Plasma diagnostics for planetary nebulae and H II regions using the N II and O II optical recombination lines

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ABSTRACT

We carry out plasma diagnostic analyses for 123 planetary nebulae (PNe) and 42 H II regions using the N II and O II optical recombination lines (ORLs). New effective recombination coefficients for the N II and O II optical recombination spectra are used. These data were calculated under the intermediate coupling scheme for a number of electron temperature ($T_e$) and density ($N_e$) cases. We use a new method to determine the $T_e$’s and $N_e$’s for the nebular sample, combining the ORLs with the most reliable measurements for each ion and the predicted intensities that are based on the new atomic data. Uncertainties of the derived $T_e$ and $N_e$ are estimated for each object.

The diagnostic results from heavy element ORLs show reasonable agreement with previous calculations in the literature. We compare the electron temperatures derived from the N II and O II ORLs, $T_e$(ORLs), and those from the collisionally excited lines (CELs), $T_e$(CELs), as well as the hydrogen Balmer jump, $T_e$(H β BJ), especially for the PNe with large abundance discrepancies. Temperatures from the He I recombination lines, $T_e$(He I), are also used for comparison if available. For all the objects included in our sample, $T_e$(ORLs) are lower than $T_e$(H β BJ), which are in turn systematically lower than $T_e$(CELs). PNe with $T_e$(He I) available show the relation $T_e$(ORLs) $\leq$ $T_e$(He I) $\leq$ $T_e$(H β BJ) $\leq$ $T_e$(CELs), which is consistent with predictions from the bi-abundance nebular model postulated by Liu et al.

Key words: atomic data—H II regions—planetary nebulae: general.

1 INTRODUCTION

Photoionized nebulae, such as planetary nebulae (PNe) and H II regions, provide much of our knowledge of elemental abundances in the Milky Way and other galaxies. Accurate measurements of electron temperatures ($T_e$) and densities ($N_e$) are essential for reliable determination of elemental abundances. Until recently, the principal means of determining elemental abundances in nebulae has been from the measurement of collisionally excited lines (CELs). The emissivities of these lines relative to a hydrogen recombinations (ORLs) are lower than $T_e$(CELs), which is consistent with predictions from the bi-abundance nebular model postulated by Liu et al.

or a heavy element with that of hydrogen. Unlike CELs, such as the [O III] and [N II] nebular lines, whose emissivities relative to a hydrogen recombination line increase exponentially with $T_e$, the emissivities of heavy element ORLs relative to a hydrogen recombination line change weakly with both $T_e$ and $N_e$, as the emissivities of recombination lines have only a similar, power-law dependence on $T_e$ and $N_e$, apart from the parentage effects to be discussed in Section 2.2, and are essentially independent of $N_e$ under typical nebular physical conditions (e.g. Liu 2006a,b). Consequently, the method based on ORLs is much less affected by temperature measurement errors, and the results in principal should be more conclusive. It is possible to use the intensity ratio of two ORLs originating from the recombination of different ion parents to estimate $N_e$ (Fang, Storey & Liu 2011), since the density sensitivity of the parent populations is partially reflected in the resultant ORL relative intensities.

While emissivities of heavy element ORLs have only a relatively weak, power-law dependence on $T_e$, this dependence varies for lines originating from levels of different orbital angular momentum quantum number $l$. Therefore, the relative intensities of ORLs can be used to derive electron temperature, provided that very accurate measurements can be secured (Liu 2003; Liu et al. 2004; Tsamis et al. 2004). Since the sensitivity of the ORL ratios to electron

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temperature and density is very weak, in order to obtain $T_e$'s and $N_e$'s of the nebular regions where ORLs arise, one needs to acquire high-precision measurements of these ORLs. However, given the low nebular abundances of heavy elements ($\sim 10^{-4}$ to $10^{-3}$ or even lower relative to hydrogen) and the relatively long time-scales for a heavy element ion to recombine with an electron under the physical conditions of gaseous nebulae (Osterbrock & Ferland 2006), the heavy element ORLs are very weak compared to the CELs and low-order hydrogen lines. Therefore, high-precision measurements of those ORLs are required.

Although several deep spectroscopic surveys have been carried out during the past decade for several dozen Galactic disc and bulge PNe (Tsamis et al. 2003b, 2004; Liu et al. 2004; Robertson-Tessi & Garnett 2005; Wesson, Liu & Barlow 2005; Wang & Liu 2007) and for a number of Galactic and extragalactic H II regions (Esteban et al. 2002, 2004; Tsamis et al. 2003a; García-Rojas et al. 2004, 2005, 2006; Peimbert et al. 2004), most of the ORL measurements are still not accurate enough for nebular analysis, due to either relatively low signal-to-noise ratios or line blending. Currently, only a limited number of objects (mainly nearby Galactic PNe) have deep enough spectra for recombination line analysis (e.g. Liu et al. 1995, 2000, 2001; Esteban et al. 1999, 2004; Sharpee, Baldwin & Williams 2004; Zhang et al. 2005a; Fang & Liu 2011).

In nebular astrophysics, there has been a long-standing dichotomy in abundance determinations and plasma diagnostics. For a given PN, the abundances of heavy element ions (C$^2+$, N$^2+$, O$^+$ and Ne$^2+$) relative to hydrogen derived from the nebular ORLs are all higher than the corresponding abundance values derived from their CEL counterparts. The electron temperatures derived from the H II Balmer jump (BJ) are also systematically lower than those derived from the CELs. A number of mechanisms have been proposed to explain this dichotomy (e.g. Peimbert 1967; Rubin 1989; Viegas & Clegg 1994), but have failed to provide a consistent interpretation of all observations, especially for those PNe with dramatically large abundance discrepancies (e.g. $>10$, 20). A bi-abundance nebular model postulated by Liu et al. (2000) provides a more natural explanation of this dichotomy. In this model, the ORLs heavy element ions arise mainly from ‘cold’ H-deficient inclusions, while the strong CELs are emitted predominantly from the warmer ambient ionized gas of ‘normal’ ($\sim$ solar values) chemical composition. Deep spectroscopic surveys (e.g. Tsamis et al. 2003b, 2004; Liu et al. 2004; Robertson-Tessi & Garnett 2005; Wesson et al. 2005; Wang & Liu 2007) and ORL analysis of individual nebulae in the past decade (e.g. Liu et al. 1995, 2000, 2001; Liu 2006a) have yielded strong evidence for the existence of such a ‘cold’ component (recent reviews on this topic are given by Liu 2003, 2006b, 2011). More recently, Nicholls, Dopita & Sutherland (2012) proposed that a $k$-distribution for electron energies, which is a departure from the conventional assumption of a Maxwell–Boltzmann equilibrium energy distribution, can explain the dichotomy in both H II regions and PNe.

Despite the paucity of high-quality observational data of the heavy element ORLs as well as the lack of the atomic data that are adequate enough for nebular analysis, plasma diagnostics have been carried out for a number of PNe and preliminary results have been obtained. Using the radiative recombination coefficients of Péquignot, Petitjean & Boisson (1991), Tsamis et al. (2004) derived electron temperatures using the O II ORLs for six Galactic PNe, and found an average $T_e$(O II ORLs) value of 2920 $\pm$ 2690 K. Liu et al. (2004) derived $T_e$(O II ORLs) for 18 PNe, and found an average value of 4910 $\pm$ 4060 K. Wesson et al. (2005) derived electron temperatures for a dozen northern Galactic PNe using the O II ORL ratios $\lambda 4075/\lambda 4089$, $\lambda 4649/\lambda 4089$, $\lambda 4072/\lambda 4089$, $\lambda 4414/\lambda 4089$ and $\lambda 1120$. Wang & Liu (2007) determined $T_e$(O II ORLs) for 11 Galactic PNe using the O II $\lambda 4089/\lambda 4649$ line ratio, and gave an average value of 4370 $\pm$ 5760 K. However, many of the recombination line intensities used for diagnostics were not corrected for line blending, and the derived electron temperatures could be problematic.

In this paper, we present a new approach to plasma diagnostics using the N II and O II ORLs for a sample of PNe and H II regions, which are selected from previous ORL surveys and individual-object studies. The most recent effective recombination coefficients are utilized for the analysis. For each ion, we use one set of detected ORLs with the most reliable measurements to constrain $T_e$ and $N_e$ simultaneously, by comparing the observed intensities with the predicted ones. The primary purpose of the current paper is to show that the plasma diagnostic method employed here is applicable to all emission nebulae, provided that more and more accurate measurements of the N II and O II ORLs are available in the future.

## 2 OPTICAL RECOMBINATION LINE DIAGNOSTICS

In this section, we compare the intensities of the observed N II and O II lines with theoretical predictions over a wide range of electron temperatures and densities.

### 2.1 Planetary nebula and H II region sample

During the past two decades, several deep optical spectroscopic surveys have been carried out and published for several dozens of Galactic disc, bulge and halo PNe and for a number of Galactic and extragalactic H II regions. These allow for detailed nebular plasma diagnostics and abundance analyses using the relatively weak hydrogen and helium recombination lines/continua and ORLs from heavy element ions. In total, over 100 PNe and 40 H II regions have been studied using ORLs. The current paper makes use of the samples from the literature listed in Tables 1 and 2.

High spectral and spatial resolution spectroscopy of PNe have been published in the literature for the detailed analysis of the physical condition and elemental abundance distributions across the nebulae. Tsamis et al. (2008) carried out a dedicated study of three Galactic PNe (NGC 5882, NGC 6153 and NGC 7009) by means of optical integral field spectroscopy using the Fibre Large Array Multi Element Spectrograph on the Very Large Telescope (VLT/FLAMES). The ratio of the abundances derived from ORLs and CELs, which is called the abundance discrepancy factor (ADF), was studied across the nebulae for doubly ionized oxygen, O$^++$. Very small values of the temperature fluctuation parameter $r^\prime$ (defined by Peimbert 1967, 1971) in the plane of the sky were found in all three objects. Most recently, Mesa-Delgado et al. (2012) published results from integral field spectroscopy of a field located near the Trapezium cluster in the Orion nebula using the Potsdam Multi-Aperture Spectrophotometer (PMAS). Detailed studies of one of the three most prominent protoplanetary discs (propyls) show that the ADF of O$^++$ is close to 1 in the propyl, while the background emission still yields the typical ADF(O$^++$) observed in the Orion nebula.

### 2.2 Atomic data

Since the 1990s, it has been possible to obtain reliable measurements of the faint ORLs emitted by heavy element ions for bright
# Table 1. Electron temperatures and densities for PNe.

| Object         | ADF (O²⁺) | From the literature | From current work |
|----------------|-----------|---------------------|-------------------|
|                | log $T_e$ (OII) (K) | log $T_e$ (NI) (K) | log $n_e$ (cm⁻³) |
| Abell 30 J3    | 766.00    | 4.25⁰⁻                             | 4.30               |
| Abell 30 J1    | 598.00    | 4.22⁰⁻                             | 4.30               |
| Abell 56       | 89.00     | 4.08                             | 4.20              |
| Hf 2-2 (2 arcsec) | 84.00    | 3.94⁰⁻                             | 2.97               |
| Hf 2-2 (4 arcsec) | 84.00    | 3.96⁰⁻                             | 2.94               |
| Hf 2-2 (8 arcsec) | 84.00    | 3.95⁰⁻                             | 2.96               |
| NGC 1501       | 32.00     | 4.04⁰⁻                             | 3.97               |
| M 1-42         | 22.00     | 3.96⁰⁻                             | 3.60               |
| NGC 40         | 17.30     | 4.03⁰⁻                             | 3.85               |
| M 2-24         | 17.00     | 4.20⁰⁻                             | 2.10               |
| NGC 2022       | 16.00     | 4.18⁰⁻                             | 4.12               |
| NGC 2022       | 16.00     | 4.18⁰⁻                             | 4.12               |
| DDm 113        | 11.80     | 4.09⁰⁻                             | 3.94               |
| VY 2-13        | 11.80     | 4.14⁰⁻                             | 3.97               |
| NGC 6153       | 9.20      | 3.86⁰⁻                             | 3.78               |
| NGC 2440       | 8.90      | 4.21⁰⁻                             | 4.15               |
| IC 2003        | 7.31      | 4.10⁰⁻                             | 4.04               |
| M 2-36         | 6.90      | 3.92⁰⁻                             | 3.78               |
| VY 1-13        | 6.17      | 4.02⁰⁻                             | 3.82               |
| NGC 3242       | 5.70      | 4.07⁰⁻                             | 4.01               |
| M 3-27         | 5.48      | 4.11⁰⁻                             | 3.96               |
| NGC 2440       | 5.40      | 4.21⁰⁻                             | 4.15               |
| NGC 7009       | 5.14      | 4.21⁰⁻                             | 4.15               |
| NGC 6818       | 4.90      | 4.12⁰⁻                             | 4.08               |
| NGC 7009       | 4.70      | 4.04⁰⁻                             | 3.86               |
| IC 3568        | 4.60      | 4.06⁰⁻                             | 3.97               |
| NGC 6210       | 4.30      | 3.99⁰⁻                             | 3.94               |
| M 3-34         | 4.23      | 4.09⁰⁻                             | 3.93               |
| NGC 6543       | 4.20      | 4.14⁰⁻                             | 3.83               |
| NGC 6543       | 4.20      | 4.14⁰⁻                             | 3.83               |
| NGC 6543 S     | 4.20      | 4.14⁰⁻                             | 3.83               |
| Hu 2-13        | 4.00      | 3.99⁰⁻                             | 4.00               |
| M 1-73         | 3.61      | 3.87⁰⁻                             | 3.74               |
| NGC 6302       | 3.60      | 4.26⁰⁻                             | 4.21               |
| NGC 3132       | 3.50      | 3.98⁰⁻                             | 4.04               |
| NGC 6302       | 3.50      | 4.26⁰⁻                             | 4.21               |
| NGC 7026       | 3.36      | 3.97⁰⁻                             | 3.87               |
| IC 1747        | 3.20      | 4.04⁰⁻                             | 3.98               |
| IC 351         | 3.14      | 4.12⁰⁻                             | 4.04               |
| NGC 6210 S     | 3.10      | 3.98⁰⁻                             | 3.94               |
| Hu 1-13        | 2.97      | 4.08⁰⁻                             | 3.92               |
| Sp 4-11        | 2.94      | 4.05⁰⁻                             | 3.95               |
| IC 4846        | 2.91      | 4.03⁰⁻                             | 3.89               |
| IC 4846        | 2.91      | 4.00⁰⁻                             | 4.26               |
| NGC 6803       | 2.71      | 3.99⁰⁻                             | 3.93               |
| Bo l l 11      | 2.63      | 4.14⁰⁻                             | 3.95               |
| NGC 6833       | 2.47      | 4.11⁰⁻                             | 4.15               |
| NGC 6879       | 2.46      | 4.02⁰⁻                             | 3.93               |
| IC 4191 fixed  | 2.40      | 4.03⁰⁻                             | 4.02               |

*Note: log $T_e$ values are given in Kelvin (K), log $n_e$ values in cm⁻³.*
Table 1 – continued

| Object         | ADF (O$^{2+}$) | From the literature | From current work |
|----------------|----------------|---------------------|-------------------|
|                | T$_{O}$ (OIII) | log T$_{e}$ (K)    | log T$_{e}$ (K)   |
| IC 4191 scanning | 2.40           | 4.03$^a$            | 2.10              |
| NGC 3132$^{10}$ | 2.40           | 3.96$^{10}$         | 3.30$^{12}$       |
| NGC 6720$^7$    | 2.40           | 4.03$^7$            | 2.10$^{0.05}$     |
| NGC 6818$^{10}$ | 2.40           | 4.12$^{10}$         | 3.20$^{0.52}$     |
| IC 4191 nebula$^{10}$ | 2.40       | 4.03$^{10}$         | 3.80$^{0.08}$     |
| IC 4191 fixed$^{10}$ | 2.40         | 4.03$^{10}$         | 3.80$^{0.08}$     |
| NGC 3918$^4$    | 2.30           | 4.10$^4$            | 2.10$^{0.01}$     |
| NGC 6884$^7$    | 2.30           | 4.04$^7$            | 3.90$^{16}$       |
| NGC 5217$^{13}$ | 2.26           | 4.05$^{13}$         | 2.10$^{0.05}$     |
| NGC 3242$^4$    | 2.20           | 4.07$^4$            | 4.00$^{0.11}$     |
| M 1-74$^{13}$   | 2.14           | 4.01$^{13}$         | 3.60$^{0.11}$     |
| NGC 5882$^{12}$ | 2.14           | 4.01$^{12}$         | 3.60$^{0.11}$     |
| NGC 5307$^3$    | 1.95           | 4.03$^3$            | 3.90$^{0.07}$     |
| IC 4406$^9$     | 1.90           | 4.00$^9$            | 3.90$^{0.07}$     |
| IC 4406$^{10}$  | 1.90           | 4.00$^{10}$         | 3.90$^{0.07}$     |
| NGC 6741$^7$    | 1.90           | 3.96$^7$            | 4.15$^7$          |
| NGC 6826$^7$    | 1.90           | 3.97$^7$            | 3.94$^7$          |
| My Cn 18$^4$    | 1.80           | 3.86$^4$            | 4.30$^7$          |
| NGC 3918$^{10}$ | 1.80           | 4.10$^{10}$         | 4.09$^{10}$       |
| NGC 7027$^{12}$ | 1.80           | 4.15$^{12}$         | 4.08$^{12}$       |
| NGC 5315$^8$    | 1.74           | 3.95$^8$            | 3.93$^8$          |
| NGC 6790$^7$    | 1.70           | 4.11$^7$            | 4.15$^7$          |
| NGC 6790$^{12}$ | 1.70           | 4.11$^{12}$         | 4.15$^{12}$       |
| Hu 1-2$^7$      | 1.60           | 4.29$^7$            | 4.30$^7$          |
| NGC 6572$^7$    | 1.60           | 4.03$^7$            | 2.50$^{0.59}$     |
| NGC 6572$^{12}$ | 1.60           | 4.01$^{12}$         | 4.09$^{12}$       |
| NGC 6572 S$^{12}$ | 1.60         | 4.01$^{12}$         | 4.09$^{12}$       |
| NGC 6891$^{13}$ | 1.52           | 3.97$^{13}$         | 3.77$^{13}$       |
| NGC 5315$^4$    | 1.40           | 3.95$^4$            | 3.93$^4$          |
| M 3-32$^{17}$   | 1.15           | 3.95$^{17}$         | 3.65$^{17}$       |
| M 3-33$^{17}$   | 1.10           | 4.02$^{17}$         | 3.77$^{17}$       |
| IC 4699$^{17}$  | 1.09           | 4.07$^{17}$         | 4.08$^{17}$       |
| NGC 6439$^{17}$ | 1.09           | 4.02$^{17}$         | 4.01$^{17}$       |
| H 1-41$^{17}$   | 1.08           | 3.99$^{17}$         | 3.95$^{17}$       |
| M 3-7$^{17}$    | 1.07           | 3.88$^{17}$         | 3.84$^{17}$       |
| NGC 6620$^{17}$ | 1.06           | 3.98$^{17}$         | 4.00$^{17}$       |
| H 1-50$^{17}$   | 1.05           | 4.04$^{17}$         | 4.10$^{17}$       |
| M 3-21$^{17}$   | 1.05           | 3.97$^{17}$         | 4.10$^{17}$       |
| H 1-35$^{17}$   | 1.04           | 3.96$^{17}$         | 4.00$^{17}$       |
nuclei. However, atomic data are necessary to analyse these spectral features, in particular the effective recombination coefficients. ORL ratios from states of different orbital angular momenta do vary since the lines are emitted (e.g. Liu 2003).

Relative populations of the fine-structure levels of the ground term of a recombining ion, such as N^2+ in the case of N II and O^2+ in the case of O II, deviate from the Boltzmann distribution and vary as a function of electron density in low-density nebulae. This variation in the level population is reflected in the emissivities, and thus in the effective recombination coefficients of the recombination lines that are formed from different parent levels. In essence, the intensity ratio of two such lines can be used to determine electron density (e.g. Fang et al. 2011).

High-precision quantum mechanical calculations of the effective recombination coefficients for N II (Fang et al. 2011) and O II lines (Storey, unpublished) have been completed. Both calculations improve those from previous work (e.g. Nussbaumer & Storey 1984; Escalante & Victor 1990; Péguignon et al. 1991; Storey 1994; Liu et al. 1995; Kisielius et al. 1998; Kisielius & Storey 2002). The strongest and best-observed lines for N II so far have been those from multiplets V3 3p 1D−3s 3P, V10 3d 1F−3p 3D and V48a,b 4f G[7/2,9/2]−3d 3P. For O II, they are from multiplets V1 3p 1D−3s 3P, V10 3d 1F−3p 3D and V48a,b 4f G[5,4]−3d 4F for O II.

Fig. 1 shows the theoretical fractional intensities of the fine-structure components of the O II multiplet V 3p 2 3P−2p 3s 3P as a function of electron density at four temperature cases. The same can be seen for the N II multiplet V 2p 3 3P−2p 3s 3P in fig. 5 from Fang et al. (2011). Fig. 2 shows the loci of the O II recombination line ratios I(λ4649)/I(λ4662) and I(λ4649)/I(λ4089) for different T_e’s and N_e’s. The same can be seen for N II recombination line ratios I(λ5767)/I(λ5666) and I(λ5769)/I(λ4041) in fig. 8 from Fang et al. (2011). Using the accurate measurements of the N II and O II line ratios, we can determine T_e’s and N_e’s simultaneously from the loci. Since different ORL ratios of N II or O II have different sensitivity on T_e and N_e, this will affect the reliability of the results.

Given the different temperature-dependence of diagnostic lines, individual diagnostic line ratios are expected to yield differing results if the nebulae are not isothermal. While the small values of r^2 appear to suggest that there are no large temperature variations where the CELs are emitted, the discrepancies between the CEL and ORL diagnostic results definitely exist, thereby suggesting that the gas is not isothermal. While the authors believe that the bi-abundance nebular model is a satisfactory explanation of the abundance (temperature) discrepancies, an alternative explanation involving non-equilibrium electron energies has also been
Electron temperatures and densities for H\n regions.

| Object            | ADF (O2\n+) | From the literature | From current work |
|-------------------|------------|---------------------|-------------------|
|                   | log \(T_e(\text{[O \textsc{ii}]})\) (K) | log \(T_e(\text{[B]})\) (K) | log \(T_e(\text{N})\) (K) | log \(T_e(\text{N})\) (cm\(^{-3}\)) | log \(T_e(\text{O})\) (K) | log \(T_e(\text{O})\) (cm\(^{-3}\)) |
| SMC N87\n\n | 2.80 | 4.09\n\n | 2.60 | 2.20 |
| M 17\n\n | 2.66 | 3.91\n\n | 4.00\n\n | 2.40\n\n | 2.60 | 2.00 |
| LMC N141\n\n | 2.60 | 4.07\n\n | 2.60 | 2.00 |
| NGC 3576\n\n | 2.21 | 3.95\n\n | 3.30\n\n | 6.00\n\n | 2.60 | 2.00 |
| 30 Doradus\n\n | 1.76 | 4.00\n\n | 3.30\n\n | 6.00\n\n | 2.60 | 2.00 |
| LMC N66\n\n | 1.61 | 4.26\n\n | 2.60\n\n | 2.00 | 2.00 |
| LMC N11 B\n\n | 1.47 | 3.97\n\n | 2.60\n\n | 2.60 | 2.00 |
| PB 6-2\n\n | 1.12 | 4.18\n\n | 2.60 | 2.00 |
| PB 6-1\n\n | 1.10 | 4.20\n\n | 2.60 | 2.00 |
| NGC 2366 NGC 2363\n\n | 1.05 | 3.84\n\n | 3.71\n\n | 2.10\n\n | 5.80\n\n | 4.20\n\n | 3.30\n\n | 6.00 |
| NGC 3603\n\n | 1.04 | 4.20\n\n | 2.85\n\n | 4.55 |
| NGC 2867-2\n\n | 1.03 | 4.06\n\n | 3.95\n\n | 2.60 | 3.80\n\n | 2.00 |
| S 311\n\n | 1.03 | 3.95\n\n | 3.98\n\n | 2.60 | 2.00 |
| M101 NGC 5461\n\n | 1.03 | 3.93\n\n | 3.15\n\n | 4.60\n\n | 2.00 |
| M101 NGC 5471\n\n | 1.03 | 4.15\n\n | 2.85\n\n | 4.90 | 1.00 |
| M 42\n\n | 1.02 | 3.92\n\n | 3.90\n\n | 4.00\n\n | 2.30\n\n | 4.20\n\n | 3.50\n\n | 1.00 |
| NGC 2867-1\n\n | 1.02 | 4.07\n\n | 3.95\n\n | 2.60 | 3.75\n\n | 7.00 |
| NGC 5253 HII-2\n\n | 1.01 | 4.08\n\n | 3.50\n\n | 3.30 |
| NGC 5253 UV-1\n\n | 1.01 | 4.04\n\n | 4.20\n\n | 2.85\n\n | 5.00 |
| NGC 5253 HII-1\n\n | 1.00 | 4.08\n\n | 3.90\n\n | 3.50\n\n | 3.50 |
| NGC 5253 UV-2\n\n | 1.00 | 4.04\n\n | 3.00\n\n | 2.50 |
| M33 NGC 604\n\n | 1.00 | 3.91\n\n | 3.00\n\n | 2.50 |
| M33 NGC 604\n\n | 1.00 | 3.91\n\n | 2.60\n\n | 2.00 |
| M101 NGC 5461\n\n | 0.99 | 3.93\n\n | 3.35\n\n | 2.80 |
| NGC 2366 NGC 2363\n\n | 0.97 | 4.20\n\n | 3.60\n\n | 2.60 |
| M101 NGC 5471\n\n | 0.97 | 3.93\n\n | 3.82\n\n | 3.05\n\n | 2.15 |
| SMC N66\n\n | 0.94 | 4.09\n\n | 2.60 | 2.00 |
| NGC 3576\n\n | 0.91 | 3.95\n\n | 3.91\n\n | 2.10 | 5.90 |
| NGC 4395 Reg 70\n\n | 0.77 | 4.03\n\n | 3.00\n\n | 2.40 |
| NGC 2403 VS 381\n\n | 0.61 | 3.94\n\n | 4.05\n\n | 2.76 |
| M 16\n\n | 0.45 | 3.88\n\n | 3.74\n\n | 3.00\n\n | 2.60 |
| NGC 2403 VS 241\n\n | 0.45 | 3.91\n\n | 3.70\n\n | 2.40 |
| NGC 1741 Zone C\n\n | 0.38 | 3.93\n\n | 3.60\n\n | 2.05 |
| M 8\n\n | 0.37 | 3.91\n\n | 3.85\n\n | 2.10 | 5.90 |
| M101 H1013\n\n | 0.36 | 3.87\n\n | 3.55\n\n | 2.50 |
| M33 NGC 595\n\n | 0.34 | 3.87\n\n | 3.70\n\n | 2.50 |
| M 20\n\n | 0.33 | 3.89\n\n | 3.78\n\n | 2.60 | 2.00 |
| NGC 2403 VS 441\n\n | 0.30 | 3.92\n\n | 3.55\n\n | 2.00 |
| M 17\n\n | 0.27 | 3.91\n\n | 4.30 | 2.00 |
| NGC 4861 BrightHII\n\n | 0.27 | 4.11\n\n | 4.20\n\n | 2.90 |
| M31 K932\n\n | 0.24 | 3.97\n\n | 3.75\n\n | 2.00 |

\(^{1}\text{Tsamis et al. (2003b); }^{2}\text{García-Rojas et al. (2008); }^{3}\text{Esteban et al. (2002); }^{4}\text{Tsamis et al. (2003a); }^{5}\text{Esteban et al. (2004); }^{6}\text{García-Rojas et al. (2004); }^{7}\text{García-Rojas et al. (2005); }^{8}\text{García-Rojas et al. (2006); }^{9}\text{García-Rojas et al. (2007); }^{10}\text{López-Sánchez et al. (2007); }^{11}\text{Esteban et al. (2009).}

proposed (Owocki & Scudder 1983; Nicholls et al. 2012). According to Nicholls et al. (2012), physical processes such as magnetic reconnection, injection of high-energy electrons through photoionization by a ‘hard’ photon spectrum may generate such non-equilibrium electrons. The role of shockwaves in generating the discrepancies also needs to be explored. The choice of abundance discrepancy model, however, does not impact on the validity of the ORL diagnostics proposed here.

The traditional method, which uses one line ratio to determine an electron temperature or a density, has the disadvantage that the density or temperature is undefined. Although a temperature-sensitive recombination line ratio is usually quite insensitive to electron
density, some density-sensitive line ratios, to a noticeable extent, may vary with electron temperature. As an example, Fig. 3 shows the O II λ4649/λ4662 ratio as a function of electron density. From the figure one can see that at low densities (log \(T_e < 3.6\)), the electron density yielded by the observed line ratio may differ by as much as 0.6 dex when the logarithmic temperature \(T_e\) increases from 2.6 to 3.6. Instead of using a single line ratio, we present our new method using one set of N II or O II ORLs to constrain the electron temperature and density simultaneously. Our method is based on the fact that the intensity ratio of two of these selected ORLs is temperature-sensitive, while the ratio of the other (two) lines of this group should be density-sensitive.

2.3 Theoretical intensities

For the N II effective recombination coefficients (Fang et al. 2011), log \(T_e\) (K) covers a range of 2.1 to 4.3, with an incremental step of 0.1, and log \(N_e\) (cm\(^{-3}\)) spans from 2.0 to 6.0, with an incremental step of 0.1. For the O II effective recombination coefficients (Storey, unpublished), log \(T_e\) (K) covers a range of 2.6 to 4.2, with an incremental step of 0.2, and log \(N_e\) (cm\(^{-3}\)) spans from 2.0 to 5.0, with an incremental step of 0.2. Both theoretical and observed intensities are normalized such that \(H\beta = 100\).

For most objects, the other O II multiplets are not as strong as those considered in Figs. 1 and 2, and are not suitable to constrain \(T_e\) and \(N_e\), due to data quality. Figs 4 and 5 show the emissivities of N II V3 and V39 ORLs (V3 λ5666, V3 λ5679, V39b λ4041 and V39a λ4035) and O II V1 and V48 ORLs (V1 λ4649, V1 λ4662, V48a λ4089 and V48c λ4087), respectively, as a function of \(T_e\) and \(N_e\), where the emissivity is derived from the effective recombination coefficients as follows:

\[
e(\lambda) = \alpha_{\text{eff}}(\lambda) \times \frac{hc}{\lambda},
\]

with \(h\) as Planck constant and \(c\) as the speed of light, all in cgs units. Due to coarse grid resolution, we performed a bilinear interpolation on the effective recombination coefficients of the O II ORLs provided by Storey (unpublished). Bilinear interpolation simply extends linear interpolation for functions of two variables, i.e. \(\alpha_{\text{eff}}(T_e, N_e)\).

Based on the calculations of the effective recombination coefficients for the N II and O II lines, theoretical intensity for an N II or O II ORL relative to \(H\beta\) can be calculated as a function of \(T_e\) and \(N_e\) as follows:

\[
I_{\text{pred}}(\lambda) = \frac{I(\lambda)}{I(H\beta)} = \frac{\alpha_{\text{eff}}(\lambda)}{\alpha_{\text{eff}}(H\beta)} \times \frac{4861 \lambda^2}{\lambda^2} \times 100,
\]

where \(I_{\text{pred}}(T_e, N_e)\) is the predicted predicted intensity of the transition \(\lambda\), normalized such that \(I(H\beta) = 100\). \(\alpha_{\text{eff}}(\lambda; T_e, N_e)\) is the effective recombination coefficient for the transition \(\lambda\), and \(\alpha_{\text{eff}}(H\beta; T_e, N_e)\) is the effective recombination coefficient for \(H\beta\), which is adopted from the hydrogenic calculations of Storey & Hummer (1995). The ionic abundances relative to hydrogen derived from ORLs, \(X^+/H^+\), can be found in the literature for each nebula.
2.4 Plasma diagnostics

2.4.1 Least-squares fit

For each PN and H II region, the aforementioned theoretical intensities were compared with the observed intensities using a least-squares minimization method as follows:

$$\chi^2 = \sum_{i=1}^{n} \left( \frac{I_{\text{obs}}(\lambda_i) - I_{\text{pred}}(\lambda_i)}{I_{\text{pred}}(\lambda_i)} \right)^2,$$

where $$\chi^2$$ is the sum over the combination of $$n$$ lines used for either N II or O II, as a function of $$T_e$$ and $$N_e$$. $$I_{\text{obs}}$$ is the observed intensity for the transition $$\lambda_i$$ and $$I_{\text{pred}}$$ is the predicted intensity. For a blended feature, the observed intensity is compared with the combined theoretical intensities of the blended lines, described in equation (3). As an example, Figs 6 and 7 show the log $$\chi^2$$ distributions for four PNe (H II 2-2, M 1-42, NGC 6153 and NGC 7009) for N II and O II, respectively. The location of the minimum log $$\chi^2$$ value can be determined for each comparison, thus providing the optimal ($$\log T_e$$, $$\log N_e$$), as shown by the white cross hairs in both figures.

2.4.2 Error estimate

For many objects, the resulting temperatures and densities are well confined. However, it is difficult to decide how reliable these diagnostic results could be. Not all of the surveys or individual detailed studies of PNe and/or H II regions provided errors along with their observational intensities. This makes estimating the uncertainties for the optimal ($$\log T_e$$, $$\log N_e$$) rather complicated. A few nebulae had observational errors presented in the literature and are adopted.
in the current analyses. For those nebulae whose measurement uncertainties are not presented in the literature, we estimated errors according to their observational conditions and data quality. In order to determine the uncertainties for the optimal \((\log T_e, \log N_e)\), we carried out an extra set of calculations that begin with random number generation.

For each line in a combination of \( \text{N} \text{II} \) or \( \text{O} \text{II} \) ORLs (e.g. \( \text{V3} \lambda 5666, \text{V3} \lambda 5679, \text{V39a} \lambda 4035 \) and \( \text{V39b} \lambda 4041 \) for \( \text{N} \text{II} \)), and \( \text{V1} \lambda 4662, \text{V1} \lambda 4649, \text{V48a} \lambda 4089 \) and \( \text{V48c} \lambda 4087 \) for \( \text{O} \text{II} \)) that are used for plasma diagnostics, we used \text{IDL} to generate a number, \( N_{\text{sim}} \), of intensity values, \( I_{\text{sim}}(\lambda_i) \), with a normal (Gaussian) distribution centred around the original observational intensity, \( I_{\text{obs}}(\lambda_i) \), along with a standard deviation obtained from the observational error, \( \sigma(I_{\text{obs}}) \), of this distribution.

The mean intensities of the simulations and standard deviations, \( I_{\text{mean}}^{\text{sim}} \) and \( \sigma(I_{\text{mean}}^{\text{sim}}) \), are calculated as follows:

\[
I_{\text{mean}}^{\text{sim}} = \frac{\sum_{i=1}^{N_{\text{sim}}} I_{\text{sim}}^{i}(\lambda)}{N_{\text{sim}}} \quad (5)
\]

and

\[
\sigma(I_{\text{mean}}^{\text{sim}}) = \sqrt{\frac{\sum_{i=1}^{N_{\text{sim}}} \left( I_{\text{sim}}^{i}(\lambda) - \langle I_{\text{sim}}^{\text{mean}} \rangle \right)^2}{N_{\text{sim}}}}. \quad (6)
\]

The standard deviation of these generated numbers also agree well with the observational errors. Each combination of the randomly generated line intensities, \( I_{\text{sim}}^{i}(\lambda) \), where \( i \) is from 1 to \( N_{\text{sim}} \), is used to determine a unique optimal \((\log T_e, \log N_e)\), following the method described in Section 2.4.1.

Errors of the optimal \( \log T_e \) and \( \log N_e \) of each nebula are then calculated from these \( N_{\text{sim}} \) temperatures and densities, which can be denoted as \( T_{e,i} \) and \( N_{e,i} \), where \( i = 1, \ldots, N_{\text{sim}} \). For those objects whose \( \log T_e - \log N_e \) diagram shows multiple peaks, standard deviations of \( T_{e,i} \) and \( N_{e,i} \) are calculated, which are centred on the mean values of these \( N_{\text{sim}} \) temperatures and densities. The random intensities of each ORL are then normally distributed within the observational errors of the observed intensities. The program would then calculate the \( \log \chi^2 \) along the entire \( T_e \) and \( N_e \) grid for each combination of the random intensities and determine the minimum value, thus deriving the unique optimal \((\log T_e, \log N_e)\) for that simulation.

Figs 8 and 9 display the frequencies of these optimal \( \log T_e \) and \( \log N_e \) distributions for the \( \text{N} \text{II} \) and \( \text{O} \text{II} \) ORLs, respectively, for the same four PNe shown in Figs 6 and 7. The errors for the optimal \( \log T_e \) and \( \log N_e \) are derived as follows:

\[
T_{\text{mean}}^{\text{sim}} = \frac{\sum_{i=1}^{N_{\text{sim}}} T_{e,i}}{N_{\text{sim}}} \quad (7)
\]
and
\[ N_{\text{mean}}^{\text{sim}} = \frac{\sum_{i=1}^{N_{\text{sim}}} N_i}{N_{\text{sim}}}, \]  
(8)
with respective standard deviations calculated as follows:
\[ \sigma(T_{\text{mean}}) = \sqrt{\frac{\sum_{i=1}^{N_{\text{sim}}} (T_i - T_{\text{mean}}^{\text{sim}})^2}{N_{\text{sim}}}} \]  
(9)
and
\[ \sigma(N_{\text{mean}}^{\text{sim}}) = \sqrt{\frac{\sum_{i=1}^{N_{\text{sim}}} (N_i - N_{\text{mean}}^{\text{sim}})^2}{N_{\text{sim}}}}. \]  
(10)

These values are represented in Figs 8 and 9 as green error bars centred at \( T_{\text{mean}}^{\text{sim}} \) and \( N_{\text{mean}}^{\text{sim}} \), but projected on the white crosshairs of the optimal \( \log T_e \) and \( \log N_e \) locations obtained from observations, thus providing upper and lower limits. The white solid contour designates the 1σ level around the peak. We project the standard deviations on to the optimal \( \log T_e \) and \( \log N_e \) locations obtained from observations because they are calculated using the 1σ errors of the observed intensities.

3 RESULTS AND DISCUSSION

3.1 Temperature structure of photoionized nebulae

Figs 10 and 11 show the electron temperatures from the literature and the current ORL diagnostics for 46 PNe and for 42 H II regions, respectively. The PNe in Fig. 10 were chosen out of the whole list because they met the following criteria: (1) \( T_e([\text{O} \text{ III}]) \), \( T_e([\text{H} \text{ I}]) \) and \( T_e(\text{He} \text{ i}) \) are available from the literature, (2) \( 2.6 < \log T_e([\text{O} \text{ III} \text{ ORLs}]) < 4.2 \) and/or (3) \( 2.1 < \log T_e([\text{N} \text{ II} \text{ ORLs}]) < 4.3 \). For the PNe plotted in Fig. 10, the mean value of \( \log T_e([\text{O} \text{ III}]) \) is 4.01 ± 0.09 K (red solid line). The mean value of \( \log T_e([\text{H} \text{ I} \text{ BJ}]) \) is 3.91 ± 0.29 K (orange dotted line). The mean value of \( \log T_e(\text{He} \text{ i} \lambda 7281/\lambda 6678) \) is 3.67 ± 0.19 K (purple dashed line). The mean value of \( \log T_e([\text{O} \text{ II} \text{ ORLs}]) \) is 3.22 ± 0.52 K (green dot–dashed line). The mean value of \( \log T_e([\text{N} \text{ II} \text{ ORLs}]) \) is 3.20 ± 0.59 K (blue solid line). In the case of the PNe, the mean values clearly show the temperature sequence: \( T_e(\text{ORLs}) \leq T_e(\text{He} \text{ i}) \leq T_e([\text{H} \text{ I}]) \leq T_e(\text{CELs}) \), which is consistent.
with predictions from the bi-abundance model (Liu et al. 2000; Liu 2003). While the physical conditions of the main nebula and cold, H-deficient component may vary from nebula to nebula, over all the nebulae there is a general trend that many contain a secondary cold, H-deficient component.

For the H\textsc{ii} regions plotted in Fig. 11, the mean value of log $T_e([\text{O} \text{III}])$ is 4.00 $\pm$ 0.11 K (red solid line). The mean value of log $T_e(\text{H} \text{I BJ})$ is 3.87 $\pm$ 0.08 K (orange dotted line). The mean value of log $T_e(\text{He} \text{I} \lambda 7281/\lambda 6678)$ is 3.91 K (purple dashed line). The mean value of log $T_e(\text{O} \text{II})$ is 3.24 $\pm$ 0.57 K (green dot-dashed line). The mean value of log $T_e(\text{N} \text{II})$ is 3.15 $\pm$ 0.87 K (blue solid line). In the case of the H\textsc{ii} regions, the mean values show the temperature relation: $T_e(\text{ORLs}) \leq T_e(\text{He I}) \leq T_e(\text{H I}) \leq T_e(\text{CELS})$, which is also seen in Fig. 10. The average $T_e$’s and standard deviations are labelled for PNe (in red) and H\textsc{ii} regions (in blue).

Many PNe and H\textsc{ii} regions in our analysis show $T_e(\text{O II})$ lower than 2.6 and $T_e(\text{N II})$ lower than 2.1, the lowest $T_e$’s for which the effective recombination coefficients are available, respectively, although only a handful of H\textsc{ii} regions had N\textsc{ii} ORLs listed in their

3.2 Physical evolution of photoionized nebulae

Fig. 12 shows histograms of $T_e([\text{O} \text{III}])$, $T_e(\text{H I BJ})$, $T_e(\text{He I})$, $T_e(\text{O II})$ and $T_e(\text{N II})$, for PNe (blue) and H\textsc{ii} regions (red). The He\textsc{i} ORL temperatures are derived from the line ratio $\lambda 7281/\lambda 6678$ using the fitting functions of Zhang et al. (2005b). Given the weakness of the N\textsc{ii} and O\textsc{ii} ORLs, the large scatter seen in the $T_e(\text{O II})$ and $T_e(\text{N II})$ distributions is most likely due to observational uncertainties. The diagram clearly shows the relation of the average temperatures for the PNe, $T_e(\text{ORLs}) \leq T_e(\text{He I}) \leq T_e(\text{H I}) \leq T_e(\text{CELS})$, which is also seen in Fig. 10. The average $T_e$’s and standard deviations are labelled for PNe (in red) and H\textsc{ii} regions (in blue). Many PNe and H\textsc{ii} regions in our analysis show $T_e(\text{O II})$ lower than 2.6 and $T_e(\text{N II})$ lower than 2.1, the lowest $T_e$’s for which the effective recombination coefficients are available, respectively, although only a handful of H\textsc{ii} regions had N\textsc{ii} ORLs listed in their
Figure 7. The same as Fig. 6 but for O\textsc{ii} V1 $\lambda$4649, V1 $\lambda$4662, V48a $\lambda$4089 and V48c $\lambda$4087 lines over the entire log $T_\text{e}$–log $N_\text{e}$ grid for the same four PNe.

literature for us to perform our diagnostics. As can be seen in Fig. 11, the temperature histograms of [O\textsc{iii}] CELs, He\textsc{i}, H\textsc{i} BJ, O\textsc{ii} and N\textsc{ii} ORLs show an increase in the width of the distributions from [O\textsc{iii}] through N\textsc{ii}. This can be either caused by the observational uncertainties arising from increasingly difficult measurements, especially for the extremely faint ORLs or possibly due to variations of physical properties of the cold H-deficient inclusions and main nebula from object to object, causing the scatter in $T_\text{e}$(N\textsc{ii}) and $T_\text{e}$(O\textsc{ii}).

Fig. 13 shows the ADF (O$^2$/H$^+$) versus $T_\text{e}$([O\textsc{iii}])–$T_\text{e}$(H\textsc{i} BJ), for PNe (blue) and H\textsc{ii} regions (red). The 123 PNe analysed here are shown as the blue open circles; amongst them the 46 PNe plotted in Fig. 10 are shown as the blue filled circles. A linear least-squares fit to the 46 PNe from Fig. 10 with reliable electron temperatures indicates a positive correlation. The PNe shown in open circles and H\textsc{ii} regions are not considered for the linear fit. This most likely indicates a tight correlation between the ADF of a nebula and the temperature discrepancy. The 42 H\textsc{ii} regions analysed here are shown as the red open squares with 40 H\textsc{ii} regions shown as red closed squares if they had positive N\textsc{ii} or O\textsc{ii} log $T_\text{e}$ results. The H\textsc{ii} regions plotted in Fig. 13 just show large scatter, making it difficult to determine if there is a correlation between the ADF value and temperature discrepancy.

The nebulae with lower ADF values might be relatively young in their evolution processes when the main nebulae are very bright, and therefore the cold, H-deficient components might be mixed with the main nebula, and the effects of the cold, H-deficient components are insignificant. However, for the nebulae with high ADF values, the nebula might have expanded such that the main nebula has a relatively low surface brightness compared to the condensed, cold H-deficient condensations, thus making it easier to distinguish the two based on the temperatures derived from CELs and those derived from ORLs. Therefore, for most PNe, there may be an evolution from younger, more compact nebulae where the cold, H-deficient component is embedded and almost indistinguishable from the main bright nebula to older, more diffuse nebulae where the fainter main nebula distinctly envelopes the cold, H-deficient condensations.
3.3 Comments on several archetypal planetary nebulae

3.3.1 Hf 2-2

There are several well-studied PNe that have been observed for many decades. Hf 2-2 is a well-known southern PN, that has an unusually high ADF (O\textsuperscript{3+}) value of 84 (Liu et al. 2006). They observed this PN with the ESO 1.52 m telescope with three separate slit widths of 2, 4, and 8 arcsec, using the 2 arcsec width for maximum spectral resolution. Hf 2-2 is known to have a close binary system with a very short orbital period (0.398 d; Lutz et al. 1998). It has also shown a very peculiar nature and may be included in the ‘born-again’ PNe scenario, relating the particularly large ADF to the phenomenon of novae, as stated in Wesson, Liu & Barlow (2003). From their 2 arcsec slit width observations, Liu et al. (2006) derived the following electron temperatures: $T_e$([O\textsc{iii}]) = 8710, $T_e$(He\textsc{i} λ6678/λ5876) = 1570 and $T_e$(H\textsc{i}) = 933 K. From the plasma diagnostics based on the N\textsc{ii} ORL lines of V3 λ5666, V3 λ5679, V39b λ4041, V39a λ4035, we derived an electron temperature of $398^{+9}_{-135}$ K, as shown in the first panels of Fig. 5 for the $\chi^2$ minimization and Fig. 8 for the standard deviations. From the plasma diagnostics based on the O\textsc{ii} ORLs of V1 λ4649, V1 λ4661, V48a λ4089 and V48c λ4087, we derived an electron temperature of $3160^{+730}_{-820}$ K, as shown in the first panels of Fig. 7 for the $\chi^2$ minimization and Fig. 9 for the standard deviations.

3.3.2 M 1-42

Another PN is the Galactic bulge nebula M 1-42, which was first discovered by Minkowsky (1946). More recently, Liu et al. (2001) observed M 1-42 along with M 2-36 using the ESO 1.52 m telescope. For M 1-42, they derived a fairly high ADF of 22. Liu et al. (2001) derived the following electron temperatures: $T_e$([O\textsc{iii}]) = 9120, $T_e$(He\textsc{i} λ7821/λ6678) = 3790 and $T_e$(H\textsc{i}) = 3980 K. From our plasma diagnostics based on the N\textsc{ii} ORLs, V3 λ5666, V3 λ5679,
V39b $\lambda 4041$ and V39a $\lambda 4035$, we derived an electron temperature of $3980^{+3100}_{-1690}$ K, as shown in the second panel of Fig. 5 for the $\chi^2$ minimization and Fig. 8 for the standard deviations. But as one can see in the two figures, the electron density of $10^6$ cm$^{-3}$ only provides an upper limit for the diagnostics. From our plasma diagnostics based on the O$\Pi$ ORLs, V1 $\lambda 4649$, V1 $\lambda 4661$, V48a $\lambda 4089$ and V48c $\lambda 4087$, we derived an electron temperature of $1410^{+150}_{-150}$ K, as shown in the second panel of Fig. 7 for the $\chi^2$ minimization and Fig. 9 for the standard deviations.

3.3.3 NGC 6153

Over the last several decades, detailed studies have been put forth on the well-known PN NGC 6153, a possibly super-metal-rich nebula and first noted by Pottasch et al. (1984). Liu et al. (2000) derived a moderately high ADF of 9.2. Deduced from their electron density distribution and the optical appearance of NGC 6153, Yuan et al. (2011) suggested that it is most likely a bipolar nebula probably with a central cavity and a density-enhanced waist, as viewed at a large angle to its polar axis. However, they also argue that a convincing physical model accounting for the full range of behaviour NGC 6153 exhibits is still to be found. From the strong emission lines, they derived the following electron temperatures: $T_e$(O$\ iii$) = 7240, $T_e$(He$\ i$ $\lambda 7821/\lambda 6678) = 3260$ and $T_e$(H$\ i$) = 6030 K. Based on the N$\ ii$ ORLs of V3 $\lambda 5666$, V3 $\lambda 5679$, V39b $\lambda 4041$ and V39a $\lambda 4035$, our plasma diagnostics derived an electron temperature of $2000^{+2080}_{-480}$ K, as shown in the third panel of Fig. 6 for the $\chi^2$ minimization and Fig. 8 for the standard deviations. Based on the O$\ ii$ ORLs of V1 $\lambda 4649$, V1 $\lambda 4661$, V48a $\lambda 4089$ and V48c $\lambda 4087$, our plasma diagnostics yields an electron temperature of $1780^{+40}_{-230}$ K, as shown in the third panel of Fig. 7 for the $\chi^2$ minimization and Fig. 9 for the standard deviations.

3.3.4 NGC 7009

NGC 7009 has had hundreds of publications over the last 50 yr or more. Also known as the Saturn nebula, this large, double-ringed, high-surface-brightness nebula is particularly well known for having an unusually rich and strong O$\ ii$ optical permitted lines ever since the early high-resolution photographic spectroscopy observations of Wyse (1942) and Aller & Kaler (1964), having observed more than 100 O$\ ii$ permitted transitions (Liu et al. 1995). A much more recent work by Fang & Liu (2011) has made use of very deep CCD spectrum of the Saturn nebula covering from 3040 to 11 000 Å. They derived a reasonably high ADF of about 5. They
identified over 1000 emission lines with 81 per cent attributed to permitted lines and more than 200 O \n permitted lines. Their observations were made using the ESO 1.52 m telescope and the William Herschel Telescope (WHT), and securing all spectra with a long slit. Table 3 in their paper lists a very complete set of electron temperatures and densities derived from numerous line ratios. They derived the following electron temperatures: \( T_e ([\text{O} \text{ III}]) = 9810 \), \( T_e (\text{He} \text{I} \lambda 7821/\lambda 6678) = 5100 \), and \( T_e (\text{H} \text{I}) = 6420 \) K. From our plasma diagnostics based on the \( [\text{O} \text{ III}] \) lines of V3, \( \lambda 5666 \), V3, \( \lambda 5679 \), V39b \( \lambda 4041 \) and V39a \( \lambda 4035 \), we derived an electron temperature of 1260 \(-235\) K, as shown in the fourth panel of Fig. 6 for the \( \chi^2 \) minimization and Fig. 8 for the standard deviations. Due to the data quality, our \( \chi^2 \) minimization technique only provides an upper limit to the electron density for \( N \text{II} \) ORLs. Wesson et al. (2008) have provided detailed optical analysis of two of the H-deficient knots (J1 and J3) in another ‘born-again’ PN Abell 30. This nebula consists of a rather large, 120 arcsec across, spherical shell of low surface brightness and several bright clumps, first discovered by Jacoby (1979) and Hazard et al. (1980), of material about 10 arcsec away from the central star. For Wesson et al. (2003), they observed Abell 30 with the 4.2 m WHT at the Observatorio del Roque de los Muchachos on La Palma, Spain, with a 0.82 arcsec slit width. These two knots have extremely high ADF values of 598 and 766 for J1 and J3, respectively.

3.3.5 Abell 58 and Abell 30

Another rather peculiar PN is Abell 58, a ‘born-again’ PN, known to contain an H-deficient knot surrounding V605 Aql, according to Wesson et al. (2008). Hidden by a thick dusty torus, V605 Aql cannot be seen directly. However, from the surface abundances derived for the star from observations of its scattered light, Clayton et al. (2006) found it to be a typical Wolf–Rayet central star of PN. Wesson et al. (2008) did not measure any \( N \text{II} \) ORLs in Abell 58. From the plasma diagnostics based on the \( [\text{O} \text{ III}] \) ORLs of V1, \( \lambda 4649 \), \( \lambda 4661 \) and V10, \( \lambda 4069.89 \) (blended with \( \lambda 4069.89 \)), we derived an upper limit to the electron temperature (~15 800 K).

Wesson et al. (2003) have provided detailed optical analysis of two of the H-deficient knots (J1 and J3) in another ‘born-again’ PN Abell 30. This nebula consists of a rather large, 120 arcsec across, spherical shell of low surface brightness and several bright clumps, first discovered by Jacoby (1979) and Hazard et al. (1980), of material about 10 arcsec away from the central star. For Wesson et al. (2003), they observed Abell 30 with the 4.2 m WHT at the Observatorio del Roque de los Muchachos on La Palma, Spain, with a 0.82 arcsec slit width. These two knots have extremely high ADF values of 598 and 766 for J1 and J3, respectively.

Wesson et al. (2003) derived the following electron temperatures for knot J1: \( T_e ([\text{O} \text{ III}]) = 20 800 \) and \( T_e (\text{He} \text{I} \lambda 5876/\lambda 4471) = 350 \) K. Wesson et al. (2003) did not measure any \( N \text{II} \) ORLs in the knot J1. From the plasma diagnostics based on the \( [\text{O} \text{ III}] \) ORLs of V1, \( \lambda 4649 \), \( \lambda 4661 \), V10, \( \lambda 4075 \) and V10, \( \lambda 4069.62 \) (blended with \( \lambda 4069.89 \)), we derived an upper limit to the electron temperature (~15 800 K). For knot J3, Wesson et al. (2003) derived the

![Figure 10. ADF(ODanny II)](https://academic.oup.com/mnras/article-abstract/428/4/3443/1001207)
Figure 11. The same as Fig. 10 but for 42 H II regions.

Figure 12. Distributions of $T_e$ derived from (a) [O III] CELs; (b) H I BJ; (c) He I line ratio $\lambda 7281/\lambda 6678$ from the literature and our own calculations; (d) O II ORL diagnostics and (e) N II ORL diagnostics for PNe (blue) and H II regions (red). For the distribution in each panel, the average $T_e$ and standard deviations are labelled for PNe (blue) and H II regions (red).

Figure 13. Log ADF (O $^3$ +/H $^+$) versus $T_e([\text{O III}]) - T_e(\text{H I BJ})$ relation for PNe (blue) and H II regions (red). The blue open circles are all 123 PNe listed in Table 1. The blue closed circles are all the 46 PNe plotted in Fig. 10. The solid black line shows a least-squares fit to the 46 PNe plotted in Fig. 10. The red open squares are all 42 H II regions listed in Table 2. The red closed squares are H II regions with either log $T_e$(O II) or log $T_e$(N II) greater than zero.

following electron temperatures: $T_e([\text{O III}]) = 17960$ and $T_e(\text{He I} \lambda 6678/\lambda 4471) = 9240$ K. They did not measure any N II ORLs in this knot either. Plasma diagnostics based on the O II ORLs yielded an upper limit to the electron temperature ($\sim 15800$ K).
Table 3. The He$^+$ diagnostics from the literature and current analysis for PNe.

| Object | ADF(O$^2$) | $T_e$(He$^+$) (K) | $\lambda_{7281}$ | $\lambda_{6678}$ | $\lambda_{6678}$ | $\lambda_{6678}$ |
|--------|-------------|------------------|---------------------------------|-----------------|-----------------|-----------------|
| Abell 30 J$^3$ | 766.0 | 9080 | 2270 | 5876 |
| Abell 30 J$^3$ | 598.0 | 5830 | 3910 |
| Abell 58$^3$ | 89.0 | 587 | 1250 |
| Hf 2-2 (2 arcsec)$^5$ | 84.0 | 1070 | 1570 |
| Hf 2-2 (4 arcsec)$^5$ | 84.0 | 840 |
| Hf 2-2 (8 arcsec)$^5$ | 84.0 | 816 | 1460 |
| NGC 1504$^6$ | 32.0 | 4800 | 5100 | 4100 |
| M 4$^2$ | 22.0 | 3790 | 2500 | 2380 |
| NGC 40$^7$ | 17.3 | 7430 | 5470 |
| M 2-2$^4$ | 17.0 | 4500 |
| NGC 2024$^7$ | 16.0 | 19700 | 6980 |
| NGC 2024$^{10}$ | 16.0 | 15 900$^{10}$ |
| DdDm 11$^3$ | 11.8 | 3500 |
| NGC 6153$^3$ | 9.2 | 3260 | 600 | 5570 |
| IC 2003$^3$ | 7.3 | 1600 | 7670 |
| M 2-3$^6$ | 6.9 | 2710 | 3420 | 4370 |
| Vy 1-2$^7$ | 6.2 | 3550 | 4430 |
| NGC 3242$^7$ | 5.7 | 4620 | 9290 | 7120 |
| NGC 2440$^{10}$ | 5.4 | 10 400 | 7430 | 4500 |
| NGC 6818$^8$ | 4.9 | 3420 | 5270 | 5880 |
| NGC 7009$^9$ | 4.7 | 5000 | 9670 | 4310 |
| IC 3568$^8$ | 4.6 | 15 900 | 10 900 |
| NGC 6210$^7$ | 4.3 | 9470 | 10 600 |
| M 3-3$^4$ | 4.2 | 17 000$^3$ |
| NGC 6543$^{11}$ | 4.2 | 6610 | 8440 | 5560 |
| M 1-7$^{13}$ | 3.6 | 8820$^{13}$ | 7960$^{13}$ |
| NGC 6302$^{10}$ | 3.6 | 15 100$^{10}$ |
| NGC 3132$^4$ | 3.5 | 10800 | 14 500 | 10 800 |
| NGC 6302$^4$ | 3.5 | 5001 |
| NGC 7026$^{13}$ | 3.4 | 4050$^{13}$ | 4120$^{13}$ |
| IC 351$^{13}$ | 3.1 | 3790$^{13}$ | 4600$^{13}$ |
| Hu 1-1$^{3}$ | 3.0 | 9550$^{13}$ | 4740$^{13}$ |
| Sp 4-1$^{3}$ | 2.9 | 3150$^{13}$ | 2400$^{13}$ |
| IC 4846$^{13}$ | 2.9 | 13 400$^{13}$ |
| IC 4846$^{17}$ | 2.9 | 8890 | 5880 | 2490 |
| NGC 6803$^{13}$ | 2.7 | 8100$^{13}$ | 4840$^{13}$ |
| BoBn 1$^{19}$ | 2.6 | 9430$^{19}$ | 5580 | 1590 |
| NGC 6833$^{13}$ | 2.5 | 14 100$^{13}$ | 2440$^{13}$ |
| NGC 6879$^{13}$ | 2.5 | 3750$^{13}$ | 2440$^{13}$ |
| IC 4191 fixed$^4$ | 2.4 | 5850 | 3080 | 2050 |
| IC 4191 scan$^4$ | 2.4 | 8340 | 2890 | 1920 |
| NGC 3132$^{10}$ | 2.4 | 13 900$^{10}$ |
| NGC 6720$^7$ | 2.4 | 13 300 | 10 300 |
| NGC 6818$^8$ | 2.4 | 5000$^{10}$ |
| IC 4191 neb$^{10}$ | 2.4 | 3000$^{10}$ |
| IC 4191 fixed$^{10}$ | 2.4 | 2800$^{10}$ |
| NGC 3918$^4$ | 2.3 | 7320 | 15 700 | 7730 |
| NGC 6884$^7$ | 2.3 | 8990 |
| IC 5217$^{13}$ | 2.3 | 5100$^{13}$ | 3000$^{13}$ |
| NGC 3242$^{10}$ | 2.2 | 10 000$^{10}$ |
| NGC 5882$^4$ | 2.2 | 5380 | 10 300 | 6840 |
| M 1-7$^{14}$ | 2.1 | 9200$^{13}$ | 3380$^{13}$ |
| NGC 5882$^4$ | 2.1 | 10 700$^{10}$ |
| NGC 7662$^2$ | 2.0 | 4830 | 3550 |
| NGC 5315$^9$ | 2.0 | 10 000$^{10}$ |
| NGC 6807$^{10}$ | 2.0 | 1000$^{10}$ |
| NGC 5507$^7$ | 1.9 | 8050 | 6040 | 4360 |

1Liu et al. (2000); 2Li et al. (2001); 3Ruiz et al. (2003); 4Tsamis et al. (2003b); 5Zhang & Liu (2003); 6Ercolano et al. (2004); 7Li et al. (2004); 8Peimbert et al. (2004); 9Sharpee et al. (2004); 10Tsamis et al. (2004); 11Wesson & Liu (2004); 12Robertson-Tessi & Garnett (2005); 13Wesson et al. (2005); 14Zhang et al. (2005a); 15Liu et al. (2006); 16Sharpee et al. (2007); 17Wang & Liu (2007); 18García-Rojas et al. (2008); 19Otsuka et al. (2010); 20Wesson et al. (2003); 21Fang & Liu (2011); 22Esteban et al. (2002); 23Tsamis et al. (2003a); 24Esteban et al. (2004); 25García-Rojas et al. (2004); 26García-Rojas et al. (2005); 27García-Rojas et al. (2006); 28García-Rojas et al. (2007); 29López-Sánchez et al. (2007); 30Esteban et al. (2009); 31Wesson et al. (2008); 32Tsamis et al. (2008).
Table 4. Electron temperatures of H II regions derived from the He I recombination lines.

| Object     | ADF(O2+) | \(T_e(\text{He}^+)(K)\) | \(T_e(\text{He}^+)(K)\) | \(T_e(\text{He}^+)(K)\) |
|------------|----------|-------------------------|-------------------------|-------------------------|
| M 17\(^a\) | 2.7      | 19 000                  | 7940                    | 5050                    |
| LMC N141\(^b\) | 2.6      | 14 900                  | 6570                    |                          |
| NGC 3576\(^c\) | 2.2      | 10 500                  | 4570                    | 6500                    |
| 30 Doradus\(^d\) | 1.8      | 12 400                  | 5550                    | 5490                    |
| LMC N66\(^e\) | 1.6      | 3630                    |                          |                          |
| LMC N11 B\(^f\) | 1.5      | 4740                    | 1930                    |                          |
| PB 6-2\(^g\) | 1.1      | 15 200                  | 15 900                  |                          |
| PB 6-1\(^h\) | 1.1      | 15 400                  |                          |                          |
| PB 5\(^i\) | 1.1      | 6760                    | 6940                    | 11 100                  |
| NGC 3603\(^j\) | 1.0      | 11 500                  | 13 900                  | 8800                    |
| NGC 2867\(^k\) | 1.0      | 9810                    | 4750                    | 3760                    |
| S 311\(^l\) | 1.0      | 9190                    | 7150                    | 7480                    |
| M 42\(^m\) | 1.0      | 6700                    | 4860                    | 2880                    |
| NGC 2867-12\(^n\) | 1.0      | 11 500                  | 6550                    | 6640                    |
| NGC 5253 HII-2\(^o\) | 1.0      | 8440                    | 6150                    | 7440                    |
| NGC 5253 UV-1\(^p\) | 1.1      | 6230                    | 7990                    | 5220                    |
| NGC 5253 HII-3\(^q\) | 1.0      | 6720                    | 4320                    |                          |
| NGC 5253 UV-2\(^r\) | 1.0      | 5210                    | 2820                    |                          |
| M33 NGC 604\(^s\) | 1.0      | 17 400                  |                          |                          |
| M101 NGC 546\(^t\) | 1.0      | 5320                    |                          |                          |
| M101 NGC 547\(^u\) | 1.0      | 10 400                  | 4770                    |                          |
| SMC N66\(^v\) | 0.9      | 12 600                  |                          |                          |
| NGC 3576\(^w\) | 0.9      | 10 300                  |                          |                          |
| M 16\(^x\) | 0.4      | 8380                    | 6350                    | 5410                    |
| M 8\(^y\) | 0.4      | 7380                    | 6240                    | 6790                    |
| M101 H1013\(^z\) | 0.4      | 4330                    |                          |                          |
| M33 NGC 595\(^{1}\) | 0.3      | 2110                    | 6560                    | 6280                    |
| M 20\(^{1}\) | 0.3      | 8870                    | 5660                    | 6180                    |
| M 17\(^{1}\) | 0.3      | 6120                    | 5590                    | 6600                    |

\(^a\)Tsamis et al. (2003b); \(^b\)García-Rojas et al. (2008); \(^c\)Tsamis et al. (2003a); \(^d\)Esteban et al. (2004); \(^e\)García-Rojas et al. (2005); \(^f\)García-Rojas et al. (2006); \(^g\)García-Rojas et al. (2007); \(^h\)López-Sánchez et al. (2007); \(^i\)Esteban et al. (2009).

3.4 Results and discussion

Tables 1 and 2 list the electron temperatures and densities for the 167 nebulae, including 123 PNe and 42 H II regions, respectively. These objects are presented in descending order of ADF (O2+/H+). Column 1 is the nebula name. Column 2 contains the ADF values from the literature. Column 3 is the log \(T_e(\text{O III})\) (K) value from the literature. Column 4 is the log \(T_e(\text{He} II)\) (K) value from the literature. Column 5 is the log \(T_e(\text{N II})\) (K) result from our plasma diagnostics. Column 6 is the log \(N_e(\text{O II})\) (cm\(^{-3}\)) result from our plasma diagnostics. Column 7 is the log \(T_e(\text{O II})\) (K) result from our plasma diagnostics. Column 8 is the log \(N_e(\text{He} II)\) (cm\(^{-3}\)) result from our plasma diagnostics.

Detailed modelling for the He I recombination lines have been computed for a couple decades now. Benjamin, Skillman & Smits (1999) combined the Smits (1996) models with collisional transitions of Sawey & Berrington (1993) producing accurate emissivities. More recently, Zhang et al. (2005b) applied computational models for the He I recombination lines for several dozen PNe. They also provide fitting functions spanning from 5000 to 20 000 K with fitting parameters that are density dependent.

Table 3 and 4 show the electron temperatures derived from the He I ORLs for PNe and H II regions, respectively. The three He I line ratios \(\lambda 7281/\lambda 6678\), \(\lambda 6678/\lambda 4471\) and \(\lambda 6678/\lambda 5876\) are used, and the objects are in descending order of the ADF values. The PN diagnostic results from Zhang et al. (2005b) are adopted. For those PNe that are not included in the sample of Zhang et al. (2005b), we derived electron temperatures using the fitting formula of Zhang et al. (2005b), and the He I line intensities are adopted from the literature. Column 1 is the nebula name. Column 2 is the ADF value from the literature. Columns 3–5 are the \(T_e\)'s derived from the He I line ratios. Most intensities provided line intensities for the four He I lines \(\lambda 7281, \lambda 6678, \lambda 5876\) and \(\lambda 4471\).

Resonance fluorescence may also have a minor effect on the strengths of ORLs. Grandi (1976) discussed in detail the effects of resonance fluorescence on permitted transitions. It has been known that the N II permitted lines from the low-lying 3d–3p and 3p–3s triplet arrays, whose upper levels are linked to the ground term 2p\(^3\)P by resonance lines, can be enhanced by fluorescence excitation. Grandi (1976) used photoionization models to study the excitation mechanisms of permitted transitions from common heavy element ions observed in the spectra of the Orion nebula and two Galactic PNe NGC 7027 and NGC 7662.

Grandi (1976) found that while the N II M28 3d\(^+\)D–3p\(^-\)3P multiplet is excited by both recombination and continuum fluorescence of the starlight, emission of the N II M3 3p\(^+\)D–3s\(^-\)3P, M5 3p\(^-\)3s\(^-\)3P and M30 4s\(^+\)3P–3p\(^-\)3P multiplets are dominated by fluorescence excitation of the N II 4s\(^+\)3P\(^0\) level by the He I 1s8p\(^+\)3P\(^0\)–1s\(^2\)S\(^0\) \(\lambda 508.643\) resonance line, which coincides in wavelength with the N II 2p4s\(^+\)3P\(^0\)–2p\(^-\)3P\(^0\) \(\lambda 508.668\) line. Fluorescence excitation cannot excite the singlet transitions or transitions from the 3d–4f configuration. Given the fact that measurements of the N II ORLs for the majority of nebulae samples are of relatively large uncertainty due to weakness and blend, the effects of fluorescence on the plasma diagnostic results are insignificant. For those nebulae with good observations, e.g., the several archetypal PNe discussed in Section 3.3, detailed modelling is needed to estimate the exact contribution of fluorescence mechanisms. Grandi (1976) showed that the dominant excitation mechanism of the strongest O II permitted lines is recombination.

4 SUMMARY

For nearly four decades, analyses of gaseous nebulae are mainly based on the bright CELs, whose emissivities are acutely sensitive to electron temperature (e.g. Osterbrock & Ferland 2006). Thus, the abundances derived from CELs could be subject to the uncertainties caused by temperature fluctuations if present in the nebulae. The ORLs of heavy element ions, although much weaker by nature, are much less sensitive to temperature, and the resultant abundances are supposed to be more reliable, provided that very accurate measurements of the heavy element ORLs can be obtained. In the current paper, we demonstrate a new plasma diagnostic method based on the N II and O II optical recombination spectra, using the new effective recombination coefficients. This is the first work devoted to nebular analysis using heavy element ORLs, with a large sample of PNe and H II regions considered. The results show systematic differences between the electron temperatures derived from CELs, the optical recombination spectra of H I and He I, and the N II and O II ORLs. The observed temperature sequence has been found in previous observations, and is in agreement with the expectation of the bi-abundance nebular model (Liu et al. 2000). Although very deep, high-resolution spectra of gaseous nebulae, especially accurate measurements of heavy element ORLs, are still rare, we need to develop plasma diagnostic tools based on the heavy element ORLs for future study, given that the most comprehensive treatment of the
N II and O II recombination under the physical conditions of gaseous nebulae (i.e. the effective recombination coefficients for the N II and O II recombination spectra) are now available. The main purpose of this paper is to demonstrate the new method of nebular analysis and show its potential application.

The method presented in the current work for determining $T_e$ and $N_e$ from multiple transitions of N II and O II is general and quite promising for the future deep spectroscopic studies of gaseous nebulae. Transitions from multiplets V3 and V39 for N II and from V1, V10 and V48 for O II tend to be the strongest and most reliable to constrain the $\chi^2$ distribution over the entire $T_e$–log $N_e$ grid. However, some other N II and O II transitions can be used as well. Therefore, this method can potentially cover broad wavelength ranges of a spectrum instead of just determining the $T_e$ and $N_e$ from just a couple of lines, depending on the data quality. Given that the heavy element ORLs are intrinsically faint, high signal-to-noise ratios are needed to improve the data quality.

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