Numerical modelling of liquid metal magnetohydrodynamic flow in an electrically and thermally coupled annulus under reduced gravity

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Abstract. For a high-precision spaceborne magnetohydrodynamic (MHD) sensor, temperature drift is troublesome for engineers. The thermal properties of liquid metal MHD (LMMHD) flow in the sensor should be considered. The thermo-fluid behaviour of LMMHHD flows and its effect on the output characteristic of the sensor were numerically investigated to explore the underlying mechanism. Based on the Boussinesq approximation, a three-dimensional model of the MHD effect in an electrically and thermally coupled annulus was simplified and implemented in a finite-element framework. The visual patterns of the velocity and electric potential depended on the type of temperature boundary, the magnitude and direction of the applied force, and the angular rate of the inlet and outlet. When the magnitude of the applied force increased from microgravity to gravity, the fluid gradually transformed from a viscous flow to a convective flow. The competitive action between the magneto- and natural convection had a substantial influence on the distribution of the electric potential, as the applied force was perpendicular to the magnetic field. The result will provide a reference for space applications and ground tests of MHD sensors.

1. Introduction

A magnetic field induces an electric field in a moving conductive fluid. The induced electric field, in turn, creates a volume force on the fluid and changes the magnetic field. When the conductive fluid is a liquid metal, the multi-physical coupling phenomenon is known as Liquid Metal Magnetohydrodynamic (LMMHD) flow. Based on LMMHD flows, various devices, such as an MHD sensor [1], MHD generator [2] and MHD micropump [3], were developed for applications in the aviation and astronautic industries. As physical properties of the fluid (including density and viscosity) are tuned to accommodate temperature fluctuations, the hydrodynamic and electrodynamic behaviours of the fluid change [4]. The problem should be simplified based on the Boussinesq approximation. A multi-fidelity analysis and a design optimization of a laminate, which was driven by an MHD pump, were successfully achieved [5]. The MHD effect between two vertical coaxial cylinders was also studied [6]. The response of an LMMHD power generation system to external inputs of a piston force was thoroughly analysed under three working fluids [7]. In the last few years, coupling in a conductive fluid has been studied by
many researchers [8-9]. However, the thermal properties of LMMHD flows under reduced gravity have seldom been investigated.

In this study, the numerical investigation of the coupled LMMHD flow with low Reynolds number under reduced gravity was carried out. The influences of several factors (temperature boundaries, angular rate and volume force) on the behaviour of the MHD effect under reduced gravity were described in detail.

2. Problem Definition and Mathematical Modelling

2.1. Problem definition

An MHD sensor is composed of permanent magnets, magnetic yokes, electrodes, a transformer, and a conditioning circuit [10], as shown in Fig. 1. The liquid metal fills the annulus of the MHD sensor. The exterior and interior walls are insulating, whereas the upper and lower walls are conductive. All walls are assumed to be stationary, owing to the difficulty that is posed by the rotating magnetic field. Hence, the liquid metal is driven by the applied velocities of an inlet and an outlet. A diagram of the LMMHD flow is shown in Fig. 2.

2.2. Governing equation

According to the Boussinesq approximation, the velocity of the liquid metal satisfies the conservation of momentum:

\[
\frac{D (\rho u)}{Dt} = -\nabla p + \mu \Delta u + f
\]

(1)

where \( D / Dt \) denotes the substantial derivative.

A body force can be expressed as

\[
f = f_a + f_b + f_l = A \left( \rho g_0 + \rho g_0 c \left( T - T_{ref} \right) \right) n_m + J \times B
\]

(2)

Where the values of \( A \) under four reduced gravities are \( 10^3, 10^2, 10^{-1}, \) and 1.
The governing equations of the present LMMHD flow can be written as follows:

\[
\begin{align*}
\rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u - \mu_j \nabla^2 u + \nabla p &= f \\
\nabla \cdot u &= 0 \\
\rho C_p \frac{\partial T}{\partial t} + \rho C_p u \cdot \nabla T - \kappa \nabla^2 T &= 0 \\
\nabla \cdot (E + u \times B) &= 0 \\
J &= \sigma (E + u \times B) \\
E &= -\sigma \nabla V_e
\end{align*}
\]  

(3)

In the equations above, \(\mu_j\), \(C_p\), \(\kappa\), \(\varepsilon\) and \(\sigma\) are functions of temperature.

2.3. Boundary conditions

The wettability of mercury on an insulating wall is dramatically different compared to a conductive wall, which results in different velocity boundaries [11]. The velocity and electric field boundary conditions on the walls can be set as

\[
\begin{align*}
|u \cdot n|_{\partial \Omega} &= 0 \\
u|_{\partial \Omega} &= 0 \\
u \times n|_{\partial \Omega} &= \omega n \\
J \cdot n|_{\partial \Omega} &= 0
\end{align*}
\]  

(4)

where \(\Omega\) represents the studied domain and \(\partial \Omega\) the boundary of the domain. At constant temperature, the temperature boundary conditions can be set to

\[T|_{\partial \Omega} = T_{\text{ref}} = T_0.\]  

(5)

Under a temperature difference, the boundary conditions should be set to

\[
\begin{align*}
|n \times (\kappa \nabla T)|_{\partial \Omega} &= 0 \\
T|_{\partial \Omega} = T_{\text{ref}} &= 293 + \Delta T \\
T|_{\partial \Omega} &= 293
\end{align*}
\]  

(6)

where \(\Delta T\) is the temperature difference.

2.4. Numerical details

The boundary layers employ six-node isoparametric elements and the non-boundary volume uses four-node isoparametric elements [12,13]. The thickness of the first layer near the conductive walls is set to \(1 \times 10^{-6}\) m. The stretching factor and the number of boundary layers are 1.2 and 20, respectively. Comsol Multiphysics® software is used on a Dell PC to implement the foregoing solution.

3. Results And Discussion

3.1. Experimental validation of the model

The MHD numerical model is validated by calibrating a test of the prototype MHD sensor. The results under angular rates of 0.01-0.12 rad/s are shown in Fig. 4. It shows that the two lines approach each other and demonstrates the validity of the numerical model.
3.2. **Influencing factors**

**Fig. 4** Velocities and electric potentials of the liquid metal at constant temperature

**Figure 5.** Velocities and electric potentials of the liquid metal under a temperature difference.

3.2.1. **Temperature condition.** Fig. 4 shows the velocities and electric potentials under the microgravity. When the convective term is zero and the viscous effect dominates in the liquid metal, the electric potentials at constant temperatures are linearly proportional to the z-coordinate.

The velocities and electric potentials under a temperature difference are shown in Fig. 5. Fig. 6 indicates the temperatures of the liquid metal under microgravity. The enhanced temperature gradients can induce larger buoyancy under the microgravity of -z.

**Figure 6.** Temperatures of the liquid metal along the –x-axis (left) and –z-axis (right).

3.2.2. **Angular rate.** Fig. 7 shows that the velocities and electric potentials gradually change when the angular rate is reduced from $1 \times 10^{-3}$ rad/s to $1 \times 10^{-5}$ rad/s. The velocities tend to be more heterogeneous as the angular rate decreases. As a result, the electric potential contours are gradually twisted into various cells.
3.3. Volume force

According to Fig. 8, the behaviour of the MHD flow shifts when the applied body force increase from $10^{-3} g_0$ to $1 g_0$. The non-uniformities become more significant. Meanwhile, the electric potential contours gradually bend and curl into various cells.

Under $10^{-3} g_0$ and $1 g_0$, the Reynolds numbers are estimated to be 10 and 1000, respectively. The inertial term (or convective term) of the momentum equation under gravity is much more important than that under microgravity.

The velocity gradients and electric field intensities are small when the applied magnetic fields are completely perpendicular to the gravitational force [14,15]. Compared with those under the above conditions, the velocity and electric field are more inhomogeneous because the applied magnetic fields are partially parallel to the gravitational force. The Lorentz force gradually increases as the reduced gravities increase, and the buoyancy soars.

![Figure 7. Velocities (left) and electric potentials (right) vs. angular rates of the liquid metal.](image)

3.4. Output characteristic under reduced gravity

When the applied forces are parallel to $-x$ and $-z$, the electric potential differences between the conductive walls under constant temperature are as shown at the top of Fig. 11. The temperatures that are experienced in a ground test of a satellite-borne device range from -20°C to 80°C. The output voltage under microgravity does not vary substantially with temperature. As the applied force increases, the voltage increasingly diverges from the lines under microgravity. When the applied force is gravity, the jitter among various temperatures is comparatively large.

The output voltages also vary with the temperature difference, which are shown on the bottom of Fig. 11. The considered maximal temperature differences are values that may be encountered in ground tests. When the temperature difference is adjusted, the voltage under microgravity remains constant. Applied forces of $10^{-2} g_0$ and $10^{-1} g_0$ have limited impacts, whereas gravity affects the output voltages.
4. Conclusion

A mathematical model of the MHD effect in an MHD sensor under reduced gravity (from $10^{-3}g_0$ to $1g_0$) was established via dimensional analysis and finite element modelling. The viscous flow played a key role under microgravity ($10^{-3}g_0$); however, the convective flow gradually dominated in the MHD behaviour when a temperature difference arose or the applied force increased. Compared with the MHD effect under microgravity, natural convection under reduce gravity suppressed the magneto-convective flow when the volumetric force was completely perpendicular to the applied magnetic field. When the...
applied force increased, the output voltage diverged from the stable line under microgravity and shifted with the temperature and temperature difference. Because the electric potential difference is proportional to the output voltage of the MHD sensor, the results provide knowledge about the temperature stability of the MHD sensor.

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