Electron excitation of the 4s4p $^3$P metastable state of calcium

Lalita Sharma$^1$, Rajesh Srivastava$^1$ and A.D. Stauffer$^2$

$^1$Department of Physics, Indian Institute of Technology-Roorkee, Roorkee 247667, India
$^2$Department of Physics and Astronomy, York University, Toronto, Canada M3J 1P3

Abstract. We have carried out calculations for the excitation by electron impact of the 4s4p $^3$P$_{0,2}$ metastable states of calcium to the fine structure levels of six higher lying triplet states. These results have been obtained using the Relativistic Distorted-Wave approximation for electron impact energies up to 100 eV. We report results for the integrated cross sections for these fine structure transitions and have also analyzed their high energy behavior.

1. Introduction

Metastable atomic states are characterized by long lifetimes and large cross sections for inelastic electron scattering. These properties make them amenable to experimental study and there have been a number of reports of measurements on such systems. We have previously applied the relativistic distorted-wave (RDW) method to inelastic electron scattering from metastable states of barium (Srivastava and Stauffer, 2005) and the noble gases (Srivastava et al, 2006). In the present work we extend our study and applied the same RDW method to the electron excitation of the 4s4p $^3$P$_{0,2}$ metastable states of calcium. Initially we consider the excitation to the fine structure levels of the first six higher lying triplet states viz. 4s5s $^3$S, 4s4d $^3$D, 3d4p $^3$D, 4p$^2$ $^3$P, 3d4p $^3$P, 4s5f $^3$F which have appreciable cross sections. The only previous study for such excitations of the metastable states to higher states of calcium was by Shafranyosh et al (1997). In the limited low energy region up to 30eV they reported measurements of the integrated cross sections for the excitation from the fine-structure unresolved 4s4p $^3$P metastable states to above said higher fine-structure unresolved states and compared these with their simple two- and six-state close coupling calculations.

In this paper we report our RDW integrated cross section results for all the transitions in a wide range of energies to 100eV and study their high energy behavior by fitting our results to known analytical formulae. In order to assess the accuracy of the wave functions used in these calculations, we also present energy differences and oscillator strengths for the fine-structure transitions studied. After averaging our results over the initial fine-structure levels and summing over the final fine-structure levels, we are also able to compare our RDW calculations with the theoretical and experimental results reported by Shafranyosh et al (1997). The RDW method is particularly suited to the calculation of cross sections for electron-induced transitions between fine-structure levels of atoms since it uses different wave functions for each of the fine-structure levels. The accuracy of the calculated oscillator strengths and energy differences for the individual fine-structure transitions along with the established reliability of our previous calculations are the primary indicators of the accuracy of our cross section results. In general, distorted-wave methods are applicable for medium- and high-energy collisions. We found from our previous work that our results were reliable for energies that are a few times the threshold energy for the transition in question. Since the threshold energy for the excitation of the metastable levels is very small, our present results should be accurate at much lower energies than for excitation of the ground states of atoms as was shown in Srivastava and Stauffer (2005) and Srivastava et al (2006).
Shafranyosh et al (1997) proved that measurements of the excitation of the metastable states of calcium are possible and thus we are presenting these calculations for individual fine-structure transitions to encourage more detailed experimental work in this area. We also give analytic fits to our cross sections to enable these to be easily incorporated in modeling studies of stellar atmospheres and the interstellar medium where calcium is an important constituent [see e.g. McWilliam (1997), Milisavljevic et al (2004) and Pottasch (1972)].

2. Theory

The RDW method calculates the distorted-wave $T$-matrix for the electron impact excitation of an atom having $N$ electrons from an initial state $i$ to a final state $f$. This can be written as (Joachain 1983) (atomic units are used throughout)

$$ T_{i \rightarrow f}^{DW} = \langle \chi_f (1, 2, \ldots, N+1) | V - U_f (N+1) | A \chi_i (1, 2, \ldots, N+1) \rangle $$  \hspace{1cm} (1)

where $V$ is the target-projectile interaction and $U_f$ is the distortion potential which is taken to be a function of the radial co-ordinates of the projectile electron only. $U_f$ is chosen to be a spherically averaged static potential of the final state of the atom. This choice of $U_f$ has been shown to yield the most consistent results (Zuo 1991).

The wave functions $\chi_{ch}^{\pm (-)}$, where 'ch' refers to the two channels, i.e. initial 'i' and final 'f', are represented as a product of the $N$-electron target wave functions $\phi_{ch}$ and a projectile electron distorted wave function $F_{i/f}^{DW, \pm (-)}$, i.e.

$$ \chi_{ch}^{\pm (-)} (1, 2, \ldots, N+1) = \phi_{ch} (1, 2, \ldots, N) F_{ch}^{DW, \pm (-)} (k_{ch}, N+1) $$  \hspace{1cm} (2)

Here ‘+’ refers to an outgoing wave while ‘-’ denotes an incoming wave. $A$ is the antisymmetrization operator that takes into account the exchange of the projectile electron with the target electrons and $k_{ch}$ are the linear momenta of the projectile electron in the initial and final state. The distorted waves $F$ are solutions of the Dirac equations including the distortion potential $U_f$ and depend on the spin of the projectile electron. More details are given in Chauhan et al (2005) where we studied electron impact excitation of the ground state of calcium.

Since our calculations are carried out in the relativistic $j-j$ coupling scheme the wave functions $\phi_{ch}$ have a definite total angular momentum and the spin of the projectile electron is specified. Thus we can write the $T$-matrix in the alternate form

$$ T_{i \rightarrow f}^{DW} = \langle J_f, M_f, \mu_f | V - U_f | J_i, M_i, \mu_i \rangle $$  \hspace{1cm} (3)

where $J$ and $M$ represent the total angular momentum of the atomic state and $\mu$ is the spin projection of the free electron. With our normalization of the distorted waves, the differential cross section (DCS) for the excitation of the atom from a fine-structure metastable level with angular momentum $J_i$ to a higher lying level $J_f$ is given by

$$ \text{DCS} = (2\pi)^4 \frac{k_f}{2(2J_i + 1)k_f} \sum_{M_f, \mu_f} | \langle J_f, M_f, \mu_f | V - U_f | J_i, M_i, \mu_i \rangle |^2 $$  \hspace{1cm} (4)

We integrate the DCS over the scattering angles of $k_f$ to obtain the integrated cross sections (ICS).
3. Wave functions
In the $j$-$j$ coupling scheme in which we represent our wave functions, the calcium atom with $N = 20$ consists of a closed-shell core plus two valence electrons. The core configuration is represented by $1s^22s^22p^63s^23p^6$ where $\overline{p}$ and $p$ indicate p-electrons with total angular momenta $j$ of $1/2$ and $3/2$, respectively. The lowest-lying metastable state of this atom is designated as $4s4p \ ^3P^o$ in the non-relativistic $LS$ coupling scheme. This is a triplet of three fine-structure levels but only the levels with total angular momentum $J = 0$ or $2$ are metastable.

| Initial level $4s4p \ ^3P^o$ | Final level | $J_i$-$J_f$ | Excitation Energy (eV) | Oscillator Strength |
|-----------------------------|-------------|-------------|------------------------|---------------------|
|                             |             |             | GRASP                 | NIST               |
| 4s5s $^1S$                  | 0-1         | 2.00456     | 2.03106               | 0.163              |
|                             | 2-1         | 1.99238     | 2.01146               | 0.164              |
| 4s4d $^3D$                  | 0-1         | 2.89659     | 2.80084               | 0.469              |
|                             | 0-2         | 2.89670     | 2.80129               | 0.439              |
|                             | 0-3         | 2.89688     | 2.80198               |                    |
|                             | 2-1         | 2.89354     | 2.78124               | 4.65E-3            |
|                             | 2-2         | 2.89365     | 2.78170               | 6.84E-2            |
|                             | 2-3         | 2.89383     | 2.78239               | 0.373              |
| 4p $^3P$                    | 0-0         | 3.32338     | 2.88383               |                    |
|                             | 0-1         | 3.41051     | 2.88969               | 0.506              |
|                             | 0-2         | 3.52522     | 2.90044               |                    |
|                             | 2-0         | 3.31672     | 2.86423               |                    |
|                             | 2-1         | 3.40385     | 2.87009               | 0.134              |
|                             | 2-2         | 3.51856     | 2.88085               | 0.349              |
| 3d4p $^3P$                  | 0-0         | 3.22936     | 2.99737               |                    |
|                             | 0-1         | 3.23211     | 2.99762               |                    |
|                             | 0-2         | 3.23959     | 2.99821               |                    |
|                             | 2-0         | 3.11196     | 2.97778               |                    |
|                             | 2-1         | 3.11471     | 2.97802               |                    |
|                             | 2-2         | 3.12219     | 2.97862               |                    |
| 3d4p $^3D$                  | 0-1         | 3.07027     | 2.85592               |                    |
|                             | 0-2         | 3.06857     | 2.85923               |                    |
|                             | 0-3         | 3.06477     | 2.86418               |                    |
|                             | 2-1         | 2.95287     | 2.83632               |                    |
|                             | 2-2         | 2.95117     | 2.83963               |                    |
|                             | 2-3         | 2.94737     | 2.84459               |                    |
| 4s5f $^3F$                  | 0-2         | 3.40745     | 3.67052               |                    |
|                             | 0-3         | 3.40737     | 3.67055               |                    |
|                             | 0-4         | 3.40696     | 3.67058               |                    |
|                             | 2-2         | 3.32398     | 3.65093               |                    |
|                             | 2-3         | 3.32390     | 3.65095               |                    |
|                             | 2-4         | 3.32349     | 3.65099               |                    |
We have calculated cross sections for excitation to the following six $LS$ terms: $4s5s \ ^3S_1$, $4s4d \ ^3D_{1,2,3}$, $3d4p \ ^3D_{1,2,3}$, $4p^2 \ ^3P_{0,1,2}$, $3d4p \ ^3P_{0,1,2}$, $4s5f \ ^3F_{2,3,4}$. The wave functions for the initial and final states are calculated within a multiconfiguration Dirac-Fock approximation using the GRASP92 program of Parpia et al (1996). For each transition we did a separate calculation to optimize the initial and final states on the basis of the transition energy and oscillator strength including the agreement between the Babushkin and Coulomb forms (equivalent to the length and velocity forms in the non-relativistic case).

The multiconfiguration wave functions for the initial $4s4p \ ^3P$, $J=0$ and 2 levels had contributions from (using non-relativistic notation) the $4s4p$, $3d4p$, $4p^2$ and $4s5s$ configurations.

The final state wave functions included the following configurations:

- $4s5s \ ^3S$: $4s5s, 4s6s, 4p^2$
- $4s4d \ ^3D$: $4s4d, 3d4s, 4p^2$
- $4p^2 \ ^3P$: $4p^2, 5p^2, 3d^2, 4s5s$
- $3d4p \ ^3P, \ ^3D$: $3d4p, 4s4p$
- $4s5f \ ^3F$: $4s5f, 4s4f, 4s4p, 3d4p$

A transition between two fine-structure levels where the change in the total angular momentum is $\Delta J$ is a dipole allowed transition only if $|\Delta J| \leq 1$ and the initial and final levels do not both have $J = 0$. From the spectroscopic designation of these terms, the first three transitions include fine-structure transitions that are clearly dipole-allowed, while the last three are odd-odd transitions and therefore all transitions are dipole-forbidden.

In table 1 we show the energy differences for the transitions studied as well as the dipole oscillator strengths for the relevant transitions. These are compared with the corresponding experimental values from the NIST database. For the majority of the transitions, the calculated energy differences agree well with the NIST values. The oscillator strengths are in close agreement with the NIST values, much closer than the accuracy estimates in the tables.

4. Results

The $T$-matrix given in equation (1) has contributions both from direct excitation as well as excitations involving the exchange of the incident electrons with one of atomic electrons. The exchange contribution is always present for the transitions studied here but there are conditions that have to be met in order that there is a non-zero direct contribution.

**Parity:** the parity of the atomic states of the calcium target is equal to $(-1)^{l+l'}$ where $l$ and $l'$ are the angular momenta of the two valence electrons. The electrostatic interaction potential $V$ can be broken down into its various multipole moments. Thus for a multipole moment of order $\lambda$ to give a non-zero contribution to the direct part of the $T$-matrix, the condition that $(-1)^{\lambda}$ equal the product of the parities of the initial and final atomic states must be met. Transitions with contributions from the dipole ($\lambda = 1$) term are known as allowed transitions whereas for all other multipoles they are referred to as forbidden.

**Angular momentum coupling:** the quantum numbers $J_i$, $J_f$ and $\lambda$ must satisfy the triangle inequality.

In addition, the independent electron Dirac-Fock wave functions used in our RDW method lead to non-zero contributions to the direct terms only for transitions involving the excitation of a single valence electron.

In the following we present details of the integrated cross sections (ICS) for all the transitions studied. As was the case for the excitation of the metastable states of the noble gases (Srivastava et al, 2006) the Bethe-Born approximation.
\[ ICS = \pi a_0^2 \frac{f_{osc}}{E \Delta E} \ln(E + b) \]  \hspace{1cm} (5)

provides a good description of the behaviour of the cross section for dipole-allowed transitions even at quite low energies (20 - 30 eV). Here \( a_0 \) is the Bohr radius, \( f_{osc} \) is the dipole oscillator strength for the transition, \( \Delta E \) is the energy of the transition and \( E \) is the energy of the incident electron in atomic units. Since the transition energies are nearly the same for all transitions between given \( LS \) terms, the magnitude of the fine-structure cross sections are determined primarily by the optical oscillator strength for allowed transitions.

The cross sections for the forbidden transitions (including exchange transitions) behave as
\[ ICS = \text{d}_f E^{\lambda \text{d}} a_0^2 \]  \hspace{1cm} (6)
at higher energies. The leading contribution to the direct term for forbidden transitions have \( \text{d}_f \) whereas \( \text{d}_f \) for the exchange terms has a negative value with greater magnitude.

4.1 The odd-even transitions

The initial 4s4p state has odd parity with \( J_i = 0 \) or 2 while the 4s5s \( ^3S_1 \), 4s4d \( ^3D_{1,2,3} \), 4p \( ^3P_{0,1,2} \) states all have even parity. Thus there are multipole contributions to the direct term only when \( \lambda \) is odd. This includes the dipole-allowed case \( \lambda = 1 \). Multipole contributions with \( \lambda \) even can only contribute via the exchange terms. In the following we will use the notation \((J_i - J_f)\) to refer to a particular fine-structure transition between states.

The two fine-structure transitions (0-1) and (2-1) to the 4s5s state are both dipole allowed and will be discussed in more detail later. For excitation of the 4s4d state, the (0-1), (2-1), (2-2) and (2-3) transitions are allowed, (0-2) is an exchange transition while (0-3) is forbidden. These are shown in figure 1 where the above analysis is reflected in the behavior of the cross sections with the exception of the (2-1) transition. This transition behaves very similarly to the (0-3) forbidden transition rather than the other allowed transitions. The reason for this is that the oscillator strength for this transition is very small. Since the oscillator strength is proportional to the matrix element of the dipole part of the interaction, this indicates that the cross section is dominated by other forbidden transitions in this energy range. At high enough energies, the dipole-allowed terms would dominate but this would occur well beyond the energy range studied here. The excitation of the 4p \( ^2 \) state involves three allowed transitions (0-1), (2-1) and (2-2) and three exchange transitions (0-0), (0-2) and (2-0). Figure 2 displays these transitions where the two different types of transitions have clearly different behaviour with energy.

The values for the coefficients in equations (5) and (6) which we obtain from our fittings are given in tables 2 and 3. The maximum error in the fittings is always less than 5% over the range from 30 to 100 eV. Note that the exponent in equation (6) given in table 3 reflect the nature of the transition (forbidden or exchange) but have not attained their expected asymptotic form over the energy range considered here.
Figure 1. Integrated cross section for the excitation of the metastable 4s4p 3P (J_i = 0, 2) states to the excited 4s4d 3D (J_f = 1, 2, 3) states. The different curves show the RDW calculations for the fine structure transitions from J_i \rightarrow J_f states. The solid curve [0→1], the dashed curve [0→2], the dotted curve [0→3], the dashed dot curve [2→1], the dashed double dot curve [2→2] and the short dashed curve [2→3].

Figure 2. Integrated cross section for the excitation of the metastable 4s4p 3P (J_i = 0, 2) states to the excited 4p 3P (J_f = 0, 1, 2) states. The different curves show the RDW calculations for the fine structure transitions from J_i \rightarrow J_f states. The solid curve [0→0], the dashed curve [0→1], the dotted curve [0→2], the dashed dot curve [2→0], the dashed double dot curve [2→1] and the short dashed curve [2→2].
Table 2. Parameters for the Bethe-Born fits [equation (5)] to the allowed fine-structure transitions from the initial 4s4p 3P levels.

| Final State | J_i-J_f | b     |
|-------------|---------|-------|
| 4s5s 3S     | 0-1     | 2.52200 |
|             | 2-1     | 2.24690 |
| 4s4d 3D     | 0-1     | 1.47373 |
|             | 2-1     | 2.62111 |
|             | 2-2     | 1.57557 |
|             | 2-3     | 1.30552 |
| 4p 3P       | 0-1     | 2.52288 |
|             | 2-1     | 2.42667 |
|             | 2-2     | 3.15200 |

4.2 The odd-odd transitions

The states 3d4p 3P_{0,1,2}, 3d4p 3D_{0,1,2,3}, 4s5f 3F_{2,3,4} have odd parity and any contributions for the direct part of the T-matrix for the excitation of these states must have \( \lambda \) even. Thus there are no allowed transitions to these states. For the excitation of the 3d4p 3P state the (0-0), (0-2), (2-0), (2-1) and (2-2) transition are all forbidden while the (0-1) transition is by exchange only. These are shown in figure 3 where the exchange transition is much smaller at large energies as expected but is similar in magnitude to the forbidden transitions near threshold where exchange effects are more important. Figure 4 shows our results for the excitation of the 3d4p 3D state and illustrates the fact that the (0-2), (2-1), (2-2) and (2-3) transitions are forbidden while the (0-1) and (0-3) transitions occur via exchange only. As in figure 3, the cross sections which proceed via exchange are comparable in magnitude to the forbidden transitions near threshold. Note that although these two states have the same basic configuration, the difference in
Figure 3. Same as Figure 2 but to the excited 3d4p $^3P$ ($J_f = 0, 1, 2$) states.

Figure 4. Same as Figure 1 but to the excited 3d4p $^3D$ ($J_f = 1, 2, 3$) states.

Figure 5. Integrated cross section for the excitation of the metastable 4s4p $^3P$ ($J_i = 0, 2$) states to the excited 4s5f $^3F$ ($J_f = 2, 3, 4$) states. The different curves show the RDW calculations for the fine structure transitions from $J_i \rightarrow J_f$ states. The solid curve [0→2], the dashed curve [0→3], the dotted curve [0→4], the dashed dot curve [2→2], the dashed double dot curve [2→3] and the short dashed curve [2→4].
angular momenta are reflected in the behaviour of the cross sections. Finally, for the excitation of the 4s5f state, the (0-2), (0-4), (2-2), (2-3) and (2-4) transitions are all forbidden and only (0-3) occurs via exchange. These results are presented in figure 5 where the behavior is similar to that for the other two odd-odd excitations.

We have fitted these cross sections to the form given in equation (6) and the results are given in table 3. as expected, the exponent $d_1$ is close to -1 for the forbidden transitions while for the exchange transitions it is more highly negative. Comparison of our results with the experimental and close coupling theoretical calculations of Shafranyosh et al (1997) which are reported in the low energy range up to 30 eV) are complicated by the fact that the RDW method may not be highly accurate in this range since it performs best at medium and high energies. However, in order to compare with the results of Shafranyosh et al (1997) who do not report cross sections for individual fine-structure transitions we have averaged our integrated cross section results of the individual fine structure transitions over the initial $J_i$ values and summed over the final $J_f$ values of the different levels of a given $LS$ term.

We compare our calculated integrated cross sections with both the experimental measurements and theoretical calculations reported by Shafranyosh et al (1997) in figures 6 and 7 for electron impact energies up to 100 eV. In all cases the initial state consists of the unresolved $J = 0$ and 2 metastable levels of the $4s4p^3P^o$ configuration. Since the experimental data in Shafranyosh et al (1997) were normalized to the close-coupling results reported in their paper, in order to make a clearer comparison here we have renormalized these measurements to our RDW results at 30 eV which is the highest energy at which the measurements were taken.

Figure 6 shows the results for the transitions to the unresolved levels of the $4s5s^3S$, $4s4d^3D$, and $4p^2^3P$ configurations. Note that for the excitation of the $4s5s$ state the cross sections for the two individual fine-structure transitions are nearly equal and are both allowed transitions. For all the three transitions we agree well in shape with the 2CC results of Shafranyosh et al. For the first two cases the RDW and 2CC results are similar in magnitude at larger energies while the 6CC results and the experimental data fall off more slowly as the energy increases. For the transition to the $4p^2^3P$ state, all three theoretical curves have a very similar shape but the RDW results are about 5% higher. Overall the agreement between the three sets of theoretical results and the experimental measurements is relatively good.
Figure 6. Integrated cross section for the excitation of the metastable 4s4p $^3P$ state to the 4s5s $^3S$, 4s4d $^3D$ and 4p $^3P$ states: solid curve, present RDW calculations; dashed curve, 2CC calculations of Shafranyosh et al (1997); dotted curve, 6CC calculations of Shafranyosh et al (1997); dots, experimental measurements of Shafranyosh et al (1997).
Figure 7. Same as figure 6 but for the excitation of the metastable 4s4p $^3P$ state to the 3d4p $^3P$, 3d4p $^3D$ and 4s5f $^3F$ states.
Figure 7 contains the results for the forbidden odd-odd transitions. These unresolved transitions have a sharp peak near threshold and an approximate $1/E$ behaviour at high energies (see below). Because of the uncertain accuracy of the RDW method at low energies, particularly for cross sections dominated by exchange, we have presented our results from 15 eV upwards. The RDW results for the $3d4p \ ^3P$ and $^3D$ transitions have a similar shape and magnitude and lie between the 2CC and 6CC results above 15 eV. They also agree well with the normalized experimental results at higher energies. For the $4s5f \ ^3F$ transitions the RDW and 2CC results are very similar above the peak of the cross section and agree with the normalized experiment at higher energies.

We have fitted our results for the unresolved odd – even transitions to the same form as given in equation (5), i.e.

$$ICS = \frac{1}{E} (c_0 + c_1 \ln(E))a_0^2$$

(7)

The coefficients $c_0$ and $c_1$ are given in table 4 and the energy is in atomic units. The cross sections for the unresolved odd – odd transitions have the form given in equation (6) at higher energies. Results of our fittings yield the values for $d_0$ and $d_1$ for these transitions as shown in table 5. The coefficient $d_1$ is close to -1 in all cases as expected.

Table 4. Parameters for equation (7) for the fitting of the unresolved allowed transitions from the initial $4s4p \ ^3P$ state.

| Final State | $c_0$  | $c_1$   |
|-------------|--------|---------|
| $4s5s \ ^3S$ | 17.36567 | 7.16706 |
| $4s4d \ ^3D$ | 20.47439 | 11.93979 |
| $4p \ ^3P$ | 33.03195 | 13.86323 |

Table 5. Parameters for equation (6) for the fitting of the unresolved forbidden transitions from the initial $4s4p \ ^3P$ state.

| Final State | $d_0$   | $d_1$   |
|-------------|---------|---------|
| $3d4p \ ^3P$ | 1.013334 | -0.942552 |
| $3d4p \ ^3D$ | 1.224218 | -0.921340 |
| $4s5f \ ^3F$ | 1.075430 | -0.822830 |

5. Conclusions

We have used our RDW approximation along with Dirac-Fock wave functions from the GRASP92 program to study electron excitation of the lowest lying metastable states of calcium to a number of higher lying triplet states. We have shown that the Dirac-Fock approximation yields reliable results for energies and oscillator strengths for most of the transitions studied. We have given a detailed analysis of the individual fine-structure transitions which are reflected in our results. We would hope that these individual fine-structure transitions will be measured in the near future as a more stringent test of our method.

Our integrated cross section results for the unresolved transitions are in satisfactory agreement with the experimental and theoretical results of Shafranyosh et al (1997). This is a further test of the RDW approximation applied to the excitation of metastable states of atoms.

Acknowledgements

ADS is grateful to the Natural Sciences and Engineering Research Council of Canada for a grant in support of this research and would like to thank the Indian Institute of Technology - Roorkee for their hospitality during a visit when much of this work was carried out. One of the authors (L.S.) is thankful to Ministry of Human Resource Development, Government of India for the award of a research fellowship.
References
[1] Chauhan R K, Srivastava R and Stauffer A D 2005 J. Phys. B: At. Mol. Opt. Phys. 38 2385
[2] Joachain C J 1983 Quantum Collision Theory (Amsterdam: North-Holland)
[3] McWilliam A, 1997 Annu. Rev. Astron. Astrophys. 35 303
[4] Milisavljević S, Šević D, V Pejčev, Filipović D M and Marinković B P 2004 J. Phys. B: At. Mol. Opt. Phys. 37 3571
[5] Parpia F A, Froese Fischer C and Grant I P 1996 Comput. Phys. Commun. 94 249
[6] Pottasch S R, 1972, Astron. & Astrophys. 17 128
[7] Shafranyosh I I, Snegurskaya T A, Margitich N A, Bogacheva S P, Lengyel, V A and Zatsarinny O I 1997 J. Phys. B: At. Mol. Opt. Phys. 30 2261
[8] Srivastava R and Stauffer A D 2005 Phys. Rev. A 71 052715
[9] Srivastava R, Stauffer A D and Sharma L 2006 Phys. Rev. A 74 012715
[10] Zuo T 1991 PhD Thesis York University, Toronto