Extracting Community Structures in Complex Networks Based on Discrete Neural Network Algorithm

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Abstract:This paper studies the problem of complex network community structure extraction, and proposes the DHNN algorithm. This algorithm shows that from the arbitrary initial value, after several iterations, it finally converges to an attractor or a limit loop of length 2, giving the energy of DHNN. The relationship between function and modularity function proves that the stable point of the network corresponds to a very large modular function value, and the example verification work is carried out. For the DHNN algorithm proposed in this paper, the simulation experiment is carried out on the actual network. The results show that the eigenvalue eigenvector algorithm of DHNN community structure extraction algorithm Newman has a large ϕ value. At the same time, the DHNN algorithm does not need to calculate the eigenvalue eigenvectors, and only needs a simple addition multiplication operation to extract the community structure in the network.

1 INTRODUCTION

Two groundbreaking work can be seen as a sign of the beginning of a new era of complex network research: one was published in the article by Watts and Strogats in Nature in 1998[3] revealing the small world characteristics of complex networks; the other is 1999 Barabasi and Albert's work on Scienc[4] reveals the scale-free nature of complex networks, arguing that this property of the network is due to two indispensable evolutionary factors, and proposes a network model accordingly. The traditional network model is based on a random graph[1]: It is considered that the connected edges between network nodes are existed with a certain probability and independently of each other, or the actual network is a regular nearest neighbor coupled network. Therefore, for a network with a sufficiently large number of nodes, the number of edges associated with each node approximates a Poisson distribution.

2 RELATED THEORIES

2.1 Hopfield neural network overview[6]

The Hopfield network was proposed by J. J. Hopfield in the 1980s. He pointed out that if the synaptic connections of the neural network were effectively symmetrical, then the system must be drilled to a fixed ordered state. Thus, it introduces the concept of an energy function, indicating that if the synaptic connections are symmetrical, the dynamic evolution of the network will reach a state of minimum energy. With this feature, the Hopfield network can perform functions such as optimization and associative memory. Hopfield network is divided into continuous type and discrete type. According to the value of node state, whether the network is continuous or discrete. The nodes of the discrete network take \{1,−1\} or \{0,1\}, and the state of the continuous network node continuously takes values in a certain interval.

Discrete Hopfield neural network method is proposed, with a symbolic function as the activation function of the network, the equation of the network can be written as:

\[ u_i(t+1) = \sum_{j=1}^{n} w_{ij} x_j(t) - \theta \]

\[ x_i(t+1) = \text{sgn}[u_i(t)] \quad (i = 1,2,\ldots,n) \tag{1} \]

Where \( w \) is the connection weight matrix, \( \theta \) is the threshold vector, and the above Hopfield network is abbreviated as \( H = (w, \Theta) \).

2.2 Module degree function in community extraction problem[7]

In the document [5], modularity function \( Q \) in the community extraction problem can be defined as:

\[ Q = \frac{1}{4m} \left( A_{ij} - k_i k_j - \frac{1}{2m} \right) \]

\[ s_i s_j = \frac{1}{4m} B_{ij} \tag{2} \]

Here \( S \) represents vector, The i elements is \( S_i \), \( i = 1,2,\ldots,n \), \( s_i \) is a constant, The symmetric matrix \( B \) is called a modular matrix, Its elements are:

\[ B_{ij} = k_i k_j - \frac{1}{2m} \tag{3} \]

We use (3) to simplify the matrix to find its row and column and are zero. Therefore, there is always a feature vector \( (1,1,\ldots,1) \) of the matrix \( B \), and the corresponding eigenvalue is 0, which is the property of the Laplasse graph matrix, which is also the basis of the graph.

3 DISCRETE HOPFIELD NEURAL NETWORK EXTRACTION COMMUNITY STRUCTURE ALGORITHM(DHNN)

For a complex network with \( n \) nodes, usually denoted by \( N = (\mathcal{V}, \mathcal{E}) \), the corresponding discrete Hopfield neural network
with \( n \) neurons can be represented as \( H = (W, \theta) \), where \( W \) represents the connection weight matrix and \( \theta \) represents the corresponding parameter vector. The state vector \( x = (x_1, x_2, \ldots, x_n) \in [+1, -1]^n \) of the neuron. The state change function of neuron \( i \) can be expressed as \( x_i \rightarrow f(v_i) \), where \( V_i \) is the activation state of neuron \( i \), defined as follows:

\[
v_i(t) = \sum_{j=1}^{n} w_{ij} x_j(t) - \theta_i
\]

When working in asynchronous mode, the state of only one neuron changes at a time, which is:

\[
x_i(t) = f(v_i(t))
\]

\[
v_i(t) = \sum_{j=1}^{n} w_{ij} x_j(t) - \theta_i
\]

\[
x_j(t+1) = x_j(t), j \neq i
\]

\( i \) can be selected sequentially or randomly.

For the case of synchronization:

\[
x(t+1) = f(v(t)), v(t) = Wx(t) - \theta
\]

Where \( f \) is the excitation function, because all the \( n \) neurons in this paper are fully parallel synchronous, so we take the discrete symbol function:

\[
f(v) = \text{sign}(v) = \begin{cases} +1, & v \geq 0 \\ -1, & \text{otherwise} \end{cases}
\]

In this way, we can get:

\[
x(t+1) = \text{sign}(Wx(t) - \theta)
\]

\[
x_j(t+1) = x_j(t) = \text{sign} \left( \sum_{j=1}^{n} w_{ij} x_i(t) - \theta_i \right)
\]

when \( i = 1, 2, \ldots, n \), when it was established, it said that the network has reached a stable state.

By combining (4) and (5), a discrete neural network model can be obtained.

\[
x(t+1) = \text{sign}(Bx(t) + x(t) - x'(t)Bx(t)x(t))
\]

For the discrete Hopfield neural network \( H = (W, \theta) \), take:

\[
\theta = x(t) - x'(t)Bx(t)x(t) = 0
\]

Then the network model can be transformed into:

\[
x(t+1) = \text{sign}(Bx(t))
\]

The module degree matrix in (8) can be obtained by disassembling:

\[
X(t+1) = \text{sign}(BX(t))
\]

\[
= \text{sign}(AX(t) - \frac{k}{2m})
\]

Where \( A_{\text{max}} \) denotes the adjacency matrix of the complex network. \( X(t) \) denotes an \( n \)-dimensional column vector of value ±1, \( k_i \) denotes the degree of node \( i \), symbol function:

\[
\text{sign}(x) = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}
\]

4 Simulation Experiment Analysis

4.1 Experimental data

In this paper, we used eight actual network datasets to test the performance of the proposed algorithm. They are the simpler Zachary’s Karate Club, the Dolphin Social Network and six other nodes. A lot of complex networks.

![Figure 1 Dolphin Social Network](image)

4.2 Experimental results and analysis

Discrete Hopfield neural network algorithm performs simulation experiments on Dolphin Social Network and analyzes its results.

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### Table 4: Discrete Hopfield Neural Network Algorithm for Extracting Classification Results of Community Structures in Dolphin Networks

| +1 | −1 | 0  |
|----|----|----|
| 0  | 1  | 2  |
| 9  | 10 | 13 |
| 22 | 25 | 26 |
| 31 | 32 | 39 |
| 54 | 56 | 57 |
| 34 | 38 | 40 |
| 42 | 47 | 48 |
| 59 | 49 | 50 |

It can be seen from Table 4 that the value of the modularity function Q is large. It can be seen from Fig. 2 that the classification result of the community structure is very close to the real network structure diagram. Whether it is from the Q value of Table 4 or the visual representation of Figure 2, it shows that the results of the discrete Hopfield network extraction community are better.

### CONCLUSION

The discrete Hopfield neural network algorithm proposed by the most important innovations in this paper has obvious advantages: First, the discrete Hopfield neural network algorithm does not need to calculate the eigenvalues and eigenvectors of the modularity matrix of complex networks in the process of extracting complex networks. Second, the value obtained by the discrete Hopfield neural network algorithm is larger than that obtained by the eigenvector algorithm and the continuous neural network algorithm.

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