Open problems in neutrino physics

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Abstract. Three open problems in neutrino physics are mentioned. A brief review of neutrino-less double beta decay (DBD) is given. The question of quenching of $g_A$ is discussed. Other possible scenarios, Majoron emission, sterile neutrinos and non-standard mechanisms, are presented and their implication to DBD discussed.

1. Introduction
Although neutrinos were introduced over 80 years ago, their properties remain to a large extent unknown. Three unanswered questions in neutrino physics are:

- What is the absolute mass scale of neutrinos? [1]
- Are neutrinos Dirac or Majorana particles? [2]
- How many neutrino species are there? [3]

An answer to the first question can be obtained by a measurement of:

1. The end point of the electron spectrum in single beta decay (KATRIN) [4]

2. The end-point of the spectrum of single electron capture (ECHO) [5]

An answer to the third question can be obtained by a study of neutrino oscillations in the range $<1$Km (Short-Baseline Neutrino, SBN, Program at FERMILAB).

An answer to all three questions can be obtained from neutrino-less double-beta decay (DBD) and related processes.

2. Double beta decay
Double beta decay is a process in which a nucleus X decays into a nucleus Y with the emission of two electrons or positrons and eventually other particles

$$^{A}_{Z} X_{N} \rightarrow^{A}_{Z+2} Y_{N-2} + 2e^+ + anything.$$ (1)

For processes not allowed by the standard model, the half-life is given by

$$r_{1/2}^{-1} = G_{0e} \left| M_{0e} \right| ^2 \left| f(m_1, U_{e1}) \right|^2.$$ (2)
where $G_{0\nu}$ is a phase-space factor, PSF, which depends on atomic physics, $M_{0\nu}$ are the nuclear matrix elements, NME, which depend on nuclear physics, and the function $f$ contains physics beyond the standard model. For processes allowed by the standard model, the half-life can be, to a good approximation, factorized in the form

$$
\left[ \frac{\tau_{1/2}^{2\nu}}{\nu_{1/2}} \right] = G_{2\nu} |M_{2\nu}|^2.
$$

For all processes and to extract physics beyond the standard model, one needs to calculate the phase space factors (PSF) and the nuclear matrix elements (NME).

2.1. Phase space factors

PSF were calculated in the 1980's by Doi et al. [6]. Also a calculation of phase-space factors is reported in the book of Böhm and Vogel [7]. These calculations use an approximate expression for the electron wave functions at the nucleus. PSF have been recently recalculated [8] with exact Dirac electron wave functions and including screening by the electron cloud. These new PSF are available from jenni.m.kotila@jyu.fi and are on the webpage nucleartheory.yale.edu.

2.2. Nuclear matrix elements

NME for neutrino-less DBD can be written as:

$$
M_{0\nu} = g_A^2 M^{(0\nu)}
$$

$$
M^{(0\nu)} = M^{(0\nu)}_{GT} - \left( \frac{g_{\nu}}{g_A} \right)^2 M^{(0\nu)}_{F} + M^{(0\nu)}_{T}.
$$

Several methods have been used for evaluation of $M_{0\nu}$: Quasiparticle Random Phase Approximation (QRPA), Shell Model (ISM), Interacting Boson Model (IBM-2), Density Functional Theory (DFT) among others. The same methods have also been used for evaluation of $M_{2\nu} = g_A^2 M^{(2\nu)}$.

For $0\nu$ processes, two scenarios have been extensively considered: (1) Emission and re-absorption of a light ($m_{light} \ll 1keV$) neutrino for which the $f$ function and the neutrino potential are $f = \langle m_{\nu} \rangle / m_{\nu}$, and $\nu(p) = \frac{2}{\pi} \frac{1}{p(p+\tilde{A})}$, where $\tilde{A}$ is the closure energy, $\tilde{A} = 1.12 A^{1/2}\text{MeV}$, also called long-range, and (2) Emission and absorption of a heavy neutrino ($m_{heavy} \gg 1GeV$) with $f_h = m_{\nu} \left( \frac{1}{m_{\nu}} \right) \left( \frac{1}{m_{\nu}} \right)$, and $\nu(p) = \frac{2}{\pi} \frac{1}{p m_{\nu} m_{\nu}}$, also called long-range. Recent results for NME for some of the models are summarized in figures 1 and 2.

By combining the PSF with the NME, one can set limits on neutrino masses. Figure 3 shows current limits on neutrino masses (light neutrino exchange) for KI-PSF [8], IBM-2 NME [9] and $g_A=1.269$.

2.3. Quenching of $g_A$

Results in figures 1 and 2 are obtained with $g_A=1.269$. It is well known from single beta decay and electron capture that $g_A$ is renormalized in models of nuclei. Two reasons for this renormalization are: (i) the limited model space in which the calculation is done, giving rise to a quenching factor $q_{N\nu}$ and (ii) the omission of non-nucleonic degrees of freedom, giving rise to a quenching factor $q_A$. 
Figure 1. NME for light neutrino exchange in IBM-2 [9], QRPA-Tü [10], ISM [11]. IBM-2 and QRPA-Tü results are with isospin restoration and Argonne short-range correlations.

Figure 2. Same as fig. 1 for heavy neutrino exchange.

Figure 3. Current limits on neutrino masses from NEMO-3 [12], CUORE-0 [13], IGEX [14], EXO [15], GERDA [16] and KamLAND-Zen [17].
Since $g_A$ appears to the second power in the NME, equation (5), and hence to the fourth power in the half-life, its quenching will have a dramatic effect on double-beta decay. Quenching of $g_A$ in $2
u\beta\beta$ double beta decay, consistent with single-beta decay, has been observed (see figure 5 of [9]). The question of whether or not $g_A$ in $0\nu\beta\beta$ is renormalized as much as in $2\nu\beta\beta$ is of much debate. The two processes differ by the momentum transferred to leptons. In $2\nu\beta\beta$ this is of the order of few MeV, while in $0\nu\beta\beta$ it is of the order of 100 MeV. The current view (2017) is that both factors, $q_A$ and $q_{Nq}$, contribute to $2\nu\beta\beta$, while only $q_A$ contributes to $0\nu\beta\beta$. This is due to the fact that, while the missing non-nucleonic degrees of freedom have a scale of excitation set by $m_A^2-m_e^2\approx 300 MeV$, the missing nuclear degrees of freedom have a scale set by $\langle m_{Nq}\rangle\approx 10 MeV$. This problem is currently being addressed from various sides. Experimentally, by measuring the matrix elements to and from the intermediate odd-odd nucleus in $2\nu\beta\beta$ decay by means of single charge exchange reactions ($^3$He, t) [18], and theoretically, by using effective field theories (EFT) to estimate the effect of non-nucleonic degrees of freedom (two-body currents) [19]. Very recently, an experimental program (NUMEN) [20] has been set up at LNS in Catania, Italy to measure both single and double charge exchange reaction intensities with heavy ions. This program will also provide useful information on the Fermi and Gamow-Teller matrix elements of interest in $0\nu\beta\beta$ and $2\nu\beta\beta$ decay.

Results in figure 3 are with $g_A=1.269$. Three scenarios for $g_A$ are [21]: (i) $g_A=1.269$ (free value obtained from neutron decay), $g_A=1$ (quark value), and (iii) $g_A=1.269A^{0.18}$ (maximal quenching obtained from $2\nu\beta\beta$ [9]). If $g_A$ is renormalized to maximal quenching, all estimates for half-lives should be increased by a factor of ~4-34 and limits on the average neutrino mass should be increased by a factor ~1.6-6, making it very difficult to reach in the foreseeable future even the inverted region.

3. Other possible scenarios
Possibilities to escape this negative conclusion are: (1) neutrino masses are degenerate and large. This possibility will be in tension with the cosmological bound on the sum of neutrino masses, $\sum_i m_i \leq 0.230 MeV$ [22]. (2) Other scenarios (Majoron emission, sterile neutrinos, …) must be considered. Here Majoron means a massless neutral boson [23] and sterile means a neutrino with no standard model interaction [3]. (3) Other non-standard mechanisms contribute [24]. Here non-standard means mechanisms other than V-A.

The scenario (2) of sterile neutrinos is currently being extensively investigated both experimentally, with planned experiments at FERMILAB and CERN-LHC, and theoretically. NME for sterile neutrinos of arbitrary mass can be calculated by using a transition operator as in Section 2, but with $f = \frac{m_s}{m_e}$ and

$$v(p) = \frac{2}{\pi} \frac{1}{\sqrt{p^2 + m_{sF}^2}} \left( \sqrt{p^2 + m_{sF}^2} + A \right),$$

where $m_{\nu}$ is the effective mass of sterile neutrinos. IBM-2 matrix elements for this scenario have been calculated [25]. PSF are the same as in Section 2. Contribution of all hypothetical neutrinos can be written in the general form [25]
\[ \left( e_{1/2}^{0n} \right)^{-1} = G_{0n} \left[ \frac{1}{m_{\nu_e}} \sum_{k=0}^{3} U_{e k}^2 m_k + \frac{1}{m_{\nu_x}} \sum_{i=1}^{3} U_{\nu_x}^2 m_i + \frac{1}{m_{\nu_y}} \sum_{j=1}^{3} U_{\nu_y}^2 m_j \right] M^{(0n)} + \left[ m_p \sum_{N} U_{e N}^2 \left( \frac{m_N}{p^2} \right) + m_p \sum_{k=1}^{3} U_{e k}^2 \right] M^{(0n)} \], \tag{6}

where the first three terms represent the contribution of light neutrinos (known with \( m_i < 0.1 \text{eV} \), unknown with \( m_i \sim 1 \text{eV} \), unknown with \( m_i \sim 1 \text{keV} \)) and the last two terms represent contributions of heavy neutrinos (unknown with mass \( m_N \sim 1 \text{MeV} \), unknown with mass \( m_j > 1 \text{GeV} \)).

Several types of sterile neutrinos have been suggested. (i) In scenario a, heavy sterile neutrinos with masses in the keV-GeV range have been suggested in [26]. (ii) In scenario b, Giunti and Laveder have suggested light sterile neutrinos with masses in the eV range to account for the reactor anomaly in oscillation experiments [27]. The presence of sterile neutrinos changes completely the picture, as shown in figure 4. With sterile neutrino (with properties of scenario b [27]) and \( \epsilon_A = 1.269 \), the inverted hierarchy is reachable by GERDA-PHASE II and CUORE.

\[ \text{Figure 4. Limits on neutrino masses when sterile neutrinos are included. Figure courtesy of J. Kotila adapted from [25].} \]

Scenario 3 of non-standard mechanisms is also being investigated both long-range, with Lagrangian [28]

\[ L_{\text{long}} = \frac{G_F}{\sqrt{2}} \left[ J_{\nu-A,\mu}^{\dagger} J_{\nu-A} + \sum_{\alpha,\beta} e_{\alpha \beta} J_{\alpha}^{\dagger} J_{\beta} \right], \tag{7} \]

and short-range, with Lagrangian [29]

\[ L_{\text{short}} = \frac{G_F^2}{\sqrt{2}} \left[ e_{\gamma} J J + e_{\gamma} J^{\mu} J_{\mu} + e_{\gamma} J^{\mu} J_{\mu} j + e_{\gamma} J^{\mu} J_{\mu} j + e_{\gamma} J^{\mu} J_{\mu} j + e_{\gamma} J^{\mu} J_{\mu} j \right]. \tag{8} \]

Predictions and limits for these mechanisms will be available in early 2018.

4. Conclusions

No matter what the mechanism of neutrino-less DBD is, its observation will answer the fundamental questions posed in the introduction. Indeed, if observed, neutrino-less DBD may provide evidence for
physics beyond the standard model other than the mass mechanism. Conversely, its non-observation will set stringent limits on other scenarios (sterile, ...), and on non-standard mechanisms. In this sense, neutrino-less DBD is a search for lepton number violation rather than a measurement of the neutrino mass.

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