Modelling coils system for generating homogeneous magnetic field

V Ogay, P Baranov and A. Stepankova
National Research Tomsk Polytechnic University, Tomsk, 634050, Russia
E-mail: bpf@tpu.ru

Abstract. Magnetometers are used for measuring the characteristics of magnetic field and magnetic properties of the material. A changeable source of a homogeneous magnetic field is necessary for verification and calibration of magnetometers. Often the Helmholtz coil is used for generating magnetic homogeneous field, but homogeneous field area generated by Helmholtz coils is confined to a small volume in the center of the coils. The paper describes result of modeling a coils system to generate a homogeneous magnetic field with increased volume in comparison to Helmholtz coils.

1. Introduction
Magnetometers are used for measuring the characteristics of magnetic field and magnetic properties of the material.

Depending on lockable physical quantities, there are magnetometers determining the field strength gradient field, the direction of the field, magnetic flux, magnetic flux density, coercive force, magnetic permeability, magnetic susceptibility, magnetic moment.

The main metrological characteristics of magnetometers are:
- Sensitivity – is the minimum value of the magnetic field induction, fixed measuring instrument. This parameter is expressed in most of cases in nT (nanoTeslas);
- Resolution – is determined by the minimal difference of induction, which can register the instrument.

There are magnetometers that measure the absolute values of the field characteristics and their relative changes in time or in area. A changeable source of a homogeneous field is necessary for verification and calibration of magnetometers [1-4].

2. The coils system for generating homogeneous field
The system of solenoid coils or Helmholtz coils [5], shown in figure 1 is often used for generating homogeneous field.

The Helmholtz coils are two concentric rings of radius $R$ arranged at a distance $R$ from each other as shown in figure 1. The coils are connected in series with the current source to create a homogeneous field. According to the law of Biot-Savart-Laplace, the resulting field of the two coils is equal to the vector sum of the fields generating by each coil. The axial field of one coil of a predetermined point can be calculated according to the equation (1):

$$B(z) = \frac{\mu_0 N I R^2}{2(R^2 + (z-h)^2)^{2/3}},$$  (1)
where $\mu_0$ – magnetic permeability of vacuum, H/m;
$N$ – the number of turns of the each coil;
$I$ – current through the coils, A;
$z$ – coordinate of the point, m;
$h$ – the distance of the coil center from the beginning of the coil center coordinates, m.

Figure 1. Helmholtz coils

Axial field of two coils can be calculated by formula (2):

$$B(z) = \frac{1}{2} \mu_0 N I R^2 \left\{ \left[ R^2 + \left( z + \frac{R}{2} \right)^2 \right]^{\frac{3}{2}} + \left[ R^2 + \left( z - \frac{R}{2} \right)^2 \right]^{\frac{3}{2}} \right\}. \quad (2)$$

Formula (2) can be expanded in a Taylor series:

$$B(z) = B(0) + \frac{1}{2} B^{(2)}(0) z^2 + \frac{1}{24} B^{(4)}(0) z^4 + \ldots. \quad (3)$$

It is easy to show that the choice of the distance between the coils is equal to their radius, similar to the condition $B^{(2)}(0) = 0$.

Homogeneous field area generating by Helmholtz coils is confined to a small volume in the center of the coils. For example, field uniformity ≤ 1% for the two coils with the radius 55 mm is achieved only for distances up to 17 mm from its center. The radiuses of the coils are often increased to increase the amount of homogeneous field. However, the same effect can be achieved by increasing the number of coils in the system [6].

Coil system offered in [6] consists of three coils of different radius with a certain ratio of number of turns in the coils. Manufacturing such a system needs a complicated frame. The system of three coils of the same radius is offered in [7]. The system of three coils requires the following conditions:

$$B^{(2)}(0) = 0; \quad B^{(4)}(0) = 0. \quad (4)$$

Further increase of the homogeneity of the field can be increased by increasing the number of coils in the system [7]. In this case, the conditions (4) are added to a growing number of even derivatives equal to zero in the center of the coil system. The axial field for multi-coil system can be calculated using the equation (5):
The first term in the expression (5) corresponds to the coil coordinate $z = 0$, the other terms correspond to the remaining pairs of coils arranged at the same distance from the origin and having the same number of turns.

For the coils system with the even number of coils, the first coil member of equation (5) is zero. In calculating such a system of coils, it is necessary to find $n/2$ of unknown distances to the center of the coils and $(n-2)/2$ of the unknown number of turns. The system of nonlinear differential equations (6) is compiled and calculated to find the unknown parameters:

$$
\begin{align*}
B^{(2)}(0) &= 0; \\
B^{(4)}(0) &= 0; \\
\vdots \\
B^{(2(n-1))}(0) &= 0.
\end{align*}
$$

The number of coils has been chosen to be eight, based on the requirements for the volume of the region of homogeneous axial field and acceptable difficulty of manufacturing the coils. The system of equations has been obtained for an integer equal to the number of turns and the distance between the centers of the coils that are multiples of 0.5 mm.

The calculation results given in Table 1.

**Table 1.** Parameters of the eight-coil system to create a homogeneous magnetic field.

| Number coil | The number of turns | The distance from the coil center to the origin, (mm) |
|-------------|---------------------|---------------------------------------------------|
| 1           | 10                  | 13,5                                              |
| 2           | 12                  | 42,5                                              |
| 3           | 17                  | 80,5                                              |
| 4           | 34                  | 146                                               |
| 5           | 10                  | −13,5                                             |
| 6           | 12                  | −42,5                                             |
| 7           | 17                  | −80,5                                             |
| 8           | 34                  | −146                                              |

The calculated value of the magnetic induction in the center of the coil system at a current of 50 mA is 22,445 µT.

**3. Modeling of the coil system**

Software COMSOL Multiphysics 4.3b is used for modeling coils system. Figure 2 shows the geometry of the coils system created in COMSOL.

The calculation of the magnetic field produced in the module mf as only it allows you to use the section Multi-Turn Coil Domain coils for modeling coils in 3D. The coils have closed geometry with a circular cross-section perpendicular to the axis z, so, Circular type of coil was chosen. The direction of
current flow in this case is modeled by specifying the border (reference edge), along which the current flows (Figure 3).

![Figure 2](image)

**Figure 2.** The system of 8 axial coils in COMSOL

![Figure 3](image)

**Figure 3.** The assignment of directing the current flow

Modeling results of the coils system at the DC current of 0.05 A are shown in figure 4 and figure 5. Inhomogeneity of the magnetic field was calculated by formula:

$$
\delta(z) = \frac{B(z) - B(0)}{B(0)} \cdot 100 \%
$$

(7)
Figure 4. The distribution of the magnetic field

Figure 5. The inhomogeneity of the magnetic field
4. Conclusion
A changeable source of a homogeneous magnetic field is necessary for calibration of magnetometers. Homogeneous field area generating by Helmholtz coils is confined to a small volume in the center of the coils.

As a result of work describes coils system to generating a homogeneous magnetic field with increased volume in comparison Helmholtz coils. Parameters of eight coils system to generating a homogeneous magnetic field are calculated. Modeling results of eight coils system showed that the homogeneity of the field created by a system of eight coils of 110 mm diameter is not less than 0.1 % at distances up to 27.5 mm from the center of the system.

The obtained results show that such a system can be used to calibrate the magnetometers.

References
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