Excitation of seismic waves at the motion of fluid in the rock massive crack

AV Azarov* and AS Serdyukov
Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia
E-mail: *antonazv@mail.ru

Abstract. The paper deals with the problem of generation of seismic radiation during the motion of a fluid in the rock massive crack. As the reason for the occurrence of seismic vibrations, a process is considered in which the interaction of a fluid and a rock causes instability of the crack walls, which leads to self-oscillations. The mathematical model describing the given process is investigated. Conclusions are made about the conditions for the occurrence of oscillations in the flow of fluid through the crack and the characteristics of the generated seismic signal.

1. Introduction
There are examples, when the recorded seismic signals are associated with the motion of the fluid through the ruptures in the rock. For example, fluid-induced oscillations of the channel walls or the excitation and resonance of fluid-filled cracks are possible causes of volcanic tremor and long-period (LP) events [1]. These phenomena are observed during seismic monitoring of volcanoes, during which steady periodic low-frequency signals with a spectral peak in the range 0.1–7 Hz are recorded. Another example is the signals observed during seismic monitoring of subglacial flows, which in its characteristics are similar to volcanic tremor. A number of studies have established a correlation between the existence of a water flow under ice and the presence of a harmonic signal [2–4]. For example, work [4] informs us about low-frequency radiation (3–11 Hz) observed in the Greenland glaciers area. The work presents observations of harmonic oscillations under the flow of moving ice in the western Antarctic. The harmonic character of the tremor is interpreted as a result of the resonance of subglacial cracks and channels filled with water. Duration, monochromatic character and tremor dynamics indicate that the source mechanism is most likely associated with the water flow in the subglacial water system, which flows from a small subglacial lake.

Also, there are examples in which harmonic low-frequency signals are observed during seismic monitoring of fracturing. In some cases, the causes of the recorded signals are associated with fluid flow within the fracture cracks [5, 6].

The mentioned above examples show that the fluid flowing through the crack in the rock can cause seismic oscillations. In this case low-frequency harmonic signals are observed. The source mechanisms of these signals may vary. In this article a process of a fluid and rock interaction causes instability of the crack walls, this leads to self-oscillations. These studies can be used to model microseismic processes [7] and the development of microseismic monitoring technologies [8].
2. Mathematical model

Let us consider the problem of the fluid flow through a crack located in an elastic space (Figure 1). In [9] a mathematical model describing the behavior of such system was proposed to explain the volcanic tremor. Similar problems were addressed in the works [10–12].

Work [9] proposes a mathematical model describing the behavior of the considered system (Figure 1) as follows. The fluid flows through a crack the walls of which are elastic half-spaces. At the crack edges pressures $p_1$ and $p_2$ are maintained. The half-space is characterized by elasticity (coefficient $k$) and viscosity (coefficient $A$). The gap has a length $L$ and a width $h(t)$. $h(t)$ does not depend on $x$, i.e. the whole crack wall either contracts or moves apart. In the $z$ direction, the crack’s length is bigger than the width, so the problem is considered to be two-dimensional. Then, the equations describing the system are as follows:

\[
\frac{3}{2} \left[ M + \frac{\rho L^3}{12h} \right] \frac{d}{dt} \left[ A + \frac{L^2}{12h} \left( \frac{12\eta}{h^2} - \frac{\rho}{2} \frac{h}{h} \right) \right] h + k (h - h_0) = L \left[ \frac{p_1 + p_2}{2} - \frac{\rho v^2}{2} \right]
\]

(1)

where $h_0$ – the crack opening without influence of external forces, $\rho$ – the fluid density, $\eta$ – the fluid viscosity, $A$ – the attenuation coefficient, and $M$ – the attached mass.

It is easy to introduce the rock pressure force into the system of equations by putting in an additional term to the right-hand side of the first equation of the system (1). Also, we specify the crack opening without influence of external forces, after which the system of equations will have the following form:

\[
\frac{3}{2} \left[ M + \frac{\rho L^3}{12h} \right] \frac{d}{dt} \left[ A + \frac{L^2}{12h} \left( \frac{12\eta}{h^2} - \frac{\rho}{2} \frac{h}{h} \right) \right] h + k h = L \left[ \frac{p_1 + p_2}{2} - \frac{\rho v^2}{2} - F \right]
\]

(2)

\[
\rho \frac{d}{dt} \left( \frac{12\eta}{h^2} v \right) = \frac{p_1 - p_2}{L}
\]
When this method of introducing rock pressure is employed in the case \( \frac{p_1 + p_2}{2} > F_r \), the model shows the crack opening and the oscillations of the walls. If \( \frac{p_1 + p_2}{2} \leq F_r \), the crack goes from the initial data state to the closed state.

3. Numerical studies of the mathematical model

Let us consider the problem of the fluid flow through a crack located in an elastic space (Figure In this paper we consider the problem in which there is a continuous uniform fluid flow through a crack in the elastic space. This system has self-oscillating properties, as a result of which oscillations of the crack walls can occur. Numerical analysis of equations (2) showed the oscillating solutions that can describe the self-oscillating properties of the system under consideration.

In [9], an analysis of system (1) was carried out in the case, when magma (a fluid with a high viscosity coefficient = 500 Pa\*s and density = 2500 kg/m^3) flows between two elastic spaces. In this paper, the system (2) is analyzed, where the rock pressure is introduced, and water is employed as a fluid with parameters = 0.001 Pa\*s and = 1000 kg/m^3. The value of the remaining parameters is taken as follows: \( M = 3 \times 10^5 \) kg/m, \( A = 10^7 \) kg/(m\*s), \( k = 600 \) MPa. The coefficient of elasticity was estimated by the formula:

\[
k = \frac{\delta f}{\delta h} = \frac{1}{\delta h} L \delta p = \frac{\mu}{w \delta p} L \delta p = \frac{L \mu}{w}
\]

where \( w \) – the crack extent in the \( z \) direction, \( \mu \) – the shear modulus. The attached mass per meter of the wall was estimated by the formula \( M = \rho \cdot w \cdot 1 \) m. For convenience, the fluid pressure in the crack is divided into two components: \( p \) – the constant part and the part that determines the pressure difference at the ends of the crack – \( \delta p \). Thus, the pressure is \( p_1 = p + \delta p \), \( p_2 = p \).

The behavior of the system of equations (2) was investigated using numerical methods. The analysis was carried out by solving a system with various parameters, such as crack length \( L \), pressure at the ends of the crack \( p_1 \) and \( p_2 \) and rock pressure \( F_r \). After that, the limits of the parameters were determined, in case of which oscillations of the crack walls appeared. The characteristics of the oscillations were determined as well.

During analysis of nonlinear systems of differential equations, it is convenient to construct phase curves presenting the results of the solution appliance. Figure 2 shows phase curves in the \((h, \dot{h})\) plane for \( L = 5 \) m, \( p = 10 \) MPa, \( F_r =11\) MPa. In Figure 2 a) the phase curve with a pressure drop \( \delta p = 3 \) MPa. In this case, the wall from the position corresponding to the initial data (0.001 m), will go into a static state with 0.0075 m opening, having made oscillations. In Figure 2 b) pressure drop \( \delta p = 4 \) MPa. Here, the crack from the position corresponding to the initial data (0.001 m) passes into the steady-state oscillation mode. The frequency of the first harmonic is 3.4 Hz. A further increase in the pressure drop leads to an increase in the frequency of the first harmonic. We note that the behavior of the numerical solutions does not depend on the initial data.
Figure 2. Phase curves in the plane (h, ẃ). Time interval for curves’ calculation is 10 seconds, \( L = 5 \text{ m} \), \( p = 10 \text{ MPa} \), \( F_r = 11 \text{ MPa} \). a) \( \delta p = 3 \text{ MPa} \); b) \( \delta p = 4 \text{ MPa} \).

Figure 2 demonstrates that there is a certain threshold value \( \delta p \). When it is reached, steady oscillations of the crack walls are observed.

When there is a fixed static part of the pressure, the length of the crack and the rock pressure, an increase in the pressure drop at the edges of the crack leads to an increase in the oscillation frequency. For example, for \( L = 5 \text{ m} \), \( p = 10 \text{ MPa} \), \( F_r = 11 \text{ MPa} \), so the dependence of the frequency of the first harmonic on the pressure drop is shown in Fig. 3a). Figure 3b) shows a dependency graph of the frequency of the first harmonic on the length of the crack.

Figure 3. a) first harmonic frequency dependence on rock pressure drop if \( L = 5 \text{ m} \), \( p = 10 \text{ MPa} \), \( F_r = 11 \text{ MPa} \); b) first harmonic frequency dependence on crack length if \( p = 10 \text{ MPa} \), \( F_r = 11 \text{ MPa} \), \( \delta p = 5 \text{ MPa} \).

Figure 3 shows that if a crack length is 5 m, oscillations with a frequency of less than 4 Hz can be generated. Increase of the crack length leads to a decrease in the oscillation frequency. An increase in the pressure difference at the ends of the crack gives an increase in the frequency of the oscillations.

There are values of the parameters under which the mathematical model (2) has a solution, where the crack wall moves chaotically. For example, at \( p = 20 \text{ MPa} \), \( \delta p = 0.3 \text{ MPa} \), \( F_r = 11 \text{ MPa} \), \( L = 5 \text{ m} \),
this mode of motion is observed. Fig. 4 a) shows the phase curve corresponding to this case, Fig. 4 b) shows the oscillations of the crack wall depending time.

Figure 4. a) phase curve и b) wall displacement in time if $p=20$ MPa, $\delta p=0.3$ MPa, $F_r=11$ MPa, $L=5$ m.

4. Conclusions
The analysis of the mathematical model (2) has shown that it can describe the oscillations that occur, when there is a uniform flow of fluid through a crack in a rock. There are parameters at which low-frequency steady-state or damped oscillations are observed. The steady periodic wall motions occur, when certain threshold values of the pressure drop at the ends of the crack are reached. The characteristics of the oscillations in this case strongly depend on the size of the crack, the pressure drop of the liquid at the end of the crack and the rock pressure. In addition, model (2) has special solutions when the motion of the crack wall is chaotic.

Therefore, the model presented in this work can be used to monitor ruptures of rock through which the fluid flows. The use of the system (2) can enable the recovery of fracture and fluid flow parameters from the observed seismic radiation.

Acknowledgements
The reported study was funded by RFBR according to the research project No. 16-35-00513 мол_а

References
[1] Konstantinou KI, Schlindwein V 2003 Nature, wavefield properties and source mechanism of volcanic tremor: a review Journal of Volcanology and Geothermal Research Vol 119 No 1 pp 161–187
[2] Winberry JP, Anandakrishnan S., Alley R. B. 2009 Seismic observations of transient subglacial water-flow beneath MacAyeal Ice Stream, West Antarctica Geophysical Research Letters Vol 36 No 11
[3] Lawrence WST, Qamar A. 1979 Hydraulic transients: A seismic source in volcanoes and glaciers Science Vol 203 No 4381 pp 654–656
[4] Röösli C et al. 2014 Sustained seismic tremors and icequakes detected in the ablation zone of the Greenland ice sheet Journal of Glaciology Vol 60 No 221 pp 563–575
[5] Bame D, Fehler M 1986 Observations of long period earthquakes accompanying hydraulic fracturing Geophysical Research Letters Vol 13 No 2 pp 149–152
[6] Tary JB, Baan M, Eaton DW 2014 Interpretation of resonance frequencies recorded during hydraulic fracturing treatments Journal of Geophysical Research: Solid Earth Vol 119 No 2
pp 1295–1315

[7] Kurlenya MV, Serdyukov AS, Azarov AV, Nikitin AA 2015 Numerical modeling of wavefields of microseismic events in underground mining *Journal of Mining Science* Vol 54 No 4 pp 689–695

[8] Serdyukov SV, Azarov AV, Dergach PA, Duchkov AA 2015 Equipment for microseismic monitoring of geodynamic processes in underground hard mineral mining *Journal of Mining Science* Vol 51 No 3 pp 634–640

[9] Julian BR 1994 Volcanic tremor: nonlinear excitation by fluid flow *Journal of Geophysical Research: Solid Earth* Vol 99 No B6 pp 11859–11877

[10] Balmforth NJ., Craster RV., Rust AC. 2005 Instability in flow through elastic conduits and volcanic tremor *Journal of Fluid Mechanics* T. 527 pp 353–377

[11] Corona-Romero P, Arciniega-Ceballos A, Sánchez-Sesma FJ 2012 Simulation of LP seismic signals modeling the fluid–rock dynamic interaction *Journal of Volcanology and Geothermal Research* Vol 211 pp 92–111

[12] Rust A. 2004 Flow-induced oscillations: A source mechanism for volcanic tremor? 2003 *Program of Study Non-Newtonian Geophysical Fluid Dynamics* p 113