Drug trading analysis model based on weight graph, correlation analysis and Leontief function

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Abstract. Our paper mainly studies the integrated application of weight graph, correlation coefficient and Leontief function to regional drug trading. To be better understood, we establish our model on the analysis of drug transaction among 5 states in America, including Ohio, Kentucky, West Virginia, Virginia and Pennsylvania. First, we build the weight graph with 5 states as vertexes to calculate the transaction strength of each state with others to predict the drug diffusion trend. And based on equilibrium analysis, we find that only the drug market in Ohio hasn’t reached balanced state, which means the drug sale will increases. Second, we select 13 socio-economic indicators, aiming to use correlation analysis to reflect their influence on drug diffusion. We find that more incomplete households, more population with bad marital status and with low educational attainment probably ascend the drug cases. Third, we apply Leontief function to analysing how the drug production change when the quantity of material, transportation, fixed capital, workforce and demand, 5 main parts in the complete industrial chain are changing. Therefore, it may help to come up with some practical ways of controlling the illegal drug transaction.

1. Introduction

1.1. Background

The United States has been experiencing the crisis regarding the use of synthetic and non-synthetic opioids. According to National Institute on Drug Abuse (NIH) [1], in 2017, there are 5 states rank first to fifth in terms of opioid-involved overdose death rates (per 100000 people): West Virginia (49.6), Ohio (39.2), Washington D.C. (34.7), New Hampshire (34.0), Maryland (32.2), all over 30. Worse still, according to the data given by National Safety Council (NSC), in 2017, the proportion of people killed because of abusing synthetic opioids is 1/96, higher than that caused by the traffic accidents, which is 1/106 [2]. How to cope with drug abuse and illegitimate drug transaction has become an issue of great concern.

Numbers of bibliographies studying the abuse of opioids in the US. For example, Cicero, Inciardi and Muñoz described the trend in opioid abuse based on RADARS\textsuperscript{®} (the Researched Abused, Diversion and Addiction-Related Surveillance system) [3]; Jones, PharmD, Muhuri, and Lurie studied the prevalence of nonmedical use of opioids among people by demographic and geographic characteristics based on logistic regression [4]; Volkow, Jones, Einstein, Wargo analysed the factors triggering the opioid crisis along with the interventions to manage and prevent opioid use disorder [5]. However, most of them made brief prediction of trend in abuse of opioids in different regions instead of scientifically and accurately figuring out the propagation characteristics of opioids based on an ocean of data they have collected. It should be pointed out that drugs are also products, having their
own markets and industrial chains. Therefore, it’s feasible to put forward efficient strategies from an economic perspective.

1.2. Brief statement of our model
Our methods can be divided into 5 steps as follows. To facilitate the narration, we let abbreviations OH, KY, WV, VA, PA respectively represent Ohio, Kentucky, West Virginia, Virginia, Pennsylvania.

- Step 1: Collating data of drug identifications in 5 states
  Data are from the DEA/National Forensic Laboratory Information System (NFLIS), containing the drug identification counts in years 2010-2017 for narcotic analgesics (synthetic opioids) and heroin in each county from these five states (OH, KY, WV, VA, PA). Since the data are given in county level, we need to sum up the numbers of drug cases in each county as the index of one state in each year for convenience. (Actually, in the following discussion, these numbers will be regarded as the amount of drug consumption/demand since more drug cases means more drug abusers or addicts needing opioids, which will ascend the drug sale.)

- Step 2: Establishment of weighted undirected graph [6] among 5 states
  Regarding 5 states as 5 vertexes, we build a weight graph, where the weight of each edge represents the drug transaction strength between 2 states, positively related to the number of drug cases but negatively related to the distance between 2 states. By calculation, we get 8 weight matrices from 2010 to 2017 about the graph, based on which the opioid diffusion strength from one state to another one can be figured out.

- Step 3: Analysis of balance between supply and demand
  Getting 8 matrices in Step 2, we’re able to figure out the ratio of drug transaction strength (In a way, the drug supply) from 5 states (including itself) to one. Then we get 8 ratio matrices to create the balance equation between supply and demand so that we can figure out the theoretical production in the balanced stage for further prediction.

- Step 4: Social factors may trigger abuse of drug
  Data are from the extracts from the U.S. Census Bureau, containing a common set of socio-economic factors collected for the counties of 5 states. Calculating the correlation coefficients [7] between drug identification counts and different social factors helps us find out those have strong connection with drug diffusion.

- Step 5: Building the Leontief function [8] of opioid industrial chain
  Apart from the social factors, the input of materials, transportation, fixed capital and workforce are vital in opioid production. Analysing the Leontief function, we can find out the dominant part in the opioid industrial chain, by destroying which can we control the drug production and cut off the diffusion.

Step 1 to 3 aims to summarize the characteristic of drug spread. Step 4 and 5 aims to search for strategies of prevent drug from diffusing. More details will be given during the realization of our model.

2. The tendency of opioid spread

2.1. Previous analysis
To get the drug identification counts of each state in each year based on the data provided by NFLIS, we let \( n'_{i,k} \) represents the number of drug cases in the \( k \)th county in state \( i \) in year \( t \). Particularly, \( i = 1 \sim 5 \) represents KY, OH, PA, VA, WV, \( t = 1 \sim 8 \) represents year 2010 to 2017. Then we get the number of drug cases of state \( i \) in year \( t \) as \( n'_i \) using the formula as follows:

\[
n'_i = \sum_k n'_{i,k}
\]  

(1)

Then we depict the image about \( t \) and \( n'_i \) (total drug case number of state \( i \) in year \( t \)) as follow.
Figure 1. Drug Case Number of 5 States from 2010 to 2017.

We can see from Figure 1 that, except for the rising trend of drug reports in Ohio, almost all drug reports in other states have been flat or falling. This result is vital for our further analysis.

2.2. Characteristic of drug trafficking based on the undirected weighted graph
To be better understood, we intercept a section containing the location of 5 states from Google Map and build an edge and vertex graph on it, shown as Figure 2.

Figure 2. Map of 5 States.

Besides, we get the distance between each two states from Google Map shown in Table 1 for the following calculation. Particularly, the distance from one state to itself is the diameter of it.

Table 1. Distance between 2 States / mile.

|     | KY  | OH  | PA  | VA  | WV  |
|-----|-----|-----|-----|-----|-----|
| KY  | 113 | 186 | 562 | 513 | 198 |
| OH  | 186 | 119 | 367 | 479 | 162 |
| PA  | 562 | 367 | 121 | 220 | 366 |
| VA  | 513 | 479 | 220 | 116 | 317 |
| WV  | 198 | 162 | 366 | 317 | 87  |

We define the weighted undirected graph \([6]\) in year \(t\) as \(G(V, E')\), in which \(V\) represents the set of 5 vertexes (also 5 states), and \(E' = (e'_{ij})\) is a \(5 \times 5\) weighted adjacency matrix \([6]\) consisting of weight between each two vertexes. Now we create a brief formula about the weight as follows.
\[ e'_{ij} = c \frac{(n'_i)^a (n'_j)^b}{d_{ij}}, (i, j = 1 - 5) \]  

\( n_i \) represents the number of drug case of state \( i \) and \( d_{ij} \) represents the distance between state \( i, j \). It’s obvious that \( E' \) is symmetric but its diagonal elements are nonzero since each state isn’t technically a vertex and drug trafficking also occurs within it. It’s reasonable because basically, with high drug production and short distance, the drug transaction between 2 states occurs more frequently. To simplify the calculation, we let 3 coefficients \( a = b = c = 1 \). In a way, \( e'_{ij} \) represents the drug trafficking strength between 2 states. Based on \( E' \), we need to calculate the total drug diffusion strength from state \( i \) to the other four ones in year \( t \). We define this index as \( m'_i \):

\[ m'_i = \sum_{j \neq i} e'_{ij} \]  

Then we depict the image about \( i \) and \( m'_i \) (total drug diffusion strength of state \( i \)) as follow.

**Table 2.** Total Diffusion Strength of 5 States from 2010 to 2017.

|    | KY            | OH            | PA            | VA            | WV            |
|----|---------------|---------------|---------------|---------------|---------------|
| 2010| 0.19718153    | 0.38646231    | 0.41233987    | 0.26628894    | 0.08358938    |
| 2011| 0.18135281    | 0.36105067    | 0.34862361    | 0.18187705    | 0.08485048    |
| 2012| 0.19513379    | 0.41639816    | 0.35676481    | 0.19958280    | 0.09264761    |
| 2013| 0.20679273    | 0.46512445    | 0.39271723    | 0.28821033    | 0.09620016    |
| 2014| 0.21139989    | 0.47299953    | 0.37895040    | 0.20579093    | 0.07541566    |
| 2015| 0.20703713    | 0.47497274    | 0.36499445    | 0.17736008    | 0.05867517    |
| 2016| 0.22317633    | 0.51093459    | 0.38252721    | 0.21411546    | 0.06210994    |
| 2017| 0.24673819    | 0.52805473    | 0.38140241    | 0.23288751    | 0.04358939    |

**Figure 3.** Total Diffusion Strength of 5 States from 2010 to 2017.

Obviously, as is shown in Table 2 and Figure 3, Ohio has an upward trend of drug transaction, also with a high propagation intensity as Pennsylvania while the drug diffusion intensity in Kentucky and Virginia is in stability and that in West Virginia is decreasing, which means the opioid is under control. Meanwhile, we notice that the variation trend of diffusion strength is similar to that of drug cases. It reflects the rationality of weight calculation in drug diffusion analysis.
2.3. Prediction of opioid spread based on equilibrium analysis

We know that in each weight matrix $E_t$, $e_{ij}^t$ represents the drug diffusion strength from state $i$ to state $j$. In fact, this value indirectly reflects the quantity of opioid supplied by state $i$ for state $j$. Based on this idea, it’s easy to create the ratio matrix in year $t$. We define it as $A'_t = (a'_{ij})$, a $5 \times 5$ matrix. Now we give a brief formula about the ratio as follows.

$$a'_{ij} = \frac{e_{ij}^t}{\sum_{k=1}^{5} e_{kj}^t}$$

(4)

The concrete meaning of $a'_{ij}$ is the drug supply from $i$ to $j$ total drug supply from 5 states to $j$. Now we apply the knowledge about the balance between supply and demand to our study. We define $X = (x_k),(k = 1 \sim 5)$, a $5 \times 1$ matrix, as the opioid production of 5 state. If the opioid supply and demand reach a balanced state, it meet the linear homogeneous equation set below.

$$A'X = X,(Demand = Supply)$$

(5)

In algebra theory, $\det(A' - I) = 0$ is equivalent to that the equation set has an infinite number of nontrivial solutions, which means there are numerous balanced state in the drug market. Therefore, we calculate the value of $\det(A' - I)$ from year 2010 to 2017 as follows.

| Year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
|------|------|------|------|------|------|------|------|------|
| det(A'-I) | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0015 | 0.001 | 0.0011 | 7.76e-04 |

As $\det(A' - I) \rightarrow 0$, the market get closer to an equilibrium state. The result in the table above shows that the drug market is reaching a balance year after year. Especially in 2017, the determinant reaches $10^{-4}$ level. Therefore, regardless of such a minute error, we postulate that the market won’t change if there’s no external interference. Using the ratio matrix $A'$ in 2017, we figure out one nonnegative solution $X = (25256.7,131275.7,66243.3,31308.5,3551.8)^T$ in 2017 as the drug quantity in equilibrium state, than compare it to the actual quantity in 2017. The result is as follow.

| State | KY | OH | PA | VA | WV |
|-------|----|----|----|----|----|
| 2017  | 28870 | 119349 | 68751 | 36994 | 3672 |
| Equilibrium | 25256.7 | 131275.7 | 66243.3 | 31308.5 | 3551.8 |
| Difference | -3613.3 | 11926.7 | -2507.7 | -5685.5 | -120.2 |

It’s clear to observe from Table 4 that the drug markets in all states have reached overbalanced state so the drug diffusion will drop off except that in Ohio, is still in an unbalanced state and is likely to absorb more opioids for higher profit, which is in line with our prediction. Unless the government carries out correct policy, the situation in Ohio will definitely get worse.

3. Finding the strategies against opioid diffusion

Having studying the characteristics of the opioid spread, we are supposed to search for practical strategies to stop the illegal drug sale and abuse of opioids from getting severe. On one hand, socio-economic factors should be taken into consideration, some of which might greatly encourage the growth of drug transactions and abuse of opioids. On the other hand, drug transactions directly depend on the whole drug industrial chain, while the industrial chain is related to the input of materials,
transportation, fix capital and labor, as we mentioned before. To study these parts of the chain, we may get some available information.

3.1. Study on social-economic factors based on correlation analysis

3.1.1. Important factor selection and data preprocessing. Data are provided by American Community Survey (ACS). It contains the information of 14 categories including educational attainment, disability status, language spoken at home, marital status, citizenship status, veteran status, place of birth, school enrollment, type of household, grandparents responsible for grandchildren, relationship, ancestry, fertility and residence 1 year ago in each county of 5 states from year 2010 to 2016. The data are the population and percent of 14 types of people we just mention.

Considering the massiveness of data, we sift out 3 categories from 14 ones provided by population subjectively for our study. The selection is reasonable since some of these categories are apparently irrelevant to drug diffusion. Besides, each category consists of several sub categories, so we need to make further selection on the sub categories as the indicators for the correlation analysis.

- **Category 1: Type of Household**
  - There are 4 types of household mentioned in data, including total household, married-couple family(with or without children), single parent family(male or female householder, with or without children), non-family household(householder living alone). We select single parent family and non-family household for our study.
  - Reason: Without a complete family, a child will probably suffer from a negative growth environment, even adults can be badly affected by the pressure from loneliness. Thus, this kind of people have the probability of going astray, for example, drug addiction.

- **Category 2: Marital Status**
  - There are 5 types of marital status in data, including married, never married, separated, divorced and widowed(male or female). We select separated, divorced and widowed for our study.
  - Reason: People suffer from bad marriage such as the separated and the divorced may take psychotropics like opioids to release the stress from their unlucky life, probably encouraging opioid diffusion.

- **Category 3: Educational Attainment**
  - There are 7 types of educational attainment in data, including less than 9th grade, 9th to 12th grade(without diploma), high school graduate, college(no degree), associate’s degree, bachelor’s degree or higher, graduate or professional degree. We select those with less than 9th grade, 9th to 12th grade(without diploma), high school graduate, and college(no degree) for our study.
  - Reason: Generally, people with high education are aware of the harm of drugs, while those with low education may take drugs out of curiosity and even be obsessed with them.

By the way, our method of preprocessing the data of each indicator we select out is simple but rather complex. To describe this procedure, we let $f_{i,k,j}$ be the value of the $j^{th}$ indicator in category $j$ of the $k^{th}$ county in state $i$ in year $t$. Then we define $f_{i,j}$ as the value of the $j^{th}$ indicator in category $j$ of state $i$ in year $t$. To sum up the data in each county, we use the following formula:

$$f_{i,j} = \sum_k f_{i,k,j}$$

(6)

Through the calculation, we eventually get the values about 13 indicators of 5 states from 2010 to 2016, which means the data volume is $13 \times 5 \times 7 = 455$. Therefore, we will not show them here.

After the selection of sub categories of 3 categories, we show the 13 indicators we need to use in the correlation analysis by the following list so as to be better understood.
Table 5. Indicators for Correlation Analysis.

| Category \( j \)       | The \( f^\text{th} \) Indicator in Category \( j \)                                                                 |
|------------------------|---------------------------------------------------------------------------------------------------------------------|
| 1. Type of House \(/ \text{unit} \) | 1. Male Householder, No Wife Present  
2. Female Householder, No Husband Present  
3. Non-family Household, Householder Living Alone |
|                        | 1. Male Separated  
2. Male Widowed  
3. Male Divorced  
4. Female Separated  
5. Female Widowed  
6. Female Divorced |
| 2. Marital Status \(/ \text{person} \) | 1. Less than 9th Grade  
2. 9th to 12th Grade, No Diploma  
3. High School Graduate  
4. Some College, No Degree |

3.1.2. Correlation analysis between drug spread and indicator above. We have defined \( n'_i \) as drug case number of state \( i \) in year \( t \). More, we define 2 new variants \( n_i \) and \( f_{i,\beta} \) respectively as drug case number of state \( i \) and the value of the \( f^\text{th} \) indicator in category \( j \) of state \( i \). Now, according to the knowledge of statistics, we calculate the correlation coefficient [7] (defined as \( r_{i,\beta} \)) about \( n_i \) and \( f_{i,\beta} \) of each state by the formula below.

\[
 r_{i,\beta}(n_i, f_{i,\beta}) = \frac{\sum_{t=1}^{7}(n'_i - \overline{n})(f'_{i,\beta} - \overline{f}_{i,\beta})}{\sqrt{\sum_{t=1}^{7}(n'_i - \overline{n})^2 \sum_{t=7}^{7}(f'_{i,\beta} - \overline{f}_{i,\beta})^2}}^{1/2} \tag{7}
\]

In the formula, \( \overline{n} \) and \( \overline{f}_{i,\beta} \) respectively represent means of drug case number and value of the \( f^\text{th} \) indicator in category \( j \) of state \( i \) within 7 years (2010 to 2016). The value of \( r_{i,\beta} \) is as follows.

Table 6. Correlation Coefficients in 5 States. Particularly, \( r_{\beta} \) represents the correlation coefficient about drug case number and the \( f^\text{th} \) indicator in category \( j \). And the values with superscript “*” are all > 0.8, which show their strong connection with drug case number in each state.

| \( r_{\beta} \) | KY | OH | PA | VA | WV |
|----------------|----|----|----|----|----|
| \( r_{11} \)  | -0.925 | 0.980* | -0.770 | -0.276 | -0.878 |
| \( r_{12} \)  | -0.948 | 0.946* | -0.799 | -0.149 | -0.758 |
| \( r_{13} \)  | -0.797 | 0.988* | -0.918 | -0.299 | -0.679 |
| \( r_{21} \)  | -0.777 | -0.195 | -0.620 | 0.197 | 0.261 |
| \( r_{22} \)  | -0.789 | 0.946* | -0.750 | -0.278 | -0.851 |
| \( r_{23} \)  | -0.916 | 0.994* | -0.794 | -0.228 | -0.712 |
| \( r_{24} \)  | -0.925 | -0.236 | -0.086 | 0.302 | 0.750 |
| \( r_{25} \)  | 0.934* | -0.988 | 0.809* | -0.151 | 0.946* |
| \( r_{26} \)  | -0.930 | 0.945* | -0.865 | -0.298 | -0.843 |
| \( r_{31} \)  | 0.917* | -0.979 | 0.836* | 0.269 | 0.873* |
| \( r_{32} \)  | 0.927* | -0.983 | 0.907* | 0.310 | 0.765 |
| \( r_{33} \)  | -0.947 | -0.956 | 0.674 | -0.234 | 0.326 |
| \( r_{34} \)  | 0.421 | -0.354 | 0.081 | 0.367 | 0.543 |
Combining Table 5 and Table 6, we notice that in different states, drug case number is related to different socio-economic factors:

- **KY**: In Kentucky, drug case number is positively correlated to the population of female widowed, people whose educational attainment are less than 9th grade, or in 9th to 12th grade but without diploma.
- **OH**: In Ohio, drug case number is positively correlated to the number of single parent households (male or female householder, no wife or husband), the population of male widowed or divorced, female divorced.
- **PA**: In Pennsylvania, drug case number is positively correlated to the population of female widowed, people whose educational attainment are less than 9th grade, or in 9th to 12th grade but without diploma.
- **VA**: In Virginia, no factors that we select out have apparent correlation to drug case numbers.
- **WV**: In West Virginia, drug case number is positively correlated to the population of female widowed, people whose educational attainment are less than 9th grade.

It’s easy to see that in OH, 3 indicators in category “Type of Household” and 3 indicators in category “Marital Status” all have strong connection with the increasing number of drug cases, while the result about “Educational Attainment” goes out of our expectation. However, the results of correlation analysis in KY, PA, WV are quite similar that the population of people with educational attainment less than 12th grade is identified as strong correlation with drug case number. Besides, intuitively observing Table 6, we notice that some indicators are weakly or even negatively correlated to drug case number. Especially in VA, the relationship between each indicator and drug case number is rather inconspicuous (-0.5 < all correlation coefficients < 0.5). It’s acceptable since we can’t completely avoid the contingency and estimation error of the data we use for correlation analysis and the number of the sample observation of each indicator is extremely limited (only 7 observations respectively from 2010 to 2016 for each indicator). In a way, these results justify the rationality of our selection in those categories. People living in incomplete family or suffering from unfortunate marriage are definitely put through mental trauma more or less. That is when opioids, the psychotropic drug come in handy. As the demand ascend, the drug spread more furiously. More importantly, we shouldn’t disregard it because those attain low education are short of precaution awareness against drugs. Above is the analysis about the results through our correlation analysis.

3.2. Study on the industrial chain based on the Leontief function

In the former part, we have just analysed the social factors related to drug cases. However, to gain more information about drug diffusion, the drug industrial chain is the most vital thing that we ought to take into consideration. To make our model understood, firstly we build the Leontief function [8] about opioid as follow:

$$Q = \min\{\frac{M}{m}, \frac{K}{k}, \frac{W}{w}, \frac{T}{t}, \frac{S}{s}\}$$  \hspace{1cm} (8)

All variants in the function are different from those we have used in former parts. In this formula, $Q$ is defined as the quantity of drug production, and the concrete implications of the other 10 parameters on the right side of the equation are showed in the following table:

| variant | (quantity)          | constant | (for 1 unit of production) |
|---------|---------------------|----------|---------------------------|
| $M$     | Material Input      | $m$      | input m units of material |
| $K$     | Fixed Capital Input | $k$      | input k units of fixed capital |
| $W$     | Workforce Input     | $w$      | input w units of workforce |
| $T$     | Transportation Input| $t$      | input t units of transportation |
| $S$     | Sales               | $s$      | s unit of production can be sold |
Specifically, the fixed capital includes factories, machines, tools used for drug production and so on, which are non-current capital. As we can see in Table 7 that the first 4 variants respectively represent 4 different production factors for drug production and accordingly the 4 constants respectively represent the input quantities of 4 factors for 1 units of drug production, except that the last variant represents the sales volume so accordingly the last constant means unit in each unit of drug production can be sold and consumed. The reason why we choose Leontief function is that drug production is determined by these 5 essential parts in the entire industrial chain. Provided the quantities of one of these factors suddenly decrease, the drug production will definitely be reduced no matter how much the quantities of the other 4 ones are. In other words, the drug production is directly related to the minimum of these 5 factors. Based on this property of Leontief function, our discussion is divided into 5 different situations as follows.

- **Situation 1:** \( Q = \frac{M}{m} \)

  In this situation, material input quantity is the minimum of 5 factors. If the local government takes actions to reduce the manufacture of the chemical substances and the cultivation of plants that are used as materials, the drug production will decrease, so will the opioid transaction.

- **Situation 2:** \( Q = \frac{K}{k} \)

  In this situation, fixed capital input quantity is the minimum of 5 factors. The best way to reduce the drug production is to close down the factories, where drugs are illegally produced. In the meantime, the government should rigorously penalize the unscrupulous businesses involved in the opioid trade.

- **Situation 3:** \( Q = \frac{W}{w} \)

  In this situation, workforce is the main factor in drug production. Similar to the measure in situation 2, putting sanctions on the illegitimate businesses and sealing up the factories so as to cut down the labors for drug production. Thus accordingly, the opioid trade will decline.

- **Situation 4:** \( Q = \frac{T}{t} \)

  In this situation, the transportation of opioids is primary in the industrial chain. Provided the road transport bureau enhances the road inspection, more illegal opioid transportation will be investigated. Therefore, the difficulty of opioid distribution will surely ascends, resulting in the increases of transport costs, which is to the disadvantage of opioid businesses. By cutting off the routes of drug transportation, the local government can control the drug diffusion more easily.

- **Situation 5:** \( Q = \frac{S}{s} \)

  In this situation, the sale(or consumption) of drugs is the key factor. On one hand, governments should crack down the illegal opioid market. On the other hand, it’s important to reduce the number of opioid consumers(or abusers). The government need to raise the awareness of each civilian about the danger and detriment of drugs to both mental and physical health of human. Once the number of drug consumers decreases, so does the demand of opioids. Then the opioid transactions will definitely decline.

Through the above analysis, we can briefly figure out different strategies to cope with drug trafficking problems contraposing 5 different situations we have discussed. However, in reality, the entire drug industrial chain is far more complicated than what we can imagine, which means that there are still various of production factors we haven’t take into account, such as water power resource, coal resource, electricity, post and telecommunications, as well as many techniques. Besides, to test the practicability of the Leontief function, we have to figure out the 5 constants \( m, k, w, t, s \) in the formula, which needs plenty of data collections, but we are clear that it’s a time-consuming and labor intensive...
work. Therefore, in this article we just come up with our idea about the application of Leontief function to finding the main factor in the industrial chain and accordingly take feasible measures to stop opioid trafficking.

4. Comparisons of our method and others

There are various of approaches used to study American opioid crisis. Therefore, it’s necessary to make a comparison among other methods and ours to strengths and weaknesses of our model. We found 2 papers both studying the same issue respectively based on Markov chain [9] model as well as SIR model [10], which are also worth learning.

The first paper is written by Bixuan Li, Kexin Zhu and Xuanlin Chen, whose analysis about opioid crisis is based on the one-step probability transition matrix of Markov chain [11]. Their model can be described as follow, rather simple and clear:

\[
P_i^{t+1} = \begin{cases} 
1, & n_i^{t+1} > n_i' \rightarrow j \\
0, & \text{else} \\
(1 - \frac{\sum_{j=1}^{5} p_{ij}^{t+1}}{\sum_{j=1}^{5} n_i^{t+1}}), & i \neq j \\
\sum_{j=1}^{5} p_{ij}^{t+1}, & i = j 
\end{cases}
\]

(9)

In the model, all symbols as well as their meanings are the same as those in our model except \(P_i^{t+1} = (p_{ij}^{t+1})\), a new symbol representing the one-step probability transition matrix from year \(t\) to year \(t+1\), where element \(p_{ij}^{t+1}\) represents the probability of the random event that drug cases transit from state \(i\) to state \(j\). If \(p_{ij}^{t+1} = 1\), state \(i\) will possibly be an attractor(or the center) of drug trafficking. Otherwise, if \(p_{ij}^{t+1} = 0, \forall i = 1 \sim 5\), the opioid crisis will traverse 5 states over time. Finally, according to Markov chain theorem [9], vector \(p\) is the stationary distribution, where each element is the probability of state \(i\) becoming the central location of opioid crisis. In this way, they can predict where opioid crisis will likely to happen. However, \(p\) is an ideal state as \(t \rightarrow \infty\), thus in reality it’s impossible to reach this limitation. Besides, this model can’t predict the probable drug identification counts in the future while our model did it and the results are shown in Table 4. Based on the idea of balance between supply and demand, we found that the drug trafficking in Ohio will be more rampant, which is similar to the prediction by Markov chain that Ohio got the highest probability to be central location of opioid crisis as time goes by.

The second one applying SIR model to analysing this problem is made by Yi Liu, Dan Li and the other three [11]. Based on SIR epidemic model, they established a system of partial differential equations to describe the relationship among the proportion of drug case number in 5 states over time. However, SIR model is a continuous model of time while the data is given in years, a discrete time span. In terms of the discreteness of the problem, our model is more suitable for the prediction about the tendency of drug diffusion in 5 states.

For Markov chain model, even though it can reasonably predict the probable areas where drug crisis exist continuously or getting ferocious, it may not always work. That’s because the calculation of the one-step probability transition matrix is based on the drug case number this and the next year, but they can’t figure out the probable drug case number in the future, which will interrupt the prediction process over time. For SIR model, although it provide efficient way for us to predict the drug diffusion trend over time, it may cause error since the original data is so sparsely distributed in time dimension that it’s not enough to build a continuous curve that can properly reflect the opioid transmission status. On the contrary, our model is effective, also simple. It’s as follows:
As we can observe in Table 3 that \( \det(A' - I) \) in each year reaches \( 10^{-3} \) level which means the market each year almost reaches the balance state. Therefore, the equilibrium data shown in Table 4 can be regarded as the drug case number in the next year 2018\((t = 9)\). By analogy, we are able to figure the results in year 2019\((t = 10)\), 2020\((t = 11)\) and so on. It’s feasible to predict the opioid diffusion in the future.

Additionally, we came up with the application of Leontief production function. To predict the drug transmission among states or counties, not only should we profoundly analyse the previous data, but also we should study the economic activity of the drug industrial chain to find out the pivotal link and get the hang of its regularity.

5. Conclusion
Much of our work focuses on the application of weighted undirected graph and its weighted adjacency matrix. It’s a simple way, but from which we do figure out the drug diffusion strength between each two states based on known data about drug identification counts. More, using these results we get the theoretical drug case number provided the market reaches a balanced state(simply transfer this problem into solve linear homogeneous equation set), comparing which with the actual data we are able to predict the drug diffusion tendency of each state year by year.

Besides, combining numbers of socio-economic factors to analyse their relationship with drug diffusion is a massive work. It might be hard to build an accurate function to describe the relationship between these factors and drug diffusion one by one, but at least through the brief correlation analysis our results effectively show the related degree between them. Therefore, we found incomplete households, population with bad marital status and with low educational attainment are probably main factors promoting the growth of drug diffusion.

The reason why we put forward the idea about Leontief function is that drugs are also merchandises, though illegal, which means to study drug diffusion, not only should we analyse the data we gain, but also we should know more about the entire drug industrial chain, such as how drugs are made up of, how they are produced, in which ways they are smuggled, and through which channels can drugs be sold.

Although our methods are from different aspects, they are disparately effective to study the drug trading from different angles. We have compared our method to others such as those based on the Markov chain model and SIR epidemic model, finding that our model is suitable for this kind of discrete data-driven problem. As a result, only by combining all these ways can we get the hang of the dynamic variation trend of the drug trading more adequately.

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