Virtual Black Holes at Linear Colliders

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Abstract

We propose that future linear colliders can create virtual black holes even though their energies are below the fundamental scale, because of the uncertainty principle. These virtual black holes provide us much information which cannot be obtained in the production of black holes at the LHC or other hadron colliders. We can observe lepton flavor and lepton (baryon) number violating processes at linear colliders by using virtual black holes. And virtual black holes can be used to the precision measurements of top and W. They can create only one top quark or one W boson, which leads to the clean signal that cannot be obtained in pair-production processes.
Introduction.

The possibility of low-energy gravity may enable us to produce black holes at future colliders, say LHC [1] or Tevatron [2]. And high-energy cosmic rays may also produce black holes [3]. But their studies can be done in the semiclassical limit, \( M_{BH} \gg M_D \), where \( M_D \) is the true fundamental scale. (See [4] for the latest lower bound of \( M_D \).) If the mass of black hole \( M_{BH} \) approaches \( M_D \), the quantum gravity drastically affect the production cross section and nobody can calculate it. Therefore future linear colliders were not considered as the candidates of black hole factories due to their low energy.

In this letter, we show that even linear colliders can produce virtual black holes, and the information given by their decay is the treasure which cannot be obtained from the decay of black holes at hadron colliders.

The Setup.

We consider virtual black holes produced at future linear colliders. As an input, we set:

\[
\sqrt{s} = 1\text{TeV}, \quad (1a) \\
n = 7, \quad (1b) \\
M_D = 1\text{TeV}, \quad (1c) \\
M_{BH} = 5\text{TeV}, \quad (1d)
\]

where \( n \) is the number of extra dimensions and \( M_{BH} \) is the mass of virtual black hole. If we approach \( M_{BH} \) to \( M_D \), sizable quantum gravity effects appear and we cannot calculate the cross section. But as stated in [5], for \( M_S/g_S < M_{BH} < M_S/g_S^2 \), where \( M_S \) is the string scale and \( g_S \) is the string coupling constant, the cross section is given by the semiclassical one. (1a-1d) satisfies this condition and we can reliably use it.

Production and Decay.

In the semiclassical limit \( M_{BH} \gg M_D \), the \((4 + n)\)-dimensional Schwarzschild
radius is given by [3]:

\[ R_S \sim \frac{1}{\sqrt{\pi} M_D} \left[ \frac{M_{BH}}{M_D} \left( \frac{8\Gamma \left( \frac{n+3}{2} \right)}{n+2} \right) \right]^{1/(n+1)}. \] (2)

And semiclassical reasoning implies that a black hole is produced when the distance between two particles fall into this Schwarzshird radius, and the cross section is given by:

\[ \sigma \sim \pi R_S^2 = \frac{1}{M_D^2} \left[ \frac{M_{BH}}{M_D} \left( \frac{8\Gamma \left( \frac{n+3}{2} \right)}{n+2} \right) \right]^{2/(n+1)} \sim 3.21 \frac{1}{M_D^2}. \] (3)

But note that we are now considering \( \sqrt{s} = 1\text{TeV} \) linear collider. Thus we cannot produce real black holes anyhow. The method to evade this problem is to use the uncertainty principle. During the time scale \( \Delta t \sim 1/(M_{BH}/2) \), we can violate the energy conservation law. For each beam bunches we should apply the uncertainty principle. This leads to the suppression factor:

\[ \left( \frac{\sqrt{s}/2}{M_{BH}/2} \right)^2 \sim \frac{1}{25}. \] (4)

But this is not the end of the story. Voloshin [4] claimed that the cross section for production of large black holes is suppressed by at least a factor \( \exp(-I_E) \), where \( I_E \) is the Gibbons-Hawking action for the black hole.

Taking into account all these results, the production cross section of virtual black holes in the assumption (1a-1d) becomes:

\[ \sigma = 3.21 \frac{1}{M_D^2} \left( \frac{1}{25} \right) \exp(-I_E) \left( 3.89 \times 10^{11} \right) = 2.1 \times 10^4 \text{ fb}. \] (5)

We can also calculate the temperature of this black hole \( T_{BH} \), the mean energies of decayed particles \( \langle E \rangle \) and the average multiplicity \( \langle N \rangle \). They are given by:

\[ T_{BH} = \frac{n + 1}{4\pi R_S} = 0.64 \text{ TeV}, \] (6a)

\[ \langle E \rangle = 2T_{BH} = 1.3 \text{ TeV}, \] (6b)

\[ \langle N \rangle = \frac{M_{BH}}{2T_{BH}} = 3.9. \] (6c)

Thus one virtual black hole emits about 4 particles. But note that the energies of their particles are not 1.3 TeV since the time scale during the uncertainty principle holds is very short, and they become \( E \sim \sqrt{s}/4 = 250\text{GeV} \).
New Physics: Lepton Flavor and Lepton (Baryon) Number Violation.

From the four decay products of one black hole, we can investigate many exciting physical results. From now we assume the integrated luminosity $L = 100 \text{fb}^{-1}$.

The first one is the test of Hawking radiation, which is enabled by the missing transverse energy carried by neutrinos. But since this issue was investigated in [1], we do not consider it in this letter.

Next, we can consider lepton flavor violating processes. The processes are described as follows:

\begin{align*}
e^+e^- \to \text{virtual blackhole} \to e^\pm \mu^\mp (q \bar{q}), & \quad (7a) \\
e^+e^- \to \text{virtual blackhole} \to e^\pm \mu^\mp (g g), & \quad (7b) \\
e^+e^- \to \text{virtual blackhole} \to e^\pm \mu^\mp (l \bar{l}), & \quad (7c) \\
e^+e^- \to \text{virtual blackhole} \to e^\pm \mu^\mp (\gamma \gamma). & \quad (7d)
\end{align*}

Black holes evaporate into the Standard Model (SM) particles without any discrimination. The number of these processes is calculated to be 590. Since the SM has no processes which mediate lepton flavor violation, $100 \text{fb}^{-1}$ operation can prove lepton flavor violation with the accuracy $1/(\sqrt{590}) \sim 4.1\%$. There are no experimental obstacles except muon tracking. JLC study [8] showed that electron calorimeter response is better than 0.3\% for 2GeV $-$ 250GeV, and the separation of two pion clusters make it possible to detect hadronic jet with energy $E \sim 250\text{GeV}$. The problem is muon tracking. The current resolution is about 1 cm, which enables us to detect muon momentum only for $p_\mu \lesssim 100 \text{GeV}$. So the detector technology is still to be upgraded.

Third, we consider lepton and baryon number violating processes.

They are:

\begin{align*}
e^+e^- \to \text{virtual blackhole} \to (\mu^-/e^-) \ u \ u \ d, & \quad (8a) \\
e^+e^- \to \text{virtual blackhole} \to (\mu^+/e^+) \ d \ d \ d. & \quad (8b)
\end{align*}
Their charge-conjugated processes also exist. Here $u$ denotes the upper sector of quark doublet and $d$ denotes the lower sector of that. The number of these processes becomes 650. Since the SM preserves lepton and baryon number, we can prove lepton (baryon) number violating processes with the accuracy $\frac{1}{\sqrt{650}} = 3.9\%$.

**Precision Measurement: Single Top, Single W**

Virtual black holes can also be used to the precision measurements. For example, consider the following processes.

\[
e^+ e^- \rightarrow \text{virtual blackhole} \rightarrow t \bar{u} (q \bar{q}), \quad (9a)
\]
\[
e^+ e^- \rightarrow \text{virtual blackhole} \rightarrow t \bar{u} (g g), \quad (9b)
\]
\[
e^+ e^- \rightarrow \text{virtual blackhole} \rightarrow t \bar{u} (l \bar{l}), \quad (9c)
\]
\[
e^+ e^- \rightarrow \text{virtual blackhole} \rightarrow t \bar{u} (\gamma \gamma), \quad (9d)
\]

with their charge-conjugated ones. The calculation show that we obtain 1300 single top quarks. Top quark immediately decays. In order to determine the mass of top quark, the following decay chain is the best.

\[
t \rightarrow W^+ b \rightarrow (e^+ / \mu^+) (\nu_e / \nu_\mu) b.
\]  
(10)

The difference from usual top-pair production is that only one neutrino is emitted, and thus the combinatorial background is completely zero. This means only the statistical and the calorimeter error dominates the error of top quark mass.

Here we estimate the error. The calorimeter error of JLC is:

\[
\frac{\sigma_E}{E} = \frac{15\%}{\sqrt{E(\text{GeV})}} \oplus 1\% \text{ for } e/\gamma, \quad (11a)
\]
\[
\frac{\sigma_E}{E} = \frac{40\%}{\sqrt{E(\text{GeV})}} \oplus 2\% \text{ for hadrons.} \quad (11b)
\]
\[
\quad \text{(11c)}
\]

The calorimeter error of b-quark with energy $250 \times 0.5\text{GeV}$ is 4.1% and it dominates the error of calorimeter. We use leptonic decay mode only and $\text{Br}(W \rightarrow (e/\mu) (\nu_e/\nu_\mu)) = 0.21$. So the statistical error is $\frac{1}{\sqrt{1300 \times 0.21}} = 6.1\%$.
Therefore the total error becomes 7.3%. This is comparable value with the error obtained by Tevatron, 5.1%. If we combine these results, final error becomes 4.5%. So virtual black holes can reduce the error of top quark mass, which means that virtual black holes can be the complement of the current electroweak precision measurements.

Next we consider single W production processes. They are:

\[ e^+e^- \rightarrow \text{virtual blackhole} \rightarrow W^+ l^- (g \bar{q}), \quad (12a) \]
\[ e^+e^- \rightarrow \text{virtual blackhole} \rightarrow W^+ l^- (g g), \quad (12b) \]
\[ e^+e^- \rightarrow \text{virtual blackhole} \rightarrow W^+ l^- (l \bar{l}), \quad (12c) \]
\[ e^+e^- \rightarrow \text{virtual blackhole} \rightarrow W^+ l^- (\gamma \gamma), \quad (12d) \]

again with their charge-conjugated ones. The calculation shows that we can obtain 450 single W bosons. Since we know that the initial energy of W-boson is about 250GeV, the mass reconstruction process from \( W^+ \rightarrow l^+ \nu \) is straightforward. The only source of the error is the calorimeter resolution. It is estimated as 1.4%, and it is suppressed by the number of single W, and the error becomes 0.066%. The current error is 0.07%, and thus again virtual black holes can play a complementary role of current measurements.

Summary.

In this letter we explored new possibility that linear colliders can be used as virtual black hole production machines. The cleanness of linear colliders enables us to analyze new physics from many resultant of black hole decay. The candidates of new physics are lepton flavor and lepton (baryon) number violation. And since black holes can create only one top quark or one W boson, black hole decay at linear colliders gives the new method of the precision measurements of top and W.

Note Added

After the submission of this letter, we learned from J. D. March-Russell that their paper was the first to think about black holes in the TeV-scale gravity and to discuss many features, say temperature and size for example.
And we also learned from K. Cheung that the large entropy is the necessity to
tell the object is truly a black hole \(S_{BH} > 25\), and their papers [10] emphasized
the fact. We calculated the entropy of black holes in our setup. The result was
\(S_{BH} = 31\), and we can say that our setup is enough to make virtual black holes.

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