Supersymmetric two-loop contributions to the anomalous magnetic moment of the muon

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Recent results of two interesting classes of supersymmetric two-loop contributions to $(g-2)_\mu$ are presented. Two-loop diagrams involving a closed sfermion loop can amount to $5 \times 10^{-10}$ or almost one standard deviation of the current experimental uncertainty, and two-loop diagrams with a closed chargino/neutralino loop can be similarly large. The dependence of these two classes on the unknown supersymmetric parameters and the sensitivity on existing experimental constraints on the parameter space are quite different. We also comment on the calculational techniques, in particular the large mass expansion and the treatment of $\gamma$ and the $\epsilon$-tensor.

1. Introduction

In the past few years, the research on the anomalous magnetic moment $a_\mu = (g-2)_\mu/2$ of the muon has made exciting progress. The Brookhaven “Muon $g-2$ Experiment” (E821) has measured $a_\mu$ with unprecedented precision to $a_\mu^{\text{exp}} = (11 659 208 \pm 6) \times 10^{-10}$ \cite{1}. Motivated by this tremendous experimental precision, many groups have contributed to an improvement of the evaluation of the hadronic contributions to $a_\mu$, the bottleneck of the Standard Model (SM) prediction \cite{2,3,4,5,6,7}. Now the theoretical precision is at the same level as the experimental one.

The comparison of the experimental value and the SM theory prediction exhibits a discrepancy of about $20 \ldots 30 \times 10^{-10}$, depending on the evaluation of the hadronic contributions to $a_\mu$, the bottleneck of the Standard Model (SM) prediction \cite{2,3,4,5,6,7}. It is an interesting question whether the observed deviation \cite{1} is due to supersymmetric effects. The supersymmetric one-loop contribution \cite{10} is approximately given by

$$a_\mu^{\text{SUSY,1L}} = 13 \times 10^{-10} \frac{\tan \beta \text{sign}(\mu)}{(M_{\text{SUSY}}/100 \text{ GeV})^2},$$

if all supersymmetric particles (the relevant ones are the smuon, sneutralino, chargino and neutralino) have a common mass $M_{\text{SUSY}}$.

This formula shows that supersymmetric effects can easily account for a $(20 \ldots 30) \times 10^{-10}$ deviation, if $\mu$ is positive and $M_{\text{SUSY}}$ lies roughly between 100 GeV (for small $\tan \beta$) and 600 GeV (for large $\tan \beta$). On the other hand, the precision of the measurement places strong bounds on the supersymmetric parameter space.

In this talk we present the results of Refs. \cite{11,12} for the Minimal Supersymmetric Standard Model (MSSM) two-loop contributions of diagrams involving either a sfermion or a chargino/neutralino subloop. These contributions constitute the class of two-loop contributions to $a_\mu$, where a supersymmetric loop is inserted into a SM (or more precisely a two-Higgs-doublet model) one-loop diagram.

These diagrams are particularly interesting since they can depend on other parameters than the supersymmetric one-loop diagrams and can

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\textsuperscript{2}This evaluation is $e^+e^-$ data driven. Recent analyses concerning $\tau$ data indicate that uncertainties due to isospin breaking effects may have been underestimated earlier \cite{13}.
therefore change the qualitative behaviour of the supersymmetric contribution to \( a_\mu \). In particular, they could even be large if the one-loop contribution is suppressed, e.g. due to heavy smuons and sneutrinos.

2. Calculation

The diagrams we have to calculate are the two-loop three-point graphs for the \( \mu \mu \gamma \) interaction with a closed sfermion or chargino/neutralino subloop (and the corresponding counterterm diagrams). The main calculational steps are the following: The amplitudes for \( a_\mu \) are generated using the program FeynArts \[13,14\], and the appropriate projector \[8\] is applied. The Dirac algebra and the conversion to a linear combination of two-loop integrals is performed using TwoCalc \[15\].

The main part of the two-loop calculation consists of the calculation of the Feynman integrals and the simplification of their coefficients, both of which is complicated by the large number of different mass scales and the involved structure of the MSSM Feynman rules.

As a first step we perform a large mass expansion \[16\] where the muon mass is taken as small and all other masses as large. The large mass expansion separates the light and heavy scales in the two-loop integrals, with the following possibilities:

(light 0-loop)\(\circ\)(heavy 2-loop): The heavy two-loop integral has no external momentum, hence it can be reduced to the two-loop vacuum master integral.

(light 1-loop)\(\circ\)(heavy 1-loop): The integrals can be reduced to the standard one-loop functions \( A_0(m) \) and \( B_0(m^2, 0, m_\mu) \).

Since all integrals contain at least one sfermion or chargino/neutralino loop, which is heavy, the third case of a light 2-loop integral does not appear. This situation changes in the calculation of the pure two-Higgs-doublet model diagrams \[12\].

A few remarks have to be made in order to justify our use of dimensional regularization with anticommuting \( \gamma_5 \). Firstly, one might expect that supersymmetry-restoring counterterms are necessary as discussed in Refs. \[17\]. This is however not the case since in the present calculation only two-Higgs-doublet model counterterms appear, which are not related by supersymmetry. Hence, generating the counterterms by multiplicative renormalization is sufficient.

Secondly, there is the well-known inconsistency of using anticommuting \( \gamma_5 \) together with the trace formula

\[
\text{Tr}(\gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) \propto \epsilon^{\mu\nu\rho\sigma},
\]

which is necessary in order to reproduce the correct four-dimensional limit of the trace.

Another problem connected with the \( \epsilon \)-tensor is that it is a purely four-dimensional object, and contractions such as \( \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma} \) or \( \epsilon^{\mu\nu\alpha\beta} \epsilon^{\rho\sigma\alpha\beta} \) have to be evaluated in four dimensions. This seems to contradict the application of the projection operator from Refs. \[8\] to extract \( a_\mu \), which relies on a purely \( D \)-dimensional covariant decomposition of the regularized \( \mu \mu \gamma \) vertex function.

In the discussion of both problems connected with \( \gamma_5 \) and the \( \epsilon \)-tensor there is one main point to be noticed. The triangle subdiagrams with three external vector bosons and external momentum \( k \), where the traces and \( \epsilon \)-tensors originate, do not lead to the covariant \( \epsilon^{\mu\nu\rho\sigma} k_\sigma \) of power-counting degree +1. This covariant is multiplied with the same prefactor as the chiral gauge anomaly, which is zero after summation over a full generation of fermions or all charginos/neutralinos. The only remaining covariants containing the \( \epsilon \)-tensor have power-counting degree < 0. If this fact is used in the two-loop diagrams with triangle subdiagram, one can show that the inconsistency of the trace does not show up. Moreover, the covariants of the \( \mu \mu \gamma \)-vertex function involving \( \epsilon \)-tensors are finite. Therefore the four-dimensional treatment of the \( \epsilon \)-tensors does not spoil the validity of the projection operator.\(^3\)

3. Results

The results for the supersymmetric contributions to \( a_\mu \) are not numbers but functions

\(^3\)For more details see the discussion in Ref. \[12\]. The situation is similar but not identical to the one of \( \mu \)-decay \[18\]. In the latter case all external momenta can be set to zero, whereas in the calculation of \( a_\mu \) the external momentum of the muon is non-zero, \( \not{p}^2 = m_\mu^2 \not{\neq} 0 \). Hence there are more momenta the \( \epsilon \)-tensor can be contracted with.
of all the unknown MSSM parameters. However, the parameter dependence and the corresponding phenomenological discussion shows important differences between the sfermion and chargino/neutralino loop contributions.

— The sfermion loop contributions depend on the Higgs sector parameters $\mu$ and $\tan \beta$ and the sfermion mass parameters in a rather complicated way. Furthermore, it turns out that experimental constraints on the MSSM parameter space significantly restrict the possible sfermion loop contributions \[11\].

— In contrast, the chargino/neutralino loop contributions depend on $\mu$, $\tan \beta$ and the gaugino mass parameter $M_2$ in a quite straightforward way, and experimental constraints on the parameter space have not much impact \[12\].

In order to understand the parameter dependence of the sfermion loop contributions in more detail, consider the diagram in Fig. 1. It has a double enhancement by the muon Yukawa coupling $\propto m_\mu \tan \beta$ and by the Higgs–stop coupling $\propto \mu m_\tilde{t}$. Its value is approximately given by \[11\]

$$ a_\mu^{\tilde{t},2L} \approx -13 \times 10^{-10} \text{sgn}(A_t) \times \left( \frac{\tan \beta}{50} \right) \left( \frac{\mu}{20 m_\tilde{t}} \right) \left( \frac{m_\tilde{t}}{M_H} \right), $$

provided that $m_\tilde{t} \lesssim M_H$ ($\tilde{t}$ stands for lightest stop mass eigenstate). For large $\tan \beta \approx 50$ and $M_H$ around 200 GeV, we find that the stop diagram contribution is mainly determined by the ratio $\mu/m_\tilde{t}$. If this ratio is very large, $\mu/m_\tilde{t} \gtrsim 20$, contributions of more than $10 \times 10^{-10}$ are possible. This was already noticed in Ref. \[19\] and would correspond to a two-loop contribution of more than $1 \ldots 2$ experimental standard deviations.

Whether or not the ratio $\mu/m_\tilde{t}$ is restricted by experimental constraints depends on yet another property of the MSSM parameters, namely the universality between the stop and sbottom mass parameters. The comparison of the experimental limit and the MSSM-prediction of the lightest Higgs-boson mass $M_h$ \[20,21\] generally constrains the ratio (off-diagonal):(diagonal entry) in the stop mass matrix. If the stop and sbottom mass parameters are universal, this translates into a limit on the ratio $\mu/m_\tilde{t} \lesssim 3$ for the relevant parameter space. As the ratio between sbottom and stop mass parameters increases, also larger values for $\mu/m_\tilde{t}$ become possible.

In Fig. 2 we show the full results for the sfermion contributions as functions of the lightest sfermion mass for universal sfermion mass pa-
rameters. The outer lines show the maximum possible results for $\tan \beta = 50$ if all experimental constraints are ignored and all MSSM mass parameters are varied universally up to 3 TeV (for the $CP$-odd Higgs-boson mass we use $M_A > 150$ GeV). The next lines show the maximum possible results if the aforementioned experimental limit on $M_h$ is taken into account. We find, in agreement with the preceding discussion, that the maximum results are drastically reduced. For a lightest sfermion mass of 100 GeV, the results are reduced from more than $15 \times 10^{-10}$ to about $5 \times 10^{-10}$. The inner lines correspond to taking into account more experimental constraints on $\Delta \rho$ [22], $\text{BR}(B_s \to \mu^+\mu^-)$ [23] and $\text{BR}(B \to Xs\gamma)$ [24]. They reduce the maximum contributions further.

The chargino/neutralino two-loop contributions have a more straightforward parameter dependence. They depend on $\tan \beta$ and the mass parameters for the Higgsinos, $\mu$, the gauginos, $M_2$, and the $CP$-odd Higgs boson, $M_A$. For the simple case that all these mass parameters are equal to a common mass scale $M_{\text{SUSY}}$, we obtain the approximation

$$a_{\mu}^{\chi,2L} = 11 \times 10^{-10} \left(\frac{\tan \beta/50}{M_{\text{SUSY}}/100 \text{ GeV}}\right)^2 \left(\frac{\text{sign}(\mu)}{50}\right).$$  

(5)

If all the masses are even equal to the smuon and sneutrino masses, this formula can be immediately compared to the one-loop contributions [2]. In this case the chargino/neutralino two-loop contributions amount to about 2% of the one-loop contributions.

If the smuon and sneutrino masses are heavier than the chargino and neutralino masses, the one-loop contributions are suppressed and the two-loop contributions can have a larger impact. Fig. 3 shows the sum $a_{\mu}^{\text{SUSY,1L}} + a_{\mu}^{\chi,2L}$ in comparison to the one-loop result $a_{\mu}^{\text{SUSY,1L}}$ alone as a contour plot in the $\mu$–$M_2$-plane. The smuon and sneutrino masses are fixed to 1 TeV and $\tan \beta = 50$, $M_A = 200$. We find that in this case the two-loop corrections from the chargino/neutralino loop diagrams can modify the $1\sigma$, $2\sigma$, … contours significantly.

4. Conclusions

Supersymmetric contributions to $a_\mu$ could easily account for the observed $(20 \ldots 30) \times 10^{-10}$ deviation between SM theory and experiment. Conversely, the precision of the experiment places stringent bounds on the MSSM parameter space.

The two-loop contributions presented here can substantially modify the supersymmetric one-loop contribution, and their knowledge reduces the theoretical uncertainty of the supersymmetric prediction for $a_\mu$. The sfermion loop contributions can have values up to $5 \times 10^{-10}$ for small sfermion masses and large $\tan \beta$; for non-universal stop and sbottom mass parameters with a large hierarchy they can be even larger. In the numerical evaluation of these contributions it is crucial to take into account all known experimental constraints on the MSSM parameter space. The chargino/neutralino loop contributions are well approximated by eq. (5) and can have values of more than $5 \times 10^{-10}$, if the chargino/neutralino masses are small.

In Refs. [11,12] also the SM/two-Higgs-doublet model-like contributions of two-loop diagrams
with fermion loops and with purely bosonic loops have been computed. The difference of the diagrams in the MSSM and the SM is smaller than $1 \times 10^{-10}$. In order to complete the full two-loop calculation of $\alpha_\mu$ in the MSSM, the two-loop corrections to the supersymmetric one-loop diagrams with smuon or sneutrino exchange have to be evaluated.

Acknowledgments

I thank the organizers and the participants of Loops & Legs 2004 for creating a pleasant and stimulating atmosphere.

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