Finite Temperature QCD Interfaces Out of Equilibrium

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The properties of interfaces in non-equilibrium situations are studied by constructing a density matrix with a space-dependent temperature. The temperature gradient gives rise to new terms in the equation for the order parameter. Surface terms induced in effective actions by abrupt temperature changes provide a natural theoretical framework for understanding the occurrence of both continuous and discontinuous behavior in the order parameter. Monte Carlo simulation of pure QCD shows both kinds of interfacial behavior. Perturbation theory predicts a universal profile in the high temperature phase, which can be tested by Monte Carlo simulation.

1. INTRODUCTION

Lattice gauge theory is a well understood theoretical approach to the equilibrium properties of finite temperature QCD, especially to bulk thermodynamic properties. Experimentally, finite temperature effects in QCD, especially to bulk thermodynamic properties. Experimentally, finite temperature effects in QCD, especially to bulk thermodynamic properties. Experimentally, finite temperature effects in QCD, especially to bulk thermodynamic properties. Experimentally, finite temperature effects in QCD, especially to bulk thermodynamic properties.

It is well established that computer simulations can give information about equilibrium interfaces, including the surface tension and the interfacial width. In principle, information about transport coefficients can be obtained from equilibrium correlation functions, although there are technical difficulties.

An alternative approach is to examine the interfacial behavior given basic information about the system at a given time. For example, we might know that the average energy density in a region of space has some given value, while the rest of space has another. Given this information, it is natural to construct a density matrix which maximizes the entropy subject to that constraint on the energy.

The local equilibrium formulation of Zubarev constructs the time evolution of the density matrix as successive changes in parameter fields like the local temperature and the local velocity. The time evolution is understood as motion from one local equilibrium state to another. In the case of a system at rest, the density matrix has the form

$$\rho = \frac{1}{Z} \exp(-\int \beta(\vec{x}) \mathcal{H}(\vec{x}) )$$

where \( \mathcal{H} \) is the energy density operator. Such a temperature profile can be imposed in a lattice simulation by noting that the inverse temperature $\beta$ is given by $N_t a$, where $N_t$ is the temporal size of the lattice in units of the lattice spacing $a$. The length of $a$ in physical units is specified by making the bare coupling constants space-dependent. In principle, this can be done in QCD in such a way that the spatial lattice spacing $a_s$ remains constant while the temporal lattice spacing $a_t$ varies \[\] in practice this is unnecessary in several cases of interest. In applications to cosmology, temperature differences may be small. This will certainly be the case if the nucleation rate is large and only small undercoolings are possible. In a first approximation to the environment in a heavy ion collision, the temperature inside the region of interest can be taken as roughly constant at a high temperature, and the region outside as very cold. Monte Carlo studies have shown that the Polyakov loop interfacial profile is not sensitive to the outer region temperature, if it is sufficiently cold.
2. BOUNDARY CONDITIONS AND SURFACE TERMS

At the equilibrium point of a first-order phase transition, a stable interface between two phase can exist. The interfacial profile for any order parameter is described by a one-dimensional kink solution which interpolates between degenerate minima of the effective potential. Out of equilibrium, the minima are no longer degenerate. In the traditional image of the ball rolling in an inverted potential, the ball starts to roll down from one hilltop, and overshoots the other. Thus, it is necessary to understand dissipative effects to study the properties of a non-equilibrium interface. As shown below, a temperature gradient is associated with dissipative mechanisms that allow the existence of an interface in non-equilibrium situations. An abrupt change in the temperature will give rise to a localized dissipation at the interface. In this way, surface terms are created in effective actions.

2.1. Semiclassical Argument

Consider a simple scalar field theory, and imagine that the temperature $T(z)$ is a given function, slowly varying over a mean free path of all particles. Semiclassical arguments show that an effective action for the one one-dimensional field theory is given by

$$S_{eff} = \int dz \frac{1}{T} \left[ \frac{1}{2} \left( \frac{d\phi}{dz} \right)^2 + V_{eff}(\phi, T) \right]$$

(2)

where $\phi$ is the order parameter, $A$ is the cross-sectional area transverse to $z$ and $V_{eff}$ is the finite temperature effective potential. If $V_{eff}$ has two minima, the equation for the interface is given by

$$- \frac{d^2\phi}{dz^2} + \left( \frac{1}{T} \frac{dT}{dz} \right) \frac{d\phi}{dz} + \frac{\partial V_{eff}}{\partial \phi} = 0$$

(3)

No stable interface can form at a constant temperature other than the critical temperature, where the two minima are degenerate. However, in the presence of a temperature gradient, this can occur, due to the dissipative term in the effective action. There is a kind of virial theorem, which relates the change in potential energy density across the barrier to the dissipative term:

$$\Delta V = \int \delta z \left( \frac{1}{T} \frac{dT}{dz} \right) \left[ \left( \frac{d\phi}{dz} \right)^2 - T \frac{\partial V_{eff}}{\partial T} \right]$$

(4)

where $\Delta V_{eff}$ is the difference in the effective potential between $z = +\infty$ and $z = -\infty$. Dissipation takes place only in those regions where the temperature is changing.

The semiclassical analysis indicates that a localized temperature gradient will produce a localized dissipation of energy. It is easier, however, to analyze the effects of a sharp temperature interface from a lattice point of view. This corresponds precisely to the role of surface terms in interfacial problems in condensed matter physics.

2.2. Renormalization Group

The renormalization group provide a simple explanation for the occurrence of surface terms. Suppose a sharp planar interface at $z=0$ is introduced into some lattice model, with couplings $J_1$ for $z < 0$ and $J_2$ for $z > 0$. Any real space renormalization group transformation applied to this model will yield new bulk couplings $J'_1$ and $J'_2$ away from the interface. At the interface, a new set of couplings $J'_s$ will be generated naturally. In this manner, even a microscopically sharp interface will naturally generate surface terms in an effective action.

2.3. Interfacial discontinuity

At the critical temperature of a first-order phase transition, the order parameter changes continuously across an interface. Out of equilibrium, an expanding bubble will evolve toward a sharp temperature discontinuity. The order parameter may or may not exhibit discontinuous behavior. It is more common in condensed matter studies of interfacial phenomena to think of bulk first-order transitions as giving rise to first-order interfacial transitions; the possibility of second-order interfacial behavior at a first-order bulk transition was first pointed out in 1982 by Lipowsky.

2.4. The interfacial profile in strong and weak coupling

The strong coupling limit is a natural limit in which to study the interfacial behavior of pure...
QCD, since the system simplifies to a system of Polyakov loop spins. Previous work using a mean field theory applied to an SU(3) spin model derived from strong coupling QCD shows that in a slab or semi-infinite geometry the surface behavior is first order. This is in reasonable agreement with Monte Carlo data \[7\].

Perturbative techniques can also be applied, using a variant of the effective action developed for order-order interfaces in high-temperature QCD \[7,8\]. This gives the interfacial profile for the Polyakov loop on the high-temperature side of the deconfinement-confinement interface. The interfacial profile is a universal function of $gTz$. However, these methods are only useful in the high temperature regime, and one cannot obtain a complete solution without an understanding of the low temperature confining regime. Surface terms can be added to the effective action to parametrize our ignorance; depending on the strength of these terms, the profile may go continuously to zero, or jump discontinuously to zero.

3. MONTE CARLO RESULTS

It is easy to simulate pure QCD at two different temperatures, one above and one below the deconfinement transition. The gas is at rest on both sides of the interface, and the location of the wall is determined by temperature profile. In the actual time evolution of the system, the interface is moving, in such a way that total energy is conserved.

Figure 1 shows the Polyakov loop profiles for simulations of hot, deconfined spherical droplets of radius $R = 7$ at $N_t = 2$. A spherical geometry was used, since it provides a detailed interfacial profile. The spherical surface introduces corrections to the droplet free energy which are proportional to the radius, but this disadvantage is mild compared to the rather coarse profile provided by a planar interface. The coupling constant $\beta_{\text{out}}$ outside the bubble was set at 1.0; no significant change in the profile was seen at $\beta_{\text{out}} = 3.0$, although boundary values changed slightly. It was necessary to go to rather large values of $\beta_{\text{in}}$, corresponding to high temperatures, to obtain an apparent sharp discontinuity in the Polyakov loop. As shown in figure 2, a mild discontinuity occurs when $\beta_{\text{in}} = 5.6$ at $N_t = 2$. Given that $\beta_c = 5.69$ at $N_t = 4$, this correspond to a temperature of almost $2T_c$. A fit to the continuum profile \[7\] for $\beta_{\text{in}} = 5.6$ gives an excellent fit; however, the fitted value for $gT$ is approximately 60% higher than the perturbative value. A test of the perturbative prediction for the profile at $N_t = 4$ or larger is indicated.

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