Mobility and density induced amplitude death in metapopulation networks of coupled oscillators

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(Dated: February 15, 2013)

We investigate the effects of mobility and density on the amplitude death of coupled oscillators in metapopulation networks, wherein each node represents a subpopulation with any number of mobile individuals. We perform stochastic simulations of the dynamical reaction-diffusion processes associated with the Landau-Stuart oscillators in scale-free networks. Interestingly, we find that, with increasing the mobility rate or density, the system may undergo phase transitions from incoherent state to amplitude death, and then to frequency synchronization. Especially, there exists an extent of intermediate mobility rate and density leading to global oscillator death. In addition, we show this nontrivial phenomenon is robust to different network topologies. Our findings may invoke further efforts and attentions to explore the underlying mechanism of collective behaviors in metapopulation coupled systems.

PACS numbers: 89.75.-k, 05.45.-a

I. INTRODUCTION

The complex dynamics of coupled nonlinear oscillators has gained significant research interest in many branches of science and technology$^1$, ranging from the modeling of biological rhythms in the heart$^2$$^4$, nervous system$^5$$^6$, intestine$^7$ and pancreas$^8$. One of the most intriguing emergent phenomena is the so-called amplitude death (AD), a phenomenon that coupled oscillators stop oscillation and settle to a quenched steady state. Since the initial work by Bar-Eli$^9$, AD has been observed in various real systems, such as the Belousov-Zhabotinsky reaction system$^{10}$, relativistic magnetrons$^{11}$, and synthetic genetic networks$^{12}$$^{13}$, to list just a few. The physical mechanism underlying AD has also been extensively investigated$^{16}$$^{19}$. For coupled oscillators on a regular lattice, mechanisms leading to AD may include oscillator mismatch$^{8}$$^{16}$, delayed coupling$^{20}$$^{21}$, asymmetric coupling$^{25}$$^{27}$, time-dependent coupling$^{28}$, conjugate coupling$^{29}$$^{31}$, nonlinear (nondiffusive) coupling$^{31}$, linear augmentation$^{32}$, and so on$^{33}$$^{35}$. See a recent review$^1$ for more details, which gives the deep insight and many clues for understanding AD induced by different scenarios.

On the other hand, AD in complex networks of coupled oscillators has also attracted much attention$^{31}$$^{36}$$^{43}$. It has been shown that network topologies, being regular$^{36}$$^{40}$, random$^{41}$, small-world$^{42}$ or scale-free$^{43}$, may influence AD in a nontrivial way, by enhancing or suppressing it with varying network randomness or heterogeneity. Nevertheless, previous studies on AD in complex networks only deal with the case of immobile individuals and each network node is occupied by one single individual. Very recently, the metapopulation network model$^{44}$, which incorporates local node population, mobility over the nodes, and a complex network structure, has drawn intensive attention. This model has been successfully exploited in different contexts, including epidemic spreading$^{45}$$^{48}$, biological pattern formation$^{49}$$^{50}$, chemical reactions$^{51}$, population evolution$^{52}$, and many other spatially distributed systems$^{53}$$^{54}$. It is shown that the density and the mobility of the individuals could have drastic impact on the emergence of collective behaviors in general$^{44}$$^{55}$, and particularly, density and mobility induced synchronization of coupled oscillators has been reported$^{56}$$^{57}$. Therefore, one may ask: How would the mobility and the density would influence AD in metapopulation networks of coupled oscillators?

In the present work, we study the emergence of AD in a metapopulation model which incorporates mobility over a complex network together with local interactions of the individuals at the network nodes. We find that, for small mobility rate and lower density, the metapopulation displays incoherent state, for intermediate mobility rate or density, all the oscillators spontaneously cease their oscillations, but for large ones, the death state can be eliminated and synchronize to a common frequency, overcoming the disorder in their natural frequencies. Furthermore, we show this nontrivial phenomenon is robust to different network topologies.

II. MODEL DESCRIPTION

We consider a system of $N$ distinct subpopulations labeled $\mu$, each corresponding to a network node. The density $\rho$ of the metapopulation is given by $\rho = \frac{1}{N} \sum_{\mu=1}^{N} N_{\mu}$,
where \( N_\mu \) is the number of individuals in node \( \mu \). Individuals inside each subpopulation run stochastically through the paradigmatic Landau-Stuart oscillators [17], whose dynamics is described by:

\[
\dot{z}_j^\mu(t) = z_j^\mu(t)(r - |z_j^\mu|^2 + i\omega_0^\mu) + K(\langle z \rangle^\mu - z_j^\mu(t))(1)
\]

where \( z_j^\mu(t) \) is the complex amplitude of the \( j = 1, ..., N_\mu \) oscillator in the \( \mu \)-th node, \( r > 0 \) denotes a homogeneous oscillation growth rate, \( K \) is the coupling strength, and \( \langle z \rangle^\mu = \frac{1}{N_\mu} \sum_{i=1}^{N_\mu} z_i^\mu(t) \) is the mean field inside node \( \mu \). \( \omega_0^\mu \) denotes the natural frequency of the \( j \)-th oscillator within node \( \mu \), which is picked up from a certain distribution. In the present work, we adopt a linear distribution \(-\gamma \leq \omega \leq \gamma \) with \( \gamma \) a constant. Without coupling, the trajectory of each single oscillator will settle to a limit cycle with frequency \( \omega_0^\mu \) and radius \( |z_j^\mu(t)| = r \). With sufficiently strong \( K \) and wide distribution of \( \omega_0^\mu \), the oscillators pull each other off their limit cycles, and collapse into a state of zero amplitude \( z_j^\mu = 0 \), i.e., an AD state.

The above equation actually defines the “reaction” process that governs the temporal behavior of each individual inside the metapopulation nodes. We now assume that the individuals can diffusion randomly among the nodes. The system evolves in time according to the following rules [14]. We introduce a discrete time step \( \tau \) representing the fixed time scale of the process. The reaction and diffusion rates are therefore converted into probabilities. In the reaction step, all the individuals are updated in parallel according to Eq.1. After that, diffusions take place by allowing each individual to move into a randomly chosen neighboring node with probability \( D\tau \), where \( D \) denotes the mobility rate. If not otherwise specified, the parameters are \( N = 1000, \tau = 0.001 \), \( r = 0.4, \gamma = 3.0 \), and \( K = 10 \). We choose the mobility rate \( D \) and the density \( \rho \) as main control parameters. Each simulation plot is obtained via averaging over 20 independent runs.

### III. RESULTS AND DISCUSSION

To begin, we consider scale-free networks generated by using the Barabási–Albert (BA) model [8] with power-law degree distribution \( p(k) \sim k^{-3} \). We fix \( \rho = 10 \) (thus we have totally \( N = N\rho \) individual oscillators) and vary \( D \) to investigate how the oscillators evolve in time. Initially, the oscillators are homogeneous distributed among the nodes. If diffusion is absent \((D = 0)\), each node will stay in an incoherent state for the above-mentioned parameters. If \( D \) is small, each node still remains incoherent, as shown in Fig.1(a) for \( D = 0.1 \), where typical time series of \( \text{Re}(z_j^\mu(t)) \) for several randomly-chosen oscillators within a random chosen node are depicted. For moderate diffusion rate \( D = 1.0 \) as shown in Fig.1(b), the time series eventually collapse into \( |Z_j^\mu(t)| \approx 0 \), which indicates the occurrence of global AD. However, for sufficiently large \( D \), say \( D = 10 \) in Fig.1(c), the oscillators are locked to a synchronized state. Therefore, we observe an interesting mobility induced transition from incoherence to AD and then to synchronization in our metapopulation oscillator network model. In addition, we have also perform simulations with fixed \( D \) and varying \( \rho \). Similar behaviors (time series not shown) are found, i.e., the system will also undergo a density induced transition from incoherence to AD and then to synchronization with increasing \( \rho \) given \( D \) is not too small.

![FIG. 1: (Color online) Time series of the real parts of the complex vector \( z_j^\mu(t) \) for several random oscillators at \( D = 0.1 \) (upper panel), \( D = 1.0 \) (middle panel) and \( D = 10 \) (lower panel). All the networks have fixed average network degree \( K \) = 6 and \( N = 1000 \). Other parameters are \( \rho = 10 \) and \( K = 10 \).](image)

To quantitatively manifest the collective oscillation intensity of the system, we choose the normalized mean "incoherent" energy \( E \) and "coherent" energy \( W \) as the characteristic variables, which are defined as [37, 42]

\[
E = \left[ \frac{\langle \sum_{\mu=1}^{N} \sum_{j=1}^{N_\mu} |z_j^\mu|^2 \rangle}{N_\rho} \right],
\]

\[
W = \left[ \frac{\langle |\sum_{\mu=1}^{N} \sum_{j=1}^{N_\mu} z_j^\mu|^2 \rangle}{N_\rho} \right].
\]

Here the brackets \( \langle \cdot \rangle \) denotes averaging over time and
stands for averaging over 20 different network realizations for each $D$ and $\rho$. A large $E$ means relatively large average oscillation amplitudes, while a larger $W$ implicates more synchronous oscillations. In the incoherent state, $W$ will be nearly zero but $E$ can be large. For AD state, both $E$ and $W$ should be nearly zero. In the synchronized state, $E$ and $W$ both have noteworthy nonzero values and they should be equal for complete synchronization.

The dependences of $E$ and $W$ on $D$ for fixed $\rho = 10.0$ and $\rho$ for fixed $D = 0.4$ are depicted in Fig. 2 wherein the curves can be divided into three stages. In stage 1, $E$ decreases monotonously with increasing $D(\rho)$, approaching nearly zero at a certain value of $D(\rho)$. In stage 2, both $E$ and $W$ remain nearly zero, demonstrating the occurrence of global AD. Both $E$ and $W$ increase with $D(\rho)$ in stage 3, corresponding to the synchronized state. In accordance with Fig. 1, the transition from incoherence to global AD and then to synchronization is clearly demonstrated. It seems that increasing density plays a similar role to increasing the mobility. Of particular interest, there exists a notable region (stage 2) for both $D$ and $\rho$ where the system remains stably in the AD state.

To further get a global picture, we have performed extensive simulations to obtain the phase diagram in the $D \sim \rho$ plane. This is shown in Fig. 3, where the contour plots of $E$ and $W$ are drawn. As expected, when both $D$ and $\rho$ are small, the system shows the incoherent state (the light region 1 of Fig. 3a)). When $D$ and $\rho$ are increased crossing certain critical values, the system finally evolves into global AD state (the dark region 2 of Fig. 3a)). For sufficiently large $D$ and $\rho$, however, the system may undergo a phase transition again from the death phase to synchronized one, as shown in the light region 3 of Fig. 3a). Thus as we keep one of the parameters fixed ($D$ or $\rho$) and increase the other, we observe transitions from the incoherent (region 1) to the amplitude death (region 2) and then to frequency synchronized state (region 3). Note that the $1 \to 2$ transition is not observable with increasing $\rho$ if the mobility rate $D$ is too small, and similarly, the $2 \to 3$ transition cannot happen if the density $\rho$ is too low.

So far, the results are all for scale-free coupled networks. One may wondering whether the interesting findings above is sensitive to the network topology or not. Thus, we have also performed similar studies on other types of networks, e.g., the small world network and random network. The phase diagrams are shown in Fig. 4 (a,b) for small-world network and (c,d) for random network, respectively. Apparently, the qualitative behaviors are the same as those observed in scale-free networks. The only difference is that the boundaries between different phase regions are slightly shifted.
IV. CONCLUSION

In summary, we have studied the collective dynamics of coupled limit cycle oscillators defined on metapopulation networks, where different subpopulations are connected by fluxes of individuals. By extensive numerical simulations, we show that mobility and density play non-trivial roles on the collective behavior of the system, by demonstrating an interesting type of density or mobility induced transition into and out from oscillation death behavior. On one hand, intermediate mobility rate and density can induce global oscillator death. On the other hand, large mobility rate and density can also eliminate oscillator death and lead to synchronization. In addition, we find that this nontrivial phenomenon is robust to the network topology. The underlying mechanisms for these interesting results are still open. An analytical theory would surely be greatly helpful, however, it is not available at the current stage and should certainly deserve more study. Since many real-life networks (cellular networks, protein networks, gene networks, etc.) inevitably involve variances in both the mobility and the density, and their collective dynamics could be modeled by coupled oscillators, these results may find a variety of applications. Our study may also stimulate further investigation on the emergent coherence of metapopulations of coupled oscillators.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grants No. 21125313, No. 20933006, No. 91027012, and No. 11205002). C.S.S. was also supported by the Key Scientific Research Fund of Anhui Provincial Education Department (Grant No.KJ2012A189).

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