Prediction of Service Life for Assembly with Time-variant Deviation

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Abstract. During operation, the time-variant deviations, such as deformation, thermal expansion, friction and wear always occur and affect the mechanical performance and service life of assembly. In this paper, a methodology for the prediction of service life for assembly with time-variant deviations is proposed. Firstly, based on the modified Unified Jacobian-Torsor model and Monte Carlo simulation, according to the distribution the geometric and dimension tolerances are randomly generated to limit the variations of surface of part. Secondly, the deformations caused by load and thermal expansion are obtained by finite element analysis, and considering the friction and wear, the prediction model of service life is constructed subject to the constraints. At last, an application is given to illustrate the prediction model, and the influences of time-variant deviations on the assembly deviation and service life are analyzed.

1. Introduction

Tolerance analysis is widely used in the management of dimensional and geometrical variations of products for engineers. These variations from the nominal dimensions are stochastic and can be described by their statistical distribution, and their assembly variations can be calculated by many tolerance analysis models, mostly with the aid of Monte Carlo simulation [1-3]. These traditional analysis models rely upon rigid body motion to model the propagation and accumulation of variations that occur in an assembly of parts. However, during the operation, it would be more realistic that the components of assembly are flexible and usually have the time-variant deviation due to inertia effect (gravity, angular velocity, angular acceleration, etc.), temperature effect and friction. In addition, with the mobility of parts, the wear between contact surfaces always exists after a few of times whether they are rigid or flexible. If ignoring these deformations and wear, it could lead to an inaccurate assembly deviation and further to affect the performance, reliability and service life of products. In this paper, a methodology is proposed to predict the service life of product caused by time-variant deviation due to the preload of bolts, centrifugal force, thermal expansion and friction.

In recently years, a few studies focus on the tolerance synthesis with considering the time-variant deviation. The thermo-mechanical strains are integrated into the tolerance analysis model to control the clearances at the tip of a high pressure turbine blade with different operating regimes of a helicopter engine [4]. A variational method of tolerance analysis using a multi-physical approach was proposed by Pierre, Teissandier[5] to validate geometric specifications, contact specifications and
thermo-mechanical specifications. An approach proposed by Benichou and Anselmetti [6] considered thermal expansion of parts integrated within functional tolerance. Based on stochastic simulations, Grandjean, Ledoux [7] analyzed the influence of form defects in assemblies subject to local deformations and mechanical loads and concluded that even if the form defects are limited, they can lead to a non-compliant assembly. As the local deformations of parts occur, the cumulative effects of component parameter drift and tolerance variation may eventually cause the output variables of interest to deviate from the design specifications. Then after deviation propagation and accumulation along the kinematic chains, the final assembly deviation would exceed the functional requirements during service life. Therefore, it would be more realistic that it could obtain a more accurate calculation result of assembly deviation with considering the time-variant deviation according to the works by Mazur [8, 9].

In this paper, based on the modified Unified Jacobian-Torsor model and Monte Carlo simulation, according to the distribution the geometric and dimension tolerances are randomly generated to limit the variations of surface of part. Secondly, the deformations caused by load and thermal expansion are obtained by finite element analysis. Considering the friction and wear, all of these deformations and wears are integrated into the tolerance analysis model to construct the service life model, and finally the influences of time-variant deviations on the assembly deviation and service life are analyzed.

The remainder of this paper is organized as follows. Section 2 gives the problem description of assembly with time-variant deviation. Section 3 gives the tolerance analysis for assembly with time-variant deviation. Section 4 gives the prediction model of service life of assembly. An application is given in section 5 to illustrate the approach of this paper and section 6 gives the discussion and conclusion.

2. Problem description

In this paper, the blade bearing of controllable pitch propeller is regarded as an application to illustrate the methodology of predicting the service life with time-variant deviation. Figure 1 is the blade bearing of the controllable pitch propeller (CPP) and Figure 1 (right) is the partial enlarged view of O-ring hydraulic seal. This assembly consists of blade foot, blade carrier, O-ring and hub. The blade carrier is mounted on the hole of hub. The O-ring is extruded to fill up the O-groove between the blade foot and hub; and it can form O-ring hydraulic seal to prevent the seawater flow into the hub and the hydraulic oils flow into the seawater. Simultaneously, the blade carrier and bladed foot are fastened with bolts and no relative displacement between them.

![Figure 1. Blade bearing of the controllable pitch propeller.](image-url)
Currently, a few researches focus on the blade bearing of controllable pitch propeller. The influence of sea state on fretting behavior of blade bearing in a CPP was described by Godjevac, Van Beek [10]. In fact, the failure of the blade bearing is mainly presented as leakage which is caused by the decreased compression ratio of O ring during the service life. During the actual service, when the wear of contact surface 3 increased, the gap (contact surface 1 in Fig. 1) between the blade foot and hub could increase, and it will lead to a rebound of O-ring. Then its compression ratio could be decreased and if it is less than the required compression ratio, it will result in leakage. However, if the initial gap is too small, the increased temperature of the hydraulic oil in the cavity of hub, bolt preload and centrifugal force may result in deformations of parts; these deformations may further reduce the gap, or result in interference fit between the blade foot and hub. Then it could be very hard to adjust the pitch of CPP, and the blade foot and blade carrier could be hard to swing when needs to change the pitch.

Therefore, in order to extend the service life, the initial gap should be small, but in order to ensure the flexibility of adjusting the pitch, the initial gap should be larger than the functional requirement. The gap during the service life is a time-varying variable, which consists of three parts. The first part regarded as \( d_{g-t} \) is formed by the dimension geometric tolerances; the second part regarded as \( d_{g-r} \) is formed by the deformations due to the centrifugal force, bolt preload and temperature; the third part regarded as \( d_{g-w} \) is caused by the friction and wear, and this part is related to the service life. Then the gap can be written as:

\[
g_{g} = d_{g-t} + d_{g-r} + d_{g-w}(T)
\]

where \( T \) is the service life of blade bearing. In this paper, these time-variant deviations are considered to predict the service life of the blade bearing, and the influences of these time-variant deviations on the assembly deviations are analyzed.

3. Tolerance analysis with time-variant deviation

3.1. Tolerance analysis with considering the geometric tolerance

A detailed mathematic deduction of the variations and constraints of planar feature specified by dimension, orientation and position tolerance has been proposed by Roy and Li [11]. In this paper, all the variations of related planes are simultaneously limited by dimension tolerance and geometric tolerance. Figure 2 (left) shows the related tolerances of parts of assembly, and the gap between the blade foot and hub is determined by the dimension tolerances \( X_3 \) of hub, \( X_2 \) of blade carrier and \( X_1 \) of blade foot as well as the their corresponding geometric tolerances \( t_1, t_2, t_3 \) and \( t_4 \). Figure 2 (right) shows the connection graph of this assembly.

![Figure 2.Parts of blade bearing (left) and its Connection graph (right)](image)
In figure 2 (right), there are two FRs, and the gap is formed by the two functional requirements. The fit of blade foot and blade carrier is perfect, and CFE3 is null. With considering the geometric tolerances, the functional requirements can be calculated by the modified method of Unified Jacobian-Torsor model proposed by Chen, Jin [12]. The more detailed description about the modified Unified Jacobian-Torsor model has been introduced in [12]. Monte Carlo method is used to generate the satisfied random values of each torsor for the components according to variations and probability distributions based on the work of Ghie, Laperrière [13].

3.2. Finite element analysis
After constructing the 3D model of the blade bearing of CPP, the next step is to develop a finite element model to determine deformation of various components and their effects on the gap between the blade foot and hub. The material properties of various elements of the analysis model are listed in Table 1. The periodic boundary conditions are used to define the symmetry surface. Figure 3 shows the finite element analysis model of this assembly, and then the next step is to determine deformation due to the temperature, centrifugal force and preload of bolt.

Table 1. The material properties of each parts of the analysis model

| Material property       | Hub     | Blade foot | Blade    | Blade carrier | Bolt    |
|-------------------------|---------|------------|----------|---------------|---------|
| Material                | CuNiAl  | CuNiAl     | CuNiAl   | 42CrMo4       | Structural steel |
| Density (kg/m$^3$)      | 7530    | 7530       | 7530     | 7380          | 7850    |
| Modulus of elasticity (MPa) | 111   | 111        | 111      | 212           | 210     |
| Poisson’s ratio (%)     | 0.32    | 0.32       | 0.32     | 0.3           | 0.3     |
| Yield strength (MPa)    | 260     | 260        | 260      | 550           | 250     |
| Tensile strength (MPa)  | 650     | 650        | 650      | 800           | 460     |
| Thermal conductivity (W/mK) | 41.9  | 41.9       | 41.9     | 44            | 60.5    |

Figure 3. The finite element analysis model of the assembly

The first step is to establish the steady-stage thermal analysis. In this assembly, the temperature of all surfaces covered by the hydraulic oil is equal to the temperature of oil including the inner surfaces of hub, blade carrier and local surface of blade foot. The temperature of outer surfaces soaked in the sea water is $19^\circ$C which are equal to the temperature of sea water. The alarm temperature of oil is $60^\circ$C and the room temperature is $25^\circ$C, the variable range is $25-60^\circ$C. The results of steady-stage thermal analysis will be imported to steady-state structural analysis for next simulation.

The centrifugal force is generated by the rotation speed of blade. The range of rotation speed is 60r/min~180r/min according. The ranges of bolt pretension of the bolts are 320 KN~390KN.

According to characterization requirement and sampling frequency, the two mating planar surfaces of contact surface 1 were discretized as a mesh. The deformation of nodes in the mesh can be obtained from the finite analysis results. The data are represented as follows:
\begin{align*}
A_{di} &= \begin{bmatrix}
    x_{i1} & y_{i1} & z_{i1} \\
    x_{i2} & y_{i2} & z_{i2} \\
    \vdots & \vdots & \vdots \\
    x_{in} & y_{in} & z_{in}
\end{bmatrix} \\
d_{g-r} &= (A_{d1} - A_{d2})
\end{align*}

where $i (i = 1, 2)$ is the number of mating surfaces, $n$ is the number of sampling points on each surface, the matrix comprised of finite element analysis data of the surface, $x_{ij}$, $y_{ij}$ and $z_{ij}$ are the coordinate values of the sampling point on the surface, respectively.

3.3. Time-variant deviations of gap

The wear of blade bearing of a CPP is caused by sliding motion and fretting motion. The wear rates of sliding motion and fretting motion both are related to the materials of blade carrier, and the related researches are given in [14].

In order to estimate the total wear of a CPP, it is necessary to know how much pitch is being used in real service conditions. When the total amount of pitch change is known, the total sliding distance in the blade bearing will be known for a certain time period. The total wear depth of blade bearings ($h$) can thus be calculated by combining the total sliding distance ($S_{\text{sliding}}$), specific wear rate ($\omega$) and load ($\sigma$) into the following equation:

$$h = \omega \cdot S \cdot \sigma$$

The total sliding distance is related to the times $n_{ij}$ of total pitch change of one day, design pitch angle $\varphi_{d}$, the blade bearing radius $r$ and the service life $T$, it can be written as:

$$S_{\text{sliding}} = n_{ij} \cdot \frac{\varphi_{d}}{360} \cdot 2 \cdot \pi \cdot r \cdot T$$

The total fretting distance is related to the fretting amplitude $d_{\text{fret}}$, design revolutions per minute $\text{rpm}$ and service life $T$, and it can be written as:

$$S_{\text{fret}} = 2 \cdot d_{\text{fret}} \cdot \text{rpm} \cdot T$$

Then the final wear depth can be written as:

$$d_{g-w}(T) = \omega_{r} \cdot S_{\text{sliding}} \cdot \sigma + \omega_{f} \cdot S_{\text{fret}} \cdot \sigma$$

4. Prediction model of service life

Service life of CPP is determined by the sealing performance of blade bearing and the flexibility of adjusting the pitch. Considering the statistical criteria, the tolerance intervals are the $\pm 3\sigma$ range, and the probability of eligible product is 99.73%. In order to ensure the flexibility of adjusting the pitch, the probability constraint in respect of the initial functional requirement can be presented as:

$$P_{\text{grap}} = P((d_{g-r} + d_{g-r}) > 0) \geq 99.73\%$$

The compression ratio range of rubber O-ring generally is 9%–15%, and the probability constraint of O-ring in respect of service life $T$ can be written as:

$$P_{r} = P(0.09 \leq \frac{d_{2} - ((d_{g-r} + d_{g-r})_{\text{max}} + d_{g-w}(T) + X_{5})}{d_{2}} \leq 0.15) \geq 99.73\%$$

Where $d_{2}$ is section diameter of O-ring in a free state; $X_{5}$ is the depth of O-groove. In this paper, two indicators are introduced to evaluate the performance of CPP, one is the probability $P_{r}(T)$ of service
life $T_i$, and the other is the actual service life $T_a$. The prediction model of service life of CPP can be written as:

$$
\begin{align*}
\text{Object:} & \quad f_1 = P(T_i), f_2 = T_a \\
& \quad \{P_{sup} \geq 99.73\%\} \\
& \quad \{P_T \geq 99.73\%\}
\end{align*}
$$

(10)

**Figure 4.** The procedure chart of prediction of service life of CPP

In this paper, an iterative method is used to predict the service life of blade bearing based on Monte Carlo simulation. The Monte Carlo simulation is used to calculate the probability of accepted assemblies. The procedure of prediction of service life of CPP is shown in Figure 4.

5. Experimental results

The related tolerance values of the blade bearing of CPP are given in Table 2. According to the Monte Carlo simulation and modified Unified Jacobian-Torsor model in [12], the first part of gap $d_{1-2}$ is shown in Table 3 including the assembly deviations of $FR_1$ and $FR_2$.

| Tolerance | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $d_2$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Value     | $2(-0.004, 0.006)$ | $71(0.00, 0.03)$ | $73(-0.09, -0.06)$ | $15(-0.1, 0)$ | $17(-0.15, 0)$ | $0.02$ | $0.02$ | $0.02$ | $0.02$ |

| Functional requirement | Tolerance | Mean | Std | Var |
|------------------------|-----------|------|-----|-----|
| $FR_1$ | $w_1$ | -0.075 | 0.0045 | $2.024 \times 10^{-5}$ |
| $FR_2$ | $w_2$ | 0.0160 | 0.00466 | $2.174 \times 10^{-5}$ |
5.1. Influence time-variant deviation on the assembly deviation

When the second part of gap $d_{g-r}$ is considered, the maximum assembly gap between the hub and blade foot could increase and the minimum assembly gap between the hub and blade foot could decrease. Because, the deformation surfaces of parts are not uniform. The changes of the maximum assembly deviation and minimum assembly deviation caused by second part of gap $d_{g-r}$ with the temperature, rotation speed and preload are shown in Figure 5, Figure 6.

In Figure 5 (left), the maximum assembly deviation increases with the temperature, but the minimum assembly deviation decreases with temperature. In Figure 5 (right), only the maximum assembly deviation increases with the rotation speed, and the minimum assembly deviation increases little when the rotation speed increases. However, in Figure 6, both the maximum and minimum assembly deviation changes little when the preload increases. Therefore, the temperature has the greatest impact on the assembly deviation, and effect both the flexibility of adjusting the pitch and the sealing performance of blade bearing. The rotation speed only effects the maximum assembly deviation, and further to effect the sealing performance, however, the preload has little influence on the assembly deviation.

![Figure 5. Second part of gap changes with the temperature (left) and rotation speed (right)](image)

![Figure 6. Second part of gap changes with the preload](image)

5.2. Service life of blade bearing

Generally, the service life of products is related to the working conditions and operating frequency. The fretting behavior of blade bearing is effected by sea state and its frequency and amplitude to
adjust the pitch[14], and the wear rate of the blade bearing is 0.54E-7 and the maximum load is 24N/mm².

In order to illustrate the methodology of predicting the service life of blade bearing, a ferry from Dover to Calais is used as an application to calculate the total amount of pitch change during one day which is introduced by Godjevac [14]. The sampling rate is 10Hz and the fretting amplitude (dfret) is assumed as 300μm. One voyage time (tvoy) is approximately 100 minutes and the total amount of pitch changes is 9.54 times of the design pitch angle (%d =30deg). The total sliding distance is obtained by multiplying the design pitch angle with the blade bearing radius (r = 0.226 m). The rated rotation speed is 150r/min. This ship makes around 13 voyages per day (nvoy) and operates approximately 340 days per year (nday).

According to the equations (5) and (6), the sliding distance and fretting distance of one year can be obtained that:

\[ S_{total} = 340 \cdot 13 \cdot \frac{30}{360} \cdot 2 \cdot 3.14 \cdot 0.226 + 2 \cdot 0.3 \cdot 10^{-3} \cdot 150 \cdot 100 = 44.772km \] (11)

Then, according to the equation (7), the final wear depth of one year can be calculated:

\[ H = 0.54 \times 10^{-7} \times 24 \times S_{total} = 58.02\mu m \] (12)

Since, the preload has little influence on the assembly deviation, and in this subsection, its influence on the service life is not analyzed. Table 4 shows the probability \(P_i(T_a)\) of service life \(T_i\) and the actual service life \(T_a\) with different temperature. In Table 4, the actual service life \(T_a\) decreases when the temperature increases as well as the probability \(P_i(T_a)\) of 5 years’ service life. In Figure 7 (left), they are the probability distribution curves of service life \(T_i\) with different temperatures, and the normal curve is the probability distribution of service life \(T_a\) without considering the real working condition. It shows that, when the temperature increases, the probability distribution curve of service life moves left, and the actual service life \(T_a\) decreases, and finally the service life is reduced almost by 8% from 25°C to 60°C.

Table 4. Service-life changes with temperature (other parameters: rpm 150r/min, preload 390KN)

| Temperature (℃) | 60   | 55   | 50   | 45   | 40   | 35   | 30   | 25   |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| of 5-year’ service life | 0.9377 | 0.9456 | 0.9543 | 0.96 | 0.9657 | 0.9713 | 0.9758 | 0.9801 |
| Actual service life   | 4.13 | 4.18 | 4.23 | 4.285 | 4.345 | 4.4 | 4.455 | 4.51 |

Table 5. Sliding and fretting distance with different rpm, and the total wear depth

| Rotation speed (Unit: r/min) | 60   | 90   | 120  | 150  | 180  |
|-----------------------------|-----|-----|-----|-----|-----|
| Total sliding and fretting distance (one year) (Unit: m) | 20898 | 28854 | 36809 | 44772 | 52723 |
| Total wear depth (one year) (Unit: um) | 27 | 37.4 | 47.7 | 58.02 | 68.33 |

Table 6. Service life changes with rpm (other parameters: temperature 30℃, preload 390KN)

| Rotation speed (Unit: r/min) | 60   | 90   | 120  | 150  | 180  |
|-----------------------------|-----|-----|-----|-----|-----|
| of 5-year’ service life | 100% | 100% | 100% | 0.9794 | 0.1899 |
| Actual service life | 11.2 | 7.5 | 5.6 | 4.25 | 3.4 |

According to the equation (6), the fretting distance is related to the rotation speed. Different rotation speed has different total wear depth, and they are given in Table 5. Table 6 is the calculated service life with different rotation speed. Figure 7 (right) is the probability distribution curve of service life \(T_i\) with different rotation speed.
Figure 7. Probability distribution of service life $T_i$ changes with temperature (left) and rpm (right)

From the Table 6 and Figure 7 (right), it can be seen that if the rated rotation speed increases, the actual service life decreases largely. When the rated rotation speed is set as 60r/min, its actual service life is 11.2 years; however, when the rated rotation speed is set as 180r/min, its actual service life is 3.4 years, almost threefold. Therefore, the temperature and rotation speed all effect the service life of the blade bearing of CPP, and if the temperature increased, the final service life would decrease by almost 8%; however, if the rated rotation speed were too large, the service life would be three times less. In actual service process, it is quite important to control the temperature and choose a reasonable rated rotation speed to extend the service life of blade bearing of CPP.

6. Conclusion

In this paper, a methodology for the prediction of service life for assembly with time-variant deviations is proposed, and the tolerance analysis model with considering the time-variant deviation is constructed. The assembly deviation of blade bearing are obtained by the modified Unified Jacobian-Torsor model with considering the geometric tolerances and dimensional tolerances; the thermo mechanical coupling deformations of blade bearing of CPP caused by centrifugal force, bolt preload and temperature are obtained by finite element analysis. Considering the friction and wear during service life, the prediction model of service life of blade bearing with considering the time-variant deviation is constructed subject to the constrains of sealing performance and flexibility of adjusting the pitch. Through this method, the influences of temperature, speed and preload of bolt on the assembly deviation are analyzed. It is also found that the thermo mechanical coupling load has great influence on the assembly deviation and service life. Especially, if the temperature and rotation speed increases, the service life decreases largely.

Based on the work of this paper, in the future, the assembly parameters would be optimized with considering the time-variant deviation, and the optimized results would be more reliable. It will also be useful for engineers to design tolerances for parts with considering the real working condition.

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