The Experimental Analysis and Parameters Influence Study on Dynamic Characteristics of Radial Hydraulically Damped Bushing

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Abstract. Radial hydraulically damped bushing (RHDB) is widely used as vibration isolator in vehicle chassis system. In this paper, new experimental process and data used to measure and describe dynamic behaviors of a RHDB are presented and discussed. Furthermore, a lumped-parameter (LP) model for calculating dynamic performances of a RHDB with one inertial track is proposed. Finally, influences of lumped-parameters on dynamic performances of a RHDB are carried out using the LP model. It is found that dynamic characteristics of a RHDB are most sensitive to equivalent pumping area of rubber spring, and are significantly influenced by its volume stiffness with high excitation frequencies above 25 Hz. But only the peak values of dynamic stiffness and loss angle are significantly affected by inertial coefficient and linear resistance of inertial track fluid.

Introduction

Hydraulically damped bushings (HDBs) can decay oscillations induced by dynamic loads of road with excitation of low-frequency and high-amplitude, and effectively decrease vibrations induced by engine with excitation of high-frequency and low-amplitude in radial direction, while conventional rubber bushing cannot satisfy the contradictory requirements for different excitations. Therefore, HDBs are widely used as a new kind of vibration isolators in engine sub-system, vehicle body and vehicle suspension since the late 1990s, which provides a relative optimum solution for stability and riding comfort of vehicles [1-5].

Experimental study is certainly the best method to investigate dynamic performances of HDBs. In the works of Refs. [6, 7], a laboratory device to investigate nonlinear dynamic characteristics of a HDB with a long inertia track and a short one is investigated. Dynamic pressures inside two fluid chambers and forces transmitted to the base of the HDB are measured with single-passage and dual-passage configurations. In order to decrease number of prototypes and tests, and to save considerable time in designing stage, lumped-parameter (LP) models have been implemented to study dynamic properties of HDBs.

In LP models, each lumped parameter represents a certain physical meaning, and dynamic characteristics of HDBs can be explicitly described as a function of lumped-parameters. The first LP model of a HDB with one inertia track is presented in Refs. [8, 9], which is used to investigate dynamic performances of a typical conventional bushing filled with internal fluid. Lumped-parameters of the LP model are identified based on experiment data of dynamic characteristics. It is concluded that the predicted resonant frequency of inertia track fluid correlates well with tested results, and the proposed LP model can be used to calculate dynamic performances of HDBs with multiple inertia tracks. However, accuracy of estimated dynamic performances of HDBs is significantly depended on accuracy of lumped-parameters [10, 11]. These parameters are generally obtained based on experiment data or approximate analytic formula [12-18]. Therefore, it is required to find a more efficiently method which can be used directly to calculate these lumped-parameters of a LP model, while nonlinearities of rubber material and hydraulic fluid should be considered.
In this work, dynamic behaviors of a RHDB by presenting new experimental process and data are described and discussed, and a LP model of a RHDB with one inertial track used to calculate its dynamic characteristics is proposed. One of the contributions in this paper includes using the least-squares method to identify lumped-parameters of a RHDB’s LP model which include radial dynamic stiffness, radial damping and equivalent piston area of rubber spring, and volumetric stiffness of chambers and resistance of inertial track fluid. Furthermore, estimated dynamic performances of a RHDB are calculated based on a LP model using lumped-parameters identified. Finally, influences of lumped-parameters on dynamic performances of a RHDB are carried.

**Experiment Analysis of Static and Dynamic Behaviors of a RHDB**

Static and dynamic behaviors of a RHDB can be evaluated by a MTS 831 servo-controlled hydraulic test rig, which can provide sweep-sine-excitation frequency ranges of 0.01 Hz to 1000 Hz. Fig. 1 illustrates the experimental setup for measuring radial force versus displacement of a RHDB with one inertial track. Excitation displacement is applied to outer sleeve which is connected to a mobile jaw with a movable exciter, while force response can be measured with a force sensor connected to a fixed jaw.

![Experimental sample](image1.png)

![Experimental setup](image2.png)

Figure 1. Experimental setup for measuring force versus displacement of a RHDB in radial direction.

Static performances of a RHDB can be characterized by static stiffness, which is defined as ratio between force response and excitation displacement. Excitation frequency of 0.01 Hz and excitation single-peak amplitude of 6 mm are used in static experiment, while sinusoidal single-peak amplitudes of 0.1 mm, 0.2 mm, 0.3 mm, 0.5 mm and 1.0 mm, and sweep frequency ranges of 0.01-150 Hz are used in dynamic experiment.

Static radial force versus displacement curves of a RHDB with one inertial track and rubber spring are shown in Fig. 2. It is seen in Fig. 2 (a) that hysteresis is existed during loading and unloading process, which means that static stiffness of the RHDB is nonlinear. Static stiffness of the RHDB can be calculated by averaged slop of loading and unloading curves, which is about 1620 N/mm. In addition, static stiffness of the rubber spring can be obtained when fluid is not filled in the chambers, which can be calculated by slop of force versus displacement curve in Fig. 2 (b) and is about 365.4 N/mm.
Fig. 2. Force versus displacement curves of a RHDB and rubber spring in radial direction.

Fig. 3 shows dynamic performances of the RHDB in radial direction with different excitation amplitudes. It is found that dynamic stiffness decreases greatly with increasing excitation amplitude when excitation frequency exceeds 25 Hz. Furthermore, the maximum loss angle with different excitation amplitudes takes place in frequency ranges of 15 Hz to 30 Hz, which increases with the decreasing of excitation amplitudes. It is obviously that frequency- and amplitude-dependency of a RHDB’s dynamic performances are strongly coupling together.

LP Model of a RHDB with One Inertial Track

Configuration and LP model of a typical RHDB with one inertial track are shown in Fig. 4. Rubber spring enclosed by inner and outer metal sleeves is used to support static load of a vehicle and provide slight damping for suspension system, which acts as a cylinder piston to pump mixture fluid between two almost identical chambers through an inertial track [1]. Under excitation of sinusoidal displacement, pressures of two chambers change in radial direction, where fluid flows back and forth through the inertial track as a tuned isolator damper to provide hydraulic damping.

In Fig. 4(b), \( K_r \) and \( B_r \) are dynamic stiffness and damping ratio of rubber spring, respectively. \( A_p \) is equivalent piston area of rubber spring. \( P_1 \) and \( P_2 \) are hydraulic pressures in upper and lower chambers, respectively. \( K_u \) and \( K_l \) are volumetric stiffness of upper and lower chambers, respectively. \( B_u \) and \( B_l \) are viscosity damping of the fluid in upper and lower chambers, respectively. \( K_v \) and \( B_v \) are volumetric stiffness and viscosity damping of both chambers, respectively, \( K_v = K_u + K_l \) and \( B_v = B_u + B_l \). \( K_{vv} \) and \( B_{vv} \) are equivalent linear stiffness and viscosity damping of \( K_v \) and \( B_v \),

\[
K_{vv} = A_p^2 \cdot K_v, B_{vv} = A_p^3 \cdot B_v
\]

(1)
Supposing inertial coefficient, linear resistance and nonlinear resistance of inertial track fluid are denoted by \( I, R_1 \) and \( R_2 \), respectively. Inertial coefficient of inertial track fluid can be obtained by

\[
I = m / A^2 = \rho l / A
\]  
(2)

where \( A \) is cross-sectional area of inertial track, \( m \) and \( \rho \) are mass and density of inertial track fluid, respectively, \( l \) is the length of inertial track. Fluid flow of inertial track fluid can be expressed by

\[
Q = A \ddot{x}
\]  
(3)

where \( x \) is excitation of dynamic displacement.

Continuity equations of inertial track fluid can be derived by Bernoulli equation for non-stationary flow,

\[
B_{vv}(\dot{x}_p - \dot{x}) + K_{vv}(x_p - x) + A_p(P_1 - P_2) = 0
\]  
(4)

\[
Q = A_p \dot{x}_p
\]  
(5)

where \( x_p \) is the displacement of equivalent piston area.

Momentum equation of inertial track fluid is

\[
P_1 - P_2 = I \ddot{Q} + (R_1 + R_2)\dot{Q}Q
\]  
(6)

where, nonlinear resistance \( R_2 \) is generally ignored compared with linear resistance \( R_1 \), Eq. (6) can be rewritten as

\[
P_1 - P_2 = I \ddot{Q} + R_1 \dot{Q}Q
\]  
(7)

Supposing variable of a RHDB’s LP model is \( X \), which is denoted by

\[
X = (x, x_p, x_r) = (\Delta P, x_p, Q)^	op
\]  
(8)

where \( \Delta P \) is pressure difference of two chambers, \( \Delta P = P_1 - P_2 \).

Combining Eqs. (4), (5) and (6), state equation can be obtained as

\[
A \ddot{X} + B X = C F(t)
\]  
(9)

where

\[
A = \begin{bmatrix} 0 & B_{vv} & 0 \\ 0 & A_p & 0 \\ 0 & 0 & -I \end{bmatrix}, \quad B = \begin{bmatrix} A_p & K_{vv} & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -(R_1 + R_2) \end{bmatrix}, \quad C = \begin{bmatrix} B_{vv} \ddot{x} + K_{vv} \dot{x} \\ 0 \\ 0 \end{bmatrix}
\]

\( F(t) \) is force response of the RHDB’s \([6,7]\), which is
\( F(t) = K_s x + B_r \ddot{x} + A_p \Delta P \) \label{eq:10}.

Complex stiffness of a RHDB can be calculated by \( K = F(t)/x(t) \), which can be derived in the Laplace domain as

\[
K(s) = K_s + B_r s + A_p s^2 \left( \frac{1s^2 + R_s s + K_v}{Is^2 + R_s s + K_v} \right) B_v s + K_v
\]

\[
= K_s + B_r s + A_p s^2 \left( \frac{1s^2 + R_s s + K_v}{Is^2 + R_s s + B_v s + K_v} \right)
\]

where \( s = j \omega \), and \( j \) is the imaginary unit.

Substituting \( s = j \omega \) into Eq. (10), the storage stiffness \( K_1 \) and loss stiffness \( K_2 \) can be obtained, respectively,

\[
K_1 = K_s + A_p \frac{\omega^2 \left[ IK_v (B_v - R_v) - K_v (IK_v - R_v) \right]}{K_v - \omega^2 I} + \omega^2 (B_v + R_v)
\]

\[
K_2 = B_r \omega + A_p \frac{\omega^2 \left[ I^2 B_v \omega^2 + B_v R_v (B_v + R_v) \omega^2 + K_v R_v \right]}{K_v - \omega^2 I} + \omega^2 (B_v + R_v)
\]

Dynamic stiffness \( K_s \), loss angle \( \phi \) and damping coefficient \( C \) of a RHDB can be obtained, respectively,

\[
K_s = \sqrt{K_1^2 + K_2^2} \quad \phi = \arctan \left( \frac{K_1}{K_2} \right) \quad C = K_s \sin(\phi) / \omega
\]

In Eq. (11), dynamic stiffness of rubber spring can be approximated by

\[
K_s = \alpha \cdot K_s
\]

where \( K_s \) is static stiffness of the rubber spring, and \( \alpha \) is the correction coefficient of stiffness, \( \alpha = 1.2 \sim 1.6 \) [14].

In addition, radial damping ratio of rubber spring in Eq. (11) can be calculated by

\[
B_r = 2 \xi \sqrt{MK_s}
\]

where \( M \) is the mass of rubber spring, \( \xi \) is damping ratio of rubber.

Therefore, dynamic stiffness \( K_s \) and radial damping ratio \( B_r \) of rubber spring can be given firstly based on the static experiment. Lumped-parameters of the proposed LP model including radial dynamic stiffness, radial damping and equivalent piston area of rubber spring, and volumetric stiffness of chambers and resistance of inertial track fluid should be identified in Eq. (11).

According to experimental dynamic performances of the RHDB with single-peak amplitude of 0.1 mm, as shown in Fig. 3, measured dynamic stiffness \( K_s(i = 1, 2, \ldots) \) and loss angle \( \phi_s(i = 1, 2, \ldots) \) of the RHDB with different frequencies can be obtained. Meanwhile, estimated dynamic stiffness \( \hat{K_s}(i = 1, 2, \ldots) \) and loss angle \( \hat{\phi_s}(i = 1, 2, \ldots) \) of the RHDB with different frequencies can be derived by Eq. (14). Therefore, lumped-parameters of the RHDB’s LP model can be identified by Eq. (17) using the least-squares method, which are \( K_s = 4.83 \times 10^8 \text{ N/m}^2 \), \( B_r = 2.77 \times 10^5 \text{ N \cdot s/m}^2 \), \( A_r = 4.61 \times 10^{-2} \text{ m}^2 \), \( R_r = 2.46 \times 10^6 \text{ N \cdot s/m}^2 \) and \( I = 1.33 \times 10^4 \text{ kg/m}^4 \).

\[
\min \frac{1}{2} \sum_r \left( K_r^{\exp} - K_r^{\hat{\text{cal}}} \right)^2 + \left( \phi_r^{\exp} - \phi_r^{\hat{\text{cal}}} \right)^2 \quad (i = 1, 2, \ldots)
\]

Fig. 5 verifies the proposed least-squares method which is an effectively method can be used for accurately identifying lumped-parameters of a RHDB LP model.
Influences of Lumped-Parameters on Dynamic Characteristics of a RHDB

In order to effectively adjust and optimize dynamic performances of a RHDB, influences of lumped-parameters including equivalent piston area of rubber spring, volumetric stiffness and volumetric viscosity damping of both chambers, and inertial coefficient and linear resistance of inertial track fluid on dynamic characteristics of a RHDB with single-peak amplitude of 0.1 mm are investigated in this section, where single parameter variation is only adopted while others parameters are identified by the least-squares method and interactions of different parameters are ignored.

**Influences of Equivalent Piston Area**

Fig. 6 shows dynamic characteristics of a RHDB with different equivalent piston area of rubber spring. Dynamic stiffness of the RHDB increases greatly with the increasing equivalent piston area in the whole excitation frequencies. The maximum dynamic stiffness and loss angle with different equivalent piston area are almost appears at excitation frequencies of 40 Hz and 20 Hz, respectively. However, loss angle of the RHDB is affected by equivalent piston area are mainly in the frequency ranges of 5 Hz to 30 Hz, as seen in Fig. 6 (b).

**Influences of Volumetric Stiffness**

Dynamic characteristics of a RHDB with different volumetric stiffness of chambers are shown in Fig. 7. Dynamic stiffness increases greatly with the increasing of volumetric stiffness when excitation frequency exceeds 23 Hz. In addition, loss angle of the RHDB increases with the increasing of volumetric stiffness mainly in the frequency ranges of 10 Hz to 60 Hz and better isolation performance can be obtained. But frequencies which correspond to the maximum dynamic stiffness and loss angle decrease slightly with the increasing of volumetric stiffness of chambers.

**Influences of Volumetric Viscosity Damping**

Influences of volumetric viscosity damping on dynamic characteristics of the RHDB are illustrated in Fig. 8. It can be seen that volumetric viscosity damping has few influence on dynamic characteristics of the RHDB, while loss angle increase slightly with the increasing excitation frequency when that exceeds 40 Hz.

**Influences of Inertial Coefficient of Inertial Track Fluid**

Fig. 9 shows dynamic characteristics of the RHDB with different inertial coefficient of inertial track fluid. It is shown that the greater inertial coefficient results in larger peak values of the dynamic stiffness and loss angle. Dynamic stiffness of the RHDB affected by the inertial coefficient is mainly in the frequency ranges of 20 Hz to 40 Hz, which increases with the increasing of the coefficient. Loss angle of the RHDB affected by the coefficient are mainly in the frequency ranges of 10 Hz to 60 Hz, which increases with the increasing of the coefficient in narrow frequency ranges.
ranges of 10 Hz to 20 Hz and decreases with the increasing of the coefficient in wide frequency ranges of 20 Hz to 60 Hz.

**Influences of Linear Resistance of Inertial Track Fluid**

Dynamic characteristics of a RHDB with different linear resistance of inertial track fluid are shown in Fig. 10, where the maximum dynamic stiffness and loss angle decrease with the increasing of the resistance. Dynamic stiffness of the RHDB increases with the increasing of the linear resistance when excitation frequency is below 23 Hz and decreases with the increasing of the resistance in the frequency ranges of 23 Hz to 60 Hz, as seen in Fig. 10 (a). Loss angle of the RHDB affected by the resistance are in the frequency ranges of 15 Hz to 40 Hz only, which increases with the increasing of the resistance in the frequency range.

![Figure 6. Dynamic characteristics of a RHDB with different equivalent piston area.](image)

![Figure 7. Dynamic characteristics of a RHDB with different volumetric stiffness of chambers.](image)

![Figure 8. Dynamic characteristics of a RHDB with different volumetric viscosity damping.](image)
Conclusions

In this work, new experimental process and data used to measure and describe dynamic behaviors of a RHDB are presented. It is concluded that dynamic stiffness of the RHDB decreases greatly with increasing of excitation amplitude, especially, when excitation frequencies are above 25 Hz. Furthermore, dynamic stiffness of the RHDB is in a static state when excitation frequency is above 40 Hz, and fluid track can be considered as a blocked fluid track. In addition, a LP model for calculating dynamic performances of a RHDB with one inertial track is proposed. The least-squares method used to identify lumped-parameters of the LP model is provided in this work. Finally, the influences of lumped-parameters on the dynamic performances of a RHDB are carried out. It is found that dynamic stiffness of the RHDB increases greatly with the increasing equivalent piston area of rubber spring in the whole excitation frequencies, while loss angle of the RHDB is affected by this parameter are mainly in the frequency ranges of 5 Hz to 30 Hz. Furthermore, dynamic characteristics of the RHDB also increase greatly with the increasing of volumetric stiffness of chambers, but better isolation performance can be obtained in the larger frequency ranges of 10 Hz to 60 Hz. In addition, inertial coefficient and linear resistance of inertial track fluid affect peak values of dynamic stiffness and loss angle only. However, volumetric viscosity damping slightly affects the dynamic characteristics of the RHDB.

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