Light Flavor Dependence of the Isgur-Wise Function

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Abstract

We present an investigation on the ligh flavor dependence of the Isgur-Wise function for $B_a \to D_a$ and $B_a \to D^*_a$ in the framework of QCD sum rules. It is found that the Isgur-Wise function for $B_s$ decay falls faster than that for $B_{u,d}$ decay, which is just contrary to the recent prediction of the heavy meson chiral perturbation theory. SU(3) symmetry breaking effects in the mass and the decay constant are also estimated.

As a heavy quark goes into the infinite mass limit, all form factors for $B \to D$ and $B \to D^*$ can be expressed in terms of a single universal function [1], the so-called Isgur-Wise function. The Isgur-Wise function represents the nonperturbative dynamics of weak decays of heavy mesons. It depends not only on the dimensionless product $v \cdot v'$ of the initial and final mesonic velocities, but also on the light quark flavor of the initial and final mesons [1-4]. Here, we apply QCD sum rule approach [3,4] to study its light quark flavor dependence.
In HQET, the low energy parameter \( F_a(\mu) \) of heavy meson \( M_a(\bar{q}Q) \) is defined by [3]
\[
\langle 0|\bar{q}\Gamma h_Q|M_a(v)\rangle = \frac{F_a(\mu)}{2} Tr[\Gamma M(v)].
\] (1)
In the leading order, the decay constant \( f_{M_a} \simeq F_a(\mu)/\sqrt{m_{M_a}} \). It should be emphasized that here and after, the subscript \( a = u, d, s \) specifies the light antiquark \( \bar{q} = \bar{u}, \bar{d}, \bar{s} \) of the heavy meson \( M_a(\bar{q}Q) \).

Starting from the two-point correlation function in HQET,
\[
\pi_5(\omega) = i \int d^4x e^{ik\cdot x} \langle 0|TA_5^{(v)}(x), A_5^{(v)+}(0)|0\rangle,
\] (2)
where \( A_5^{(v)} = \bar{q}\gamma_5 h_Q \) and \( \omega = 2k \cdot v \), one obtains the sum rule for \( F_a(\mu) \)
\[
F_a^2(\mu)e^{-2\Lambda_a/T} = \frac{3}{8\pi^2} \int_{2m_q}^{\omega_c^a} ds \sqrt{s^2 - 4m_q^2(2m_q + s)} e^{-s/T} \left[<0|\bar{q}q|0>|1 - \frac{m_q}{2T} + \frac{m_q^2}{2T^2} - \frac{<0|\bar{q}q|0 >}{2T^2} - \frac{m_q}{4T^2}[\gamma - 0.5 - \ln\frac{T}{\mu}] + \frac{4\pi\alpha_s}{81T^3} <0|\bar{q}q|0 >^2 \right.
\]
(3)
with \( \gamma = 0.5772 \). Taking the derivative with respect to the inverse of \( T \), one can obtain the corresponding sum rule for \( \Lambda_a \).

The Isgur-Wise function \( \xi_a(v\cdot v', \mu) \) is defined by the matrix element at the leading order in \( \frac{1}{m_q} \)[3],
\[
< M_a(v')|\bar{h}_{Q_2}(v')\Gamma h_{Q_1}(v)|M_a(v) > = -\xi_a(v\cdot v', \mu)Tr[\bar{M}(v')\Gamma M(v)].
\] (4)
Same as above, it is not difficult to get the sum rule for the Isgur-Wise function
\[
\xi_a(y, \mu) = \frac{K(T, \omega_a^c, y)}{K(T, \omega_a^c, 1)}
\] (5)
where
\[
K(T, \omega_a^c, y) = \frac{3}{8\pi^2}(\frac{2}{1+y})^2 \int_{m_q}^{\omega_c^a} d\alpha \frac{1}{\sqrt{2(1+y)m_q}} \sqrt{\alpha^2 - 2(1+y)m_q^2 e^{-\alpha/T}}
\]
\[\begin{align*}
&- <0|\bar{q}q|0> \left[1 - \frac{m_q}{2F} + \frac{m_q^2}{4F^2} (1 + y)\right] \\
&+ <0|\bar{q}q|0> \left[\frac{y-1}{48F(1+y)} - \frac{m_q}{4F^2} (\gamma - 0.5 - \ln \frac{T}{\mu})\right] \\
&+ \frac{g_\sigma^2 <0|\bar{q}q|0>^2}{4F^2} \frac{2y+1}{3} + \frac{4\pi g_\sigma^2 <0|\bar{q}q|0>^2}{81F^3} y.
\end{align*}\]  

(6)

In the above derivation, we have used the sum rule for \(F_a(\mu)\).

In the numerical analysis of sum rules, we take the parameters such as condensates and \(m_q\) as in [3-4] and set the scale \(\mu = 1GeV\). For the continuum model \(\omega^c = \sigma(y)\omega^c\), we use the experiment preferred model \(\sigma(y) = \frac{y+1}{2y}\) as in [3].

Evaluations of sum rules for \(F_a\) and \(\bar{\Lambda}_a\) give

\[\bar{\Lambda}_s \simeq 0.62 \pm 0.07GeV, \quad F_s \simeq 0.36 \pm 0.05GeV^{3/2},\]  

(7)

\[\bar{\Lambda}_{u,d} \simeq 0.55 \pm 0.07GeV, \quad F_{u,d} \simeq 0.32 \pm 0.05GeV^{3/2}.\]  

(8)

However, in order to reduce the errors, writing the mass difference \(\Delta M = m_{M_s} - m_{M_{u,d}} = \bar{\Lambda}_s - \bar{\Lambda}_{u,d}\) and the ratio \(R_F = F_s/F_{u,d}\) with the corresponding sum rules, one gets \(\Delta M = 69 \pm 5MeV\), which is in good agreement with the recent experiment results [5,6] \(m_{B_s} - m_B = 90 \pm 6MeV, m_{D_s} - m_D = 99.5 \pm 0.6MeV\), and the ratio \(R_F = 1.13 \pm 0.01\).

In Fig.1, the Isgur-Wise function \(\xi_s\) is shown as a function of \(y\). Obviously, the dependence on the parameters \(\omega^c_s\) and \(T\) is very weak. At the center of the sum rule window \(T=0.8GeV\), we obtain the slope parameter \(\varrho^2_a\) defined as \(\varrho^2_a = -\xi'_a(\mu = 1, \mu)\)

\[\varrho^2_a = 1.09 \pm 0.04,\]  

(9)

the uncertainty is due to the variation of \(\omega^c_s\). One can compare with

\[\varrho^2_{u,d} = 1.01 \pm 0.02.\]  

(10)
and find that SU(3) breaking effects in the slope parameter is not large but the important thing is

$$\tilde{q}_s^2 > \tilde{q}_{u,d}^2.$$  (11)

This result just indicates that the Isgur-Wise function $\xi_s$ falls faster than the Isgur-Wise function $\xi_{u,d}$ as shown below.

In Fig.2, we show $R_{IW} = \xi_s/\xi_{u,d}$ as a function of $y$ at $T = 0.8 GeV$ for different $\omega_{u,d}^c = 1.7 \sim 2.3 GeV$ and $\omega_s^c = \omega_{u,d}^c + 0.1 GeV$. One can find that the ratio $R_{IW}$ displays a soft dependence on $\omega_{u,d,s}^c$. At $y = 1.6$ ($q^2 = 0$ for $B_{u,d} \rightarrow D_{u,d} + l\nu$), we get from the sum rule

$$R_{IW} \simeq (95 \pm 2)\%,$$  (12)

where the uncertainty is ascribed to the uncertainty in $\omega_{u,d,s}^c$ and $T$.

In the evaluations of sum rules for $\xi_a$ and $R_{IW}$, the continuum model is chosen as $\sigma(y) = \frac{y+1}{2y}$. This may cause large errors in $\xi_a$ and $R_{IW}$. As discussed in [3], one knows

$$\sigma_{min} = \frac{y+1 - \sqrt{y^2 - 1}}{2} \leq \sigma(y) \leq \sigma_{max} = 1,$$  (13)

and the model $\sigma_{max}$ and $\sigma_{min}$ respectively constitutes the upper bound and the lower bound for $\xi_a$. However, for $R_{IW}$, the model $\sigma_{max}$ and $\sigma_{min}$ just give the lower bound and the upper bound respectively. Although different continuum model gives different value for $R_{IW}$, one can find that all of these values clearly give

$$R_{IW} < 1\ ,\ for\ y \neq 1.$$  (14)

Therefore we conclude that $R_{IW} < 1$ (for $y \neq 1$) is independent of the model choice $\sigma(y)$.

In summary, It is very interesting to find that the Isgur-Wise function for $B_s \rightarrow D_s$ falls faster than the Isgur-Wise function for $B_{u,d} \rightarrow D_{u,d}$, which is just contrary to the
prediction of the heavy meson chiral perturbation theory where only SU(3) breaking chiral loops are calculated [2]. Our result $R_{IW} \leq 1$ agrees with that of other calculations [7]. It is expected that the future experiments can test this result and reveal the underlying mechanism of SU(3) breaking effects.

References

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Figure Captions

Fig.1: The Isgur-Wise function $\xi_s$ as a function of $y$. The band corresponds to variations of $\omega_s^c$ in $1.8GeV \sim 2.4GeV$ and $T$ in $0.7GeV \sim 0.9GeV$. 
Fig.2: The ratio $R_{IW} = \xi_s/\xi_{u,d}$ as a function of $y$ at $T=0.8\text{GeV}$ ($\omega_c^e = \omega_{u,d}^c + 0.1\text{GeV}$): Dashed line: $\omega_{u,d}^c = 1.7\text{GeV}$, Solid line: $\omega_{u,d}^c = 2.0\text{GeV}$, Dotted line: $\omega_{u,d}^c = 2.3\text{GeV}$. 
This figure "fig1-1.png" is available in "png" format from:

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