Forecasting system using single exponential smoothing with golden section optimization

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Abstract. Puri Wira Mahkota Ltd is a distributor company in the automotive parts industry. Puri Wira Mahkota has an obstacle in preparing weekly order on stock to the primary production unit. They predict total sales of product in the coming period just by using an expectation from stock administration staff then it will be compared with sales data in the previous period without any calculations using a definitive formula. Forecasting calculation could support Puri Wira Mahkota to predict the quantity of product. The purpose of this research is to apply the calculation process that can predict the quantity of product should be produced by the primary production unit every month and also to prevent over stock and out of stock. The method used in the forecasting calculation is Single Exponential Smoothing. To optimize Single Exponential Smoothing we used the Golden Section. The principle of the Golden Section is to reduce the alpha area boundary so it will produce an ideal forecast value with the minimum MAPE (Mean Absolute Percentage Error) level. This research shows the results that the Golden Section finds the optimal forecasting value with a level of MAPE (Mean Absolute Percentage Error) of 43.39%.

1. Introduction
Puri Wira Mahkota Ltd is one of the major distributor companies in automotive parts industry. This company does not only distribute but also produces the products that will be sold to the customer. This company needs a calculation process that can help the operational activities of company in forecasting the quantity of products that must be ordered by the branch office, and in the future could help the decision making process according to the conditions and ongoing market segmentation. Predicting the quantity of products that must be ordered in every period can help the management in the process of making decision to determine the quantity of products that should be produced by the primary production unit every month and also to prevent overload stock and out of stock in branch warehouse.

In this study the method that used in forecasting calculations is Single Exponential Smoothing. Single Exponential Smoothing is an improvement method of the Single Moving Average where the forecasting method is done by repeating calculations continuously using the latest data. The calculation of the Single Exponential Smoothing requires parameter values (alpha) between 0 until 1. In order to produce the minimum MAPE (Mean Absolute Percentage Error) value, it must use an appropriate alpha value. The obstacle in the process of calculation Single Exponential Smoothing is the alpha value, which is still determined in a trial-error, so it will take a long time in forecasting...
calculations process. That is why this study will discuss about the calculations that can optimize the determination of alpha values in the process of implementing the Single Exponential Smoothing Method. The calculation is the Golden Section.

There are some previous research that the authors use as references. The first research is entitled "Drug Stock Forecasting System Using the Exponential Smoothing Method" [1]. This study discusses the condition of drug storage in the Pharmacy Installation of the Syarifah Ambami Rato Ebu Bangkalan Regional General Hospital. The hospital must provide enough medicine for the patient. However, these storage activities cannot be carried out for a long time because the drugs consists of several chemicals with very strict expiry dates, because drugs should not be used after they come to their expiry date. The obstacle revealed in this study is the current system of the Pharmacy Installation of the Syarifah Ambami Rato Ebu Bangkalan Regional General Hospital, which was only limited to making a recap of drug sales without data processing so the pharmacy staff had a difficulty in the process of predicting the stock of drugs that would appear in the coming period. The Pharmacy Installation of this Hospital did not have any calculations process to forecast the drug stock that must be provided in the hospital, and it caused tardiness in the drugs distribution to patients. So the patients still had to get out of the hospital area to buy a medicine at the nearest pharmacy.

Another related research is entitled "Inventory Forecasting System with Weight Moving Average Method in The Kids 24 Store" [2]. The study discusses the activity of calculating the quantity of product that must be purchased by The Kids 24 Store in the coming period based on sales data of the previous period. The obstacles identified in this study was in the process of product selling The Kids 24 Store still used the manual recording in the cash book every day by the finance staff, the product that had been sold would be recorded in the ledger. Then the shopkeeper at the end of every month would directly count the quantity of the remaining products in the store and then reported to the owner. With the operational system as described above, the manager was often confused in determining the quantity of products that had to be ordered or purchased in the next period. As an impact, the store manager only guessed about the kind and the quantity of the product that must be purchased without a definite calculation process and without calculating the real demand of customers in the next period.

Another research is entitled "Forecasting System Using Single Exponential Smoothing With Golden Section Optimization" has similarities with previous research. The process of determining product that must be ordered in the next period was still carried out without a definite calculation system. This causes a mismatch between the quantity of product that had been ordered or produced with the actual customer demand. The difference between this research with previous research is in terms of the methods that is used in the forecasting calculations and in terms of the method that is used to optimize parameter value in forecasting calculations. In this study, for the process of forecasting calculations, the Single Exponential Smoothing Method was used, which would then be optimized using the Golden Section.

2. Research Method

2.1. Forecasting
Forecasting is an activity to achieve an optimal decision, requires an appropriate, systematic and accountable way. The component is needed by management and is an integral part of the decision-making process for forecasting methods. Forecasting methods will be used to measure or estimate the situation in the future. Forecasting is not only done to determine the quantity of products that must be made or the capacity of services that must be provided, but is also needed for various other fields (such as in procurement, sales, personnel section, resources, including for forecasting technology, economics, or socio-cultural changes). In the structure of company, one section always has a relationship with other sections, so good or bad forecast result will affect the company as a whole [3]. Forecasting produces important data, because the forecasting data can help in the process of determining decisions to take forward. Forecasting will provide information about what might happen in the future. The old data are used as reference data in predicting the future data [4].
Forecasting is an art and science in predicting future events. Forecasting will involve taking historical data (such as last year's sales) and projecting them into the future with a mathematical model [5]. Forecasting (forecast) is a statement about the value that will come from variables such as demand. This means that forecasting are predictions about the future [6].

2.2. Single Exponential Smoothing
Single Exponential Smoothing Forecasting is one of the categories of time series methods that use past weighting of data to forecast. The magnitude of the weight changes exponentially depending on historical data. Forecasting with the Single Exponential Smoothing Method is done by entering the current demand forecast with actual demand data into the Exponential Smoothing formula [5].

To calculate single exponential smoothing, the equation can be used (1):

\[ F_{t+1} = \alpha A_t + (1 - \alpha)F_{t-1} \]  

Where:
- \( F_{t+1} \) = forecasting value in the coming period
- \( \alpha \) = parameter value between 0 to 1 (0 < \( \alpha < 1 \))
- \( A_t \) = actual data
- \( F_{t-1} \) = forecasting value in the previous period

2.3. Golden Section
In this study, the parameter values are determined using the Golden Section calculation. Golden Section Optimization is a classical solution to the single peak optimizing problem. A search interval \([a, b]\) of length \( L \) is divided by two points \( X_1 \) and \( X_2 \), which must satisfy equation (2):

\[ ax_2 = x_1 b = \lambda L \]  

Where the value of \( \lambda \) is equal to 0.618. After comparing the corresponding function values \( f(X_1) \) and \( f(X_2) \), Golden Section Optimization selects the next search space according to the following rules. If \( f(X_1) < f(X_2) \), the maximum must lie in the range of \([X_1, b]\), which is taken as a new interval for the next iteration. On the other hand, if \( f(X_1) > f(X_2) \), \([a, X_2]\) is taken in the next iteration. The new interval is always 0.618 times the original interval. The process is repeated continuously until the distance between \( X_1 \) and \( X_2 \) is less than a certain chosen precision [7]. The steps taken to explain the Golden Section function interventions carried out in stages as follows:

1. Optimization of the Golden Section in the Single Exponential Smoothing (SES) Model.
   - Determine the lower limit (a), the upper limit (b) and the tolerance value of cessation of iteration (\( \text{eps} \)). For the Exponential Smoothing Method the lower limit is 0 and the upper limit is 1.
2. Calculate the value of Golden Ratio (R).
3. Determine the initial value for the parameter, \( r_1 = \frac{-b + \sqrt{b^2 - 4ab}}{2} = 0.618 \) and \( r_2 = \frac{-b - \sqrt{b^2 - 4ab}}{2} = -1.618 \). Because the parameter value is \( 0 < \alpha < 1 \), then (r) value use 0.618. Determine the initial values for the parameters \( \alpha_1 = r \times a + (1 - r) \times b \) and \( \alpha_2 = (1 - r) \times a + r \times b \).
4. Find the maximum value \( f(x) \) between combinations \( a, b, \alpha_1 \) and \( \alpha_2 \).
5. Reduce the interval based on the Golden Section criteria.
6. Repeat steps “5.” until \( |\alpha_2 - \alpha_1| \leq \text{eps} \).
7. Determine the minimum value between a combination of calculations \( a, b, \alpha_1 \) and \( \alpha_2 \). Alpha produced is optimal.
3. Design and Implementation

3.1. Data
The data that used in this research are monthly sales report for each products from the stock administration staff of Puri Wira Mahkota Ltd Branch Office Denpasar. The data obtained were 81 (eighty one) data of products with sales period starting from January 2016 until June 2019.

3.2. General Overview
The process started from the stock administration staff who would recapitalize the data of products and track list of sales in the previous period. Then after the recapitalize process was done, the data of the product selling details were produced. In the next process, the data of product selling detail would be use to carry out the process of calculating forecast parameter values using the Golden Section. After Golden Section calculation is done, it will get the results of the forecast parameter values which is then used to do the Single Exponential Smoothing calculations. The calculating of Single Exponential Smoothing will produce forecasting results, which in the next steps would be continued with MAPE (Mean Absolute Percentage Error) calculation. Forecasting results and MAPE (Mean Absolute Percentage Error) values would be used to reduce the interval boundary according to the Golden Section criteria. If the iteration received a convergent value, the process would continue to save the data store of forecasting which later could be checked by the area manager in the form of forecasting reports. However, if the iteration did not find a convergent point, the process would return to the Golden Section calculation and continue to the next process until it found the point of iteration (convergent). The implementation of the explanation above can be seen in the Figure 1:

![Figure 1. General Overview of Forecasting Calculation.](image)

3.3. Implementation
Implementation of Single Exponential Smoothing calculations for Vario Headlamps products without Golden Section optimization.
1. The calculation of Single Exponential Smoothing will be applied using actual sales data and forecast data in the previous period, October 2016 and November 2016. For October 2016 actual sales data is 555 (five hundred fifty five) and forecast value is 234.51 (two hundred thirty four point fifty one). Data from October 2016 will used to calculate forecast value in
November 2016. After the calculation is done, for November 2016 actual sales data is 407 (four hundred and seven) and get the forecast value 266.56 (two hundred sixty six point fifty six). Has the same process with the previous period, data from November 2016 will be used to calculate forecast value in December 2016. For the equation of Single Exponential Smoothing calculation (3):

\[ F_{t+1} = \alpha A_t + (1 - \alpha)F_t \]
\[ F_{t+1} = (0.1 \times 407) + (1 - 0.1) \times 266.56 \]
\[ F_{t+1} = 40.7 + (0.9 \times 266.56) \]
\[ F_{t+1} = 40.7 + 239.90 \]
\[ F_{t+1} = 280.60 \]

2. Mean Absolute Percentage Error (MAPE) is calculated using the absolute error in each period divided by the observed values that are evident for that period. Then, averaging those fixed percentages. This approach is useful when the size of a prediction variable is significant in evaluating the accuracy of a prediction [8]. Calculate the accuracy of the forecast with an alpha value of 0.1 using the MAPE (Mean Absolute Percentage Error) calculation. For the equation can be seen as follows:

Example of calculating the forecast error value in December 2016 (4):

\[ Error \ Value = Actual \ Data - Forecast \ Value \]
\[ et = A_t - F_t \]
\[ et = 457 - 280.60 = 176.40 \]

Example of calculating the percentage of forecast errors in December 2016 (5):

\[ Percentage \ Error = \frac{Error \ Value}{Actual \ Data} \times 100 \]
\[ e/y = (et/A_t) \times 100 \]
\[ e/y = (176.40/457) \times 100 \]
\[ e/y = 38.60\% \]

Example of calculating MAPE (Mean Absolute Percentage Error) of forecast (6):

\[ MAPE = \frac{\sum [Actual - forecast] \times 100}{Actual} \]
\[ MAPE = (1050.29/42) \times 100\% \]
\[ MAPE = 25.01\% \]

Implementation of Single Exponential Smoothing calculations for Vario Headlamps products using Golden Section optimization.

Determine the lower limit of iteration (7):

\[ alpha1 = r \times a + (1 - r) \times b \]
\[ alpha1 = 0.618 \times 0 + (1 - 0.618) \times 1 \]
\[ alpha1 = 0.382 \]

Determine the upper limit of iteration (8):

\[ alpha2 = (1 - r) \times a + r \times b \]
\[ alpha2 = (1 - 0.618) \times 0 + 0.618 \times 1 \]
\[ alpha2 = 0.618 \]
The example of iteration limit calculation was implemented to find the optimum alpha value by reducing the interval limit according to the Golden Section criteria. If the alpha value was found then it would be implemented in Single Exponential Smoothing calculation to find the forecast value, and the next step was to determine the accuracy of the forecast, so it would produce an ideal MAPE value with a minimum error rate.

Results of the Golden Section iteration calculation used the sample Vario Headlamps product: Optimization of the Single Exponential Smoothing parameter on the Vario Headlamps product was obtained at the 4th iteration with alpha2 - alpha1 = 0.0000 ≤ ε, meaning that the iteration stopped at that point. Furthermore, the alpha1 and alpha2 values were convergent at 0.6180 and the f(alpha1) and f(alpha2) convergent values were obtained for a Mean Absolute Percentage Error value of 24.1319%. If it is displayed in table form, then as can be seen in Table 1.

### Table 1. Forecasting Iteration of Vario Headlamps Product.

| Iteration | Alpha1  | Alpha2  | MAPE 1 | MAPE 2 | Alpha2 – Alpha1 |
|-----------|---------|---------|--------|--------|-----------------|
| 1         | 0.3820  | 0.6180  | 24,4499| 24,1317| 0.2360          |
| 2         | 0.6181  | 0.7639  | 24,1320| 24,8674| 0.1458          |
| 3         | 0.5279  | 0.6180  | 24,0264| 24,1318| 0.0901          |
| 4         | 0.6181  | 0.6180  | 24,1320| 24,1319| 0.0000          |

### 4. Result and Analysis

After the Golden Section calculation was implemented in the inventory forecasting system, iteration summaries and MAPE convergent values were obtained for each product. Based on the optimum alpha value, then the process continue with the forecasting process of product using Single Exponential Smoothing to get an average result of Mean Absolute Percentage Error for the all product of 43.39%, as can be seen in Table 2.

### Table 2. All Product Convergent Results.

| PRODUCT NAME                   | ITERATION CONVERGENT | MAPE CONVERGENT |
|--------------------------------|-----------------------|-----------------|
| LEGSHIELD LUAR SUPRA X 125 07 HITAM | 19                    | 49,3194         |
| LEGSHIELD LUAR GRAND            | 3                     | 42,0739         |
| LEGSHIELD LUAR JUPITER MX HITAM | 3                     | 48,2700         |
| LEGSHIELD TENGAH ATAS SUPRA X 125 07 | 7                   | 54,4435         |
| LEGSHIELD DALAM SUPRA HITAM    | 3                     | 36,6072         |
| COVER BODY SUPRA HITAM + COVER STOP | 3                  | 40,4834         |
| COVER BODY VEGA R HITAM + COVER STOP | 3                  | 54,8891         |
| FRONT FENDER SCOOPY FI HITAM   | 3                     | 38,9640         |
| FRONT FENDER BEAT FI HITAM     | 3                     | 54,4224         |
| FRONT FENDER BEAT FI PUTIH     | 3                     | 50,5681         |
| FRONT FENDER BEAT HITAM        | 3                     | 45,8184         |
| FRONT FENDER A SUPRA X 125 07 HITAM | 3                  | 44,5157         |
| FRONT FENDER VARIO HITAM       | 3                     | 31,3497         |
| FRONT FENDER VARIO PUTIH       | 3                     | 44,4836         |
| FRONT FENDER A SUPRA FIT NEW HITAM | 3                 | 53,7495         |
| FRONT FENDER SUPRA X HITAM     | 3                     | 34,9918         |
| FRONT FENDER A GRAND HITAM     | 3                     | 50,9872         |
| FRONT FENDER A SMASH HITAM     | 3                     | 51,2546         |
| FRONT FENDER JUPITER MX HITAM  | 3                     | 49,8996         |
| FRONT FENDER JUPITER Z HITAM   | 3                     | 41,2328         |
| PANEL BEAT FI HITAM            | 3                     | 46,3879         |
| PANEL BEAT FI PUTIH            | 3                     | 42,7729         |
| PANEL BEAT HITAM               | 3                     | 47,6756         |
| PANEL VARIO HITAM              | 3                     | 30,4406         |
| PANEL VARIO PUTIH              | 3                     | 26,9666         |
| PANEL SUPRA X HITAM            | 3                     | 48,2736         |
| FRONT HANDLE COVER ABSOLUTE REVO HITAM | 3             | 45,5257         |
| PRODUCT NAME | ITERATION | CONVERGENT | MAPE \CONVERGENT |
|-------------|-----------|------------|-----------------|
| FRONT HANDLE COVER SUPRA X 125 07 HITAM | 3 | 42,1601 |
| FRONT HANDLE COVER VARIO HITAM | 3 | 49,5842 |
| FRONT HANDLE COVER SUPRA X 125 HITAM CAKRAM | 3 | 49,8949 |
| FRONT HANDLE COVER SUPRA FIT HITAM CAKRAM | 4 | 51,5459 |
| FRONT HANDLE COVER GRAND HITAM | 3 | 48,9082 |
| FRONT HANDLE COVER JUPITER Z 06 HITAM | 3 | 50,7211 |
| FRONT HANDLE COVER JUPITER MX HITAM (KOPLING) | 3 | 48,3468 |
| REAR HANDLE COVER VARIO | 3 | 33,8835 |
| REAR HANDLE COVER SUPRA X 125 | 3 | 49,2241 |
| REAR HANDLE COVER SUPRA FIT | 3 | 46,5796 |
| REAR HANDLE COVER SUPRA X | 3 | 30,5728 |
| BOX SAMPING SUPRA X HITAM | 7 | 55,3718 |
| LAMPU DEPAN VARIO TECHNO 125 | 3 | 34,1278 |
| LAMPU DEPAN BEAT FI | 3 | 34,2005 |
| LAMPU DEPAN BEAT | 3 | 30,9301 |
| LAMPU DEPAN SUPRA X 125 07 | 3 | 36,4982 |
| LAMPU DEPAN VARIO | 4 | 24,1320 |
| LAMPU DEPAN SUPRA X 125 | 3 | 47,8484 |
| LAMPU DEPAN SUPRA FIT NEW | 3 | 34,0315 |
| LAMPU DEPAN KARISMA | 3 | 51,2604 |
| LAMPU DEPAN SUPRA FIT | 3 | 45,8401 |
| LAMPU DEPAN SUPRA X | 3 | 28,8088 |
| LAMPU DEPAN SUPRA | 3 | 46,0945 |
| LAMPU DEPAN MIO M3 | 3 | 42,5587 |
| LAMPU DEPAN JUPITER MX 11 | 3 | 46,9593 |
| LAMPU DEPAN VEGA ZR | 3 | 37,6352 |
| LAMPU DEPAN MIO 08 | 3 | 46,0674 |
| LAMPU DEPAN JUPITER Z 06 | 3 | 35,7506 |
| LAMPU DEPAN JUPITER MX | 7 | 31,4294 |
| LAMPU DEPAN FIZ-R | 6 | 57,6826 |
| MIKA LAMPU VARIO 110 FI | 3 | 28,3293 |
| MIKA LAMPU VARIO TECHNO 125 | 3 | 22,0457 |
| MIKA LAMPU BEAT FI | 4 | 51,2673 |
| MIKA LAMPU VARIO | 3 | 28,7579 |
| MIKA LAMPU MIO SOUL GT | 5 | 48,5018 |
| MIKA LAMPU MIO 08 | 3 | 45,3529 |
| MIKA LAMPU MIO SOUL | 5 | 53,3563 |
| MIKA LAMPU JUPITER Z 06 + INNER | 3 | 50,1900 |
| MIKA LAMPU MIO | 4 | 59,5105 |
| MIKA SPEEDOMETER BEAT FI | 5 | 46,5758 |
| MIKA SPEEDOMETER BEAT | 3 | 45,9651 |
| MIKA SPEEDOMETER VARIO | 3 | 46,0542 |
| MIKA SPEEDOMETER SUPRA X 125 + INNER | 3 | 52,5856 |
| MIKA SPEEDOMETER SUPRA | 3 | 31,7265 |
| MIKA SPEEDOMETER GRAND | 5 | 46,2670 |
| MIKA SEN Fr VARIO (P) | 3 | 50,1744 |
| MIKA SEN Rr + STOP VARIO (P/M) KO | 3 | 34,5615 |
| FRONT WINKER ASSY VARIO | 4 | 32,8902 |
| FRONT WINKER ASSY GRAND (C/P) | 3 | 54,1146 |
| Fr / Rr WINKER ASSY V-IXION | 3 | 26,2413 |
| STOP LAMP ASSY VARIO | 3 | 42,4764 |
| STANDARD SAMPING SUPRA | 3 | 50,8912 |
| HANDFAT BEAT / BEAT 10 / VARIO TECHNO / VARIO / SPACY | 3 | 42,3613 |
| PIPA GAS SUPRA / SUPRA X / SUPRA FIT / SUPRA V / SUPRA XX | 3 | 49,5755 |

The performance of a naïve forecasting model should be the baseline for determining whether the forecasting values are good. It is irresponsible to set arbitrary forecasting performance targets (such as MAPE < 10% is Excellent, MAPE < 20% is Good) without the context of the forecast ability from the observation data. If the forecasting result is worse than a naïve forecast (It means could call this as a “bad” result), then clearly the forecasting process needs an improvement [9]. Forecasting result in this research has been improved from the result with the naïve forecast.
5. Conclusion
According to the result and analysis above, it can be concluded that after calculating the Golden Section, an optimum parameter value is obtained which will then be implemented in the Single Exponential Smoothing calculation to find the ideal MAPE. The average MAPE obtained using 81 product data and sales data from January 2016 until June 2019 was 43.39%. With the details of the smallest MAPE value is 22.04% and the largest MAPE value is 59.51%.

6. Future Research
In this research, the Mean Absolute Percentage Error rate was 43.39%. For the future research will be conducted research on the use of methods other than Single Exponential Smoothing so that the rate of Mean Absolute Percentage Error obtained is lower than the research that has been done.

Acknowledgments
The authors would like to thank the lecturer of STIKI Indonesia for supporting this research. The author would also like to thank the operational staff of Puri Wira Mahkota Ltd for supplying product selling data for this study.

References
[1] Mala Sari R E, Kustiyahningsih Y, Sugiharto R. Sistem Peramalan Stok Obat Menggunakan Metode Exponential Smoothing. Konf Nas Sist Inform 2015. 2015;216–21.
[2] Sundari SS, Susanto, Reviandi W. Sistem Peramalan Persediaan Barang Dengan Weight Moving Average Di Toko The Kids 24. Konf Nas Sist dan Inform. 2015;598–603.
[3] Herjanto E. Manajemen Operasi Edisi Ketiga. Jakarta: Grasindo; 2015. 77-100 p.
[4] Kurniasih N, Ahmar AS, Hidayat DR, Agustin H, Rizal E. Forecasting Infant Mortality Rate for China: A Comparison between α-Sutte Indicator, ARIMA, and Holt-Winters. J Phys Conf Ser. 2018;1028(1):6–12.
[5] Heizer J, Render B. Manajemen Operasi Manajemen Keberlangsungan dan Rantai Pasokan. Edisi 11. Jakarta: Salemba Empat; 2015.
[6] Stevenson WJ, Chuong SC. Operations Management an Asian Perspective. Jakarta: Salemba Empat; 2013.
[7] Shi JY, Zhang DY, Ling LT, Xue F, Li YJ, Qin ZJ, et al. Dual-algorithm maximum power point tracking control method for photovoltaic systems based on grey wolf optimization and golden-section optimization. J Power Electron. 2018;18(3):841–52.
[8] Khair U, Fahmi H, Hakim S Al, Rahim R. Forecasting Error Calculation with Mean Absolute Deviation and Mean Absolute Percentage Error. J Phys Conf Ser. 2017;930(1).
[9] Gilliland M. Forecasting FAQs (The Business Forecasting Deal: Exposing Myths, Eliminating Bad Practices, Providing Practical Solutions). 2010;193–246.