QCD corrections to the charged Higgs decay of a heavy quark

Andrzej Czarnecki

Department of Physics, University of Alberta, Edmonton, Canada T6G 2J1

and

Sacha Davidson

Center for Particle Astrophysics, University of California at Berkeley, Berkeley, CA 94720, U.S.A.

Abstract

We present an analytic formula for the $O(\alpha_s)$ corrections to the decay $t \to H^+b$ for nonzero $m_H$ and $m_b$. The Equivalence Theorem is used to relate these corrections to those for the process $t \to W^+b$. Contrary to previously published results we find that the effect of the $b$ quark mass is negligible for a wide range of $m_t$, $m_H$ and $\tan \beta$.

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1 Introduction

Many extensions of the Standard Model contain more than one Higgs doublet. The electroweak gauge bosons only absorb one of the charged Higgs fields, leaving the others as physical charged scalars, so that in a two-doublet model

the top could decay to $H^+ b$ (if $m_t > m_{H^+} + m_b$). For certain choices of parameters, this process dominates over the expected $t \to W^+ b$ decay, so it is of interest to correctly calculate its QCD corrections.

It is well known that the QCD corrections to the decay rate of a heavy quark into a $W$ are of order 10% in the Standard Model \cite{1,2}. Several groups have undertaken to calculate these corrections for the decay into a charged Higgs. The effect of the soft gluons has been calculated in ref. \cite{3}, and the decay $t \to H^+ b g$, with real gluons only, has been studied in ref. \cite{4}. The full one-loop QCD corrections have been computed, at first by neglecting the mass of the $b$-quark in ref. \cite{5}, and then with a nonzero $m_b$ in ref. \cite{6} (in the framework of what is called the Model I - see discussion below). The claim of the latter paper is that the QCD correction for $m_t = 150$ GeV, $m_b = 4.5$ GeV and $\alpha_s = 0.1$ is as large as $-15\%$ in the (unphysical) limit of the massless Higgs. However, as will be argued in the following section, according to the Equivalence Theorem (see \cite{7,8,9} as well as \cite{10} and references therein) the correction in this limit should be the same as the correction to the decay $t \to W^+ b$, which for the above values of parameters is $-8.6\%$ \cite{11,11}. The purpose of the present paper is to reevaluate the first order QCD corrections with non-zero $b$-quark mass. We use dimensional regularization for both the ultraviolet and the infrared divergences, which leads to simpler algebra than if one assigns a finite mass to the gluon, as was done in \cite{3,6,10}. Our result is consistent with the Equivalence Theorem.

In a model with two Higgs doublets and generic couplings to all the quarks, it is difficult to avoid flavour-changing neutral currents. We therefore limit ourselves to models that naturally side-step these problems by restricting the Higgs couplings \cite{12}. The first possibility is to have the doublet $H_2$ coupling to all the quarks, and the $H_1$ doublet interacting with none of them. The vacuum expectation value of $H_1 = v_1$ will nonetheless contribute to the $W$ mass, leading to an $H^- t \bar{b}$ vertex of the form

$$
\frac{g V_{tb}}{\sqrt{2} m_W} H^- \bar{b} \{ m_t \cot \beta R - m_b \cot \beta L \} t \quad \text{(model I)}
$$

(1)
where $H^-$ is the physical charged Higgs, $V_{tb}$ is the ‘33’ element of the CKM matrix, L and R are the chiral projection operators, and $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs. The second possibility is to have $H_2$ couple to the right-handed up-type quarks ($u_R, c_R, t_R$), and the $H_1$ couple to the right-handed down-type quarks. This is what happens in the Minimal Supersymmetric Standard Model. It is easy to show that the interaction Lagrangian

$$H_2 u^{(u)}_R h_{ij} q^j_L + H_1 d^{(d)}_R h_{ij} q^j_L + h.c.$$ (2)

leads to the vertex

$$\frac{g V_{tb}}{\sqrt{2} m_W} H^- \bar{b} \{m_t \cot \beta R - m_b \tan \beta L\} t \quad \text{(model II)}$$ (3)

where we have numbered the models in accordance with [12].

In order to simplify the following formulas we introduce dimensionless parameters for the scaled masses:

$$\epsilon = \frac{m_b}{m_t}, \quad \chi = \frac{m_H}{m_t}, \quad w = \frac{m_W}{m_t},$$ (4)

and write the vertex $t \rightarrow H^+ b$ as

$$i \frac{g}{2 \sqrt{2} w} V_{tb} (a + b \gamma_5) t.$$ (5)

where from (1) and (3)

Model I: \[
\begin{align*}
    a &= \cot \beta (1 - \epsilon) \\
    b &= \cot \beta (1 + \epsilon)
\end{align*}
\]

Model II: \[
\begin{align*}
    a &= \cot \beta + \epsilon \tan \beta \\
    b &= \cot \beta - \epsilon \tan \beta
\end{align*}
\]

The next section contains our result, which is examined in the Discussion. The Appendix contains some details of the calculation.

2 QCD Corrections

The notation we use is similar to that used in the analysis of semileptonic decays [1, 2]. In terms of the dimensionless parameters (4), we define the
following kinematic variables:

\[ \bar{P}_0 \equiv \frac{1}{2}(1 - \chi^2 + \epsilon^2) \]
\[ \bar{P}_3 \equiv \frac{1}{2}\sqrt{1 + \chi^4 + \epsilon^4 - 2(\chi^2 + \epsilon^2 + \chi^2 \epsilon^2)} \]
\[ P_\pm \equiv \bar{P}_0 \pm \bar{P}_3 \]
\[ \bar{Y}_p \equiv \frac{1}{2} \ln \frac{\bar{P}_+}{\bar{P}_-} \]
\[ \bar{W}_0 \equiv \frac{1}{2}(1 + \chi^2 - \epsilon^2) \]
\[ \bar{W}_\pm \equiv \bar{W}_0 \pm \bar{P}_3 \]
\[ \bar{Y}_w \equiv \frac{1}{2} \ln \frac{\bar{W}_+}{\bar{W}_-} \]

The tree level decay rate is

\[ \Gamma^0 (t \rightarrow H^+b) = \frac{G_F m_t^3}{4\sqrt{2}\pi} |V_{tb}|^2 \left[ \bar{P}_0 (a^2 + b^2) + \epsilon (a^2 - b^2) \right] \bar{P}_3 \]  

and the \( O(\alpha_s) \) correction is

\[ \Gamma^{(1)} = \frac{\alpha_s}{6\pi^2} \frac{G_F m_t^3 |V_{tb}|^2}{\sqrt{2}} \left[ (a^2 + b^2) G_+ + (a^2 - b^2) \epsilon G_- + ab G_0 \right] \]

with

\[ G_+ = \bar{P}_0 \mathcal{H} + \bar{P}_0 \bar{P}_3 \left[ \frac{9}{2} - 4 \ln \left( \frac{4\bar{P}_3^2}{\epsilon \chi} \right) \right] + \frac{1}{4\chi^2} \bar{Y}_p \left( 2 - \chi^2 - 4\chi^4 + 3\chi^6 - 2\epsilon^2 - 2\epsilon^4 + 2\epsilon^6 - 4\chi^2 \epsilon^2 - 5\chi^2 \epsilon^4 \right), \]
\[ G_- = \mathcal{H} + \bar{P}_3 \left[ 6 - 4 \ln \left( \frac{4\bar{P}_3^2}{\epsilon \chi} \right) \right] + \frac{1}{\chi^2} \bar{Y}_p \left( 1 - \chi^2 - 2\epsilon^2 + \epsilon^4 - 3\chi^2 \epsilon^2 \right), \]
\[ G_0 = -6 \bar{P}_0 \bar{P}_3 \ln \epsilon, \]

and

\[ \mathcal{H} = 4\bar{P}_0 \left[ \text{Li}_2 \left( \frac{\bar{P}_+}{\bar{P}_3} \right) - \text{Li}_2 \left( \frac{\bar{P}_-}{\bar{P}_3} \right) - 2\text{Li}_2 \left( 1 - \frac{\bar{P}_-}{\bar{P}_+} \right) \right] \]
\[ + \bar{Y}_p \left( \frac{4\bar{P}_3^2 \chi}{\bar{P}_+^2} \right) - \bar{Y}_w \ln \epsilon \left[ 2\bar{Y}_w \left( 1 - \epsilon^2 \right) + \frac{\bar{P}_3^2}{\chi^2} \ln \epsilon \left( 1 + \chi^2 - \epsilon^2 \right) \right]. \]
In the limit of the zero mass of the $b$ quark the QCD correction becomes

$$\lim_{\epsilon \to 0} \Gamma^{(1)} = \frac{\alpha_s}{6\pi^2} \frac{G_F m_t^3 |V_{tb}|^2}{\sqrt{2}} \cot^2 \beta \left( 2\tilde{G}_+ + \tilde{G}_0 \right),$$

(10)

where

$$\tilde{G}_+ = (1 - \chi^2)^2 \left[ \text{Li}_2 \left( 1 - \chi^2 \right) - \frac{\chi^2}{1 - \chi^2} \ln \chi ight. + \ln \chi \ln \left( 1 - \chi^2 \right) + \frac{1}{2\chi^2} \left( 1 - \frac{5}{2}\chi^2 \right) \ln \left( 1 - \chi^2 \right) - \frac{\pi^2}{3} + \frac{9}{8} + \frac{3}{4} \ln \epsilon \bigg],$$

$$\tilde{G}_0 = -\frac{3}{2} \left( 1 - \chi^2 \right)^2 \ln \epsilon,$$

(11)

and we see that the mass singularities $\sim \ln \epsilon$ cancel in the expression for the total rate (10). Our result in this limit is identical to the one obtained by Liu and Yao [10] and is in agreement with the corrected version of ref. [5].

If we further take the limit $m_H \to 0$ the rate becomes:

$$\lim_{\epsilon, \chi \to 0} \Gamma^{(1)} = \frac{\alpha_s}{6\pi^2} \frac{G_F m_t^3 |V_{tb}|^2}{\sqrt{2}} \cot^2 \beta \left( \frac{5}{4} - \frac{\pi^2}{3} \right),$$

(12)

which is in agreement with the conclusion of the ref. [10] as well as with our previous result [13].

Now we would like to compare the corrections to the decay width $\Gamma (t \to H^+ b)$ with those to $\Gamma (t \to W^+ b)$. For simplicity we now take $m_b = 0$ and $\cot \beta = 1$, and examine the ratio of the first order correction to the Born rate:

$$f_H(\chi) = \frac{\Gamma^{(1)} (t \to H^+ b)}{\Gamma^{(0)} (t \to H^+ b)},$$

$$f_W(w) = \frac{\Gamma^{(1)} (t \to W^+ b)}{\Gamma^{(0)} (t \to W^+ b)}.$$ 

(13)

It has been noted in [10] that in the limit of the infinite top mass these ratios are equal: $f_H(0) = f_W(0)$. On the other hand, when $m_H$ approaches $m_t$, we have:

$$f_H(\chi) \xrightarrow{\chi \to 1} \frac{\alpha_s}{3\pi} \left[ -6 \ln(1 - \chi^2) - \frac{8}{3}\pi^2 + 13 \right].$$

(14)
By comparison with \[1\] we see that:

$$\lim_{x \to 1} \frac{f_H(x)}{f_W(x)} = 1.$$  
(15)

Finally, we examine the corrections in the limiting case where the mass of the charged Higgs is zero but the $b$ quark mass is finite. This is of course unphysical, but serves as a useful check on our equations. If we choose the parameters $a$ and $b$ from (3) to correspond to the couplings of the single Standard Model Higgs, then the Equivalence Theorem implies that the corrections are the same as in the process $t \to bW$ in the limit of massless $W$ boson and nonzero $m_b$. The latter can be obtained by taking the limit of the relevant formula [1, 11]:

$$\lim_{m_W \to 0} \Gamma^{(1)}(t \to bH^+) = \frac{\alpha_s}{6\pi^2} \frac{G_F m_t^3 |V_{tb}|^2}{\sqrt{2}} \left\{ 8(1 - \epsilon^2)^2(1 + \epsilon^2) \left[ \text{Li}_2(\epsilon^2) - \frac{\pi^2}{6} + \ln(\epsilon) \ln(1 - \epsilon^2) \right] 
- 4\epsilon^2 \left( 7 - 5\epsilon^2 + 4\epsilon^4 \right) \ln(\epsilon) - 8 \left( 1 - \epsilon^2 \right)^3 \ln(1 - \epsilon^2) 
- (1 - \epsilon^2)(-5 + 22\epsilon^2 - 5\epsilon^4) \right\}. \quad (16)$$

The couplings of the Goldstone boson charged Higgs of the Standard Model (longitudinal $W$) to $t$ and $b$ can easily be calculated to be those of Model I, with $\cot \beta = 1$. In this case, the corrections to the decay $t \to bH^+$ are, in the limit $m_H \to 0$:

$$\lim_{m_H \to 0} \Gamma^{(1)}(t \to bH^+) = \frac{\alpha_s}{6\pi^2} \frac{G_F m_t^3 |V_{tb}|^2}{\sqrt{2}} \left[ 2(1 + \epsilon^2)G_0^0 - 4\epsilon^2 G_0^0 + (1 - \epsilon^2)G_0^0 \right]. \quad (17)$$

where $G_i^0$ are limits of corresponding functions $G_i$ for $m_H = 0$:  

6
Inserting these expressions into equation (17) we obtain the same formula as (16).

\[ G_+^0 = (1 + \epsilon^2)^2 \left[ \text{Li}_2 (\epsilon^2) - \frac{\pi^2}{6} + \ln (\epsilon) \ln (1 - \epsilon^2) \right] \\
+ \left( \frac{3}{4} + \epsilon^2 - \frac{5}{4} \epsilon^4 \right) \ln (\epsilon) - \left( 1 - \epsilon^4 \right) \ln (1 - \epsilon^2) + \frac{5}{8} (1 - \epsilon^4) \\
G_-^0 = 2(1 + \epsilon^2) \left[ \text{Li}_2 (\epsilon^2) - \frac{\pi^2}{6} + \ln (\epsilon) \ln (1 - \epsilon^2) \right] \\
+ \left( 3 - \epsilon^2 \right) \ln (\epsilon) - 2 \left( 1 - \epsilon^2 \right) \ln (1 - \epsilon^2) + 2(1 - \epsilon^2) \\
G_0^0 = -\frac{3}{2} (1 - \epsilon^4) \ln (\epsilon) \tag{18} \]

3 Discussion

In Figure 1 the ratio of the first order QCD correction to the Born rate for the decay \( t \to H^+b \) is plotted as the function of the ratio of masses \( \chi = m_H/m_t \). We have chosen the set of parameters \( \cot \beta = 1, \ m_t = 150 \text{ GeV} \) for an easy comparison with the analogous diagram in ref. [6]. It can be seen that the effect of the mass of the \( b \)-quark is negligible, except in the case of \( m_t - m_H \sim m_b \).

There are also logarithmic corrections (\( \sim \epsilon^2 \ln \epsilon \)) to the decay rate in model II, as can be seen from Figure 2. Here we compare the branching ratios of the decays \( t \to H^+b \) and \( t \to W^+b \), taking \( m_t = 100 \text{ GeV} \) so that this plot can be easily compared with a similar one published in ref. [12]. Our graph is different from theirs in that we now include QCD corrections to both decay rates. These corrections modify the diagram significantly only in the case of large values of \( \tan \beta \) in Model II. It must be noted however, that although the corrections are relatively large, the top decays principally to \( W^+b \) in this region of \( \tan \beta \). In model I, both \( a \) and \( b \) are proportional to \( \cot \beta \), so the log of the branching ratio as a function of \( \tan \beta \) decreases with a slope of -2. As can be seen from figure 2, the \( \ln \epsilon \) corrections cancel to order \( \epsilon^2 \) among \( G_+, G_- \) and \( G_0 \). However in model II, the decay rate is a polynomial in \( \tan \beta \) with exponents -2, 0 and 2, and the \( \ln \epsilon \) does not cancel in the “0” and “2” terms.
Summary

We have calculated an analytic expression for the $O(\alpha_s)$ corrections to the decay $t \to H^+ b$ for non-zero $m_H$ and $m_b$. The $m_b = 0$ limit of our result is the same as in [10] and the corrected version of [5]; however, our full expression disagrees with the $m_b \neq 0$ result presented in [6]. To check the $m_b$ dependence of our results we have compared it with the corresponding formula for the decay $t \to W^+ b$.

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A Details of the calculation

Throughout this calculation we have used dimensional regularization, working in $d = 4 - 2\omega$ dimensions. The counterterm for the vertex function has been calculated according to ref. [5, 14]:

\[
\delta \Lambda = a \left\{ \frac{1}{2} (Z_t - 1) + \frac{1}{2} (Z_b - 1) \right\} - \frac{a \, \delta m_t}{2 m_t} - \frac{a - b \, \delta m_b}{2 m_b}
+ \left[ b \left\{ \frac{1}{2} (Z_t - 1) + \frac{1}{2} (Z_b - 1) \right\} - \frac{a + b \, \delta m_t}{2 m_t} + \frac{a - b \, \delta m_b}{2 m_b} \right] \gamma_5
\]

and the renormalization constants are (we take the renormalization scale equal to the mass of the decaying quark):

\[
Z_t - 1 = -\frac{\delta m_t}{m_t} = \frac{\alpha_s}{3\pi} \left( -\frac{3}{\omega} + 3\gamma - 3 \ln \frac{4\pi}{\epsilon} - 4 \right),
\]

\[
Z_b - 1 = -\frac{\delta m_b}{m_b} = \frac{\alpha_s}{3\pi} \left( -\frac{3}{\omega} + 3\gamma - 3 \ln \frac{4\pi}{\epsilon^2} - 4 \right). \quad (19)
\]
The contribution of the virtual corrections to the total decay rate is:

$$\Gamma_{\text{virt}}^{(1)} = \frac{\alpha_s}{6\pi^2} \frac{G_F m_t^3 |V_{tb}|^2}{\sqrt{2}} \left[ (a^2 + b^2) V_+ + (a^2 - b^2) \epsilon V_- + ab G_0 \right],$$  \hspace{1cm} (20)

where

\begin{align*}
V_+ &= \bar{P}_0 \mathcal{V} + \frac{\bar{Y}_p}{2\chi^2} \left[ (1 - \epsilon^2)^2 (1 + \epsilon^2) - \chi^4 (3 + 3\epsilon^2 - 2\chi^2) \right], \\
V_- &= \mathcal{V} + \bar{Y}_p \frac{(1 - \epsilon^2)^2 - \chi^4}{\chi^2},
\end{align*}  \hspace{1cm} (21)

with

\begin{align*}
\mathcal{V} &= 2\bar{P}_3 \left[ -\frac{1}{\omega} + 2\gamma + 2 \ln \frac{\bar{P}_3}{2\pi} - 3 + \frac{1}{\chi^2} (1 - \epsilon^2 + \chi^2) \ln \epsilon \right] \\
&\quad + 2\bar{P}_0 \left[ \text{Li}_2 \left( \frac{\bar{P}_p}{\bar{P}_+} \right) - \text{Li}_2 \left( \frac{\bar{P}_p}{\bar{P}_-} \right) - \text{Li}_2 \left( 1 - \frac{\bar{P}_p}{\bar{P}_+} \right) - \bar{Y}_p^2 - 2 \ln \epsilon \bar{Y}_w \right] \\
&\quad - 2\bar{Y}_p \left( -\frac{1}{2\omega} + \gamma + \ln \frac{P_3 \epsilon}{2\pi \chi} \right).
\end{align*}

The real gluon radiation calculation requires an integration over three body phase space \( \Phi(t; H, b, g) \). This leads to divergences due to the emission of soft and collinear gluons, for which we use dimensional regularization (see ref. [15] and references quoted therein). The phase space integration we are considering now is analogous to the decay \( t \to Wbg \), for which the relevant integrals have been listed in [11]. Adding the result to the virtual gluon contribution (21) yields the final formula (8).

We would like to add that this calculation was greatly facilitated by the use of algebraic manipulation programs FORM [16] and Mathematica [17].

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Figure 1: Ratio of the first order correction to the Born rate for $\tan \beta = 1$ and $m_t = 150$ GeV: $m_b = 4.5$ GeV (Model I - solid, Model II - dotted), $m_b = 0$ (dash-dotted line).

Figure 2: Ratio of the branching ratios of $t \to bH^+$ and $t \to bW^+$ for $m_b = 4.5$ GeV: in the Model I with and without QCD corrections (solid and dashed lines, undistinguishable) and in the Model II (dash-dotted and dotted lines, respectively).