Jet-Supercavity Interaction: Insights from Physics Analysis

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Abstract. Various closure conditions of a ventilated cavity enveloping all or part of a high-speed underwater body are introduced, including those involving a propulsion jet. The flow regimes for the latter are described based on Efros-Paryshev theory, which is extended to estimate the efficiency and fundamental limitations of a rocket-type propulsor.

Introduction

The closure regions of nominally axisymmetric cavities have been the subject of extensive research. Emphasis has been placed on the closure of a supercavity. Since such a cavity closes downstream of the body that it envelopes, it is not subject to interference from a solid surface. For practical operation of self-propelled vehicles other closure conditions are of interest. Several closure conditions are shown in figure 1 as images acquired in experiments more fully described in Moeny, et al., (2015).

![Figure 1](image1.png)

Figure 1. Some regimes of cavity closure: a – Re-entrant jet off body; b – Twin-vortex off body; c – Collapsed closure on body; d – Closure on propulsion jet. Approximate location of cavity closure (either in or downstream of the field of view) is indicated.

![Figure 2](image2.png)

Figure 2. Efros and Paryshev closure models: a – Efros re-entrant jet model of supercavity closure; b – Extension of Efros model to closure on a “hard” jet; c – Paryshev’s model for closure on a “soft” jet; d and e – Recent water tunnel results corresponding to b and c, respectively. Images a-c from Paryshev (2006).

Other regimes of interest not shown in figure 1 include closure on a lifting or planing afterbody, which generates a vortex pair. Cavitator lift also generates trailing vortex structures. Also of interest are dynamical effects of cavity auto-oscillation. This article primarily addresses closure of nominally axisymmetric, steady cavities on a propulsion plume. That can have a strong effect on entrainment of ventilation gas from the cavity, and therefore direct consequences for design and operations.
Methods

As described in Paryshev (2006), classical re-entrant jet cavity closure theory (Efros, 1946) has been extended to cavity closure on a solid afterbody, and then later to cavity closure on a high-pressure gas jet. In the latter, the model is applied to flow systems for which the stagnation pressure of the jet is greater than that of the freestream flow of ambient liquid, a case referred to herein as a "hard" jet, since it is dynamically equivalent to closure on a solid body. In that case, high jet stagnation pressure forces ambient liquid to form re-entrant flow upstream along the jet into the cavity. A complimentary case is that of a "soft" jet, with stagnation pressure less than the ambient liquid flow. In that case, the central jet stagnates on the ambient liquid, forcing re-entrant flow of gas upstream along the cavity boundary. The original Efros model, its application to closure on a "hard" jet, and the case of cavity closure on a "soft" jet are depicted in figure 2, along with some high-speed video frames from recent experiments performed to explore these effects. The latter show subtle but important changes in the cavity-jet system. For the "hard" jet (figures 2b and c), the cavity remains convex well downstream of the field of view, and does so until the liquid just outside the cavity boundary stagnates on the central jet. For the "soft" jet, inflection occurs near cavity closure, due to rapid transition of cavity pressure from a nearly constant value, through a high pressure region near jet stagnation on the cavity boundary, followed by a rapid pressure drop to ambient conditions once the jet exits the cavity. In the short region of super-ambient pressure, the cavity boundary is concave. Downstream geometry is dominated by turbulent jet spreading after penetration of the liquid layer. Thus, the topology of closure on a jet is governed by the ratio of stagnation pressures of the ambient liquid and the jet: 

\[ \frac{P}{\rho V^2} = \frac{\rho_j V_j}{\rho_l V_l} \]

where \( \rho_l \) and \( \rho_j \) are the densities of the liquid and the jet, respectively, and where \( V_l \) and \( V_j \) are the associated velocities.

A model for supercavity closure on a "soft" jet was offered by Paryshev (2006). The simplifying assumptions may be understood via figure 3a, and may be summarized as follows: (1) the flow system is slender; (2) the cavity sections obey the principle of independence of expansion (Logvinovich, 1969); such that, (3) standard approximations for cavity shape may be applied (for example, Logvinovich & Serebryakov, 1975); (4) the jet stagnates on the cavity boundary, such that all or part of the gas in the jet re-enters the cavity along the inner surface of the cavity boundary; and, (5) the region of high pressure associated with jet stagnation is confined to a short axial segment of the system as described above. Paryshev applies the model to deduce the condition required for part of the jet to penetrate the cavity boundary in terms of the momentum flux of the jet, \( \rho_j V_j^2 S_j \) (where \( S_j \) is the nominal area of the propulsion jet), made dimensionless with respect to the cavitator drag, \( D_c \), resulting in the second dimensionless parameter of importance: \( \tau = \frac{\rho_j V_j^2 S_j}{D_c} \). Paryshev’s model results in a condition for penetration that \( \tau > 1/2 \): for lower values, the “soft” jet is “weak,” none of the gas penetrates the cavity, and it only supplements the ventilation gas; for values increasing above the cut-off, the “soft” jet is “strong,” and an increasing fraction penetrates the cavity.

The three regimes demarcated by Efros-Paryshev theory are depicted in figure 3b. Neither model accounts for turbulent mixing and entrainment as depicted in figure 3c, which shows a budget for the volumetric flow of gas through the system in the Paryshev regime. In that figure, \( q_v \) represents the basic flow of ventilation gas supplied to the cavity independent of the propulsion gas, whereas \( q_p \) represents the flow of gas through the rocket nozzle. The Efros-Paryshev model captures the re-entrant flow of propulsion gas into the cavity, \( q_i \), and any direct penetration of propulsion gas through the cavity boundary, \( q_j \); but it does not account for entrainment due to turbulent mixing or other diffusive processes of the ventilation gas by the jet, \( q_v \), nor entrainment by the ambient liquid, \( q_l \). Paryshev’s model estimates the supplemental ventilation of the cavity by the propulsion gas in the “soft jet” regime of closure. This is plotted as a function of both the stagnation pressure ratio and the dimensionless jet momentum flux in figure 4, and is complementary to the flow of gas exiting the cavity: the two flow rates sum to the total volumetric flow rate from the propulsion nozzle.
Figure 3. Paryshev’s closure model and the resulting regimes of flow: a – Model simplifications (from Paryshev, 2006); b – Flow regimes for cavity closure on a propulsion jet; c – Gas flow budget. The regime of efficient propulsion is discussed in a later section.

When applied in the context of a rocket free body diagram, Paryshev’s model can be extended to estimate the thrust and propulsive efficiency of the rocket as a function of the same quantities. As is well known, the efficiency of a rocket-propulsion system is maximized when the jet exits the nozzle downstream at vehicle speed (in the vehicle reference frame), such that the gas is simply left behind the vehicle, and thrust is derived solely from the relative momentum flux of the ejecta (Sutton & Biblarz, 2001). A gas-based underwater rocket must overcome drag associated with the much heavier ambient liquid. Typically, such a system must eject mass much faster than the optimal rate, and efficiency is reduced.

The following expressions for dimensionless thrust and propulsion efficiency have been derived and cast in the context of parameters characterizing supercavity closure on a jet:

\[ T = \frac{T}{P_c} \left[ 1 + \frac{\Delta \sigma}{\sigma} \right] \] and \[ \eta = \frac{2 \sqrt{\rho_c}}{1 + \frac{\Delta \sigma}{\sigma} + P_c} \]  \hspace{1cm} (1)

where \( T \) is the thrust, \( \rho_c = \rho / \rho \) is the dimensionless jet density, and \( \Delta \sigma = \sigma - \sigma_c \) is the difference between the cavitation number, \( \sigma = (p_c - p) / (1/2 \rho \nu^2) \), and the jet over-expansion index, \( \sigma_c = (p_c - p) / (1/2 \rho \nu^2) \). Here \( p_c \) is the depth pressure well upstream of the cavitator, \( p \) is the ventilated cavity pressure, and \( \rho \) is the pressure in the jet. Assuming the jet exits a round nozzle, its diameter can be estimated for specified operating conditions and dimensionless thrust from the first of equations (1) and the definition of \( T \).

Assuming jet closure downstream of the nozzle, it must also fit within the cavity at the exit plane. Thus the jet diameter must be less than the maximum cavity diameter (or even smaller, if the nozzle is located closer to closure). This geometric constraint may be estimated using well-known approximations of cavity geometry. Also, the kinetic energy of the jet must be positive semi-definite, and it can be shown that this requires that \( P \leq 1/\sigma \). Thus there are two bounds on the operation of a rocket nozzle enveloped in a supercavity that limit the maximum value of stagnation pressure ratio that can be achieved.

**Results**

Example results for the required jet diameter are plotted as a function of the stagnation pressure ratio in figure 5 up to the upper bounds introduced above. Thrust and propulsion efficiency are plotted in...

\[ \eta = \frac{2 \sqrt{\rho_c}}{1 + \frac{\Delta \sigma}{\sigma} + P_c} \]
figure 6 as functions of dimensionless jet momentum flux and stagnation pressure ratio. In both figures, the conditions are $\sigma = O(0.05)$, $\bar{p} = O(0.001)$, and $\sigma / \sigma = 1.5$.

**Figure 4.** Dimensionless ventilating volumetric flow rate from jet for $\sigma = O(0.05)$

**Figure 5.** Limitations on rocket propulsion operation for conditions stated in text.

**Discussion**

For the conditions of figure 6, propulsive efficiency asymptotes to a maximum value for stagnation pressure ratios greater than approximately five, and becomes nearly independent of the stagnation pressure ratio. As can be seen from the second of equations (1), efficiency is also independent of the jet momentum flux and the dimensionless thrust. The dimensionless thrust is a function of both the stagnation pressure ratio and the dimensionless jet momentum flux. A well-designed supercavitating high-speed body with minimal parasitic drag due to appendages and other effects will operate with a dimensionless thrust slightly exceeding unity.

**Summary and Conclusions**

Efros-Paryshev theory of supercavity closure on a propulsion jet has been extended to explore propulsion performance of rocket-propelled supercavitating bodies. Propulsion efficiency is limited by the low density ratio of the jet relative to the ambient liquid. Efficiency can be maximized by operating in the Paryshev “soft strong” jet regime. Although it is desirable to maximize the stagnation pressure ratio, upper bounds on this quantity have been identified.

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