Generating a Fermion Mass Hierarchy in a Composite Supersymmetric Standard Model

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A mechanism is suggested by which the dynamics of confinement could be responsible for the fermion mass matrix. In this approach the large top quark Yukawa coupling is generated naturally during confinement, while those of the other quarks and leptons stem from non-renormalizable couplings at the Planck scale and are suppressed. Below the confinement scale(s) the effective theory is minimal supersymmetric $SU(5)$ or the supersymmetric standard model. Particles in the 5 representations of $SU(5)$ are fundamental while those in the 10 and 5 are composite. The standard model gauge group is weakly coupled and predictions of unification can be preserved. A hierarchy in confinement scales helps generate a hierarchical spectrum of quark and lepton masses and ensures the Kobayashi-Maskawa matrix is nearly diagonal. However, the most natural outcome is that the strange quark is heavier than the charm quark; additional structure is required to evade this conclusion. No attempt has been made to address the issues of $SU(5)$ breaking, SUSY breaking, doublet/triplet splitting or the $\mu$ parameter. While the models presented here are neither elegant nor complete, they are remarkable in that they can be analyzed without uncontrollable dynamical assumptions.
Many authors over the years have proposed scenarios in which some or all of the particles of the standard model are composite. The work of 't Hooft [1] on anomaly matching provided some important consistency conditions on compositeness, but most approaches have been limited by the need to make assumptions about strongly coupled gauge theories. Within the context of supersymmetric model building, the idea that quarks and leptons might be the supersymmetric partners of composite pseudo-Nambu-Goldstone bosons was studied in detail during the 1980’s; Ref. [2] provides a review of the extensive literature.

In the last couple of years, our understanding of the non-perturbative dynamics of supersymmetric gauge theories has improved. The new methods are very powerful and can be used for model building. In [3] a variant of the missing partner mechanism was shown to be valid even at strong coupling. In [4], following [5], it was suggested that the $SU(3)$ color group was the dual of another $SU(3)$, which implies magnetic quarks and numerous composite Higgs doublets; unfortunately it also implies $\Lambda_{QCD} \gg m_Z$.

In this letter I propose to use the dynamics of confinement as discussed in [6] to generate the fermion mass hierarchy in a supersymmetric version of the standard model.* Because of recent developments involving duality of N=1 supersymmetric theories (see [8] for a review) the results of [6] now satisfy a large number of consistency checks far beyond simple matching of global anomalies. In this sense the scenario presented here is free of uncontrollable dynamical assumptions.

It is convenient to present the new mechanism within the context of minimal $SU(5)$ supersymmetric grand unified theories, though it can also be applied directly to the standard model gauge group. I will first present a toy one-generation model in which the basic physics is explained; I will show that an up-type Yukawa coupling can be large, while down-type Yukawa couplings are naturally smaller. From there I turn to theories with three generations. The attractive features of the mechanism are displayed in a simple model. The top quark is naturally heavy, the splittings of up-type quark masses are generically larger than those of down-type quark masses, and the Kobayashi-Maskawa matrix is naturally close to unity. Unfortunately, the charm quark is naturally too light relative to the strange quark and additional physics must be invoked to avoid this conclusion. I present two approaches to achieving a reasonable mass spectrum, though neither is especially elegant. However, many other variants of these models can be constructed.

* When this paper was complete I learned that this mechanism was previously studied by A. Nelson [7].
important unresolved issue involves the doublet-triplet splitting problem, for which I have presented no solution here. I have also left unaddressed the questions of SU(5) breaking and supersymmetry breaking. Despite the weaknesses of these models, I hope that the reader will find them amusing and thought-provoking.

1. A toy model with one generation

The main issue is to generate a 10 representation of SU(5) with a coupling to the Higgs boson in the 5 representation. Antisymmetric tensors of SU(N) can be generated via the Berkooz trick \[9,10\], using an SU(N) × Sp(M) model, in which they emerge as bound states of two Sp(M) quarks; the Sp group confines for appropriate choice of M and only the SU(N) group remains at low energies. A non-perturbative superpotential is also generated \[9,10\]; we will see that for SU(5) × SU(2) (recall Sp(1) ≈ SU(2)) that this corresponds to the top-quark Yukawa coupling. The bottom-quark Yukawa coupling will have its source in a non-renormalizable operator of dimension four* and will be suppressed.

At the Planck scale** m_{pl} consider a theory with gauge group SU(5) × SU(2). The coupling of SU(5) is weak while that of SU(2) is large and blows up at the confinement scale \[Λ_c \equiv \eta m_{pl}\]. The matter content of the theory, in terms of SU(5) × SU(2) multiplets, consists of a field \(X\) in the (5, 2) representation, a field \(S\) in the (1, 2), two fields \(\bar{H}_i\) in the (\(\bar{5}, 1\)), and a massive field \(\phi\) in the adjoint of SU(5) whose sole purpose is to break SU(5) to the standard model. All couplings in the superpotential which are consistent with the symmetries are assumed to be present in the effective Lagrangian; all dimensions are assumed to be \(m_{pl}\) and all dimensionless coefficients are order one. The lowest-dimension gauge-invariant operators which do not involve \(\phi\) are \(\bar{H}_i XS\), \(\bar{H}_1 XX H_2\), \(X^5 S\), \((\bar{H}_i XS)(\bar{H}_j XS)\), etc. Additional powers of \(\phi\) can always be inserted to make new operators. (The fact that this model breaks supersymmetry \[12\] when SU(5) becomes strongly coupled is irrelevant.

* Here operator dimensions are given as appropriate for the superpotential, not the Lagrangian; thus the bottom-quark Yukawa comes from an quartic operator in the superpotential which is dimension five in the Lagrangian.

** Throughout this letter I write \(m_{pl}\) to signify whatever scale is the appropriate ultraviolet boundary condition at which all non-renormalizable couplings are generated at order one. This may be \(m_{pl}\) or \(m_{pl}/4\pi\) or the string scale; the general mechanism presented here is insensitive to the exact value of the scale.
for the analysis near $\Lambda_c$; models with three generations will not break supersymmetry by themselves.)

Dynamics of strong coupling [3] will drive the $SU(2)$ group (which has six doublets) to confine near the scale $\Lambda_c$. Below this scale, $SU(2)$ with six doublets $q_i$ has \textit{composite massless degrees of freedom:} the mesons $V_{ij} = q_i q_j$, which are antisymmetric in flavor [13, 6]. Classically this field satisfies the constraint $\partial(Pf V)/\partial V_{ij} = 0$; quantum mechanically the constraint is unmodified and is implemented by the superpotential $W = \Lambda_c^{-3} Pf V$ [3].

In this model, the $SU(2)$ dynamics can be analyzed similarly; the $SU(5)$, whether broken or not at the scale $\Lambda_c$, is weakly coupled and is a spectator to the confining dynamics of $SU(2)$. The six doublets of $SU(2)$ consist of five from $X$ and one from $S$; after confinement the low-energy massless fields are $A^{[\alpha \beta]} = X^\alpha X^\beta/\Lambda_c$ in the 10 of $SU(5)$ and $H^\alpha = X^\alpha S/\Lambda_c$ in the 5 of $SU(5)$, where $\alpha, \beta$ are $SU(5)$ indices. (I have defined $A$ and $H$ as canonically normalized fields.) As above a superpotential is generated of the form

$$W_L = (X^5 S) \Lambda_c^{-3} = AAH$$

(1.1)

This term has the structure of the top-quark–Higgs-boson Yukawa coupling. The coefficient of this term is order one; additional contributions to this coefficient from tree-level dimension-six terms at the Planck scale will be be suppressed by $\eta^3$.

Let us now analyze the theory below the scales of the strong dynamics. For the moment, and for simplicity only, let us assume that $M_{GUT} < \Lambda_c = \eta m_{pl}$, so that $SU(5)$ is unbroken at $\Lambda_c$.* The low energy theory consists of the fields $A, H, \bar{H}, \phi$ in the 10, 5, $\bar{5}$, 24 of $SU(5)$ — a standard model generation, along with a pair of up- and down-type Higgs multiplets. The light fields are coupled by the renormalizable superpotential

$$W = m \bar{H} H + Y AAH + y A\bar{H} \bar{Q}$$

(1.2)

where I have defined $\bar{H}$ as the linear combination of the $\bar{H}_i$ which couples to $H$, and $\bar{Q}$ as the orthogonal combination. Of course there are many higher dimension terms. The mass $m$ is of order $\Lambda_c$; if we want to forbid it we may do so by adding a discrete gauge symmetry under which $\bar{H}, \bar{Q}$ change sign. The coupling $Y$, which was generated dynamically, is order one. The coupling $y$, which stems from a dimension-four term $1/m_{pl} \bar{H}_1 XX \bar{H}_2$ in the tree-level superpotential at $m_{pl}$, is of order $\eta$. After $\phi$ condenses at the scale $M_{GUT}$, the theory

* An interesting challenge would be to relate the scale $M_{GUT}$ to the dynamical scale $\Lambda_c$, though as yet I know of no specific mechanism for doing so.
has a single standard model generation; the top quark Yukawa coupling is order one, while
the bottom quark and tau lepton couplings are order $\eta$. (Recall that the actual masses of
the top and bottom quarks involve the additional parameter $\langle H \rangle / \langle \bar{H} \rangle \equiv \tan \beta$, so their
ratio is not predicted.)

It is essential to note that this result depends crucially on the choice of gauge group.
A top quark coupling of order one can only be generated in this way if the weak gauge
group is $SU(5)$ or one if its subgroups and if the confining gauge group is $SU(2)$. Of course
the mechanism can be embedded in a larger gauge group which breaks at some scale to
$SU(5) \times SU(2)$; thus we can use $SU(k) \times Sp(m)$ for $k > 5$, for example.

The confinement must take place at energies near the Planck scale; otherwise the
dimension-four operator which becomes the bottom-quark Yukawa coupling after confine-
ment will have too small a coefficient. However, it need not be that $M_{\text{GUT}} < \Lambda_c$. As we
take $\Lambda_c$ equal to or smaller than $M_{\text{GUT}}$, the low-energy model changes little, since the
analysis of the superpotential and of the size of its couplings is insensitive to the breaking
of the perturbative $SU(5)$ gauge group. For this reason the analysis will also work if the
weakly coupled gauge group is $SU(3) \times SU(2) \times U(1)$.

2. A model with three generations

Next, I turn to a three-generation model. The simplest implementation involves replicat-
ing the previous structure three times. At the Planck scale $m_{\text{pl}}$, the theory has gauge
group $SU(5) \times [SU(2)]^3$, where the gauge coupling of the first factor is small while those
of the last three are larger and diverge at the scale $\Lambda_c$.

The matter content of the theory, in terms of $SU(5) \times SU(2) \times SU(2) \times SU(2)$
multiplets, is as follows. There are three fields $X_1, X_2, X_3$ which are in the representa-
tions $(5, 2, 1, 1), (5, 1, 2, 1)$ and $(5, 1, 1, 2)$ respectively. There are fields $S_1, S_2, S_3$ in the
$(1, 2, 1, 1), (1, 1, 2, 1)$ and $(1, 1, 1, 2)$, and fields $\bar{H}_i, \bar{Q}_i, i = 1, 2, 3$, in the $(\bar{5}, 1, 1, 1)$. There is
also a field $\phi$ in the adjoint of $SU(5)$ which breaks $SU(5)$ to the standard model. Avoid-
ing proton decay and keeping the Higgs bosons light requires a gauged discrete symmetry,
which I take to be a $Z_6$ under which the fields $X_i, S_i, \bar{H}_i, \bar{Q}_i, \phi$ have charge $1, 1, 1, 3, 0$. (This
is anomaly-free under $SU(5) \times [SU(2)]^3$; instantons of each group leave it unbroken.) The
lowest-dimension gauge-invariant operators which can appear in the superpotential (and
do not depend on $\phi$) are $\bar{H}_i X_j X_j \bar{Q}_k, [(X_i^2)(X_j^2)(X_k S_k)], [(\bar{H}_i X_j S_j)^2]$, etc.
Each $SU(2)$ subgroup confines, generating fields $A_i$ and $H_i$ in the 10 and 5 representations of $SU(5)$ and a superpotential $W = A_i A_i H_i$ with coefficient of order one. In terms of $SU(5)$ representations, the low energy theory consists of three standard model generations along with three pairs of up- and down-type Higgs doublets and their color-triplet partners. The light fields are coupled by the renormalizable superpotential

$$W = Y^{ijk} A_i A_j H_k + y^{ijk} A_i \bar{Q}_j \bar{H}_k$$

(2.1)

where all repeated indices are summed. Of course there are many non-renormalizable terms. The structure of these terms is as before: $Y^{iii}$ is of order one, all other $Y^{ijk}$ are of order $\eta^3$, and the $y^{ijk}$ are all of order $\eta$.

However, since all three pairs of Higgs bosons are light, and since all up-type Yukawa couplings are large, one must explain why only one up-type Higgs boson gets a large vacuum expectation value, why it couples mostly to the top quark, and why the vacuum expectation value of the down-type Higgs bosons couples mostly to the bottom quark. Without treating the third generation differently from the other two, this seems challenging at best.

3. A more realistic and less flavor-symmetric model.

If one does treat the third generation differently, the number of Higgs boson pairs can be reduced to one. This can easily be done, at the cost of simplicity, by altering the discrete symmetries. One should take care, however, that $V_{tb}$ be close to unity. This is not trivial to guarantee. Suppose we permit $H_1, H_2, \bar{H}_1, \bar{H}_2$ to become massive as a result of a discrete symmetry, while $H_3$ and $\bar{H}_3$ remain light. We have ensured dynamically that $H_3$ couples only to $A_3$ at leading order, so we may identify $A_3$ as containing the left-handed top and bottom quarks. The mass matrix of the down-type quarks is $y^{ij3}$; we may use the $SU(3)$ acting on $\bar{Q}_i$ (assuming it is not broken by the discrete symmetry) to set $y^{313}$ and $y^{323}$ to zero. To ensure that the bottom-quark is the most massive down-type quark and that $V_{tb}$ is order one, we must have $y^{333}$ much larger than any other element of the matrix. But no symmetry guarantees this, and since the couplings $y$ were assumed to be generated at the Planck scale, they are in fact naturally all of the same order. Fortunately, a hierarchy in confinement scales will assure this automatically.

One path to a reasonable model, treating the third generation differently from the others, is to change the gauge symmetry. To create a model with three composite 10 representations of $SU(5)$, it is natural to generalize the group to $SU(5) \times Sp(m_1) \times Sp(m_2) \times \ldots$
$Sp(m_3) \times G_D$, where $G_D$ is a discrete symmetry.* For each choice of $m_i$ the dynamical superpotentials and the available anomaly-free discrete symmetries are different, and thus each case has its own features and problems. One reasonable choice of gauge group is $SU(5) \times Sp(3)_1 \times Sp(3)_2 \times SU(2)_3 \times Z_{12}$; the first group is weakly coupled and the last three are strongly coupled, with dynamical scales labelled $\Lambda_1$, $\Lambda_2$, $\Lambda_3$. Define $\eta_i \equiv \Lambda_i/m_{pl}$.

One reason for this choice of group is that, like $SU(2)$ with six doublets, $Sp(3)$ with ten fields $q_i$ in the fundamental representation confines its quarks into a gauge singlet $V_{ij} = q_i q_j$ which is antisymmetric in flavor [1]; it generates a superpotential $W = Pf V_{ij}/\Lambda^7$, which is non-renormalizable even in terms of the low-energy degrees of freedom. In the present theory each $Sp(3)$ will therefore generate a 10 and several 5 representations of $SU(5)$ along with some singlets, but they will not have order-one up-type Yukawa couplings. For this gauge group there is a discrete symmetry under which the third Higgs $H_3$ is special but under which the $A_i$ fields have the same charge. By using this structure we can allow all $A_i A_j H_3$ couplings with only the top quark Yukawa coupling large.

The matter content of the model, in terms of $SU(5) \times Sp(3)_1 \times Sp(3)_2 \times SU(2)_3 \times Z_3 \times Z_4$ multiplets, is as follows:

$SU(5) \quad Sp(3)_1 \quad Sp(3)_2 \quad SU(2)_3 \quad Z_3 \quad Z_4$

|       | 5   | 2   | 1   | 1   | $z^2$ | $-i$ |
|-------|-----|-----|-----|-----|-------|------|
| $x_1$ | 5   | 2   | 1   | 1   | $z^2$ | $-i$ |
| $x_2$ | 5   | 1   | 2   | 1   | $z^2$ | $-i$ |
| $X$   | 5   | 1   | 1   | 2   | $z^2$ | $-i$ |
| $s_1^r$ | 1   | 2   | 1   | 1   | $z$  | $i$  |
| $s_2^r$ | 1   | 1   | 2   | 1   | $z$  | $i$  |
| $S$   | 1   | 1   | 1   | 2   | $z^2$| $i$  |
| $\bar{H}$ | 5   | 1   | 1   | 1   | $z^2$| $-1$|
| $Q_u$ | 5   | 1   | 1   | 1   | 1    | 1    |
| $\phi$ | 24  | 1   | 1   | 1   | 1    | 1    |

Here the discrete charges are represented by the phase acquired by a field under the gauge transformation; $z$ is a third root of unity. The Planck scale superpotential can contain the operators $Q_u x_a s_a^r$, $\bar{H} X X Q_u$, $\bar{H} x_a x_a Q_u$, $[X^5 S]$, $[(x_a^2)(X^2)(XS)]$, $[(x_a^2)(x_b^2)(XS)]$, $[(Q_u x_a s_a^r)^2]$, $[(\bar{H} X S)^2]$, etc., and terms dependent on $\phi$.

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* Here $Sp(n)$ is the symplectic group whose fundamental representation is of dimension $2n$; the group is also confusingly referred to as $Sp(2n)$. Recall that $Sp(1) \approx SU(2)$. 

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The confining dynamics leaves us at low energies with the composite fields $A_3^{[\alpha\beta]} = X^\alpha X^\beta / \Lambda_3$, $H^\alpha = X^\alpha S / \Lambda_3$, and, for $a = 1, 2$, $r, s = 1, \ldots, 5$, $A_a^{[\alpha\beta]} = x_a^\alpha x_a^\beta / \Lambda_a$, $H_a^{\alpha r} = x_a^\alpha s_a^r / \Lambda_a$, $K_a^{rs} = s_a^r s_a^s / \Lambda_a$; here $\alpha, \beta$ are $SU(5)$ indices. Their charges are

$SU(5) \quad Z_3 \quad Z_4$

\[
A_a \quad 10 \quad z \quad -1 \\
H_a^r \quad 5 \quad 1 \quad 1 \quad (a = 1, 2, r = 1, \ldots, 5) \\
K_a^{rs} \quad 1 \quad z^2 \quad -1 \quad (a = 1, 2, r, s = 1, \ldots, 5) \\
A_3 \quad 10 \quad z \quad -1 \\
H \quad 5 \quad z \quad 1
\]

These are all coupled together in the dynamical superpotential

$$W_{dyn} = A_3 A_3 H + \sum_{a=1,2} \frac{1}{\Lambda_a^2} \text{Pf} \begin{bmatrix} A_a^{\alpha\beta} & H_a^{\alpha r} \\ -H_a^{\alpha r} & K_a^{rs} \end{bmatrix}$$

(3.3)

In addition, the superpotential at the Planck scale contributes to the low-energy theory. After confinement the terms $\bar{Q}_u x_a s_a$ generate masses of order $\Lambda_a$ for all ten $H_a^r$, $a = 1, 2$, $r = 1, \ldots, 5$, and for ten of the thirteen fields $\bar{Q}_a$. I will refer to the leftover fields as $\bar{Q}_i$, $i = 1, 2, 3$. The renormalizable superpotential governing the light fields is

$$W = \sum_{i,j=1}^3 \left[ Y^{ij} A_i A_j H + y^{ij} A_i \bar{Q}_j \bar{H} \right]$$

(3.4)

The spectrum of couplings is now approaching that of the real world. Recall that $\eta_i = \Lambda_i / m_{pl}$ and take $\eta_1 < \eta_2 < \eta_3$. The coupling $Y^{33}$ is order one, so we identify $A_3$ as containing the left-handed top and bottom quark. The other couplings $Y^{ij}$ are naturally of order $\eta_3 \eta_i \eta_j$, which establishes a hierarchy between the top quark and the other up-type quarks. Meanwhile, the couplings $y^{ij}$ are of order $\eta_i$. The resulting mass matrices, which should be compared with the matrices of Yukawa couplings in supersymmetric models at the unification scale [14], are therefore

$$M_u \sim \begin{bmatrix} \eta_1^2 \eta_2 \eta_3 & \eta_1 \eta_2 \eta_3 & \eta_1 \eta_3^2 \\ \eta_1 \eta_2 \eta_3 & \eta_2^2 \eta_3 & \eta_2 \eta_3^2 \\ \eta_1 \eta_3^2 & \eta_2 \eta_3^2 & 1 \end{bmatrix} \sin \beta ; \quad M_d \sim \begin{bmatrix} \eta_1 & \eta_1 & \eta_1 \\ \eta_2 & \eta_2 & \eta_2 \\ \eta_3 & \eta_3 & \eta_3 \end{bmatrix} \cos \beta$$

(3.5)

This spectrum is interesting, though unacceptable. Its general form is appealing in that it naturally implies three observed properties of the mass matrices: the top quark is heavy, the splittings between down-type quarks are smaller than those between up-type quarks, and the Kobayashi-Maskawa matrix is close to unity when the quark mass splittings
are large. However, the devil is in the details; it is not possible to get the bottom, strange and charm quark masses in the correct ratios. In particular, fixing the bottom and strange quark masses leaves the charm quark too light. Even allowing that unknown coefficients of order one might be as small as .2, no reasonable tuning brings the masses and mixing angles into agreement with data. One could attempt to fix this problem by generating the charm quark mass radiatively at low energies. In the next two sections I will describe two other approaches which repair the situation at the cost of an additional parameter.

It should be noted that this mechanism does not require $\Lambda_i > M_{GUT}$. While for sufficiently small $\Lambda_i$ the unification of gauge coupling constants will be disrupted, the confinement physics will be unaffected; the weakly coupled $SU(5)$, whether intact or broken, is a spectator to the important dynamics. Indeed one may give up unification and take the standard model gauge group up to $m_{pl}$; the confinement physics is insensitive to this choice.

4. Improving the model

In this section, I modify the above model slightly in order to achieve a reasonable fermion mass spectrum. This particular method has the by-product that the $SU(5)$ relations between the strange and down quark masses and those of the muon and electron can be altered. The trick is to adjust the discrete symmetry so that $A_1$ and $A_2$ have opposite $Z_{12}$ charge to the choices given in (3.2); I also assign $Z_3 \times Z_4$ charge $(1, -1)$ to the adjoint $\phi$. (Although it is not necessary to do so, I will also include a singlet $S$ which has the same charge as $\phi$.) The effect is that the strange and down quark masses are only generated when $SU(5)$ is broken and are somewhat suppressed relative to the bottom quark mass.

Since the $A_a$, $a = 1, 2$, now have opposite charge to the previous case, the coupling $A_a Q_j \bar{H}$ is now forbidden from appearing in the superpotential, and so the strange and down quark masses are set to zero. However, previously ignored operators, such as $\bar{H} \phi H$ ($\bar{H} S H$) and $A_a \phi Q_j \bar{H}$ ($A_a S Q_j \bar{H}$), can now give masses to the Higgs bosons and to the down and strange quarks. We may forbid the Higgs mass terms, if desired, by adding yet another discrete symmetry; but let us keep them for the moment. The superpotential includes the terms

$$W = \sum_{i,j=1}^{3} Y^{ij} A_i A_j H + \sum_{j=1}^{3} y^{3j} A_3 \bar{Q}_j \bar{H} + \bar{H} (h \phi + h' S) H$$

$$+ \frac{1}{m_{pl}} \sum_{a=1}^{2} \sum_{j=1}^{3} \bar{H} A_a [t^{a j} \phi + t'^{a j} S] \bar{Q}_j$$

(4.1)
The couplings $h, h'$ are of order $\eta_3$ while $t^{aj}, t'^{aj}$ are of order $\eta_a$.

Let us first assume that only $\phi$ gets a vacuum expectation value equal to $M_{GUT} \equiv \zeta m_{pl}$ while $\langle S \rangle = 0$. Then, ignoring the fact that the Higgs bosons are given masses, we find predictions (at $M_{GUT}$) of the following sort:

$$ M_u \sim \begin{pmatrix} \eta_1^2 \eta_3 & \eta_1 \eta_2 \eta_3 & \eta_1 \eta_3^2 \\ \eta_1 \eta_2 \eta_3 & \eta_2^2 \eta_3 & \eta_2 \eta_3^2 \\ \eta_1 \eta_3^2 & \eta_2 \eta_3^2 & 1 \end{pmatrix} \sin \beta ; \quad M_d \sim \begin{pmatrix} \eta_1 \zeta & \eta_1 \zeta & \eta_1 \zeta \\ \eta_2 \zeta & \eta_2 \zeta & \eta_2 \zeta \\ \eta_3 & \eta_3 & \eta_3 \end{pmatrix} \cos \beta \quad (4.2) $$

Again the Kobayashi-Maskawa matrix is nearly diagonal as a result of the confinement hierarchy and the discrete symmetries. Notice that to get the down quark masses in the correct proportion while maintaining a reasonable charm quark mass we actually need $\eta_2 > \eta_3 > \eta_1$ for small $\tan \beta$, while for large $\tan \beta$ the hierarchy requires $\eta_3 > \eta_2 > \eta_1$. It appears that $\tan \beta \sim 1$ cannot be accommodated.* Also, $\zeta$ cannot be too small without driving $\eta_2$ close to one, at which point a field theoretic discussion of confinement breaks down. The mixing angles are of the right order of magnitude; the Cabbibo angle tends to be too small but is the most sensitive of the angles to the specific values of the coefficients in the two mass matrices.

Note also that the standard $SU(5)$ relations for lepton and down-quark masses have been altered by this mechanism, though the direction of the effect should be the same for both generations, while in fact the ratios $m_s/m_d$ and $m_{\mu}/m_e$ are far from equal. When the singlet $S$ also acquires a vacuum expectation value, we have a hope of killing two quarks with one stone. Suppose that a version of the sliding singlet mechanism [15] could be used here, solving the doublet-triplet splitting problem by making $\langle S \rangle$ and $\langle \phi \rangle$ proportional. (Recall that such a mechanism can be stable if supersymmetry breaking occurs at a low scale [16].) Simultaneously, if the coefficients $t^{aj}, t'^{aj}$ have no particular symmetry, the $SU(5)$ relations for the two light generations would be broken, and even the ratios $m_s/m_d$ and $m_{\mu}/m_e$ would be unrelated to one another.

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* If the Higgs boson $\bar{H}$ or the fields $\bar{Q}_i$ are also composite, then additional suppression factors will reduce the entire down-quark matrix uniformly, allowing smaller values for $\tan \beta$, and having no easily observable effect at low energy.
5. A second model with an acceptable spectrum

Another way to build a theory which can lead to acceptable fermion masses is to restrict the couplings $y$ by a symmetry so that the down-type Higgs boson masses is to massless fields are neutral under $H$ relabel $X$ to symmetry. The operators $\bar{Q}_a$, $S_a$ are assigned charge $(1, i)$, $(1, -i)$ for $a = 1, 2$; another five 5 representations $\bar{Q}_u$ are neutral under $Z_{12}$. After confinement, $H_1$ and $H_2$ are neutral under the discrete symmetry and become massive along with two of the $\bar{Q}_u$; label the remaining three fields $\bar{Q}_i$, and relabel $H \equiv H_3$. The fields $A_1, A_2$ have discrete charge $(1, -1)$, while the charges for $A_3, \bar{H}, H$ are $(z, -1), (z^2, -1), (z, 1)$. The renormalizable terms in the superpotential for the massless fields are

$$W = Y A_3 A_3 H + y A_3 \bar{H} Q_3$$

where the flavor index of the $Q_i$ was rotated so that only $Q_3$ appears in the above formula. This model is just the one-generation model we started with, plus two massless generations.

Giving masses to the other quarks and leptons requires partially breaking the discrete symmetry. The operators $A_a \bar{Q}_j H$ and $A_a H$, where $a, b = 1, 2$ and $j = 1, 2, 3$, have charge $(z^2, 1)$ and $(z, 1)$ respectively under the $Z_3 \times Z_4$. If a gauge singlet $S$ with discrete charge $(z^2, 1)$ acquires a vacuum expectation value $\langle S \rangle = \xi m_{pl}$, then it will allow light quark masses to be generated. In particular the charm and up quark masses will be suppressed by $\xi$ and those of the strange and down quarks will be suppressed by $\xi^2$. Dangerous terms like $HH$ and $A_i \bar{Q}_j Q_k$ and allowed terms of the form $A_3 A_2 H$ cannot be generated, since their charges are not multiples of $(z, 1)$. Unfortunately the term $\bar{Q}_i H$ cannot be generated; this can only be forbidden by adding yet another discrete symmetry (R-parity) under which $A_i, Q_i$ change sign while $H, \bar{H}$ do not.

The $SU(2)$ groups for the three generations become strongly coupled at scales $\Lambda_1, \Lambda_2, \Lambda_3$; define $\eta_i \equiv \Lambda_i/m_{pl}$. The high-energy mass matrices, up to factors of order one, are then

$$M_u \sim \begin{bmatrix} \eta_1^2 \eta_3 \xi & \eta_1 \eta_2 \eta_3 \xi & \eta_1 \eta_3^2 \xi^2 \\ \eta_1 \eta_2 \eta_3 \xi & \eta_2^2 \eta_3 \xi & \eta_2 \eta_3^2 \xi^2 \\ \eta_1 \eta_3 \xi^2 & \eta_2 \eta_3 \xi^2 & 1 \end{bmatrix} \sin \beta ; \ M_d \sim \begin{bmatrix} \eta_1 \xi^2 & \eta_1 \xi^2 & \eta_1 \xi^2 \\ \eta_2 \xi^2 & \eta_2 \xi^2 & \eta_2 \xi^2 \\ \eta_3 & \eta_3 & \eta_3 \end{bmatrix} \cos \beta$$

Again the Kobayashi-Maskawa matrix will be close to diagonal, as has been guaranteed by the hierarchy in the confinement scales and the discrete symmetry. For this model $\tan \beta$ must be large, with $\eta_2 \sim \eta_3 > \eta_1$ and $\xi \sim .2$. (As before, $\tan \beta$ can be smaller if $\bar{Q}_i$ or $\bar{H}$ are also composite.) Again the mixing angles are of the right order of magnitude, with the Cabibbo angle tending to be too small but varying rapidly as coefficients are adjusted.
6. Summary

While these models in their present form win no prizes for elegance, do not by themselves break $SU(5)$ and supersymmetry, and do not consistently generate a small $\mu$ term and a large mass for the color-triplet Higgs bosons, they have a number of interesting features which can perhaps be used in more complete and successful models.

(1) The generations of the standard emerge in a curious way. Those particles which are contained in $\mathbf{5}$ representations of $SU(5)$ — the down-type antiquarks and the down-type Higgs and lepton doublets — are present as fundamental fields, while all other particles, in the 5 and 10 representations, arise as massless composites below the scales $\Lambda_i$ of the confining gauge groups.

(2) The confining dynamics has implications for the masses of the quarks. The top quark gets its mass from an operator which is generated dynamically during confinement; its coefficient is of order one. The other quarks and the leptons get their masses from operators which at $m_{pl}$ have dimension at least four in the superpotential (five in the Lagrangian); their Yukawa couplings to the Higgs bosons are of order $\Lambda/m_{pl}$ to a positive power. A hierarchy of confinement scales is inherited by the Yukawa couplings, ensuring that the Kobayashi-Maskawa matrix is close to the unit matrix when the quark masses are very different from one another. Furthermore, the tendency is for the splittings of masses of adjacent generations to be larger in the up-quark sector than in the down-quark sector.

(3) In the simplest illustration of the mechanism, the spectrum of predicted quark mass relations is difficult to reconcile with the observed masses. Two models are proposed in which certain quark masses and mixings are only generated when a discrete symmetry is broken at a lower scale; this introduces a new parameter into the theory and improves the spectrum at the cost of predictivity. In one version the strange and down quark masses are generated during $SU(5)$ breaking, potentially destroying their relations with lepton masses.

(4) In these models, the predictions of $SU(5)$ grand unification are naturally preserved, since the $SU(5)$ group is a weakly coupled spectator to the dramatic events of confinement. This is true even when $M_{GUT}$ lies at or somewhat above the confinement scale(s). If one gives up on $SU(5)$ unification the mechanism will still work with the standard model gauge group. Since the left-handed down quark is composite while the right-handed down-quark is not, this mechanism cannot be directly transplanted to models with $SO(10)$ or $E_6$ unified gauge groups.
Finally, it is worth commenting on the remarkable fact that the analysis of this scenario is firmly based on developments in our understanding of strongly coupled supersymmetric gauge theories. Most composite models have relied on questionable dynamical assumptions. The dynamics discussed here are known to be consistent with a wide variety of phenomena found in supersymmetric gauge theories [8]. Note, in particular, that the choice of gauge group $SU(2)$ was essential. For example, if $[SU(2)]^3$ were simply replaced with $[Sp(2)]^3\ast$, the low energy $SU(5)$ composite representations would still be $A_i$ and $H_i$, but no top-quark–Higgs-boson Yukawa coupling would be generated, and a quantum mechanical constraint $\langle A^2H \rangle = \Lambda^3$ would break $SU(5)$ to at most $SU(2) \times SU(2)$ at the confinement scale [6]. Adding two additional fundamental representations of each $Sp(2)$ to the model would leave $SU(5)$ unbroken but still would not lead to a top-quark–Higgs-boson Yukawa coupling. Larger groups would be even more unstable to symmetry breaking. Thus, the dynamics of this model is quite special.

In summary, the dynamics of confinement as understood in [8] have been applied to an extension of the minimal supersymmetric standard model, with the fields and couplings of the standard model emerging only at low energy. The successful predictions of $SU(5)$ grand unification for the gauge couplings and the tau-lepton and bottom-quark Yukawa couplings can be preserved despite the strong coupling phenomena. The mass hierarchies and diagonal Kobayashi-Maskawa matrix are explained as due to a hierarchy in the confinement scales of the three generations in conjunction with a discrete symmetry. The large top quark mass and the larger mass splittings in the up-quark sector versus the down-quark sector are natural predictions of the mechanism, though to get the bottom, charm and strange quark masses to be consistent apparently requires fine tuning or additional structure. Several implementations of this mechanism with a minimal number of Higgs bosons have been presented. One simple model generates a spectrum which has many good features but is probably ruled out; two other variants give reasonable fermion mass spectra, though both are complicated and incomplete. Many other variants are possible, so perhaps more successful and elegant models using this mechanism can be found, or perhaps other theories can be invented which contain the special features of this scenario. Even should it prove to be a dead end, this work demonstrates that our improved understanding of gauge theories makes it possible to build strongly coupled models which have interesting dynamics, can be analyzed reliably, and resemble the real world.

* $Sp(2)$ is the symplectic group with a four-dimensional fundamental representation; it is often called $Sp(4)$. 
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References

[1] G. ’t Hooft, in *Cargese Summer Inst. on Recent Developments in Gauge Theories*, Cargese, France, 1979.
[2] R.R. Volkas and G.C. Joshi, Phys. Rep. 78 (1987) 1437.
[3] T. Hotta, K.-I. Izawa and T. Yanagida, *Dynamical Models for Light Higgs Doublets in Supersymmetric Grand Unified Theories*, hep-ph/9509201.
[4] N. Maekawa, *Duality of a Supersymmetric Standard Model*, KUNS-1361, hep-ph/9509407.
[5] N. Seiberg, Nucl. Phys. B435 (1995) 129, hep-th/9411149.
[6] N. Seiberg, Phys. Rev. D49 (1994) 6857, hep-th/9402042.
[7] A.E. Nelson, unpublished.
[8] K. Intriligator and N. Seiberg, *Lectures on Supersymmetric Gauge Theories and Electric-Magnetic Duality*, RU-95-48, hep-th/9509066.
[9] M. Berkooz, *The Dual of Supersymmetric SU(2k) with an Antisymmetric Tensor and Composite Dualities*, RU-95-29, hep-th/9505088.
[10] K. Intriligator, R.G. Leigh, M.J. Strassler, *New Examples of Duality in Chiral and Non-chiral Supersymmetric Gauge Theories*, RU-95-38, hep-th/9506148, to appear in Nucl. Phys. B.
[11] K. Intriligator and P. Pouliot, Phys. Lett. 353B (1995) 471, hep-th/9505006.
[12] M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, *New Tools For Low-Energy Dynamical Supersymmetry Breaking*, SCIPP-95-32, hep-ph/9507378.
[13] K. Konishi and K. Shizuya, Nuovo Cim. 90A (1985) 111; D. Amati, K. Konishi, Y. Meurice, G.C. Ross and G. Veneziano, Phys. Rep. 162 (1988) 169.
[14] see, for example, V. Barger, M.S. Berger, P. Ohmann, Phys. Rev. D47 (1993) 1093; Phys. Rev. D47 (1993) 2038.
[15] E. Witten, Phys. Lett. 105B (1981) 267.
[16] D. Nemeschansky, Nucl. Phys. B234 (1984) 379.