Multi-Group Formation Tracking Control for Second-Order Nonlinear Multi-Agent Systems Using Adaptive Neural Networks

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ABSTRACT This paper investigates the multi-group formation tracking (MGFT) control problem for second-order nonlinear multi-agent systems (MASs) with unknown dynamics. The objective of the MGFT control is to divide all agents into several subgroups to form different desired sub-formations while following their respective leaders. Firstly, the neural network (NN) approximator is constructed to solve the problem of unknown dynamics. Then, the distributed adaptive control protocol is designed based on the NN approximator. According to the Lyapunov stability theory and algebraic graph theory, sufficient criteria are obtained to realize the MGFT control. The semi-globally uniformly ultimately boundedness of formation errors is proved in detail. Finally, a numerical simulation example is given to confirm the validity of our theoretical results.

INDEX TERMS Nonlinear multi-agent systems, formation control, unknown dynamics, adaptive neural networks.

I. INTRODUCTION

As one of the most active and attractive research topics in the coordinated control for MASs, formation control has received increasing attention in the last two decades because of its wide applications in practical systems [1–9]. MASs are composed of multiple interacting intelligent agents which generate complex cluster behaviors through local information interaction. For formation control, it is essential to design suitable control protocols that enable the agents to reach and maintain desired geometry to accomplish the task.

Several common formation control strategies of MASs are the virtual structure strategy [10], graph based approach [11,12], behavior based approach [13] and leader-follower strategy [14]. Among these methods, the leader-follower method has been widely used by many researchers due to its simplicity, reliability and scalability [15–17]. Han et al. [18] studied the formation tracking problem using a fast terminal sliding mode control approach under a linear MAS. Hashim et al. [19] proposed a distributed robust neuro-adaptive cooperative tracking controller for higher-order nonlinear MASs with prescribed performance, where the uncertain parameters and external disturbances were considered. Safaei [20] investigated consensus and formation-tracking problems of networked agents with completely unknown nonlinear dynamics, and proposed the distributed adaptive model-free control algorithm. Yang et al. [21] studied the formation control with collision, obstacle avoidance and connectivity maintenance problems for nonlinear MASs under external disturbances, and proposed a novel control protocol based on adaptive neural networks. Wen et al. [22] studied the leader-follower formation tracking problem for nonlinear MASs on the basis of neural network techniques. The above studies merely considered the normal formation tracking problem of single target, where all the followers form the same desired geometry and track the leader. However, multi-objective searching and cooperative enclosing for multiple targets ubiquitously exist in reality, such as the surveillance operation of satellites cluster for multi-target, predators’ formation in multi-prey hunting, and division of labor in society for different interests and conflicts, which implies that MASs can be decomposed into several subgroups, each subgroup can achieve different distributed tasks in each own desired sub-formation geometry. Therefore, it is vital to investigate the MGFT control problem.
Currently, owing to the aforementioned superiorities, a lot of effort has been made to investigate the formation problems for multiple groups, e.g., cluster formation control under communication delays and aperiodic sampling [23], time-varying multi-formation acquisition [24], multi-formation control for nonlinear MASs [25], multi-target tracking of heterogeneous collaborative-robots with external disturbances [26] and multi-formation tracking using impulsive control methods [27]. However, these works on the MGFT problem of MASs have not considered the impact of the unknown dynamics. In fact, on account of the influence coming from environmental disturbance and agent’s own intrinsic dynamics, there are unknown nonlinear dynamics in practical engineering systems. By applying the adaptive learning and nonlinear continuous function approximation ability of NNs, the control strategy combining NNs with adaptive control technology can effectively solve the problem of unknown dynamics. Inspired by the above analysis, this paper investigates the MGFT control problem for second-order nonlinear MASs with unknown dynamics using adaptive NN control method.

This paper is organized as follows. In Section 2, the necessary preliminaries, related lemmas and assumptions are given. Section 3 designs the MGFT control protocol and proves the system’s stability for second-order nonlinear MASs with unknown dynamics. In Section 4, a numerical simulation example is presented to illustrate the effectiveness of the proposed protocol. The conclusions are summarized in Section 5.

Notations: Let $\mathbb{R}^n$ be the $n$-dimensional Euclidean space and $\mathbb{R}^{m \times n}$ be the space of $m \times n$ real matrices. Give a matrix $A$, $A^T$ denotes its transpose. $A > 0 (A \geq 0)$ if matrix is positive definite (positive semi-definite). $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and the maximum eigenvalue of the matrix $A$, respectively. $\otimes$ is the Kronecker product operator. $\| \cdot \|$ denotes the Euclidean norm for vectors.

II. PRELIMINARIES

A. GRAPH THEORY

To describe the communications among the $N$ follower agents, we consider a weighted undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{L}\}$, where $\mathcal{V} = \{a_i\} \subseteq \mathbb{R}^{N \times N}$ represents a nonnegative weighted adjacency matrix, $\mathcal{V}(\mathcal{G}) = \{1, \ldots, N\}$ denotes the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. If an edge $(i, j) \in \mathcal{E}$, then it implies that the information can be exchanged between node $i$ and node $j$. The elements of the adjacency matrix $\mathcal{A}$ are positive if and only if $(i, j) \in \mathcal{E}$, i.e., $a_{ij} = a_{ji} > 0$ if $e_{ij} = (i, j) \in \mathcal{E}$. If $e_{ij} \notin \mathcal{E}$, then $a_{ij} = 0$. For all $i \in \mathcal{V}$, we assume that $a_{ii} = 0$. Define the degree of node $i \in \mathcal{V}$ as $d_i = \sum_{j=1}^{N} a_{ij}$. The degree matrix for the graph $\mathcal{G}$ is $\mathcal{D} = \text{diag}(d_1, d_2, \ldots, d_N)$, then $\mathcal{L} = [l_{ij}]_{N \times N} = \mathcal{D} - \mathcal{A}$ denotes the corresponding Laplacian matrix of the graph $\mathcal{G}$, where $l_i = \sum_{j=1}^{N} a_{ij}$ and $l_j = -a_{ij}$, $i \neq j$. The undirected graph $\mathcal{G}$ is called to be connected if there exists an undirected path between any two nodes, otherwise it is disconnected.

Then, to investigate the MGFT problem, a graph $\mathcal{G}$ is used to denote the communication topology of the MAS with $M$ leaders and $N$ followers. Define $\mathcal{V}_l$ as the set of $M$ leaders (marked 0 ) and $\mathcal{V}_f$ as the set of the $N$ followers (marked 1, $\ldots$, $N$), respectively. Then, the leader adjacency matrix is defined as $\mathcal{B} = \text{diag}(b_1, b_2, \ldots, b_N)$, where if there exists a connection between the $i$ th follower and the leader, then $b_i > 0$; otherwise $b_i = 0$, it describes the communication weight between the $i$ th follower and the leader. For the graph $\mathcal{G}$, the corresponding Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{H} = \mathcal{B} + \mathcal{L}$.

Without loss of generality, assume that all agents can be split up into $M$ ($M \geq 1$) subgroups. Each subgroup can be denoted by $\mathcal{G}_l = \{\mathcal{A}_l, \mathcal{V}_l, \mathcal{E}_l\}$, $l \in \{1, \ldots, M\}$, where $\mathcal{V}_l \cap \mathcal{V}_f = \emptyset$ ($l, r \in \{1, \ldots, M\}$, $l \neq r$). $\bigcup_{l=1}^{M} \mathcal{V}_l = \mathcal{V}$, $\bigcup_{l=1}^{M} \mathcal{E}_l \subseteq \mathcal{E}$. For the $l$ th subgroup, let $n_l$ be the number of the agents, where $\sum_{l=1}^{M} n_l = N$. Define $\sigma_l = \sum_{n_{l-1}}^{n_l}$, then the agent index of the $l$ th subgroup is $\mathcal{V}_l = \{1 + \sigma_1, 2 + \sigma_1, \ldots, n_l + \sigma_1\}$ with $n_0 = 0$. For the $M$ subgroups, the weighted adjacency matrix $\mathcal{A}$ can be described as the following block form:

$$
\mathcal{A} = \begin{bmatrix}
\mathcal{A}_1 & \mathcal{A}_{12} & \cdots & \mathcal{A}_{1M} \\
\mathcal{A}_{12} & \mathcal{A}_2 & \cdots & \mathcal{A}_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
\mathcal{A}_{1M} & \mathcal{A}_{2M} & \cdots & \mathcal{A}_{MM}
\end{bmatrix}.
$$

B. NEURAL NETWORKS

In this paper, due to the excellent approximation ability of neural networks (NNs), NNs are selected to approximate the unknown nonlinear functions which represent the agents’ inherent dynamics. A continuous function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ can be approximated by $f_{\Theta}(x)$ as follows:

$$
f_{\Theta}(x) = W^T \Phi(x)
$$

where $W \in \mathbb{R}^{m \times n}$ is the adjustable weight matrix. $p$ represents the neuron number. $x \in \Omega \subset \mathbb{R}^n$ is the input vector of NNs. The basis function vector is denoted by $\Phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_p(x)]^T \in \mathbb{R}^p$ with $\phi_i(x) = e^{-\frac{(x - \mu_i)^2}{2\sigma_i^2}}$, $i = 1, 2, \ldots, p$, where $\mu_i = [\mu_{i1}, \ldots, \mu_{im}]^T$ is the center of the receptive field.
Based on the above fact, for any smooth continuous function \( f(x) \), one has the optimal weight matrix \( W^* \in \mathbb{R}^{n \times n} \) expressed as

\[
W^* := \arg \min_{W \in \mathbb{R}^{n \times n}} \| f(x) - W^T \Phi(x) \|,
\]

then \( f(x) \) can be rewritten as \( f(x) = W^T \Phi(x) + \varepsilon(x) \), where \( \varepsilon(x) \in \mathbb{R}^n \) is the error of approximation, which is bounded by a positive constant \( \rho \) such that \( \| \varepsilon(x) \| \leq \rho \).

In order to minimize the possible deviation between \( W^T \Phi(x) \) and \( f(x) \), the optimal weight matrix \( W^* \) must be obtained. However, since \( W^* \) is just an “artificial” quantity for analysis, it is unavailable for the actual control design. Usually, the estimation of \( W^* \) is employed to construct the actual control, which is obtained by online adaptive tuning.

C. SOME USEFUL LEMMAS AND ASSUMPTIONS

Lemma 1. [28] If there is at least one follower communicated to the leader in the connected undirected graph \( G \), then \( B + \mathcal{L} \) is a positive definite matrix.

Lemma 2. [29] The matrix inequality that

\[
\begin{bmatrix}
M_1(x) & M_2(x) \\
M_1^T(x) & M_2(x)
\end{bmatrix} > 0,
\]

where \( M_1(x) \) and \( M_2(x) \) are symmetric matrices, is equivalent to any conditions as follows:

1) \( M_1(x) > 0, M_2(x) - M_1^T(x)M_1^{-1}(x)M_2(x) > 0 \);
2) \( M_2(x) > 0, M_1(x) - M_1^T(x)M_2^{-1}(x)M_2(x) > 0 \).

Lemma 3. [30] For all \( t \geq 0 \) and bounded initial condition, let function \( V(t) \) be a continuous function. If \( \dot{V}(t) \leq -\kappa \dot{V}(t) + \omega \) holds, where \( \kappa \) and \( \omega \) are two positive constants, the following one can be obtained:

\[
V(t) \leq e^{-\kappa t} V(0) + \frac{\omega}{\kappa} (1 - e^{-\kappa t}).
\]

Assumption 1. [25] \( \sum_{j=1}^{N} a_{ij} = 0, \forall i \in V / V_j, l \in \{1, \ldots, M\} \).

Assumption 2. In the MGFT problem, the undirected subgraph \( G \), for the followers in the l th subgroup is connected, and there is at least one follower connected to the leader 0, that is, \( B, \neq 0, l \in \{1, \ldots, M\} \).

Remark 1. Although the cooperation weight between different subgroups can be positive or negative, its sum is 0. Therefore, for one subgroup, \( \text{Assumption 1} \) indicates that the total information coming from all other subgroups is 0, which enables each subgroup to accomplish the task as a whole.

III. MAIN RESULTS

A. PROBLEM FORMULATION

Consider a second-order nonlinear MAS containing \( M + N \) agents. The nonlinear dynamics of the \( i \) th follower is given by the following form

\[
\begin{bmatrix}
p_i(t) \\
p_i(\dot{t})
\end{bmatrix} = \bar{f}_i(t),
\]

\[
\begin{bmatrix}
p_i(t) \\
p_i(\dot{t})
\end{bmatrix} = \bar{f}_i(t) + f_i(p_i(t), v_i(t)),
\]

where \( p_i(t) = [p_{i1}, \ldots, p_{iM}]^T \in \mathbb{R}^d, v_i(t) = [v_{i1}, \ldots, v_{id}]^T \in \mathbb{R}^d \) and \( u_i(t) = [u_{i1}, \ldots, u_{id}]^T \in \mathbb{R}^d \) denote the position, velocity and control input of the \( i \) th follower, respectively.

The dynamics of the \( j \) th leader can be described as

\[
\begin{bmatrix}
p_j^0(t) \\
p_j^0(\dot{t})
\end{bmatrix} = \bar{f}_j^0(t),
\]

\[
\begin{bmatrix}
p_j^0(t) \\
p_j^0(\dot{t})
\end{bmatrix} = a_j^0(t),
\]

where \( p_j^0(t) \in \mathbb{R}^d \) and \( v_j^0(t) \in \mathbb{R}^d \) are the position and velocity vector of the \( j \) th leader, respectively.

The control objective. Design a distributed adaptive formation control scheme to solve the MGFT problem of the second-order nonlinear MASs with unknown dynamics such that

1) the position and velocity errors can converge to the desired accuracy;
2) all formation errors are semi-globally uniformly ultimately bounded (SGUUB).

B. CONTROL PROTOCOL DESIGN

In order to realize the MGFT control for the nonlinear
second-order MASs (1) and (2), a distributed adaptive formation control protocol is designed in this section.

Because the nonlinear vector-value function \( f_i(p_i(t), v_i(t)) \) is unknown but continuous, it is unable to be applied directly in the control protocol design. Define a compact set \( \Omega \subset \mathbb{R}^{2d} \), for \( p_i^1, p_i^2 \in \Omega \), the unknown nonlinear function \( f_i(p_i(t), v_i(t)) \), \( \forall i \in \mathcal{V}, l \in \{1, \ldots, M\} \) can be approximated by NNs as

\[
f_i(p_i, v_i) = W_i^\top \Phi_i(p_i, v_i) + e_i(p_i, v_i),
\]

where the basis function vector is denoted by \( \Phi_i(p_i, v_i) \in \mathbb{R}^n \). \( W_i^* \in \mathbb{R}^{n \times d} \) is the optimal NN weight matrix consisting of \( n \) neurons. \( e_i(p_i, v_i) \in \mathbb{R}^n \) is the error of approximation satisfying \( \|e_i(p_i, v_i)\| \leq \rho_i \), where \( \rho_i \) is a positive constant.

Define the position error vector \( \varphi_i (t) \) and velocity error vector \( \omega_i (t) \) as

\[
\varphi_i (t) = p_i(t) - p_i^* - p_i^\dagger, \quad \omega_i (t) = v_i(t) - v_i^* - v_i^\dagger,
\]

\( \forall i \in \mathcal{V}, l \in \{1, \ldots, M\} \).

According to the systems (1) and (2), the error dynamics can be described as

\[
\dot{\varphi}_i (t) = -\gamma_i \varphi_i (t) + f_i(p_i, v_i) - a_i^\dagger (t), \quad \forall i \in \mathcal{V}, l \in \{1, \ldots, M\}.
\]

For the \( l \) th subgroup, let \( \sigma_l = \sum_{n=l}^\infty n^{n-1} \), then \( \mathcal{V}_l = \sigma_l + 1, \sigma_l + 2, \ldots, \sigma_l + n_l \) . Denote \( \varphi_i^l(t) = [\varphi_i^l(t), \varphi_i^l(t), \ldots, \varphi_i^l(t))]^\top \in \mathbb{R}^{n_l \times d} \), \( \varphi_i^l(t) = [\varphi_i^l(t), \varphi_i^l(t), \ldots, \varphi_i^l(t))]^\top \in \mathbb{R}^{n_l \times d} \), \( u_i = [u_i^n, u_i^{n+1}, \ldots, u_i^n + \ldots ]^\top \in \mathbb{R}^{n \times d} \), \( F_i(\varphi) = [f_i^n, f_i^{n+1}, \ldots, f_i^n + \ldots ]^\top \in \mathbb{R}^{n \times d} \), where \( l \in \{1, \ldots, M\} \).

The compact form of the error dynamics (6) can be obtained as

\[
\dot{\varphi}_i (t) = \Lambda_i \varphi_i (t)
\]

where

\[
\Phi_i (t) = \left[ \begin{array}{c} \varphi_i^n (t) \\ u_i^n + F_i (\varphi_i) - a_i^n (t) \otimes 1_n \end{array} \right],
\]

\( 1_n = [1, \ldots, 1]^\top \in \mathbb{R}^n \), \( \otimes \) is the Kronecker product.

Since not all followers can directly connect with the leader in each subgroup, the formation tracking errors are defined as

\[
e_{p_i} (t) = \sum_{j \in \mathcal{N}_{ij}} a_{ij} (p_j(t) - p_i^* - p_i^\dagger - p_j^\dagger) + b_{ij} (p_j(t) - p_i^* - p_i^\dagger - p_j^\dagger),
\]

\[
e_{v_i} (t) = \sum_{j \in \mathcal{N}_{ij}} a_{ij} (v_j(t) - v_i^* - v_i^\dagger - v_j^\dagger) + b_{ij} (v_j(t) - v_i^* - v_i^\dagger - v_j^\dagger),
\]

\( \forall i \in \mathcal{V}, l \in \{1, \ldots, M\} \),

where \( a_i \) and \( b_i \) are the elements of the adjacency matrix \( A \) and the leader adjacency matrix \( B \), \( \mathcal{N}_{ij} = \{ j | j \in \mathcal{V}_i; (i, j) \in \mathcal{E}_i \} \) is the set of neighbors of agent \( i \) in the \( j \) th subgroup.

On the basis of (5), then the formation tracking error dynamics can be rewritten as

\[
e_{p_i} (t) = \sum_{j \in \mathcal{N}_{ij}} a_{ij} (p_j(t) - p_i^* - p_i^\dagger) + b_{ij} (p_j(t) - p_i^* - p_i^\dagger),
\]

\[
e_{v_i} (t) = \sum_{j \in \mathcal{N}_{ij}} a_{ij} (v_j(t) - v_i^* - v_i^\dagger) + b_{ij} (v_j(t) - v_i^* - v_i^\dagger),
\]

\( \forall i \in \mathcal{V}, l \in \{1, \ldots, M\} \).

Based on the NN approximation (4), by applying the estimation \( \hat{W}_i (t) \) of the optimal NN weight matrix \( W_i^* \), the adaptive MGFT control protocol is proposed as

\[
u_i (t) = \sum_{j \in \mathcal{N}_{ij}} a_{ij} k_e [(p_j(t) - p_i^* - p_i^\dagger) - (p_j^\dagger - p_i^\dagger)],
\]

\[
u_i (t) = \sum_{j \in \mathcal{N}_{ij}} a_{ij} k_v [(v_j(t) - v_i^* - v_i^\dagger) - (v_j^\dagger - v_i^\dagger)],
\]

\( \forall i \in \mathcal{V}, l \in \{1, \ldots, M\} \),

where \( k_e > 0 \), \( k_v > 0 \) denote the position damping gain and velocity damping gain, respectively. \( \hat{W}_i (t) \in \mathbb{R}^{n \times d} \) is the estimation of \( W_i^* \).

Choose the NN updating law for \( \hat{W}_i (t) \) as follows

\[
\dot{\hat{W}}_i (t) = A (\Phi_i (p_i, v_i) e_i (t) + e_i (t)) - \gamma_i \hat{W}_i (t),
\]

\( \forall i \in \mathcal{V}, l \in \{1, \ldots, M\} \),

where \( \gamma_i > 0 \) is a positive design constant, \( \Lambda_i \in \mathbb{R}^{n \times n} \) is a positive definite matrix. 

C. STABILITY ANALYSIS

Theorem 1. Suppose that Assumptions 1-2 hold. Consider the nonlinear MASs (1) and (2) with bounded initial conditions. If the design parameters are selected to satisfy

\[
k_e > 1, \quad k_v > \frac{1}{2 (\lambda_{\min} (H_e))}, \quad k_e + k_v > \frac{1}{\lambda_{\min} (H_e)},
\]

\( l \in \{1, \ldots, M\} \),

where \( \lambda_{\min} (H_e) \) is the minimum eigenvalue of matrix \( H_e = E + L_e \), then the MGFT control objectives can be achieved under the distributed adaptive control protocol (10) with the NN weight updating law (11).

Proof. For the \( l \) th subgroup, define the following Lyapunov function candidate for the system (7) as

\[
V_i (t) = \frac{1}{2} \varphi_i^n (t) \left[ (k_e + k_v) H_e H_e H_e \right] \varphi_i^n (t) + \frac{1}{2} \sum_{j \in \mathcal{N}_{ij}} \varphi_j^n (t) A_i \varphi_j^n (t),
\]
where $\hat{W}(t) = \hat{W}(t) - W^*_t$, $I_n \in \mathbb{R}^{n \times n}$ is an identity matrix, $l \in \{1, \ldots, M\}$.

Due to Assumption 2 holds, the condition (12) and the Lemmas 1-2 are satisfied, which means $V_i(t)$ is positive definite. Then the time derivative of $V_i(t)$ along the trajectory of system (7) and (11) is

$$
V_i(t) = \phi_i^T(t) \left[ \begin{array}{c} k + k_i \end{array} \right] H \left[ \begin{array}{c} H_i \end{array} \right] + \sum_{i=1}^{n} \left( e_i^T(t) + e_i(t) \right) + \sum_{i=1}^{n} \left( e_i^T(t) - e_i(t) \right)^T \gamma \hat{W}(t) \right].
$$

Using the above equation (9), we can obtain

$$
V_i(t) = \sum_{i=1}^{n} \left( e_i^T(t) + e_i(t) \right) \phi_i(t) + \sum_{i=1}^{n} \left( e_i^T(t) - e_i(t) \right) \phi_i(t)
$$

Substituting the above equation (9), we can obtain

$$
V_i(t) = \sum_{i=1}^{n} \left( e_i^T(t) + e_i(t) \right) \phi_i(t) + \sum_{i=1}^{n} \left( e_i^T(t) - e_i(t) \right) \phi_i(t)
$$

By the property of trace operation, $\text{Tr} \{ab^T\} = \text{Tr} \{ba^T\} = d^Tb$, where $a, b \in \mathbb{R}^d$, one gets

$$
\text{Tr} \{ (\hat{W}(t) - W^*_t)^T \Phi_i(p, v_i) (e_i^T(t) + e_i(t))^T \} = (e_i^T(t) + e_i(t))^T (\hat{W}(t) - W^*_t)^T \Phi_i(p, v_i).
$$

Substituting the above equation (21) into (20), one has

$$
V_i(t) \leq \sum_{i=1}^{n} \left( e_i^T(t) + e_i(t) \right)^T - \frac{\gamma}{2} \text{Tr} \{ (\hat{W}(t) - W^*_t)^T \Phi_i(p, v_i) (e_i^T(t) + e_i(t))^T \} + \Theta(t).
$$

By the definition of $\Theta(t)$, there is

$$
\Theta(t) = \sum_{i=1}^{n} \left( e_i^T(t) + e_i(t) \right)^T (\hat{W}(t) - W^*_t)^T \Phi_i(p, v_i).
$$

where $\Theta(t)$ is a bounded, it is bounded by a constant $\omega$, i.e., $\|\Theta(t)\| \leq \omega$.
Then, the following inequality can be obtained
\[
V_i(t) \leq -\lambda_{\text{min}} \phi_i(t) + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{\gamma_j}{\lambda_{\text{min}}} \mathcal{L} \left( W_i(t) A_i^{-1} W_i(t) \right) + \omega_i, 
\]
where \( \lambda_{\text{min}} \) is the minimal eigenvalue of 
\[
\left( k_i - 1 \right) \mathcal{H}_i \mathcal{H}_i^T \\
0 \\
\left( k_i - 1 \right) \mathcal{H}_i \mathcal{H}_i^T 
\]
and \( \lambda_{\text{max}}^\beta \) is the maximal eigenvalue of 
\[
\left( k_i + k_j \right) \mathcal{H}_i \mathcal{H}_i^T \\
0 \\
\left( k_i + k_j \right) \mathcal{H}_i \mathcal{H}_i^T 
\]
and \( \lambda_{\text{max}}^\alpha \) is the maximal eigenvalue of \( A_i^{-1} \).

Let \( \kappa = \min \{ \lambda_{\text{min}}^\beta, \lambda_{\text{max}}^\alpha, \lambda_{\text{max}}^\beta, \lambda_{\text{max}}^\alpha \} \), (23) can become
\[
V_i(t) \leq -\kappa \phi_i(t) + \omega_i. 
\]

According to Lemma 3, the following fact can be yielded from (24)
\[
V_i(t) \leq e^{-\kappa t} V_i(0) + \frac{\omega}{\kappa} (1 - e^{-\kappa t}). 
\]

In the control protocol (10), it is obvious that only the neighboring states in the same subgroup are applied. Then, the Laplacian matrix \( \mathcal{H} \) matched with the graph \( \mathcal{G} \) can be denoted as \( \mathcal{H} = \mathcal{B} + \mathcal{L} \) where \( \mathcal{B} = \mathcal{L} \).

Then, rewrite the system (7) to a compact form
\[
\phi(t) = \begin{bmatrix} \phi_i(t) \\ u + F(\phi) - a^\phi(\phi) \mathcal{L} \right \}, 
\end{bmatrix}
\]
where
\[
\phi(t) = [\phi^T, \phi^T, \ldots, \phi^T]^T \in \mathbb{R}^{2N}, \\
\phi^T(t) = [\phi^T, \phi^T, \ldots, \phi^T]^T \in \mathbb{R}^{N}, \\
\phi^T(t) = [\phi^T, \phi^T, \ldots, \phi^T]^T \in \mathbb{R}^{N}, \\
u = [u^T, u^T, \ldots, u^T]^T \in \mathbb{R}^{N}, \\
F = \left[ F^T, F^T, \ldots, F^T \right]^T \in \mathbb{R}^{N}
\]
and \( a^\phi(\phi) = [a^T, a^T, \ldots, a^T]^T \in \mathbb{R}^{N} \).

Likewise, the Lyapunov candidate for the whole multi-agent system can be constructed as follows
\[
V(t) = \frac{1}{2} \phi(t) \left( k_i + k_j \right) \mathcal{H} \mathcal{H}^T \mathcal{H} \mathcal{H}^T \mathcal{L} \right \), \\
\times \phi(t) + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{\gamma_j}{\lambda_{\text{min}}} \mathcal{L} \left( W_i(t) A_i^{-1} W_i(t) \right) 
\]

Using the same method proved above, we can obtain the following fact
\[
V(t) \leq e^{-\kappa t} V(0) + \frac{\omega}{\kappa} (1 - e^{-\kappa t}). 
\]

The above inequality means that all formation errors \( \phi_i(t), \phi_j(t), \bar{W}_i(t), \forall i \in \mathcal{V}, \forall j \in \{1, \ldots, M\} \) are SGUUB and the position and velocity errors \( \psi_i(t), \psi_j(t), \bar{W}_i(t), \forall i \in \mathcal{V}, \forall j \in \{1, \ldots, M\} \) can converge to the desired accuracy under the condition (12). It implies that the MGFT control can be achieved for second-order nonlinear MASs with unknown dynamics.

IV. NUMERICAL SIMULATION

In this section, a numerical simulation example is provided to demonstrate the effectiveness of the theoretical analysis and the feasibility of the control protocol in three-dimensional (3D) space.

In this simulation example, a nonlinear MAS is considered with three leaders and sixteen followers in three subgroups. As shown in Fig.1, the leaders are labelled as the pentagons, and the followers are labelled as the circles (marked from 1 to 16). Evidently, the sixteen followers are divided into three subgroups, namely \( N = 16, \ M = 3, \ \mathcal{V}_1 = \{1, 2, \ldots, 8\}, \ \mathcal{V}_2 = \{9, 10, \ldots, 12\} \) and \( \mathcal{V}_3 = \{13, 14, 15\} \). In each subgroup, the nonlinear dynamics for followers are described as
\[
f_i(\psi_i(t), v_i(t)) = [c_{1i} \cos^2(\psi_i), c_{2i} \sin^2(\psi_i), c_{3i} \cos^2(\psi_i)]^T, \\
\forall i \in \mathcal{V}_1, l \in \{1, 2, 3\}
\]
and \( c_{1i}, c_{2i}, c_{3i} \) are randomly selected from -1 to 1. The desired motion of leaders can be depicted by
\[
a_{1i}^0(\psi) = \begin{bmatrix} -0.15 \sin(0.15t), -0.15 \cos(0.15t), 0 \end{bmatrix}^T, \ j = 1, \\
a_{2i}^0(\psi) = \begin{bmatrix} -0.2 \sin(0.2t), -0.2 \sin(0.2t), 0 \end{bmatrix}^T, \ j = 2, \\
a_{3i}^0(\psi) = \begin{bmatrix} -0.1 \sin(0.1t), -0.1 \cos(0.1t), 0 \end{bmatrix}^T, \ j = 3.
\]

The initial position states and velocity states of leaders are
\[
p_i^0(0) = [0, 0, 0]^T, \ \ p_i^0(0) = [-18, 18, 0]^T, \\
\ c_i^0(0) = [18, -18, 0]^T \ \ \text{and} \ \ c_i^0(0) = [1, 0, 1]^T, \\
\ v_i^0(0) = [0, 1, 1]^T, \ v_i^0(0) = [1, 0, 1]^T, \ \text{respectively.} \ \text{The initial positions} \ p_i(0) \ \text{of all followers are random variables uniformly distributed on [-20, 20]. For the distributed control protocol (10), the design parameters are selected as} \ k_p = 55, \ k_i = 35. \ \text{The NN is designed to contain 12 neurons with centers} \ u_i \ \text{evenly spaced from -8 to 8. Then the parameters designed for the updating law (11) are chosen as} \ A = \begin{bmatrix} 1.6 \end{bmatrix} I_{12}, \ \gamma_i = 0.5 \ \text{and the initial values are} \ \bar{W}_i(0) = [0.8]_{2 \times 2}, \ \forall i \in \mathcal{V}_1, l \in \{1, 2, 3\}.\]
Algorithm 1 Distributed MGFT Control Implementation

1: **Initialization**: Give the initial position $p_i, p_i^0$ and the initial velocity $v_i, v_i^0$ for all agents $i \in \{1,2,\ldots,N\}, j \in \{1,2,\ldots,M\}$. Set $t$, time step $h$, simulation time $T$ and the initial values of $\hat{W}_i$.

2: Choose a set of parameters $k_p, k_v, A, \gamma, \mu$;

3: if conditions $k_p > 1, k_v > \frac{3}{2} + \frac{1}{2(\lambda_{\text{min}}(H_i))}$ and $k_p + k_v > \frac{1}{\lambda_{\text{min}}(H_i)}$ hold simultaneously, then

4: go to Step 8

5: else

6: go to Step 2

7: end if

8: while $0 \leq t \leq T$ do

9: Obtain the position errors $\phi_i$, the velocity errors $\phi_v$ and the formation tracking errors $e_p, e_v$;

10: Compute the control $u_i(t)$ in (10) and the updating law $\dot{\hat{W}}(t)$ in (11);

11: $t = t + h$;

12: end while

---

**FIGURE 1.** The communication topology $\mathcal{G}$ of three subgroups in 3D space.

It can be seen from the Fig. 1, the desired sub-formation shapes are selected as a cube, a tetrahedron and a rhombus. Without loss of generality, set all weight value of intra-subgroup coupling edges as one.

**FIGURE 2.** Trajectories of sixteen followers tracking three different leaders in three subgroups.

**FIGURE 3.** The norm of position errors $\phi_{pi}(t), \forall i \in \mathcal{I}, j \in \{1,2,3\}$.

**FIGURE 4.** The norm of velocity errors $\phi_{vi}(t), \forall i \in \mathcal{I}, j \in \{1,2,3\}$.
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velocity errors between the followers and the respective leader along X, Y and Z axes, evolutions of the norms of position errors between the followers track their respective leaders. Fig. 3 depicts the subgroups in the desired sub-formation shapes, and the presented in Fig. 2, all agents are divided into three with the theoretical result obtained in Theorem 1. As MASs (1) and (2) is accomplished, which also confirms the multi-agent systems were proved to achieve the multi-group formation tracking control under the proposed control scheme, where all the followers can reach the desired sub-formation and the geometric center of the followers can track the corresponding leader in each subgroup. Finally, a numerical simulation example in 3D space has been provided to verify the effectiveness of the theoretical results.

In the future work, we will further investigate the multi-group formation tracking control problem with data packet loss and communication delays.

It is easy to learn from Figs. 2-6 that the MGFT control of the MASs (1) and (2) is accomplished, which also confirms with the theoretical result obtained in Theorem 1. As presented in Fig. 2, all agents are divided into three subgroups in the desired sub-formation shapes, and the followers track their respective leaders. Fig. 3 depicts the evolutions of the norms of position errors between the followers and the respective leader along X, Y and Z axes, respectively. Fig. 4 depicts the evolutions of the norms of velocity errors between the followers and the respective leader along X, Y and Z axes, respectively. As presented in Figs. 3 and 4, the position and velocity errors can converge to the desired accuracy in three subgroups. Figs. 5 and 6 describes the evolutions of followers’ position and velocity states along X, Y and Z axes, respectively. As presented in Figs. 5 and 6, the position and velocity states of followers can converge to the desired formation and maintain it in three subgroups.

V. CONCLUSION

In this paper, the multi-group formation tracking control problem for second-order nonlinear multi-agent systems with unknown dynamics was studied. The neural network approximator was used to compensate the unknown dynamics. Based on adaptive neural network control method, the multi-group formation tracking control protocol was proposed and analyzed in detail. Using the knowledge of algebraic graph theory and Lyapunov stability theory, sufficient conditions were obtained such that the multi-agent systems were proved to achieve the multi-group formation tracking control under the proposed control scheme, where all the followers can reach the desired sub-formation and the geometric center of the followers can track the corresponding leader in each subgroup. Finally, a numerical simulation example in 3D space has been provided to verify the effectiveness of the theoretical results.

In the future work, we will further investigate the multi-group formation tracking control problem with data packet loss and communication delays.

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