Strong decays of higher excited heavy-light mesons in a chiral quark model

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The strong decay properties of the higher excited heavy-light mesons from the first radially excited states up to the first \( P \)-wave states are studied in a constituent quark model. It is found that many missing excitations have good potentials to be found in future experiments for their narrow widths, and some of them dominantly decay into the first orbital excitations rather than into ground states. In future observations, one should focus on the decay processes not only into the ground states, but also into the low-lying \( P \)-wave excitations with \( J^P = 0^+ \) as well. Furthermore, the nature of the newly observed states \( D_s(3000) \), \( D'_s(3000) \) and \( B(5970) \) is discussed. It is predicted that \( D_s(3000) \) seems to be a partner of \( D_{sJ}(3040) \), which could be identified as the low-mass mixed state \([2P_{1J}]_{Ls} \) via the \( 2^1P_1-2^3P_1 \) mixing. The \( D'_s(3000) \) state seems to favor the \( 1^3F_4 \) state, while the quantum numbers \( J^P = 0^+ \) and \( 2^+ \) can not be excluded completely, more experimental observations are needed to determine its \( J^P \) values. The \( B(5970) \) resonance is most likely to be the \( 1^3D_1 \) with \( J^P = 3^- \).

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I. INTRODUCTION

During the past several years, significant progress has been made in the observation of the heavy-light mesons. More and more higher excitations have been found in experiments. In the \( D \)-meson family, four new members, the \( D(2550) \), \( D(2600) \), \( D(2750) \) and \( D(2760) \) were discovered by BABAR Collaboration \cite{1}, which were confirmed by LHCb Collaboration with slightly different masses \cite{2}. Furthermore, the LHCb Collaboration observed two new higher \( D \)-meson excitations \( D'_s(3000) \) and \( D'_s(3000) \) with natural and unnatural parities, respectively \cite{2}. In the \( D_s \)-meson family, except for the \( D_{sJ}(2700) \) and \( D_{sJ}(2860) \) observed by BABAR and Belle before, \cite{3} recently a new broad higher \( D_s \)-meson excitation \( D_{sJ}(3040) \) was observed by BABAR as well \cite{5}. Very recently, progress has been obtained in the search for the excited \( B \)-meson as well. The CDF Collaboration observed some evidence of a new narrow excited state \( B(5970) \) in the \( B\pi \) channel \cite{6}.

About the newly observed states \( D(2550) \), \( D(2600) \), \( D(2750) \), \( D(2760) \), \( D_{sJ}(2700) \), \( D_{sJ}(2860) \) and \( D_{sJ}(3040) \), their strong decays had been analyzed in our previous papers \cite{7–9}. We found that the \( D(2760) \) could be identified as the \( 1^3D_3 \) with \( J^P = 3^- \), while the \( D(2750) \) is most likely to be the mixed state \( [1D'_s]_{Ls} \) \( J^P = 2^- \) via the \( 1^1D_2-1^3D_2 \) mixing. The \( D(2600) \) favors the mixed state \( [1^2D_s]_{L} \) \( J^P = 1^- \) via the \( 1^1D_1-2^1S_1 \) mixing. The \( D(2550) \) could be assigned as the \( 2^1S_0 \) assignment although its predicted width is obviously narrower than that given in experiments. For the newly observed \( D_{sJ} \) mesons, we found that the \( D_{sJ}(2700) \) is most likely to be the strange partner of the \( D(2600) \). There might exist two-state scenario for the \( D_{sJ}(2860) \), one resonance corresponds to the strange partner of the \( D(2760) \) denoted by \( D_{sJ}(2860) \) and the other resonance is the strange partner of the \( D(2750) \) denoted by \( D_{sJ}(2860) \). The \( D_{sJ}(3040) \) could be identified as the low-mass physical state \( [2P_{1J}]_{L} \) via the \( 2^1P_1-2^3P_1 \) mixing. More discussions of these newly observed \( D \) and \( D_s \) mesons can be found in the literature \cite{10–30}.

On the other hand, about the higher excitations \( D'_s(3000) \), \( D'_s(3000) \) and \( B(5970) \) observed very recently, a few studies can be found in the literature \cite{31–37}. For example, the \( D'_s(3000) \) and \( D'_s(3000) \) states were explained as the first \( P \)-wave states in the \( D \) meson family with \( J^P = 0^+ \) and \( 1^+ \), respectively \cite{32}. As a candidate of the \( 2^1S_1 \) \cite{36,37} \( 1^3D_1 \) and \( 1^3D_3 \) states \cite{36}, the strong decay properties of the \( B(5970) \) were studied with the heavy meson effective theory.

These newly observed higher excitations provide us a good chance to establish a fairly abundance \( D \)- and \( D_s \)-meson spectroscopy, which has been summarized in Tab. \ref{tab:1}. For comparison, some interested predictions of the heavy-light meson spectroscopy in theory are also included \cite{39,40}. From the table, it is seen that the knowledge about the \( B \)- and \( B_s \)-meson spectroscopy is very poor. Except for the ground states, only two low-lying orbital excitations in the \( P \)-waves are found in the \( B \)- and \( B_s \)-meson spectroscopy. The discovery of the \( B(5970) \) enhances our confidence in the search for more higher excitations in the \( B \)- and \( B_s \)-meson families. From now on, we might have a golden time to study the higher excitations of the heavy-light mesons.

To provide helpful information for the experimental search for more excited heavy-light mesons and to gain a unified understanding of the newly observed resonances, in this work we continue to study the strong decay properties of the higher excitations of heavy-light mesons up to the \( F \)-waves (See Tab. \ref{tab:1}) with the chiral quark model as well. This model has been developed and successfully used to deal with the strong decays of heavy-light mesons, charmed and strange baryons \cite{7–9,41,43}.

This paper is organized as follows. In the subsequent section, a brief review of the model is given. The numerical results are presented and discussed in Sec. \ref{sec:3}. Finally, a summary is given in Sec. \ref{sec:4}.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Resonance} & \textbf{Mass (MeV)} & \textbf{Notation} \\
\hline
\end{tabular}
\caption{Review of \( D \)-mesons.} 
\end{table}
In this section, we give a brief review of the chiral quark model. The details of this model can be found in our previous papers [4, 5]. In this model, the low energy quark-pseudoscalar-meson and quark-vector-meson interactions in the SU(3) flavor basis might be described by the effective Lagrangians

\[ \mathcal{L}_{\text{qqq}} = \sum_{j} \frac{1}{f_m} \bar{\psi}_j y_\mu^j \gamma_\mu \gamma_5 \psi_j \partial^\mu \phi_m, \]

\[ \mathcal{L}_{\text{qqq}} = \sum_{j} \bar{\psi}_j (4\eta^j + \frac{ib}{2m_j} \tau_{\mu\nu} q^\nu) V^\mu \psi_j, \]

respectively, where \( \psi_j \) represents the \( j \)-th quark field in the hadron, \( \phi_m \) is the pseudoscalar meson field, \( f_m \) is the pseudoscalar meson decay constant, and \( V^\mu \) represents the vector meson field. Parameters \( a \) and \( b \) denote the tensor and vector coupling strength, respectively.

In this model, the wave function of a heavy-light meson is adopted the non-relativistic harmonic oscillator wave function, i.e., \( \psi_{nm}^\mu = R_{nm} Y_{nm} \). To match the non-relativistic wave functions of the heavy-light mesons, we should adopt the non-relativistic form of Eq. (1) in the calculations, which is given by [47, 49]

\[ H_m = \sum_j \left[ A \sigma_j \cdot \mathbf{q} + \frac{\alpha_m}{2\mu_q} \sigma_j \cdot \mathbf{p}_j \right] I_j \phi_m, \]

in the center-of-mass system of the initial meson. Where we have defined \( A \equiv -(1 + \frac{a}{2\mu_q}) \). On the other hand, from Eq. (2), one can easily derive the non-relativistic transition operators for the emission of a transversely or longitudinally polarized vector meson, which are given by [50, 52]

\[ H_m^T = \sum_j \left\{ i \frac{\mathbf{b} \cdot \sigma_j}{2m_q} (\mathbf{q} \times \mathbf{e}) + \frac{\alpha}{2\mu_q} \mathbf{p}_j \cdot \mathbf{e} \right\} I_j \phi_m, \]

and

\[ H_m^L = \sum_j \frac{\alpha M_{\mathbf{q}}}{|\mathbf{q}|} I_j \phi_m. \]

In the above equations, \( \mathbf{q} \) and \( \alpha_m \) are the three-vector momentum and energy of the final-state light meson, respectively; \( \mathbf{p}_j \) is the internal momentum operator of the \( j \)-th quark in the heavy-light meson rest frame; \( \sigma_j \) is the spin operator corresponding to the \( j \)-th quark of the heavy-light system; and \( \mu_q \) is a reduced mass given by \( 1/\mu_q = 1/m_j + 1/m_j' \) with \( m_j \) and \( m_j' \) for the masses of the \( j \)-th quark in the initial and final mesons.

### TABLE I: The heavy-light meson spectroscopy predicted in theory compared with the observations in experiments.

| States | \( n^S L \) | Predicted Mass | Observed state | Predicted Mass | Observed state | Predicted Mass | Observed state | Predicted Mass | Observed state |
|--------|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( 1^1S_0 \) | 2900 | 2460 | 2535 | 5279/5280 | \( B(5280) \) | 5373/5372 | \( B(5372) \) | 5421/5414 | \( B'(5414) \) |
| \( 1^1S_1 \) | 2490 | 2460 | 2535 | 5742/5774 | \( B(5774) \) | 5802/5842 | \( B(5842) \) | 5805/5831 | \( B(5831) \) |
| \( 1^1P_0 \) | 2775 | 2460 | 2535 | 5886/5890 | \( B(5890) \) | 5985/5976 | \( B(5976) \) | 6049/6092 | \( B(6092) \) |
| \( 1^1P_1 \) | 2883 | 2850 | 2970 | 6025/6119 | \( B(60119) \) | 6103/6192 | \( B(6192) \) | 619/6192 | \( B(6192) \) |
| \( 1^1P_2 \) | 2979 | 2863 | 2970 | 6264/6412 | \( B(6412) \) | 6369/6501 | \( B(6501) \) | 6363/6468 | \( B(6468) \) |
| \( 2^1P_0 \) | 3045 | 3021 | 3155 | 6163/6221 | \( B(6221) \) | 6264/6318 | \( B(6318) \) | 6316/6378 | \( B(6378) \) |
| \( 2^1P_1 \) | 3101 | 3090 | 3224 | 6175/6209 | \( B(6209) \) | 6278/6321 | \( B(6321) \) | 6296/6345 | \( B(6345) \) |
| \( 1^3F_0 \) | 3074 | 3129 | 3203 | 6194/6281 | \( B(6281) \) | 6296/6345 | \( B(6345) \) | 6329/6439 | \( B(6439) \) |
| \( 1^3F_1 \) | 3123 | 3145 | 3247 | 6200/6260 | \( B(6260) \) | 6296/6345 | \( B(6345) \) | 6329/6439 | \( B(6439) \) |
| \( 1^3F_2 \) | 3091 | 3187 | 3220 | 6226/6380 | \( B(6380) \) | 6337/6475 | \( B(6475) \) | 6337/6475 | \( B(6475) \) |
respectively. $M_i$ is the mass of the emitted vector meson. The plane wave part of the emitted light meson is $\varphi_i = e^{-iq \cdot \Gamma}$, and $I_j$ is the flavor operator defined for the transitions in the SU(3) flavor space \,[43–52]. The parameter $b'$ in Eq. (4) is defined as $b' \equiv b - a$.

For a light pseudoscalar meson emission in a heavy-light meson strong decays, the partial decay width can be calculated with \,[7, 41]

$$\Gamma = \left( \frac{\delta}{f_m} \right)^2 \frac{(E_f + M_f)|q|}{4\pi M_i(2J_f + 1)} \sum_{I_{j_1}I_{j_2}} |M_{I_{j_1}I_{j_2}}|^2, \quad (6)$$

where $M_{I_{j_1}I_{j_2}}$ is the transition amplitude, $J_{I_{j_1}}$ and $J_{I_{j_2}}$ stand for the third components of the total angular momenta of the initial and final heavy-light mesons, respectively. $\delta$ as a global parameter accounts for the strength of the quark-meson couplings. It has been determined in our previous study of the strong decays of the charmed baryons and heavy-light mesons \,[7, 41]. Here, we take the same value as that determined in Refs. \,[7, 41], i.e. $\delta = 0.557$.

In the calculation, the standard quark model parameters are adopted. Namely, we set $m_u = m_d = 330$ MeV, $m_s = 450$ MeV, $m_c = 1700$ MeV, and $m_b = 5100$ MeV for the constituent quark masses. The harmonic oscillator parameter $\beta$ in the wave functions of heavy-light mesons is taken as $\beta = 0.40$ GeV. The decay constants for $\pi$, $K$ and $\eta$ mesons are taken as $f_\pi = 132$ MeV, $f_K = f_\eta = 160$ MeV, respectively. For the vector-meson coupling strength which still suffers relatively large uncertainties, we adopt the values extracted from vector meson photoproduction, i.e. $a = -3$ and $b' = 5$ \,[50–52]. The masses of the mesons used in the calculations are adopted from the Review of Particle Physics \,[53]. With these parameters, the strong decay properties of the well known heavy-light mesons and charmed baryons have been described reasonably \,[7, 43, 44, 42].

III. CALCULATIONS AND RESULTS

A. The $2^1S_0$ states

Recently, the BABAR Collaboration observed a excited D-meson state $D(2550)$ in $D^+\pi$ channel with a width of $\Gamma = 130$ MeV \,[1], which was also observed by the LHCb Collaboration lately \,[2]. In theory, the first radially excited D-meson state $2^1S_0$ has a mass $\sim 2.58$ GeV \,[39, 40, 44]. Furthermore, the decay mode and the BABAR analysis of angle distributions indicate that the $D(2550)$ should be classified as the radially excited state $2^1S_0$. We have studied the strong decays of the $D(2550)$ as the first radially excited D-meson state $2^1S_0$ in Refs. \,[7, 9]. The strong decays of $D(2550)$ are dominated by the $D^+\pi$ and $D_0(2400)\pi$ modes. In present work we have improved our predictions according to the newly observations from LHCb Collaboration \,[2]. The results are listed in Tab. \,[11].

The predicted partial decay width ratio is

$$\frac{\Gamma[D^+\pi]}{\Gamma[D_0(2400)\pi]} \approx 0.28, \quad (7)$$

which strongly depends on the mass of $D_0(2400)$. Our predicted width, $\Gamma = 68$ MeV, is about a factor of 2 narrower than the data. The $^3P_0$ model \,[23] and relativistic quark model \,[39] calculations also obtained a narrow width for the $D(2^1S_0)$ state.

If the $D(2550)$ is the $2^1S_0$ assignment indeed, the same excitation in the $D_s^+, B^-$, and $B_s^-$ meson spectroscopy might be found in the future experiments. To obtain more information of these missing states, we further study the strong decay properties of these radially excited states $D_s(2^1S_0)$, $B(2^1S_0)$ and $B_s(2^1S_0)$ in the followings.

TABLE II: The strong decay width (MeV) of the $D(2550)$ as the $2^1S_0$ with a mass of 2580 MeV.

| Mode          | $D^+\pi$ | $D_0(2400)\pi$ | Total |
|---------------|----------|----------------|-------|
| Width         | 15       | 53             | 68    |

In the $D_s$-meson family, the predicted mass of the first radially excited state $2^1S_0$ is around $2.7$ GeV \,[39, 40, 44]. There is no experimental information about this state. The strong decay properties of $D_s(2^1S_0)$ have been studied in our previous work \,[7]. Its strong decays are governed by the $D^*K$ channel. Based on our model, the $D_s(2^1S_0)$ is a very narrow state with a width of $\Gamma \approx 10$ MeV. Such a narrow state should be observed in the $D^*K$ final states if it exists indeed.

In the $B$-meson family, the predicted mass of $B(2^1S_0)$ is around $5.9$ GeV \,[39, 40, 44]. We calculate its strong decay properties, which are listed in Tab. \,[11]. It is seen that the strong decays are dominated by the $B^*\pi$ and $B(1^3P_0)\pi$ channels, and the predicted partial width ratio is

$$\frac{\Gamma[B^*\pi]}{\Gamma[B(1^3P_0)\pi]} \approx 1.3. \quad (8)$$

The $B(2^1S_0)$ is also a narrow state with a width of $\Gamma \approx 40$ MeV, which is slightly narrower than that of $D(2^1S_0)$. The decay width predicted by us is roughly compatible with the recent calculations with $^3P_0$ model \,[33].

TABLE III: The strong decay width (MeV) of the $B(2^1S_0)$ with a mass of 5886 MeV. The mass of $B(1^3P_0)$ is taken as 5706 MeV.

| Mode          | $B^*\pi$ | $B(1^3P_0)\pi$ | $B(1^3P_2)\pi$ | $B^*\eta$ | Total |
|---------------|----------|----------------|----------------|-----------|-------|
| Width         | 16       | 24             | 0.01           | 0.25      | 40    |

In the $B_s$-meson family, the predicted mass of $B_s(2^1S_0)$ is around $6.0$ GeV \,[35, 40, 44]. We calculate the strong decay properties of this state with a mass of $M = 5985$ MeV. This state dominantly decays into the $B^*\eta$ channel. The total decay width is $\Gamma \approx 26$ MeV, which is a little narrower than that of the $^3P_0$ calculations \,[33].

As a whole, the first radially excited states $2^1S_0$ in the $D^*, D_s^+, B^-$ and $B_s^-$ meson spectroscopy might have a fairly narrow width. However, up to now none of them have been established. It is still a puzzle why such narrow radially excited states are still missing. More theoretical and experimental studies are needed.
The mixing angle for the $D(2600)$ and $D(2700)$ may correspond to the $D(3600)$ in the range of $(2.65 \sim 2.80)$ GeV, we predict that the $D(2700)$ is a broad state with a width of $\Gamma \approx (360 \pm 120)$ MeV. Its strong decays are dominated by $D\pi$ and $D(2420)\pi$. On the other hand, the mass of $D(2700)$ in the range of $(2.72 \sim 2.88)$ GeV, the total width is $\Gamma \approx (120 \pm 10)$ MeV. The predicted width for the $D(2700)$ is not very broad. This state might be observed in the $D\pi$ channel. It is interestingly found that LHCb Collaboration had observed a new resonance $D(2860)$ with a width of $\Gamma = (159 \pm 122)$ MeV very recently [53, 54]. Their analysis shows that the spin-parity should be $J^P = 1^-$. This newly observed resonance $D(2860)$ favors to be assigned as the physical partners of the $D(2600)$ and $D(2700)$, i.e., $D(2860)$, predicted in our previous work [8].

If both $D(2600)$ and $D(2700)$ correspond to the mixed state $|SD\rangle_{L}$, indeed, the $2S \cdot 1^3D_1$ mixing might be a common character in the heavy-light mesons. The future search for these missing mixing states $|SD\rangle_{L}$ and $|SD\rangle_{H}$ in the heavy-light meson spectroscopy becomes interesting and important.

Following the mixing scheme in Eq. (9), the strong decay properties of the physical partners of the $D(2600)$ and $D(2700)$, i.e., $|SD\rangle_{H}$, have been obtained as well. Taking the mass of $D(2700)$ in the range of $(2.65 \sim 2.80)$ GeV, we predict that the $D(2700)$ is a broad state with a width of $\Gamma \approx (360 \pm 120)$ MeV. Its strong decays are dominated by $D\pi$ and $D(2420)\pi$. On the other hand, the mass of $D(2700)$ in the range of $(2.72 \sim 2.88)$ GeV, the total width is $\Gamma \approx (120 \pm 10)$ MeV. The predicted width for the $D(2700)$ is not very broad. This state might be observed in the $D\pi$ channel. It is interestingly found that LHCb Collaboration had observed a new resonance $D(2860)$ with a width of $\Gamma = (159 \pm 122)$ MeV very recently [53, 54]. Their analysis shows that the spin-parity should be $J^P = 1^-$. This newly observed resonance $D(2860)$ favors to be assigned as the physical partners of the $D(2600)$ and $D(2700)$, i.e., $D(2860)$, predicted in our previous work [8].

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B. The $2S \cdot 1^3D_1$ mixing

In 2010, the BABAR Collaboration discovered a new excited $D$-meson state $D(2600)$ in $D\pi$ and $D^*\pi$ decay channels with a mass of $M = 2612 \pm 6$ MeV and a total decay width of $\Gamma = 93 \pm 6 \pm 13$ MeV [1], which might be a partner of the $D(2700)$ observed previously by BABAR and Belle [2, 3]. The $D(2600)$ may correspond to the $D(2650)$ observed by LHCb very recently [2].

In our previous work [8, 9], we have carefully studied the strong decay properties of the $D(2600)$ and $D(2700)$. According to our analysis, both $D(2600)$ and $D(2700)$ could be explained as the mixed state $|(SD)\rangle_{T}$ via the $2S \cdot 1^3D_1$ mixing:

$$
\begin{pmatrix}
|SD\rangle_{T} \\
|SD\rangle_{H}
\end{pmatrix} = \begin{pmatrix}
\cos \phi & \sin \phi \\
-\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
|2S\rangle \\
|1^3D_1\rangle
\end{pmatrix}.
$$

The mixing angle for the $D(2600)$ is $\phi = (-36 \pm 6)^\circ$, while that for the $D(2700)$ is $\phi = (-54 \pm 7)^\circ$. To explain the strong decay properties of the $D(2600)$ and/or $D(2700)$, configuration mixing between $2S$ and $1^3D_1$ is also suggested in the literature [17, 26, 28].

FIG. 1: The partial decay width and total decay width for the mixed states via $2S \cdot 1^3D_1$ mixing in the $B$- and $B_s$-meson families as functions of the mixing angle $\phi$. The shaded bands correspond to possible mixing angle region derived from the strong decay properties of the $D(2600)$ and $D(2700)$. The masses for the mixed states in the $B$-meson family, $B(|SD\rangle_{L})$ and $B(|SD\rangle_{H})$, are adopted as 5920 and 6025 MeV, respectively. While, the masses for the mixed states in the $B_s$-meson family, $B_s(|SD\rangle_{L})$ and $B_s(|SD\rangle_{H})$, are adopted as 6000 and 6100 MeV, respectively. The mass of $B(1P_1)$ is taken as 5720 MeV. In the figure, we have hidden some decay channels for their small partial decay widths.
Firstly, we consider the $2^3S_1-1^3D_1$ mixing in the $B$-meson family. According to the theoretical calculations of the B-meson spectroscopy, the mass of the low-mass state $|\langle SD \rangle_{1L}|$ is $M \sim 5.9$ GeV \cite{39,40}. Thus, we set the mass of the low-mass mixed state $|\langle SD \rangle_{1L}|$ with $M = 5.9$ GeV, and plot its partial decay widths and total decay width as functions of the mixing angle $\phi$ in Fig. 1. Theoretically, the mixing angle $\phi$ between $2^3S_1$ and $1^3D_1$ in the $D_s^-$, $D_s^+$, $B_s$- and $B_{sJ}$-meson families should be the same in the heavy quark symmetry limit. Combining our previous analysis of the strong decay properties of the $D(2600)$ and $D_{sJ}(2700)$ \cite{8,9}, the mixing angle might be $\phi \approx (45 \pm 16)^\circ$. From Fig. 1, it is seen that the mixed state $B(|\langle SD \rangle_{1L}|)$ has a width of $\Gamma \approx (150 \pm 50)$ MeV. Its decays are dominated by the $B\pi$ and $B'\pi$, the partial width ratio between $B\pi$ and $B'\pi$ is

$$\frac{\Gamma[\pi]}{\Gamma[\pi']} \approx 1.0 \pm 0.3. \quad (10)$$

While for the high-mass mixed state $B(|\langle SD \rangle_{1H}|)$, we take its mass with $M = 6.0$ GeV, and plot its partial decay widths and total decay width as functions of the mixing angle $\phi$ in Fig. 1 as well, where we can see that this resonance is a broad state with a width of $\Gamma \approx (310 \pm 90)$ MeV. It decays mainly through the $B\pi$ and $B_{1J}(5721)\pi$ channels.

Finally, we study the $2^3S_1-1^3D_1$ mixing in the $B_s$-meson family. According to the theoretical predictions in various models, the masses for the mixed state $|\langle SD \rangle_{1H}|$ are around 6.0 and 6.1 GeV, respectively \cite{39,40}. With these predicted masses, we plot the partial decay widths and total decay widths for the mixed states $|\langle SD \rangle_{1L}|$ and $|\langle SD \rangle_{1H}|$ as functions of the mixing angle $\phi$ in Fig. 1, respectively. Adopting the mixing angle $\phi \approx (45 \pm 16)^\circ$, from Fig. 1 we see that the low-mass state $|\langle SD \rangle_{1L}|$ might have a width of $\Gamma \approx (130 \pm 50)$ MeV. Its strong decays are dominated by the $BK$ and $B'K$ channels. While, the high-mass state $|\langle SD \rangle_{1H}|$ is a slightly broader state with a width of $\Gamma \approx (140 \pm 60)$ MeV. The decay widths for the $|\langle SD \rangle_{1L}|$ and $|\langle SD \rangle_{1H}|$ are sensitive to the mixing angle.

The predicted masses of these mixed states $|\langle SD \rangle_{1L}|$ and $|\langle SD \rangle_{1H}|$ in $B$- and $B_s$-meson spectroscopy have a large uncertainty, which may bring uncertainties to the predicted decay widths. To investigate this effect, we plot the decay width as a function of the mass in Fig. 2 with the mixing angle $\phi = (45 \pm 16)^\circ$. It shows that there is an uncertainty of about $(50 \sim 200)$ MeV in the total decay width for the uncertainties of the mass. The sensitivity of different decay modes to the mass can also be seen clearly in the plot.

In a summary, configuration mixing might exist between $2^3S_1$ and $1^3D_1$ according to our model prediction. The decay properties are sensitive to the mixing angle. As flavor partners of the $D(2600)$ and $D_{sJ}(2700)$, the low-mass mixed states $B(|\langle SD \rangle_{1L}|)$ and $B(|\langle SD \rangle_{1H}|)$ have a relatively narrow width, which are most likely to be observed in future experiments. In the high-mass mixed states, the widths of charmed-strange state $D_s(|\langle SD \rangle_{1H}|)$ and bottom-strange state $B_s(|\langle SD \rangle_{1H}|)$ are as comparable as those of the low-mass mixed states, which are possibly to be found in experiments as well. While, the bottom state $B(|\langle SD \rangle_{1H}|)$ is a broad state, which might be hard to be found in experiments. However, it should be
pointed out that in some Refs. [10, 11, 18–20], one believes that configuration mixing might be weak between $2^3S_1$ and $1^3D_1$. Within their models, the strong decay properties of the $D(2600)$ and/or $D_{sJ}(2700)$ can be well explained by taking them as a pure $2^3S_1$ state. To uncover the puzzles in the $2^3S_1$ and $1^3D_1$ states, more accurate measurements for the $D(2600)$ and $D_{sJ}(2700)$ as well as a further search for the missing resonances with $J^P = 1^−$ in the heavy-light meson spectroscopy in experiment is needed.

C. The $1^1D_2$-1$^3D_2$ mixing

The $D(2750)$ is a new excitation observed by BABAR Collaboration in the $D\pi$ channel with a narrow width of $\Gamma \approx 71 \pm 17$ MeV [1]. This state might correspond to the $D(2740)$ observed by the LHCb Collaboration in the same channel recently [2]. Their helicity analysis indicates that this state should have an unnatural parity, i.e., $J^P = 0^-, 1^+, 2^-, \cdots$. In our previous work [3], the new resonance $D(2750)$ had been discussed carefully. We concluded that the spin and parity numbers of the $D(2750)$ might be $J^P = 2^−$, this state is most likely to be the high-mass mixed state $|1D_2^s\rangle_H$ via the $1^1D_2$-1$^3D_2$ mixing:

$$
\begin{pmatrix}
|1D_2^s\rangle_L \\
|1D_2^s\rangle_H
\end{pmatrix} = \begin{pmatrix}
\cos \phi_{1D} & \sin \phi_{1D} \\
-\sin \phi_{1D} & \cos \phi_{1D}
\end{pmatrix} \begin{pmatrix}
|1D_2^s\rangle_L \\
|1^3D_2\rangle
\end{pmatrix},
$$

(11)

with a mixing angle $\phi_{1D} = -(51 \pm 18)^\circ$. The mixing angle is consistent with that $\phi_{1D} = -50.8^\circ$ obtained in the heavy quark symmetry limit [27]. Our predictions are in compatible with the recent analysis in Ref. [10]. It should be mentioned that with the mixing angle $\phi_{1D} = -50.8^\circ$, the low-mass state $|1D_2^s\rangle_L$ and high-mass state $|1D_2^s\rangle_H$ will correspond to a broad state and a narrow state, respectively [27, 29].

The strong decay properties of the low-mass state $D(|1D_2^s\rangle_L)$ have been studied in [9] as well. The mass of $D(|1D_2^s\rangle_L)$ is likely to be $\sim 2.7$ GeV. This state is a broad state with a width of $\Gamma \approx (250 \sim 500)$ MeV, and its strong decays are dominated by the $D^*\pi$ and $D(2460)\pi$ channels. The $D(|1D_2^s\rangle_L)$ might be hard to be observed in experiments for its broad width.

In our previous study [8], we also found that the $1^1D_2$-1$^3D_2$ mixing might be crucial to uncover the longstanding puzzles in the $D_{sJ}(2860)$. Many people believe that the $D_{sJ}(2860)$ might be the $1^3D_3$ state. However, considering the $D_{sJ}(2860)$ as the $1^3D_3$ state only, one can not understand the important partial width ratio of $\Gamma(DK)/\Gamma(D^*K)$ measured by BABAR [5] at all. Considering there are two $D_s$ states with masses around 2.86 GeV, one resonance corresponds to the $1^3D_3$ [denoted by $D_{sJ}(2860)$] and the other resonance is the mixed state $|1D_2^s\rangle_H$ [denoted by $D_{sJ}(2860)$] via the $1^1D_2$-1$^3D_2$ mixing with the same mixing angle, we can explain the present strong decay properties of the $D_{sJ}(2860)$ observed in experiments naturally [8]. Our conclusion is compatible with the recent theoretical analysis in Ref. [11].

If the $D_{sJ}(2860)$ could be assigned as the high-mass state $|1D_2^s\rangle_H$ indeed, its low-mass partner $|1D_2^s\rangle_L$ might be observed in experiments as well. The mass of low-mass partner $|1D_2^s\rangle_L$ is estimated to be $30 \sim 50$ MeV lighter than that of the high-mass state $|1D_2^s\rangle_H$ [39, 40, 44]. Thus, the mass of $|1D_2^s\rangle_L$ might be in the range of $(2.80 \sim 2.83)$ GeV. Its strong decay properties have been analyzed in our previous work [9]. It is found that $|1D_2^s\rangle_L$ is a broad state with a width of $\Gamma \approx (260 \pm 30)$ MeV. The strong decays are dominated by the $D^*K$ channel.

It should be emphasized that if both $D(2750)$ and $D_{sJ}(2860)$ correspond to the mixed state $|1D_2^s\rangle_H$ indeed, the $1^1D_2$-1$^3D_2$ mixing might exist in the $B$- and $B_s$-meson spectroscopy as well. To provide useful clues for the future experimental search for these mixed states $|1D_2^s\rangle_H$ and $|1D_2^s\rangle_L$ in $B$- and $B_s$-meson spectroscopy, we study their strong decay properties in the followings.

Firstly, we study the $1^1D_2$-1$^3D_2$ mixing in the $B$-meson family. According to the predictions in Refs. [39, 40, 44], the mass of the high-mass mixed state $|1D_2^s\rangle_H$ is in the range of $(6.04 \sim 6.12)$ GeV. However, we find the masses of the $|1D_2^s\rangle_H$ resonances might be systemically overestimated by $\sim 100$ MeV in theory, if the $D(2750)$ and $D_{sJ}(2860)$ correspond to the mixed state $|1D_2^s\rangle_H$. Thus, the mass of the $|1D_2^s\rangle_H$ in $B$-meson family might be $M \approx (5.94 \sim 6.02)$ GeV.

Taking the mass of $B(|1D_2^s\rangle_H)$ with $M = 5.98$ GeV, we plot the partial decay widths and total decay width as functions of the mixing angle $\phi_{1D}$ in Fig. 6. It is shown that the decay width of the high-mass state $|1D_2^s\rangle_H$ is about $\Gamma \approx (40 \sim 90)$ MeV, when we take the mixing angle with $\phi_{1D} = -(51 \pm 18)^\circ$. The main decay channels are $B^*\pi$ and $B_2(5747)\pi$. The predicted branching ratio is

$$
\frac{\Gamma (B^*\pi)}{\Gamma_{\text{total}}} \approx 70\% \sim 93\%.
$$

(12)
is a fairly narrow state with a width of $\phi$. The shaded region corresponds to possible mixing angle region derived from the strong decay properties of the $D(2750)$. The masses of the low-mass mixed states $|D_2\rangle_L$ in the $B-$ and $B_s$-meson families are adopted as 5.95 GeV and 6.05 GeV, respectively. The masses of the high-mass mixed states $|D_2\rangle_H$ in the $B-$ and $B_s$-meson families are adopted as 5.98 GeV and 6.08 GeV, respectively. In the figure, we have hidden some decay channels for their small partial decay widths.

The partial widths for the $B(|1^3P_0\rangle \pi, B(|1^1P_1\rangle \pi, B(|1^3P'_1\rangle \pi, B'\eta$ and $B'K$ channels are tiny, which are not shown in the figure. Such a narrow state, $B(|1D_2\rangle_H)$, is most likely to be observed in the $B'\pi$ channel.

While for the low-mass mixed state $|1D_2\rangle_L$, its mass might be $(20 \sim 50)$ MeV lighter than that of the $|1D_2\rangle_H$. Thus, the mass of $|1D_2\rangle_L$ might be in the range $M = (5.9 \sim 6.0)$ GeV. The dependence of the strong decay width of the $|1D_2\rangle_L$ on the mass is shown in Fig. 4 as well. From the figure, we can see that the low-mass state $B(|1D_2\rangle_L)$ is a very broad state with a width of $\Gamma = (400 \pm 20)$ MeV. Its strong decays are governed by the $B'\pi$ and $B_2(5747)\pi$ channels. This broad state might be hard to be observed in experiments.

Secondly, we study the $1^1D_2 - 1^3D_2$ mixing in the $B_s$-meson family. According to the theoretical predictions, the mass gap between the $B_s$ resonances and the $B$ resonances is about 100 MeV [39,40,41]. Thus, the high-mass state $B_s(|1D_2\rangle_H$ might have a mass of $M \approx (6.04 \sim 6.12)$ GeV, while the mass of the low-mass state $B_s(|1D_2\rangle_L$ might be $M \approx (5.99 \sim 6.10)$ GeV.

Taking the mass of the $B_s(|1D_2\rangle_H$ with $M = 6.08$ GeV, we plot the partial widths and total decay width as functions of the mixing angle in Fig. 5. Adopting the physical mixing angle $\phi_{1D} = (-51 \pm 18)^\circ$ determined before, we find the $B_s(|1D_2\rangle_H$ is a fairly narrow state with a width of $\Gamma \approx (13 \sim 50)$ MeV. Its decays are dominated by the $B'K$ channel. The predicted branching ratio is

$$\frac{\Gamma[B'K]}{\Gamma_{total}} \approx 100\%.$$  (13)

This narrow state is most likely to be discovered in the $B'K$ channel in future experiments.

Furthermore, we analyze the strong decay properties of the low-mass state $B_s(|1D_2\rangle_L$). Its partial decay widths and total width as functions of the mixing angle are plotted in Fig. 5 as well, where we have fixed the mass with $M = 6.05$ GeV. From the figure, we find that the low-mass state $B_s(|1D_2\rangle_L$ has a broad width of $\Gamma \approx 240$ MeV, and its decays are governed by the $B'K$ channel.

Finally, we study the effects of the mass uncertainties of the mixing states $|1D_2\rangle_L$ and $|1D_2\rangle_H$ in $B$ and $B_s$ spectroscopy on the decay widths. To investigate this effect, we plot the decay width as a function of the mass in Fig. 6 with the mixing angle fixed at $\phi_{1D} = -50.8^\circ$. The sensitivity of different decay modes to the mass can also be seen clearly in the plot.

As a whole, the high-mass mixed states $B(|1D_2\rangle_H$ and $B_s(|1D_2\rangle_H$ might be narrow states, which might be observed in the $B'\pi$ and $B'K$ channels, respectively. However, the low-mass mixed state $|1D_2\rangle_L$ should be a broad state. The typical total decay widths for the low-mass mixed states $D(|1D_2\rangle_L$ and $B(|1D_2\rangle_L$ are usually larger than 300 MeV, which might be too broad to be observed in experiments. However, those low-mass mixed states containing a strange quark, $D_s(|1D_2\rangle_L$
and $B_s(1D_2)$, have a relatively narrower width, ~250 MeV, which have some possibilities to be observed in future experiments.

**D. The $1^3D_3$ states**

The $D(2760)$ is a new excitation in the $D$-meson family observed by BABAR in the $D\pi$ channel with a mass of $M = 2763.3 \pm 4.6$ MeV and a width of $\Gamma = 60.9 \pm 8.7$ MeV. Although its mass and width are very close to those of the $D(2750)$, they might be two different resonances due to several reasons, which have been explained in Refs. [9]. Recently, this state was confirmed by the LHCb Collaboration in the $D\pi$ and $D^*\pi$ channels, and their helicity analysis indicates that this state should have a natural parity, i.e., $J^P = 0^+, 1^-2^+, 3^-\cdots$. In our previous work [8], this new resonance $D(2760)$ was studied carefully. It is predicted that the $D(2760)$ could be assigned to the $1^3D_3(J^P = 3^-)$ state. Our conclusion is in consistent with that in Refs. [10] [12] [23]. Taking the $D(2760)$ as the assignment of the $1^3D_3$, the total decay width is $\Gamma \approx 68$ MeV, which consists with the data. Its strong decays are dominated by the $D\pi$ and $D^*\pi$ channels, and the partial width ratio is

$$\frac{\Gamma[D\pi]}{\Gamma[D^*\pi]} \approx 0.65,$$

which can be tested in future experiments. Furthermore, in our previous work [8], we suggested there are two largely overlapping resonances at 2.86 GeV in the $D$-meson family. One state corresponds to the $1^3D_3$ $(J^P = 3^-)$ state [denoted by $D_{sJ}(2860)$], which dominantly decays into $DK$ channel. While the other one corresponds to the mixed state $|1D_3\rangle_H$ with $J^P = 2^-$ [denoted by $D_sJ(2860)$], which dominantly decays into $D^*K$ channel. Considering the $D_sJ(2860)$ as the $1^3D_3$ state, we have studied its strong decay properties. The predicted decay width is $\Gamma \approx 45$ MeV, which is good comparable with the experimental data ($\Gamma = 58 \pm 11$ MeV). This state has two main decay channels $DK$ and $D^*K$, and the predicted partial width ratio is

$$\frac{\Gamma[D^*K]}{\Gamma[DK]} \approx 0.5,$$ (15)

which can be tested in future experiments. The present experimental measurement of the ratio, $\Gamma[D^*K]/\Gamma[DK] = 1.10 \pm 0.15 \pm 0.19$, might include the contributions of both $D_{sJ}(2860)$ and $D_sJ(2860)$. Thus, its value is obviously larger than the theoretical predictions by assuming the $D_{sJ}(2860)$ as the $1^3D_3$ state only. If both $D(2760)$ and $D_{sJ}(2860)$ correspond to the $1^3D_3$ excitation indeed, their flavor partners in $B$ and $B_s$ spectroscopy should be observed in future experiments as well.

In the $B$-meson family, the predicted mass of $1^3D_3$ is around $6.0 \sim 6.1$ GeV [33] [39] [40] [44]. Considering the $D(2760)$ and $D_{sJ}(2860)$ as the $1^3D_3$ assignment, their mass is systematically overestimated by $50 \sim 100$ MeV in theory. Thus, we estimate the mass of $B(1^3D_3)$ is in the range of

FIG. 4: The partial decay width and total decay width for the mixed states via $1^3D_s-1^3D_3$ mixing in the $B$- and $B_s$-meson families as functions of the mass. The mixing angle is adopted as $\phi_{13} = -50.8^0$. In the figure, we have hidden some decay channels for their small partial decay widths.
From the figure it is seen that the \( B(5970) \) is a narrow state with a width of \( \Gamma \approx 70 \text{ MeV} \). From Fig. 5 it is interestingly found that the \( B(5970) \) is most likely to be the \( 1^3D_3 \) assignment. Considering the \( B(5970) \) as the \( 1^3D_3 \) state, the predicted decay width is \( \Gamma \approx 60 \text{ MeV} \), and the \( B\pi \) and \( B^*\pi \) decay channels are its dominant decay modes. Our predictions are in a good agreement with the observations. Although the mass of the \( B(5970) \) is close to the estimated masses of the mixed states \( B((5S)D_{1/2}) \) and \( B((1D)_{3/2}) \), they are not good candidates of the \( B(5970) \). For the \( B((5S)D_{1/2}) \), when we set its mass with ~ 5.97 GeV, its theoretical width is ~ 300 MeV, which is too large to compared with the width of the \( B(5970) \). While for the \( B((1D)_{3/2}) \), its strong decays are governed by the \( B^*\pi \), and the \( B\pi \) channel is forbidden. Thus, it could not be assigned to the \( B(5970) \).

Finally, we study the strong decays of \( 1^3D_3 \) in the \( B_s \)-meson family. Usually, the mass of the \( B_s \) resonances is about 100 MeV larger than that of the \( B \) resonances \([33, 35, 39, 43]\). Thus, we estimate the mass of \( B_s(1^3D_3) \) might be in the range of \( (6.05 \sim 6.15) \text{ GeV} \). We have studied the strong decay properties of \( B_s(1^3D_3) \), which are shown in Fig. 5. It is found that the \( B_s(1^3D_3) \) might be a narrow state with a width of \( \Gamma \approx (25 \sim 75) \text{ MeV} \), and its strong decays are dominated by the \( B K \) and \( B^*K \) channels. If the \( B(5970) \) corresponds to the \( B(1^3D_3) \) indeed, the mass of the \( B_s(1^3D_3) \) might be \( M \approx 6.07 \text{ GeV} \). With this mass, the predicted width for the \( B_s(1^3D_3) \) is

\[ \Gamma \approx 30 \text{ MeV}, \]  

and the predicted ratio between \( BK \) and \( B^*K \) is

\[ \frac{\Gamma[BK]}{\Gamma[B^*K]} \approx 1.4. \]  

We hope the experimentalists can carry out a search for the \( B_s(1^3D_3) \) in the \( BK \) and \( B^*K \) channels, which is also helpful to clarify the nature of the newly observed state \( B(5970) \).

In a brief, the \( D(2760), D_{s0}(2860) \) and \( B(5970) \) might be classified as the low-lying \( D \)-wave excitations with \( J^P = 3^- \) (i.e. \( 1^3D_3 \)). The last unobserved one in the \( B_s \)-meson family should be a narrow state, which is most likely to be found in the \( BK \) and \( B^*K \) channels.

---

### E. The \( 2^3P_0 \) states

In the \( D \)-meson family, the predicted mass for the \( 2^3P_0 \) state is ~ 2.95 GeV \([34, 39, 40]\). For the uncertainties of the predicted mass, we vary the mass of \( D(2^3P_0) \) in the range of \( M = (2.9 \sim 3.0) \text{ GeV} \), and plot the strong decay properties in Fig. 6. From the figure it is seen that the \( D(2^3P_0) \) might be a very broad state with a width of \( \Gamma \approx 600 \sim 800 \text{ MeV} \). This state dominantly decays into the \( D(2550)\pi \), \( D\pi \) and \( D(2430)\pi \) channels. A very broad width for the \( D(2^3P_0) \) is also predicted in the recent study with a \( 3^3P_0 \) model \([10]\). Such a broad resonance might be difficult to be observed in experiments.

Recently, LHCb Collaboration observed a new excited \( D \)-meson state \( D'_s(3000) \) in the \( D\pi \) channel with a rather narrow width \( \Gamma \approx 110 \text{ MeV} \) \([2]\). Their further helicity analysis indicates that this new state should have a natural parity, that is, the spin-parity number of the \( D'_s(3000) \) should be \( J^P = 0^+, 1^- , 2^+ , \cdots \). In Ref. [34], Sun \etal. believed that the \( D'_s(3000) \) could be explained as the \( D(2^3P_0) \). If we exclude the contribution of the \( D(2550)\pi \) channel to the de-
FIG. 6: The partial decay width and total decay widths of the $2^1P_0$ states in the heavy-light mesons as functions of the mass. In the figure, we have hidden some decay channels for their small partial decay widths. The mass of the $B(2^1S_0)$ is adopted as $M = 5886$ MeV.

cay width, our results agree with the predictions in Ref. [34]. However, including the contribution of $D(2550)\pi$ decay channel, our calculations show that the decay width, $\Gamma \approx 800$ MeV, is too broad to compare with the data. Thus, the $D_s' (3000)$ as the $D(2^3P_0)$ assignment still bears controversies, although the parity and decay modes consist with the observations.

In the $D_s'$-meson family, the predicted mass of the $2^3P_0$ is about 3.05 GeV, which is ~ 100 MeV heavier than that of $D(2^3P_0)$ [39, 40]. Considering the uncertainties of the predicted mass, we plot the strong decay properties of $D_s(2^3P_0)$ as functions of the mass in Fig. 6. It is shown that if the mass of $D_s(2^3P_0)$ is less than 3.05 GeV, the $D_s(2^3P_0)$ has a moderate width of $\Gamma \approx 150 \sim 200$ MeV, and its strong decays are governed by the $DK$ channel. In this case, the $D_s(2^3P_0)$ is most likely to be observed in the $DK$ channel. On the other hand, if the mass of $D_s(2^3P_0)$ is larger than 3.05 GeV, the decay channel $D(2550)K$ will be opened, which dominates the strong decays. Then, the $D_s(2^3P_0)$ might be a broad state with a width of $\Gamma \approx 400 \sim 800$ MeV. In this case, the $D_s(2^3P_0)$ might be too broad to be observed in experiments.

In the $B$-meson family, the predicted mass of the $2^3P_0$ is about 6.2 GeV [33, 39, 40]. To know about the decay properties of the $B(2^3P_0)$, in Fig. 6 we plot its decay width as a function of mass in the range of $M = (6.1 \sim 6.25)$ GeV. From the figure it is seen that the decay width of $B_s(2^3P_0)$ is $\Gamma \approx 240$ MeV with $M = 6.27$ GeV, which is much narrower than that of $B(2^3P_0)$. The strong decays of $B_s(2^3P_0)$ are governed by the $BK$ channel. The other decay modes $B(1P_1)K$ and $B(1P_1')K$ also contribute to the decays obviously, if the mass of $B_s(2^3P_0)$ is larger than 6.25 GeV. Since the predicted width of $B_s(2^3P_0)$ is not very broad, this resonance might be observed in the $BK$ channel in future experiments.

As a whole, in the $2^3P_0$ states, the widths of $D(2^3P_0)$ and $B(2^3P_0)$ might be too broad to be observed in experiments. However, the widths of the resonances $D_s(2^3P_0)$ and $B_s(2^3P_0)$ are relatively narrower, thus, they might be observed in the $DK$ and $BK$ channels, respectively.

F. The $2^1P_1$-$2^3P_1$ mixing

The $D_sJ(3040)$ is a new broad state in the $D_s$-meson family observed by BABAR in the $D^*K$ channel [5]. It has a mass of $M = 3044 \pm 8^{+38}_{-5}$ MeV and a broad width of $\Gamma = 239 \pm 35^{+46}_{-42}$ MeV. According to our previous study [8], the $D_sJ(3040)$ can
be naturally explained as the low-mass mixed state $|2P_1⟩_L$ via the $2^1P_1-2^3P_1$ mixing:

$$
\begin{pmatrix}
|2P_1⟩_L \\
|2P'_1⟩_H
\end{pmatrix} = \begin{pmatrix}
\cos \phi_{2P} & -\sin \phi_{2P} \\
\sin \phi_{2P} & \cos \phi_{2P}
\end{pmatrix} \begin{pmatrix}
|2^1P_1⟩ \\
|2^3P_1⟩
\end{pmatrix},
$$

(19)

with a mixing angle $\phi_{2P} \approx -(51 \pm 27)^\circ$. This mixing angle is consistent with that ($\phi_{2P} = -54.7^\circ$) derived in the heavy quark symmetry limit [27]. It should be mentioned that with the mixing angle $\phi_{2P} = -54.7^\circ$, the low-mass state $|2P_1⟩_L$ usually has a broad width, while the high-mass state $|2P'_1⟩_H$ has a narrower width [27, 29]. As a broad mixed state, the strong decays of the $D_s(3040)$ are dominated by the $D^*K$ channel. The partial widths of $D_0(2400)K, D_1(2430)K$ and $D_2(2460)K$ are sizeable as well. Our conclusion is consistent with that obtained in Refs. [14, 21, 40].

As the physical partner of $|2P_1⟩_L$, the high mass state $|2P'_1⟩_H$ might be observed in experiments. Assuming the mass of the $|2P'_1⟩_H$ is in range of $(3.04 \sim 3.2)$ GeV, its strong decay properties have been analyzed in our previous work [8]. The decay width of $|2P'_1⟩_H$ is narrower than that of the $D_s(3040)$, which increases fast with the increasing mass. The main decay channels might be $D_0(2400)K, D_1(2430)K$ and $DK^*$. If the $D_{2s}(3040)$ could be assigned as a mixed state via the $2^1P_1-2^3P_1$ mixing indeed, the other mixed states between $2^1P_1$ and $2^3P_1$ in the $D$, $B$- and $B_s$-meson spectroscopy might be found in future experiments.

Firstly, we study the mixed states of $2^1P_1-2^3P_1$ in the $D$-meson family. According to the predictions in Refs. [39, 40, 44], the mass of the low-mass state $D(2P_1⟩_L)$ might be in the range of $(2.9 \sim 3.0)$ GeV, while the mass of the high-mass state $D(2P'_1⟩_H)$ might be in the range of $(3.0 \sim 3.1)$ GeV. Adopting the mixing angle $\phi_{2P} = -54.7^\circ$ predicted in the heavy quark symmetry limit, we have plotted their partial decay widths and total decay width as functions of the mass in Fig. 7. From the figure, it is seen that the low-mass state $D(2P_1⟩_L)$ is a broad state with a width of $\Gamma \approx (370 \pm 90)$ MeV, its strong decays are dominated by the $D^*\pi$ and $D(2600)\pi$ channels, and the partial decay widths of the $D_0(2400)\pi, D_1(2430)\pi$ and $D_sK$ channels are sizeable as well.

Although the high-mass state $D(2P'_1⟩_H)$ has a relatively narrower width than the low-mass state $D(2P_1⟩_L)$, the $D(2P'_1⟩_H)$ is also a very broad state with a width of $\Gamma \approx (300 \pm 90)$ MeV when we adopt its mass in the range of $(3.0 \sim 3.1)$ GeV. Its strong decays are governed by the $D(2600)\pi$ channel. The other channels $D_0, D_0(2400)\pi, D_0(2317)K$ and $D_1(2430)\pi$ also have sizable contributions to the decays.

Recently, a new resonance $D_{3s}(3000)$ was observed by LHCb. Its measured mass and width are $M \approx 2972$ MeV and $\Gamma \approx 188 \pm 45$ MeV, respectively. Furthermore, the helicity analysis from LHCb indicates this state has an unnatural parity, that is, its spin-parity should be $J^P = 0^+, 1^+, 2^+, \cdots$. According to the observed mass, the observed $D^*\pi$ decay mode and possible $J^P$ numbers of the $D_{3s}(3000)$, we predict that the $D_{3s}(3000)$ might be a good candidate of the low-mass state $D(2P_1⟩_L)$. As the $D(2P_1⟩_L)$ candidate with the mixing angle $\phi_{2P} = -54.7^\circ$ derived in the heavy quark symmetry limit, the $D_{3s}(3000)$ might have a broad width of $\Gamma \approx 360$ MeV (see Fig. 7), which is about a factor 2 larger than the data ($\Gamma \approx 188 \pm 45$ MeV). The predicted partial decay width ratio of the main two decay channels $D^*\pi$ and $D(2600)\pi$ is

$$
\frac{\Gamma[D^*\pi]}{\Gamma[D(2600)\pi]} \approx 0.7.
$$

(20)

However, we have noticed that the mixing angle $\phi_{2P}$ deter-
mined by the strong decay properties of the $D_{sJ}(3040)$ bares a large uncertainty. Taking the $D_J(3000)$ as a candidate of the $D((2P_1)_L)$ state, we have plotted the partial widths and total decay width as functions of the mixing angle $\phi_{2P}$ in Fig. 8. From the figure, it is found that the decay width is very sensitive to the mixing angle. If we adopt a mixing angle in the range of $\phi_{2P} = -(3 \sim 26)^\circ$, the decay width of the $D_J(3000)$ can be explained. The mixing angle for $D_J(3000)$ has an overlap with that for the $D_{sJ}(3040)$ in the range of $\phi_{2P} = -(20 \sim 26)^\circ$. It is found that the mixing angle $\phi_{2P} = -(20 \sim 26)^\circ$ obtained in present work is close to the mixing angle $\phi_{2P} = -30.4^\circ$ suggested in [11].

If both $D_{sJ}(3040)$ and $D_J(3000)$ can be assigned as the mixed state $|2P_1_L\rangle$, they should share nearly the same mixing angle. Taking the same mixing angle we predict that the width of the $D_J(3000)$ should be a little larger than that of the $D_{sJ}(3040)$ for the larger decay phase space and more decay channels of the $D_J(3000)$.

Recently, the strong decay properties of the $D_J(3000)$ were studied in Ref. [34] as well. Considering the $D_J(3000)$ as the $D((2P_1)_L)$ with the mixing angle $\phi_{2P} = -54.7^\circ$, they could well explain the measured width of the $D_J(3000)$. However, it should be pointed out that they did not include the $D(2600)\pi$ decay mode in their calculations. Excluding the $D(2600)\pi$, our prediction is in agreement with that in Ref. [34]. To understand the nature of the $D_J(3000)$, more observations are needed in future experiments.

FIG. 9: The decay width of the mixed states via the $2^1P_1$-$2^3P_1$ mixing in the $B$- and $B_s$-meson families as a function of the mass. Where, the mixing angle $\phi_{2P} = -54.7^\circ$ is adopted. The $B(5920)$ stands for the mixed state $B((S D)_L)\pi$. In the figure, we have hidden some decay channels for their small partial decay widths.

In the $B$-meson family, the predicted mass for the low-mass state $B((2P_1)_L)$ is $M \approx 6.2$ GeV [35, 40, 44], the mass of the high-mass state $B((2P_1')_H)$ is about $(20 \sim 80)$ MeV heavier than that of the low-mass state. Considering the uncertainty of the predicted masses for these states, in Fig. 9 we plot the partial decay widths and total decay width of $B((2P_1)_L)$ and $B((2P_1')_H)$ as functions of the mass in the possible range, where we adopt the mixing angle $\phi_{2P} = -54.7^\circ$ derived in the heavy quark symmetry limit. From the figure, we find that the low-mass state $B((2P_1)_L)$ is a broad state with a width of $\Gamma \approx (450 \pm 150)$ MeV. The $B^*\pi$ decay mode governs its strong decays, while the other decay channels $B((S D)_L)\pi$, $B(1P_1)\pi$, $B(1^3P_2)\pi$, $B^*K$, $B(1^3P_0)\pi$ and $B^*\eta$ also have sizable partial decay widths.

The width of the high-mass state $B((2P_1')_H)$, $\Gamma \approx (50 \sim 450)$ MeV, is much narrower than that of the low-mass state $B((2P_1)_L)$. The $B((2P_1')_H)$ dominantly decays into $B((S D)_L)\pi$, $B(1^3P_0)\pi$, $B(1P_1)\pi$ and $B(1^3P_2)\pi$. It might be
a challenge to observe the $B(2P_1^s)_H$ in these final states with higher excitations of $B$ meson.

Since the $D_{sJ}(3040)$ as a broad state has been observed in experiments, the broad resonance $B(2P_1^s)_L$ might be observed in the $B^*\pi$ as well. To further know about the effects of the uncertainties of the mixing angle on the strong decays of $B(2P_1^s)_L$, we have plotted the partial widths and total decay width of $B(2P_1^s)_L$ as functions of the mixing angle $\phi_{2P}$ in Fig. 10 where we have fixed the mass of $B(2P_1^s)_L$ with $M = 6175$ MeV predicted in [39]. From the figure, it is seen that even the mixing angle $\phi_{2P}$ varies in a large range $\phi_{2P} \approx -20^\circ \sim -80^\circ$, the strong decays of $B(2P_1^s)_L$ are still dominated by the $B^*\pi$ channel, and the uncertainty of the decay width is no more than 60 MeV.

In the $B_s$-meson family, the predicted mass for the low-mass state $B_s(2P_1^s)_L$ is $M \approx (6.28 \sim 6.32)$ GeV [39, 44]. The mass of the high-mass state $B_s(2P_1^s)_H$ is about (20 \sim 30) MeV heavier than that of the low-mass state.

We have plotted the partial decay widths and total decay width of $B_s(2P_1^s)_L$ and $B_s(2P_1^s)_H$ as functions of the mixing angle $\phi_{2P} = -54.7^\circ$ in Fig. 10. From the figure, we find that the low-mass state $B_s(2P_1^s)_L$ has a broad width of $\Gamma \approx (220 \pm 70)$ MeV within the predicted mass region $M \approx (6.28 \sim 6.32)$ GeV. The $B^*K$ decay channel governs its strong decays, while the other decay channels $B(1P_1)K$, $B(1^3P_2)K$, $B(1^3P_0)K$ also have sizable partial decay widths. The width of the high-mass state $B_s(2P_1^s)_H$ is $\Gamma \approx (95 \pm 33)$ MeV within the possible mass region $M \approx (6.30 \sim 6.35)$ GeV, which is much narrower than that of $B_s(2P_1^s)_L$. The $B_s(2P_1^s)_H$ dominantly decays into the $B(1^3P_0)K$, $B(1P_1)K$, and $BK^*$ channels. Although the $B_s(2P_1^s)_H$ has a relatively narrow width, it might be a great challenge to observe this resonance in these final states with higher $B_s$ resonances.

The decay width of $B_s(2P_1^s)_L$ is comparable with that of the $D_{sJ}(3040)$. Thus, the $B_s(2P_1^s)_L$ is most likely to be observed in the $B^*K$ channel. To further know about the effects of the uncertainties of the mixing angle on the strong decays of $B_s(2P_1^s)_L$, we have plotted the partial widths and total decay width of $B_s(2P_1^s)_L$ as functions of the mixing angle $\phi_{2P}$ in Fig. 10 where we have fixed the mass of $B_s(2P_1^s)_L$ with $M = 6275$ MeV. From the figure, it is seen that even the mixing angle $\phi_{2P}$ is changed in a large range $\phi_{2P} \approx -20^\circ \sim -80^\circ$, the strong decays of $B_s(2P_1^s)_L$ are still dominated by the $B^*K$ channel, and the uncertainty of the decay width is no more
From the figure, it is found that the hidden some decay channels for their small partial decay widths. The mass of the about 3 into the 110 MeV) and natural parity of the B*K channel. If the mass of the B(1D2) state, it should be found in the D(2430)πK channel as well.

Considering the newly observed state D*(3000) as a candidate of D(23P2), we find that the observed mass, width (Γ ≈ 110 MeV) and natural parity of the D*(3000) can be well explained. However, the Dπ decay channel is not the main decay channel of D(23P2), which is tiny compared with the dominant decay mode D1(2430)π. The predicted branching fraction is

$$\frac{\Gamma[D\pi]}{\Gamma_{\text{total}}} \approx 1 - 2\%,$$

which is compatible with the predictions in Refs. [10, 34]. To clarify the nature of the D*(3000), it is suggested to further observe this state in the D1(2430)π channel. If the D*(3000) corresponds to the D(23P2) state, it should be found in the D1(2430)π channel as well.

In the D-meson family, the predicted mass of the 23P2 is about 3.02 GeV [33, 39, 40]. To know about the decay properties of the D(23P2), in Fig. 11 we plot its decay width as a function of mass in the range of M = (2.95 ~ 3.05) GeV. From the figure, it is found that the D(23P2) mainly decays into the D1(2430)π, D(2550)π and D(2600)π channels. The predicted total decay width of D(23P2) is Γ ≈ 150 MeV, if we adopt the mass ~ 3.01 GeV as predicted in Ref. [40].

Considering the newly observed state D*(3000) as a candidate of D(23P2), we find that the observed mass, width (Γ ≈ 110 MeV) and natural parity of the D*(3000) can be well explained. However, the Dπ decay channel is not the main decay channel of D(23P2), which is tiny compared with the dominant decay mode D1(2430)π. The predicted branching fraction is

$$\frac{\Gamma[D\pi]}{\Gamma_{\text{total}}} \approx 1 - 2\%,$$

which is compatible with the predictions in Refs. [10, 34]. To clarify the nature of the D*(3000), it is suggested to further observe this state in the D1(2430)π channel. If the D*(3000) corresponds to the D(23P2) state, it should be found in the D1(2430)π channel as well.

In the D-meson family, the predicted mass of the 23P2 is about 3.02 GeV [33, 39, 40]. To know about the decay properties of the D(23P2), in Fig. 11 we plot its decay width as a function of mass in the range of M = (2.95 ~ 3.05) GeV. From
the figure, it is found that the $D_s(23^1P_2)$ mainly decays into the $D_1(2430)\Lambda$ channel. The total decay width of $D_s(23^1P_2)$ is $\Gamma \approx 120 \pm 60$ MeV. This resonance might be observed in the $D_1(2430)\Lambda$ final states.

In the $B$-meson family, the predicted mass of the $23^1P_2$ is about 6.19 ~ 6.26 GeV [33, 39, 40]. In Fig. 11, we have plotted its decay width as a function of mass in the range of $M = (6.15 ~ 6.3)$ GeV. From the figure it is found that the total decay width of $B(23^1P_2)$ is $\Gamma \approx (50 ~ 300)$ MeV. The $B(23^1P_2)$ mainly decays into the $B_s(5721)\pi$ channel.

In the $B_s$-meson family, the predicted mass of the $23^1P_2$ is about 6.29 ~ 6.36 GeV [33, 39, 40]. The decay properties of the $B(23^1P_2)$ have been shown in Fig. 11. From the figure, it is found that the $B_s(23^1P_2)$ mainly decays into $B_s(5721)\Lambda$. The total decay width of $B_s(23^1P_2)$ is $\Gamma \approx (30 ~ 180)$ MeV. This state might be observed in the $B_s(5721)\Lambda$ channel.

In a brief, the decay widths of the $23^1P_2$ excitations in the heavy-light spectroscopy are not very broad, they have good potentials to be observed in their dominant decay channels.

**FIG. 12:** The partial decay width and total decay widths of the $1^3F_2$ states in the heavy-light mesons as functions of the mass. In the figure, we have hidden some decay channels for their small partial decay widths. The mass of the $B(1D'_2)$ is adopted as $M = 6025$ MeV.

H. The $1^3F_2$ states

In the $D$-meson family, the predicted mass for the $1^3F_2$ state is $\sim 3.1$ GeV [33, 39, 40]. Considering the uncertainties of the mass, in Fig. 12 we plot the partial decay widths and total width as functions of mass in the range of $M = (3.05 ~ 3.15)$ GeV. It is found that the $1^3F_2$ state in the $D$-meson family is a very broad state with a width of $\Gamma \approx 900 \pm 200$ MeV. This state dominantly decays into the $D(2420)\pi$, $D(2750)\pi$ and $D\eta$ channels. The partial widths for the $D_s(2460)\pi$, $D^*\pi$ and $D(2760)\eta$ are also sizable.

In the $D_s$-meson family, the predicted mass for the $1^3F_2$ state is $\sim 3.23$ GeV [33, 39, 40]. Considering the uncertainties of the mass, we vary the mass in the range of $M = (3.15 ~ 3.30)$ GeV, the strong decay properties of the $1^3F_2$ state have been shown in Fig. 12 as well. From the figure it is seen that the decay width of the $D_s(1^3F_2)$ state is $\Gamma \sim 500$ MeV. The main decay channels are $D(2420)\Lambda$, $D\pi$ and $D^*\pi$. If the $D(2750)\Lambda$ channel is opened, the $D(2750)\Lambda$ mode will dominate the decays, the decay width of the $D_s(1^3F_2)$ state will become very broad with a width of $\Gamma > 700$ MeV.

In the $B$-meson family, the predicted mass for the $1^3F_2$ state
is 6.26 - 6.41 GeV [33, 39, 40]. In this mass region, we show the strong decay properties of the \( B(1^3F_2) \) state in Fig. 12. It is found that this state is also a very broad state with a width of \( \Gamma \approx 1200 \pm 400 \) MeV. It mainly decays into the \( B(1P_1)\pi \), \( B(1D_1)\pi \), \( B\pi \) and \( B'\pi \) channels.

In the \( B_s \)-meson family, the predicted mass for the \( 1^3F_2 \) state is 6.37 - 6.50 GeV [39, 40]. In this mass region, we study the strong decays of the \( B_s(1^3F_2) \) state, our results are shown in Fig. 12 as well. It is seen that this state is a very broad state with a width of \( \Gamma \approx 400 - 900 \) MeV. The strong decays are governed by the \( B(1P_1)K \). The other main decay channels are \( B K, B'K \) and \( B(1^3P_2)K \).

As a whole, the \( 1^3F_2 \) excitations in the heavy-light mesons are very broad resonances, whose width is larger than 400 MeV. Such broad states might be difficult to be observed in experiments, which might explain why these states are still missing in the heavy-light meson spectroscopy.

\[
\begin{array}{c}
\text{FIG. 13: The partial decay width and total decay width for the mixed states via the } 1^3F_2-1^3F_3 \text{ mixing as functions of the mass. The mixing angle is adopted as } \phi_{1F} = -49.1^\circ. \text{ In the figure, we have hidden some decay channels for their small partial decay widths.}
\end{array}
\]

I. The \( 1^3F_2-1^3F_3 \) mixing

Since the heavy-light mesons are not charge conjugation eigenstates, state mixing between spin \( S = 0 \) and \( S = 1 \) states with the same \( J^P \) can occur via the spin-orbit interac-
The physical states with $J^P = 3^+$ can then be described as

$$
\begin{pmatrix}
|1F_3\rangle_L \\
|1F_3\rangle_H
\end{pmatrix} = \begin{pmatrix}
\cos \phi_{1F} & \sin \phi_{1F} \\
-\sin \phi_{1F} & \cos \phi_{1F}
\end{pmatrix}
\begin{pmatrix}
|1F_\pi\rangle \\
|1F_\rho\rangle
\end{pmatrix},
$$

(22)

where the subscripts $L$ and $H$ stand for the low mass and high mass of the physical states after the mixing. In the heavy quark symmetry limit, the mixing angle $\phi_{1F}$ is predicted to be $\phi_{1F} \approx -49.1^\circ$ [40]. It should be mentioned that with the mixing angle $\phi_{1F} \approx -49.1^\circ$, the low-mass state $|1F_3\rangle_L$ usually has a broad width, while the high-mass state $|1F_3\rangle_H$ has a narrower width.

In the $D$-meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3\rangle_H$ are about $3.07 \sim 3.13$ GeV and $3.12 \sim 3.15$ GeV, respectively [39, 40]. With the mixing angle $\phi_{1F} \approx -49.1^\circ$ derived in the heavy quark symmetry limit, we predicted the strong decay properties of the mixed states $D(|1F_3\rangle_L)$ and $D(|1F_3\rangle_H)$ in their possible mass region. The results are shown in Fig. 13. It is found that the $D(|1F_3\rangle_L)$ is a very broad state with a width of $\Gamma \approx 600 \sim 900$ MeV. Its strong decays are dominated by the $D(2760)\pi$, $D(2460)\pi$ and $D^*\pi$ channels. While the width of the $D(|1F_3\rangle_H)$ is $\Gamma \approx 200 \sim 300$ MeV, which is much narrower than that of the low-mass state. The $D(|1F_3\rangle_H)$ mainly decays into the $D(2460)\pi, D^*\pi, D^0, D(2420)\pi$ and $D(2750)\pi$ channels.

In the $D_s$-meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3\rangle_H$ are about $3.20 \sim 3.26$ GeV and $3.25 \sim 3.27$ GeV, respectively [39, 40]. The strong decay properties of these mixed states are also studied in their possible mass region. Our predictions are shown in Fig. 13. It is seen that the width of the low-mass mixed state $D_s(|1F_3\rangle_L)$ is very broad, which is in the range of $\Gamma \approx 400 \sim 800$ MeV. Its strong decays are governed by the $D(2460)K$ and $D^*K$ channels. While the high-mass state $D_s(|1F_3\rangle_H)$ has a much narrower decay width, i.e., $\Gamma \approx 100 \sim 250$ MeV. This high-mass state dominantly decays into the $D(2460)K, D^*K$ and $DK^*$ channels.

In the $B$-meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3\rangle_H$ are about $6.22 \sim 6.39$ GeV and $6.27 \sim 6.42$ GeV, respectively [39, 40]. In the possible mass region, we study the strong decay properties of these mixed states in the $B$-meson family. The results are shown in Fig. 13. It is found that the low-mass mixed state $B(|1F_3\rangle_L)$ is a very broad state with a width of $\Gamma \approx 650 \sim 1400$ MeV. This state dominantly decays into the $B_2(5747)\pi, B(1^{3}D_3)\pi$ and $B^*\pi$ channels. While for the $B(|1F_3\rangle_H)$, the decay width is $\Gamma \approx 200 \sim 650$ MeV, which is sensitive to the mass. If the $B(|1F_3\rangle_H)$ has a smaller mass as predicted in Ref. [39], it might be observed in its main decay channels $B^*\pi$ and $B_2(5747)\pi$.

In the $B_s$-meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3\rangle_H$ are about $6.33 \sim 6.47$ GeV and $6.38 \sim 6.52$ GeV, respectively [39, 40]. Considering the uncertainties of the mass, we have plotted the strong decay properties of these mixed states as functions of the mass in Fig. 13. From the figure, it is seen that the $|1F_3\rangle_L$ state in the $B_s$-meson family has a width of $\Gamma \approx 200 \sim 700$ MeV, whose strong decays are dominated by the $B_2(5747)K$ and $B^*K$ channels. While the high-mass state $|1F_3\rangle_H$ has a relatively smaller width, which is $\Gamma \approx 100 \sim 500$ MeV. Its strong decays are governed by the $B^*K, BK^*$ and $B_2(5747)K$ channels. These mixed states $|1F_3\rangle_L$ and $|1F_3\rangle_H$ in the $B_s$-meson family might be observed in the $B^*K$ channel, if they have a smaller mass as predicted in Ref. [39].

In the calculations we have adopted the ideal mixing angle $\phi_{1F} \approx -49.1^\circ$ extracted from the heavy quark symmetry limit. This ideal mixing angle might have some uncertainties. To see the effects of the uncertainties of the mixing angle on the strong decay properties, we have plotted the partial decay widths and total decay width as functions of the mixing angle in Fig. 13. From the figure, it is seen that strong decay properties of the mixed states do not change obviously, if we consider an uncertainty $\pm 20^\circ$ around the ideal mixing angle $\phi_{1F} \approx -49.1^\circ$.

In a summary, the mixed states $|1F_3\rangle_L$ in the light-heavy mesons are usually broad states, they should be difficult to be found in experiments. While the decay width of the high-mass mixed states $|1F_3\rangle_H$ are relatively narrower than that of the low-mass states. These high-mass states $|1F_3\rangle_H$ might be observed in future experiments if they have a smaller mass as predicted in Ref. [39]. However, these high-mass states $|1F_3\rangle_H$ might be very broad states if their masses are as large as those predicted in Ref. [40].

### J. The $1^3F_4$ states

In the $D$-meson family, the predicted mass for the $1^3F_4$ state is $3.09 \sim 3.19$ GeV [33, 39, 40]. In this mass region, the predicted strong properties are shown in Fig. 15. The width of the $D(1^3F_4)$ state is $\Gamma \approx 230 \pm 70$ MeV. It dominantly decays into the $D\rho, D\pi, D(2430)\pi, D(2460)\pi, D^*\pi$ and $D(2760)\pi$ channels. The branching ratios for $D^*\pi$ and $D\pi$ are 10% and
However, the predicted mass is about 100 MeV larger than the data. If the $D_f^*(3000)$ corresponds to the $D(1^3F_2)$ state indeed, it should be observed in both $D\pi$ and $D^*\pi$ channels. To clarify the puzzles in the $D_f^*(3000)$, more observations are needed.
In the \( D_\tau \)-meson family, the predicted mass for the \( 1^3F_4 \) state is \( 3.22 \sim 3.30 \text{ GeV} \) \([33, 39, 40]\). The strong decay properties are shown in Fig. 15 as well. In the possible mass region, the predicted width of the \( D_\tau (1^3F_4) \) state is \( \Gamma \approx 200 \pm 100 \text{ MeV} \). The main decay channels are \( D\pi, D^*\pi, D, D^*K, D(2430)K, \) and \( D(2460)K \). If the mass of the \( D_\tau (1^3F_4) \) state is larger than the threshold of \( D(2760)K \), the \( D(2760)K \) channel will become a dominant decay channel. The predicted partial width ratio of \( DK \) and \( D^*K \) is

\[
\frac{\Gamma[DK]}{\Gamma[D^*K]} \approx 1.2. \tag{23}
\]

This state might be observed in both \( DK \) and \( D^*K \) channels.

In the \( B \)-meson family, the predicted mass for the \( 1^3F_4 \) state is \( 6.22 \sim 6.38 \text{ GeV} \) \([33, 39, 40]\). In this mass region, we study the strong decay properties of the \( B(1^3F_4) \) state, which are shown in Fig. 15. From the figure, it is seen that the strong decays of the \( B(1^3F_4) \) state are dominated by the \( B\pi \) and \( B^*\pi \) channels.

The partial width ratio is

\[
\frac{\Gamma[B\pi]}{\Gamma[B^*\pi]} \approx 1.1, \tag{24}
\]

which is less sensitive to the mass. Furthermore, the other decay channels \( B'\rho, B(1^3D_3)\pi, B(1^3P_3)\pi \) and \( B(1P_1)\pi \) also have

---

**FIG. 15:** The partial decay width and total decay width for the \( 1^3F_4 \) excitations as functions of the mass. In the figure, we have hidden some decay channels for their small partial decay widths.

**TABLE IV:** The strong partial decay widths (MeV) and total width (MeV) of \( D'(3000) \) as a candidate of \( D(1^3F_4) \).

| Mode | Mode | Width  |
|------|------|--------|
| \( D\eta \) | \( D\eta' \) | 1.0    |
| \( D^*\eta \) | \( D^*\eta' \) | 0.5    |
| \( (2420)\eta \) | \( (2420)\eta' \) | 8.9 \times 10^{-5} |
| \( (2420)\pi \) | \( (2420)\pi' \) | 5.6 \times 10^{-4} |
| \( (2550)\pi \) | \( (2550)\pi' \) | 1.0 \times 10^{-3} |
| \( (2750)\pi \) | \( (2750)\pi' \) | 8.1 \times 10^{-12} |
| \( (2760)\pi \) | \( (2760)\pi' \) | 3.6 \times 10^{-3} |
| \( (2760)\eta \) | \( (2760)\eta' \) | 0.8    |

| Mode | Mode | Width  |
|------|------|--------|
| \( D(1^1F_3) \) | \( D(1^1F_3) \) | 1.0    |
| \( D(1^3F_4) \) | \( D(1^3F_4) \) | 2.3    |
| \( D(2430)\pi \) | \( D(2430)\pi' \) | 9.2    |
| \( D(2460)\pi \) | \( D(2460)\pi' \) | 7.0    |
| \( D(2550)\pi \) | \( D(2550)\pi' \) | 0.9    |
| \( D(2600)\pi \) | \( D(2600)\pi' \) | 5.9 \times 10^{-2} |
| \( D(2750)\pi \) | \( D(2750)\pi' \) | 4.6    |
| \( D(2760)\pi \) | \( D(2760)\pi' \) | 11.4   |
| \( D(2430)K \) | \( D(2430)K \) | 1.0    |
| \( D(2460)K \) | \( D(2460)K \) | 9.2    |
| \( D(2430)\pi \) | \( D(2430)\pi' \) | 2.3    |
| \( D(2460)\pi \) | \( D(2460)\pi' \) | 9.2    |
| \( D(2550)\pi \) | \( D(2550)\pi' \) | 7.0    |
| \( D(2600)\pi \) | \( D(2600)\pi' \) | 0.9    |
| \( D(2750)\pi \) | \( D(2750)\pi' \) | 5.9 \times 10^{-2} |
| \( D(2760)\pi \) | \( D(2760)\pi' \) | 4.6    |
| \( D(2760)\eta \) | \( D(2760)\eta' \) | 11.4   |
| \( D(2430)\pi \) | \( D(2430)\pi' \) | 2.3    |
| \( D(2460)\pi \) | \( D(2460)\pi' \) | 9.2    |
| \( D(2550)\pi \) | \( D(2550)\pi' \) | 7.0    |
| \( D(2600)\pi \) | \( D(2600)\pi' \) | 0.9    |
| \( D(2750)\pi \) | \( D(2750)\pi' \) | 5.9 \times 10^{-2} |
| \( D(2760)\pi \) | \( D(2760)\pi' \) | 4.6    |
| \( D(2760)\eta \) | \( D(2760)\eta' \) | 11.4   |

\[
\frac{\Gamma[B\pi]}{\Gamma[B^*\pi]} \approx 1.1,
\]

which is less sensitive to the mass. Furthermore, the other decay channels \( B'\rho, B(1^3D_3)\pi, B(1^3P_3)\pi \) and \( B(1P_1)\pi \) also have
obvious contributions to the strong decays. The decay width is sensitive to the mass. In the possible mass region, the predicted width of the $B(1^{3}F_{4})$ state is $\Gamma \approx 120 \sim 480$ MeV. If this state has a mass of $\sim 6.2$ GeV as predicted in Ref. [39], it might be observed in future experiments. However, if the mass is close to $\sim 6.4$ GeV as predicted in Ref. [40], the $B(1^{3}F_{4})$ state might be difficult to be found for its broad width.

In the $B_{s}$-meson family, the predicted mass for the $1^{3}F_{4}$ state is $6.33 \sim 6.48$ GeV [39, 39, 40]. In this mass region, we have shown the strong decay properties in Fig. [13]. The decay width of the $B_{s}(1^{3}F_{4})$ is sensitive to the mass. The total decay width is $\Gamma \approx 60 \sim 400$ MeV, which bares a large uncertainty. Its strong decays are dominated by the $BK$, $B^{'K}$ and $B^{'K}^{\ast}$ channels. The predicted partial width ratio of the $BK$ and $B^{'K}$ is

$$\frac{\Gamma(BK)}{\Gamma(B^{'K})} \approx 1.2$$

(25)

which is insensitive to the mass. If this state has a smaller mass of $\sim 6.34$ GeV as predicted in Ref. [39], it should be a narrow state with a width of $\Gamma \approx 80$ MeV. In this case, the $B_{s}(1^{3}F_{4})$ might be observed in both $BK$ and $B^{'K}$ channels.

As a whole the decay widths of the $1^{3}F_{4}$ excitations in the heavy-light mesons are sensitive to the mass. If these states have a smaller mass as predicted in Ref. [39], they might have relatively narrow width $\Gamma \sim 100$ MeV. In this case, these excitations have good discovery potentials in future experiments. However, if these states have a larger mass as predicted in Ref. [40], they might be difficult to be found in experiments for their broad widths. Finally, it should be pointed out that the newly observed natural parity resonance $D_{s}(3000)$ seems to be a good candidate of the $D(1^{3}F_{4})$, although its mass is about $100 \sim 200$ MeV less than the model predictions.

### IV. SUMMARY

In the chiral quark model framework, we systematically study the strong decays of the higher excited heavy-light mesons from the first radially excited states up to the first $F$-wave states. We summarize our major results as follows.

The first radially excited states $2^{1}S_{0}$ in the $D_{s}$, $D_{s^{'}}$, $B$- and $B_{s}$-meson spectroscopy are narrow states, and their predicted width might be less than 100 MeV. If the $D(2550)$ corresponds to this excitation instead, the other radially excited states $D_{s}(2^{1}S_{0})$, $B(2^{1}S_{0})$ and $B_{s}(2^{1}S_{0})$ are most likely to be also observed in the $D^{'K}$, $B^{'\pi}$ and $B^{'K}$ final states, respectively, for their narrower widths.

Configuration mixing might exist between the low-lying $J^{P} = 1^-$ states $2^{3}S_{1}$ and $1^{3}D_{1}$. Both $D(2600)$ and $D_{s}(2700)$ could be assigned as the low-mass mixed state $|(S D_{1})_{1}\rangle$ via the $2^{1}S_{1} \rightarrow 1^{3}D_{1}$ mixing. As flavor partners of the $D(2600)$ and $D_{s}(2700)$, the low-mass mixed states in the $B$- and $B_{s}$-meson families, $B(|(S D_{1})_{1}\rangle)$ and $B_{s}(|(S D_{1})_{1}\rangle)$, should have a relatively narrow width. They are most likely to be observed in future experiments. In the high-mass mixed states, the widths of the charmed-strange state $D_{s}(|(S D_{1})_{1}\rangle)$ and the bottom-strange state $B_{s}(|(S D_{1})_{1}\rangle)$ are as comparable as the low-mass mixed states, which might be observed in the $DK$ and $BK$ channels, respectively. While, the bottom state $B(|(S D_{1})_{1}\rangle)$ is a broad state, which might be difficult to be found in experiments. It should be pointed out that the newly observed resonance $D_{s}(2860)$ by LHCb Collaboration might correspond to the physical partners of the $D_{s}(2700)$, i.e., the high-mass state $D_{s}(|(S D_{1})_{1}\rangle)$.

In the low-lying $D$-wave states with $J^{P} = 2^-$, $1^{1}D_{2}$ and $1^{3}D_{2}$, there might be configuration mixing as well. The narrow resonances $D(2750)$ and $D_{s}(2860)$ might be classified as the high-mass mixed state $|(L D_{2})_{1}\rangle$ via the $1^{1}D_{2} \rightarrow 1^{3}D_{2}$ mixing. The other high-mass mixed states in the $B$- and $B_{s}$-meson families, $B(|(L D_{2})_{1}\rangle)$ and $B_{s}(|(L D_{2})_{1}\rangle)$ have a narrow width as well. These two narrow states might be observed in the $B^{'\pi}$ and $B^{'K}$, respectively. However, the low-mass mixed state $|(L D_{2})_{1}\rangle$ should be a broad state. The typical total decay widths for the mixed states $D(|(L D_{2})_{1}\rangle)$ and $B(|(L D_{2})_{1}\rangle)$ in the $D$- and $B$-meson families are usually larger than 300 MeV, which might be too broad to be observed in experiments. While the widths for the mixed states $D_{s}(|(L D_{2})_{1}\rangle)$ and $B_{s}(|(L D_{2})_{1}\rangle)$ containing a strange quark are $\sim 250$ MeV, which still have some possibilities to be observed in future experiments.

For the low-lying $D$-wave excitations with $J^{P} = 3^-$, $1^{1}D_{3}$, in the $D_{s}$, $D_{s^{'}}$, $B$- and $B_{s}$-meson spectroscopy, we might have observed three excitations $D(2760)$, $D_{s}(2860)$ and $B(5970)$ in recent experiments. The last unobserved one in the $B_{s}$-meson family should be a narrow state with a width of $\Gamma \approx (25 \sim 75)$ MeV, which is most likely to be found in the $BK$ and $B^{'K}$ channels.

No evidence of the second $P$-wave excitations with $J^{P} = 0^+$ is found in experiments. In the $D$- and $B$-meson families, the $D(2^{3}P_{0})$ and $B(2^{3}P_{0})$ excitations should be very broad states, whose widths are larger than 400 MeV. It might be difficult to be found in experiments. However, the widths of the resonances $D_{s}(2^{3}P_{0})$ and $B(2^{3}P_{0})$ in the $D_{s}$ and $B_{s}$-meson spectroscopy are relatively narrower, which might be observed in the $DK$ and $BK$ channels, respectively.

The physical states of the second $P$-wave states with $J^{P} = 1^+$ might be mixed states between the $2^{3}P_{1}$ and $2^{1}P_{1}$. The strong decay properties of both $D_{s}(3040)$ and $D_{s}(3000)$ can be explained by assigning them as the low-mass mixed state $|(2P_{1})_{L}\rangle$ via the $2^{1}P_{1} \rightarrow 2^{3}P_{1}$ mixing with a mixing angle in the range of $\phi_{2P} = -(20 \sim 26)^{\circ}$. The mixed state $B_{s}(|(2P_{1})_{L}\rangle)$ in the $B_{s}$-meson family is the narrowest state in these low-mass mixed states, thus, it is most likely to be observed in the $B^{'K}$ channel. On the contrary, the mixed state $B(|(2P_{1})_{L}\rangle)$ in the $B$-meson family might have the broadest width in these low-mass mixed states, thus, its discovery potentials might be small. The high-mass mixed states $D(|(2P_{1})_{L}\rangle)$, $D_{s}(|(2P_{1})_{L}\rangle)$, $B(|(2P_{1})_{L}\rangle)$ and $B_{s}(|(2P_{1})_{L}\rangle)$ are usually narrower than those of low-mass states. These states dominantly decay into the first $P$-wave states by emitting a light pseudoscalar meson. However, experimental analysis of these decay channels is still absent, which might explain why these broader mixed states $D_{s}(3040)$ and $D_{s}(3000)$ have been first found in experiments. It is strongly suggested to carry out experimental analysis of the final states containing a low-lying $P$-wave state with $J^{P} = 0^+, 1^+$. 
No evidence of the second $P$-wave excitations with $J^P = 2^+$ in the $D_\ast$, $D_{\ast S}$, $B$- and $B_{\ast}$-meson spectroscopy is found in experiments. Our calculations indicate that these $P$-wave excitations $D(2^3P_2)$, $D_{\ast}(2^3P_2)$, $B(2^3P_2)$ and $B_{\ast}(2^3P_2)$ have a relatively narrow width, their strong decays are governed by the $D_{1}(2430)\pi$, $D_{1}(2430)K$, $B(1P_{1})\pi$ and $B(1P_{1})K$, respectively. Thus, it is might be a good chance for us to find the $D(2^3P_2)$ and $D_{\ast}(2^3P_2)$ excitations by analyzing the data in the $D_{1}(2430)\pi$ and $D_{1}(2430)K$ final states, respectively.

The $1^1F_2$ excitations in the heavy-light mesons are most likely to be very broad resonances, whose widths are larger than 400 MeV. Such broad states might be difficult to be observed in experiments, which might explain why these states are still missing in the heavy-light meson spectroscopy. Considering configuration mixing in the first $F$-wave states with $J^P = 3^+$, we predict that the low-mass mixed states $|1F_3^\ast\rangle_H$ in the heavy-light mesons are usually broad states, they should be difficult to be found in experiments. While the decay widths of the high-mass mixed states $|1F_3^\ast\rangle_H$ are relatively narrower than those of the low-mass states. These high-mass states $|1F_3^\ast\rangle_H$ might be observed in future experiments if they have a smaller mass as predicted in Ref.[39]. The optimal observed channels for the $D(|1F_3^\ast\rangle_H)$, $D_{\ast}(|1F_3^\ast\rangle_H)$ and $B(|1F_3^\ast\rangle_H)$ and $B_{\ast}(|1F_3^\ast\rangle_H)$ are $D\pi$, $D\pi^\ast K$, $B\pi$ and $B\pi^\ast K$, respectively. However, these high-mass states $|1F_3^\ast\rangle_H$ might be very broad states if their mass is as large as that predicted in Ref.[40].

For the $1^3F_4$ excitations, their decay widths are sensitive to the mass. If these states have a smaller mass as predicted in Ref. [39], they might have relatively narrow width $\Gamma \sim 100$ MeV. In this case, these excitations have good observation potentials in future experiments. The optimal observed channels for the $D(1^3F_4)$, $D_{\ast}(1^3F_4)$, $B(1^3F_4)$ and $B_{\ast}(1^3F_4)$ are $D\pi/D\pi^\ast$, $DK/DK^\ast$, $B\pi/B\pi^\ast$ and $BK/BK^\ast$, respectively. However, if these states have a larger mass as predicted in Ref. [40], they might be difficult to be found in experiments for their broad widths. Taking the newly observed natural parity resonance $D_{\ast J}^*(3000)$ as a candidate of the $D(1^3F_4)$, we find that the theoretical predictions of the strong decay properties are consistent with the observation. However, the predicted mass is inconsistent with the observation, which is about 100 $\sim$ 200 MeV larger than the data. If the $D_{\ast J}^*(3000)$ corresponds to the $D(1^3F_4)$ state indeed, it should be observed in both $D\pi$ and $D\pi^\ast$ channels. To clarify the puzzles in the $D_{\ast J}^*(3000)$, more observations are needed.

Finally, it should be pointed out that the chiral quark model still has some limitations. For example, the simple harmonic oscillator wave functions of the resonances have been adopted in the calculations, which should be different from the realistic wave functions more or less. Thus, the wave functions might bring some uncertainties to our predictions. Furthermore, our model is a non-relativistic model, the relativistic effects could bring some uncertainties to the decay widths as well. Thus, we might only give a qualitative prediction of the strong decay properties for some resonances. Acknowledgements

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