Random processes imitation in the problem of fatigue under variable loading

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Abstract. There is strong evidence, that the extrema and their sequence play a major role in metal fatigue. Due to this random testing with extrema control might be preferable, compared with the testing with generating by the spectral densities. The method is proposed allowing to generate random processes with the stable integral characteristics of cumulative distributions, and at the same time, being random by their nature. The algorithm consists of two steps. The first one is creating the random sequence of extrema, basing on the target Markov matrix, filled up on the base of the loading process, recorded during exploitation. The next step is the creation of the random process with its continuous first derivate. The special concatenation procedure was proposed and had been proved to create the continuous function reconstruction out of the point-wise specification (local maximums and minimums sequence). To judge the frequency content a regression analysis of two random variables was performed. The obtained continuous process might be undergone by the spectral analysis to get the spectral density function, which might be required in applying some methods for random fatigue estimation. The main trait of the proposed method is the coincidence of the rain-flow cycle counting characteristics of the initial and the resulting processes.

1. Introduction and reverence of the problem

Due to the fact, that the service loading is random in most cases, testing under irregular loading is getting more and more widespread. It concerns also the machine design stage. Testing under irregular loading reveals important peculiarities, connected with the randomness of impact. The randomness of loading is also one of the sources of the scatter of experimental longevities N and influence upon the variance of log(N). Historically after testing only under sinus-like loading for about 70 years, scientists set up into testing under so-called program loading [1], that is a sequence of sinus-like blocks with varied amplitude and repetition number. This type of testing was a first step to approaching the real, random character of the loading. With testing and controlling technique progress, the second step was inevitable: that was the random testing. At first, it was the random shakers which performed testing with a given spectral density of acceleration.

After developing the servo-hydraulic testing equipment, the researchers got the opportunity of the total control on loading using the feed-back technique. Since then, technically any peculiarities of the random process might be fulfilled (the backside of this event is loss of the productivity at some extend). The manufactures of such systems together with the researches developed the standards for testing for irregular loading for the wide number of scientific and industrial specific tasks [2].
In [3] the detailed review concerning the random load sequences is presented. More specifically, it is distinguished between “load-time history” for processes with actual-time information (e.g. real-time processes) and “load sequence” as a succession of loading events without time information (e.g. load turning points).

One of the disadvantages of using the standard loading sequences is in the fact, that they are not ever “random” after being recorded. Markov matrix simulation [4], and especially target matrix simulation [5] allows in a justified way add randomness in the tests building the sequence of the turning points. To fill up this sequence with the time component, the method of regeneration of the continuous processes was developed. This last step might be required while estimation the spectral densities of the smooth process and for some other problems, where the continuality of the first derivative of the process is the must.

2. Method. Step 1.
In [4] the earlier developed method for random sequence imitiation by the Markov’s matrix is described in details. One of the limitations of the standard approach with Markov’s matrix is its non-consistency with the real examples of loading. We propose a target Markov matrix imitation. On the base of a real process, 32x32 matrix for upward and downward half-cycles repetition is filled down. The example of such a matrix, shortened for simplicity down to 12x12 cells, is shown in Figure 1.

![Figure 1](image)

**Figure 1.** The Markov’s matrix (simplified) filled up after the TWIST sequence

The digits in the cells are the repetitions numbers of the half-cycles in the TWIST sequence. Ones on the upper right triangle are the upward half-cycles. Left downside triangle consists of the repetitions numbers of downward ranges. At the main diagonal are zeros. The cells, which are closer to the main diagonal include the bigger number of repetition. That is explainable due to the fact, that in each random process, the cycles with small amplitudes are prevailing.

Before filling up the matrix, the random process is subjected to the procedure of the division into classes (levels). This procedure is similar to rounding. It corresponds to the digit number in digital recorders. The engineering practice confirmed, that to the task of fatigue investigation, 32 levels are enough. The procedure, which performs rounding the initial process into 32 levels, as well all the other program codes, used in this paper, are written in R [6]. Figure 2 shows the initial realization (shown red) and 5 modelled replicas. The realization and its replicas, modelled by the target Markov’s matrix using the special technique [5] are shown in the form of the turning points sequence. This the initial sequence is built on the base of the real realization loading in the machine, recorded during 1 hour.

The basic parameters of the random sequences (1 initial + 5 replicas) are shown in Table 1. Here: mean and sd are the evident statistical indicators; $\sigma_{\text{max}}$ is the maximum stress amplitude after the rain-flow cycle counting; $I$ is irregularity factor, $V$ is the fullness ratio (depends on the distribution of the rain-flow amplitudes and on $m$- the fatigue exponent). From the Figure 2 and Table 1, it is evident, that the parameters vary, since during their generation not only the information of repetition from the Markov’s matrix is used, but also the random number generator.
3. Method. Step 2.

The replicas, shown in Figure 2 and described in the Table 1, are well suited for the fatigue testing and damage evaluation with cycle counting methods (the rain-flow). On the other hand, they are not smooth and do not possess the quality of continuality of their first derivative. They also lack any information.

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**Figure 2.** The plots of the initial sample and some replicas

**Table 1.** Some characteristics of the initial sample and its replicas

| Sample (88 cycles)       | Initial | 1-st replica | 2-nd replica | 3-rd replica | 4-th replica | 5-th replica |
|--------------------------|---------|--------------|--------------|--------------|--------------|--------------|
| Mean [MPa]               | 543     | 532          | 528          | 539          | 527          | 560          |
| sd [MPa]                 | 106     | 113          | 109          | 96.3         | 115          | 120          |
| \(\sigma_{\text{amax}}\) [MPa] \(^1\) | 301     | 301          | 301          | 301          | 301          | 292          |
| Irregularity factor \(I\) | 0.62    | 0.54         | 0.61         | 0.65         | 0.66         | 0.65         |
| Fullness ratio \(V\)     | 0.5232  | 0.5511       | 0.5568       | 0.5225       | 0.5568       | 0.5770       |

\(^1\) Maximum amplitude in the loading block

\(^2\) \(I=n_o/n_e\). \(n_o\) – is the number of process crossing the mean level, \(n_e\) – extremum number

\(^3\) \(V = \frac{m}{n} \sum h_i \left(\frac{\sigma_{ai}}{\sigma_{\text{amax}}}\right)^m\) \(m\)– fatigue exponent; \(n=88\), total number of cycles in block; \(h_i\) – cycles number on the \(i\)-th step; \(\sigma_{ai}\) – stress amplitude value on the \(i\)-th step.
about time. Therefore, it’s impossible to estimate spectral density of the replicas, which is needed sometimes for applying the spectral methods for longevity estimation.

In Figure 3, left, a short piece of the ‘saw’-like sequence, generated according to the Step 1 is shown.

![Figure 3. A fragment of the realization a) ‘saw’-like sequence of extrema; b) cosine extrapolation](image)

To overcome the problem of not smoothness of the ‘saw-like’ process, Figure 3(a), the closest peaks are proposed to be connected by half-cosines (Figure 3(b)). These half-cosines are concatenated later on. The sequence of reading form the continuous random process with continuous first derivate. The main idea of this modelling – that is the values of the turning points and their sequence remain unchanged while transferring from (a) to (b). This point is paramount for fatigue estimation not only on the stage of crack initiation but also at crack propagation stage [5].

The equations for half-cosines:

\[ x(t) = A \cos (w t + \phi) \]  

where \( x(t) \) – is the part of the continuous extrapolating function, which is described on the domain \( t=0... \pi/w \), because the period of the cosine function is \( T=2\pi/w \), s.

For each half-wave starting from the successive extremum \( MAX \) or \( MIN \), the parameters \( A \), \( w \), and \( \phi \) are unique. The stress amplitude \( A \), MPa is defined as half of the range (modulus) of successive extremes:

\[ A = \text{mod}(MAX_i - MIN_{i+1})/2 \lor A = \text{mod}(MAX_i - MIN_i)/2 \]  

In the example shown in Figure 3(b) for each half-wave 10 readings have been taken.

Usually, the frequency component is not defined in known standards of random sequences. So there is a need to define it. Intuitively one can imagine, that the greater the oscillation, the more time will be required to its executing. That fact is well-known to the personal, who perform the testing under random loading. In Figure 4 the dependence of the range \( Raz \) of oscillation on the period \( t \) of the exemplar service sequence is shown. Although the scatter on the plot in Figure 4 is significant enough (correlation factor is only \( K=0.6664 < 1 \)), one could still notice a tendency of increasing \( Raz \) with \( t \) increasing. This fact was utilized during the imitation of the smooth random process.
For one of the trials of the modelled process was underdone the spectral analysis. The graph of spectral density, in this case, is shown in Figure 5. For other trials, the graph might differ because of the ambiguity of choosing the time component.

4. Discussion and conclusions
Some researchers insist on using spectral densities for fatigue estimation [7]. So, the proposed method might have helped in a situation, when there is a need to apply the spectral density approach in case of utilization some standard SLH (standardized load-time histories, [3]), which do not possess time component. An example of such situation might be found in [8], where the author speculated about the spectral density of WHISPER sequence [3], which is the specific sequence for the loading of wind turbines.

On the other hand, there is serious doubt about the necessity and even applicability of spectral approaches in case of uniaxial machine parts loading at the post-processing stage, that is when the machine already exists. There is an abundance of the scientific papers, highly pricing the spectral approach comparing with the rain-flow approach. That might have confused the engineers. It seems that the time has come to clarify this argue. Although the best argument will be the experimental one, the design modelling with proposed method might have been also valuable.
Markov’s matrix imitation will help the researchers to investigate the longevities scatter and to make the important step towards random loading nature understanding in the machines life cycle.

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