Leader–follower formation source seeking control of multiple ships using sliding mode active disturbance rejection observer

Zhicheng Yuan¹, Benchao Wu¹, Jiayi He², Xingchen Fu¹ and Hua Chen³,⁴

Abstract
In this paper, the control of multiple ships for unknown scalar field source seeking problem with unknown external disturbances is considered. The sliding mode active disturbance rejection observers are designed first to converge to fixed multiple of the unknown external disturbances in finite time, respectively, and a least square method is adopted to estimate the gradient of the unknown scalar field at the position of the leading ship. Second, the surge, sway and angle velocity of the leading ship can converge to the virtual kinematic controllers through the input control of the dynamic controllers using force and torque in finite time. Third, the virtual controllers and dynamic controllers of the following ships are developed to urge the following ships to accomplish the source seeking problem from the perspective of dynamics. Finally, theoretical proofs and simulations are provided to prove the effectiveness of the strategy proposed.

Keywords
Leader–follower formation, dynamic feedback, active disturbance rejection control, sliding mode

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Introduction
In the past few decades, large quantities of research works on source seeking have been done.¹–⁴ In Li et al.,¹ the authors have coped with the source seeking problem of an autonomous underwater vehicle (AUV) embedded with multiple sensors. The gradient estimation based on the specified sensors whose configuration designed as a semicircle is proposed and the controller making use of the estimated gradient eventually lead the AUV to the signal source. This search is characterized using the large number of sensors in special formation. In Yang et al.,² a distributed stochastic source seeking algorithm is presented based on the method of stochastic extremum seeking. The measurements of the signal field at each vehicle, together with their relative distances are utilized to navigate all the vehicles to the signal source. The danger of the signal source is considered by some researchers³ and a sliding mode controller is proposed to drive the robot to reach an area with a specified distance away from the signal source and afterwards the robot will stay in the region continuously. On the basis of a new sliding mode strategy, a control algorithm without gradient estimation is proposed,⁴ which is simple in calculation and can seek the source with good performances. However, so far, there has been few research works relative to the source seeking of a pack of ships in spite of the fact that the control of ships is of great significance especially in the stability and impact strength of various ships in some important occasions.

As for the control of ships, there exists lots of research works on the underactuated ships⁵–⁷ together with full-actuated ships⁸,⁹ with respect to tracking and control. In Demg et al.,⁵ an adaptive bounded term is added to distribute the errors into actuated motion and the fuzzy logic system is employed to estimate the uncertainties adopting the methodology of robust damping. And with the help of Gauss error function, the underactuated ship’s trajectory can converge to the desired one. In Pettersen and Nijmeijer,⁶ a coordinate transformation first proposed by Pettersen and Egeland (1996) with a dynamic function considered additionally

¹School of Mechanical and Electrical Engineering, Hohai University, Changzhou, China
²School of Internet of Things Hohai University, Changzhou, China
³College of Science, Hohai University, Nanjing, China
⁴Engineering Research Center of Dredging Technology of Ministry of Education, Hohai University, Changzhou, China

Corresponding author:
Hua Chen, College of Science, Hohai University, Nanjing 210098, China.
Email: chenhua112@163.com

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is developed, and with the assist of averaging method, a unique technique involved in feedback control law is given to realize the stabilization and tracking problem. Input and velocity are considered restricted and kinematic along with dynamic functions are studied together with the help of backstepping method, and using the dynamic surface control method, the global tracking control problem is then accomplished. Differing from the underactuated ships, full-actuated ship's research is relatively not that difficult. Similar to Chwa, dynamic surface control is adopted to avoid the traditional “explosion of complexity” problem and based on the minimal learning parameterization, the measurable propeller is selected as the control inputs with the uncertainty estimated by radial-basis-function neural network. In another paper, with external disturbances and without velocity measurement, a finite-time control law is developed with the velocity and disturbances measured by finite-time observer approximately to lead the ship to the desired direction and position.

Viewing research works about the source seeking of ships, most of the controllers designed to solve the source seeking problem are kinematic ones, which cannot meet the stability demands in some cases influenced by external disturbances and are not that realistic to some extent to be taken as control inputs compared with dynamic controllers using torque and force as control inputs. Besides, the signal source can be hazardous sometimes and it is not appropriate to lead the ships to the signal source directly.

For the sliding mode active disturbance rejection control (ADRC), Li et al. proposed an ADRC method based on output prediction using the principle of smith predictor to resist the negative effects originating from the phase hysteresis to cope with the multivariable systems with the structural vibration as an example. In Li and Li, consider the internal and external disturbances in wind energy conversion systems (WECS) and put forward a predictive active disturbance rejection control (PADRC) strategy for a direct-driven permanent magnet synchronous generator (PMSG)-based WECS to maximize the wind power extraction. Ding et al. construct a new barrier sliding mode (SOSM) control algorithm which enables the output variable to keep in the boundary of the constraint region, thus finishing the output constraint problem successfully. Mei and Ding propose a Lyapunov-based SOSM controller with a power integrator technique to stabilize the sliding variables and develop a novel SOSM controller with a saturation level with the local SOSM controller combined. In Li et al., put forward a maximum power point tracking strategy based on model predictive controller (MPC) to track the maximum power in a direct-driven PMSG-based WECS with the disturbances and uncertainties observed by the extended state observer (ESO). Huang et al. constructed an $H_\infty$ observer for discrete-time switched systems with the $H_\infty$ observer guaranteed by sufficient conditions derived and utilized the zonotope method to estimate the observer-based states. Chen et al. compared the ADRC method with other disturbance-observer-based control methods and presented their applications in industrial sections and future directions. Zheng et al. combined an adaptive feedforward mechanism using an inverse plant model with the ADRC method to guarantee fast tracking performance with low bandwidth, which can receive ideal inverse model given appropriate initial conditions. This paper provides the adaptive controllers of multiple full-actuated ships from the perspective of dynamic level. By virtue of the gradient estimation in Zhu et al., we have had a new understanding of the least square method and it is made good use of to estimate the gradient. Unlike common control methods, this paper focus on utilizing force and torque instead of the constraints on motion as control inputs directly, enabling the system to earn better robustness despite of the external disturbances since the uncertainty can be diminished by the actual force or torque given. Moreover, we take advantage of the sliding mode disturbance observers to estimate the unknown disturbances in finite time and the Lyapunov stability theorem is employed to testify that the kinematic parameters can converge to the virtual controller asymptotically. Based on the virtual controller designed, the system tracking error will converge to zero asymptotically at last. In addition, owing to Zhu et al., the kinematic controllers of their research are adopted as our virtual controllers for the dynamic feedback system. The main innovations and contributions of our research can be summarized as the following perspectives:

1. The gradient of the unknown scalar field at the position of the leading ship is estimated with the aid of least square method under the circumstance that each ship can only measure the scalar field value of its own position.
2. The source seeking errors of the leader–follower system converge to zero asymptotically with time growing to positive infinity based on the proposed virtual controllers while the disturbance observers converge to fixed multiple of the unknown external disturbances in finite time by the sliding mode control.
3. The rigorous mathematical deduction and proof affirm the reliability of the control laws proposed in this paper. Further simulation results additionally demonstrate the effectiveness and stability of the control law.
4. This paper concentrates on research works on full-actuated ships whose control can be studied more comprehensively and considerably compared with underactuated ships and the control problem can be solved more perfectly and easily.

This paper is organized as follows. In section “Problem statement,” the formalization of the source
seeking problem considered is given. Section “Main results” presents our main results including gradient estimation, controller design and stability analysis. Simulation results are provided and relative analysis is given in section “Simulation.” Finally, the conclusions are announced in section “Conclusion.”

Problem statement

In this paper, we adopt a nonlinear ship model with six degrees of freedom which can be studied from surge, sway and yaw control, respectively. The kinematic model of the ith ship is given as follows

\[
\begin{align*}
\dot{x}_i &= u_i \cos \phi_i - v_i \sin \phi_i \\
\dot{y}_i &= u_i \sin \phi_i + v_i \cos \phi_i (i = 1, 2, 3, \ldots, n) \\
\dot{\phi}_i &= w_i
\end{align*}
\]

(1)

As is shown in Figure 1, \((x_i, y_i)\) is the center of the ith ship, \(\phi_i\) is the yaw angle of the ith ship, \(u_i\) and \(w_i\) denote the surge and sway velocity of the ith ship.

And the dynamic model of the ith ship is described as

\[
\begin{align*}
\dot{u}_i &= \frac{m_i}{m_1} v_i w_i - \frac{d_i}{m_i} u_i + \frac{1}{m_1} (\tau_{ui} + \tau_{uid}) \\
\dot{v}_i &= -\frac{m_i}{m_1} u_i w_i - \frac{d_i}{m_i} v_i + \frac{1}{m_1} (\tau_{vi} + \tau_{vid}) \\
\dot{w}_i &= \frac{m_i}{m_1} u_i v_i - \frac{d_i}{m_i} w_i + \frac{1}{m_1} (\tau_{wi} + \tau_{wid})
\end{align*}
\]

(2)

where parameters \(m_i > 0\) are the ship inertia including mass effects, and \(d_i > 0\) denote the hydrodynamic damping in surge, sway and yaw, and \(\tau_{ui}, \tau_{vi}, \tau_{wi}\) are the surge force, sway force and yaw torque as control inputs, and \(\tau_{uid}, \tau_{vid}, \tau_{wid}\) are the unknown bounded disturbances.

Here, we assume that the source center \(r^*\) which can also be described as \((x^*, y^*)\) has the only maximal value in the scalar field. The unknown scalar field can be represented by the following equation

\[
F(r_i) = F(||r_i - r^*||)
\]

(3)

where \(r_i = (x_i, y_i)\) denotes the ith ship’s position. It is obvious that the scalar field value depends only on the distance between the ship and the signal source.

In this paper, our control task is to enable all ships to reach an area with a safe distance from the signal source and construct a specified leader–follower formation with external disturbances considered.

Main results

In this section, we will present how all ships reach an area with a safe distance from the signal source. We will start this by giving some lemmas and assumptions.

Lemma 1. Consider the first-order system \(\dot{\sigma} = \lambda\), where \(\sigma\) is a positive value function with respect of time.\(^{20}\) There is a positive number \(\lambda_m\) satisfying \(|\lambda| \leq \lambda_m\). For a certain number \(\sigma(0)\), there is always a positive number \(C_0\) satisfying \(|\sigma(0)| \leq C_0\). Set Lyapunov function \(\lambda = -z \text{sgn}(\sigma)\sigma^s\), where \(z\) and \(\sigma\) are constants and satisfy \(z \leq (\lambda_m/C_0)^s\), \(0 < \sigma < 1\). There is a finite time \(T_0 = (|\sigma(0)|^{1-\sigma})/(z(1 - \sigma)) \leq (C_0^{-1-\sigma})/(z(1 - \sigma))\) such that, \(\lim_{t \to T_0} \sigma = 0, \sigma = 0(t \geq T_0)\).

\[\text{Lemma 2.} \quad \text{Consider a continuous positive definite function } \kappa(t) \text{ satisfying the following two conditions}\]\n
1. \(\kappa(t_0) \geq 0\)
2. \(\kappa(t) \leq -\mu \kappa(t), \forall t \geq T_0\)

where \(t_0\) is the initial value, \(\mu\) is a positive constant and \(s \in (0, 1)\). \(\kappa(t)\) can converge to zero in finite time.

Assumption 1. Each ship is equipped with a sensor at the center which cannot measure the position and relevant parameters of the signal source directly.

Assumption 2. The scalar value of the source center is the strongest and it declines as the distance farther away.

Assumption 3. Each ship can get the position and velocity of the leading ship and the leading ship can get the position and the measurement of the scalar filed value of all following ships.

To finish the source seeking problem of the leading ship, we take this assumption as a premise to utilize the least square method mentioned in section “Estimation of the gradient of the leading ship,” or we will be restrained by the gradient estimation.

Considering system equations (1) and (2), the sliding mode surface of disturbances can be designed as

\[
\begin{align*}
G_{ui} &= P_{ui} - u_i \\
G_{vi} &= P_{vi} - v_i \\
G_{wi} &= P_{wi} - w_i
\end{align*}
\]

(4)
The sliding mode disturbance observers are designed as

\[ P_{ui} = m_i \dot{w}_i + \frac{d_i}{m_i} u_i - c_{ui} G_{ui} - \nu_{ui} \text{sgn}(G_{ui}) \]

\[ \dot{P}_i = -m_i \dot{w}_i - \frac{d_i}{m_i} v_i - c_{vi} G_{vi} - \nu_{vi} \text{sgn}(G_{vi}) \]

\[ \dot{P}_w = -m_i \dot{w}_i - \frac{d_i}{m_i} v_i - c_{wi} G_{wi} - \nu_{wi} \text{sgn}(G_{wi}) \]

where parameters \( c_{ui}, c_{vi}, c_{wi}, \nu_{ui}, \nu_{vi}, \nu_{wi}, f_{ui}, f_{vi}, f_{wi}, k_{ui}, k_{vi}, k_{wi} \) satisfy following conditions

\[
\begin{align*}
&c_{ui}, c_{vi}, c_{wi} > 0 \\
&\nu_{ui}, \nu_{vi}, \nu_{wi} > 0 \\
&f_{ui}, f_{vi}, f_{wi}, k_{ui}, k_{vi}, k_{wi} > 0 \\
&f_{ui} k_{ui} < 1, f_{vi} k_{vi} < 1, f_{wi} k_{wi} < 1
\end{align*}
\]

The sliding mode disturbance observers are designed as

\[
\dot{\tau}_{uid} = -c_{ui} G_{ui} - \nu_{ui} \text{sgn}(G_{ui}) - \nu_{ui} G_{ui} \dot{\tau}_{uid}/k_{ui}
\]

\[
\dot{\tau}_{vid} = -c_{vi} G_{vi} - \nu_{vi} \text{sgn}(G_{vi}) - \nu_{vi} G_{vi} \dot{\tau}_{vid}/k_{vi}
\]

\[
\dot{\tau}_{wid} = -c_{wi} G_{wi} - \nu_{wi} \text{sgn}(G_{wi}) - \nu_{wi} G_{wi} \dot{\tau}_{wid}/k_{wi}
\]

By combining equations (2), (4), (5), (7), we can get

\[
\dot{G}_{ui} = \dot{P}_{ui} - \dot{u}_i = \frac{m_i}{m_i} \dot{w}_i - \frac{d_i}{m_i} \dot{u}_i - c_{ui} G_{ui} - \nu_{ui} \text{sgn}(G_{ui})
\]

\[
- \nu_{ui} G_{ui} \dot{\tau}_{uid}/k_{ui} + \frac{1}{m_i} \tau_{uid}
\]

\[
- \left( \frac{m_i}{m_i} \dot{w}_i - \frac{d_i}{m_i} \dot{u}_i - c_{ui} G_{ui} - \nu_{ui} \text{sgn}(G_{ui}) \right)
\]

\[
- \frac{1}{m_i} \tau_{uid} - \frac{1}{m_i} \tau_{uid}
\]

Similarly, it can be obtained that

\[
\dot{G}_{vi} = \dot{P}_{vi} - \dot{u}_i = \frac{m_i}{m_i} \dot{w}_i - \frac{d_i}{m_i} \dot{u}_i - c_{vi} G_{vi} - \nu_{vi} \text{sgn}(G_{vi})
\]

\[
- \nu_{vi} G_{vi} \dot{\tau}_{vid}/k_{vi} + \frac{1}{m_i} \tau_{vid}
\]

\[
- \left( \frac{m_i}{m_i} \dot{w}_i - \frac{d_i}{m_i} \dot{u}_i - c_{vi} G_{vi} - \nu_{vi} \text{sgn}(G_{vi}) \right)
\]

\[
- \frac{1}{m_i} \tau_{vid} - \frac{1}{m_i} \tau_{vid}
\]

**Proof.** Consider the Lyapunov function

\[
\lambda_1(G) = \frac{1}{2} G^2
\]
Substituting equations (1)–(4) into the derivative of equation (10) with respect to time and it can be obtained
\[
\dot{\lambda}_1(G_u) = G_u \dot{G}_u = G_u \left( -c_u G_u + t_u \text{sgn}(G_u) \right) - \nu_u \frac{G_u}{m_1} - \frac{1}{m_1} \tau_{nul} G_u
\]
(11)

In the same way, we can get that
\[
\begin{align*}
\dot{\lambda}_1(G_u) &= G_u \dot{G}_u = G_u \left( -c_u G_u + t_u \text{sgn}(G_u) \right) - \nu_u \frac{G_u}{m_1} - \frac{1}{m_2} \tau_{nul} G_u \\
\dot{\lambda}_1(G_{G_u}) &= G_{G_u} \dot{G}_{G_u} = G_{G_u} \left( -c_{G_u} G_{G_u} + t_v G_{G_u} \right) - \nu_{G_u} \frac{G_{G_u}}{m_2} - \frac{1}{m_2} \tau_{nul} G_{G_u} \\
\dot{\lambda}_1(G_{G_{G_u}}) &= G_{G_{G_u}} \dot{G}_{G_{G_u}} = G_{G_{G_u}} \left( -c_{G_{G_u}} G_{G_{G_u}} + t_v G_{G_{G_u}} \right) - \nu_{G_{G_u}} \frac{G_{G_{G_u}}}{m_3} - \frac{1}{m_3} \tau_{nul} G_{G_{G_u}}
\end{align*}
\]
(12)

According to equations (7) and (12), it can be obtained
\[
\begin{align*}
\dot{\lambda}_1(G_u) &\leq -c_u G_u - \nu_u \frac{G_u}{(l_u + k_u)/k_u} - 2c_u \lambda_1(G_u) - 2(l_u + k_u)/2k_u \cdot \nu_u G_u \\
\dot{\lambda}_1(G_{G_u}) &\leq -c_{G_u} G_{G_u} - \nu_{G_u} \frac{G_{G_u}}{(l_{G_u} + k_{G_u})/k_{G_u}} - 2c_{G_u} \lambda_1(G_{G_u}) - 2(l_{G_u} + k_{G_u})/2k_{G_u} \cdot \nu_{G_u} G_{G_u} \\
\dot{\lambda}_1(G_{G_{G_u}}) &\leq -c_{G_{G_u}} G_{G_{G_u}} - \nu_{G_{G_u}} \frac{G_{G_{G_u}}}{(l_{G_{G_u}} + k_{G_{G_u}})/k_{G_{G_u}}} - 2c_{G_{G_u}} \lambda_1(G_{G_{G_u}}) - 2(l_{G_{G_u}} + k_{G_{G_u}})/2k_{G_{G_u}} \cdot \nu_{G_{G_u}} G_{G_{G_u}}
\end{align*}
\]
(13)

Based on Lemma 2 and inequality equation (13), we can conclude that the sliding mode disturbance surface can converge to zero in finite time. Considering equation (9) and it is obvious that the sliding mode disturbance observers can converge to fixed multiple of the unknown external disturbances in finite time. This completes the proof.

**Estimation of the gradient of the leading ship**

By system equation (1), we can get the position of the \(i\)th ship \((x_i, y_i)\), and based on equation (3) and Assumption 3, the leading ship can get the scalar field value measured by \(i\)th ship \(F(x_i, y_i)\). Using Taylor expansion around the leading ship’s position, we can get the first-order approximation of the scalar field at the \(i\)th following ship’s position as
\[
F(x_i, y_i) \approx F(x_i, y_i) + \left. \frac{\partial F(x_i, y_i)}{\partial x_j} \right|_{j=1} (x_j - x_i) + \left. \frac{\partial F(x_i, y_i)}{\partial y_j} \right|_{j=1} (y_j - y_i)
\]
(14)

Make a difference between the field value of all the following ships and that of the leading ship and it can be obtained
\[
\begin{align*}
F(x_2, y_2) - F(x_1, y_1) &\approx \left[ x_2 - x_1, y_2 - y_1 \right] \cdot \left[ \begin{array}{c} \frac{\partial F(x_i, y_i)}{\partial x_j} \left|_{j=1} \right. \\ \frac{\partial F(x_i, y_i)}{\partial y_j} \left|_{j=1} \right. \end{array} \right] \\
F(x_3, y_3) - F(x_1, y_1) &\approx \left[ x_3 - x_1, y_3 - y_1 \right] \cdot \left[ \begin{array}{c} \frac{\partial F(x_i, y_i)}{\partial x_j} \left|_{j=1} \right. \\ \frac{\partial F(x_i, y_i)}{\partial y_j} \left|_{j=1} \right. \end{array} \right]
\end{align*}
\]
(15)

For the sake of simplifying the following description, we define that
\[
E = \left[ \begin{array}{c} F(x_2, y_2) - F(x_1, y_1) \\ \vdots \\ F(x_n, y_n) - F(x_1, y_1) \end{array} \right]
\]
(16)

\[
Z = \left[ \begin{array}{c} x_2 - x_1 \\ \vdots \\ x_n - x_1 \end{array} \right]
\]
(17)

\[
\beta = \left[ \begin{array}{c} \frac{\partial F(x_i, y_i)}{\partial x_j} \left|_{j=1} \right. \\ \frac{\partial F(x_i, y_i)}{\partial y_j} \left|_{j=1} \right. \end{array} \right]
\]
(18)

Based on equations (15)–(18), we can get that
\[
E = Z \beta
\]
(19)

To estimate \( \beta = [\beta_x, \beta_y]^T \), a least square formula is adopted\(^{10}\) when \( Z \cdot Z^T \) is nonsingular, and we can get the estimation value as
\[
\hat{\beta} = (Z \cdot Z^T)^{-1} \cdot Z^T \cdot E
\]
(20)

where \( \hat{\beta} = [\hat{\beta}_x, \hat{\beta}_y]^T \) is the estimated gradient at the leader’s position.

Based on the estimation of the gradient, the virtual controllers of the leading ship \( \dot{u}_i, \dot{v}_i \) and \( \ddot{w}_i \) is then proposed to meet source seeking mission. Worth noting, according to the characteristics of Taylor expansion, we can ignore the higher order terms as long as the distances among following ships and leading ship are small enough, thus making it feasible to substitute \( \hat{\beta} \) for \( \beta \). Therefore, it is necessary to keep a formation which meets the condition mentioned above and the following ship’s virtual controllers and dynamic controllers will be proposed in sequel.

**Controller design of the leading ship**

In this sub-section, virtual controllers for the leading ship are proposed to finish the source seeking task, and then the dynamic controllers for the leading ship are given to make the kinematic parameters converge to the virtual controllers.
Theorem 2. The virtual controllers of the leading ship are given as follows

\[
\begin{align*}
\dot{u}_1 &= b_0 \\
\dot{v}_1 &= -\alpha \frac{(\varphi_d - \varphi_i) \sin(\varphi_d)}{2\pi} - \varphi_d \\
v_1 &= 0
\end{align*}
\]  
(21)

where \(\alpha, b_0\) are positive constants, \(\varphi_{id}\) denotes desired yaw angle, and \(|\varphi_{id}| < \alpha\), whose expression is given as

\[
\varphi_{id} = \arctan(2(\beta_x, \beta_y) - \pi + 2\arctan \left( \frac{F(x_d, y_d)}{F(x, y)} \right))
\]  
(22)

where \(F(x_d, y_d)\) denotes the specified scalar field value which is the so-called safe area’s measurement since we suppose that the signal source is hazardous and the ships cannot move too close to the signal source to avoid danger. Under the control of equation (21), it can be proved that the leading ship will finally arrive at the desired area.

Proof. Take a Lyapunov function \(\lambda_2 = 1/2(\varphi_1 - \varphi_{id})^2\), then the derivative of \(\lambda_2\) with respect to time is as follows

\[
\dot{\lambda}_2 = (\varphi_1 - \varphi_{id})(\varphi_1 - \varphi_{id}) = (\varphi_1 - \varphi_{id}) - \alpha \frac{(\varphi_1 - \varphi_{id}) \sin(\varphi_d)}{2\pi} - \varphi_d
\]  
(23)

where \(\forall \varphi_1, \varphi_{id}, \exists \theta > 0, -\alpha((\varphi_1 - \varphi_{id})/2\pi)^{5/3} = -\varphi_1 + \varphi_{id}\) can always be right. Therefore, it is obvious that \(\lambda_2 \leq 0\) is always right and if and only if \(\varphi_1 - \varphi_{id} = 0\), which means \(\lambda_2 = 0\), then \(\lambda_3 = 0\). By LaSalle invariance principle, we can get that \(\varphi_1\) will asymptotically converge to \(\varphi_{id}\). Next, we will continue to prove that with the desired yaw angle, the leading ship will finally reach the goal described above.

Take a Lyapunov function \(\lambda_3 = 1/2(F(x_1, y_1) - F(x_d, y_d))^2\), then we can get the derivative of \(\lambda_3\) with respect of time as follows

\[
\dot{\lambda}_3 = (F(x_1, y_1) - F(x_d, y_d)) \cdot (F(x_1, y_1) - F(x_d, y_d))
\]  
(24)

We can conclude from equation (24) and Assumption 2 that \(\lambda_3 \leq 0\) is always correct and only if \(F(x_1, y_1) - F(x_d, y_d) = 0\), which means \(\lambda_3 = 0\), then \(\lambda_3 = 0\). Similarly, it is clear that \(F(x_1, y_1)\) will converge to \(F(x_d, y_d)\) asymptotically. Consequently, if \(\varphi_1\) equals to \(\varphi_{id}\), the leading ship will move along the desired level curve eventually. This completes the proof.

We define controller error system as

\[
\begin{align*}
\Delta u_i &= u_i - \hat{u}_i \\
\Delta v_i &= v_i - \hat{v}_i \\
\Delta w_i &= w_i - \hat{w}_i
\end{align*}
\]  
(25)

The derivatives of \(\Delta u_i, \Delta v_i, \Delta w_i\) with respect to time are

\[
\begin{align*}
\dot{\Delta u}_i &= \frac{u_m}{m_1}v_iw_i - \frac{m_1}{m_2}u_i + \frac{1}{m_2}(\tau_u + m_1\hat{\tau}_{uid}) - \hat{u}_i \\
\dot{\Delta v}_i &= -\frac{m_1}{m_2}u_iw_i - \frac{m_1}{m_2}v_i + \frac{1}{m_2}(\tau_u + m_1\hat{\tau}_{vid}) - \hat{v}_i \\
\dot{\Delta w}_i &= \frac{(m_1 - m_2)}{m_2}u_iw_i - \frac{1}{m_2}(\tau_u + m_1\hat{\tau}_{wid}) - \hat{w}_i
\end{align*}
\]  
(26)

The derivatives of leading ship’s virtual controllers equation (2) with respect of time are

\[
\begin{align*}
\dot{\hat{u}}_1 &= 0 \\
\dot{\hat{v}}_1 &= \frac{5\pi}{6\pi}(\varphi - \varphi_d)^{5/3} - \frac{(\varphi_d - \varphi_i) \sin(\varphi_d)}{2\pi} - \varphi_d \\
\end{align*}
\]  
(28)

We design the dynamic controllers as follows

\[
\begin{align*}
\tau_{u1} &= -m_2v_1w_1 + d_1u_1 - m_1\hat{k}_{u1}\text{sgn}(\Delta u_1)\text{sgn}(\Delta u_1)
\end{align*}
\]  
(29)

We can conclude from equation (24) and Assumption 2 that \(\lambda_3 \leq 0\) is always correct and only if \(F(x_1, y_1) - F(x_d, y_d) = 0\), which means \(\lambda_3 = 0\), then \(\lambda_3 = 0\). Similarly, it is clear that \(F(x_1, y_1)\) will converge to \(F(x_d, y_d)\) asymptotically. Consequently, if \(\varphi_1\) equals to \(\varphi_{id}\), the leading ship will move along the desired level curve eventually. This completes the proof.

Theorem 3. Considering the closed-loop controller error system equation (25) and dynamic controllers equation (29), based on Lemma 1, the controller error system will converge to zero in finite time, thus making \(u_1 \rightarrow \hat{u}_1, v_1 \rightarrow \hat{v}_1\) and \(w_1 \rightarrow \hat{w}_1\) in finite time.

Proof. Substituting equation (29) into the error system equation (27), it can be obtained that
\[ \Delta u_1 = \frac{m_2}{m_1} v_1 w_1 - \frac{d_1}{m_1} u_1 + \frac{1}{m_1} (\tau_{u_1} + m_1 \dot{\tau}_{u_1}) - \dot{u}_1 \]

\[ = \frac{m_2}{m_1} v_1 w_1 - \frac{d_1}{m_1} u_1 + \frac{1}{m_1} (-m_2 v_1 w_1 + d_1 u_1) \]

\[ - m_1 \dot{k}_u \text{sgn}(\Delta u_1) \Delta u_1 |^{\text{est}} - m_1 \dot{\tau}_{u_1} + m_1 \dot{\tau}_{u_1} \]

\[ = - \dot{k}_u \text{sgn}(\Delta u_1) |^{\text{est}} \]

(30)

In the same measure, we get

\[ \begin{aligned}
\Delta \dot{u}_1 &= - \dot{k}_u \text{sgn}(\Delta u_1) |^{\text{est}} \\
\Delta \dot{\tau}_1 &= - \dot{k}_u \text{sgn}(\Delta u_1) |^{\text{est}} \\
\Delta \dot{\tau}_{u_1} &= - \dot{k}_u \text{sgn}(\Delta u_1) |^{\text{est}} 
\end{aligned} \]

(31)

To simplify the analysis, we take the equation (31) as an example to indicate our conclusion. Set Lyapunov stability function \( \lambda_{u_1} = 1/2(\Delta u_1)^2 \). The derivative of \( \lambda_{u_1} \) with respect of time is

\[ \dot{\lambda}_{u_1} = \Delta u_1 \Delta \dot{u}_1 \]

\[ = - \dot{k}_u |^{\text{est}} + 1 = - \dot{k}_u |^{\text{est}} + 1/2 \]

(32)

Equation (32) can be rewritten as

\[ \frac{d \lambda_{u_1}}{(\lambda_{u_1})|^{\text{est}} + 1/2} = - 2(\lambda_{u_1})|^{\text{est}} + 1/2 \dot{k}_u dt \]

(33)

Integrating both sides of the equation, we can get that

\[ \int_{\lambda_{u_1}(0)}^{\lambda_{u_1}} \frac{1}{1 - ((\epsilon_{u_1} + 1/2)(\lambda_{u_1}(0)))^{1-(\epsilon_{u_1} + 1/2)}} d \lambda_{u_1} = \int_{0}^{t} - 2(\lambda_{u_1})|^{\text{est}} + 1/2 \dot{k}_u dt \]

(34)

Finally, after some transformation, we can get

\[ \lambda_{u_1}|^{1-(\epsilon_{u_1})/2} = - 2(\epsilon_{u_1}-1)/2 \dot{k}_u (1 - \epsilon_{u_1})t + (\lambda_{u_1}(0))^{1-\epsilon_{u_1})/2 \]

(35)

Apparently, \( \lambda_{u_1} \) decreases with time exponentially until it decreases to zero and it will keep zero state from then on. Based on the Lyapunov function \( \lambda_{u_1} = 1/2(\Delta u_1)^2 \), we can conclude that \( \Delta u_1 \) will converge to zero in finite time which means the actual velocity \( u_1 \) will converge to the virtual controller \( \dot{u}_1 \) eventually. Consequently, \( v_1 \) and \( w_1 \) will also converge to \( \dot{v}_1 \) and \( \dot{w}_1 \) ultimately. This completes the proof.

**Controller design of the following ships**

In this sub-section, virtual controllers for the following ships are proposed directly or indirectly (equivalent transformation) to enable following ships to finally move to the small annulus of the safe distance from the signal source and be in a dynamic circular formation and then dynamic controllers \( \tau_{u_1}, \tau_{v_1}, \tau_{w_1} \) (\( i = 1, 2, 3, 4, \ldots, n \)) are proposed to make kinematic parameters \( u_i, v_i, w_i \) converge to the virtual controllers \( \dot{u}_i, \dot{v}_i, \dot{w}_i \).

For all the following ships, the relative position of whom with the leading ship can be described as

\[ x_{il} = x_i - x_l, y_{il} = y_i - y_l, \quad i = 2, 3, 4, \ldots, n \]

(36)

To facilitate the solving progress, we first give two of the virtual controllers \( \dot{u}_i, \dot{v}_i \) as

\[ \begin{aligned}
\dot{u}_i &= b_1 (i = 2, 3, 4, \ldots, n) \\
\dot{v}_i &= 0
\end{aligned} \]

(37)

where \( b_1 > b_0 \) is a positive constant shared by all following ships.

Only under the circumstances that \( u_i = b_1, v_i = 0 \) can we continue the following design. Here, we make the hypothesis that we have already finish the periodical target that \( u_i = b_1, v_i = 0 \), and we will prove this in sequel.

In this case, the derivative of the relative position can be given as

\[ \begin{aligned}
\dot{x}_{il} &= \dot{x}_i - \dot{x}_l = u_i \cos \phi_i - u_l \cos \phi_l \\
\dot{y}_{il} &= \dot{y}_i - \dot{y}_l = u_i \sin \phi_i - u_l \sin \phi_l
\end{aligned} \]

(38)

To simplify the relative dynamics, \( x_i \) is introduced to denote the angle of the relative yaw rate and then the equation (38) can be transformed as

\[ \begin{aligned}
\dot{x}_i &= u_l \cos x_i \\
\dot{y}_i &= u_l \sin x_i
\end{aligned} \]

(39)

where \( u_l = \sqrt{(\dot{x}_i - \dot{x}_l)^2 + (\dot{y}_i - \dot{y}_l)^2} \)

\( x_i = \arctan 2 (\dot{y}_i - \dot{x}_i, \dot{x}_i - \dot{x}_l) \).

Differentiating \( x_i \) with respect of time and it can be obtained that

\[ \dot{x}_i \]

\[ = - (\dot{x}_i - \dot{x}_l)(\dot{y}_i - \dot{y}_l) + (\dot{y}_i - \dot{y}_l)(\dot{x}_i - \dot{x}_l) \]

\[ = (\dot{x}_i - \dot{x}_l)^2 + (\dot{y}_i - \dot{y}_l)^2 \]

(40)

Combining system equation (1), controllers equation (21) and equation (37), we can get

\[ \begin{aligned}
\dot{\phi}_i &= \frac{\dot{y}_i - \dot{y}_l}{\dot{x}_i - \dot{x}_l} \\
u_i &= \sqrt{\dot{x}_i^2 + \dot{y}_i^2} \quad i = 1, 2, 3, \ldots, n
\end{aligned} \]

(41)

According to \( u_{il} \) and equation (40), it can be obtained that


and we will prove that \( p_i \) will converge to \((x_i, y_i)\) asymptotically.

The dynamic point \((x_{ic}, y_{ic})\) that all the following ships move around can be expressed as\(^{31-32}\)

\[
x_{ic} = x_i - r_0 \sin \chi_i, \quad y_{ic} = y_i - r_0 \cos \chi_i
\]

(45)

Considering Lyapunov stability function,

\[
\lambda_4 = \frac{1}{2} (x_{ic} - x_i)^2 + \frac{1}{2} (y_{ic} - y_i)^2
\]

(46)

The derivative of \( \lambda_4 \) with respect of time is

\[
\dot{\lambda}_4 = (x_{ic} - x_i) \dot{x}_i + (y_{ic} - y_i) \dot{y}_i
\]

(47)

Based on the virtual controllers mentioned in equation (44), here, we directly substitute \( \dot{u}_i, \dot{x}_i, \dot{v}_i \) for \( u_i, x_i, v_i \) it can be obtained that

\[
\dot{\lambda}_4 = (x_{ic} - x_i) (u_i \cos \chi_i - r_0 \cos \chi_i) + (y_{ic} - y_i) (u_i \sin \chi_i - r_0 \sin \chi_i)
\]

(48)

\[
= \delta ((x_{ic} - x_i) \cos \chi_i + (y_{ic} - y_i) \sin \chi_i) (u_i \cos \chi_i + y_{ic} \sin \chi_i)
\]

It is clear that \( \dot{\lambda}_4 \leq 0 \) is always right and when \( x_{ic} \cos \chi_i + y_{ic} \sin \chi_i = 0 \), \( \dot{\lambda}_4 = 0 \). However, under the condition that \( (x_{ic} - x_i) \cos \chi_i = - (y_{ic} - y_i) \sin \chi_i \), \[x_{ic} - x_i = 0 \] and \[y_{ic} - y_i \cos \chi_i \neq 0 \], this becomes an unstable equilibrium point. As to the invariant set \{\((x_{ic}, y_{ic})|x_{ic} - x_i = 0, y_{ic} + r_0 \cos \chi_i = 0\}\), this means the formation center \((x_{ic}, y_{ic})\) coincides with leading ship's position \((x_i, y_i)\), which instead implies that the following ships will finally abide the circular orbit centered on the leading ship taking the leading ship as the dynamic reference system.

Above indicates that the virtual controllers \( \dot{u}_i, \dot{x}_i, \dot{v}_i \) can finish the source seeking task without doubt, and we will give the dynamic controllers as follows to make \( u_i, x_i, v_i \) finally converge to \( \dot{u}_i, \dot{x}_i, \dot{v}_i \). Differentiating equation (44) with respect of time and we can get

\[
\begin{aligned}
\dot{\hat{u}}_i &= 0 \\
\dot{\hat{v}} &= 0 \\
\dot{\hat{x}_i} &= \frac{1}{\sqrt{(x_i-x)^2+(y_i-y)^2}} \\
&\quad \left(u_i \sin (\phi_i - \phi) - \hat{u}_i \cos (\phi - \chi_i) \right) \\
&\quad - \chi_i \hat{x}_i \sin \chi_i + y_{ic} \cos \chi_i)
\end{aligned}
\]

(49)

Differentiating equation (43) with respect of time, it can be obtained that

Theorem 4. With the help of the transformation given in equation (43), the virtual controllers are proposed as follows which can enable the following ships to be distributed on the dynamic circle with the leading ship as the center

\[
\begin{align*}
\dot{u}_i &= b_1 \\
\dot{x}_i &= \frac{1}{w_i} (u_i \cos \chi_i + y_{ic} \sin \chi_i) &i = 2, 3, 4, \ldots, n \\
\dot{v}_i &= 0
\end{align*}
\]

(44)

where \( r_0 \) denotes the distance between each following ship and their kinematic center, \( \delta \) is a positive constant.

It can be proved that all the following ships can keep the desired formation whose center are the leading ship.

Proof. To realize the goal, we first let all the following ships to move around a specified point \( p_i = (x_{ic}, y_{ic}) \), and we will prove that \( p_i \) will converge to \((x_i, y_i)\) asymptotically.
\[ \ddot{\phi}_i = \dot{\theta}_i = \left( \frac{2u_{ij}}{u_i^2} \dot{X}_i + \frac{u_{ij}^2}{u_i^2} \ddot{X}_i - \frac{u_{ij}^2}{u_i^2} \dddot{w}_i + \frac{u_i}{u_i} \left( -\sin(\phi_i - \phi_1)(w_i - w_1) \right) + \cos(\phi_i - \phi_1) \dot{w}_i \right) \left( 1 - \frac{u_i}{u_i} \cos(\phi_i - \phi_1) \right) + \frac{u_i}{u_i} \sin(\phi_i - \phi_1) \left( \frac{u_i^2}{u_i^2} \ddot{X}_i \right) \left( 1 - \frac{u_i}{u_i} \cos(\phi_i - \phi_1) \right)^2 \]  

\[ \text{(50)} \]

In order to finish the control task and based on equation (50), we give

\[ \ddot{\phi}_i = \dot{\theta}_i = \left( \frac{2u_{ij}}{u_i^2} \dot{X}_i + \frac{u_{ij}^2}{u_i^2} \ddot{X}_i - \frac{u_{ij}^2}{u_i^2} \dddot{w}_i + \frac{u_i}{u_i} \left( -\sin(\phi_i - \phi_1)(w_i - w_1) \right) + \cos(\phi_i - \phi_1) \dot{w}_i \right) \left( 1 - \frac{u_i}{u_i} \cos(\phi_i - \phi_1) \right) + \frac{u_i}{u_i} \sin(\phi_i - \phi_1) \left( \frac{u_i^2}{u_i^2} \ddot{X}_i \right) \left( 1 - \frac{u_i}{u_i} \cos(\phi_i - \phi_1) \right)^2 \]  

\[ \text{(51)} \]

The dynamic controllers \( \tau_{vi}, \tau_{ui}, \tau_{wi} \) are given as\(^{33-34}\)

\[
\begin{align*}
\tau_{ui} &= -m_2 v_i w_i + d_1 u_i - m_1 \dot{k}_{ui} \text{sgn} (\Delta u_i) |\Delta u_i|^{\epsilon_{ui}} - m_1 \dot{\tau}_{ui} \\
\tau_{vi} &= m_1 u_i v_i + d_2 v_i - m_3 \dot{k}_{vi} \text{sgn} (\Delta v_i) |\Delta v_i|^{\epsilon_{vi}} - m_3 \dot{\tau}_{vi} \\
\tau_{wi} &= -(m_1 - m_2) u_i v_i + d_3 w_i - m_3 \dot{k}_{wi} \text{sgn} (\Delta w_i) |\Delta w_i|^{\epsilon_{wi}} - m_3 \dot{\tau}_{wi} + m_3 \dot{w}_i 
\end{align*}
\]  

\[ \text{(52)} \]

Substituting equation (51) into the error system equation (25), it can be obtained that

\[
\begin{align*}
\dot{u}_i &= -\dot{k}_{ui} \text{sgn}(u_i) |u_i|^{\epsilon_{ui}} \\
\dot{v}_i &= -\dot{k}_{vi} \text{sgn}(v_i) |v_i|^{\epsilon_{vi}} \\
\dot{w}_i &= -\dot{k}_{wi} \text{sgn}(w_i) |w_i|^{\epsilon_{wi}} 
\end{align*}
\]  

\[ \text{(53)} \]

According to Theorem 3, we can finally conclude that the kinematic parameters \( u_i, v_i, w_i \) converge to the virtual controller \( \ddot{u}_i, \ddot{v}_i, \ddot{w}_i \) in finite time. This completes the proof.

**Simulation**

In this section, the source seeking method with virtual controllers will be proved by the MATLAB simulation results in the case of the multiple ships with external disturbances. We use four ships including one leading ship and three following ships for simulation, the initial position of the leading ship is \((5, 5)\). And in our scenario, the radius of the circular formation is 200 m. The initial position of other three following ships are as follows: \((200, 325)\), \((5, 205)\) and \((50, 95)\).

And the scalar field \( F(r) = F_{\text{max}} \left[ 1 - \tanh(\|r - r^*\|) \right] \) is used, and the parameters are set as \( F_{\text{max}} = 6, \alpha = 1 \). As described in equation (2), the dynamic model of the ships is disturbed and the disturbances are measured by actual observers. The observational and truth error systems are as follows, we design \( m_1 = 12, m_2 = 16, m_3 = 60, d_1 = 22, d_2 = 10, d_3 = 80 \).

And the scalar mode disturbance observers \( \dot{e}_{uid}, \dot{e}_{vid}, \dot{e}_{wid} \) are designed as equation (7), choosing \( c_{uid} = c_{vid} = c_{wid} = 1 \), \( v_{uid} = v_{vid} = v_{wid} = 2 \). According to Theorem 2, the virtual controllers of the leading ship in equation (21), we design \( b_0 = 10, \alpha = 0.5 \). And some parameters in dynamic controllers equation (29) are state \( \dot{k}_{ui} = \dot{k}_{vi} = \dot{k}_{wi} = 1 \), \( \epsilon_{ui} = \epsilon_{vi} = \epsilon_{wi} = 0.2 \). Combined with formulas (25) and (26), we simulate the controller error system.

The simulation results are shown as follows. It can be observed from Figure 5 that following ships converge to the circular orbit with a desired distance to the leading ship, as claimed in Theorem 4. Figure 5 shows the Lyapunov function \( \lambda_1(G) \) can converge to zero in finite time, easily confirm that the disturbance observers converge to the unknown external disturbances. It also can be confirmed in Figure 6, the observers error system converges to zero dynamically. And it can be observed from Figure 7 that the controller error system \( u, v, w \) converge to zero in finite time, thus urging the kinematic parameters \( \Delta u_i, \Delta v_i, \Delta w_i \) converge to the virtual controllers \( \ddot{u}_i, \ddot{v}_i, \ddot{w}_i \). Figure 8 shows the trajectory of the leading ship, from which we can see that the leading ship approaches to the signal source and remains the proximity of the source position in finite time.

**Conclusion**

This paper studies the problem of the source seeking of the leader–follower formation control of the full-actuated ships. Based on the least square method, a gradient estimation method is proposed to measure the
gradient of the scalar field at the position of the leading ship. Virtual controllers are designed as force or torque inputs and the surge, yaw and angle velocity can converge to the virtual controllers in finite time with the help of them. In addition, the virtual controllers can ensure that the system tracking errors will converge to zero in finite time, thus making the source seeking successfully.

The source seeking of the underactuated ships, which is more complicated since the uncertain degree, that is, the sway velocity have to be dealt with from the perspective of the dynamic function which is controlled by the surge and yaw velocity indirectly, thus making the construction of the dynamic controllers which control the kinematic parameters directly difficult. Consequently, in the coming time, we will further consider the source seeking of the underactuated ships.

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ORCID iD
Hua Chen https://orcid.org/0000-0002-4737-6780

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