Improved sensitivity of interferometric gravitational wave detectors to ultralight vector dark matter from the finite light-traveling time

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Recently several studies have pointed out that gravitational-wave detectors are sensitive to ultralight vector dark matter and can improve the current best constraints given by the Equivalence Principle tests. While a gravitational-wave detector is a highly precise measuring tool of the length difference of its arms, its sensitivity is limited because the displacements of its test mass mirrors caused by vector dark matter are almost common. In this Letter we point out that the sensitivity is significantly improved if the effect of finite light-traveling time in the detector’s arms is taken into account. This effect enables advanced LIGO to improve the constraints on the $U(1)_B - L$ gauge coupling by an order of magnitude compared with the current best constraints. It also makes the sensitivities of the future gravitational-wave detectors overwhelmingly better than the current ones. The factor by which the constraints are improved due to the new effect depends on the mass of the vector dark matter, and the maximum improvement factors are 470, 880, 1600, 180 and 1400 for advanced LIGO, Einstein Telescope, Cosmic Explorer, DECIGO and LISA respectively. Including the new effect, we update the constraints given by the first observing run of advanced LIGO and improve the constraints on the $U(1)_B$ gauge coupling by an order of magnitude compared with the current best constraints.

INTRODUCTION

While the existence of dark matter has been firmly established by the observations, its identity is still unknown. Weakly Interacting Massive Particles are promising candidates of dark matter, and most of the searches have focused on the electro-weak mass scale [1–4]. However, despite the extensive efforts, they have not been detected, which motivates us to search for dark matter candidates in different mass range.

Among them is an ultralight boson, whose mass can be down to $\sim 10^{-22}$ eV [5]. Due to the large occupation number, it behaves as classical waves in our Galaxy, whose angular frequency is almost equal to its mass. A lot of searches have been proposed and conducted to detect this type of dark matter [9–30]. Some of them search for the oscillation of fundamental constants such as the fine-structure constant, which may be caused through its coupling to the Standard Model particles [10–12, 17, 18]. The metric perturbations generated by it can be detected in the pulsar timing array experiments [6–9]. If it has the axion-type coupling, it differentiates the phase velocities of the circular-polarized photons and may be detected with an optical cavity [21–25] or astronomical observations [31,33].

Recently, it was pointed out that gravitational-wave detectors are sensitive to ultralight vector dark matter arising as a gauge boson of $U(1)_B$ or $U(1)_B - L$ gauge symmetry [27], where $B$ and $L$ are the baryon and lepton numbers, respectively. The vector dark matter oscillates the test mass mirrors of the detectors though its coupling with baryons or leptons. Since the gravitational-wave detectors are highly precise measuring tools of the length difference of their arms, they are sensitive to tiny oscillations, and they can be used to probe the parameter space which has not been excluded by the Equivalence Principle (EP) tests [34,37]. The actual search was also conducted with the data from the first observing run (O1) of the LIGO detectors [35], and the constraints better than that from the Eötvös torsion pendulum experiment [34,35] was obtained for the $U(1)_B$ case [28].

What limits the sensitivity of gravitational-wave detectors is that the displacements of the test mass mirrors caused by the vector dark matter are almost common. It makes the length between the mirrors almost constant over the time, and the amplitude of the signal due to the length change is suppressed by a factor of the velocity of dark matter, which is in the order of $10^{-3}$. In this Letter we point out that the effect of the finite light-traveling time is crucial in this case. Even if the displacements are completely common, the optical path length of the laser light changes, as the test mass mirrors oscillate while the light is traveling in the arm. While it is suppressed by the product of oscillation frequency and the arm length, it can be more important than the contribution from the length change. It becomes more pronounced for the future gravitational-wave detectors, which have longer arms. This effect was taken into account in the previous studies for scalar dark matter [15,20] but never done before for vector dark matter.

This Letter is organized as follows. First we introduce the model we consider and the force exerted by the vector dark matter. Then we calculate the signal produced by the vector dark matter in a gravitational-wave detector.
taking into account the finite light-traveling time. Next we estimate the future constraints and show how much they are improved due to the new contribution. We also update the current constraints from the O1 data of the advanced LIGO detectors. Finally we summarize the results we have obtained. Throughout this Letter we apply the natural unit system, \( h = c = \epsilon_0 = 1. \)

**VECTOR DARK MATTER**

We consider a massive vector field, \( A^\mu, \) which couples to \( B \) or \( B - L \) current \( J^\mu_D \) (\( D = B \) or \( B - L \)), as dark matter. The Lagrangian is given by

\[
\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_A^2 A^\mu A_\mu - \epsilon_D e J^\mu_D A_\mu, \tag{1}
\]

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \) \( m_A \) is the mass of the vector field and \( \epsilon_D e \) is the coupling constant normalized to the electromagnetic one \( e. \)

The spatial components of the vector dark matter in our Galaxy can be modeled as \( 39 \)

\[
A = \sum_i A_i e_i \cos(\omega_i t - k_i \cdot x + \phi_i), \tag{2}
\]

where \( i \) is an index to identify each dark matter particle and we sum over their vector potentials. \( A_i \) is the amplitude, \( e_i \) the polarization unit vector, \( \omega_i \) is the angular frequency, \( k_i \) is the wave number and \( \phi_i \) is the constant phase of the \( i \)-th particle. The equation of motion gives the following dispersion relation,

\[
\omega_i = \sqrt{k_i^2 + m_A^2}. \tag{3}
\]

The norms of the wave numbers in our Galaxy are in the order of \( m_A v_{DM} \sim 10^{-3} m_A, \) where \( v_{DM} \) is the dark matter velocity dispersion in our Galaxy. Substituting it into \( \ref{dispersion} \) leads to

\[
\omega_i - m_A \sim m_A v_{DM}^2 \sim 10^{-6} m_A. \tag{4}
\]

This means the vector field, and hence the signal we observe, can be treated as monochromatic waves with frequency of \( m_A/2\pi \) over the coherence time, which is given by

\[
\tau \equiv \frac{2\pi}{m_A v_{DM}^2} \sim \frac{10^7}{m_A}, \tag{5}
\]

and the coherence is lost for a longer time interval.

The force exerted by the vector dark matter on a test mass mirror located at \( x_0 \) is given by

\[
F \simeq -\epsilon_D e Q_D A \simeq m_A \epsilon_D e Q_D \sum_i A_i e_i \sin(\omega_i t - k_i \cdot x_0 + \phi_i), \tag{6}
\]

where \( Q_D \) is the \( B \) or \( B - L \) charge of the test mass mirror. The test mass mirror oscillates around \( x_0 \) due to the force, and its position is given by \( x = x_0 + \delta x(t, x_0), \)

\[
\delta x(t, x_0) \simeq -\frac{\epsilon_D e Q_D}{m_A} \sum_i A_i e_i \sin(\omega_i t - k_i \cdot x_0 + \phi_i). \tag{7}
\]

\( Q_D/M \) is approximately given by

\[
\frac{Q_D}{M} \sim \begin{cases} 1/m_n, & (D = B) \\ 0.5/m_n, & (D = B - L) \end{cases} \tag{8}
\]

where \( m_n \) is the neutron mass.

**SIGNAL IN A GRAVITATIONAL-WAVE DETECTOR**

The signal in a gravitational-wave detector is given by

\[
h(t) = \frac{\varphi(t; n) - \varphi(t; m)}{4\pi\nu L}, \tag{9}
\]

where \( \nu \) is the laser frequency of the detector, \( L \) is the arm length, and \( n \) and \( m \) are unit vectors along the two arms of the interferometer. \( \varphi(t; n) \) is the phase of laser light returning back from the arm after the round trip.

The phase of laser light returning back at the time \( t \) is the same as that of laser light entering the arm at the time \( t - T_r \), where \( T_r \) is the round-trip time. Thus, we have

\[
\varphi(t; n) = 2\pi\nu(t - T_r) + \phi_0, \tag{10}
\]

where \( \phi_0 \) is a constant phase. The round-trip time is given by

\[
T_r = -x_1(t) + 2x_e(t - L) - x_1(t - 2L), \tag{11}
\]

where \( x_1(t) \) and \( x_e(t) \) represent the positions of the input and end test mass mirrors of the arm. With the coordinate system where the input test mass mirror is at \( x = 0 \) in the absence of vector dark matter, \( x_1(t) \) and \( x_e(t) \) are given by

\[
x_1(t) = n \cdot \delta x(t, 0), \quad x_e(t) = L + n \cdot \delta x(t, Ln). \tag{12}
\]

Substituting \( \ref{round-trip} \) and \( \ref{phi_0} \) into \( \ref{signal} \), we obtain

\[
\varphi(t; n) = -2\pi\nu (\delta L_1 + \delta L_2) + 2\pi\nu(t - 2L) + \phi_0, \tag{13}
\]

where

\[
\delta L_1 \equiv n \cdot (-\delta x(t, 0) + 2\delta x(t - L, 0) - \delta x(t - 2L, 0)) = -\frac{4\epsilon_D e Q_D}{m_A} \sin^2 \left( \frac{m_AL}{2} \right)
\]

\[
\delta L_2 = -\frac{4\epsilon_D e Q_D}{m_A} \sin^2 \left( \frac{m_AL}{2} \right)
\]
\[ \delta L_2 \equiv 2\mathbf{n} \cdot (\delta x(t - L, Ln) - \delta x(t - L, 0)) \]
\[ \approx \frac{2\epsilon_D c L Q_D}{m_A M} \times \sum_i A_i (\mathbf{n} \cdot \mathbf{e}_i) \cos (\omega_i (t - L) + \phi_i). \]

To derive the approximate expression of \( \delta L_2 \), we assume \( L|k| \ll 1 \), which is valid for the frequency range and the arm length of the gravitational-wave detectors we consider.

As can be seen in the definition of \( \delta L_2 \), it is from the deviation of the arm length from \( L \), and it has been taken into account in the previous studies. Compared to the gravitational waves with the same frequency, the wavelength of the vector dark matter is longer by a factor of \( 1/v_{DM} \sim 10^3 \). This makes force acting on the two test mass mirrors oscillate, as the test mass mirrors oscillate while light is traveling. This contribution is significant only when the oscillation frequency is comparable to the inverse of the round-trip time, and it is suppressed by a factor of \( v_{DM} \sim 10^{-3} \) through \( k_i \).

On the other hand, \( \delta L_1 \) is the new contribution we point out, which arises due to the finite light-traveling time in the arm. Even if the displacements are completely common and the arm length is constant, the optical path length can oscillate, as the test mass mirrors oscillate while light is traveling. This contribution is significant only when the oscillation frequency is comparable to the inverse of the round-trip time, and it is suppressed by \( m_A L \). Nevertheless, \( \delta L_1 \) is important in this case as \( \delta L_2 \) is suppressed more significantly. For the advanced LIGO detector, whose arm length is 4 km and frequency band is 10 – 1000 Hz, the ratio between \( \delta L_1 \) and \( \delta L_2 \) is given by

\[ \frac{\delta L_1}{\delta L_2} \sim \frac{m_A L}{v_{DM}} \sim 8 \left( \frac{m_A}{2\pi \times 100\text{Hz}} \right), \]

which indicates \( \delta L_1 \) is more significant in most of the frequency range. The ratio becomes larger for the future detectors, whose arms are longer, and the improvements due to \( \delta L_1 \) are more pronounced as shown in the next section.

\( \varphi(t; \mathbf{m}) \) can be calculated just by replacing \( \mathbf{n} \) by \( \mathbf{m} \) in \( \varphi(t; \mathbf{n}) \), and the signal is given by

\[ h(t) = h_1(t) + h_2(t), \]

where

\[ h_1(t) = \frac{2\epsilon_D c Q_D}{m_A M} \sin^2 \left( \frac{m_A L}{2} \right) \times \sum_i A_i (\mathbf{n} \cdot \mathbf{e}_i - \mathbf{m} \cdot \mathbf{e}_i) \sin (\omega_i (t - L) + \phi_i), \]

\[ h_2(t) = -\frac{\epsilon_D c Q_D}{m_A M} \sum_i A_i (\mathbf{n} \cdot \mathbf{e}_i) (\mathbf{n} \cdot \mathbf{k}_i) \cos (\omega_i (t - L) + \phi_i). \]

FUTURE PROSPECTS

Finally we estimate the sensitivities achieved by the future gravitational-wave experiments, taking into account the new contribution \( h_1 \). Here we consider advanced LIGO (aLIGO), Einstein telescope (ET) [40], Cosmic Explorer (CE) [41], DECIGO [42] and LISA [43] as representative gravitational-wave detectors.

The signal keeps its coherence only for the finite time of \( \tau \). One of the detection methods suitable for this type of signal is the semi-coherent method [20 59], where the whole data are split into segments whose lengths are \( \sim \tau \) and the squares of the Fourier components calculated with the segments are summed up incoherently. The detection threshold of the signal’s amplitude with this detection method can be estimated with

\[ \langle h^2 \rangle = \frac{S (m_A)^2}{T_{\text{eff}}}. \]

While the previous study [27] considered a different detection method, which correlates data from multiple detectors, the difference of the threshold amplitude is within an \( O(1) \) factor [20].

\( S(f) \) is the one-sided power spectral density (PSD) of noise in the \( h(t) \) channel. The PSDs for the representative detectors are shown in Fig. 1. \( T_{\text{eff}} \) is the effective observation time given by

\[ T_{\text{eff}} = \begin{cases} T_{\text{obs}}, & (T_{\text{obs}} < \tau) \\ \sqrt{T_{\text{obs}}}, & (T_{\text{obs}} \geq \tau) \end{cases} \]

where \( T_{\text{obs}} \) is the observational time. \( \langle h^2 \rangle \) is \( h^2(t) \) averaged over time. Averaging over random polarization and propagation directions, we can estimate it as follows,

\[ \langle h^2 \rangle = \langle h_1^2 \rangle + \langle h_2^2 \rangle, \]

\[ \langle h_1^2 \rangle = \frac{8\epsilon_D^2 c^2 v_{DM} Q_D^2}{3m_A^2 L^2 M^2} \sin^4 \left( \frac{m_A L}{2} \right) (1 - \mathbf{n} \cdot \mathbf{m}), \]

\[ \langle h_2^2 \rangle = \frac{2\epsilon_D^2 c^2 v_{DM}^2 Q_D^2}{9m_A^2 M^2} (1 - (\mathbf{n} \cdot \mathbf{m})^2). \]

The values of the arm length, \( L \), for the representative detectors are listed in Tab. 1. \( \mathbf{n} \cdot \mathbf{m} = 0 \) for aLIGO and CE, and \( \mathbf{n} \cdot \mathbf{m} = 1/2 \) for ET, DECIGO and LISA.

The future constraints on \( |\epsilon_D| \) estimated with [20] are shown in Fig. 2. Here, we assume the observation time of
2 years and apply $v_{\text{DM}} = 230 \text{ km} \text{s}^{-1}$, which is taken from [14] and applied in [27]. For comparison, the constraints without the contribution from $h_1$ are shown as dashed lines. The figure shows that the inclusion of $h_1$ significantly improves the constraints. The factor by which the constraints are improved depends on the mass, and the maximum improvement factors are 470, 880, 1600, 180 and 1400 for aLIGO, ET, CE, DECIGO and LISA respectively. The improvements are more significant for ET and CE compared to aLIGO because they have longer arms. The relatively significant improvement for LISA is due to its long arm length. The contribution from $h_1$ is canceled at $m_A \simeq 5 \times 10^{-16} \text{ eV}$ for LISA, where the frequency of the signal is equal to the inverse of the one-way-trip time of the light.

The current best constraints given by the EP tests are also shown as blue and orange lines in Fig. 2. The figure shows that the $h_1$ contribution makes the future constraints better than the current best constraints by orders of magnitude for both $U(1)_B$ and $U(1)_{B-L}$ cases. For reference, the constraints are improved by factors of 13000, 1200 and 15000 at $m_A = 10^{-16} \text{ eV}$, $10^{-14} \text{ eV}$ and $10^{-12} \text{ eV}$ respectively for the $U(1)_B$ case, and 1400, 130 and 1500 respectively for the $U(1)_{B-L}$ case. Notably the inclusion of $h_1$ enables aLIGO to improve the constraints on the $U(1)_{B-L}$ gauge coupling by an order of magnitude.

Next, we update the constraints given by the O1 data of aLIGO by incorporating $h_1$. The inclusion of $h_1$ improves the constraints by a factor of $\sqrt{\langle h_1^2 \rangle + \langle h_2^2 \rangle + \langle h_3^2 \rangle}$, and the improved constraints are shown as red lines in Fig. 2. The previously calculated constraints are also shown as green lines. As seen in the figure, the inclusion of $h_1$ makes the O1 constraints on the $U(1)_B$ gauge coupling better than the current best constraints by an order of magnitude at $m_A \gtrsim 2 \times 10^{-13} \text{ eV}$. The improved O1 constraints on the $U(1)_{B-L}$ gauge coupling are comparable to the current best constraints at $7 \times 10^{-13} \text{ eV} \lesssim m_A \lesssim 5 \times 10^{-12} \text{ eV}$.
Finally, we have updated the constraints given by the aLIGO O1 data incorporating the new contribution. The updated constraints on the $U(1)_B$ gauge coupling are better than the current best constraints by an order of magnitude.

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CONCLUSION

In this Letter we have pointed out that the effect of the finite light-traveling time is crucial for calculating the signal produced by ultralight vector dark matter in a gravitational-wave detector. By taking it into account properly we have calculated the new contribution to the signal. Then we have estimated the future constraints on the gauge coupling given by gravitational-wave detectors incorporating the new contribution. As a result, we have found that the new contribution significantly improves the future constraints given by gravitational-wave detectors. The factor by which the constraints are improved depends on the mass of the vector dark matter, and the maximum improvement factors are 470, 880, 1600, 180 and 1400 for aLIGO, ET, CE, DECIGO and LISA respectively. These improvements make the future constraints better than the current best constraints from the EP tests by orders of magnitude. Notably, it enables aLIGO to improve the constraints on the $U(1)_{B-L}$ gauge coupling by an order of magnitude.

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