Original Research Paper

Dynamic adaptive spectrum allocation in flexible grid optical network with multi-path routing

Ujjwal Ujjwal | Neha Mahala | Jaisingh Thangaraj

Department of Electronics Engineering, Indian Institute of Technology (Indian School of Mines), Dhanbad, India

Correspondence
Jaisingh Thangaraj, Department of Electronics Engineering, Indian Institute of Technology (Indian School of Mines), Dhanbad-826004, India.
Email: t.jaisingh@gmail.com

Abstract
Fragmentation is one of the major issues in elastic optical network (EON) due to its dynamic characteristics which may possibly degrade the spectrum efficiency. Several techniques have been presented in the literature to get the optimal solution of this problem while multi-path provisioning comes out to be very compelling among them. In multi-path provisioning enable networks, multi-path routing and spectrum assignment (MPRSA) creates further complications due to the hardware availability and more spectrum consumption. This paper presents a novel technique termed as adaptive service provisioning (ASP) to come up with a solution of the MPRSA problem and it minimizes the spectrum fragments in the network by precisely managing the scattered spectrum slots. This technique performs the adaptive traffic demand splitting on disjoint paths in case it is unable to be served by k-shortest routing paths based on the parameters such as hardware support, capacity available and demand. An integer linear programming (ILP) model as well as a heuristic algorithm for solving MPRSA problem in large optical networks where ILP is not compliant is developed. Numerical simulations are performed to investigate the performance of the algorithm in terms of blocking probability and spectrum utilization.

1 | INTRODUCTION

The unceasing growth in network bandwidth approximately 40% per year due to worldwide communication and speedy adoption of internet services (cloud computing, video on demand) requires higher data rates and efficient utilization of spectral resources. Conventional dense wavelength division multiplexing (DWDM) network split up the respective optical spectrum ranging from 1530 to 1565 nm (referred as C-band) into discrete bands, typically spaced by 50 or 100 GHz (standardized by the International Telecommunication Union (ITU)). The frequency spacing between two adjacent channels is quite large, thereby in case of lesser bandwidth requirement; a large portion of spectrum goes unutilized. However, higher data rates (>100 Gb/s) does not exactly fit in the existing grid, even if adequate wide spectrum is provided, the transmission of high-data rate signals over long optical reach with good spectral efficiency is challenging [1]. Therefore, researchers have focused on finding the solution for fixed grid and spectral usage in a more efficient manner [2]. Recently, elastic optical network (EON) has emerged out as a promising solution to this challenge through flexible grid technology by providing flexible spectrum allocation depending on the demand. The flexible characteristic of the EON allows the handling of heterogeneous traffic demands by precisely allocating the spectral resources according to the requirement through which it also improves the spectrum efficiency. In case of flexible grid technology, a 50 GHz fixed grid channel is divided into narrow width frequency slots (FSs) and a set of contiguous slots are allocated to the request according to the requested bandwidth and modulation format. It accomplishes spectrum allocation in such a way that it improves the flexibility and efficiency of the resource utilization. Although it leads to the increment in the hardware complexity, but that can be compensated through the photonic integration technology [3].

The problem of searching an appropriate route and assigned spectral resources on the route is termed as routing and spectrum assignment (RSA). RSA has received an intensive attention in recent years due to its huge impact on the operation of EON. Some researchers have considered RSA as a single problem, whereas others have divided it into two subsections: a routing subsection and spectrum assignment subsection. RSA...
in EON is similar to routing and wavelength assignment (RWA) in WDM with an exemption in the allocation of the spectrum. RSA is more challenging than RWA as it considers spectrum contiguity, spectrum continuity and non-overlapping spectrum constraint where a connection request should be assigned similar positional and contiguous FSs on each fibre link along its route. If a connection request requires similar links, then it must be allocated to a non-overlapping portion of the spectrum. In static optical network, slots are assigned and remain unchanged, whereas in dynamic scenario FSs are assigned and released, according to the holding time of the connection request [4, 5]. However, when a large bandwidth request is terminated, released FSs may be assigned to a small connection request and remaining slots may remain unoccupied due to the spectrum continuity and spectrum contiguity constraint. This is known as spectrum fragmentation. In this, a connection request may not get serviced even though sufficient bandwidth is available on the links, but scattered at different range in whole spectrum. This problem of spectrum fragmentation causes insufficient utilization of spectral resources and result in to poor blocking performance. Several works on the spectrum fragmentation issue has reported in the literature. In [6], authors investigated the viability of applying survivable virtual network embedding (SVNE) scheme with multi-path allocation for software defined network (SDN) and network function virtualization (NFV) technology. This proposed SVNE achieves considerable improvement in terms of spectral efficiency and survivability. The authors in [7] proposed three algorithms to address the spectrum fragmentation issue in the EON named as Most Fragmented Path per Core (MFPC), Fragmentation Measure Metric Aware with Routing and Spectrum and Core Assignment (FMMA-RSCA) and Spectrum Block Multi-pathing per Core (SBMC). These algorithms provide a reduction in blocking probability, by managing fragmentation in the core. SBMC achieves best blocking performance among all as it is based on multi-path service provisioning and classified course.

In [8], authors proposed various fragmentation metrics for spectrally/spatially flexible optical networks (SS-FONs), as extended evaluation techniques of fragmentation in EONs. Moreover, the new concept of bordering super-channels (BSChs) was introduced. Six new algorithms have been proposed in [9], to resolve the fragmentation issue in the network and improves blocking performance. An Intelligent Fragmentation Aware Routing, Spectrum and Core Allocation (IF-RSCA) is proposed in [10], it utilizes core classifications based on connection requests holding times and selects the best route, spectrum, and core for each arriving connection request to decrease the fragmentation level and connection blocking probabilities. The authors in [11] presented a novel approach that utilizes machine learning to predict future connection requests to accomplish efficient spectral resource allocation, which decreases both crosstalk and fragmentation in space division multiplexing elastic optical networks (SDM-EONs). Recent developments on spectrum fragmentation management is presented in [12]. The fragmentation management is usually classified into two techniques, namely, non-defragmentation and defragmentation. The non-defragmentation technique prevents spectrum fragmenta-

tion when a light-path is established, and the defragmentation technique resolves the spectrum fragmentation issue by reconfiguring and rearranging the spectrum allocated for established light-paths.

In research work [13], spectrum reconfiguration techniques to reassigned spectrum resources by solving auxiliary graph is presented. In reference [14], authors presented ‘utilization entropy’ of link, route and network to determine fragmentation level of an optical network or estimates ‘how well the spectrum resources are used’. A low value of utilization entropy signifies that the spectrum resources are used uniformly with lesser gaps on the slot utilization map and low fragmentation level. When large gaps come up between occupied slots, then utilization entropy value is getting higher, signifying high level of fragmentation. The value of utilization entropy is normalized between 0 and 1, thereby it acts as a general indicator for evaluating and comparing the fragmentation level in the network. Such kind of indicator is particularly advantageous for network optimization purposes, viz monitoring the network efficiency, determining the timing of triggering network reoptimization and estimating the efficiency of reoptimization. Besides all the techniques which directly solve the spectrum fragmentation issue, we can utilize the scattered, the small, isolated and non-contiguous slots by multi-path routing. Multi-path routing provides a solution to the scarcity of resources and the survivability issue in EON. The main concerned about using multi-path routing is the extra consumption of spectral resources by guard-bands. Extra guard-bands on the multiple paths leads to a spectrum under utilization. Many multi-path routing techniques have been presented in the literature. An adaptive multi-path RSA approach based on the relative cost technique is proposed in [15]. The relative cost evaluates the effect of established calls on network resources at a specific state. A pre-split multi-flow routing, modulation and spectrum assignment (RMSA) algorithm is proposed in [16]. Three novel measures are adopted in this approach, viz frequency slices are divided into standard blocks, connection requests are pre-split up into flows precisely using the standard blocks and free standard blocks are searched for the flows with a proposed modified-most-used rule. These measures can align both the occupied and available FSs, thereby expected to improve the blocking performance. Two energy-efficient multi-path based survivable routing, spectrum, and core assignment (RSCA) approaches, viz inter-core crosstalk-aware (CMDE-RSCA) and fragmentation-aware (FMDE-RSCA) have been proposed in [17]. The first one lessens the inter-core crosstalk in the network as far as it can. The latter one mainly decreases the spectrum fragmentation, while keeping inter-core crosstalk within admissible limit. In [18], authors proposed a multi-path routing algorithm for the protection of SDM-EONs using shared-backup path protection. In [19], authors proposed a multi-path service provisioning with differential delay constraint for orthogonal frequency-division multiplexing (OFDM)-based EON. Dynamic service provisioning with hybrid/multi-path routing has been proposed in [20]. Results indicate that a significant decrement in bandwidth fragmentation ratio is obtained. In [21], author proposed a multi-flow virtual concatenation (MFVC)
model in EONs for achieving a better spectral efficiency using non-contiguous FSs. In this split multi flow (SMF), RSA algorithm is proposed based on the MFVC model for the advance determination of the distribution and size of spectral fragments. In [22], author presented multi-path fragmentation aware RSA technique for Advance Reservation (AR) and Immediate Reservation (IR) of connection requests for the improvement of blocking performance. In [23], authors deal with the multi-path service provisioning provided by the Multi-path Provisioning with Content Connectivity (MPCC) to improve the survivability of EON. In this, a dynamic content placement strategy is proposed for observing the variation in connection request distribution and perform the adjustment of content location. In [24], authors presented a multi-path protection scheme using hybrid and Failure-Independent Path Protecting (FIPP). The authors in [25] proposed multi-path routing with traffic grooming to enhance blocking performance and spectral efficiency. Overall, no work has been presented considering shortest disjoint multi-paths with adaptive fragmentation of the connection request for the reduction of blocking probability. This motivates us to improve the blocking performance and spectrum utilization by considering shortest disjoint paths with pre-computed capacity and hardware (transponder) availability for adaptive fragmentation of the connection request.

This paper proposes a novel scheme termed as Adaptive Service Provisioning (ASP) which aims to provide the solution of Multi-Path Routing and Spectrum Assignment (MPRSA) problem in EON and quantifying the improvement in the blocking performance and spectrum utilization. The primary objective of this scheme is to improve blocking performance and reduces the extra spectrum consumption by guard-bands in the multi-path provisioning by adaptive fragmentation of connection request on disjoint paths. Initially, ASP scheme performs dynamic service provisioning using $k$-shortest routing paths, in case if it is unable to get served by $k$-shortest paths, then it is shifted to the adaptive fragmentation of connection request on shortest disjoint paths based on the parameters, such as hardware impairment (bandwidth variable transponders (BVTs))/slice-able bandwidth variable transponders (SBVTs)), pre-computed capacity available and demand. It is noteworthy that, at the receiving end, transceiver assembles the inverse multiplexed multiple sub-traffic demands into original traffic demands using an electronic buffer when differential delay is addressed. ('Note: we do not address the differential delay issue among units (sub-traffic demands), which is left out of scope for this paper.')

The adaptive fragmentation of connection request utilizes minimum number of disjoint paths and scattered spectrum slots which in turn can possibly resolve the potential contention condition without penalizing subsequent demands due to the over-consumption of transponders and guard-bands. We develop an Integer Linear Programming (ILP) model as well as heuristic algorithm for solving MPRSA problem in large optical networks where ILP is not compliant. It is evident from the numerical results that the spectrum utilization is improved by the proposed scheme, and it improves the admissible traffic compared to the existing schemes. The performance of our proposed algorithm is evaluated in terms of blocking probability, spectrum utilization and dynamic connection establishment percentage. It is to be noted that in multi-path routing, the differential delay occurs due to the routing paths disparity give rise to the requirement of additional bandwidth and buffers at the destination node intending to indemnify the transmission [26]. However, addressing of differential delay do not come under the scope of this paper. While, to rectify this problem following techniques can be used: (1) Split spectrum mechanism which limits all sub-flows of a traffic demand to be routed over the same routing path, (2) Multi-path service provisioning approach which considers differential delay constraint [27].

The rest of the paper is organized as follows: Section 2 describes a split spectrum approach followed by the system model and problem statement for the dynamic RSA in Section 3. The ILP formulation of the RSA problem is presented in Section 4 and the proposed ASP algorithm is discussed in Section 5. In Section 6, we present the simulation platform and results of the performance evaluation. Finally, Section 7 concludes the paper.

## 2 SPLIT SPECTRUM APPROACH

Split spectrum is a promising way to mitigate the effect of spectrum fragmentation in EON. It increases the spectrum assignment possibilities of a traffic demand by splitting into multiple sub-traffic demands and each sub-traffic demand is routed over different routing paths. This approach is mainly implemented in two cases:

- When enough contiguous FSs are not available to serve the demand;
- When the modulation format used to transmit the desired bit rates become in-feasible due to transmission impairments which restricts the optical reach.

From the above-mentioned scenario, the traffic demand can be splitted into several sub-traffic demands to get the benefits of available spectral resources [27–29].

The split spectrum approach offers two instant benefits:

- More appropriate modulation technique can be implemented as sub-traffic demands require lower bit rates for the transmission;
- This helps in recovering the loss caused by the spectrum fragmentation by utilizing a small set of contiguous or isolated FSs.

The implementation of the split spectrum approach involves BVTs or SBVT, one per sub-traffic demand after the splitting process. In BVT-based implementation, each sub-traffic demand requires an independent BVT for transmission as shown in Figure 1. Therefore, the number of divisions significantly affects the performance of the split spectrum approach. Large number of divisions may completely exhaust the available BVTs at the node which results in further blocking of the request due to unavailability of hardware. Therefore, this issue has to be considered during the implementation. Furthermore,
this implementation does not provide any increment in hardware system complexity and cost due to the utilization of the existing hardware in the network.

Next, SBVT or Multi-Flow (MF) transponder is able to assign its capacity into one or multiple optical flows which are then transmitted to single or multiple destinations. In the MF transponder-based implementation, once the traffic demand has gone through the splitting process, the resulting sub-traffic demands are directed to an MF transponder for transmission on the corresponding disjoint paths (the bandwidth of each sub-traffic demand depends on the available FSs on the corresponding disjoint paths). An MF transponder is capable of transmission and reception of multiple optical flows, each optical flow could be rate-adaptive and reach-adaptive depending on the data rate and optical reach of the data to be transmitted over the EONs as shown in Figure 2. Therefore, SSA can be realized employing a single MF transponder to transmit all the sub-traffic demands resulted from the splitting process [27]. MF transponder–based implementation is advantageous over BVTs as a single transponder is employed per traffic demand, whether it is to be split up or not, therefore potential shortage of transponders is not noticeable in this scenario. Conversely, hardware requirements and complexity are higher in MF transponder–based implementation, which may result in higher network cost.

3 | SYSTEM MODEL AND PROBLEM STATEMENT

Here, we state the problem of MPRSA in the EON. The main objective of MPRSA is to determine the most appropriate resources (routing paths, spectrum and modulation format) for the multi-path provisioning, according to the current state of the network, so that blocking probability can be minimized. To stimulate the minimization of blocking probability, two principal sub-objectives are carried out: (1) Reduction in the spectral resources during connection provisioning, and (2) Reduction in the spectrum fragmentation during allocation of resources in the EON.

In order to achieve the first sub-objective, two action lines are defined. First, a traffic demand should be divided into minimum number of sub-traffic demands as more number of divisions (splitting of traffic) exhausts the available transponders at the node. Moreover, each sub-traffic demand requires guard-bands, on the either side, so using more sub-traffic demands results extra spectrum consumption in the network. Second, summation of the bit rate of all sub-traffic demands should be at least equal to the bit rate of original traffic demand. The main purpose of doing so is to reduce the spectrum requirement of a traffic demand as higher bit-rate modulation format requires larger spectrum.

For the second sub-objective, small, isolated and non-contiguous FSs are utilized which usually remains unoccupied in the dynamic network scenario.

3.1 | System model, definition and assumptions

EON topology can be expressed as a graph $G(N,E,S)$, where $N$ denotes the number of nodes, $E$ denotes the set of links and $S$ denotes the FSs per fibre link. The data carrying capacity of the FSs can be defined as $M_k * l_{slot}$, where $M_k$ is the modulation level ($M_k$ is 1, 2, 3 and 4 for BPSK, QPSK, 8-QAM and 16-QAM, respectively) and $l_{slot}$ is the spectrum width of the slot. Modulation technique is assigned to each connection request based on its transmission distance, such as BPSK for 5000 km, QPSK for 2500 km, 8-QAM for 1250 km and 16-QAM for 625 km. A connection request is expressed as $C_t(s,d,t_s,t_h,Q)$, where $s$ and $d$ are source and destination node, respectively, $t_s$ is the start time, $t_h$ is the holding time and $Q$ is the demand of the connection request (Gb/s). In order to serve a connection request, a path has to be searched which can accommodate its capacity $Q$ within the given service time $[t_s, t_s + t_h]$. On the selected routing path, (we assume that there is no spectrum conversion in the network) the capacity $Q$ is mapped into $N_w$ contiguous FSs which are available at a given service time on each link. Here, $N_w$ is determined according to Distance Adaptive Modulation (DAM). It is given by Equation (1).

$$N_w = \left\lfloor \frac{Q}{M_k * l_{slot}} \right\rfloor + N_g,$$

where $N_g$ is the number of FSs occupied by the guard-band.
We assume that the impairments that are associated to dispersion can be significantly decreased or fully compensated by coherent detection and digital signal processing at the receiver. For every connection request \( C \), the proposed service provisioning algorithm’s main goal is to search a low cost RSA subject to the following constraints:

- **Spectrum continuity constraint**: In order to establish a light-path, a similar set of FSs must be allocated for each link along the route.
- **Spectrum contiguity constraint**: For every sub-traffic demand, FSs must be adjacent to each other in the spectrum domain.
- **Demand constraint**: The cumulative bit rate of modulation formats employed by BVTs to serve the sub-traffic demands must be at least equal to the bit rate of original traffic demand.
- **Hardware constraint**: The split sub-traffic demands should not exceed the number of available transponders at nodes.

The proposed mechanism tries to utilize the small set of non-contiguous and isolated FSs, when a connection request is blocked due to the unavailability of enough contiguous FSs to serve the complete bandwidth. This approach forms the foundation for the proposed technique. If a traffic demand cannot get served due to the unavailability of enough FSs on the links along the route, then it can be served by splitting into multiple lower rate sub-traffic demand and routed over different disjoint paths. Routing of sub-traffic demands over multiple paths requires transponders at the source and destination nodes for each path. Therefore, it is necessary to find out the hardware support before splitting a traffic demand on multiple light-paths. This helps in controlling the number of divisions done to the traffic demand, which directly affect the number of transponder usage and guard-band consumption in the network. However, this verification allows blocking of the connection request at the initial of the mechanism when source or the destination node does not have at least one transponder available, even though sufficient spectrum resources are available. Therefore, hardware (transponder) availability is the main concerned faced during experimentation, as every part into which a connection request is split employs an independent transponder. Although, the analysis of the extra hardware that would allow carrying out the splitting and the integrating of the connection request into the physical nodes is out of the scope of this paper and left for future work.

### 3.1.1 Hardware support

It is very essential to consider the available BVT and maximum independent flows supported by SBVT at the nodes while serving a connection request in EON. Even though we allocate the bandwidth through FSs \( (l_{dis}) \), one BVT or one independent flow in case of SBVT is to be assigned to every connection request for transmission. In addition, when traffic is sent over different BVTs or SBVTs, the Ethernet to Elastic Optical Networks (ETH-EON) interface adopts different modulation techniques or baud rates to optimize transmission performance. Hence, data aggregation is essential for optimum utilization of BVT or SBVT per transmitter for every node between any \( s \rightarrow d \) node pair. Before slicing and checking of FSs available, it is essential to check for Hardware Support (BVT or SBVT) for every node between \( s \rightarrow d \) pair. A connection request should be fragmented according to the number of BVTs available at the nodes or the number of independent flows supported by SBVT.

Let, \( b_i \) denote the available BVT for every node along the route. It is to be calculated for all nodes along the route of a \( s \rightarrow d \) connection. \( b_i = 1 \), when at least one BVT or SBVT is available on node \( i \)th route, else it is 0. Hence, \( b_i \) is a combined result of the cumulative outcome of path computation. For \( i \)th path it is essential to consider the BVTs or SBVTs available for every node between \( s \rightarrow d \) connection during service time \( (t_i, t_i + t_b) \). The availability of FSs along with \( 'b_d' \) decides the transmission of data along the route. Hence, among \( p \) disjoint paths, \( s_{dis \_joint} \) paths with \( b_i = 1 \) are used for data transmission according to the availability of available FSs.

In the following, we come up with ILP formulation to address the stated MPRSA problem as well as heuristic algorithm for the scenario when ILP is not compliant.

### 4 ILP FORMULATION OF THE MPRSA PROBLEM

In this section, we formulate the MPRSA problem in EON as an ILP model, with the objective of reducing the sub-carrier slot requirement of a traffic demand. In this formulation, \( G(N,E) \) represents the physical network topology where \( N \) denotes the set of nodes \( (n_1, n_2, n_3, ..., n_s) \) and \( E \) is the set of links \( (e_1, e_2, e_3, ..., e_m) \). Given a connection request \( C_i(s, d, n_1) \), where \( s \) and \( d \) denote the source and destination \( (s, d \in N) \) and \( (n_1) \) represents the bandwidth requirement in terms of number of FSs. The ILP model of the connection request \( C_i(s, d, n_1) \) is given below.

#### Notations and variables

1. \( S \): Set of FSs \( (f_1, f_2, f_3, ..., f_M) \) on each link.
2. \( r \): Number of pre-determined shortest paths for every \( s \rightarrow d \) in \( G(N,E) \).
3. \( R_{sd} \): Set of \( r \) shortest paths for every \( s \rightarrow d \) pair in \( G(N,E) \).
4. \( R_k \): The \( k \)th shortest path in \( R_{sd} \), \( k \in [1, r] \).
5. \( r' \): Number of the pre-determined link disjoint path for every \( s \rightarrow d \) pair.
6. \( R'_{sd} \): Set of \( r' \) link disjoint path for every \( s \rightarrow d \) pair in \( G(N,E) \).
7. \( R'_k \): The \( g \)th link disjoint path in \( R'_{sd} \), \( g \in [1, r'] \).
8. \( x_{fg} \): \( 0 \leq x_{fg} \leq 1 \) if FS \( f \in S \) on link \( e \) is occupied by the connection request \( C_i \), else, it is 0.
9. \( W_{fg} \): \( 0 \leq W_{fg} \leq 1 \) if shortest path \( R_k \) is selected for the connection request \( C_i \), else, it is 0.
10. \( x'_{fg} \): \( 0 \leq x'_{fg} \leq 1 \) if path \( R'_k \) is allocated sub-carrier \( f \), else, it is 0.
11. \[ x'_{g} \in \{0,1\} \text{ if path } R'_{g} \text{ is used to accommodate request } C_{g}, \text{ else, it is } 0. \]

12. \[ U'_{g} \in \{0,1\} \text{ if sub-carrier slot } f(1 \leq f \leq S) \text{ is vacant on the path } R'_{g}, \text{ and } U'_{g} = 1, \text{ if FS } f \text{ is unavailable on the path } R'_{g}. \]

13. \( G_{b} \): Number of FS acting as guard-band.

\[ y_{f} \in \{0,1\} \text{ if FS } f \text{ is assigned to at least one connection request on any link in the network, else, it is } 0. \]

### 4.1 Objective function

\[ \text{minimize } \sum_{f \in S} y_{f} \quad (2) \]

Our objective is to minimize the total allocated FSs for all connection requests.

### 4.2 Constraints

\[ (x'_{g} - x'_{g+1} - 1) \times (-S) \geq \sum_{f' \in [f+2,S]} x'_{g} \quad \forall f, g \quad (3) \]

Equation (3) shows the spectrum contiguity constraint that indicates the adjacent FSs are allocated on a path.

\[ U'_{g} \cdot x'_{g} \leq 0 \quad \forall g, f. \quad (4) \]

Equation (4) shows the spectrum non-overlapping constraint which specifies any slot \( f \) cannot be allocated on a path if it is already occupied on the path.

\[ \sum_{f \in \Lambda} x'_{g} = (n_{e} + G_{b})W_{k} \quad \forall e \in E. \quad (5) \]

Equation (5) shows the connection request accommodation constraint. It ensures that allocation of adequate number of FSs including requested bandwidth \( n_{e} \) and guard-band \( G_{b} \) along the shortest routing path \( R_{k} \) to the connection request \( C_{e} \). Here, \( W_{k} \) is set to 1 while sufficient FSs are available. When sufficient FSs are unavailable, \( W_{k} \) is set to 0 and the connection request is adaptively fragmented on the disjoint path subject to following constraint (6):

\[ \sum_{g \in \Lambda} \sum_{f \in [1, S]} x'_{g} \geq n_{e} + \sum_{g \in [1, r'] G_{b}. \quad (6) \]

Equation (6) ensures that bandwidth condition of the demand is fulfilled.

\[ \sum_{g \in [1, r']} \sum_{f \in [1, S]} x'_{g} - \left( n_{e} + \sum_{g \in [1, r']} x'_{g}G_{b} \right) = 0. \quad (7) \]

Equation (7) ensures that the slot allocation on disjoint paths as per the available number of FSs till the requested FSs capacity is fully provisioned. Since MPRSA problem is NP-complete, considering the limitations in ILPs, a four-node topology is considered as mentioned in Figure 3 for our computation.

In this formulation, each spectrum link consists of 17 FSs and nine connection are being active: \( C_{1}(A, C, 3), C_{2}(A, B, 4), C_{3}(B, C, 5), C_{4}(A, C, 8), C_{5}(A, G, 3), C_{6}(A, C, 5), C_{7}(A, D, 7), C_{8}(D, C, 5) \) and \( C_{9}(A, C, 7) \). After solving the ILP, eight light-paths are established to successfully provisioning the eighth connection requests. While provisioning ninth connection request, it gets rejected on the shortest path, and hence it is provisioned by splitting it with two disjoint paths. Hence, it is found that ILP formulation can be successfully implemented on the small-scale network, but it is not the same while it is applied on large-scale network. Thereby, the following heuristic algorithm is proposed in Section 5.
5 | ASP ALGORITHM

In a real scenario, optical networks are absolutely wide-reaching such as PAN European and NSFNET, which arises difficulty in solving RSA optimization problem due to the computation complexity of ILP formulation. In dynamic scenario, connection setup time and tear down time cannot be predicted in advanced. Therefore, ILP formulation is not appropriate for RSA problem. To this end, heuristic algorithm is designed to provide the solution in a dynamic traffic scenario. This paper presents a heuristic algorithm, termed as ASP to effectively utilize the spectrum. The detailed procedure of ASP algorithm is illustrated with an example network topology as shown in the Figure 3. The spectrum on every fibre link is divided into 17 FSs, where the label of the FSs are arranged in the increasing order from 1 to 17 as shown in the Figure 4. In the given topology, we assumed that connection requests \( C_i(s, d, Q) \) arrive in time order is shown by different colours. Initially, all spectral resources are unoccupied so first seven requests get service provisioning on shortest paths from node \( s \) to \( d \) are \( C_{i1}(A \rightarrow B \rightarrow C), C_{i2}(A \rightarrow B), C_{i3}(B \rightarrow C), C_{i4}(A \rightarrow C), C_{i5}(A \rightarrow C), C_{i6}(A \rightarrow D \rightarrow C) \) and \( C_{i7}(A \rightarrow D) \). On further arriving of connection requests \( C_{i8}(D, C, 5) \) and \( C_{i9}(A, C, 7) \) and \( C_{i10}(B, C, 5) \). The \( C_{i8} \) is provisioned successfully by \( (D \rightarrow C) \) but request \( C_{i9} \) and \( C_{i10} \) has been rejected due to the unavailability of seven and five contiguous FSs along any shortest path considering the spectrum continuity and contiguity constraint. But requested amount of FSs are scattered on the whole spectrum due to fragmentation as shown in Figure 4. To this end, we have proposed an ASP algorithm in EON. The procedure and benefits of the ASP for routing and spectrum assignment can be reflected from Figures 3 to 5. The small set of available and useless spectrum fragments can be assigned to sub-traffic demands when an original demand is blocked or reroute under the control of the current network scenario. The service provisioning of connection request \( C_{i9}(A, C, 7), C_{i10}(B, C, 5) \) are performed by ASP algorithm using adaptively fragmented flows on the shortest disjoint paths \( t_{d_{i}j} \): \( t_{disjoint1} (A \rightarrow B \rightarrow C), t_{disjoint2} (A \rightarrow C) \), and \( t_{disjoint1} (B \rightarrow C), t_{disjoint2} (B \rightarrow A \rightarrow D \rightarrow C) \) which are determined through the port availability, respectively. The evaluation of disjoint path considering port availability helps in allowing the blocking of traffic demand at very early stage when no transponder is available. The descriptive methodology of the ASP algorithm is defined along the lines. Next, the \( C_a \) for each disjoint path at a given service time is determined based on the similar positional and contiguous FSs \( (N_{j}) \) available on each link along the route at given service time.

\[
C_a = (N_j - 1)M_k * l_{dot}, \tag{8}
\]

\[
N_j(R, t_s, t_e) = \sum_{i=t_s}^{t_e} L(R, t). \tag{9}
\]

Here, \( L(R, t) \) is defined as the number of contiguous slots available on route \( R \) at service time \( [t_s, t_e] \).

According to the defined Equations (8) and (9) the cumulative capacity available at \( t_{disjoint} \) paths are determined.
The connection request gets blocked when Equation (10) is satisfied.

\[ \sum_{i=1}^{t_{\text{disjoint}}} C_{ai} < Q. \]  

(10)

This is the process of identifying that incoming traffic demand can be accommodated by the disjoint paths or not. In other cases, when \( \sum C_{ai} \geq Q \), the algorithm performs the adaptive fragmentation of the traffic demand on disjoint paths. In the context of the given example \((C_{9}, C_{10})\), the cumulative capacity at corresponding disjoint paths in terms of sub-carrier slots is greater than or equal to the requested bandwidth. So the incoming traffic demands \( C_{9} \) and \( C_{10} \) are served by ASP algorithm using adaptively fragmented flows on the corresponding disjoint paths as shown in Figure 5. Initially, the capacity available at the first disjoint path is allocated to the traffic demand, then, this process is performed repeatedly with increment in the number of sub-traffic demands by one every time until the \( X \geq 0 \).

\[ X = Q - \sum_{i=1}^{t_{\text{disjoint}}} C_{ai}. \]  

(11)

The bandwidth of sub-traffic demands is initially uncertain and it depends on the capacity available at each disjoint path. At the end when \( X \) reaches to zero that means the traffic demand is successfully served using different sub-traffic demands. The detailed procedure of the proposed algorithm is shown in the Flowchart (Figure 6) and Algorithm 1.

### 5.1 Slot saving ratio

In multi-path routing, a high data rate signal is split up into multiple lower rate signals and each signal is routed over different routing paths. However, it would not be efficient to divide a high data rate signal into lower data rate ones as guard-bands are needed along each routing path to avoid interference from adjacent ones. As a result, high spectrum occupancy unfairly reduces the spectral efficiency of the network. Therefore, it is essential to employ multi-path routing in such a way that it turn down the spectrum consumption by guard-bands. To this end, we introduced a metric termed as the slot saving ratio to validate the performance of multi-path routing. Ideally, it should be approaches to one. Slot saving ratio of a connection request is determined using Equation (12).

\[ \text{Slot saving ratio} = \frac{N_{r}}{N_{s}}, \]  

(12)

where \( N_{r} \) is the total number of FSs required by a traffic demand, \( N_{s} \) is the total number of FSs assigned to the sub-traffic demands on multiple paths. When a connection is set up, the total number of FSs required by a traffic demand consumes one additional slot on either side as guard-band.

\[ N_{r} = t_{fl} + G_{b}. \]  

(13)

where \( t_{fl} \) is the number of slots corresponding to the actually required bandwidth of traffic demand, \( G_{b} \) is the number of guard-bands placed on the either side of traffic demand.

\[ N_{s} = \sum_{m=1}^{L} (t_{m} + G_{bm}), \]  

(14)

where \( t_{m} \) is the number of slots corresponding to the actually required bandwidth of \( m \)th sub-traffic demand. \( L \) is the number of sub-traffic demand.

Aforementioned equations illustrate that the cumulative effect of guard-band consumption by each sub-traffic demand
ALGORITHM 1 Adaptive Service Provisioning Algorithm

INPUT: The network topology $G(N, E, S)$, the connection request $C_i(s, d, t_s, t_f, Q)$, slot width ($t_{slot}$)

OUTPUT: RSA for each application request or blocked

1: Begin
2: for each $(s, d) \in G(N, E, S)$ do
3: determine all feasible shortest routing paths
4: store the paths in $R_{sd}$
5: end for
6: while a connection request arrives do
7: free the FSs used by expired requests
8: get parameters $(s, d, Q, t_s, t_f)$
9: for the start time $t_s$ do
10: for the $k$-shortest routing paths $(R_{sd,k})$ do
11: calculate the transmission distance for every $(R_{sd,k})$
12: determine the modulation technique and modulation level $(M_k)$ based on transmission distance
13: calculate the number of FSs $N_k$ based on $Q, M_k$ and slot width ($t_{slot}$)
14: examine the availability of contiguous FSs on $(R_{sd,k})$ for service interval $(t_s, t_s + t_f)$
15: if the FSs are available on the routing path between the time interval $(t_s, t_s + t_f)$ then
16: serve the connection request
17: else
18: calculate $\ell'$ disjoint paths
19: end if
20: end for
21: for ($i = 1; i \leq P; i + +$) do
22: Calculate the hardware support $\ell$ disjoint paths
23: if hardware support is available then
24: store $i$th path in $t_{disjoint}$ set
25: else
26: blocked
27: end if
28: end for
29: for ($i = 1; i \leq t_{disjoint}; i + +$) do
30: compute $c_{id}$ using $c_{id} = (N - 1)M_k * t_{slot}$
31: end for
32: if ($\sum_{i=1}^{P} c_{id} \geq Q$) then
33: accept the connection request
34: for ($i = 1; i \leq t_{disjoint}; i + +$) do
35: while ($Q - \sum_{i=1}^{P} c_{id} \geq 0$) do
36: allocate $c_{id}$ to the connection request
37: else
38: blocked
39: end while
40: end for
41: end if
42: end for
43: end while

increases the overall spectrum requirement of a traffic demand. This result in decrementing the slot saving ratio of the traffic demand. The adaptive feature of the ASP algorithm performs an optimal number of divisions on traffic demand, as a result, it would turn down the spectrum consumption by guard-bands which come up with a significant improvement in the slot saving ratio.

5.2 Complexity analysis

The computational complexity of serving a request $C_i(s, d, t_s, t_f, Q)$ can be derived from the procedure Algorithm. The complexity of finding $k$-shortest path (Lines 3–4) depends on the algorithm used to find it. If we assume yen’s algorithm for finding $k$-shortest path and Dijkstra algorithm for implementing yen’s algorithm, the worst-case complexity comes out to be $O(\sum_{i=1}^{P} N(E + N * \lg(N)))$. In Lines 2–5, we have to calculate $k$-shortest path for all source-destination pair. For $N$ nodes we have $N(N - 1)/2$ source-destination pairs. So complexity of Lines 2–5 comes out to be $O(K * N^3(E + N * \lg(N)))$. To free the FSs used by the expired requests we have to visit each and every slot of all the nodes present, so complexity of Line 7 is $O(N * S)$. To calculate the transmission distance and checking the number of available FSs on a path, in the worst case takes $O(N * S)$. So in the worst-case complexity of Lines 10–16 is $O(K * N * S)$. For computing $P$-disjoint path, Dijkstra algorithm is called $P$ times, this gives the complexity of Line 19 as $O(P * (E + N * \lg(N)))$. In the worst case, Lines (23–25) iterate $N * S$ times. So complexity of Lines (22–29) is $O(P * N * S)$. If hardware support is available on every disjoint path then $t_{disjoint} = P$. So in the worst-case complexity of Lines (30–43) is $O(P * N * S)$. Therefore, the complexity of serving a request turns out to be $O(K * N^3(E + N * \lg(N))) + O(N * S) + O(K * N * S) + O(P * (E + N * \lg(N))) + O(P * N * S)$. Since in general $K \gg P$ ($P = edgenessity$), so $O(N * S) + O(K * N * S) + O(P * N * S)$ tends to $O(K * N * S)$ and $O(K * N^3(E + N * \lg(N))) + O(P * (E + N * \lg(N)))$ tends to $O(K * N^3(E + N * \lg(N))) + O(K * N * S)$. Hence, final complexity is $O(K * N^3(E + N * \lg(N))) + O(K * N * S)$.

6 PERFORMANCE EVALUATION

We have evaluated the performance of the proposed RSA algorithm for stochastic traffic in terms of blocking probability, spectrum utilization and the percentage dynamic connection establishment by simulating PAN-European network with 28 nodes, 41 links and NSFNET with 14 nodes, 21 links as shown in Figure 7. Blocking probability is the ratio blocked connection requests to the arriving connection requests in the network while spectrum utilization can be defined as the ratio of the sum of the total FSs occupied by the connection requests to the total number of FSs on all fibre links. The higher spectrum utilization results better spectrum efficiency. In our simulation platform, we assume that the spectrum width of a slot...
is 12.5 GHz and data carrying capacity of each fibre link to be 4 THz which accommodates 320 slots on each fibre link. To evaluate the performance of proposed algorithm, we have generated 10^5 bidirectional connection requests per execution such connection requests arrive following the Poisson distribution with a call arrival rate of \( \lambda \) and stays for a holding time with an exponential distribution and with service rate \( \mu \). The capacity of each connection request is evenly distributed within 12.5–200 Gbps. We considered the average duration or call holding time to be 10 time units with an inter-arrival time of one time unit. The traffic load (\( \text{total transmission time} \)) is simulated for a range of 100 to 800 Erlang. For comparison purpose, our proposed algorithm sets as a standard against the Shortest-Path with Exact Fit (SPEF) spectrum assignment and multi-path routing algorithm [30]. SPEF algorithm assigns the spectral resources without managing the spectrum fragment, while algorithms in [30] manage the spectrum fragments with equi-distributed splitting on multi-paths. We have created the network platform through C++ on HP workstation equipped with Xeon 2.67 GHz processor with 16 GB RAM to evaluate our proposed strategy. Table 1 shows the simulation parameters which have been used.

![Figure 7](image.png)

**Figure 7** Network topology which have been used (a) NSFNET and (b) Pan European topology.

**Table 1** Simulation parameter

| Parameter                      | Value   |
|-------------------------------|---------|
| Spectrum width of slot (GHz)  | 12.5    |
| Link capacity (THz)           | 4       |
| FSs per fibre link            | 320     |
| Average holding time (seconds)| 10      |
| Transmission reach of BPSK (km)| 5000   |
| Transmission reach of QPSK (km)| 2500   |
| Transmission reach of 8-QAM (km)| 1250   |
| Transmission reach of 16-QAM (km)| 625    |
| Range of connection request   | 12.5–200 Gbps |

Figure 8 illustrates the simulation results of the blocking probability versus traffic load of ASP algorithm and reference algorithms, namely, SPEF and multi-path routing with the NSFNET and PAN-European topology, respectively. At lower values of traffic load, connection requests are absolutely provisioned by single path routing. But on increasing the traffic load, single path routing remain inadequate to set up the connection, therefore multi-path routing algorithms such as ASP and [30] are the foremost techniques to successfully provision the connection request in the network. On further increment in the traffic load, multi-path routing algorithm [30] becomes in-efficacious due to the unavailability of the same set of contiguous FSs on the disjoint paths whereas, ASP algorithm is found to be more efficient to establish the connections due to adaptive connection request fragmentation. In this way, ASP accomplishes 34% improvement in the connection established over the earlier mentioned techniques.
Thereafter, we look over the influence of guard-band size on the blocking performance while accommodating the connection request. Each connection request requires guard-bands, on the either sides to avoid the interference from the adjacent requests and the size of the guard-bands, mainly rely on the extent of crosstalk and quality of service required for the service provisioning. It may be more distinguishable while provisioning the sub-traffic demands as overall guard-band count proportionally to the number of divisions. In this scenario, the number of guard-bands has been modified from 1 (ideal) to 2. Figure 11 illustrates the simulation results of the blocking probability versus traffic load at different guard-band size of ASP algorithm and reference SPEF algorithm with the NSFNET and PAN-European topology, respectively. Analysing the above results, we can examine that increment in the guard-band size give rise to the extra spectrum occupancy which result in poor blocking performance. We can observe that both SPEF and ASP algorithms show similar response in this regard, whereas being a single path routing algorithm performance of the SPEF is insusceptible to the guard-band size in contrast to ASP algorithm and the small guard-band size improves the blocking probability in case of the ASP algorithm as it is inevitable to increase the number of guard-bands with the increment of sub-traffic demands on the multi-paths.


6.1 Time and hardware complexity

In our proposed algorithm, adaptive fragmentation is performed on the connection requests, thereby the run time of the algorithm will be directly proportional to the number of disjoint paths out of $t_{\text{disjoint}}$ paths on which adaptive fragmentation is done. Only in worst case all the $t_{\text{disjoint}}$ paths will be used, whereas multi-path algorithm [30], exploits equi-distributed splitting and all the $t_{\text{disjoint}}$ paths are being utilized. In worst-case scenario, both the Algorithm will have same run time. But in practice, the probability of hitting worst case is extremely low in our proposed algorithm due to the adaptive fragmentation. Let $t_{\text{disjoint}} = 3$.

$\alpha =$ Probability of serving connection request using only one path.

$\beta =$ Probability of serving connection request using two paths.

$\gamma =$ Probability of hitting worst case, i.e. using all the three paths.

$N = 1,000,000$ (Total number of requests in simulation).

In case of algorithm [30], run time will be proportional to $t_{\text{disjoint}} \times N$, whereas in case proposed algorithm run time will be proportional to $t_{\text{disjoint, effective}} \times N$.

$t_{\text{disjoint, effective}} = (1 * \alpha + 2 * \beta + 3 * \gamma)$, $\alpha + \beta + \gamma = 1$. Since probability of hitting worst case is very low, thereby $\gamma$ will be much less than one and $t_{\text{disjoint, effective}} < t_{\text{disjoint}}$. In the simulation, we found $\alpha$ to be around 0.2, $\beta$ to be around 0.6 and $\gamma$ to be around 0.2.

This gives $t_{\text{disjoint, effective}} = (0.2 + 1.2 + 0.6) = 2$, thereby $\frac{(t_{\text{disjoint}} - t_{\text{disjoint, effective}})}{t_{\text{disjoint}}} \times 100 = 33.33\%$ proposed algorithm is faster than algorithm [30].
Further moving to the hardware complexity, the establishment of multiple light-paths for a connection request signifies that a transponder at the source and destination nodes must be assigned to each one of them. Thereby, the number of parts in which connection request is split up takes an essential role in the performance of multi-path routing, since more number of parts will entail higher hardware requirement (transponders) and control plane complexity in terms of framing, management and release operations. Referring to my previous point \( \left( t_{\text{split}, \text{path}} < t_{\text{split}, \text{joint}} \right) \), our proposed algorithm utilizes least possible number of paths, leading to lesser transponders usage and control plane complexity as compared with algorithm \( [30] \), highlighting the viability and scalability of the proposed approach.

7 | CONCLUSION

This paper proposed a novel technique to solve MPRSA problem in EON. Our proposed strategy termed as ASP enhances the number of connections served through the adaptive fragmentation of connection request on the disjoint paths based on their pre-computed capacity and hardware (BVTs/SBVTs) availability. We develop an ILP model as well as heuristic algorithm for solving MPRSA problem in large optical networks where ILP is not compliant. The performance of the proposed algorithm has been evaluated through simulation results where a single path and multi-path algorithm has been considered as a benchmark. Through our proposed technique blocking performance, the spectrum utilization and slot saving ratio has also been improved significantly.

REFERENCES

1. Sector, I.T.S.: ITU-T Recommendation G. 694.1: Spectral grids for WDM applications: DWDM frequency grid (2002)
2. Jinno, M., et al.: Spectrum-efficient and scalable elastic optical path network: Architecture, benefits, and enabling technologies. IEEE Commun. Mag. 50(2), s12–s20 (2012)
3. Gersel, O., et al.: Elastic optical networking: A new dawn for the optical layer. IEEE Commun. Mag. 50(2) (2012)
4. Christodoulopoulos, K., et al.: Elastic bandwidth allocation in flexible OFDM-based optical networks. J. Lightwave Technol. 9, 1354–1366 (2011)
5. Wang, Y., et al.: A study of the routing and spectrum allocation in spectrum sliced elastic optical path networks. In: Proceedings IEEE INFOCOM, pp. 1503–1511. IEEE (2011)
6. Yin, S., et al.: Survivable multipath virtual network embedding against multiple failures for SDN/NFV. IEEE Access 6, 76909–796923 (2018)
7. Yousefi, F., Rahbar, A.G.: Novel fragmentation-aware algorithms for multipath routing and spectrum assignment in elastic optical networks-space division multiplexing (EON-SDM). Opt. Fiber Technol. 46, 287–296 (2018)
8. Lochowicz, P., et al.: Fragmentation metrics and fragmentation-aware algorithm for spectrally/spatially flexible optical networks. Jo. Opt. Commun. Networking 12(5), 133–145 (2020)
9. Yousefi, F., et al.: Fragmentation-aware algorithms for multipath routing and spectrum assignment in elastic optical networks. Opt. Fiber Technol. 53, 102019 (2019)
10. Beyragh, A.A., et al.: IF-RSCA: Intelligent fragmentation-aware method for routing, spectrum and core assignment in space division multiplexing elastic optical networks (SDM-EON). Opt. Fiber Technol. 50, 284–301 (2019)
11. Xiong, Y., et al.: A machine learning approach to mitigating fragmentation and cross-talk in space division multiplexing elastic optical networks. Opt. Fiber Technol. 50, 99–107 (2019)
12. Oki, E., et al.: Spectrum fragmentation management in elastic optical networks. In: 21st International Conference on Transparent Optical Networks (ICTON), pp. 1–4. IEEE (2019)
13. Yin, Y., et al.: Dynamic on-demand defragmentation in flexible bandwidth elastic optical networks. Opt. Express 20(2), 1798–1804 (2012)
14. Wang, X., et al.: Utilization entropy for assessing resource fragmentation in optical networks. In: Optical Fiber Communication Conference, OTh1A-2, Optical Society of America (2012)
15. Alyatama, A.: Multi-path routing based on relative cost in elastic optical networks. In: 7th International Conference on Electrical and Electronics Engineering (ICEEE), pp. 226–231. IEEE (2020)
16. Yuan, J., et al.: A pre-split multi-flow RMSA algorithm in elastic optical networks. Opt. Fiber Technol. 52, 101993 (2019)
17. Paira, S., et al.: On energy efficient survivable multipath based approaches in space division multiplexing elastic optical network: Crosstalk-aware and fragmentation-aware. In: IEEE Global Communications Conference (GLOBECOM), pp. 1–6. IEEE (2019)
18. Oliveira, H.M., da Fonseca, N.L.: Multipath routing, spectrum and core allocation in protected SDM elastic optical networks. In: IEEE Global Communications Conference (GLOBECOM), pp. 1–6. IEEE (2019)
19. Lu, W., et al.: Dynamic multi-path service provisioning under differential delay constraint in elastic optical networks. IEEE Commun. Lett. 171, 158–161 (2013)
20. Zhu, Z., et al.: Dynamic service provisioning in elastic optical networks with hybrid single-/multi-path routing. J. Lightwave Technol. 31(1), 15–22 (2013)
21. Yang, H., et al.: Multi-flow virtual concatenation triggered by path cascading degree in flexi-grid optical networks. Opt. Fiber Technol. 19(6), 604–613 (2013)
22. Zhu, R., et al.: Multi-path fragmentation-aware advance reservation provisioning in elastic optical networks. In: IEEE Global Communications Conference (GLOBECOM). IEEE (2016)
23. Gao, T., et al.: Spectrum-efficient multipath provisioning with content connectivity for the survivability of elastic optical datacenter networks. Opt. Fiber Technol. 36, 353–365 (2017)
24. Oliveira, H.M., da Fonseca, N.L.: Spectrum overlap and traffic grooming in p-cycle algorithm protected SDM optical networks. In: IEEE International Conference on Communications (ICC), pp. 1–6. IEEE (2018)
25. Dharmaweera, M.N., et al.: Traffic-grooming and multipath-routing-enabled impairment-aware elastic optical networks. IEEE/OSA J. Opt. Commun. Networking 8(2), 58–70 (2016)
26. Srivastava, A., Srivastava, A.: Flow aware differential delay routing for next-generation Ethernet over SONET/SDH. In: IEEE International Conference on Communications, vol. 1. IEEE (2006)
27. Xia, M., et al.: Split spectrum: A multi-channel approach to elastic optical networking. Opt. Express 20(28), 29143–29148 (2012)
28. Pages, A., et al.: Optimal route, spectrum, and modulation level assignment in split-spectrum-enabled dynamic elastic optical networks. IEEE/OSA J. Opt. Commun. Networking 6(2), 114–126 (2014)
29. Dabhi, S., et al.: Split spectrum approach to elastic optical networking. In: 38th European Conference and Exhibition on Optical Communications, pp. 1–3. IEEE (2012)
30. Thangaraj, J.: Limitation of Erlang B Traffic Model in elastic optical networks. J. Opt. Commun. Networking 12(5), 133–145 (2020)

How to cite this article: Ujjwal U, Mahala N, Thangaraj J. Dynamic adaptive spectrum allocation in flexible grid optical network with multi-path routing. IET Commun. 2021;15:211–223. https://doi.org/10.1049/cm2.12046