THE MACHO PROJECT LARGE MAGELLANIC CLOUD VARIABLE STAR INVENTORY. III. MULTIMODE RR LYRAE STARS, DISTANCE TO THE LARGE MAGELLANIC CLOUD, AND AGE OF THE OLDEST STARS

C. Alcock,1,2 R. A. Allsman,3 D. Alves,1,2 T. S. Axelrod,4 A. C. Becker,5 D. P. Bennett,1,2 K. H. Cook,1,2 K. C. Freeman,4 K. Griest,2,6 J. Guern,2,6 M. J. Lehner,2,5 S. L. Marshall,1,2 D. Minniti,1,2 B. A. Peterson,1,2 M. R. Pratt,2,5 P. J. Quinn,4,8 A. W. Rodgers,4 W. Sutherland,9 and D. L. Welch10
(The MACHO Collaboration)

Received 1996 August 16; accepted 1997 January 8

ABSTRACT

We report the discovery of 73 double-mode RR Lyrae (RRd) stars in fields near the bar of the Large Magellanic Cloud (LMC). The stars are detected among the MACHO database of short-period variables that currently contains about 7900 RR Lyrae stars. Fundamental periods (Pf) for these stars are found in the range 0.46–0.55 days, and first-overtone–fundamental period ratios are found to be in the range 0.742 < P1/Pf < 0.748. The range in period ratios is unexpectedly large, and a significant fraction of our current sample has period ratios smaller than any previously discovered RRd variables. We present mean magnitudes, colors, and light-curve properties for all LMC RRd stars detected to date. We undertake a determination of the absolute magnitudes for these stars based primarily on pulsation theory and on the assumption that all observed stars are at the fundamental blue edge of the instability strip. Comparison of the calibrated MACHO V and Rc photometry with these derived absolute magnitudes yields an absorption-corrected distance modulus to the LMC of 18.48 ± 0.19 mag that is in good agreement with that found (18.5) through comparison of Galactic and LMC Cepheids.

Exploring this luminosity calibration, we derive an increase in the distance modulus, and thus a reduction in the age found via isochrone fitting, for M15 of about 30% and discuss some implications for cosmology.

Subject headings: galaxies: distances and redshifts — Magellanic Clouds — stars: oscillations — stars: variables: other (RR Lyrae)

1. INTRODUCTION

Multimode RR Lyrae stars, also known as RRd-type stars, are especially interesting tests of pulsation and evolutionary theory because precise period ratios allow an estimate of the mass of each star. They also can be examined to indicate systematic differences in the properties of the horizontal branch (HB) between and within stellar systems, e.g., the Oosterhoff groups of Galactic globular clusters are differentiated in their period ratio properties. In principle, the masses can be compared with structural and evolutionary models of HB stars, and furthermore the masses can be used in the derivation of pulsation-based luminosities of HB stars in clusters containing multimode RR Lyrae stars. This paper reports properties of a total of 73 such stars in the Large Magellanic Cloud (LMC), nearly twice as large as the number of all known RRd stars prior to this work.

The MACHO Project LMC RR Lyrae database is described in Alcock et al. (1996), where accumulated V and R photometry for approximately 7900 RR Lyrae stars in 22 intensively observed fields has been analyzed to determine the distributions of period and amplitude. In that paper, a random selection of 500 light curves was examined, and two multimode stars were identified. The ratios of first-overtone–fundamental period, P1/Pf, of those stars were found to be lower than in any previously discovered RRd stars.

This paper reports the discovery of an additional 71 RRd stars. We first discuss the observational properties of this new sample. Next, we estimate absolute magnitudes using pulsation theory. Finally, we note implications of the derived luminosities for cosmology.

2. RESULTS

2.1. Photometry

The photometric reduction procedures for the MACHO images are described in Alcock et al. (1996). The MACHO instrumental magnitudes have been placed on the standard
photometric system of Kron-Cousins $V$ and $R$ through comparison with Landolt (1992) standard stars that were transferred to the 22 high-priority MACHO fields. While gross zero-point differences between fields have been removed, small offsets within each field exist as an artifact of the MACHO photometry reduction procedure. We conservatively adopt an error in $V$ or $R$ of $\pm 0.10$ mag. The transformation between MACHO instrumental color and Kron-Cousins ($V-R$) is less affected by the internal zero-point offsets, and therefore the color for each star in Table 1 has an estimated uncertainty of $\pm 0.05$ mag. We arrive at these estimates of the photometric uncertainties through numerous consistency checks within the MACHO database and by comparisons with published photometry of LMC stars.

The light curves used in this analysis represent all data acquired for these stars during an interval of approximately 1400 days and typically represent 400–700 points per color. In Figure 1, we show the mean apparent $V$ and $(V-R)$ for LMC double-mode RR Lyrae stars and illustrate their position relative to other LMC field stars. The light curves for these stars are available on the MACHO project home pages (http://www.macho.anu.edu.au/ and http://www.macho.mcmaster.ca/). Improved calibrations for the MACHO photometry are a matter of work in progress; fortunately, due to the large number of RRd stars discovered, the individual photometric uncertainties do not affect any of the conclusions drawn in this paper. As discussed further on in §2.4.4, we find our mean photometry to be in excellent agreement with published photometry of LMC RR Lyrae stars.

### 2.2. Selection Criteria

Double-mode RR Lyrae candidates were selected from our color-magnitude diagram–based RR Lyrae sample by two techniques. The first, described in Alcock et al. (1996), relied on visual examination of 500 light curves using the IRAF11 PDM task. More recently, candidate RRd stars were selected based on their measure of fit to a single periodic light curve and their period and light-curve amplitude.

The strong period dependence of our selection criteria deserves mention. For photometric periods between 0.35 and 0.40 days, corresponding to fundamental periods between 0.46 and 0.50 days, our selection criteria identify double-mode RR Lyrae stars very efficiently. Longward of this range, our yield is reduced by the increase in false positives with period estimates of one-half of a day. The measure-of-fit statistic is also less effective at long periods because of the presence of Blazhko variation in some RRab stars.

| $P_0$ (days) | $P_0^*$ (days) | $\Delta t^*$ (mag) | $\Delta t^*$ (mag) |
|-------------|----------------|-------------------|-------------------|
| 0.008       | 0.011          | 0.08              | 0.08              |
| 0.011       | 0.014          | 0.11              | 0.11              |
| 0.014       | 0.017          | 0.11              | 0.11              |
| 0.017       | 0.020          | 0.11              | 0.11              |
| 0.020       | 0.023          | 0.11              | 0.11              |

**Table 1:** Properties of LMC Double-Mode RR Lyrae Stars

---

11 Developed at the National Optical Astronomical Observatories.
pulsators. Since, in Alcock et al. (1996), we found the shortest period fundamental pulsators were of $P = 0.457$ days, we might not expect to find multimode stars with fundamental periods shorter than this. Accordingly, we note that our current sample is not necessarily complete and that the distribution in period (and therefore mass) is possibly unrepresentative.

2.3. Observed Properties of the Sample

We summarize our discovery of LMC RRd stars in Table 1. This table lists, from left to right, the equinox J2000.0 right ascension and declination, the internal database identifier, the number of measurements and mean magnitude in $V$ and $R$, respectively, the estimate of the fundamental period $P_0$ (in days) and the period ratio, $P_1/P_0$, the semi-amplitude of the Fourier component for the frequency corresponding to $P_1$ (in magnitudes), and the ratio of the amplitudes for the first Fourier components of the first-overtone and fundamental modes. We estimate the combined systematic and individual errors of the positions to be $\pm 0.005$.

The periods of the RR Lyrae stars were initially estimated using the IRAF PDM task as described in Alcock et al. (1996). The periods, period ratios, and modal amplitudes reported in Table 1 are the result of a weighted least-squares fit to the RRd light curves for the first three Fourier terms in each mode and all coupling terms.

We note that the selection criteria used above are biased against stars with fundamental periods shorter than 0.46 days, recalling that there appear to be no type of RR Lyrae stars in the LMC field with $P < 0.457$ days and that amplitude ratios smaller than unity may therefore be underrepresented or do not occur.

2.4. Derived Properties of the LMC Multimode Stars

In theory, it is possible to estimate absolute magnitudes of RRd stars by combining the Stefan-Boltzmann equation and the period-density relations. The result of such an exercise is an expression relating period to luminosity, effective temperature, and mass. Unlike singly periodic RR Lyrae stars, the period ratio allows us to estimate the mass based on atmospheric pulsation models. Provided that we can determine the effective temperature, the set of required geometric parameters, and the period-density relations. The result of such an exercise is an expression relating period to luminosity, effective temperature, and mass. Unlike singly periodic RR Lyrae stars, the period ratio allows us to estimate the mass based on atmospheric pulsation models.
the first-overtone and fundamental modes, $\Delta_1/\Delta_0$, are plotted against the fundamental mode period $P_0$. Almost without exception, the amplitude of the first-overtone mode is greater than that of the fundamental—a property that this sample shares with previously discovered RRd stars in globular clusters and dwarf spheroidal galaxies. We caution, however, that our sample is incomplete because of period-dependent selection biases and that more RRd stars with $\Delta_1/\Delta_0 < 1$ may yet be discovered in our LMC sample.

In Figure 2, we show the diagram of $P_1/P_0$ plotted against $P_0$ for stars in M15 (Nemec 1985b), Draco (Nemec 1985a), IC 4499 (Clement et al. 1986; Walker & Nemec 1996), M68 (Walker 1994), M3 (Nemec & Clement 1989), and the LMC. We see in Figure 2 that most of the detected multimode stars in the LMC field have $P_1/P_0$ less than 0.7445, i.e., lower than any of their Galactic counterparts. Petersen (1973) and, later, Cox (1991, 1995) and Bono et al. (1996a) have calculated model pulsating envelopes of differing mass, composition, opacities, and modeling strategies to derive the dependence of the relation between the periods of each mode as a function of these parameters. Such diagrams relating $P_1/P_0$ to $P_0$ have become known as Petersen diagrams.

The detailed discussions by Cox (1995) and Bono et al. (1996b) show that the assignment of a mass to a multimode star in the Petersen diagram is dependent on the computational approximations made, e.g., linear or nonlinear pulsation models, model structure resolution, and opacities. For example, Cox (1995) models would indicate that a different and erroneous mass range is derived if, say, a star of Oosterhoff group I (OoI) composition, $[\text{Fe/H}] \sim -1.5$, is analyzed relative to a grid of models with assumed Oosterhoff group II (OoII) composition, $[\text{Fe/H}] \sim -2.0$. On the other hand, Bono et al. (1996a) show the significant changes made to derived masses if nonlinear pulsation, rather than linear pulsation, models are used. Bono et al. (1996b) find characteristic masses for OoI cluster stars shown in Figure 2 with $P_0 = 0.485$ days to be $0.65 \pm 0.05 \, M_\odot$. For the OoII cluster stars, comparison with the nonlinear models yields masses of $0.82 \pm 0.05 \, M_\odot$. It could be argued that in order to ascribe masses to the LMC stars, it is necessary to have an indication of the composition of the population represented in Figure 2. In Alcock et al. (1996), we showed that the spectroscopically determined abundance distribution of a few of the LMC field RR Lyrae stars was not dissimilar to the general Galactic halo with a modal value of $[\text{Fe/H}] = -1.7$. In addition, the amplitude-period distribution of the LMC Bailey type $ab$ field stars indicated a population with a characteristic composition between the Galactic globular clusters M3 and M15 with a mean $[\text{Fe/H}]$ of $-1.7$. Assuming this composition to apply to the LMC multimode stars, we derive a mass of $0.55 \pm 0.05 \, M_\odot$ for those at the shortest period of the range at $P_0 = 0.46$ days. An alternative view would be that the masses of the LMC stars with low $P_1/P_0$ are not so low, but that many are a more metal-rich component of the LMC HB and that observables is complete. Such a relationship is well known for Cepheid variables, where the existence of the instability strip boundaries gives rise to an obvious period-luminosity relation. For double-mode RR Lyrae stars, we expect a similar bounded region in the color-magnitude diagram that will also give rise to a period-luminosity relationship.

We will now proceed through the steps necessary to obtain absolute magnitude estimates. First, the estimation of masses from periods and period ratios will be described. Second, the estimation of effective temperature and the relationship of these stars with the fundamental mode is greater than that of the fundamental--a property that this sample shares with previously discovered RRd stars in globular clusters and dwarf spheroidal galaxies. We caution, however, that our sample is incomplete because of period-dependent selection biases and that more RRd stars with $\Delta_1/\Delta_0 < 1$ may yet be discovered in our LMC sample.
comparison with a relatively metal-rich grid of models in the Petersen diagram would yield masses as high as 0.60 \( M_\odot \) or higher. To achieve this, the assumed metal abundance of these stars would have to be very high. Cox (1995), discussing the reality of the mass differential between OoI and OoII stars, states that the derived OoI mass could be as large as the OoII mass only if the iron abundance in the \( Z\)-mixture is increased by more than 30\% over that for the solar mixture. Bono et al. (1996b) suggest further that with the use of full-amplitude, well-resolved, nonlinear, nonlocal convective models with OPAL95 opacities, the mass differential between OoI and OoII stars is preserved even if they are derived relative to a grid of common composition models. We therefore think that the metal-rich scenario is both unlikely, as such stars would be largely of lower temperature than the instability strip FBE, and ineffective in increasing the derived mass. Additionally, the LMC field shows (Alcock et al. 1996) a red HB morphology for its composition, so that blue metal-rich HB stars are unlikely to be a major component of the multimode population of the LMC HB.

The distribution of multimode stars in Figure 2 shows them to occupy a strip almost orthogonal to the loci of constant mass and indicates, assuming they are representative of the halo field in composition, a global dependence of mass on the fundamental pulsation period. From Figure 2, it can be shown that a relation of the form

\[
\log P_0 = 0.440 \log M - 0.231
\]  

applies across the indicated mass range of the multimode stars in the LMC and the Galactic Oosterhoff groups I and II.

The shortest period of Bailey type \( ab \) stars among the total sample of LMC RR Lyrae stars is 0.457 days, and, additionally, with a period ratio of about 0.7421, the overtone periods are close to one-third of a day and are thus more difficult to discover because of aliasing. We mark these period regions as hatched areas in Figure 2 to emphasize the degree to which the period distribution of the multimode stars is affected by selection biases in this work. Nevertheless, it is clear that the tight trend of observed period and period ratio in Figure 2 for RRd stars born in a wide range of environments is consistent with a tight relationship between luminosity, effective temperature, and mass.

2.4.2. Pulsation Models

The pulsation equation for model envelopes of RR Lyrae stars has been characterized, using OPAL95 opacities by Bono et al. (1996a), as

\[
\log P_{0, \text{tr}} = 11.627 + 0.823 \log L_{\text{tr}} - 3.506 \log T_{e, \text{tr}} - 0.582 \log M_{\text{tr}} ,
\]  

representing nonlinear pulsation behavior over the mass range 0.65–0.75 \( M_\odot \). The validity of the extrapolation of this equation to assumed masses as low as 0.55 \( M_\odot \) is supported by the previous linear pulsation calculations of van Albada & Baker (1971) and Cox (1995). We have annotated the variables to reflect the assumption that the multimode stars are in the transition zone (the region between the fundamental blue edge and the first-overtone red edge) of the instability strip. Their equation (2) is a linear fit to the results of the period determinations for RR Lyrae star models using the most recent composition parameters in a study of the nonlinear pulsation properties. The fit to the pulsation equation is in the spirit of the influential study of RR Lyrae star pulsation by van Albada & Baker (1971), and we note in passing that the use of their form of the pulsation equation, or that of Cox (1995), does not affect the results and conclusions drawn in this paper.

We make the further assumption (see, for example, Caputo 1990 and Bono & Stellingwerf 1994) that there is a close similarity between the temperature of the blue edge (FBE) of the fundamental instability strip and the transition zone occupied by the multimode stars. \( P_{\text{tr}} \) is thus taken as the period of the fundamental mode at the blue edge of the instability strip. For the FBE, we can derive a relationship between the fundamental period and the effective temperature by combining equation (8) of Sandage (1993a), the relation between the shortest period for RRab stars and [Fe/H], and equation (5) of Sandage (1993b), the relation between \( \log L_{\text{tr}} \) and [Fe/H] for RRab stars, to obtain

\[
\log T_{e, \text{tr}} = 3.816 - 0.0984 \log P_{\text{tr}} .
\]  

We see that the FBE is very nearly vertical in the \( (\log L_{\text{tr}}, \log T_e) \)-plane. Sandage (1993a, 1993b) expresses both the FBE period and the FBE temperature in terms of abundance [Fe/H], but since the range of [Fe/H] in the LMC field RR Lyrae stars is the same as in the calibrating Galactic RR Lyrae clusters and field, we have written the FBE temperature directly in terms of period. In the range of \( P_{\text{tr}} \) of the beat RR Lyrae stars, this calibration suggests that \( T_{e, \text{tr}} \) varies from 6940 to 7080 K. A check on equation (3) is possible through a comparison of our prediction of the temperature of the RRd stars in the Galactic cluster M15 with an independent determination of that temperature. Equation (3) suggests a mean temperature for the M15 stars of 6940 K. This is in good agreement with the result of the careful discussion by Silbermann & Smith (1995), who find the M15 RRd stars to have a mean temperature of 6900 K. The calibration is also consistent with calculations of the fundamental and first-overtone instability strip boundaries (and the zone where both fundamental and first overtone are unstable) by Bono & Stellingwerf (1994). Their models show that the temperature of the strip, in which pulsation in either the fundamental mode or the first overtone is allowed, becomes narrower and bluer as models of higher luminosity are considered, and at \( \log L \) near 1.8, the double-mode strip extends from 6750 to 7000 K. The assumption that the double-mode stars lie near this temperature is also supported by the close similarity of the periods of the shortest period LMC Bailey type \( ab \) stars at \( P_0 = 0.457 \) days to the most common fundamental periods of the multimode stars at \( P_0 = 0.48 \) days.

The luminosity of the multimode stars is then

\[
\log \left( \frac{L_{\text{tr}}}{L_\odot} \right) = 2.506 + 2.405 \log P_{\text{tr}} .
\]  

Over the range of the transition periods covering the majority of the LMC multimode stars shown in Figure 2, at \( P_0 = 0.46 \) days, \( \log L_{\text{tr}}/L_\odot = 1.69 \), and at \( P_0 = 0.50 \) days, \( \log L_{\text{tr}}/L_\odot = 1.78 \). In deriving absolute visual magnitudes for these stars, we follow Silbermann & Smith (1995) in their detailed discussion of the appropriate values of the solar and RR Lyrae star bolometric corrections. Our adopted bolometric correction is 0.09 mag, which places us on the same system as Vanden Berg & Bell (1985). We find the
metallicity dependence of the bolometric correction to be small, as evidenced by the inspection of R. A. Bell’s model atmosphere calculations published in Butler, Dickens, & Epps (1978). At $P_0 = 0.46$ and 0.50 days, we find the LMC multimode stars to have $M_V = +0.59$ and $+0.38$, respectively. For the longer fundamental periods of the multimode stars in the metal-weak Galactic globular cluster M15 with a characteristic $P_0 = 0.545$ days, the derived luminosity corresponds to $M_V = +0.13$ mag.

Most of the other calibrations of RR Lyrae star luminosities have been expressed in terms of composition, and there has been considerable debate about the slope of the coefficient of the term in $[\text{Fe/H}]$. The calibration presented here is in terms of the shortest period of the $ab$-type RR Lyrae stars in a given system. We also note that our calibration depends on the theoretical pulsation equation that, as we will show in a following section, appears to be supported by the luminosities of the LMC RR Lyrae stars. Our calibration of RR Lyrae star luminosities is brighter than many previously derived. This calibration receives support from the independent study of the Bailey type $c$ RR Lyrae stars by Simon & Clement (1993), who compared the Fourier phase parameters and periods of the first-overtone pulsators with linear and hydrodynamic pulsation models in order to determine temperatures and a luminosity calibration. The result of their calibration applied to the Reticulum cluster of the LMC yields roughly the same distance modulus as we derive below for the field LMC stars. Their discussion of the Galactic multimode stars as checks on their calibration of the RR$c$ variables additionally supports the assumptions we made above concerning the location of the multimode instability strip in the luminosity-temperature plane.

Further indirect support of this calibration comes from the fact that there is a growing body of data concerning the spectroscopic determination of the effective temperature and surface gravity, leading directly to the stellar mass-luminosity ratio, of blue HB stars in Galactic globular clusters. Much of this recent work is reviewed by Moehler, Heber, & Rupprecht (1996), who show that with the previous assignment of luminosities to the HB, the derived HB masses are implausibly low (at $0.35 M_\odot$), and the effect appears more marked among the more metal-weak clusters such as NGC 6397, M92, and M15. Our proposed revision for such clusters can involve a 40% increase in the HB luminosity and a consequent increase in the derived mass of HB stars in these clusters. The resulting derived mass is $\sim 0.48 M_\odot$, which is close to the predicted mass for blue HB stars and provides indirect support for the revised HB luminosities proposed here.

In his recent monograph, Smith (1995) reviews at least six methods, both fully and semiempirical, which yield luminosities of Galactic RR Lyrae stars roughly 0.4 mag fainter than those derived here. Conversely, all these methods would predict the LMC RR Lyrae stars to be too faint by the same amount. This discrepancy may be less marked than it appears at first sight since the populations of solar neighborhood RR Lyrae stars that can be studied by statistical parallax or Baade-Wesselink techniques may be more representative of those systems in which the FBE occurs at shorter periods. In this paper, we adopt the position that the characteristics of RR Lyrae stars in the LMC and the Galactic halo clusters are properly comparable, and we explore the consequences of that comparison.

### 2.4.3. Error Analysis

The application of the luminosity calibration derived in the previous section to the photometry of our multimode RR Lyrae stars in the LMC necessitates a careful examination of the uncertainties. We adopt a mass uncertainty of $\pm 0.05 M_\odot$ (Kovacs et al. 1992) in our (log $P_0$, log $M_f$)-relation. Assuming double-mode RR Lyrae stars lie at the FBE, the uncertainty in equation (3), our (log $T_{e,u}$, log $P_0$)-relation derived from Sandage (1993a, 1993b) is dominated by the uncertainty of the $([B - V], \log T_{e,u})$-relation used to define the temperatures of Galactic RR Lyrae stars along the FBE. We return to the unpublished model atmosphere calculations of R. A. Bell as seen in Butler et al. (1978), the model atmospheres of Kurucz (1979), and the instability strip analysis of Bonino et al. (1996a, 1996b), and by intercomparison we adopt at the FBE an uncertainty of 0.01 in log $T_{e,u}$. Propagating the errors through to equation (4), we find that at a given period, the uncertainty in log $L_V$ is $\pm 0.065$, resulting in an uncertainty in $M_V$ of $\pm 0.16$ mag. We have assumed that equation (4) is an exact representation. Uncertainty in the bolometric correction is systematic and difficult to quantify. We adopt the system of Vandenberg & Bell (1985) as convincingly argued by Silbermann & Smith (1995) and, as already noted, find (Alcock et al. 1996) a negligible dependence of the correction on metallicity for the range of LMC RR Lyrae abundances observed. We find the differences in bolometric corrections adopted by different authors to be smaller than the uncertainty in $M_V$ introduced by the uncertainty in the adopted temperatures, and we will not discuss it further.

### 2.4.4. LMC Distance Modulus

In Figure 3, we show the multimode stars in the $V, (V-R)$ color-magnitude diagram (CMD). We adopt the reddening vector with $R = A_V/E(V-R) = 5.0$ (Bessell 1996) that is illustrated in Figure 3. The adopted photometric uncertainty for each point is also shown. The insert shows the multimode stars in relation to a representative MACHO LMC CMD. Using the temperatures from equation (3), for stars with log $g = 2.6$, we adopt the $([B - V], (V-R))$-relation (Bessell 1996),

$$V - R = 0.004 + 0.566(B - V), \quad (5)$$

with an uncertainty of $\pm 0.017$ in the zero point. From Butler et al. (1978), we interpolate the $T$ (our adopted temperature for the FBE), $B - V$ relations between $[\text{Fe/H}] = \pm 1$ and $-2$ for $[\text{Fe/H}] = -1.7$, a value appropriate for the LMC RR Lyrae stars (Alcock et al. 1996), and derive

$$V - R_0 = 0.19 + 0.15 \log P_{u}, \quad (6)$$

with an uncertainty in the zero point of $\pm 0.02$ mag derived from the uncertainties in equations (3) and (5) as discussed previously. Adopting our FBE temperature, the intrinsic color, $(V-R)_0$, ranges from 0.14 to 0.15 mag for our multimode stars. Furthermore, we note that the mean reddening, $\langle E(V-R) \rangle = 0.049$ mag or $\langle E(B-V) \rangle = 0.086$ mag, agrees well with other measurements of the reddening toward the LMC (Bessell 1991). In Figure 4, we show the dereddened $V$ magnitudes plotted against $P_0$. The error bars noted are derived from the uncertainty in $V$ and in $(V-R)$ scaled by the ratio of total to selective absorption $(R = 5)$, equivalent to $\pm 0.27$ mag. The contribution of the
No. 1, 1997 LMC VARIABLE STAR INVENTORY. III. 95

FIG. 3.—Observed median $V$ magnitudes plotted against derived $(V-R)$ colors for the multimode RR Lyrae stars. The reddening vector is marked with an arrow. The insert shows the CMD of the multimode stars (filled circles), in relation to a representative CMD in an LMC field.

FIG. 4.—The absorption-corrected magnitudes, $V_0$, plotted against the fundamental period $P_0$. The line has the slope given by eq. (4).

uncertainty in our adopted intrinsic color is systematic, and we have not included it in the uncertainty of each individual dereddened magnitude. Our multimode RR Lyrae stars have a mean period $\langle P_0 \rangle = 0.482$ corresponding to $\langle M_V \rangle = 0.46 \pm 0.16$ mag. The mean dereddened magnitude of our multimode RR Lyrae sample is $\langle V_0 \rangle = 18.94 \pm 0.03$ mag, in excellent agreement with the mean dereddened $V$ magnitude of nearly 180 RR Lyrae stars found in LMC clusters (Walker 1992). The distance modulus of the LMC naturally follows $(m-M)_{LMC} = 18.48 \pm 0.19$ mag, where the final error is a combination of the uncertainty in $(V-R)_0$ (scaled by $R = 5$), the uncertainty in $\langle V_0 \rangle$, and the uncertainty in $\langle M_V \rangle$. Fitting the data of Figure 4 to the $(M_V, \log P_0)$-calibration with a fixed slope yields the same LMC distance modulus. The best-fit distance modulus is shown as a horizontal line in Figure 4. The corresponding $(V_0, \log P_0)$-calibration is also shown. The distance modulus for the LMC multimode RR Lyrae stars derived here is consistent with that $18.58 \pm 0.04$ mag derived from the Galactic calibration of LMC Cepheids through the use of $VJHK$ Baade-Wesselink luminosities discussed by Laney & Stobie (1994, 1995), and through a comparison they make with Galactic Cepheids in clusters and LMC stars that leads to a modulus of $18.53 \pm 0.04$ mag.
3. DISCUSSION

Walker (1989, 1992) has emphasized the apparent discrepancy of 0.3 mag between RR Lyrae star luminosities derived from Baade-Wesselink analyses of Galactic stars and the luminosities of LMC cluster variables if the Cepheid distance calibration is correct. We have shown here that there is a consistency between the luminosities derived from the pulsation equation and the independent luminosity measures, such as the LMC Cepheid distance calibration and the SN 1987A ring distance (Gould 1995). Consequently, the pulsation properties of the multimode RR Lyrae stars appear to be a useful tool in distance studies through their calibration of the more common and easily discovered RRab stars.

The unusually large range in $P_t/P_0$ and the range in implied masses appear to present new questions about the evolution from the zero-age HB. The Lee & Demarque (1990) and Sweigert (1987) HB evolutionary tracks do not carry the evolution of stars with masses as low as 0.55 $M_\odot$ as far as the instability strip, but it does not appear that such stars could be of similar luminosity to the HB when they reach the instability strip following redward evolution. Cox (1991) goes further and states that stars with a mass of 0.55 $M_\odot$ would never reach the instability strip at all. However, as we see in Figure 2, there appear to be LMC multimode RR Lyrae stars with this mass as derived from pulsation theory. In Alcock et al. (1996), we argued that the field HB of the LMC has a red morphology and that, as a result, the beat RR Lyrae stars are rare. Our more recent detections still show fewer of these stars among the LMC field population than, say, in the OoII cluster M15. We remind the reader that our survey is incomplete and that this fraction may rise. In the same paper, we also argued that the LMC field RR Lyrae star abundance distribution was similar to the halo Galactic distribution with a modal [Fe/H] $= -1.7$. These findings led us to conclude that the LMC field was younger than most Galactic globular clusters.

An implication of Figure 4 and the period-luminosity relation contained in equation (4) is a differential rescaling of the distances to Galactic globular clusters (and the Galactocentric distance) when HB luminosities are the criteria. Oosterhoff group II clusters are the most affected.

Walker (1992) speculated that the distance to the LMC was defined correctly by the Galactic Cepheid distance scale and inferred that the LMC RR Lyrae stars were 0.3 mag brighter than previously suggested. He examined the consequences of a global increase in HB star luminosities on the age and distance of Galactic globular clusters. In this paper, having introduced a period-luminosity relation for the multimode RR Lyrae stars, we have determined the RR Lyrae star luminosities to be significantly brighter than previously thought and have shown that this revision is most marked for the OoII clusters in the Galaxy.

For illustrative purposes, we discuss the archetypal Galactic cluster M15, which has multimode RR Lyrae stars with a mean fundamental period of $P_0 = 0.545$ days.

Equation (4) implies that the HB $M_V = +0.13$ mag. When the data of Silbermann & Smith (1995) are reanalyzed with the pulsation equation given in equation (2), and our masses defined in equation (1), we find that $M_V(M15) = +0.20$, which is consistent, within their claimed errors of $\pm 0.12$, with the value we have derived above. Buonanno, Corsi, & Fusi-Pecchi (1989) list the brightness difference between the main-sequence turnoff (MSTO) and the HB as 3.54 mag for this cluster, leading to $M_{V,\text{to}} = 3.67$ mag. These authors extract from evolutionary calculations the relation

$$\log t_0 = -0.41 + 0.37M_{V,\text{to}} - 0.43Y - 0.13[\text{Fe/H}]$$  \hspace{1cm} (7)

between age, turnoff luminosity, and composition. For M15, we take $Y = 0.23$ and $[\text{Fe/H}] = -2.23$ (see Buonanno et al. 1989 and Minniti et al. 1993). This equation yields an evolutionary age of 13.4 Gyr. This estimate does not include the effect of any $\alpha$-element enrichment. Minniti et al. (1996) have found [O/Fe] = 0.45 for M15, and following the results of the discussion of VandenBerg & Stetson (1991) on the impact of [O/Fe] enrichment on derived cluster ages, the age of M15 becomes 12.6 Gyr. The age reduction of 0.8 Gyr that results from the consideration of $\alpha$-element enrichment is confirmed in the calculations of Salaris, Chieffi, & Straniero (1993). The age estimate of 12.6 Gyr has the characteristic uncertainty of $\pm 1.5$ Gyr associated with MSTO ages. Undoubtedly, these age estimates can be refined by the fitting of evolutionary tracks to the recalibrated CMDs. We note that compared with a HB absolute magnitude of $M_V = +0.49$ mag for M15 (Armandroff 1989), our calibration produces a reduction in age of 30% that is relatively independent of the assumed value of $\alpha$-element enrichment.

A fuller exploration of the parameter space of cosmological models defined by the age of the oldest stars is beyond the scope of this paper, but we note that a 30% reduction in the estimated age of the oldest stars will help reconcile globular cluster ages with recent estimates of the Hubble constant, $H_0$.

We are grateful for the skilled support given our project by the technical staff at Mount Stromlo Observatory (MSO). Work performed at Lawrence Livermore National Laboratory (LLNL) is supported by the Department of Energy (DOE) under contract W7405-ENG-48. Work performed by the Center for Particle Astrophysics (CPA) on the University of California campuses is supported in part by the Office of Science and Technology Centers of the National Science Foundation (NSF) under cooperative agreement (AST-8809616). Work performed at MSO is supported by the Bilateral Science and Technology Program of the Australian Department of Industry, Technology and Regional Development. K. G. acknowledges a DOE OJI grant, and the support of the Sloan Foundation. D. L. W. was a Natural Sciences and Engineering Research Council (NSERC) University Research Fellow during this work.
REFERENCES

Alcock, C., Allsman, R. A., Axelrod, T. S., Bennett, D. P., Cook, K. H., Freeman, K. C., & Griest, K. 1996, AJ, 111, 1146
Armandroff, T. 1989, AJ, 97, 375
Bessell, M. S. 1991, A&A, 242, L17
——— 1996, private communication
Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1996a, preprint (astro-ph/9606030)
——— 1996b, preprint (astro-ph/9606030)
Bono, G., & Stellingwerf, R. F. 1994, ApJS, 93, 223
Buonanno, R., Corsi, C., & Fusi-Pecci, F. 1989, A&A, 216, 80
Butler, D., Dickens, R. J., & Epds, E. 1978, ApJ, 225, 148
Caputo, F. 1990, in ASP Conf. Ser. 11, Confrontation between Stellar Pulsation and Evolution, ed. C. Cacciari & G. Clementini (San Francisco: ASP), 22
Clement, C. M., Nemec, J. M., Robert, N., Wells, T., Dickens, R. J., & Bingham, E. A. 1986, AJ, 92, 825
Cox, A. N. 1991, ApJ, 381, L71
——— 1995, in ASP Conf. Ser. 78, Astrophysical Applications of Powerful New Databases, ed. S. J. Adelman & W. L. Wiese (San Francisco: ASP), 243
Gould, A. 1995, ApJ, 452, 189
Kovacs, G., Buchler, J. R., Marom, A., Iglesias, C. A., & Rogers, F. J. 1992, A&A, 259, L46
Kurucz, R. L. 1979, ApJS, 40, 1
Landolt, A. U. 1992, AJ, 104, 340
Lanev, C. D., & Stobie, R. S. 1994, MNRAS, 266, 441
Laney, C. D., & Stobie, R. S. 1995, in ASP Conf. Ser. 83, Astrophysical Applications of Stellar Pulsation, ed. R. S. Stobie & P. A. Whitelock (San Francisco: ASP), 254
Lee, Y.-W., & Demarque, P. 1990, ApJS, 73, 709
Minniti, D., Geisler, D., Peterson, R. C., & Clariá, J. J. 1993, ApJ, 413, 548
Minniti, D., Peterson, R. C., Geisler, D., & Clariá, J. J. 1996, ApJ, 470, 953
Moehler, S., Heber, U., & Rupprecht, G. 1996, ESO preprint 1183
Nemec, J. M. 1985a, AJ, 90, 204
——— 1985b, AJ, 90, 240
Nemec, J. M., & Clement, C. M. 1989, AJ, 98, 860
Petersen, J. O. 1973, A&A, 27, 89
Salaris, M., Chieffi, A., & Straniero, O. 1993, ApJ, 414, 580
Sandage, A. R. 1993a, AJ, 106, 687
——— 1993b, AJ, 106, 703
Silbermann, N. A., & Smith, H. A. 1995, AJ, 110, 704
Simon, N. R., & Clement, C. M. 1993, ApJ, 410, 526
Smith, H. A. 1995, in Cambridge Astrophysics Ser. 27, RR Lyrae Stars (Cambridge: Cambridge Univ. Press)
Sweigert, A. V. 1987, ApJS, 65, 95
van Albada, T. S., & Baker, N. 1971, ApJ, 169, 311
VandenBerg, D. A., & Bell, R. A. 1985, ApJS, 58, 561
VandenBerg, D. A., & Stetson, P. B. 1991, AJ, 102, 1043
Walker, A. R. 1989, AJ, 98, 2086
——— 1992, ApJ, 390, L81
——— 1994, AJ, 108, 555
Walker, A. R., & Nemec, J. M. 1996, AJ, 112, 2026