Gravitational Collider Physics via Pulsar–Black Hole Binaries II: Fine and Hyperfine Structures Are Favored

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Abstract

A rotating black hole can be clouded by light bosons via superradiance and thus acquire an atom-like structure. If such a gravitational atom system is accompanied by a pulsar, the pulsar can trigger transitions between energy levels of the gravitational atom, and these transitions can be detected by pulsar timing. We show that in such pulsar–black hole systems, fine and hyperfine structure transitions are more likely to be probed than the Bohr transition. Also, the calculation of these fine and hyperfine structure transitions are better under analytic control. Thus, these fine and hyperfine structure transitions are more ideal probes in the search for gravitational collider signals in pulsar–black hole systems.

Unification Astronomy Thesaurus concepts: Black hole physics (159); High energy astrophysics (739); Pulses (1306)

1. Introduction

Compact objects play an important role in astrophysics and particle cosmology. With their astronomical mass squeezed into a compact region of spacetime, these objects provide an ideal platform for high-energy astrophysical phenomena as well as a test ground for gravity (Shapiro & Teukolsky 1983; Camenzind 2007).

Arguably, the most notable compact objects are pulsars (PSRs) and black holes (BHs). Pulsars are usually rotating neutron stars (NSs) that emit beams of electromagnetic radiation from their magnetic poles. The beam sweeps across the earth periodically due to the rotation. To a distant observer, despite its complex internal dynamics, a pulsar can be simply viewed as a clock that ticks by emitting periodic radio pulses. The precise periodicity of the pulsar makes it an excellent timing tool. For instance, pulsar timing is used to measure the orbital decay of binary systems (Hulse & Taylor 1975; Weisberg & Taylor 2005), to detect low-frequency gravitational waves (GWs; Hobbs et al. 2010; Dewdney et al. 2009; Arzoumanian et al. 2020), to probe gravity in the strong-field regime (Angélil et al. 2010), and even to serve as autonomous space navigation beacons (Becker et al. 2013).

Black holes also play a central role in modern physics. Although isolated stationary black holes are classically characterized only by three parameters, they are known to carry more structures in the presence of perturbations (Konoplya & Zhidenko 2011; Hawking et al. 2016). Not only does the horizon emit Hawking radiation quantum mechanically (Hawking 1974), which inspired numerous studies on the long-standing information paradox (Hawking 1975), a rotating black hole carries a dissipative ergoregion capable of radiating particles on a classical level (Zel’Dovich 1971; Press & Teukolsky 1972). This phenomenon, known as superradiance, has also been widely studied for over half a century (see a comprehensive review given by Brito et al. 2015b). For bosonic particles with mass $\mu \lesssim (GM_B)^{-1}$, where $M_B$ is the mass of the black hole, superradiance triggers instability in the spectrum of the black hole bound states (Damour et al. 1976). This leads to the formation of a bosonic cloud of size $r_1 \sim O(10^{-1} - 10^{3})GM_B$ around the black hole, with an energy spectrum similar to that of the hydrogen atom. In isolation, such a gravitational atom emits monochromatic GWs through pair annihilations as well as spontaneous level transitions (Arvanitaki & Dubovsky 2011). When a binary companion is introduced, the periodic orbital motion may hit the resonance band of the gravitational atom and induce Landau–Zener transitions (Landau 1932; Zener 1932) between different energy levels. The backreaction effect produces floating or sinking/kicked orbits observable from the GW signatures emitted by the binary. This recently proposed framework aimed at probing ultralight bosons is known as Gravitational Collider Physics (GCP; Baumann et al. 2019a, 2019b, 2020).

However, GWs (Ng et al. 2020) are not the only observation channel for GCP resonances and ultralight bosons. The backreaction on the binary can be naturally viewed as a characteristic feature in the time dependence of the orbital period derivative, i.e., a timing problem. Given the simple yet accurate time periodicity of the pulsar, it is natural to consider the pulsar–black hole binary as a viable probe of the GCP resonances. This PSR–BH radio observation channel has recently been verified in Ding et al. (2021) for Bohr transitions of the gravitational atom.

Bohr transitions change the principal quantum number; hence the corresponding resonance frequencies are relatively high. For instance, at resonance, a typical pulsar orbiting around a five-solar-mass black hole has an orbital period as short as $P \sim O(1)$ s. Such a PSR–BH system rapidly emits GWs, leading to a significant orbital decay. Thus the pulsar...
timing accuracy proves to be always sufficient (Ding et al. 2021). However, there are still several problems faced by the PSR–BH radio channel for Bohr transitions. (i) The short binary period during Bohr transitions suggests that the binary is near the end of the inspiral process. Binaries with such a short period are statistically disfavored. This can be understood if one approximates the PSR–BH binary population distribution by the time spent in the inspiral for a single PSR–BH binary. For a typical PSR–BH binary lifetime $T^{\text{life}}$, the duration $\Delta t_{\text{rot}}$ of a Bohr transition (which is comparable to the time left until merger $T^{\text{merger}}$) satisfy $\Delta t_{\text{rot}} \ll T^{\text{life}}$. Thus the probability of observing such a Bohr transition is suppressed by $\Delta t_{\text{rot}}/T^{\text{life}} \ll 1$. Given the fact that the number of observable PSR–BH binaries in our Galaxy is limited (Faucher-Giguère & Loeb 2011; Shao & Li 2018; Chattopadhyay et al. 2020), the event rate for Bohr transitions in the PSR–BH binaries may be extremely low. (ii) During a Bohr transition, the binary separation is comparable to the size of the boson cloud. This may threaten the validity of the quadrupole approximation, leading to the inadequacy of considering a narrow resonance with $\Delta n = 2$ only. Additional effects such as dynamical friction (Zhang & Yang 2020), upscattering effects (Wong 2020), and the emergence of additional molecular states (Ikeda et al. 2021) may also dramatically change the prediction.

Compared to the Bohr transition, fine/hyperfine GCP transitions are observationally more probable and theoretically cleaner to analyze. In this work, we set out to analyze the fine/hyperfine GCP transitions of PSR–BH binaries for ultralight scalar bosons. The advantages of probing GCP with fine/hyperfine transition are: (i) The energy differences of fine and hyperfine transition are suppressed by extra factors of $\alpha^2$ and $\alpha^6$, respectively, where $\alpha \equiv GM_p \mu \ll 1$ is the gravitational fine structure constant. Therefore, they have a much longer resonant orbital period. In addition, some fine/hyperfine transitions give floating orbits that enjoy an extremely long duration. This dramatically increases the event rate. (ii) The increase in the resonant orbital period is accompanied by the increase in the binary separation, which is now much greater than the cloud radius. This ensures the validity of the quadrupole approximation and the narrow resonance.

Apparently, a disadvantage of fine/hyperfine transitions is that the signal, namely the change of period derivative, is smaller than that of Bohr transitions. The long orbital period gives rise to an orbital decay that may be too tiny to be detected by the first generation of space GW detectors (Amaro-Seoane et al. 2017; Luo et al. 2020; Mei et al. 2021; Ando et al. 2009). However, thanks to the well-established timing accuracy of pulsars, long-term observation is sufficient to capture the fine/hyperfine resonances, as we will show below.

This paper is organized as follows. In Section 2, we review some technical details of the gravitational atom and GCP transitions. In Section 3, we discuss the problems faced by the Bohr transitions and motivate our study for fine/hyperfine transitions. Then, in Section 4, we focus on the major fine/hyperfine transitions induced by the quadrupole moment of the orbiting pulsar and analyze their observational feasibility. We conclude and give future prospects in Section 5.

### 2. The Gravitational Atom

In this section, we briefly review the basic aspects of the gravitational atom and GCP following Baumann et al. (2019a) and Baumann et al. (2020). A Kerr black hole is equipped with a dissipative ergosphere that can amplify incoming waves (Penrose 1969; Žel’Dovich 1971). For a massive bosonic field, its mass serves as a natural mirror that reflects the amplified modes (Press & Teukolsky 1972; Cardoso et al. 2004), from which an instability is generated (Damour et al. 1976). This superradiance instability leads to the growth of a bosonic cloud, whose behavior is governed by the Schrödinger equation with corrections for Kerr spacetime,

$$i \partial_t \psi(t, x) = \left( -\frac{1}{2} \partial_x^2 - \frac{2}{r} + \mathcal{O}(\alpha^2) \right) \psi(t, x),$$  \hspace{1cm} (1)

where $\alpha \ll 1$ guarantees the validity of nonrelativistic expansion. The solutions of the Schrödinger equation with ingoing boundary condition at the horizon are atomic states $|nlm\rangle$ labeled by the principal, angular, and magnetic quantum numbers. The frequency of each eigenstate is in general complex: $\omega_{nlm} = E_{nlm} + i \Gamma_{nlm}$, with

$$E_{nlm} = \mu \left(1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{8n^4} - \frac{3(n - 2l - 1)\alpha^4}{n^4(l + 1/2)} \right) + \frac{2a_{nlm}}{n^2l(l + 1/2)(l + 1)} + O(\alpha^6)$$

and

$$\Gamma_{nlm} = 2\bar{r} C_{nlm}(m\Omega_H - \omega_{nlm}) \alpha^{4l + 5},$$ \hspace{1cm} (3)

where $\bar{r} \equiv r/M_B$ and $\Omega_H \equiv \frac{\dot{a}}{2M_B(1 + \sqrt{1 - \beta^2})}$ is the angular velocity of the BH outer horizon. Here $\bar{a} \equiv a/M_B \lesssim 1$ is the dimensionless spin parameter ($a$ is the BH spin). Note that the spin distribution of an astrophysical BH is currently still under debate. Although BHs in binaries were expected to have relatively large spins (O’Shaughnessy et al. 2005; Nielsen 2016), LIGO’s results seem to favor BHs with small spins or an isotropic spin distribution (Farr et al. 2017). Considering this uncertainty and for simplicity, we will assume $\bar{a} \approx 1$ throughout this work. This allows the maximal population of atomic states with $m > 0$. For BHs with a smaller spin parameter, the $m > 0$ states that possess a negative $\Gamma_{nlm}$ will not be superradiant. The coefficient $C_{nlm}$ can be found in Baumann et al. (2019a). Notice that although Equation (3) is derived under Detweiler’s approximation (Detweiler 1980) with $\alpha \ll 1$, numerical studies have confirmed it validity for $\alpha < 0.5$ (Brito et al. 2015b). We also point out that the atomic spectrum in Equation (2) is only valid in the linear regime where the self-gravity of the bosonic cloud is negligible. For simplicity, we assume weak self-gravity throughout this work and leave a nonlinear treatment for future studies. For a positive $\Gamma_{nlm}$, the cloud mass $M_{nlm}$ grows at a timescale $T_{nlm}^{\text{growth}} \equiv \Gamma_{nlm}^{-1}$ until saturation, and then it slowly depletes via the emission of GWs. The depletion power of a highest helicity state with $l = m$ is

$$M_{nlm} \equiv -B_{nlm} \left( \frac{M_{nlm}}{M_B} \right)^2 \alpha^{4l + 10}.$$  \hspace{1cm} (4)

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5 Throughout this paper, we assume the ultralight boson is a (pseudo)scalar.
The coefficient $B_{nl}$ is computed by Yoshino & Kodama (2014) in the flat spacetime limit as

$$
B_{nl} = \frac{16^{l+1}l(2l - 1)\Gamma(2l - 1)^2\Gamma(l + n + 1)^2}{n^{3l+8}(l + 1)^4\Gamma(l + 1)^4\Gamma(4l + 3)^3\Gamma(n - l)^2}.
$$

(5)

However, we caution the reader that numerical analysis shows that Equation (5) underestimates the depletion power by approximately one order of magnitude (Yoshino & Kodama 2014; Brito et al. 2015a). The scaling powers of $M_{nlm}$ and $\alpha$, in contrast, are robust for $\alpha \ll 1$. Nevertheless, due to its analytical generality for all $n$ and $l$, we will adopt Equation (5) to estimate the depletion timescale,

$$
T_{nlm}^{(deplete)} \approx B_{nl}^{-1} \frac{M_B^2}{M_{nlm,0}} \alpha^{-4l-10},
$$

(6)

and warn the reader about the potential $O(10)$ uncertainty due to Equation (5). Here $M_{nlm,0}$ is the initial mass of the cloud state, whose value at saturation can be estimated using angular momentum conservation (see Table 1 in Baumann et al. 2020).

Now let us consider the binary case system. The motion of a binary companion will generate a periodic tidal perturbation on the gravitational axis, leading to level crossings in the atomic spectrum. At a large binary separation, the tidal perturbation is solely dependent on the mass of the binary companion. Thus it can be any astronomical object compact enough to fit into the orbit. In particular, we consider a pulsar of mass $M_P$. GCP transitions happen when the frequency of pulsar revolution matches the energy difference between two atomic states. After expanding the tidal perturbation into multipole moments and calculating its matrix elements between atomic states, one obtains the following selection rule for a process $|nlm\rangle \rightarrow |n'l'm'\rangle$ (Baumann et al. 2019a):

$$
\begin{align*}
-m' + m_X + m & = 0 \\
l + l_X + l' & = 2p, \text{ for } p \in \mathbb{Z} \\
|l-l'| & \leq l_X \leq l + l'.
\end{align*}
$$

(7)

For large circular equatorial orbits, the resonant orbital period is

$$
P = 2\pi \sqrt{\frac{\Delta m}{\Delta E}},
$$

(8)

with $\Delta E \equiv E_{nlm} - E_{nlm'}$. $\Delta m \equiv m' - m$. We see that a larger $\Delta E$ generically corresponds to shorter orbital periods.

The atomic transition is accompanied by an exchange of angular momentum between the cloud and the binary. This backreaction on the binary motion produces floating orbits and sinking orbits. We call transitions with $\Delta E < 0$ floating orbits, where the boson cloud loses its energy, delaying the orbital decay. In contrast, $\Delta E > 0$ gives sinking orbits, where the period of the binary system decreases faster with their energy given to the cloud. The total transition time is $\Delta t_{tot} \approx \Delta t + \Delta t_\gamma$, where $\Delta t$ is the transition time without backreaction, and $\Delta t_\gamma$ is the extra time caused by the backreaction. Here $\Delta t_\gamma$ is positive for floating orbits and negative for sinking orbits. The detailed expressions of $\Delta t$ and $\Delta t_\gamma$ can be found in Baumann et al. (2020). A GCP transition can sometimes turn a superradiant state into a nonsuperradiant state with negative $l_{nlm}r_{nlm}$, where the cloud is absorbed into the BH. Interestingly, it has been recently pointed out that this can be avoided under certain conditions (Takahashi & Tanaka 2021).

### 3. Bohr Transitions versus Fine and Hyperfine Transitions

For Bohr transitions among the lowest a few states, the energy difference is typically large, hence a short binary period. For instance, a Bohr transition from $n=3$ to $n=2$ gives a resonant orbital period

$$
P_{3\rightarrow2} = \frac{288GM_B}{5\alpha^3} = (2.6) s \times \left(\frac{M_P}{5M_\odot}\right) \left(\frac{\alpha}{0.12}\right)^{-3}.
$$

(9)

Denoting $q \equiv \frac{M_B}{M_\odot}$, the time left until the merger is then

$$
T_{3\rightarrow2}^{(merger)} = (1 \text{ day}) \times \left(1 + q^{1/3}\right) \left(\frac{M_B}{5M_\odot}\right) \left(\frac{\alpha}{0.12}\right)^{-8},
$$

(10)

suggesting that the PSR–BH binary is near the end of the inspiral phase. Although the orbital decay at this stage is significant for both pulsar timing and GW detectors, the likelihood of encountering a binary at this stage is smaller than that in the middle of the inspiral phase by at least several orders of magnitude. Yet the total number of observable PSR–BH binaries in our Galaxy is estimated to be $O(10^2–10^3)$ (Fauczker-Giguère & Loeb 2011; Shao & Li 2018; Chattopadhyay et al. 2020). Therefore, the event rate for Bohr transitions may be extremely low.

Another problem for Bohr transitions comes from the validity of multipole expansion. The binary separation for a typical $n = 3$ to $n = 2$ Bohr transition is

$$
R_{3\rightarrow2} = (4.8 \times 10^3 \text{ km}) \times (1 + q^{1/3}) \left(\frac{M_B}{5M_\odot}\right) \left(\frac{\alpha}{0.12}\right)^{-2}.
$$

(11)

The size of the bosonic cloud for the state with principal quantum number $n$ is $r_n = n^2 r_1$, with $r_1 = M_B \alpha^{-1}$ being the Bohr radius. For $n = 3$, we have

$$
r_3 = 9M_B \alpha^{-2} = (4.6 \times 10^3 \text{ km}) \left(\frac{M_B}{5M_\odot}\right) \left(\frac{\alpha}{0.12}\right)^{-2}.
$$

(12)

Hence the multipole expansion, in particular, the narrow resonant approximation of the $l_{9,m} = m = 2$ quadrupole moment (Baumann et al. 2020) may be questionable for $q \sim 1$. This is because when $R_{3\rightarrow2}$ is close to $r_3$, the pulsar is already moving inside the cloud, and higher multipole moments that mediate other transitions with $|\Delta m| > 2$ are nonnegligible. In addition, the dynamical friction of the cloud (Zhang & Yang 2020), upscattering effects (Wong 2020) and the formation of molecular states (Ikeda et al. 2021) can also have important impacts on the transition. As a result, an accurate account for Bohr transitions may require a nonperturbative treatment.

For fine ($\Delta n = 0$, $\Delta l = 0$) and hyperfine ($\Delta n = \Delta l = 0$, $\Delta m = 0$) transitions, however, both difficulties can be evaded. From Equation (2), we see that the energy difference in a fine (hyperfine) transition is smaller by a factor of $\alpha^2$ ($\alpha^3$) than a Bohr transition, leading to a much longer resonant orbital period. This means they can happen for PSR–BH binaries in the middle phase of inspiral, which is statistically more favored. The binary separation is also enlarged by a factor of $\alpha^{-4/3}$ ($\alpha^{-2}$) for a fine (hyperfine) transition, making the
Hyperpulsars will be stuck on the constraint and the selection rules in Equation 3, further divides these two sets into parity-odd multipole expansion well-defined and the $|\Delta m| = m_k = 2$ approximation accuracy.

Moreover, for $n \leq 3$, fine/hyperfine transitions always occur with $\Delta E < 0$. This suggests that these phenomenologically interesting fine/hyperfine transitions give rise to floating orbits. Thus the time spent on the GCP resonance is extended by $\Delta t_s$, further increasing the likelihood of detection. If $\Delta t_s \gg T_{\text{deplete}}$, once the binary hits the resonance band, the pulsar will be stuck on the floating orbit until the cloud depletes, which typically takes $10^8$ yr. This greatly enhances the detection likelihood.

A quantitative comparison of Bohr transitions and fine/hyperfine transitions is shown in Table 1, where the mass parameters are fixed to be $\alpha = 0.12$, $M_B = 5M_\odot$ and $M_p = 1.4M_\odot$. Here we have enumerated all GCP transitions that involve states with $n, n' \leq 3$, and that are mediated by the $l = m = 2$ quadrupole moment. Because of the uncertainty in the $T_{\text{deplete}}$ formula (6), we have only kept its order of magnitude. Also note that the depletion time for the state $321$ has not yet been computed in the literature to our best knowledge. Therefore, we only give its possible range estimated by $T_{322}^{\text{deplete}}$ and $T_{311}^{\text{deplete}}$. It is clear from Table 1 that fine/hyperfine transitions solve all issues aforementioned, by having a much larger $P_r$, a much longer $\Delta t_s, T_{\text{merge}}$, and a much smaller ratio $r_n/R_s$.

Going beyond the lowest a few states, we can find more interesting structures emerging. Given the $l = m = |\Delta m| = 2$ constraint and the selection rules in Equation (7), one can find all possible quadrupole-mediated GCP transitions with $n, n' \leq n_{\max}$. The formula for the total number of allowed transitions is

$$N_{\text{tot,ne,ext}}^{(HF/FF)} = \frac{1}{2}(n_{\max} - 2)n_{\max}(n_{\max}^2 - 2n_{\max} + 3),$$

(13)

where $n_{\max} \geq 4$. Within the nth energy level, the number of allowed fine transitions and hyperfine transitions are given by

$$N_n^{(F)} = 2n^2 - 10n + 14,$$

$$N_n^{(HF)} = n^2 - 3n + 3,$$

(14)

We have enumerated all allowed transitions with $n_{\max} = 4$ and $n_{\max} = 8$ in Figure 1. The transition graph with $n_{\max} \geq 4$ neatly factorize into the direct product of four connected subgraphs. This can be understood as the consequence of the quadrupole approximation. Because $\Delta m = 2$, states with odd $m$ cannot jump to states with even $n'$ and vice versa. In addition, parity conservation further divides these two sets into parity-odd (odd $l$ and $l'$) families and parity-even (even $l$ and $l'$) families, leading to the four disjoint sectors. Note that although higher multipole moments ($l \geq 3$) are able to mediate Bohr transitions between these sectors, they are in general too small to influence fine/hyperfine transitions.

The advantage of fine/hyperfine transitions over Bohr transitions persists as more GCP transitions are included. In Figure 2, we have shown the time left until the merger as well as the ratio of cloud size and binary separation for all transitions up to $n_{\max} = 10$. It is clear that the three types of transitions occupy different regions in the parameter space. Most Bohr transitions lie in the gray region where the multipole expansion breaks down, while fine/hyperfine transitions are far safer. Fine/hyperfine transitions also correspond to a much longer time before the merger, hence a higher event rate.

### 4. Uncovering Fine and Hyperfine Structures: Pulsar Timing Accuracy

The direct observation of GCP transitions for a PSR–BH binary relies on an accurate measurement of the orbital motion, which is recorded as modulations in the $\text{R}^\text{ot}$ delay of pulsar time-of-arrivals. Unlike Bohr transitions, fine/hyperfine transitions occur at a much longer orbital period, where the GW emission is still weak. The resonance frequency lies in the range of space-based GW detectors such as LISA. At a low orbital frequency, the corresponding orbital decay is much slower. Such a weak effect may require a long-term observation that lasts more than a decade. As a comparison, LISA only has a lifetime of 4–6 years (Amaro-Seoane et al. 2017). Therefore, it is questionable whether the first generation of space GW detectors (Amaro-Seoane et al. 2017; Luo et al. 2020; Mei et al. 2021; Ando et al. 2009) are precise enough to probe the GCP transitions in due time. In contrast, radio telescopes are earth-based and can last many decades. For instance, the Arecibo telescope built in 1963 had been functioning for 57 years before its tragic collapse in 2020. Thus a long-term observation of a PSR–BH binary may reveal the tiny deviations of orbital decay and uncover the fine/hyperfine structure of the gravitational atom.

The orbital decay of the PSR–BH binary, according to Hulse & Taylor (1975) and Weisberg & Taylor (2005), can be observed by recording the periastron time shift

$$\Delta P = P(0) \int_0^{t_c} \frac{1}{P(t)} \, dt,$$

(15)
If we consider a small timescale with respect to that of a significant orbital decay, we can linearize the period change by $P(t) \approx P(0) + \dot{P}t$. The periastron time shift then increases quadratically with observation time:

$$\Delta_P = \frac{1}{2} \frac{\dot{P}}{P} t^2.$$  

(16)

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**Figure 1.** The transition graph for states with $n, n' \leq n_{\text{max}}$. The purple, blue, and orange arrows represent Bohr transitions, fine transitions, and hyperfine transitions, respectively. The size of the vertices indicates the superradiance growth rate, i.e., a large vertex corresponds to a fast-growing cloud.
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Therefore, we demand
\[
\frac{2R_c}{1 + q} > \tau,
\]  
(18)

which automatically implies \( P_f > \tau \). The error during one continuous observation window is also \( w \sim \tau \). Suppose we can observe the pulsar for \( t_{\text{obs}} \) every day, which means we can measure \( t_{\text{obs}} / P \) periods every day. That is, for every single continuous measurement, the error can be estimated by
\[
\min(t_{\text{obs}}) / P.
\]

If we observe for \( 0 < t \leq T_{\text{obs}} \), where \( T_{\text{obs}} \) is the maximal observation time, then the number of independent measurement is \( \lfloor t/1 \text{day} \rfloor \), where \( \lfloor \rfloor \) denote the ceil function. In summary, the uncertainty for Periastron time shift is
\[
\sigma_{\Delta \rho} = \frac{1}{\sqrt{\lfloor t/1 \text{day} \rfloor}} \frac{\tau}{\min(t_{\text{obs}}) / P}.
\]

(19)

We can detect the GCP transitions only if the difference between the periastron time shift with transition backreaction and that without it is greater than the observation uncertainty,
\[
|\Delta \rho_{\text{GCP}} - \Delta \rho_{\text{GR}}| > \sigma_{\Delta \rho}.
\]

(20)

In addition, there are two more constraints on the model parameters. Namely, the superradiant timescale of the boson cloud should be short enough to observe, and the cloud should be stable on an astrophysical timescale (Baumann et al. 2020),
\[
T(\text{growth}) \lesssim 10^6 \text{ yrs}, \quad T(\text{deplete}) \gtrsim 10^8 \text{ yrs}.
\]

(21)

Combining the constraints in Equations (18), (20), and (21), we obtain the feasible parameter region for the fine/hyperfine transitions shown in Figure 3. Overall, transitions starting with \(|22\rangle\) give a wide range of the parameter region that covers \(10^{-3} M_\odot < M_B < 10^3 M_\odot\) and \(0.06 < \alpha < 0.5\). In contrast, transitions starting with \(|31\rangle\) and \(|21\rangle\) allow a relatively limited parameter space with significantly smaller \(\alpha\). Notice that the right edge of the parameter space is constrained by the depletion time, which is subjected to an \(\mathcal{O}(10)\) uncertainty. The lower edge is constrained by the Rømer delay resolvability (Equation (18)). The upper edge is constrained by the timing accuracy of periapsis time deviation (Equation (20)). The left edge is either constrained by the timing accuracy (for hyperfine transitions) or the cloud growth time (for fine transitions).

Thus we see that timing accuracy plays an important role in probing hyperfine transitions. Naturally, pulsars with shorter rotation periods \(\tau\) provide a finer resolution of the orbital motion, thereby increasing the timing accuracy. Alternatively, for a given PSR–BH system, one can also extend the observation time \(T_{\text{obs}}\) to increase the timing accuracy. This fact is demonstrated in Figure 4 for the hyperfine transition \(|32\rangle \rightarrow |30\rangle\). Some parameter choices may require decades of observation for a clear detection, a task suitable only for ground-based apparatus such as radio telescopes.

The fine/hyperfine transitions for states with \(n \geq 4\) are qualitatively similar. Since the superradiance growth rate and the GW depletion rate are not very sensitive to the principal quantum number, the feasible parameter regions for \(|n\rangle_{22} \rightarrow |n\rangle_{20}\) and \(|n\rangle_{22} \rightarrow |n\rangle_{00}\) are similar to those plotted in Figure 3. However, we caution that these results for states with \(l, m < n - 1\) are obtained under the assumption of

\[6\) Note that the transition \(|32\rangle \rightarrow |31\rangle\) is not shown due to our lack of information on \(T_{321}^{\text{deplete}}\).}
individual full occupation. In reality, only the leading superradiant states with $l = m = n - 1$ are dominant, and they extract the most portion of energy from the BH. However, the other states may also be populated via some previous atomic transitions that start with the dominant states. Another notable fact is that transitions starting with higher angular quantum
number $l$ generally require a larger $\alpha$, since their growth rate is further suppressed by powers of $\alpha$.

5. Conclusion

In this paper, we focused on probing fine/hyperfine GCP transitions in a PSR–BH binary with pulsar timing. Starting from a general review of the gravitational atom and GCP, we pointed out the problems faced by Bohr transitions. Namely, the Bohr transitions may be extremely rare because they happen near the end of the inspiral phase. The quadrupole narrow resonance approximation may also become invalid when the binary separation is comparable to the cloud size. Then, we showed that these problems can be evaded in the fine/hyperfine transitions, which typically enjoy a longer resonant orbital period and greater binary separation. All fine/hyperfine transitions with $n \leq 3$ lead to floating orbits that delay the orbital decay, further enhancing the likelihood of observation. The advantage persists for higher energy levels, as the three types of transitions occupy distinct regions in the parameter space. The subsequent analysis of pulsar timing accuracy demonstrates the feasibility of detecting fine/hyperfine transitions in GCP. In particular, we find that, assuming full occupation, the fine transition $|n22\rangle \rightarrow |n00\rangle$ and hyperfine transitions $|n22\rangle \rightarrow |n20\rangle$ give wide parameter regions that can be probed via pulsar timing. Increasing the total observation time also increases the timing accuracy, making the detection of transitions with large black hole mass possible. In the spirit of multimessenger astronomy, due to the long lifetime of radio telescopes and the stringent accuracy requirement, the observation of fine/hyperfine transitions via this PSR–BH radio channel should serve as a complement to observing Bohr transitions via the BH–GW channel.

However, there are still many questions left unanswered in the current work, and we hope to address them in the future. We conclude this paper by mentioning a few of them. First, our estimation of boson cloud depletion time is based on an approximate formula with a considerable amount of error, especially at large $\alpha$ and when $l = m$. We hope to improve our constraints in the future using more accurate numerical solutions of cloud depletion. Second, the self-gravity of bosonic cloud is neglected in this work. Yet for copiously populated cloud states, self-gravity and nonlinearity may qualitatively change the resonance picture, which is interesting to explore in our next step. Third, we have argued that the event rate of fine/hyperfine transitions is greatly enhanced by their long orbital periods and floating orbits, yet we did not give any explicit estimation of the likelihood of observation. To perform such an analysis, one needs to consider the initial distributions of BHs and pulsars in the Galaxy as well as their evolution histories. Fourth, in addition to pulsar timing, the measurement of the Doppler effect and the ellipsoidal modulation of a white dwarf (WD) in a binary is nowadays accurate enough for detecting the orbital decay (Hermes et al. 2012; Burdge et al. 2019a, 2019b). Since there are more WDs than pulsars in the Galaxy, it is interesting to consider probing GCP resonances in WD–BH binaries, whose event rate can be further enhanced. The question is, of course, whether the accuracy can reach the requirement of detecting the deviations in the orbital decay due to GCP resonances. We leave a detailed analysis to future works.

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