Remarks on a $B \wedge F$ model with topological mass from gauging spin

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Abstract – Aspects of screening and confinement are reassessed for a $B \wedge F$ model with topological mass with the gauging of spin. Our discussion is carried out using the gauge-invariant, but path-dependent, variables formalism. We explicitly show that the static potential profile is the sum of a Yukawa and a linear potential, leading to the confinement of static external charges. Interestingly enough, similar results are obtained in a theory of antisymmetric tensor fields that results from the condensation of topological defects as a consequence of the Julia-Toulouse mechanism.

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Introduction. – The mass generation mechanism in gauge theories is one of the most striking and fascinating quantum phenomenon, which has been of great interest since the work of Stückelberg [1]. Mention should be made, at this point, to the renowned Higgs mass generation mechanism, which was theoretically predicted in 1964 [2–4] and experimentally confirmed in 2012 [5,6].

In this connection, it becomes of interest, in particular, to recall the Schwinger model or Quantum Electrodynamics in $(1 + 1)$ dimensions [7] due to some very interesting and peculiar properties that it possesses, like fermion confinement and the energy spectrum contains a massive mode in spite of the gauge invariance of the original Lagrangian. We also recall that $B \wedge F$ models, a Chern-Simons–like formulation with antisymmetric tensor fields, also experience mass generation [8–10].

Meanwhile, in previous works [11,12], we have considered the confinement vs. screening properties of some theories of massless antisymmetric tensors, magnetically and electrically coupled to topological defects that eventually condensate, as a consequence of the topological sector of quantum chromodynamics (QCD). Our results show that the static potential profile contains a linear term leading to the confinement of probe charges, exactly as in the original model in $(1 + 1)$ dimensions. It should be further noticed that the same 4-dimensional model also appears as one version of the $B \wedge F$ models in $(3 + 1)$ dimensions under dualization of Stückelberg-like massive gauge theories [16]. Interestingly, this particular model is characterized by the mixing between a $U(1)$ field and an Abelian 3-form field of the type that appears in the topological sector of quantum chromodynamics (QCD).

On the other hand, we further note that recently a novel way to induce the $B \wedge F$ term has been considered in [17]. The crucial ingredient of this development is that the $B \wedge F$ term is generated at one loop by coupling the antisymmetric gauge field to the vorticity of charged fermions.

Let us also recall here that the confining phase of gauge theories can be formulated in the context of string theories. In fact, this confining string theory has been first derived for a 4D compact $U(1)$ gauge theory, where the key ingredient is the contribution of a condensate of magnetic monopoles to the partition function of the theory [18]. However, in the present work the mechanism of confinement is not based on the condensation of topological excitations, but from gauging spin. This makes the...
In such a situation, and letting \( Z_\sigma \equiv A_\sigma + \frac{e}{m} \partial_\sigma \chi \), with \( Z_{\mu\nu} = F_{\mu\nu} \), eq. (3) reduces to

\[
\mathcal{L} = -\frac{1}{4} Z_{\mu\nu}^2 - \frac{1}{2} \tilde{H}_\sigma \tilde{H}^\sigma - \frac{m}{6} \tilde{H}^\sigma Z_\sigma.
\]  

Then, eq. (1) can be written in the form

\[
\mathcal{L} = -\frac{1}{4} Z_{\mu\nu}^2 + \frac{1}{2} \mu^2 Z_{\mu\nu}^2,
\]

where we have made use of \( W_\sigma \equiv \tilde{H}_\sigma + \frac{m}{\mu} Z_\sigma \) and \( \mu^2 \equiv m^2/|e| \). Thus, finally we end up with a Maxwell-Proca theory.

Accordingly, this effective model provides us with a suitable starting point to study the interaction energy. Next, we also notice that before proceeding with the determination of the energy, it is necessary to restore the gauge invariance in (5). Following an earlier procedure [11,12], we may express eq. (5) as

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} \left( 1 + \frac{\mu^2}{\Delta} \right) F^{\mu\nu},
\]

where \( \Delta = \partial_\mu \partial^\mu \).

Having established the new effective Lagrangian, we can now compute the interaction energy. To this end, we first consider the Hamiltonian framework of this new effective theory. The canonical momenta are found to be \( \Pi^\mu = -(1 + \frac{\mu^2}{\Delta}) F^{0\mu} \). This yields the usual primary constraint \( \Pi^0 = 0 \), and \( \Pi^i = -(1 + \frac{\mu^2}{\Delta}) F^{0i} \). Therefore the canonical Hamiltonian is

\[
H_C = \int d^3 x \left\{ -A_0 \partial_0 \Pi^i - \frac{1}{2} \Pi_i \left( 1 + \frac{\mu^2}{\Delta} \right)^{-1} \Pi^i \right\}
\]

\[
+ \int d^3 x \left\{ \frac{1}{4} F_{ij} F^{ij} \right\}.
\]

Temporal conservation of the primary constraint \( \Pi_0 \) leads to the secondary constraint \( \Gamma_1(x) \equiv \partial_0 \Pi = 0 \). It can be easily seen that there are no further constraints in the theory. According to the general theory, we obtain the extended Hamiltonian by adding all the first-class constraints with arbitrary constraints. We thus write \( H = H_C + \int d^3 x (c_0(x) \Pi_0(x) + c_1(x) \Gamma_1(x)) \), where \( c_0(x) \) and \( c_1(x) \) are the Lagrange multipliers. Moreover, it follows from this Hamiltonian that \( A_0(x) = [A_0(x), H] = c_0(x) \), which is an arbitrary function. Since \( \Pi_0 = 0 \), neither \( A^0 \) nor \( \Pi^0 \) are of interest in describing the system and may be discarded from the theory. In fact, the term containing \( A_0 \) is redundant, because it can be absorbed by redefining the function \( c'(x) \). Therefore, the Hamiltonian is now given as

\[
H = \int d^3 x \left\{ -\frac{1}{2} \Pi_i \left( 1 + \frac{\mu^2}{\Delta} \right)^{-1} \Pi^i + \frac{1}{4} F_{ij} F^{ij} \right\}
\]

\[
+ \int d^3 x \{ c'(\partial_0 \Pi) \},
\]

where \( c'(x) = c_1(x) - A_0(x) \).
Since there is one first-class constraint $\Gamma_1(x)$, we choose one gauge condition that makes the full set of constraints to become second class. A particularly convenient choice is

$$\Gamma_2(x) \equiv \int_{C_x} dz^\nu A_\nu(z) \equiv \int_0^1 d\lambda x^i A_i(\lambda x) = 0. \quad (9)$$

where $\lambda (0 \leq \lambda \leq 1)$ is the parameter describing the spacelike straight path $z^i = \xi^i + \lambda (x - \xi)^i$, and $\xi$ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^0 = 0$. As a consequence, the only nontrivial Dirac bracket for the canonical variables is given by

$$\{A_i(x), \Pi^j(y)\}^* = \delta_i^j \delta^{(3)}(x - y) - \partial_i^* \int_0^1 d\lambda x^i \delta^{(3)}(\lambda x - y). \quad (10)$$

We are now in the position to calculate the interaction energy for the model under consideration. For this purpose, we shall compute the expectation value of the energy operator $H$ in the physical state $|\Phi\rangle$. In this context, we recall that the physical state $|\Phi\rangle$ can be written as $|20]$:

$$|\Phi\rangle \equiv |\Psi(y)\rangle |\Psi(y')\rangle = \overline{\psi}(y) \exp \left(iq \int_y^{y'} dz^i A_i(z)\right) \psi(y') |0\rangle, \quad (11)$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a spacelike path starting at $y'$ and ending at $y$, on a fixed time slice.

Taking the above Hamiltonian structure into account, we see that

$$\Pi_i(x) = \overline{\Psi}(yt) \Psi(y') |\Pi_i(xt)|0\rangle + q \int_y^{y'} dz^i \delta^{(3)}(z - x) \langle \Phi \rangle. \quad (12)$$

Having pointed out this observation, and since the fermions are taken to be infinitely massive (static sources), we can substitute $\Delta = -\nabla \cdot \nabla$ in eq. (8). Therefore, the interaction energy takes the form

$$\langle H \rangle_\Phi = \langle H \rangle_0 + V_1, \quad (13)$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$. The $V_1$ term is given by

$$V_1 = \langle \Phi | \int d^3x \left[-\frac{1}{2} \Pi_i \left(1 - \frac{\mu^2}{\nabla^2}\right)^{-1} \Pi_i\right] |\Phi\rangle. \quad (14)$$

Following our earlier procedure $[11,12]$, we see that the static potential takes the form

$$V = \frac{q^2}{4\pi} \varepsilon^{-\mu L}, \quad (15)$$

where $L = |y - y'|$.

An alternative way of stating the previous result is by considering the expression $[19]

$$V \equiv q(A_0(0) - A_0(L)), \quad (16)$$

where the physical scalar potential is given by

$$A_0(t, r) = \int_0^1 d\lambda x^i E_i(t, \lambda r). \quad (17)$$

This equation follows from the vector gauge-invariant field expression

$$A_\mu(x) \equiv A_\mu(x) + \partial_\mu \left( -\int_\xi^3 dz^\nu A_\nu(z) \right), \quad (18)$$

where the line integral is along a spacelike path from the point $\xi$ to $x$, on a fixed slice time. It should again be stressed here that the gauge-invariant variables (17) commute with the sole first constraint (Gauss' law), showing in this way that these fields are physical variables.

We also recall that Gauss' law for the present model reads

$$\partial_\nu \Pi^\nu = J^0, \quad (19)$$

where, $J^0$, is the external source. It should be further recalled that, $E = \nabla \times A$, and for, $J^0(x) = q\delta^{(3)}(x)$, we can express (17) as

$$A_\mu(t, x) = \int_0^1 d\lambda x^i \left(1 - \frac{\mu^2}{\nabla^2}\right)^{-1} \Pi_i(\lambda x). \quad (20)$$

With the aid of eqs. (16) and (20), we readily find that the interaction energy for a pair of static point-like opposite charges located at $0$ and $L$ is given by

$$V = \frac{q^2}{4\pi} \varepsilon^{-\mu L}, \quad (21)$$

after subtracting a self-energy term.

$B \wedge F$ model with topological mass from gauging spin. – As already stated, our next undertaking is to use the ideas of the previous section in order to consider $B \wedge F$ model with topological mass with the gauging of spin. For this purpose, the authors of ref. [17] consider the four-dimensional space-time Lagrangian density:

$$\mathcal{L} = \bar{\psi} \gamma^\mu (i\partial_\mu + eA_\mu)\psi - m\bar{\psi}\psi + gbB_{\mu\nu}J^{\mu\nu} - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda}. \quad (22)$$

The crucial idea underlying this suggestion consists in proposing the interaction Lagrangian in the form

$$\mathcal{L}^{int} \equiv gbB_{\mu\nu}J^{\mu\nu} = \frac{2mg}{\Delta} F_{\mu\nu}J_\mu, \quad (23)$$

where $F_\mu = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} B^{\nu\alpha\beta}$. Given its relevance, it is of interest to study the effect of the above scenario on a physical observable. We also note here that integrating out the
fermionic field in (21) induces an effective model for the $A_\mu$ and $B_{\mu\nu}$ fields. Hence, one gets the following effective Lagrangian density:

$$\mathcal{L} = -\frac{1}{4e_{ph}^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} + \frac{mg_{ph}}{2\pi} A_\mu F^\mu$$

$$- \frac{6m^2a_{ph}}{\ln \frac{\Lambda^2}{m^2}} F_{\mu}^\mu \frac{1}{\Delta} F^\mu + \chi \partial_{\mu} F^\mu,$$

(24)

where $\chi$ is the Lagrange multiplier to take into account the Bianchi identity. Whereas $\frac{1}{\gamma_{ph}} = e^2(1 + \frac{\Lambda}{\ln \frac{\Lambda^2}{m^2}})$ and $g_{ph} = \frac{4\pi}{\ln \frac{\Lambda^2}{m^2}}$. Here, we have made use of the same notation as in ref. [17].

Before we proceed further, we shall pause to mention that the non-local $F_{\mu}^\mu F^\nu$ term, which arises from the coupling between the fermionic spin current and the rank-2 field, $B_{\mu\nu}$,

$$g B_{\mu\nu} J^{\mu\nu} = -\frac{g}{\Delta} F_{\mu}^\mu I_{\mu},$$

(25)

may actually be seen as corresponding to a non-minimal coupling between the fermion and the dual of the field-strength $H_{\mu\nu\rho}$. We could, from the very beginning, have started off with the non-minimal coupling accommodated in the $U(1)$ electromagnetic covariant derivative

$$D_\mu \Psi = \partial_\mu \Psi + ieA_\mu \Psi + \frac{g}{m} F_{\mu\nu} \gamma^\nu \Psi,$$

(26)

with $g$ dimensionless. This however would not generate the $F_{\mu}^\mu F^\nu$ term in its non-local form $F_{\mu}^\mu F^\nu$, which is crucial for the confining behavior of the potential, as we shall see more explicitly below. We then agree with the choice of the non-local formulation, as the authors of ref. [17] propose.

Next, in the same way as was done in the previous section, after integrating out the $B_{\mu\nu}$ field in favor of the $A_\mu$ field, the effective Lagrangian density reduces to

$$\mathcal{L} = -\frac{1}{4e_{ph}^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left( \frac{mg_{ph}}{2\pi} \right)^2 A_\mu \left( \frac{1}{\Delta + \gamma} \right) A^\mu,$$

(27)

where $\gamma = \frac{12m^2a_{ph}}{\ln \frac{\Lambda^2}{m^2}}$.

As in our preceding discussion, we now restore the gauge symmetry in eq. (24). This allows us to write the Lagrangian density as

$$L = -\frac{1}{4} F_{\mu\nu} \left( 1 + \frac{m^2a_{ph}^2}{4\pi^2} \gamma \right) F^{\mu\nu}.$$

(28)

Following the same steps that lead to (15), the static potential for two opposite charges located at $\mathbf{y}$ and $\mathbf{y}'$ becomes

$$V = -\frac{g^2}{4\pi} e^{-M L} L + g^2 \gamma \ln \left( 1 + \frac{R^2}{M^2} \right) L,$$

(29)

where $L = |\mathbf{y} - \mathbf{y}'|$, $M^2 = m^2g_{ph}^2 \left( \frac{\gamma^2}{\Delta^2} + \frac{\Delta}{\ln \frac{\Lambda^2}{m^2}} \right)$ and $\Gamma$ is an ultraviolet cutoff. It is of interest also to notice that Lagrangians (24) and (27) are effective descriptions with cutoff $\Lambda$. So, our results are valid up to the energy scale $\Lambda$. Now, the potential (29) must also be restricted to the same cutoff ($\Lambda$); therefore, it is sensible to identify the cutoff $\Gamma$, which appears in the derivation of the potential, with the cutoff $\Lambda$ ($\Lambda, \Gamma \gg m$). Thus, we finally obtain that the static potential is given by

$$V = -\frac{g^2}{4\pi} e^{-\frac{ML}{L}} + g^2 \gamma \ln \left( 1 + \frac{\Lambda^2}{M^2} \right) L.$$

(30)

The above static potential profile displays the conventional screening part, encoded in the Yukawa potential, and the linear confining potential. Accordingly, one of the most startling predictions of the interaction Lagrangian (23) is the existence of a confining potential. Incidentally, it is of interest to notice that in the limit $m \rightarrow 0$ the confinement disappears, which clearly shows the role played by the fermions. It may be noted here that the confining potential of this model has been reported before [21]. However, in spite of their relevance, this result was obtained in a gauge-fixed scheme, and we think that this result should be corroborated by a gauge-independent analysis.

We may parenthetically note here that we can recover the previous result via a path-integral approach. In fact, the initial point of our analysis is the Lagrangian density:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \left( 1 + \frac{\mu^2}{\Delta + \gamma} \right) F^{\mu\nu} - A_\mu J^\mu,$$

(31)

where $\mu^2 = \frac{m^2a_{ph}^2}{4\pi^2}$ and $\gamma = \frac{12m^2a_{ph}}{\ln \frac{\Lambda^2}{m^2}}$. We now re-examine the interaction energy between static point-like sources for the present electrodynamics. For this purpose, we begin by writing down the functional generator of the Green's functions, that is,

$$Z[J] = \exp \left( -\frac{i}{2} \int d^4x d^4y J^\mu (x) D_{\mu\nu} (x,y) J^\nu (y) \right),$$

(32)

where $D_{\mu\nu} (x,y) = \int \frac{dk}{(2\pi)^4} D_{\mu\nu} (k) e^{-ikx}$, is the propagator in the Feynman gauge. In this case, the corresponding propagator is given by

$$D_{\mu\nu} (k) = -\frac{1}{k^2(k^2 - \gamma - \mu^2)} \left\{ (k^2 - \gamma) \eta_{\mu\nu} - \mu^2 \frac{k_\mu k_\nu}{k^2} \right\}.$$  

(33)

By means of expression $Z = e^{iW[J]}$, and employing eq. (32), $W[J]$ takes the form

$$W[J] = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} J^\mu (k) \left[ -\frac{(k^2 - \gamma)}{k^2(k^2 - \gamma - \mu^2)} \right] J_\nu (k) - \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} J^\mu (k) \left[ \frac{\mu^2}{(k^2 - \gamma - \mu^2)} \right] J_\nu (k).$$  

(34)
Since the current $J^\mu(k)$ is conserved, expression (34) then becomes

$$W[J] = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} J^\mu(k) \frac{\left(k^2 - \gamma \eta_{\mu\nu}\right)}{k^2(k^2 - \gamma - \mu^2)} J^\nu(k).$$

(35)

For $J^\mu(x) = Q\delta^{(3)}(x - x^{(1)})\delta_0^\mu + Q\delta^{(3)}(x - x^{(2)})\delta_0^\mu$, and using standard functional techniques, we obtain that the interaction energy of the system is given by

$$U(r) = QQ' \int \frac{d^3k}{(2\pi)^3} \frac{(k^2 + \gamma)(k^2 + \gamma + \mu^2)}{k^2} e^{ikr},$$

(36)

where $r \equiv x^{(1)} - x^{(2)}$.

a) For $Q' = -Q$ and $\gamma = 0$, we readily verify that the interaction energy is given by

$$U(r) = -Q^2\frac{\epsilon^{-\mu r}}{4\pi r}.$$  

(37)

with $r = |r|$. An immediate consequence of this result is that in the limit, $\mu r \ll 1$, the interaction energy transforms into a Coulombic one. It is worth noting that this approach, despite being completely different, leads to the same result obtained above.

b) We shall now consider the $\gamma \neq 0$ case, namely,

$$U(r) = -Q^2\frac{\epsilon^{-\mu r}}{4\pi r} - \gamma Q^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2(k^2 + \gamma + \mu^2)} e^{ikr}.$$  

(38)

Considering the limit, $\mu r \ll 1$, we can write

$$U(r) = -Q^2\frac{1}{4\pi r} - \gamma Q^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} e^{ikr}.$$  

(39)

We may accordingly rewrite eq. (39) in the form

$$U(r) = -\frac{Q^2}{4\pi r} + \gamma Q^2 \frac{\epsilon^{-\mu r}}{2\pi r},$$

(40)

which is similar to the result obtained in ref. [21].

Finally, there is a further strand about this result when compared to the result obtained above. It is clear that, under the assumed condition, the logarithmic term of our paper does not appear. The reason is simple, the method used in our paper naturally incorporates a regime where a cut-off appears, with the parameters of the model, providing an effective result that absorbs the effects of the previous physical cut-off. In summary then, notice that we can choose $\Gamma = 0.2014 M$, where $M^2 = m^2 g_{ph} (\frac{\epsilon}{4\pi} + \frac{12}{16\pi^2} M^2)$. As $\Gamma$ is arbitrary, we should choose this value and, in this way, we show the equivalence between both calculations.

Interestingly enough, the above static potential profile is analogous to that encountered in a theory of antisymmetric tensor fields that results from the condensation of topological defects as a consequence of the Julia-Toulouse mechanism [11,12]. It is worth recalling here that the Julia-Toulouse mechanism is a condensation process dual to the Higgs mechanism proposed in [14], which describes phenomenologically the electromagnetic behavior of antisymmetric tensors in the presence of magnetic branes (topological defects) that eventually condensate due to thermal and quantum fluctuations. Exploiting the previous phenomenology, we have studied in [11,12] the dynamics of the extended charges ($p$-branes) inside the new vacuum provided by the condensate. More specifically, in [11] we have considered the topological defects coupled both longitudinally and transversely to two different tensor potentials, $A_p$ and $B_q$, such that $p + q = 2D$, where $D = d + 1$ space-time dimensions. To be more precise, after the condensation the phenomenological Lagrangian density [11] is given by

$$\mathcal{L} = \frac{(-1)^q}{2(q+1)!} [H_{q+1}(B_q)]^2 + c B_q \varepsilon^{\alpha \beta \gamma \delta} \partial_\alpha A_{p+1}$$

$$+ \frac{(-1)^{p+1}}{2(p+2)!} [F_{p+2}(A_{p+1})]^2$$

$$- \frac{(-1)^{p+1} (p+1)!}{2} m^2 A_{p+1}^2,$$

(41)

which reveals a $B \wedge F$ type of coupling between the $B_q$ potential with the tensor $A_{p+1}$ carrying the degrees of freedom of the condensate. In fact, following the procedure described in [11], we can further obtain the effective theory that results from integrating out the fields representing the vacuum condensate in the manner

$$\mathcal{L} = \frac{(-1)^{q+1}}{2(q+1)!} H_{q+1}(B_q) \left(1 + \frac{e^2}{\Delta + m^2}\right) H^{q+1}(B_q).$$

(42)

From eq. (42) it now follows that for $p = 1$ and $q = 1$, the effective theory can be brought to the form

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} (A) \left(1 + \frac{e^2}{\Delta + m^2}\right) F^{\mu\nu} (A).$$

(43)

It is a simple matter to verify that eq. (42) reduces to eq. (28). With this then, we now see that the model studied here (28) may be considered as some kind of effective theory that incorporates automatically the contribution of the condensate of topological defects to the vacuum of the model.

It follows from the above discussion a new connection among different effective theories, which are of interest from the point of view of providing unifications among diverse models as well as exploiting their equivalence in explicit calculations, as we have illustrated in this work.

**Final remarks.** – In summary, we have considered aspects of screening and confinement for a $B \wedge F$ model with topological mass with gauging spin. It was shown that the static potential profile is the sum of a Yukawa and a linear potential, leading to the confinement of static external charges. To do this, once again we have exploited a key aspect for understanding the physical contents of gauge theories, that is, the correct identification of field degrees.
of freedom with observable quantities. We point out that our analysis reveals that similar results are obtained in a theory of antisymmetric tensor fields that results from the condensation of topological defects as a consequence of the Julia-Toulouse mechanism. Finally, it should be emphasized that an interesting feature of the present approach is to provide connections among different models. As we have already noticed, these connections show us a new sort of “duality” among diverse models and allow us to use this equivalence in concrete calculations, as we have illustrated in the present work.

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