A study of hybrid quarkonium using lattice QCD

K.J. Juge, J. Kuti, and C.J. Morningstar

Dept. of Physics, University of California at San Diego, La Jolla, California 92093-0319

Abstract. The hybrid quarkonium states are studied using the Born-Oppenheimer expansion. The first step in this expansion is the determination of the energy levels of the gluons in the presence of a static quark-antiquark pair as a function of the quark-antiquark separation. The spectrum of such gluonic excitations is determined from first principles using lattice QCD.

INTRODUCTION

In addition to conventional hadrons, QCD suggests the existence of states containing excited glue, such as glueballs and hybrid hadrons. Although conventional hadrons are reasonably well described by the constituent quark model, states comprised of gluonic excitations are still poorly understood. As experiments begin to focus on the search for glueballs and hybrid mesons, a better understanding of these states is needed. Lattice QCD simulations presently offer the best means of achieving this goal.

A great advantage in studying hybrid mesons comprised of heavy quarks is that such systems can be studied not only by direct numerical simulation, but also using the Born-Oppenheimer (BO) expansion. In this approach, the hybrid meson is treated analogous to a diatomic molecule: the slow heavy quarks correspond to the nuclei and the fast gluon field corresponds to the electrons[1]. First, one treats the quark $Q$ and antiquark $\bar{Q}$ as spatially-fixed colour sources and determines the energy levels of the glue as a function of the $Q\bar{Q}$ separation $r$; each of these energy levels defines an adiabatic potential $V_{Q\bar{Q}}(r)$. The quark motion is then restored by solving the Schrödinger equation in each of these potentials. Conventional quarkonia are based on the lowest-lying static potential; hybrid quarkonium states emerge from the excited potentials. Once the static potentials have been determined (via lattice simulations), it is a simple matter to determine the complete conventional and hybrid quarkonium spectrum in the leading Born-Oppenheimer (LBO) approximation. This is a distinct advantage over meson simulations which yield only the very lowest-lying states, often with large statistical uncertainties.
Here, we present results for the spectrum of gluonic excitations in the presence of a static quark-antiquark pair. This study is the first to comprehensively survey the spectrum in SU(3) gauge theory. Using our potentials, we also determine the hybrid quarkonium spectrum.

**HYBRID QUARKONIUM**

The first step in the Born-Oppenheimer expansion is the determination of the energy levels of the gluons in the presence of the quark and antiquark, fixed in space some distance \( r \) apart. At this point in the approximation, the quark and antiquark simply act as static colour sources. The gluonic energies (or static potentials) may be labelled by the magnitude (denoted by \( \Lambda \)) of the projection of the total angular momentum of the gluons onto the molecular axis, by the sign of this projection (chirality or handedness), and by the behaviour under the combined operations of charge conjugation and spatial inversion about the midpoint between the quark and the antiquark. States with \( \Lambda = 0, 1, 2, \ldots \) are typically denoted by the capital Greek letters \( \Sigma, \Pi, \Delta, \ldots \), respectively. States which are even (odd) under the above-mentioned parity–charge-conjugation operation are denoted by the subscripts \( g \) (\( u \)). The energy of the gluons is unaffected by reflections in a plane containing the molecular axis; since such a reflection interchanges states of opposite handedness, such states must necessarily be degenerate (\( \Lambda \) doubling). However, this doubling does not apply to the \( \Sigma \) states; \( \Sigma \) states which are even (odd) under a reflection in a plane containing the molecular axis are denoted by a superscript \( + \) (\( - \)). Hence, the low-lying levels are labelled \( \Sigma_g^+, \Sigma_g^-, \Sigma_u^+, \Sigma_u^-, \Pi_g, \Pi_u, \Delta_g, \Delta_u \), and so on.

Static potentials were extracted from Monte Carlo estimates of generalized Wilson loops; some of our results are shown in Fig. 1 (for a complete presentation of our results, see Ref. \[2\]). The results are expressed in terms of the hadronic scale parameter \( r_0 \). The definition of this parameter and a description of its calculation are given in Ref. \[3\]. The familiar static potential is shown as \( \Sigma_g^+ \). For all \( r \) studied, the first-excited potential is the \( \Pi_u \); hence, the lowest lying hybrid mesons should be based on this potential.

As \( r \) becomes very large, the linearly rising \( \Sigma_g^+ \) potential suggests that the ground state of the glue may be modelled as a fluctuating tube of colour flux; in this picture, the gluonic excitations are expected to be phonon-like with energy gaps proportional to \( 1/r \). However, it appears that for \( r \) below about 1.5 fm, the gluonic spectrum cannot be explained in terms of a simple string model. In Ref. \[1\], a QCD motivated bag model was successfully used to describe both the \( \Sigma_g^+ \) and \( \Pi_u \) potentials for this range of \( r \). In this picture, the strong chromoelectric fields of the quark and antiquark repel the physical vacuum (dual Meissner effect), creating a bubble inside which perturbation theory is applicable. In the ground state, the inward pressure on the bubble from the physical vacuum balances the outward chromostatic force in such a way to produce a linearly confining potential. The addition of one or more transverse gluons into the bag produces the excited potentials;
FIGURE 1. The static quark potential $V_{QQ}^\pm(r)$ and some of its gluonic excitations in terms of the hadronic scale parameter $r_0$ against the quark-antiquark separation $r$.

The kinetic energy of the gluons inside the bubble is a key factor in determining the form of these potentials. This model has recently been revisited and results (in the ellipsoidal approximation) for almost all of the potentials studied here are in remarkable agreement with our findings from the lattice simulations (see Ref. [4]).

The next step in the BO expansion is to restore the quark motion by solving the radial Schrödinger equation using the static potentials. Results for the $b\bar{b}$ spectrum are shown in Fig. 2. The heavy quark mass $M_b$ was tuned in order to reproduce the experimentally-known $\Upsilon(1S)$ mass: $M_\Upsilon = 2M_b + E_0$, where $E_0$ is the energy of the lowest-lying state in the $\Sigma_g^+$ potential. In the LBO approximation, many mesons of different $J^{PC}$ are degenerate, such as $0^+,0^{++},1^{++},1^{+-},1^{--},1^{--}$ from the $\Pi_u$ potential. Below the $B\bar{B}$ threshold, the LBO results are in very good agreement with the spin-averaged experimental measurements. Note that these results make use of the quenched potentials (which ignore the light quarks) and do not include spin, retardation, and other relativistic effects. Above the threshold, agreement with experiment is lost, suggesting significant corrections from either the light quarks, relativistic effects, or possibly mixings between the states from the different adiabatic potentials. Note that the mass of the lowest-lying hybrid (from the $\Pi_u$ potential) is about 10.8 GeV. Above 11 GeV, the LBO approximation based on the quenched $V_{Q\bar{Q}}$ potentials predicts a very dense population of hybrid states.
FIGURE 2. Spin-averaged $b\bar{b}$ spectrum in the leading Born-Oppenheimer and quenched approximations. Solid lines indicate experimental measurements. Short dashed lines indicate the $S$ and $P$ state masses obtained by solving the appropriate Schrödinger equation in the $\Sigma_g^+$ potential using $r_0^{-1} = 0.430$ GeV and $M_b = 4.60$ GeV for the heavy quark mass. Long dashed and dashed-dotted lines indicate the hybrid quarkonium states obtained from the $\Pi_u$ and $\Sigma_u^-$ potentials, respectively.

CONCLUSION

A first comprehensive survey of the spectrum of quenched SU(3) gluonic excitations in the presence of a static $Q\bar{Q}$ pair was presented. The hybrid quarkonium states were calculated in the leading Born-Oppenheimer approximation. This work was supported by the U.S. DOE, Grant No. DE-FG03-90ER40546.

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