Adequacy assessment of the simulation of construction machines’ dynamic transmission systems

V A Zhulai¹, Yu F Ustinov¹, M A Romanovich²,

¹ Voronezh State Technical University, 14, Moskovsky Avenue, Voronezh, 394026, Russian Federation
² BSTU named after V G Shukhov, 46, Kostyukova Street, Belgorod, 308012, Russian Federation

E-mail: bel31rm@yandex.ru

Abstract. In the study of vibroacoustic processes in mechanical gears of construction and road machines, one of their important tasks is to assess the compliance of the developed mathematical models with real objects. The article presents the methodology and theoretical dependencies for numerical methods for assessing the adequacy of the models of gears of mechanical transmissions of construction machines, which are linear oscillatory systems. The developed technique is based on the test effect of a single impulse perturbation on the model and the real object and comparing the spectra of their own oscillations. The results of theoretical and experimental studies of the primary shaft of a manual transmission of a medium-class earth-moving machinery are presented.

1. Introduction

One of the main ways to solve the problem of maintaining the performance of construction machines is the development of effective systems and methods for monitoring the parameters of the technical condition of machines and mechanisms under operating conditions without disassembling their components and aggregates. Domestic and foreign experience shows that a significant reduction in labor and material costs for the maintenance and repair of machines can be achieved through the use of vibroacoustic diagnostic systems [1-6].

Vibroacoustic diagnostics of machines and mechanisms is currently a new independent scientific area of technical diagnostics. The subject of study are the patterns of binding parameters of vibroacoustic processes in the elements of machines with the parameters of the technical state of kinematic pairs. The purpose of the study is the creation of scientific basis for determining the parameters of the technical condition of indirect evidence. The principal difference in vibroacoustic diagnostic methods is the determination of the technical condition of assemblies and mechanisms in working condition in contrast to the traditional element-by-element control of individual parts and assembly units. It makes possible to use these methods not only under operating conditions, but also to control the quality of finished products at manufacturing plants. Successfully solving the problems of vibroacoustic diagnostics makes it possible to provide for the enormous information content of vibroacoustic processes that inevitably accompany the work of any machines and mechanisms [1-6].

It is especially difficult to make a diagnosis when the disturbance source is removed a considerable distance from the installation place of the primary signal converter. Then the source signal distorted
when passing through various internal structures of the mechanism contains components from a large number of interconnected sources. It is typical for gears, internal combustion engines, hydraulic units [1–4, 6].

Elastic vibrations of the body elements of the units are a source of reliable information about the dynamic processes occurring inside them. Therefore, to solve the problems of vibroacoustic diagnostics and prediction of the parameters of noise and vibration of gears, it is necessary to determine with maximum accuracy the response of external hull elements to internal force disturbances acting in gearing [1].

The analysis showed [1–3] that the study and analytical description of the process of formation and transformation of elastic vibrations is an important step in the development of a common methodology for vibro-acoustic diagnostics of the technical condition of gears.

2. Methods

The most common section in the transmission systems of construction machines is cylindrical straight-toothed gears because of their advantages over other types of mechanical transmission systems [1].

Dynamic forces arising in gears as a result of the action of various defects are disturbing effects on the dynamic transmission system. Thus, to calculate its vibroacoustic parameters, it is necessary to have a mathematical model of this dynamic system. The accuracy of determining the parameters of vibrations of gears depends on the type of models of the dynamic system and the disturbance [1]. Therefore, the choice of the model type should correspond to the objectives of the research carried out with its help [1–3].

Any mechanical design is a dynamic system that converts the input action into an output reaction. For linear systems with constant parameters, the connection between the input signal \( x(t) \) and the output signal \( y(t) \) is given by the pulse characteristic \( h(t) \) and the frequency characteristic \( H(f) \). The frequency characteristic \( H(f) \), describing the properties of a linear system in the frequency domain, is defined as the Fourier transform of the function \( h(t) \) [4]:

\[
H(f) = \int_{0}^{\infty} h(\tau)e^{-j2\pi f \tau} d\tau.
\]  

Frequency response is a complex function that can be represented through a module and an argument:

\[
H(f) = |H(f)|e^{-j\phi(f)}.
\]

The frequency response module \( |H(f)| \) is called amplitude or amplitude-frequency response, and the argument \( \phi(f) \) is called phase response.

For a linear system with constant parameters, the frequency response \( H(f) \) depends only on the frequency and does not depend on the time and type of the input signal.

When assessing the adequacy of the modeling of the mechanical oscillatory system, we have the coincidence of values of own frequencies and their amplitude, i.e. the amplitude-frequency characteristics of the real object and its model.

To describe the behavior of linear systems in the time domain, Green’s functions are used to express the reaction of the system to external action using the convolution equation (Duhamel integral). Green’s functions are the kernel of the Fredholm integral equation:

\[
y(t) = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau
\]

where \( h(\tau) = 0 \) at \( \tau<0 \) if the system is physically feasible.

Substituting in equation (3) \( x(t)=\delta(t) \), where \( \delta(t) \) is a delta function, we obtain, in accordance with its filtering properties, \( y(t)=h(t) \). That is, the kernel of the Fredholm integral equation is a reaction to the Delta momentum of a linear system.

Fourier transform of Delta function:
\[
\int_{0}^{\infty} \delta(\tau) e^{-j2\pi \tau} d\tau = 1. \tag{4}
\]

Therefore, when applying the input of a single pulse \( x(t) = \delta(t) \), given that in this case \( X(f) = 1 \), the expression for the Fourier transform of the response takes the form:

\[
Y(f) = H(f) \tag{5}
\]

where \( Y(f) \) and \( H(f) \) are the Fourier transforms of the input \( x(t) \) and output \( y(t) \) signals, respectively.

The physical expression of the above equation is: when a single pulse or impulse applied to a dynamic perturbation system as input, the Fourier transform of the response will be frequency characteristic of the system. Thus, it is possible to reveal the internal dynamic properties of the mechanical oscillatory system by acting on its input with a shock pulse – a disturbance closest in nature to the Delta function. At the same time, having determined the spectral response density, it is possible to obtain the amplitude characteristic of the linear oscillatory system as a “black box”, which will allow a comparative analysis and assessment of the adequacy of modeling of various mechanical dynamic systems.

An important engineering application of the frequency response can be obtained by taking the Fourier transform from both parts of the equation (3):

\[
Y(f) = H(f)X(f). \tag{5}
\]

After some transformations of equation (5) taking into account (2), it is possible to obtain one more very important for practical application ratio of characteristics of dynamic systems [7]:

\[
S_2(f) = |H(f)|^2 S_1(f) \tag{6}
\]

where \( S_1(f) \) and \( S_2(f) \) are spectral densities of input and output signals of stationary processes, respectively.

This ratio allows one to calculate \(|H(f)|^2 \) for known \( S_1(f) \) and \( S_2(f) \), i.e. to uniquely determine the frequency response of linear mechanical dynamic systems.

There are several ways of experimental study of the dynamic properties of mechanical oscillatory systems, consisting in the supply of a known input action and registration of the response at the output [8]. One of the most versatile and effective types of input action, as shown above, is a pulse, exciting in the oscillatory system complex damped oscillations in a wide range of frequencies. The width of the spectrum of the pulse is determined by its duration, which is the most important characteristic. To fully identify and evaluate the internal properties of the mechanical oscillatory system, it is necessary that the perturbation spectrum is not already the spectrum of the system’s own oscillations, in which all the essential frequencies lie.

At shock interaction of various metal details with curvilinear surfaces there is a shock pulse having sinusoidal or cosine form.

For this form of impulse [9, 10] 95 to 99% of energy are concentrated in the interval \( 0 < f_b \leq \frac{(1.5 \div 1.7)}{\tau_b} \), where \( \tau_b \) – the duration of the shock pulse. For example in the oscillatory systems, the system can be excited by oscillations with a frequency of not more than \( f_b = \frac{(1.5 \div 1.7)}{\tau_b} \).

In addition to the bandwidth of the excited oscillations, the pulse duration has a direct impact on the uniformity of the spectrum in this frequency band. In this case, the amplitude of the input action and, accordingly, the vibrations at the output do not have a fundamental value. Their values are selected within the linearity of the characteristics of the object, but much higher than the noise level.

Thus, for qualitative and quantitative evaluation of internal Eigen properties of mechanical oscillatory systems and their models that determine the dynamic characteristics of these systems and assess the adequacy of the description of the relationship between the input and output processes, information is needed on the parameters of their amplitude characteristics.
3. Results and discussion

The above theoretical prerequisites were practically implemented in the study of vibrations of elements of straight-toothed gears of mechanical transmissions of construction machines.

In the dynamic gear system, the main element that directly perceives the external disturbance, converts it and transmits it to the housing elements, is the shafts with gears. The development of a mathematical model of such a shaft based on the classical theory of oscillations is extremely difficult due to the inability to take into account all factors affecting the propagation of different types of waves and their interaction. This led to the use of one of the most effective and modern numerical methods – the finite element method (FEM) for modeling the dynamic system of the gear shaft. FEM is successfully used to solve many spatial problems associated with the calculation of structures.

Numerical investigation of vibroacoustic parameters of the shaft with the gear were carried out using the program complex “IMPULS”, intended for the calculation of rapidly varying processes in complex spatial structures [11].

Experimental studies were carried out on the developed laboratory installation with a set of measuring and recording equipment. The object of theoretical and experimental research was the primary shaft of the mechanical gearbox of the middle-class earth-moving machines. The purpose of these studies was to verify the adequacy of the developed mathematical model of the real structure.

To determine and compare the internal dynamic characteristics of the vibrating system of the shaft with the gear as the input power of influence in the model was adopted by a sinusoidal pulse of unit amplitude with duration of \( \tau_b = 0.2 \) m/s, and in a laboratory setting – blow hammer with the initial velocity of \( V_0 = 0.3 \) m/s. These parameters provide the input excitation of the natural frequencies with the upper limit to 7.5 – 8.5 kHz and uniformity of the spectrum equivalent to the spectrum of the delta function with a maximum error of up to 5\% of the bandwidth \( f = 0..6000 \) Hz, and 10\% for \( f = 0..8500 \) Hz [9].

Comparison and analysis of the dynamic characteristics and verification of the adequacy of the mathematical model were carried out on the values of vibration accelerations of the gear tooth directed along the horizontal axis perpendicular to the shaft axis.

Impact on the wheel tooth shock pulse leads to the emergence and spread in the shaft of transverse and longitudinal waves, causing different types of vibrations of the shaft. Thus, a characteristic feature of shock excitation is the occurrence of complex damped oscillations, which, like the input action, are not a stationary process. That is, there is a non-stationary input process \( x(t) \) physically existing only for a finite time interval \( T \) and the corresponding output process \( y(t) \) different from zero in the interval of \( 0 \leq t \leq T \).

It is shown in [7] that the relations between input and output characteristics of transients are identical to the relation (6) for stationary processes. Only spectral densities of "energy" are used instead of spectral densities of "power". Theoretically, this implies that the averaging required to obtain estimates of the energy spectra of transients can be obtained as a result of multiple repetition of the experiment. However, [7] shows the possibility of obtaining significant results with sufficient accuracy for practical calculations in a single experiment, if the signal/noise ratio is large, which is the case in many real vibrational systems under shock excitation.

According to the definition, the spectral energy density function for frequencies \( f \) in the range from \( f = 0 \) to \( f = f_b \), calculated using the finite Fourier transform, is:

\[
S_e(f) = 2E \left[ X_T(f)^2 \right]
\]

where \( E \left[ \right] \) means averaging over an existing ensemble of \( n_d \) (transient) realizations \( X_T(f)^2 \) at a fixed frequency \( f \). The value \( X_T(f) \) is a finite Fourier transform of the original function \( x(t) \) given over a time interval of length \( \tau_b \).

The functions of the spectral energy density and the spectral power density of the same transient
with duration T are related by the ratio [7]:

\[ G_x(f) = TS_x(f). \]  

(8)

The spectral analysis of the discretized realization of the transient process was carried out using the discrete Fourier transform of the equidistant sequence of samples \( x_n \) with the sampling interval \( \Delta t \) and zero mean.

This sequence is given by the formula \( x_n = x(n, \Delta t) \), where \( n = 0, 1, 2, \ldots, N - 1 \) at the beginning of the reference \( t_0 = 0 \).

The transformed sequence is defined by the expression [12]:

\[ X_k = \Delta t \sum_{n=0}^{N} x_n \exp \left( -\frac{j2\pi kn}{N} \right); \]  

(9)

for discrete frequency values:

\[ f_k = \frac{k}{T} = \frac{k}{N\Delta t}; \quad k = 0, 1, 2, \ldots, N - 1. \]

And the value of the spectral energy density is then written as:

\[ S(f_k) = |X_k|^2. \]  

(10)

If you change the amplitude of the signal in volts, the unit of measurement \( S(f_k) \) will be \( \frac{V^2}{Hz} \).

Sampling rate was \( f_d = \frac{1}{\Delta t} \) : for experimental studies \( f_d = 25 \) kHz; for theoretical \( f_d = 100 \) kHz. The figure 1 shows the one-way spectral energy densities of vibration accelerations of free damped oscillations of a shaft with a gear, obtained by the above formulas.

![Figure 1. Spectral energy densities of vibration accelerations of free damped oscillations of a shaft with a gear: a) theoretical; b) experimental](image)

From the analysis of the results it can be seen that:

- the main frequencies of different types and forms of natural vibrations of the investigated shaft with the gear are in the low-frequency- 250 ÷ 1000 Hz, mid-frequency- 3000 ÷ 4500 Hz and high – frequency- 7000 ÷ 8500 Hz ranges;

- the smallest difference in the obtained estimates of the frequency characteristics of the mathematical model and the experiment is observed in the mid-frequency region. So the peaks of spectral density are: for the model \( f_{p3}^m = 3.13 \) kHz and \( f_{p4}^m = 4.24 \) kHz; for the experiment \( f_{p3}^e = 3.08 \) kHz.
kHz and $f_{p4}^* = 4.22$ kHz, respectively, the difference is $\Delta f_3 = 1.6\%$; $\Delta f_4 = 0.5\%$. The difference in the values of other peak frequencies is more significant.

Thus, based on the results of a comparative analysis of the characteristics of the model and the object, an assessment of its adequacy, as well as refinement of the model parameters for improvement can be carried out.

4. Conclusion

The technique and theoretical dependence for numerical methods of adequacy estimation of models of gear transmissions in mechanical transmissions of construction machinery with the help of test inputs and the spectral analysis of the response makes it possible to study any linear dynamic system as a “black box” with no description of their internal structure.

Regarding the basis of the finite element method, the mathematical model of the shaft with the gear box of the construction machines adequately describes the problems of vibroacoustic diagnostics of damages of multi-shaft drives gears. The problems of vibroacoustic diagnostics describe the dynamic properties of its mechanical oscillatory system and allow you to replace expensive full-scale tests with studies on the model.

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