)

\[ h\nu_{com} \sim m_{e}c^{2}, \] where \( m_{e} \) is the electron rest mass, the scattering can no longer be approximated as elastic because recoil effect becomes non-negligible (Klein–Nishina effect). In this case, the acceleration efficiency is significantly reduced.

In Figure 3, we display the obtained result for the case of a stratified jet with \( \eta_{0} = 400 \) and \( L_{0} = 10^{53} \) erg s\(^{-1} \) for the spine and \( \eta_{1} = 200 \) and \( L_{1} = (\eta_{in}/\eta_{0})L_{0} = 5 \times 10^{52} \) erg s\(^{-1} \) for the sheath. As mentioned in Section 2, the injection radius is set at a position where a velocity sheath between the two regions develops (\( r_{inj} = r_{s1} \)). The corresponding optical depth is \( \tau_{inj} \approx 100 \) for the spine and \( \tau_{inj} \approx 1 \) for the sheath. The various lines in the figure show the cases where the observer angle with respect to the jet axis is \( \theta_{obs} = 0^\circ \) (red), \( 0.25^\circ \) (green), \( 0.5^\circ \) (blue), \( 0.75^\circ \) (purple), \( 0.6^\circ \) (light blue) and \( 0.75^\circ \) (black). The spectrum varies sensitively with the observer angle. The spectrum for \( \theta_{obs} = 0^\circ \) is thermal-like and nearly identical to that obtained in the case of a uniform jet (Figure 2). The reason for this is simple. Because most of the scattered photons propagate in a direction within a cone of half-opening angle \( \approx \Gamma \sim 0.14(\Gamma/400)^{-1} \), the majority of the observed photons are from a region of \( \theta \ll 0.14 \). Hence, only a small fraction of photons from the sheath region and the boundary \( \theta \gg \theta_{0} = 0.5^\circ \) can reach the observer, so that the spectrum does not deviate largely from the case of a uniform jet. In contrast, if the observer angle is larger, photons from the sheath and boundary layer become observable. As a result, a non-thermal component appears above the peak energy of the thermal spectrum as a result of the photon acceleration in the boundary layer. The non-thermal component is hardest when the observer angle is aligned to the boundary layer \( \theta_{obs} = \theta_{b} = 0.5^\circ \) and becomes softer as the deviation between \( \theta_{obs} \) and \( \theta_{b} \) becomes larger, because the boundary layer corresponds to the site of photon acceleration. As mentioned earlier, the photon acceleration becomes inefficient when the photon energy becomes large enough so that the recoil of electrons cannot be neglected (Klein–Nishina effect). Hence, in all cases, the spectrum does not extend up to energies higher than \( h\nu \sim \Gamma_{0}m_{e}c^{2} \approx 200(\Gamma_{0}/400) \) MeV.
The peak energy and luminosity of the thermal component differs enormously for $\theta_{\text{obs}} > \theta_0$ and for $\theta_{\text{obs}} < \theta_0$ because of the differences in the assumed parameters in the spine and sheath regions. For an observer at $\theta_{\text{obs}} < \theta_0$, the thermal component is determined mainly by photons that have propagated through the spine region. Therefore, the observed spectrum is nearly identical to the case of the uniform jet considered earlier, in which a same set of parameters ($\eta_0, L_0$ and $r_0$) is assumed. In contrast, for an observer at $\theta_{\text{obs}} > \theta_0$, photons that have propagated through the sheath region dominate the thermal component. Accordingly, the peak energy and luminosity are lower by a factor $\sim (\eta_0/\eta_1)^{3/5}(L_0/L_1)^{-5/12} \sim 4.7$ and $\sim (\eta_0/\eta_1)^{8/5}(L_0/L_1)^{1/3} \sim 8$, respectively.

In Figure 4, we display the results obtained for $\eta_0 = 200$ and $L_0 = 10^{52} \text{ erg s}^{-1}$ for the spine and $\eta_1 = 100$ and $L_0 = 5 \times 10^{51} \text{ erg s}^{-1}$ for the sheath. The terminal Lorentz factor of the outflow is smaller by a factor of two for both the spine and the sheath than those assumed in the previous case. The optical depths at the injection radius are $\tau(r_{\text{inj}}) \sim 170$ for the spine and $\tau(r_{\text{inj}}) \sim 290$ for the sheath.

As in the previous case, the non-thermal component is hardest when $\theta_{\text{obs}} = \theta_0 = 0.5$ and becomes softer as the deviation between $\theta_{\text{obs}}$ and $\theta_0$ becomes larger. However, the major difference with the previous case is that the observer dependence of the hardness is weaker. For example, as shown in Figure 4, the non-thermal component can be prominent even for $\theta_{\text{obs}} = 0^\circ$. This is because the beaming effect is weaker than in the previous case as a result of the smaller values of the Lorentz factor, so that the photons can spread out in wider angles.

Figure 5 shows the dependence of the spectrum on the difference between the dimensionless entropies (terminal Lorentz factor) of the spine and sheath. Each panel corresponds to the result for the observer angle fixed at $\theta_{\text{obs}} = 0.25^\circ$ (top panel), $\theta_{\text{obs}} = 0.4^\circ$ (middle left), $\theta_{\text{obs}} = 0.5^\circ$ (middle right), $\theta_{\text{obs}} = 0.6^\circ$ (bottom left), and $\theta_{\text{obs}} = 0.75^\circ$ (bottom right). In all cases, the dimensionless entropy and kinetic luminosity of the spine are chosen to be $\eta_0 = 400$ and $L_0 = 10^{53} \text{ erg s}^{-1}$, respectively. The red line shows the case for a uniform jet, and the green, blue, purple, light blue, and black lines show the cases for a sheath with dimensionless entropies of $\eta_1 = 300, \eta_1 = 250, \eta_1 = 200, \eta_1 = 150$, and $\eta_1 = 100$, respectively. The kinetic luminosity of the sheath is determined by $L_1 = (\eta_0/\eta_1)L_0$.

As mentioned earlier, the peak energy and luminosity of the thermal-component depend on the dimensionless entropy and the kinetic luminosity as $E_p \propto \eta^{5/3} L^{-5/12}$ and luminosity $L_p \propto \eta^{8/5} L^{1/3}$. Hence, for an observer who sees mainly the photons from the sheath region ($\theta_{\text{obs}} > \theta_0$), these values show considerable decrease in models assuming smaller $\eta_1$, as shown in Figure 5.

There are two reasons that the tendency of the non-thermal component becomes significant as $\eta_1$ becomes smaller. First, the bulk Lorentz factor of the sheath becomes smaller for smaller $\eta_1$. As a result, the ratio between the bulk Lorentz factor of the spine and sheath $\Gamma_0/\Gamma_1$ becomes larger, which, in turn, leads to an increase in the energy gain per each crossing, as explained earlier in this section. In addition, the wider spreading of the photons propagating in the sheath region caused by the increase in the beaming angle $\sim \Gamma_1^{-1}$ increases the chance for the photons to cross the boundary layer from the sheath region to the spine region. Second, for smaller value of $\eta_1$, the radius where the velocity shear begins to develop $r_{\text{inj}} = \eta_1 r_0$ becomes smaller. This also leads to an increase in the probability for the photons to be accelerated because the optical depth of the acceleration region ($r > r_{\text{inj}}$) increases (see Equation (3)).

3.3. Relation between Photon Energy and Number of Crossings

To demonstrate that photons accelerated via the multiple crossing of the spine and sheath boundary layer are the origin of the non-thermal component, we analyzed the relation between the photon energy and the number of crossings that the corresponding photons have experienced. Here, the number of crossings, $n_{\text{cr}}$, is defined as the total number of events that the photon has crossed the boundary layer (either from spine to sheath or sheath to spine) before it reaches the outer boundary $r_{\text{out}}$.

In Figure 6, we show the distribution of the average number of crossings for a given observed photon energy, $\langle n_{\text{cr}} \rangle$. In contrast, Figure 7 shows the distribution of the average observed energy for a given number of crossings, $\langle \nu \rangle_{n_{\text{cr}}}$. The two cases of stratified jet that are displayed in the figures by the purple (Case I: $\eta_0 = 400$ and $\eta_1 = 200$) and black lines (Case II: $\eta_0 = 400$ and $\eta_1 = 100$) correspond to the analysis of photons shown in Figure 5 using same colors.

From Figure 6, it is confirmed that the photons at higher energies tend to have a larger number of crossings. While the photons below the thermal peak energy ($h\nu \lesssim 1 \text{ MeV}$) do not require multiple crossings, the prominent non-thermal component extending above $\sim 1 \text{ MeV}$ is produced by the photons that cross the boundary layer $\sim 10$–$15$ times on average. Comparing the two cases, the overall distribution of $\langle n_{\text{cr}} \rangle_{\nu}$ does not vary much. The difference in the average number of crossing is within $\sim 2$ in all energies up to $\sim 100 \text{ MeV}$. However, this does not imply that the average energy for a given number of crossings $\langle \nu \rangle_{n_{\text{cr}}}$ does not vary much in the two cases. Conversely, the difference in $\langle \nu \rangle_{n_{\text{cr}}}$ is large between the two cases, as shown in Figure 7, because the average energy gain per crossing is quite sensitive to the ratio in the terminal Lorentz factor $\Gamma_0/\Gamma_1$ (see Equations (12) and (13)). However, because of the large dispersion in the energy gain per crossing, the energy distribution of photons does not show a sharp peak at the average energy $\langle \nu \rangle_{n_{\text{cr}}}$, but rather extends to energies below and above $\langle \nu \rangle_{n_{\text{cr}}}$ by many orders of magnitude.

As a result, the distribution of $\langle \nu \rangle_{n_{\text{cr}}}$ does not directly reflect the distribution of $\langle n_{\text{cr}} \rangle_{\nu}$, because the photons that have crossed the boundary layer a certain number of times can dominate the
other population of photons in wide energy ranges. To clarify this phenomenon, we show in Figure 8 the energy distribution of the photon number count rate, $\nu N_{\nu}$ [photons s$^{-1}$], for a given number of crossings. The figure confirms that although there is a significant discrepancy in $\langle \nu \rangle_{n_{\nu}}$, the photons with number of crossing $n_{\nu} \sim 10$ tend to dominate the population in the energy range in which non-thermal component becomes prominent ($h\nu \gtrsim \text{few MeV}$) in both cases. For this reason, the resultant distribution of $\langle n_{\nu} \rangle_{n_{\nu}}$ does not vary much in the two cases.

Regarding the distribution of the average energy, $\langle \nu \rangle_{n_{\nu}}$ tends to increase with the increasing number of crossing $n_{\nu}$ initially and then approaches a constant value. This asymptotic behavior is due to the Klein–Nishina effect. As the photon energy becomes large and exceeds 100 keV in the comoving frame,
acceleration efficiency is reduced by the effect (see Section 3.2 for details), and the average energy can no longer increase. As Figure 7 shows, the dependence of \((v)_{n_{cr}}\) on \(n_{cr}\) is not smooth but rather bumpy. This reflects the fact that the photons tend to be upscattered when crossing occurs from the sheath region to the spine region (see Equation (12)), but tend to be downscattered when crossing occurs in the opposite direction (see Equation (13)). Therefore, the energy of individual photons is not a monotonically increasing function of \(n_{cr}\) but it shows a bumpy dependence because upscattering and downscattering occurs alternately in each crossing events. Although somewhat reduced, this feature remains even after averaging up and leads to the appearance of the bumpy feature (wiggles) in the distribution of \((v)_{n_{cr}}\).

To quantify the energy gain and loss in each crossing, we display the ratio of the average energy for photons with \(n_{cr}\) crossings to that for photons with \(n_{cr} - 1\) crossings \((\langle v \rangle_{n_{cr}}/\langle v \rangle_{n_{cr}-1})\) in Figure 9. In addition to the analysis of the total photons, we also display the results of the analysis for the two populations of photons that were initially injected in the spine region (\(\theta_{inj} \leq \theta_0\); blue line) and sheath regions (\(\theta_{inj} > \theta_0\); green line). In the former case \((\theta_{inj} \leq \theta_0)\), if \(n_{cr}\) is an odd (even) number, the number of crossings from the spine region (sheath) to the sheath (spine) region is greater by 1 than that for \(n_{cr} - 1\) crossings. Hence, the number of downscattering (upscattering) event is greater by 1 for the photons with an odd (even) number of \(n_{cr}\) than for those with \(n_{cr} - 1\) crossings. As a result, \((\langle v \rangle_{n_{cr}}/\langle v \rangle_{n_{cr}-1})\) is less (greater) than unity for the odd (even) number of \(n_{cr}\). The opposite is true for the latter case \((\theta_{inj} > \theta_0)\). Jets with a larger difference in the terminal Lorentz factor (Case II; right panel) show a larger range of energy ratio \((\langle v \rangle_{n_{cr}}/\langle v \rangle_{n_{cr}-1})\) than those with a smaller difference (Case I; left panel) because the efficiency of the single upscattering (downscattering) increases (decreases) as the relative difference in the Lorentz factor becomes larger.

Regarding the upscatterings, the energy ratio \((\langle v \rangle_{n_{cr}}/\langle v \rangle_{n_{cr}-1})\) for photons that cross the boundary from the sheath region to the spine region (green line) only once \((n_{cr} = 1)\) is relatively small because a large fraction of these photons experience the crossing when the velocity shear is not fully developed \((r < r_0)\). For photons with a larger number of crossings \((n_{cr} \geq 2)\), a large fraction of the photons experience the last crossing at \(r > r_0\), where the velocity shear is fully developed. Therefore, the energy ratio is larger than that for \(n_{cr} = 1\) and is roughly constant as long as the Klein–Nishina effect is negligible. When the Klein–Nishina effect becomes important \((n_{cr} \geq 10)\) for Case I and \(n_{cr} \geq 5\) for Case II, \((\langle v \rangle_{n_{cr}}/\langle v \rangle_{n_{cr}-1})\) decreases as \(n_{cr}\) increases, and again asymptotically approaches a constant value. Regarding the case of downscatterings, the energy ratio is relatively insensitive to the number of crossings in both cases and is roughly in the range \(\sim 0.25\) to 0.6. The values of energy ratio are consistent within a factor of \(\sim 2\), with the rough estimations given by Equations (12) and (13). Above the saturation radius \((r \geq r_0)\), the equations predict an upscattering (downscattering) with energy ratio of \(\sim 2.5\) to 0.6) and \(\sim 8.5\) to 0.5) for Cases I and II, respectively. The energy ratio of the total photons is larger and smaller when the photons from the spine correspond to the upscattering (even \(n_{cr}\)) and downscattering (even \(n_{cr}\)), respectively, because more photons originate from the spine region than from the sheath region.

Last, to obtain a further insight into the relation between the photon acceleration and number of crossing, we show the evolution of the average photon energy evaluated in the comoving frame, \(v_{cmf}\), with radius in Figure 10. The red, green, blue, purple, light blue, yellow, and black lines display the photons that have experienced \(n_{cr} = 0, 3, 5, 10, 15, 20\), and 25 crossings, respectively. For comparison, we also plot the curve of \(v_{cmf} \propto r^{-2/3}\) with a thin black line, which corresponds to the adiabatic cooling expected above the saturation radius \((r \geq r_0)\). Regarding the photons that do not experience any crossings \((n_{cr} = 0)\), the overall evolution of the energy is determined solely by the adiabatic cooling as a result of the expansion of the jet, given that photon acceleration does not take place. Below the saturation radius of the spine \(r \leq r_0\), the cooling
Figure 8. Energy distribution of photons for a given number of crossing events in the case of a jet in which the spine region with half-opening angle of $\theta_0 = 0.5$ is embedded in a wider sheath outflow with half-opening angle of $\theta_1 = 1$. The red, green, blue, purple, light blue, yellow, and black lines display the photons that have experienced $n_{cr} = 0$, $3$, $5$, $10$, $15$, $20$, and $25$ crossings, respectively. The values used for dimensionless entropy (terminal Lorentz factor) and kinetic luminosity are chosen as $\eta_0 = 400$ and $L_0 = 10^{53}$ erg $s^{-1}$ for the spine. The left and right panels show the cases where the dimensionless entropy of the sheath is given by $\eta_1 = 200$ and $\eta_1 = 100$, respectively. In each case, the kinetic luminosity of the sheath is given by $L_1 = (\eta_0/\eta_1)L_0$. (A color version of this figure is available in the online journal.)

Figure 9. Ratio of the average observed energy for photons that cross the spine-sheath boundary layer ($\theta = \theta_0$) $n_{cr}$ times to those that experience $n_{cr} - 1$ crossings for a jet in which a spine region with half-opening angle of $\theta_0 = 0.5$ is embedded in a wider sheath outflow with half-opening angle of $\theta_1 = 1$. The values used for dimensionless entropy (terminal Lorentz factor) and kinetic luminosity are chosen as $\eta_0 = 400$ and $L_0 = 10^{53}$ erg $s^{-1}$ for the spine. The left and right panels show the cases of the dimensionless entropy of the sheath given by $\eta_1 = 200$ and $\eta_1 = 100$, respectively. In each case, the kinetic luminosity of the sheath is given by $L_1 = (\eta_0/\eta_1)L_0$. The energy ratio for total photons are displayed in the left and right panels with the purple and black lines, respectively. In addition, the energy ratio for the photons that were initially injected in the spine region ($\theta_{inj} \leq \theta_0$) and in the sheath region ($\theta_{inj} > \theta_0$) are shown by blue and green lines, respectively, in both panels. (A color version of this figure is available in the online journal.)

The evolution of the comoving energy cannot be described only by the adiabatic cooling caused by the presence of the photon acceleration. As the number of the crossings increases, the departure from the simple adiabatic cooling becomes significant and the comoving energies tend to be larger. For a given number of crossings, the departure is more prominent in Case II than in Case I because of the increase in the acceleration efficiencies. When the number of the crossings is sufficiently large so that the effect of Klein–Nishina becomes non-negligible, the evolution of the average comoving energy asymptotically approaches a single curve because the acceleration saturates. This tendency is clearly seen in Figure 10 (for example, see lines that display the evolution of photons with $n_{cr} \geq 10$ in Case II).
different emission components such as synchrotron emission as discussed in Pe’er et al. (2012) might be required in order to explain the low-energy end of the power-law component. To extend the power-law component up to GeV energies, large Lorentz factors are required because the non-thermal component extends only up to energy of $h\nu \lesssim (200\eta/400)$ MeV because of the Klein–Nishina effect. We discuss this issue in detail in Section 4.6.

The spectral slope below the peak energy is only moderately sensitive to the values of the parameters and can be well approximated by a blackbody emission from the off-axis region ($|\theta - \theta_{obs}| \gtrsim \Gamma^{-1}$) of a sphere at a radius $r \sim r_{ph}/5$, as in the case of a uniform jet. In all cases, the low-energy photon indices are roughly $\alpha_{\text{ph}} \sim 0.5$, which is harder than the typical observed value ($\alpha_{\text{ph}} \sim -1$). We note, however, that this does not imply that the low-energy slope cannot be reproduced in this scenario. For example, while the instantaneous spectrum is hard, the time integration can lead to a softer spectrum if the evolution history of the outflow is considered. To demonstrate this, we consider the case when the dimensionless entropies of the spine-sheath jet have evolved from $\eta_0 = 400$ and $\eta_1 = 200$ (initial stage) to $\eta_0 = 200$ and $\eta_1 = 100$ (later stage), while the kinetic luminosity is fixed as $L_0 = 10^{53}$ erg s$^{-1}$ and $L_1 = 5 \times 10^{52}$ erg s$^{-1}$. We illustrate the resultant spectrum in Figure 11 under the assumption that a nearly equal time period is spent in the two stages. The black solid line shows the overall spectrum, and the red and green solid lines show the contribution from the initial and later stages, respectively. Because the peak energy and the luminosity vary with the dimensionless entropy of the flow roughly as $\propto \eta^{1/3}$, they are smaller by a factor $\sim 6$, in the later stage. As a result, while the peak energy and luminosity of the overall spectra is determined by the initial stage, at lower energies, contribution from the later stage becomes dominant. Because of the superposition of the two components, the spectrum below the peak becomes softer than that of the individual component, so that a typical observed spectrum ($\nu L_\nu \propto \nu$) is reproduced. The high-energy part of the overall spectrum largely resembles typical ones from observations. Hence, we conclude that typical observed
spectra can be successfully reproduced when a time evolution is considered.\(^{10}\)

\[4.2. \text{On the Observer Angle Dependence and Structure of the Jet}\]

As shown in the previous section, our results have a strong dependence on the observer angle \(\theta_{\text{obs}}\). While strong non-thermal emission can be seen when the observer angle is close to the angle in which a velocity shear is present (\(|\theta_0 - \theta_{\text{obs}}| \lesssim \Gamma^{-1}\)), the non-thermal feature tends to be weaker for observer angles far from \(\theta_0\). This tendency seems to contradict the fact that most of the observed GRBs have non-thermal features in their spectra. However, this difficulty can be overcome if the jet possesses a more complex structure. For example, if the jet has more than two components and velocity shear is present at multiple angles more closely spaced than \(\Gamma^{-1}\), photons from the acceleration regions will be prominent for all observers lying within the opening angle of the jet (\(\theta_{\text{obs}} \lesssim \theta_1\)). Even in the case of a simple spine-sheath jet, if the angular boundary of the spine and sheath (\(\theta_0\)) varied rapidly with time, the accelerated non-thermal photons would be observable across a broad range of angles. The structure need not be in the direction of \(\theta\). Strong velocity shear in the azimuthal (\(\phi\)) direction and/or radial direction caused by the presence of turbulence or shocks can also provide an acceleration sites for the photons (Bromberg et al. 2011; Ioka et al. 2011). Any structure showing strong velocity shear within an angle \(\Gamma^{-1}\) from the line-of-sight can give rise to a non-thermal component above the thermal peak. Hence, we expect that a jet that has a rich structure and/or rapid time variability will be naturally accompanied by non-thermal emission, irrespective of the observer angle. The origin of the structure and variability could be due to the nature of the central engine (McKee 2006; Nagataki et al. 2007; Nagataki 2009, 2011; McKee & Blandford 2009) and/or the propagation of a jet through the envelope of the progenitor star (Zhang et al. 2003; Mizuta et al. 2006, 2011; Morsony et al. 2007; Lazzati et al. 2009; Nagakura et al. 2011).

\[4.3. \text{On the } E_p-L_p \text{ Relation}\]

Our calculations show that the peak energy of the observed spectra can be roughly approximated as the thermal peak of a blackbody emission from the surface of optical depth \(\tau(r) \sim 2\) (\(r \sim r_{\text{phot}}/2\)). In contrast, the peak luminosity roughly agrees with that of the emission from \(\tau(r) \sim 1\) (\(r \sim r_{\text{phot}}\)). This result is valid for an adiabatic fireball that has a photosphere above the saturation radius (\(r_{\text{phot}} > r_s\)) (see Section 3.1 for details). As a result, the peak energy and luminosity are given by the following:

\[E_p \sim 800 r_{i,8}^{1/6} \eta_{i,401}^{8/3} L_{53}^{-5/12} \text{ keV},\]

\[L_p \sim 10^{52} r_{i,8}^{2/3} \eta_{i,401}^{8/3} L_{53}^{-1/3} \text{ erg s}^{-1},\]

respectively.\(^{11}\) Derived from observations, there is an empirical relation between the peak energy and luminosity (Yonetoku et al. 2004; Kodama et al. 2008; Nava et al. 2011; Amati et al. 2002; Wei & Gao 2003) that is roughly given by

\[E_p \approx 600 \left( \frac{L_p}{10^{33} \text{ erg s}^{-1}} \right)^{1/2} \text{ keV}.\]

Therefore, in order to reproduce the empirical relation from the photospheric emission, the parameters of the fireball \((r_i, \eta_i, \text{ and } L)\) must satisfy the following:

\[L_{53} \sim 12 r_{i,8}^{-2/7} \eta_{i,401}^{16/7} \text{ erg s}^{-1}.\]

Another important ingredient for comparison with observations is the emission efficiency \(\eta_r = L_p/L_.\) Observations of the afterglows suggest a high GRB efficiency in the range \(0.01 \lesssim \eta_r \lesssim 1\), with a typical value at \(\eta_r \sim 0.1–0.2\) (e.g., Fan & Piran 2006; Zhang et al. 2007). This gives another constraint on the fireball parameters, which can be written as follows:

\[0.01 \lesssim 0.1 r_{i,8}^{2/3} \eta_{i,401}^{8/3} L_{53}^{-2/3} \lesssim 1.\]

To sum up, the observed \(E_p-L_p\) relation as well as the high efficiency can be reproduced when Equations (17) and (18) are satisfied.

In Figure 12, we show the calculated spectra for a uniform jet (\(\theta_1 = \theta_0 = 1^\circ\)) in which both conditions are fulfilled. The adopted values of the parameters are summarized in the figure. The red, green, blue, and purple lines correspond to the cases \(\eta_0 = 100, \eta_0 = 200, \eta_0 = 400\), and \(\eta_0 = 500\), respectively. The remaining parameters \((L_0\) and \(r_i\)\) are determined so that an efficiency of \(\eta_r \sim 0.1 r_{i,8}^{2/3} \eta_{i,401}^{8/3} L_{53}^{-2/3} \sim 0.2\) is realized. The thick light blue line displays the observed \(E_p-L_p\) relation. From the figure, we can confirm that the photospheric emission can reproduce the empirical \(E_p-L_p\) relation while at the same time retaining a high efficiency when the two conditions are satisfied.

\(^{11}\) For cases in which efficient energy dissipation is present within the flow, the dependence of \(E_p\) and \(L_p\) on the fireball parameters can be significantly different (e.g., Rees & Mészáros 2005; Giannios 2012; Lazzati et al. 2013; Comparison of Equations (14) and (15) with the observed peak energy and luminosity will enable us to constrain the properties (fireball parameters) of the GRB jet (e.g., Pe’er et al. 2007; Fan et al. 2012).
4.4. Dependence on the Width of the Spine–sheath Boundary Layer

In the present study, we have assumed an infinitesimal width for the boundary layer of the spine and sheath region. However, in reality, the boundary layer is expected to possess a finite width because of the interaction between the two regions. First, by definition, photons are closely coupled to the matter below the photosphere, and, therefore, will couple the two regions as a result of Compton friction (radiative viscosity). In particular, this effect will be important in the regions where the energy is dominated by radiation ($r \lesssim r_s$) (e.g., Arav & Begelman 1992). Second, even in the absence of radiation coupling, the Kelvin–Helmholtz instability should grow whenever velocity shear is present (e.g., Turland & Scheuer 1976; Bodo et al. 2004). These effects will relax the discontinuous change in the velocity and lead to broadening of the boundary layer. Although the detail analysis of the resultant jet structure is beyond the scope of the present study, these effects will reduce the acceleration of the photons. To quantify the effect of the broadening of the boundary layer on the spectra, here we compute the photon propagation within a spine-sheath jet having a boundary layer with finite width in transverse direction. We explore the dependence on the width of the boundary layer by considering cases with various widths.

In modeling the jet structure, we added slight modifications in the original spine–sheath jet model considered in the present study. The boundary layer is defined as a region having finite transverse width $d\theta_B$ that is located in the range $\theta_i < \theta < \theta_B$, where $\theta_i = \theta_B - d\theta_B/2$ and $\theta_B = \theta_B + d\theta_B/2$. Correspondingly, the spine and sheath regions are limited to the ranges $0 < \theta < \theta_i$ and $\theta_i < \theta < \theta_B$, respectively. While the fluid properties (fireball parameters) of the spine and sheath region are determined in the same way as in the cases of infinitesimal boundary layer (Section 2.1.2), the properties of the boundary layer are determined by simply imposing a linear interpolation of the fireball parameters from the two regions. Hence, the initial radius of the fireball is fixed in all regions ($r_i = 10^8$ cm), and the transverse distribution of the dimensionless entropy and kinetic luminosity within is given by the following:

$$\eta(\theta) = \begin{cases} \eta_0, & \text{for } 0 \leq \theta \leq \theta_i, \\ \frac{d\theta_B}{d\theta} \eta_0 + \left(\frac{\theta - \theta_i}{\theta_B - \theta_i}\right) \eta_0, & \text{for } \theta_i < \theta < \theta_B, \\ \eta_1, & \text{for } \theta_B \leq \theta \leq \theta_B, \end{cases}$$

and

$$L(\theta) = \begin{cases} \frac{L_0}{\eta_0} + \left(1 - \frac{\theta - \theta_i}{\theta_B - \theta_i}\right) \eta_0, & \text{for } 0 \leq \theta \leq \theta_i, \\ \frac{L_0}{\eta_0} + \left(1 - \frac{\theta - \theta_i}{\theta_B - \theta_i}\right) \eta_0, & \text{for } \theta_i < \theta < \theta_B, \\ \frac{L_1}{\eta_0}, & \text{for } \theta_B \leq \theta \leq \theta_B, \end{cases}$$

respectively. Hence, the profile of the velocity (Lorentz factor) and density within the jet is continuous in all regions.

Having determined the background fluid properties, the propagation of photons is calculated in the same way as in the case of an infinitesimal boundary layer. Blackbody emission is injected at the surface of a fixed radius $r_{in} = r_{in}$ with a comoving temperature determined from the fireball parameters (Section 2.2.1). Then, photons are propagated until they reach the outer boundary of the calculation range (see Section 2.2 for details).

In Figures 13 and 14, we show the obtained spectra for a stratified jet in which the fireball parameters for the spine and sheath regions are identical to those used in Case I ($\eta_0 = 400$ and $\eta_1 = 200$) and Case II ($\eta_0 = 400$ and $\eta_1 = 100$) discussed in Section 3.3. The various lines reflect the width of the boundary layer $d\theta_B$. The thick red line represents the case for the infinitesimal width ($d\theta_B = 0$); and the thin green, blue, purple, light blue, yellow, black, and red lines represent the cases for the finite widths of $d\theta_B = (100\eta_0)^{-1}$, $(20\eta_0)^{-1}$, $(10\eta_0)^{-1}$, $(5\eta_0)^{-1}$, $(2\eta_0)^{-1}$, $(\eta_0/2)^{-1}$, and $(\eta_0/4)^{-1}$, respectively. In all cases, the observer angle is fixed at $\theta_{obs} = \theta_0 = 0.5$. As expected, the non-thermal component becomes softer as the width of the boundary layer broadens as a result of the reduction in efficiency of photon acceleration. While the broadening of the boundary layer causes the spectra to depart from the high-energy tail of the Band function for Case I, good agreement is found at $d\theta_B \sim (5\eta_0)^{-1}$ for Case II. Therefore, although a large gradient in the Lorentz factor is required, the broadening of the boundary layer does not rule out the photon acceleration mechanism in a stratified jet as the origin of the high-energy spectra in the prompt emission.

The figures show that, to provide an efficient acceleration site comparable to the case of boundary layer with infinitesimal with, the bulk Lorentz factor must vary by a factor of few roughly within an angle $d\theta_B \sim (100\eta_0)^{-1} \ll \Gamma^{-1}$. As is expected, this condition is roughly equivalent to the condition for the boundary layer to be optically thin above the radius where the velocity shear begins to develop ($r \gtrsim r_{in}$). This can be shown as follows. The typical angle between the photon-propagation direction and the fluid-velocity (radial) direction is roughly $(\theta_0) \sim \Gamma^{-1}$. 

![Figure 12](https://example.com/figure12.png)

Figure 12. Observed spectra in the case of a uniform jet that reproduces the observed $E_p-L_p$ relation and radiative efficiency of $\eta_p \sim 0.2$. The red, green, blue, and purple lines correspond to the cases of $\eta_0 = 100$, $\eta_0 = 200$, $\eta_0 = 400$, and $\eta_0 = 500$. The thick light blue line represents the observed $E_p-L_p$ relation. (A color version of this figure is available in the online journal.)
Hence, at a given radius $r$, the typical distance that the photons must propagate to cross the boundary layer is roughly given as $l_{\text{cross}} \sim r(d_{\text{th}}/\eta_0) \sim r/\tau_{\text{th}}$. The aforementioned estimate is valid as long as $d_{\text{th}} \ll \eta_0^{-1}$. In contrast, the mean free path for the typical photon at a given radius $r$ is roughly given by $l_{\text{mfp}} \sim D(\theta_0)\sigma_{\text{ff}} r/\tau(r)$. Thus, the typical optical depth for a photon to cross a boundary layer of finite width can be estimated as $\tau_{\text{core}} \sim l_{\text{mfp}}/l_{\text{fin}} \sim \tau(r)\Gamma_{\text{th}}/\Gamma$. Because $\Gamma_{\text{fin}}$ for Case I (Case II), respectively, $\tau_{\text{core}} \lesssim 1$ is obtained at $r = r_{\text{inj}}$ for $d_{\text{th}} \ll (100\theta_0)^{-1}$. As a result, because $\tau_{\text{core}}$ decreases as the radius increases, when the condition $d_{\text{th}} \ll (100\theta_0)^{-1}$ is satisfied, the boundary layer is optically thin to crossings at $r \gtrsim r_{\text{inj}}$.

Last, we briefly comment on a similar calculation performed by Lundman et al. (2013). They explore photon propagation within a jet having a continuously decaying velocity profile $\Gamma \propto \theta^{-\alpha}$ at the outer region ($\theta \gtrsim \theta_0$). Because the efficiency of photon acceleration increases as the difference in velocity (Lorentz factor) between the spine and sheath regions increases, their calculation should also show non-thermal high-energy photons when a large velocity gradient is considered. Although not so prominent as is shown by our results, signs of photon acceleration are also reported in their study. While only a thermal component is present when a relatively small velocity gradient ($p = 2$) is assumed, power-law excess above the thermal peak energy appears in the calculated spectra when a large velocity gradient ($p = 4$) is assumed. Therefore, as the velocity gradient enlarges, we expect to find more efficient acceleration, as shown in the present study.

4.5. On the Absorption Processes

Because the photon number density is significant inside the jet ($r \lesssim r_{\text{ph}}$), photons are subject to $\gamma\gamma$ attenuation once they exceed the threshold energy for the process. The threshold energy is given by $E_{\text{th}} = 2(m_c^2c^2)/(1 - \cos\theta_{\text{ph}})\epsilon$, where $\epsilon$ and $\theta_{\text{ph}}$ are the energy of the target photons and angle between the propagation direction of the target and incident photons, respectively. Because the photons are advected in the radial direction within a small angle $\Gamma^{-1}$, the collision angle of the photons is expected to be in a range $\theta_{\text{ph}} \lesssim \Gamma^{-1}$. Hence, for photons confined in an angle $\Gamma^{-1}$, the minimum value of the threshold energy ($\theta_{\text{th}} \sim \Gamma^{-1}$) can be estimated as roughly $E_{\text{th, min}} \sim 100(\Gamma/100)^2(\epsilon/100\text{MeV})^{-1}\text{MeV}$. For the cases considered in the present study ($\Gamma \geq 100$), the photon energies do not exceed 100 MeV by much and, therefore, the pair creation process is negligible for the majority of the photons. However, for the small fraction of photons that propagate with a large angle ($\gtrsim \Gamma^{-1}$) with respect to the radial direction, this effect can become non-negligible. In particular, pair annihilation might play an important role for the high-energy photons that are being accelerated, given that scattering with a large angle with respect to the fluid-velocity (radial) direction is favored for gaining photon energy (see Section 2 for details). To check this, we compared our results with those obtained by discarding the photons that have exceeded the threshold energy obtained by substituting the angle between the photon-propagation direction and radial direction for $\theta_{\text{ph}}$ and the highest photon energy appear in the calculation $\sim \Gamma\eta_{\text{m},e}c^2/\epsilon$ for $\epsilon$. We find that only a small fraction of photons are absorbed, and the change in the spectrum is negligible. Therefore, we conclude that $\gamma\gamma$ attenuation effect does not affect the obtained results.

The flow within the fireball is expected to be fully ionized, and the effect of free-free absorption should also be discussed. For an electron–proton plasma, the frequency averaged free-free opacity can be written as roughly $\kappa_{\text{ff}}(r) \approx 1.7 \times 10^{-25} \Gamma(r)^{-7/2}n_e(c^2)^{1/2}\text{cm}^{-1}$ (Rybicki & Lightman 1979) in the fluid rest-frame. In the laboratory frame, by using a Doppler factor, the opacity can be expressed as $\kappa_{\text{ff,lab}} = D^{-1}\kappa_{\text{ff}}$. Because photons advect in the radial direction within an angle $\Gamma^{-1}$ ($D \sim \Gamma$), the optical depth for photons that have propagated from $r = r_{\text{finj}}$ to the observer (infinity) can be estimated as roughly $\tau_{\text{ff}} \sim r_{\text{finj}}\kappa_{\text{ff,lab}}(r) \sim 8.3 \times 10^{-6} L/(10^{53}\text{ergs s}^{-1})^{9/8}(\eta/400)^{-2}(r_i/10^8\text{cm})^{5/4}(r_{\text{finj}}/10^{10}\text{cm})^{-5/2}$, where we assumed $r_{\text{finj}} \lesssim r_s$ and used Equations (2) and (5) in the last equality. Therefore, we can conclude that free–free absorption is also negligible.
4.6. On the GeV Gamma-Ray

Fermi observations have shown that a fraction (∼8%) of GRBs are accompanied by significant emissions at energies well above ∼100 MeV (e.g., Zhang et al. 2011; The Fermi Large Area Telescope Team et al. 2012). Within the framework of our model, the energy of the photons is limited by the bulk Lorentz factor (Telescope Team et al. 2012). Within the framework of our model, the present study.

Following is a summary of the main results and conclusions of the study. High-energy photons are independently produced at the boundary layer, lower values are allowed for the bulk Lorentz factor. By denoting the maximum Lorentz factor of the electrons measured in the rest-frame of the fluid as \( \gamma_{\text{max}} \), the lower limit on the bulk Lorentz factor to produce GeV photons decreases as \( \Gamma > 2000 \). Therefore, within this picture, GRBs with intense GeV emissions might imply the presence of fluid components with very high bulk Lorentz factors (\( \Gamma > 2000 \)) or a dissipative process producing relativistic electrons within the flow. In either case, pair cascades caused by \( \gamma \gamma \) attenuation might play an important role (Ioka et al. 2011), and the details of the process and its effect on the observed photon spectra are beyond the scope of the present study.

Photons with energies above and below ∼100 MeV might have distinct origins. For example, it is widely discussed that \( \sim \)GeV emission can result from energy dissipation by external shocks similar to afterglow emission (Kumar & Barniol Duran 2009, 2010; Ghisellini et al. 2010), as a result of the smooth temporal decay seen in lightcurve of many LAT detected GRBs. Therefore, one possible interpretation is that, while emission at \( \lesssim 100 \) MeV has a photospheric origin as discussed in the present study, higher energy photons are independently produced at the external shock.

5. SUMMARY AND CONCLUSIONS

In the present study, we have explored photospheric emission from ultra-relativistic jets that have a velocity shear in the transverse direction. For the jet structure, we considered a two-component outflow in which a fast spine jet is embedded in a slower sheath region. The fluid properties such as electron number density \( n_e(r) \) and bulk Lorentz factor \( \Gamma(r) \) are determined by applying the adiabatic fireball model in each region independently. Thermal photons are initially injected at a radius far below the photosphere \( (r_{\text{inj}} = \eta_1 r_s \ll r_{ph}) \) in which the velocity shear begins to develop. Using a Monte Carlo technique, injected photons propagate until they reach the outer boundary, located at a radius where the optical depth is small \( (\tau \ll 1) \). The following is a summary of the main results and conclusions of the present study.

1. Because of the presence of velocity shear, photons that cross the boundary between the spine and sheath \( (\theta_0) \) multiple times can gain energy through a Fermi-like acceleration mechanism. The acceleration process at the boundary layer can proceed efficiently until the photon reaches an energy where the Klein–Nishina effect becomes important. As a result, the maximum energy of the accelerated photons is limited by the bulk Lorentz factor of the outflow as \( \sim \Gamma_0 m_e c^2 \approx 200(\Gamma_0/400) \) MeV. These accelerated photons produce a non-thermal component above the thermal peak in the observed spectra. Although it depends strongly on the flow profile, the non-thermal component can reproduce the high-energy spectra of typical GRBs \( (\nu L_\nu \propto \nu^{-0.5}) \). The accelerated photons might also be capable of explaining the extra-hard-power-law component above the bump of the thermal-like peak seen in some peculiar bursts (GRB 090510, 090902B, 090926A).

2. The efficiency of the acceleration is sensitive to the relative difference of the bulk Lorentz factor in the two regions \( (\Gamma_0/\Gamma_1) \), the optical depth at the radius where the velocity shear begins to develop \( (\tau(r_{\text{inj}})) \) and the optical depth for a photon to cross the boundary transition layer of the two regions \( (\tau_{\text{cr}}) \). For an efficient acceleration, larger values are favored for \( \Gamma_0/\Gamma_1 \) and \( \tau(r_{\text{inj}}) \), while smaller value is favored for \( \tau_{\text{cr}} \), since both the energy gain per crossing and probability for the photons to cross the boundary layer increase. The increase in the efficiency leads to a harder high-energy non-thermal component in the observed spectra.

3. The observed spectra strongly depend on the observer angle \( \theta_{\text{obs}} \). The high-energy non-thermal component is hardest when the observer is aligned to the boundary layer \( (\theta_{\text{obs}} = \theta_0) \) and becomes softer as the difference between \( \theta_{\text{obs}} \) and \( \theta_0 \) becomes larger. Additionally, the non-thermal component is most prominent for an observer located near the boundary layer \( (|\theta_0 - \theta_{\text{obs}}| \lesssim \Gamma^{-1}) \), and it becomes significantly weaker or absent when the observer angle is far from the boundary layer \( (|\theta_0 - \theta_{\text{obs}}| > \Gamma^{-1}) \). In order for the intense non-thermal component to be seen for all observers in a range \( \theta_{\text{obs}} \lesssim \theta_1 \), a multi-component jet in which velocity shears are present in an interval of angles smaller than \( \Gamma^{-1} \) is required.

4. The observed spectra below the peak energy are determined by the majority of thermal photons that have not experienced acceleration. The spectra \( \nu L_\nu \propto \nu^{-2.4} \) are somewhat softer than that expected from the Rayleigh–Jeans part of a single-temperature blackbody emission \( \nu L_\nu \propto \nu^3 \), given that the emission is a superposition of photons released at various angles that have different observed temperatures. This is still much harder than the typical low-energy spectral index of the observed GRBs \( (\nu L_\nu \propto \nu) \), implying that the steady outflow component has difficulty in reproducing the overall spectra. Hence, time evolution of outflow properties seems to be required. We have shown that time-integrated spectra of an unsteady outflow can reproduce the low-energy spectra as a result of the multi-temperature effect.

5. Photons begin to decouple from the matter below the photosphere typically at \( r \sim r_{ph}/5 \), irrespective of the imposed fireball parameters \( (r_{\text{inj}}, L \) and \( \eta) \) in the background fluid. The resultant peak energy \( E_p \) and luminosity \( L_p \) can be roughly approximated by the corresponding values of a blackbody emission from the surface of \( r \sim r_{ph}/2 \) and \( r \sim r_{ph} \), respectively. The empirical \( E_p^{-1} - L_p \) relation can be well reproduced by considering the difference in the outflow properties of individual sources.

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