ABSTRACT
Various tensor decomposition methods have been proposed for data compression. In real world applications of the tensor decomposition, selecting the tensor shape for the given data poses a challenge and the shape of the tensor may affect the error and the compression ratio. In this work, we study the effect of the tensor shape on the tensor decomposition and propose an optimization model to find an optimum shape for the tensor train (TT) decomposition. The proposed optimization model maximizes the compression ratio of the TT decomposition given an error bound. We implement a genetic algorithm (GA) linked with the TT-SVD algorithm to solve the optimization model. We apply the proposed method for the compression of RGB images. The results demonstrate the effectiveness of the proposed evolutionary tensor shape search for the TT decomposition.

Index Terms— Tensor decomposition, tensor train decomposition, data compression, genetic algorithm, evolutionary algorithm

1. INTRODUCTION
Compressing high-volume data via tensor decomposition has gained great success in the recent years. The tensor decomposition is applied to approximate high dimensional problems in different domains including scientific computation, machine learning, and visual processing [1, 2, 3, 4, 5]. In order to decompose a high order tensor into the low-dimensional parameters, different methods have been successfully applied including the CANDECOMP/PARAFAC (CP) decomposition [6], the Tucker decomposition [7], and the tensor train (TT) decomposition [8]. The tensor decomposition was also extended to a more general form called the tensor networks which leads to various decomposition formats [9]. In recent years, Bayesian methods have also been developed for automatic rank determination in various tensor problems including tensor completion and tensorized neural network training [10, 11, 12].

Regardless of the specific choice of a tensor decomposition method, the data or the model parameters are represented as a d-way tensor prior to the decomposition. This often involves a reshaping step which changes the dimension and mode size of a tensor without changing the total number of its elements. The shape of a tensor affects the accuracy and the compression ratio of the subsequent tensor decomposition. Despite the importance of finding an optimum shape for the tensor decomposition, studies on this domain remains very sparse.

In this study we investigate the effect of the tensor shape on the compression ratio achieved by the tensor decomposition. We formulate the task of finding the best shape for the tensor decomposition as an optimization model. We present a genetic algorithm (GA) for solving the problem. Specifically, we narrow down the study to the TT decomposition, but our proposed technique can be extended to other tensor decomposition methods.

2. RELATED WORKS
2.1. Tensor Decomposition and Applications
A detailed review of tensor decomposition and its application in different fields (e.g., signal processing, computer vision, data mining, scientific computing and neuroscience) is provided in [11]. Furthermore, the tensor decomposition has been recently shown promising in various areas including neural network (NN) compression [13, 14, 15, 16, 17, 18, 19, 20], uncertainty quantification [19, 2, 3, 4], and tensor completion/recovery [20, 10, 21] to name a few. In many real-world applications, deciding about some hyper-parameters (such as tensor ranks and tensor shapes) of the tensor decomposition can be very challenging. There have been some recent studies which addressed the tensor rank determination problem [10, 20, 12] [14, 11]. The recent work [11] determines the tensor ranks automatically in a neural network training, enabling on-device training of neural networks with limited computing resources [5]. However, the study of the effect of the shape on the tensor decomposition has been rarely reported in the literature. Therefore, in this work we investigate how reshaping may affect the result of the decomposition and how an optimum shape can be found if there exists one.

2.2. Evolutionary Algorithms
The origin of evolutionary computation dates back to mid 1950s when it was applied in mathematical programming, machine learning, and industrial manufacturing and notably the invention of evolutionary strategies (ES), evolutionary programming (EP) and genetic algorithms (GAs) [22]. [23] presented the early version of the genetic algorithm (GA). Over the past years, variations of evolutionary algorithms (i.e., GAs) have been developed and have been extensively applied to solve problems in various fields where the problems were not approachable with other optimization methods. [24] presented a wide range of evolutionary algorithms including GAs and their applications in engineering domains. Particularly, [25] applied an evolutionary algorithm to find optimal hyper parameters of the singular value decomposition for the neural network compression.

2.3. Evolutionary Algorithms and Tensor Decomposition
At the intersection of evolutionary algorithms and tensor decompositions, [26] suggested tensor decomposition-based mutation in the neuroevolution of augmenting topologies (NEAT) algorithm. [27]
applied the CP decomposition to solve high-dimensional optimization problems with evolutionary algorithms. \cite{23} formulated the CP decomposition of non-negative tensors as a stochastic problem and solved it using an evolutionary algorithm. Also, \cite{9} applied an evolutionary search for determining an optimum tensor network topology. To the best of our knowledge, the present study is the first endeavor which applies an evolutionary tensor shape search with the tensor decomposition for an optimum data compression.

3. BACKGROUND

Throughout this manuscript, capital calligraphic letters (e.g., $A$) are used to denote tensors, boldface capital letters (e.g., $A$) are used for matrices, boldface lower case letters (e.g., $a$) are used for vectors, and Roman (e.g., $a$) or Greek (e.g., $\alpha$) letters are used for scalars. $\mathcal{A}[i_1, \ldots, i_d]$ refers to the element $t_{i_1, \ldots, i_d}$ of the tensor $\mathcal{A}$.

3.1. Tensor Shape and Reshaping

An order-$k$ ($k$-way) tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times \ldots \times I_k}$ denotes a $k$-dimensional data array. The order of a tensor is the number of its dimensions. We use $\Theta = (I_1, I_2, \ldots, I_k)$ to specify the shape of a tensor, where $I_j \in \mathbb{N}$ is the size of dimension $j$. Therefore, the shape of a tensor determines the order and the number of elements on each dimension. Reshaping refers to changing the order and the number of elements on each dimension. For example, a $k$-way tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times \ldots \times I_k}$ can be reshaped to a $d$-way tensor-like $\mathcal{Y} \in \mathbb{R}^{n_1 \times \ldots \times n_d}$. Reshaping a tensor may change its cardinality. If the cardinality of $\mathcal{Y}$, $|\mathcal{Y}|$, is greater than that of $\mathcal{X}$, $|\mathcal{X}|$, then dummy elements (e.g., zeros) are entered. Throughout the manuscript, we apply two different functions for reshaping: (1) reshape($\mathcal{X}, \Theta$) is used when reshaping does not change the cardinality, and (2) $\Phi(\mathcal{X}, \Theta)$ is used to denote reshaping a given tensor $\mathcal{X}$ to a new shape $\Theta$ if reshaping can change the cardinality.

3.2. Tensor Train (TT) Decomposition

In the tensor train (TT) format \cite{8}, a $d$-way tensor $\mathcal{Y} \in \mathbb{R}^{n_1 \times \ldots \times n_d}$ is approximated with a set of $d$ cores $\hat{\mathcal{G}} = \{\hat{\mathcal{G}}_1, \hat{\mathcal{G}}_2, \ldots, \hat{\mathcal{G}}_d\}$ where $\hat{\mathcal{G}}_j \in \mathbb{R}^{r_j \times n_{j-1} \times n_j}$, $r_j$'s for $j = 1, \ldots, d - 1$ are the ranks, $r_0 = r_d = 1$, and each element of $\mathcal{Y}$ is approximated by Eq. (1):

$$\hat{\mathcal{Y}}[i_1, \ldots, i_d] = \sum_{i_0, \ldots, i_{d-1}} \hat{\mathcal{G}}_1[i_0, i_1, i_2] \hat{\mathcal{G}}_2[i_1, i_2, i_3] \ldots \hat{\mathcal{G}}_d[i_{d-1}, i_d, i_d].$$

(1)

Given an error bound $\epsilon$, the core factors, $\hat{\mathcal{G}}_j$'s, are computed using $(d - 1)$ sequential singular value decomposition (SVD) of the auxiliary matrices formed by unfolding tensor $\mathcal{Y}$ along different axes. This decomposition process which is called the TT-SVD is presented in Algorithm 1.

In this work, we only apply the proposed evolutionary tensor shape search for the TT-SVD. However, it is possible to extend this framework to other tensor decomposition methods such as the CP decomposition, the Tucker decomposition, and generally to the tensor networks.

4. TENSOR SHAPE OPTIMIZATION

Let $\mathcal{X} \in \mathbb{R}^{I_1 \times \ldots \times I_k}$ be the original data given to be compressed using the TT decomposition and $\tilde{\mathcal{X}} \in \mathbb{R}^{\tilde{I}_1 \times \ldots \times \tilde{I}_k}$ is the approximation of the given $\mathcal{X}$. For example, $\mathcal{X}$ can be an RGB image where $k = 3$. In order to compress the given data, the first step is reshaping the given $\mathcal{X}$ into a $d$-way tensor (usually $d \geq k$) like $\mathcal{Y} \in \mathbb{R}^{n_1 \times n_2 \times \ldots \times n_d}$. If the cardinality of $\mathcal{Y}$, $|\mathcal{Y}|$, is greater than that of $\mathcal{X}$, $|\mathcal{X}|$, then dummy elements (e.g., zeros) are entered. Next, $\mathcal{Y}$ is approximated using the TT decomposition where $\mathcal{Y} \in \mathbb{R}^{n_1 \times n_2 \times \ldots \times n_d}$ is the approximation of $\mathcal{Y}$.

Given $d$ (the order of $\mathcal{Y}$), let us define $\Theta = (n_1, n_2, \ldots, n_d)$ as a possible shape; $\mathcal{Y}_\Theta$ and $\tilde{\mathcal{Y}}_\Theta$ refer to the tensor with shape $\Theta$ and its approximation, respectively; and let $S$ be the shape made of all possible $\Theta$'s such that $n_1 \in \mathbb{N}$ and $l \leq n_i \leq u$ for $i = 1, 2, \ldots, d$ and $l, u \in \mathbb{N}$. If $l = d$, $d$ is the maximum order because when $n_1 = 1$ dimension $i$ becomes ineffective, practically. We are looking for a tensor shape which maximizes the compression ratio of the TT decomposition given an error bound $\epsilon$ as below:

$$\max_{\Theta \in \bar{S}} C(\Theta) = 1 - \frac{|\tilde{\mathcal{Y}}_\Theta|}{|\mathcal{X}|}$$

subject to

$$\tilde{\mathcal{Y}}_\Theta = f(\mathcal{Y}_\Theta, \epsilon)$$

$$\mathcal{Y}_\Theta = \Phi(\mathcal{X}, \Theta)$$

$$\Theta = \{\Theta | \Theta \in S, |\mathcal{Y}_\Theta| \geq |\mathcal{X}|\}$$

$$S = \{\Theta = (n_1, n_2, \ldots, n_d) | n_i \in \mathbb{N}, l \leq n_i \leq u\}$$

(2)

where $\Theta \subset S$ and the sub-space $\Theta$ refers to the feasible domain of the decision space $S$. $f(\mathcal{Y}_\Theta, \epsilon)$ generates the factors of $\mathcal{Y}_\Theta$, $\tilde{\mathcal{Y}}_\Theta$, using the TT-SVD algorithm based on the error bound $\epsilon$. $\Phi(\mathcal{X}, \Theta)$ re-sizes the given tensor $\mathcal{X}$ to the shape $\Theta$ and enter zero values (dummy elements) if $|\mathcal{Y}_\Theta| > |\mathcal{X}|$ to fill the rest of the reshaped tensor. The upper limit of the $C(\Theta)$ is 1. When $0 < C(\Theta) < 1$ the cardinality of the factors are less than that of the data but when $C(\Theta) \leq 0$ the memory requirement is inflated and there is no data compression.

Any shape that results in a tensor $\mathcal{Y}_\Theta$ whose cardinality is smaller than the cardinality of the original given data $\mathcal{X}$ ($|\mathcal{Y}_\Theta| < |\mathcal{X}|$) is infeasible because some data is missed. Furthermore, we allow dummy elements (e.g., zeros) when a possible $\Theta$ results in a tensor whose cardinality is greater than that of the original data. Any shape which results in an unnecessary large cardinality is undesirably because it makes the compression less efficient. The objective function defined in Eq. (2) maximizes the compression ratio considering the effect of the added dummy elements. Therefore, the objective function guides the search toward a shape whose cardinality is the closest to that of the data. The definition of the feasible

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Algorithm 1: TT-SVD

**Input:** $d$-way tensor $\mathcal{Y}$, error bound $\epsilon$.

**Output:** $\tilde{\mathcal{G}} = \{\tilde{G}_1, \tilde{G}_2, \ldots, \tilde{G}_d\}$

$$\sigma = \frac{\epsilon}{1 - \epsilon} \|\mathcal{Y}\|_F$$

$r_0 = 1, r_d = 1$. $W = \text{reshape}(\mathcal{Y}, (n_1, \frac{|\mathcal{Y}|}{r_1}))$

for $j = 1$ to $j = d - 1$ do

$$W = \text{reshape}(W, (r_{j-1}n_j, n_j, r_{j-1}n_j))$$

Compute $\sigma$-truncated SVD: $W = USV^T + E$, where $\|E\|_F \leq \sigma$

$r_j = \text{the rank of matrix } W$ based on $\sigma$-truncated SVD

$$\tilde{G}_j = \text{reshape}(U, (r_{j-1}, n_j, r_j))$$

$W = SV^T$

$$\tilde{G}_d = \text{reshape}(W, (r_{d-1}, n_d, r_d))$$

Return $\tilde{G} = \{\tilde{G}_1, \tilde{G}_2, \ldots, \tilde{G}_d\}$
subspace $\Theta$ prevents shapes which have a cardinality smaller than that of the original tensor.

Let us define $E(\theta)$ as the relative error measured by the Frobenius norm:

$$E(\theta) = \frac{\|X - \hat{X}_{\theta}\|_F}{\|X\|_F},$$

with $\hat{X} = \Phi^{-1}(\hat{Y}_\theta)$ and $\hat{Y}_\theta = \Psi(G_\theta)$.

where $\Phi(\cdot)^{-1}$ resizes the tensor to the original shape and removes dummy elements if there are any. $\Psi(\cdot)$ generates the approximation tensor $\hat{Y}_\theta$ from the factors, and $G_\theta$ refers to the decomposed factors. Since the added dummy elements are zero, then $\|X\|_F = \|Y\|_F$ and $\|X - \hat{X}_{\theta}\|_F \leq \|Y - \hat{Y}_{\theta}\|_F$. Also, the TT-SVD guarantees that $\|Y - \hat{Y}_{\theta}\|_F \leq \varepsilon$. Therefore, if the TT-SVD (described in Algorithm 1) is applied for the decomposition of the reshaped tensor, $E(\theta) \leq \varepsilon$ and it is not required to consider the error bound as a constraint in the optimization model.

5. GENETIC ALGORITHM FOR TENSOR SHAPE SEARCH

We apply a genetic algorithm (GA) to solve the defined optimization model and to find the optimum shape for the tensor decomposition. A pseudo code of the GA for tensor shape search is presented in Algorithm 2 and its key steps are described below.

5.1. Initialization

The GA starts with generating a set of random shapes (solutions) $I = \{\theta_1, \theta_2, ..., \theta_m\}$ as an initial population. The initial population is generated by applying a discrete uniform distribution [specifically, $\text{unif}(l, u)$] on each variable $(n_i, k = 1, 2, ..., d)$ of $\theta_j = (n_1, n_2, ..., n_d)$ for $j = 1, 2, ..., m$. Next for each shape $\theta_j$, the TT-SVD is called, and the compression ratio $C(\theta_j)$ is calculated by Eq. (2).

5.2. Selection

Proportional to the compression ratio, a selection probability is assigned to each shape using the equation below:

$$\Pi(\theta_j) = \frac{C(\theta_j)}{\sum_{j=1}^{m} C(\theta_j)}, \quad j = 1, 2, ..., m,$$

where $\Pi(\theta_j)$ is the selection probability of shape $\theta_j$. In the selection process of the GA, $p$ ($p < m$) shapes are selected as parents. $(p-1)$ solutions are selected based on the probability distribution $\Pi$ (calculated above) with replacement such that the shapes with higher probability ($\Pi$) have more chance to be selected to enter to the parent set. If a solution is selected several times, then several copies of that exist in the parent set. An elitism operation is also applied so that the best shape of the current population (the shape with the maximum compression ratio) is moved to the parent set with probability 1.

5.3. Reproduction

During the reproduction process, first the crossover operator is applied. Based on the crossover operator two shapes like $\theta = (n_1, ..., n_d)$ and $\theta' = (n'_1, ..., n'_d)$ are randomly selected from the

parent set and a new trial shape is generated by exchanging the variables of the two selected solution as below:

$$\theta^\text{new} = (n_1, ..., n_c, n'_c+1, ..., n'_d) \quad (5)$$

where $c$ is the crossover point. Next, the mutation operator is applied on the newly generated solution. Based on the mutation operator some of the dimensions (variables) of the newly generated shapes are randomly replaced with applying a discrete uniform distribution $\text{unif}(l, u)$. If $\theta = (n_1, ..., n_c, ..., n_d)$ is a newly generated shape by the crossover, the mutated shape is $\theta^\text{new} = (n_1, ..., n'_c, ..., n_d)$ where dimension $i$ is muted. The procedure of selecting parents and generating new solutions continue until $m - p$ new shapes are generated. The compression ratio of the newly generated shapes (new population) are calculated and the selection probabilities are updated.

5.4. Iteration and Convergence

The process of selection and reproduction repeats for $T$ iterations. The best final shape is reported as the best (optimum) solution. There is no guarantee that the GA finds an optimum solution but experimental results have shown the effectiveness of the GA in finding a near optimum solution. [24], [29] presented the stochastic convergence of the elitist GA.

6. EXPERIMENTAL RESULTS

We apply the proposed evolutionary tensor shape search linked with the TT-SVD algorithm to decompose some arbitrary RGB images from the Microsoft common objects in context (COCO) data set [30] depicted in Fig. 1. In the experiments, the images are resized such that the longest dimension has 320 pixels with a fixed aspect ratio of the original image. Fig. 3 also shows the original shape (height,
width, depth) of the data arrays of the images below them. We compare the decomposition results of the reshaped data with that of the original shapes. In order to have a fair comparison, we set $d = 3$ and $l = 2$. Therefore, all the optimum shapes are of order three similar to the original shapes. Allowing the GA to change the order may even lead to a better result depending on the data. For each image the GA run for 50 iterations with a population size of 100 and a parent size of 30. Fig. 2 shows the convergence curve of the GA runs for the studied images. The result of the experiments for the error bound equal to 0.1 is listed in Table 1. In Table 1 $\theta^*$ refers to the optimum shape found by the GA.

In Table 1 it is seen that for all images the compression ratio of the optimum shape ($\theta^*$) found by the GA is superior to that of the original shape ($\theta$). Also, all the errors are smaller than the error bound $\epsilon$. Although the error slightly increases by improving the compression ratio, the change in the error is negligible and is bounded whereas the improvement in the compression ratio is significant. In Table 1 it is also seen that the compression ratios of different images vary and it is because the images have different ranks. Regardless of the rank of the images, the proposed method improved the compression ratio of all the studied images. We can conclude that the compression results of the optimum shapes were significantly improved in comparison with that of the original shapes.

### Table 1. The result of the compression of the studied images with their original shape ($\theta$) and the optimum shape ($\theta^*$) for $\epsilon = 0.1$

| Image | $C(\theta)\%$ | $E(\theta)$ | Optimum shape ($\theta^*$) | $C(\theta^*)\%$ | $E(\theta^*)$ |
|-------|---------------|-------------|-----------------------------|-----------------|----------------|
| 1     | 72.98         | 0.0553      | (222,16,60)                | 89.78           | 0.0694         |
| 2     | 75.14         | 0.0505      | (437,8,60)                 | 88.88           | 0.0701         |
| 3     | 94.18         | 0.0526      | (428,10,48)                | 98.31           | 0.0680         |
| 4     | 82.89         | 0.0559      | (107,16,120)               | 92.35           | 0.0696         |
| 5     | 62.58         | 0.0646      | (471,8,60)                 | 75.46           | 0.0702         |
| 6     | 17.70         | 0.0495      | (1920,4,30)                | 58.46           | 0.0694         |
| 7     | 36.07         | 0.0499      | (2270,3,30)                | 65.71           | 0.0697         |
| 8     | 97.13         | 0.0583      | (71,320,9)                 | 98.65           | 0.0695         |
| 9     | 79.61         | 0.0505      | (193,51,21)                | 85.61           | 0.0694         |
| 10    | 80.21         | 0.0504      | (349,12,60)                | 88.52           | 0.0685         |

### 7. CONCLUSION AND FUTURE WORK

In this work, we have studied the possible effect of the shape of the tensor in data compression using the tensor decomposition. We have formulated the task of finding the optimum shape for the tensor train (TT) decomposition as an optimization model which maximizes the compression ratio subject to an error bound. We have solved the proposed optimization model using a genetic algorithm (GA) linked with the TT-SVD algorithm. The results have demonstrated the effectiveness of the proposed method in efficiently compressing the given data while keeping the error bounded.

The study of the effectiveness of the proposed evolutionary tensor shape search for other decomposition methods and improving the efficiency of the optimization algorithm are the subjects of the future studies.
8. REFERENCES

[1] T. G. Kolda and B. W. Bader, “A fast learning algorithm for deep belief nets,” SIAM review, vol. 51(3), pp. 455–500, 2009.

[2] Z. Zhang, X. Yang, I. V. Oseledets, G. E. Karniadakis, and L. Daniel, “Enabling high-dimensional hierarchical uncertainty quantification by ANOVA and tensor-train decomposition,” IEEE Transactions on Computer-aided Design of Integrated Circuits and Systems, vol. 34(1), pp. 63–76, 2015.

[3] Z. Zhang, W. T. Weng, and L. Daniel, “Big-data tensor recovery for high-dimensional uncertainty quantification of process variations,” IEEE Transactions on Components, Packaging and Manufacturing Technology, vol. 7(5), pp. 687–697, 2017.

[4] Z. Zhang, K. Batselier, H. Liu, L. Daniel, and N. Wong, “Tensor computation: a new framework for high-dimensional problems in EDA,” IEEE Transactions on Computer-aided Design of Integrated Circuits and Systems, vol. 36(4), pp. 521–536, 2017.

[5] K. Zhang, C. Hawkins, X. Zhang, C. Hao, and Z. Zhang, “On-FPGA training with ultra memory reduction: A low-precision tensor method,” arXiv preprint arXiv:2104.03420, 2021.

[6] R. Bro, “Parafac. tutorial and applications,” Intelligent Laboratory Systems, vol. 38(2), pp. 149–171, 1997.

[7] L. R. Tucker, “Some mathematical notes on three-mode factor analysis,” Psychometrika, vol. 31(3), pp. 279–311, 1966.

[8] V. Oseledets, “Tensor train decomposition,” SIAM journal on Scientific Computation (SISC), vol. 33(5), pp. 2295–2317, 2011.

[9] C. Li and Z. Sun, “Evolutionary topology search for tensor network decomposition,” Proc. International Conference on Machine Learning, vol. 119, pp. 5947–5957, 2020.

[10] Q. Zhao, L. Zhang, and A. Cichocki, “Bayesian CP factorization of incomplete tensors with automatic rank determination,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 37(9), pp. 1751–1763, 2015.

[11] C. Hawkins, X. Liu, and Z. Zhang, “Towards compact neural networks via end-to-end training: A Bayesian tensor approach with automatic rank determination,” arXiv preprint arXiv:2010.08689, 2020.

[12] C. Hawkins and Z. Zhang, “Bayesian tensorized neural networks with automatic rank selection,” Neurocomputing, vol. 453, pp. 172–180, 2021.

[13] Y. D. Kim, E. Park, S. Yoo, T. Choi, L. Yang, and D. Shin, “Compression of deep convolutional neural networks for fast and low power mobile applications,” arXiv:1511.06530, 2015.

[14] V. Lebedev, Y. Ganin, M. Rakhuba, I. Oseledets, and V. Lempitsky, “Speeding-up convolutional neural networks using fine-tuned cp-decomposition,” arXiv:1412.6533, 2015.

[15] A. Nikov, D. Podoprikhin, A. Osokin, and D. Vetrov, “Tensorizing neural networks,” arXiv:1509.06569, 2015.

[16] W. Wang, Y. Eriksson B. Sun, and Wang W., “Wide compression: tensor ring nets,” arXiv:1802.09052, 2018.

[17] G. Shupeng, H. N. Wang, H. Yang, C. Yu, J. Wang, and J. Liu, “Model compression with adversarial robustness: a unified optimization framework,” Advances in Neural Information Processing Systems, vol. 32, pp. 1285–1296, 2019.

[18] Jingling Li, Y. Sun, J. Su, T. Suzuki, and F. Huang, “Understanding generalization in deep learning via tensor methods,” International Conference on Artificial Intelligence and Statistics, pp. 504–515, 2020.

[19] Z. He and Z. Zhang, “High-dimensional uncertainty quantification via tensor regression with rank determination and adaptive sampling,” IEEE Transactions on Components, Packaging and Manufacturing Technology, vol. 11, no. 9, pp. 1317–1328, 2021.

[20] P. Rai, Y. Wang, S. Guo, G. Chen, D. Dunson, and L. Carin, “Scalable Bayesian low-rank decomposition of incomplete multiway tensors,” Proceedings of the 31st International Conference on Machine Learning, vol. 32(2), pp. 1800–1808, 2014.

[21] Q. Zhao, L. Zhang, and A. Cichocki, “Bayesian sparse Tucker models for dimension reduction and tensor completion,” arXiv:1505.02343, 2015.

[22] R. Solgi and H. A. Loaiciga, “Bee-inspired metaheuristics for global optimization: a performance comparison,” Artificial Intelligence Review, 2021.

[23] J. H. Holland, “Adaptations in natural and artificial systems,” University of Michigan Press, Ann Arbor, MI, 1975.

[24] O. Bozorg-Haddad, M. Solgi, and H. A. Loaiciga, Metaheuristic and Evolutionary Algorithms for Engineering Optimization, Wiley, 2017.

[25] J. Huang, W. Sun, and L. Huang, “Deep neural networks compression learning based on multiojective evolutionary algorithms,” Neurocomputing, vol. 378, pp. 260–269, 2020.

[26] A. Marzullo, C. Stamioli, F. Calimeri, and S. Van Huffel, “A tensor-based mutation operator for neuroevolution of augmenting topologies (NEAT),” 2017 IEEE Congress on Evolutionary Computation (CEC), pp. 681–687, 2017.

[27] Q. Wang, L. Zhang, S. Wei, and B. Li, “Tensor decomposition-based alternate sub-population evolution for large-scale many-objective optimization,” Information Sciences, vol. 569, pp. 376–399, 2021.

[28] S. Laura, C. Prissette, S. Maire, and N. Thirion-Moreau, “A parallel strategy for an evolutionary stochastic algorithm: application to the CP decomposition of nonnegative n-th order tensors,” 28th European Signal Processing Conference (EUSIPCO), pp. 1956–1960, 2021.

[29] R. R. Sharapov and A. V. Lapshin, “Convergence of genetic algorithms,” Pattern Recognition and Image Analysis, vol. 16, pp. 392–397, 2006.

[30] T. Y. Lin, M. Maire, S. Belongie, L. Bourdev, D. Girshick, J. Hays, P. Perona, D. Ramanan, C. L. Zitnick, and A. Dollar, “Microsoft COCO: common objects in context,” arXiv:1405.0312, 2015.