The Tensor Part of the Skyrme Energy Density Functional

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Abstract

We systematically study the effect of the \( J^2 \) tensor terms in the Skyrme energy functional on properties of spherical nuclei. We build a set of 36 parameterizations covering a wide range of the corresponding parameter space. We analyze the impact of the tensor terms on the evolution of single-particle-level splittings along chains of semi-magic nuclei in spherical calculations. We find that positive values of the coupling constants of proton-neutron and like-particle tensor terms allow for a qualitative description of the evolution of neutron and proton single-particle level splittings in chains of Ca, Ni and Sn isotopes.

Key words: Nuclear energy density functional, tensor interaction, single-particle energies
PACS: 21.10.Pc, 21.30.Fe, 21.60.Jz

The tensor force has been identified very early as an important part of the nucleon-nucleon interaction, and its effects, e.g. the specific correlations it generates and their importance for the binding of nuclear systems, have been studied in infinite and few-body systems. However, it is only recently that energy density functional (EDF) practitioners have renewed their interest in the various “tensor terms” occuring in the nuclear EDF [1234567]. We attempt in this contribution to sketch a systematic study of the variation of tensor-term parameters and constrain their values [8].

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1 Work performed in the framework of the Espace de Structure Nucléaire Théorique.
In spherical symmetry, a zero-range tensor force added on top of the usual Skyrme effective vertex \[ \text{[9]} \] contributes to the EDF through terms proportional to \( J^2 \) \[ \text{[10,11]} \] where \( J \) is the spin-orbit current density vector, here defined through its radial component:

\[
J_q(r) = \frac{1}{4\pi r^3} \sum_{n,j,\ell} (2j+1) v_{n,j,\ell}^2 \left[ j(j+1) - \ell(\ell+1) - \frac{3}{4} \right] \psi_{n,j,\ell}^2(r).
\] (1)

The resulting total spin-orbit field for neutrons reads (invert \( n \) and \( p \) for protons)

\[
W_n(r) = \frac{W_0}{2} (2\nabla \rho_n + \nabla \rho_p) + \alpha J_n + \beta J_p,
\] (2)

where the first term comes from the zero-range spin-orbit vertex and the two others from the tensor vertex. When the functional is derived from such a Skyrme-tensor vertex the coupling parameters \( \alpha \) and \( \beta \) can be chosen independently of the more standard force or functional parameters. In order to study their effects, we build a series of parameterizations, for each of which \( (\alpha, \beta) \) are fixed and all other parameters are fitted according to a protocol \[ \text{[8]} \] similar to the one used for the construction of the Saclay-Lyon parameterizations. They are labelled TJJ, with indices \( I \) and \( J \) related to \( \alpha \) and \( \beta \) through \( \alpha = 60 (J - 2) \) MeV fm\(^5\) and \( \beta = 60 (I - 2) \) MeV fm\(^5\).

Tensor terms alter the strength and shape of the spin-orbit potential, Eq. \[ \text{(2)} \], when \( J \) varies due to the filling of a single-particle state. The tensor contribution to \( W_q(r) \) thus depends on details of the relative placement of the levels, and is subject to much sharper relative variations than the spin-orbit contribution. As such, it can be constrained by examining the variation of the relative placement of single-particle states in a series of nuclei differing by the filling of levels which significantly contribute to \( J \).

The first example, displayed on the left panel of Fig. \[ \text{I} \] is the tin chain, along which the \( h_{1/2} \) neutron level is filled (between \( N = 64 \) and \( 82 \)) yielding a large contribution to \( J_n \) and thus to the proton spin-orbit field due to the \( \beta J_n \cdot J_p \) coupling. This has been previously identified as a possible source of the change of slope (as a function of \( N \)) in the spacing of proton 1\( g_7/2 \) and 1\( h_{11/2} \) levels \[ \text{[12]} \]. The spacing of 2\( d_5/2 \) and 1\( g_7/2 \) levels is affected in a similar way. We can reproduce the magnitude of the single-particle-level spacing shifts by setting the np-coupling \( \beta \) to 120 MeV fm\(^5\), however, the effect appears to occur at too large a neutron number, owing to the incorrect placement of the neutron 1\( h_{11/2} \) level relative to the 3\( s_{1/2} \) and 2\( d_{3/2} \) ones.

The neutron-neutron coupling can be constrained by examining the spacings of neutron levels in nuclei of the same isotopic chain. In the Ca chain, for example, the filling of the 1\( f_{7/2} \) level between \( ^{40}\text{Ca} \) and \( ^{48}\text{Ca} \) affects the splitting of the neutron 1\( d \) shell, yielding a shift of the 1\( d_{3/2} \) relative to the 2\( s_{1/2} \) one. Similarly, the filling of the 1\( f_{5/2} \) level between \( ^{56}\text{Ni} \) and \( ^{68}\text{Ni} \) acts on the 2\( p \) and 1\( f \) states and produces a relative shift of the 1\( f_{5/2} \) and 2\( p_{1/2} \) levels. The right panel of Fig. \[ \text{I} \] displays the evolution of level splittings related to the latter effects, as a function of the like-particle coupling constant \( \alpha \):

\[
\delta^{\text{Ca}} = \left( \varepsilon^{48\text{Ca}}_{1d_{3/2}} - \varepsilon^{48\text{Ca}}_{2s_{1/2}} \right) - \left( \varepsilon^{40\text{Ca}}_{1d_{3/2}} - \varepsilon^{40\text{Ca}}_{2s_{1/2}} \right),
\] (3)
\[
\delta^{\text{Ni}} = \left( \varepsilon^{68\text{Ni}}_{1f_{5/2}} - \varepsilon^{68\text{Ni}}_{2p_{1/2}} \right) - \left( \varepsilon^{56\text{Ni}}_{1f_{5/2}} - \varepsilon^{56\text{Ni}}_{2p_{1/2}} \right). \] (4)

The two cases are consistent with each other, as a satisfactory comparison to experiment is obtained for values in the range \( \alpha \simeq 120 - 150 \) MeV fm\(^5\).
Fig. 1. Incidence of a variation of $J^2$ coupling constants on single-particle level shifts. Left panel: Distance of the proton $1h_{11/2}$ and $1g_{7/2}$ levels (top) and of the proton $2d_{5/2}$ and $1g_{7/2}$ levels (bottom), for the chain of tin isotopes. Right panel: Shift of the distance between the neutron $1d_{3/2}$ and $2s_{1/2}$ levels when going from $^{40}$Ca to $^{48}$Ca, Eq. (3) (top) and of the neutron $1f_{5/2}$ and $2p_{1/2}$ levels when going from $^{56}$Ni to $^{68}$Ni, Eq. (4) (bottom).

To conclude, we have constrained the tensor-term parameters, yielding $\alpha \sim \beta \sim 120\text{ MeV fm}^5$. However, the addition of these terms does not provide a global improvement of single-particle spectra, as has been expected from more limited studies, and even deteriorates some aspects of single-particle spectra in doubly-magic nuclei [8]. Complementary work on the central and spin-orbit parts should thus be performed.

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