Friendship Paradox Redux: Your Friends Are More Interesting Than You

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Abstract
Feld’s friendship paradox states that “your friends have more friends than you, on average.” This paradox arises because extremely popular people, despite being rare, are overrepresented when averaging over friends. Using a sample of the Twitter firehose, we confirm that the friendship paradox holds for >98% of Twitter users. Because of the directed nature of the follower graph on Twitter, we are further able to confirm more detailed forms of the friendship paradox: everyone you follow or who follows you has more friends and followers than you. This is likely caused by a correlation we demonstrate between Twitter activity, number of friends, and number of followers. In addition, we discover two new paradoxes: the vitality paradox that states “your friends receive more viral content than you, on average,” and the activity paradox, which states “your friends are more active than you, on average.” The latter paradox is important in regulating online communication. It may result in users having difficulty maintaining optimal incoming information rates, because following additional users causes the volume of incoming tweets to increase super-linearly. While users may compensate for increased information flow by increasing their own activity, users become information overloaded when they receive more information than they are able or willing to process. We compare the average size of cascades that are sent and received by overloaded and underloaded users. And we show that overloaded users post and receive larger cascades and they are poor detector of small cascades.

Introduction
The so-called “Friendship Paradox” or Feld’s Paradox, states that, on average, your friends have more friends than you do. This is due to the overrepresentation of extremely popular individuals in the average of friends (Feld 1991). The paradox has been empirically demonstrated both online, such as Facebook (Ugander et al. 2011), and offline (Feld 1991; Zuckerman and Jost 2001) social networks. Because people use their local network to assess themselves and as sources of information about the greater world (Zuckerman and Jost 2001; Sgourev 2006; Wolfson 2000; Yogarasimhan 2012; Kanai et al. 2012a), the friendship paradox leads to systematic biases in our perceptions. For example, a majority of people believe they possess above average driving skill (McKenna, Stanier, and Lewis 1991; Groeger and Brown 1989). Furthermore, many personal characteristics correlate with high network degree, such as the incidence of drug and alcohol use (Tucker et al. 2011; 2012), wealth (Morselli and Tremblay 2004; Amuedo-Dorantes and Mundra 2007; Van Groenou and Van Tilburg 2003), and extraversion (Pollet, Roberts, and Dunbar 2011; Quercia et al. 2012), which may further effect our perceptions. Interestingly, your friends’ superior social connectivity puts them at a greater risk, in aggregate, of an infection by a biological pathogen. This fact has been used as a principle for establishing epidemiological early-warning networks, because your friends will be more heavily exposed to pathogens in aggregate (Christakis and Fowler 2010). Managing one’s social network requires cognitive effort, which has been linked directly to physiological attributes within the brain (Dunbar 1993; Powell et al. 2012; Bickart et al. 2012; Kanai et al. 2012b). However, the effect of the friendship paradox on our cognitive limitations is not well examined.

In online social networks, the friendship paradox has a surprising twist. If we wish to receive more information, we can usually choose to incorporate more individuals into our online social networks, e.g., by following them on Twitter. However, as we grow our social network, we dramatically increase the volume of incoming information, since, as we show in this paper, not only are your friends better connected than you, they also tend to be more active, producing more information on average than you are willing to consume. Thus, increase in information flow collides with our innate cognitive limitations and does not increase our ability to appreciate the totality of our relationships. By increasing the incoming flow of information, we dilute our attention and reduce the visibility of any individual tweet (Hodas and Lerman 2012). Receiving too much information may exceed our ability and desire to maintain existing social connections, even if they are unreciprocated (Kwak, Chun, and Moon 2011). Thus, users will naturally attempt to regulate the amount of incoming information by tuning the number of users they follow.

In the present work, we consider the evidence for and the consequences of the friendship paradox on Twitter, which, as a directional network, presents an opportunity to study the paradox in more detail. In the first part, we demonstrate the present evidence that the friendship paradox holds, as
expected, on Twitter. We expand this analysis to other properties of the friendship network, presenting a full reciprocity friendship paradox: your friends (followees) and followers have more friends and followers than you do. We then document new behavioral paradoxes. The friend activity paradox states that your friends tend to be more active than you are. Thus, the behavioral traits that lead one to be well connected will also have direct influence on information overload. Your friends also send and receive content that has higher virality than you do, what we call the virality paradox. These facts together suggest the glib expression “your friends are more interesting than you are”. In the second part, we explore how the relative information load caused by the activity paradox alters user behavior, comparing underloaded users with overloaded users. We show that, compared to underloaded users, overloaded users both post and receive more viral URLs and are less sensitive to smaller outbreaks of less popular URLs.

A Variety of Paradoxes on Twitter

The friendship paradox, as formulated by Feld, is applicable to offline relationships, which are undirected, and it has also been observed in the undirected social network of Facebook (Ugander et al. 2011). We demonstrate empirically that the friend paradox also exists on Twitter. Unlike the friendship relations of the offline world and Facebook, the relations on Twitter are directed. When user a follows the activity of user b, he or she can see the posts tweeted by b but not vice versa. We refer to user a as the follower of b, and b as a friend or followee of a. Note that here friendship is a directed relationship.

Figure 1 illustrates a directed social network of a social media site, such as Twitter. The user receives information from friends and, in turn, posts information to her or his followers. The friends may themselves receive broadcasts from their friends, whom we call friends-of-friends and post tweets to their own followers, whom we call followers-of-friends.

Data

We use the Twitter dataset presented by (Yang and Leskovec 2011), which contains 476 million tweets that are 20-30% of all tweets posted from June to December 2009. We also used the Twitter social network gathered by Kwak et al. (2010), which includes links between all users who joined Twitter before August 2009. Since we need both tweets and social links, we only consider users who have posted at least one tweet. The subgraph of such users includes 5.8M users and 193.9M links between them. This graph is used for showing the friendship paradox on Twitter.

Friendship Paradox

The friendship paradox can be stated in four different ways on a directed graph:

i) On average, your friends (followees) have more friends than you do.

ii) On average, your followers have more friends than you do.

iii) On average, your friends have more followers than you do.

iv) On average, your followers have more followers than you do.

We empirically validate each statement above. The first statement says that, on average, a user’s friends are better connected than he or she is, i.e., they follow more people than he or she does. To validate this statement, for each user in the dataset we count how many friends she has, i.e., how many other users she follows. Then, for each friend, we count how many other users the friend follows, and average over all friends. Top Figure 2(i) plots the average number of friends-of-followers (ordinate axis) vs the number of friends (abscissa) a user follows for the users with fewer than 1000 friends. About 99.7% of users had fewer than 1000 friends. The line of unit slope shows equality of connectedness. The probability density function (PDF) of the ratio of the average friend’s connectivity to a user’s connectivity, shown in bottom Figure 2(i), is > 1 for 98% of the users, peaking around 10. In other words, in the Twitter follower graph, a typical friend of a user is ten times better connected than the user.

Not only are a user’s friends better connected, but so are the user’s followers. Top Figure 2(ii) plots the average number of friends a user’s followers have vs the number of friends the user has for users with fewer than 1000 followers (99.6% of all users). Bottom Figure 2(ii) shows the PDF of the ratio of the friends-of-followers to user’s friends. Again, for 98% of users, this ratio is above one, indicating that the average follower is better connected than the user. In fact, a typical follower is almost 20 times better connected than the
user is.

The last two variants of the friendship paradox deal with user’s popularity, i.e., the number of followers he or she has. It appears that on Twitter, user’s both friends and followers are more popular than the user himself or herself. This is shown in Figures 2(iii) and 2(iv). In our data set, 99% and 98% of users were respectively less popular than their friends and followers. While a typical follower is about 10 times more popular than the user (Fig. 2(iv) bottom), the ratio of the friend’s average popularity to the user’s popularity shows a bimodal distribution (Fig. 2(iii) bottom). While some of a user’s friends are ten times more popular, some friends are about 10,000 times more popular, showing a tendency of Twitter users to follow highly popular celebrities.

**Friend Activity Paradox**

In addition to connectivity and popularity paradoxes, we also demonstrate a novel activity paradox on Twitter.

*Friend activity paradox:* On average, your friends are more active than you are.

To empirically validate this paradox, we measure user activity, i.e., the number of tweets posted by a user during a given time period; we exclude users who joined Twitter after the start of the time period. After windowing by a two-months time period we are left with 37M tweets from 3.4M users and 144.5M links among these users. Note that the dataset contains a random sample of all tweets; therefore, the number of tweets posted by the user in our sample is an unbiased measure of his or her overall activity. At the same time, we measure the number of sampled tweets posted by user’s friends during the same time interval. Figure 3(a) shows the average activity (number of posted tweets) per friend of users who each have same level of activity, i.e., mean average friend activity as a function of user activity. The unit slope $y = x$ line is shown for comparison. 88% of all users are less active than their typical friend. Figure 3(b) shows the probability distribution of the ratio of average per friend activity over user activity. For the vast majority of users, the friend activity paradox holds: their friends are more active than they are.

It is known that some users become inactive after some time. To ensure that our results are not affected by inactive users, we checked the same paradox for a shorter time period of one week, during which time fewer users may have become inactive. Activity paradox still holds. In fact, a much larger fraction of users are in the paradox regime: 99% of users are less active than their friends. Also, note that in all the analyses that we are comparing users with their friends (followers) we exclude users who don’t have any friends (followers), because there is no one for the comparison.

**Virality Paradox**

Your friends’ superior social connectivity puts them in a better position to monitor, in aggregate, the flow of information, thereby mediating the information you receive via the social network. Perhaps this also puts them in a position to receive higher quality content. As a measure of quality, we investigate virality of URLs tweeted by users, i.e., number of times a URL was posted by any user over some time period.

![Figure 2: Variants of the friend paradox on Twitter showing that your (i) friends and (ii) followers are better connected than you are (i.e., have more friends on average) and (iii, iv) are more popular than you are (i.e., have more followers on average). Top row shows the average connectivity (popularity) of user’s network neighbors vs user’s connectivity (popularity). Data residing above the dashed $y = x$ line indicates “paradox” conditions. Bottom row shows the probability distribution of the ratio of the average neighbor’s connectivity (or popularity) to user’s connectivity (or popularity). Although some users are systematically not in paradox, indicated in the top row, they are a tiny fraction of total users.](image)
Virality paradox: On average, your friends spread more highly viral content than you do.

To confirm this paradox, we calculate average size of posted URL cascades for each user and compare this value with the average size of posted cascades of friends. We observe that 32% of users haven’t posted any URLs (average cascade size of 0), while their friends did. Therefore, these inactive users have posted fewer viral cascades than their friends. For the remaining 68% of users, Figure 4(a) shows the probability distribution of the ratio of average size of cascades posted by friends to the average size of cascades posted by user. We find that 79% of users have ratio of greater than 1, which means that their friends have posted more viral content. Considering the users who haven’t post any URLs, 86% of all users have posted less viral content than their friends.

Users not only post less popular URLs than their friends, but also receive less viral content than their friends do, on average. Figure 4(b) shows the probability distribution of the ratio of the average size of cascades friends receive to the average size of cascades received by the user. Here again 76% of users receive smaller (less viral) cascades than their friends (15% of users have received URLs with same level of virality as their friends).

Spam Filtering

One trivial explanation of our results could be the presence of spammers in our sample. Spammers generate more tweets than normal users, so their presence in our sample could bias our estimates of user activity. To validate that the paradoxes don’t exist because of the spammers, we eliminated spammers from the dataset in two different ways. First, we use the set of spammers from (Ghosh et al. 2012). These users’ profile was suspended by Twitter authorities and also the users posted at least one blacklisted URL. Second, we took the approach of (Ghosh, Surachawala, and Lerman 2011) and classified users as spammers based on entropy of content generated...
and entropy of time intervals between tweets (spammers tend to have low entropy of content and tweeting time intervals).

In both cases, after removing all spammers from the network and excluding their tweets, all paradoxes still hold. In fact, in some cases the paradox becomes even stronger. For example, if we exclude users based on their content and activity entropy, 93% of users would be less active than their friends (instead 88% before spam filtering).

**Friend Paradox and Information Overload**

The friend activity paradox in directed social networks of online social media is not a mere statistical curiosity — it has surprising implications for how social media users process information. As social media users become more active on the site, they may want to grow their social networks to receive more novel information. Clearly, adding more friends will increase the amount of information a user has to process. However, according to the friend activity paradox, an average new friend is more active than the user is herself; therefore, the volume of new information in a user’s stream will grow super-linearly as new connections are added. Sometimes the volume of new information will exceed user’s ability to process it, pushing the user into information overload regime. Overloaded users are less sensitive detectors of information.

**User Activity and Incoming Information Volume**

We study how the volume of incoming information, measured by the number of tweets received by a user, grows with the size of a user’s social network. Figure 5(a) shows the average number of tweets received by users who follow a given number of friends. The data is shown for users with up to 2000 friends, and has surprisingly low dispersion. This data is best fit by a power-law function with exponent 1.14 ($R^2 = 0.9865$). The best linear fit has slope of 71 ($R^2 = 0.8915$), while the best quadratic fit has slope of 60 ($R^2 = 0.8930$). The lines in Figure 5(a) show the best power-law and linear fits, where the linear fit was shifted
Information Overload

In the section above, we showed that the volume of incoming information in a user’s stream quickly increases with the number of friends the user follows. While the user may attempt to compensate for this growth by increasing her own activity, this may not be enough. As a result, the user may receive more posts than she can read or otherwise process. We include them, because their activity impacts the information load of people who choose to follow them.

Finally, we look at the correlation between user activity and the number of friends and followers. Figure 6 shows user activity, measured by the number of tweets posted during the time interval, as a function of the number of followers and friends the user has. There is a significant correlation between user’s activity, connectivity, and popularity (p-value < 0.01). The correlation between user activity and the number of followers appears especially strong. This correlation could, in fact, explain the friend activity paradox, because highly active users contribute to the average friend activity of their many followers, causing overrepresentation when averaging over friend’s activity. The detailed mechanism for this correlation is not yet clear. It is conceivable that as the user becomes more active, she begins to follow more and more people. Being active leads her to acquire new followers as her posts become visible to others, for example, by being retweeted. This will lead to a correlation between the number of friends and followers that goes beyond simple reciprocation of links. We leave these questions for future research.

Related Work

The friendship paradox describes the phenomenon that most people have fewer friends than their friends have (Feld

| Category | Underloaded | Overloaded |
|----------|-------------|------------|
| Group 1  | 12.56       | 104.96     |
| Group 2  | 40.78       | 132.94     |
| Group 3  | 119.75      | 160.99     |
| Group 4  | 145.44      | 202.86     |

Table 1: Median of average size of received cascades for under- and overloaded users. Overloaded users have larger median across all four groups, sending, respectively, 1) <5 tweets, 2) 5–19, 3) 20–59, and 4) >60 tweets.
The high correlation between activity and degree suggests that most friends are discovered via Twitter, on average. This fact will cause users who have more friends to receive even more tweets per friend, leading to a super-linear growth in incoming information. Receiving a surplus of tweets reduces the visibility of each tweet and also it divides users’ attention across different topics. Hodas and Lerman show that visibility and divided attention play a considerable role in social contagion (Hodas and Lerman 2012).

The present work demonstrates that a clear model of how users discover friends and manage existing friendships is essential for mitigating any undesirable consequences of the high correlation between activity and connectivity. For example, among children, this can result in “popular” kids having undue influence on others regarding the perception of peer alcohol and drug abuse (Tucker et al. 2011; Wolfson 2000). Furthermore, better understanding the activity paradox can help online social networks identify and recommend interesting users to follow that will account for any undesired information overload.

**Conclusion**

The present work has demonstrated that the friendship paradox exists on Twitter for over 98% of users, although this is not surprising, given the underlying mathematical foundation developed by Feld (Feld 1991). However, we have demonstrated a new paradox, the activity paradox, whereby your friends are more active than you are. They also receive more viral content than you, on average, and send out more viral content than you. We have shown a large correlation between activity and both in- and out-degree in the follower graph on Twitter. Hence, we propose that the activity paradox is not a fluke particular to Twitter; it results from active users generating more visibility for themselves, leading to more followers. Active users are also more interested in consuming content, on average, causing them to follow more users as they grow more active. For the puta-

Figure 7: Comparison of size of posted and received cascades of overloaded and underloaded users, grouped by their activity. Group (i) consists of users who posted fewer than 5 tweets, (ii) users who posted 5–19 tweets, (iii) users who posted 19–59 tweets, and (iv) users who posted more than 60 tweets during two months time period.
tive user choosing whom to follow, it is not surprising that active users are more likely to appear in the feed via retweets of others. Hence, the key relationship can be hypothesized to be that activity causes connectivity, leading to the more detailed friendship paradox we report: your friends and followers have more friends and followers than you do.

If you have ever felt like your friends are more interesting or more active than you are, it seems the statistics confirm this to be true for the vast majority of us. The consequence, beyond the psychological implication of comparing oneself to one’s friends, is that we will receive more incoming information than we prefer, i.e., information overload. We make contacts with people who are easiest to discover—who are the most active—but we have a finite budget for communication. The present work shows that the resulting super-linear increase in information arising from following additional users could be a significant cognitive load (Sweller, Merrienboer, and Paas 1998). Those users who become overloaded, measured by receiving far more incoming messages than they send out, are contending with more tweets than they can handle. Controlling for activity, they are more likely to participate in viral cascades, likely due to receiving the popular cascades multiple times. Any individual tweet’s visibility is greatly diluted for overloaded users, because overloaded users receive so many more tweets than they can handle. Because of the connection between cognitive load and managing information overload, the present results suggest that users will dynamically adjust their social network to maintain some optimal individual level of information flux. Future work will elucidate how the activity paradox can be used to model the dynamics of growing and shrinking our social networks over time.

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