Using frustrated internal reflection as an analog to quantum tunneling

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Abstract. Also in teaching quantum physics one would like to bring experiments to the secondary school classroom, if not for “proofing” theories, then at least for illustrating and visualizing concepts. When EM waves are totally internally reflected from the inside of a prism, so called evanescent waves can tunnel through a narrow slit to another prism [1] and the intensity shows the exponential dependence on the width of the slit typical for tunneling. This is an example for quantum tunneling as quantum particles can tunnel as well due to their wave properties. This paper describes our progress from an ineffective microwave tunneling demonstration which we tried out with 5 teachers and 112 students, to a well-known optical demonstration but newly mounted in a small for loan suitcase with an educational script based on our try-outs and the PhET tunneling applet to link quantum concepts to tunneling phenomena.

1. Introduction

The inside of a prism can be used as a mirror. When light hits the inside of the prism with an angle of incidence greater than the critical angle, all light will be reflected. From a geometric optics point of view the laser beam is totally reflected. From a wave optics perspective, it is different as waves cannot stop abruptly but will decay over a certain distance. This total internal reflection can also be observed when looking from down below at the surface of a glass of water, the water surface seen from below acts like a mirror. However, if we use two prisms (figure 1) with a slit or wedge with a width of some wavelengths or less, some waves might “tunnel” through. This is called frustrated internal reflection and the waves are called evanescent waves. This phenomenon can be observed with water waves, microwaves, and light [1] but it can also happen with quantum particles because of their wave behavior: quantum tunneling. Therefore, classical wave frustrated internal reflection demonstrations are used as analogs to visualize quantum tunneling. Albion et al [2] describe an experiment using microwaves with prisms.

McKagan et al [4] as well as other researchers [3] reported that many students have a great deal of trouble understanding even the most basic aspects of tunneling and in particular “students fail to grasp the basic models that we are using to describe the world as anything more than abstract model systems. These models include wave functions as descriptions of physical objects, potential energy graphs as descriptions of the interaction of those objects with their environments, and total energy as a delocalized property of an entire wave function that is a function of position.” Assisting teaching of quantum physics at the secondary level rather than at the tertiary level like McKagan, our goals are modest but we did want to show tunneling as normal wave behavior in an experiment and then connect this to a particle tunneling simulation developed by McKagan and PhET colleagues and assess student understanding after this experience. In this paper we first describe some teaching results and problems with a microwave tunneling experiment and then we describe the tunneling experiment with prisms which fits in a small suitcase available for loan to schools.
2. Experiments
We incorporated the microwave demonstration in a lesson series on quantum physics which was used in 5 secondary schools with 5 teachers and 112 grade 12 students who were enrolled in the pre-university science stream. After the lesson series students took a posttest focused on duality and on tunneling and 24 students at 4 schools were interviewed about duality and tunneling. Here we report on the results of tunneling.

2.1. Interviews
In spite of a brief explanation of total internal reflection at the start of the lesson, students had not picked up that idea and thought that some microwaves would go straight instead of being reflected. Total internal reflection is not part of the current Dutch physics curriculum and geometric optics is only taught in grade 8 or 9 and not repeated in senior secondary, thus largely forgotten by grade 12 students. Furthermore, students thought that if some waves would go straight, then all waves would go straight and none would be reflected. So students seem to think that either all waves are reflected or all tunnel instead of some waves tunneling and the rest being reflected. When asked for everyday examples of total internal reflection, students had no idea until we showed them a laser beam from below reflecting off the water surface in a glass of water. So we had to drastically change our presentation of the demo.

Because of problems with understanding total internal reflection, students did not really make the link between quantum tunneling and the microwave demonstration. They did learn about tunneling though, mainly from the PhET tunneling applet which was demonstrated in detail and on which students were questioned in the interviews.

![Figure 1](image1.png) Prisms with a wedge. In our earlier microwave version microwave source was used instead of the laser and the prisms were carton containers with polystyrene beads.

![Figure 2](image2.png) The laser beam comes from the right and is partly reflected (to the right) and partly "tunneled" (straight). Intensity is measured with a photodiode.

We found two main student misunderstandings about the set-up:
1) students do not understand the phenomenon of total internal reflection
2) students think that the microwaves either are totally internally reflected or 100% or the beam tunnels through and goes straight.

Even though there was a short explanation at the start of the demonstration, students did not understand that the backside of the prism acts as a perfect mirror.
From an interview:

Student 1 (S1):  Those microwaves are refracted and at a particular angle it reflects completely in the prism.

Interviewer (I):  So under a particular angle it is completely reflected? So when I would measure behind it [behind the prism], would I find something there?

S1:  Yes I think so.

Somehow students got the idea that with the second prism in place, all microwaves will go straight and none or little will be detected at a right angle.

Interviewer (I):  Then a second prism was added and then the teacher measured behind it, what did he find?

Student 2 (S2):  That there is a signal because the microwave can get through the small barrier between the prisms. Without such a prism there is a very big barrier. So now it does not reflect and it passes through the second prism.

I:  And if I shift this one [2nd prism] farther away, what change do I see?

S2:  According to me it does not receive anymore. Then they [the microwaves] do not go through the barrier anymore.

I:  And if I put it [the 2nd prism] very near?

S2:  Then they do.

I:  Which similarities do you see with the model that was discussed? Because what was tunneling about?

S2:  About the chance that a particle goes through the surface of a barrier. Here we also see a barrier and the chance that it goes through.

I:  and if you increase the barrier for a particle, what is that here?

S2:  That is the distance between the two prisms.

This student seems to think that the tunneling is all or nothing, either 100% transmission or 100% reflection. However, he does perceive the intended similarity between the barrier in microwave tunneling and the barrier in particle tunneling.

Interviewer (I):  If I make the slit between the prisms larger, what do I see over there [interviewer points to a place roughly perpendicular to the original beam]?

S3:  But now you don’t measure anything anymore, because the light now ....

I:  You do not measure anything there [perpendicular]?

S3:  Now perhaps you will, if you ask it like that, but I thought that you would not measure anything there because the light or the microwaves now will not bend, but apparently, they will?

Also S3 initially thought that the tunneling will be all or nothing. The student later gives correct explanations about the PhET tunneling applet about the influence of changing barrier width or height on the probability of tunneling, connection of wave length with kinetic energy, and same kinetic energy before and after the barrier. Like S3 there were students who in a first reaction thought that with the second prism all microwave intensity would cross the barrier but corrected themselves when asked whether we would measure anything at the reflection position. But they thought the tunneled intensity would be greater than the reflected intensity. With these misinterpretations, the demonstration loses part of its educational value. Most students had great trouble with the question about what this demonstration had to do with quantum particles. They thought it must have to do something with waves, but most were not aware that the gap between the prisms was within one wavelength of the microwave radiation.

2.2. Recommendations

So there are some clear recommendations for modifying the microwave demonstration:

- Explain total internal reflection and show some simple everyday examples with light such as the water surface as mirror in a glass of water or aquarium.
• In every measurement the beam intensity should be measured both sideways (reflection) and straight ahead (transmission) and the two should be compared.

• Take measurements for a few different values of slit width and express slit width in terms of the wavelength.

With regard to the PhET applet (figure 3) most interviewed students could distinguish the energy, wave function, and probability density graphs, link wavelength to kinetic energy, and amplitude to intensity. They could also show the different ways in which the tunneling probability could be decreased or increased. Even a pair of students who did not remember the word “tunneling” (“we have no idea Sir”), remembered the basic facts and interpretations when they saw the applet and mentioned the correct changes to variables to increase or decrease the probability of tunneling. Of course, deeper problems with quantum representations are likely to still be present [4]. Students were also interviewed about a textbook problem showing the potential energy graph with alpha decay. They could apply their tunneling knowledge correctly but surprisingly many students could not give the cause for acceleration of the alpha particle outside the nucleus, they forgot about Coulomb repulsion between nucleus and alpha particle!

3. Results
On the posttest (112 students, 5 schools) 40-50% of the students still felt that tunneling costs energy in spite of ample warnings during the lessons as the teachers were aware of this popular misconception.
20. An electron tunnels from left to right through the barrier below.

What can you say about the relationship between kinetic energy of this particle before and after tunneling?

A. 46% $E_{\text{before}} > E_{\text{after}}$
B. 40% $E_{\text{before}} = E_{\text{after}}$ (correct)
C. 10% $E_{\text{before}} < E_{\text{after}}$

21. A particle with a certain energy state has a chance to tunnel through the barrier. Then the height of the barrier is increased. What is the effect of increasing the height of the barrier?

A. 7% The kinetic energy of the particle that passes the barrier will become smaller.
B. 48% The probability of tunneling becomes smaller. (correct)
C. 30% Both A and B are true.
D. 11% None of the above.

22. A particle with a certain state of energy has a chance to tunnel through the barrier. Then the barrier is widened. What is the effect of widening the barrier?

A. 4% The kinetic energy of the particle that passes the barrier will become smaller.
B. 40% The probability of tunneling becomes smaller. (correct)
C. 46% Both A and B are true.
D. 5% None of the above.

In item 21 half the students expect that tunneling costs energy even though the lessons emphasized that energy on the left is energy on the right. Similar percentages go for options A or C in items 22 and 23. Through cross tabulations in SPSS we found that 53% of those answering A in item 21, answered A or C in item 22 and 76% did so in item 23, and 70% of those answering C on item 22 also answered C on item 23, confirming consistency in the view that tunneling costs energy [3]. In the interviews students did better on energy questions, perhaps because of the visual support of the PhET applet.

4. Discussion
The microwave experiment can be described and explained with classical wave physics without any need to invoke quantum physics. Due to wave-particle duality, particles can also tunnel through barriers with the same exponential factor in transmission. But does the experiment have educational value? What are or could be the educational objectives of the experiment within quantum physics?

- Showing an interesting demonstration?
- Showing that tunneling is essentially a wave property and quantum particles can tunnel due to wave-particle duality?
- Demonstrating the exponential fall off of the intensity with the gap width?
Showing a scanning tunneling microscope in real or simulation would come much closer to typical quantum tunneling. From our interview results we have to conclude that in our teaching, the microwave version of the tunneling demonstration did not contribute much to understanding the tunneling phenomenon. On the other hand, students gave very good explanations in response to our questions about the quantum-tunneling PhET simulation. In interviews they correctly explained the different features of the three graphs and the effects of increasing width and height of the barrier and total energy of the particle on the amplitude and wavelength of the transmission graph.

Another problem concerned the equipment for the microwave experiment. Albion et al [2] used prisms with methacrylate pellets, we used prism shapes filled with polystyrene beads. Set-ups with light instead of microwaves are well-known [5], but initially we were not able to see frustrated internal reflection trying many prisms. Finally, we ordered some high-grade prisms and then it all worked perfectly including the measurement of the exponential fall-off of the intensity over several orders of magnitude (figures 1 & 2 & 5). The intensity was measured by a photodiode but can also be observed directly on a paper screen (figure 2), possibly avoiding the student belief that light rays will either tunnel or be reflected but not both. This experiment has now been mounted in a small suitcase which can be borrowed by schools from the university just like our suitcase for single photon interference with a double slit (figure 4).

Figure 4 Set-up for the optics tunneling experiment. Although there is only one light sensor, both the reflected and the tunneled beam can be projected on paper simultaneously.

Figure 5 The logarithm of intensity of tunneled light versus slit width as measured in turns to shift the laser beam to the wider part of the wedge.
5. Conclusion

With these experiences and information about typical misconceptions about tunneling [3] we came to the following sequence of steps in teaching tunneling which we now use in our suitcase experiment:

a) Setting the stage, protons fuse in the Sun to Helium and give off energy. Why don’t all the protons fuse at once, why can the process run for 10 billion years? The question is raised but will only be answered at the end of the sequence.

b) Demonstrate total internal reflection with a prism and a laser. Turn the prism and show that the intensity of the refracted beam becomes zero when the angle of incidence is greater than the critical angle.

c) Show this also with a glass of water or with an aquarium. With the latter one can show the water surface as a mirror when looking from below.

d) Now what if we have a second prism and a very small gap (figure 1)? Demonstrate that the intensity is now divided between a reflected beam and a beam that goes straight (tunnels through the wedge). Compare these intensities (figure 2). The smaller the gap, the greater the intensity of the tunnelled beam.

e) Present the graph of intensity versus width which has been recorded before.

f) Now think of photons as a wave-particle, it is not allowed in the gap, but waves have uncertainty in location and that makes it possible to cross the gap if it is narrow.

g) Discuss tunneling situations with electrons which can behave like waves.

h) Discuss tunneling with graphs of the energy, wave function, and probability using the PhET quantum tunneling applet and teaching suggestions [6].

i) Now explain the tunneling of protons in the Sun without going into too detailed reaction equations. One could also go into alpha decay as tunneling explains the huge range in half-lives from less than nano-seconds to billions of years [1].

6. References

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