Real-time Pricing Mechanism in Smart Grid based on System of Incentive and Penalty

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Abstract. In the smart grid, users, power suppliers and Power Market Scheduling Center are the main participants. In view of the increasing demand response ability of users, an identification mechanism of punishing malicious users and unstable power providers while incenting non-malicious users is proposed, and an optimization model to maximize social welfare is considered to study the real-time pricing problem in smart grid managed by PMSC. The model is analyzed by Lagrange dual method and finally solved by heuristic algorithm to obtain the optimal electricity price and power demand. The simulation results show that the algorithm can converge well and the reliability and stability of power system are optimized under the dual regulation of penalty and incentive. In addition, there is a positive correlation between the incentive factor and users’ utility and social welfare within proper range.

Keywords: PMSC; real-time pricing; penalty mechanism; incentive factor; heuristic algorithm.

1. Introduction

Smart grid is a complete power system that can realize real-time monitoring of users and equipment. Compared with the traditional grid, it can not only improve the reliability, economy and flexibility of the grid, but also provide a more complete and convenient grid status display interface for the grid operation and management personnel. So the smart grid is a new type of grid that helps power grid to operate intelligently[1]. Users are an important part of smart grid system and encouraging users to participate in power system operation and management is one of the significant features of smart grid. In order to improve the safety operation level of power system, smart grid provides optimal power quality and ensures power supply reliability to meet users’ requirements through intelligent interaction with them. Meanwhile, power market participants can better participate in grid security management by market transactions[2].

In smart grid, demand response is one of the main solutions to manage the electricity price and the power consumption of users as well as the power generation capacity of suppliers[3]. Demand response refers to when the price of power wholesale market rises or system reliability is threatened, users change their original electricity consumption mode to reduce or transfer the load requirement of a certain time slot to respond to the power supply after receiving the direct compensation notice of induced load reduction or the electricity price rise signal from power suppliers, which ensures the stability of the grid and restrain the short-term behavior of electricity price rise [3-4]. Users can change the power requirement according to their own power demand and the balance of the power system’s ability to meet their demand, so that the power company can reduce capital and operating expenses and obtain greater profits[5]. There are many types of demand response, but they are mainly divided into two types: demand response based on price and demand response based on incentive[6-8]. In the demand response based on price, the electricity price is used to adjust the demand of customers to avoid high price period and maximize users’ utility [3]. In the incentive-based demand response, the utility system provides fixed or time-varying incentives to users to reduce their power consumption when the power system is under pressure [9].

Real-time pricing is an important pricing mechanism in demand response based on price. In the power market based on real-time electricity price, electricity price can adjust and optimize the operation reliability and economy of power system[10-13]. The research on smart grid real-time pricing is mainly carried out from two aspects: one is to propose various game models for RTP research, the other is considering social welfare maximization model. For example, the authors analyzed the price competition among different power suppliers through Stackelberg game model...
and found the equilibrium solution in [10]. In [11], a two-layer game model is considered to study RTP and search for a unique Nash equilibrium solution. In [12], a game model with the goal of maximizing user utility is proposed to study the real-time electricity price. As for the study of real-time electricity price in the social welfare maximization model in [13], the social welfare maximization model is proposed and the augmented Lagrangian multiplier method is used to calculate the real-time electricity price. In [14], the authors propose a social welfare maximization model to study the real-time pricing mechanism of smart grid from the total power consumption of users and users’ household appliances respectively. However, with the opening of the power market, a single power supplier may not be able to meet the needs of multiple users at the same time and multiple power suppliers are required to provide power jointly. Moreover, some literatures consider the irrational behavior of users and power suppliers. Therefore, the authors proposed the Mechanism of Identification and Processing (MIP) to identify and process malicious users and unstable power suppliers in the case of multiple power suppliers and users, and designed heuristic algorithm to solve problems in [15]. However, the maximum consumption of malicious users is replaced with the average amount of power consumption of non-malicious as punishment and the penalty is relatively small. Moreover, without considering the incentives for non-malicious users and stable power suppliers, it cannot effectively prevent them from turning into malicious users and unstable suppliers. In this paper, we not only increase penalties for malicious users, but also provide incentives to non-malicious users in order to reduce the number of unstable power suppliers. Therefore, the social welfare function is updated and the MIP is improved. We solve the social welfare maximization model by the proposed algorithm. Finally, the simulation results show that the reliability and stability of power system are optimized and the algorithm can converge rapidly under the dual regulation of penalty and incentive. In addition, we obtain the optimal electricity price and the maximum social welfare in different time slots by changing the value of power consumption willing. Moreover, there is a positive correlation between incentive factors and users’ utility and social welfare. Through comparison, we find that the renewed model is better than DPAMU.

The rest of this paper is organized as follows. The system model is introduced in Section II. In Section III, the heuristic algorithm is presented. We simulate the results in Section IV. In Section V, we give the conclusion.

2. System Model

2.1 Problem Description

In real life, the load power capacity of one power supplier cannot meet the demand of multiple users, therefore, multiple power suppliers are required to provide sufficient power for users [15]. We consider smart grid with multiple power suppliers and multiple users. If all the suppliers announce the price of electricity respectively in each time slot, then there will be price competition, which may cause the loss of power supplier in the buyer’s market or the user may pay higher price in the seller’s market. Therefore, in order to maximize social welfare, we consider the third party supervisory organization, namely the power market dispatch center or PMSC[15]. PMSC manages all the power suppliers, making them offer users a uniform price and an optimal power supply for maximum utility. However, some users may provide wrong data to PMSC, as a result, PMSC cannot provide optimal electricity price and social welfare cannot be maximized. In addition, if the data provided by the power supplier is tampered with into a random number, then the electricity price and total load may not converge, that is, the electricity price is not optimal. Therefore, PMSC needs a security mechanism to identify malicious users and unstable power suppliers so as to keep the data of participants safe, so MIP is proposed as a security mechanism [15].

The entire time cycle is divided into 24 time slots on average representing the 24 hours of one day. We refer to the users and power suppliers providing wrong data as malicious users and unstable power suppliers. The users and power suppliers providing real data are called non-malicious users and stable power suppliers. PMSC identifies the malicious users, non-malicious users, stable power suppliers and unstable power suppliers according to the data received.
2.2 Utility Function and Cost Function

In this paper, we assume that the users are risk-averse. The utility function is non-minus and the marginal benefit function is non-plus, thus the utility function is defined as follows [16]:

\[
U(x, w) = \begin{cases} 
wx - \frac{\alpha}{2}x^2, & 0 \leq x \leq \frac{w}{\alpha} \\
\frac{w^2}{2\alpha}, & \frac{w}{\alpha} \leq x.
\end{cases}
\]

(1)

where \( \alpha \) is a parameter pre-defined for every user and is known by PMSC. The parameter \( w \) denotes the power consumption willing of users and different users have different consumption willing, and \( x \) denotes the actual power consumption of users.

We consider the cost function \( C_p^k(l_p^k) \)[15][17], \( p = 1, 2, \ldots, M \) denoting the cost of power supplier \( p \) in time slot \( k \).

\[
C_p^k(l_p^k) = a_p^k l_p^k + b_p^k l_p^k + c_p^k
\]

(2)

where \( a_p^k, b_p^k \) and \( c_p^k \) are positive constants predetermined by the power supplier \( p \).

2.3 Malicious Users and Unstable Power Suppliers

In this paper, we assume that every user in each time slot has the same power consumption range \( [x_{\min}^k, x_{\max}^k] \). The malicious user calculates the power requirement according to the following formula:

\[
x_i^k = x_{\min}^k + \text{rand} \cdot (x_{\max}^k - x_{\min}^k), i \in N_{\text{mal}}
\]

(3)

where \( \text{rand} \) denotes a random constant in the interval \((0,1)\) and \( x_i^k \) denotes the actual power consumption of malicious user \( i \) in time slot \( k \). \( N_{\text{mal}} \) is the collection of malicious users, \( x_{\min}^k \) is the minimum power consumption of all users in time slot \( k \) and \( x_{\max}^k \) denotes the maximum power consumption of all users in time slot \( k \).

Malicious users randomly select data from the power consumption range and send them to PMSC, which may damage the stability of the model, causing the non-convergence of electricity price. And users’ utility cannot be maximized, causing the loss of social welfare.

PMSC can identify and punish malicious users according to the energy requirement data received from malicious users. Based on [15], we increase penalties by replacing the wrong data provided by malicious users with the minimum power consumption data of all users. Because the minimum power consumption can only meet users’ basic demand, it cannot meet users’ normal demand or higher quality demand. Therefore, the power available to malicious users is limited and it cannot meet the needs of daily life. In order to avoid the loss of their own utility, users are more inclined to send real power consumption data to PMSC, which helps to prevent non-malicious users from turning into malicious users and maintain the stability of the power system. Therefore, PMSC set the minimum power requirement of all users to the maximum power consumption of malicious users as penalty in time slot \( k \):

\[
x_i^k = x_{\min}^k, i \in N_{\text{mal}}
\]

(4)

In other words, malicious users’ actual power consumption is \( x_{\min}^k \).

Besides, we assume that all power suppliers have the same load capacity, and the maximum load capacity of each power supplier can meet the total requirements of all users, the load capacity of each
power supplier $p$ in time slot $k$ belongs to the following interval $t_p^k \in [0, N\cdot x_{\text{max}}^k]$. Unstable power suppliers provide wrong data, PMSC can set load capacity of them to 0 as penalty.

### 2.4 Incentive Function and Social Welfare Function

In this paper, PMSC identifies malicious users and non-malicious users based on the data received. For non-malicious users, PMSC gives monetary incentives to prevent them from turning into malicious users and improve the stability of the power system. The incentive formula is defined as follows:

$$I = \pi x^i, \quad i \in N - N_{\text{mal}}$$  \hfill (5)

where $\pi$ is a positive constant and denotes the incentive factor, because the users’ utility should be greater than or equal to the monetary incentive, that is, $wx - \frac{\alpha}{2} x^2 \geq \pi x$, reduction to $w - \pi \geq \frac{\alpha}{2} x$, so the incentive factor should be less than or equal to $w$.

$x_i^k$ is non-malicious users’ power consumption in time slot $k$ and $x_i^k \in [x_{\text{min}}^k, x_{\text{max}}^k]$.

Due to monetary incentives, the utility of non-malicious users is also increased. According to (1) and (5), the social welfare function can be redefined as follows:

$$U(x_i^k, w_i^k) = \begin{cases} w_i^k x_i^k - \frac{\alpha}{2} (x_i^k)^2 + \pi(x_i^k - x_{\text{min}}^k), & 0 \leq x_i^k \leq \frac{w + \pi}{\alpha} \\ \frac{(w_i^k + \pi)^2}{2\alpha} - \pi x_{\text{min}}^k, & x_i^k > \frac{w + \pi}{\alpha} \end{cases}$$  \hfill (6)

where $\pi(x - x_{\text{min}})$ denotes the monetary incentives that PMSC provides to non-malicious users.

### 2.5 Optimal Model

In this paper, we consider a smart grid real-time pricing model in demand response based on price. And we divide a day into 24 time slots on average and assume that each period is independent from each other. We set up the following optimization model to maximize social welfare:

$$\max \left[ \sum_{k \in K} \sum_{i \in N - N_{\text{mal}}} U(x_i^k, w_i^k) - \sum_{p \in M - M_{\text{mal}}} C_p^k(t_p^k) \right]$$  \hfill (7)

s.t. \quad $\sum_{i \in N - N_{\text{mal}}} x_i^k + \sum_{j \in N_{\text{mal}}} x_j^k \leq \sum_{p \in M - M_{\text{mal}}} t_p^k$

where $M$ is the set of power suppliers, $M_{\text{mal}}$ is the set of unstable power suppliers and $x_i^k$ is the actual power consumption of user $i$ in time slot $k$.

According to (4), (7) can be converted to the following problem:

$$\max \left[ \sum_{k \in K} \sum_{i \in N - N_{\text{mal}}} U(x_i^k, w_i^k) - \sum_{p \in M - M_{\text{mal}}} C_p^k(t_p^k) \right]$$  \hfill (8)

s.t. \quad $\sum_{i \in N - N_{\text{mal}}} x_i^k + \sum_{j \in N_{\text{mal}}} x_j^k \leq \sum_{p \in M - M_{\text{mal}}} t_p^k$

Because different time slots are independent of each other, (8) is equivalent to the following formulas:

$$\max \left[ \sum_{i \in N - N_{\text{mal}}} U(x_i^k, w_i^k) - \sum_{p \in M - M_{\text{mal}}} C_p(t_p) \right]$$  \hfill (9)

s.t. \quad $\sum_{i \in N - N_{\text{mal}}} x_i^k + \sum_{j \in N_{\text{mal}}} x_j^k \leq \sum_{p \in M - M_{\text{mal}}} t_p^k$
We let $\beta = \frac{N_{mal}}{N-N_{mal}}$ and use the Lagrangian dual decomposition method to obtain the following Lagrangian function:

$$L(x, l, \lambda) = \sum_{i \in N-N_{mal}} U(x_i^k, w_i^k) - \sum_{p \in M-M_{sup}} C_p^k(l_p^k) - \lambda^k \left( \sum_{i \in N-N_{mal}} x_i^k + \beta \sum_{i \in N-N_{mal}} x_i^k - \sum_{p \in M-M_{sup}} l_p^k \right)$$

(10)

where $\lambda^k$ is the Lagrangian multiplier which denotes the electricity price in time slot $k$ [15][18]. According to [19], (10) has the following subproblems:

$$D(\lambda) = \max_{i \in N-N_{mal}} B_i^k(\lambda^k) + \sum_{p \in M-M_{sup}} S_p^k(\lambda^k)$$

(11)

$$B_i^k(\lambda^k) = \max \ U(x_i^k, w_i^k) - \lambda^k \left( x_i^k + \beta x_i^k \right)$$

(12)

$$S_p^k(\lambda^k) = \max \lambda^k l_p^k - C_p^k(l_p^k)$$

(13)

The dual problem is:

$$\min D(\lambda)$$

(14)

when the PMSC provides the electricity price to users and power suppliers, users can calculate the optimal power consumption according to (12) and power suppliers can calculate the optimal power load according to (13).

The electricity price in time slot $k$ can be calculated by the following iterative formula:

$$\lambda_{t+1}^k = \lambda_t^k + \gamma \left[ dD(\lambda) \right] = \lambda_t^k + \gamma \left( \sum_{i \in N-N_{mal}} \left( x_i^k(\lambda_t^k) + \beta x_i^k \right) - \sum_{p \in M-M_{sup}} l_p^k(\lambda_t^k) \right)$$

(15)

where $\gamma$ is the step size, $\lambda_t^k$ is the electricity price at the iteration $t$ of the time slot $k$. $x_i^{k^*}$ denotes the optimal power consumption of user $i$ in time slot $k$ and $l_p^{k^*}$ denotes the optimal power load of the power supplier $p$ in time slot $k$.

### 2.6 Identifying and Punishing Malicious Users

According to the identification mechanism in [15], malicious users randomly send electricity data within a certain power consumption range to PMSC, while non-malicious users calculate the optimal power consumption data in sections and then send them to the PMSC. According to social welfare function (7) and (12), we get the following two optimization problems respectively:

$$x_i^{k^*}(\lambda_t^k) = \arg \max w_i^k x_i^k - \frac{\alpha}{2} \left( x_i^k \right)^2 + \pi \left( x_i^k - x_i^{min} \right) - \lambda_t^k \left( x_i^k + \beta x_i^{min} \right), \quad x_i^k \leq \frac{w_i^k + \pi}{\alpha}$$

(16)

$$x_i^{k^*}(\lambda_t^k) = \arg \max \frac{\left( w_i^k + \pi \right)^2}{2\alpha} - \pi x_i^{min} - \lambda_t^k \left( x_i^k + \beta x_i^{min} \right), \quad x_i^k > \frac{w_i^k + \pi}{\alpha}$$

(17)

where $w_i$ is the power consumption willing of user $i$ and it is constant in all the time slots.

According to (17), the optimal power consumption of user $i$ is:

$$x_i^{k^*}(\lambda_t^k) = \frac{w_i - \lambda_t^k + \pi}{\alpha}$$

(18)
According to (18), the optimal power consumption of user $i$ is:

$$x_i^*(\lambda_i^k) = \frac{w_i^k + \pi}{\alpha}$$  \hspace{1cm} (19)

We assume that the power consumption range of all the users is $[x_{i,\min}^k, x_{i,\max}^k]$ in time slot $k$, then the optimal power consumption can be calculated in the following three cases:

1. If $x_{i,\max}^k \leq \frac{w_i^k + \pi}{\alpha}$, the optimal power consumption can be calculated according to (18) and it must be within $[x_{i,\min}^k, x_{i,\max}^k]$.

so, the user’s optimal power consumption is: $x_i^*(\lambda_i^k) = \max \left\{ \min \left( \frac{w_i^k - \lambda_i^k + \pi}{\alpha}, x_{i,\max}^k \right), x_{i,\min}^k \right\}$.

2. If $x_{i,\min}^k \leq \frac{w_i^k + \pi}{\alpha} \leq x_{i,\max}^k$, we should calculate the optimal power consumption in sections. Within the range $\left[ x_{i,\min}^k, \frac{w_i^k + \pi}{\alpha} \right]$, the optimal power consumption is:

$$x_i^*(\lambda_i^k) = \max \left\{ \min \left( \frac{w_i^k - \lambda_i^k + \pi}{\alpha}, x_{i,\max}^k \right), x_{i,\min}^k \right\}. \text{ Within the range } \left[ \frac{w_i^k + \pi}{\alpha}, x_{i,\max}^k \right], \text{ the optimal power consumption is } \frac{w_i^k + \pi}{\alpha}$.

3. If $x_{i,\min}^k \geq \frac{w_i^k + \pi}{\alpha}$, the user’s optimal power consumption is $x_{i,\min}^k$.

According to the above analysis, the optimal power consumption of non-malicious users can be $x_{i,\min}^k$, $x_{i,\max}^k$, $\frac{w_i^k - \lambda_i^k + \pi}{\alpha}$ or $\frac{w_i^k + \pi}{\alpha}$. When PMSC receives the optimal power consumption from user $i$, it should first decide whether it is equal to $x_{i,\min}^k$ or $x_{i,\max}^k$. If it is neither $x_{i,\min}^k$ nor $x_{i,\max}^k$, PMSC should calculate and store the user’s power consumption willing $w_i^{t+1}$ and $w_i^{t+1}$ according to (18) and (19).

In the following iterations, we continue to calculate the user’s power consumption willing. For instance, in the iteration $t+1$, if the user’s optimal power consumption is neither $x_{i,\min}^k$ nor $x_{i,\max}^k$, we calculate $w_i^{t+1}$ and $w_i^{t+1}$ respectively. Then we compare the four power consumption willing to identify whether the user is malicious.

The steps of PMSC to identify and punish malicious users is as follows:

**2.7 Identifying and Punishing Unstable Power Suppliers**

If the data provided to PMSC by some power suppliers is not optimal, it will disturb the pricing process. Stable power suppliers will calculate the optimal load capacity according to (2) and (13). The optimal load capacity of the power supplier $p$ is:

$$l_p^*(\lambda_p^k) = \arg\max_{p \in \mathcal{P}} \lambda_p^k l_p^k - a_p^k \left( \frac{\lambda_p^k}{2} \right)^2 - b_p^k t_p^k - c_p^k$$  \hspace{1cm} (20)

In the case that the load capacity range of power suppliers is not taken into account, the optimal load capacity can be calculated as follows:

$$l_p^*(\lambda_p^k) = \frac{\lambda_p^k - b_p^k}{2a_p^k}$$  \hspace{1cm} (21)

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Step1 PMSC initializes each user’s power consumption willing. For instance, the user’s willing set is \( w_i^0 = (w_{i1}^0, w_{i2}^0) \) and the initial values of \( w_{i1}^0 \) and \( w_{i2}^0 \) are set to 0.

Step2 If all user’s data can proceed, exit. Otherwise, PMSC gets the data at the iteration \( t \) in time slot \( k \) and determines whether it is in the range \( [x_{i_{\text{min}}}^k, x_{i_{\text{max}}}^k] \).

Step3 If all the user’s data can proceed, exit. Otherwise, PMSC gets the data in the iteration \( t \) in time slot \( k \) and determines whether it is within the range \( [x_{i_{\text{min}}}^k, x_{i_{\text{max}}}^k] \).

Step4 If \( x_i^t \neq x_{i_{\text{min}}}^k \) and \( x_i^t \neq x_{i_{\text{max}}}^k \), then calculate the user’s power consumption willing according to (19) and (20) and compare them with \( w_{i1}^0 \) and \( w_{i2}^0 \) respectively.

Step5 If \( w_{i1}^0 = w_{i2}^0 = 0 \), replace \( w_{i1}^t \) and \( w_{i2}^t \) with \( w_{i1}^0 \) and \( w_{i2}^0 \) respectively. If \( w_{i1}^0 \neq 0 \) or \( w_{i2}^0 \neq 0 \), then determine which of the following four equations is true: \( w_{i1}^0 = w_{i1}^t \), \( w_{i2}^0 = w_{i2}^t \), \( w_{i1}^0 = w_{i2}^t \), \( w_{i2}^0 = w_{i1}^t \). If none of the four equations is true, the user is malicious. If one of the four equations is true, the user is non-malicious, then incent the user according to (5) and return to Step2.

Step6 PMSC replaces the load of malicious users according to (4), and return to Step2.

Since the load capacity of each power supplier has a certain value range, the optimal load capacity of the power supplier is:

\[
L_p^*(\lambda^k_t) = \max \left\{ \min \left( \frac{\lambda^k_t - b^k_p}{2a^k_p} [N^* x_{\text{max}}^k], 0 \right) \right\}
\]  
(22)

In this paper, \( a^k_p \), \( b^k_p \) and \( c^k_p \) of power supplier \( p \) are different constants in different time slots. Therefore, if two different optimal load capacity values are obtained in the iteration \( t \) and \( t+1 \): \( L_p^*(\lambda^k_t) \) and \( L_p^*(\lambda^k_{t+1}) \), if \( L_p^*(\lambda^k_t) \) and \( L_p^*(\lambda^k_{t+1}) \) are neither equal to 0 nor equal to \( [N^* x_{\text{max}}^k] \), then PMSC can solve the following equations to obtain the parameters \( a_p^* \) and \( b_p^* \) of the power supplier \( p \) in time slot \( k \).

\[
\begin{align*}
\lambda^k_t - b^k_p - 2a^k_pl_p^*(\lambda^k_t) &= 0 \\
\lambda^k_{t+1} - b^k_p - 2a^k_pl_p^*(\lambda^k_{t+1}) &= 0
\end{align*}
\]  
(23)

If PMSC obtained the third optimal load capacity data in the iteration \( t+2 \) which is neither equal to 0 nor equal to \( [N^* x_{\text{max}}^k] \), then PMSC only needs to verify whether the following formula is equal to zero:

\[
\lambda^k_{t+2} - b^k_p - 2a^k_pl_p^*(\lambda^k_{t+2}) = 0
\]  
(24)

If (24) is equal to zero, the power supplier \( p \) is stable, otherwise \( p \) is unstable.

The steps to identify and punish unstable power suppliers are as follows:
Step 1
PMSC initializes the parameter values of each power supplier. For instance, the parameter set of the power supplier $p$ is $\{a_p^k, b_p^k\}$, the initial values of $a_p^k$ and $b_p^k$ are -1.

Step 2
If PMSC receives two optimal load capacity values from the power supplier $p$ and neither of them is equal to 0 and $N^k * x_{\text{max}}^k$, then the cost function parameters of the supplier $p$ can be calculated according to (23).

Step 3
If PMSC receives the third optimal load capacity value from the power supplier $p$ which is not equal to 0 and $N^k * x_{\text{max}}^k$, then verify (24) is equal to 0 or not, if (24) is equal to 0, the supplier $p$ is stable, otherwise $p$ is unstable.

3. Heuristic Algorithm

In this paper, the optimal model consists of three parts: power suppliers, users and PMSC. PMSC updates the electricity price. The optimal power consumption of the users is calculated by users and the optimal load capacity of the power suppliers is calculated by power suppliers. So, we can get the following flow chart:

![Flow Chart](image)

In Fig.1, PMSC initializes the electricity price and sends it to users and power suppliers. After receiving the data, non-malicious users will calculate the optimal power consumption and send it to PMSC while malicious users will send PMSC random power consumption belonging to $x_{\text{min}}^k + \text{rand} * (x_{\text{max}}^k - x_{\text{min}}^k)$). For power suppliers, stable power suppliers will calculate the optimal load capacity according to (21) while unstable power suppliers update their load capacities randomly which belong to the range $[0, N^k * x_{\text{max}}^k]$. All the power suppliers will send the optimal load capacity to PMSC. If PMSC receives the data, it will firstly identify and punish malicious users and unstable power suppliers by using the above mechanism. And then PMSC will update the electricity price according to (16) and send it to the users and power suppliers, and $\beta$ is also sent to users.
4. Numerical Simulation

We respectively define PMSC, users and power suppliers for simulation. In each time slot, PMSC randomly initializes the electricity price and sends it to power suppliers and users. Power suppliers and users update their power supplies and consumptions according to the received electricity price and send them to PMSC and PMSC updates the electricity price according to the received data and identifies malicious users and unstable power suppliers. After several iterations, the price tends to be stable and the total load value is equal to the total demand value, so the simulation ends.

4.1 Convergence Performance

In this paper, we divide the day into 24 time slots on average representing 24 hours a day and assume that the power consumption willing \( w \) of users can be selected in the range \([2,4]\) and remains fixed throughout the day[15]. We let \( N = 100 \) and \( M = 10 \), and the parameter \( \alpha \) is set as 0.5. The parameters of the cost function of power suppliers are defined as follows:

\[
\begin{align*}
    a_p^k &= 0.01 + 0.002p, \quad p = 1,2,...,M \\
    b_p^k &= 1 - 0.02p, \quad p = 1,2,...,M \\
    c_p^k &= \text{rand}(0,1)
\end{align*}
\]

We assume that the maximum and minimum power requirements of different users are the same in each time slot which is \([0.05,15]\). And the load capacity of each power supplier is \( |N| \cdot x_{\text{max}}^k \). First of all, we set the proportion of malicious users and unstable suppliers as \([0,0]\), \([0.3,0.3]\) and \([0.6,0.6]\) to simulate the convergence of electricity price. The simulation result is shown in the following figure:

![Fig.2 Convergence of electricity price](image)

From the Fig.2, we can see that the price converges to a stable value in the end. And if the proportion of malicious users and unstable power suppliers changes, the stable price will also change. According to Fig.2, the price is unstable and decreasing in the initial iteration process, which indicates that the total load capacities of power suppliers are much bigger than the total requirements of the users, resulting in oversupply. Therefore, the electricity price is not optimal in theory until it converges to a stable value.

To illustrate the convergences of load requirements of the users and the load capacities of the power suppliers, we set the proportion of malicious users and unstable power suppliers as \([0.3,0.3]\) and \([0.6,0.6]\). The simulation result is shown in Fig.3.
According to Fig.3, both the load requirements of the users and the load capacities of power suppliers converge eventually. In Fig.3, when the proportion of unstable power suppliers and malicious users increases, the number of stable suppliers and non-malicious users decreases, which results in the reduction of load capacities and load requirements.

4.2 Social Welfare

We set the proportion of malicious users and unstable power suppliers as [0.3, 0.3], the incentive factor is set as 0.5. The power consumption willing will change over time, in other words, it is larger during the peak period, but it is always within the range of [2, 4]. The social welfare of this paper is compared with that of DPAMU in Fig.4.

From the Fig.4, the social welfare in this paper is superior to that of DPAMU, because we incent non-malicious users. Moreover, the power consumption willing is greater during the peak period, so the load requirement of the users and the optimal load capacity of power suppliers will change. Therefore, the model in this paper is more beneficial to users and power suppliers.
4.3 Electricity Price

In this paper, we set the proportion of unstable power suppliers and malicious users as [0.3,0.3] respectively. The incentive factor is set as 0.8. In the case that the power consumption willing varies with time slots, the electricity price of each time slot is as follows:

![Electricity Price Graph]

Fig.5 Changes in electricity price in 24 time slots

4.4 Incentive Factor

We assume that the consumption willing is time varying within the range [2,4], then analyze the effect of incentive factors on users’ utility and social welfare. From the above, we can calculate the range $0 < \pi < w^i_k - \frac{\alpha}{2} x^i_{\max}$. Since all users have the same power consumption scope, we can get users’ utility and social welfare in different time slots. The incentive factor is set as 0, 0.3, 0.5 and 1, and the proportion of malicious users and unstable power suppliers is set as [0.3,0.3]. The users’ utility of each time slot is shown in Fig.6. The social welfare of each time slot is shown in Fig.7.

![Incentive Factor Graph]

Fig.6 Effect of incentive factors on users’ utility
According to the above, users’ utility and social welfare increase with the increase of incentive factors. Therefore, the incentive factor should be set to a larger value in the range $0 < \pi < \frac{\alpha}{2} x_{\max}^k$.

5. Conclusion

This paper adopts the Price-based DR and modifies the social welfare maximization model to identify and punish malicious users and unstable power suppliers. On one hand, we increase the penalty for malicious users. Under this regulation, the reliability of power system can be adjusted and optimized and the real-time interaction between users and power suppliers is realized. On the other hand, we provide monetary incentives for non-malicious users. The simulation results show that incentive factors are positively correlated with users’ utility and social welfare within a reasonable range. Considering that the consumption willing is time-varying, when the proportion of suppliers and users is fixed, we can get the variation of social welfare and electricity price of one day, which proves that the proposed model is more beneficial to power suppliers and users than DPAMU. Future research direction includes providing subsidies for stable suppliers.

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