Why the astrophysical black hole candidates may not be black holes at all

Abhas Mitra
Nuclear Research Laboratory, Bhabha Atomic Research Center
Mumbai- 400 085, India
Email: amitra@apsara.barc.ernet.in, abhasmitra@rediffmail.com

In a recent paper[1], it has been shown that, there cannot be any rotating (Kerr) Black Hole (BH) with finite mass in order that the generic properties associated with the symmetries of stationary axisymmetric Einstein equations are obeyed, i.e, in order to have \( m \geq 0 \), we must have \( a = 0 \). In other words, all observed chargeless BHs with finite masses must be non-rotating Schwarzschild BHs. Here, by comparing the invariant 4 volume associated with original Kerr metric [2] with the Boyer-Lindquist version of the same [3], we further find that, stationary axisymmetric vacuum Einstein solutions actually correspond to \( m = 0 \) in addition to \( a = 0 \)!

This means that if the Kerr solution is a unique one, the Schwarzschild BHs too correspond to only \( m = 0 \) and therefore the observed BH candidates (BHs) with \( m > 0 \) are not BHs at all. This is in agreement some detailed analysis of recent observations[4-7] which suggest the the BHCs have strong intrinsic magnetic moment rather than any Event Horizon. If one would derive the Boyer-Lindquist metric in a straightforward manner by using the Backlund transformation, it would follow that \( a = m \sin \phi \), where \( \phi \) is the azimuth angle. This relationship directly confirms the result that for a supposed rotating BH, actually, both \( a = m = 0 \).

For all 4-D curvilinear coordinate transformations, it is known that \( \sqrt{-g} d^4x = \text{Invariant} \), where \( g \) is the corresponding metric determinant. By using this basic mathematical tool, it will be seen in the following that all Kerr Black Holes have the unique mass \( m = 0 \) in addition to the unique rotation parameter \( a = 0[1] \). And since all observed astrophysical objects in particular the Black Hole Candidates (BHCs) necessarily have finite mass, \( m > 0 \), it would follow that they may not be BHs at all. And we point out that the BHcs could be Ultra Compact Objects with very high surface gravitational redshift \( z \gg 1 \) and touch upon several generic properties of them.

We first recall the original Kerr metric [2]

\[
\begin{align*}
ds^2 &= \hat{\rho}^2(d\bar{t}^2 + \sin^2 \bar{\theta}d\bar{\phi}^2) + 2(d\bar{r} + d\bar{t} + a \sin^2 \bar{\theta}d\bar{\phi})(d\bar{r} + a \sin^2 \bar{\theta}d\bar{\phi}) \\
&\quad - (1 - 2m\bar{r}/\hat{\rho}^2)(d\bar{r} + a \sin^2 \bar{\theta}d\bar{\phi})^2,
\end{align*}
\]

(1)

where \( \bar{t} \) is time, \( \bar{r} \) is radial coordinate, \( \bar{\phi} \) is azimuth angle, \( \bar{\theta} \) is polar angle, \( a \) is an integration constant, interpreted as angular momentum per unit mass; and \( m \) too is an integration constant. And the parameter

\[
\hat{\rho}^2 = \bar{r}^2 + a^2 \cos^2 \bar{\theta}
\]

(2)
It is implicitly assumed that the integration constants \( a \) and \( m \) are positive.

By means of a straightforward but lengthy algebra, it can be seen that the determinant associated with this metric is (see Appendix I)

\[
\bar{g} = -\bar{\rho}^4 \sin^2 \bar{\theta}
\]  

(3)

By using the following coordinate transformations

\[
dt = d\bar{t} - \frac{2mr}{\Delta}d\bar{r},
\]

(4)

\[
d\phi = d\bar{\phi} + \frac{a}{\Delta}d\bar{r}
\]

(5)

and

\[
\theta = \bar{\theta}; \quad r = \bar{r}, \quad \rho^2 = r^2 + a^2 \cos^2 \theta = \bar{r}^2 + a^2 \cos \bar{\theta}^2 = \bar{\rho}^2
\]  

(6)

Boyer and Lindquist[3] rewrote metric (1) as

\[
ds^2 = \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + [r^2 + a^2 + \frac{2mr}{\rho^2}a^2 \sin^2 \theta] \sin^2 \theta d\phi^2
\]

\[
+ \frac{4amr}{\rho^2} \sin^2 \theta d\phi dt - (1 - \frac{2mr}{\rho^2})dt^2
\]

(8)

The Boyer- Lindquist form of the Kerr metric is now considered as the “standard” form for Kerr BHs and in this case, it is already well known[8] that the associated determinant is

\[
g = -\rho^4 \sin^2 \theta = -\bar{\rho}^4 \sin^2 \bar{\theta} = \bar{g}
\]  

(9)

Since for all stationary axisymmetric solutions of Einstein equations in the “standard” form, \( g_{\phi\phi} = \sin^2 \theta g_{\theta\theta} \) when \( \theta \) is uniquely defined and measured from the axis of symmetry, it was found in Paper[1] that either

\[
a = 0; \quad m \geq 0
\]  

(10)

or,

\[
a \geq 0; \quad 2mr = -\rho^2; \quad m \leq 0, \quad \text{if } r \geq 0
\]  

(11)

The latter solution involving negative mass can be seen to be unphysical in the following manner:

For a given value of \( m \) and \( a \), Eqs. (6) and (11) would yield

\[
r = -m \pm \sqrt{m^2 - a^2 \cos^2 \theta}
\]  

(12)

This is the equation of a single surface and the 4-D spacetime, thus, would collapse to a 3-D spacetime if Eqs.(11-12) would be valid solutions. If so, then it would be possible to choose a coordinate system in which \( dx^1 = 0 \) and to set the invariant 4-volume \([8,9]\) \( \sqrt{-g} \ d^4x = 0 \). But since for a 4-D problem, \( \sqrt{-g} \ d^4x > 0 \) and invariant too, we must reject this branch corresponding to negative mass. In any
case, we are interested in the observed BH Candidates (BHCs) having $m > 0$ and hence this $m \leq 0$ solution is irrelevant here.

When $a = 0$, the transformation equation (5) trivially becomes

$$d\phi = d\tilde{\phi}$$  \hspace{1cm} (13)

Now if we again use the principle of invariance of 4-volume\[8,9\] to the coordinate systems $\tilde{t}, \tilde{r}, \tilde{\theta}, \tilde{\phi}$ and $t, r, \theta, \phi$, we will have

$$\sqrt{-\tilde{g}} \; d\tilde{t} \; d\tilde{r} \; d\tilde{\theta} \; d\tilde{\phi} = \sqrt{-g} \; dt \; dr \; d\theta \; d\phi$$  \hspace{1cm} (14)

But since, now, $d\tilde{\phi} = d\phi$ and already $\tilde{r} = r$, $\tilde{\theta} = \theta$, $\tilde{g} = g$, we also have

$$d\tilde{t} = dt$$  \hspace{1cm} (15)

Then using Eq.(15) in Eq.(4), it trivially follows that, in order that stationary axisymmetric vacuum Einstein equations obey all the symmetry constraints (Paper I) and allowed transformations, the integration constant

$$m = 0$$  \hspace{1cm} (16)

in addition to $a = 0$. The fact that $a = m = 0$ for all Kerr BHs actually follows from a previous work:

Neugebauer\[10\] obtained the Kerr solution from a “seed solution” by using Backlund transformations; the 4th line from the bottom of p.(73) of Ref.\[10\] clearly shows that $a$ and $m$ are related through

$$a = m \sin \phi$$  \hspace{1cm} (17)

Since $\sin \phi$ is a variable and not identically zero, the foregoing relation can be satisfied only when $a = m = 0$! However, it is astonishing to see that Neugebauer failed to realize this. Neugebauer’s Eq.(17) also confirms both the present result ($m = 0$) as well as the result of Paper\[1\] that $a = 0$ for Kerr BHs. (Scanned image (pdf) of this page of Ref[10] is given as Supporting online material II) Therefore if the Schwarzschild BHs are seen to be arising from a solution of algebraically special stationary axisymmetric vacuum solutions of Einstein equations, then mass of all Schwarzschild BHs would be zero. However if it would be argued that spherically symmetric vacuum solution exists independent of general stationary axisymmetric solutions, it would appear that, despite the present exact result, there could be finite mass Schwarzschild BHs. But this would be highly unlikely because a spherically symmetric vacuum solution indeed follows as a subset of the rotating solution with $a = 0$. Furthermore, the Kerr solution is considered to be the unique solution.

Therefore the observed BHCs, rotating or non-rotating, may not be even Schwarzschild BHs. Thus the observed BHCs cannot be BHs at all and they cannot have any Event Horizon (EH), the exclusive hallmark of a BH. Whenever there is no EH, any astrophysical plasma, even when it is macroscopically neutral, is likely to develop strong intrinsic magnetic moment upon compactification \textit{a la} the pulsars following approximate magnetic flux conservation. Thus as per our analysis, the stellar mass BHCs may have locally measured intrinsic surface magnetic field $B_{\text{local}}$ which could
be even much stronger than the typical pulsar magnetic fields. If a compact object has a gravitational surface redshift $z$, it can be shown that a distant observer would perceive a surface magnetic field\[4-7\]

$$B = B(\infty) = (1 + z)^{-1}B_{\text{local}}$$  \hspace{1cm} (18)

However, in case, $z \gg 1$, the value of $B$ for such Ultra Compact Objects (UCOs) could appear weaker than typical pulsar values (which have $z \sim 0.1 - 0.2$). We recall here that, for isotropic self-gravitating objects in strict hydrostatic equilibrium $z < 2^{[9,11]}$. But the BHCs may not be in strict hydrostatic equilibrium; they may be in quasistatic equilibrium at best and slowly contracting like primordial clouds and pre-main sequence stars. If so, there need not be any upper limit on $z$. It may be also recalled here that primordial clouds or pre-main sequence stars may be in unstable quasiequilibrium (though actually collapsing) for hundreds of million years without any nuclear burning at their cores. Whenever they are not in strict equilibrium, they may be contracting at unimaginably slow rate and by virtue of virial theorem, while part of the gravitational energy released by contraction goes into generating internal energy (and attendant pressure gradient), part of it is radiated out\[9,11\].

One may wonder here “what about the upper mass limit of $\sim 3m_\odot$ of compact objects? This upper mass limit corresponds to cold baryonic objects in strict hydrostatic equilibrium. Even when the object is baryonic but not cold, this upper limit has little relevance. As an extreme case of hot baryonic objects, one may recall the theoretical possibility of existence of Supermassive Stars whose masses could be as large as $10^{10}m_\odot$ or even higher\[9,11\]. When such “hot” objects are not in strict equilibrium, they can generate their own pressure gradient by virtue of virial theorem. Also even for an almost cold self-gravitating object like a primordial gas cloud, there is no upper mass limit when it is not in strict hydrostatic equilibrium.

We may think of some generic properties of UCOs with $z \gg 1$:

It is known that compact objects with local surface magnetic field considerably higher than $10^{9-10}$ G, do not display Type I X-ray burst even when they possess a physical surface and this is the reason that Her X-1 or many other X-ray pulsars do not show Type I X-ray burst. Thus high $z$ UCOs also may not show any Type I burst activity despite possessing a physical surface.

When $z \gg 1$, any signal generated on the surface would propagate out through a gravitational field with extremely steep spatial gradient. Then the temporal properties of the original signal would be constantly distorted; for example a proper time interval of $\sim 1$s on the surface may appear as $\sim 10$s at few Schwarzschild radii away in case $z \sim 10$. Thus no spin pulsation would be directly seen!

On the other hand, strong local intrinsic magnetic field would focus both inflows and outflows. This focusing of outflow coupled with rotation is likely to generate strong collimated jets; thus “jets” could be a generic feature of such UCOs. As to the generation of Ultra Relativistic jets, so far the popular idea has been that the speed of the jet may be bounded by the “escape velocity” from the central object\[12\]. Thus it was generally believed that jets associated with neutron stars $(1 + z \sim 1.1 - 1.2)$ may not have bulk Lorentz factor higher than $\Gamma \sim 1.3$ or so whereas jets associated with BHs $(1 + z = \infty)$ could have $\Gamma \gg 1$. However as of now, observationally, in a strict sense, no jet or no outflow can be directly associated with any EH or any “Ergosphere” because no “EH” has been detected so far\[13\].
On the other hand, contrary to the popular idea, it is now known that Cir X-1, an object *with a physical surface* and strong *intrinsic* magnetic field does give rise to ultrarelativistic outflow with $\Gamma \sim 10^{12}$. Thus it should not be surprising if it is found that microquasars or quasars with strong outflows contain central objects having physical surface and intrinsic magnetic field because we have found that objects with EHs have the unique property $a = m = 0$. In fact what could be surprising is the notion that ultrarelativistic or any outflow at all could originate from objects *from which even light cannot escape*.

There could be yet another generic property of an object with high $z$. It is known that the last stable circular orbit around a compact object lies at $r = 6m$ for material particles and at $r = 3m$ for photons or other massless particles. The latter value corresponds to $z \sim 0.72$ which means that once a collapsing object attains $z > 0.72$, photons and neutrinos would find it extremely difficult to diffuse out from its core. In other words such collapsing would be extremely “hot” and would tend to remain so for long durations.

In fact the detailed analysis of X-ray and radio data of the stellar mass BHCs indeed suggest that these objects have strong intrinsic magnetic moment and they are in *quasistatic* state probably because of strong magnetic and radiation pressure[4-7]. Robertson & Leiter have inferred that the intrinsic luminosity of these objects are close to corresponding Eddington values yet some of them appear so faint because in GR, the Eddington luminosity seen by a distant observer[11] is

$$L_{ed}(\infty) = 1.3 \times 10^{38} (1 + z)^{-1} \left( \frac{m}{1 m_\odot} \right) \text{ erg/s}$$

and these objects might have $z \sim 10^{-8}$. Such a range of value of $z$ may appear outlandish but here it should be remembered that these range of $z$ or any finite value of $z$ are infinitely smaller than the corresponding value of $z = \infty$ for a BH! Another explanation for the extremely weak quiescent luminosities of several BHCs and Neutron Stars as well as the difference in the luminosities between the two categories could be that the quiescent states are dominated by jet outflows rather than any advection dominated inflow[14]. Irrespective of such tentative explanations and uncertainties about the precise nature of BHCs, the fact remains that they have $m > 0$ and hence they may not be BHs which are characterized by $a = m = 0$. As a final comment, the result mentioned here, has been obtained independently in Paper I and the present paper by starting from the equation[15,16].

$$g_{\phi\phi} = g_{\theta\theta} \sin^2 \theta$$

As discussed in Paper I, this relationship is valid for any stationary axisymmetric metric which is in *standard form*, i.e., the only cross term in the metric is $d\phi dt$ and when $\theta$ is uniquely defined by measuring it from the axis of symmetry The html file for Ref 16 showing this result as well as the pdf file showing the Eq $a = m \sin \phi$ can be obtained either from the editor or the author.
References

[1] Mitra, A. Why the astrophysical black hole candidates are not rotating black holes, (astro-ph/0407501).

[2] Kerr, R.P. Gravitational field of a spinning mass as an example of algebrically special metrics, Phys. Rev. Lett. 11, 237 (1963).

[3] Boyer, R.H. and Lindquist, R.W. Maximal Analytical Extension of the Kerr Metric, J. Math. Phys. 8, 265 (1967).

[4] Robertson, S. & Leiter, D. Evidence for intrinsic magnetic moment in black hole candidates, Astrophys. J. 565, 447 (2002) (astro-ph/0102381).

[5] Leiter, D. & Robertson, S. Does principle of equivalence prevent trapped surfaces from being formed in general relativistic collapse process Found. Phys. Lett. 16, 143 (2003) (astro-ph/0111421).

[6] Robertson, S. & Leiter, D. On the intrinsic magnetic moment in black hole candidates Astrophys. J. 569, L203 (2003) (astro-ph/0310078).

[7] Robertson, S. & Leiter, D. On the origin of the radio/X-ray luminosity correlation in black hole candidates, Mon. Not. Roy. Astr. Soc. 350, 1391 (2004) (astro-ph/0402445).

[8] Landau, L.D. & Lifshitz, E.M., Classical Theory of Fields, Vol. 2 (Pergamon Press, Oxford 1985).

[10] Neugebauer, G. in General Relativity (eds. G.S. Hall & J.R. Pulham) (SUSSP, Edinburg and IOP, London, 1996) (see p. 73).

[11] Shapiro, S.L. & Teukolsky, S.A. Black Holes, White Dwarfs and Neutron Stars (John Wiley, New York 1983).

[12] Fender, R. et al. An ultra-relativistic outflow from a neutron star accreting gas from a companion, Nature 427, 222-224 (2004) (astro-ph/0401290).

[13] Abramowicz, M.A. & Kluzniak, W. No observational proof of the black hole Event Horizon Astron. Astrophys. 396, L31 (2002) (astro-ph/0207270).

[14] Fender, R.P., Gallo, E. and Jonker, P.G. Jet dominated states an alternative to advection across black hole Event Horizon in ‘quiescent’ X-ray binaries Mon. Not. R. Astr. Soc. 343, 199 (2003) (astro-ph/0306614).

[15] Hartle, J.B. and Sharp, D.H. Variational principle for the equilibrium of a relativistic rotating star, Astrophys. J. 147, 317 (1967) (see Eq. 74).

[16] Thorne, K.S. in Proc. of international school of Physics (Enrico Fermi), General Relativity, Gravitation & Cosmology (ed. B.K. Scahs) 241 (eq. 3.3) (Academic Press, New York, 1971).
1 Appendix 1. Determinant for the original Kerr Metric

In its original form, the Kerr metric is

\[ ds^2 = \rho^2(d\theta^2 + \sin^2 \theta d\phi^2) + 2(dr + dt + a \sin^2 \theta d\phi)(dr + a \sin^2 \theta d\phi) \\
- (1 - 2mr/\rho^2)(dr + dt + a \sin^2 \theta d\phi)^2 \]  \hspace{1cm} (21)

In the parent manuscript, we indicated the original Kerr coordinates by an overbar which we have removed here. Here the parameter

\[ \rho^2 = r^2 + a^2 \cos^2 \theta \]  \hspace{1cm} (22)

Let us call

\[ x = 1 + \frac{2mr}{\rho^2} \]  \hspace{1cm} (23)

Then it can be seen that the various non-vanishing components of the metric tensor are

\[ g_{\theta\theta} = \rho^2 \]  \hspace{1cm} (24)
\[ g_{\phi\phi} = (\rho^2 + xa^2 \sin^2 \theta) \sin^2 \theta \]  \hspace{1cm} (25)
\[ g_{rr} = x \]  \hspace{1cm} (26)
\[ g_{tt} = -(1 - 2mr/\rho^2) = x - 2 \]  \hspace{1cm} (27)
\[ g_{r\phi} = g_{\phi r} = ax \sin^2 \theta \]  \hspace{1cm} (28)
\[ g_{\phi t} = g_{t\phi} = \frac{2amr \sin^2 \theta}{\rho^2} = a(x - 1) \sin^2 \theta \]  \hspace{1cm} (29)
\[ g_{rt} = g_{tr} = \frac{2mr}{\rho^2} = x - 1 \]  \hspace{1cm} (30)

The determinant of the metric tensor can be found to be

\[ g = g_{\theta\theta}[g_{tt}(g_{rr}g_{\phi\phi} - g_{\phi r}^2) - g_{r\phi}(g_{rr}g_{\phi\phi} - g_{\phi r}g_{t\phi})] \\
+ g_{t\phi}(g_{rr}g_{t\phi} - g_{r\phi}g_{t\phi}) \]  \hspace{1cm} (31)

First note that

\[ (g_{rr}g_{t\phi} - g_{t\phi}g_{r\phi}) = a(x - 1)x \sin^2 \theta - (x - 1)a \sin^2 \theta = 0 \]  \hspace{1cm} (32)

so that

\[ g = g_{\theta\theta}[g_{tt}(g_{rr}g_{\phi\phi} - g_{\phi r}^2) - g_{r\phi}(g_{rr}g_{\phi\phi} - g_{\phi r}g_{t\phi})] \]  \hspace{1cm} (33)
Then using Eqs.(22-29) in (32), we have,

\[
\frac{g}{\rho^2} = (x - 2)\left[ x\rho^2 \sin^2 \theta + x^2 a^2 \sin^2 \theta - a^2 x \sin^\theta \right] \\
- (x - 1)\left[ (x - 1) \sin^2 \theta (\rho^2 + x a^2 \sin^2 \theta) - a^2 x (x - 1) \sin^4 \theta \right] \\
= (x - 2) x \rho^2 \sin^2 \theta \\
- (x - 1)^2 \sin^2 \theta \left[ \rho^2 + x a^2 \sin^2 \theta - x a^2 \sin^2 \theta \right] \\
= (x - 2) x \rho^2 \sin^2 \theta - (x - 1)^2 \sin^2 \theta \rho^2 \\
= \rho^2 \sin^2 \theta \left[ x(x - 2) - (x - 1)^2 \right] \\
= -\rho^2 \sin^2 \theta \\
\]

Therefore the determinant

\[
g = -\rho^4 \sin^2 \theta \\
\]

Now introducing overbar for the original Kerr metric, as has been done in the parent manuscript, we obtain

\[
\bar{g} = -\bar{\rho}^4 \sin^2 \bar{\theta} \\
\]

References

[1] Kerr, R.P. Gravitational field of a spinning mass as an example of algebraically special metrics, Phys. Rev. Lett. 11, 237 (1963).