Heavy Quark Parton Distribution Functions,

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Abstract. We present the CTEQ6HQ parton distribution set which is determined in the general
variable flavor number scheme which incorporates heavy flavor mass effects; hence, this set pro-
vides advantages for precision observables which are sensitive to charm and bottom quark masses.
We describe the analysis procedure, examine the predominant features of the new distributions,
and compare with previous distributions. We also examine the uncertainties of the strange quark
distribution and how the the recent NuTeV dimuon data constrains this quantity.

Parton distributions functions (PDFs) provide the essential link between the theo-
retically calculated partonic cross-sections, and the experimentally measured physical
cross-sections involving hadrons and mesons. This link is crucial if we are to make inci-
sive tests of the standard model, and search for subtle deviations which might signal new
physics. The choice of the renormalization scheme is an important issue to address if we
are to make the most efficient use of our fixed-order perturbation expansion. Separately,
it is important to know the uncertainty range of the PDFs, and properly fold these into
the overall uncertainty estimates. We will address both of these issues in turn.

GLOBAL ANALYSIS AND THE CTEQ6HQ PDFS:

The CTEQ6HQ (or C6HQ for short) PDFs[1] are obtained by performing a global
analysis using the generalized (non-zero quark-mass) MS perturbative QCD framework
of Refs. [2, 3], which we label the general-mass variable-flavor-number scheme (GM-
VFNS). When matched to the corresponding hard-scattering cross-sections calculated
in the same scheme, the combination should provide a more accurate description of the
precision DIS structure function data, as well as other processes which are sensitive to
charm and bottom mass effects.

The C6HQ global fitting follows the same procedure as that of the earlier CTEQ6
analysis.[4] The data sets used before are supplemented by the H1 and ZEUS data sets

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1 The research presented here was performed in collaboration with W.K. Tung, P. Nadolsky, J.F. Owens,
J. Pumplin, D. Stump, J. Huston, & H.L. Lai.
2 Contribution to the proceedings of the XIII International Workshop on Deep Inelastic Scattering (DIS
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for the structure function $F_2^c$ with tagged charm particles in the final state. The $F_2^c$ data sets are quite relevant for this analysis since $F_2^c$ is sensitive to the charm and gluon distributions, which are tightly coupled in the generalized $\overline{\text{MS}}$ formalism.

The C6HQ set is the best fit obtained with these inputs. A broad measure of the quality of this fit is provided by the overall $\chi^2$ of 2008 for a total number of 1925 data points ($\chi^2$/DOF = 1.04). This is to be compared to a $\chi^2$ of 2037 for 1925 points ($\chi^2$/DOF = 1.07) in the case of CTEQ6M (or C6M for short). The new C6HQ fit reduces the overall $\chi^2$ by 29 out of $\sim 2000$ as compared to the C6M fit. The improvement of this generalized $\overline{\text{MS}}$ result over the zero-mass $\overline{\text{MS}}$ result is encouraging, since the generalized $\overline{\text{MS}}$ formalism represents a more accurate formulation of PQCD. However, a difference of $\chi^2$ of 29 is within the current estimated range of uncertainty of PDF analysis. [4] Therefore, the significance of this difference is arguable. We also note that the improvement in $\chi^2$ is spread over most of the data sets: there is no “smoking gun” for the overall difference.

Since perturbative calculations are renormalization scheme dependent, it is important to use properly matched hard-scattering cross-sections and PDFs when evaluating factorized cross-sections for physical applications. This issue is particularly relevant for applications involving heavy quarks, since the heavy quark introduces a new mass scale which leads to complications of the PQCD formalism. To illustrate this point, we compare the above results with two possible uses of the PDFs that represent a mis-use of PQCD in principle, but occur frequently in the literature in practice, perhaps out of necessity. These involve using PDFs obtained in the general-mass scheme convoluted with hard-scattering cross-sections (Wilson coefficients) defined in the zero-mass scheme, and vice versa. For example, if we use the C6M PDFs which are derived in the ZM-VFN scheme with the GM-VFN hard-scattering cross-sections, we obtain a total $\chi^2$ of 2431 for 1925 data points ($\chi^2$/DOF = 1.26). Conversely, if we use the C6HQ PDFs which are derived in the GM-VFN scheme with the ZM-VFN hard-scattering cross-sections, we obtain a total $\chi^2$ of 2496 for 1925 data points ($\chi^2$/DOF = 1.30). For the same data sets, these mis-matched schemes result in an overall $\chi^2$ difference of 420--490 compared to the C6HQ set ($\chi^2$ of 2008). These are quite large differences relative to the tolerances discussed in Refs. [5, 6, 7, 4], and result in obvious discrepancies with some of the precision DIS data sets. Clearly, for quantitative applications it is imperative to maintain consistency between the PDFs and the hard-scattering cross-sections.

**Comparison with Related PDFs:**

The C6HQ and C6M fits provide comparable descriptions of the global QCD data in two different schemes. Some of the differences in the PDFs arise purely from the choice of scheme. We are particularly interested in the differences for the gluon distribution, which will strongly influence the closely correlated charm distribution (since it is generated via the $g \rightarrow c\bar{c}$ process). It is also interesting to compare the differences between the earlier CTEQ5HQ (C5HQ) distributions with the new C6HQ distributions; differences between these PDFs are attributable both to new data, and to minor differences in the way the theoretical inputs are implemented.
Fig. 1a) shows the comparison of the gluon distribution at $Q_0$. Here the difference between C5HQ and the CTEQ6 generation of gluon distributions is pronounced. The change in this least-well-determined parton distribution is due to the recent precision DIS data (most influential in the small $x$ region) in conjunction with the greatly improved inclusive jet data from the Tevatron (critical for the medium to large $x$ regions). The differences between C6HQ and C6M gluons at large $x$ are due to a combination of scheme-dependence, and the inherent uncertainty range of the current analysis.

Fig. 1b) shows the comparison of the strange distributions at the same $Q_0$. The noticeable difference between the C5HQ curve and the others, in this case, is largely the result of different theoretical inputs: namely, the $\kappa$ parameter which determines the ratio of strange to non-strange sea quarks at the initial scale $Q_0$. This $\kappa$ factor, known only approximately, was chosen to be $1/2$ both in the CTEQ5 and CTEQ6 analyses, but for slightly different values of $Q_0$: 1.0 GeV for C5HQ, and 1.3 GeV for the CTEQ6 sets. We now look at the uncertainty of the strange quark in further detail; this has received increased attention recently since this is an important ingredient in the NuTeV determination of $\sin \theta_W$. 

**Figure 1.** Comparison of the a) gluon and b) strange-quark distributions at $Q_0 = m_c = 1.3$ GeV for the CTEQ5HQ, CTEQ6M, and CTEQ6HQ sets. The axes are scaled to highlight the valence components of these distributions.

**Figure 2.** a) Ratio of the 40 CTEQ6M PDF sets compared to the central set for the strange-quark vs $x$. b) Same as previous, with additional sets included (with a more general $s(x)$ parameterization). In both figures, guide-lines are drawn at $\pm 5\%$. 

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**Strange-quark PDF and Uncertainties:**

Using the set of 40 C6M PDF sets, we can produce a band of distributions which, in principle, should characterize the uncertainty of the s-quark, cf., Fig. 2a). Based on this plot, one would be tempted to conclude that the uncertainty on the strange quark is better than 5% over much of the x range. Because the parameterization of s(x) in the global fit is constrained to be κ[\bar{u}(x) + d(x)] at Q_0, the figure actually reflects the uncertainty not on s(x) but instead on [\bar{u}(x) + d(x)]. This constraint is imposed because none of the data in the global analysis directly measures the s(x) distribution; hence, Fig. 2a) is by no means a representation of the true uncertainty.

Recent measurements of the charged-current charm production (νs → cμ → μ±μ∓X) from CCFR and NuTeV [8, 9] have the potential to determine \{s(x), \bar{s}(x)\} with much more precision. In Fig. 2b) we again show the 40 C6M PDF sets combined with a number of additional fits that relax the parameterization of s(x). This is not an exhaustive collection designed to span the full parameter space, but rather an illustration that the implied uncertainty of Fig. 2a) is much too conservative; the dimuon data will go a long way toward improving our knowledge of the strange PDF. An accurate determination of the strangeness of the proton, as well as the strangeness asymmetry [s(x) − \bar{s}(x)] will have important implications for a number of measurements, including the NuTeV sinθ_W measurement.

**Concluding Remarks:**

The C6HQ PDFs presented here complement the CTEQ6 sets by providing distributions which can be used in the generalized \overline{MS} scheme with massive partons. This analysis includes the complete set of NLO processes including the real and virtual quark-initiated terms.

While the zero-mass parton scheme is sufficient for many purposes, the fully massive scheme can be important when physical quantities are sufficiently sensitive to heavy quark contributions. This is evident when comparing the C6HQ and C6M fits to the mis-matched sets where the precise DIS data from HERA highlights the discrepancies.

The C6HQ fits also provide the basis for a series of further studies involving more quantitative analysis of strange, charm, and bottom quark distributions inside the nucleon. For example, the C6HQ PDFs are necessary for a consistent analysis of resumed differential distributions for heavy quark production.[10] Using the full range of data from both the charged and neutral current processes, these distributions can reduce the uncertainties in the calculations; hence, they have significant implications for charm and bottom production, and can help resolve questions about intrinsic heavy quark constituents inside the proton, the ΔxF_3 structure function, and the extraction of sinθ_W.
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