Abstract

We calculate the leading twist contribution to near-forward proton-proton (and proton-antiproton) elastic scattering with large momentum transfer, in the multiple scattering (Landshoff) mechanism. The amplitude in the near-forward region is dominated by singlet exchange for all three valence quark-quark scatterings. We assume the existence of a hard singlet quark-quark amplitude, which we estimate to be $\mathcal{O}(\alpha_s^2/t)$. For a three-quark state whose transverse size is less than $1/\Lambda_{\text{QCD}}$, Sudakov resummation accounts for both approximate $d\sigma_{pp}/dt \sim t^{-8}$ at moderate $t$, and $d\sigma_{pp}/dt \sim t^{-10}$ at larger $t$. The transition from approximate $t^{-8}$ to $t^{-10}$ behavior is strongly correlated with the transverse size of the valence three-quark state in the proton.
1 Introduction

Perturbative QCD (pQCD) offers a formalism in which to study high energy exclusive hadronic processes [1, 2, 3]. Factorization theorems make possible the separation of long distance effects, describing the formation of color singlet hadronic bound states, from the short distance interactions of quarks and gluons. Once long distance interactions have been modeled by hadronic wave functions, the remaining hard process can be calculated perturbatively. At leading power, the relevant partons are the valence quarks that carry the momenta of the incoming and outgoing hadrons.

Fixed-angle hadron-hadron elastic scattering has been discussed in this formalism, for both a single hard scattering [1, 2, 3] and for multiple, independent (‘Landshoff’), scatterings of the valence quarks [4, 5]. To calculate the amplitude, however, it is necessary to incorporate systematically the effect of soft gluon exchange, as described in refs. [6, 7]. This extended factorization can also be useful in the treatment of hadronic form factors [8, 9] and proton-antiproton annihilation [10]. The resulting suppression of spatially separated partonic scatterings accounts for approximate “quark counting” power behavior [1] for fixed angle hadron-hadron elastic scattering. In this paper, we will show that the improved perturbative formalism, including the transverse structure of the proton, can naturally account for the approximate $t^{-8}$ behavior of $d\sigma^{pp}/dt$ at small angles. In addition, we suggest a relatively sharp transition to dimensional counting behavior, $d\sigma^{pp}/dt \sim t^{-10}$, as $t$ increases. The position of this transition is directly related to the transverse size of the quark valence state of the proton.

Large angle scattering ($\theta = \mathcal{O}(\pi/2)$) involves two scales, the momentum transfer $-t \sim s$, and the scale of hadronic masses. In contrast, near-forward scattering ($\theta \to 0$) is a three scale problem, involving the center of mass energy, $\sqrt{s}$, the momentum transfer $-t$ and the scale of hadronic masses. Correspondingly, the near-forward amplitude at each order has both “Sudakov” double logarithms in momentum transfer and “Regge” logarithms of the energy [11, 12]. In a recent
paper, we have studied the relationship between these two phenomena [13].

In this paper, we use the results of Refs. [13] and [6] to investigate the interplay between soft gluon corrections and the transverse structure of the proton in a range of momentum transfers, $3\text{ GeV}^2 \lesssim -t \lesssim 40\text{ GeV}^2$. For the description of the hadronic bound state we use quark distribution amplitudes derived from QCD sum rules [14, 15, 16, 17]. To leading power in $\alpha_s$, the three valence quarks from each proton scatter independently. Of course, at the lower end of the $t$-range considered here not all individual q-q scatterings can be really hard. Nevertheless, the Sudakov-improved pQCD approach remains well defined, and affords a model for the amplitude over the entire range.

In the $s \to \infty$, $t$ fixed, limit q-q elastic scattering is dominated by singlet exchange in the $t$-channel. The suppression of octet exchange may be described as gluon reggeization [12]. In [13] we have studied this process as the $\theta \to 0$ limit of fixed angle q-q scattering. There, we rederived the reggeization of octet exchange from the energy-dependence of its Sudakov suppression, while the corresponding factor for the singlet is $s$-independent in the leading logarithmic approximation. Of course, even with singlet exchange, the elastic scattering of colored objects is IR divergent. In [13] we have suggested ways of separating dimensionally regularized IR singular contributions from the hard part of singlet exchange. The renormalization group can be used to resum the leading and non-leading $\ln(t)$ dependence of the q-q amplitude, employing one-loop anomalous dimensions [6, 18, 19].

In the following, we shall treat hard singlet exchange as an $\alpha_s^2$ process [20] only (denoted $H_s^m(Q_m)$ below), where $m = 1, 2, 3$ labels the hard q-q scattering of momentum transfer $Q_m$. Once the q-q scatterings are embedded in the full hadronic amplitude, IR divergences cancel, due to the color singlet nature of the external hadrons. The IR cutoff (denoted $1/\tilde{b}$ below), although arbitrary when q-q scattering is considered in isolation [13], has a physical meaning in the case of proton-proton scattering, as the transverse separation between independent hard scatterings.

The explicit form of the hard singlet to lowest order, $\alpha_s^2$, is ambiguous, because the singlet amplitude is already IR divergent to this order. As pointed out in ref.
different IR subtractions procedures yield different expressions for $H^m_s$. The IR subtractions that define the hard singlet to lowest order can be fixed only by their effect on the amplitude at higher orders in $\alpha_s$. Without facing this difficult question here, we estimate the lowest order hard singlet in section 3 below.

Because, as noted above, the transverse separation between hard scatterings plays the role of an IR cutoff, the transverse structure of the proton valence state wave function is an essential feature of our calculation. The perturbative (small $\tilde{b}$) behavior of the wave function is available from [6], but for the range of $t$ considered here we will find it necessary to include as well a “nonperturbative” or “intrinsic” transverse structure as well as the familiar light cone wave functions. The importance of transverse structure has been stressed in [21, 22, 23].

Our discussion is organized as follows. In section 2 we describe in some detail the factorized form of the p-p amplitude in the multiple scattering scenario [6] for fixed scattering angle $\theta$, stressing the roles of hadronic wave functions and flavor flows. In section 3, we take this amplitude to the forward region, considering the contribution from singlet exchange in all three q-q scatterings. In section 4, we calculate the lowest order three-singlet amplitude. This calculation determines the combination of the quark distribution amplitudes that appears in the p-p amplitude. Leading and non-leading $\ln(t)$ corrections from all orders are then included through soft gluon resummation factors. In section 5, we define the kinematic cut-offs and the choice of scales that turn the closed form of the p-p amplitude into a numerically stable four dimensional integral, which we compute for a range of momentum transfers. We examine the rate of convergence of the amplitude in impact parameter space and its sensitivity to scale choices, and study the effect of the transverse structure of the valence three-quark state. We summarize our results in the final section.
2 The proton-proton amplitude in the multiple scattering scenario

We consider the leading twist contribution of the valence three-quark Fock state for each proton to the proton-proton elastic scattering amplitude. The p-p amplitude, \( A \), is shown in fig. 1 for multiple scattering [4]. The valence quark momenta are parameterized in the CM frame as

\[
k_i^\mu = \sqrt{\frac{s}{2}} x_i v_i^\mu + \kappa_i v_i^\prime \mu + k_i^\mu, \tag{1}
\]

where \( i \) is the proton label, the \( v_i \) are lightlike vectors along the proton directions, with \( P_i = \sqrt{s/2} v_i \), and the \( v_i^\prime \) are lightlike vectors in the directions opposite to \( v_i \) \((v_i \cdot v_i^\prime = 1\)). \( k_i^{\mu} \equiv k_i^{\mu\perp} \) denotes the components of \( k_i \) in the plane perpendicular to the spacelike component of \( v_i \). Using the notation \( v_{ij} = v_i \cdot v_j \), we parameterize the invariants of the scattering as

\[
t = -sv_{13} = -sv_{24}, \quad u = -sv_{14} = -sv_{23}. \tag{2}
\]

Since each proton contains two \( u \)-quarks and one \( d \)-quark in the valence state, there is more than one flavor-distinguishable way of connecting the quarks to the three hard scatterings. We label the inequivalent flavor flows as follows. We call \( H^3 \) the hard scattering blob in which the \( d \)-quark from proton 2 participates, and \( H^2 \) the hard scattering in which the \( d \)-quark from proton 1 participates, if \( H^2 \neq H^3 \). Then we obtain the three inequivalent flavor flows shown in fig. 1, labeled by \( f = 1, 2, 3 \). Notice that for \( f = 1 \), \( H^1 \) and \( H^2 \) are flavor indistinguishable.

For identical quarks, both \( t \) and \( u \) gluon exchange channels are available. For \( f = 2 \) the gluonic exchange channels in the u-d hard scatterings \((H^2 \text{ and } H^3)\) are fixed to be \( t \), and for \( f = 3 \) they are fixed to be \( u \). Table 1 summarizes the channel configurations of the hard scatterings for each flavor flow.
2.1 The factorized amplitude

According to ref. [6], the leading twist elastic p-p amplitude may be written in the following factorized form,

\[
\mathcal{A}(s,t,h_i) = (-i) \left( \frac{2\pi^8}{\sin^2 \theta} \right) \int_0^1 d\xi_1 d\xi_2 \int \prod_{i=1}^4 d\kappa_i d^2k_i d\hat{\kappa}_i d^2\hat{k}_i \int d\ell d\hat{\ell} \\
\times \sum_{f=1}^3 \sum_{a_i,b_i,c_i} Y^{(f)}_{\alpha_i \beta_i \gamma_i}(k_i, \hat{k}_i; P_i, h_i) \\
\times U_{\{a_i,b_i,c_i\}}(\ell, \hat{\ell}) \delta(\eta \cdot \sum_i k_i + \ell) \delta(\eta \cdot \sum_i \hat{k}_i + \hat{\ell}) \\
\times H_1^{1}(\xi_1 P_1) H_2^{2}(\xi_2 P_1) H_3^{3}(1 - \xi_1 - \xi_2) P_i + R, \tag{3}
\]

where \(\alpha_i, a_i\) are Dirac and SU\((N_c)\) indices respectively, \(h_i\) are proton helicities, \(\theta\) is the scattering angle in the CM frame, \(\eta\) is the spacelike unit vector perpendicular to the scattering plane and \(R\) is a remainder suppressed by a power of \(\sqrt{s}\). The \(Y\)’s in eq. (3) represent the valence structure of the external protons, the \(H^m\)’s represent the three hard scatterings and \(U\) summarizes the effects of soft gluons, in terms of two transverse momentum components normal to the scattering plane, \(\ell\) and \(\hat{\ell}\). From the kinematics of q-q scattering when all the valence quarks are on shell, \(x_i = \xi_i \geq 0, \hat{x}_i = \xi_2 \geq 0\) in eq. (3) for every \(i = 1, 2, 3, 4\). The antisymmetric color structure of each wave function has been separated from the \(Y\)’s and absorbed into the soft-gluon function \(U\). \(U\) has remaining free color indices that are summed against those of the hard scatterings, \(H^m\), as in Ref. [13].

The hard scatterings depend only on the large components of the quark momenta. For this reason we may integrate the wave functions over the small components \(\kappa_i\) and \(\hat{\kappa}_i\). The transverse momentum integrals in eq. (3), however, are linked by delta functions. The transverse delta functions in the factorized form of the amplitude reflect the physical picture of p-p scattering, as it proceeds in the multiple scattering mechanism. The incoming and the outgoing proton wave functions are highly
Lorentz contracted along their respective light cone directions. Therefore, thought of as discs, they intersect along the direction perpendicular to the scattering plane \[6\]. The hard scatterings may take place anywhere along this line of intersection. Thus, in configuration space the wave functions are unlocalized in the normal to the scattering plane. When their separations are large, soft gluons moving in the normal direction can resolve the color of the quarks. These effects are summarized in \[U\]. The amplitude is conveniently represented by rewriting eq. (3) in “impact parameter” space in the transverse components,

\[
A(s, t, h_i) = (-i) \left(\frac{2\pi}{\sin^2 \theta}\right)^6 \int_0^1 d\xi_1 d\xi_2 \int db_1 db_2 \times \sum_{f=1}^3 \sum_{\alpha_i, \beta_i, \gamma_i} \tilde{Y}^{(f)}_{\alpha_i \beta_i \gamma_i}(\xi_1, \xi_2, b_1, b_2; P_i, h_i) \\
\times \tilde{U}_{\{a_i b_i c_i\}}(b_1, b_2, \mu) \prod_{m=1}^3 H_{\{a_i a_i\}}(\xi_i, P_i, \mu),
\]

where \(\mu\) is the renormalization scale and where \(\tilde{Y}\), defined by

\[
\tilde{Y}^{(f)}_{\alpha_i \beta_i \gamma_i}(\xi_1, \xi_2, b_1, b_2; P_i, h_i) = \int d\kappa_i d\hat{k}_i \int d^2k_i d^2\hat{k}_i e^{-i(b_1 \cdot k_1 + b_2 \cdot k_2)} \\
\times Y^{(f)}_{\alpha_i \beta_i \gamma_i}(k_i, \hat{k}_i; P_i, h_i),
\]

is \(Y\) integrated over the \(\kappa\)’s and Fourier transformed with respect to its transverse momentum components, in terms of the vectors \(b_1 = \tilde{b}_i \eta\), with \(\eta\) the normal to the scattering plane. This is the factorized expression we shall employ below.

Let us now discuss the various functions in eq. (3) in more detail.

### 2.2 Wave functions on and off the light cone

Before assigning quarks to the independent scatterings we review the description of the \((uud)\) Fock state. We omit the flavor flow index \((f)\) in \(Y\) until we reinsert it in \(A\), eq. (3). It is convenient to discuss the wave functions first in momentum space.
As usual, the three-quark wave function is the Fourier transformation of the three-quark operator $[24]$. For a proton moving in the + direction, with energy $E$, we have

$$Y_{\alpha\beta\gamma}(k_{u_1}, k_{u_2}; P, h, \mu) = \frac{(2E^2)^{1/4}}{N_c!} \int \frac{d^4y_1}{(2\pi)^4} e^{ik_{u_1} \cdot y_1} \frac{d^4y_2}{(2\pi)^4} e^{ik_{u_2} \cdot y_2} \times \langle 0\vert T\left[u_\alpha^a(y_1)u_\beta^b(y_2)d_\gamma^c(0)\right]\vert P, h\rangle \epsilon_{abc}.$$  \hspace{1cm} (6)

In the limit $y_\mu^i \rightarrow y_+ y^\mu$, the three operators approach the light cone, and $Y$ is related to a normal light-cone wave function $[24]$. $\mu$ is the factorization scale. As mentioned above, the $Y$’s are defined as color-invariant functions (appropriate ordered exponentials between the quark fields are suppressed).

We have already seen that the fields in eq. (6) are not on the light cone in general. Nevertheless, the ratio of transverse to longitudinal momenta of the quarks will, in all the wave functions discussed below, vanish in the high-energy limit. As a result, the leading twist contribution to the amplitude, eq. (3), may still be given by a spinor basis on the light cone.

Permutation symmetry between the two $u$-quarks and the requirement of total isospin 1/2 imply that $Y_{\alpha\beta\gamma}$ can be expanded directly in terms of products of spinors with definite helicity,

$$\mathcal{M}_{\alpha\beta\gamma}^{(1)} = (E_1 E_2 E_3/2)^{-1/2} u_\alpha(k_{u_1}, +) u_\beta(k_{u_2}, -) d_\gamma(k_d, +),$$

$$\mathcal{M}_{\alpha\beta\gamma}^{(2)} = (E_1 E_2 E_3/2)^{-1/2} u_\alpha(k_{u_1}, -) u_\beta(k_{u_2}, +) d_\gamma(k_d, +),$$

$$\mathcal{M}_{\alpha\beta\gamma}^{(3)} = -(E_1 E_2 E_3/2)^{-1/2} u_\alpha(k_{u_1}, +) u_\beta(k_{u_2}, +) d_\gamma(k_d, -),$$ \hspace{1cm} (7)

as

$$Y_{\alpha\beta\gamma} = \frac{1}{2^{1/4} 8 N_c!} f_N(\mu) \left[ \Psi_{132} \mathcal{M}_{\alpha\beta\gamma}^{(1)} + \Psi_{213} \mathcal{M}_{\alpha\beta\gamma}^{(2)} + (\Psi_{132} + \Psi_{231}) \mathcal{M}_{\alpha\beta\gamma}^{(2)} \right],$$ \hspace{1cm} (8)

where the subscripts on the scalar function $\Psi$ refer to the order of momentum arguments, with $k_3 = k_d$, for example, $\Psi(k_1, k_2, k_3) \equiv \Psi_{132}$. In (7), $E_1$, $E_2$ and $E_3$ are the energies of the first and second $u$-quarks and the $d$-quark respectively. For
a proton with (−) helicity, it is only necessary to flip the spinor helicities in eq. (7). Here and below, spinors are normalized according to

\[ \mathbf{N}(P, h) \gamma^\mu \mathbf{N}(P, h) = 2P^\mu. \]  

A commonly-used alternate basis for \( Y \) is described in the Appendix.

We are now ready to discuss the transverse structure of the three quark state, since this is crucial to the factorized amplitude as given in eq. (4). At high energy, the dominant radiative corrections come from soft gluons, whose couplings are independent of the quarks’ spin states [6]. The effects of these gluons on the wave functions factorize from the light cone quark distribution amplitude (referred to as LCDA below) and exponentiate in the impact parameter (\( b \)) space representation. That is, \( \tilde{Y} \), eq. (5), the Fourier transform of the function \( \Psi(k_{u_1}, k_{u_2}, k_d; \mu) \) may be written as

\[
\tilde{Y}_{\alpha\beta\gamma}(\xi_1, \xi_2, b_1, b_2; P_i, h_i) = \frac{1}{2^{1/4} N_c} f_N(\mu) \left[ \mathcal{P}_{123} \mathcal{M}_{\alpha\beta\gamma}^{(1)} + \mathcal{P}_{213} \mathcal{M}_{\alpha\beta\gamma}^{(2)} + (\mathcal{P}_{132} + \mathcal{P}_{231}) \mathcal{M}_{\alpha\beta\gamma}^{(2)} \right],
\]

where

\[
\mathcal{P}(\xi_1, \xi_2, \xi_3, \tilde{b}_1, \tilde{b}_2; \mu) = \phi(\xi_1, \xi_2, \xi_3; \mu) \tilde{\psi}(\xi_1, \xi_2, \tilde{b}_1, \tilde{b}_2)
\]

\[
= \int d\kappa_1 \int d\kappa_2 \int d^2 k_1 e^{-i\kappa_1 \cdot \eta(b_1 - b_3)} \int d^2 k_2 e^{-i\kappa_2 \cdot \eta(b_2 - b_3)} \times \Psi(k_1, k_2, k_3; \mu),
\]

with \( \tilde{\psi} \) is a dimensionless function of the transverse separations. To be specific, we denote by \( b_m \) the positions of the hard scatterings \( H^m \) along the \( \eta \) direction and by \( \tilde{b}_m \) their mutual transverse separations defined as

\[
\tilde{b}_1 = b_2 - b_3, \quad \tilde{b}_2 = b_1 - b_3, \quad \tilde{b}_3 = \tilde{b}_2 - \tilde{b}_1.
\]

Another connection between the function \( \mathcal{P} \) and the LCDA \( \phi \) is [8]

\[
\mathcal{P}(\xi_1, \xi_2, \xi_3, \tilde{b}_1 = \tilde{b}_2 = 0; \mu) = \phi(\xi_1, \xi_2, \xi_3; \mu) + \mathcal{O}(\alpha_s(\mu)),
\]
that is, neglecting perturbative corrections for large \( \mu \),

\[
\bar{\psi}(\xi_1, \xi_2, 0, 0) = 1. \tag{14}
\]

Alternately, in momentum space, \( \Psi \) is related to the LCDA by

\[
\int d\kappa_1 \int d\kappa_2 \Psi(k_{u1}, k_{u2}, k_d; \mu) = \phi(\xi_1, \xi_2, \xi_3; \mu) \psi(\xi_1, \xi_2, \xi_3, k_{u1}, k_{u2}), \tag{15}
\]

with

\[
\int d^2k_1 d^2k_2 \psi(\xi_1, \xi_2, \xi_3, k_1, k_2; \mu) = 1. \tag{16}
\]

At very high energy, the behavior of \( P \) in the \( \tilde{b}_i \) is a computable exponential \[6\], which vanishes when any of the \( \tilde{b}_i \) equal \( 1/\Lambda \). The perturbative transverse wave function, then, is of finite size. Rather than give its explicit form here, we have simply absorbed it into the resummed soft-gluon factor specified in eq. (23) below. In the following, we shall assume that this has been done, and consider the function \( \psi(\xi_i, \tilde{b}_i) \) as a measure of the remaining “nonperturbative” transverse structure of the proton. As we shall see in Sect. 5 below, this structure has strong influence on near-forward elastic scattering. In a sense, we may think of \( \psi \) as the source of the “intrinsic” transverse momentum of valence quarks in the proton. We should keep in mind, however, that the distinction between perturbative and nonperturbative, or process-dependent and intrinsic, is somewhat arbitrary, and must depend, in particular, on the order to which the perturbation expansion has been carried out.

As a last topic in this subsection, we briefly review the evolution of the light cone distributions. In the standard analysis \[3\], the LCDA is expanded in the basis of the Appel polynomials \( A_j \) as

\[
\phi(x_1, x_2, x_3; \mu) = \phi_{as}(x_1, x_2, x_3) \sum_{j=0} N_j \left( \frac{\ln \mu^2 / \Lambda^2}{\ln \mu_0^2 / \Lambda^2} \right)^{-\beta_1 j} a_j A_j(x_1, x_2, x_3). \tag{17}
\]

\( \phi_{as}(x_1, x_2, x_3) = 120 x_1 x_2 x_3 \) is the asymptotic limit of \( \phi(\mu) \) for \( \mu \to \infty. \mu_0 \approx 1\text{GeV}, \beta_1 = (11/3)N_c - (2/3)n_f \) is the one-loop coefficient of the QCD \( \beta \)-function and
Λ = Λ_{QCD} is the QCD scale parameter. The normalization coefficients $N_j$, defined by

$$N_j \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \phi_{as}(x) A_j^2(x) = 1,$$

and the anomalous dimensions $b_j/\beta_1$ are the same for all models. Model dependence of $\phi$ comes through the coefficients $a_j$. For a list of the constants and functions $N_j, b_j, a_j, A_j(x)$ see table 2 and [10, 25]. The dimensional parameter $f_N(\mu)$ is given by

$$f_N(\mu) = f_N(\mu_0) \left( \frac{\ln \mu^2/\Lambda^2}{\ln \mu_0^2/\Lambda^2} \right)^{-\gamma_{1N}}, \quad f_N(\mu_0) = (5.2 \pm 0.3) \times 10^{-3}\text{GeV}^2. \quad (19)$$

Both $\phi$ and $f_N$ depend mildly on $\mu$ due to QCD evolution.

The quark distribution amplitudes identified above contain much of the soft physics of the problem. However, information about soft gluon exchange is also contained in the perturbative resummation factors. As mentioned above, these will include the perturbative $b$ dependence of the wave functions [3]. To organize this dependence, we must analyze color flow in hard and soft exchange.

### 2.3 Color flow, hard and soft exchange

According to the conventions of fig. 1, $k_i, \hat{k}_i, P_i - k_i - \hat{k}_i$ are the momenta entering ($i = 1, 2$) or leaving ($i = 3, 4$) the blobs $H^1, H^2$ and $H^3$ respectively. The identification of these momenta with $(k_{u_1})_i, (k_{u_2})_i, (k_d)_i$, is obtained once the flavor flow $f$ is fixed. Similarly, the third index $\gamma$ of the proton wave function, eq. (6), is identified with the Dirac index $\beta_i$ or $\gamma_i$ in eq. (3) according to the hard scattering $H^m, m = 2, 3$, in which the $d$-quark of the corresponding proton participates. Each hard-scattering function $H^m$ consists of lines with virtuality $\xi_m^2 t$, with $\xi_m, m = 1, 2, 3$, the longitudinal momentum fraction carried by the quarks in scattering $m$ (they are all the same for a given $m$).

The color flow basis suitable for our treatment is the octet-singlet basis,

$$H_{\{\alpha_i a_i\}}^m = (H_s^m)_{\{\alpha_i\}} (c_s)_{a_i} + (H_s^m)_{\{\alpha_i\}} (c_s)_{a_i}.$$
\[(c_{\text{adj}})_{\{a_i\}} \equiv \left(T_m\right)_{a_3a_1}\left(T_m\right)_{a_1a_2}, \quad (c_s)_{\{a_i\}} \equiv \delta_{a_1a_3}\delta_{a_2a_4}. \quad (20)\]

For near-forward scattering the anomalous dimension matrix associated with \(H^m\) becomes diagonal in this basis \[18,13,19\]. More detail will be given in the following section.

The color tensor \(U\) includes soft radiative corrections that cannot be absorbed into the proton wave functions. As such, its perturbative expansion begins at zeroth order in the coupling,

\[\tilde{U}_{\{a_ib_ic_i\}}(b_1,b_2,\mu) = 4\prod_{i=1}^{4} \epsilon_{a_ib_ic_i} + \mathcal{O}(\alpha_s). \quad (21)\]

At lowest order, \(U\) consists simply of the \(\epsilon\) color tensors associated with the external protons. In ref. \[6\] the one-loop radiative corrections for fixed angle p-p scattering were analyzed, and the corresponding renormalization group equations identified. The IR safety of the p-p process was shown to arise from the cancellation of IR divergences among graphs of the form shown in fig. 2. The underlying physical principle is that there is an effective IR cut-off \(1/R\) for the transverse momentum of the exchanged gluons, with \(R\) the distance between adjacent \(H^m\)'s. Below this cut-off, long wavelength gluons cannot couple to the color singlet hadronic state.

Leading logarithms in \(s/\mu^2\), \(t/\mu^2\) and \(b_i^2/\mu^2\) may be resummed into exponential factors, \(\exp(-s_{I})\), one for each hard scattering function \(H_{I}^m\). As above, \(I = s, \text{adj}\) labels the color flow of \(H^m\) in the basis \(29\). Beyond one loop in the corresponding anomalous dimensions, the exponentials for each hard scattering no longer factorize from each other in general. Once leading logarithms have been organized in this fashion, the renormalization scale \(\mu\) in \(H^m\) is replaced by the hard-scattering scale

\[Q_{m} = \xi_{m}C_{2}\sqrt{-t}, \quad (22)\]

where \(C_2\) is a parameter chosen to optimize higher order corrections. Similarly, in \(\tilde{U}\), \(\mu\) may be replaced by a scale of the order of the \(1/\tilde{b}_{i}\)'s. In the following, we shall neglect higher orders in \(\alpha_s(1/\tilde{b}_{i})\), and retain only the first term in the expansion of \(\tilde{U}\), eq. \(21\).
The exponents that are generated by the one-loop radiative corrections to the hard scattering \( H^m \), with color flow \( I \), are given by \([6]\)

\[
s_I(Q_m, \tilde{b}_m) = c_1 \left[ q_m \ln \left( \frac{q_m}{\zeta_m} \right) - q_m + \zeta_m \right] + c_2 \left[ \frac{q_m}{\zeta_m} - \ln \left( \frac{q_m}{\zeta_m} \right) - 1 \right] + c_3 \left[ 2(\ln(2q_m) + 1) - 2\frac{q_m}{\zeta_m}(\ln(2\zeta_m) + 1) - \ln^2(2q_m) + \ln^2(2\zeta_m) \right] + \frac{2}{\beta_1} B_I \ln \left( \frac{q_m}{\zeta_m} \right),
\]

with

\[
q_m \equiv \ln(Q_m/\Lambda) = \ln(\xi_m C_2 \sqrt{-t}/\Lambda), \quad \zeta_m \equiv -\ln(\tilde{b}_m/\Lambda).
\]

The positive constants \( c_1, c_2, c_3, \beta_1 \) and \( \beta_2 \) are

\[
c_1 = \frac{8C_F}{\beta_1}, \quad c_2 = \frac{8}{\beta_1^2} \left[ N_c C_F \left( \frac{67}{18} - \zeta(2) \right) - \frac{5}{9} C_F n_f + C_F \beta_1 (\gamma_E - \ln 2) \right], \\
c_3 = \frac{C_F \beta_2}{2\beta_1^3}, \quad \beta_1 = \frac{11}{3} N_c - \frac{2}{3} n_f, \\
\beta_2 = \frac{34}{3} N_c^2 - \left( \frac{20}{3} N_c^2 + 4C_F \right) T_F n_f.
\]

For \( N_c = 3 \) and normalization \( T_F = 1/2 \), we have \( C_F = 4/3 \) and we take \( n_f = 3 \). \( \beta_2 \) is the two-loop coefficient of the QCD \( \beta \)-function and \( \gamma_E \) is the Euler constant.

Of particular interest is the last term \( B_I \) in eq. \((23)\), which contains the color flow information of the hard scattering,

\[
B_I = \lambda_I + 2C_F \ln \left( \frac{e^{2\gamma_E} - 1}{4C_F^2} \right) - N_c C_F.
\]

Here, \( \lambda_I \) is the eigenvalue of the anomalous dimension matrix of \( H^m \) along the color flow direction \( I \), and the term \(-N_c C_F \) comes from the anomalous dimensions of the participating quarks, \( 4\gamma_q = -N_c C_F \alpha_s/\pi \) in the axial gauge. The \( \lambda_I \) will be specified in the next section.
To summarize our results so far, the p-p elastic amplitude may be expressed in terms of integrals in $b$-space in the general form

$$A(s,t,h_i) = (-i) \frac{(2\pi)^6}{\sin^2 \theta} \int_0^1 d\xi_1 d\xi_2 \int d\tilde{b}_1 d\tilde{b}_2 \times \sum_{f=1}^{3} \sum_{\alpha_i, \beta_i, \gamma_i} \tilde{Y}_{\alpha_i \beta_i \gamma_i}(\xi_1, \xi_2, b_1, b_2; P_i, h_i) \times U_{\{\alpha_i \beta_i \gamma_i\}}(\tilde{b}_i, \alpha_s(1/\tilde{b}_i)) (c_{I_1})_{\alpha_i} (c_{I_2})_{\beta_i} (c_{I_3})_{\gamma_i} \times \exp \left( -s_{I_1}(Q_1, \tilde{b}_1) - s_{I_2}(Q_1, \tilde{b}_2) - s_{I_3}(Q_1, \tilde{b}_3) \right) \times \prod_{m=1}^{3} (H_{I_m})_{\{\alpha_i\}}(\xi_i, P_i, \alpha_s(Q_m)) \right) . \quad (27)$$

3 The quark-quark hard amplitude in the near-forward region

In the kinematic region of near forward scattering with large momentum transfer,

$$s \gg -t \gg \Lambda^2 , \quad (28)$$

the leading power contribution to the p-p elastic amplitude comes from the “ttt” gluon exchange channel for $f = 1$ and $f = 2$ (see table 1). The information about color flow in each hard scattering $H^m$ enters through the eigenvalue $\lambda_i$ in the $B_i$ term of the exponent, eq. (23). In [13] we have calculated the one-loop octet and singlet eigenvalues. The result is

$$\lambda_1 \equiv \lambda_{adj} = N_c \ln \left( \frac{s}{-t} \right) - \frac{2\pi i}{N_c} + \lambda_s , \quad \lambda_s = 2C_F , \quad (29)$$

giving

$$B_{adj} - B_s = N_c \ln \left( \frac{s}{-t} \right) + O(\ln^0 s) . \quad (30)$$

Consequently, the contribution to the p-p amplitude from octet exchange in a single hard scattering is suppressed relative to singlet exchange by a factor

$$\left( \frac{s}{-t} \right)^{-\frac{2N_c}{M} \ln(q_\alpha/\zeta_m)} , \quad (31)$$
with $q_m, \zeta_m$ defined in eq. (24). We note that the $\lambda_I$ in eq. (29) are gauge dependent, but that the full result in eq. (23) is gauge invariant [6]. Due to eq. (31), we expect that the dominant contribution to $A$ in the near-forward region comes from singlet exchange in all three hard scatterings, which we denote by $A_{3s}$.

For the octet, $H_{adj}^m$ begins at single gluon exchange, but the lowest order contribution to the singlet, $H_s^m$ is at the one-loop level, and the contributing on-shell graphs are IR divergent. The IR finite hard part of $H_s^m$ can be put in the form [13]

$$ H_{s\{\alpha_i\}}^m (c_s)_{\{a_i\}} = \tau^m(\alpha_s(Q_m^2), \xi_m^2 t)(\gamma^\mu)_{\alpha_3a_1}(\gamma^\mu)_{\alpha_4a_2} C_s (c_s)_{\{a_i\}}, $$

$$ \tau^m(\alpha_s(Q_m^2), \xi_m^2 t) = \frac{4\pi\alpha_s^2(Q_m^2)}{\xi_m^2 t} \ln \left( \frac{Q_m^2}{\xi_m^2 Q^2} \right), \quad (32) $$

where

$$ C_s = \frac{N_c^2 - 1}{4N_c^2} $$

is the color coefficient coming from the projection of the lowest order color structures along the $t$-channel color singlet, and where we define

$$ Q^2 = \frac{e^{\gamma_E}}{4\pi} \rho \left( -t \right). \quad (34) $$

$\rho$ parameterizes the ambiguity in the IR subtraction to lowest order. We expect an optimal $\rho$ to be determined by an examination of higher orders.

Inspection of eqs. (22), for $Q_m$, and (23), for $\tau_m$, shows that an “$\overline{MS}$” choice for the parameter $C_2$,

$$ C_2^2 = \frac{e^{\gamma_E}}{4\pi}, \quad (35) $$

gives a hard singlet that is proportional to $\rho$. We take for $C_2$ this $\overline{MS}$ choice for the rest of this paper and denote

$$ \tau^m(\alpha_s(Q_m^2), \xi_m^2 t) = -i \frac{4\pi\alpha_s^2(Q_m^2)}{\xi_m^2 t} \rho. \quad (36) $$

With the choice (35) for $C_2$, the $B_s$ coefficient of the Sudakov exponent, eq. (26), using eq. (29) for $\lambda_s$, becomes

$$ B_s = 2C_F(\gamma_E + \ln \pi) - N_c C_F. \quad (37) $$

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4 Calculation of the p-p amplitude

At this stage, all the necessary ingredients are available for the calculation of $A_{3s}$. First we calculate the lowest order ($\alpha^6$) contribution to the hard-scattering amplitude. The calculation is straightforward using eqs. (7), (8), and (9). To present the result in a compact form we define

$$T_{123} \equiv \frac{1}{2} [P_{132} + P_{231}],$$

(38)

for each external wave function, with $P$ defined in eq. (11). The result in the impact parameter representation, eq. (27), is found from

$$\sum_{f=1}^{2} \sum_{a_i,b_i,c_i} Y_{\alpha_i\beta_i\gamma_i}(\xi_1, \xi_2, b_1, b_2; P_i, h_i) \tilde{U}_{a_i,b_i,c_i}(b_i, \alpha_s(1/b_i)) (c_{i_1})_{a_i} (c_{i_2})_{b_i} (c_{i_3})_{c_i}$$

$$\times \prod_{m=1}^{3} (H_{I_m})_{\{a_i\}} (\xi_i, P_i, \alpha_s(Q_m))$$

$$= -\tau^3 \mathcal{C} \delta_h \delta_{h_2} \delta_{h_4} f_N^4(\mu) \frac{1}{4} \left\{ [P_{123}^2 + P_{213}^2 + 4T_{123}^2]^2 + [P_{123}^2 + P_{213}^2 + 4T_{123}^2]^2 \right\} \left\{ [P_{123}^2 + P_{213}^2 + 4T_{123}^2][P_{132}^2 + P_{312}^2 + 4T_{132}^2] \right\}.$$ (39)

Notice that the first term inside the braces, from $f = 1$, is symmetric under $1 \leftrightarrow 2$, because for $f = 1$, $H^1$ and $H^2$ represent the same flow of flavor.

The color factor $\mathcal{C}$ is determined by the normalization $N_c!$ of the $Y$’s, eq. (8), the eikonal tensor $\tilde{U}$, eq. (21), and the color structure of the $H^m$’s, eqs. (32), (33). It is

$$\mathcal{C} = \frac{C^3_s}{(N_c!)^4} \left( \prod_{i=1}^{4} \epsilon_{a_i,b_i,c_i} \right) (c_s)_{\{a_i\}} (c_s)_{\{b_i\}} (c_s)_{\{c_i\} = (N_c^2 - 1) \frac{1}{4N_c^2}} \left( \frac{1}{(N_c!)^2} \right),$$

(40)

yielding $\mathcal{C} = 2/3^8$ for $N_c = 3$.

Including the soft-gluon factors for singlet exchange and using

$$\sin^2 \theta = -4t/s(1 + O(t/s))$$

we obtain

$$A_{3s}(s,t) = -\frac{1}{(4\pi)^2} s \int_0^1 d\xi_1 d\xi_2 \int_{-\infty}^{\infty} d\tilde{b}_1 d\tilde{b}_2 [T_{3s}]^{(1)} R(\xi_1, \xi_2, \tilde{b}_1, \tilde{b}_2; \mu)$$

$$\times \exp[-s_s(Q_1, |\tilde{b}_1|) - s_s(Q_2, |\tilde{b}_2|) - s_s(Q_3, |\tilde{b}_1 - \tilde{b}_2|)],$$

(41)
with the lowest order contribution \([T_{3s}]^{(1)}\) given by

\[
[T_{3s}]^{(1)} = i(2\pi)^8 T^{2r3} C \delta_{h_1h_3}\delta_{h_2h_4}
\]

\[
= -\frac{2}{3^8}(2\pi)^8 \left\{ \frac{(4\pi)^3}{\xi_1^2 \xi_2^2 (1 - \xi_1 - \xi_2)^2 f^3} \right\} \rho^3 \delta_{h_1h_3}\delta_{h_2h_4}.
\]

The second line in the above equation follows from eq. (36), which defines the hard scatterings, and we denote by \(w_m\) the hard scale at which each hard scattering \(H^m\) is evaluated. The wave function combination \(R\) is

\[
R = f_N^4(\mu) \frac{1}{4} \left\{ \left[ P_{123}^2(\mu) + P_{213}^2(\mu) + 4T_{123}^2(\mu) \right] + P_{123}^2(\mu) + P_{213}^2(\mu) + 4T_{123}^2(\mu) \right\}.
\]

In summary, the three singlet amplitude is given by eqs. (41)-(43), with soft gluon resummation exponents for singlet exchange given in eqs. (23) and (37). These results are the same for proton-antiproton near-forward hard elastic scattering, since the corresponding amplitude is obtainable from \(s \leftrightarrow u\) crossing of p-p scattering, and in the forward direction \(u^2 \approx s^2\).

5 Numerical results

The experimental study \[26, 27\] of proton-proton near-forward elastic scattering reveals two characteristic features in the momentum transfer range of interest here. The first is the independence of the differential cross section \(d\sigma/dt\) from the energy, for an energy range \(\sqrt{s} \gtrsim 25\) GeV up to 62 GeV. The second is the power behavior \(d\sigma/dt \sim t^{-8}\). The lowest order multiple scattering model \[\[\]\] predicts precisely this scaling and power behavior. Landshoff and Donnachie \[\[\]\] were able to fit the experimental data in the range \(3\) GeV\(^2 \lesssim -t \lesssim 15\) GeV\(^2\) by

\[
\frac{d\sigma_{pp}}{dt} = Ct^{-8}, \quad C = 0.09\ \text{mb(GeV)}^6.
\]

First, we examine the factorized amplitude eq. (27) in this region without transverse structure for the wave functions beyond the soft gluon exponentials \(\exp[-s_s]\), that
is, with $\psi(\xi_i, \tilde{b}_i) = 1$ in eq. (11). Then we shall include a model for nonperturbative transverse structure and study its influence.

5.1 Perturbative transverse structure

Upon retaining only the LCDA’s in the proton wave-function (see eq. (13)), the function $R$ eq. (43) becomes

$$R = f_N^4(\mu) \frac{1}{4} \left\{ \left[ \phi_{123}^2(\mu) + \phi_{213}^2(\mu) + 4T_{123}^2(\mu) \right]^2 + \left[ \phi_{123}^2(\mu) + \phi_{213}^2(\mu) + 4T_{123}^2(\mu) \right] \left[ \phi_{132}^2(\mu) + \phi_{312}^2(\mu) + 4T_{132}^2(\mu) \right] \right\},$$

(45)

with $T_{123} \equiv (1/2)(\phi_{321} + \phi_{132})$.

Energy-independence of the cross section follows from eqs. (41), (42) and the relation

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |\tilde{A}|^2,$$

(46)

in which the $1/s^2$ cancels with the $s^2$ in $|\tilde{A}|^2$ from eq. (11). Also, the singlet eigenvalue $\lambda_s$, appearing in the exponents $s_s$, eqs. (26), (29), is energy independent, unlike the octet, $\lambda_{adj}$. Exact $s$-independence is not anticipated to persist at very high energies, where the energy dependence of the singlet exchange hard scatterings should become important [12].

The $t^{-8}$-dependence is characteristic of the multiple scattering model if we assume that the integral over the suppression factors $\exp(-s_s)$ yields negligible $t$-dependence. Indeed, then $A_{3s} \sim s/t^4$ and, from eq. (46), $d\sigma_{pp}/dt \sim t^{-8}$. This power dependence cannot persist at high enough $t$, where the strong Sudakov suppression in the exponents leads to the onset of dimensional counting behavior of the form $A_{3s} \sim s/t^5$, and hence to $d\sigma_{pp}/dt \sim t^{-10}$.

In the numerical study that follows we plot the quantity $N^{-1}|A_{3s}|t^4/s$ as a function of $-t$. The dimensionless normalization coefficient $N$ is

$$N = (2\pi)^8(4\pi)^7 \frac{1}{\beta_1^6} C = 6.895 \times 10^4.$$

(47)
It has been defined without the factor $\rho^3$, and we use the one-loop running coupling $\alpha_s(w^2_m) = 4\pi/(\beta_1 \ln(w^2_m/\Lambda^2))$ in $[T_{3s}]^{(1)}$, eq. (42). The numerical results for the amplitude $A_{3s}$ are in units of $|\rho^3|$. The straight line labeled ‘DL’ in the figures corresponds to the fit of eq. (44). In order to compute $A_{3s}$ from eq. (41) we need to fix the $\tilde{b}_1, \tilde{b}_2$ integration limits and the scales $\mu, w_1, w_2, w_3$. This we do as follows.

i) Lower limits for $\tilde{b}_1, \tilde{b}_2$. The exponents $s_s$ have a root at $\overline{Q}_m = 1/|\tilde{b}_m|$ (see eq. (23)). We refer to the region $0 \leq |\tilde{b}_m| \leq 1/\overline{Q}_m$ as the small-$b$ region. Formally in this region the factor $\exp[-s_s]$ in eq. (41) enhances instead of suppressing. We take its value in the small-$b$ region to be 1, i.e.,

$$s_s(\overline{Q}_m, |\tilde{b}_m|) \rightarrow s_s(\overline{Q}_m, |\tilde{b}_m|) \theta(\overline{Q}_m - 1/|\tilde{b}_m|)$$

and integrate down to $|\tilde{b}_m| \rightarrow 0$ [8].

ii) Upper limits for $\tilde{b}_1, \tilde{b}_2$. $|\tilde{b}_m|$ are strictly limited to be less than $1/\Lambda$, as past this value the Sudakov exponents diverge. We impose upper cut-off $\tilde{b}_c$ in the $\tilde{b}_1, \tilde{b}_2$ integrals and examine the behavior of $A_{3s}$ as $\tilde{b}_c \rightarrow 1/\Lambda$.

iii) For the scales $\mu, w_m$ we make the following choices. The factorization scale $\mu$, separating wave functions from the remainder of the amplitude, (eqs. (17), (19)), is taken equal to the minimum average transverse momentum of the valence quarks, corresponding to the maximum transverse separation between the hard scatterings,

$$\mu = \min \left\{ \frac{1}{|\tilde{b}_1|}, \frac{1}{|\tilde{b}_2|}, \frac{1}{|\tilde{b}_1 - \tilde{b}_2|} \right\}.$$  

(49)

In the soft quark region, $\xi_m \rightarrow 0$, the hard scales $w_m$ shift from $\overline{Q}_m$ to $\mu$ as

$$w_m = \max\{\overline{Q}_m, \mu\}.$$  

(50)

In addition to the conditions (18), (51), we impose a lower bound $\mu_{\min}$ for all the scales

$$\mu, w_1, w_2, w_3 \geq \mu_{\min},$$  

(51)

and examine the sensitivity of the amplitude as $\mu_{\min} \rightarrow \Lambda$.  

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The result depends, in principle, on the values of $\mu_{\text{min}}$ and $\Lambda$, the model for $\phi(x_i; \mu)$ and the value of the undetermined parameter $\rho$ in eq. (12). In this numerical study we take $|\rho| = 1$, but it should be noted that the results for the amplitude scale with $|\rho|^3$. We start with $\mu_{\text{min}} = 1 \text{ GeV}$ and the LCDA model of Ref. [14] and look at the rate of convergence of $A_{3s}$ as $b_c \to 1/\Lambda$, as well as its sensitivity to the choice of the scale parameter $\Lambda$. The results are shown in figs. 3, 4. Note that for $\Lambda = 0.1 \text{ GeV}$, $\mu_{\text{min}} = 10\Lambda$ corresponds to $b = 1/\mu_{\text{min}} = 0.1/\Lambda$ and for $\Lambda = 0.2 \text{ GeV}$, $\mu_{\text{min}} = 5\Lambda$ corresponds to $b = 1/\mu_{\text{min}} = 0.2/\Lambda$. As the figures show, up to these values for $b_c$ the Sudakov suppression has hardly started affecting the amplitude. Therefore, in the region of the impact parameter space where Sudakov effects become important the coupling constant is effectively fixed at $\alpha_s(\mu^2_{\text{min}})$ due to condition (51).

We tentatively divide the $t$-range into three regions corresponding to small (a), intermediate (b), and large (c), absolute slopes. We observe approximate linear dependence of $\ln(A_{3s} | t^4/s)$ on $\ln (t)$ in each of the above regions. Table 3 lists the slopes of the integral for selected values of $b_c$. The slopes on the logarithmic plot correspond to $\langle t \rangle$ deviation of the amplitude from the $t^{-4}$ behavior. The onset of dimensional counting in region (c) is apparent, where strong Sudakov suppression stem the growth of the amplitude. Note also the relatively small deviations from the $t^{-4}$ behavior in region (a), less pronounced in region (b), where the transition to dimensional counting occurs.

The dependence of the amplitude on $\Lambda$ is also of interest. We observe that, for this choice of $\mu_{\text{min}}$ and LCDA model, $A_{3s}$ decreases with increasing $\Lambda$ for $b_c \lesssim 0.3/\Lambda$ while it increases past this value of $b_c$. This is due to the competition between two counteracting effects. The first is the decrease of the impact parameter space volume with increasing $\Lambda$ due to the Jacobian factor $1/\Lambda^2$ in

$$\int_0^{b_c} \, \mathrm{d}b_1 \, \mathrm{d}b_2 = \frac{1}{\Lambda^2} \int_0^{b_c \Lambda} \, \mathrm{d}b'_1 \, \mathrm{d}b'_2,$$

noting that the integrand is a function of $b_c \Lambda$ only. The second is the increase of the coupling constant with $\Lambda$. From $\Lambda = 0.1 \text{ GeV}$ to $0.2 \text{ GeV}$, $\alpha_s(\mu^2_{\text{min}})$ increases by a factor $\sim 1.4$. This gives a substantial effect because the lowest order contribution

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to the amplitude is $O(\alpha_s^6)$. In the case of the proton form factor \[9\], where the lowest order contribution is $O(\alpha_s^2)$, the shrinking of the impact parameter space is always stronger than the increase of $\alpha_s$ with increasing $\Lambda$, even when $\mu_{\text{min}} \to \Lambda$. Proton-proton near-forward scattering is much more sensitive to changes in $\alpha_s$, even though the Sudakov suppression here is stronger relative to the form factor, due to the presence of the positive eigenvalue $\lambda_s = 2C_F$ (eq. (29)) in the exponent $s_s$.

Decreasing the lower bound $\mu_{\text{min}}$ down to 0.7 GeV, we recompute the amplitude for the same combination of parameters as before. The results are shown in figs. 5, 6. Again here, for $b_c \gtrsim 0.3/\Lambda$ the coupling is essentially fixed at $\alpha_s(\mu_{\text{min}}^2)$. The main feature is the marked increase of the amplitude (notice the scale shift on the $A_{3s}$ axis). For $\Lambda = 0.1$ GeV, $A_{3s}$ increases by a factor $\sim 3.0$ in region (a) and by a factor $\sim 2.3$ in region (c). For $\Lambda = 0.2$ GeV the increase is by a factor $\sim 5.5$ in (a) and $\sim 3.4$ in (c). This shows that the calculation is quite sensitive to the lower bound $\mu_{\text{min}}$. Although Sudakov suppression always makes the integral finite, removing the condition (51) (equivalent to letting $\mu_{\text{min}} \to \Lambda$) leads to an increase by orders of magnitude.

For fixed $t$, the relative logarithmic distance between the $b_c$ curves remains almost constant with varying $\mu_{\text{min}}$ at fixed $\Lambda$. This is a safe diagnostic that the dependence of the amplitude on $\mu_{\text{min}}$ is due to the coupling constant, since for the largest part of the impact parameter space $\alpha_s = \alpha_s(\mu_{\text{min}}^2) \propto \ln^{-1}(\mu_{\text{min}}/\Lambda)$. The drift of $A_{3s}$ to higher values with decreasing $\mu_{\text{min}}$ does not affect the slope of the $b_c$ curves in region (c). Dimensional counting is bound to set in for high enough $t$. But the transition region from approximate $t^{-4}$ to dimensional counting behavior shifts towards smaller $t$ with decreasing $\mu_{\text{min}}$. This is not due to any change in Sudakov effects, but to the fact that the smaller $t$ region receives larger corrections from $\alpha_s(\mu_{\text{min}}^2)$.

The general characteristics of the $A_{3s}$ behavior described above are the same for all LCDA models, and the following conclusion can be drawn. Sudakov suppression is not strong enough by itself to stabilize the pQCD calculation of the p-p near-forward amplitude against contributions from soft regions. This is not to be interpreted as a failure, but rather as an indication of the importance of the trans-

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verse structure of the three-quark Fock state modeling the proton. It is apparent
that in the $t$-region where $t^{-4}$ behavior is observed, the Sudakov suppression sets
in at unphysical scales, larger than the transverse size of the proton. $t^{-4}$ behavior
for the amplitude is due to “geometrical” scattering, to which the entire size of the
proton can contribute, and the transition to dimensional counting occurs only once
the Sudakov suppression is strong enough to confine the independent scatterings to
regions smaller than the average transverse size of the three quark state. In this
way we can assign physical meaning to the scale $\mu_{\text{min}}$, as being of the order of the
average transverse momentum of the valence quarks inside the proton. Following
this reasoning, we proceed by including in the calculation the transverse structure
of the proton.

5.2 Intrinsic transverse structure

Returning to the factorized form of the hadronic wave function, eq. (15),
\[
\Psi(k_u, k_d; \mu) = \phi(x_1, x_2, x_3; \mu) \psi(x, k),
\]
we consider a transverse wave function of the form [3, 21, 22, 23, 10]
\[
\psi(x, k) = N \exp \left[-R_{3q}^2 \sum_{i=1}^{3} \frac{k_i^2}{x_i} \right],
\]
where $N = R_{3q}^4/(\pi^2 x_1 x_2 x_3)$ and $R_{3q}$ is a mean impact parameter. The impact
parameter representation of $\psi(x_i, k_i)$ is defined in eq. (11). The result is
\[
\psi(x_i, \tilde{b}_i) = \exp \left[-\frac{1}{4R_{3q}^2} (x_1 x_2 \tilde{b}_3^2 + x_1 x_3 \tilde{b}_2^2 + x_2 x_3 \tilde{b}_1^2) \right].
\]
The value of $N$ is fixed by the requirement $\psi(\tilde{b}_i = 0) = 1$ (eq. (14)). In this paper
we do not advocate any particular choice of $\psi(x_i, k_i)$, but use (54) because it has
received much previous attention.
The assignment of quarks to hard scatterings according to the flavor flow $f$ does not affect $\psi(x_i, \tilde{b}_i)$, because it is totally symmetric in its indices. The expression of the amplitude, eq. (41), can be readily obtained in this case from $\mathcal{R}$, eq. (43), which now reads

$$\mathcal{R} = f_N^4(\mu) \frac{1}{4} \psi^4(x_i, \tilde{b}_i) \left\{ \left[ \phi_{123}^2(\mu) + \phi_{213}^2(\mu) + 4T_{123}^2(\mu) \right]^2 \\
+ \left[ \phi_{123}^2(\mu) + \phi_{213}^2(\mu) + 4T_{123}^2(\mu) \right] \left[ \phi_{132}^2(\mu) + \phi_{312}^2(\mu) + 4T_{132}^2(\mu) \right] \right\}. \quad (56)$$

The parameter $R_{3q}$ is a measure of the transverse size of the three-quark Fock state. Brodsky, Huang, Lepage and Mackenzie [21] have presented arguments on how to estimate it. Here, we keep it as a free phenomenological parameter and examine the behavior of the p-p amplitude within a range of values of $R_{3q} \approx 1/k_{\text{rms}}$. A crucial point is that $R_{3q} < 1/\Lambda$. Since $\psi(x_i, \tilde{b}_i)$ contains the transverse size information we set the maximum of the $b$ integrals to $b_c = 1/\Lambda$.

The only, yet conceptually important, modification in the choice of scales $\mu, w_m$ is in condition (51), where $\mu_{\text{min}}$ is replaced by $1/R_{3q}$,

$$\mu, w_1, w_2, w_3 \geq \frac{1}{R_{3q}}, \quad (57)$$

since $1/R_{3q}$ is a typical transverse momentum.

The numerical results for $A_{3s}$ are shown in figs. 7-10 for the various LCDA models and for fixed $\Lambda = 0.20$ GeV. The numbers on the $R_{3q}$ curves are the slopes in their respective linear regions.

Before drawing conclusions from the numerical results, we list some of the approximations made in the course of this calculation. (i) we have integrated only along the direction perpendicular to the scattering plane, where the two Lorentz contracted disks intersect. This picture becomes less clear-cut as $\theta \to 0$ and contributions from the integration over the azimuthal angle in the impact parameter space will have to be considered; (ii) we have assumed that the gaussian, eq. (54), correctly describes the transverse momentum distribution; (iii) we have neglected all hadronic mass scales (proton and constituent quark masses) that would normally
show up in the transverse wave function $\psi$; (iv) we have neglected the transverse momenta in the hard q-q scatterings $H_m$ by assuming that their contribution in the soft quark region can be captured by taking $\alpha_s(w_m^2) = \alpha_s(1/R_{3q}^2)$. We suggest, however, that these and other shortcomings, such as the neglect of higher-order corrections in $U$ and the $H^m$, do not affect the main features observed in figs. 7-10 and shared by all LCDA models.

In figs. 7-10, we see: (i) approximate, but not exact, $t^{-4}$, behavior for the amplitude for moderately large $t$, where we find $A_{3s} \sim s/t^{4.2}$. We note the fit $d\sigma^{pp}/dt \sim t^{-8.4}$ of the FNAL data, suggested by the authors of ref. [27]. (ii) A transition from Landshoff to dimensional counting behavior, whose position is a sensitive function of the transverse size $R_{3q}$. Since the experiment is decisive about the existence of Landshoff behavior at least up to $-t = 15$ GeV$^2$, we can infer that our calculations imply a small transverse size for the valence state, $R_{3q} \lesssim 0.3$ fm [21]. A much larger value of $R_{3q}$ would exhibit $t^{-10}$ behavior in the cross section at lower $t$.

Our results so far are encouraging, but we recognize that the energies and momentum transfers involved are not so high that the perturbative picture is applicable without reservation. In particular, we would like to get an idea of the importance of soft quarks, and consequently low momentum transfers to our results. When one of the scales $\xi_i$ becomes small, the corresponding momentum transfer decreases as $\xi_i^2$, and at some point the transverse size of the “hard” scattering may be as large as the separation between hard scatterings.

Thus, as an additional test of the independent scattering method, we have recalculated the amplitude with a cutoff $\xi_i > \lambda$ on soft interactions. Fig. 11 shows the effect of demanding that all three of the $\xi_i$ be larger than $\lambda$. The $\lambda = 0.0$ curve is the reference, and corresponds to the calculations in the preceding figures. The contributions of soft quarks are substantial, but not overwhelming. Demanding that the softest quark have 1/20th of each proton’s momentum leaves more than half the amplitude, while demanding 1/10th reduces the amplitude by a factor of about three. We should keep in mind, however, that at collider energies, even a quark with
\( \xi = 0.05 \) is quite energetic. Of course, at such scales, the quark-quark momentum transfer is no longer very large, and a treatment that includes the transverse momenta of the wave functions as part of the hard scattering \[8\] is probably necessary. Fig. 12 shows the same calculation, but now with the restriction imposed only on the two more energetic quarks. Here we note that most of the time the middle quark has at least one-tenth of the energy, and it has more than 1/5th about one-third of the time. This shows, we believe, that the independent scattering picture, in which there are more than one, and sometimes three, independent and well-localized scatterings, describes the basic picture of proton-proton elastic scattering near the forward direction.

6 Conclusions

We have described proton-proton near-forward elastic scattering with an improved pQCD factorization formalism, which describes both short-distance \( t^{-10} \) and Landshoff \( t^{-8} \) power behavior for moderate momentum transfers with \( s \gg -t \). The presence of \( t^{-8} \) behavior in the cross section implies the relevance of a new scale, of the order of the average transverse momentum of the constituent quarks. The inclusion of a nonperturbative, or intrinsic, transverse size somewhat smaller than \( 1/\Lambda \) seems to be required for the description valence state. Indeed, were the proton three-quark much larger than \( 1/\Lambda \), Sudakov effects would produce \( t^{-10} \) behavior in the kinematic region where \( t^{-8} \) has been observed. All of these results require color singlet exchange for the independent quark-quark hard scatterings.

We note that it is most natural for us to compare the calculated amplitude to experiment, rather than the cross section directly, since in elastic scattering, it is the amplitude and not the cross section that is the object of computation. Of course, the range of uncertainty in the cross section is much larger than in the amplitude. Nevertheless, it is quite striking that for \( |\rho| = 1 \) in eq. \[34\] the normalization of the amplitude works so well, considering the high powers involved; see eqs. \[19\], \[17\].

The transition from Landshoff to short-distance power behavior is strongly cor-
related with the transverse size of the three-quark state modeling the proton. We believe that further understanding and technical improvements on the theoretical level combined with measurements of the position and the width of the transition region could enable us to determine the average transverse size and estimate the form of the transverse structure from the experimental data. Evidently, near-forward elastic scattering involves spatially-separated hard scatterings of colored objects. It is clearly of interest to investigate a role for such phenomena in diffractive jet and other semi-inclusive cross sections.

Despite the numerical significance of soft-quark contributions, the model described above displays a satisfying self-consistency. In fact, we know of no other model for p - p elastic scattering that describes the $t^{-8}$ behavior of the elastic scattering cross section. We also think it more than ever of interest to determine the normalization and $s$ dependence of the singlet scattering amplitude.

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A Matrix wave functions

As an alternative to eq. (7), \(Y_{\alpha\beta\gamma}\) may be expanded in terms of spin matrix tensors \(E^{(d)}\), \(d = 1, 2, 3\) \[28\],

\[
Y_{\alpha\beta\gamma} = \sum_{d=1}^{3} E^{(d)}_{\alpha\beta\gamma}(P, h) Y^{(d)}(k_{u_1}, k_{u_2}, k_d; \mu),
\]

(58)

for incoming protons \((i = 1, 2)\) and in terms of a conjugate basis

\[
\bar{Y}_{\alpha\beta\gamma} = \sum_{d=1}^{3} \bar{E}^{(d)}_{\alpha\beta\gamma}(P, h) Y^{(d)}(k_{u_1}, k_{u_2}, k_d; \mu),
\]

(59)

for outgoing protons \((i = 3, 4)\). The \(E\)'s and \(\bar{E}\)'s are given by

\[
E^{(1)}_{\alpha\beta\gamma} = E^{-1/2}(\gamma_5 C)_{\alpha\beta}(\gamma_5 N)_{\gamma}, \quad \bar{E}^{(1)}_{\alpha\beta\gamma} = E^{-1/2}(C^{-1}\gamma_5)_{\alpha\beta}(\gamma_5 N)^{\gamma},
\]

\[
E^{(2)}_{\alpha\beta\gamma} = E^{-1/2}(\gamma_5 C)_{\alpha\beta} N_{\gamma}, \quad \bar{E}^{(2)}_{\alpha\beta\gamma} = -E^{-1/2}(C^{-1}\gamma_5)_{\alpha\beta} \gamma_{\gamma},
\]

\[
E^{(3)}_{\alpha\beta\gamma} = iE^{-1/2}(\sigma_{\mu\nu} v^\nu C)_{\alpha\beta} (\gamma^{\mu}
\gamma_{5} N)^{\gamma}, \quad \bar{E}^{(3)}_{\alpha\beta\gamma} = -iE^{-1/2}(C^{-1}\sigma_{\mu\nu} v^\nu)_{\alpha\beta} (\gamma_{5}\gamma^{\mu})_{\gamma}.
\]

(60)

We have defined these spin structures to be dimensionless. Here, \(C\) is the charge conjugation matrix, while

\[
\sigma_{\mu\nu} = \frac{i}{2} \left[ \gamma_{\mu}, \gamma_{\nu} \right], \quad \gamma_5 = \frac{i}{4!} (\gamma_{\mu}\gamma_{\nu}\gamma_{\lambda}\gamma_{5} C),
\]

(61)

with \(\epsilon_{0123} = 1\) and \(N(P, h)\) the massless proton spinor, normalized, as above, to

\[
N(P, h)\gamma^{\mu} N(P, h) = 2P^{\mu}.
\]

(62)

The real functions \(Y^{(d)}\) in this basis are conveniently normalized in momentum space as

\[
Y^{(d)}(k_{u_1}, k_{u_2}, k_d; \mu) = \frac{1}{2^{1/4} N_c^{1/4} \gamma} \tilde{f}_N(\mu) \bar{\Psi}^{(d)}(k_{u_1}, k_{u_2}, k_d; \mu),
\]

(63)

where, in a variation of the notation of ref. \[14\] for instance,

\[
\Psi^{(1)} \equiv V', \quad \Psi^{(2)} \equiv A', \quad \Psi^{(3)} \equiv T',
\]

(64)
\( f_N(\mu) \) is a standard normalization constant, specified below. The primes indicate that these wave functions still depend on all four components \( k_i^\mu \) and have non-zero mass dimension. The various symmetries of the wave function require that \( V', A' \) and \( T' \) may be themselves obtained from a single function \( \Psi(k_{u_1}, k_{u_2}, k_d; \mu) \), as

\[
V'(k_{u_1}, k_{u_2}, k_d; \mu) = \frac{1}{2} [\Psi(k_{u_1}, k_{u_2}, k_d; \mu) + \Psi(k_{u_2}, k_{u_1}, k_d; \mu)] ,
\]

\[
A'(k_{u_1}, k_{u_2}, k_d; \mu) = \frac{1}{2} [\Psi(k_{u_1}, k_{u_2}, k_d; \mu) - \Psi(k_{u_1}, k_{u_2}, k_d; \mu)] ,
\]

\[
T'(k_{u_1}, k_{u_2}, k_d; \mu) = \frac{1}{2} [\Psi(k_{u_1}, k_d, k_{u_2}; \mu) + \Psi(k_{u_2}, k_d, k_{u_1}; \mu)] .
\] (65)

Eq. (11) enables us to relate \( V', A' \) and \( T' \) to dimensionless functions of the light-cone variables \( \xi_i \) only (the functions \( V, A \) and \( T \) of Ref. [14]).

Linear combinations of the spin structures \( E^{(d)} \) parameterize the on-shell states with definite quark helicity defined by eq. (7). The correspondence for our normalization choice, and for a proton with (+) helicity, is

\[
\mathcal{M}_{\alpha\beta\gamma}^{(1)} = E_{\alpha\beta\gamma}^{(1)} - E_{\alpha\beta\gamma}^{(2)} = (E_1 E_2 E_3/2)^{-1/2} u_\alpha(k_{u_1}, +) u_\beta(k_{u_2}, -) d_\gamma(k_d, +) ,
\]

\[
\mathcal{M}_{\alpha\beta\gamma}^{(2)} = E_{\alpha\beta\gamma}^{(1)} + E_{\alpha\beta\gamma}^{(2)} = (E_1 E_2 E_3/2)^{-1/2} u_\alpha(k_{u_1}, -) u_\beta(k_{u_2}, +) d_\gamma(k_d, +) ,
\]

\[
\mathcal{M}_{\alpha\beta\gamma}^{(3)} = E_{\alpha\beta\gamma}^{(3)} = -(E_1 E_2 E_3/2)^{-1/2} u_\alpha(k_{u_1}, +) u_\beta(k_{u_2}, +) d_\gamma(k_d, -) ,
\] (66)

where, as in Sec. 2, \( E_1, E_2, E_3 \) are the energies of the first and second \( u \)-quarks and the \( d \)-quark respectively.

With this form of the wave function, the calculation of the p-p amplitude involves Dirac traces over the spin structures \( E^{(d)} \), which are rather lengthy but straightforward. The following relations are helpful \( (\eta = (+, -, -, -)) \):

\[
(\overline{N}_3 \gamma^\mu N_1)(\overline{N}_4 \gamma_\mu N_2) = -2s \delta_{h_1 h_3} \delta_{h_2 h_4}
\] (67)

\[
\frac{i}{16} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma_5 = \gamma_\nu \gamma_\rho \gamma_\sigma - \eta_{\nu\rho} \gamma_\sigma + \eta_{\nu\sigma} \gamma_\rho - \eta_{\rho\sigma} \gamma_\nu ,
\]

\[
\frac{1}{16} \mathrm{tr}\{[\gamma_\mu, \gamma_\nu] \gamma^\alpha [\gamma_\rho, \gamma_\sigma] \gamma^\beta\} = \eta_{\mu\rho} \delta_\alpha^\delta_\beta + \eta_{\mu\sigma} \delta_\alpha^\delta_\rho - \eta_{\nu\rho} \eta_{\sigma\rho} \eta_{\alpha\beta} .
\] (68)

The result, of course, is the same as in eqs. (11), (12) and (13) above.
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Table 1.
The gluonic exchange channels in the hard scatterings for each flavor flow.

| $f$ | Channels in $H^1H^2H^3$ | No. of channels |
|-----|--------------------------|-----------------|
| 1   | $ttt, ttu, tut, tuu, + t \leftrightarrow u$ | 8               |
| 2   | $ttt, utt$                | 2               |
| 3   | $tuu, uuu$                | 2               |

Table 2.
The decomposition coefficients in terms of Appel polynomials, and the anomalous dimensions of $\phi(x_i; \mu)$, for the models of Chernyak and Zhitnitsky (CZ) \cite{14}, Chernyak, Ogloblin and Zhitnitsky (COZ) \cite{15}, King and Sachrajda (KS) \cite{16} and Gary and Stefanis (GS) \cite{17}.

| $j$ | $b_j$ | $a_j$ (CZ) | $a_j$ (COZ) | $a_j$ (KS) | $a_j$ (GS) | $N_j$ | $A_j(x)$            |
|-----|-------|------------|-------------|------------|------------|-------|---------------------|
| 0   | 0     | 1.00       | 1.00        | 1.00       | 1.00       | 1     | 1                   |
| 1   | 20/9  | 0.410      | 0.350       | 0.310      | 0.391      | 21/2  | $x_1 - x_3$         |
| 2   | 24/9  | -0.550     | -0.424      | -0.370     | -0.588     | 7/2   | $3x_2 - 1$          |
| 3   | 32/9  | 0.357      | 0.460       | 0.630      | -0.749     | 63/10 | $3(x_1 - x_3)^2 + x_2(5x_2 - 3)$ |
| 4   | 40/9  | -0.0122    | -0.00259    | 0.00333    | 0.0176     | 567/2 | $(1/3)(x_1 - x_3)(4x_2 - 1)$ |
| 5   | 42/9  | 0.00106    | 0.0633      | 0.0632     | 0.574      | 81/5  | $7x_2 - 5 + (14/3)(x_1^2 + x_2^2 + 3x_1x_3)$ |
Table 3.
The slopes of the $b_c$-lines in figs. 3, 4 considered approximately straight in the $t$-regions (a), (b), (c).

| $b_c\Lambda$ | slopes for $\Lambda = 0.1$ GeV | slopes for $\Lambda = 0.2$ GeV |
|--------------|-------------------------------|-------------------------------|
|              | (a)  | (b)  | (c)  | (a)  | (b)  | (c)  |
| 1.0          | -0.38 | -0.61 | -0.84 | -0.23 | -0.52 | -0.89 |
| 0.8          | -0.28 | -0.52 | -0.81 | -0.17 | -0.42 | -0.79 |
| 0.6          | -0.19 | -0.39 | -0.70 | -0.10 | -0.33 | -0.69 |
| 0.4          | -0.09 | -0.27 | -0.56 | -0.02 | -0.23 | -0.58 |
Figure Captions

1. Proton-proton elastic scattering in the multiple scattering scenario, and flavor routings for each flavor flow \( f \). The dashed lines represent the \( d \)-quarks. All momenta flow from left to right.

2. Graphs whose IR divergences mutually cancel.

3. The three-singlet amplitude \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the CZ model [14], without transverse structure and for \( \Lambda = 0.1 \) GeV, \( \mu_{\text{min}} = 1.0 \) GeV.

4. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the CZ model [14], without transverse structure and for \( \Lambda = 0.2 \) GeV, \( \mu_{\text{min}} = 1.0 \) GeV.

5. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the CZ model [14], without transverse structure and for \( \Lambda = 0.1 \) GeV, \( \mu_{\text{min}} = 0.7 \) GeV.

6. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the CZ model [14], without transverse structure and for \( \Lambda = 0.2 \) GeV, \( \mu_{\text{min}} = 0.7 \) GeV.

7. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the CZ model [14], with transverse structure and for \( \Lambda = 0.2 \) GeV. The ISR data are from ref. [26] at \( \sqrt{s} = 52.8 \) GeV, and the FNAL data from ref. [27] at \( \sqrt{s} = 27.4 \) GeV. The error bars include both statistical and normalization errors.

8. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the COZ model [13], with transverse structure and for \( \Lambda = 0.2 \) GeV.

9. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the KS model [16], with transverse structure and for \( \Lambda = 0.2 \) GeV.

10. \(|A_{3s}|\), modulo \( \rho^3 N s / t^4 \), as a function of \(-t\) for the GS model [17], with transverse structure and for \( \Lambda = 0.2 \) GeV.
11. $|A_{3s}|$, modulo $\rho^3 N s/t^4$, with lower cutoff $\lambda$ on the scale $\xi_{\text{med}}$ of each of the
hard scatterings.

12. $|A_{3s}|$, modulo $\rho^3 N s/t^4$, with lower cutoff $\lambda$ on the scale $\xi_{\text{med}}$ of the intermediate
hard scattering.
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