Charge-Density-Wave Formation in the Doped Two-Leg Extended Hubbard Ladder

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We investigate electronic properties of the doped two-leg Hubbard ladder with both the onsite and the nearest-neighbor Coulomb repulsions, by using the the weak-coupling renormalization-group method. It is shown that, for strong nearest-neighbor repulsions, the charge-density-wave state coexisting with the $p$-density-wave state becomes dominant fluctuation where spins form intrachain singlets. By increasing doping rate, we have also shown that the effects of the nearest-neighbor repulsions are reduced and the system exhibits a quantum phase transition into the $d$-wave-like (or rung-singlet) superconducting state. We derive the effective fermion theory which describes the critical properties of the transition point with the gapless excitation of magnon. The phase diagram of the two-leg ladder compound, Sr$_{14-2x}$Ca$_{2x}$Cu$_2$O$_{41}$, is discussed.

KEYWORDS: doped Hubbard ladder, intersite Coulomb repulsion, spin gap, charge density wave, superconductivity

Electronic properties on ladder systems have been studied intensively both theoretically and experimentally, since the superconducting (SC) state was discovered in the self-doped two-leg ladder material Sr$_{14-2x}$Ca$_{2x}$Cu$_2$O$_{41}$ with $x \geq 12$ under pressure over 3 GPa. The substitution of Ca for Sr changes the hole-doping rate in the ladder Cu sites, where the rate varies monotonically from 0.07 to 0.25 with increasing $x$ from 0 to 12. A characteristic feature is the presence of a gap in magnetic excitations at temperature much higher than the SC transition temperature. Besides the SC state, recent experimental studies have focused on the charge dynamics in the slightly doped materials and verified collective modes from the sliding of the charge-density-wave (CDW) developed on ladder systems.

A global phase diagram is obtained on the plane of $x$ and temperature showing that the hole doping suppresses the CDW state followed by the insulating state without the CDW order, and then the high doping leads to the SC state under pressure. Quite recently the CDW collective modes are also suggested in the highly doped material Sr$_2$Ca$_{12}$Cu$_{23}$O$_{41}$. Therefore it is of particular interest to investigate the competition between the SC state and the CDW state in doped ladder systems.

From a theoretical point of view, the origin of the spin gap in the ladder compounds seems to be explained successfully for both the undoped $^{11,12}$ and doped $^{13}$ cases, and it is known that the $d$-wave-like SC (SCd) state appears in doped ladder systems. In addition, the charge-ordered state is also suggested when intersite interactions are included. Further the competition between the SCd state and the charge-ordered or charge-density-wave (CDW) state has been examined. However, critical behavior is not yet fully understood. In the present paper, the possible scenario of the instability of the CDW state and the competition between the CDW state and the SC state are proposed in the doped two-leg ladder of the extended Hubbard model (EHM) with nearest-neighbor repulsive interactions by extending the previous analytical calculations $^{14,15}$ to analyze the critical behavior in more detail.

We consider the two-leg EHM given by $H = H_0 + H_{\text{int}}$. The first term describes the hopping energies along and between legs:

$$H_0 = -t_{\parallel} \sum_{j,\sigma \sigma'} \left( c_{j,\sigma}^\dagger c_{j+1,\sigma'} + \text{H.c.} \right) - t_{\perp} \sum_{j,\sigma \sigma'} \left( c_{j,\sigma}^\dagger c_{j,\sigma+1} + \text{H.c.} \right),$$

where $c_{j,\sigma}$ annihilates an electron of spin $\sigma (=\uparrow, \downarrow)$ on rung $j$ and leg $l (=1, 2)$. The Hamiltonian $H_{\text{int}}$ denotes interactions between electrons:

$$H_{\text{int}} = U \sum_j n_{j,\uparrow} n_{j,\downarrow} + V_{\parallel} \sum_{j,\sigma} n_{j,\sigma} n_{j+1,\sigma} + V_{\perp} \sum_{j} n_{j,\uparrow} n_{j,\downarrow},$$

where $U$ represents on-site repulsion and $V_{\parallel}$ ($V_{\perp}$) represents intrachain (interchain) nearest-neighbor repulsion with $n_{j,\sigma} = c_{j,\sigma}^\dagger c_{j,\sigma}$ and $n_{j,\sigma} = n_{j,\sigma+1} + n_{j,\sigma+1}$. The $H_0$ term is diagonalized by using the Fourier transform of $c_{\sigma}(k)$ where $k = (k_\parallel, k_\perp)$ with $k_\perp = 0$ or $\pi$. The energy dispersion is given by $E(k) = -2t_{\perp} \cos k_\perp - t_{\parallel} \cos k_\parallel$. Here we consider the case with finite hole doping $\delta$ satisfying $t_{\perp} < 2t_{\perp} \cos^2 \delta$, in which both the bonding $(k_\perp = 0)$ and the antibonding $(k_\perp = \pi)$ energy bands are partially filled and the Fermi points are located at $k_{F,0} = \frac{\pi}{2}(1 - \delta) + \lambda$ and at $k_{F,\pi} = \frac{\pi}{2}(1 - \delta) - \lambda$ with $\lambda \equiv \sin^{-1}\left[ \frac{t_{\perp}}{(2t_{\parallel} \cos^2 \delta)} \right]$. We examine the case of small $\delta$ by neglecting the differences in the Fermi velocities of the bonding/antibonding band, i.e., $v_{F,0} = v_{F,\pi} (\equiv v_\lambda)$.

Following the standard weak-coupling approach ($g$-ology), the linearized kinetic energy is given by $H_0 = \sum_{k,\rho,\sigma} g_{k,\parallel} (p_{k,\parallel}) - k_{F,\rho,\delta} \phi_{k,\rho,\sigma}^\dagger \phi_{k,\rho,\sigma} (k)$, where the index $\rho = +/-$ denotes the right-/left-moving electron. By introducing field operators by $\phi_{\rho,\sigma,\zeta}(x) = L^{-1} \sum_k e^{ib_{\rho,\sigma}^\dagger k_{\parallel} x} c_{\rho,\sigma}(k_{\parallel}, k_\perp)$ with $\zeta = (+/-)$ for $k_\perp = 0(\pi)$ and $L$ being the system size, the interactions near the Fermi points are rewritten as $H_{\text{int}} = (1/4) \int dx \sum_{\rho,\sigma} \sum_{\zeta = \pm} H_{\text{int},\zeta}$, where $H_{\text{int},\zeta}$ is given by

$$H_{\text{int},\zeta} = g_{1(2)\zeta} \left\{ \psi_{\rho,\sigma,\zeta,\uparrow}^\dagger \psi_{\rho,\sigma,\zeta,\downarrow} + \psi_{\rho,\sigma,\zeta,\downarrow}^\dagger \psi_{\rho,\sigma,\zeta,\uparrow} \right\} + g_{1(2)\zeta} \left\{ \psi_{\rho,\sigma,\zeta,\uparrow}^\dagger \psi_{\rho,\sigma,\zeta,\downarrow} + \psi_{\rho,\sigma,\zeta,\downarrow}^\dagger \psi_{\rho,\sigma,\zeta,\uparrow} \right\},$$

and $\bar{\sigma} = \uparrow (\downarrow)$ for $\sigma = \uparrow (\downarrow)$, $\epsilon = \zeta_1 \xi_1$ and $\bar{\epsilon} = \zeta_1 \xi_2$. The summation of the band index $\zeta_i (i = 1, 2, \ldots, 4)$ is taken under the Nucl. Phys. Lett. 142, 1 (1978).
condition $\zeta_1\zeta_2\zeta_3\zeta_4 = +1$. The coupling constants $g_{i\ell}^{\pm\pm}$ and $g_{i\ell}^{\pm\pm}$ with $i = 1(2)$ corresponding to the backward (forward) scattering are given by $g_{i\ell}^{\pm\pm} = (l_{1V_1} + m_{1V_1}^\pm)$ and $g_{i\ell}^{\pm\pm} = (U + l_{2V_1} + m_{2V_1}^\pm)$ where $l_{1\pm} = \pm 1$, $m_{1\pm} = -2\cos\phi\delta\cos2\lambda$, $m_{2\pm} = -2\cos\phi\delta\cos2\lambda$, and $m_{2\pm} = +2\cos2\lambda$. We neglect the umklapp scattering processes because they are given by term with $\cos^2\lambda$. In Eq. (3), the phase field $\phi$ of the SC

As possible states, we consider the singlet $d$-wave superconducting (SCd) state, the CDW state, and the $p$-density-wave (PDW) state. The PDW state corresponds to the spin-Peierls state in the limit of $\delta \to 0$. The order parameter of the SCd state is given by $O_{\text{SCd}} = N^-1\sum_n^N(c_{j,1,1}c_{j,2,1} - c_{j,1,1}c_{j,1,2})$, while those of the density waves are $O_{\text{CDW}} = N^-1\sum_n^N f_n(k)c_{j,n}(k)c_{j,(k + Q)}$, with $Q = (\pi(1 - \delta), \pi)$. $f_{\text{CDW}} = 1$ and $f_{\text{PDW}} = \sin\phi_{ij}$. These operators are rewritten in terms of bosonic phase fields by applying the Abelian bosonization method.\(^{16,17}\) The field operators of the right- and left-moving electrons are as $\psi_{\sigma,\rho}(\pi) = \eta_{\sigma,\rho}(2\pi a)^{-1/2}\exp[ipk_{\rho,\pi}x + ip\phi_{\rho,\pi}(x)]$ where $s = +$ for $\sigma = +$ and $s = -$ for $\sigma = -$. The fields satisfy the commutation relations: $[\phi_{\rho,\pi}(x), \phi_{\sigma,\pi}(x')] = i\epsilon_{\rho,\pi}\sigma_{\rho,\pi}(x - x')\delta_{\rho,\pi}\delta_{\rho,\pi}'$ and $[\phi_{\rho,\pi}(x), \phi_{\rho,\pi}^\dagger(x')] = i\epsilon_{\rho,\pi}\sigma_{\rho,\pi}(x - x')\delta_{\rho,\pi}\delta_{\rho,\pi}'$. The Klein factors $\eta_{\sigma,\rho}$ are introduced in order to return the correct anticommutation relations.\(^{14}\) For calculating physical quantities, the field $\phi_{\rho,\pi}(x)$ is replaced by new bosonic fields: $\phi_{\rho,\pi} = (\phi_{\rho,\pi}^a + \delta_{\rho,\pi})$ and $\psi_{\rho,\pi}^a = \phi_{\rho,\pi}^a + \xi_{\rho,\pi}^a + \eta_{\rho,\pi}^a = (\phi_{\rho,+}^a + \phi_{\rho,-}^a)$ with $\phi_{\rho,\pi}^0 = \phi_{\rho,\pi}$ and $\phi_{\rho,\pi}^a$ represent charge and spin fluctuations, respectively, and the suffixes refer to the even and odd sectors. They satisfy $[\phi_{\rho,\pi}^a(x), \psi_{\rho,\pi}^b(x')] = -i\epsilon_{\rho,\pi}\sigma_{\rho,\pi}(x - x')\delta_{\rho,\pi}'$ with $\Theta(x)$ being the Heaviside step function. In terms of $\phi_{\rho,\pi}$ and $\psi_{\rho,\pi}^a$, the order parameters $O = \int d\mathbf{x}H$ are given by

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We can also rewrite the Hamiltonian in terms of the bosonic phase variables. In Eq. (3), the phase field $\phi_{\rho}$ appears in the form $\cos2\phi_{\rho} + 4\lambda\phi_{\rho}$. Since we can safely assume that $t_\perp$ is a relevant perturbation for $t_\perp$ being not so small,\(^{18,19}\) the term with $\cos2\phi_{\rho} + 4\lambda\phi_{\rho}$ would become irrelevant, and thus we discard it in the following. We also neglect the $\cos2\phi_{\rho} - \cos2\phi_{\rho}$ term since its scaling dimension is larger than 2. Then our Hamiltonian reduces to $H = \int d\mathbf{x}H$ with

where the coupling constants of the harmonic terms are given by $g_{\rho,\pi} = \sum_{\pi = \pm}^\pm f_{\rho,\pi}(g_{\rho,\pi}^{\pm\pm} + (-g_{\rho,\pi}^{\pm\pm}/g_{\rho,\pi}^{\pm\pm}))$ with $r = \pm$, $f_{\rho,\pi}^r = 1$, $f_{\rho,\pi}^r = \epsilon$, and $f_{\rho,\pi}^r = -1$ and $f_{\rho,\pi}^r = -\epsilon$. The coupling constants of the nonlinear terms are $g_{\rho,\pi} = (\pm g_{\rho,\pi} - g_{\rho,\pi}) = (g_{\rho,\pi} - g_{\rho,\pi} - g_{\rho,\pi} - g_{\rho,\pi}) = 0$. We choose following six coupling constants:

The present model and the above treatment are quite similar to those in Ref. 15. However, the application of the renormalization-group (RG) method to Eq. (5) is complicated to estimate excitation gaps of spin modes properly. Therefore, we fermionize the spin part of Eq. (5)\(^{14}\) by introducing spinless fermion fields $\psi_{\rho,\pi}(x) = \eta_{\rho,\pi}(2\pi a)^{-1/2}\exp[\pm i\phi_{\rho,\pi}(x)]$ where $r = \pm$ and $[\eta_{\rho,\pi}, \eta_{\rho,\pi}] = 2\delta_{\rho,\pi}$. By using the SU(2) constraints and the Majorana fermions $\xi_{\rho}^n (n = 1 \sim 4)$, the equation (5) is rewritten as

where $\psi_{\rho,\pi} = (\xi_{\rho,\pi}^{1,2} + i\xi_{\rho,\pi}^{3,4})/\sqrt{2}$, $\psi_{\rho,\pi} = (\xi_{\rho,\pi}^{1,2} + i\xi_{\rho,\pi}^{3,4})/\sqrt{2}$, and $\xi_{\rho} = (\xi_{\rho,\pi}^{1,2} + i\xi_{\rho,\pi}^{3,4})/\sqrt{2}$. Thus the effective theory for the spin sector becomes $O(3) \times Z_2$ symmetric, as seen in the isotropic Heisenberg\(^{17}\) and half-filled Hubbard ladder.\(^{14}\)

We investigate the low-energy behavior by using the perturbative RG method with the lattice constant $a \to ae^{d/2}$. Following six scaling equations are obtained:

\begin{align}
\frac{d}{dt}G_{\rho,\rho} &= -3\frac{G_{\rho,\rho}}{4G_{\rho,\rho}} - \frac{1}{4G_{\rho,\rho}}, \\
\frac{d}{dt}G_{\sigma,\sigma} &= -2G_{\sigma,\sigma} - G_{\sigma,\sigma} - \frac{1}{2G_{\sigma,\sigma}}, \\
\frac{d}{dt}G_{\sigma,\rho} &= -2G_{\sigma,\rho} + G_{\sigma,\rho} - \frac{1}{2G_{\sigma,\rho}}, \\
\frac{d}{dt}G_{\rho,\sigma} &= -G_{\rho,\sigma} - 2G_{\rho,\sigma} - G_{\rho,\sigma} - G_{\rho,\sigma}, \\
\frac{d}{dt}G_{\rho,\rho} &= -G_{\rho,\rho} - G_{\rho,\rho} - G_{\rho,\rho} - G_{\rho,\rho}, \\
\frac{d}{dt}G_{\rho,\rho} &= -G_{\rho,\rho} - G_{\rho,\rho} - G_{\rho,\rho} - G_{\rho,\rho},
\end{align}
and \( \frac{d G_{\rho \nu}}{dt} = 0 \) where \( G(0) = g/(2\pi V_F) \). We noted that these RG equations can be also derived directly from Eq. (5). We analyze the RG equations numerically for \( U > 0, V_i > 0 \) and \( V_{i,l} > 0 \). For small \( V_{i,l}/U \) and \( V_l/U \), the limiting behavior of RG equations is given by \((G_{\rho \nu}, G_{\sigma \nu}, G_{\sigma \sigma}, G_{\sigma \sigma}, G^*_{\sigma \sigma}) = (-, -, +, +, +)\) which corresponds to \((g_{\rho \nu}^2, g_{\sigma \nu}^2, g_{\sigma \sigma}^2, g_{\sigma \sigma}^2, g_{\sigma \sigma}^* g_{\sigma \sigma}^*)\) = \((-\!, 0, 0, 0\!\), 0\) in Eq. (5). The relevant behavior of coupling constants implies that the phases are locked in order to minimize the cosine potential in Eq. (5). The positions of phase locking and the corresponding ground states are summarized in Table I. Since the \( \theta_{\rho \nu} \)-field is conjugate to \( \phi_{\sigma \nu} \), these two fields cannot be locked at the same time. From Eq. (4), the nonvanishing order parameter is \( \phi_{\rho \nu} \). Since the correlation function of the operator \( \rho_{\sigma \nu} \) exhibits power-law behavior, we obtain that the SCd fluctuation becomes quasi-long-range ordered (quasi-LRO) in this case. We note that the SCd state moves to the D-Mott or \( D'\)-Mott state in the limit of \( \delta \to 0 \).14 For large \( V_{i,l}/U \) and \( V_l/U \), the limiting behavior of RG equations is now given by \((G_{\rho \nu}, G_{\sigma \nu}, G_{\sigma \sigma}, G_{\sigma \sigma}, G^*_{\sigma \sigma}) = (-, +, +, +, +)\), corresponding to \((g_{\rho \nu}^2, g_{\sigma \nu}^2, g_{\sigma \sigma}^2, g_{\sigma \sigma}^2, g_{\sigma \sigma}^* g_{\sigma \sigma}^*)\) = \((+, +, 0, 0, 0\) in this case, the dominant order parameters are given by \( \phi_{\rho \nu} \) and \( \phi_{\rho \rho} \) both of which lead to the quasi-LRO with the same exponent of the correlation functions. We call this coexisting state the CDW+PDW state.

In order to analyze the properties near the critical point of the transition between the SCd state and the CDW+PDW state, we restrict ourselves to the case where the mass of the charge mode \( \rho \) is larger than those of the spin modes (\( \sigma \)). The \( \theta_{\rho \nu} \)-field is locked by the cosine potential below the mass scale of the charge mode \( m_{\rho \nu} \). By replacing \( \cos 2\theta_{\rho \nu} \) with its average value \( \cos 2\theta_{\rho \nu} \) in Eq. (5), the effective low-energy Hamiltonian for the spin degrees of freedom is obtained as14,17

\[
H_{\sigma} = -\frac{\sigma V_F}{2} (\xi^+ \cdot \partial_i \xi^4 - \xi^- \cdot \partial_i \xi^-) - m_0^2 \xi^+ \cdot \xi^- - m_0^2 \xi^-\xi^+ + \frac{g_{\sigma \nu}^2}{2} (\xi^+ \cdot \xi^-)^2 - g_{\sigma \nu}^2 (\xi^+ \cdot \xi^-)\xi^+ \cdot \xi^- - g_{\sigma \sigma}^2 \xi^+ \cdot \xi^- + 2 g_{\sigma \sigma}^* g_{\sigma \sigma}^* \xi^+ \cdot \xi^-,
\]

where \( m_0^2 \) and \( m_0^1 \) represent bare masses of the Majorana triplet and singlet sector: \( m_0^2 = (\sigma V_F/2\pi a)(U - V_{i,l} - 2U_{i,l} \cos \pi \delta) \) and \( m_0^1 = (\sigma V_F/2\pi a)(U - V_{i,l} + 2V_{i,l} \cos (2\pi \delta) + 2U_{i,l} \cos 2\delta) \). The quantity \( m_0^2 \) (\( m_0^1 \)) has physical meanings of the gap in the magnon (soliton) excitation in the spin modes of the ladder.17 Equation (9) is further analyzed in terms of the following scaling equations for coupling constants:

\[
\frac{d G_l}{dl} = G_i - 2G_l G_{\sigma \nu} - G_{\nu} G_{\sigma \nu}, \quad (10a)
\]
\[
\frac{d G_s}{dl} = G_s - 3G_l G_{\sigma \nu}, \quad (10b)
\]
\[
\frac{d G_{\sigma \nu}}{dl} = -G_{\sigma \nu}^2 - G_{\sigma \nu}^2 - G_{\sigma \nu}^2, \quad (10c)
\]
\[
\frac{d G_{\sigma \nu}}{dl} = -2G_{\sigma \nu} G_{\sigma \nu} - G_{\sigma \nu}, \quad (10d)
\]

where \( G_i = m_0^2/\nu_F, G_s = m_0^1/\nu_F, \) and \( G_{\sigma \nu} = g_{\sigma \nu}/2\pi V_F. \) The couplings \( G_i \) and \( G_s \) are relevant, while \( G_{\sigma \nu} \) is marginal. In Eq. (10), the \( G_s \) term as a function of \( l \) increases rapidly compared with other \( G_i \) and becomes relevant at \( l = l_i \) corresponding to the energy scale of a gap in the Majorana singlet mode \( m_s \approx t_i e^{-l_i} \), where we stop the calculation of Eq. (10). The mode remained below the energy scale of \( m_s \) is the Majorana triplet sector. The effective theory for this mode is given by \( H_{\sigma \nu} = -t_i^2 V_F (\xi^+ \cdot \partial_i \xi^4 - \xi^- \cdot \partial_i \xi^-) - m_0^2 \xi^+ \cdot \xi^- - g_{\sigma \nu}^2 (\xi^+ \cdot \xi^-)^2 \) with \( m_0^2 = \nu_F (l_s) \) and \( g_{\sigma \nu}^2 = 2 \nu_F (l_s) \). Then we solve the RG equations given by \( dG_i/\nu_F = G_i - 2G_i G_{\sigma \nu} + 2G_{\sigma \nu}^2 = -2G_{\sigma \nu}^2 + G_l^2 \) with the initial conditions \( G_i(l_s) = m_0^2/\nu_F \) and \( G_{\sigma \nu}(l_s) = g_{\sigma \nu}/2\pi V_F. \) We easily find that these RG equations have two stable fixed points \( G_i, G_{\sigma \nu} = (+\infty, -\infty) \) and \( (-\infty, -\infty) \), corresponding to the SCd state and the CDW+PDW state, respectively, where the magnitude of the gap in the Majorana triplet sector can be estimated from \( m_s \approx t_i e^{-l_i} \) \( sgn(G_i) \) where \( l_s \) is determined by \( G_i(l_s) = 1 \) (see Table I). There are also two unstable fixed points \( G_i, G_{\sigma \nu} = (0, 0) \) and \( (0, -\infty) \), corresponding to the second-order and first-order phase transitions,14 while only the former transition is obtained in the present numerical calculation.

From the numerical integration of the RG equations, we obtain the ground-state phase diagram shown in Fig. 1. The SCd state (the CDW+PDW state) is obtained for \( V_{i,l}/U + V_{i,l}/U \gtrless 0.4 (\gtrless 0.4) \). The SCd state, on the one hand, is stabilized by the on-site repulsive interaction, which segregates up-spin from down-spin on the same site and leads to the singlet pairing on a rung. On the other hand, the CDW+PDW state is obtained due to the nearest-neighbor repulsive interactions, which induce density wave leading to the singlet state on the same site or chain. The effect of \( V_{i,l} \) is slightly larger than that of \( V_{i,l} \), although both the intersite interactions have essentially the same effect of inducing the CDW+PDW state. In Fig. 2, the change from the CDW+PDW state to the SCd state is shown with increasing the doping \( \delta \) (\( > 0.05 \)). The novel aspect of the present paper is the competition induced by the doping which reduces the effect of only \( V_{i,l} \) as shown in Eq. (6). In the inset, we show the respective masses estimated from \( m_{\sigma \nu} = t_i \exp(-l_{\sigma \nu}) \) \( (a = t, s, \rho) \) by noting that the corresponding coupling constant \( G_{\sigma \nu} \) becomes of the order of unity at \( l = l_s \). Our system exhibits a second-order phase transition and the magnon excitation gap vanishes at the quantum critical point (QCP). The critical property for the Majorana triplet sector, which differs from that the conventional Tomonaga-Luttinger liquid, is described by the SU(2) Wess-Zumino-Novikov-Witten model with the central charge \( c = 3/2.17 \).

In the present paper, by applying the weak-coupling RG method to the EHM on two-leg ladder, we have shown that
the doping $\delta$ suppresses the CDW+PDW quasi-LRO state and yields the system to the QCP, and that the SCd quasi-LRO state is stabilized at further doping. Here we discuss the experimental results of the two-leg ladder compound Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$. The phase diagram of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$ obtained in Ref. 9 resembles our phase diagram of Fig. 2 if the magnitude of gap $|m_t|$ is regarded as the transition temperature. On closer look, our phase diagram is contrast to the features of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$, that the resistivity above the transition temperatures shows an insulating behavior and there is no experimental evidence of the QCP between the CDW state and the SC state. In order to explain the phase diagram of Sr$_{14-x}$Ca$_x$Cu$_{24}$O$_{41}$, the dimensionality effect and/or the disorder effect has been discussed. The quantum critical behavior would be smeared out by these effects, which are not taken into account in the present paper. However it will be still interesting to examine the competing region in the sense that the magnon gap would become extremely small and anomalous behavior can be expected at temperatures higher than characteristic energies of the disorder and the dimensionality. We note that the origin of the high temperature insulating phase is still unknown and the analysis is left for a future study.

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