Mechanical analysis of ribbed floor using continuum model

Jiejiang Zhu*, Bolun Zhou

Department of Civil Engineering, Shanghai University, Shanghai, 200444, China
*Corresponding author’s e-mail: zhujjt@126.com

Abstract. An approximate method based on a continuum model is proposed for analysis of floor system. A continuum model equivalent to the original ribbed floor is derived and the theoretical analytical solution of the deflection and force of the floor system is calculated. The calculation of rectangular slab whose size is 8m and 6m is carried out in the paper. Finite element analysis program SAP2000 is used for structural calculation to verify the correctness of the sample. The continuum method can result in considerable simplicity in computation analysis and provide a basis for subsequent optimization design.

1. Introduction

With the rapid development of computer technology, matrix displacement approach has been widely used. By far, the most common method for structure design is discrete finite element method, which is based on matrix displacement method[1-3]. However, with the building structure becoming more complex, finite element method used to calculation greatly increases the total computing time. Thus, some specific simplified methods are definitely essential for preliminary design stage[4,5].

Beam and slab always act on each other and have the same distortion in the floor system. However, such collaborative working condition is usually neglected in traditional design method. Some assumptions like rigid and elastic floor slab assumption are taken to simulate the effect of slab to the stiffness of the whole structure. When the section of beam is small, the vertical deformation of the beam is relatively large, which has significant effect on the internal force distribution of beam and slab. Moreover, the stiffness in two directions is different for floor system because of the existence of secondary beams. In the paper, the floor system is equivalent to the orthotropic slab by analyzing the stiffness of beam and slab, so that the internal force and the deformation of the structure are obtained.

2. Analytical solution of orthotropic slab

2.1. Governing equations and boundary conditions

A rectangular slab of length \(l_x\) and \(l_y\) is clamped on four edges under any transverse loading condition \(q(x,y)\). The calculation diagram is shown in figure 1. The governing equation is given by[6]
where $w$ is deflection function, $D_x$ and $D_y$ are flexural rigidities of X and Y direction of the slab, $D_t$ is conversion stiffness, $q(x,y)$ is the transverse load.

$$\frac{D_t}{D_x} \frac{\partial^4 w}{\partial x^4} + 2D_t \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_t}{D_y} \frac{\partial^4 w}{\partial y^4} = q(x,y) \quad (1)$$

where $w$ is deflection function, $D_x$ and $D_y$ are flexural rigidities of X and Y direction of the slab, $D_t$ is conversion stiffness, $q(x,y)$ is the transverse load.

$$D_x = \frac{E_x h^3}{12(1-\nu_{xx}\nu_{yy})} \quad (2)$$

$$D_y = \frac{E_y h^3}{12(1-\nu_{xy}\nu_{yy})} \quad (3)$$

$$D_t = D_x \nu_{yy} + 2D_y \nu_{xx} + 2D_{xy} \quad (4)$$

$$D_{xy} = \frac{G h^3}{12} \quad (5)$$

Where $E_x$ and $E_y$ are moduli of elasticity of X and Y direction of the slab, $h$ is the thickness of the slab, $\nu_x$ and $\nu_y$ are Poisson’s ratios of X and Y direction of the slab, $D_{xy}$ is the torsion stiffness, $G$ is the shear modulus of rigidity.

The four edges of the slab are clamped. By the coordinate axis of figure 1, on the edges $x=0$, $x=l_x$, $y=0$, $y=l_y$, the geometric boundary conditions can be expressed as follows:

$$w = 0 \quad (6)$$

$$\frac{\partial w}{\partial x} = 0 \text{ or } \frac{\partial w}{\partial y} = 0 \quad (7)$$

### 2.2. Solution to differential equation

The deflection function can be expressed as follows:

$$w(x,y) = w_0(x,y) + w^*(x,y) \quad (8)$$

Where $w_0(x,y)$ is the general solution corresponding to the equation (1), $w^*(x,y)$ is the particular solution corresponding to the equation (1).

We usually assure that $D^2_t = D_xD_y$ [7], thus,

$$w_0(x,y) = \sum_{m,n} \left[ A_{mn} \sin \alpha x \sin \beta y + B_{mn} \sin \alpha x \sin \beta y \right] \sin \alpha x + \sum_{m,n} \left[ E_{mn} \sin \alpha x \sin \beta y + F_{mn} \sin \alpha x \sin \beta y \right] \sin \beta y \quad (9)$$

Where $A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}, F_{mn}, G_{mn}, H_{mn}$ is undetermined coefficient, $\alpha = \frac{m \pi}{l_x}$, $\beta = \frac{(2n-1) \pi}{2l_y}$.

$$\gamma = \sqrt{\frac{D_x}{D_y}} \quad (10)$$

The particular solution $w^*(x,y)$ is related to different loading conditions and it could be expressed by series or polynomial. Actually, the solution expressed by the series can be generally applied, while it is hard to find the form by polynomial corresponding to arbitrary load only for some particular situation.

The particular solution under any transverse loading condition can be expressed as:

$$w^*(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \alpha x \sin \beta y \quad (10)$$
Where $A_{mn}$ is undetermined coefficient.

Transverse load can also be expressed as:

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \alpha x \sin \beta y$$  \hspace{1cm} (11)

where

$$q_{mn} = \frac{4}{l_x \cdot l_y} \int_{0}^{l_x} \int_{0}^{l_y} q(x, y) \sin \alpha x \sin \beta y \, dy \, dx$$  \hspace{1cm} (12)

Substituting equations (10) and (11) into equation (1), we arrive at

$$A_{mn} = \frac{q_{mn}}{D_{x} \alpha^4 + 2D_{y} \alpha^2 \beta^2 + D_{y} \beta^4}$$  \hspace{1cm} (13)

If the slab is under uniform load, we know that

$$q(x, y) = q_0$$  \hspace{1cm} (14)

Thus, one obtains that

$$q_{mn} = \frac{8}{\pi^2 m (2n-1)} q_0 \left(1 - \cos m \pi \right) \left(1 - \cos \left(\frac{2n-1}{2}\pi\right) \right) = \begin{cases} 0 (m, n = 2, 4, 6, ...) \\ \frac{16q_0}{\pi^2 m (2n-1)} (m, n = 1, 3, 5, ...) \end{cases}$$  \hspace{1cm} (15)

Substituting equations (9) and (10) into boundary conditions. The number of terms of the series in the deflection function is taken as $k$ terms. By use of Fourier series, all the coefficients in the deflection function are obtained.

3. Solutions to internal forces

The corresponding internal forces can be obtained by the transformation of deflection function. The bending moments for beam and slab and the shear force for beam are given by [8,9]

$$M_x = -D_x \left( \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right)$$  \hspace{1cm} (16)

$$M_y = -D_y \left( \frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2} \right)$$  \hspace{1cm} (17)

$$Q_x = -\left( D_x \frac{\partial^3 w}{\partial x^2 \partial y} + D_y \frac{\partial^3 w}{\partial x \partial y^2} \right)$$  \hspace{1cm} (18)

$$Q_y = -\left( D_y \frac{\partial^3 w}{\partial x^2 \partial y} + D_y \frac{\partial^3 w}{\partial x \partial y^2} \right)$$  \hspace{1cm} (19)

Where $M_x, M_y$ are the bending moments of X and Y direction; $Q_x, Q_y$ are shear forces of X and Y directions.

The internal force of the beam is taken as the line integral under the load range in the support and mid-span position. The internal force of the slab is taken as the line integral under the range of 1m in the support and mid-span position.

4. Numerical analysis

The plane layout is shown in figure 2. The floor has a constant thickness of 0.1m, which is subjected to uniform dead load of 2kN/m² and live load of 3kN/m². For the concrete used, Young’s modulus $E = 3 \times 10^7$ kN/m² and Poisson’s ratio of two directions $\nu_x, \nu_y = 0.2$. There are one secondary beam in X direction and two secondary beams in Y direction. The cross section of each beam is 200×400mm.
4.1. Stiffness calculation

(1) Stiffness of the slab $D_s$ and secondary beam $D_b$

$$D_s = \frac{Eh^3}{12(1-\nu_x\nu_y)} = \frac{3 \times 10^7 \times 0.1^3}{12(1-0.2 \times 0.2)} = 2604.17 \text{kNm/m}$$

$$D_b = \frac{E_b(h_b-h)^3}{12(1-\nu_x\nu_y)} = \frac{3 \times 10^7 \times 0.2 \times (0.4-0.1)^3}{12(1-0.2 \times 0.2)} = 1.35 \times 10^4 \text{kNm}$$

(2) Equivalent stiffness in X and Y directions and conversion stiffness $D_k$

$$D_s = D_s l_x = 3.433 \times 10^4 \text{kNm}$$

$$D_y = 2D_s l_x + D_b = 4.26 \times 10^4 \text{kNm}$$

$$D_k = \sqrt{D_x^2 + D_y^2} = 3.82 \times 10^4 \text{kNm}$$

4.2. Convergence analysis

The deflection solution is infinite series, so that the degree of convergence is judged by the term of series $k$. The results of internal force of the beam and slab are shown in figures 3 to 5. We conclude that when the term of series $k$ is 10, the results are basic convergences.

4.3. Example verification

The results of the analytical solutions are compared with those made by finite element analysis software SAP2000. Young’s modulus is changed in two directions of the slab so as to simulate the condition of orthotropic slab in SAP2000. The results of the internal force are compared in table 1. Actually, deviation is basically controlled around 3%~5%.

| Calculation result | Secondary beam in X direction | Secondary beam in Y direction | Slab (bending moment) |
|--------------------|-------------------------------|-------------------------------|-----------------------|
|                    | Support moment | Midspan moment | Shear force | Support moment | Midspan moment | Shear force | Bottom in X and Y | Top in X and Y |
| Analytical solution| 32.56            | 12.41            | 33.28       | 58.52          | 27.96          | 48.61       | 2.31/6.43          | 4.11/12.92      |
| Finite element result| 33.36            | 12.87            | 34.48       | 56.2           | 30.1           | 48.31       | 2.4/6.06           | 3.9/14.13       |

5. Conclusions

In traditional design, the calculation of internal force is segmented for beam and slab. In this paper, a new calculation method is proposed, which shows that the rib floor system is equivalent to the orthotropic slab working together. From the example above, we know that when the term of series is 10, the internal force function is basic convergent.

The continuum method for floor system is of high accuracy. It can result in considerable simplicity in the analysis of the large ribbed floor. Further studies should analyze different boundary conditions, such as mixed boundary conditions.
References

[1] Akin, J.E. (1994) Finite Elements for Analysis and Design. Academic Press, London.
[2] Budak, V.D., Grigorenko, A.Y., Borisenko, M.Y., Boichuk, E. V. (2016) Determination of the natural frequencies of an elliptic shell of constant thickness by the finite-element method. Journal of Mathematical Sciences, 212(2): 182-192.
[3] Zienkiewicz, O.C. (1989) The Finite Element Method. McGraw-Hill, London.
[4] Balic, I., Trogrlic, B., Mihanovic, A. (2017). Simplified multimodal pushover target acceleration method for seismic resistance analysis of medium-rise RC structures. KSCE Journal of Civil Engineering, 21(1): 378-388.
[5] Zhu, J., Xu, D., Wang, Y. (2001) Analysis of frame lateral displacement using continuum model. Journal of Hohai University (Natural Sciences), 29(1): 12-16.
[6] Leknitskii, S.G. (1968). Anisotropic Plates. Gordon & Breach, London.
[7] Szilard, R. (2004) Theories and Applications of Plate Analysis: Classical, Numerical and Engineering Methods. John Wiley & Sons, Inc.
[8] Karkon, M., Rezaiee-Pajand, M. (2019) Finite element analysis of orthotropic thin plates using analytical solution. Iranian Journal of Science and Technology, Transactions of Civil Engineering, 43(2):125-135.
[9] Reddy, J.N. (2007) Theory and Analysis of Elastic Plates and Shells. CRC Press/Taylor & Francis, Boca Raton-London-New York.