WIMPy Leptogenesis With Absorptive Final State Interactions

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Abstract

We consider a class of leptogenesis models in which the lepton asymmetry arises from dark matter annihilation processes which violate $CP$ and lepton number. Importantly, a necessary one-loop contribution to the annihilation matrix element arises from absorptive final state interactions. We elucidate the relationship between this one-loop contribution and the $CP$-violating phase. As we show, the branching fraction for dark matter annihilation to leptons may be small in these models, while still generating the necessary asymmetry.
I. INTRODUCTION

The baryon asymmetry of the universe (BAU) is both evident in observation [1, 2] and well-motivated theoretically at a variety of scales and epochs. The existence of non-baryonic dark matter (DM) is also well established by observational signatures of its gravitational interactions at several different scales [1, 3–5]. Calculations of primordial light element abundances predicted by big bang nucleosynthesis (BBN) [6], recent observations of cosmic microwave background (CMB) anisotropy by the Wilkinson Microwave Anisotropy Probe (WMAP) [3] and Planck satellite [7], the galaxy power spectrum obtained by the Sloan Digital Sky Survey (SDSS) [4], and a variety of other data create a combined picture indicating that the densities of baryonic and dark matter in our universe are

\[
\Omega_b h^2 \sim 0.022,
\]

\[
\Omega_{DM} h^2 \sim 0.12.
\]

Both BAU and DM are strong motivations for physics beyond the Standard Model (SM), as neither significant baryon number (B) violation nor appropriate non-luminous gravitationally interacting fields exist within the SM. The most common models that explain DM provide a Weakly Interacting Massive Particle (WIMP) to account for the observed density [5]. A typical WIMP, with weak scale mass and couplings, will depart from equilibrium in the early universe when self-annihilation freezes out, yielding roughly the correct relic DM density. It is natural to wonder if the annihilation process which determines the dark matter density can also yield a baryon asymmetry.

Many models have been proposed to explain BAU [2], but generally, in order for a process to produce the observed baryon asymmetry, the Sakharov conditions [8] for baryogenesis must be satisfied. The Sakharov conditions require:

- a violation of baryon number (or lepton number (L), if the asymmetry is generated above the electroweak phase transition (EWPT)),
- a violation of \(C\) and \(CP\),
- a departure from thermal equilibrium.

Although decaying dark matter with a weak scale mass has been suggested as a model for baryogenesis [9], a WIMP framework which satisfies the first two conditions would automatically accommodate the third. This mechanism of “WIMPy baryogenesis” was recently proposed by Cui, Randall and Shuve [10]. In a typical WIMPy model, dark matter, denoted \(X\) with \(m_X \sim \text{TeV}\), annihilates to an additional new weak scale field, denoted \(H\),
and a quark (or lepton) via CP-violating interactions. The CP-violating phase arises from interference between tree-level and loop diagrams. $H$ then subsequently decays into light particles, including particles which are uncharged under Standard Model gauge symmetries, thus sequestering any negative baryon (or lepton) asymmetry. Between the time $X$ begins to depart from equilibrium, $T \sim m_X$, and when $X$ freezes out with the correct relic density, the correct baryon asymmetry can also be produced. If DM annihilates to leptons, then any lepton asymmetry is transferred to baryons through electroweak sphalerons at temperatures above the EWPT.

While in Ref. [10], UV-complete models mediated by weak scale pseudoscalars are presented, generalization to an effective field theory (EFT) was more recently shown by Bernal, Josse-Michaux and Ubaldi [11]. The EFT WIMPy model achieves results similar to a UV-complete model, with DM annihilating to $H$ and quarks through all possible dimension six operators. This allows one to focus on aspects of the generation of a baryon asymmetry which can arise from many different UV completions.

Subsequent work [12] provided a general analysis of how washout processes can erase an asymmetry. The key constraint on WIMPy baryogenesis models, either constructed in a UV-complete or EFT formalism, is that the couplings which must be added to generate needed one-loop contributions also introduce dangerous tree-level washout processes. These processes are “dangerous” because they are not Boltzmann-suppressed, even when some of the external particles are heavy. As a result, features of the model must be chosen to render these processes less dangerous. This work focused on the particular case where $m_H \sim m_X$, and found a class of models in which $H$ may decay entirely into Standard Model particles, while still generating the required baryon asymmetry. For these processes, the asymmetry is largely generated after the washout processes are frozen out.

In this work, we consider WIMPy leptogenesis [13], in which dark matter annihilation directly produces a lepton asymmetry which is converted into a baryon asymmetry by electroweak sphalerons. We focus on an effective field theory approach, in which dark matter annihilation can be parametrized in terms of a set of effective operators. In contrast to previous work, here we will obtain a CP-violating phase at tree-level from the interference between dimension six operators. We will find that one-loop corrections effects are necessary for generating an asymmetry, but they can be sequestered in absorptive terms in the final state interactions of $H$. Because these terms do not directly affect either the Standard Model or dark sector, we will be able to relax some of the parameter space constraints typically

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1 If dark matter annihilation directly produces a baryon asymmetry, then $H$ would have to be charged under $SU(3)_{\text{qcd}}$. Models of this form are constrained by direct searches for color-charged particles at colliders [14], and Standard Model quarks produced by the decay of $H$ would affect the baryon asymmetry. Though such models are certainly possible, we will for simplicity focus on the case where dark matter annihilation directly produces a lepton asymmetry, along with a particle $H$ whose decay does not affect the baryon or lepton asymmetry.
found in WIMPy baryogenesis models.

We will find that some dangerous tree-level washout processes, which are often introduced by the interactions needed to obtain CP-violation, are not present in the models we consider. Moreover, the ratio of CP-violating to CP-conserving interactions is not controlled by the scale of new physics. As a result, one can generate the correct baryon asymmetry for \( \sim 1.5 \) TeV dark matter even if the scale of new physics is as large as 10 TeV.

The outline of the paper is as follows. In section II, we review the reason why a one-loop contribution is necessary, and describe our class of models. In section III we describe the solution of the Boltzmann equations which govern the dark matter and lepton densities. In section IV, we present our results and compare to those of other models of WIMPy baryogenesis. We conclude with a discussion of our results in section V.

II. OUR MODEL

We will consider models with a single dark matter candidate \( X \) (stabilized by a \( Z_2 \) symmetry) whose annihilation can be modeled with a set of effective four-point operators. Each effective operator can be written as a product of an initial state dark matter bilinear and a final state bilinear. This final state bilinear couples a Standard Model lepton (\( L \)) and an exotic fermion (\( H \)). In order for the annihilation process to contribute to a lepton asymmetry, the effective operators must violate \( C, CP \) and \( L \).

We can already constrain the set of effective operators which are relevant. For dark matter annihilation to contribute to a baryon asymmetry, there must be interference between two matrix element terms with different phases (otherwise, any \( CP \)-violating phase would cancel in the squared matrix element). At lowest dimension, there are only two sets of fermionic dark matter bilinears which can interfere in an annihilation process [16]:

- \( iX\gamma^5X \) and \( X\gamma^0\gamma^5X \) can both annihilate an \( S = 0, L = 0, J = 0 \) (\( CP \)-odd) initial state.
- \( \bar{X}\gamma^iX \) and \( \bar{X}\sigma^{0i}X \) can both annihilate an \( S = 1, L = 0, J = 1 \) (\( CP \)-even) initial state.

If the dark matter is spin-0, then there are no dimension 2 or dimension 3 bilinears which can interfere. Moreover, if dark matter is a Majorana fermion, the second set of bilinears above vanish.

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2 In some models of [11] it is necessary to use a \( Z_4 \) symmetry, since one of the final state particles is also charged under this stabilizing symmetry. That is not the case here, unless the effective operator respects \( SU(2)_L \). In that case, we would also need a \( Z_4 \) symmetry under which \( X \) and \( H \) are charged, in order to protect \( H \) from Standard Model decays which could wash out the asymmetry.
We will thus consider two sets of effective operators:

\[ O_{S=0} = \frac{\lambda_1}{2M_*^2}(i\vec{X}\gamma^5 X)(\bar{H}P_L L) + \frac{\lambda_1^*}{2M_*^2}(i\vec{X}\gamma^5 X)(\bar{L}P_R H) \]

\[ + \frac{\lambda_2}{2M_*^2}(\vec{X}_\gamma\gamma^5 X)(\bar{H}\gamma^\mu P_L L) + \frac{\lambda_2^*}{2M_*^2}(\vec{X}_\gamma\gamma^5 X)(\bar{L}\gamma^\mu P_L H) \]

\[ O_{S=1} = \frac{\lambda_3}{M_*^2}(\vec{X}_\gamma\mu X)(\bar{H}\gamma_\mu P_L L) + \frac{\lambda_3^*}{M_*^2}(\vec{X}_\gamma\mu X)(\bar{L}\gamma_\mu P_L H) \]

\[ + \frac{\lambda_4}{M_*^2}(\vec{X}\sigma^{\mu\nu} X)(\bar{H}\sigma_{\mu\nu} P_L L) + \frac{\lambda_4^*}{M_*^2}(\vec{X}\sigma^{\mu\nu} X)(\bar{L}\sigma_{\mu\nu} P_R H) \] (2)

where \( L \) is a Standard Model lepton and \( H \) is an exotic field with no lepton number. Note that these operators cannot interfere with each other, as they annihilate initial states with different spin and/or orbital angular momentum. We may thus treat each operator separately. Both operators are maximally \( C \)-violating. Since both operators will yield similar results, we focus on the case of \( O_{S=0} \).

The quantum numbers of \( H \) depend on whether dark matter annihilates to a charged or neutral lepton (we have assumed for simplicity that dark matter annihilates to a left-handed lepton; similar results can be obtained if dark matter annihilates to a charged right-handed lepton). The quantum numbers of the new fields for all cases are summarized in Table I.

We assume our effective operators do not respect \( SU(2)_L \) and \( U(1)_Y \), in order to ensure that lepton number is not washed out by electroweak scale interactions between \( H \) and Standard Model particles. Thus, our effective operator couplings must scale with the vev of the Higgs field.

| Fields | \( SU(2)_L \) | \( Q_{U(1)_Y} \) | \( Q_{U(1)_L} \) | \( Z_2 \) |
|--------|--------------|----------------|----------------|---------|
| \( X \) | 1            | 0              | 0              | -       |
| \( P_L L = l_L \) | \( \Box \) | -1/2           | +1            | +       |
| \( H \) | 1            | 0              | 0              | +       |
| \( P_L L = \nu_L \) | \( \Box \) | -1/2           | +1            | +       |
| \( H \) | 1            | 0              | 0              | +       |

TABLE I. Particle Content

The relative phase between the left-handed and right-handed components of \( X, L \) and \( H \) can be fixed by requiring that they all have real mass eigenvalues. The only phase rotations left are non-chiral rotations of these fields, which can be used to absorb any overall phase of the coefficients \( \lambda_1, \ldots, \lambda_4 \). Note that the top line of eq. 2 is \( CP \)-invariant if \( \lambda_1 \) is purely imaginary, while the last three lines are \( CP \)-invariant if \( \lambda_{2,3,4} \) are purely real. We thus see that a \( CP \)-violating term must be proportional to \( Re(\lambda_1\lambda_2^*) \) or \( Im(\lambda_3\lambda_4^*) \).

The authors are grateful to B. Garbrecht for pointing out these potentially large washout terms, discussed in [15]. Note, these washout terms can also be forbidden if the discrete symmetry group is enlarged to a \( Z_4 \) under which \( H \) is charged.
A. One-loop corrections and absorptive interactions

Consider the annihilation processes $XX \to Y$ and $XX \to \bar{Y}$, where $Y$ represents any multi-particle final state, and $\bar{Y}$ is the $CP$-conjugate final state. We may write the quantum matrix element for these processes as

$$\mathcal{M}_{XX \to Y} = \mathcal{M}_{XX \to Y}^{CP} + \mathcal{M}_{XX \to Y}^{CPV},$$
$$\mathcal{M}_{XX \to \bar{Y}} = \pm \left( \mathcal{M}_{XX \to Y}^{CP} - \mathcal{M}_{XX \to Y}^{CPV} \right),$$

(3)

where $\mathcal{M}_{X}^{CP}$ and $\mathcal{M}_{X}^{CPV}$ are the $CP$-invariant and $CP$-violating terms in the matrix element, respectively. The sign of $\mathcal{M}_{XX \to \bar{Y}}$ is determined by the $CP$ transformation properties of the initial state. The final state asymmetry is then governed by the relation

$$\sigma_{XX \to Y} - \sigma_{XX \to \bar{Y}} \propto \text{Re} \left[ \mathcal{M}_{XX \to Y}^{CP} \mathcal{M}_{XX \to Y}^{CPV} \right].$$

(4)

In order to generate a final state asymmetry, it is necessary that:

- there exist both $CP$-invariant and $CP$-violating contributions to the matrix element.
- the relative phase between the $CP$-invariant and $CP$-violating amplitudes differs from $\pm \pi/2$.

The first requirement above is satisfied by interference between two terms (parametrized by coefficients $\lambda_1$ and $\lambda_2$) in the operator $O_{S=0}$.

But for the $XX \to \bar{H}L$ matrix element generated by the operators in eq. 2, the $CP$-invariant part is purely real and the $CP$-violating part is purely imaginary. This is a result of the optical theorem, and as the second point above indicates, implies that there will be no observable consequence to $CP$-violation.

The above line of reasoning leads to the usual result indicating that the generation of an asymmetry from dark matter annihilation requires interference between tree-level and one-loop diagrams. The one-loop diagrams then generate a relative phase from regions of phase space where the intermediate particles go on-shell, again as a result of the optical theorem. The important point, however, is that the one-loop contribution is not needed to provide a $CP$-violating matrix element; it is needed to generate the correct phase between the $CP$-violating and $CP$-invariant terms in the matrix element.

But complex matrix element phases can also arise from final state absorptive interactions. Within the effective operator approach, this one-loop correction is already present in the $H$ external leg correction. Assuming $H$ is unstable, its fully-corrected propagator will have an imaginary contribution which is proportional to the total decay width, $\Gamma_H$. This imaginary
contribution will be sufficient to generate a relative phase between the $CP$-invariant and $CP$-violating amplitudes which differs from $\pm \pi/2$, yielding a lepton asymmetry.

To be concrete, we will consider the case where $H$ is unstable and decays through $H \to H'\phi$, where $H'$ is a fermion and $\phi$ is a scalar (for simplicity we will assume that $m_{H'}, m_\phi \ll m_H$, and that the Standard Model lepton $L$ is either stable, or has a much longer lifetime than $H$). It is easy to see why treating $H$ as an unstable particle allows us to generate an asymmetry between the total cross sections for the process $XX \to \phi^*\bar{H}'L$ and $XX \to \phi\bar{L}H'$. Consider the operator $O_{S=0}$ where we assume $\lambda_{1,2}$ are real. In this case, $\lambda_1$ is the coefficient of the $CP$-violating operator which couples to the right-handed Weyl spinor $H_R$, while $\lambda_2$ is the coefficient of the $CP$-invariant operator which couples to the left-handed Weyl spinor $H_L$. We will, for simplicity, assume that $H$ can only decay from the left-handed helicity (as with Standard Model fermions), through a $CP$-invariant operator

$$O_H = |g|(\phi^*\bar{H}'P_LH + \phi\bar{H}P_RH').$$

If $\Gamma_H/m_H$ is sufficiently small, the dark matter annihilation amplitude will have an intermediate $H$ which will be approximately on-shell. We then see that the $CP$-violating amplitude for $XX \to \bar{H}_R L_L \to \phi\bar{H}_R'L_L$ depends on the helicity-flip term in the propagator of $H$, while the $CP$-invariant amplitude for $XX \to \bar{H}_L L_L \to \phi\bar{H}_R'L_L$ depends on the helicity-preserving term in the $H$ propagator. We can write the corrected $H$ propagator as

$$S(p) = \frac{\hat{p}_H + (m_H - i\Gamma_H/2)}{p_H^2 - m_H^2 - im_H\Gamma_H},$$

where we see that the relative phase between the $CP$-violating and $CP$-invariant matrix elements arises from the $-i\Gamma_H/2$ contribution to the helicity-flip term of the propagator.

The necessity of a one-loop contribution is already familiar from previous work on WIMPy baryogenesis. The difference in this work is that, unlike previous cases, here the one-loop correction is not the source of $CP$-violation; $CP$-violation arises from the interference of two tree-level effective operators, and the one-loop propagator correction only changes the relative phase between those terms in the matrix element.

This difference has important phenomenological consequences. In models where the $CP$-violating phase is generated from loop diagrams, the required additional field content and vertices typically introduce new tree-level washout processes which can erase the asymmetry. This typically results in a more constrained parameter space. In our example, however, since the one-loop contributions are sequestered from the Standard Model and dark sectors, no new tree-level washout processes are introduced. Moreover, the absorptive terms can arise from strongly-coupled physics, even if the actual dark matter-Standard Model matter interactions are perturbatively calculable.
### III. Calculation of the Baryon Asymmetry

The tree-level cross section for dark matter annihilation is given by

\[
\sigma_{\text{tree}}^{XX \rightarrow H L \rightarrow \phi^* H'L} = \frac{s}{16\pi M_*^4} \left\{ |\lambda_1|^2 + \frac{4Im(\lambda_1^* \lambda_2) m_H m_X}{s} + |\lambda_2|^2 \left[ \frac{4}{3} \left( 1 - \frac{4m_X^2}{s} \right) + \frac{2m_H^2}{s} + \frac{4m_H^2 m_X^2}{3s^2} \right] \right\} \left[ 1 - \frac{m_H^2}{s} \right]^2, \tag{7}
\]

where \(\sqrt{s}\) is the energy in center-of-mass frame and for simplicity we assume \(m_{H'}, m_\phi, m_L \ll m_X\). At tree-level, the cross section for the conjugate process \(XX \rightarrow \bar{L}H \rightarrow \phi \bar{L}H'\) is the same. But when one includes loop-corrections to the \(H\) propagator, one finds an asymmetry in the annihilation cross sections:

\[
(\sigma^{XX \rightarrow \phi^* H'L} - \sigma^{XX \rightarrow \phi L H'}) \propto \Gamma_H \frac{Re(\lambda_1^* \lambda_2^*)}{M_*^4} \left[ 1 - \frac{m_H^2}{s} \right]^2 \tag{8}
\]

where \(\Gamma_H\) is the decay width of \(H\), and we have assumed the narrow-width approximation. If we define \(\epsilon\) as the ratio of the cross section asymmetry to symmetric part:

\[
\epsilon \equiv \frac{\sigma^{XX \rightarrow \phi^* H'L} - \sigma^{XX \rightarrow \phi L H'}}{\sigma^{XX \rightarrow \phi^* H'L} + \sigma^{XX \rightarrow \phi L H'}} \tag{9}
\]

then we find \(\epsilon \sim \Gamma_H/m_{H,X}\). Assuming \(m_X\) and \(m_H\) are comparable, the narrow-width approximation would be largely valid even for models with a cross section asymmetry as large as \(O(10\%)\).

The effective operator approximation will be largely valid if \(m_X \ll M_*\). To keep the heavy mediator effectively decoupled from low-energy physics, we set \(M_* = 10\ \text{TeV}\).

#### A. The Boltzmann equation

We can write the Boltzmann equations in terms of dimensionless variables \(x = m_X/T\) and \(Y = n/s\), where \(n\) is the number density and \(s\) is the entropy density. Assuming an adiabatic process, the entropy \(S\) should be constant, and \(Y\) is essentially a comoving number density.

We will assume that \(H'\) and \(\phi\) are light particles which remain in equilibrium throughout the relevant cosmological epoch, allowing us to make the approximation \(Y_\phi = Y_{\phi^*} = Y_{\phi_{\text{eq}}}\), \(Y_{H'} = Y_{H''} = Y_{H'_{\text{eq}}}\). \(L\) is a Standard Model lepton which is also light, but it will depart from equilibrium due to the generated lepton asymmetry. But this departure from equilibrium will be small when the lepton asymmetry is small compared to the total lepton density. We define \(Y_{\Delta L} \equiv Y_L - Y_{\bar{L}}\) as the asymmetry in \(L\), which is either a charged lepton or neutrino of a single generation, and we assume all generations have the same asymmetry. We can assume \(Y_L + Y_{\bar{L}} \approx 2Y_{L_{\text{eq}}}\). For dark matter annihilation to any Standard Model fermion/antifermion
pair, including the subdominant \(CP\)- and \(L\)-violating annihilations that are the source of leptogenesis, the coupled Boltzmann equations are [1]:

\[
\frac{x^2 H(m_X)}{s(m_X)} \frac{dY_X}{dx} = -\langle \sigma_A v \rangle (Y_X^2 - Y_{X_{eq}}^2),
\]

\[
\frac{x^2 H(m_X)}{s(m_X)} \frac{dY_{\text{eq}}^L}{dx} = \frac{1}{2} \left[ \langle \sigma_{XX \rightarrow \phi^* H' L^v} \rangle (Y_X^2 - Y_{X_{eq}}^2) - \langle \sigma_{XX \rightarrow \phi L^H v} \rangle \right] Y_L/Y_{L_{eq}}
\]

\[
-\langle \sigma_{XL \rightarrow \phi^* H' XH^v} \rangle Y_Y(Y_Y - Y_{L_{eq}}) + \langle \sigma_{XL \rightarrow \phi^* H^v} \rangle Y_Y(Y_Y - Y_{L_{eq}})
\]

\[+ ..., \]

(11)

where we have assumed the dark matter is a Majorana fermion and the “+...” terms involved suppressed \(2 \rightarrow 3\) processes in which there is no on-shell resonance. \(H(T)\) is the Hubble parameter at temperature \(T\) given a flat, radiation-dominated early universe. The equilibrium rates for the relevant \(3 \rightarrow 2\) processes are equal to the equilibrium rates for the reverse \(2 \rightarrow 3\) processes as a result of detailed balance. The actual rates for out-of-equilibrium \(3 \rightarrow 2\) processes are determined by rescaling the equilibrium rates by the ratio of the actual incoming particle densities to the equilibrium densities. \(dY_{\text{eq}}^L/dx\) is the rate at which a lepton asymmetry is injected by annihilation processes, not including the effects of electroweak sphalerons [4].

We can then rewrite the second equation Boltzmann equation as

\[
\frac{x^2 H(m_X)}{s(m_X)} \frac{dY_{\text{eq}}^L}{dx} \sim \langle \sigma^{CPV}_{XX} v \rangle (Y_Y^2 - Y_{X_{eq}}^2) - \langle \sigma^{CP}_{XX} v \rangle Y_{X_{eq}} Y_{L_{eq}} - \langle \sigma^{CP}_{XL} v \rangle Y_Y Y_{\Delta L},
\]

(12)

where

\[
\langle \sigma^{CP}_{XX} v \rangle \equiv \frac{1}{2} \left[ \langle \sigma_{XX \rightarrow \phi^* H' L^v} \rangle + \langle \sigma_{XX \rightarrow \phi L^H v} \rangle \right],
\]

\[
\langle \sigma^{CPV}_{XX} v \rangle \equiv \frac{1}{2} \left[ \langle \sigma_{XX \rightarrow \phi^* H' L^v} \rangle - \langle \sigma_{XX \rightarrow \phi L^H v} \rangle \right],
\]

\[
\langle \sigma^{CP}_{XL} v \rangle \equiv \frac{1}{2} \left[ \langle \sigma_{XL \rightarrow \phi^* XH^v} \rangle + \langle \sigma_{XL \rightarrow \phi^* H^v} \rangle \right].
\]

(13)

We can see that the first term on the right-hand side of eq. (12) drives the asymmetry, while the last two terms tend to wash it out. Note that these washout terms are both Boltzmann-suppressed.

\[4\] Note, if the coupling is proportional to the Higgs vev, then there can also be \(2 \leftrightarrow 4\) processes in which a Higgs boson is produced. We disregard these processes for simplicity, but they will not change the result significantly because they only rescale the inclusive cross-section by a factor proportional to the phase space integration. This rescaling can be absorbed into the couplings. Alternatively, the charges of \(H\) could be chosen such that the contacts operators respect \(SU(2)_L\) and \(U(1)_Y\), in which case there is no Higgs coupling. In this case, \(H\) and \(X\) could both be protected from contact with the Standard Model with charges under a \(Z_4\) discrete symmetry [11], but \(X\) would have to be Dirac due to its imaginary charges. Our numerical results do not change appreciably in this case.
In this scenario, the only washout processes are those generated from the original four-point effective operators via crossing symmetry. Since the necessary loop contribution arises only from the correction to the $H$ propagator, the introduction of the particles which appear in the loop does not yield new tree-level washout processes. As a result, there are no dangerous washout processes (in the sense of [12]) which are not Boltzmann suppressed.

Note that there is an asymmetry in the process $\phi \bar{L} H' \leftrightarrow \phi^* \bar{H}' L$ after one subtracts the real intermediate state (RIS) one-loop diagrams in which an intermediate $XX$ two-particle state goes on-shell (this RIS contribution is already accounted for in process $XX \leftrightarrow \phi \bar{L} H', \phi^* \bar{H}' L$).

The rates for the processes $\phi \bar{L} H' \leftrightarrow \phi^* \bar{H}' L$ can be related to the rates for the processes $XX \leftrightarrow \phi L H', \phi^* H' L$ using the CPT-theorem, which implies that the rates for the inclusive processes $\phi \bar{L} H' \rightarrow \text{anything}$ and $\phi^* \bar{H}' L \rightarrow \text{anything}$ are identical. We have implicitly included the rate asymmetries for these $3 \leftrightarrow 3$ processes in the Boltzmann equation, which thus satisfies detailed balance. We therefore generate no lepton asymmetry when the dark matter is in equilibrium, in contrast to previous models of leptogenesis [15]. Although we can safely ignore finite number density corrections to our calculation, we could equivalently use the CTP formalism [18] to manifestly demonstrate the generation of the asymmetry, but this is beyond the scope of our paper.

We have not specified the high-energy Lagrangian which generates the effective operators described. A particular UV model which generates these effective operators at low-energy may also generate other effective operators which contribute to washout processes, for example, operators of the form $(\bar{H} P_L L)^2$. However, this is a model-dependent question; there will exist UV-completions (for example, models where the mediating particle is exchanged in the $t$- or $u$-channel) in which such operators are not generated at tree-level. In keeping with our use of effective field theory, we will not assume the existence of any additional effective operators beyond the ones we have introduced.

Although it is necessary to assume that $H$ is unstable in order to generate a lepton asymmetry, we nevertheless were able to assume that the width of $H$ is relatively narrow. This implies that we should be able write equivalent Boltzmann equations in which we treat $H$ as a metastable particle which is initially in thermal equilibrium.

B. The effect of electroweak sphalerons

The expression for $d Y_{\Delta L}^{\text{inj}} / d x$ in the Boltzmann equation provides the source term in the differential equation for the lepton number density, which is coupled to the baryon number density through sphalerons. Given a number of generations $N_G$ and arbitrary lepton number sources $f_i$ for each generation $i$, we can write the evolution of the baryon and lepton number
The functions \( \eta(T) \) and \( \gamma(T) \) are defined in terms of the temperature \( T \), the temperature-dependent Higgs field expectation value \( v_{\text{min}} \), and the Chern-Simons diffusion rate \( \Gamma_{\text{diff}}(T) \):

\[
\eta(T) = \frac{\chi(T)}{1 - \chi(T)},
\]

\[
\gamma(T) = N_G^2 \rho(T) [1 - \chi(T)] \frac{\Gamma_{\text{diff}}(T)}{T^3},
\]

\[
\rho(T) = \frac{3 \left[ 65 + 136 N_G + 44 N_G^2 + (117 + 72 N_G) \left( \frac{v_{\text{min}}}{T} \right)^2 \right]}{2 N_G \left[ 30 + 62 N_G + 20 N_G^2 + (54 + 33 N_G) \left( \frac{v_{\text{min}}}{T} \right)^2 \right]},
\]

\[
\chi(T) = \frac{4 \left[ 5 + 12 N_G + 4 N_G^2 + (9 + 6 N_G) \left( \frac{v_{\text{min}}}{T} \right)^2 \right]}{65 + 136 N_G + 44 N_G^2 + (117 + 72 N_G) \left( \frac{v_{\text{min}}}{T} \right)^2}.
\]  

The lattice and analytical calculations of \( v_{\text{min}} \) are consistent. The lattice and analytical calculation of \( \Gamma_{\text{diff}} \) differ by an order of magnitude in the range of overlap \( (T = 140 - 155 \text{ GeV}) \), but exhibit the same logarithmic slope. We will use the \( \Gamma_{\text{diff}}(T) \) determined from lattice calculations for \( T \geq 140 \text{ GeV} \) and use the analytical result for \( T \leq 140 \text{ GeV} \) after rescaling the analytical result by a constant factor to provide consistency with lattice calculations in the region of overlap. The \( v_{\text{min}}(T) \) and \( \Gamma_{\text{diff}}(T) \) which we use are plotted in figure [1].

Now we recast the coupled equations for the comoving baryon and lepton numbers in terms of our dimensionless source variables, noting \( N_G = 3 \) and assuming all lepton generations evolve equivalently.

\[
xH(T) \frac{dY_{\Delta B}}{dx} = -\gamma(T) [Y_{\Delta B} + 3 \eta(T) Y_{\Delta L}]
\]

\[
xH(T) \frac{dY_{\Delta L}}{dx} = -\frac{1}{3} \gamma(T) [Y_{\Delta B} + \eta(T) Y_{\Delta L}] + xH(T) \frac{dY_{\text{inj}}}{dx}
\]  

(16)
FIG. 1. Lattice calculation of $v_{\text{min}}^2/(T^2 g_{\text{weak}}^2)$ (left) and $\log(\Gamma_{\text{diff}}/T^4)$ (right) through the transition region [20]. For $T < 140$ GeV, $\log(\Gamma_{\text{diff}}/T^4)$ is extrapolated from analytical calculations deep in the broken phase [19], with a constant rescaling to provide consistency with the lattice calculation for $T = 140 - 155$ GeV.

IV. RESULTS

We will assume that $m_H \leq 2m_X$, so that the process $\bar{X}X \rightarrow \bar{H}L$ is kinematically allowed. As in [10–12], we will assume $m_H \sim m_X$. We assume that the reheating temperature of the universe is large enough that the dark matter was in relativistic thermal equilibrium in the early universe ($x < 1$). We then numerically solve the coupled Boltzmann/sphaleron rate equations, using equilibrium at $x = 1$ as a boundary condition.

Sphaleron processes will start to decouple for temperatures $T \lesssim \mathcal{O}(100)$ GeV. We will thus find that, for $m_X \gg 100$ GeV, the lepton asymmetry is generated when sphalerons are active, and the magnitude of the baryon asymmetry is roughly the same as that of the lepton asymmetry at late times.

For all of the models we consider, we take $\langle \sigma A v \rangle = 1$ pb $\gg \langle \sigma_{\bar{X}X \rightarrow \bar{H}LV} \rangle$. As a result, $X$ will annihilate rapidly enough to ensure that its density does not exceed observational bounds, while the annihilation process $\bar{X}X \rightarrow \bar{H}L, \bar{L}H$ will not significantly affect the dark matter density (though it will impact the baryon asymmetry). For the case where only the operator $\mathcal{O}_{S=0}$ is present, the relevant parameters of the model are $m_X$, $m_H/m_X$, $\Gamma_H/m_H$ and $\text{Re}(\lambda_1 \lambda_2^*)$ (the last parameter is replaced by $\text{Im}(\lambda_3 \lambda_4^*)$ in the case where only $\mathcal{O}_{S=1}$ is present).

We can then illustrate our results with some benchmark points. In figure we plot the thermally-averaged cross sections for the processes $XX \rightarrow \phi^* \bar{H}'L$ (both $CP$-invariant and $CP$-violating terms) and $XL \rightarrow \phi H'X$ (the $CP$-invariant term) as a function of $x = m_X/T$. We also plot the contribution of these terms to the lepton source injection rate, as well as $Y_B$, $Y_X$ and $Y_{Xeq}$. We have chosen the “high-mass” benchmark parameters $m_X = 5$ TeV,
As we expected, the process $XL \rightarrow \phi H' X$ is kinematically-suppressed at low temperature. Note, however, that we have assumed the narrow-width approximation; this approximation will break down at sufficiently low temperatures, when the Boltzmann suppression required for the production of an on-shell $H$ is larger than the cross section suppression when $H$ is off-shell. But if we instead take $H'$ and/or $\phi$ to be massive, then even this off-shell process can be suppressed, while on-shell processes will be unaffected.

In figure 3 we plot the $Y_B$, $Y_X$, $Y_{X_{eq}}$ and the lepton injection rates (both source and washout terms) as a function of $x = m_X/T$ for a narrower-width benchmark model with $m_X = 5$ TeV, $m_H = 7$ TeV, $\lambda_1 = \lambda_2 = 0.5$, $\langle \sigma_A v \rangle = 1$ pb, $\Gamma_H/m_H = 0.05$ (left panel) and for a low-mass benchmark model with $m_X = 1.5$ TeV, $m_H = 2.2$ TeV, $\lambda_1 = \lambda_2 = 1$, $\langle \sigma_A v \rangle = 1$ pb, $\Gamma_H/m_H = 0.1$ (right panel). For all benchmark models, the couplings are chosen so that the final baryon asymmetry matches observation. Note that the narrower-width benchmark is nearly identical to the high mass benchmark, only washout processes freeze out slightly later. For the low-mass benchmark, sphalerons begin to decouple around when washout processes freeze out, thus forcing a sharper freeze out of baryon number. The parameters of these benchmark models are summarized in Table II.

It is interesting to note that the $CP$-violating part of the process $XX \leftrightarrow \bar{L}H, \bar{H}L$ begins to drive the asymmetry near $x \sim 1$, i.e. as soon as the dark matter becomes non-relativistic. This may seem counterintuitive, since the Sakharov conditions require a departure from thermal equilibrium and dark matter freeze out occurs much later, near $x \sim 20 - 30$. The
FIG. 3. Rate contributions and Boltzmann equation solutions for our narrower-width (left panel) and low-mass (right panel) benchmarks (colors same as in right panel of figure 2). Parameters are summarized in Table II.

| benchmark       | $m_X$  | $m_H$  | $\Gamma_H/m_H$ | $\lambda_1 = \lambda_2$ | $\epsilon$ | $\langle \sigma_{XX \rightarrow \phi^* H^* L^v} \rangle / \langle \sigma_A v \rangle$ |
|-----------------|--------|--------|-----------------|--------------------------|------------|--------------------------------------------------|
| low-mass        | 1.5 TeV | 2.2 TeV | 0.10            | 1.0                      | 0.045      | 0.002                                             |
| high-mass       | 5.0 TeV | 7.0 TeV | 0.10            | 0.5                      | 0.045      | 0.008                                             |
| narrower-width  | 5.0 TeV | 7.0 TeV | 0.05            | 1.0                      | 0.022      | 0.033                                             |

TABLE II. Benchmarks

point is that the dark matter density departs slightly from the equilibrium density as soon as dark matter becomes non-relativistic. At freeze-out, dark matter stops tracking the equilibrium density and the departure from equilibrium becomes large. One can see this simply by considering the Boltzmann equation (eq. 10); if the dark matter density is equal to the equilibrium density, then $dY/dx = 0$. This relation is satisfied if dark matter is relativistic ($Y \sim \text{const}$), but is violated when dark matter is non-relativistic ($Y \sim x^{-3/2}e^{-x}$).

A slight departure from equilibrium is necessary to provide the excess annihilation which drives $Y$ to smaller values. Although the departure from equilibrium is very small before freeze out, the dark matter density at $x \sim 1$ is so much larger than at $x \sim 20$ that the driving contribution to the lepton asymmetry is largest at small $x$. However, at small $x$ the washout processes are also at their strongest and the net asymmetry is quite small. A large asymmetry begins to be generated as soon as the washout processes begin to freeze out, which for these models typically happens near $x \sim 10$, as in [12].

There are a few features which distinguish our results from those of other WIMPy baryogenesis models. First of all, in previous works, the interactions necessary to produce the needed one-loop diagrams also introduce tree-level washout diagrams in which $X$ does not appear as an initial or final state. These washout processes are essentially processes of the form $\bar{H}L \leftrightarrow LH$, and can be significant even when $T < m_X$. On the other hand, in the
class of models we consider, all washout diagrams have $X$ as a final state.

Note also that $\epsilon \sim \Gamma_H/2m_{H,X}$, and is independent of the mediator scale $M_*$. This is in contrast with other WIMPy baryogenesis models, where one typically finds $\epsilon \propto m_X^2/M_*^2$. This difference has some interesting effects. If we assume that $X$ constitutes all of the dark matter, then the total cross section for the annihilation process $XX \rightarrow \bar{H}L, \bar{L}H$ is bounded by $\sim 1$ pb. In order to generate a large enough asymmetry, $\epsilon$ cannot be too small; it appears that one would need $\epsilon \sim \mathcal{O}(0.01 - 0.1)$. If $\epsilon \sim (\lambda^2/4\pi)m_X^2/M_*^2$ and $\lambda \sim 1$, this would require that $M_* \lesssim 3m_X$. By contrast, in our low-mass benchmark model, the new physics scale $M_*$ can be much larger, since $\epsilon$ is independent of the scale of the new physics in the effective operator. As a result, the total $XX \rightarrow \bar{H}L, \bar{L}H$ cross section can be much smaller than the $\langle \sigma_A v \rangle \sim 1$ pb.

In all of the models we have considered, we have chosen $m_H \sim m_X$. It is difficult to find a successful model if one instead chooses $m_H \ll m_X$. The reason is because we have assumed the narrow-width approximation, in which $H$ acts as a resonance, and thus require $\Gamma_H \ll m_H$. In such a model, since we would also still require $\epsilon \propto \Gamma_H/m_X$, a light $H$ field would imply a small cross section asymmetry.

V. CONCLUSIONS

We have considered a model of WIMPy leptogenesis, in which a lepton asymmetry is generated by dark matter annihilation processes which violate $C$, $CP$ and lepton number. We have studied in detail the necessity for one-loop contributions to the annihilation process. In particular, we have found the $CP$-violating terms may all arise at tree-level, while one-loop diagrams may arise only in absorptive final state interactions in a sterile sector.

The advantage of this type of model is that it allows one to sequester the one-loop suppression from the generation of a $CP$-violating phase. In WIMPy baryogenesis models where this sequestration does not occur, the introduction of one-loop terms implies the presence of new tree-level process which are not Boltzmann suppressed and which can washout the baryon asymmetry. In the class of models we have considered, these dangerous washout processes do not occur. Moreover, because the ratio of $CP$-violating to $CP$-conserving terms in the cross section is independent of the scale of new physics, we find that WIMPy models with $m_X \sim 1.5$ TeV can work with a new physics scale $M_*$ as large as 10 TeV, and where the annihilation cross sections relevant for WIMPy leptogenesis are much smaller than 1 pb.

It is interesting to consider prospects for probing these dark matter interactions experimentally. Indirect detection may be feasible for WIMPy leptogenesis models in general, but
given our specific $CP$-violating processes, primary dark matter annihilation channels would likely dominate over any measurable imprint our subdominant channel would leave on the cosmic ray spectrum. Better prospects may lie with high energy/luminosity $e^+/e^-$ colliders, which may be able to probe lepton number- or lepton flavor-violating processes to which these operators could contribute.

For simplicity, we have focused on the approximation where, despite the presence of final state absorptive interactions, the sterile particle $H$ can be treated as a narrow resonance produced on-shell. But this assumption is not required, and if the narrow-width approximation does not hold, then all of the asymmetry-generating and washout processes would have to be fully treated as $2 \to 3$ processes. In this case, one would expect that one-loop suppression required for $CP$-violating annihilation rates would be significantly reduced. It would be interesting to study this scenario concretely.

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