I. INTRODUCTION

Complex systems with a large number of individual constituents are ubiquitous in physics. Examples range from standard many-particle systems in thermodynamics [1–3] over the structure of complex networks [4–6] to the dynamics of human mobility [7–8]. Often, such systems exhibit intriguing collective dynamics emerging from apparently simple interactions [9]. In the context of mobility, statistical physics approaches have helped to explain, for example, congestion phenomena in car traffic [10–13], trail and lane formation in self-organized pedestrian movement [14–15] as well as universal aspects of large-scale commuting patterns [16–18].

In recent years, on-demand ride-hailing and ride-sharing (also referred to as ride-pooling) has grown to be a major part of urban mobility [19], building on the widespread access to smartphones and mobile communication [20]. Like with conventional taxi services, users request a ride to a desired destination and are then picked up and delivered with door-to-door service. However, unlike with standard taxi rides, users of ride-sharing services may be matched into a shared ride with other riders and delivered to their destination in the same vehicle. The efficiency of such a service strongly depends on the complex interactions between customer requests and the positions and routes of the vehicles [21–24]. Importantly, a high degree of self-organization of the drivers and the routes of the vehicle fleet may lead to collective dynamics that limit the efficiency of the ride-sharing service or the urban mobility system as a whole [25, 26]. Still, the combination of multiple trips into one vehicle potentially reduces the total distance driven compared to individual mobility options and makes ride-sharing a promising option to reduce congestion and emissions with growing urban mobility demands [21–24, 29]. However, ride-sharing can only improve urban mobility if ride-hailing users actually adopt sharing [30–32].

Here, we study this complementary perspective. We analyze how interactions between users shape the adoption of ride-sharing compared to the adoption of single taxi rides. Based on a recently proposed game-theoretic model of ride-sharing adoption dynamics [30], we demonstrate how strategic interactions between the users give rise to spontaneous symmetry breaking, pattern formation dynamics, and bistability, potentially limiting the adoption of ride-sharing. Our results may provide a framework to understand real-world adoption patterns of ride-sharing in complex urban settings and (re)design ride-sharing services and incentives to promote sustainable shared mobility.

II. MODEL

A. Ride-sharing incentives

The decision of a ride-hailing user to request a single or a shared ride depends on the possible benefits of the two options [30, 31, 33–36] (see Fig. 1). Users requesting a ride balance two competing incentives: (i) Service providers incentivize shared rides through financial discounts. These discounts are often percentage discounts of the single trip fare, proportional to the direct trip distance \(d_{\text{req}}\) of the requested trip, and are granted independently of whether the ride is actually matched with another user. (ii) Potential detours \(d_{\text{det}}\) from successfully matched rides discourage requesting shared rides. Detours mediate repulsive strategic interactions between the users’ decisions and underlie the collective phenomena observed in the adoption dynamics.

We combine both the financial and the detour incentives into an expected utility difference \(\mathbb{E}[\Delta u(x)]\) between the utility \(u(x|\text{share})\) of a discounted shared ride with a potential detour and the constant utility
u (x | single) of a direct single ride for the full price for a user with destination x,

\[ E[\Delta u(x)] = E[u(x | single)] - E[u(x | share)] \]

where we take the utilities to be proportional to the relevant distances and \( E[X | Y] \) denotes the expectation value of the random variable \( X \) conditional on the user’s sharing decision \( Y \). The first term in the second equation denotes the financial discount for a shared ride request to destination \( x \), proportional to the direct trip distance \( d_{req} \). The second term denotes the expected detour \( E[d_{det}(x) | share] \) when requesting a shared ride. The proportionality factors \( a \) and \( b \) denote the importance of financial incentives and detour, respectively. The financial discount is constant since the direct trip distance \( d_{req} \) depends only on the destination \( x \). The (expected) detour depends on the destinations and sharing decisions of other users, affecting which requests are matched with each other. Over time, users learn the expected utility difference between requesting a shared or a single ride for a given trip and update their sharing decisions accordingly [see Eq. (3) below].

B. Ride-sharing model

In the most basic setting, ride-sharing combines several concurrent ride-hailing users with homogeneous preferences \( a \) and \( b \) in such a simplified one-to-many demand setting with a set of equidistant destinations on a ring with radius \( R \) around the origin \( o \) (see Fig. 1b). In this setting, the direct trip distance is identical for all destinations, \( d_{req} = R \), and we identify the possible destinations \( x \) by the angle \( \phi \) in polar coordinates. We consider a uniform distribution of the destinations of the \( N \) users and further assume that at most two users are matched into a shared ride. If two users with destinations \( \phi_1 \) and \( \phi_2 \) are matched into a shared ride, the user dropped off second experiences a detour given by the direct geometric distance between the two destinations (see Fig. 1b). This detour depends only on the difference \( \phi_1 - \phi_2 \) and can be expressed via the law of cosines as

\[ d_{det}(\phi_1, \phi_2) = R \sqrt{2 - 2 \cos (\phi_2 - \phi_1)}. \]

Shared ride requests are matched to minimize the total distance driven including the return trip to the origin. With this matching strategy the provider also naturally minimizes the cumulative detour for all users. However, the provider will match two shared ride requests regardless of their destination on the ring (i.e. also if they are on opposite sides) since one shared trip is always shorter than two single trips.

C. Replicator dynamics

In such a basic one-to-many setting, the adoption of ride-sharing is characterized by the probability \( p(\phi, t) \in [0, 1] \) of users with destination \( \phi \) to request a shared ride. We model the evolution of this sharing adoption with time-continuous replicator dynamics

\[ \frac{\partial p(\phi, t)}{\partial t} = p(\phi, t) (1 - p(\phi, t)) E[\Delta u(\phi, t)]. \]
If the average utility of a shared ride is larger than the utility of a single ride, $E[\Delta u(\varphi, t)] > 0$, users increase their sharing adoption $p(\varphi, t)$. If the utility of a shared ride is smaller, $E[\Delta u(\varphi, t)] < 0$, they decrease it. In the stationary states of Eq. (3), no user group (identified by their destination $\varphi$) can increase their utility by changing their decision unilaterally.

The absolute magnitude of the utility differences $\Delta u$ determines the time scale of the dynamics in Eq. (2) but does not change the stationary states. The stationary sharing patterns only depend on the relative importance of detour compared to financial incentives $\beta = b/a$. In the following we thus set $a = 1$ and $R = 1$ without loss of generality. The expected utility difference becomes

$$E[\Delta u(\varphi, t)] = 1 - \beta E[d_{\text{det}}(\varphi, t) | \text{share}].$$

(4)

D. Simulation approach

To simulate the dynamics, we discretize the set of destinations $\varphi \in [0, 2\pi)$ into 360 distinct destinations. We compute estimates for $E[\Delta u(\varphi, t)]$ by simulating the ride-sharing decisions, the matching of shared rides, and the resulting utilities over 1000 realizations per destination. For each realization, we fix one ride-hailing user $j = 1$ requesting a shared ride to destination $\phi_1 = \varphi$ and select the destinations $\phi_j$, $j \in \{2, \ldots, N\}$, of the remaining $N - 1$ users uniformly randomly from all destinations. Each user realizes their sharing decision according to their current adoption probability, requesting a shared ride with probability $p(\phi_j, t)$ and a single ride with probability $1 - p(\phi_j, t)$. We then match the shared ride requests to minimize the total distance driven, selecting uniformly between identical options (e.g. when three users request a shared ride to the same destination).

We compute the utility for the focal user $j = 1$ and estimate the expected utility $E[\Delta u(\varphi, t)]$ as the average over all 1000 realizations. After computing the expected utility for all destinations $\varphi$, we advance the sharing adoption by one Euler-step $\Delta t$ following Eq. (3) and repeat the process. All simulations use a time step $\Delta t = 1$ unless explicitly stated otherwise.

III. RESULTS

A. Two concurrent users

We first consider the simplest setting with exactly $N = 2$ concurrent users who request a ride at the same time. The problem of matching rides is trivial in this case and the users are matched into a shared ride if and only if both users request a shared ride. Otherwise, a user making a shared ride request receives the financial discount but is served in a single ride without incurring any detour. With the detour Eq. (2), we explicitly write the expected value of the utility difference $E[\Delta u(\varphi, t)]$

between a shared and a single ride for a user with destination $\varphi$ as the integral over all destinations $\phi$ of the other user with the corresponding detour if they also request a shared ride with probability $p(\phi, t)$.

$$\frac{\partial p(\varphi, t)}{\partial t} = p(\varphi, t) (1 - p(\varphi, t))$$

$$\times \frac{1}{2\pi} \int_{0}^{2\pi} \left[ 1 - p(\phi, t) \frac{\beta \sqrt{2 - 2 \cos(\varphi - \phi)}}{2} \right] d\phi. \quad (5)$$

FIG. 2. Pattern formation in ride-sharing adoption dynamics. (a) The sharing adoption $p(\varphi)$ with $N = 2$ concurrent users evolves from an initially homogeneous sharing adoption $[\text{Eq. (6)}$, light blue$]$ towards a stable single-peak sharing pattern (dark blue) where some users always request a shared ride while others never do. The plot shows the sharing adoption $p(\varphi, t)$ at times $t \in \{0, 20, 30, 40, 50, 100, 150\}$ (from light to dark) with a relative detour importance $\beta = 3$. The sharing adoption evolves following the replicator dynamics $[\text{Eq. (5)}$ with a time step $\Delta t = 0.05$. We center the sharing peak at $\varphi = \pi$ for visualization. (b) Stable steady states characterized by the width of the single-peak adoption pattern. Data points are created by adiabatically varying the relative detour importance $\beta$ and tracking the width of the sharing peak in the steady state. (inset) A linear stability analysis of the replicator dynamics $[\text{Eq. (5)}$, see appendix for details$]$ with relative detour importance $\beta = 3$ predicts a single peak as the most unstable mode, consistent with the observations from the direct numerical simulations (panel a). Crosses denote results of the analytical stability analysis with a continuous destination space, circles denote results from the exact evaluation in discretized destination space.
The factor $1/2$ in the detour term captures the expected value given that only one (randomly chosen) of the two users in a shared ride experiences a detour while the other is driven directly to their destination (compare also Fig. 4b).

Solving the two-player replicator dynamics, Eq. (5), for a homogeneous steady state $p(\varphi, t) = p^*$ such that $\frac{\partial p(\varphi, t)}{\partial t}|_{p^*} = 0$ we find

$$p^* = \frac{\pi}{2 \beta}.$$ (6)

We provide the full calculation in the appendix. As expected, the homogeneous steady state sharing adoption increases as the relative detour importance $\beta$ decreases. However, simulating the replicator dynamics from this steady state (Fig. 2a) reveals that this state is indeed unstable. The symmetry in the system is broken by random fluctuations in the estimation of the expected utility differences. Over time, the ride-sharing adoption increases on one side of the ring and decreases on the other until finally a single-peak sharing/non-sharing pattern emerges. Figure 2a illustrates how the total sharing adoption, characterized by the width $\theta$ of the sharing peak, changes with the relative detour importance $\beta$. While the total sharing adoption still increases as the relative detour importance decreases, it does so with a spatially heterogeneous pattern until the relative detour importance decreases below a critical value $\beta_c = \pi/2$ where $p^* = 1$.

A stability analysis of the two-player replicator dynamics confirms this observation (Fig. 2a inset). While the homogeneous steady state is stable with respect to spatially homogeneous changes of the sharing adoption, it is unstable to spatially dependent perturbations. In particular, the most unstable mode with the largest growth rate $\lambda(k) > 0$ is a single-period sinusoidal perturbation, $p(\varphi, t) = p^* + \delta \cos(k \varphi) e^{\lambda(k)t} + \mathcal{O}(\delta^2)$ with $k = 1$, increasing the adoption on one side of the ring and decreasing it on the opposite side (compare Fig. 2a, see appendix for the full calculation). Effectively, increasing the sharing adoption in one location strongly increases the expected detour for users with far-away destinations while the expected detour for close-by destinations is only weakly affected (see appendix, Fig. A1). The detour-mediated interaction of the users thus exhibits long-range-inhibition, promoting the formation of a single-peak pattern where users with certain destinations always request shared rides while users with other destinations never request shared rides.

**B. $N$ concurrent users**

With more than two players, the qualitative interaction between users and their sharing adoption $p(\varphi, t)$ at different locations $\varphi$ remains robust. Figure 3a illustrates the emergence of a ride-sharing adoption pattern with a single-peak for $N = 16$ concurrent users starting from a homogeneous state with low sharing adoption. Interestingly, Fig. 3b suggests that, in the same setting, the full sharing state $p(\varphi, t) = 1$ is stable as well. An initially high, homogeneous sharing adoption evolves to this full sharing state instead of breaking down into a single-peak sharing pattern. In essence, a single user outside an existing sharing peak does not want to share due to a large expected detour as there are no other users requesting a
shared ride with a similar destination. At the same time, however, if all users adopt ride-sharing, the expected detours become small due to the large number of users (i.e. users are more likely matched with another user with a close-by destination).

This bistability is also directly predicted by the expected utility: We assume that the sharing adoption \( p_\theta^* \) takes the form of a single sharing peak with width \( \theta \), i.e. \( p_\theta^*(\varphi) = 1 \) if \( 0 \leq \varphi \leq \theta \) and \( p_\theta^*(\varphi) = 0 \) otherwise. The stability of such a pattern is determined by the decisions of users on the edge of the sharing peak given by the expected utility difference \( E[\Delta u(\theta)]_{p_\theta^*} \). Unfortunately, a direct analytical calculation would involve a large number of case distinction to discern which users are actually matched. Here we make use of Eq. (4) and numerically evaluate the expected detour over \( 10^5 \) realizations of the destinations and sharing decisions of the other users to compute the expected utility.

In continuous destination space, the sharing peak width remains unchanged if the user on the edge is indifferent between shared and single rides, i.e. if the utility difference

\[
E[\Delta u(\theta)]_{p_\theta^*} = 1 - \beta E[d_{\text{det}}(\theta) | \text{share}]_{p_\theta^*} = 0 , \quad (7)
\]

giving the critical detour importance

\[
\beta_c(\theta) = \frac{1}{E[d_{\text{det}}(\theta) | \text{share}]_{p_\theta^*}} . \quad (8)
\]

This critical detour importance also predicts where the full sharing state with \( \theta = 2\pi \) becomes unstable. In discrete destination space, we compute the expected detour for a user directly outside the sharing peak and thus find the critical detour importance where the width of the sharing peak would not grow further. This correctly predicts the width of the sharing peak as we slowly decrease the relative detour importance \( \beta \). However, due to subtle differences in a discrete destination setting (the probability for two users to go to the same destination is non-zero), it is slightly different from the stability condition for an existing sharing peak breaking down when we slowly increase the relative detour importance, explaining the local bistability observed in Fig. 3c.

Figure 3 shows the bifurcation diagram for \( N = 16 \) users, illustrating the bistability of the partial \((\theta < 2\pi)\) and full \((\theta = 2\pi)\) sharing state as well as the saddle-node bifurcation where the partial sharing state disappears for decreasing relative detour importance \( \beta \). For small peak widths \( \theta \) the sharing peak is always stable. Due to the uniform request distribution, users with destinations inside the sharing peak always experience shorter expected detours (and thus prefer sharing) while users further outside always experience longer detours (and thus prefer single rides). However, for sufficiently large peak widths \( \theta \), detours become smaller as the peak width increases since users are matched around the circle. The sharing peak is unstable.

The critical detour importance increases as the number of users increases (and the expected detour decreases), making it easier to achieve full sharing adoption. However, at the same time, the width of the bistability region also increases with the number of users. Figure 4 shows the full bifurcation diagram as a function of the relative detour importance and the number of users. Intriguingly, the width \( \theta^* \) of the sharing peak where the partial sharing regime undergoes the saddle-node bifurcation is minimal for intermediate numbers of ride-hailing users (see Fig. 3 inset). This suggests a potentially larger loss of ride-sharing adoption in these intermediate-demand settings compared to the extreme cases of low or very high demand.

IV. DISCUSSION

The basic mechanism underlying the bistability between partial and full adoption observed in our model is similar to the bistability in classic models of technology adoption with network effects [38-41], i.e. if the utility of the technology or service relies on the wide-spread adoption, for example communication technologies or online social platforms. However, the inherent spatial compo-
nent of ride-sharing services and the strategic interactions between users with different destinations give rise to spatially heterogeneous adoption patterns. This bistability may lead to a loss in efficiency of ride-sharing services due to lower than expected adoption and is particularly relevant if only a small fraction of the potential user base adopts the service. Similar effects may partially underlie currently low ride-sharing adoption in metropolitan areas such as New York City [30].

We have studied the dynamics of ride-sharing adoption in a highly simplified setting. While the qualitative, fundamental interactions are likely persistent also in real-world ride-sharing applications, additional influences from alternative mode choice options, matching strategies, or quickly evolving regulatory boundary conditions add to the dynamics [31] [39] [30]. In particular, adoption patterns in real ride-sharing applications seem to be dominated by socio-economic heterogeneities and the interaction with other (public) transport modes [30] [30]. While this observation suggests that the pattern formation dynamics found in the simplified model may not be dominant, the spatially heterogeneous willingness to share may result in higher difficulty to achieve large-scale ride-sharing adoption due to the inherent bistability of partial ride-sharing adoption patterns and full sharing adoption. Moreover, additional disincentives from inconvenience and loss of privacy during a shared ride may further limit adoption of ride-sharing.

More complex urban settings such as two-dimensional street networks, heterogeneous demand distributions with many-to-many routing, and more complex matching schemes with on-route pickups may give rise to more complex interactions between the users and to more intricate adoption patterns. Unfortunately, the complexity of evaluating the expected utility and the dependence on the realization of the matching of shared rides renders the (numerical) evaluation of the dynamics a non-trivial problem already in our simplified system.

Overall, our results demonstrate the possibility of spontaneous symmetry breaking and self-organized pattern formation dynamics in shared urban mobility. Importantly, these complex dynamics need not be rooted in the algorithms underlying new types of mobility services, spatially distributed vehicle fleets, or the inherent heterogeneities, but may ultimately result from interactions among the users alone. These findings emphasize the importance of understanding the collective dynamics of both the supply [29] and demand sides [30] of ride-sharing, and the interactions between them, to enable more sustainable mobility [12]. Revealing the fundamental detour-mediated interactions between the users as the cause of pattern formation dynamics of ride-sharing adoption, our modeling approach may contribute to a better understanding of the (non)adoption of ride-sharing services. This may ultimately help to (re)design services and incentives that promote the adoption of ride-sharing and other forms of sustainable shared mobility.

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APPENDIX

*N = 2* player model with continuous destinations

The full replicator equation for \( N = 2 \) players is given by Eq. (5) in the main text:

\[
\frac{\partial p(\phi, t)}{\partial t} = p(\phi, t) \left( 1 - p(\phi, t) \right) \times \frac{1}{2\pi} \int_0^{2\pi} \left[ 1 - p(\phi, t) \beta \sqrt{2 - 2 \cos(\phi - \phi)} \right] d\phi
\]  

(A1)

Homogeneous steady state

We look for homogeneous steady states \( p(\phi, t) = p^* = \text{const.} \) defined by \( \frac{\partial p(\phi, t)}{\partial t} \bigg|_{p^*} = 0 \). From the structure of Eq. (A1), we quickly find two trivial fixed points at \( p^* = 0 \) and \( p^* = 1 \). These fixed points are the result of the mathematical structure of the replicator equation (if one strategy dies out completely, it cannot reproduce). We find another fixed point \( 0 < p^* < 1 \) by setting the integral in Eq. (A1) to zero:

\[
1 - \frac{\sqrt{2}}{4\pi} \int_0^{2\pi} \sqrt{1 - \cos(\phi - \phi)} d\phi = 0
\]  

(A2)

The remaining integral can be solved by applying the half-angle identity \( \sin^2(x/2) = \frac{1 - \cos(x)}{2} \) such that the integrand becomes \( \sqrt{1 - \cos(\phi - \phi)} = \sqrt{2} \left| \sin \left( \frac{\phi - \phi}{2} \right) \right| \). Using the \( 2\pi \) periodicity and the symmetry of our solution, we set \( \phi = 0 \) without loss of generality. The value of \( \sin \left( \frac{-\phi}{2} \right) \) is negative over the whole integration range. We use the antisymmetry of the sine-function to replace the absolute value by changing the sign of the argument and compute the integral as

\[
\int_0^{2\pi} \sqrt{1 - \cos(\phi - \phi)} d\phi = \sqrt{2} \int_0^{2\pi} \sin \left( \frac{\phi}{2} \right) d\phi = \sqrt{8} \int_0^\pi \sin(y) dy = \sqrt{32} \cdot (A3)
\]

Inserting this result in Eq. (A2) leads to

\[
1 - \frac{\sqrt{2}}{4\pi} \frac{\beta p^*}{\sqrt{32}} = 0 ,
\]  

(A4)

which yields the fixed point

\[
p^* = \frac{\pi}{2\beta} \;
\]

reproducing Eq. (6) in the main text.

Linear stability analysis

To analyze the linear stability of the two player system, we make the ansatz

\[
p(\phi, t) = p^* + \delta \cos(k\phi) e^{\lambda t} + O(\delta^2)
\]

(A6)

for the sharing adoption in Eq. (A1). Inserting the ansatz gives

\[
\frac{\partial}{\partial t} \left[ p^* + \delta \cos(k\phi)e^{\lambda t} + O(\delta^2) \right] = \left( p^* + \delta \cos(k\phi)e^{\lambda t} + O(\delta^2) \right) \left( 1 - \left[ p^* + \delta \cos(k\phi)e^{\lambda t} + O(\delta^2) \right] \right) \left( 1 - \int_0^{2\pi} \frac{1}{2\pi} \left( \delta \cos(k\phi)e^{\lambda t} + O(\delta^2) \right) \left( \frac{\beta}{2} \sqrt{2 - 2 \cos(\phi - \phi)} \right) d\phi \right)
\]

(A7)
The evaluation of the last part of Eq. (A7) leads to two integrals:

\[ 1 - \int_0^{2\pi} \frac{1}{2\pi} \cdot (p^* + \delta \cos(k\phi) e^{\lambda t}) \left( \frac{\beta}{2} \sqrt{2 - 2\cos(\varphi - \phi)} \right) d\phi \]

\[ = 1 - \frac{p^*}{2\pi} \left[ \sqrt{\frac{2\beta}{2}} \int_0^{2\pi} \sqrt{1 - \cos(\varphi - \phi)} d\phi \right] - \frac{\delta e^{\lambda t}}{2\pi} \left[ \sqrt{\frac{2\beta}{2}} \int_0^{2\pi} \cos(k\phi) \sqrt{1 - \cos(\varphi - \phi)} d\phi \right] \quad (A8) \]

The first integral is already known from Eq. (A3) and cancels with the 1 when evaluated at the homogeneous steady state Eq. (A5). To evaluate the second integral, we repeatedly apply integration by parts, assuming the integrand to be of the shape \( f' \cdot g \) with \( f' = \cos(k\varphi) \) and \( g = \sqrt{1 - \cos(\varphi - \phi)} \), which leads to:

\[ \int_0^{2\pi} f'(\varphi) g(\varphi, \phi) d\phi = \frac{1}{k} \sin(k\phi) \sqrt{1 - \cos(\varphi - \phi)} \bigg|_0^{2\pi} - \frac{1}{k} \int_0^{2\pi} \sin(k\phi) \cdot \frac{1}{2\sqrt{1 - \cos(\varphi - \phi)}} d\phi \quad (A9) \]

where the first part vanishes when inserting the boundaries. To evaluate the remaining integral, we once again use the half angle identity \( \sqrt{1 - \cos(x)} = \sqrt{2} \sin \left( \frac{x}{2} \right) \) to simplify the expression we substitute \( x = \frac{\varphi - \phi}{2} \) to find:

\[ \frac{\sqrt{2}}{k} \int_{\frac{\pi}{2} - \frac{\pi}{2}}^{\frac{\pi}{2}} \sin(k(\varphi - x)) \cos(x) \, dx = \frac{\sqrt{2}}{k} \left( \int_0^{\frac{\pi}{2}} \sin(k(\varphi - x)) \cdot \cos(x) \, dx - \int_{\frac{\pi}{2}}^{\pi} \sin(k(\varphi - x)) \cdot \cos(x) \, dx \right) \quad (A11) \]

These two integrals are of the same form, and have to be integrated by parts with \( f'(x) = \cos(x) \) and \( g(x, \varphi) = \sin(k(\varphi - x)) \):

\[ \int_a^b \sin(k(\varphi - x)) \cdot \cos(x) \, dx = [\sin(k(\varphi - x)) \sin(x)]_a^b + 2k \int_a^b \sin(x) \cos(k(\varphi - x)) \, dx \quad (A12) \]

The resulting integral has to be integrated by parts once more, this time with the substitutions \( f'(x) = \sin(x) \) and \( g(x, \varphi) = \cos(k(\varphi - x)) \). We get:

\[ \int_a^b \sin(k(\varphi - x)) \cos(x) \, dx = [\sin(k(\varphi - x)) \sin(x)]_a^b + 2k \left( -\cos(x) \cos(k(\varphi - x)) \right)_a^b + 2k \int_a^b \cos(x) \sin(k(\varphi - x)) \, dx \quad (A13) \]

which we solve for the integral:

\[ \int_a^b \sin(k(\varphi - x)) \cos(x) \, dx = \frac{[\sin(k(\varphi - x)) \sin(x)]_a^b - 2k [\cos(x) \cos(k(\varphi - x))]_a^b}{1 - 4k^2} \quad (A14) \]

Plugging this result back into Eq. (A11) and evaluating the integrals with the corresponding boundaries give the result for the integral:

\[ \frac{\sqrt{2}}{k} \left( \frac{-2k [\cos(\frac{k\varphi}{2}) - \cos(k\varphi)]}{1 - 4k^2} \right) - \frac{2k [\cos(k\varphi) - \cos(\frac{k\varphi}{2} - \pi)]}{1 - 4k^2} \right) = \frac{\sqrt{2} \cos(k\varphi)}{1 - 4k^2} \quad (A15) \]

Inserting this result into Eq. (A8) and keeping only terms of order \( \delta \) gives the linearized equation around the homogeneous steady state Eq. (A5):
Finally, we solve this equation for the growth rate $\lambda_k$ which is possible for all $\cos(k\varphi) \neq 0$:

$$\lambda_k = \left(p^* - p^{*2}\right) \left(-\frac{2\beta}{\pi} \frac{1}{1 - 4k^2}\right) = \frac{p^* - 1}{1 - 4k^2}$$

(A17)

which shows that the homogeneous steady state is stable with respect to spatially homogeneous perturbations, $\lambda_0 < 0$, but unstable for all inhomogeneous perturbation, $\lambda_k > 0$ for $k > 0$.

**Derivation of the theoretical sharing peak width**

To predict the final steady state sharing pattern, we make an ansatz for the shape of the sharing adoption pattern based on the single peak pattern observed in the simulations. This sharing adoption pattern $p^*_\varphi(\varphi)$ takes the form of a single sharing peak with width $\theta$, i.e. $p^*_\varphi(\varphi) = 1$ if $0 \leq \varphi \leq \theta$ and $p^*_\varphi(\varphi) = 0$ otherwise. We now calculate the expected utility for an individual with a destination just at the border where the sharing regime and the non-sharing regime meet. As described in equation $[7]$ in the main text, a user with destination $\varphi = 0$ or $\varphi = \theta$ at the border between the sharing regimes should be indifferent between both options in the steady state, $E[\Delta u(0)]|_{p^*_\varphi} = E[\Delta u(\theta)]|_{p^*_\varphi} = 0$. Otherwise the sharing probabilities at this point would either increase or decrease over time.

The sharing adoption

$$p^*_\varphi(\varphi) = \Theta(\theta - \varphi)$$

(A18)

with $\varphi \in [0, 2\pi)$ where $\Theta$ denotes the Heavyside function plugged into the expected utility integral yields

$$E[\Delta u(\theta)]|_{p^*_\varphi} = \int_0^{2\pi} \frac{1}{2\pi} \left[ 1 - \Theta(\theta - \varphi) \frac{\beta \sqrt{2 - 2 \cos(\theta - \varphi)}}{2} \right] \, d\varphi = 0.$$  

(A19)

The integral can be calculated by once again using the halve angle identity $\sqrt{1 - \cos(x)} = \sqrt{2} \left| \sin \left( \frac{x}{2} \right) \right|$:

$$1 - \frac{\sqrt{2}\beta}{4\pi} \int_0^\theta \sqrt{1 - \cos(\theta - \varphi)} \, d\varphi = 1 - \frac{\beta}{2\pi} \int_0^\theta \left| \sin \left( \frac{x}{2} \right) \right| \, dx$$  

(A20)

as $0 \leq \theta \leq 2\pi$ we neglect the absolute value and calculate the integral, resulting in:

$$1 - \frac{\beta}{2\pi} \int_0^\theta \sin \left( \frac{x}{2} \right) \, dx = 1 - \frac{\beta}{\pi} \left( 1 - \cos \left( \frac{\theta}{2} \right) \right) = 0$$  

(A21)

which is then solved for $\theta \in [0, 2\pi]$ to get the theoretical prediction for the width of the sharing peak in a system with two concurrent users (compare Fig. 2b in the main text):

$$\theta = \min \left[ 2\pi, 2 \arccos \left( 1 - \frac{\pi}{2\beta} \right) \right]$$  

(A22)

**Response function of ride-sharing adoption**

To understand the mechanism behind the pattern formation dynamics, we quantify the interactions of the two users via the response to a small, localized change of the adoption probability

$$p(\varphi) = p^*(\varphi) + \delta p(\varphi; \epsilon, \phi) = p^*(\varphi) + \epsilon \Theta(\varphi - \phi) \Theta(\phi + \delta \varphi - \varphi)$$  

(A23)

around a reference state $p^*(\varphi)$, where $\Theta(\cdot)$ denotes the Heaviside-function. We compute the response function

$$\chi(\varphi, \phi) = \lim_{\epsilon \to 0} \lim_{\delta \phi \to 0} \frac{E[\Delta u(\varphi)]|_{p^*(\varphi) + \delta p(\varphi; \epsilon, \phi)} - E[\Delta u(\varphi)]|_{p^*(\varphi)}}{\epsilon \delta \varphi}.$$  

(A24)

This response function $\chi(\varphi, \phi)$ measures the effect of a change of sharing adoption by users with destination $\phi$ on the expected utility (and thereby the sharing adoption) of a user with destination $\varphi$. 
In the two-user setting, the response function is independent of the reference state since both users are matched if and only if both request a shared ride. We directly evaluate the response function in continuous destination space evaluating the limits in the expected utility integral in Eq. (A1). We find

$$\chi(\varphi, \phi) = \beta \frac{\sqrt{2} - \sqrt{2} \cos (\varphi - \phi)}{2}.$$  

We also evaluate the response function numerically by comparing the expected detour in the homogeneous steady state and increasing the sharing adoption of a single destination $\phi$ by $\epsilon = 0.01$.

Figure A1 illustrates the response to an increase of the sharing adoption of users with destination $\phi = \pi$ in the center of the plot. The expected utility of shared rides decreases for destinations on the opposite side of the circle ($\varphi = 0$ or $\varphi = 2\pi$) as expected detours become larger. For locations close to $\phi = \pi$, the utility remains almost constant. The interactions between the adoption decisions of users with different destinations exhibit long range inhibition, thereby promoting the formation of a single peak adoption pattern as observed in the main text.

![Diagram](https://via.placeholder.com/150)

**FIG. A1.** **Effective interaction explains pattern formation dynamics.** Response function $\chi(\varphi, \pi)$ [Eq. (A24)] for $N = 2$ players with relative detour importance $\beta = 3$ around the homogeneous steady state $p^*(\varphi) = \pi/(2\beta)$. The dashed line shows the result of the direct evaluation of the response function in continuous destination space [Eq. (A25)]. The circled show the result of the numerical evaluation in discretized destination space with a increase $\epsilon = 0.01$ of the sharing probability at destination $\pi$. Far away destinations ($\varphi = 0$ or $2\pi$) are disincentivized to share by larger expected detours. Close by destinations around $\varphi^* = \pi$ are less affected. The interactions between the adoption decisions of users with different destinations exhibits long range inhibition.