Distinct antiblockade features of strongly interacting Rydberg atoms under a two-color weak excitation scheme

Suying Bai, Xuedong Tian, Xiaoxuan Han, Yuechun Jiao, Jinhui Wu, Jianming Zhao and Suotang Jia

Abstract

We present distinct antiblockade features of strongly interacting 64D5/2 Rydberg atoms employing a two-color excitation scheme. The first color (pulse A) is set to resonantly excite a few seed Rydberg atoms, each of which establishes a blockade region due to the long-range multipole interactions. The second color (pulse B) is blue detuned so that the multipole-interaction-induced shifts of certain atoms are well compensated to result in the antiblockade effect. We find in particular that a few seed atoms can result in a remarkable difference of the Rydberg excitation in the presence of pulse B for a wide range of blue detuning. Relevant dynamics of this antiblockade excitation is also investigated by varying the pulse-B duration for a fixed blue detuning, further confirming the facilitation process of Rydberg excitation accompanied by a saturation effect. These experimental results can be well recovered by theoretical simulations based on a multilevel two-body model.

1. Introduction

Creating ultracold gases in the μK regime has opened a new avenue for the investigation of strongly interacting systems. For nondegenerate gases in a magneto-optical trap (MOT), the interaction between ground state atoms is typically very weak. However, Rydberg atoms with principal quantum numbers \( n \gtrsim 10 \) can strongly interact, even in a dilute gas, due to their long-range dipole–dipole (\( \sim n^4 \)) and Van der Waals (vdw) interactions (\( \sim n^{11} \)) [1]. These scalings allow for accurately controlling the interactions over a large range by varying \( n \). The strong interaction between Rydberg atoms shifts the energy level and suppresses the further excitation of nearby atoms, leading to a blockade effect [2, 3]. The blockade radius \( R_B \) is usually defined as the distance between two atoms at which the interaction-induced shift equals the excitation linewidth. Within the region of \( R < R_B \), only one Rydberg atom can be excited, yielding thus the blockade volume of a single Rydberg excitation. In this case, a superatom is formed as the superposition of one Rydberg atom and many ground-state atoms [4, 5]. Based on the blockade effect, Rydberg atoms can be employed to implement quantum information processing [6, 7], quantum registers [8–11], single-photon sources [12, 13] and transistors [14–16]. Rydberg atoms are also an ideal platform for investigating excitation transfer [17–20] and many-body dynamics [21] and implementation of atomic arrays [22, 23] beyond the blockade regime.

But we note that it is also viable to have two or more Rydberg atoms in a blockade region under appropriate driving schemes. This so-called antiblockade effect has been investigated by several groups to consider different topics. In particular, the evidence of antiblockade has been observed as an asymmetric Autler–Townes structure when cold rubidium atoms are modeled by a three-level system with the lower transition resonantly driven by a strong laser [24, 25]. The antiblockade effect may also manifest itself in enhanced excitation probabilities only on the blue side of resonance when full counting statistics is investigated for an effective two-level model of cold Rydberg atoms [26, 27]. Dynamics of the aggregation or facilitation process is further verified as a result of the successive antiblockade excitation of \( nS \) states in room-temperature gases with a strong inhomogeneous
2. Long-range interaction model

We now briefly introduce the theoretical model for calculating the multipole interaction potential of \( nD_j \) Rydberg states, while details can be found in our previous works [31, 32]. As shown in figure 1(a), for two \( nD_j \) Rydberg atoms, denoted by \( a \) and \( b \), with an interatomic separation \( R \) along the quantization \( z \)-axis, we first assume that (i) their Rydberg electrons have positions \( r_a \) and \( r_b \) and (ii) the interatomic distance \( R \) is larger than the LeRoy radius \( R_{LR} \) [33]. Then the Hamiltonian for such a pair of Rydberg atoms can be written as

\[
\hat{H} = \hat{H}_a + \hat{H}_b + \hat{V}_{int},
\]

where \( \hat{H}_{a(b)} \) is the Hamiltonian of atom \( a(b) \), and \( \hat{V}_{int} \) denotes the multipole interaction [31, 32, 34, 35]

\[
\hat{V}_{int} = \sum_{q=2}^{\infty} \frac{1}{R^{q+1}} \sum_{L_a=0}^{q-1} \sum_{L_b=0}^{q-1} \sum_{\Omega=-L_a}^{L_a} f_{ab}^{L_a L_b} \hat{Q}_{a} \hat{Q}_{b},
\]

\[
f_{ab}^{L_a L_b} = \frac{(-1)^{q} (L_a + L_b)!}{\sqrt{(L_a + \Omega)!(L_a - \Omega)!(L_b + \Omega)!(L_b - \Omega)!}},
\]

where \( L_{a(b)} \) is the multipole order of atom \( a(b) \), and \( L_\cdot \) is the lesser of \( L_a \) and \( L_b \). The sum over \( q = L_a + L_b \) starts at 2 (because atoms are neutral and have no monopole moment) and is truncated at a maximal order \( q_{\text{max}} \). The factor \( f_{ab}^{L_a L_b} \) depends on the multipole orders \( L_a \) and \( L_b \) and the counting index \( \Omega \) in the third sum. The operators \( \hat{Q}_{a(b)} = \sum_{q=2}^{\infty} \frac{1}{2^{q+1} q_{ab}^{q+1}} \hat{r}_{a(b)} Y_{qab}^{\Omega} \hat{r}_{a(b)} \) include the radial matrix elements \( \hat{r}_{a(b)} \) and the angular-dependent spherical harmonics \( Y_{qab}^{\Omega} \).

We then diagonalize the Hamiltonian of the Rydberg-atom pair on a dense grid of \( R \) in the \( \{2.0, 5.5\} \) \( \mu \)m range with 400 steps. To improve the plot quality of the potential curves at small \( R \), the radial steps have been chosen equidistant in \( R^{-3} \). Because of global azimuthal symmetry, the projection of the sum of the electronic angular momenta, \( M = m_a + m_b \), is conserved. In figure 1(b), we present the calculated interaction potential.

Figure 1. (a) An illustration of two Rydberg atoms \( a \) and \( b \), separated by \( R \) on the \( z \)-axis with \( r_a \) and \( r_b \) being the relative positions of their Rydberg electrons. (b) Calculated multipole potential curves for 64 \( D_{3/2} \) states of cesium atoms with \( M = \{0, 5\} \), for a maximal order \( q_{\text{max}} = 6 \) and an energy defect 30 GHz. A vertical dashed line is used to indicate the atom-pair interactions at \( R = 4 \) \( \mu \)m.
controlled by another AOM to attain the two-photon resonant.

3. Experimental setup

Our antiblockade experiments are performed in a standard cesium MOT, the atomic cloud has a diameter ~600 μm, density ~ 3 x 10^10 cm⁻³ and temperature ~ 100 μK, measured by a shadow imaging technique. After switching off the MOT beams, we successively switch on a resonant pulse A and a detuned pulse B in the timing sequence as sketched in figure 2(c). Both pulses are applied in the two-photon excitation scheme (see figure 2(a)) by stabilizing a ‘Topica DLP’ laser (852 nm wavelength and 100 kHz linewidth) to the [6S₁/2, F = 4] (|g⟩) → [6P₃/2, F' = 5] (|e⟩) lower transition via a polarization spectroscopy technique [36] and a ‘Topica TA SHG10’ laser (510 nm wavelength and 1 MHz linewidth) to the [6P₃/2, F' = 5] (|e⟩) → [6D₃/2] (|r⟩) upper transition with a F–P cavity of finesse 15000. The 852 nm laser has a fixed 360 MHz blue detuning relative to the lower transition for both pulse A and pulse B as controlled by a double-pass acousto-optic modulator (AOM). The 510 nm laser, however, is controlled by another AOM to attain the two-photon resonant (detuned) excitation of 64D₃/2 state for pulse A (pulse B). During the scan of two-photon detuning for pulse B, the laser power is held fixed using a PID (proportional integral derivative) feedback loop that controls the RF power supplied to the 510 nm AOM. To be more specific, the 852 and 510 nm lasers have powers ~270 μW and ~13.3 mW, respectively, corresponding to single-photon Rabi frequencies \( \Omega_{852} = 2\pi \times 48.5 \text{ MHz} \) on the lower transition and \( \Omega_{510} = 2\pi \times 6.4 \text{ MHz} \) on the upper transition. Then it is easy to get the two-photon effective Rabi frequency \( \Omega_{eff} = 2\pi \times 0.43 \text{ MHz} \), which will be used for theoretical simulations in section 5.

The 852 nm and 510 nm laser beams of waists 180 μm and 40 μm, respectively, cross over with an angle about 40° at the MOT center, yielding thus an elliptical excitation region. This cross-beam scheme allows the atomic density within the excitation region to maintain nearly identical. The excitation region is surrounded by three pairs of field-compensation electrodes so that stray electric fields can be reduced to less than 50 mV cm⁻¹ via the Stark spectroscopy. Rydberg atoms are detected using a ramp ionization field (ramp time = 3 μs), which is
large enough to ionize all Rydberg atoms in $64D_{5/2}$ and nearby states transferred due to blackbody radiation. The Rydberg ions are detected with a microchannel plate (MCP) detector, which has detection efficiency of about 20%. The detected ion signals are amplified with an amplifier and analyzed with a boxcar integrator (SRS-250) and then recorded with a computer. Before the measurements, we first calibrate the MCP ions detection system with two shadow images taken before and after the laser excitation. From the difference of two shadow images, we can obtain the number of Rydberg excitations and therefore the gain factor $f_{\text{gain}}$ of the MCP detection system, $f_{\text{gain}} \approx 3.67$ mV in this work. The actual number of Rydberg excitations will be taken as $R_{\text{sig}}/f_{\text{gain}}$ ($R_{\text{sig}}$ is the measured Rydberg signal).

Figure 2(b) shows the basic idea of our antiblockade excitation scheme. Once pulse A resonantly excites a Rydberg-atom $a$, its neighbor atoms in the blockade region, black line in the left part of figure 2(b), will undergo level shifts due to strong Rydberg interactions as shown in figure 1(b). The subsequent detuned pulse B results in the resonant Rydberg excitation restricted to specific distances and therefore the formation of so-called antiblockade or the facilitated excitation. The facilitated excitation mainly occurs at the boundary of a blockade region, denoted with $b$ in the middle part of figure 2(b). The Rydberg atom $b$ is thought as a new seed atom that expands the blockade region and leads to an aggregation process of Rydberg excitation. The number of Rydberg excitations increases during the aggregation process and saturates after a certain time, as shown in the right part of figure 2(b).

4. Experimental observation of antiblockade

We first study how a few seed atoms excited by pulse A affect the Rydberg excitation due to pulse B. We consider three cases where only pulse A, only pulse B, or both pulses are switched on. Figure 3 presents the Rydberg excitation spectra as a function of pulse B detuning when pulse A and pulse B have the 0.4 $\mu$s and 6.0 $\mu$s durations, respectively. It is clear that pulse A itself prepares a few seed Rydberg atoms, whose number is small and keeps unchanged as we scan the pulse-B detuning. Pulse-B itself results in a 5 times larger signal of Rydberg excitation at the zero detuning, and the signal exhibits a Lorentzian profile of linewidth $\approx$8 MHz. Outside this Lorentzian profile, no atoms can be excited to the Rydberg state when only pulse B is applied. However, if we apply a 0.4 $\mu$s pulse A before pulse B, the Rydberg excitation process becomes very different. That is, the Rydberg excitation is slightly suppressed at resonance, $\sim$20% less than that with only pulse B, due to the blockade effect of pulse A. It is interesting that the Rydberg excitation is largely enhanced in a wide range (10–50 MHz here) of blue detuning due to the facilitated antiblockade effect. To be more specific, the Rydberg excitation is vanishing at the 20 MHz blue detuning when only pulse B is applied, but is clearly enhanced by a factor of 2.5 when pulse A is applied before pulse B. At the 30 MHz blue detuning, as marked with the gray squares, the enhanced Rydberg excitation is less than that at the 20 MHz blue detuning with an enhanced factor,
of 2 instead. The large difference between the three curves demonstrates that the existence of a small amount of seed atoms a greatly facilitates the excitation probability of pulse B at the blue detuned side.

Note, however, that the Rydberg excitation also shows the antiblockade effect at the red detuned side. But the increased excitation is much weaker, e.g. at the −10 MHz position, than that at the blue detuned side. This is because only one potential curve shows attractive interaction for every molecule state M, whose strength is much less than the repulsive interactions as shown in figure 1(b). A Rydberg molecular signal will be obtained at the pulse B detuning ~50 MHz, which is beyond the scope of this work (see [31, 32] for details) and thus not shown.

To gain a deeper insight into the Rydberg facilitation process, we have also done a series of dynamic measurements on the Rydberg excitation by varying the pulse-B duration while keeping the zero (30 MHz) detuning for pulse A (B), as shown in figure 4. In the case where only pulse A or B is turned on, the Rydberg population does not change when the pulse-B duration increases, as denoted by blue triangles (only pulse A) and black squares (only pulse B). This is because the Rydberg excitation due to pulse A is irrelevant to the pulse-B duration in the absence of pulse B, and pulse B excites nothing as it is far detuned from the Rydberg transition in the absence of seed Rydberg atoms. But when both pulses are turned on, as denoted by red circles, the Rydberg excitation demonstrates completely different characteristics. That is, the number of Rydberg excitation first quickly increases when the pulse-B duration is increased until 5 µs and then gradually approaches a saturation value.

We also note that initially prepared Rydberg population in 64D₅/₂ can be redistributed to nearby dipole allowed levels by blackbody radiation [37]. The time-of-flight spectra in our experiments prominently demonstrate the excitation of 64D₅/₂ state while a small fraction of the signal can be attributed to the nearby P states when pulse B has a duration >6 µs. Meanwhile, for a short pulse-A duration and a small number of 64D₅/₂ atoms, we do not see any free-ion signals due to the redistribution of initially prepared Rydberg atoms [37]. So the P-state signals induced by the blackbody radiation can be neglected in our work.

5. Simulations based on multilevel two-body model

We now try to simulate the above experimental observations with a multilevel two-body theoretical model [38] by including different multipole-interaction-induced shifts of all magnetic sublevels of the Rydberg state based on the following two considerations. First, the lower (852 nm) and upper (510 nm) excitation lasers have Rabi frequencies Ω₈52 = 2π × 48.5 MHz, Ω₅10 = 2π × 6.4 MHz, and single-photon detuning Δ = Δ₈52 ≈ Δ₅10 ≈ 2π × 360 MHz for pulse j ∈ {A,B} as taken from the experiment, so we can eliminate level |e⟩ to have an effective Rabi frequency Ωₑeff = Ω₈52 / 2Δj ≈ 2π × 0.43 MHz that couples levels |g⟩ and |r⟩. Second, the average atomic distance is R ~ 4.0 µm from the averaged atomic density N ~ 10¹⁰ cm⁻³, so we know from figure 1(b) that the multipole interaction between two nearest-neighbor (next-nearest-neighbor) atoms is much stronger (smaller) than Ωₑeff. Then, roughly two atoms can be found in each blockade volume so that we just need to consider the multipole interactions...
interactions of atomic pairs. Aiming to attain more accurate results, however, we write down the effective Hamiltonian without neglecting level $|e_i\rangle$

$$H_{\text{eff}} = \sum_{i=a,b} \left[ \Delta_0 |e_i\rangle \langle e_i| + (\Delta_{0s} + \Delta_{10}) |r_i\rangle \langle r_i| + \Delta_{\text{int}} r_i |r_i\rangle \langle r_i| \right] + \sum_{i=a,b} \left[ \Omega_0 |e_i\rangle \langle g_i| + \Omega_{10} |r_i\rangle \langle e_i| + \Omega_{0s} |g_i\rangle \langle e_i| + \Omega_{10s} |e_i\rangle \langle r_i| \right],$$

for atoms $a$ and $b$ in each blockade volume ($\hbar = 1$).

Taking $H_{\text{eff}}$ into master equation $i\partial_t \rho = -i[H_{\text{eff}}, \rho]$ of density operator $\rho$ and adding the decay ($\gamma_{j\mu}$) and dephasing ($\gamma_{j\mu}^{\text{ph}}$) rates, we obtain a set of dynamical equations for $9 \times 9$ density matrix elements $\rho_{j\mu,\nu}^{\text{eff}}$ with the former (latter) $\mu$ and $\nu$ referring to the three levels of atom $a$ ($b$) in the excitation process of pulses $j$. Then we use this set of equations to simulate the pulse A (B) excitation process by applying the resonant (detuned) pulse A (B) with $\Delta_{\text{int}} = [-4.0 \sim 34.0] \times 2\pi$ MHz, taken from figure 1(b), for 21 interaction potentials formed with different magnetic sublevels with $M \in \{0, 1, 2, 3, 4, 5\}$ of weight factor $\zeta \in \{6, 5, 4, 3, 2, 1\}$. We further note that (i) pulse A has a fixed duration $t_a = 0.4$ ms while the duration $t_b$ for pulse B will be varied in the range $[0 \sim 12]$ ms in our simulations; (ii) our numerical results are attained as a sum of 21 independent numerical realizations with different magnetic sublevels serving as level $|r_i\rangle$; (iii) the resonant A excitation process will result in a blockade effect while the off-resonant B excitation process is expected to yield an antiblockade effect.

Numerical simulations on the excitation spectra are displayed with solid lines in figure 3 under three indicated conditions with regard to the application of pulse-A and pulse-B. It is seen that our calculations can well reproduce the experimental results when only pulse-A or pulse-B is switched on. Whereas in the case where both pulses are applied, the calculated spectrum is just basically consistent with its experimental counterpart. To well reproduce the experimental results when only pulse-A or pulse-B is switched on. Whereas in the case where both pulses are applied, the calculated spectrum is just basically consistent with its experimental counterpart. To be more concrete, our calculations seem to slightly underestimate the experimental results around $-10$ MHz and 20 MHz detuning. This may be attributed to the fact that we have assumed for simplicity the seed Rydberg atoms excited by pulse-A have a homogeneous distribution in the whole sample. In figure 4, we further simulate the Rydberg excitation dynamics when pulse-B is fixed at 30 MHz detuning, marked with the gray square in figure 3. Our calculations well reproduce the experimental results in all three cases, consistent with corresponding steady-state results in figure 3. In particular, we find that the Rydberg excitation tends to a saturation value with numerical calculations and experimental results being more consistent when pulse-B has a long enough duration (e.g. for $t_B > 8$ ms). This means that the assumption of homogeneously distributed seed Rydberg atoms is more reasonable for a more persistent excitation due to pulse-B. Finally, we note that the calculations in figures 3 and 4 display in fact the two-color excitation probabilities, which have been multiplied by a factor of 3.8 so that they could be quantitatively consistent with relevant experimental results.

6. Conclusions

In summary, we have investigated the long-range multipole interactions of cesium atoms in the $64D_{5/2}$ state based on a two-color excitation scheme. The first color (pulse-B) is resonant with the Rydberg transition for preparing a few seed atoms in the blockade regime. The second color (pulse-B) is blue detuned from the Rydberg resonance for facilitating the antiblockade excitation. The two-color excitation scheme is found to work well for manipulating the antiblockade and facilitation processes of $64D_{5/2}$ Rydberg state. That is, pulse-B excites atoms from the ground state to the Rydberg state only on resonance without the seed atoms, but enables the characterization of facilitation effect over a wide blue detuned range in the presence of seed atoms. We also measured the evolution dynamics of the Rydberg excitation and further found an aggregation-like saturation effect, which has been discussed in [30] for $70S$ Rydberg state of rubidium atoms. To understand the experimental observations, we have developed a multilevel two-body model by including multipole-interaction-induced shifts for all magnetic sublevels of different excitation weights. The simulations reproduce successfully the measurements on both excitation spectra and dynamic evolutions.

Acknowledgments

We thank G Raithel for discussions on the calculation of multipole Rydberg potentials. The work was supported by the National Key R&D Program of China (Grant No. 2017YFA0304203), the National Natural Science Foundation of China (Grants Nos. 61675123, 61775124 and 11804202, 11847018), the State Key Program of National Natural Science of China (Grant No. 11434007 and 61850007), and Changjiang Scholars and Innovative Research Team in University of Ministry of Education of China (Grant No. IRT_17R70).
ORCID iDs

Jianming Zhao @ https://orcid.org/0000-0001-8420-9319

References

[1] Gallagher T F 1994 *Rydberg Atoms* (Cambridge: Cambridge University Press)
[2] Tong D, Farooqui S M, Stanojevic J, Krishnan S, Zhang Y P, Côté R, Eyler E E and Gould P L 2004 *Phys. Rev. Lett.* **93** 063001
[3] Singer K, Reitz-Lamour M, Amthor T, Marcassa L G and Weidemüller M 2004 *Phys. Rev. Lett.* **93** 163001
[4] Liu Y M, Yan D, Tian X D, Cui C L and Wu J H 2014 *Phys. Rev. A* **89** 033839
[5] Petrovsky D, Otterbach J and Fleischhauer M 2011 *Phys. Rev. Lett.* **107** 213601
[6] Safmann M, Walker T G and Mølmer K 2010 *Rev. Mod. Phys.* **82** 2313–63
[7] Lukin M D, Fleischhauer M, Cote R, Duan L M, Jaksh D, Carac I J and Zoller P 2001 *Phys. Rev. Lett.* **87** 037901
[8] Endres M, Bernien H, Keesling A, Levine H, Anschuetz E R, Krajenbrink A, Senko C, Vuletic V, Greiner M and Lukin M D 2016 *Science* **354** 1024
[9] Xia T, Lichtman M, Maller K, Carr A W, Piotrowicz M J, Isenhower L and Saffman M 2015 *Phys. Rev. Lett.* **114** 100503
[10] Kim H, Lee W, Lee H G, Jo H, Song Y and Ahn J 2016 *Phys. Rev. Lett.* **115** 073003
[11] Lester B J, Luick N, Kaufman A M, Reynolds C M and Regal C A 2015 *Phys. Rev. Lett.* **115** 073003
[12] Dudin Y O and Kuzmich A 2012 *Science* **336** 887
[13] Peyronel T, Firstenberg O, Liang Q Y, Hofferberth S, Gorshkov A V, Pohl T, Lukin M D and Vuletic V 2012 *Nature* **488** 57
[14] Li W B, Viscom D, Hofferberth S and Lesanovsky I 2014 *Phys. Rev. Lett.* **112** 243601
[15] Gorniaczyk H, Tresp C, Schmidt J, Fedder H and Hofferberth S 2014 *Phys. Rev. Lett.* **113** 035601
[16] Tiarks D, Baur S, Schneider K, Durr S and Rempe G 2014 *Phys. Rev. Lett.* **113** 053602
[17] Günter G, Schenpp H, Robert-de Saint-Vincent M, Gavryusev V, Helmrich S, Hofmann C S, Whitlock S and Weidemüller M 2013 *Science* **342** 954–6
[18] Barredo D, Ravets S, Labuhn H, Réquin L, Vernier A, Ngorette F, Lahaye T and Browaeys A 2014 *Phys. Rev. Lett.* **112** 183602
[19] Barredo D, Labuhn H, Ravets S, Lahaye T, Browaeys A and Adams C S 2015 *Phys. Rev. Lett.* **114** 113002
[20] Mainelli W, Pelle B, Faroo R, Arimondo E, Pillet P and Cheinet P 2016 *Phys. B: At. Mol. Opt. Phys.* **49** 214001
[21] Ates C, Pohl T, Pattard T and Rost J M 2007 *Phys. Rev. A* **76** 013413
[22] Barredo D, Lienhard V, Léseleuc S, Lahaye T and Browaeys A 2018 *Nature* **561** 79
[23] Mello D O, Schaffner D, Werkmann J, Preuschoff T, Kohfahl L, Schlosser M and Birkl G 2019 *Phys. Rev. Lett.* **122** 203601
[24] Ates C, Pohl T, Pattard T and Rost J M 2007 *Phys. Rev. Lett.* **98** 023002
[25] Amthor T, Giese G, Hofmann C S and Weidemüller M 2010 *Phys. Rev. Lett.* **104** 013601
[26] Schenpp H et al 2014 *Phys. Rev. Lett.* **112** 013602
[27] Malossi N, Valado M M, Scotto S, Huillery P, Pillet P, Ciampini D, Arimondo E and Morsch O 2014 *Phys. Rev. Lett.* **113** 023606
[28] Urvoy A, Ripka F, Lesanovsky I, Booth D, Shaffer J P, Pfau T and Low R 2015 *Phys. Rev. Lett.* **114** 203002
[29] Letcher F, Thomas O, Niederprüm T, Ott H and Fleischhauer M 2017 *Phys. Rev. A* **95** 023410
[30] Simonelli C, Valado M M, Masella G, Asteria L, Arimondo E, Ciampini D and Morsch O 2016 *J. Phys. B: At. Mol. Opt. Phys.* **49** 154002
[31] Han X, Bai S Y, Yao J C, Hao L P, Xue Y M, Zhao J M, Jia S T and Raithel G 2018 *Phys. Rev. A* **97** 031403
[32] Han X, Bai S Y, Yao J C, Raithel G, Zhao J M and Jia S T 2019 *J. Phys. B: At. Mol. Opt. Phys.* **52** 1535102
[33] Le Roy R J 1974 *Can. J. Phys.* **52** 246
[34] Schwettmann A, Crawford J, Overstreet K R and Shaffer J P 2006 *Phys. Rev. A* **74** 020701
[35] Deiglmayr J, Salmannshausen H, Pillet P and Merkt F 2014 *Phys. Rev. Lett.* **113** 193601
[36] Pearsan C P, Adams C S, Cox S G, Griffin P F, Smith D A and Hughes I G 2002 *J. Phys. B: At. Mol. Opt. Phys.* **35** 5141
[37] Caliri L L and Marcassa L G 2007 *Phys. Rev. A* **75** 066503
[38] Kondo J M, Booth D, Gonçalves I F, Shaffer J P and Marcassa L G 2016 *Phys. Rev. A* **93** 012703