Vacuum less global monopole in Brans-Dicke theory

F. Rahaman*, M. Kalam, R. Mukherjee, S. Das and T. Roy

Abstract

In the present work, the gravitational field of a vacuum less global monopole has been investigated in Brans-Dicke theory under weak field assumption of the field equations. It has been shown that the vacuum less global monopole exerts attractive gravitational effects on a test particle. It is dissimilar to the case studied in general relativity.

1. INTRODUCTION

Topological defects could be produced at a phase transition in the early Universe. The study of topological defects has wide applicability in many areas of physics. In the cosmological arena, defects have been put forward as a possible mechanism for structure formation. Monopole is one of the topological defects which arises when the vacuum manifold contains surfaces which can not be shrunk continuously to a point[1]. A typical symmetry breaking model is described by the Lagrangian,

$$L = \frac{1}{2} \partial_\mu \Phi^a \partial^\mu \Phi^a - V(f)$$

(1)

Where $\Phi^a$ is a set of scalar fields, $a = 1, 2, \ldots, N$, $f = (\Phi^a \Phi^a)^{1/2}$ and $V(f)$ has a minimum at a non zero value of $f$. The model has $O(N)$ symmetry and admits domain wall, string and monopole solutions for $N = 1, 2$ and $3$ respectively.

It has been recently suggested by Cho and Vilenkin(CV)[2,3] that topological defects can also be formed in the models where $V(f)$ is maximum at $f = 0$ and it decreases monotonically to zero for $f \rightarrow \infty$ without having any minima. For example,

$$V(f) = \lambda M^{4+n}(M^n + f^n)^{-1}$$

(2)

Where $M, \lambda$ and $n$ are positive constants.

Pacs Nos : 98.90 cq, 04.20 Jb, 04.50

Key words and phrases : Vacuum less global monopole, Brans-Dicke theory, Gravitational field.

*Dept. of Mathematics, Jadavpur University, Kolkata-700 032, India
E-Mail: farookrahaman@yahoo.com
This type of potential can arise in non-perturbative super string models. Defects arising in this model are termed as vacuum less.

The recent extensive search for a matter field which can give rise to an accelerated expansion for the Universe stems from the observational data regarding the luminosity-red shift relation of type Ia supernova up to about \( z \sim 1 \) \([4]\). This matter field is called ”Quintessence” or Q-matter. The most popular candidate for Q-matter has so far been a present epoch \([5]\). Example of Q-matter are fundamental fields or macroscopic objects and network of vacuum less defects may be one such good examples as scalar field with potential like (2) can act as Quintessence model \([6]\).

CV have studied the gravitational field of topological defects in the above models within the frame work of general relativity \([3]\). Also, Rahaman et al \([7]\) have studied vacuum-less global monopole and cosmic string in Einstein-Cartan theory. But at sufficiently high energy scales, it seems likely that gravity is not given by Einstein’s action. One of the important modification to Einstein’s theory of gravitation has been proposed by Brans-Dicke \([8]\). In the gravitational theory, in addition to the space time metric a scalar field \( \phi \) is introduced as dynamical variables. This theory can be thought of a minimal extension of general relativity designed to properly accommodate both Mach’s principle and Dirac’s large number hypothesis. Here, the gravitational effects are in part geometrical and in part due to a scalar interaction. Also the gravitational constant ‘\( G \)’ is a variable scalar and is related to the scalar field \( \phi \sim G^{-1} \). In recent year, the Brans-Dicke theory has a lot interest as power law inflation is possible for this theory with constant vacuum energy density. For this type of extended inflation, it is possible to ‘slow roll over’ for the Universe during the phase transitions. The motivation for studying gravitational properties of defects in Brans-Dicke theory is that only defects we can hope to observe now are those formed after or near end of inflation.

2. The Basic Equations

A global monopole is described by a triplet of scalar fields \( \Phi^a, a = 1, 2, 3 \). The monopole ansatz is \( \Phi^a = f(r) \frac{x^a}{r} \), where \( r \) is the distance from the monopole center. For the power law potential (2), it can be verified that the field equation for \( f(r) \) admits a solution of the form \([2,3]\)

\[
f(r) = aM \left( \frac{r}{\delta} \right)^{\frac{2}{n+2}}
\]

Where \( \delta = \frac{1}{M \sqrt{\lambda}} \) is the core radius of the monopole, \( r \) is the distance from the monopole center and \( a = \frac{n+2}{n+4} \frac{n}{n+2} \frac{1}{n+2} \).

The solution (3) applies for

\[
\delta \ll r \ll R
\]

where the cut off radius \( R \) is set by the distance to the nearest anti monopole.
The Brans-Dicke equations are taken as solutions for monopole. For global vacuum less monopole, one can use the flat space approximation of the corresponding line element as

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2d\Omega_2^2$$  \hspace{1cm} (5)$$

The general energy momentum tensor for the vacuum less monopole is given by

$$T^i_i = \frac{(f')^2}{2A} + \frac{f^2w^2}{r^2} + \frac{1}{2c^2r^2}\left[\frac{(w')^2}{A} + \frac{(1-w^2)^2}{2r^2}\right] + V(f)$$  \hspace{1cm} (6)$$

$$T^r_r = -\frac{(f')^2}{2A} + \frac{f^2w^2}{r^2} + \frac{1}{2c^2r^2}\left[\frac{(w')^2}{A} + \frac{(1-w^2)^2}{2r^2}\right] + V(f)$$  \hspace{1cm} (7)$$

$$T^\theta_\theta = T^\phi_\phi = \frac{(f')^2}{2A} + \frac{1}{2c^2r^2}\left[\frac{(w')^2}{A} + \frac{(1-w^2)^2}{2r^2}\right] + V(f)$$  \hspace{1cm} (8)$$

For monopole, the gauge field is $A^a_i(r) = \frac{1-\omega(r)}{\epsilon r^2}$. $T^a_i$'s with $w = 1$ are that for global monopole. For global vacuum less monopole, one can use the flat space approximation for $f(r)$ in (3) for $r \gg \delta$ and the form of $V(f)$ given in (2).

The Brans-Dicke equations are taken as

$$R_{ab} = \frac{8\pi}{\phi} [T_{ab} - \frac{1}{2}g_{ab}(\frac{2\omega + 2}{2\omega + 3})T + \frac{\omega}{\phi^2}\phi_{,a}\phi_{,b} + \frac{1}{\phi}\phi_{,a;b}]$$  \hspace{1cm} (9)$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^a}[\sqrt{-g}g^{\alpha\beta}\frac{\partial}{\partial x^\beta}]\phi = \frac{8\pi}{2\omega + 3}T$$  \hspace{1cm} (10)$$

where $T = \text{Trace of } T_{ab}$

Hence, The field equations become

$$\frac{B''}{2A} - \frac{B' A'}{4A} + \frac{B'}{B} - \frac{B'}{rB} = \frac{FB}{\phi r^b} - \frac{B'\phi'}{2A\phi}$$  \hspace{1cm} (11)$$

$$-\frac{B''}{2B} + \frac{B' A'}{4B} + \frac{B'}{rA} = -\frac{GB}{\phi r^b} - \frac{\omega(\phi')^2}{\phi^2} + \frac{1}{\phi}\left[\phi'' - \frac{A\phi'}{2A}\right]$$  \hspace{1cm} (12)$$

$$1 - \frac{r}{2A} \left[\frac{B'}{B} - \frac{A'}{A}\right] = \frac{HB}{\phi r^b} - \frac{r\phi'}{A\phi}$$  \hspace{1cm} (13)$$

$$\phi'' + \frac{1}{\phi'}\left[\frac{B'}{B} - \frac{A'}{A} + \frac{4}{r}\right] = \frac{AL}{r^b}$$  \hspace{1cm} (14)$$

where

$$F = \frac{8\pi a^2 M^2}{2\omega + 3} [-1 + \frac{\omega}{\omega a^2} - \frac{2(\omega+2)}{(n+2)^2}] \delta^{n+2},$$

$$G = \frac{8\pi a^2 M^2}{2\omega + 3} [1 - \frac{\omega}{\omega a^2} - \frac{2(3\omega+4)}{(n+2)^2}] \delta^{n+2},$$

$$H = \frac{8\pi a^2 M^2}{2\omega + 3} \left[\frac{\omega}{\omega a^2} + 2(\omega + 1) - \frac{2(\omega+2)}{(n+2)^2}\right] \delta^{n+2},$$

$$L = \frac{8\pi a^2 M^2}{2\omega + 3} \left[\frac{3}{\omega a^2} + 2 + \frac{2}{(n+2)^2}\right] \delta^{n+2},$$

$$b = \frac{2n}{(n+2)}.$$
3. Solutions in the weak field approximations:

Under the weak field approximations one can write

\[ A(r) = 1 + f(r), \quad B(r) = 1 + g(r), \quad \phi = \phi_0 + \epsilon(r). \]

where \( \phi_0 \) is a constant which may be identified with \( \frac{1}{G} \) when \( \omega \to \infty \) ( \( G \) being the Newtonian gravitational constant).

\[
\frac{\phi'}{\phi} = \frac{\epsilon'}{\phi_0}, \quad \frac{\phi''}{\phi} = \frac{\epsilon''}{\phi_0}, \quad \frac{B'}{B} = \frac{g'}{g}, \quad \frac{A'}{A} = f'
\]

(15)

where \( f, g \ll 1 \).

In this approximations eqs. (9) – (12) take the following forms as

\[
\frac{1}{2} g'' + \frac{g'}{r} = Fr^{-b}
\]

(16)

\[-\frac{1}{2} g'' + \frac{f'}{r} = Gr^{-b} + \frac{\epsilon''}{\phi_0}\]

(17)

\[ f + \frac{1}{2} (f' - g')r = Hr^{-b} + r \frac{\epsilon'}{\phi_0}\]

(18)

\[ \epsilon'' + 2 \frac{\epsilon'}{r} = Lr^{-b}\]

(19)

Solving these equations, we get

\[
g = \frac{2F}{\phi_0(3 - b)(2 - b)r^{2-b}}
\]

(20)

\[ f = \frac{F(1 - b)}{\phi_0(3 - b)(2 - b)r^{2-b}} + \frac{L(1 - b)}{\phi_0(3 - b)(2 - b)r^{2-b}} + \frac{G}{\phi_0(2 - b)r^{2-b}}\]

(21)

\[ \epsilon = \frac{L}{(3 - b)(2 - b)r^{2-b}}\]

(22)

As \( \omega \to \infty \), we come back to the general relativity solution as Cho and Vilenkin [3].
4. Gravitational effects on test particles:

Let us consider a relativistic particle of mass \(m\), moving in the gravitational field of a monopole described by equation (5) using the formalism of Hamilton and Jacobi (H-J). According to the H-J equation is [9]

\[
\frac{1}{B(r)} \left( \frac{\partial S}{\partial t} \right)^2 - \frac{1}{A(r)} \left( \frac{\partial S}{\partial r} \right)^2 - \frac{1}{r^2} \left( \frac{\partial S}{\partial \theta} \right)^2 - \frac{1}{r^2 \sin^2 \theta} \left( \frac{\partial S}{\partial \phi} \right)^2 + m^2 = 0 \tag{23}
\]

where

\[
A(r) = 1 + \frac{E(1-b)}{\phi_0(3-b)(2-b)} r^{2-b} + \frac{L(1-b)}{\phi_0(3-b)(2-b)} r^{2-b} + \frac{G}{\phi_0(2-b)} r^{2-b}
\]

and

\[
B(r) = 1 + \frac{2F}{\phi_0(3-b)(2-b)} r^{2-b}
\]

In order to solve the particle differential equation, let us use the separation of variables for the H-J function \(S\) as follows [9].

\[
S(t, r, \theta, \phi) = Et - S_1(r) - S_2(\theta) - J\phi \tag{24}
\]

Here the constants \(E\) and \(J\) are identified as the energy and angular momentum of the particle.

The radial velocity of the particle is (For detail calculations see Ref. [9]).

\[
\frac{dr}{dt} = B \sqrt{E^2 \frac{1}{B} + m^2 - \frac{p^2}{r^2}} \tag{25}
\]

where \(p\) is the separation constant. The turning points of the trajectory are given by \(\frac{dr}{dt} = 0\) and as a consequence the potential curves are

\[
\frac{E}{m} = \sqrt{[1 + \frac{2F}{\phi_0(3-b)(2-b)} r^{2-b}] \left[ \frac{p^2}{m^2 r^2} - 1 \right]} \tag{26}
\]

In this case the extremals of the potential curve are the solutions of the equation

\[
m^2 Y (2-b) r^{4-b} + b Y r^{2-b} - 2p^2 = 0 \tag{27}
\]

where \(Y = \frac{2F}{\phi_0(3-b)(2-b)}\)

This equation has at least one positive real root provided \((-b+2)\) is a positive integer. So it is possible to have bound orbit for the test particle. Thus the gravitational field of the global monopole is shown to be attractive in nature but here we have to imposed some restriction on the constant "n".
5. Concluding Remarks:

The recent extensive search for a matter which can give rise to an accelerated expansion for the Universe is quintessence matter or 'Q' matter. Examples of Q-matter are fundamental fields or macroscopic objects and network of vacuum less defects may be one such good examples as scalar field with potential(2) can act as quintessence models. In this paper, we have extended the earlier work of CV regarding the gravitational field of vacuum less global monopole to the scalar tensor theory. Our study of the motion of the test particle reveals that the vacuum less global monopole in Brans-Dicke theory exerts gravitational field which is attractive in nature. It is dissimilar to the case studied in general relativity. Finally we see that when $\omega \to \infty$, CV solution is recovered.

Acknowledgements

We are grateful to Dr. A. A. Sen for helpful discussions. We are also thankful to UGC for financial support and IUCAA for sending papers and preprints.

References

[1] Kibble, T.W.B.J. Phys.A 9, 1387 (1976) A. Vilenkin and E.P.S. Shellard (1994), Cosmic String and other Topological Defects (Camb. Univ. Press)

[2] I. Cho and A. Vilenkin, Phys. Rev. D 59, 021701 (1999)

[3] I. Cho and A. Vilenkin, Phys. Rev. D 59, 063510 (1999)

[4] Perlmutter S et al, (1999) Astrophys. J 517, 565

[5] Caldwell R R et al, (1998) Phys. Rev. Lett. 80, 1582

[6] Peebles P J E and A Vilenkin (1999) Phys. Rev. D 59, 063505

[7] F. Rahaman, S. Mandal and B.C. Bhui, Fizika B12, 291 (2003); F. Rahaman, B.C. Bhui, A Ghosh and R. Mondal, gr-qc/0610086

[8] Brans C and Dicke R.H (1961), Phys. Rev. 124, 925

[9] S. Chakraborty, Gen. Rel. Grav. (1996), 28, 1115; S. Chakraborty and F. Rahaman, Pramana 51, 689 (1998)