HADRONIZATION IN HEAVY-ION REACTIONS

T. S. BIRÓ, P. LÉVAI, J. ZIMÁNYI
Research Institute for Particle and Nuclear Physics
H-1525 Budapest P.O.Box 49, Hungary
E-mail: tsbiro@sunserv.kfki.hu

AND

C. T. TRAXLER
Institute für Theoretische Physik
D-35392 Giessen, Heinrich-Buff-Ring 15, Germany
E-mail: Chris.Traxler@theo.physik.uni-giessen.de

We present a model of fast hadronization of constituent quark matter in relativistic heavy ion collisions based on rate equations and capture cross sections in non-relativistic potential. We utilize a thermodynamically consistent approach with a non-ideal equation of state including correlation terms based on string phenomenology. We investigate strange and non-strange particle ratios observed in CERN SPS experiments.

1 Correlations, Clusters, Confinement

Quark matter in the moment of hadronization is one of the most strongly correlated system we deal with in physics. Confinement forces are long range forces, so no “ionization” of bound clusters of constituent quarks is possible below the deconfinement temperature. In order to work with an “effective” theory of the strongly interacting quark matter, even in its simplest form of constructing an descriptive phenomenological equation of state, we have to take into account effects of correlation.

A comparison with well known hard core problem in nuclear matter may be enlightening. Although the underlying pair potential diverges at zero distance ($r \to 0$), the relative wave function is zero at this point due to exactly this repulsive core. As a consequence the effective pair potential is finite at small distance giving rise to finite corrections to the mean field energy (correlation energy). In case of the confining static potential between massive quarks the relative wave function vanishes beyond a characteristic distance (the Bohr radius of hadronic bound state). Here also the divergence at $r \to \infty$ is regularized by the pair correlations and the modification to the mean field energy and pressure is finite.

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The above consideration results in an effective equation of state at the characteristic particle density determined by the minimum of the effective pair potential. This concept can be, however, extended to all densities: the correlation improved pair potential results in a better high-density behavior of nuclear and in a better low-density behavior of quark matter. Eventually a certain, nonlinear density-dependence in the equation of state takes such correlations into account in a phenomenological way.

In our present model massive quarks and their hadronic (i.e. color neutral) clusters are constituents of a mixture. We describe this mixture with a quasi particle Hamiltonian including a density dependent background energy density,

\[ \mathcal{H} = \sum_i (\omega_i(k) + U_i(n)) n_i + U(n). \]  

(1)

The self-consistency of the definition of the chemical potential,

\[ \mu_i = \frac{\partial \mathcal{H}}{\partial n_i} = \omega_i + U_i, \]  

(2)

requires

\[ \sum_j n_j \frac{\partial U_j}{\partial n_i} + \frac{\partial U}{\partial n_i} = 0, \]  

(3)

and the variational minimum with respect to the temperature leads to

\[ \frac{\partial \mathcal{H}}{\partial T} = \sum_j n_j \frac{\partial U_j}{\partial T} + \frac{\partial U}{\partial T}. \]  

(4)

These thermodynamical consistency relations are satisfied by the following general form of the free energy

\[ F(N_1, N_2, \ldots, V, T) = \sum_i F_{\text{id}}(N_i, V, T) + V b(N_1/V, N_2/V, \ldots). \]  

(5)

Here \( N_i = V n_i \) is the number of constituents, \( V \) the reaction volume and \( T \) the temperature. We choose the free energy as the thermodynamical potential, because for a system of ”quark to hadron transition” - chemical, shortly ”transchemical”, equations the time evolution of the numbers can be obtained relatively easy, contrary to that of the chemical potentials \( \mu_i \). Note that the \( b(n_1, n_2, \ldots) \) correction does not depend on the temperature or on the volume explicitly. The derived thermodynamical quantities are

\[ \mu_i = \mu_{i\text{id}} + \frac{\partial b}{\partial n_i}, \]  

2
\[ S = S^{id}, \]
\[ E = E^{id} + Vb, \]
\[ p = p^{id} - b + \sum_i n_i \frac{\partial b}{\partial n_i}. \]  

(6)

The relation to the vector mean field \( U_i \) and to the background (bag) energy density is given by

\[ U_i = \frac{\partial b}{\partial n_i}, \]
\[ U = b - \sum_i n_i \frac{\partial b}{\partial n_i}. \]  

(7)

Let us consider a few examples. For the good old MIT Bag model \( b = B \) is constant, leading to \( U = B \) and vanishing vector mean field for the constituents \( U_i = 0 \). The pressure \( p = n_q T - B = aT^4 - B \) is negative below a critical \( T = T_c \), where the quark matter is unstable.

In a pure mean field approach \( b \) is linear, \( b = \sum_i B_i n_i \), leading to \( U = 0 \) (no bag constant) and \( U_i = B_i \). This system is stable at all temperatures (\( p = p^{id} \)).

Finally we consider a simple system of massive quarks interacting via a string-like potential. In this case the correction to the free energy is proportional to the average distance of quarks,

\[ b = \kappa n \langle \ell \rangle = \sigma n^{2/3}. \]  

(8)

In this case one obtains corrections to both mean fields, \( U = (1/3)\sigma n^{2/3} \) and \( U_q = (2/3)\sigma n^{2/3} \). The correlation correction is distributed between the background ("bag" constant) and the vector mean field for the quarks. The pressure of this system also becomes negative below a certain temperature:

\[ p = nT - \frac{1}{3} \sigma n^{2/3} \approx aT^4 - bT^2. \]  

(9)

This property is common with the bag model. The string EOS, however, results zero pressure at zero density (so no vacuum renormalization is needed). Furthermore - due to the vector mean field correction - the string EOS shows a non-ideal behavior of the chemistry, which is absent in the bag model.

Fig.1 shows the dependence of the quark chemical potential on the density,

\[ \mu = d \ln \frac{n}{cT^3} + \frac{2}{3} \sigma n^{-1/3}. \]  

(10)
Since the entropy production in an isolated expanding system is given by

\[
\dot{S} = -\sum_i \frac{\mu_i}{T} \dot{N}_i \geq 0,
\]  

(11)

any physical description of the coalescence rate of quarks possesses the property that \( \mu \) and \( \dot{N} \) have opposite signs. In the particular two-body fusion process studied in the transchemistry model of next Section, one has

\[
\dot{N} \propto -N^2 \left(1 - e^{-\mu/T}\right),
\]

(12)
satisfying the above condition.

Due to this often cited entropy growth principle, the quark density in a constant volume decreases whenever \( \mu > 0 \) and increases whenever \( \mu < 0 \). The isotherms of the \( \mu(n) \) relation depicted in Fig. 1 reveal that below a given temperature \( T_{\text{chem}} \) no chemical equilibrium is possible in a pure quark matter with string EOS. In this case the quark chemical potential is overall positive at any density — so quarks will be eliminated from the system. The only natural way to find a thermodynamically stable state is to form color neutral clusters which makes no contribution to the string correction in the equation of state. This is what happens by the hadronization of colored constituent quark matter and this is the basis of the transchemistry model discussed in the next section.

![Figure 1: Isotherms of the chemical potential are plotted against the quark density with a non-ideal equation of state with string-like interaction. The infinite temperature case coincides with the ideal gas form, while below a critical temperature there is no chemical equilibrium possible: the quark density tends towards zero.](image)

Finally we would like to show some results of a molecular dynamical calculation of massive quark matter in the framework of the chromodielectric model.
Figure 2: Simulation snapshots after initialization of a central-rapidity slice of a QGP tube with Lorentz-invariant flow (“Bjorken scenario”) in the chromodielectric model. Colored objects (quarks, gluons) are drawn in dark grey, hadrons in light grey.

This model describes the confining strings due to an abelian Gauss law but with a nontrivial dielectric constant. Inspecting the snapshots of quark matter evolution in a Bjorken scenario (Fig. 2) the ramification is clearly seen in the intermediate stage of hadronization.

2 The transchemistry model

A detailed description of the transchemistry model can be found in 1. The equation of state used for the description of the evolution of constituent quark matter into hadrons during a sudden expansion utilizes a generalization of the above discussed string-like correlation energy. In the hadronizing mixture we deal with quarks, anti-quarks, diquarks, anti-diquarks carrying color charge and with mesons, baryons and anti-baryons being color neutral. The number of strings is assumed to be proportional to a weighted sum of color particles

\[ Q = \sum_i q_i N_i. \]

We take \( q_i = 0 \) for hadrons, \( q_i = 1 \) for quarks and \( q_i = 3/2 \) for diquarks. The average length, \( \langle \ell \rangle = n_c^{-1/3} \) is determined by the color density \( n_c = \sum_{i=c} N_i/V \) excluding hadrons from the sum. The correction to the free energy of a mixture of massive ideal gas constituents without inner degrees of freedom is then given by

\[ \Delta F = \sigma n_c^{-1/3} Q. \]
The modification of the non-hadronic chemical potentials becomes

$$\mu_i = \mu_i^{id} + \sigma n_c^{-1/3} \left( q_i - \frac{1}{3} \frac{Q}{V n_c} \right).$$

(14)

The non-ideal cooling law, equivalent to \( \partial \mu T^{\mu \nu} = 0 \) for a perfect fluid of the mixture, becomes

$$\frac{\dot{T}}{T} = -\frac{2}{3} \frac{\dot{V}}{V} \frac{\sum_i \dot{N}_i}{\sum_i N_i} - \frac{2}{3} \frac{\sum_i (m_i/T) \dot{N}_i}{\sum_i N_i},$$

(15)

with the additional term

$$\sum_i (\Delta \mu_i/T) \dot{N}_i = \sigma \sum_{i\in c} n_c^{-1/3} \left( q_i - \frac{1}{3} \frac{Q}{V n_c} \right) \dot{N}_i,$$

(16)

re-heating the system while eliminating color charges. The cooling is due to the expansion (\( \dot{V} > 0 \)) and due to rest mass production by making hadronic resonances.

The initial temperature and volume was obtained in a one dimensional Bjorken scenario from the bombarding energy and the stopping power. At the CERN SPS \( Pb + Pb \) experiment we obtain \( V_0 \approx 400 \text{ fm}^3 \) and \( T_0 \approx 180 \text{ MeV} \). The initial quark numbers include direct and gluon fragmentation pair creation and sum up to about 3000 with a strangeness ratio of \( f_s = 0.21 \).

The general two body fusion process is supported by the in-medium cross section of massive constituents in a Coulomb potential in medium. This is proportional with a screening volume, \( \rho^3 \), which also depends on the presence of color charges. We make the assumption that \( \rho^3 n_c \) is constant obtaining a dynamical confinement effect of increasing hadronization cross sections in the low color density stage. These phenomenologically amounts to the string-like correlation correction to the hadronization cross sections and influences the dynamics of the transchemical evolution.

Each chemical reaction is of type \( i + j \leftrightarrow k \), accounting for the changes \( dN_i = dN_j = -Adt \) and \( dN_k = Adt \) in the particle numbers. The general form,

$$A = R_{ij\rightarrow k} N_i N_j \left( 1 - e^{\frac{-\mu k}{T} - \frac{-\mu i}{T} - \frac{-\mu j}{T}} \right),$$

(17)

assures the increase of entropy while approaching chemical equilibrium. Finally, in order to get the final hadron numbers, hadronic decays are taken into account with the dominant branching ratios obtained from Particle Data Table 2.
3 Constituent Quark Matter at SPS

In this section we present results of the numerical solution of 43 coupled differential equations describing the chemical and thermal evolution of the quark, diquark - hadron mixture. In Fig.3 the evolution of the temperature, entropy, pressure and energy density is shown for a 168 GeV/nucleon Pb + Pb collision. At the beginning there is a rapid drop of the temperature (cf. Fig.3a) due to heavy hadron resonance formation. It is followed by a mild re-heating as an effect of the color confinement (by producing a colorless hadron the kinetic energy of the relative motion and the string energy is removed and replaced by the rest mass of the hadron). The total entropy monotonically increases during the hadronization while the contribution of colored particles is gradually eliminated (Fig.3b). The partial pressure of the constituent quark plasma rapidly decreases, it becomes even negative as the color density drops (Fig.3c). The total pressure, however, remains positive - so the mixture is stable and continues the adiabatic expansion. The partial and total energy densities evolve in the expected way going over to the $\epsilon \propto 1/\tau$ behavior at very late times only, when $p = 0$.

Figure 3: Evolution of thermodynamical quantities in the transchemistry model

Fig.4 presents the time evolution of constituent particle numbers: quarks, diquarks (left column from top to bottom), mesons and baryons (right column). The evolution of anti-particles is qualitatively similar, their absolute numbers
are only smaller. The hadron numbers shown here are those before hadronic resonance decays. It is clear from these figures that mesons are formed faster than baryons, the reason being that in this model the baryons are produced by a two step process and the initial diquark content was zero in these particular calculations.

Table 1: hadron multiplicities for $Pb + Pb$ collision at 168 GeV/nucleon bombarding energy.

|                  | Pb+Pb    | NA49    | TRCHEM. | ALCOR    | RQMD    |
|------------------|----------|---------|---------|----------|---------|
| $K^+$            | 76       | 76.14   | 78.06   | 79.0     |         |
| $K^-$            | 32       | 38.36   | 34.66   | 50.4     |         |
| $K^0_S$          | 54       | 57.25   | 56.36   | 63.5     |         |
| $p - \bar{p}$   | 145      | 151.6   | 147.03  | 171.8    |         |
| $\Lambda^0$-like| 50±10    | 62.30   | 69.07   | 56.8     |         |
| $\bar{\Lambda}^0$-like | 8±1.5 | 8.14    | 8.12    | 19.3     |         |

In the ALCOR model, the ratio of hadronic species are determined by the steepness of these curves (i.e. by $\dot{N}_i(0)$). Since the $N_i(t)$ curves do not cross, the algebraic approach of ALCOR to the solution of the underlying rate equations works well.

Figure 4: Evolution of thermodynamical quantities in the transchemistry model

Finally in Table 1 some hadron numbers obtained in the transchemistry model are compared with experimental data of the NA49 collaboration and
with two other theoretical models: ALCOR and RQMD. A more detailed comparison can be found in [1].

4 Conclusion

We presented a new model for the hadronization of constituent quark plasma based on a string-like correlation energy in the equation of state and on rate equations in a quark matter - hadron matter mixture. Due to large hadronization rates and due to a very off-equilibrium initial state, over-saturated with (massive) quarks, the hadronization process is fast: its time-scale is a few fm/c. We also found that in the particular situation of 168 GeV/nucleon heavy-ion collisions the algebraic ALCOR model is a good approximation to the numerical solution of the transchemical rate equations. The comparison with presently available experimental data indicate, that in such collisions a piece of matter is formed, inside which massive quarks, diquarks and their anti particles interact with a string-like mean field.

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hard core

confinement