A New Numerical Method of Estimates of Temperatures along a Thick Steel Slab and Concentrations of Alcohol along a Hollow Tube

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Abstract

A new continuous numerical method based on the approximation of polynomials is here proposed for solving the equation arising from heat transfer along a thick steel slab and a hollow tube subject to initial and boundary conditions. The method results from discretization of the heat equation which leads to the production of a system of algebraic equations. By solving the system of algebraic equations we obtain the problem approximate solutions.

Keywords: Polynomials; Interpolation; Multistep collocation; Heat conduction

Introduction

The development of numerical techniques for solving heat conduction equation in science and engineering subject to initial and boundary conditions is a subject of considerable interest. In this paper, we develop a new continuous numerical method which is based on interpolation and collocation at some point along the coordinates (Odekunle, 2008). To do this we set U(x,t) represents the temperature at any point in the slab and the tube. Heat is flowing from one end to another under the influence of the temperature gradient ∂U/∂x. To set up the solution method we select an integer \( N > 0 \) such that \( N \) is the number of interpolation points along the space axis and \( k > 0 \) the number of interpolation points along the time axis.

The new method strives to provide solutions to the heat flow eqns. arising from heat transfer along a thick steel slab and a hollow tube subject to initial and boundary conditions. The method results from discretization of the heat equation which leads to the production of a system of algebraic equations. By solving the system of algebraic equations we obtain the problem approximate solutions.

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Where \( z = i + \frac{\beta - 1}{\rho} \) and \( W^{-1} \) exists (Odekunle, 2008). Hence, by equation (2.2) we obtain
\[
a = \bar{a}E, \quad \bar{a} = W^{-1}
\]
(2.3)
The vector \( \bar{a} = (a_0, \ldots, a_{p-1})^\top \) is now determined in terms of known parameters in \( \bar{a}E \). If \( \bar{a}_{r,i} \) is the \((r, i)^{th}\) row of \( \bar{a} \) then
\[
a_r = \bar{a}_{r,1}E
\]
(2.4)
Eqn. (2.4) determines the values [8-17]. Let us take first and second derivatives of eqn. (2.0) with respect to \( x \),
\[
\tilde{U}'(x,t) = \sum_{r=0}^{n-1} a_r \left[ Q'_r(x,t) \right]
\]
(2.5)
Substituting eqn. (2.4) into eqn. (2.5), we obtain
\[
\tilde{U}'(x,t) = \sum_{r=0}^{n-1} a_r \left[ Q''_r(x,t) \right]
\]
(2.6)
We reverse the roles of \( x \) and \( t \) in eqn. (2.1) and we arbitrarily set \( k = 0 \left[ 1 - \frac{b}{a} \right] \) and \( h = 0 \), by Crammer’s rule eqn. (2.1) becomes
\[
Y a = E, \quad E = \left[ \begin{array}{cccc}
\tilde{U}_0(x_j), & \tilde{U}_1(x_j), & \ldots, & \tilde{U}_{n-1}(x_j)
\end{array} \right]^\top
\]
(2.7)
and [18-20]
\[
\begin{bmatrix}
Q_0(x_i, t_j),& Q_1(x_i, t_j),& \ldots, & Q_{p-1}(x_i, t_j) \\
\vdots, & \vdots, & \ddots, & \vdots \\
Q_0(x_i, t_j),& Q_1(x_i, t_j),& \ldots, & Q_{p-1}(x_i, t_j)
\end{bmatrix}
\]
(2.8)
Where \( \eta = j + \frac{1}{4} \), \( \gamma = j + b \left( \frac{a_0 - 1}{a} \right) \), and \( Y^{-1} \) exists (Odekunle, 2008).
\[
a = LE, \quad L = Y^{-1}
\]
(2.9)
The vector \( a = (a_0, \ldots, a_{p-1})^\top \) is now determined in terms of known parameters in \( LE \). If \( L_{r,i} \) is the \((r, i)^{th}\) row of \( L \) then
\[
a_r = L_{r,1}E
\]
(2.10)
Also, eqn. (2.9) determines the values of \( a_r \). Taking the first derivatives of eqn. (2.0) with respect to \( t \), we obtain
\[
\tilde{U}'(x,t) = \sum_{r=0}^{n-1} a_r \left[ Q'_r(x,t) \right]
\]
(2.11)
But by eqn. (1.0) or (1.1) it is obvious that eqn. (2.11) is equal to eqn. (2.6), therefore,
\[
\sum_{r=0}^{n-1} L_{r,1} E \left[ Q'_r(x,t) \right] - \sum_{r=0}^{n-1} L_{r,1} E \left[ Q''_r(x,t) \right] = 0
\]
(2.12)
Collocating eqn. (2.12) at \( x=x_i \) and \( t=t_j \) we obtain a new numerical scheme that solves eqns. (1.0) and (1.1) explicitly.

**Numerical Examples**

In this section we give some numerical examples to compute approximate solutions for equations (1.0) and (1.1) by the method discussed in this paper [5]. This is in order to test the numerical accuracy of the new method. To achieve this, we truncate the Taylor’s polynomial after second degree and use it as the basis function in the computation. The resultant scheme is used to solve the following two problems.

**Example 1 (Eyaya, 2010)**

Given a 2 cm thick steel slab, solve for the temperatures as a function of \( x \) and \( t \) at \( t=2.062 \) seconds if the initial temperatures are given by the relation [24-26]. \( U(x,0) = 100 \sin \left( \frac{\pi x}{4} \right) \) where \( k \) for steel is 0.13 cal/sec °cm, \( c=0.11 \) cal/g°C and \( p=7.8 \) g/cm³.

**Solution**

By simplification eqn. (1.0) becomes \( \frac{\partial^2 U}{\partial x^2} = cp \frac{\partial U}{\partial t} \). To solve this equation we take \( \Delta x=0.25 \) cm, then we find \( \Delta t \) by the relation \( k \Delta t = \frac{\pi}{4} \), \( \Delta t = 0.825 \) sec. We let \( \beta=4, a=64 \) arbitrarily which implies that \( \gamma = \frac{1}{4} \). Taking two interpolation points along space coordinates and one along time implies that \( g=2, b=1, p=3 \) and for \( i = 1, 4, 7, \ldots, and j = 1, 2, \ldots \), we obtain \( h = \frac{1}{4}, 0, -\frac{1}{4} \), then the calculated temperatures are tabulated as shown in Table 1 [27].

**Example 2 (Eyaya, 2010)**

A hollow tube 25 cm long is initially filled with air containing 2% of ethyl alcohol vapors. At the bottom of the tube is a pool of alcohol which evaporates in to the stagnant gas above [6]. (Heat transfers to the alcohol from the surroundings to maintain a constant temperature of, 30°C at which temperature the vapor pressure is 0.1 atm.). At the upper end of the tube, the alcohol vapors dissipate to the outside air, so the concentration is essentially zero. Considering only the effects of molecular diffusion, determine the concentration of alcohol as a function of time and distance measured from the top of the tube.

**Solution**
Table 2: Concentrations of alcohol.

| T    | x=0 | x=5 | x=10 | x=15 | x=20 | x=25 |
|------|-----|-----|------|------|------|------|
| 0    | 0   | 3.00| 3.00 | 3.00 | 3.00 | 15   |
| 21.11| 0   | 2.63| 3.00 | 3.00 | 4.50 | 15   |
| 42.22| 0   | 2.35| 2.95 | 3.19 | 5.63 | 15   |
| 63.33| 0   | 2.13| 2.91 | 3.47 | 6.50 | 15   |
| 84.44| 0   | 1.96| 2.88 | 3.78 | 7.18 | 15   |
| 105.55| 0 | 1.83 | 2.88 | 4.09 | 7.73 | 15 |
| 126.66| 0 | 1.73 | 2.90 | 4.39 | 8.19 | 15 |
| 147.77| 0 | 1.66 | 2.94 | 4.68 | 8.56 | 15 |
| 168.88| 0 | 1.61 | 2.99 | 4.95 | 8.88 | 15 |

Molecular diffusion follows the law \( \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \) where \( D \) is the diffusion coefficient, with units in \( \text{cm}^2/\text{sec} \). For ethyl alcohol \( D=0.111 \text{ cm}^2/\text{sec} \) at \( 30^\circ \text{C} \), and the vapor pressure is such that 10 volume percent alcohol in air is present at the surface [7].

If we take \( \beta=4 \), \( \alpha=3 \) arbitrarily then \( \nu = i - \frac{1}{4} \), \( z = i + \frac{1}{3} \) and \( \eta = \gamma = i + \frac{1}{3} \). Taking two interpolation points along space coordinates and one along time implies that \( g=2 \), \( b=1 \), implies that \( p=3 \) and for \( i = \frac{1}{4} \), \( j = \frac{3}{8} \), we obtain \( h = -\frac{1}{4} \), \( k = 0 \), then the calculated concentrations of alcohol are tabulated as shown in Table 2.

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