Economic consideration of optimal vaccination distribution for epidemic Spreads in complex networks

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Abstract. The main concern of epidemiological modeling is to implement an economical vaccine allocation to the population. Here, we investigate the optimal vaccination allocation in complex networks. We find that the optimal vaccine coverage depends not only on the relative cost of treatment to vaccination but also on the vaccine efficacy. Especially with a high cost of treatment, nodes with high degree are prioritized to vaccinate. These results may help us understand factors that may impact the optimal vaccination distribution in the control of epidemic dynamics.

1. Introduction

The concept of basic reproduction number $R_0$, defined as the average number of secondary cases generated by a ‘typical’ infectious case, throughout the infectious period, in a totally susceptible population [1], is often used to evaluate the ability of a disease to invade the population. If $R_0 < 1$, the infectious individual is unable to sustain itself in the population; if $R_0 > 1$, there is a positive probability for a large epidemic to occur [2]. In order to efficiently control the outbreak of an infectious disease, a vaccine strategy has to be planned and the efficacy of it has to be evaluated. Designing a vaccine strategy for the control of an outbreak is a key goal for epidemiological modeling. Traditionally, such kind of strategy has been formed to stop an outbreak as quickly as possible regardless of the costs that will be taken, yielding the critical vaccination coverage $p_c$, i.e., the fraction of population that one needs to vaccinate in order to avoid a major outbreak, $p_c = 1 - \frac{1}{R_0}$. If $p > p_c$, the prevalence will be eradicated. However, from an economic point of view, there is a growing requirement for a successful strategy that minimizes the total cost as lower as possible [3, 4, 5].

Our purpose in this paper is to investigate the optimal vaccination distribution in complex networks with the consideration of cost, which is assumed to be the sum of the cost for treating infected individuals and that for vaccinating susceptible individuals. Since vaccines rarely provide full protection from diseases [6, 7, 8], imperfect vaccines are assumed in the present paper, as observed in the vaccines that are currently being developed against malaria [9, 10]. We find that for homogeneous networks, the minimal cost is relevant to the relative cost of treatment to vaccination. For heterogeneous networks, since it is difficult to determine
the fraction of susceptible and infected individuals at equilibrium due to the uncertainty of vaccine allocation and the increased number of variables, optimal vaccination distributions are numerically determined with an optimization routine. In light of our findings, optimal vaccination distributions are found closely relevant to the relative cost. For example, for a high cost of treatment, nodes with high degree have to be prioritized to vaccinate. The results have significant implications for the success control of infectious diseases that spread via contact and help us understand nodes’ roles in the implement of an economical vaccine allocation strategy for public health service.

An SIR model with imperfect vaccines in homogeneous networks

To carry out the epidemiological analysis, a classical SIR model with birth and death processes is used. Assume totally $x_0$ percent of susceptible individuals can receive vaccines. Unvaccinated susceptible individuals get infected by contacting infected individuals with rate $\beta I$, while vaccinated susceptible individuals get part protection from infection with a decreased infection rate $\beta_v$, described by $\beta_v = (1 - \alpha)\beta I$, while infected individuals recover with rate $r$. Without economic consideration, the critical vaccine coverage, i.e., the fraction of population, necessary to vaccinate in order to avoid a major outbreak is given by $1 = R_0(1 - \alpha x_0)$, where $R_0$ is the basic reproduction number defined as $R_0 = \frac{\beta I}{\mu + r}$ [1]. Thus, the critical vaccine coverage $x_{oc}$ is given by $x_{oc} = \frac{1}{\alpha}(1 - \frac{1}{R_0})$.

Let us consider the cost required for the model. The total cost is assumed to be the sum of the cost for vaccinating susceptible individuals and that for treating infected individuals. For simplicity, we assume that the cost of vaccination is exponentially dependent on the vaccine efficacy $\alpha$ and that for treatment is proportional to the prevalence at equilibrium. Thus, the total cost is defined as

$$f(x_0) = c_v e^{-\gamma x_0}S + c_I I,$$

where $S$ and $I$ are the stationary fractions of susceptible and infected individuals, $c_v$ and $c_I$ are per-capita costs of vaccination and treatment. Usually we have $c_v < c_I$ described by $c_I = \gamma c_v$ with $\gamma > 1$. The optimal solution of Eq. (1), $x_{oc}^*$, is achieved by checking the first-order condition $\frac{df(x_0)}{dx_0} = 0$ and the second order condition $\frac{d^2f(x_0)}{dx_0^2} = 0$.

The total costs as a function of $\gamma$ and $\alpha$ can are shown in Fig. 1. We compared the total costs for different vaccine efficacy $\alpha = 1.0$ and $\alpha = 0.2$. We see that with a perfect vaccine efficacy $\alpha = 1$, if the cost of treatment $\gamma$ is not so high, i.e., $\gamma \leq 6$, then it is unnecessary to vaccinate any one at all. By increasing $\gamma$, the optimal vaccine coverage $x_0$ increases up to the value of $x_{oc}$, during which process, a smaller optimal value $x_{oc}^*$ is observed. For a lower vaccine efficacy $\alpha = 0.2$, vaccine allocation is unnecessary due to its low efficacy. Therefore, the results suggest us that the optimal vaccine coverage is determined by a number of factors such as vaccine efficacy and the cost of treatment, which should be taken into account in the design of vaccine allocation by the public health service.

2. The SIR model with imperfect vaccines in heterogeneous networks

In order to stress the role of heterogeneity on epidemic process with vaccination, we consider the SIR model with imperfect vaccines in heterogeneous networks, which is described by a given degree distribution $p(k) \sim k^{-3}$ with a finite average connectivity $\langle k \rangle = \sum k p(k)$. Thus, the problem is equivalent to allocate $x_k$ to nodes with degree $k$ with the minimal total cost constrained by the total amount of vaccine resources $\sum_k p(k) x_k = x_0$.

To find the optimal vaccination distribution, we here directly implement an optimization routine to determine the optimal vaccination distribution $x_k$ in degree $k$. With the tabu
Figure 1. Total costs versus $\gamma$ and $x_0$ for different vaccine efficacies $\alpha$ in homogeneous networks. (a)$\alpha = 1.0$; (b) $\alpha = 0.2$. Parameters are set as $R_0 = 1.5$, $\mu = \frac{1}{80}$, and $r = \frac{1}{14}$. The basic per-capita cost is set as $c_v = 1$.

search [12], we determine the optimal vaccination distribution $x_k$ with the minimal cost with combinations of parameters such as the relative cost $\gamma$ and the vaccine efficacy $\alpha$. Incorporating these factors into the mode, we implement the tabu search for more than 10 times with different initial solutions, and take the one with the lowest cost as the optimal solution.

Figure 2. Optimal distributions of $x_k$ versus $k$ for different combinations of $\gamma$ and $\alpha$ with very limited amount of vaccine resources $x_0 = 10\%$ that is not large enough to eradicate the disease. (a) $\gamma = 1$ and $\alpha = 0.5$; (b) $\gamma = 10$ and $\alpha = 0.5$; (c) $\gamma = 1$ and $\alpha = 1.0$; (d) $\gamma = 10$ and $\alpha = 1.0$. Parameters are set as $c_v = 1$, $R_0 = 1.5$, $\mu = 1/80$, and $r = 1/14$. Nodes with degree $k > k_c = 11$ are taken as one group.

Figure 2 shows the optimal vaccination distribution $x_k$, as a function of $k$, at the level of a very limited vaccine resources with $x_0 = 1\%$ and $R_0 = 1.5$, for the comparison between lower and higher relative costs ($\gamma = 1$ in panels (a) and $\gamma = 10$ in panels (b)). It demonstrates that the role of high degree nodes since all nodes with degree larger than $k_c = 11$ are almost prioritized to vaccinate. The role of nodes with medium degrees are found fundamentally important to the
decrease of the cost at a high relative cost \((\gamma = 10)\), in which the optimal vaccination distribution switches to vaccinate more nodes with medium degrees. For example, degrees less than 5 are not found to vaccinate at all (panels (c) and (d)). At a high relative cost, the optimal vaccination distribution favors the nodes that could efficiently reduce the prevalence at equilibrium, and thereby reducing the cost of treatment.

The results presented above indicate that whether a vaccination strategy is optimal or not depends on some factors such as the available amount of vaccine resources and the cost of treatment as we tested. From a realistic point of view, these results have important implications for the success of economical allocations of vaccines for the diseases that spread via contact, typically characterized by sexually transmitted diseases (STDs), by which people get infected through sexual contact with an infected person. Since the human sexual contact network has been found to follow a power-law distribution \[11\], according to our results, we guess that if STDs vaccines are very limited, then persons who have broad sexual partners have to be prioritized to vaccinate in order to reduce the perceived cost possibly caused by the potential infection of them.

Conclusions
In this study, we investigate the optimal vaccine allocation to the population with the consideration of economic factors. The study is applicable to diseases that spread through air, such as influenza, and diseases that spread through contact, such as STDs. Our study demonstrates that the design of economical vaccination strategies should be incorporated with a number of factors that involved in the model such as the cost of treatment as we tested and the vaccine efficacy. Achieving more complete control of STDs may be possible if advances in our understanding of nodes’ roles in the epidemiology and transmission dynamics can be integrated into new interventional strategies.

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