Chiral Liquids in 1-d: A New Class of NFL-Fixed Points

Natan Andrei\textsuperscript{1}, Michael R. Douglas\textsuperscript{1} and Andrés Jerez\textsuperscript{1,2}\textsuperscript{4}

\textsuperscript{1} Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855
\textsuperscript{2} University of Oxford, Department of Physics, Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, United Kingdom

(November 15, 2018)

We identify a non Fermi Liquid (NFL) class of fixed points describing the infrared behaviour of interacting chiral fermions in one dimension. The thermodynamic properties and asymptotic correlation functions are characterized by universal exponents, which we determine by means of conformal and Bethe-Ansatz techniques. The mechanism leading to the breakdown of Fermi-Liquid theory is quite general and can be expected to be realized in systems with broken $T$-invariance. As an example we study the edge states of interacting QHE systems. We calculate the universal frequency dependence of the spin conductance in these systems as well as the NMR response.

71.27.+a

Models displaying Non-Fermi-Liquid (NFL) behavior have been intensely studied following the suggestion that the normal phase of the cuprate superconductors cannot be described in terms of the conventional Landau theory. A much studied example are the Luttinger liquids, a class of one dimensional models whose behavior in the infrared (IR) limit is given by the Luttinger model (in the language of conformal field theory a $c = 1$ bosonic field theory) characterized by single particle correlation functions having only cuts with non-universal exponents.

In this article we identify a large new class of IR fixed points, to be referred to as chiral fluids, available in one dimension to interacting fermions with different numbers of left and right moving degrees of freedom. The new fixed points describe NFL physics, and are universal in character, independent of the strength of the interaction.

Systems described by chiral fluids break $T$-invariance; this may occur explicitly, in the presence of a magnetic field, or spontaneously by the interactions. The edge states in a paired sample of integer QHE systems furnish a concrete example: Thus consider two integer QHE systems side by side with filling factors $\nu_L = f_L = 2$, and $\nu_R = f_R = 3$. The lines represent the edge states.

Fixed points of one dimensional quantum field theories can always be related to two dimensional statistical models, and thus the general theory of such fixed points is conformal field theory (CFT). A general classification of fixed points (likely to arise physically) is known, as are calculational techniques for determining arbitrary Green’s functions. Various approaches to the subject exist; one can study an action which flows to the fixed point of interest, or define a model by its symmetries and use techniques of ‘current algebra’ to compute correlation functions. We will do both in this note. A direct way to search for and classify NFL behavior is to use algebraic results for correlation functions. However, merely showing that a certain fixed point exists abstractly does not make it a physical model. The main points of our work are first, that we have explicit models of interacting fermions which realize these fixed points, and second, that they illustrate a very general mechanism for producing NFL behavior.

We start with the hamiltonian,

$$H_0 = -i v_F \int dx \left( \sum_{r=1}^{f_R} \psi_{R,a,r}^*(x) \partial_x \psi_{R,a,r}(x) - \sum_{l=1}^{f_L} \psi_{L,a,l}^*(x) \partial_x \psi_{L,a,l}(x) \right).$$

(1)

The fields $\psi_{R,a,r}(x)$ ($\psi_{L,a,l}^*(x)$), with $a = \pm 1$ the spin index, and $r = 1...f_R$, $l = 1...f_L$ the right (left) flavor index, create right-(left-) moving particles with a linearized dispersion $\epsilon = \pm v_F (k - k_F)$. Since the flavor and spin
indices enter the Hamiltonian on equal footing, it has the symmetry $U(2f_L) \otimes U(2f_R)$ under independent unitary transformations on $\psi_L$ and $\psi_R$.

The first interacting model we consider, to illustrate some of the ideas, is the familiar example of the Luttinger model obtained by adding interactions in the charge sector. The Hamiltonian is (here we choose $f_R = f_L$),

$$H = H_0 + g_{2c} \int dx \rho_L(x) \rho_R(x) + g_{4c} \int dx (\rho_L(x) \rho_L(x) + \rho_R(x) \rho_R(x)),$$

(2)

where the right-moving charge-density is $\rho_R(x) = \sum_{\gamma=1}^{f_R} \psi_{R,a,r}(x) \psi_{R,a,r}(x)$, and a similar definition holds for the left movers. Renormalization group calculations indicate that the model is conformally invariant (in the infinite cut-off limit). As is well known, upon expressing the Hamiltonian in terms of bosonic fields it becomes quadratic and can be solved by a Bogoliubov rotation. The $g_{4c}$ term modifies the charge velocity (without destroying the FL property at the Fermi surface), while the effect of the $g_{2c}$ term is to modify the exponents of the fermionic correlation functions destroying the pole structure characteristic of a FL.

In the language of CFT, the essential features of this model which make it an NFL are as follows. First, the model can be decomposed into two non-interacting sectors, a spin/flavor sector and a charge sector. The decomposition is

$$SU(2f_L) \times U(1) \otimes SU(2f_R) \times U(1)$$

in a notation we proceed to describe. The notation “$SU(N)_k$” stands for a level-$k$ chiral WZW model. It is the unique chiral CFT containing $SU(N)$ current algebra of central charge $k$ and no other degrees of freedom. The ‘central charge’ $k$ determines the two-point function of the symmetry currents, $\langle J^a(x_1) J^b(x_2) \rangle = k/(x_1-x_2)^2$. We normalize the currents so that they have the same commutation relations as generators of the fundamental representation $f^a$ satisfying $[t^a, t^b] = i\delta^{ab}$ (for $SU(2)$, $t^a = i\sigma^a$). Then, $f$ flavors of complex fermions have $SU(f)_k$ current algebra. In a CFT the currents and stress tensor are chiral, so there are independent left and right central charges $k_L$ and $k_R$.

A similar quantity is the conformal central charge $c$, associated with the two-point function of the stress tensor. We will always use the word ‘conformal’ when referring to it. Here it takes the value,

$$c = \frac{(N^2-1)k}{(k+N)}.$$

(4)

General results of CFT show that the IR limit of the specific heat of any gapless model will be linear, $c_V = \frac{1}{2}(c_L + c_R)T$. Similarly the IR limit of the magnetic susceptibility of a spin system is the constant $\chi = (k_L + k_R)\nu_0$, ($\nu_0 = 1/\pi v_F$ the density of states per unit length) whether the fixed point describes a FL or not.

The “$U(1)$” also denotes a chiral CFT; it contains the $U(1)$ current algebra and has $c = 1$. We have indicated by “$\chi$” a product of theories which are essentially decoupled: “$\otimes$” has the same meaning but separates the left and right moving subsectors.

To prove that this product of decoupled theories is equivalent to the Fermi theory (3), one first observes that the Fermi theory has the stated current algebra, showing that it must at least contain this product of chiral WZW models. One then computes the sum of conformal central charges of (3) and observes that it is equal to that of the Fermi theory. Thus any degrees of freedom not accounted for would form a unitary CFT of central charge zero, which is trivial. The (right-moving) fermion creation operator is a product of operators from the two sectors, $\psi_{R,a,r} = e^{i\phi_R} g_R^{ar}$, where $g^{ar}$ is a field in the spin/flavor sector, transforming in the fundamental representation of $SU(2f_R)$, and $\phi_R$ is the charge field.

The $U(1) \otimes U(1)$ charge sector contains a marginal operator which changes the central charge. As a result the dimension of $e^{i\phi}$ will change, and so will dimension of $\psi$ from 1/2 to (say) $\Delta$. One must show by explicit computation that $g_{4c}$ couples to this operator (it clearly does not affect the spin/flavor sector). The reward for doing this is that since there is no scale at the IR fixed point, the IR limit of the single particle Green’s function is uniquely determined to be

$$\langle \psi(x_1) \psi^\dagger(x_2) \rangle = (x_1-x_2)^{-2\Delta}$$

(5)

and $\Delta \neq \frac{1}{2}$ directly implies NFL behavior.

It is this strategy which we will generalize to find new fixed points. The kinematics of one dimension permits many decompositions of the sort we just used, because a linear combination of any number of left (right)-movers is again a left (right)-mover. A powerful way to find them is to choose other decompositions of the symmetry group of the theory. One can then build general states and operators using the action of the symmetry currents on a finite set of primary fields.

We will discuss models with interactions between the spin-densities (currents, in the CFT language), $S_{L}^i$ and $S_{R}^i$, where $S_{L}^i(x) = \sum_l S_{L,l}^i(x) \equiv \sum_{a,b,l} \psi_{L,a,l}^\dagger \sigma_{a,b}^l \psi_{L,b,l}$ (similarly for $S_{R}^i$). These satisfy two commuting $SU(2)$ current algebras (Kac-Moody algebras) with central charges $f_L$ and $f_R$.

Now the appropriate decomposition to make is into a product of three sectors, containing the spin, flavor, and charge symmetries. The free fermion theory (1) is equivalent to the model

$$SU(2)_{f_L} \times SU(f_L)_{2} \times U(1) \otimes SU(2)_{f_R} \times SU(f_R)_{2} \times U(1)$$

(6)

One proves this the same way as for (3), by comparing the symmetries and central charges. The currents
spanning the left-spin $SU(2)_{L}$ theory are $S_{i}^{J}(x)$, and the flavor currents spanning the left-flavor $SU(f_{L})_{2}$ theory are $F_{A}^{f}(x) = \sum_{\alpha} F_{A,\alpha}^{f}(x) \equiv \sum_{\alpha,A} \psi_{\alpha,k}^{*} \lambda_{A,k}^{f} \psi_{n,k}^{f}$ with $\lambda^{A}$ in the fundamental representation of $SU(f_{L})$. Similar comments apply to the right hand sector. The fermion creation operator now becomes $\psi_{R,a,r}(x) = g_{a}(x) h_{R}^{*}(x) e^{i\phi_{R}}(x)$ where $g^{a}$ and $h^{r}$ are fields in the spin and flavor sectors, respectively, transforming in the fundamental representation of $SU(2)$ and $SU(f_{R})$, and $\phi_{R}$ is the charge field. The dynamics of the three sectors decouples.

We proceed to study IR fixed points of models obtained by adding spin-exchange interactions to $H_{0}$,

$$H = H_{0} + \sum_{r,l,i} \int dx \ (g_{a})^{Rl} S_{R,a,r}^{i}(x) S_{L,i}^{I}(x). \quad (7)$$

with $(g_{a})$ a matrix of couplings. The models are isotropic when the matrix elements are all equal, $(g_{a})^{Rl} = g_{a}$. The interaction breaks the separate left and right $SU(2)$ spin symmetries but preserves a global $SU(2)$.

Standard perturbative calculations show that this perturbation is marginally relevant and destabilizes the weak coupling fixed point. We will first give general arguments from CFT to identify the resulting IR fixed point. We will then describe thermodynamic Bethe Ansatz (TBA) results which confirm this identification and allow studying the flow in detail.

We begin by discussing the flavor-isotropic model, characterized by one coupling $g_{a}$. For $f_{R} = f_{L}$, the model is chirally invariant and flows to a strong coupling fixed point generating a mass gap. If we use the decomposition $\psi_{R,a,r}(x) = g_{a}(x) h_{R}^{*}(x) e^{i\phi_{R}}(x)$, the interaction will affect only the spin sector, and thus the IR fixed point will still be non-trivial, describing gapless flavor and charge excitations. However the single particle Green’s function necessarily couples to the spin sector as well and thus sees the gap.

We now argue that for $f_{R} > f_{L}$, the spin sector of the model is gapless, and flows to a non-trivial fixed point. We use the following result: under any flow, the difference of conformal central charges (here referring to the spin sector only) between left and right, $c_{R} - c_{L}$, and the difference of Kac-Moody central charges, $f_{R} - f_{L}$, must be preserved. The simplest proof uses the concept of ‘anomaly matching,’ familiar in the particle physics literature. Let us make the argument for the difference of the Kac-Moody central charges $f_{R} - f_{L}$. This is the coefficient of the two-point function of the spin current, and if it is non-zero this current cannot be consistently coupled to an $SU(2)$ gauge field. The theory can be made consistent by adding decoupled chiral degrees of freedom to bring the total Kac-Moody charge to zero. Once we cancel the anomaly, whatever theory we flow to in the IR can manifestly be coupled to a gauge field. Since the extra chiral degrees of freedom do not participate in the flow, the original $f_{R} - f_{L}$ must be preserved. The same argument can be made for $c_{R} - c_{L}$ by considering the coupling of the stress-tensor to a two-dimensional metric.

Since $c_{R} - c_{L} \neq 0$ the IR fixed point is non-trivial, and since $f_{R} - f_{L} > 0$ it has a non-trivial $SU(2)$ spin symmetry. On general grounds one expects to get the theory with the lowest possible conformal central charge $c_{R} + c_{L}$, consistent with the known $c_{R} - c_{L}$, $f_{R} - f_{L}$ and the symmetries. This is because $c = c_{R} + c_{L}$ always decreases under RG flow (the $c$-theorem) and thus a generic perturbation will send us to this theory.

In fact there is a unique such fixed point theory. It is

$$\frac{SU(2)_{f_{L}} \times SU(2)_{f_{R} - f_{L}}}{SU(2)_{f_{R}}} \otimes SU(2)_{f_{R} - f_{L}}. \quad (8)$$

The right movers are the chiral WZW model $SU(2)_{f_{R} - f_{L}}$ while on the left, we introduced a new notation for ‘quotient,’ which denotes a chiral coset CFT. This sector is spin singlet and has conformal central charge which is the difference of those of the numerator and denominator, so this ansatz matches $c_{R} - c_{L}$.

We will confirm this prediction of the IR fixed point for the model (8) below by appealing to TBA results. It is important to realize however that the result is more general than these specific models and follows from the two assumptions of chirality and non-abelian (in our case $SU(2)$ spin) symmetry. (Although the same argument can be made for $U(1)$, the fixed points one then obtains are not really new.) We refer to this mechanism as ‘chiral stabilization.’

The UV and IR limits of the specific heat and the magnetic susceptibility can be determined from the central charges. The specific heat will be linear in $T$ in the UV and in the IR (with corrections), and will undergo the flow:

\begin{equation}
\begin{aligned}
c_{\nu}^{\nu} &= \pi \left( f_{L} + f_{R} \right) T \\
&\rightarrow c_{\nu}^{\nu} &= \pi \left( f_{L} + f_{R} + \frac{3(f_{R} - f_{L})}{f_{R} - f_{L} + 2} - \frac{3f_{R}}{f_{R} + 2} \right) T \quad (9)
\end{aligned}
\end{equation}

where we also included charge and flavor contributions. The flow in the susceptibility will be, $\chi^{uv} = (f_{R} + f_{L})\nu_{0} \rightarrow \chi^{uv} = (f_{R} - f_{L})\nu_{0}$, leading to a Wilson ratio $R_{W} = \left( f_{L} + f_{R} + \frac{3(f_{R} - f_{L})}{f_{R} - f_{L} + 2} - \frac{3f_{R}}{f_{R} + 2} \right) / (f_{R} - f_{L})$.

We discuss now the operators around the IR fixed point. A $SU(2)_{k}$ theory contains primary fields $\phi_{j}^{m,j} (x, t)$ transforming under a particular left and right representation of the symmetry. There is a finite number of operators allowed: $0 \leq j, j' \leq k/2$, and their dimension is $h = \frac{j(j+1)}{2}$. For the coset theory there is a single primary $\phi_{j}^{m,j}$ for each choice $0 \leq j \leq f_{L}/2$, $0 \leq j' \leq (f_{R} - f_{L})/2$ and $0 \leq j'' \leq f_{R}/2$. The dimension of the primary is the difference of the dimensions of the group primaries, up to an integer. We can thus match the physical fields with the operator basis around the fixed point and read off the IR behavior of the correlation functions,
<ψ_{L,a,t}^* (x,t) ψ_{L,a',t'} (0,0) > → δ^{aa'} δ^{t't'} (x-v_{FT})^{-(1+\delta_L)} (x+v_{FT})^{-\delta_L}
<ψ_{R,a,t}^* (x,t) ψ_{R,a',t'} (0,0) > → δ^{aa'} δ^{t't'} (x-v_{FT})^{-(1+\delta_R)} (x+v_{FT})^{-\delta_R}
<S_L^1 (x,t) S_L^j (0,0) > → \delta^{ij} (x+v_{FT})^{-2} (x^2-v_{FT}^2)^{-4/((f_R-f_L+2)}
<S_R^j (x,t) S_R^1 (0,0) > → \delta^{ij} (x-v_{FT})^{-2}

with $\delta_L = 3/2(f_R-f_L + 2)$ and $\delta_R = 3f_L/2(f_R-f_L + 2)(f_R-f_L+1)$. This holds for any value of $f_L < f_R$ except when $f_R-f_L = 1$, in which case the anomalous dimension in the $S_L$ correlation function is 2.

These results establish that the model (3) is a NFL. The momentum distributions for small momenta are $n_{\alpha}(k) \sim |k-k_F|^{2\delta_{\alpha}}$. In these expressions the left and right components move with velocity $v_F$. One can include as well interactions in the charge degrees of freedom of the form $g_{4\epsilon} \langle \bar{\psi}_L^\epsilon \gamma^\mu \psi_R^\epsilon \ast \rangle$. The coupling $g_{4\epsilon}$ would modify $v_F$, while the coupling $g_{2\epsilon}$ would again vary the central charge in the $U(1)$ charge sector. Thus each of our models can flow to a line of fixed points, generalizing the Luttinger liquid. See Fig. 3.

Universal fixed points in the spin sector (we have indicated some possible RG flows to available fixed points. The particular fixed point to which the model will flow depends on the value of $f_L-f_R$.)

Lines of fixed points in the charge sector.

The Luttinger liquid.

FIG. 2. Fixed point structure of the model.

These fixed points are robust against small exchange asymmetry, given by perturbations of the form $t_{ij} S_B^i S_B^j$. The IR limit of the perturbation is expected to contain the operator of lowest dimension with the same quantum numbers. For $SU(2)$, the symmetry breaking operators have a spin 1 ($t_{ij} = -t_{ji}$) component and a spin 2 ($t_{ij} = t_{ji}$) component. These will flow to operators with the $SU(2)$ quantum numbers realized entirely on the right (for $f_R > f_L$).

The simplest case to analyze is $f_R - f_L = 1$, because the chiral WZW model appearing in (3) has no primary fields of spin $j > \frac{1}{2}$, and thus the leading operators with $SU(2)$ spins 1 and 2 are $f^{ijk} f^{jk}$ and $J^i J^i$. The perturbation is a scalar operator and if there also exists an operator in the coset sector with the same dimension, their product will be the IR limit. The stress-tensor is dimension 2 and thus the spin 2 perturbation flows to $J^i J^i$, an irrelevant operator. It is a non-trivial fact that the cosets appearing in (3) (the BPZ minimal models) do not contain dimension 1 operators, and thus the spin 1 perturbation must flow to an operator of higher dimension, also irrelevant.

Given both the UV and IR results, it is very plausible that small symmetry breaking perturbations are irrelevant all along the flow. For $f_R - f_L > 1$, primary fields of $SU(2)$ spin 1 will exist, and then the IR limit of the symmetry breaking perturbation can be relevant.

By contrast, flavor anisotropy tends to be relevant. Here we discuss the various limits of extreme anisotropy. These can be modeled as a sequence of flows, each of the type described above. Consider for example a coupling $g^{r\ell} = g_1$ for $r \leq f_R$ and $g^{r\ell} = g_2$ for $r > f_R$. This breaks the $SU(f_R) \times U(1)$ right flavor and charge symmetry down to $SU(f_R) \times SU(1) \times SU(f_R) \times U(1)$ with $f_R + f_R = f_R$. Clearly we want to bosonize the two groups of right fermions separately, introducing spin densities $S_B^1$ and $S_B^2$ generating $SU(2)$ Kac-Moody algebras of level $f_R$ and $f_R$, so that the interaction will again involve only the spin sector of the theory.

In the limit $g_1 \gg g_2$, we can regard the $g_1$ interaction as generating precisely the flow described above, approaching arbitrarily closely to the IR fixed point described above. We can then identify the $g_2$ interaction as a specific perturbation of this IR fixed point using our earlier results. If it is still marginally relevant, this will produce a flow to a final IR fixed point. Of course this analysis would be reversed for $g_2 \ll g_1$. For the intermediate regime, we can make a guess as to the likely behavior by appealing to the c-theorem: if one of the two final IR fixed points reached by the two limits of extreme anisotropy has higher $c$, it is likely that any finite anisotropy will cause the flow to continue to the other IR fixed point.

There are several patterns which can arise in our example. If $f_R \geq f_L \geq f_R$, the $g_1 \gg g_2$ limit will start with the flow $SU(2)_{f_L} \otimes SU(2)_{f_R} \rightarrow SU(2)_{f_R} \times SU(2)_{f_R} \otimes SU(2)_{f_R} \times SU(2)_{f_R}$. The remaining interaction is irrelevant at this fixed point. The $g_2 \gg g_1$ limit will follow a different sequence: $SU(2)_{f_L} \otimes SU(2)_{f_R} \rightarrow SU(2)_{f_L} - f_R \otimes SU(2)_{f_R} - f_R \rightarrow SU(2)_{f_R} - f_R \otimes SU(2)_{f_R} - f_R$...
given in a subsequent work.

One may also drive the charge sector to universal NFL fixed points. Indeed, the charge sector of \( H_0 \) has a particle-hole \( SU(2) \) symmetry with the (right) charge being the third component \( C_{\Gamma R, r} = \frac{1}{2} \int \psi^*_{\Gamma R, a, r}(x) \psi_{\Gamma R, a, r}(x) \), \( C_{\Gamma R, r} = \int \psi^*_{\Gamma R, r}(x) \psi_{\Gamma R, r}(x) \) and \( C^{-} = (C^+)^* \). Then, the previous arguments indicate that adding an interaction term (an umklapp term) \( \sum \int dx (g_{\Gamma})^{m} C_{\Gamma R, r}(x) C_{\Gamma L, l}(x) \) would drive the model to a chiral NFL fixed point in the charge sector. Again, the resulting charge correlation functions would be characterized by the exponents discussed above.

We can actually follow the flow at any scale by solving the model exactly. The model exhibits dynamical fusion (a different approach was given in \( \text{[3]} \)), and allows a solution by a method developed in the context of the anisotropic multichannel Kondo model \( \text{[4]} \). We find that the model generates scales \( m_L^e, m_R^e : l \leq f_L, r \leq f_R \), parameterizing the patterns of flavor symmetry breaking, and setting the scales of the excitation energies and momenta. The free energy in the spin sector, the only sector modified by the interaction with the impurity, is given by,

\[
F(T, h) = -\frac{TL}{2\pi} \int_{-\infty}^{\infty} d\xi \left( \sum_r m_r^e e^{-\xi} \ln(1 + \eta_r(\xi, \frac{h}{T})) + \sum_l m_l^e e^{\xi} \ln(1 + \eta_l(\xi, \frac{h}{T})) \right), \tag{10}
\]

where the functions \( \{\eta_r(\xi, \frac{h}{T})\} \) are the solution of the following system of coupled integral equations (TBA-equations, see Figs. \( \text{[5]} \)):

\[
\ln \eta_r = \frac{2m_r^e}{T} e^{\xi} - \frac{2m_r^e}{T} e^{-\xi} + G \ln(1 + \eta_{n+1}) + G \ln(1 + \eta_{n-1}), \quad n = 1, ..., \infty, \ \eta_0 \equiv 0, \tag{11}
\]

with boundary condition: \( \ln \eta_0 \to 2n\mu h / T \). The integral operator \( G \) is defined by the kernel \( 1/(2\pi \cosh(\xi - \xi')) \). In the isotropic case, \( m_L^i = m_L^e \delta_{f_L, l}, m_R^i = m_R^e \delta_{f_R, r} \).

![Diagram](https://via.placeholder.com/150)

**FIG. 3.** Diagrammatic representation of the TBA equations. Each circle corresponds to a function \( \eta_l \), and the links indicate which functions are connected by the equations. The equations for \( \ln \eta_L \) and \( \ln \eta_R \) have driving terms, which are represented by filled circles.

The forms of the driving terms \( m_L^e e^{\xi} \) and \( m_R^e e^{-\xi} \) are characteristic of massless left and right moving excitations, and when both occur at the same level (namely, when \( f_L = f_R \)) a driving term \( m \cosh \xi \) results, indicating a mass gap. To be more explicit, consider the case of two-channels of right movers and one-channel of left movers. The two couplings \( g_1, g_2 \), are obtained by diagonalizing the matrix of couplings \( g_{\Gamma l} \). Choose \( g_1 < g_2 \), \( \phi = g_1 / g_2 \). The physical scales then are, \( m_R^1 = 2D_R \cos(\phi \pi / r), m_R^2 = D_R e^{-\pi / r}, m_R^3 = 2D_R e^{-2\pi / r} \), with \( D_L, D_R \) the densities of left and right movers.

**FIG. 4.** Same as the previous figure, but for a system with channel anisotropy.

From the TBA-equations both the IR and the UV limits can be read off using TBA-rules: the IR limit of the left movers is obtained from the equations by considering the right mass as infinitely heavy and truncating the equations at the level it was inserted. An analogous rule holds for the right movers. The UV limit is obtained, on the other hand, by considering the masses as vanishing. Applying these rules we deduce the IR limit, and find accord with the conformal considerations. The solution of the equations provides in addition the full interpolation between the UV and IR limits.

We have solved the TBA equations (10,11) numerically for \( f_L + f_R = 2, 3, 6, 7 \), and for several values of \( f_L - f_R \). The results are contained in Figs. \( \text{[6]} \).

\[
f_L + f_R = 6.7
\]

**FIG. 5.** Numerical Solution of the TBA equations: \( C / T \) vs. \( T \) for systems with \( f_L + f_R = 6.7 \)

**Fig. 5** corresponds to the linear coefficient of the specific heat for \( f_R + f_L = 6.7 \). Since the TBA equations describe only the spin sector, we have added to each curve the charge and flavor contributions. This is a temperature independent term that can be read off eq. (10) leading
Therefore, the CFT analysis yields $\gamma_{SU(2)}$ sector in the UV regime correspond to analysis in detail for the case left movers to the thermodynamics. We have made such well as the individual contributions of the right and the
contribute to $\gamma_{SU(2)}$ to strong coupling. The values confirm the CFT predictions with anomalous exponents.

Hence both right and left movers have correlation functions with anomalous exponents.

The calculation of the magnetic susceptibility, Fig. 7, indicates a crossover from a high-T regime where there
are three kinds of spinons (two right movers and one left movers) to a completely different low-T regime. In the IR fixed point, the left movers form a singlet, as can be seen in Fig. 8 by the vanishing of $\chi^{(f=1)}$ at low-T, while the right movers contribution corresponds to just one spinon. The Wilson ratio is
$f_R = 2, f_L = 1$ with channel anisotropy.

$\chi^{(f=1)}, f_R = 2, f_L = 1$ versus $T$ for $f_R = 2, f_L = 1$.

We consider also the effects of channel anisotropy in the $f_R = 2, f_L = 1$ case, Figs. 8-10. Channel anisotropy

FIG. 7. Magnetic susceptibility $\chi$ vs. $T$ for $f_R = 2, f_L = 1$.

FIG. 8. $C_V/T$ vs. $T$ for $f_R + f_L = 2, 3$ and different values of Channel anisotropy. The legend indicates the values of $m$ which are not zero. The isotropic cases correspond to the cases with only two nonvanishing $m$: $m^L_L = m^R_L = 1$, and $m^L_R = m^R_R = 1$.

We consider also the effects of channel anisotropy in the $f_R = 2, f_L = 1$ case, Figs. 8-10. Channel anisotropy
in the right moving sector becomes manifest in the TBA equations by the appearance of additional driving terms at levels below \( f_R \) (equivalently for the left movers). For small anisotropy (when \( m_R^{r<f_R} \ll m_R^{f_R} \)), the \( T \ll m_R^{r<f_R} \) behavior of the system corresponds to a new fixed point, characterized by the value of \( r \) (the level of the new driving term in TBA and in Fig. 3). There is also an intermediate regime, \( m_R^{r<f_R} \ll T \ll m_R^{f_R} \), where the behavior of the system is similar to the isotropic fixed point.

The behavior of \( \gamma \) is shown for several values of channel anisotropy in Fig. 3. We have also included the isotropic case, and the \( f_L = f_R = 1 \) case. At high temperatures, the behavior is the characteristic of the free model, with \( \gamma^{uv} \equiv \pi (f_R + f_L) \). As the temperature is lowered, there is a crossover to the strong coupling regime. If the anisotropy is small enough, the system has an intermediate regime characterized by the same central charge as the isotropic fixed point. As the anisotropy increases, the intermediate region disappears. For low enough temperatures, \( T \ll m_R^{f_R} \), the system flows to a different fixed point, with the same central charge as the high-T, \( f_R = f_L = 1 \) case. Thus, even though the flavor symmetry has been broken, there are still massless modes in the spin sector. Notice that the low-T behavior of the \( f_L = f_R = 1 \) is characterized by the opening of a gap in the spin sector and a drop in the value of \( C/V \). We also observe a peak in \( C/V \) at the crossover region for any anisotropy. Fig. 3 corresponds to the spin part of \( C/V \), which has been divided into right and left contributions in Fig. 10. The first thing to notice in Fig. 3 is that the value of \( \gamma^{uv}_{\text{spin}} \) in the cases with anisotropy is larger than the isotropic value. This is so because, as indicated in the CFT analysis, one starts from different symmetries in the weak coupling region. On the one hand the isotropic case has a symmetry \((SU(2)_2 \times SU(2)_2 \times U(1)) \otimes (SU(2)_1 \times U(1))\) which yields \( \gamma^{uv}_{\text{spin}} = \frac{\pi}{72} (\frac{1}{2} + 1) \) at high-T. On the other hand, the anisotropic case has a symmetry \((U(1) \times U(1)) \otimes (SU(2)_1 \times SU(2)_1) \otimes (SU(2)_1 \times U(1))\). This yields \( \gamma^{uv}_{\text{spin}} = \frac{\pi}{72} (2 + 1) \) at high-T. This can also be seen in Fig. 3. For small anisotropy there is a crossover to an intermediate regime where \( \gamma^{\text{intermediate}}_{\text{spin}} = \frac{\pi}{72} (\frac{1}{2} + 1) \) whereas for the isotropic case \( \gamma^{\text{spin}} = \frac{\pi}{72} (1 + \frac{1}{2}) \). Even though there is a difference in the spin sector, the total values of \( \gamma^{\text{spin}} \) coincide, as we saw in Fig. 8. Notice also that the total spin part in the intermediate regime coincide with the high-T value for \( f_L = f_R = 1 \). However, in the latter case, there is the same spinon contribution from right and left movers, whereas in the former case we have contributions from objects with central charge \( \frac{3}{2} \), respectively.

Finally, at low enough temperatures, the left modes become massive, as in the \( f_L = f_R = 1 \) case, and their contribution to \( \gamma^{\text{spin}} \) vanishes. All the specific heat from spin modes comes from the right movers, with a contribution \( \gamma^{\text{spin}} = \frac{\pi}{72} \), corresponding to a single spinon mode.

We return now to the example of the paired QHE samples presented earlier. Virtual hopping between the edges will generate interaction terms of the form \( \| \) which will drive spin sector to exhibit a universal behavior characteristic of a chiral liquid. This fixed point will determine the behavior of the system down to energy scales \( \sim E_z \) below which the system will flow to more conventional fixed points. The effect of the chiral fixed point will show up in the spin susceptibility, in the spin conductance \( G_s \) or in the NMR response encoded in the spin correlation function \( \langle S^i(t,0)S^r(0,0) \rangle = \langle S^i_L(t,0)S^r_L(0,0) \rangle + \langle \).
\[ S_{R}(t,0)S_{R}'(0,0) >. \] From our previous results we thus have

\[ \frac{1}{TT} = \chi'' = \text{constant} + \omega^\alpha \]

where \( \alpha = 4/(f_R - f_L + 2) \), except when \( f_R - f_L = 1 \), in which case \( \alpha = 2 \).

We have thus proposed and studied models of interacting chiral fermions which realize a new class of non-Fermi liquids in one dimension. A general mechanism, chiral stabilization, was found to be the source of this behavior. It would apply in any model of chiral fermions with non-abelian symmetry, and in some cases even with weakly broken non-abelian symmetry.

The generality of this mechanism suggests that many other experimental realizations will be found and also new fixed points will be identified. Higher hierarchy edge states in QHE systems furnish an example for both. When the filling factor is \( \nu = n/(np + 1) \) with \( p \) a negative even number one edge state mode moves in a direction opposite to the rest\(^1\) providing a chiral imbalance. These modes are no longer fermions but chiral luttinger excitations and when allowed to interact in the manner discussed above new fixed points are expected to arise which would be observable in very clean samples.

Acknowledgments: It is a pleasure to thank E. Y. Andrei, P. Coleman, K. Intrilligator, L. Ioffe, J. Jain, A. Lopez, A. Ruckenstein and A. J. Schofield for comments, suggestions and enlightening discussions. AJ was supported by EPSRC grant GR/K97783.

---

\(^1\) Present address: Institut Laue-Langevin.

1. P. W. Anderson, Cargese Lectures 1988; Varma et al. Phys. Rev. Lett. \textbf{63} (1989) 1996.
2. A. Schmeller, J. P. Eisenstein, L. N. Pfeiffer, K. W. West, Phys. Rev. Lett. \textbf{75} (1995) 4290.
3. A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys. \textbf{B241} (1984) 333, reprinted in C. Itzykson, H. Saleur and J.-B. Zuber, Conformal Invariance and Applications to Statistical Mechanics, World 1988.
4. J. Solyom, Adv. in Physics \textbf{28} (1979) 201; V. Emery in \textit{Highly conducting One-dimensional Solids} ed. J.T. Devreese (1979).
5. J. Voit, Phys. Rev. B \textbf{47} (1992) 6740.
6. F.D.M. Haldane, J. Phys. C \textbf{14}, 2585 (1981).
7. E. Witten, Comm. Math. Phys. \textbf{92} (1984) 455. V. G. Knizhnik and A. B. Zamolodchikov, Nucl. Phys. \textbf{B247} (1984) 83
8. I. Affleck and A. Ludwig, Nucl. Phys. \textbf{B360} (1991) 641.
9. N. Andrei, Integrable Models in Condensed Matter Physics, in \textit{Series on Condensed Matter Physics-Vol. 6}, 458-551, World Scientific, Lecture Notes of ICTP Summer course, September 1992. Editors: S. Lundquist, G. Morandi, and Yu Lu.
10. G. ’t Hooft, Lecture at the NATO Advanced Study Institute on \textit{Recent Developments in Gauge Theory}, Cargèse (1979).
11. A. B. Zamolodchikov, JETP Lett. \textbf{43} (1986) 730.
12. P. Goddard, A. Kent and D. Olive, Comm. Math. Phys. \textbf{103} (1986) \textbf{105}; M. R. Douglas, Caltech Ph. D. thesis, CALT-1453; K. Gawedzki and A. Kupiainen, Nucl. Phys. \textbf{B320} (1989) 625.
13. The leading correction (in the infinite cut-off limit) to the linear term in the specific heat is \( T^2+4/(f_R-f_L+2) \). It comes from the leading singlet irrelevant operator at the IR fixed point \( \phi_0^1 J^a g^\alpha \).
14. N. Andrei and C. Destri, Phys. Rev. Lett. \textbf{52}, (1984) 364.
15. A. Polyakov and P. Wiegmann, Phys. Lett. \textbf{141B} (1984) 223; A. Tsvelik and P. Wiegmann Z. Phys. \textbf{B54} (1985) 201.
16. N. Andrei and A. Jerez, Phys. Rev. Lett. \textbf{74}, (1995) 4507.
17. K. Intrilligator, private communication. A.B. Zamolodchikov and Al. B. Zamolodchikov, Nucl. Phys. \textbf{B379} (1992) 603.
18. X.G. Wen, Phys. Rev. Lett. \textbf{64}, (1990) 2206; C. L. Kane and M.F.A. Fisher, Phys. Rev. \textbf{B 51}, (1995), 13449.