Influence of a thin interlayer and a distribution of microdefects on guided wave dispersion properties in an elastic laminate plate

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Abstract. Adequate mathematical and computational models for damaged interfaces and thin interlayers between two solids are necessary for the non-destructive ultrasonic evaluation of the reliability and safety of bonded structures in service. However, the complexity of real interfaces does not allow to describe their mechanical behaviour precisely. In this paper, the problem of elastic guided wave propagation in a multi-layered structure with thin polymeric plies with possible weakened adhesion is treated uniformly employing effective boundary conditions at sublayer interfaces. While for the pristine structure continuity of the displacements and stresses at the interfaces is assumed, for the damaged interfaces, an approach relying on spring-like boundary conditions is used. The effect of the internal layer and damaged interfaces between sub-layers on an elastodynamic behaviour of a laminate is analysed. Their influence on the dispersion properties of guided waves propagating in a laminated composite structure is illustrated both theoretically and experimentally.

1. Introduction

Ultrasonic guided waves are widely applied in the nondestructive evaluation and structural health monitoring for an inspection of laminate thin-walled composite structures. They are sensitive to possible faults such as cracks, pitting corrosion, voids etc. The accuracy of damage identification and detection might be improved employing relevant computational models of wave propagation, which take into account intrinsic mechanical properties of the inspected waveguide structure [1, 2, 3]. Among the latter, the models taking into account the presence and the influence of adhesive connections (which, in turn, could be either pristine or damaged) between the plies of a multi-layered composite are of particular interest [4, 5, 6, 7]. Typically, the complexity of interfaces does not allow for an explicit description of their mechanical behaviour. Thus, the derivation of boundary conditions that approximate the interfacial properties of the contact between two solid bodies has been the subject of extensive investigations [8, 9]. One of the basic approaches is to treat the interface and its vicinity as an additional thin elastic layer typically with specific, probably viscoelastic, properties and to assume its ideal contact with surrounding substrates. As an alternative, various phenomenological approaches that intend characterizing the overall strength of the joint are considered. For example, within the well-developed quasi-static approach [10, 11] imperfect interface due to the presence of cracks is treated as a set of distributed springs connecting one material to another. Parameters for this model in the case
of isotropic materials are spring stiffnesses, which bind the normal and transverse displacement to the traction vector at the interface. Approximate boundary conditions derived from these “spring-like” ideas could be also used for describing guided wave propagation in a multi-layered medium with imperfect contact between substrates [12, 13].

In the present study, the influence of thin intermediate adhesive-like films between laminate sub-layers on elastodynamic properties of the whole waveguide structure is considered both experimentally and theoretically. To address this problem, laminate specimens have been manufactured from two isotropic aluminium plates glued together using two-sided thin epoxy tape. Since the adhesion quality could not be assured in advance and is influenced by various external factors (i.e., poor surface preparation, non-uniform pressure during attachment, etc.), along with the basic elastodynamic problem for a three-layered structure with an ideal contact between plies, a more sophisticated formulation is considered. The latter takes into account the presence of partially debonded interfaces or zones of imperfect contact between sub-layers in such a laminate employing the distributed spring model. For this purpose, analytic frequency-dependent expressions for spring stiffnesses derived in [14, 15, 16] using the ensemble average technique and the boundary integral equation method [17, 18]. The results of corresponding numerical evaluations have revealed the strong influence of polymeric interlayer and possible adhesion imperfections on the wave dynamic properties of the considered laminate structure. Employing frequency-wavenumber analysis applied to the out-of-plane velocities of piezo-induced wave motion acquired with the laser Doppler vibrometry, the wavenumbers of propagating guided waves in a wide frequency range have been evaluated as well. The strong influence of the adhesive sub-layer is confirmed manifesting itself in remarkable changes in dispersion curves of both the fundamental and the high-order guided waves being in a good coincidence with corresponding theoretically predicted wavenumbers.

2. Boundary conditions
Let us consider two isotropic and homogeneous elastic plates \( D_1 \) and \( D_2 \) of thicknesses \( h_1 \) and \( h_2 \) and a thin film \( D_0 \) of thickness \( h_0 \) between them as shown in figure 1. Materials of the

![Diagram of the problem](image)

**Figure 1.** Geometry of the problem.

layers are characterized by the mass density \( \rho_j \) and Lame constants \( \lambda_j, \mu_j \). The latter can be also expressed in terms of wave velocities \( c_{js} \) (here subscript \( s \) corresponds to longitudinal L
and transversal T waves correspondingly). For time-harmonic in-plane motion with the angular frequency $\omega$, the displacement vector $u^{(j)} = \{u_1^{(j)}, u_2^{(j)}\}$ obeys Lamé equation

$$k_{jL}^2 \nabla \nabla \cdot u^{(j)} - k_{jL}^2 \nabla \times (\nabla \times u^{(j)}) + u^{(j)} = 0$$

written in terms of wave numbers $k_{jL} = \omega/c_{jL}$. The stress tensor components $\tau^{(j)}$ by the Hooke’s law are expressed through the components of the displacement vector.

### 2.1. Thin interlayers

As it is mentioned in the introduction, a thin elastic layer the interface between two different solids can be substituted by an interface with effective properties. For this case, the asymptotic approach was developed in [9] for a thin curved layer, and the numerical analysis of its application was provided for a thin spherical layer. As a result, the authors conclude that this approach is quite accurate if the layer thickness is small enough relatively to the wavelength.

Thus, for the calculation of the dispersion properties of a three-layered plate of thickness $H = h_1 + h_0 + h_2$, the continuity of the traction and displacement vectors at the interfaces is assumed:

$$\tau^{(1)} - \tau^{(0)} = u^{(1)} - u^{(0)} = 0, \quad x_2 = -h_1,$$

$$\tau^{(2)} - \tau^{(0)} = u^{(2)} - u^{(0)} = 0, \quad x_2 = -h_1 - h_0.$$  

Here $\tau^{(j)}$ are components of the traction vector $\{\sigma_{12}^{(j)}, \sigma_{22}^{(j)}\}$. At the surfaces of the waveguide, stress-free boundary conditions

$$\tau^{(1)} = 0, \quad x_2 = 0; \quad \tau^{(2)} = 0, \quad x_2 = -H.$$  

are assumed.

The effective boundary conditions derived in [9] can be applied for the simulation of dynamic behaviour of a thin interlayer. In this case, (4) and total thickness $H = h_1 + h_0 + h_2$ of the waveguide should be kept, two thicker sub-layers should have thicknesses $h_1 + h_0/2$ and $h_2 + h_0/2$ and the thin interlayer is substituted by the effective boundary conditions, i.e. boundary conditions (2) and (3) are replaced by the relations:

$$
\begin{align*}
&u_1^{(1)} - u_1^{(2)} = s_{13}^{(1)} \tau_1^{(1)} + s_{13}^{(2)} \tau_1^{(2)}, \\
&u_2^{(1)} - u_2^{(2)} = s_{21}^{(1)} u_1^{(1)} + s_{21}^{(2)} u_1^{(2)} + s_{24}^{(1)} \tau_2^{(1)} + s_{24}^{(2)} \tau_2^{(2)}, \\
&\tau_1^{(1)} - \tau_1^{(2)} = s_{31}^{(1)} u_1^{(1)} + s_{31}^{(2)} u_1^{(2)} + s_{34}^{(1)} \tau_2^{(1)} + s_{34}^{(2)} \tau_2^{(2)}, \\
&\tau_2^{(1)} - \tau_2^{(2)} = s_{42}^{(1)} u_2^{(1)} + s_{42}^{(2)} u_2^{(2)}. \\
&x_2 = -h_1 - h_0/2,
\end{align*}
$$

where

$$s_{13}^{(j)} = \frac{h_0}{2} \left( \frac{1}{\mu_0} - \frac{1}{\mu_j} \right), \quad s_{34}^{(j)} = h_0(\lambda_j \mu_0 - \lambda_0 \mu_j) b_0 b_j \frac{\partial}{\partial x_1}, \quad s_{24}^{(j)} = \frac{h_0}{2} (b_0 - b_j),$$

$$s_{31}^{(j)} = h_0(\mu_j - \mu_0 + \lambda_j \mu_j - \lambda_0 \mu_0) \frac{\partial^2}{\partial x_1^2} + \frac{h_0}{2} (\mu_j - \mu_0) \omega^2, \quad b_j = \frac{1}{\lambda_j + 2 \mu_j}.$$
2.2. Concentration of micro-defects or damaged interface

On the other hand, a damaged interface can be described by a distributed spring model [10], where stiffness characterizes the weakness of adhesion. The effective elastic moduli of elastic solids containing randomly distributed micro-cracks can be estimated in terms of crack density. Consider a random distribution of cracks at the interface, where the distribution is assumed translationally invariant, and all the cracks are of the same size. The wave interaction between the cracks is neglected by assuming small cracks compared to the wavelength. Interface damage is defined through crack density $C = \frac{S_{\text{dom}}}{S_{\text{total}}}$. The ensemble average technique is employing for the build of the scattered field, which is represented in the form of plane waves in a far-field zone from the interface [14, 19] and finally, the total transmission coefficient for distribution of cracks is derived. The random distribution of cracks is then compared with the distributed spring model, where the damaged interface (for instance, between media $D_0$ and $D_1$) is described via spring boundary conditions

$$\tau^{(1)} = \tau^{(0)} = \begin{pmatrix} \kappa_T & 0 \\ 0 & \kappa_N \end{pmatrix} \cdot (u^{(1)} - u^{(0)})$$ (7)

written in terms of tangential $\kappa_T$ and normal $\kappa_N$ spring stiffnesses. The spring boundary conditions demand that the stresses are continuous, while the displacement jump is proportional to the stresses. According to [14], normal ($s = N$) and tangential ($s = T$) spring stiffnesses, which define the severity of damage, can be represented as follows

$$\kappa_s = \frac{f_s}{C(p_s \cdot \Delta u_s)} - \frac{f_s}{2}, \quad f_s = \frac{ic_{1s} k_{1s} c_{0s} k_{0s}}{c_{1s} k_{1s} + c_{0s} k_{0s}}.$$ (8)

Here $p_s$ is a unit wave vector, and $\Delta u_s$ is the average of the crack opening displacement (COD).

Relying on the asymptotic analysis in the frequency domain and assuming a certain shape of micro-cracks, approximate analytical representations for SBCs could be derived [16]. The substitution of asymptotic average COD into (8) results in a closed-form relations for tangential $\kappa_T$ and normal $\kappa_N$ spring stiffnesses. In the case of a distribution of identical strip-like cracks of width $a$, the spring stiffnesses can be expressed in terms of the circular frequency $\omega$ as follows

$$\kappa_T = \kappa_N = \frac{8}{C \pi a \beta_1},$$ (9)

where

$$\beta_1 = \frac{\lambda_1 + \mu_1}{(\lambda_1 + \mu_1) \mu_1} + \frac{\lambda_0 + 2 \mu_0}{(\lambda_0 + \mu_0) \mu_0}.$$ 

Thus, if a damaged interface is assumed, then dispersion relations for the considered three-layered waveguide can be composed of (4) and (7) at interfaces $x_2 = -h_1$ and $x_2 = -h_1 - h_0$ instead of (2) and (3).

3. Results of numerical and experimental analysis

Further on, the dispersion properties of GWs propagating in a three-layered plate composed of two 2-mm thickness plates made of aluminium ($\lambda_1 = \mu_1 = 50.5$ GPa, $\rho_1 = 26.3$ GPa, $\rho_1 = 2700$ kg m$^{-3}$) and two-sided tape between them ($\lambda_0 = 1.39$ GPa, $\mu_0 = 0.227$ GPa, $\rho_0 = 930$ kg m$^{-3}$) of 0.05 mm thickness are investigated. Possible contact imperfections between the layers are simulated employing SBCs according to the theory presented above. Spring stiffnesses in normal and tangential directions are assumed to be equal and frequency-independent. In the case under consideration, the 3D dispersion equation for modes of an infinite plate allows decomposition into two independent ones.
The wavenumbers of propagating Lamb waves (LWs) are shown in Figure 2. First of all, this plot demonstrates that the presence of a thin interlayer sufficiently changes the dispersion properties of the waveguide. Namely, there is almost no coincidence between dispersion curves for a three-layered structure and a monolithic aluminium layer of the same thickness. On the contrary, the behaviour of certain normal modes for a laminate structure coincides with the dispersion curves for a single 2-mm aluminium plate, which could be explained by a softness of the polymeric interlayer, i.e., aluminium substrates though being attached remain “quasi-decoupled” for these particular guided waves. Taking possible contact imperfections at interfaces into account provides additional changes into dispersion curves, and the sensitivity to adhesion disturbances varies depending on the frequency and the number of the mode. The fundamental modes in the low-frequency range (much lower than the first cut-off frequency) are practically insensitive to the weakened adhesion of considered severity, whereas its effect is pronounced at $f \in (0.25, 1)$ MHz, especially for the fundamental symmetric Lamb wave $S_0$. Two next LWs also show good sensitivity in this region.

For the experimental observation, a corresponding laminate sample was made from two aluminium plates with a thickness of 2 mm attached together with a thin 0.05 mm double-sided epoxy tape. The out-of-plane velocities of piezo-induced wave packages were measured on the specimen surface by laser Doppler vibrometry and were post-processed via the frequency-
wavenumber analysis (FWA) [20]. To minimize parasitic side-wall reflections and to improve the FWA quality the specimen edges were extensively covered by plasticine. In figure 3, the FWA results (i.e., dark-coloured areas indicate experimental dispersion curves of LWs) and corresponding numerical data for three-layered waveguide as well as for a single 2 mm aluminium layer are presented. A detailed analysis shows that it is essential to consider a thin interlayer in simulations, otherwise, the experimental frequency dependencies of LW wavenumbers are not captured adequately. Moreover, assuming contact imperfections at sublayer interfaces via boundary conditions (6) improves the coincidence between predicted and experimental dispersion curves of higher LWs. Surprisingly, however, is that in figure 3 at low frequencies experimental wavenumbers of the fundamental antisymmetric Lamb wave $A_0$ agree with the $A_0$ dispersion curve for a single-layered 2 mm aluminium waveguide rather than with the results for the three-layered structure. An adequate explanation of this phenomena would be a subject of further research.

![Figure 3](image)

**Figure 3.** Results of the FWA and wavenumbers of GWs propagating in a three-layered pristine (thin solid line) and damaged simulated via the SBC (dashed line) plates made from aluminium and epoxy tape and in 2 mm thickness aluminium plate (thick solid line).

4. Conclusions
Computational models aiming at describing the influence of thin interlayers between two solids with possible contact imperfections are proposed. They are essential for predictive simulations in ultrasonic based non-destructive and structural health monitoring of adhesively bonded
constructions in service to ensure their reliability and safety. Within these models, the effect of the internal thin polymeric layer and imperfect interfaces between sub-layers in a laminate composite structure is analysed revealing their non-neglectable influence on dispersion properties of propagating Lamb waves, which, in turn, is verified experimentally with laser Doppler vibrometry measurements of piezoelectric induced LWs.

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References

[1] Tamborrino R, Palumbo D, Galietti U, Aversa P, Chiozzi S and Luprano V 2016 Compos. Part B-Eng. 91 337–45
[2] Attar L, Leduc D, Ech Cherif El Kettani M, Predoi M, Galy J and Pareige P 2020 NDT&E Int. 111 102213
[3] Yilmaz B and Jasiuniene E 2020 Int. J. Adhes. Adhes. 102 102675
[4] Jiao D and Rose J L 1991 J. Adhes. Sci. Tech. 5 631–46
[5] Siryabe E, Renier M, Meziane A, Galy J and Castaings M 2017 Ultrasonics 79 34–51
[6] Rucka M, Wojtczak E and Lachowicz J 2018 Appl. Sci. 8 522
[7] Dahmen S 2019 Ultrasonics 94 37–49
[8] Rokhlin S I and Wang Y J 1991 J. Acoust. Soc. Am. 89 503–15
[9] Boström A, Bövik P and Olsson P 1992 J. Nondestruct. Eval. 11 175–84
[10] Baik J M and Thompson R B 1984 J. Nondestruct. Eval. 4 177–96
[11] Margetan F, Thompson R, Rose J and Gray T 1992 J. Nondestruct. Eval. 11 109–26
[12] Mal A 1988 Int. J. Eng. Sci. 26 873–81
[13] Balvaintin A, Baltazar A and Aranda-Sanchez J I 2012 Int. J. Mech. Sci. 63 66–73
[14] Golub M V and Boström A 2011 Wave Motion 48(2) 105–15
[15] Golub M V and Doroshenko O V 2019 Int. J. Sol. Struct. 165 115–26
[16] Golub M V and Doroshenko O V 2020 Eur. J. Mech. A-Sol. 81 103894
[17] Krenk S and Schmidt H 1982 Philos. Trans. R. Soc. A 308 167–98
[18] Glushkov Y V and Glushkova N V 1996 J. Appl. Math. Mech. 60 277–83
[19] Glushkov E V and Glushkova N V 2001 J. Comp. Acoust. 9(3) 889–98
[20] Wilde M V, Golub M V and Eremin A A 2019 Ultrasonics 98 88–93