A white-light trap for Bose-Einstein condensates

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Abstract.
We propose a novel method for trapping Bose-condensed atoms using a white-light interference fringe. Confinement frequencies of tens of kHz can be achieved in conjunction with trap depths of only a few µK. We estimate that lifetimes on the order of 10 s can be achieved for small numbers of atoms. The tight confinement and shallow depth permit tunneling processes to be used for studying interaction effects and for applications in quantum information.

1. Introduction
The growth of interest in quantum computation has spurred investigations of few-particle interaction effects in a variety of systems. Bose-condensed atomic gases are no exception, and are in fact a natural candidate for quantum information studies [1], since they are good sources of particles in a completely well-defined quantum state. However, because interactions in a condensate are weak, they typically can only be observed through mean-field effects with large numbers of atoms. In contrast, few-particle phenomena have been observed in several experiments using condensates confined in optical lattices [2, 3]. These experiments rely on confining atoms relatively tightly within a lattice site to maximize interaction strength, and also maintain a relatively shallow lattice depth so that tunneling processes can provide sensitive dependence on the weak interaction energy. These two requirements, tight confinement and shallow depth, suggest that an important parameter is the spatial size of the trap. Optical lattices work for observing effects like the Mott-insulator transition [4] because the size of their lattice sites, typically about 500 nm, is sufficiently small.

Although atoms trapped in an optical lattice are interesting, they are not ideally suited for quantum computing and related studies because a large number of sites are typically occupied, with spatial inhomogeneities making each site different and the close spacing making individual addressing difficult. We propose here a novel trapping method that solves these problems by providing a confining potential similar to that of one single site in an optical lattice. This should allow detailed study of interaction effects, perhaps controllable at the single-particle level. The trapping mechanism is similar to that of optical lattices, but instead of using a laser beam to generate a standing wave, we propose to use a broadband source to generate a white-light interference fringe.

A similar confining potential can be more conventionally obtained in a dipole-force trap generated by a single tightly focused laser beam. At CNRS [5], a trap of this type has been demonstrated with a beam waist of about 1 µm, comparable to
Figure 1. Layout for white-light trap. Light from a lamp is collected by a condenser lens, filtered appropriately, and divided by a beamsplitter. The resulting beams are directed counter-propagating and focused to achieve high intensity. The angle $\theta$ parametrizes the focusing. Interference occurs in the plane where the distance to the source along both paths is equal, indicated by the dashed line. The location of this plane can be adjusted using translating mirrors, as shown. The two small sets of axes represent the coordinate systems used to calculate the interference term in equation (7).

The principle of white-light interference is well known [6]. In an arrangement such as in figure 1, there exists a plane $z = 0$ in which the optical path lengths from the source are identical for both possible paths. So, regardless of the frequency of the light $\omega$, an interference maximum or minimum will occur in this plane, with the sign of the interference depending on the number of mirror reflections occurring in the two arms. However, if the source emits a range of frequencies $\Delta \omega$, then at distances $z \gg c/\Delta \omega$ the interference maxima and minima for different frequencies will be uncorrelated and the total intensity will be nearly constant. If $\Delta \omega$ is sufficiently large, the interference will extend over only one oscillation and only a single trapping site results.

White light has not previously been used for atom trapping because the available light intensity permits only relatively weak traps. However, a condensate provides a source of atoms in the absolute ground state of their potential, so the white-light trap can work as long as sufficient intensity can be generated to provide at least one

the confinement size proposed here. However, obtaining such a tight beam requires complex focusing optics with high numerical aperture, which is expensive in terms of both money and optical access. In contrast, the white-light trap can use optics of modest numerical aperture with substantial aberrations. Also, the dipole trap is relatively weak in the direction along the beam axis, while the white-light trap can give tight confinement in one, two, or three dimensions as desired. However, the dipole force approach can provide much stronger trapping potentials, can more readily provide and manipulate multiple traps, and relies on more familiar laser physics. Thus both approaches have unique features which could prove useful.

2. White-Light Trap

The principle of white-light interference is well known [6]. In an arrangement such as in figure 1, there exists a plane $z = 0$ in which the optical path lengths from the source are identical for both possible paths. So, regardless of the frequency of the light $\omega$, an interference maximum or minimum will occur in this plane, with the sign of the interference depending on the number of mirror reflections occurring in the two arms. However, if the source emits a range of frequencies $\Delta \omega$, then at distances $z \gg c/\Delta \omega$ the interference maxima and minima for different frequencies will be uncorrelated and the total intensity will be nearly constant. If $\Delta \omega$ is sufficiently large, the interference will extend over only one oscillation and only a single trapping site results.

White light has not previously been used for atom trapping because the available light intensity permits only relatively weak traps. However, a condensate provides a source of atoms in the absolute ground state of their potential, so the white-light trap can work as long as sufficient intensity can be generated to provide at least one
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bound state. The brightest commercially available broadband source is a mercury arc lamp, which emits light ranging from the ultraviolet to about 2.5 µm wavelength [7]. This spectrum can be filtered to remove light with frequencies higher than an atom’s principle transition, so that atoms will be attracted to a region of high intensity. For instance, Rb atoms have principle transitions to the 5P_{1/2} state at 794 nm and to the 5P_{3/2} state at 780 nm, so light with wavelengths from about 800 nm to 2.5 µm could be used. The confining potential $V$ depends on the ac polarizabilty of the atoms, $\alpha$, through [8]

$$V(r) = -\frac{2\pi}{c} \int \alpha(\omega) S(r, \omega) d\omega$$

where $S(r, \omega)$ is the spectral density of the light at position $r$. At sufficiently low frequencies, the polarizability is well approximated by the static polarizability $\alpha_s$,

$$\alpha_s = 4.7 \times 10^{-29} \text{ m}^3$$

for Rb [9]. The dominant contribution to $\alpha_s$ is from

$$\alpha_r(\omega) = \frac{1}{2} \frac{\Gamma c^3}{\omega_{1/2}^2} \left( \frac{\omega_{1/2}^2 - \omega^2}{\omega_{1/2}^2 - \omega^2} + \frac{2\omega_{3/2}^2}{\omega_{3/2}^2 - \omega^2} \right)$$

where $\Gamma = 2\pi \times 5.9 \text{ MHz}$ is the transition linewidth of the P_{1/2} transition and $\omega_{1/2}$ and $\omega_{3/2}$ are the respective transition frequencies. The difference between $\alpha_s$ and $\alpha_r(0)$ is about 15%. Across the whole frequency range, we therefore approximate the polarizability by

$$\alpha(\omega) = \alpha_s - \alpha_r(0) + \alpha_r(\omega).$$

Our approximation slightly underestimates the polarizability since it neglects the growing resonant contributions of higher-lying states. To bound the error, we compared to an overestimate obtained by assuming that difference $\alpha_s - \alpha_r$ was entirely due to the next-lowest dipole allowed transition at 420 nm. The difference in $\alpha(\omega)$ was below 2% over the relevant frequency range.

To maximize the light intensity, the arc should be imaged onto the location of the atoms, as shown in figure 1. The spectral density for a single beam is then given by [10]

$$S_0(\omega) = \frac{1}{2} \pi \theta^2 B(\omega)t(\omega)$$

where $\theta$ is the final focusing angle as shown, $B(\omega)$ is the spectral radiance of the source, $t(\omega)$ accounts for the imperfect transmission of any filters or other optical elements, and the factor of 1/2 accounts for the beam splitter. In the infrared region, the Hg arc lamp spectrum is fairly flat as a function of wavelength. For instance, the Oriel Instruments 100 W lamp model 6281 has a reported brightness $B_\lambda$ of about $5 \times 10^5$ W sr^{-1} m^{-2} nm^{-1} [7]. The spectral radiance therefore increases in the infrared,

$$B(\omega) = \frac{2\pi c}{\omega^2} B_\lambda$$

which helps to compensate for the decreasing polarizability.

The interference pattern produced by a spatially incoherent source has the form

$$S(r, \omega) = 2S_0[1 + \mu_{12} \cos k(z_1 - z_2)]$$

where $k = \omega/c$ and $\mu_{12}(r_1, r_2, \omega)$ is the spectral degree of coherence function. Here $r_1 = r - p_1$ is the displacement of the field point $r$ from an arbitrary reference point $p_1$ in one of the beams and $r_2$ is the displacement from the corresponding reference in the
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Figure 2. Potential energy of white-light trap. The solid trace gives the potential along the $z$-axis and the dashed trace along the $x$-axis, as defined in equation (8). The focusing angle is $\theta = 0.15$ and the filter transmission is $t = 0.133$. The calculation uses the measured spectral radiance curve from the Oriel Instruments catalogue for the model 6281 lamp. The filter function was assumed to increase smoothly from 0 to $t$ between wavelengths of 800 nm and 810 nm, be constant between 810 nm and 2.4 $\mu$m, and drop smoothly to zero at 2.5 $\mu$m.

other beam, illustrated by the two coordinate systems in figure 1. In particular, $z_1$ and $z_2$ are the distances along the beam axes for the two paths. The van Cittert–Zernike theorem gives the spectral degree of coherence

$$\mu_{12} = \frac{2J_1(k\rho\theta)}{k\rho\theta}. \quad (7)$$

Here $\rho^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ for $x$ and $y$ in the transverse plane.

In order to obtain constructive interference at the centre of the pattern, it is clear that both light beams must encounter a similarly even or odd number of mirror reflections so that the reflection phase shifts cancel. This means that the two images will have the same parity, making it impossible to align both the $(x_1, x_2)$ and $(y_1, y_2)$ axis pairs. The Bessel function in equation (7) therefore contributes nontrivially. In the simplest situation, the $y$ axes can be taken aligned, so that $\rho = 2x$. (More generally, new coordinates $(x', y')$ can always be found in which the $y'$ axes are aligned and the $x'$ axes are opposed.) The spectral density is then given by

$$S(r, \omega) = 2S_0 \left(1 + \cos 2kz \frac{J_1(2kx\theta)}{kx\theta}\right). \quad (8)$$

The resulting potential along the $x$ and $z$ axes is shown in figure 2. In addition, $S_0$ itself varies with position in a way that depends on the details of the imaging system. The Oriel lamp has a nominal source size of about 250 $\mu$m, but a wide range
of imaging magnifications is possible, depending on experimental convenience. The final image size $\Sigma$ gives the length scale for transverse variation of $S_0$, while the scale for longitudinal variation is $\Sigma/\theta$. We assume that both of these scales are sufficiently large to neglect hereafter.

A single pair of interfering beams thus generates tight confinement in one direction, looser confinement in another, and typically weak confinement in the third. To obtain tight confinement in all three dimensions, three beam pairs can be used. These could be obtained either from three independent lamps or by splitting the light from a single lamp. Even if a single lamp is used, mutual incoherence of the beam pairs can be assumed, since the total optical path length for beams in different pairs is unlikely to be equal within the accuracy required for white-light interference. The three-dimensional potential will thus be separable, with the potential along each axis given by the sum of the curves in figure 2. Figure 3 shows the result along with some of the single-particle bound states, for light from one lamp with $\theta = 0.15$ and $t = \frac{1}{3} \times 0.4$. If desired, deeper traps can be obtained using multiple lamps and larger focusing angles. The parameters shown support about 70 bound states per dimension, with two states confined in the narrow part of the potential. The presence of at least one tightly bound central state is robust, persisting down to about a factor of ten lower intensity.
The lifetime of atoms in the white-light trap will be limited by various losses, some of which are novel and others generic to tight optical traps. One loss mechanism is spontaneous emission, which leads to heating and dephasing of the trapped atoms. For both off- and on-resonant light, the scattering rate is given by the Rayleigh formula [9],

$$R_s = \frac{8\pi}{3\hbar c^4} \int S(\omega)\alpha(\omega)^2\omega^3 d\omega.$$  \hspace{1cm} (9)

with $\alpha$ from (3). Here $S$ is the total spectral density at the location of the atoms. The parameters from figure 3 give a scattering time $R_s^{-1} \approx 30$ s, but to achieve this rate it is essential that light near the atomic transitions be filtered very effectively. For the calculation shown, the filter function $t(\omega)$ was taken to rise smoothly from zero at $\lambda = 800$ nm to its full value at $\lambda = 810$ nm. In reality, some amount of resonant light will be present and can easily dominate the scattering rate. Before filtering, the spectral density near 790 nm is $12 S_0 \approx 5 \times 10^{-8} (W/m^2)/Hz$. This would give a scattering time of about 200 ns, so a filter attenuation of about $10^8$ is needed.

Holographic notch filters can provide attenuations greater than $10^6$ with bandwidths of about 20 nm [7], and additional attenuation might be obtained by passing the light through an optically thick Rb gas cell. An edge filter can be used to remove light with frequencies above the principle transitions.

The broadband nature of the light source also permits resonant two-photon excitation to higher-lying excited states. For the range of frequencies applied, transitions are possible to the 6S and 4D states of Rb, both of which lie about 20 000 cm$^{-1}$ above the 5S ground state. Second-order perturbation theory gives the total transition rate for these processes to be

$$R_2 = \sum_{b,c} \frac{\pi \Gamma_{ab} \Gamma_{bc} \sigma_{ab} \sigma_{bc}}{\hbar^2} \left( \frac{1}{\omega_{bc}} - \frac{1}{\omega_{ab}} \right) \int \frac{S(\omega)S(\omega_{ac} - \omega)}{(\omega - \omega_{ab})(\omega - \omega_{bc})} d\omega.$$  \hspace{1cm} (10)

where $a$ labels the ground state, $b$ the intermediate 5P states, and $c$ the final excited states. Then $\omega_{ij}$, $\sigma_{ij}$, and $\Gamma_{ij}$ are respectively the transition frequency, resonant scattering cross section, and total linewidth of the $i \rightarrow j$ transition. The parameters used here result in a negligibly low rate of about $5 \times 10^{-3}$ s$^{-1}$. A similar expression gives the rate for stimulated Raman transitions between atomic ground states, which is again small compared to the Rayleigh scattering rate.

Heating and trap loss can also arise from noise in the trap potential, just as in laser-based optical traps. Intensity fluctuations with fractional noise spectrum $S_I$ lead to heating at a rate [11]

$$\Gamma_c \approx \frac{\pi}{2} \Omega^2 S_I(2\Omega)$$  \hspace{1cm} (11)

where $\Omega$ is the classical oscillation frequency, about $2\pi \times 15$ kHz for the parameters used above. A 10 s heating time is then obtained for $S_I(f = 30kHz)^{1/2} \approx 10^{-5}$ Hz$^{-1/2}$. Typical free-running noise for research-grade arc lamps is specified to be better than $5 \times 10^{-3}$ rms in a 0.1 to 100 Hz bandwidth [7]. At higher frequencies, noise power is typically observed to fall as $1/f$ [12], leading to an expected noise density at 30 kHz below $10^{-6}$ Hz$^{-1/2}$.

Difficulties can also be expected from the phenomenon of arc wander, in which the bright spot of the arc changes position over time [7, 12, 13]. The time scale for
this motion is again slow, on the order of Hz, with an amplitude typically a small fraction of the size of the arc [13]. Arc wander will not directly change the interference pattern, but can alter the potential strength as the images of the source move relative to the atoms. The motion is slow enough for the atoms to adiabatically follow, so heating should be negligible. However, the noise introduced would be undesirable for tunneling experiments such as those proposed below. One solution is to use a large demagnification ratio when imaging the light onto the atoms. If the geometrical image of the arc is smaller than the aberration-limited spot size of the system, then the relative wander of the images will be reduced. This does, however, lower the intensity below the geometrical value (4), so quantitative measurements of arc wander in a given lamp would be a necessary step in designing a trap system.

The presence of aberrations in the imaging systems of figure 1 might also be of concern. Certainly, producing a clean interference signal will require the imaging elements of both beams in a pair to be identical to high degree. However, the interference pattern does not in fact depend on aberrations common to both beams, so long as the geometrical image of the source is large compared to the diffraction-limited spot size $\Sigma_{DL}$ of the imaging system [14]. Here $\Sigma_{DL} \approx 0.6\lambda_{\text{max}}/\theta$ is about 10 $\mu$m. It is thus desirable to have a relatively large, aberration-limited image. This is consistent with the earlier assumption of a large image, and also beneficial because diffraction-limited imaging optics with the wavelength range and numerical aperture required would be complex and expensive.

Finally, losses may result from inelastic interactions between the trapped atoms. In this regard, the white-light trap is no different from an optical lattice with comparable confinement strength. The peak density for the ground state shown in figure 3 is $n_1 \approx 2 \times 10^{14}$ cm$^{-3}$ per atom, and at this density, three-body recombination occurs at a significant rate. The rate coefficient for condensed $^{87}$Rb atoms in the $F = 1, m_F = -1$ hyperfine state is $K \approx 6 \times 10^{-30}$ cm$^6$/s [15], defined for large numbers of atoms $N$ with density $n(r)$ by

$$\frac{dN}{dt} = -K \int n(r)^3 d^3r.$$  

(12)

Allowing for the loss of three atoms per collision, counting actual triplets of atoms, and assuming a Gaussian density distribution, the collision rate $R_c$ is

$$R_c \approx \frac{K n_1^2}{3^{5/2}} N(N-1)(N-2),$$  

(13)

so three atoms would last for about 10 s before recombining.

Two-body dipolar decay is suppressed for this hyperfine state, with an expected rate constant on the order of $10^{-17}$ cm$^3$/s [16]. More significant will be light-induced losses due to photoassociation (PA) of atom pairs. Accurate estimation of the PA rate for far off-resonant light is difficult, as it depends on Frank-Condon overlap factors between the atomic and molecular states that are not readily available. In reference [17], PA was observed in a laser-cooled Rb gas, with about 100 transitions occurring in the relevant wavelength range. Loss coefficients ranging from $10^{-11}$ to $10^{-10}$ cm$^3$/s were measured for an optical intensity of $10^{10}$ W/m$^2$. The linewidth of the laser used in that experiment was about $10^{10}$ Hz, so the equivalent white-light intensity is about $10^9$ W/m$^2$. Assuming a linear scaling with intensity [18], this yields an estimated total loss coefficient of about $5 \times 10^{-16}$ cm$^3$/s. This again limits the sample lifetime to about 10 s for three atoms. If necessary, PA losses can be further suppressed by increasing the short-wavelength cutoff of the white-light.
An additional experimental issue that we consider is alignment of the beams and interference fringe. This might be accomplished using the condensate itself as a target. Even without the interference fringe, each beam produces an optical trap deep enough to confine many atoms (though a magnetic gradient would also be needed to support them against gravity). By transferring atoms to a white-light beam trap and observing their resulting location, the positions of the white-light beam foci can be determined and made coincident. The location of the interference fringes can then be positioned using translation stages, as illustrated in figure 1. The stages can be roughly positioned by direct measurement of the beam paths, but final adjustment will likely require each stage to be finely stepped until its fringe is located near the condensate. When this is achieved, the atoms will be more tightly confined, leading to changes in either direct images of the condensate or images of ballistic expansion. If one beam pair is aligned at a time, only one dimension will be tightly confined. This permits a relatively large number of atoms to be trapped at the fringe, which would aid the alignment process. Once aligned, the location of the fringes may vary with mechanical vibrations and thermal drifts, but as long as this variation is slow, no heating or loss should result.

4. Tunneling

From these arguments, we conclude that the white-light trap is a feasible method for tightly confining small numbers of trapped atoms for times on the order of 10 s. This could permit a variety of interesting experiments. In particular, probing elastic interactions between the atoms is interesting for quantum information and related applications. The leading elastic effect is a mean-field energy shift. For low occupation numbers, $N$ interacting atoms have a chemical potential

$$\mu \approx -E_0 + (N-1)\frac{4\pi\hbar^2a}{m} \int |\psi_0|^4 d^3r$$

(14)

where $E_0$ is the single-particle binding energy, $\psi_0$ is the single-particle ground state, $m$ is the mass and $a$ is the scattering length. For larger $N$, the actual wave function must be determined from the nonlinear Schrödinger equation, which will also alter the collisional loss rate. Equation (14) will be an adequate approximation as long as the interaction energy is small compared to the excitation energy of the first excited state. For $^{87}$Rb, the scattering length is 5.77 nm, and the parameters of figure 3 give an excitation energy of 620 nK and interaction strength $g \equiv 4\pi\hbar^2a \int d^3r|\psi_0|^4/mk_B = 32$ nK per atom. Thus the simple expressions above should hold for $N \approx 10$ atoms or less; interaction losses will likely limit $N$ to this regime in any case.

One way to probe the mean-field shift is to allow the atoms to escape the trap through tunneling and to measure the number left behind. In an applied potential gradient $U^\prime$, the single-particle tunneling rate is approximately

$$R_T = \Omega \exp \left( -\frac{2}{\hbar} \int \sqrt{2m[V(z) + U^\prime z - \mu]} dz \right)$$

(15)

The exponential dependence on $\mu$ provides sensitive energy resolution. In gravity, for instance, the tunneling rate at the highest-lying energy shown in figure 3 is about $7 \times 10^3$ s$^{-1}$, while for the next highest it is $2.6 \times 10^{-2}$ s$^{-1}$. By applying a stronger gradient, the interaction shift itself can be resolved: in a gradient of $k_B \times 1.05$ K/m, a single particle in the trap ground state has a lifetime of 0.7 s while a second particle
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escapes in 85 ms. In this case, if the gradient is applied for 200 ms then the probability for two atoms to remain is about 0.05, for one is 0.75, and for none is 0.20. By measuring these probabilities as a function of the gradient strength and duration, the dependence of $\mu$ on $N$ can be experimentally determined. Evidently, tunneling can also be used to prepare atomic samples with sub-Poissonian accuracy in $N$, a result useful for many quantum information applications.

A variety of other tunneling experiments can be considered. For instance, the white-light trap could be positioned at the edge of a large conventionally trapped condensate. For close spacings, atoms can be expected to tunnel between the condensate and the white-light trap when the chemical potentials of the two systems are equal. So, as the chemical potential in the white light trap is varied by, for instance, changing the intensity, a series of tunneling steps will occur that are reminiscent of the Coulomb blockade effect in quantum dots. This idea is explored further in Ref. [19].

A final interesting possibility is to study atoms confined in the white-light trap in the vicinity of a Feshbach resonance. At an average density of $10^{14}$ cm$^{-3}$ per atom, the strongly interacting regime $na^3 \sim 1$ can be reached with $N \approx 5$ atoms if the scattering length is increased to about 100 nm. Enhancements of this size and larger have been readily observed in $^{85}$Rb [20]. In this regime, calculation of the energies, excitations, and loss rates of the system is not trivial, so the white-light trap could provide a testing ground for a variety of many-body quantum techniques.

5. Conclusions

In conclusion we have outlined a proposal for trapping ultracold atoms with a white-light interference fringe. Using a relatively modest apparatus, this technique can provide a single trap with oscillation frequencies of tens of kHz and total depth of a few $\mu$K. We estimate the lifetime of small numbers of atoms in the trap to be of order 10 s. Although this paper has considered the specific example of Rb atoms, the white-light method is considerably more general. Any atomic species should be trappable, and even a variety of molecules if the absorption of infrared light can be avoided. An important limitation, of course, is the initial need for an ultracold source.

We believe the white-light technique to be well-suited for studying interaction effects in few-atom systems. Tunneling processes can be used to probe the energy spectrum of the atoms with nearly single-particle resolution. This would provide a unique test of theory for weakly interacting bosons and offers the opportunity to observe correlation effects in more strongly interacting systems. In some respects, the trap would serve as the bosonic version of a quantum dot, in which interactions play an important, nontrivial, but calculable role in determining the state of the system.

A variety of applications relevant to quantum information can be foreseen. One interesting possibility is the use of the white-light trap as a very small optical tweezer, capable of positioning atoms at specific sites within a conventional optical lattice. Although translating the white-light trap may be difficult, in this case the lattice itself could be translated instead. This could overcome one of the chief obstacles to quantum computing in an optical lattice, the difficulty of individual addressing. We are enthusiastic about such applications, and believe the white-light trap to be worth further study.
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