Parameterization of the antiproton inclusive production cross section on nuclei

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A new parameterization of the $\bar{p}$ inclusive production cross section in proton-proton and proton-nucleus collisions is proposed. A sample of consistent $pA \rightarrow \bar{p}X$ experimental data sets measured on $1 \leq A \leq 208$ nuclei, from 12 GeV up to 400 GeV incident energy, have been used to constrain the parameters. A broader energy domain is covered for the $pp \rightarrow \bar{p}X$ reaction and with a simplified functional form used in the fits. The agreement obtained with the data is good. The results are discussed.

PACS numbers: 13.85.Ni Inclusive production with identified hadrons

I. INTRODUCTION

An accurate description of the inclusive antiproton production cross section in proton-nucleus collisions, necessarily relies on the empirical approach to the experimental data since theoretical calculations can provide at best approximate values in the current stage of the theory. The aim of the present work was to develop a handy analytical parameterization for the description of the inclusive $\bar{p}$ production cross section in proton-proton ($pp$) and proton-nucleus ($pA$) collisions on the basis of the existing body of data, updating the former works on the subject.

The motivations of the work have their origin in the needs of Cosmic Ray (CR) physics where a good knowledge and a good description of the $\bar{p}$ inclusive production cross section is a key requirement for a detailed understanding of the production and propagation of secondary galactic and atmospheric antiprotons. The $\bar{p}$ component of the CR flux is an important window for cosmology. The main contribution to this flux originates from the evaporation products of primordial black holes [2], both by the dark matter neutralino annihilation [1] or by the $\bar{p}$A → $\bar{p}X$ reaction. In addition to the secondary products, a primary component could exist, undergone for example, by the dark matter neutralino annihilation [1] or by the evaporation products of primordial black holes [2], both being of major physical and astrophysical interest. Such signatures could be obtained only if the basic processes of galactic and atmospheric $\bar{p}$ production cross section in $pp$ and $pA$ collisions are known with a good enough accuracy over a momentum range extending from around threshold up to a few hundreds of GeV where the CR flux becomes vanishingly small.

The approach used here closely follows the forms used by Kalinovskii, Mokhov and Nikitin [8] – referred to as KMN in the following – for the description of the $pA \rightarrow \bar{p}X$ cross section. The functional form used in this reference has been modified in order to reproduce a much larger sample of data sets over a much larger dynamical range and for a larger range of nuclear mass, than in the original work. This work extends a previous effort covering a more limited domain of incident momentum and of nuclear mass [4, 5].

II. INCLUSIVE CROSS SECTIONS IN HADRON COLLISIONS

In high energy hadron collisions the final state is often complex, many particles being produced in the collision. The inclusive single particle production cross section is a quantity of interest in many physics studies, for a reaction $ab \rightarrow cX$, where $c$ is the particle of interest and where $X$ represents all the other particles produced in any quantum final state allowed in the collision. The invariant inclusive single particle distribution is defined by:

$$f (ab \rightarrow cX) = \frac{d^3 \sigma}{d p^2_c} = \frac{E_c}{\pi} \frac{d^2 \sigma}{dp_\parallel d p^2_\perp} = \frac{d^2 \sigma}{\pi dyd(p^2_\perp)},$$

(1)

where $d^3 \sigma/d p^2_c$ is the triple differential cross section for detecting particle $c$ within the phase-space volume element $d^3 p_c$. $E_c$ is the total energy of $c$, while $p_\parallel$ and $p_\perp$ are the longitudinal and transverse components of $p_c$, respectively. The rapidity variable $y = 0.5 \ln((E + p_\parallel)/(E - p_\parallel))$ is often used to describe the $p_\parallel$ dependence of the cross section because of its interesting properties in Lorentz transformations [14]. To obtain two last expression in (1), an azimuthal symmetry of the differentiel cross section was used. It is also convenient to introduce the following dimensionless variables:

$$x_f = \frac{p^*_f}{p^*_\parallel_{\text{max}}} \quad \text{and} \quad x_R = \frac{E^*}{E^*_{\text{max}}},$$

(2)

where $x_f$ is the Feynman scaling variable and $x_R$ the radial scaling variable (which depends only on the radial distance from the kinematic boundary [14]), with $p^*_f$ and $p^*_\parallel_{\text{max}}$ being the longitudinal momentum of the particle

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and its maximum possible value in the center of mass (cm) frame respectively, while similarly $E^*$ and $E^*_{\text{max}}$ are the total energy of the inclusive particle and its maximum possible value in the cm frame respectively. The latter can written as $E^*_{\text{max}} = (s - M^2_{X,\text{min}} + m_p^2) / 2\sqrt{s}$, with $M^2_{X,\text{min}} = 2m_p + m/A$ being the minimum possible mass of the recoiling particle in the considered process and $\sqrt{s}$ the total cm energy. Note that for any nuclear collision, the kinematical variables used here will always be expressed in the nucleon-nucleon (NN) rather than in the nucleon-nucleus cm frame, since the NN cm frame is the relevant physical system, the incident nucleon energies being on the scale of 10 GeV while the average binding energy of the nucleon in the nucleus is about 8 MeV. Bound nucleons can be considered as free particles for the incident protons.

A parameterization of the inclusive production cross section can be guided by some general phenomenological features of hadron collisions (See [8, 9] for the general Physics context).

- All experimental hadronic production cross sections show a strong exponential decrease in transverse momentum, the exponential slope being more or less incident energy and recoil mass $M_X$ dependent.

- Hadronic scaling: The inclusive distribution $f(\alpha b \rightarrow cX)$ of particle $c$, is to a good approximation, a function only on $p_\perp$ and $x_f$ (or $x_R$) at the high energy limit $\sqrt{s} \rightarrow \infty$. Furthermore, a large number of slow particles is produced (low $x_f$ values), the distribution decreasing rapidly to zero as $x_f \rightarrow 1$, like $(1 - x_f)^n$. This form can be explained by the counting rules in parton model. These features are predicted qualitatively by the Regge poles phenomenology and the parton model.

- For a given $p_\perp$, the inclusive distribution of produced particles is (to a good approximation for $pA$ collisions) symmetric in the rapidity space with $f(p_\perp, y - y_{cm}) = f(p_\perp, y + y_{cm})$, where $y_{cm}$ is the rapidity of the cm in the laboratory frame (Lab), or in the cm frame $f(p_\perp, y^*) = f(p_\perp, -y^*)$. A forward-backward symmetry of the cross section is expected from first principles (symmetry of the wave function and of the interaction) for the NN system (In $pA$ collisions however, the absorption of low energy particles in the nuclear medium may distort the natural symmetry). In the central region where $y^* = 0$, the inclusive distribution consists of a plateau which width increases slowly with the incident energy. This plateau reduces to a simple maximum over the energy range considered here. The inclusive distribution rises again in the fragmentation region where $y^* \rightarrow \pm y^*_{\text{max}}$ for particles which can be produced diffractively but it is not the case for $\bar{p}$ and the inclusive distribution falls simply in the fragmentation region.

### III. PARAMETERIZATION OF THE $p + A \rightarrow \bar{p} + X$ CROSS SECTION

Following the approach proposed in [3], the former phenomenological features of hadron collisions have been used to constrain the parameters of a functional form describing the inclusive $\bar{p}$ production cross section, which could reproduce all the relevant experimental data available from $pp$ and $pA$ collisions. The data used are listed in Table I. The measurements on nuclear targets cover basically the whole range of nuclear mass, from proton to lead, over a range of incident energies from 12 GeV up to 400 GeV, matching the useful range for CR studies.

The KMN parameterization used previously [3] is in very poor agreement with the data listed in Table I and a reexamination of the analytical approach, better constrained by recent data was necessary. The larger incident energy domain used here required some energy dependence to be introduced in the parameterization following the general features described above as (loose) guidelines.

In this study, the $\bar{p}$ inclusive cross section will be expressed as a function of the three variables, $\sqrt{s}$, $p_\perp$ and $x_R$ (see for example [7] for the relevance of the choice of these variables):

$$\frac{E d^3\sigma}{dp^3} = f(\sqrt{s}, p_\perp, x_R).$$

(3)

The following functional form used to describe the $\bar{p}$ production cross section is an evolved version of the KMN formula:

$$\frac{E d^3\sigma}{dp^3} = \sigma_{\text{in}} A^{C_1 \ln(\sqrt{s})} p_\perp (1 - x_R)^{C_3 \ln(\sqrt{s})} e^{-C_4 x_R}$$

$$\left[ C_5 (\sqrt{s})^{C_6} e^{-C_7 p_\perp} + C_8 (\sqrt{s})^{C_9} e^{-C_{10} p_\perp^2} \right]$$

(4)

where $A$ is the target mass. The total inelastic cross section $\sigma_{\text{in}}$ for $pA$ collisions was borrowed from [10]:

$$\sigma_{\text{in}} (mb) = \sigma_0 [1 - 0.62 \exp(-E_{\text{inc}} / 200)]$$

$$\sin (10.9 E_{\text{inc}}^{0.28})$$

$$\sigma_0 (mb) = 45 A^{0.7} [1 + 0.016 \sin (5.3 - 2.63 \ln A)]$$

(5)

Where $E_{\text{inc}}$ is the incident kinetic energy in MeV.

The 10 parameters $C_1 - C_{10}$ have been fitted to the set of experimental data listed in Table I by a standard $\chi^2$ minimization procedure using the code MINUIT [11].

In relation (4), the term $(1 - x_R)^{C_3}$ originates from the hadronic scaling properties, namely, the quark counting rules of the parton model of hadronic interactions [8] (see Sect. II). It was found empirically in this study, that a significantly better result is obtain if the exponent is energy dependent. The $\ln(\sqrt{s})$ factor multiplying the $C_3$ coefficient was found to give the best result. The term $e^{-C_4 x_R}$ is induced by the Regge regime [3]. The last factor of relation (4) accounts for the transverse momentum dependence of the cross section (see Sect. III). The analysis of the experimental data (Table I) showed that the
term of angular dependence $e^{-C_{\perp}p_{\perp}^2}$ is dominant at low energy, $E_{p}^{lab} \approx 10$ GeV, while the term $e^{-C_{\perp}p_{\perp}^2}$ dominates at high energies, $E_{p}^{lab} > 100$ GeV. The $\sqrt{s}$ dependence has been introduced to allow the transition from the $p_{\perp}$ to the $p^2_{\perp}$ dependence. The target mass dependence was accounted for by the factor $A_{C_{1}, \ln}(\sqrt{s})p_{\perp}$, with an energy dependent exponent introduced for the same reason as above, at variance with the constant exponent used in the KMN parameterization. The energy dependence used (linear in $\sqrt{s}$) accounts for the experimental increase of this coefficient with the incident energy found using the KMN approach. For incident energies $E_{p}^{lab} < 55$ GeV, this coefficient becomes negative.

In Sect. 2, it was mentioned that one of the features of the inclusive distribution is its symmetry in the rapidity space. By construction our parameterization (relation 4) satisfies this symmetry property since it depends only on $\sqrt{s}$, $p_{\perp}$ and $y_{cm}$. This is illustrated in Fig.1 which shows the fit to the 23.1 GeV/c cross section data versus particle momentum $p_{lab}$ (left), and rapidity $y$ (right). In this latter case the distribution is symmetric around the value of the cm rapidity $y_{cm} = 1.9$, corresponding to $p_{0} \approx 3.3$ GeV/c in the Lab. The fit to the cross section data in the upper $p > p_{0}$ region of the momentum region also determines the values of the cross section for $p < p_{0}$ where no data is available.

![Figure 1](image1.png)

**FIG. 1:** Experimental $\bar{p}$ production cross section for $p + Pb$ collisions at 23.1 GeV/c and at 0 degree scattering angle compared with the parameterization 4, plotted against particle momentum $p_{lab}$ (left), and rapidity $y$ (right). In this latter case the distribution is symmetric around the value of the cm rapidity $y_{cm} = 1.9$, corresponding to $p_{0} \approx 3.3$ GeV/c in the Lab. The fit to the cross section data in the upper $p > p_{0}$ region of the momentum region also determines the values of the cross section for $p < p_{0}$ where no data is available.

**IV. RESULTS FOR NUCLEAR TARGETS**

The data used in the fit procedure are summarized in Table I. The fit sample included measurements from 12 GeV up to 400 GeV incident proton Lab energy on nuclear targets going from Deuterium up to Pb nuclei, and for momentum transfers up to 6.91 GeV/c. For $pp$ collisions, the incident cm energy $\sqrt{s}$ extended from about 6 up to 63 GeV.

The $\chi^2$ per point obtained with the parameterized relation 4 is 5.32 (Table III) for 654 experimental points (see list in Table I). The values of the parameters obtained in the fit are given in Table III together with error. The correlation coefficients between the parameters determined in the search are given in appendix. The results for nuclear targets ($A \geq 2$) are shown on Figs.2 to 6 where the data points are compared with the calculated values are given in appendix. The values of the $\chi^2$ per point of each set are given in the figures. In each case, some basic informations (Authors, beam energy, target nuclei, and $\chi^2$ per point obtained for the considered set) are given on the figures. In all the figures, the top distribution cor-
As it can be seen on the figures, the quality of the fits varies from fair to excellent. A poor fit is obtained however for the 24–26 GeV data from [18], the calculations underestimating the data by a factor of about 2. Nevertheless, this set has been kept in the fit procedure since its contribution is small and not hardly affects the results (which is not the case for the data listed in Table IV). To the opposite, outstandingly good fits have been obtained consistently and simultaneously for the CERN data from [15, 17] in the 20–25 GeV incident energy range, and for the high energy and large momentum transfer data from [23, 24].

Table III compares the values of the $\chi^2$ per point obtained in the present study, with that obtained using the

| system | parameterization | $\chi^2$ per point |
|--------|-----------------|--------------------|
| $pp, pA$ | KMN [3] | 80.0 |
| $pp, pA$ | this work [4] | 5.3 |
| $pp$ | Tan and Ng [31] | 28.1 |
| $pp$ | this work [2] | 3.6 |

TABLE III: Comparison between this work and the other parameterizations.
FIG. 3: Experimental data points from [15] compared to the best fit calculations.

FIG. 4: Experimental data points from [16, 18, 19] compared to the best fit calculations. The KMN relation [3] for the same data. The latter is seen to be more than one order of magnitude larger than the value obtained using (4). This gives the scale of the improvements achieved by the present study on the issue.

These results demonstrate the ability of the proposed parameterization to describe the inclusive \( \bar{p} \) production cross section on nuclei over the quoted ranges of incident energy, momentum transfer, and target mass, with a good accuracy.

FIG. 5: Experimental data points from [17] compared to the best fit calculations.

FIG. 6: Experimental data points from [20, 22, 23, 24] compared to the best fit calculations.
TABLE IV: Antiproton production cross section data not taken into account in the $\chi^2$ minimization procedure, classified by increasing energy. See text for explanations.

| Experience             | target | $\frac{p_{inc}}{E_{inc}}$ (GeV/c) | $\pi^-$ kinematical range (GeV/c) | $\theta_{lab}$ (mrad) |
|------------------------|--------|----------------------------------|---------------------------------|----------------------|
| Yu.M.Antipov et al     | Al     | 70                               | $p_{lab}$: 10–60               | 0                    |
| IHEP 1971 [27]         |        |                                  |                                 |                      |
| V.V.A.Abramov et al    | C, Al, Cu, Sn, Pb | 70                              | $p_{lab}$: 0.99–4.65          | 160                  |
| IHEP 1984 [28]         |        |                                  |                                 |                      |
| W.Bozzi et al          | Be, Al, Pb | 200                            | $p_{lab}$: 20–37              | 0                    |
| CERN 1978 [29]         |        |                                  |                                 |                      |
| I.G.Bearden et al      | Be, S, Pb | 450                            | $p_{lab}$: 4–8.5              | 37, 131              |
| CERN 1998 [30]         |        |                                  |                                 |                      |

A. Data discarded from the selection

The data listed in Table IV were not included in the fit sample, because of their obvious inconsistency with the other data. This is illustrated on Figs. 7 and 8 where they are compared to the best fit calculations obtained in the previous step on the selected sample. As it can be seen, the difference between data and calculated values amounts up to about one order of magnitude. The ratio goes from 2 to 10 for the Serpukhov experiments [27, 28]. For [27], it is about 5, and consistent with a simple normalization problem.

A larger and more surprising disagreement is found with some recent CERN data from NA44 [30], in particular for the measurements in the small rapidity bin. Note also that the parameterization describes quite well the data from [24] obtained on the same targets as [30] over a wider kinematical region (see Fig. 6).

The $\pi^-$ cross section data from [24] appearing in the table, were given in the original works in units of the corresponding $\pi^-$ production cross section measured at the same momentum. Although the absolute value could be obtained using the known $\pi^-$ cross section, the results were considered too inaccurate however, and discarded from the selected sample.

B. Analysis of the $pp \rightarrow pX$ data

This reaction is the dominant contribution to the secondary $p$ production induced by Cosmic Rays, since the interstellar gas is mainly constituted of hydrogen gas. It is thus important to obtain as accurate a description as possible for the cross section.

Considering separately the $p + p$ collision data in Table II the parameterization gives for the best fit a value of the $\chi^2$ per point of 7.08. For the same data, the well known parameterization of Tan and Ng [31] gives a value of 28.1. In addition, this latter parameterization is valid only for $p_{\perp} = 0 - 0.8$ GeV/c and is not able to reproduce the large $p_{\perp}$ data such as those from [23] and [24] where $p_{\perp} = 0.76 - 6.91$ GeV/c. Note also that Tan and Ng’s parameterization contains 8 parameters for $\sqrt{s} > 10$ GeV ($p_{lab} > 50$ GeV/c) and 17 for $\sqrt{s} < 10$ GeV.

However, in the course of the study, it appeared that some of the parameters of relation had no incidence on the resulting fits. The parameterization has then been revisited and simplified from some of its parame-
TABLE V: Values of the parameters of relation (6) obtained by fitting the experimental production cross sections list in Table 3 for proton-proton collisions and the corresponding error following the PDG standard conventions.

| parameter | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $D_6$ | $D_7$ |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| value     | 3.4610(20) | 4.340(20) | 0.007855(3) | 0.5121(27) | 3.6620(5) | 0.023070(1) | 3.2540(77) |

In comparison with relation (4), the dependence with the mass of the target has been removed since the only proton target is considered in this case. In addition, the energy dependent factors $\sqrt{s}$ in front of $D_1$ and $D_7$ in (4) have been also removed because of their ineffectiveness in the minimization procedure.

The parameters $D_1$ to $D_7$ have been adjusted by the same $\chi^2$ minimization procedure as previously [11], to the set of experimental data listed in Table I restricted to pp collisions. With formula (6), the $\chi^2$ per point obtained for the best fit is 3.59, for 228 experimental point, instead of 7.08 with relation (4). The values of the fit parameters obtained with (6) are given in Table V. The correlation coefficients between the parameters determined in the search are given in appendix. Note that the values of the coefficients $C_3$ and $D_2$, $C_5$ and $D_3$, respectively, are of the same order of magnitude. This was expected since they describe the same physics in the relations (4) and (6).

Figs. 9 and 10 show the pp → pX data analyzed, compared with the best fit results obtained for the whole pp and pA data sets from Table I using relation (4) (dashed line) and with those obtained for the pp data only using relation (6) (solid line). The simplified form (6) clearly provides a significantly better account of the measured cross sections, the $\chi^2$ value obtained being better by a factor of about 2 (about 3.6 against about 7).

The results obtained in this work have been used in the calculations of the $d$, $t$ and $\text{He}$ production from $p + p$ and $p + A$ collisions in the atmosphere and in the galaxy [32, 33, 34].

V. ANTIPROTON MEAN MULTIPLICITY

In this section, the antiproton mean multiplicity, defined as

$$\langle n_{\bar{p}} \rangle = \frac{1}{\sigma_{in}} \int \frac{d^3p}{E},$$

and depending only on $\sqrt{s}$, has been computed by means of relations (6) and (6) and compared with the experi-
mental data in $pp$ collision \cite{35}. Note that the original data from \cite{35} have been corrected from the single-diffractive contribution to the total inelastic cross section $\sigma_{\text{in}}$ \cite{36}. The corrected antiproton mean multiplicity should thus be somewhat smaller than the measured values (by $\sim 15$–20\%).

FIG. 11: Antiproton mean multiplicity distribution in the whole phase space, calculated using relation (4) (solid line), and relation (6) (dashed line), compared with experimental data \cite{35}, uncorrected (full circles) and corrected (open squares) from diffractive contribution. See text for details.

The results, shown on Fig. 11 are in good agreement with the experimental data. Note that below $\sqrt{s} \approx 15$ GeV/$c$, the results given by the relation (4) and (6) become significantly different (about a factor 2 at the maximum). As expected, the simplified form (6) gives a little bit better results since the experimental data are from $pp$ collisions.

VI. CONCLUSION

The parameterization of the $\bar{p}$ inclusive production cross section on nuclei has been revisited by investigating a broad collection of data sets available, covering a large dynamical domain of incident energy and of momentum transfer, for a broad range of nuclear masses. Good results have been obtained but for a small sample of data sets inconsistent with the other data. The experimental $\bar{p}$ inclusive production cross sections can be reproduced to within a few tens of percent over this range, i.e., for incident energies from 12 GeV up to 400 GeV, and for target mass $1 \leq A \leq 208$. These results constitute a significant improvement with respect to the former KMN parameterization, decreasing by a factor of about 15 the value of the $\chi^2$ per point obtained using the latter. A simplified version of the functional form has been developed for $pp$ collisions giving also good results up to very high energies, much beyond the range dictated by the Cosmic Ray Physics requirements which motivated the study. This also constitutes an improvement, consistent with the data on nuclei, of the Tan and Ng formula used so far as a standard in the calculations.

The parameterisations (4) and (6) are also able to reproduce the experimental antiproton mean multiplicity measured in $pp$ collision with a good accuracy.

A point to be emphasized is that because of the symmetry of the cross section in the rapidity space, the fitted range in the Lab momentum of the particle, usually measured above the cm rapidity, also determines the cross section at low momenta, a range of major importance for Cosmic Ray antiprotons where accuracy is extremely important.

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VII. APPENDIX

The symmetrical matrix (8) and (9) give respectively the correlation coefficients for the parameters $C_1-C_{10}$ and $D_1-D_7$ of equations (4) and (6) respectively.

In relation (4), the coefficients $C_1$ and $C_2$ appear to be strongly correlated (correlation coefficient 0.961), as it could be expected from their functional dependence. On the contrary, coefficient $C_5$ and $C_6$ are not correlated (correlation coefficient 0.232), since effective in different energy ranges (see Sect. III). The same remarks apply to the coefficients $D_1-D_7$. 
\[
\begin{pmatrix}
1.000 & 0.961 & 0.120 & -0.200 & -0.148 & 0.128 & 0.086 & -0.067 & -0.048 & -0.165 \\
1.000 & 0.131 & -0.209 & -0.199 & 0.157 & -0.098 & -0.067 & -0.060 & -0.148 \\
1.000 & -0.937 & -0.321 & 0.228 & -0.049 & -0.655 & -0.620 & -0.289 \\
1.000 & 0.282 & -0.180 & -0.042 & 0.834 & 0.784 & 0.311 \\
1.000 & -0.962 & 0.358 & -0.110 & -0.128 & 0.239 \\
1.000 & -0.164 & 0.232 & 0.286 & -0.202 \\
1.000 & -0.127 & 0.007 & -0.028 \\
1.000 & 0.979 & 0.210 \\
1.000 & 0.148 \\
1.000 &
\end{pmatrix}
\]

Correlation coefficients for the parameters \( D_1 - D_{10} \) given in Tables \ref{tab1}.

\[
\begin{pmatrix}
1.000 & -0.933 & -0.557 & 0.604 & 0.080 & -0.311 & 0.087 \\
1.000 & 0.654 & -0.730 & -0.141 & 0.435 & -0.135 \\
1.000 & -0.979 & 0.502 & -0.212 & -0.241 \\
1.000 & -0.336 & 0.033 & 0.228 \\
1.000 & -0.833 & -0.213 \\
1.000 & 0.282 \\
1.000 &
\end{pmatrix}
\]

Correlation coefficients for the parameters \( C_1 - C_7 \) given in Tables \ref{tab2}.

\[\text{(8)}\]

\[\text{(9)}\]

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