Environmentally Friendly Renormalization Group and Phase Transitions

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We discuss an environmentally friendly renormalization group approach to analyze phase transitions. We intend to apply this method to the Electroweak Phase Transition. This work is in progress. We present some previously obtained results concerning a $\lambda \phi^4$ theory, where the main features of this algorithm are introduced.

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1 Introduction

The actual mechanism of baryogenesis is still elusive. Although several scenarios which generate an excess of matter over antimatter have been proposed, we still lack evidence which favours any of them. However, constraints on inputs of the models have started ruling out some of these. This seems to be the case for the Electroweak Phase Transition (EWPT) within the Standard Model where a strong enough first order transition is not achieved for realistic Higgs masses [1]. Even in the supersymmetric extension, the parameter space for the desired transition is getting reduced.

Lattice calculations have played an important role in reaching these conclusions and even though different approaches have been used to study the EWPT, we believe there is still some physical insight to gain from analyzing this transition by means of an environmentally friendly renormalization group running with the temperature. In what follows, we will describe the main steps in the application of this method to $\lambda \phi^4$. The details can be found somewhere else [2].

2 Analysis

The Renormalization Group Equation can be seen as a simple consequence of the fact that the physics is independent of the arbitrary renormalization scale $\mu$ at which we choose to define our parameters

$$\mu \frac{d\Gamma(N)}{d\mu} = 0$$

An environmentally friendly renormalization group realises that the effective degrees of freedom of a system may change qualitatively during its evolution. Such is the case, for instance, of the confinement- deconfinement transition where we have the change between quark-gluon degrees of freedom and hadron- meson degrees of freedom. Hence, a suitable choice of parameters to describe this evolution is convenient, otherwise we end up with a description which does not fit the new degrees of freedom. In the case of finite temperature field theory, a renormalization group running with the temperature seems to be an adequate method of tracking the evolution of the system.

For $\lambda \phi^4$, the Euclidean action is written as follows in terms of bare quantities and the inverse temperature $\beta = 1/T$

$$S[\phi_B] = \int_0^\beta dt \int d^{d-1}x \left[ \frac{1}{2} (\nabla \phi_B)^2 + \frac{1}{2} M_B^2 \phi_B^2 + \frac{\lambda_B}{4} \phi_B^4 \right]$$

The renormalized parameters are specified by the normalization conditions at an arbitrary temperature scale $\tau$

$$\frac{\partial}{\partial p^2} \Gamma_\tau^{(2)}(p, \tilde{\phi}_H(\tau), M(\tau), \lambda(\tau), T = \tau) \big|_{p=0} = 1$$
\[
\Gamma^{(2)}(p = 0, \phi_H(\tau), M(\tau), \lambda(\tau), T = \tau) = M^2(\tau)
\]
\[
\Gamma^{(4)}_I(p = 0, \phi_H(\tau), M(\tau), \lambda(\tau), T = \tau) = \lambda(\tau)
\]
where \( \phi_H \) is the renormalized field corresponding to a reference external current \( H \).

From here, the flow equations describing the running of the mass and the coupling are

\[
\tau \frac{dM^2(\tau)}{d\tau} = \beta_M, \quad \tau \frac{d\lambda(\tau)}{d\tau} = \beta_\lambda
\]

where the \( \beta \)-functions \( \beta_M \) and \( \beta_\lambda \) are to be calculated at each order of approximation.

To one loop and taking the external current \( H = 0 \) these take the following form

\[
\beta_M = \begin{cases} 
\frac{\lambda^2}{2} \tau \frac{d}{d\tau} O_1 & \tau > T_c \\
-\lambda(\tau) \frac{d}{d\tau} O_1 + \frac{3}{2} M^2 \tau \frac{d}{d\tau} O_2 & \tau < T_c
\end{cases}
\]

\[
\beta_\lambda = -\frac{3}{2} \lambda^2 \tau \frac{d}{d\tau} O_2
\]

In this expression, \( O_n \) corresponds to the one loop diagram with \( n \) propagators, without vertex factors, and at zero external momentum.

The flow equation for the coupling \( \lambda \) can be solved analytically and substituted into the equation for the mass \( M \), which needs to be solved numerically. Once this is done, the critical temperature \( T_c \) separating the two phases is obtained. Further discussion of this case and comparison of this approach and the work of others can be found in [2]. The techniques of environmentally friendly renormalization can be revised in [3].

### 3 Conclusions

Environmentally friendly renormalization groups seem to be a fine probe to analyze the electroweak phase transition, better equipped than other approaches, though some physical intuition of the dynamics is needed. We hope to gain some more insight into the details of the EWPT using this method. This will add to the knowledge of the phase transition we have up to now.

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The analysis of the electroweak phase transition by means of an enviromentally friendly renormalization group is currently being carried out in collaboration with Chris Stephens, Axel Weber and Carlos Mendoza.
References

[1] See Laine’s talk in this conference.

[2] M. A. van Eijck, D. O’Connor and C. R. Stephens, Int. J. Mod. Phys. 10 (1995) 3343.

[3] D. O’Connor and C. R. Stephens, Nucl. Phys. B360 (1991) 297; J. Phys. A25 (1992) 101; Int. J. Mod. Phys. A9 (1994) 2805; Phys. Rev. 72 (1994) 506.