Multi-stable Deployment of Unitary Tensegrity Structures

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Abstract. Tensegrities are lightweight structures that, by actuating some or all its components, can become deployable systems that can expand from an initial package to a full-size. This characteristic gives them a great advantage in aerospace applications. In this paper, we present a formulation for the coordinates of tensegrity single units whose only variables are the number of bars and the height of the structure, allowing to know the geometrical configuration of the unit for any given height. We prove that the set of these stable configurations forms a stable path along which a tensegrity unit can be deployed. Numerical simulations using MATLAB, and real models, are used to verify the strategy.

1. Introduction
Tensegrity structures are a special kind of spatial system, composed of bars and strings. Because of the way these members are arranged, the internal forces are distributed only along the axes of them in the form of tension in the strings, and compression in the bars. Thanks to this arrangement, other forces such as torsion or torque are not present. This condition complements being lightweight, deployable and the possibility of applying control methods as the main characteristics of the tensegrity structures. Their introduction to the world was in the shape of sculptures; however, in recent years, the engineering has found applications for them in the form of bridges, antennas, masts and robots. The aerospace engineering has taken advantage of the deployability of these structures to develop tensegrity antennas and masts that can be packed into smaller packages than conventional structures, thus reducing costs and weight.

On this topic, Tibert[1] described several applications of deployable tensegrities, Zolesi et al. [2] developed a large deployable antenna reflector based on a double layer tensegrity structure, Sultan and Skelton[3] presented a deployment strategy for tensegrities based on an equilibrium manifold of configurations, Pinaud et al.[4] explored the deployment capabilities of class 2 tensegrity structures by designing and building a prototype model, Rhode-Barbarigos[5] proposed a stochastic search algorithm as the deployment strategy for a tensegrity bridge.

Recently, Sultan[6] used the infinitesimal mechanisms to find paths that produce no loss of energy due to kinematic damping, Porta and Hernández-Juan[7] used differential geometry to propose a quasi-static motion path planning method, Veuve et al.[8] used a stochastic algorithm to develop a bio-inspired learning strategy. However, these approaches are based on advanced mathematical models that demand high computational resources. This makes them applicable mostly to multistage, asymmetric and complex tensegrity structures, that can’t be modeled for obtaining an analytical solution.
Concerning the equilibrium of single units, Motro[9] demonstrated that the rotation angle between top and bottom planes of a simplex depends only on the number of compressive members; Skelton and De Oliveira[10] expanded this result to derive relationships for the internal force density of the different components of a tensegrity unit that ensure a stable configuration. These relationships, a function of the radius and number of bars of the structure, are widely used in the literature for the form finding of static tensegrities.

In this paper we propose a multi-stable deployment strategy for unitary tensegrity structures. We prove that, making use of the static equilibrium conditions of a tensegrity unit, it is possible to obtain a deployment path along which the structure is stable for any desired height. The nodal coordinates of the structure are reformulated as a function of the height , keeping the length of the bars constant. With the rotation angle between planes as the restriction, the coordinates for any desired height can be obtained. This method assumes that the nodes travel only along stable positions, making unnecessary the use of dynamic equations and reducing the computational cost.

2. Tensegrity Unit

Complex tensegrity systems can be modeled as the composition of several single units, each unit with \( n \) number of nodes, \( b \) bars and \( s \) strings. In a 3-dimensional structure, the simplest composition has 3 bars and 9 strings. Tensegrity units can be modelled as cylinders of radius \( r \), but not every arrangement gives a prestressable configuration; it is necessary then to obtain the range of values that allows the structure to be in equilibrium.

![Figure 1. Geometrical parameters of a tensegrity unit.](image)

A tensegrity unit can be defined as two polygons located on parallel planes whose nodes are interconnected. The bottom and top planes have \( nb \) nodes each one; the nodes on each plane are connected to their corresponding adjacent nodes by a string, as can be observed in figure 1. Another string connects a bottom node to a top node, located in a position rotated an angle \( \alpha \) along Z-axis. A compressive element connects a bottom node \( n_i \) with a corresponding top node \( n_{i+1} \), rotated an angle \((\theta+\alpha)\), where \( \theta \) is \( 2\pi/b \). It has been demonstrated by Motro [9] and Skelton and De Oliveira [10] that tensegrity simplex are stable only when \( \alpha \) is \( \pi/2-\pi/b \). In this paper we only consider the structures known as class 1 tensegrities [10], described as systems where not two or more bars coincide at any node; in other words, bars are only connected by tensile members.
It is straightforward to define the location of the nodes using polar coordinates, as a function of the polygonal angle $\theta$, the twist angle $\alpha$, the height $h$ and the radius $r$. In a generic way, the bottom nodes $n_b$ of the structure can be described as

$$n_{\text{bottom}} = \begin{bmatrix} r \cos((n_i - 1)\theta) \\ r \sin((n_i - 1)\theta) \\ 0 \end{bmatrix}$$

(1)

Using the same notation, and introducing the parameters $\alpha$ and $h$, the top nodes $n_t$ are described as

$$n_{\text{top}} = \begin{bmatrix} r \cos((n_i - 1)\theta + \alpha) \\ r \sin((n_i - 1)\theta + \alpha) \\ h \end{bmatrix}$$

(2)

3. Deployment Strategy

The deployment process consists of rising the structure from an initial almost flat configuration to the desired final position. It is assumed that both the collapsed and the deployed arrangements are symmetrical prestressable configurations; it is also assumed that all the tendons are maintained under tension during the process, and the bars don’t come in contact between each other. There are no external forces or torque acting throughout the deployment.

During the deployment process, it is desirable to control the sequence in function of the length, or height $h$, of the structure. For this reason, it is useful to rewrite the dependent parameters of equation (1) and equation (2) in terms of the independent parameter $h$. Tendon control is chosen over bar control, because the task of changing the active length of strings can be easily accomplished by motors attached at the end of the bars, while modifying the length of bars requires more complex equipment. In this way, the length $L$ of the bar is fixed as constant. A new variable is introduced, the polar angle $\varphi$, as the right angle between the $Z$-axis and any of the bars, as detailed in figure 2, and given by the relation

$$\varphi = \cos^{-1}\left(\frac{h}{L}\right)$$

(3)

Through the deployment course, the lengths of the top and bottom strings change symmetrically, resulting in a variation of the radius $r$ that describes the cylinder. The radius of the circumscribed circle is then described as a function of the polar angle $\varphi$ as

$$r = \frac{1}{2}L\sin(\varphi)$$

(4)
If the relationship of equation (4) is input into the expressions in equation (1) and equation (2), the bottom nodal coordinates become

\[
\begin{bmatrix}
\frac{L}{2} \sin(\varphi) \cos((n_i - 1)\theta)
\\
\frac{L}{2} \sin(\varphi) \sin((n_i - 1)\theta)
\\
0
\end{bmatrix}
\]  \quad (5)

and the coordinates corresponding to the top nodes are also transformed into

\[
\begin{bmatrix}
\frac{L}{2} \sin(\varphi) \cos((n_i - 1)\theta + \alpha)
\\
\frac{L}{2} \sin(\varphi) \sin((n_i - 1)\theta + \alpha)
\\
h
\end{bmatrix}
\]  \quad (6)

The polygonal angle \( \theta \) is a constant geometric parameter for a given number of bars \( b \); it has been proven\[9\] that \( \alpha \) is also a constant parameter that depends only on the number of bars, and \( L \) is a design constant representing the length of the compressive elements. Under these conditions, equation (5) and equation (6) give the nodal coordinates as a function where the only variable is the desired \( h \), given that \( b \) is defined a priori. For such a configuration, the lengths of the horizontal \( S_h \) and vertical strings \( S_v \) are given by

\[
S_h = \sqrt{L^2 \sin^2(\varphi) \sin^2\left(\frac{\theta}{2}\right)}  \quad (7)
\]
\[
S_r = \sqrt{L^2 \sin^2 \left( \frac{\alpha}{2} \right) \sin^2 (\varphi) + h^2}
\] (8)

4. Numerical Example

To validate the results, the deployment courses of a tensegrity structures is presented. Firstly, the software MATLAB is used for simulating the deployment of a single unit, including the nodal coordinates and the force densities. The analytical solution of the problem has been done for a structure with any number \( b \) of bars, so the simplest tensegrity structure, composed of 3 bars, is chosen. Next, real models of the positions calculated here are shown to verify the results.

The 3-bar single unit has been widely studied by Skelton and de Oliveira [10] and Zhang, Guest and Ohsaki [11], among several other authors. For this example, a bar of length \( L = 240 \) mm and a final structural height \( h \) of 200 mm are chosen as design parameters.

An arbitrary initial height \( h \) of 20 mm is determined, and the deployment course is divided into 4 steps: initial position, two intermediate positions and the final height. The total deployment distance is of 180 mm, divided into steps of 60 units each one. The nodal coordinates for each step are obtained using the equations derived in equation (5) and equation (6). The values of the coordinates for the initial position are shown in table 1, and the graphic of this configuration appears in figure 3(a).

Table 1. Coordinates for the configuration of \( h = 20 \) mm.

| AXIS | NODES [mm] |
|------|------------|
|      | 1 | 2 | 3 | 4 | 5 | 6 |
| x    | 119.6 | -59.8 | -59.8 | -103.0 | 0.0 | 103.6 |
| y    | 0.0 | 103.6 | -103.0 | 59.8 | -119.0 | 59.8 |
| z    | 0.0 | 0.0 | 0.0 | 20.0 | 20.0 | 20.0 |

\[ \text{Figure 3. Initial configuration of the tensegrity unit,} \]
\[ \text{(a) } h = 20 \text{ mm, (b) } h = 80 \text{ mm, (c) } h = 140 \text{ mm, (d) } h = 200 \text{ mm.} \]

For this simulation, the initial height is not considered as zero, as a collapsed tensegrity can’t have a zero-height because of the thickness of the bars. However, the condition of the minimum height is not studied in this work since it doesn’t affect the overall deployment strategy.

Table 2 shows the coordinates for the second position. The value of the \( x \) coordinate of node 1 represents the value of the radius \( r \) of the cylinder that models the structure. It is noticed how this value
is 6.5 mm smaller than the one of the initial configuration; as expected, the radius of the structure is reduced while the height increases. This effect is also noticed from the graphic in figure 3(b).

Table 2. Coordinates for the configuration of $h=80$ mm.

| AXIS | NODES [mm] |
|------|------------|
|      | 1  2   3   4  5   6  |
| x    | 113.1 -56.6 -56.6 -97.9 0.0 97.9 |
| y    | 0.0 97.9 -97.9 56.6 -113.56.6 |
| z    | 0.0 0.0 0.0 80.0 80.0 80.0 |

The coordinates for the third position are also calculated and shown in table 3. In a similar situation as the $x$ coordinate of the node 1, the value of the $y$ coordinate of the node 5 also represents the value of the radius $r$ of the structure. Comparing these values, as seen in figure 3©, allows to confirm that the deployment strategy maintains the overall cylindrical shape throughout the steps.

Table 3. Coordinates for the configuration of $h=140$ mm.

| AXIS | NODES [mm] |
|------|------------|
|      | 1  2   3   4  5   6  |
| x    | 97.5 -48.7 -48.7 -84.4 0.0 84.4 |
| y    | 0.0 84.4 -84.4 48.7 -97.5 48.7 |
| z    | 0.0 0.0 0.0 140.0 140.0 140.0 |

The final configuration, as seen in figure 3(d), has the desired height of 200 mm. From the table 4 can be confirmed that the bottom and top radius have the same value. Also, by calculating the distance between nodes 1 and 4, it is confirmed that the length of the bars is unchanged, at a length of 240 mm.

Figure 4. Initial configuration of the real tensegrity unit.
(a) $h=20$ mm, (b) $h=80$ mm, (c) $h=140$ mm, (d) $h=200$ mm.

Figure 4 shows real models built with the same parameters as the simulated and shown in figure 4. These structures, made of nylon strings and wooden bars, verify that the analytical and numerical results can be applied in real applications. The length of the strings of each structure are calculated from the coordinates in table of the corresponding configuration using equation (7) and equation (8).
Table 4. Coordinates for the configuration of \( h=200 \) mm.

| AXIS | NODES [mm] |
|------|------------|
|      | 1          | 2          | 3          | 4          | 5          | 6          |
| x    | 66.3       | -33.2      | -33.2      | -57.5      | 0.0        | 57.5       |
| y    | 0.0        | 57.5       | -57.5      | 33.2       | -66.3      | 33.2       |
| z    | 0.0        | 0.0        | 0.0        | 200.0      | 200.0      | 200.0      |

5. Conclusions
In this paper, a multi-stable deployment strategy for tensegrity structures has been presented. The nodal coordinates as a function of the height of the units were derived, based on the static equilibrium condition for tensegrity units. It was proven that, based on these two sets of information, it is possible to obtain a deployment course along which the tensegrity structure is always stable. Examples of a prismatic single unit, simulated in MATLAB and real models, were given for verification purposes.

The advantage of this deployment strategy is that it avoids the use of dynamic equations, simplifying the algorithm and reducing the computational cost. Further research should consider the deployment of multi-stage tensegrity structures.

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