Exact effective action for N=1 supersymmetric theories.

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Abstract

We investigate nonperturbative effects in N=1 supersymmetric theories and propose a new expression for the effective action, which correctly reproduces quantum anomalies and agrees with the transformation law of instanton measure. Actually the result is a nonperturbative extension of Veneziano-Yankielowitch effective Lagrangian. The possibility of integrating out the gluino condensate is discussed.

1 Introduction

Investigation of nonperturbative dynamics is a very important problem of quantum field theory. It is well known [1, 2], that except for perturbative corrections there is a series of instanton contributions. Their sum was found exactly in [3] for N=2 supersymmetric Yang-Mills theory with $SU(2)$ gauge group in the constant field limit. It was checked, that the asymptotic of exact result reproduced one-instanton contribution correctly [4, 5].

Attempts to construct exact results for N=1 SUSY theories were made in [6, 7]. However, the corresponding results do not agree with instanton calculations. (In the case $N_f = N_c - 1$ the agreement takes place only at the one-instanton level [8], while higher instanton corrections break it.) Moreover, Affleck-Dine-Seiberg results do not produce correct expressions for anomalies, because they do not contain any gauge degrees of freedom.

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In principle, at the perturbative level the anomalies is correctly reproduced by the Veneziano-Yankielowitch effective Lagrangian \[8\], containing gluino condensate. However, this result is not applicable beyond the frames of perturbation theory.

The purpose of the present paper is to obtain the exact (nonperturbative) effective Lagrangian for \(N=1\) SUSY Yang-Mills theory with matter. It should correctly reproduce anomalies, be in agreement with instanton calculations (at least with the transformation law of the collective coordinate measure) and have a structure similar to the \(N=2\) case (because \(N=2\) SUSY theories have also \(N=1\) SUSY).

In order to do it we will use the method, based on the consideration of quantum anomalies \[9\] beyond the frames of perturbation theory. The first exact expression was found recently for R-anomaly in \(N=2\) supersymmetric Yang-Mills theory \[10, 11, 12, 13\]. Due to the instanton contributions it differs from the perturbative result. \[1\]

The derivation was based on the Seiberg and Witten exact expression \[3\], but the result appeared to have a very simple interpretation: exact anomaly is a vacuum expectation value of the perturbative one. Nevertheless, for checking this relation one should essentially use the exact prepotential, found in \[3\] by completely different methods. Thus, we come to the question, whether it is possible to solve the inverse problem, i.e. to derive exact results from the form of anomalies. This idea really allows to derive Seiberg-Witten solution in \(N=2\) SUSY \(SU(2)\) Yang-Mills theory and the general structure of Picard-Fuchs equations in other \(N=2\) SUSY theories (first obtained in \[15, 16\] by other methods) \[17\].

In the present paper the corresponding approach is applied to \(N=1\) supersymmetric Yang-Mills theories.

Our paper is organized as follows:

In Section 2 we briefly discuss the relation between perturbative and exact anomalies, which is used in Section 3 to derive the exact effective Lagrangian. First, in Subsections 3.1 we apply it (together with the results of Appendix B and D) to investigate the general structure of the superpotential. The exact result is obtained in Subsection 3.2. In Section 4 we discuss the possibility of integrating out the gluino condensate and equivalence of the obtained expression and Seiberg’s exact results. Conclusion is devoted to the brief review and discussion of the results. In the Appendix A we review the necessary information concerning supersymmetric theories and summarize our notations. Then, in Appendix B we investigate the structure of nonperturbative corrections, that agrees with the transformation law of the collective coordinate measure under \(U(1)_x\) transformations. In Appendix C we briefly review the structure of moduli space for \(N=1\) supersymmetric theories, that is used in Appendix D to rewrite the effective action in the gauge invariant form.

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1 Such possibility was pointed rather long ago \[14\], but a series of instanton corrections with unknown coefficients produced considerable difficulties.
2 The relation between perturbative and exact anomalies

Now let us consider \( N=1 \) supersymmetric \( SU(N_c) \) Yang-Mills theory with \( N_f \) matter supermultiplets and find the general structure of the effective superpotential and anomalies. (Our notation are summarized in Appendix A.)

The effective Lagrangian can be split into the following parts \[ L_{\text{eff}} = L_k + L_a + L_m, \] (1)

where

\[ L_k = \int d^4\theta \ K(S, S^*, \Phi, \Phi^*); \]
\[ L_a = \text{Re} \int d^2\theta \ w(S, \Phi) \] (2)

and \( S \equiv \text{tr} W^2 \) is the gluino condensate.

Here \( L_k \) denotes kinetic term, that does not contribute to the anomaly, \( L_a \) is a holomorphic part of the superpotential and \( L_m \) is a mass term. Below we will consider only massless case (\( L_m = 0 \)). Therefore, the only nontrivial contribution to anomalies comes from \( L_a \) and it is the only part, that we are able to investigate. (Our method cannot give any information about a possible kinetic term.)

In order to define \( L_a \) we consider the anomaly of \( U(1)_x \)-symmetry (for more details see Appendix A)

\[ U(1)_x : \quad W(\theta) \rightarrow e^{i\alpha}W(e^{-i\alpha \gamma_5} \theta); \]
\[ \phi(\theta) \rightarrow e^{i\alpha} \phi(e^{-i\alpha \gamma_5} \theta); \]
\[ \tilde{\phi}(\theta) \rightarrow e^{i\alpha} \tilde{\phi}(e^{-i\alpha \gamma_5} \theta) \] (3)

beyond the frames of perturbation theory. The result can be found by using the relation between perturbative and exact anomalies.

Of course, exact anomalies are quite different from perturbative ones. For example, the exact expression for R-anomaly in the \( N=2 \) SUSY \( SU(2) \) Yang-Mills theory, found by transforming Seiberg-Witten effective action [10, 18], is

\[ \langle \partial_{\mu} j_{R}^{\mu} \rangle = \frac{1}{16\pi} \text{Re} \int d^2\theta_1 d^2\theta_2 (F + F_D) = \frac{1}{8\pi^2} \text{Im} \int d^2\theta_1 d^2\theta_2 u. \] (4)

where \( u \equiv \frac{1}{2} \langle \Phi^2 \rangle \) and \( \Phi \) is \( N=2 \) superfield

\[ \Phi(y, \theta_1, \theta_2) = \phi(y, \theta_1) - i\tilde{\theta_2}(1 + \gamma_5)W(y, \theta_1) + \frac{1}{2} \tilde{\theta_2}(1 + \gamma_5)\theta_2 G(y, \theta_1); \]
\[ y^\mu = x^\mu + \frac{i}{2} \bar{\theta}_i \gamma^\mu \gamma_5 \theta_i; \]
\[ G(y, \theta_1) = \frac{1}{2} \int d^2 \bar{\theta}_1 e^{2V} \phi^+ e^{-2V}. \] (5)

Here we should attract attention to the easily verified identity
\[ F + F_D = -\frac{2i}{\pi} u \] (6)

that will be used below.

From the other side, in the perturbation theory
\[ A \equiv \langle \partial_\mu j_R^\mu \rangle_{\text{pert}} = \frac{1}{16\pi^2} \text{Im tr} \int d^2 \theta_1 d^2 \theta_2 \Phi^2. \] (7)

Of course, the expressions (4) and (7) are quite different. The former is a series over \( \Lambda^4 \) produced by instanton contributions. In particular, taking into account one instanton correction we have [4, 5]
\[ \langle \partial_\mu j_R^\mu \rangle = \frac{1}{16\pi^2} \text{Im tr} \int d^2 \theta_1 d^2 \theta_2 \left[ \Phi^2 + \frac{\Lambda^4}{2}\Phi^2 + O(\Lambda^8) \right], \] (8)

that in components can be written as
\[ \langle \partial_\mu j_R^\mu \rangle = \frac{e^2}{4\pi^2} \left( 1 + \frac{3\Lambda^4}{2e^2\varphi^2} \right) F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{3\Lambda^4}{e^2\pi^2 \varphi^5} F_{\mu\nu} \bar{\Psi}_D \Sigma_{\mu\nu} \gamma_5 \Psi_D \
\quad + \frac{60\Lambda^4}{e^2\pi^2 \varphi^6} (\bar{\Psi}_D \Psi_D) (\bar{\Psi}_D \gamma_5 \Psi_D) + O(\Lambda^8), \] (9)

where
\[ \tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \] (10)

and we introduced a Dirac spinor
\[ \Psi_D = \frac{1}{2} (1 + \gamma_5) \psi_1 + \frac{1}{2} (1 - \gamma_5) \psi_2. \] (11)

And nevertheless, the nonperturbative result is only a vacuum expectation value of the perturbative one, that in particular produces a natural solution of anomalies cancellation problem in the realistic models.

This result is not unexpected. Really, performing, for example, chiral transformation in the generating functional we have
\[ 0 = \left. \left[ \frac{\delta Z}{Z} \frac{\delta}{\delta \alpha} \right]_{\alpha=0} \right|_{\alpha=0} = \left. \frac{\delta}{\delta \alpha} \int D A D \bar{\psi}^j D \psi^j \exp \left( iS - \partial_\mu \alpha j_5^\mu \right) \right|_{\alpha=0} \]
\[ = \left. \frac{\delta}{\delta \alpha} \int D A D \bar{\psi}^j D \psi \exp \left( iS - \partial_\mu \alpha j_5^\mu - \alpha A \right) \right|_{\alpha=0} = \langle \partial_\mu j_5^\mu - A \rangle, \] (12)
where $A$ denotes the perturbative anomaly, produced by the measure noninvariance [19]. Finally

$$\langle \partial_\mu J^\mu_5 \rangle = \langle A \rangle. \quad (13)$$

(R-transformation are considered similarly).

It is just the relation, mentioned above. Of course, it is valid for a wide range of models and is really a point to start with. Let us note, that the derivation presented in [10] essentially used the form of exact results. So, we are tempted to reverse the arguments. In the next section we will try to apply this approach to N=1 SUSY theories.

3 Exact effective Lagrangian

3.1 General structure

Let us apply (13) to N=1 supersymmetric $SU(N_c)$ Yang-Mills theory with $N_f$ matter supermultiplets. At the perturbative level the anomaly of $U(1)_x$-symmetry has the following form

$$\partial_\mu J^\mu_x = \left( -N_f + N_c + xN_f \right) \frac{1}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{tr} F_{\mu\nu} F_{\alpha\beta}$$

$$= -\left( N_f - N_c - xN_f \right) \frac{1}{16\pi^2} \text{Im} \int d^2\theta \ W^2, \quad (14)$$

so that the exact anomaly is

$$\langle \partial_\mu J^\mu_x \rangle = -\left( N_f - N_c - xN_f \right) \frac{1}{16\pi^2} \text{Im} \int d^2\theta \ u \quad (15)$$

where $u \equiv \langle \text{tr} W^2 \rangle$. Therefore, the effective Lagrangian should depend in particular on $S = \text{tr} W^2$. This result is not new. At the perturbative level the similar investigation was made in [3]. However, in this paper we do not intend to restrict ourselves by the frames of perturbation theory. Therefore, we can not assume, that $u = \text{tr} W^2$ (Here we would like to remind [3]).

Taking into account the dependence on the gluino condensate and performing $U(1)_x$ transformation in the effective action, from the other side we obtain

$$\langle \partial_\mu J^\mu_x \rangle = -\frac{\partial \Gamma}{\partial \alpha} = -\text{Im} \int d^2\theta \left( 2w - 2 \frac{\partial w}{\partial S} S - x \frac{\partial w}{\partial v} v \right). \quad (16)$$

Comparing (15) with (16) and taking into account, that the equality should be satisfied for all $x$, we obtain
\[ 2w - 2 \frac{\partial w}{\partial S} S = \frac{1}{16\pi^2} (N_f - N_c) u; \]
\[ \frac{\partial w}{\partial v} v = \frac{1}{16\pi^2} N_f u. \]  

(17)

This equation is very similar to the results of [8]. Nevertheless, there is a crucial difference: \( u \neq S \). Therefore, one can only conclude that

\[ 2w - 2 \frac{\partial w}{\partial S} S - \frac{N_f - N_c}{N_f} \frac{\partial w}{\partial v} = 0. \]  

(18)

This equation corresponds to the exact conservation of R-symmetry at nonperturbative level. The similar condition was used in [3, 4], although the dependence \( w = w(S) \) was ignored. Of course, it is quite clear, that integrating \( S \) out yields ADS superpotential [20] and corresponds to imposing the condition

\[ \frac{\partial w}{\partial S} = 0. \]  

(19)

Nevertheless, this equation can have no solutions. In this case the gluino condensate can not be integrated out of the effective action and it is impossible to obtain Seiberg’s exact results. Below we will discuss equation (19) in details.

It is desirable, that the solution of (18) agrees with instanton calculations. The necessary condition of it is the agreement with the transformation law of the instanton measure. The presence of the gluino condensate in the effective action allows to achieve it. The possible structure of instanton corrections is analysed in the Appendix [3]. The result for n-instanton correction has the following form

\[ w^{(n)} = S g_n \left( v^{3 \frac{N_c}{S}} \right) \left( \frac{\Lambda}{v} \right)^{n(3N_c-N_f)} \]  

(20)

where \( g_n \) is an arbitrary function.

It can be easily verified, that the only solution of (18), agreeing with (20), is

\[ w = -\frac{1}{32\pi^2} S f(z); \quad f(z) = f_{\text{pert}}(z) + \sum_{n=1}^{\infty} c_n z^n, \]  

(21)

where

\[ z \equiv \frac{\Lambda^{3N_c-N_f}}{v^{2N_f} S^{N_c-N_f}} \]  

(22)

is a dimensionless parameter. [3]

\[ ^2 \text{It corresponds to } g_n(x) = \frac{1}{32\pi^2} c_n x^{n(N_c-N_f)}, \quad n \geq 1 \text{ in (20).} \]

\[ ^3 \text{Therefore, for } N_c = N_f + 1 \text{ first instanton correction does not contain gluino condensate, that allows to compare it with ADS-superpotential [4]. However, higher corrections depend on } S \text{ and destroy the agreement. So, the statement, that in this case instantons generate ADS-superpotential is not correct.} \]
In the final result $z$ should be written in terms of gauge invariant variables. Of course, the result will depend on the structure of moduli space, that is briefly reviewed in Appendix C. The derivation, made in Appendix D, gives

$$z = \frac{\Lambda^{3N_{c}-N_{f}}}{\det M S^{N_{c}-N_{f}}}, \quad N_{f} < N_{c};$$

$$z = \frac{\Lambda^{3N_{c}-N_{f}} S^{N_{f}-N_{c}}}{\det M - (\tilde{B}^{A_{1}A_{2}...A_{N_{f}-N_{c}}} M_{A_{1}} B_{1} M_{A_{2}} B_{2} ... M_{A_{N_{f}-N_{c}}} B_{N_{f}-N_{c}} B_{B_{1}B_{2}...B_{N_{f}-N_{c}}})},$$

$$N_{f} \geq N_{c}. \quad (23)$$

In order to define $f_{pert}$ we note, that at the perturbative level $u = S$. Therefore, in this case (17) gives

$$\frac{\partial f_{pert}}{\partial z} z = 1, \quad (24)$$

so that

$$w = -\frac{1}{32\pi^{2}} S \left( \ln z + \sum_{n=1}^{\infty} c_{n} z^{n} \right). \quad (25)$$

Substituting it to (17), we obtain

$$u = S \left( 1 + \sum_{n=1}^{\infty} n c_{n} z^{n} \right), \quad (26)$$

that defines all anomalies in the theory according to (15). At the perturbative level both (23) and (26) are certainly in agreement with [8].

3.2 Exact result

Let us define $f$ exactly. The general structure of the holomorphic superpotential, found in section 3.1, is similar to the structure of the nonperturbative prepotential in the N=2 supersymmetric Yang-Mills theory [21]. In the latter case the relation between perturbative and nonperturbative anomalies leads to Picard-Fuchs equations [17], that can be used for derivation of exact results. Is it possible to extend this approach to the case of N=1 supersymmetry?

First we substitute (21) into (17), that gives

$$S \frac{df}{dz} z = u \quad (27)$$

(and therefore $u/S$ depends only on $z$).

The way to solve this equation is indicated by the analogy with N=2 supersymmetric $SU(2)$ Yang-Mills theory. In terms of N=1 superfields its action is written as
\[
\frac{1}{16\pi} \text{Im} \int d^4x d^2\theta \left( \frac{d^2F}{d\phi^2} W^2 + \frac{1}{2} \int d^2\theta \frac{dF}{d\phi} \phi^+ \right).
\] (28)

Let us compare it with

\[
S_a = -\frac{1}{32\pi^2} \text{Re} \int d^4x d^2\theta \ S f(z)
\] (29)

and introduce \( a \equiv z^{-1/4} \) (this choice of the power will be explained below). The first term in (28) will coincide with (29) if

\[
2\pi i \frac{d^2F}{da^2} \equiv f; \quad \frac{d^2U}{da^2} \equiv \frac{u}{S}.
\] (30)

Then (27) takes the form

\[
F + F_D = -\frac{2i}{\pi} U,
\] (31)

where

\[
F_D = F - aa_D; \quad a_D = \frac{dF}{da}.
\] (32)

This equation coincides with (3) and, therefore, we are tempted to identify \( F \) with Seiberg-Witten solution. It is so. Really, differentiating (31) with respect to \( U \), we obtain

\[
a_D \frac{da}{du} - a \frac{da_D}{da} = -\frac{2i}{\pi}.
\] (33)

It means, that \( a \) and \( a_D \) are 2 independent solutions of the Picard-Fuchs equation

\[
\left( \frac{d^2}{da^2} + L(U) \right) \begin{pmatrix} a \\ a_D \end{pmatrix} = 0,
\] (34)

where \( L(U) \) is an undefined function.

At the perturbative level (see (24))

\[
f_{\text{pert}} = -4 \ln a; \quad \text{so that} \quad F = \frac{i}{\pi} a^2 \left( \frac{3}{2} - \ln a \right);
\]

\[
u_{\text{pert}} = W^2 = S, \quad U = a^2/2
\] (35)

and, therefore

\[
a = \sqrt{2U}; \quad a_D = -\frac{2i}{\pi} (a \ln a - a) = -\frac{i}{\pi} \sqrt{2U} \left( \ln(2U) - 2 \right)
\] (36)

satisfy
\[
\left( \frac{d^2}{dU^2} + \frac{1}{4U^2} \right) \left( \begin{array}{c} a \\ a_D \end{array} \right) = 0. \tag{37}
\]

However, the perturbative solution does not satisfy the requirement \[3, 22\]
\[\text{Im } \tau > 0, \quad \text{where } \tau = \frac{d^2 F}{da^2} = \frac{da_D}{da} = \frac{1}{2\pi i} f, \tag{38}\]
that is derived exactly as in the N=2 case. Therefore, two singularities (at \( U = 0 \) and \( U = \infty \)) are impossible.

To find the structure of singularities let us note, that the solution (25) should contain all positive powers of \( z \) and, therefore, is invariant under \( Z_4 \) transformations \( a \to e^{i\pi k/2} a \).

Taking into account (30) and (26) we conclude, that the corresponding transformations in the \( U \)-plane are \( U \to e^{i\pi k} U \). Thus, singularities of \( L(U) \) in the Picard-Fuchs equation (34) should come in pairs: for each singularity at \( U = U_0 \) there is another one at \( U = -U_0 \).

Therefore, the considered model is completely equivalent to N=2 supersymmetric SU(2) Yang-Mills theory without matter and the only possible form of Picard-Fuchs equation (up to the redefinition of \( \Lambda \)) is
\[
\left( \frac{d^2}{dU^2} + \frac{1}{4(U^2 - 1)} \right) \left( \begin{array}{c} a \\ a_D \end{array} \right) = 0 \tag{39}
\]
with the solution \[3, 22\]
\[
a(U) = \sqrt{\frac{2}{\pi}} \int_{-1}^{1} dx \frac{\sqrt{x - U}}{\sqrt{x^2 - 1}}; \quad a_D(U) = \sqrt{\frac{2}{\pi}} \int_1^U dx \frac{\sqrt{x - U}}{\sqrt{x^2 - 1}}. \tag{40}\]

Its uniqueness and, therefore, the uniqueness of the choice (39) was proven in [23].

The function \( F \) can be found by
\[
\frac{dF}{dU} = a_D \frac{da}{dU}. \tag{41}\]

Its general structure is well known to be
\[
F = -\frac{i}{\pi} a^2 \left( \ln a + \sum_{n=0}^{\infty} F_n a^{-4n} \right), \tag{42}\]
so that
\[
f = 2\pi i \frac{d^2 F}{da^2} = -4 \ln a + \sum_{n=0}^{\infty} f_n a^{-4n} = \ln z + \sum_{n=0}^{\infty} f_n z^n. \tag{43}\]

And now it is quite clear, that the choice \( a = z^{-1/4} \) was made to obtain the true structure of instanton corrections (21).

So, our main result is
\[ L_a = \frac{1}{16\pi} \text{Im} \int d^2 \theta \, S \tau (z^{-1/4}) \]  

(44)

where \( \tau(a) \) is Seiberg-Witten solution and \( z \) is given by (23).

4 On the impossibility to integrate out the gluino condensate for \( N_c > N_f \)

In this Section we will discuss the possibility of integrating the gluino condensate out of the exact superpotential. Substituting (44) into the condition

\[
\frac{\partial w}{\partial S} = 0 \tag{45}
\]

we can rewrite the latter in the following form (\( a = z^{-1/4} \))

\[
\frac{d \ln a}{d \ln \tau} = \frac{1}{4} (N_f - N_c) \tag{46}
\]

Taking the perturbative asymptotic of the exact result we find, that

\[
\ln(2a) = \frac{1}{4} (N_f - N_c) \tag{47}
\]

Of course, this equation has solution for all values of \( N_f \) and \( N_c \). Therefore, at the perturbative level the gluino condensate can be always integrate out, as it is usually assumed [20]. However, the situation is quite different beyond the frames of the perturbation theory. For the investigation it is very convenient to rewrite Seiberg-Witten solution in terms of elliptic functions [24]

\[
a(u) = \frac{4}{\pi k} E(k); \quad a_D(u) = \frac{4}{i \pi k} \left( E'(k) - K'(k) \right) \tag{48}
\]

where \( k^2 = 2/(1 + u) \). The functions \( E, K, E' \) and \( K' \) are defined as

\[
E(k) = \frac{\pi}{2} F\left(-\frac{1}{2}, \frac{1}{2}; 1, k^2\right) = \int_0^1 dt \frac{\sqrt{1-k^2 t^2}}{\sqrt{1-t^2}}
\]

\[
K(k) = \frac{\pi}{2} F\left(\frac{1}{2}, \frac{1}{2}; 1, k^2\right) = \int_0^1 dt \frac{1}{\sqrt{(1-k^2 t^2)(1-t^2)}}
\]

\[
E'(k) = E(\sqrt{1-k^2}); \quad K'(k) = K(\sqrt{1-k^2}) \tag{49}
\]

where \( F \) is a hypergeometric function. The function \( \tau \) is then given by
\[ \tau = \frac{iK'}{K} \] (50)

This form is very convenient for the computer research. Performing this work we used the analytical calculation system ”MAPPLE”.

The function

\[ \frac{d \ln a}{d \ln \tau} \] (51)

is plotted at the Fig.1 (curve 1.). For comparisons at this figure we also present its perturbative asymptotic (curve 2.).

For \( a \to \infty \) the perturbative result almost coincides with the exact one. However, there are crucial differences for small \( a \). Elliptic functions are real only for \( k^2 = 2/(1 + u) < 1 \). In a region, where the perturbation theory is not applicable \( (k^2 \to 0) \), (51) is also positive. It is equal to 0 for \( k^2 = 0 \), which corresponds \( a = 4/\pi \). Therefore, the range of values of (51) is \([0, \infty)\) and equation (46) has a solution only for a positive RHS, i.e. \( N_c \leq N_f \).

For \( N_c = N_f \) the classical constrain \( \text{det}M - \tilde{B}B = 0 \) is broken by instanton corrections [7] as

\[ \text{det}M - \tilde{B}B = \text{const} \Lambda^{2N_f} \] (52)

In the frames of our approach it is produced automatically, because in this case

\[ L_a = \frac{1}{16\pi} \text{Im} \int d^2 \theta S_{\tau} \left( \left( \frac{\Lambda^{2N_f}}{\text{det}M - \tilde{B}B} \right)^{-1/4} \right) \] (53)

and, therefore, the gluino condensate is a natural Lagrange multiplier. Integrating it out, we obtain the equation \( \tau(a) = 0 \) which has a solution

\[ a = \left( \frac{\Lambda^{2N_f}}{\text{det}M - \tilde{B}B} \right)^{-1/4} = \frac{4}{\pi} \] (54)

Finally, we obtain, that in this case

\[ \text{det}M - \tilde{B}B = \left( \frac{4}{\pi} \right)^4 \Lambda^{2N_f} \] (55)

5 Conclusion.

In the present paper we obtain the exact (nonperturbative) effective Lagrangian (44) for N=1 SUSY Yang-Mills theories with matter. Our result has the following differences from the ones, found in [6, 7]: Firstly, it agrees with the transformation law of the collective coordinate measure under the chiral symmetries, due to the presence of gluino condensate \( S \). Secondly, the result correctly reproduces anomalies beyond the frames
of perturbation theory. And, finally, it has the same structure as the Seiberg-Witten solution.

To derive the exact expression we developed a method, based on the relation between perturbative and exact anomalies. Here we should mention, that the similar approach was presented very long ago by Veneziano and Yankielowitch [3]. Nevertheless, their derivation is valid only in the perturbation theory. We would like to attract the attention, that Veneziano-Yankielowitch effective Lagrangian is not applicable at the nonperturbative level.

So, our result can be considered as a syntheses of Veneziano-Yankielowitch effective Lagrangian and Seiberg’s exact results, which is free from some of their shortcomings.

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**Appendix**

A **N=1 supersymmetric Yang-Mills theories**

The massless N=1 supersymmetric Yang-Mills theory with $SU(N_c)$ gauge group and $N_f$ matter multiplets is described by the action

$$S = \frac{1}{16\pi} \text{tr} \text{Im} \left( \tau \int d^4xd^2\theta \ W^2 \right) + \frac{1}{4} \int d^4xd^4\theta \sum_{A=1}^{N_f} \left( \phi_A e^{-2V} \phi^A + \tilde{\phi}_A e^{2V} \tilde{\phi}_A \right)$$  \hspace{1cm} (56)

where the matter superfields $\phi$ and $\tilde{\phi}$ belong to fundamental and antifundamental representations of the gauge group $SU(N_c)$.

Here we use the following notations

$$A_\mu = e A_\mu^a T^a \quad \text{and so on,} \quad \text{tr} T^a T^b = \delta^{ab};$$

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2};$$  \hspace{1cm} (57)

$$V(x, \theta) = -\frac{i}{2} \bar{\theta} \gamma^\mu \gamma_5 \theta A_\mu(x) + i \sqrt{2} (\bar{\theta} \theta)(\bar{\theta} \gamma_5 \lambda(x)) + \frac{i}{4} (\bar{\theta} \theta)^2 D;$$
\[ W(y, \theta) = \frac{1}{2}(1 + \gamma_5)(i\sqrt{2}\lambda(y) + i\theta D(y)) + \frac{1}{2}\sum_{\mu\nu} \theta F_{\mu\nu}(y) + \frac{1}{\sqrt{2}}(1 + \gamma_5)\theta \gamma^\mu D_\mu \lambda(y); \]
\[ \phi(y, \theta) = \varphi(y) + \sqrt{2} \bar{\theta}(1 + \gamma_5)\psi(y) + \frac{1}{2}\bar{\theta}_1(1 + \gamma_5)\theta f(y); \]
\[ y^\mu = x^\mu + \frac{i}{2}\bar{\theta} \gamma^\mu \gamma_5 \theta - \frac{\bar{\theta}}{2} \gamma^\mu \gamma_5 \theta. \quad (58) \]

Eliminating auxiliary fields we find that in components the action \( (56) \) is written as

\[ S = \frac{1}{e^2} \text{Re} \text{ tr} \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i\bar{\lambda}(1 + \gamma_5)\gamma^\mu D_\mu \lambda + \frac{\theta e^2}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu} \right) + \sum_A \int d^4x \left\{ D_\mu \varphi^+_A \varphi^A + D_\mu \varphi^0_A \varphi^A + i\bar{\Psi}_A \gamma^\mu D_\mu \Psi - i\bar{\phi}_A \gamma^\mu D_\mu \phi^A - i\bar{\tilde{\phi}}_A \gamma^\mu D_\mu \tilde{\phi}^A + \frac{1}{2} \left( \varphi^+_A T^a \varphi^A - \varphi^+_A T^a \tilde{\varphi}^A \right)^2 \right\} \quad (59) \]

where we introduced the Dirac spinor
\[ \Psi \equiv \frac{1}{2} \left[ (1 + \gamma_5)\psi + (1 - \gamma_5)\tilde{\psi} \right] \quad (60) \]

In the massless case the action is invariant under the transformations

\[ U(1)_1 : \quad W(\theta) \to e^{i\alpha} W(e^{-i\alpha\gamma_5}) \quad \phi(\theta) \to \phi(e^{-i\alpha\gamma_5}), \quad \tilde{\phi}(\theta) \to \tilde{\phi}(e^{-i\alpha\gamma_5}); \]
\[ U(1)_2 : \quad W(\theta) \to W(\theta) \quad \phi(\theta) \to e^{i\beta} \phi(\theta) \quad \tilde{\phi}(\theta) \to e^{i\beta} \tilde{\phi}(\theta). \quad (61) \]

that in components are written as

\[ U(1)_1 : \quad A_\mu \to A_\mu; \quad \varphi \to \varphi; \quad \tilde{\varphi} \to \tilde{\varphi}; \]
\[ \lambda \to e^{i\alpha\gamma_5} \lambda; \quad \Psi \to e^{-i\alpha\gamma_5} \Psi. \]

\[ U(1)_2 : \quad A_\mu \to A_\mu; \quad \varphi \to e^{i\beta} \varphi; \quad \tilde{\varphi} \to e^{i\beta} \tilde{\varphi}; \]
\[ \lambda \to \lambda; \quad \Psi \to e^{i\beta\gamma_5} \Psi. \quad (62) \]

The conservation of corresponding currents

\[ J_1^\mu = \bar{\lambda}^\mu(1 + \gamma_5) \gamma^\mu \lambda^a + \sum_A \bar{\Psi}_A \gamma^\mu \gamma_5 \Psi_A; \]
\[ J_2^\mu = -\sum_A \bar{\Psi}_A \gamma^\mu \gamma_5 \Psi_A - i \sum_A \left( \varphi^*_A D^\mu \varphi_A - D^\mu \varphi^*_A \varphi_A + \tilde{\varphi}^*_A D^\mu \tilde{\varphi}_A - D^\mu \tilde{\varphi}^*_A \tilde{\varphi}_A \right). \quad (63) \]
is destroyed at the quantum level by anomalies. In the perturbation theory

\[
\partial_\mu J^\mu_1 = (-N_f + N_c) \frac{1}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{tr} F_{\mu\nu} F_{\alpha\beta} = (N_f - N_c) \frac{1}{16\pi^2} \text{Im} \int d^2 \theta \ W^2;
\]

\[
\partial_\mu J^\mu_2 = N_f \frac{1}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{tr} F_{\mu\nu} F_{\alpha\beta} = -N_f \frac{1}{16\pi^2} \text{Im} \int d^2 \theta \ W^2.
\]  \hspace{1cm} (64)

Nevertheless, it is possible to construct an anomaly free symmetry. Really, from (64) we conclude, that

\[
J^\mu_R \equiv J^\mu_1 + N_f - N_c \frac{N_f}{N_f} J^\mu_2 = \bar{\lambda}^a (1 + \gamma_5) \gamma^\mu \lambda^a + \frac{N_c}{N_f} \sum_A \bar{\Psi}_A \gamma^\mu \gamma_5 \Psi_A - i \sum_A \left( 1 - \frac{N_f}{N_f} \right) \left( \varphi_A^* D^\mu \varphi_A - D^\mu \varphi_A^* \varphi_A + \bar{\varphi}_A^* D^\mu \bar{\varphi}_A - D^\mu \bar{\varphi}_A^* \bar{\varphi}_A \right)
\]  \hspace{1cm} (65)

is conserved even at the quantum level.

This current is produced by the transformations

\[
U(1)_R : \quad W(\theta) \rightarrow e^{i\alpha R} W(e^{-i\alpha R \gamma_5} \theta);
\]

\[
\phi(\theta) \rightarrow \exp \left( i\alpha R \frac{N_f - N_c}{N_f} \right) \phi(e^{-i\alpha R \gamma_5} \theta);
\]

\[
\bar{\phi}(\theta) \rightarrow \exp \left( i\alpha R \frac{N_f - N_c}{N_f} \right) \bar{\phi}(e^{-i\alpha R \gamma_5} \theta).
\]  \hspace{1cm} (66)

Below we will also use the combination of $U(1)_1$ and $U(1)_2$ with $\beta = x \alpha$ in (62), i.e.

\[
U(1)_x : \quad W(\theta) \rightarrow e^{i\alpha} W(e^{-i\alpha \gamma_5} \theta);
\]

\[
\phi(\theta) \rightarrow e^{ix\alpha} \phi(e^{-i\alpha \gamma_5} \theta);
\]

\[
\bar{\phi}(\theta) \rightarrow e^{ix\alpha} \bar{\phi}(e^{-i\alpha \gamma_5} \theta).
\]  \hspace{1cm} (67)

where $x$ is an arbitrary constant.

In particular, for $x = (N_f - N_c)/N_f$ we obtain $U(1)_R$ transformations; for $x = 0$ - $U(1)_1$ and for $x \rightarrow \infty$ (after redefinition $\alpha \rightarrow \alpha/x$) $U(1)_2$.

The corresponding current is

\[
J^\mu_x \equiv J^\mu_1 + x J^\mu_2 = \bar{\lambda}^a (1 + \gamma_5) \gamma^\mu \lambda^a + (1 - x) \sum_A \bar{\Psi}_A \gamma^\mu \gamma_5 \Psi_A - i \sum_A x \left( \varphi_A^* D^\mu \varphi_A - D^\mu \varphi_A^* \varphi_A + \bar{\varphi}_A^* D^\mu \bar{\varphi}_A - D^\mu \bar{\varphi}_A^* \bar{\varphi}_A \right)
\]  \hspace{1cm} (68)

In the perturbation theory
\[ \partial_{\mu} J_{x}^{\mu} = \left( -N_{f} + N_{c} + xN_{f} \right) \frac{1}{16\pi^{2}} \varepsilon^{\mu\nu\alpha\beta} \text{tr} F_{\mu\nu} F_{\alpha\beta} \]

\[ = \left( N_{f} - N_{c} - xN_{f} \right) \frac{1}{16\pi^{2}} \text{Im} \int d^{2}\theta W^{2}. \] \hspace{1cm} (69)

### B Structure of instanton corrections versus collective coordinate measure transformation law

In order to define a structure of effective potential we will calculate anomalies by 2 different ways and compare the results. The action is invariant under \(U(1)_{1} \times U(1)_{2}\) group. However, it is more convenient to investigate the anomaly of \(U(1)_{x}\) symmetry, constructed in Appendix A.

Performing \(U(1)_{x}\) transformation in the effective action we obtain

\[ \langle \partial_{\mu} J_{x}^{\mu} \rangle = -\frac{\partial \Gamma}{\partial \alpha} = -\text{Im} \int d^{2}\theta \left( 2w - 2 \frac{\partial w}{\partial S} S - x \frac{\partial w}{\partial v} v \right), \] \hspace{1cm} (70)

where we substituted \(\phi\) and \(\tilde{\phi}\) by their vacuum expectation values \(v\). (For simplicity we assume, that all \(v_{i}\) are equal; a brief review of the moduli space structure is given in the Appendix A.)

On the other hand, the anomaly can be found from the transformation law of the collective coordinate measure.

At the one-instanton level in this case there are \(4 + N_{c}^{2}\) bose zero modes, \(2N_{c}\) gluino zero modes (corresponding to supersymmetric \((\epsilon_{a})\) and superconformal \((\beta_{a})\) transformations) and \(2N_{f}\) zero modes for matter multiplets (supersymmetry \(\epsilon_{A}\)). Each zero mode should be removed by integration over the corresponding collective coordinate. The measure is written as

\[ d\mu = \]

\[ = \text{const} \int d^{4}a \frac{d\rho}{\rho^{5}} (M\rho)^{4N_{c}} \text{d}(\text{gauge}) \frac{1}{M^{N_{c}+N_{f}} \rho^{N_{c}}} \prod_{a=1}^{N_{c}} d\epsilon_{a} d\bar{\beta}_{a} \prod_{A=1}^{N_{f}} d\epsilon_{A} d\bar{\epsilon}_{A} \frac{1}{\rho^{2}v^{2}} \exp \left( -\frac{8\pi^{2}}{e^{2}} \right) \]

\[ = \text{const}A^{3N_{c}-N_{f}} \int d^{4}a d\rho \frac{3N_{c}-2N_{f}-5}{v^{2N_{f}}} \text{d}(\text{gauge}) \prod_{a=1}^{N_{c}} d\epsilon_{a} d\bar{\beta}_{a} \prod_{A=1}^{N_{f}} d\epsilon_{A} d\bar{\epsilon}_{A}, \] \hspace{1cm} (71)

where we take into account normalization of all zero modes. The gauge part and constant factors are written only schematically, because they are not important in our discussion. As above we need not know the explicit form of the action in the constant field limit. We should only emphasize, that it is a dimensionless function of collective coordinates, \(\phi\) and, in principle, \(W\). Of course, it is not invariant under \(U(1)_{x}\)-transformations

\(^{4}\)This approach was first used in \(25\) for \(N=2\) supersymmetric SU(2) Yang-Mills theory.
\[
\begin{align*}
W & \rightarrow e^{ia_5}W; & \theta & \rightarrow e^{-ia_5}\theta; \\
\phi & \rightarrow e^{i\alpha}\phi; & \tilde{\phi} & \rightarrow e^{i\alpha}\tilde{\phi}; \\
\epsilon_a & \rightarrow e^{i\alpha_5}\epsilon_a; & \beta_a & \rightarrow e^{i\alpha_5}\beta_a; \\
\epsilon_A & \rightarrow e^{i(x-1)a_5}\epsilon_A; & \tilde{\epsilon}_A & \rightarrow e^{i(x-1)a_5}\tilde{\epsilon}_A; \\
\rho & \rightarrow \rho; & a^\mu & \rightarrow a^\mu
\end{align*}
\]

as above.

Let us perform an additional substitution

\[
\begin{align*}
\theta & \rightarrow e^{-i\alpha_5}\theta; & x^\mu & \rightarrow e^{-2i\alpha\mu}; \\
\epsilon_A & \rightarrow e^{-i\alpha_5}\epsilon_A; & \tilde{\epsilon}_A & \rightarrow e^{-i\alpha_5}\tilde{\epsilon}_A; \\
\rho & \rightarrow e^{-2i\alpha}\rho; & a^\mu & \rightarrow e^{-2i\alpha}a^\mu,
\end{align*}
\]

so that the final transformations

\[
\begin{align*}
(1 + \gamma_5)\epsilon_a & \rightarrow e^{ia}(1 + \gamma_5)\epsilon_a; & (1 - \gamma_5)\beta_a & \rightarrow e^{-ia}(1 - \gamma_5)\beta_a; \\
(1 + \gamma_5)\epsilon_A & \rightarrow e^{-i\alpha}(1 + \gamma_5)\epsilon_A; & (1 + \gamma_5)\tilde{\epsilon}_A & \rightarrow e^{-i\alpha_5}(1 + \gamma_5)\tilde{\epsilon}_A; \\
\rho & \rightarrow e^{-2i\alpha}\rho; & a^\mu & \rightarrow e^{-2i\alpha}a^\mu; & \theta & \rightarrow e^{-2i\alpha_5}\theta
\end{align*}
\]

(except for \(\theta\)) correspond to dimension of the fields. The dimensionless action would have been invariant, if we had made additional rotation

\[
v \rightarrow e^{i(2-x)\alpha}v; \quad W \rightarrow e^{2i\alpha_5}W.
\]

However, we can not make it because \(v\) and \(W\) are not collective coordinates (and, therefore, integration variables). It means, that under (74)

\[
\begin{align*}
S(v, W) & \rightarrow S(e^{i(x-2)\alpha}v, e^{-2i\alpha_5}W); \\
d\mu(v) & \rightarrow \exp \left[i\alpha \left(-2(3N_c - 2N_f) - 2N_f(x - 2) - 2N_f\right)\right] d\mu(e^{i(x-2)\alpha}v) \\
& = \exp \left[i\alpha \left(-2(3N_c - N_f)\right)\right] d\mu(e^{i(x-2)\alpha}v).
\end{align*}
\]

It is quite evident, that the \(n\)-instanton collective coordinate measure is transformed as

\[
d\mu(v) \rightarrow \exp \left[i\alpha \left(-2n(3N_c - N_f)\right)\right] d\mu(e^{i(x-2)\alpha}v).
\]

Moreover, we should also perform the inverse substitution in the remaining integral (see the definition of the superpotential)
\[
\int d^4x d^2\theta \to e^{4i\alpha} \int d^4x d^2\theta,
\]
so that finally from (73), (77) and (78) we conclude, that

\[
w(v, W) \to \exp \left[ i\alpha \left( -2n(3N_c - N_f) + 4 \right) \right] w(e^{i(x-2)\alpha}v, e^{-2i\alpha\gamma_5}W).
\]

Taking into account that the action contains only \((1 + \gamma_5)W\), we find the anomaly to be

\[
\langle \partial_\mu J^\mu_R \rangle = -\frac{\partial\Gamma}{\partial\alpha}\bigg|_{\alpha=0} = \text{Im} \int d^2\theta \left( -2n(3N_c - N_f) + (-2 + x)v \frac{\partial}{\partial v} - 2W \frac{\partial}{\partial W} + 4 \right) w. \tag{80}
\]

Comparing (70) and (80), we obtain the following equation for \(n\)-instanton contribution to the superpotential:

\[
\left( 2v \frac{\partial}{\partial v} + 3W \frac{\partial}{\partial W} - 6 \right) w^{(n)} = -2n(3N_c - N_f)w^{(n)}. \tag{81}
\]

It is easily verified, that the solution is

\[
w^{(n)} = W^2 g_n \left( \frac{v^3}{W^2} \right) \left( \frac{\Lambda}{v} \right)^{n(3N_c - N_f)} = S g_n \left( \frac{v^3}{S} \right) \left( \frac{\Lambda}{v} \right)^{n(3N_c - N_f)} \tag{82}
\]
where \(g_n\) is an arbitrary function. Its explicit form can be found from the relation between perturbative and exact anomalies.

Of course, the result (82) is in a complete agreement with dimensional arguments and does not depend on the particular choice of symmetry (i.e. \(x\)).

\section{The classical moduli spaces of N=1 supersymmetric theories}

To describe the vacuum states it is convenient to introduce two \(N_f \times N_c\) matrixes of the form

\[
\phi \equiv \left( \phi^1, \phi^2, \ldots, \phi^{N_f} \right); \quad \tilde{\phi} \equiv \left( \tilde{\phi}_1, \tilde{\phi}_2, \ldots, \tilde{\phi}_{N_f} \right). \tag{83}
\]
(Their rows correspond to different values of color index.) The energy is minimal if \(\phi = \tilde{\phi} \equiv v\). Performing rotations in the color and flavor spaces we can always reduce the matrix \(v\) to the form

\[\]
$$v = \begin{pmatrix} v_1 & 0 & \ldots & 0 \\ 0 & v_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & v_{N_f} \\ 0 & 0 & \ldots & 0 \end{pmatrix}$$ \hspace{1cm} (84)$$

if $N_f < N_c$ and

$$v = \begin{pmatrix} v_1 & 0 & \ldots & 0 & 0 & \ldots \\ 0 & v_2 & \ldots & 0 & 0 & \ldots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & v_{N_c} & 0 & \ldots \end{pmatrix}$$ \hspace{1cm} (85)$$

if $N_f > N_c$.

1. $N_f < N_c$.

In the generic point the gauge group $SU(N_c)$ is broken down to $SU(N_f - N_c)$. Therefore,

$$\left(N_c^2 - 1\right) - \left((N_c - N_f)^2 - 1\right) = 2N_c N_f - N_f^2$$ \hspace{1cm} (86)$$

chiral superfields are eaten up by super-Higgs mechanism. Taking into account that originally there are $2N_cN_f$ chiral matter superfields, we conclude that only

$$2N_c N_f - \left(2N_c N_f - N_f^2\right) = N_f^2$$ \hspace{1cm} (87)$$

ones remain massless.

The flat direction can be described in the gauge invariant way by $N_f^2$ composite chiral superfields

$$M^A_B = \tilde{\phi}_{Aa} \phi^{Ba}.$$ \hspace{1cm} (88)$$

(Here $a$ denotes a color index.)

2. $N_f \geq N_c$.

If the number of flavors is equal to or larger than the number of colors, the original gauge group is completely broken in the generic point. Therefore, the number of remaining massless chiral superfields is

$$2N_c N_f - \left(N_c^2 - 1\right) = 2N_c N_f - N_c^2 + 1.$$ \hspace{1cm} (89)$$

In this case the gauge invariant description is provided by "mesons"

$$M^A_B = \tilde{\phi}_{Aa} \phi^{Ba}.$$ \hspace{1cm} (90)$$

and "barions"
\[ B_{AN_{c+1}A_{N_{c+2}}...A_{N_f}} = \frac{1}{N_{c}!} \epsilon^{A_{1}A_{2}...A_{N_f}} \epsilon^{a_{1}a_{2}...a_{N_{c}}} \phi^{A_{1}a_{1}} \phi^{A_{2}a_{2}}...\phi^{A_{N_{c}}a_{N_{c}}}; \]
\[ \tilde{B}_{AN_{c+1}A_{N_{c+2}}...A_{N_f}} = \frac{1}{N_{c}!} \epsilon^{A_{1}A_{2}...A_{N_f}} \epsilon^{a_{1}a_{2}...a_{N_{c}}} \phi^{A_{1}a_{1}} \phi^{A_{2}a_{2}}...\phi^{A_{N_{c}}a_{N_{c}}}. \] (91)

However, their overall number is greater than \(2N_{c}N_{f} - N_{c}^2 + 1\). The matter is that at the classical level these fields are not independent and satisfy some constraints. For example, if \(N_{f} = N_{c}\) the number of massless superfields is \(N_{f}^2 + 1\) while \(N_{M} + N_{B} = N_{f}^2 + 2\). The constraint eliminating the redundant chiral variable is

\[ \det M = \tilde{B}B. \] (92)

Similarly, for \(N_{f} = N_{c} + 1\)

\[ B_{A}M_{A}^{B} = M_{B}^{A} \tilde{B}_{A} = 0; \]
\[ \det M \left( M^{-1} \right)^{B} = B_{A} \tilde{B}_{B}. \] (93)

However, at the quantum level these constraints are violated by instanton corrections and are no longer valid.

**D The gauge invariant form of parameter z**

If \(N_{f} < N_{c}\), the only gauge invariant parameter of \(v^{2N_{f}}\) order is \(\det M\), so that

\[ z = \frac{\Lambda^{3N_{c}-N_{f}}}{\det M S^{N_{c}-N_{f}}}. \] (94)

For \(N_{f} \geq N_{c}\) the moduli space is parametrized by mesons \(M_{A}^{B}\) and barions \(B_{B_{1}...B_{N_{f}-N_{c}}}, \tilde{B}_{A_{1}...A_{N_{f}-N_{c}}},\) satisfying some classical constrains. At the quantum level these constrains are broken by instanton corrections. In the effective action approach the modifications should be produced automatically. It can be achieved by integrating out the \(S\)-superfield. (In particular, for \(N_{f} = N_{c}\) \(S\) is a natural Lagrange multiplier). The result should have the following form:

\[ \det M - \tilde{B}B = \text{const} \, \Lambda^{2N_{f}}, \quad N_{f} = N_{c}; \]
\[ w_{\text{eff}} = \text{const} \, \Lambda \left( \frac{3N_{c}-N_{f}}{N_{f}-N_{c}} \right) \left( \det M - (\tilde{B}_{A_{1}A_{2}...A_{N_{f}-N_{c}}} M_{A_{1}} B_{1} M_{A_{2}} B_{2} ... M_{A_{N_{f}-N_{c}}} B_{N_{f}-N_{c}}) \times B_{B_{1}B_{2}...B_{N_{f}-N_{c}}})^{-\frac{1}{N_{f}-N_{c}}} + h.c., \quad N_{f} > N_{c}. \] (95)
It can be achieved if and only if $v^{2N_f}$ is substituted by
\[ \det M - (\tilde{B}A_1A_2...A_{N_f-N_c} M_{A_1} B_1 M_{A_2} B_2 ... M_{A_{N_f-N_c}} B_{B_1B_2...B_{N_f-N_c}}), \] (96)
so that finally
\[ z = \frac{\Lambda^{3N_c-N_f} S^{N_f-N_c}}{\det M - (\tilde{B}A_1A_2...A_{N_f-N_c} M_{A_1} B_1 M_{A_2} B_2 ... M_{A_{N_f-N_c}} B_{B_1B_2...B_{N_f-N_c}})}, \] (97)

We would like to mention, that in the presented approach \[95\] certainly contains multiinstanton corrections, that contribute to the overall constant factor in the RHS.

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Figure 1: Plots of the left hand side of the equation \( \frac{d \ln a}{d \ln \tau} = \frac{1}{4} (N_f - N_c) \) as a functions of the variable \( a \). Curve 1. corresponds to the exact result and curve 2. to the perturbative one.