Numerical modeling the dynamics of plasma flows interaction in magnetic field

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Abstract. On the basis of numerical simulation, the processes of whistler, Alfven and collisionless shock waves generation are investigated in the interaction of high-speed plasma flows in a magnetic field. The statement of the problem corresponds to the conditions of laboratory experiments at the K1-1 facility (ILP SB RAS) [1]. The numerical model is based on the kinetic description of the ion plasma component and the gas-dynamic approximation for magnetized electrons (hybrid model) [2]. To solve the Vlasov kinetic equation, the author's modification of the particle method in the cell (PIC) is used. The structure of the generated perturbations is analyzed as a function of the plasma parameters and a magnetic field. The characteristics of shock waves for the regimes of a multistream flow formation and the mechanisms of ion acceleration in the front of a shock wave are studied.

1. Introduction

The study of plasma flow collisionless interaction mechanisms is one of the fundamental problems of plasma physics and arises in the study of a wide range of phenomena in the laboratory and in space plasmas, where high-speed plasma flows play a decisive role. These include solar flares, solar wind flow around the magnetosphere of the planets, and the grandiose cosmic catastrophes such as supernova stars explosions and giant jet ejection from the nuclei of some galaxies. Similar phenomena that have significantly smaller energy and space-time scales occur during active experiments in the tail of the Earth's magnetosphere, in the laboratory simulation of astrophysical processes, in a magnetic plasma traps. All these phenomena are accompanied by the release of energy and the formation of high-speed plasma structures, the generation of shock waves and the particles acceleration in the wave front.

A common property of the processes under consideration, both in the laboratory and in the space plasma, is their collisionless character with respect to Coulomb collisions, i.e. the interaction of the flows occurs at distances considerably less than the classical mean free paths. In contrast to ordinary gas, where the shock wave profile is formed under the influence of nonlinear and dissipative processes, in collisionless plasma the structure of shock waves is determined by the collective interaction of the electromagnetic field and plasma particles. Quasi-stationary shock waves with a front width much shorter than the mean free path can be formed in a plasma due to the competition of nonlinearity, the dispersion of the phase velocity of waves, and the dissipation of energy in the front due to the development of small-scale turbulence.
Despite numerous laboratory studies [1,3], and a huge amount of observational data obtained over almost half a century of satellite observations [4], many fundamental questions of the theory of collisionless shock waves have no answer. First of all, this refers to the mechanism of acceleration of charged particles on the front of a shock wave, one of the possible manifestations of which are galactic cosmic rays. Laboratory experiment on modeling of non-stationary space processes is carried out at the KI-1 facility, where the processes of interaction of a dense plasma cloud with a magnetized plasma background are considered. Experimental studies have shown that the collisionless energy transfer from the cloud to the background plasma occurs on the scale of the gas dynamic braking radius, even for the super-Alfven velocities of the cloud due to the action of the vortex electric fields, which arise when the magnetic field is displaced by the plasma piston. These processes are accompanied by the generation of collisionless shock waves by a plasma piston. As a result of the experiments carried out, the conditions of the cloud-background independent of the energy exchange mechanism due to the vortex electric fields were found. The physical model of such an interaction is based on the displacement of the magnetic field by an expanding cloud and the formation of a compressed magnetic field generating on the cloud boundary. A more detailed study of the generation of a shock wave due to the expansion of a plasma cloud in a background plasma is possible on the basis of numerical simulation, which currently plays an important role in the study of various phenomena in high-temperature plasmas, supplementing the results of the linear theory and laboratory experiments. Collisionless shock waves at amplitudes below critical can be described on the base of the gas-dynamic type equations. The dissipation necessary for the formation of such waves is due to the interaction of the particles with the fluctuation fields of various plasma instabilities. However, when the amplitude of the waves reaches a critical value, a complete reflection of the incident flow occurs (hydrodynamic overturning of the wave profile).

The most general approach for solving the problem of generation and structure of shock waves in this case is based on the use of a nonlinear system of kinetic equations for ions and electrons (Vlasov’s equations) and Maxwell’s equations with a self-consistent electromagnetic field. However, the large difference in spatial and temporal scales, caused by the difference in the masses of electrons and ions, makes it difficult to use the fully kinetic approximation. The most promising are the combined models, where the magnetohydrodynamics approximation is used for magnetized electrons, the kinetic approximation for the ions, which takes into account the Larmor rotation of the reflected ions at the foot of the shock wave. In this paper, a numerical simulation of shock waves in a plasma was carried out on the basis of a two-dimensional hybrid model using PIC method to solve the Vlasov kinetic equation. The statement of the problem corresponds to the conditions of laboratory experiments at the KI-1 facility is analyzed. The wave characteristics for the regimes of formation of a multistream flow associated with the overturning of its front as well as the dependence of the amplitude, velocity, and structure of the shock waves on the parameters of the plasma and the magnetic field are investigated. We note that the solution of the problem of expanding the plasma cloud in the background medium, taking into account all angles of propagation of the generated shock wave with respect to the external magnetic field, has an advantage in comparison with the traditional formulation of the problem of the formation of a shock wave in the reflection of a high-speed plasma flow from a conducting wall [5].

2. The statement of the problem

Let us consider the problem of expanding a dense plasma bunch in a magnetized background in the following formulation. At the initial time \( t = 0 \) in the center \( r = 0, z = 0 \) of a cylindrical domain \( 0 < r < r_{max}, z_{min} < z < z_{max} \) with the uniform magnetic field \( \vec{B} = (0, 0, B_0) \) and a background plasma with density \( n_0 \), there is a spherical plasma cluster (“cloud”) of radius \( R_0 = (r_0^2 + z_0^2)^{1/2} \) with the number of particles \( N \) and kinetic energy \( W = m_i N V_0^2 \). The density of the background plasma \( n(r, z) = n_0 = \text{const} \), or the initial distribution of the background density is nonuniform with gradient directed along the magnetic field:
\[ n(r, z) = \begin{cases} 
\frac{n_2 - n_1}{\Delta} z + n_0, & z \leq -\Delta/2, \\
\frac{\Delta}{2} < z < \Delta, \\
\frac{n_1}{\Delta}, & z > \Delta/2, 
\end{cases} \]

where \( \Delta = 0.1R_1 \), \( R_1 = (3N/4\pi n_0)^{1/3} \) is the gasdynamical radius of deceleration according to the "snow plow" model [6], \( (n_1 + n_2)/2 = n_0 \).

This statement of the problem is most approximate to the conditions of laboratory experiments [1], where the processes of collisionless cloud-background energy exchange are considered in the absence of dissipation mechanisms associated with the electrical conductivity and thermal conductivity of the plasma. The experimental data were used to test the models created, and the results of the calculations were used to interpret the results of laboratory experiments.

The numerical model of the collisionless expansion of a dense plasma cloud into a plasma background in the presence of a magnetic fields based on a hybrid model, which consists of the Vlasov kinetic equation for plasma ions, hydrodynamic equations for electrons:

\[
\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{r}} + \frac{e}{m_i} \left( \vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) \frac{\partial f}{\partial \vec{v}} = 0,
\]

\[
m_e \left( \frac{\partial \vec{V}_e}{\partial t} + (\vec{V}_i \cdot \vec{v}) \vec{V}_e \right) = -e \left( \vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right).
\]

Self-consistent electromagnetic fields are governed by the Maxwell equations:

\[
\text{rot}\vec{B} = \frac{4\pi e}{c} \left( \int \vec{v} f(\vec{r}, \vec{v}, t) d\vec{v} - n\vec{V}_e \right),
\]

\[
\text{rot}\vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}.
\]

Here \( f(\vec{r}, \vec{v}, t) \) is the ion distribution function, \( \vec{v} \) and \( \vec{V}_e \) are the velocities of ion and electron plasma components, \( \vec{B} \) is the magnetic field, \( \vec{E} \) is the electric field, \( e \) is the electrical charge, \( n \) is the plasma density, \( m_e \) and \( m_i \) are the masses of electron and ion, \( c \) is the speed of light.

We assume charge neutrality of plasma, which is valid, if the typical spatial scales are much greater than the Debye length. Considering low-frequency plasma flows we neglect the displacement currents in the Maxwell equations. On the symmetry axis \( \vec{r} = 0 \), the natural boundary conditions for the symmetry of the solution are set; on the remaining boundaries, the boundary conditions coincide with the initial condition. The algorithm for solving the problem is based on a combination of the particle method with splitting schemes for the magnetic field [7]. We note that in the calculated region the motion of \( 10^7 \) particles on the grid [120x240] was considered; the spatial steps \( h_r = h_z = 0.02 R_1 \).

### 3. Results of computer simulation

During expansion a dense plasma cloud can slow down because it pushes out the magnetic field from the cloud volume due to diamagnetic currents and creates a magnetic cavity with the size \( R_2 = (3W/B_0^2)^{1/3} \), where the average magnetic field magnitude is lower than in the ambient background [6]. Another possible factor determining cloud deceleration is an interaction between the plasma cloud and an ambient plasma. One of the criteria for the similarity of the processes under consideration is the Alfvén-Mach number \( M_A = V_0/V_A \), which characterizes the rate of expansion of the plasma cloud with velocity \( V_0 \) respect to the Alfvén velocity \( V_A = B_0/(4\pi n_0) \)^{1/2}. It is easy to obtain that \( R_2/R_1 = M_A^{2/3} \). Hence at large Alfvén-Mach numbers \( (M_A > 1) \) a slowing down of the cloud is caused by cloud-background interactions, while for sub-Alfvénic flows \( (M_A < 1) \) a cloud expands energy.
on ousting and deforming the magnetic field. The results of numerical simulation made it possible to distinguish three regimes that differ in the type and structure of the generated in background plasma disturbances: $M_A<1$, $1<M_A<2.5$ and $M_A>2.5$. The main channel for transferring the kinetic energy of the cloud also depends on $M_A$: for $M_A>1$ the kinetic energy of the cloud is transferred mainly to the background plasma, and for $M_A<1$ to the electromagnetic field.

When the cloud expands at distances $R<R_1$ and times shorter than the deceleration time $t^* = R_1/V_0$ for a wide range of angles relative to a magnetic field $\theta = 0 \div 180^\circ$, the energy is exchanged between the streams and the perturbations of density and magnetic field are generated. In this case, a shell of increased concentration is formed in the form of an axially symmetric plasma layer propagating together with the shock of the compressed magnetic field. This process leads to the formation of a plasma cavity with a radius $R = R_1$. Plasma density inside cavity much smaller than the initial background density. As a result of the displacement of the magnetic field by the expanding plasma, a magnetic cavity of a similar size is formed. At the distances $R > R_1$, propagation of waves generated as a result of the cloud-phonon interaction is observed.

Let us consider the expansion of a cloud in a homogeneous background for the Alfven-Mach numbers $1<M_A<2.5$ for the initial size of the cloud $R_0 = 0.1 R_1$. Figures 1, 2 show the magnetic field lines and the contour plots of background plasma density at time $t = 2t^*$, $t = 3t^*$ for $M_A = 2$ illustrating the phenomena described above. The magnetic cavity, which is practically symmetrical with respect to the plane $z = 0$ at time $t = 0.5t^*$, extends with time along the magnetic field and at the moment of deceleration of the cloud $t = 1t^*$, the ratio of its longitudinal and transverse dimensions is 1.2. During the time $t = 2t^*$ the cloud lost 80% of its initial energy, 35% of which was converted to the kinetic energy of the background plasma.

![Figure 1](image1.png)

**Figure 1.** Projections of magnetic field lines to the plane $(r, z)$ at time $t = 2t^*$ - (a) and $t = 3t^*$ - (b). Coordinates $(r, z)$ are expressed in units of the radius of deceleration of the cloud by the background plasma $R_1$.

![Figure 2](image2.png)

**Figure 2.** The isolines of the density of the background plasma at time $t = 2t^*$ - (a) and $t = 3t^*$ - (b) for $M_A = 2$. The coordinates $(r, z)$ are expressed in units of $R_1$, and density - in units of $n_0$. 


The structure of the perturbation propagating in the background plasma depends on the propagation angle with respect to the magnetic field $\vec{B}_0$. The curvature of the magnetic field lines is related to the oscillatory structure of the shock waves propagating in the background plasma in a quasi-longitudinal (relative to $\vec{B}_0$) direction. The oscillatory structure of the waves is seen on the isolines of the plasma density (Fig. 2) and the background ions phase space (Fig. 3). Figure 3 gives a visual representation of the shape and velocity of the shock wave front and the higher speed of the oscillator precursor. The size of the oscillations, corresponding to the wave lengths for which the dispersion of the phase velocity appears is $\delta = c/\omega_0i$. $\omega_0i = (4\pi n_0 e^2/m_i)^{1/2}$.

The characteristic features of the observed wave structures - right-handed polarization and dispersion properties - make it possible to identify the perturbation generated by the expansion of the plasma cloud as an electronic whistler, the oscillatory structure of the wave packet of which is due to the manifestation of a positive dispersion-the faster propagation of short-wave oscillations in accordance with the law $\omega/k = V_A + (1 + \theta^2k^2c^2/\omega_0i^2)^{1/2}$ [8].

![Figure 3](image)

**Figure 3.** The phase spaces of the ions of the background plasma ($R, u_R$) at time $t = 3t^*$ for (a) $\theta = 26^\circ$, (b) $\theta = 45^\circ$, (c) $\theta = 60^\circ$, (d) $\theta = 90^\circ$. The particle coordinate $R = (r^2 + z^2)^{1/2}$ is expressed in units of $R_1$, the speed is expressed in units of $V_A$. We note that the whistler perturbations, characteristic of oblique collisionless shock waves, were first observed in laboratory experiments [9], and later in the head shock wave of the Earth's magnetosphere [10]. With increasing speed, i.e. the Alfvén-Mach number of the shock wave $M_A$, the number and amplitude of the oscillations propagating ahead of the main jump of the magnetic field decrease. Such a rearrangement of the structure of the front of the shock wave is characteristic when it approaches the stage of breaking. Typical for large numbers of Alfvén-Mach $M_A > 2.5$ is the formation of a magnetic cavity, the size of which almost coincides with the size of the expanding cloud. The deceleration of the cloud is accompanied by an acceleration of the background plasma to velocities $V_0$. The perturbation propagating in the background plasma has the form of a thin shell with a thickness of the Larmor radius, recorded in a quasi-transverse direction at angles $\theta > 45^\circ$ at distances $R \gg R_1$. The wave generated in this case is a collisionless shock wave whose amplitude and velocity (Alfvén-Mach number) can exceed values critical for shock waves in a plasma when the incoming stream reflects from the wave front with the formation of a multistream flow. For example, for $\theta = 90^\circ$ and $\theta = 30^\circ$ the critical values of the Alfvén-Mach number are equal 2.76 and 1.91, respectively. Figure 4 shows the phase spaces ($R, u_R$) of the background plasma for different values of the angle $\theta$. The propagation of a wave whose Alfvén-Mach number is equal to $M_A \approx 4.96$ is accompanied by the reflection of ions and their rotation in a magnetic field. The most intense interaction of the cloud with the background, accompanied by the deceleration of the cloud, occurs at an angle of $\theta = 90^\circ$. Thus, at the time $t = t^*$, the reflected ions exist only for the angle $\theta = 90^\circ$, while for the angles $\theta = 60^\circ$ and $\theta = 45^\circ$ the formation of a multistream flow happens at $t = 2t^*$. 

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Figure 4. Phase spaces of the ions of the background plasma \((R, u_R)\) at times \(t = 1t^*\) (I), \(t = 2t^*\) (II) for (a) \(-\theta = 90^\circ\), (b) \(-\theta = 60^\circ\), (c) \(-\theta = 45^\circ\), (d) \(-\theta = 26^\circ\), (\(M_A = 5\)).

The particle coordinate \(R = (r^2 + z^2)^{1/2}\) is expressed in units of \(R_1\), the speed is expressed in units of \(V_A\).

The spatial size of the wave is equal to the Larmor radius of the cloud ions. The main energy flux of the cloud is directed across the magnetic field, 60% of the energy of the directed motion of the cloud is transferred to the kinetic energy of the background, 20% of the energy of the cloud transmitted to the magnetic field is carried by a transverse wave.

Figure 5. Phase spaces of the ions of the background plasma \((R, u_R)\) at \(t = 2t^*\) for (a) \(-\theta = 30^\circ\), (b) \(-\theta = 150^\circ\), and (c) the surface of the background plasma density at \(t = 2t^*\). The particle coordinate \(R = (r^2 + z^2)^{1/2}\) is expressed in units of \(R_1\), the speed is expressed in units of \(V_A\), a plasma density is expressed in units of \(n_0\).

Let us consider the features of the expansion of a cloud in an inhomogeneous plasma background with a density gradient directed along the initial magnetic field \((\nabla n \parallel \vec{B}_0)\). We note that practically in all astrophysical phenomena and laboratory experiments an inhomogeneous of the background plasma plays an important role, determining both the deceleration mechanisms and the structure of the generated perturbations. The structure of the perturbations and the type of wave generated in an inhomogeneous magnetized plasma under the action of a spherically-expanding plasma piston, similar to the case of a homogeneous background, depends on the viewing angle \(\theta\) and the magnitude of the initial Alfvén-Mach number \(M_A\), which in a non-homogeneous background depends on \(\theta\). The ion phase spaces and spatial distribution of the density of the background plasma for the ratio of the densities \(n_2/n_1 = 5\), \(\Delta = 0.1R_1\), at time \(t = 2t^*\) is shown in Fig. 5. In the region of a high dense background plasma a solitary compression wave is generated (Fig. 5, a), whereas in a less dense plasma (under the condition \(M_A < 2.5\)) the intense magnetic field and density oscillations are excited with a characteristic oscillation size, \(\delta = c/\omega_{qi}\), associated with the propagation of electronic whistlers (Fig. 5, b). A feature
of the considered mode of cloud expansion in an inhomogeneous background is the displacement of the center of its mass in the direction of a less dense background, i.e. the effect of the "floating" of the cloud is observed.

Conclusion
Based on the liquid-kinetic hybrid numerical model, the processes of collisionless interaction of magnetized plasma flows due to the expansion of a plasma cloud in a background plasma are investigated. It is shown that the structure of the generated perturbations depends on the Alfvén-Mach number $M_A$ of the expanding plasma bunch. For the regime with $1<M_A<2.5$, the shock wave has an oscillatory precursor, which is ahead of the wave front. For $M_A>2.5$, the breaking of shock wave and the generation of accelerated ions was observed. Computer simulation was performed for parameters of laboratory experiments at KI-1 facility: $n_0 = 10^{11} - 10^{14}$ cm$^{-3}$, $B_0 = 100$ G, $N = 10^{18}$, $V_0 = 1.4 \times 10^7$ cm/c.

Acknowledgements
The research is carried out using the equipment of the shared research facilities of HPC computing resources at Siberian Supercomputer Center of ICMMG SB RAS. This work has been carried out within the framework of the budget project 0315-2016-0009 for ICMMG SB RAS and has been supported by the RFBR under grant 16-01-00209.

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