Nonlocality of Hardy type in experiments using independent particle sources

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Abstract

By applying Hardy’s argument, we demonstrate the violation of local realism in a gedanken experiment using independent and separated particle sources.

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The nonlocal nature of quantum systems arising from entanglement has played a central role in quantum information science. Discussions about quantum nonlocality were initiated by Einstein, Podolsky, and Rosen [1] and extended by Bell [2]. Although the violation of Bell’s inequality predicted by quantum mechanics has been experimentally verified [3], there have been arguments about the detection loopholes [4, 5]. Greenberger, Horne, and Zeilinger (GHZ) [6] demonstrated quantum mechanical violation of local realism without using the Bell’s inequality for more than three particles. Hardy proved the nonlocality without using the Bell’s inequality for all entangled states (except maximally entangled states) of two spin-$\frac{1}{2}$ particles [7]. Considerable theoretical and experimental effort has been devoted to testing this Hardy type nonlocality [8, 9, 10, 11]. An attempt to extend Hardy’s theorem to cover maximally entangled states was made by Wu et al. using a quantum optical setting [10]. Recently Yurke and Stoler demonstrated violation of local realism in an experimental configuration involving independent sources [11, 12, 13]. Specifically, they showed that: (1) in the fermion case the Pauli exclusion principle can be exploited in a local realism experiment of the Hardy type [11]; (2) GHZ type nonlocality can arise even when the particles come from independent widely separated sources [12]; and (3) violation of the Bell’s inequality can be demonstrated by a quantum optical setting using independent particle sources [13].

The aim of our paper is to demonstrate nonlocality of Hardy type in experiments using independent particle sources. A schematic of the apparatus for the Gedanken experiment is shown in Fig. 1, which is similar to the setup proposed by Yurke and Stoler [13], except that the four beam splitters $B_i$ have transmittance $T_i$ and reflectivity $R_i = 1 - T_i$ where $T_i \neq R_i$ and $i = 1..4$. We parameterize $T_i$ and $R_i$ as $T_i = \sin^2(\theta_i) \equiv S_i^2$ and $R_i = \cos^2(\theta_i) \equiv C_i^2$, respectively.
FIG. 1. Schematic of the apparatus used in our gedanken experiment for Hardy’s nonlocality. See text for detailed explanations.

Two independent particles radiated from the source $S_1$ and $S_2$ are incident on the input ports of the beam splitters $B_1$ and $B_2$, respectively. Vacuum ($|0\rangle$) enters the other input ports of $B_1$ and $B_2$. The outputs of these beam splitters propagate to two detectors. Detector 1 consists of the phase shifter $\phi_3$, the beam splitter $B_3$, and the particle counters E and F. Similarly, detector 2 consists of the phase shifter $\phi_4$, the beam splitter $B_4$, and the particle counters G and H. The beam path labels appearing in Fig. 1 will also be used to denote the annihilation operators for modes propagating along these beam paths.

The analysis of the firing statistics at each particle counter is carried out as follows. The beam splitters $B_1, B_2, B_3,$ and $B_4$ perform the mode transformation

$$
\begin{align*}
(a') &= \begin{pmatrix} S_1 & iC_1 \\ iC_1 & S_1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \\
(b') &= \begin{pmatrix} S_2 & iC_2 \\ iC_2 & S_2 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix}, \\
(e) &= \begin{pmatrix} S_3 & iC_3 \\ iC_3 & S_3 \end{pmatrix} \begin{pmatrix} e^{-i\phi_3}b' \\ c' \end{pmatrix},
\end{align*}
$$

(1)
From the mode transformation shown in Eq. (1), it follows that the annihilation operators for the modes \( a, b, c \) and \( d \) can be expressed in terms of those for \( e, f, g \) and \( h \) as follows:

\[
\begin{align*}
    a &= S_1S_4g - iS_1C_4h - ie^{i\phi_3}C_1S_3e - e^{i\phi_3}C_1C_3f \\
    b &= -iC_1S_4g - C_1C_4h + e^{i\phi_3}S_1S_3e - ie^{i\phi_3}S_1C_3f \\
    c &= -iS_2C_3e + S_2S_3f - e^{i\phi_4}C_2C_4g - ie^{i\phi_4}C_2S_4h \\
    d &= -C_2C_3e - iC_2S_3f - ie^{i\phi_4}S_2C_4g + e^{i\phi_4}S_2S_4h.
\end{align*}
\]

The state vector for two identical bosons injected into the interferometer can be expressed as the direct product of the individual state vectors. In second quantized notation, the input state vector is therefore given by

\[
|\psi\rangle = a^\dagger e^\dagger |0\rangle. \quad (3)
\]

By substituting Eq. (2) into Eq. (3), we obtain the output state vector. This vector can be divided into two parts

\[
|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle, \quad (4)
\]

where

\[
|\psi_1\rangle = i[S_1S_2C_3S_4 - e^{-i\phi_3 - i\phi_4}C_1C_2S_3C_4]|1_e|1_g
+ [S_1S_2S_3S_4 + e^{-i\phi_3 - i\phi_4}C_1C_2C_3C_4]|1_f|1_g
- [S_1S_2C_3C_4 + e^{-i\phi_3 - i\phi_4}C_1C_2S_3S_4]|1_e|1_h
+ i[S_1S_2S_3C_4 - e^{-i\phi_3 - i\phi_4}C_1C_2C_3S_4]|1_f|1_h,
\]

\[
|\psi_2\rangle = ie^{-i\phi_4}C_1S_2(S_3^2 - C_3^2)|1_e|1_f
+ ie^{-i\phi_4}S_1C_2(S_4^2 - C_4^2)|1_g|1_h
- 2e^{-i\phi_3}C_1S_2S_3C_3[|2_e + |2_f]
- 2e^{-i\phi_4}S_1C_2S_4S_4[|2_g + |2_h].
\]
Here \(|n\rangle_e\) denotes the \(n\) particle state of the mode \(e\). Now consider detector 1 and let \(E\) (\(F\)) denote the event in which the counter \(E\) (\(F\)) counts a single particle and the counter \(F\) (\(E\)) counts no particle. Similarly for detector 2, we define events \(\bar{G}\) (\(\bar{H}\)) in which the counter \(G\) (\(H\)) counts one particle and the counter \(H\) (\(G\)) counts no particle. Among all the possible events, we are interested only in the events \(\bar{E}, \bar{F}, \bar{G}\) and \(\bar{H}\). Thus, we do not need to pay attention to the evolution of the state \(|\psi_2\rangle\), because \(\bar{E}, \bar{F}, \bar{G}\) and \(\bar{H}\) are not reflected in \(|\psi_2\rangle\). Hence, below we consider only the evolution of the state \(|\psi_1\rangle\).

Let us consider the following four cases.

a) Set \(\phi_3 = \phi_4 = \pi/2\) and choose

\[ S_3 = S_4 = \sqrt{\frac{C_1C_2}{C_1C_2 + S_1S_2}} \equiv \tau. \]  \hspace{1cm} (6)

Then we obtain

\[ |\psi_1\rangle = i\sqrt{C_1C_2S_1S_2}|1\rangle_e|1\rangle_g + |1\rangle_f|1\rangle_h \]
\[ + [C_1C_2 - S_1S_2]|1\rangle_e|1\rangle_h. \] \hspace{1cm} (7)

Let \(FG(S_3 = \tau, S_4 = \tau, \phi_3 = \frac{\pi}{2}, \phi_4 = \frac{\pi}{2})\) denote the probability of the simultaneous appearance of events \(\bar{F}\) and \(\bar{G}\) for the following experimental settings. The transmittance of \(B_3\) is set to \(S_3^2 = \tau^2\) and that of \(B_4\) is set to \(S_4^2 = \tau^2\). Since there is no \(|1\rangle_f|1\rangle_g\) term in Eq. (7), we obtain

\[ FG(S_3 = \tau, S_4 = \tau, \phi_3 = \frac{\pi}{2}, \phi_4 = \frac{\pi}{2}) = 0. \] \hspace{1cm} (8)

b) If \(\phi_3 = \frac{\pi}{2}, \phi_4 = \frac{3\pi}{2}\), and

\[ S_3 = \tau, \]
\[ S_4 = \sqrt{\frac{(C_1C_2)^3}{(S_1S_2)^3 + (C_1C_2)^3}} \equiv \tau', \] \hspace{1cm} (9)

we have
\[ |\psi_1\rangle = \frac{S_1 S_2 C_1 C_2}{\sqrt{(C_1 C_2)^2 + (S_1 S_2)^2 - S_1 S_2 C_1 C_2}} |1\rangle_f |1\rangle_g \]
\[ - \sqrt{(C_1 C_2)^2 + (S_1 S_2)^2 - S_1 S_2 C_1 C_2} |1\rangle_c |1\rangle_h \]
\[ + i \frac{\sqrt{S_1 S_2 C_1 C_2 (S_1 S_2 - C_1 C_2)}}{\sqrt{(C_1 C_2)^2 + (S_1 S_2)^2 - S_1 S_2 C_1 C_2}} |1\rangle_f |1\rangle_h. \]

Thus we have the following quantum prediction:
\[ F(S_3 = \tau, S_4 = \tau', \phi_3 = \frac{\pi}{2}, \phi_4 = \frac{3\pi}{2}) = 1, \] if
\[ G(S_3 = \tau, S_4 = \tau', \phi_3 = \frac{\pi}{2}, \phi_4 = \frac{3\pi}{2}) = 1, \] (11)
since there is only one term \(|1\rangle_f |1\rangle_g\) containing \(|1\rangle_g\).

c) Setting \(\phi_3 = \frac{3\pi}{2}, \phi_4 = \frac{\pi}{2}\) and
\[ S_3 = \tau', \]
\[ S_4 = \tau, \] (12)
we obtain
\[ |\psi_1\rangle = \frac{S_1 S_2 C_1 C_2}{\sqrt{(C_1 C_2)^2 + (S_1 S_2)^2 - S_1 S_2 C_1 C_2}} |1\rangle_f |1\rangle_g \]
\[ - \frac{\sqrt{S_1 S_2 C_1 C_2}}{\sqrt{(C_1 C_2)^2 + (S_1 S_2)^2 - S_1 S_2 C_1 C_2}} (|1\rangle_f |1\rangle_g) \]
\[ + i \frac{\sqrt{S_1 S_2 C_1 C_2 (S_1 S_2 - C_1 C_2)}}{\sqrt{(C_1 C_2)^2 + (S_1 S_2)^2 - S_1 S_2 C_1 C_2}} |1\rangle_f |1\rangle_h. \]

Thus, if
\[ F(S_3 = \tau', S_4 = \tau, \phi_3 = \frac{3\pi}{2}, \phi_4 = \frac{\pi}{2}) = 1, \] then
\[ G(S_3 = \tau', S_4 = \tau, \phi_3 = \frac{3\pi}{2}, \phi_4 = \frac{\pi}{2}) = 1. \] (14)

d) Setting \(\phi_3 = 3\pi/2, \phi_4 = 3\pi/2,\) and choosing \(S_3 = S_4 = \tau',\) we obtain
\[ |\psi_1\rangle = i \frac{\sqrt{(C_1 C_2 S_1 S_2)^3}}{(S_1 S_2)^2 + (C_1 C_2)^2 - S_1 S_2 C_1 C_2} (|1\rangle_e |1\rangle_g) \]
\[ + |1\rangle_f |1\rangle_h) + \frac{S_1 S_2 C_1 C_2 (C_1 C_2 - S_1 S_2)}{(S_1 S_2)^2 + (C_1 C_2)^2 - S_1 S_2 C_1 C_2} (|1\rangle_f |1\rangle_g) \]
\[ + \frac{[(C_1 C_2)^2 + (S_1 S_2)^2](C_1 C_2 - S_1 S_2)}{(S_1 S_2)^2 + (C_1 C_2)^2 - S_1 S_2 C_1 C_2} (|1\rangle_e |1\rangle_h). \]

Thus the following quantum prediction is obtained
\[ FG(S_3 = \tau', S_4 = \tau', \phi_3 = \frac{3\pi}{2}, \phi_4 = \frac{3\pi}{2}) = 1 \]
with a nonzero probability
\[ P = \frac{(S_1 S_2 C_1 C_2)^2 (S_1 S_2 - C_1 C_2)^2}{[(S_1 S_2)^2 + (C_1 C_2)^2 - S_1 S_2 C_1 C_2]^2}, \]
where \( C_1 C_2 \neq S_1 S_2 \).

Finally, we demonstrate that, following Hardy’s [1] and Wu et al.’s argument [10], local realism and quantum mechanics are incompatible using an experimental setting with independent and separated particle sources. The notion of local realism is introduced by assuming that there exist some hidden variables \( \lambda \) that describe the state of individual particles. According to the assumption of locality, the choice of the measurement at detector 1 would not influence the outcome of the measurement at detector 2, which means that, for a specified \( \lambda \), the probability of the event \( \bar{F} \) is uniquely determined by the transmittance of \( B_3 \) and \( \phi_3 \), whereas that of \( \bar{G} \) is determined solely by the transmittance of \( B_4 \) and \( \phi_4 \). Let us denote the probabilities of the events \( \bar{F} \) and \( \bar{G} \) for a value of hidden variable \( \lambda \) by \( F(\lambda, S_3, \phi_3) \) and \( G(\lambda, S_4, \phi_4) \), respectively. Using Eq. (16), for some values of hidden variable \( \lambda \), we expect simultaneous occurrence of events \( \bar{F} \) and \( \bar{G} \) when \( S_3 = \tau', S_4 = \tau', \phi_3 = \frac{3\pi}{2} \) and \( \phi_4 = \frac{3\pi}{2} \) and thus obtain \( F(\lambda, S_3 = \tau', \phi_3 = \frac{3\pi}{2}) = G(\lambda, S_4 = \tau', \phi_4 = \frac{3\pi}{2}) = 1 \). On the other hand, from Eqs. (11) and (14), we have \( G(\lambda, S_4 = \tau, \phi_4 = \frac{\pi}{2}) = 1 \), since \( F(\lambda, S_3 = \tau, \phi_3 = \frac{3\pi}{2}) = 1 \); and \( F(\lambda, S_3 = \tau, \phi_3 = \frac{\pi}{2}) = 1 \) since \( G(\lambda, S_4 = \tau', \phi_4 = \frac{3\pi}{2}) = 1 \) for the same values of \( \lambda \). Therefore, we should have \( F(\lambda, S_3 = \tau, \phi_3 = \frac{\pi}{2}) = G(\lambda, S_4 = \tau, \phi_4 = \frac{\pi}{2}) = 1 \). But this contradicts the quantum prediction of Eq. (8) that the probability of simultaneous occurrence
of the events $F$ and $G$ is zero when $S_3 = \tau, S_4 = \tau, \phi_3 = \frac{\pi}{2}$ and $\phi_4 = \frac{\pi}{2}$.

In summary, we have shown the violation of local realism of EPR type without using Bell’s inequality for the case of two particles originating from independent sources.

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