The maximum spectral radius of $C_4$-free graphs of given order and size

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Abstract

Suppose that $G$ is a graph with $n$ vertices and $m$ edges, and let $\mu$ be the spectral radius of its adjacency matrix.

Recently we showed that if $G$ has no 4-cycle, then $\mu^2 - \mu \leq n - 1$, with equality if and only if $G$ is the friendship graph.

Here we prove that if $m \geq 9$ and $G$ has no 4-cycle, then $\mu^2 \leq m$, with equality if $G$ is a star. For $4 \leq m \leq 8$ this assertion fails.

Keywords: 4-cycles; graph spectral radius; graphs with no 4-cycles; friendship graph.

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This note is part of an ongoing project aiming to build extremal graph theory on spectral grounds, see, e.g., [3] and [6, 14].

Suppose $G$ is a graph with $n$ vertices and $m$ edges and let $\mu(G)$ be the spectral radius of its adjacency matrix. How large can $\mu(G)$ be if $G$ has no cycles of length 4? This question was partially answered in [10], Theorem 3:

Let $G$ be a graph of order $n$ with $\mu(G) = \mu$. If $G$ has no 4-cycles, then

$$\mu^2 - \mu \leq n - 1. \quad (1)$$

Equality holds if and only if every two vertices of $G$ have exactly one common neighbor.

The condition for equality in (1) is a popular topic: as shown in [4] and [5], the only graph satisfying this condition is the friendship graph - a set of $\lfloor n/2 \rfloor$ triangles sharing a single common vertex. Thus equality is possible only for $n$ odd, and (1) may be improved for even $n$.

Conjecture 1 Let $G$ be a graph of even order $n$ with $\mu(G) = \mu$. If $G$ has no 4-cycles, then

$$\mu^3 - \mu^2 - (n - 1) \mu + 1 \leq 0. \quad (2)$$

Equality holds if and only if $G$ is a star of order $n$ with $n/2 - 1$ disjoint additional edges.
Note that the number of edges of $G$ is missing in (1) and (2). In contrast, Nosal [15] showed that if $\mu(G) > \sqrt{m}$, then $G$ has triangles. Our main result here is a similar assertion for 4-cycles:

**Theorem 2** Let $m \geq 9$ and $G$ be a graph with $m$ edges. If $\mu(G) > \sqrt{m}$, then $G$ has a 4-cycle.

Note that Theorem 2 is tight, for all stars are $C_4$-free graphs with $\mu(G) = \sqrt{m}$. Also, let $S_{n,1}$ be the star of order $n$ with an edge within its independent set: $S_{n,1}$ is $C_4$-free and has $n$ edges, but $\mu(G) > \sqrt{n}$ for $4 \leq n \leq 8$, as shown in Lemma 6 below.

Observe that the original result of Nosal was sharpened in [12], Theorem 2, (i):

If $\mu(G) \geq \sqrt{m}$, then $G$ has a triangle, unless $G$ is a complete bipartite graph with possibly some isolated vertices.

It turns out that Theorem 2 can be sharpened likewise, at the price of a considerably longer proof, which we omit.

**Theorem 3** Let $m \geq 9$ and $G$ be a graph with $m$ edges. If $\mu(G) \geq \sqrt{m}$, then $G$ has a 4-cycle unless $G$ is a star or $S_{9,1}$ with possibly some isolated vertices.

**Proofs**

Our notation follows [2]; thus, if $G$ is a graph $G$, and $X$ and $Y$ are disjoint sets of vertices of $G$, we write:

- $E(G)$ for the edge set of $G$ and $e(G)$ for $|E(G)|$;
- $G[X]$ for the graph induced by $X$, $E(X)$ for $E(G[X])$, and $e(X)$ for $|E(X)|$;
- $e(X,Y)$ for the number of edges joining vertices in $X$ to vertices in $Y$;
- $G - uv$ for the graph obtained by removing the edge $uv \in E(G)$;
- $\Gamma_G(u)$ for the set of neighbors of a vertex $u$ and $d_G(u)$ for $|\Gamma_G(u)|$;
- $\Gamma_X(u)$ for $\Gamma_G(u) \cap X$ and $d_X(u)$ for $|\Gamma_X(u)|$.

We drop the subscript in $\Gamma_G(u)$ and $d_G(u)$ when it is understood.

Define $S_{n,k}$ to be the star of order $n$ with $k$ disjoint edges within its independent set.

Next we give some facts, needed in the proof of Theorem 2.

First, a fact implied by Theorem 1 in [16]:

**Fact 4** Let $x$ be a unit eigenvector to the spectral radius of a graph with some edges. Then the entries of $x$ do not exceed $2^{-1/2}$.

Next, a known fact, proved here for completeness:

**Lemma 5** Let $A$ and $A'$ be the adjacency matrices of two graphs $G$ and $G'$ on the same vertex set. Suppose that $\Gamma_G(u) \subseteq \Gamma_{G'}(u)$ for some vertex $u$. If some positive eigenvector $x$ to $\mu(G)$ satisfies $\langle A'x, x \rangle \geq \langle Ax, x \rangle$, then $\mu(G') > \mu(G)$.
Proof Since \( \langle A' x, x \rangle \geq \langle A x, x \rangle \), the Rayleigh principle implies that \( \mu(G') \geq \mu(G) \). If \( \mu(G') = \mu(G) \), then \( \langle A' x, x \rangle = \langle A x, x \rangle \), and, again by the Rayleigh principle, \( x \) is an eigenvector to \( \mu(G') \). But this is impossible, for
\[
\mu(G') x_u = \sum_{uv \in E(G')} x_v \geq \sum_{uv \in E(G)} x_v = \mu(G) x_u.
\]
We use above that \( \Gamma_G(u) \subset \Gamma_{G'}(u) \), but there is some \( v \in \Gamma_{G'}(u) \) such that \( v / \in \Gamma_G(u) \). This completes the proof of Lemma 5.

Finally, some facts about \( \mu(S_{n,k}) \):

Lemma 6

(a) \( \mu(S_{n,k}) \) is the largest root of the equation
\[
x^3 - x^2 - (n - 1) x + n - 1 - 2k = 0;
\]
(b) \( \mu(S_{n,k}) \leq \sqrt{n - 1 + k} \) for \( n - 1 + k \geq 9 \), and \( \mu(S_{n,1}) > \sqrt{n} \) for \( 4 \leq n \leq 8 \).

Proof Suppose that 1 is the dominating vertex of \( S_{n,k} \), and \( \{2,3\}, \ldots, \{2k,2k+1\} \) are its \( k \) additional edges. Set \( \mu = \mu(S_{n,k}) \) and let \( (x_1, \ldots, x_n) \) be an eigenvector to \( \mu \). By symmetry,
\[
x_2 = x_3 = \cdots = x_{2k+1} \quad \text{and} \quad x_{2k+2} = x_{2k+3} = \cdots = x_n.
\]
Setting \( x_1 = x, x_2 = y, x_n = z \), we see that
\[
\mu z = x,
\mu y = y + x,
\mu x = 2ky + (n - 2k - 1) z.
\]
Solving this system, we find that \( \mu \) is a root of the equation
\[
x^3 - x^2 - (n - 1) x + n - 1 - 2k = 0.
\]
If \( \mu \) is not the largest root of this equation, then it has to be smaller than
\[
x_{\min} = 1/3 + \sqrt{1/9 + (n - 1)/3},
\]
the point where the function
\[
f_k(x) = x^3 - x^2 - (n - 1) x + n - 1 - 2k
\]
has a local minimum. This, however, is not possible since
\[
\mu > \sqrt{n - 1} > 1/3 + \sqrt{1/9 + (n - 1)/3}.
\]
This completes the proof of (a).

To prove (b) note that
\[
f_k(\sqrt{n - 1 + k}) = (\sqrt{n - 1 + k})^3 - (\sqrt{n - 1 + k})^2 - (n - 1) \sqrt{n - 1 + k} + n - 2k
\]
\[= k \left( \sqrt{n - 1 + k - 3} \right),
\]
implying the assertion since \( \sqrt{n - 1 + k} > x_{\min} \) and \( f_k(x) \) is increasing for \( x > x_{\min} \).
Proof of Theorem \[2\]

Let $m \geq 9$, and assume for a contradiction that $G$ is a $C_4$-free graph with $m$ edges, satisfying $\mu(G) > \sqrt{m}$. Set $\mu = \mu(G)$, and suppose that

$$\mu = \max \{ \mu(G) : G \text{ is a } C_4\text{-free graph with } e(G) = m \}.$$  \hspace{1cm} (3)

Also, for the purposes of the proof we may and shall suppose that $G$ has no isolated vertices. This implies that $G$ is connected.

Indeed, let $G_1$ be a component of $G$ with $\mu(G_1) = \mu(G)$, and let $G_2$ be the nonempty union of the remaining components of $G$. Remove an edge from $G_2$, and add an edge between $G_1$ and $G_2$. The resulting graph is $C_4$-free with $m$ edges, but its spectral radius is larger than $\mu$, contradicting (3). Hence, $G$ is connected.

The essentially part of the proof is induction on $m$, but it needs some preparation. We first introduce some structure in $G$ and settle several cases with direct arguments, in particular the case $m \leq 13$. Then, having restricted the structure of $G$, we prove the induction step. Now the details.

Let $\{1, \ldots, n\}$ be the vertices of $G$, and let $x = (x_1, \ldots, x_n)$ be a positive unit eigenvector to $\mu$, i.e.,

$$\mu = 2 \sum_{ij \in E(G)} x_ix_j.$$  \hspace{1cm}

By symmetry, suppose that $x_1 \geq \cdots \geq x_n$. We claim that all vertices of degree 1 are joined to vertex 1.

Indeed, assume for a contradiction that there exists a vertex $u \neq 1$ such that $d(u) = 1$ and $u$ is joined to $v \neq 1$. Remove the edge $uv$ and join $u$ to vertex 1. The resulting graph $G'$ is $C_4$-free and has $m$ edges. Also, we see that

$$\sum_{ij \in E(G')} x_ix_j = \sum_{ij \in E(G)} x_ix_j + x_u (x_1 - x_v) \geq \sum_{ij \in E(G)} x_ix_j.$$  \hspace{1cm}

Since $\Gamma_G(1) \subseteq \Gamma_{G'}(1)$, Lemma [5] implies that $\mu(G') > \mu$, contradicting (3). Hence, all vertices of degree 1 are joined to vertex 1.

Let $A = (a_{ij})$ be the adjacency matrix of $G$ and $A^2 = B = (b_{ij})$. Since $x$ is an eigenvector of $B$ to $\mu^2$, we have

$$x_1\mu^2 = \sum_{i=1}^{n} b_{1i}x_i \leq x_1 \sum_{i=1}^{n} b_{1i} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}a_{ji} = x_1 \sum_{v \in \Gamma(1)} d(v).$$  \hspace{1cm} (4)

Set

$$U = \Gamma(1), \quad W = \{2, 3, \ldots, n\} \setminus \Gamma(1),$$

and let $t = e(U)$ and $q = e(W)$. We see that

$$\sum_{v \in U} d(v) = d(1) + 2e(U) + e(U,W) = e(G) - e(W) + e(U) = m - q + t.$$  \hspace{1cm}

Thus (4) gives $\mu^2 \leq m + t - q$, and from $\mu^2 > m$, we get the crucial inequality $t \geq q + 1$.  \hspace{1cm} 4
Since all vertices of degree 1 belong to $U$, we have $d(u) \geq 2$ for all $u \in W$. Also, since $G$ is $C_4$-free, a vertex in $W$ can be joined to at most one vertex in $U$. Thus, for all $w \in W$ we have $d_W(w) \geq d(w) - 1 \geq 1$, and consequently,

$$2q = \sum_{w \in W} d_W(w) \geq \sum_{w \in W} 1 = |W|.$$

Suppose first that $q = 0$. Then $|W| = 0$, and so, $e(U, W) = 0$. Therefore, vertex 1 is dominating and $G = S_{m+1-t,t}$. By Lemma 6,

$$\mu = \mu(S_{m+1-t,t}) \leq \sqrt{m}$$

for $m \geq 9$, contradicting the hypothesis. Therefore, $q \geq 1$.

The next claim gives a useful property of $G[W]$, and, in particular, settles the case $q = 1$.

**Claim 1** The graph $G[W]$ contains no isolated edges.

**Proof** Let $uv \in E(W)$ be an isolated edge. Since $d(u) \geq 2$ and $d(v) \geq 2$, we see that $d_U(u) = d_U(v) = 1$. Let $\{k\} = \Gamma_U(u)$ and $\{l\} = \Gamma_U(v)$. Remove the edges $uk, vl$, and join $u$ and $v$ to the vertex 1. The resulting graph $G'$ is $C_4$-free and has $m$ edges. Also, we see that

$$\sum_{ij \in E(G')} x_ix_j = \sum_{ij \in E(G)} x_ix_j + x_u(x_1 - x_k) + x_v(x_1 - x_l) \geq \sum_{ij \in E(G)} x_ix_j.$$

Since $\Gamma_G(1) \not\subseteq \Gamma_{G'}(1)$, Lemma 5 implies that $\mu(G') > \mu$, contradicting (3), and completing the proof of Claim 1.

Claim 1 implies that $q \geq 2$. Our next goal is to obtain a contradiction for $m \leq 13$. Indeed, suppose that $m \leq 13$; then $q \geq 2$ gives

$$13 \geq m = 3t + e(U, W) + q \geq 4q + 3 + e(U, W) \geq 11 + e(U, W),$$

which is possible only if $q = 2$, $e(U, W) \leq 2$, and $t = 3$.

The graph $G[W]$ has 2 non-isolated edges, and thus is a path of order 3. Let $u, v, w$ be the vertices of this path and suppose that $uv \in E(W)$ and $vw \in E(W)$. Since $d(u) \geq 2$ and $d(w) \geq 2$, we find that $d_U(u) = d_U(w) = 1$. This, in view of $e(U, W) \leq 2$, gives $e(U, W) = 2$, and so, $v$ has no neighbors in $U$.

Let $\{k\} = \Gamma_U(u)$ and $\{l\} = \Gamma_U(w)$. Remove the edges $uk, wl, uv$, and join $u, v, w$ to the vertex 1. The resulting graph $G'$ is $C_4$-free and has $m$ edges. Also, we see that

$$\sum_{ij \in E(G')} x_ix_j = \sum_{ij \in E(G)} x_ix_j + x_u(x_1 - x_k) + x_w(x_1 - x_l) + x_v(x_1 - x_u) \geq \sum_{ij \in E(G)} x_ix_j.$$

Since $\Gamma_G(1) \not\subseteq \Gamma_{G'}(1)$, Lemma 5 implies that $\mu(G') > \mu$, contradicting (3).

At this point we have proved the theorem for $9 \leq m \leq 13$. Assume now that $m \geq 14$ and that the theorem holds for $m - 1$; we shall prove it for $m$. The induction step is based on three claims.
Claim 2 If an edge $uv \in E(G)$ satisfies $d(u) = d(v) = 2$, then $x_u x_v < 1/4\mu$.

Proof Let $\{i, u\} = \Gamma(v)$ and $\{j, v\} = \Gamma(u)$. From

$$\mu x_u = x_i + x_v \leq x_1 + x_v \quad \text{and} \quad \mu x_u = x_i + x_v \leq x_1 + x_v$$

we see that $x_u + x_v = 2x_1/(\mu - 1)$. Hence, using the AM-QM inequality and Fact 4, we obtain

$$x_u x_v \leq \left(\frac{x_u + x_v}{2}\right)^2 = \frac{x_i^2}{(\mu - 1)^2} \leq \frac{1}{2(\mu - 1)^2} \leq \frac{1}{4\mu}$$

whenever $\mu^2 \geq 14$. This completes the proof of Claim 2. \hfill $\Box$

Claim 3 Let $m \geq 20$. Let the vertices $u, v, w$ satisfy $d(u) = d(w) = 2$ and $d(v) = 3$, and let $v$ be joined to $u$ and $w$. Then either $x_u x_v < 1/4\mu$ or $x_w x_v < 1/4\mu$.

Proof We first note that if $x \geq \sqrt{20}$, then

$$\frac{(x^2 - 2)^2}{x(x + 1)(x + 2)} > \frac{x^4 - 4x^2}{x(x + 1)(x + 2)} = \frac{x(x - 2)}{x + 1} = \frac{x^2 - 4x - 2}{x + 1} + 2 > 2. \quad (5)$$

Next, letting $\Gamma(u) = \{i, v\}$, $\Gamma(w) = \{j, v\}$, and $\Gamma(v) = \{k, u, w\}$, we see that

$$\mu x_u = x_i + x_v \leq x_1 + x_v,$$

$$\mu x_w = x_j + x_v \leq x_1 + x_v,$$

$$\mu x_v = x_k + x_u + x_w \leq x_1 + x_u + x_w,$$

and therefore,

$$\mu (x_u + x_w) \leq x_1 + 2x_v,$$

$$\mu x_v \leq x_1 + x_u + x_w.$$ 

The solution of this system is

$$x_u + x_w \leq \frac{2\mu + 1}{\mu^2 - 2} x_1, \quad x_v \leq \frac{\mu + 2}{\mu^2 - 2} x_1.$$

Now, assuming $x_u \geq x_w$, and using Fact 4 we obtain

$$x_u x_v \leq \frac{(\mu + 1)(\mu + 2)}{(\mu^2 - 2)^2} \frac{x_1^2}{x_1^2} \leq \frac{(\mu + 1)(\mu + 2)}{2(\mu^2 - 2)^2}.$$ 

Finally, inequality (5) implies that

$$x_u x_v \leq \frac{(\mu + 1)(\mu + 2)}{2(\mu^2 - 2)^2} \leq \frac{1}{4\mu}$$

whenever $\mu^2 \geq 20$. This completes the proof of Claim 3. \hfill $\Box$
Claim 4: If there exists $uv \in E(G)$ satisfying $x_u x_v \leq 1/4\mu$, then $\mu^2 (G - uv) > \mu^2 - 1$.

Proof: For every edge $uv \in E(G)$, by the Rayleigh principle, we have

$$\mu^2 (G - uv) \geq \left( 2 \sum_{ij \in E(G-uv)} x_i x_j \right)^2 = (\mu - 2x_u x_v)^2 > \mu^2 - 4\mu x_u x_v \geq \mu^2 - 1,$$

completing the proof of Claim 4. \(\square\)

Having proved the claims, we proceed with the induction step. If there exists $uv \in E(U)$ with $d(u) = d(v) = 2$, then by Claims 2 and 4 we obtain $\mu (G - uv) > \sqrt{m - 1}$; by the induction hypothesis $G$ contains a $C_4$, a contradiction.

Hereafter, we assume that $d(u) + d(v) \geq 5$ for all $uv \in E(U)$. For every edge $uv \in E(U)$, let $W_{uv} = \Gamma_W(u) \cup \Gamma_W(v)$. Since a vertex in $W$ can be joined to at most one vertex in $U$, the sets $W_{uv}$, $uv \in E(U)$ are disjoint. From

$$2q = 2e(W) = \sum_{w \in W} d_W(w) \geq \sum_{uv \in E(U)} \sum_{w \in W_{uv}} d_W(w) \geq t \min_{uv \in E(U)} \sum_{w \in W_{uv}} d_W(w)$$

we see that there is an edge $uv \in E(U)$ such that $\sum_{w \in W_{uv}} d_W(w) \leq 1$. Then from

$$|W_{uv}| = d(u) + d(v) - 4 \geq 1$$

we conclude that $W_{uv}$ contains a single vertex $w$, and that $d_W(w) = 1$.

Assume, by symmetry, that $w$ is joined to $v$. Then, $d(u) = 2$, $d(w) = 2$, and $d(v) = 3$. Now, if $m \geq 20$, Claims 3 and 4 imply either $\mu (G - uw) > \sqrt{m - 1}$ or $\mu (G - uv) > \sqrt{m - 1}$; by the induction hypothesis $G$ contains a $C_4$, contradiction.

To complete the proof we have to settle the case when $15 \leq m \leq 19$ and $d(u) + d(v) \geq 5$ holds for all $uv \in E(U)$. We shall show that these conditions also lead to a contradiction.

From

$$e(U,W) = \sum_{uv \in E(U)} d_W(u) + d_W(v) \geq \sum_{uv \in E(U)} (5 - 4) = t$$

and

$$19 \geq m = 3t + e(U,W) + q \geq 3t + t + q \geq 5q + 4 \quad (6)$$

we see that $q \leq 3$ and $t \leq 4$.

Consider first the case $q = 3$. From (6) we find that this is possible only if $m = 19$, $t = 4$, $e(U,W) = 4$. This implies also that $|W| \geq e(U,W) \geq 4$.

$G[W]$ has no isolated vertices and, by Claim 1, it has no isolated edges either. Thus, from $e(W) = 3$ we see that $G[W]$ is a tree of order 4. Now the structure of $G$ is determined: $G$ consists of 4 triangles sharing vertex 1, a tree $T$ of order 4, and a 4-matching joining every vertex of $T$ to a separate triangle.
Select \( u \in W \) to be with \( d_W (u) = 1 \) and let \( \{ v \} = \Gamma_W (u) \), \( \{ k \} = \Gamma_U (u) \), \( \{ l \} = \Gamma_U (v) \). Suppose that \( x_k \geq x_l \), remove the edge \( vl \), and add the edge \( vk \). The resulting graph \( G' \) is \( C_4 \)-free and has \( m \) edges. Also, we see that

\[
\sum_{ij \in E(G')} x_i x_j = \sum_{ij \in E(G)} x_i x_j + x_v (x_k - x_l) \geq \sum_{ij \in E(G)} x_i x_j.
\]

Since \( \Gamma_G (k) \subsetneq \Gamma_G' (k) \), Lemma \( 5 \) implies that \( \mu (G') > \mu \), contradicting \( (3) \).

The same argument applies when \( x_k < x_l \), completing the proof in this case.

Let now \( q = 2 \). If \( t = 4 \), then \( |W| \geq e(U,W) \geq t = 4 \), and so \( W \) contains isolated edges, contradicting Claim \( \text{[1]} \). Hence, \( t = 3 \), \( |W| = 3 \), and \( G[W] \) is a path of order \( 3 \). Now, the structure of \( G \) is determined: \( G \) consists of the graph \( S_{m-4,3} \), a path \( P \) of order \( 3 \), and a 3-matching, joining every vertex of \( T \) to a separate triangle of \( S_{m-4,3} \).

At this point we apply again the above argument, completing the proof of Theorem \( 2 \). \( \square \)

Concluding remarks

Theorem 3 in \( [10] \) gives a result more general than just inequality \( (1) \):

**Theorem 7** Let \( G \) be a graph of order \( n \) with \( \mu (G) = \mu \). If \( G \) has no \( K_{2,k+1} \) for some \( k \geq 1 \), then

\[
\mu^2 - \mu \leq t(n - 1).
\]

Equality holds if and only if every two vertices of \( G \) have exactly \( k \) common neighbors.

This theorem is sharper than Theorem 3 in \( [1] \), and for some values of \( n \) and \( k \) it is as good as one can get. However, in general, the maximal \( \mu (G) \) of \( K_{2,k+1} \)-free graphs \( G \) of order \( n \) is not known at present.

Note that for \( k > 1 \), there may exist regular graphs with every two vertices having exactly \( k \) common neighbors: here is a small selection from \( [17] \):

| \( k \) | \( n \) | \( \mu \) |
|------|------|------|
| 2    | 16   | 6    |
| 3    | 45   | 12   |
| 4    | 96   | 20   |
| 5    | 175  | 30   |
| 6    | 36   | 15   |

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