I. INTRODUCTION

First seen in $D_{s0}^*(2317) \rightarrow D_s\pi^0$ by BABAR (2003) [1], $D_{s1}(2460) \rightarrow D_{s1}^*\pi^0$ by CLEO (2003) [2] and confirmed by BELLE (2004) [3], a clear experimental proof for the inner structure of $D_{s0}^*(2317)$ is still unavailable [4]. The experiments observe a narrow mass below $DK$ threshold for the $D_{s0}^*$ state as shown in Figure 1 while a number of models such as Quark Model [5] and Lattice Theory find it above close to $DK$ threshold [6]. It also has very small width and only the upper limit has been measured with the following experimental mass value:

$$M_{D_{s0}^*}(2317) = (2317.7 \pm 0.6) \text{ MeV}, \quad \Gamma_{D_{s0}^*} < 3.8 \text{ MeV}.$$  

However, this width value is in disagreement with the Heavy Quark Symmetry (HQS) estimation expecting to create a broad 1/2 doublet with $J^P = 0^+, 1^+$. The decay modes of $D_{s0}^*(2317)$ was reported respectively by the CLEO and BABAR collaborations [2, 10] at the 90% confidence level:

$$\frac{\Gamma(D_{s0}^*(2317) \rightarrow D_s^*(2112)\gamma)}{\Gamma(D_{s0}^*(2317) \rightarrow D_s\pi^0)} < 0.052 \quad \text{CLEO} \quad [2]$$

and the branching ratio as follows:

$$\frac{B(D_{s0}^*(2317) \rightarrow D_s^*(2112)\gamma)}{B(D_{s0}^*(2317) \rightarrow D_s\pi^0)} < 0.059 \quad \text{CLEO} \quad [2],$$

$$< 0.18 \quad \text{BELLE} \quad [11],$$

$$< 0.16 \quad \text{BABAR} \quad [12].$$

On the other hand, in 2018 the absolute branching fraction $B(D_{s0}^*(2317) \rightarrow D_s^*\pi^0)$ is measured as $1.00^{+0.09}_{-0.11}(\text{stat})^{+0.06}_{-0.14}(\text{syst})$ with a statistical significance of $5.8\sigma$ in the BESIII detector at a center-of-mass energy $\sqrt{s} = 4.6$ GeV for the first time [13]. It predicts that $D_{s0}(2317)$ should have a branching fraction of $\gamma D_{s0}^*$ at around 15% or even larger, but agrees well with the calculation in the molecular picture [14] which shows that the branching fraction of $\pi^0 D_{s0}^*$ is between $(93 - 100)\%$.

Flood of works are made on heavy-light, narrow open-charm systems both in conventional and non-conventional frameworks such as using Chiral Unitarity Model (CUM) [15, 16], Light-cone QCD Sum Rules (LCQCDSR) [17], Effective Lagrangian Approach...
(ELA) [14, 18], QCD Sum Rules (QCDSR) [19–22], Lattice QCD (LQCD) [23, 24], Nonrelativistic Constituent Quark Model (NRCQM) [25], etc.

Nevertheless many of these mesons in the open-charm sector can not be well described by the Quark Model. This situation has opened a discussion on their inner structures. The uncertainty with conventional $c\bar{s}$ interpretation motivates many authors to hypothesize that $D_{s0}^*(2317)$ state might be a molecule [15, 26] or a diquark-antidiquark state [27, 28]. Further the existence of a $DK$ pole at this energy has been recently confirmed from Lattice calculations of scattering amplitudes. This state can be responsible of the bump near the $DK$ threshold around 2.4 GeV [29].

As the mass of $D_{s0}^*(2317)$ is about 40 MeV below the threshold of $DK$, it is thought to be most likely a $DK$ hadronic molecule [26]. They pointed out that such a selection naturally manifests the anomalous mass around 2 GeV for such structures. The uncertainty with conventional $c\bar{s}$ interpretation motivates many authors to hypothesize that $D_{s0}^*(2317)$ state might be a molecule [15, 26] or a diquark-antidiquark state [27, 28]. Further the existence of a $DK$ pole at this energy has been recently confirmed from Lattice calculations of scattering amplitudes. This state can be responsible of the bump near the $DK$ threshold around 2.4 GeV [29].

Howbeit there is no well-defined separation of phases by means of the crossover nature of transition, thermal properties of hadrons change expeditiously in the vicinity of $T_c$. As the temperature raise, the quark condensate values are predicted to lessen from a non-vanishing value in vacuum to $(q\bar{q}) \approx 0$ which coincides with chiral symmetry restoration. Therefore it is vital to find out experimental observables which are sensitive to the quark condensates since $q\bar{q}$ is not a measurable quantity. In this context, to get clear signals from QGP many heavy-ion experiments will be operated at the Nuclotron-based Ion Collider facility (NICA), future Facility for Antiproton and Ion Research (FAIR), as well as RHIC at Large Hadron Collider (LHC).

This paper, adopting the QCD Sum Rule approach to finite temperature we evaluate the mass and pole residue of $D_{s0}^*(2317)$ treating it has four-quark content taking into account quark, gluon and quark-gluon mixed condensates up to dimension six assuming the quark-hadron duality is valid as well. Note that the vacuum condensate expressions are displaced with their thermal versions. This analysis can give us some hints on the nature of the $D_{s0}^*(2317)$ and also provide knowledge about the systematics of strong interactions in hot medium supplying information about the beginning stage of the universe which is believed to full of free quarks and gluons.

This article is arranged as follows. In Section II we obtain the TQCDSR to evaluate the mass and pole residue of $D_{s0}^*(2317)$. Section III is devoted to numerical analysis where we first present input parameters used in the computation and give our numerical results for the mass and pole residue of the considered state. These results are confronted with the data obtained from other theoretical models and available experimental data in the vacuum limit. Lastly, Appendix consists of the explicit expressions of the obtained two-point thermal spectral densities $\rho^{QCD}(s,T)$ in the TQCDSR theory.

II. FINITE TEMPERATURE SUM RULES FOR THE $D_{s0}^*(2317)$ STATE

To explore the deviations of mass and pole residue of $D_{s0}^*(2317)$ depending on increasing temperature, we extend the QCDSR technique to TQCDSR. Calculation is initiated by writing down the correlation function as in QCDSR model [42, 45]:

$$\Pi(q,T) = i \int d^4x \, e^{i q \cdot x} \langle \omega | T \{ J(x) J^\dagger(0) \} | \omega \rangle,$$  \hspace{1cm} (3)

where $T$ represents the time ordering operator, $\omega$ denotes the hot medium, $T$ is the temperature and $J(x)$ is the interpolating current accompanying to $D_{s0}^*(2317)$ resonance. In thermal equilibrium, thermal average of an operator $\mathcal{A}$ can be written as follows:

$$\langle \mathcal{A} \rangle = Tr (e^{-\beta\mathcal{H}} \mathcal{A})/Tr (e^{-\beta \mathcal{H}}),$$  \hspace{1cm} (4)

here $\beta = 1/T$ and $\mathcal{H}$ is the QCD Hamiltonian.
QCDSR technique which is very effective and practical can be used to obtain the mass and pole residue of the $D_{s0}^*(2317)$ state. In QCDSR, correlation function’s dependence on momentum provides us to extract the physical properties of any hadron by calculating the correlator in two different momentum regions. After getting correlation functions by converging these two regions to each other, we equalize these two expressions under the assumption quark-hadron duality which is sweeps the contributions of higher states under the carpet. In this context, we will introduce the two window. The first is called physical side which is described by hadronic observables such as mass, residue, coupling constant of the considered hadrons. The second is QCDSR side calculated in terms of QCD parameters such as masses of quarks, quark-gluon condensates.

A. Physical Side

At low momentum ($q^2 \leq 0$ for $q^2$ space-like) or large distance, in Eq. [3] the interpolating current $J$ and its conjugate $J^\dagger$ are interpreted as annihilation and creation operators of the hadron. The correlation function is saturated with a complete set of hadrons having the same quark content and quantum numbers. This interpretation of the correlator is called “Physical side”.

To extract the QCDSR expressions we initially compute the correlation function in connection with the physical degrees of freedom. By integrating Eq. [3] with respect to $x$, the following equality is obtained (for brevity, we will use $D$ to represent $D_{s0}^*(2317)$ state in formulas):

$$\Pi^{\text{Phys}}(q,T) = \frac{\langle \omega | J | D(q) \rangle \langle D(q) | J^\dagger | \omega \rangle}{m_D^2(T) - q^2} + \ldots, \quad (5)$$

where $m_D(T)$ is the temperature-dependent ground state mass of scalar $D_{s0}^*(2317)$ particle and dots symbolize contributions of the higher states and continuum which are parametrized by means of the continuum threshold parameter $s_0$. The definition of the temperature-dependent pole residue with the matrix element is:

$$\langle \omega | J | D(q) \rangle = \sqrt{2} f_D^8(T) m_D^4(T). \quad (6)$$

So the correlation function for the physical side can be written with respect to the ground state mass and pole residue in the form below:

$$\Pi^{\text{Phys}}(q^2, T) = \frac{2m_D^8(T) f_D^8(T)}{m_D^2(T) - q^2}. \quad (7)$$

After isolating the ground state contributions from the pole terms by taking derivative, i.e applying Borel transformation, the physical side is found as:

$$\tilde{B}(q^2) \Pi^{\text{Phys}}(q^2, T) = 2m_D^8(T) f_D^8(T) e^{-m_D^2(T)/M^2}, \quad (8)$$

here $M$ is a sum rule parameter called as Borel parameter providing elimination of the contributions from excited resonances and continuum states.

B. QCD Side

For high momentum ($q^2 \geq 0$), or short distance, the correlation function is evaluated with the help of Wilson’s Operator Product Expansion (OPE) due to the complex structure of QCD vacuum. Employing OPE, contributions coming from quark, gluon and mixed condensates can be included in the calculation of the correlator and this second way of getting the correlation function is called “QCD side”.

Now, our propose is to define the QCD side in which the correlation function is determined in terms of quark and gluon degrees of freedom. In the first step of the calculation, selecting the appropriate currents heavy and light quark fields are contracted and after some lengthy calculation, selecting the appropriate currents heavy and antidiquark and the current $J(x)$ can be used to obtain the mass and pole residue of the $D_{s0}^*(2317)$ state can be regarded as diquark-antidiquark and the current $J(x)$ can be defined by the following expression [28]:

$$J(x) = \left( i\partial^\gamma \gamma_5 e^\nu \right) \left( i\partial^\gamma \gamma_5 s \right) \quad (9)$$

where $i$ and $j$ are color indices.

Also $D_{s0}^*(2317)$ state can be regarded as diquark-antidiquark and the current $J(x)$ can be defined by the following expression [28]:

$$J(x) = \frac{\varepsilon_{ijk\varepsilon m nk}}{\sqrt{2}} \left[ (u_1^\gamma C_\gamma C_{\gamma 5} \bar{\pi}_m \gamma_5 C_{\gamma 5} T ) + u \rightarrow d \right], \quad (10)$$

where $i, j, k, m, n$ are color indexes, $\varepsilon$ is the polarization tensor, $C$ is the charge conjugation matrix.

We can formulate the correlation function $\Pi^{\text{QCD}}(q^2, T)$ as dispersion integral:

$$\Pi^{\text{QCD}}(q^2, T) = \int_{M^2}^{\infty} \rho(s,T) \frac{ds}{s - q^2} + \bar{\Gamma}(q^2, T), \quad (11)$$

where $M^2 = (m_c + m_s + 2m_q^2)$ with $q = u$ or $d$ quark, $\rho^{\text{QCD}}(s,T)$ is the spectral density and $\bar{\Gamma}(q^2, T)$ symbolize the contributions coming from the nonperturbative part. The spectral densities are calculated as an imaginary part of the correlation function with the relation $\rho^{\text{QCD}}(s, T) = \frac{1}{2} Im[\Pi^{\text{QCD}}(s, T)]$.

After some manipulations including the contraction of the quark fields, we obtain the QCD side of the correlation function in terms of the heavy and light quark propagators in the molecular and diquark-antidiquark scenarios, respectively:

$$\Pi^{\text{QCD}}(q^2, T) = \int d^4x e^{iq\cdot x} \left[ - Tr \left( S_{d,a}^b(x) \gamma_5 S_{c,a'}^a(x) \right) \times \gamma_5 S_{d,a}^b(x) \gamma_5 S_{b}^{a'}(x) \gamma_5 \right] + Tr \left( S_{d,a}^b(x) \gamma_5 S_{c,a'}^a(x) \gamma_5 \right) \times Tr \left( S_{d,b}^b(x) \gamma_5 S_{c}^{b'}(x) \gamma_5 \right), \quad (12)$$

$$\Pi^{\text{QCD}}(q^2, T) = iAA' \int d^4x e^{iq\cdot x} \left[ Tr \left( \bar{\gamma}_5 S_{d,a}^{i} (x) \gamma_5 \right) \times S_{c}^{k b'}(x) \right] + Tr \left( \bar{\gamma}_5 S_{d,a}^{i} (x) \gamma_5 \right) \times S_{c}^{k b'}(x) \right] + Tr \left( \bar{\gamma}_5 S_{u}^{n m} (x) \gamma_5 \right) S_{u}^{n m} (x) \right] + u \rightarrow d \right]. \quad (13)$$
and we employed short-hand notation \( \mathcal{S}_{ij}(x) = C \mathcal{S}^{ijT}(x)C \) in Eq. (13). Meanwhile, at finite temperature, due to failure of the Lorentz invariance with the preferred reference frame and unveiling of the residual \( O(3) \) symmetry, the additional operators emerge in the short distance expansion of the product of two quark bilinear operators and consequently, the thermal heavy and light quark propagators contain new terms compared with the vacuum quark propagators [47]. Also we replaced the vacuum condensates by their thermal averages. The general definition of the thermal heavy (charm) quark propagator \( \mathcal{S}^c_{ij}(x) \) can be written as [33]:

\[
\mathcal{S}^c_{ij}(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left[ \frac{\Lambda^2}{2} \delta_{ij} - \frac{g^{\alpha\beta}}{4} \left( \vec{k} + m_c \right) \sigma_{\alpha\beta} \left( \vec{k} + m_c \right) + \frac{g^2}{12} G_{ij}^{A} \delta_{ij} m_c^2 \right] + \ldots ,
\]

(15)

here for the external gluon field \( G_{ij}^{A} \) the below short-hand notation is used:

\[
G_{ij}^{A} \equiv G^{\alpha\beta} \lambda_{ij}^{A}/2 ,
\]

where \( \lambda_{ij}^{A} \) are the standard Gell-Mann matrices with the number of gluon flavours \( A = 1, 2 \ldots 8 \) being \( i, j \) are color indices. In Eq. (15) the first term gives the perturbative contribution to the considered parameter while the others, i.e nonperturbative terms contain gluonic additives. In the nonperturbative terms the gluon field strength tensor \( G_{ij}^{A} G_{A}^{\alpha\beta}(0) \) is fixed at \( x = 0 \) and the thermal light quark propagator \( \mathcal{S}_{q}^{ij}(x) \) is expressed as [38-49]:

\[
\mathcal{S}_{q}^{ij}(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left[ \frac{\Lambda^2}{2} \delta_{ij} - \frac{m_q}{4\pi^2 x^2} \delta_{ij} - \frac{\langle \bar{q}q \rangle_T}{12} \delta_{ij} \right. \\
- \frac{x^2}{192} m_q \langle \bar{q}q \rangle_T \left[ 1 - i \frac{m_q}{6} \right] \delta_{ij} \\
+ \frac{i}{3} \left( \frac{m_q}{16} \langle \bar{q}q \rangle_T - \frac{1}{12} \langle u^\alpha \Theta_{\mu\nu} u^\nu \rangle \right) \\
+ \frac{1}{3} \langle u \cdot x \rangle \langle u^\alpha \Theta_{\mu\nu} u^\nu \rangle \delta_{ij} \\
- \frac{ig q^{\alpha\beta}}{32\pi^2 x^2} \left( \frac{x^2}{1176} \langle \bar{q}q \rangle_T^2 \right) \\
- \frac{ig_{ij}}{4} \left( \frac{x^2}{1176} \langle \bar{q}q \rangle_T^2 \right) \\
\left. + \ldots \right] ,
\]

(16)

where \( m_q \) indicates the light quark mass, \( \langle \bar{q}q \rangle_T \) is the light quark condensate as a function of temperature, \( u_\mu \) is the four-velocity of matter in hot medium which is \( u_\mu = (1, 0, 0, 0) \) in the matter rest frame and \( \Theta_{\mu\nu}^\prime \) is the fermionic part of the energy momentum tensor. Additionally the following gluon condensate expression depending on the gluonic part of the energy-momentum tensor \( \Theta_{\mu\nu}^g \) is used [47]:

\[
\langle \bar{q}q \rangle_T^g = \frac{1}{4} \left( g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} \right) \langle G_{\alpha\beta} G^{\mu\nu} \rangle \\
+ \frac{1}{6} \left[ g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} - 2 \langle u^\alpha u^\mu \rangle g_{\beta\nu} - \langle u^\alpha \Theta_{\mu\nu} u^\nu \rangle \right] \\
- u_\beta u_\mu g_{\alpha\nu} + u_\beta u_\nu g_{\alpha\mu} \right) \langle u^\alpha \Theta_{\lambda\nu}^g u^\nu \rangle.
\]

(17)

The next step to extract the thermal mass and pole residue sum rules of the \( D_s^*(2317) \) state is to impose the Borel transformation (i.e. omitting the continuum contribution taking derivative) to the invariant amplitude \( \Pi_{QCD}(q^2, T) \) and selecting the same structures in both physical and QCD sides, then equalizing the obtained expressions with the related part of \( B(q^2) \) \( \Pi_{Phys}(q, T) \), lastly thermal pole residue sum rule is found as:

\[
f_D(T) = \sqrt{n_1/d_1}
\]

(18)

here,

\[
n_1 = \int_{M^2}^{s_0(T)} \rho_{QCD}(s, T) e^{-s/M^2} + \hat{B}_T(q^2, T),
\]

(19)

\[
d_1 = 2M^s_{D}(T)e^{-m_{D}^2(T)/M^2}.
\]

(20)

To determine the thermal mass sum rule of \( D_s^*(2317) \) state, one must take the derivative of Eq. (18) according to \( 1/M^2 \) and so we get the following analytic expression for the mass sum rule depending on temperature:

\[
m_D(T) = \sqrt{n_2/d_2}
\]

(21)

where,

\[
d_2 = \int_{M^2}^{s_0(T)} \rho_{QCD}(s, T) e^{-s/M^2} + \hat{B}_T(q^2, T),
\]

(22)

\[
n_2 = \frac{d}{d(-1/M^2)} d_2.
\]

(23)

In Eqs. (19) and (22) \( s_0(T) \) represent the thermal continuum threshold parameter which is related with \( s_0(0) \) as introduced in Eq. (31) whose task is to separate the contributions coming from the ground state and higher states. The two auxiliary parameters \( s_0 \) and Borel mass of the sum rule method will be explained later in detail in Section [111].

Now we will perform numerical analysis to achieve to conclusion considering \( D_s^*(2317) \) state according to both molecular and diquark-antidiquark structure scenarios.
### III. NUMERICAL CALCULATION AND ANALYSIS

The TQCDSR calculations for the mass and pole residue of open-charm system $D_s^*(2317)$ involve some parameters e.g. quark, gluon and mixed vacuum condensates and quark masses as well. Values of these input parameters are compiled in the Table I.

| Parameters | Values          |
|------------|----------------|
| $m_u$      | 2.16 ± 0.25 MeV [8] |
| $m_d$      | 4.67 ± 0.45 MeV [8] |
| $m_s$      | 93.6 MeV [8] |
| $m_c$      | 1.27 ± 0.02 GeV [8] |
| $\langle 0|\bar{q}q|00\rangle$ | $\langle 0\rangle$ [9] |
| $\langle 0|\bar{s}s|00\rangle$ | $0.8 \times \langle 0|\bar{q}q|00\rangle$ GeV [42, 43] |
| $\langle \pi\bar{G}^2 \rangle$ | $0.028(3)$ GeV [42, 43] |
| $m_0^2$    | $0.8 \pm 0.1$ GeV [22, 43] |

In addition to these parameters, to continue the calculations the temperature-dependent quark, gluon condensates and also energy density must be determined. As for the quark condensate, the fit function provided from Ref. [51], which intersects with the Lattice QCD data is utilized representing the $u$ and $d$ quarks with $q$ :

$$\langle \bar{q}q \rangle_T = C_1 e^{aT} + C_2$$

and for the $s$ quark

$$\langle \bar{s}s \rangle_T = C_3 e^{bT} + C_4,$$

where $a = 0.040$ MeV$^{-1}$, $b = 0.516$ MeV$^{-1}$, $C_1 = -6.534 \times 10^{-4}$, $C_2 = 1.015$, $C_3 = -2.169 \times 10^{-5}$, $C_4 = 1.002$ are coefficients of the fit function [52] and is valid up to a temperature $T = 180$ MeV and $\langle 0|\bar{q}q|00\rangle$ represents the condensate of the light quarks at vacuum.

The gluonic and fermionic pieces of the energy density can be parametrized as in Ref. [49] using the Lattice QCD data presented in Ref. [58]:

$$\langle u^\mu g_{\mu\nu} u^\nu \rangle_T = (\xi_1 e^{aT} + C_5) T^4,$$

$$\langle u^\mu g_{\mu\nu} u^\nu \rangle_T = (\xi_2 e^{bT} - C_6) T^4,$$

where the vacuum subtracted values of the considered quantities are used as $\delta f(T) = f(T) - f(0)$ and $\delta T_\mu(T) = \varepsilon(T) - 3p(T)$: $\varepsilon(T)$ is the energy density and $p(T)$ is the pressure. Taking into account the recent Lattice calculations [50] we get the fit function of $\delta T_\mu(T)$ as [52]

$$\frac{\delta T_\mu(T)}{T^4} = (C_7 e^{gT} + C_8)$$

with $C_7 = 0.020$, $h = 29.412$ GeV$^{-1}$, $C_8 = 0.115$. Moreover we use the following expression for the temperature-dependent strong coupling [53, 54]

$$g_s^{-2}(T) = \frac{11}{8\pi^2} \ln \left( \frac{2\pi T}{\Lambda_{MS}} \right) + \frac{51}{8\pi^2} \ln \left[ 2 \ln \left( \frac{2\pi T}{\Lambda_{MS}} \right) \right]$$

where $\Lambda_{MS} \simeq T_c/1.14$.

Continuum threshold as a function of temperature belonging to $D_s^*(2317)$ state is another auxiliary parameter that needs to be determined. The expression of the continuum threshold in terms of temperature is used as follows (for details see [55–57]) :

$$s_0(T) = \left[ \frac{\langle \bar{q}q \rangle_T}{\langle 0|\bar{q}q|00\rangle} \right]^{2/3}$$

where $s_0(T)$ is defined with $s_0(0)$ in vacuum threshold. It is not random and relies on the mass of the first excited state of the $D_s^*(2317)$. That’s why the chosen interval for the $s_0$ is to be relatively weak dependent from physical quantities for the $D_s^*(2317)$ state. As stated in the philosophy of the QCDSSR technique, the physical quantities shouldn’t be connected with the auxiliary parameters $M^2$ and $s_0$. But in real calculations these quantities nevertheless are sensitive to the choice both of $M^2$ and $s_0$. Therefore, the parameters $M^2$ and $s_0$ should be settled to minimize the dependence of $m_D$ and $f_D$ on them. Convergence of the OPE plus suppression of the contributions coming from the higher states and continuum are a must for the sake of fixing the working region of the Borel parameter $M^2$. As shown in Figure 2 $M^2_{max}$ should be 1.6 GeV$^2$. To determine the working region of continuum threshold we use $s_0 = m_D + (0.3; 0.5)$ in the classical conjecture.

It is clear that $m_D$ and $f_D$ should not depend on the auxiliary parameters $M^2$ and $s_0$. The analysis carried out by taking into account all of aforementioned constraints allow us to fix the continuum threshold and Borel parameter as $1.2$ GeV$^2 \leq M^2 \leq 1.6$ GeV$^2$ and $6.8$ GeV$^2 \leq s_0 \leq 7.9$ GeV$^2$

Note that the dependence of the mass and pole residue on $M^2$ is stable in this interval, so we can trust that the obtained sum rules will give accurate results. To show the independence of physical quantities from $M^2$ and $s_0$, we present the 3D plot of mass versus continuum threshold and Borel parameter $M^2$ in the diquark-antidiquark picture in Figure 3 and we see the stability of mass sum rule in terms of these parameters which is the main criterion of QCDSSR.
At $T = 0$ open-charm system $D_{s0}^{*}(2317)$ is investigated with many theoretical model both in the molecular and diquark-antidiquark pictures [20–35]. Our results in $T = 0$ limit TQCDSR model are presented below in both molecular and diquark-antidiquark scenarios:

$$m_{D}^{mol.} = 2306^{+33}_{-34} \text{ MeV}, \quad f_{D}^{mol.} = 73.3^{+1.8}_{-1.7} \text{ keV},$$

$$m_{B}^{d}_{s} = 2317^{+33}_{-34} \text{ MeV}, \quad f_{B}^{d}_{s} = 93.1^{+2.3}_{-2.4} \text{ keV}.$$  

The last step is to look for changing of the mass and pole residue of the $D_{s0}^{*}(2317)$ state in terms of temperature. In this context the ratio of changing the mass and pole residue graphs are drawn as a function of the temperature for the molecular and diquark-antidiquark assumptions in Figures 4 and 5 respectively. As you see from Figures 4 and 5, three different plots and their labels are overlapped.

As a by product we interchange the $c$ and $b$ quarks in the interpolating current and deduce from SU(2) symmetry that the bottom partner of $D_{s0}^{*}(2317)$ which we name this resonance as $B_{sJ}$ can possibly between the following mass and pole residue values in both molecular and diquark-antidiquark pictures, respectively:

$$m_{B_{sJ}}^{mol.} = (5062 - 5446) \text{ MeV}, \quad f_{B_{sJ}}^{mol.} = (3.2 - 5.9) \text{ keV},$$

$$m_{B_{sJ}}^{d}_{s} = (5052 - 5460) \text{ MeV}, \quad f_{B_{sJ}}^{d}_{s} = (33.3 - 61.5) \text{ keV}$$

which are consistent with the estimations in Ref. [14] within the limits of uncertainties.

## IV. Conclusion

Theoretical and experimental disagreement on mass and decay width of open-charm meson $D_{s0}^{*}(2317)$ stimulates several non-conventional (exotic) interpretation
of this state, such as in the molecular and diquark-antidiquark structures predominantly in the literature. Its measured mass and width does not match the predictions from potential-based quark models. Therefore it is so important to make further studies to finalize this uncertainties.

In this sense we examined the \( D_{s0}^*(2317) \) state in the molecular and diquark-antidiquark pictures by calculating its spectroscopic parameters using TQCDSR method. We can summarize our results as follows:

- Our numerical calculations show that mass and pole residue parameters are practically independent of temperature at least up to a temperature of 100 MeV, but right after that point they begin to drop by growing temperature.
- Near the critical temperature, the pole residue arises at 29% and 32% of its value in vacuum in molecular and diquark-antidiquark pictures, respectively while the masses decrease by 7% for both pictures. As for the mass, our result is in good agreement with Ref. 3.
- Our results do not give any definite information as to whether \( D_{s0}^*(2317) \) resonance is molecular or diquark-antidiquark structure since they are within the uncertainties of the TQCDSR theory.
- Although there is no scalar particle yet detected in the bottom-strange meson list of the PDG, we predict that it could be found in experiments in the near future.

As a result, the noticeable decline in the values of mass and pole residue can be conceivable as a manifestation of the QGP phase transition. The QGP provides a very good ground to understand the nonperturbative aspects of strong interactions, early universe, and astrophysical processes such as neutron stars. In short, the investigation of QGP is quite important to reveal the nature of both micro and macro scale events in our universe. For this reason searches related with thermal properties of hadrons and also exotic candidates can provide valuable hints and information for the future experiments such as CMS, LHCb and PANDA.

**Appendix A: The two-point thermal spectral densities**

Here, we present the results of our evaluations for the spectral density

\[
\rho^{QCD}(s, T) = \rho^{pert.}(s) + \sum_{k=3}^{6} \rho_{k}(s, T),
\]

essential for our calculations of the mass and pole residue as a function of the temperature belonging to the \( D_{s0}^*(2317) \) resonance via TQCDSR in the diquark-antidiquark picture as an example. \( \rho_{k}(s, T) \) denote the nonperturbative contributions to \( \rho^{QCD}(s, T) \) and \( g^2 = 4\pi\alpha_s \). The explicit form for \( \rho^{pert.}(s) \) and \( \rho_{k}(s, T) \) are expressed with the integrals over the Feynman parameter \( z \) as follows:

\[
\rho^{pert.}(s) = \int_{0}^{1} dz \frac{z^{2} (m_{c}^{2} + s\beta)^{2}}{3 \times 2! \pi^{6} \beta^{3}} \left( m_{c}^{4}z^{2} - 4m_{c}^{3}z(m_{d} + m_{u}) + 4m_{c}^{2}z^{2}[2m_{s}(m_{d} + m_{u}) + sz] - 4m_{c}^{2}z^{2} \beta \right) \times \Theta[L(s, z)] \quad (A2)
\]
\[ \rho^{(q)}(s, T) = \int_0^1 \frac{dz}{2\pi \frac{m_c^2}{2}} \left[ \langle \bar{d}d \rangle \left( m_c^2 z + 2m_b^2 \beta z(m_d - m_s) + 2m_s^2 \beta(-m_c^2 + 4m_d m_s + sz) + 2m_b^2 \beta^2 \left( 2m_c^2 m_s + 3sz(m_d - m_s) \right) \right) + m_c^5 \langle \bar{u}u \rangle z \right. \\
\left. + 3sz(m_d - m_s) \right] + m_c^s \beta^2 \left( -2m_c^2 + 8m_d m_s + sz \right) + 2m_s^2 \beta \left( 3m_c^2 m_s + 2sz(m_d - m_s) \right) + m_c^5 \langle \bar{u}u \rangle z \\
- 2m_b^4 \beta \left( \langle \bar{s}s \rangle(m_d - m_s + m_u) + \langle \bar{u}u \rangle(m_s - m_u) \right) + 2m_s^3 \beta \left( 2m_c^2 \langle \bar{s}s \rangle - 2m_d m_s \langle \bar{s}s \rangle + m_u \left[ -m_s \langle \bar{s}s \rangle \right. \\
\left. + 4m_s \langle \bar{u}u \rangle + 2m_u \langle \bar{s}s \rangle - m_u \langle \bar{u}u \rangle + s \langle \bar{u}u \rangle z \right) \right) + 2m^2 \beta^2 \left( 2m_c m_u \langle \bar{u}u \rangle - 3sz \langle \bar{s}s \rangle(m_d - m_s + m_u) \right) \\
+ \langle \bar{u}u \rangle(m_s - m_u)) \right] + m_c s \beta^2 \left( 4m_2 \langle \bar{s}s \rangle - 2m_d m_s \langle \bar{s}s \rangle - 2m_u \left[ m_s \langle \bar{s}s \rangle - 4\langle \bar{u}u \rangle + m_u \langle \bar{u}u \rangle - 2\langle \bar{s}s \rangle \right] \right) \\
+ s \langle \bar{u}u \rangle z \right) - 2s \beta^3 \left( 2sz \langle \bar{s}s \rangle(m_d - m_s + m_u) + \langle \bar{u}u \rangle(m_s - m_u) \right) - 3m_s^2 \langle \bar{u}u \rangle \right] \right] \Theta[L(s, z)] \tag{A3} \]

\[ \rho^{G^{2+}(q)}(s, T) = \int_0^1 \frac{dz}{3 \times 21 \beta^2} \left[ 6szm^2 \left( 3\langle \bar{u}u \rangle z \right) + (\bar{d}d) \left( 3z m^2 + 2 \beta z m^2 \left( 2m_d - 3m_s \right) + \beta m_c \left( 3s - m^2 + 6m_d m_s \right) + \beta^2 \right. \\
\left. \times (6szm_d - 9szm_u + 2m_d^2 m_s) \right) - 2z \beta m^2 \left( 3\langle \bar{s}s \rangle m_d - 2\langle \bar{s}s \rangle m_s + 3\langle \bar{u}u \rangle m_s + 3\langle \bar{s}s \rangle m_u - 2\langle \bar{u}u \rangle m_u \right) \right] \\
+ \left[ \frac{1}{g^2} \left( \frac{1}{2} \right) \left[ -4 \langle \bar{s}s \rangle z \right. \left. + 6szm_d - 9szm_u + 2m_d^2 m_s \right] \right] \right] \right] \Theta[L(s, z)] \tag{A4} \]

where \( \Theta \) denotes the unit step function, \( L(s, z) = sz(1 - z) - zm^2 \) and \( \beta = z - 1 \).
