Charge form factor of $\pi$ and $K$ mesons

F. Cardarelli$^{(a)}$, I.L. Grach$^{(b)}$, I.M. Narodetskii$^{(b)}$
E. Pace$^{(a,c)}$, G. Salmè$^{(d)}$, S. Simula$^{(d)}$

$^{(a)}$Istituto Nazionale di Fisica Nucleare, Sezione Tor Vergata
Via della Ricerca Scientifica, I-00133 Roma, Italy
$^{(b)}$Institute of Theoretical and Experimental Physics
Moscow 117259, Russia
$^{(c)}$Department of Physics, University of Rome "Tor Vergata"
Via della Ricerca Scientifica, I-00133 Roma, Italy
$^{(d)}$Istituto Nazionale di Fisica Nucleare, Sezione Sanità,
Viale Regina Elena 299, I-00161 Roma, Italy

Abstract

The charge form factor of $\pi$ and $K$ mesons is evaluated adopting a relativistic constituent quark model based on the light-front formalism. The relevance of the high-momentum components of the meson wave function, for values of the momentum transfer accessible to $CEBAF$ energies, is illustrated. The predictions for the elastic form factor of $\pi$ and $K$ mesons are compared with the results of different relativistic approaches, showing that the measurements of the pion and kaon form factors planned at $CEBAF$ could provide information for discriminating among various models of the meson structure.

PACS number(s): 12.39.Ki, 13.40.Gp, 14.40.Aq, 14.40.Lb
The evaluation of the electromagnetic (e.m.) properties of $\pi$ and $K$ mesons has recently received a renewed interest, because measurements of the pion and kaon charge form factors are planned at CEBAF [1]. In the past few years, light-front constituent quark models have been extensively applied to relativistic calculations of various electroweak properties of mesons [2] - [5] and baryons [6]. In most of these applications ([4] - [6]) it is assumed that: i) the hadron wave function is given by a harmonic oscillator (HO) ansatz, which is expected to describe the effects of the confinement scale only, or has a power-law (PL) behaviour, dictated at large momenta by the perturbative QCD theory [7]; ii) the constituent quarks are point-like objects as far as their e.m. properties are concerned. In Ref. [2] a different approach is adopted, namely: i) a light-front mass operator, constructed from the effective $q\bar{q}$ Hamiltonian of Ref. [8] reproducing the meson mass spectra, is considered and the corresponding eigenfunctions are used to describe the dynamics of the constituent quarks inside the meson; ii) a non-vanishing size of the constituent quarks is assumed and a simple monopole charge form factor for the constituent quarks is introduced. Within this approach existing pion data both at low and high values of the squared four-momentum transfer $Q^2$ are reproduced. Moreover, it has been shown that the high-momentum components, generated in the wave function by the one-gluon-exchange (OGE) part of the effective $q\bar{q}$ interaction of Ref. [8], sharply affect the pion charge form factor for values of $Q^2$ up to few (GeV/c)$^2$, i.e. in a range of values accessible to CEBAF energies. Differently, in Ref. [4] it has been claimed that the charge form factor of pseudoscalar mesons is insensitive to a large class of wave functions, and, moreover, that the high-momentum tail of the wave function does not matter for energies accessible to present experiments. The aims of this brief report are i) to point out that our wave functions do not belong to the limited class of wave functions considered in [4], and ii) to clarify the relevance of the high-momentum components of the meson wave function, particularly for values of $Q^2 \sim$ few (GeV/c)$^2$, by analyzing in detail the structure of the expression of the pion form factor used in Refs. [2] and [4]. Moreover, our theoretical predictions for the elastic form factor of $\pi^+$, $K^+$ and $K^0$ mesons are compared with the results obtained within different sophisticated relativistic approaches, showing that the measurements of the pion and kaon form factors planned at CEBAF [1] could provide information for discriminating among various models of the meson structure.

We will start directly from the general expression of the charge form factor of a pseudoscalar meson, $F^{PS}(Q^2)$, obtained within the light-front constituent quark model (see, e.g., Ref. [2]), viz.

$$F^{PS}(Q^2) = e_q f^q(Q^2) H^{PS}(Q^2; m_q, m_{\bar{q}}) + e_{\bar{q}} f^{\bar{q}}(Q^2) H^{PS}(Q^2; m_{\bar{q}}, m_q)$$  \hspace{1cm} (1)

where $e_q$ ($m_q$) is the charge (mass) of the constituent quark and $f^q(Q^2)$ its charge form factor. In Eq. (1) the body form factor $H^{PS}(Q^2; m_1, m_2)$ is given explicitly by

$$H^{PS}(Q^2; m_1, m_2) = \int d\vec{k}_\perp d\xi \frac{\sqrt{M_0 M_0'}}{4\xi(1-\xi)} \sqrt{ \left[ 1 - \left( \frac{m_1^2 - M_0'^2}{M_0^2} \right)^2 \right] \left[ 1 - \left( \frac{m_2^2 - M_0'^2}{M_0^2} \right)^2 \right] }$$

2
where the free mass operator \( M_0 \) and the intrinsic light-front variables \( \vec{k}_\perp, \langle \vec{k}_\perp \rangle \), \( \xi \) are defined as

\[
M_0^2 = \frac{m_1^2 + k_1^2}{\xi} + \frac{m_2^2 + k_1^2}{(1-\xi)} , \quad M_0^2 = \frac{m_1^2 + k_1^2}{\xi} + \frac{m_2^2 + k_2^2}{(1-\xi)}
\]

\[
\vec{k}_\perp = \vec{p}_\perp - \xi \vec{P}_\perp = -\vec{p}_\perp + (1-\xi)\vec{P}_\perp , \quad \vec{k}_\perp = \vec{k}_\perp + (1-\xi)\vec{Q}_\perp
\]

\[
\xi = p_+^1 / P^+ = 1 - p_2^+ / P^+
\]

In Eqs. (2) - (4) the subscript \( \perp \) indicates the projection perpendicular to the spin quantization axis, defined by the vector \( \hat{n} = (0,0,1) \), and the plus component of a four-vector \( p \equiv (p^0, \vec{p}) \) is given by \( p^+ = p^0 + \hat{n} \cdot \vec{p} \). Moreover, \( \vec{P} \equiv (P^+, \vec{P}_\perp) = \vec{p}_1 + \vec{p}_2 \) is the light-front momentum of the meson, \( k^2 \equiv k_\perp^2 + k_n^2, k_\perp^2 \equiv k_\perp^2 + k_\perp^2, k_n \equiv (\xi - 1/2)M_0 + (m_2^2 - m_1^2)/M_0 \) and \( k_n' \equiv (\xi - 1/2)M_0' + (m_2^2 - m_1^2)/M_0' \).

Following Ref. 4, the radial wave function \( w^{PS}(k^2) \) appearing in Eq. (2) can be identified with the equal-time radial wave function in the meson rest-frame. In what follows, we will make use of the eigenfunctions of the effective \( q\bar{q} \) Hamiltonian, developed by Godfrey and Isgur (GI) 8 to reproduce the meson mass spectra. In case of pseudoscalar mesons one has

\[
H_{q\bar{q}} w^{PS}(k^2)|00\rangle \equiv \sqrt{m_q^2 + k^2 + m_\bar{q}^2 + k^2 + V_{q\bar{q}}} \ w^{PS}(k^2)|00\rangle = M_{q\bar{q}} w^{PS}(k^2)|00\rangle \quad (4)
\]

where \( M_{q\bar{q}} \) is the mass of the meson, \( |00\rangle = \sum_{\nu \bar{\nu}} \langle \frac{1}{2} \nu \frac{1}{2} \bar{\nu} |00\rangle \chi_{\nu} \bar{\chi}_{\bar{\nu}} \) is the usual quark-spin wave function of a pseudoscalar meson and \( V_{q\bar{q}} \) is the effective \( q\bar{q} \) potential. The GI interaction, \( V_{(GI)} \), is composed by an OGE term (dominant at short separations) and a linear-confining term (dominant at large separations). We will consider two types of wave functions: the first one is given by the solution of Eq. (4) obtained when the OGE part of \( V_{(GI)} \) is switched off, i.e., when only its linear confining term, \( V_{(conf)} \), is retained, whereas the second choice is obtained by solving Eq. (4) with the full GI interaction. The two different forms of \( w^{PS}(k^2) \) will be denoted hereafter by \( w^{PS}_{(conf)} \) and \( w^{PS}_{(GI)} \) corresponding to \( V_{(conf)} \) and \( V_{(GI)} \), respectively. Note that the pion mass corresponding to \( V_{(conf)} \) in Eq. (4) is 1.024 GeV, whereas the one using \( V_{(GI)} \) is 0.149 GeV. The pion wave functions \( w^{\pi}_{(conf)} \) and \( w^{\pi}_{(GI)} \) are shown in Fig. 1 and compared with the HO \( w^{\pi}_{(HO)} \propto \exp(-k^2/2\alpha^2) \) and PL \( w^{\pi}_{(PL)} \propto (1 + k^2/\beta^2)^{-2} \) wave functions used in Ref. 4. It should be stressed that the latter ones are constrained by imposing the reproduction of the leptonic decay constants of \( \pi \) and \( \rho \) mesons and by assuming a point-like quark electroweak (e.w.) current. It can clearly be seen that: i) the momentum behaviours of \( w^{\pi}_{(conf)} \) and \( w^{\pi}_{(GI)} \) are sharply different, because of the configuration mixing induced by the OGE part of the effective \( q\bar{q} \) interaction; ii) for \( k < 1 \) GeV/c \( w^{\pi}_{(HO)} \) and \( w^{\pi}_{(PL)} \) are quite similar (possibly because they have to fulfil the above-mentioned constraints) and do not differ significantly from \( w^{\pi}_{(conf)} \), which takes
into account the effects of the confinement scale only; iii) the high-momentum tail of \( w_{(GI)}^\pi \), while exhibiting a nominal power-law fall off at large momenta, is much higher than the one pertaining to \( w_{(PL)}^\pi \). The average transverse momentum \( \bar{k}_\perp \equiv \sqrt{< k_\perp^2 >} \) turns out to be \( \simeq 0.8 \text{ GeV}/c \) in case of \( w_{(GI)}^\pi \) and \( \simeq 0.3 \text{ GeV}/c \) for \( w_{(HO)}^\pi, w_{(PL)}^\pi \) and \( w_{(conf)}^\pi \). Thus, the HO and PL wave functions adopted in Refs. [4] and [8] (c) can hardly be considered representative of the range of uncertainty of the momentum behaviour of the wave function. As a matter of fact, our \( w_{(GI)}^\pi \) wave function, which is eigenfunction of a mass operator reproducing the meson mass spectra, does not belong to the limited class of wave functions considered in Refs. [4] and [8] (c), since it gives rise to an overestimation of the leptonic decay constants when a point-like quark e.w. current is adopted (cf. [2]).

The relevance of the high-momentum components of the wave function in the calculation of the pion form factor can be investigated by considering in Eq. (2) different values of the upper limit of integration over \( k_\perp \equiv |\vec{k}_\perp| \) (denoted hereafter by \( k_\perp^{\text{conf}} \)). The results of the calculations, obtained assuming \( f^a = 1 \) in Eq. (1) and using in Eq. (2) both \( w_{(conf)}^\pi \) and \( w_{(GI)}^\pi \), are shown in Fig. 2 for values of \( Q^2 \) up to 10 \((\text{GeV}/c)^2\). In what follows we will limit ourselves to consider the wave functions \( w_{(conf)}^\pi \) and \( w_{(GI)}^\pi \), because for \( Q^2 < 10 \((\text{GeV}/c)^2\) the results obtained using the HO and PL wave functions of Ref. [4] do not differ significantly from those calculated with \( w_{(conf)}^\pi \). From Fig. 2 it can clearly be seen that, both for \( w_{(conf)}^\pi \) and \( w_{(GI)}^\pi \), the calculation of the pion charge form factor is strongly affected by components of the wave function corresponding to \( k_\perp > \bar{k}_\perp \). As a matter of fact, in case of \( w_{(conf)}^\pi \), \( \sim 90\% \) of the form factor at \( Q^2 > 0.5 \((\text{GeV}/c)^2\) is due to components of the wave functions with \( k_\perp > 0.3 \text{ GeV}/c \) \((\sim \bar{k}_\perp^{\text{conf}})\), and, moreover, the saturation is almost reached only when \( k_\perp^{\text{conf}} \simeq 1.5 \text{ GeV}/c \) \((\sim 5 \bar{k}_\perp^{\text{conf}})\). In case of \( w_{(GI)}^\pi \), the high-momentum tail corresponding to \( k_\perp > 0.8 \text{ GeV}/c \) \((\sim \bar{k}_\perp^{GI})\) is responsible for \( \sim 50\% \) of the pion form factor at \( Q^2 > 0.5 \((\text{GeV}/c)^2\) and the saturation at high values of \( Q^2 \) is almost reached only when \( k_\perp^{GI} \simeq 2.5 \text{ GeV}/c \) \((\sim 3 \bar{k}_\perp^{GI})\). Such results are simply related to the fact that, for \( Q \sim \text{few GeV}/c \), values of \( k_\perp \sim 1 \text{ GeV}/c \) can give rise to low values of \( k_\perp^{conf} \) \((= |\vec{k}_\perp + (1 - \xi)\vec{Q}_\perp|)\), when \( \vec{k}_\perp \) is antiparallel to \( \vec{Q}_\perp \) and the struck quark carries an average fraction of the momentum of the meson \((\text{i.e., } \xi \sim \bar{\xi} = 0.5 \text{ in the pion})\) [4]. This means that for \( Q \sim \text{few GeV}/c \) configurations both at short and large transverse \( qq \) separations are relevant (see the product \( w^{PS}(k^2)w^{PS}(k'^2) \) in the integrand of Eq. (2)). To sum up, the results reported show that: i) the momentum behaviour of the wave function at \( k > 1 \text{ (GeV}/c) \) can play a relevant role in determining the pion form factor for values of \( Q^2 \) accessible to CEBAF energies; ii) according to the findings of Ref. [2] the pion form factor is sharply overestimated due to the effects of the high-momentum components generated in the wave function by the OGE part of the GI interaction (compare solid lines in Fig. 2(a) and 2(b)), which, as known, nicely explains the \( \pi - \rho \) mass splitting. In this work we have checked that the same conclusions hold as well for the charge form factor of \( K \) meson, whereas they are no longer true in case of heavy pseudoscalar mesons, like the \( D \) and \( B \) mesons. As a matter of fact, the explicit

\footnote{Note that the \( \xi \)-distribution corresponding both to \( w_{(conf)}^\pi \) and \( w_{(GI)}^\pi \) exhibits a flat maximum in the region \( 0.2 < \xi < 0.8 \).}
calculations of Eqs. (1-2) (assuming \( f_q = 1 \)) yield almost the same results in a wide range of values of \( Q^2 \) (\( Q^2 \gg 1 \) (GeV/c)\(^2\)) both for \( \psi_{D(B)}^{\text{conf}} \) and \( \psi_{D(B)}^{D(B)} \) wave functions. We will limit ourselves to comment that such a result can be ascribed to the fact that: i) the body form factor (\( H_P^S \)) corresponding to the virtual photon absorption by the heavy \( c (b) \) quark in the \( D (B) \) meson is dominant; ii) the average fraction of the momentum of the meson carried by the heavy quark is very close to 1, leading to \( k'_{\perp} \simeq k_{\perp} \), which implies a weak dependence of the calculated form factor on the heavy meson wave function in a wide range of values of \( Q^2 \).

The results reported in Fig. 2 suggest that, if the constituent quarks are assumed to be point-like particles (i.e., if \( f_q = 1 \)), the pion form factor calculated with wave functions having \( \langle r^2 \rangle_{\pi} \sim 0.3 \) GeV/c (like, e.g., \( \psi_{\text{conf}}^{\pi}, \psi_{\text{HO}}^{\pi} \) and \( \psi_{\text{PL}}^{\pi} \)) is in fairly good agreement with existing data, whereas the one obtained using \( \psi_{\text{GI}}^{\pi} \) is not. However, once the assumption \( f_q = 1 \) is made, the parameters which unavoidably appear in the hadron wave function are usually adjusted in order to fit e.m. (or, more generally, electroweak) hadron properties (see, e.g., Ref. \([6]\)(c)). In this way the relativistic constituent quark model (RCQM) looses (at least partially) its predictive power, for the wave function is not completely independent of the e.m. observable under investigation. A different approach is to adopt the eigenfunctions of a (light-front) mass operator able to reproduce correctly the hadron mass spectra, so that the hadron wave functions do not depend upon any observable but the hadron energy levels. In this way, the momentum behaviour of the hadron wave functions is dictated by the features of the effective \( q\bar{q} \) interaction appearing in the mass operator and the investigation of the e.m. properties of hadrons could provide information on those of the constituent quarks. Thus, in order to recover the predictive power of the \( RCQM \), the same e.m. one-body current should be used for all the hadrons. Following this strategy, a simple monopole ansatz for the charge form factor of the constituent u and d quarks has been considered in Ref. \([2]\), viz.

\[
f_q(Q^2) = \frac{1}{1 + Q^2 < r^2 >_q / 6}
\]

(5)

When the wave function \( \psi_{\text{GI}}^{\pi} \) is adopted in Eq. (2), the value \( < r^2 >_u = < r^2 >_d = (0.48 \text{ fm})^2 \) has to be chosen in order to reproduce the experimental value of the pion charge radius \( < r^2 >_{\text{exp}} = (0.660 \pm 0.024 \text{ fm})^2 \). It should be pointed out that such a value of the constituent quark radius is in nice agreement with the ansatz \( < r^2 >_q = \kappa / m_q^2 \), suggested in Ref. \([4]\) from the analysis of the so-called strong interaction radius of hadrons, when the values \( \kappa \simeq 0.3 \), extracted from the chiral quark model of Ref. \([4]\), and \( m_u = m_d = 0.220 \text{ GeV} \) \([3]\) are adopted \([4]\). Moreover, it should be stressed that, though the u(d)-quark charge radius is fixed only by the pion data at very low values of \( Q^2 \), the predictions of our RCQM compare very favourably with the data also at high values of \( Q^2 \) (see Ref. \([2]\)). This is illustrated in

\[\text{Note that the difference between the } \rho \text{ and } \pi \text{ radii found in Ref. } \([1]\) (i.e., } < r^2 >_{(\rho)} - < r^2 >_{(\pi)} = 0.11 \pm 0.06 \text{ fm}^2 \) is independent of the constituent quark radius and is nicely explained by the configuration mixing due to the spin-dependent part of the effective \( q\bar{q} \) interaction, as it can be inferred from the results of Refs. \([2,3]\) yielding } < r^2 >_{(\rho)} - < r^2 >_{(\pi)} = 0.14 \text{ fm}^2 \).
Fig. 3, where our results for the pion charge form factor are compared with the experimental data [12] and also with the predictions of different sophisticated relativistic approaches, like the covariant Bethe-Salpeter approach of Ref. [13] and the QCD sum rule technique of Ref. [14]. The predictions of the simple Vector Meson Dominance (VMD) model, including the \( \rho \)-meson pole only, are also shown in the same figure. It can be seen that existing pion data do not discriminate among calculations based on different models of the pion structure.

By using in Eq. (2) the appropriate eigenfunctions of the GI Hamiltonian [4], the elastic form factors of charged \( K^+ \) and neutral \( K^0 \) mesons have been calculated. In Fig. 4 the results of our calculations, performed adopting different choices of the charge radius of the constituent s quark \( \langle r^2 \rangle_s \), are reported and compared with the predictions of Ref. [13], based on a covariant Bethe-Salpeter approach. It can be seen that for \( Q^2 > 1 \ (GeV/c)^2 \) the calculated charge form factors of \( K^+ \) and \( K^0 \) mesons are remarkably sensitive to the value used for \( \langle r^2 \rangle_s \), so that their experimental investigation could provide information on the e.m. structure of light constituent quarks. From Fig. 4 it can also be seen that, unlike the case of the pion, the measurement of the kaon form factor at \( Q^2 > 1 \ (GeV/c)^2 \) could discriminate among different models of the meson structure.

In conclusion, the charge form factor of \( \pi \) and \( K \) mesons has been evaluated within a light-front constituent quark model. The use of the eigenfunctions of a mass operator, constructed from the effective \( q\bar{q} \) Hamiltonian of Ref. [8] reproducing the meson mass spectra, and the introduction of a phenomenological charge form factor for the constituent quarks have been briefly discussed. It has been shown that the high-momentum tail of the meson wave function (namely, \( k > 1 \ GeV/c \)) is essential in determining the behaviour of the form factor already at \( Q^2 > 0.5 \ (GeV/c)^2 \). Thus, the investigation of \( \pi \) and \( K \) form factors at CEBAF represents a powerful tool to study the short-range structure of mesons. The predictions of our relativistic constituent quark model for the charge form factor of \( \pi \) and \( K \) mesons have been compared with those of different sophisticated relativistic approaches, showing that the planned experiments at CEBAF [1], aimed at measuring independently the pion and kaon form factor for \( Q^2 < 3 \ (GeV/c)^2 \), could provide relevant information on the electromagnetic structure of light constituent quarks and could represent an interesting tool to discriminate among different models of the meson structure.

We gratefully acknowledge S. Brodsky for helpful discussions and R.A. Williams for supplying us with the numerical output of the \( K^+ \) and \( K^0 \) calculations of Ref. [13].

References

[1] CEBAF Proposal E-93-012: Electroproduction of Light Quark Mesons (M. Kossov, spokesman); CEBAF Proposal E-93-018: Separation of Longitudinal and Transverse Amplitudes in Kaon Electroproduction (O.K. Baker, spokesman); CEBAF Proposal E-93-021: The Charged Pion Form Factor (D. Mack, spokesman).
[2] F. Cardarelli, I.L. Grach, I.M. Narodetskii, E. Pace, G. Salmé and S. Simula: Phys. Lett. 332B (1994) 1.

[3] F. Cardarelli, I.L. Grach, I.M. Narodetskii, G. Salmé and S. Simula: Phys. Lett. 349B (1995) 393.

[4] F. Schlumpf: Phys. Rev. D50 (1994) 6895.

[5] P.L. Chung, F. Coester and W.N. Polyzou: Phys. Lett. B205 (1988) 545; W. Jaus: Phys. Rev. D44 (1991) 2851 and references therein quoted; B.D. Keister: Phys. Rev. D49 (1994) 1500.

[6] (a) P.L. Chung and F. Coester: Phys. Rev. D44 (1991) 229; (b) M. Yabu, M. Takizawa and W. Weise: Z. Phys. A345 (1993) 193; (c) F. Schlumpf: Phys. Rev. D47 (1993) 4114 (erratum, ib. D49 (1994) 6246); S. Brodsky and F. Schlumpf: Phys. Lett. 329B (1994) 111.

[7] G.P. Lepage and S.J. Brodsky: Phys. Rev. D22 (1980) 2157; S. J. Brodsky and G.P. Lepage: Exclusive Processes in Quantum Chromodynamics, in Perturbative Quantum Chromodynamics, A.H. Mueller ed., World Scientific Publishing (Singapore, 1989), p. 93.

[8] S. Godfrey and N. Isgur: Phys. Rev. D32 (1985) 185.

[9] S.R. Amendolia et al.: Phys. Lett. 146B (1984) 116.

[10] B. Povh and J. Hüfner: Phys. Lett. 245B (1990) 653; see also S.M. Troshin and N.E. Tyurin: Phys. Rev. D49 (1994) 4427.

[11] U. Vogl, M. Lutz, S. Klimt and W. Weise: Nucl. Phys. A516 (1990) 469.

[12] C.N. Brown et al.: Phys. Rev. D8 (1973) 92; C.J. Bebek et al.: Phys. Rev. D9 (1974) 1229, Phys. Rev. D13 (1976) 25, Phys. Rev. D17 (1978) 1693.

[13] W.W. Buck, R. A. Williams and H. Ito: Report No. CEBAF-TH-94-02 (unpublished); H. Ito, W.W. Buck and F. Gross: Phys. Rev. C45 (1992) 1918; Phys. Lett. 287B (1992) 23.

[14] V.A. Nesterenko and A.V. Radyushkin: Phys. Lett. 115B (1982) 410.
Figure Captions

Fig. 1. Pion wave functions \( (k \cdot w^\pi (k^2))^2 \), calculated using in Eq. (4) different effective \( q\bar{q} \) interactions, as a function of the relative momentum \( k \). Dotted line: \( w^\pi_{\text{(conf)}} \), corresponding to the case in which only the linear confining part of the \( GI q\bar{q} \) interaction \([8]\) is considered. Solid line: \( w^\pi_{\text{(GI)}} \), corresponding to the solution of Eq. (4) obtained using the full \( GI q\bar{q} \) interaction. The dot-dashed and dashed lines correspond to the harmonic oscillator \( (w^\pi_{\text{(HO)}} \propto \exp(-k^2/2\alpha^2)) \) and power-law \( (w^\pi_{\text{(PL)}} \propto (1 + k^2/\beta^2)^{-2}) \) wave functions introduced in Ref. \([4]\), respectively.

Fig. 2. Charge form factor of the pion, \( Q^2 F^\pi (Q^2) \), calculated assuming \( f^q = 1 \) in Eq. (1) and using in Eq. (2) the wave functions \( w^\pi_{\text{(conf)}} \) (a) and \( w^\pi_{\text{(GI)}} \) (b). The various lines correspond to the results obtained assuming in Eq. (2) different values of \( k_U^\perp \), the upper limit of integration over \(|\vec{k}^\perp|\). The dotted and dashed lines correspond to \( k_U^\perp = 0.3 \) and \( 0.8 \) \( GeV/c \), respectively, whereas the dot-dashed lines correspond to \( k_U^\perp = 1.5 \) \( GeV/c \) in (a) and \( 2.5 \) \( GeV/c \) in (b). The solid lines represent the full calculations of the elastic form factor (i.e., when \( k_U^\perp \to \infty \)). The experimental data are taken from Ref. \([12]\).

Fig. 3. Elastic form factor of the charged pion, times \( Q^2 \), as a function of \( Q^2 \). The solid line represents the results of our relativistic constituent quark model (RCQM), obtained using in Eq. (2) the appropriate eigenfunction of the effective \( q\bar{q} \) Hamiltonian of Ref. \([8]\) (see Eq. (4)) and adopting in Eq. (1) the monopole charge form factor (Eq. (5) with the quark charge radius equal to \( \langle r^2 \rangle_u = \langle r^2 \rangle_d = (0.48 \ fm)^2 \). The dashed and dot-dashed lines represent the predictions of the covariant Bethe-Salpeter approach of Ref. \([13]\) and of the QCD sum rule technique of Ref. \([14]\), respectively. The dotted line is the prediction of a simple VMD model, which includes the \( \rho \) - meson pole only (i.e., \( F^\pi = (1 + Q^2/m_{\rho}^2)^{-1} \)).

Fig. 4. Elastic form factor of charged \( K^+ \) (a) and neutral \( K^0 \) (b) mesons, times \( Q^2 \), as a function of \( Q^2 \). The solid line represents the results of our relativistic constituent quark model (RCQM), obtained using in Eq. (2) the appropriate eigenfunctions of the effective \( q\bar{q} \) Hamiltonian of Ref. \([8]\) (see Eq. (4)) and adopting in Eq. (1) a SU(3) symmetric (monopole) charge form factor for the constituent quarks (i.e., \( f^u = f^d = f^s \)) with the charge radius \( \langle r^2 \rangle_u = \langle r^2 \rangle_d = \langle r^2 \rangle_s = (0.48 \ fm)^2 \). The dot - dashed lines are the results of the calculations of Ref. \([13]\), based on a covariant Bethe-Salpeter approach. The dashed lines represent the predictions of our RCQM, calculated using different values for the charge radius of the constituent \( s \) and \( u(d) \) quarks, namely \( \langle r^2 \rangle_s = (0.25 \ fm)^2 \) and \( \langle r^2 \rangle_u = \langle r^2 \rangle_d = (0.48 \ fm)^2 \). Note that these values correspond to the ansatz \( \langle r^2 \rangle_q = \kappa/m_q^2 \) \([10]\), adopting \( \kappa \simeq 0.3 \) \([11]\), \( m_u = m_d = 0.220 \ GeV \) and \( m_s = 0.419 \ GeV \) \([8]\). Eventually, the dotted line in (a) is the prediction of the VMD model including the \( \rho \) - meson pole only.
F. Cardarelli et al., Physical Review D (Brief Report): fig. 2
F. Cardarelli et al., Physical Review D (Brief Report): fig. 3.
F. Cardarelli et al., Physical Review D (Brief Report): fig. 4.