Capacity optimization through sensing threshold adaptation for cognitive radio networks

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Abstract We propose capacity optimization through sensing threshold adaptation for sensing-based cognitive radio networks. The objective function of the proposed optimization is the maximization of the capacity at the secondary user subject to transmit power and sensing threshold constraints for protecting the primary user. After proving the concavity of capacity on sensing threshold, the problem is solved using the Lagrange duality decomposition method in conjunction with a subgradient iterative algorithm. The numerical results show that the proposed optimization can lead to significant capacity maximization for the secondary user as long as this is affordable to the primary user.

Keywords Capacity optimization · Cognitive radio network · Convex optimization · Lagrange duality · Subgradient method

1 Introduction

The cognitive radio (CR) concept brought about the idea to exploit spectrum holes (i.e. bands) which result from the proven underutilization of the electromagnetic spectrum by modern wireless communication and broadcasting technologies [1]. A cognitive
radio network (CRN) consists of a primary (i.e. licensed) network (PN) and a secondary one (SN). The main objective of CRNs is to provide the means that allow the secondary users (SUs) to access the resources of the PN without causing harmful interference to the primary users (PUs). A SU may exploit spectrum bands of the PN relying either on opportunistic spectrum access (OSA) or on spectrum sharing (SS) methodologies [2]. In OSA, PN’s bands can be used by a SU only if they are vacant while in spectrum sharing both SU and PU can utilize the same frequency band simultaneously. For achieving concurrent spectrum usage power control (PoC) and spectrum sensing (SpSe) are employed at the SU in a SS CRN known as sensing-based SS CRN, which allow for the exploitation of spectrum bands that are originally allocated to a PN by a SU under the constraints for protecting the PU [3]. PoC aims at protecting the PU from possible harmful interference from the SU by constraining its transmission power [4–6]. SpSe at the SU specifies the probability of the PU’s detection through a specific sensing threshold which impacts both the achievable capacity of the SU as well as the degree of protection offered to the PU [2]. The SU’s capacity optimization over transmit power has been studied for both CRNs [4,5] as well as for conventional wireless networks [13,14]. However, the SU’s capacity optimization over the sensing threshold has not been recognized and studied. In [7], the authors have pointed out the importance of incorporating a sensing threshold adaptation in CRNs but, however, they do not provide any details on how the SU’s capacity can be optimized over it. Hence, in this paper, the problem of maximizing the capacity of the SU over the sensing threshold in CRNs is formulated and solved. The problem formulation results in a convex optimization problem. After proving the concavity of the capacity on sensing threshold, the optimization problem is solved using a Lagrange dual decomposition method in conjunction with a subgradient iterative algorithm [11,12,15,16,18].

The rest of this paper is organized as follows. Section 2 provides the system model of the sensing-based SS CRN with the details of the SpSe model. In Sect. 3, we first formulate the convex optimization problem, in the sequel we prove the concavity of the SU’s capacity on the sensing threshold and then we provide its solution. Section 4 discusses the obtained numerical results that show the achievable SU’s capacity maximization and the PU’s protection through the sensing threshold adaptation and we conclude this work with Sect. 5.

2 System model

2.1 Cognitive radio network model

We consider a sensing-based SS CRN with one PN and one SN which one provides a primary and secondary link, respectively (Fig. 1). Both links consist of a transmitter and a receiver where for the PN are denoted as PU-Tx (Primary User Transmitter) and PU-Rx (Primary User Receiver) and as SU-Tx (Secondary User Transmitter) and SU-Rx (Secondary User Receiver) for the SN, respectively. The links are assumed to be flat fading channels (i.e. all frequency components of a signal will experience the same magnitude of degradation) with additive white Gaussian noise (AWGN) [13]. The independent random variables of the AWGN are denoted with $n_p$ and $n_s$ for the
primary and secondary link, respectively, which are assumed with mean zero and variance $N_0$. The PoC at the SU’s transmitter aiming to protect the PU’s receiver and for this reason the transmit power $P_t$ is denoted as $P_0^t$ when the PU is idle or as $P_1^t$ when the PU is active and $P_0^t > P_1^t$ holds \[3\]. Furthermore, perfect channel state information (CSI) is assumed to be available at the SU-Tx from the SU-Rx through a feedback channel.

### 2.2 Spectrum sensing model

The PU’s activity is identified via a spectrum sensor that is employed at the SU-Tx. We assume an energy detector for SpSe which is able to sense the signal-to-noise-ratio (SNR) $\gamma$ for a specific time interval $\tau$ within the sampling frequency $f_s$ \[8\]. Spectrum sensing indicates whether the PU is active or idle by comparing the sensed SNR $\gamma$ with a sensing threshold $\eta$. The SpSe procedure results in four possible outcomes, i.e. detection, missed detection, false alarm and no false alarm with probabilities $P_d$, $(1 - P_d)$, $P_f$, $(1 - P_f)$, respectively. The probabilities of false alarm and detection are defined as follows in relation with the hypotheses that the PU is idle or active denoted as $h_0$ and $h_1$, respectively

\[
\begin{align*}
P_f &= \Pr[\gamma > \eta \mid h_0] \\
P_d &= \Pr[\gamma > \eta \mid h_1]
\end{align*}
\]

Throughout this paper, we suppose a SpSe model with circularly symmetric complex Gaussian noise with mean zero and variance $\sigma^2$. Then the corresponding probabilities of false alarm and detection are defined as follows \[8\]

\[
P_d(\eta, N) = Q\left(\frac{\eta}{\sigma^2} - \gamma - 1\right)\sqrt{\frac{N}{2\gamma + 1}}
\]

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Fig. 2 Operating characteristics of spectrum sensing for different values of sensing threshold $\eta$, number of sensed samples $N$ and sensed SNRs $\gamma$ ($\gamma = -12$ dB (circle marker) and $\gamma = -15$ dB (no marker))

$$P_f (\eta, N) = Q \left( \left( \frac{\eta}{\sigma^2} - 1 \right) \sqrt{N} \right)$$  \hspace{1cm} (3)

where $Q(\cdot)$ is the complementary standard Gaussian distribution function\(^1\) and $N$ is the number of samples taken from SpSe that is equal to $N = \tau f_s$.

Figure 2 illustrates the complementary Receiver Operating Characteristic (ROC) of the considered SpSe model which plots the probability of missed detection $P_m = 1 - P_d$ versus the probability of false alarm $P_f$ for different values of the sensing threshold $\eta$. We depict the results obtained for different number of samples $N$ and variance equal to $\sigma^2 = 1$. The lines without marker are obtained for $\gamma = -15$ dB and the lines with circle marker illustrate the results obtained with $\gamma = -12$ dB. Obviously, the SpSe behaves totally different by changing the sensing threshold $\eta$ and the number of the sensed samples $N$ for a given sensed SNR $\gamma$. Therefore, in order to retain a specific behavior for the SpSe mechanism in a sensing-based CRN, a proper sensing threshold value $\eta$ and number of samples $N$ for a given sensed SNR $\gamma$ must be selected. In [3], the authors have proposed the throughput optimization over sensing time $\tau$ when frame duration $T$ is considered and thus an optimal value of number of samples $N$ is obtained. In this paper, we investigate the capacity optimization over sensing threshold $\eta$ which is more generic and can lead to substantial capacity maximization regardless of the frame duration.

\(^1\) $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du.$
3 Capacity optimization over sensing threshold

In this section, we present the capacity optimization over the sensing threshold subject to PoC and SpSe constraints for protecting the PU. To this end, an optimization problem is formulated that is proved to be convex. The problem’s solution is provided by a Lagrange dual decomposition method in conjunction with a subgradient iterative algorithm.

3.1 Optimization problem formulation

Based on the probabilities that the PU is idle or active denoted as $\pi_0$ and $\pi_1$, respectively, and the four results of SpSe, the following possible transmission scenarios are identified for the SU with their corresponding capacities:

- the PU is idle with no false alarm denoted as $C_0(P_0^t)\pi_0(1 - P_f(\eta))$,
- the PU is idle with false alarm denoted as $C_0(P_0^t)\pi_0 P_f(\eta)$,
- the PU is active and detected denoted as $C_1(P_1^t)\pi_1 P_d(\eta)$, and
- the PU is active and misdetected denoted as $C_0(P_0^t)\pi_1 (1 - P_d(\eta))$.

Thus, the overall SU’s capacity is obtained as follows [3]

$$C_s(P_0^t, P_1^t, \eta) = C_0(P_0^t)\pi_0 (1 - P_f(\eta)) + C_1(P_1^t)\pi_0 P_f(\eta) + C_1(P_1^t)\pi_1 P_d(\eta) + C_0(P_0^t)\pi_1 (1 - P_d(\eta)) \quad (4)$$

The expression given in equation (4) is the objective function that we maximize over the transmit powers $P_0^t$ and $P_1^t$ and the sensing threshold $\eta$ for a sensed SNR $\gamma$ and a number of sensed samples $N$. The constraints on the transmit powers regulate the average transmit power at the SU-Tx denoted as $H(P_0^t, P_1^t, \eta)$ and the interference power at the PU-Rx denoted as $I(P_0^t, P_1^t, \eta)$, respectively

$$H(P_0^t, P_1^t, \eta) = \pi_0 E(P_0^t) (1 - P_f(\eta)) + \pi_0 E(P_1^t) P_f(\eta) + \pi_1 E(P_1^t) P_d(\eta) + \pi_1 E(P_0^t) (1 - P_d(\eta)) \quad (5)$$

$$I(P_0^t, P_1^t, \eta) = G_{sp} \pi_0 P_0^t (1 - P_f(\eta)) + G_{sp} \pi_0 P_1^t P_f(\eta) + G_{sp} \pi_1 P_0^t P_d(\eta) + G_{sp} \pi_1 P_1^t (1 - P_d(\eta)) \quad (6)$$

where $G_{sp}$ is the channel gain at the link between the SU-Tx and PU-Rx and $E(\cdot)$ is the expectation over the probability density function (pdf) $p(\gamma_s)$ of the fading channel at the SU link. The constraint on the sensing threshold regulates the operating characteristics of SpSe and henceforth a target probability of detection has been used so far [2,3]. However, in this work we choose to regulate the sensing threshold by using a more factual parameter than the target probability of detection. To this end, we define

$$E(P_t) = \int_{-\infty}^{+\infty} P_t(\gamma_s) p(\gamma_s) d\gamma_s.$$
a level for the PU’s capacity loss $C_{p,loss}$ that the PU can afford and thus the PU’s CSI is involved [9].

Assuming that $P_{av}$ is the maximum average transmit power at the SU-Tx, $I_{pk}$ is the maximum peak interference power constraint that the PU-Rx can tolerate and that the PU’s capacity loss $C_{p,loss}$ is less than some prescribed percentage $q$ over the maximum PU’s capacity $C_{p,max}$, then the SU’s capacity optimization is formed as follows

$$\text{maximize} \quad C_s \left( P_1^0, P_1^1, \eta \right)$$

subject to

$$H \left( P_1^0, P_1^1, \eta \right) \leq P_{av}$$

$$I \left( P_1^0, P_1^1, \eta \right) \leq I_{pk}$$

$$C_{p,loss} \leq qC_{p,max}$$

with $P_1^0 \geq 0, P_1^1 \geq 0, \eta \in [0, \infty]$ (7)

for a constant $N \in [0, T_{fs}]$, where $T$ is the frame duration and constant probabilities $\pi_0$ and $\pi_1$.

3.2 Concavity on sensing threshold

It is easily observed that the problem is a convex optimization one with respect to transmit powers $P_1^0$ and $P_1^1$. However, it is unclear whether this problem is a convex optimization problem with respect to sensing threshold $\eta$. In the following we show that the SU’s capacity $C_s$ is concave on sensing threshold $\eta$.

**Proposition 1** For the range of $\eta$ such that $P_d(\eta)$ is increasing and concave on $\eta$ and $P_f(\eta)$ is increasing and concave on $\eta$, the capacity $C_s$ is concave on $\eta$.

**Proof** Differentiating both $P_f(\eta)$ and $P_d(\eta)$ with respect to $\eta$ gives:

$$P_f' (\eta) = \frac{dP_f (\eta)}{d\eta} = \frac{1}{\sqrt{2\pi}} \left( \frac{\sqrt{N}}{\sigma^2} \right) \exp \left( - \left( \frac{\eta}{\sigma^2} - 1 \right)^2 / 2 \right)$$

(9)

$$P_d' (\eta) = \frac{dP_d (\eta)}{d\eta} = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma^2} \left( \frac{\sqrt{N}}{\sqrt{2\gamma + 1}} \right) \exp \left( - \left( \frac{\eta}{\sigma^2} - \gamma - 1 \right)^2 / 2 \right)$$

(10)

Figure 3 depicts the first derivatives $P_f' (\eta)$ and $P_d' (\eta)$ assuming $\gamma = -15dB$, $\sigma=1$, $f_{fs} = 6$ MHz and $\tau = 2$ ms. The question we need to answer is if the first derivative of a function is increasing or decreasing or staying constant on the parameter of interest. Obviously, for $\eta > 0$, it is clear that $P_f' (\eta) > 0$ and $P_d' (\eta) > 0$. Thus, it can be said that both $P_f$ and $P_d$ are concave on $\eta$. However, there are critical values that could be identified as local extreme values, i.e. local minima and local maxima [11]. These
values are related with term $|\eta/\sigma^2 - 1|$ for the probability of false alarm $P_f$ and with term $|\eta/\sigma^2 - \gamma - 1|$ for the probability of detection $P_d$. Figure 4 shows these values $|\eta/\sigma^2 - 1|$ and $|\eta/\sigma^2 - \gamma - 1|$ where the concavity of $P_d$ and $P_f$ on $\eta$ is proved.

We now check the concavity of SU’s capacity $C_s$ and thus we derive the first derivative of SU’s capacity $C_s$ with respect to $\eta$

$$C'_s(\eta) = \frac{dC_s(\eta)}{d\eta} = [C_1\pi_0 - C_0\pi_0] \frac{dP_f}{d\eta} + [C_1\pi_1 - C_0\pi_1] \frac{dP_d}{d\eta}$$ \hspace{1cm} (11)

and then

$$C'_s(\eta) = \pi_0 P'_f(\eta) (C_1 - C_0) + \pi_1 P'_d(\eta) (C_1 - C_0)$$ \hspace{1cm} (12)

Since both $P'_f(\eta)$ and $P'_d(\eta)$ are concave on $\eta$ and $(C_1 - C_0) < 0$ holds, the first derivative of capacity is decreasing on $\eta$, i.e. $C'_s(\eta) < 0$. This implies that the SU’s capacity $C_s$ is concave on sensing threshold $\eta$ and thus the optimization problem over sensing threshold $\eta$ is concave either.

3.3 Lagrange duality and subgradient iterative algorithm

In order to solve the convex optimization problem defined in equation (4), a dual decomposition method can be applied for a value for the sensing threshold $\eta$ [16,18].
The Lagrangian of (4) is defined as

$$L \left( P^0_t, P^1_t, \lambda \right) = L \left( P^0_t, P^1_t, \eta \right) - \lambda \left( H \left( P^0_t, P^1_t, \eta \right) - P_{av} \right)$$  (13)

where $\lambda$ is the nonnegative Lagrange dual variable associated with the constraint $H(P^0_t, P^1_t, \eta) \leq P_{av}$. The Lagrangian dual function is now defined as

$$q \left( \lambda \right) = \sup_{\left\{ P^0_t, P^1_t \right\}} \left\{ L \left( P^0_t, P^1_t, \lambda \right) \bigg| I \left( P^0_t, P^1_t, \eta \right) \leq I_{pk} \right\}$$  (14)

The dual function can then be minimized to obtain an upper bound on the optimal value $C^*_s$ of the optimization problem in (4)

$$\min_{\lambda} q \left( \lambda \right)$$  (15)

where the optimal dual objective $q^*$ forms the duality gap $C^*_s - q^*$ which is indeed zero since the Karush–Kuhn–Tucker (KKT) conditions are satisfied.

Based on the above problem formulation, the optimal solution of the transmit power is obtained as follows [4]

$$P^0_t = \left( \frac{1}{\lambda} - \frac{N_0}{\gamma_s} \right)^+, \text{ for } \gamma_s \geq \lambda$$  (16)
The subgradient of \( q(\lambda) \) is obtained as follows:

**Proposition 2** The subgradient of \( q(\lambda) \) is \( g(P_0^0, P_1^1) = P_{av} - H(P_0^0, P_1^1) \) for the \( i \)th iteration (i.e. a given sensing threshold \( \eta \)) and \( g(P_0^0, P_1^1, \lambda) \) is an element of \( \partial q(\lambda) \).

**Proof** For any \( \mu \in \text{dom}(q) \), since \( q(\mu) \) is obtained by maximizing \( L(P_0^0, P_1^1, \mu) \) over \( P_0^0, P_1^1 \in \text{dom}(C) \), we have \( q(\mu) \geq L(P_0^0, P_1^1, \mu) \) \[15,16\]. Moreover, since \( P_0^0, P_1^1 \) achieves the maximum, we have \( q(\lambda) = L(P_0^0, P_1^1, \lambda, \mu) \). Combining the pieces, we obtain

\[
q(\mu) \geq L(P_0^0, P_1^1, \lambda, \mu) = L(P_0^0, P_1^1, \lambda) + [L(P_0^0, P_1^1, \lambda, \mu) - L(P_0^0, P_1^1, \lambda)]
\]

Then, the problem can be solved by algorithm 1, which requires the calculation of the subgradient \( g \) at each iteration.

In the subgradient iterative algorithm described above, the sensing threshold \( \eta \) is matched to a specific capacity loss \( C_{P,\text{loss}} \) using the separation principle in wireless networking \[10,17\]. According to this principle, a probability of missed detection \( P_m = 1 - P_d \) at the SU-Tx will result in an outage probability for the PU-Rx that presents the probability that the transmission is decoded with a large error probability at the PU-Rx \[13\]. In particular, if the received SNR at the PU-Rx is below \( \gamma_{p,\text{min}} \), then the bursty transmitted bits are decoded with an error probability approaching one, and

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3 \( \partial q(\lambda) \) denotes the set of all subgradients at \( \lambda \) that is called the subdifferential.

4 Where \( \{P_0^0, P_1^1\} \lambda \) it the pair values of the transmit powers \( P_0^0 \) and \( P_1^1 \) for the Lagrange multiplier value \( \lambda \).
thus the PU-Rx declares an outage with the following probability

\[ P_{\text{out}} = \Pr[\gamma_p < \gamma_{p,\text{min}}] \]  

(19)

In this case, the capacity loss at the PU’s link is obtained as follows

\[ C_{p,\text{loss}} = C_{p,\text{max}} - \int_{\gamma_{p,\text{min}}}^{\infty} \log_2(\gamma_p) p(\gamma_p) d\gamma_p \]  

(20)

where \( p(\gamma_p) \) is the probability density function (pdf) of the fading distribution at the PU’s link, while the maximum achievable capacity at the PU’s link is equal to \( C_{p,\text{max}} = \int_0^{\infty} \log_2(\gamma_p) p(\gamma_p) d\gamma_p \). Thus, a capacity loss \( C_{p,\text{loss}} \) which depends on the outage probability \( P_{\text{out}} \) will dictate the missed detection probability \( P_m \) that matches to a sensing threshold \( \eta \) value.

4 Numerical results

Figure 5 depicts the results obtained for the SU’s capacity versus the sensing threshold \( \eta \) for different values of sensed SNR \( \gamma \) at the spectrum sensor and an average transmit power \( P_{av} \) of the SU-Tx. We assume a number of samples equal to \( N = 12,000 \) corresponding to a sensing time equal to \( \tau = 2 \) ms for a sampling frequency \( f_s = 6 \) MHz and variance equal to \( \sigma^2 = 1 \). The optimal power allocation at the SU’s link is taking place over a Rayleigh fading channel with unit mean and AWGN with variance \( N_0 = 1 \) [5]. Besides, we assume an interference power constraint equal to \( I_{pk} = 0 \) db while the PU’s activity is considered as \( \pi_1 = 0.4 \). It is observed that a proper power allocation and sensing threshold adaptation for a given sensed SNR \( \gamma \) results in significant capacity increase for the SU especially for large values of the average transmit power e.g. \( P_{av} = 15 \) db. This maximization is getting lower for lower values of \( P_{av} \), e.g. \( P_{av} = 5 \) db and becomes negligible when \( P_{av} \) is lower than the considered peak interference power constraint \( I_{pk} \), i.e. when \( P_{av} < I_{pk} \). This is due to the fact that a missed detection do not affects the system’s behavior since the transmit powers are now equal, i.e. \( P_0^1 = P_1^0 \). Moreover, low values of sensed SNR \( \gamma \) will lead to further capacity maximization for a given sensing threshold \( \eta \).

Figure 6 illustrates the PU’s capacity loss \( C_{p,\text{loss}} \) versus the sensing threshold \( \eta \) assuming the same CRN setup as previously in terms of implementation details for the PoC and the SpSe at the SU-Tx. We depict the results for different average transmit powers \( P_{av} \) of the PoC and sensed SNRs \( \gamma \) at the SpSe of the SU-Tx either. The results are obtained assuming that the probability of missed detection \( P_m \) at the SU-Tx yields an outage probability, i.e. \( P_m = P_{\text{out}} \) at the PU-Rx. Thus, a specific sensing threshold \( \eta \) value represents a specific missed detection probability \( P_m \) which is matched next into a CSI at the Pu-Rx \( \gamma_p \) that yields an outage. Figure 6 shows that the higher the sensing threshold \( \eta \), the higher the capacity loss \( C_{p,\text{loss}} \) is become. This is expected since the probability of missed detection \( P_m \) is getting higher. In particular, the probability of missed detection \( P_m \) leads to lower values in CSI at the Pu-Rx \( \gamma_p \) and
thereafter in lower achievable capacities at the PU, i.e. higher capacity loss $C_{p, loss}$. Obviously, there exists a fundamental tradeoff between the achievable capacity maximization and the affordable capacity loss $C_{p, loss}$ at the PU-Rx when different sensing
threshold values are considered. Hence, the PU’s capacity loss $C_{p,\text{loss}}$ can act as a factual quality of service metric that can be satisfied by properly adapting the sensing threshold. Finally, the lower the sensed SNR $\gamma$, the lower the sensing threshold is become which brings about maximum capacity loss.

5 Conclusion

In this paper, we study the capacity optimization for sensing-based cognitive radio networks over sensing threshold. In particular, we consider a sensing-based spectrum sharing CRN in which both power control and spectrum sensing are employed for the PU’s protection. The proposed optimization is proved to be a convex optimization problem that we solve using the Lagrange dual decomposition method. A subgradient iterative algorithm provides the optimum values for both transmit power and sensing threshold of the power control and spectrum sensing, respectively. The numerical results show the SU’s capacity maximization achieved through sensing threshold adaptation and the corresponding capacity loss that can be afforded at the PU.

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