SOFT TERMS FROM DILATON/MODULI SECTORS

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We study the structure of the soft SUSY-breaking terms obtained from large classes of 4-D Strings under the assumption of dilaton/moduli dominance in the process of SUSY-breaking. In particular, we first analyze in detail the dilaton-dominated limit because of its finiteness properties and phenomenological predictivity, and second, we consider the new features appearing when several moduli fields contribute to SUSY breaking. Although some qualitative features indeed change in the multimoduli case with respect to the dilaton dominance one, the most natural mass relations at low-energy \( m_l < m_q \approx M_g \) are still similar. We also study the presence of tachyons pointing out that their possible existence may be, in some cases, an interesting advantage in order to break extra gauge symmetries. Finally, we find that the mechanism for generating a “\( \mu \)-term” by the Kähler potential, as naturally implemented in orbifolds, leads to the prediction \( |tg\beta| = 1 \) at the String scale, independently of the Goldstino direction. In this connection, it is worth noticing that in the dilaton-dominated case we obtain the remarkable result that the whole SUSY spectrum is completely determined with no free parameters.

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1 Introduction

Recently there has been some activity in trying to obtain information about the structure of soft Supersymmetry (SUSY)-breaking terms in effective \( N = 1 \) theories coming from four-dimensional Strings. The basic idea is to identify some \( N = 1 \) chiral fields whose auxiliary components could break SUSY by acquiring a vacuum expectation value (vev). No special assumption is made about the possible origin of SUSY-breaking. Natural candidates in four-dimensional Strings are 1) the complex dilaton field \( S = \frac{1}{g} + ia \) which is present in any four-dimensional String and 2) the moduli fields \( T^i, U^i \) which parametrize the size and shape of the compactified variety in models obtained by compactification of a ten-dimensional heterotic String.

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The important point in this assumption of locating the seed of SUSY-breaking in the dilaton/moduli sectors, is that it leads to some interesting relationships among different soft terms which could perhaps be experimentally tested.

In ref. a systematic discussion of the structure of soft terms which may be obtained under the assumption of dilaton/moduli dominated SUSY breaking in some classes of four-dimensional Strings was presented, with particular emphasis on the case of Abelian (0, 2) orbifold models. It was mostly considered a situation in which only the dilaton $S$ and an “overall modulus $T$” field contribute to SUSY-breaking. In fact, actual four-dimensional Strings like orbifolds contain several $T_i$ and $U_i$ moduli. Thus it is natural to ask what changes if one relaxes the overall modulus hypothesis and works with the multimoduli case. This is one of the purposes of the present talk. The second one is to analyze in more detail the dilaton-dominated limit, where only the dilaton field contributes to SUSY breaking. This is a very interesting possibility not only due to phenomenological reasons, as universality of the soft terms, but also to theoretical arguments. In this connection it is worth noticing that the boundary conditions $-A = M_{1/2} = \sqrt{3}m$ of dilaton dominance coincide with some boundary conditions considered by Jones, Mezincescu and Yau in 1984. They found that those same boundary conditions maintain the (two-loop) finiteness properties of certain $N = 1$ SUSY theories. This could perhaps be an indication that at least some of the possible soft terms obtained in the present scheme could have a more general relevance.

In section 2 we present an analysis of the effects of several moduli on the results obtained for soft terms. In the multimoduli case several parameters are needed to specify the Goldstino direction in the dilaton/moduli space, in contrast with the overall modulus case where the relevant information is contained in just one angular parameter $\theta$. The presence of more free parameters leads to some loss of predictivity for the soft terms. This predictivity is recovered and increased in the case of dilaton dominance, where the soft terms eq.12 are independent of the 4-D String considered and fulfill the low-energy mass relations given by eq.13. Also we show that, even in the multimoduli case, in some schemes there are certain sum-rules among soft terms eq.16 which hold independently of the Goldstino direction. The presence of these sum rules cause that, on average the qualitative results in the dilaton-dominated case still apply. Specifically, if one insists e.g. in obtaining scalar masses heavier than gauginos (something not possible in the dilaton-dominated scenario), this is possible in the multimoduli case, but the sum-rules often force some of the scalars to get negative squared mass. If we want to avoid this, we have to stick to gaugino masses bigger than (or of order) the scalar masses. This would
lead us back to the qualitative results obtained in dilaton dominance. In the case of standard model 4-D Strings this tachyonic behaviour may be particularly problematic, since charge and/or colour could be broken. In the case of GUTs constructed from Strings, it may just be the signal of GUT symmetry breaking. However, even in this case one expects the same order of magnitude results for observable scalar and gaugino masses and hence the most natural mass relations at low-energy are still similar to the dilaton dominance ones.

Another topic of interest is the $B$-parameter, the soft mass term which is associated to a SUSY mass term $\mu H_1 H_2$ for the pair of Higgses $H_{1,2}$ in the Minimal Supersymmetric Standard Model (MSSM). Compared to the other soft terms, the result for the $B$-parameter is more model-dependent. Indeed, it depends not only on the dilaton/moduli dominance assumption but also on the particular mechanism which could generate the associated “$\mu$-term”).

An interesting possibility to generate such a term is the one suggested in ref. in which it was pointed out that in the presence of certain bilinear terms in the Kähler potential an effective $\mu$-term of order the gravitino mass, $m_{3/2}$, is naturally generated. Interestingly enough, such bilinear terms in the Kähler potential do appear in String models and particularly in Abelian orbifolds. In section 3 we compute the $\mu$ and $B$ parameters as well as the soft scalar masses of the charged fields which could play the role of Higgs particle in such Abelian orbifold schemes. We find the interesting result that, independently of the Goldstino direction in the dilaton/moduli space, one gets the prediction $|t g \beta| = 1$ at the String scale. On the other hand, if we consider the interesting Goldstino direction where only the dilaton breaks SUSY, the whole soft terms and the $\mu$-parameter depend only on the gravitino mass. Imposing the phenomenological requirement of correct electroweak breaking we arrive to the remarkable result that the whole SUSY spectrum is completely determined with no free parameters.

2 Soft terms

2.1 General structure of soft terms: the multimoduli case

We are going to consider $N = 1$ SUSY 4-D Strings with $m$ moduli $T_i$, $i = 1, \ldots, m$. Such notation refers to both $T$-type and $U$-type (Kähler class and complex structure in the Calabi-Yau language) fields. In addition there will be charged matter fields $C_\alpha$ and the complex dilaton field $S$. In general we will

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Footnote: The results for $B$ corresponding to other sources for the $\mu$-term can also be found, for the multimoduli case under consideration, in ref. In particular, the possibility of generating a small $\mu$-term from the superpotential was studied.
be considering \((0, 2)\) compactifications and thus the charged fields do not need to correspond to 27s of \(E_6\).

Before further specifying the class of theories that we are going to consider a comment about the total number of moduli is in order. We are used to think of large numbers of \(T\) and \(U\)-like moduli due to the fact that in \((2, 2)\) \((E_6)\) compactifications there is a one to one correspondence between moduli and charged fields. However, in the case of \((0, 2)\) models with arbitrary gauge group (which is the case of phenomenological interest) the number of moduli is drastically reduced. For example, in the standard \((2, 2)\) \(Z_3\) orbifold there are 36 moduli \(T_i\), 9 associated to the untwisted sector and 27 to the fixed points of the orbifold. In the thousands of \((0, 2)\) \(Z_3\) orbifolds one can construct by adding different gauge backgrounds or doing different gauge embeddings, only the 9 untwisted moduli remain in the spectrum. The same applies to models with \(U\)-fields. This is also the case for compactifications using \((2, 2)\) minimal superconformal models. Here all singlets associated to twisted sectors are projected out when proceeding to \((0, 2)\). So, as these examples show, in the case of \((0, 2)\) compactifications the number of moduli is drastically reduced to a few fields. In the case of generic Abelian orbifolds one is in fact left with only three \(T\)-type moduli \(T_i\) \((i = 1, 2, 3)\), the only exceptions being \(Z_3\), \(Z_4\) and \(Z_6\), where such number is 9, 5 and 5 respectively. The number of \(U\)-type fields in these \((0, 2)\) orbifolds oscillates between 0 and 3, depending on the specific example. Specifically, \((0, 2)\) \(Z_2 \times Z_2\) orbifolds have 3 \(U\) fields, the orbifolds of type \(Z_4, Z_6, Z_8, Z_2 \times Z_4, Z_2 \times Z_6\) and \(Z_1^2\) have just one \(U\) field and the rest have no untwisted \(U\)-fields. Thus, apart from the three exceptions mentioned above, this class of models has at most 6 moduli, three of \(T\)-type (always present) and at most three of \(U\)-type. In the case of models obtained from Calabi-Yau type of compactifications a similar effect is expected and only one \(T\)-field associated to the overall modulus is guaranteed to exist in \((0, 2)\) models.

We will consider effective \(N = 1\) supergravity (SUGRA) Kähler potentials of the type:

\[
K(S, S^*, T_i, T_i^*, C_\alpha, C_\alpha^*) = -\log(S + S^*) + \hat{K}(T_i, T_i^*) + \hat{K}_{\alpha\beta}(T_i, T_i^*)C_\alpha^*C_\beta + (Z_{\alpha\beta}(T_i, T_i^*)C_\alpha^*C_\beta + h.c.) .
\]

The first piece is the usual term corresponding to the complex dilaton \(S\) which is present for any compactification whereas the second is the Kähler potential of the moduli fields, where we recall that we are denoting the \(T\)- and \(U\)-type moduli collectively by \(T_i\). The greek indices label the matter fields and their kinetic term functions are given by \(\hat{K}_{\alpha\beta}\) and \(Z_{\alpha\beta}\) to lowest order in the matter fields. The last piece is often forbidden by gauge invariance in specific models although it may be relevant in some cases as discussed in section 3.
In this section we are going to consider the case of diagonal metric both for
the moduli and the matter fields \( \tilde{K} \). Then \( \hat{K}(T_i, T_i^\tau) \) will be a sum of contri-
butions (one for each \( T_i \)), whereas \( \hat{K}_{ij} \) will be taken of the diagonal form
\( \hat{K}_{ij} \equiv \delta_{ij} \hat{K}_n \). The complete \( N = 1 \) SUGRA Lagrangian is determined by the
Kähler potential \( K(\phi_M, \phi_M^\star) \), the superpotential \( W(\phi_M) \) and the gauge kinetic
functions \( f_a(\phi_M) \), where \( \phi_M \) generically denotes the chiral fields \( S, T_i, C_{\alpha} \). As
is well known, \( K \) and \( W \) appear in the Lagrangian only in the combination
\( G = K + \log |W|^2 \). In particular, the (F-part of the) scalar potential is given
by
\[
V(\phi_M, \phi_M^\star) = e^G \left( G_M K_M^N S_N - 3 \right),
\]
where \( G_M \equiv \partial_M G \equiv \partial G / \partial \phi_M \) and \( K_M^N \) is the inverse of the Kähler metric
\( K_N^M = \partial_N \partial_M K \).

The crucial assumption now is to locate the origin of SUSY-breaking in
the dilaton/moduli sector. Let us take the following parametrization for the
vev’s of the dilaton and moduli auxiliary fields
\[
F_S = e^{G/2} \left( G_{SS} - \frac{1}{2} \Theta_S \right), \quad F_i = e^{G/2} G_{ii}^{1/2} G_i,
\]
where \( \sum_i \Theta_i^2 = 1 \) and \( e^G = m^{3/2} \) is the gravitino mass-squared. The angle
\( \theta \) and the \( \Theta_i \) just parametrize the direction of the goldstino in the \( S, T_i \) field
space. We have also allowed for the possibility of some complex phases \( \gamma_S, \gamma_i \)
which could be relevant for the CP structure of the theory. This parametriza-
tion has the virtue that when we plug it in the general form of the SUGRA
scalar potential eq.(2), its vev (the cosmological constant) vanishes by construc-
tion. Notice that such a phenomenological approach allows us to ‘reabsorb’ (or
circumvent) our ignorance about the (nonperturbative) \( S \)- and \( T_i \)- dependent
part of the superpotential, which is responsible for SUSY-breaking. It is now
a straightforward exercise to compute the bosonic soft SUSY-breaking terms
in this class of theories. Plugging eqs.\( \text{(3)} \) and \( \text{(1)} \) into eq.\( \text{(2)} \) one finds the
following results (we recall that we are considering here a diagonal metric for
the matter fields):
\[
m^2 = m^{2/3} \left[ 1 - 3 \cos^2 \theta \left( \hat{K}^{-1/2}_{ii} \Theta_i e^{i\gamma_i} (\log \hat{K}_{ii} \hat{K}^{-1/2}_{ij} \Theta_j e^{-i\gamma_j}) \right) \right],
\]
\[
A_{\alpha\beta\gamma} = -\sqrt{3}m^{3/2} \left[ e^{-i\gamma_S} \sin \theta \right.

- \left. e^{-i\gamma_i} \cos \theta \Theta_i (\hat{K}^{-1/2}_{ii} \right)

\left. \left( \hat{K}_i - \sum_{\delta=\alpha,\beta,\gamma} (\log \hat{K}_{\delta}) i + (\log h_{\alpha\beta\gamma}) i \right) \right].
\]
\( ^d \)An extensive analysis of the off-diagonal case, including the calculation of the soft terms
and their effects on flavour changing neutral currents (FCNC), can be found in ref.\( \text{(4)} \).
The above scalar masses and trilinear scalar couplings correspond to charged fields which have already been canonically normalized. Here \( h_{\alpha\beta\gamma} \) is a renormalizable Yukawa coupling involving three charged chiral fields and \( A_{\alpha\beta\gamma} \) is its corresponding trilinear soft term.

Physical gaugino masses \( M_a \) for the canonically normalized gaugino fields are given by
\[
M_a = \frac{1}{2}(\text{Re} f_a)^{-1} e^{G/2} f_a M^{MN} G_N.
\]
Since the tree-level gauge kinetic function is given for any 4-D String by \( f_a = k_a S \), where \( k_a \) is the Kac-Moody level of the gauge factor, the result for tree-level gaugino masses is independent of the moduli sector and is simply given by:
\[
M_a \equiv m_{3/2} \sqrt{3} \sin \theta e^{-i\gamma_S}.
\]

The soft term formulae above are in general valid for any compactification as long we are considering diagonal metrics. In addition one is tacitly assuming that the tree-level Kähler potential and \( f_a \)-functions constitute a good approximation. The Kähler potentials for the moduli are in general complicated functions. Before going into specific classes of Superstring models, it is worth studying the interesting limit \( \cos \theta = 0 \), corresponding to the case where the dilaton sector is the source of all the SUSY-breaking (see eq. (3)).

2.2 The \( \cos \theta = 0 \) (dilaton-dominated) limit

Since the dilaton couples in an universal manner to all particles, this limit is quite model independent. Using eqs. (4,5) and neglecting phases one finds the following simple expressions for the soft terms which are independent of the 4-D String considered
\[
m_\alpha = m_{3/2},
A_{\alpha\beta\gamma} = -\sqrt{3} m_{3/2},
M_a = \sqrt{3} m_{3/2}.
\]

This dilaton-dominated scenario is attractive for its simplicity and for the natural explanation that it offers to the universality of the soft terms. Actually, universality is a desirable property not only to reduce the number of independent parameters in the MSSM, but also for phenomenological reasons, particularly to avoid FCNC.

Because of the simplicity of this scenario, the low-energy predictions are quite precise. Since scalars are lighter than gauginos at the String scale, at low-energy (~ \( M_Z \)), gluino, slepton and (first and second generation) squark mass relations turn out to be
\[
M_g : m_Q : m_u : m_L : M_e \simeq 1 : 0.94 : 0.92 : 0.32 : 0.24.
\]
Although squarks and sleptons have the same soft mass, at low-energy the
former are much heavier than the latter because of the gluino contribution to
the renormalization of their masses.

In section 3 we will show that even a stronger result than that of eq. (7)
is obtained in the context of a natural mechanism for solving the µ prob-
lem, namely the whole SUSY spectrum (gluino, squarks, sleptons, Higgses,
charginos, neutralinos) is completely determined with no free parameters.

2.3 Orbifold compactifications

To illustrate some general features of the multimoduli case we will concentrate
here on the case of generic (0, 2) symmetric Abelian orbifolds. As we mentioned
above, this class of models contains three $T$-type moduli and (at most) three
$U$-type moduli. We will denote them collectively by $T_i$, where e.g. $T_i = U_{i-3}$;
i = 4, 5, 6. For this class of models the Kähler potential has the form

$$K(\phi, \phi^*) = - \log(S + S^*) - \sum_i \log(T_i + T_i^*) + \sum_\alpha |C_\alpha|^2 \Pi_i (T_i + T_i^*)^{n_i^\alpha}.$$  \hspace{1cm} (8)

Here $n_i^\alpha$ are fractional numbers usually called “modular weights” of the matt-
the fields $C_\alpha$. For each given Abelian orbifold, independently of the gauge group
or particle content, the possible values of the modular weights are very re-
stricted. For a classification of modular weights for all Abelian orbifolds see ref.2. Using the particular form (8) of the Kähler potential and eqs.(4,5 ) we
obtain the following results\footnote{This analysis was also carried out, for the particular case of the three diagonal moduli $T_i$, in order to obtain unification of gauge coupling constants. Some particular multimoduli examples were also considered in ref.9.} for the scalar masses, gaugino masses and soft
trilinear couplings:

$$m_{\alpha}^2 = m_3^{3/2}(1 + 3 \cos^2 \theta \, n_{\alpha}^i \, \Theta^2) ,$$

$$M = \sqrt{3}m_3^{3/2} \sin \theta e^{-i\gamma S} ,$$

$$A_{\alpha\beta\gamma} = -\sqrt{3}m_3^{3/2} (\sin \theta e^{-i\gamma S} + \cos \theta \sum_{i=1}^6 e^{-i\gamma_i T_i} \omega_{i\alpha\beta\gamma} ) ,$$ \hspace{1cm} (9)

where we have defined :

$$\omega_{i\alpha\beta\gamma} = (1 + n_{\alpha}^i + n_{\beta}^i + n_{\gamma}^i - Y_{i\alpha\beta\gamma}) ; \quad Y_{i\alpha\beta\gamma} = \frac{h_{i\alpha\beta\gamma}}{k_{i\alpha\beta\gamma}} 2 Re T_i .$$ \hspace{1cm} (10)

Notice that neither the scalar nor the gaugino masses have any explicit depen-
dence on $S$ or $T_1$, they only depend on the gravitino mass and the goldstino
angles. This is one of the advantages of a parametrization in terms of such angles. Although in the case of the $A$-parameter an explicit $T_i$-dependence may appear in the term proportional to $Y_{\alpha\beta\gamma}$, it disappears in several interesting cases.

With the above information we can now analyze the different structure of soft terms available for each Abelian orbifold.

1) **Universality of soft terms**

   In the dilaton-dominated case ($\cos \theta = 0$) the whole soft terms are universal. However, in general, they show a lack of universality due to the modular weight dependence (see eqs. (9,10)).

2) **Soft masses**

   In the multimoduli case, depending on the goldstino direction, tachyons may appear. For $\cos^2 \theta \geq 1/3$, one has to be very careful with the goldstino direction if one is interested in avoiding tachyons. Nevertheless, as we will discuss below, having a tachyonic sector is not necessarily a problem, it may even be an advantage, so one should not disregard this possibility at this point.

   Consider now three particles $C_\alpha, C_\beta, C_\gamma$ coupling through a Yukawa $h_{\alpha\beta\gamma}$. They may belong both to the untwisted (U) sector or to a twisted (T) sector, i.e. couplings of the type UUU, UTT, TTT. Then, using the above formulae, one finds that in general for any choice of goldstino direction

   \[ m^2_\alpha + m^2_\beta + m^2_\gamma \leq |M|^2 = 3m_{3/2}^2 \sin^2 \theta \]  

   (11)

   Notice that if we insist in having a vanishing gaugino mass, the sum-rule (11) forces the scalars to be either all massless or at least one of them tachyonic. Nevertheless we should not forget that tachyons, as we already mentioned above, are not necessarily a problem, but may just show us an instability.

3) **Gaugino versus scalar masses**

   In the multimoduli case on average the scalars are lighter than gauginos but there may be scalars with mass bigger than gauginos. Eq. (11) tells us that this can only be true at the cost of having some of the other three scalars with negative squared mass. This may have diverse phenomenological implications depending what is the particle content of the model, as we now explain in some detail:

   3-a) **Gaugino versus scalar masses in standard model 4-D Strings**

   Let us suppose we insist in e.g., having tree-level gaugino masses lighter than the scalar masses. If we are dealing with a String model with gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times G$ this is potentially a disaster. Some observable particles, like Higgses, squarks or sleptons would be forced to acquire large vev’s (of order the String scale). For example, the scalars associated through
the Yukawa coupling $H_2 Q_L u^c_L$, which generates the mass of the $u$-quark, must fulfill the above sum-rule (11). If we allow e.g. the scalars $H_2, Q_L$ to be heavier than gauginos, then $u^c_L$ will become tachyonic breaking charge and color. However, tachyons may be helpful if the particular Yukawa coupling does not involve observable particles. They could break extra gauge symmetries and generate large masses for extra particles. We recall that standard-like models in Strings usually have too many extra particles and many extra $U(1)$ interactions. Although the Fayet-Iliopoulos mechanism helps to cure the problem, the existence of tachyons is a complementary solution.

We thus see that, for standard model Strings, if we want to avoid charge and colour-breaking minima (or vev’s of order the String scale for the Higgses), we should grosso modo come back to a situation with gauginos heavier than scalars. Thus the low-energy phenomenological predictions of the multimoduli case are similar to those of the dilaton-dominated scenario (see subsect.2.2): due to the sum-rule the tree-level observable scalars are always lighter than gauginos

$$m_\alpha < M$$

Now, at low-energy ($\sim M_Z$), gluino, slepton and (first and second generation) squark mass relations turn out to be

$$m_t < m_q \simeq M_g$$

where gluinos are slightly heavier than scalars. This result is qualitatively similar to the dilaton dominance one, in spite of the different set of (non-universal) soft scalar masses, because the low-energy scalar masses are mainly determined by the gaugino loop contributions. The only exception are the sleptons masses, which do not feel the important gluino contribution, and therefore can get some deviation from the result of eq.(11).

Although String loop corrections, in the particular case that at tree-level $M_\alpha \to 0$ and $m_\alpha \to 0$, can yield scalars heavier than gauginos, it was shown in ref. that this possibility is a sort of fine-tuning. Non-renormalizable Yukawa couplings can also yield scalars heavier than gauginos. However it was shown in ref. that still at low-energy eq.(13) is typically valid, the only difference being that now squarks will be slightly heavier than gluinos.

3-b) Gaugino versus scalar masses in GUT 4-D Strings

What it turned out to be a potential disaster in the case of standard model Strings may be an interesting advantage in the case of String-GUTs. In this case it could well be that the negative squared mass may just induce gauge symmetry breaking by forcing a vev for a particular scalar (GUT-Higgs field) in the model. The latter possibility provides us with interesting phenomenological
consequences. Here the breaking of SUSY would directly induce further gauge symmetry breaking. An explicit example of this situation can be found in ref.\textsuperscript{2}.

Let us finally remark that, in spite of the different possibilities of soft masses in the multimoduli case, the most natural (slepton-squark-gluino) mass relations \textit{at low-energy} will be similar to the ones of the dilaton-dominated case eq.(13) as showed in point 3-a.

3 The B-parameter and the $\mu$ problem

It was pointed out in ref.\textsuperscript{5} that terms in a Kähler potential like the one proportional to $Z_{\alpha\beta}$ in eq.(1) can naturally induce a $\mu$-term for the $C_\alpha$ fields of order $m_{3/2}$ after SUSY-breaking, thus providing a rationale for the size of $\mu$. Recently it has been suggested that such type of terms may appear in the Kähler potential of some Calabi-Yau type compactifications\textsuperscript{7}. It has also been explicitly shown\textsuperscript{12} that the untwisted sector of orbifolds with at least one complex-structure field $U$ possesses the required structure $Z(T_i, T^*_i)C_1C_2 + h.c.$ in their Kähler potentials. Specifically, the $Z_N$ orbifolds based on $Z_4, Z_6, Z_8, Z_{12}$ and the $Z_N \times Z_M$ orbifolds based on $Z_2 \times Z_4$ and $Z_2 \times Z_6$ do all have a $U$-type field in (say) the third complex plane. In addition the $Z_2 \times Z_2$ orbifold has $U$ fields in the three complex planes. In all these models the piece of the Kähler potential involving the moduli and the untwisted matter fields $C_1, C_2$ in the third complex plane has the form

$$K = -\log(T_3 + T^*_3) - \log(U_3 + U^*_3) + \frac{(C_1 + C^*_2)(C^*_1 + C_2)}{(T_3 + T^*_3)(U_3 + U^*_3)} \quad (14)$$

where one can easily identify the functions $Z, \tilde{K}_1, \tilde{K}_2$ associated to $C_1$ and $C_2$:

$$Z = \tilde{K}_1 = \tilde{K}_2 = \frac{1}{(T_3 + T^*_3)(U_3 + U^*_3)} \quad (15)$$

Plugging back these expressions in eqs.(13) one can compute $\mu$ and $B$ for this interesting class of models\textsuperscript{2,3}:

$$\mu = m_{3/2} \left(1 + \sqrt{3} \cos \theta (e^{i\gamma_3} \Theta_3 + e^{i\gamma_6} \Theta_6)\right) \quad (16)$$

$$B\mu = 2m_{3/2} \left(1 + \sqrt{3} \cos \theta (\cos \gamma_3 \Theta_3 + \cos \gamma_6 \Theta_6) + 3 \cos^2 \theta \cos(\gamma_3 - \gamma_6) \Theta_3 \Theta_6\right) \quad (17)$$

In addition, we recall from eq.(10) that the soft masses are

$$m_{C_1}^2 = m_{C_2}^2 = m_{3/2}^2 \left(1 - 3 \cos^2 \theta (\Theta_3^2 + \Theta_6^2)\right) \quad (18)$$
In general, the dimension-two scalar potential for $C_{1,2}$ after SUSY-breaking has the form

$$V_2(C_1, C_2) = (m_{C_1}^2 + |\mu|^2)|C_1|^2 + (m_{C_2}^2 + |\mu|^2)|C_2|^2 + (B\mu C_1 C_2 + h.c.) \quad (19)$$

In the specific case under consideration, from eqs. (16,17,18) we find the remarkable result that the three coefficients in $V_2(C_1, C_2)$ are equal, i.e.

$$m_{C_1}^2 + |\mu|^2 = m_{C_2}^2 + |\mu|^2 = B\mu \quad (20)$$

Although the common value of the three coefficients in eq.(20) depends on the Goldstino direction via the parameters $\cos\theta, \Theta_3, \Theta_6, \ldots$ (see expression of $B\mu$ in eq.(17)), we stress that the equality itself holds *independently of the Goldstino direction*.

It is well known that, for a potential of the generic form (19) (+D-terms), the minimization conditions yield

$$\sin 2\beta = - \frac{2B\mu}{m_{C_1}^2 + m_{C_2}^2 + 2|\mu|^2}. \quad (21)$$

In particular, this relation embodies the boundedness requirement: if the absolute value of the right-hand side becomes bigger than one, this would indicate that the potential becomes unbounded from below. As we have seen, in the class of models under consideration the particular expressions of the mass parameters lead to the equality (20), which in turn implies $\sin 2\beta = -1$. Thus one finds $\tan \beta = \frac{<C_2>}{<C_1>} = -1$ for any value of $\cos\theta, \Theta_3, \Theta_6$ (and of the other $\Theta_i$'s of course), i.e. for any Goldstino direction.

As an additional comment, it is worth recalling that in previous analyses of the above mechanism for generating $\mu$ and $B$ in the String context\(^7\), the value of $\mu$ was left as a free parameter since one did not have an explicit expression for the function $Z$. However, if the explicit orbifold formulae for $Z$ are used, one is able to predict both $\mu$ and $B$ reaching the above conclusion\(^7\).

Now that we have computed explicitly the whole soft terms and the $\mu$ parameter, it would be interesting to analyze the dilaton-dominated scenario ($\cos\theta = 0$) because of its predictivity. In particular, from eqs. (16,17) we obtain

$$m_\alpha = m_{3/2},$$

$$A_{\alpha\beta\gamma} = -\sqrt{3}m_{3/2}.$$  

\(^7\)We should add that situations are conceivable where the above result may be evaded, for example if the physical Higgs doublets are a mixture of the above fields with some other doublets coming from other sectors (e.g. twisted) of the theory.
\[ M_a = \sqrt{3} m_{3/2} , \]
\[ B = 2 m_{3/2} , \]
\[ \mu = m_{3/2} . \]  

(22)

and therefore the whole SUSY spectrum depends only on one parameter \((m_{3/2})\). If we would know the particular mechanism which breaks SUSY, then we would be able of computing the superpotential and hence \(m_{3/2} \equiv e^K |W|\). However, still this parameter can be fixed from the phenomenological requirement of correct electroweak breaking \(2M_W/g_2^2 = <H_1>^2 + <H_2>^2\). Thus at the end of the day we are left with no free parameters. The whole SUSY spectrum is completely determined in this scenario. This is a remarkable result which deserves further investigation\(^\text{13}\). Of course, if in the next future the mechanism which breaks SUSY is known (i.e. \(m_{3/2}\) can be explicitly calculated) and the above scenario is the correct one, the value of \(m_{3/2}\) should coincide with the one obtained from the phenomenological constraint.

It is worth noticing here that although the value of \(\mu\) is compactification dependent even in this dilaton-dominated scenario \(\mu = m_{3/2}(\tilde{K}_1\tilde{K}_2)^{-1/2}Z\), the result of eq.\((22)\), \(\mu = m_{3/2}\), will be obtained in any compactification scheme with the following property: \(\tilde{K}_1 = \tilde{K}_2 = Z\). Of course, this is the case of orbifolds as was shown in eq.\((13)\).

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