Covariant Lagrange multiplier constrained higher derivative gravity with scalar projectors

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We formulate higher derivative gravity with Lagrange multiplier constraint and scalar projectors. Its gauge-fixed formulation as well as vector fields formulation is developed and corresponding spontaneous Lorentz symmetry breaking is investigated. We show that the only propagating mode is higher derivative graviton while scalar and vector modes do not propagate. Despite to higher derivatives structure of the action, its first FRW equation is the first order differential equation which admits the inflationary universe solution.

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I. INTRODUCTION

The study of the evolution of early-time and late-time universe indicates that General Relativity is not complete theory of gravitational interaction. At best, it may be the effective theory to describe the classical gravity at intermediate energies. Indeed, there are quite strong proposals on the unified description of the early-time inflation and late-time acceleration in terms of modified gravity (for review, see [1]). It is desirable to extend such modified gravity till Planck scale where it should be renormalizable or somehow consistent (finite?) one at UV. Otherwise, such proposals which pass local/cosmological tests (for recent review, see [2]) remain to be phenomenological ones.

It is quite well-known that higher derivative gravity may be multiplicatively renormalizable (for a review, see [3]). However, the unitarity issue in such approach is so far the open problem. Recently, very interesting attempt to formulation of power-counting renormalizable gravity has been presented in terms of gravity [4] which also uses effectively higher derivative propagator. Unfortunately, such approach leads to explicit breaking of Lorentz invariance from the very beginning. Nevertheless, such formulation may be extended to covariant theory [5] where Lorentz invariance is broken spontaneously, in the same way as celebrated gauge invariance. The construction of higher-derivative power-counting model with consistent graviton spectrum in this direction may suggest the new perspectives towards to multiplicative renormalizability as well as resolution of some problems which appear in Lorentz non-invariant gravities.

In the present work we propose covariant Lagrange multiplier constrained higher derivative gravity with scalar projectors. Its gauge-fixed formulation is developed. It is demonstrated that only higher derivative graviton degree of freedom is propagating at tree level while scalar and vector degrees of freedom do not propagate. The theory turns out to be power-counting (super-)renormalizable. The spontaneous Lorentz symmetry breaking is investigated. The equivalent representation in terms of vectors is also developed. Finally, FRW cosmology is studied. It is shown that theory looks very similar to $R^2$ gravity. The possibility of eternal inflation is demonstrated.

II. THE MODEL OF POWER-COUNTING RENORMALIZABLE, COVARIANT HIGHER DERIVATIVE GRAVITY

It is expected that power-counting renormalizable covariant gravity should be higher derivative theory, for instance, of the sort recently proposed in refs. [3]. In the present section we propose new formulation of such theory and derive its gauge-fixed formulation and propagator. It will be shown that it may have very good UV behavior in gauge-fixed formulation. Different versions of Lagrange multiplier gravity were discussed in refs. [1].

In general, the unitarity is broken in the higher derivative theories. In order to guarantee the unitarity, the models, where Lorentz symmetry and/or the full general covariance is explicitly broken, were proposed by Hořava [4]. Although

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the models are very interesting, due to the lacking of the Lorentz symmetry and/or the full general covariance, there appears an extra propagating and problematic scalar mode (see [3], for example). In the models proposed below, there is the Lorentz symmetry and/or the full general covariance in the actions but the symmetry and/or the covariance is broken spontaneously. As a result, we obtain the models where the UV behavior of the graviton propagator is improved but any extra mode, like scalar mode, does not appear.

We now start with the action including the Lagrange multiplier field $\lambda$ [2] and the scalar field $\phi$:

$$S_{\text{Lag}} = -\int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right),$$

which gives a constraint

$$\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 = 0,$$

that is, the vector $(\partial_\mu \phi)$ is time-like. Therefore the Lorentz symmetry and/or the full general covariance is broken spontaneously. The detailed discussion about the spontaneous breakdown of the symmetry and/or the covariance will be discussed in detail in the next section. For the spontaneous breakdown, $U_0$ needs not to be a constant but some non-vanishing and positive function of $\phi$. Just for the simplicity, we consider the case that $U_0$ is a constant. At least locally, one can choose the direction of time to be parallel to $(\partial_\mu \phi)$. Then Eq. (2) has the following form:

$$\frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 = U_0.$$

Therefore the spatial region becomes a hypersurface where $\phi$ is a constant since the hypersurface is orthogonal to the vector $(\partial_\mu \phi)$.

We are interesting in the fluctuations over the flat background:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}.$$  

Note that [3] gives

$$\phi = \sqrt{2U_0}t.$$

Then one can define a projection operator

$$P_\mu^\nu \equiv \delta_\mu^\nu + \frac{\partial_\mu \phi \partial^\nu \phi}{2U_0},$$

and it follows

$$P_i^\mu P_j^\nu R_{\mu\nu} = \frac{1}{2} \left( h_{i,j}^{\mu} + h_{j,i}^{\mu} - \partial_\mu h_{ij} - \partial_i \partial_j (h_{\rho}^{\mu}) \right),$$

$$\frac{1}{2U_0} P_i^\mu P_j^\nu \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi = 0,$$

$$\partial^\mu \phi \partial^\rho \phi \nabla_\mu \nabla_\rho + 2U_0 \partial^\mu \phi \partial^\rho \phi = 2U_0 \partial_\mu \phi \partial^\rho \phi,$$

and therefore

$$P_i^\mu P_j^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) = \frac{1}{2} \left( h_{kj,i}^k + h_{kj,i}^k - h_{ij,k}^k - \partial_i \partial_j (h_{\rho}^{\mu}) \right).$$

Note that $P_0^\mu = 0$. Then we can propose the action of the power-counting renormalizable, covariant higher derivative gravity with scalar projector as

$$S_{2n+2} = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \alpha \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho \nabla_\nu \phi)^n \right\} P_{\alpha}^{\mu} P_{\beta}^{\nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right] \times \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_\mu \nabla_\nu - \partial_\mu \phi \partial^\mu \phi \nabla^\rho \nabla_\rho \nabla_\nu \phi)^n \right\} P^{\alpha \mu} P^{\beta \nu} \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_\rho \phi \nabla^\rho \nabla_\mu \nabla_\nu \phi \right) \right] \right] + \lambda \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U_0 \right),$$

1 We have merely found the expressions of the actions [11] and [12] so that the propagator of the graviton could be given in [17]. We have not found any deep physical principle to choose the actions in the forms of [11] and [12] but we believe that there might exist some hidden symmetry to construct such theories.
for $z = 2n + 2$ model ($n = 0, 1, 2, \cdots$) \textsuperscript{2} and

$$S_{2n+3} = \int d^4 x \sqrt{-g} \left[ \frac{R}{2k^2} - \alpha \left\{ (\partial^\mu \phi \partial^\nu \phi \partial_{\mu} \partial^\nu \phi)^n P_{\alpha}^\mu P_{\beta}^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_{\mu} \phi \partial^\nu \phi \right) \right\} \right. $$

$$\times \left\{ (\partial^\mu \phi \partial^\nu \phi \partial_{\mu} \partial^\nu \phi)^{n+1} P_{\alpha}^\mu P_{\beta}^\nu \left( R_{\mu\nu} - \frac{1}{2U_0} \partial_{\mu} \phi \partial^\nu \phi \right) \right\}$$

$$- \lambda \left( \frac{1}{2} \partial_{\mu} \phi \partial^\mu \phi + U_0 \right) \right] .$$

(12)

for $z = 2n + 3$ model ($n = 0, 1, 2, \cdots$). Here the quantity $z$ is introduced to express the anisotropy between the time coordinate and spacial coordinates in (11).

One can confirm that the actions admit a flat space vacuum solution. Indeed, field equations are:

$$0 = \frac{1}{2k^2} \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + G_{\mu\nu}^{\text{higher}} - \frac{\lambda}{2} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{2} g_{\mu\nu} \left( \frac{1}{2} \partial_{\rho} \phi \partial^\rho \phi + U_0 \right) .$$

(13)

Here $G_{\mu\nu}^{\text{higher}}$ comes from the higher derivative term (the second term) in the actions (11) and (12). Assuming the flat vacuum solution, Eq. (2) given by the variation over $\lambda$ has a form (3). Since for the flat space solution all the curvatures and $\nabla_{\mu} \nabla_{\nu} \phi$ vanish, Eq. (15) reduces to

$$0 = \lambda \partial_{\mu} \phi \partial_{\nu} \phi ,$$

(14)

whose solution is $\lambda = 0$ since $\partial_{\mu} \phi \neq 0$ due to the constraint equation (2) (if the coordinate system is chosen properly, we have $\partial_{\mu} \phi = \sqrt{2U_0}$ and $\partial_{\mu} \phi = 0$). Hence, the actions (11) and (12) admit the flat space vacuum solution with $\lambda = 0$.

Let us investigate the perturbation from the flat background (11) with $\lambda = 0$ in more detail.

First, by using the diffeomorphism invariance with respect to time coordinate, we choose Eq. (15) as a (unitary) gauge condition. Then the actions (11) and (12) have the following form:

$$S_{2n+2} = \int d^4 x \left[ -\frac{1}{8k^2} \left( -2h_{ij} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} + 2h_{ij} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} \right. \right.$$

$$\left. + h_{ij} \left( 2\delta^{jk} \partial^i - \delta^{ik} \partial^j - \delta^{jk} \partial^i \right) \partial_t h_{jk} + h_{ij} \left( \left( \delta^{ij} \partial^k - \frac{1}{2} \delta^{ik} \delta^{jl} - \frac{1}{2} \delta^{il} \delta^{jk} \right) \left( -\partial^2_k + \partial_k \partial^k \right) \right) \right]$$

$$- \frac{2^{n-2}}{\alpha} U_0^{2n-2} \left\{ (\partial_k \partial^k)^n h_{k,i,j,k} - \partial_i \partial_j \left( h_{\mu}^{\mu} \right) \right\} \times \left\{ (\partial_k \partial^k)^n \left( h_{k,i,j,k} + h_{i,j,k} \right) \right\} ,$$

(15)

$$S_{2n+3} = \int d^4 x \left[ -\frac{1}{8k^2} \left( -2h_{ij} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} + 2h_{ij} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} \right. \right.$$

$$\left. + h_{ij} \left( 2\delta^{jk} \partial^i - \delta^{ik} \partial^j - \delta^{jk} \partial^i \right) \partial_t h_{jk} + h_{ij} \left( \left( \delta^{ij} \partial^k - \frac{1}{2} \delta^{ik} \delta^{jl} - \frac{1}{2} \delta^{il} \delta^{jk} \right) \left( -\partial^2_k + \partial_k \partial^k \right) \right) \right]$$

$$- \frac{2^{n-1}}{\alpha} U_0^{2n-1} \left\{ (\partial_k \partial^k)^n h_{k,i,j,k} - \partial_i \partial_j \left( h_{\mu}^{\mu} \right) \right\} \times \left\{ (\partial_k \partial^k)^n \left( h_{k,i,j,k} + h_{i,j,k} \right) \right\} + U_0 \lambda h_{tt} \right] .$$

(16)

Here, only the terms quadratic with respect to the perturbation are kept. Note that there remains the diffeomorphism invariance with respect to the spatial coordinates. It will be fixed later in (25). An important thing is that there does not appear $h_{tt}$ in the higher derivative term with a coefficient $\alpha$. The constraint equation (2) shows

$$h_{tt} = 0 .$$

(17)

\textsuperscript{2} The idea proposed in ref. [3] for quantum gravity is to modify the ultraviolet behavior of the graviton propagator in Lorentz non-invariant way as $1/|k|^{2z}$, where $k$ is the spatial momenta and $z$ could be 2, 3 or larger integers. They are defined by the scaling properties of space-time coordinates $(x, t)$ as $x \to bx$ and $t \to b^z t$. When $z = 3$, the theory seems to be UV renormalizable.
The variation of $h_{tt}$ can be solved with respect to $\lambda$:

$$\lambda = -\frac{1}{4\kappa^2 U_0} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} + 2^{2n-1} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n} \partial^i \partial^j \left( h_{ki,j}^\mu + h_{kji}^\mu - h_{ij,k} - \partial_i \partial_j \left( h_{\mu}^\mu \right) \right),$$  \hspace{1cm} (18)

for the action (11) and

$$\lambda = -\frac{1}{4\kappa^2 U_0} \left( \delta^{ij} \partial_k \partial^k - \partial^i \partial^j \right) h_{ij} + 2^{2n} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+1} \partial^i \partial^j \left( h_{ki,j}^\mu + h_{kji}^\mu - h_{ij,k} + \partial_i \partial_j \left( h_{\mu}^\mu \right) \right),$$  \hspace{1cm} (19)

for the action (12). The linearized equations given by the variation over $\phi$ are:

$$0 = \partial_t \left\{ \lambda + 2^{2n-1} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n} \partial^i \partial^j \left( h_{ki,j}^\mu + h_{kji}^\mu - h_{ij,k} - \partial_i \partial_j \left( h_{\mu}^\mu \right) \right) \right\},$$  \hspace{1cm} (20)

for the action (11) and

$$0 = \partial_t \left\{ \lambda + 2^{2n} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+1} \partial^i \partial^j \left( h_{ki,j}^\mu + h_{kji}^\mu - h_{ij,k} + \partial_i \partial_j \left( h_{\mu}^\mu \right) \right) \right\},$$  \hspace{1cm} (21)

for the action (12).

We now decompose $h_{ti}$, which corresponds to the fluctuation of the shift function $N_i$, as follows

$$h_{ti} = \partial_t s + v_i, \quad \partial^i v_i = 0.$$  \hspace{1cm} (22)

Here $s$ is the spatial scalar. We further write the linearized diffeomorphism invariance transformations with respect to the spatial coordinates as follows

$$\delta x^i = \partial^i u + w^i, \quad \partial_i w^i = 0.$$  \hspace{1cm} (23)

Then under the diffeomorphism, $s$ and $v_i$ in (22) are transformed as

$$\delta s = \partial_t u, \quad \delta v_i = \partial_t w_i,$$  \hspace{1cm} (24)

and therefore one can choose the gauge condition $s = v^i = 0$, that is,

$$h_{ti} = 0.$$  \hspace{1cm} (25)

Furthermore the variation of $h_{ti}$ gives

$$\partial_t \left( -2\delta^{jk} \partial^i + \delta^{ik} \partial^j + \delta^{ij} \partial^k \right) h_{jk} = 0,$$  \hspace{1cm} (26)

which is identical with that in the usual Einstein gravity since the higher derivative terms in the actions (15) and (16) do not contain $h_{ti}$. We decompose $h_{ij}$ as

$$h_{ij} = \delta_{ij} A + \partial_i B_j + \partial_j B_i + C_{ij} + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \partial_k \partial^k \right) E,$$  \hspace{1cm} (27)

with

$$\partial^i B_i = 0, \quad \partial^i C_{ij} = \partial^j C_{ij} = 0, \quad C^i_i = 0.$$  \hspace{1cm} (28)

Then substituting (27) into (20), we obtain

$$0 = \partial_t \left( -4\partial_t A + 2\partial_k \partial^k B_i + \frac{4}{3} \partial_i \partial_k \partial^k E \right).$$  \hspace{1cm} (29)

By multiplying with $\partial^t$, one gets

$$\partial_t \partial_t \partial^i \left( -4A + \frac{4}{3} \partial_k \partial^k E \right) = 0,$$  \hspace{1cm} (30)

which shows

$$A = \frac{1}{3} \partial_k \partial^k E,$$  \hspace{1cm} (31)
under the boundary condition that \( A \) and \( E \) should vanish at spatial infinity. Then the equation given by substituting (31) into (29) gives
\[
\partial_i \partial_j \partial^j B_i = 0 ,
\]
which also indicates that \( B_i = 0 \) under the boundary condition that \( B_i \) should vanish at spatial infinity.

Using (27) with (28), Eqs. (18) and (20) have the following forms:
\[
\lambda = \frac{1}{2\kappa^2 U_0} \partial_k \partial^k \left( -A + \frac{1}{3} \partial_i \partial^i E \right) - 2^{2n} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n+2} \left( -A + \frac{1}{3} \partial_i \partial^i E \right) ,
\]
\[
0 = \partial_i \left\{ \lambda + 2^{2n} \alpha U_0^{2n-1} \left( \partial_k \partial^k \right)^{2n+2} \left( -A + \frac{1}{3} \partial_i \partial^i E \right) \right\} ,
\]
and (19) and (21)
\[
\lambda = \frac{1}{2\kappa^2 U_0} \partial_k \partial^k \left( -A + \frac{1}{3} \partial_i \partial^i E \right) - 2^{2n+1} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+3} \left( -A + \frac{1}{3} \partial_i \partial^i E \right) ,
\]
\[
0 = \partial_i \left\{ \lambda + 2^{2n+1} \alpha U_0^{2n} \left( \partial_k \partial^k \right)^{2n+3} \left( -A + \frac{1}{3} \partial_i \partial^i E \right) \right\} .
\]
Combining (31) with the above equations, we find
\[
\lambda = B_i = 0 ,
\]
and therefore the scalar modes \( \lambda \) and the vector mode \( B_i \) do not propagate.

After the gauge fixing and using (37) etc., the actions (11) and (12) have the following forms:
\[
S_{2n+2} = \int d^4 x \left\{ \frac{1}{8\kappa^2} \left( C_{ij} \left( -\partial_i^2 + \partial_k \partial^k \right) C^{ij} \right) - 2^{2n-2} \alpha U_0^{2n} \left\{ \left( \partial_k \partial^k \right)^{n+1} C_{ij} \right\} \left\{ \left( \partial_k \partial^k \right)^{n+1} C^{ij} \right\} \right. 
\]
\[
+ \frac{1}{8\kappa^2} \left\{ -6A \left( \partial_i^2 + \partial_k \partial^k \right) A + \frac{2}{3} \partial_k \partial^k E \left( -\partial_i^2 + \partial_k \partial^k \right) \partial_k \partial^k E \right. 
\]
\[
+ 4A \partial_k \partial^k A + \frac{4}{3} A \left( \partial_k \partial^k \right)^2 E - \frac{8}{9} \partial_k \partial^k E \left( \partial_k \partial^k \right)^2 E \right\} 
\]
\[
- 2^{2n-2} \alpha U_0^{2n} \left\{ \left( \partial_k \partial^k \right)^n \left( -\partial_i \partial_j A - \delta_{ij} \partial_k \partial^k A + \frac{1}{3} \partial_i \partial_j \partial_k \partial^k E + \frac{1}{3} \delta_{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right.
\]
\[
\times \left\{ \left( \partial_k \partial^k \right)^n \left( -\partial^i \partial^j A - \delta_{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta_{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right\} ,
\]
\[
S_{2n+3} = \int d^4 x \left\{ \frac{1}{8\kappa^2} \left( C_{ij} \left( -\partial_i^2 + \partial_k \partial^k \right) C^{ij} \right) - 2^{2n-1} \alpha U_0^{2n+1} \left\{ \left( \partial_k \partial^k \right)^{n+1} C_{ij} \right\} \left\{ \left( \partial_k \partial^k \right)^{n+2} C^{ij} \right\} \right. 
\]
\[
+ \frac{1}{8\kappa^2} \left\{ -6A \left( \partial_i^2 + \partial_k \partial^k \right) A + \frac{2}{3} \partial_k \partial^k E \left( -\partial_i^2 + \partial_k \partial^k \right) \partial_k \partial^k E \right. 
\]
\[
+ 4A \partial_k \partial^k A + \frac{4}{3} A \left( \partial_k \partial^k \right)^2 E - \frac{8}{9} \partial_k \partial^k E \left( \partial_k \partial^k \right)^2 E \right\} 
\]
\[
- 2^{2n-1} \alpha U_0^{2n+1} \left\{ \left( \partial_k \partial^k \right)^{n+1} \left( -\partial_i \partial_j A - \delta_{ij} \partial_k \partial^k A + \frac{1}{3} \partial_i \partial_j \partial_k \partial^k E + \frac{1}{3} \delta_{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right.
\]
\[
\times \left\{ \left( \partial_k \partial^k \right)^{n+1} \left( -\partial^i \partial^j A - \delta_{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta_{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right\} .
\]

Then by the variation of \( A \), we obtain
\[
0 = 1 \frac{1}{8\kappa^2} \left\{ -12 \left( \partial_i^2 + \partial_k \partial^k \right) A + 8 \partial_k \partial^k A + \frac{4}{3} \left( \partial_k \partial^k \right)^2 E \right\} 
\]
\[
- 2^{2n-1} \alpha U_0^{2n} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n} \left( -\partial^i \partial^j A - \delta_{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta_{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} ,
\]
\[
3 \text{ It is interesting that the structure of the actions (38) and (39) reminds the one of U(1) invariant F(R) Hořava-Lifshitz gravity [6].}

for the action \( \text{(38)} \) and

\[
0 = + \frac{1}{8\kappa^2} \left\{ -12 \left( -\partial^2 + \partial_k \partial^k \right) A + 8\partial_k \partial^k A + \frac{4}{3} \left( \partial_k \partial^k \right)^2 E \right\} \\
-2^{2n} a U_{0}^{2n+1} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n+1} \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\}, \tag{41}
\]

for the action \( \text{(39)} \). On the other hand, by the variation over \( E \), one gets

\[
0 = \partial_k \partial^k \left[ \frac{1}{8\kappa^2} \left\{ \frac{4}{3} \left( -\partial^2 + \partial_k \partial^k \right) \partial_k \partial^k E + \frac{4}{3} \partial_k \partial^k A + \frac{16}{9} \left( \partial_k \partial^k \right)^2 E \right\} + \frac{2^{2n-1}}{3} a U_{0}^{2n} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n} \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right], \tag{42}
\]

for the action \( \text{(38)} \) and

\[
0 = \partial_k \partial^k \left[ \frac{1}{8\kappa^2} \left\{ \frac{4}{3} \left( -\partial^2 + \partial_k \partial^k \right) \partial_k \partial^k E + \frac{4}{3} \partial_k \partial^k A + \frac{16}{9} \left( \partial_k \partial^k \right)^2 E \right\} + \frac{2^{2n}}{3} a U_{0}^{2n+1} \left( -\partial_i \partial_j - \delta_{ij} \partial_k \partial^k \right) \left\{ \left( \partial_k \partial^k \right)^{2n+1} \left( -\partial^i \partial^j A - \delta^{ij} \partial_k \partial^k A + \frac{1}{3} \partial^i \partial^j \partial_k \partial^k E + \frac{1}{3} \delta^{ij} \left( \partial_k \partial^k \right)^2 E \right) \right\} \right], \tag{43}
\]

for the action \( \text{(39)} \). By using \( \text{(31)} \), both of Eqs. \( \text{(40)} \), \( \text{(41)} \), \( \text{(42)} \), and \( \text{(43)} \) give the same expression:

\[
0 = \partial_i^2 A. \tag{44}
\]

Therefore \( A \) and \( E \) only depend on the spacial coordinate and therefore they do not propagate. Then we have shown that all the scalar modes \( \phi, \lambda, h_{\mu}, s, A, \) and \( E \) and all the vector modes \( v_i \) and \( B_i \) in Eqs. \( \text{(22)} \) and \( \text{(27)} \) do not propagate and the only propagating mode is massless graviton corresponding to the tensor mode \( C_{ij} \), which should be distinguished from Hořava quantum gravity \( \text{(4)} \) where Lorentz invariance is explicitly broken. Then the actions \( \text{(11)} \) and \( \text{(12)} \) and therefore \( \text{(38)} \) and \( \text{(39)} \) reduce to the simple forms:

\[
S_{2n+2} = \int d^4x \left[ \frac{1}{8\kappa^2} \left\{ C_{ij} \left( -\partial^2 + \partial_k \partial^k \right) C^{ij} \right\} - 2^{2n-2} a U_{0}^{2n} \left\{ \left( \partial_k \partial^k \right)^{n+1} C_{ij} \} \left\{ \left( \partial_k \partial^k \right)^{n+1} C^{ij} \right\} \right], \tag{45}
\]

\[
S_{2n+3} = \int d^4x \left[ \frac{1}{8\kappa^2} \left\{ C_{ij} \left( -\partial^2 + \partial_k \partial^k \right) C^{ij} \right\} - 2^{2n-1} a U_{0}^{2n+1} \left\{ \left( \partial_k \partial^k \right)^{n+1} C_{ij} \} \left\{ \left( \partial_k \partial^k \right)^{n+2} C^{ij} \right\} \right]. \tag{46}
\]

In the original models \( \text{(4)} \), due to the traceless and transverse conditions for \( C_{ij} \) in \( \text{(28)} \), the higher order terms do not contribute to the propagator of the graviton and therefore the UV behavior has not been really improved.

Thus, the propagator has the following form in the momentum space:

\[
\langle h_{ij}(p) h_{kl}(-p) \rangle = \langle C_{ij}(p) C_{kl}(-p) \rangle \\
= \frac{1}{2} \left\{ \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) \left( \delta_{kl} - \frac{p_k p_l}{p^2} \right) - \left( \delta_{ik} - \frac{p_i p_k}{p^2} \right) \left( \delta_{jl} - \frac{p_j p_l}{p^2} \right) - \left( \delta_{il} - \frac{p_i p_l}{p^2} \right) \left( \delta_{jk} - \frac{p_j p_k}{p^2} \right) \right\} \times \left\{ \left( p_i^2 - 2^{2n} \alpha \kappa^2 U_{0}^{2n} p_{i(n+1)} \right)^{-1}, \right. \tag{47}
\]

\[
\left. \left( p_i^2 - 2^{2n-1} \alpha \kappa^2 U_{0}^{2n+1} p_{i(2n+3)} \right)^{-1}, \right. \tag{47}
\]

Here \( p_i = \sum_{i=1}^{3} (p_i')^2 \) and \( p^2 = - (p^0)^2 + \sum p_i'^2 \). If \( \alpha > 0 \), there appears the tachyonic pole when

\[
1 = 2^{2n} \alpha \kappa^2 U_{0}^{2n} p_{i(n+2)}^{4n+2}, \quad z = 2n + 2 \text{ case}
\]

\[
1 = 2^{2n-1} \alpha \kappa^2 U_{0}^{2n+1} p_{i(n+1)}^{6n+6}, \quad z = 2n + 3 \text{ case}, \tag{48}
\]

with \( p^0 = 0 \) and therefore at least the flat vacuum becomes unstable. On the other hand, there exist a stable flat vacuum when \( \alpha < 0 \).

In the present model, there is no propagating vector or scalar mode at least on the tree level. The change of the tensor structure of the propagator in \( \text{(47)} \) means that the vector or scalar mode could appear, that is, the vector or scalar mode must correspond to a composite state, which usually does not appear at any perturbative level. Therefore, it is expected the tensor structure should not be changed by the quantum corrections.
In the ultraviolet region, where \( k \) is large, the propagator behaves as \( 1/|k|^4 \) for \( z = 2 (n = 0) \) case in [45] and therefore the ultraviolet behavior is rendered. For \( z = 3 (n = 0) \) case in [45], the propagator behaves as \( 1/|k|^6 \) and therefore the model becomes power-counting renormalizable. For \( z = 2n + 2 (n \geq 1) \) case in [45] or \( z = 2n + 3 (n \geq 1) \) case in [45], the model becomes power-counting super-renormalizable. The dispersion relation of the graviton is then given by

\[
\omega = c_0 k^z ,
\]

(49)
in the high energy region. Here \( c_0 \) is a constant, \( \omega \) is the angular frequency corresponding to the energy and \( k \) is the wave number corresponding to momentum. If \( c_0 < 0 \), the dispersion relation becomes inconsistent and therefore \( c_0 \) should be positive.

Let us discuss the generality of the expression [10], which appears in the actions [11] and [12]. We now require for this expression, when a flat background is chosen

1. The expression does not vanish. This condition is trivial but this often occurs due to some identity and the condition without torsion \( \nabla_\mu g_{\nu\mu} \).

2. The expression is given by the second rank symmetric tensor as in [10].

3. Each term contains the second derivative of perturbed metric \( h_{\mu\nu} \) like \( \partial_\nu \partial_\sigma h_{\mu\nu} \). This condition plays a role of the dimensionality condition. Since \( \partial_\nu \phi \) can be regarded as a constant vector, the third derivative of \( \phi \) like \( \nabla_\mu \nabla_\nu \nabla_\rho \phi \) contains the second derivative of the perturbed metric like \( \partial_\nu \partial_\sigma h_{\mu\nu} \). In order to obtain second rank symmetric tensor from the third rank tensor, which is now the third derivative of \( \phi \) like \( \nabla_\mu \nabla_\nu \nabla_\rho \phi \), we need one more derivative of \( \phi \) as in the second term of [111] like \( \nabla^\rho \phi \nabla_\mu \nabla_\nu \nabla_\rho \phi \), \( \nabla^\rho (\nabla_\mu \nabla_\nu \nabla_\rho \phi + \nabla_\nu \nabla_\mu \nabla_\rho \phi) \), or \( \nabla^\rho (\nabla_\mu \nabla_\nu \nabla_\rho \phi + \nabla_\nu \nabla_\rho \nabla_\mu \phi) \). This kind of terms was not considered in [5].

4. The expression does not contain \( h_{ij} \), so that the traceless and transverse conditions \( C_i^j = \partial^i C_{ij} = 0 \) for \( C_{ij} \) in [25] should be kept as in the Einstein gravity. This condition ensures the absence of the extra modes.

5. The expression should contain the graviton, which is the traceless and transverse part \( C_{ij} \) in [27] of the perturbed metric.

The combination of the terms in [10] satisfies the above conditions but the combination in [10] is not, however, unique. More generally, one can propose the combination like

\[
Q_{\alpha\beta} \equiv P_{\alpha\beta} (R_{\mu\nu} - \frac{1}{2 U_0} \partial_\mu \phi \partial_\nu \phi + \partial_\mu \phi \partial_\nu \phi + \partial_\nu \phi \partial_\mu \phi + \partial_\nu \phi \partial_\mu \phi + \partial_\mu \phi \partial_\nu \phi + \partial_\nu \phi \partial_\mu \phi + \partial_\nu \phi \partial_\mu \phi + \partial_\nu \phi \partial_\mu \phi + \partial_\nu \phi \partial_\mu \phi)
\]

(50)

Here \( c_1, c_2, c_3, c_4, c_5, \) and \( c_6 \) are constants. When we consider the fluctuation from the flat metric in [4],

\[ \nabla^\rho (\nabla_\mu \nabla_\nu \nabla_\rho \phi + \nabla_\nu \nabla_\rho \nabla_\mu \phi) \sim \nabla^\rho (\nabla_\mu \nabla_\nu \nabla_\rho \phi + \nabla_\nu \nabla_\rho \nabla_\mu \phi) \sim -h_{tt,\mu\nu} , \]

(51)

Then by using [17] and [22] with [28] and [37], we find the terms with the coefficients \( c_1, c_2, c_3, c_4, c_5, \) and \( c_6 \) vanish and therefore these terms do not affect the propagator of graviton although they may give rise some interaction terms. Having in mind the above considerations, one may speculate that proposed theory is multiplicatively-renormalizable in gauge-fixed formulation (after spontaneous breaking of Lorentz symmetry).

In the above analysis, we have chosen Eq. [5] as a gauge condition. Then Eq. [17] follows from the constraint equation [2]. As another gauge condition, instead of [5], we may choose [17]. When we write the fluctuation of \( \phi \) as

\[
\phi = \sqrt{2U_0} t + \delta \phi ,
\]

(52)

the constraint equation [2] gives

\[
\frac{\partial \delta \phi}{\partial t} = 0 .
\]

(53)

Then \( \delta \phi \) does not depend of the time coordinate:

\[
\delta \phi = \Phi(\mathbf{x}) .
\]

(54)
The gauge condition (17) has the residual gauge symmetry for the diffeomorphism with respect to the time coordinate \( t \). The residual gauge symmetry is the shift of \( t \) by a function independent of \( t \)

\[
t \rightarrow t + f(x),
\]

(55)

Therefore by using the residual gauge transformation given by

\[
f(x) = -\frac{\Phi(x)}{\sqrt{2U_0}},
\]

(56)

we obtain (55) from (52).

Thus, we presented gauge-fixed formulation of the theory which leads to only propagating higher derivative graviton. Scalar and vector degrees of freedom do not propagate.

III. SPONTANEOUS LORENTZ SYMMETRY BREAKING

Due to the constraint equation (2), vector \( \partial_\mu \phi \) has nontrivial value and therefore the Lorentz symmetry is broken. Note that the Lorentz symmetry breaking is spontaneous. Let us compare this situation with that in the usual field theory like the Higgs model. The usual \( U(1) \) Higgs model, whose potential is given by

\[
V_{\text{Higgs}} = -\frac{m^2}{2} \phi^* \phi + \frac{\lambda_0^2}{4} (\phi^* \phi)^2,
\]

(57)

has a global \( U(1) \) symmetry, which is the invariance under the transformation

\[
\phi \rightarrow e^{i\theta_0} \phi,
\]

(58)

with a constant real parameter \( \theta_0 \). In (57), \( \phi \) is a complex scalar field and \( m \) and \( \lambda_0 \) are positive parameters. The minimum of the potential is given by

\[
\phi = \frac{e^{i\varphi} m}{\lambda_0}.
\]

(59)

Here \( \varphi \) is a constant phase. The value of \( \varphi \) can be arbitrary. If one chooses specific value of \( \varphi \), the value of \( \varphi \) is changed under the \( U(1) \) transformation (58) as \( \varphi \rightarrow \varphi + \theta \), and therefore the ground state is not invariant under the \( U(1) \) transformation (58) and the \( U(1) \) symmetry breaks spontaneously. One can always choose the real axis of the complex \( \phi \)-plane to be parallel with the value of \( \phi \) in the ground state so that \( \varphi = 0 \).

In our model, the constraint equation (2) shows that the value of the vector \( (\partial_\mu \phi) \) is located on the hyperboloid defined by

\[
- x^\mu x_\mu \equiv t^2 - x^2 = 2U_0.
\]

(60)

The value of the vector \( (\partial_\mu \phi) \) changes on the hyperboloid under the Lorentz transformation. If we choose a value of \( (\partial_\mu \phi) \), the Lorentz symmetry is broken spontaneously. After that one can always choose the time axis to be parallel to the vector \( (\partial_\mu \phi) \).

We should also note that the actions (11) and (12) have a shift symmetry

\[
\phi \rightarrow \phi + \phi_0.
\]

(61)

Here \( \phi_0 \) is a constant. In the flat vacuum background, the actions (11) and (12) are invariant under the time translation:

\[
t \rightarrow t + t_0.
\]

(62)

Here \( t_0 \) is a constant. Since the solution (55) of the constraint equation (3) depends on the time coordinate \( t \), the solution spontaneously breaks the symmetry under the time translation (62). The solution (55) also breaks the shift symmetry in (61). Note that the diagonal symmetry of the time translation (62) and the shift symmetry (61) is not broken. In fact, if \( \phi_0 \) in (61) is chosen as

\[
\phi_0 = -\sqrt{2U_0} t_0,
\]

(63)

under the simultaneous transformation, the solution (55) is invariant. The diagonal symmetry effectively plays the role of the time translation and the flat vacuum solution is effectively invariant under the time translation.
Furthermore, by introducing a new scalar field $\phi$ using the vector field $A_A$, the combination

$$0 = F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu,$$

(64)

the vector field $A_\mu$ becomes pure gauge field and can be rewritten as

$$A_\mu = \partial_\mu \phi.$$

(65)

Furthermore, by introducing a new scalar field $\varphi$ and considering the combination

$$A^\mu_\mu \equiv A_\mu + \partial_\mu \varphi,$$

(66)

the combination $A^\mu_\mu$ is invariant under the gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \epsilon(x^\mu), \quad \varphi \to \varphi - \epsilon.$$

(67)

Then we rewrite the actions (11) and (12), which are invariant under the gauge transformation (67) as

$$S^A_{2n+2} = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \alpha \left\{ (A^\mu_\nu A^{\nu\mu} \nabla_\nu \nabla_\mu - 2U_0 \nabla^\rho \nabla_\rho)^n P^A_\alpha P^A_\beta \left( R_{\mu\nu} - \frac{1}{2U_0} A^\rho_\mu \nabla_\rho \nabla_\nu A^\varphi_\nu \right) \right\} \right.$$

$$\times \left\{ \left( A^\mu_\nu A^{\nu\mu} \nabla_\nu \nabla_\mu - 2U_0 \nabla^\rho \nabla_\rho \right)^n P^A_\alpha P^A_\beta \left( R_{\mu\nu} - \frac{1}{2U_0} A^\rho_\mu \nabla_\rho \nabla_\nu A^\varphi_\nu \right) \right\}$$

$$\left. - \lambda \left( \frac{1}{2} A^\mu_\alpha A^{\alpha\mu} + U_0 \right) - B^{\mu\nu} F_{\mu\nu} \right],$$

(68)

for $z = 2n + 2$ model ($n = 0, 1, 2, \cdots$), and

$$S^A_{2n+3} = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \alpha \left\{ (A^\mu_\nu A^{\nu\mu} \nabla_\nu \nabla_\mu - 2U_0 \nabla^\rho \nabla_\rho)^n P^A_\alpha P^A_\beta \left( R_{\mu\nu} - \frac{1}{2U_0} A^\rho_\mu \nabla_\rho \nabla_\nu A^\varphi_\nu \right) \right\} \right.$$

$$\times \left\{ \left( A^\mu_\nu A^{\nu\mu} \nabla_\nu \nabla_\mu - 2U_0 \nabla^\rho \nabla_\rho \right)^{n+1} P^A_\alpha P^A_\beta \left( R_{\mu\nu} - \frac{1}{2U_0} A^\rho_\mu \nabla_\rho \nabla_\nu A^\varphi_\nu \right) \right\}$$

$$\left. - \lambda \left( \frac{1}{2} A^\mu_\alpha A^{\alpha\mu} + U_0 \right) - B^{\mu\nu} F_{\mu\nu} \right].$$

(69)

for $z = 2n + 3$ model ($n = 0, 1, 2, \cdots$). Here $B^{\mu\nu}$ is the anti-symmetric tensor field whose variation gives a constraint (64). In the actions (68) and (69), $P^A_\mu$ is defined by

$$P^A_\mu \equiv \delta^\mu_\nu + \frac{A^\rho_\mu A^{\rho\nu}}{2U_0},$$

(70)

instead of (63).

The actions (68) and (69) are classically equivalent to the actions (11) and (12). To be sure, let us check the propagating modes on the flat background. For this purpose, we decompose $B_{\mu\nu}$ and $A_\mu$ as follows,

$$B_{ij} = \partial_i f + e_i, \quad B_{ij} = \partial_i m_j - \partial_j m_i + \epsilon_{ijk} \partial^k k, \quad A_i = \partial_i l_i,$$

$$\partial^i e_i = \partial^i m_i = \partial^i l_i.$$

(71)

Then one gets

$$B^{\mu\nu} F_{\mu\nu} = 2f \partial_k \partial^k (-\partial_i \phi + A_i) - 2m^2 \partial_k \partial^k l_j + 2e^2 \partial_k l_i + \text{total derivative terms}.$$

(72)

Note that in the above expression and therefore even in the total action, there does not appear $e_i$ and $k$. The variation over $f$ gives

$$A_i = \partial_i k.$$

(73)
and the variation over \( m_i \) gives
\[
l_i = 0. 
\]

The variation over \( e_i \) gives \( \partial_i l_i = 0 \), which is consistent with (74) and does not lead to new constraint. On the other hand, the equation given by the variation over \( A_0 \) can be solved with respect to \( f \) and therefore \( f \) becomes auxiliary field. Similarly, the equation given by the variation over \( \mu \) can be solved with respect to \( m_i \), which also becomes auxiliary field. Then from (71) and (73), we find \( A_\mu \) is pure gauge field \( A_\mu = \partial_\mu \phi \). Hence, in the actions (68) and (69), the scalar mode \( \phi \) and the scalar field \( \varphi \) appear only in the combination of \( \partial_\mu (\phi + \varphi) \). Therefore the variations over \( \phi \) and \( \varphi \) give the identical equations corresponding to (20) and (21). Hence, using the same arguments as in Section III we obtain the same graviton propagator (47).

V. FRW COSMOLOGY

In this section, we discuss simple accelerating FRW cosmology for higher-derivative gravity under discussion. We start with a little bit different but general action:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2k^2} - \sum_{n=0}^{n_{\text{max}}} \alpha_n \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_{\mu} \nabla_{\nu} - \partial_{\mu} \phi \partial^\nu \phi \nabla_{\nu} \nabla_{\nu} \phi \right\} + P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \right] \\
\times \sum_{m=0}^{n_{\text{max}}} \alpha_m \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_{\mu} \nabla_{\nu} - \partial_{\mu} \phi \partial^\nu \phi \nabla_{\nu} \nabla_{\nu} \phi \right\} P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \\
- \sum_{m=0}^{n_{\text{max}}} \alpha_m \left\{ (\partial^\mu \phi \partial^\nu \phi \nabla_{\mu} \nabla_{\nu} - \partial_{\mu} \phi \partial^\nu \phi \nabla_{\nu} \nabla_{\nu} \phi \right\} P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \\
- \lambda \left( \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + U_0 \right) \right].
\]

In the low energy, one only needs to consider the contribution from the first Einstein-Hilbert term. Since the \( n = 0 \) term contributes in the next-to-leading order, we now consider the simplified model given by the Einstein-Hilbert term and the \( n = 0 \) term:

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2k^2} - \alpha_0 P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \right] P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \\
- \lambda \left( \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + U_0 \right) \right].
\]

It is interesting that even in the FRW background, we can choose the local Lorentz frame and obtain the same graviton propagator (47) at short distances, where space-time can be regarded to be flat.

In order to consider the cosmology, we assume the FRW-like metric:
\[
ds^2 = -e^{2b(t)} \, dt^2 + a(t)^2 \sum_{i=1,2,3} (dx^i)^2.
\]

Here \( a(t) \) is called a scale factor. The variation of the action with respect to \( b(t) \) gives the equation corresponding to the first FRW equation and the variation with respect to \( a(t) \) gives the equation corresponding to the second FRW equation. Since we have
\[
\Gamma^t_{tt} = \dot{b}, \quad \Gamma^t_{ij} = e^{-2b} a^2 H \delta_{ij}, \quad \Gamma^i_{tj} = \Gamma^i_{jt} = H \delta^i_j, \quad \Gamma^i_{tt} = \Gamma^i_{ti} = \Gamma^i_{tt} = \Gamma^i_{jk} = 0,
\]
and
\[
R_{tt} = -3 \left( \dot{H} + H^2 - bH \right), \quad R_{ij} = \left( \dot{H} + 3H^2 - bH \right) a^2 e^{-2b} \delta_{ij}, \quad R_{tt} = R_{tt} = 0,
\]
\[
R = 6 \left( \dot{H} + 2H^2 - bH \right) e^{-2b},
\]

where

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{2k^2} - \alpha_0 P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \right] P_{\alpha\beta} \left( \partial_{\alpha} \phi \nabla_{\beta} \nabla_{\nu} \phi \right) \\
- \lambda \left( \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + U_0 \right) \right].
\]
the action (76) has the following form:

$$S = \int d^4 x a^3 e^b \left\{ \frac{3}{\kappa^2} \left( \dot{H} + 2H^2 - b\dot{H} \right) e^{-2b} - 27\alpha_0 H^4 e^{-4b} \right\}. \tag{80}$$

We should note $\partial_t \phi = e^b \sqrt{2U_0}$ and $\partial_i \phi = 0$. It is remarkable that this action is very similar to the one of well-known $R^2$ gravity. By the variation of the action with respect to $b$, one obtains an equation corresponding to the first FRW equation:

$$\frac{3}{\kappa^2} H^2 + 81\alpha_0 H^4 = \rho_{\text{matter}}. \tag{81}$$

Here $\rho_{\text{matter}}$ is the energy-density of the matter which was not explicitly written before. One puts $b = 0$ after the variation over $b$. By the variation over $a$, one also obtains the equation corresponding to the second FRW equation, which can be evaluated by using the first FRW equation and matter conservation law. We should note that the equation (81) is the first order differential equation with respect to the scale factor $a(t)$, which should be distinguished from the usual higher derivative gravity like $F(R)$ gravity, where the equation corresponding to the first FRW equation is the third order differential equation with respect to $a(t)$. Note that $H = 0$, which expresses the flat solution, is a trivial solution of (81) when there is no matter. We also note that there is a de Sitter solution, where $H$ is a constant, given by

$$H^2 = -\frac{1}{27\alpha_0 \kappa^2}, \tag{82}$$

which may express the inflation in the early universe. The existence of the de Sitter solution in (82) requires $\alpha_0 < 0$, which does not always conflict with the condition to avoid the tachyon in (48). Now we only kept the term with $n = 0$ in the action (75), just for simplicity. If we include the higher term with $n > 0$, the de Sitter solution might become unstable and there can occur the instable inflationary solution.

### VI. DISCUSSION

In summary, we formulated covariant higher derivative gravity with Lagrange multiplier constraint and scalar projectors. It is demonstrated that such theory admits flat space solution. Its gauge-fixing formulation is fully developed. The study of spectrum shows that the only propagating mode is (higher derivative) graviton, while scalar and vector modes do not propagate. Eventually, scalar and vector modes correspond to composite states at any perturbative level.

Furthermore, we show that Lorentz symmetry breaking in the theory under discussion is spontaneous. The equivalent formulation in terms of vector fields is developed. The preliminary study of FRW cosmology indicates to the possibility of inflationary universe solution. It is interesting that first FRW equation in the theory turns out to be the first order differential equation which is quite unusual for higher derivative gravity which normally leads to third order differential equation with respect to scale factor. This may indicate to presence of some hidden symmetry in the higher derivative gravity under consideration.

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