Probabilistic analysis of tunnel excavation with spatially correlated Young’s modulus

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Abstract. Understanding the responses of tunnel excavation is essential in urbanization. Tunnel constructions are subject to uncertainties originated from complex geological conditions, including the spatial variability of soil properties. Therefore, it is necessary to adopt probabilistic methods that can analyse the spatial variability of geotechnical parameters in tunnel construction. Previously few studies considered plastic strain of tunnels considering the spatial variability of geotechnical properties. In this study, the effects of the statistics (correlation lengths and Coefficients of Variation (CoV)) of spatially correlated Young’s modulus on horizontal/vertical displacements and maximum plastic strain are studied. The mean and median values of the vertical displacement are insensitive to the correlation lengths, while the standard deviation is positively correlated with the correlation lengths. With the increase of CoV, the distribution of maximum plastic strain becomes dispersed, but the change is no longer significant when the CoV is small.

1. Introduction
Understanding the responses of tunnel excavation is essential in urbanization. Tunneling plays a significant role in the transportation and development of underground space. The construction of the tunnel is in the ascendant, and it is critical to understand the displacement and stress caused by tunnel excavation. Tunnel constructions are subject to uncertainties originated from complex geological conditions, including the spatial variability of geotechnical properties. Therefore, it is necessary to adopt probabilistic methods that can analyze soil spatial variability in tunnel construction.

Many scholars have studied the problem of tunnel excavation and construction using probabilistic methods. Lü et al. [1] assessed the tunnel convergence considering spatial variability in rock mass properties using interpolated autocorrelation and response surface method. Chen et al. [2] employed and compared three artificial neural network methods for the prediction of maximum surface settlement caused by EPB shield tunneling. Cheng et al. [3,4] provided a method to analyze the stability of tunnels in a three-dimensional model and investigated soil parameters on tunnel-induced ground movements considering soil spatial variability. Huang et al. [5] studied the influence of spatial variability of soil Young's modulus on tunnel convergence in soft soils. These studies show that the probabilistic method can be a sensible way to quantify the uncertainty of geotechnical properties in
tunnel engineering [6,7]. However, previously few studies considered plastic strain of tunnels considering the spatial variability of geotechnical properties.

In this study, a numerical model of tunnel excavation is firstly established. The effects of the statistics (correlation lengths and Coefficients of Variation (CoV)) of spatially correlated Young’s modulus on horizontal/vertical displacements and maximum plastic deformation are studied.

2. Methods

2.1. Random field generation for probabilistic analysis
The spatial variability of soil properties can be represented by the random field. The mean value, standard deviation, and correlation length are demand to simulate a random field with a given covariance function. In this study, Karhunen–Loève expansion method is chosen to simulate the random field. The natural logarithm of Young’s modulus InE can be expressed as [8]:

\[ \ln E(x) \approx \mu_{\ln E}(x) + \sum_{i=1}^{n} \lambda_i \theta_i \phi_i(x) \]  

where \( \mu_{\ln E} \) is the mean value of log-normally distributed \( E \); \( \lambda_i \) and \( \theta_i \phi_i(x) \) are the \( i \)th eigenvalues and eigenvectors of the covariance function \( C(x) \); \( \theta_i \) are Gaussian random numbers; \( n \) is the truncation level; \( x = [x_i, z_j] \) are the coordinates and \( j \) is the index of location. The covariance function \( C(x) \) can be described by empirical models. In this study, the exponential covariance function is adopted:

\[ C(x) = \sigma_{\ln E}^2 \exp \left( -\left( \frac{(x_1-x_2)^2}{l_x^2} + \frac{(x_1-x_2)^2}{l_z^2} \right)^{1/2} \right) \]

where \( l_x \) and \( l_z \) are the correlation lengths; \( \sigma_{\ln E} \) is the standard deviation of log-normally distributed \( E \).

Normally, to ensure the soil properties are positive values, it is generally assumed that soil parameters follow log-normal distributions. The mean value \( \mu_{\ln E} \) and the standard deviation \( \sigma_{\ln E} \) to simulate log-normal distribution can be calculated as:

\[ \sigma_{\ln E}^2 = \ln(1 + \sigma_E^2 / \mu_E^2) \]

\[ \mu_{\ln E} = \ln(\mu_E) - \frac{1}{2} \sigma_E^2 \]

where \( E \) represents the elastic modulus; \( \mu_E \) and \( \sigma_E \) are the mean value and the standard deviation of \( E \), respectively. Log-normally distributed \( E_{\text{log-normal}} \) can be computed as:

\[ \ln E = a \mu_{\ln E} + \sigma_{\ln E} \]

\[ E_{\text{log-normal}} = \exp(\ln E) \]

where \( a \) is a random number of standard normal distribution.

2.2. Simulation of soil behaviour during a tunnel excavation
For tunnel construction, surface deformation and plastic region are essential indicators for reinforcements during the excavation. Therefore, the soil behaviour during a tunnel excavation is simulated with the elastoplastic model by COMSOL Multiphysics [9] (Figure 1(a)) and unsupported excavation is considered for simplification. In the initial step, the stress state before excavation is computed for ground stress equilibration as elastic (Figure 1(b)). In the second step, elastoplastic behaviour is simulated after the excavation incorporating the stress calculated in the first step. The Drucker-Prager yield criterion is adopted to simulate soil plastic behaviour:

\[ J_2^{1/2} + a I_1 - \kappa = 0 \]

where \( a \) is a positive parameter that controls the influence of the pressure on the yield limit; \( \kappa \) is the yield stress under pure shear as a constant; \( J_2 \) is the second invariant of stress deviator; \( I_1 \) is the first invariant of the stress tensor. The coefficients in the Drucker-Prager model can be matched to the coefficients in the Mohr-Coulomb criterion by \( a = \frac{2}{\sqrt{3}} \frac{\sin \phi}{(2 - \sin \phi)} \) and \( \kappa = \frac{2}{\sqrt{3}} \frac{\cos \phi}{(2 - \sin \phi)} \).

The geometry and boundary conditions are shown in Figure 1(a). The study domain is 45 m in depth and 90 m in width. The tunnel is 5 m in radius and the centre point is located 20 m below the...
The bottom displacement is constrained in a vertical direction. For the boundary conditions on both sides, a roller boundary is applied to realize the infinite extension of the soil in a lateral direction. Young’s modulus is viewed as a spatially varied parameter with a mean value of $12 \times 10^6$ Pa. The material is assumed to be silty soil. The Poisson’s ratio, density, cohesion, and angle of internal friction are 0.495, 2000 kg/m$^3$, $130 \times 10^3$ Pa, and 30°, respectively.

The deterministic results are shown in Figure 1(c) ~ Figure 1(e). The largest vertical deformation about (0.24m) appears at the top and bottom of the tunnel. The horizontal deformation above the tunnel is more considerable than below. The plasticity region is around the tunnel with a width of around 1 m and the area is estimated to be 17 m$^2$.

Figure 1. Tunnel excavation model with homogenous soil: (a) Model geometry and mesh grid; (b) stress state before excavation (ground stress equilibration); (c) vertical displacement; (d) horizontal displacement; (e) plastic strain.

3. Results and discussions

3.1. Effects of correlation length

Figure 2 shows the effects of correlation lengths on the statistics of horizontal displacement. The horizontal/vertical correlation lengths are increased from 1/0.5 m to 10/5 m and then to 90/45 m with
the same ratio. The median values, mean values, and standard deviations of the horizontal displacement are also increased. Among them, the variations of median values and mean values are not significant, but the standard deviations around the sides of the tunnel are increased notably from 0.02 to 0.04 m.

Figure 3 explains the effects of correlation lengths on the statistics of vertical displacement. Similar to the case of horizontal deformation, with the increase of the correlation distance, the mean and median values increase trivially, and the standard deviation increases significantly. The above research shows that the mean and median values of the vertical displacement are insensitive to the correlation lengths, while the standard deviation is positively correlated with the correlation lengths.

Figure 2. Effects of correlation lengths on horizontal displacement: (a) Median values; (b) mean values and (c) standard deviations. (HL and VL denote horizontal and vertical correlation length, respectively).

Figure 3. Effects of correlation lengths on vertical displacement: (a) Median values; (b) mean values and (c) standard deviations. (HL and VL denote horizontal and vertical correlation length, respectively).
The distributions of maximum plastic strain with different correlation lengths are displayed in Figure 4. The maximum plastic strain according to different correlation lengths has small deviations. With the increase of correlation lengths, the distributions of maxima of plastic strain become scatter, which illustrates that it suffers larger uncertainty.

![Figure 4. Effects of correlation lengths on distributions of maximum plastic strain.](image)

3.2. Effects of CoV

The effect of Coefficients of Variation (CoV) on the statistics of horizontal displacement is shown in Figure 5. It is obvious that the statistics, including median value, mean value and standard deviation are all growing with the increase of CoV. In Figure 5(a) and Figure 5(b), as CoV increases, the area enclosed by the contours of -0.05 gradually increases. It shows that the shrinkage of soil near the tunnel is more obvious with the increasing CoV. In Figure 5(c), with the growth of CoV, the fluctuated area of horizontal displacement gradually increases. When CoV = 0.8 is adopted, the standard deviation of the horizontal displacement on both sides of the tunnel is as high as 0.2 m, which denotes significant uncertainty.

Figure 6 displays the effect of CoV on the statistics of vertical displacement. Although the median and mean values of vertical displacement increase with larger CoV, the changes are not significant compared to horizontal displacement (Figure 6(a) and Figure 6(b)). It is signified that the vertical displacement is not sensitive to the variability of Young's modulus. As for the standard deviation of the vertical displacement, the trend is the same as the horizontal displacement, but the upper and lower sides of the tunnel are mainly affected.

Figure 7 demonstrates the effects of CoV on distributions of maximum plastic strain. The maximum plastic strain according to different CoVs is all centralized around the same value of 0.1 m. With the increase of CoV, the distribution becomes dispersed, but the change is no longer significant when the CoV is small.
Figure 5. Effects of Coefficients of Variation (CoV) on horizontal displacement: (a) Median values; (b) mean values and (c) standard deviations.

Figure 6. Effects of Coefficients of Variation (CoV) on vertical displacement: (a) Median values; (b) mean values and (c) standard deviations.
4. Conclusions
In this study, a numerical model of tunnel excavation is first established. The effects of correlated
Young’s modulus on horizontal/vertical displacements and maximum plastic strain are studied. Major
conclusions are summarized below:
1. The mean and median values of the vertical displacement are insensitive to the correlation
lengths, while the standard deviation is positively correlated with the correlation lengths. With
the increase of correlation lengths, the distributions of maxima of plastic strain become scatter,
which illustrates that it suffers larger uncertainty.
2. The statistics of horizontal displacement deviation are all growing with the increase of CoV.
With the increase of CoV, the distribution of maximum plastic strain becomes dispersed, but
the change is no longer significant when the CoV is small.

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Figure 7. Effects of Coefficients of Variation (CoV) on distributions of maximum plastic strain.