Research Article

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A Galerkin strategy for tri-hybridized mixture in ethylene glycol comprising variable diffusion and thermal conductivity using non-Fourier’s theory

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Abstract: This research is conducted to investigate heat and mass transport past over a stretched surface having pores in a pseudo-plastic model. To study porosity effect, Darcy Forchheimer relation is used. Thermal and mass transport expressions are derived by engaging the double diffusion theories as extensively used by researchers proposed by Cattaneo and Christov. Furthermore, the thermal performance is studied by mixing the tri-hybrid nanoparticles in a pseudo-plastic material. The phenomenon of boundary layer is used to derive the complex model. The correlation for tri-hybrid nanoparticles is used to convert the model partial differential equations into ordinary differential equations (ODE) along with appropriate similarity transformation. The transfigured ODEs are coupled nonlinear in nature, and the exact solution is not possible. To approximate the solution numerically, finite element scheme (FES) is used and code is developed in MAPLE 18.0 for the graphical results, grid independent survey, and tabular results. The obtained results are compared with the published findings that confirm the accuracy and authenticity of the solution and engaged scheme. From the performed analysis, it is concluded that FES can be applied to complex engineering problems. Furthermore, it is monitored that nanoparticles are essential to boost the thermal performance and higher estimation of Schmidt number control the mass diffusion.

Keywords: Darcy’s Forchheimer medium, variable diffusion and thermal conductivity, generalized diffusion models, grid independent investigation, finite element scheme

1 Introduction

Fluid flows past over a stretched and rotated surfaces have gained much attention of the researchers due to their wider applications. Diffusion theories play a vital role to control heat and mass transportation in fluid flows. Researchers have much attraction to study physical problems discussing thermal transport. To keep thermal stability and maintain high thermal performance, mixing of nanoparticles is essential, and it boosts the heat transfer in fluid flows. Several models of nanofluids have been proposed and extensively used. Buongiorno’s model discusses the involvement of Brownian motion and thermophoresis, whereas the single phase model contains the involvement of volume fraction and also researchers have addressed numerous relations that present the involvement of particle shapes. Several remarkable attempts have been made to discuss the utilization of nanoparticles in fluid flows. For instance, Koriko et al. [1] discussed the growth of energy transfer inserting nanoparticles along with bio-convection flow in the presence of magnetic field and gyrotactic microorganisms towards a vertical surface. Ali et al. [2] studied the role of hybrid nanoparticles in the energy transfer under the action of viscous dissipation.
Saleem and Heidarshenas [3] investigated the study of various nanoparticles in wavy wall in view of heat transfer phenomena. Pop et al. [4] discussed the role of magneto-hydrodynamic in stagnation point flow along with hybrid nanoparticles considering melting aspects. Tili et al. [5] estimated features of nanoparticles in an unsteady flow using various shapes of nanoparticles considering Oldroyd-B ferrofluid under the action of magnetic field. Qin [6] estimated the heat transfer features including the role of nanostructures in chamber. Nawaz et al. [7] discussed the comparative analysis among hybrid nanoparticles and nanoparticles into dust particles considering hyperbolic tangent liquid adopting the finite element approach. Nazir et al. [8] used the finite element approach to analyse comparative investigation between hybrid nanostructures and nanostructure in base liquid called ethylene glycol in Carreau Yasuda liquid. Nazir et al. [9] used non-Fourier’s law in rheology of hyperbolic tangent liquid inserting vital impact of hybrid nanostructures towards a stretching surface. Nazir et al. [10] discussed the thermal character of hybrid nanostructures in hyperbolic tangent rheology in the base fluid (ethylene glycol) via finite element approach. Sheikholeslami and Farshad [11] concluded thermal aspects of the solar collector system inserting hybrid nanostructures. Swain et al. [12] used an exponentially surface to analyse a role of hybrid nanostructures in the presence of chemical reaction via slip conditions. Faraji et al. [13] developed a model related to phase change liquid in view of hybrid nanomaterials along with heat source in a rectangular enclosure. Abdelmalek et al. [14] investigated an inclination into heat energy using nanoparticles and hybrid nanostructures in view of base fluid in wedge. They have used finite element scheme (FES) to know comparison among hybrid nanostructures and nanostructures including chemical reaction into solute particles. Nawaz et al. [15] analysed rheology of power law liquid into solute particles and thermal energy inserting an impact of hybrid nanoparticles over a heated surface via finite element approach. Latocha et al. [16] estimated prediction of hydroxyapatite in nanoparticles towards 3D-printed reactors. Haider et al. [17] discussed the role of hybrid nanoparticles in Williamson liquid using heat flux model. They adopted heated surface to captures comparison performance of hybrid nanoparticles and hybrid nanoparticles in ethylene glycol. Nazir et al. [18] discussed features of thermal energy using nanoparticles towards a surface in Casson liquid. Dadheech et al. [19] analysed the comparison of heat energy performance among SiO₂–MoS₂ hybrid nanofluid and nanofluid (MoS₂) in ethylene glycol under the presence of magnetic field. Shafiq et al. [20] discussed the heat energy features in Walters’ B’ nanofluid over a Riga plate via statistical approach. Marzougui et al. [21] used the lid-driven cavity to capture the impacts of entropy generation and thermal transfer in the attendance of non-variable magnetic field. Pushpa et al. [22] captured flow and convective thermal transfer inserting nanoparticles in thin baffle. Dhif et al. [23] studied the performance of hybrid nanoparticles in a solar collector system. Abdelsalam et al. [24] discussed the study related to Rabinowitsch suspension in the presence of leveraging elasticity in a wavelike conduit. Abdelsalam et al. [25] conducted results of swimming sperms in the presence of electro-magnetically using a wavelike conduit. Raza et al. [26] adopted curved surface to achieve consequences in Williamson liquid under thermal radiation. Eldesoky et al. [27] integrated thermal features into conjunction along with slip conditions in view of peristaltic motion using a catheterized pipe. Bhatti and Abdelsalam [28] captured peristaltic motion adding hybrid nanoparticles along with magnetic field. Elkoumy et al. [29] performed the role of Maxwell liquid in terms of peristaltic motion inserting magnetic field and Hall force. Bhatti et al. [30] simulated the heat transfer based on intra-uterine motion in channel. Some important contributions covering the modelling aspects are reported in earlier studies [31–36].

Current model is developed using the rheology of pseudo-plastic material along with the presence of tri-hybrid nanoparticles. The theories regarding non-Fourier’s and Darcy’s Forchheimer within heat generation and chemical reaction are investigated. FES is considered to find numerical simulations. Investigations regarding current model is not achieved yet.

Figure 1: Prepared approach regarding mixtures of ternary hybrid nanoparticles.
Section 1 presents literature survey; Section 2 presents modelling; Section 3 contains methodology procedure and mesh-free investigations; and Section 4 reports detailed discussion and description of obtained solution against numerous involved parameters, and Section 5 lists important findings. Developing approach of tri-hybrid nanoparticles is shown in Figure 1.

2 Flow analysis of tri-hybrid nanoparticles

The flow of tri-hybrid nanoparticles in pseudo-plastic liquid towards the heated surface is addressed. Solute particles and heat energy of particles are analysed considering the theory of Cattaneo–Christov model (CCM). Processes of chemical reaction and heat generation are implemented in the presence of CCM. The concept of Forchheimer porous is modelled in the momentum equation. Layers associated with momentum and thermal are generated because of stretching of plate. Composite relation among nanofluid, nanoparticles, hybrid nanomaterials, and tri-hybrid nanoparticles is considered in equations (12)–(14). Physical configuration of current model is captured in Figure 2 (Table 1).

System of formulated partial differential equations [33–35] is as follows:

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{1}
\]

\[
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \nu_{\text{Thnf}} \frac{\partial}{\partial y} \left( \frac{\partial U}{\partial y} - \frac{1}{\rho C_p \text{Thnf}} \frac{\partial T}{\partial y} \right) - \frac{\nu_{\text{Thnf}}}{k^2} F_D U - \frac{F_n}{(k^2)\nu^2}, \tag{2}
\]

\[
U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{1}{(\rho C_p \text{Thnf})} \frac{\partial}{\partial y} \left( k_{\text{Thnf}} \frac{\partial T}{\partial y} \right) + \frac{Q}{(\rho C_p \text{Thnf})} (T - T_{\infty}), \tag{3}
\]

\[
U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} + D_{\text{Thnf}} \frac{\partial C}{\partial y} = \frac{\partial}{\partial y} \left( D_{\text{Thnf}} \frac{\partial C}{\partial y} \right) + K_a (C - C_{\infty}). \tag{4}
\]

Where heat generation is \( Q \), temperature is \( T \), thermal conductivity is \( k \), fluid density is \( \rho \), velocities are \( (U, V) \), time relaxation is \( \lambda_t \), specific heat capacitance is denoted by \( C_p \), kinematic viscosity is \( \nu \), power law number is \( m \), space coordinates are \( y, x \), tri-hybrid nanoparticles are represented by Thnf, ambient temperature is \( T_{\infty} \), permeability within porous medium is \( k \), inertia coefficient in term of porous medium is \( F_D \), \( C \) is concentration, ambient concentration is \( C_{\infty} \), chemical reaction \( K_a \), and mass diffusion is \( D \). No-slip theory is used to generate boundary conditions (BCs) and required conditions are as follows:

\[
U = u_w, \quad V = -v_w, \quad C = c_w, \quad T = T_w \text{ at } y = 0,
\]

\[
U \rightarrow u_{\infty}, \quad C \rightarrow C_{\infty}, \quad T \rightarrow T_{\infty} \text{ when } y \rightarrow \infty. \tag{5}
\]

Transformations of model are as follows:
Variable thermal conductivity and variable mass diffusion in view of tri-hybrid nanoparticles [36] are defined as follows:

\[ k_{T_{\text{Thnf}}}^l = k_{T_{\text{Thnf}}} \left[ 1 + \varepsilon_l \left( \frac{T - T_{\text{co}}}{T_w - T_{\text{co}}} \right) \right], \]

\[ D_{T_{\text{Thnf}}}^l = D_{T_{\text{Thnf}}} \left[ 1 + \varepsilon_l \left( \frac{T - T_{\text{co}}}{T_w - T_{\text{co}}} \right) \right]. \]

Transformations deliver system of ODEs and non-linear ODEs are as follows:

\[ (|F|^m-1F') + \frac{1}{m + 1}F^mF - \varepsilon F' - \frac{v}{v_{T_{\text{Thnf}}}}F_p(F^{1/2}) = 0, \]  

Properties associated with thermal for correlations among tri-hybrid nanoparticles are as follows:

\[ \rho_{T_{\text{Thnf}}} = (1 - \varphi_f)\rho_l[(1 - \varphi_3)\rho_3 + \varphi_2\rho_3] + \varphi_1, \]

\[ \mu_l = \frac{(1 - \varphi_f)^2(1 - \varphi_3)\rho_l + \varphi_2\rho_3}{\rho_{T_{\text{Thnf}}} - \rho_3 - \rho_f}, \]

\[ K_{T_{\text{Thnf}}} = K_l + 2K_{nf} - 2\varphi_f(K_{Thnf} - K_l), \]

\[ K_{T_{\text{Thnf}}} = K_l + 2K_{Thnf} - 2\varphi_f(K_{Thnf} - K_l), \]  

\[ K_{T_{Thnf}} = K_l + 2K_{Thnf} - 2\varphi_f(K_{Thnf} - K_l), \] 

\[ K_{T_{Thnf}} = K_l + 2K_{Thnf} - 2\varphi_f(K_{Thnf} - K_l). \]
The formulated skin friction coefficient is defined as follows:

$$ C_f = \frac{2(\tau_w)}{U \rho_f}, \quad (17) $$

The temperature gradient is modelled as follows:

$$ \frac{1}{(Re)^{\frac{1}{2}}} \int C_f = - \frac{(1 - \phi_2)^{-2.5}}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}} [F'/(0)|F''/(0)|^{m-1}]. \quad (18) $$

The rate of mass diffusion is as follows:

$$ S_c = \frac{x M_w}{(C_w - C_o) D_f}, \quad (Re)^{\frac{1}{2}} S_c = - \frac{(1 - \phi_2)^{-2.5}}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}} \phi'(0). \quad (20) $$

The formulation of the weak form is imposed to obtain weak form in view of the shape functions. The residuals are as follows:

$$ \int_{Q_1} \left[ (1 + \epsilon \theta)\theta'' + \epsilon \theta'(\theta')^2 + \frac{Pr}{m+1} F' F' + \frac{k_l}{k_{hf}} H_0 Pr \theta \right] d\xi = 0, \quad (21) $$

$$ \int_{Q_2} \left[ \frac{k_l}{k_{hf}} H_0 Pr \Omega_a [F'\theta' + F'\theta'' + H_0 F\theta'] \right] d\xi = 0, \quad (23) $$

$$ \int_{Q_3} \left[ - \frac{(1 - \phi_2)^{-2.5} S_c}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}} K_c \phi + \frac{(1 - \phi_1)^{-2.5} S_c}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}} F\phi' \right] d\xi = 0. \quad (24) $$

Here, $Q_1$, $Q_2$, $Q_3$, and $Q_4$ are named as weight functions. The shape functions are as follows:

$$ \pi_i = (-1)^{i-1} \left( \frac{-\xi + \xi_{i-1}}{-\xi + \xi_{i+1}} \right), \quad i = 1, 2. \quad (25) $$

**Step I:** An approach associated with Galerkin finite element is imposed to obtain weak form in view of shape functions.

**Step II:** The assembly approach is utilized for the development of stiffness element, whereas assembly approach is performed via assembly procedure of FEA. Stiffness elements are as follows:

$$ K_{11}^{ij} = \int_{Q_j} \left[ \frac{d\pi_i}{d\eta} \right] d\xi, \quad K_{12}^{ij} = 0, \quad K_{13}^{ij} = 0, \quad K_{14}^{ij} = B_i^1 = 0, \quad (26) $$

$$ K_{22}^{ij} = \int_{Q_j} \left[ \left| S \right|^{m-1} \frac{d\pi_i}{d\eta} \right] d\xi, \quad (27) $$

Reynolds number is $Re = \left( \frac{v^{m-1} - \eta}{\nu} \right)$.  

### 3 Numerical scheme

FES is utilized to simulate numerical results. FES is very capable of simulating CFD problems. Detail steps are described below.

**Step I:** Equations (6) and (7) within BCs are called the strong form. It is noticed that collecting all terms of equations (6) and (7) on one side and integrating it over each elements of domain are residuals. Such procedure is known as weighted (residual method) for the development of weak forms. The residuals are as follows:

$$ \int_{Q_{i+1}} Q_i (F' - S) d\xi = 0, \quad (21) $$

$$ \int_{Q_2} \left[ (|F'|^{m-1} F') + \frac{1}{m+1} F'' F - \epsilon F' \right] d\xi = 0, \quad (22) $$

$$ \int_{Q_3} \left[ \frac{k_l}{k_{hf}} H_0 Pr \Omega_a [F'\theta' + F'\theta'' + H_0 F\theta'] \right] d\xi = 0, \quad (23) $$

$$ \int_{Q_4} \left[ - \frac{(1 - \phi_2)^{-2.5} S_c}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}} K_c \phi + \frac{(1 - \phi_1)^{-2.5} S_c}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}} F\phi' \right] d\xi = 0. \quad (24) $$

Here, $Q_1$, $Q_2$, $Q_3$, and $Q_4$ are named as weight functions. The shape functions are as follows:

$$ \pi_i = (-1)^{i-1} \left( \frac{-\xi + \xi_{i-1}}{-\xi + \xi_{i+1}} \right), \quad i = 1, 2. \quad (25) $$

**Step II:** An approach associated with Galerkin finite element is imposed to obtain weak form in view of shape functions.

**Step III:** The assembly approach is utilized for the development of stiffness element, whereas assembly approach is performed via assembly procedure of FEA. Stiffness elements are as follows:

$$ K_{11}^{ij} = \int_{Q_j} \left[ \frac{d\pi_i}{d\eta} \right] d\xi, \quad K_{12}^{ij} = 0, \quad K_{13}^{ij} = 0, \quad K_{14}^{ij} = B_i^1 = 0, \quad (26) $$

$$ K_{22}^{ij} = \int_{Q_j} \left[ \left| S \right|^{m-1} \frac{d\pi_i}{d\eta} \right] d\xi, \quad (27) $$

Reynolds number is $Re = \left( \frac{v^{m-1} - \eta}{\nu} \right)$.
Characteristics related to solute and thermal are established inserting comprehensive role of tri-hybrid nanoparticles via Darcy’s Forchheimer law. Cattaneo–Christov law is used along with the dual role of thermal energy based on phenomena generation and absorption of heat energy. Diffusion of mass species is manufacturing in the presence non-Fourier’s theory. Base fluid (ethylene glycol) is considered along with pseudo-plastic liquid. A strong approach is called finite element method to visualize graphical impacts of various parameters on concentration, velocity, and temperature profiles. Further, coding of finite element approach is designed by MAPLE 18 software. Current code is developed to simulate complex ODEs along with boundary conditions. Detail outcomes regarding the measurement of velocity, concentration, and thermal energy are mentioned below.

3.1 Validation of results

The comparison among present analysis and published results [33–35] are derived in Table 3. It is noticed that the present problem is reduced into published problems [33–35] by considering \( \varphi_a = \varphi_b = \varphi_c = 0, \) \( F_r = \varepsilon = 0. \) A good agreement between the results of the present problem and published works is noticed.

4 Graphical discussion and outcomes

Characteristics related to solute and thermal are established inserting comprehensive role of tri-hybrid nanoparticles via Darcy’s Forchheimer law. Cattaneo–Christov
particles. Hence, the frictional force makes a reduction into motion among fluid layers. Hence, inverse proportional relation is existed among flow and \( \epsilon \). Impact of \( F_r \) on velocity curves is measured by Figure 3b, inserting the role of ternary hybrid nanoparticles. Appearance of \( F_r \) is model out due to impact of Darcy’s Forchheimer law. A declination role into motion of particles is investigated when \( F_r \) is enhanced. Layers associated with the momentum boundary are investigated decreasing function against impact of \( F_r \) model related to Darcy–Forchheimer is based on Darcian motion into fluid particles. Mathematically, it appeared as a velocity squared in motion equations. A retardation motion is developed due to appearance of Forchheimer. Momentum layers have a decreasing function versus impact of \( F_r \). Hence, fluid is termed as thick when \( F_r \) is increased. An effect of \( m \) on velocity curves is established in Figure 3c. It is estimated that the description of \( m \) is modelled out due to inserting impact of pseudo-plastic liquid. Moreover, the category of shear thinning, shear thickening, and Newtonian liquid is based on values \( m \). For \( m < 1 \), fluid appeared as shear thickening among fluid particles. It is noticed that layers along with momentum layers are declined when \( m \) is increased.

Table 2: Mesh-free study for temperature gradient, concentration gradient, and Sherwood number via 300 elements

| Number of elements | \( F_r(\frac{\theta_{\max}}{2}) \) | \( \theta(\frac{\theta_{\max}}{2}) \) | \( \phi(\frac{\phi_{\max}}{2}) \) |
|-------------------|------------------|------------------|------------------|
| 30                | 0.006299584904   | 0.5916150941     | 0.4046811155     |
| 60                | 0.00477218361    | 0.5635673948     | 0.6701679329     |
| 90                | 0.00433296367    | 0.5530486985     | 0.3494690039     |
| 120               | 0.004125211694   | 0.5475247702     | 0.3450328235     |
| 150               | 0.004004033797   | 0.544183135      | 0.3440517986     |
| 180               | 0.003924751415   | 0.5418082729     | 0.3448555416     |
| 210               | 0.003868847656   | 0.5401363276     | 0.3436001995     |
| 240               | 0.00382731579    | 0.538730488      | 0.3492091727     |
| 270               | 0.003795243660   | 0.5378847796     | 0.3522432276     |
| 300               | 0.003769731588   | 0.5370874271     | 0.3556147229     |

Table 3: Validation of numerical results for skin friction coefficient

| Skin friction coefficient | Present work skin friction coefficient |
|---------------------------|----------------------------------------|
| Sakiadis [33]             | -0.44375                               | -0.446169          |
| Fox et al. [34]           | -0.4437                                | -0.442930          |
| Chen [35]                 | -0.4438                                | -0.443731          |

Figure 3: (a) Role of \( \epsilon \) on velocity curves. (b) Role of \( F_r \) on velocity curves. (c) Role of \( m \) on velocity curves.
4.2 Analysis of temperature curves against physical parameters

Thermal aspects including ternary hybrid nanoparticles are observed against variation in $H_h$, $\epsilon_1$, and $\Omega_a$. These thermal aspects are measured with respect to change in $H_h$, $\epsilon_1$, and $\Omega_a$ by Figure 4a–c. The impact of variable thermal conductivity ($\epsilon_1$) on temperature curves is examined by Figure 4a. It is noticed that thermal energy of particles is boosted when $\epsilon_1$ is increased. The parameter related to $\epsilon_1$ is produced because of variable thermal conductivity. Mathematically, $\epsilon_1$ has a relationship related to direct proportional temperature. Hence, an increment into thermal energy is based on distribution in $\epsilon_1$. Moreover, layers of thermal at boundary are increased versus higher values of $\epsilon_1$. Physically, $\epsilon_1$ has a directly proportional relation versus temperature difference. Hence, an increment in $\epsilon_1$ results in a significant ability of thermal energy to conduct more heat energy into fluid particles is observed. The ability to conduct temperature into hybrid and nanoparticles is also increased when $\epsilon_1$ is increased.

Figure 4b demonstrates influence of $H_h$ on temperature curves. Dual role of heat energy is visualized on aspects of thermal energy while negative values of $H_h$ are due to the role of heat absorption and positive values of $H_h$ are due to the role of heat energy generation. Production into heat energy of particles is inclined into nanoparticles and tri-hybrid nanoparticles based on variation in $H_h$. Mathematically, $H_h$ appeared in energy equation, which is a product of temperature difference. Basically, it occurred because of an external heat source. Thermal energy is boosted when an external heat source is placed at the wall. Boundary layers based on the thermal impact are increasing behaviour. This increasing effect of thermal energy is produced due to applying an external source of heat energy, which is placed at the wall of surface. Hence, the heat energy is established according to values of $H_h$. The prediction of heat energy against distribution in $\Omega_a$ is addressed by Figure 4c. Argumentation into heat energy is measured when $\Omega_a$ is increased. The parameter is called $\Omega_a$ is made because of non-Fourier’s theory. Non-Fourier’s theory is implemented in current analysis. Hence, heat energy is boosted when $\Omega_a$ is increased. An increasing function is investigated among $\Omega_a$ and thermal layers. Moreover, fluid particles take more time to restore more heat energy when $\Omega_a$ is increased. The layers associated within thermal are increasing inserting higher values of $\Omega_a$. Figure 5 is observed as a vital impact on comparison of tri-hybrid nanoparticles, nanoparticles,

![Figure 4: (a) Role of $\epsilon_1$ on temperature curves. (b) Role of $H_h$ on temperature curves. (c) Role of $\Omega_a$ on temperature curves.](image-url)
4.3 Analysis of concentration curves against physical parameters

Diffusion of solute particles is observed versus impacts of $K_c$, $\Omega_c$, $Sc$, and $\varepsilon_2$ considering Figure 6a–d. Figure 6a illustrates that the variation into thermal energy is verified against chemical reaction number ($K_c$). In this figure,

Figure 5: Comparison study among fluid, nanoparticles, hybrid nanoparticles and tri-hybrid nanoparticles.

Figure 6: (a) Role of $K_c$ on concentration curves. (b) Role of $Sc$ on concentration curves. (c) Role of $\Omega_c$ on concentration curves. (d) Role of $\varepsilon_2$ on concentration curves.
Table 4: Numerical simulations of skin friction coefficient, concentration gradient and temperature gradient versus $F_r$, $H_h$, $Sc$, and $K_c$

| $(Re)^{1/7} C_f$ | $(Re)^{1/7} Nu$ | $(Re)^{1/7} Sc$ |
|------------------|------------------|------------------|
| 0.0              | 0.9831194688     | 1.723048742      | 0.1738269697 |
| $F_r$            | 0.3              | 0.994870276      | 1.63799787   | 0.1636186839 |
| 0.6              | 0.5              | 1.0055424430     | 1.244665792  | 0.1434299640 |
| $-1.5$           | 1.048480792      | 2.9889080937     | 0.1582282354 |
| $H_h$            | 0.3              | 1.021340103      | 2.184136239  | 0.1468662589 |
| 0.7              | 1.010480792      | 2.110134671      | 0.1380771772 |
| $-1.4$           | 1.018480792      | 1.275290361      | 0.1059774514 |
| $Sc$             | 0.3              | 1.018480792      | 1.275290361  | 0.2836633481 |
| 0.6              | 1.018480792      | 1.275290361      | 0.6105156610 |
| $-1.4$           | 1.018480792      | 1.275290361      | 0.0831204767 |
| $K_c$            | 0.0              | 1.018480792      | 1.310199330  | 0.0912766237 |
| 0.5              | 1.018480792      | 1.501284349      | 0.2961188108 |

4.4 Analysis of Sherwood number, temperature gradient, and drag force coefficient

Temperature gradient, Sherwood number, and drag force coefficient are simulated by incorporating influences of Forchheimer number, heat generation number, chemical reaction number, and $Sc$ including the study of ternary hybrid nanoparticles. Impacts related to these parameters on temperature gradient, Sherwood number, and drag force coefficient are simulated as shown in Table 4. Role of $F_r$ boosts surface force but the concentration and temperature gradients are declined when $F_r$ is increased. $H_h$ diminishes the rate of thermal energy, mass diffusion, and surface force. It is estimated that the dual trends (heat absorption and heat generation) are addressed. In both cases for $H_h$ are observed significantly for establish maximum production in rate of concentration and temperature gradients. A declination is predicted in view of surface force versus an impact of chemical reaction (generative and destructive) but in terms of Sherwood and Nusselt, numbers are inclined. For the case of $Sc$, rates of mass diffusion and thermal energy are significantly increased.

5 Conclusion

Solute and thermal characterizations in pseudo-plastic liquid are investigated along with the ternary hybrid nanoparticles. An expanding surface is considered to measure the flow, thermal energy, and solute particles into particles against the distribution of various physical parameters. Darcy’s Forchheimer law is incorporated along with non-Fourier’s law in the presence of heat generation and heat absorption. A term related to chemical reaction is also considered into solute particles. The finite element approach has used to characterize graphical outcomes. Main findings are addressed below.

- Flow slows down versus higher values of Darcy’s number, power law number, and Forchheimer number;
- Argumentation is investigated into heat energy of particles for the case of ternary hybrid nanoparticles rather than for case of fluid, nanoparticles, and hybrid nanoparticles.
- Maximum achievement of heat energy for tri-hybrid nanoparticles as compared than nanofluid and hybrid nanoparticles.
- Diffusion of species slows down when $Sc$ is enhanced.
- Applications of ternary hybrid nanoparticles based on present problem are utilized in medicines, cancer cells,
memory devices, materials, electronics, systems, and devices.

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