On the gravitational seesaw and light bending

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Abstract. Local gravitational theories with more than four derivatives are superrenormalizable, and also may be unitary in the Lee-Wick sense. It makes sense to study low-energy properties of these theories, e.g., identify observables which might be useful for experimental detection of higher derivatives. Using an analogy with neutrino Physics, we explore the possibility of a gravitational seesaw mechanism, in which several dimensional parameters of the same order of magnitude produce a hierarchy in the masses of propagating particles and make a relatively light degree of freedom detectable by frequency dependence in the gravitational light bending. It turns out that such a seesaw mechanism in the six- and more-derivative theories is unable to reduce the lightest mass more than in the simplest four-derivative model. Adding more derivatives can only make heavier masses even larger. This fact may be favorable for protecting the theory from instabilities, but makes experimental detection of higher derivatives more difficult.

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1 Introduction

The role of higher derivatives in quantum and classical gravity theories is important, complicated and ambiguous. On the one hand it is well-known that semiclassical [1] and quantum [2] gravity can be formulated as renormalizable theories only with the four-derivative terms in the action (see [3, 4] for an introduction and [5] for a recent review). On the other hand, by adding higher-derivative terms to the Einstein-Hilbert action we introduce massive unphysical ghosts, related instabilities and (in the quantum gravity case) non-unitary $S$-matrix. Recently, it was shown that in a theory with six or more derivatives one can have all massive poles complex, and then the $S$-matrix becomes unitary in the Lee-Wick sense [6].

Let us remember that higher derivatives emerge also in the gravitational effective action in string theory. The corresponding terms are removed by means of the Zweibach reparametrization of the background metric in target space [7]. However, this procedure is ambiguous, since the no-ghost condition does not fix many terms in the higher derivative sector [8]. Furthermore, another source of ambiguity is that the problem may be solved not only by completely removing all potentially dangerous terms, but also by reducing the effective action to a ghost-free non-local form [9].

It is important to note that in both these approaches the removal of massive ghosts requires an absolutely precise fine-tuning of the action. Nevertheless, any small violation here should leads to destructive instabilities and, moreover, these instabilities are even stronger for smaller violations [10]. This means that the ghost-killing procedure in string theory [7] (or [9]) demands an absolutely precise fine-tuning of infinitely many parameters. On the other hand, violations of the fine tuning can not be avoided if the loop contributions are taken into account [11] in the effective field theory framework (see, e.g., [12]). The most reasonable position is all in all to assume the existence of higher derivatives and try to understand why they do not produce a total destruction of the classical gravitational solutions [13].

Assuming that there is no fine-tuning, do we have a chance to see the effect of higher-derivative terms at low energies? This question is particularly relevant, because already at the semiclassical level the loop corrections produce non-local form factors in the quadratic curvature terms. At low energies it is natural to assume a truncation of the infinite series
in d’Alembert operator, leading to an effective polynomial theory of the type [14]

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R - 2\Lambda \right) + \int d^4x \sqrt{-g} \left\{ c_1 R_{\mu\nu\alpha\beta}^2 + c_2 R_{\mu\nu}^2 + c_3 R^2 + d_1 R_{\mu\nu\alpha\beta}^2 R_{\mu\nu}^2 + d_2 R_{\mu\nu} R_{\mu\nu}^2 + d_3 R_{\mu\nu\alpha\beta}^2 + d_4 R_{\mu\nu}^4 + d_5 R_{\mu\nu\alpha\beta} R_{\mu\nu}^2 R_{\mu\nu} + \ldots 
\]

\[ + f_1 R_{\mu\nu\alpha\beta}^2 \kappa R_{\mu\nu\alpha\beta}^2 + f_2 R_{\mu\nu} R_{\mu\nu}^2 \kappa R_{\mu\nu} + f_3 R_{\mu\nu} R_{\mu\nu}^2 + \ldots + f_{..} R_{\mu\nu}^{k+2} \right\}, \quad (1) \]

where we have used the same sign conventions as in [15].

In what follows we will be interested in the bending of light by a weak gravitational field. In this spirit, we can disregard the cosmological constant term and those terms which are third- or higher-order in curvature. According to the Refs. [14, 6], the last condition means that we can also drop the terms which are quadratic on the Riemann tensor. Furthermore, for the sake of simplicity we restrict the analysis to the \( k = 1 \) case. As a consequence, the relevant part of the action can be cast into the form

\[ S = S_{grav} + \int d^4x \sqrt{-g} L_m, \quad (2) \]

\[ S_{grav} = \int d^4x \sqrt{-g} \left\{ \frac{\kappa^2}{2} R + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R_{\mu\nu}^2 + \frac{A}{2} R_{\mu\nu} R + \frac{B}{2} R_{\mu\nu} R_{\mu\nu} \right\}, \quad (3) \]

where an additional matter action was introduced; besides, the notations were adjusted for the sake of consequent calculations. Here \( \alpha, \beta, A \) and \( B \) are free parameters, where the first two are dimensionless while \( A \) and \( B \) carry dimension of (mass)\(^{-2}\). The notation \( \kappa^2/2 = 16\pi G = M_P^{-2} \) is conventional in the quantum gravity literature.

In string theory all massive parameters are constructed from the single dimensional parameter \( \alpha' \), and hence all masses in the action are supposed to have the same (typically Planck) order of magnitude. However, our experience with the seesaw mechanism in neutrino Physics shows that this does not rule out a situation where several huge massive parameters combine into one particle of light mass, with the other masses becoming even greater. In our case the quantities in the action must satisfy \( A^{-1}, B^{-1}, \kappa^{-2} \sim M_P^2 \). In what follows we discuss the possibility of a seesaw-like mechanism. As we shall see, in the gravitational case it enables one to have a parameter \( B^{-1} \) much smaller than \( M_P^2 \), and still have an associate mass of the order of \( M_P \). This scenario can be achieved by reducing the lighter mass of the tensor excitation, which is the well-known ghost mode. After discussing gravitational seesaw, we shall explore the possibility to meet a potentially observable low-energy effect owed to this lighter ghost.

The paper is organized as follows. In Sect. 2 we discuss the new gravitational seesaw mechanism, which is possible in the theory with more than four derivatives, such as in (3). In Sect. 3 gravitational bending of light is considered. We remark that in this first
communication many technical details will be omitted, to be presented in the parallel work [16], which is mainly devoted to the complex poles case. Finally, in Sect. 4 we draw our conclusions.

2 Gravitational seesaw in higher derivative theories

The conventional point of view is that higher derivatives are not observable at low energies because of the Planck suppression. In order to have the Planck suppression in forth-order gravity, the coefficients of the higher-order terms have to be of order one or at least not too many orders of magnitude greater. However, what is correct as far as the four-derivative model is concerned, is not necessary right for theories exhibiting sixth-derivatives or more. Since there are several massive parameters, one can imagine a specific seesaw like mechanism, which enables two (or more) large-mass parameters to combine in such a way that they produce a much smaller physical mass. Let us examine the theory (3) in this respect.

Introducing a suitable gauge condition, in the weak-field limit, i.e. \( g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \) and \( |\kappa h_{\mu\nu}| \ll 1 \), the linearized field equations can be cast into the form

\[
\left( \frac{2}{\kappa^2} - \frac{\beta}{2} \Box - \frac{B}{2} \Box^2 \right) \left( R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R \right) \\
- \left( \alpha + \frac{\beta}{2} + A \Box + \frac{B}{2} \Box \right) \left( \eta_{\mu\nu} \Box R - \partial_\mu \partial_\nu R \right) = -\frac{T_{\mu\nu}}{2}.
\]

(4)

It is possible to show that the weak gravitational field generated by a static point-like mass, \( T_{\mu\nu}(r) = M \eta_{\mu0} \eta_{\nu0} \delta^{(3)}(r) \), has non-zero components given by (one can find more detailed and general results in [17])

\[
h_{00} = \frac{M\kappa}{16\pi} \left( -\frac{1}{r} + \frac{4}{3} F_2 - \frac{1}{3} F_0 \right), \\
h_{11} = h_{22} = h_{33} = \frac{M\kappa}{16\pi} \left( -\frac{1}{r} + \frac{1}{3} F_2 + \frac{2}{3} F_0 \right),
\]

(5)

where

\[
F_k = \frac{m_{k+}^2 e^{-m_{k+} r}}{m_{k+}^2 - m_{k-}^2} - \frac{m_{k-}^2 e^{-m_{k-} r}}{m_{k-}^2 - m_{k+}^2}.
\]

Here \( k = 0, 2 \) labels the spin of the particles, whose masses are defined by the positions of the poles of the propagator,

\[
m_{2\pm}^2 = \frac{\beta \pm \sqrt{\beta^2 + \frac{16}{\kappa^2} B}}{2B}, \quad m_{0\pm}^2 = \frac{\sigma_1 \pm \sqrt{\sigma_1^2 - \frac{8\sigma_2}{\kappa^2}}}{2\sigma_2}, \quad \sigma_1 = 3\alpha + \beta, \quad \sigma_2 = 3A + B.
\]

(6)
One can observe that in the sixth-order gravity massive particles occur in dependent pairs with the same spin.

It was shown (see, e.g., [15]) that the deflection of light in four-derivative gravity depends only on the tensors modes of the metric perturbation $h_{\mu\nu}$. The reason is that the scalar mode does not enter into the calculations concerning the bending owed to the fact that linearized gravity and linearized $R + R^2$ gravity are conformally related. The overall effect is a sum of the contributions of the massless graviton and the massive tensor mode. The proof can be easily generalized for the six-derivative case. Hence in what follows we shall consider only these massive modes.

As mentioned before, the quantities $m_{2\pm}$ in Eq. (6) can be real or complex depending on the parameters $\beta$ and $B$. Real poles occur provided that $\beta, B < 0$ and $16|B| \leq \kappa^2 \beta^2$. Under these circumstances, the lighter one is a massive ghost while the other is a healthy massive tensor field [14]. A moment’s reflection shows the existence of a very special relation between $\beta$ and $B$ such that one of the masses is much smaller than the other,

$$m_{2+}^2 \ll m_{2-}^2$$

which has the form $16|B| \ll \kappa^2 \beta^2$. In the theory where this condition is satisfied the masses $m_{2\pm}$ can be approximated by

$$m_{2+}^2 \approx \frac{4}{\kappa^2 |\beta|} \ll m_{2-}^2 \approx \frac{\beta}{B}.$$  

As in the original neutrino’s seesaw mechanism one of the masses depends, roughly, on only one parameter, while the other depends on both. Moreover, this relation occurs in such a manner that if the lighter mass is reduced, then the larger mass is augmented. A remarkable difference with respect to the neutrino’s mechanism is that while in the neutrino case it works to make the lightest mass even lighter, in the gravitational model the effect is to turn the largest mass even larger, according to Eq. (8). This happens due to the presence of the parameter $B$ in the denominator of Eq. (6), making the lightest mass to depend only on $\beta$ while the largest one depends on both parameters.

In this vein, there are two possible ways of having $m_{2-}$ of the order of the Planck mass: to have a small $|B|$ or a larger $|\beta|$. The first choice is the standard one, since it prescribes that $\beta \sim 1$ and $B \sim M_P^2$ so as to have all the masses to the order of $M_P$. The second possibility, which relies on the seesaw mechanism, allows one to have $|B| \gg M_P^2$ and still have $m_{2-} \sim M_P$. Of course, having a large $|B|$ still yielding one large mass can only be achieved by means of the ghost mass reduction through a parameter $\beta \gg 1$. The final result is that the much lighter mass of the first ghost depends only on the second-
fourth-derivative terms. By the end of the day, the six-derivative terms are not capable
to produce an efficient seesaw mechanism working like in the case of neutrino mass.

One can present a strong argument in favor of the non-possibility of the strong seesaw
mechanism for even higher, e.g. eight- and more-derivative gravity theories\(^1\). For instance,
consider the action (1) with \(k = 2\), that means eight-derivative theory. One can write the
equation for the massive poles in the propagator in the form

\[
\frac{1}{m_0^4} k^6 - \frac{3}{m_1^4} k^4 + 3\beta k^2 - m_2^2 = 0 .
\]  

(9)

Here \(m_{0,1,2}^2\) are positive massive parameters coming from the action. In string theory one
can assume that all of them are of the same order of magnitude, say

\[
m_0^2 \sim m_1^2 \sim m_2^2 \sim M_P^2.
\]  

(10)

Let us assume that this is the case. One can rewrite (9) in the more simple form

\[
k^6 - \frac{3m_0^4}{m_1^4} k^4 + 3\beta m_0^4 k^2 - m_0^2 m_2^2 = 0 .
\]  

(11)

The roots of this equation are defined by Cardano formula and can be either real or
complex. Consider the particular case of real positive roots which satisfy the hierarchy
\(\mu_1^2 \ll \mu_2^2 \sim \mu_3^2\). Then the equation becomes

\[
k^6 - (\mu_1^2 + \mu_2^2 + \mu_3^2) k^4 + (\mu_1^4 \mu_2^2 + \mu_1^2 \mu_3^2 + \mu_2^4 \mu_3^2) k^2 - \mu_1^2 \mu_2^2 \mu_3^2 = 0 .
\]  

(12)

Using the hierarchy \(\mu_1^2 \ll \mu_2^2 \sim \mu_3^2\), the last equation boils down to

\[
k^6 - (\mu_2^2 + \mu_3^2) k^4 + \mu_2^2 \mu_3^2 k^2 - \mu_1^2 \mu_2^2 \mu_3^2 = 0 .
\]  

(13)

It is easy to see that there is a contradiction between Eq. (11) with (10) and Eq. (13).
According to (11) we have

\[
\frac{3m_0^4}{m_1^4} \sim M_P^2, \quad 3\beta m_0^4 \sim M_P^4 \quad \text{and} \quad m_0^2 m_2^2 \sim M_P^6 .
\]  

(14)

However, this does not fit Eq. (13), because the last requires

\[
\mu_2^2 + \mu_3^2 \sim M_P^2, \quad \mu_2^2 \mu_3^2 \sim M_P^4, \quad \text{but} \quad \mu_1^2 \mu_2^2 \mu_3^2 \ll M_P^6 .
\]  

(15)

\(^1\)We refer to the mechanism which allows the reduction of the lightest mass as the 'strong seesaw
mechanism'; while the mechanism that occurs in higher-derivative gravity can be called 'weak seesaw',
only allowing a larger mass to become even larger.
This consideration can be easily extended to the higher number of derivatives, and the result will be always the same. We leave it as an exercise to the interested reader (who can start from the six-derivative case, indeed).

The main conclusion is that the real poles of the propagator can not provide a much smaller mass of the lightest ghost constructed from the coefficients which are all of the Planck order of magnitude. Is it bad or not, from the Physics side? We know that the presence of ghost means potential instability, but in the case of gravity the situation may be different [13], for instance because of the singular nature of non-polynomial theory which escapes the Ostrogradsky instability [10]. Since a consistent theory of quantum or semiclassical gravity without higher derivatives looks impossible, the general situation with stability looks unclear and it makes sense to assume that ghosts exist but for some reason they do not lead to a fast decay of the vacuum and other type of instabilities. The existing explanation for this is related to the huge mass of the ghost [13] (not tachyon! - see [18]) which does not permit the creation of a ghost particle from vacuum without generating Planck-order density of gravitons. From this perspective it is important that the mass of the lightest ghost is protected from the seesaw mechanism even if more derivatives are added to the action (1).

It remains to see what would be phenomenological consequences of the light ghost within the the seesaw mechanism (7). In the next section we consider an example of this kind.

3 Bending of light and seesaw

In the year of 1919 two expeditions, one to Sobral (Brazil) and another to the island of Príncipe (Africa), measured the position of stars close to solar disk during an eclipse, confirming the prediction of general relativity, that gravity affects the propagation of light. In the fourth derivative gravity the light bending is more complicated, since there is a qualitatively new dependence on the energy of the photon [15] (see also further references therein). Let us consider the light deflection in the context of the sixth-order gravity model (3). We shall use the semiclassical approach, by considering the photon as a quantum particle which interacts with the classical external gravitational field. As we have already noted, in the weak-field regime only $\beta$ and $B$ are relevant, for they describe the tensor mode.

The scattering of a photon by a classical external gravitational field, as depicted in
Figure 1: Photon scattering by an external gravitational field. Here $|p| \approx |p'|$.

Fig. 1 is described by the vertex function

$$V_{\mu\nu}(p, p') = \frac{\kappa}{2} h^{\lambda\delta}_{\text{ext}}(k) [-\eta_{\mu\nu}\eta_{\lambda\rho} p \cdot p' + \eta_{\lambda\rho} p_\mu p_\nu + 2 (\eta_{\mu\nu} p_\lambda p'_\rho - \eta_{\nu\rho} p_\lambda p'_\mu - \eta_{\mu\lambda} \eta_{\nu\rho} p \cdot p')] .$$

(16)

For the photon $p_\mu p^\mu = E^2 - p^2 = 0 = p'_\mu p'^\mu$. Assuming a very small energy exchange between the photon and the gravitational field, it follows that $|p| \approx |p'|$.

The Feynman amplitude for this process is

$$\mathcal{M}_{rr'} = V_{\mu\nu}(p, p') e^\mu_r(p) e^{r'}_{r'}(p'),$$

(17)

where $e^\mu_r(p)$ and $e^{r'}_{r'}(p')$ are the polarization vectors for the ingoing and outgoing photons, respectively, which satisfy the completeness relation ($n^2 = 1$)

$$\sum_{r=1}^2 e^\mu_r(p) e^{r'}_r(p) = -\eta^{\mu\nu} - \frac{p_\mu p'^\nu}{(p \cdot n)^2} + \frac{p_\mu n^n + p'^\nu n^\nu}{p \cdot n} .$$

(18)

Taking (17) and (18) into account, the unpolarized cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{2(4\pi)^2} V_{\mu\nu} V^{\mu\nu} .$$

(19)

The simple calculation provides the expression

$$\frac{d\sigma}{d\Omega} = \frac{\kappa^4 M^2 E^4 (1 + \cos \theta)^2}{(16\pi)^2} \left[ \frac{1}{k^2} - \frac{1}{m^2_{2-} - m^2_{2+}} \left( \frac{m^2_{2-}}{k^2 + m^2_{2+}} - \frac{m^2_{2+}}{k^2 + m^2_{2-}} \right) \right]^2 ,$$

(20)

where $E = E'$ is the energy of the photon and $\theta$ is the deflection angle, i.e. the angle between $p$ and $p'$. For small angles one can set $k^2 \approx 2p^2 (1 - \cos \theta) \approx E^2 \theta^2$ and the previous expression reduces to

$$\frac{d\sigma}{d\Omega} = 16G^2 M^2 \left[ \frac{1}{\theta^2} + \frac{E^2}{m^2_{2-} - m^2_{2+}} \left( \frac{m^2_{2+}}{E^2 \theta^2 + m^2_{2-}} - \frac{m^2_{2-}}{E^2 \theta^2 + m^2_{2+}} \right) \right]^2 = -b \frac{db}{\theta d\theta} .$$

(21)
In the last expression \( b \) is the impact parameter of the photon. The last equation can be integrated yielding

\[
\frac{1}{\theta_E^2} = \frac{1}{\theta^2} + \frac{E^2}{(m_{2-}^2 - m_{2+}^2)^2} \left( \frac{m_{2-}^4}{E^2 \theta^2 + m_{2+}^2} + \frac{m_{2+}^4}{E^2 \theta^2 + m_{2-}^2} \right)
\]

\[
+ \frac{2E^2}{m_{2-}^2 - m_{2+}^2} \ln \left( \frac{E^2 \theta^2}{E^2 \theta^2 + m_{2+}^2} \right) - \frac{m_{2+}^2}{m_{2-}^2} \ln \left( \frac{E^2 \theta^2}{E^2 \theta^2 + m_{2-}^2} \right)
\]

\[
- \frac{m_{2-}^2 m_{2+}^2}{(m_{2-}^2 - m_{2+}^2)^2} \ln \left( \frac{E^2 \theta^2 + m_{2-}^2}{E^2 \theta^2 + m_{2+}^2} \right),
\]

where \( \theta_E = 4GM/b \) is the scattering angle in Einstein’s gravity.

The quantitative analysis of the light deflection in the sixth-order gravity can be performed by solving Eq. (22) with respect to \( \theta \), or directly through the cross-section formula (21), which is technically simpler. The first observation is that the propagation of photons in this model is dispersive, i.e., it depends on the energy of the photon, in the same way as in fourth-derivative gravity [15]. There are, however, some new aspects which are typical for the six-derivative models. The ghost \( m_{2+} \) gives opposite-sign effect compared to the massless graviton and to the normal massive particle \( m_{2-} \). As far as \( m_{2+} < m_{2-} \), the total effect is a smaller deflection compared to the Einstein gravity.

The effect of both particles is related to the ratio \( E/m_{\pm 2} \) and one can distinguish different cases.

(i) For the scattering of photons with transplanckian energies \( E \gg m_{2-} \), there will not be deflection at all, since the effect of the repulsive interaction with the ghost cancels that of the healthy tensor modes. One can note that this is similar to the situation with the removal of singularity in the modified Newtonian potential in the same theories [17].

(ii) On the other extreme of the energy scale, for a low-energy photon the effect of massive modes is completely negligible. The light bending is exactly like in general relativity without higher derivatives.

(iii) A non-trivial effect of massive modes can be seen only for an intermediate scale of energies. In the six-order model one may have a very special situation characterized by the hierarchy (7). Then there will be an interval of energies \( E \) where the effect of normal massive mode \( m_{2-} \sim M_P \) is negligible and only a much lighter ghost is relevant.

In the situation where the seesaw mechanism provides a strong hierarchy, the effect depends on the ratio \( E/m_+ \). For a sufficiently large ratio one can observe that light deflects less than in general relativity; moreover, more energetic photons undergo less
deflection than less energetic ones. Indeed, the following chain of inequalities holds

\[
16G^2M^2 \left( \frac{1}{\theta^2} - \frac{E^2}{E^2\theta^2 + m_{2+}^2} \right)^2 < \frac{d\sigma}{d\Omega} < \frac{16G^2M^2}{\theta^2} = \left( \frac{d\sigma}{d\Omega} \right)_{GR}
\]

between the cross-sections for the sixth-order gravity with and without seesaw (7), and the one in general relativity.

In order to have an idea about the magnitude of the effect, let us consider the phenomenology for the seesaw case with \( m_{2-} \sim M_P \gg m_{2+} \). In this situation one can disregard Yukawa terms of mass \( m_{2-} \) and Eq. (22) simplifies to

\[
\frac{1}{\theta_E^2} = \frac{1}{\theta^2} + \frac{1}{\theta^2 + \frac{m_{2-}^2}{E^2}} + 2E^2 \ln \frac{\theta^2 + \frac{m_{2-}^2}{E^2}}{\theta^2 + \frac{m_{2+}^2}{E^2}},
\]

which is the same that occurs in the fourth-order gravity [15]. Following the analysis of the aforementioned work, comparison of Eq. (24) with observational data collected during solar eclipses [19, 20] implies that \( m_{2+} \gtrsim 10^{-13} GeV \).

Since the scattering of photons depends on their energy, this limit can be increased if measurements of light deflection close to the solar limb are carried out in higher frequency region of the electromagnetic spectrum.

Let us note that the measurements which lead to the aforementioned limit are those carried out in the visible spectrum. From Eddington’s 1919 pioneer expedition up to the 1960s, astrometry of stars close to the solar disk during eclipses were based on photographic plates, which had a broad spectral sensibility window near the visible (closer to the violet) and yielded the determination of deflection angles with the uncertainty of 0.1” at best. Precision and accuracy of this kind of measurement could only be increased after the 1960s, when new interferometric techniques of radio waves were developed making it possible to determine deflection angles up to the precision of 0.01”. Notwithstanding their much better precision, these measurements cannot perhaps yield a sufficiently good lower-bound on \( m_{2+} \), because they are carried out in the radio frequencies of the order of 10 GHz [21, 22].

In order to arrive at numerical estimates one can use Eq. (24) to express the ratio \((m_{2+}/E)^2\) in terms of a fixed angle \( \theta \). If we set, for example, \( \theta = \theta_E - 0.1" = 1.65" \) for a photon grazing the Sun, Eq. (24) yields

\[
\frac{m_{2+}^2}{E^2} = 4.30 \times 10^{-9}.
\]

By means of this relation it is possible to quantify how energetic the observed photons should be in order to detect a deviation of 0.1” caused by a ghost excitation of a fixed
mass $m_{2+}$. Also, if the light deflection data are available at a frequency $\nu$, it is possible to determine a lower bound to the mass of the ghost. Indeed, a deviation of 0.1” from the prediction of general relativity for the deflection of photons grazing the Sun could only be detected at the frequency of $7.5 \times 10^{14}$ Hz (visible spectrum, violet) if $m_{2+} \gtrsim 10^{-13}$GeV. For the radio frequencies, to the order of 10 GHz (see, e.g., Refs. [21, 22]), such detection would occur if $m_{2+} \gtrsim 10^{-18}$GeV. Both these limits are completely our of realistic situations and only show that one has to deal with much higher frequencies to have a chance to detect massive modes in the higher-derivative gravity.

Conversely, in order to detect, with an accuracy of 0.1”, a deviation from Einstein’s prediction caused by a ghost with mass $m_{2+} = 10^{-6}$GeV it would be necessary to observe deflection of photons whose frequency is at least to the order of $10^{21}$ Hz. If deflection close to the solar limb could be measured with such an accuracy at the frequency of $10^{27}$ Hz (on the order of 10 TeV, the same order as the higher energy emissions by the Crab Pulsar [23]), the corresponding lower-bound would be $m_{2+} \gtrsim 10^{-1}$GeV – for instance, the Planck mass is on the order of $M_P \sim 10^{19}$GeV. We note, however, that obtaining such an astrometric accuracy at this frequency is a tough experimental challenge; not to mention the difficulties which arise from making measurements close to the solar disk. Furthermore, an important issue that has to be taken into account is the morphology of the emission region, which may extend to a few dozens of arcseconds [23], making the measurement of light deflection for these sources an extremely difficult task. At the same time, detecting the frequency-dependence of the light bending is the most promising way to detect higher derivatives and one can hope to achieve the corresponding data in the (maybe not very close) future.

Finally, the possibility to detect an effect in the case without seesaw, with the lightest ghost of the Planck-order mass, is far beyond the expectations for the experimental capabilities nowadays and in the near future. Anyway, we consider important to have the theoretical estimates for the corresponding measurements, which we obtained here.

4 Conclusions and discussions

We have described a qualitatively new gravitational seesaw mechanism which might be possible in the higher derivative gravity models with the number of derivatives $\geq 6$. These theories are characterized by a discrete spectrum of “masses” which may be real or complex.

If the dimensional parameters of the action have Planck order of magnitude, they could combine, in principle, in such a way that one of the masses is still on the order
of $M_P$ while another is many orders of magnitude smaller. As we have seen above, such a strong gravitational seesaw is not possible. An essential reduction of the mass of the lightest ghost can be achieved only by adjusting the four-derivative term in the action. Adding the six-derivative terms does not modify the situation in this part.

A strong reduction of mass of the lightest tensor ghost can be achieved by taking a huge value of the dimensionless parameter $\beta$, exactly like in the four-derivative gravity. This situation is qualitatively similar to the one in the extra-dimensional theories. The difference is that here the reduction of Planck suppression occurs due to choice of $\beta$ and not because of the incomplete compactification of some extra dimensions. For the sake of completeness we explored the bending of light in this kind of theory in the Sect. 3.

In the more realistic case of complex poles the situation is more complicated, in particular there may exist a strong dependence not only on the energy of the photon, but also on the impact parameter. Let us note that the seesaw mechanism may be possible in this case, albeit its nature is quite different and detailed calculations and analysis require much more efforts [16]. Taking into account the possibility to have a continuous mass spectrum of the models such as [9], the main conclusions which we can draw at the moment are as follows:

(i) A strong gravitational seesaw does not work in the same way like in neutrino Physics. One can not reduce the lightest ghost mass by tuning the parameters of the higher derivative action (1), except the dimensionless parameter $\beta$.

(ii) Since a huge value of the dimensionless parameter $\beta$ can not be completely ruled out theoretically, it is important to derive the corresponding upper limits from experimental and observational sides. In the present work we made some step in this direction.

(iii) Our results indicate the importance of developing experimental facilities for higher precision measurements of the gravitational bending of light. It is especially relevant to explore the frequency-dependence, including for the high-energy photons. On the other hand, the observation of frequency-dependent gravitational lensing would provide useful information about higher derivatives in gravity.

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