Generalized Passarino-Veltman reduction scheme in the absence of Lorentz invariance

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Passarino-Veltman reduction scheme (PVRS) offers an efficient procedure to reduce the one-loop Feynman diagrams (OLFDs). It is well established and widely applied in the system of particle physics where the Lorentz invariance is present, but there have been only sparse attempts to generalize it to other subfields in which the Lorentz invariance is lacking, such as nuclear physics and condensed matter physics. We report a systematic theoretical framework by presenting two equivalent generalized PVRS (GPVRS) in the absence of Lorentz invariance. We apply the GPVRS to simplify several typical OLFDs in different important models of relativistic particles or pseudo-relativistic quasi-particles at finite temperature and/or finite density. We verify our results by gauge invariance and various limiting cases. The GPVRS can analyze the OLFDs in a wide range of physical systems without Lorentz invariance, such as quark-gluon plasma in nuclear physics and Dirac topological materials in condensed matter physics.

\textbf{Introduction.}—One-loop Feynman diagrams (OLFDs) play an extremely important role in calculating the transition amplitudes and/or correlation functions of many subfields of physics, including particle physics, nuclear physics and condensed matter physics \cite{1–5}. The brute-force evaluation of OLFDs is often a time-consuming and error-prone process, therefore a very significant mathematical physics problem is how to most efficiently perform the OLFD calculations. Based on the Lorentz invariance, Passarino-Veltman reduction scheme (PVRS) \cite{6}, which elaborated on an idea of Brown and Feynman \cite{7}, provides a powerful framework for simplifying the OLFDs \cite{8–9}. With the help of PVRS, a huge number of OLFDs can be automatically assembled from a small number of basic building blocks \cite{6, 8, 9}. Owing to this advantage, the PVRS triggered many variants \cite{9–20} and program packages \cite{20–25} for automatic algebraic calculation, and has been widely applied in numerous processes of high energy physics.

PVRS \cite{6} and its variants \cite{8–20} are well established for particle physics which is the Lorentz-invariant system, however, there have been only sparse attempts \cite{26–28} to generalize the PVRS from particle physics to other subfields in which the Lorentz invariance is absent, such as nuclear physics and condensed matter physics. On the other hand, the OLFDs in these two subfields are more complicated than that in particle physics due to finite temperature, finite density (chemical potential), or pseudo-relativistic dispersion. Therefore a systematic generalized PVRS (GPVRS) to evaluate the OLFDs therein with great efficiency is in urgent need, which is the motivation of the work described here.

In this work, we propose two equivalent GPVRS in the absence of Lorentz invariance that can boil the OLFDs down to a set of basic building blocks. We employ the GPVRS to several important models of relativistic particles or pseudo-relativistic quasi-particles at finite temperature and/or finite density. The GPVRS is verified by gauge invariance and various limiting cases. The GPVRS can be used to simplify the OLFDs in the systems of quark-gluon plasma \cite{29, 30}, graphene \cite{31}, silicene \cite{32}, topological insulators \cite{33, 34}, and topological semimetals \cite{35}, which are the frontiers of nuclear physics and condensed matter physics.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{ } & \textbf{ } & \textbf{ } \\
\hline
$E(\lambda, p, m) = \pm \sqrt{\lambda^2 p^2 + m^2}$ & $\lambda = 1$ & $0 < \lambda < 1$ \\
\hline
\textbf{T = 0, } & \textbf{PVRS} & \textbf{GPVRS} \\
$\mu = 0$ & Lorentz-invariant & not Lorentz-invariant \\
\hline
\textbf{T = 0, } & \textbf{GPVRS} & \textbf{GPVRS} \\
$\mu \neq 0$ & Lorentz-invariant & not Lorentz-invariant \\
\hline
\end{tabular}
\caption{PVRS in the presence of Lorentz invariance (already done) v.s. GPVRS (generalized PVRS) in the absence of Lorentz invariance (to be done in this work). It is Lorentz-invariant only when the relativistic particles ($\lambda = 1$) interact at zero temperature ($T = 0$) and zero density ($\mu = 0$). For the pseudo-relativistic quasi-particles ($0 < \lambda < 1$) in graphene, silicene, topological insulator and topological semimetals, the velocity parameter $\lambda$, defined as $\lambda = v_F/c$, is typically of the order 1/100.}
\end{table}

\textbf{One-loop generic scalar and tensor integrals.}—As is shown in the Supplemental Material \cite{36}, up to three-point diagram (see Fig. (1) for details), the OLFDs in the systems of relativistic particles or pseudo-relativistic quasi-particles can be decomposed into linear combinations of a series of one-loop generic scalar integrals $A_0(1), B_0(p_1; 1; 2), C_0(p_1, p_2; 1; 2; 3)$ and generic tensor integrals $B^p; B^p(p_1; 1; 2), C^p; C^{p\sigma}; C^{\sigma\tau}(p_1, p_2; 1; 2; 3)$, where
\[ A_0(1) = \int \frac{d^Dl}{i\pi^{D/2}} \frac{1}{\mathcal{P}(l, 0; 1)} \]  

\[ B_0; B^\rho; B^{\rho\sigma}(p_1; 1; 2) = \int \frac{d^Dl}{i\pi^{D/2}} \frac{1; l^\rho, l^\rho l^\sigma}{\mathcal{P}(l, 0; 1)\mathcal{P}(l, p_1; 2)} \]  

\[ C_0; C^\rho; C^{\rho\sigma\tau}(p_1, p_2; 1; 2; 3) = \int \frac{d^Dl}{i\pi^{D/2}} \frac{1; l^\rho, l^\rho l^\sigma, l^\rho l^\sigma l^\tau}{\mathcal{P}(l, 0; 1)\mathcal{P}(l, p_1; 2)\mathcal{P}(l, p_1 + p_2; 3)} \]

It is noted that in the denominator of Eqs.(1)-(3), \( \mathcal{P}(l, p; n) = (l^0 + p^0 + \mu_n)^2 - [\lambda_n^2(l + p)^2 + m_n^2] \), where \( p_n = (p_n^0, p_n) = (p_n^0, p_n^1) \) are the external momenta while \( l = (l^0, l) = (l^0, l') \) is the momentum of the first internal propagator. In the arguments, \((n) \equiv (m_n, \mu_n, \lambda_n)\), where \( m_n, \mu_n, \) and \( \lambda_n \) denote the mass, density, and velocity parameter of the \( n \)-th internal propagators, respectively. In the Matsubara formalism [37], the temperature and density are encoded into the time-component of internal momentum. Throughout this work, we work in natural units where \( \hbar = c = \kappa_B = 1 \) and use the Greek (Roman) letters to denote the spacetime (spatial) components.

![FIG. 1. Generic n-point one-loop Feynman diagram with external momenta \( p_n = (p_n^0, p_n) = (p_n^0, p_n^1) \). The momentum of the first internal propagator is denoted by \( l = (l^0, l) = (l^0, l') \). In addition, \( m_n, \mu_n, \) and \( \lambda_n \) represent the mass, density, and velocity parameter of the \( n \)-th propagator, respectively.](image)

For the Lorentz-invariant systems where relativistic particles interact at zero temperature and zero density, owing to Lorentz covariance the generic tensor integrals \( B^\rho; B^{\rho\sigma}(p_1; 1; 2) \) and \( C^\rho; C^{\rho\sigma\tau}(p_1, p_2; 1; 2; 3) \) boil down to the Lorentz-covariant tensor structures and the Lorentz-invariant linear combinations of generic scalar integrals, such as \( A_0(1), B_0(p_1; 1; 2) \), and \( C_0(p_1, p_2; 1; 2; 3) \) [6, 8, 9]. In other words, with the help of PVRS, only a small number of generic scalar integrals are needed. If these basic building blocks are evaluated ahead of time, all the OLFDs can be automatically assembled.

However, any of the finite temperature, finite density, and pseudo-relativistic dispersion, can explicitly break the Lorentz invariance. More concretely, finite temperatures and/or finite densities specify a rest reference frame of the many-body system in which the temperatures and/or densities are measured [2], rendering the Lorentz invariance broken. On the other hand, for the pseudo-relativistic systems, such as graphene, silicene, topological insulators, and topological semimetals, the largest continuous spacetime symmetry in \( D \) dimension is no more than the \( SO(D - 1) \) spatial rotation symmetry, instead of the \( SO(1, D - 1) \) (proper normal) Lorentz symmetry. Therefore, the Lorentz invariance is broken by nature in the pseudo-relativistic systems, even at zero temperature and zero density, letting it alone at finite temperature and/or finite density. This is opposite to what is claimed in the condensed matter physics community, which is why we term such kind of quasi-particle dispersion in graphene, silicene, topological insulators, and topological semimetals as “pseudo-relativistic” rather than follow the term “relativistic” in most related literatures.

Consequently, PVRS can not be directly employed to simplify the OLFDs in the relativistic systems at finite temperature and/or finite density, or in the pseudo-relativistic systems at arbitrary temperature and density. In the following we present two GPVRS to reduce the generic tensor integrals in the absence of Lorentz invariance by illustrating the reduction procedure of \( B^\rho(p_1; 1; 2) \) and \( B^{\rho\sigma}(p_1; 1; 2) \), leaving the details of reducing \( C^\rho; C^{\rho\sigma\tau}(p_1, p_2; 1; 2; 3) \) to the Supplemental Material [36].

**First GPVRS.**—Due to the absence of Lorentz invariance, the generic tensor integrals \( B^\rho(p_1; 1; 2) \) and \( B^{\rho\sigma}(p_1; 1; 2) \) are not Lorentz-covariant such that the Lorentz-covariant tensor structures \( p_1^\rho, p_1^\rho p_2^\rho, \) and \( g^{\rho\sigma} \) are no longer complete to expand them. This incompleteness can be complemented by introducing an extra \( D \)-dimensional constant vector \( u^\rho = (1; 0, \cdots, 0) \). This constant vector plays a twofold role in accounting not only for the absence of Lorentz invariance due to pseudo-relativistic quasi-particles but also for the presence of a rest reference frame due to finite temperature and/or finite density [2]. Treating the spacetime components of generic tensor integrals on the same footing [6, 8, 9] and adding the effect of Lorentz-symmetry-breaking in terms of the constant vector, we express the generic tensor in-
tegrals $B^p(p_1; 1; 2)$ and $B^{ps}(p_1; 1; 2)$ as

$$B^p(p_1; 1; 2) = u^p B_1 + p_1^p B_2,$$

$$B^{ps}(p_1; 1; 2) = u^p u^p B_{11} + (u^p p_1^p + p_1^u u^p) B_{12} + g^{ps} B_{00} + p_1^p p_1 B_{22}. \tag{5}$$

Compared with the PVRS [6], four remarks concerning the first GPVRS are in order (see the Supplemental Material [36]). First, besides the Lorentz-covariant tensor structures $p_1^p$, $g^{ps}$, and $p_1^p p_1^p$ appeared in the PVRS, there are three extra tensor structures $u^p$, $u^p u^p$ and $(u^p p_1^p + p_1^u u^p)$, which are not Lorentz-covariant. Second, besides two generic scalar integrals $A_0$ and $B_0$, one must introduce generic tensor integrals $B^0$ and $B^{00}$, acting as two extra basic building blocks, to account for the absence of Lorentz invariance, where the 0-component of $l$ is defined as $l^0 = u \cdot l$. This indicates that the generic scalar integrals in Refs. [26, 27] are not yet suitable for expanding the generic tensor integrals because they are incomplete. Third, $A_0(1)$, $B_0(p_1; 1; 2)$, $B^0(p_1; 1; 2)$, and $B^{00}(p_1; 1; 2)$, depending separately on $p_1^0 = u \cdot p_1$ and $|p_1| = \sqrt{(u \cdot p_1)^2 - p_1^2}$, are not Lorentz-invariant.

The form factors $B_1$, $B_2$, $B_{00}$, $B_{11}$, $B_{12}$, and $B_{22}$, as different linear combinations of $A_0(1)$, $B_0(p_1; 1; 2)$, $B^0(p_1; 1; 2)$, and $B^{00}(p_1; 1; 2)$, are not Lorentz-invariant either. Fourth, for the relativistic particles interacting at zero temperature and zero density, there is no preferred rest reference frame such that the constant vector $u^p = (1; 0, \ldots, 0)$ can not play any role and hence vanishes [2]. As a result, the Lorentz invariance is restored and the first GPVRS goes back to the PVRS.

Second GPVRS.—The essential spirit of PVRS is to express the generic tensor integrals in terms of Lorentz-covariant tensor structures imposed by Lorentz symmetry [6, 8, 9]. This crucial idea turns out also applicable to other continuous spacetime symmetries, such as the SO($D - 1$) spatial rotation symmetry. In the presence of spatial rotation invariance, the spatial component of generic tensor integrals can be similarly written as that in the PVRS [6, 8, 9]. In a spatial rotation covariant manner, the spatial components of the generic tensor integrals $B^p(p_1; 1; 2)$ and $B^{ps}(p_1; 1; 2)$ can be reduced on the same footing as

$$B^i(p_1; 1; 2) = p_1^i B_2,$$  \tag{6}

$$B^{0i}(p_1; 1; 2) = p_1^i B_{12}, \tag{7}$$

$$B^{ij}(p_1; 1; 2) = \delta^{ij} B_{00} + p_1^i p_1^j B_{22}. \tag{8}$$

Note that other two components of the generic tensor integrals $B^p(p_1; 1; 2)$ and $B^{ps}(p_1; 1; 2)$ are nothing but $B^0(p_1; 1; 2)$ and $B^{00}(p_1; 1; 2)$, which we have previously selected out as extra generic tensor integrals in the first GPVRS. Consequently, one can express four form factors $B_2$, $B_{12}$, $B_{00}$ and $B_{22}$ as different linear combinations of $A_0(1)$, $B_0(p_1; 1; 2)$, $B^0(p_1; 1; 2)$, and $B^{00}(p_1; 1; 2)$. More details are found in the Supplemental Material [36].

Compared with the first GPVRS which treats the spacetime components of generic tensor integrals on the same footing, the second GPVRS restricts the reduction only to spatial component of the generic tensor integrals. From Eqs. (4) and (5) in the first GPVRS and Eqs. (6)- (8) in the second GPVRS, one obtains the correspondence between two GPVRS in terms of the relations between two sets of form factors. These relations are satisfied by explicit expressions of the form factor in terms of $A_0(1)$, $B_0(p_1; 1; 2)$, $B^0(p_1; 1; 2)$, and $B^{00}(p_1; 1; 2)$ in the first GPVRS and the second GPVRS. This observation confirms the equivalence between these two GPVRS (see the Supplemental Material [36]). In addition, the second GPVRS is more convenient to apply than the first GPVRS, and is more systematic than the naive generalization of PVRS in Ref. [28] proposed by the author of the present paper.

**Application and verification.**—As illustrations, we employ the GPVRS to perform several typical OLFs in $\phi^4$-theory, Yukawa theory, Quantum Electrodynamics (QED), Nambu-Jona-Lasino model, and Quantum Chromodynamics (see the Supplemental Material [36]). To be concrete, we show the reduction procedure of one-loop photon self-energy at finite temperature and/or finite density in $D$-dimensional QED of relativistic/pseudo-relativistic fermion. The Lagrangian reads

$$\mathcal{L} = \bar{\psi}_f [i \gamma^0 (\partial_0 + i e A_0) + i \lambda_f \gamma^j (\partial_j + i e A_j) - m_f] \psi_f + \bar{\psi}_f \gamma^0 \mu_f \psi_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial^\mu A_\mu)^2, \tag{9}$$

where $e$, $m_f$, and $\lambda_f$ denote the charge, mass, and velocity parameter of fermionic particle/quasi-particle with flavor $f$. At finite temperature and/or finite density, it can describe various systems, including hot and dense quark matter, neutrino gas, the early Universe at large lepton chemical potential [29, 30] when $\lambda_f = 1$, and graphene [31], silicene [32], topological insulators [33, 34], topological semimetals [35] when $0 < \lambda_f < 1$.

The one-loop photon self-energy $i\Pi^{\mu\nu}(q)$ [Fig. (2)] can be decomposed into linear combinations of a generic scalar integral $B_0(q; 1; 1)$ and two generic tensor integrals $B^p; B^{ps}(q; 1; 1)$ as (see the Supplemental Material [36]):

![FIG. 2. One-loop photon self-energy $i\Pi^{\mu\nu}(q)$ in the Quantum Electrodynamics of relativistic/pseudo-relativistic fermion.](image-url)
where the argument \((q; 1; 1)\) in \(B^\rho(q; 1; 1)\) and \(B^{\rho\sigma}(q; 1; 1)\) was omitted without causing any confusion, in which \((1) \equiv (m_1, \mu_1, \lambda_1)\). After performing \(A_0(1), B_0(q; 1; 1), B^\rho(q; 1; 1), \) and \(B^{0\rho}(q; 1; 1)\) in the Matsubara formalism, one expresses the generic tensor integrals \(B^\rho(q; 1; 1)\) and \(B^{\rho\sigma}(q; 1; 1)\) via either Eqs. (4) and (5) in the first GPVRS, or Eqs.(6)-(8) in the second GPVRS. Consequently, the one-loop electron self-energy \(i\Pi^{\mu\nu}(q)\) can be directly assembled. In addition, the one-loop electron self-energy and vertex correction at finite temperature and/or finite density can be similarly obtained via the GPVRS (see the Supplemental Material [36]).

The result of one-loop photon self-energy is verified by gauge invariance \(q_\mu i\Pi^{\mu\nu}(q) = 0\) at finite temperature and/or finite density in the Supplemental Material [36] and at zero temperature and finite density in Ref. [28]. Furthermore, the result is verified by its limiting cases. In the context of QED for Weyl fermion \((m_1 = 0\) in the Lagrangian) in \(D = 4\) dimension, the author of the present paper explicitly calculated [28] analytical expressions of photon self-energy \(i\Pi^{\mu\nu}(q)\) at zero temperature and finite density. The exact analytical expressions in Ref. [28] automatically give rise to \(i\Pi^{00}(q)\) [38], \(i\Pi^{i0}(q)\) [diagonal component of \(i\Pi^{ij}(q)\)] [39, 40], \(i\Pi^{0i}(q)/i\Pi^{00}(q)\) [41], the parity-odd part of \(i\Pi^{ij}(q)\) [42], the parity-even part of \(i\Pi^{ij}(q)\). Besides, under the hard dense loop approximation \((\mu \gg q_0 |q|)\), it restores the results in Ref. [43]. All of these indicate that it is more efficient and general to calculate the OLFDS with the help of GPVRS [28] than by a brute-force [38–43]. In addition, the GPVRS can be applied to graphene, silicene, and topological insulators \((D = 3)\) to systematically obtain the polarization function [44–48], current-current correlation function [49], and spin susceptibility [50, 51].

**Symmetry-breaking and dimension.**—Both PVRS and GPVRS are based on continuous spacetime symmetry of the system. For the PVRS in the presence of Lorentz invariance, it is the \(SO(1, D - 1)\) (proper normal) Lorentz symmetry; while for the GPVRS in the absence of Lorentz invariance, it is the \(SO(D - 1)\) spatial rotation symmetry broken down from Lorentz symmetry. If the \(SO(D - 1)\) symmetry further breaks down to the \(SO(D - 2)\) symmetry, one can introduce another extra \(D\)-dimensional constant vector \(v^\rho = (0; 0, \ldots, 0, 1)\) in the first GPVRS, or an extra \((D - 1)\)-dimensional constant vector \(v^i = (0, \ldots, 0, 1)\) in the second GPVRS. Following a similar procedure, one can then reduce the generic tensor integrals after the symmetry-breaking of \(SO(D - 1)\) spatial rotation.

The dimension of spacetime \(D\) does not affect the GPVRS in this work, for example, it is valid for two distinct dimensions of physical interest, \(D = 3\) or \(D = 4\). However, if the continuous spacetime symmetry of a system is less than \(SO(2)\) in \(D\) dimension, there is no advantage of applying PVRS or GPVRS to simplify the generic tensor integrals, and hence a singularity appears in the form factors when the dimension of spacetime is \(D = 2\) (see Supplemental Material [36]).

**Summary and outlook.**—In summary, we established two equivalent GPVRS in the absence of Lorentz invariance to systematically reduce the OLFDS for relativistic particles or pseudo-relativistic quasi-particles at arbitrary temperature and density. For physics satisfying the Lorentz invariance, the GPVRS automatically goes back to the well-known PVRS. We verified the GPVRS via the gauge invariance and various limiting cases. Additionally, the roles of symmetry-breaking and dimension were analyzed. The GPVRS is suitable for efficient calculations of the OLFDS in the systems without Lorentz invariance, such as hot and dense quark matter, neutrino gas, the early Universe at large lepton chemical potential, graphene, silicene, topological insulators, and topological semimetals.

Based on the GPVRS in this work, computer program packages can be developed for automatic algebraic calculation of the transition amplitudes and/or correlation functions just like that in the Lorentz-invariant systems [20–25]. In addition, this work opens up a new realm for reduction schemes in the absence of Lorentz invariance. The generalizations include possible extensions for non-relativistic systems and two-loop diagrams. However, these interesting problems are beyond the focus of this work and deserve further study in the future.

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