Effect of temperature on stability behaviour of functionally graded spherical panel

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Abstract. In the present article, stability of functionally graded spherical panel under thermal environment is examined. The effective material properties are evaluated through Voigt’s micromechanical model and continuous gradation is achieved using power-law distribution of the volume fraction of constituents. In addition, material properties are taken as temperature dependent. Finite element solutions are obtained through commercially available tool ANSYS using an eight node serendipity element. The linear eigenvalue buckling problem is solved using Block Lanczos method. Comparison study is made with the available published literatures. Finally, the effect of different geometry and material parameters such as thickness ratio, curvature ratio and power-law index on the critical buckling temperature of functionally graded spherical shell panel under thermal environment is demonstrated.

1. Introduction

Researches on advanced composites are now being increased due to their compatibility and durability in severe environment conditions. Functionally graded material (FGM) is known for its tailor-made properties which are achieved through the continuous gradation of material phase from one surface to another surface. FGMs are made from the designed combination of metal/alloy and ceramic materials. Metals or metal alloys are known for their mechanical strength and fracture toughness whereas ceramics hold good heat and corrosion resistance. This kind of in-homogenous material eliminates delamination as well as inter-laminar thermal stress concentration which can be generally seen in laminated structures. These novel characteristics make FGM most trustworthy for different sectors like aerospace, defence, nuclear plant, biomedical etc. Due to this, many researchers have shown their interest on modelling and analysis of FGM structures.

Na and Kim [1],[2] examined the buckling behaviour of functionally graded (FG) panel under uniform and non-uniform thermal field across the transverse direction. Woo et al. [3] investigated the post-buckling behaviour of FG plate and cylindrical panel under axial loading conditions. Zhao et al. [4] approached element-free kp-Ritz method in conjunction with first order shear deformation theory (FSDT) to analyse the thermo-mechanical buckling behaviour of FG flat panel. This work has been extended by Zhao and Liew [5] for the cylindrical. Tung and Duc [6] employed classical plate theory and von Karman strain terms to obtain the thermo-mechanical buckling and post-buckling behaviour of FG plate. Lee et al. [7] examined the thermo-mechanical post-buckling behaviour of FG plates based on the element-free kp-Ritz method in the FSDT framework. Thai and Choi [8] proposed an efficient refined theory to investigate the buckling behaviour FG plate. Ghannadpour et al. [9] applied finite strip
method to obtain the buckling responses of FG plate under various uniform and non-uniform thermal fields across the thickness direction.

It is noted that very few work has been reported on stability analysis of FGM panel under thermal environment. Here, authors have attempted the thermal buckling analysis of FG spherical shell panel under thermal environment with temperature dependent material properties. Finite element (FE) solutions have been presented for FG spherical panel through commercially available FE tool ANSYS. For the discretisation purpose, an eight node serendipity element is considered. The convergence of present finite element model has been carried out and subsequently validation of the proposed model with the previous published results is performed. The effects of thickness ratio, curvature ratio, volume fraction and temperature on stability behaviour of FG spherical shell panel are presented and discussed.

2. Effective material properties of FGM

In this study, FGM is the composition of gradual variation of metal phase (at bottom surface) to ceramic phase (at top surface) across the thickness direction. The FGM constituents are considered as cubic function of temperature $T$ as in [10].

$$ P_{c,m}(T) = P_0(P_{c}T^{-1} + 1 + P_2T^2 + P_3T^3) \tag{1} $$

where, $P_{c,m}$ denotes material properties of metal/ceramic, $P_0$, $P_1$, $P_2$ and $P_3$ are the temperature coefficients.

In order to evaluate the effective material properties of FGM ($P$), Voigt’s micromechanical model is used [11].

$$ P(T,z) = (P_c(T) - P_m(T))V_c(z) + P_m(T) \tag{2} $$

where, $V_c(z)$ is the ceramic volume fraction which is the function of thickness coordinate $z$. This can be achieved through power-law distribution [12] and expressed as

$$ V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \quad (0 \leq n < \infty) \tag{3} $$

where, $n$ is the power-law index which exhibits the material profile across the thickness. Different material profiles are shown in Figure 1 for volume fractions corresponds to different values of power-law index along with the dimensionless thickness ($Z=z/h$).

By substituting Eq. (3) in Eq. (2), the effective material properties of FGM can be expressed for Young’s modulus $E(T,z)$ and thermal expansion coefficient $\alpha(T,z)$ as

$$ E(T,z) = \left[E_c(T,z) - E_m(T,z)\right]\left(\frac{z}{h} + \frac{1}{2}\right)^n + E_m(T,z) \tag{4} $$

$$ \alpha(T,z) = \left[\alpha_c(T,z) - \alpha_m(T,z)\right]\left(\frac{z}{h} + \frac{1}{2}\right)^n + \alpha_m(T,z) \tag{5} $$

Here, Poisson’s ratio $\nu$ is taken constant throughout the thickness of the FG panel. The temperature dependent material properties of ceramic and metal are shown in table 1.
In this study, an FG spherical panel of radius $R$ with uniform thickness $h$ and sides $a$ and $b$ is considered as shown in Figure 2. Here, FG spherical shell panel is modelled and analyse in ANSYS through ANSYS parametric design language (APDL) code. An eight noded serendipity shell element (SHELL281), defined in the ANSYS library, is utilised to discretise the FG spherical panel. This shell element has total six degrees of freedom per node i.e., translations and rotations in the $x$, $y$ and $z$ directions [13].

**Table 1.** Temperature-dependent material properties of ceramic and metal [10]

| Material   | Properties | $P_0$       | $P_1$       | $P_1$       | $P_2$       | $P_3$       | TID       |
|------------|------------|-------------|-------------|-------------|-------------|-------------|-----------|
| Si$_3$N$_4$| E (Pa)     | 3.48e+11    | 0           | -3.07e-04   | 2.16e-07    | -8.95e-11  | 3.22e+11  |
|            | $\alpha$ (K$^{-1}$) | 5.87e-06    | 0           | 9.10e-04    | 0           | 0           | 7.47e-06  |
| Ti6Al4V    | E (Pa)     | 1.23e+11    | 0           | -4.59e-04   | 0           | 0           | 1.06e+11  |
|            | $\alpha$ (K$^{-1}$) | 7.58e-06    | 0           | 6.64e-04    | -3.15e-06   | 0           | 6.94e-06  |

**Figure 1.** Different material distributions along the non-dimensional thickness coordinate.
4. Results and discussions
In this section, the stability behaviour of FG spherical panel is performed under uniform temperature field. Here, FG panel is raised uniformly from the reference temperature ($T_0 = 300 \, ^\circ K$) to the final temperatures ($T = 400, 500, 600$ and $700 \, ^\circ K$). The FG spherical shell panel is discretised and solved using finite element steps in ANSYS APDL platform. Block Lanczos method is used to obtain the eigenvalue buckling responses.

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**Figure 2.** A discretised FG spherical shell panel model.

**Figure 3.** Convergence behaviour of simply-supported FG ($Ti-6Al-4V/Si_3N_4$) spherical panel with $R/a=5$, $a/h=100$ and $n=0.2$ under thermal environment.
Figure 4. Critical buckling temperature rise of clamped (Al/Al₂O₃) FG flat panel (a/h=50).

Figure 5. Effect of thickness ratio on critical buckling temperature rise of simply-supported FG spherical panel under thermal environment.
**Figure 6.** Effect of curvature ratio on critical buckling temperature rise of simply-supported FG spherical panel under thermal environment.

**Figure 7.** Effect of power-law index on critical buckling temperature rise of simply-supported FG spherical panel under thermal environment.

4.1. Convergence and comparison

The convergence study is performed through the proper mesh refinement of the present FG panel. Buckling responses of simply-supported FG spherical shell panel ($a=b=1$, $R/a=5$, $n=0.2$ and $a/h=100$)
under three different temperature field (ΔT=0, 100 and 200 °K) are obtained for different meshes as shown in Figure 3. Material properties are taken same as given in Table 1. It is clear from the figure that a (18×18) mesh is sufficient enough to obtain the buckling response throughout in the analysis.

In order to check the efficacy of the present model, a comparison study has been made with the previous published results of [4]. Geometry and material parameters are taken as in [4]. In this problem, a square FG (Al/Al₂O₃) plate (a/h=50) is analysed for five different power-law indices (n=0, 0.5, 1, 2, 5) and presented in Figure 4. It is observed from the figure that the difference between the present results and the reference results are very nominal.

4.2. Numerical illustrations
Some numerical problems are carried out to show the robustness of present FG model. Titanium alloy (Ti–6Al–4V) as metal and silicon nitrated (Si₃N₄) as ceramic are considered at the bottom and the top surfaces, respectively. The influences of thickness ratios (a/h), curvature ratios (R/a), power-law indices (n) and temperature on the buckling behaviour of FG spherical panel are discussed in the following paragraphs.

Figure 5 exhibits the effect of thickness ratio on the critical buckling temperature rise of simply-supported FG spherical shell panel (a/b=1, R/a=5, n=2) at different elevated temperatures. It is noted that that the critical buckling temperature decreases with the increase in thickness ratio i.e., thin shell panels have less buckling load.

Figure 6 represents the variation of critical buckling parameter of simply-supported FG spherical panel (a/b=1, n=2, a/h=100) along with the different temperature values for three different values of curvature ratio (R/a=10, 20, 50). It is clearly observed that as the curvature ratio increases the critical buckling temperature rise decreases i.e., curved panel is having relatively higher buckling temperature.

Figure 7 exhibits the influence of power-law index on the critical buckling temperature rise of simply-supported FG spherical shell panel (a/b=1, R/a=5, a/h=100) along with the elevated temperature values. It is found that the critical buckling temperature rise is increasing with the increase in power-law index. It is also interesting to note that as the elevated temperature rise increases the critical temperature reduces in all the cases considered.

5. Conclusions
In this study, the thermal bucking behaviour of FG spherical shell panel under uniform temperature field is investigated. The effective material properties of FGM are evaluated through Voigt’s model and power-law distribution. In addition, temperature dependent material properties of FGM constituents are considered. Finite element solution for the buckling behaviour of present FG model is proposed using Block Lanczos method. The present model is compared with the previous reported results. The influences of different material and geometrical parameters on the thermal buckling of FG spherical panel are illustrated.

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