Probing $CP$ Violation in $\Omega \to \Lambda K \to p\pi K$ Decay

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Abstract

The sum of the $CP$-violating asymmetries $A_{\Omega}$ and $A_{\Lambda}$ in the decay sequence $\Omega \to \Lambda K$, $\Lambda \to p\pi$ is presently being measured by the E871 experiment. We evaluate contributions to $A_{\Omega}$ from the standard model and from possible new physics, and find them to be smaller than the corresponding contributions to $A_{\Lambda}$, although not negligibly so. We also show that the partial-rate asymmetry in $\Omega \to \Lambda K$ is nonvanishing due to final-state interactions. Taking into account constraints from kaon data, we discuss how the upcoming result of E871 and future measurements may probe the various contributions to the observables.

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I. INTRODUCTION

The question of the origin of \( CP \) violation remains one of the outstanding puzzles in particle physics. Although \( CP \) violation has now been seen in a number of processes in the kaon and \( B \)-meson systems \([1]\), it is still far from clear whether its explanation lies exclusively within the picture provided by the standard model \([2]\). To pin down the sources of \( CP \) violation, it is essential to observe it in many other processes.

Hyperon nonleptonic decays provide an environment where it is possible to make additional observations of \( CP \) violation \([3, 4]\). Currently, there are \( CP \)-violation searches in such processes being conducted by the HyperCP (E871) Collaboration at Fermilab. Its main reactions of interest are the decay chain \( \Xi^- \rightarrow \Lambda \pi^- \), \( \Lambda \rightarrow p \pi^- \) and its antiparticle counterpart \([4]\). A different, but related, system also being studied by HyperCP involves the spin-\( \frac{3}{2} \) hyperon \( \Omega^- \), namely the sequence \( \Omega^- \rightarrow \Lambda K^- \), \( \Lambda \rightarrow p \pi^- \) and its antiparticle process \([5]\). For each of these decays, the decay distribution in the rest frame of the parent hyperon with known polarization \( w \) has the form

\[
\frac{d\Gamma}{d\Omega} \sim 1 + \alpha w \cdot \hat{p} ,
\]

where \( d\Omega \) is the final-state solid angle, \( \hat{p} \) is the unit vector of the daughter-baryon momentum, and \( \alpha \) is the parameter relevant to the \( CP \) violation of interest. In the case of \( \Omega \rightarrow \Lambda K \rightarrow p \pi K \), the HyperCP experiment is sensitive to the sum of \( CP \) violation in the \( \Omega \) decay and \( CP \) violation in the \( \Lambda \) decay, measuring \([5]\)

\[
A_{\Omega\Lambda} = \frac{\alpha_{\Omega} \alpha_{\Lambda} - \alpha_{\pi} \alpha_{K}}{\alpha_{\Omega} \alpha_{\Lambda} + \alpha_{\pi} \alpha_{K}} \simeq A_{\Omega} + A_{\Lambda} ,
\]

where

\[
A_{\Omega} \equiv \frac{\alpha_{\Omega} + \alpha_{\pi}}{\alpha_{\Omega} - \alpha_{\pi}} , \quad A_{\Lambda} \equiv \frac{\alpha_{\Lambda} + \alpha_{K}}{\alpha_{\Lambda} - \alpha_{K}}
\]

are the \( CP \)-violating asymmetries in \( \Omega \rightarrow \Lambda K \) and \( \Lambda \rightarrow p \pi \), respectively. Similarly, the observable it measures in \( \Xi \rightarrow \Lambda \pi \rightarrow p \pi \pi \) is \( A_{\Xi\Lambda} \simeq A_{\Lambda} + A_{\Xi} \) \([4]\).

On the theoretical side, \( CP \) violation in \( \Lambda \rightarrow p \pi \) and \( \Xi \rightarrow \Lambda \pi \) has been extensively studied \([3, 6–10]\). In contrast, the literature on \( CP \) violation in \( \Omega \) decays is minimal, perhaps the only study being Ref. \([11]\) which deals with the partial-rate asymmetry in \( \Omega \rightarrow \Xi \pi \). There is presently no data available or experiment being done on this rate asymmetry. In view of the upcoming measurement of \( A_{\Omega\Lambda} \) by HyperCP, it is important to have theoretical expectations of this observable. Clearly, the information to be gained from \( A_{\Omega\Lambda} \) will complement that from \( A_{\Xi\Lambda} \). Since the estimates of \( A_{\Lambda} \) and \( A_{\Xi} \) within and beyond the standard model (SM) have been updated very recently in Refs. \([9, 10]\), in this paper we focus on \( A_{\Omega} \).

We begin in Sec. II by relating the observables of interest in \( \Omega \rightarrow \Lambda K \) to the strong and \( CP \)-violating weak phases in the decay amplitudes. We discuss the role played by final-state interactions in this decay, which not only affect \( A_{\Omega} \), but also cause its partial-rate asymmetry to be nonvanishing, thereby providing another \( CP \)-violating observable. In Sec. III, we employ heavy-baryon chiral perturbation theory (\( \chiPT \)) to calculate \( P \) - and \( D \)-wave amplitudes for baryon-meson scattering in channels with isospin \( I = \frac{1}{2} \) and strangeness \( S = -2 \). We use the derived amplitudes in a
coupled-channel \( K \)-matrix formalism to determine the strong parameters needed in evaluating the \( CP \)-violating asymmetries. In Sec. IV, we estimate the asymmetries within the standard model. Working in the framework of \( \chi PT \), we calculate the weak phases by considering factorizable and nonfactorizable contributions to the matrix elements of the leading penguin operator. Subsequently, we compare the resulting \( A_\Omega \) with \( A_\Lambda \), which was previously evaluated, as both asymmetries appear in \( A_{\Omega\Lambda} \). In Sec. V, we address contributions to the \( CP \)-violating asymmetries from possible new physics, taking into account constraints from \( CP \) violation in the kaon system. Specifically, we consider contributions induced by chromomagnetic-penguin operators, which in certain models can be enhanced compared to the SM effects. Sec. VI contains our conclusions.

II. OBSERVABLES AND PHASES

The amplitudes for \( \Omega^- \to \Lambda K^- \) and \( \bar{\Omega}^+ \to \bar{\Lambda} K^+ \) each contain parity-conserving \( P \)-wave and parity-violating \( D \)-wave components, with the former being empirically known to be dominant [12]. They are related to the parameters \( \alpha_\Omega \) and \( \alpha_{\Omega\Pi} \) by

\[
\alpha_\Omega = \frac{2 \text{Re}(p^*d)}{|p|^2 + |d|^2}, \quad \alpha_{\Omega\Pi} = \frac{2 \text{Re}(\bar{p}^*\bar{d})}{|\bar{p}|^2 + |\bar{d}|^2},
\]

where \( p \) and \( d \) (\( \bar{p} \) and \( \bar{d} \)) are the \( P \)- and \( D \)-wave components, respectively, for the \( \Omega^- \) (\( \bar{\Omega}^+ \)) decay. Since both \( \Omega \) and \( \Lambda \) have \( I = 0 \), each of these decays is an exclusively \( |\Delta I| = \frac{1}{2} \) transition.

Before writing down the amplitudes in terms their phases, we note that the strong phases in \( \Omega \to \Lambda K \) are not generated by the strong rescattering of \( \Lambda K \) alone. Watson’s theorem for elastic unitarity [13] does not apply here, though it does in the cases of \( \Lambda \to p\pi \) and \( \Xi \to \Lambda\pi \). Final-state interactions also allow \( \Omega \to \Xi\pi \to \Lambda K \) to contribute, yielding additional strong phases as well as weak ones, because the channel \( \Xi\pi \leftrightarrow \Lambda K \) is open at the scattering energy \( \sqrt{s} = m_\Omega \). Since the \( \Omega \to \Xi\pi, \Lambda K \) rates overwhelmingly dominate the \( \Omega \) width [12], we expect other contributions via final-state rescattering to be negligible.

The requirements of \( CPT \) invariance and unitarity provide us with a relationship between the amplitudes for \( \Omega \to B\phi \) and its antiparticle counterpart. Thus, with \( \mathcal{M}_{\Omega\to B\phi}^{(L)} \) denoting the amplitude corresponding to \( B\phi \) being in a state with orbital angular momentum \( L \), we have

\[
(-1)^{L+1} \mathcal{M}_{\Omega\to B\phi}^{(L)} = S_{\Lambda\Lambda}^{(L)} \mathcal{M}_{\Omega\to \Lambda K}^{(L)*} + S_{\Lambda\Xi}^{(L)} \mathcal{M}_{\Omega\to \Xi\pi}^{(L)*},
\]

where \( S_{BB'}^{(L)} \) is the element of the strong \( S \)-matrix associated with the \( L \) partial-wave of \( B\phi \to B'\phi' \), and only the \( I = \frac{1}{2} \) component of the \( \Xi\pi \) state is involved in the second term. Assuming that the \( \Xi\pi \) and \( \Lambda K \) channels are the only ones open, we can express the \( S \)-matrix as [14]

\[
S = \begin{pmatrix}
S_{\Xi\Xi} & S_{\Xi\Lambda} \\
S_{\Lambda\Xi} & S_{\Lambda\Lambda}
\end{pmatrix} = \begin{pmatrix}
\hat{\eta} e^{2i\delta_{\Xi\Xi}} & i\sqrt{1-\hat{\eta}^2} \ e^{i(\delta_{\Xi\pi}+\delta_{\Lambda K})} \\
i\sqrt{1-\hat{\eta}^2} \ e^{i(\delta_{\Xi\pi}+\delta_{\Lambda K})} & \hat{\eta} e^{2i\delta_{\Lambda K}}
\end{pmatrix},
\]

where \( \hat{\eta} \) is the inelasticity factor and \( \delta_{B\phi} \) denotes the phase shift in \( B\phi \to B\phi \). Clearly \( S \) is unitary, and each partial-wave has its own \( S \). Now, since \( \hat{\eta} \) is expected to be close to and smaller than 1, it
is convenient to introduce a parameter $\varepsilon$ defined by

$$\hat{\eta} = 1 - 2\varepsilon ,$$

and so $\varepsilon$ is positive and small. Consequently, for $L = 1$ and 2, to first order in $\sqrt{\varepsilon}$ we have [15]

$$p = e^{i\delta_{P}^{K}} \left( p_{\Lambda} e^{i\phi_{P}^{\Lambda}} + i\sqrt{\varepsilon_{P}} p_{\Xi} e^{i\phi_{P}^{\Xi}} \right) , \quad d = e^{i\delta_{D}^{K}} \left( d_{\Lambda} e^{i\phi_{D}^{\Lambda}} + i\sqrt{\varepsilon_{D}} d_{\Xi} e^{i\phi_{D}^{\Xi}} \right) ,$$

where $p_{B}$ and $d_{B}$ are real, associated with $\Omega \to B\phi$, and $\phi_{B}^{P,D}$ denote the corresponding weak phases in the $|\Delta I| = \frac{1}{2}$ amplitudes.

Putting together the results above, and keeping only the terms at lowest order in small quantities, we obtain

$$A_{\Omega} = -\tan(\delta_{P}^{\Lambda} - \delta_{D}^{\Lambda}) \sin(\phi_{P}^{\Lambda} - \phi_{D}^{\Lambda}) \frac{p_{\Xi}}{p_{\Lambda}} \sqrt{\varepsilon_{P}} \sin(2\phi_{P}^{\Lambda} - \phi_{P}^{\Xi} - \phi_{D}^{\Xi}) + \frac{d_{\Xi}}{d_{\Lambda}} \sqrt{\varepsilon_{D}} \sin(\phi_{P}^{\Lambda} - \phi_{D}^{\Xi}) ,$$

where we have made use of the expectation that $\delta_{P,D}^{\Lambda,\Xi}$, and $d_{B}/p_{B}$ are also small. Unlike the strong phases in $\Lambda$ and $\Xi$ decays, there are no data currently available for $\delta_{P,D}^{\Lambda}$, and so we will calculate them here. To estimate the weak phases $\phi_{A,\Xi}$, we will consider contributions coming from the SM as well as from possible new physics. As for $p_{B}$ and $d_{B}$, we will extract their approximate values from data shortly, under the assumption of no final-state interactions and no $CP$ violation.

Now, the presence of the $\sqrt{\varepsilon}$ terms with additional weak and strong phases in the decay amplitudes in Eq. (8) implies that the rate of $\Omega \to \Lambda\bar{K}$,

$$\Gamma_{\Omega \to \Lambda\bar{K}} = \frac{|k_{\Lambda}| (E_{\Lambda} + m_{\Lambda})}{12\pi m_{\Omega}} (|p|^{2} + |d|^{2}) ,$$

evaluated in the rest frame of $\Omega$, is no longer identical to that of $\bar{\Omega} \to \bar{\Lambda}K$. Hence these decays yield another $CP$-violating observable, namely the partial-rate asymmetry

$$\Delta_{\Omega} = \frac{\Gamma_{\Omega \to \Lambda\bar{K}} - \Gamma_{\bar{\Omega} \to \bar{\Lambda}K}}{\Gamma_{\Omega \to \Lambda\bar{K}} + \Gamma_{\bar{\Omega} \to \bar{\Lambda}K}} .$$

It follows that to leading order

$$\Delta_{\Omega} = \frac{2p_{\Xi}}{p_{\Lambda}} \sqrt{\varepsilon_{P}} \sin(\phi_{P}^{\Lambda} - \phi_{P}^{\Xi}) .$$

We will also estimate this asymmetry below.\(^{1}\) Since $\Delta_{\Omega}$ results from the interference of $P$-wave amplitudes, a future measurement of it will probe $CP$ violation in the underlying parity-conserving interactions. We note that the strong parameters entering Eq. (12), and the second and third terms in Eq. (9), are not the strong phases, but $\varepsilon_{P,D}$.

\(^{1}\) In Ref. [11] the partial-rate asymmetry in $\Omega \to \Xi\pi$ was evaluated under the assumption that $\varepsilon = 0$. 4
Before ending this section, we determine the values of $p_B$ and $d_B$ which are needed in Eqs. (9) and (12), and also in evaluating the weak phases. To do so, we apply the measured values of $\alpha$ and $\Gamma$, as well as of the masses involved, in the corresponding formulas, as those in Eqs. (4) and (10), assuming that the strong and weak phases are zero. The experimental values of $\Gamma$ for $\Omega \to \Lambda \bar{K}, \Xi \pi$ are well determined, but those of $\alpha$ are not [12]. HyperCP is currently also measuring $\alpha_{\Omega}$, in $\Omega \to \Lambda \bar{K}, \Xi \pi$, with much better precision, and has reported [5] preliminary results of $\alpha_{\Omega} = (1.84 \pm 0.46 \pm 0.04) \times 10^{-2}$ and $\alpha_{\Omega} = (2.01 \pm 0.17 \pm 0.04) \times 10^{-2}$. Applying the PDG averaging procedure [12] to all the experimental results, including the preliminary ones from HyperCP, yields the average $\alpha_{\Omega} = 0.020 \pm 0.002$, which we adopt in the following. In the case of $\Omega \to \Xi \pi$, we use the data given by the PDG [12], and also

\[ |\Xi\pi\rangle = \sqrt{\frac{2}{3}} |\Xi^0\pi^-\rangle + \frac{1}{\sqrt{3}} |\Xi^-\pi^0\rangle \]  

(13)

to project out the $|\Delta I| = \frac{1}{2}$ amplitudes. Thus we extract

\[ p_{\Lambda} = 3.73 \pm 0.03 \text{ , } \quad d_{\Lambda} = 0.037 \pm 0.004 \text{ , } \]

\[ p_{\Xi} = 2.00 \pm 0.03 \text{ , } \quad d_{\Xi} = 0.08 \pm 0.12 \text{ , } \]

(14)

all in units of $G_F m_{\pi}^2$, with $G_F$ being the Fermi coupling constant.

III. STRONG PHASES AND INELASTICITY FACTORS

To calculate the strong parameters needed in Eq. (9), we take a $K$-matrix approach [14]. Furthermore, we include the contributions of other $B\phi$ states with $I = \frac{1}{2}$ and $S = -2$, namely $\Sigma \bar{K}$ and $\Xi \eta$, which are coupled to $\Lambda \bar{K}$ and $\Xi \pi$ through unitarity constraints. Although at $\sqrt{s} = m_{\Omega}$ the $\Sigma \bar{K}$ and $\Xi \eta$ channels are below their thresholds, it is important to incorporate their contributions to the open ones. Such kinematically closed channels have been shown to have sizable influence on the open ones in some other cases [16, 17].

The $K$ matrix for the four coupled channels can be written as

\[ K = K^T = \begin{pmatrix} K_{oo} & K_{oc} \\ K_{co} & K_{cc} \end{pmatrix}, \]

(15)

where the subscripts “o” and “c” refer to open and closed channels, respectively, at $\sqrt{s} = m_{\Omega}$. Thus $K_{oo,oc,co,cc}$ are all $2 \times 2$ matrices in this case and $K_{co} = K_{oc}^T$. Now, it is convenient to introduce the matrix

\[ K_r = K_{oo} + iK_{oc}(\mathbb{I} - iq_c K_{cc})^{-1}q_c K_{co}, \]

(16)

where $\mathbb{I}$ is the $2 \times 2$ unit matrix and $q_c = \text{diag}(k_{\Sigma \bar{K}}, k_{\Xi \eta})$, with $k_{B\phi}$ being the magnitude of the CM three-momentum in $B\phi$ scattering, implying that $k_{\Sigma \bar{K}}$ and $k_{\Xi \eta}$ are purely imaginary at $\sqrt{s} = m_{\Omega}$. The elements of $S$ in Eq. (6) can then be evaluated using [14]

\[ S = \mathbb{I} + 2i q_o^{1/2} K_r (\mathbb{I} - iq_c K_r)^{-1} q_o^{1/2}, \]

(17)
where \( q_o = q_0^{1/2} q_{0o}^{1/2} = \text{diag}(k_{\Xi\pi}, k_{\Lambda\bar{K}}) \). For the \( K \)-matrix elements, we make the simplest approximation by adopting the partial-wave amplitudes \( f_{B\phi \to B'\phi'} \) at leading order in chiral perturbation theory, namely

\[
K_{oo} = \left( \begin{array}{cc} f_{\Xi\pi \to \Xi\pi} & f_{\Xi\pi \to \Lambda\bar{K}} \\ f_{\Lambda\bar{K} \to \Xi\pi} & f_{\Lambda\bar{K} \to \Lambda\bar{K}} \end{array} \right), \\
K_{oc} = K_{co}^T = \left( \begin{array}{cc} f_{\Xi\pi \to \Sigma\bar{K}} & f_{\Xi\pi \to \Xi\eta} \\ f_{\Lambda\bar{K} \to \Sigma\bar{K}} & f_{\Lambda\bar{K} \to \Xi\eta} \end{array} \right), \\
K_{cc} = \left( \begin{array}{cc} f_{\Sigma\bar{K} \to \Sigma\bar{K}} & f_{\Sigma\bar{K} \to \Xi\eta} \\ f_{\Xi\eta \to \Sigma\bar{K}} & f_{\Xi\eta \to \Xi\eta} \end{array} \right).
\]

(18)

Before deriving them, we remark that time-reversal invariance of the strong interaction implies \( f_{B\phi \to B'\phi'} = f_{B'\phi' \to B\phi} \).

The chiral Lagrangian that describes the interactions of the lowest-lying mesons and baryons is written down in terms of the lightest meson-octet, baryon-octet, and baryon-decuplet fields [18, 19]. The meson and baryon octets are collected into 3 matrices \( \phi \) and \( B \), respectively, and the decuplet fields are represented by the Rarita-Schwinger tensor \( T_{\mu\nu}^a \), which is completely symmetric in its SU(3) indices \((a, b, c)\). The octet mesons enter through the exponential \( \Sigma = \xi^2 = \exp(i\phi/f) \), where \( f \) is the pion-decay constant.

In the heavy-baryon formalism [19], the baryons in the chiral Lagrangian are described by velocity-dependent fields, \( B_v \) and \( T^{\mu}_v \). For the strong interactions, the Lagrangian at lowest order in the derivative and \( m_s \) expansions is given by [19, 20]

\[
\mathcal{L}_s = \langle \bar{B}_v i v \cdot DB_v \rangle + 2D \langle \bar{B}_v S^{\mu}_v \{ A_\mu, B_v \} \rangle + 2F \langle \bar{B}_v S^{\mu}_v \{ A_\mu, B_v \} \rangle \\
- T^{\mu}_v iv \cdot DT_{\nu\mu} + \Delta m T^{\mu}_v T_{\nu\mu} + C (T^{\mu}_v A_\mu B_v + \bar{B}_v A_\mu T^{\mu}_v) \\
+ \frac{b_D}{2B_0} \langle \bar{B}_v \{ \chi_+, B_v \} \rangle + \frac{b_F}{2B_0} \langle \bar{B}_v \{ \chi_+, B_v \} \rangle + \frac{b_0}{2B_0} \langle \chi_+ \rangle \langle \bar{B}_v B_v \rangle \\
+ \frac{c}{2B_0} T^{\mu}_v \chi_+ T_{\nu\mu} - \frac{c_0}{2B_0} \langle \chi_+ \rangle \bar{T}^{\mu}_v T_{\nu\mu} + \frac{1}{4} f^2 \langle \chi_+ \rangle + \cdots,
\]

(19)

where \( \langle \cdots \rangle \) denotes Tr(\( \cdots \)) in flavor-SU(3) space, and we have shown only the relevant terms. In the first two lines, \( S^{\mu}_v \) is the spin operator and \( A_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi - \xi^\dagger \partial_\mu \xi \right) \), with further details given in Ref. [21]. The last two lines of \( \mathcal{L}_s \) contain \( \chi_+ = \xi^\dagger \chi \xi^\dagger + \xi^\dagger \chi \xi + \xi \chi \xi \), with \( \chi = 2B_0 M = 2B_0 \text{diag}(m_u, m_d, m_s) \), which explicitly breaks chiral symmetry. We will take the isospin limit \( m_u = m_d \equiv \hat{m} \) and consequently \( \chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2) \). The constants \( D, F, C, B_0, b_D, b_F, c, c_0 \) are free parameters which can be fixed from data.

In the center-of-mass (CM) frame, the \( P \)-wave amplitude for \( B\phi \to B'\phi' \) with total angular-momentum \( J \) has the form

\[
\mathcal{M} = -8\pi \sqrt{s} \chi^{\dagger}_{B'} \left\{ \left[ f^{(P,J=\frac{1}{2})}_{B\phi \to B'\phi'} + 2f^{(P,J=\frac{3}{2})}_{B\phi \to B'\phi'} \right] \hat{k} \cdot \hat{k} + \left[ f^{(P,J=\frac{3}{2})}_{B\phi \to B'\phi'} - f^{(P,J=\frac{1}{2})}_{B\phi \to B'\phi'} \right] i\sigma \cdot \hat{k}' \times \hat{k} \right\} \chi_B,
\]

(20)
where $\sqrt{s}$ is the CM energy, $\chi_B$ and $\chi_{B'}$ are the Pauli spinors of the baryons, $\hat{k}$ and $\hat{k}'$ denote the unit vectors of the momenta of $B$ and $B'$, respectively, and $f_{B\phi \rightarrow B'\phi'}^{(P,J)}$ are the partial-wave amplitudes. At lowest order in $\chi$PT, the $J = \frac{3}{2}$ amplitude arises from the Lagrangian in Eq. (19), and the pertinent diagrams are displayed in Fig. 1. The amplitudes in the $I = \frac{1}{2}$ channels are then extracted using the $I = \frac{1}{2}$ states in Eq. (13) and

$$|\Lambda K\rangle = |\Lambda K^+\rangle, \quad |\Sigma K\rangle = \sqrt{\frac{2}{3}} |\Sigma^- K^0\rangle + \frac{1}{\sqrt{2}} |\Sigma^0 K^-\rangle, \quad |\Xi\eta\rangle = |\Xi^- \eta\rangle,$$

(21)

which follow a phase convention consistent with the structure of the $\phi$ and $B_v$ matrices. We write the results as

$$f_{B\phi \rightarrow B'\phi'}^{(P,J=\frac{3}{2})} = -\mathcal{P}_{B\phi,B'\phi'} \frac{k_{B\phi} k_{B'\phi'} \sqrt{m_B m_{B'}}}{4\pi f^2 \sqrt{s}},$$

(22)

where the expressions for $\mathcal{P}_{B\phi,B'\phi'}$ corresponding to the four channels have been collected in Appendix A.

![FIG. 1: Diagrams contributing to the $P$-wave $J = \frac{3}{2}$ amplitude for $B\phi \rightarrow B'\phi'$ at leading order in $\chi$PT. In all figures, a dashed line denotes a meson field, a single (double) solid-line denotes an octet-baryon (decuplet-baryon) field, and each solid vertex is generated by $\mathcal{L}_a$ in Eq. (19).](image)

Since a $D$-wave amplitude has to be at least of second order in momentum, $O(k^2)$, it cannot arise from the Lagrangian in Eq. (19) alone. Also required is the Lagrangian involving baryons at second order in the derivative expansion, namely

$$\mathcal{L}'_s = -\frac{1}{2m_0} \bar{B}_v \left[D^2 - (v \cdot D)^2\right] B_v + \frac{1}{2m_0} \bar{T}_v^\mu \left[D^2 - (v \cdot D)^2\right] T_{\mu v} + \cdots,$$

(23)

where $m_0$ is the octet-baryon mass in the chiral limit, and we have shown only the relevant terms. These are two of the relativistic-correction terms in the $O(k^2)$ Lagrangian, and so their coefficients are fixed.

In the CM frame, the $D$-wave amplitude for $B\phi \rightarrow B'\phi'$ has the form

$$\mathcal{M}'_{B\phi \rightarrow B'\phi'} = -8\pi \sqrt{s} \chi_B^\dagger \left\{ \left[ 2 f_{B\phi \rightarrow B'\phi'}^{(D,J=\frac{3}{2})} + 3 f_{B\phi \rightarrow B'\phi'}^{(D,J=\frac{5}{2})} \right] \left[ \frac{3}{2} (\hat{k}' \cdot \hat{k})^2 - \frac{1}{2} \right] \\
+ \left[ f_{B\phi \rightarrow B'\phi'}^{(D,J=\frac{3}{2})} - f_{B\phi \rightarrow B'\phi'}^{(D,J=\frac{5}{2})} \right] (3\hat{k}' \cdot \hat{k}) i \sigma \cdot \hat{k}' \times \hat{k} \right\} \chi_B.$$

(24)

The leading nonzero contribution to this amplitude for $J = \frac{3}{2}$ comes from diagrams shown in Fig. 2. The resulting $I = \frac{1}{2}$ partial-wave amplitudes are given by

$$f_{B\phi \rightarrow B'\phi'}^{(D,J=\frac{3}{2})} = -\mathcal{D}_{B\phi,B'\phi'} \frac{k_{B\phi}^2 k_{B'\phi'}^2 \sqrt{m_B m_{B'}}}{4\pi f^2 m_0 \sqrt{s}},$$

(25)
where the expressions for $D_{B\phi,B'\phi}$ corresponding to the four channels have also been collected in Appendix A.

Numerically, we adopt the tree-level values $D = 0.80$ and $F = 0.50$, extracted from hyperon semileptonic decays [19], as well as $C = -1.7$, from the strong decays $T \rightarrow B\phi$.\(^2\) We also employ $f = f_\pi = 92.4\text{ MeV}$, $m_0 = 0.7\text{ GeV},^3$ and the isospin-averaged masses

$$
\begin{align*}
    m_\pi &= 137.3, & m_K &= 495.7, & m_\eta &= 547.3, \\
    m_N &= 938.9, & m_\Lambda &= 1115.7, & m_\Sigma &= 1193.2, & m_\Xi &= 1318.1, \\
    m_\Delta &= 1232.0, & m_{\Sigma^*} &= 1384.6, & m_{\Xi^*} &= 1533.4, & m_\Omega &= 1672.5,
\end{align*}
$$

all in units of MeV. Thus, putting together all the results above and setting $\sqrt{s} = m_\Omega$, from the $P$- and $D$-wave $S$-matrices we obtain

$$
\delta_{\Lambda K}^P = -0.65^\circ, \quad \sqrt{\varepsilon_P} = 0.013, \quad \delta_{\Lambda K}^D = +0.05^\circ, \quad \sqrt{\varepsilon_D} = 0.0009, \quad (27)
$$

which are pertinent to Eqs. (9) and (12). The effects of the closed channels turn out to be significant on $\delta_{\Lambda K}^P$ and $\varepsilon_P$. Excluding the $\Sigma K$ and $\Xi\eta$ channels would lead to $\delta_{\Lambda K}^P = -2.7^\circ$ and $\sqrt{\varepsilon_P} = 0.065$. The closed channels have minor effects on the $D$-wave parameters.

Since the numbers in Eq. (27) proceed from the leading nonzero amplitudes in $\chi$PT, part of the uncertainties in these predictions comes from our lack of knowledge about the higher-order contributions, which are presently incalculable. To get an idea of how they might affect our results, we redo the calculation using the one-loop values $D = 0.61$, $F = 0.40$, and $C = -1.2$ [19, 23], finding $\delta_{\Lambda K}^P = -0.47^\circ$, $\sqrt{\varepsilon_P} = 0.010$, $\delta_{\Lambda K}^D = +0.03^\circ$, and $\sqrt{\varepsilon_D} = 0.0003$. The differences between the two sets of results then provide an indication of the size of this part of the uncertainties. Another part is due to our lack of knowledge about the reliability of our $K$-matrix approximation. A comparison of $K$-matrix results in $\Lambda\pi$ scattering with experiment suggests that this approach gives results with the correct order-of-magnitude and sign [17, 24]. For these reasons, we may conclude that

$$
-0.9^\circ \leq \delta_{\Lambda K}^P - \delta_{\Lambda K}^D \leq -0.5^\circ, \quad 0.01 \leq \sqrt{\varepsilon_P} \leq 0.02, \quad 0.0003 \leq \sqrt{\varepsilon_D} \leq 0.002. \quad (28)
$$

We will employ these numbers in evaluating the asymmetries.

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\(^2\) We have chosen the sign of $C$ after nonrelativistic quark models [19], which predict $3F = 2D$ and $C = -2D$, both well satisfied by the adopted $D$, $F$, and $C$ values.

\(^3\) This $m_0$ value comes from simultaneously fitting the tree-level formulas for the octet-baryon masses and the sigma term, $\sigma_N = -2(b_D + b_P + 2b_0) \hat{m}$, all derived from Eq. (19), to the measured masses and the empirical value [22] $\sigma_N \approx 45\text{ MeV}$. 

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FIG. 2: Diagrams for the leading nonzero contribution to the $D$-wave $J = \frac{3}{2}$ amplitude for $B\phi \rightarrow B'\phi'$. Each hollow vertex is generated by $L'_s$ in Eq. (23).
IV. \textit{CP}-VIOLATING ASYMMETRIES WITHIN STANDARD MODEL

To calculate the \textit{CP}-violating phases, we will work in the framework of heavy-baryon $\chi$PT. The amplitude for the weak decay $\Omega \rightarrow B\phi$ in the heavy-baryon approach has the general form

$$i\mathcal{M}_{\Omega \rightarrow B\phi} = -\langle B\phi |\mathcal{L}|\Omega \rangle = \bar{u}_B \left( A^{(P)}_{B\phi} + 2S_v \cdot k_\phi A^{(D)}_{B\phi} \right) k_\phi \cdot u_\Omega. \quad (29)$$

where $k_\phi$ is the four-momentum of $\phi$, and the superscripts refer to the $P$- and $D$-wave components of the amplitude. In the rest frame of $\Omega$, these components are related to the $p$ and $d$ amplitudes by

$$p = |k_\phi| A^{(P)}, \quad d = k_\phi^2 A^{(D)}. \quad (30)$$

We will follow the usual prescription for estimating a weak phase [6, 7, 9], namely, first calculating the imaginary part of the amplitude and then dividing it by the real part of the amplitude extracted from experiment under the assumption of no strong phases and no \textit{CP} violation.

Within the SM, the weak interactions responsible for hyperon nonleptonic decays are described by the short-distance effective $|\Delta S|=1$ Hamiltonian [25]

$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} V^*_{us} V_{ud} \sum_{i=1}^{10} C_i Q_i + \text{H.c}, \quad (31)$$

where $V_{kl}$ are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [26],

$$C_i \equiv z_i + \tau y_i \equiv z_i - \frac{V^*_{td} V_{ts}}{V^*_{ud} V_{us}} y_i \quad (32)$$

are the Wilson coefficients, and $Q_i$ are four-quark operators whose expressions can be found in Ref. [25]. In this case, the weak phases $\phi^{P,D}$ of Eq. (9) proceed from the $\text{CP}$-violating phase residing in the CKM matrix, and its elements appearing in $C_i$ above can be expressed in the Wolfenstein parametrization [27] as

$$V^*_{ud} V_{us} = \lambda, \quad V^*_{td} V_{ts} = -A^2 \lambda^5 (1 - \rho + i\eta) \quad (33)$$

at lowest order in $\lambda$. As is well known, $\mathcal{H}_w$ transforms mainly as $(8_L, 1_R) \oplus (27_L, 1_R)$ under SU(3)$_L \times$SU(3)$_R$ rotations. It is also known from experiment that the octet term dominates the 27-plet term [28]. We, therefore, assume in what follows that within the SM the decays of interest are completely characterized by the $(8_L, 1_R), \ |\Delta I| = \frac{1}{2}$ interactions. The leading-order chiral Lagrangian for such interactions is [18, 29]

$$\mathcal{L}_w = h_D \langle \bar{B}_v \left\{ \xi^+ h, \xi \right\}, B_v \rangle + h_F \langle \bar{B}_v \left\{ \xi^+ h, B_v \right\} \rangle + h_C \bar{T}_\nu \xi^+ h \xi T_{\nu\mu} + \text{H.c.}, \quad (34)$$

where the $3 \times 3$-matrix $h$ selects out $s \rightarrow d$ transitions, having elements $h_{kl} = \delta_{k2}\delta_{3l}$, and the parameters $h_{D,F,C}$ contain the weak phases of interest. These phases are induced primarily by the imaginary part of $C_6$ associated with the penguin operator $Q_6$, and this is due to its chiral structure and the relative size of $\text{Im} C_6$. In order to relate the imaginary part of $h_{D,F,C}$ to $\text{Im} C_6$, we use the
results of Ref. [9], obtained from factorizable and nonfactorizable contributions. Accordingly, we have

\[ \text{Im } h_D = 5.14 \, y_6, \quad \text{Im } h_F = -14.3 \, y_6, \quad \text{Im } h_C = 32.5 \, y_6, \quad (35) \]

all in units of \( \sqrt{\frac{\alpha}{2}} f, G_F, m_\pi, A^2 \lambda^5 \eta \).

From \( L_w \) together with \( L_s \), we can derive the diagrams displayed in Fig. 3, which represent the leading-order contributions to the \( P \)-wave transitions in \( \Omega^- \to \Lambda \bar{K}, \Xi \pi \) and yield the amplitudes

\[
\mathcal{A}_{\Lambda \bar{K}}^{(P)} = \frac{\mathcal{C} (h_D - 3h_F)}{2\sqrt{3} f (m_\Xi - E_\Lambda)} - \frac{\mathcal{C} h_C}{2\sqrt{3} f (m_\Omega - m_\Xi)},
\]

\[
\mathcal{A}_{\Xi \pi}^{(P)} = \sqrt{\frac{2}{3}} \mathcal{A}_{\Xi \pi}^{(P)} + \frac{1}{\sqrt{3}} \mathcal{A}_{\Xi \pi}^{(P)} = \frac{-\mathcal{C} h_C}{2\sqrt{3} f (m_\Omega - m_\Xi)}. \quad (36)
\]

Applying Eq. (35) in \( p_{B\phi} = |k_\phi| A_{B\phi}^{(P)} \) then leads to

\[
\text{Im} \frac{p_{\Lambda \bar{K}}^{\text{expt}}}{p_\Lambda} = -1.15 \ A^2 \lambda^5 \eta \ y_6, \quad \frac{\text{Im} p_{\Xi \pi}^{\text{expt}}}{p_\Xi} = +23.6 \ A^2 \lambda^5 \eta \ y_6, \quad (37)
\]

where \( p_{\Lambda \bar{K}}^{\text{expt}} \) and \( p_{\Xi \pi}^{\text{expt}} \) are the central values of \( p_{\Lambda \Xi} \) in Eq. (14). The uncertainties in these predictions are due to our neglect of higher-order terms that are presently incalculable and to our lack of knowledge on the reliability of the matrix-element calculation. Therefore, we assign an error of 100\% to these ratios, as was similarly done in Ref. [9] for the weak phases in \( \Lambda \to p\pi \) and \( \Xi \to \Lambda \pi \). Thus, using \( A^2 \lambda^5 \eta = 1.26 \times 10^{-4} \) and \( y_6 = -0.096 \), as in Ref. [9], we obtain

\[
\phi_\Lambda^P = (1.4 \pm 1.4) \times 10^{-5}, \quad \phi_\Xi^P = (-2.9 \pm 2.9) \times 10^{-4}. \quad (38)
\]

The \( \phi_\Xi^P \) result is comparable in size to that estimated in Ref. [11] using the vacuum-saturation method.\footnote{The numerical differences between the estimates also arise from the use of a positive \( \mathcal{C} \) value in Ref. [11].}

Turning now to the \( D \)-wave phases, we note that the expression for the \( \mathcal{A}^{(D)} \) term in Eq. (29) implies that \( L_w \), in conjunction with \( L_s \) and \( L'_s \), cannot solely give rise to diagrams for the \( D \)-wave components. Rather, the weak Lagrangian that can generate the leading nonzero contributions to this term must have the Dirac structure \( \bar{B}_v S^\mu_v \partial_\mu \mathcal{A}_\alpha T^\alpha \), which is of \( \mathcal{O}(k^2) \). The \( D \)-wave amplitude

\[ \text{FIG. 3: Diagrams representing standard-model contributions to the leading-order } P \text{-wave amplitude for } \Omega^- \to B\phi. \text{ Each square represents a weak vertex generated by } L_w \text{ in Eq. (34).} \]
at $\mathcal{O}(k^2)$ can also receive contributions from so-called tadpole diagrams, each being a combination of a strong $\Omega B\phi K$ vertex, generated by a Lagrangian having the structure $B_v S_\mu^a A_\mu A_\alpha T^a_\alpha$, and a $K$-vacuum vertex coming from a weak Lagrangian of $\mathcal{O}(m_s)$. Unfortunately, at present the parameters of these strong and weak Lagrangians of $\mathcal{O}(k^2)$ are incalculable. The best that we can do is to make a crude estimate based on naive dimensional analysis [30]. Thus, since the lowest-order chiral Lagrangian yielding $p_{B\phi}$ is of $\mathcal{O}(1)$, whereas that yielding $d_{B\phi}$ is of $\mathcal{O}(k^2)$, and since $k \sim m_s$ in hyperon nonleptonic decays, we expect that

$$\frac{d_{B\phi}}{p_{B\phi}} \sim \frac{m_s^2}{\Lambda^2_\chi},$$

where $\Lambda_\chi \sim 4\pi f$ is the chiral-symmetry breaking scale. It is worth remarking here that for $m_s \sim 0.12$ GeV [31] this naive expectation is compatible with the value of $d_\Lambda/p_\Lambda$ from Eq. (14), in which the $d_\Lambda$ number is determined largely by the preliminary data from HyperCP [5]. For these reasons, we make the approximation

$$\phi^P = \text{Im} \frac{d}{d_{\text{expt}}} = \frac{m_s^2}{\Lambda^2_\chi} \frac{p_{\text{expt}}}{d_{\text{expt}}} \phi^P$$

for the magnitude of the phase, where $\phi^P$ comes from Eq. (38). Since $d_\Xi$ as quoted in Eq. (14) is poorly determined, we take the further approximation $d_\Xi = p_\Xi d_\Lambda/p_\Lambda$ for its magnitude in order to estimate $\phi^D_\Xi$. All this leads to

$$\phi^D_\Lambda = (0 \pm 3) \times 10^{-5}, \quad \phi^D_\Xi = (0 \pm 6) \times 10^{-4}.$$  

The errors that we quote in $\phi^P,D_\Lambda$ are obviously not Gaussian and simply indicate the ranges resulting from our calculation.

Putting together the numbers from Eqs. (14), (28), (38), and (41) in Eq. (9) yields

$$-9 \times 10^{-6} \leq A_\Omega \leq +2 \times 10^{-6}.$$  

We note that the second term on the right-hand side of Eq. (9), which would vanish if the $\Xi\pi \leftrightarrow \Lambda\bar{K}$ rescattering were ignored, has turned out to be the largest one. This is due to $\phi^P_\Xi$ and $\varepsilon_P$ being much larger than $\phi^P_\Lambda$ and $\varepsilon_D$, respectively, as well as to $\delta^P_{\Lambda K}$ being small. For the partial-rate asymmetry in Eq. (12), we find

$$0 \leq \Delta_\Omega \leq 13 \times 10^{-6}.$$  

This is comparable to the corresponding asymmetry in $\Omega \to \Xi\pi$ [11], but larger than those in octet-hyperon decays [6].

Since the asymmetry measured by HyperCP is the sum $A_{\Omega A} = A_\Omega + A_\Lambda$, it is important to know how $A_\Omega$ compares with $A_\Lambda$. The SM contribution to $A_\Lambda$ has been evaluated most recently to be $-3 \times 10^{-5} \leq A_\Lambda \leq 4 \times 10^{-5}$ [9]. Thus within the standard model $A_\Omega$ is smaller than $A_\Lambda$, but not negligibly so, and the resulting $A_{\Omega A}$ has a value within the range

$$-4 \times 10^{-5} \leq A_{\Omega A} \leq 4 \times 10^{-5}.$$  

For this observable, HyperCP expects to have a statistical precision of $9 \times 10^{-2}$ [5], and so its measurement will unlikely be sensitive to the SM effects.
V. \textit{CP-Violating Asymmetries Due to New Physics}

Here we evaluate $A_\Omega$ and $\Delta_\Omega$ arising from possible physics beyond the standard model. In particular, we consider contributions generated by the chromomagnetic-penguin operators (CMO), which in some new-physics models could be significantly larger than their SM counterparts [8, 32, 33]. The relevant effective Hamiltonian can be written as [33]

$$\mathcal{H}_{w,g} = C_g Q_g + \tilde{C}_g \tilde{Q}_g + \text{H.c.},$$

(45)

where $C_g$ and $\tilde{C}_g$ are the Wilson coefficients, and

$$Q_g = \frac{g_s}{16\pi^2} \bar{d} \sigma^{\mu\nu} t^a (1 + \gamma_5) s G_{\mu\nu}^a,$$

$$\tilde{Q}_g = \frac{g_s}{16\pi^2} \bar{d} \sigma^{\mu\nu} t^a (1 - \gamma_5) s G_{\mu\nu}^a$$

(46)

are the CMO, with $G_{\mu\nu}^a$ being the gluon field-strength tensor, $g_s$ the gluon coupling constant, and $\text{Tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$. Since various new-physics scenarios may contribute differently to the coefficients of the operators, we will not focus on specific models, but will instead adopt a model-independent approach, only assuming that the contributions are potentially sizable, in order to estimate bounds on the resulting asymmetries as allowed by constraints from kaon measurements.

The chiral Lagrangian proceeding from the CMO has to respect their symmetry properties. Under SU(3)$_L \times$SU(3)$_R$ rotations $Q_g$ and $\tilde{Q}_g$ transform as $(3_L, 3_R)$ and $(\bar{3}_L, \bar{3}_R)$, respectively. Moreover, under a CPS transformation (a CP operation followed by interchanging the $s$ and $d$ quarks) $Q_g$ and $\tilde{Q}_g$ change into each other. These symmetry properties are also those of the quark densities $\bar{q}_q$ and $\tilde{q}_q$, of which the lowest-order chiral realization has been derived in Ref. [9]. From this realization, we can infer the leading-order chiral Lagrangian induced by the CMO, namely

$$\mathcal{L}_{w,g} = \beta_D \langle \bar{B}_v \left\{ \xi^\dagger h \xi^\dagger, B_v \right\} \rangle + \beta_F \langle \bar{B}_v \left[ \xi^\dagger h \xi^\dagger, B_v \right] \rangle + \beta_0 \langle h \Sigma \rangle \langle \bar{B}_v B_v \rangle$$

$$+ \beta_D \langle \bar{B}_v \left\{ \xi h \xi, B_v \right\} \rangle + \beta_F \langle \bar{B}_v \left[ \xi h \xi, B_v \right] \rangle + \bar{\beta}_0 \langle h \bar{\Sigma} \rangle \langle \bar{B}_v B_v \rangle$$

$$+ \beta_C T^a \xi h \xi T_{va} - \beta_0 \langle h \bar{\Sigma} \rangle \langle \bar{T}^a T_{va} \rangle + \bar{\beta}_C T^a \xi h \xi T_{va} - \bar{\beta}_0 \langle h \Sigma \rangle \langle \bar{T}^a T_{va} \rangle$$

$$+ \beta_\varphi f^2 B_0 \langle h \Sigma \rangle + \tilde{\beta}_\varphi f^2 B_0 \langle h \bar{\Sigma} \rangle + \text{H.c.},$$

(47)

where $\beta_i$ ($\tilde{\beta}_i$) are parameters containing the coefficient $C_g$ ($\tilde{C}_g$). The part of this Lagrangian without the decuplet-baryon fields was first written down in Ref. [10].

From $\mathcal{L}_{w,g}$ along with $\mathcal{L}_s$, we derive the diagrams shown in Fig. 4, which represent the lowest-order contributions induced by the CMO to the $P$-wave transitions in $\Omega \rightarrow \Lambda K, \Xi \pi$. We remark that each of the three diagrams in the figure is of $\mathcal{O}(1)$ in the $m_s$ expansion, and that Fig. 3 does not include the meson-pole diagram because within the SM it contributes only at next-to-leading order. The amplitudes following from Fig. 4 are

$$A^{(P)g}_{\Lambda K} = \frac{c}{2\sqrt{3} f (m_\Xi - E_\Lambda)} - \frac{c \beta_C^+}{2\sqrt{3} f (m_\Omega - m_\Xi)},$$

$$A^{(P)g}_{\Xi \pi} = \frac{-c \beta_C^+}{2\sqrt{3} f (m_\Omega - m_\Xi)} + \frac{\sqrt{3} c \beta_\varphi^+}{2 f (m_s - m)},$$

(48)
where $\beta_i^+ \equiv \beta_i + \tilde{\beta}_i$ and we have used $m_K^2 - m_n^2 = B_0 (m_s - \hat{m})$, derived from Eq. (23).\footnote{It is worth noting here that, as in the $\Lambda \rightarrow p\pi$ and $\Xi \rightarrow \Lambda\pi$ cases \cite{10}, each of the two amplitudes in Eq. (48) vanishes if we set $\beta_{D,F}^+ = \kappa b_D, \beta_C^+ = \kappa c$, and $\beta_{\varphi}^+ = \kappa / 2$, with $\kappa^+$ being a constant, take the limit $E_\Lambda = m_\Lambda$, and use the relations $m_\Xi - m_\Lambda = 2 \frac{3}{2} (b_D - 3b_F) (m_s - \hat{m})$ and $m_\Xi - m_\Xi = \frac{2}{3} c (m_s - \hat{m})$, both derived from Eq. (19). This satisfies the requirement implied by the Feinberg-Kabir-Weinberg theorem \cite{34} that the operator $\bar{d}s$ cannot contribute to physical decay amplitudes \cite{35}, and thus serves as a check for the formulas in Eq. (48).}

In order to estimate the weak phases in $A_\Omega$, we need to determine the parameters $\beta_i^+$ in terms of the underlying coefficient $C_g^+ \equiv C_g + \tilde{C}_g$, which is the combination corresponding to parity-conserving transitions. From the effective Hamiltonian in Eq. (45) and the chiral Lagrangian in Eq. (47), we can derive the one-particle matrix elements

\begin{equation}
\langle n | \mathcal{H}_{w,g} | \Lambda \rangle = \frac{\beta_D^+ + 3\beta_F^+}{\sqrt{6}} \bar{u}_n u_\Lambda , \quad \langle \Lambda | \mathcal{H}_{w,g} | \Xi^0 \rangle = \frac{\beta_D^+ - 3\beta_F^+}{\sqrt{6}} \bar{u}_\Lambda u_\Xi , \\
\langle \Xi^- | \mathcal{H}_{w,g} | \Omega^- \rangle = -\frac{\beta_D^+}{\sqrt{3}} \bar{u}_\Xi u_\Omega , \quad \langle \pi^- | \mathcal{H}_{w,g} | K^- \rangle = \beta_{\varphi}^+ B_0 .
\end{equation}

(49)

Since there is presently no reliable way to determine these matrix elements from first principles, we employ the MIT bag model to estimate them. The results for $\beta_{D,F,\varphi}^+$ have already been derived in Ref. \cite{10} using the bag-model calculations of Ref. \cite{36} and are given by

\begin{equation}
\beta_D^+ = -\frac{3}{7} \beta_F^+ = \frac{2 I_M N^4}{\pi R^2} C_g^+ , \quad \beta_C^+ = \frac{-8 I_M N^4 \sqrt{2m_K^2}}{\pi B_0 R^2} C_g^+ ,
\end{equation}

(50)

where $N, R,$ and $I_M$ are bag parameters. For $\beta_{\varphi}^+$, extending the work of Ref. \cite{36} we find

\begin{equation}
\beta_{\varphi}^+ = \frac{-8 I_M N^4}{\pi R^2} C_g^+ .
\end{equation}

(51)

Numerically, we take $R = 5.0 \text{ GeV}^{-1}$ for the octet baryons, $R = 5.4 \text{ GeV}^{-1}$ for the decuplet baryons, and $R = 3.3 \text{ GeV}^{-1}$ for the mesons, after Refs. \cite{36, 37}. In addition, as in Ref. \cite{10}, we have $N = 2.27$ and $I_M = 1.63 \times 10^{-3}$ for both the baryons and mesons. It follows that

\begin{equation}
\beta_D^+ = -\frac{3}{7} \beta_F^+ = 1.10 \times 10^{-3} C_g^+ \text{ GeV}^2 , \quad \beta_C^+ = -3.78 \times 10^{-3} C_g^+ \text{ GeV}^2 , \quad \beta_{\varphi}^+ B_0 = -7.09 \times 10^{-3} C_g^+ \text{ GeV}^3 ,
\end{equation}

(52)
We note that $C_g^+$ here is the Wilson coefficient at the low scale $\mu = \mathcal{O}(1 \text{ GeV})$ and hence already contains the QCD running from the new-physics scales. We also note that the bag-model numbers in Eq. (52) are comparable in magnitude to the natural values of the parameters as obtained from naive dimensional analysis [30],

$$\beta_{D,F,C}^{\text{NDA}} = \frac{C_g g_s \Lambda^2}{16\pi^2} \equiv 0.0024 C_g \text{ GeV}^2, \quad \beta_{\varphi}^{\text{NDA}} B_0 = \frac{C_g g_s \Lambda^3}{16\pi^2} \equiv 0.0028 C_g \text{ GeV}^3, \quad (53)$$

where we have chosen $g_s = \sqrt{\frac{4\pi}{\Lambda^2}}$. The differences between the two sets of numbers provide an indication of the level of uncertainty in estimating the matrix elements. This will be taken into account in our results below.

Applying Eq. (52) in $p_{B\phi} = |k_\phi| A^{(P)}_{B\phi}$ then leads to the CMO contributions

$$\left(\phi^P_\Lambda\right)_g = (-1.0 \pm 2.0) \times 10^5 \text{ GeV Im } C_g^+, \quad \left(\phi^P_\Xi\right)_g = (2.3 \pm 4.6) \times 10^5 \text{ GeV Im } C_g^+. \quad (54)$$

where, as in the $\Lambda \to p\pi$ and $\Xi \to \Lambda\pi$ cases [10], we have assigned an error of $200\%$ to each of these numbers to reflect the uncertainty due to our neglect of higher-order terms that are presently incalculable and the uncertainty in estimating the matrix elements above. For the $D$-wave phases, we have here the same problem in estimating them as in the standard-model case, and so we have to resort again to dimensional arguments. Thus, since the $D$-wave amplitude is parity violating, we have

$$\left(\phi^D_\Lambda\right)_g = (0 \pm 3) \times 10^5 \text{ GeV Im } C_g^-, \quad \left(\phi^D_\Xi\right)_g = (0 \pm 8) \times 10^5 \text{ GeV Im } C_g^-,$$

where $C_g^- \equiv C_g - \tilde{C}_g$ is the combination corresponding to parity-violating transitions.

Putting together the numbers from Eqs. (14), (28), (54), and (55) in Eq. (9), we find

$$10^{-4} \text{ GeV}^{-1} (A_\Omega)_g = (0.3 \pm 1.3) \text{ Im } C_g^+ + (0 \pm 1) \text{ Im } C_g^-.$$

As in the SM result, the second term in $A_\Omega$ dominates these numbers. For the partial-rate asymmetry, we obtain

$$\left(\Delta_\Omega\right)_g = (-0.7 \pm 1.4) \times 10^4 \text{ GeV Im } C_g^+.$$

We can now write down the contribution of the CMO to the sum of asymmetries $A_{\Omega A} = A_\Omega + A_\Lambda$ being measured by HyperCP. The most recent evaluation of their contribution to $A_\Lambda$ has been done in Ref. [10], the result being $10^{-4} \text{ GeV}^{-1} (A_\Lambda)_g = (-4.2 \pm 8.3) \text{ Im } C_g^+ + (3.5 \pm 7.0) \text{ Im } C_g^-$. Evidently, $(A_\Omega)_g$ is much smaller than, though still not negligible compared to, $(A_\Lambda)_g$. Summing the two asymmetries yields

$$10^{-4} \text{ GeV}^{-1} (A_{\Omega A})_g = (-4 \pm 10) \text{ Im } C_g^+ + (4 \pm 8) \text{ Im } C_g^-.$$

Since the CMO also contribute to the $CP$-violating parameters $\epsilon$ in kaon mixing and $\epsilon'$ in kaon decay, which are now well measured, it is possible to obtain bounds on $(A_{\Omega A})_g$ and $(\Delta_\Omega)_g$ using
the $\epsilon$ and $\epsilon'$ data. As discussed in Ref. [10], the experimental values $|\epsilon| = (22.80 \pm 0.13) \times 10^{-4}$ and $\text{Re}(\epsilon'/\epsilon) = (16.6 \pm 1.6) \times 10^{-4}$ [12, 31] imply that

$$|\text{Im}C^+_g| < 5.0 \times 10^{-8} \text{GeV}^{-1}, \quad |\text{Im}C^-_g| < 7.4 \times 10^{-9} \text{GeV}^{-1}. \quad (59)$$

Then, from Eqs. (57) and (58), it follows that

$$|A_{\Omega\Lambda}| < 8 \times 10^{-3}, \quad |\Delta_\Omega| < 1 \times 10^{-3}. \quad (60)$$

The upper limits of these ranges well exceed those within the SM in Eqs. (43) and (44), but the largest size of $(A_{\Omega\Lambda})_g$ is still an order of magnitude below the expected sensitivity of HyperCP [5]. This, nevertheless, implies that a nonzero measurement by HyperCP would be an unmistakable signal of new physics.

VI. CONCLUSION

We have evaluated the sum of the $CP$-violating asymmetries $A_\Omega$ and $A_\Lambda$ occurring in the decay chain $\Omega \rightarrow \Lambda K \rightarrow p\pi K$, which is currently being studied by the HyperCP experiment. The dominant contribution to $A_\Omega$ has turned out to be due to final-state interactions via $\Omega \rightarrow \Xi\pi \rightarrow \Lambda K$. We have found that both within and beyond the standard model $A_\Omega$ is smaller than $A_\Lambda$, but not negligibly so. Taking a model-independent approach, we have also found that contributions to $A_{\Omega\Lambda} = A_\Omega + A_\Lambda$ from possible new-physics through the chromomagnetic-penguin operators are allowed by constraints from kaon data to exceed the SM effects by up to two orders of magnitude. In summary,

$$|A_{\Omega\Lambda}|_{\text{SM}} \leq 4 \times 10^{-5}, \quad |A_{\Omega\Lambda}|_g < 8 \times 10^{-3}. \quad (61)$$

Since the SM contribution is well beyond the expected reach of HyperCP, a finding of nonzero asymmetry would definitely indicate the presence of new physics. In any case, the upcoming data on $A_{\Omega\Lambda}$ will yield information which complements that to be gained from the measurement of $A_{\Xi\Lambda}$ in $\Xi \rightarrow \Lambda\pi \rightarrow p\pi\pi$.

Finally, we have shown that the contribution of $\Omega \rightarrow \Xi\pi \rightarrow \Lambda K$ also causes the partial-rate asymmetry $\Delta_\Omega$ in $\Omega \rightarrow \Lambda K$ to be nonvanishing, thereby providing another means to observe $CP$ violation in this decay. This asymmetry and that in $\Omega \rightarrow \Xi\pi$ tend to be larger than the corresponding asymmetries in octet-hyperon decays and hence are potentially useful probes of $CP$ violation in future experiments. Since $\Delta_\Omega$ results from the interference of $P$-wave amplitudes, a measurement of it will probe the underlying parity-conserving interactions. Numerically, we have found

$$0 \leq (\Delta_\Omega)_{\text{SM}} \leq 1 \times 10^{-5}, \quad |\Delta_\Omega|_g < 1 \times 10^{-3}. \quad (62)$$

where the bound on the contribution of the CMO arises from the constraint imposed by $\epsilon$ data.
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APPENDIX A: $\mathcal{P}$ AND $\mathcal{D}$ FACTORS IN $P$-WAVE AND $D$-WAVE $J = \frac{3}{2}$ AMPLITUDES FOR $B \phi \to B' \phi'$ IN $I = \frac{1}{2}$, $S = -2$ CHANNELS

For the four coupled channels, the $\mathcal{P}$ factors are

$$\mathcal{P}_{\Xi, \Xi} = \frac{-\frac{1}{6}(D - F)^2}{E_\Xi - E_\Xi' - m_\Xi} + \frac{\frac{1}{12}C^2}{\sqrt{s} - m_\Xi},$$

$$\mathcal{P}_{\Xi, A} = \frac{1}{\Xi} \frac{D + F}{E_\Xi - E_\Xi' - m_A} + \frac{\frac{1}{12}C^2}{\sqrt{s} - m_\Xi},$$

$$\mathcal{P}_{\Xi, \Sigma, \Xi} = \frac{-\frac{1}{6}D(D - 3F)}{E_\Xi - E_\Xi' - m_\Lambda} + \frac{-\frac{1}{3}(D + F)F}{E_\Xi - E_\Xi' - m_\Sigma} + \frac{-\frac{1}{12}C^2}{\sqrt{s} - m_\Xi} + \frac{\frac{1}{36}C^2}{E_\Xi - E_\Xi' - m_\Xi},$$

$$\mathcal{P}_{\Xi, \Xi} = \frac{-\frac{1}{6}(D - F)(D + 3F)}{E_\Xi - E_\Xi' - m_\Xi} + \frac{-\frac{1}{12}C^2}{\sqrt{s} - m_\Xi} + \frac{-\frac{1}{36}C^2}{E_\Xi - E_\Xi' - m_\Xi},$$

$$\mathcal{P}_{\Lambda, \Lambda} = \frac{\frac{1}{18}(D + 3F)^2}{E_\Lambda - E_\Lambda' - m_N} + \frac{\frac{1}{12}C^2}{\sqrt{s} - m_\Xi},$$

$$\mathcal{P}_{\Lambda, \Sigma, \Xi} = \frac{-\frac{1}{6}(D - F)(D + 3F)}{E_\Lambda - E_\Lambda' - m_\Lambda} + \frac{-\frac{1}{12}C^2}{\sqrt{s} - m_\Xi},$$

$$\mathcal{P}_{\Lambda, \Xi} = \frac{\frac{1}{3}D(D - 3F)}{E_\Lambda - E_\Xi' - m_\Lambda} + \frac{-\frac{1}{12}C^2}{\sqrt{s} - m_\Xi},$$

$$\mathcal{P}_{\Sigma, \Sigma, \Xi} = \frac{-\frac{1}{6}(D - F)^2}{E_\Xi - E_\Xi' - m_N} + \frac{\frac{1}{12}C^2}{\sqrt{s} - m_\Xi} + \frac{\frac{2}{15}C^2}{E_\Xi - E_\Xi' - m_\Xi},$$

$$\mathcal{P}_{\Sigma, \Xi} = \frac{\frac{1}{3}D(D + F)}{E_\Xi - E_\Xi' - m_\Sigma} + \frac{\frac{1}{12}C^2}{\sqrt{s} - m_\Xi} + \frac{-\frac{1}{36}C^2}{E_\Xi - E_\Xi' - m_\Xi},$$

$$\mathcal{P}_{\Xi, \Xi} = \frac{\frac{1}{6}(D + 3F)^2}{E_\Xi - E_\Xi' - m_\Xi} + \frac{\frac{1}{12}C^2}{\sqrt{s} - m_\Xi} + \frac{\frac{1}{36}C^2}{E_\Xi - E_\Xi' - m_\Xi},$$

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and the $\mathcal{D}$ factors

\[
\mathcal{D}_{\pi, \pi} = \frac{(D - F)^2}{60(E_{\pi} - E_{\pi}' - m_{\pi})^2} - \frac{7C^2}{540(E_{\pi} - E_{\pi}' - m_{\pi})^2},
\]
\[
\mathcal{D}_{\pi, \Lambda K} = \frac{-D(D + F)}{30(E_{\pi} - E_{K}' - m_{\Lambda})^2} + \frac{7C^2}{180(E_{\pi} - E_{K}' - m_{\pi})^2},
\]
\[
\mathcal{D}_{\pi, \Sigma K} = \frac{D(D - 3F)}{90(E_{\pi} - E_{K}' - m_{\Lambda})^2} + \frac{(D + F)F}{15(E_{\pi} - E_{K}' - m_{\Sigma})^2} + \frac{7C^2}{270(E_{\pi} - E_{K}' - m_{\pi})^2},
\]
\[
\mathcal{D}_{\eta, \pi} = \frac{(D - F)(D + 3F)}{60(E_{\eta} - E_{\pi}' - m_{\pi})^2} + \frac{-7C^2}{180(E_{\pi} - E_{\eta}' - m_{\pi})^2},
\]
\[
\mathcal{D}_{\Lambda K, \Lambda K} = \frac{-(D + 3F)^2}{180(E_{\Lambda} - E_{K}' - m_{\Lambda})^2},
\]
\[
\mathcal{D}_{\Sigma K, \Sigma K} = \frac{(D - F)(D + 3F)}{60(E_{\Sigma} - E_{K}' - m_{\Sigma})^2},
\]
\[
\mathcal{D}_{\Lambda K, \Sigma K} = \frac{-D(D - 3F)}{90(E_{\Lambda} - E_{\eta}' - m_{\Lambda})^2},
\]
\[
\mathcal{D}_{\Sigma K, \Sigma K} = \frac{(D - F)^2}{60(E_{\Sigma} - E_{K}' - m_{\Sigma})^2} + \frac{14C^2}{135(E_{\Sigma} - E_{K}' - m_{\Delta})^2},
\]
\[
\mathcal{D}_{\Sigma K, \Sigma K} = \frac{-D(D + F)}{30(E_{\Sigma} - E_{\eta}' - m_{\Sigma})^2} + \frac{-7C^2}{180(E_{\Sigma} - E_{\eta}' - m_{\pi})^2},
\]
\[
\mathcal{D}_{\Sigma K, \Sigma K} = \frac{-(D + 3F)^2}{180(E_{\Sigma} - E_{\eta}' - m_{\Sigma})^2} + \frac{7C^2}{180(E_{\Sigma} - E_{\eta}' - m_{\pi})^2},
\]

where $E_{\phi}'$ is the energy of $\phi$ in the final state. We note that contributions to the propagators from the $\Delta m$ and quark-mass terms in Eq. (19) have been implicitly included in these results.

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