GRavitational recoil velocities from eccentric binary Black hole mergers

Carlos F. Sopuerta,1 Nicolás Yunes,2 and Pablo Laguna2

Center for Gravitational Wave Physics, Pennsylvania State University, University Park, PA.

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Abstract

The formation and growth of supermassive black holes is a key issue to unveil the secrets of galaxy formation. In particular, the gravitational recoil produced in the merger of unequal mass black hole binaries could have a number of astrophysical implications, such as the ejection of black holes from the host galaxy or globular cluster. We present estimates of the recoil velocity that include the effect of small eccentricities. The approach is especially suited for the last stage of the merger, where most of the emission of linear momentum in gravitational waves takes place. Supplementing our estimates with post-Newtonian approximations, we obtain lower and upper bounds that constrain previous recoil velocity estimates, as well as a best estimate that agrees with numerical simulations in the quasi-circular case. For eccentricities $e \leq 0.1$, the maximum recoil is found for a mass ratio of $M_1/M_2 \sim 0.38$ with velocities in the range 79–216 km s$^{-1}$ $(1 + e)$ and a best estimate of 167 km s$^{-1}$ $(1 + e)$.

Subject headings: black hole physics — cosmology: theory — galaxies: nuclei — gravitation — gravitational waves — relativity

Online material: color figures

1. Introduction

In present hierarchical cold dark matter cosmogonies, large-scale structures are formed by the merger of small (subgalactic) structures that originated from small primordial density perturbations (Press & Schechter 1974). In the case of galaxies, there is evidence that most of the nearby ones host supermassive black holes (SMBHs) at their centers (Richstone et al. 1998; Magorrian et al. 1998) with masses in the range $10^5$–$10^9 M_\odot$. Moreover, observations have revealed tight relations between the SMBH and the bulge of the host galaxy (Ferrarese & Merritt 2000), and indicate also that the SMBH mass may be determined by the mass of the host dark matter halo (Ferrarese 2002). This suggests that there must be a deep relation between the formation mechanism of the SMBH and the host galaxy that is not yet completely understood. It has been suggested (Volonteri et al. 2003) that SMBHs grow as a combination of gas accretion and mergers with other SMBHs that come together as a result of the merger of their host dark matter halos and dynamical friction (Begelman et al. 1980).

The last stages of the merger of SMBHs will be driven by the emission of gravitational radiation in the low-frequency band. These extremely energetic events will be observable by the planned space-based gravitational-wave antenna LISA (Danzmann 2003; Prince 2003). In addition to energy and angular momentum, in the case of unequal-mass SMBHs there is a net flux of linear momentum carried away from the system by the gravitational waves (Peres 1962; Bekenstein 1973). Thus, momentum conservation implies that the final SMBH remnant will experience a recoil. There may be observational evidence of an ejected SMBH from an ongoing galaxy merger (Haehnelt et al. 2006) either from gravitational recoil or gravitational slingshot of three or more SMBHs in the merger. The knowledge of the magnitude of the recoil velocity is crucial to understanding the demography of SMBHs at the centers of galaxies and in the interstellar and intergalactic media, and their apparent absence in dwarf galaxies and stellar clusters (Madau & Quataert 2004; Merrit et al. 2004). An estimate of the recoil can also be used to constrain theories in which SMBHs grow at the center of dark matter halos (Haiman 2004) and to estimate SMBH merger rates (Miccic et al. 2006).

In this Letter, we present the results of a calculation of the recoil velocity based on an approximation scheme that is valid when the black holes (BHs) are very close to each other, that is, in the last stage of the binary merger. This is the stage at which most of the recoil accumulates. This contrasts with previous calculations that use approximations valid when the BHs are well separated, except for recent fully numerical relativistic calculations (Baker et al. 2006; Herrmann et al. 2006; Gonzalez et al. 2006). We include the effect of eccentricity in the recoil velocity since there are indications that SMBH binaries may have not completely circularized by the time of merger (Aarseth 2003; Armitage & Natarajan 2005; Dotti et al. 2006).

2. Recoil velocities and the close-limit approximation

General relativity predicts that an unequal-mass binary system in coalescence produces anisotropic emission of gravitational waves that carry linear momentum away from the binary. As a consequence, the center of mass of the system experiences a recoil velocity $v_r$ (Wiseman 1992). The first estimates, done by Fitchett (1983) using a quasi-Newtonian approach, yielded a maximum $v_r \sim 1480$ km s$^{-1}$ $(r_f/r_i)^8$ for a symmetric mass ratio of $\eta = M_i M_2 / M^2 \sim 0.2$, where $r_f = 2GM/c^2$ is the gravitational radius, $M_i$ and $M$ are the individual and total masses, respectively, and $r_i$ is the orbital separation at which gravitational-wave emission ends. Similar results up to the innermost stable circular orbit (ISCO) were found using a relativistic perturbative method (Fitchett & Detweiler 1984). This velocity estimate is large compared to galactic escape velocities. In contrast, the first post-Newtonian (PN) calculation (Wiseman 1992) for neutron star binaries produced a much lower result: $v_r < 1$ km s$^{-1}$.

Motivated by the astrophysical impact of the gravitational recoil, there has been recently a number of efforts to obtain better estimates of the recoil velocity. Using a relativistic perturbative scheme for the description of extreme mass ratio
also that the maximum recoil takes place at with a magnitude of \(p\). Baker et al. (2006) reported for a symmetric mass ratio general relativistic numerical simulations of kicks. This estimate covers the last part of the merger, plunge, and ring-down. One can complement this result with PN estimates from the inspiral phase. If we consider the contribution from PN approximations up to ISCO, we obtain a lower bound for the maximum recoil of \(v_r \sim 80\) km s\(^{-1}\), whereas if we push the PN approximations up to the point where we start the CLA scheme, we get an upper limit in the maximum recoil of \(v_r \sim 215\) km s\(^{-1}\) (it is known that PN and perturbative schemes overestimate the linear-momentum flux in the strong field region). Alternatively, if instead of pushing the PN method toward small separations, we push the CLA scheme toward larger separations around \(5GM/c^2\), we get a slightly larger upper bound.

In this Letter, we improve and extend these estimates for initial configurations corresponding to BBHs in eccentric orbits. The most important point in the implementation of the CLA scheme is to establish a correspondence between the initial configuration representing a BBH and a perturbed single BH. This correspondence allows us to identify, in the initial BBH configuration, a background (a nonspinning Schwarzschild BH) and perturbative multipolar gravitational modes (Sopuerta et al. 2006). We then evolve these modes by using the machinery of BH perturbation theory, and from the results of the evolution, we can compute the fluxes of energy, angular momentum, and linear momentum \((P_{GW}^\prime)\) emitted by the BBH system during the merger. Thus the recoil velocity is given by

\[
v_r = \| v'_r \|, \quad v'_r = -M^{-1} \int_{t_1}^{t_2} dt P_{GW}^\prime, \tag{1}
\]

where \(t_1\) is the time corresponding to the initial separation of the BBH, and \(t_2\) is some time after merger, when the gravitational-wave emission becomes negligible. In our calculations, we use an initial BBH configuration (Brill & Lindquist 1963; Bowen & York 1980) that has been shown to accurately represent binaries to Newtonian order. They are characterized by four parameters (Sopuerta et al. 2006): the total mass of the system \((M)\), the symmetric mass ratio of the binary \((\eta)\), the initial separation \((a)\), and the initial linear momentum \((P)\) of each BH. Here \(M\) and \(\eta\) enter as a scale and input parameters, respectively. Given \(\eta\), the type of orbit is determined by the pair \((d, P)\). For a quasi-circular orbit we find a relation between \(P\) and \(d\) by minimizing the gravitational binding energy of the black hole binary. This relation turns out to be analogous to the Newtonian one, although it has to be interpreted within the framework of general relativity.

In this work, in order to study the effect of eccentricity in the gravitational recoil, we take the distance parameter to be \(d = a(1 - e)\), where \(a\) is a parameter distance that plays the role of the semimajor axis and \(e\) is the eccentricity. Thus the linear momentum parameter is given by

\[
P = P_e(a) \sqrt{\frac{1 + e}{1 - e}}, \tag{2}
\]

where \(P_e(a)\) is the linear momentum parameter for quasi-circular configurations (Sopuerta et al. 2006) with radius \(a\).

For our kick estimates, we have kept the semimajor axis fixed \((a = 4GM/c^2)\) and only varied the symmetric mass ratio and eccentricity parameters \((\eta, e)\). Figure 1 shows the recoil velocity \(v_r\) as a function of the symmetric mass ratio \(\eta\) for several values of the eccentricity: \(e = 0 - 0.25\). We observe that the maximum recoil occurs at \(\eta \sim 0.19\), consistent with pre-
vious results. As we increase the eccentricity, the recoil velocity also increases by approximately a factor of $1 + e$ for eccentricities in the range $e < 0.1$. This increase can be qualitatively understood by expanding equation (2) for small eccentricities, which leads to an increase by a factor $1 + e$ in the linear momentum parameter. Thus, it seems that an increase in the plunge velocity leads to roughly the same increase in the recoil velocity. If we consider the entire range of eccentricities that we considered, the numerical results can be fitted to the following two-parameter nonlinear function:

$$v_c^{\text{CLA}}(\eta, e) = \alpha(1 - \frac{4\eta}{1 - e^2})(1 + \beta_1 \eta + \beta_2 \eta^2) \times \frac{1 + e}{1 - e}(1 + \gamma_1 e + \gamma_2 e^2 + \gamma_3 e^3), \quad (3)$$

where $\alpha$ (km s$^{-1}$), $\beta_1$, $\beta_2$, $\gamma_1$, $\gamma_2$, and $\gamma_3$ are fitting parameters given in Table 1 in the CLA row. The fit was performed using numerical results in the range $\eta = 0–0.25$ and $e = 0–0.5$ with an average error $<1$ km s$^{-1}$. The fitting function (3) can be shown to be functionally equivalent to that presented by Fitchett (1983) for small eccentricities, but with different coefficients that lead to significantly smaller magnitudes for the recoil.

3. ESTIMATING THE TOTAL RECOIL

The estimates of the recoil velocities for BBH mergers provided by equation (3) and the first row of Table 1 are incomplete because we have not taken into account the contribution from the inspiral ($\sim 20$ km s$^{-1}$) and the beginning of the merger, where fully nonlinear computations are needed. In order to obtain an estimate of the total accumulated recoil velocity, we can supplement our calculations with estimates from approximation techniques that are accurate for moderate and large separations. In this Letter, we use a PN approximation that provides the accumulated kick up to some minimum separation. For quasi-circular orbits (Blanchet et al. 2005) and assuming that the angular frequency is given by the Newtonian estimate $\omega = (Md^3) \times 10^{-2}$, the accumulated recoil up to minimum separations of $4GM/c^2$ and $6GM/c^2$ can be fitted by equation (3) with the second and third rows, respectively, of Table 1. As in Sopuerta et al. (2006), upper and lower limits can be obtained by combining these PN estimates with the CLA calculation.

Our best estimate consists of adding to the CLA estimates, obtained by starting at a separation of $4GM/c^2$, the PN accumulated recoil from infinity to that separation. However, to compensate for using the PN approximation in a regime where it becomes less accurate, we use a PN expression for the angular velocity with errors of $O(v/c^3)$, instead of the Newtonian value used in Sopuerta et al. (2006) to obtain an upper limit. Our results for the quasi-circular case are shown in Figure 2, with error bars derived just from the PN approximate error. In this figure, we also compare our results to the full relativistic results of Gonzalez et al. (2006) and find that they agree remarkably well.

### Table 1

| Model          | $\alpha$ | $\beta_1$ | $\beta_2$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ |
|----------------|----------|-----------|-----------|------------|------------|------------|
| CLA            | 5232     | -2.621    | 3.199     | -0.942     | 0.808      | -0.405     |
| PN ($r^{\text{min}}_{\text{out}}$) | 8226     | 0.347     | 0.083     | 0          | 0          | 0          |
| PN ($r^{\text{min}}_{\text{in}}$)  | 1206     | 0.1057    | 0.05      | 0          | 0          | 0          |

*a Here $r^{\text{min}}_{\text{out}} = 4GM/c^2$ and $r^{\text{min}}_{\text{in}} = 6GM/c^2$ are the minimum separations up to which the PN approximation is applied.

The main caveat of supplementing the CLA with a PN approximation is that currently the latter is only available for quasi-circular orbits (Blanchet et al. 2005). However, by comparing the contribution from the CLA scheme with that provided by the PN approximation for the quasi-circular case, we find that the total recoil velocity can be accurately described by the CLA contribution times a factor

$$v_c^{\text{total}} = v_c^{\text{CLA}}(1 + \cal{E}), \quad (4)$$

so that $\cal{E}$ can be written in terms of the symmetric mass ratio $\eta$ as

$$\cal{E} = \kappa(1 + \lambda_1 \eta + \lambda_2 \eta^2), \quad (5)$$

where the coefficients $\kappa$, $\lambda_1$, and $\lambda_2$ depend on the minimum distance that we use in the PN approximation. When the minimum distance is taken to be $r^{\text{min}}_{\text{in}} = 4GM/c^2$, we obtain $\kappa = 1.357 (0.648)$, $\lambda_1 = 4.418 (5.612)$, and $\lambda_2 = 5.33 (23.63)$ (the values in parentheses correspond to using the 2PN angular velocity instead of the Keplerian one); whereas when we take it to be $r^{\text{min}}_{\text{in}} = 6GM/c^2$ the coefficients are $\kappa = 0.199$, $\lambda_1 = 4.223$, and $\lambda_2 = 3.926$.

We can then obtain lower and upper bounds as well as a best estimate for the total recoil velocities from eccentric BBH mergers if we assume that expressions (4) and (5) can be extrapolated to the range of eccentricities under consideration, $e \leq 0.25$. When we use $r^{\text{min}}_{\text{in}} = 6GM/c^2$ and the Keplerian estimate for the angular velocity, the total recoil obtained is a lower limit because we do not account for the contribution in the region (4–6)GM/c$^2$. On the other hand, if we use $r^{\text{min}}_{\text{in}} = 4GM/c^2$ and the Keplerian angular velocity, equation (4) provides an upper limit, since it is well known that the PN approximation overestimates the recoil in the (4–6)GM/c$^2$ range. Our best estimate is obtained with $r^{\text{min}}_{\text{in}} = 4GM/c^2$ and the 2PN angular velocity. These estimates, lower and upper bounds and best estimate, are shown in Figure 3.

4. SUMMARY AND DISCUSSION

We applied the CLA scheme to the computation of recoil velocities for BBH mergers including the effect of eccentricity.

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![Figure 2](image-url)
For small eccentricities, $e < 0.1$, we find a generic increase in the recoil of the order of 10% with respect to the quasi-circular case. This increase is related to the fact that for slightly eccentric orbits, the magnitude of the initial velocity of each BH increases roughly by a factor of $1 + e$. Since the CLA scheme is valid for separations smaller than the corresponding one for the ISCO, we have supplemented our results with estimates obtained with a 2PN approximation, which work very well for large separations. Combining appropriately both approximation schemes, we have produced lower and upper bounds, as well as a best estimate, for the recoil velocity.

Our calculations indicate that the maximum recoil velocity takes place at a symmetric mass ratio $\eta$ of $\sim 0.19$ and its magnitude can be as low as $\sim 79$–$88$ km s$^{-1}$ and as high as $\sim 216$–$242$ km s$^{-1}$, with a best estimate in the range $\sim 167$–$187$ km s$^{-1}$. The variation in these estimates is due to the eccentricity of the orbit. All our estimates can be fitted by the nonlinear function of $\eta$ and $e$ given by equation (3). This formula is expected to work well for small eccentricities, and it reduces to the expression given by Fitchett (1983) with a significantly smaller magnitude, when $e \ll 1$. These results also significantly narrow previous calculations (Favata et al. 2004; Blanchet et al. 2005; Damour & Gopakumar 2006) and agree remarkably well with full relativistic computations (Gonzalez et al. 2006) in the quasi-circular case.

One of the main conclusions that can be extracted from our and recent estimates of the gravitational recoil from BBH mergers is that the main contribution to the recoil comes from a narrow interval of separations that include the ISCO. Not surprisingly, this coincides with the nonlinear gravitational regime, where perturbative approximation schemes tend to break down. In this sense, the attractive feature of our approach is that it uses an approximation method, the CLA scheme, valid for small separations, and complements the estimate with another approximation, the PN scheme, valid for large to moderate separations.

As mentioned before, SMBH merger recoil velocities are relevant for hierarchical dark matter scenarios of structure formation, as well as in the understanding of the displacement of nuclear structure in dense galaxies and the distortion of X-shaped radio sources. Our results do not significantly alter the astrophysical conclusions obtained previously by Merritt et al. (2004). Instead, our results confirm and sharpen these conclusions, by providing narrower lower and upper bounds for the possible recoil velocities.

Future work can assess the robustness of our estimates by studying their dependence on the initial configuration and by the inclusion of higher gravitational multipoles in the calculation. In the future, when precise numerical relativistic simulations of eccentric orbits become available, it would then be possible to compare results and to determine with better precision the distribution of recoil velocities in the parameter space ($\eta, e$). A very important issue is the effect of spins, which has not yet been dealt with with enough generality to ascertain the impact of the additional scales introduced by spinning BHs.

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